I discuss some theoretical ideas concerning the representation of quantum gravity as a Lorentz-symmetry-violating ‘medium’ with non-trivial optical properties, which include a refractive index in ‘vacuo’ and stochastic effects associated with a spread in the arrival times of photons, growing linearly with the photon energy. Some of these properties may be experimentally detectable in future satellite facilities (e.g. GLAST or AMS), using as probes light from distant astrophysical sources such as gamma ray bursters. I also argue that such linear violations of Lorentz symmetry may not always be constrained by ultra-high-energy cosmic-ray data, as seems to be the case with a specific (stringy) model of space-time foam.

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1 Introduction

The suggestion made by J.A. Wheeler that space time may acquire a discrete foamy structure at sub-Planckian scales has received considerable attention. The relevant works span a wide range of research fields, from phenomenological approaches to theoretical modelling of quantum gravity and/or string theory. The purpose of this talk is to focus on a recent scenario on the emergence of a foamy space-time structure in the context of string theory, and to discuss briefly its possible phenomenological consequences, especially in an astrophysical context using gamma-ray-bursters (GRB) as the relevant probes.

An important feature of most models of quantum space time foam is the breaking of Lorentz Invariance (LI) by quantum gravity effects. In such an approach LI is only an approximate symmetry of the low-energy world. In the context of the specific model of (stringy) space time foam proposed in, the basic idea may be summarised as follows: consider a closed-string state propagating in a \((D + 1)\)-dimensional space-time, which impacts a very massive \(D\) particle embedded in this space-time. In the modern view of string theory, \(D\) particles must be included in the consistent formulation of the ground-state vacuum configuration. We argue that the scattering of the closed-string state on the \(D\) particle induces recoil of the latter, which
distorts the surrounding space-time in a stochastic manner.

From a string-theory point of view, the essential feature is the deviation from conformal invariance of the relevant world-sheet \( \sigma \) model that describes the recoil. This is compensated by the introduction of a Liouville field \( \Phi \), which in turn is identified with the target time in the approach adopted here.

2 Non-Trivial Optical Properties of Space-Time Foam

It has been pointed out in [9], based on some models of space-time foam in the context of Liouville strings, that the quantum-gravity ‘medium’ may affect the optical properties of the vacuum by inducing, among others, effects associated with a non-trivial refractive index.

Similar phenomena have also been found [10] within the so-called ‘loop gravity’ approach to the dynamics of quantum space-time [11]. In what follows we shall describe briefly the phenomenon by restricting ourselves, for definiteness, to a specific stringy model of quantum space-time foam, in which one encounters massive \( D \)-brane defects [5].

In such a picture, the recoil of the massive space-time defect, during the scattering with the low-energy probe (e.g. photon or neutrino), curves the surrounding space time, giving rise to a gravitational field of the form \( G_{ij} \sim \eta_{ij} + \mathcal{O} \left( \frac{E}{M_D c^2} \right) \), where \( c \) is the velocity of light in empty space, \( E \ll M_D \) is the photon energy, and \( M_D \) the gravitational scale of the defect. In string theory, \( M_D = M_s / g_s \), where \( M_s \) is the string scale, and \( g_s \ll 1 \) is the string coupling (assumed weak). One may identify \( M_D \) with the Planck scale \( 10^{19} \) GeV in four space-time dimensions, or keep the \( M_D \) as a phenomenological parameter to be constrained by observations [5, 7].

An important effect of such a distortion of space-time is the appearance of an induced index of refraction: the effective (group) velocity \( v \) of photons in the quantum-gravitational ‘medium’ depends in a way proportional to the energy of the particle probe:

\[
v = c \left( 1 - \mathcal{O} \left( \frac{E}{M_D c^2} \right) \right)
\]

where the minus sign reflects the fact that there are no superluminal propagation in the \( D \)-brane recoil approach to space time foam. This latter property has to be contrasted with some models in the loop gravity approach [11], where superluminal effects are present, leading to a dependence on the helicity of the photon state and thus characteristic birefringence effects. Notice that, since the space-time foam effects [6] are proportional to the energy of the particle...
probe, the phenomenon is quite distinct from conventional electromagnetic plasma effects, which attenuate with increasing energy. As emphasised in \[1\], the effect (1) appears as a mean-field effect. Such an effect may be either severely constrained \[13\] by ultra-high-energy \((10^{20} \text{ eV})\) cosmic-ray (UHECR) data \[14\] or be viewed as implying that the standard assumptions on the maximum distance travelled by very-high-energy radiation have to be revised \[13\].

In addition to the above mean-field effects, there are stochastic fluctuations about this mean value, which manifest themselves as light-cone fluctuations, leading to a stochastic spread in the arrival times of photons from a source at distance \(L\) of the form \[12\]

\[
\Delta t = \frac{\sqrt{\langle \sigma^2 \rangle - \langle \sigma_0 \rangle^2}}{L}
\]

where \(\sigma = \sum_{n=0}^{\infty} \sigma_n\), with \(\sigma_n\) denoting the \(n\)th order term, with respect to an expansion in powers of the gravitational fluctuations \(h_{\mu\nu}\) about flat space time, of the squared geodesic separation \(2\sigma(x, x')\) between two space-time points \(x\) and \(x'\). The calculation for the recoil case, then, yields \[5\]

\[
\Delta t = \mathcal{O}\left( \frac{g_s E_L M_{\odot}}{c^3} \right)
\]

where the suppression by the extra power of \(g_s\), as compared with the mean-field effect (1), is due to the fact that the effect (2) represents quantum fluctuations about the mean value.

Notice that the effect (2) is not associated with any modification of the dispersion relation of the particle probes, but pertains strictly to fluctuations in the arrival times of photons \[5\], \[12\]. In fact, from its construction, the effect is associated with the quantum uncertainties (notably probe-energy dependent \[1\]) about the mean value of the recoil velocity along the incident direction, say \(U_x\). These are non zero even in the case of a vanishing mean-field \(U_x\) that may characterize models of isotropic space time foam. In such isotropic scenarios one averages \(U_x\) over all possible directions, which leads to a vanishing refractive index effect (1). This in fact provides a possible counterargument on recent claims \[13\] that violations of Lorentz symmetry which grow linearly with the particle energy may be incompatible with UHECR data. Incidentally, if one adopts the point of view that UHECR may come from sources within a 50 Mpc radius from us, as seems to have been suggested by the photon-pion production off the cosmic-background radiation \[14\], then one would obtain a temporal dispersion of UHECR due to such effects over a period of \(10^8\) sec.
GRB 970508: BATSE data Ch. 1 and Ch. 3

Time (s)  Photon rate (counts/s)

Gauss fit
Ch. 1: \( t_p = 1.33(9) \text{ s}, \sigma_p = 1.41(9) \text{ s} \)
Ch. 3: \( t_p = 0.99(6) \text{ s}, \sigma_p = 1.46(6) \text{ s} \)

Lorentz fit
Ch. 1: \( t_p = 1.29(9) \text{ s}, \sigma_p = 1.2(1) \text{ s} \)
Ch. 3: \( t_p = 0.99(7) \text{ s}, \sigma_p = 1.33(8) \text{ s} \)

tail fit
Ch. 1: \( t_p = 1.1(8) \text{ s}, \sigma_p = 2(1) \text{ s} \)
Ch. 3: \( t_p = 0.8(2) \text{ s}, \sigma_p = 1.6(3) \text{ s} \)

pulse fit
Ch. 1: \( t_p = 0.7(2) \text{ s}, \sigma_p = 2.0(3) \text{ s} \)
Ch. 3: \( t_p = 0.7(2) \text{ s}, \sigma_p = 2.1(3) \text{ s} \)

Figure 1. Time distribution of the number of photons observed by BATSE in Channels 1 and 3 for GRB 970508, compared with the following fitting functions: (a) Gaussian, (b) Lorentzian, (c) ‘tail’ function, and (d) ‘pulse’ function. We list below each panel the positions \( t_p \) and widths \( \sigma_p \) (with statistical errors) found for each peak in each fit.

3 Astro-particle Physics Phenomenology of Stringy Space-Time Foam

Although the effects (1) and (2) are tiny, however they are enhanced the further the photon travels. Hence, data from distant cosmological sources, such as GRB, should provide a stringent constraint on these effects.

We presented in [7] a detailed analysis of the astrophysical data for a sample of GRB whose redshifts \( z \) are known (see fig. [1] for the data of a typical burst: GRB 970508). We looked (without success) for a correlation with the redshift, calculating a regression measure for the stochastic effect (3) (c.f. figure (3)), but also for the refractive index effect (4).

We determined limits on the quantum gravity scale \( M_D \) by constraining the possible magnitudes of the slopes in linear-regression analyses of the dif-
Figure 2. Values of the changes ($\Delta\sigma_f$) in the widths of the peaks fitted for each GRB studied using BATSE and OSSE data, plotted versus $\tilde{z} = 1 - (1 + z)^{-1/2}$, where $z$ is the redshift. The indicated errors are the statistical errors in the ‘pulse’ fits provided by the fitting routine, combined with systematic error estimates obtained by comparing the results obtained using the ‘tail’ fitting function. The values obtained by comparing OSSE with BATSE Channel 3 data. The solid line is the best linear fit.

4 Conclusions

We have discussed here some possible low-energy astrophysical probes of quantum gravity, concentrating on the possibility that the velocity of light might depend on its frequency, i.e., the corresponding photon energy. This idea is very speculative, and the model calculations that we have reviewed require
justification and refinement. However, we feel that the suggestion is well motivated by the basic fact that gravity abhors rigid bodies, and the related intuition that the vacuum should exhibit back-reaction effects and act as a non-trivial medium. These features have appeared in several approaches to quantum gravity, including the canonical approach and ideas based on extra dimensions.

As could be expected, we have found no significant effect in the data available on GRBs, either in the possible delay times of photons of higher energies, or in the possible stochastic spreads of velocities of photons with the same energy. However, it has been established that such probes may be sensitive to scales approaching the Planck mass, if these effects are linear in the photon energy. We expect that the redshifts of many more GRBs will become known in the near future.

Future observations of higher-energy photons from GRBs would be very valuable, since they would provide a longer lever arm in the search for energy-dependent effects on photon propagation. As emphasised in the text, such observations are essential for the stochastic quantum gravity effect, and hence should be considered complementary to cosmic ray or other astrophysical data, which may not be relevant for constraining this effect.

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