Non-Hermitian Quantum Physics of Open Systems

Ingrid Rotter*

Max Planck Institute for the Physics of Complex Systems, D-01187 Dresden, Germany
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Abstract

Information on quantum systems can be obtained only when they are open (or opened) in relation to a certain environment. As a matter of fact, realistic open quantum systems appear in very different shape. We sketch the theoretical description of open quantum systems by means of a projection operator formalism elaborated many years ago, and applied by now to the description of different open quantum systems. The Hamiltonian describing the open quantum system is non-Hermitian. Most studied are the eigenvalues of the non-Hermitian Hamiltonian of many-particle systems embedded in one environment. We point to the unsolved problems of this method when applied to the description of realistic many-body systems. We then underline the role played by the eigenfunctions of the non-Hermitian Hamiltonian. Very interesting results originate from the fluctuations of the eigenfunctions in systems with gain and loss of excitons. They occur with an efficiency of nearly 100%. An example is the photosynthesis.

* email: rotter@pks.mpg.de
1. Introduction.

What have, for example, nuclear physics and photosynthesis in common? The quick answer is: nothing. That is however only half the truth: both are open quantum systems. The atomic nuclei are embedded in the continuum of scattering wavefunctions whereby their states get, generally, a finite lifetime. The photosynthesis occurs in the light-harvesting complex, in which light energy is converted in chemical energy. These two physical processes are completely different from one another. The only common feature is that the two systems have a finite extension and are embedded in an infinitely extended environment. In the following, we will consider similarities and differences between these two (and many other) so unequal open quantum systems. [1].

2. Projection operator formalism.

A convenient method to describe the different states of a nucleus is to use the projection operator formalism elaborated many years ago [2], according to which the whole function space is divided into the two subspaces $\mathcal{P}$ and $\mathcal{Q}$ with $\mathcal{P} + \mathcal{Q} = 1$. The subspace $\mathcal{Q}$ describes the finite system (e.g. the nucleus), while the subspace $\mathcal{P}$ contains the continuum of scattering wavefunctions. The solution for the nucleus can be found. According to Feshbach, it is, however, a formal solution since the number of states of a heavy nucleus is very large and does not allow to obtain numerical results without introducing approximations. Mostly, statistical methods are used. This resulted finally not only in the description of nuclear states, but in numerous studies on quantum chaos (see e.g. [3]).

The situation is different in light nuclei where the level density is small and the properties of individual nuclear states can be investigated. Here, the solution obtained for the description of the nucleus is no longer a formal solution. It can rather be found numerically. It is the basic equation of the so-called continuum shell model that describes the open nuclear system. The Hamiltonian of the continuum shell model is non-Hermitian.

Practical problems in finding reliable solutions for light nuclei in the framework of the continuum shell model arise from the fact that the interaction between the nucleons in nuclei is different from that between free nucleons. It is a residual interaction which remains after the central mean field of the nucleus has been created by the nucleons themselves. The residual interaction is badly known.

These problems do not occur when dealing with atoms. In this case, the electrons move
in the field that is created mainly by the atomic nucleus. Thus, the interaction between the electrons is quite well known. Here, the continuum shell model has been applied successfully to the description of different interference processes, for references see the review [4].

3. Effective non-Hermitian Hamilton operator.

Many numerical calculations for open many-body quantum systems are performed successfully by using the non-Hermitian Hamilton operator [4]

\[ H^{\text{eff}} = H_0 + \sum_c V_{0c} \frac{1}{E^+ - H_c} V_{c0}. \]

Here, \( H_0 \) is the Hamiltonian describing the corresponding closed system with discrete states, \((E^+ - H_c)^{-1}\) is the Green function in the continuum with the Hamiltonian \( H_c \) describing the environment of decay channels, and \( V_{0c}, V_{c0} \) stand for the coupling of the closed system to the different channels \( c \) of the environment. The non-Hermiticity of \( H^{\text{eff}} \) arises from the second term of \( H^{\text{eff}} \), i.e. from the perturbation of the system occurring under the influence of its coupling to the environment. It is complex: the real part arises from the principal value integral and the imaginary one from the residuum (for details see [4]).

Sometimes, the effective Hamiltonian is approximated by \( H^{\text{eff}} = H_0 + \alpha W \) where \( W \) is imaginary, and the properties of the system are studied as a function of \( \alpha \) (for examples see the review [4]). In [2] the non-Hermitian operator is assumed to be \( \hat{\mathcal{H}} = \hat{H} - i\hat{\Gamma} \). This operator allows us to study the general meaning of the imaginary part of the non-Hermitian Hamiltonian. The information on the considered physical system, involved in \( H^{\text{eff}} \), is however lost. In any case, the non-Hermiticity of \( H^{\text{eff}} \) arises from the second term which is added to \( H_0 \) as a perturbation.

Some experimentally well-known results can be understood when the system is described by the Hamiltonian \( H^{\text{eff}} \). An example is the variation of the cross section picture at low level density to that at high level density. At low level density isolated resonances can be identified, while this is impossible at high level density where the individual resonances overlap and cause a fluctuation picture of the cross section. This transition has been traced successfully in nuclear physics, many years ago, by means of \( H^{\text{eff}} \) [6]. Further examples from recent studies on mesoscopic and other systems can be found in the review [7].

The advantage of using \( H^{\text{eff}} \) is, above all, that the results for the closed system obtained with the Hermitian Hamiltonian \( H_0 \), can be used. With vanishing coupling strength between
system and environment, the second term of $H^{\text{eff}}$ vanishes, i.e. $H^{\text{eff}} \rightarrow H_0$.

In the non-Hermitian formalism there are singular points. From mathematical studies, their existence has been known for quite a long time \cite{8}. At these points, called mostly \textit{exceptional points} (EPs), two eigenvalues of the non-Hermitian operator coalesce causing interesting topological effects. These effects are interesting in themselves, and are studied not only theoretically but also experimentally. The influence of EPs onto the dynamics of open quantum systems is studied in many papers, see e.g. the reviews \cite{4,7,9} and the recent papers \cite{10}.

As a result of the many studies in different fields of physics, the following can be stated. Far from EPs, Hermitian quantum physics provides usually good results. Under the influence of EPs, however, interesting new features appear (which are mostly counterintuitive).

Some problems with $H^{\text{eff}}$ are the following.

(i) According to numerical studies, EPs do not play any role when the system is coupled to only one well-defined environment (called usually \textit{one-channel case}). This is due to nonlinear effects involved in the non-Hermitian Hamiltonian \cite{11}. This result agrees with the experience obtained from the successful description of one-channel processes without taking into account EPs.

(ii) EPs are defined mathematically for systems coupled to one channel \cite{8}, while most physically interesting processes (such as transfer processes) occur when the system is coupled to at least two channels.

4. Eigenfunctions of the non-Hermitian operator.

In order to study the influence of EPs on the properties of physical systems one has, first of all, to know the eigenvalues of the non-Hermitian Hamiltonian. Accordingly, the properties of the eigenfunctions of $H^{\text{eff}}$ are seldom studied although they influence the system properties not only in a larger parameter range than the eigenvalues. Much more important (and almost unknown in current literature) is that the eigenfunctions incorporate more sensitive signatures of the EPs than the eigenvalues.

The eigenfunctions $\Phi_i$ of the non-Hermitian Hamiltonian are, generally, biorthogonal and should be normalized according to \cite{4}

$$\langle \Phi_i^* | \Phi_j \rangle = \delta_{ij}$$
in order to guarantee a smooth transition from an open quantum system to a nearly closed one. Further, the $\Phi_i$ are (almost) orthogonal far from EPs and differ from one another only by a phase in approaching an EP. Thus, the phases are not rigid, generally. The phase rigidity defined by

$$r_k \equiv \frac{\langle \Phi_k^* | \Phi_k \rangle}{\langle \Phi_k | \Phi_k \rangle}$$

characterizes the distance of the state $k$ from an EP: $r_k \to 0$ in approaching an EP while $r_k \approx 1$ far from an EP. Since, moreover, every state of the system is coupled to the common environment, all states may mix via the environment. This so-called external mixing of the eigenfunctions $\Phi_i$ is a second-order effect which becomes infinitely large at an EP.

The eigenfunctions $\Phi_i$ are therefore not only different from the eigenfunctions $\Phi_0^i$ of the Hermitian Hamiltonian $H_0$. They contain, moreover, valuable information on the possible influence of an EP on the dynamics of an open quantum system which is determined by the distance of the considered eigenstates $i$ from an EP. This information is involved, above all, in the phase rigidity and in the external mixing of the eigenfunctions $\Phi_i$ of the non-Hermitian Hamiltonian.

One of many examples on the role of the eigenfunctions of the non-Hermitian Hamiltonian in physics is that higher-order EPs (at which more than two eigenvalues coalesce) cannot be observed. They can be found mathematically by considering the eigenvalues. However, the external mixing of the eigenfunctions via the environment causes some energy shift of all the eigenstates. As a consequence, the possibility to observe higher-order EPs is prevented.

5. Genuine non-Hermitian Hamilton operator.

The description of the properties of open quantum systems by means of $H^{\text{eff}}$ has provided very many interesting results which contribute to a better understanding of non-Hermitian many-body quantum physics. Effects which are caused by interferences between the eigenfunctions $\Phi_i$ of the non-Hermitian Hamiltonian are however more difficult to extract or are not at all describable with $H^{\text{eff}}$.

As has been shown in [12], interferences between the eigenfunctions $\Phi_i$ of the non-Hermitian Hamilton operator may become important in physical processes. For example, they explain the so-called phase lapses observed experimentally a few years ago in the transmission through quantum dots [13]. These experimental results remained puzzling for many
Furthermore, the results of non-Hermitian quantum physics are always different from those of Hermitian quantum physics as has been shown theoretically as well as experimentally [14]. This is incompatible with $H_{\text{eff}} \rightarrow H_0$ occurring with vanishing coupling strength between system and environment. It means rather that the non-Hermitian Hamiltonian $\mathcal{H}$ must be a genuine non-Hermitian Hamiltonian (instead of $H_{\text{eff}}$).

A genuine non-Hermitian operator involves the complex energies $\varepsilon_i = e_i + i/2 \gamma_i$ of all $N$ states of the system. For $N = 2$, it reads

$$\mathcal{H} = \begin{pmatrix} \varepsilon_1 & \omega \\ \omega & \varepsilon_2 \end{pmatrix} = \begin{pmatrix} e_1 + i/2 \gamma_1 & \omega \\ \omega & e_2 + i/2 \gamma_2 \end{pmatrix}. $$

The operator $\mathcal{H}$ allows us to describe not only quantum systems with excitation of eigenstates but also those which occur without excitation of any eigenstates of $\mathcal{H}$. In the last case, fluctuations of the eigenfunctions cause measurable effects. The fluctuations are large, above all, near to EPs. Since eigenstates of $\mathcal{H}$ are not excited in such a case, the process occurs with an efficiency of nearly 100% [16].

This theoretical result allows us to obtain an understanding for the experimental results obtained for the photosynthesis [17–20]. These experimental results cannot be described by means of the standard Hermitian operator $H_0$ or by using the effective Hamiltonian $H_{\text{eff}}$ which both describe the many-particle properties of the system with excitation of eigenstates of the Hamiltonian. Fluctuations being basic in the process of photosynthesis, are related exclusively to the eigenfunctions which fluctuate around singular (exceptional) points.

6. Concluding remarks.

Numerical investigations of concrete problems are seldom simpler in the genuine non-Hermitian formalism than those based on the Hermitian Hamiltonian $H_0$ and even on the effective Hamiltonian $H_{\text{eff}}$. There are however many interesting problems of the many-body physics to which the standard methods, including those based on $H_{\text{eff}}$, cannot be applied. These are, e.g., questions related to the role of EPs in many-channel problems, or to the existence of other types of singular points. As a matter of principle, the Hamiltonians $H_0$ or $H_{\text{eff}}$ can be used only for the description of many-particle problems in which internal degrees of freedom of the system (i.e. the eigenvalues) are excited.

Interesting questions are related to processes which occur without excitation of internal
degrees of freedom of the system, i.e. without excitation of the eigenstates of the non-Hermitian Hamiltonian. These questions can be studied only on the basis of the genuine Hamiltonian $\mathcal{H}$, in which not only the properties of the eigenstates but also the fluctuations of the eigenfunctions around EPs are involved. Photosynthesis is surely not the only example of such a process. These problems cannot be described neither by $H_0$ nor by $H^{\text{eff}}$. In literature, the high sensitivity of the eigenfunctions of the non-Hermitian Hamilton operator $\mathcal{H}$ and its relation to the dynamics of open quantum systems is almost not studied up to now.

Another interesting result is that reordering processes under the influence of EPs occur also in optics. The Dicke phenomenon of superradiance is well known for many years. Only recently, the corresponding long-living subradiant modes could be identified experimentally \[21\].

A further open question in non-Hermitian quantum physics is the existence of singularities which are different from the EPs. Their mathematical and physical meaning should be clarified. An example are the points at which the eigenfunctions $\Phi_i$ of the non-Hermitian Hamilton operator are orthogonal (and not biorthogonal).

Summarizing, it can be stated that the attractiveness of non-Hermitian quantum physics for the description of open quantum systems consists, above all, in the fact that some basic problems of the standard Hermitian quantum physics of closed systems do not appear in the non-Hermitian quantum physics of open systems. One of many examples is that the "Schrödinger cat" is not shielded in an open quantum system from a direct observation. More examples can be found in \[11\]. Studying concrete physical systems, it has been possible to explain some “puzzling” experimental results in the framework of the non-Hermitian quantum physics. Other results will surely provide the basis for further interesting studies, including applications.

[1] We underline that we consider open quantum systems. This should not be confused with the consideration of PT-symmetric systems the nature of which is different, as stated in, e.g., C.M. Bender, Journal of Physics: Conference Series 631, 012002 (2015).

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[15] In contrast to the definition that is used in, for example, nuclear physics, we define the complex energies before and after diagonalization of the non-Hermitian Hamiltonian by $\varepsilon_k = e_k + \frac{i}{2}\gamma_k$ and $E_k = E_k + \frac{i}{2}\Gamma_k$, respectively, with $\gamma_k \leq 0$ and $\Gamma_k \leq 0$ for decaying states. This definition will be useful when discussing systems with gain (positive widths) and loss (negative widths), see e.g. [16].

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