Study on the analogy between velocity and temperature fluctuations in the turbulent rotating channel flows

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Abstract Turbulent rotating channel flow is of great interest in engineering practice, such as the cooling of the blades in turbo machinery. This paper investigates the turbulence characteristics of temperature fluctuations in rotating turbulent channel flow by direct numerical simulation. It is known that the analogy exists between temperature and velocity fluctuations in non-rotating channel flows. In this paper the analogous property is investigated between temperature and streamwise velocity fluctuations in rotating turbulent channel flows and it has been found that the analogy exists at pressures side in near wall region but the analogy is violated at the suction wall. The influence of the large scale flow structure in rotating channel, known as the Taylor-Görtler vortex, on the characteristics of temperature fluctuations are revealed.

1. Introduction

The turbulent channel flow subject to spanwise rotation is a prototype of cooling passages in turbo-machinery and generator. It has been found that the heat transfer is reduced at the suction wall of the rotating channel; hence the investigation of the turbulent heat flux on the suction side is significant in design of cooling system.

The heat transfer in the turbulent channel flow without rotation has been investigated by various authors, such as Kim and Moin (1989), Kasagi et al. (1992, 1993) and Kawamura et al. (1998, 2000, 2009). A prominent feature of temperature fluctuations was found in the experiment of turbulent boundary layer with heat transfer by Fulachier et al. (1976) that there is an analogy between temperature and streamwise velocity fluctuations. Antonia et al. (2009) analyzed the analogous property with the high resolution data bank provided by Kozuka et al. (2009) and found that the analogy between root mean square of temperature fluctuations and turbulence kinetic energy is better than that between root mean square of temperature and streamwise velocity fluctuations. The analogy between temperature and velocity fluctuations is not only of academic interest but also of practical significance that it can be used in the modeling of turbulent heat flux.

The investigation of turbulent channel flows subject to spanwise rotation has been carried out by experimental measurements (Johnston et al. 1972) and direct numerical simulation (Kristoffersen et al. 1993). Both experimental and numerical studies have found the enhancement
of turbulence on the pressure side and reduction of turbulence on the suction side. A peculiar large scale structure, known as Taylor-Görtler vortex, similar to that appearing in the turbulent curved channel flow, was discovered in direct numerical simulation (Kristoffersen et al. 1993). As far as temperature fluctuations in rotating channel is concerned, Nagano et al. (2003) performed direct numerical simulation of temperature fluctuations in a rotating turbulent channel flow. They focused on the turbulence modeling for heat flux and proposed a new RANS model. They found that their model can predict mean temperature profile with success while the prediction of higher statistics of temperature fluctuations is unsatisfactory.

Does the analogous property exist between temperature and streamwise velocity fluctuations in rotating turbulent channel flows? In this paper we investigate the analogous property between velocity and temperature fluctuations in details by quantitative analysis of the DNS data in a rotating channel flow with constant streamwise temperature gradient. It has been found that the analogy still exists at pressure wall but is violated at suction wall. It has been further discovered that the large scale structure, i.e. Taylor-Görtler vortex, has significant effect on the analogous property. The reason for the violation of analogous property at the suction side is attributed to the different transport properties between turbulent heat energy and streamwise kinetic energy in the rotating channel due to the Coriolis force.

The paper is organized as follows. In section 2 the numerical method and flow parameters are introduced and the primary statistics properties of temperature and flow field in the rotating channel are presented in Section 3. The effect of Taylor-Görtler vortex on the analogy is explored in Section 4. A brief analysis of the budget in turbulent heat and kinetic energy transport is presented in Section 5 with discussions on the violation of analogous property on the suction side. Some discussions followed by concluding remarks are presented in Section 6.

2. The governing equations and numerical methods

The fluid motion is governed by Navier-Stokes equation with Boussinesq hypothesis as follows

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} - 2 \epsilon_{ij} u_k \Omega \frac{\partial u_i}{\partial x_k} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

(1)

$$\frac{\partial u_i}{\partial x_i} = 0$$

(2)

in which \((x_1, x_2, x_3)\) or \((x, y, z)\) stands for the streamwise, vertical and spanwise directions respectively and \(\Omega\) is the spanwise rotation rate.

The temperature \(T\) is decomposed into a constant mean streamwise temperature gradient and a residual part:

$$T(x_1, x_2, x_3, t) = x_1 \frac{d\Theta}{dx} + \Theta(x_1, x_2, x_3, t)$$

(3)

The heat balance equation can then be written as follows

$$\frac{\partial \Theta}{\partial t} + u_j \frac{\partial \Theta}{\partial x_j} + Q = \kappa \frac{\partial^2 \Theta}{\partial x_j \partial x_j}$$

(4)

in which \(Q\) is the streamwise heat flux

$$Q = u_i \frac{d\Theta}{dx_i}$$

(5)

The flow configuration is illustrated in Figure 1. The pseudo spectral method is applied in numerical computation with periodic condition for both velocity and temperature in streamwise and spanwise directions. The non-slip condition and zero residual temperature are prescribed at channel wall, i.e.

$$u_i(x, \pm 1, z, t) = 0 \text{ and } \Theta(x, \pm 1, z, t) = 0$$

(6)
The flow parameters are $Re=2666$, $Ro=0.0, 0.1, 0.2, 0.3$ and $Pr=1.0$ in computation with grids $128 \times 129 \times 128$ in streamwise, normal and spanwise direction respectively. The Reynolds number is defined as $Re=U_m h/\nu$, in which $U_m$ is the bulk velocity and $h$ is the half width of the channel. The rotation number is defined as $Ro=2\Omega h/U_m$.

The ensemble average, denoted by $\langle \cdot \rangle$, is performed in streamwise, spanwise directions and in time as follows

$$\langle \cdot \rangle = \int_0^{L_x} \int_0^{L_z} \int_0^{L_z} \kappa dx dy dz / L_x L_z L_e$$ (7)

The turbulent fluctuations are then obtained as the difference between the instantaneous and averaged values that

$$u'_i = u_i - \langle u_i \rangle, \quad \theta' = \theta - \langle \theta \rangle$$ (8)

The large scale fluctuations is identified by the streamwise average that

$$\bar{u}'_i = \int_0^{L_x} u'_i dx / L_x, \quad \bar{\theta}' = \int_0^{L_x} \theta' dx / L_x$$ (9)

The large scale velocity fluctuations defined above reveal the Taylor-Görtler vortex in rotating turbulent channel flows, shown in Figure 2, and will be called TG fluctuation hereafter. Note that the defined TG fluctuation is averaged in the streamwise direction so that it is independent of $x$.

Figure 2. Taylor-Görtler vortex identified by TG fluctuation (9) at Re=2666, Ro=0.1

3. Statistical properties of flow and temperature fields in rotating channel

The mean velocity and Reynolds stress profiles are presented in Figure 3 (a) and (b) that the turbulence is enhanced at pressure wall and suppressed at suction wall in rotating turbulent channel flows, see Figure 3 (b), and that the mean velocity is smaller at pressure wall than that at suction wall, see Figure 3 (a), in which a peculiar feature of the rotating turbulent channel flow is also shown that the mean shear rate equals the rotation rate at the center of the channel.

The root mean square of temperature and streamwise velocity fluctuations are presented in Figure 4. It can be seen that the turbulent statistics are not changed so much with the rotation numbers at the pressure wall while the statistical properties at suction wall are changed considerably with rotation numbers.
Figure 3. Mean velocity and Reynolds stress at Re=2666. The arrows indicate the increasing rotation number.

Figure 4. Root mean square of temperature and streamwise velocity fluctuations at Re=2666. The arrows indicate the increasing rotation number. Symbol: Ro=0, Solid lines: pressure side, Dashed line: suction side, Dot dashed line: linear approximation at the wall.

For quantitative examination of the analogy between temperature and streamwise velocity fluctuations, the correlation coefficient between temperature and streamwise velocity fluctuations is a critical parameter which is defined as

$$R_{u\theta} = \frac{\langle u'\theta' \rangle}{\sqrt{\langle u'^2 \rangle} \sqrt{\langle \theta'^2 \rangle}}$$  \hspace{1cm} (10)

Figure 5 (a) and (b) show the correlation coefficients on suction and pressure sides respectively. It is clearly shown that the correlation coefficient $R_{u\theta}$ is over 0.8 near the pressure wall ($y^+<50$) for all rotation numbers, see Figure 5 (b), while the correlation coefficients are less than 0.6 near the suction wall when rotation number is larger than 0.2, see Figure 5 (a), the correlation coefficient drops considerably far from the suction wall. This indicates that there exists analogy between velocity and temperature fluctuations on the pressure side in rotating turbulent channel flows but
the analogous property is violated at suction wall.

![Figure 5](image)

**Figure 5.** Correlation coefficients. Solid line: Ro=0.0, Dashed line: Ro=0.1, Dot dashed line: Ro=0.2, Double dot dashed line: Ro=0.3

The turbulent Prandtl numbers, an important parameter for turbulent heat flux, can be calculated as follows

$$Pr_t = \frac{\nu_t}{\kappa_t} = \frac{\langle \{u'v'\}d\langle \theta \rangle/dy \rangle}{\langle \{v'\theta'\}d\langle u \rangle/dy \rangle}$$

(11)

which is presented in Figure 6 (c) and (d).

![Figure 6](image)

**Figure 6.** Turbulent Prandtl numbers. Solid line: Ro=0.0, Dashed line: Ro=0.1, Dot dashed line: Ro=0.2, Double dot dashed line: Ro=0.3

4. The effects of Taylor-Görtler vortex on the analogous property

The iso-contour lines of temperature and streamwise velocity fluctuations are shown in figure 7. In non-rotating channel, low speed streak structure can be clearly seen from the iso-contour lines, and good analogy in the structure is observed between velocity and temperature fluctuations in Figure
7 (a) and (b). This is in accordance with previous studies (Kawamura, et al. 1998, Antonia, et al. 2009). The streaks on the pressure side of the rotating channel can still be found, but they are squeezed in some areas due to the existence of Taylor-Görtler vortex. The iso-contour lines of $\theta'$ and $u'$ are similar on the pressure side at $Ro=0.3$, see Figure 7 (c), (d), and this is consistent with the high correlation coefficients observed at pressure wall in last section. The squeezing phenomenon can also be found on the suction side of the rotating channel at $Ro=0.3$. The turbulence is suppressed considerably on the suction side at higher rotation numbers, so streak structures are nearly not found at $Ro=0.3$ (see Figure 7 (e) and (f)) and the analogy between velocity and temperature fluctuations is violated. The squeezing effect of streak lines, shown in Figure 7, is caused by the Taylor-Görtler vortex, demonstrated in Figure 8 where it is clearly shown that streaks exist at the pressure wall while they are reduced at the suction wall.

Figure 7. Iso-contours of $u'$ and $\theta'$ at $y^+=15$, $u''<-1.2$ or $\theta''<-1.2$ at $Re=2666$, $Pr=1.0$

Figure 8. The TG fluctuations on y-z plane at $Re=2666$, $Ro=0.1$, $Pr=1.0$, upper wall:suction; lower wall: pressure. Solid lines: $\tilde{\theta}' > \bar{\theta}_r$ or $\tilde{u}' > \bar{u}_r$, Dashed lines: $\tilde{\theta}' < -\bar{\theta}_r$ or $\tilde{u}' < -\bar{u}_r$.
The quadrant analysis of heat and momentum flux is a useful tool for understanding transport properties in turbulent shear flows. Replacing streamwise velocity fluctuation by temperature fluctuation in the quadrant analysis of velocity fluctuations one can investigate the property of turbulent heat flux. For instance the ejection of low temperature is represented in the second quadrant and sweep of high temperature is in the fourth quadrant. It is well known in non-rotating channel that the ejection of streamwise momentum is dominant in the region $y^+ > 15$ while the sweep is overwhelming in the region $y^+ < 15$. The similar situation occurs in the turbulent heat flux as shown in Figure 9 (a) because of the analogy between $\theta'$ and $u'$ in the non-rotating channel. On the pressure side of the rotating channel, the quadrant contributions of heat and momentum flux are almost unchanged in comparison with those in non-rotating channel (see Figure 9 (b) (d)) since the analogy between $\theta'$ and $u'$ is quite good at pressure wall. On the suction side, the second quadrant contribution to the vertical heat and monument fluxes decreases evidently and the quadrant distribution of turbulent heat flux is different from that of Reynolds stress.

For better understanding of the analogous property a brief analysis of turbulent heat and kinetic energy transport in rotating channel flow is helpful.

5. Brief analysis of the budget in turbulent heat and kinetic energy transport

Fulachier et al. (1984) and Kawamura et al. (2009) argued that the existence of analogy in statistics between velocity and temperature fluctuations in wall bounded turbulence without rotation can be interpreted by the transport equations for turbulent heat energy $k_{\theta} = \langle \theta' \theta' \rangle / 2$ and turbulent streamwise kinetic energy $k_{u} = \langle uu' \rangle / 2$. Following this idea the transport equation for turbulent heat energy and kinetic energy will be analyzed in rotating channel flow below.
The transport equations for \( k_\theta \) and \( k_u \) can be written in rotating channel as follows,

\[
\frac{Dk_\theta}{Dt} = -\langle v'\theta' \rangle \frac{d\langle \theta \rangle}{dy} - \langle u'\theta' \rangle Q - \frac{1}{2} \frac{d}{dy} \left( \frac{1}{\operatorname{Re} \operatorname{Pr}} \frac{\partial k_\theta}{\partial y} \right) + \frac{1}{\operatorname{Re} \operatorname{Pr}} \frac{d^2 k_\theta}{dy^2} \left( \frac{1}{\operatorname{Re} \operatorname{Pr}} \frac{d \langle \theta' \theta' \rangle}{d\xi_j d\xi_j} \right)
\]

(12)

\[
\frac{Dk_u}{Dt} = -\langle u'v' \rangle \frac{d\langle u \rangle}{dy} - \frac{1}{2} \frac{d}{dy} \left( \frac{1}{\operatorname{Re} \operatorname{Pr}} \frac{\partial k_u}{\partial y} \right) + \frac{1}{\operatorname{Re} \operatorname{Pr}} \frac{d^2 k_u}{dy^2} \left( \frac{1}{\operatorname{Re} \operatorname{Pr}} \frac{d \langle u' u' \rangle}{d\xi_j d\xi_j} + \langle \frac{\partial u'}{\partial \xi} \cdot \frac{\partial u'}{\partial \xi} \rangle + \operatorname{Ro} \langle u'v' \rangle \right)
\]

(13)

In both equations (12) and (13) there are similar terms in transport of turbulent heat and kinetic energy, such as production \( P_\theta \) (\( P_u \)), turbulent diffusion \( T_\theta \) (\( T_u \)), molecular diffusion \( D_\theta \) (\( D_u \)) and dissipation \( \varepsilon_\theta \) (\( \varepsilon_u \)) while there are additional terms in turbulent kinetic energy equation, such as the redistribution \( \Gamma_u \) and the Coriolis force \( C_u \). Figure 10 shows the budget of \( k_\theta \) and \( k_u \) in rotating and non-rotating turbulent channel.

In non-rotating wall bounded turbulent flow, Fulachier et al. (1984) argued that the analogous property exists because both production terms are dominant in transport of turbulent heat and kinetic energy. Figure 10 (a) and (b) show that both production terms are dominant in the near wall region, in particular both take peak values at \( y^+ = 20 \) in non-rotating channel flows. In rotating channel Fulachier’s argument is still true on pressure sides in near wall region where the production term are dominant around \( y^+ = 20 \) in both transport equation for \( k_\theta \) and \( k_u \), see Figure 10 (c), (d), and correlation coefficients between temperature and velocity fluctuations are over 0.9, see Figure 5 (b). Far from the wall the correlation coefficients gradually reduced since Coriolis term is in the same order of magnitude with production term in \( k_u \) equation while there is no Coriolis term in \( k_\theta \) equation. On suction side of rotating channel the production term is much greater than Coriolis term in \( k_u \) equation whereas the budget in \( k_\theta \) equation is nearly unchanged in comparison with that at lower rotation number. As the result the analogy between temperature and streamwise velocity fluctuations is violated at the suction side at higher rotation numbers. Another difference in the budget between \( k_u \) and \( k_\theta \) equations is the redistribution term which is absent in \( k_\theta \) equation. The redistribution term, denoted by triangles, is quite small at pressure wall in comparison with production term at all rotation numbers. The redistribution term is also small at suction wall at \( \operatorname{Ro} = 0.1 \) while it increases considerably at \( \operatorname{Ro} = 0.3 \).

6. Concluding remarks

The above discussion indicates that in rotating turbulent channel flows the property of turbulent heat flux near the suction wall is greatly different form that near the wall of non-rotating turbulent channel flows while the turbulent heat flux near the pressure wall is similar to that in non-rotating turbulent channel flows. The violation of analogy between temperature and streamwise velocity fluctuations at the suction side has been explained in preceding sections through detailed analyzing the budget of transport equations for turbulent heat and streamwise kinetic energy that the Coriolis force plays significant role in the transport equation for \( k_u \) on the suction side while the Coriolis force is absent in the transport equation for \( k_\theta \). It has been also found that the Taylor-Görtler vortex, the peculiar large scale fluctuation induced by Coriolis force in rotating channel flows, directly influences the analogous property between velocity and temperature fluctuations. It is
expected that the reconstruction of an appropriate RANS or LES model for turbulent heat flux at suction wall of rotating channel is hopeful with consideration of Coriolis force and TG vortex.

![Graphs showing the budget of transport of $k_\theta$ and $k_u$ at Re=2666 and Pr=1.0. Solid lines: $P_\theta(P_u)$, Dotted dash lines: $T_\theta(T_u)$, Square: $D_\theta(D_u)$, Dashed lines: $-\varepsilon_\theta(\varepsilon_u)$, Cross: $C_u$, Triangle: $\Gamma_u$.](image)

**Figure 10.** The budget of transport of $k_\theta$ and $k_u$ at Re=2666 and Pr=1.0. Solid lines: $P_\theta(P_u)$, Dotted dash lines: $T_\theta(T_u)$, Square: $D_\theta(D_u)$, Dashed lines: $-\varepsilon_\theta(\varepsilon_u)$, Cross: $C_u$, Triangle: $\Gamma_u$.}
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