Gravity in the Einstein-Gauss-Bonnet Theory with the Randall-Sundrum Background

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Abstract

We obtain the full 5D graviton propagator in the Randall-Sundrum model with the Gauss-Bonnet interaction. From the decomposition of the graviton propagator on the brane, we show that localization of gravity arises in the presence of the Gauss-Bonnet term. We also obtain the metric perturbation for observers on the brane with considering the brane bending and compute the amplitude of one massless graviton exchange. For the positive definite amplitude or no ghost states, the sign of the Gauss-Bonnet coefficient should be negative in our convention, which is compatible with string amplitude computations. In that case, the ghost-free condition is sufficient for obtaining the Newtonian gravity. For a vanishing Gauss-Bonnet coefficient, the brane bending allows us to reproduce the correct graviton polarizations for the effective 4D Einstein gravity.

Keywords: [Randall-Sundrum compactification, Gauss-Bonnet interaction, Classical the-
ories of gravity]

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I. INTRODUCTION

The Randall-Sundrum (RS) [1,2] model has drawn much attention recently because of several fascinating features related to the new attempt toward the hierarchy solution. The RS I model [1] is composed of two branes of the opposite tensions located at the orbifold fixed points in 5D non-factorizable geometry (or a slice of $AdS_5$), which was proposed to explain the gauge hierarchy between the Planck and the electroweak scales. Here, the negative tension brane is considered as the visible brane. On the other hand, the RS II model [2] has a single brane with a positive tension in the 5D warped geometry (or a glued patch of two $AdS_5$ spaces with a UV cutoff), where it is possible to get the localization of gravity on the brane even for non-compact extra dimension.

However, the RS I model is cosmologically problematic for several reasons. Firstly, the Hubble parameter, $H \propto \rho^{1/2}$, in the early universe and the negative tension brane as the visible brane may be in conflict, since our universe would give rise to an imaginary Hubble parameter in a later cosmology at low temperatures [3]. Second, there is no mechanism of radius stabilization in the RS I model itself [4]. Third, it is not obvious whether the universe with matter tends to the RS limit necessarily. For solving some of these problems, there was a proposal of radius stabilization with a bulk scalar coupled to branes [5,6], and introduction of the Gauss-Bonnet term [7]. Also, there exists a proposal interpreting the positive tension brane as the visible brane and the negative tension brane as the intermediate scale brane [8] to obtain the $\mu$-term in supergravity through nonrenormalizable superpotential [8]. In the last case, supersymmetry is necessary, which is redundant for a solution of the gauge hierarchy problem but maybe inevitable if some problems such as the unification of of the particle forces is required.

For the RS II model [2], it has been shown that gravity is localized on the brane, which is shown by decomposing the full graviton propagator [10,11]. Here, a localized source induces a localized field, which diminishes as one goes toward the AdS horizon [10,12]. And, the brane bending effect in the existence of matter on the brane is shown to be crucial.
for consistency of the linearized approximation [11] and is necessary to reproduce the 4D Einstein gravity on the brane [10][11]. Discussions related to the AdS/CFT correspondence in the RS II model also have been studied in the literature [14,11,13,15–17] from which one can see that the RS II model is described by the visible matter gravitationally coupled to CFT with a cutoff.

It is important to check the consistency of linearized gravity after the inclusion of the Gauss-Bonnet term which is called the Einstein-Gauss-Bonnet (EGB) theory. The Gauss-Bonnet term is a topological term in $D = 4$ [18,19] and it does not affect the graviton propagator even for the $D > 4$ flat spacetime background [18]. Since there are no higher order derivatives induced from variations of the Gauss-Bonnet term other than the second, it seems that there is no ghost problem in the low energy limit of the EGB theory. Therefore, it is reasonable to take the next leading curvature terms in the action as the Gauss-Bonnet term. For the RS II model in the EGB theory, it is shown that localization of gravity also arises around the RS type static background in view of the graviton mass spectrum [7]. The difference from the original RS II model is that it seems that the localized gravity appears on the negative tension brane for the static backgrounds allowed [7]. In this paper, considering the full graviton propagator and the brane bending effect in the existence of the Gauss-Bonnet term, we show that excitation of ghost particles on the brane is a universal phenomenon in the low energy limit, regardless of our static solution backgrounds. This result is different from that in [20], where it was shown that only one of static solutions could excite ghosts in the EGB theory. Several extensions of the EGB theory with a 5D dilaton field and the general higher order curvature terms were considered for discrete or smooth domain wall solutions in the subsequent papers [16,22–24].

In this paper, we obtain the full 5D graviton propagator in the RS II model with the Gauss-Bonnet term and a manifest expression for localization of gravity which can be shown by the decomposition of the graviton propagator. Because the gauge choice of the metric perturbation is non-trivial with matter on the brane, the simplest gauge condition (the so-called Randall-Sundrum gauge condition [2]) considered without matter is not valid any
more. Nonetheless, in order to maintain the RS gauge even with matter on the brane, it has been shown that the brane is seen to be bent along the extra dimension \[10,25,11,26\]. Without the RS condition the gauge degrees of freedom are not completely removed, and in this case the introduction of the brane bending mode \[10,25,11,26\] becomes crucial to choose the RS gauge and cancel the unphysical scalar degree of freedom of the tensor structure of 5D massless graviton. However, elimination of the scalar degree of freedom is incomplete in the existence of the Gauss-Bonnet term, and thus the 4D Einstein gravity should be modified by the residual scalar degree. In that case, the bending of light travelling near the Sun results in the different value as the factor \((1 - \frac{2}{3} \beta)^{-1}\) of that in the 4D Einstein gravity. (Here we notice that \(\beta\) is the dimensionless quantity given in terms of the Gauss-Bonnet coefficient and bulk cosmological constant as we will see later.) And we also investigate the stability of the static backgrounds from the amplitude of one massless graviton exchange and the Newtonian potential on the brane. As a result, the additional RS type solution is always unstable under perturbations because it excites ghosts giving rise to repulsive gravitational potentials between positive matter sources. On the other hand, for the static solution connected with the original RS solution, if the source relation is given as \(S_{11} + S_{22} \neq 0\), the ghost-free condition requires that the sign of the Gauss-Bonnet coefficient(\(\alpha\)) should be negative in our convention, which is consistent with string amplitude computations \[31\], but otherwise it also allows the positive sign of \(\alpha\). And the condition of obtaining the Newtonian gravity is necessary for the ghost-free condition if \(S_{11} + S_{22} \neq 0\), but otherwise just sufficient.

In Sec. II, we present equations of motion of the EGB theory in the RS background. In Secs. III and IV we obtain Green’s function and gravitational potential, respectively. In Sec. V solutions of the metric components are derived, and in Sec. VI we comment on the possibility of the light bending effect in the EGB theory. Sec. VII is a conclusion.
II. THE EINSTEIN-GAUSS-BONNET THEORY IN THE RANDALL-SUNDRUM BACKGROUND

The RS metric of 5D warped spacetime consistent with the orbifold symmetry \( y \to -y \) is given by

\[
ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2
\]

\[\equiv \bar{g}_{MN} dx^M dx^N\]

where \( y \) is the fifth coordinate with \( y \in (-\infty, \infty) \), of which just the half \([0, \infty)\) will be considered and \( k \) is the AdS curvature scale. Even with the Gauss-Bonnet term added in the 5D Einstein gravity (see Appendix A for the model setup), the static solutions are shown to have the same form as the RS metric, except the definition of \( k \): \[k = k_\pm \equiv \left( \frac{M^2}{4\alpha} \left[ 1 \pm \sqrt{1 + \frac{4\alpha \Lambda_b}{3M^5}} \right] \right)^{1/2}\]

where \( M, \alpha \) and \( \Lambda_b \) are the 5D fundamental scale, the dimensionless parameter of the Gauss-Bonnet term and the bulk cosmological constant, respectively.

Let us expand the EGB theory around the RS metric,

\[g_{MN} = \bar{g}_{MN} + h_{MN},\]

with the RS gauge condition \[h_{55} = h_{5\mu} = 0, \quad \text{(Gaussian normal(GN) condition)}\]

and

\[h_{\mu} = \partial^\mu h_{\mu\nu} = 0 \quad \text{(4D transverse traceless(TT) condition).}\]

Then we obtain the following linearized equations of motion for the case of a single brane without matter,

\[
\left[ -\frac{1}{2} \left( 1 - \frac{4\alpha k^2}{M^2} + \frac{8\alpha k^2}{M^2} \delta(y) \right) d^2 e^{2k|y|} - \frac{1}{2} \left( 1 - \frac{4\alpha k^2}{M^2} \right) \partial_y^2 + \frac{8\alpha k^2}{M^2} \text{sgn}(y) \delta(y) \partial_y \\
+ 2k^2 \left( 1 - \frac{4\alpha k^2}{M^2} \right) - 2k \left( 1 - \frac{12\alpha k^2}{M^2} \right) \delta(y) \right] h_{\mu\nu}(x, y) = 0.
\]
The general higher curvature terms tend to give rise to the delocalization of gravity \[21, 23\], but investigation of the graviton mass spectrum of the above equation shows \[7\] that there is one bound state of massless graviton on the brane which is identified as the 4D graviton state, while the Kaluza-Klein modes give rise to small corrections to the Newtonian gravity for the length scale larger than the fundamental length scale. The fact that the localization of gravity is allowed in the EGB theory has been shown previously in \[21, 23\] by integrating the action with respect to the extra dimension. In general, however, with or without the Gauss-Bonnet term, the metric does not satisfy the RS gauge condition on the brane with matter unless the matter energy-momentum tensor satisfies a specific condition, \(T_{\mu}{}^\mu = 0\), as we will see later. Thus, it is necessary to start with the more general gauge other than the RS gauge \[11, 27, 28, 21\] or find out the metric junction condition on the brane by rewriting the Einstein’s equations in terms of the extrinsic curvature tensor \[10, 29\].

There exists an ambiguity of gauge choice for the metric perturbation around the RS background. For the massless state in the Kaluza-Klein reduction of the RS model, the 5D TT condition, \(\partial^Mh_{MN} = 0 = h_M{}^M\), leads us to be left with 5 degrees of freedom (DOF) of massless state after the residual gauge transformations, but with 6 DOFs without the trace free condition. The 5(6) DOFs are composed of two massless traceless symmetric tensor modes, two massless vector modes and one (two) massless scalar mode(s) in the 4D sense. Particularly, for the Gaussian normal condition, two vector modes and one scalar mode are decoupled while two tensor modes remain for 5D TT condition [Case (1)], and two tensor modes and one scalar mode remain for 5D transverse condition [Case (2)]. Case (1) does not satisfy the consistent boundary condition for the existence of matter on the brane, but it may do so if we make loose the GN condition to: \(h_{5\mu} = 0\) and \(h_{55}(= -h_\mu{}^\mu) \neq 0\). On the other hand, for Case (2), which can be shown to satisfy the boundary condition with matter on the brane, there exists an additional unphysical scalar degree of freedom, which is shown to be cancelled by the fifth coordinate transformation (brane bending) maintaining the metric as a GN form \[10, 26\].

When we choose just the Gaussian normal condition without the 4D transverse traceless
condition, the linearized equations of motion for \( h_{\mu\nu} \) and \( h \equiv h^\mu_{\mu} \) with matter on the brane are given as follows (see Appendix A for the details),

\[
\begin{align*}
\left[ -\frac{1}{2} \left( 1 - \frac{4\alpha k^2}{M^2} + \frac{8\alpha k^2}{M^2} \delta(y) \right) \partial_y^2 - \frac{1}{2} \left( 1 - \frac{4\alpha k^2}{M^2} \right) \partial_y^2 + \frac{8\alpha k^2}{M^2} \text{sgn}(y) \delta(y) \partial_y \\
+ 2k \left( 1 - \frac{4\alpha k^2}{M^2} \right) - 2k \left( 1 - \frac{12\alpha k^2}{M^2} \right) \delta(y) \right] (h_{\mu\nu} - \eta_{\mu\nu} h) \\
+ \frac{1}{2} \left( 1 - \frac{4\alpha k^2}{M^2} + \frac{8\alpha k^2}{M^2} \delta(y) \right) e^{2k|y|} (2\partial_\mu \partial_\lambda h_{\nu\lambda} - \partial_\mu \partial_\nu h - \eta_{\mu\nu} \partial_\lambda h^{\lambda\rho}) = M^{-3} T_{\mu\nu},
\end{align*}
\]

(8)

\[
\left( 1 - \frac{4\alpha k^2}{M^2} \right) \delta^{(1)} R_{5\mu} = M^{-3} T_{5\mu},
\]

(9)

\[
\left( 1 - \frac{4\alpha k^2}{M^2} \right) \delta^{(1)} \left( R_{55} - \frac{1}{2} g_{55} R \right) = M^{-3} T_{55},
\]

(10)

where

\[
T_{\mu\nu} \equiv S_{\mu\nu}(x) \delta(y),
\]

(11)

\[
\delta^{(1)} R_{5\mu} = \frac{1}{2} \partial_y \left[ e^{2k|y|} (\partial_\lambda h_{\lambda\mu} - \partial_\mu h) \right],
\]

(12)

\[
\delta^{(1)} \left( R_{55} - \frac{1}{2} g_{55} R \right) = -\frac{1}{2} e^{4k|y|} \partial^\mu \partial^\nu (h_{\mu\nu} - \eta_{\mu\nu} h) - \frac{3}{2} k \text{sgn}(y) \partial_y (e^{2k|y|} h).
\]

(13)

Here, \( \delta^{(1)} \) denotes the linear perturbation. If we take the trace on both sides of the \((\mu\nu)\)-component of Eq. (8) and use the \((55)\)-component of Eq. (10), we obtain the equation of motion for the region \( y > 0 \) and the boundary condition at \( y = 0 \) for \( h \) as follows,

\[
\partial_y \left[ e^{-2ky} \partial_y (e^{2ky} h) \right] = \frac{2}{3} M^{-3} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \left( T_{\mu\mu} - 2 e^{-2ky} T_{55} \right),
\]

(14)

\[
e^{-2ky} \partial_y (e^{2ky} h)|_{y=0} = \frac{1}{3} M^{-3} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \left[ S_{\mu\mu}(x) - 2 \left( \frac{8\alpha k^2}{1 - 4\alpha k^2/M^2} \right) T_{55}(0) \right].
\]

(15)

The \( h \) equation can be rewritten by using the conservation of \( T \), i.e. \( \nabla_M T^{M5} = 0 \) (where \( \nabla \) denotes the action on the background),

\[
\partial_y \left[ e^{-2ky} \left( \partial_y (e^{2ky} h) + \frac{2}{3k} M^{-3} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} T_{55} \right) \right] = -\frac{2}{3k} M^{-3} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \partial^\mu T_{\mu5}.
\]

(16)
III. GREEN’S FUNCTION

From Eq. (14) or Eq. (16), if $T_\mu^\mu \neq 0$ or $\partial^\mu T_\mu^5 \neq 0$, the trace $h$ has the exponentially growing component, in which case the linear approximation breaks down. So, to cancel the growing component, let us consider a deformation of coordinates maintaining the metric as a Gaussian normal form [10,25,11]:

$$y' = y - \xi^5(x),$$
$$x'^\mu = x^\mu - \xi^\mu(x, y)$$
$$= x^\mu + \frac{1}{2k} e^{2ky} \eta^{\mu\lambda} \partial_\lambda \xi^5(x) - \hat{\xi}^\mu(x),$$

such that

$$ds^2 = g_{\mu\nu}(x, y) dx^\mu dx^\nu + dy^2 = g'_{\mu\nu}(x', y') dx'^\mu dx'^\nu + dy'^2.$$  \hspace{1cm} (17)

Then, the metric perturbation transforms as

$$h_{\mu\nu}(x, y) \rightarrow h'_{\mu\nu}(x, y) = h_{\mu\nu}(x, y) - \frac{1}{k} \partial_\mu \partial_\nu \xi^5(x) + e^{-2ky}[2\partial_{(\mu} \hat{\xi}_{\nu)}(x) - 2k\xi^5(x)]\eta_{\mu\nu}. \hspace{1cm} (19)$$

When we integrate both sides of the Eq. (16) with respect to $y$ with the initial condition at $y = 0$, Eq. (15), we have the exponentially growing component of $h$ eliminated by the gauge choice with

$$\partial_\mu \partial^\mu \xi^5(x) = \frac{1}{3} M^{-3} \left(1 - \frac{4\alpha k^2}{M^2}\right)^{-1} \left[\frac{S_\mu^\mu(x)}{2} + \frac{1}{k} \left(1 - \frac{12\alpha k^2}{M^2}\right) T_5^5(0) - \frac{1}{k} \int_0^{y_m} dy \partial^\mu T_\mu^5\right] \hspace{1cm} (20)$$

where $y_m$ denotes the range of the matter distribution along the $y$ coordinate. For the matter localized on the brane, we have the solution for $\xi^5$ as

$$\xi^5(x) = \frac{1}{6} M^{-3} \left(1 - \frac{4\alpha k^2}{M^2}\right)^{-1} \int d^4 x' G_4(x, x') S_\chi^\lambda(x') \hspace{1cm} (21)$$

where $G_4(x, x')$ is Green’s function for the massless scalar in the 4D Minkowski space. Then, in the primed coordinate, integrating Eq. (16) with respect to $y$, we obtain
\[ \partial_y (e^{2ky} h') = -\frac{2}{3k} M^{-3} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \left[ T_{55}(y) - e^{2ky} \int^y \! dy \partial^\mu T_{\mu 5} \right]. \]  

(22)

On the other hand, we obtain the \((5\mu)\)-component in the primed coordinate from Eq. (9),

\[ \partial_y (e^{2ky} \partial^\lambda h'_{\mu \lambda}) = \partial_y (e^{2ky} \partial^\mu h') + 2M^{-3} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} T_{5\mu}. \]  

(23)

Then, with the definition of the following metric,

\[ \bar{h}_{\mu \nu} \equiv h_{\mu \nu} - \frac{1}{2} \eta_{\mu \nu} h, \]  

(24)

from Eq. (23) and Eq. (22), we always have

\[ \partial_y (e^{2ky} \partial^\lambda \bar{h}'_{\mu \lambda}) = 0. \text{ for } y > y_m. \]  

(25)

[Note that matter does not exist beyond \(y_m\) and \(T_{55}(y > y_m) = 0\).] Moreover, we can obtain the de Donder gauge condition \(\partial^\lambda \bar{h}'_{\mu \lambda} = 0\) just outside the matter distribution by the remaining gauge invariance \(\xi_\mu\) and the same for \(y > y_m\) using Eq. (25). Therefore, when there exists the matter source only on the brane (i.e, \(T_{5\mu} = T_{55} = 0\)), we can always choose the 4D transverse traceless gauge \(\partial^\mu \bar{h}'_{\mu \lambda} = 0\) both in the bulk and on the brane.

By the way, the boundary condition of the \((\mu \nu)\)-component on the brane is given from the Eq. (8) as follows,

\[ \partial_y \bar{h}_{\mu \nu} = \frac{12\alpha k^2}{M^2} \left( \partial_y + 2k \right) \left( \bar{h}_{\mu \nu} + \frac{1}{2} \eta_{\mu \nu} \bar{h} \right) \bigg|_{y=+0} \]

\[ - \frac{4\alpha k}{M^2} e^{2ky} \left( \partial^\lambda \bar{h}'_{\mu \lambda} - \partial^\mu \partial^\lambda \bar{h}'_{\nu \lambda} - \partial^\nu \partial^\lambda \bar{h}'_{\mu \lambda} + \eta_{\mu \nu} \partial^\lambda \partial^\rho \bar{h}'_{\lambda \rho} \right) \bigg|_{y=+0} = -M^{-3} S_{\mu \nu}. \]  

(26)

Then, with the metric \(\bar{h}_{\mu \nu}\), we can rewrite the above boundary condition as follows,

\[ \left( 1 - \frac{12\alpha k^2}{M^2} \right) \left( \partial_y + 2k \right) \left( \bar{h}_{\mu \nu} + \frac{1}{2} \eta_{\mu \nu} \bar{h} \right) \bigg|_{y=+0} \]

\[ + \frac{4\alpha k}{M^2} e^{2ky} \left( \partial^\lambda \bar{h}'_{\mu \lambda} - \partial^\mu \partial^\lambda \bar{h}'_{\nu \lambda} - \partial^\nu \partial^\lambda \bar{h}'_{\mu \lambda} + \eta_{\mu \nu} \partial^\lambda \partial^\rho \bar{h}'_{\lambda \rho} \right) \bigg|_{y=+0} = -M^{-3} S_{\mu \nu}. \]  

(27)

which becomes in terms of \(\bar{h}'_{\mu \nu}\),

\[ \left( 1 - \frac{12\alpha k^2}{M^2} \right) \left( \partial_y + 2k \right) \left( \bar{h}'_{\mu \nu} + \frac{1}{2} \eta_{\mu \nu} \bar{h}' \right) \bigg|_{y=+0} \]

\[ + \frac{4\alpha k}{M^2} e^{2ky} \left( \partial^\lambda \bar{h}'_{\mu \lambda} - \partial^\mu \partial^\lambda \bar{h}'_{\nu \lambda} - \partial^\nu \partial^\lambda \bar{h}'_{\mu \lambda} + \eta_{\mu \nu} \partial^\lambda \partial^\rho \bar{h}'_{\lambda \rho} \right) \bigg|_{y=+0} \]

\[ = -M^{-3} S_{\mu \nu} - 2 \left( 1 - \frac{12\alpha k^2}{M^2} \right) \left( \partial^\mu \partial^\nu - \eta_{\mu \nu} \partial^\lambda \right) \xi^5. \]  

(28)
When $\partial^\mu h'_\mu = 0 = h'$ [Note the definitions given in Eqs. (19) and (24).] is chosen on the brane by the gauge shift of $\hat{\xi}_\mu$, we obtain the boundary condition in the primed coordinate,

\[
(1 - \frac{12\alpha k^2}{M^2})(\partial_y + 2k)h'_\mu|_{y=+0} + \frac{4\alpha k}{M^2}2k\eta \partial_\lambda^2 h'_\mu|_{y=+0} = -M^{-3}S_{\mu\nu} - 2\left(1 - \frac{4\alpha k^2}{M^2}\right)(\partial_\mu \partial_\nu - \eta_{\mu\nu}\partial_\lambda^2)\xi^5.
\]

(29)

Then, by using the Eq. (21), it is shown that the trace of the above equation is consistent with the traceless condition $h' = 0$. Note that the transverse condition $\partial^\mu h'_\mu = 0$ implies the conservation of the 4D energy-momentum tensor: $\partial^\mu S_{\mu\nu} = 0$.

Consequently, considering the Eq. (7) in the RS gauge for the case without matter on the brane, the linearized equations of motion for the brane with matter becomes in the RS gauge,

\[
\left[\left(1 - \frac{4\alpha k^2}{M^2} + \frac{8\alpha k}{M^2}\delta(y)\right)\partial^2_{\lambda} e^{2k|y|} + \left(1 - \frac{4\alpha k^2}{M^2}\right)\partial^2_y - \frac{16\alpha k^2}{M^2} \text{sgn}(y)\delta(y)\partial_y - 4k^2\left(1 - \frac{4\alpha k^2}{M^2}\right) + 4k\left(1 - \frac{12\alpha k^2}{M^2}\right)\delta(y)\right]h'_\mu\nu(x, y) = -2M^{-3}\Sigma_{\mu\nu}(x)\delta(y),
\]

(30)

where

\[
\Sigma_{\mu\nu} \equiv S_{\mu\nu} + 2M^2\left(1 - \frac{4\alpha k^2}{M^2}\right)(\partial_\mu \partial_\nu - \eta_{\mu\nu}\partial_\lambda^2)\xi^5.
\]

(31)

Therefore, in the RS gauge for which the brane is located at $y = -\xi^5(x)$, we can obtain the metric perturbation in terms of the graviton propagator as

\[
h'_\mu\nu(x, y) = -2M^{-3} \int d^4x' \int_0^{\infty} dy' G_5(x, y; x', y')\Sigma_{\mu\nu}(x')\delta(y')
\]

\[
= -M^{-3} \int d^4x' G_5(x, y; x', 0)\Sigma_{\mu\nu}(x')
\]

\[
= -M^{-3} \int d^4x' G_5(x, y; x', 0)\left[S_{\mu\nu}(x') - \frac{1}{3}\eta_{\mu\nu}S_{\lambda}(x') + \frac{1}{3}\partial_{\mu}\partial_{\nu}\frac{1}{2}\partial_\rho S_{\lambda}(x')\right],
\]

(32)

where Eq. (21) is used in the last line and we note that the graviton propagator is well defined for either side of the brane, i.e. valid for the half of the bulk space $y > 0$ and the other half by the $Z_2$ symmetry. However, the metric $h'_\mu\nu$ is not appropriate for observers on the brane since the brane becomes bent as $y = -\xi^5(x)$ in the RS gauge. Nonetheless, there exists a simpler gauge for observers on the brane in which case the brane is located at $y = 0$.
as before the coordinate transformation. It will be shown in Sec. V. Then, the equation of motion for the graviton propagator is given by

\[
\left[ \left( 1 - \frac{4\alpha k^2}{M^2} \right) - 4k^2 \left( 1 - \frac{4\alpha k^2}{M^2} \right) \right] \delta(y) \partial^2_e - \frac{16\alpha k^2}{M^2} \text{sgn}(y) \delta(y) \partial y
\]

\[
G_5(x, y; x', y') = \delta^4(x - x') \delta(y - y'). \tag{33}
\]

Now we can decompose the graviton propagator into Fourier modes as

\[
G_5(x, y; x', y') = \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} G_p(y, y'), \tag{34}
\]

and the equation of motion for the Fourier mode \( G_p(y, y') \) is given by

\[
\left[ \left( 1 - \frac{4\alpha k^2}{M^2} \right) - 4k^2 \left( 1 - \frac{4\alpha k^2}{M^2} \right) \right] \delta(y) \partial^2_e - \frac{16\alpha k^2}{M^2} \text{sgn}(y) \delta(y) \partial y
\]

\[
G_p(y, y') = \delta(y - y'). \tag{35}
\]

### IV. GRAVITATIONAL POTENTIAL

By a change of variable \( z \equiv \frac{ky}{k} \), the following equation of motion and the boundary condition are satisfied by the propagator,

\[
(z^2 \partial^2_z + z \partial_z - p^2 z^2 - 4)G_p(z, z') = \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \frac{1}{k} \delta(z - z') \tag{36}
\]

\[
(z \partial_z + 2 - \beta p^2 z^2)G_p(z, z') \big|_{z=1/k} = 0 \tag{37}
\]

where

\[
\beta \equiv \frac{4\alpha k^2/M^2}{1 - 12\alpha k^2/M^2}. \tag{38}
\]

The solution for the graviton propagator is composed of the Bessel functions of order 2, \( J_2(q/k) \) and \( Y_2(q/k), q^2 = -p^2 \) and it must satisfy Eq. \( \text{(37)} \) and the boundary conditions at \( z = z' \),

\[
G_5|_{z=z'} = G_5|_{z=z'}, \tag{39}
\]

\[
\partial_z(G_\geq - G_\leq)|_{z=z'} = \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \frac{1}{k z'} \tag{40}
\]
where > (<) indicates \( z \) larger (lesser) than \( z' \).

For \( z < z' \), taking into account the Eq. (37), we have

\[
G_<(z, z') = A(z') \left[ \left( Y_1(q/k) + \frac{\beta q}{k} Y_2(q/k) \right) J_2(qz) - \left( J_1(q/k) + \frac{\beta q}{k} J_2(q/k) \right) Y_2(qz) \right] = iA(z') \left[ \left( J_1(q/k) + \frac{\beta q}{k} J_2(q/k) \right) H_2^{(1)}(qz) - \left( H_1^{(1)}(q/k) + \frac{\beta q}{k} H_2^{(1)}(q/k) \right) J_2(qz) \right]
\]

(41)

where \( H_1^{(1)} = J_{1,2} + iY_{1,2} \) is the first Hankel function. On the other hand, let us turn to the range of \( z > z' \). Requiring that positive frequency gravitational waves be ingoing into the AdS horizon [11], which implies that there is no radiation reemitted from the AdS horizon, we also obtain

\[
G_>(z, z') = B(z') H_2^{(1)}(qz).
\]

(42)

From the matching conditions Eq. (33) and Eq. (40), we get the graviton propagator in momentum space as follows,

\[
G_p(z, z') = \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \left( \frac{i\pi}{2k} \right) \left[ \left( \frac{J_1(q/k) + \frac{\beta q}{k} J_2(q/k)}{H_1^{(1)}(q/k) + \frac{\beta q}{k} H_2^{(1)}(q/k)} \right) H_2^{(1)}(qz_<) H_2^{(1)}(qz_>) - J_2(qz_<) H_2^{(1)}(qz_>) \right].
\]

(43)

Then, for the source on the brane, i.e., \( z' = \frac{1}{k} \), the graviton propagator in coordinate space is given by

\[
G_5(x, z; z', \frac{1}{k}) = \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \left[ \int \frac{H_2^{(1)}(qz)}{q \left( H_1^{(1)}(q/k) + \frac{\beta q}{k} H_2^{(1)}(q/k) \right)} \right].
\]

(44)

Furthermore, using the Bessel recursion relation \( H_2^{(1)}(q/k) = \frac{2k}{q} H_1^{(1)}(q/k) - H_0^{(1)}(q/k) \), we have the graviton propagator on the brane decomposed into two terms, corresponding to the massless mode and the Kaluza-Klein (KK) states, respectively,

\[
G_5(x, \frac{1}{k}; x', \frac{1}{k}) = \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \left[ \left( \frac{2k}{1 + 2\beta} \right) G_4(x, x') \right] + G_{KK}(x, x'),
\]

(45)

where
\[ G_4(x, x') \equiv \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{q^2}, \quad (46) \]
\[ G_{KK}(x, x') \equiv -\left(1 - \frac{4\alpha k^2}{M^2}\right)^{-1} \left(\frac{1}{1 + 2\beta}\right)^2 \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \cdot \frac{1}{q} \left[ \frac{H_0^{(1)}(q/k)}{H_1^{(1)}(q/k) - \left(\frac{\beta q/k}{1+2\beta}\right) H_0^{(1)}(q/k)} \right]. \quad (47) \]

That is, from the above decomposition of the graviton propagator, we confirm the localization of gravity on the brane even in the existence of the Gauss-Bonnet term, irrespective of the static solution backgrounds. However, since there appears the non-trivial factor in front of the 4D Minkowski Green function, we should be careful with the ghost-like behavior of the resultant Newtonian potential, which will be dealt with later. And, we can see that the Gauss-Bonnet term affects both massless graviton and KK graviton propagators unlike the case in the flat background spacetime [18].

From the exact form of graviton propagator we found above, let us consider the asymptotics of the graviton propagator for a static source on the brane. At large distances \( r \equiv |x - x'| \gg k^{-1} \) or low energies \( p/k \ll 1 \), the graviton propagator on the brane has the following limiting behavior,

\[ V(r) = \int dt G_5(x, \frac{1}{k}; x', \frac{1}{k}) \]
\[ \simeq \left(1 - \frac{4\alpha k^2}{M^2}\right)^{-1} \int \frac{d^3p}{(2\pi)^3} e^{ip(x-x')} \left[ -\left(\frac{1}{1 + 2\beta}\right) \frac{2k}{p^2} + \left(\frac{1}{1 + 2\beta}\right)^2 \frac{1}{k} \ln(p/2k) \right] \quad (48) \]
\[ = -\left(1 - \frac{4\alpha k^2}{M^2}\right)^{-1} \frac{k}{2\pi r} \left[ \frac{1}{1 + 2\beta} + \left(\frac{1}{1 + 2\beta}\right)^2 \frac{1}{2(kr)^2} \right]. \]

We note that the above static limit of the graviton propagator is very similar to the result of Ref. [8]. Nonetheless, the non-trivial factor \((1+2\beta)^{-1}\) as well as the overall factor \((1 - \frac{4\alpha k^2}{M^2})^{-1}\) in front of the Newton potential, \(1/r\), show the crucial point such as the ghost-like behavior in the existence of the Gauss-Bonnet term. However, even if the equation of the graviton propagator does not allow the mode sum in our case because of the non-hermitianity of the differential operator, we may regard the graviton propagator as the sum of massless mode and massive KK modes just as in the RS case [3].
On the other hand, for \( z \gg k^{-1} \) and/or \( r \equiv |x - x'| \gg k^{-1} \), another static limit of the graviton propagator off the brane is given by

\[
V(r, z) = \int dtG_5(x, z; x', \frac{1}{k}) \quad (49)
\]

\[
\simeq \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \left( \frac{1}{1 + 2\beta} \right) \frac{\pi i}{2k} \int \frac{d^3p}{(2\pi)^3} e^{ip(x-x')} H_2^{(1)}(ipz) \quad (50)
\]

\[
\simeq -\left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1} \left( \frac{1}{1 + 2\beta} \right) \frac{3}{4\pi k} \frac{1}{z^3} \frac{1 + \frac{2r^2}{3z^2}}{(1 + \frac{r^2}{z^2})^{3/2}} \quad (51)
\]

As a result, the metric perturbation falls off as \( h \sim \frac{1}{z^3} \) (for \( z \gg r \)), which ensures that the perturbative expansion is valid towards the AdS horizon \([11]\). And, a localized gravitational source produces a localized field, not strong at the horizon.

V. SOLUTIONS FOR THE METRIC COMPONENTS

To find solutions of the initial metric perturbation \( h_{\mu\nu} \), by using the additional freedom \( \hat{\xi}_\mu(x) \) in the inverse transformation of the metric in Eq. (19) to set

\[
\hat{\xi}_\mu(x) = \partial_\mu \left( \frac{1}{2k} \xi^5(x) - \frac{1}{3} M^{-3} \int d^4x' G_5(x, \frac{1}{k}; x', \frac{1}{k}) \frac{1}{\partial_\rho} S_\lambda(x') \right), \quad (52)
\]

we can have a simple expression for the metric perturbation on the brane at \( z = \frac{1}{k} \) whose 4D hypersurface is perpendicular to the AdS horizon,

\[
h_{\mu\nu}(x) = h_{\mu\nu}^{(m)}(x) + h_{\mu\nu}^{(b)}(x) \quad (53)
\]

where \(^{(m)}\) denotes the matter contribution and \(^{(b)}\) denotes the brane bending effect,

\[
h_{\mu\nu}^{(m)}(x) = -M^{-3} \int d^4x' G_5(x, \frac{1}{k}; x', \frac{1}{k}) \left( S_{\mu\nu}(x') - \frac{1}{3} \eta_{\mu\nu} S_\lambda(x') \right) \quad (54)
\]

\[
h_{\mu\nu}^{(b)}(x) = 2k \eta_{\mu\nu} \xi^5(x). \quad (55)
\]

The second term of the metric perturbation \( h_{\mu\nu}^{(b)} \) in Eq. (53) is due to the brane bending \([10,23,11,26]\), which is a scalar mode on the brane coming from the trace part of matter energy-momentum tensor in Eq. (21). We can rewrite Eq. (53) by using Eq. (43) and Eq. (21) as follows,
\[ h_{\mu\nu}(x) = -M_P^{-2} \int d^4x' G_4(x, x') \left[ S_{\mu\nu}(x') - \left( \frac{1}{2} + \frac{1}{3}\beta \right) \eta_{\mu\nu} S^\lambda_\lambda(x') \right] \]

\[ - M^{-3} \int d^4x' G_{KK}(x, x') \left[ S_{\mu\nu}(x') - \frac{1}{3} \eta_{\mu\nu} S^\lambda_\lambda(x') \right] \]

(56)

where brane bending effect contributes only to the trace part. The effective 4D Planck scale \( M_P \) in the limit of vanishing \( \beta \) or \( \alpha \) could be identified as

\[ M_P^2 = \frac{M^3}{2k} \left( 1 - \frac{4\alpha k^2}{M^2} \right) \left( 1 + 2\beta \right) = \frac{1}{16\pi G_N} \]

\[ = \frac{M^3}{2k} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^2 \left( 1 - \frac{12\alpha k^2}{M^2} \right)^{-1}. \]

(57)

From the above expression for the metric perturbation, we can see that as \( \beta \to 0 \) or \( \alpha \to 0 \), the brane bending effect yields the usual factor \( \frac{1}{2} \) of the trace part of the 4D graviton propagator, but otherwise the 4D Einstein gravity could be modified by the additional polarization factor \( \frac{1}{3}\beta \). The case for the bending of light will be treated in Sec. VI.

Let us consider the amplitude for one massless graviton exchange in the presence of the source \( S_{\mu\nu} \). In momentum space, it is given from the first part of the metric perturbation Eq. (56) as follows,

\[ \mathcal{L}_{\text{massless}} = \frac{1}{4} h_{\mu\nu} S^{\mu\nu} \]

\[ = \frac{4\pi G_N}{p^2} \left[ S_{\mu\nu} S^{\mu\nu} - \left( \frac{1}{2} + \frac{1}{3}\beta \right) (S^\lambda_\lambda)^2 \right]. \]

(58)

If we take a Lorentz frame in which \( p^\mu \) is given as a null vector such as

\[ p^0 = p^3, \quad p^1 = p^2 = 0, \]

(59)

we obtain the following source relations, from the transverse condition, \( \partial^\mu S_{\mu\nu} = 0 \),

\[ S_{00} = S_{03} = S_{33}, \quad S_{01} = S_{13}, \quad S_{02} = S_{23}. \]

(60)

Therefore, we obtain the amplitude as

\[ \mathcal{L}_{\text{massless}} = \frac{4\pi G_N}{-p_0^2 + p_3^2} \left[ |S_{+2}|^2 + |S_{-2}|^2 - \frac{1}{3}\beta (S_{11} + S_{22})^2 \right] \]

(61)

where the first two terms are due to the exchange of massless spin-2 graviton, \( S_{\pm2} \equiv \frac{1}{2}(S_{11} - S_{22}) \pm iS_{12} \), and the third term comes from the residual massless graviscalar. To
obtain a positive definite amplitude, the conditions, $G_N > 0$ and $\beta \leq 0$, should be satisfied simultaneously.

For the $k_+$ solution, the amplitude is always negative since $G_N < 0$ so that ghost states could be excited near the background. That is, it means that the $k_+$ solution is unstable under perturbations and therefore we have to exclude it at the perturbative level.

On the other hand, for the $k_-$ solution, if $S_{11} + S_{22} \neq 0$, there should be a bound on the bulk parameters such as $-\frac{1}{3} < \beta \leq 0$ (equivalently, $\alpha \leq 0$) in order to have only the positive norm states. Therefore, the allowed sign of $\alpha$, i.e., $\alpha < 0$ is compatible with string tree amplitude computations [31]. If we require that $S_{11} + S_{22} = 0$, the bound on the bulk parameters becomes $\beta > -\frac{1}{3}; -(5/9) < \frac{4\alpha k^2}{3M^3} \leq 0$ for $\alpha > 0$ and any value for $\alpha < 0$.

By the way, because the tensor structure of the metric perturbation will be modified, the above identification of the effective 4D Planck scale should appear a little different. For instance, in case of a static point source with mass $m$ on the brane, i.e., for the energy-momentum tensor $S_{\mu\nu} = m\delta_{\mu0}\delta_{\nu0}\delta^{(3)}(x)$, components of the metric perturbation due to the matter and the brane bending mode are given by, in accord with Eq. (54) and Eq. (55),

$$h_{00}^{(m)}(x) = -\frac{2}{3}M^{-3}mV(r),$$

$$h_{ij}^{(m)}(x) = -\frac{1}{3}M^{-3}mV(r)\delta_{ij},$$

$$\xi^5(x) = \left(1 - \frac{4\alpha k^2}{M^2}\right)^{-1}\frac{M^{-3}m}{24\pi r}.\tag{64}$$

where $V(r)$ is the same as given in Eq. (49) and $i, j$ run from 1 to 3. Therefore, we obtain the approximate metric perturbation in the unprimed coordinate for a static point source on the brane as the following,

$$h_{00}(x) \simeq \frac{2G_Nm}{r}\left[1 - \frac{2}{3}\beta + \frac{2}{3}\left(\frac{1}{1+2\beta}\right)^2\frac{1}{(kr)^2}\right],$$

$$h_{ij}(x) \simeq \frac{2G_Nm}{r}\left[\left(1 + \frac{2}{3}\beta\right)\left(1 - \frac{2}{3}\beta\right)^{-1}\left(\frac{1}{1+2\beta}\right)^2\frac{1}{(kr)^2}\right]\delta_{ij},\tag{67}$$

where by the Newton potential $\Phi_N = -\frac{1}{2}h_{00}$, the Newton constant is identified as
where the correct effective 4D Planck scale $\bar{M}_P^2$ is given by

$$M^2_P \equiv \left( 1 - \frac{2}{3} \beta \right)^{-1} M_P^2.$$  \hspace{1cm} (69)

In the Eq. (65), the first term of $\frac{1}{r}$ potential stems from the spin-2 graviton while the second term does from the residual graviscalar. But, since the two contributions are of the same behavior as $\frac{1}{r}$, we should identify the Newton constant from the sum of the two as in Eq. (66).

For the $k_+$ solution ($k = k_+$ in Eq. (68)), the Newton constant $G_N$ would be negative and $1 - \frac{2}{3} \beta > 0$ always, which might give rise to repulsive forces of massless and massive gravitons. That is, the $k_+$ solution would give rise to the anti-gravity so that it is shown not to be allowed under perturbations again.

The instability of the $k_+$ solution can be shown from the weak energy condition on the background matter: $T^{(0)}_{MN} \xi^M \xi^N \geq 0$ for any null vector $\xi^M$ [30, 28]. We assume that the background matter $T^{(0)}_{MN}$ localized on the brane (i.e., $T^{(0)}_{55} = T^{(0)}_{5\mu} = 0$) produces the metric with 4D Poincaré invariance as follows,

$$ds^2 = \bar{g}_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2.$$  \hspace{1cm} (70)

From the weak energy condition and the modified Einstein’s equation, $T^{(0)}_{MN} = M^3 (\bar{G}_{MN} + \bar{H}_{MN}) + \Lambda_b \bar{g}_{MN}$, we have the following non-negative condition for the background metric (compare with the result given in Ref. [23]),

$$\sigma'' \left( 1 - \frac{4\alpha (\sigma')^2}{M^2} \right) \geq 0.$$  \hspace{1cm} (71)

Therefore, for $\sigma'' \geq 0$ for the localization of gravity as in the RS case, we have the condition $1 - \frac{4\alpha (\sigma')^2}{M^2} = \mp \sqrt{1 + \frac{4\alpha \Lambda_b}{3M^5}} \geq 0$, for the $k_\pm$ solutions, respectively. Therefore, the $k_-$
solution is the only 4D flat solution allowed with localized gravity that satisfies the weak energy condition. On the other hand, the $k_+$ solution violates the weak energy condition for localized gravity.

On the other hand, for the $k_-$ solution, the attractive normal gravitational potential is allowed for the range of $\alpha$ as $-0.47 < \frac{4\alpha\Lambda_b}{3M^5} \leq 0$ for $\alpha > 0$ and any value for $\alpha < 0$. Therefore, even the $k_-$ solution could give rise to a repulsive force in some bulk parameter space with $\alpha > 0$ even though the weak energy condition is satisfied. Recalling that there is a lower bound in the allowed static solution space itself such as $-1 \leq \frac{4\alpha\Lambda_b}{3M^5} \leq 0$ for $\alpha > 0$ for the $k_-$ solution, we see that the Gauss-Bonnet interaction should be weaker than the bound given in Ref. [7] to avoid the anti-gravity by about the half. However, from the consistency for no ghost states in case of $S_{11} + S_{22} \neq 0$, the sign of $\alpha$ should be negative. And, if $S_{11} + S_{22} = 0$, the ghost-free condition is just necessary for the attractive gravity.

**VI. BENDING OF LIGHT AND MORE ON THE LOCALIZATION PROBLEM**

We can also show that the bending of light passing by the Sun could be modified with the Gauss-Bonnet term. For a source in the $xz$ plane on the brane, the bending of light travelling in the $z$ direction is described by a Newton-like force law, $\ddot{x} = \frac{1}{2}(h_{00} + h_{zz})_{,x}$. Note that corrections to $h_{00}$ and $h_{zz}$ are different [viz. Eqs. (66) and (67)], which modifies the bending of light. If the metric perturbations due to the Sun are approximated by those from a point source, the bending of light is $\left(1 - \frac{2}{3}\beta\right)^{-1}$ of that predicted from the 4D Einstein gravity. Therefore, since the maximum deflection angle of light is given as $\delta \theta_{max} = (0.95 \pm 0.11)1.75''$ from the experimental measurements [22], we can get another bound on the Gauss-Bonnet coefficient as $-0.20 < \frac{4\alpha\Lambda_b}{3M^5} < 1.2$ for the static background connected to the RS solution. As a result, we can see that the ghost bound is compatible with the experimental bound from the bending of light.

Now let us make some comments on quasi-localized gravity theories [33] with the Gauss-Bonnet term. In the quasi-localized gravity, the linearized 4D Newtonian gravity can be
reproduced at the intermediate scale up to some correction proportional to the decay width of metastable graviton while the 5D Einstein gravity remains itself both at the short and large distances \[33\]. But, because the 4D Newtonian gravity is reproduced by the massive 5D graviton, the unwanted polarizations in the tensor structure of the 5D graviton propagator cannot give rise to the correct 4D Einstein gravity \[34\] and even if the brane bending effect cancels the extra polarization at the intermediate scale \[26\], it gives rise to the scalar anti-gravity at the ultra-large distances \[35\]. For our case with the Gauss-Bonnet interaction, we cannot escape such a ghost problem \[33,36,27,28\]. That is, the gravitational potential corresponding to the brane bending mode gives a repulsive force even with the Gauss-Bonnet term as follows,

\[
V(r) = \int dt \left( -\frac{1}{2} h^{(b)}_{00}(x) \right) = \frac{kM^{-3}}{24\pi r} \left( 1 - \frac{4\alpha k^2}{M^2} \right)^{-1}.
\]

Even though the scalar potential could be attractive for the \(k_+\) solution, there should exist the more severe ghost problem in the 4D graviton sector as already discussed in the previous section and moreover the weak energy condition of Eq. (71) would be violated. So we argue that there still appears the scalar anti-gravity in quasi-localized gravity even with the Gauss-Bonnet interaction.

VII. CONCLUSION

In conclusion, we obtained the full 5D graviton propagator in the Einstein-Gauss-Bonnet theory with the Randall-Sundrum background. From the decomposition of the graviton propagator on the brane, we confirmed the localized gravity in the Einstein-Gauss-Bonnet theory. The brane bending effect, even if it is crucial to reproduce the right 4D Einstein gravity in the RS model, does not cancel the extra polarization of the tensor structure of 5D massless graviton completely for the case with the Gauss-Bonnet term. As a result, we can have the bound on the Gauss-Bonnet coefficient from relativistic effects such as the bending of light. And furthermore, if \(S_{11} + S_{22} \neq 0\), the ghost-free condition from the amplitude of one
graviton exchange requires $\alpha \leq 0$, which is compatible with string amplitude computations. If $S_{11} + S_{22} \neq 0$, the ghost-free condition is sufficient for giving the correct Newtonian gravity.

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APPENDIX A: METRIC EXPANSIONS WITH THE GAUSS-BONNET TERM

When the Gauss-Bonnet curvature squared interaction is added as the additional effective interaction to the Einstein-Hilbert term in the RS II model, the 5D action is given by the following,

\[ S = S_0 + S_m \]  \hspace{1cm} (A1)

\[ S_0 = \int d^5 x \sqrt{-g} \left( \frac{M^3}{2} R - \Lambda_b + \frac{1}{2} \alpha M R^2 - 2 \alpha M R_{MN} R^{MN} + \frac{1}{2} \alpha M R_{MPNQ} R^{MPNQ} \right) \]

\[ + \int d^4 x dy \delta(y) \sqrt{-g^{(4)}} (-\Lambda), \]  \hspace{1cm} (A2)

\[ S_m = \int d^5 x \sqrt{-g} L_m \]  \hspace{1cm} (A3)

where \( g, g^{(4)} \) are the determinants of the metrics in the bulk and the brane, \( M \) is the five dimensional gravitational constant, \( \Lambda_b \) and \( \Lambda \) are the bulk and brane cosmological constants, \( \alpha \) is the effective coupling, and \( L_m \) is the matter Lagrangian on the brane or in the bulk.

Equations of motion in this EGB theory are,

\[ G_{MN} + H_{MN} = M^{-3} (-\Lambda_b g_{MN} + T_{MN}) \]  \hspace{1cm} (A4)

where

\[ G_{MN} \equiv R_{MN} - \frac{1}{2} g_{MN} R, \]  \hspace{1cm} (A5)

\[ H_{MN} \equiv \frac{\alpha}{M^2} \left[ - \frac{1}{2} g_{MN} (R^2 - 4 R^2_{PQ} + R_{PQST} R^{PQST}) \right. \]

\[ + 2 R R_{MN} - 4 R_{MP} R^{P}_N - 4 R^K_{MPN} R^{P}_K + 2 R_{MQSP} R^{QSP}_N \right], \]  \hspace{1cm} (A6)

\[ T_{MN} \equiv T^{(0)}_{MN} + T^{(m)}_{MN}, \]  \hspace{1cm} (A7)

\[ T^{(0)}_{MN} \equiv \frac{\sqrt{-g^{(4)}}}{\sqrt{-g}} \delta(y) \text{diag}(-\Lambda, -\Lambda, -\Lambda, -\Lambda, 0), \]  \hspace{1cm} (A8)

\[ T^{(m)}_{MN} \equiv - \frac{2}{\sqrt{-g}} \delta S_m / \delta g^{MN}. \]  \hspace{1cm} (A9)

For the background metric Eq. (70), the nonvanishing components of the Riemann tensor and related things are given by
\[ R_{\mu\nu\lambda\rho} = - (\sigma')^2 (g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}), \]  
(A10)

\[ R_{5\mu\nu} = (\sigma'' - (\sigma')^2) \bar{g}_{\mu\nu}, \]  
(A11)

\[ R_{\mu\nu} = (\sigma'' - 4(\sigma')^2) \bar{g}_{\mu\nu}, \]  
(A12)

\[ R_{55} = 4(\sigma'' - (\sigma')^2), \]  
(A13)

\[ \bar{R} = 8\sigma'' - 20(\sigma')^2. \]  
(A14)

Then, considering the metric fluctuation around the RS background Eq. (7) such as \( g_{MN} = \bar{g}_{MN} + h_{MN}, \) from the general formulae for linearizing the curvature tensors,

\[ \delta^{(1)} R_{MN} = -\frac{1}{2} \bar{\nabla}^2 h_{MN} - \bar{R}_{MNPQ} h^{PQ} + \bar{R}_{(MP} h_{N)P} + \bar{\nabla}_{(M} \bar{\nabla}^{P} h_{N)P} - \frac{1}{2} \bar{\nabla}_{M} \bar{\nabla}_{N} h^{P}_{P} \]  
(A15)

\[ \delta^{(1)} R = - R_{MN} h^{MN} - \bar{\nabla}^2 h_{P}^{P} + \bar{\nabla}_{M} \bar{\nabla}_{N} h^{MN} \]  
(A16)

\[ \delta^{(1)} R^{K}_{MPN} = \frac{1}{2} (\bar{R}^{K}_{QPN} h_{M}^{Q} - \bar{R}^{Q}_{MPN} h_{K}^{Q}) + \bar{\nabla}_{P} \bar{\nabla}_{[M} h_{N]}^{K} - \bar{\nabla}_{(P} \bar{\nabla}^{K} h_{N)M}, \]  
(A17)

we get the following linear expansions with the Gaussian normal condition, \( h_{5\mu} = h_{55} = 0; \)

\[ \delta^{(1)} R_{\mu\nu} = -\frac{1}{2} \bar{g}^{PQ} \partial_{P} \partial_{Q} h_{\mu\nu} - 2(\sigma')^2 h_{\mu\nu} \]

\[ + \epsilon^{2\sigma} \left( \partial_{(\mu} \partial_{(\lambda} h_{\nu\lambda)} - \partial_{\mu} \partial_{\nu} h \right) + \frac{1}{2} \eta_{\mu\nu} \sigma' \partial_{5} h + (\sigma')^2 \eta_{\mu\nu} h \]  
(A18)

\[ \delta^{(1)} R_{5\mu} = \frac{1}{2} \epsilon^{2\sigma} \partial_{5} (\partial^{(\lambda} h_{\mu\lambda)} - \partial_{\mu} h) + \epsilon^{2\sigma} \sigma' (\partial^{(\lambda} h_{\mu\lambda)} - \partial_{\mu} h) = \frac{1}{2} \partial_{5} \left[ \epsilon^{2\sigma} (\partial^{(\lambda} h_{\mu\lambda)} - \partial_{\mu} h) \right] \]  
(A19)

\[ \delta^{(1)} R_{55} = \epsilon^{2\sigma} \left( - \frac{1}{2} \partial_{5}^2 h - \sigma' \partial_{5} h - (\sigma')^2 h \right) \]  
(A20)

\[ \delta^{(1)} R = \epsilon^{2\sigma} \left( \epsilon^{2\sigma} \partial_{\mu} \partial_{5} h_{\mu\nu} - \bar{g}^{PQ} \partial_{P} \partial_{Q} h + \sigma' \partial_{5} h + (-2\sigma'' + 6(\sigma')^2) h \right), \]  
(A21)

and

\[ \delta^{(1)} R^{\lambda}_{\mu
u\rho} = \frac{1}{2} (\bar{R}^{\lambda}_{\mu\nu\rho} h^{Q} - \bar{R}^{Q}_{\mu\nu\rho} h^{\lambda}) + \bar{\nabla}_{\rho} \bar{\nabla}_{\mu} h_{\nu}^{\lambda} - \bar{\nabla}_{\nu} \bar{\nabla}_{\mu} h_{\rho}^{\lambda} - \bar{\nabla}_{\rho} \bar{\nabla}_{\nu} h_{\mu}^{\lambda} + \bar{\nabla}_{\nu} \bar{\nabla}_{\rho} h_{\mu}^{\lambda}) \]

\[ = \frac{1}{2} \left[ \epsilon^{2\sigma} (\partial_{\rho} \partial_{\mu} h_{\nu}^{\lambda} - \partial_{\mu} \partial_{\nu} h_{\rho}^{\lambda}) + \bar{g}^{\lambda\sigma} (-\partial_{\rho} \partial_{\sigma} h_{\mu\nu} + \partial_{\nu} \partial_{\sigma} h_{\mu\rho}) 
\right. \]

\[ + \sigma' (-\eta_{\mu\nu} \partial_{5} h_{\lambda}^{\lambda} + \eta_{\mu\nu} \partial_{5} h_{\rho}^{\lambda} + \bar{g}^{\lambda\rho} \partial_{5} h_{\mu\nu} - \bar{g}^{\lambda\nu} \partial_{5} h_{\mu\rho}) + 2(\sigma')^2 (-\eta_{\mu\nu} h_{\lambda}^{\lambda} + \eta_{\mu\nu} h_{\rho}^{\lambda}) + 2(\sigma')^2 (-\eta_{\mu\nu} h_{\lambda}^{\lambda} + \eta_{\mu\nu} h_{\rho}^{\lambda}) \]  
(A22)

\[ \delta^{(1)} R^{5}_{\mu\nu5} = \frac{1}{2} (\bar{R}^{5}_{\mu\nu5} h^{Q} - \bar{R}^{Q}_{\mu\nu5} h^{5}) + \bar{\nabla}_{5} \bar{\nabla}_{\mu} h_{\nu}^{5} - \bar{\nabla}_{\nu} \bar{\nabla}_{\mu} h_{5}^{5} - \bar{\nabla}_{\rho} \bar{\nabla}_{\nu} h_{\mu}^{5} + \bar{\nabla}_{\nu} \bar{\nabla}_{\rho} h_{\mu}^{5}) \]

\[ = -\frac{1}{2} \partial_{5}^2 h_{\mu\nu} - \sigma' \partial_{5} h_{\mu\nu} - (\sigma')^2 h_{\mu\nu} \]  
(A23)

\[ \delta^{(1)} R^{\lambda}_{5\rho5} = \frac{1}{2} (\bar{R}^{\lambda}_{5\rho5} h^{5} - \bar{R}^{5}_{5\rho5} h^{\lambda}) + \bar{\nabla}_{\rho} \bar{\nabla}_{5} h_{\mu}^{\lambda} - \bar{\nabla}_{\mu} \bar{\nabla}_{\rho} h_{5}^{\lambda} - \bar{\nabla}_{\rho} \bar{\nabla}_{\lambda} h_{55} + \bar{\nabla}_{5} \bar{\nabla}_{\rho} h_{\rho5}) \]
The general formulae for the higher curvature terms are,

\[ \delta^{(1)} R^\lambda_{\mu\nu} = 2 \bar{R} \delta^{(1)} R \]
\[ \delta^{(1)} (R_{MN} R^{MN}) = 2 \bar{R} P Q \delta^{(1)} R_{PQ} - 2 h^{PM} \bar{R}_{PQ} \bar{R}^Q_M \]
\[ \delta^{(1)} (R_{MNPQ} R^{MNPQ}) = \bar{g}^{MT} \delta^{(1)} (R_{MNPQ} R_T^{NPQ}) - h^{MT} \bar{R}_{MNPQ} \bar{R}^P_T \]
\[ \delta^{(1)} (RR_{MN}) = \bar{R}_{MN} \delta^{(1)} R + \bar{R} \delta^{(1)} R_{MN} \]
\[ \delta^{(1)} (R_{MP} R_N P^P) = \bar{R}_M P \delta^{(1)} R_{NP} + \bar{R}_N P \delta^{(1)} R_{MP} - h^{PQ} \bar{R}_{MP} \bar{R}_{NQ} \]
\[ \delta^{(1)} (R^K_{MPN} R^K_{P^P}) = \bar{R}_K P \delta^{(1)} R^K_{MPN} + \bar{R}_M P \delta^{(1)} R^K_{NP} - h^{PQ} \bar{R}^K_{MPN} \bar{R}^{KQ} \]
\[ \delta^{(1)} (R_{MQSP} R_N^{Q^P}) = R_S P^M Q \delta^{(1)} R^S_{P^P Q^P} + R_S P^N Q \delta^{(1)} R^S_{P^P Q^P} - h^{QT} \bar{R}_{SPMQ} \bar{R}^{SP}_{NT} \]

Then, imposing the Gaussian normal condition only, \( h_{55} = h_{5\mu} = 0 \), linear expansions of the Einstein tensor \( G_{MN} \), the higher curvature tensor \( H_{MN} \) and the energy-momentum tensor \( T_{MN} \) are given by,

\[ \delta^{(1)} G_{\mu\nu} = -\frac{1}{2} \bar{g}^{PQ} \partial_P \partial_Q (h_{\mu\nu} - \eta_{\mu\nu} h) + (-4 \sigma'' + 8 (\sigma')^2) h_{\mu\nu} + (\sigma'' - 2 (\sigma')^2) \eta_{\mu\nu} h \]
\[ + \frac{1}{2} e^{2\sigma} (2 \partial_{(\mu} \partial_{\lambda)} h_{\nu)} - \partial_{\mu} \partial_{\nu} h - \eta_{\mu\nu} \partial_{\lambda} \partial_{\rho} h^\rho \lambda), \]

\[ \delta^{(1)} G_{5\mu} = \frac{1}{2} \partial_y \left[ e^{2\sigma} (\partial^\lambda h_{\mu\lambda} - \partial_{\mu} h) \right], \]

\[ \delta^{(1)} G_{55} = -\frac{1}{2} e^{4\sigma} \partial^\mu \partial^\nu (h_{\mu\nu} - \eta_{\mu\nu} h) - \frac{3}{2} \sigma' \partial_y (e^{2\sigma} h), \]

and

\[ \delta^{(1)} H_{\mu\nu} = \frac{\alpha}{M^2} \left[ \left( -2 \sigma'' + 2 (\sigma')^2 \bar{g}^{\rho\sigma} \partial_\rho \partial_\sigma + 2 (\sigma')^2 \partial_y^2 + 4 \sigma'' \sigma' \partial_y \right) (h_{\mu\nu} - \eta_{\mu\nu} h) + (24 \sigma'' (\sigma')^2 - 20 (\sigma')^4) h_{\mu\nu} + (-12 \sigma'' (\sigma')^2 + 8 (\sigma')^4) \eta_{\mu\nu} h \right. \]

\[ \left. + (2 \sigma'' - 2 (\sigma')^2) e^{2\sigma} (2 \partial_{(\mu} \partial_{\lambda) h_{\nu)} - \partial_{\mu} \partial_{\nu} h - \eta_{\mu\nu} \partial_{\lambda} \partial_{\rho} h^\rho \lambda) \right], \]

\[ \delta^{(1)} H_{5\mu} = -\frac{4 \alpha (\sigma')^2}{M^2} \delta^{(1)} G_{5\mu}, \]
\[ \delta^{(1)} H_{55} = -\frac{4\alpha(\sigma')^2}{M^2} \delta^{(1)} G_{55}, \quad (A39) \]

and

\[ \delta^{(1)} T_{\mu\nu} = (-\Lambda_b - \Lambda \delta(y)) h_{\mu\nu} + T_{\mu\nu}^{(m)} \]
\[ = M^3 \left[ 6(\sigma')^2 \left( 1 - \frac{2\alpha(\sigma')^2}{M^2} \right) - 3\sigma'' \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} \right) \right] h_{\mu\nu} + T_{\mu\nu}^{(m)}, \quad (A40) \]
\[ \delta^{(1)} T_{5\mu} = T_{5\mu}^{(m)}, \quad (A41) \]
\[ \delta^{(1)} T_{55} = T_{55}^{(m)}, \quad (A42) \]

where we expressed the bulk and brane cosmological constants in terms of the background metric function \( \sigma \). Consequently, from the modified Einstein’s equations, \( G_{MN} + H_{MN} = M^{-3} T_{MN} \), the linearized equations of motion become,

\[ \left[ -\frac{1}{2} \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} + \frac{4\alpha\sigma''}{M^2} \right) \partial^2 e^{2\sigma} - \frac{1}{2} \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} \right) \partial^2_y + \frac{4\alpha\sigma'' \sigma'}{M^2} \partial_y \right. \]
\[ + 2(\sigma')^2 \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} \right) - \sigma'' \left( 1 - \frac{12\alpha(\sigma')^2}{M^2} \right) \left( h_{\mu\nu} - \eta_{\mu\nu} h \right) \]
\[ + \frac{1}{2} \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} + \frac{4\alpha\sigma''}{M^2} \right) e^{2\sigma} \left( 2\partial_\mu \partial^\lambda h_{\nu\lambda} - \partial_\mu \partial_\nu h - \eta_{\mu\nu} \partial_\lambda \partial_\rho h^{\lambda\rho} \right) = M^{-3} T_{\mu\nu}^{(m)}, \quad (A43) \]

\[ \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} \right) \delta^{(1)} G_{6\mu} = M^{-3} T_{6\mu}^{(m)}, \quad (A44) \]
\[ \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} \right) \delta^{(1)} G_{55} = M^{-3} T_{55}^{(m)}. \quad (A45) \]

Apart from terms including \( \sigma'' \), which corresponds to the delta function source, the linearized Einstein’s equation remains intact up to the overall factor \( \left( 1 - \frac{4\alpha(\sigma')^2}{M^2} \right) \) on its left hand side even in the existence of the Gauss-Bonnet term. But the overall factor is not a positive definite quantity, which implies that a test particle might feel a repulsive force (or exchange of ghost particles of negative energy) from arbitrary matter source.
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