A brief compendium of correlations and analytical formulae for the thermal field generated by a heat source embedded in porous and purely-conductive media

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Abstract. This work reviews and compares suitable models for the thermal analysis of forced convection over a heat source in a porous medium. The set of available models refers to an infinite medium in which a fluid moves over different three heat source geometries: i.e. the moving infinite line source, the moving finite line source, and the moving infinite cylindrical source. In this perspective, the present work presents a plain and handy compendium of the above-mentioned models for forced external convection in porous media; besides, we propose a dimensionless analysis to figure out the reciprocal deviation among available models, helping the selection of the most suitable one in the specific case of interest. Under specific conditions, the advection term becomes ineffective in terms of heat transfer performances, allowing the use of purely-conductive models. For that reason, available analytical and numerical solutions for purely-conductive media are also reviewed and compared, again, by dimensionless criteria. Therefore, one can choose the simplest solution, with significant benefits in terms of computational effort and interpretation of the results. The main outcomes presented in the paper are: the conditions under which the system can be considered subject to a Darcy flow, the minimal distance beyond which the finite dimension of the heat source does not affect the thermal field, and the critical fluid velocity needed to have a significant contribution of the advection term in the overall heat transfer process.

1. Introduction
The heat transfer over a heat source embedded in a saturated porous medium represents the reference model for many engineering problems in different contexts, e.g., geothermal energy, ground-source heat pump applications, nuclear engineering, storage of nuclear waste material, groundwater flows, solar power collectors, compact heat exchangers, food industries and many others [1–3].

A proper design of the heat source (e.g., the dimension of the heat exchanger apparatus) is strictly related to a sufficiently-accurate modeling of the thermal field evolution in the surrounding medium. Many works have been published on this topic, including both numerical and analytical methods [4–8]. Despite the recent development of computer science, we believe that numerical methods are particularly appropriate for a detailed design and the analysis of a specific case or device, but they do not seem able to provide fast and general indications on the physical phenomena involved in the heat
transfer process. In this perspective, analytical models represent a very suitable tool for a more general analysis of the problem, preliminary sizing, and feasibility analysis, because of their short computational time and flexibility in parametric designs.

This notwithstanding, analytical models are always affected by some simplifying hypotheses to make the problem solvable. In the context of heat sources in porous media, typical assumptions refer to:

- the relevance of the fluid movement in terms of heat transfer performance, i.e. purely-conductive medium vs. saturated porous medium;
- the radial dimension of the heat source, i.e. line vs. cylindrical heat source models;
- temperature gradients in the axial direction of the heat source, i.e. infinite vs. finite axial extension.

We classify the solutions proposed in literature as shown in Table 1, in accordance with the above-mentioned hypotheses.

| Geometry                      | Purely-conductive media | Porous media\(^a\) |
|-------------------------------|-------------------------|---------------------|
| Infinite axial extension      | ILS                     | MILS                |
|                               | Infinite line source    | Moving infinite line source |
|                               | FLS                     | MFLS                |
|                               | Finite line source      | Moving finite line source |
| Finite axial extension        | ICS                     | MICS                |
|                               | Infinite cylindrical source | Moving infinite cylindrical source |
|                               | FCS                     | MFCS                |
|                               | Finite cylindrical source | Moving finite cylindrical source (not yet developed) |

\(^a\)Generally, the porous medium is considered saturated.

Purely-conductive models consist of the limit solution of porous media when the fluid velocity is sufficiently low. Similarly, line models represent the limit of cylindrical solutions when the radial dimension of the heat source is negligible with respect to the space scales of the problem. Finally, the deviation between finite and infinite axial models is mainly related to the time scales of interest, resulting practically equivalent at short times [9,10].

These preliminary considerations suggest that specific conditions exist, in terms of time and space scales, at which the mentioned models result practically equivalent to each other. Therefore, one should be able to readily choose the most appropriate model for the specific analysis, in order to achieve sound and interpretable results with the least amount of computational effort.

To this aim, we started a systematic work aimed at developing a handy compendium of formulae for the analysis of the thermal field generated by a heat source embedded in purely-conductive and porous media. Our goal consists of developing analytical correlations and maps of the solutions, together with some quantitative, general and dimensionless criteria to assess when one model is practically equivalent to another one. Purely-conductive models have already been reviewed and discussed in [9]. In this work, we introduce the discussion on porous media, addressing the main features of the two analytical models currently available in literature: the MILS and the MFLS. A numerical model for the MICS has been recently developed by the authors in [11], while the analysis of the MFCS model is still ongoing.

In section 2, we will present the geometries and the constitutive equations of the discussed problems, together with a dimensional analysis aimed at identifying the dimensionless groups governing the solutions. In section 3, we will present the compendium of formulae and maps for the above-
mentioned porous models. Finally, in section 4, we briefly recall purely-conductive models that can be used as reference when the advection effects become negligible (see, for instance, Fig. 1).

2. Momentum and energy equations

The discussed models (see Table 1) differ from each other in terms of geometry and boundary conditions. However, the mathematical structure and the constitutive equations of the problem are always the same. The momentum equation consists of the classical Darcy’s formulation for a porous medium, namely:

\[ u = -\frac{K}{\mu} \nabla p \]  

We recall that Eq. 1 is valid when the so-called *local pore Reynolds number*, \(Re_K\), is lower than one [12].

The formulation of the energy balance (Eq. 2) is based upon the *local thermal equilibrium* hypothesis, i.e. both solid and fluid phases have the same local temperature, \(\theta\) [12].

\[ \frac{\partial \theta}{\partial t} + u \cdot \nabla \theta = \alpha_w \nabla^2 \theta \]  

where:

\[ \alpha_w = \frac{\lambda_w}{(\rho c)_w} \quad u = \frac{(\rho c)_f}{(\rho c)_w} u \]

\[ \lambda_w = (1 - \varepsilon_p) \lambda_s + \varepsilon_p \lambda_f \quad (\rho c)_w = (1 - \varepsilon_p)(\rho c)_s + \varepsilon_p (\rho c)_f \]

and \(u\) is evaluated through Eq. 1. We can make the following remarks:

- Eq. 2 is equivalent to Fourier’s law when \(u\) is zero;
- infinite-axial-extension models (e.g., MILS) neglect the temperature gradient over the z-axis, therefore the problem becomes a 2D problem in the \(x-y\) (or \(r-\phi\)) plane;
- the boundary conditions needed to set the thermal problem depend on the specific geometry; Table 2 shows the full set of equations for all the models included in Table 1;
- the porosity \(\varepsilon_p\) is the main parameter affecting the effective thermo-physical properties of the porous medium; all the parameters are volume-average quantities depending on \(\varepsilon_p\).

| Geometry                  | Line models                                      | Cylindrical models                                     |
|---------------------------|--------------------------------------------------|--------------------------------------------------------|
| B.C.:                     | \[ \theta (r \to \infty, \phi, t) = \theta_s \]  | \[ \theta (r \to \infty, \phi, t) = \theta_s \]  |
| In infinite axial extension | \[ (2\pi r) \frac{\partial \theta}{\partial r} \bigg|_{r=r} = q_s \quad r \to 0 \] | \[ (2\pi r_s) \frac{\partial \theta}{\partial r} \bigg|_{r=r_s} = q_s \] |
| I.C.:                     | \[ \theta (r, \phi, t = 0) = \theta_s \]  | \[ \theta (r, \phi, t = 0) = \theta_s \]  |
| Finite axial extension    | B.C.:                                            | B.C.:                                                  |
2.1 Dimensional analysis

According to the Buckingham theorem [13], the thermal problem can be rewritten as a function of 6 dimensionless parameters given by the combination of 10 physical variables: the temperature variation, $\theta - \theta_\infty$, the time, $t$, the radial coordinate, $r$, the angular coordinate, $\phi$, the effective fluid velocity, $u_\infty$, the effective thermal conductivity, $\lambda_\infty$, the effective thermal diffusivity, $\alpha_\infty$, the source radius, $r_s$, the source depth, $H$, and the source magnitude, $q_s$.

The resulting dimensionless quantities are: the dimensionless temperature, $\Theta$, the Fourier number, $F_o$, the Péclet number, $P_e$, the dimensionless radial coordinate, $R$, the angular coordinate, $\Phi$, the dimensionless axial coordinate, $Z$. The specific length to define the just-mentioned groups depends on the specific reference model, as well as the actual number of groups needed to evaluate the dimensionless temperature $\Theta$.

According to [14], it is possible to define the overall characteristic length and time of the advection-diffusion problems and the corresponding dimensionless coordinate and time as:

$$L = \frac{\alpha}{u_\infty}, \quad T = \frac{L}{u_\infty}, \quad X = \frac{s}{L} = P_e, \quad \tau = \frac{t}{T} = F_o P_e \frac{r_s}{n}$$

However, as we will show in Table 3, sometimes it is more convenient to refer to the Péclet and Fourier numbers distinctly, especially when we want to compare porous models with purely-conductive ones.

3. Compendium of analytical solutions for a heat source in Darcy flow

Table 3 shows the analytical solutions of the three models already available in literature, i.e. the MILS, the MFLS, and the MICS. The MFCS model will be presented in next works. Table 3 also shows some dimensionless criteria to evaluate when a given model is practically equivalent to another one. In this work, the term “practically equivalent” refers to a relative deviation of 5% in terms of $\Theta$.

**Table 3.** Available solutions of heat sources in porous media.

| Model References | Description                                                                 | Solution                                                                 |
|------------------|-----------------------------------------------------------------------------|--------------------------------------------------------------------------|
| MILS [6,14,15]   | Infinite line heat source subject to a Darcy crossflow. The source is located in $(0,0)$             | $\Theta(\phi, P_e, F_o) = \frac{1}{4\pi} \exp \left( \frac{P_e}{2} \cos \phi \right) \Gamma \left( 0, \frac{1}{4 F_o} \frac{P_e}{n} \right)$  |
|                  |                                                                             | $\Theta = \frac{(\theta - \theta_\infty) \lambda_\infty}{q_s} - \frac{u_\infty}{\alpha_\infty} F_o \frac{\alpha_\infty}{P_e} F_o \frac{r_s}{n}$ |
|                  |                                                                             | $\Gamma(u, x, h) = \int_u^h \beta^{r+1} \exp \left( - \beta \frac{h}{\beta} \right) d\beta$ |
Steady-state solution (i.e. $\tau > \tau_c$)

$$\tau_c = 6.14 Pe_c^{-0.42}, \quad 10^{-1} \leq Pe_c \leq 10$$

$$\Theta_s (\phi, Pe_c) = \frac{1}{2\pi} \exp \left( \frac{Pe_c}{2} \cos \phi \right) K_1 \left( \frac{Pe_c}{2} \right)$$

Equivalence with other models
- MILS is practically equivalent to the ILS when:

$$F_o < F_{o,s}$$ (see Fig. 1)

MFLS [16] Finite line source subject to a Darcy crossflow. The source is located in $(0, 0)$.

$$\phi (R, Z, \phi, F_{o,s}, Pe_c) = \frac{1}{2\pi} \exp \left( \frac{Pe_c}{2} R \cos \phi \right) \times$$

$$\left[ \int_0^1 f (R, Z, Z', F_{o,s}, Pe_c) dZ' - \int_0^1 f (R, Z, Z', F_{o,s}, Pe_c) dZ' \right]$$

$$f (R, Z, Z', F_{o,s}, Pe_c) = \frac{1}{4R} \left\{ 1 + \exp \left( \frac{Pe_c}{2} R \right) \operatorname{erfc} \left( \frac{R + Pe_c F_{o,s}}{2\sqrt{F_{o,s}}} \right) \right\}$$

$$\Theta = \left( \frac{\theta - \theta_o}{q_s} \right) P_{e,s} = \frac{u_s H}{a_s} F_{o,s} \frac{a_t}{H} R - \frac{r}{H}$$

$$R - \sqrt{R^2 + (Z-Z')^2}$$

Steady-state solution (i.e. $\tau > \tau_c$)

$\tau_c$ expression is under development

$$\Theta_s (R, Z, \phi, Pe_c) = \frac{1}{4\pi} \exp \left( \frac{Pe_c}{2} R \cos \phi \right) \times$$

$$\left[ \int_0^1 \exp \left( \frac{Pe_c}{2} R \right) dZ' - \int_0^1 \exp \left( \frac{Pe_c}{2} R \right) dZ' \right]$$

MICS [11] Infinite cylindrical heat source subject to a Darcy crossflow. The centre of the source is located in $(0, 0)$.

$$\tilde{\Theta}_{s,s} (Pe_{c,s}, F_{o,s}) = 10^{-8} \left[ \frac{p_c \tilde{t} + p_o \tilde{t} + p_s}{\tilde{t} + q_c \tilde{t} + q_s} \right]$$

$\tilde{t}$ is $\theta$-averaged temperature of the heat source, $\tau = \tau_c$

The values of the coefficients $p$ and $q$ are shown in [11]

$$\tilde{\Theta}_{s,s} (Pe_{c,s}, F_{o,s}) = \left( \frac{\theta - \theta_o}{q_s} \right) Pe_{c,s} = \frac{u_s H}{a_s} F_{o,s} \frac{a_t}{H} \tau = F_{o,s} Pe_{c,s}^2$$

Steady-state solution (i.e. $\tau > \tau_c$)

$$\tau_c = 1.99 Pe_{c,s} + 3.56, \quad 5.0 \times 10^{-1} \leq Pe_{c,s} \leq 10^3$$

$$\Theta_{s,s} (Pe_{c,s}) = \left\{ \begin{array}{ll}
0.291 Pe_{c,s}^{-0.52}, & Pe_{c,s} \leq 0.1 \\
0.1745 Pe_{c,s}^{-0.33}, & Pe_{c,s} > 0.1
\end{array} \right.$$
Figure 1. MILS model: \( F_{o_{r,c}} \) as a function of Péclet number, \( P_{e_{r}} \), and angular coordinate, \( \phi \).

4. Purely-conductive models
In this section, we briefly recall the analytical expressions of the ILS, ICS, FLS, and FCS solutions. The reader can refer to [9] for a comprehensive analysis of the just-mentioned models.

| Model | References | Description | Solution |
|-------|------------|-------------|----------|
| ILS   | [6,9]      | Infinite line heat source in a semi-infinite medium. The source is located at \((0,0)\). | \[ \Theta(F_{o_{r}}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\exp(-\beta^2)}{\beta} d\beta = \frac{1}{4\pi} E_{i} \left( \frac{1}{4F_{o_{r}}} \right) \] |

\( E_{i}(x) \) is the exponential integral function.

**Equivalence with other models**
- ILS is practically equivalent to the MILS when:
  \[ F_{o_{r}} < F_{o_{r_{c}}} \] (see Fig. 1)
- ILS is practically equivalent to the ICS when:
  \[ F_{o_{r}} \geq 3.1 (r/r_{s}) + 7.1 \]
- ILS is practically equivalent to the FLS when:
  \[ F_{o_{r}} \leq 3.0 \times 10^{-5} R^{0.5} \]

\( r / r_{s} > \begin{cases} 10, & P_{e_{r}} < 1, \forall t \\ 5, & P_{e_{r}} > 1, \forall t \end{cases} \)

- MICS is practically equivalent to the ICS in the range
  \[ \tau_{d_{r_{1}}} \leq \tau \leq \tau_{d_{r_{2}}} \]
  where:
  \[ \tau_{d_{r_{1}}} = 3.6 \times 10^{-1} P_{e_{r}^{2.12}} \]
  \[ \tau_{d_{r_{2}}} = \begin{cases} 0.69 P_{e_{r}}^{0.21}, & P_{e_{r}} < 0.5 \\ 0.72 P_{e_{r}} + 0.88, & P_{e_{r}} \geq 0.5 \end{cases} \]
ICS [9,10,17] Infinite cylindrical heat source in a semi-infinite medium. The centre of the source is located in \((0,0)\).

\[
\Theta \left( F_0 \omega , R, Z \right) - \frac{1}{4 \pi} \int \left[ \frac{1}{d/H} \text{erfc} \left( \frac{d/H}{2 \sqrt{F_0 \omega}} \right) - \frac{1}{d'/H} \text{erfc} \left( \frac{d'/H}{2 \sqrt{F_0 \omega}} \right) \right] dZ ,
\]

where:

\[
d / H = \sqrt{R^2 + (Z - Z')^2}, \quad d' / H = \sqrt{R^2 + (Z + Z')^2}.
\]

Equivalence with other models
- ICS is practically equivalent to the ILS when:
\[
F_0 \omega \approx 3.1 (r / r_s) + 7.1.
\]
- ICS is practically equivalent to the FCS when:
\[
F_0 \omega \approx 10^{-1}.
\]

FLS [6,9,18] Finite line heat source in a semi-infinite medium. The source is located in \((0,0)\).

\[
\Theta \left( F_0 \omega , R, Z \right) - \frac{1}{4 \pi} \int \left[ \frac{1}{d/H} \text{erfc} \left( \frac{d/H}{2 \sqrt{F_0 \omega}} \right) - \frac{1}{d'/H} \text{erfc} \left( \frac{d'/H}{2 \sqrt{F_0 \omega}} \right) \right] dZ ,
\]

where:

\[
derf \left( 4 \pi \right) = -0.36 \log_{10} R - 0.13 \quad 10^{-1} \leq R \leq 10^{-1}
\]

Axial-average temperature

\[
\Theta \left( F_0 \omega , R \right) - \int_0^1 \Theta \left( F_0 \omega , R, Z \right) dZ = \frac{1}{4 \pi} \int \left[ \frac{1}{d/H} \text{erf} \left( \frac{d/H}{2 \sqrt{F_0 \omega}} \right) \right] dZ.
\]

\[
\text{erf} (x) = \int e^{x^2} \text{d}x = \frac{1}{\sqrt{\pi}} \left[ 1 - \exp \left( -x^2 \right) \right]
\]

Steady-state solution (i.e. \(F_0 \omega > F_0 \omega , s\))

\[
F_0 \omega , s = 7.83 R + 3.84 R^2 + 10 \quad 10^{-1} \leq R \leq 10^{-1}
\]

\[
\Theta , s \left( R \right) = -0.36 \log_{10} R - 0.13
\]

\[
\Theta , s \left( R, Z \right) = \frac{1}{4 \pi} \left[ \frac{1}{d/H} - \frac{1}{d'/H} \right] dZ
\]

Equivalence with other models
- FLS is practically equivalent to the ILS when:
\[
F_0 \omega \approx 3.0 \times 10^{-1} (r / r_s)^{3/4}, \quad 10^{-1} \leq R \leq 10^{-1}
\]
- FLS is practically equivalent to the FCS when:
\[
F_0 \omega \approx 6.3 \times 10^{-1} (r / r_s)^{3/4} + 5.4 \times 10^{-1} (r / r_s) + 17.8, \quad 1 \leq r / r_s \leq 100
\]

FCS [9,19] Finite cylindrical heat source in a semi-infinite medium. The axis of the source is located in \(r = 0\).

Analytical solution is not available. The reader can refer to the dimensionless maps in [9].

Steady-state solution (i.e. \(F_0 \omega > F_0 \omega , s\))
5. Conclusions

In this work, we reviewed the available analytical solutions for the problem of a heat source embedded in a porous medium, i.e. MILS, MFLS and MICS models. Specifically, we provided a handy compendium of formulae to evaluate the thermal field evolution around a constant heat source at any space and time. The assumptions and the characteristics of each model were presented and discussed. The main references were selected for each model, to provide a fast bibliographic tool to the reader. Purely-conductive models were also reviewed and presented for the same bibliographic purposes.

Together with the general solution of the thermal field, we presented some dimensionless expressions to evaluate the dimensionless time needed to reach the steady-state condition and the corresponding temperature values. We investigated the conditions under which the fluid movement does not produce significant effects in terms of heat transfer. In other words, we presented some dimensionless criteria to evaluate the threshold Péclet numbers at which MILS and MICS are practically equivalent to the ILS and ICS, respectively.

We suggest using the MICS solution to evaluate the temperature of the heat source in the presence of a significant fluid movement. In fact, the MILS model is not able to account for the actual velocity field in proximity of the source, thus it overestimates the heat exchange between the source and the fluid (the reader can refer to [11] for further details). On the other hand, the MILS model can be used to evaluate the temperature at a sufficient distance from the source, namely when the actual radial dimension of the source does not affect the velocity field.

We believe that the present compendium, with possible additional developments on the MFCS model, represents a fast and useful tool for researchers and professional operators, helping the design and the performance analysis of such heat transfer systems, especially when coupled to models of engineering devices – such as ground-coupled borehole heat exchangers [20], energy piles [21], and heat pump units [22] – in lifetime simulations of the overall system [23].

Nomenclature

- $H$: heat source depth [m]
- $K$: permeability [m$^{-2}$]
- $L = \alpha \cdot \frac{u}{\alpha}$: characteristic length of advection-diffusion problems [m]
- $T = L / u$: characteristic time of advection-diffusion problems [s]
- $c$: specific heat [J kg$^{-1}$ K$^{-1}$]
- $p$: pressure [Pa]
- $q$: linear heat source strength [J s$^{-1}$ m$^{-1}$]
- $r$: radial coordinate [m]
\( r_s \) \quad \text{heat source radius [m]}
\( t \) \quad \text{time [s]}
\( u \) \quad \text{Darcy velocity or seepage velocity vector [m s}^{-1}]
\( u \) \quad \text{magnitude of the Darcy velocity or seepage velocity [m s}^{-1}]
\( x \) \quad \text{coordinate [m]}
\( y \) \quad \text{coordinate [m]}

**Greek letters**

\( \Gamma \) \quad \text{generalized incomplete gamma function}
\( \phi \) \quad \text{angular coordinate [rad]}
\( \alpha \) \quad \text{thermal diffusivity [m}^2\text{s}^{-1}]
\( \beta \) \quad \text{auxiliary variable [-]}
\( \epsilon_r \) \quad \text{porosity [-]}
\( \theta \) \quad \text{temperature [K]}
\( \lambda \) \quad \text{thermal conductivity [W m}^{-1}\text{K}^{-1}]
\( \mu \) \quad \text{dynamic viscosity [Pa s]}
\( \rho \) \quad \text{density [kg m}^3]\)

**Dimensionless numbers and groups**

\[ \Theta = \left( \frac{\theta - \theta_\infty}{\theta_b - \theta_\infty} \right) \lambda \]
\( \text{dimensionless temperature} \)
\[ \tau = \frac{t}{T} = F_o \cdot P_e \cdot \epsilon_r \]
\( \text{dimensionless time} \)
\[ F_o = \frac{\alpha t}{x^3} \]
\( \text{generic Fourier number referred to the length } x. \)
\[ P_e = \frac{x}{L} = \frac{u x}{\alpha} \]
\( \text{generic Péclet number referred to the length } x \)
\[ R = \frac{r}{H} \]
\( \text{dimensionless radial coordinate} \)
\[ R e_x = \frac{\rho \cdot u \cdot K^{1.5}}{\mu} \]
\( \text{local pore Reynolds number} \)
\[ Z = \frac{z}{H} \]
\( \text{dimensionless axial coordinate} \)

**Subscripts**

\( \infty \) \quad \text{undisturbed condition}
\( b \) \quad \text{heat source}
\( c \) \quad \text{threshold value from a purely-conductive to a Darcy’s regime}
\( f \) \quad \text{liquid phase}
\( m \) \quad \text{volume-averaged/effective quantity}
\( s \) \quad \text{solid phase or steady-state quantity}

**Acronyms**

ILS \quad \text{Infinite line source model}
ICS \quad \text{Infinite cylindrical source model}
FLS   Finite line source model
FCS   Finite cylindrical source model
MILS  Moving infinite line source model
MICS  Moving infinite cylindrical source model
MFLS  Moving finite line source model
MFCS  Moving finite cylindrical source model

References
[1] Nield D A and Bejan A 2006 Convection in Porous Media (New York, NY: Springer Science+Business Media, Inc)
[2] Thevenin J 1995 Int. Commun. Heat Mass Transf. 22 507–16
[3] Grassi W, Conti P, Schito E and Testi D 2015 J. Phys. Conf. Ser. 655 12003
[4] Layeghi M and Nouri-Borujerdi A 2004 J. Porous Media 7 239–47
[5] Zeng Z, Brown J M B and Vardy A E 1997 Heat Mass Transf. 33 41–9
[6] Carslaw H S and Jeager J C 1959 Conduction of heat in solids (Oxford, UK: Clarendon Press)
[7] Layeghi M and Nouri-Borujerdi A 2006 Int. J. Comput. Methods Eng. Sci. Mech. 7 323–9
[8] Kimura S 1990 J. Geotherm. Res. Soc. Japan 12 79–90
[9] Conti P 2016 Energies 9 890
[10] Philippe M, Bernier M and Marchio D 2009 Geothermics 38 407–13
[11] Conti P, Testi D and Grassi W 2017 Int. J. Heat Mass Tranfer 117C 154–66
[12] Bejan A 2003 Porous Media Heat Transfer Handbook ed A Bejan and A D Kraus (Hoboken, NJ: John Wiley & Sons Incorporated) chapter 15 pp 1131–80
[13] Buckingham E 1915 Nature 96 396–7
[14] Diao N, Li Q and Fang Z 2004 Int. J. Therm. Sci. 43 1203–11
[15] Sutton M G, Nutter D W and Couvillion R J 2003 J. Energy Resour. Technol. 125 183–9
[16] Molina-Giraldo N, Blum P, Zhu K, Bayer P and Fang Z 2011 Int. J. Therm. Sci. 50 2506–13
[17] Ingersoll L R, Zobel O J and Ingersoll A C 1954 Heat conduction with engineering, geological and other applications (New York, NY: McGraw-Hill)
[18] Claesson J and Javed S 2011 ASHRAE Trans. 117 279–88
[19] Priarone A and Fossa M 2016 Appl. Therm. Eng. 103 934–44
[20] Conti P, Testi D and Grassi W 2016 Appl. Therm. Eng. 106 1257–67
[21] Batini N, Rotta Loria A F, Conti P, Testi D, Grassi W and Laloui L 2015 Appl. Therm. Eng. 86 199–213
[22] Casarosa C, Conti P, Franco A, Grassi W and Testi D 2014 J. Phys. Conf. Ser. 547 12006
[23] Conti P 2015 Proc. 5th Int. Youth Conf. on Energy (Pisa, IT) (IEEE) pp 8