EFFICIENT MERGER OF BINARY SUPERMASSIVE BLACK HOLES IN NONAXISYMMETRIC GALAXIES

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Received 2006 February 2; accepted 2006 March 22; published 2006 April 10

ABSTRACT

Binary supermassive black holes (SBHs) form naturally in galaxy mergers, but their long-term evolution is uncertain. In spherical galaxies, N-body simulations show that binary evolution stalls at separations much too large for significant emission of gravitational waves (the “final-parsec problem”). Here we follow the long-term evolution of a massive binary in more realistic, triaxial and rotating galaxy models with particle numbers as high as $10^6$. We find that the binary does not stall. The binary hardening rates that we observe are sufficient to allow complete coalescence of binary SBHs in 10 Gyr or less, even in the absence of collisional loss-cone refilling or gasdynamical torques, thus providing a potential solution to the final-parsec problem.

Subject headings: black hole physics — galaxies: evolution — galaxies: interactions

1. INTRODUCTION

When two galaxies containing supermassive black holes (SBHs) merge, a binary SBH forms at the center of the new galaxy. The two SBHs can eventually coalesce, but only after stellar- or gasdynamical processes bring them close enough together ($<10^{-5}$ pc) that gravitational radiation is emitted. There is strong circumstantial evidence that rapid coalescence is the norm. For instance, in a galaxy containing an uncoalesced binary, mergers will eventually bring a third SBH into the nucleus, precipitating a gravitational slingshot interaction that ejects one or more of the SBHs from the nucleus (Mikkola & Valtonen 1990; Iwasawa et al. 2005). This would weaken the tight correlations that are observed between SBH mass and galaxy properties (Ferrarese & Merritt 2000; Graham et al. 2001; Marconi & Hunt 2003).

Unless the binary mass ratio is extreme, dynamical friction rapidly brings the smaller SBH in to a distance $\sim G\mu/\sigma^2$ from the larger SBH, where $\mu = M_1M_2/(M_1 + M_2)$ is the binary’s reduced mass and $\sigma$ is the one-dimensional velocity dispersion of the stars. At this separation—on the order of 1 pc—the two SBHs begin to act like a “hard” binary, ejecting passing stars with velocities large enough to remove them from the nucleus. N-body simulations (Makino & Funato 2004; Szell et al. 2005; Berczik et al. 2005) show that continued hardening of the binary takes place at a rate that depends strongly on the number $N$ of “star” particles used in the simulation. As $N$ increases, the hardening rate falls, as expected if the binary’s loss cone is repopulated by star-star gravitational encounters (Yu 2002; Milosavljević & Merritt 2003). When extrapolated to the much larger $N$ of real galaxies, these results suggest that binary evolution would generally stall (the “final-parsec problem”).

To date, N-body simulations of the long-term evolution of binary SBHs have only been carried out using spherical or nearly spherical galaxy models. But it has been suggested (Merritt & Poon 2004) that binary hardening might be much more efficient in nonaxisymmetric galaxies because of the qualitatively different character of the stellar orbits. Here we test that suggestion by carrying out the first N-body simulations of massive binaries in strongly nonaxisymmetric galaxy models. We find that the hardening rate is independent of $N$ for particle numbers up to at least $10^6$. To the extent that our galaxy models are similar to real merger remnants, these results imply that binary SBHs can efficiently harden through purely stellar-dynamical interactions in many galaxies, thus providing a plausible solution to the final-parsec problem.

2. METHOD

Our N-body models were generated from the phase-space distribution function

$$f(E, L_z) = \text{const} \times (e^{-\beta E} - 1)e^{-\beta|L_z|}$$

(Lagoute & Longaretti 1996). Here $E = v^2/2 + \Phi$ is the energy per unit mass of a star, $\Phi(\sigma, z)$ is the gravitational potential in the meridional plane, and $L_z$ is the angular momentum per unit mass in the direction of the symmetry ($z$) axis. The $\beta^{-1}$-dependent factor has the effect of flattening the models and simultaneously giving them a net rotation. The degree of flattening is specified by the dimensionless parameter $\omega_0 = [9/(4\pi G\rho)]^{1/2}c_0$, with $\rho_c$ the central mass density. The parameter $\beta$ determines the central concentration and was chosen such that the spherical model generated from equation (1) had a central concentration $W_0 = 6$ (King 1966). Here and below we adopt standard N-body units, that is, the gravitational constant and total mass of the galaxy are 1, and the galaxy’s energy is $-\frac{1}{2}$.

A pair of massive particles representing the two SBHs were introduced into the models at $t = 0$. The two particles were given equal masses, $M_1 = M_2 = M_{\bullet}/2$, and were placed on coplanar, circular orbits at distances $\pm 0.3$ from the galaxy center in the equatorial plane. In most of the simulations described below, $M_{\bullet} = 0.04$. This is rather larger than the typical ratio, $\sim 1 \times 10^{-3}$, observed between SBH mass and galaxy mass; such a large mass for the SBH particles was chosen in order to minimize the rate of relaxation-driven loss-cone refilling, which occurs more rapidly for smaller $M_{\bullet}$ (Berczik et al. 2005), and to come as close as possible to the “empty loss cone” regime that characterizes real (axisymmetric) galaxies. In order to estimate the dependence of the binary decay rate on $M_{\bullet}$, we carried out a limited set of additional simulations with different values of $M_{\bullet}$, as described below.

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Integrations of the particle equations of motion were carried out using a high-accuracy, direct summation N-body code (Berczik et al. 2005) on two parallel supercomputers incorporating special-purpose GRAPE (Fukushige et al. 2005) accelerator boards: gravitySimulator\(^5\) and GRACE.\(^6\) Integration parameters were similar to those adopted in Berczik et al. (2005), and we refer the reader to that paper for details about the performance of the code. Integrations were carried out for various values of \(N\) in the range \(0.025 \times 10^6 \leq N \leq 0.4 \times 10^6\), and for various values of the galaxy rotation parameter \(q_0\) in the range \(0 \leq q_0 \leq 1.8\). In addition, each model was integrated with two choices for the orientation of the binary’s angular momentum, either parallel to that of the galaxy (“prograde”) or counter to it (“retrograde”). In order to verify the large-N dependence of the binary hardening rate, a single integration of the \(q_0 = 1.8\) model was carried out using \(N = 10^6\).

3. RESULTS

Figure 1a shows the evolution of \(1/a\), the binary inverse semimajor axis, in a set of simulations with \(q_0 = 0\) and various \(N\)-values. These spherical, nonrotating models are very similar to the models considered in Berczik et al. (2005), and the binary evolution found here exhibits the same strong \(N\)-dependence that was observed in that study: the hardening rate, \(s(t) \equiv (dldt)(1/a)\), is approximately constant with time and decreases roughly as \(N^{-1}\). This behavior has been described quantitatively (Milosavljević & Merritt 2003) on the basis of loss-cone theory: stars ejected by the binary are replaced in a time that scales as the two-body relaxation time, and the latter increases roughly as \(N\) in a galaxy of fixed mass and size.

When the rotation parameter \(q_0\) is increased to \(\sim 0.6\), the initially axisymmetric models become unstable to the formation of a bar, yielding a slowly tumbling, triaxial spheroid. Figure 2 illustrates the instability with snapshots of the \(q_0 = 1.8\) model integration. This model is moderately flattened initially, with mean short-to-long axis ratio of \(\sim 0.46\), and strongly rotating, with roughly 40% of the total kinetic energy in the form of streaming motion. Movies based on the simulation\(^7\) reveal that the two SBH particles initially come together by falling inward along the bar before forming a bound pair.

To determine the dependence of the binary’s evolution rate on the properties of the galaxy model, a suite of N-body integrations were carried out for various \((q_0, N)\) and for \(M_* = 0.04\). Axis ratios of the galaxy were computed using its moment-of-inertia tensor, as described by Dubinski & Carlberg (1991). The results are summarized in Figure 3. After a strong bar forms at \(t \approx 10\) in the unstable models, it evolves gradually toward rounder shapes, but the system maintains a significant triaxiality until the end of the simulation. This slow evolution appears similar to that in the simulations of Theis & Spurzem (1999), where two-body relaxation was identified as the driving force.

\(^5\) See http://www.cs.rit.edu/~grapecluster/clusterInfo/grapeClusterInfo.shtml.
\(^6\) See http://www.ari.uni-heidelberg.de/grace.
\(^7\) See http://www.cs.rit.edu/~grapecluster/BinaryEvolution.
mechanism. The presence of a massive binary in our simulations might also tend to destroy the triaxiality (Athanassoula et al. 2005), although an integration excluding the binary showed a similar degree of evolution (see Fig. 3, left).

The steep, $\sim N^{-1}$, dependence of the binary hardening rate in the spherical model changes to an essentially constant hardening rate for $\omega_{0} \geq 1.2$ (Fig. 3, right). The hardening rate was found not to depend on the initial sense (prograde vs. retrograde) of the binary orbit.

Since $M_{2}/M_{gal} = 0.04$ is considerably larger than the ratio $\sim 1 \times 10^{-3}$ observed in real galaxies, we carried out an additional set of simulations in order to evaluate the $M_{2}$-dependence of the binary hardening rate. These integrations used $\omega_{0} = 1.8$, $N = 0.1 \times 10^{5}$, and $0.01 \leq M_{2} \leq 0.08$. We found that the hardening rate increased with decreasing $M_{2}$. At $t = 100$, the hardening rates $s$ were 20.0, 13.0, 8.2, and 3.4 for $M_{2}$ of 0.01, 0.02, 0.04, 0.08, respectively. These results should be interpreted with caution, since we did not vary $N$ and therefore cannot state with certainty whether the $s$-values are independent of $N$.

A question for future study is whether the results from our galaxy model, with its very strong bar-mode instability, are characteristic of more realistic models of merging galaxies.

4. DYNAMICAL INTERPRETATION

A hardening rate that is independent of $N$ implies a collisionless, that is, relaxation-independent, mode of loss-cone refilling. Just such a mode is expected in triaxial galaxies: the lack of an axis of symmetry implies that stellar orbits need not conserve any component of the angular momentum, and hence they can pass arbitrarily close to the center after a finite time and interact with a central object (Norman & Silk 1983; Gerhard & Binney 1985).

A full derivation of the expected rate of supply of stars to the binary in these models would be very difficult, but we can do an approximate calculation. The rate per unit of orbital energy at which centrophilic orbits supply mass (i.e., stars) to a region of radius $r_{c}$ at the center of a galaxy is

$$\dot{M}(E)dE = r_{A}(E)M_{*}(E)dE,$$

(2)

where $A(E)d$ is the rate at which a single star on a centrophilic (e.g., box or chaotic) orbit of energy $E$ experiences near-center passages with pericenter distances $\leq d$ and $M_{*}(E)dE$ is the mass in stars on centrophilic orbits with energies from $E$ to $E + dE$ (Merritt & Poon 2004). Setting $r_{c} = Ka$, with $K \approx 1$, gives the mass flux into the galaxy’s sphere of influence; the implied hardening rate is

$$s \approx \frac{d}{dt} \left( \frac{1}{a} \right) \approx \frac{2\langle C \rangle}{aM_{*}} \int M(E)dE$$

(3)

(Berczik et al. 2005). Here $\langle C \rangle \approx 1.25$ is the average value of the dimensionless energy change during a single star-binary encounter, $C \equiv (M_{*}/2m_{*})(\Delta E/E)$.

Our N-body models have density $\rho \sim r^{-2}$ beyond the core radius $r_{c} \approx 0.25$. In a $\rho \propto r^{-2}$ galaxy,

$$A(E) \approx \frac{\sigma}{r_{c}^{2}} e^{-(E-E_{0})/\sigma}, \quad M_{*}(E) = f_{c}(E) \frac{2\sigma}{G} \int e^{-(E-E_{0})/\sigma} dE$$

(4)

with $r_{c} = GM_{*}/\sigma^{2}$, $E_{0} = \Phi(r_{c})$, and $f_{c}(E)$ the fraction of the orbits at energy $E$ that are centrophilic (Merritt & Poon 2004). The implied binary hardening rate is

$$s \approx \frac{4\langle C \rangle}{G} \frac{\sigma}{\sigma_{c}^{2}} \int e^{-(E-E_{0})/\sigma} dE \approx 2.5\frac{\sigma}{r_{c}^{2}} \int e^{-(E-E_{0})/2\sigma^{2}} dE \approx 2.5\frac{\sigma}{r_{c}^{2}} \int e^{-(E-E_{0})/\sigma} dE$$

(5)

where $\langle C \rangle \approx 1.25$. This integral may be evaluated analytically as

$$\int e^{-(E-E_{0})/\sigma} dE \approx \frac{1}{\sigma}$$

For $\langle C \rangle \approx 1.25$ and $\sigma = 0.50$, this value is $\approx 2.5$, in agreement with the value calculated from (3)
Here \( \dot{f} \) is an energy-weighted, mean fraction of centrophilic orbits, and the lower integration limit was set to \( E_{c}^{j} \); the latter can only be approximate, since the true density of our galaxy models departs from \( r^{-2} \) at \( r < r_{c} \approx r_{g} \). Substituting \( \sigma \approx 0.47 \) and \( r_{c} \approx 0.18 \) from the galaxy models gives \( s \approx 40f_{j} \). By comparison, the hardening rates in the \( N \)-body models reach a peak value at \( r \approx 20 \) of \( s \approx 16 \), consistent with the derived expression if \( \dot{f}_{j} \approx \dot{s} \). The gradual drop observed in the hardening rate at later times, \( s(t) \approx 16 - 5.2 \ln (t/20), \) \( 20 < t < 250 \), suggests that the number of centrophilic stars is becoming smaller, as a result of depletion by the binary and the gradual change in the galaxy’s shape.

Taken at face value, equation (5) implies \( s \propto M_{*}^{-2} \); however, for small \( M_{*} \), \( r_{c} \ll r_{g} \) and the assumption that \( \rho \sim r^{-2} \) for \( r > r_{c} \) breaks down. In any case, the observed dependence of \( s \) on \( M_{*} \) is slightly weaker, \( s \propto M_{*}^{-1} \) (§ 3).

5. IMPLICATIONS

The timescale for gravitational wave emission by a binary black hole is

\[
\tau_{gw} = \frac{5}{16} F(e) \sigma^{3} M_{*}^{-2} \left( \frac{a_{h}}{l} \right)^{4}
\]

(Peters 1964). Here \( M_{*} = M_{1} + M_{2}, \mu = M_{1}M_{2}/M_{*}^{2} \) is the reduced mass of the binary, \( \sigma \) is the one-dimensional central velocity dispersion of stars in the nucleus, and \( a_{h} = GM_{*}/4\sigma^{2} \) is the semimajor axis of the binary when it first becomes "hard," that is, tightly bound; the factor \( F(e) \) depends on the binary’s orbital eccentricity and \( F(0) = 1 \). In order that gravitational wave–driven coalescence take place in less than \( 10^{10} \) yr, an equal-mass, circular-orbit binary with \( M_{*} = 10^{3} M_{\odot} \) must first reach a separation \( a \approx 0.05 a_{h} \) (Merritt & Milosavljević 2005). This is just achieved in our simulations: \( a_{i} \approx 1.1 \times 10^{-2} \), and the final value of \( a \) in the bar-unstable models is \( \approx 6 \times 10^{-5} \). This is a conservative interpretation, since (1) for reasonable scalings of our galaxy model to real galaxies (e.g., total mass \( 10^{11} M_{\odot} \), half-mass radius \( 10^{3} \) pc), an elapsed time of 250 in \( N \)-body units corresponds to \( \approx 1 \) Gyr; (2) the binary is continuing to harden at the final time step in our simulations (Fig. 1b); (3) our experiments with different \( M_{*} \) found \( s \propto M_{*}^{-1} \), implying substantially more rapid hardening in the case \( M_{1}/M_{\text{gal}} \approx 10^{-3} \); and (4) the binary had nonzero eccentricity in our simulations. In addition, gas is a significant component of disk galaxies, and in many mergers, the final hardening of the binary would be accelerated by gasdynamical torques (Escala et al. 2005; Dotti et al. 2006).

We simulations of binary evolution are substantially more realistic than existing ones based on spherical or nearly spherical galaxy models. Even more realistic simulations, which follow both the early and late stages of a merger between two galaxies, are probably beyond the capabilities of current algorithms and hardware, because of the need to accurately treat both large (\( \approx 10 \) kpc) and small (\( \approx 0.01 \) pc) spatial scales. However, our galaxy models (slowly tumbling triaxial spheroids) are similar to those produced in full merger simulations (Bournaud et al. 2005; Naab et al. 2006), suggesting that our results for the long-term evolution of the binary are probably fairly generic despite the rather artificial initial conditions.

Uncertainties about the resolution of the "final-parsec problem" have been a major impediment to predicting the frequency of SBH mergers in galactic nuclei and, hence, to computing event rates for proposed gravitational wave interferometers such as LISA. If binary coalescence rates are assumed to be similar to galaxy merger rates, gravitational wave events integrated over the observable universe could be as frequent as \( 10^{2} \) yr\(^{-1} \) (Haehnelt 1994; Sesana et al. 2004). Our results, combined with the indirect evidence that binary SBH coalescence is efficient, suggest that such high event rates should be taken seriously.

We thank M. Milosavljević and S. Harfst for comments on the manuscript. This work was supported by grants AST 02-06031, AST 04-20920, and AST 04-37519 from the National Science Foundation, grant NNG04GJ48G from NASA, grant HST-AR-09519.01-A from STScI, grant I80 041 GRACE from the Volkswagen Foundation, by SFB439 of the Deutsche Forschungsgemeinschaft, and by INTAS grant IA-03-59-11. We thank the Center for the Advancement of the Study of Cyber-infrastructure at Rochester Institute of Technology for their support.

8 See http://lisa.jpl.nasa.gov.

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