Research on Magnetic localization method of underwater Magnetometer in Single component

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Abstract. In order to solve the problem of single component magnetometer’s localization, a new method is proposed using auxiliary magnetic source. The localization model is designed and multi-direction moving path of magnetic source is optimized, then DE algorithm is adopted to calculate the coordinates of magnetometer. Simulation indicated the method is right and efficient, meanwhile, the result is got in high precision and the influence of marine noise is analysed in localization. The method supplies a novel path for single component magnetometer’s localization in practice.

1. Introduction
Magnetometer in single component is applied widely in magnetic monitoring station and stationary degaussing instruments[1-2]. Single component magnetometers are placed equidistantly and well-distributed on seabed, which could measure varieties of ship and submarine built by iron, and their magnetic characters can be analysed and degaussed. Magnetic measured data is got by ship magnetic measurement in special depth, thus magnetic stretch would be adopted to get ship's complete magnetic character by relative position between ship and magnetometer. Position of magnetometer is got by buoy traditionally. Buoy is fastened with magnetometer in common, and its horizontal coordinate is considered as magnetometer's, so sensor's 3D coordinates could be got while its depth is measured by water depth sensor. However, the method is not accurate, buoy's horizontal position would be changed by ocean wave and current, so error of magnetometer's coordinate would occur, which will have a negative impact on magnetic stretch, therefore localization is necessary before ship magnetic measurement.

Auxiliary magnetic source is usually used in localization of underwater magnetometer. The nonlinear magnitude model is established between magnetic data and relative position, then 3D coordinates of magnetometer can be calculated reversely. For example, electrified solenoid coil is fixed on magnetometer and its position is considered as magnetometer's in document[3], and the coil's coordinates can be calculated by excitation magnetic field; electrified solenoid coil is used as magnetic dipole on the sea in document[4], and localization of sensor is converted to nonlinear unconstrained optimization problem on the basis of knowing data in depth. The result is ideal even attitude angle of coil is more than 5º; analytic formula of underwater magnetometer is calculated in document[5]. Vector magnetometer is studied in above documents, but magnetometer in single component is not discussed fully. Data of single component includes fewer data and characters than vector data, thus more difficulties happen in localization. In order to solve the problem, multi-movements are
constructed to get more information about single component magnetometer, and calculated model is built by measured equation to realize the localization of underwater magnetometer

2. Localization principle of magnetometer

2.1 Model of magnetic measurement

Electrified solenoid coils are used as active moving magnetic source in localization and the produced magnetic field is thought as known condition. The magnetic field should be calculated accurately which would determine the precision of localization. At first the principal coordinate system of is set up, and the beginning of path is taken as origin of coordinate when coils move on the sea. The moving direction is X axis; Y axis' direction is defined from left to right horizontal; Z axis' direction is vertical down, and the detail of coordinate system is shown in Figure 1.

\[
\mathbf{B}_z = \frac{\mu NIS (2z_p^2 - (x_p - x_i)^2 - (y_p - y_i)^2)}{4\pi \left[ (x_p - x_i)^2 + (y_p - y_i)^2 + z_p^2 \right]^{3/2}}
\]  

(2.1.2)

\(\mu\) is vacuum permeability in above formula, \(\mathbf{m}\) is magnetic torque of solenoid coils. The coils is placed upright and \(\mathbf{m}\) only has vertical component in localization, so \(\mathbf{m}=[0,0,NIS]\), \(N\) is number of coils, \(I\) is the value of electric current, and \(S\) is area of coils \((S=\pi R^2, R\) is radius); \(\mathbf{r}\) is the shifting vector from coils centre to magnetometer. \(\mathbf{r}=[x_i-x_p, y_i-y_p, z_i-z_p]\), \(\mathbf{y}=[x_p, y_p, z_p]\) is the coordinate of \(P\) point, \((x_i, y_i, z_i)\) is the coordinate of magnetometer in \(i\)th point and \(z_i=0\) because coils move on the sea, thus the measured value \(B_z\) of single component magnetometer can be described below when all information are substituted to formula(2.1.1)

2.2 Calculated model of localization

Figure 1 Coordinate of magnetic source’s movement

Solenoid coil can be equivalent to magnetic dipole for its small cubage compared to long distance away from magnetometer, thus the measured value of magnetic field \(\mathbf{B}\) is calculated by equation of magnetic dipole in principal coordinate system[6].
The functional relation is as follows between magnetic field and coordinate of $P$ point according to formula (2.1.2):

$$B_z = f(x_p, y_p, z_p)$$  \hspace{1cm} (2.2.1)

Point $P$'s coordinate is the unknown parameter to be solved and the measured value $B_z$ can be got, therefore calculated result is thought to be right if the fitting error is approximate to 0 between calculated value $B_{zc}$ and measured value $B_{zm}$, thus the object function $F$ is described below:

$$F(p_x, p_y, p_z) = \frac{1}{n} \sum_{i=1}^{n} |B_{zc}^i - B_{zm}^i|$$ \hspace{1cm} (2.2.2)

Related literatures use various algorithms to compute coordinate of magnetometer, such as LMM, NSGA and so on[7,8]. DE algorithm is adopted in this paper for its high efficiency and stability.

2.3 Principle of DE algorithm
Differential evolution algorithm (DE) borrows the idea of genetic algorithm (GA), and it is simple and has better convergence[9]. Its operating steps is listed as follows:

1) Initialization. $N$ is the individual number in population, which represents the 3D coordinates of magnetometer. The vector of $i$th unit is $x_i = [x_{i1}, x_{i2}, x_{i3}]$, and search space range is $[x_{min}, x_{max}]$, thus the initialization formula is shown below.

$$x_{ij} = x_{min} + rand(x_{max} - x_{min})$$ \hspace{1cm} (2.3.1)

2) Variation. Three different units are chose in population randomly, then two of them are split and weighted with the third one to get evolved unit using the formula below[10].

$$v^{G+1}_{ij} = x^G_i + F(x^G_{x2} - x^G_{x3})$$ \hspace{1cm} (2.3.2)

In above formula, $G$ is the generation and $F$ is scaled factor which can control differential vector's effect to evolved unit, $F \in (0,2)$.

3) Crossover. Binomial distribution is used to get new individual $u^{G+1}_i$ in this step.

$$u^{G+1}_{ij} = \begin{cases} 
    v^{G+1}_{ij}, & \text{rand}(j) \leq CR \text{ or } j = randn(i) \\
    x^G_{ij}, & \text{rand}(j) > CR \text{ or } j \neq randn(i)
\end{cases}$$ \hspace{1cm} (2.3.3)

CR is the crossover probability. The greater value is good for local search and lower value is good for variety of population. $\text{rand}(j)$ is a random value evenly distributed from 0 to 1. while $\text{randn}(i)$ is a random integer from 1 to D.

4) Selection. DE uses rapacious selecting principle and $u^{G+1}_i$ compares with $x^G_i$ in order to get new individual having higher fitness value.

$$x^{G+1}_i = \begin{cases} 
    u^{G+1}_i, & f(u^{G+1}_i) \leq f(x^G_i) \\
    x^G_i, & f(u^{G+1}_i) \geq f(x^G_i)
\end{cases}$$ \hspace{1cm} (2.3.4)

3. Model experiment and analysis
3.1 Experiment simulation
Electrified solenoid coils move from $(0,0,0)$m to $(1.2,0,0)$ along line $l_1$ in principal coordinate system, and magnetic field is measured every 0.05m. There are 25 measured values in all while the $i$th is $c_i$. Only vertical magnetic field is produced by solenoid coils and distributes symmetrically, thus double symmetrical results would occur along x axis when only measured data of line $l_1$ is adopted, therefore the real position of magnetometer can't be got. Another line $l_2$ is designed parallel with $l_1$ and the two line is $d$ apart vertically in y axis($d=0.1$m), and measured value is also recorded every 0.05m. There are 50 measured data gathered in $l_1$ and $l_2$, the schematic diagram of magnetic source is shown in figure.2.
Radius of solenoid coils is setting as 0.05m\((r=0.05m)\), and the number of coils \(N=100\), electric current \(I=2.5A\). In order to emphasize the effect of method, magnetometer array is designed in localization. There are 9 magnetometers distributed evenly along \(x\) and \(y\) axis in all, and their coordinates are listed: 
\[P_1=(0.3,0.25,0.514), \quad P_2=(0.3,0,0.514), \quad P_3=(0.3,-0.25,0.514), \quad P_4=(0.6,0.25,0.514), \quad P_5=(0.6,0,0.514), \quad P_6=(0.6,-0.25,0.514), \quad P_7=(0.9,0.25,0.514), \quad P_8=(0.9,0,0.514), \quad P_9=(0.9,-0.25,0.514),\]
while \(P_i\) represents the coordinate of \(i\)th magnetometer. Therefore the magnetic value can be calculated by formula(2.2.2).

The specific steps are listed as follows:

1) First, geomagnetic field is measured every \(T\) seconds, and 50 values are recorded, then the data is averaged to take as exact value of geomagnetic field.
2) When the current is on, measure the magnetic field evenly and subtract the geomagnetic field of step 1, so every magnetometer has \(2n\) data.
3) Build the fitness function and calculate the coordinates of magnetometer array using DE algorithm.

\(D_e\) is defined as distance error between calculated and measured value, while \(r_e\) is the average relative error of different positions. Their formulas are as follows:
\[
D_e = \sqrt{(x_c-x_m)^2+(y_c-y_m)^2+(z_c-z_m)^2}, \quad r_e = \frac{1}{n} \sum_{i=1}^{n} D_i/r
\]

\([x_c,y_c,z_c]\) is the calculated result and \([x_m,y_m,z_m]\) is the real position in upper expression, \(r_i\) is the distance from \(i\)th measured point to magnetometer. The localization result is shown below.

### Table 1. Localization of magnetometer array

| \(P_i\) | \(D_e\) (mm) | \(r_e\) (%) |
|--------|-------------|------------|
| \(P_1\) | 0.011       | 0.001      |
| \(P_2\) | 0.015       | 0.002      |
| \(P_3\) | 0.325       | 0.019      |
| \(P_4\) | 0.124       | 0.011      |
| \(P_5\) | 0.068       | 0.007      |
| \(P_6\) | 0.053       | 0.043      |
| \(P_7\) | 0.297       | 0.015      |
| \(P_8\) | 0.112       | 0.025      |
| \(P_9\) | 0.181       |            |

The 3D localization result is listed in Table 1 and Figure 3. It indicates that the result is identical with setting value. All located errors of array are no more than 1mm and \(r_e\) is within 1%. Simulation
shows that the method is correct and efficient, accurate result is got and it can be used in real localization of magnetometer in single component.

3.2 Interference impact analysis

Because ship magnetic measurement is carried out in marine condition, the magnetic disturbance would reduce measuring accuracy and has adverse effect on magnetometer's localization. The vertical component of marine noise is setting as $B_z$, thus the expression of measured value is shown below.

\[ B_z = f(r) + B_w \]  \hspace{1cm} (3.2.1)

Gauss white noise is simulated as marine magnetic noise and its range is set as 5nT, 10nT, 20nT, 30nT, 50nT separately. The signal-to-noise's formula is defined as follows:

\[ SNR = 10\log\left(\frac{P_{Bn}}{P_n}\right) \]  \hspace{1cm} (3.2.2)

$P_{Bn}$ is the signal power of magnetic noise and $P_n$ is the power of magnetic field produced by solenoid coils. The analysed result is listed below by DE.

| $B_z$(nT) | SNR  | $D_e$(mm) | $r_e$(%) |
|----------|------|-----------|---------|
| 5        | 33.4491 | 2.2113   | 0.3264 |
| 10       | 27.3848 | 3.7096   | 0.5476 |
| 20       | 20.9878 | 8.4764   | 1.2512 |
| 30       | 17.5113 | 15.7147  | 2.3197 |
| 50       | 13.5302 | 25.7789  | 3.8053 |

The result indicated SNR drops and locating error increases continually when noise range rises. Because the range of marine noise is no more than 30nT in practice, so $D_e$(mm) is 15mm and $r_e$ is less than 3% in real condition, which means that noise has apparently effect on localization. It is necessary to reduce the influence for assuring the precision of calculated result before actual localization.

4. Conclusion

A new method is proposed for magnetic localization of underwater magnetometer in the paper. Electrified solenoid coils are used as auxiliary magnetic source, which is equivalent to magnetic dipole for computing the magnetic field. Multi-direction paths are adopted to get more differential measured data in order to be beneficial for calculation and DE algorithm is used to compute the coordinate of magnetic sensor. The result indicates the method is efficient and ideal, which has important meaning for magnetic localization of magnet sensor in practice.

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