Analysis of an unsteady quasi-capillary channel flow with time-resolved PIV and RBF-based super-resolution

Manuel Ratz, Domenico Fiorini, Alessia Simonini, Christian Cierpka, Miguel A. Mendez

Abstract

We investigate the interface dynamics in an unsteady quasi-capillary channel flow. The configuration consists of a liquid column that moves along a vertical 2D channel, open to the atmosphere and driven by a controlled pressure head. Both advancing and receding contact lines were analyzed to test the validity of classic models for dynamic wetting and to study the flow field near the interface. The operating conditions are characterized by a large acceleration, thus dominated by inertia. The shape of the moving meniscus was retrieved using Laser-Induced Fluorescence-based image processing, while the flow field near was analyzed via Time-Resolved Particle Image Velocimetry (TR-PIV). The TR-PIV measurements were enhanced in the post-processing, using a combination of Proper Orthogonal Decomposition and Radial Basis Functions to achieve super-resolution of the velocity field. Large counter-rotating vortices were observed, and their evolution was monitored in terms of the maximum intensity of the $Q$-field. The results show that classic contact angle models based on interface velocity cannot describe the evolution of the contact angle at a macroscopic scale. Moreover, the impact of the interface dynamics on the flow field is considerable and extends to several capillary lengths below the interface.

Keywords

Unsteady quasi-capillary channel, Time-resolved PIV, RBF super resolution, Dynamic contact angles, Dynamic wetting

Introduction

The dynamics of a gas–liquid interface moving along a wall play an essential role in many industrial applications such as wetting and dewetting processes (e.g., inkjet printing, see Eral et al.9), coating applications by Kistler and Schweizer (e.g., slot die coating, see reference (18)) or capillary driven flows (e.g., liquid absorption in porous media, see Ma et al.21).

Because the contact angle cannot be predicted in the framework of continuous mechanics, one must resort to nonequilibrium statistical mechanics, ad hoc assumptions in the continuous framework10,16,39 or empirical laws that link the contact angle to macroscopic quantities such as the contact line velocity (e.g., Hoffman13,17) or acceleration (e.g., Bian et al.2,37). While these approaches have been successful in ‘quasi-steady’ conditions and very viscous fluids,31,43 their generalization to accelerating contact lines and low-viscosity fluids remain an open research topic for both capillary or quasi-capillary flows (see Fiorini et al.;10 Quéré;31 Shardt et al.;35 Willmott et al.42) and impacting droplets.1,40
A well-known challenge in the modeling of flows near contact lines is the singularity that arises at the contact line when combining the notion of no-slip, i.e., zero velocity at the wall, with the notion of dynamic contact line. A possible resolution to this paradox is the splitting streamline pattern proposed by Huh and Scriven. This pattern can be modeled as a creeping flow obeying a bi-harmonic equation in the streamfunction and is schematically illustrated in Fig. 1 for advancing and receding contact lines. In an advancing configuration (Fig. 1a), the fluid is pushed from the bulk of the flow toward the wall, while in a receding configuration (Fig. 1b), the opposite pattern is expected. In both cases, the splitting streamlines pattern produces a rolling motion near the contact line and has been observed in various experimental campaigns by Chen et al.; Fuentes and Cerro; Nasarek et al.; Zimmerman et al. In an open channel flow, such as those encountered in capillary-rise problems by Washburn, Levine et al., the presence of the rolling motion collides with the assumption of fully developed flows. It is thus relevant to analyze how far from the interface one might expect to see the impact of capillary forces near the contact line.

The scope of this work is twofold. On the one hand, we aim at analyzing the well-posedness of a correlation linking the macroscopic contact angle to the kinematics of the contact line in inertia dominated conditions, i.e., in the presence of large accelerations and large velocities. On the other hand, we aim to analyze the flow field in the proximity of the contact line and measure up to which distance from the interface the rolling motion is present. Both aspects are analyzed considering a 2D, quasi-capillary channel (i.e., with width δ ≈ 2lc, with lc = \sqrt{\sigma / \rho g} the capillary length, \sigma the surface tension, \rho the liquid density and g the gravitational acceleration). The experimental setup and measurement conditions are recalled in section “Experimental set-up and test cases”. The configuration of interest is a narrow rectangular channel in which water moves along a solid wall forming two dynamic contact angles. The flow is sustained by a controlled pressure head that reproduces both advancing and receding contact lines.

The shape of the interface was tracked using Laser-Induced Fluorescence (LIF)-based image visualization and edge detection, while the contact line was characterized via an inverse method by fitting a simple model for the interface shape. The setup for the LIF-based flow visualization and the related image processing is described in section “LIF visualization: set-up and processing”. The velocity field was characterized via Time-Resolved Particle Image Velocimetry (TR-PIV). Vortices near the contact line were visualized and their strength quantified in terms of Q-fields.

To accurately compute the spatial derivatives required in the Q-field definition, we propose a super-resolution method that combines Proper Orthogonal Decomposition (POD) and Radial Basis Function (RBF) regression. The setup for the TR-PIV measurements and the post-processing of the velocity fields is described in section “TR-PIV: set-up”. The results of both experimental campaigns are presented in section “Results”, while conclusions and perspectives are collected in section “Conclusions and perspectives”.

**Experimental setup and test cases**

A sketch of the experimental setup for the measurements is shown in Fig. 2a. This consists of a rectangular channel with width δ = 5 mm, in the figure depth L = 250 mm and height H = 150 mm.

This channel is open to atmosphere at the top and connected to a pressurized chamber on the bottom. The chamber sustains the flow via a (time varying) gauge pressure Δp(t). To reproduce an advancing
contact line (i.e., interface moving upwards, along the $y$-axis), a pressure step of amplitude $D_P$ is introduced by the opening of a fast electronic valve connected to a pressure line. To reproduce a receding contact line (i.e., interface moving downward), the same procedure is followed in reverse: the chamber is initially pressurized, i.e., $D_P(0) = 0$ and the release valve is opened to the atmosphere.

In the current setup, the opening of the valve is carried out manually. Therefore, the experimental conditions for the receding contact line can slightly differ between experiments. Nevertheless, the time evolution of the pressure inside the tank is monitored with a pressure gauge and can thus be linked to the dynamics of the interface motion as described in section “Modeling and parameter definition”. For the advancing contact angle problem, the fast opening of the valve (which occurs linearly in about 0.2 s) makes the evolution of the pressure in the channel more repeatable, as shown in Fig. 3.

A picture of the experimental setup is shown in Fig. 2b. The high-speed camera is placed perpendicularly to the channel’s cross section, which is illuminated with a laser sheet from the right side. The camera and the laser sources changed between the LIF-visualization and the PIV experiments, as described in sections “LIF visualization: set-up and processing” and “TR-PIV: set-up”, respectively.

The injection/removal of air during the pressurization/depressurization used in the advancing/receding experiments was performed on two sides of the chamber to ensure the symmetry of the flow and minimize the entry effects. Together with the camera (1), the picture shows the channel (2) and the pressurized chamber (3), the fast opening valve (4), the two pressure ports (5 and 6) and the pressure gauge (7).

The walls of the channel were made in acrylic glass and the experiments were carried out using demineralized water (density $\rho = 997$ kg/m$^3$, dynamic viscosity $\mu = 0.001$ Pa s, and surface tension $\sigma = 0.072$ mN/m). The liquid was introduced into the facility via a lateral port, and all experiments were carried out on an initially dry surface. The influence of the surface preconditioning was tested by repeating the advancing experiments after intervals of approximately 15 min. Since no noticeable difference was observed in the interface dynamics (nor in the contact evolution) between the first and the following experiments, we conclude that this time is long enough to have dry surfaces.

The static contact angle was measured using the same LIF-based image processing approach described in section “LIF visualization: set-up and processing” on a quiescent meniscus. This approach is analogous to the Meniscus Profile Method (MPM) as proposed by Petrov and Sedev. The interface was positioned in the camera’s field of view by gently rising the chamber pressure, and the measurement was taken after waiting.
long enough to have a static interface. The static contact angle was found to be $\Theta_0 = 33^\circ \pm 2^\circ$.

Finally, we emphasize that the need for splitting the experimental campaigns into a high-speed LIF and a TR-PIV stems from the different objectives and the different technical constraints for the two measurements. The LIF-based visualization aims to analyze the motion of the gas–liquid interface during its rise and oscillation, while the TR-PIV seeks to explore the flow field during the passage of the interface. Consequently, the LIF-based visualization required videos of longer duration, lower frame rate and larger field of view than the TR-PIV campaign. Before proceeding with the description of the measurement setup, it is worth elaborating on a simple attempt to model the response of the liquid column in the next section; this also allows to better understand the test conditions investigated.

**Modeling and parameter definition**

Models of the capillary rise have been proposed by Levine et al.\(^{19}\) and Washburn\(^{41}\) and were used for the current campaign to have a preliminary evaluation of the interface displacement as a function of the imposed pressure head.

Denoting as $m$ the mass of the water column in motion and as $h(t)$ the liquid height as a function of time, the force balance acting at the inlet of the channel sets

$$\frac{d}{dt}(mh) = F_g + F_{\Delta p} + F_s + F_v \quad (1)$$

where $\dot{h}$ is the time derivative of the column height (thus the mean velocity of the flow), $F_g$ is the force due to gravity, $F_{\Delta p}$ is the force induced by the gauge pressure in the chamber, $F_s$ is the force due to capillary forces at the interface, and $F_v$ is the viscous force exerted at the channel walls.

Assuming that the contact line moves at the same velocity as the average liquid height in the channel, the inertial term on the left-hand side of (1) can be further expanded as:

$$\frac{d}{dt}(mh) = \frac{d}{dt}(\rho L \dot{h} \dot{h}) = \rho L \dot{h} (\dot{h}^2 + \ddot{h} \dot{h}), \quad (2)$$

where the dots denote differentiation in time.

The first two terms on the right-hand side do not require particular assumption except for the uniformity of the pressure head and the liquid height. These are:

$$F_g = -mg = -\rho \dot{h} \rho L g \quad (3)$$

$$F_{\Delta p} = \Delta p(t) A_{\text{proj}} = \Delta p(t) L \delta, \quad (4)$$

where $A_{\text{proj}}$ is the channel cross section and the liquid mass is $m = \rho \delta L h$.

The surface tension contribution $F_s$ in equation (1) depends on the shape of the meniscus, which in turn depends on the (unknown) contact angle. Denoting as $f(x)$ the interface shape with respect to the interface location at the center of the channel, any curvature of the interface in the span-wise direction is neglected and the Young–Laplace equation gives the capillary induced pressure difference at the interface:

$$\Delta p_s = \frac{\sigma}{\delta} \int_{-\delta/2}^{\delta/2} \kappa(x) \, dx,$$

$$\kappa(x) = \frac{f''}{(1 + f'^2)^{3/2}} \quad (5)$$

where $\kappa$ is the interface curvature, averaged across the channel width, $f$ denotes the interface shape and the prime denotes spatial derivatives (see also section “LIF visualization: set-up and processing”). The computation of this term can be done once a model of the interface shape is considered. The model used in this work is described in detail in section “LIF visualization: set-up and processing”, and the capillary contribution to equation (1) is $F_s = p_s A_{\text{proj}}$.

Finally, the contribution of the viscous forces $F_v$ in equation (1) depends on the shape of the stream-wise velocity profile near the wall. Assuming that this profile remains parabolic at all times, a shear stress can be estimated:

$$\tau = -\mu \dot{h} \dot{u} = -\frac{4 \mu \mu_{\text{max}}}{\delta} = -\frac{12 \mu}{\delta} \dot{h} \dot{h} \quad (6)$$

where the maximum velocity is linked to the mean velocity ($\dot{h}$) using the assumption of parabolic velocity profile. The contribution of the wall shear stress in equation (1) is thus $F_{v\text{isc}} = \tau A_{\text{wet}} = -\dot{h} \dot{h} \mu_1 L \mu / \delta$, with $A_{\text{wet}}$ being the wetted surface of the channel walls.

Given a model for the interface shape $f(x)$ (which depends on the evolution of the dynamic contact angle $\Theta(t)$), and given the time-varying pressure in the chamber $\Delta p(t)$, it is possible to integrate equation (1) and compute the evolution of the liquid height under the aforementioned assumption. While the presented model is oversimplified (it does not account, among other things, for the pressure losses at the channel entrance), it is possible to analyze the relative impact of the various terms to the interface dynamics. Moreover, this model was used to link, at least approximately, the pressure in the chamber to the velocity and acceleration of the contact line during the various testing conditions.
Experimental conditions and test cases

In the LIF-based flow visualization, the experiments were conducted in four conditions, viz. three advancing cases and one descending case. The advancing test cases were carried out setting $\Delta p(0) = 0$ and $\Delta P = 1000$, 1200 and 1500 Pa. These led to $\Delta p(t \to \infty) = 660$, 825 and 985 Pa, respectively. The descending case was carried out with $\Delta p(0) = 1200$ Pa and $\Delta p(t \to \infty) = 0$. The relevant experimental parameters are listed in Table 1. In these experiments, the camera position was not varied: this was placed at the top of the channel, with the upper edge of the image at approximately 15 mm from the upper edge.

All experiments were repeated three times. Since no appreciable differences were observed in the interface dynamics, we report in this article only the result of a single run per test case. Concerning the descending test case, the choice of presenting only one case is due to the limited impact observed by the initial step $\Delta p(0)$ on the evolution of the interface shape: during the descent, the interface tends to reach a constant velocity after a certain distance and this velocity is fairly independent of the initial pressure step. The chosen value is the one that allows for observing the interface over a large field of view, providing enough space for the video acquisition at multiple heights.

For the PIV measurements, two rising cases and three descending cases were considered. The experimental parameters are listed in Table 2. The ‘Test Case 1’ in the PIV campaign corresponds to the conditions of ‘Test Case 3’ in the LIF-visualization campaign. In the Test Case 2, the acceleration was reduced by reducing the pressure step setting $\Delta p(0) = 200$ Pa. The other test cases are in the same conditions as ‘Test Case 4’ in the LIF-visualization campaign, and the only difference is in the position of the camera which is moved downward by 40 mm and 120 mm with respect to the reference value. As for the LIF visualization experiments, the TR-PIV experiments were repeated three times and no significant variability was observed.

Measurement techniques

LIF visualization: setup and processing

For the LIF measurements, rhodamine B from Sigma-Aldrich is dissolved in the water. Images were captured with a Phantom v2012 high-speed camera which has a resolution of $1200 \times 800$ px. The particles were illuminated with a continuous Stabilite 2017 ion laser system with a wavelength of 515 nm. The power of the laser was set to 1.9 W. An objective lens with a focal length of 105 mm was used. The camera was placed at a larger distance from the channel compared to the PIV experiments, to allow the tracking of the interface over a large vertical distance. The achieved optical magnification was 30 px/mm. Images were acquired in single frame mode, with a frame rate of 500 Hz and an exposure time of 20 $\mu$s. In total, 1500 images were acquired, resulting in a measurement duration of 3 s.

A zoom view of the LIF-based visualization of the dynamic meniscus is shown in Fig. 4. The images were processed using the nonlocal means denoising by Buades et al.3 and then high-pass filtered to highlight the gas–liquid interface. The result of this step, for the image in Fig. 4a, is shown in Fig. 4b. Only the right side of the symmetric meniscus is shown. The interface is detected by calculating the vertical gradient of the image intensity, similarly to Mendez et al.,22, 24 by searching for the peak in the intensity gradient in each

| Table 1: Experiments in the LIF-based visualization |
|-----------------------------------------------|
| LIF  | $\Delta p(0)$ | $\Delta P$ | $\Delta p(t \to \infty)$ | Camera pos. |
|------|---------------|------------|--------------------------|-------------|
| Test case 1 | 0             | 1000       | 660                      | 0           |
| Test case 2 | 0             | 1200       | 825                      | 0           |
| Test case 3 | 0             | 1500       | 985                      | 0           |
| Test case 4 | 1200         | –          | 0                        | 0           |

| Table 2: Experiments in the TR-PIV campaign |
|-----------------------------------------------|
| TR-PIV | $\Delta p(0)$ | $\Delta P$ | $\Delta p(t \to \infty)$ | Camera pos. |
|--------|---------------|------------|--------------------------|-------------|
| Test case 1 | 0             | 1500       | 985                      | 0           |
| Test case 2 | 200          | 1200       | 985                      | 0           |
| Test case 3 | 1200         | –          | 0                        | 0           |
| Test case 4 | 1200         | –          | 0                        | 40          |
| Test case 5 | 1200         | –          | 0                        | 120         |

Fig. 4: Figure (a), Raw image of the meniscus obtained from the experiments. Figure (b), Meniscus after nonlocal means denoising and high-pass filtering
image column. At the end of this process, outliers were removed using a local median filter.

A simple model for the interface shape is then fitted to the detected points. This model was heuristically adapted from the analytical solution of a meniscus in stationary conditions and reads:

$$f(x) = \cosh \left( \frac{x^2}{b} \right) - 1,$$

where $y = f(x)$ is the vertical position of the interface with respect to the interface at the center of the image (see Fig. 5) and $a, b \in \mathbb{R}$ are constants to be fit at each time step.

The fitting is performed using the `minimize` function from Python’s Scipy library to minimize the $l_2$ norm of the prediction error and the time evolution of each parameter is further smoothed using a low-pass filter to eliminate outliers.

Equation (7) is used to (1) compute the contact angle by solving $\tan(\Theta) = f'(\delta/2)$, with $f'$ denoting the spatial derivative, (2) to compute the location of the contact line as $h(\delta/2, t) = f(\delta/2) + h(t)$, with $h(t)$ the liquid height at the center of the channel and (3) compute the contact line velocity as $u_c(t) = h'(\delta/2, t)$.

**TR-PIV: setup**

For the PIV measurements, the fluid was seeded with Red Fluorescent Polymer Microspheres from Thermo Fisher Scientific with a diameter of 12 μm. The flow was observed with a TR-PIV system from Dantec Dynamics. The particles were illuminated with the laser (DM40-527-DH Nd:YLF from Photonics Industries) with a wavelength of 527 nm and a maximum pulse energy of 40 mJ at a frequency of 1 kHz. Images were recorded with a SpeedSense Ethernet M310 camera having a resolution of 1280x800 px (at a maximum frame rate of 3260 Hz). The acquisition frequency of both laser and camera, operating in single frame mode, was 1.2 kHz. An objective lens with a focal length of 105 mm was used to get an optical magnification of 60 px/mm.

Figure 6 shows two snapshots from the PIV acquisition before (a,c) and after (b,d) the image prepro-

cessing, which was carried out using the POD-based background removal by Mendez et al. Figure 6a shows a snapshot in which the interface is not within the field of view; in this case the illumination and the seeding were fairly homogeneous. Figure 6c shows a snapshot in which the interface is in the field of view; in this case the interface shape and the light refraction hinder the interface detection and the velocimetry in its proximity.

This is why the PIV interrogation was carried out in a cropped region of the FOV moving with the interface and having its upper boundary (edge 4-3 in Fig. 2a) approximately 1.5 mm below the expected liquid level. This distance was measured in several of the snapshots for which the interface is visible and was kept constant by a tracking algorithm.

In particular, the Region of Interest (ROI) in which the PIV was carried out, is displaced for each image pair $k$, at time $t_k$, using the mean velocity computed along the bottom edge 1–2 (denoted as $y_{1,2}$), i.e.,

$$\tau_k = \int_{0}^{\delta/2} v(x, y_{1,2}, t_k) \, dx.$$  

The vertical displacement of the FOV is thus $\Delta z = \tau \Delta t$, with $\Delta t$ the sampling frequency of the PIV. The velocity of the ROI was found to closely match the mean velocity of the interface $h(t)$.

In the ROI, the PIV interrogation was carried out with the OpenPIV software by using windows of large aspect ratio, justified by the fact that velocity component $v$ (along $y$) is much larger than the velocity component $u$ (along $x$). The rectangular interrogation windows allow for a better sampling of the flow in the direction where the largest gradients are expected, without deteriorating the signal-to-noise ratio in the cross-correlation maps.

**Fig. 5:** Location of the reference frame moving with the meniscus, and with respect to which the model in equation (7) is defined.

**Fig. 6:** PIV snapshots before and after the preprocessing. The interface is not in the field of view in figure, (a) while it is in the field of view in (c). Figures (b) and (d) result from the eigenbackground removal from (a) and (c), respectively.
Adaptive iterative multigrid interrogation\textsuperscript{34} was used to calculate the velocity fields. For the advancing contact line experiments, the initial window size is set to 256×64 px and reduced over two steps to 64×16 px. For the receding contact line experiments, the initial size is 64×64 px, and reduced to 24×24 px in two iterations. Outliers were removed based on the signal-to-noise ratio measured in terms of peak to standard deviation in the cross-correlation map.

Super-resolution of PIV fields

The post-processing of the PIV fields aimed to increase the resolution of the PIV data and enable accurate computation of \( Q \)-fields. The approach consists of using a Radial Basis Function (RBFs, see Fornberg and Flyer\textsuperscript{[11]}) expansion of the velocity field to compute derivatives analytically. A similar approach has been used for robust interpolation\textsuperscript{[8]} to compute derivatives near walls\textsuperscript{[6]} and compute pressure fields from PIV\textsuperscript{[36]}. In this work, we use it to compute the \( Q \)-fields analytically and thus accurately measure the intensity of the vortices near the gas–liquid interface.

To reduce the computational cost of the RBF regression, we combine it with a classic Proper Orthogonal Decomposition (POD). The main idea behind the POD-RBF-super-resolution strategy is to perform RBF regression of the spatial and temporal structures of the decomposition, then use these to reconstruct high-resolution POD modes and finally rebuild high-resolution velocity fields. The procedure is herein briefly summarized. Let

\[
\tilde{u}(x_0, t_0) = \sum_{i=1}^{R} \sigma_i \tilde{u}_i(x_0) \phi_i(t_0),
\]

\( R \ll n_t \) denotes the truncation index, and \( \sigma_i \) is the amplitude of the \( i \)-th POD mode with spatial structure \( \tilde{u}_i(x_0) \) and temporal structure \( \phi_i(t_0) \).

The details of the POD computation can be found elsewhere (e.g., Mendez et al.\textsuperscript{[25,26]}) and are omitted here. We denote as:

\[
\tilde{u}_i(x_0, x_0, t_0) = \sum_{j=1}^{n_w} w_{ij}^u \gamma_i^u(x_0|t_j, \Sigma_{\phi_u}),
\]

\( n_w \) is the number of contact line experiments, the initial spatial grid \( x_0 \) containing \( n_t \) points. Here \( R \ll n_t \) denotes the truncation index, and \( \sigma_i \) is the amplitude of the \( i \)-th POD mode with spatial structure \( \tilde{u}_i(x_0) \) and temporal structure \( \phi_i(t_0) \).

The RBF expansion of the spatial and temporal structures, respectively. Therefore, \( w_{ij}^u \) and \( w_{ij}^v \) are the set of weights defining the regression functions, \( \gamma_i^u(x_0|t_j, \Sigma_{\phi_u}) \) are the \( n_u \) radial basis functions in space and \( \gamma_i^v(t_j|t, \Sigma_{\phi_v}) \) are the \( n_v \) radial basis functions in time. These have collocation points \( x_0 \) and \( t_j \) and shape parameters \( \Sigma_{\phi_u} \) and \( \Sigma_{\phi_v} \). Note that the coefficients \( w_{ij}^u \) must be defined as vectors, i.e., \( w_{ij}^u = (w_{ij}^u, w_{ij}^v) \) since the same radial basis \( \gamma_i^\phi \) is used for both components \( \phi^u \) and \( \phi^v \). However, because every snapshot is reshaped as a column vector, regardless of whether this collects a vector or a scalar field, we keep the same notation as for the interpolation of the temporal structures. We consider Gaussian RBFs both in space and in time. The spatial RBFs \( \gamma_i^\phi(x_0|x, \Sigma_{\phi_i}) \) are defined as:

\[
\gamma_i^\phi(x_0|x, \Sigma_{\phi_i}) = \exp(-\|x-x_0\|^2 / \Sigma_{\phi_i}),
\]

\( \Sigma_{\phi_i} \) is a diagonal matrix with entries \( \sigma_i \).

Fig. 7: LIF results for Test Case 1. Figure (a) corresponds to the interface above the velocity field in Fig. 10d. The three images are marked by the three red squares in Fig. 9e. The titles recall the Ca and G numbers for each snapshot as well as the time \( t \) at which the meniscus came into the FOV of the camera.
\[ \gamma \phi_i = \exp \left( - (x_0 - x_i)^T \Sigma_{\phi_i}^{-1} (x_0 - x_i) \right) \]  
\tag{11}

with \( \Sigma_{\phi_i}^{-1} = \text{diag}(1/2\sigma_x^2, 1/2\sigma_y^2) \), while the temporal RBFs \( \gamma \phi_i(t_0|t_j, \Sigma_{\phi_i}) \) are defined as:

\[ \gamma \phi_i = \exp \left( -(t_0 - t_j)^2/(2\sigma_i^2) \right) \]  
\tag{12}

with \( \Sigma_{\phi_i} = 1/2\sigma_i^2 \).

For both space and time regressions, the collocation point and the shape parameters are defined a priori and the regression is solved once the weights are identified. Reshaping all bases functions as columns of matrices \( \Gamma_{\phi}(x_0) \in \mathbb{R}^{n_x \times n_\phi} \) and \( \Gamma_{\phi}(t_0) \in \mathbb{R}^{n_t \times n_\phi} \), collecting all the unknown weights into column vectors \( w_{\phi} \in \mathbb{R}^{n_\phi} \), \( w_{\phi} \in \mathbb{R}^{n_\phi} \), and collecting all the entries of \( \phi_0(t_0) \) and \( \phi_i(t_0) \) into column vectors \( \phi_0 \in \mathbb{R}^{n_\phi} \) and \( \phi_0 \in \mathbb{R}^{n_\phi} \), the weights are the solution of classic least square problems. Using a Tikhonov regularization to mitigate the risks of over-fitting, the solution is:

\[ \mathbf{Ax} = \mathbf{b} \rightarrow \mathbf{x} = \left( \mathbf{A}^T \mathbf{A} + \varepsilon \mathbf{I} \right)^{-1} \mathbf{A}^T \mathbf{b}, \]  
\tag{13}

where \( \mathbf{A} = \Gamma_{\phi}(x_0) \), \( \mathbf{x} = \mathbf{w}_{\phi} \) and \( \mathbf{b} = \mathbf{w}_{\phi} \) for the regression in space and \( \mathbf{A} = \Gamma_{\phi}(t_0) \), \( \mathbf{x} = \mathbf{w}_{\phi} \) and \( \mathbf{b} = \mathbf{w}_{\phi} \) for the regression in time. The regularization parameter \( \varepsilon \) is taken as \( \varepsilon ||\mathbf{A}||_F \), with \( \varepsilon = 10^{-11} \) and \( ||\cdot||_F \) the Frobenius norm. Note that the inverse in (13) can be precomputed and the system solved for all the modes in one single step.

Given the weights, equations (9) and (10) can be used on any arbitrary mesh grid \( \mathbf{x} \) and any time discretization \( t \). Then, the resulting high resolution mode can be introduced in the expansion (8).

The analytic description of the flow allows for the computation of derivatives by simply replacing the RBF with the required (analytic) derivatives. For example, \( \partial_t u \) can be computed by replacing \( \phi_i(x) \) with \( \partial_t \phi_i(x) \) in (8) and this can be computed by replacing \( \gamma \phi_i(x|x_i, \Sigma_{\phi_i}) \) with \( \partial_t \gamma \phi_i(x|x_i, \Sigma_{\phi_i}) \) in (9). These derivatives are analytically available because \( \partial_t \gamma \phi_i \) is analytically available by differentiating (11).

Finally, using the available derivatives it is possible to compute the strength of vortices in the PIV fields in terms of \( Q \)-field, which for a 2D incompressible flow reads

\[ Q = \frac{1}{2} \left( ||\Omega||_F^2 - ||S||_F^2 \right) \]  
\tag{14}

\[ = -\frac{1}{2} \left( (\partial_x u)^2 + 2\partial_x u \partial_x v + (\partial_y v)^2 \right) \]

where \( S = 1/2(\nabla V + \nabla V^T) \) and \( \Omega = 1/2(\nabla V - \nabla V^T) \) the symmetric and anti-symmetric portions of the velocity gradient tensor.

\section*{Results}

\subsection*{LIF visualization results}

We begin by illustrating the results of the interface detection via high-speed LIF-based visualization. The results of the repeated experiments yielded similar results, thus the results always show the case of the first repetition. Figure 7 shows three snapshots of Test Case 1, each taken at different points of the oscillation. Figure 8 shows three snapshots of Test Case 4. In each figure, the text in the image recalls the dimensionless characteristics conditioning each snapshot, viz. the dimensionless velocity of the contact line in terms of capillary number \( Ca = \mu u_0/c \) and dimensionless acceleration \( G = a/g \) as well as the time passed after the meniscus entered the FOV. The signs of \( Ca \) and \( G \) follow the coordinate system defined in Fig. 2, i.e., a positive sign indicates an upwards motion or an acceleration upwards. The result of the interface fitting, following equation (7) is shown using a dashed line.

Despite the widely different conditions in terms of velocity and acceleration, the shape of the interface is well described by the model in equation (7). The interface remains symmetric with respect to the vertical axis, while the contact angle in dynamic conditions largely varies between the rising and the descending conditions (Fig. 9).

For Test Case 1 to 3, Fig. 9 plots on the left the evolution of the liquid height as a function of time (blue dashed lines) together with the prediction of the simple model in section “Modeling and parameter definition”. These test cases differ in the amplitude of the pressure step introduced, thus on the overall level of acceleration. Despite the simplifications involved, the model correctly predicts the interface’s position and velocity in the first 0.5 s, i.e., at the end of the first cycle. Afterward, as the interface oscillates around its equilibrium conditions, the model prediction loses accuracy.

Among the various terms in equation (1), it was found that the inertial contribution (left-hand side term) was the dominant one, followed by the viscous and the surface tension terms. Given the good performances of the simple model in equation (7) in representing the shape of the interface and thus the accurate prediction of the capillary contribution, it is clear that the main limitation of the model arises from the modeling of the viscous term. Although this was not within the scope of this work, this result should be considered for future works on unsteady capillary or quasi capillary channels (e.g., see references (41), (19)).
Finally, on the right side of Fig. 9, maps for every test case of the evolution of the contact angle \( \Theta \) as a function of the capillary number \( Ca \) are shown. The markers are colored by the level of acceleration \( G \) in each point. The range of contact angles spanned within each experiment is \( \Theta \in [20, 110]^{\circ} \). The largest values are produced during the first rise of the interface when the acceleration is also the largest (\( G = -0.24 \)). The range of capillary number observed during the experiments is \( Ca \in [-1.25, 5.30] \times 10^{-3} \) and is approximately the same in all test cases. Large capillary numbers are not shown in the plots to focus on the regions with small capillary numbers. During the descending phase, the capillary number varies between \( Ca = -1.25 \times 10^{-3} \) to \( Ca \approx 0 \); yet the contact angle remains approximately constant and equal to \( \Theta \approx 22^\circ \). On the contrary, during the final phase of the experiment, when the interface oscillation are vanishing and the contact line is almost at rest, the contact angle exhibits the most prominent variation (in the range \( \Theta \in [20, 80]^{\circ} \)). In this phase, also the acceleration is approximately constant. While these results suggest that the history of the contact line dynamic might play a role in predicting the contact angle, it also brings the question of whether it is possible to use classic contact line models of the form \( \Theta = f(Ca, G) \) for low viscous fluids and in the presence of acceleration.

**TR-PIV experiment results**

The PIV velocity fields were processed retaining a total of \( R = 25 \) POD modes in equation (8). The temporal structures of the modes were approximated with \( n_o = 500 \) RBFs with \( \sigma_x = 0.01 \) while \( n_b = 900 \) RBFs were used for the spatial structures (see equation 11). For the regression in space, the RBFs were collocated over a regular grid of \( n_{Rx} \times n_{Ry} = 60 \times 15 \) and anisotropic kernels with \( \sigma_x = 10 \) and \( \sigma_y = 15 \) were selected. This allows to efficiently account for the high aspect ratio of the flow. No resampling was performed in the time domain since the available time resolution was sufficient for the scope of this work.

Figure 10 shows the results of Test Cases 1 and 2, i.e., the advancing contact line. Figures 10a and 10b show the gravitational acceleration \( G \) and the maximum of the \( Q \)-field representing the vortex strength within the ROI as a function of the capillary number \( Ca \). The labels in the legend refer to the operating conditions described in section “Modeling and parameter definition”. The capillary number in these plots is different from the one used in the LIF campaign, i.e., it is not based on the velocity of the contact line but on the mean velocity computed by zero-padding the velocity field at the walls and taking the mean of the average velocity of the bottom five rows. This operation acts as smoothing and serves for plotting purposes. We distinguish these quantities with an over-bar in the labels of figure axes.

A clear trend is visible: the \( Q \)-field increases with decreasing mean flow velocity.

Three instantaneous velocity fields are shown in Figs. 10c, 10d and 10e, corresponding to the three points marked in Figs. 10a and 10b. For plotting purposes, the velocity fields are slightly high-pass filtered by using a Gaussian filter with \( \sigma_x = \sigma_y = 15 \) and a truncation after 2.5 standard deviations. The quiver plot only shows every third vector along \( x \) and every second one along \( y \) for better visibility. The \( Q \)-field is also shown in each snapshot for quantitative analysis, and the axis aspect ratio is set to one. We recall that the upper horizontal boundary of the ROI is at approximately 1.5 mm from the interface at the center of the channel. In each case, two counter-rotating vortices are visible close to the walls (the one on the right rotates clockwise and the one on the left counterclockwise). The vortices extend considerably along the vertical direction but only over a small distance in the cross-stream direction. Therefore, they...
have no remarkable influence on the velocity field at a distance of 1 mm from the wall.

The initial pressure was high enough to produce a strong acceleration, as shown in Fig. 10a. While the rolling motion of the flow is still present, the large acceleration and high velocities push the stream-line split injection pattern and its vortices toward the wall. It is worth noticing that the velocity fields look similar

Fig. 9: LIF results of the oscillating interface for different test cases. Figure (a), (c) and (e) temporal evolution of the mean interface height inside the channel for Test Case 1, 2 and 3, respectively. The dashed line represents the height taken from the images, the dotted line the predicted height from the model in section “Modeling and parameter definition”. Figure (b), (d) and (f) show the measured contact angle as a function of capillary number $Ca = \frac{\mu u_c}{\sigma}$. The markers are colored according to the acceleration number $G$. The capillary number at $t = 0$ s is $Ca = 5.3 \times 10^{-3}$, but the axis is limited in the range $[-1.25, 1.25] \times 10^{-3}$ to better visualize the final region at $Ca \approx 0$. The red squares in Fig. (f) correspond to images in Fig. 7 (c.f the times label in the captions of that figure)
even though Figs. 10c and 10d have largely different accelerations. Referring to the expected theoretical flow field from Fig. 1a, it appears that the stream-line split injection forms a smaller angle with the walls than what pictured in the schematic of Fig. 1a. As a result, the rolling motion expected by the viscous-capillary balance is confined to a narrow region of the flow.

Figure 11 shows the same results for Test Case 3 to 5. The capillary and acceleration numbers in Fig. 11a and b have a different sign because the interface moves downward. A similar trend as for the rising interface can be observed, with the magnitude of the $Q$-field increasing as the velocity decreases. However, the reader should note that a decreasing velocity, in this case, means that the absolute value of the velocity is increasing. Another observation is the increased strength of the $Q$-field, with the maximum value being almost an order of magnitude above the maximum value for the rising interface.

The vortices in Figs. 11c, 10d and 10e show a different behavior than the ones for the rising interface. Besides the reversed rotation, as expected from the theoretical flow topology in Fig. 1, their centers are much closer to the center of the channel, and the rolling motion extends much further in both the $x$ and $y$ directions. In the case of the receding contact line, the stream-line split ejection (see Fig. 1b) forms a larger angle with the wall than for the advancing contact line in the investigated configuration ($\Theta_0 = 33 \pm 2^\circ$).

The large difference cannot be justified by the different velocity, which in the slowest rising test cases is comparable to the fastest falling test cases, nor by its acceleration, which is also similar in the cases in Fig. 11c and the one in Fig. 10c. Instead, the main difference between the rising and descending test cases is in the shape of the interface (cf. Fig. 8a) for a snapshot during the rise and Fig. 8c for a snapshot during the fall. This impacts the velocity flow field in the entire ROI, thus more than 5 mm below the interface.

![Figure 10: PIV results for the oscillating interface. Figure (a), (b) dimensionless acceleration ($G$) and maximum $Q$-field as a function of the capillary number $Ca$ for two test cases. The blue circles correspond to Test Case 1 and the orange crosses to Test Case 2. Figure (c)–(e) $Q$-field contour and high-pass filtered velocity field. The aspect ratio of the plots is one](image-url)
Conclusions and perspectives

The dynamics of accelerating menisci was investigated using high-speed LIF-based visualization and Time-Resolved PIV. The first was used to track the evolution of the interface shape and the dynamic contact angle, while the second was used to measure the flow field in the fluid adjacent to the interface. Both advancing and receding contact lines were investigated for a wetting configuration (static contact angle $\Theta_0 = 33^\circ \pm 2^\circ$) consisting of water flowing over acrylic glass.

A simple model was used to describe the interface shape over the full range of operating conditions, with capillary numbers in the range $Ca \in [-1.25, 5.30] \times 10^{-3}$ and contact lines acceleration in the range $G \in [-0.24, 0.07]$. This model was also used to measure the dynamic contact angle which reaches values of $\Theta = 110^\circ$ during a rise (at large accelerations) and $\Theta = 20^\circ$ during a descent. Within the entire range of investigated conditions, it appeared impossible to relate the dynamic contact angle to the contact line velocity and/or acceleration: during some phases of the experiment, the contact angle remains constant over a wide range of $Ca$ and $G$, while in others it varies significantly over small intervals of $Ca$ and $G$. The quest for identifying a functional dependency $\theta = f(Ca, G)$ in inertia-dominated conditions might thus not be well posed.

Concerning the velocity field underneath the interface, two counter-rotating vortices were observed below the menisci in both advancing and receding cases. These were detected, and their intensity quantified in terms of $Q$-field, using a super-resolution approach combining RBFs and POD. The topology of these vortices complies with the well-known splitting streamline pattern postulated to solve the apparent incompatibility of contact line motion versus no-slip condition at the wall. The strength of these vortices depends on the mean velocity and the acceleration of the flow, and significantly differs between advancing and receding conditions. In the receding conditions, the splitting streamline forms a much larger angle with the wall and has a much larger impact on the flow field. Significant differences were observed between advanc-
ing/receding wetting configurations even when the modulus of velocity and acceleration of the contact lines were comparable. This highlights the relevance of capillary forces near the interface, since the main difference between the two cases was found in the interface’s shape: This appeared nearly flat in the advancing cases and was characterized by large menisci during the receding test cases.

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