Extra dimensions, orthopositronium decay, and stellar cooling

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Abstract

In a class of extra dimensional models with a warped metric and a single brane the photon can be localized on the brane by gravity only. An intriguing feature of these models is the possibility of the photon escaping into the extra dimensions. The search for this effect has motivated the present round of precision orthopositronium decay experiments. We point out that in this framework a photon in plasma should be metastable. We consider the astrophysical consequences of this observation, in particular, what it implies for the plasmon decay rate in globular cluster stars and for the core-collapse supernova cooling rate. The resulting bounds on the model parameter exceed the possible reach of orthopositronium experiments by many orders of magnitude.

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I. INTRODUCTION

Theories with extra dimensions\[1, 2, 3, 4, 5\] have been very popular in the last decade\[6, 7, 8, 9\]. In a large class of such theories the extra-dimensional space is “warped”, i.e. the metric scales exponentially along one of the additional dimensions (see Eq. (1) later). The scaling arises naturally as a solution of Einstein’s equations in the extra-dimensional “bulk” filled with a negative cosmological constant. This solution has the same origin as the inflationary solution, $a(t) \sim \exp(\sqrt{\Lambda} t / M_{\text{pl}})$, but is “aligned” to scale along one of the spatial directions, rather than the time direction. This “inflation along a spatial direction” can be arranged by introducing one or more domain walls (branes), tuning their tension(s), and replacing the positive cosmological constant of inflation with a negative value. In a model with two branes, this setup holds promise for solving the hierarchy problem, as the exponents can reduce ratios of vastly different scales to relatively modest numbers\[7\]. The same reasoning may explain the smallness of the Yukawa couplings\[10\]. In a model with a single brane, this setup offers an alternative to compactification\[8\] and, through the AdS/CFT (holographic\[11\]) connection\[12\], could in fact describe a four-dimensional world with a new conformal sector.

Of course, in any such theory, one faces the problem of explaining why we see only four space-time dimensions. One possible line of argument is that the Standard Model fields could be dynamically confined to the four-dimensional Minkowsky defect (brane), as discussed already two decades ago (RS0)\[1\]. In models with warping, it is possible that the fields are localized to the brane by the metric itself, i.e., by gravity. The localization of the graviton by this mechanism in the model with a single warped extra dimension was discussed already in a seminal paper\[8\] (RSII). A scalar field can be similarly localized\[13\]. Gauge fields are not localized in the minimal setup of RSII, but can be localized if the model is extended with additional compact extra dimensions\[14\].

The states localized in this way act most of the time as “normal” four-dimensional massless particles. Under some circumstances, however, they can tunnel into the extra dimensions, “disappearing” from our world. The tunneling can happen if the state is given a nonzero mass, or if it is produced as a virtual state with time-like momentum. Not only is

\[1\] For an expanded set of references, see, e.g.,\[3, 5\].
this suggestion intriguing, but, more importantly, potentially experimentally testable.

We will focus on the possibility of photon tunneling. This effect would manifest itself as unexplained missing energy events at an $e^+e^-$ collider. The measurements of the $Z$ boson resonance provide considerable bounds on the allowed curvature of the extra dimension (see later). Another interesting experimental direction that is being actively pursued is the search for an invisible mode in the orthopositronium decay [15]. The orthopositronium serves as an $e^+e^-$ collider with a hermetic detector. Compared to the $Z$ resonance measurement, one obviously loses on the center-of-mass energy, but gains considerably on the sensitivity to the branching ratio into the invisible mode. The recently published results [15] find the bound $\text{Br}(\text{oPs} \to \text{extra dim}) \leq 4.2 \times 10^{-7}$, with $\text{Br}(\text{oPs} \to \text{extra dim}) \leq 10^{-8} - 10^{-9}$ expected in the future [16].

In this paper, we point out that in the same framework photons in plasma (plasmons) should also be subject to the invisible decay. Indeed, plasma modifies the photon dispersion relation, in a sense providing it with a mass, thereby opening up the decay channel. In what follows, we consider the effects of the additional cooling on the cores of low-mass red giants, horizontal branch stars, and core-collapse supernovae. The bounds we find on the model parameter exceed the possible reach of the orthopositronium by many orders of magnitude.

II. TUNNELING INTO EXTRA DIMENSIONS: OVERVIEW

As already mentioned, the existence of the photon mode localized on the positive tension brane in the scenarios with warped extra dimension(s) is well established. Following [14], let us consider a space with the metric

$$ds^2 = a(z)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - \delta_{ij} d\theta_i d\theta_j) - dz^2.$$  

(1)

Here $z$ labels the infinite warped extra dimension, $a(z) = \exp (-k|z|)$. At $z = 0$ we have a domain wall (brane) with positive tension. The variables $\theta_i \in [0, 2\pi R_i]$ label $n \geq 1$ additional compact dimensions, with radii $R_i$. The fields are assumed to be independent of $\theta_i$.

The action for the electromagnetic field $A_C(x, z)$ in this space is

$$S = \int \sqrt{|g|} d^4x \, dz \prod_{i=1}^{n} \frac{d\theta_i}{2\pi R_i} \mathcal{L},$$  

(2)

$$\mathcal{L} = -\frac{\Lambda}{4} F_{CD} F^{CD}.$$  

(3)
In Eq. (2), \( F_{CD} \equiv \partial_C A_D - \partial_D A_C \) and \( \Lambda \) is a constant with mass dimension 1, which will be determined later from the requirement that the standard four-dimensional coupling is reproduced. The Latin indices are assumed to run over all coordinates, including the extra dimensions.

The equations of motion in vacuum is \( \partial_C (\sqrt{g} F^{CD}) = 0 \). In the \( A_5 = 0 \) gauge this reads

\[
\eta^{\lambda\nu} \partial_\lambda F_{\nu\mu} = \partial_z (a(z)^{n+2} \partial_z A_\mu)/a(z)^n, \tag{4}
\]

\[
\partial_z (\eta^{\mu\nu} \partial_\mu A_\nu) = 0. \tag{5}
\]

The Greek indices run through 0, 1, 2, 3 (our space-time) and \( \eta^{\mu\nu} \) is the usual Minkowsky metric. As pointed out in [5], this system of equations has an obvious solution that is independent of \( z \), \( A_\mu(x,z) \rightarrow A_\mu(x) \). Eq. (5) is trivially satisfied in this case, while Eq. (4) becomes the usual Maxwell’s equation for a massless photon. This solution describes the zero mode localized on the brane. The reason this is so is because the eigenfunctions are normalized with the integration measure \( \int dza^n \) (hence the need to introduce the compact dimensions).

In general, the eigenfunctions are plane waves, \( e^{-ipx} \), as a function of \( x = 0, 1, 2, 3 \), owing to the fact that the Poincare invariance along the brane is preserved. The \( z \) dependence of the eigenfunctions is given by the eigenmodes of the operator on the right hand side of Eq. (4). Denoting the eigenvalue by \( m^2 \), we find that \( m^2 = p^2 \)

\[
-\partial_z^2 A_\mu(z) + (2 + n) k \text{sign}(z) \partial_z A_\mu(z) = e^{2k|z|} m^2 A_\mu(z). \tag{6}
\]

From the four-dimensional point of view, the higher modes behave as massive photons (Eq. (4) for a given value of \( m^2 \neq 0 \) takes the form of the Proca equation). As we will see shortly, their eigenfunctions are strongly suppressed on the brane.

Further physical insight can be gained by recasting this equation in the Schrödinger form. After changing the variable \( z \rightarrow s = \text{sign}(z)[\exp(k|z|) - 1] \) and the redefinition of the fields as \( A_\mu(z) \rightarrow \phi_\mu(z) = A_\mu(z) \exp[-k|z|(n+1)/2] \) we get

\[
\left[ -\frac{1}{2} \frac{\partial^2}{\partial s^2} + \frac{(n + 1)(n + 3)}{8(|s| + 1)^2} - \frac{n + 1}{2} \delta(s) \right] \phi_\mu = \frac{m^2}{2k^2} \phi_\mu. \tag{7}
\]

This transformation is similar to what was done in [8] for the graviton. Not only has the first derivative term disappeared, but also the measure with which \( \phi \) is normalized is trivial,
\[ \int ds \] Eq. (7) thus describes a non-relativistic Schrödinger problem and the usual physical intuition fully applies here. We have a particle of unit mass in a “volcano” potential\(^2\), with a confining \( \delta \)-function at the origin and a positive barrier outside that slopes off to zero as \( |s| \to \infty \). This potential can support a single bound state of zero energy with the wave function \( \phi_0(s) = \sqrt{n/2}(1+|s|)^{-(n+1)/2} \), which corresponds to the flat solution of the original equation.

It is clear in this picture that the spectrum of states residing away from the origin starts from zero energy and is continuous. This means that the localized state is only marginally bound: an infinitely small perturbation to this setup that lifts the zero mode, \( 0 \to E' \equiv m^2/2k^2 \) (for example by decreasing in absolute value the coefficient of the \( \delta \)-function) of the localized state makes it metastable. The particle can then tunnel through the potential barrier and escape from the brane \([5]\). The eigenvalue in this case becomes complex and the eigenfunction at \( |z| \to \infty \) has an asymptotic form of outgoing plane waves.

The decay rate due to tunneling for this class of problems can be estimated as follows. The turning points of the tunneling on either side of the brane are given by the condition \( (n+1)(n+3)/8(|s_0|+1)^2 = E' \). For \( s \gtrsim |s_0| \) the solution asymptotes to the plane wave, \( a(E')e^{-i\sqrt{2E'}|s|} \), while for \( s \lesssim |s_0| \) it can be approximated by the unperturbed function, \( \sqrt{n/2}(1+|s|)^{-(n+1)/2} \). The amplitude of the plane wave \( a(E') \) (the barrier penetration factor) can then be estimated as roughly the unperturbed solution at the turning point, \( |a(E')|^2 \sim (E')^{(n+1)/2} \). The flux away from the brane computed at large \( |s| \) equals \( 2|a(E')|^2\sqrt{2E'} \sim (E')^{(n+2)/2} \). The ratio of the decay rate \( \Gamma' \) to the energy of the metastable state \( E' \) is \( \sim (E')^{n/2} \), true in any system of units. In the normal units in which the energy is \( m \), we thus obtain for the decay rate in the rest frame \([17]\)

\[ \Gamma_0^{\text{vac}} = c_n m (m/k)^n. \] (8)

The numerical coefficient \( c_n \) can be found by considering the properties of the exact solution given by the Hankel functions \([17]\). We find

\[ c_n = (\pi n)/(2^{n+1}\Gamma|n/2 + 1|^2), \] (9)

\(^{2}\) Notice that the coefficients in Eq. (7) are different from those for the graviton, which has a localized solution even for \( n = 0 \).
where $\Gamma$ denotes the gamma function. Numerically, $c_n = (1, \pi/4, 1/3, \pi/32, 1/45, \ldots)$ for $n = (1, 2, 3, 4, 5, \ldots)$.

We can also now easily see that the continuum modes residing in the bulk are suppressed on the brane. Indeed, they have to tunnel to the brane from the outside. This suppresses the wave functions by the barrier penetration factor $\sim (m/k)^{(n+1)/2}$, making the model phenomenologically viable for energies $\ll k$.

Another way to describe the escape into the extra dimensions is by inspecting the propagator between two points on the brane $[17]$. As shown in $[17]$, the Fourier transform of this propagator for a massless localized scalar in the Randall-Sundrum background ($n = 0$) is

$$\frac{p H_1^{(1)}(p/k)}{k H_2^{(1)}(p/k)} - 1 \approx 2k^2 \left[ p^2 + i \frac{\pi}{(\Gamma(2))^2} p^2 \left( \frac{p}{2k} \right)^2 \right]^{-1},$$

where $H^{(1)}$ denotes the Hankel function of the first kind and $p \equiv \sqrt{p^2}$, where $p^2$ is the square of the four dimensional momentum. This approach makes it very clear that time-like virtual particles are also subject to tunneling. For $p^2 > 0$, up to the overall normalization factor, this propagator has a standard Breit-Wigner form $\left(p^2 + i p \Gamma\right)^{-1}$ with the imaginary part giving the decay rate, $\Gamma(\gamma^* \rightarrow \text{extra dim}) = (\pi/4)\sqrt{p^2}(p^2/k^2)$.

The time-like virtual photon is formed, e.g., in $e^+e^-$ annihilation at colliders. The bound from the measurements of the $Z$ width at LEP $[16]$ is $k \gtrsim m_Z(c_n m_Z/\Delta \Gamma_Z^{\text{inv}})^{1/n}$, where $\Delta \Gamma_Z^{\text{inv}} < 2.0$ MeV is the limit on the additional invisible decay width of $Z$. For $n = 2$ this yields $[16] k \gtrsim 17$ TeV. Clearly, getting a tighter bound on $\Gamma^{\text{inv}}$ would improve the bound, which is the idea behind looking for this process in orthopositronium decay. The invisible width is $\Gamma(o\text{Ps} \rightarrow \text{extra dim}) \sim c_n m_{o\text{Ps}}(m_{o\text{Ps}}/k)^n \alpha^4$ (one power of $\alpha$ comes from the photon vertex, and three more from the wavefunction of $o\text{Ps}$ at the origin), compared to the standard three-photon width, $\Gamma(o\text{Ps} \rightarrow 3\gamma) \sim m_{o\text{Ps}} \alpha^6$ $[18]$. One gets a bound $k \gtrsim m_{o\text{Ps}}(c_n m_{o\text{Ps}}/\Delta \Gamma_{o\text{Ps}}^{\text{inv}})^{1/n} \alpha^{4/n} \approx m_{o\text{Ps}}(c_n/BR_{o\text{Ps}}^{\text{inv}})^{1/n} \alpha^{-2/n}$. Compared to the LEP bound, one trades a factor of $m_Z/m_{o\text{Ps}} \sim 10^5$ for a factor of $(\Delta \Gamma_Z^{\text{inv}}/(m_Z \alpha^2 BR_{o\text{Ps}}^{\text{inv}}))^{1/n}$. Properly keeping track of all coefficients, one finds $[16]$, for $n = 2$, $k \gtrsim 0.5$ TeV with the present accuracy $BR(o\text{Ps} \rightarrow \text{extra dim}) \leq 4.2 \times 10^{-7}$. If the bounds on $BR(o\text{Ps} \rightarrow \text{extra dim})$ are improved to the $10^{-10}$ level, the orthopositronium bound for $n = 2$ would surpass that of LEP. Note that for larger $n$ LEP has a bigger advantage, in particular $k_{\text{LEP}}(n \rightarrow \infty) > m_Z$, while $k_{o\text{Ps}}(n \rightarrow \infty) > m_{o\text{Ps}}$. 


III. PLASMON DECAY TO EXTRA DIMENSION

Let us now consider the effect of plasma on the zero mode. Strictly speaking, one needs to specify how the electrons in plasma are localized to our brane, for example with a domain wall in a new scalar field. While the fine details will be model dependent, the essential features can be obtained by assuming the localization “by hand”, with a delta function, in the spirit of [19].

The effective photon Lagrangian, Eq. (3), gains an additional term, 

$$-(1/2)A_C\Pi^{CD}A_D\delta(z),$$

where $\Pi^{CD} = \langle j_C j_D \rangle$ is the photon self-energy in plasma, or truncated forward scattering matrix element [20]. The presence of this term changes the equation of motion to

$$\Lambda \partial^2_{\sigma} \left( \sqrt{|g|} F^{CD} \right) = -\Pi^{CD} A_C \delta(z).$$

Let us describe the main properties of $\Pi$. First of all, since $\Pi_{55} = \Pi_{5C} = 0$, it has a block diagonal form. In addition we can see from the above equations that $\Pi_{55}$ plays no role in our gauge ($A_5 = 0$). Let us therefore concentrate on the 4–dimensional part $\Pi_{\mu\nu}$. In the hypothesis of isotropic plasma the tensor $\Pi_{\mu\nu}$ is diagonalizable. Because of the gauge invariance, one of the eigenvectors is directed along the photon 4-momentum $q$, and has zero eigenvalue. The others define the directions $\epsilon^{(i)}$ of the different physical polarizations, and have in general non-vanishing eigenvalues $\pi^{(i)}$. If we assume parity invariance the two transverse modes have the same eigenvalue $\pi^T$, whereas the eigenvalue of the longitudinal mode, $\pi^L$, is in general different.

In the basis spanned by $\epsilon^{(i)}$, the equation of motion is diagonal. Eq. (7) in the presence of plasma generalizes to

$$\left[ -\frac{1}{2} \frac{\partial^2}{\partial s^2} + \frac{(n+1)(n+3)}{8(|s|+1)^2} - \left( \frac{n+1}{2} - \frac{\pi^{(i)}}{2k\Lambda} \right) \delta(s) \right] \phi^{(i)} = \frac{m^2}{2k^2} \phi^{(i)},$$

where $\phi^{(i)}$ denotes the components of $\phi$ along $\epsilon^{(i)}$. Treating the plasma term as a perturbation, in the lowest order of perturbation theory we can write that the zero mode is lifted by the energy $\delta E' = \langle \phi_0 | \pi^{(i)} \delta(s) / (2k\Lambda) | \phi_0 \rangle = \pi^{(i)} (n/2) / (2k\Lambda)$. To reproduce the four-dimensional phenomenology, we write $\Lambda = kn/2$, $m^2 = \pi^{(i)}$. The rest of the argument proceeds analogously to the vacuum case considered earlier. The bound state becomes metastable and the corresponding decay rate into extra dimensions in the rest frame is given by

$$\Gamma^{pl}_{0}^{(i)} = c_n \sqrt{\pi^{(i)}} (\sqrt{\pi^{(i)}} / k)^n.$$
The quantities $\pi^{(i)}$ are related to the plasma frequency, $\sqrt{\pi^{(i)}} = \zeta^{(i)} \omega_{\text{pl}}$. Here $\zeta^{(i)}$ is in general a function of the photon energy and momentum. Fortunately, for transverse photons it can be shown \[20, 21\] to be always close to one, $1 \leq \zeta^T \leq \sqrt{3}/2$. Moreover, the contribution of longitudinal photons to stellar cooling rates in all cases of interest to us can be neglected.

Lastly, for the purpose of computing the cooling rate we need the decay rate of a moving plasmon, $\Gamma_{\omega}^{pl}(i)$. The latter is related to the one given in Eq. (12) by the Lorenz factor,

$$\Gamma_{\omega}^{pl}(i) = \Gamma_{0}^{pl}(i) \sqrt{\pi^{(i)}} / \omega = c_n (\zeta^{(i)})^{n+2} \omega_{\text{pl}} / (\omega_{\text{pl}} / k)^n,$$

where $\omega$ is the energy of the plasmon.

**IV. IMPLICATIONS**

**A. Astrophysical Bounds**

The energy loss rate per unit volume in a star is computed as (decay rate) $\times$ (energy loss) $\times$ (photon number density), i.e., as the integral of $\omega \Gamma_{\omega}^{pl}(i)$ over the phase space (e.g., \[20\]),

$$Q_T = \frac{\Gamma \omega}{\pi^2} (\zeta \omega_{\text{pl}})^2 g(\zeta \omega_{\text{pl}} / T),$$

where $g(x) = \int_1^{\infty} (\xi (\xi^2 - 1)) / (\exp (\xi x) - 1) d\xi$, and the subscript $T$ reminds that this is the contribution from transverse photons only.

Let us consider, first, the stars on the Red Giant (RG) branch. For RG stars (at the helium flash) the internal temperature is about $T \simeq 10^8 \text{K}$ and the density $\rho \simeq 10^6 \text{g cm}^{-3}$. In these conditions, the main standard cooling mechanism is the plasmon decay into neutrinos (see, e.g., \[20, 22\]). The rate for this decay is

$$\Gamma_{\text{SM}}^{(i)} = \frac{1}{48 \pi^2 \alpha} \frac{Z^{(i)} C_V^2 G_F^2 \zeta^{(i)6} \omega_{\text{pl}}^6}{\omega},$$

where $Z^{(i)}$ is a renormalization constant whose value is $\simeq 1$ for transverse photons and between $0$ and $1$ for longitudinal photons \[21\], $C_V = 0.96$ is the vector-current coupling constant and $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$ is the Fermi constant. The contribution of longitudinal photons to this cooling is always less than $10\%$ \[23\] and we will neglect it in what follows.
We will also neglect the longitudinal contribution to the non-standard cooling, since this is certainly a conservative assumption.

Stellar models with the cooling rate in Eq. (15) are in good agreement with observations of globular cluster populations [24]. To maintain this agreement, we need to constrain any additional energy loss to not exceed about twice the standard neutrino luminosity [23]. From (14), (15) and (13), we find

\[
\frac{Q_{\text{ED}}}{Q_{\text{SM}}} = \frac{\Gamma_{\text{ED}}}{\Gamma_{\text{SM}}} = \frac{c_n (\zeta T)^{n-4}}{u} \left( \frac{M_W}{\omega_{\text{pl}}} \right)^4 \left( \frac{\omega_{\text{pl}}}{k} \right)^n
\]

where \( u = (C_V^2 g^4)/(1536 \pi^2 \alpha) \simeq 1.5 \times 10^{-3} \), \( g \simeq 0.65 \) is the weak coupling constant, and we set \( Z^T = 1 \). If we impose that this does not exceeds about 2 we find

\[
k \geq \zeta^T c_n^{1/n} \omega_{\text{pl}} B^{1/n}
\]

where for RG stars

\[
B = \frac{1}{2u (\zeta T)^4} \left( \frac{M_W}{\omega_{\text{pl}}} \right)^4.
\]

In the nonrelativistic limit, the plasma frequency is given by \( \omega_{\text{pl}} = 28.7 \text{eV}(Y_e \rho)^{1/2}(1 + (1.0 \times 10^{-6} Y_e \rho)^2)^{1/4} \) [20], where \( \rho \) is in units of g cm\(^{-3}\) and \( Y_e \) is the electron fraction. To be conservative, we take for \( \zeta T \) its largest value and for \( \omega_{\text{pl}} \) its value in the center of the star just before helium flash, \( \omega_{\text{pl}} \simeq 17.8 \text{keV} \) [23], corresponding to \( \rho \simeq 10^6 \text{g cm}^{-3} \). This choice leads to \( B = 6.2 \times 10^{28} \) and to the bounds

\[
k \gtrsim 183 M_W \left( \frac{M_W}{\omega_{\text{pl}}} \right)^3 \simeq 1.4 \times 10^{21} \text{TeV}, \quad (n = 1),
\]

\[
k \gtrsim 13 M_W \left( \frac{M_W}{\omega_{\text{pl}}} \right) \simeq 5 \times 10^6 \text{TeV}, \quad (n = 2),
\]

\[
k \gtrsim 4.5 M_W \left( \frac{M_W}{\omega_{\text{pl}}} \right)^{1/3} \simeq 60 \text{TeV}, \quad (n = 3),
\]

which are many orders of magnitude stronger than the direct laboratory bounds\(^3\).

Next, let us consider the effects of the extra cooling on the supernova SN1987A. It is known that after the explosion this anomalous energy loss cannot significantly exceed \( Q_{\text{Max}} =

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\(^3\) The bound for \( n = 1 \) should be interpreted to mean that the model is excluded for all energies for which it is a valid effective description, possibly up to the Planck scale.
$3 \times 10^{32} \text{erg}^{-1} \text{cm}^{-3} \text{s}^{-1}$ (e.g., [20]), which corresponds to the energy released in neutrinos. The plasma frequency in a SN core is approximately given by $\omega_p^2 \simeq 4\alpha \mu^2 / 3\pi \sim (10\text{MeV})^2$, where $\mu \sim 200\text{MeV}$ [25] is the electron chemical potential. Therefore $\omega_p < T \simeq 30\text{MeV}$, in which case one can approximate $g(x) \simeq 2/x^3$, giving $Q_T \simeq 2\Gamma \omega^3 / \pi^2$. We find again a bound as in Eq. (17), but with

$$B = \frac{2(\zeta^T \omega_p)^2 T^3}{\pi^2 Q_{Max}} \simeq 5.8 \times 10^{19} \left( \frac{T}{30\text{MeV}} \right)^3 \left( \frac{\omega_p}{10\text{MeV}} \right)^2,$$

which implies

$$k \gtrsim 6 \times 10^{14} \text{TeV}, \quad (n = 1),$$

$$k \gtrsim 7 \times 10^{4} \text{TeV}, \quad (n = 2),$$

$$k \gtrsim 27 \text{TeV}, \quad (n = 3),$$

where, to be conservative, we have used $\zeta^T = 1$.

Finally, a similar argument applies to stars on the horizontal branch (HB). Typically HB stars have an average temperature in the Helium core of $T \simeq 0.8 \times 10^8 \text{K}$ and density $\rho \simeq 0.5 \times 10^4 \text{g cm}^{-3}$ [20, 26]. This implies, $\omega_p \simeq 1.5\text{keV} < T$. In this case, an anomalous energy loss cannot be larger than $Q_{Max}/\rho \simeq 10 \text{erg g}^{-1} \text{s}^{-1}$ in order to have good agreement between the predicted and observed number ratio of HB and RG stars [20]. In this case we find ($\zeta^T = 1$) $B = 1.7 \times 10^{30} T_8^3 (\omega_p/1.5\text{keV})^2$, where $T_8 = T / 10^8 \text{K}$. The corresponding bounds are

$$k \gtrsim 1.1 \times 10^{21} \text{TeV}, \quad (n = 1),$$

$$k \gtrsim 1.1 \times 10^6 \text{TeV}, \quad (n = 2),$$

$$k \gtrsim 9 \text{TeV}, \quad (n = 3).$$

### B. Implications for Orthopositronium Decay

The bounds we just found can be directly translated into the value of the branching ratio (BR) necessary to have an analogous bound from the orthopositronium experiment:

$$\text{BR} = \frac{\Gamma(o_{Ps} \rightarrow \text{extra dim})}{\Gamma(o_{Ps} \rightarrow 3\gamma)} \simeq 1.5 \times 10^5 c_n \left( \frac{m_{o_{Ps}}}{k} \right)^n$$

$$< \frac{1.5 \times 10^5}{B} \left( \frac{m_{o_{Ps}}}{\omega_p} \right)^n,$$
where, in the last step, we set $\zeta^T = 1$ for simplicity. Approximately this means $\text{BR} \lesssim 2 \times 10^{-24+1.75n}$, from RG, $\text{BR} \lesssim 2 \times 10^{-15-1.48n}$, from SN87A, $\text{BR} \lesssim 2 \times 10^{-25+2.8n}$, from HB stars. Thus the astrophysical bounds on the allowed branching ratio of oPs to extra dimensions for $n = 2$ are some 14 orders of magnitude stronger than the present sensitivity of the oPs experiments. Moreover, the bound from supernova cooling is at least 8 orders of magnitude more stringent than the present experiments for any value of $n$.

V. CONCLUSIONS

We have seen that the models in which the photon is gravitationally trapped on the brane face significant constraints from astrophysical considerations. The exact constraint depends on the number $n$ of extra compact dimensions. For $n = 2$ or 3 the AdS curvature $k$ is constrained to be orders of magnitude above the electroweak scale. For $n = 1$ the bound extends all the way to the Planck scale. For $n \geq 4$, the astrophysical bounds are weaker than those coming from the LEP measurement of the $Z$. For any $n$, the astrophysical bounds imply the rate of orthopositronium decay into extra dimensions that is at least eight orders of magnitude smaller than the present experimental sensitivity. It is this implication for the ongoing and planned experiments that provides the main motivation for our work.

A detailed discussion of the implications for the models is beyond the scope of this paper. Briefly, our bounds do not exclude the models, but provide significant constraints on them. One way to keep the scales in the model close to the electroweak scale is by having a large ($n \gtrsim 4$) number of extra dimensions. Another possibility is to arrange for an additional binding mechanism for the photon, besides gravity. The binding energy in the latter case needs to significantly exceed the plasma frequency in the proto-neutron star inside a core-collapse supernova ($\sim 10$ MeV).

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