Production Spectra of $^3\text{He}(\pi, K)$ Reactions with Continuum Discretized Coupled Channels

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Abstract

We investigate theoretically $\Lambda$ production spectra of $^3\text{He}(\pi, K)$ reactions at $p_\pi = 1.05$–1.20 GeV/c in the distorted-wave impulse approximation, using the continuum-discretized coupled-channel method. The production cross section of a $^3\Lambda\text{H}(1/2^+)$ ground state is also discussed.

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I. INTRODUCTION

It has been believed that hypertriton ($^3\Lambda\text{H}$) has lifetime within a few % of the free $\Lambda$ lifetime of $\tau_\Lambda = 263.2\pm2.0$ ps because $^3\Lambda\text{H}$ ($T = 0, J^\pi = 1/2^+$) is composed of a deuteron ($I = 0, ^3S_1$) and a $\Lambda$ hyperon loosely bound with the small $\Lambda$ binding energy of $B_\Lambda = 130\pm50$ keV. It is supported by several theoretical calculations [1]; see however [2]. Recently, unexpected short lifetime of $^3\Lambda\text{H}$ was measured in hypernuclear production by high-energy heavy-ion collisions [3]; the world average lifetime of $\tau(\text{av})(^3\Lambda\text{H}) = 185^{+23}_{-28}$ ps is shorter than $\tau_\Lambda$ by about 30%. (ALICE Collaboration [4] newly reported the preliminary result of $\tau(^3\Lambda\text{H}) = 237^{+33}_{-36}$ ps which is moderately closer to $\tau_\Lambda$.) To solve the hypertriton lifetime puzzle, experimental measurements of the $^3\Lambda\text{H}$ lifetime are planned by $^3\text{He}(K^-, \pi^0)$ and $^3\text{He}(\pi^-, K^0)$ reactions at J-PARC. Moreover, HypHI Collaboration [5] found a bound $nn\Lambda$ system, whereas theoretical calculations suggest that no $^3\Lambda n$ ($T = 1, J^\pi = 1/2^+$) bound state exists [6]. Therefore, it is important to investigate theoretically production of such $NN\Lambda$ systems, e.g., $(\pi, K)$ reactions on a $^3\text{He}$ target, in order to settle the current problems related to three-body hypernuclei.

In this paper, we focus on the $\Lambda$ production spectra of $^3\text{He}(\pi, K)$ reactions at $p_\pi = 1.05$–1.20 GeV/c in the distorted-wave impulse approximation (DWIA), using the continuum-discretized coupled-channel (CDCC) method [7] in order to well describe the $NN$ continuum states above the $N + N + \Lambda$ breakup threshold. We also discuss the production cross section of $^3\Lambda\text{H}(1/2^+)$ in $^3\text{He}(\pi^-, K^0)$ reactions, considering nuclear medium effects of the $\pi N \to \Lambda K$ amplitude and recoil effects.

II. CALCULATIONS

Inclusive differential cross sections for nuclear $(\pi, K)$ reactions in the laboratory frame within the DWIA [7] are given by (in units $\hbar = c = 1$)

\[
\frac{d^2\sigma}{dE_Kd\Omega_K} = \beta \frac{1}{[J_A]} \sum_{M_A} \sum_B |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \delta(E_K + E_B - E_\pi - E_A),
\]

(1)

where $[J] = 2J + 1$, $\beta$ is a kinematical factor, and $E_K$, $E_\pi$, $E_B$ and $E_A$ are energies of outgoing $K$, incoming $\pi$, hypernuclear states and the target nucleus, respectively; $\Psi_B$ and $\Psi_A$ are wavefunctions of hypernuclear states and the target nucleus, respectively. $\hat{F}$ is a
FIG. 1: Momentum transfer to the final state, $q_{\Lambda}$, for the $(\pi, K)$ reactions on a $^3\text{He}$ target at scattering $K$ angles of $\theta_{\text{lab}} = 3^\circ$, $7^\circ$, and $11^\circ$ in the laboratory frame, as a function of the incident pion momentum $p_\pi$.

The strangeness-exchange external operator given by

$$\hat{F} = \int d\mathbf{r} \chi_K^{(-)*}(p_K, \mathbf{r})\chi_\pi^{(+)}(p_\pi, \mathbf{r}) \sum_{j=1}^A \overline{f}_{\pi N \rightarrow \Lambda K} \delta(\mathbf{r} - \mathbf{r}_j) \hat{O}_j,$$

where $\chi_K^{(-)*}$ and $\chi_\pi^{(+)}$ are distorted waves for outgoing $K$ and incoming $\pi$, respectively, which are calculated with the help of the eikonal approximation. $\hat{O}_j$ is a baryon operator changing $j$th nucleon into a $\Lambda$ hyperon in the nucleus. Figure 1 displays the momentum transfer to the final state, $q_{\Lambda} = |p_\pi - p_K|$, as a function of the incident pion momentum $p_\pi$, where $p_\pi$ and $p_K$ are the laboratory momenta of $\pi$ and $K$ in the nuclear reaction, respectively. $\overline{f}_{\pi N \rightarrow \Lambda K}$ is the $\pi N \rightarrow \Lambda K$ amplitude in nuclear medium, which is obtained by the optimal Fermi-averaging method [8]. It should be noticed that strong $E_\Lambda$ dependence appears in $\overline{f}_{\pi N \rightarrow \Lambda K}$ because the elementary cross sections for the $\pi N \rightarrow \Lambda K$ reactions depend on the incident pion momentum [9]; we confirm that the optimal Fermi-averaged cross sections of $d\sigma/d\Omega = \beta |\overline{f}_{\pi N \rightarrow \Lambda K}|^2$ at $p_\pi = 1.05$ and $1.20$ GeV/c have strong $E_\Lambda$ dependence [8], as shown
FIG. 2: Calculated laboratory cross sections for the $\pi N \rightarrow \Lambda K$ reactions in nuclear medium at $p_\pi = 1.05$ and 1.20 GeV/c with $\theta_{\text{lab}} = 3^\circ$, 7$^\circ$, and 11$^\circ$, as a function of $E_\Lambda$. The optimal Fermi averaging $^8$ is used.

Hypernuclear final states are considered as three-body $NN\Lambda$ systems in the $^3\text{He}(\pi, K)$ reactions, involving continuum states above the $N + N + \Lambda$ threshold. Here we employ the CDCC method $^10$ in order to well describe the $NN$ continuum states as breakup channels. Thus the wavefunctions $\Psi_B$ with $J^\pi$ in the $LS$-coupling scheme can be written as

$$
\Psi_B \simeq \Psi_B^{\text{CDCC}}(r, R) = \sum_{\alpha=1}^{N_{\text{max}}} \sum_{\ell_2=0}^{\ell_{\text{max}}} \left[ \tilde{\phi}^{(2N)}_{\alpha,\ell_2}(r) \otimes \varphi_{\alpha,\ell_\Lambda}^{(A)}(R) \right]_{LB} \otimes X_{I_{\alpha},S_{\alpha}}^{B} \gamma_{MB},
$$

where $\tilde{\phi}^{(2N)}_{\alpha,\ell_2}(r)$ is the $NN$ wavefunction having bound and continuum-discretized states for angular momentum $\ell_2$, spin $^1S_0$ or $^3S_1$ in channel $\alpha$, $\varphi_{\alpha,\ell_\Lambda}^{(A)}(R)$ is the relative wavefunction between $NN$ and $\Lambda$ with $\ell_\Lambda$, and $X_{I_{\alpha},S_{\alpha}}^{B}$ is the isospin-spin function for $NN\Lambda$. Because we omit spin-flip processes in the $(\pi, K)$ reactions on the $^3\text{He} (1/2^+)$ target, the final states on $J^\pi = |L^\pi \pm 1/2|$ with $L^\pi = 0^+, 1^-, 2^+, \cdots$, and $S = 1/2$ can be populated. We obtain $\varphi_{\alpha,\ell_\Lambda}$, solving a coupled-channel equation with the potential $U_{\alpha\alpha'}$ given by the microscopic $2N$-$\Lambda$
FIG. 3: Calculated inclusive $K^+$ spectrum in the $^3\text{He}(\pi^+,K^+)$ reaction at 1.20 GeV/c, $\theta_{\text{lab}} = 3^\circ$, together with partial-wave components $L^\pi$ and $S = 1/2$, as a function of the missing mass $M_x$. The spectrum is taken into account a detector resolution of 4 MeV FWHM.

folding model;

$$U_{\alpha\alpha'}(R) = \int \rho_{\alpha\alpha'}(r) \left( \bar{v}_{\Lambda N}(R - r/2) + \bar{v}_{\Lambda N}(R + r/2) \right) dr,$$

where $\rho_{\alpha\alpha'}(r)$ is the nucleon or transition density, and $\bar{v}_{\Lambda N}$ is the spin-averaged $\Lambda N$ potential. For the $\Lambda N$ potential $v_{\Lambda N}$, we assume a single Gaussian form which reproduces the scattering length and the effective range in $\Lambda p$ scattering at low energies, fitting into those of NSC97f. We can also reproduce the experimental value of $B_\Lambda = 0.13$ MeV for $^3\Lambda\text{H}(1/2^+)$ when we slightly modify the strengths of $v_{\Lambda N}$ by a factor of 0.92. We rewrite a sum over the final states in Eq. (1) as

$$\sum_B |\Psi_B\rangle \langle \Psi_B| \delta(E_B - E_A - \omega) = -\frac{1}{\pi} \text{Im} \hat{G}(\omega),$$

where $\hat{G}(\omega)$ is a complete Green’s function for the $2N$-$(\Lambda\Lambda)$ systems given by CDCC wavefunctions. Therefore, the inclusive differential cross sections are obtained by the Green’s function method [11].
FIG. 4: Calculated inclusive $K^0$ spectrum in the $^3$He ($\pi^-, K^0$) reaction at 1.20 GeV/c, $\theta_{\text{lab}} = 3^\circ$. The bin with a finite width of 1 MeV at $M_x = 2991.2$ MeV/c$^2$ denotes the integrated cross section of $^3\Lambda H (1/2^+)$, which is denoted by $d + \Lambda$ in the figure. See also the caption of Fig. 3.

III. RESULTS AND DISCUSSION

In the $^3$He($\pi^+, K^+$) reactions, we consider the production of the $pp\Lambda$ states with only $NN$, $^1S_0$ components. Figure 3 displays the calculated inclusive $K^+$ spectrum of the $^3$He($\pi^+, K^+$) reaction at $p_\pi = 1.20$ GeV/c, $\theta_{\text{lab}} = 3^\circ$, together with partial-wave components of the spectrum. Considering large momentum transfer of $q_\Lambda \simeq 360$ MeV/c in exothermic ($\pi, K$) reactions, we find that many partial waves moderately contribute to the spectrum; there appears an enhancement of the $T = 1, J^\pi = 1/2^+$ ($L^\pi = 0^+, S = 1/2$) component just above the $p + p + \Lambda$ threshold. This enhancement may indicate that a pole of the s-wave $pp\Lambda$ resonance or virtual state resides near the $p + p + \Lambda$ threshold, as suggested by several three-body calculations.

In the $^3$He($\pi^-, K^0$) reactions, we study the production of the $pn\Lambda$ states with $NN$, $^3S_1$ and $^1S_0$ components, which include the $^3\Lambda H (T = 0, J^\pi = 1/2^+)$ ground state. Figure 4
TABLE I: Calculated results of the integrated production cross sections of the \(^3\text{Λ}H\) ground state, \(d\sigma/d\Omega (\text{\(^3\text{Λ}H\)})\), in the \(^3\text{He}(\pi^-, K^0)\) reactions at \(p_\pi = 1.05\) and 1.20 GeV/c.

| \(p_\pi\) (GeV/c) | \(\theta_{\text{lab}}\) (degree) | \((d\sigma/d\Omega)_{\pi^N\rightarrow\Lambda K}^{\text{\(3\text{Λ}H\)}}\) \((\text{µb/sr})\) | \(q_\Lambda\) \((\text{MeV/c})\) | \(q_{\text{eff}}^{\Lambda}\) \((\text{MeV/c})\) | \(d\sigma/d\Omega (\text{\(^3\text{Λ}H\)})\) \((\text{µb/sr})\) |
|------------------|-----------------|-----------------|----------------|-----------------|-----------------|
| 1.05             | 3               | 463             | 344            | 354             | 236             | 0.15            | 3.07            |
|                  | 7               | 459             | 340            | 369             | 246             | 0.10            | 2.42            |
|                  | 11              | 450             | 332            | 393             | 262             | 0.05            | 1.59            |
| 1.20             | 3               | 292             | 225            | 326             | 217             | 0.22            | 3.13            |
|                  | 7               | 287             | 235            | 350             | 233             | 0.13            | 2.24            |
|                  | 11              | 277             | 236            | 383             | 255             | 0.05            | 1.33            |

\(^a\beta|f_{\pi^-p\rightarrow\Lambda K}\nu|^2\) is used.

\(^b\beta|\bar{f}_{\pi^-p\rightarrow\Lambda K}\nu|^2\) is used.

\(^c q_{\text{eff}}^{\Lambda} \approx (M_C/M_\Lambda)q_\Lambda\) where a recoil factor of \(M_C/M_\Lambda\) is equal to 2/3 for the \(^3\text{He}\) target.

displays the calculated inclusive \(K^0\) spectrum of the \(^3\text{He}(\pi^-, K^0)\) reaction at \(p_\pi = 1.20\) GeV/c, \(\theta_{\text{lab}} = 3^\circ\), together with partial-wave components of the spectrum. We find that the integrated production cross section of \(^3\text{Λ}H\) amounts to \(d\sigma/d\Omega (\text{\(^3\text{Λ}H\)}) = 3.13\) µb/sr. In Table II we show the values of \(d\sigma/d\Omega (\text{\(^3\text{Λ}H\)})\) at \(p_\pi = 1.05\) and 1.20 GeV/c with \(\theta_{\text{lab}} = 3^\circ, 7^\circ,\) and \(11^\circ\) in order to see the sensitivity to the in-medium \(\pi N \rightarrow \Lambda K\) amplitudes and to momentum transfers. We find that the values of \(d\sigma/d\Omega (\text{\(^3\text{Λ}H\)})\) at 1.05 and 1.20 GeV/c are similar, although the values of \((d\sigma/d\Omega)_{\pi^N\rightarrow\Lambda K}^{\text{\(3\text{Λ}H\)}}\) near the \(\Lambda\) threshold at 1.05 GeV/c are about 1.5 times as large as those at 1.20 GeV/c. This is caused by the fact that the former is larger than the latter in terms of \(q_{\text{eff}}^{\Lambda}\). If the recoil effects are switched off \((M_C/M_\Lambda = 2/3\) is replaced by 1), the values of \(d\sigma/d\Omega (\text{\(^3\text{Λ}H\)})\) are reduced by an order of magnitude or more. Hence we recognize that the recoil effects for production on the light target as \(^3\text{He}\) are important in the nuclear \((\pi, K)\) reactions.

IV. SUMMARY

We have shown the calculated \(\Lambda\) production spectra of the \(NN\Lambda\) systems in the \(^3\text{He}(\pi, K)\) reactions at 1.05–1.20 GeV/c with CDCC which describes the \(NN\) continuum states above
the $N+N+\Lambda$ breakup threshold. The production cross section of $^3\Lambda H(1/2^+)$ in the $(\pi^-, K^0)$ reaction is evaluated, e.g., $d\sigma/d\Omega (^3\Lambda H) \simeq 3 \mu b/sr$ at 1.05–1.20 GeV/$c$, $\theta_{\text{lab}} = 3^\circ$. The recoil effects are very important to the production with the light nuclear target as $^3\text{He}$, as well as the medium effects of the $\pi N \rightarrow \Lambda K$ amplitudes for nuclear $(\pi, K)$ reactions. More precise analysis on convergence of the CDCC model space depending on $(k_{\text{max}}, \ell_{\text{max}})$ should be needed. This investigation is in progress.

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