Economic optimization of acceptance interval in conformity assessment: 1. Process with no systematic effect

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Abstract
In inspections for conformity assessment, an acceptance interval smaller than the tolerance interval is often determined in order to reduce the risk of consumers obtaining non-conforming items in the market. The presence of non-conforming items in the market impairs the evaluation of items by customers and may have an impact on revenue by decreasing prices. However, setting too small an acceptance interval reduces the revenue from the process by decreasing the number of the items available in the market. We thus propose a method to determine the optimum acceptance interval in conformity assessment by means of maximization of the revenue from processes. For this purpose, we give a mathematical model for the price of an item and its cost in the production process. Through theoretical analysis and simulations, it is shown that a parameter in the price model is the key in the optimization. In this paper we report a method for processes where no systematic effect component of measurement uncertainty exists, and in part 2 of this series we will report an extended method in which systematic effects are taken into consideration.

Keywords: conformity assessment, uncertainty, global consumer’s risk, acceptance interval, guard band

Supplementary material for this article is available online

(Some figures may appear in colour only in the online journal)

1. Introduction
The demand for quality from customers is increasing in various industries. One means of indicating the quality of products is conformity assessment, which is defined as ‘demonstration that specified requirements (5.1) are fulfilled’ in ISO/IEC 17000:2020 [1, 2]. Inspections provide key evidence to support conformity assessment from the technical standpoint. We discuss inspections in production processes that have measurement uncertainty. There have been many suggestions regarding how to incorporate measurement uncertainty in the inspection of production processes. For example, JCGM 106 [2], published by the Joint Committee for Guides in Metrology (JCGM) of the Bureau International des Poids et Mesures (BIPM) and other seven organizations, sets forth important guidelines for taking measurement uncertainty into consideration in conformity assessment. It should be noted that, in this study, the term of measurand in JCGM 106, which is the quantity intended to be measured in an inspection, is referred to as quality characteristic.
In inspections in this study, we consider the tolerance interval. The tolerance interval is defined in JCGM 106 as the interval of permissible values of a quality characteristic. In this study, we use the permissible difference for the tolerance in accordance with in ISO 16337 [3] to mean the upper tolerance limit less the target value of the quality characteristic. Items with a quality characteristic within or outside the tolerance interval are referred to as conforming or non-conforming items, respectively.

To increase the proportion of conforming items in the market, a guard band is sometimes introduced. When using a guard band, we accept an item as eligible to be available in the market only when the measured value for the item is within the smaller interval than the tolerance interval by the guard band. The acceptable difference and acceptance interval in this study indicate the permissible difference for the tolerance less the guard band which corresponds to the upper acceptance limit less the target value of the quality characteristic and the interval of permissible measured values with which the item is eligible to be available in the market, respectively. To accept or reject an item means to make an item available or unavailable in the market, respectively, based on the inspection result.

It might happen that an accepted item is, in fact, non-conforming, which corresponds to false acceptance. On the contrary, false rejection means that a rejected item is, in fact, conforming in this study. We cannot avoid false acceptances and rejections with a certain probability when the inspection has measurement uncertainty. False acceptance and rejection increase the loss of the consumers and the producer, respectively. Thus, the risks involving false acceptance and rejection are often called the consumer’s risk (CR) and the producer’s risk (PR), respectively. In this study, the term of risk is used to mean the probability of an undesirable event’s occurring as those.

Furthermore, it is worth noting that JCGM 106 defines specific and global risks. When there is concern regarding the false acceptance or rejection of an item, we refer to these risks as the specific CR or PR. On the other hand, when such concern arises in relation to a process, we refer to these risks as the global CR or PR. Mass production processes are usually analyzed in terms of the global risks. The global CR is abbreviated as gCR hereafter.

There have been many studies involving risk control or cost analysis in conformity assessments. In terms of industrial applications, Deaver’s proposals [4, 5] are often employed when inspection processes are designed. Deaver has suggested several types of CRs, as explained in section 2, which were revisited in some studies including those by Dobbert [6, 7] and Castrup [8]. Pendrill [9–11] reviewed studies involving the application of uncertainty in conformity assessments. For example, Pendrill himself reported a series of studies mainly focusing on the optimization of uncertainty both using specific risks [12, 13] and global risks [14, 15]. Furthermore, Rossi and Crenna [16] investigated specific and global consumer’s and producer’s risks comprehensively and proposed a concept to optimize the acceptance interval in order to minimize the total cost of false acceptance and rejection. Forbes [17] not only suggested a way to determine acceptance interval based on the Bayesian decision-making approach but also showed the influence of unknown systematic effects associated with the measurement system. Other interesting studies [18–22] were introduced in Pendrill’s reviews or reported recently.

Some of the above studies were incorporated in JCGM 106. For example, JCGM 106 applies the definition of gCR suggested by Deaver [4]. However, it has sometimes been pointed out that gCR is not a true risk that consumers incur from an item in the market. Castrup [8] defined $FA_2$ in his paper to denote gCR in JCGM 106, and noted that

Oddly enough, when false accept risk (or CR) is discussed in journal articles, conference papers and in some risk analysis software, it is $FA_2$ that is being discussed. This may be somewhat puzzling in that $FA_2$ is not referenced to the consumer’s perspective.

In this study, we therefore use gCR as defined by Castrup [8] to express the risk in terms of the consumer’s perspective. (Castrup originally presented the gCR used in this study in reference [23].)

We believe that the redefined gCR can be used to express the customer’s evaluation of the items produced in a process, which is naturally linked to the price. A mathematical expression of the price is thus assumed in this study. Moreover, the cost of production is also expressed in a mathematical way, employing a microeconomic approach [24]. These expressions help us to develop an economic model of the production process. Our idea is to determine the acceptance interval in the inspection in order to maximize the revenue. While we can derive some economic indices using the economic model, the revenue from a process must be the most simple and important index for producers.

This study is not the first study labeled ‘economic’. We believe that all studies on conformity assessments involve economic optimization in their own way. For example, in the report by Pendrill [12], we can find the realistic economic influence of the measurement uncertainty in the petroleum dispensers. In this and the relevant studies by the same author [11, section 6], the relationship between the cost beyond the percentage risks and the measurement uncertainty was considered. Some other studies [17, 18, 22] discussed the costs of false acceptance and rejection. However, since the cost of the false acceptance in these previous studies is basically defined as the consumer’s loss, it seems unclear how a producer is motivated to reduce the cost. In this study, the cost of false acceptance is indirectly reflected in the price of the item, then it is considered to maximize the consequent revenue of the producer. It can be said that one objective of this study is to specify the relationship between the cost and the revenue using the basic concept of the microeconomics.

In this study, we discuss only processes where all of the manufactured items are inspected, similarly to the discussion in JCGM 106 to derive the gCR. Moreover, we focus here on the case where no effect component of the measurement uncertainty caused by an unknown bias is taken into consideration, and will report on an unknown systematic effect in our next report [25] because a discussion of these two topics is too lengthy for one paper. Although part of our idea was
described in our previous report [26] as an academic conference proceeding, here we present the full information required to establish the proposal in this study. This paper is organized as follows: sections 2 and 3 explain the gCR in this study and the price model using it, respectively. Section 4 presents the mathematical expressions of the production cost and the revenue. Section 5 elucidates the computational approach to obtain the optimum acceptance interval. In section 6, a simulation study to characterize the proposed method is reported. Section 7 provides a summary. Online appendices are also provided to present the derivation of some of the mathematical expressions, the numerical computation procedure, and the comparison of the present study with previous ones.

2. Expression of global consumer’s risk

2.1. Global consumer’s risk in this study

In JCGM 106, the following definition of the gCR is given as a nomenclature:

\[ \text{probability that a non-conforming item will be accepted based on a future measurement result } \]  
\[ \text{[2, 3.3.15]} \]

In addition, the following is also noted:

Such calculations have traditionally been performed using measured frequency distributions of the various outcomes when a large sample of nominally identical items are measured. The global consumer’s risk, in such an approach, is equal to the fraction of items in a measured sample that are accepted for use but do not conform with a specified requirement [2, 9.5.1.1].

Expression (17) in JCGM 106 is given to denote the gCR, which can be expressed with the symbols used in this study as follows:

\[ \theta_A = \int_{C \cup A} p_0(x) p_m(x|\xi) d\xi dx, \]  
\[ (1) \]

where, defining \( \xi \) and \( x \) as the quality characteristic of an item and the measured value, \( p_0(\xi) \) and \( p_m(x|\xi) \) are the probability density functions for \( \xi \) of the manufacturing process and \( x \) given \( \xi \) of the measurement process, respectively. \( C \) and \( A \) are the sets for \( \xi \) outside the tolerance interval and \( x \) within the acceptance interval, respectively. We define \( C \) and \( A \) as the sets for \( \xi \) within the tolerance interval and \( x \) outside the acceptance interval, respectively.

The following three types of items are considered: (1) items with \( \xi \in C \) and \( x \in A \) (2) items with \( \xi \in C \) and \( x \in A \) and (3) items with \( x \in A \). We define \( N_1 \) to \( N_3 \) to express the (large) number of items attributed to the first to third types above, respectively. \( \theta_A \) in expression (1) can then be interpreted as

\[ \frac{N_2}{N_1 + N_2 + N_3}. \]  
\[ (2) \]

The denominator denotes the number of all the items. The gCR \( \theta_A \) in expression (1) was originally proposed by Deaver [4]. This definition of the gCR has become widely accepted in industrial circles through various studies and practices including that by Dobbert [6, 7].

We would like to emphasize that only items of the first and second types are available in the market. The consumers evaluate the quality based only on the market-available items. Thus, we believe that the following risk may be appropriate to express the gCR:

\[ \frac{N_2}{N_1 + N_2}. \]  
\[ (3) \]

Expression (3) is eventually reduced to the gCR \( \theta \) expressed with

\[ \theta = \frac{\int_{C \cup A} p_0(x) p_m(x|\xi) d\xi dx}{\int_{C \cup C \cup A} p_0(x) p_m(x|\xi) d\xi dx}, \]  
\[ (4) \]

using probability density functions. Expression (4) has already been presented prior to this study. As mentioned in section 1, Castrup [8] presented the gCR used in this study in reference [23]. Furthermore, Dobbert [6] proposed three expressions of the gCR, and labeled them as risks A, B, and C. Risks A and B correspond to \( \theta_A \) and \( \theta \) in expressions (1) and (4), respectively. For later discussion, the definition of risk C is also given using

\[ \theta_C(x) = \frac{\int_{C \cup A} p_0(x) p_m(x|\xi) d\xi dx}{\int_{C \cup C \cup A} p_0(x) p_m(x|\xi) d\xi dx}. \]  
\[ (5) \]

Risk C is given as \( \theta_C(A) \). \( \theta_C(A) \) is not the gCR but the specific CR when \( x = A \). It was originally proposed by Deaver [5].

The gCR in expression (4) can be naturally linked to the price, which is the evaluation of the item by the customers. This feature allows us to develop applications of the measurement uncertainty from an economic perspective. One of the applications will be the determination of the acceptance interval.

2.2. Specific expression of the global consumer’s risk in this study

For simplicity, we assume that the target value of the quality characteristic of items in this study is zero. \( T \) and \( A \) denote the permissible difference for the tolerance and the acceptable difference so that the tolerance and the acceptance intervals as ranges are given as \( C = \{ \xi | -T \leq \xi \leq +T \} \) and \( A = \{ x | -A \leq x \leq +A \} \), respectively. (Note that in JCGM 106 [2], the symbols \( T \) and \( A \) are employed to mean the magnitudes of the tolerance and the acceptance intervals, respectively, instead of the corresponding differences.) The normal distribution of

\[ \xi \sim N(0, \sigma^2) \]  
\[ (6) \]

is given for \( \xi \), where \( \sigma \) denotes the standard deviation of the quality characteristic. The assumption that the mean value of \( \xi \) is zero can be interpreted that the quality characteristic of an item in this study indicates the difference between the quantity intended to be measured on the item and the target value. We can apply this assumption to any case where we have only one quality characteristic to be checked with symmetric tolerance and acceptance intervals to the target value without loss of generality.

Considering the measurement uncertainty, we assume the normal distribution for the conditional distribution measured value \( x \) given \( \xi \) as

\[ x \mid \xi \sim N(\xi, \sigma^2). \]  
\[ (7) \]
where \( u_t \) is the random effect component of the measurement uncertainty as the standard deviation. In this study, \( u_t \) is a constant for a process. The systematic effect component is not discussed in this paper, but will be in our next report [25]. Because of the measurement uncertainty, false acceptance and rejection can happen. Figure 1 shows the scheme of the decision making based on the assumed conditions.

The gCR defined by expression (4) is given in a specific manner for the process characterized by the models shown in expressions (6) and (7). For this purpose, we need the rejection rate \( R \). The rejection rate is defined as the fraction of the rejected items over all manufactured items. To obtain the unconditional distribution of \( x \), integrating out \( \xi \) from expressions (6) and (7) results in

\[
x \sim N(0, \sigma_t^2),
\]

where

\[
\sigma_t = \sqrt{\sigma^2 + u_t^2}.
\]

Since \( R \) is the probability such that \( x < -A \) or \( x > +A \),

\[
R = \Phi \left( \frac{-A}{\sigma_t} \right) + \left[ 1 - \Phi \left( \frac{A}{\sigma_t} \right) \right] = 2\Phi \left( -\frac{A}{\sigma_t} \right),
\]

where \( \Phi(y) \) is the cumulative distribution function of the standardized normal distribution for variable \( y \). Using \( R \), the gCR given by expression (4) based on the models given by expressions (6) and (7) is expressed as follows:

\[
\theta = \frac{H(-\infty, -T) + H(T, +\infty)}{1 - R} = \frac{2H(-\infty, -T)}{1 - R},
\]

where

\[
H(a, b) = \int_a^b \left[ \Phi \left( \frac{A - \xi}{u_t} \right) - \Phi \left( \frac{-A - \xi}{u_t} \right) \right] \frac{1}{\sigma} \phi \left( \frac{\xi}{\sigma} \right) \, d\xi,
\]

and \( \phi(y) \) denotes the probability density function of the standardized normal distribution for variable \( y \). Furthermore, for \( \theta_A \) given by expression (1) and \( \theta_C(A) \) given by expression (6), we obtain

\[
\theta_A = H(-\infty, -T) + H(T, \infty),
\]

\[
\theta_C(A) = \Phi \left( \frac{-T - A_w}{u_w} \right) + \left[ 1 - \Phi \left( \frac{T + A_w}{u_w} \right) \right]
= \Phi \left( \frac{-T - A_w}{u_w} \right) + \Phi \left( \frac{T + A_w}{u_w} \right),
\]

where

\[
u_w = \sqrt{\frac{1}{\sigma^2} + \frac{u_t^2}{u_t^2}} = \frac{\sigma}{u_t}, \quad A_w = \frac{u_t^2}{u_t^2} A.
\]

To calculate \( \theta \), numerical computation is necessary. We applied the procedures using the splitting method [27] explained in online appendix B.

3. Modelling of the price

3.1. Assumed function for the price

We apply the following function for the price \( P \) of an item:

\[
P = P_0(1 - \beta \theta).
\]

Figure 2 shows the variation of \( P \) respect to \( \theta \). Expression (16) with \( \theta = 0 \) implies that \( P_0 \) can be interpreted as the price for a conforming item. Similarly, \( P = P_0(1 - \beta) \) can be the price for a non-conforming item. \( P_0 \beta \) can hence mean the magnitude of the loss caused by a non-conforming item so that \( \beta \) shows
the ratio between the loss and the price of a conforming item. \( \beta \) is referred to as the degree of loss in this study.

### 3.2. Parameter determination for the proposed price

The determinations of \( P_0 \) and \( \beta \) are not simple tasks. For \( \beta \), the model given by expression (16) implies that the price of an item which can be non-conforming with the probability of 10% is given to be zero when \( \beta = 10 \). When the prices of a conforming item and the loss caused by the non-conforming device using it are respectively $1000 and $1000000, \( \beta \) is given as 1000. These cases seem realistic depending on the situations. We suggest \( \beta = 10 \) and 1250 for the items of which the prices are scarcely and highly sensitive, respectively, as shown in table 1.

For the determination of \( P_0 \), we consider the reference condition, which is determined by \( A = A_{\text{ref}} \). We can compute the gCR \( \theta = \theta_{\text{ref}} \) for \( A = A_{\text{ref}} \). It is assumed we know the price with \( \theta = \theta_{\text{ref}} \) as \( P = P_{\text{ref}} \). The assumption that we can determine the reference condition where we know \( P_{\text{ref}} \) and \( \theta_{\text{ref}} \) may be helpful to determine \( P_0 \). When we can assess \( \beta \) from certain information, \( P_0 \) is readily available as

\[
P_0 = \frac{P_{\text{ref}}}{1 - \beta \theta_{\text{ref}}}. \tag{17}
\]

### 3.3. Possible interpretation of the proposed price

It is possible to interpret expression (16) as a linear approximation of the complicated price function under the reference condition. In that case, the magnitude of \( \beta \) may be larger than its realistic ratio of the price difference and the price of a conforming item. \( P \) as a convex function of \( \theta \) seems to be typically considered, as shown in figure 3. It is reasonable to approximate the curve for \( P \) with the tangent line at \( (\theta_{\text{ref}}, P_{\text{ref}}) \), if the best acceptable difference is expected to be close to \( A_{\text{ref}} \). It is found from figure 3 that for \( \theta > \theta_{\text{ref}} \), the approximate linear function varies more than the true price difference. It can hence be justified to give \( \beta \) so that \( P_0 \beta \) is much larger than the true price difference between \( \theta = 0 \) and 1.

The linear approximation of the curve of the price is not the only interpretation of expression (16). An alternative interpretation may be that an item is sold at the price of \( P_0 \) irrespective of its conformity. Suppose that the producer pays an average cost of \( P_0 \beta \) for a non-conforming item in response to complaints from customers who obtain non-conforming items. \( P_0 \beta \) may reflect the loss in future sales through the reputational damage caused by the customer dissatisfaction on a non-conforming item suggested by Pendrill [11]. The assessed revenue for this case is the same as that in the case where an item is sold at the price given in expression (16). Our proposal in this study can hence also be applied to this alternative interpretation, where \( P \) is no longer the actual price but the contribution to sales by an item in the market. The slight computational difference is that not \( P_{\text{ref}} \) but \( P_0 \) is a known value and \( P_{\text{ref}} \) is computed as \( P_{\text{ref}} = P_0 (1 - \beta \theta_{\text{ref}}) \).

Other price functions are possible to be employed for the same purpose. However, we believe that the percentage risk is now a key control factor for the manufacturing business. In the price model given by expression (16), the proposed gCR can directly influence the price of an item. When other properties of items are important factors in some industries, the price model may have to be adjusted to take the fact into consideration. For example, the price model as the function of the variance of the quality characteristics may be considered. In the quality engineering studies [28] including our previous one [29], the cost of poor quality is often expressed by the function of the variance.

### 4. Modelling of the production cost and the revenue

#### 4.1. Assumed functions for the costs

We discriminate between production and manufacturing in this manuscript. Suppose that an item is manufactured and inspected, then discarded or sold when rejected or accepted, respectively. The process consisting of inspection and discarding or selling is referred to as the non-manufacturing process. Selling here indicates any processes after inspection involving accepted items being sold to customers. A production process consists of the manufacturing and non-manufacturing processes. \( Q \) and \( W \) denote the output and manufactured amounts, respectively. The output amount \( Q \), which may be referred to as the output quantity in the microeconomics, is defined as the amount of items that will be available in the market after acceptance. The manufactured amount \( W \) includes the amount of
items unavailable in the market after rejection in addition to \( Q \). With the rejection rate \( R \), the relationship

\[ Q = (1 - R)W \] (18)

holds between \( Q \) and \( W \).

The production cost can be divided into the fixed cost and the variable cost, which are respectively expressed by \( F \) and \( V \). While \( F \) is a constant irrespective of the output amount \( Q \), \( V \) is a function of it. Specifically, the variable cost \( V \) consists of four components in this study; namely, the variable costs of (i) manufacturing, (ii) inspection, (iii) discarding, and (iv) selling, as mentioned above.

Letting \( G \) denote the variable cost of manufacturing, we discuss the mathematical expression of \( G \). Here, we define the marginal manufacturing cost \( g(W) \) by the relationship

\[ G = \int_0^W g(w)dw. \] (19)

A key assumption in this study is that we give the following rectified linear function for \( g(W) \):

\[ g(W) = \begin{cases} \gamma W + g_0 & \text{for } W > g_0/\gamma, \\ 0 & \text{Otherwise,} \end{cases} \] (20)

to prevent a negative value of \( g(W) \), where \( \gamma \) is a positive constant.

Suppose that \( c_I, c_D, \) and \( c_S \) are the cost of inspecting an item, the cost of discarding a rejected item, and the cost of selling an accepted item, respectively. Then we define

\[ \eta = \frac{1}{1 - R} [c_I + c_DR + c_S(1 - R)]. \] (21)

as the non-manufacturing cost of an accepted item.

The variable cost \( V \) can be expressed as follows:

\[ V = \int_0^W g(w)dw + \eta Q, \] (22)
as the sum of the manufacturing and the non-manufacturing costs, or

\[
V = \int_0^Q g \left( \frac{q}{1 - R} \right) \frac{dq}{1 - R} + \eta Q \\
= \int_0^Q \frac{1}{1 - R} \cdot g \left( \frac{q}{1 - R} \right) + \eta dq,
\] (23)
as the integration over the output amount. Defining \( h(Q) \) as the integrant of the right side of expression (23) for \( q = Q \) as

\[ h(Q) = \frac{1}{1 - R} \cdot g \left( \frac{Q}{1 - R} \right) + \eta, \] (24)

\( h(Q) \) is interpreted as the marginal cost in terms of microeconomics. Note that we discriminate between the marginal cost \( h(Q) \) and the marginal manufacturing cost \( g(W) \) in this report.

![Figure 4](https://via.placeholder.com/150)

**Figure 4.** Relationship between the price and the marginal cost \( h(Q) \) as a function of the output amount \( Q \) for the cases (a) \( g_0 \geq 0 \) and (b) \( g_0 < 0 \).

### 4.2. Analysis of revenue with fixing the acceptable difference

The revenue \( M \) is given as the difference between the sales \( PQ \) and the cost \( V + F \) as follows:

\[ M = PQ - V - F. \] (25)

In this subsection, we consider that the acceptable difference \( A \) is a constant and that the production process is fixed. Then, \( P \) is given by expression (16), \( F \) is the fixed cost, and \( V \) is a function of \( Q \) as shown in expression (23). Thus, \( M \) in expression (25) is regarded as a function of \( Q \).

In this subsection, we determine \( Q \) to maximize \( M \) with fixing \( A \). In other words, \( Q \) can be optimized through

\[
\frac{dM}{dQ} = P - \frac{dV}{dQ} = P - h(Q) = 0.
\] (26)

Expression (26) implies an important relationship in microeconomics [24]; namely, that the output amount \( Q \) is optimum
when the price $P$ is equal to the marginal cost $h(Q)$. Figure 4 shows the variation of $h(Q)$ as a function of $Q$ for cases where (a) $g_0 \geq 0$ and (b) $g_0 < 0$. Figures 4(a) and (b) reveal that the optimum output amount $Q = Q_1$ and $Q_2$ when the price $P = P_1$ and $P_2$, respectively.

Substituting expressions (18), (20) and (24) to expression (26), we obtain the following equation for $W$:

$$P = \left[ \frac{\gamma W + g_0}{1 - R} + \eta \right] = 0. \quad (27)$$

The solution is given as

$$W = \frac{1}{\gamma} [(1 - R)(P - \eta) - g_0]. \quad (28)$$

However, $W$ in expression (28) is not always the solution. Figures 4(a) and (b) show the two cases when expression (28) does not give the optimum $W$.

As shown in figure 4(a), $Q = Q_2$ for $P = P_2$ is negative and cannot be the optimum output amount. In this case, expression (28) cannot give the optimum $W$. This happens when $g_0 \geq 0$ and $P \leq h(0) = 1/(1 - R) \cdot g_0 + \eta$. The latter condition can be transformed to

$$(1 - R)(P - \eta) \leq g_0. \quad (29)$$

If these conditions hold, the best solution is to produce no items; in other words, $W = Q = 0$.

As shown in figure 4(b), $Q = Q_4$ for $P = P_4$ cannot be the solution. When $g_0 < 0$ and $P \leq \eta$, no items can be made with the cost less than the price. The best solution is $W = Q = 0$ in this case as well. The condition $P \leq \eta$ is essentially identical to

$$(1 - R)(P - \eta) \leq 0, \quad (30)$$

because $(1 - R) > 0$.

The above discussion provides the optimum manufactured amount $W$ as follows:

$$W = \begin{cases} 
0 & \text{for } g_0 \geq 0 \text{ and } (1 - R)(P - \eta) \leq g_0, \\
0 & \text{for } g_0 < 0 \text{ and } (1 - R)(P - \eta) \leq 0, \\
\frac{1}{\gamma} [(1 - R)(P - \eta) - g_0] & \text{otherwise}. 
\end{cases} \quad (31)$$

The optimum $Q$ can be given by expression (18) with $W$ in expression (31). The revenue in this study is $M$ computed by expression (25) with the optimum $Q$.

4.3. Parameter determination for the proposed manufacturing cost

We consider the reference condition determined by giving $A_{ref}$ and $P_{ref}$ as described in section 3.2. $R_{ref}$ denotes the rejection rate under the reference condition. Using $R_{ref}$, a variable $\eta_{ref}$ is defined:

$$\eta_{ref} = \frac{1}{1 - R_{ref}} [c_1 + c_3 R_{ref} + c_3 (1 - R_{ref})]. \quad (32)$$

Moreover, the output and manufactured amounts are defined as $Q_{ref}$ and $W_{ref}$, respectively. Expression (31) yields the expression of $W_{ref}$ as

$$W_{ref} = \frac{1}{\gamma} [(1 - R_{ref})(P_{ref} - \eta_{ref}) - g_0], \quad (33)$$

assuming that the manufactured amount is the optimum value. $g_{ref} = g(W_{ref})$ is hence given as

$$g_{ref} = \gamma W_{ref} + g_0 = (1 - R_{ref})(P_{ref} - \eta_{ref}). \quad (34)$$

It is the usual practice to assess the elasticity of supply when considering the marginal cost in microeconomics [24]. The elasticity of supply means the relative increase of the output in response to changes in the price, and is equivalent to that of the manufactured amount in this study. Its mathematical expression under the reference condition is given as follows:

$$E_{ref} = \frac{\partial Q/Q}{\partial P/P} \bigg|_{r = r_{ref}} = \frac{\partial W/W}{\partial P/P} \bigg|_{r = r_{ref}} = \frac{\partial W}{\partial P} \bigg|_{r = r_{ref}} \frac{1 - R_{ref}}{\gamma}. \quad (35)$$

Inserting the following relationship obtained from expression (31)

$$\frac{\partial W}{\partial P} = \frac{1 - R_{ref}}{\gamma}, \quad (36)$$

into expression (35), we obtain

$$E_{ref} = \frac{P_{ref} (1 - R_{ref})}{\gamma W_{ref}}. \quad (37)$$

Expression (37) implies that $\gamma$ can be assessed from $E_{ref}$ as follows:

$$\gamma = \frac{P_{ref} (1 - R_{ref})}{W_{ref} E_{ref}}. \quad (38)$$

Further, $g_0$ can be given as follows:

$$g_0 = g_{ref} - \gamma W_{ref}. \quad (39)$$
4.4. Possible interpretation of the proposed manufacturing cost

The linear approximation of an actual marginal manufacturing cost under the reference condition, as shown in figure 5, is a possible interpretation of expression (20). Although it may be a difficult task to obtain the actual marginal manufacturing cost, the relationships given by expressions (38) and (39) show that we can determine $g(W)$ as the linear approximation of the marginal manufacturing cost when we know $P_{\text{ref}}$, $W_{\text{ref}}$, and $E_{\text{ref}}$ in addition to $A_{\text{ref}}$. However, it is shown in the later discussion that we do not need the specific values of $W_{\text{ref}}$ and $E_{\text{ref}}$ to optimize the acceptance interval. In other words, the specific form of $g(W)$ is not required for the purpose of this study. The explanation is given in section 5.

5. Maximization of the revenue

5.1. Simple expression of the revenue

In this subsection, we review the mathematical formula for the revenue. In the later discussion, we assume the existence of the reference condition under which we can obtain positive revenue. We hereafter use $P_{\text{ref}}$, $W_{\text{ref}}$, and $E_{\text{ref}}$ as input valuables rather than $P_0$, $\gamma$, and $g_0$. $\gamma$ and $g_0$ are used only temporarily in the later analysis so that they are not needed to be obtained explicitly. $P_0$ obtained through expression (17) is not always but usually required (see subsection 5.3).

We introduce the revenue under the reference condition, $M_{\text{ref}}$. The difference of $(M - M_{\text{ref}})$ is considered. Online appendix A.1 provides it for the case of $W > 0$,

$$M - M_{\text{ref}} = g_{\text{ref}}W_{\text{ref}} \left[ K - 1 + \frac{\chi_{\text{ref}}}{2}(K - 1)^2 \right],$$

and online appendix A.2 provides it for the case of $W = 0$,

$$M - M_{\text{ref}} = \begin{cases} 
\frac{g_{\text{ref}}W_{\text{ref}}}{2\chi_{\text{ref}}} & \text{for } g_0 \geq 0, \\
\frac{g_{\text{ref}}W_{\text{ref}}}{\chi_{\text{ref}}}(\chi_{\text{ref}} - 1) & \text{for } g_0 < 0,
\end{cases}$$

where

$$K = \frac{(1 - R)(P - \eta)}{(1 - R_{\text{ref}})(P_{\text{ref}} - \eta_{\text{ref}})},$$

and

$$\chi_{\text{ref}} = E_{\text{ref}} \left( 1 - \frac{\eta_{\text{ref}}}{P_{\text{ref}}} \right).$$
Figure 7. Optimized acceptable difference $A_{\text{opt}}$ as a function of the measurement uncertainty $u_r$ for (a) $T = 2$ and (b) $T = 3$. The detailed conditions are given in subsection 5.3. Note that no systematic effect component of the measurement uncertainty is considered.

We define $m = (M - M_{\text{ref}})/g_{\text{ref}}W_{\text{ref}}$, which can be concisely expressed as follows:

$$m = \begin{cases} 
-\frac{1}{2\chi_{\text{ref}}} 
& \text{for } \chi_{\text{ref}} \geq 1 \text{ and } K \leq 1 - \frac{1}{\chi_{\text{ref}}}, \quad (44) \\
\frac{\chi_{\text{ref}}}{2} - 1 
& \text{for } \chi_{\text{ref}} < 1 \text{ and } K \leq 0, \quad (45) \\
(K - 1) + \frac{\chi_{\text{ref}}}{2} (K - 1)^2 
& \text{Otherwise}, \quad (46)
\end{cases}$$

$m$ is referred to as the standardized revenue in the latter part of the manuscript. Online appendix A.3 shows the case of $W = 0$ under the condition that $\chi_{\text{ref}} = 1$ and $K \leq 1 - 1/\chi_{\text{ref}}$ for $g_0 \geq 0$, and $0 < \chi_{\text{ref}} < 1$ and $K \leq 0$ for $g_0 < 0$, as given in expressions (44) and (45), respectively. Since $\chi_{\text{ref}}$ is positive when $\gamma$ is a positive constant, we do not discuss cases for $\chi_{\text{ref}} \leq 0$.

We need $K$ and $\chi_{\text{ref}}$ to compute $m$. Figure 6 shows the relationships between the variables. $K$ is a function of the inputs $T$, $\sigma$, $u_r$, $A$, $A_{\text{ref}}$, $\beta$, $P_{\text{ref}}$, $c_1$, $c_D$, and $c_S$. $\chi_{\text{ref}}$ can be assessed from these inputs and $E_{\text{ref}}$. It is found that we do not need to know $W_{\text{ref}}$ for the computation of $m$.

$A$ is the only parameter to be optimized in this study. Thus, we can express the problem in this study as follows:

$$A_{\text{opt}} = \arg \max_A m,$$  

where $A_{\text{opt}}$ is the optimum acceptable difference. Although numerical approaches are applicable to solve this problem, we found an approximate closed-form solution under a limited condition. See subsections 5.2 and 5.3 for the analytical discussion and the approximate solution, respectively.

5.2. Analytical discussion of the revenue optimization

As mentioned in subsection 5.1, we assume the existence of the reference condition under which we can obtain positive revenue. When $A = 0$, we discard all items and never obtain positive revenue. The existence of the reference condition assures that we can find the optimum acceptable difference $A_{\text{opt}}$ in the range of $[A_{\text{opt}} | 0 < A_{\text{opt}} \leq +\infty]$. Moreover, for a case in which expression (44) or (45) is applied, $m$ is given as negative because no items are manufactured. Considering the positive revenue under the reference condition, the solution must be $A = A_{\text{opt}}$ with which expression (46) is applied. We focus only
on the maximization of $m$ in expression (46) in the discussion in this subsection.

The derivative of $m$ in expression (46) with respect to $A$ is

$$\frac{dm}{dA} = \frac{dK}{dA} [1 + \chi_{ref}(K - 1)]$$  \hspace{1cm} (48)

online appendix A.4 shows that $1 + \chi_{ref}(K - 1) > 0$ when expression (46) is applied. Thus,

$$\frac{dK}{dA} \bigg|_{A=A_{opt}} = 0.$$  \hspace{1cm} (49)

Expression (49) denotes the equation to obtain $A_{opt}$. Since $K$ is not a function of $E_{ref}$, expression (49) implies that we do not need the value of $E_{ref}$ in the determination of $A_{opt}$. The determination of $E_{ref}$ is sometimes a difficult task, and the problem shown by expression (49) makes the proposed method realistically applicable. Interestingly, $K$ can be linked to the cost expressed in some previous studies. The details on the relationship between the present and the previous studies are shown in online appendix C.

In accordance with online appendix A.5, we can transform expression (49) into the following specific expression of the problem:

$$\left[ \Phi \left( \frac{-T - A_w}{u_w} \right) + \Phi \left( \frac{-T + A_w}{u_w} \right) \right]_{A=A_{opt}} = \frac{P_0 + (c_D - c_S)}{\beta P_0},$$  \hspace{1cm} (50)

where $A_w$ and $u_w$ are defined by expression (15). Expression (50) is a practical expression of the problem to be solved in this study. The number of solutions for expression (50) is zero or one for $A_{opt} > 0$. If there is no solution, $A_{opt} = +\infty$ is the solution.

Interestingly, the left side of expression (50) is $\theta(A_{opt})$ in expression (14). By redefining the $\text{gCR}$ and considering the price and production models, we consequently obtain expression (50), in which the specific CR of risk $C$ is focused on. However, the right side of expression (50), does not give a probability but a specific quantity computed by the price of a conforming item $P_0$, the degree of loss $\beta$, the discarding cost of a rejected item $c_D$, and the selling cost of an accepted item $c_S$.

5.3. Approximate closed-form solution for the optimized acceptable difference

The specific value of $P_0$ may not be necessary by assuming that $c_D$ and $c_S$ are negligibly smaller than $P_0$, because

$$\frac{P_0 + (c_D - c_S)}{\beta P_0} = \beta^{-1},$$  \hspace{1cm} (51)

under the condition that $c_D/P_0 \approx 0$ and $c_S/P_0 \approx 0$. The condition is sufficiently realistic considering that $c_D$ and $c_S$ are not the total costs of the discarding and selling but only the variable costs.

| Table 2. Summary of the inputs and the analysis results. |
|---|---|---|---|---|
| Case | 1 | 2 | 3 | 4 |
| Inputs | | | | |
| $\beta$ | 50 | 50 | 10 | 0.1 |
| $E_{ref}$ | 1 | 3 | 1 | 1 |
| Price of a conforming item | $P_0/10^3$ | 1.73 | 1.73 | 1.09 | 1.00 |
| Optimum acceptable difference | $A_{opt}$ | 1.59 | 1.59 | 1.78 | $+\infty$ |
| Values at the optimum acceptable difference | $R/\%$ | 12.3 | 12.3 | 8.4 | 0 |
| | $\theta/10^{-3}$ | 0.54 | 0.54 | 2.5 | 4.6 |
| | $\eta$ | 225 | 225 | 220 | 210 |
| | $P/10^3$ | 1.69 | 1.69 | 1.07 | 1.00 |
| | $K$ | 1.72 | 1.72 | 1.04 | 1.06 |
| | $m$ | 0.93 | 1.34 | 0.04 | 0.06 |

Moreover, we may be able to give an approximative closed-form solution for the problem shown by expression (50) when $u_t \ll T$. Since $u_w < u_t \ll T$ when $u_t \ll T$,

$$\phi \left( \frac{-T - A_w}{u_w} \right) \ll \phi \left( \frac{-T + A_w}{u_w} \right).$$  \hspace{1cm} (52)

In particular, when $c_D/P_0 \approx 0$, $c_S/P_0 \approx 0$, and $u_t \ll T$, expressions (50) to (52) result in

$$\phi \left( \frac{-T + A_w}{u_w} \right)_{|A=A_{opt}} = \beta^{-1}.$$  \hspace{1cm} (53)

Thus, the following relationship can be approximately obtained:

$$A_w|_{A=A_{opt}} = T + u_w \phi^{-1} (\beta^{-1}),$$  \hspace{1cm} (54)

where $\phi^{-1}$ is the inverse function of the cumulative distribution function of the standardized normal distribution. Defining

$$r = \frac{u_t}{\sigma},$$  \hspace{1cm} (55)

$A_{opt}$ satisfying the relationship shown by expression (54) can be given as follows:

$$A_{opt} = (1 + r^2) T + \sqrt{1 + r^2} \cdot u_t \cdot \Phi^{-1} (\beta^{-1}).$$  \hspace{1cm} (56)

To show the accuracy of $A_{opt}$ with expression (56), we compare it with $A_{opt}$ obtained by numerical computation based on expression (50) under the following conditions:

$$T = 2 \text{ and } 3, \quad \sigma = 1, \quad \beta = 50.$$  \hspace{1cm} (57)
Figure 8. Variation of the standardized revenue m as a function of the acceptable difference A for (a) the cases where $E_{\text{ref}} = 1$ (case 1) and 3 (case 3) while fixing $\beta = 50$, and (b) the cases where $\beta = 50$ (case 1), 10 (case 2), and 0.1 (case 3) while fixing $E_{\text{ref}} = 1$.

Figure 7(a) shows the results with $T = 2$. The difference between the numerical and approximate solutions is quite small when $u_r \leq 1$. The difference cannot be negligible, however, when $u_r > 1$. Figure 7(b) shows the results with $T = 3$. We cannot find a practical difference between the two results for the case of $u_r < 1.6$.

6. Characterization through simulations

In this section, we characterize the proposed approach through the application of simulated data. ANSI/NCSL Z 540.3 [30] is applied in some calibration industries. This standard implies the condition where no guard band is required. Specifically, the condition in the standard is that the measurement uncertainty should be less than one-fourth the width of the tolerance interval or less. No guard band is accepted in JCGM 106 as well based on the concept of ‘shared risk’, which means that the risks of false acceptance and false rejection are ‘shared’ by the consumers and the producer.

We apply the proposed approach to cases where no guard band is given, in accordance with ANSI/NCSL Z 540.3. It is hence assumed that the expanded uncertainty $k_u$ with the expansion coefficient of $k = 2$ is equal to $T/4$. To emphasize the significance of the acceptance interval, we suppose cases where the process capability is not sufficient as $T = 2$ and $\sigma = 1$ (so the process capability index is 0.67). Specifically, the following conditions are given:

$$T = 2, \quad \sigma = 1, \quad u_r = 0.25, \quad A_{\text{ref}} = 2. \quad (58)$$

The units are not shown, but these values should have an identical physical unit.

The other conditions are given as follows:

$$P_{\text{ref}} = 1000, \quad c_1 = 10, \quad c_D = 100, \quad c_S = 200. \quad (59)$$

These values should have an identical financial unit. We consider four cases with varying $\beta$ and $E_{\text{ref}}$, as shown in table 2. $\beta$ and $E_{\text{ref}}$ are dimensionless.

The gCR and the rejection rate $R$ under the reference condition are given as

$$R_{\text{ref}} = 5.23 \%, \quad \theta_{\text{ref}} = 0.84 \%. \quad (60)$$

which are computed using the procedure described in online appendix B. Further, $\theta_A = 0.80 \%$ and $\theta_C(A) = 31.4 \%$. The computed values of $P_0$ are shown in table 2.

Figure 8(a) shows a comparison between $E_{\text{ref}} = 1$ (case 1) and 3 (case 2) while fixing $\beta = 50$. We first focus on the variation of $m$ with $E_{\text{ref}} = 1$. $m$ is assessed as about $-0.6$ for $A = 0$, with which the output amount from the process is zero. $m$ increases with increasing $A$ because the rejection rate
decreases. The optimum acceptable difference \( A_{\text{opt}} \) is found to be 1.59 and the maximum value of \( m \) is 0.93. Considering that \( \gamma_{\text{ref}} W_{\text{ref}} \) is quite close to the sales under the reference condition \((P_{\text{ref}}, \gamma_{\text{ref}})\), the additional revenue of \(0.93 \times \gamma_{\text{ref}} W_{\text{ref}}\) is not negligible in terms of the business. For \( A > A_{\text{opt}} \), \( m \) decreases with increasing \( A \) because of the reduction of the price \( P \). Since no manufacturing is the optimum decision when \( P \) is smaller than a certain value, \( m \) becomes a constant value of \(-0.6\) when \( A \) is greater than a certain value around 2.2.

We then focus on the variation of \( m \) with \( E_{\text{ref}} = 3 \). It is found that when \( A \) is close to zero, \( m \) is constant with respect to \( A \), because the high rejection rate makes the marginal costs larger and manufacturing no items is the optimum decision. In the range of \( A = 0.4 \) to 2.1, the variation of \( m \) is qualitatively the same as that with \( E_{\text{ref}} = 1 \). The value of \( A_{\text{opt}} \) is not a function of \( E_{\text{ref}} \) and is found to be 1.59, while the optimum standardized revenue of \( m = 1.34 \) is larger than that with \( E_{\text{ref}} = 1 \) at \( A = A_{\text{opt}} \). Thus, \( E_{\text{ref}} \) does not involve the optimization of the acceptance interval but the maximum value of the standardized revenue. When \( A \) is larger than 2.1, \( m \) is constant because manufacturing no items is the optimum decision as well.

Figure 8(b) shows a comparison between \( \beta = 50 \) (case 1), 10 (case 3), and 0.1 (case 4) while fixing \( E_{\text{ref}} = 1 \). The variation of \( m \) with \( \beta = 50 \) (case 1) is the same as that in figure 8(a). In case 3 with \( \beta = 10 \), \( m \) is about \(-0.6\) for \( A = 0 \), similarly to case 1 with \( \beta = 50 \). While the variation of \( m \) for \( 0 < A \leq A_{\text{opt}} \) is qualitatively similar to that in case 1, the optimum acceptable difference and standardized revenue there are \( A_{\text{opt}} = 1.78 \) and \( m = 0.04 \), which are different from \( A_{\text{opt}} = 1.59 \) and \( m = 0.93 \) in case 1. In the range of \( A > A_{\text{opt}} \), \( m \) decreases monotonically.

In case 4, \( m \) with \( \beta = 0.1 \) is evaluated using \( P_0 = 1.0008 \times 10^3 \) with a more accurate expression than that in table 2. Expression (50) does not give \( A_{\text{opt}} < +\infty \), because

\[
\frac{P_0 + c_D - c_S}{\beta P_0} = 9.0.
\]

In this case, \( A_{\text{opt}} = +\infty \), for which the standardized revenue \( m \) is 0.06. This result implies that no inspection is the best decision. We could reduce not only the variable cost but also the fixed cost involving the inspection if the inspection was omitted in an actual process. The improvement of the revenue may thus be larger than that evaluated from \( m = 0.06 \) because of the reduction of the fixed and variable costs of the inspection.

These results mean that the magnitude of \( \beta \) is a key in the optimization of the acceptance interval and the evaluation of the revenue. \( \beta \) should be determined carefully and realistically with as much information as possible.

7. Summary

In this study, we discuss the optimization of acceptance interval in conformity assessment with a tolerance interval from an economic point of view. The variance in the manufacturing process and the measurement uncertainty in the inspection are taken into consideration. We give a price model that reflects the probability of non-conforming items among all of the market-available items. In the production process, the inspection, discarding, selling, and manufacturing costs are mathematically modelled with the manufactured amount assessed by classical microeconomic theory. The optimization of acceptance interval is thus mathematically expressed as the maximization of the revenue. We give an approximate closed-form solution for cases where the measurement uncertainty in the inspection is much smaller than the tolerance interval, and the discarding and selling costs are negligible. Through characterization by simulations, we found that a parameter that we call the ‘degree of loss’ is the key in the determination of the acceptance interval. The degree of loss means the ratio of the price difference between conforming and non-conforming items to the price of a conforming item. Only a random effect component is considered in the variance in the inspection in this study, and we will present a discussion of cases where a systematic effect component exists in our next report.

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13