Economic-Mathematical Model of Telecommunication System, Which is Being Developed for a High-Tech Company

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Abstract. The article proposes a model of a telecommunication system that allows to conduct a joint study of its functional and cost characteristics. For this model, three technical indicators and three economic metrics are used. They allow to explore the basic characteristics of a telecommunication network under normal operating conditions and when a critical situation of a different nature occurs. In this case, the numerical values of the attributes included in the bundle of the three technical indicators and the three economic metrics differ only. Consequently, the model itself remains unchanged. A methodology for analyzing the proposed model is developed. Two examples of the application of the proposed methodology are given. It is shown that in critical situations, which are often associated with a sharp decrease in the available resources of the telecommunication network with a simultaneous sharp increase in traffic, it is advisable to use methods of interdisciplinary approach. Such methods, as a rule, will significantly improve the effectiveness of further research.

1. Introduction
Telecommunication system plays a key role in the success of any company [1, 2, 3]. For a high-tech company, the value of a telecommunication system increases tremendously. This growth has quantitative and qualitative aspects, which are not always possible to translate into the language of mathematics. Nevertheless, it remains relevant to develop and study the model of a telecommunication system, which is being developed for a high-tech company, to solve a number of theoretical and practical problems.

In this paper, we propose an economic-mathematical model of a telecommunication system that allows to conduct a joint study of its functional and cost characteristics. These characteristics are related to each other. This explains the chosen approach to formation of the telecommunication system model.

2. Proposed model of the telecommunication system
Telecommunication system must be viewed from at least two points of view: functional capabilities and economic indicators. They are closely linked [4]. For this reason, it seems logical to develop an economic-mathematical model that allows to take into account basic properties of the object under
study and the processes occurring in it. At the macro level, this model can be represented as a "black box" [5]. The meaning of the functions $A(t)$, $B(t)$, $C(t)$ and $D(t)$ will be provided in the following section.

Queuing system (QS) in Kendall's classification [6, 7] is often indicated by three symbols that characterize the flow of calls, servicing durability and level of resources respectively. For the model in the form of a "black box", experimental QS is represented in this form $GI/GI/V$. The symbol GI in the first position indicates the fact that the time intervals between moments of calls arrival in the QS are independent random variables with an arbitrary distribution law [8, 9]. A similar symbol in the second position indicates that the call servicing durability is an independent random variable with an arbitrary distribution law. The symbol V determines the level of resources, usually expressed by integer, which is equal to or greater than one.

The economic attributes of the model can also be determined by three symbols: $C/P/E$ at the macro level. They are selected as the first letters in the English-language names of the following characteristics: cost parameters ($Cost – C$), penalty for violation of established qualitative indicators ($Penalty – P$), efficiency of telecommunication system ($Effectiveness – E$).

Thus, the proposed model of the telecommunication system can be designated by the bundle of this kind at the macro level: $GI/GI/V ↔ C/P/E$. In fact, the "black box" consists of a set of QSs forming a network of queuing [10, 11]. In this case, each QS can be designated using three symbols from Kendall's classification. From the point of view of economic attributes, the situation looks different. The characteristics forming the triple $C/P/E$, it is appropriate to detail through transition to specific economic indicators. The proposed bundle $GI/GI/V ↔ C/P/E$ reflects the traditional approach to planning telecommunication networks [12, 13], but also it allows to take into account highly unpredictable events, which often are described by the phrase "Black Swan" [14].

3. Methodological approach to study the model

The methodology of the model study is based on obtaining results, which are typical for each level of detail. In particular, "black box" can be analyzed by refining the bundle parameters $GI/GI/V ↔ C/P/E$. For example, this kind of analysis could include establishing a link between the amount of capital costs (one of the most important indicators for the attribute $C$) and IP packet flow intensity (one of the most important indicators for the attribute $GI$ in the first position of Kendall's classification), processed by telecommunication system.

Next, the model should be detailed in the form of a queuing network, which requires knowledge of its structure, if it is already defined, or the introduction of reasonable assumptions about the topology that will be built. This detail provides specification of the model in accordance with the hierarchical principles of building a telecommunication system. In this case, static shape of the model under study is formed.

Then, the dynamic shape of the model under study is formed, based on obtaining predictive estimates. At the same time, the "black box" itself and the hierarchical model of the telecommunication system are considered. It stands to reason that predictive assessments (especially, for the long term) are not characterized by high integrity.

The next step is development of scenarios for evolution of the telecommunication system, taking into account various factors, among which an important role is assigned to the evolution of tasks facing a high-tech company. Essentially, that at the stage of the model study, unlikely events such as the "Black Swan" are also considered. Scenario approach allows to minimize the emerging technical and economic risks inherent in the evolution of complex conservative systems.

If necessary, other options for detailing the model of a telecommunication system can be used. First of all, the expediency of such a decision will depend on interdisciplinary researches, which are typical for high-tech companies.

In any case, to obtain numerical estimates, it is appropriate to introduce at least the four functions mentioned above – $A(t)$, $B(t)$, $C(t)$ and $D(t)$. The parameter $t$ (time) is important, but it is the only argument of all functions. Essentially, that all four functions can change their nature depending on
subject of research. Nevertheless, they can be given a universal character by using a couple of "take and transfer" operations, often used in robotics [15]:

- \( A(t) \) – what you need to take (content, set of IP packets, etc.) for later transfer for processing, the essence of which could not be of practical use from the point of view of the process under consideration;
- \( B(t) \) – how to transfer, in order to solve the task in accordance with the specified requirements;
- \( C(t) \) – how to manage the transfer to solve the task in accordance with specified requirements or with the current capabilities of the object under study, caused by emergency situations;
- \( D(t) \) – what constitutes a useful output as a result of performing operations "take and transfer".

4. Example of model research results at macro level

As an example, it is appropriate to consider the simplest QS model with a Poisson input flow, an exponential distribution of call servicing time and one processing device. It is denoted as \( M/M/1 \) [5]. If the intensity values for the incoming call flow \( \lambda \) are known and for their servicing \( \mu \), the estimation of the necessary resources for pre-defined quality indicators is calculated according to the known formulas [5]. These resources certainly determine the capital costs included in the set of indicators \( C \) as one of the main parameters. Efficiency of telecommunication system \( E \) can be estimated by the ratio of \( \lambda \) to \( \mu \), which in the theory of teletraffic is called QS loading.

The situation with assessing fines for violation of established qualitative indicators \( P \) is somewhat more complicated. In the simplest case, you can set a penalty for exceeding the time delay of calls of a certain threshold \( X \). The average delay time of calls for the model in question is \((\mu-\lambda)^{-1}\). Suppose that \( X \) is triple average delay time of calls. This means that the penalty is levied when the call delay exceeds the amount \( 3(\mu-\lambda)^{-1} \).

Of course, for more sophisticated matching models in the bundle \( GI/GI/V \leftrightarrow C/P/E \) will not be so trivial. Moreover, for some models necessary correspondences can not be obtained analytically. When the proposed model is detailed, the solution of the arising problems becomes even more complicated.

5. Example of model research results related to throughput of network resources

This example relates to dependence of expected revenues on the amount of network resources. Theoretically, the greater the amount of resources, the higher the potential income. This fact is illustrated by the monotonically increasing curve in the Fig. 1 This type of allegation is explained by the improvement of the quality of servicing of multi-service traffic, through which fines for violation of SLA (Service Level Agreement) are minimized and new customers are attracted.

![Figure 1. Curve of dependence of expected revenues on the amount of network resources.](image-url)
In practice, there is a certain threshold of performance $T_{\text{opt}}$, above which it does not lead to Service provider's income rise. This statement can be explained in at least two ways. The first method is based on Theory of Constraints [16]. The second method is based on the so-called principle of "practical certainty" [8], which allows ignoring the highly unlikely (but existing in theory) events. For example, at performance $T_{\text{opt}}$ theoretically, the probability of losing IP packets is $10^{-10}$. Obviously, such probability in practice can be neglected.

The last statement is true for the usual operating conditions of the telecommunication network. The situation can change dramatically under the influence of external conditions, typical example of which is an emergency situation, leading to a reduction in the available resources of the telecommunication network with a simultaneous sharp increase in traffic [17, 18]. In similar cases the bundle of the form $\text{GI/\text{GI/V} \leftrightarrow C/P/E}$ remains the same. Of course, the values of the attributes included in this bundle change considerably. Then even theoretically, occurrence of events, the probability of which was around $10^{-10}$, it may turn out to be quite a real situation.

It is obvious that calculation of resources when planning a telecommunication network taking into account possible events such as emergency situations is not reasonable. On the other hand, Service provider should be ready to solve the problems associated with the loss of a part of the resources due to external destructive influences and traffic growth caused by reaction of subscribers to recent event. From this point of view, it is appropriate to introduce another bundle, which is distinguished by the subscript "x", highlighting the fact that we are talking about the network in critical conditions: $\text{GI/\text{GI/V}_x \leftrightarrow C/P/E}_x$. Both bundles are very important for Service provider. They will allow to develop solutions that ensure the effective operation of a telecommunication network in ordinary and critical (extreme) conditions.

Analysis of the bundle $\text{GI/\text{GI/V} \leftrightarrow C/P/E}$ is carried out through the use of well-known economic and mathematical methods. To analyse the bundle $\text{GI/\text{GI/V}_x \leftrightarrow C/P/E}_x$ it should be used an interdisciplinary approach [19, 20], because it is necessary to take into account, for example, economic damage caused by low level of quality of traffic servicing and even failures of telecommunications network fragments. In such cases, the solution of the task is significantly complicated. It should be noted that the use of interdisciplinary approaches will yield results that more accurately reflect the real operating conditions of the telecommunication network and assess its economic performance adequately.

6. Conclusion
The economic-mathematical model of a telecommunication system, which is being developed for a high-tech company, can be represented by the two bundles $\text{GI/\text{GI/V} \leftrightarrow C/P/E}$ and $\text{GI/\text{GI/V}_x \leftrightarrow C/P/E}_x$, which allow to conduct a joint study of technical characteristics and financial indicators. The proposed bundle can be supplemented and detailed depending on the complexity of the problem being solved. In addition, it is appropriate to decompose the model on different grounds, among which the special relevance is inherent in the hierarchical division of telecommunication system, dynamics of the development of the complex of the used technical means, the scenarios for evolution of the high-tech company.

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