INTRODUCTION

Sr₂RuO₄ stands out among the unconventional superconductors as one of the few materials with a chiral order parameter. The tetragonal crystal structure allows five unitary representations for a p-wave pairing symmetry. One of these is the chiral order parameter, of the form $k_y \pm i k_x$, which is strongly suggested by muon spin relaxation and high-resolution polar Kerr effect measurements. Very recently, nuclear magnetic resonance experiments demonstrated that the $d$-vector is not parallel to the $c$-axis and suggested possible chiral $d$-wave states. Such chiral states are attracting renewed attention due to the possibility of hosting Majorana bound states, which in turn are of interest for hosting fermionic topological states. A complication in the physics of Sr₂RuO₄ is the presence of multi-component phases with $T_c \approx 3$ K known as the “extrinsic phase”. The vast majority of experiments in the past two decades have been limited to bulk crystals, typically hundreds of microns in dimension. This is partly due to the unavailability of superconducting Sr₂RuO₄ films. The chiral domains, however, are expected to be no more than a few microns in size.

Spontaneous emergence of Josephson junctions in homogeneous rings of single-crystal Sr₂RuO₄

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The chiral $p$-wave order parameter in Sr₂RuO₄ would make it a special case amongst the unconventional superconductors. A consequence of this symmetry is the possible existence of superconducting domains of opposite chirality. At the boundary of such domains, the locally suppressed condensate can produce an intrinsic Josephson junction. Here, we provide evidence of such junctions using mesoscopic rings, structured from Sr₂RuO₄ single crystals. Our order parameter simulations predict such rings to host stable domain walls across their arms. This is verified with transport experiments on loops, with a sharp transition at 1.5 K, which show distinct critical current oscillations with periodicity corresponding to the flux quantum. In contrast, loops with broadened transitions at around 3 K are void of such junctions and show standard Little–Parks oscillations. Our analysis demonstrates the junctions are of intrinsic origin and makes a compelling case for the existence of superconducting domains.

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Single crystals of Sr$_2$RuO$_4$ were grown with the floating zone method$^{23}$ and structured into microrings using Ga-based FIB etching. Figure 1b–d shows scanning electron microscope (SEM) images of Rings A and B. The inner and outer radii of Ring A are $r_{in} \approx 0.21 \mu m$ and $r_{out} \approx 0.55 \mu m$, respectively. Similar dimensions are used in Ring B: $r_{in} \approx 0.3 \mu m$ and $r_{out} \approx 0.54 \mu m$. Both crystals have a thickness of around 0.7 $\mu m$.

The temperature-dependent resistance $R(T)$ of both rings (presented in Fig. 2a, b) shows sharp superconducting transitions similar to that of bulk Sr$_2$RuO$_4$. The apparent enhancement of the resistance just above $T_c$ in Fig. 2b could be attributed to changes in the current path$^{26}$. The high quality of the sample is also evident by their particularly high residual resistivity ratio; RRR = $R$(300 K)/$R$(3 K) = 238 for Ring A and RRR = 177 for Ring B. To demonstrate that FIB milling does not alter the intrinsic characteristics of Sr$_2$RuO$_4$, we compare the $R(T)$ of Ring A with the one measured before milling the crystal in Supplementary Fig. 1, which shows that $T_c$ and the overall transport properties remain unchanged under structuring. Figure 2c shows the typical current–voltage $V(I)$ behaviour at different temperatures. For both rings, the $V(I)$ measurements exhibit negligibly small hysteresis even at temperatures far below $T_c$.

Insights from theoretical simulations

Before presenting the results of transport measurements under a magnetic field, we examine the expected chiral-domain configurations in our structure. This is accomplished by performing detailed
distinct chiral domains, separated by a pair of ChDW. Within the domain wall the order parameter is reduced to about half of its original amplitude in the banks on each side, resulting in the formation of two parallel Josephson weak links. While the suppressed order parameter is unfavourable in terms of the condensation energy, the formation of such ChDW is favoured by the second term of the free energy in Supplementary Eq. (S11). Since the order parameter is suppressed at the sample edge, the second term gains importance with reducing sample size and may further enhance an inhomogeneous order-parameter state with a ChDW. The ChDW region extends over a length of the order of $\xi$. As shown in Fig. 3b, the presence of a magnetic field along the ring axis makes the positions of the ChDWs shift away from the middle of the arms since one of the chiral components is favoured by the magnetic field. The ChDWs, however, remain in the arms of the ring due to the strong pinning by the restricted dimensions.

Figure 3c shows the calculated chiral-domain configuration for Ring B, which also applies to Ring A at temperatures near $T_c$. This is obtained by setting $\zeta_{in} = 1.3$ and $\zeta_{out} = 3.6$ (corresponding to $T \approx 1.45$ K for Ring A). As the arms of the ring are now considerably narrower on the scale of $\xi(T)$, the contribution of the edge regions dominates the configuration of the order parameter. As a consequence, it becomes energetically favourable for the two chiral components to coexist over the entire ring. This state also produces a pair of parallel weak links due to the suppression of the order parameter $|\Psi|$, which extend over the arms of the rings. Figure 3d presents a phase diagram of the lowest energy states, calculated for various $\zeta_{in}$ and $\zeta_{out}$. Amongst these, mono-domain “Meissner” state can be stabilised by increasing the $r_{out}/r_{in}$ ratio. In this state, the arms of the ring are unable to provide effective pinning of ChDWs. This scenario is explored in Supplementary Note 4 and Supplementary Fig. 5, where a ring with relatively wide arms (Ring E) approaches the mono-domain state at low temperatures. The evolution of the equilibrium domain configuration as a function of temperature for Rings A and B are represented by the dashed lines in Fig. 3d. This suggests that the rings are in one of the domain states shown in Fig. 3a and c at all temperatures below $T_c$ except in a narrow range around 1–1.2 K, where additional domain walls could appear in Ring A. As a general finding, our GL calculations show that ChDWs could spontaneously emerge in our mesoscopic rings and behave as stable Josephson junctions over a broad temperature range, resulting in a DC SQUID of intrinsic origin. The change of chirality across such junctions and its influence on their transport characteristics remain open questions and are worthy of further studies. Note that the GL formalisms for chiral $p$-wave and chiral $d$-wave superconductors have analogous form, and the segregation of chiral domains as discussed above is applicable to both cases.

Critical current oscillations
We examined the supercurrent interference of the rings by measuring $I_c$ at each magnetic field $H$. The results are presented in Fig. 4, where we observe the same behaviour in both Rings A and B. Figure 4a, b shows the $I_c$ of Ring A, measured for positive ($I_c^+$) and at negative ($I_c^-$) bias currents, taken at temperatures deep inside the superconducting state and close to $T_c$, respectively. For both temperatures, we observe distinct critical current oscillations, with the period corresponding to the fluxoid quantisation over the ring area. This interference pattern corresponds to that of a DC SQUID with a pair of parallel Josephson junctions. The junctions would also need to be symmetric each other; an imbalance in $I_c$ could not produce the cusp-shaped minima of the patterns. The figure also shows $-I_c^-(H)$ overlaid on its time-reversed counterpart, $+I_c^+(H)$. Figure 4c shows that the same SQUID oscillations appear in Ring B, only with a slightly smaller period (consistent with its slightly larger inner radius). The oscillations emerge
spontaneously at the onset of superconductivity and continue down to $T \ll T_c$. More importantly, we find that the patterns are not distorted, despite the substantial variations in $I_c(T)$ and $E(T)$.

It is worth noting that, unlike the polar Kerr experiments, we find field cooling and zero-field cooling of the samples to yield the same results in our measurements. This, however, is to be expected in mesoscopic structures, where domain walls are strongly pinned to the confined regions in order to lower the free energy of the system (see Supplementary Note 2 and Supplementary Fig. 2 for more details). Such pinning mechanism is absent in the polar Kerr experiments, which are performed on bulk crystals.

To demonstrate the robustness of the SQUID behaviour further, in Fig. 5a, b, we plot the magnetoresistance of Ring A produced by the $I_c$ oscillations over a wide range of temperatures. These are measured by applying a constant DC current $I$ while sweeping the magnetic field $H$ along the ring axis. Here, the resistance $R$ is defined by the average of two voltages before and after current reversal at each magnetic field: $R = (V(H) - V(-H))/2I$. When the measurement current exceeds the critical current $I_c(H)$, the system is driven out of the zero-voltage regime of the $V(H)$ and produces a finite resistance. Combining the results of a wide range of temperatures, Fig. 5a, b reveals that the SQUID oscillations emerge together with $I_c$ at the onset of the superconducting transition. In Fig. 5c, d, we describe the shape of $R(H)$, where in some cases the peaks can appear to be split or broadened. This is clearly due to a slight difference in the values of $I_{c}\text{Flux}$, which causes the voltage peaks for $\pm I$ to appear asymmetrically. We observed a similar asymmetry in rings showing the LP effect.

The magnetovoltage and field-dependent $V(I)$ measurements are crucial in resolving an outstanding issue regarding previous reports of unconventional behaviour of $Sr_2RuO_4$ rings. Cai et al. have consistently observed magnetoresistance oscillations with unexpectedly large amplitude\(^{28,29}\), very similar to the data presented in Fig. 5. The reported magnetoresistance oscillations are also stable over a wide range of temperatures and, in some cases, show small dips around $\Phi_0/2$. As Fig. 5 demonstrates, however, the averaged resistance $R$ could produce a very similar effect even when there is no splitting of the peaks in the raw magnetovoltage signal.

### $T_c$ oscillations in rings with an extrinsic phase

We already mentioned that ChDWs can produce the observed $I_c(H)$ oscillations by acting as Josephson junctions. This should be contrasted with the fluxoid-periodic behaviour of structures with a partial or full extrinsic phase, characterised by a noticeably broader transition which begins near 3 K (see Fig. 6e and Supplementary Fig. 4). We recently reported observations of the LP oscillations in such $Sr_2RuO_4$ microrings\(^{27}\), and here we demonstrate that those are of a fundamentally different nature than the $I_c$ oscillations discussed in this report. For this, we compare the data from Ring A with those of Ring C (sample B in ref. \(^{27}\)), where the transition is considerably broader (Fig. 6e). This ring was prepared from a 2-μm-thick crystal with a $T_c$ of 1.5 K. After microstructuring, however, the ring was found to have a higher $T_c$ with its transition already starting at 2.7 K. The magnetotransport measurements reveal that the ring itself is predominantly in the extrinsic phase, introduced by microstructuring (most likely due to a strain induced by FIB milling of the thick crystal). Compared to Rings A (RRR = 238) and B (RRR = 177), this structure has a smaller residual resistivity ratio RRR = 129. Nevertheless, the value of RRR is still substantial, indicating strong metallicity for Ring C. Figure 6a, b shows $R(H)$ for temperatures within the resistive-transitions of Rings A and C (taken 1.67 K and 2.3 K, respectively). In both cases, we find inhomogeneous oscillations, which we compare with simulated LP oscillations (the red curves).

The change of the transition temperature due to the LP oscillations is given by\(^{20}\):

$$\frac{\Delta T_c}{T_c(0)} = -\left(\frac{\mu_0 R_{Intrinsic}}{\Delta \Phi_0/2\pi}ight)^2 - \frac{\xi_0^2}{w^2} \left(\frac{n - \Delta \Phi_0}{\Phi_0 - \Delta \Phi_0/2\pi}\right)^2,$$

where $\Phi_0 = h/2e$ is the flux quantum with the Planck constant $h$ and the elementary charge $e$, and $w = r_{out} - r_{in}$ is the width of the ring arm. The first term represents the effect of the Meissner shielding, and the second term corresponds to fluxoid quantisation. To convert the change of the transition temperature to the resistance variation, we assume that the $R(T)$ curve does not change its shape under magnetic field and shifts horizontally by $\Delta T_c(H) = T_c(H) - T_c(0)$. For the simulations in Fig. 6a, b, we used $\xi(0) = 66$ nm, $2r_{in} = 0.55$ μm, $2r_{out} = 1.1$ μm for Ring A, and $2r_{in} = 0.7$ μm, $2r_{out} = 1.0$ μm for Ring C. Both the period and amplitude of the oscillations for Ring C agree with those of the simulation. We therefore consider these to be the LP oscillations, driven by variations in $T_c$. For Ring A, however, the oscillation amplitude is substantially larger than what $T_c$ variations can produce. Such large-amplitude magnetoresistance is driven by the $I_c(H)$ oscillations instead. In Fig. 6c, d, we compare the $I_c(H)$ of both rings at lower temperatures. In contrast to Rings A and B, the SQUID
The corresponding magnetovoltage when the measurement current is applied to one direction $V_+$ and to the other direction $V_-$. The peaks (dips) in $V_+$ ($V_-$) appear at different field values and hence double peaks appear in the resistance.

**DISCUSSION**

Before adopting ChDW scenario as the origin of the observed $I_c$ oscillations, we consider other known mechanisms for $I_c$ oscillations. Firstly, even in a homogeneous loop SQUID-like behaviour may emerge depending on the size of the ring with respective to either the penetration depth $\lambda$ or the coherence length $\xi$. $I_c$ can be modulated by the circulating persistent current $I_p$, which varies linearly with the flux, and switches its direction at every increment of $\Phi_0/2$. This mostly results in a sawtooth-like modulation of $I_c^{31}$, which cannot account for non-linear form of the patterns shown in Fig. 4. Furthermore, the magnitude of $I_p$ is inversely proportional to the kinetic inductance $L_K$, which depends on the penetration depth $I_c \propto \lambda^2(T)$. If the $I_c$ oscillations were driven by circulating currents, their amplitude $\Delta I_c$ would grow larger by lowering the temperature since $\Delta I_c \propto I_p \propto 1/\xi^2(T)$.

This is clearly not the case for the Sr$_2$RuO$_4$ rings, where oscillation amplitude is unaffected by temperature (e.g. $\Delta I_c \approx 12 \mu A$ at both temperatures shown in Fig. 4a). SQUID oscillations can also emerge in loops without weak link, if the dimensions are much smaller than $\xi(T)$ and $\lambda(T)$. However, this is not applicable to our structures, where the radii and the width of the arms are several times larger than the characteristic length scales for $T < T_c$ (e.g. for Ring A, $\xi(T) \approx 0.07 \mu m$ and $\lambda(T) \approx 0.19 \mu m$ at $T = 0.78K$).

Secondly, Cai et al. attributed the large-amplitude magnetoresistance of their Sr$_2$RuO$_4$ rings to current-excited moving vortices$^{28,29}$. As demonstrated by Berdiyorov et al.$^{33}$ this mechanism can only produce large-amplitude oscillations over a finite temperature range, typically down to $T \approx 0.95T_c$ (e.g. see Fig. 6b of ref.$^{33}$ and Fig. 2 of ref.$^{34}$). This is not the case for the Sr$_2$RuO$_4$ rings that are in the intrinsic (1.5-K) phase, as the magnetoresistance oscillations appear for all $T < T_c$ (see Fig. 5a, b and Fig. 3a in ref.$^{28}$).

Thirdly, geometrical constrictions (e.g. bridges and nanowires) can serve as Josephson junctions, as long as their dimensions are comparable to $\xi$. The current-phase relation (CPR) of such junctions is defined by the ratio of $\xi(T)$ to the length of the weak link $L$. Since $\xi(T)$ varies with temperature while $L$ remains fixed, the CPR of such weak links is strongly temperature dependent. Generally, lowering the temperature transforms the CPR from sinusoidal to a sawtooth-like function, which ultimately turns into a hysteretic $V(I)$ relation. This is in direct contrast to the $V(I)$ curves of the Sr$_2$RuO$_4$ rings, which show negligible hysteresis for temperatures as low as 0.2$T_c$ (see Fig. 2c). Furthermore, the interference patterns taken at over wide range of temperatures show the same overall shape, with characteristically round lobes (Fig. 4). This could not be produced by constriction junctions, as the interference pattern would be heavily deformed by the pronounced changes in $\xi(T)/L$ with temperature. In case of ChDWs, however, the length of the junction barrier is determined by the coherence length and therefore has a temperature dependence similar to $\xi(T)$. Hence, a ChDW junction can maintain a relatively fixed $\xi(T)/L$ ratio for different temperatures. This would agree with the lack of hysteresis in our $V(I)$ measurements (Fig. 2c) and the unperturbed shape of the interference patterns (Fig. 4).

Lastly, we exclude the possibility of forming accidental proximity junctions by Ru inclusions or any other normal metal.
within the Sr$_2$RuO$_4$ crystal. Apart from their absence in the SEM images taken while the milling of the rings, inclusions would induce an extrinsic 3-K phase. The crystals, however, show no such enhancement of $T_c$ either before or after FIB processing. Moreover, the (single) sharp resistive transitions of Rings A and B could not be produced in the presence of normal metal weak links. Accidental tunnel junctions, formed by nanocracks or grain boundaries, can also be excluded due to the high metallicity of our samples. In summary, the Josephson effect found in Sr$_2$RuO$_4$ microrings cannot be attributed to conventional types of weak link such as constriction junctions, kinematic vortices (phase-slip lines), proximity and tunnel junctions.

To summarise, our simulations of a chiral $p$-wave order parameter show that a mesoscopic loop with nanostructured transport leads can host a multi-domain state. The degenerate chiral states are separated by ChDWs located in the arms of the ring, where a pair of parallel Josephson junctions is formed due to the local suppression of both chiral states. We examined the existence of such junctions by performing transport experiments on Sr$_2$RuO$_4$ microrings. The rings with a sharp transition near 1.5 K show distinct $I_e$ oscillations, similar to that of a DC SQUID with a pair of Josephson junctions with matching $I_c$. The junctions emerge together with the superconducting transition and are present for all temperatures below $T_c$. In contrast, for Sr$_2$RuO$_4$ rings with an extrinsic (3-K) phase, the Josephson junctions are entirely absent. Such rings show standard Little–Parks oscillations near $T_c$, which can be properly modelled, but no critical current oscillations. Our findings suggest that the Josephson junctions are an inherent property of the order parameter, and make a compelling case for the existence of ChDWs in the intrinsic (1.5-K) phase of Sr$_2$RuO$_4$. We should note that our present results formally do not distinguish the type of degenerate states responsible for the formation of the junctions; our transport measurements would also be consistent with domain walls of helical states, as well as of spin-singlet chiral states. This work also demonstrates that the combination order parameter simulations with mesoscopic structures can be instrumental in the study of superconducting domains and will, in coming experiments, allow for detailed design and understanding of a system before the actual fabrication.

**METHODS**

**Microring fabrication**
Sr$_2$RuO$_4$ single crystals were prepared with the floating zone method$^{23}$, and their transition temperature $T_c$ before the sample fabrication was confirmed to be 1.50 K using a compact AC susceptometer$^{24}$ in a Quantum Design PPMS. We crush the crystal into small pieces to obtain thin crystals with the thickness of approximately 1 μm. Although Sr$_2$RuO$_4$ is chemically stable in the ambient condition, we find that small crystals can degrade in the air. Therefore, freshly crushed crystals were used. The crystal is placed on a SrTiO$_3$ substrate, where it is contacted by either gold or silver for transport measurements. For Rings A, C and D, two pads of high-temperature-cure silver paint (6838, Dupont) are attached to the two sides of the crystal. The paint is then cured at 500 °C for 20 min. In case of Rings B and E however, the crystals are contacted using a combination of electron-beam lithography and sputter deposition of gold. Once a crystal is contacted by the gold or silver paint, a 100-nm-thick layer of SiO$_2$ is deposited using electron beam evaporation to protect the crystal during structuring. The contacts and the crystal underneath are then cut with a Gallium FIB to produce a four-wire arrangement. Lastly, the microrings are structured using the FIB (30 kV, 20 pA).

**Measurements**
Transport measurements were performed in a $^3$He refrigerator (Heliox, Oxford Instruments) down to 0.3 K. In the DC resistance measurement, we flip the direction of the measurement current to subtract the contribution of the thermoelectric voltage, and the resistance $R$ is defined to be $R = |V/I| - |-(V-ΔV)/2I|$. The transition temperature shift due to the LP oscillations is calculated to be approximately 10 mK by using Eq. (1). Therefore, temperature stability during the magnetoresistance measurement must be much smaller than this value. By putting a 80-Ω by-pass resistor in parallel to the heater and by tuning the PID values of the temperature controller, we achieved a temperature stability of 100 μK. Current–voltage (iV) measurements are performed under constant temperature and magnetic field with triangular current waves of frequency 2 mHz.
Simulations
For details of the Ginzburg–Landau simulations, we refer to the formalism of ref. 20, and the additional discussion in the Supplementary information.

DATA AVAILABILITY
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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AUTHOR CONTRIBUTIONS

The authors declare no competing interests.

ADDITIONAL INFORMATION

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