Scaling in $\gamma^* p$ total cross sections and the generalized vector dominance/color dipole picture

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December 17, 2021

Abstract

The scaling in $\sigma_{\gamma p}$ cross sections (for $Q^2/W^2 << 1$) in terms of the scaling variable $\eta = (Q^2 + m_0^2)/\Lambda^2(W^2)$ is interpreted in the generalized vector dominance/color-dipole picture (GVD/CDP). The quantity $\Lambda^2(W^2)$ is identified as the average gluon transverse momentum absorbed by the $q\bar{q}$ state, $\langle \vec{l}^2 \rangle = (1/6)\Lambda^2(W^2)$. At any $Q^2$, for $W^2 \to \infty$, the cross sections for virtual and real photons became universal, $\sigma_{\gamma^* p}(W^2, Q^2)/\sigma_{\gamma p}(W^2) \to 1$.

Two important observations were made on deep inelastic scattering (DIS) at low values of the Bjorken scaling variable $x Bj \approx Q^2/W^2 << 1$, since HERA started running in 1993:

i) The diffractive production of high-mass states (of masses $M_X \approx 30 GeV$) at an appreciable rate relative to the total virtual-photon-proton cross section, $\sigma_{\gamma^* p}(W^2, Q^2)$. The sphericity and thrust analysis of the diffractively produced states revealed (approximate) agreement in shape with the final state found in $e^+e^-$ annihilation at $\sqrt{s} = M_X$. This observation of high-mass diffractive production confirms the conceptual basis of generalized vector dominance (GVD) that extends the role of the low-lying vector mesons in photoproduction to DIS at arbitrary $Q^2$, provided $x Bj << 1$.

ii) An increase of $\sigma_{\gamma^* p}(W^2, Q^2)$ with increasing energy considerably stronger than the smooth “soft-pomeron” behavior known from photoproduction and hadron-hadron scattering.

We have recently shown that the data for total photon-proton cross sections,

*Supported by BMBF under Contract 05HT9PBA2
Presented at DIS 2001, Bologna, Italy, April 27 to May 1, 2001.
including virtual as well as real photons, show a scaling behavior. In good approximation,
\[ \sigma_{\gamma^*p}(W^2, Q^2) = \sigma_{\gamma^*p}(\eta), \]
with
\[ \eta = \frac{Q^2 + m_0^2}{\Lambda^2(W^2)} \]
(1)

Compare Fig. 1. The scale \( \Lambda^2(W^2) \), of dimension GeV\(^2\), turned out to be an increasing function of the \( \gamma^*p \) energy, \( W^2 \), and may be represented by a power law or a logarithmic function of \( W^2 \),
\[ \Lambda^2(W^2) = \begin{cases} c_1(W^2 + W_0^2)^{c_2}, \\ c_1' \ln(W^2/W_0^2 + c_2'). \end{cases} \]
(2)

In a model-independent fit to the experimental data, the threshold mass, \( m_0^2 < m_1^2 \), and the two parameters \( c_2(c'_2) \) and \( W_0^2(W'_0^2) \) were found to be given by \( m_0^2 = 0.125 \pm 0.027 \text{GeV}^2, \ c_2 = 0.28 \pm 0.06, \ W_0^2 = 439 \pm 94 \text{GeV}^2 \) with \( \chi^2/ndf = 1.15 \), and \( m_0^2 = 0.12 \pm 0.04 \text{GeV}^2, \ c_2' = 3.5 \pm 0.6, \ W_0^2 = 1535 \pm 582 \text{GeV}^2 \), with \( \chi^2/ndf = 1.18 \). The overall normalization, \( c_1(c_1') \) in (3) is irrelevant for the scaling behavior.

For the interpretation of the scaling law (1), we turn to the generalized vector dominance/color-dipole picture (GVD/CDP)\([6, 5]\), of deep-inelastic scattering at low \( x \ll 1 \). It rests on \( \gamma^* (q\bar{q}) \) transitions from \( e^+e^- \) annihilation, forward scattering of the \( (q\bar{q}) \) states of mass \( M_{q\bar{q}} \) via (the generic structure of) two-gluon exchange\([7]\) and transition to spacelike \( Q^2 \) via propagators of the \( (q\bar{q}) \) states of mass \( M_{q\bar{q}} \). In the transverse-position-space representation\([8]\), we have
\[
\sigma_{\gamma^*p}(W^2, Q^2) = \int dz \int d^2r_\perp |\psi|^2(r_\perp^2 Q^2 z(1-z), Q^2 z(1-z), z) \cdot \sigma_{(q\bar{q})p}(r_\perp^2, z(1-z), W^2). \]
(4)
We refer to ref. [8] for the explicit representation of the square of the photon wave function, $|\psi|^2$. The ansatz (4) for the total cross section must be read in conjunction with the Fourier representation of the color-dipole cross section,

$$
\sigma_{(q\bar{q})p}(r_\perp^2, z(1 - z), W^2) = \int d^2l_\perp \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, z(1 - z), W^2) \cdot (1 - e^{-\vec{l}_\perp \cdot \vec{r}_\perp}).
$$

Upon insertion of (5) into (4), together with the Fourier representation of the photon wave function, one indeed recovers [6] the expression for $\sigma_{\gamma^*p}$ that displays the $x \to 0$ generic structure of two-gluon exchange:

The resulting expression for $\sigma_{\gamma^*p}$ is characterized by the difference of a diagonal and an off-diagonal term with respect to the transverse momenta (or masses) of the ingoing and outgoing $q\bar{q}$ states.

From (5), the color-dipole cross section, in the two limiting cases of vanishing and infinite interquark separation, becomes, respectively,

$$
\sigma_{(q\bar{q})p}(r_\perp^2, z(1 - z), W^2) = \sigma^{(\infty)} \cdot \begin{cases} 
\frac{1}{4}r_\perp^2 (\vec{l}^2)_{W^2,z}, & \text{for } r_\perp^2 \to 0, \\
1, & \text{for } r_\perp^2 \to \infty.
\end{cases}
$$

The proportionality to $r_\perp^2$ for small interquark separation is known as “color transparency” [8]. For large interquark separation, the color-dipole cross section should behave as an ordinary hadronic one. Accordingly,

$$
\sigma^{(\infty)} = \pi \int dl_\perp^2 \tilde{\sigma}(\vec{l}_\perp^2, z(1 - z), W^2)
$$

must be independent of the configuration variable $z$ and has to fulfill the restrictions from unitarity on its energy dependence. The average gluon transverse momentum $\langle \vec{l}^2 \rangle_{W^2,z}$ in (6), is defined by

$$
\langle \vec{l}^2 \rangle_{W^2,z} = \frac{\int dl_\perp^2 \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, z(1 - z), W^2)}{\int dl_\perp^2 \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, z(1 - z), W^2)}.
$$

Replacing the integration variable $r_\perp^2$ in (4) by the dimensionless variable

$$
u \equiv r_\perp^2 \Lambda^2(W^2) z(1 - z),
$$

the photon wave function becomes a function $|\psi|^2(u, Q^2, \Lambda^2, z)$. The requirement of scaling (1), in particular for $Q^2 >> m_0^2$, then implies that the color-dipole cross sections be a function of $u$,

$$
\sigma_{(q\bar{q})p}(r_\perp^2, z(1 - z), W^2) = \sigma_{(q\bar{q})p}(u).
$$

Taking into account (6), we find

$$
\langle \vec{l}^2 \rangle_{W^2,z} = \Lambda^2(W^2) z(1 - z),
$$

It is precisely the identical structure [8] that justifies the GVD/CDP (4), (5) from QCD.
and upon averaging over $z$,

$$\langle \vec{l}^2 \rangle_{W^2} = \frac{1}{6} \Lambda^2(W^2).$$

(12)

The quantity $\Lambda^2(W^2)$ in the scaling variable (2) is accordingly identified as the average gluon transverse momentum, apart from the factor $1/6$ due to the averaging over $z$.

Inserting $\langle \vec{l}^2 \rangle_{W^2,z}$ from (11) into (6), we have

$$\sigma_{q\bar{q}p} = \sigma^{(\infty)} \cdot \left\{ \begin{array}{ll}
\frac{1}{4} r^2 \Lambda^2(W^2) z(1-z), & \text{for } \Lambda^2 \cdot r^2_\perp \to 0, \\
1, & \text{for } \Lambda^2 \cdot r^2_\perp \to \infty.
\end{array} \right.$$  

(13)

The dependence of the photon wave function in (4) on $r^2_\perp Q^2$ requires small $r_\perp$ at large $Q^2$, in order to develop appreciable strength; for large $Q^2$, the $r^2_\perp \to 0$ behavior in (13), with its associated strong $W$ dependence, becomes relevant until, finally, for sufficiently large $W$, the soft $W$ dependence of $\sigma^{(\infty)}$ will be reached.

Thus, by interpreting the empirically established scaling, $\sigma_{\gamma^*p} = \sigma_{\gamma p}(\eta)$, in the GVD/CDP, we have obtained the dependence of the color-dipole cross section on the dimensionless variable $u$ in (10) and, consequently, with (13), qualitatively, the dependence on $\eta$ shown in fig. 1. Conversely, assuming a functional form for the color-dipole cross section according to (10), one recovers the scaling behavior (1).

In [5], we have shown that approximating the distribution in the gluon momentum transfer by its average value, (11),

$$\bar{\sigma}_{(q\bar{q})p} = \sigma^{(\infty)} \cdot \frac{1}{\pi} \delta (\vec{l}^2_\perp - \Lambda^2(W^2) z(1-z)), $$

(14)

allows one to analytically evaluate the expression for $\sigma_{\gamma^*p}$ in (4) in momentum space. The threshold mass $m_0 < m_p$ enters via the lower limit of the integration over the masses of the ingoing and outgoing $q\bar{q}$ states. For details we refer to [5], and only note the approximate result

$$\sigma_{\gamma^*p}(\eta) \simeq \frac{2\alpha}{3\pi} \sigma^{(\infty)} \cdot \left\{ \begin{array}{ll}
\ln(1/|\eta|), & \text{for } \eta \to \eta_{\min} = \frac{m_0^2}{\Lambda^2(W_0)}, \\
1/2\eta, & \text{for } \eta >> 1.
\end{array} \right.$$  

(15)

Note that for any fixed value of $Q^2$, with $W^2 \to \infty$, the soft logarithmic dependence as a function of $\eta^{-1}$ is reached. We arrive at the important conclusion that in the $W^2 \to \infty$ limit virtual and real photons become equivalent,

$$\lim_{W^2 \to \infty} \frac{\sigma_{\gamma^*p}(W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = 1.$$  

(16)

Even though convergence towards unity is extremely slow, such that it may be difficult to ever be verified experimentally, the universality of real and virtual...
photons contained in (16) is remarkable. It is an outgrowth of the HERA results which are consistent with the scaling law (1) with $\eta$ from (2) and $\Lambda^2(W^2)$ from (3). Note that the alternative of $\Lambda^2 = \text{const}$ that implies Bjorken scaling of the structure function $F_2 \sim Q^2 \sigma_{\gamma^* p}$ for sufficiently large $Q^2$, leads to a result entirely different from (16),

$$\lim_{W^2 \to \infty} \frac{\sigma_{\gamma^* p}(W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = \frac{\Lambda^2}{2Q^2 \ln \frac{\Lambda}{\mu}} \quad \text{(assuming } \Lambda = \text{const.})$$

(17)
i.e. a suppression of the virtual-photon cross section by a power of $Q^2$.

![Figure 2](image1.png)  
**Figure 2:** The dependence of $\Lambda^2$ on $W^2$, as determined by a fit of the GVD/CDP predictions for $\sigma_{\gamma p}$ to the experimental data.

![Figure 3](image2.png)  
**Figure 3:** The GVD/CDP scaling curve for $\sigma_{\gamma^* p}$ compared with the experimental data for $x < 0.01$.

In Fig. 2, we show $\Lambda^2(W^2)$ as obtained from the fit of $\sigma_{\gamma^* p}$ to the experimental data. The figure shows the result of fits based on the power law and the logarithm in (3), as well as the results of a pointlike fit, $\Lambda^2(W^2)$. Using (12), one finds that the average gluon transverse momentum increases from $\langle \vec{l}^2 \rangle \simeq 0.5 GeV^2$ to $\langle \vec{l}^2 \rangle \simeq 1.25 GeV^2$ for $W$ from $W \simeq 30 GeV$ to $W \simeq 300 GeV$. In Fig. 3, we show the agreement between theory and experiment for $\sigma_{\gamma^* p}$ as a function of $\eta$. For further details we refer to ref. [5].

In summary, we have shown that the HERA data on DIS in the low-$x$ diffraction region find a natural interpretation in the GVD/CDP. The scale $\Lambda^2(W^2)$ entering the scaling variable $\eta$, was found to be proportional to the average gluon transverse momentum absorbed by the incoming (outgoing) $q\bar{q}$ state in the virtual-forward-Compton amplitude. The cross sections for real and virtual photons on protons become identical in the limit of infinite energy.
Acknowledgments

It is a pleasure to thank G. Cvetic, B. Surrow and M. Tentyukov for a fruitful collaboration that led to the results reported here.

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