A signature of anisotropic bubble collisions

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Our universe may have formed via bubble nucleation in an eternally-inflating background. Furthermore, the background may have a compact dimension—the modulus of which tunnels out of a metastable minimum during bubble nucleation—which subsequently grows to become one of our three large spatial dimensions. When in this scenario our bubble universe collides with other ones like it, the collision geometry is constrained by the reduced symmetry of the tunneling instanton. While the regions affected by such bubble collisions still appear (to leading order) as disks in an observer’s sky, the centers of these disks all lie on a single great circle, providing a distinct signature of anisotropic bubble nucleation.

I. INTRODUCTION

String theory argues for the existence of an enormous landscape of metastable, positive-energy vacua (in addition to other states) [1, 2]. If any such vacuum is obtained in spacetime, it expands exponentially and without bound: while one expects quantum transitions, in the form of bubble nucleation, from any such vacuum to another [3], in metastable states the resulting bubbles do not percolate [4]. Thus emerges a view of cosmology that sees our universe as part of the inside of a bubble, which nucleated within some eternally-inflating background, in which other bubbles endlessly nucleate and collide [5].

The string landscape corresponds in part to the various ways to compactify the higher-dimensional fundamental theory down to a (3+1)-dimensional spacetime such as we observe. Yet there should also exist metastable compactifications with fewer and more large spatial dimensions, in which case we expect these vacua to also play a part in the above cosmology. The consequences of this have only just begun to be explored [6–12]. We here focus on the scenario where the vacuum in which our bubble nucleates has an additional compactified dimension—the modulus of which tunnels out of a metastable minimum during bubble nucleation—which subsequently grows to become one of our three large spatial dimensions. Then the reduced symmetries of the parent vacuum (relative to the local, SO(3,1)-symmetric asymptotic geometry in the bubble) imply anisotropic initial conditions for the evolution of our universe.

Some signatures of this scenario have been explored in [1, 11, 12], where ultimately it is found that an induced quadrupole of order the present curvature parameter, $Ω_0^0$, places severe observational constraints on such effects as statistically-anisotropic perturbations in the cosmic microwave background (CMB), and spatial variations in the observed luminosity of standard candles, both of which are suppressed relative to leading-order terms by $Ω_0^0$. Yet these are not the only possible observational signatures of anisotropic bubble nucleation.

We here study a certain class of bubble collisions, namely those involving two bubbles of the type that contains our universe, assuming such bubbles nucleate as described above, via metastable modulus decay. With such bubbles the tunneling instanton is independent of the coordinate of the compact dimension, so that when two such bubbles collide, the locus of events at the “point” of first contact spans the entire compact dimension. This, along with the reduced symmetry of the parent vacuum, constrains the geometry of such bubble collisions. Setting aside order $Ω_0^0$ affects due to background anisotropy, the regions affected by these collisions project onto disks on the CMB sky. However, unlike the case with a (3+1)-dimensional parent vacuum, the centers of these disks all lie on a single great circle. In the circumstance where at least three such collisions are observed, this provides a distinct signature of anisotropic bubble nucleation.

Detection of effects of bubble collisions requires that the number of e-folds $N_e$ of inflation in our bubble not be too large. In the model of anisotropic bubble nucleation that we consider here, observational constraints on $N_e$ are tighter than in the isotropic scenario, since absent any fortuitous cancellations coming from other (still unexplored) large-scale effects, the induced quadrupole mentioned above requires $Ω_0^0 \lesssim 10^{-5}$. However, because the initial effects of bubble collisions can be much larger than the spatial curvature on scales now entering the horizon, there is hope to detect signals of bubble collisions even when the present-day curvature parameter is well below the level of cosmic variance [13, 14].

The study of (3+1)-dimensional bubble collisions in a (3+1)-dimensional parent vacuum has a long history, for an excellent summary see [15] (and references therein). Most aspects of the present analysis follow directly from ideas and techniques developed in that scenario; this work in particular benefits from [15–19].

The remainder of this paper is organized as follows. In Section II we briefly review a toy model of modulus stabilization, providing for an effectively (2+1)-dimensional parent vacuum, and describe the tunneling instanton that connects to our asymptotically (3+1)-dimensional daughter bubble. The basic features of a collision between two such daughter bubbles in such a parent vacuum is described in Section III, where we work out the collision-affected region in an observer’s CMB sky. In Section IV we discuss the distribution of positions.
II. MODULUS (META)STABILIZATION AND TUNNELING INSTANTON

A toy model to implement anisotropic bubble nucleation is described in [3]. We here briefly review the model, and discuss the tunneling instanton, to motivate essential features of the geometry used in Section III. We have introduced the following notation. Any quantity “kinetic” function specified by $K$, where $\Psi$ represents the modulus field. The effective potential is described in [9]. We here briefly review the essential features of the geometry used in Section III.

$$S_{4d} = \int \sqrt{-g} d^4x \left[ \frac{1}{16\pi G} (R - 2\Lambda) + \mathcal{L}_\varphi \right],$$

(1)

where $g$ is the determinant of the $(3+1)$-dimensional metric $g_{\mu\nu}$, $R$ is the corresponding Ricci scalar, $\Lambda$ is a cosmological constant, and

$$\mathcal{L}_\varphi = -\frac{1}{2} K (\partial_\mu \varphi^* \partial^\mu \varphi) - \frac{\lambda}{4} (|\varphi|^2 - m^2)^2,$$

(2)

where $\varphi$ is the scalar, for which we allow a non-canonical “kinetic” function specified by $K$. The other terms are constants. Other degrees of freedom, for instance the inflaton and the matter fields of the Standard Model, are assumed to be unimportant during the tunneling process, and are absorbed into $\Lambda$ (and/or $g$ and $R$).

The stabilization of the volume modulus is studied by starting with a metric ansatz with line element

$$ds^2 = e^{-\Psi} g_{ab} dx^a dx^b + L^2 e^{\Psi} dz^2,$$

(3)

where $\Psi$ represents the modulus field. The effective $(2+1)$-dimensional metric $g_{ab}$ and the modulus $\Psi$ are both taken to be independent of $z$. Meanwhile, the compact dimension $z$ is defined using periodic boundary conditions, with $-\pi < z < \pi$, so that it has the topology of a circle with physical circumference $2\pi L e^{\Psi/2}$. Note that we have introduced the following notation. Any quantity defined explicitly within the effective $(2+1)$-dimensional theory (the theory with the $z$ dimension integrated out), such as the $(2+1)$-dimensional metric, is marked with an overline. Whereas Greek indices are understood to run over all dimensions, Latin indices are understood to run over all but the $z$ dimension.

The equations of motion permit solutions of the form

$$\varphi \approx \left( m^2 - \frac{n^2}{\Lambda L^2} K' e^{-\Psi} \right)^{1/2} e^{inz},$$

(4)

if we take

$$m^2 > \frac{n^2}{\Lambda L^2} K' e^{-\Psi}.$$

Here $n$ is an integer and $K' \equiv dK(X)/dX$, with $X \equiv \partial_\mu \varphi^* \partial^\mu \varphi = (n^2 m^2 / L^2) e^{-\Psi}$. After integrating by parts, we find the effective $(2+1)$-dimensional action

$$S_{3d} = \int \sqrt{-\bar{g}} d^3x \left[ \frac{1}{16\pi G} \bar{R} - \frac{1}{2} \bar{\partial}_a \bar{\partial}^a \bar{\psi} - \overline{\mathcal{V}(\bar{\psi})} \right],$$

(6)

where $\bar{G} \equiv G/(2\pi L)$, $\bar{\psi} \equiv \Psi/\alpha$, with $\alpha = \sqrt{16\pi G}$, and

$$\overline{\mathcal{V}(\bar{\psi})} = \frac{\Lambda}{8\pi G} e^{-\alpha \bar{\psi}} + \frac{1}{2} e^{-\alpha \bar{\psi}} \mathcal{K}(X(\bar{\psi})).$$

(7)

To stabilize the modulus it is necessary to introduce at least three terms into (a polynomial) $\mathcal{K}(X)$; since we are here simply interested in producing a viable model, we use the ad hoc model

$$\mathcal{K}(X) = 2\pi L \left( X + \kappa_2 X^2 + \kappa_3 X^3 \right),$$

(8)

where $\kappa_2$ and $\kappa_3$ are constants. The resulting effective potential $\overline{\mathcal{V}(\bar{\psi})}$ is displayed in Figure 1 (using the same parameter values as in [3]). There is a metastable minimum at some value $\psi = \psi_p$, corresponding to the parent vacuum state, which appears as $(2+1)$-dimensional de Sitter space on scales much larger than $2\pi L e^{\alpha \psi_p/2}$. The daughter vacuum is created when $\psi$ tunnels through the barrier, to some value $\psi = \psi_d$, after which $\psi$ accelerates from rest and rolls down the potential, with $\psi \to \infty$ as time $x^0 \to \infty$. For more details see [3].

The semi-classical theory of vacuum decay via bubble nucleation in $(3+1)$ dimensions is laid out in [3]; bubble nucleation in our scenario proceeds analogously. The essential difference is the reduced symmetries in the parent vacuum. In our scenario the tunneling field is also the modulus, which is independent of the compact $z$ dimension (by hypothesis). Thus the instanton must be independent of $z$, and the compact dimension acts as a bystander to the tunneling process. The remaining geometry is SO(2,1) invariant, allowing one to follow the steps in Section III.

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1 String theory indicates that the $(3+1)$-dimensional vacuum itself has six or seven compact dimensions, however throughout this paper we consider the associated moduli fields to be non-dynamical spectators in all of the processes of interest.
methods of [3], which starts with an SO(3,1) invariant geometry, by merely changing some numerical factors. Let us briefly sketch the procedure to help describe the results. The tunneling instanton is found by studying the O(3)-symmetric Euclidean line element (ansatz) of the (2+1)-dimensional parent vacuum,

\[ ds^2 = d\chi^2 + \rho^2(\chi) \left[ d\xi^2 + \cos^2(\xi) d\phi^2 \right]. \tag{9} \]

The scale factor \( \rho \) and the tunneling field \( \psi \) obey the “inverted-potential” Euclidean equations of motion,

\[ \frac{\dot{\rho}^2}{\rho^2} - 1 = 8\pi G \left( \frac{1}{2} \dot{\psi}^2 - \nabla^2 \right) \tag{10} \]

where the dot and prime denote differentiation with respect to \( \chi \) and \( \psi \) respectively. In the tunneling solution, as \( \chi \) runs from zero to some maximum value \( \chi_{\text{max}} \), \( \psi \) interpolates from \( \psi_q \) to \( \psi_p \) not far from the parent vacuum state \( \psi_p \), with \( \psi \rightarrow 0 \) in both limits. In the limit \( \chi \rightarrow \chi_{\text{max}} \), the scale factor approaches

\[ \rho(\chi) \rightarrow H_p^{-1} \sin(H_p\chi + \delta_p), \tag{12} \]

where \( H_p^2 \equiv 8\pi G \bar{V}(\tilde{\psi}_p) \) and the phase \( \delta_p \) is determined by boundary conditions. An analogous solution applies in the limit \( \chi \rightarrow 0 \).

The Lorentzian solution is determined by analytic continuation, \( \xi \rightarrow i\chi \), which gives the line element

\[ ds^2 = d\chi^2 + \rho^2(\chi) \left[ -d\xi^2 + \cosh^2(\xi) d\phi^2 \right]. \tag{13} \]

In the appropriate limits of \( \chi \) the above solution corresponds to a slicing of de Sitter space. The instanton boundary \( \chi = 0 \) is now the future lightcone of the origin of coordinates, and serves as the boundary of the nucleated bubble geometry. The initial conditions for the bubble are determined by matching, and can be obtained by analytic continuation of (12). Taking \( \chi \rightarrow i\chi \) and \( \xi \rightarrow \xi - i\pi/2 \) gives the line element

\[ ds^2 = -e^{-\Phi(\chi)} d\chi^2 + e^{-\Psi(\chi)} \rho^2(\chi) \left[ d\xi^2 + \sinh^2(\xi) d\phi^2 \right] + L^2 e^{\Phi(\chi)} dz^2, \tag{14} \]

where we have restored the conformal factor \( e^{-\Psi} = e^{-\alpha \psi} \) of the original metric ansatz [3], and have revealed the compact dimension \( z \). Note that while the analytically-continued instanton solutions for \( \Psi = \psi/\alpha \) and \( \rho \) describe the bubble geometry near the instanton boundary, the future evolution of the metric components of (14) is determined by the matter content of the bubble.

III. ANISOTROPIC BUBBLE COLLISIONS

We find it convenient to parameterize the geometry of the parent vacuum using a flat de Sitter slicing on the inflating submanifold, with line element of the form

\[ ds^2 = A^2(\sigma) \left( -d\sigma^2 + d\rho^2 + \rho^2 d\phi^2 \right) + B^2 dz^2, \tag{15} \]

where \( B \) is a constant, setting the size of the compact \( z \) dimension \((-\pi < z < \pi)\), and \( A(\sigma) = -1/H_p\sigma \), with \( H_p \) being the de Sitter Hubble rate of the parent vacuum (we everywhere ignore the back-reaction of bubble walls on the metric). Meanwhile, the interior of a given bubble can be described with a line element of the form

\[ ds^2 = a^2(\eta) \left[ -d\eta^2 + d\xi^2 + \sin^2(\xi) d\phi^2 \right] + b^2(\eta) dz^2, \tag{16} \]

where \( \eta \) foliating hypersurfaces of constant density. The scale factors \( a \) and \( b \) are determined by the matter content of the bubble, with the instanton boundary conditions setting \( a \rightarrow 0 \) and \( b \rightarrow B \) as \( \eta \rightarrow -\infty \).

We assume the energy density in the bubble is initially dominated by the inflaton potential, and that it behaves essentially as cosmological constant, with asymptotic Hubble rate \( H_A \). The solution for the scale factors, up until the end of inflation, is then

\[ a(\eta) = H_A^{-1} \cosh \left( \ln(H_p/H_A) - \eta \right) \tag{17} \]

\[ b(\eta) = B \coth \left( \ln(H_p/H_A) - \eta \right), \tag{18} \]

where the offset \( \ln(H_p/H_A) \) is introduced to facilitate matching at the instanton boundary. In particular, at very early times, \(-\eta \gg \ln(H_p/H_A)\), this solution becomes \( a(\eta) \approx (2/H_A)^{\eta} \), which corresponds to the early-time limit of \( a(\eta) = H_p^{-1} \cosh(-\eta) \), the scale factor solution for open de Sitter spacetime with asymptotic Hubble rate \( H_p \). This means that we can extend the flat de Sitter chart of the parent vacuum (slightly) into the bubble, and match coordinates using the (early-time limit of the) standard flat–to–open de Sitter coordinate transformations. For matching at the future lightcone of a bubble nucleating at \((\sigma, \rho) = (-H_p^{-1}, 0)\), this gives

\[ -H_p\sigma = (1 + e^{\xi+\eta})^{-1} \tag{19} \]

\[ H_p\rho = (1 + e^{-\xi-\eta})^{-1}, \tag{20} \]

where we have also used \( \xi + \eta = \text{constant} \), and therefore \( \xi \rightarrow \infty \) as \( \eta \rightarrow -\infty \), as will be appropriate when we apply these transformations later. The coordinates \( \phi \) and \( z \) are matched trivially at the boundary.

Note that the evolution of the bubble begins with a period of curvature domination, with the subsequent period of inflation beginning at \( \eta \sim \ln(H_p/H_A) \approx -1 \). During inflation \( \dot{a}/a \) and \( b/b \) rapidly converge to generate a locally SO(3,1)-symmetric geometry, after which the bubble interior may follow the standard big bang cosmology. (While it is possible for effects of the background anisotropy to become significant at late times [20], observational constraints limit such effects to be very small, and we ignore them here.) Empirically, the number of e-folds of inflation is large, meaning inflation ends at time \( \eta_H = \ln(H_p/H_A) \). The subsequent evolution of \( \eta_H \) is dominated by its growth between recombination and the present, during which \( \eta_H \) changes by [3]

\[ \Delta \eta \equiv \eta_H \approx 6.1 \sqrt{\frac{\Omega_c}{3}}, \tag{21} \]
where $\Omega_0^c$ is the present-day curvature parameter. This is observationally constrained to be small, $\Omega_0^c \lesssim 6.3 \times 10^{-3}$ [21], and in the context of anisotropic bubble nucleation, absent fortuitous cancellations from other large-scale effects, an induced quadrupole constrains it to be even smaller, $\Omega_0^c \lesssim 10^{-5}$ [20]. All this is to obtain the time of a present observer,

$$\eta_0 \approx \ln(H_p/H_d) + r_*. \quad (22)$$

Before proceeding to calculations, let us discuss the qualitative features of the bubble collision geometry. Figure 2 represents two anisotropic bubbles nearing collision. The largest torus represents a spatial section of the parent vacuum: the coordinate that wraps around the vertical axis is the compact $z$ dimension, while the radial/vertical cross sections are subsets of the $(\rho, \phi)$ plane (one should imagine all tori as flat, $S^1 \times S^1$). The two smaller tori represent bubbles of daughter vacuum, with the coordinates $\phi$ and $z$ aligned with those of the parent vacuum at the boundary (note that the radial/vertical slices are not spatial slices of the open-FRW coordinates inside the bubbles). Distances in the figure reflect comoving coordinate separations: on subsequent spatial slices the metric on the radial/vertical cross sections of the parent vacuum expands according to the scale factor $A$, while the azimuthal metric component of the larger torus is a constant, $B$. The azimuthal metric inside the bubbles expands with the scale factor $b$, and radial/vertical cross sections inside the bubbles expand with $a$.

The bubbles expand into the parent vacuum at a rate approaching the speed of light; however the intervening parent vacuum also expands. In comoving coordinates, this translates to bubble radii that asymptotically approach bubble-nucleation-time-dependent constants. Two bubbles collide if they nucleate sufficiently close to each other, or at sufficiently early times, for their asymptotic cross sections to overlap.

A crucial feature of Figure 2 is that the bubble walls are independent of the compact dimension, i.e. the bubbles reflect the toroidal symmetry of the diagram. This is a property of the tunneling instanton discussed in Section 4.1, and it implies that the “point” of first contact between the two bubbles is in fact a ring, spanning the length of the compact dimension. Our primary interest is to understand the effect of this.

A more traditional representation of a bubble collision is given in Figure 3. Here we suppress the $(\phi, z)$ torus at every point, displaying the $(\sigma, \rho)$ plane of the parent vacuum and $(\eta, \xi)$ planes of the bubbles. (The radial/vertical cross sections of Figure 2 correspond to horizontal lines in this figure.) Here the solid dot specifies the position of the observer, the light dotted line the observer’s past lightcone, the dark dotted lines the future lightcones of the bubble nucleation events (technically, the tunneling instanton boundaries), the dark solid lines bubble domain walls, and the solid curves in the observer’s bubble surfaces of equal bubble time $\eta$. The bubble collision occurs at the intersection of the dark solid lines in Figure 3. However, it is much simpler to treat the collision as if it occurs at the intersection of the future lightcones of the nucleation events, corresponding to the intersection of dark dotted lines in Figure 3. This is an accurate approximate for thin bubble walls and/or late bubble collisions. We focus on the possibility that the bubble collision leaves some imprint on the CMB, in which case we are interested in the intersection of the past lightcone of the observer, the hypersurface of recombination, and the future lightcone of the colliding bubble nucleation event.

Note that we consider only those bubble collisions that create small perturbations to the bubble geometry within the observer’s past lightcone. One can consider, for instance, the collision between two bubbles of the same type as our own, where after the collision the energy in the bubble walls is converted into radiation, which rapidly redshifts away during inflation within the bubble. Meanwhile the collision may affect, for instance, the starting value of the inflaton, allowing effects of the collision to persist until late times. One can also imagine collisions between a bubble like ours and a bubble with much larger vacuum energy, if the post-collision domain wall between bubbles rapidly accelerates away from the observer, leaving the interior of the observer’s bubble largely only weakly perturbed by the collision. For more
The intersection of the past lightcone of the observer with the hypersurface of recombination corresponds to the surface of last scattering. As we neglect late-time anisotropies, we can describe the relevant late-time geometry using the line element

\[ ds^2 = a^2(\eta) \left[ -d\eta^2 + d\sigma^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2 \right], \]

the coordinates of which can be related to the global bubble coordinate system (in the spacetime region where both charts are accurate) according to

\[ \xi = r \sin(\theta), \quad \phi = \phi, \quad BH_d z = r \cos(\theta), \]

where we have used \( b(\eta)/a(\eta) = BH_d \), as indicated by the asymptotic behavior of \( a(\eta) \) and \( b(\eta) \). Here the angular coordinate \( \theta \) covers the interval \( 0 < \theta < \pi \).

Exploiting some symmetries of the collision geometry, we can use boost and \( z \)-translation invariance to place the observer at \((\xi, z) = (0, 0)\). The surface of last scattering then corresponds to the two-sphere with radial coordinate \( r_* = \eta_0 - \eta_* \), on the hypersurface \( \eta = \eta_* \), where \( \eta_0 \) is the time of the observer and \( \eta_* \) is the time of recombination. Here we have assumed \( \eta_0 - \eta_* < \pi BH_d \), so that the surface of last scattering does not wrap entirely around the closed \( z \) dimension, as indicated by unsuccessful searches in the CMB for a closed spatial dimension \( \gamma \).

Note that the surface of last scattering covers only the region \( 0 \leq \xi_* \leq r_* \ll 1 \).

In addition to the above boost, we can use \( \sigma \)- and \( \rho \)-translation invariance to place the nucleation of the observer’s bubble at \((\sigma, \rho) = (-H_*^{-1}, 0)\). Meanwhile we denote the location of the colliding bubble nucleation event as \((\sigma, \rho, \phi) = (\sigma_n, \rho_n, 0)\), where

\[ H_p(\rho_n - \sigma_n) > 1, \quad H_p(\rho_n + \sigma_n) > -1, \]

so that the colliding bubble nucleates outside of the observer’s bubble, and vice versa. The boundary of the future lightcone of the colliding bubble is then

\[ \rho_c(\sigma, \phi, z) = \rho_n \cos(\phi) \pm \sqrt{(\sigma - \sigma_n)^2 - \rho_n^2 \sin^2(\phi)}, \]

where it is implicit that the lightcone exists only where \( \rho_c \) is real. Not that \( \rho_c \) is independent of \( z \), due to the bubble collision occurring simultaneously across all \( z \).

As we have remarked, the instanton boundary conditions allow us to extend the coordinates of the parent vacuum into the bubble, where at some fiducial time \( -\eta_1 \gg \ln(H_p/H_d) \) we can transform the coordinates of the lightcone using \( (19) \) and \( (20) \). The result is a curve in the \((\xi, \phi)\) plane,

\[ \xi(\phi) = \ln \left[ \frac{1 + 2H_p \sigma_n - H_p^2 (\rho_n^2 - \sigma_n^2)}{1 - 2H_p \rho_n \cos(\phi) + H_p^2 (\rho_n^2 - \sigma_n^2)} \right] - \eta_1, \]

valid for all \( -\eta_1 \gg \ln(H_p/H_d) \), but beyond which the future lightcone is complicated to develop. On the other hand, the local radius of curvature of (27) is of order the radius of curvature of the bubble; meanwhile we are only interested in the portion of the future lightcone that will intersect the surface of last scattering, the radius of which is empirically much smaller than this radius of curvature, \( 0 \leq \xi_* \ll 1 \). We can therefore approximate the relevant portion of the lightcone as a line, corresponding to the tangent of the future lightcone evaluated at \( \phi = 0 \). In terms of the bubble coordinates this gives

\[ \xi_c(\eta, \phi, z) = \frac{2 \tanh^{-1}[H_p(\rho_n + \sigma_n)] - \eta}{\cos(\phi)} - \eta_0. \]

Again, the future lightcone spans all values of \( z \).

The intersection of the surface of last scattering and the future lightcone of the colliding bubble nucleation event then corresponds to the region

\[ \xi_c(\eta_*, \phi, z) \geq \xi_c(\eta_*, \phi, z), \]

where as before \( \xi_* = r_* \cos(\theta) \), and \( \eta_* = \eta_0 - r_* \). The angular coordinates \((\theta, \phi)\) on the two-sphere of last scattering therefore satisfy

\[ \cos(\phi) \sin(\theta) \geq \frac{2}{r_*} \tanh^{-1}[H_p(\rho_n + \sigma_n)] - \eta_0 + 1, \]

which has solutions when \( \eta_0 > 2 \tanh^{-1}[H_p(\rho_n + \sigma_n)] \).

Note that the angular coordinates satisfying (30) correspond to a disk centered at \((\theta, \phi) = (\pi/2, 0)\).

The disk-shaped profile of the region satisfying (30) is in part a consequence of large-colliding-bubble-lightcone approximation of (28), and in part a consequence of isotropy approximation of (23). In both cases, however, the corrections to these leading-order results are suppressed by the present day curvature parameter \( \Omega_0^c \), which is observationally constrained to be very small, and in the context of anisotropic bubble nucleation likely to be much smaller; see the discussion surrounding (21).

Furthermore, the precise shape of a given collision-affected region on the CMB will be blurred by superimposed inflationary perturbations. For these reasons we consider the above approximations to be sufficient.

While we have so far taken the colliding bubble to nucleate at \( \phi = 0 \), with more than one such bubble we cannot freely choose the \( \phi \) coordinate of each nucleation event. Nevertheless, rotational symmetry indicates that the solution for a colliding bubble nucleating at \( \phi = \phi_n \) simply corresponds to taking \( \phi \to \phi - \phi_n \) in (30).

Note that, crucially, there is no corresponding generalization of the \( \theta \)-dependence of (30), since each colliding bubble nucleation event occurs simultaneously across all values of \( z \). This implies that the regions affected by such bubble collisions are always centered at \( \theta = \pi/2 \), a great circle on the CMB sky of the observer. (The local isotropy of (28) is broken by the choice of preferred coordinate system (21); hence the preferred value \( \theta = \pi/2 \).)

This is illustrated in Figure 3. Absent any other observations breaking the local background isotropy seen by
the observer, two bubble collisions are required to determine the preferred \( z \) direction (the plane with \( \theta = \pi/2 \)), and thus three collisions are necessary to verify/falsify the anisotropic bubble nucleation hypothesis.

**IV. DISTRIBUTION OF BUBBLE COLLISIONS**

The distribution of the number, sizes, and positions of bubble collisions in an observer’s sky has already been computed in the case of \((3+1)\)-dimensional parent and daughter vacua (see e.g. \([9, 10]\)), and it is straightforward to apply the relevant techniques to the present scenario. In particular, our analysis follows very closely \([18]\), and the reader is directed to there and to \([15]\) for details that are glossed over in the interest of brevity below. Note that with respect to the distribution of positions of regions affected by bubble collisions, we have already deduced that they are all centered at \( \theta = \pi/2 \) (see Section III). Therefore we here focus on the distribution with respect to the \( \phi \) coordinate.

The calculation involves integrating over all possible nucleation sites of a colliding bubble, with the parent vacuum idealized to cover all of the spacetime outside of the bubbles. To regulate the diverging spacetime volume of the parent vacuum, it is customary to draw a spacelike hypersurface at \( \sigma = \sigma_{\text{in}} \), below which it is assumed there are no bubble nucleations, and in the end take \( \sigma_{\text{in}} \to -\infty \) \([10]\). The translations that were used to place the observer’s bubble nucleation event at \( (\sigma, \rho) = (-H_p^{-1}, 0) \) merely shift the hypersurface \( \sigma = \sigma_{\text{in}} \). Furthermore, rotational symmetry allows us to define the origin of the \( \phi \) coordinate so that the observer sits at \( \phi = 0 \) (a colliding bubble then in general nucleates at some azimuthal angle \( \phi = \phi_0 \)). On the other hand, the boost that was used to translate the observer from an arbitrary position \( (\xi, \phi) = (\xi_0, 0) \) to \((0, 0)\) corresponds to a non-trivial rotation of the hypersurface \( \sigma = \sigma_{\text{in}} \).

To understand the effects of this rotation, note that the line element of the bubble geometry \([10]\) is induced on a four-dimensional hypersurface embedded in a \((5+1)\)-dimensional Minkowski space,

\[
ds^2 = -dt^2 + du^2 + dv^2 + dw^2 + dx^2 + dy^2 , \quad (31)
\]
given the constraint equations

\[
t = a(\eta) \cosh(\xi) \quad (32)
\]
\[
u = b(\eta) \cos(\zeta) \quad (33)
\]
\[
v = b(\eta) \sin(\zeta) \quad (34)
\]
\[
w = f(\eta) \quad (35)
\]
\[
x = a(\eta) \sinh(\xi) \cos(\phi) \quad (36)
\]
\[
y = a(\eta) \sinh(\xi) \sin(\phi) \quad (37)
\]

where the function \( f(\eta) \) corresponds to a solution of the differential equation

\[
f^2 = a^2 + b^2 - \frac{b^2}{2} \cdot (38)
\]

Then the arbitrary point \( (\xi, \phi) = (\xi_0, 0) \) is translated to the point \((0, 0)\) by the boost

\[
t' = \gamma(t - \beta x) \quad (39)
\]
\[
x' = \gamma(x - \beta t) \quad (40)
\]

with the other embedding coordinates unchanged, if we take \( \gamma = \cosh(\xi_0) \) and \( \beta = \tanh(\xi_0) \).

To understand the effect of this boost on the hypersurface \( \sigma = \sigma_{\text{in}} \), we should embed the coordinates of the parent vacuum in the above \((5+1)\)-dimensional Minkowski space. However, the above boost will take the hypersurface \( \sigma = \sigma_{\text{in}} \to -\infty \) to points on the de Sitter hyperboloid not covered by the flat slicing, so we first transform to a new coordinate system. A particularly convenient slicing of the de Sitter submanifold has line element

\[
ds^2 = A^2(X)[-dT^2 + dX^2 + \cosh^2(T) d\phi^2] + B^2 dz^2 , \quad (41)
\]

where the “scale factor” is now \( A(X) = H_p^{-1} \text{sech}(X) \), and the coordinates \( T \) and \( X \) run from \(-\infty \to \infty \). Where the two charts overlap, the above coordinates are related to the flat de Sitter ones by

\[
-H_p \sigma = \frac{\cosh(X)}{\sinh(T) - \sinh(X)} \quad (42)
\]
\[
H_p \rho = \frac{\cosh(T)}{\sinh(T) - \sinh(X)} , \quad (43)
\]

with trivial identification of \( \phi \). The \((5+1)\)-dimensional Minkowski embedding is determined by the constraints

\[
t = H_p^{-1} \sinh(T) \text{sech}(X) \quad (44)
\]
\[
u = B \cos(\zeta) \quad (45)
\]
\[
v = B \sin(\zeta) \quad (46)
\]
\[
w = -H_p^{-1} \tan(\zeta) \quad (47)
\]
\[
x = H_p^{-1} \cosh(T) \text{sech}(X) \cos(\phi) \quad (48)
\]
\[
y = H_p^{-1} \cosh(T) \text{sech}(X) \sin(\phi) . \quad (49)
\]

The slicing \((41)\) of the de Sitter submanifold is displayed in Figure 3 where it can be seen that while the chart does not cover the entire de Sitter hyperboloid, it

FIG. 4: Disks corresponding to the regions affected by four bubble collisions, on the two-sphere of an observer’s CMB sky.
Therefore for simplicity we here ignore it. Bubbles do not percolate during eternal inflation \[17, 18, 23\]. The error is small, for essentially the same reason that bubble nucleation rate, \(\Gamma\), is small. However, given the expected count rate of nucleating the observer’s bubble, and \(X_n\), is large. Integrating over \(X_n\) in the plane \((\psi, X_n)\) involves moving down such a null ray until one hits the cutoff, which now corresponds to the intersection of (50) and (53).

\[
X_n = \frac{1}{2} \ln \left[ \frac{\rho(\psi) + \gamma \beta^2 (\psi - \phi_n) \cos(\phi_n)}{\rho(\psi) + \gamma + \beta \cos(\phi_n)} \right],
\tag{54}
\]

where we have defined \(\rho(\psi) \equiv \exp [r_\ast \cos(\psi) + \eta_0 - r_\ast]\) and, as before, \(\gamma = \cosh(\xi_0)\) and \(\beta = \tanh(\xi_0)\). Putting everything together gives

\[
\frac{dN}{d\psi d\phi_n} = \frac{r_\ast \Gamma}{H_3^3} \sin(\psi) \int_{-\infty}^{X_n(\psi, \phi_n)} \frac{\cosh[T_n(\psi, X_n)]}{\cosh^3(X_n)} dX_n
\]

\[
= \frac{r_\ast \Gamma}{H_3^3} \sin(\psi) \left[ 1 + \theta^2(\psi) \left[ 1 + 2e^{-2X_n(\psi, \phi_n)} \right] \right].
\tag{55}
\]

Before proceeding, note that the distribution (55) in general depends on the angular position of the colliding bubble nucleation event, \(\phi_n\). This is in addition to the anisotropy due to the reduced symmetry of the parent vacuum, which expresses itself in the \(\theta\)-alignment of the regions affected by bubble collisions, as described in Section III. Instead the anisotropy with respect to \(\phi_n\) indicates the same “persistence of memory” effect found by [10] in the case of a \((3+1)\)-dimensional parent vacuum, which stems from the cutoff hypersurface (50) breaking the de Sitter symmetries of the parent vacuum. In the case of a \((3+1)\)-dimensional parent vacuum, the anisotropy is only significant if \(H_3 \approx H_0\), and even then it appears to be unobservable [18]. Below we find the same to hold with our result.

While the distribution function (55) may seem complicated, under reasonable assumptions it gives way to...
a very simple phenomenology. Setting aside for the moment
the constant prefactor \( r_s \Gamma / H^2 \)
and we obtain
dependence of the distribution on \( \phi \).

\[ \xi \]
sions with angular scale smaller than the CMB sky, which
we take one step further and assume
does not allow for much simplification of (55), however if
\( \eta_0 \) is small, \( \eta_0 \approx r_s \), in which case \( \varrho(\psi) \approx e^{\eta_0} \approx H_p / H_d \) and

\[ dN / d\psi \propto \sin(\psi) \quad \text{(56)} \]

This agrees with the distribution of angular scales in the
case of a (3+1)-dimensional parent vacuum [17, 18].

The observer could in principle sit at any coordinate
\( 0 \leq \xi \leq \infty \) (before the boost that translates the ob-
servers to reside at \( \xi = 0 \)). The location of a typ-
cical observer in an eternally-inflating multiverse depends
on how one regulates the diverging spacetime volume.
What spacetime measure is appropriate to this or any
other prediction is still an open question, however the
measures receiving the most recent attention all predict
that typical observers reside at \( \xi_0 \sim O(0)^{1/2} \). This by itself
does not allow for much simplification of (55) however if
we take one step further and assume \( H_p / H_d \gg e^{\xi_0} \), the
dependence of the distribution on \( \phi_n \) becomes negligible,
and we obtain

\[ dN / d\psi \propto r_s \Gamma H_p / H_d \sin(\psi). \quad \text{(57)} \]

Indeed, even in the case where \( \xi_0 \) is large, careful study of (55) indicates that the distribution is only significantly
changed from (57) within a narrow region of width \( \Delta \phi_n / H_d \sim H_d / H_p \) about the point \( \phi_n = 0 \).

We can now consider the total number of bubble col-
lisions with angular scale smaller than the CMB sky, which
is found by integrating (55) over the intervals \( 0 < \psi < \pi \)
and \( \pi < \phi_n < \pi \). When \( H_p / H_d \gg e^{\xi_0} \), the angular
dependence is simple, and we find

\[ \frac{N}{N} = 4\pi r_s \frac{\Gamma H_p}{H_d^2}, \quad \text{(58)} \]

This result is not significantly changed in the case where
\( \xi_0 \) is large, so long as \( H_p / H_d \gg 1 \), because the number of
bubble collisions coming from the affected part of the
distribution is suppressed by a factor \( \Delta \phi_n / 2\pi \sim H_d / H_p \).
The case \( H_p \approx H_d \) is more complicated, but (58) captures
the overall scale of the result.

The factor \( \Gamma / H^3 \) is the dimensionless decay rate of
the parent vacuum, and can be crudely approximated as

\[ \frac{\Gamma}{H^3} \approx e^{-S_E}, \quad \text{(59)} \]

where \( S_E \) is the Euclidean action of the parent vacuum [3]. It is not hard to show that

\[ S_E = \frac{\sqrt{\pi^2 B^2}}{32 G^3 V}, \quad \text{(60)} \]

where \( V \) is the parent vacuum energy (of the (3+1)-
dimensional Lagrangian, see Section II). Thus while
\( S_E \) could potentially be very large—and \( \Gamma / H^3 \) strongly
exponentially suppressed—there is hope for vacuum en-
ergies and compactification radii not too far below the
Planck scale, in which case the suppression would not be
so severe (and in which case the prefactor neglected in
(58) could also become important).

Meanwhile, \( r_s \) is empirically constrained to be much
less than unity, and is itself exponentially suppressed by
the degree to which inflation in the bubble exceeds the
\( \sim 60 \) \( e \)-folds necessary to satisfy the observational con-
straints. At the same time there are indications that long
periods of slow-roll inflation may be difficult to achieve
in string theory, and a toy model of inflation in the land-
scape, combined with anthropic selection, gives reason-
able hope that inflation may not have lasted too long
within our bubble. [24–33].

Countering these two small factors is the ratio \( H_p / H_d \),
which could in principle be very large, for instance \( \sim 10^{30} \)
in the case of TeV-scale inflation with a near Planck-scale
parent vacuum. Notice however that without this factor
the number of bubble collisions is necessarily typically
much less than unity. Thus it seems the effects of such
bubble collisions are likely to be observed only if the par-
ent vacuum energy is very large, the scale of inflation in
our bubble is relatively small, and inflation in our bubble
does not last too long beyond what is necessary to make
the present geometry approximately flat.

Finally, recall that the number of bubble collisions
coming from the region \( \Delta \phi_n \) affected by anisotropy with
respect to \( \phi_n \) is suppressed by a factor \( \sim H_d / H_p \). We can
now see that this factor cancels the necessary enhancement to \( N \) mentioned above, which is why the “persis-
tence of memory” anisotropy is considered unobservable.

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2 By “typical” we mean randomly selected from a reference class
of similar observers (with similar environments), including other
observers in what we have called the observer’s bubble, and ob-
servers in other bubbles of the same type as the observer’s bubble
but nucleating elsewhere in spacetime. For the purposes of this
paper a “similar” observer is any one residing at time \( \eta_0 \) in a
bubble consistent with the cosmological assumptions above.

3 One can discriminate from among the various measure propos-
as for searching for phenomenological pathologies. Two patho-
logies in particular, Boltzmann brain domination [24,25] and run-
away inflation [52–54], seemingly rule out measures that predict
a uniform distribution of observers over the coordinate \( \xi \). The
measures known to survive these pathologies, the causal patch
measure [35], scale-factor cutoff measure [31,32], and comoving
probability measure [34], all distribute observers roughly uni-
f ormly according to the flat de Sitter comoving congruence
of the parent vacuum, which corresponds to bubble coordinates \( \xi \)
pre dominantly of order unity.
V. CONCLUSIONS

We have studied the properties of bubble collisions in an eternally-inflating parent vacuum where one of our three large spatial dimensions is compact. In particular, we have focused on bubbles formed via tunneling of a metastable modulus, corresponding to decompactifying a spatial dimension of the parent vacuum, producing bubbles with internal geometries that, after another period of inflation, resemble our own FRW cosmology. Such collisions are not unexpected in a multiverse populated by the string landscape, though our universe may or may not reside in a bubble of this type.

We find that such collisions may leave a distinct signature on the CMB sky, as their effects are limited to disks centered on a single great circle on the two-sphere of last scattering. This is very different than the case of a (3+1)-dimensional parent vacuum, where the corresponding disks can appear anywhere on the CMB sky. The distribution of angular scales of such regions, and of their azimuthal positions, qualitatively resemble those in the case of a (3+1)-dimensional parent vacuum. In particular, the distribution of angular scales \( \psi \) follows \( dN/d\psi \propto \sin(\psi) \). The probability that a bubble collision both resides in our past lightcone and has effects that do not cover the entire CMB sky depends on properties of the parent vacuum, the tunneling instanton, and inflation in the bubble. It could be much less than unity but there is hope for parameter values that make observing the effects of bubble collisions more likely.

There are many remaining interesting questions. Perhaps the most pressing issue, pertaining to signatures of bubble collisions in this and other scenarios, is to determine what such collisions would actually look like in terms of CMB observables (for instance, what is the temperature profile of an affected region). Also, in the context of anisotropic bubble nucleation, we have explored only one type of bubble collision—that between two bubbles with one more large spatial dimension than an effectively (2+1)-dimensional parent vacuum—but one can also consider collisions between bubbles like ours and lower- and higher-dimensional bubbles, or different parent-vacuum geometries altogether.

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