Optimal design of foundations by means of nonlinear calculation methods

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Abstract. This paper proposes using the defining equations from the theory of adaptive evolution of mechanical systems (which is based on the variational principles of nonlinear structural mechanics) to design the shape and size of foundations. It presents an expression for finding the potential energy of a system and the deformation energy density, as well as the variational Lagrange equation. The paper formulates a nonlinear boundary problem solved by finite-element analysis. The solution imposes a constraint on the modulus of elasticity to take into account the physico-mechanical properties of the materials. A calculation algorithm and an ADPL program are written for ANSYS. The paper also presents a solution to the problem of finding the rational foundation shaped for the case of plain strain. The solution-derived rational foundation shape is shown. The authors plot the stresses and energy densities as a function of evolution at the onset and finish of iterative processes. Note that the resulting foundation shape is more stable, more accurately positioned in the soil, and can carry a greater load compared to more conventional shapes.

1. Introduction
The choice of a foundation design for a building depends on a number of factors, including geotechnical conditions, the presence or absence of groundwater, the design of the structure itself, the loads it has to sustain, and the calculation methods. The foundation shape determines the bearing capacity, the cost-effectiveness, the constructability, and the conditions of further use.

Advancements in the methods for optimizing the foundation design must be reflected in the effective standards; any such advancement is imperative today.

As of today, foundation design uses calculations of two groups of limit states: those by strength, and those by strain. Conventionally, the foundation shape is designed and calculation-verified to suit the conditions of further use and the geotechnical conditions. Shall it be necessary to increase the bearing capacity or stability, engineers usually use variational design of specific elements, enlarge the cross-sections, or adjust the material properties.

The existing method for calculating settling and subsidence per SNiP 2.02.01-83* [1] uses a few conventions and assumptions. The disadvantages of the conventional foundation-sizing method are as follows: it produces a profile with an uneven distribution of material stiffness; it cannot precisely identify the capacity of the compressible stratum; it uses linear calculation models.
2. Defining Equations of a Nonlinear Structural System per TAEMS

Optimal structural design is a matter covered in a number of papers [1-10]. The theory of adaptive evolution of mechanical systems ("the TAEMS") [11] is based on synergetic principles [12,13] and can be used to rationally configure this or that object while predicting their behavior in this or that application.

This paper proposes using G.V. Vasilkov’s theory of adaptive evolution of mechanical systems [11] (which is based on the variational principles of nonlinear structural mechanics) to design the shape and size of foundations [14-17].

The variational principle of nonlinear mechanics is formulated in [11]. The total potential energy of the system $\Pi_1$:

$$\Pi_1 = U - \int_V u^T \rho dV - \int_S u^T g dS;$$

$$U = \frac{1}{2} \int_V (Au)^T DAu dV;$$  \hspace{1cm} (1)

Where

$$dV' = \varepsilon^{-1} \varepsilon dV; \Rightarrow V' = \int \varepsilon^{-1} \varepsilon dV; \varepsilon \leq \varepsilon_0; \varepsilon_0 \leq \left[ e_{ij} \right]; u_i \leq \left[ u_i \right];$$

$\varepsilon$ is the current strain energy density; $\varepsilon_0$ is the standardized value of the same.

While evolving, the system is in equilibrium. At the (n+1) step, the variational Lagrange equation is written as (2):

$$\delta \Pi^n = \int_V \delta \varepsilon^n \varepsilon^{-1} \varepsilon^n dV - \int_V \delta u_i^n \rho_i dV - \int_S \left( \delta u_i^n \right) g_i dS = 0;$$  \hspace{1cm} (2)

where $\Pi^n$ is the full potential system at the nth step.

The defining value of obtaining a more uniformly strong foundation design the better meets the strength requirements is such value, at which the condition holds (3)

$$E^{n+1} = \left( \varepsilon^n \cdot \varepsilon_0^{-1} \right) E^n;$$  \hspace{1cm} (3)

To find the optimal foundation design, state a nonlinear boundary problem soluble by nonlinear iterative methods. To solve such practical problems, use finite-element analysis [18,19].

The basic ratios of finite-element analysis are as follows: $u = \Phi^T q$ is the vector function of displacements; $e = Au$ are the finite-element deformations; $\sigma = D e$ are the stresses; $A$ is the matrix of differentiation operation; $D$ is the matrix of the medium moduli; $q$ is the vector of nodal displacements.

Equation of the onset of possible displacements:

$$\delta \Pi = \int_V \delta e^T \sigma dV - \int_V \delta u_i^n \rho_i dV - \int_S \left( \delta u_i^n \right) g_i dS = 0;$$  \hspace{1cm} (4)

Here $\rho_i$ is the vector of bulk forces, $g_i$ is the vector of surface forces.

The finite-element stiffness matrix $k_i = \int_V \Phi^T D \Phi dV$ and the nodal-force vector $P_i = \int_V \Phi q dV - \int_S \Phi g_i dS$ are constructed by standard methods. The general stiffness matrix ($K$) and the general nodal-force vector ($P$) are linked by the ratio $K \cdot q = P$. 


Moduli of elasticity are constant within a finite element; however, they are different for different finite elements. When solving the nonlinear problem, each iteration imposes the constraint $E_{k,n} \in \left( E_{SO}, E_{CON} \right)$ on the modulus of elasticity of the medium.

3. Algorithm to Find the Rational Foundation Structure

To solve practical problems, the author has compiled a calculation algorithm that comprises the following operations:

- introduce the dimensions of the analyzed area, construct a grid of triangular elements of optimal shape for the case of plain strain, the initial moduli of soil ($E_{SO}$) and concrete ($E_{CON}$) and Poisson’s ratios of soil ($\nu_{SO}$) and concrete ($\nu_{CON}$);
- recalculate the moduli of strain and Poisson’s ratios in the context of the physico-mechanical properties of soil and concrete

$$
E_{SO} = \frac{E_{SO}}{1 - \nu_{SO}^2}; \quad \nu_{SO} = \frac{\nu_{SO}}{1 - \nu_{SO}^2}; \quad E_{CON} = \frac{E_{CON}}{1 - \nu_{CON}^2}; \quad \nu_{CON} = \frac{\nu_{CON}}{1 - \nu_{CON}^2};
$$

(5)

- adjust the energy density; calculate the modulus of elasticity at the n+1 step

$$
E_{FE} = E_{SO}; \quad \nu_{FE} = \nu_{SO};
$$

$$
\nu_{n+1} = \frac{R_{n,SO}^2}{2E_{SO}^n}; \quad E_{n+1} = \left\{ E^n + \left[ \frac{\nu_{SO}}{\nu_{CON}} E^n \right] \right\}/2
$$

(6)

- calculate the coefficient to account for the presence of reinforcements in the foundation

$$
k_{arm} = \frac{E_{CON} - E^n}{E_{SO} - E^n};
$$

$$
\nu_{n+1} = \nu_{SO} \left( 1 - k_{arm} \right) + \nu_{CON} k_{arm};
$$

(7)

(8)

- solve the system of equations

$$
K^n q = P^n; \quad \Rightarrow U_n = 0.5 \left( q_{\eta}^n \right)^T k_n q_{\eta}^n \quad \Rightarrow \varepsilon_{\eta}^n = U_{\eta}^n / V_{\eta};
$$

$$
E_{n+1} = (S \cdot \varepsilon_{\eta}^n) E^n; \quad n = 1, 2, ..., S_1; \quad r = 1, 2, ..., S_2
$$

(9)

- plot the displacement and stress curves.

Stop calculating when the relative error of calculating the total potential energy is below the value $\mu$:

$$
\frac{\sum U_{n+1} - \sum U_n}{\sum U_{min}} \leq \mu = 0.05%;
$$

(10)
4. Problem. Find the rational shape of a continuous footing

Source data: the soil area is 18m x10m; the distributed surficial load is \( q_{\text{conv}} = 315.0 \text{kN/m} \) 1.2 wide in the middle of the area; for the soil \( E_{SO} = 6.5 \text{ MPa}, \; \nu_{SO} = 0.25 \), for the concrete \( E_{CON} = 23.0 \text{ GPa}, \; \nu_{CON} = 0.2 \).

Calculate by means of finite elements using the TAEMS defining equations [11]. The calculation algorithm and an ADPL program are written for ANSYS [20]. In this case, there are 18,372 triangular elements and 9,384 nodes. The required accuracy is attained after 270 iterations.

The solution-derived rational continuous-footing shape is shown in Figures 1 to 4. The displacement of the rationally shaped foundation totals 3.2 cm; cf. the 5.3 cm vertical displacement of a rectangular foundation of an equally sized foundation. That being said, a TAEMS-derived optimized foundation will under the same conditions settle to a lesser degree, be more stable, have better carrying capacity, and feature more accurate depth and cross-section profile. The shape of the foundation greatly depends on the order of load application.

After finding the foundation shape, the engineers finalize the design and make calculations for limit state groups I and II (strength and strain limit states). The foundation is either a standard design or is based on the reinforced-concrete structures calculations.

5. Summary
The variational principles of the structural mechanics (the TAEMS) have been used to develop a method to optimize the foundation shape and to propose a foundation cross-section shape. The model features better bearing capacity, stability, and more accurate positioning in the soil.

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