TMDs for spin-1 hadrons

S. KUMANO1,2 and Qin-Tao SONG3,4

1 KEK Theory Center, IPNS, KEK, Oho 1-1, Tsukuba, Ibaraki, 305-0801, Japan
2 J-PARC Branch, KEK Theory Center, IPNS, KEK, and Theory Group, Particle and Nuclear Physics Division, J-PARC Center, Shirakata 203-1, Tokai, Ibaraki, 319-1106, Japan
3 School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China
4 CPHT, CNRS, Ecole Polytechnique, Institut Polytechnique de Paris, Route de Saclay, 91128 Palaiseau, France
E-mail: shunzo.kumano@kek.jp, songqintao@zzu.edu.cn

(Received January 14, 2022)

Transverse-momentum-dependent parton distribution functions (TMDs) were investigated at the twists 3 and 4 for spin-1 hadrons in addition to the twist-2 ones. They were found by studying all the possible decomposition of a quark correlation function in a Lorentz-invariant way with the Hermiticity and parity invariance. The time-reversal invariance was not imposed for the TMDs due to an active role of gauge links; however, they were used for collinear parton distribution functions (PDFs) by integrating the TMDs over the transverse momentum. We found that 30 TMDs exist in the tensor-polarized spin-1 hadron at the twists 3 and 4, whereas there are 10 TMDs at the twist 2. We also showed that there are 3 collinear PDFs at the twists 3 and 4. The corresponding TMD fragmentation functions exist at the twists 2, 3, and 4 simply by changing function names and variables. Since the time-reversal invariance is valid in the collinear PDFs, the integrals of time-reversal-odd TMDs over the transverse momentum should vanish. It leads to the sum rules

\[ \int d^2 k_T h_{1LT}(x, k_T^2) = \int d^2 k_T g_{1LT}(x, k_T^2) = \int d^2 k_T h_{2LT}(x, k_T^2) = \int d^2 k_T h_{3LT}(x, k_T^2) = 0 \]

on the time-reversal-odd TMDs at the twists 3 and 4.

KEYWORDS: QCD, quark, gluon, spin-1 hadron, TMD, higher twist

1. Introduction

In the field of nucleon structure, recent interests tend to focus on three-dimensional structure functions, namely generalized parton distributions (GPDs), generalized distribution amplitudes (GDAs or timelike GPDs), and transverse-momentum-dependent parton distribution functions (TMDs). These functions play a crucial role in determining the origins of hadron spins and masses. Among them, the TMD physics is unique for finding explicit color degrees of freedom in terms of color flow within hadrons. It is an interesting interdisciplinary physics field, for example, in connection with the gluon condensate, the color Aharonov-Bohm effect, and the color entanglement.

On the other hand, structure functions of spin-1 hadrons could become an interesting field of high-energy spin physics because of additional functions to the ones of the spin-1/2 nucleons. The spin-1 deuteron has been described as a simple bound system of a proton and a neutron in traditional nuclear physics. However, new hadron-physics aspects could be probed by tensor-polarized structure functions, such as \( b_1 \) [1] and the gluon transversity [2]. Such aspects will be investigated experimentally in the near future at various accelerator facilities, such as the Thomas Jefferson National Accelerator Facility (JLab), Fermilab (Fermi National Accelerator Laboratory), Nuclotron-based Ion Collider fAcility (NICA), LHC (Large Hadron Collider)-spin, and electron-ion colliders (EIC, EicC). Considering this situation, we investigated possible TMDs, collinear parton distribution functions (PDFs), and fragmentation functions up to the twist 4 for spin-1 hadrons [3], so that the spin-1 TMDs, the PDFs, and the fragmentation functions can be studied at the same level with the spin-1/2 structure functions by including higher-twist effects. In this report, we explain our recent results.
2. Decomposition of a quark correlation function in a tensor-polarized spin-1 hadron

The TMDs and PDFs are defined from a correlation function

\[ \Phi_{ij}^{[c]}(k, P, T | n) = \int \frac{d^4 \xi}{(2\pi)^4} e^{ik\xi} \langle P, T | \tilde{\psi}_j(0) W^{[c]}(0, \xi) \psi_i(\xi) | P, T \rangle, \]  

(1)

which is the amplitude to extract a parton from a hadron and then to insert it into the hadron at a different space-time point. Since only the difference from the spin-1/2 nucleon case is the tensor-polarization part, we restrict our studies to the tensor polarization \( T^{\mu\nu} \) described by the polarization parameters \( (S_{LL}, S_{LT}^T, S_{TT}^{\mu\nu}) \) as

\[ T^{\mu\nu} = \frac{1}{2} \left[ \frac{4}{3} S_{LL} \frac{(P^+)^2}{M^2} \tilde{n}^\mu \tilde{n}^\nu - \frac{2}{3} S_{LL}(\tilde{n}^\mu n^\nu - g^{\mu\nu}) \right. \\
+ \frac{1}{3} S_{LL} \frac{M^2}{(P^+)^2} n^\mu n^\nu + \frac{P^+}{M} \tilde{n}^\mu S_{LT}^\nu - \frac{M}{2P^+} n^\mu S_{LT}^\nu + S_{TT}^{\mu\nu} \right]. \]  

(2)

In these equations, \( k \) and \( P \) are quark and hadron momenta, \( \psi \) is the quark field, \( \xi \) is the space-time coordinate, \( W^{[c]}(0, \xi) \) is the gauge link with the integral path \( c, n \) and \( \tilde{n} \) are the lightcone vectors \( n^\mu = (1, 0, 0, -1)/\sqrt{2} \) and \( \tilde{n}^\mu = (1, 0, 0, 1)/\sqrt{2} \). \( g^{\mu\nu}_T \) is given by \( g^{11}_T = g^{22}_T = -1 \) and the others \( = 0 \), \( a^{\mu\nu} \) indicates the symmetrized combination \( a^{\mu\nu} = a^{\alpha\beta} b^{\mu\nu} + a^{\alpha\nu} b^{\mu\beta} \), \( M \) is the hadron mass, and \( P^+ \) is the lightcone momentum given by \( P^+ = (P^0 + P^T)/\sqrt{2} \). In Eq. (1), the vector polarization \( S^\mu \) is not explicitly written because only the tensor polarization is investigated in this paper.

Taking into account the constraints of the Hermiticity and parity invariance, we decompose the quark correlation function in a Lorentz-invariant way as

\[ \Phi(k, P, T | n) = A_{13} \frac{M}{T} T_{kk} + A_{14} \frac{M}{T} T_{kk} P + \ldots + A_{20} \frac{M^2}{T} g_{\mu\nu} P T_{kk}, \]

\[ + B_{21} \frac{M}{P \cdot n} T_{kn} + B_{22} \frac{M^3}{(P \cdot n)^2} T_{mn} + \ldots + B_{52} \frac{M}{P \cdot n} \sigma_{\mu\nu} T_{kn}. \]  

(3)

Its full expression is given in Eq. (20) of Ref. [3]. Here, \( A_i \) and \( B_j \) are expansion coefficients, and the contraction \( X_{\mu k} \equiv X_{\mu k} k^k \) is used. This expansion is an extension of the work in Ref. [4] by including the additional \( n \) terms so as to find twist 3 and twist 4 functions, as it was done in the spin-1/2 nucleons [5]. The TMD and collinear correlation functions are given by integrating over the quark momenta as

\[ \Phi^{[c]}(x, k_T, P, T) = \int dk^+ dk^- \Phi^{[c]}(k, P, T | n) \delta(k^+ - xP^+), \]  

(4)

\[ \Phi(x, P, T) = \int dk T^{[c]}(x, k_T, P, T). \]  

(5)

3. TMDs and PDFs for a spin-1 hadron up to twist 4

The TMDs and collinear PDFs are defined by taking traces of the TMD and collinear correlation functions in Eqs. (4) and (5) with various \( \gamma \) matrices (\( \Gamma \)) as \( \Phi^{[\Gamma]} \equiv \frac{1}{\sqrt{2}} \text{Tr}[\Gamma \Phi^{[\Gamma]}] \). The twist-2 functions were studied by calculating \( \Phi^{[\gamma]}, \Phi^{[\gamma \gamma]}, \Phi^{[\gamma \mu \gamma \nu]}, \Phi^{[\sigma \sigma]}, \Phi^{[\sigma \gamma]}, \Phi^{[\sigma, \gamma]} \) (or \( \Phi^{[\sigma]} \)) [4]. The twist-2 TMDs and PDFs are listed in Tables I and II. The twist-3 and 4 results are our studies [3] in Tables III, IV, V, and VI. The twist-3 TMDs were obtained by taking the traces, \( \Phi^{[\gamma]}, \Phi^{[\gamma \gamma]}, \Phi^{[\gamma \mu \gamma \nu]}, \Phi^{[\sigma \sigma]}, \Phi^{[\sigma \gamma]}, \Phi^{[\sigma, \gamma]} \), and \( \Phi^{[\sigma \mu \sigma]} \). The \( i \) and \( j \) are transverse indices \( (i, j = 1 \text{ or } 2) \). The twist-4 TMDs were obtained by \( \Phi^{[\gamma]}, \Phi^{[\gamma \gamma]}, \Phi^{[\gamma \mu \gamma \nu]} \), and \( \Phi^{[\sigma \sigma]} \). For example, some of twist-3 TMDs are defined in the trace \( \Phi^{[\gamma]} \) as

\[ \Phi^{[\gamma]}(x, k_T, T) = \frac{M}{P^+} \left[ f_{LT}(x, k_T^2) S_{LT}^i \frac{k_T^i}{M} + f_{LT}(x, k_T^2) S_{LT}^j \frac{k_T^j}{M} - f_{LT}(x, k_T^2) \frac{k_T^i S_{LT}^j k_T^j}{M^2} - f_{LT}(x, k_T^2) \frac{k_T^i S_{LT}^j k_T^j}{M^2} + f_{LT}(x, k_T^2) \frac{k_T^i S_{LT}^j k_T^j}{M^2} \right]. \]  

(6)
The TMDs with the prime (′) are redefined as \( F(x, k_T^2) \equiv F′(x, k_T^2) - (k_T^2/(2M^2)) F′(x, k_T^2) \) with \( k_T^2 = -K_T^2 \), so that the functions without the primes are shown in the tables.

In the tables, the polarizations U, L, and T indicate unpolarized, longitudinally polarized, and transversely polarized, respectively, and they also exist in the spin-1/2 nucleons. The tensor polarizations (LL, LT, TT) are additional in the spin-1 hadrons. The time-reversal even (T-even) and odd (T-odd) distributions are classified in the tables. Chiral-odd distributions are shown with the square brackets [ ], and the distributions without the bracket are chiral-even ones. The asterisks *1, *2, *3, *4 indicate that the collinear PDFs \( h_{1LT}(x), g_{LT}(x), h_{LL}(x), h_{3LT}(x) \) vanish, respectively, due to the time-reversal invariance; however, the corresponding fragmentation functions \( H_{1LT}(z), G_{LT}(z), H_{LL}(z), H_{3LT}(z) \) should exist as collinear fragmentation functions [3,6]. Furthermore, finite transverse-momentum moments could exist even for the T-odd TMDs [7]. In the tensor-polarized spin-1 hadron, there are 40 TMDs in total, and there are 20 TMDs at the twist 3 and 10 TMDs at the twist 4. There are four PDFs in total, and there are two at the twist 3 and one at the twist 4 in the tensor polarizations. Since the time-reversal invariance is valid in the collinear PDFs, we have the sum rules for the T-odd TMDs as

\[
\int d^2k_T h_{1LT}(x, k_T^2) = \int d^2k_T g_{LT}(x, k_T^2) = \int d^2k_T h_{LL}(x, k_T^2) = \int d^2k_T h_{3LT}(x, k_T^2) = 0. \tag{7}
\]

There are an additional sum rule and useful relations in these PDFs and multiparton distribution functions [7,8]. Now, it became possible to investigate structure functions of spin-1 hadrons up to twist 4 in the same way with the spin-1/2 nucleons.

---

Table I. Twist-2 TMDs.

| Quark \(\gamma^*\) | U (\(\gamma\)) | L (\(\gamma\)) | T (\(\delta\gamma, \sigma\)) |
|-------------------|----------------|----------------|------------------|
| Hadron            | Teven Todd | Teven Todd | Teven Todd |
| U                 | \(f_1\)    | \([A]\)     | \([A]\)         |
| L                 | \(f_1\)    | \([A]\)     | \([A]\)         |
| T                 | \(f_1\)    | \([A]\)     | \([A]\)         |
| LL                | \(f_{g,1}\) | \([g,1]\)  | \([g,1]\)       |
| LT                | \(f_{g,1}\) | \([g,1]\)  | \([g,1]\)       |
| TT                | \(f_{g,1}\) | \([g,1]\)  | \([g,1]\)       |

Table II. Twist-2 PDFs.

| Quark \(\gamma^*\) | U (\(\gamma\)) | L (\(\gamma\)) | T (\(\delta\gamma, \sigma\)) |
|-------------------|----------------|----------------|------------------|
| Hadron            | Teven Todd | Teven Todd | Teven Todd |
| U                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| L                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| T                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LL                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| TT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |

Table III. Twist-3 TMDs.

| Quark \(\gamma^*\) | U (\(\gamma\)) | L (\(\gamma\)) | T (\(\delta\gamma, \sigma\)) |
|-------------------|----------------|----------------|------------------|
| Hadron            | Teven Todd | Teven Todd | Teven Todd |
| U                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| L                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| T                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LL                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| TT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |

Table IV. Twist-3 PDFs.

| Quark \(\gamma^*\) | U (\(\gamma\)) | L (\(\gamma\)) | T (\(\delta\gamma, \sigma\)) |
|-------------------|----------------|----------------|------------------|
| Hadron            | Teven Todd | Teven Todd | Teven Todd |
| U                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| L                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| T                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LL                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| TT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |

Table V. Twist-4 TMDs.

| Quark \(\gamma^*\) | U (\(\gamma\)) | L (\(\gamma\)) | T (\(\delta\gamma, \sigma\)) |
|-------------------|----------------|----------------|------------------|
| Hadron            | Teven Todd | Teven Todd | Teven Todd |
| U                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| L                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| T                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LL                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| TT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |

Table VI. Twist-4 PDFs.

| Quark \(\gamma^*\) | U (\(\gamma\)) | L (\(\gamma\)) | T (\(\delta\gamma, \sigma\)) |
|-------------------|----------------|----------------|------------------|
| Hadron            | Teven Todd | Teven Todd | Teven Todd |
| U                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| L                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| T                 | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LL                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| LT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
| TT                | \([\gamma]\) | \([\gamma]\) | \([\gamma]\) |
4. TMD and collinear fragmentation functions for a spin-1 hadron up to twist 4

Fragmentation functions of spin-1 hadrons can be investigated in the same way up to the twist-4. The collinear fragmentation functions were investigated up to the twist 4 in Ref. [6]; however, consistent studies on the TMDs of the spin-1 hadrons were restricted to the twist-2 level until recently. Since we have obtained the twist-3 and 4 TMDs, it became possible to investigate the TMD fragmentation functions up to the twist 4. It is not necessary to investigate the TMD fragmentation functions from the beginning. Simply changing the kinematical variables and function names as follows

Kinematical variables: \( x, k_T, S, T, M, n, \gamma^+, \sigma^{\pm} \Rightarrow z, k_T, S_h, T_h, M_h, \bar{n}, \gamma^-, \sigma^{\pm} \),

Distribution functions: \( f, g, h, e \Rightarrow \) Fragmentation functions: \( D, G, H, E \),

we obtained the corresponding TMD fragmentation functions [3]. The twist-2 TMD and collinear fragmentation functions are listed in Tables VII and VIII. The twist-3 and twist-4 functions are given in Tables IX, X, XI, and XII. We may note that the T-odd collinear fragmentation functions exist, although the T-odd PDFs do not exist, because the time-reversal invariance does not have to be imposed. Since the spin-1 fragmentation functions can be measured, for example, for the \( \rho \) meson, these TMD and collinear fragmentation functions are interesting quantities for future measurements.

### Table VII. Twist-2 TMD fragmentation functions.

| Quark | \( U(\gamma^+) \) | \( L(\gamma^+) \) | \( T(\sigma^+ \sigma^-) \) |
|-------|-----------------|-----------------|------------------|
| U     |                  |                  |                  |
| L     |                  |                  |                  |
| T     |                  |                  |                  |

### Table VIII. Twist-2 fragmentation functions.

| Hadron | \( U(\gamma^+) \) | \( L(\gamma^+) \) | \( T(\sigma^+ \sigma^-) \) |
|--------|-----------------|-----------------|------------------|
| U      |                  |                  |                  |
| L      |                  |                  |                  |
| T      |                  |                  |                  |

### Table IX. Twist-3 TMD fragmentation functions.

| Quark | \( T^- \) | \( T^- \gamma^- \) | \( \sigma^- \gamma^+ \) |
|-------|-----------|-----------------|------------------|
| U     |           |                 |                  |
| L     |           |                 |                  |
| T     |           |                 |                  |

### Table X. Twist-3 fragmentation functions.

| Hadron | \( T^- \) | \( T^- \gamma^- \) | \( \sigma^- \gamma^+ \) |
|--------|-----------|-----------------|------------------|
| U      |           |                 |                  |
| L      |           |                 |                  |
| T      |           |                 |                  |

### Table XI. Twist-4 TMD fragmentation functions.

| Quark | \( T^- \) | \( T^- \gamma^- \) | \( \sigma^- \gamma^+ \) |
|-------|-----------|-----------------|------------------|
| U     |           |                 |                  |
| L     |           |                 |                  |
| T     |           |                 |                  |

### Table XII. Twist-4 fragmentation functions.

| Hadron | \( T^- \) | \( T^- \gamma^- \) | \( \sigma^- \gamma^+ \) |
|--------|-----------|-----------------|------------------|
| U      |           |                 |                  |
| L      |           |                 |                  |
| T      |           |                 |                  |
5. Prospects and summary

The TMD physics is one of popular hadron physics topics nowadays partly because the TMD distributions are rare quantities to probe the explicit color flow within hadrons. They have been investigated for the nucleons so far. However, since the polarized spin-1 deuteron targets and beams will be available in the 2020’s and 2030’s at world accelerator facilities, we expect that the TMDs of the polarized deuteron should become an interesting topic. Considering this situation, we showed all the possible TMDs, collinear PDFs, and fragmentation functions up to the twist 4 in our theoretical studies. Now, it became possible to investigate them at the same level with the nucleon structure functions including higher-twist effects before the polarized-deuteron experiments. Since there are additional polarization quantities in the spin-1 deuteron, namely the tensor polarizations, our studies will become valuable along with future experimental investigations.

Appendix

S. Kumano was partially supported by Japan Society for the Promotion of Science (JSPS) Grants-in-Aid for Scientific Research (KAKENHI) Grant Number 19K03830. Qin-Tao Song was supported by the National Natural Science Foundation of China under Grant Number 12005191, the Academic Improvement Project of Zhengzhou University, and the China Scholarship Council for visiting Ecole Polytechnique.

References

[1] F. E. Close and S. Kumano, Phys. Rev. D 42, 2377 (1990); S. Kumano, Phys. Rev. D 82, 017501 (2010); S. Kumano and Qin-Tao Song, Phys. Rev. D 94, 054022 (2016); W. Cosyn, Yu-Bing Dong, S. Kumano, and M. Sargsian, Phys. Rev. D 95, 074036 (2017).
[2] S. Kumano, Qin-Tao Song, Phys. Rev. D 101, 054011 (2020); 101, 094013 (2020); S. Hino and S. Kumano, Phys. Rev. D 59, 094026 (1999); 60, 054018 (1999).
[3] S. Kumano and Qin-Tao Song, Phys. Rev. D 103, 014025 (2021).
[4] A. Bacchetta and P. Mulders, Phys. Rev. D 62, 114004 (2000).
[5] K. Goeke, A. Metz and M. Schlegel, Phys. Lett. B 618, 90 (2005); A. Metz, P. Schweitzer and T. Teckenbrup, Phys. Lett. B 680, 141 (2009).
[6] X. Ji, Phys. Rev. D 49, 114 (1994).
[7] S. Kumano and Qin-Tao Song, arXiv:2112.13218, submitted for publication.
[8] S. Kumano and Qin-Tao Song, JHEP 09, 141 (2021).
[9] J. P. Ma, C. Wang, and G. P. Zhang, arXiv:1306.6693 (unpublished, 2013).