Gauge Unified Theories without the Cosmological Constant Problem

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We study gauge theories in the context of a gravitational theory without the cosmological constant problem (CCP). The theory is based on the requirement that the measure of integration in the action is not necessarily $\sqrt{-g}$ but it is determined dynamically through additional degrees of freedom. Realization of these ideas in the framework of the first order formalism solves the CCP. Incorporation of a condensate of a four index field strength allows, after a conformal transformation to the Einstein frame, to represent the system of gravity and matter in the standard GR form. Now, however, the effective potential vanishes at a vacuum state due to the exact balance to zero of the gauge fields condensate and the original scalar fields potential. As a result it is possible to combine the solution of the CCP with: a) inflation and transition to a $\Lambda = 0$ phase without fine tuning after a reheating period; b) spontaneously broken gauge unified theories (including fermions). The model opens new possibilities for a solution of the hierarchy problem.

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I. INTRODUCTION

The cosmological constant problem in the context of general relativity (GR) can be explained as follows. In GR one can introduce such a constant or one may set it to zero. The problem is that after further investigation of elementary particle theory, we discover new phenomena like radiative corrections, the existence of condensates, etc. each of which contributes to the vacuum energy. In order to have a resulting zero or extremely small cosmological constant as required by observations of the present day universe one would have to carefully fine tune parameters in the Lagrangian so that all of these contributions more or less exactly cancel. This question has captured the attention of many authors because, among other things, it could be a serious indication that something fundamental has been missed in our standard way of thinking about field theory and the way it must couple to gravity. For a review of this problem see [1].

The situation is made even more serious if one believe in the existence of an inflationary phase for the early universe, where the vacuum energy plays an essential role. The question is then: what is so special about the present vacuum state which was not present in the early universe? In this paper we are going to give an answer this question.

As it is well known, in nongravitational physics the origin from which we measure energy is not important. For example in nonrelativistic mechanics a shift in the potential $V \rightarrow V + \text{constant}$ does not lead to any consequence in the equations of motion. In the GR the situation changes dramatically. There all the energy density, including the origin from which we measure, affects the gravitational dynamics.

This is quite apparent when GR is formulated from a variational approach. There the action is

$$S = \int \sqrt{-g} L d^4 x$$

$$L = -\frac{1}{\kappa} R(g) + L_m$$

where $\kappa = 16\pi G$, $R(g)$ is the Riemannian scalar curvature of the 4-dimensional space-time with metric $g_{\mu \nu}$, $g \equiv \text{Det}(g_{\mu \nu})$ and $L_m$ is the matter Lagrangian density. It is apparent now that the shift of the Lagrangian density $L$, $L \rightarrow L + C$, $C = \text{const}$ is not a symmetry of the action (1). Instead, it leads to an additional piece in the action of the form $C \int \sqrt{-g} d^4 x$ which contributes to the equations of motion and in particular generates a so called “cosmological constant term” in the equations of the gravitational field.

In Refs. [2]-[4] we have developed an approach where the cosmological constant problem is treated as the absence of gravitational effects of a possible constant part of the Lagrangian density. The basic idea is that the measure of integration in the action principle is not necessarily $\sqrt{-g}$ but it is allowed to "float" and to be determined dynamically through additional degrees of freedom. In other words the floating measure is not from first principles related to $g_{\mu \nu}$, although relevant equations will in general allow to solve for the new measure in terms of other fields of the theory ($g_{\mu \nu}$ and matter fields). This theory is based on the demand that such measure respects the principle of non gravitating vacuum energy (NGVE principle) which states that the Lagrangian density $L$ can be changed to $L + \text{constant}$ without affecting the dynamics. This requirement is imposed in order to offer a new approach for the solution of the cosmological constant problem. Concerning the theories based on the NGVE principle we will refer to them as NGVE-theories.

The invariance $L \rightarrow L + \text{constant}$ for the action is achieved if the measure of integration in the action is a total derivative, so that to an infinitesimal hypercube in 4-dimensional space-time $x_0^\mu \leq x^\mu \leq x_0^\mu + dx^\mu$, $\mu = 0, 1, 2, 3$ we associate a volume element $dV$ which is: (i) a total derivative, (ii) it is proportional to $d^4 x$ and (iii) $dV$ is a general coordinate invariant. The usual choice $\sqrt{-g} d^4 x$ does not satisfy condition (i).

The conditions (i)-(iii) are satisfied [3], [4] if the measure corresponds to the integration in the space of the four scalar fields $\varphi_a, (a = 1, 2, 3, 4)$, that is

$$dV = d\varphi_1 \land d\varphi_2 \land d\varphi_3 \land d\varphi_4 \equiv \Phi \frac{1}{4!} d^4 x$$

where

$$\Phi \equiv \varepsilon_{a_1 a_2 a_3 a_4} \varepsilon^{\mu \nu \lambda \sigma} \left( \partial_\mu \varphi_{a_1} \right) \left( \partial_\nu \varphi_{a_2} \right) \left( \partial_\lambda \varphi_{a_3} \right) \left( \partial_\sigma \varphi_{a_4} \right).$$

(4)

Notice that the measure which was discussed in Introduction, is a particular realization of the NGVE-principle (for other possible realization which leads actually to the same results, see Sec.I of Ref [4]). For deeper discussion of the geometrical meaning of this realization of the measure see Ref. [3].

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The total action is defined as follows

\[ S = \int L \Phi d^4x \] (5)

where \( L \) is a total Lagrangian density. We assume in what follows that \( L \) does not contain explicitly the measure fields, that is the fields \( \varphi_a \) by means of which \( \Phi \) is defined. If this is satisfied, an infinite dimensional symmetry appears (see Sec.II).

Introducing independent degrees of freedom related to the measure we arrive naturally at a conception that all possible degrees of freedom that can appear should be considered as such. This is why we expect that the first order formalism, where the affine connection is not assumed to be the Christoffel coefficients in general should be preferable to the second order formalism where this assumption is made.

In fact, it is found that the NGVE theory in the context of the first order formalism does indeed provide a solution of the cosmological constant problem \( \Phi \), while this is not the case when using the second order formalism.

The simplest example (Sec. IIIB) where these ideas can be tested is that of a matter Lagrangian described by a single scalar field with a nontrivial potential. In this case the variational principle leads to a constraint which implies the vanishing of the effective vacuum energy in any possible allowed configuration of the scalar field. These allowed configurations are however constant values at the extrema of the scalar field potential and an integration constant that results from the equations of motion has the effect of exactly canceling the value of the potential at these points. So, the scalar field is forced to be a constant and hence the theory has no nontrivial dynamics for the scalar field.

In this case the measure (4) is not determined by the equations of motion. In fact a local symmetry (called "Local Einstein Symmetry" (LES)) exists which allows us to choose the measure \( \Phi \) to be \( \alpha \) whatever we want. In particular \( \Phi = \sqrt{-g} \) can be chosen and in this case the theory coincides in the vacuum with GR with \( \Lambda = 0 \).

A richer structure is obtained if a four index field strength which derives from a three index potential is allowed in the theory. The introduction of this term breaks the LES mentioned above. In this case, the constraint that the theory provides, allows to solve for the measure in terms of \( \sqrt{-g} \) and the matter fields of the theory. The equations can be written in a form that resemble those of the Einstein theory by the use of a conformal transformation (or in equivalent language by going to the effective Einstein frame).

Then the theory which contains a scalar field shows a remarkable feature: the effective potential of the scalar field that one obtains in the Einstein conformal frame is that which generally allows for an inflationary phase which evolves at a later stage, without fine tuning, to a vacuum of the theory with zero cosmological constant. This will be discussed in Sec.IVA.

The 4-index field strength also allows for a Maxwell-type dynamics of gauge fields and of massive fermions. In this case the effective coupling constants of gauge theories and the fermion masses depend on a dimensionless integration constant \( \omega \) associated with the integration of the equation of motion for the 4-index field strength. This integration constant \( \omega \), together with another integration constant \( M \) (that goes together with the original scalar field potential \( V(\varphi) \), to form the combination \( V + M \)) determines the scalar field potential which has an absolute minimum with zero effective vacuum energy (fine tuning is not necessary). Furthermore, in the context of cosmology, where the scalar field plays the role of inflaton, \( \omega \) and \( M \) also govern the parameters of the inflationary picture present in this model.

In the most simple version of the theory with a scalar field and 4-index field strength, the scalar field has nontrivial cosmological dynamics, however at the absolute minimum \( V + M = 0 \) (if no fine tuning is made), the mass of the scalar is generically infinite. Additional problems in this simplest model are related to an appearance of nonrenormalizable couplings between gauge and scalar fields. All these difficulties can be overcome (without changing the advantages described above) by a certain sort of unification of all gauge fields, together with modification of the kinetic term of the gauge sector in the original Lagrangian density. It is argued that some of these modifications are natural if we believe in the existence of gauge fields condensates in the vacuum. In Sec. V, we implement this in the framework of two different models: "model with a critical limit" and "model with persistent condensate". These models allow to include fermions and to provide mass generation for them (see Sec. VI). In the model with persistent condensate in the \( \Lambda = 0 \) vacuum there is an exact balance between the integration constant \( M \), the original potential and the contribution from the gauge condensate. An explicit construction of unified gauge theory \( SU(2) \times U(1) \) as an example) based on these ideas and keeping all the above mentioned advantages is presented in Sec.VII.

II. GENERAL FEATURES OF THE NGVE THEORY IN THE FIRST ORDER FORMALISM

We assume that the total Lagrangian density \( L \) in Eq. (3) does not contain the measure fields \( \varphi_a \), that is the fields by means of which the measure \( \Phi \) is defined. If this condition is satisfied then the theory has an additional symmetry.
In fact, the action \( (5) \) is invariant under the infinitesimal shift of the fields \( \varphi_a \) by an arbitrary infinitesimal function of the total Lagrangian density \( L \), that is \[ (6) \]

\[ \varphi_a' = \varphi_a + \epsilon g_a(L), \quad \epsilon \ll 1 \]

Our choice for the total Lagrangian density is

\[ L = \frac{1}{\kappa} R(\Gamma, G) + L_m \]

where \( L_m \) is the matter Lagrangian density and \( R(\Gamma, G) \) is the scalar curvature which in the first order formalism in the framework of the Metric-Affine theory \[ (7) \] is defined as follows

\[ R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma) \]

\[ R_{\mu\nu}(\Gamma) = R^\lambda_{\mu\nu\lambda}(\Gamma) \]

\[ R^\lambda_{\mu\nu\lambda}(\Gamma) \equiv \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma} \Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu} \Gamma^\alpha_{\mu\sigma} \]

where \( \Gamma^\lambda_{\mu\nu} \) are the connection coefficients which have to be obtained from the variational principle.

Equations that originate from the variation of the action \[ (3) \] with respect to the measure fields \( \varphi_a \), are

\[ A^a_{\mu} \partial_{\mu} \left[ -\frac{1}{\kappa} R(\Gamma, g) + L_m \right] = 0 \]

where \( A^a_{\mu} = \varepsilon_{bcda} \varepsilon^{\nu\lambda\sigma\mu} (\partial_{\nu} \varphi_b)(\partial_{\lambda} \varphi_c)(\partial_{\sigma} \varphi_d) \). Since \( A^a_{\mu} \partial_{\mu} \varphi_a' = 4^{-1} \delta_{aa'} \Phi \) it follows that \( \text{Det}(A^a_{\mu}) = 4^{-4} \Phi^3 \), so that if \( \Phi \neq 0 \), it follows from Eq.\( (11) \)

\[ -\frac{1}{\kappa} R(\Gamma, g) + L_m = M = \text{constant} \]

Let us now study equations that originate from variation with respect to \( g^{\mu\nu} \). For simplicity we present here the calculations for the case where there are no fermions. Performing the variation with respect to \( g^{\mu\nu} \) we get

\[ -\frac{1}{\kappa} R_{\mu\nu}(\Gamma) + \frac{\partial L_m}{\partial g^{\mu\nu}} = 0 \]

Contracting Eq.\( (13) \) with \( g^{\mu\nu} \) and making use Eq.\( (12) \) we get

\[ g^{\mu\nu} \left( \frac{\partial (L_m - M)}{\partial g^{\mu\nu}} - (L_m - M) \right) = 0 \]

This equation is a constraint since generically \( L_m \) contains only the fields and their first derivatives. A similar constraint is achieved by using the vierbein - spin-connection (VSC) formalism (see Appendix B). Notice that if \( L_m - M \) is homogeneous of degree one in \( g^{\mu\nu} \) it satisfies the constraint \[ (14) \] automatically (that is without using equations of motion).

### III. THE THEORY FOR THE VACUUM, FOR SOME SIMPLE MODELS AND LOCAL EINSTEIN SYMMETRY

#### A. The vacuum case

In the vacuum, when we choose \( L_m = 0 \) in Eq.\( (5) \), it follows from Eq.\( (13) \)

\[ R^{\mu\nu}(\Gamma) = 0 \]
Eq. (12) implies then that the integration constant $M = 0$. Adding an arbitrary constant $C$ to $L_m$ does not change the resulting Eq. (15) since, as we see from Eqs. (12) and (13) or from the constraint (14), the integration constant $M$ has to compensate the constant $C$.

To clarify the sense of Eq. (15) we need the connection coefficients $\Gamma^\lambda_{\mu\nu}$. Varying the action (5) with $L = -\frac{1}{\kappa} R(\Gamma, g)$ with respect to $\Gamma^\lambda_{\mu\nu}$, we get

\begin{equation}
- \Gamma^\lambda_{\mu\nu} - \Gamma^\alpha_{\beta\mu} g^\beta_{\alpha\nu} + \delta^\lambda_{\nu} \Gamma^\alpha_{\mu} + \delta^\lambda_{\mu} g^\alpha_{\beta} \Gamma^\gamma_{\alpha\beta\gamma} g^\nu_{\lambda} - g_{\alpha\nu} \partial^\mu g^\alpha_{\lambda} + \delta^\lambda_{\mu} g_{\alpha\nu} \partial^\beta g^\alpha_{\beta} - \delta^\nu_{\mu} \Phi^\mu_{\nu} + \Phi^\lambda_{\mu} = 0.
\end{equation}

We will look for the solution of the form

\begin{equation}
\Gamma^\lambda_{\mu\nu} = \{^\lambda_{\mu\nu}\} + \Sigma^\lambda_{\mu\nu}
\end{equation}

where $\{^\lambda_{\mu\nu}\}$ are the Christoffel’s connection coefficients. Then $\Sigma^\lambda_{\mu\nu}$, satisfies the equation

\begin{equation}
- \sigma_{\lambda\mu} g_{\nu\lambda} + \sigma_{\mu\nu} g_{\nu\lambda} - g_{\nu\alpha} \Sigma^\lambda_{\mu\alpha} - g_{\mu\alpha} \Sigma^\lambda_{\nu\alpha} + g_{\mu\lambda} \Sigma^\alpha_{\nu\lambda} + g_{\nu\alpha} g_{\alpha\mu} g^\beta_{\gamma} \Sigma^\alpha_{\beta\gamma} = 0
\end{equation}

where

\begin{equation}
\sigma \equiv \ln \chi, \quad \chi \equiv \frac{\Phi}{\sqrt{-g}}
\end{equation}

The general solution of eq. (15) is

\begin{equation}
\Sigma^\alpha_{\mu\nu} = \delta^\alpha_{\mu} \Sigma^\lambda_{\nu\lambda} + \frac{1}{2} (\sigma_{\mu\nu} - \sigma_{\nu\mu} g_{\mu\nu} g^\alpha_{\beta})
\end{equation}

where $\lambda$ is an arbitrary function which appears due to the existence of the Einstein - Kaufman $\lambda$-symmetry (see [8], [9] and Appendix A): the curvature tensor (10) is invariant under the $\lambda$- transformation

\begin{equation}
\Gamma^{\alpha}_{\mu\nu}(\lambda, \sigma) = \Gamma^{\alpha}_{\mu\nu} + \delta^\alpha_{\mu} \lambda_{\nu}
\end{equation}

Although this symmetry was discussed in Ref. [8] in a very specific unified theory, it turns out that $\lambda$- symmetry has a wider range of validity and in particular it is useful in our case.

If we choose the gauge $\lambda = \frac{1}{2}$, then the antisymmetric part of $\Sigma^\alpha_{\mu\nu}$ disappears and we get finally

\begin{equation}
\Sigma^\alpha_{\mu\nu}(\sigma) = \frac{1}{2} (\delta^\alpha_{\mu} \sigma_{\nu} + \delta^\alpha_{\nu} \sigma_{\mu} - \sigma_{\nu\mu} g_{\mu\nu} g^\alpha_{\beta})
\end{equation}

In the vacuum, the $\sigma$-contribution (22) to the nonmetricity (see Appendix A) can be eliminated. This is because in the vacuum the action (5), (7) is invariant under the local Einstein symmetry (LES)

\begin{equation}
g_{\mu\nu}(x) = J^{-1}(x) g'_{\mu\nu}(x)
\end{equation}

\begin{equation}
\Phi(x) = J^{-1}(x) \Phi'(x)
\end{equation}

The transformation (24) can be the result of a diffeomorphism $\varphi_a \rightarrow \varphi'_a = \varphi'_a(\varphi_b)$ in the space of the scalar fields $\varphi_a$ (see Ref. [9]). Then $J = \text{Det}(\frac{\partial \varphi'_a}{\partial \varphi_b})$.

Notice that even when we are not in the vacuum, but the matter Lagrangian density $L_m$ satisfies the constraint [9] automatically (that is $L_m$ is homogeneous of degree one in $g^{\mu\nu}$, up to irreverent additive constant) then the total action (5), (6) possesses LES too. For examples see [3], [4].

For $J = \chi$ we get $\chi' \equiv 1$, $\Sigma^\alpha_{\mu\nu}(\sigma) \equiv 0$ and $\Gamma^{\alpha}_{\mu\nu} = \{^\alpha_{\mu\nu}\}$, where $\{^\alpha_{\mu\nu}\}$ are the Christoffel’s coefficients corresponding to the new metric $g'_{\mu\nu}$. In terms of the new metric $g'_{\mu\nu}$, the curvature (10) becomes the Riemannian curvature and therefore Eq. (15) is equivalent to the vacuum Einstein’s equation with zero cosmological constant.
B. Single scalar field with a nontrivial potential

Now let us consider the cases when the constraint (14) is not satisfied without restrictions on the dynamics of the matter fields. Nevertheless, the constraint (14) holds as a consequence of the variational principle in any situation.

A simple case where the constraint (14) is not automatic is the case of a single scalar field with a nontrivial potential $V(\phi)$

$$L_m = \frac{1}{2} \phi,_{\alpha} \phi^{,\alpha} - V(\phi)$$

(25)

In this model, the kinetic part of the action possesses LES and satisfies the constraint automatically since $\frac{1}{2} \phi,_{\alpha} \phi^{,\alpha}$ is homogeneous of degree one in $g^{\mu\nu}$. The potential part apparently does not satisfy the LES and as a result of this the constraint (14) implies

$$V(\phi) + M = 0$$

(26)

Therefore we conclude that, provided $\Phi \neq 0$, there is no dynamics for the theory of a single scalar field, since constraint (26) forces this scalar field to be a constant.

The constraint (26) has to be solved together with the equation of motion

$$\Box \phi + \sigma,_{\mu} \phi^{,\mu} + \frac{\partial V}{\partial \phi} = 0,$$

(27)

where $\sigma = \ln \chi$. From Eqs. (26) and (27) we conclude that the $\phi$-field has to be located at an extremum of the potential $V(\phi)$. Since the constraint (14) eliminates the dynamics of the scalar field $\phi$, we cannot really say that we have a situation where the LES (23), (24) is actually broken, since after solving the constraint together with the equation of motion (i.e. on the mass shell) the symmetry remains true.

Taking into account that $\phi = \text{constant}$ and $L_m - M = -(V(\phi) + M) = 0$, we see from Eqs. (12)-(14) that $R_{\mu\nu}(\Gamma, g) = 0$. As we have seen in Sec. IIIA, the $\sigma$ contribution to the connection can be eliminated in the vacuum by the transformations (23), (24). Notice that since $\phi = \text{constant}$, the single scalar field $\phi$ part of the Lagrangian density acts as an arbitrary constant. As we have seen also in Sec. IIIA, this situation is indistinguishable from the vacuum case. Then repeating the LES transformation of the end of Sec. IIIA, we see that in terms of the new metric $g_{\mu\nu}'$, the tensor $R_{\mu\nu}(\Gamma, g)$ becomes the usual Ricci tensor $R_{\mu\nu}(g')$ of the Riemannian space-time with the metric $g_{\mu\nu}'$. Therefore we conclude that for the case of a single scalar field with a nontrivial potential, the theory is equivalent to the Einstein’s GR with the zero cosmological constant. As a consequence, in this simple model, among maximally symmetric solutions, only Minkowski space is a solution. The absence of deSitter space as a solution makes us suspect that the NGVE theory is inconsistent with the idea of inflation. This however is not true as we will see in Sec.IV.

IV. FOUR INDEX FIELD STRENGTH CONDENSATE AS AN UNIVERSAL GOVERNOR.
SIMPLE MODELS.

As we have mentioned, one of the biggest puzzles of modern physics is what is referred to as the "cosmological constant problem", i.e. the absence of a possible constant part of the vacuum energy in the present day universe [1]. On the other hand, many questions in modern cosmology appear to be solved by the so called "inflationary model" which makes use of a big effective cosmological constant in the early universe [9]. A possible conflict between a successful resolution of the cosmological constant problem and the existence of an inflationary phase could be a "potential Achilles heel for the scenario" as has been pointed out [10]. Here we will show (see also [6]) that indeed there is no conflict between the existence of an inflationary phase and the disappearance of the cosmological constant in the later phases of cosmological evolution (without the need of fine tuning). The four index field strength condensate plays a crucial role for this.

Another problem related to the NGVE theory consists of the very strong restriction which constraint (14) dictates on the matter models which generally do not satisfy the LES. This makes the incorporation of fermion masses and gauge fields not straightforward (see Refs. [3], [4]). In what follows we will see, however, that the incorporation of the four index-field strength in four dimensional space-time turns the constraint into an equation for $\chi$-field. After solving this constraint we obtain well defined matter models. However, as we will see, undesirable problems appear nevertheless even in this model: (a) the mass of the scalar field turns out to be infinite; (b) nonminimal
nonrenormalizable couplings appear at very high energies. In Sec.V we will see that treating all gauge field strengths, including the four-index one in a unified fashion, leads to the resolution of the mentioned above problems. So, the aim of this section is to demonstrate first a mechanism where the four-index field strength condensate provides nontrivial dynamics while leaving more sophisticated improvements of these ideas to later sections.

A. Scalar field and the cosmological model of the very early universe

As it follows from our analysis above (see Sec.III), a model with only a scalar field although solves the cosmological constant problem, it cannot give rise to inflation since the gravitational effects of the scalar field potential is always canceled by the integration constant $M$. We will see that nontrivial dynamics of a single scalar field including the possibility of inflation can be obtained by considering a model with an additional degree of freedom described by a three-index potential $A_{\beta\mu\nu}$ as in the Lagrangian density

$$L = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} \phi, \alpha \phi^\alpha - V(\phi) + \frac{1}{4!} F_{\alpha\beta\mu\nu} F^{\alpha\beta\mu\nu}. \quad (28)$$

Here

$$F_{\alpha\beta\mu\nu} \equiv \partial_{[\alpha} A_{\beta\mu\nu]} \quad (29)$$

is the field strength which is invariant under the gauge transformation

$$A_{\beta\mu\nu} \rightarrow A_{\beta\mu\nu} + \partial_{[\beta} f_{\mu\nu]} \quad (30)$$

In ordinary 4-dimensional GR, the $F_{\alpha\beta\mu\nu} F^{\alpha\beta\mu\nu}$ term gives rise to a cosmological constant depending on an integration constant $[11], [12]$. In our case, due to the constraint (14), the degrees of freedom contained in $F_{\alpha\beta\mu\nu}$ and those of the scalar field $\phi$ will be intimately correlated. The sign in front of the $F_{\alpha\beta\mu\nu} F^{\alpha\beta\mu\nu}$ term is chosen so that in this model the resulting expression for the energy density of the scalar field $\phi$ is a positive definite one for any possible space-time dependence of $\phi$ in an effective ”Einstein picture” (see below). Notice also that two last terms in the action with the Lagrangian (28) break explicitly the LES.

The gravitational equations (13) take now the form

$$-\frac{1}{\kappa} R_{\mu\nu}(\Gamma) + \frac{1}{2} \phi, \mu \phi, \nu + \frac{1}{6} F_{\mu\alpha\beta\gamma} F^{\alpha\beta\gamma\nu} = 0. \quad (31)$$

Notice that the scalar field potential $V(\phi)$ does not appear explicitly in Eqs. (31). However, the constraint (14), which takes now the form

$$V(\phi) + M = -\frac{1}{8} F_{\alpha\beta\mu\nu} F^{\alpha\beta\mu\nu}, \quad (32)$$

allows us to express the last term in (31) in terms of the potential $V(\phi)$ (using the fact that $F_{\alpha\beta\mu\nu} F^{\alpha\beta\mu\nu} \propto \epsilon^{\alpha\beta\mu\nu}$ in 4-dimensional space-time).

Varying the action with respect to $A_{\nu\alpha\beta}$, we get the equation

$$\partial_\nu (\Phi F^{\mu\nu\alpha\beta}) = 0 \quad (33)$$

Its general solution has a form

$$F_{\alpha\beta\mu\nu} = \frac{\lambda}{\Phi} \epsilon_{\alpha\beta\mu\nu} = \frac{\lambda}{\lambda \sqrt{-g}} \epsilon_{\alpha\beta\mu\nu}, \quad (34)$$

where $\lambda$ is an integration constant. Then $F_{\alpha\beta\mu\nu} F^{\alpha\beta\mu\nu} = -\lambda^2 4! / \lambda^2$ is not a constant now as opposed to the GR case [11], [12] and therefore

$$V(\phi) + M = 3\lambda^2 / \lambda^2 \quad (35)$$

and
\[ F_{\mu\alpha\beta}F^{\rho\beta\gamma} = -(6\lambda^2/\chi^2)g_{\mu\nu} = -2[V(\varphi) + M]g_{\mu\nu} \tag{36} \]

This shows how the potential \( V(\varphi) \) appears in Eq. \( (31) \), spontaneously violating the symmetry of the action \( V(\varphi) \rightarrow V(\varphi) + \text{constant} \), which now corresponds to a redefinition of the integration constant \( M \).

The equation of motion of the scalar field \( \varphi \) is

\[ (-g)^{-1/2}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\varphi) + \sigma_{\mu\nu} \varphi^\mu + V'(\varphi) = 0, \tag{37} \]

where \( V' = \frac{dV}{d\varphi} \).

The derivatives of the field \( \sigma \) enter both in the gravitational Eqs. \( (31) \) (through the connection) and in the scalar field equation \( (37) \). In order to see easily the physical content of this model, we have to perform a conformal transformation

\[ g_{\mu\nu}(x) = \chi g_{\mu\nu}(x); \quad \varphi \rightarrow \varphi \tag{38} \]

to obtain an "Einstein picture". Notice that now this transformation is not a symmetry and indeed changes the form of equations. In this new frame, the gravitational equations become

\[ G_{\mu\nu}(\overline{g}_{\alpha\beta}) = \frac{\kappa}{2} T_{\mu\nu}^{\text{eff}}(\varphi) \tag{39} \]

where

\[ G_{\mu\nu}(\overline{g}_{\alpha\beta}) = R_{\mu\nu}(\overline{g}_{\alpha\beta}) - \frac{1}{2} \overline{g}_{\mu\nu} R(\overline{g}_{\alpha\beta}) \tag{40} \]

is the Einstein tensor in the Riemannian space-time with metric \( \overline{g}_{\mu\nu} \), and the source is the minimally coupled scalar field \( \varphi \)

\[ T_{\mu\nu}^{\text{eff}}(\varphi) = \varphi_{,\mu} \varphi_{,\nu} - \frac{1}{2} \overline{g}_{\mu\nu} \varphi_{,\alpha} \varphi_{,\alpha} + \overline{g}_{\mu\nu} V_{\text{eff}}(\varphi) \tag{41} \]

with the new effective potential

\[ V_{\text{eff}} = \frac{2}{3} \chi^{-1}(V + M) \tag{42} \]

The scalar field equation \( (37) \) in the Einstein picture takes a form

\[ \frac{1}{\sqrt{-g}} \partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\varphi) + \chi^{-1}V'(\varphi) = 0. \tag{43} \]

For the possible expression for \( \chi^{-1} \) we have from Eq. \( (35) \)

\[ \frac{1}{\chi} = \pm \frac{1}{\lambda \sqrt{3}} (V + M)^{1/2} \tag{44} \]

Independently of the sign chosen for \( \chi \) in Eq. \( (44) \), the gravitational and scalar field equations have the same physical content expressed in different space-time signatures. In what follows we simply take the + sign in \( (44) \). Therefore, the effective scalar field potential \( (44) \) has the form

\[ V_{\text{eff}}(\varphi) = \frac{2}{\lambda^3 \sqrt{3}} (V + M)^{3/2} \tag{45} \]

and the scalar field Eq. \( (37) \) becomes a conventional general relativistic scalar field equation with the potential \( V_{\text{eff}}(\varphi) \):

\[ \frac{1}{\sqrt{-g}} \partial_{\mu}(\sqrt{-g} \overline{g}^{\mu\nu}\partial_{\nu}\varphi) + V_{\text{eff}}'(\varphi) = 0. \tag{46} \]

We see that in the Einstein picture, for any analytic \( V(\varphi) \), \( V_{\text{eff}}(\varphi) \) has an extremum, that is \( V_{\text{eff}}'(\varphi) = 0 \), either when \( V' = 0 \) or \( V + M = 0 \). The extremum when \( V + M = 0 \) corresponds to an absolute minimum (since \( V_{\text{eff}}'(\varphi) \) is non negative) and therefore it is a vacuum with zero effective cosmological constant. It should be emphasized that all what is required is that \( V + M \) touches zero at some point \( \varphi_0 \) but \( V' \) at this point does not need to vanish. Therefore no
fine tuning in the usual sense, of adjusting a minimum of a potential to coincide with the point where this potential itself vanishes, is required. And the situation is even more favorable since even if $V + M$ happens not to touch zero for any value of $\phi$, we always have an infinite set of other values of the integration constant $M$ where this will happen.

In the context of the cosmology, for the Friedmann-Robertson-Walker universe where in the Einstein picture $\phi = \varphi(t)$ and

$$d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - \tau^2(t)dt^2, \quad dl^2 = [d\tau^2/(1 - k\tau^2) + r^2d\Omega^2],$$

(47)

we notice that due to the positivity of $V_{\text{eff}}$ constructed from an arbitrary analytic $V(\phi)$ according to Eq. (43), most of the known inflationary scenarios can for the very early universe can be implemented depending on the choice of the potential $V(\phi)$. It is very interesting that the parameters ruling the inflation are controlled by the integration constants $M$ and $\lambda$.

After inflation, when the scalar field $\phi$ approaches the position $\phi_0$ of the absolute minimum of the potential $V_{\text{eff}}$, i.e. when $V(\phi) + M \to 0$, the $\chi$-field approaches infinity as it seen from the constraint (35). To clarify the meaning of this effect, let us go back to the picture with the original $g_{\mu\nu}$ while still using the cosmic time $t$ that was defined in the Einstein picture. Then equation for $\phi$ is

$$\ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} \left( - \frac{3V'}{4(V + M)} \dot{\phi}^2 + \frac{\sqrt{V + M}}{\lambda}\frac{\chi}{\phi} V' \right) = 0,$$

(48)

where $a^2(t) = \tau^2(t)/\chi(t)$, $a_0(t) = 1/\chi(t)$ and constraint (35) have been used.

Generally, $\dot{\phi}$ does not go to zero as $\phi \to \phi_0$ (and $V(\phi) + M \to 0$). In this asymptotical region we can find the first integral of Eq. (48). Assuming that $V'(\phi_0) \neq 0$, i.e. without fine tuning, we get

$$\dot{\phi}a^3(t) \simeq c|V(\phi) + M|^{3/4}, \quad c = \text{const} \quad (\text{as} \quad \phi \to \phi_0),$$

(49)

which means that $a(t) \to 0$ as $\phi \to \phi_0$ (notice that if we would have chosen a coordinate frame in the original picture such that $ds^2 = dt^2 - a^2(t')dt^2$, then instead of (49) we would have gotten $a^3(t)\dot{\phi}/dt' \simeq c|V(\phi) + M|^{1/2}$ as $\phi \to \phi_0$). Then integrating the gravitational equations we get asymptotically (as $\phi \to \phi_0$) $a^2(t) = a_0^2/\chi(t)$, $a_0 = \text{const}$, that is in the original frame there is a collapse of the universe from a finite a to $a = 0$ in a finite time and therefore the Riemannian curvature goes to infinity as $\phi \to \phi_0$. This pathology is not seen in the Einstein frame due to the singularity of the conformal transformation $\tau^2 = \chi a^2$ at $\phi = \phi_0$. In fact, this is not a problem from the point of view of physics, since as $\phi \to \phi_0$ (and $V(\phi) + M \to 0$), the LES becomes restored at the critical point $\phi \equiv \phi_0$ where $V(\phi_0) + M = 0$. In the presence of the LES, the conformal transformation $\tau^2 = \chi g_{\mu\nu}(x)$ becomes part of the LES transformation and represents a nonsingular gauge choice for the metric $\tau^2$.

In contrast to this, there is a real problem in the scenario discussed above. Although the point $\phi_0$ where $V(\phi) + M = 0$ satisfies $V_{\text{eff}}(\phi_0) = 0$, $V_{\text{eff}}(\phi_0) = 0$ and furthermore it is an absolute minimum of the effective potential $V_{\text{eff}}(\phi)$, we can see however, that the second derivative of $V_{\text{eff}}(\phi)$ is equal to infinity at this point $\phi_0$. It means that there are no physical excitations of the scalar field $\phi$ around this minimum. This causes problems both in the cosmological picture when considering the possibility of small oscillations around the minimum and in the associated particle physics, since the mass of a scalar, like for example, the Higgs mass, will appear infinite. In Sec. V we will show how this problem is solved.

B. Gauge fields and the Higgs mechanism in the NGVE theory

Let us consider now a model including gravity (formulated in the first order formalism), four index field strength $F_{\alpha\beta\mu\nu}$, a gauge field $\hat{A}_\mu$ and a complex scalar field $\phi$ minimally coupled to the gauge field with the action

$$S = \int d^4x \left[ - \frac{1}{16\pi G} R(G, \phi) + \frac{1}{4} F_{\alpha\beta\mu\nu} F^{\alpha\beta\mu\nu} + \frac{1}{m^4} (\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right. + g^{\mu\nu}(\partial_\mu - i\tilde{e} \hat{A}_\mu) \phi (\partial_\nu + i\tilde{e} \hat{A}_\nu) \phi^* - V(|\phi|) \left. \right]$$

(50)

where $\tilde{F}_{\mu\nu} \equiv \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$.

Notice that the kinetic term of the gauge field $\hat{A}_\mu$ is chosen in an unusual way where an additional parameter $m$ with dimensionality of mass is introduced to provide the canonical dimensionality for the gauge field $\hat{A}_\mu$. The reason
for such a choice of the kinetic term of the gauge field is to achieve for it the same degree of homogeneity in $g^{\mu \nu}$ in the Lagrangian density as we have for the four index field strength $F_{\alpha \beta \mu \nu}$. As we will see, for such a choice, after solving the constraint we obtain the standard effective low energy physics. For example, in the absence of other interactions, the gauge field equations possess conformal invariance or, what is the same, they have the standard Maxwell form.

By making use of the gauge invariance we choose the unitary gauge (where $Im \phi(x) = 0$) and then the Lagrangian density takes the form

$$L = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{4!} F_{\alpha \beta \mu \nu} F^{\alpha \beta \mu \nu} + \frac{1}{m^4} (\ddot{F}_{\mu \nu} \ddot{F}^{\mu \nu})^2 + \frac{1}{2} g^{\mu \nu} \varphi_{, \mu} \varphi_{, \nu} - V(\varphi) + \frac{1}{2} \varphi^2 g^{\mu \nu} \dddot{A}_\mu \dddot{A}_\nu$$

(51)

where we have defined $|\phi| = \frac{1}{\sqrt{2}} \varphi$.

The constraint (14) corresponding to the Lagrangian density (51) is

$$\frac{1}{8} F_{\alpha \beta \mu \nu} F^{\alpha \beta \mu \nu} + \frac{3}{m^2} (\dddot{F}_{\mu \nu} \dddot{F}^{\mu \nu})^2 + V(\varphi) + M = 0$$

(52)

Similar to the first and the fourth terms, the last term in Eq. (51) does not contribute to the constraint since it is homogeneous of degree one in $g_{\mu \nu}$.

The equation for $F_{\alpha \beta \mu \nu}$ and its solution are still the same as in Eqs. (33), (34) which bring constraint (52) to the following equation:

$$\frac{\omega^2 m^4}{\chi^2} = \frac{1}{m^4} (\dddot{F}_{\mu \nu} \dddot{F}^{\mu \nu})^2 + \frac{1}{3} (V(\varphi) + M)$$

(53)

where we have defined $\lambda$ in terms of the mass parameter $m^2$

$$\lambda = \omega m^2$$

(54)

$\omega$ being a dimensionless constant.

Varying the action with respect to $\dddot{A}_\mu$ we get

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \chi (\dddot{F}_{\alpha \beta} \dddot{F}^{\alpha \beta}) g^{\mu \nu} g^{\rho \delta} \dddot{F}_{\rho \delta}) - \frac{\omega^2 m^4}{8} \varphi^2 g^{\mu \nu} A_\alpha = 0$$

(55)

Looking at gauge field fluctuations around the true vacuum $\varphi = \varphi_0$ where $V(\varphi_0) + M = 0$ and $V'_\text{eff}(\varphi_0) = 0$ (and ignoring the scalar field fluctuations around the true vacuum $\varphi_0$, see the previous subsection), we get from (53)

$$\chi(\dddot{F}_{\mu \nu} \dddot{F}^{\mu \nu}) = \pm \omega m^4$$

(56)

First of all notice that in the case where is no coupling to the scalar field ($\dddot{c} = 0$), Eq. (55) after making use Eq. (56) becomes the Maxwell’s equations

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} g^{\rho \delta} \dddot{F}_{\rho \delta}) = 0$$

(57)

which are indeed conformally invariant.

In the presence of interactions with scalar field ($\dddot{c} \neq 0$), Eqs. (55) and (56) lead to the singularity in the second term when $\dddot{F}_{\alpha \beta} \dddot{F}^{\alpha \beta} = 0$.

It is worthwhile to remind that in the context of cosmology we studied in Subsection IVA, the $\chi$-field becomes divergent as $\varphi$ approaches the absolute minimum $\varphi_0$. Therefore it is not a surprise that a singularity also occurs in the case where $\dddot{F}_{\alpha \beta} \dddot{F}^{\alpha \beta} = 0$, that is when electric dominated field evolves into magnetic dominated one (or vice versa). Similar to what happens in the cosmological scenario, here we will see that this singularity is eliminated by the same conformal transformation (38), that is by transforming to the Einstein frame.

In fact, performing the conformal transformation (38) and taking into account the constraint (56) we obtain for the gauge field equation in the Einstein frame

$$\frac{1}{\sqrt{-g}} \partial_\mu (\pm \sqrt{-g} g^{\mu \nu} g^{\rho \delta} \dddot{F}_{\rho \delta}) - \frac{\omega^2}{8} \varphi^2 g^{\mu \nu} A_\alpha = 0$$

(58)
Notice that we have not taken the sign ± in Eq. (58) outside the derivative operator. This makes it apparent that if we pick one of the two branches displayed in Eq. (58), it cannot evolve continuously to the another branch. Therefore the requirement of the analyticity of the resulting equations exclude for example the alternative $\chi|\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}| = \omega m^4$ instead of (53) as a solution of the constraint (53) at the point $\varphi = \varphi_0$.

When choosing one of the branches in Eq. (58) we note that the conformal transformation (38) changes the relative signatures of the original metric $g_{\mu\nu}$ and the metric $\tilde{g}_{\mu\nu}$ in the Einstein frame when the gauge field evolves from electric dominated to magnetic dominated (and vice versa).

Similar to what happens in the case of cosmology (see Subsection IVA), Eq. (58) shows that in the Einstein frame there is no singularity when $\chi^{-1} \to \pm 0$. Therefore, changes of the signature of the metric take place only in the original frame and not in the Einstein frame.

For the choice of the signature $(+ - - -)$ in the Einstein frame we have to choose the branch ($-$) in Eq. (58) in order to avoid tachyonic behavior. After the change of notations $e = 2\sqrt{2}\omega; A_{\mu} = 2\sqrt{2}\omega \tilde{A}_{\mu}$; (59)

we get the canonical form of equations for the vector field

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g} \tilde{g}^{\nu\rho} \tilde{F}_{\rho\nu}) + m_A^2 A^\nu = 0$$

where

$$m_A^2 = e^2 \varphi_0^2$$

is the mass of the vector boson $A_{\mu}$ which is generated by the spontaneous symmetry breaking (SSB) of the gauge invariance when the scalar field $\phi$ is located at the absolute minimum $|\phi| = \frac{1}{\sqrt{2}} \varphi = \frac{1}{\sqrt{2}} \varphi_0$ of the effective potential (45). We have used the notations

$$A^\mu = \tilde{g}^{\mu\nu} A_\nu; \; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu; \; F^{\mu\nu} = \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} F_{\alpha\beta}$$

The appropriate gravitational equations at the absolute minimum $\varphi_0$ in the Einstein frame, when the source is the vector field $A_{\mu}$ takes the standard form

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{\kappa}{2} T_{\mu\nu}(A_\alpha)$$

where

$$T_{\mu\nu}(A_\alpha) = \frac{1}{4} \tilde{g}_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - F_{\mu\alpha} F_{\nu\beta} \tilde{g}^{\alpha\beta} + \frac{1}{2} m_A^2 (A_{\mu} A_{\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} A_{\alpha} A^\alpha)$$

For the scalar field equation in the Einstein frame we obtain

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g} \tilde{g}^{\mu\nu} \partial_\nu \varphi) - e^2 A_{\mu} A^\mu \varphi + V_{eff} = 0$$

with $V_{eff}$ given by Eq. (45).

As we see, the rescaling (59) provides the canonical normalization of the gauge field so to reproduce the standard form of the energy-momentum tensor (64) and standard interaction of the vector field $A_{\mu}$ to the scalar field after SSB. A very interesting feature of the theory in the Einstein picture is the fact that the gauge coupling constant $e$ depends on the integration constant $\omega$ which appears in the solution for the four index field condensate (34) (see also the definition (54).

Finally we have to notice that it is possible to improve the model discussed in this Sec.IV (see for example [6]) in such a way that the mass of the scalar field becomes finite (compare with discussion at the end of Subsec.IVA). Then, however, at the very high energies where the scalar field fluctuations around the vacuum $\varphi = \varphi_0$ have to be taken into account, it follows from the constraint (53) and Eq. (55) that the nonminimal nonrenormalizable interaction of the gauge field with the scalar field appears.
V. UNIFIED GAUGE SECTOR AND MODELS WITH REALISTIC PARTICLE FIELDS DYNAMICS AND COSMOLOGY

As we have seen in Sec. IV, it is possible to realize a nontrivial dynamics of a scalar field and a gauge field while solving the cosmological constant problem. However, the simple standard models presented in Sec. IV give rise to nonrenormalizable coupling between gauge and scalar fields. We are going to show in this section that by a certain sort of unification of all gauge fields we can get rid of the above mentioned problem.

The basic idea consists of demanding that the dependence of the Lagrangian density on the gradients of the gauge field potentials is only through a single variable which is the sum of all possible kinetic terms and a corresponding four index field strength term. The fact that all gauge fields must come together is automatic in a unified gauge models where the Lagrangian density must depend only on $F^a_{\mu \nu}$, where $a$ is for example an $SU(5)$ index. In addition we insist also in introducing the three index potential $A_{\mu \nu \alpha}$ into the game in a similar way.

$\frac{\delta y}{\sqrt{-g}} \partial_{\mu} A_{\nu \alpha \beta}$

We will call $y$ the gauge complex. Here $m$ is a parameter with the dimensions of mass.

The demand that the term which depends on the condensate of $A_{\mu \nu \alpha}$, has to have the same transformation under $g_{\mu \nu} \rightarrow \Omega g_{\mu \nu}$ as the ordinary gauge fields, finds a simple analogy in a related higher dimensional picture where the components of the gauge field strengths in the direction of the extra dimensions can play a similar role to that of the $A_{\mu \nu \alpha}$ field in 4-dimensional space-time. In 6-dimensional case for example, $F_{\mu \nu} F^{\mu \nu} \equiv F_{ab} F_{ab} + 2 F_{a \mu} F_{a \mu} + F_{ab} F_{ab}$, where $\mu, \nu = 0, 1, 2, 3$; $a, b = 4, 5$ and $F_{ab} F_{ab}$ plays then the role of $\sqrt{-g} \partial_{\mu} A_{\nu \alpha \beta}$ in the condensate state $|13\rangle$ (here $F_{ab}$ takes a "magnetic monopole" expectation value). Then the requirement of equal behavior under conformal transformation is of course automatic.

Another independent problem we have discussed in Sec. IVA, is the infinite value of masses of scalar fields. Concerning to the resolution of this problem we have device two approaches: a) The first one (see Subsection VA) uses a critical limit of a family of Lagrangians. In this limit the scalar field acquires a finite mass $|6\rangle$. b) The second approach (see Subsection VB), which seems more generic, is based on the appearance of a "persistent condensate".

A. Model with a critical limit

Following the guidelines described above we consider Lagrangians which depend on the ordinary gauge field $\tilde{A}_\mu$ and the three index gauge field $A_{\mu \nu \alpha}$ only through the gauge complex $y$, Eq.(66). At first we consider the interesting family of Lagrangians with a power low dependence on $y$. Therefore, in the unitary gauge for $\tilde{A}_\mu$, instead of dealing with (50),(51), we have the action

$$S = \int \Phi d^4x \left[ -\frac{1}{\kappa} R(\Gamma, g) - \frac{1}{pm^{4(p-1)}y^p} + \frac{1}{2} g^{\mu \nu} \varphi_{,\mu} \varphi_{,\nu} - V(\varphi) + \frac{1}{2} e^2 \varphi^2 g^{\mu \nu} \tilde{A}_\mu \tilde{A}_\nu \right]$$

where dimensionless parameter $p$ is a real number. As we will see later, the physically interesting case is achieved in the critical limit $p \rightarrow \infty$.

Constraint (14) takes now the form

$$\frac{2p - 1}{pm^{4(p-1)}y^p} = V(\varphi) + M$$

$^1$The other possibility including $\sqrt{-F_{\mu \nu \alpha \beta} F^{\mu \nu \alpha \beta}} \equiv \frac{\sqrt{-g}}{\sqrt{-g}} \partial_{\mu} A_{\nu \alpha \beta}$ is not analytic one.
which defines \( y \) as a function of \( \phi \).

From variation with respect to \( A_{\nu \alpha \beta} \) we obtain the equation

\[
\partial_\mu (\chi y^{p-1} e^{\nu \alpha \beta}) = 0
\]  

(69)

the solution of which can be written as

\[
\chi y^{p-1} = \omega m^{4(p-1)}
\]  

(70)

where \( \omega \) is a dimensionless integration constant.

Varying with respect to the scalar field \( \phi \) we get

\[
(\omega g)^{-1/2} \partial_\mu (\sqrt{-g} g^\mu \nu \partial_\nu \phi) + \sigma_{\mu \nu} \varphi^\mu + V'(\phi) + e^2 \varphi^\alpha A_\alpha A_\beta = 0.
\]  

(71)

The equation for the gauge field \( \tilde{A}_\alpha \) is

\[
\frac{1}{\sqrt{-g}} \partial_\mu [\chi y^{p-1} \sqrt{-g} \tilde{F}^\mu \nu] + \tilde{e}^2 \varphi^2 \chi \tilde{A}_\nu = 0
\]  

(72)

which becomes

\[
\frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} \tilde{F}^\mu \nu] + \frac{\tilde{e}^2}{4} \varphi^2 \chi \tilde{A}_\nu = 0
\]  

(73)

due to Eq. (70). And finally, the variation of \( g^{\mu \nu} \) leads to the gravitational equations

\[
\frac{1}{\kappa} R_{\mu \nu} (g, g) = - \frac{y^{p-1}}{2 m^{4(p-1)}} g_{\mu \nu} + \frac{y^{p-1}}{m^{4(p-1)}} \left[ \frac{1}{2} \tilde{F}^\alpha \beta F_{\alpha \beta} g_{\mu \nu} - 2 \tilde{F}_{\mu \alpha} \tilde{F}_{\nu \beta} g^{\alpha \beta} \right] + \frac{1}{2} \varphi^\mu \varphi^\nu + \tilde{e}^2 \varphi^2 \tilde{A}_\mu \tilde{A}_\nu
\]  

(74)

where Eq. (13) has been used.

For the same reasons as those explained in Sec. IV, we have to perform the conformal transformation (38) which provides a formulation of the theory in the Einstein picture. Eqs. (71), (73) and (74) become then correspondingly

\[
\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^\mu \nu \partial_\nu \phi) + \frac{dV^{(p)}}{d\phi} + e^2 \varphi^\alpha A_\alpha A_\beta = 0
\]  

(75)

\[
\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^\mu \nu \partial_\nu \phi) + e^2 \varphi^2 \tilde{g} A_\alpha A_\beta = 0
\]  

(76)

\[
G_{\mu \nu} (\tilde{g}_{\alpha \beta}) = \frac{\kappa}{2} T_{\mu \nu}
\]  

(77)

\[
T_{\mu \nu} = \varphi_\mu \varphi_\nu - \frac{1}{2} g_{\mu \nu} \varphi^\alpha \varphi_\alpha + V^{(p)} (\phi) g_{\mu \nu} + \frac{1}{4} g_{\mu \nu} F_{\alpha \beta} F^{\alpha \beta} - F_{\mu \alpha} F_{\nu \beta} \tilde{g}^{\alpha \beta} + e^2 \varphi^2 (A_\mu A_\nu - \frac{1}{2} g_{\mu \nu} A_\alpha A_\alpha)
\]  

(78)

where

\[
V^{(p)} (\phi) \equiv \left[ \omega m^{4(1-1/p)} \right]^{-1} \left( \frac{p}{2 p - 1} \right)^{2-1/p} (V(\phi) + M)^{2-1/p}
\]  

(79)

and the rescalings

\[
A_\mu \equiv 2 \sqrt{\omega} \tilde{A}_\mu; \quad e = \frac{\tilde{e}}{2 \sqrt{\omega}}
\]  

(80)

have been performed. It is assumed that \( \omega > 0 \).

Equations (75)-(78) describe the family of canonical equations of GR (parametrized by the parameter \( p \)) for a gauge model (in the unitary gauge) including gauge field \( A_\mu \) minimally coupled to the scalar field \( \phi \) with the potential (79) and the coupling constant \( e \). For \( p = 1 \) (which has to be studied by itself), the theory reproduces the Einstein GR
with the original potential \( V(\varphi) \) and with a cosmological constant \( M \) (see Ref. [3]). In contrast, for any \( p > 1 \), the effective potential \( V^{(p)}_{\text{eff}}(\varphi) \), given by Eq. (79), has an absolute minimum at the point \( \varphi_0 \) where \( V(\varphi_0) + M = 0 \), which generalizes the results of Sec. IV.

However an additional remarkable feature appears when \( p \to \infty \). In this case

\[
V_{\text{eff}}(\varphi) \equiv \lim_{p \to \infty} V^{(p)}_{\text{eff}}(\varphi) = \frac{1}{4\omega m^4} (V + M)^2
\]

and for any analytical function \( V(\varphi) \), all derivatives of the effective potential \( V_{\text{eff}}(\varphi) \) are finite at the absolute minimum \( \varphi = \varphi_0 \) where \( V(\varphi_0) + M = 0 \). In particular, \( V_{\text{eff}}(\varphi_0) \propto [V'(\varphi_0)]^2 \) is finite (and nonzero if we do not carry out the fine tuning \( V'(\varphi_0) = 0 \)). Therefore the Higgs boson, in particular, can reappear as a physical particle of the theory. In the context of cosmology where \( V_{\text{eff}}(\varphi) \) plays the role of the inflaton potential, a finite mass of the inflaton allows to recover the usual oscillatory regime of the reheating period after inflation that are usually considered.

Notice that again (as it was in Sec. IV) the gauge coupling constant \( e \) theory. In the context of cosmology where \( V_{\text{eff}}(\varphi) \) generalizes the results of Sec. IVB and VA and for any analytical function \( V(\varphi) \), all derivatives of the effective potential \( V_{\text{eff}}(\varphi) \) are finite at the absolute minimum \( \varphi = \varphi_0 \) where \( V(\varphi_0) + M = 0 \). In particular, \( V_{\text{eff}}(\varphi_0) \propto [V'(\varphi_0)]^2 \) is finite (and nonzero if we do not carry out the fine tuning \( V'(\varphi_0) = 0 \)). Therefore the Higgs boson, in particular, can reappear as a physical particle of the theory. In the context of cosmology where \( V_{\text{eff}}(\varphi) \) plays the role of the inflaton potential, a finite mass of the inflaton allows to recover the usual oscillatory regime of the reheating period after inflation that are usually considered.

Notice that again (as it was in Sec. IV) the gauge coupling constant \( e \) depends on the integration constant \( \omega \) which appears in the solution for the four index field condensate (70).

From Eqs. (85) and (71) it follows that

\[
\chi = \left(2 - \frac{1}{p}\right)^{1-1/p} \omega m^{4(1-1/p)} (V + M)^{1+1/p}
\]

so that if \( p > 1 \) or \( p < 0 \) we obtain that \( \chi \to \infty \) as \( V + M \to 0 \) which generalizes the situation described in Subsection IVA.

It is very instructive to look at what happens to the condensate \( y \) when we approach the true vacuum \( \varphi = \varphi_0 \) where \( V(\varphi_0) + M = 0 \). For any finite \( p > 1 \) we see from Eq. (85) that \( y = (V + M)^{1/p} m^{4-4/p} (\frac{p-1}{p})^{1/p} \) and \( y \to 0 \) in this limit. We notice however the very interesting effect which consists of the fact that as \( p \) becomes big, \( y \) approaches zero but at a very slow rate. In the limit \( p \to \infty \) we can indeed argue that \( y \) does not necessarily approaches zero but rather to an undetermined constant since \( y \sim 0^0 \) which is not defined. This suggests that the possible existence of a condensate \( y \) that survives even in the true vacuum (which we will call "persistent condensate") is the cause of the remarkable feature which allows \( V_{\text{eff}} \) to be of the form (83). In the next subsection we will indeed verify this explicitly.

**B. Model with persistent condensate**

To implement the suggestion discussed at the end of the previous subsection let us consider a model with the action (expressed in the unitary gauge)

\[
S = \int \Phi d^4x \left[ -\frac{1}{\kappa} R(\Gamma, g) - m^4 f(u) + \frac{1}{2} g_{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - V(\varphi) + \frac{1}{2} \varphi^2 g_{\mu\nu} \hat{A}_\mu \hat{A}_\nu \right]
\]

(83)

where we have used the notations of Secs. IVB and VA and \( f(u) \) is a function of the dimensionless argument \( u = y/m^4 \).

We will see that the only requirement condition on the function \( f(u) \), that provides a persistent condensate with physically reasonable consequences, is that \( f'(u) \equiv \frac{df}{du} = 0 \) for some \( y = y_0 > 0 \).

The constraint (14) has now the form

\[
-2u f'(u) + f(u) + \frac{1}{m^4} [V(\varphi) + M] = 0.
\]

(84)

Varying with respect to \( A_{\nu\alpha\beta} \) we get

\[
\partial_{\mu}(\chi f' \varepsilon^{\mu\nu\alpha\beta}) = 0
\]

(85)

which gives

\[
\chi f' = \omega
\]

(86)

where \( \omega \) is a dimensionless integration constant and Eq. (85) replaces Eq. (70).

Equation for the gauge field \( \hat{A}_\mu \)
\[
\frac{1}{\sqrt{-g}} \partial_\mu [\chi' \sqrt{-g} F^{\mu \nu}] + \frac{\tilde{e}^2}{4} \varphi^2 \chi' A^\nu = 0
\]  
(87)

is reduced exactly to the form of Eq. (73) due to Eq. (86).

The gravitational equations originated by the variation of \( g^{\mu \nu} \) take the form

\[
\frac{1}{\kappa} R_{\mu \nu}(\Gamma, g) = -\frac{1}{2} y f' g_{\mu \nu} + \frac{1}{2} f' \left[ \tilde{F}^{\alpha \beta} \tilde{F}_{\alpha \beta} g_{\mu \nu} - 4 \tilde{F}_{\mu \alpha} \tilde{F}^{\alpha \beta} g_{\nu \beta} \right] + \frac{1}{2} \tilde{e}_\mu \varphi \varphi_{,\nu} + \frac{\tilde{e}^2}{2} \varphi^2 A_{\mu} A_{\nu}
\]  
(88)

after using Eq. (66).

The scalar field equation has the same form as Eq. (71).

Performing the conformal transformation (38) to the Einstein frame and rescaling \( \tilde{A}_\mu \) and \( \tilde{e}_2 \) to \( A_\mu \) and \( e_2 \) by making use Eqs. (80) with \( \omega > 0 \), we obtain equations of the form of Eqs. (75)-(78) where

\[
\frac{d V_{\text{eff}}}{d \varphi} \equiv V_{\text{eff}}' = \frac{1}{\omega} \left. \frac{d f}{d u} \right|_{u = u_0} \frac{d V}{d \varphi}
\]  
(89)

and instead of \( V_{\text{eff}}^p \) in the gravitational Eqs. (78), the effective scalar field potential appears of the form

\[
V_{\text{eff}}(\varphi) = \frac{y}{\omega} (f(u))^2
\]  
(90)

Here \( y \equiv y/m^4 \) and \( f(u) \) are functions of \( \varphi \) due to the constraint (84). As in the previous examples, it can be shown that two different form of appearance of \( V_{\text{eff}} \) and \( V_{\text{eff}}' \) in the gravitational field equations and in the scalar field equation correspondingly, are selfconsistent (the reason for this is the existence of Bianchi identities). This consistency also may be shown by taking the derivative of Eq. (90) with respect to \( \varphi \) and using the derivative of the constraint (84) with respect to \( \varphi \). As the result we obtain Eq. (89).

Looking at Eq. (89) we see that there are two ways of obtaining an extremum of \( V_{\text{eff}}(\varphi) \):

(a) The first one is when \( \frac{d V_{\text{eff}}}{d \varphi} = 0 \) which corresponds to an extremum of the original potential \( V(\varphi) \). In this case there is no reason for the vanishing \( V_{\text{eff}}(\varphi_0) = 0 \) if we do not resort to some kind unnatural fine tuning.

(b) The second way is to consider the situation where

\[
\left. \frac{d f}{d u} \right|_{u = u_0 \neq 0} = 0
\]  
(91)

and the appropriate value of \( \varphi_0 \) is related to \( u_0 \) by the constraint (84). For this extremum of the effective potential we see immediately from Eq. (90) that \( V_{\text{eff}}(\varphi_0) = 0 \).

If we assume that \( y_0 = \frac{m^4}{M} \) is positive then it is clear also from (90) that \( \varphi_0 \) (where \( V_{\text{eff}}(\varphi_0) = 0 \), \( \frac{d V_{\text{eff}}}{d \varphi} \big|_{\varphi = \varphi_0} = 0 \), is a minimum since any small fluctuations bring us to a higher positive value of \( V_{\text{eff}} \). In this case the vacuum is defined both by value of the gauge complex condensate \( u = u_0 \) and by the scalar condensate \( \varphi = \varphi_0 \) satisfying the condition

\[
f(u_0) + \frac{1}{m^4} [V(\varphi_0) + M] = 0.
\]  
(92)

which follows from the constraint (84) and Eq. (91). Notice that transition to the Einstein frame does not change the value of the gauge complex condensate \( u_0 \) since it is defined by the value of \( \varphi_0 \) due to Eq. (12).

It is very important to notice that Eq. (92) represents the exact mutual cancellation of the contributions to the vacuum energy of the integration constant \( M \), the scalar field condensate and the gauge complex condensate.

It can be shown explicitly (by using the constraint (84), its derivative and Eq. (89)) that the mass square of the scalar particle is

\[
V_{\text{eff}}'(\varphi_0) = \frac{1}{2 \omega y_0} [V'(\varphi_0)]^2.
\]  
(93)

which is positive if both \( \omega > 0 \) and \( y_0 > 0 \).
VI. THE INCLUSION OF FERMIONS

A. Fermions in the NGVE theory

To present a complete enough picture let us consider a model including gravity, gauge and scalar field sectors (as in the action (83), once again in the unitary gauge) and, in addition, the fermionic sector (see Appendix B):

\[ S = \int \Phi d^4x \left\{ -\frac{1}{\kappa} V^{a\mu} V_{b\nu} R_{\mu\nu a\omega}(\omega) - m^4 f(u) + \frac{1}{2} g^{\mu\nu} \varphi_{\mu \nu} \varphi_{\mu \nu} - V(\varphi) + \frac{1}{2} \varepsilon^2 \varphi^2 g^{\mu\nu} \tilde{A}_\mu \tilde{A}_\nu \right. \\
+ \left. \frac{i}{2} \bar{\Psi} \left\{ \gamma^a \gamma^\mu \left( \partial_\mu - i e A_\mu \right) + \frac{1}{4} \omega^{\mu \nu} \left( \sigma^{\nu a} + \sigma_{a b} \gamma^b \right) V_a^\mu \right\} \Psi + U(\bar{\Psi} \Psi) \right\} (94) \]

where the selfinteraction term \( U(\bar{\Psi} \Psi) \) depending on the argument \( \bar{\Psi} \Psi \) remains unspecified in this subsection.

Equation for \( \Psi \) which follows from the action (94) is

\[ \left\{ i \left[ V_a^\mu \gamma^a \left( \partial_\mu - i e A_\mu \right) + \gamma^a C_{ab} + \frac{1}{4} \omega^{\mu \nu} \left( \sigma^{\nu a} + \sigma_{a b} \gamma^b \right) V_a^\mu \right] + \frac{1}{2} \gamma^a V_a^\mu \sigma_{a \mu} + U'(\bar{\Psi} \Psi) \right\} \Psi = 0 \] (95)

where

\[ C_{ab}^\mu = \frac{1}{2 \sqrt{-g}} \partial_\mu \left( \sqrt{-g} V_a^\mu \right) \] (96)

is the trace of the so-called Ricci rotation coefficients \( \frac{\mu}{4} \) and \( U' \) is derivative of \( U \) with respect to its argument \( \bar{\Psi} \Psi \). Spin-connection \( \omega^{\mu \rho}_{\rho} \) is defined by Eqs. (C4)-(C7). Remind that \( \sigma \)-field is defined by Eq. (19).

After the transition to the Einstein frame by means of the conformal transformations

\[ V_{a\mu}^\prime(x) = \chi^{1/2}(x) V_{a\mu}(x) \] (97)

\[ \Psi^\prime(x) = \chi^{-1/4}(x) \Psi(x); \quad \bar{\Psi}^\prime(x) = \chi^{-1/4}(x) \bar{\Psi}(x) \] (98)

\[ \varphi \rightarrow \varphi; \quad \tilde{A}_\mu \rightarrow \tilde{A}_\mu \] (99)

and performing the rescalings (80), Eq. (83) is reduced to the form

\[ \left\{ i \left[ V_a^\mu \gamma^a \left( \partial_\mu - i e A_\mu \right) + \gamma^a C_{ab} + \frac{1}{4} \omega^{\mu \nu} \left( \sigma^{\nu a} + \sigma_{a b} \gamma^b \right) V_a^\mu \right] + \chi^{-1/2} U'(\bar{\Psi} \Psi) \right\} \Psi' = 0 \] (100)

where \( U'(\bar{\Psi} \Psi) = U'(\bar{\Psi} \Psi)\psi = \chi^{-1/4} \psi; \quad \bar{\Psi} = \chi^{1/4} \bar{\Psi} \) and \( \chi \) enters only as a factor in front of \( U'(\bar{\Psi} \Psi) \). In Eq. (100),

\[ C_{ab}^\mu = \frac{1}{2 \sqrt{-g}} \partial_\mu \left( \sqrt{-g} V_a^\mu \right) \] (101)

and the spin-connection \( \omega^{\mu \rho}_{\rho} \) is

\[ \omega^{\mu \rho}_{\rho} = \omega^{\mu \rho}_{\mu}(V_a^\rho) + K^{\mu \rho}(V_a^\rho, \Psi^\prime, \bar{\Psi}^\prime) \] (102)

Notice that after the conformal transformation (87)-(89), the \( \sigma \)-contribution to the spin-connection (the second term in the r.h.s. of Eq. (C4)) is canceled.

Notice also that once again we performed the rescaling (80) in order to provide the standard form of the appropriate equations for the gauge and gravitational fields.
As it is shown in Appendix B, the famous Nambu - Jona-Lasinio (NJL) model \[15\] works in a special way in the context of the NGVE theory: in the theory with only fermionic matter, the constraint does not impose restrictions on dynamics (see also \[3\]).

If we generalize the model towards a realistic theory by the consideration both bosonic and fermionic sectors as in the previous subsection, then still the fermionic part does not contribute to the constraint if we restrict ourself to the NJL - type model. In this case the constraint works exactly as in Sec. V where bosonic sector breaks the LES explicitly and the constraint becomes an equation for $\chi$.

In the Einstein frame, the NJL term (see the last term in Eq. (100)) remains its original form in terms of the transformed fields $\Psi'$ and $\overline{\Psi}$.

As it is well known \[16\], the the NJL mechanism allows for a dynamical mass generation in realistic models. Since in the Einstein frame the theory takes the canonical form, therefore the same mechanisms of the dynamical mass generation can be applied also here.

C. Classical model for generating mass of fermions

As we have discussed in the previous subsection, there is a possibility to generate masses of fermions in the NGVE theory due to the quantum effect in the NJL model. Now we are going to show that the NGVE theory in the context of the model with the action (94) allows for a purely classical mechanism of obtaining fermion masses.

The constraint \[137\] corresponding to the model \[14\] is

$$\nabla\Psi U' (\nabla\Psi) - 2U(\nabla\Psi) - 2 [m^4 (2u f''(u) - f(u)) - (V(\varphi) + M)] = 0 \quad (103)$$

where Eq. \[110\] has been used.

Directing our attention to the situation where the scalar field excitations around the vacuum $\{u = u_0, \varphi = \varphi_0$ with condition \[123\], (see Sec.VB) are ignored, we get the constraint for the first order fluctuations of $\Psi$ and $\pi \equiv u - u_0$

$$\nabla\Psi U' (\nabla\Psi) - 2U(\nabla\Psi) - 4m^4u_0f''(u_0)\pi = 0 \quad (104)$$

where Eq. \[121\] has been used.

We consider now the model with fermionic selfinteraction of the form

$$U(\nabla\Psi) = -C(\nabla\Psi)^q, \quad C = km^{4-3q} \quad (105)$$

where $k > 0$ is a dimensionless parameter.

Remembering that our aim is to study the theory in the Einstein frame, we first calculate $\chi$. From Eq. \[130\] we have $\chi = \frac{\omega}{\chi}$ and expanding $f'$ around $u = u_0$ we get $f' = f'(u_0) + f''(u_0)\pi + \ldots = f''(u_0)\pi$. $\pi$ can be obtained from Eq. \[104\] and for special form \[105\] we obtain, after a simple computation, the following equation for $\chi$

$$\frac{\omega}{\chi} = \frac{C}{4u_0m^4} \left[ (2 - q)\chi^{q/2} (\nabla' \Psi')^q \right] \quad (106)$$

in terms of the fermion field $\Psi'$ in the Einstein frame (see Eq. \[128\]) that is

$$\chi = \left( \frac{C(2 - q)}{4\omega u_0 m^4} \right)^{q/2} \left( \nabla' \Psi' \right)^{-\frac{2q}{q-2}} \quad (107)$$

Finally, we observe that in the Einstein frame, the last term in Eq. \[104\] has the form of a mass term, with a mass $m_f$ given by

$$m_f = Cq\chi^{-1/2} (\nabla\Psi)^{q-1} = Cq\chi^{\frac{q-2}{q-2}} (\nabla' \Psi')^{q-1} \quad (108)$$

Such a term will be a legitimate mass term only if $m_f$ as given by Eq. \[108\] is a genuine constant. From Eqs. \[107\] and \[108\] we obtain that $m_f$ is given by
\[ m_f = C q \left( \frac{C(2 - q)}{4\omega u_0 m^4} \right)^{\frac{2(q - 1)}{q - 2}} \left( \overline{\psi} \psi' \right)^{\frac{q - 2}{q - 2}} \] (109)

We observe therefore that \( m_f \) is indeed a genuine constant if \( q = 2/3 \):

\[ m_f = \frac{m L^{3/2}}{\sqrt{3\omega u_0}} \] (110)

We conclude therefore that if we start with a fermion selfinteraction term in the original Lagrangian density (see Eq. (19)) of the form

\[ U(\overline{\psi}\psi) = -C(\overline{\psi}\psi)^{2/3}, \] (111)

we obtain normal propagation of a massive fermion in the (physical) Einstein frame.

One should notice at this point that a selfinteraction of the form \( (111) \) has a remarkable feature: such a term comes in the action with a coupling constant \( C = km^2 \) with dimensionality \( mass^2 \), just as we are used in the case of bosonic masses, like in the case of a vector meson mass term for example.

Notice that the same result, that is the fact that the form of the fermion selfinteraction \( (111) \) provides mass generation of fermions in the Einstein frame, is obtained also in the model with critical limit (see Sec. VA). In fact, if we start with the Lagrangian density similar to Eq. (11) and reformulate it in the vierbein - spin-connection formalism with adding the fermionic sector as it was done in Sec. VIA, then in the limit \( p \rightarrow \infty \), the selfinteraction \( (111) \) in the Einstein frame turns out to be the mass term of fermion where again \( m_f \propto \frac{m}{\sqrt{\omega}} \).

VII. UNIFIED GAUGE THEORIES IN THE CONTEXT OF NGVE THEORY

Now we able to formulate a realistic gauge theory. For the illustration of this we take \( SU(2) \times U(1) \) model of electroweak interaction. However, there are no obstacles to formulate QCD, GUT or any other spontaneously broken gauge model in the context of the NGVE theory. The common feature of such models in the NGVE theory is that the spontaneous symmetry breaking (SSB) in the vacuum \( \{ \varphi_0, y_0 \} \) does not generate the vacuum energy and therefore the cosmological constant is equal to zero in this vacuum.

The \( SU(2) \times U(1) \) model of electroweak interaction in the NGVE theory has to be considered together with gravitational interaction and the Lagrangian density is the following

\[ L = -\frac{1}{k} V^{\mu\nu} V_{\mu\nu} R_{\mu\nu} - m^4 f(u) + i \overline{\psi} \not{D} \psi + i \overline{\tau} R \not{D} \psi_R + i \overline{\tau} R \not{D} \psi_R \\
+|D_\mu \varphi|^2 - V(|\varphi|) - \lambda \sqrt{2} m^{4/3} (\overline{\tau} R \varphi + h.c.)^{2/3} \] (112)

In Eq. (112) we used notations of Ref. 17: \( SU(2) \) vector gauge field \( \tilde{A}_\mu = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3) \); \( U(1) \) abelian gauge field \( \tilde{B}_\mu \); Left lepton dublet

\[ L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \]

Right \( SU(2) \) singlets \( \nu_R \) and \( e_R ; SU(2) \) scalar fields dublet

\[ \varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \]

and corresponding antiparticles \( \varphi^\dagger = (\varphi^-, \varphi^0) \). The left and right components of fermions are defined by \( \Psi_L = \frac{1}{2}(1 + \gamma_5) \psi \) and \( \Psi_R = \frac{1}{2}(1 - \gamma_5) \psi \) correspondingly; \( D_\mu = \partial_\mu - ig \tilde{T} A_\mu - ig' \tilde{Y} \tilde{B}_\mu \) where the hypercharge \( Y \) is: \( Y = -1 \) for \( \nu_L \) and \( e_L \); \( Y = -2 \) for \( e_R \). \( \tilde{T} = \frac{1}{2} \tilde{t} \) for \( \varphi \) and \( L \). For isoscalar fields \( e_R \) and \( \nu_R \), \( T = 0 \). The last term in (113) is written in the form which provides us the mechanism for the fermion mass generation described in Sec. VIC and accompanied by SSB. Parameter \( \lambda \) in the last term of Eq. (112) is dimensionless coupling constant.

Operator \( \not{D} \) in the third term \( \overline{\tau} R \not{D} \psi \) of Eq. (112) is defined as follows:
\[ \mathcal{D} = \overline{\mathcal{D}}_L - \overline{\mathcal{D}}_L \]

where

\[ \overline{\mathcal{D}}_L = \frac{1}{2} V_\alpha^\mu \gamma^a \left( \partial_\mu + \frac{1}{2} \sigma^a_{cd} I_{cd} - \frac{i}{2} \tilde{g} \tilde{A}_\mu + \frac{i}{2} \tilde{g} \tilde{B}_\mu \right) \]

\[ \overline{\mathcal{D}}_L = \frac{1}{2} \left( \partial_\mu - \frac{1}{2} \sigma^a_{cd} I_{cd} + \frac{i}{2} \tilde{g} \tilde{A}_\mu - \frac{i}{2} \tilde{g} \tilde{B}_\mu \right) \gamma^a V_\alpha^\mu \]

where \( I \) is \( 2 \times 2 \) unit matrix in the isospin space. The forth and the fifth terms in Eq. (112) are defined by equations:

\[ \overline{\mathcal{D}} e_R = \frac{1}{2} V_\alpha^\mu \gamma^a \left( \partial_\mu + \frac{1}{2} \sigma^a_{cd} I_{cd} + i \tilde{g} \tilde{A}_\mu \right) \]

\[ \overline{\mathcal{D}} e_R = \frac{1}{2} \left( \partial_\mu - \frac{1}{2} \sigma^a_{cd} I_{cd} - i \tilde{g} \tilde{B}_\mu \right) \gamma^a V_\alpha^\mu \]

\[ \overline{\mathcal{D}} \nu R = \frac{1}{2} V_\alpha^\mu \gamma^a \left( \partial_\mu + \frac{1}{2} \sigma^a_{cd} I_{cd} \right) \]

\[ \overline{\mathcal{D}} \nu R = \frac{1}{2} \left( \partial_\mu - \frac{1}{2} \sigma^a_{cd} I_{cd} \right) \gamma^a V_\alpha^\mu \]

In the kinetic term of the scalar fields in Eq. (112) the notations are the following:

\[ |D_\mu \phi|^2 = (D_\mu \phi)^\dagger (D_\nu \phi)^\dagger g^{\mu \nu} \]

\[ D_\mu \phi \equiv \left( \partial_\mu - \frac{i}{2} \tilde{g} \tilde{A}_\mu - \frac{i}{2} \tilde{g} \tilde{B}_\mu \right) \phi \]

The gauge complex \( u \), which enters into Eq. (112), as an argument of the function \( f \) in the second term, is defined now as follows:

\[ u \equiv \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} Tr G_{\mu \nu} G^{\mu \nu} + \frac{m^2}{\sqrt{g}} \epsilon^{\mu \nu \alpha \beta} \partial_\mu A_\nu A_\alpha \beta \]

where \( F_{\mu \nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu, \ G_{\mu \nu} \equiv \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu - i \tilde{g} [A_\mu A_\nu - A_\nu A_\mu] \) where \( A_\mu = \tilde{A}_\mu \tilde{T} \).

From the action (112) we will come to equations of motion and the constraint similar to what we got before but with the appropriate modifications related to the \( SU(2) \times U(1) \) theory. Nonabelian structure of the tensor \( G_{\mu \nu} \) does not change the constraint since nonlinear term of \( G_{\mu \nu} \) has the same degree of homogeneity in \( g^{\mu \nu} \) as the linear terms. After we will get Eqs. (84), (89) and (90), we see that

\[ \frac{dV_{\text{eff}}}{d\phi} = 0 \]

may be realized when

\[ |\phi|^2 = \phi^2_0 = \frac{1}{2} \eta^2 \]

for some vacuum expectation value

\[ \langle \phi \rangle = \frac{1}{\sqrt{2}} \left( 0 \ \eta \right) \]

Then the appropriate gauge complex condensate \( u_0 \) is determined by Eq. (122) in such a way that the effective cosmological constant defined by Eq. (90) is equal to zero.
Around this vacuum the constraint and equations of motion take the standard form. It is important to note a few remarkable features of the theory. First, in spite of the fact that more than one gauge vector field enter in the gauge complex $u$ (Eq. (124)), the equation similar to Eq. (86) again is enough to provide in the Einstein frame, the form similar to Eq. (76) for all of the gauge field equations. Second, once again, in order to provide the form (76) of the gauge field equations, we have to perform the rescaling

$$\tilde{A}_\mu = 2\sqrt{\omega} \tilde{A}_\mu; \quad B_\mu = 2\sqrt{\omega} \tilde{B}_\mu;$$

$$g = \frac{\tilde{g}}{2\sqrt{\omega}}; \quad g' = \frac{\tilde{g}'}{2\sqrt{\omega}}$$

(125)

where the integration constant $\omega$ appears as the universal parameter. It is interesting that with this rescaling, where $\tilde{g} \tilde{A}_\mu = g A_\mu$, $\tilde{g}' B_\mu = g' B_\mu$, we obtain the canonical form of all of the equations similar to Eqs. (75) - (78) with the appropriate modification to the $SU(2) \times U(1)$ theory. From equations similar to Eq. (76) we obtain masses of vector bosons. After the standard field redefinition from $\tilde{A}_\mu, B_\mu$ to intermediate vector bosons $W^+_\mu, W^-_\mu, Z_\mu$ and electromagnetic field $A_\mu$, we obtain the following expressions for their masses:

$$m_W = \frac{1}{2} g' \eta = \frac{\tilde{g}}{4\sqrt{\omega}} \eta; \quad m_Z = \frac{1}{2} \tilde{g} \eta; \quad m_A = 0,$$

(126)

where

$$\tilde{g} = \sqrt{g^2 + g'^2} = \frac{1}{2\sqrt{\omega}} \sqrt{g^2 + g'^2}$$

(127)

Turning now to the fermionic sector, once again we can find the spin-connection $\omega^{cd}_\mu$. Varying the action with respect to $\omega^{cd}_\mu$ we obtain an equation similar to Eq. (C2) and the appropriate modification of Eqs. (C4)-(C7) as the solution of it. In the Einstein frame the $\sigma$-contribution $K^{cd}(\sigma)$ disappears as we discussed at the end of Sec. VIA. Contribution of the fermions selfinteraction to $\omega^{ab}_\mu$, like $\propto \kappa \eta c V_{di} e^{a\delta c a} e_L \gamma^5 e'_L$ ($e'_L$ is the left electron spinor in the Einstein frame) which are supressed by factor $\kappa = 16\pi G$ in comparison to the first term in Eq. (103), may be neglected if we are interested in particle physics at energies much less than the Planck energy scale. Therefore neglecting the last term in Eq. (102) we remain only with the Riemannian contribution to the spin-connection.

Concerning the mass generation of fermions, we have to point out that the choice of the last term in Eq. (112) is related to our intention to use the mechanism for the electron mass generation developed in Sec.VIC, where we have found out that the exponent $q$ must be equal to 2/3. Comparing with notations of Sec.VIC we see that after the SSB, the factor $C$ in Eq. (111) has to be identified in the low energy theory as

$$km^2 = C = \lambda e \left( \frac{\eta}{\sqrt{2}} \right)^{2/3} m^{4/3}$$

(128)

As a result we get from Eq. (110):

$$m_e = \frac{\lambda e^{3/2}}{\sqrt{6u_0 \omega}} \eta$$

(129)

Notice that ratios between masses of all particles of the model (see Eq. (13) for the Higgs boson mass, Eqs. (126), (127) for $W^\pm_\mu$ and $Z_\mu$ masses and the electron mass, Eq. (129)) are $\omega$-independent. The same is true for the weak angle

$$\sin \theta_W = \frac{g'}{\tilde{g}}$$

(130)

It is very interesting that a big value of the integration constant $\omega$ pushes gauge coupling constants and all masses to small values. In addition, the constant $g_Y$ of the effective Yukawa coupling $g_Y \bar{e} e' \varphi$

$$g_Y = \frac{\lambda e^{3/2}}{\sqrt{3u_0 \omega}}$$

(131)
has an additional factor $u_0^{-1/2}$, which can explain (if the gauge complex condensate $u_0 = \frac{\sqrt{2}}{m}$ is also big) the observed further suppression of the effective Yukawa coupling and therefore the appropriate suppression of the observed lepton masses. We can think about all these effects related to a big values of $\omega$ and $u_0$ as a "cosmological seesaw mechanism", where masses are driven to small values due to the appearance of large number in a denominators.

As we pointed out at the beginning of this section, other gauge unified theories can be formulated in the same fashion. If for example we would consider also QCD, then the same effect of additional suppression would be obtained for the masses of quarks.

**VIII. DISCUSSION AND CONCLUSIONS**

In this paper we have seen that the idea to allow the measure to be determined dynamically rather than postulating it to be $\sqrt{-g}$ from the beginning, has deep consequences. In fact, in the context of the first order formalism the NGVE theory does not have a cosmological constant problem.

In this theory, if a four index field strength is introduced it develops a condensate which turns out to be expressed in terms of other fields. A consequence of this is the possibility to produce the standard particle physics dynamics. The resulting dynamics has then interesting consequences in what concerns to the hierarchy problem. As we have seen, all masses and gauge coupling constants are driven to small values if the integration constant $\omega$, that parametrizes the condensate strength (see Eq. (68)), is big. In addition to this, masses of fermions are driven to small values in comparison with masses of bosons as the gauge complex condensate $y_0$ becomes big (see Eq. (131)).

The appearance of the parameter $\omega$ in the relation between the original coupling constants and the effective ones suggests an idea that it may be possible to think in different ways concerning renormalization theory. It seems to promise allure prospects since the strength of the condensate specifies a boundary condition or state of the Universe.

It is very interesting that a theory designed in order to solve the cosmological constant problem tell us about an apparently unrelated subject, like what determines the effective coupling constants and masses of the theory. One should recall that the wormhole approach to the cosmological constant problem ends up claiming that wormholes determine all couplings of the theory also [19].

Other important consequence of the theory described in this paper is that one can obtain scalar field dynamics which allows for an inflationary era, the possibility of reheating after scalar field oscillations and the setting down to a zero cosmological constant phase at the later stages of cosmological evolution, without fine tuning. It is interesting that the theory not just reproduces all possibilities well known in the cosmology of the very early universe solving at the same time the cosmological constant problem. In addition to this, the effective scalar field potential includes integration constants - the feature which let us to hope that by an appropriate choice of those constants, the correct density perturbations and reheating could be obtained naturally. Moreover, the integration constant $\omega$ enters both in the effective potential and in the effective coupling constants and masses. This means that may be a strong correlation between coupling constants and masses of particles and some of the cosmological parameters.

Some open problems are of course apparent. For example, in what appears to be the most attractive scheme for generating standard gauge field dynamics, scalar potentials and fermion masses, namely the "persistent gauge field condensate scenario" (see Sec. VB) has to be understood in a deeper way. There a nonspecified function $f$ of a special combination $y$ (Eqs. (66) or (124) as examples) of all gauge fields, including 3-index potential, is introduced. For this function we require only the existence of an extremum at some point $y = y_0 > 0$ and this is enough to get the effective action of electro-weak, QCD and other gauge unified models. The possible origin of such structure as well as the choice of function has to be studied.

The fact that similar type of function appear for example in the QCD effective action as result of radiative corrections [24], is encouraging. In such a case no four index field strength is introduced however and the effective action is a function of $F_{a\mu\nu}F^{a\mu\nu}$ there. Notice that appearance of a four index field strength condensate in the framework of the theory developed in this paper, makes the Lorentz invariance of the vacuum in QCD not a problem, as opposed to the situation where only regular gauge fields are present as the argument of the nontrivial function, leading to an expectation value of $F_{a\mu\nu}F^{a\mu\nu}$.

As we have seen, the four index field strength plays the central role in the model described in this paper. In this connection, one has to recall that four index field strength plays a fundamental role in some supergravity models, in particular the $D = 11$, $N = 1$ supergravity and in the $D = 4$, $N = 8$ supergravity theories. The possibility of incorporating some versions of supergravity into the framework developed in this paper seems therefore a subject which could be a potentially fruitful one.

Finally, another subject that can certainly be studied concerns the possibility of exploiting the correlations between the masses and coupling constants and the strength of the condensate. If we allow for a coupling between the 3-index
potential $A_{\mu\nu\alpha}$ and 2 + 1 dimensional membranes of the form $\int \hat{T} \ W \ A_{\mu\nu\alpha} \ dx^\mu \wedge dx^\nu \wedge dx^\alpha$, one can see that the condensate strength can suffer a discontinuous change across the membrane. This effect can be exploited to construct a "bag" containing particles which are very light and weakly coupled inside the bag while being very heavy and strongly coupled outside the bag. The analogy with the famous MIT bag - model [21] is self-evident of course.

Finally, an important question that has to be addressed is of course the quantization of the theory and whether quantum corrections will respect the basic structure of the theory which enables the theory not to have a cosmological constant problem. As we have seen, the key of the resolution of the cosmological constant problem is based on the study of an action of the form (5), where the measure fields $\varphi_a$ does not enter in the Lagrangian. This form appears then to be associated with the existence of the infinite dimensional symmetry (6) which is valid if and only if the structure (5) is maintained. We interpret this as a strong indication that the resolution of the cosmological constant problem discussed in this paper will survive quantum corrections, provided no quantum anomalies of the symmetry (6) are found.

IX. ACKNOWLEDGMENTS

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APPENDIX A: METRIC - AFFINE FORMALISM IN THE NGVE THEORY AND $\lambda$-SYMMETRY

In Sec. IIIA we have shown that in the NGVE theory there is a contribution of the $\chi$-field to the connection (see Eqs. (17) and (20)). This contribution is defined up to the $\lambda$-symmetry transformation (21). By using this symmetry, in Sec.IIIA we have chosen the gauge where the antisymmetric part of the connection (that is a $\chi$-contribution into the torsion) disappears.

It is interesting to see what is the geometrical meaning of the $\lambda$-gauge dependent contribution to the connection (17), (20). For this we calculate the covariant derivative of the metric tensor $g_{\mu\nu}$ with the connection defined by (17) and (20) and we get

$$g_{\mu\nu:\alpha} = -2g_{\mu\nu} \lambda_{,\alpha} \equiv N_{\mu\nu\alpha}.$$

(A1)

This means that the $\lambda$ dependent contribution to the connection is responsible for the appearance of the nonmetricity tensor (18).

With the choice $\lambda = \frac{1}{2} \sigma$, in Sec. IIIA, we have eliminated the $\chi$-contribution into the torsion keeping the nonmetricity tensor $N_{\mu\nu\alpha}$ which in this "$\sigma$-torsionless" gauge is equal to $-g_{\mu\nu} \sigma_{,\alpha}$. However we have the freedom to choose for example the "$\sigma$-metric - compatible" gauge where the nonmetricity disappears: $\lambda = \text{constant}$. In such a case, the torsion is not eliminated from the connection (17), (20).

Notice that these peculiarities of the $\lambda$-symmetry concerning the possibility of eliminating the $\chi$-contribution to the torsion or, alternatively, of eliminating the $\chi$-contribution to the nonmetricity appear to be a very interesting feature of the NGVE theory in the metric - affine formalism. This feature results from the basic assumption that not only metric and connection are independent dynamical variables (as it is in the case of the Metric - Affine theory), but also the measure degrees of freedom are independent variables when varying the action. This is why we will call this theory the Metric - Affine - Measure (MAM) theory.

APPENDIX B: CONSTRAINT AND LES IN THE VIERBEIN - SPIN-CONNECTON FORMALISM

To incorporate fermions into the NGVE theory we have to use the vierbein - spin-connection (VSC) formalism (see, however a purely affine approach due to Ne’eman [22]). In this Appendix we review in a short form how the NGVE principle works in the VSC-formalism.

In this case we define [14]
\[ R(\omega, V) = V^{a\mu} V^{b\nu} R_{\mu\nu\alpha}(\omega), \]  
\[ R_{\mu\nu\alpha}(\omega) = \partial_\mu \omega_{\nu\alpha} - \partial_\nu \omega_{\mu\alpha} + (\omega^c_{\mu\alpha} \omega_{c\nu} - \omega^c_{\nu\alpha} \omega_{c\mu}) \]  
where \( V^{a\mu} = \eta^{ab} V^{b\mu} \), \( \eta^{ab} \) is the diagonal \( 4 \times 4 \) matrix with elements \( (1, -1, -1, -1) \) on the diagonal, \( V^{b\mu}_a \) are the vierbeins and \( \omega^a_{\mu\nu} = -\omega^a_{\nu\mu} \) \( (a, b = 0, 1, 2, 3) \) is the spin connection. The matter Lagrangian \( L_m \) that appears in Eq. \( (B1) \) is now a function of matter fields, vierbeins and spin connection, considered as independent fields. We assume for simplicity that \( L_m \) does not depend on the derivatives of vierbeins and spin connection.

We are now going to study the theory defined by the action \( (B1) \) in the case that the scalar curvature is defined by \( (B1), (B2) \).

As in Sec. II, variation with respect to the scalar fields \( \phi_a \) leads to the equations

\[ A^a_\mu \partial_\mu (-\frac{1}{\kappa} R(\omega, V) + L_m(V, \omega, \text{matter fields})) = 0 \]  
which implies, if \( \Phi \neq 0 \), that

\[ -\frac{1}{\kappa} R(\omega, V) + L_m(V, \omega, \text{matter fields}) = M \]  
On the other hand, considering the equations obtained from the variation of the vierbeins, we get if \( \Phi \neq 0 \)

\[ -\frac{2}{\kappa} R_{\mu\alpha}(V, \omega) + \partial L_m \partial V^{\alpha\mu} = 0, \]  
where

\[ R_{\mu\alpha}(V, \omega) \equiv V^{b\nu} R_{\mu\nu\alpha}(\omega). \]  
Notice that eq. \( (B5) \) is indeed invariant under the shift \( L_m \to L_m + \text{const} \). By using Eq. \( (B1) \) we can eliminate \( R(\omega) \) from the equations \( (B4) \) and \( (B5) \) after contracting the last one with \( V^{a\mu} \). As a result we obtain the nontrivial constraint in the form

\[ V^{a\mu} \partial(L_m - M) \partial V^{\alpha\mu} - 2(L_m - M) = 0 \]  
which replaces Eq. \( (14) \) (see Sec. II) and in the absence of fermions, is equivalent to the constraint \( (14) \).

The simplest example of a fermion is that of spin 1/2 particles. In this case we regard the spinor field \( \Psi \) as a general coordinate scalar and transforming nontrivially with respect to local Lorentz transformation according to the spin \( 1/2 \) representation of the Lorentz group.

The NGVE principle prescribes for the fermionic hermitian action (which allows for the possibility of fermion self interactions) to be of the form

\[ S_f = \int L_f \Phi d^4x \]  
where

\[ L_f = \frac{i}{2} \bar{\Psi} \gamma^{a\mu} V^{a\mu}_c (\partial_\mu - \frac{1}{2} \omega^c_{\mu\sigma} \sigma_{cd}) \gamma^d V^{c\mu} \psi + U(\bar{\Psi} \Psi) \]  
Here \( \sigma_{cd} \equiv \frac{1}{4} [\gamma_c, \gamma_d] \). Spin-connection \( \omega^c_{\mu\sigma} \) should be determined by the equation obtained from the variation of the full action with respect to \( \omega^c_{\mu\sigma} \) (see Appendix C).

From \( (B8) \) and using the equations of motion derived from the action \( (B8), (B9) \), we get

\[ V^{a\mu}_c \frac{\partial L_f}{\partial V^{a\mu}_c} - 2L_f = \bar{\Psi} \Psi U' - 2U, \]  
where \( U' \) is the derivative of \( U \) with respect to its argument \( \bar{\Psi} \Psi \). We see that the constraint \( (B7) \) is satisfied on the mass shell (since the fermion equations of motion are used) with \( M = 0 \) for \( L_f \) defined by eq. \( (B8) \) if, for example, \( U = c(\bar{\Psi} \Psi)^2 \). Any other quartic interaction, like \( \bar{\Psi} \gamma_a \Psi \gamma^a \Psi, \bar{\Psi} \sigma_{ab} \Psi \sigma^{ab} \Psi, (\bar{\Psi} \gamma_5 \Psi)^2 \), etc. would also satisfy the
constraint (B7) on the mass shell with \( M = 0 \). In particular, the Nambu - Jona-Lasinio model \(^{15}\) would also satisfy the constraint (B7) on the mass shell with \( M = 0 \).

In the presence of Dirac fermions with the Lagrangian density (B9), the LES (23), (24) is appropriately generalized to

\[
V_a^\mu(x) = J^{-1/2}(x)V_\mu^a(x); \quad V_a^\mu(x) = J^{1/2}(x)V_\mu^a(x)
\]  
(B11)

\[
\Phi(x) = J^{-1}(x)\Phi'(x)
\]  
(B12)

\[
\psi(x) = J^{1/4}(x)\psi'(x); \quad \overline{\psi}(x) = J^{1/4}(x)\overline{\psi'}(x)
\]  
(B13)

provided that \( V(\overline{\psi}\psi) \) has a form of one of the mentioned above quartic interactions. Notice that in this case the condition for the invariance of the action with the matter Lagrangian (B9) under the transformations (B11)-(B13) is not just the simple homogeneity of degree 1 in \( g_{\mu\nu} \) or degree 2 in \( V_\mu^a \), because of the presence of the fermion transformation (B13).

**APPENDIX C: CONNECTION IN THE VSC-FORMALISM**

We analyze here what is the dependence of the spin connection \( \omega^{ab}_{\mu} \) on \( V_\mu^a \), \( \chi \), \( \Psi \) and \( \overline{\Psi} \). Varying the action (B8), (B9) with respect to \( \omega^{ab}_{\mu} \) and making use that

\[
R(V, \omega) \equiv -\frac{1}{4\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}V^c_\alpha V^d_\beta K_{\mu\nu}^{ab}(\omega)
\]  
(C1)

we obtain

\[
\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}[\chi V^\alpha_\alpha D_\nu V^\beta_\beta + \frac{1}{2} V^a_\alpha \varepsilon^d_{\beta\gamma\delta} + \kappa \frac{1}{4\sqrt{-g}} V^c_\alpha \varepsilon_{abcd} \overline{\Psi}\gamma^5 \gamma^d \Psi = 0,
\]  
(C2)

where

\[
D_\nu V^a_\beta \equiv \partial_\nu V^a_\beta + \omega^a_{\nu\alpha}(V)\partial_\nu V^a_\alpha
\]  
(C3)

The solution of Eq. (C2) is represented in the form

\[
\omega^{ab}_{\mu} = \omega^{ab}_{\mu}(V) + K^{ab}_{\mu}(\sigma) + K^{ab}_{\mu}(V, \overline{\Psi}, \Psi)
\]  
(C4)

where

\[
\omega^{ab}_{\mu}(V) = V^a_\alpha V^b_\nu \{_{\mu\nu}^{\alpha\beta}\} - V^b_\sigma \partial_\mu V^a_\nu
\]  
(C5)

is the Riemannian part of the connection,

\[
K^{ab}_{\mu}(\sigma) = \frac{1}{2} \sigma_{\alpha\beta}(V^a_\alpha V^b_\nu - V^b_\sigma \partial_\mu V^a_\nu)
\]  
(C6)

and

\[
K^{ab}_{\mu}(V, \overline{\Psi}, \Psi) = \frac{\kappa}{8} \eta_{ci} V^a_{dm} \varepsilon^{abcd} \overline{\Psi}\gamma^5 \gamma^d \Psi.
\]  
(C7)

Notice that the spin-connection \( \omega^{ab}_{\mu} \) defined by Eqs. (C4)-(C7) is invariant under the LES transformations (B11)-(B13).
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