A New Approach to The Quantum Mechanics

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Abstract

In this paper, we try to give a new approach to the quantum mechanics(QM) on the framework of quantum field theory(QFT). Firstly, we make a detail study on the (non-relativistic) Schrödinger field theory, obtaining the Schrödinger equation as a field equation, after field quantization, the Heisenberg equations for the momentum and position operators of the particles excited from the (Schrödinger) field and the Feynman path integral formula of QM are also obtained. We then give the probability concepts of quantum mechanics in terms of a statistical ensemble, realizing the ensemble(or statistical) interpretation. With these, we make a series of conceptual modifications to the standard quantum mechanics, especially propose a new assumption about the quantum measurement theory which can solve the EPR paradox from the view of the QFT. Besides, a field theoretical description to the double-slit interference experiment is developed, obtaining the required particle number distribution. In the end, we extend all the above concepts to the relativistic case so that the ensemble interpretation is still proper.

Two extra topics are added, in the first one, an operable experiment is proposed to distinguish the Copenhagen interpretation from the ensemble one via very different experimental results. While the second topic concerns with the extensions of the concept of coherent state to both the Bosonic and Fermionic field cases, to obtain the corresponding classical fields. And in the concluding section, we make some general comparisons between the standard QM and the one derived from the QFT, from which we claim that the QFT is the fundamental theory.

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I. INTRODUCTION

Quantum theory is well known as one of the most powerful theory in the last century. Although it provides an elegant way to describe the physics of the micro-world, its explanation is so complicated and obscure, that the Nobel Prize-winning physicist Richard P. Feynman said that ”I can safely say that nobody understands quantum mechanics”[1].

From the standard point of view, quantum theory includes two parts, one is the quantum mechanics (QM) which focuses on the behavior of quantum particles, for instance, the electrons, the photons; the other part is the quantum field theory (QFT) which gives the rules for the fields, such as the electromagnetic field. Usually, these two parts are considered to be independent from each other, by treating particles and fields as two kinds of independent physical objects sharing the same quantization scheme. However, from the QFT[2], it’s easy to find that the particles can be treated as quantum excitations of the corresponding fields, for example, the electron as excitation of electron field or Dirac field. Since particles are quantum excitations of fields, then whether QM could be obtained from QFT seems to be an interesting question[3]. We will show below that this is possible, and even it provides a new and natural interpretation to QM.

The paper are roughly divided into three major parts. In the first one, we study in details a non-relativistic field, the so called Schrödinger field, which is certainly relevant to the ordinary non-relativistic QM. All of the three standard formula of QM are obtained from this field theory, the Schrödinger equation as field equation, the Heisenberg equations for the momentum and position operators of the particles after the field quantization, and the Feynman path integral formula[4] of QM for particles. Then, in the second part, the probability concepts of QM are given in terms of a statistical ensemble, realizing the ensemble interpretation of QM[5]. With these, we further make a series of conceptual modifications to the standard quantum mechanics (SQM, the ”Copenhagen Interpretation”), especially propose a new assumption about the quantum measurement theory which can solve the EPR paradox from the view of the QFT. In the end of this part, a field theoretical description to the double-slit interference experiment is developed, obtaining the required particle number distribution. In the last part, an extension to the relativistic QFT is developed, with a method of separating the particle field from the anti-particle field, so that the ensemble interpretation is still proper. There are also two additive topics. In the first one, an oper-
able experiment is proposed to distinguish the Copenhagen interpretation from the ensemble one via very different experimental results. While the second topic concerns with the extensions of the concept of coherent state for the oscillator to both the Bosonic and Fermionic field cases, to obtain the corresponding classical fields.

In the concluding section, we make some general analysis on the basic rules of the standard QM, especially we show that the single particle operators are not fundamental, but only as derivations of the QFT. Therefore, we conclude that QFT is the fundamental theory.

II. NON-RELATIVISTIC SCHRÖDINGER FIELD

The concept of Schrödinger field[6] is useful or practical in (low energy) many-particle physics in which the particle number $N$ is invariant. Theoretically, this concept is related to the so called secondary quantization by treating the QM for particles as a fundamental theory. However, if the field was treated as a basic element, and the QFT as the fundamental theory, then what would happen? In this section, we will show the answer to this question.

The action for the Schrödinger field can be

$$S = \int dt d^3x \left[ i \psi^* \dot{\psi}(t, x) - \frac{1}{2m} \nabla \psi^* \nabla \psi(t, x) - \psi^* \psi(t, x) V(x) \right],$$

with $V(x)$ an external potential, for example the Coulomb potential. For simplicity, we don’t consider the field self-interactions $V(\psi)$ which is necessary in most real physical situations. In fact, in QFT, relativistic or not, interactions are already well developed.

From eq.(1), it’s easy to find that the field equation is just the standard Schrödinger equation

$$i \frac{\partial \psi}{\partial t} = -\frac{\nabla^2 \psi}{2m} + V(x)\psi,$$

which can also be obtained from the relativistic equations like the Dirac equation in the non-relativistic limit. In the SQM, the Schrödinger equation is known as the quantized equation of a particle, with the wave property. However, in QFT, after the field quantization, the particles manifest themselves, and satisfy the field equation automatically.

With the canonical quantization, the Schrödinger field will satisfy the communicative relations

$$[\psi(x), \psi^*(y)]_\pm = \delta^3(x - y),$$

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with the minus for Bosonic case, plus for Fermionic case. In field theory, we need to consider the space-time symmetries of the Lagrangian, for example, the symmetry under space-time translation

\[ x^\mu \rightarrow x^\mu + d^\mu, \]  

(4)

from which we obtain the energy and momentum operators

\[ H = \int d^3x \left[ \frac{1}{2m} \nabla \psi^\dagger (x) \nabla \psi(x) + \psi^\dagger (x) V(x) \psi(x) \right] \]  

\[ P = \int d^3x \psi^\dagger (x) (-i \nabla) \psi(x). \]  

(5)

(6)

In addition, we can define another two operators

\[ X = \int d^3x \psi^\dagger (x) x \psi(x) \]  

\[ N = \int d^3x \psi^\dagger (x) \psi(x), \]  

(7)

(8)

i.e. the position and particle number operators.

Among these operators, there are the following communicative relations

\[ [H, N] = 0, \]  

(9)

\[ [H, P] = i \int d^3x \psi^\dagger (x) \nabla V(x) \psi(x), \]  

(10)

\[ [H, X] = -\frac{1}{m} \int d^3x \psi^\dagger (x) \nabla \psi(x), \]  

(11)

by using the communicative relations in eq.(3) for both the Bosonic and Fermionic cases. As is known, \( H \) generates the time translation for an arbitrary operator \( O \) constructed with the fields

\[ [H, O] = -i \dot{O}, \]  

(12)

so are those in eqs.(9)-(11). Eq.(9) says that the particle number is invariant, while the other two are the familiar Heisenberg equations of the momentum and position operators. Since all these operators can be represented with the creators and annihilators of particles in the Fock space, we can denote a single-particle state as \( |1> \), and let eqs.(10) and (11) operate on it, we then have the QM for single particle. In fact, eqs.(10) and (11) tell us that all the particles satisfy the QM.

\(^1\) The general form of eq.(12) is actually \( O(t) = e^{iHt} O(0) e^{-iHt} \) in the QFT.
Up to now, we have obtained two main QM equations, one is the Schrödinger equation (2) as field equation, the other is the system of the Heisenberg equations (10) and (11). Further, we could obtain the Heisenberg uncertainty relation which is believed to be the most important property of QM from the communicative relation

\[ [X_i, P_j] = i\delta_{ij} \int d^3x \psi^\dagger(x)\psi(x). \] (13)

Similarly, we can let it operate on \(|1\rangle\) to get the relation for single particle in QM.

Now we make some studies in the free field case for simplicity

\[ \psi(x) = \int \frac{d^3p}{(2\pi)^3} a(p)e^{ipx}, \psi^\dagger(x) = \int \frac{d^3p}{(2\pi)^3} a^\dagger(p)e^{-ipx}, \] (14)

then the (free field) energy and momentum operators become

\[ H = \int \frac{d^3p}{(2\pi)^3} E_p a^\dagger(p)a(p) = \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{2m} a^\dagger(p)a(p) \] (15)

\[ P = \int \frac{d^3p}{(2\pi)^3} pa^\dagger(p)a(p), \] (16)

while the position and particle number operators will be

\[ X = \int \frac{d^3p}{(2\pi)^3} a^\dagger(p)i\partial_p a(p) \] (17)

\[ N = \int \frac{d^3p}{(2\pi)^3} a^\dagger(p)a(p). \] (18)

From eq.(12), we can also have a velocity operator

\[ V \equiv \dot{X} = \int \frac{d^3p}{(2\pi)^3} \partial_p E_p a^\dagger(p)a(p) = \int \frac{d^3p}{(2\pi)^3} \frac{p}{m} a^\dagger(p)a(p), \] (19)

which is similar to the velocity of the non-relativistic particle. With these operators(or physical observables) obtained, what we then need are their eigenstates. Obviously, the momentum state can be easily defined as \(|p\rangle = a^\dagger(p)|0\rangle\) with normalization \(<p|q> = \delta^3(p - q)\), then what is the position state \(|x\rangle\)? Let’s define it as follows

\[ |x\rangle = \psi^\dagger(x)|0\rangle, \] (20)

and with eq.(3), it’s easy to verify that \(|x\rangle\) is just the eigenstate of operator \(X\) with eigenvalue \(x\), with normalization condition

\[ <y|x>=<0|\psi(y)\psi^\dagger(x)|0> = \delta^3(x - y). \] (21)
Now, the Feynman path integral formula of QM can be obtained as usual in the textbook. With the Heisenberg picture operator

\[ X^H(t) = e^{iHt} X^S(0)e^{-iHt}, \] (22)

and a moving basis \(|x, t>\) which satisfies

\[ X^H(t)|x, t> = x|x, t>, \] (23)

we then have

\[ |x, t> = e^{iHt} |x> = e^{iHt} \psi^S(x)e^{-iHt} e^{iHt}|0> = \psi^H(x, t)|0>. \] (24)

Thus the transition amplitude is

\[ <x_2, t_2|x_1, t_1> = <0|\psi^H(x_2, t_2)\psi^H(x_1, t_1)|0>, \] (25)

with \(\hat{H}\) the familiar single-particle Hamiltonian. Obviously, eq.(25) is the starting point of Feynman path integral formula of QM.

In fact, the Feynman path integral formula can be derived in a pure field theoretical way as follows. From eq.(24), eq.(25) could be rewritten as

\[ <x_2, t_2|x_1, t_1> = <0|\psi^H(x_2, t_2)\psi^H(x_1, t_1)|0>, \] (26)

which is just the propagator of the field in QFT[2]. Now we can compute this propagator with the standard field method, i.e. define the interaction picture

\[ \psi^iI(x, t) = e^{iH_0t}\psi^S(x)e^{-iH_0t}, \] (27)

with the free field Hamiltonian \(H_0\) constructed from the fields in the Schrödinger picture. Then we have the following relation

\[ \psi^H(x, t) = e^{iHt} e^{-iH_0t} \psi^iI(x, t)e^{iH_0t} e^{-iHt} = U^\dagger(t, 0)\psi^iI(x, t)U(t, 0), \] (28)

where the time-evolution operator \(U(t, 0) = e^{iH_0t}e^{-iHt}\) satisfying the equation

\[ i\frac{\partial U(t, 0)}{\partial t} = H_{int}(t)U(t, 0), \] (29)

with a time-ordering formal exponential solution

\[ U(t, 0) = T \exp[-i \int_0^t dt'H_{int}(t')]]. \] (30)
With eq.(28), the right-hand-side of eq.(26) will become

\[ < 0 | U^\dagger(t_2, t_0) \psi^I(x_2, t_2) U(t_2, t_0) U^\dagger(t_1, t_0) \psi^I(x_1, t_1) U(t_1, t_0) | 0 >, \]  

(31)

for a general reference time \( t_0 \). With the condition \( U(t_1, t_0) | 0 > = | 0 > \) for the Schrödinger field, and the following property of \( U[2] \)

\[ U(t_2, t_0) U^\dagger(t_1, t_0) = U(t_2, t_1), \]  

(32)
eq(31) will be further simplified as

\[ < 0 | \psi^I(x_2, t_2) U(t_2, t_1) \psi^I(x_1, t_1) | 0 > = < 0 | T \psi^I(x_2, t_2) \psi^I(x_1, t_1) e^{-iC_{t_2}^t_{t_1} H_{0}(t')} | 0 >. \]  

(33)

To compute eq.(33), the element is the propagator \( K(x_2, t_2; x_1, t_1) \) of free field defined as

\[ K(x_2, t_2; x_1, t_1) = < 0 | \psi^I(x_2, t_2) \psi^I(x_1, t_1) | 0 >. \]  

(34)

With eqs.(14) and (27), the free propagator is

\[ K(x_2, t_2; x_1, t_1) = \int \frac{d^3p}{(2\pi)^3} e^{-iE_p(t_2-t_1)+ip(x_2-x_1)}, \]  

(35)

which can be solved by the Gaussian integral formula, and we thus have

\[ K(x_2, t_2; x_1, t_1) = \left[ \frac{m}{2\pi i(t_2-t_1)} \right]^2 \exp\left\{ i \frac{m}{2} \left( \frac{x_2-x_1}{t_2-t_1} \right)^2 (t_2-t_1) \right\}. \]  

(36)

Furthermore, we can infer from eq.(34) that

\[ K(x_3, t_3; x_2, t_2) K(x_2, t_2; x_1, t_1), \]  

(37)

where the completeness relation

\[ \int d^3x \psi^I(x, t) | 0 > = \int d^3x \psi^I(x, t) = I, \]  

(38)

of the Schrödinger field has been used. Now just like the case in the standard derivation of the path integral formula, the time interval \((t_2, t_1)\) can be split up to many small slices \( \epsilon \), for example \( N \), then eq.(36) can be rewritten as

\[ K(x_2, t_2; x_1, t_1) = \left( \frac{m}{2\pi i \epsilon} \right)^{3N} \int \prod_i d^3x_i \exp\left\{ i \sum_i \frac{m}{2} \left( \frac{x_{i+1} - x_i}{\epsilon} \right)^2 \epsilon \right\}. \]  

(39)
Eq. (33) can be computed in perturbative power series, in which the zero order is just the free field propagator in eq. (34), while the first order is

\[ (-i) \int_{t_1}^{t_2} dt d^3x K(x_2, t_2; x, t)V(x)K(x, t; x_1, t_1), \]  

(40)
similar for larger order. In order to compare with the path integral formula, we give here the standard path integral formula[2]

\[ \lim_{N \to \infty} \left[ \frac{m}{2 \pi i \epsilon} \right]^{\frac{3N}{2}} \int \prod_i d^3x_i \exp i \sum_i \left[ \frac{m}{2} \frac{(x_{i+1} - x_i)^2}{\epsilon} - \epsilon V\left(\frac{x_{i+1} + x_i}{2}\right) \right]. \]  

(41)

 Obviously, we only need to compare the potential terms, which can also be rewritten order by order. The zero order is the free case, just like eq. (39), while the first order is

\[ (-i) \left[ \frac{m}{2 \pi i \epsilon} \right]^{\frac{3N}{2}} \int \prod_i d^3x_i \exp i \sum_i \left[ \frac{m}{2} \frac{(x_{i+1} - x_i)^2}{\epsilon} \right] \sum_i \epsilon V\left(\frac{x_{i+1} + x_i}{2}\right), \]  

(42)

which in the large N limit is just eq. (40) with the sum over the positions and time slices replaced with the integral \( \int_{t_1}^{t_2} dt d^3x \). Now, let’s see the second order, for the field case, we have

\[ \frac{(-i)^2}{2} \int_{t_1}^{t_2} dt d^3x \int_{t_1}^{t_2} dt' d^3x' K(x_2, t_2; x, t)V(x)K(x, t; x', t')V(x')K(x', t'; x_1, t_1), \]  

(43)

while from eq. (41), we have

\[ (-i)^2 \sum_i \epsilon^2 V^2\left(\frac{x_{i+1} + x_i}{2}\right) + \frac{(-i)^2}{2} \sum_{i \neq j} \epsilon^2 V\left(\frac{x_{i+1} + x_i}{2}\right) V\left(\frac{x_{j+1} + x_j}{2}\right), \]  

(44)

where the second term is easily to be identified, then what about the first one? Noting that from eq. (34), we have

\[ K(x_2, t; x_1, t) = \langle 0 | \psi'(x_2, t) \psi^\dagger(x_1, t) | 0 \rangle = \delta^3(x_2 - x_1), \]  

(45)

this means that the first term in eq. (44) can be easily from the \( t \to t' \) limit of eq. (43). Then order by order, we can find that eqs. (33) and (41) are actually identical, confirming the eq. (25) which give the path integral formula of QM from the Schrödinger field theory.

Up to now, we have obtained all the three equivalent approaches to the non-relativistic QM on the framework of the quantum Schrödinger field theory, with all the physical observables completely constructed with the quantized fields, especially the position operator in eq. (7) whose meaning is obscure for field. In the next section, we will show some further physical results of the Schrödinger field theory, especially the possible modifications to the SQM.
III. MODIFICATIONS TO THE SQM

A. Probability Concepts From QFT

Since the particle number is invariant as in eq.(9), the statistical property of some collection of particles in the field theory could be transferred to the probability property of single particle. Supposing that $\psi(t, x)$ has the energy expansion

$$\psi(t, x) = \sum_n a_n \psi_n(x) \exp(-iE_n t), \quad (46)$$

from it, we can obtain that, in QM, the probability for a particle to be at state $n$ is

$$P_n = \frac{\left| \int d^3x \psi_n^*(x) \psi(x) \right|^2}{\int d^3x \psi^*(x) \psi(x)} = \frac{|a_n|^2}{\int d^3x \psi^*(x) \psi(x)}. \quad (47)$$

A possible field theoretical generalization could be

$$P_n = \frac{\langle \int d^3x \psi^*(x) \psi_n(x) \int d^3y \psi_n^*(y) \psi(y) \rangle}{\langle \int d^3x \psi^*(x) \psi(x) \rangle} = \frac{\langle a_n^\dagger a_n \rangle}{\langle \sum_n a_n^\dagger a_n \rangle}. \quad (48)$$

with notation $>$ standing for particle state in Fock space. It seems that eqs.(47) and (48) could be identical, with the meaning that, among a collection of particles excited from the field, the probability for the particle picked up to be at state $n$ is in fact the ratio of the particle number at this state by the total particle number in this collection. With this identification, the mean value of some physical quantity, for example, the energy, is

$$\bar{E} \equiv \frac{\langle H \rangle}{\langle \int d^3x \psi^*(x) \psi(x) \rangle} = \frac{\langle a_n^\dagger a_n \rangle}{\langle \sum_n a_n^\dagger a_n \rangle}, \quad (49)$$

with $H$ the field energy operator defined in eq.(5).

Obviously, eqs. (48)and (49) are more familiar to us conceptually, based on the traditional probability concepts which come from the statistical property of a collection of particles for single particle or collections of a collection of particles for many-particle system, i.e. some statistical

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$^2$ For simplicity, we assume that there is no state degeneration and the expansion eq.(46) can be treated as either the wave function or the field.

$^3$ Here, the time is ignored because they are stationary states, the same below.

$^4$ For the continuous case like the energy density, we have the probability density $P(\omega) d\omega \equiv \frac{\langle a_n^\dagger(\omega) a_n(\omega) \rangle}{\int d\omega \langle a_n^\dagger(\omega) a_n(\omega) \rangle}.$

$^5$ In fact, as we will show below, the particle state $>$ is usually standing for some ensemble, i.e. copies of a single particle or many-particle systems conceptually.
ensemble of systems, made up of the particles excited from the field. However, in order for the identity for eqs. (47) and (48), we have to make an important assumption that, the state of each single particle is definite and unique (but unknown to us if without any measurement), as long as there’s no disturb, which is very different from that in SQM. This is an extension of the Newton’s first law with the velocity replaced by the state. Then the state > is usually standing for some ensemble made up of particles with different kinds of states so that eq. (48) is proper.

One important example of the above ensemble is a sample of particles in an experiment about some physical process, in other words, all the particles in the sample come from a physical process such as a scattering. It is certain that the sample must contain some information about the physical process, for example the scattering angle distribution, which can be studied by the ratios of the particles in all the angles. And what the scattering distribution can tell us is just the probability for one particle to be observed in one angle, which can be solved by QM or QFT. Therefore, in this sense, the QM and QFT should be physically identical. Since the QM could be obtained from QFT as shown in the last section, the probability concept could be realized with the use of the sample or ensemble specific for some property of the particle, for instance, the scattering angle distribution.

Now, let’s consider a simple example. Suppose that in the SQM, there is a state vector

\[ |\phi> = \sqrt{\frac{1}{3}}|1> + e^{i\delta(t,x)} \sqrt{\frac{2}{3}}|2> . \]  

If this was a state for a single particle, it would be a so called pure state in SQM, and the density matrix is

\[ |\phi><\phi| = \frac{1}{3}|1><1| + \frac{2}{3}|2><2| + \frac{\sqrt{2}}{3}(e^{-i\delta(t,x)}|1><2| + e^{i\delta(t,x)}|2><1|). \]  

The wave function for this state can be resulting from

\[ <0|\psi(x)|\phi> = \sqrt{\frac{1}{3}}\psi_1(x) + e^{i\delta(t,x)} \sqrt{\frac{2}{3}}\psi_2(x), \]  

with the use of field expansion in eq.(46). Then from eq.(47), this state says that, the probability for this particle to be at state 1 is \( \frac{1}{3} \), and to be at state 2 is \( \frac{2}{3} \). According to the above discussions, the

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6 Here \( |1> = a^\dagger_1|0> \), and we have assigned an arbitrary phase term which could be space-time dependent, and we will see below that this important phase term usually comes from interactions, such as a measurement.

7 Noting that by substituting \( |\phi> \) into eq.(48), we will obtain the form of eq.(47), and state in eq.(53) below has the same physical results as \( |\phi> \).
probability properties of single particle could be from the statistical properties of a collection of particles. Then, there must be a collection of particles which consists of copies of a single particle \[ [1, 1, \cdots 1; 2, 2, \cdots 2]. \]

The meaning is obvious, i.e. within the \( N \) particles, \( \frac{N}{N} \) are at state 1, the rest are at state 2, i.e. an ensemble for the single particle\(^8\). And the density matrix for single particle should be

\[
\rho = \frac{1}{3}|1><1| + \frac{2}{3}|2><2|.
\]

Comparing with eq.(51), the off-diagonal terms disappear, which is one of the important distinctions between our statistical ensemble and the SQM. In fact, this example involves the problem of superposition state which will be discussed in details in the next subsection, here what we only need to know is that the state in eq.(50) would hardly be a pure state for a single particle, but should be identified with a so called mixed state in SQM, with density matrix in eq.(54).

In fact, if we "normalize" the field function, a possible field expression may be as follows

\[
\phi^\dagger(x)|0> = \sqrt{\frac{1}{3}}\psi_1^\dagger(x)|1> + \sqrt{\frac{2}{3}}e^{-i\delta(t,x)}\psi_2^\dagger(x)|2>.
\]

with the "re-normalized" field

\[
\phi^\dagger(x) = \sqrt{\frac{1}{3}}a_1^\dagger\psi_1^\dagger(x) + \sqrt{\frac{2}{3}}e^{-i\delta(t,x)}a_2^\dagger\psi_2^\dagger(x),
\]

which incorporates both the field and wave function properties. Then let's consider the following expression

\[
\int d^3x \phi^\dagger(x)|0><0|\phi(x),
\]

which is formally the intermediate section of two propagators

\[
\int dy_0d^3y < 0|\psi(x_0, x)\phi^\dagger(y_0, y)|0> < 0|\psi(y_0, y)\phi^\dagger(z_0, z)|0>.
\]

With the orthonormalization conditions, eq.(57) is just the density matrix in eq.(54)\(^9\)! All these confirm our ideas above. We can also obtain the mean value of a physical quantity, for example,

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\(^8\) Do not confuse with the many-particle state which is a real N-particle system. In fact, it’s easy to distinguish them by noting that it’s impossible to include so many particles with the same state for the Fermionic case.

\(^9\) The density matrix formula in eq.(57) has a time evolution \( \rho(t) = e^{iHt}\rho(0)e^{-iHt} \), different from the one in SQM, because it’s made up of fields.
the energy

\[ \hat{E} = Tr(\rho H) = \sum_n \langle n | \int d^3x \phi^\dagger(x)0 > < 0 | \phi(x)H|n \rangle = \frac{1}{3} E_1 + \frac{2}{3} E_2. \]  

(59)

Considering an interaction \( B(x) \), we can also have a general density matrix

\[ \int d^3x \psi^\dagger(x)|0 > B(x) < 0 | \psi(x) = \sum_{mn} B_{mn}|m > < n|, \]  

(60)

with \( B_{mn} \) the transition amplitude

\[ B_{mn} = \int d^3x \psi^*_m(x)B(x)\psi_n(x). \]  

(61)

Here is a note about \( N \), the particle number in the single particle ensemble. From the above example, it appears that \( N \) could be any number, and it’s indeed so. The reason is that the ratios of particles with different kinds of states are almost fixed for an ensemble corresponding with some physical process, and just like the law of large numbers in probability theory, we need to take \( N \to \infty \) in the real situations. The above ideal example is only for showing how to transform the ensemble (53) into the QM-like formalism in eq.(55)or (56), i.e. ”re-normalize” the field functions, and obtain the density matrix for a single particle in eq.(57).

The above single-particle ensemble is a simple one with some fixed probabilities. A general one can be described as follows, treating the particle as one system, and we don’t know the exact state of this system without any measurement, so we have to list all the possibilities, i.e. the particles at state 1, or 2, or \ldots etc. For every possibility, there will be a corresponding (variable) probability(i.e. the ratios of the particle numbers), \( P_1, P_2, \ldots \) etc. Then a state for the ensemble(or ensemble state) will be

\[ e^{i\delta_1} \sqrt{P_1}|1 > + e^{i\delta_2} \sqrt{P_2}|2 > + \cdots + e^{i\delta_n} \sqrt{P_n}|n > + \cdots = \sum_n \alpha_n|n >, \]  

(62)

an extension of eq.(50), and easily to see it has the same form with a general state vector in SQM. If we would like to know the exact state of the particle, we have to observe it, and obtain that the particle is in fact at some definite state, for example, \( k \). It appears that there is the so called quantum collapse here, as in SQM, but no!

Noting that the state in eq.(62) is for the single-particle ensemble, not for the particle, this means that there must be a realization to the ensemble, and the simple example in eq.(50) or (53) is one with definite probabilities for each state. We could also realize the ensemble artificially by
collecting particles with arbitrary unknown ratios at will. And no matter whether the probabilities are already known (fixed) or unknown, the essential feature is the same. The collapse due to an observation on the state in eq.(62) can be explained with a familiar example. Supposing there are three balls with red or blue colors in a bag, and further we know that there are one red ball and two blue ones. Then from these, we know that if we pick a ball arbitrarily, it can be red or blue, and the probability for it to be red is \( \frac{1}{3} \), \( \frac{2}{3} \) for blue, then this ”state” about the color of a single ball could be described by the state in eq.(50). However, if we observe the ball and find that it is blue, then how to explain this observation? Is there also a collapse classically? The only reason is that the state in eq.(50) is an ensemble state, which is just a useful tool, and QM or QFT is a realization of it physically by collecting the copies of the particle conceptually. The reason for using an ensemble is that we can’t obtain the exact information of a system without any measurement and can only list all the possibilities with the corresponding probabilities. Therefore, to a certain extent, QM is much more consistent with the ensemble interpretation.

We can extend the single-particle ensemble to a general N-particle one which is usually seen in the statistical mechanics or many-particle physics. Considering \( N \) (identical) particles within \( k \) states (without degenerations), we can start with

\[
\psi^\dagger(x_1)\psi^\dagger(x_2)\cdots\psi^\dagger(x_N)|0>, (63)
\]

up to some constant, and it can also be rewritten in Fock space with a recombination as, for example the Bosonic case

\[
\sum_{n_1+n_2+\cdots+n_k=N} \sqrt{\frac{P_{[n_j]}}{S}} e^{i\delta_{\nu_j}}(a_1^{\dagger})^{n_1}(a_2^{\dagger})^{n_2}\cdots(a_k^{\dagger})^{n_k}|0>, (64)
\]

with \( S \) a symmetry factor which is \( (n_1)! (n_2)! \cdots (n_k)! \) for Bosonic case and 1 for the Fermionic case, \( P_{[n_j]} \) the probability for a possible distributions \([n_j]\) with \( n_1 \) particles in state 1, \( n_2 \) particles in state 2, etc, specially \( n_j = 0, 1 \) for the Fermionic case due to the Pauli exclusion principle, and the sum is over all the possibilities. With some other extra specific conditions for bosons and fermions, we could further obtain the Bose and Fermi statistics. It’s easy to see that eq.(64) is just a state for an N-particle ensemble, the extension of eq.(62). And the reason for using an ensemble is the same with the single-particle case, i.e. we don’t know the states for all the particles to specify the state of the system without measurements. Further, in this case, the situation is much more complicated than the single-particle case because of the large number of the particles and interactions among
them. Therefore, we can only describe the system with the method of statistical mechanics by finding out the most possible distribution in eq.(64).

In order to understand these, let’s take a look at the above simple example in eq.(53) again, with \( N = 3 \), that is three particles within two states. We still take the Bosonic case, and it’s easy to see that there are four possibilities with density matrixes

\[
\rho_1 = |1><1| \quad (65)
\]
\[
\rho_2 = \frac{1}{3}|1><1| + \frac{2}{3}|2><2| \quad (66)
\]
\[
\rho_3 = \frac{2}{3}|1><1| + \frac{1}{3}|2><2| \quad (67)
\]
\[
\rho_4 = |2><2|. \quad (68)
\]

Recall that that example is originally a single particle ensemble, so the above four density matrixes are all for single particle, that is we can only obtain the information about single particle from them. While the density matrixes for a real 3-particle system, from which we could obtain the information about the whole system, are respectively (up to some normalization constants)

\[
\sigma_1 = (a_1^\dagger)^3|0><0|(a_1)^3 \quad (69)
\]
\[
\sigma_2 = a_1^\dagger(a_2^\dagger)^2|0><0|(a_2)^2a_1 \quad (70)
\]
\[
\sigma_3 = a_2^\dagger(a_1^\dagger)^2|0><0|(a_1)^2a_2 \quad (71)
\]
\[
\sigma_4 = (a_2^\dagger)^3|0><0|(a_2)^3, \quad (72)
\]

corresponding to the ensemble state in eq.(64). If the four possibilities in eqs.(65)-(68) have equal probability\(^{10} \), i.e. 1/4, then the final result for single particle is

\[
\rho = \frac{1}{4}(\rho_1 + \rho_2 + \rho_3 + \rho_4) = \frac{1}{2}(|1><1| + |2><2|), \quad (73)
\]

which can also obtain via the ordinary probability computations. In fact, there exists a class of \( N \)-particle ensemble which can be made up with the single-particle ones for each particle, which can be seen from eq.(63), with each field function \( \psi^j(x_j) \) standing for a single-particle ensemble, i.e. the ”re-normalized” field in the form of eq.(56). Then the probability \( P_{[\rho]} \) is the multiplication of the corresponding probabilities of single particle ensembles and some symmetry factors.

---

\(^{10}\) This is only an assumption, and for large \( N \) we could obtain the most possible distribution from the statistical mechanics.
In SQM, the state in eq.(50) is a superposition state, which is related with the principle of superposition of states. However, from the view of ensemble, the state in (50) is not physical, but a state for an ensemble(or mixed state in SQM), so is the one in eq.(62). Therefore, it appears that the principle of superposition is suitable for the field, in other words, the collection of particles with different states i.e. the ensemble. We will make some detail discussions on these in the next subsection, which give some modifications to the SQM.

B. Conceptual Modifications to The SQM

Based on the field theoretical descriptions above, in this subsection, we will give a series of modifications to the SQM, and make some detail discussions on the superposition principle and quantum measurement theory.

(1) The wave function, or the probability amplitude in QM is not a fundamental element, but a derivation of the field $\psi(t, x)$, a distribution in space-time, which is real in nature. In addition, the original Schrödinger equation is the non-relativistic field equation, as shown in eq.(2).

This is the result of section II., where the Schrödinger equation (wave form) and Heisenberg equations (particle form) are both derived from the field theory. The meaning for the latter is clear, while the Schrödinger equation is ambiguous because in SQM it’s rewritten in the following form

$$i\frac{\partial}{\partial t}|\phi(t)\rangle = \hat{H}|\phi(t)\rangle,$$

with $\hat{H}$ the Hamiltonian operator for single particle. What this equation can tell us is the evolution of the state of the particle, somewhat deterministic, that is given the state at some time $t_0$, it could determine the state thereafter. However, the original Schrödinger equation (2) can also be considered to be the time evolution of the quantized field

$$i\frac{\partial}{\partial t}\psi(t, x) = -[H, \psi(t, x)],$$

with $H$ the Hamiltonian of the field as in eq.(5), or in a more compact form

$$\psi(t, x) = e^{iHt}\psi(0, x)e^{-iHt}.$$  

Thus, with a state vector $|\phi\rangle$, and by using eqs.(20) and (52), we have the wave function

$$\phi(t, x) = <0|\psi(t, x)|\phi > = <0|e^{iHt}\psi(x)e^{-iHt}|\phi >= x|\phi(t) >,$$
with
\[|\phi(t) > = e^{-i\hat{H}t}|\phi > \rightarrow e^{-i\hat{\tilde{H}}t}|\phi > , \] (78)

where we have reduced the field Hamiltonian \( H \) into single particle one \( \hat{H} \) because of the state \( < x | \) in eq.(77), just like the case in eq.(25). Therefore, field equation (75) is much more fundamental than the state evolution equation (74).

(2) The probability \( \int d^3x \phi^* \phi(t,x) \) in SQM corresponds to the particle number operator \( \int d^3x \psi^\dagger \psi(t,x) \), then the probability conservation in the SQM is in fact the conservation of total particle number in non-relativistic QFT as shown in eq.(9).

As described in the last subsection, the probability concepts for single particle comes from the statistical concepts of a statistical ensemble like the one in eq.(53) or (62). And in order for the identity of eqs.(47) and (48), we have made an important assumption, the state of every single particle is definite and unique (but unknown to us if without any measurement)\(^{11}\). Of course, this is very different from the assumption in SQM, where the state for single particle could be the superposition of states in the following form
\[|\phi > = \sum_n \alpha_n |n > , \] (79)

which is the same form as the ensemble state in eq.(62). Because eq.(74) is not a fundamental equation, then the state in eq.(79) loses its physical meaning as a state of single particle, so does the wave function \( \phi(t,x) \). All these involve the so called ”the superposition principle” in QM, and now let’s see how to interpret it properly.

In SQM, the superposition principle generally says that, if \( \psi_1 \) and \( \psi_2 \) are both the states of a system, then the linear combination \( \alpha \psi_1 + \beta \psi_2 \) (with \( \alpha \) and \( \beta \) arbitrary complex numbers) is also a possible state of the system. This principle can be proved loosely by the linearity of the Schrödinger equation. It also could be seen roughly from the expansion (46) or (79), which could be interpreted to be superposition for single particle state in SQM. But the field theoretical form (46) may also be interpreted as followed, the states are all the possibilities for the particles excited from the field, and since we have assumed the definite and unique for single particle, the concept of ensemble is needed. Then we could have the following modification

(3) The superposition principle is suitable for the field, that is an ensemble of particles, or ensemble states in eqs.(62) and (64), not for a single particle.

\(^{11}\) We could know the state only if we had observed it, i.e. interacted with it.
In fact, the superposition principle in QM is so strongly dependent on the linearity of the Schrödinger equation that if we include the self-interaction terms into the action in eq.(1), the resulting equation is non-linear and hard to solve, and the expansion in eq.(46) is useless, and we could only use the free theory expansion to obtain the perturbative power series for the interactions, as shown in eq.(33). Therefore, we could not decide which principle (SQM’s or ours) is much more physical, because it’s necessary to combine the assumption of the quantum measurement. As is known to us, in SQM, there is the so called mysterious quantum collapse owing to the superposition principle. However, there is nothing abnormal with our ensemble concepts, which will be discussed in the next subsection\textsuperscript{12}.

In the rest of this section, we will introduce some examples about the superposition principle suitable for both the SQM and QFT mathematically. First of all, let’s distinguish two concepts, superposition state and superposition of states\textsuperscript{13}. Easily to see, the former is included in the latter. In fact, superposition of states can be generally expressed mathematically as $\alpha\psi_1 + \beta\psi_2 + \cdots$. However, the superposition state as state of single particle must be physical in nature, though mathematically has the form of superposition of states. In other words, a physical state $\psi$ which can be expressed as

$$\psi = \alpha\psi_1 + \beta\psi_2,$$

with some fixed numbers $\alpha$ and $\beta$(up to some overall normalization constant) to specify the physical properties, and this mathematical expression is just a convenient relation for some analysis.

Let’s introduce a class of superposition state. The first example is the eigenstates of momentum $e^{i p \cdot x}$ which can be expressed mathematically as the combinations of some special functions like spheric harmonics functions(Rayleigh expansion), and vice versa. In QM language, these are the transformations between momentum and angular momentum states

$$\{|p_1, p_2, p_3\rangle \equiv |p, l, m\rangle\}. \quad (81)$$

The second example will be used in the next subsection. It involves the spin states of electrons, i.e. $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$, and for any direction $\hat{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$, $|\uparrow_n\rangle$ and $|\downarrow_n\rangle$. There are transformations between them, for example,

$$|\uparrow_n\rangle = e^{-i\varphi} \cos \frac{\theta}{2} |\uparrow_z\rangle + e^{i\varphi} \sin \frac{\theta}{2} |\downarrow_z\rangle. \quad (82)$$

\textsuperscript{12} We have already a simple explanation below eq.(62) in the last subsection.

\textsuperscript{13} These concepts may be different from those in SQM, but the discussions below will be self-consistent.
with fixed coefficients to specify the direction \( \hat{n} \), thus, we have similarly

\[ \{ | \uparrow_z \rangle, | \downarrow_z \rangle \} \Leftrightarrow \{ | \uparrow_n \rangle, | \downarrow_n \rangle \}. \tag{83} \]

The last example is about the coupling of angular momenta. In order to understand it, we first give a simple example which involves the same essential feature with the coupling of angular momenta. Suppose that there are two particles 1 and 2 with states \( \{(E_1, p_1)\} \) and \( \{(E_2, p_2)\} \) respectively. Easily to see, this description is proper in the lab frame, and we can also describe them in the center-of-mass frame with states \( \{(E_c, P), (E, p)\} \), representing the energy of the center-of-mass frame, total momentum, relative energy and momentum, respectively. Then there should be some relations for these two classes of states as eqs.(81) and (83). In fact, the coupling of angular momenta operates similarly, and we will have the following correspondence for two angular momenta \( J_1, J_2 \)

\[ \{ | j_1, m_1; j_2, m_2 \rangle \} \Leftrightarrow \{ | j, m \rangle \}, \tag{84} \]

with the CG coefficients in the relations between them.

Obviously, the above three examples satisfy the conditions of superposition state, i.e. physical and superposition of states. The reason for putting them into one class is that there are states transformations for all of them, i.e. eqs.(81),(83) and (84). Furthermore, we can see that all these transformations are related to some coordinate transformations, (81) for Cartesian coordinate and spherical coordinate, (83) for rotations on a sphere, and (84) for transformations between lab frame and center-of-mass frame\(^{14}\). And according to the Wigner theorem, all these transformations between the states are all unitary with fixed coefficients(up to some overall phase terms), the last key condition for the superposition state. In fact, this class of superposition state can be defined for any two complete states, for instance \( \{ | n \rangle \} \Leftrightarrow \{ | i \rangle \} \), with the relations \( | n \rangle = \sum | i \rangle | i \rangle < i | n \rangle \) using the completeness relation \( I = \sum | i \rangle | i \rangle < i, \) vice versa. Obviously, these satisfy those conditions of the superposition state, and with eq.(84), it’s possible for states of any many-particle system, as long as they have some definite quantum numbers, that is they are physical states.

Now let’s see the state in eq.(50) again, and easily to see, it’s superposition of states. However, if it was also a superposition state, then what its quantum numbers are? Further, the coefficients are not fixed, especially the possible arbitrary phase difference \( \exp(i\delta) \) between them. Therefore, this state is not a physical one for single particle, just an ensemble state. The same things happen

\(^{14}\) With frame transformation \( X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \).
to the state in eq.(62) which describes a general statistical ensemble. Therefore, we can conclude that the ensemble state as in eqs.(62) and (64) are not superposition states, only superposition of states.

In one word, in our familiar examples, superposition state appears only in some cases like eqs.(81),(83) and (84). And the expressions like eq.(82) are only the mathematical relations between the corresponding states. When there are interactions, the coefficients of those expressions would obtain some arbitrary phase terms, then the condition of fixed coefficients is broken, and the superposition state will change into ensemble states. All these will be shown in the next subsection, where the interaction is the quantum measurement, and then which superposition principle (SQM’s or QFT’s) is much more proper will also be clear.

C. Quantum Measurement Theory

As is well known, there’s a so called quantum collapse in the quantum measurement theory of the SQM. The reason for this concept is the superposition of states for single particle. Suppose that the initial state of a particle is of the form eq.(79), then after a quantum measurement, the state will collapse into one of the states, \( n \) for example. And according to the SQM, the whole process will be instantaneous and irreversible. In fact, we have show below eq.(62) that, the states usually used in SQM are not physical for single particle but as single-particle ensemble state, and the so called ”collapse” happens only metaphysically or logically as in the probability theory, not real in nature. However, there are still some superposition states analyzed in the last subsection, as in eq.(81),(83) and (84). According to SQM, collapse happens still for them, but as we will show in this subsection, there is also a consistent quantum measurement theory for these superposition states, assuring that nothing unusual will happen.

Now, let’s consider a quantum measurement, the famous Stern-Gerlach experiment for measuring the spins of electrons. However, let’s first replace the non-uniform magnetic field with a uniform one\(^{15}\). Let the electrons with spin \(| \uparrow_z \rangle\) travel in this uniform magnetic field, obviously there’s no deflections. With the interaction \( B \cdot \hat{\mathbf{n}} \), \( \hat{\mathbf{n}} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \) for any direction,

\(^{15}\) In section V.A., we will propose an operable experiment to test which interpretation is more proper, the Copenhagen or ensemble interpretation.
the final state will be\(^{16}\)
\[
e^{-i\omega t} \cos \frac{\theta}{2} \uparrow_{n} > + e^{i\omega t} \sin \frac{\theta}{2} \downarrow_{n} > ,
\]
a superposition of states. Then, let another sample of electrons with spin \(\uparrow_{z} \rangle\) travel in a non-uniform magnetic field, i.e. the Stern-Gerlach apparatus. As is known, they will deflect into two directions, with definite spins \(\uparrow_{n} \rangle\) and \(\downarrow_{n} \rangle\) respectively. These two situations are similar physical processes, but according to the SQM, the conclusions for the state of a single electron are completely different.

In fact, the above two situations can be described in the unique way with the field theoretical languages. Here, the field expansion could be\(^{17}\)
\[
\psi(t, x) = \psi_{\uparrow z}(t, x) + \psi_{\downarrow z}(t, x) = \int \frac{d^{3} p}{(2\pi)^{3}} \left[ a_{p \uparrow z} u_{\uparrow z} + a_{p \downarrow z} u_{\downarrow z} \right] e^{i(p x - E p t)} ,
\]
with spinor representation
\[
u_{\uparrow z} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad u_{\downarrow z} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .
\]
And the interaction term is
\[
H_{I} = -\frac{e}{2m_{e}} \int d^{3} x \psi^{\dagger}(x) \vec{\sigma} \cdot B(x) \psi(x) ,
\]
then the evolution is
\[
\exp(-iH_{I}t) \uparrow_{z} > ,
\]
with eq.(85) specific for the uniform magnetic field\(^{18}\). Furthermore, with \(e^{i(p x - E p t)}\) substituted, eq.(85) can be rewritten as\(^{19}\)
\[
e^{i[p x -(E+\omega)t]} \cos \frac{\theta}{2} \uparrow_{n} > + e^{i[p x -(E-\omega)t]} \sin \frac{\theta}{2} \downarrow_{n} > ,
\]
\(^{16}\) \(\omega \equiv |e|B/2m_{e}\). We can see that, the relation between \(\uparrow_{z} >\) and \(\uparrow_{n} \rangle \downarrow_{n} \rangle\) is changed by the interaction with the phase terms \(e^{i\omega t}\) added in, which are discussed in the end of last subsection. Then, the state in eq.(85) is not a superposition state, although it was before the interaction.

\(^{17}\) The expansion (86) is only a non-relativistic form because of the constant spinor \(u\) in eq.(87). And if the particles are in states \(\uparrow_{n} \rangle \downarrow_{n} \rangle\), \(a_{\uparrow z} u_{\uparrow z} + a_{\downarrow z} u_{\downarrow z}\) is replaced with \(a_{\uparrow z} u_{\uparrow z} + a_{\downarrow z} u_{\downarrow z}\).

\(^{18}\) The general frequency is of the form \(\omega(z)\) with \(z\) the direction of the magnetic field, and when the magnetic field is uniform, the frequency will be a constant, as in eq.(85). Further, \(\omega(z)t \approx (z - z_{0})\partial \omega(z_{0})t \approx p_{z}(z - z_{0})\) i.e. the phase in time can be transformed into phase in space, representing the deflection in the \(z\) direction.

\(^{19}\) The exact expression is the eq.(215) in section V.A..
i.e. the superposition of states with energies $E + \omega$ and $E - \omega$ for the electrons.

There are two interpretations to eq.(85), one is the SQM version, assigning a state vector $|\phi(t)\rangle$ describing the evolution of the state around the sphere; the other one is the QFT version in eq.(90), in which the time phase terms are parts of the plane waves, $e^{i(p_2 - (E + \omega)t)}$. In the SQM, $|\uparrow_z\rangle$ collapses irreversibly into $|\uparrow_n\rangle$ or $|\downarrow_n\rangle$, with the probabilities $|\cos \frac{\theta}{2}|^2$ and $|\sin \frac{\theta}{2}|^2$ respectively. However, with the expression (82), under another measurement, $|\uparrow_n\rangle$ may collapses irreversibly back into $|\uparrow_z\rangle$ again. It appears that a combination of two irreversible processes could be reversible. In one word, these statements are a little obscure. However, with the field theoretical language, whatever the magnetic field is uniform or not, the descriptions and the conclusions are definite and unique. From eq.(85) or (90), and according to the subsection A., we can conclude that, among the sample of electrons with spin $|\uparrow_z\rangle$, $|\cos \frac{\theta}{2}|^2$ of them whose states will become $|\uparrow_n\rangle$, while the others will be $|\downarrow_n\rangle$. Then for one single electron, the probability for its state to become $|\uparrow_n\rangle$ is $|\cos \frac{\theta}{2}|^2$, which is just the conclusion of the SQM. Remind that we have assumed in subsection A. that the state for single particle is definite and unique, so the processes from $|\uparrow_z\rangle$ to $|\uparrow_n\rangle$, and $|\uparrow_n\rangle$ back to $|\uparrow_z\rangle$ are all about the state transitions which are unitary without any collapse. In fact, the essential reason is still that the state in eq.(85) or (90) is an ensemble state not for single particle. Therefore, we have the conclusion that the superposition state will change into ensemble states under the interactions (measurements).

Noting that the evolution in eq.(89) has a similar form as eq.(78), then one may say that this evolution is the SQM version. In fact, it is not! Recall the computations of cross sections in QFT, or the formula of S matrix\[2\]

\[
\text{out} < p_1, p_2 | k_1, k_2 >_{in} \equiv < p_1, p_2 | S | k_1, k_2 > = \lim_{T \to \infty} < p_1, p_2 | \exp(-iH2T) | k_1, k_2 >, \tag{91}
\]

then in our case, it is about a single particle

\[
\text{out} < \uparrow_n | \uparrow_z >_{in} = \lim_{T \to \infty} < \uparrow_n | \exp(-iH2T) | \uparrow_z >, \tag{92}
\]

similarly for $|\downarrow_n\rangle$. And easily to see, eq.(89) is just the right hand part of eq.(92), the results are still the transition amplitudes $<\uparrow_n | \uparrow_z>$ essentially, for the evolution in eq.(89) is of the form of phase factors as in eq.(85) or (90).

Which description is more proper is now clear, and we can extend the above discussions to all of the superposition states. As for other superposition of states, the field theoretical descriptions (or ensemble concepts) are already proper. According to the example above, we can give the following new quantum measurement assumption

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Quantum measurement is one kind of unitary field interactions. Under the measurement, the states of particles are unchange if they were just the eigenstates of the measured physical quantity already, or changed into some of the eigenstates if they were not before measurement. We could identify the states by some apparent different macroscopic behaviors, such as deflections in the above example. What we could obtain is just the probability of different processes, which is the task of QFT.\(^{20}\)

With the modifications above, quantum collapse is completely avoided for the superposition states. After a measurement, the original superposition state would change into the ensemble state due to the arbitrary phase terms resulting from interactions, as in eq.(85). And the so called collapses occur only metaphysically, not physically. In addition, the description is the unique (non-relativistic) QFT, which is a space-time local theory. Therefore, the so called non-locality in EPR paradox may also be avoided.

### D. EPR Paradox [7]

We consider the example advocated by Bohm and Aharonov [8]. Let a pair of spin one-half particles formed in the singlet spin state\(^{21}\)

\[
|\Psi >_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow >_{AB} - |\downarrow\uparrow >_{AB}),
\]

move freely in opposite directions. Assume that we make a measurement for the particle A, then according to the SQM, there will be quantum collapse, if A is found to be at |\uparrow >_A, the state of B will be collapsed into |\downarrow >_B. This collapse is instantaneous, so that we can construct two events in space-time, which are separated by a space-like interval, one is the measurement for A, the other is the one for B. Then the relativity causality and locality are violated.

Obviously, the violation of causality and locality is also owing to the quantum collapse, which, as we have described above, could be completely avoided in the field theoretical language. In fact, as we have analyzed in subsection B., the state in eq.(93) is an example of superposition states as in eq.(84), i.e. the coupling of angular moentua. In addition to the state in eq.(93), there are another

\(^{20}\)The changes of the states of the particles manifest themselves in the change of the field function, i.e.\(\delta \phi(x)\).

\(^{21}\)Here, |\uparrow > can be at any direction, because of the total spin is zero for |\Psi >_{AB}, so we can treat it as |\uparrow >_z. This is only for this spin singlet, not for other Bell states, for example, \(\frac{1}{\sqrt{2}}(|\uparrow\uparrow >_{AB} + |\downarrow\downarrow >_{AB}).\)
three, one of which is

$$|\Phi >_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow >_{AB} + |\downarrow\uparrow >_{AB}).$$  \hspace{1cm} (94)$$

With these two states, we can express $|\uparrow\downarrow >_{AB}$ as follows

$$|\uparrow\downarrow >_{AB} = \frac{1}{\sqrt{2}}(|\Phi >_{AB} + |\Psi >_{AB}).$$  \hspace{1cm} (95)$$

similarly for the other one. Then according to the quantum measurement in SQM, with a special quantum measurement, we could obtain the so called quantum entangled states, just like the case in the last subsection for the single spin states. These are also obscure, so we need the field theoretical language.

All the things are already studied in the last subsection. What we need are eqs.(88) and (89) for the measurements. First, we make a measurement for A, after that, the state in eq.(93) will become

$$\frac{1}{\sqrt{2}}(e^{-i\alpha t}|\uparrow_n\downarrow_n >_{AB} - e^{i\alpha t}|\downarrow_n\uparrow_n >_{AB}).$$  \hspace{1cm} (96)$$

Then for B, we have

$$\frac{1}{\sqrt{2}}(e^{-i\beta t}e^{i\alpha t}'|\uparrow_n\downarrow_n >_{AB} - e^{i\alpha t}'e^{-i\beta t}'|\downarrow_n\uparrow_n >_{AB}).$$  \hspace{1cm} (97)$$

Of course, we have assumed that the directions of the magnetic field are the same for both the measurements, and for different directions, the expression will be complicated.

From eqs.(96) and (97), we can claim that:

(1) The total spin is not conserved during the measurements, which is easily to understand, because the interactions are spin dependent, there are angular momentum exchanges between the particles and the magnetic field (or the photons). Only if the directions of the magnetic field were the same for both the measurements, the spin in that direction would be conserved.

(2) The measurements for A and B are independent, because they are field interactions, so we cannot construct two events which violate the relativity causality and locality.

(3) According to our ideas, the states in eqs.(96) and (97) are not superposition states because of the arbitrary phase terms, and from eq.(97), we could obtain that, the probability for the transition from $|\Psi >_{AB}$ to $|\uparrow_n\downarrow_n >_{AB}$ is one-half.

---

22 Notice that eqs.(96) and (97) are of the forms of eq.(64) for two particles within two states.
Here is one important note about eq.(96), from which one may say that, the state of B is changed
instantaneously. In fact, the state of B is still the initial one $|\uparrow_B\rangle$ or $|\downarrow_B\rangle$, the expression for
eq.(96) is just for convenient mathematically. In fact, with the following representation of the state
in eq.(93)

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow_z\downarrow_z\rangle_{AB} - |\downarrow_z\uparrow_z\rangle_{AB}), \quad (98)$$

the original eq.(96) will be

$$\frac{1}{\sqrt{2}}[e^{-i\omega t}|\uparrow_n\rangle_A (e^{-i\hat{z}} \sin \frac{\theta}{2} |\uparrow_z\rangle_B - e^{i\hat{z}} \cos \frac{\theta}{2} |\downarrow_z\rangle_B)$$

$$- e^{i\omega t}|\downarrow_n\rangle_A (e^{i\hat{z}} \cos \frac{\theta}{2} |\uparrow_z\rangle_B + e^{-i\hat{z}} \sin \frac{\theta}{2} |\downarrow_z\rangle_B)]. \quad (99)$$

Although the parts for B can be rewritten compactly as in eq.(96), we could obtain the physical
results easily and consistently from eq.(99), for instance, with a measurement only for A, we
should sum over all the possibilities about B. For example, the probability for A to be at state
$|\uparrow_n\rangle_A$ is

$$P_{|\uparrow_n\rangle_A} + P_{|\downarrow_n\rangle_A} = \frac{1}{2} \times (|\cos \frac{\theta}{2}|^2 + |\sin \frac{\theta}{2}|^2) = \frac{1}{2}, \quad (100)$$

the same for the other one. If the magnetic field for measuring B is different from that of A, eq.(99)
will be a good starting point.

All the above descriptions can be written in more field theoretical forms. For example, the state
in eq.(93) is

$$\Psi_{AB}(x_1, x_2) = \psi_\uparrow(x_1)_A \psi_\downarrow(x_2)_B - \psi_\downarrow(x_1)_A \psi_\uparrow(x_2)_B, \quad (101)$$

and the variation due to the measurement for A is (first order)

$$\delta \Psi_{AB}(x_1, x_2) = \delta \psi_\uparrow(x_1)_A \psi_\downarrow(x_2)_B - \delta \psi_\downarrow(x_1)_A \psi_\uparrow(x_2)_B, \quad (102)$$

or

$$\delta \Psi_{AB} = iT[\frac{-e}{2m_e} \int d^3 y B(y) \psi_\uparrow^\dagger_A (\sigma_A \cdot \hat{n}) \psi_A(y), \Psi_{AB}]. \quad (103)$$

In Bell’s paper [9], there is a correlation function

$$P(a, b) =_{AB} < \Psi|(\sigma_A \cdot a)(\sigma_B \cdot b)|\Psi >_{AB}, \quad (104)$$
with eq.(49), the corresponding field theoretical expression is

\[
\begin{align*}
&\text{AB} < \Psi | \int d^3x_1 \psi_A^\dagger(\sigma_A \cdot a)\psi_A(x_1) \int d^3x_2 \psi_B^\dagger(\sigma_B \cdot b)\psi_B(x_2) | \Psi >_{AB} \\
&\text{AB} < \Psi | \int d^3x_1 \psi_A^\dagger \psi_A(x_1) \int d^3x_2 \psi_B^\dagger \psi_B(x_2) | \Psi >_{AB}
\end{align*}
\]

or the one similar to eq.(59).23

There is still a problem in the analysis above, the field function in eq.(101) is not identical to the following expression

\[
\psi_{\uparrow n}(x_1)A \psi_{\downarrow n}(x_2)B - \psi_{\downarrow n}(x_1)A \psi_{\uparrow n}(x_2)B.
\]

The difference between them can be canceled in an artificial way, and the exact expression should be relativistic. After all, the EPR paradox is solved, it’s just a misunderstanding.

Furthermore, with our ideas, the so called quantum entanglement is also not real in nature, and among the four Bell states in SQM, eqs.(93) and (94) are physical states, while the other two

\[
\frac{1}{\sqrt{2}}(|\uparrow\uparrow>_{AB} \pm |\downarrow\downarrow>_{AB}),
\]

are only superposition of states $|\uparrow\uparrow>_{AB}$ and $|\downarrow\downarrow>_{AB}$. And the example shown in the original EPR paper can also be solved since it just involves the transformations between the lab frame and center-of-mass frame, as described below eq.(83) in subsection B..

E. Double-slit Interference Experiment

In QM, the most famous experiment must be the double-slit interference experiment which is believed to contain the essential features of QM. In this subsection, we will study this experiment in details by using the concepts described previously, and obtain the required particle number distribution. The experiment is sketched in Fig.1. From the original point $O$, there will be a beam of particles moving to the double-slit screen, if the two slits are both open, then we will obtain interference fringes on the receiving screen. However, if we control the slits so that they are open not at the same time, then the interference fringes would disappear.

There is a rough QM description with the use of the wave properties of quantized particles as follows. For an arbitrary point $x$ on the receiving screen, there will be two waves $\psi_1(x)$ and $\psi_2(x)$

---

23 In other words, the particle pair should be considered as one system, just like the single-particle case with states $|\uparrow_n\downarrow_n>_{AB}$, $|\downarrow_n\uparrow_n>_{AB}$, etc.
coming from the two slits respectively, then the total wave will be

$$\psi(x) = \psi_1(x) + \psi_2(x),$$ \hspace{1cm} (108)

and according to QM, we should compute the probability $|\psi(x)|^2$, then there will be crossing interference terms. In fact, the interference can also be considered to be from the path difference as sketched in Fig.1, from the point of view of pure classical waves, such as the light waves.

Now, let’s give a field theoretical description. Eq.(108) is still proper, with the wave functions interpreted as fields. The path difference in wave theory is actually phase difference which can be resulting from the interactions of the particles with the double-slit screen. The interaction can be considered to be elastic collision, and under the interaction, the energies of the particles are unchange while the momenta are changed. We can describe this process with the following form in first order

$$\psi(x, x_0) = \int d^4y \int d^4z K(x, x_0; y, y_0) V(y) K(y, y_0; z, z_0) \psi(z, z_0),$$ \hspace{1cm} (109)

with the propagator defined as in eq.(34)

$$K(x, x_0; y, y_0) = \langle 0 | \psi(x, x_0) \psi^\dagger(y, y_0) | 0 \rangle.$$ \hspace{1cm} (110)

We assume the following interactions\(^{24}\)

$$V(y) = \delta^3(y - x_1) + \delta^3(y - x_2),$$ \hspace{1cm} (111)

\(^{24}\) Notice that if we had infinite slits on the screen, then the interactions would be $\delta^3(y - x_1) + \delta^3(y - x_2) + \cdots = \int d^3x \delta^3(y - x) = 1$, and eq.(109) would be just the combination of two propagators.
and after simple computations, we will obtain the dependence of the field function on the $x_1$ and $x_2$. In fact, we can obtain them in a much simpler way, note that the two slits are actually two sources as in eq.(111), and the field equation will be that of propagator with the source terms. In one word, we can use the propagator in eq.(110) as a basis. Since the propagator is for the free particle, the field function in the interval $[x, x + dx]$ will be

$$
\psi(x) = a_{p_1} e^{ip_1 (x-x_1)} + a_{p_2} e^{ip_2 (x-x_2)},
$$

(112)

then the particle number density $N(x) = \psi^\dagger(x) \psi(x)$ is

$$
N(x) = a_{p_1}^\dagger a_{p_1} + a_{p_2}^\dagger a_{p_2} + e^{i\alpha(x,x_1,x_2)} a_{p_1}^\dagger a_{p_2} + e^{-i\alpha(x,x_1,x_2)} a_{p_2}^\dagger a_{p_1},
$$

(113)

where we have collected the phase terms in a compact form, and easily to see they will cause the interference.

The next task is to find out the state of ensemble for the two momentum states, for example the Bosonic case in eq.(64) with $k = 2$

$$
\sum_{j=0}^{n} \sqrt{\frac{P_j}{j!(n-j)!}} (a_{p_1}^\dagger)^j (a_{p_2}^\dagger)^{n-j} |0>,
$$

(114)

where the arbitrary phase terms are already absorbed into eq.(113). We need the mean value of particle number density $< N(x) >$ with the state in eq.(114) substituted. For the diagonal term, the result is just $n$, while for the off-diagonal terms we will have

$$
\sum_{j=0}^{n} 2 \sqrt{P_j P_{j+1}} \sqrt{(j+1)(n-j)} \cos \alpha.
$$

(115)

For simplicity, we assume that the probability is

$$
P_j = \left(\frac{1}{2}\right)^n \frac{n!}{j!(n-j)!},
$$

(116)

which is related to the binomial coefficients, and substituting it into eq.(115), we then have

$$
n \cos \alpha,
$$

(117)

which is the interference term! Therefore, the total particle number distribution is

$$
V < N(x) >= n (1 + \cos \alpha(x,x_1,x_2)),
$$

(118)

---

25 The momenta of the particles will roughly be constant in this case.
with $V$ the space volume, the normalization of plane wave which is ignored for convenient previously. Obviously, eq.(118) is also proper for the single particle ensemble state, for instance, $(|p_1 > + |p_2 >)/\sqrt{2}$, with single particle in the whole space, i.e. $n = 1$. In this sense, we could also obtain the above special result in a simpler way, by noting that the above $n$-particle ensemble with probabilities in eq.(116) is in fact made up of single particle ensemble as noted in the end of subsection A.. Now, we rewrite the filed in the ordinary form

$$\psi(x) = a_{p_1}e^{ip_{1}x} + a_{p_2}e^{ip_{2}x} + \cdots, \quad (119)$$

and in order to obtain the exact interference term, we should have the following single particle ensemble state

$$| > = \frac{1}{\sqrt{2}}(e^{-ip_{1}x_{1}}|p_{1} > + e^{-ip_{2}x_{2}}|p_{2} >), \quad (120)$$

with the respective phase terms added. Then the $<N(x)>$ is

$$V <N(x)> = \frac{1}{2}(2 + e^{i\alpha(x,x_{1},x_{2})} + e^{-i\alpha(x,x_{1},x_{2})}) = 1 + \cos \alpha(x,x_{1},x_{2}), \quad (121)$$

which is just the $n = 1$ case of eq.(118)! To obtain the general formula eq.(118), we construct the $n$-particle ensemble out of the state in eq.(120), obtaining the state in eq.(114) with the phase terms already absorbed into eq.(113), and the probability condition in eq.(116). Now, if we control the slits so that they are open not at the same time, so that the source of each particle at the interval $[x, x + dx]$ are definite, in other words, the distribution $[n_{j}]$ in eq.(64) is determined in this case, then the state may be, for example

$$\frac{1}{\sqrt{(n_{1})!(n_{2})!}}(a_{p_{1}}^{+})^{n_{1}}(a_{p_{2}}^{+})^{n_{2}}|0>, \quad n_{1} + n_{2} = n, \quad (122)$$

i.e. a measurement which causes a collapse of the ensemble state in eq.(114), then the off-diagonal terms in $<N(x)>$ disappear, so do the interferences.

Notice that, this field theoretical description includes the QM description in eq.(108), by using the wave functions defined in eq.(77) with the single particle ensemble state in eq.(120). Even it can include the classical field case with the coherent state defined in eqs.(228) and (238).

---

26 This is also for the Fermionic case with the Pauli exclusion principle, ignoring the spins.

27 Notice that $<p_{1}|e^{iP_{1}x}\psi_{1}^{*}(x)\psi(x)e^{-iP_{1}x_{1}}|p_{1} >= <p_{1}|e^{iP_{1}x}\psi_{1}^{*}(x)\psi(x)e^{-iP_{1}x_{1}}|p_{1} >= <p_{1}|\psi_{1}(x_{1})\psi(x - x_{1})p_{1} >$. 

28
IV. RELATIVISTIC EXTENSIONS

In section II., we have studied in details the non-relativistic (quantum) Schrödinger field theory which can be considered to be much more fundamental than the QM, for all the QM can be derived from this field theory. With this new approach to QM, modifications to the SQM is developed in section III., where the ensemble interpretation is realized by treating most of the states in QM as ensemble states, such as the ones in eqs.(62) and (64), while the rest as a class of superposition state as shown in eqs.(81)(83) and (84). In this section, we will extend these concepts to the relativistic QFT, indicating that fields are the fundamental elements of the physical world and QFT is the unique consistent theory by now.

As demonstrated in section II., the most important elements are the ordinary QM physical operators (about particles) made up of fields, such as the operators in eqs.(5)-(8), and the corresponding eigenstates. Since the energy and momentum operators can be obtained from transformations of the action under the space-time translation, what we need are only the rest two, the particle number and position operators. Notice from eqs.(7) and (8) that, the position operator may be considered to be followed by substituting a space coordinate into the formula of particle number operator, so the only thing we need is to find out the general rule for the particle number operator. Easily to see, there is a gauge symmetry of the action (1) with the fields transform as, in quantized form

$$\psi \rightarrow e^{-i\alpha} \psi, \quad \psi^\dagger \rightarrow e^{i\alpha} \psi^\dagger,$$

then there is a physical quantity corresponding to this symmetry, just like the ordinary $U(1)$ gauge, and obviously this physical quantity is the particle number!

However, as is well known, in the ordinary relativistic QFT formula, it seems to be impossible to impose this symmetry, for the particle and anti-particle fields are written in some special combined form, for example a free charged scalar field

$$\phi(x) = \phi_1(x) + \phi_2(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a(p)e^{ipx} + b^\dagger(p)e^{-ipx}),$$

(124)

to guarantee the communicative relation $[\phi(x), \phi^\dagger(y)] = 0$. But if we treat the fields $\phi_1(x)$ and $\phi_2(x)$ as independent, we could construct another combined field in the following form

$$\phi'(x) = i(\phi_1(x) - \phi_2(x)),$$

(125)

28 In this section and below, we use the bold face letters to denote the vector form of the three space dimensional coordinates and momenta.
which obviously also satisfies the corresponding communicative relation \( [\phi'(x), \phi'^*(y)] = 0 \). Now, we write down the field action (in quantized form)

\[
S = \int d^4x (\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi),
\]

(126)

then substituting the two fields in eqs. (124) and (125) into it, we have

\[
S_\phi = \int d^4x [(\partial_\mu \phi_1^\dagger \partial^\mu \phi_1 - m^2 \phi_1^\dagger \phi_1) + (\partial_\mu \phi_2^\dagger \partial^\mu \phi_2^\dagger - m^2 \phi_2^\dagger \phi_2^\dagger ) + (\partial_\mu \phi_1^\dagger \partial^\mu \phi_2^\dagger - m^2 \phi_1^\dagger \phi_2^\dagger ) + (\partial_\mu \phi_2 \partial^\mu \phi_1 - m^2 \phi_2 \phi_1 )],
\]

(127)

and

\[
S'_\phi = \int d^4x [(\partial_\mu \phi_1^\dagger \partial^\mu \phi_1 - m^2 \phi_1^\dagger \phi_1) + (\partial_\mu \phi_2^\dagger \partial^\mu \phi_2^\dagger - m^2 \phi_2^\dagger \phi_2^\dagger ) - (\partial_\mu \phi_1^\dagger \partial^\mu \phi_2^\dagger - m^2 \phi_1^\dagger \phi_2^\dagger ) - (\partial_\mu \phi_2 \partial^\mu \phi_1 - m^2 \phi_2 \phi_1 )].
\]

(128)

Easily to see, the combination \( \frac{1}{2}(S_\phi + S'_\phi) \) is what we need

\[
S' = \int d^4x [(\partial_\mu \phi_1^\dagger \partial^\mu \phi_1 - m^2 \phi_1^\dagger \phi_1) + (\partial_\mu \phi_2^\dagger \partial^\mu \phi_2^\dagger - m^2 \phi_2^\dagger \phi_2^\dagger )],
\]

(129)

from which the fields \( \phi_1 \) and \( \phi_2 \) are independent from each other, thus we could consider separately the particle field and anti-particle field. Let’s consider the particle field \( \phi_1 \) and its action \( S_\phi_1 \), the canonical momenta are\(^{29}\)

\[
\pi_{\phi_1} = \phi_1^\dagger , \quad \pi_{\phi_1'} = \phi_1,
\]

(130)

then the Hamiltonian density is

\[
\mathcal{H}_{\phi_1} = \pi_{\phi_1} \phi_1 + \pi_{\phi_1'} \phi_1^\dagger - \mathcal{L}_{\phi_1} = \pi_{\phi_1} \phi_{\phi_1} + \phi_{\phi_1}^\dagger \phi_{\phi_1} + m^2 \phi_{\phi_1} \phi_{\phi_1},
\]

(131)

after substituting the expansion of \( \phi_1 \) in eq. (124), we will have the energy of the field

\[
H_{\phi_1} = \int d^3x \mathcal{H}_{\phi_1} = \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{2} (a(p)a(p)^\dagger + a(p)^\dagger a(p)),
\]

(132)

similarly, the momentum of the field is

\[
P_{\phi_1} = - \int d^3x (\pi_{\phi_1} \nabla \phi_1 + \pi_{\phi_1'} \nabla \phi_1^\dagger) = \int \frac{d^3p}{(2\pi)^3} \frac{p}{2} (a(p)a(p)^\dagger + a(p)^\dagger a(p)).
\]

(133)

\(^{29}\) The communicative relations among the four quantities, \( \phi_1, \phi_1^\dagger, \pi_{\phi_1}, \) and \( \pi_{\phi_1'} \) can be computed by using the expansion in eq. (124), which are different from the relations among \( \phi, \phi^\dagger, \pi_{\phi}, \) and \( \pi_{\phi'} \).
Since the action $S_{\phi_1}$ has a similar form as the action in eq.(1), there is also a gauge transformation of $\phi_1$

$$\phi_1 \rightarrow e^{-i\alpha} \phi_1, \phi_1^\dagger \rightarrow e^{i\alpha} \phi_1^\dagger,$$

(134)

from which we obtain a physical quantity, i.e. the particle number

$$N_{\phi_1} = -i \int d^3x (\pi_{\phi_1} \phi_1 - \pi_{\phi_1^\dagger} \phi_1^\dagger) = \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} (a(p)a^\dagger(p) + a^\dagger(p)a(p)).$$

(135)

Thus, a possible position operator can be defined as

$$X_{\phi_1} = -i \int d^3x (\pi_{\phi_1} \phi_1 - \pi_{\phi_1^\dagger} \phi_1^\dagger),$$

(136)

and after a simple computation we have

$$X_{\phi_1} = i \int \frac{d^3p}{(2\pi)^3} \left( \frac{1}{2} a^\dagger(p) \partial_p a(p) + \frac{1}{2} \partial_p a(p) a^\dagger(p) \right) a(p) a^\dagger(p) \sqrt{E_p \frac{1}{\sqrt{2E_p}}} \partial_p \left( \frac{1}{\sqrt{2E_p}} \frac{1}{\sqrt{E_p}} \right),$$

(137)

where the last two terms will vanish, using the communicative relation $[a(p), a^\dagger(p)] = (2\pi)^3 \delta^3(0)$ and $\int d^3p F(p) = 0(F(p) = F(-p))$. Then the position operator become

$$X_{\phi_1} = i \int \frac{d^3p}{(2\pi)^3} a^\dagger(p) \partial_p a(p),$$

(138)

which is just the eq.(17)! Up to the orders of the creators and annihilators, we have obtained all the required physical operators for particles.

The next task is to find out the eigenstates of these operators as in section II. The momentum state is already defined well in QFT[2]

$$|p> = \sqrt{2E_p} a^\dagger(p)|0>,$$

(139)

with normalization

$$<p|q> = 2E_p(2\pi)^3 \delta^3(p - q).$$

(140)

Noting that this definition has a good Lorentz transformation property, for $E_p \delta^3(p - q)$ is Lorentz invariant. However, it seems to be impossible for this property to be imposed on space-time[^30], that is we can only have the following normalization

$$<x|y> = \delta^3(x - y).$$

(141)

[^30]: For a real particle, its energy and momentum satisfy the condition $E_p^2 = p^2 + m^2$, but there is not a general relation between space and time coordinates of the particles.
One may simply thought that $\phi_1^\dagger(x)|0>$ be the required position eigenstate, but the factor $1/\sqrt{2E_p}$ in the field expansion would make the problems more complicated. Recalling the state in eq.(20) and the filed expansion in eq.(14), we can define a new "field"

$$\psi_{\phi_1}(x) = \int \frac{d^3p}{(2\pi)^3} a(p)e^{ipx},$$

(142)

with the communicative relation

$$[\psi_{\phi_1}(x), \psi_{\phi_1}^\dagger(y)] = \delta^3(x-y),$$

(143)

then the position eigenstate $|x>$ can be defined as

$$|x> \equiv \psi_{\phi_1}^\dagger(x)|0>,$$

(144)

with the required normalization in eq.(141) by using eq.(143).

Here, let's have a look at the "field" $\psi_{\phi_1}(x)$ defined in eq.(142), obviously it is not a well defined filed, because its Lorentz transformation is obscure. But it's indeed useful for us, with it, we could redefine all the above physical operators as

$$H_{\phi_1} = \int d^3x \psi_{\phi_1}^\dagger(x)\hat{H}\psi_{\phi_1}(x) = \int \frac{d^3p}{(2\pi)^3} E_p a^\dagger(p)a(p),$$

(145)

$$P_{\phi_1} = \int d^3x \psi_{\phi_1}^\dagger(x)\hat{p}\psi_{\phi_1}(x) = \int \frac{d^3p}{(2\pi)^3} pa^\dagger(p)a(p),$$

(146)

$$N_{\phi_1} = \int d^3x \psi_{\phi_1}^\dagger(x)\psi_{\phi_1}(x) = \int \frac{d^3p}{(2\pi)^3} a^\dagger(p)a(p),$$

(147)

$$X_{\phi_1} = \int d^3x \psi_{\phi_1}^\dagger(x)\hat{x}\psi_{\phi_1}(x) = i \int \frac{d^3p}{(2\pi)^3} a^\dagger(p)\partial_p a(p),$$

(148)

with the single particle operators defined as

$$\hat{H} = \sqrt{\hat{p}^2 + m^2}, \quad \hat{p} = -i\nabla, \quad \hat{x} = x,$$

(149)

which are the familiar operators of QM in the relativistic forms.

Now, let's study the Lorentz transformation $\Lambda$, which will be implemented as some unitary operator $U(\Lambda)$, and for the momentum state in eq.(139), we have[2]

$$U(\Lambda)|p> = |\Lambda p>, \quad U(\Lambda)a^\dagger(p)U^{-1}(\Lambda) = \sqrt{\frac{E_{\Lambda p}}{E_p}}a^\dagger(\Lambda p).$$

(150)

(151)
Considering a boost in the 3-direction $p_3' = \gamma(p_3 + \beta E)$, $E' = \gamma(E + \beta p_3)$, the operators in eqs.(145)-(148) should have the ordinary transformation properties, noting that $\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p}$ is Lorentz invariant, for example, the energy and momentum operators transform as

$$UHU^{-1} = \gamma(H - \beta P_3), \quad UP_3U^{-1} = \gamma(P_3 - \beta H), \quad (152)$$

while the particle number operator is Lorentz invariant. The last one is the transformation of the position operator $UX_3U^{-1}$, where the eq.(151) makes the problem complicated. To solve it, we introduce operators $\hat{a}^\dagger$ and $\hat{a}$

$$\hat{a}^\dagger(p) = \sqrt{2E_p} a^\dagger(p), \quad \hat{a}(p) = \sqrt{2E_p} a(p). \quad (153)$$

with the communicative relation

$$[\hat{a}(p), \hat{a}^\dagger(q)] = 2E_p(2\pi)^3 \delta^3(p - q), \quad (154)$$

then

$$X = \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} [a^\dagger(p) \partial_p a(p) - \partial_p a^\dagger(p) a(p)]$$

$$= \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} [\hat{a}^\dagger(p) \partial_p \hat{a}(p) - \partial_p \hat{a}^\dagger(p) \hat{a}(p)]. \quad (155)$$

Thus we have

$$UX_3U^{-1} = \frac{i}{2} \int \frac{d^3p'}{(2\pi)^3} \frac{1}{2E_{p'}} [\hat{a}^\dagger(p') \gamma(\partial_{p_3'} + \beta \partial_E) \hat{a}(p')$$

$$- \gamma(\partial_{p_3'} + \beta \partial_E) \hat{a}^\dagger(p') \hat{a}(p')] = \gamma(X_3 - \beta T), \quad (156)$$

where we have use the transformation property

$$U(\Lambda) \hat{a}^\dagger(p) U^{-1}(\Lambda) = \hat{a}^\dagger(\Lambda p), \quad (157)$$

and defined a time operator

$$T = -\frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} [\hat{a}^\dagger(p) \partial_E \hat{a}(p) - \partial_E \hat{a}^\dagger(p) \hat{a}(p)]$$

$$= -\frac{i}{2} \int \frac{d^3p}{(2\pi)^3} [a^\dagger(p) \partial_E a(p) - \partial_E a^\dagger(p) a(p)], \quad (158)$$

which transforms as

$$UTU^{-1} = -\frac{i}{2} \int \frac{d^3p'}{(2\pi)^3} \frac{1}{2E_{p'}} [\hat{a}^\dagger(p') \gamma(\partial_{p_3'} + \beta \partial_E) \hat{a}(p')$$

$$- \gamma(\partial_{p_3'} + \beta \partial_E) \hat{a}^\dagger(p') \hat{a}(p')] = \gamma(T - \beta X_3), \quad (159)$$
consistent with the above transformations, in other words, \( P_\mu = (H, -P) \) and \( X^\mu = (T, X) \) are two 4-vector operators. Further, we have the following communicative relation between \( H \) and \( T \)

\[
[T, H] = -i \int \frac{d^3p}{(2\pi)^3} a^\dagger(p)a(p) = -iN,
\]

which combined with eq.(13) makes up the diagonal terms \(-i\eta^{\mu\nu}N\) of the communicative relation \([X^\mu, P^\nu]\), leaving the terms \([X, H]\) and \([T, P]\).

Now, we have to find out the eigenstate of time operator \( T \), and after a few tedious calculations, we have

\[
[X, T] = \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} [a^\dagger(p)[\partial_p, \partial_{E_p}]a(p) - [\partial_p, \partial_{E_p}]a^\dagger(p)a(p)],
\]

and since

\[
[\partial_p, \partial_{E_p}] = \sum_j \partial_p(\frac{\partial}{\partial E_p})\partial_{p_j},
\]

thus the position and time operators have different eigenstates. To find it, let’s propose the following form

\[
|t> = \Phi^\dagger(t, x = 0)|0> = \int \frac{d^3p}{(2\pi)^3} \phi(p)e^{iE_p't}a^\dagger(p)|0>,
\]

with an undetermined factor \( \phi(p) \), and this state should satisfy \( T|t> = t|t> \), or the communicative relation

\[
[T, \Phi^\dagger(t, x = 0)] = t\Phi^\dagger(t, x = 0),
\]

from which we get a differential equation about \( \phi(p) \)

\[
2\partial_{E_p}\phi(p) + \sum_j \partial_p(\frac{\partial}{\partial E_p})\phi(p) = 0,
\]

with a special solution

\[
\phi(p) = \frac{1}{2} \sqrt{\frac{1}{E_p \sqrt{E_p^2 - m^2}}},
\]

which is completely different from the “field” in eq.(142).

Though we have defined a time operator and find out its eigenstate, its physical meaning is still obscure, thus let’s focus on the \( X \) and its eigenstate \(|x>\), and consider the transition amplitude for free particles as in eq.(25)

\[
<x_2, t_2|x_1, t_1> = <x_2|e^{-iH(t_2-t_1)}|x_1> = <0|\psi_{\phi_1}(x_2)e^{-iH(t_2-t_1)}\psi_{\phi_1}(x_1)|0>,
\]
by using eqs. (142), (143) and (145), we have further
\[
\langle x_2, t_2 | x_1, t_1 \rangle = \langle x_2 | e^{-i\hat{H}(t_2-t_1)} | x_1 \rangle = \int \frac{d^3p}{(2\pi)^3} e^{i[p(x_2-x_1)-E_p(t_2-t_1)]},
\] (168)
where we have used a re-defined momentum eigenstate
\[
|p\rangle = a_\dagger(p)|0\rangle = \frac{1}{\sqrt{2E_p}}|p\rangle, \quad (\hat{p}|p\rangle = p|p\rangle),
\] (169)
and the following single particle completeness relation
\[
I = \int \frac{d^3p}{(2\pi)^3} |p\rangle <\langle p| = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} |p\rangle <\langle p|.
\] (170)
The integration in eq. (168) is a little complicated compared to eq. (35), thus we make some approximations by expanding the phase term \(\chi = i[p(x_2-x_1) - E_p(t_2-t_1)]\) near its extreme point \(\partial_p \chi = 0\) which gives
\[
x_2 - x_1 = \frac{\partial E_p}{\partial p}(t_2 - t_1) = \frac{p}{E_p}(t_2 - t_1) = v(t_2 - t_1) = v\Delta t.
\] (171)
Then we have
\[
\langle x_2, t_2 | x_1, t_1 \rangle \propto \exp(-im\sqrt{1 - v^2}\Delta t) \rightarrow \exp(-im \int_{t_1}^{t_2} d\tau),
\] (172)
which is similar to eq. (36).

Here, let’s make some discussions about the two fields \(\phi_1(x)\) and \(\psi_{\phi_1}(x)\). Obviously, \(\phi_1(x)\) is a real field which is a scalar representation of the Lorentz group, while \(\psi_{\phi_1}(x)\) is ill defined. However, as studies above, \(\psi_{\phi_1}(x)\) is useful for describing the physics of particles, for example the eqs. (145)-(148). In other words, it’s much more like the non-relativistic Schrödinger field as analyzed in section II., so it can also be used to construct the QM for particles by noting that \(\psi_{\phi_1}(x)\) can be inserted into eq. (48) directly, thus the ensemble interpretation to QM is still proper in the relativistic case. However, when treating some physical processes with interactions added in, we should use the real field \(\phi_1(x)\), that is the QFT combining with the anti-particle field \(\phi_2(x)\) for which the above analyses are still applicable.

Now, let’s consider some other kinds of fields, for example the Dirac spinor field \(\psi_a(x)\) and the electromagnetic field \(A_\mu(x)\). And as we will show, for both of these two fields, there are some problems with the position operator. First, let’s see the free Dirac field with action
\[
S = \int d^4x \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi,
\] (173)
with the field expansion[2]

\[
\psi(x) = \psi_1(x) + \psi_2^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a(p, s)u(p, s)e^{ipx} + b(p, s)v(p, s)e^{-ipx})
\]

\[
\psi^\dagger(x) = \psi_1^+(x) + \psi_2(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a^+(p, s)u^+(p, s)e^{-ipx} + b(p, s)v^+(p, s)e^{ipx}).
\]

As in eq.(125), we can define another field and its conjugate

\[
\psi'(x) = i(\psi_1(x) - \psi_2^+(x))
\]

\[
\psi'^\dagger(x) = -i(\psi_1^+(x) - \psi_2(x)),
\]

and we have similarly

\[
S' = \frac{1}{2}(S_{\psi} + S_{\psi'}) = \int d^3x[\bar{\psi}_1^0(i\gamma^\mu \partial_\mu - m)\psi_1 + \psi_2\gamma^0(i\gamma^\mu \partial_\mu - m)\psi_2^+],
\]

that is, we separate the electron field from the positron field, and up to the orders of the field operators, the two fields should have the same structure, just like the case of the previous charged scalar fields. Thus, it's also possible to define the particle number and position operators

\[
N_{\psi_1} = \int d^3x\bar{\psi}_1^+(x)\psi_1(x) = \int \frac{d^3p}{(2\pi)^3} \sum_s a^+(p, s)a(p, s)
\]

\[
X_{\psi_1} = \int d^3x\bar{\psi}_1^+(x)x\psi_1(x),
\]

where we have used the normalization[2]

\[
u^+(p, r)u(p, s) = 2E_p\delta^{rs}, \quad u(p, s) = \left(\frac{\sqrt{P \cdot \sigma E^s}}{\sqrt{P \cdot \sigma E^s}}\right), (\xi^{\dagger\dagger}\xi^s = \delta^{rs}).
\]

As for the position operator, after some computations we will have

\[
X_{\psi_1} = \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \sum_s [a^+(p, s)\partial_p a(p, s) - \partial_p a^+(p, s)a(p, s)]
\]

\[
+ \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \sum_{sr} \frac{1}{2E_p} a^+(p, s)a(p, r)[u^+(p, s)\partial_p u(p, r) - \partial_p u^+(p, s)u(p, r)],
\]

where the first term is the familiar operator, while the second term can be simplified in the following way. We rewrite the spinor in eq.(181) in a new form which is easily to compute[10]

\[
u(p, s) = \frac{p_\mu \gamma^\mu + m}{\sqrt{E_p + m}}u(0, s), \quad u(0, s) = \frac{1}{\sqrt{2}}(\xi^s),
\]
then after tedious computations, the second term in eq.(182) will become
\[
\frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \sum_{sr} \frac{1}{2E_p} a^\dagger(p, s) a(p, r) K^{sr}_i, \quad K^{sr}_i = 2\xi_i^{\dagger s} \frac{i\epsilon_{ijk} p_j \sigma_k}{\sqrt{E_p + m}} \xi^r
\]
with the index \( i \) in \( K^{sr}_i \) denoted the \( \partial_p \) term. Though eq.(184) is not vanishing, it is commuting with the energy and momentum operators, then the velocity operator and the uncertainty relation in eq.(13) are still well defined. However, since \( \{\psi_1(x)_a, \psi_1^\dagger(y)_b\} \neq \delta_{ab} \delta^3(x - y) \), thus the position state can not be constructed from this field, either. Therefore, we have to define a new "field" as
\[
\psi(x)_a = \int \frac{d^3p}{(2\pi)^3} \sum_s a(p, s) \xi^s e^{i p x},
\]
with the communicative relation
\[
\{\psi(x)_a, \psi^\dagger(y)_b\} = \delta_{ab} \delta^3(x - y), \quad (\sum_s \xi^s \xi^{\dagger s} = I).
\]

Then, we could define the position operator and its eigenstate as
\[
X_{\psi_1} = \int d^3x \psi^\dagger_1(x)x \psi_1(x), \quad |x>_a = \psi^\dagger_1(x)_a |0>,
\]
and the previous discussions for the scalar field apply here, too. And with this ill-defined "field", we could obtain operators as those in eqs.(145)-(149).

Now, let’s consider the free electromagnetic field with action in the vector form[10]
\[
S = \frac{1}{2} \int d^4x (E^2 - B^2),
\]
and with the Coulomb gauge \( \nabla \cdot A = 0 \), we could work completely with the following transverse field expansions[10]
\[
A(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=1}^2 \epsilon(p, s) (a(p, s) e^{ipx} + a^\dagger(p, s) e^{-ipx})
\]
\[
E(x) = -\dot{A}(x),
\]
with only two transverse components. Though the photon can be considered as either particle or anti-particle, we could still separate the field and the action formally, and the particle number operator is defined as
\[
N_A = -\frac{i}{2} \int d^3x [\dot{A}^\dagger(x) \cdot A(x) - A(x) \cdot A^\dagger(x)]
\]
\[
= \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \sum_{s=1}^2 (a(p, s) a^\dagger(p, s) + a^\dagger(p, s) a(p, s)),
\]
\[\text{Notice that eq.(86) is in the form of eq.(185), and we could replace the } \xi^s \text{ with a general } u(0, s) \text{ in eq.(183).}\]
where the factor 1/2 is due to the fact that particle and anti-particle are the same. Thus the position operator is

\[
X_A = -\frac{i}{2} \int d^3x [\hat{A}^\dagger(x) \cdot A(x) - \hat{A}(x) \cdot \hat{A}^\dagger(x)]
\]

\[
= \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \sum_{s=1}^2 (a^\dagger(p, s)\partial_p a(p, s) - \partial_p a^\dagger(p, s)a(p, s))
\]

\[
+ \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \sum_{sr} a^\dagger(p, s)a(p, r) \sum_i [\epsilon_i(p, s)\partial_p \epsilon_i(p, r) - \partial_p \epsilon_i(p, s)\epsilon_i(p, r)],
\]

which is similar to the case of Dirac field in eq.(182), and the last term is not vanishing because of the momentum dependence of the polarization vectors, due to the Coulomb gauge in the form \(p \cdot \epsilon(p, s) = 0\). However, we could also define a new "field" with some fixed frame in which \(n \cdot \epsilon(n, s) = 0\), and an arbitrary chosen vector \(n = (n_1, n_2, n_3)\)

\[
\Lambda(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{s=1}^2 \epsilon(n, s)a(p, s)e^{ipx},
\]

with the communicative relation

\[
\{\Lambda(x), \Lambda^\dagger(x)\} = (\delta_{ij} - n_in_j)\delta^3(x - y), \sum_s \epsilon_i(n, s)\epsilon_j(n, s) = \delta_{ij} - n_in_j.
\]

then the particle number and position operators are

\[
N_A = \int d^3x \Lambda^\dagger(x) \cdot \Lambda(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{s=1}^2 a^\dagger(p, s)a(p, s)
\]

\[
X_A = \int d^3x \Lambda^\dagger(x) \cdot \Lambda(x) = \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \sum_{s=1}^2 (a^\dagger(p, s)\partial_p a(p, s) - \partial_p a^\dagger(p, s)a(p, s)),
\]

and the eigenstate of position operator is

\[
|\mathbf{x} \rangle = \Lambda^\dagger(x)|0 \rangle,
\]

although its meaning is also not clear.

In the end of this section, we try to give a somewhat systematical study about the separation of the field, such as eq.(125). For simplicity, we will still take the charged scalar field case. Notice that in the action eq.(126), up to some differentials, there is a general form of the fields

\[
\phi^\dagger \phi = \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_1^\dagger \phi_2^\dagger + \phi_2^\dagger \phi_1 = (\phi_1^\dagger, \phi_2) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \Phi^\dagger (I + \sigma_1) \Phi,
\]

38
similarly for the field in eq.(125)

\[
\phi' \phi' = \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 - \phi_1^\dagger \phi_2^\dagger - \phi_2 \phi_1 = (\phi_1^\dagger, \phi_2^\dagger)
\left( \begin{array}{cc}
1 & -1 \\
-1 & 1 \\
\end{array} \right)
\left( \begin{array}{c}
\phi_1 \\
\phi_2 \\
\end{array} \right) = \Phi^\dagger (I - \sigma_1) \Phi,
\]

(199)

that is \(\phi\) and \(\phi'\) seem to be in two different "chirality" representations! To understand these, let’s start from a general case with \(\sigma_1\) replaced by \(\sigma_n\). First, notice that

\[
\Phi^\dagger \Phi = \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2,
\]

(200)
is just of the form of action in eq.(129), with

\[
\phi = \sqrt{2} u_\uparrow_1 \downarrow \Phi = \left(1, 1\right) \left(\begin{array}{c}
\phi_1 \\
\phi_2 \\
\end{array} \right), \quad (I + \sigma_1 = 2 u_\uparrow_1 u_1^\dagger),
\]

(201)

then we could define a general field

\[
\phi_{\uparrow_n} = \sqrt{2} u_\uparrow_n \downarrow \Phi = \sqrt{2} (e^{i\phi_{\uparrow_n}} \cos \theta/2 \phi_1 + e^{-i\phi_{\uparrow_n}} \sin \theta/2 \phi_2), (I + \sigma_n = 2 u_\uparrow_n u_n^\dagger),
\]

(202)

and the communicative relation gives

\[
\left[\phi_{\uparrow_n}(x), \phi_{\uparrow_n}^\dagger(y)\right] = 2 \left[\cos^2 \frac{\theta}{2} \left[\phi_1(x), \phi_{\uparrow_n}^\dagger(y)\right] - \sin^2 \frac{\theta}{2} \left[\phi_2(x), \phi_{\uparrow_n}^\dagger(y)\right]\right],
\]

(203)

to restrict it to be zero, we have the condition \(\theta = \pi/2\). In this way, we obtain two projectors

\[
P_\uparrow = \frac{I + \sigma_{\theta=\pi/2}}{2}, \quad P_\downarrow = \frac{I - \sigma_{\theta=\pi/2}}{2},
\]

(204)

from which we have two different "chirality" representations

\[
\Phi_\uparrow = \frac{I + \sigma_{\theta=\pi/2}}{2} \Phi, \quad \Phi_\downarrow = \frac{I - \sigma_{\theta=\pi/2}}{2} \Phi,
\]

(205)

and the action constitution will be

\[
\Phi_\uparrow^\dagger \Phi_\uparrow = \Phi\left(\frac{I + \sigma_{\theta=\pi/2}}{2}\right)^\dagger \Phi = \Phi_\uparrow^\dagger u_{\uparrow_n} u_{\uparrow_n}^\dagger \Phi = \frac{1}{2} \phi_{\uparrow_n}^\dagger \phi_{\uparrow_n}^\dagger \Phi_{\uparrow_n}^\dagger \Phi_{\uparrow_n}^\dagger,
\]

(206)

by using the eq.(202), similarly for other component. Easily to see, if we further restrict \(\varphi = 0\), we then obtain the field \(\phi\) and \(\phi'\), with \(i\) added in \(\phi'\) to insure the hermitian for the special real scalar field. There is a residue "chiral" symmetry in eq.(206) under the following transformation\(^\text{32}\)

\[
\Phi \rightarrow e^{-i\sigma_{\theta=\pi/2}} \Phi,
\]

(207)

\(^\text{32}\) The full form in eq.(200) is symmetric under a general two-component transformation \(e^{-i\sigma_{\theta}} \Phi\), especially, \(e^{-i\sigma_{\theta}} \Phi\) is the transformation in eq.(134).
which induces
\[
\phi_{\uparrow \pi/2} \rightarrow \sqrt{2} u_{\uparrow \pi/2} e^{-i\alpha \sigma_\theta} \Phi = e^{-i\alpha} \phi_{\uparrow \pi/2},
\]
the global gauge transformation! And the other component has an opposite transformation
\[
\phi_{\downarrow \pi/2} \rightarrow \sqrt{2} u_{\downarrow \pi/2} e^{-i\alpha \sigma_\theta} \Phi = e^{i\alpha} \phi_{\downarrow \pi/2}.
\]
Though whether this gauge transformation is the usual one is still unknown, the above studies indeed give a systematical view on the separations of the fields and actions.

V. TWO ADDITIVE TOPICS

In this section, we will consider two extra topics, one is about an operable experiment to distinguish the Copenhagen interpretation from the ensemble one by very different experimental results, while the other is concerned with a special ensemble state, the coherent state.

A. An Operable Experimental Test

There are already many interpretations to the QM, for example, the standard Copenhagen interpretation (CI), the ensemble interpretation (EI) revived in this paper. However, it seems that all of them are somewhat metaphysical, and one could choose any interpretation at will. As is well known, with the CI, there is the so called quantum collapse in the quantum measurement theory. However, according to the EI developed previously, nothing unusual happens. Here, we try to give an operable (quantum measurement) experiment to test which interpretation is much more proper, via the possible different experimental phenomena owing to the two different interpretations.

We use the famous Stern-Gerlach experiment[11] as our basis, which involves the quantum measurement of spin. Besides, we add another apparatus to the original one, which may be considered to be a variant of the Stern-Gerlach apparatus with the non-uniform magnetic field replaced
by a uniform one, as shown in section III.C. The combined apparatuses are sketched in Fig. 2. Now, let a beam of electrons with specific spin \( |\uparrow_x\rangle \) travel into the first magnetic field, obviously there’s no deflections because of the uniform magnetic field. According to QM, the final state will be

\[
|\phi\rangle = \frac{1}{\sqrt{2}}(e^{-i\omega T} |\uparrow_z\rangle + e^{i\omega T} |\downarrow_z\rangle),
\]  

(210)

with \( \omega \equiv |e|B/2m_e \), and \( T \) the period of the electrons traveling in the first magnetic field. According to CI, the state in eq.(210) is a superposition state for a single electron. Then, let these electrons travel into the second magnetic field, i.e. making measurements on the spins, and the Stern-Gerlach apparatus would split the beam into two distinct components. According to CI, the probability for observing the \( |\uparrow_x\rangle \) is

\[
|\langle \uparrow_x | \phi \rangle|^2 = \cos^2 \omega T,
\]  

(211)

similarly for \( |\downarrow_x\rangle\),

\[
|\langle \downarrow_x | \phi \rangle|^2 = \sin^2 \omega T.
\]  

(212)

However, according to EI, we should treat the state in eq.(210) as an ensemble state, which briefly says that the electrons in the beam are roughly divided into two parts with almost the same particle number, one part with state \( |\uparrow_z\rangle\), while the other \( |\downarrow_z\rangle\), and with \( e^{\pm i\omega T} \) some irrelevant phase terms\(^{33}\). This means that, when we make measurements with the Stern-Gerlach apparatus, the probability for observing the \( |\uparrow_x\rangle \) will be

\[
|\langle \uparrow_z | \phi \rangle|^2 |\langle \uparrow_x | \uparrow_z\rangle|^2 + |\langle \downarrow_z | \phi \rangle|^2 |\langle \uparrow_x | \downarrow_z\rangle|^2 = \frac{1}{2},
\]  

(213)

with the same probability for \( |\downarrow_x\rangle\).

Since in the experiment, the probabilities are relevant to the ratios of particle numbers, then there are obvious differences between CI and EI, comparing eqs.(211)(212) with (213). That is, by tuning the strength of magnetic filed or the length of the first apparatus, we could vary the phases in eqs.(211) and (212) so as to obtain alterable probabilities, correspondingly the particle numbers in the two components. We even could obtain nothing in one of the two components when

\[
\omega T = \frac{1}{2}n\pi \quad n = 0, \pm 1, \pm 2, \ldots
\]  

(214)

\(^{33}\)The phase terms could have their effects in some physical process, for example the Young’s double-slit experiment in the section III.E. Here, the measurement is about spin, so those phase terms may have no effects.
However, according to EI, there are always two components with equal particle numbers up to some admissible experimental errors. Therefore, with the possible results of the above experiment, we could obtain the following conclusions:

(1) The notable differences between the particle numbers are observed by tuning the possible parameters, then CI is more proper, and furthermore, we obtain the strict evidences of the superposition state.

(2) The particle numbers of the two components are always equal up to admissible experimental errors, this means that CI is wrong, while EI is more proper, and the state in eq.(210) is not a superposition state.

(3) We observe a complete new phenomenon which can not be explained by either CI or EI, then we have to find out another proper interpretation to QM.

Although the CI to QM is familiar to us, there are still some corrections, which can be seen by adding the wave functions to the state in eq.(210)$^{34}$

$$\frac{1}{\sqrt{2}} (e^{i[pL-(E+\omega)T]}|p, \uparrow_z> + e^{i[p'L-(E'-\omega)T']}|p', \downarrow_z>), \quad E + \omega = \bar{E} = E' - \omega, \quad (215)$$

with $L$ the length of the first apparatus and $\bar{E}$ the energy of the electron before entering into the first apparatus. In short, the above function can be rewritten as

$$\frac{1}{\sqrt{2}} (|E + \omega, p, \uparrow_z> + e^{i\delta}|E' - \omega, p', \downarrow_z>), \quad (216)$$

with the meaning of superposition of two different states. To obtain the phase terms in eq.(215), we have used a space-time translation

$$\exp i\Delta t(\dot{X}P - H), \quad (217)$$

with the velocity operator defined in eq.(19), for the electrons are still free within a constant potentials $V \sim \pm \omega$.

From eq.(215), we can see that the complete probabilities corresponding to eqs.(211) and (212) will be more complicated due to $p'$ and $T'$. However, if the frequency $\omega$ is much smaller than the energy of the electron, i.e. $\omega \ll \bar{E}$, we can obtain the corrections of eqs.(211) and (212) up to first order. These can be seen as follows by computing the phase difference classically

$$(p - p')L - \bar{E}(T - T') \approx -3\omega \bar{T}, \quad (218)$$

$^{34}$ Notice that the state in eq.(215) is of the result of eq.(92) with the operator in eq.(217) acting on the state.
with $\bar{T} \approx Lm_e/\bar{p}$, $\bar{p} \approx \sqrt{2m_eE}$. Then the probabilities in eqs.(211) and (212) will approximately be

$$\cos^2 \frac{3}{2} \omega \bar{T} \quad \sin^2 \frac{3}{2} \omega \bar{T}. \quad (219)$$

We can even construct the real wave-packet for each state

$$\int dk \phi(k)e^{ikL-(E+\omega)T} \int dk' \phi(k')e^{ik'L-(E'-\omega)T'}, \quad (220)$$

with the respective probability densities $|\phi(k)|^2$ and $|\phi(k')|^2$ which are mainly valued near $k = p$ and $k' = p'$. Furthermore, noticing that the average momentum $\bar{k}$ should be constant in the process, then we have

$$\phi(k) = \phi(k')e^{i\delta}, \quad (221)$$

and the phase term can simply be ignored. With these, eq.(218) is still valid. In fact, within the CI, since the phase terms always affect the probabilities, the results of the experiment are always tenable. As for EI, the phase terms are irrelevant to the physical results, so the conclusions above are unchanged.

### B. Coherent State: From Quantum Field to Classical Field

In the quantization of oscillator, there is a special state, the so called coherent state defined as

$$|z> = e^{-\frac{|z|^2}{2}} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |n>, \quad <z|z> = 1. \quad (222)$$

Treating the oscillator as an 1-dimensional field, then coherent state is a superposition of infinitely many states with different particle numbers, and according to section III., it’s actually an ensemble state. Besides, this state can be related to the classical oscillator via

$$a|z> = z|z>. \quad (223)$$

In this subsection, we extend this coherent state to the general field case, including the Bosonic and Fermionic cases, indicating that the classical field could be obtained from the the quantized field with the filed operator acting on the extended coherent states.
For Bosonic case, we take the scalar field as examples, and since the communicative relation \([a(p), a^\dagger(q)] = (2\pi)^3 \delta^3(0)\) is a divergence, then re-normalizing these operators by a factor \(1/\sqrt{(2\pi)^3 \delta^3(0)}\) and discretizing them formally, so that we have

\[ [a_p, a^\dagger_q] = \delta_{pq}. \quad (224) \]

Now, the conditions are almost the same as those of the oscillator case, then we could define the coherent state for each momentum state \(p\) as

\[ |\phi(p)\rangle = e^{-1/2|\phi(p)|^2} \sum_{n=0}^{\infty} \frac{\phi(p)^{n} (a^\dagger_p)^n |0\rangle}{\sqrt{n!}}, \quad |\phi(p)\rangle|\phi(p)\rangle = 1. \quad (225) \]

For different momentum states, we have

\[ <\phi(q)|\phi(p)> = e^{-1/2|\phi(p)|^2 - 1/2|\phi(q)|^2}, (p \neq q). \quad (226) \]

Further, like eq.(223), we have

\[
\begin{cases}
    a_p|\phi(p)\rangle = \phi(p)|\phi(p)\rangle \\
    a_p|\phi(q)\rangle = 0 \quad (p \neq q)
\end{cases} \quad (227)
\]

Now we collect all the momentum states altogether, and define a state

\[ |\psi\rangle = \prod_p |\phi(p)\rangle, \quad (228) \]

with normalization

\[ <\psi|\psi\rangle = \prod_{p,q} <\phi(q)|\phi(p)\rangle = \prod_p <\phi(p)|\phi(p)\rangle = 1, \quad (229) \]

and the equation

\[ a_p|\psi\rangle = \prod_q a_p|\phi(q)\rangle = \prod_q \phi(p)|\phi(q)\rangle = \phi(p)|\psi\rangle, \quad (230) \]

then the classical filed derived from the quantized one is

\[ <\psi|\hat{\psi}(x)|\psi\rangle = <\psi| \sum_p a_p e^{ipx} |\psi\rangle = \sum_p \phi(p) e^{ipx} = \psi(x). \quad (231) \]

Now, let’s consider the Fermionic case, the first step is the same as that of the Bosonic case, that is the discretion of the operators

\[ [a_p, a^\dagger_q] = \delta_{pq}. \quad (232) \]
Then we have to define the coherent state, unlike the Bosonic case where $\phi(p)$ is a c-number, here, we should deal with Grassmann numbers satisfying $\phi_1\phi_2 = -\phi_2\phi_1$, further the complex conjugate is defined as

$$(\phi_1\phi_2)^* \equiv \phi_2^*\phi_1^* = -\phi_1^*\phi_2^*. \quad (233)$$

Thus we can define the coherent state as

$$|\phi(p)\rangle = e^{-\frac{1}{2}\phi^*(p)(1 + \phi(p))}\hat{a}_p\hat{a}_0^\dagger|0\rangle, \quad (234)$$

with normalization

$$<\phi(p)|\phi(p)> = e^{-\frac{1}{2}\phi^*(p)(1 + \phi(p))} = 1, \quad (235)$$

where we have used the relations $e^{-\phi^*(p)} = 1 - \phi^*\phi(p)$ and $\phi^*\phi^*\phi = -\phi^*\phi\phi = 0$. Then with the annihilation acting on the state, we have

$$a_p|\phi(p)\rangle = e^{-\frac{1}{2}\phi^*(p)}\phi(p)|0\rangle = \phi(p)|0\rangle, \quad (236)$$

or

$$<\phi(p)|a_p|\phi(p)\rangle = \phi(p). \quad (237)$$

As the Bosonic case, we could also define a state by noting that for different momentum states, the $|\phi(p)\rangle$'s are commuting

$$|\psi\rangle = \prod_p |\phi(p)\rangle, \quad <\psi|\psi\rangle = 1, \quad (238)$$

and

$$<\psi|a_p|\psi\rangle = \prod_q <\phi(q)|\phi(q)\rangle <\phi(p)|a_p|\phi(p)\rangle = \phi(p). \quad (239)$$

With these, we then have the classical field, ignoring some spinor structures

$$<\psi|\hat{\phi}(x)|\psi\rangle = \sum_p a_pe^{ipx}|\psi\rangle = \sum_p \phi(p)e^{ipx} = \psi(x). \quad (240)$$

The reason for the direct product structure of the state $|\psi\rangle$ for both cases is mainly because that different momentum states are independent from each other in the free field case. If not, there would be some states such as $a|1\rangle + b|1,2\rangle$, in which different states are interrelated with each
other, so that the probability in eq.(48) is not valid. However, with the direct product structure, we can still have

\[ P_p = \frac{\langle \psi | a_p^\dagger a_p | \psi >}{\langle \psi | \sum_p a_p^\dagger a_p | \psi >} = \frac{\phi^*(p)\phi(p)}{\sum_p \phi^*(p)\phi(p)}, \]  

(241)

and for Bosonic case, its meaning is easy to understand, with \( |\phi(p)|^2 \) treated as some classical intensity strength, while for the Fermionic case unclear. In fact, the state in eq.(234) is not a real ensemble state in the usual sense, due to the character of Grassmann numbers. And a real ensemble state for the Fermionic case should be of the form

\[ |\phi(p)\rangle = (1 + |\phi(p)|^2)^{-1/2}(1 + \phi(p)a_p^\dagger)|0\rangle, \]  

(242)

with the c-number \( \phi(p) \), but if so, we would not obtain the classical anticommuting fields.

VI. CONCLUSIONS AND DISSUASIONS

In this paper, we develop in details a new approach to the QM. From the non-relativistic Schrödinger field theory, the main three approaches to QM are obtained consistently, the Schrödinger equation (2) as field equation, the Heisenberg equations (10) and (11) for the momentum and position operators of the particles, and the Feynman path integral formula eq.(25). With the identity of eqs.(47) and (48), the probability concepts of QM can be induced from the statistical properties of some collection of particles, with the use of concepts of ensemble states, such as the states in eqs.(62) and (64). Therefore, the modifications to the SQM is inevitable, for example, the Schrödinger equation (74) which is believed to be fundamental in SQM can be derived from general quantized field equation (75). The most important modification is the concept of superposition state which in our view belong to a class of states with the form of eqs.(81),(83) and (84), while the rest are almost the ensemble states. Then, the quantum collapse in SQM measurement is just misunderstanding, and the EPR paradox is also solved in eqs.(96) and (97). In addition, the most famous experiment, the double-slit interference experiment is interpreted in field theoretical languages, too, with the particle number distribution eq.(118) obtained.

When considering the relativistic field theory, a method of separating the particle field from the anti-particle field is developed in eqs.(124)-(129), so that the operators which are physical observables of particles are possible to be defined, see for examples, eqs.(145)-(148). This method is useful for the scalar field well, while for the Dirac field and the gauge field, there are some problems with their position operators, and to resolve them, we introduce some ill-defined "fields" so
that the ensemble interpretation is still proper. An operable experiment is proposed in section V.A. to distinguish the Copenhagen interpretation from the ensemble one via very different experimental results, see eqs.(211)-(214). We also make some extensions of the concepts of coherent state for the oscillator to both the Bosonic and Fermionic fields, obtaining the corresponding classical fields in eqs.(231) and (240).

Now, let’s make some general discussions, especially about the differences between the QM in the standard form and the one derived from the QFT, on the framework of the derivations in sections II. and IV.. First, let’s list some familiar rules about the standard QM,

1. The states of particles or systems are described by the wave function formulism or the Dirac’s bra-ket formalism in Hilbert space. The most familiar and important states of a single particle are its positions $|x>$ and momentum $|p>$. 

2. There are single particle operators which are some physical observables whose eigenvalues can be measured in experiments, for examples the energy $\hat{H}$, the momentum $\hat{p}$, the position $\hat{x}$ for single particle, and the communicative relations among them.

3. The state of the system $|\phi(t)>$ satisfies the time evolution equation (74).

For the non-relativistic case, the above three rules are perfectly realized, which are described well in field theoretical languages in section II., with the field operators in eqs.(5)-(7), and the state in eq.(20), we can further induce the single particle operators

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}), \quad \hat{p} = -i\nabla, \quad \hat{x} = x, \quad \text{(243)}$$

and further the mean value of some operator $\hat{O}$ within some state $|\phi>$ is

$$<\phi|\int d^3x\psi^\dagger(x)\hat{O}\psi(x)|\phi>. \quad \text{(244)}$$

If $|\phi> = \sum_n \alpha_n|n>$, $|n> = a_n^\dagger|0>$, $\sum_n |\alpha_n|^2 = 1$, then we have

$$\psi(x)|\phi> = \sum_n \alpha_n\psi_n(x)|\phi> = \sum_n \alpha_n\psi_n(x)|0> = \phi(x)|0>.$$

from which we obtain the wave function $\phi(x)^{35}$, then eq.(244) reduces to

$$\int d^3x\phi^\dagger(x)\hat{O}\phi(x), \quad \text{(246)}$$

which is the familiar QM formalism, specially for $\hat{O} = I$, the probability assumption in QM, which is 1 in this case, confirming the eqs.(47)-(49).

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35 Don’t confuse with the classical field in eq.(231).
However, for the relativistic case, the above three rules are not always proper, even for the free field case. As shown in section IV, we find out a method to separate the particle field from the anti-particle field, then one may simply believe that the above rules should be satisfied, too. For the Dirac field, rules 2. and 3. are realized, while for the first one, the position state is not well defined. The single particle operators for Dirac field are

\[ \hat{H} = \gamma^0 \gamma^i \hat{p} + m \gamma^0, \quad \hat{p} = -i \nabla, \quad \hat{x} = x. \]  

(247)

However, for the scalar and electromagnetic fields, there are not single particle operators formally in the original field formula, because of the twice differentials about time. These can also be seen in the following way, supporting a state \(|\phi\rangle\)

\[ |\phi\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{\beta(p)}{\sqrt{2E_p}} |p\rangle, \quad <\phi| = \int \frac{d^3p}{(2\pi)^3} |\beta(p)|^2 = 1, \]  

(248)

then for the scalar particle field \(\phi_1\) in eq.(124), eq.(245) will be

\[ \phi_1(x)|\phi\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{\beta(p)}{\sqrt{2E_p}} e^{ipx} |0\rangle, \]  

(249)

which is not normalized to 1, due to the factor \(1/\sqrt{2E_p}\). This is also the case for the electromagnetic field. There are also some other problems, for example the energy and momentum operators of the scalar field defined in eqs.(131)-(133) are completely different from the non-relativistic case formally, but they are all resulting from the principle of space-time transformations.

Therefore, to achieve the above three rules, we have to define some ill-defined "fields" as in eqs.(142),(185) and (193), which are similar to the non-relativistic field. And with these "fields", we can also introduce single particle operators with the forms in eq.(149), then the above three rules of QM are all satisfied, especially the wave functions are well defined, for example for the electromagnetic field(or photon) case, we can extend the state \(|\phi\rangle\) by including the polarizations

\[ |\phi\rangle = \int \frac{d^3p}{(2\pi)^3} \sum_s \frac{\beta(p, s)}{\sqrt{2E_p}} |p, s\rangle, \quad <\phi|\phi\rangle = \int \frac{d^3p}{(2\pi)^3} \sum_s |\beta(p, s)|^2 = 1, \]  

(250)

then the wave function will be from

\[ \Lambda_i(x)|\phi\rangle = \int \frac{d^3p}{(2\pi)^3} \sum_s \beta(p, s) \epsilon_i(n, s) e^{ipx} |0\rangle, \]  

(251)

with normalization

\[ <\phi| \int d^3x \Lambda^\dagger(x) \cdot \Lambda(x)|\phi\rangle = \int \frac{d^3p}{(2\pi)^3} \sum_s |\beta(p, s)|^2 = 1. \]  

(252)
Though the QM with standard form is realized with those ill-defined "field", the Lorentz group is broken owing to the bad transformation properties of those "fields", just like the non-relativistic case. Therefore, when considering some general physical processes which should be Lorentz invariant, the standard form of QM in which single particle operators can be defined, is not enough and even wrong, instead QFT is the most proper description. In this sense, the position operators which can be defined well with the ill-defined "field", as in eqs.(148),(187) and (196), together with the corresponding position eigenstates are actually not real physical, and the only physical observables are all those which can be obtained from the invariance of the action under some transformations, for examples, the energy and momentum, the charge and the particle number with transformations in eq.(134).

Here is a note about the relationships between the single particle operators, defined in eqs.(149), (243) and (247), and the corresponding ones constructed with fields, for example the operators in eqs.(5)-(8) for the non-relativistic case, and those in eqs.(132),(133),(135) and (136) for the scalar field case, or those defined in eqs.(145)-(148), with the use of ill-defined "fields". For the non-relativistic case, the communicative relations among the operators constructed with fields seem to be determined completely by the structure of the single particle operators, as long as the field communicative relations in eq.(3) for both Bosonic and Fermionic cases are imposed, so do the operators defined with the ill-defined "field", with the communicative relations among these ill-defined "field" satisfied, such as those in eqs.(143), (186) and (194). However, for the well-defined relativistic fields, the above structure is not always enough. This can be seen generally as follows. Consider two single particle operators $\hat{O}_1$ and $\hat{O}_2$, which in field theory may be of the forms

$$O_1 = \int d^3x \phi^\dagger(x) \hat{O}_1 \phi(x), \quad O_2 = \int d^3x \phi^\dagger(x) \hat{O}_2 \phi(x),$$

then the problem is

$$[O_1, O_2] = \int d^3x \phi^\dagger(x)[\hat{O}_1, \hat{O}_2] \phi(x),$$

which is obviously true for the non-relativistic field case and ill-defined "field” case, but not for all the relativistic fields generally. Taking the charged scalar field as example, if $\phi(x)$ is treated as the full field, then with $[\phi(x), \phi^\dagger(y)] = 0$, the left hand side of eq.(254) is 0 identically, while the right hand side is not. If $\phi(x)$ is only as the particle field, and considering the operators $\hat{p}$ and $\hat{x}$, then after some computations, we have

$$[X_i, P_j] = \int \frac{d^3p}{(2\pi)^3} \left( \frac{1}{2E_p} \right)^2 (\hat{a}_{ij} - \frac{p_i p_j}{E_p^2}) a^\dagger(p) a(p),$$
which is completely different from $\int d^3x \phi^\dagger(x)[\hat{\mathbf{x}}, \hat{\mathbf{p}}_j] \phi(x)$. In fact in this case, the momentum operator is not of the above form at all, but should be constructed with the canonical momentum field $\pi(x)$, as in eq.(133), so do other operators, since $[\phi(x), \pi(y)] = i\delta^3(x - y)$ for the whole field (including both the particle and anti-particle field). From these, we can conclude that, in general, the single particle operators in QM are not enough to determine the structure of the operators constructed with fields. In fact, we can construct a lot of operators with the fields, the space-time coordinates and their differentials, among which only a few have some physical meanings, i.e. those which can be derived from the symmetries of the actions under some transformations. In this sense, the single particle operators are not fundamental, but instead, the quantized fields are! Even, we can treat the single particle operators as just the induced formal results of the corresponding field operators resulting from the transformations, of course, the position operator is not of this kind, and it doesn’t exit physically at all.

However, just like the non-relativistic field case, there are indeed some special fields and some special cases, where eq.(254) is true. The operators in eq.(247) for the Dirac field are of this special kind, owing to its spinor structure, and their communicative relations can determine the whole structure via the communicative relation of the full Dirac field

$$\{\psi(x)_a, \psi^\dagger(y)_b\} = \delta_{ab}\delta^3(x - y).$$

Taking the position operator $\int d^3x \psi^\dagger(x)\mathbf{x}\psi(x)$, for example, which is in fact not a real position operator for it has no corresponding eigenstate because of $\psi(x)|0 > \neq 0$, and with eq.(256), we could obtain the velocity operator, $\int d^3x \psi^\dagger(x)\gamma^0\gamma^i\psi(x)$, which in the sense of single particle operator, can also be derived from the equation

$$\dot{\mathbf{x}}^i = i[\hat{H}, \mathbf{x}^i] = \gamma^0\gamma^i,$$

i.e. eq.(254) is realized in this case. Even for the electron field $\psi_1(x)$ in eq.(174), the eq.(257) is still proper, since the commutating of position operator in eq.(182) with the corresponding energy operator is just the velocity operator, which can also be verified directly with $\psi_1(x)$ substituted, after some tedious computations.

With the above discussions about the single particle operators, let’s study generally the statistical properties of the ensemble which were briefly exhibited in section II.A., such as eqs.(48) and (49). For simplicity, we work still in the non-relativistic case. For a general operator $O = \int d^3x \psi^\dagger(x)\hat{O}\psi(x)$, a single particle ensemble state $|\phi >$ contains almost all the statistical
information about some specific property (such as the energy state) of the single particle system, for example the expectation value \( \langle \phi | O | \phi \rangle \), which can also be considered to be the mean value of a collection of particles which realize that ensemble. Furthermore, when considering the fluctuations, we need the expectation value of \( O^2 \). After some computations, we have

\[
O^2 = \int d^3x \psi^\dagger(x) \hat{O}^2 \psi(x) = \int d^3x \psi^\dagger(x) \left( \int d^3y \psi^\dagger(y) \hat{O}_y \psi(y) \right) \hat{O}_x \psi(x),
\]

where the communicative relations in eq.(3) are used. Then for the single particle ensemble state \( |\phi\rangle \), the last term vanishes by using eq.(245), and QM formula is fulfilled, and the fluctuations for single particle can be derived in the familiar way. However, for a N-particle ensemble, the last term in eq.(258) will not vanish, since there are correlations among those particles in the N-particle system. We can see these with a simple example, for instance, the energy operator together with its eigenstates, obviously, for this case, \( \langle H^2 \rangle \) is \( E_i^2 \), that is the expectation value of the square of the total energy, while the first term in eq.(258) is \( \sum_i E_i^2 \), which lacks the correlations between different energy states. This is easy to understand, by noting that the N-particle system is as a whole just like a single particle. Therefore, QFT is much useful than QM when treating the many-particle systems, and also in this sense, QFT is a fundamental theory.

Though there may be some special cases, the QM with the standard form of the above three rules is indeed not a fundamental theory generally, not only because QM can be consistently derived from QFT both non-relativistically and relativistically, but also because of the non-universality of those assumed rules as a general quantization scheme, for we could not measure or determine theoretically the physical states of the whole (or global) field in general\(^{36}\), but only describe them formally mathematically. What we can obtain or measure are only the states of the particles excited from those fields, and the corresponding local properties. Thus in this sense, QFT is the unique fundamental theory in principle, in which fields are fundamental elements of our physical world, in the nowadays experimental limit.

\(^{36}\) In the standard quantization scheme, the field operator, just like the position operator, should satisfy the eigenvalue equation \( \hat{\phi}(x) |\phi\rangle = \phi(x) |\phi\rangle \)\(^2\), but it seems impossible to realize physically. We can (classically) measure exactly the static field, such as the electrostatic field, with a test particle, by observing the motion of the particle, but not possible for a general dynamical field.
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