Dynamical response of a rotating cantilever pipe conveying fluid based on the absolute nodal coordinate formulation

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ABSTRACT

A dynamical model of a rotating cantilever pipe conveying fluid is derived based on the absolute nodal coordinate formulation. The free vibration and dynamical response of the system are investigated in this paper. Based on the absolute nodal coordinate method and the extended Lagrangian equation proposed by Irschik for the nonmaterial system, the motion equation of the rotating flexible cantilever pipe conveying fluid is built. The influence of the rotational angular velocity and flow velocity on the natural frequency of the system is analyzed. The critical nondimensional circular frequency of in-plane vibration and critical nondimensional flow velocity are investigated. The static deformation is shown under different flow velocities and angular velocities. The nonlinear transient analyses of a rotating flexible cantilever pipe are completed with the variation of parameters. During the rotation, the Coriolis force of fluid acting on the pipe has a great effect on the static deformation.

KEYWORDS: absolute nodal coordinate formulation, rotating pipe conveying fluid, critical flow velocity, dynamical response

1. INTRODUCTION

A pipe conveying fluid is an important part of engineering structures; it is extensively applied in nuclear reactors, ocean mining, heat exchangers, drug delivery, and microfluidic and nanofluidic devices. So far, the research on the pipe conveying fluid has obtained substantial research results. Early studies on pipes conveying fluid have revealed the linear dynamical character of a cantilever and a simply supported pipe [1]. A cantilever pipe conveying fluid undergoes flutter instability, and a simply supported pipe conveying fluid may generate buckling instability beyond the critical flow velocity. Nonlinear dynamical models of fluid-conveying pipes under different circumstances have been developed in recent decades and many scholars have reported complex and interesting nonlinear dynamical behavior. Semler et al. [2] applied the Hamilton principle to build the nonlinear equation of motion of an inextensible cantilever pipe conveying fluid. Based on Semler’s model theory, Ghayesh [3], Paidoussis et al. [4], Modarres-Sadeghi et al. [5] and Wang et al. [6] established the 3D nonlinear dynamical character of cantilever pipes conveying fluid and used it to explore the nonlinear dynamical phenomenon with different types of motion constraints and a mass attached at the tip end. Ghayesh et al. [7] investigated an extensible cantilever pipe by using an extended Lagrangian equation with nonmaterial bodies. The effects of different system parameters on the static elongation and dynamical behavior of the system were analyzed. Stangl et al. [8] used extended Lagrange equations in combination with the Ritz method and directly built a set of nonlinear ordinary differential equations of motion. Zhang et al. [9] derived 3D coupled equations of motion in a compact matrix form by using the extended Hamilton principle and considered the effect of extra linear springs and lumped masses at arbitrary positions along the pipe in these equations. Based on the Hamilton principle, Chen et al. [10] derived a geometrically exact model that considers full geometric nonlinearities caused by curvature effects; they also provided a new version of the governing equation for possible pipe oscillations with extremely large amplitudes. All of these models are impossible to present in detail here. For further information, interested readers can refer to the reviews of Paidoussis [1] and Ibrahim [11]. Their reviews summarize fruitful research on pipes conveying fluid.

A rotating cantilever pipe is one of the structure types of pipes conveying fluid, and its vibration has elicited the attention of several scholars. The famous scholar Panussis [12] reported that the pipe conveying fluid and the rotating uniform cantilever beam without flow are the closest prior art to the rotating flexible pipe conveying fluid with cantilever: Panussis [12] used the Newton method to derive lateral motion equations of the cantilever tube on the horizontal plane of rotation and out of the plane. The critical nondimensional circular frequency of lateral vibration and the critical nondimensional speed of the rotating cantilever system have also been examined under rotating and nonrotating cases. Bogdević [13] studied the vibration problem of a rotating pipe conveying fluid by using the ordinary finite element method. A nonlinear response problem that includes subharmonic and superharmonic resonances was presented. Yoon and

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Son [14] used mixed deformation variable modeling and applied the Lagrange equation to derive the equation of motion of a rotating cantilever tube with mass at the end. Then, they analyzed the effect of the end mass and fluid and angular velocities on the dynamics of the cantilever pipe. Khajehpour and Azadi [15] introduced the governing equations of a rotating cantilever pipe conveying fluid, and the longitudinal and lateral induced vibrations were controlled using piezoelectric layers. The present authors built a nonlinear dynamical system model by using the absolute nodal coordinate formulation (ANCF). The ANCF was proposed by Shabana [16] based on the finite element method and continuum theory to effectively deal with large deformation and rotation multibody dynamics. This method is considered to be a multibody dynamics study. The ANCF presented numerous advantages when it was used to study the dynamics of large deformation and rotation because the ANCF considers the axial strain of the beam element. Berzeri and Shabana [17] utilized the ANCF to study the centrifugal stiffening effect on rotating 2D beams. They analytically showed that the resulting solution does not exhibit any instability as the angular velocity of the beam increases. Zhang et al. [18] applied the ANCF to a new dynamical model of a rotating flexible beam with a concentrated mass located at an arbitrary position. Frequency veering and associated mode-shift phenomena were observed and discussed for different simulation cases. Gerstmayr et al. [19] conducted a comprehensive review of the finite element ANCF, which can be used to obtain efficient solutions to large deformation problems in multibody systems. The ANCF can also be utilized in the vibration analysis of pipes conveying fluid. Stangl et al. [20] showed that the ANCF method was suitable for modeling large deformations and rotations in the framework of Bernoulli–Euler beam theory for pipes conveying fluid. Chun et al. [21] applied the ANCF method to study the nonlinear dynamical behavior of a cantilever pipe and a simply supported pipe. The numerical results of the pipes were compared with those in the existing reference, and the authors found that the ANCF provided more accurate results. Based on the ANCF and the transfer matrix method (TMM), Rong et al. [22] used a novel efficient Riccati ANC–TMM for nonlinear dynamical analysis of a pipe conveying fluid with large deformations. The review of literature on pipes conveying fluid, especially rotating cantilever pipes conveying fluid, indicated that scholars focused on the analysis of linear problems in the rotating conveying fluid. Meanwhile, e of nonlinear response has rarely been investigated. The vibration of cantilever pipes entails large deformation and rotation. Hence, due to the superiority of the ANCF, this method is suitable for modeling rotating pipes that convey fluid. The ANCF is considered an effective method in the investigation of flexible multibody systems. Meijsaad [23] presented its integration with flexible multibody systems for simulating pipes that can undergo arbitrarily large overall motions. However, this field remains open. To the author’s knowledge, the nonlinear system response and the application of the ANCF to rotating pipes conveying fluid have not been examined. Thus, the current study is the first to investigate a rotating cantilever pipe conveying fluid by the ANCF. A nonlinear analysis of the pipe conveying fluid can be performed by applying the ANCF, as indicated previously.

However, a linear analysis of this system based on the ANCF has not been performed. A linear analysis of a rotating pipe conveying fluid is conducted in the current study under the modeling framework of the ANCF. The fully nonlinear Green strain and second Piola–Kirchhoff tensor are applied without considering any order approximation. The present study is an extension of [18, 20]. In [20], the nonlinear response of a pipe conveying fluid was analyzed without considering the rotation, and a linear analysis was not performed. In [18], only the vibration character of the rotating cantilever beam was investigated.

The present study is concerned with the linear analysis and nonlinear response of the rotating pipe conveying fluid by using the ANCF. A 2D two-node Euler beam element based on the ANCF is introduced as the modeling method of the system. Under the rotating condition, the equations of motion of the pipe finite element of the ANCF are derived using an extended version of Lagrange’s equations. The effects of angular velocity and fluid velocity on the natural frequency and the critical flow velocity are analyzed. Finally, the nonlinear dynamical response is shown with the effect on the variable rotating angular velocity and flow velocity.

2. DYNAMICAL EQUATION

A rotating cantilever pipe conveying fluid is considered in Fig. 1. When it does not deform, the length of the pipe is \( L \) and the pipe is attached to a hub of radius \( \delta \), and the center axis of the hub is the axis of rotation in the center rigid body with rotating counterclockwise velocity \( \omega \). The present system studies only in-plane vibration, so the effect of gravity is not considered. A simplified diagram of the model in the \( X_1O_1Y_1 \) plane is shown in Fig. 2. \( X_1O_1Y_1 \) is the inertial coordinate system. \( X_2O_2Y_2 \) is the local coordinate system rotating with the pipe; \( O_2 \) is the connection point between the pipe and the central rigid body section. When the pipe does not deform, \( X_2 \) denotes the axial direction of the pipe, and \( Y_2 \) is perpendicular to the \( X_2 \) axis, which satisfies the right-hand rule. In the \( X_2O_2Y_2 \) system, the ANCF is used to establish the model of the pipe with the theory of Euler–Bernoulli beam, and regardless of the moment of inertia and shear deformation of the cross section.

The following section is a brief introduction to the 2D two-node model under the ANCF method, as seen in Fig. 3. The pipe is divided into \( n \) elements, which is similar to the traditional
finite element method. Each element node is represented by the displacement and absolute slope of the node, where $L$ represents the pipe length and $L_e = L/n$ represents the element length. The absolute position vector $r$ of an arbitrary point $P$ on the pipe element is represented as follows:

$$r = \begin{bmatrix} r_{X_2} \\ r_{Y_2} \end{bmatrix} = \begin{bmatrix} a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \\ b_0 + b_1 x^1 + b_2 x^2 + b_3 x^3 \end{bmatrix} = S e,$$

(1)

where $r_{X_2}$ and $r_{Y_2}$ represent the absolute positions in the $X_2$ and $Y_2$ directions, respectively, $x \in [0, L_e]$ represents an arbitrary point on the pipe element before deformation, which is the coordinate of the pipe element nodes:

$$e = [e_1 \ e_2 \ e_3 \ e_4 \ e_5 \ e_6 \ e_7 \ e_8]^T.$$

(2)

The absolute displacement of the node is

$$e_1 = \left. r_{X_2} \right|_{x=0}, \quad e_2 = \left. r_{Y_2} \right|_{x=0}, \quad e_5 = \left. r_{X_2} \right|_{x=L_e}, \quad e_6 = \left. r_{Y_2} \right|_{x=L_e}.$$

(3)

The absolute position slope of the node is

$$e_3 = \left. \frac{\partial r_{X_2}}{\partial x} \right|_{x=0}, \quad e_4 = \left. \frac{\partial r_{X_2}}{\partial x} \right|_{x=0}, \quad e_7 = \left. \frac{\partial r_{Y_2}}{\partial x} \right|_{x=L_e}, \quad e_8 = \left. \frac{\partial r_{Y_2}}{\partial x} \right|_{x=L_e}.$$

(4)

The shape function of the displacement field of the describing unit is obtained by the absolute displacement and the absolute slope vector interpolation of the node as follows:

$$S = [S_1 I \ S_2 I \ S_3 I \ S_4 I].$$

(5)

where

$$S_1 = \frac{1}{2} - \frac{1}{2} \xi + \frac{1}{4} \xi^3, \quad S_2 = \frac{L_e}{8} (1 - \xi - \xi^2 - \xi^3),$$

$$S_3 = \frac{1}{2} + \frac{1}{2} \xi - \frac{1}{4} \xi^3, \quad S_4 = \frac{L_e}{8} (-1 + \xi^2 + \xi^3),$$

where $\xi = 2x/L_e - 1, x \in [0, L_e]$ and $\xi \in [-1, 1]$.

Irschik and Holl [24] deduced the extended Lagrangian equation suitable for controlling the movement of the body along the surface of the material body, which can better deal with the inflow and outflow of the control body. In this paper, the nonlinear equation of motion of a rotating pipe is derived based on the extended Lagrange equation. The equations of motion are as follows:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \int_{\Gamma} da \cdot (V_{E} - V_{P}) \frac{\partial T}{\partial \dot{q}_i} - \int_{\Gamma} da \cdot \left( \frac{\partial V_{E}}{\partial \dot{q}_i} - \frac{\partial V_{P}}{\partial \dot{q}_i} \right) \dot{T} - Q_i = 0.$$

(6)

In the above formula, $T$ is the total kinetic energy of the system, $q_i$ is the generalized coordinate and $Q_i$ is the generalized force of the system. The two surface points in the equation are used to describe the mass flowing into the outflow control. $\Gamma$ represents the boundary of the fluid inlet and outlet system, and $da$ describes the micro-element direction on the curved surface. $V_E$ and $V_P$ represent the velocity of the fluid and the velocity of the pipe, respectively. $\dot{T}$ denotes the kinetic energy of the unit fluid. Table 1 shows the system parameters under the rotation of the conveying fluid pipe; the vector of $O_1 O_2$ in the floating coordinate system $X_2 O_2 Y_2$ is expressed as

$$r_e = \begin{bmatrix} \delta \\ 0 \end{bmatrix}^T.$$

(7)

The absolute coordinate vector of an arbitrary point on the beam element in the floating coordinate system $X_2 O_2 Y_2$ is

$$r = [X \ Y]^T = S e.$$

(8)

When the fluid-conveying pipe rotates, the arbitrary point on the pipe in the floating coordinate system is expressed as

$$r_1 = r + r_e.$$

(9)

The speed of an arbitrary point on the pipe unit in the base coordinate system $X_1 O_1 Y_1$ is

$$V_p = \Phi \dot{r} + \Phi \times \Theta \cdot \dot{r}_1,$$

(10)

where $\Phi$ represents the direction cosine matrix of the floating coordinate system with respect to the base point coordinate system:

$$\Phi = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$ 

(11)

$\Theta$ represents the angular velocity matrix:

$$\Theta = \begin{bmatrix} 0 & -\dot{\theta} & \dot{\theta} \\ \dot{\theta} & 0 & 0 \\ -\omega & 0 & 0 \end{bmatrix} = \omega \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

(12)

Then, the kinetic energy of the unit pipe is

$$T_p = \frac{1}{2} m \int_0^{L_e} V_{p}^T V_{p} dx.$$ 

(13)
In this work, it is considered that the pipe is extensible and the elongation of the pipe is

\[ f = \sqrt{\mathbf{r}^T \mathbf{r}}. \]  

(14)

Then, the relationship between the pipe micro-element before and after deformation is as follows:

\[ ds = f \, dx. \]  

(15)

Assuming that the volume of the pipe remains unchanged and that the internal fluid is incompressible,

\[ dx_0 A_0 = ds A_1 = f \, dx A_1, \quad U_0 A_0 = U_1 A_1. \]  

(16)

In Eq. (16), \( A_0 \) and \( A_1 \), respectively, indicate the cross-sectional area before and after deformation; \( U_0 \) and \( U_1 \) indicate the flow velocity of the fluid before and after the extension of the pipe, respectively. Therefore,

\[ U_1 = U_0 A_0 / A_1 = U_0 f. \]  

(17)

The tangential direction vector of an arbitrary point on the axis of the pipeline after the deformation of the pipeline is

\[ \mathbf{r} = \mathbf{r}^t / f. \]  

(18)

The absolute velocity of the pipeline fluid in the \( X_2 O_2 Y_2 \) coordinate system is

\[ \mathbf{V}_F = \mathbf{i} + U f \mathbf{r} = \mathbf{i} + U \mathbf{r}^t. \]  

(19)

When the pipe conveying fluid is rotated, the fluid velocity in the \( X_1 O_1 Y_1 \) coordinate system is

\[ \mathbf{V}_F = \Phi \mathbf{V}_F + \Phi \mathbf{\Theta} \cdot \mathbf{r}_1 = \Phi (\mathbf{i} + U \mathbf{r}^t) + \Phi \mathbf{\Theta} \cdot \mathbf{r}_1. \]  

(20)

Then, the kinetic energy of the fluid is

\[ T_F = \frac{1}{2} M \int_0^{L_0} \mathbf{V}_F^T \mathbf{V}_F \, dx, \]  

(21)

where \( M = \rho_F A_F \). The kinetic energy of a unit volume of fluid is as follows:

\[ T_F = \frac{1}{2} \rho_F V_F^T \mathbf{V}_F = \frac{1}{2} \rho_F (\Phi \mathbf{V}_F + \Phi \mathbf{\Theta} \cdot \mathbf{r}_1)^T \times (\Phi \mathbf{V}_F + \Phi \mathbf{\Theta} \cdot \mathbf{r}_1). \]  

(22)

The above formula is substituted into the first surface integral of Eq. (6), and the following results are obtained:

\[ \int_{A_1} d\mathbf{a} \cdot (\mathbf{V}_F - \mathbf{V}_F) \frac{\partial \mathbf{T}_F}{\partial \mathbf{\xi}} = A_1 d\Phi \mathbf{r} \cdot U \mathbf{\Theta} \frac{\partial \mathbf{T}_F}{\partial \mathbf{\xi}} = \frac{A_F}{f} \mathbf{r}^t \cdot U \mathbf{r} \frac{\partial \mathbf{T}_F}{\partial \mathbf{\xi}} = a A_F U \frac{\partial \mathbf{T}_F}{\partial \mathbf{\xi}}. \]  

(23)

Thus,

\[ a A_F U \frac{\partial \mathbf{T}_F}{\partial \mathbf{\xi}} = a A_F U \rho_F S^T \dot{\mathbf{e}} + a A_F U \rho_F S^T (U S' + \Theta S) \mathbf{e} + a A_F U \rho_F S^T \dot{\Theta} \mathbf{r}. \]  

(24)

In the above formula, \( a \) indicates the direction of fluid flow: the inflow is \(-1\) and the outflow is \(1\). Each pipe element is connected to one another, so the two surface integrals cancel out each other at the pipe node. In cantilever conditions, the end of the pipe is a free term, at the point \( \mathbf{r} \neq 0 \), so only the free end contributes to the system. The second term surface integral in Eq. (6), \( \mathbf{V}_F - \mathbf{V}_F = \Phi \mathbf{U} \mathbf{r}^t \) does not contain the generalized velocity term \( \mathbf{e} \); therefore,

\[ \int_{A_1} d\mathbf{a} \cdot \left( \frac{\partial \mathbf{V}_F}{\partial \mathbf{\xi}} - \frac{\partial \mathbf{V}_F}{\partial \mathbf{\xi}} \right) \dot{\mathbf{e}} = 0. \]  

(25)

The axial strain of the pipe is given as

\[ \varepsilon_1 = \frac{1}{2} (f^2 - 1) = \frac{1}{2} \left( \mathbf{r}^T \mathbf{r} - 1 \right). \]  

(26)

The curvature of the pipe is given as

\[ \kappa = \left| \frac{d^2 \mathbf{r}}{dx^2} \right| = \left| \frac{d^2 \mathbf{r}}{dx^2} \cdot d\mathbf{a} \right| = \frac{1}{f^2} \left| \mathbf{r}^t \right|. \]  

(27)

Then, the total potential energy of the system is given as

\[ V = \int_0^{L_0} \left[ EAp_1 e_1^2 + Elk^2 \right] dx. \]  

(28)

The generalized force of the system can be written as

\[ Q_i = -\frac{\partial V}{\partial \mathbf{e}_i}. \]  

(29)

Substituting Eqs (13), (21), (24) and (29) into Eq. (6), the following dimensionless variables and parameters are introduced:

\[ q_i = \frac{e_i}{L}, \quad u = \sqrt{\frac{M}{EI}} L_0, \quad \beta = \frac{M}{M + m}, \quad \xi = \left( \frac{EI}{m + M} \right)^{1/2} \frac{t}{L^2}, \quad \Pi = \frac{EA_p L}{EI} \frac{L}{L_0}, \quad \Omega = \frac{\omega L^2}{EI}, \quad \Omega_t = (M + m) \frac{L^4}{EI} \frac{\dot{\omega}}{\omega}, \quad \delta_a = \frac{r_a}{L} \left[ \frac{\delta}{L} \right] = \left[ \delta_1 \right]^T. \]  

(30)

It is convenient to write the following expressions:

\[ S_{00} = \frac{L_0}{2} \int_{-1}^{1} S^T S \, d\xi, \quad S_{01} = \int_{-1}^{1} S^T S' \, d\xi, \]  

\[ \]
In the above formula, 

\[ m_{ii} = \int_{-1}^{1} \mathbf{s}^T \mathbf{s} \, d\xi, \quad \mathbf{s}_{ii} = \frac{2}{L_e} \int_{-1}^{1} \mathbf{s}^T \mathbf{s}' \, d\xi, \]

\[ \mathbf{s}_{22} = \left( \frac{2}{L_e} \right)^3 \int_{-1}^{1} \mathbf{s}^T \mathbf{s}'' \, d\xi, \quad \mathbf{s} \mathbf{S} = \mathbf{S}^T \mathbf{S} \]

\[ \mathbf{s}_{00} (1) = \mathbf{S}^T \mathbf{S} |_{\xi = 1}, \quad \mathbf{S}_{11} = \left( \frac{2}{L_e} \right)^2 \mathbf{S}^T \mathbf{S}' \]

\[ \mathbf{S}_{22} = \left( \frac{2}{L_e} \right)^4 \mathbf{S}^T \mathbf{S}'' \],

\[ \mathbf{B}_{11} = \frac{L_e}{2} \int_{-1}^{1} \mathbf{S}^T \mathbf{H} \mathbf{S} \, d\xi, \quad \mathbf{B}_{12} = \frac{L_e}{2} \int_{-1}^{1} \mathbf{S}^T \mathbf{H}^T \mathbf{S} \, d\xi, \]

\[ \mathbf{B}_{01} = \int_{-1}^{1} \mathbf{S}^T \mathbf{H} \mathbf{S} \, d\xi, \quad \mathbf{B}_{10} = \int_{-1}^{1} \mathbf{S}^T \mathbf{H}^T \mathbf{S} \, d\xi, \]

\[ \mathbf{B}_{12} (1) = \omega \mathbf{S}^T \mathbf{H} \mathbf{S} |_{\xi = 1}, \quad \mathbf{B}_{20} = \frac{L_e}{2} \int_{-1}^{1} \mathbf{S}^T \, d\xi, \]

\[ \mathbf{B}_{21} = \frac{L_e}{2} \int_{-1}^{1} \mathbf{S}^T \, d\xi, \quad \mathbf{Q}_{1} = \mathbf{B}_{20} \mathbf{H}_{r}, \]

\[ \mathbf{Q}_{2} = \mathbf{B}_{21} \mathbf{H}_{r}, \quad \mathbf{Q}_{3} = \mathbf{B}_{20} \mathbf{H}_{r}, \]

where \( \mathbf{H} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \).

The dimensionless motion element equation of the rotating pipe conveying fluid is obtained as follows:

\[ \mathbf{m}^e \dot{\mathbf{q}}_i + \mathbf{c}^e \mathbf{q}_i + (\mathbf{k}^e + \mathbf{k}^{ne}) \mathbf{q}_i = \mathbf{f}_i^e. \quad (31) \]

In the above formula:

\[ (\frac{\partial}{\partial \tau}, \mathbf{B}_{12} (1) = \omega \mathbf{S}^T \mathbf{H} \mathbf{S} |_{\xi = 1}, \quad \mathbf{m}^e = \mathbf{S}_{00}, \]

\[ \mathbf{c}^e = \mathbf{u}^e \mathbf{S}^T \mathbf{H} \mathbf{S} |_{\xi = 1}, \quad \mathbf{m}^e = \mathbf{S}_{00}, \]

\[ \mathbf{k}^e = \mathbf{u}^2 \mathbf{S}^T \mathbf{H} \mathbf{S} |_{\xi = 1}, \quad \mathbf{m}^e = \mathbf{S}_{00}, \]

\[ \mathbf{k}^{ne} = \Pi_0 \mathbf{L} \int_{-1}^{1} \mathbf{q}^T \mathbf{S} \mathbf{S} \, d\xi \]

\[ + \frac{L_e}{2} \left[ \int_{-1}^{1} \frac{1}{(\mathbf{q}^T \mathbf{S} \mathbf{S})^3} \mathbf{S}_{22} \, d\xi \right. \]

\[ - \int_{-1}^{1} \frac{2 \mathbf{q}^T \mathbf{S}_{22} \mathbf{q} \mathbf{S}_{11} \, d\xi}{(\mathbf{q}^T \mathbf{S} \mathbf{S})^3} \left. \right], \]

\[ \mathbf{f}_i^e = -\omega \mathbf{Q}_1 + \omega^2 \mathbf{Q}_2 + \omega u \mathbf{S}^T \mathbf{H} \mathbf{S} \mathbf{Q}_3 \]

Here, \( \mathbf{m}^e \) is the mass matrix of the rotating pipe element, \( \mathbf{c}^e \) is the corresponding damping matrix, \( \mathbf{k}^e \) is the linear stiffness matrix, \( \mathbf{k}^{ne} \) is the nonlinear stiffness matrix and \( \mathbf{f}_i^e \) is the node load vector of the element. According to the traditional finite element method, the matrix of those elements can be assembled into the corresponding global mass, damping, stiffness and joint load matrices.

For the convenience of calculation later, the element dimensionless tangent stiffness matrix \( \mathbf{K}_{Te} \) is given below:

\[ \mathbf{K}_{Te} = \int_{0}^{L_e} \mathbf{S}^T \mathbf{S} \mathbf{q} \cdot \mathbf{q}^T \mathbf{S}^T \mathbf{S} \, dx \]

\[ -4E \int_{0}^{L_e} \mathbf{S}^T \mathbf{S} \mathbf{q} \cdot \mathbf{q}^T \mathbf{S}^T \mathbf{S} \, dx \]

\[ -E \int_{0}^{L_e} \left[ 4 \mathbf{S}^T \mathbf{S} \mathbf{q} \cdot \mathbf{q}^T \mathbf{S}^T \mathbf{S} \right. \]

\[ \left. \left( \mathbf{q}^T \mathbf{S}_{11} \mathbf{q} \right)^4 \right] \, dx. \quad (32) \]

3. LINEAR ANALYSIS

The global differential equation of a rotating pipe conveying fluid can be expressed as

\[ \mathbf{M} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{F}, \quad (33) \]

where \( \mathbf{M} \) is the global mass matrix, \( \mathbf{C} \) is the global damping matrix and \( \mathbf{F} \) is the global load vector; \( \mathbf{K} \) is the global stiffness matrix:

\[ \mathbf{K} = \mathbf{K}^L + \mathbf{K}^{Ne}, \quad (34) \]

where \( \mathbf{K}^L \) and \( \mathbf{K}^{Ne} \) are the global linear stiffness matrix and nonlinear stiffness matrix that are composed of the linear stiffness matrix and the nonlinear stiffness matrix of an element in Eq. (31), respectively. Because the system has a rotating character, the effects of the centrifugal forces lead to a particular static deformation \( \mathbf{q}_s \) that can be obtained by using the Newton–Raphson algorithm to solve Eq. (33). If \( \mathbf{q}_s \) is substituted into formula (8), the coordinates of each point on the pipe axis in \( X_2 O_2 Y_2 \) can be obtained when the pipe reaches static stability under the corresponding angular velocity and flow velocity, so the static configuration of the pipe can be obtained in \( X_2 O_2 Y_2 \). To analyze the free vibration during the rotation of the conveying pipe, the equation of motion needs to be linearized. In the static equilibrium position, the perturbation form of the equation of motion is given as follows:

\[ \mathbf{M} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + \mathbf{K}_T (\mathbf{q}_s) \mathbf{q} = 0, \quad (35) \]

where \( \mathbf{K}_T \) is the tangent stiffness matrix of the system at the current static equilibrium position:

\[ \frac{\partial \mathbf{K}_T}{\partial \mathbf{q}} = \frac{\partial (\mathbf{K}^L + \mathbf{K}^{Ne} (\mathbf{q}_s))}{\partial \mathbf{q}} \]

\[ \mathbf{K}_T = \mathbf{K}^L + \mathbf{K}^{Ne} (\mathbf{q}_s) + \frac{\partial (\mathbf{K}^{Ne} (\mathbf{q}_s))}{\partial \mathbf{q}} \mathbf{q} \]

\[ \mathbf{K}_T^L + \frac{\partial (\mathbf{K}^{Ne} (\mathbf{q}_s))}{\partial \mathbf{q}} \mathbf{q}. \]
The first two natural frequencies increase as the angular velocity increases. At the same angular velocity, the fifth-order frequency grows rapidly at first. However, the fifth-order natural frequency in the case of fluid velocity is greater than that in the case without fluid velocity. With the increase of angular velocity, the fifth-order frequency grows rapidly at first.

Figure 4 shows the effect of angular velocity on the first five natural frequencies at a certain flow velocity. The numerical results are calculated at the parameter values of $\Pi_0 = 4900, \beta = 0.2, \delta = 0.0$ and $L = 1.0$. The natural frequencies obtained by removing the Coriolis terms from the present model. The present value 2 denotes the actual natural frequency of the model with Coriolis terms. As can be seen from Table 2, at fluid velocity $u = 0.0$, the natural frequency obtained by removing the Coriolis terms from the present model. The present value 2 denotes the actual natural frequency of the model with Coriolis terms. As can be seen from Table 2, at fluid velocity $u = 0.0$, the natural frequency is consistent with that in [18], which further shows that the model can degenerate into a rotating flexible beam without Coriolis terms. The first two natural frequencies increase as the angular velocity and radius of the central rigid body increase. At the same radius, the first two natural frequencies increase as the rotational velocity increases. Similarly, at the same angular velocity, the first two natural frequencies increase as the radius of the central rigid body increases. Comparison of the results of present values 1 and 2 shows that the Coriolis terms have an important influence on the dynamics of the system. In particular, when the frequencies of the system are high, the difference between the two cases becomes obvious. In Table 3, the effect of the number of elements on natural frequencies and tip axial deformation is shown. It can be seen that seven elements can be used to obtain good results.

Table 2. Comparison of the first two natural frequencies ($\beta = 0.0, \Pi_0 = 4900, u = 0.0$ and $L = 1$).

| $\delta$ | First natural frequency | Second natural frequency |
|---|---|---|
| | Value 1 | Value 2 | Value 1 | Value 2 |
| $\delta = 0$ | | | | |
| 2 | 3.6221 | 3.6197 | 3.6176 | 22.5183 | 22.5237 | 22.5160 |
| 10 | 5.0551 | 5.0126 | 4.9389 | 31.9654 | 32.1525 | 31.8981 |
| 20 | 6.7862 | 6.6758 | 6.3185 | 51.1725 | 51.5636 | 50.8500 |
| 30 | 8.1661 | 8.0430 | 7.2122 | 72.6640 | 73.1607 | 71.6204 |
| 40 | 9.3079 | 9.2676 | 7.8502 | 95.1558 | 96.6674 | 92.9963 |
| 50 | 10.2897 | 10.4249 | 8.3712 | 118.3231 | 118.7771 | 114.6138 |
| $\delta = 1$ | | | | |
| 2 | 4.4009 | 4.3957 | 4.3931 | 23.2570 | 23.2820 | 23.2546 |
| 10 | 13.1651 | 13.1301 | 12.9418 | 42.8510 | 43.1484 | 42.7630 |
| 20 | 24.5228 | 24.5061 | 23.3079 | 75.4436 | 75.8975 | 74.9216 |
| 30 | 35.0590 | 35.1228 | 32.0182 | 108.8587 | 109.2717 | 107.4180 |
| 40 | 44.7026 | 44.8791 | 39.2990 | 141.9650 | 142.2019 | 139.1250 |
| 50 | 53.5357 | 53.8393 | 45.5174 | 174.4671 | 174.4505 | 169.8159 |
| $\delta = 5$ | | | | |
| 2 | 6.6439 | 6.6321 | 6.6281 | 25.9785 | 26.0661 | 25.9850 |
| 10 | 26.9924 | 26.9791 | 26.6250 | 69.8884 | 70.3076 | 69.7558 |
| 20 | 50.0709 | 50.2034 | 48.2709 | 128.6763 | 128.9543 | 127.8837 |
| 30 | 70.0706 | 70.4004 | 65.9930 | 183.7893 | 183.6017 | 180.8105 |
| 40 | 87.7582 | 88.2911 | 80.9789 | 235.0837 | 234.3352 | 218.4708 |
| 50 | 103.7545 | 104.4848 | 94.0792 | 283.1207 | 281.8216 | 252.0178 |

where

$$\frac{\partial (K_{Ne} (q))}{\partial q} q = \sum_{i=1}^{n} B_i^T K_T B_i,$$

$q$ is the global generalized coordinate and $B_i = [0_{8 \times 4(n-1)} \ I_{8 \times 8} 0_{8 \times 4(n-1)}]_{8 \times 4(n+1)}$ denotes the element transformation matrix. Thus, $q_i = B_i q$ is obtained, $q_i$ is the element generalized coordinate.

Assuming $\chi_j = q_j$ and $\chi_j = q_j^T$, Eq. (35) can be transformed into the matrix form:

$$\begin{bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -M^{-1} K_T & -M^{-1} C \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix}, \tag{36}$$

where $I$ is the identity matrix; the eigenvalue problem of a rotating pipe conveying fluid can be analyzed from Eq. (36) by knowledge of linear algebra.

In the current work, seven elements are used to calculate the frequency. When the fluid flow velocity in the pipe is $u = 0.0$, the rotating pipe model degenerates into a rotating flexible beam model. The parameters are set to $\Pi_0 = 4900, \beta = 0.2$ and $L = 1.0$. The radius of the center rigid body, $\delta$, is set to 0, 1 and 5. The dimensionless angular velocity, $\Omega$, is set to 2, 10, 20, 30, 40 and 50. The first two natural frequencies of the flexible beam are compared with those in [18] in Table 2. In [18], the equation of motion of a rotating cantilever flexible beam was obtained using the principle of virtual work without considering the Coriolis terms. In Table 2, the present value 1 denotes the natural frequency obtained by removing the Coriolis terms from the present model. The present value 2 denotes the actual natural frequency of the model with Coriolis terms. As can be seen from Table 2, at fluid velocity $u = 0.0$, the natural frequency is consistent with that in [18], which further shows that the model can degenerate into a rotating flexible beam without Coriolis terms. The first two natural frequencies increase as the angular velocity and radius of the central rigid body increase. At the same radius, the first two natural frequencies increase as the rotational velocity increases. Similarly, at the same angular velocity, the first two natural frequencies increase as the radius of the central rigid body increases.
Table 3. The effect of the number of elements on natural frequencies and tip axial deformation.

| δ   | Ω    | ω₁     | ω₂     | δ₁     | ω₁     | ω₂     | δ₁     |
|-----|------|--------|--------|--------|--------|--------|--------|
| δ = 0 | 2    | 3.6204 | 22.5449 | 2.59E-04 | 3.62   | 22.5281 | 2.61E-04 |
|     | 10   | 5.0629 | 32.0748 | 6.44E-03 | 5.0336 | 32.0131 | 6.50E-03 |
|     | 20   | 7.0105 | 51.4899 | 2.55E-02 | 6.8299 | 51.3233 | 2.57E-02 |
|     | 30   | 8.882  | 73.2361 | 5.61E-02 | 8.4558 | 72.9495 | 5.67E-02 |
|     | 40   | 10.7214| 95.9816 | 9.73E-02 | 10.0139| 95.5807 | 9.83E-02 |
|     | 50   | 12.5224| 119.3803| 1.48E-01 | 11.5320| 118.8787| 1.49E-01 |
| δ = 1 | 2    | 4.3975 | 23.2911 | 6.40E-04 | 4.3964 | 23.2700 | 6.47E-04 |
|     | 10   | 13.2065| 43.0742 | 1.57E-02 | 13.1642| 42.9551 | 1.58E-02 |
|     | 20   | 24.7874| 76.0115 | 5.90E-02 | 24.6435| 75.7306 | 5.96E-02 |
|     | 30   | 35.6332| 109.7736| 1.23E-01 | 35.3817| 109.3364| 1.24E-01 |
|     | 40   | 45.6105| 143.1982| 1.99E-01 | 45.2575| 142.6217| 2.02E-01 |
|     | 50   | 54.7766| 175.9834| 2.85E-01 | 54.3293| 175.2859| 2.89E-01 |
| δ = 5 | 2    | 6.6386 | 26.0490 | 2.16E-03 | 6.6347 | 26.0129 | 2.18E-03 |
|     | 10   | 27.1706| 70.3840 | 4.96E-02 | 27.0721| 70.1379 | 5.02E-02 |
|     | 20   | 50.7125| 129.7747| 1.65E-01 | 50.4658| 129.2557| 1.67E-01 |
|     | 30   | 71.1987| 185.4289| 3.07E-01 | 70.8191| 184.6727| 3.10E-01 |
|     | 40   | 89.3463| 237.2011| 4.57E-01 | 88.8482| 236.2394| 4.63E-01 |
|     | 50   | 105.7736| 285.6716| 6.10E-01 | 105.1673| 284.5232| 6.17E-01 |

Figure 4 The first five natural frequencies under the influence of nondimensional angular velocity at (a) ω₀ = 0.0, (b) ω₀ = 2.0, (c) ω₀ = 4.0 and (d) ω₀ = 5.0.

flow velocity the larger the angular velocity, the larger the first five natural frequencies.

Figure 6a and b shows the effect of angular velocity on the first-order and second-order frequencies with different fluid velocities, respectively. Under the same angular velocity, the larger the flow velocity, the first two natural frequencies are naturally smaller. The first-order natural frequency shows two situations: (1) with the velocity u₀ = 0.0, 1.0, 2.0 and 3.0, the greater the angular velocity, the greater the frequency value; and (2) with the flow velocity u₀ = 4.0 and 5.0, the greater the angular velocity,
Figure 5  The first five natural frequencies under the influence of flow velocity when (a) $\Omega = 2.0$, (b) $\Omega = 4.0$, (c) $\Omega = 6.0$ and (d) $\Omega = 10.0$.

Figure 6  Effect of dimensionless angular velocity on frequencies with different fluid velocities: (a) first-order natural frequencies and (b) second-order natural frequencies.

The smaller the frequency value. It seems that there is a common point of intersection at different flow velocities, but from the local magnification, they do not meet at one point. With the constant flow velocity, as the angular velocity increases, the second natural frequencies also increase.

The critical flow velocity and the corresponding unstable frequency of the rotating pipe conveying fluid under different fluid pipe mass ratios $\beta$. In Table 4, $C = \Omega \sqrt{\beta}/u_i$ and C is determined by the dimensionless angular velocity, flow velocity and mass ratio. It can be seen from Table 4 that with $C = 0.0$ the critical flow velocity and frequency obtained in this paper are very close to those in [12]. That is to say, when $C = 0.0$, the relative errors of the critical velocity for $\beta = 0.2$, 0.5 and 0.8 relative to [12] are 0.01%, 0.17% and 0.51%, and the relative errors of frequency are 0.08%, 0.22% and 1.12%. $C \neq 0$ indicates that the pipe has an angular velocity. At this time, the results obtained for $\beta = 0.5$ and $\beta = 0.8$ are close to those in [1]. When $\beta = 0.5$ and $C = 0.5$, the relative errors of critical velocity and frequency are 1.98% and 9.18%, respectively, and when $\beta = 0.8$ and $C = 0.3$, the relative errors of critical velocity and frequency are 3.26% and 5.37%, respectively. When $\beta = 0.2$ and $C = 0.5$, the relative errors of critical velocity and frequency are 15.27% and 10.90%, respectively.

Figure 7 shows the variation of the critical flow velocity of the rotating pipe as the parameter C increases with different mass ratios $\beta$. When the mass ratio is $\beta = 0.5$ and $\beta = 0.8$, as the parameter C increases, the overall trend is that the critical velocity increases smoothly. When the mass ratio is $\beta = 0.2$, the critical flow velocity increases with the parameter increasing from $C = 0.0$ to $C = 0.3$. However, the critical velocity hardly changes...
Table 4. Critical velocity $u_{cr}$ and instability frequency $\omega_{cr}$ under parameter $C$ and mass ratio $\beta$.

| $\beta$ | $C = 0$ | $C = 0.5$ | $C = 0$ | $C = 0.5$ | $C = 0$ | $C = 0.3$ |
|---------|---------|------------|---------|------------|---------|------------|
| $u_{cr}$ | Present | 5.59269    | 9.2303  | 9.33959    | 13.4831 | 13.6038    | 14.8350    |
|         | Ref. [24] | 5.59221    | 10.8943 | 9.32357    | 13.7558 | 13.5348    | 14.3663    |
| Error   | 0.01%   | 15.27%     | 0.17%   | 1.98%      | 0.51%   | 3.26%      |             |

| $\omega_{cr}$ | Present | 13.7220 | 40.2259 | 26.5993 | 56.4966 | 45.6347 | 50.4428 |
|               | Ref. [24] | 13.7111 | 36.2709 | 26.5414 | 51.7445 | 45.1306 | 47.8712 |
| Error         | 0.08% | 10.90% | 0.22% | 9.18% | 1.12% | 5.37% |

Figure 7 Critical velocity of a rotating cantilever pipe conveying fluid.

Figure 8 Comparison of the dynamical responses with the parameters $\Omega = 0.0$ and $u = 6.6$.

Figure 9 Tip lateral dynamical response at $u = 0.0$: (a) the dimensionless chordwise and (b) the dimensionless axial.

4. RESPONSE ANALYSIS

In this section, the response of a rotating cantilever pipe under rotational angular velocity and flow velocity is analyzed. The number of elements used in the calculation of nonlinear responses is given as 7. For the parameter values of $\beta = 0.2$, $\Omega = 0.0$, $u = 6.6$, $\Pi_0 = 1000.0$ and $L = 1.0$, a comparison of the dynamical response is provided in Fig. 8. The present model is simulated using the generalized $\alpha$ method. From Fig. 8, the results are consistent with those in [21]. When the pipe is in the process of rotation and without fluid, the system parameters are $L = 1.0$, $\beta = 0.2$ and $\Pi_0 = (447.22)^2$. According to the variation of the angular velocity in [25], the angular velocity is given as follows:

$$\Omega = \begin{cases} \Omega_0 \left( t - \frac{1}{2\pi} \sin \left( \frac{2\pi t}{t_0} \right) \right), & 0 \leq t \leq t_0, \\ \Omega_0, & t \geq t_0. \end{cases} \quad (37)$$

In Eq. (37), $\Omega_0$ is the angular velocity at a stable time and $t_0$ is the time at which the angular velocity reaches a steady state. The dimensionless angular velocity increases from zero at a certain acceleration to $\Omega_0$ and then rotates at a constant angular velocity $\Omega_0$. When there is no fluid in the pipe and $t_0 = 15.0$ and $\Omega_0 = 6.0$, Fig. 9 shows the lateral response of the rotating cantilever pipe at the tip and the difference between the present
Figure 10 (a) The tip lateral response of the rotating pipe and (b) the tip axial response of the rotating pipe ($u = 0.5$ and $t_0 = 6.0$).

Figure 11 The effect of flow velocity with $u = 0.5$ and $t_0 = 6.0$: (a) the lateral displacement and (b) the axial displacement.

theory and [25]. The amplitude of response under the present theory is also consistent with that of [25].

Figure 10a and b shows the lateral and axial responses of the tip with the dimensionless fluid velocity $u = 0.5$, dimensionless time $t_0 = 6.0$ and dimensionless angular velocity $\Omega_0$ of 3.0, 4.0 and 5.0. From Fig. 10, as the dimensionless angular velocity increases, the lateral and axial deflections of the tip increase. At $u = 0.5$, based on the method of calculating static deflection in Eq. (37), the lateral static displacement of the tip is obtained as $-0.1453$, $-0.1786$ and $-0.2041$, and the axial displacement is obtained as $-0.0115$, $-0.0169$ and $-0.0217$, respectively.

From Fig. 10a and b, the stable position of response is consistent with the calculation of the static result. Figure 10a and b further shows that the greater the angular velocity, the greater the lateral and axial displacements of the pipe conveying fluid.

Figure 11a and b shows the lateral and axial responses of the tip with dimensionless flow velocity $u = 0.5$, 1.0 and 2.0, dimensionless angular velocity $\Omega_0 = 5.0$ and stable time $t_0 = 10.0$, respectively. Under the same fluid velocity, the transverse and axial displacement trends of the tip are the same as those described above. The transverse and axial displacements of the tip increase at first, and then decrease, and are finally stable near the static displacement. When the dimensionless angular velocity $\Omega_0$ is 5.0, stable time $t_0 = 10.0$ and the dimensionless flow velocity $u = 0.5$, 1.0 and 2.0, the transverse static displacement of the tip is obtained as $-0.20410$, $-0.3877$ and $-0.6418$, and the axial static displacement is obtained as $-0.0218$, $-0.0860$ and $-0.2713$, respectively, which is also consistent with the displacement by calculation of response. It can be further seen from Fig. 11a and b that the influence of velocity on the response of pipe tip can be seen. When the stability time $t_0$ and stable angular velocity $\Omega_0$ are the same, the transverse and axial displacements of the tip increase with the increase of velocity.

By comparing Figs 10 and 11 with Fig. 8, it can be found that the lateral and axial displacement equilibrium position of the tip is no longer zero when the system has fluid velocity and angular velocity.

Figure 12a–d shows the configuration of the rotating pipe conveying fluid at $t_0 = 5.0$ and $\Omega_0 = 5.0$ with the dimensionless fluid velocity $u = 0.0, 0.5, 2.0$ and 4.0, and after the angular velocity reaching the steady state. Through the previous calculation in Section 3, when the dimensionless angular velocity is $\Omega_0 = 5.0$, the critical flow velocity of the rotating pipe conveying fluid is $u_{cr} = 8.7$. Therefore, when $u = 4.0$, the rotating pipe conveying fluid is in a stable state and finally reaches the static equilibrium position. Figure 12a shows the configuration of the rotating conveying pipe at different time points at the flow velocity $u = 0.0$. It can be seen from the figure that after reaching the stabilization time point, the conveying pipe has almost no bending deformation, which coincides with the previously calculated rotational flexible beam response, further illustrating that at the flow velocity $u = 0.0$, the model degenerates into a rotating flexible beam. Comparing Fig. 12a–d, as the flow velocity is increased, the bending deformation at the time of reaching the stable angular velocity is larger, and the results of the displacement changes in the lateral and axial directions of the tip obtained in Fig. 11a and b are identical. It is concluded that the fluid will have a great effect on the deformation of the rotating pipe with the fluid velocity.

Figure 13a and b shows the configuration of the time point with the dimensionless flow velocity $u = 5.0, 8.0$, steady...
Dynamical response of a rotating cantilever pipe conveying fluid

Figure 12 Rotating pipe configuration at (a) $t_0 = 0.0$, $u = 0.0$ and $\Omega_0 = 0.0$; (b) $t_0 = 5.0$, $u = 0.5$ and $\Omega_0 = 5.0$; (c) $t_0 = 5.0$, $u = 2.0$ and $\Omega_0 = 5.0$; and (d) $t_0 = 5.0$, $u = 4.0$ and $\Omega_0 = 5.0$.

Figure 13 Configuration of the rotating pipe at (a) $t_0 = 5.0$, $u = 5.0$ and $\Omega_0 = 2.0$; and (b) $t_0 = 5.0$, $u = 8.0$ and $\Omega_0 = 2.0$.

Figure 14 Tip transverse dynamical response at $t_0 = 5.0$, $u = 8.0$ and $\Omega_0 = 2.0$.

According to the previous linear analysis, the critical flow velocity of the rotating cantilever pipe with a dimensionless rotational angular velocity $\Omega_0 = 2.0$ is 6.0. Figure 13a) shows a steady state at the current flow velocity $u = 5.0$. Figure 13b shows an unstable state at the flow velocity $u = 8.0$. As can be seen from Fig. 13b, the configuration of the rotating cantilever pipe at every time point is very large, and it appears to vibrate back and forth near a certain position. Figure 14 shows the lateral deformation of the tip of the rotating cantilever pipe under the conditions of Fig. 13b; it can also be seen that the tip shows the periodic vibration near a certain position.

The cause of the large deformation of the pipe conveying fluid during the rotation is further analyzed. As shown in Fig. 15,
the pipe is fixedly connected to the central rigid body and rotates counterclockwise according to a certain angular velocity, the fluid in the pipe moves relatively to the pipe at a certain speed, the fluid mass at any point Q in the pipe is $M_1$, the velocity of the pipe is $U$ and $r'$ is the tangent direction of the pipe; at this time, the movement speed of the pipe is $\dot{r}$ and the rotation radius at Q is $r_1$, so that the actual movement speed in the rotation coordinate system $X_2OY_2$ is $u = U r' + \dot{r}$. The expression $F = 2M_1 u \times \omega$ is the Coriolis force of the fluid acting on the pipe, where $\omega$ is the speed of rotation of the relative central rigid body of the pipe conveying fluid, as shown in Fig. 15.

During the rotation of the pipe conveying fluid, the Coriolis force of fluid acting on the pipe is analyzed, and the direction of the Coriolis force is opposite to the direction of rotation. Coriolis force acts on the pipe, causing the pipe to deform in a direction opposite to the angular velocity. From the expression of the Coriolis force, it can be seen that the magnitude of the Coriolis force is directly proportional to the actual velocity of the fluid and proportional to the rotational angular velocity. It can be concluded that the larger the flow velocity, the greater the deformation degree of the pipe, and the greater the rotational angular velocity, the greater the deformation of the pipe. In [12], the effect of the Coriolis force is omitted, and the influence of Coriolis force on the system is considered in this article. Figure 16a and b shows the effect of the flow velocity $u$ at the dimensionless angular velocity $\Omega = 4.0$ on the deformation of the pipe conveying fluid and the effect of the angular velocity at the dimensionless flow velocity $u = 4.0$ on the deformation of the pipe conveying fluid, with the system parameters $\Pi_0 = 4900.0$ and $\beta = 0.2$. In Fig. 16a, when $u = 0.0$, the pipe conveying fluid is the initial horizontal state, indicating that the angular velocity does not cause lateral deformation to the pipe conveying fluid without the flow velocity. There is no relative movement between the fluid and the pipe so that there is no effect of Coriolis force on the lateral deformation of the pipe. As the flow velocity increases, it can be seen that the degree of pipe deformation increases. In Fig. 16b, when the dimensionless angular velocity is $\Omega_0 = 0.0$, and the flow velocity is $u = 4.0$ and does not exceed the critical flow velocity, no deformation occurs in the transverse direction.

The larger the angular velocity, the larger the Coriolis force, and the greater the deformation of the pipe. The results obtained...
in Fig. 16a and b are consistent with the results obtained in the previous analysis of the pipe Coriolis force.

Figure 17a and b shows the lateral and axial responses of the tip with the flow velocity \( u = 2.0 \), angular velocity \( \Omega_0 = 2.0 \), settling time \( t_0 = 6.0 \) and \( \beta = 0.2 \). From Fig. 17, as \( \Pi_0 \) increases, the tip deformation in the axial direction becomes small and the lateral deformation also becomes small.

Figure 18a and b shows the lateral and axial responses of the tip under different \( \Pi_0 \) at the flow velocity \( u = 2.0 \), angular velocity \( \Omega_0 = 5.0 \) and \( \beta = 0.2 \). It is clear from the figure that as \( \Pi_0 \) decreases, the deformation degree becomes greater at the tip, as seen in Fig. 18a and b.

From Figs 17 and 18, the axial stiffness \( \Pi_0 \) has an important effect on the static deformation. Under the same axial stiffness \( \Pi_0 \), the different angular and flow velocities have different static deformation.

5. CONCLUSION

Based on the ANCF method, the nonlinear motion equation of the rotating cantilever pipe conveying fluid is derived by using the extended Lagrangian equation. Linear analysis and dynamical response analysis were performed on the system by introducing dimensionless parameters. The linear analysis mainly analyzes the natural frequency, critical flow velocity and static deformation of the rotating cantilever pipe system. The response analysis primarily analyzes the effect of flow rate and rotating angular velocity on the lateral and axial responses of the tip, as well as the configuration of the rotating pipe at different times.

Through numerical calculations, the following conclusions can be obtained: (1) The model degenerates into a rotating flexible beam when the flow velocity is \( u = 0.0 \). The dimensionless natural frequency obtained through numerical calculation is consistent with that in [18]. The model degenerates into a pipe conveying fluid without rotation, i.e. \( \Omega = 0.0 \). The dimensionless natural frequency is obtained through numerical calculation and is also consistent with those in [1, 21]. (2) The dimensionless natural frequency of the rotating pipe conveying fluid is determined by the angular velocity and the flow velocity. (3) When the pipe is rotating, due to the relative velocity between the fluid and the pipe, the Coriolis force will cause the pipe to deform. This result is quite different from the classic model. (4) The axial stiffness of the pipe system also influences the deformation of the pipe. (5) The pipe conveying fluid and the rotating beam had been analyzed by the ANCF in [18, 20]. The present study expands the application range of the ANCF in dynamical modeling.

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Figure 18 (a) Tip transverse response and (b) tip axial response (\( \Omega = 5.0 \) and \( u = 2.0 \)).
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