Voltage-controlled Berry phases in two vertically coupled InGaAs/GaAs quantum dots

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Abstract – The voltage-controlled Berry phases in two vertically coupled InGaAs/GaAs quantum dots are investigated theoretically. It is found that Berry phases can be changed dramatically from 0 to 2π (or 2π to 0) only by simply applying an external voltage. Under realistic conditions, as the tunneling is varied from 0.8 eV to 0.9 eV via a bias voltage, the Berry phases are altered obviously, and this can be detected in an interference experiment. The scheme is expected to be useful in constructing quantum computation based on geometric phases in an asymmetrical double quantum dot controlled by voltage.

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Introduction. – Recently with the advent of quantum information and communication [1], the phase of a wave function plays an important role in numerous quantum information protocols. The state vector of a quantum system can rotate as it undergoes a cyclic evolution in state space, such that it returns to its initial physical state, its wave function can acquire a geometric phase factor in addition to the familiar dynamic phase [2,3]. If the cyclic change of the system is adiabatic, this additional factor is known as Berry’s phase [4]. Since it has potential applications in the implementation of quantum computation by geometric means [5,6] which are less susceptible to noise from the environment, the study of Berry’s phase is becoming more and more important. Fuentes-Guridi et al. [7] calculated the Berry phase of a particle in a magnetic field considering the quantum nature of the field. Yi et al. [8] studied the Berry phase in a composite system and showed how the Berry phases depend on the coupling between the two subsystems. San-Jose et al. [9] have described the effect of geometric phases induced by either classical or quantum electric fields acting on single-electron spins in quantum dots. Yuan and Zhu [10] have shown that the Berry phases of two coupled quantum dots depend on the environmental temperatures. Most recently, observations of Berry phases in solid-state materials are reported [11–13]. Leck et al. [13] demonstrated the controlled Berry phase in a superconducting qubit. They manipulate the qubit geometrically using microwave radiation and observe the Berry phase in an interference experiment. In this letter, we theoretically present a scheme where the Berry phases can be controlled by a bias voltage in a double quantum dot (QD). The Berry phases can be changed dramatically from 0 to 2π (or from 2π to 0) only by simply applying an external voltage. This scheme is expected to be useful in constructing quantum computation based on the geometric phase in an asymmetrical double quantum dot controlled by voltage.

Quantum-dot system. – A vertically coupled InGaAs/GaAs asymmetrical quantum-dot molecule consisting of two layers of dots (the upper one and the lower one) with different band structures coupled by tunneling is shown in fig. 1(a). Samples are arrays of InGaAs dots in a matrix of GaAs which are vertically stacked and electrically coupled in the growth direction. Dots in two different layers show a strong tendency to align vertically. The coupling is mainly determined by the separation distance of two layers. In this quantum-dot system, the lower QD is slightly small, so its energy difference between ground state and first excited state is larger than that of the upper one. From ref. [14], we know that for QD separation d > 9 nm the tunneling coupling between the two dots is weak and the QD system can be discussed in terms of a simplified single-particle picture [15–18]. Applying an electromagnetical field, we can excite one electron from the valence to the conduction band in the lower dot which can in turn tunnel to the upper dot. Figure 1(a) gives a schematic of

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the system. The experimental results [18] show that for weak tunneling and strong lateral quantization, the basic physics is well described by this simple model. Within this picture, two species of excitons can be distinguished: direct exciton, in which the electron and hole are localized in the same dot, and indirect excitons, in which electron and hole are in different dots. In order to ensure that only single-exciton species are generated, the excitation power must be very low. The electron component of the wave function becomes delocalized over both dots while that of the hole remains localized due to the much larger effective mass. The tunnel barrier in an asymmetric quantum-dot molecule can be controlled by applying a bias voltage between the \( n^+ \) - contact and the Schottky gate. Figure 1(b) depicts an energy level diagram of an asymmetric quantum-dot molecule. The ground state \( |0\rangle \) is the system without excitations, the direct exciton state \( |1\rangle \) is a system with a pair of electron and hole bound in the lower dot, and the indirect exciton state \( |2\rangle \) is a system with one hole in the lower dot and an electron in the upper dot. Using this configuration the Hamiltonian of the system reads as follows \( \hbar = 1 \) [17]:

\[
H_1 = \sum_{j=0}^{2} \varepsilon_j |j\rangle \langle j| + \omega_c a_c^\dagger a_c + g_c (|1\rangle \langle 0| a_c + |0\rangle \langle 1| a_c^\dagger) + T_c (|1\rangle \langle 2| + |2\rangle \langle 1|),
\]

(1)

where \( \varepsilon_j \) is the energy of state \( |j\rangle \), \( T_c \) is the electron tunneling matrix element between two dots, \( a_c^\dagger \) and \( a_c \) are, respectively, the creation and annihilation operators of the quantized field with frequency \( \omega_c \). \( g_c \) is the coupling constant of the quantized field and of the direct exciton (the state of \( |0\rangle \) and \( |1\rangle \)). The Hamiltonian \( H_1 \) is rather simple, but it is not complete. Since the double quantum dots is embedded in the macroscopic crystal, the single electron is unavoidably scattered by phonons while tunneling between two dots. Considering the coupling between electron and phonons, the Hamiltonian can be written as

\[
H = H_1 + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2} (|1\rangle \langle 1| - |2\rangle \langle 2|) \sum_k g_k (b_k^\dagger + b_k),
\]

(2)

where \( b_k^\dagger (b_k) \) and \( \omega_k \) are the creation (annihilation) operator and energy for the \( k \)-th phonon mode, respectively, \( g_k \) is the coupling constant determined by the crystal material and the geometry of the coupling quantum dots. The electron-phonon interaction in Hamiltonian (2) contains only the diagonal elements, because the role of off-diagonal ones is suppressed at low temperatures. Applying a canonical transformation with the generator \[19,20]\]

\[
S = (|1\rangle \langle 1| - |2\rangle \langle 2|) \sum_k \frac{g_k}{2\omega_k} (b_k^\dagger - b_k),
\]

(3)

the transformed Hamiltonian is given by

\[
H' = e^\delta H e^{-\delta} = H'_0 + H'_f,
\]

(4)

where

\[
H'_0 = \varepsilon_0 |0\rangle \langle 0| + (\varepsilon_1 - \Delta) |1\rangle \langle 1| + (\varepsilon_2 - \Delta) |2\rangle \langle 2|
\]

\[
+ \sum_k \omega_k b_k^\dagger b_k + \omega_c a_c^\dagger a_c,
\]

(5)

\[
H'_f = g_c (|1\rangle \langle 0| a_c + |0\rangle \langle 1| a_c^\dagger) + T_c (|2\rangle \langle 1| X^2 + |1\rangle \langle 2| X^2^\dagger),
\]

(6)

\[
\Delta = \sum_k \frac{g_k^2}{4\omega_k}, \ X = \exp \left[ - \sum_k \frac{g_k}{2\omega_k} (b_k^\dagger - b_k) \right].
\]

(7)

Here we assume that the relaxing time of the environment (phonon fields) is so short that the excitons do not have time to exchange energy and information with the environment before the environment returns to its equilibrium state. The excitons interact weakly with the environment so that the equilibrium thermal properties of the environment are preserved. Therefore it is reasonable to replace the operator \( X \) with its expectation value over the phonons number states which are determined by a thermal average and write the Hamiltonian as \[21-23\]

\[
H' = \varepsilon_0 |0\rangle \langle 0| + (\varepsilon_1 - \Delta) |1\rangle \langle 1| + (\varepsilon_2 - \Delta) |2\rangle \langle 2|
\]

\[
+ \sum_k \omega_k b_k^\dagger b_k + \omega_c a_c^\dagger a_c
\]

\[
+ g_c e^{-\lambda (N^{ph} + 1)/2} (|1\rangle \langle 0| a_c + |0\rangle \langle 1| a_c^\dagger)
\]

\[
+ T_c e^{-2\lambda (N^{ph} + 1)/2} (|2\rangle \langle 1| + |1\rangle \langle 2|),
\]

(8)

where \( \lambda = \sum_k (g_k / 2\omega_k)^2 \) is the Huang-Rhys factor. Here for the sake of simplicity we only perform the analysis for the simplest case in which only the longitudinal-optical (LO) phonon is considered, i.e. all the phonons have the same frequency \[10,22,23\]. We anticipate that this is sufficient to illustrate the main physics in the
more complicated case of the acoustic phonons [20]. In such a case $\omega_0 = \omega_0$ is irrelevant to the wave vector $k$ of
the phonon, and the phonon populations can be written as $N_{ph} = \frac{1}{e^{\frac{\omega_0}{kT}} - 1}$ [21,22], where $k_B$ is the Boltzmann
constant and $T$ is the temperature of the environment.
After using the operator $\Lambda = e^{i\sum \omega_0 b_k^\dagger b_k}$ to transform to a
frame rotating at the frequency $\omega_0$, we can cancel the term
$\sum_k \omega_0 b_k^\dagger b_k$ in eq. (8). Then we assume the eigenfunctions
of the Hamiltonian equation (8) as follows:
$$|\psi\rangle^l = C_0^l|0, n+1\rangle + C_1^l|1, n\rangle + C_2^l|2, n\rangle,$$
where $n$ is the photon number of the quantized field.
Substituting the eigenstates, eq. (9), to the Schrödinger
equation $H^l|\psi\rangle = \eta|\psi\rangle^l$, we can get the eigenvalue equation
as
$$\eta^3 - (e_1 + e_2 + e_3)\eta^2 + (e_1 e_2 + e_1 e_3 + e_2 e_3 - e_4^2 - e_5^2)\eta$$
$$+ (e_1 e_2^2 + e_2^2 e_3 - e_1 e_2 e_3) = 0.$$ (10)
From eq. (10) we can obtain the eigenvalues $\eta_l (l = 1, 2, 3)$
and then we will get the coefficients of eq. (9): $C_0^l = \frac{1}{\sqrt{1 + \xi_1^2 + \xi_2^2}} b_1^l$, $C_1^l = \frac{a_l}{\sqrt{1 + \xi_1^2 + \xi_2^2}}$, and $C_2^l = \frac{b_l}{\sqrt{1 + \xi_1^2 + \xi_2^2}}$.
Here
$$a_l = \frac{\eta_l - e_1}{e_4},$$
$$b_l = \frac{e_5 \eta_l - e_1}{e_4 \eta_l - e_3},$$
with $e_4 = e_0 + (n+1)\omega_c$, $e_2 = e_1 - \Delta + n\omega_c$, $e_3 = e_2 - \Delta + n\omega_c$, $e_4 = g_e e^{-\lambda(N_{ph}+1/2)}$, $e_5 = T_c e^{-2\lambda(N_{ph}+1/2)}$.

Adiabatic approximation and Berry phase. —

According to ref. [7], the phase-shift operator $U(\varphi) = e^{-\varphi L^a}$ is introduced. Applied adiabatically to the Hamiltonian equation (8), the phase-shift operator alters the states of the field and gives rise to the following eigenstates:
$$|\psi\rangle^l_p = C_0^l e^{-i(n+1)\varphi}|0, n+1\rangle + C_1^l e^{-in\varphi}|1, n\rangle$$
$$+ C_2^l e^{-i\varphi}|2, n\rangle.$$ (13)

Changing $\varphi$ slowly from $0$ to $2\pi$, the Berry phase is calculated as $\gamma_l = \int_0^{2\pi} p^l \left( \frac{\partial}{\partial \varphi} |\psi\rangle^l_p \right) d\varphi$, it gives $\gamma_l = 2\pi [n + (C_0^l)^2]$.

For the illustration of the numerical results, we choose the vertically coupled InGaAs/GaAs quantum dots as an example for experiments. For such a double dot, we assume that $\omega_0 = 0 eV$, $\omega_c = 1.3 eV$ and $\omega_c = 1.0 eV$ [24], $\Delta = 0.01 eV$ and $\lambda = 0.02$ [25]. The LO-phonon energy ($\omega_0$) of GaAs is $36 meV$ [21]. We apply a quantized field with frequency $\omega_c = 0.3 eV$ which is just resonant with the bonding state of the level $|1\rangle$ and the level $|2\rangle$ via the tunnel coupling as shown in fig. 1(a). Figure 2 shows the Berry phases as a function of tunneling ($T_c$) via a bias voltage for three coupling constants ($g_e$) at $T = 50 mK$. It is obvious that the Berry phase is different from zero even for the driving field in the vacuum state ($n = 0$), which is in agreement with the results obtained by Fuentes-Guridi et al. [7]. As the bias voltage is turned, the Berry phases can be changed dramatically from $2\pi$ to $0$ as the parameter $g_e$ is fixed. From fig. 2 we can see that the Berry phase is approximately equal to $2\pi$ for the tunneling $T_c < 0.8 eV$, but as the tunneling continuously increases to $T_c > 0.9 eV$ the Berry phase is suddenly down to zero. The range of $T_c$ where the Berry phase changes obviously from $2m\pi$ to $2(m \pm 1)\pi (m$ is an integer) is related to the different coupling constants ($g_e$). As the coupling constants increase, the change of the Berry phase becomes slow. From fig. 2 it is evident that the Berry phases are altered more sharply at $g_e = 0.02 eV$ than that at $g_e = 0.1 eV$. Figure 3 shows two kinds of Berry phases ($\gamma_1, \gamma_2$) as a function of tunneling $T_c$. In general, there are three Berry phases in all, here we just give two typical kinds of them ($\gamma_1, \gamma_2$).
The $\gamma_3$ phase does not give a new result. Figure 3 shows that the two curves ($\gamma_1$ and $\gamma_2$) are symmetrical at $T_c \approx 0.83$ eV. This turning point of the curves is due to the presence of the resonance with the quantized field. To understand the resonance properties due to the quantized field, we analyze the general features of the system. Without the single-mode field, it is evident that the eigenenergies of the system are $E_1 = 0$, $E_2 = \frac{1}{2} (\varepsilon_1 - \Delta + \varepsilon_2 - \Delta) - \frac{i}{2} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4T_c^2 e^{-2\lambda(N_{ph}+1/2)}}$ and $E_3 = \frac{1}{2} (\varepsilon_1 - \Delta + \varepsilon_2 - \Delta) + \frac{i}{2} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4T_c^2 e^{-2\lambda(N_{ph}+1/2)}}$. There are two different resonant frequencies here. One of the resonant frequencies is $\omega_c = \frac{1}{2} (\varepsilon_1 - \Delta + \varepsilon_2 - \Delta) - \frac{i}{2} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4T_c^2 e^{-2\lambda(N_{ph}+1/2)}}$. Taking the quantized field into consideration, if the coupling constant $g_c$ is relatively small, the resonant condition will not alter. In fig. 2 and fig. 3, we assume $\omega_c = 0.3$ eV, then from the above resonant condition it is easy to find a critical tunneling value $T_{c0} \approx 0.83$ eV which is just the turning point of the curves in fig. 2 and fig. 3. The present scheme can also be applied to the double–quantum-dot system realized by Gustavsson et al. [26], where the driving field is operated at microwave frequency. In such a case, the tunneling $T_c$ can be reached to meV or even to $\mu$eV through varying the gate voltage.

In order to observe the Berry phase, recently Giuliano et al. [27] have proposed an experimental scheme where a quantum dot is capacitively coupled to one arm of a double-path electron interferometer and the electrons passing through the arm of the interferometer coupled to the dot may pick up a finite phase which can be detected by the double-path electron interferometer. In the latest experiment, Leek et al. [13] measured the Berry phase in a Ramsey fringe interference experiment. We anticipate that the Berry phase in our system can also be observed and measured in an experimental setup analogous to that of Leek’s. For our experimental setup, we can fix the incident light through a single-mode fiber and slowly change the bias field, if the Berry phase is suddenly altered, then we will detect the variation of the Berry phase in the Ramsey interference setup [13,28]. Slow and accurate control of the bias field in this setup can be achieved by assisting a special electric circuit. The quantum-dot system is parallely coupled in the circuit. We can control the capacitor or other devices so as to slowly control the bias field and cycle on the dot system.

In conclusion, we have theoretically investigated the voltage-controlled Berry phases in an asymmetric semiconductor double quantum dots. It is found that Berry phases can be changed suddenly from 0 to $2\pi$ (or $2\pi$ to 0) by simply applying an external voltage. Under realistic experimental conditions, as the tunneling is varied from 0.8 eV to 0.9 eV via a bias voltage, the Berry phase can be altered dramatically. The range of $T_c$ where the Berry phase changes obviously from $2m\pi$ to $2(m+1)\pi$ is related to different coupling constant ($g_c$). As the coupling constant $g_c$ increases, the change of the Berry phases become slow. It is should be noted here that the role of the phonons in our model is not essential to the result. The main contribution of the phonons is the suppression of the tunneling strength in our model. If the phonons are excluded from the model, the results obtained in the present paper are still suitable. Finally, we hope that the scheme proposed here will open up the electrical controllability of the Berry phases which is expected to be useful in constructing quantum computer based on geometric phases in an asymmetric double quantum dot controlled by voltage.

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