A KIND OF PREDICTION FROM STRING PHENOMENOLOGY: EXTRA MATTER AT LOW ENERGY

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We review the possibility that the Supersymmetric Standard Model arises from orbifold constructions of the \( E_8 \times E_8 \) Heterotic Superstring, and the phenomenological properties that such a model should have. In particular, trying to solve the discrepancy between the unification scale predicted by the Heterotic Superstring (\( \approx g_{\text{GUT}} \times 5.27 \times 10^{17} \text{ GeV} \)) and the value deduced from LEP experiments (\( \approx 2 \times 10^{16} \text{ GeV} \)), we will predict the presence at low energies of three families of Higgses and vector-like colour triplets. Our approach relies on the Fayet-Iliopoulos breaking, and this is also a crucial ingredient, together with having three Higgs families, to obtain in these models an interesting pattern of fermion masses and mixing angles at the renormalizable level. Namely, after the gauge breaking some physical particles appear combined with other states, and the Yukawa couplings are modified in a well controlled way. On the other hand, dangerous flavour-changing neutral currents may appear when fermions of a given charge receive their mass through couplings with several Higgs doublets. We will address this potential problem, finding that viable scenarios can be obtained for a reasonable light Higgs spectrum.

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1. Introduction

In SuperString Theory the elementary particles are not point-like objects but extended, string-like objects. It is surprising that this apparently small change allows us to answer fundamental questions that in the context of the quantum field theory of point-like particles cannot even be posed. For example: Why is the Standard Model gauge group \( SU(3) \times SU(2)_L \times U(1)_Y \)? Why are there three families of particles? Why is the mass of the electron \( m_e = 0.5 \text{ MeV} \)? Why is the fine structure constant \( \alpha = 1/137 \)? In addition, only SuperString Theory has the potential to unify all gauge interactions with gravity in a consistent way. In this sense, it is a crucial step in the construction of the fundamental theory of particle physics to find a consistent SuperString model in four dimensions accommodating the observed Standard Model (SM), i.e. we need to find the SuperString Standard Model (SSSM). Actually, this is the main task of what we call String Phenomenology.

In the late eighties, the compactification of the \( E_8 \times E_8 \) Heterotic String on
six-dimensional orbifolds proved to be an interesting method to carry out this task\(^a\) (for a brief historical account see the Introduction in Ref. 1 and references therein).

For example, it was shown that the use of two Wilson lines on the torus defining the symmetric $Z_3$ orbifold can give rise to four-dimensional supersymmetric models with gauge group $SU(3) \times SU(2) \times U(1) \times G_{	ext{hidden}}$ and, automatically, three generations of chiral particles.\(^b\) In addition, it was also shown that the Fayet–Iliopoulos (FI) D-term, which appears because of the presence of an anomalous $U(1)$, can give rise to the breaking of the extra $U(1)$’s. In this way it was possible to construct supersymmetric models with gauge group $SU(3) \times SU(2) \times U(1)_Y$, three generations of particles in the observable sector, and absence of dangerous baryon- and lepton-number-violating operators.\(^c\)

Unfortunately, we cannot claim that one of these $Z_3$ orbifold models is the SSSM, since several problems are always present. For example, the initially large number of extra particles, which are generically present in these constructions, is highly reduced through the FI mechanism, since many of them get a high mass ($\approx 10^{16−17}$ GeV). However, in general, some extra $SU(3)$ triplets, $SU(2)$ doublets and $SU(3) \times SU(2)$ singlets still remain at low energy. On the other hand, given the predicted value for the unification scale in the Heterotic String,\(^d\) $M_{\text{GUT}} \approx g_{\text{GUT}} \times 5.27 \times 10^{17}$ GeV, the values of the gauge couplings deduced from LEP experiments cannot be obtained.\(^e\) It was also not possible to obtain in these models the necessary Yukawa couplings reproducing the observed fermion masses.\(^f\)

At this point, it is fair to say that almost 20 years have gone by since String Phenomenology started, and the SSSM has not been found yet.\(^d\) As acquittal on the charge we should remark that there are thousands of models (vacua) that can be built. Some of them have the gauge group of the SM or GUT groups, three families of particles, and other interesting properties, but many others have a number of families different from three, no appropriate gauge groups, no appropriate matter, etc. A perfect way of solving this problem would be to use a dynamical mechanism to select the correct model (vacuum). Such a mechanism should be able to determine a point in the parameter space of the Heterotic String determining the correct compactification producing the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, three families of the known particles, the correct Yukawa couplings, etc. The problem is that such a mechanism has not been discovered yet.

So, for the moment, the best we can do is keep trying, i.e. to use the experimental

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\(^a\)Other interesting attempts at model building used Calabi–Yau spaces and fermionic constructions.

\(^b\)Recently, other interesting models in the context of the $Z_3$ orbifold\(^6\) as well as in the context of the $Z_4$ orbifold\(^7\) and $Z_{12}$ orbifold\(^8\) have been analysed.

\(^c\)Recall that this is only possible in the context of the Minimal Supersymmetric Standard Model (MSSM) for $M_{GUT} \approx 2 \times 10^{16}$ GeV.

\(^d\)And this sentence can also be applied to any of the interesting models constructed in more recent years using D-brane technology\(^9\). Actually, the probability of obtaining an MSSM like gauge group with three generation in the context of intersecting D-branes in an orientifold background seems to be extremely small\(^1\) of about $10^{-9}$. 

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results available (such as the SM gauge group, three families, fermion masses, mixing angles, etc.), to discard models. Although the model space is in principle huge, a detailed analysis can reduce this to a reasonable size. For example, within the $Z_3$ orbifold with two Wilson lines, one can construct in principle a number of order $50000$ of three-generation models with the $SU(3) \times SU(2) \times U(1)^3$ gauge group associated to the first $E_8$ of the Heterotic String. However, a study implied that most of them are equivalent \cite{18}, and in fact, at the end of the day, only 192 different models were found \cite{19}. This reduction is remarkable, but we should keep in mind that the analysis of each one of these models is really complicated.

Nevertheless, a certain degree of optimism is important when working in String Phenomenology, and one can argue that if the SM arises from SuperString Theory there must exist one model with the right properties. In the present review we will adopt this viewpoint, and will assume that the SM arises from orbifolds constructions. Instead of the painful work of searching for the correct orbifold model, we will try to deduce the phenomenological properties that such a model must have in order to solve the crucial problems mentioned above, with the hope that this analysis will allow us to make predictions that can be tested at the LHC.

In fact, all those problems, extra matter, gauge coupling unification, and correct Yukawa couplings, are closely related. The first two because the evolution of the gauge couplings from high to low energy through the renormalization group equations (RGEs), depends on the existing matter \cite{20}. In Section 2 we will discuss a solution to the gauge coupling unification problem implying the prediction of three generations of supersymmetric Higgses and vector-like colour triplets at low energies. In this solution the FI scale plays an important role.

Concerning the third problem, how to obtain the observed structure of fermion masses and mixing angles, this is in our opinion the most difficult task in String Phenomenology. For example, the right model must reproduce also the correct mass hierarchy for quarks and leptons, $\frac{m_t}{m_u} \sim 10^5$, $\frac{m_t}{m_e} \sim 10^3$, etc., and this is not a trivial task, although it is true that one can find interesting results in the literature. In particular, orbifold spaces have a beautiful mechanism to generate a mass hierarchy at the renormalizable level. Namely, Yukawa couplings between twisted matter can be explicitly computed and they get suppression factors, which depend on the distance between the fixed points to which the relevant fields are attached \cite{21}-\cite{26}. The couplings can be schematically written as $\lambda \sim e^{-\sum_i^2 c_i T_i}$, with $Re T_i \sim R_i^2$, and the $T_i$ are the moduli fields associated to the size and shape of the orbifold. The distances can be varied by giving different vacuum expectation values (VEVs) to these moduli, implying that one can span in principle five orders of magnitude the Yukawa couplings \cite{24}-\cite{26}. Unfortunately, this is not the end of the story, since Nature tells us that a weak coupling matrix exists with weird magnitudes for the entries, and that therefore we must arrange our up- and down-quark Yukawa couplings in order to have specific off diagonal elements. In Section 3 we will see that

\footnote{Needless to say, the recent experimental confirmation of neutrino masses makes the task even
to obtain this at the renormalizable level is possible if three Higgs families and the FI breaking are present. Thus we have a common solution for the three problems mentioned above.

On the other hand, it is well known that dangerous flavour-changing neutral currents (FCNCs) may appear when fermions of a given charge receive their mass through couplings with several Higgs doublets. This situation might be present here since we have three generations of supersymmetric Higgs. In Section 4 we will address this potential problem, finding that viable scenarios can be obtained.

2. Predictions from gauge coupling unification

Since we are interested in the analysis of gauge couplings, we need to first clarify which is the relevant scale for the running between the supersymmetric scale $M_S$ and the unification point. Let us recall that in heterotic compactifications some scalars singlets $C_i$ develop vacuum expectation values (VEVs) in order to cancel the FI $D$-term, without breaking the SM gauge group. An estimate about their VEVs can be done with the average result $\langle C_i \rangle \sim 10^{16-17}$ GeV (see e.g. Ref. 6). After the breaking, many particles, say $\xi$, acquire a high mass because of the generation of effective mass terms. These come for example from operators of the type $C_i \xi \xi$.

In this way extra vector-like triplets and doublets and also singlets become very heavy. We will use the above value as our relevant scale, the so-called FI scale $M_{FI} \approx 10^{16-17}$ GeV.

As discussed in the Introduction, we are interested in the unification of the gauge couplings at $M_{GUT} \approx g_{GUT} \times 5.27 \times 10^{17}$ GeV. This is not a simple issue, and various approaches towards understanding it have been proposed in the literature. Some of these proposals consist of using string GUT models, extra matter at intermediate scales, heavy string threshold corrections, non-standard hypercharge normalizations, etc. In our case, we will try to obtain this value by using first the existence of extra matter at the scale $M_S$. We will see that this is not sufficient and, as a consequence, the FI scale must be included. Let us concentrate for the moment on $\alpha_3$ and $\alpha_2$. Recalling that three generations appear automatically for all the matter in $Z_3$ orbifold scenarios with two Wilson lines, the most natural possibility is to assume the presence of three light generations of supersymmetric Higgses. This implies that we have four extra Higgs doublets, $n_2 = 4$, with respect to the case of the MSSM. Unfortunately, this goes wrong. Whereas $\alpha_3^{-1}$ remains unchanged, since the number of extra triplets is $n_3 = 0$, the line for $\alpha_2^{-1}$ is pushed down with respect to the case of the MSSM. As a consequence, the two couplings cross at a very low scale ($\approx 10^{12}$ GeV). We could try to improve this situation by assuming the presence of extra triplets in addition to the four extra doublets. Then the line for $\alpha_3^{-1}$ is also pushed down and therefore the crossing might be obtained for larger scales. However, even more involved. We have to explain also the weak coupling matrix with the charged leptons. Besides, in addition to the hierarchies shown above, we have to explain others such as $m_\nu \geq 10^6$. 
Fig. 1. Unification of the gauge couplings at $M_{\text{GUT}} \approx g_{\text{GUT}} \times 5.27 \cdot 10^{17}$ GeV with three light generations of supersymmetric Higgses and vector-like colour triplets. In this example we show one of the four possible patterns of heavy matter in eq. (2), in particular that with a) $n_{F I}^3 = 0$. The line corresponding to $\alpha_1$ is just one of the many possible examples.

for the minimum number of extra triplets that can be naturally obtained in our scenario, $3 \times \{(3,1) + (\bar{3},1)\}$, i.e. $n_3 = 6$, the “unification” scale turns out to be too large ($\approx 10^{21}$ GeV). One can check that other possibilities including more extra doublets and/or triplets do not work. Thus, using extra matter at $M_S$ we are not able to obtain the Heterotic String unification scale, since $\alpha_3$ never crosses $\alpha_2$ at $M_{\text{GUT}} \approx g_{\text{GUT}} \times 5.27 \cdot 10^{17}$ GeV. Fortunately, this is not the end of the story. As we will show now, the FI scale $M_{FI}$ is going to play an important role in the analysis.

In order to determine whether or not the Heterotic String unification scale can be obtained, we need to know the number of doublets $n_{F I}^2$ and triplets $n_{F I}^3$ in our construction with masses of order the FI scale $M_{FI}$. It is possible to show that within the $Z_3$ orbifold with two Wilson lines, three-generation standard-like models must fulfil the following relation for the extra matter: $2 + n_2 + n_{F I}^2 = n_3 + n_{F I}^3 + 12$. Then, it is now straightforward to check that only models with $n_2 = 4, n_3 = 6$, and therefore $n_{F I}^2 - n_{F I}^3 = 12$, may give rise to the Heterotic String unification scale (other possibilities for $n_2, n_3$ do not even produce the crossing of $\alpha_3$ and $\alpha_2$). This is shown in Fig. 1 for an example with $n_{F I}^3 = 0$, and assuming $M_S = 500$ GeV. There we are using $M_{FI} = 2 \cdot 10^{16}$ GeV as will be discussed below.

Note that at low energy we then have (excluding singlets)

$$3 \times \{(3,2) + 2(\bar{3},1) + (1,2)\} + 3 \times \{(3,1) + (\bar{3},1) + 2(1,2)\},$$

i.e. the matter content of the Supersymmetric SM with three generations of Higgses and vector-like colour triplets.

Let us remark that in these constructions only the following patterns of matter
with masses of order $M_{FI}$ are allowed:

\begin{align}
& a) \quad n_3^{FI} = 0, \quad n_2^{FI} = 12 \rightarrow 3 \times \{4(1, 2)\}, \\
& b) \quad n_3^{FI} = 12, \quad n_2^{FI} = 18 \rightarrow 3 \times \{3(3, 1) + (\bar{3}, 1) + 6(1, 2)\}, \\
& c) \quad n_3^{FI} = 12, \quad n_2^{FI} = 24 \rightarrow 3 \times \{2(3, 1) + (\bar{3}, 1) + 8(1, 2)\}, \\
& d) \quad n_3^{FI} = 18, \quad n_2^{FI} = 30 \rightarrow 3 \times \{3(3, 1) + (\bar{3}, 1) + 10(1, 2)\}. \quad (2)
\end{align}

Thus for a given FI scale, $M_{FI}$, each one of the four patterns in eq. (2) will give rise to a different value for $g_{GUT}$. Adjusting $M_{FI}$ appropriately, we can always get $M_{GUT} \approx g_{GUT} \times 5.27 \cdot 10^{17}$ GeV. In particular this is so for $M_{FI} \approx 2 \times 10^{16}$ GeV as shown in Fig. 1. It is remarkable that this number is within the allowed range for the FI breaking scale as discussed above. For the pattern in Fig. 1 corresponding to case $a)$ we have $g_{GUT} \approx 1.1$, and therefore $M_{GUT} \approx 5.8 \cdot 10^{17}$ GeV.

Of course, we cannot claim to have obtained the Heterotic String unification scale until we have shown that the coupling $\alpha_1$ joins the other two couplings at $M_{GUT}$. The analysis becomes more involved now and a detailed account of this issue can be found in Ref. 1. Let us just mention that the fact that the normalization constant, $C$, of the $U(1)_Y$ hypercharge generator is not fixed in these constructions as in the case of GUTs (e.g., for $SU(5)$, $C^2 = 3/5$) is crucial in order to obtain the unification with the other couplings.

Summarizing, the main characteristic of this scenario is the presence at low energy of extra matter. In particular, we have obtained that three generations of Higgses and vector-like colour triplets are necessary.

Since more Higgs particles than in the MSSM are present, there will be of course a much richer phenomenology. Note for instance that the presence of six Higgs doublets implies the existence of sixteen physical Higgs bosons, where eleven of them are neutral and five charged.

Concerning the three generations of vector-like colour triplets, say $D$ and $\bar{D}$, they should acquire masses above the experimental limit $\mathcal{O}(200 \text{ GeV})$. This is possible, in principle, through couplings with some of the extra singlets with vanishing hypercharge, say $N_i$, which are usually left at low energies, even after the FI breaking. For example, in the model of Ref. 3 there are 13 of these singlets. Thus couplings $N_i D \bar{D}$ might be present. From the electroweak symmetry breaking, the fields $N_i$ a VEV might develop. Note in this sense that the Giudice–Masiero mechanism to generate a $\mu$ term through the Kähler potential is not available in prime orbifolds as $Z_3$. Thus an interesting possibility to generate VEVs, given the large number of singlets present in orbifold models, is to consider couplings of the type $N_i H_u H_d$, similarly to the Next-to-Minimal Supersymmetric Standard Model (NMSSM). It is also worth noticing that some of these singlets might not have the necessary couplings to develop VEVs and then their fermionic partners might be candidates for
right-handed neutrinos.$^4$

For the models studied in Refs. 4, 6 the extra colour triplets have non-standard fractional electric charge, $\pm 1/15$ and $\pm 1/6$ respectively. In fact, the existence of this kind of matter is a generic property of the massless spectrum of supersymmetric models. This means that they have necessarily colour-neutral fractionally charged states, since the triplets bind with the ordinary quarks. For example, the model with triplets with electric charge $\pm 1/6$ will have mesons and baryons with charges $\pm 1/2$ and $\pm 3/2$. On the other hand, the model studied in Ref. 3 has ‘standard’ extra triplets, i.e. with electric charges $\mp 1/3$ and $\mp 2/3$; these will therefore give rise to colour-neutral integrally charged states. For example, a $d$-like quark $D$ forms states of the type $uD$, $uuD$, etc.

Let us finally mention that a detailed discussion about the stability of these charged states, how to solve possible conflicts with cosmological bounds, and their production modes can be found in Ref. 1.

3. Quark and lepton masses and mixing angles

Crucial ingredients in the above analysis were that all three generations of supersymmetric Higgses remain light ($H_u^i, H_d^i, i = 1, 2, 3$), and the FI breaking. And, precisely, both ingredients favour to obtain the correct Yukawa couplings at the renormalizable level.$^2$ Namely, having three families of Higgses introduces more Yukawa couplings, and after the FI breaking some physical particles appear combined with other states, and the Yukawa couplings are modified in a well controlled way. This, of course introduces more flexibility in the computation of the mass matrices.

Let us recall that the $Z_3$ orbifold is constructed by dividing $R^6$ by the $[SU(3)]^3$ root lattice modded by the point group $(P)$ with generator $\theta$, where the action of $\theta$ on the lattice basis is $\theta e_i = e_{i+1}$, $\theta e_i+1 = -e_{i} - e_{i+1}$, with $i = 1, 3, 5$. The two-dimensional sublattices associated to $[SU(3)]^3$ are shown in Fig. 2. In orbifold constructions, twisted strings appear attached to fixed points under the point group. In the case of the $Z_3$ orbifold there are 27 fixed points under $P$, and therefore there are 27 twisted sectors. We will denote the three fixed points of each two-dimensional sublattice as shown in Fig. 2. Thus the three generations arise because in addition to the overall factor of 3 coming from the right-moving part of the untwisted matter, the twisted matter come in 9 sets with 3 equivalent sectors on each one. Let us

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$^4$Let us remark however, that right-handed neutrino superfields with R-parity breaking couplings of the type $N_i H_u H_d$ have been proposed recently$^{31}$ to solve the $\mu$ problem.

$^2$Let us recall that the major problem that one encounters when trying to obtain models with entirely renormalizable Yukawas lies at the phenomenological level, and is deeply related to obtaining the correct quark mixing. Summarizing the analyses of Refs. 24, 25, for prime orbifolds with the minimal Higgs content the space selection rules and the need for a fermion hierarchy forces the fermion mass matrices to be diagonal at the renormalizable level. Thus, in these cases the CKM parameters must arise at the non-renormalizable level. For analyses of non-prime orbifolds see Refs. 24, 25, 32.
suppose that the two Wilson lines correspond to the first and second sublattices. The three generations correspond to move the third sublattice component \((\mathbf{x} \cdot \mathbf{o})\) of the fixed point keeping the other two fixed.

As mentioned in the Introduction, we must arrange our up-and down-quark Yukawa couplings in order to have specific off diagonal elements,

\[
H_u \bar{u}_L \lambda_{u}^{\beta \gamma} u_R \gamma + H_d \bar{d}_L \lambda_{d}^{\beta \gamma} d_R \gamma .
\]

In principle this property arises naturally in the \(Z_3\) orbifold with two Wilson lines\(^{23,26}\). For example, if the SU(2) doublet \(H_u\) corresponds to \((\mathbf{o} \quad \mathbf{o} \quad \mathbf{o})\), the three generations of \((3,2)\) quarks to \((\mathbf{o} \quad \mathbf{o} \quad \mathbf{o} \quad \mathbf{x} \quad \mathbf{\cdot} \quad \mathbf{\cdot})\) and the three generations of \((\bar{3},1)\) up-quarks to \((\mathbf{o} \quad \mathbf{o} \quad \mathbf{o} \quad \mathbf{x} \quad \mathbf{\cdot} \quad \mathbf{\cdot})\), then there are three couplings allowed from the space group selection rule (the components of the three fixed points in each sublattice must be either equal or different): \(\lambda_{tt} H_u \bar{t}_L t_R\) associated to \((\mathbf{o} \quad \mathbf{o} \quad \mathbf{o})\) with \(\lambda_{tt} \sim 1\), \(\lambda_{cu} H_u \bar{c}_L c_R\) associated to \((\mathbf{o} \quad \mathbf{o} \quad \mathbf{x})\) with \(\lambda_{cu} \sim e^{-T_5}\), and \(\lambda_{uc} H_u \bar{u}_L c_R\) associated to \((\mathbf{o} \quad \mathbf{o} \quad \mathbf{\cdot})\) with \(\lambda_{uc} \sim e^{-T_5}\). In this simple example one gets one diagonal Yukawa coupling without suppression factor and two off diagonal degenerate ones \(\sim e^{-T_5}\), but other more realistic examples producing the observed structure of quark and lepton masses and mixing angles can be obtained using three generations of Higgses\(^{26,27}\).

Let us first study the situation before taking into account the effect of the FI breaking. Consider for example the following assignments of observable matter to fixed point components in the first two sublattices;

\[
\begin{align*}
Q & \quad \mathbf{o} \quad \mathbf{o} \quad \mathbf{u}^c \quad \mathbf{o} \quad \mathbf{o} \quad \mathbf{d}^c \quad \mathbf{x} \quad \mathbf{o} \\
H^u & \quad \mathbf{o} \quad \mathbf{o} \quad H^d & \quad \mathbf{\cdot} \quad \mathbf{o}
\end{align*}
\]

In this case the up- and down-quark mass matrices, assuming three different radii, are given by

\[
M^u = gN A^u, \quad M^d = gN \varepsilon_1 A^d,
\]

where \(g\) is the gauge coupling constant, \(N\) is proportional to the square root of
volume of the unit cell for the $Z_3$ lattice, and

$$A^u = \begin{pmatrix} v_1^u & v_3^u \varepsilon_5 & v_2^u \varepsilon_5 \\ v_3^u \varepsilon_5 & v_2^u & v_1^u \varepsilon_5 \\ v_2^u \varepsilon_5 & v_1^u \varepsilon_5 & v_3^u \end{pmatrix}, \quad A^d = \begin{pmatrix} v_1^d & v_3^d \varepsilon_5 & v_2^d \varepsilon_5 \\ v_3^d \varepsilon_5 & v_2^d & v_1^d \varepsilon_5 \\ v_2^d \varepsilon_5 & v_1^d \varepsilon_5 & v_3^d \end{pmatrix}. \tag{6}$$

Here $v_i^u$, $v_i^d$ denote the VEVs of the Higgses $H_i^u$, $H_i^d$ respectively, and $\varepsilon_i = 3 e^{-\frac{2\pi}{3} T_i}$. For example, for $T_5 \sim 1.95$ one has $\varepsilon_5 \sim 0.05$.

The elements in the above matrices can be obtained straightforwardly. For example, if the Higgs $H_1^u$ corresponds to $(o,o,o)$, then since the three generations of $(3,2)$ quarks $Q$ correspond to $(o,o,(o,x,\cdot))$ and the three generations of $(\bar{3},1)$ quarks $u_c$ to $(o,o,(o,x,\cdot))$, there are only three allowed couplings,

$$(o,o,o)(o,o,o)(o,o,o), \quad (o,o,o)(o,o,x)(o,o,\cdot), \quad (o,o,o)(o,o,\cdot)(o,o,x).$$

The corresponding suppression factors are given by $1$, $\varepsilon_5$, $\varepsilon_5$ respectively, and are associated with the elements $11$, $23$, $32$ in the matrix $M^u$.

These matrices clearly improve the result obtained with only one Higgs family. However, it is possible to show that although the observed quark mass ratios and Cabbibo angle can be reproduced correctly, the $13$ and $23$ elements of the CKM matrix cannot be obtained. Fortunately, this is not the end of the story because the previous result is modified when one takes into account the FI breaking. In particular, it will be possible to get the right spectrum and a CKM with the right form.

As discussed in the Introduction and Section 2, some scalars $C_i$ develop large VEVs in order to cancel the FI $D$-term generated by the anomalous $U(1)$. Thus many particles $\xi$ are expected to acquire a high mass because of the generation of effective mass terms, and in this way vector-like triplets and doublets and also singlets become heavy and disappear from the low-energy spectrum. This is the type of extra matter that typically appears in orbifold constructions. The remarkable point is that the SM matter remain massless, surviving through certain combinations with other states. Let us consider the simplest example, a model with the Yukawa couplings

$$C_1 \xi_1 f, \quad C_2 \xi_2, \tag{7}$$

where $f$ denotes a SM field, $\xi_{1,2}$ denote two extra matter fields (triplets, doublets or singlets), and $C_{1,2}$ are the fields developing large VEVs denoted by $\langle C_{1,2} \rangle = c_{1,2}$. It is worth noting here that $f$ can be an $u^c$, $d^c$, $L$, $\nu^c$ or $e^c$ field, but not a $Q$ field. This is because in these orbifold models no extra $(3,2)$ representations are present, and therefore the Standard Model field $Q$ cannot mix with other representations through Yukawas.

Clearly the ‘old’ physical particle $f$ will combine with $\xi_{1,2}$. It is now straightforward to diagonalise the mass matrix arising from the mass terms in eq. (7) to find
two very massive and one massless combination. The latter is given by
\[ f' \equiv \frac{1}{\sqrt{|c_1|^2 + |c_2|^2}} (c_2 f - c_1^* \xi_2). \] (8)

Notice for example that the mass terms (7) can be rewritten as
\[ \sqrt{|c_1|^2 + |c_2|^2} \xi_1', \]
where \( \xi_1' \equiv \frac{1}{\sqrt{|c_1|^2 + |c_2|^2}} (c_1 f + c_2 \xi_2). \) Indeed the unitary combination is the massless field in eq. (8). The Yukawa couplings and hence mass matrices of the effective low energy theory are modified accordingly. For example, consider a model where we begin with a Yukawa coupling \( H Q f \). Since we have
\[ f = \frac{1}{\sqrt{|c_1|^2 + |c_2|^2}} (c_2 f' + c_1^* \xi_2'), \] (9)
then the ‘new’ coupling (involving the light state) will be\[ H Q f'. \]

The situation in realistic models is more involved since the fields appear in three copies. All these effects modify the mass matrices of the low-energy effective theory (see Eq. (5)), which, for the example studied in Ref. 26, are now given by
\[ M_u = g N a^u A^u B^{u*}, \quad M_d = g N \varepsilon_1 a^d A^d B^{d*}, \]
where
\[ A B = \begin{pmatrix} v_1 \varepsilon_5 \beta & v_3 \varepsilon_5 & v_2 \alpha \\ v_3 \varepsilon_5^2 \beta & v_2 & v_1 \alpha \\ v_2 \varepsilon_5^2 \beta & v_1 \varepsilon_5 & v_3 \alpha / \varepsilon_5 \end{pmatrix}, \] (11)
and the parameters \( a^f, \alpha, \) and \( \beta \) depend on \( c_{1,2}, \varepsilon_{1,3}, \) and their possible values are discussed in Ref. 26. As shown in Ref. 27 for natural values of those parameters and the VEVs, one can find configurations that obey the electroweak symmetry breaking conditions, and can account for the correct quark masses and mixings.

In addition to the magnitudes of the CKM matrix elements we also require a CP violating phase. Although it has been shown that observable CP violation cannot be obtained at the renormalizable level in odd order orbifolds\[34,33\] for a minimal Higgs sector, the above matrices having in addition to the ‘mixing’ of states three families of Higgses avoid this problem\[35]. Thus one possibility here (in addition to the one already mentioned in footnote i) is to assume that the VEVs of the moduli...

\(^{b}\text{We should add that the coupling } HQ\xi_2, \text{ which would induce another contribution to } HQf', \text{ is not in fact allowed. For this to be the case the fields } \xi_2 \text{ and } f \text{ would have had to have exactly the same } U(1)^n \text{ charges. This is not possible since different particles all have different gauge quantum numbers.} \)

\(^{i}\text{Note that the } c_i \text{ are in general complex VEVs, and therefore they can give rise to a contribution to the CP phase. This mechanism to generate the CP phase through the VEVs of the fields cancelling the FI } D\text{-term was used first, in the context of non-renormalisable couplings, in Ref. 24. For a recent analysis, see Ref. 33.} \)
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Fig. 3. Feynman diagrams contributing to $\Delta m_K$ at tree-level. $h^{s(p)}$ denote scalar (pseudoscalar) Higgses.

have an imaginary phase, which can occur when the flat moduli directions are lifted by supersymmetry breaking and find their minimum where the phases are non-zero\[35,36,37]. Such a phase feeds directly into $\varepsilon_5$. It is easy to check that this phase is physically observable, and leads to a non-zero $\delta$ phase for the CKM matrix which is of order one.

Let us finally mention that the correct masses for charged leptons can be obtained following a similar approach, as discussed in Ref. [26]. For neutrinos this turns out to be not sufficient, but a see-saw mechanism arising in a natural way in orbifolds might solve the problem [38].

4. Phenomenological viability of orbifold models with three Higgs families

The most challenging implication of an extended Higgs sector is perhaps the occurrence of tree-level FCNCs mediated by the exchange of neutral Higgs states. Clearly, having six Higgs doublets (and thus six quark Yukawa couplings) the transformations diagonalising the fermion mass matrices do not diagonalise the Yukawa interactions. Since experimental data is in good agreement with the SM predictions, where such an effect is not present, the potentially large contributions arising from the tree-level interactions must be suppressed in order to have a model which is experimentally viable. In general, the most stringent limit on the flavour-changing processes emerges from the small value of the $K_L - K_S$ mass difference\[39].

A detailed discussion of FCNCs in multi-Higgs doublet models was presented in Ref. [40] (see also the references therein). We summarise here some relevant points and apply the method to the orbifold case\[27], focusing on the neutral kaon sector and investigating the tree-level contributions to $\Delta m_K$. The latter is simply defined as the mass difference between the long- and short-lived kaon masses,

$$\Delta m_K = m_{K_L} - m_{K_S} \simeq 2 |M_{12}^{K}| = 2 \left| \langle K^0 | H_{\text{eff}}^{\Delta S = 2} | K^0 \rangle \right|, \tag{12}$$

where $H_{\text{eff}}^{\Delta S = 2}$ is the effective Hamiltonian for the diagrams in Fig. 3. Once all
the contributions to $M_{\nu}^2$ have been taken into account, the prediction of this orbifold model regarding $\Delta m_K$ should be compared with the experimental value, $(\Delta m_K)_{\exp} \approx 3.49 \times 10^{-12}$ MeV.

In Ref. [27] the numerical approach was divided in two steps. Firstly, one focus on the string sector of the model, and for each point in the space generated by the free parameters of the orbifold ($\varepsilon_5, \alpha^f$), one derives the up- and down-quark mass matrices and computes the CKM matrix. Further imposing the conditions associated with electroweak symmetry breaking, and fixing a value for $\tan \beta$, one can then determine the values of $g N$ and $\varepsilon_1$. A secondary step requires specifying the several Higgs parameters, which must obey the minimum criteria. Finally, the last step comprehends the analysis of how each of the Yukawa patterns constrains the Higgs parameters in order to have compatibility with the FCNC data. In particular, we want to investigate how heavy the scalar and pseudoscalar eigenstates are required to be in order to accommodate the observed value of $\Delta m_K$.

Let us summarize the analysis of the orbifold parameter space by commenting on the relative number of input parameters and number of observables fitted. Working with the six Higgs VEVs ($u_i^{u,d}$), and the orbifold parameters $\varepsilon_1, \varepsilon_5, \alpha^u$ and $\alpha^d$, one can obtain the correct electroweak symmetry breaking ($M_Z$), as well as the correct quark masses and mixings (six masses and three mixing angles).

In order to discuss now the tree-level FCNCs, let us remark that the present orbifold model does not include a specific prediction regarding the Higgs sector. For instance, we have no hint regarding the value of the several bilinear terms, nor towards their origin. Concerning the soft breaking terms, the situation is similar. In the absence of further information, we merely assume that the structure of the soft breaking terms is the usual one (see Ref. [27] for further details), taking the Higgs soft breaking masses and the $B\mu$-terms as free parameters (provided that the electroweak symmetry breaking and minimisation conditions are verified).

In the absence of orbifold predictions for the Higgs sector parameters, and motivated by an argument of simplicity, we begin our analysis by considering textures for the soft parameters as simple as possible. In particular, we arrive to four representative cases with the following associated scalar and pseudoscalar Higgs spectra:

(a) $m^s = \{82.5, 190.6, 493.9, 515.9, 744.4, 760.2\}$ GeV;
$m^p = \{186.8, 493.9, 515.9, 744.4, 760.2\}$ GeV.
(b) $m^s = \{84.6, 213.9, 387.4, 560.8, 785.9, 879.1\}$ GeV;
$m^p = \{215.2, 387.3, 560.5, 785.9, 878.9\}$ GeV.
(c) $m^s = \{83.6, 292.9, 733.6, 785.9, 987.6, 1057.0\}$ GeV;
$m^p = \{291.1, 733.6, 785.9, 987.6, 1057.0\}$ GeV.
(d) $m^s = \{79.4, 121.5, 296.9, 354.3, 794.6, 808.8\}$ GeV;
$m^p = \{114.8, 296.9, 353.7, 794.6, 808.8\}$ GeV.

In Fig. 4 we plot the ratio $\Delta m_K/(\Delta m_K)_{\exp}$ versus $\varepsilon_5$, for cases (a)-(d), and $\tan \beta = 5$. All the points displayed comply with the bounds from the CKM matrix.
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From Fig. 4, it is clear that it is quite easy for the orbifold model to accommodate the current experimental values for $\Delta m_K$. Even though the model presents the possibility of important tree-level contributions to the kaon mass difference, all the textures considered give rise to contributions very close to the experimental value. Although (a) and (b) are not in agreement with the measured value of $\Delta m_K$, their contribution is within order of magnitude of $(\Delta m_K)^{\text{exp}}$. As seen from Fig. 4 with a considerably light Higgs spectrum (i.e. $m_{h_0} < 1$ TeV), one is safely below the experimental bound, as exhibited by cases (c) and (d). This is not entirely unexpected given the strongly hierarchical structure of the Yukawa couplings (notice from Eq. (11) that $\lambda_{d_{11}}^{2}$ is suppressed by $\varepsilon_{5}^{2}$).

Let us finally mention that the analysis for other neutral meson systems, $B_d$, $B_s$ and $D^0$, can be carried out in an analogous way\textsuperscript{27}.

Additionally, and given the existence of flavour violating neutral Higgs couplings, and the possibility of having complex Yukawa couplings, it is natural to have tree-level contributions to CP violation. In the kaon sector, indirect CP violation is parameterised by $\varepsilon_K$. From experiment one has $\varepsilon_K = (2.284 \pm 0.014) \times 10^{-3}$. A comparison of this quantity with the theoretical result in orbifold models can be found in Ref. [27].

5. Conclusions

We have attacked the problem of the unification of gauge couplings in Heterotic String constructions. In particular, we have obtained that due to the Fayet-Iliopoulos scale, $\alpha_3$ and $\alpha_2$ cross at the right scale when a certain type of extra
matter is present. In this sense three families of supersymmetric Higgses and vector-like colour triplets might be observed in forthcoming experiments. The unification with $\alpha_1$ is obtained if the model has the appropriate normalization factor of the hypercharge. Let us recall that although we have been working with explicit orbifold examples, our arguments are quite general and can be used for other schemes where the Standard Model gauge group with three generations of particles is obtained, since extra matter and anomalous $U(1)$’s are generically present in string compactifications.

Another advantage of these models is that they naturally predict three generations, and also that the three generations of Higgs fields give enough freedom to allow an entirely geometric explanation of masses and mixings. The Fayet-Iliopoulos mechanism plays also an important role here. Namely, after the gauge breaking some physical particles appear combined with other states, and the Yukawa couplings are modified in a well controlled way.

On the other hand, the presence of six Higgs doublets poses the potential problem of having tree-level FCNCs. By assuming simple textures for the Higgs free parameters, we have verified for example that the experimental data on the neutral kaon mass difference can be easily accommodated for a quite light Higgs spectra, namely $m_{h^0} \lesssim 1$ TeV.

The presence of a fairly light Higgs spectrum, composed by a total of 21 physical states, may provide abundant experimental signatures at future colliders, like the Tevatron or the LHC. In fact, flavour violating decays of the form $h_i \rightarrow q\bar{q}$, or $h_i \rightarrow l^+l^-$ may provide the first clear evidence of this class of models.

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