Exact $S_3$ symmetry solving the supersymmetric flavor problem

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Abstract

The exact discrete flavor symmetry, $S_3$ in the hadronic sector and $S_3 \times Z_2$ in the leptonic sector, which has been recently found, is introduced in a supersymmetric extension of the standard model. We investigate the supersymmetric flavor problem, and explicitly find that thanks to the flavor symmetry the dangerous FCNC processes and CP-violating phases are sufficiently suppressed.

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The minimal supersymmetric standard model (MSSM) [1] contains more than 100 free parameters [2]. Most of them belong to the supersymmetry breaking (SSB) sector. The problem is not only this huge number, but also the fact that these parameters have to be highly fine tuned so that they do not cause problems with experimental observations on the flavor changing neutral current (FCNC) processes and CP-violation phenomena [3]-[7]. This problem, called the SUSY flavor problem, is not new, but has existed ever, since supersymmetry found phenomenological applications. There are several approaches to overcome this problem [8]-[15].

The possibility of introducing more than 100 independent parameters into the MSSM is closely related to the fact that the degrees of freedom inherent in the Yukawa sector of the standard model (SM) are much more than the observable degrees of freedom. Since an exact flavor symmetry in the Yukawa sector can reduce this redundancy of the SM, the same symmetry can reduce that huge number of the independent parameters of the MSSM, and could suppress the dangerous FCNC processes and CP-violating phases. Recently, it has been found [16] that a nonabelian, discrete flavor symmetry based on the permutation group $S_3$ is consistent with the present experimental knowledge. That is, it is spontaneously broken, only because the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken $^1$. The idea to use nonabelian discrete symmetries to soften the SUSY flavor problem has been in fact proposed in [17, 18]. However, the symmetries considered in [17, 18] are explicitly broken at the supersymmetry breaking scale. Therefore, there is no convincing reason why they should be intact in the SSB sector. In this letter we will demonstrate that the exact flavor symmetry of [16], $S_3$ in the hadronic sector and $S_3 \times Z_2$ in the leptonic sector, can considerably soften the SUSY flavor problem.

Let us begin by supersymmetrizing the $S_3$ invariant SM. The three generations of quark and lepton chiral superfields in MSSM are assumed to belong in a reducible representation of $S_3$ equivalent to $1_S + 2$. We also introduce the $S_3$-doublet Higgs pair, $H^U_I, H^D_I (I = 1, 2)$, as well as the $S_3$-singlet Higgs pair, $H^U_3, H^D_3$. The same R-parity is assigned to these fields as in MSSM. Then we assume that the superpotential in MSSM, $W = W_D + W_U + W_E + W_\nu$, is invariant under the $S_3$-symmetry. The each part is given explicitly as follows (the neutrino superpotential is abbreviated for the present purpose):

$$W_D = Y_D^{1} Q_1 H^D_3 D_{IR} + Y_D^{3} Q_3 H^D_3 D_{3R} + Y_2^{1} \left[ Q_1 \kappa_{IJ} H^D_1 D_{IR} + Q_3 \eta_{IJ} H^D_2 D_{IR} \right]$$

$$W_U = Y_U^{1} Q_1 H^U_3 U_{IR} + Y_U^{3} Q_3 H^U_3 U_{3R} + Y_2^{1} \left[ Q_1 \kappa_{IJ} H^U_1 U_{JR} + Q_3 \eta_{IJ} H^U_2 U_{JR} \right]$$

$$W_E = Y_E^{1} L_1 H^E_3 E_{IR} + Y_E^{3} L_3 H^E_3 E_{3R} + Y_2^{1} \left[ L_1 \kappa_{IJ} H^E_1 E_{JR} + L_3 \eta_{IJ} H^E_2 E_{JR} \right]$$

$$+ Y_4^{1} L_3 H^E_3 E_{IR} + Y_5^{1} L_1 H^E_3 E_{3R},$$

\(^1\)The permutation symmetries have been considered in [17], [19]-[23].
where\
\[ \kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]
(4)

We will discuss the Higgs part of the superpotential later on. It will be shown that Higgs VEV’s can satisfy \( \langle H_1 \rangle = \langle H_2 \rangle \), which we shall assume throughout this letter. Consequently, the quark and lepton mass matrices take the general form
\[ M_a = \begin{pmatrix} m_{a1}^2 + m_{a2}^2 & m_a^2 & m_{a5}^2 \\ m_a^2 & m_{a1}^4 - m_{a2}^4 & m_{a5}^4 \\ m_{a5}^2 & m_{a5}^4 & m_{a3}^2 \end{pmatrix}, \]
(5)
where \( a = u, d, e \). It has been found in [16] that the exact \( S_3 \) flavor symmetry (along with an additional exact \( Z_2 \) in the leptonic sector) is consistent with all the observed quark and lepton masses and mixing angles. Since this result can remain valid after supersymmetrization of the model, we would like to focus on the flavor structure in the superpartner sector, assuming the same flavor symmetry in the SSB sector.

Now we come to discuss the SUSY flavor problem in the slepton sector. The \( Z_2 \)-parity assignment is the same as in [16]:
\[ + \quad \text{for} \quad H_{3}^{U,D}, \quad L_3, \quad L_I, \quad E_{3R}, \quad E_{IR} \quad \text{and} \quad - \quad \text{for} \quad H_I^{U,D}. \]
(6)

Note that \( m_1^e = m_3^e = 0 \) due to this discrete symmetry. Therefore, the lepton mass matrix can be written as
\[ M_e = m_2^e \begin{pmatrix} 1 & 1 & x \\ 1 & -1 & x \\ y & y & 0 \end{pmatrix}, \]
(7)
where \( x = m_5^e/m_2^e = 16.78 \) and \( y = m_4^e/m_2^e = 0.006836 \) [16].

In order to evaluate the amount of the flavor changing processes generated in this model, we need to know the unitary matrices \( U_{eL} \) and \( U_{eR} \) to the mass eigenstates, which satisfy
\[ U_{eL}^\dagger M_e U_{eR} = \text{diag}(m_e, m_\mu, m_\tau). \]
(8)

It is noticed that \( x \simeq m_\tau/m_\mu \) and \( y \simeq \sqrt{2}m_e/m_\mu \). Since \( y \) and \( 1/x \) are small numbers, let us evaluate the rotation matrices by expanding them with respect to these parameters. Here we evaluate the matrices at \( O(y^2/x^2) \). Then the rotation matrices are found to be
\[ U_{eL} = \begin{pmatrix} \frac{y}{2} \left(1 + \frac{1}{x^2}\right) & \frac{1}{\sqrt{2}} \left(1 - \frac{y^2}{4} - \frac{y^2}{2x^2}\right) & \frac{1}{\sqrt{2}} \\ -\frac{y}{2} \left(1 - \frac{1}{x^2}\right) & -\frac{1}{\sqrt{2}} \left(1 - \frac{y^2}{4} + \frac{y^2}{2x^2}\right) & \frac{1}{\sqrt{2}} \\ -1 + \frac{y^2}{4} & \frac{y}{\sqrt{2}} & \frac{y}{\sqrt{2}x^2} \end{pmatrix}, \]
(9)
\[ U_{eR} = \begin{pmatrix} -1 + \frac{y^2}{2x^2} & \frac{y^2}{2} \left(1 - \frac{1}{x^2}\right) & \frac{1}{x} \\ \frac{y^2}{2} \left(1 - \frac{1}{x^2}\right) & 1 & 0 \\ \frac{1}{x} & -\frac{y^2}{2x} & 1 - \frac{1}{2x^2} \end{pmatrix}. \]
(10)

The MNS matrix \( V_{\text{MNS}} \equiv U_{eL}^\dagger U_\nu \) is also found by using the rotation matrix of the neutrino sector \( U_\nu \) given in ref. [16]. Note that the maximal mixing appearing in the left handed
rotation matrix is responsible for the atmospheric neutrino mixing. While the solar neutrino mixing is explained by the large mixing in the rotation matrix of the neutrino mass matrix. That is, the elements, \((U_{eL})_{21}\) and \((U_{eL})_{23}\) become, respectively, the (1,3) and (3,3) elements of the mixing matrix \(V_{\text{MNS}}\):

\[
U_{e\beta} = -y (1 - 1/x^2)/2 \simeq -m_\beta/\sqrt{2}m_\mu, \quad \cos \theta_{\text{atm}} = 1/\sqrt{2}.
\]  

The \(S_3\) symmetry restricts also the soft scalar mass matrices so that the off-diagonal elements are forbidden by the symmetry and two of three are completely degenerate. Explicitly the matrices are given in the form of

\[
\hat{m}^2_{LL} = m^2_\ell \begin{pmatrix} a_L & 0 & 0 \\ 0 & a_L & 0 \\ 0 & 0 & b_L \end{pmatrix}, \quad \hat{m}^2_{RR} = m^2_\ell \begin{pmatrix} a_R & 0 & 0 \\ 0 & a_R & 0 \\ 0 & 0 & b_R \end{pmatrix},
\]

where \(m_\ell\) denotes the average of the slepton masses and \((a_{L(R)}, b_{L(R)})\) are free parameters.

The branching ratio of a LFV event, e.g. \(Br(\mu \rightarrow e + \gamma)\), is proportional to the off-diagonal elements of the mass matrix with respect to the lepton mass eigenstates, which is given by \(U^\dagger_{eL} \hat{m}^2_{LL} U_{eL}\). By using the expression given is by eq. (9), this matrix may be evaluated as

\[
U^\dagger_{eL} \hat{m}^2_{LL} U_{eL} \simeq m^2_\ell \begin{pmatrix} b_L - (y^2/2)\Delta_L & -(y/\sqrt{2})\Delta_L & -(y/\sqrt{2}x^2)\Delta_L \\ -(y/\sqrt{2})\Delta_L & a_L + (y^2/2)\Delta_L & (y^2/2x^2)\Delta_L \\ -(y/\sqrt{2}x^2)\Delta_L & (y^2/2x^2)\Delta_L & a_L \end{pmatrix},
\]

where \(\Delta_L = b_L - a_L\). It is convenient to see the ratio of the off-diagonal term to average of the diagonal terms, \((\delta^\ell_{ij})_{LL}\), whose upper bounds from experimental results are given in the literatures [7]. The explicit numbers of \(\delta^\ell\)'s in this model are predicted as follows;

\[
(\delta^\ell_{12})_{LL} \simeq -4.8 \times 10^{-3} \Delta_L, \quad (\delta^\ell_{13})_{LL} \simeq -1.7 \times 10^{-5} \Delta_L, \quad (\delta^\ell_{23})_{LL} \simeq 8.3 \times 10^{-8} \Delta_L.
\]

The upper bounds for ratios of the off-diagonal elements \((\delta_{ij})_{LL(RR)}\) [7] are shown in Table 1. It is seen that all the off-diagonal elements are small enough to satisfy the LFV constraints. It is noted that the (1,2)- and (1,3)-elements are suppressed due to the small mixing. On the other hand the slepton mass matrix for the \(S_3\)-doublets, which are roughly regarded as smu and stau, is proportional to the identity matrix. Therefore the (2,3)-element is almost vanishing, although the mixing angle of these bases is maximum.

Similarly, for the right-handed slepton the off-diagonal elements of the mass matrix may be evaluated by using \(U_{eR}\) given by eq. (10) and found to be

\[
U^\dagger_{eR} \hat{m}^2_{RR} U_{eR} \simeq m^2_\ell \begin{pmatrix} a_R + (1/x^2)\Delta_R & -(y^2/2x^2)\Delta_R & (1/x)\Delta_R \\ -(y^2/2x^2)\Delta_R & a_R & -y^2/2x^2\Delta_R \\ (1/x)\Delta_R & -y^2/2x^2\Delta_R & b_R - (1/x^2)\Delta_R \end{pmatrix},
\]
Table 1: Limits on the $|\delta_{ij}|$ from $\ell_i \to \ell_i\gamma$ decays for $m_{\tilde{\ell}}^2/m_{\tilde{\ell}}^2 = 1$. Here the parameter $\tilde{m}_{\tilde{\ell}}$ denotes $m_{\tilde{\ell}}$(GeV)/100. See [7] for details.

|   | $(\delta_{12})_{LL}$ | $(\delta_{12})_{LR}$ |
|---|-----------------|-----------------|
|   | $7.7 \times 10^{-3}$ $\tilde{m}_{\tilde{\ell}}$ | $1.7 \times 10^{-6}$ $\tilde{m}_{\tilde{\ell}}$ |
|   | $(\delta_{13})_{LL}$ | $(\delta_{13})_{LR}$ |
|   | $29$ $\tilde{m}_{\tilde{\ell}}$ | $1.1 \times 10^{-1}$ $\tilde{m}_{\tilde{\ell}}$ |
|   | $(\delta_{23})_{LL}$ | $(\delta_{23})_{LR}$ |
|   | $5.3$ $\tilde{m}_{\tilde{\ell}}$ | $2.0 \times 10^{-2}$ $\tilde{m}_{\tilde{\ell}}$ |

where $\Delta_R = b_R - a_R$ The explicit ratios of the off-diagonal elements to the diagonal elements are found to be

$$
(\delta_{12})_{RR} \simeq -8.28 \times 10^{-8} \Delta_R, \quad (18)
$$

$$
(\delta_{13})_{RR} \simeq 5.49 \times 10^{-2} \Delta_R, \quad (19)
$$

$$
(\delta_{23})_{RR} \simeq -1.39 \times 10^{-6} \Delta_R. \quad (20)
$$

Thus the flavor mixing in the right-handed sector is also very small.

It is also necessary for the mass matrix between the left-handed and the right-handed sleptons, which is generated through the so-called A-terms, to satisfy the FCNC constraints. Since the Yukawa interactions in the superpotential must be invariant under the $S_3$-symmetry, the left-right mass matrix may be parametrized as

$$
\tilde{m}_{LR}^2 = \begin{pmatrix}
  m_2 A_2 & m_2 A_2 & m_5 A_5 \\
  m_2 A_2 & -m_2 A_2 & m_5 A_5 \\
  m_4 A_4 & m_4 A_4 & 0
\end{pmatrix},
$$

(21)

where $A_i$ are free parameters. Here, however, we also assume them to be on the order of the gaugino masses. After rotating to the bases of lepton mass eigenstates, the mass matrix is found to be

$$
U_{eL}^\dagger \tilde{m}_{LR}^2 U_{eR} \simeq m_2 \begin{pmatrix}
  y A_4 & y(A_2 - A_4) & -(y/x)(A_4 - A_5) \\
  -(y^2/\sqrt{2})(A_2 - A_4) & \sqrt{2} A_2 & (y^2/\sqrt{2}) (A_4 - A_5) \\
  -\sqrt{2} (A_2 - A_5) & (y^2/\sqrt{2})(A_2 - A_5) & \sqrt{2} x A_5
\end{pmatrix}, \quad (22)
$$

The ratio of the off-diagonal term to the average slepton mass $m_{\tilde{\ell}}$ are explicitly given by

$$
(\delta_{12})_{LR} \simeq 5.1 \times 10^{-6} (\tilde{A}_2 - \tilde{A}_4) \left( \frac{100}{m_{\tilde{\ell}}(\text{GeV})} \right)^2, \quad (23)
$$

$$
(\delta_{13})_{LR} \simeq 1.1 \times 10^{-2} (\tilde{A}_2 - \tilde{A}_5) \left( \frac{100}{m_{\tilde{\ell}}(\text{GeV})} \right)^2, \quad (24)
$$

$$
(\delta_{23})_{LR} \simeq 2.5 \times 10^{-8} (\tilde{A}_2 - \tilde{A}_5) \left( \frac{100}{m_{\tilde{\ell}}(\text{GeV})} \right)^2, \quad (25)
$$
where $\tilde{A}_i$ denotes $A_i$(GeV)/100. These results should be compared with the limits for $|\delta_{i}^{LR}|$ shown in Table 1. It is seen that $(\delta_{12}^{LR})_{LR}$ saturates nearly the experimental bound from $\mu \to e \gamma$ process, unless $A_2 - A_4$ is somewhat suppressed with some reason. The other off-diagonal elements are sufficiently smaller than the experimental bounds. Indeed alignment of the A-terms to the Yukawa couplings is fairly good in this model. The origin of this remarkable feature is that the structure of A-terms are also restricted by the $S_3$-symmetry. 

The additional discrete symmetry $Z_2$ (which turns out be is anomaly-free) forbids CP-violations in the leptonic as well as in the hadronic sector. Consequently, $Z_2$ should be explicitly broken in the hadronic sector to accommodate CP violations. Therefore, the mass matrices for the up (down) sector quarks are parametrized as given by eq. (5). As in the leptonic case, we introduce the unitary matrices $U_{u(d)L}$ and $U_{u(d)R}$ satisfying

$$U_{u(d)L}^\dagger M_{u(d)} U_{u(d)R} = \text{diag}(m_{u(d)}, m_{c(s)}, m_{t(b)}) \textbf{.}$$

In [16], it has been found that the hierarchical quark masses in the up and down sectors, 

$$m_u/m_t = 1.33 \times 10^{-5}, \quad m_c/m_t = 2.99 \times 10^{-3},$$
$$m_d/m_b = 1.31 \times 10^{-3}, \quad m_s/m_b = 1.17 \times 10^{-2},$$

as well as the parameters in the CKM matrix may be obtained by choosing the parameters in the mass matrices as follows;

$$m_1^u/m_3^u = -0.000293, \quad m_2^u/m_3^u = -0.00028,$$
$$m_4^u/m_3^u = 0.031, \quad m_5^u/m_3^u = 0.0386,$$
$$m_1^d/\text{Re}(m_3^d) = 0.0004, \quad m_2^d/\text{Re}(m_3^d) = 0.0227,$$
$$m_3^d/\text{Re}(m_3^d) = 1 + 1.2 I, \quad m_4^d/\text{Re}(m_3^d) = 0.283,$$
$$m_5^d/\text{Re}(m_3^d) = 0.058.$$  

It should be noted that there are 10 real parameters and one phase in mass matrices to fine tune to produce six quark masses, three mixing angles and one CP-violating phase. Therefore the unitary matrices may be determined without ambiguity in this tuning process. The explicit unitary matrices are found to be:

$$U_{uL} \approx \begin{pmatrix} 0.64 & -0.77 & 0.038 \\ -0.77 & -0.64 & 0.038 \\ 0.0051 & 0.054 & 1.0 \end{pmatrix},$$

$$U_{uR} \approx \begin{pmatrix} 0.64 & 0.77 & 0.031 \\ -0.77 & 0.64 & 0.031 \\ 0.0041 & -0.043 & 1.0 \end{pmatrix},$$

$$U_{dL} \approx \begin{pmatrix} 0.77 & 0.62 - 0.15 I & 0.023 - 0.027 I \\ -0.62 - 0.15 I & 0.77 & 0.022 - 0.027 I \\ -0.0079 - 0.00063 I & -0.036 - 0.034 I & 1.0 \end{pmatrix},$$

$$U_{dR} \approx \begin{pmatrix} 0.77 & 0.59 + 0.16 I & 0.11 + 0.13 I \\ -0.62 + 0.15 I & 0.75 & 0.11 + 0.13 I \\ -0.039 + 0.0031 I & -0.18 + 0.17 I & 1.0 \end{pmatrix}.$$
Note that the off-diagonal elements in $U_{dL}$ and $U_{dR}$ carry large complex phases.

The $S_3$-symmetry restricts also the squark mass matrices to the same forms as given by eq. (12). These matrices should be rotated to bases of the quark mass eigenstates by using the unitary matrices given by eqs. (29-32). As for the down-type squark mass matrix, the ratios of the off-diagonal elements to the average of squark mass squared turn out to be as follows;

\begin{align}
(\delta^d_{12})_{LL} &\approx (3.0 \times 10^{-4} + 2.4 \times 10^{-4} I) \Delta_L, \\
(\delta^d_{13})_{LL} &\approx (-7.9 \times 10^{-3} + 6.3 \times 10^{-4} I) \Delta_L, \\
(\delta^d_{23})_{LL} &\approx (-3.5 \times 10^{-2} + 3.4 \times 10^{-2} I) \Delta_L.
\end{align}

and

\begin{align}
(\delta^d_{12})_{RR} &\approx (7.3 \times 10^{-3} - 6.0 \times 10^{-3} I) \Delta_R, \\
(\delta^d_{13})_{RR} &\approx (-3.7 \times 10^{-2} - 3.0 \times 10^{-3} I) \Delta_R, \\
(\delta^d_{23})_{RR} &\approx (-1.7 \times 10^{-1} - 1.6 \times 10^{-1} I) \Delta_R.
\end{align}

where $\Delta_{L(R)} = b_{L(R)} - a_{L(R)}$. The upper bounds for the parameters from measurements of $K - \bar{K}, D - \bar{D}, B_d - \bar{B}_d$ mixing, $\epsilon_K, b \rightarrow s \gamma$ and $\epsilon'/\epsilon$ [7] are shown in Table 2. The imaginary parts are constrained by CP-violating processes. Interestingly enough these mixings satisfy the experimental bounds not only for the FCNC but also for CP-violation. Also the off diagonal elements of the up-type squark mass matrix are found to be on the same order of those for the down-type squarks. Therefore mixing in the up-type squark sector is sufficiently smaller than the experimental bounds.

| $\sqrt{|\text{Re}(\delta^d_{12})^2_{LL,RR}|}$ | $\sqrt{|\text{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}|}$ | $\sqrt{|\text{Re}(\delta^d_{12})^2_{LR}|}$ |
|-----------------|-----------------|-----------------|
| $4.0 \times 10^{-2}$ | $2.8 \times 10^{-3}$ | $4.4 \times 10^{-3}$ |

| $\sqrt{|\text{Re}(\delta^d_{13})^2_{LL,RR}|}$ | $\sqrt{|\text{Re}(\delta^d_{13})_{LL}(\delta^d_{13})_{RR}|}$ | $\sqrt{|\text{Re}(\delta^d_{13})^2_{LR}|}$ |
|-----------------|-----------------|-----------------|
| $9.8 \times 10^{-2}$ | $1.8 \times 10^{-2}$ | $3.3 \times 10^{-3}$ |

| $\sqrt{|\text{Re}(\delta^d_{12})^2_{LL,RR}|}$ | $\sqrt{|\text{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}|}$ | $\sqrt{|\text{Re}(\delta^d_{12})^2_{LR}|}$ |
|-----------------|-----------------|-----------------|
| $1.0 \times 10^{-1}$ | $1.7 \times 10^{-2}$ | $3.1 \times 10^{-3}$ |

| $\sqrt{|\text{Im}(\delta^d_{12})^2_{LL,RR}|}$ | $\sqrt{|\text{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}|}$ | $\sqrt{|\text{Im}(\delta^d_{12})^2_{LR}|}$ |
|-----------------|-----------------|-----------------|
| $3.2 \times 10^{-3}$ | $2.2 \times 10^{-4}$ | $3.5 \times 10^{-4}$ |

| $|\delta^d_{23}|_{LL,RR}$ | $|\delta^d_{23}|_{LR}$ |
|-----------------|-----------------|
| $8.2 \bar{m}_q$ | $1.6 \times 10^{-2}$ |

| $|\text{Im}(\delta^d_{12})_{LL,RR}|$ | $|\text{Im}(\delta^d_{12})_{LR}|$ |
|-----------------|-----------------|
| $4.8 \times 10^{-1}$ | $2.0 \times 10^{-5}$ |

Table 2: Limits on the $|\sigma^d_{ij}|$ from $K - \bar{K}, D - \bar{D}, B_d - \bar{B}_d$ mixing, $\epsilon_K, b \rightarrow s \gamma$ and $\epsilon'/\epsilon$ for $m_q/m_\tilde{q} = 1$ [7]. Here the parameter $\bar{m}_q$ denotes $m_\tilde{q}$(GeV)/500.
Mixing between the left-handed and the right-handed squarks and also their effects to FCNC and CP-violation may be evaluated just as done for the slepton sector. Again it is crucial that the A-terms are constrained by the $S_3$-symmetry. The left-right mass matrix is parametrized as

$$\tilde{m}^2_{LR} = \begin{pmatrix}
m_1 A_1 + m_2 A_2 & m_2 A_2 & m_5 A_5 \\
m_2 A_2 & m_1 A_1 - m_2 A_2 & m_5 A_5 \\
m_4 A_4 & m_4 A_4 & m_3 A_3
\end{pmatrix},$$

(39)

with assuming $A_i$ are on the same order of the gaugino mass. Now the off-diagonal elements on the bases of quark mass eigenstates may be explicitly obtained, since the rotation matrices $U_{uL(R)}$ and $U_{dL(R)}$ are fixed. Their ratio to the average squark masses, whose experimental constraints are shown also in Table 2, are found to be

$$\langle \delta_{12}^u \rangle_{LR} \sim O(10^{-5}) \left( \frac{500}{m_{\tilde{q}}(\text{GeV})} \right)^2,$$

(40)

$$\langle \delta_{13}^u \rangle_{LR} \sim O(10^{-4}) \left( \frac{500}{m_{\tilde{q}}(\text{GeV})} \right)^2,$$

(41)

$$\langle \delta_{23}^u \rangle_{LR} \sim O(10^{-3}) \left( \frac{500}{m_{\tilde{q}}(\text{GeV})} \right)^2,$$

(42)

$$\langle \delta_{12}^d \rangle_{LR} \sim \left( O(10^{-6}) + O(10^{-8}) I \right) \left( \frac{500}{m_{\tilde{q}}(\text{GeV})} \right)^2,$$

(43)

$$\langle \delta_{13}^d \rangle_{LR} \sim \left( O(10^{-5}) + O(10^{-6}) I \right) \left( \frac{500}{m_{\tilde{q}}(\text{GeV})} \right)^2,$$

(44)

$$\langle \delta_{23}^d \rangle_{LR} \sim \left( O(10^{-4}) + O(10^{-5}) I \right) \left( \frac{500}{m_{\tilde{q}}(\text{GeV})} \right)^2,$$

(45)

where the A-parameters $A_i$ are assumed to be $O(500\text{GeV})$. It is seen that alignment of the LR-mass matrix to the Yukawa matrix is remarkably good. This is because (1,2)-elements of the matrix $\tilde{m}^2_{LR}(\tilde{m}^2_{LR})^\dagger$, or $(\tilde{m}^2_{LR})^\dagger \tilde{m}^2_{LR}$, are almost of the democratic type and mixing with the 3rd elements are tiny. Thus it has been found that the FCNC as well as CP constraints for supersymmetric extension of the standard model are resolved solely by the exact flavor symmetry of $S_3$.

One of the important features of this model is to have a pair of $S_3$-doublet Higgs fields $H_i^{U(D)}(I = 1, 2)$ as well as a pair of $S_3$-singlet Higgs $H_3^{U(D)}$. Also all the Higgs fields must acquire non-vanishing VEV. Moreover $\langle H_1^{U(D)} \rangle = \langle H_2^{U(D)} \rangle$ should be satisfied, which has been assumed so far. On the other hand the $S_3$-invariant supersymmetric mass terms for these Higgs fields may be given by

$$W = \mu_D H_1^U H_1^D + \mu_S H_3^U H_3^D,$$

(46)

as extension of the so-called $\mu$-term in the MSSM. Then, however, it is found that the global symmetry of the tree-level Higgs potential is enhanced to the complex extension of $U(2)$. Consequently, there appear extra massless goldstone particles and this naive
extension cannot be accepted phenomenologically. Note, however, that the whole theory does not have this enhanced symmetry and their masses are not protected from radiative corrections in the presence of soft supersymmetry breaking terms. Therefore, the dangerous light Higgs particles are not exactly massless (pseudo-goldstones), although they will be still much lighter than the lightest Higgs in the MSSM.

Thus it is necessary to extend the Higgs sector so that the tree level potential does not have continuous symmetries enhanced from $S_3$. For this purpose let us introduce SM-gauge singlets $N_I (I = 1, 2)$ and $N_3$ belonging to $2 + 1_S$ of $S_3$, where we assume that $N$'s have even parity of $Z_2$. ($S_3$-doublet (singlet) Higgs fields have been assigned to be odd (even)under the $Z_2$-parity.) Then we consider the $S_3$-invariant superpotential

$$W = \mu_D H_I^U H_D^I + \mu_S H_3^U H_3^D + m_D N_I N_I + m_S N_3 N_3$$
$$+ \lambda_1 H_I^U H_D^J N_3 + \lambda_3 H_3^U H_3^D N_3 + \lambda_2 H_I^U H_D^J (\kappa_{IJ} N_1 + \eta_{IJ} N_2)$$
$$+ \lambda_4 (N_3)^3 + \lambda_5 N_I N_I N_3,$$  \hspace{1cm} (47)

where the matrices $\kappa$ and $\eta$ are given by (4). The superpotential (47) has no enhanced continuous symmetry. Observe that the superpotential (47) is also invariant under

$$H_1 \leftrightarrow \pm H_2, \ N_1 \rightarrow N_1, \ N_2 \rightarrow -N_2.$$  \hspace{1cm} (48)

Furthermore, the $D$ terms and also the relevant soft supersymmetry breaking terms have the same discrete symmetry. This implies that $\langle H_1 \rangle = \langle H_2 \rangle, \ \langle N_2 \rangle = 0$ correspond to a stationary point of the scalar potential 2. We, therefore, can expect that the desired properties (avoiding the pseudo Goldstones and $\langle H_1 \rangle = \langle H_2 \rangle$) can be satisfied in a wide range of the parameter space. The masses of the Higgs multiplets depend on the parameters in the scalar potential, and the presence of multiple Higgs particles give rise to FCNCs and CP-violations at the tree-level. In multi-Higgs models, the tree-level FCNCs and CP-violations may cause problems in general. However, various dangerous processes in the nonsupersymmetric case have been considered in [16], and it has been found that these processes satisfy the severe experimental upper bounds. So we may assume that the situation does not change by supersymmetrization. A complete analysis on this problem will go beyond the scope of the present paper, and we would like to leave it for future problems.

The idea to use nonabelian discrete symmetries to soften the SUSY flavor problem is not new [17, 18]. The only problem is that the symmetries considered in [17, 18] are explicitly broken at the supersymmetry breaking scale. Therefore, there is no convincing reason why they should be intact in the SSB sector. The flavor symmetry recently found in [16] seems to be exact so far: It is spontaneously broken, only because the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken. It is therefore the most natural to assume that the flavor symmetry is intact in the SSB sector, too. We indeed found in this letter that the flavor symmetry of [16] can soften considerably the SUSY flavor problem.

2The other stationary point $\langle H_1 \rangle = -\langle H_2 \rangle$ is physically equivalent to $\langle H_1 \rangle = \langle H_2 \rangle$, because they can be related by a phase rotation of the matter supermultiplets.
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