A model for nulling and mode changing in pulsars

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ABSTRACT

We propose that in some pulsars the magnetosphere has different states with different geometries or different distributions of currents; it occasionally switches between them. These states have different spin-down rates and emission beams; in some of the states no radio emission is produced at all. Switching into a different state manifests as a mode change when we see different parts of the emission beam or the beams in different states have significantly different geometries; it manifests as nulling when either we miss the new beam or no radio emission is generated in the new state. We show that modest variations in the beam shape can be accompanied by large variations in the pulsar spin-down rate $W$ – the dependence of $W$ on the opening angle of the emission beam $\alpha$ can be as strong as $W \propto \alpha^4$. We speculate about physical mechanisms which may cause reconfiguration of the magnetosphere.

Key words: stars: magnetic field – stars: neutron – pulsars: general – pulsars: individual: PSR B1931+24 – pulsars: individual: PSR J1832+0029.

1 INTRODUCTION

Rotationally powered pulsars are observed mostly in the radio band because of the very high sensitivity of radio telescopes. Pulsar radio emission shows a very rich set of phenomena. Examples of such phenomena are mode changing and nulling, which are observed in many pulsars. There are hints that mode changing and nulling may be manifestations of the same phenomenon (e.g. Wang, Manchester & Johnston 2007). It is not clear yet whether nulling and/or mode changes are ‘uniform’ phenomena, i.e. whether there is a single class of reasons why pulsars null or change modes. There are nulling pulsars which are quiet for only a few periods and there are pulsars which spend a very long time in each of the states. There are pulsars for which the emission intensity during nulls drops at least by several orders of magnitude or switches off completely so that they cannot be detected even in a deep search, while for some pulsars, nulls seem to be just mode(s) with low emission intensity (e.g. Biggs 1992; Wang et al. 2007).

It was not clear for a long time whether nulling is due to the ‘microphysics’ of the radio emission mechanism or whether changes in the pulsar magnetosphere as a whole play a role. The discovery of two nulling pulsars, PSR B1931+24 and PSR J1832+0029, with different spin-down rates in ON and OFF phases (Kramer et al. 2006; Lyne 2009) provides strong evidence that at least some sub-class of nulling may be related to the global properties of pulsars and not merely a property of radio emission mechanism(s) alone. In this short Letter we will focus on pulsars where global processes involving changes in the whole magnetosphere seem to be at play. We propose a qualitative model which could explain nulling and mode changing as manifestations of the same phenomenon and account for the observed variation of the spin-down rate.

2 THE MODEL

The radio emission of a pulsar is negligible in the overall energy budget, usually constituting a very tiny fraction of the pulsar spin-down rate, less then $10^{-3}$. Most of the energy flux is carried away by the relativistic pulsar wind and does not reveal itself in the emission. Therefore, merely the fact whether the radio emission is produced or for some reason is switched off or strongly suppressed cannot change the pulsar spin-down such that it would be measurable. However, there are at least two pulsars – PSR B1931+24 and PSR J1832+0029 – which exhibit nulling and at the same time their spin-down rates are substantially less when they are non-visible (Kramer et al. 2006; Lyne 2009). In the case of PSR B1931+24 the difference in the spin-down rate between the ON and OFF modes is about 50 per cent. Nulling in these pulsars takes an extreme form; they spend a very long time in both states (days), whereas many other nulling pulsars switch off for much shorter times – hours or less (e.g. Biggs 1992; Wang et al. 2007). Taking into account the smallness of the pulsar period derivatives, $P \sim 10^{-15}$, it is very hard to detect variations of spin-down rate between modes lasting only hundreds or thousands of pulsar periods, even if these variations are large. We may speculate that other nulling pulsars, or at least some of them, can have non-negligible variations of the spin-down rate too, but they are difficult to measure and are not detected.

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The behaviour of PSR B1931+24 and PSR J1832+0029 hints that nulling, at least in some pulsars, is a manifestation of some global process which changes the spin-down rate of the pulsar. This global process should involve changes in the structure of the pulsar magnetosphere. The most popular radio pulsar model, introduced by Goldreich & Julian (1969), implies that the pulsar magnetosphere is filled with plasma, and accelerating electric field is present only in geometrically very small regions – in the pulsar polar cap and in regions around and inside the current sheet carrying the bulk of the return current. The rest of the magnetosphere should be force-free. In the force-free magnetosphere the energy losses of the pulsar are determined by the configuration of the magnetosphere. Within this configuration we understand both the geometrical relation between the open and closed magnetic field line zones as well as the distribution of the current density in the open field line zone. In other words, in the force-free model, changes in the pulsar spin-down rate are possible only if the configuration of the magnetosphere changes. Recently the force-free pulsar magnetosphere model has been studied in great detail (e.g. Contopoulos, Kazanas & Fendt 1999; Contopoulos 2005; Gruzinov 2005; Spitkovsky 2006; Timokhin 2006, 2007a). For an aligned rotator it was shown that there exist (i) stationary magnetospheric configurations with different sizes of the closed magnetic field line zone (Timokhin 2006) and (ii) configurations with different current density distributions in the open field line zone (Contopoulos 2005; Timokhin 2007a,b). These configurations have different spin-down rates.

Having the above-described facts, hints and speculations in mind, we make the following conjecture. Magnetospheres of some pulsars at some stages of their evolution can have sets of quasi-stable states with different sizes of the closed field line zone or different current density distributions in the open field line zone, or both. Occasionally a magnetosphere switches between these states.

Currently there is no reliable theory which can explain pulsar radio emission. The radio emission is coherent and it seems that conditions for generation of coherent emission in pulsar magnetospheres may be not always fulfilled - at least there are pulsars with periods and magnetic fields typical for radio-emitting pulsars which are visible in gamma-rays but show no detectable radio emission (Abdo et al. 2010). If, at least for some range of pulsar parameters, the conditions for generation of coherent emission are very sensitive to the properties of the magnetospheric plasma - like current and plasma densities or/and their gradients - then it can be that in some of the magnetospheric states the coherent emission is not generated at all or is generated at different places. Each state then has unique radio emission properties and switching between different magnetospheric states will manifest as mode changing or nulls.

Another reason for changes of the pulsar mean profile can be due to self-similar changes of the emission beam when the radio emission zone shrinks or expands, i.e. radio emission is generated in the same way in all states but the beam scales self-similarly with the size of the emission zone. Then the emission beam in some states is wider than in another. Depending on the orientation of our line of sight relative to the emission beam we can see either mode change or null depending on whether our line of sight still crosses the narrower emission beam or not.

We note that in the proposed model the magnetosphere is always filled with plasma and qualitatively works in the same way, i.e. particles are accelerated, gamma-rays are emitted, etc. But changes in the beam pattern due to current redistribution or shrinking of the corotating zone are always accompanied by changes in the spin-down rate. Below we try to quantify the proposed model. We defer speculations about reasons causing such behaviour of the magnetosphere to the discussion.

We do not know what the conditions for coherent emission of radio waves in the pulsar magnetosphere are. Hence, we cannot make any quantitative statements about what changes in the magnetospheric configuration can cause failure of the emission mechanism or what is the necessary condition for large changes in the beam pattern when the magnetospheric configuration changes. However, we can estimate how the emission beam changes when the emission patterns in all configurations are similar – when nulling/mode changes are due to pure geometrical effects. Even if contraction/expansion of the emission beam takes place only in some nulling/mode changing pulsars, detection of a correlation between the width of the mean profile and the spin-down rate in a mode changing pulsar could be a strong argument in favour of the proposed model. On the other hand, for pulsars where nulling/mode changes cannot be (fully) explained by contraction/expansion of the emission beam, such estimates could provide useful limits on variation of the spin-down rate.

For simplicity we consider only the case of an aligned rotator with a dipole magnetic field. The main point we wish to demonstrate with these crude estimations is that large variations of the spin-down rate can be accompanied by modest changes in the geometry of the radio beam, or, reversing the statement, if changes of the beamwidth due to reconfiguration of the force-free magnetosphere are observed, then changes in the spin-down rate would be large and could be detected.

### 2.1 Shrinking–expanding corotating zone

Let us assume that a pulsar magnetosphere has different states with different sizes of the corotating zone, the zone with closed magnetic field lines. Let us consider two states: state A with the corotating zone smaller than that in state B (see Fig. 1). The size of the polar cap, limited by the last closed field line, is smaller in state B. The current density distribution in the polar cap of the pulsar for such

![Figure 1. Schematic view of the pulsar magnetosphere when the closed field line zone changes its size. Radio emission beams are shown by the blue lines; the last closed field lines are shown by the black lines. The lines of sight for which the pulsar will demonstrate mode change and null are shown by magenta and red arrows, respectively.](https://example.com/pulsar_magnetosphere.png)
states does not differ dramatically (see fig. 5 in Timokhin 2006). It is natural to assume that all processes in regions not far from the neutron star (NS) for small changes in the size of the polar cap behave similarly, i.e. the beaming pattern in different configurations can be approximately described by a shrinking/contraction proportional to the changes in the size of the polar cap.

If radio emission is generated close to the NS, then the emission beam in state A has a larger opening angle. When the pulsar switches from state A to state B, depending on our line of sight, we either see different parts of the emission zone, which might result in a mode change, or miss the emission cone entirely and conclude that the pulsar is in the ‘null’ state. Configurations with a smaller corotating zone have more open magnetic field lines and, hence, have higher energy flux to infinity. Therefore, during the ON state the spin-down rate of the pulsar is higher.

If emission is directed along magnetic field lines, then the opening angle of the emission cone \( \alpha \) is

\[
\alpha = \theta + \arctan\left(\frac{B_0}{B_r}\right) \simeq 3 \theta/2 \propto \theta_{pc},
\]

where \( \theta \) is the colatitude of the emission zone, \( B_0, B_r \) are components of the magnetic field in spherical coordinates and \( \theta_{pc} \) is the colatitude of the polar cap boundary. The last step in equation (1) comes from our assumption about similarities of emission zones in different configurations. Energy losses of the aligned rotator depend on the size of the corotating zone as (equation 62 in Timokhin 2006)

\[
W \simeq W_{md}\left(\frac{R_F}{R_{LC}}\right)^{-2},
\]

where \( R_F \) is the actual size of the corotating zone, \( R_{LC} \) is the size of the light cylinder. \( W_{md} \) are the magnetodipolar energy losses \( W_{md} = B_0^2 R_F^4 \Omega_{NS}^2/\pi c^3 \), where \( B_0 \) is the magnetic field in the polar cap, \( R_{NS} \) is the NS radius, \( \Omega_{NS} \) is the pulsar angular velocity and \( c \) is the speed of light. For dipolar magnetic field \( \theta_{pc} \simeq \sqrt{R_{NS}/R_F} \). Expressing \( R_F \) through \( \theta_{pc} \) we get

\[
W \simeq W_{md}\left(\frac{R_{LC}}{R_{PC}}\right)^2 \theta_{pc}^4.
\]

Taking into account relation (1) we finally get

\[
W \propto \alpha^4.
\]

We see that reconfiguration of the magnetosphere can indeed cause much smaller changes in the beam opening angle than in the spin-down rate, and so small changes in the emission geometry can be accompanied by measurable changes of the spin-down rate.

### 2.2 Changing current density distribution

Let us now consider the case when the polar cap size is nearly the same (\( R_F \) does not change), but instead there are magnetospheric states with different current density distributions in the open field line zone. A model of such type was qualitatively discussed by Arons (1983) in relation to mode changing in PSR B0809+74.

The force-free magnetosphere starts at the top of the cascade zone in the polar cap, above the pair-formation front, where there is enough plasma to short out the accelerating electric field. The angular velocity in the magnetosphere is constant along magnetic field lines. Each open magnetic field line rotates with the angular velocity \( \Omega_e \) it has at the top of the cascade zone in the polar cap. \( \Omega_e \) depends on the distribution of the potential drop in the cascade zone \( V_{j1} \) – the potential drop between the base of the force-free magnetosphere and the NS surface. In the force-free magnetosphere the current density distribution is set by the requirement that the open magnetic field lines pass smoothly through the light cylinder. For magnetospheric states with different distributions of the accelerating potential \( V_{j1} \) the distributions of \( \Omega_e \) will be also different, and so will be the positions and the shapes of the light cylinder. Hence, such states will have different current density distributions in the open field line zone as well.

Pulsar energy losses are given by (cf. equation 40 in Timokhin 2007a)

\[
W \sim \frac{J_{\Omega_e}}{2\pi c} \Phi_{pc},
\]

where \( J \) is the total current flowing through the polar cap, \( \Phi_{pc} = B_0 r_{pc}/c \) is the total magnetic flux in the open field line zone and \( r_{pc} \) is the polar cap radius. The dependence of \( J \) on \( \Omega_e \) could be approximated by that dependence in the magnetosphere of a rotating split-monopole \( J \propto \Omega_e \) (Blandford & Znajek 1977). And so we have for the energy losses

\[
W \propto \eta^2,
\]

the spin-down rate is proportional to the square of the total electric current flowing through the polar cap, a dependence typical for power dissipated in an electric circuit.

To change the spin-down rate the total current must change; small local changes in the current density which do not change the total current cannot account for changes in \( W \). As a toy model let us consider two states A and B, where the shape of the current distribution remains the same but the current density \( j \) in state B is less than that in state A by some factor \( \eta \) slightly less than 1,

\[
j_B = \eta j_A.
\]

The total current changes then as \( J_B = \eta J_A \), and for changes in the energy losses we have

\[
W \propto \eta^2.
\]

There is no reliable model for pulsar radio emission. We can only speculate what sets the pattern for the emission beam. Let us assume that in state B the emission zone shifts to the field line where the current density is equal to the current density which flowed through the emission zone in state A, i.e.

\[
j_B(\theta_B) = j_A(\theta_A).
\]

This toy model results in changes of the beam geometry similar to that shown in Fig. 1.

The current density in the polar cap of the aligned rotator with negligible differential rotation of the open magnetic field lines is close to the Michel (1973) current density (Timokhin 2006),

\[
\frac{\delta \theta}{\theta} \simeq -1(1 - \theta^2/\theta_{pc}^2)
\]

with \( \delta \) the angle between the field line and the axis of the pulsar. Using relations (7) and (9) with the current density given by equation (10), we get the following expression for variation of the emission zone colatitude in the case of small changes in the current density:

\[
\delta \frac{\theta}{\theta} \simeq -1(1 - \theta^2/\theta_{pc}^2)
\]

From equation (8) we have \( \delta W/W = 2(\eta - 1) \), and small variations of the energy losses are related to variation of the emission zone colatitude as

\[
\delta \frac{W}{W} \simeq 4\zeta \delta \theta.
\]

where \( \zeta = (\theta^2/\theta_{pc}^2)/(1 - \theta^2/\theta_{pc}^2) \). If the emission zone is close to the edge of the polar cap, \( \theta > \theta_{pc}/\sqrt{2} \simeq 0.71 \theta_{pc} \), then

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\[ \zeta > 1. \] Taking into account the relation between the colatitude of the emission zone and the beam opening angle (equation 1) we get the power law
\[ W \propto \alpha^4, \] (13)
which is valid for small variations of \( W \) and \( \alpha \). We see that in this case changes in the spin-down rate could be even larger than in the case of a changing spin-orbit angle discussed before. Although the model considered here is very simple, it demonstrates nevertheless that the dependence of the energy losses on the beam opening angle can be rather strong.

3 DISCUSSION
Let us summarize the reasoning behind the proposed model. It seems that the force-free model agrees well with observations: pulses are narrow, pointing to smallness of the regions with accelerating electric field; as inferred from observations of pulsar wind nebulae, there should be more than enough plasma in the magnetosphere to screen the accelerating electric field. If pulsar magnetosphere is essentially force-free, then changes of the spin-down rate are possible only if the configuration of the magnetosphere changes. So, in the case of PSR B1931+24 and PSR J1832+0029 we must conclude that we have to do with changes in the magnetosphere configuration, or the force-free model fails. Different configurations should have different emission-beam geometries.

We do not suggest that all nulling pulsars switch between states with different emission beams, nor that changes in the emission beam could always be attributed to the pure geometrical effect of shrinking/expansion; the latter can work only for pulsars where the line of sight crosses the outer parts of the emission beam. In the case of PSR B1931+24 the line of sight seems to make a central traverse of the beam (Rankin, private communication), and so the simple geometrical models of Sections 2.1 and 2.2 do not work for it. In the frame of the proposed model, nulls in this pulsar should be then attributed to cessation of the radio emission due to changed parameters of magnetospheric plasma in the OFF state. But if the pulsar magnetosphere can switch between different states, shrinking/expansion of the emission beam can take place in some pulsars – at least in the mode changing pulsar PSR B0943+10 with tangential sightline trajectory the modes have different widths of the mean profile (Rankin & Suleymanova 2006). The point we wish to emphasize here is that reconfiguration of the magnetosphere can result in modest changes in the emission beam and in substantial changes in the spin-down rate at the same time, i.e. the emission beams in different states can look similar while the energy-loss rate changes a lot.

An important consequence of the proposed model is that there must be mode changing pulsars showing substantial variations of spin-down rate, so to say, the mode changing ‘twins’ of PSR B1931+24 and PSR J1832+0029. If the magnetospheres can have different states, switching between the states should manifest as mode changing or nulls. The spin-down rate variation in the above-mentioned nulling pulsars strongly suggests that such states exist. Hence, there also must be pulsars where states with different spin-down rates manifest as different emission modes. The proposed model also implies that nulling and mode changing pulsars should be among the pulsars with high timing noise.

Let us now discuss what could be the underlying physical mechanisms for the proposed model. It is not a question whether for a given pulsar period and magnetic field strength different configurations of the magnetosphere are admitted or not. At least for the aligned rotator it has been shown explicitly that such configurations exist, and we see no reasons why this cannot be the case for the inclined rotator. The real question is why the magnetosphere can have a set of metastable configurations, i.e. configurations where the magnetosphere can stay for times much longer than the pulsar period.

Although the properties of the force-free magnetosphere are quite well known, we cannot say what is the configuration of a pulsar magnetosphere. The reason for this is that the configuration of the magnetosphere depends on the ‘boundary conditions’ – physics of the polar cap cascade zone and physics of magnetic reconnection in the current sheet, especially at the so-called Y-point where the current sheet of the outer magnetosphere merges with that along the boundary between open and closed magnetic field lines. The distribution of the potential drop in the polar cap cascade zone sets the angular velocity of the open magnetic field lines \( \Omega_F \) (e.g. Contopoulos 2005; Timokhin 2007a,b); the reconnection rate at the ‘Y’-point might influence the size of the corotating zone (e.g. Bucciantini et al. 2006; Contopoulos & Spitkovsky 2006). The physics of both of these regions is poorly understood, so we can only speculate about what properties the whole system consisting of the force-free magnetosphere + the cascade zone + the current sheet can have. Here we suggest one possible scenario for the existence of several metastable magnetospheric configurations.

The total energy of the magnetosphere of the aligned rotator with \( \Omega_P = \Omega_{NS} \) monotonically depends on the size of the corotating zone; it decreases with the increase of \( R_Y \) (Timokhin 2006). The total energy of the split-monopole magnetosphere with different current densities monotonically increases with the increase of the total current (Timokhin 2007a). It can be that the combining effect of different current density distributions and different sizes of the corotating zone, especially in the inclined rotator, results in a set of metastable magnetospheric configurations, in the sense that such configurations represent local minima of the total energy – any small deviations of the current density and/or the size of the corotating zone from that in a given ‘minimal’ configuration would result in a larger value of the total energy.

Changes in the current density distribution require variations of the potential drop in the polar cap. These variations should be of the order of several per cent of the potential drop across the polar cap in order to produce noticeable changes in \( \Omega_F \) (Timokhin 2007a). In young pulsars, where a very small fraction of the vacuum potential drop would be enough to short out the accelerating electric field, these variations should be small. With the aging of a pulsar, the freedom in the current density distribution will increase, thus making the existence of multiple metastable configurations of the magnetosphere possible. In the frame of the proposed model this might be the reason why mode changes and nulling are seen in older pulsars.

The pulsar magnetosphere represents a case of a highly non-linear system with a complicated means of adjustment between its subcomponents. For example, the polar cap cascade zone should adjust the current density flowing through it to that required by the

\footnote{In the time-dependent numerical simulations of the inclined rotator magnetosphere by Spitkovsky (2006) and Kalapotharakos & Contopoulos (2009), a constant \( \Omega_P = \Omega_{NS} \) and a fast reconnection rate at the Y-point were implicitly assumed. Therefore, the fact that the only stable configuration that was found in these simulations was the configuration with \( R_Y = R_{LC} \) does not exclude the possibility that other stable configurations with different \( \Omega_P \) and \( R_Y \) exist.}
magnetosphere. It is not clear now how this adjustment works. The characteristic time-scale of the cascade zone is of the order of microseconds, while the magnetospheric characteristic time-scale is of the order of the pulsar rotation period. The pulsar magnetosphere, being a highly non-linear system with very different characteristic time-scales, might exhibit rich dynamical behaviour. In particular, some of these metastable configurations may be a sort of strange attractor, where a system spends substantial time and then suddenly changes to a different state. The time intervals between these changes may be very large, much larger than any characteristic time-scale of the system, and transitions can happen quasi-periodically. This might account for quasi-periodicity in changes between ON and OFF states in PSR B1931+24.

Rotating radio transients (RRATs) – recently discovered transient radio sources – are thought to be rotating NSs too (McLaughlin et al. 2006). The discovery of pulsar PSR J0941−39 which switches between a RRAT-like isolated burst-emitting mode and a mode with emission typical for a nulling pulsar might imply that RRATs are an extreme case of nulling pulsars (Burke-Spolaor & Bailes 2010). In the frame of the proposed model it means that RRATs are nulling pulsars which spend most of their time in a state(s) where our line of sight does not cross the beam or no radio emission is generated. The existence of such pulsars seems to be a natural consequence of our model.

As a direct hint that the pulsar magnetosphere can indeed evolve on time-scales much larger than the rotational period, we can consider the evolution of the subpulse drift rate in PSR B0943+10 (Rankin & Suleymanova 2006). In this mode changing pulsar the subpulse drift rate evolves after the offset of the ‘B’ mode with a characteristic time-scale of some ~4000 rotational periods. This time is larger than any characteristic time-scale the force-free magnetosphere can have. It might be that in this case we directly observe manifestations of some non-linear process(es) involving adjustment between the magnetosphere, the polar cap cascades and/or the current sheet. If the pulsar magnetosphere is indeed a highly non-linear system, then the existence of different metastable states seems to be a quite natural assumption.

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