How to Complete the Description of Physical Reality by Non-local Hidden Variables?

Timur F. Kamalov
Physics Department
Moscow State Open University
107996 Moscow, 22, P. Korchagn str., Russia
E-mail: ykamalov@rambler.ru
www.TimKamalov.narod.ru

Abstract
Which non-local hidden variables could complement the description of physical reality? The present model of extended Newtonian dynamics (MEND) is generalize but not alternative to Newtonian Dynamics because its extended Newtonian Dynamics to arbitrary reference frames. It is Physics of Arbitrary Reference Frames. Generalize and alternative is not the same. MEND describes the dynamics of mechanical systems for arbitrary reference frames and not only for inertial reference frames as Newtonian Dynamics. Newtonian Dynamics can describe non-inertial reference frames as well introducing fiction forces. In MEND we have fiction forces naturally and automatically from new axiomatic and we needn’t have inertial reference frame. MEND is differs from Newtonian Dynamics in the case of micro-objects description.

PACS 45.20.d-, 45.40.-f

Keywords: Model of Extended Newtonian Dynamics (MEND), non-local hidden variables.

1 Introduction
Classical Newtonian mechanics is essentially the simplest way of mechanical system description with second-order differential equations, when higher order time derivatives of coordinates can be neglected. The extended model of Newtonian mechanics with higher time derivatives of coordinates is based on generalization of Newton’s classical axiomatics onto arbitrary reference frames (both inertial and non-inertial ones) with body dynamics being described with higher order differential equations. Newton’s Laws, constituting, from the mathematical viewpoint, the axiomatics of classical physics, actually postulate the assertion that the equations describing the dynamics of bodies in inertial frames are second-order differential equations. However, the actual time-space is almost without exception non-inertial, as it is almost without exception that there exist (at least weak) fields, waves, or forces perturbing an ideal inertial frame. Non-inertial nature of the actual time-space is also supported by observations of the practical astronomy that expansion of the reality occurs with an acceleration. In other words, actually any real reference frame is a non-inertial one; and such physical reality can be described with a differential equation with time
derivatives of coordinates of the order exceeding two, which play the role of additional variables. This is evidently beyond the scope of Newtonian axiomatics. Aristotle's physics considered velocity to be proportional to the applied force, hence the body dynamics was described by first derivative differential equation. Newtonian axioms postulates reference frames, where a free body maintains the constant velocity of translational motion. In this case the body dynamics is described with a second order differential equation, with acceleration being proportional to force [1]. This corresponds to the Lagrangian depending on coordinates and their first derivatives (velocities) of the body, and Euler-Lagrange equation resulting from the principle of the least action. This model of the physical reality describes macrocosm fairly good, but it fails to describe micro particles. Both Newtonian axioms and the Second Law of Newton are invalid in microcosm. Only averaged values of observable physical quantities yield in the microcosm the approximate analog of the Second Law of Newton; this is the so-called Ehrenfest’s theorem. The Ehrenfest’s equation yields the averaged, rather than precise, ration between the second time derivative of coordinate and the force, while to describe the scatter of quantum observables the probability theory apparatus is required. As the Newtonian dynamics is restricted to the second order derivatives, while micro-objects must be described with equations with additional variables, tending Planck’s constant to zero corresponds to neglecting these variables. Hence, offering the model of extended Newtonian dynamics, we consider classical and quantum theories with additional variables, describing the body dynamics with higher order differential equations. In our model the Lagrangian shall be considered depending not only on coordinates and their first time derivatives, but also on higher-order time derivatives of coordinates. Classical dynamics of test particle motion with higher-order time derivatives of coordinates was first described in 1850 by M.Ostrogradskii [2] and is known as Ostrogradskii’s Canonical Formalism. Being a mathematician, M. Ostrogradskii considered coordinate systems rather than reference frames. This is just the case corresponding to a real reference frame comprising both inertial and non-inertial reference frames. In a general case, the Lagrangian takes on the form \((n \to \infty)\)

\[ L = L(t, q, \dot{q}, \ddot{q}, ..., \dot{q}^{(n)}). \]  

2 Theory of Extended Newtonian Dynamics

Let us consider in more detail this precise description of the dynamics of body motion, taking into account of real reference frames. To describe the extended dynamics of a body in an any coordinate system (corresponding to arbitrary reference frame) let us introduce concepts of kinematic state and kinematic invariant of an arbitrary reference frame.

**Definition:** Kinematic state of a body is set by \(n\)-th time derivative of coordinate. The kinematic state of the body is defined provided the \(n\)-th time derivative of body coordinate is zero, the \((n - 1)\)-th time derivative of body
coordinate being constant. In other words, we consider the kinematic state of the body defined if \((n-1)\)-th time derivative of body coordinate is finite. Let us note that a reference frame performing harmonic oscillations with respect to an inertial reference frame does not possess any definite kinematic state. Considering the dynamics of particles in arbitrary reference frames, we suggest the following two postulates.

**Postulate 1.** Kinematic state of a free body is invariable. This means that if the \(N\)-th time derivative of a free body coordinate is zero, the \((n-1)\)-th time derivative of body coordinate is constant. That is,

\[
\frac{d^n q}{dt^n} = 0, \quad \frac{d^{n-1} q}{dt^{n-1}} = \text{const.} \tag{2}
\]

In the extended model of dynamics, conversion from a reference frame to another one will be defined as:

\[
q' = q_0 + \dot{q}t + \frac{1}{2!}\ddot{q}t^2 + \ldots + \frac{1}{n!}q^{(n)}t^n \tag{3}
\]

\[
t' = t. \tag{4}
\]

**Postulate 2.** If the kinematic invariant of a reference frame is \(N\)-th time derivative of body coordinate, then the body dynamics is described with the differential equation of the order \(2N\):

\[
\alpha_{2N}q^{(2N)} + \ldots + \alpha_0 q = F(t, q, \dot{q}, \ddot{q}, ..., q^{(N)}). \tag{5}
\]

This means that the Lagrangian depends on \(N\)-th time derivative of coordinate, so variation when applying the least action principle will yield the order higher by a unity. Therefore, the dynamics of a free body in a reference frame with \(N\)-th order derivative being invariant shall be described with a differential equation of the order \(2N\). To consider dynamics of a body with an observer in an arbitrary coordinate system (which corresponds to the case of any reference frame), we apply the least action principle, varying the action function for \(N\)-th order kinematic invariant, we obtain the equation of the order \(2N\):

\[
\delta S = \delta \int L(t, q', q)dt = \int \sum_{n=0}^{N} (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial \dot{q}^{(n)}} \delta \dot{q}^{(n)} dt = 0. \tag{6}
\]

Then the equation describing the dynamics of a body with \(N\)-invariant is a \(2N\)-order differential equation, and for the case of irreversible time arrow we shall retain only even components. Expanding into Taylor’s series the function \(q = q(t)\) yields:

\[
q = q_0 + \dot{q}t + \frac{1}{2!}\ddot{q}t^2 + \ldots + \frac{1}{n!}q^{(n)}t^n. \tag{7}
\]

It is well known that the kinematic equation in inertial reference frames of Newtonian physics contains the second time derivative of coordinate, that is, acceleration:
\[ q_{\text{Newton}} = q_0 + vt + \frac{1}{2}at^2. \]  
(8)

Let us denote the additional terms with higher derivatives as

\[ q_r = \frac{1}{3!}q^{(3)}t^3 + \ldots + \frac{1}{n!}q^{(n)}t^n. \]  
(9)

Then

\[ q = q_{\text{Newton}} + q_r. \]  
(10)

In our case, the discrepancy between descriptions of the two models is the difference between the description of test particles in the model of extended Newtonian dynamics with Lagrangian \( L(t, q, \dot{q}, \ddot{q}, \ldots, \dot{q}^{(n)}; \ldots) \) and Newtonian dynamics in inertial reference frames with the Lagrangian \( L(t, q, \dot{q}) \):

\[ \int [L(t, q, \dot{q}, \ddot{q}, \ldots, \dot{q}^{(n)}) - L(t, q, \dot{q})] dt = h, \]  
(11)

\( h \) being the discrepancy (error) between descriptions by the two models. Comparing this value with the uncertainty of measurement in inertial reference frames, expressed by the Heisenberg uncertainty relation, the equation (11) can be rewritten as

\[ S(t, q, \dot{q}, \ldots, \dot{q}^{(n)}) - S(t, q, \dot{q}) = h. \]  
(12)

In the classical mechanics, in inertial reference frames, the Lagrangian depends only on the coordinates and their first time derivatives. In the extended models, in real reference frames, the Lagrangian depends not only on the coordinates and their first time derivatives, but also on their higher derivatives. Applying the least action principle [3], we obtain Euler-Lagrange equation for the extended Newtonian dynamics model:

\[ \sum_{n=0}^{N} (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial \dot{q}^{(n)}} = 0, \]  
(13)

or

\[ \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} - \ldots + (-1)^N \frac{d^N}{dt^N} \frac{\partial L}{\partial \dot{q}^{(N)}} = 0. \]  
(14)

The Lagrangian will be expressed through quadratic functions of variables:

\[ L = kq^2 - k_1 \dot{q}^2 + k_2 \ddot{q}^2 - \ldots + (-1)^\alpha k_{\alpha} \dot{q}^{(\alpha)2} = \sum_{\alpha=0}^{\infty} (-1)^\alpha k_{\alpha} \dot{q}^{(\alpha)2}. \]  
(15)

For our case, the action function will be:

\[ S = \int q \frac{\partial L}{\partial q} - \dot{q} \frac{\partial L}{\partial \dot{q}} + \ldots + (-1)^\alpha \dot{q}^{(\alpha)} \frac{\partial L^{(\alpha)}}{\partial \dot{q}^{(\alpha)}} + \ldots = \sum_{\alpha=0}^{\infty} (-1)^\alpha \dot{q}^{(\alpha)} \frac{d^\alpha}{dt^\alpha} \frac{\partial L}{\partial \dot{q}^{(\alpha)}}. \]  
(16)
Or

\[ S = 2kq^2 - 2k_1\dot{q}^2 + 2k_2q^2 + \ldots + 2k_\alpha\dot{q}^{(\alpha)}2 = 2 \sum_{\alpha=0}^{\infty} (-1)^\alpha k_\alpha\dot{q}^{(\alpha)}2. \]  

(17)

Introducing the notation

\[ F = \frac{\partial L}{\partial q}, p = \frac{\partial L}{\partial \dot{q}} \]

(18)

\[ F^2 = \frac{\partial L}{\partial q}, p^3 = \frac{\partial L}{\partial q^{(3)}} \]

(19)

\[ F^4 = \frac{\partial L}{\partial q^{(4)}}, p^5 = \frac{\partial L}{\partial q^{(5)}} \]

(20)

\[ \ldots \]

\[ F^{2n} = \frac{\partial L}{\partial q^{(2n)}}, p^{2n+1} = \frac{\partial L}{\partial q^{(2n+1)}}, \]

(21)

we obtain the description of inertial forces for the extended Newtonian dynamics model. The value of the resulting force accounting for inertial forces can be expressed through momentums and their derivatives, expressing the Second Law of Newton for the extended Newtonian dynamics model:

\[ F - \frac{dp}{dt} + \frac{d^2}{dt^2}(F^2 - \frac{dp^3}{dt}) + \frac{d^4}{dt^4}(F^4 - \frac{dp^5}{dt}) + \ldots \frac{d^n}{dt^n}(F^n - \frac{dp^{n+1}}{dt}) = 0. \]

(22)

Expanding the force into Taylor series, we obtain:

\[ F(t) = F_0 + \dot{F}t + \frac{1}{2!}\ddot{F}t^2 + \ldots \]

(23)

In other words, (22) can be written as

\[ \sum_{n=0}^{\infty} \frac{d^{2n}}{dt^{2n}}(F^{2n} - \frac{d^{2n}p^{2n+1}}{dt^{2n}}) = 0. \]

(24)

The action function takes on the form

\[ S = \sum_{n=0}^{\infty} (-1)^n q^{(n)}p^{n+1} = \sum_{n=0}^{N} (-1)^n q^{(n)}\frac{\partial L}{\partial q^{(n+1)}}. \]

(25)

For this case, energy can be expressed as

\[ E = \alpha_0q^2 + \alpha_1\dot{q}^2 + \alpha_2q^2 + \ldots + \alpha_n\dot{q}^{(n)}2 + \ldots \]

(26)
Denoting the Appel’s energy of acceleration [4] as $Q$, $\alpha_n$ being constant factors, we obtain for kinetic energy and potential energy, respectively,

$$E = V + W + Q \quad (27)$$
$$V = \alpha_0 q^2, \quad (28)$$
$$W = \alpha_1 \dot{q}^2 \quad (29)$$
$$Q = \alpha_2 \ddot{q}^2 + \ldots + \alpha_n q^{(n)2} + \ldots \quad (30)$$

The Hamilton-Jacobi equation for the action function will take on the form

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q, \quad (31)$$

The first addend in (30) is the so-called Appel’s energy of acceleration [4]. Let us compare $Q$ with the quantum potential [5] and complement the equation (31) with the continuity equation. If $Q \approx \alpha_2 \frac{\nabla^2 S}{m^2}$ (here, the value of the constant is chosen $\alpha_2 = \frac{\hbar m}{2}$). Hence, in the first approximation we obtain for the function

$$\psi = e^{\frac{i}{\hbar}S}, \quad (32)$$

the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi + V \psi. \quad (33)$$

3 Conclusions

Our case corresponds to Lagrangian $L(t, q, \dot{q}, \ddot{q}, \ldots, \dot{q}^{(n)}, \ldots)$, depending on coordinates, velocities and higher time derivatives, which we call additional variables, extra addends, or hidden variables. In arbitrary reference frames (including non-inertial ones) additional variables (addends) appear in the form of higher time derivatives of coordinates, which complement both classical and quantum physics. We call these additional addends, or variables, constituting the higher time derivatives of coordinates, addition variables or hidden variables, complementing the description of particles. It should be noted that these hidden variables can be used to complement the quantum description without violating von Neumann theorem, as this theorem is not applied for non-linear reference frames, while the extended Newtonian dynamics model assumes employing any reference frames, including non-linear ones. Comparing the generalized Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q, \quad (34)$$

$Q$ being the additional variables with higher derivatives, with the quantum Bohm’s potential, one can conclude that neglecting higher-order time derivatives of coordinates brings about incompleteness of physical reality description.
The coordinate derivative of $Q$ determines the quantum force. This means that complete description of physical reality requires considering differential equations of the order exceeding second; uncertainty of the position of the particle under investigation shall be attributed to fluctuations of the reference body and reference frame associated with it. Hence, the differential equation describing this case shall be of the order exceeding second. In this case, uncertainty of a micro objects description is follow by incompleteness of the description of the physical reality by Newtonian physics, that is, the lack of a complete description with additional variables in the form of higher time derivatives of coordinates. The contemporary physics presupposes employment of predominantly inertial reference frames; however, such a frame is very hard to obtain, as there always exist external perturbative effects, for example, gravitational forces, fields, or waves. In this case, the relativity principle enables transfer from the gravitational forces or waves to inertial forces. For example, if we consider a spaceship with two observers in different cabins, one can see that this system is non-ideal, the inertial forces (or pseudo-forces) could constitute additional variables here. In this case, superposition of the two distributions obtained by the observers could yield a non-zero correlation factor, though each of the two observations has a seemingly random nature. If the fact that the reference frame is non-inertial and hence there exist additional variables in the form of inertial effects is ignored, then non-local correlation of seemingly independent observations would seem surprising. This example could visualize not only the interference of corpuscle particles, but also the non-local character of quantum correlations when considering the effects of entanglement.

References

[1] Newton I. Philosophiae naturalis principia mathematica. London, 1687. 220 p.

[2] M. V. Ostrogradskii Memoire sur les equations differentielles relatives aux problemes des isoperim'etres // Memoires de l’Academie Imperiale des Sciences de Saint-Peterbourg, v. 6, 1850. P. 385.

[3] Lagrange J.I. Mecanique analitique. Paris, De Saint, 1788. 131 p.

[4] Appel P., Traite’ de Me’caique Rationelle, Paris, Ganthier-Villars e’diteur, 1953.

[5] Bohm D. A suggested interpretation of the quantum theory in terms of “hidden” variables I //Physical Review. 1952. v. 85. P. 166.