Remarks on the static potential in theories with Lorentz violation terms

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Abstract – We study the impact of Lorentz-violating terms on a physical observable for both electrodynamics of chiral matter and an Abelian Higgs-like model in 3 + 1 dimensions. Our calculation is done within the framework of the gauge-invariant, but path-dependent, variables formalism. Interestingly enough, for electrodynamics of chiral matter we obtain a logarithmic correction to the usual static Coulomb potential. Whereas for an Abelian Higgs model with a Lorentz-breaking term, our result displays new corrections to the Yukawa potential.

Introduction. – The study of the physical consequences of topology or, more precisely, topological terms has considerably increased over the last years. In all these studies, the $\theta$-term (or axion-term) $\sim \theta E \cdot B$, has been the focus of interest. As is well known, the axion is a hypothetical pseudo-scalar particle introduced to explain the CP nonviolation problem in QCD [1–3]. In this respect, we also recall that the axion term provides a consistent framework for the Witten effect [4–6] as well as for the topological magneto-electric effect [7,8]. Particularly impressive is that the effects of this topological term have been materialized through the discovery of new materials [9].

On the other hand, in recent times a great deal of attention has been devoted to the study of quantum-anomaly–induced effects with chiral fermions [10]. An example that illustrates this is the chiral magnetic effect (CME), which is the generation of vector current by an external magnetic field in the presence of imbalance between the chemical potentials of right-handed and left-handed fermions [11–14]. Along the same line, we also mention the anomalous Hall effect [15,16]. However, it should be emphasized that the crucial feature of these quantum-anomaly–induced effects is to change the electromagnetic response of chiral matter. Interestingly, these systems (electrodynamics of chiral matter) are described by a Maxwell-Chern-Simons electrodynamics with a constant four-vector, which breaks the Lorentz invariance. Let us also mention here that the issue of Lorentz invariance violation in quantum field theories has been a subject of intense study [17–20], where the most studied framework is the standard model extension, which consists of the minimal standard model plus small Lorentz-violating terms. Particularly significant from this point of view are the Lorentz invariance violation electrodynamics, including either even or odd violating terms. It is worth recalling at this stage that theories with a topological term in (2 + 1) dimensions, where the physical excitations obeying it are called anyons, have been widely discussed in the literature [21–24]. Accordingly, the three-dimensional Chern-Simons gauge theory offers a natural setting so that Wilczek’s charge-flux composite model of the anyon can be realized [25].

In this context we also point out that an Abelian Higgs model with a Lorentz-breaking term has been considered in refs. [26–28], where aspects of causality, unitarity, spontaneous gauge-symmetry breaking, and vortex formation were investigated. As a result, it was shown that unitarity is always violated for an external vector time-like or null. However, whenever the external vector is space-like, physically consistent excitations are found. Also, it was found a physical feature analogous of what happens in (2 + 1)D, namely, the electrostatic and magnetostatic fields are not independent.

With these ideas in mind, in this work we examine another aspect of these theories, that is, the impact of the Lorentz-violating terms on a physical observable. To this end we will study the confinement vs. screening issue
for both electrodynamics of chiral matter and an Abelian Higgs model with a Lorentz-breaking term. Our calculation is accomplished by making use of the gauge-invariant, but path-dependent, variables formalism along the lines of [29,30]. As we shall see, in the case of electrodynamics of chiral matter, by adopting a purely space-like vector $v^\mu$, our result shows that the static potential is a logarithmic correction to the usual static Coulomb potential. On the other hand, in the case of the Abelian Higgs model with a Lorentz-breaking term, the static potential displays new corrections to the Yukawa potential.

**Interaction energy.**

_Electrodynamics of chiral matter._ We turn now to the problem of obtaining the interaction energy between static point-like sources for the two models we shall consider in this work. With this purpose, let us consider first the Hamiltonian analysis for the electrodynamics of chiral matter coupled to an external source $J^0$. We start from the four-dimensional spacetime Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\mu}{4} \theta \tilde{F}_{\mu\nu} F_{\mu\nu} - A_0 J^0, \quad (1)$$

where $\theta = \theta(t, x)$. Note that the Lagrangian (1) can be written alternatively in the form

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\mu}{4} \partial_\mu \theta \varepsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma} - A_0 J^0. \quad (2)$$

Letting $v_\mu = \partial_\mu \theta$, we can, therefore, write

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\mu}{4} v_\mu \varepsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma} - A_0 J^0. \quad (3)$$

It may be noted here that the choice of the background four-vector $v_\mu$ as the four-gradient of the scalar $\theta$ automatically ensures gauge invariance of the Carroll-Field-Jackiw term. Moreover, in a supersymmetric scenario with Lorentz-symmetry violation, supersymmetry imposes that $v_\mu$ must necessarily be a gradient. And, this does not mean that the background becomes dynamical: the fact that $v_\mu$ stems from a scalar does not impose that a $v_\mu$-kinetic term must be introduced into the Carroll-Field-Jackiw Lagrangian.

Having characterized the new effective Lagrangian, we can now examine the Hamiltonian structure of the theory under consideration when $v_\mu = (0, v_i)$. The canonical momenta are

$$\Pi^\lambda = F^{\lambda 0} + \frac{\mu}{2} \varepsilon^{\lambda 00\alpha} v_i A_\alpha, \quad (4)$$

which results in the usual primary constraint, $\Pi^0 = 0$ and $\Pi^\lambda = F^{\lambda 0} + \frac{\mu}{2} \varepsilon^{\lambda 00\alpha} v_i A_\alpha$. This allows us to write the following canonical Hamiltonian $H_C$:

$$H_C = \int d^3x \left\{ -\frac{1}{2} F_{0i} F^{0i} + \frac{1}{4} F_{ij} F^{ij} \right\} + \int d^3x \left\{ -A_0 \left( \partial_\mu \Pi^\mu - \frac{\mu}{2} \varepsilon^{0ijk} v_i \partial_j A_k + J^0 \right) \right\}. \quad (5)$$

The secondary constraint generated by the time preservation of the primary constraint, $\Pi^0 = 0$, is now $\Gamma_1 \equiv \partial_\mu \Pi^\mu - \frac{\mu}{2} \varepsilon^{0ijk} v_i \partial_j A_k + J^0 = 0$. The above constraints are the first-class constraints of the theory since no more constraints are generated by the preservation of the secondary constraint. The corresponding total (first-class) Hamiltonian that generates the time evolution of the dynamical variables then reads $H = H_C + \int d^3x (u_0(x) \Pi_0(x) + u_1(x) \Gamma_1(x))$, where $u_0(x)$ and $u_1(x)$ are arbitrary Lagrange multipliers to implement the constraints. Since $\Pi^0 = 0$ always and $A_0(x) = [A_0(x), H] = u_0(x)$, which is completely arbitrary, we eliminate $A^0$ and $\Pi^0$ because they add nothing to the description of the system. The Hamiltonian then takes the form

$$H = \int d^3x \left\{ -\frac{1}{2} F_{0i} F^{0i} + \frac{1}{4} F_{ij} F^{ij} \right\} + \int d^3x \left\{ w(x) \left( \partial_\mu \Pi^\mu - \frac{\mu}{2} \varepsilon^{0ijk} v_i \partial_j A_k + J^0 \right) \right\}, \quad (6)$$

where $w(x) = u_1(x) - A_0(x)$.

In order to break the gauge freedom of the theory, we introduce a gauge condition such that the full set of constraints becomes second class, so we choose

$$\Gamma_2(x) \equiv \int_0^1 d\lambda x^\lambda A_\lambda(x) = 0, \quad (7)$$

where $0 \leq \lambda \leq 1$ is the parameter describing the space-like straight path $x^i = \xi^i + \lambda(x - \xi^i)$, and $\xi$ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. With this, the only nontrivial Dirac bracket is given by

$$\{ A_i(x), \Pi_j(y) \} = \delta_{ij} \delta^{(3)}(x - y)$$

$$- \partial_t \int_0^1 d\lambda x^\lambda \delta^{(3)}(\lambda x - y). \quad (8)$$

From this expression we readily obtain the Dirac brackets in terms of the magnetic $(B^i = \varepsilon^{ijk} \partial_j A_k)$ and electric $(E^i = \Pi^i - \frac{\mu}{2} \varepsilon^{0ijk} v_j A_k)$ fields as

$$\{ E_i(x), B_j(y) \}^* = \varepsilon_{ijk} \delta^{(3)}(x - y) \quad (9)$$

$$\{ B(x), B(y) \}^* = 0, \quad (10)$$

and

$$\{ E_i(x), E_j(y) \}^* = \mu \varepsilon_{ijk} \delta^{(3)}(x - y). \quad (11)$$

This allows us to derive the equations of motion for the electric and magnetic fields, that is,

$$\dot{B}_i(x) = \varepsilon_{ijk} \partial_k E_j(x), \quad (12)$$

$$\dot{E}_i(x) = -\mu \varepsilon_{ijk} v_k E_j(x) + \varepsilon_{ijk} \partial_k B_j(x). \quad (13)$$

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Similarly, we see that Gauss’s law takes the form
\[
\left(\nabla^2 + \mu^2 v^2\right) E_i = 0 \quad (\text{14})
\]

We also note that under the assumed conditions of static fields, eqs. (12) and (13) must vanish, which, then, yields
\[
E_i = \partial_i \Phi, \quad (\text{15})
\]

where
\[
\Phi = \frac{\nabla^2}{\left[\nabla^2 - \mu^2 v^2 \nabla^2 - \mu^2 (\mathbf{v} \cdot \nabla)^2\right]} \Phi \quad (\text{16})
\]

For \(J^0 (x) = q \Phi (x)\), it follows that
\[
\Phi = \frac{q}{2\mu v} \int \frac{dk}{(2\pi)^2} \frac{k^2}{\left(k^2 - \mu^2 v^2 k^2 + \mu^2 (\mathbf{v} \cdot \mathbf{k})^2\right)} e^{i\mathbf{k} \cdot \mathbf{x}}. \quad (\text{17})
\]

Now, considering \(\mathbf{v} = v \hat{z}\), we find that the foregoing equation can be brought to the form
\[
\Phi = \frac{q}{2\mu v} \int \frac{dk}{(2\pi)^2} \frac{k^2}{k_\perp} e^{i k_\perp \cdot \mathbf{x}_\perp} \times \left(\frac{k_\perp^2}{(k_\perp^2 - \alpha^2)} - \frac{1}{(k_\perp^2 - \beta^2)}\right) e^{i k_\parallel z} + \frac{q}{2\mu v} \int \frac{dk}{(2\pi)^2} \frac{k^2}{k_\perp} e^{i k_\perp \cdot \mathbf{x}_\perp} \times \left(\frac{k_\perp^2}{(k_\perp^2 - \alpha^2)} - \frac{k_\perp^2}{(k_\perp^2 - \beta^2)}\right) e^{i k_\parallel z}, \quad (\text{18})
\]

where \(\alpha^2 = -k_\perp^2 + \mu v k_\perp\) and \(\beta^2 = -k_\perp^2 - \mu v k_\perp\). It should be further noted that
\[
\int \frac{dk}{(2\pi)^2} \frac{1}{(k_\perp^2 - \alpha^2)} \frac{1}{(k_\perp^2 - \beta^2)} e^{i k_\parallel z} = \frac{1}{2} \left\{ e^{-\sqrt{k_\perp^2 - \mu v k_\perp} z} - e^{-\sqrt{k_\perp^2 + \mu v k_\perp} z} \right\}, \quad (\text{19})
\]

and
\[
\int \frac{dk}{(2\pi)^2} \frac{k_\perp^2}{(k_\perp^2 - \alpha^2)} - \frac{k_\perp^2}{(k_\perp^2 - \beta^2)} e^{i k_\parallel z} = \frac{1}{2} \sqrt{k_\perp^2 - \mu v k_\perp} e^{-\sqrt{k_\perp^2 - \mu v k_\perp} z} - \frac{1}{2} \sqrt{k_\perp^2 + \mu v k_\perp} e^{-\sqrt{k_\perp^2 + \mu v k_\perp} z}. \quad (\text{20})
\]

Finally, making use of the preceding results, we can write the electric field as the sum of two parts:
\[
E_i = \frac{q}{8\pi \mu v} \partial_i \Phi^{(1)} + \frac{q}{4\pi \mu v} \partial_i \Phi^{(2)}, \quad (\text{21})
\]

where
\[
\Phi^{(1)} = \int_0^\infty dk_\perp k_\perp^2 \int_0^\infty d\mathbf{k}_\perp \Phi (\mathbf{k}_\perp | \mathbf{x}_\perp |) e^{-\sqrt{k_\perp^2 - \mu v k_\perp} z} \left(\frac{1}{k_\perp^2 - \mu v k_\perp}\right)^{3/2}, \quad (\text{22})
\]

and
\[
\Phi^{(2)} = \frac{1}{2} \int_0^\infty dk_\perp \int_0^\infty d\mathbf{k}_\perp \Phi (\mathbf{k}_\perp | \mathbf{x}_\perp |) e^{-\sqrt{k_\perp^2 + \mu v k_\perp} z} \left(\frac{1}{k_\perp^2 + \mu v k_\perp}\right)^{3/2}. \quad (\text{23})
\]

Here \(J_0 (k_\perp | \mathbf{x}_\perp |)\) is a Bessel function of the first kind, where \(k_\perp\) and \(\mathbf{x}_\perp\) denote the momentum component and coordinate component perpendicular to \(\mathbf{v}\). To get the above expressions we used \(J_0 (x) = \frac{1}{x} \int_0^\infty e^{ix \cos \theta} d\theta\).

We now are in a position to calculate the energy interaction between static point-like sources, by using the gauge-invariant but path-dependent variables formalism. This is accomplished by making use of \([29]\)
\[
V \equiv q (A_0 (0) - A_0 (y)), \quad (\text{24})
\]

where the physical scalar potential is given by
\[
A_0 (x) = \int_0^1 d\mu x^i E_i (\lambda x), \quad (\text{25})
\]

with \(i = 1, 2, 3\). We also recall that \(25\) follows from the vector gauge-invariant field expression \([29]\)
\[
A_\mu (x) \equiv A_\mu (x) + \partial_\mu \left( - \int_\xi^x d\mu A_\mu (z) \right), \quad (\text{26})
\]

where the line integral is along a space-like path from \(\xi\) to \(x\), on a fixed time slice. Interestingly, these variables \(26\) commute with the sole first-class constraint (Gauss’s law), showing that these fields are physical variables.

Now making use of eqs. \(21\) and \(25\), we readily find that
\[
A_0 (x) = \frac{q}{8\pi \mu v} \Phi^{(1)} (x) + \frac{q}{4\pi \mu v} \Phi^{(2)} (x), \quad (\text{27})
\]

after subtracting the self-energy terms.

From eq. \(24\), the corresponding static potential for two opposite charges located at \(0\) and \(y\) it should be calculated.

One may gain further insight into the overall structure of the interaction energy by examining eqs. \(22\) and \(23\) in some limit. With this in mind, we shall introduce a cutoff \(\Lambda\) in eqs. \(22\) and \(23\). It may be noted here that according to the current estimates in the literature on the parameters associated to the \(v\) Lorentz-symmetry-violating operators \([31]\), the product of the parameters \(\mu\) and \(v\) must be upper-bounded as follows: \(\mu, v < 10^{-42}\) GeV. Since we are considering our cutoff, \(\Lambda\), to be very small, this means that we are confined to the regime of low-frequency electromagnetic waves (radio waves, for instance). Therefore, we may safely take the modulus of the wave vector (in natural units) in the range \(k \sim 10^{-22}\) GeV–10–15 GeV. Then, \(\Lambda\) must be of this order, too. As a consequence, with the values estimated above for \(\mu, v, k\) and \(\Lambda\), we can ensure that \(\mu v k_\perp \ll k_\perp^2, \Lambda^2\).
In such a case, we can rewrite expression (16) in the form
\[ \Phi = \frac{q}{8\pi \mu v} \lim_{\lambda \to 0} \bar{\Phi}^{(1)} + \frac{q}{4\pi \mu v} \lim_{\lambda \to 0} \bar{\Phi}^{(2)}, \] (28)
where
\[ \bar{\Phi}^{(1)} = \int_0^\infty dk_1 k_1^2 J_0(k_1|x_1|) \frac{e^{-k_1^2 + \Lambda^2 - \mu v k_1 z}}{k_1^2 + \Lambda^2 - \mu v k_1} \]
\[ - \int_0^\infty dk_1 k_1^2 J_0(k_1|x_1|) \frac{e^{-k_1^2 + \Lambda^2 + \mu v k_1 z}}{k_1^2 + \Lambda^2 + \mu v k_1}, \] (29)
and
\[ \bar{\Phi}^{(2)} = -\frac{1}{2} \int_0^\infty dk_1 J_0(k_1|x_1|) \frac{e^{-k_1^2 + \Lambda^2 - \mu v k_1 z}}{(k_1^2 + \Lambda^2 - \mu v k_1)^{3/2}} \]
\[ + \frac{1}{2} \int_0^\infty dk_1 J_0(k_1|x_1|) \frac{e^{-k_1^2 + \Lambda^2 + \mu v k_1 z}}{(k_1^2 + \Lambda^2 + \mu v k_1)^{3/2}}. \] (30)

We shall now examine the potential &ll; A case. As a consequence of this the \( \Phi \) function reads
\[ \Phi = \frac{q}{4\pi} \int_0^\infty dk_1 J_0(k_1|x_1|) e^{-k_1^2 z} \]
\[ + \frac{1}{4\pi} q^4 v^2 z \int_0^\infty dk_1 \frac{1}{k_1} J_0(k_1|x_1|) e^{-k_1^2 z}. \] (31)

Now, making use of eqs. (24), (25) and (31), we find that the potential for two opposite charges located at 0 and \( r \) takes the form
\[ V = -\frac{q^2}{4\pi} \frac{1}{r} + \frac{q^2}{4\pi} \frac{\mu v^2}{16} \ln \left( \frac{z + r}{2z} \right), \] (32)
after subtracting divergent terms, and \( r = |r| \). Evidently, by considering the limit \( \mu v \to 0 \), we obtain a Coulombic potential.

The Abelian Higgs model with a Lorentz-breaking term
We now extend what we have done to a Lorentz-violating Higgs model. However, before going to the derivation of the interaction potential, we shall summarize very quickly the principal features of this model. For this purpose, we start from the four-dimensional space-time Lagrangian density [26]:
\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu}^2 + |D_\mu \varphi|^2 - m^2 |\varphi|^2 + \lambda |\varphi|^4 \]
\[ - \frac{\mu}{4} \varepsilon_{\mu \nu \alpha \lambda} v_\alpha A_\lambda, \] (33)
where \( D_\mu \equiv \partial_\mu + i e QA_\mu \). As before, \( v_\mu \) is an arbitrary four-vector which selects a preferred direction in the space-time. Now we recall that when the gauge symmetry is spontaneously broken by means the new vacuum \( <0|\varphi|0> = a \), where \( a = (\frac{m^2}{\Lambda^2})^{1/2} \) and \( m^2 < 0 \), the corresponding effective Lagrangian density reads
\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{\mu}{4} v_\alpha A_\lambda \varepsilon^{\mu \nu \alpha \beta} + \frac{M^2}{2} A_\mu A^\mu - A_0 J^0, \] (34)
where \( M^2 \equiv 2e^2 Q^2 a^2 \) and \( J^0 \) is an external source. To get the last expression we adopted a polar parametrization and used the unitary gauge.

This new effective theory provides us with a suitable starting point to study the interaction energy. Nevertheless, to carry out such study one would need to restore the gauge invariance in eq. (34). It is with this goal that, by making use of standard techniques for constrained systems, we find that eq. (34) reduces to
\[ \mathcal{L} = \frac{1}{4} F_{\mu \nu} \left( 1 + \frac{M^2}{\Delta} \right) F^{\mu \nu} + \frac{\mu}{2} \varepsilon^{\mu \nu \alpha \beta} v_\alpha A_\beta (\partial_\alpha A_\beta) - A_0 J^0, \] (35)
where \( \Delta \equiv \partial_\mu \partial^\mu \). Notice that, for notational convenience, we have maintained \( \Delta \) in eq. (35), but it should be borne in mind that we are considering the static case.

With the foregoing information, we proceed to obtain the Hamiltonian. The canonical momenta are \( \Pi^\mu = - (1 + \frac{M^2}{\Delta}) F^{\mu \nu} + \frac{\mu}{2} \varepsilon^{\mu \nu \alpha \beta} v_\alpha A_\beta + \partial_\mu A_0 \), and one immediately identifies the primary constraint \( \Pi^0 = 0 \), while the remaining nonzero momenta are \( \Pi^I = - (1 + \frac{M^2}{\Delta}) F^{0 I} + \frac{\mu}{2} \varepsilon^{0 I K} v_K A_K \).
The canonical Hamiltonian is now obtained in the usual way and is given by
\[ H_C = \int d^3 x \left\{ - A_0 \left( \partial_0 \Pi^I - \frac{\mu}{2} \varepsilon^{0 I K} v_K \partial_0 A_K + J^0 \right) \right\} \]
\[ + \int d^3 x \left\{ - \frac{1}{2} F_{0I} \left( 1 + \frac{M^2}{\Delta} \right) F^{0I} \right\} \]
\[ + \int d^3 x \left\{ \frac{1}{4} F_{0I} \left( 1 + \frac{M^2}{\Delta} \right) F^{0I} \right\}. \] (36)
As before, requiring the primary constraint \( \Pi^0 \) to be preserved in time yields the secondary constraint (Gauss's law) \( \Gamma_1 \equiv \partial_\mu \Pi^\mu - \frac{\mu}{2} \varepsilon^{0 I K} v_K \partial_0 A_K + J^0 = 0 \). Thus, the Hamiltonian is now given as
\[ H = \int d^3 x \left\{ \omega(x) \left( \partial_0 \Pi^I - \frac{\mu}{2} \varepsilon^{0 I K} v_K \partial_0 A_K + J^0 \right) \right\} \]
\[ + \int d^3 x \left\{ - \frac{1}{2} F_{0I} \left( 1 + \frac{M^2}{\Delta} \right) F^{0I} \right\} \]
\[ + \int d^3 x \left\{ \frac{1}{4} F_{0I} \left( 1 + \frac{M^2}{\Delta} \right) F^{0I} \right\}, \] (37)
where, as before, \( \omega(x) = u_I(x) - A_0(x) \).

Since our goal is to compute the static potential for the theory under consideration, we shall use the same gauge-fixing condition that was used in our preceding calculation. In view of this situation, we now write the Dirac brackets in terms of the magnetic and electric fields as
\[ \{ E_i(x), B_j(y) \}^* = - \left( 1 + \frac{M^2}{\Delta} \right)^{-1} \varepsilon^{k j} \partial_\delta (3)(x - y), \] (38)
\[ \{ B(x), B(y) \}^* = 0, \] (39)
\[ \{ E_i(x), E_j(y) \}^* = - \left( 1 + \frac{M^2}{\Delta} \right)^{-2} \mu \varepsilon_{ij k} \delta^{(3)}(x - y). \] (40)
It gives rise to the following equations of motion for the magnetic and electric fields:

\[ \dot{E}_i(x) = -\mu \left(1 + \frac{M^2}{\Delta}\right)^{-1} \varepsilon_{ijk} v_k E_j(x) + \varepsilon_{ijk} \partial_k B_j(x), \] (41)

and

\[ \dot{B}_i(x) = +\varepsilon_{ijk} \partial_k E_j(x). \] (42)

It follows from the above discussion that Gauss’s law for the present theory reads

\[ \left(1 + \frac{M^2}{\Delta}\right) \partial_i \Pi^i - \mu v_i B^i + J^0 = 0. \] (43)

Again, as in the previous subsection, we shall consider static fields. Thus, we obtain

\[ E_i = \partial_i \Phi, \] (44)

where

\[ \Phi = \frac{(\nabla^2 - M^2)}{(\nabla^2 - M^2)^2 + \mu^2 \omega^2 \nabla^2 - \mu^2 (\nabla \cdot \nabla)^2} (-J^0). \] (45)

For \( J^0(x) = \Phi^{(3)}(x) \), the foregoing expression becomes

\[ \Phi = \frac{q}{8\pi\mu v} \Phi^{(1)} + \frac{q}{16\pi\mu v} \Phi^{(2)} + \frac{qM^2}{8\pi\mu v} \Phi^{(3)}, \] (46)

where

\[ \Phi^{(1)} = \int_0^\infty dk_1 k_1^2 J_0(k_1|x_1|) e^{-\sqrt{k_1^2 + M^2 - \mu v k_1}} \] (47)

\[ \Phi^{(2)} = \int_0^\infty dk_1 k_1^2 J_0(k_1|x_1|) e^{-\sqrt{k_1^2 + M^2 + \mu v k_1}} \] (48)

and

\[ \Phi^{(3)} = \int_0^\infty dk_1 k_1^2 J_0(k_1|x_1|) e^{-\sqrt{k_1^2 + M^2 + \mu v k_1}} \] (49)

We now have all the information required to compute the potential energy for static charges in this theory. Thus, by employing eq. (25), the gauge-invariant scalar potential may be rewritten as

\[ A_0(x) = \frac{q}{8\pi\mu v} \Phi^{(1)} + \frac{q}{16\pi\mu v} \Phi^{(2)} + \frac{qM^2}{8\pi\mu v} \Phi^{(3)}, \] (50)

after subtracting the self-energy terms.

As was explained before, from eq. (24), the corresponding static potential for two opposite charges located at \( \mathbf{0} \) and \( \mathbf{r} \) it should be calculated.

However, following our earlier line of argument, we shall now consider the background which is small compared with the mass term \((\mu^2 v^2 \ll M^2)\). Accordingly, expression (46) can be simplified,

\[ \Phi = \frac{q}{8\pi\mu v} \Phi^{(1)} + \frac{q}{16\pi\mu v} \Phi^{(2)} + \frac{qM^2}{8\pi\mu v} \Phi^{(3)}, \] (51)

where

\[ \Phi^{(1)} = \mu v \int_0^\infty dk_1 k_1^3 J_0(k_1|x_1|) e^{-\sqrt{k_1^2 + M^2}} \] (52)

\[ \Phi^{(2)} = \mu v \int_0^\infty dk_1 k_1^3 J_0(k_1|x_1|) e^{-\sqrt{k_1^2 + M^2}} \] (53)

and

\[ \Phi^{(3)} = \mu v \int_0^\infty dk_1 k_1^3 J_0(k_1|x_1|) e^{-\sqrt{k_1^2 + M^2}} \] (54)

After some further manipulations, eq. (51) reduces to

\[ \Phi = \frac{3q}{16\pi} \frac{e^{-Mr}}{r} - \frac{q}{16\pi} \frac{11}{16} \frac{M^2}{v^2} \left(1 + \frac{M^2}{2}\right) e^{-Mr} \] (55)

Here \( K_0 \) and \( K_1 \) are modified Bessel functions.

Once again, following our earlier procedure, we get the interaction energy as

\[ V = -\frac{3q^2}{16\pi} \frac{e^{-Mr}}{r} + \frac{q^2}{16\pi} \frac{11}{16} \frac{M^2}{v^2} \left(1 + \frac{M^2}{2}\right) e^{-Mr} \] (56)

\[ + \frac{q^2}{8\pi} \frac{11}{16} \frac{M^2}{v^2} \left(1 + \frac{M^2}{2}\right) K_0(M|x_1|) \]
\[
- \frac{q^2}{8\pi}\frac{1}{16}\mu^2 v^2 \left\{ \frac{M^2z^2}{6} \right\} \int_{1}^{\infty} du \frac{e^{-M|x|u}}{\sqrt{u^2-1}} \\
- \frac{q^2}{8\pi}\frac{1}{16}\mu^2 v^2 \left\{ \frac{Mz|x|}{2} K_1(M|x|) \right\} \\
+ \frac{q^2}{8\pi}\frac{1}{16}\mu^2 v^2 M^2z|x|^2 \int_{1}^{\infty} du e^{-M|x|u} \sqrt{u^2-1}.
\]

(56)

In this way our calculation shows new corrections to the Yukawa potential. Evidently, by considering the limit \(\mu v \to 0\), we obtain a Proca-like theory. Finally, it is worth mentioning that the previous calculation generalizes the results presented in ref. [32], where electric and magnetic fields were calculated for the special case where the background \(v\) and \(x\) are parallel.

**Final remarks.** – Finally, by exploiting the gauge-invariant but path-dependent variables formalism, we have addressed the confinement versus screening issue for both electrodynamics of chiral matter and an Abelian Higgs-like model in 3 + 1 dimensions. An important feature of this framework is a correct identification of physical degrees of freedom for understanding the physics hidden in gauge theories. As a consequence, in the case of electrodynamics of chiral matter and a purely space-like vector, \(v^\mu\), we have obtained a logarithmic correction to the usual Coulomb potential. On the other hand, in the case of the Abelian Higgs model with a Lorentz-breaking term and a purely space-like vector, \(v^\mu\), the static potential displays new corrections to the Yukawa potential.

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