MEMS-based Inertial Navigation on Dynamically Positioned Ships: Dead Reckoning

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Abstract: Dead reckoning capabilities are vital in ship navigation if position and heading references are unrealizable or lost. In safety critical marine operations such as dynamic positioning, the International Maritime Organization and classification societies require that the vessel possesses dead reckoning capabilities and position reference redundancy. In this paper, we conduct a full-scale experimental validation and comparison of the dead reckoning capabilities using two different high-rate and low-cost micro-electro-mechanical inertial measurement units. The full-scale experimental validation is achieved with two nonlinear observers, aided by gyrocompasses and position reference systems, in a dynamic positioning operation carried out by an offshore vessel in the North Sea. The dead reckoning performance is evaluated after ten minutes without aiding from position reference system measurements.

Keywords: Inertial navigation; Navigation systems, Position estimation, Dead Reckoning; Nonlinear observers; Inertial Measurement Unit; Marine systems

1. INTRODUCTION

The dead reckoning (DR) term is used in the navigation community to describe the process of estimating position relative some departure point by keeping track of the distance covered and direction of travel. The origin of DR as an expression is not known, but one possibility is that it stems from ded reckoning, short for deduced reckoning, Misra and Enge (2011, Ch. 1). Inertial navigation systems (INS) are DR based, by deducing position, velocity and attitude (PVA) from previously known states by integration of angular rate measurements and accelerometer readings, once and twice, respectively. Due to errors in the inertial sensor, such as noise and biases, DR is insufficient to maintain accurate estimates over time. To counter INS drift over time, aiding is introduced. Aided INS are often referred to as integrated navigation systems.

INS can be aided by a number of sensors and position reference (PosRef) systems such as radio, laser, hydroacoustic and satellite-based systems. The latter types have the benefit of world-wide coverage, known as Global Navigation Satellite System (GNSS). However, these systems are exposed to both natural degradation and deliberate outages. Natural degradation can be caused by signal distortion from reflection of nearby objects, known as multipath, loss of signal due to sun storms or loss of line of sight to the satellite. Deliberately outages can be caused by signal jamming. During loss of reference, good DR capabilities is vital for the INS to provide accurate PVA estimates to the user. Recent works on aided INS using nonlinear observers (NLO), such as Hua (2010), Grip et al. (2013), Bryne et al. (2014; 2015), Grip et al. (2015), and Rogne et al. (2016), have not discussed the DR capabilities using such observer designs. An exception is Fusini et al. (2016), however this result uses visually aided INS to improve the heading estimates and provide velocity aiding, which again is used to improve the DR capabilities. For marine surfaces vessel, particular in dynamic positioning (DP), multiple heading references are available as a result of class requirements, such as DNV GL (2011), leaving the DR capabilities using IMUs and NLOs unanswered for ship navigation.

INS applied in DP is far from novel, as proposed in an industrial context almost 20 years ago (Vickery, 1999). Other similar products were introduced to the market in the years that followed, Faugstadmo and Jacobsen (2003) and Paturel (2004). All these products have been developed with high-end IMUs, based on ring laser gyro (RLG) or fiber optic gyro (FOG) technology. Furthermore, these products have been developed to improve or filter existing PosRef signals before applying them as PosRef measurements in the DP estimator, where the integrated INS solution of Faugstadmo and Jacobsen (2003) is only interfaced with hydroacoustic position reference (HPR) systems. Moreover, some of these INS products are also subjected to export restrictions, limiting the market potential and increasing the cost of installation due to a possibly lengthy approval process before installation.
Even though the cost of installation is quite high, INS integration in DP has received considerable attention in the industry the last years, Stephens et al. (2008), Carter (2011; 2014), Russell (2012) and Willumsen and Hals (2013). From the latter, it is stated that export restrictions are rarely a problem since “acoustic systems are covered by the same rules”. However, this statement does not apply to GNSS technology, which is not subjected to such restrictions. Furthermore, there currently exist numerous MEMS-based IMUs on the market not subjected to export licenses. Such units have a great potential to be used in the maritime and offshore markets. Therefore, studies on DR capabilities of GNSS aided INS, using high-rate MEMS IMUs, utilized in ship navigation are of great interest.

1.1 Main Contributions

This paper presents an initial study of the DR capabilities in DP obtained using GNSS-aided INS based on high-rate MEMS IMUs. The study is carried out:

- Based on two NLO designs, Mahony et al. (2008) and Rogne et al. (2016), respectively, interconnected with translational motion observers (TMO).
- Employing two types of MEMS IMUs.
- Evaluating the DR capabilities in both position and heading.

For a study on attitude determination using NLOs and high-rate MEMS IMU, Byrne et al. (2016) can be advised.

2. PRELIMINARIES

2.1 Notation

The Euclidean vector norm is denoted \( \| \cdot \|_2 \). The \( n \times n \) identity matrix is denoted \( I_n \), while the transpose of a vector or a matrix is denoted with \( (\cdot)^\top \). Coordinate frames are denoted with \( \{ \cdot \} \). \( S(\cdot) \in SS(3) \) represents the skew symmetric matrix such that \( S(z_1)z_2 = z_1 \times z_2 \) for two vectors \( z_1, z_2 \in \mathbb{R}^3 \). In addition, \( z_n^c \in \mathbb{R}^3 \) denotes a vector \( z \), to frame \( \{ c \} \), relative \( \{ b \} \), decomposed in \( \{ a \} \). Moreover, \( \odot \) denotes the Hamiltonian quaternion product. Saturation is represented by \( \text{sat} \), where the subscript indicates the saturation limit.

The rotation matrix describes the rotation between two given frames \( \{ a \} \) and \( \{ b \} \) and is denoted \( R_{ab} \in SO(3) \). Similar to the rotation matrix, the rotation between \( \{ a \} \) and \( \{ b \} \) may be represented using the unit quaternion \( q_{ab} = (s, r)^\top \) where \( s \in \mathbb{R}^3 \) is the real part of the quaternion and \( r \in \mathbb{R}^3 \) is the vector part. Roll, pitch and yaw are denoted \( \phi \), \( \theta \), and \( \psi \), respectively.

2.2 Coordinate Reference Frames

This paper uses four reference frames: The Earth Centered Inertial (ECI) frame, the Earth Centered Earth Fixed (ECEF) frame, a tangent frame equivalent of a Earth-fixed North-East-Down (NED) frame, and the BODY reference frame, denoted \( \{ t \} \), \( \{ e \} \), \( \{ n \} \) and \( \{ b \} \) respectively (see Fig. 1). ECI is an assumed inertial frame following the Earth as it rotates around the sun, where the x-axis points towards vernal equinox, the z-axis is pointing along the Earth’s rotational axis and the y-axis completes the right hand frame. Regarding the ECEF, the x-axis points towards the zero meridian, the z-axis points along the Earth’s rotational axis, while the y-axis completes the right hand frame. The Earth’s rotation rate \( \omega_{ie} = 7292115 \times 10^{-11} \text{ rad/s} \) is given by the WGS-84 datum. It is further decomposed in the ECEF and NED frame as

\[
\omega_{ie} = \begin{pmatrix}
0 \\
0 \\
\mu 
\end{pmatrix},
\omega_{ie}^n = \begin{pmatrix}
\cos(\mu) \\
0 \\
-\sin(\mu) 
\end{pmatrix} \omega_{ie},
\tag{1}
\]

where \( \mu \) is the latitude on the Earth and \( \omega_{ie}^n \) represents angular velocity. The longitude is denoted \( \lambda \). Furthermore, the navigation frame is a local Earth-fixed tangent frame, \( \{ n \} \), where the x-axis points towards north, the y-axis points towards east, and the z-axis points downwards. The BODY frame is fixed to the vessel, and the origin of \( \{ b \} \) is located at the vessel’s nominal center of gravity. The \( x \)-axis is directed from aft to fore, the \( y \)-axis is directed to starboard and the \( z \)-axis points downwards.

2.3 Kinematic Strapdown Equations

Estimating position when using the tangent frame as the navigation frame, results in implementation of the strapdown equations,

\[
\dot{p}_{nb}^n = v_{nb}^n, \tag{2}
\]

\[
\dot{v}_{nb}^n = -2S(\omega_{ie}^n)v_{nb}^n + R_{ib}^n f_{ib}^n + g_b^k, \tag{3}
\]

where \( p_{nb}^n \in \mathbb{R}^3 \) is the position, relative a defined origin of the tangent frame, \( p_{nb}^n(0) \) based on \( \mu(0) \) and \( \lambda(0) \). Furthermore, \( v_{nb}^n \in \mathbb{R}^3 \) is the linear velocity. It follows that \( g_b^k(\mu, \lambda) \in \mathbb{R}^3 \) is the local gravity vector which may be obtained using a gravity model based on the vessel’s latitude and longitude. \( R_{ib}^n \in SO3 \) is the rotation matrix. See Byrne et al. (2016) for the rotational kinematic equations used to obtained \( R_{ib}^n \). Moreover, \( f_{ib}^n = (R_{ib}^n)^\top (a_{ib}^n - g_b^k) \in \mathbb{R}^3 \) is the specific force decomposed in \( \{ b \} \) and where \( a_{ib}^n \) are the accelerations decomposed in the tangent frame.

Bryne et al. (2014; 2015) extended the translational motion kinematics further, by including a state of integrated vertical(down) position/heave, i.e,

\[
\dot{p}^n = p_{nb, z}^n, \tag{4}
\]

and is motivated by the fact that the mean vertical position of the vessel is zero over time, since the wave-induced motion of the craft in heave oscillates about the mean sea surface. This augmentation of (2)–(3) can be exploited in the INS by incorporating the virtual vertical reference (VVR) of Bryne et al. (2014; 2015; 2016).
3. IMU AND SHIP SENSOR CONFIGURATION

3.1 IMU and Error Sources

A strapdown IMU is a sensor unit measuring tri-axis angular velocity and tri-axis specific force of the unit in BODY frame relative the inertial frame,

\[ f_{IMU}^b = (f_x^b, f_y^b, f_z^b)^T, \quad \omega_{IMU}^b = (\omega_x^b, \omega_y^b, \omega_z^b)^T, \]

where the subscripts \(x, y\) and \(z\) denote the forward, starboard and downways axes in the BODY frame. In addition to the specific forces and angular velocity, each measurement is contaminated with sensor biases, errors and noise. Sensor errors may consist of nonlinearity, scale factors, cross-coupling and g-sensitivity errors, where the latter errors only influence the angular rate sensors reading. In addition to internal noise sources, external noise may be due to e.g. electrical and magnetic interference or stem from mechanical sources in the form of vibrations. In this paper, we assume that error sources related to sensor nonlinearity, scale factors, cross-coupling and g-sensitivity are compensated for in calibration by the manufacturer, or otherwise are neglectable. Sensor biases may also be calibrated for by the manufacturer, however some time-varying bias instability and run-to-run instability is often present with MEMS IMUs. Therefore, we model the angular rate and accelerometer measurements as

\[ \omega_{IMU}^b = \omega_b^b + b_b + w_b^b, \]

\[ f_{IMU}^b = f_b^b + b_a + w_a^b, \]

where \(\omega_b^b\) and \(f_b^b\) are the true angular rates and specific forces, respectively. Moreover, the corresponding sensors biases are denoted \(b_b^b\) and \(b_a^b\), while \(w_b^b\) and \(w_a^b\) represent the sensor noise and vibration induced noise contained in the respective measurements. Both the angular rate rate gyro and accelerometer biases are assumed constant,

\[ b_g^b = 0_{3 \times 1}, \quad b_a^b = 0_{3 \times 1}. \]

3.2 Ship Sensor Configuration

Several IMUs were installed on an offshore vessel, operating in the North sea, equipped with a Rolls-Royce Marine DP system. The ship in question is owned and operated by Farstad Shipping. In this paper we will present results obtained using one STIM300 and one ADIS16485 MEMS IMU. The sensor configuration utilized in the aided strapdown INS, based on the kinematic formulation of Bryne et al. (2016), is the same as NLO A and Rogne et al. (2016) as NLO C. The attitude determination performance using both A and C is evaluated in Bryne et al. (2016), therefore a similar naming convention is chosen here. An overview of the modular nonlinear observer structure used to estimate PVA is shown in Fig. 2, applicable for both NLO A and C, where A has no aiding TMO. The main difference between NLO A and C is outlined in Tab. 2, see Bryne et al. (2016) for details. Both NLOs are based on the same observer equations,

\[ \Sigma_1 : \begin{align*}
\dot{\hat{q}}_b^n &= \frac{1}{2} \hat{q}_b^n \odot \left( \begin{array}{c} 0 \\ \omega_b^b \end{array} \right) - \frac{1}{2} \left( \begin{array}{c} 0 \\ \omega_m^n \end{array} \right) \odot \hat{q}_b^n, \\
\dot{\hat{\omega}}_b^{IMU} &= \omega_b^{IMU} - \hat{b}_g^b + \hat{\sigma}_i, \\
\dot{\hat{b}}_g^b &= \text{Proj} \left( \hat{b}_g^b, -k_1 \hat{\sigma}_i \right),
\end{align*} \]

where \(\text{Proj}\) denotes the angular rate bias projection algorithm of Grip et al. (2012) and the reference therein, and \(k_1\) is the gain assorted with the angular rate bias estimation. The difference between observers A and C, as indicated in Tab. 2 lie in the injection terms, \(\hat{\sigma}_i\), given as

\[ \hat{\sigma}_i = k_1 \left( \psi_i \right) \times R(\hat{q}_b^i) \hat{u}_2^i + k_2 \hat{\psi}_b^i \times R(\hat{q}_b^i) \hat{v}_2^i, \]

(9)

depending on the observer setup in question, where \(i \in [A, C]\), and where \(\psi_i\) is the true orientation and \(\hat{\psi}_b^i\) and \(\hat{\omega}_b^{IMU}\) are the measurement vectors and \(\hat{\omega}_b^{IMU}\) and \(\hat{\omega}_m^n\) are the reference vectors, calculated using

\[ \hat{\omega}_b^{IMU} = f_b^b, \quad \hat{\omega}_m^n = f_m^n \times c^n. \]

For both NLO A and C, \(c^n = (\cos(\psi), -\sin(\psi), 0)^T\) and \(c^n = (1, 0, 0)^T\) as stated in Bryne et al. (2014).  

### Table 1. IMU specifications

|                      | ADIS16485 | STIM300 |
|----------------------|-----------|---------|
| In-run Gyro Rate Bias Stability | 6.25 deg/s | 0.5 deg/s |
| Angular Random Walk   | 0.3 deg/s | 0.15 deg/s |
| In-run Accelerometer Bias Stability | 0.032 mg | 0.05 mg |
| Velocity Random Walk  | 0.023 m/s | 0.06 m/s |

**Table 2. NLO reference vectors configuration**

| NLO | Vector \(e^n\) | Vector \(f^n\) |
|-----|----------------|----------------|
| NLO A | Unit vector North | \(-g_b^n/\|g_b^n\|_2\) |
| NLO C | Unit vector North | \(f_b^n/\|f_b^n\|_2\) through feedback |

- 3x yaw measurements from gyrocompasses, \(\psi\), at 5 Hz.

The specifications of the IMUs installed on the offshore vessel are presented in Tab. 1.

### 4. NONLINEAR OBSERVERS

In this papers we use two NLOs, each in cascade with a TMO to evaluate the DR capabilities using the two MEMS IMUs. The NLOs in question are the observer of Mahony et al. (2008) and Rogne et al. (2016), where the latter is based upon the results of Mahony et al. (2008), Grip et al. (2012; 2013) and Bryne et al. (2014; 2015). In this paper we will refer to Mahony et al. (2008) as NLO A and Rogne et al. (2016) as NLO C. The attitude determination performance using both A and C is evaluated in Bryne et al. (2016), therefore a similar naming convention is chosen here. An overview of the modular nonlinear observer structure used to estimate PVA is shown in Fig. 2, applicable for both NLO A and C, where A has no aiding TMO. The main difference between NLO A and C is outlined in Tab. 2, see Bryne et al. (2016) for details. Both NLOs are based on the same observer equations,

\[ \Sigma_1 : \begin{align*}
\dot{\hat{q}}_b^n &= \frac{1}{2} \hat{q}_b^n \odot \left( \begin{array}{c} 0 \\ \omega_b^b \end{array} \right) - \frac{1}{2} \left( \begin{array}{c} 0 \\ \omega_m^n \end{array} \right) \odot \hat{q}_b^n, \\
\dot{\hat{\omega}}_b^{IMU} &= \omega_b^{IMU} - \hat{b}_g^b + \hat{\sigma}_i, \\
\dot{\hat{b}}_g^b &= \text{Proj} \left( \hat{b}_g^b, -k_1 \hat{\sigma}_i \right),
\end{align*} \]

(8a)

(8b)

(8c)

where \(\text{Proj}\) denotes the angular rate bias projection algorithm of Grip et al. (2012) and the reference therein, and \(k_1\) is the gain assorted with the angular rate bias estimation. The difference between observers A and C, as indicated in Tab. 2 lie in the injection terms, \(\hat{\sigma}_i\), given as

\[ \hat{\sigma}_i = k_1 \left( \psi_i \right) \times R(\hat{q}_b^i) \hat{u}_2^i + k_2 \hat{\psi}_b^i \times R(\hat{q}_b^i) \hat{v}_2^i, \]

(9)

depending on the observer setup in question, where \(i \in [A, C]\), and where \(\psi_i\) is the true orientation and \(\hat{\psi}_b^i\) and \(\hat{\omega}_b^{IMU}\) are the measurement vectors and \(\hat{\omega}_b^{IMU}\) and \(\hat{\omega}_m^n\) are the reference vectors, calculated using

\[ \hat{\omega}_b^{IMU} = f_b^b, \quad \hat{\omega}_m^n = f_m^n \times c^n. \]

For both NLO A and C, \(c^n = (\cos(\psi), -\sin(\psi), 0)^T\) and \(c^n = (1, 0, 0)^T\) as stated in Bryne et al. (2014).

1 Engineering sample used, not necessarily in compliance with spec.
Fig. 2. Complete observer structure. The attitude observer estimates the quaternion \( \hat{q}_b^n \) and angular rate/gyro \( \hat{b}_a^n \). The TMO estimate position, \( \hat{p}_b^n \), and linear velocity, \( \hat{v}_b^n \). Estimated accelerometer bias is denoted \( \hat{b}_b^n \). An aiding translational motion observer may be utilized to aid the attitude observer. Dashed lines indicate optional feedback.

### 4.1 Attitude observer A

For NLO A, \( \hat{\sigma}_A \) is implemented with \( \hat{f}_b^n \) and \( \hat{f}_n^n \) is based on the injection term of Mahony et al. (2008) choosing

\[
\hat{f}_b^n = \frac{\hat{f}_{\text{IMU}}^b - b_b^n}{\| \hat{f}_{\text{IMU}}^b - b_b^n \|_2}, \quad \hat{f}_n^n = \frac{-g_b^n}{\| g_b^n \|_2},
\]

where the local gravity vector is utilized as reference vector based on the assumption that the specific force in the navigation frame is dominated by \(-g_b^n\) and \(b_a^n\) is the estimated bias, either statically from calibration or online estimation. In this part of the work we apply static accelerometer bias compensation.

### 4.2 Attitude Observer C

Regarding NLO C, the reference vector \( f_b^n \), in the calculation of \( \hat{\sigma}_C \), is chosen as

\[
f_n^n = \text{sat}_{Mf}(\hat{f}_{\text{IMU}}^b),
\]

where \( \hat{f}_{\text{IMU}}^b \) is estimated in \( \Sigma_2 \), \( k_1 \), \( k_2 \) and \( k_f \) are gains. See, Bryne et al. (2016) for details.

The TMO used to aid NLO C is obtained from Rogne et al. (2016), and is applicable in DP to estimate \( \dot{f}_b^n \) in order to improve the quaternion estimates of NLO C since \( \dot{f}_b^n \neq -g_b^n \) due to the wave-induced motions of the vessel. The observer takes the form of

\[
\Sigma_2 : \begin{cases}
\dot{\hat{p}}_b = \hat{p}_{nb,b} + \theta_{bg} K_{pp,1} \hat{p}_1, \\
\dot{\hat{v}}_b = \hat{v}_{nb,b} + \theta_{bg} K_{pp,2} \hat{p}_1, \\
\dot{\hat{v}}_{nb} = -2 S(\omega_v^b) \hat{v}_{nb} + \dot{f}_b^n + g_b^n \\
& \quad + \theta_{bg} \left( \begin{array}{c} 0_{2 \times 1} \\ K_{vv} \\ 0_{1 \times 2} \end{array} \right) \hat{p}_1, \\
\dot{\xi} = -R(q_b^n) S(\sigma) \left( \begin{array}{c} \hat{f}_{\text{IMU}}^b - b_b^n \\ \hat{p}_1 \\ \hat{v}_1 \end{array} \right), \\
\dot{f}_b = R(q_b^n) \left( \hat{f}_{\text{IMU}}^b - b_b^n \right) + \xi,
\end{cases}
\]

where \( \hat{p}_1 = p_{1f} - \hat{p}_f \), and \( \hat{v} = v_{vir} - \begin{pmatrix} \hat{v}_{nb,x} \hat{v}_{nb,y} \end{pmatrix}^T \cdot K|_{|p} \) and \( K_{|p} \) are gains, while \( \theta_{bg} \geq 1 \) is a high-gain tuning parameter used to guarantee stability. In DP, the velocity of the ship is approximately zero, hence in that case,

\[
\hat{v}_v = (0, 0)^T.
\]

Consider the observer structure in Fig. 2, by using the virtual velocity measurements in \( \Sigma_2 \), instead of measurements obtained from PosRefs, any errors in the PosRef is prevented from entering \( \Sigma_1 \), making the attitude estimation more fault tolerant, Rogne et al.(2015; 2016). Any errors in the lever arm from PosRef to IMU is also prevented from affecting the attitude estimates using (11) as aiding measurement. The origin of the error dynamics of observer NLO A and C, (see Tab. 2 for details on A and C) is almost globally exponentially stable and uniformly semiglobal exponentially stable (USGES) in Mahony et al. (2008) and Rogne et al. (2016), respectively.

### 4.3 Accelerometer Bias Compensation for DR

Even though the angular rate and accelerometer biases are not mutually uniformly observable, Farrell (2008, Ch. 11.9), without the vessel accelerating and rotating, some accelerometer bias compensation have to be done in order to obtain an INS with reasonable DR capabilities. In Bryne et al. (2016), a fixed pre-compensated accelerometer bias was successfully applied for attitude estimation. However, some acceleration errors may be present owing to some in-run bias instability, w.r.t. Tab. 1. To atone for this, we extend NLO A with an additional observer, based on the observer of Fossen (2011, Ch. 11.5.1) with globally exponentially stable (GES) error dynamics. In this paper however, only the horizontal plane is considered since vertical DR is not relevant for ships. Another difference is that the attitude is not known, but estimated. This results in the observer,

\[
\Sigma_3 : \begin{cases}
\dot{\hat{p}}_{nb,xy} = \hat{v}_{nb,xy} + K_{pp} \hat{p}_{xy}, \\
\dot{\hat{v}}_{nb,xy} = -2 S_{xy} \hat{v}_{nb,xy} + \hat{R}_{xy} \left( \hat{f}_{\text{IMU},xy} - b_{xy} \right), \\
\dot{\hat{b}}_{a,r} = K_{ap} \hat{R}_{xy} \hat{p}_{xy}, \\
\dot{b}_a = \hat{b}_a + \hat{b}_{a,r},
\end{cases}
\]

where \( \hat{b}_{b,r} \) is the estimate of residual uncompensated accelerometer bias and

\[
S_{xy} = \begin{pmatrix} 0 & \sin(\omega_{xy}) \\ -\sin(\omega_{xy}) & 0 \end{pmatrix}, \quad R_{xy} = \begin{pmatrix} R(q_b^n)_{11} R(q_b^n)_{12} \\ R(q_b^n)_{21} R(q_b^n)_{22} \end{pmatrix},
\]
and where $K_{sp}$ are gains chosen such that $A - KC$ is Hurwitz when writing (12) in matrix form,
\[
\dot{x} = Ax + Bu + K(y - C\dot{x}) \tag{13}
\]
with $x = \begin{bmatrix} p_{o2x2}^n, p_{o2x2}^b, v_{o2x2}^b, y_{o2x2}^b, (b_{o2x2}^n, b_{o2x2}^b) \end{bmatrix}^T$, $y = (p_{o2x2}^{\text{GNSS}}, v_{o2x2}^{\text{GNSS}})$ \^T
and $u = f_{b,xy}^b$ where $x$ and $y$ represent the horizontal components of the GNSS and accelerometer measurements, and
\[
A = \begin{bmatrix} 0_{2x2} & I_2 & 0_{2x2} \\ 0_{2x2} & -2S_{xy} & -R_{xy} \\ 0_{2x2} & 0_{2x2} & 0_{2x2} \end{bmatrix}, B = \begin{bmatrix} 0_{2x2} \\ R_{xy} \\ 0_{2x2} \end{bmatrix}, C = (I_2 \ 0_{2x2} \ 0_{2x2})^T, K = (K_{sp}^T K_{ip}^T K_{ip}^T)^T.
\]
Equations (12b)–(12c) are used in an attempt to estimate $\tilde{b}_{o,r}$ in order to compensate for any acceleration errors in $\tilde{R}_{xy}(f_{b,xy}^b - \tilde{b}_{o,r}^b)$, such that velocity error growth of $\tilde{v} = v_{o,b}^n - \tilde{v}_{o,n}^n$ in (12b) is limited, which in turn improves the DR performance of (12a). Since the accelerometer bias is decomposed in BODY, the error dynamics of $\Sigma_1 - \Sigma_2 - \Sigma_3$ cannot be proven globally stable due to the nonlinear state dependent term, $R(\tilde{q}_o^b)$, from the NLO enters the error dynamics of (13) both through the $A$ matrix and the injection term and not only through the input w.r.t. to the USGES result of Rogne et al. (2016, Theorem 1) applying a cascades system argument. However, a local stability result is obtainable through linearization.

For observer C we base the estimation of the NLO acceleration error on the signal $\xi$, in $f_{b,xy}^b$ from $\Sigma_2$, (10e) to perform the same task as (12b)–(12c) by replacing the term $\tilde{R}_{xy}(f_{b,xy}^b - \tilde{b}_{o,r}^b)$ with $\tilde{R}_{xy}^n(f_{b,xy}^b - \tilde{b}_{o,r}^b)$ with $f_{b,xy}^n$ from $\Sigma_2$.

5. FULL-SCALE VERIFICATION OF DR CAPABILITY IN DP APPLYING MEMS IMUs

In this section, the evaluation of the DR properties using an ADIS16485 and a STIM300 MEMS IMU is presented. The DR performance evaluation is carried out with data collected during a DP operation performed in the North Sea during the fall of 2015. The GNSS track of the vessel in DP, obtained from the ship’s dGNSS receiver, is shown in Fig. 3.

![Fig. 3. The path track of the DP operation used to evaluate the DR performance of the respective IMUs. The path track is obtained from the on board dGNSS solution.](image)

First, the heading DR performance when using the IMUs available is discussed, and illustrated with an example. Then, the position DR performance during the particular DP operation is evaluated, applying both IMUs and NLO A and C. The resulting DR statistics are based on a multitude of estimation runs, where NLO A and C were tuned with similar gains.

5.1 Heading Angle DR Capabilities

The heading angle DR capabilities using the IMUs available were found to be in compliance with the IMUs’ the angular rate specifications, presented in Tab. 1. 12 heading DR evolutions of the absolute yaw angle error
\[
|\tilde{\psi}| = |\psi - \dot{\psi}|, \tag{14}
\]
relative the averaged ship gyrocompass measurements for both sensors, for one hour in heading DR after disabling the observer injection from the compass by setting $k_2 = 0$, are shown in Fig. 4. The observer initialization time was 15 minutes. In addition, the average DR error, of the 12 runs, is highlighted in Fig. 4. Examples of typical angular rate bias estimates are shown in Fig. 5, where the yaw rate bias is frozen from $t = 15$ minutes.

![Fig. 4. DR performance in yaw obtained using the ADIS16485 and STIM300 IMUs. Highlighted evolution indicates average DR error.](image)

5.2 Position DR Capabilities

Evaluation of the DR capabilities in position is more elaborate than for heading since the theoretical growth of errors are a combination of higher order terms, (Groves, 2013, Ch. 5.7), as opposed to linear growth for heading. In order to obtain statistically significant results related to the position drift while performing DR, each combination of IMU and NLO was evaluated 50 times by comparing the DR errors accumulated when disabling GNSS feedback at arbitrary times. The DR evaluation is done by taking the norm of the difference between the horizontal components of $p_{o,GNSS}^n$ and $\hat{p}_{o,GNSS}^n$, defined $\hat{p}_{o,GNSS} := p_{o,GNSS}^n - \tilde{p}_{o,GNSS}^n$ where,
\[
\tilde{p}_{o,GNSS} = \tilde{p}^n + R(\tilde{q}_o^n)r^b, \tag{15}
\]
and where $r^b$ being the lever arm from the IMU to the GNSS such that
\[
\|\hat{p}_{o,GNSS}\|_2 = \|p^n + R(\tilde{q}_o^n)r^b - \tilde{p}^n - R(\tilde{q}_o^n)r^b\|_2,
\]
\[
= \|\tilde{p} + (R(\tilde{q}_o^n) - R(\tilde{q}_o^n))r^b\|_2. \tag{16}
\]

NLO A: Fig. 6 shows the aggregated drift errors over 10 minutes, after PosRef injection is disabled, applying NLO A for both the ADIS16485 and the STIM300 IMU. The
Typical angular rate bias estimates of the ADIS16485

(b) Typical angular rate bias estimates of the STIM300

Fig. 5. Angular rate bias estimates after heading DR test.

Table 3. Position DR error statistics using NLO A as attitude observer in DP.

|                   | ADIS16485 | STIM300 |
|-------------------|-----------|---------|
| Mean error [m] 1 min | 4.1631    | 3.9816  |
| Mean error [m] 5 min | 35.2529   | 31.5438 |
| Mean error [m] 10 min | 116.3540  | 102.3643|
| Min error [m] after 10 min | 12.1275   | 14.6419 |
| Max error [m] after 10 min | 314.9137  | 283.8679|
| RMSE [m] after 10 min    | 132.6765  | 119.2748|

Table 4. Position DR error statistics using NLO C as attitude observer in DP.

|                   | ADIS16485 | STIM300 |
|-------------------|-----------|---------|
| Mean error [m] 1 min | 7.2513    | 3.9651  |
| Mean error [m] 5 min | 67.0432   | 28.7058 |
| Mean error [m] 10 min | 167.4041  | 93.9744 |
| Min error [m] after 10 min | 14.5668   | 36.3566 |
| Max error [m] after 10 min | 402.2314  | 172.0885|
| RMSE [m] after 10 min    | 190.3469  | 98.8203 |

Statistical results based on the 50 DR runs are presented in Tab. 3 using A as attitude observer.

NLO C: Fig. 7 shows the aggregated drift errors over 10 minutes, after PosRef injection is disabled, applying NLO C for both the ADIS16485 and the STIM300 IMU. The statistical results based on the 50 DR runs are presented in Tab. 4 using C as attitude observer.

5.3 Discussions

Heading DR: It is evident, w.r.t. to Fig. 5, that the gyro bias estimates using the STIM300 is smoother and more in-run stable than those obtained using the ADIS16485, resulting in the performance difference seen in Fig. 4. This is in compliance with the sensor specifications presented in Tab. 1. The asymptotic angular rate bias estimation performances seen in Fig. 5, is representative of what is seen from run to run.

Position DR: As seen from Tabs. 3–4 and Figs. 6–7, the two main conclusions from the 4 times 50 DR runs performed over the data set collected during DP is that:

- using the STIM300 results in better DR performance, than using the ADIS16485,
- using observer structure A often resulted in better DR performance than using observer structure C for ADIS16485.

The first result is as expected considering the angular rate in-run bias stability presented in Tab. 1 and the results from Sec. 5.1 where the STIM300 provides the best attitude estimates. However, why observer structure A gave better DR performance than using structure C for ADIS16485, is not evident since in Bryne et al. (2016) C outperformed A when it came to attitude estimation.
Fig. 7. Aggregated DR error over 50 runs using NLO C. Red indicates the mean DR error.

One possible answer could lie in that the mean attitude error of NLO A, obtained in Bryne et al. (2016), was small such that the DR position estimates based on (12) were obtained with a reasonably accurate averaged attitude. The results indicate a large spread of the DR error over 10 minutes. Especially when using the ADIS16485 and NLO structure C. This might be due to noise, mechanical disturbance such as vibration, or insufficient tuning of the observers, or that the virtual velocity measurement employed in NLO structure C is not accurate enough when estimating \( \mathbf{f}_{\mathbf{n}}^{\mathbf{b}} \) for DR purposes. More probable however is that the assumption of constant bias done in Sec. 3.1 is not sufficient over time for the NLO to provide the same DR performance as NLO structure A, even though the same assumption yielded satisfactory results related to attitude estimation, (Bryne et al., 2016).

Considering the quality of the results obtained compared to the results in Paturel (2004), using either of the two MEMS-based IMUs available in this work, gave worse results than in the cited works where an FOG-based IMU was applied. In the results presented in Paturel (2004), a position accuracy during GNSS outage stayed within GNSS accuracy for a period exceeding two and a half minutes. The mean position drift after a 50 seconds GNSS outage was less than half a meter. These results are considerably better than the approximately 4 meters error obtained using the STIM300 IMU after one minute DR for both NLOs. However, in Paturel (2004) only 10 runs are presented, making a definite statistical comparison difficult due to the few DR trajectories presented. The FOG product in question is currently advertised to have a 20 m error with a 50 % circular error probability after five minutes of unaided navigation, whereas we obtain approximately 30 meters averaged error in the same time frame, being 50 % worse.

As depicted in Figs. 6–7, a MEMS-based INS may provide relatively stable position estimates (less than four meters error) for half a minute, without PosRef injection. From a fault-tolerance perspective, such as Rogne et al. (2016), the results obtained here indicate what kind of PosRef errors one might detect based on MEMS IMUs. For instance a PosRef drift of 10 centimeters per second results in a PosRef error of 3 meters after half a minute, which might be possible to detect with the results obtained, considering the average DR error is two meters with either of the two observers and the STIM300 unit. Moreover, in the situation of PosRef failure during DP, if four meters is an acceptable error margin, 30 seconds is available to the DP operator to decide whether the operation should be aborted or not. This might be sufficient time for PosRef recovery e.g. if line-of-sight is established with one more satellite, resulting in a complete GNSS solution.

Since the ship was in DP with small roll and pitch motion (between ±3 degrees), the x- and y-axis accelerometer biases were probably not entirely observable, which will affect the DR performance even though \( \mathbf{f}_{\mathbf{n}}^{\mathbf{b}} \) is estimated directly and indirectly using NLO A and C, respectively. The DR performance properties are not only dependent on the sensor biases, but also on the velocity-random walk and the sensed vibrations on the ship. Integrating these over time, results in a large error even when averaging them out using high-rate integration (1000 Hz). Regarding tuning, approximately the same tuning as in Rogne et al. (2015; 2016) was used. With more emphasis on tuning for a DR application, better results may be accomplished. Also, time-synchronization errors between our IMUs and the on-board dGNSS system may result in small errors in velocity and specific force at the time of disabling dGNSS injection, resulting in a steeper error slope than otherwise obtained if the position and inertial measurements were synchronized. It is also difficult to conclude with certainty that the results found using the STIM300 are representative, since we used an engineering sample provided by Sensonor. This is a test unit, not necessarily in compliance with the standard specification.

6. CONCLUDING REMARKS

Full-scale assessment of dead reckoning capabilities, in conjunction with the comparison and validation of two nonlinear observers, has been carried out. The observers were driven by two MEMS based IMUs, aided by GNSS and gyrocompasses, and evaluated in a DP operation...
executed by an offshore vessel in the North Sea. For heading, 12 cases were run for the purpose of showcasing typical observer and IMU behavior were presented. For position, a more elaborate study was conducted, running each combination of observer and IMU 50 times each for assessing statistics regarding their performance. The results showed that STIM300 was better suited for dead reckoning than the ADIS16485 unit, and that NLO A was the preferred observer. An in-depth study on the effects of ship vibrations on the DR performance has to be undertaken. Further works on optimal tuning should also be carried out.

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