A perspective on gear meshing quality based on transmission error analysis

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Abstract. Gears are one of the most important elements in power transmission systems. Their use is present in many industries due to their various advantages. Durability, constant transmission ratio, reduced size, high efficiency, suitability for a wide range of powers are some of the benefits. But gears also feature a number of drawbacks like vibration of the gear meshing system generating an undesirable noise. The main source of such a noise is the transmission error that result from misalignment of the gear, tooth profile errors and tooth deflections. This article is a state of the art on the theory of transmission error components that influences gear meshing. The errors of static transmission in gears may have several causes. One originates in the function of the gear mechanism which transmits the rotation and torque between axes and generates forces on the teeth, thus modifying their geometry due to elastic deformation. Another undesirable consequence that would be present even if tooth deflections were insensitive is caused by manufacturing errors such as profile, eccentricity, pitch or even assembling errors. One of the best known methods of reducing transmission error of a gear pair is the profile correction. This paper describes the main differences between static and dynamic transmission errors. It will be shown the most common methods of detecting the transmission error using analytical methodology, optimization algorithms, hybrid numerical/ Finite-Element Analysis and experimental researches in order to establish the dynamic behaviour of the gear pairs under different operating conditions.

1. Introduction

Gears are one of the most important elements in power transmission systems. Their use is present in many industries due to a number of advantages. Durability, constant transmission ratio, reduced size, high efficiency, suitability for a wide range of powers are some of them. However, gears may present drawbacks as well, such as the need of high precision machining and assembly, the use of limited transmission ratio and vibration of the gear meshing system which generates an undesired noise especially with high speeds. The main source of this noise is the transmission error that results from the gear misalignment, tooth profile errors, tooth deflections and gear dynamics.

The present paper represents a review on the theory of transmission error components that influences gear meshing.

The most common profile of the gear teeth is the involute curve. This type of profile was first used by Euler and it is the most widespread, being used for cylindrical spur, helical or conical gears. Another type is the cycloid used in the Wildhaber-Novikov gears. Utterly different from involute gears, cycloid are highly sensitive to axial distance variation and do not allow overloads.

Environmental noise has always been a problem engineers had to face and solve. Walker [1] was the first in trying to improve the quality of the power transmission gears by studying changes in the teeth profile of spur gears in order to reduce the noise level and vibration. Gear noise is represented by
high amplitude acoustic signals and it has three major components: gear whine, gear rattle and gear clatter. The source of gear whine is considered to be the gear tooth mesh and originates in loaded gears. A steady harmonic displacement excitation causes the whine feature which is determined by faults in current tooth place relative to the ideal position [2]. It can show a non-linear action mostly because of contact loss that is represented by the effect of the developed mesh stiffness at higher loads [3]. Contact noises of gear pairs under load generate the whine noises [4]. Gear whine is caused by tooth stiffness variation of the meshing gear. The velocity variation influences the frequency of the gear whine noise, which is considered a pure tonal noise [5]. Usually loaded gears generate a higher noise level than unloaded gears. In the gear mesh process, the static transmission error is considered the main cause for the occurrence of the whining noise [6, 7, 8]. Gear rattling occurs when unloaded gearwheels are affected by torsional oscillation [4]. Gear rattling is usually induced by gear backlash. It is formed when gear mesh is slightly loaded and is believed to be a noise, vibration, harshness matter [9]. Many authors considered that gear rattle appears when the initial torque of the unloaded gear surpasses the drag torque [10]. The source of the gear rattle is based on the separation and impact of the gear teeth. The level of noise becomes more and more powerful as the number of teeth and module increases. Gear clattering is also produced by torsional vibration, but unlike the gear rattle, is a result of loaded gears. According to Y. Miura and S. Nakamura [11], the crankshaft and camshaft of gear teeth detach at the moment when the maximum phase difference produced by gear meshing error fit with the edge of the engine revolution vibration. The action of the impact takes place when the relative velocity is too high. Gears with identical characteristics can reach a level of noise of 10dB at high rotational speeds. It is concluded that noise can be reduced by optimizing the macro geometry of the gear teeth.

The concept of transmission error was first introduced by Harris [12] in 1958, and was implemented on profile changes by creating a special chart named Harris map [13]. A number of authors [14] make a direct link between gearing noise and transmission error. From a theoretical point of view, gears have perfect involute teeth flanks and infinite stiffness and the rotation of the input shaft would represent a known function of the output shaft with the gear ratio. In industry it is impossible to obtain gears with no transmission error, mainly because errors occur during manufacturing and assembly processes. Transmission error (TE) [7, 8, 15] can be described as the difference between the actual position of the output gear and the theoretical one if the gears were perfectly conjugate, and it represents an angular displacement. It arises when gears tooth flanks collide and it depends on the parameters of the elastic deformation, gear tooth geometry faults and inaccurate assembling. Other less important parameters are the geometric changes caused by high operating speed and temperature modification, improper lubrication and high speed dynamic instability [16]. TE is caused by deliberate teeth profile modification and manufacturing errors like gear eccentricity and tooth surfaces faults. Tooth surface errors may be caused by wear while eccentricity may be caused by manufacturing process, shaft misalignment or defects in the bearings [17]. TE can be divided into two main components: Static Transmission Error (STE) and Dynamic Transmission Error (DTE). Some authors also include other two elements: Manufacturing Transmission Error (MTE) and Kinematic Transmission Error (KTE). Generally the value of the transmission error is low, but even under these conditions it produces noise at a frequency close to the resonance frequency of the shafts and the gear housing which generates an increased system noise level. Depending on the author, the TE can be represented as angular rotation measured in radians (rad), angular displacement measured in arc sec or µrad, or is transformed into a linear displacement measured in µm [5]. Gregory, Harris and Munro [15] describe that gear transmissions having ideal involute flanks may present TE on intervals. This may occur due to a change in mesh stiffness. They also state that TE can be split up into a constant element and a changeable element. The constant element influences the tooth meshing frequency and the harmonics, while the variable element takes effect on the rest of the frequency spectrum.

2. Classes and particulars of transmission error

2.1. Manufacturing Transmission Error

In his work Bonori [18] considers manufacturing errors as part of one-degree-of-freedom paradigm having backlash and changeable stiffness and also emphasizes the importance of tooth profile errors in
the fabrication process. He presented an iterative routine to provide an incidental profile within the known tolerances. Because the profile geometry could be changed, it was used a special chart (K-chart) to maintain the tolerance of the tooth profile. Existing tolerances must be taken into account in order to apply the conditions of micro and macro geometry of the gear and also to influence gear mesh stiffness and TE [19]. Robust optimization methods are used to determine the TE response function using the most suitable gear design parameters. Nevertheless tolerances create a changeable behavior. Manufacturing errors (ME) produce a significant vibration growth at all frequencies but do not influence the dynamic behavior. This behavior is highlighted at low speeds.

High contact ratio gears are usually used to reduce vibration. The macro-geometric characteristics of gears are: number of teeth, pressure angle, diameters, backlash and clearance [20]. Deliberate elimination of material from gear tooth flank that alter the ideal involute geometry represent the micro-geometric changes. Macro-geometry parameters are an effect of manufacturing tools, while micro-geometry characteristics refers to the rectification of resources. Manufacturing and assembly errors have a strong impact on mesh stiffness variation and STE. Manufacturing error parameters are profile, longitudinal and parallelism errors. Likewise, dispersion is appreciated to be constant above a range according to the tolerance class [8].

When low contact ratio gears are used, the connection of a pair of tooth is made throughout the line of contact considering the active profile. Thus, gear mesh stiffness and static transmission error present considerable fluctuations. Some authors also considered that manufacturing errors represent the same feature as geometrical errors [21].

Machining errors are frequently established as profile, pitch or helix deviations inside of a two dimensional coordinate system. Nevertheless, machining errors can also be considered as 3D shape deflection of the machined tooth area by the theoretical form over the line of action [22].

Changes of the gear tooth geometry have effect on the position of the output gear relative to the input gear. MTE is a direct result of the manufacturing process and is related to a single gear, not to the gear mesh. MTE is calculated with no load at all or at a light load and it is a helpful parameter to verify the precision of a production line. A particular case of MTE is the Kinematic Transmission Error (KTE) which also takes into consideration current roughness and ruggedness on the tooth face [5]. According to J. Astoul et al. [23] article on spiral bevel gear, KTE is a class of TE without load.

2.2. Static Transmission Error (STE)

STE is considered to be part of the motion equation of the gear pairs as a displacement-type excitation and it is taken into account as a transmission error established in quasi-static regime. The expression of the function is imposed by the mesh stiffness and the geometry of the tooth profile, elastic deflections, misalignments and manufacturing errors.

STE is a periodic function, according to the shifting characteristic of the gear mesh stiffness, at a mesh and shaft rotational frequency period. It also represents the source of excitation inside gear dynamics. First harmonic mesh frequency is considered to be the main element of the noise amplification [24].

STE is influenced by the size of the gear torque and sometimes is considered a loaded transmission error (LTE) [25]. It is deeply altered by the profile reliefs and for reduced loading torque, at certain intervals of gear meshing, may present an absence of teeth contact. Micro-geometry changes are made in order to reduce the peak-to-peak static transmission error (PPSTE) value [26]. Usually, profile correction has a small impact on the average value of STE but it strongly influences the peak-to-peak value [27].

Frequency research of STE can be carried out through Fourier series technique [28]. When STE is subject to harmonic analysis, it can be split into two main elements: the mean component and a random component. The mean component includes the load submissive element produced by elastic tooth deformations and a medium deflection element of the tooth surface of the meshing gear. The random component is formed by incidental components of meshing gears [29]. The researches performed by Ulrich Kissling [30] establish that peak-to-peak transmission error is proportional with the torque and its value reduces throughout higher transverse contact ratio.
2.3. Dynamic Transmission Error (DTE)
Dynamic transmission error is generated in the transmission systems that take into account both weight and fluctuating stiffness. STE represents the main parameter that stimulates DTE. Gears in rotation and having weight produce inertial forces that induce dynamic mesh forces inside the transmission system.

DTE is influenced by speed and it can be considered as a function consisting of multiplication of the static transmission error function and a transfer function [5].

According to Hotait and Kahraman [31] DTE is determined by the transmission error function throughout the action line of the meshing gear and it can be represented as a sum between the base radius of the drive gear and base radius of the driven gear multiplied by their corresponding angular displacement. It is concluded that there is a linear connection between the dynamic factor and dynamic transmission error.

Considering the elastic deflection and other errors, in order to achieve a soft movement when sufficient force is sent to the gear mesh, profile modification has to be made [32]. In Song He et al. article [33] DTE was studied through the impact of profile changes and considering the case of sliding friction. Applying numerical methods it has been determined that DTE is not affected by profile modification when frictional case is considered. Nevertheless, an extra vibration is inserted when the contact teeth cross over the pitch point. In the absence of sliding friction and considering low speed, DTE almost has the same value as STE. They have reached the conclusion that DTE is determined by the gear geometry, mesh stiffness and bearing stiffness.

DTE is also influenced by eccentricity, backlash and tooth elastic deformation. Eccentricity is a result of assembly errors and it can be defined as a displacement between the center of the gear and the center of rotation. Such class of error determines a change towards the line of action that influences the place where the tooth pair starts and finishes the contact [34]. It is considered that shaft assembly is equable and the mass center corresponds with the center of rotation. Sufficient force to generate noise into the transmission system is produced by a small eccentricity [35]. No load transmission error (NLTE) represents the type of transmission error produced by gear eccentricity. The backlash element can be divided into two main components: steady backlash and time-varying backlash. It is considered that periodic time-varying backlash better depicts the main effect of the gear eccentricity. The amplitude of DTE increases with the amplification of the loaded torque. For a gear mesh with eccentricity, DTE consists of NLTE caused by eccentricity and the transmission error generated by tooth elastic deformation. Considering that load is varied with the rotation frequency, the high frequency element of DTE modifies into the shape of mesh stiffness and the low frequency element takes the image of a harmonic function. When teeth are no longer in contact, the DTE shape tends to become stable [36].

The high frequency element of TE is particularly related to the dynamic load when considering the low speed case. The shaft frequency element of TE is especially linked to the dynamic load [37].

The study of the nonlinear behaviour created by backlash is carried out on samples with few degrees-of-freedom [38]. In order to limit the maximum period of time used for calculation, a spectral and iterative procedure was established [39].

In M. Bozca [40] article the equation of TE is achieved using torsional oscillation equations through state space equation. Larger dynamic forces are caused by higher dynamic transmission error amplitudes. DTE raise noise and vibration behavior. Bozca also noticed a close relationship between TE and gear rattle. In his research he concludes that a 95% decrease in the TE value corresponds to a 12% decrease in the rattle noise level.

3. Mathematical model
Many mathematical theories have been developed in order to calculate the transmission error. An important mathematical model for gear dynamics has been created by Özgüven and Houser [41].

Transmission error can be regarded as a minimized objective function. According to Marco Barbieri et al [42], the dynamics equation of motion has the following form:

\[ m_i \ddot{x}(t) + c(\dot{x}(t) - \dot{e}(t)) + k(t)f_1(x(t) - e(t)) + k_{bs}(t)f_2(x(t) - e(t)) = T_g(t) \] (1)
where \( m_e \) is the equivalent mass, \( c \) is constant mesh damping, \( T_g \) is the equivalent applied load, \( k(t) \) is the time varying mesh stiffness function in direct contact and \( k_{bs}(t) \) represents time varying mesh stiffness function in backside contact. Function \( x(t) \) represents the DTE. \( f(t) \) represents the backlash function and \( e(t) \) is manufacturing error. The expression \( x(t) - e(t) \) is the current TE.

The TE equation of a gear pair throughout the line of action can be written as [43]:

\[
TE(t) = r_p \theta_p(t) + r_g \theta_g(t)
\]

where \( \theta_p, \theta_g \) are pinion and gear rotational displacements and \( r_p, r_g \) represent the base circle radii.

Transmission error function is periodic with a period \( 2\pi/N_1 \) and angular position of the input gear, where \( N_1 \) represents the number of teeth of the gear drive. The most unfavorable configuration of a transmission error function is represented by a linear function. In order to achieve a better solution Litvin and Fuentes [44] developed a second order parabolic function. They have presented the approach of how the noise and vibration can be decreased. Use has been made of a parabolic function of transmission errors that accepts the taking of slightly linear discontinuous functions that are produced by misalignments. Theoretically transmission of gears is accomplished by a linear function. In a real work environment, because of the misalignment, the transmission function is actually piecewise slightly linear while the gear meshing lasts between two pairs of teeth. The new transmission function (Fig 1) is achieved after the parabolic function of TE assimilates the linear function. In order to achieve the non-linear transmission error function, a second order polynomial function is implemented. The parabolic function of transmission errors is implemented into the transmission function.

The second order parabolic function was also studied by Litvin and Gonzalez-Perez [45] in order to diminish vibration and noise. In their article the TE function is given at every meshing period as an outcome of three sections. The marginal meshing period sections are in tangency with the medium section that has no transmission errors. Cheng-Kang Lee [46] developed a verifiable fourth order polynomial function of TE to study the cylindrical crown gear drive. He has come to the conclusion that gear drive can become stronger against unavoidable casual errors if a predesigned parabolic function of TE is attributed. Su Jinzhan et al. [47] focus their research on the seventh order polynomial TE function. Their work was applied to spiral bevel gears that can be achieved with high order coefficients. A mixture of tooth contact analysis methods was provided to compare with seventh order TE functions of gears tooth resistance.

High order polynomial TE functions were also investigated by Jiang Jinke and Fang Zongde [48]. They applied this type of functions to model tooth face changes of cylindrical gears. They explained why high order polynomial TE functions gave better results than the second order functions.

Figure 1. Transmission function as a sum of a linear function and a predesigned parabolic function of transmission errors [44]
The mathematical equation of the fourth-order polynomial TE function can be established with the formula [46]:

\[
\Delta \phi_2 (\phi_1) = c_0 + c_1 \phi_1 + c_2 \phi_1^2 + c_3 \phi_1^3 + c_4 \phi_1^4 = XY^T
\]

where \( \Phi_1, \Phi_2 \) are the rotation angles; \( X \) and \( Y \) are single column matrices whose elements are coefficients \( c_0\cdots c_4 \) and the angular position of the input gear.

Implementing the generalized Pithagora's theorem we can determine the pitch circle radius. Taking into account the eccentricity of TE throughout the line of action, raising the center distance results in an improvement of the line. The driven gear needs to displace the distance modification and also its particular tooth thickness throughout the line of action in order to stay in contact with the driving gear. This situation causes a sinusoidal relative movement among gears at every revolution. Thus, the static transmission error influenced by eccentricity can be estimated as [49]:

\[
e(t) = U \sin(\theta_1 + d), \quad U = [r_b^2(R^2)^{-1}][AR(R^2 - r_b^2)^{-1/2}]
\]

where \( U \) is the amplitude variation of STE influenced by eccentricity; \( \theta \) is the rotational displacement; \( d \) represents the phase variation of centre distance; \( A \) is the amplitude variation of centre distance; \( R \) represents the centre distance between gears and \( r_b \) is the base circle radius.

4. Analytical methods for estimating the transmission error

4.1. Fourier Transform

The Fourier Transform represents the main instrument of signal processing. Wide vibrations are produced when tooth mesh frequency reach the resonance frequency of a gearbox. The specter elements of the signal derive from the modulation of amplitude and phase. Time Fourier Transform is generated to assess the spectrum [50].

According to W.D. Mark [51] STE tooth meshing harmonic of a helical gear has two parts: the join of the tooth pair elastic deformations and the changes in the geometric tooth pair stiffness and single tooth pair stiffness. Tooth meshing harmonics are the prevailing STE harmonics of gearing systems.

Low-order discontinuity of converted functions leads to the high frequency asymptotic conduct of the Fourier transforms. The discontinuous character occurs at the start and end of the gear meshing process which is determined by the behavior of the loaded teeth at initialization, and termination and by the changes in the working surface generating tooth meshing harmonic Fourier series coefficients. The harmonic amplitudes occur inside or close to the high frequency asymptotic region. Additional harmonic elements are not being produced through changes in gear mesh stiffness.

In Mark’s article the following formula of STE Fourier series coefficients is obtained [51]:

\[
\alpha_p[\zeta] \approx \alpha_0[\zeta_1]p[\alpha_p[\zeta_1]/\alpha_0[\zeta_1] - \alpha_p[\delta K_M/\bar{K}_M]]p = \pm 1, \pm 2, ...
\]

where \( \zeta(x) \) is the lineal transmission error; \( \bar{K}_M \) is the average component and \( \delta K_M(x) \) is the variable component; \( \alpha_p[\zeta_1] \) determine the complex Fourier series coefficients of the TE share \( \zeta_1(x) \); \( \alpha_0[\zeta_1] \) show the real value of \( \alpha_0[\zeta_1] \). The first element inside the accolade of the equation represents the Fourier factor of the stiffness weighted changes/ elastic deformation while the second element describes the role of the gear mesh stiffness fluctuations.

The decrease in the transmission error is necessary for minimizing the discontinuities that occur at the beginning and end of the gear tooth mesh. It has been noticed that the contact ratio has a close relationship with the loading.

Because engaging teeth are not always in contact it can be seen that, when mesh frequency increases, subharmonic edge is inserted by parabolic function of transmission error [52].

Martin Zajiček and Jan Dupal focus on an analytical periodic solution constant condition by applying Green’s function using the shape of a garbled Fourier series function. This process allows to discover the border of stability of the system matrix. The Green’s function is determined as a reply of the fixed side to a Dirac string of unit impulses. An analytical harmonic balanced method (HBM) was
implemented in order to model the gears as a linear transmission system with a periodical stiffness [53].

The gear mesh presents a uniform release over the line of action. Since mesh stiffness is a periodic function, its analytical description can be made by development in Fourier series [54]. The same approach can be resorted to in order to represent the manufacturing error [19].

\[ k(t) = k \left[ 1 + 2 \sum_{i=1}^{L} \varepsilon_i \cos(\omega_i t) \right] \]  
(6)

\[ e(t) = \sum_j E_j \cos( j \tilde{\omega}_m t - \gamma_j) \]  
(7)

where \( k \) represent stiffness, \( \varepsilon_i \) is the coefficient of time-varying stiffness, \( \omega \) is the meshing frequency and \( t \) is time; \( \omega_m = \omega_1 N_1 = \omega_2 N_2 \) represent mesh angular frequency; \( \omega_1, \omega_2 \) represent the medium rotational speeds of pinion and gear; \( \tilde{\omega}_m = \omega_m / (N_1 N_2) \). Applying the Discrete Fourier Transform (DFT) we can determine the amplitudes \( k \), \( E_j \) and phases \( \omega_i, \gamma_j \).

4.2. Hilbert Transform

Hilbert transform (HT) is one of the most popular methods used in vibration analysis and signal processing [55, 56]. It is used in both time and frequency domain establishing a quick and efficient way of identifying the non-linear behavior using assessed frequency response functions and bringing a close perspective of their type. HT discovery techniques are time efficient comparing to classical spectral analysis methods. The techniques developed with the HT provide information on instantaneous rotation angle. Nevertheless, like other detecting procedures, HT cannot determine whether the deviations identified in the frequency response function are statistically meaningful. The use of artificial neural networks can exceed this restrictions [55]. A standard way of processing signals contain a spectral research through the Fourier transform and also a statistical review. An incidental signal can be expressed as a product of two independent functions [56]. These functions are the amplitude (envelope) and cosine of the instantaneous phase. When a signal is represented in a Cartesian coordinate system, the second projection throughout vertical axis is a product of amplitude and the sinus phase function. Relations can also be expressed using the Fourier series approach. Modulated signals research is based on Euler’s definition of harmonic functions. Gabor improved Euler’s representation and made a generalization for complex functions. According to that generalization, since time is the independent variable, HT represents the imaginary side of the analytic signal and is also called the quadrature or conjugate of the genuine function. An analytic signal is a complex signal where HT represents the imaginary part of the real part. Applying the classic description an analytic signal can be written as [55]:

\[ X(t) = |X(t)| \left[ \cos \psi(t) + j \sin \psi(t) \right] = A(t) e^{j\psi(t)} = x(t) + j\tilde{x}(t) \]  
(8)

Bendat recommends that Bruel and Kjær channel analyser should contain HT as a standard signal method. Huang refresh HT into an additional signal analysis method called Hilbert-Huang transform (HHT) [57]. This new method comprises also Empirical Mode Decomposition (EMD). EMD procedure can divide the complex signal into several intrinsic mode functions (IMF), being described by the narrow time scale feature of the signal. The specific feature information of the genuine signal can be taken more accurately by investigating the IMF element that requires the narrow feature of the signal. Nevertheless, EMD has also some disadvantages such as sensitivity to signal noise, the need for improvement of spline fitting, end effects and mode mixing, especially when modes are at similar frequencies [58]. Another decomposition method is the Hilbert Vibration Decomposition (HVD). This method is used for complex vibration signals with non-stationary components whose amplitudes and frequencies change. HVD method is developed based on the HT description of the instantaneous frequency. Unlike the EMD method, HVD does not take into consideration the spline interpolation algorithm and it has a higher frequency resolution. HVD theory is developed from the assumption that the primary signal results from superposition of harmonic functions [59]. For multi-degree-of-
freedom transmission systems, a number of restrictions of the Hilbert transform theory can be avoided using the continuous wavelet transform theory (CWT) [60].

5. Optimization methods used to estimate the transmission error

5.1. Statistical methods
Driot and Perret-Liaudet suggested that the contact line location between the gear teeth is achieved using the kinematic research of the gear meshing. Many times the STE is presented as a suitable gear mesh displacement made as a difference between the product of the base radius and the angular position of the pinion and gear. In dynamic mode, under the influence of STE, a gear mesh causes dynamic mesh forces that are delivered to the entire transmission system. STE creates the outward force vector and spectral mesh stiffness oscillation as suggested by the procedure. Many differential equations with recurrent factors can be solved by this procedure into an acceptable period of time. The oscillation response to every degree-of-freedom of the system is determined by the procedure employed. The execution time is about 60 percent lower in contrast to the standard numerical time of the synthesized model.

Monte Carlo procedure describes a category of computational algorithms that are used to achieve numerical results with the help of random sampling. All samples must contain information about mesh stiffness, STE, modal base and response. The calculation time is considerably large because Monte Carlo simulation requires many samples. The Box-Muller method is applied to obtain Gaussian variables. For a good accuracy with a relative error less than 1%, the Monte Carlo method requires at least ten thousand samples and the accuracy grows relative to the function $1/n^{1/2}$ [19].

Monte Carlo simulation was also presented by Pierre Garambois et al. [9] in order to examine the gears sturdiness influenced by manufacturing errors. For all gears static transmission error and mesh stiffness changes are assessed by means of the probability density functions (PDF) of the root mean square value (RMS). It has been considered the most unfavourable situation of the uniform manufacturing errors dispersion through a related tolerance class series. One thousand incidental manufacturing error samples from all gears have been simulated according to Monte Carlo procedure. The optimization targets are diminishing the static transmission error mean value of the probability density function as described by the changes in the mesh stiffness and by the static transmission error assessment.

Another statistical method is Tagucci procedure [19]. Random variables and functions characterized by a probability density function can assess the statistical moments with this procedure. D’Errico developed Tagucci solution in order to emphasize the non-linear result. The simplicity of numerical execution and high computing efficiency are the fundamental benefits of this method, but at the same time probability density functions of the response function cannot be given.

5.2. Genetic Algorithms
Genetic Algorithm (GA) theory determines the progress of species above different generations. Along with Particle Swarm Optimization and other algorithms, GA is part of the Evolutionary Algorithms.

GA is a stochastic optimization method suggested by the environment and appropriate when dealing with complex technical problems [61].

In Giorgio Bonori et al. [20] research an optimized Genetic Algorithm is determined in order to increase gear dynamic results. An application of genetic algorithms was expanded to acquire the most suited profile changes to reduce peak-to-peak STE and also the harmonics. The GA is achieved from binary encoding characteristics, which detect the profile changes on gear pair. A STE numerical estimation is performed when a population of strings is set. Two types of binary code are considered. First the standard binary code is evaluated and then a Grey code is established. The standard binary code is used to perform substring decoding, while the Grey encoding technique present the integers in the numerical base two. The value of PPSTE is considered the objective function and it is the subject of three optimization types. By applying the seventh order harmonic of the STE function a fourth optimization can be achieved. Acceptable outcome is obtained by using linear profile changes and Grey encoding.
Genetic algorithms have also been studied by Ph. Velex et al. [62]. The approach of the evolutionary algorithm uses the following steps. First a population of individuals is spread throughout the investigation area (four identical children of profile change characteristics) is produced and assessed. Then a new population of individuals is generated through different operators and it is implemented to the prior string of samples and then reassessed. Last, the procedure is reiterated until convergence. Three series of potential profile changes are optimized: short, intermediate and long reliefs. An important decrease of PPSTE is also obtained.

In Pierre Garambois et al. [8] article, a Non-Dominated Sorting GA II (NSGA-II) was used. The main purpose of the algorithm is to generate a population that developed along generations. The algorithm accomplishes crossover and mutations to obtain a new generation. The law of mutation is to build a new candidate with random original characteristics. Usually the mutation probability is 10%. The crossover rule is to generate other two candidates that have a random combination of characteristics of two random parents. Crossover is important for all decision variables of candidates and has a 90% probability.

Jakub A. Korta and Domenico Mundo [26] used in their work an updated version of NSGA II algorithm named Controlled Elitist GA (CEGA). This algorithm enables the evolving of the non-dominant candidates. This characteristic gives the opportunity of discovering the global Pareto line.

5.3. Simplex optimization

Marcello Faggioni et al. [63] developed in their research a Random-Simplex method. In order to obtain an optimum value for a nonlinear function a random sampling is executed following a constant distribution. Simplex method determines the optimum function. The Simplex procedure is an iterative routine based on the idea of an active polyhedron with N+1 vertices. The objective function is calculated in each vertex. A static and dynamic optimization is realized with the random sampling method and then a simplex procedure is applied to determine the optimal parameters that are not influenced by a tip relief of the gear pair. A random-simplex method was implemented to an appropriate high contact ratio gear.

The load distribution problem of the gear teeth was settled using the method of modified simplex algorithm. However, modified simplex procedures need to solve tooth contact patterns that imply solving higher contact problems.

6. Numerical methods for determining the transmission error

A common procedure to determine the transmission errors is the tooth contact analysis (TCA) method. Nevertheless TCA does not take into account the load. C. Jia et al. article [64] studies the relationship between peek-to-peek transmission error (PPTE), which represents the size of load transmission error oscillation, and TCA under load. He has established a procedure that requests a verifiable transmission function to guide tooth surface change. In order to obtain an accurate measure of correction of the transmission function, a deliberate TE curve is developed. This is obtained by way of the load transmission error (LTE) curve of the normal involute gears pairs under load. The deliberate TE curve is created in such a way as to counterbalance the LTE. An important decrease of PPTE is achieved by tooth modification. He concludes that tooth modification is particularly influenced by transmission function and less by individual geometry.

Tooth areas that entry and exit the meshing teeth are adjusted from ideal involute surfaces whereas is necessary to bypass collision loading and TE discontinuities. Some research can be developed by studying the changes and tooth elastic deformations that produce the STE [65]. Some authors [66] considered that there are two different classes of gear mesh stiffness: rectilinear gear mesh stiffness and torsional gear mesh stiffness. Rectilinear mesh stiffness is measured over the line of action of the gear mesh and torsional mesh stiffness is determined as a ratio of a torque, placed over the gear, and the adequate angular displacement. Sometimes in gears with acknowledged stiff structure, torsional mesh stiffness may generate rectilinear mesh stiffness. Chi, Howard and Wang [67] focus their research on the examination of the specific torsional gear mesh stiffness in order to determine the STE of meshing gears. They measure the STE by the instrumentality of a number of experiments and also establish the theoretical part and the specific torsional gear mesh stiffness using numerical analysis tools like Finite-Element Analysis (FEA). The experiments were conducted by making use of a test rig
where the gear mesh was formed from a nylon gear and a permanent aluminum gear under different torques.

The simulation of the dynamic behaviour or LTE of the gear teeth needs accurate modeling of the mesh stiffness [68]. Stiffness represents more than a function of the connection on the involute, depending on the rest of the tooth part and being determined by the profile shift and the cutting tool [69].

X. Lian et al. [70] have introduced three different models to assess gear mesh stiffness with the FEA process. The first model is made by assigning a rigid gear bore area and provides an accurate gear stiffness outcome. The second model uses angular deflection at various circumferential angles at the end surface circle of the gear bore. The third model is validated only for gears with identical tooth profile and involves the angular deflection at an aleatory circumferential angle. The last two models have been made without taking into consideration the assigned presumption. Gear mesh stiffness has also been studied by Munro et al. [71] who describes a method that benefits from the features of transmission error plots of average and fluctuating elements, through a series of tooth loads. This is accomplished using predicted Harris maps.

Zehua Hu et al. [72] expend in their article a dynamic node finite element example of gear rotor transmission system having various ranges of crack and considering as parameters the TE excitation, time-varying mesh stiffness, bearing support, shaft flexibility and backlash.

Shuling Li [73] apply the mathematical programing method (MPM) and Finite element method (FEM) to detect the spur gear loaded tooth contact analysis (LTCA), deformation and stress computation having various contact ratios and annexes. FEM is usually used to evaluate the deformation and also to determine the stress of different structures. Knowing tooth load distribution in LTCA, FEM is used to compute the root bending stress.

In other researches Load Distribution Program (LDP) is used to calculate the load distribution over the contact line of helical and spur gears. LDP is a good instrument to foresee TE regardless of the type of gear axes [74]. LDP is also used by J. Bruyère et al. [75] to evaluate the results of static transmission error.

7. Experimental Methods used to establish the transmission error

Angular vibration is the source that generates the TE. There are a number of techniques to determine the angular vibration throughout rotation movement, such as: tangentially assembled accelerometers, laser torsional vibration meter using the Doppler effect and incremental rotary encoders. Time length period is established by the following methods: sample number and interpolation, high frequency oscillator and impulse counter, and phase demodulation [76].

In practice the most common method of measuring TE is the technique involving incremental rotary encoders. With their assistance, an electrical pulse string is delivered. Two encoders are fitted on each shaft. The generated signals are compared in order to obtain the TE. The average speed encoder attached to the shaft is inversely proportional to the time lapse of two consecutive pulses. This technique is also used to assess the effect of tooth profile errors and gear eccentricity on the dynamic conduct of a low speed spur gear [9]. A generic research method for signal processing is the wavelet analysis which provides information on both frequency and time [77]. The easiest way of acquiring TE is to analyse two torsional vibration signals, dropping one signal from the other and taking into account the transmission ratio. DTE can be determined measuring torsional acceleration of the shafts, comparing them and incorporating the signal twice. The accelerometer model has some advantages over the encoder model as it is able to measure high frequencies with fidelity and is more flexible. However, its capacity to low frequency reply is poor. The encoder model presents an overhead frequency threshold due to mechanical resonance limitations. Still it is easier to put it into practice. Phase demodulation procedure is used for all incremental encoder models. Bandwidth is the most important characteristic of demodulation. In order for the neutral carrier to be demodulated correctly, the one side bandwidth of the modulated signal must have a lower value than the carrier frequency.

The classic Fourier series approach presents the signal as a multitude of sine waves. A complex demodulation analysis can be performed with the HT. Torsional vibration signals can be read from the rotary encoder through a number of techniques, such as carrier tracking, pulse timing, frequency domain block-shift transforms and zoom demodulation and also Hilbert transform. Carrier tracking
method pursues the carrier frequency and is related to the use of PLL circuits. This method has one major disadvantage, since the transmission error measurement is not performed accurately because there is a time lag in the PLL circuit, thus the difference signal will be altered by phase fluctuation through channels. Time lapses between pulses are determined with a high-speed clock. A sinusoidal output sensor produces the pulses. A fine track torsional vibration signal is obtained from digital processing of the timing information. This technique is very precise for all frequencies and can analyze a large number of records, but the processing time is usually extensive. The main advantage of zoom demodulation method is the possibility of recording for a long period of time without the need of a large buffer [16].

In Rocca’s [17] article TE is verified by analysing theoretical and experimental outcome. The experimental rig consists of a gear pair, each gear having an optical sensor fixed. A servomotor is used to move the drive shaft. The driven shaft has no outward load and testing is conducted in idle operating condition. The distance between the axes is varied with a micrometric linear guide linked to a chassis assisting the driven shaft, in order to change the gear backlash value. In the experiment two Baumer Electric encoders are fitted at the end of both encoders. A data counter is settled to decipher the signals.

In Murat Inalpolat et al. article [78] quasi-static transmission error of spur gears having various indexing errors was measured with an open architecture gearbox. The test rig was developed so that it could function at low speed and under high load conditions. Angular position of the gear pair was established by measuring the transmission error with two encoders attached at each shaft. Encoder conditioners took the signals from the encoders and then a consumer product transmission error measurement device processed the signal and calculated the value of the transmission error.

Ma Ru Kang and Ahmet Kahraman [43] present in their article an accelerometer measurement technique to experimentally examine the torsional, translational and rotational vibrations of helical and spur gear mesh. They apply this technique to a number of gear pairs sustained by shafts of different diameters and lengths in order to underline the efficiency of the measurement method. They used uniaxial tangentially fitted accelerometers. A series of triaxial accelerometers were applied to allow for the measurement of movement of gears in all directions.

An example of experimental test rig is presented in figure 2:

![Figure 2. Determination of TE using incremental optical encoders [79]](image)

8. Conclusion
The current paper reviews the basic types of transmission errors and also the most common methods for detecting the mean and peak-to-peak values by using analytical methodology, optimization algorithms, numerical analysis and experimental researches.

Transmission error represents the main cause for the occurrence of noise and vibration in the gears, thus affecting the gear meshing quality.

Static transmission error results from manufacturing and assembly errors and also from design inaccuracies. Manufacturing errors, elastic energy and input load has a powerful impact on the probability density functions of time-varying mesh stiffness and PPSTE.

Peak-to-peak STE is considered the objective function and has a powerful correlation with the gear dynamics transmission system. Thus, minimizing PPSTE determines a better gear oscillation behaviour.
Tooth contact analysis method influences the shape of transmission error. Mesh frequency harmonics represents the prevailing STE harmonics of the gear pairs and its value decreases with increased load. Profile changes do not influence the PPTE fluctuation as the loads increase. Dynamic load is particularly linked to the shaft frequency component of TE when low speed case is considered. When tooth flank precision is increased, high frequency amplitude of TE is removed.

STE can be estimated as a Fourier transform series. Hilbert transform techniques provides better time testing solutions without altering data precision. Fourier transform requires a longer time for data processing, and measures the frequency points near resonance frequency. First two harmonics of Fourier Transform are enlarged by increasing the pressure angle. Using Fourier transform and digital filters, Hilbert transform performance can be estimated. Mesh frequency and its first harmonic represents the prevailing source of gear noise.

Monte Carlo simulation is usually used to achieve predictions for testing different statistical techniques. Probability density functions are obtained when simulation ends. Under the influence of the manufacturing errors, Monte Carlo simulation checks the gear robustness.

Taguchi technique has an important benefit due to computational performance and numerical implementation.

A Genetic Algorithm application is expended for spur gear optimization. STE influenced Genetic algorithms objective functions. Also, linear profile changes are well determined using Grey encoding. Genetic algorithm technique establishes the optimum series of abstract profile changes and it is not an appropriate tool to determine the lowest value since it usually operates in a high fitness region. Non-Dominated Sorting GA II algorithm is successfully used to determine multi-objective optimization problems. Gear pair design method can be extended to estimate the optimum value by using genetic algorithm.

Tooth stiffness computation is realized with the finite element analysis tool which delivers an improved level of precision.

Experimental researches are usually carried out by an incremental rotary encoder method. This method is preferred because it represents a less costly and efficient solution for transmission error determination.

This article represents the first step of a future research. Different analytical, optimization, numerical and experimental methods will be established. New approaches will be achieved by using the statistical ANOVA test and we will try to implement other less used methods, such as the Particle Swarm Optimization technique. For experimental determination a rotary encoder will be used and also a laser torsional vibration meter will be considered in order to detect the displacement that represents the transmission error value. The possible innovative experimental element will be the use of non-circular gears.

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9. References
[1] Walker H 1938 Gear Tooth Deflection and Profile Modification Engineer 166 pp 410-435
[2] Tosun M, Yildiz M and Ozkan A 2018 Investigation and refinement of gearbox whine noise Applied Acoustics 130 pp 305–311
[3] Henriksson M 2009 On noise generation and dynamic transmission error of gears (Stockholm: KTH Engineering Sciences - Doctoral Thesis) pp 1-15
[4] Fietkau P, Bertsche B 2013 Influence of tribological and geometrical parameters on lubrication conditions and noise of gear transmissions Mechanism and Machine Theory 69 pp 303–320
[5] Tharmakulasingam R 2009 Transmission Error in Spur Gears: Static and Dynamic Finite-Element Modeling and Design Optimization (London: School of Engineering and Design Brunel University – Doctoral Thesis) pp 6-50
[6] Gregory R W, Harris S L, Munro R G 1963 Dynamic behaviour of spur gears Proceedings of the Institution of Mechanical Engineers 178 pp 207-218
[7] Welbourn D B 1979 Fundamental Knowledge of Gear Noise Proceedings of the Institution of Mechanical Engineers pp 9-14
[8] Garambois P, Perret-Liaudet J, Rigaud E 2017 NVH robust optimization of gear macro and microgeometries using an efficient tooth contact model *Mechanism and Machine Theory* **117** pp 78–95
[9] Ottewill J R, Neild S A, Wilson R E 2010 An investigation into the effect of tooth profile errors on gear rattle *Journal of Sound and Vibration* **329** pp 3495–3506
[10] Halse C K Wilson R E Bernardo M, Homer M E 2007 Coexisting solutions and bifurcations in mechanical oscillators with backlash *Journal of Sound and Vibration* **305** pp 854–885
[11] Miura Y and Nakamura S 1998 Gear rattling noise analysis for a diesel engine *European conference on vehicle noise and vibration* pp. 3-11
[12] S L Harris 1958 Dynamic loads on the teeth of spur gears *Proceedings of the Institution of Mechanical Engineers* **172** pp 87–112
[13] Palmer D, Fish M 2012 Evaluation of Methods for Calculating Effects of Tip Relief on Transmission Error, Noise and Stress in Loaded Spur Gears *Geartechnology* **January/February 2012** pp 56-67
[14] Dudley D W, Townsend D P 1996 *Dudley’s Gear Handbook* (New York - McGraw-Hill Inc)
[15] Gregory R W, Harris S L, Munro R G 1963 A Method of Measuring Transmission Error in Spur Gears of 1:1 Ratio, *Journal of Scientific Instruments* **40** pp 5-9
[16] Sweeney P J, Randal R B 1996 Gear transmission error measurement using phase demodulation *Proceedings of the Institution of Mechanical Engineers Part C-Journal of Mechanical Engineering Science* **210** pp 201-213
[17] Rocca E, Russo R 2011 Theoretical and experimental investigation into the influence of the periodic backlash fluctuations on the gear rattle *Journal of Sound and Vibration* **330** pp 4738–4752
[18] Bonori G, Pellicano F 2007 Non-smooth dynamics of spur gears with manufacturing errors *Journal of Sound and Vibration* **306** pp 271–283
[19] Driot N, Perret-Liaudet J 2006 Variability of modal behavior in terms of critical speeds of a gear pair due to manufacturing errors and shaft misalignments *Journal of Sound and Vibration* **292** pp 824–843
[20] Bonori G, Barbieri M and Pellicano F 2008 Optimum profile modifications of spur gears by means of genetic algorithms *Journal of Sound and Vibration* **313** pp 603 – 616
[21] Ghosh S S, Chakraborty G 2016 On optimal tooth profile modification for reduction of vibration and noise in spur gear pairs *Mechanism and Machine Theory* **105** pp 145–163
[22] Li S 2007 Finite element analyses for contact strength and bending strength of a pair of spur gears with machining errors, assembly errors and tooth modifications *Mechanism and Machine Theory* **42** pp 88–114
[23] Astoul J, Mermoz E, Sartor M, Linares J M, Bernard A 2014 New methodology to reduce the transmission error of the spiral bevel gears, *CIRP Annals - Manufacturing Technology* **63** pp 165–168
[24] Karpat F, Ekwaro-Osire S, Cavdar K, Babalik F C 2008 Dynamic analysis of involute spur gears with asymmetric teeth *International Journal of Mechanical Sciences* **50** pp 1598–1610
[25] Fernandez del Rincon A, Viadero F, Iglesias M, García P, de-Juan A, Sancibrian R 2013 A model for the study of meshing stiffness in spur gear transmissions *Mechanism and Machine Theory* **61** pp 30–58
[26] Korta J A, Mundo D 2017 Multi-objective micro-geometry optimization of gear tooth supported by response surface methodology *Mechanism and Machine Theory* **109** pp 278–295
[27] Lin T, He Z 2017 Analytical method for coupled transmission error of helical gear system with machining errors, assembly errors and tooth modifications *Mechanical Systems and Signal Processing* **91** pp 167–182
[28] Siyu C, Jinyuan T, Lijuan W 2014 Dynamics analysis of a crowned gear transmission system with impact damping: Based on experimental transmission error *Mechanism and Machine Theory* **74** pp 354–369
[29] Mark W D 1978 Analysis of the vibratory excitation of gear systems: Basic theory *The Journal of the Acoustical Society of America* **63** pp 1409-1430
[30] Kissling U 2010 Effects of profile corrections on peak-to-peak transmission error
[31] Hotait M A, Kahraman A 2013 Experiments on the relationship between the dynamic transmission error and the dynamic stress factor of spur gear pairs *Mechanism and Machine Theory* **70** pp 116–128

[32] Bruyère J, Velex P 2014 A simplified multi-objective analysis of optimum profile modifications in spur and helical gears *Mechanism and Machine Theory* **80** pp 70–83

[33] He S, Gunda R, Singh R 2007 Effect of sliding friction on the dynamics of spur gear pair with realistic time-varying stiffness *Journal of Sound and Vibration* **301** pp 927–949

[34] Fernández-del-Rincón A, Iglesias M, de-Juan A, Diez-Ibarbia A, García P, Viadero F 2016 Gear transmission dynamics: Effects of index and run out errors *Applied Acoustics* **108** pp 63–83

[35] Halse C K, Wilson R E, di Bernardo M, Homer M E 2007 Coexisting solutions and bifurcations in mechanical oscillators with backlash *Journal of Sound and Vibration* **305** pp 854–885

[36] Guangjian W, Lin C, Li Y, Shuaiqiong Z 2017 Research on the dynamic transmission error of a spur gear pair with eccentricities by finite element method *Mechanism and Machine Theory* **109** pp 1–13

[37] Siyu C, Jinyuan T, Lijuan W 2014 Dynamics analysis of a crowned gear transmission system with impact damping: Based on experimental transmission error *Mechanism and Machine Theory* **74** pp 354–369

[38] Karahan A, Singh R 1991 Interactions between time-varying mesh stiffness and clearance nonlinearity in a gear pair *Journal of Sound and Vibration* **146** pp 135–156

[39] Perret-Liaudet J 1996 An original method for computing the response of a parametrically excited forced system *Journal of Sound and Vibration* **196** pp 165–177

[40] Bozca M 2018 Transmission error model-based optimisation of the geometric design parameters of an automotive transmission gearbox to reduce gear-rattle noise *Applied Acoustics* **130** pp 247–259

[41] Ozguven H N, Houser D R 1988 Mathematical models used in gear dynamics – a review, *Journal of Sound and Vibration* **121** pp 383–411

[42] Barbieri M, Bonori G, Pellicano F 2012 Corrigendum to: Optimum profile modifications of spur gears by means of genetic algorithms *Journal of Sound and Vibration* **331** pp 4825–4829

[43] Kang M R, Kahraman A 2012 Measurement of vibratory motions of gears supported by compliant shafts *Mechanical Systems and Signal Processing* **29** pp 391–403

[44] Litvin F L, Fuentes A 2009 *Gear Geometry an Applied Theory – Second Edition* (New York: Cambridge University Press) Chapter 9.2

[45] Litvin F L, Gonzalez-Perez I, Fuentes A, Hayasaka K and Yukishima K 2005 Topology of Modified Surfaces of Involute Helical Gears with Line Contact Developed for Improvement of Bearing Contact, Reduction of Transmission Errors, and Stress Analysis *Mathematical and Computer Modelling* **42** pp 1063-1078

[46] Lee C K 2009 Manufacturing process for a cylindrical crown gear drive with a controllable fourth order polynomial function of transmission error *Journal of Materials Processing Technology* **209** pp 3–13

[47] Jinzhuan S, Zongde F, Xiangwei C 2013 Design and analysis of spiral bevel gears with seventh-order function of transmission error *Chinese Journal of Aeronautics* **26** pp 1310–1316

[48] Jinke J, Zongde F 2015 Design and analysis of modified cylindrical gears with a higher-order transmission error *Mechanism and Machine Theory* **88** pp 141–152

[49] Ottewill J R, Neild S A, Wilson R E 2009 Intermittent gear rattle due to interactions between forcing and manufacturing errors *Journal of Sound and Vibration* **321** pp 913–935

[50] Tuma J 2009 Gearbox Noise and Vibration Prediction and Control *International Journal of Acoustics and Vibration* **14** pp 1-11

[51] Mark W D 2018 Tooth-meshing-harmonic static-transmission-error amplitudes of helical gears *Mechanical Systems and Signal Processing* **98** pp 506–533

[52] Yang J J, Shi Z H, Zhang H, Li T X, Nie S W, Wei B Y 2018 Dynamic analysis of spiral bevel and hypoid gears with high-order transmission errors *Journal of Sound and Vibration* **417** pp 149-164
[53] Zajiček M, Dupal J 2017 Analytical solution of spur gear mesh using linear model Mechanism and Machine Theory 118 pp 154–167
[54] Shen Y, Yang S, Liu X 2006 Nonlinear dynamics of a spur gear pair with time-varying stiffness and backlash based on incremental harmonic balance method International Journal of Mechanical Sciences 48 pp 1256–1263
[55] Feldman M 2011 Hilbert transform in vibration analysis Mechanical Systems and Signal Processing 25 pp 735–802
[56] Yang Y, He Y, Cheng J, Yu D 2009 A gear fault diagnosis using Hilbert spectrum based on MODWPT and a comparison with EMD approach Measurement 42 pp 542–551
[57] Ondra V, Sever I A, Schwingshackl C W 2017 A method for detection and characterisation of structural non-linearities using the Hilbert transform and neural networks Mechanical Systems and Signal Processing 83 pp 210–227
[58] Li Z, Jiang Y, Hu C, Peng Z 2016 Recent progress on decoupling diagnosis of hybrid failures in gear transmission systems using vibration sensor signal: A review Measurement 90 pp 4–19
[59] Feldman M, Braun S 2017 Nonlinear vibrating system identification via Hilbert decomposition Mechanical Systems and Signal Processing 84 pp 65–96
[60] Yousfi N, Zghal B, Akrout A, Walha L, Haddar M 2018 Damping models identification of a spur gear pair Mechanism and Machine Theory 122 pp 371–388
[61] Miler D, Loncar A, Zezelj D, Domitran Z 2017 Influence of profile shift on the spur gear pair optimization Mechanism and Machine Theory 117 pp 189–197
[62] Velex P, Chapron M, Fakhfakh H, Bruyère J, Becquerelle S 2016 On transmission errors and profile modifications minimizing dynamic tooth loads in multi-mesh gears Journal of Sound and Vibration 379 pp 28–52
[63] Faggioni M, Samani F S, Bertacchi G, Pellicano F 2011 Dynamic optimization of spur gears Mechanism and Machine Theory 46 pp 544–557
[64] Jia C, Fang Z, Zhang Y 2017 Topography of modified surfaces based on compensated conjugation for the minimization of transmission errors of cylindrical gears Mechanism and Machine Theory 116 pp 145–161
[65] Velex P, Ajmi M 2006 On the modelling of excitations in geared systems by transmission errors Journal of Sound and Vibration 290 pp 882–909
[66] Howard I, Jia S, Wang J 2001 The dynamic modelling of a spur gear in mesh including friction and a crack Mechanical Systems and Signal Processing 15 pp 831–853
[67] Chi C W, Howard I, De Wang J 2007 An experimental investigation of the static transmission error and torsional mesh stiffness of nylon gears ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference 7 pp 207–216
[68] Sánchez M B, Pleguezuelos M, Pedrero J I 2017 Approximate equations for the meshing stiffness and the load sharing ratio of spur gears including hertzian effects Mechanism and Machine Theory 109 pp 231–249
[69] Pedersen N L, Jørgensen M F 2014 On gear tooth stiffness evaluation Computers and Structures 135 pp 109–117
[70] Liang X, Zhang H, Zuo M J, Qin Y 2018 Three new models for evaluation of standard involute spur gear mesh stiffness Mechanical Systems and Signal Processing 101 pp 424–434
[71] Munro R G, Palmer D, Morrish L 2001 An experimental method to measure gear tooth stiffness throughout and beyond the path of contact Proceedings of the Institution of Mechanical Engineers Part C - Journal of Mechanical Engineering Science 215 pp 793-803
[72] Hu Z, Tang J, Zhong J, Chen S 2016 Frequency spectrum and vibration analysis of high speed gear-rotor system with tooth root crack considering transmission error excitation Engineering Failure Analysis 60 pp 405–441
[73] Li S 2008 Effect of addendum on contact strength, bending strength and basic performance parameters of a pair of spur gears Mechanism and Machine Theory 43 pp 1557–1584
[74] Army Research Laboratory Technical Report ARL-TR-493 1994 Comparison of Transmission Error Predictions With Noise Measurements for Several Spur and Helical Gears NASA Technical Memorandum 106647 AIAA-94-3366 pp 1-10
[75] Bruyère J, Gu X, Velex P 2015 On the analytical definition of profile modifications minimizing transmission error variations in narrow-faced spur helical gears *Mechanism and Machine Theory* 92 pp 257–272

[76] Tuma J 2006 Dynamic Transmission Error Measurements *Engineering Mechanics* pp 1-15

[77] Peng Z K, Chu F L 2004 Application of the wavelet transforms in machine condition monitoring and fault diagnostics: a review with bibliography *Mechanical Systems and Signal Processing* 18 pp 199–221

[78] Inalpolat M, Handschu M and Kahraman A 2015 Impact of Indexing Errors on Spur Gear Dynamics, *Geartechnology* July 2015 pp 64-70

[79] Akerblom M 2008 Gear noise and vibration – A literature survey *Technical Report Volvo CE Component Division* pp 1-25