I propose a new class of interpretations, real world interpretations, of the quantum theory of closed systems. These interpretations postulate a preferred factorization of Hilbert space and preferred projective measurements on one factor. They give a mathematical characterisation of the different possible worlds arising in an evolving closed quantum system, in which each possible world corresponds to a (generally mixed) evolving quantum state. In a realistic model, the states corresponding to different worlds should be expected to tend towards orthogonality as different possible quasiclassical structures emerge or as measurement-like interactions produce different classical outcomes. However, as the worlds have a precise mathematical definition, real world interpretations need no definition of quasiclassicality, measurement, or other concepts whose imprecision is problematic in other interpretational approaches. It is natural to postulate that precisely one world is chosen randomly, using the natural probability distribution, as the world realised in Nature, and that this world’s mathematical characterisation is a complete description of reality.

INTRODUCTION

The aim of this paper is to define a new class of interpretations of quantum theory. These interpretations are defined in their own terms and stand or fall on their own merits. However, one way to motivate them, which I find helpful and hope readers also may, is to start from the idea that the unitarily evolving quantum state of a closed system somehow branches into different possible worlds, to note some of the problems with this idea as traditionally understood, and then to consider a new approach to these problems.

The idea of branching worlds is, of course, traditionally associated with many-worlds interpretations of quantum theory. According to a traditionally popular view of many-worlds interpretations, a branch is supposed to separate into a number of sub-branches whenever a quantum measurement takes place, with these sub-branches corresponding to different measurement outcomes. More generally, different branches are supposed to be defined by different classical events, whether or not these events are associated with quantum measurements as traditionally defined.

Some Everettians, it should be noted, argue that the universal wave function can and should be interpreted differently, and in particular that a many-worlds interpretation need not invoke the notion of branching at a fundamental level. This paper does not engage in that debate, since it is not relevant to our discussion. The interpretation of quantum theory proposed here is fundamentally different from Everett’s and other previously proposed interpretational ideas (for example [1, 2, 3, 4, 5, 6, 7, 8]), though it has some features in common with some of them.

That said, the mathematical definitions of branching given below should also be of interest even to a convinced believer in purely unitary quantum mechanics who sees no fundamental role for branchings. They can be interpreted purely phenomenologically, as attempts to define a branching structure for a quantum theory of cosmology, in such a way that the structure tends to contain quasiclassical worlds, to the extent that the cosmological theory allows. Even those who are not persuaded that a quasiclassical world need be defined by fundamental laws acknowledge that, at any rate, we experience such a world, and that we need to be able to characterise such worlds in order to test fundamental theory against our experience. To do this, we need a procedure which starts from the theory and generates descriptions of such worlds, thus allowing us to derive statements of the form “A quasiclassical world has property X with probability p, according to our theory.” It would be very useful if this procedure could be defined by a mathematical algorithm. Our definitions of branching structures are proposals for algorithms of this type.

INTERPRETING A BRANCHING STRUCTURE

Many worlds or one?

As Bell and others have noted, if the branching of worlds were objectively defined, a many worlds interpretation would seem unnecessarily extravagant. Given a precisely defined branching structure, we can just as easily define a one world interpretation of quantum theory.
To flesh this out, let us suppose that we indeed have an algorithm for defining a branching structure for the universal wave function, given a formulation of quantum theory, an initial state \(\psi(0)\) at \(t = 0\), and a Hamiltonian \(H\). For the purposes of this illustration, we follow a traditional view of a branching structure, taking it to be a process by which the unitarily evolving wave function is divided into a sum of components at some time \(t_1\), each component then unitarily evolves but may further sub-divide at some later time, from which point the sub-components then unitarily evolve until such time as they sub-divide into sub-sub-components, and so on. Branches are not allowed to merge again after dividing, on this view. Suppose too that the branching structure becomes asymptotically constant – in other words that the number of branches becomes constant – as \(t \to \infty\), and that we can apply the algorithm in the asymptotic limit to obtain a complete list of branches \(B_j\).

Each branch is then uniquely identified by a final component, that is, one which will undergo no further sub-division: the branch can be identified throughout time by following the branching diagram backwards from the final states. Each branch \(B_j\) thus defines an unnormalised pure quantum state \(\psi_j(t)\) at each time \(t\), which can be identified from the final component by following the branch backwards and taking the component corresponding to the branch at any given time. The branch states need not be, and generally will not be, orthogonal at every time \(t\): in particular, before any given sub-division, all branches whose final components ultimately arise from that sub-division will have the same branch state. However, we suppose that asymptotically the branch states define an orthogonal decomposition of \(\psi(t)\), so that
\[
\lim_{t \to \infty} (\psi(t) - \sum_j \psi_j(t)) = 0
\]
and
\[
\lim_{t \to \infty} (\psi_j(t), \psi_j'(t)) = p_j \delta_{jj'},
\]
where the \(p_j\) are constants.

We can then define the probability \(p(B_j)\) of each branch \(B_j\) by the Born rule. If the relevant states also have asymptotic limits, so that we can define
\[
\psi^\infty = \lim_{t \to \infty} \exp(-iHt/\hbar)\psi(0),
\]
and
\[
\psi_j^\infty = \lim_{t \to \infty} \psi_j(t),
\]
we have
\[
p(B_j) = (\psi^\infty, \psi_j^\infty) = p_j.
\]

More generally, so long as the relevant inner product has an asymptotic limit, we can define
\[
p(B_j) = \lim_{t \to \infty} (\psi(t), \psi_j(t)),
\]
where \(\psi(t) = \exp(-iHt/\hbar)\psi(0)\) is the unitarily evolved state at time \(t\).

If these assumptions — the existence of a precisely defined branching structure and the existence of the relevant asymptotic limits — held, then a perfectly sensible interpretation of the mathematics would be to postulate that precisely one branch is randomly chosen, via the Born rule probability distribution, and that this branch alone is realised in Nature.

Clearly, this is more economical than a many-worlds interpretation, in the sense that just one branch is realised in Nature, rather than all. It also makes clear what the Born probabilities are probabilities of. In contrast, it is (at the very least) less immediately clear that many-worlds interpretations can find a sensible role for Born probabilities: whatever \(p(B_j)\) might possibly be in a many-worlds interpretation, it clearly cannot be the probability of reality being described by the branch \(B_j\), since all the branches are equally real.

That said, the aim here is not to argue that, given our starting assumptions, the one-world view is demonstrably correct and the many-worlds view completely indefensible; it is simply to point out that the one-world view appears natural. The one-world view is not actually logically necessary to motivate all the interpretational ideas that we set out below. However, it will be adopted from here on, on the grounds that it is both pedagogically helpful and fundamentally appealing.

Next we examine some standard intuitions about the form of the branching structure.
Do measurement theory and decoherence allow us to define a satisfactory one-world interpretation?

It might perhaps (though naively) be argued that we can go on to sketch a satisfactory interpretation of quantum theory, by defining the branching structure as follows.

First, suppose that each sub-branching — each point at which a branch splits into two or more — corresponds to a quantum measurement type interaction. Then in principle we can define the entire branching tree in a cosmological theory by listing all the quantum measurement-type interactions. Each elementary branch can then be defined by listing a consistent set of possible outcomes of all the measurements encountered on some branching path from the initial to the final time. If we apply the Born probability rule as above, then precisely one branch is picked out as corresponding to reality, and hence precisely one set of consistent measurement outcomes is realised.

Now, standard decoherence arguments tell us that we can calculate the probabilities of quantum measurement outcomes at any point after they have been classically recorded, and that the answers will be essentially constant after that point. In particular, the final time Born probability rule for branches gives essentially the same probability for the set of measurement outcomes as that predicted by Copenhagen quantum mechanics. Thus, it might be argued, we can derive the standard Copenhagen Born rule probabilities for individual measurement events from the final time Born rule for branches, since we have postulated that branchings correspond to quantum measurements. Hence, the argument runs, we have outlined an interpretation of quantum theory which treats the universe as a closed quantum system and which explains both why we see a quasiclassical world and why the Copenhagen interpretation of quantum theory describes the outcomes of our experiments within that world.

Problems with naive definitions of branching

This is too glib, though. Invoking a “quantum measurement type interaction” as a fundamental notion replicates one of the unsatisfactory features of the Copenhagen interpretation. We know roughly what we mean by the term, but we don’t have a precise definition. Is a cosmic committee meant to survey the evolution of the universe and decide case by case whether, and if so precisely when, a measurement took place?

In any case, the notion of quantum measurement isn’t sufficiently general to produce a realistic interpretation of a quantum theory of cosmology. All the quasi-classical objects and structures in our universe, including all measuring devices, natural and artificial, emerged from a purely quantum initial state. The emergence of quasiclassical structure from a quantum state doesn’t correspond to a quantum measurement interaction in any standard sense of the phrase. Yet, without an account of this process, we can’t begin to justify an account of the world as it now appears or the measurement devices and classical records it now contains.

This is a deeper problem than seems generally to be recognised. The set of possible quasi-classical structures that could have emerged from the initial state of our universe is, presumably, a continuous set, not a discrete one, at least if space-time itself is continuous. The peak of the Moon’s centre of mass wave function could have been epsilonically further away from that of the Earth, our galaxy’s relative momentum to its neighbours could have been slightly different, and so on. All these possible quasi-classical structures are ultimately defined by quantum states. We could perhaps address this problem mathematically by defining a complete set of mutually exclusive quasi-classical states, from which precisely one is realised, if we had some natural mathematical prescription; for instance, a natural projective decomposition on the subspace spanned by all the quasi-classical states. However, the notion of quasi-classicality doesn’t, per se, supply such a prescription.

Another consequence of the multiplicity of quasi-classical structures is that, even when we want to consider a particular measurement event, labelling Everett branches by its outcomes is much more problematic than it first appears. Though we may think we have a particular experiment in mind, it is harder than it initially seems to pin down exactly what we are or should be referring to. Would an instantiation of the experiment in an alternative quasiclassical world with the Moon’s centre-of-mass wave function peak very slightly further away from the Earth’s count as the same experiment or a different one? What about an instantiation in which the experimental apparatus is in a slightly different quasiclassical state? Again, resolving these and similar questions requires mathematical criteria, and the notions of quasiclassicality and measurement don’t seem, per se, adequate to define suitable criteria.

CHARACTERISING BRANCHES MATHEMATICALLY

We are thus motivated to propose a mathematical characterisation of possible branches. First, we need some assumptions.
Suppose we are given a quantum theory of a closed system with Hilbert space $\mathcal{H}$, Hamiltonian $H$ and initial state $\psi_I = \psi(0)$. In non-relativistic quantum mechanics, we can obtain the state at any later time $t$ as $\psi(t) = \exp(-iHt/\hbar)\psi_I$. In relativistic quantum field theory with a suitable fixed background space-time, for example Minkowski space, we can obtain the quantum state on any spacelike hypersurface by the Tomonaga-Schwinger formalism, assuming (as we do) that we have a field theory for which the Tomonaga-Schwinger solutions are mathematically well defined on all hypersurfaces. To apply our discussion to a hypothetical quantum theory of gravity, we assume for now – without worrying about the details – that the quantum gravity theory allows something analogous, i.e. that we can evolve the initial state to produce sequences of states of the matter and gravitational fields that characterise the evolving physical system.

It might be objected that we cannot presently justify assuming the existence of a mathematically well defined quantum field theory that characterises the non-gravitational interactions, let alone a mathematically well defined quantum theory of gravity. Granted, theoretical physics has not produced such theories as yet (or at least not demonstrably so). But our aim here is to offer a new way of interpreting of quantum theories, not to solve all the current problems of theoretical physics. We need to assume that we have a mathematically well defined theory in order for there to be any sense in trying to interpret it. We also need to postulate some properties of the theory. These postulates may not all necessarily turn out to be valid for the final quantum theory of everything, if indeed there is one. Perhaps the best case that can be made for them at present is that they seem reasonably plausible, are not particularly ad hoc, and do in fact apply to some interesting theories.

So, we adopt the following assumptions:

**Postulate 1** The Hilbert space $\mathcal{H}$ of the system has a natural preferred representation as a tensor product $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, which is defined at every time (or on every spacelike hypersurface, or for every possible evolved state in a quantum gravity theory).

**Postulate 2** There is a natural preferred complete set of orthogonal projections $\{P_i\}_{i \in I}$ defined on the Hilbert space $\mathcal{H}_B$ at every time. (Or on every spacelike hypersurface, etc.: to save repetition and fix our notation we henceforth state the postulates for the case of non-relativistic quantum mechanics and take the (more fundamentally interesting) alternatives of relativistic quantum field theory and quantum gravity as understood.)

For quantum field theory in Minkowski space, examples of possible natural tensor products are given by taking $\mathcal{H}_A$ and $\mathcal{H}_B$ to be the subspaces corresponding respectively to fermions and bosons, or massive and massless particles. (Or vice versa – see the discussion below.) For a quantum gravity theory, possible examples might be for $\mathcal{H}_A$ and $\mathcal{H}_B$ to correspond respectively to the gravitational and matter degrees of freedom, or perhaps to the massless particles (including gravitons and photons) and the massive particles. (Or perhaps vice versa – again see the discussion below.)

An example of a possible natural projective decomposition (defined via a limit) is the set of projections onto the simultaneous eigenstates of single-particle position operators. Other examples are the sets of projections onto simultaneous eigenstates of momentum operators or of energy operators. Another example, which depends on the state $\psi(t)$ at any given time (or the state on any given hypersurface, etc.) is determined by the Schmidt decomposition

$$\psi(t) = \sum_j (p_j(t))^{1/2}e_j(t) \otimes f_j(t),$$

where for each time $t$ the sets $\{e_j(t)\}$ and $\{f_j(t)\}$ are subsets of orthonormal bases of $\mathcal{H}_A$ and $\mathcal{H}_B$ respectively. In this case we can take the projectors $P_i$ at time $t$ to be projections onto the subspaces $\langle f_j(t) : p_j(t) = p_k(t) \rangle$ spanned by vectors $f_j(t)$ whose Schmidt coefficients are equal.

Given the chosen projective decomposition, we can decompose the state at any time as

$$\psi(t) = \sum_k (I \otimes P_k) \psi(t) = \sum_k \psi_k(t),$$

where $\psi_k(t) = (I \otimes P_k) \psi(t)$. Here the range of $k$ may depend on time $t$: for instance, if the $P_k$ are Schmidt projections, the size of their set depends on the number of degeneracies in the Schmidt decomposition. The notation $\sum_k$ should be read as implicitly allowing this, and also as allowing integrals over continuous ranges instead of, or combined with, a discrete sum. (Note that we have not yet defined a branching structure: the $\psi_k(t)$ will not, according to our definitions, generally define states corresponding to different branches at time $t$.)

Suppose now that we have some hypothetical fundamental physical theory which stipulates that time runs only
from 0 to $T$, so that we have initial state $\psi_I = \psi(0)$ and final state $\psi_F = \psi(T)$. We have the final state decomposition

$$\psi(T) = \sum_l (I \otimes P_l)\psi(T) = \sum_l \psi_l(T).$$

We will define a set of branches with a one-to-one correspondence between the branches $B_l$ and those states $\psi_l(T)$ in equation (3) that are nonzero.

Define

$$q_l^T(k, t) = |(\psi_l(T), \exp(-i H(T - t)/\hbar)\psi_k(t))|^2$$

and

$$p_l^T(k, t) = \frac{q_l^T(k, t)}{\sum_k q_l^T(k, t)}.$$

Define

$$\rho_l^T(t) = \sum_k \text{Tr}_{H_B}(\psi_k(t)^\dagger \psi_k(t)) p_l^T(k, t)$$

(4)

where the sum is over indices $k$ for which $\psi_k(t)$ is nonzero. We call $\rho_l^T(t)$ the real state of branch $B_l$ at each time $t$. Note that $\rho_l^T(t)$ is generally a mixed quantum state and is defined on $H_A$ alone.

**Postulate 3** In a closed quantum system for which a fundamental physical theory stipulates that time runs only from 0 to $T$, precisely one of the branches $B_l$ is realised. The branch $B_l$ is realised with probability

$$p_l^T = (\psi(T), \psi_l(T)) = (\psi_l(T), \psi(T)).$$

If $B_l$ is realised, then reality at each time $t$ in the range $0 \leq t \leq T$ is described by the real state $\rho_l^T(t)$.

**Asymptotic Hypothesis** In the fundamental theories of closed quantum systems that we consider, the quantities defined above have asymptotic limits as $T \to \infty$. That is, the probabilities $p_l^T$ tend to constants $p_l^\infty$ as $T \to \infty$ and the real state $\rho_l^T(t)$, for any finite $t$, tends to a constant state $\rho_l^\infty(t)$ as $T \to \infty$.

**Postulate 4** In a closed quantum system, described by a fundamental theory satisfying the asymptotic hypothesis, for which time runs from 0 to $\infty$, precisely one of the branches $B_l$ is realised. The branch $B_l$ is realised with probability given by the asymptotic value

$$p_l^\infty = \lim_{T \to \infty} p_l^T.$$

If $B_l$ is realised, then reality at each time $t \geq 0$ is described by the real state defined by the asymptotic limit

$$\rho_l^\infty(t) = \lim_{T \to \infty} \rho_l^T(t).$$

**DISCUSSION**

**Ontology**

The real quantum state plays a crucial role in real world interpretations, directly representing physical reality. Indeed, perhaps the most natural version of the interpretation is to take the real quantum state as the primary physical quantity, and to relegate the standard quantum state vector to the status of an auxiliary mathematical quantity, useful for calculating the possible realised branches and their probabilities, but not necessarily itself corresponding to anything in reality. In Bell’s terminology, on this view, the real state corresponds to a beable, while the standard quantum state need not.

The factorization of $\mathcal{H}$ into $\mathcal{H}_A \otimes \mathcal{H}_B$ is meant to be encoded into the laws of nature; not an arbitrary choice, nor an observer-dependent construction, nor a split into system and environment defined for a particular experiment. Moreover, the degrees of freedom characterised by $\mathcal{H}_B$ play a different role to those characterised by $\mathcal{H}_A$: the $\mathcal{H}_B$ degrees of freedom are auxiliary mathematical objects, which do not necessarily have any direct correspondence to reality, whereas the $\mathcal{H}_A$ degrees of freedom do directly represent reality.
Correspondence to reality

Why might one hope such a picture can accurately represent the quasiclassical world we perceive?

Role of the fundamental factorization

Firstly, we require that the factorization $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ allows us to characterise quasiclassical physics entirely in terms of degrees of freedom corresponding to $\mathcal{H}_A$.

In the case of quantum field theory in Minkowski space, it seems fairly clear that this can be done. If $\mathcal{H}_A$ and $\mathcal{H}_B$ are the subspaces corresponding respectively to fermions and bosons, or massive and massless particles, then in either case we can represent the local densities of chemical species in terms of $\mathcal{H}_A$ degrees of freedom. We can then express quasiclassical variables — describing, for instance, the orbit of the Moon around the Earth, or the behaviour of apparatus and measuring devices in a two-slit experiment — in terms of these local densities.

Though it may seem slightly less intuitive, this is also presumably true if we swap the roles of $\mathcal{H}_A$ and $\mathcal{H}_B$. In that case, the photon degrees of freedom in $\mathcal{H}_A$ allow us to characterise atoms and larger aggregations of matter in terms of their associated electromagnetic fields, and hence to recover a description of the quasiclassical world.

In the case of quantum gravity, the question is, as usual, less clear. Taking $\mathcal{H}_A$ and $\mathcal{H}_B$ correspond respectively to the gravitational and matter degrees of freedom allows the spacetime to be characterised entirely by $\mathcal{H}_A$ degrees of freedom. Since any significant quasiclassical event involving matter eventually has a significant effect on the gravitational field, this might perhaps suffice (though the scale of the timelag and its ontological significance would need, at least, to be discussed). If $\mathcal{H}_A$ corresponds to massless particles, including gravitons and photons, then, as above, we can characterise matter distributions in terms of their associated electromagnetic fields, and thus one would expect to be able to give a direct description of both matter and metric at classical scales.

As we lack both a well defined quantum gravity theory and a clear understanding of the relevant physics as the universe tends towards its final state, we can offer nothing better than reasoned speculation. Still, these possibilities suggest it is reasonable to hypothesise that an appropriate factorization might exist in quantum gravity theories.

Given an appropriate choice of factorization, we can think of the degrees of freedom in $\mathcal{H}_A$ as characterising a quasiclassical system, and those in $\mathcal{H}_B$ an environment interacting with that system. We can also think of ourselves as, in a certain sense, contained within the system and distinct from the environment. The system and environment here are not disjoint and separated physical entities: it is not the case that every physical degree of freedom within the region defined by our bodies belongs to $\mathcal{H}_A$. Nonetheless, all the quasiclassical variables which we assume define our experiences can be defined by degrees of freedom in $\mathcal{H}_A$ and, in that looser but natural sense, one may say we belong to $\mathcal{H}_A$.

Role of the final projective decomposition

Part of the intuition behind defining branches via the final state decomposition is that, for a realistic model with realistic dynamics, an appropriate factorization, and an appropriate final projective decomposition, the possible final states of the environment characterise the possible quasiclassical histories of the system. In particular, the effects of any significant quasiclassical event are irreversibly recorded in the environment, in such a way that the record can be “read” by an appropriate abstractly specified final projective decomposition, such as one of those discussed above. The idea here is that any two significantly distinct alternative quasiclassical outcomes — of cosmological evolution, or of a quantum experiment — should produce final states which are very nearly orthogonal and which are distinguished with near certainty by an appropriate natural asymptotic projective decomposition on $\mathcal{H}_B$. For example, if the projective decomposition is onto photon momentum states, the intuition is that the final cosmological states resulting from two different possible outcomes of a measurement are distinguished by photon momenta, because the background electromagnetic field some time after the experiment effectively carries a record of the result which will persist to the end of time.

We use the final projective decomposition to define an effective post-selection on the ensemble of possible evolutions. One way of picturing this is that Nature first simulates the evolution of the universe using standard quantum theory, with the initial state $\psi_I = \psi(0)$, carries out a measurement at $t = \infty$, defined by the final state decomposition, on this simulated system, and then uses the result of this measurement to determine the branch that She proceeds to realise in the actual universe. A less teleological and less anthropomorphic view is simply to think of the branches as natural structures which define a sample space for a fundamentally probabilistic theory, which has a natural probability
distribution defined by the Born rule. The theory simply says that one element of the sample space is randomly selected, and this element determines reality.

Role of the finite time projective decompositions

Suppose we now that we adopt the view just mentioned, namely that our universe is effectively post-selected by the outcome of a final projective measurement at \( t = \infty \), defined by the asymptotic projective decomposition. One would then expect, if we aim for an interpretation that produces a description of reality at times between 0 and \( \infty \), that this description should be determined by the post-selected final state as well as the initial state.

If we live in such a universe, then we have no direct knowledge of the post-selected measurement outcome, of course. But, given a theory which fixes the initial state and the Hamiltonian, we could in principle list all the possible outcomes, and use the standard quantum pre- and post-selection rules to infer the probabilities of our observations (of our own experiments or of other events) conditioned on any given outcome of this final measurement. These calculations would be consistent with standard quantum theory: postulating a final measurement does not affect the intervening dynamics or the overall probabilities of any intervening event.

Let us imagine further that a measurement were carried out at time \( t \) on \( \mathcal{H}_B \), defined by the projective decomposition at time \( t \). If our experience is characterised by \( \mathcal{H}_A \) — i.e. if the only degrees of freedom we need to describe it are those of \( \mathcal{H}_A \) — then we learn nothing directly about the outcome of the measurement on \( \mathcal{H}_B \). However, supposing for the moment that this is the only measurement intervening between the initial and final states, we can infer the probabilities of the various outcomes from the initial state and any possible final state. We can further calculate a density matrix describing the state of the \( \mathcal{H}_A \) degrees of freedom at time \( t \), using these probabilities: this density matrix depends on the final state \( \psi_f \), and hence (since we define the branch by the final state) on the branch \( B_f \). This is precisely the calculation by which we arrive at the real state \([11]\).

Now we take a further step. We calculate density matrices on \( \mathcal{H}_A \) at every time \( t \) using the above procedure, and we suppose that we can give a coherent time-dependent description of evolving physical worlds using the density matrices \( \rho(t) \) corresponding to branch \( B_l \) at each time \( t \).[14] This postulate does not follow from standard quantum theory. It can only be justified empirically.

What could constitute empirical justification? Roughly speaking, we would need to show that, given a realistic model, the proposed interpretation explains why we experience a persistently quasiclassical world of the type we do. One good test would be whether that it predicts with high probability that the realised world will be persistently quasiclassical and that our world is in some sense typical in the set of realised worlds.[15] A weaker, but still significant, and arguably necessary, criterion is whether it predicts, with high probability, that if the realised world is quasiclassical, with large-scale structure, for some reasonably long time interval, then it will continue in that state for a further long interval.[16]

These properties will clearly not hold for worlds defined by a generic choice of time-dependent projective decomposition on \( \mathcal{H}_B \), nor will the asymptotic hypothesis hold true for generic choices. The particular choices suggested above are chosen because it seems plausible that one or more of them might indeed define persistently quasiclassical worlds in realistic models, and that (for essentially the same reason) the asymptotic hypothesis might indeed hold true in such models.

The idea here is that the split \( \mathcal{H}_A \otimes \mathcal{H}_B \) and the postulated natural projective decompositions at each time (on each hypersurface, etc) are chosen so that (to good approximation) the projective decomposition at any given time characterises quasiclassical information about \( \mathcal{H}_A \) irreversibly recorded in degrees of freedom of \( \mathcal{H}_B \). Each possible outcome of the final projective decomposition characterises a possible branch, in which all the information necessarily to characterise the final quasiclassical state of \( \mathcal{H}_A \) is encoded in the corresponding outcome state in \( \mathcal{H}_B \). For each possible branch, we can define the real state at any given time \( t \), a mixed state on \( \mathcal{H}_A \) defined by the pre- and post-selected measurement probabilities for the natural projective decomposition on \( \mathcal{H}_B \) at time \( t \). This state approximately characterises the quasiclassical information about \( \mathcal{H}_A \) that is already, at time \( t \), irreversibly encoded in \( \mathcal{H}_B \).

The real state of any given branch is thus defined via a procedure that is defined by rules based on standard unitary quantum dynamics (with a specified Hamiltonian) and on the projection postulate — though the real state itself neither follows standard unitary quantum dynamics nor standard projection postulate induced state collapse.

Can we, in fact, show that for one of the natural factorizations and natural sets of projective decompositions described above, it is indeed the case that, to good approximation, in realistic cosmological models, the projective decomposition at any given time characterises quasiclassical information about \( \mathcal{H}_A \) irreversibly recorded in degrees of freedom of \( \mathcal{H}_B \), and the asymptotic hypothesis holds? The question should be, at any rate, decidable, since there are rather few choices that have any serious claim to be regarded as natural. The projective decomposition most studied
to date in other contexts — in particular in investigations of modal interpretations of quantum theory — is the Schmidt decomposition. Reasons have been identified for doubting that it generally identifies quasi-classical degrees of freedom in realistic models. It seems fair to say, though, that the question is not yet definitively resolved. In any case, the alternatives of projective decompositions defined by the position and momentum eigenstates of fields in $\mathcal{H}_B$ look very promising.

In short, there are good intuitive reasons to take the hypothesis seriously. We may, in fact, have a relatively natural solution to the measurement problem, which essentially respects unitary quantum dynamics and Lorentz or general covariance. The form of the real state also means that it is at least not obvious that this interpretation has the sort of potentially worrisome “tails problem” that arises in modified dynamical theories in which suppressed components of the state vector corresponding to unselected quasi-classical worlds acquire exponentially small amplitudes but nonetheless retain (something like) the same mathematical structure and information content of the selected component.

Detailed calculations in realistic models remain to be carried out; they should illuminate these questions further and, one might reasonably hope, give persuasive evidence that a well defined real world interpretation can adequately describe the quasi-classical reality we experience.

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