Why Quantum Information Processing

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Abstract
In this brief note, I will consider the following questions: (1) What is QIP? (2) Why QIP is interesting? (3) What QIP can do? (4) What QIP cannot do? (5) What are the major challenges in QIP?

1 Organization of this course
Chapter 1: This Introduction: Logistics, Motivation and Introduction.
Chapter 2: Basics of quantum mechanics and quantum information.
Chapter 3: Elements of quantum computing and quantum error correction
Chapter 4: Quantum source coding
Chapter 5: Quantum channel coding
Chapter 6: Quantum cryptography I: Quantum key distribution
Chapter 7: Quantum cryptography II: Bit commitment, secret sharing, etc.
Some guest lecturers may also be invited for special topics on the subject.

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1
2 Introduction

[N.B. This brief note summarizes my personal perspective on the subject. Comments and criticisms are most welcome.]

Quantum information processing (QIP) is a new and exciting area of interdisciplinary research. It combines ideas in physics, mathematics, computer science and engineering. The main goal of QIP is to harness the fundamental laws of quantum mechanics to improve dramatically all aspects (e.g. acquisition, transmission, and processing) of information processing. In this introductory chapter, I will survey the motivation and power of QIP. More specifically, I will answer the following questions:

1. What is QIP?
2. Why QIP is interesting?
3. What QIP can do?
4. What QIP cannot do?
5. What are the major challenges in QIP?

2.1 What is QIP?

Let me begin with the first question: What is QIP? One can think of QIP as a new quantization program. Its goal is combine and unify quantum mechanics with other areas of research such as theory of computing, cryptography, information theory and error-correcting codes. In the synthesis of the ideas of quantum mechanics with other subjects, new research problems appear. As a result of tackling those problems, new insights are gained and a more complete and unified theory emerge.

In some cases, such a synthesis can lead to the questioning of the very foundation of the original subject. Take the example of the theory of computation. It was commonly accepted for a long time that the Church-Turing thesis—that any computational model can be efficiently simulated by a conceptual device called the Turing machine—is a valid assumption/foundation of the subject. The upshot is that it does not really matter how a classical computer operates: Classical computers of different architectural designs essentially give equivalent computational power. However, quantum computing challenges the Church-Turing thesis: There is no known efficient classical algorithm for simulating the evolution of a quantum system. Moreover, in 1994, Peter Shor of AT&T made a remarkable discovery [1] that quantum computers can, in principle, factor large integers efficiently (i.e., in polynomi-
nal time in the size of the inputs). In contrast, no efficient classical algorithm for factoring is known.

Therefore, we have to ask the question: Does the Church-Turing thesis apply to a quantum computer? This is an important question. If we can answer this question one way or another, we are making an important conceptual breakthrough. Why is this so? Suppose the answer is yes. Then, there must exist an efficient classical algorithm for simulating any quantum system. Such a discovery will be a big surprise to quantum physicists and will make physicists and computational chemists who are interested in simulating quantum systems very happy. I have been told that much of the world’s computational power is used in computational chemistry. And an important part of computational chemistry is quantum chemistry. Added to this, our colleagues in lattice QCD or condensed matter physics or nanotechnologists will be delighted to learn about such classical algorithms! On the other hand, if the answer is no, then there will be a burning desire for us to work on the construction of a quantum computer, our ultimate computational machine.

Moreover, I remark that Shor’s discovery is far more than an intellectual curiosity. Indeed, the presumed hardness of factoring is the foundation of security of RSA encryption scheme, which is well-known standard encryption scheme. Since RSA is widely used to guarantee the security in electronic commerce and business, if a quantum computer is ever built, the whole foundation of security of electronic commerce and business will fall apart.

In the context of information theory, our viewpoint is that there is only one information theory—quantum information theory. Conventional information theory is simply a special case of quantum information theory. We hope that quantum information theory will make conventional information theory complete in an analogous way that the theory of complex numbers makes real numbers complete.

2.2 Why QIP is interesting?

2.2.1 Applications of the fundamental laws of nature have been the driving force of economic growth.

I am impressed by the viewpoint of Prof. Michael Berry. Throughout history, life has been transformed by the conscious application of physical laws. In the nineteenth century, life was transformed by the Newton’s laws and later thermodynamics through heat engines, which were the driving force
of the whole industrial revolution. Incidentally, Newton’s laws and thermodynamics are the foundations of mechanical engineering. In the twentieth century, life was similarly transformed by the application of Maxwell’s equations, through electricity and communicating words and pictures at the speed of light through electromagnetic waves. Incidentally, Maxwell’s equations can be regarded as an important foundation of electrical engineering. Looking forward to the twenty-first century, Michael Berry confidently proclaimed that: “It is easy to predict that in the twenty-first century, it will be quantum mechanics that influences all our lives.”

He went to write: "There is a sense in which quantum mechanics is already having profound effects. Leon Lederman claims that a large part of the gross national product of the industrial countries stems from quantum mechanics. I suppose he is referring to transistors—the ‘fundamental particles’ of modern electronics—that depend on properties of semiconducting materials designed by applying quantum mechanics to electrons in solids, and to lasers, where the Bose-Einstein statistics of identical particles generate coherent avalanches of photons to read the bar-codes in our supermarkets and guide delicate surgery in our eyes.”

2.2.2 2. Predicted Demise of Moore’s Law

A main motivation of quantum information processing is the predicted demise of the Moore’s law [3]. Gordon Moore—Co-founder of Intel—made an empirical observation that the number of transistors in a single chip has been growing exponentially with time. (It doubles every 18 to 24 months.) As a consequence, the computing power in a small computer, such as our PC, also grows exponentially with time. This exponentially grow is tied to the exponential grow in the chip market, which has been doubling about every five years.

Moore’s law cannot continue forever. There are two factors that will bring Moore’s law to an end. The first factor is economic. The cost of a fabrication facility has also been increasing exponentially with time, doubling every chip generation. This is called Moore’s second law. Currently, a single fabrication facility will take billions of dollars to build. By 2010, according to Moore’s second law, it may cost about $40 billion, which is roughly 10 percent of the total annual market at that time.

The second factor is technological. Incidentally, the Semiconductor Industry Association has published a National Technological Roadmap that
studies the continuation of the current exponential growth in computational power until the year 2012. The number of technological obstacles for achieving such exponential growth has been growing. Some of those obstacles are commonly believed to of fundamental, rather than technical origin. Indeed, one can consider the microscopic implication of Moore’s law. If one plots the number of impurities in a single transistor, one sees an exponential decrease with time. By extrapolation, one sees that by the year 2020 or so, the number of impurities per transistor is about one. That would mean that we will be effectively working with a single electron transistor. The operation of such a single-impurity/electron transistor must be described directly by quantum mechanics. Moreover, since electrons are indivisible, naive extrapolation of (the microscopic implication of) the Moore’s law will make no sense at all beyond the year 2020. The revelation is that quantum effects are going to play a dominant role in the operations of future transistors. A fundamental understanding of quantum effects will, therefore, be necessary for the continued increase in computing power.

Information revolution of the twentieth century was based on bulk quantum effects. There has been growing appreciation that this might only be the first phase of the information revolution. In the second phase of the information revolution, which will take place in the twenty-first century, it has been suggested that quantum physics may play a direct role in the operation of computing devices.

2.2.3 3. QIP offers dramatic improvement over conventional information processing

Quantum information processing is interesting because it offers dramatic improvement over conventional information processing. See Section 3.

2.2.4 4. QIP as a proof technique

One argument against QIP is that large-scale quantum computation may prove to be impractical. Moreover, even quantum mechanics might turn out only to be an approximation of nature. However, even in those cases, QIP would remain an important proof technique in solving mathematical problems. For instance, in [4], QIP is used a proof technique to obtain a better bound on conventional coding theory problem. Indeed, this supports the viewpoint is one should consider quantum information theory is a natural generalization
of classical information theory and as such it provides us with new proof techniques in problems strictly in conventional information theory.

An analogy of quantum information processing is complex numbers. Even though all measured quantities are real (and we have yet to find a measurement that will give directly a complex output!), the usefulness of complex analysis is never in doubt!

2.2.5 5. QIP is exciting

The final reason why QIP is interesting is that people often find QIP counter-intuitive, fascinating, provocative, and, most importantly, fun to work on. It is a young and rapidly evolving subject which may have lots of surprises waiting to be discovered.

Quantum mechanics is intellectually interesting because it challenges our way of thinking about the world. It changes the rules of what is possible or what is impossible. For instance, in conventional information theory, we accept without question that information can be copied and observed without change. However, in quantum mechanics, there is a well-known principle called the Heisenberg’s uncertainty principle which asserts that it is fundamentally impossible to know two conjugate observables—e.g. its position and momentum—simultaneously. As we will discuss later, an implication of the Heisenberg’s uncertainty principle is that quantum information cannot be copied. This is contrary to our classical intuition and is, in fact, the foundation of security of a new type of cryptographic schemes based on quantum mechanics—quantum cryptography. Heisenberg’s uncertainty principle ensures that an eavesdropper cannot copy quantum signals with disturbing them. In other words, an eavesdropper will necessarily leave her traces behind.

Quantum mechanics is one of the best battle-tested theory of the physical world. It has routinely been used to describe microscopic phenomena of nature inside atoms, lasers and solids. So far, it has agreed prefectly with experiments. Of course, every physical law is only a model of nature. When extrapolated to domains far beyond its original conception, it may eventually break down. Here, we are not concerned with this potential breakdown. We will regard quantum mechanics as a mathematical model and see how it influences our thinking of information theory.
2.3 What QIP can do?

Quantum computing can efficiently solve some hard problems that are intractable with conventional computers. A standard example is Shor’s efficient quantum algorithm for factoring, which is a hard problem studied by mathematicians for centuries and is also the foundation of security of standard RSA crypto-system. Therefore, “if a quantum computer is ever built, much of conventional cryptography will fall apart!” (Brassard).

Fortunately, quantum mechanics can be used to make codes as well as breaking them. Indeed, quantum cryptography (the art of code-making), in principle, allows perfectly secure communication between two parties in the presence of a technologically arbitrarily advanced eavesdropper. Quantum code-making will be discussed in Chapter 6. In summary, quantum mechanics has the potential to revolutionize both code-making and code-breaking.

Quantum information processing also allows other novel types of information processing. For instance, in a process called “quantum teleportation”, a quantum state is decomposed locally in one spatial location and remotely reconstructed in another location through the communication of a classical message and the prior sharing of some quantum resource called ”entanglement”. While the communication of the classical message ensures that the process does not violate causality, the very fact that the state of an object can be decomposed in one place and re-constructed in another even without knowing its complete description is a rather surprising phenomenon. It reminds people of science fiction and captures people’s imagination in the same way as teleportation in the TV series Star Trek.

Also, there are various multi-party computational or cryptographic tasks that are impossible in the conventional setting. Yet, by allowing the parties to share prior quantum correlations, they become possible.

2.4 What QIP cannot do?

There are tasks that even quantum information processing cannot do. The first example is to compute a “non-computable function” defined in the standard Church-Turing model. So, while quantum computing can speed up (sometimes dramatically) some computations, it can always be simulated by a classical computer—but perhaps with an exponential overhead in some resource—time or space or resolution.

A second example is a cryptographic task called quantum bit commit-
ment. This will be discussed in Chapter 7.

2.5 What are the major challenges of QIP?

A first major challenge of QIP is to discover new tasks where QIP gives a dramatic advantage. For instance, invent a useful new quantum algorithm. One important open problem is the graph isomorphism problem. Quantum algorithms will be briefly discussed in Chapter 3.

A second major challenge of QIP is to construct a large-scale quantum information processing in the real world. While writing equations on paper is easy, building a real large-scale quantum information processing is a major technological challenge that many groups in the world have been working very hard on. See Chapter 3 for a brief discussion and [5] for details.

A third major challenge of QIP is to solve some of the well-known open problems in the subject. An example is to find an efficient way to compute the various quantum channel capacities defined in the literature. See Chapter 4.

A fourth major challenge of QIP is to continue the quantization program and apply it to other subjects. If one takes the quantization program seriously, one would believe that it is meaningful to pick almost any scientific subject and ask if and how it can be combined with quantum mechanics. For instance, one can start with control theory and ask how quantum control theory can be formulated. [A caveat: It has been suggested that some subjects such as game theory may not have a particularly useful or natural quantum version. So, there may be limits to the viewpoint that every subject can be quantized.]

A fifth major challenge is to use QIP as a proof technique to tackle problems strictly in classical information theory. The power and limitation of such a proof technique deserves investigations.

References

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[3] See, for example, Joel Birnbaum and R. Stanley Williams, ”Physics and the Information Revolution” available at http://www.physicstoday.org/vol-53/iss-1/p38.html

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