ALMOST STRONG PROPERNESS

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Abstract. We introduce the forcing property “almost strong properness” which sits between properness and strong properness. As an application, we introduce a simple forcing with finite conditions to force MRP.

Introduction

The Mapping Reflection Principle (MRP) was discovered and shown to be a consequence of PFA by Moore [1] using a proper forcing with countable conditions. This is a strong reflection principle which decides the value of the continuum to be \( \aleph_2 \), and implies, among other things, both the Singular Cardinal Hypothesis and the failure of square principle, see [2]. In this note, we introduce a subclass of proper forcing notions which contains strongly proper forcings, we shall observe that almost strongly proper forcings preserve c.c.c.-ness and have the \( \omega_1 \)-approximation property, and hence they preserve Suslin trees. We then demonstrate how to force MRP using an almost strongly proper forcing with finite conditions. Consequently, MRP holds under Todorčević's forcing axiom PFA(S). This fact was first proved by Teruyuki Yorioka and Tadatoshi Miyamoto in [3]. In the same paper, they introduced also the notion of an almost strongly proper forcing for another purpose, but it is apparently different from ours.

1. Almost Strong Properness

Let us recall the definition of a strongly generic condition due to Mitchell [4]. Suppose \( \mathbb{P} \) is a forcing, and that \( X \) is a set. A condition \( p \in \mathbb{P} \) is called \((X, \mathbb{P})\)-strongly generic if and only if for every \( q \leq p \) there is \( q \restriction_{X} \in X \cap \mathbb{P} \) such that every \( r \in X \cap \mathbb{P} \) with \( r \leq q \restriction_{X} \) is compatible with \( q \). The notion of strong properness is defined in an obvious way.

Definition 1.1 (Almost Strong Genericity). Suppose \( \mathbb{P} \) is a forcing, and \( M \prec H_{\theta} \) contains \( \mathbb{P} \). A condition \( p \in \mathbb{P} \) is called \((M, \mathbb{P})\)-almost strongly generic if and only if for every \( q \leq p \),

\[
S_q(M) := \{X \in [\mathbb{P}]^{\omega} : \exists q \restriction_{X} \in X \ such \ that \ \forall r \in X, r \leq q \restriction_{X} \Rightarrow r \approx q\}
\]

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is $M$-stationary i.e for every algebra\footnote{Recall that an algebra over a set $Z$ is a function from $[Z]^<\omega$ to $Z$.} $F \in M$ over $\mathbb{P}$, there is some $A \in M \cap S_q(M)$ which is $F$-closed.

It is clear that every $(M, \mathbb{P})$-strongly generic condition is $(M, \mathbb{P})$-almost strongly generic, and that every $(M, \mathbb{P})$-almost strongly generic condition is $(M, \mathbb{P})$-generic.

**Definition 1.2 (Almost Strong Properness).** A forcing notion $\mathbb{P}$ is called almost strongly proper (a.s.p. for short), if for every sufficiently large regular cardinal $\theta$, there is a club of countable models $M \prec H_\theta$ containing $\mathbb{P}$ such that every condition $p \in M \cap \mathbb{P}$ can be extended to an $(M, \mathbb{P})$-almost strongly generic condition.

The following easy lemma appears frequently in applications.

**Lemma 1.3.** A condition $p \in \mathbb{P}$ is $(M, \mathbb{P})$-almost strongly generic if and only if for every $U \in M$ which is unbounded in $[\mathbb{P}]^\omega$, we have that $U \cap S_p(M) \cap M \neq \emptyset$.

**Proof.** Suppose $U \in M$ is an unbounded subset of $[\mathbb{P}]^\omega$. Let $\overline{U}$ be the closure of $U$ under $\subseteq$-increasing sequences. Thus $\overline{U}$ is a club in $[\mathbb{P}]^\omega$ belonging to $M$. Since $p$ is $(M, \mathbb{P})$-almost strongly generic, and $\overline{U}$ is in $M$, there is some $X \in \overline{U} \cap M \cap S_p(M)$. Thus there is $p \upharpoonright X \in X$ with the property that all its extensions in $X$ are compatible with $p$. Since $X \in \overline{U}$, there is a $\subseteq$-increasing sequence $(X_n)_{n<\omega}$ of elements in $U$ such that $\bigcup_{n<\omega} X_n = X$, moreover such a sequence can be chosen form $M$, by elementarity. Now, $p \upharpoonright X \in X_n$ for some $n < \omega$. Obviously, every extension of $p \upharpoonright X$ in $X_n$ is compatible with $p$, and that $X_n \in M \cap U$. Consequently, $X_n \in U \cap M \cap S_p(M)$. The other direction is trivial.

Recall that a forcing notion $\mathbb{P}$ has the $\omega_1$-approximation property if for every $V$-generic filter $G \subseteq \mathbb{P}$, the pair $(V, V[G])$ has the $\omega_1$-approximation property i.e every set $x \in V[G]$ with the property that $x \cap a \in V$ for every countable $a \in V$, should be in $V$ (see [5]).

The following is well-known and easy to prove.

**Lemma 1.4.** No forcing with the $\omega_1$-approximation property can add new cofinal branches through tress of height $\omega_1$.

**Proposition 1.5.** Every a.s.p. forcing has the $\omega_1$-approximation property.

**Proof.** Assume that $\mathbb{P}$ is an a.s.p. forcing. Suppose that $p_0 \in \mathbb{P}$ forces $\dot{f} : \gamma \to 2$ to be a countably approximated function, where $\gamma$ is an ordinal. We shall show that the set of conditions below $p_0$ deciding $\dot{f}$ is dense below $p$. Fix $p \leq p_0$. Suppose that $\theta < \theta^*$ are
sufficiently large regular cardinals. Let $M \prec H_{\theta^*}$ be countable and contain the relevant objects, in particular $p$. Let $q \leq p$ be an $(M, \mathbb{P})$-almost strongly generic condition. One can extend $q$ further to make sure that $q \Vdash \dot{f} \upharpoonright M \in V$. Therefore, we may assume without loss of generality that there is, in $V$, some function $g : M \cap \gamma \to 2$ such that $q \Vdash g = \dot{f} \upharpoonright M$. Let

$$\mathcal{U} = \{ N \cap \mathbb{P} : \mathbb{P}, \gamma, \dot{f} \in N \prec H_\theta \text{ and } N \text{ is countable} \}.$$ 

$\mathcal{U}$ is unbounded in $[\mathbb{P}]^\omega$ and belongs to $M$. We now use the almost strong genericity of $q$ and Lemma 1.3 to pick some $N \prec H_\theta$ in $M$ with $\dot{f}, \gamma, \mathbb{P} \in N$ for which there exists $q \upharpoonright N$ so that every condition in $N$ extending $q \upharpoonright N$ is compatible with $q$. We are done if $q \upharpoonright N$ decides $\dot{f}$. Suppose this is not the case thus there are, by elementarity, $\zeta \in \gamma \cap N$ (and hence in $M$) and $q_0, q_1 \in N$ extending $q \upharpoonright N$ such that $q_0 \Vdash \dot{f}(\zeta) = 0$ and $q_1 \Vdash \dot{f}(\zeta) = 1$, but this is impossible since $q \Vdash \dot{f}(\zeta) = g(\zeta)$; otherwise either $q_0$ or $q_1$ is incompatible with $q$; a contradiction!

Proposition 1.6. Every a.s.p. forcing preserves c.c.c-ness.

Proof. Suppose $\mathbb{Q}$ is a c.c.c forcing, and that $\mathbb{P}$ is an a.s.p. forcing. Choose a sufficiently large regular cardinals $\theta < \theta^*$ so that $\mathbb{P}, \mathbb{Q} \in H_\theta$. Suppose $\mathcal{C}$ is a club in $[H_\theta]^\omega$ witnessing the almost strong properness of $\mathbb{P}$. Let $M \prec H_{\theta^*}$ in $\mathcal{C}$ contain $\mathbb{P}$ and $\mathbb{Q}$. It is enough to show that for every $q \in \mathbb{Q}$ and every $(M, \mathbb{P})$-almost strongly generic condition $p \in \mathbb{P}$, $(p, q)$ is $(M, \mathbb{P} \times \mathbb{Q})$-generic$^2$. Let $D \in M$ be a dense subset of $\mathbb{P} \times \mathbb{Q}$. Consider

$$\mathcal{U} = \{ N \cap \mathbb{P} : \mathbb{P}, \mathbb{Q}, D \in N \prec H_\theta \text{ and } N \text{ is countable} \}.$$ 

$\mathcal{U}$ is unbounded in $[\mathbb{P}]^\omega$ and belongs to $M$. By the almost strong genericity of $p$ and Lemma 1.3, one can choose $N \prec H_\theta$ in $M$ with $\mathbb{P}, \mathbb{Q}, D \in M$ so that there is $p \upharpoonright N \in N$, for which every stronger condition in $N \cap \mathbb{P}$ is compatible with $p$. Set 

$$E = \{ q' \in \mathbb{Q} : \exists p' \leq p \upharpoonright X \text{ with } (p', q') \in D \}.$$ 

It is easily seen that $E$ is a dense subset of $\mathbb{Q}$ in $\mathcal{N}$. Clearly $E$ is in $N$, and hence in $M$. Now since $\mathbb{Q}$ is c.c.c., there is $q' \in N \cap E$ such that $q'$ is compatible with $q$, by the definition of $E$ and the elementarity of $N$, there is $p' \leq p \upharpoonright X$ in $N$ so that $(p', q') \in D \cap N$, but then $(p', q')$ is compatible with $(p, q)$. On the other hand $(p', q') \in N \cap D \subseteq M \cap D$, and thus $(p, q)$ is $(M, \mathbb{P})$-generic.

Corollary 1.7. Every a.s.p. forcing preserves Suslinity.

$^2$Recall that a forcing notion is c.c.c if and only if the maximal condition is generic for unboundedly many elementary submodel in some $H_\theta$ big enough.
Proof. Assume that $\mathbb{P}$ is an a.s.p. forcing. Suppose $S$ is a Suslin tree. Let $G \subseteq \mathbb{P}$ be $V$-generic filter. By Lemma 1.4 and Proposition 1.5, $S$ does not have cofinal branches in $V[G]$. Now, Consider the corresponding Suslin forcing of $S$, say $\mathbb{S}$ which is $\mathbb{S}$ is c.c.c. By Proposition 1.5, $\mathbb{S}$ remains c.c.c., and hence Suslin in $V[G]$.

2. Mapping Reflection Principle

Definition 2.1 (Ellentuck Topology). Let $X$ be an uncountable set. The Ellentuck topology on $[X]^\leq\omega$ is the topology generated by the following sets as basic open sets:

\[ [a, A] := \{ x \subseteq X : a \subseteq x \subseteq A \}, \text{ where } a \text{ is finite and } A \subseteq X \text{ is countable}. \]

The following definitions are due to Moore, [1].

Definition 2.2 ($M$-stationarity). Suppose $M$ and $X$ are sets. A set $\Sigma \subseteq [X]^\leq\omega$ is called $M$-stationary if for every algebra $F \in M$ over $X$, there is some countable set $A \in M \cap \Sigma$ closed under $F$.

Definition 2.3 (Open and Stationary Mapping). A function $\Sigma$ is called open and stationary mapping if there are a regular cardinal $\theta = \theta_\Sigma$ and an uncountable set $X = X_\Sigma$ with $X \in H_\theta$ such that:

1. $\text{dom}(\Sigma)$ is the collection of countable elementary submodels of $H_\theta$ containing $X$.  
2. For each $M \in \text{dom}(\Sigma)$, $\Sigma(M)$ is $M$-stationary, and also open in $[X]^\leq\omega$ with respect to the Ellentuck topology.

Definition 2.4 (Reflection). An open stationary mapping $\Sigma$ reflects if there is a continuous $\in$-chain $\langle M_\xi : \xi < \omega_1 \rangle$ of models in $\text{dom}(\Sigma)$ such that for every $\xi < \omega_1$, there is $\zeta < \xi$ so that for every $\eta \in \xi \setminus \zeta$, $M_\eta \cap X \in \Sigma(M_\xi)$. The sequence $\langle M_\xi : \xi < \omega_1 \rangle$ is called a reflecting sequence for $\Sigma$.

Definition 2.5 (MRP). The Mapping Reflection Principle (MRP) states that every open stationary mapping reflects.

The following can be proved in the same way as Lemma 1.3.

Lemma 2.6. Suppose $\Sigma \subseteq [X]^\leq\omega$ is open. Then $\Sigma$ is $M$-stationary if and only if for every $U \in M$ which is unbounded in $[X]^\omega$, we have that $U \cap \Sigma \cap M \neq \emptyset$.

Theorem 2.7. Suppose that $\Sigma$ is an open stationary mapping. Then there is an a.s.p. forcing $\mathbb{P}_\Sigma$ with finite conditions which adds a reflecting sequence for $\Sigma$. 

The rest of this section is devoted to the proof of Theorem 2.7. Suppose $\Sigma$ is an open stationary mapping. Let $X = X_\Sigma$ and $\theta = \theta_\Sigma$.

**Definition 2.8 (Forcing Poset).** We let $\mathbb{P}_\Sigma$ consist of triples $p = (\mathcal{M}_p, d_p, f_p)$, where

1. $\mathcal{M}_p$ is a finite $\in$-chain of models in $\text{dom}(\Sigma)$.
2. $d_p : \mathcal{M}_p \to [H_\theta]^{<\omega}$ is a function such that if $M \in N$, then $d_p(M) \in N$.
3. $f_p$ is a regressive function on $\mathcal{M}_p$, i.e. for every $M \in \mathcal{M}_p$, $f_p(M) \in [M]^{<\omega}$, such that whenever $P \in M$ are $\mathcal{M}_p$ and $f_p(M) \in P$, then $P \cap X \in \Sigma(M)$.

We equip $\mathbb{P}_\Sigma$ with the following ordering. We say $q$ is stronger than $p$ and write $q \leq p$, if

1. $\mathcal{M}_p \subseteq \mathcal{M}_q$.
2. For each $M \in \mathcal{M}_p$, $d_p(M) \subseteq d_q(M)$.
3. $f_p \subseteq f_q$.

For convention, we let $\emptyset$ be in $\mathcal{M}_p$, for every $p \in \mathcal{M}_p$, and leave $f_p(\emptyset)$ undefined.

**Lemma 2.9.** Suppose $p$ is a condition in $\mathbb{P}_\Sigma$. Let $M$ be an element of $\text{dom}(\Sigma)$ containing $p$. Then there is a condition $p^M \leq p$ such that $M \in \mathcal{M}_p^M$.

**Proof.** We let $p^M$ be defined as follows. Let $\mathcal{M}_p^M$ be just $\mathcal{M}_p \cup \{M\}$, extend $d_p$ as a function by letting $d_{p^M}(M) = \emptyset$, and also extend $f_p$ as a function by letting $f_{p^M}(M)$ be some finite set in $M \setminus \mathcal{M}_p$. It is easily seen $p^M$ is a condition and extends $p$.

**Proposition 2.10.** Suppose $\theta^* > \theta$ is a sufficiently large regular cardinal. Assume $M^* \prec H_\theta^*$ is countable and contains $\Sigma, X, \mathcal{P}(\theta)$. Let $M := H_\theta \cap M^*$. Suppose $p_0 \in \mathbb{P}_\Sigma$ is such that $M \in \mathcal{M}_{p_0}$. Then $p_0$ is $(M^*, \mathbb{P}_\Sigma)$-almost strongly generic.

**Proof.** Fix $p \leq p_0$. We aim to show that $S_p(M^*)$ is $M^*$-stationary, thus suppose $F \in M^*$ is an algebra on $\mathbb{P}_\Sigma$. Let $p \upharpoonright M = (\mathcal{M}_p \cap M, d_p \upharpoonright M, f_p \upharpoonright M)$. It is clear that $p \upharpoonright M$ is a condition in $M$. Set

$$X = \{(P, Q) \in \mathcal{M}_p \times \mathcal{M}_p : P \subseteq M, M \in Q \text{ and } f_p(Q) \in P \in Q\}.$$ 

If $(P, Q) \in X$, then there is a finite set $b_p^Q$ in $P$, and hence in $M$, such that $[b_p^Q, P \cap X] \subseteq \Sigma(Q)$. Fix such sets. Set $b_p = \bigcup\{b_p^Q : (P, Q) \in X\}$ and $B = \{b_P : P \in \mathcal{M}_p\}$. Pick some regular cardinal $\mu$ with $\theta < \mu < \theta^*$ and consider the following set which is easily verified to be unbounded in $[X]^{<\omega}$ and belongs to $M^*$.

$$U = \{R \cap X : \{B, X, F, p \upharpoonright M, \Sigma, \theta\} \subseteq R \prec H_\mu \text{ and } R \text{ is countable }\}.$$ 

Since $X \in H_\theta$, every algebra on $X$ belongs to $H_\theta$, and hence $\Sigma(M)$ is also $M^*$-stationary.

Now by the $M^*$-stationarity of $\Sigma(M)$ and Lemma 2.6, one can find $A \in U \cap M^* \cap \Sigma(M)$. Since $M^* \prec H_\theta^*$, there is $R \in M^*$ with $\{B, X, F, p \upharpoonright M, \Sigma, \theta\} \subseteq R$ such that $A = R \cap X$. 
Fix such $R$. Now using the openness of $\Sigma(M)$, there exists a finite set $b^M_R \subseteq A$ such that

$$[b^M_R, A]$$

is included in $\Sigma(M)$. We may assume that $b_R$ contains, as subsets, all $b_M^Q$ for $(M, Q) \in X$. We need to extend $p \upharpoonright M$ to a condition in $R$ so that its extensions in $R$ do not violate the third condition of Definition 2.8. To this end, let $p^*$ be the same as $p \upharpoonright M$ except about $d_{p^*}$, where we let it be defined on $M_p \cap M$ by $d_{p^*}(P) = d_p(P) \cup b_{p^*}$, where $P^+$ is the next model of $P$ in $(M_p \cap M) \cup \{R\}$. It is clear that $p^*$ is a condition in $R$ since $p \upharpoonright M, B, b_R \in H_\emptyset \cap R$. Now, suppose that $q \in R$ extends $p^*$. We shall show that $q$ is compatible with $p$. Put $M_q = M_q \cup M_p$. Let also $d_r$ be defined on $M_q$ as follows

$$d_r(P) = d_q(P)$$

if $P \in M$, and $d_r(P) = d_p(P)$ otherwise. we simply put $f_r = f_p \cup f_q$. What remains to be shown is that whenever $P \in Q$ are in $M_r$ with $f_r(Q) \in P$, we have that $P \cap X \in \Sigma(Q)$. To avoid the trivial case, we may assume that $Q \notin M$, and $P \in M_q \setminus M_p$. In this case there is $S \in M_p \cap M$ such that $S \in P \in S^+$. Now, if $(S^+, Q) \in X$, then

$$b^Q_{S^+} \subseteq b_{S^+} \in P.$$ 

Consequently, we have that $P \cap X \in [b^Q_{S^+}, S^+ \cap X] \subseteq \Sigma(Q)$. Otherwise

$$S^+ = R;$$

in this case if $Q = M$, then $P \cap X \in [b_R, A] \subseteq \Sigma(M)$, and if $M \neq Q$, then

$$P \cap X \in [b_R, M \cap X] \subseteq [b^Q_M, M \cap X] \subseteq \Sigma(Q).$$

Recall that $F \in R$, and hence $R \cap \mathbb{P}_\Sigma$ is closed under $F$, and that $R \cap \mathbb{P}_\Sigma$ is in $M^* \cap S_p(M^*)$. This concludes the proof.

Corollary 2.11. $\mathbb{P}_\Sigma$ is a.s.p.

Proof. It is clear from Lemma 2.9 and Proposition 2.10.

For a $V$-generic filter $G \subseteq \mathbb{P}_\Sigma$, we let $M_G = \{M : \exists p \in G$ such that $M \in M_p\}$.

Lemma 2.12. Suppose $G$ is a $V$-generic filter on $\mathbb{P}_\Sigma$. Then $M_G$ is a continuous $\in$-chain.

Proof. We show that if $(M_n)_{n<\omega}$ is a sequence in $M_G$, then $\bigcup_{n<\omega} M_n$ is also in $M_G$. This is equivalent to saying that if $M \in M_G$ is not the minimal member and is such that for every $P \in M_G$ below $M$, there is a model in $M_G$ between $P$ and $M$. Then $M$ is the union of models below $M$ in $M_G$. Thus suppose countable model $M \prec H_\emptyset$ is forced, by a condition $p$, to be in $M_G$ with the above property. Without loss of generality, we may assume that $M_p$ contains $M$ and some model below $M$ in $M_G$ as well. Now let $x \in M$. If $q \leq p$ is an arbitrary condition, then one can extend $q$ to a condition $q_x$ such that $x \in d_{q_x}(Q_x)$ where $Q_x$ is the largest model below $M$ in $M_{q_x}$. It then implies that any extension of $q_x$ which has a model above $Q_x$ should contain $x$. This is possible as there is some model below $M$ in $M_q$. This shows that the set of conditions $r$ such that $x$ belongs to some model below $M_r \cap M_r$ is dense below $p$. Thus for every $x \in M$, $p$ forces that there is a model below $M$ in $M_G$ containing $x$. Therefore, $M$ is the union of its predecessors in $M_G$, whenever $G$ is a $\mathbb{P}_\Sigma$-generic filter containing $p$. 

\[2.10\]
Let $f_G$ be defined on $\mathcal{M}_G$ by letting $f_G(M) = f_p(M)$ for some, or equivalently all, $p \in G$ with $M \in \mathcal{M}_p$.

**Proposition 2.13.** $\mathbb{P}_\Sigma$ adds a reflecting sequence for $\Sigma$.

**Proof.** Let $G$ be a $V$-generic filter on $\mathbb{P}_\Sigma$. By Lemma 2.12, $\mathcal{M}_G$ is a continuous $\in$-chain of models. Let $\langle M_\xi : \xi < \omega_1 \rangle$ be the natural enumeration of $\mathcal{M}_G$, i.e. for each $\xi < \omega_1$, $M_\xi \in M_{\xi+1}$. We claim that this is a reflecting sequence for $\Sigma$. If $\xi < \omega_1$ is a limit ordinal, then $f_G(M_\xi)$ belongs to $M_\xi$, and thus by the continuity, there is $\zeta < \xi$ so that $f_G(M_\xi) \in M_\zeta$, and thus for each $\eta \in \xi \setminus \zeta$, $f_G(M_\xi) \in M_\eta$. It is enough to pick $q \in G$ such that $M_\xi, M_\eta \in M_q$, and hence $M_\eta \cap X \in \Sigma(M_\xi)$.

3. Conclusion

In [6], Todorčević introduced and studied the forcing axiom PFA(S), and showed its consistency as well. This forcing axiom states that $S$ is a coherent Suslin tree, and if $\mathbb{P}$ is a proper forcing which preserves the c.c.c-ness of $S$, and that if $\mathcal{D}$ is a collection of dense subsets of $\mathbb{P}$ with $|\mathcal{D}| \leq \aleph_1$, then there is a $\mathcal{D}$-generic filter on $\mathbb{P}$. Now by Proposition 1.6 alone, $\mathbb{P}_\Sigma$ preserves the c.c.c-ness of $S$. Using standard arguments and Corollary 2.11, one can show that MRP holds under PFA(S).

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