Response spectra analysis of the modal summation technique verified by observed seismometer and accelerometer waveform data of the M6.5 Pidie Jaya Earthquake

Irwandi13, Ibnu Rusydy23, Umar Muksin13, Ariska Rudyanto4, Daryono4
1Physics Department, Syiah Kuala University, Banda Aceh, Indonesia
2Geology Department, Syiah Kuala University, Banda Aceh, Indonesia
3Tsunami and Disaster Mitigation Research Center, Syiah Kuala University, Banda Aceh, Indonesia
4Indonesian Agency for Meteorology, Climatology and Geophysics

E-mail: irwandi@unsyiah.ac.id

Abstract. Wave vibration confined in the boundary will produce stationary wave solution in discrete states called modes. There are many physics applications related to modal solutions such as air column resonance, string vibration, and emission spectrum of the atomic Hydrogen. Naturally, energy is distributed in several modes so that the complete calculation is obtained from the sum of the whole modes called modal summation. The modal summation technique was applied to simulate the surface wave propagation above crustal structure of the earth. The method is computational because it uses 1D structural model which is not necessary to calculate the overall wave propagation. The simulation results of the magnitude 6.5 Pidie Jaya earthquake show the response spectral of the Summation Technique has a good correlation to the observed seismometer and accelerometer waveform data, especially at the KCSI (Kotacane) station. On the other hand, at the LASI (Langsa) station shows the modal simulation result of response is relatively lower than observation. The lower value of the reaction spectral estimation is obtained because the station is located in the thick sedimentary basin causing the amplification effect. This is the limitation of modal summation technique, and therefore it should be combined with different finite simulation on the 2D local structural model of the basin.

1. Introduction
Most of the physics application is to find the solutions by implementing differential equations. The initial value and boundary condition will lead to unique solutions with specific patterns. For instance, when a solution of the wave equation in boundary value condition is confined to a finite region of spaces of the boundary, some pattern will happen. Space that is filled up by vibrating patterns is called standing wave. There are many physics applications related to modal solutions such as air column resonance, string vibration, and emission spectrum of the atomic Hydrogen. Confining the wave could quantize the frequency of the vibration into several modes. The quantized frequency is distributed into spectral of energy. Therefore, if we know the spectral of energy we can find the complete solution by summing the modes of the wave. Thus, the similar method is used to find seismic wave propagation on the surface of earth structure known as the surface wave.
In the framework of the linear theory of elastic media, the balance of the forces such as inertia, body forces, and surface forces acting on a cubic element within the continuum is considered. Therefore Newton’s laws of motion could be expressed as follows,

\[ \rho \frac{\partial^2 u_x}{\partial t^2} = \rho x + \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{yx}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z} \]

\[ \rho \frac{\partial^2 u_y}{\partial t^2} = \rho y + \frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\sigma_{yz}}{\partial z} \]

\[ \rho \frac{\partial^2 u_z}{\partial t^2} = \rho z + \frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} \]

Where a Cartesian coordinate system \((x, y, z)\) is adopted. \(s_{ij}(x,t) (i=x, y, z; j=x, y, z)\) indicates the second-order stress tensor, \(\rho\) is the density of the material, and \(X, Y, Z\) are the components of body forces for a unit mass.

In general, the relation between stress and deformation can be complex because it is influenced by several parameters including pressure, temperature, and the amount of the variability of the stress. Nevertheless, by assuming the deformations and the stresses in short duration (the conditions mostly satisfy the problems in the ground motion estimation) are small, we can assume that solid behaves linearly, and the constitutive relation is linking to the stresses and the deformation could be based on Hooke’s laws of the stress tensor. After applying some symmetrical properties and assuming the locally isotropic stress, we obtain,

\[ \sigma_{ij} = \lambda \epsilon_{kk}\delta_{ij} + 2\mu \epsilon_{ij} \]

where \(\lambda\) is the quantities and \(\mu\) are called Lamé parameters.

Let us consider a halfspace system where the vertical \(z\)-axis is positive downward from the free surface. The vertical stresses are null which is defined by the plane \(z=0\) (Figure 1).

By assuming that \(l, m,\) and \(r\) are continuous functions of \(z\) and the body wave velocities are

\[ \alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \text{and} \quad \beta = \sqrt{\frac{\mu}{\rho}} \]

the values of \(\alpha_{ll}\) and \(\beta_{ll}\) are the largest when \(z > H\) remain constant for greater depths. Here, the elastic parameters depend only on the vertical coordinate [1-2].

2. Method of Modal Summation Technique

By neglecting the body forces, we can consider solutions of Equation 1 to form harmonic wave plane propagating along the positive \(x\)-axis:
The seismic wave source is introduced in the medium representing the fault which is supposed to be planar, as a discontinuity in the displacement and shear stresses fields, concerning the fault plane. On the contrary, the normal stresses are supposed to be continuous across the fault plane. Maruyama [3] and Burridge and Knopoff [4] demonstrated the representation theorem the rigorous equivalence of the effects between a faulted medium with a discontinuity in the displacements and shear stress fields and an unfaulted medium where the proper body forces are applied.

Following the procedure proposed by Kausel and Schwab [5], we assume that the periods and the wavelengths, which we are interested in, are large compared to the rise time and the dimensions of the source. Therefore, the source function, describing the discontinuity of the displacement across the fault, can be approximated by a step function in time and a point source in space. Furthermore, if the normal stress is continuous across the fault, then based on the representation theorem the equivalent body force in an unfaulted medium is double-couple with a total null moment. With this assumption, the eigenvalues and eigenfunctions of the problem are already determined. We can write the expression for the displacement with varying time, i.e., the synthetic seismogram, for the three components of motion. The asymptotic expression of the Fourier transforms of the displacement \( U = (U_x, U_y, U_z) \), at a distance \( r \) from the source, can be written as

\[
U_r = \sum_{m=1}^{\infty} m U_m(r, z, \omega)
\]

The suffixes R and L refer to the quantities associated with Rayleigh and Love modes, respectively. In Equation 4, \( S(\omega) = |S(\omega)| \exp[i \cdot \arg(S(\omega))] \) is the FT of the source time function while \( \chi(h_s, \varphi) \) represents the azimuthal dependence of the excitation factor expressed by [6]:

\[
\chi_R(h_s, \varphi) = d_0 + i \chi_L(h_s, \varphi) = i
\]

with:

\[
\begin{align*}
d_0 &= \frac{1}{2} B(h_s) \sin \lambda \sin 2\delta d_{1R} \\
&= -C(h_s) \sin \lambda \cos 2\delta d_{2R} \\
&= -C(h_s) \cos \lambda \cos \delta d_{3R} \\
&= A(h_s) \sin \lambda \sin \delta d_{4R} \\
&= -\frac{1}{2} A(h_s) \sin \lambda \sin 2\delta \\
d_{1L} &= G(h_s) \cos \lambda \sin \delta d_{2L} \\
&= -G(h_s) \sin \lambda \cos 2\delta d_{3L} \\
&= \frac{1}{2} V(h_s) \sin \lambda \sin 2\delta d_{4L} \\
&= V(h_s) \cos \lambda \sin \delta
\end{align*}
\]

Where \( \varphi \) is the angle between the strike of the fault and the strike-receiver angle that is measured anticlockwise, \( h_s \) is the focal depth, \( \delta \) is the dip angle and \( \lambda \) is the rake angle (see Figure 2). The function of \( h_s \) that appear in Equation 6 depends on the values assumed by the eigenfunctions at the
hypocentre. The details in proving the above equation was done by Panza et al. [2]. The seismogram is computed by summing the time series radiated by the single point-sources with the appropriate time-shifts that are defined by the rupture process. The resultant time series shows great influence on the directivity and the distribution of energy released in time may have influenced on the synthesized ground motion [2, 7].

At each site, the horizontal components (P–SV radial and SH transverse) synthetic seismograms are first computed for a seismic moment of $10^{-7}$ N m and then scaled to the magnitude of the earthquake using the moment–magnitude relation from Kanamori [8]. The finiteness of the source is accounted by scaling the spectrum using the spectral scaling law proposed by Gusev [9] as reported by Aki [10]. For the period between 1 and 2 s, the Gusev spectral fall-off produces higher spectral values than the $\omega^{-2}$ spectral fall-off, and thus guarantees a conservative hazard computation [11].

![Figure 2. Angle conventions used for the seismic source system.](image)

### 3. Theoretical Comparison and Observation of the Response Spectral

Parameters for a seismic source system are origin time, source location, strike, dip, rake, and magnitude. The value is taken from the Pidie Jaya earthquake data with origin time 2016-12-06 22:03:33 UTC or 2016-12-07 05:03:33 local time, at 5.283°N, 96.168°E 13.5 km depth, focal mechanism parameters: Strike=243°, dip=81°, rake=33°, and magnitude = 6.5 Mw. The source information of the Pidie Jaya earthquake is obtained from USGS [12]. The fault that generated the Pidie Jaya earthquake seems to be the branch of the Northern part of the Great Sumatran Fault [13-14]. Figure 3 shows the seismic source location, the focal mechanism on the map, and the available station of BMKG.
Figure 3. The source Location of the M6.5 Pidie Jaya and the seismic stations closed to the epicenter (blue stars).

Figure 4. (a) The synthetic waveform and observed accelerogram from the KCSI station recorded during the Pidie Jaya earthquake (b) related response spectral of the waveform.

The waveform solutions expressed by equation 4 depend on the 1D geological structure \( u_x(z, \omega) \) that form dispersion curve as a function of depth. The accelerogram waveform solution for the KCSI station from the Pidie Jaya earthquake is shown in Figure 4a. Because the waveform depends on the geological structure, the spectrum in the frequency domain is also called response spectra as shown in Figure 4b.

We investigate the response spectra of the observed and computed signals for accelerogram and seismogram from the two stations KCSI and LASI as shown in Figure 5 and 6. The Response Spectral of Acceleration (RSA) and the Response Spectra of Velocity (RSV) analyses are useful for practical engineering purposes.

Figures 5a, 5c, and 5e show the comparison of the accelerogram spectral response for the KCSI of EW, NS, and Z components, respectively. The low-frequency accelerograms have relatively good suitability between synthetic and observation compared to those of high frequency. The seismogram waveforms which could be interpreted as velocities have good suitability not only at low frequency but also for low high frequency as shown in figure 5b, 5d, and 5f. The coda and noise waves which
naturally work at high-frequency acceleration contribute the difference between the synthetic and the observed.

Figure 6 shows the comparison between accelerogram (figure 6a, 6c, and 6e) and seismogram (figure 6b, 6d, and 6f) for the LASI station. For the LASI station, the observed waveform responds spectra is higher than the synthetics seismogram. The location of the LASI station is located in Langsa Aceh at 4.457°N and 97.970°E which is near the coastline on top of the thick sedimentary layer.

![Figure 6](image1)

Figure 6. The synthetic and the observed accelerogram (left) and seismogram (right) respond spectral of the KCSI station which the epicenter distance of 263 km and located at 3.522°N and 97.772°E in the Kutacane City. Geologically, the station is located on the thin sediment of the Paleozoic metamorphic bed rock of Alas formation.
Figure 6. The synthetic and the observed accelerogram (left) and seismogram (right) response spectra of the LASI station with epicenter distance of 219.78 km at 4.457°N and 97.970°E in the Langsa City. Geologically, the station is located on top of the thick sedimentary basin of younger alluvium formation.

4. Conclusions and Suggestion
The respond spectral of seismogram and accelerogram has successfully produced by modal summation technique for thin sedimentary layer located on bedrock as shown at the KCSI station during the Pidie Jaya earthquake. However, the modal summation technique has limitation to produce appropriate respond spectra for thick sedimentary basin because the 1D approach cannot handle the amplification effect of the 2D sedimentary basin. To reach the appropriate respond spectra for civil engineering purposes, the 1D model of summation technique must be hybridized with a 2D finite different method to handle the amplification effects. Research on the synthetic seismogram needs to be compared to
several other methods to increase the precision of the ground response spectral calculation for the geotechnical purposes.

Acknowledgments
I wish to acknowledge the assistance from the Department of Geoscience, Trieste University, and International Centre for Theoretical Physics ICTP. I would like to thank colleagues from the Physics Department, the Geophysics Engineering Department, TDMRC and BMKG for encouragement.

References
[1] Irwandi 2017 Advantages of realistic model based on computational method: NDSHA versus Standard PSHA IOP Conf. Series: Earth and Environmental Science 56 (doi:10.1088/1755-1315/56/1/012007)
[2] Panza G F, Romanelli F and Vaccari F 2001 Seismic wave propagation in laterally heterogeneous anelastic media: theory and applications to seismic zonation Adv. Geophys. 43 1–95
[3] Maruyama T 1963 On the Force Equivalents of Dynamical elastic dislocations with reference to the earthquake Mechanism. Bull. Earthquake Res. Inst. Tokyo Univ. 41 467-86
[4] Burridge R and Knopoff L 1964 Body force equivalents for seismic dislocations Bulletin of the Seismological Society of America 54 1875-88
[5] Kausel E and Schwab F 1973 Contributions to love-wave transformation theory: Earth-flattening transformation for love waves from a point source in a sphere Bulletin of the Seismological Society of America 63 983-93
[6] Ben-Menahem A and Harkrider D G 1964 Radiation patterns of seismic surface waves from buried dipolar point sources in a flat stratified earth Journal of Geophysical Research 69 2605-20
[7] Hassan H M, Romanelli F, Panza G F, ElGabry M N and Magrin A 2017 Update and sensitivity analysis of the neo-deterministic seismic hazard assessment for Egypt Engineering Geology 218 77–89
[8] Kanamori H 1977 The Energy Release in Great Earthquakes Journal of Geophysical Research 82 2981-7
[9] Gusev A A 1983 Descriptive statistical model of earthquake source radiation and its application to an estimation of short-period strong motion Geophysical Journal International 74 787-808
[10] Aki K 1987 Strong Motion Seismology Strong Ground Motion Seismology (Springer Netherlands) 3-39
[11] Vaccari F 1995 LP-Displacement Hazard Evaluation in Italy Proc. 24th General Assembly of the European Seismological Commission 1489-98
[12] USGS (United Stated Geological Survey) 2016 Earthquake Hazard program M 6.5-14km WNW of Reuleuet, Indonesia https://earthquake.usgs.gov/earthquakes/eventpage/us10007ghm#moment-tensor
[13] Tabei T, Kimata F, Ito T, Gunawan E, Tsutsumi H, Ohta Y, Yamashina T, Soeda Y, Ismail N, Nurdin I, Sugiyanto D and Meilano I 2015 Geodetic and Geomorphic Evaluations of Earthquake Generation Potential of the Northern Sumatran Fault, Indonesia GENAH 21-28
[14] Irwandi, Marwan, Muksin, and Fashbir 2017 Applications of the VLF-EM method for rapid Sumatran fault identification in Leuser national park, Aceh AIP Conference Proceedings 1861 030050 (doi: 10.1063/1.4990937)