Catadioptric lenses in Visible Light Communications

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Abstract. Since few years ago, visible light communications (VLC) have experience an accelerated interest from a research point of view. The beginning of this decade has seen many improvements in VLC at an electronic level. High rates of transmission at low bit error ratios (BER) have been reported. A few numbers of start-ups have initiated activities to offer a variety of applications ranging from indoor geo-localization to internet, but in spite of these advancements, some other problems arise. Long-range transmissions mean a high BER which reduce the number of applications. In this sense, new redesigned optical collectors or in some cases, optical reflectors must be considered to ensure a low BER at higher distance transmissions. Here we also expose a preliminary design of a catadioptric and monolithical lens for a LI-FI receiver with two rotationally symmetrical main piecewise surfaces za and zb. These surfaces are represented in a system of cylindrical coordinates with an anterior surface za with a central and refractive sector surrounded by a peripheral reflective sector and a back piecewise surface zb with a central reflective sector and a reflective sector, both characterized as ideal for capturing light within large acceptance angles.

1. Introduction

Visible Light Communications (VLC) or LiFi (for Light Fidelity) is a relatively new technology that will complement HiFi communications and is intended to be used in places where HiFi interferes with EM sensible equipment. For instance we can name airplanes, hospitals, and places that will remain as radio frequency (RF) free communication environments. This technology can, in a certain measure, relieve pressure on the saturated RF broadband and can also be used in places where communications must be secured. VLC can benefit from the huge network of LED based lamps worldwide as LED lamps are found in street illumination, buildings, homes, street fires, airplanes and vehicles. Applications ranges from downlink –where the emitter lamp broadcast information and the receiver only can download this information– to bidirectional communications where both the lamps and mobile devices are equipped with transmitter (Tx) and receiver (Rx) units to uplink and downlink purposes. Examples of the first are train stations or museums where light can transmit information while examples of the last are the communication between the light of two vehicles or the communication between cars and road markers or street lights; internet or machine to machine (M2M) communications.

VLC is passing from a development stage to a commercial stage; the number of publications can assess its increasing relevance. Nevertheless, the optical issues have not been taken into consideration yet. As a consequence the optical elements used to spread, direct or concentrate the light from LEDs or the light collectors in photodiodes should be redesigned for they to become useful not only for...
illumination but also for communications. The new optic elements will accomplished different tasks for different applications, i.e. for some configurations the light must cover a broad cone while for others a sharp cone is the better solution. In the first case, we can think for example, in a large space in a metro or train station where a powerful LED based lamp illuminates the platform as well as at the same moment it spread information concerning traffic, arrivals or ambience music. In the second case, lamps in a museum can illuminate an artwork or a masterpiece as well as spread useful concerning information on any mobile device. Here, the illumination cone will meet design specifications provided by museum specialists. From the receiver point of view, the same questions can be made. Probably the most important question is related to the use and applications that this technology will serve. Is it better to design multipurpose lenses, surely rendering expensive this technology, or to design specific purpose lenses that will reduce production costs? The same question is valid for reflectors in transmitters. It can be seen that a designer has several restrictions to consider and they will meet low costs, illumination purposes and data communications transfer.

From the electronics point of view, a transmission rate exceeds 1 Gb/s [1,2] at an ambience illumination up to 1200 lux. Typical distance between the transmitter (Tx) and the receiver (Rx) is 1 m, nevertheless distances up to 3 m have been reported [1]. Bi-directional communication with RGB LEDs for downlink and phosphor based LEDs for uplink, of some hundreds of Mb/s have recently been reported [3]. Different schemes have been also proposed to transmit a downstream at wide field-of-view [4]. Multi-Gb/s parallel data transmission using CMOS-controlled micro-LEDs will surely be a reality in the near future [5]. High transmission rates, broader field-of-transmission, longer transmission distances, robustness against ambience illumination are the main challenges of VLC. On-off keying modulation (OOK) is, by far, the most simple modulation scheme used in VLC. It consists in turning on and off the LED depending on the bit value where off is the logic state 0 and on the state 1. The IEEE 802.17.7 standard offers three physical (PHY) modulation schemes for VLC [6]. According to it, the PHY II operating mode in OOK modulation scheme has a data rate of 96 Mb/s.

In this article, we will refer to this modulation scheme to show a bi-directional uplink/downlink system to transmit Ethernet at 96 Mb/s. The maximum measured distance is 3 m when the environmental luminous intensity measured on the test table equals 3,000 lux (2,400 lux on the detector). LEDs used in the transmission are red (650 nm peak emission wavelength). Design considerations have taken into account this electronics choice.

2. Electronics

An Ethernet network is connected via an Ethernet-to-VLC transceiver (figure 1) to the LED driver for transmission, switching the LED at a cut-off frequency of 70 MHz.

2.1. Receiver

In reception mode a photodetector if fully covered by the LED beam collected by a concentrator lens. This receiver module consists in a silicon PIN photo-diode mounted with a silicon bipolar trans-impedance preamplifier integrated circuit (IC) in an optical subassembly (OSA). This OSA is mated to a custom silicon bipolar circuit that provides post-amplification and quantization. The module utilizes an InP PIN photodiode in the same configuration. The post-amplifier also includes a signal detection circuit that provides a positive emitter-coupled logic (PECL) output upon detection of a usable input optical signal level.

2.2. Transmitter

The transmitter section of the system consists of seven 650 nm high speed LEDs in an optical subassembly (OSA) for the uplink, and a lamp made of seven 650 nm high speed LEDs for the downlink, which mates to the lens. The OSA is driven by custom, silicon bipolar IC which converts differential PECL logic signals (ECL referenced to a +5 Volt supply) into an analogue LED diode drive current.
The LED Driver is a high speed LED Driver/current switch specifically targeted for use Fibre Channel 266 Mb/s optical transmitters. The device incorporates open collector outputs with a capability of driving peak currents of 100 mA. Since the output current switching circuitry simply switches current between the complementary outputs, the dynamic switching demands on the system power supply are greatly reduced. In addition, because the design is pure bipolar, the device current drain is insensitive to the data pattern and frequency of operation. In addition to this, to allow for open loop compensation for the LED’s negative optical output power tracking over temperature, a circuit is included to provide an adjustable positive temperature tracking coefficient to the LED drive current. The driver is intended either to be integrated into high performance fiber optic modules or to be used stand-alone to drive a packaged optical LED device. The wide frequency response of the device allows it to be used to support 100 Mb/s that permits us to operate in PHY II.

**Figure 1.** Ethernet to VLC transceiver.

### 3. Optical Parameters

There are several restrictions when designing optical systems for VLC. The most important ones are related with the cost of fabrication, the size, the weight and the particular application intended for. Here we concentrate our efforts in the design of a collector lens that permits the photodiode the correct interpretation of the transmitted signal. The light transmitted by a LED lamp carries the modulated signal; this light beam has a distribution typical from LED illumination lamps. As a consequence, the signal will be degraded rapidly with distance and with angle and this degradation will eventually induce errors in the receptor expressed as bit losses. Even if BER is a central concern for electronic engineers, optics will offer a low-cost solution to its reduction. Our proposal [7] consists in collecting the light reflected by walls, windows or objects located around the receiver by a grand acceptance angle collector, which should reduce BER. It means that the light and signal emitted will better arrive onto the photodetector.

The optical system will not display an image but uniformly distribute the intensity at the entrance pupil; it will have a huge acceptance angle $\alpha$ and low internal (or external) reflection loses. There are several possible solutions to obtain a low cost optical system. Yet another lens parameter consists in coupling the lens to a new generation mobile device such as a mobile telephone or ultra-compact computer devices. Ergonomic and functionality condition the lens to have a flat first surface and small dimensions even at the trade of reducing the intensity collection surface.

The optical system is only intended for capturing light and not to form an image; it has to be similar to a Galilean or Keplerian telescope, with or without a field lens, with an inverted configuration –
absolute angular augmentation or magnification $|M|$ less than the unit or absolute reduction $m = 1/|M|$. It can or cannot invert the image, and it will have preferably a high acceptance degree $\alpha$ – the angular semi field of a field-angular lens. A uniform distribution of intensity in the entrance that permits the system to obtain a uniform intensity distribution at the exit is desired; a system that minimizes internal reflexions or refractions is then preferred. The coupled diaphragm is set at the first surface. As there are lots of possible solutions, it will always be possible to find one with good specifications and not expensive.

There are many examples of optical designs using condenser lenses with highly refractive power that can be either spherical or aspherical; usually they are hemispheres or spheres. Since our main purpose is to design an optical system that can be easily linked with portable devices, like cellular phones and portable computers, the following restrictions can be specified: For ergonomic and functional reasons, the first surface must be flat, the total length of the system and the volume must be minimal and finally, its production cost very low.

**Proposal**

Based on several concepts presented previously in [7-8], we show here a design of a catadioptric and monolithic optical system, assuming the object is at infinity, which form an non-magnified and inverted image, with multiple applications like: magnifiers, illumination systems, surveillance systems, solar collectors without heliostat and light detectors, and antennae, among many more.

The catadioptric-monolithic system is composed by a first lenticular flat parabolic surface of revolution in a cylindrical coordinate system $(x, z)$ having itself two sections: The first one is flat, occupy the center of the surface and is perpendicular to the optical axis $z$ and has a diameter $d_a$. The second is peripheral (or marginal) and corresponds to an annular section of a revolution parabola around the optic axis, with a positive focal distance $f_a$. Thus, the first two sections of this surface –the central-flat and the peripheral-convex-parabola– have a representation in the meridional plane that corresponds to

\[
 z_a = \begin{cases} 
 0, & \forall x \setminus x^2 \leq \frac{d_a^2}{4} \quad \text{refractive} \rightarrow n_a = n > 1. \\
 \frac{x^2}{4f_a} - t_a, & \forall x \setminus x^2 > \frac{d_a^2}{4} \quad \text{reflective} \rightarrow n_a = -1. 
\end{cases}
\]

the vertex of the parabola is placed at an axial distance $t_a$ from the coordinate’s origin. The flat central sector corresponds to a refractive interface towards a medium with a relative refraction index $n_a = n$ greater than the unit, and the parabolic-peripheral sector corresponds to a reflective interface, that is equivalent to $n_a = -1$, notation usually used for reflective optical systems.

As an approximation to the final design, we assume a total central thickness $t$, where a second surface is created, lenticular as well, flat-parabolic, concentric and inverted with respect to the first surface, and can be specified within the same coordinated system with a two-section function: The first is located in the central region, is flat and is perpendicular to the optical axis $z$ at the point $(0, t)$ and has a diameter $d_b$; the second is peripheral and corresponds to an annular-section of another revolution-parabola with a negative focal distance $f_b$; thus, the second surface has a representation on the meridional plane corresponding to
\[
\tilde{z}_b = \begin{cases} 
    t, & \forall x \backslash x^2 \leq \frac{d_b^2}{4} \text{ refractive } \rightarrow \ n_b = \frac{1}{n} < 1. \\
    t + t_b + \frac{x^2}{4f_b}, & \forall x \backslash x^2 > \frac{d_b^2}{4} \text{ reflective } \rightarrow \ n_b = -1.
\end{cases}
\tag{2}
\]

The parabolic surface is placed at an axial distance \( t_b \) from the point \((0, t)\) to the point \((0, t + t_b)\). The flat-central sector corresponds to a refractive interface towards a medium with a relative refractive index equivalent to \( n_b = -1 \).

Since both parabolas are opposite they must intercept at a point in space, limiting the frontier of the mirror-lens that makes it possible to form a monolithic optical system; thus, the maximum distance \( d \) of the monolithic bi-convex-bi-plane system can be determined by equaling sections in Eqs (1) and (2) evaluated at the semi diameter \( d/2 \):

\[
\frac{d^2}{16f_a} - t_a = t + t_b + \frac{d^2}{16f_b},
\tag{3}
\]

with a solution for \( d \)

\[
d = 4 \sqrt{f_a f_b (t_a + t + t_b)} \left/ f_b - f_a \right. \nonumber \tag{4}
\]

By assuming that the focus of the parabola of the second surface is found in the coordinate origin of the catadioptric-monolithic system (refractive surface of entry), the next condition must be satisfied

\[
-f_b = t + t_b, \tag{5}
\]

given that \( f_b \) is negative. Moreover, by making the parabolic focus of the first surface coincide with the point \((0, t)\), the following condition must also be accomplished.

\[
f_a = t_a + t \tag{6}
\]

The sagittae of the parabola can be calculated as a function of the cord diameters that corresponds to the diameter of the refractive windows, with

\[
t_a = \frac{d_a^2}{16f_a} \tag{7}
\]

and

\[
t_b = -\frac{d_b^2}{16f_b} \tag{8}
\]

Substituting Eqs. (7 and 8) in Eqs. (5 and 6), and solving the quadratic equations for the valid focal distances it is easy to obtain.
\[ f_a = \frac{2t + \sqrt{d_a^2 + 4t^2}}{4}, \]  
\[ (9) \]

and

\[ f_b = -\frac{2t + \sqrt{d_b^2 + 4t^2}}{4}, \]  
\[ (10) \]
given that \( f_a > t/2 \) and \( f_b < t/2 \).

Thus, according to standard ISO [9], the parabolas can be represented using the canonical Schwarzschild's formula, with a conical constant \( K = -1 \) and curvatures in the vertex \( c_a = 1/(2f_a) \) and \( c_b = 1/(2f_b) \).

Substituting Eqs. (9-10) in Eqs. (7-8) the sagittae can be represented as a function of the diameters of the refractive windows diameter

\[ t_a = \frac{d_a^2}{4\left(2t + \sqrt{d_a^2 + 4t^2}\right)} = \frac{-2t + \sqrt{d_a^2 + 4t^2}}{4}, \]  
\[ (11) \]

and by symmetry

\[ t_b = \frac{d_b^2}{4\left(2t + \sqrt{d_b^2 + 4t^2}\right)} = \frac{-2t + \sqrt{d_b^2 + 4t^2}}{4}. \]  
\[ (12) \]

The representation with the piecewise function of the two surfaces given by equations (1-2) can also be expressed as a function of the windows diameter and the central thickness. Substituting Eqs. (11-12) in Eqs. (1-2), rationalizing and simplifying, to obtain

\[ z_a = \begin{cases} 
0, & \forall x \ \ x^2 \leq \frac{d_a^2}{4} \ \ \ \ \text{refractive} \rightarrow \ n_a = n > 1, \\
\left(2t - \sqrt{d_a^2 + 4t^2}\right)\left(\frac{1}{4} - \frac{x^2}{d_a^2}\right), & \forall x \ \ \frac{d_a^2}{4} < x^2 < \frac{d_a^2}{4} \ \ \ \text{reflective} \rightarrow \ n_a = -1.
\end{cases} \]  
\[ (13) \]

and

\[ z_b = \begin{cases} 
t, & \forall x \ \ x^2 \leq \frac{d_b^2}{4} \ \ \ \ \text{refractive} \rightarrow \ n_b = \frac{1}{n} < 1, \\
t - \left(2t - \sqrt{d_b^2 + 4t^2}\right)\left(\frac{1}{4} - \frac{x^2}{d_b^2}\right), & \forall x \ \ \frac{d_b^2}{4} < x^2 < \frac{d_b^2}{4} \ \ \ \text{reflective} \rightarrow \ n_b = -1.
\end{cases} \]  
\[ (14) \]
Figure 2 shows an example of this design, showing the preset values for a productive process.

![Figure 2](image)

**Figure 2.** Here we show a catadioptric monolithic bi-flat bi-parabolic lens, with the parabolic sectors to be metalized with a thin layer of reflective metal, usually made of silver, aluminum, gold or platinum. The entrance window diameter corresponds to the diaphragm’s diameter coupled to the first flat surface \( d_a = 16 \) mm, the exit window diameter \( d_b = 8 \) mm and the central thickness \( t = 10 \) mm. The total diameter is \( d = 31.0179 \) mm.

Figure 3 shows the ray tracing to show that the lens has image forming properties that are not dependent of the refractive index of the lens, and that it presents zero spherical aberration, minimal coma at the edge of the field, and low field curvature. The object is assumed to be flat and the image plane coincides with the entrance and exit windows respectively.

![Figure 3](image)

**Figure 3.** The tracing of rays in three catadioptric monolithic bi-flat bi-parabolic lenses is shown for the following parameters: Equal diameters in the entrance and exit windows \( d_a = d_b = 4, 6 \) and 8 mm for lenses (Figs. 3a-3c respectively), and central thickness \( t = 10 \) mm, the total calculated diameter is \( d = 31.018 \) mm. The trace of rays that depart from an object point O is shown on the border of the bottom window’s field, to try to form an image I in the upper window. One can also observe that the meridional coma aberration grows when the proportion \( d_a / t = d_b / t \) increases. For rays to emerge rapidly from the top window, it is convenient to cut off the upper plane that corresponds to the lower parabolic focal plane with defocus; thus, diminishing the noise in the detector and avoiding lens heating. All dimensions are in millimetres. No magnification is observed, but there is an apparent magnification increase due to the fact that the detector seems to be closer to an observer.

Thus, analytically, it can be inferred that:

1. The proposed catadioptric design with lenticular flat-parabolic surfaces forms an inverted image with almost the same size, regardless of its windows’ diameter ratio \( d_b/d_a \).
2. That radiation losses by reflected rays decrease to a minimum when there’s no magnification.
3. That radiation losses by reflected rays decrease when thickness increases.
4. A thick lens is necessary to avoid lens heating and noise in the detector.
5. That the image does not depend on the lens refractive index.
6. Aberrations are small.

In figure 4, we show a solution that can be used in signal detections or as a LiFi collector assuming, an object far away or at infinity. Another advantageous property of this proposed system can be observed: It drive collimated beams towards the exit window over the entry window with extreme acceptance angles. Here only the frontal (bottom) part of the lens is visible in mobile apparatus. The visible part of the lens is flat and can be touched. Great acceptance angles will permit the collector to trap light reflected by reflecting surfaces in the vicinity of the photodetector rendering, in this way, the receiver more robust and decreasing the BER at the same time.

**Figure 4.** Wide acceptance angle receiver-lens. Here we show the tracing of rays up to the second reflection in six catadioptric monolithic bi-flat-parabolic lenses with an external object in infinity, with different acceptance angles $\alpha = 5, 10, 25, 45, 65$ and $85$ degrees; 4a-4f, respectively. Incoming rays enter from bottom to top. Prescribed parameters for all the lenses: detection zone diameter $d_b = 8$ mm, entrance window diameter $d_a = 8$ mm, central thickness $t = 16$ mm and refractive index $n = 1.7$. The total preliminary diameter was calculated with $d = 46.298$ mm. The tracing of rays for a collimated beam that comes from an object O in infinity (upwards) try to form an image $I$ in the upper window. The tracing of emerging rays is suppressed because we assume a detector in direct contact with the exit window. A small loss of intensity is observed when $\alpha > 25^\circ$.

**4. Conclusions**

In this article we have shown the perspectives of applications of VLC systems and the electronics for a state-of-the-art Ethernet LED-based commercial system known as VLC or LiFi. Our system enters now in production and is robust in indoor ambience. We have also introduced to a new incoming optical design technology that will be charged to reduce BER in VLC systems as well as given some
optical receiver wide acceptance angle examples to reduce it. The lens here briefly describes is a part of a complete family of new great acceptance angle, compact and low fabrication cost collecting lenses to be used in photoreceivers for LiFi application. A similar design approach can be also used to design optical reflectors for several different applications of LiFi technology. We hope this article will motivate optical designers as well as specialists in photometry and radiometry to contribute to this emerging technology.

5. References

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