Qualia as physical measurements: a mathematical model of qualia and pure concepts

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Abstract

A space of qualia is defined to be a sober topological space whose points are the qualia and whose open sets are the pure concepts in the sense of Lewis, carrying additional algebraic structure that conveys the conscious experience of subjective time and logical abstraction. This structure is analogous to that of a space of physical measurements. It is conjectured that qualia and measurements have the same nature, corresponding to fundamental processes via which classical information is produced and physically stored, and that therefore the hard problem of consciousness and the measurement problem are two facets of the same problem. The space of qualia is independent from any preexisting notions of spacetime and conscious agent, but its structure caters for a derived geometric model of observer. Intersubjectivity is based on relating different observers in a way that leads to a logical version of quantum superposition.

Keywords: Models of consciousness, qualia, pure concepts, hard problem of consciousness, measurement problem, measurement spaces.

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1 Introduction

The hard problem of consciousness is indissociable from the word “qualia” \[5, 6\] but little is known about qualia in a precise mathematical sense. The term was originally coined by Lewis in his 1929 book “Mind and the World-Order” \[13\], in which he writes:

“There are recognizable qualitative characters of the given, which may be repeated in different experiences, and are thus a sort of universals; I call these ‘qualia.’ But although such qualia are universals, in the sense of being recognized from one to another experience, they must be distinguished from the properties of objects.”

A strong motivation behind the work presented in this paper is the sentiment that qualia cannot be dissociated from fundamental physics: if one regards physics as a mathematical theory about the phenomena that are observable in the real world, then one must admit that consciousness should belong to the realm of physics because, essentially by definition, qualia are observable — for instance by naive introspection, or by the more skilled methods of trained meditators. Perhaps one can even argue that nothing else is truly observable, on the grounds that any phenomenon in nature is ultimately perceived and communicated through conscious experience. So it is a bit perplexing that so far physics has had relatively little to say about this, despite the fact that vexing questions concerning measurements and observers have crept unavoidably into physics with the discovery of quantum mechanics. A symptom of the difficulties is that no consensual solution to the measurement problem has yet been found despite the steady proliferation of interpretations and modifications of quantum mechanics. See, e.g., \[1, 2, 12\]. In the words of Mermin \[16\]: “New interpretations appear every year. None ever disappear.”

While there are intrinsic descriptions of qualia by terms such as “ineffable,” “private,” and so on, these tend to carry little, if any, physical content. One way to circumvent this is by acknowledging that the process through which we come to feel that we understand a physical phenomenon is often synonym with becoming familiar with a mathematical description of that phenomenon. Hence, the aim of this paper is to suggest a mathematical approach to qualia, in particular in the style often employed in abstract mathematics through which it is not the entities themselves that are given a mathematical definition, but rather the spaces they inhabit — such as when, for instance, a vector is defined to be a point of a vector space.

We note that an approach of this type has been put forward by Stanley \[27\], who proposed that spaces of qualia are closed pointed cones in infinite dimensional separable topological real vector spaces. But this is quite different from the model of the present paper, in which another crucial aspect is illustrated by the following passage from Lewis’ book where he introduces the notion of “pure concept”:
“Because it is our main interest here to isolate that element of knowledge which we can with certainty maintain to be objective and impersonal, we shall define the pure concept as ‘that meaning which must be common to two minds when they understand each other by the use of a substantive or its equivalent.’”

In this paper, qualia and pure concepts will be treated on equal footing, as interdependent aspects of consciousness: pure concepts are the communicable aspects of experience, consisting of finite amounts of classical information that can be carried physically through communication channels, whereas qualia are the ineffable qualities of the moments of conscious experience. Then the gist of the paper lies in a mathematical similarity between this interdependence of qualia and pure concepts, on one hand, and, on the other, the interdependence of physical measurements and measurable physical properties as described in [23]. Explicitly, the model of qualia presented here is an example of a measurement space, whose points are abstract physical measurements and whose open sets correspond to measurable physical properties.

The analogy between qualia and measurements adds plausibility to the thesis that the hard problem of consciousness is closely related to the measurement problem of quantum mechanics, and that the solution to both may lie in treating measurements and qualia as primitive notions that require no preexisting “macroscopic entities,” biological or otherwise. Indeed, an important aspect of the space of qualia of this paper is that no prior notion of “conscious agent” is assumed beforehand: qualia (and measurements) do not belong à priori to entities of any kind. This is important because the dependence on a prior existence of such entities is rife with conceptual complications, as pointed out forcefully by Bell in his “attack” against measurements [4], for instance in the following amusing passage:

“It would seem that the theory is exclusively concerned about ‘results of measurement’, and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of ‘measurer’? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system... with a PhD?”

However, despite not being required beforehand in the theory, observers emerge as derived notions. Concretely, following [23], entities such as agents, observers, etc., correspond to certain well behaved subcollections of qualia, and physical mechanisms that enable communication between different such entities need to be in place in order to account for intersubjectivity. So, mathematically, suitable collections of qualia equipped with additional structure for intersubjectivity are the “derived agents.” In a related vein, no underlying classical space, or spacetime, is assumed to exist beforehand, but spaces of “points,” or “states,” emerge from the structure of observers, corresponding to geometric mental models of the world. Moreover, the structure of the qualia space caters for subjective experience associated to such mental models.

Such an emergence of agents and spatial information from qualia space provides a way in which to address the combination problem of panpsychism [7], since it describes the appearance of rich conscious experience from an “everywhere present” proto-consciousness. But it must be kept in mind that the word “everywhere” is not written in relation to a preexisting notion of space, and that, as mentioned above, classical space itself may emerge from the qualia space. This means that, à priori, the theory
of qualia (or measurement) spaces presented in this paper is background independent, which can be argued to be an advantage [25].

The presentation of this paper will be from scratch, trying to motivate the structure of qualia spaces in a way which is as independent as possible from the rationale for measurements of [23]. Although the exposition will mostly be non-technical, the reader may benefit from basic knowledge of topology and lattice theory, for which standard references are [9, 19]. See also [11, 21] for locales and sober spaces, and [23, 24] for measurement spaces.

2 The space of qualia

“Is it possible to introduce a “space of elements of consciousness,” and investigate the possibility that consciousness may exist by itself, even in the absence of matter, just like gravitational waves, excitations of space, may exist in the absence of protons and electrons?”
— Andrei Linde [14]

2.1 The set of qualia

A somewhat vague definition of quale — plural qualia — is that it is the atomic quality of an individual instance of conscious experience. One may be tempted to make this more precise by postulating intrinsic properties of qualia such as ineffability, privacy, etc., but a precise understanding of these terms is no easy task, especially if seeking to adopt a mathematical perspective. Besides, some properties, such as privacy, seem to overload ontology unnecessarily by suggesting that there should be a notion of external conscious agent with respect to which qualia are private.

Alternatively, one may describe patterns or laws that concern the collection of all qualia — meant to be a space whose points correspond to all the possible qualities of instances of subjective experience regardless of how or where they arise. This is a similar route to that often taken in mathematics where it is not the objects per se that are defined but rather the collections of similar objects.

So in what follows a quale does not necessarily pertain to an a priori defined notion of observer, human or otherwise. This issue will be revisited below, where “observers” will be seen to arise as emerging phenomena. To begin let us assume that qualia form a set:

PRINCIPLE 0 (Qualia ensemble). The collection of all the qualia is represented by a set $Q$.

Notice that the definition of qualia, according to Lewis and adopted in this paper, refers to the quality of an instance of experience, rather than the instance itself, so it is by definition repeatable; that is, the same quality of conscious experience, i.e., the same quale, can occur over and over again. This is an important aspect to keep in mind in everything that follows.

2.2 Physically realized concepts

Concepts are about thoughts and the ability to communicate them. Both can be regarded as instances of some form of information processing: either within a single
being’s mind or across different beings. I shall keep in line with the view that information is physical, hence regarding the storage and the processing of information as physical processes in the course of which energy is transmitted. This suggests an interpretation of Lewis’ notion of pure concept according to which concepts are recorded in physical devices (e.g., brain synapses) in response to experiences of a subjective nature, in other words qualia.

Note that concepts are not meant to be themselves qualia; that is, although experiences associated to concepts may exist (this will be addressed in section 4.2), such experiences are to be distinguished from the concepts themselves. To some degree this introduces a separation between two worlds: the physical world of concepts and the mental world of qualia. However, as will be seen momentarily, this separation is more subtle than the presentation so far would have one believe, and in particular it does not lead to dualism.

The physical character of concepts will be translated abstractly to a very simple constraint, namely that the number of experiences and resources (e.g., energy) involved in forming a concept must be finite:

**PRINCIPLE 1** (Concept formation). *Each concept is physically recorded, using finite resources, in response to a finite number of occurrences of qualia.*

In order to obtain an abstract mathematical theory that avoids the nitty-gritty of the physics, chemistry and biology involved in storing and processing concepts, let us simply identify each concept with the set of all the qualia that are associated with it. For instance, the concept *Alice’s RED* will contain all of Alice’s experiences that have some redness about them, such as seeing a glorious sunset, seeing Bob’s face blushing, etc. Similarly, there is a concept *Bob’s RED*, and while talking to each other Alice and Bob may agree that their concepts of red are “the same.” Mathematically, this means that there is a more abstract concept that contains both Alice’s and Bob’s experiences of red:

\[ \text{Alice+Bob’s RED} = \text{Alice’s RED} \cup \text{Bob’s RED}. \]

Now the question arises of which subsets of \( Q \) can be concepts. This is the only point where the finiteness assumptions of Principle 1 appear. Due to such finiteness, Alice cannot always be sure about whether each of her experiences qualifies as being red or not, and one way to convey this uncertainty is to take the concepts to form a topology \( \Omega(Q) \) on \( Q \) (see section 2.4 below for more details). Then, if a quale \( \alpha \) belongs to a concept \( C \) it must be an interior point of \( C \), which is taken as meaning that \( \alpha \) definitely belongs to \( C \), whereas if \( \alpha \notin C \) then \( \alpha \) may be a boundary point and it may be impossible to decide, using finite means, whether or not \( \alpha \) belongs to \( C \).

As a simple example of this idea, let us pretend that \( Q \) is the space \( \mathbb{R} \) of real numbers, and suppose that \( C \) is the open interval \((1, 2) \subset \mathbb{R} \). Suppose also that \( \alpha \) is the point 2 marked with an infinitely sharp pen on an infinitely precise ruler. Unless our eyesight is infinitely sharp it will be impossible, just by looking at the marking, to rule out the possibility that \( \alpha \) may be either interior or exterior to \( C \). We may look closer by halving the distance to the ruler, but again we shall not be able to decide. No matter how often we repeat this procedure the situation will be the same. This means that the problem “\( \alpha \in C? \)” is only semi-decidable: we are able to decide by finite means that it is true precisely in those situations where indeed it is true; that is, when \( 1 < \alpha < 2 \). This idea is reminiscent of work in computer science where open sets
of the space of states of a computer that runs a program) represent semi-decidable properties [26], although important differences exist — beginning with the fact that qualia are not meant to be regarded as states of a system.

Summarizing, Principle 1 is mathematically translated to the existence of a topology \( \Omega(\mathcal{Q}) \) whose open sets are the physically realized concepts.

### 2.3 Interdependence of qualia and concepts

Let us go back to the apparent separation between the mental world of qualia and the physical world of concepts, by means of a toy example featuring baby Alice shortly after she was born and exposed to light for the first time. For the sake of the argument let us assume that all that she initially experienced was brightness, without any associated experience of color, but that after additional exposure to light flashes her brain synapses changed and because of this she started to notice that there are different colors.

The modified synaptic structures encode information about what for Alice is red, blue, etc., so they are examples of physically realized concepts which formed in response to visual stimuli that initially did not carry color qualia. So this toy example describes how Alice’s subjective experience of color was enabled by the formation of her color concepts, hence reversing the causal role that is suggested by Principle 1. In other words, it is also true that qualia arise only when corresponding concepts are in place:

**PRINCIPLE 2** (Dependent origination). *Qualia cannot arise except in relation to concepts.*

According to principles 1 and 2 neither qualia nor concepts are regarded as independently existing entities, but rather as entities which arise interdependently like two facets of the same reality — subjective experience entails physical reactions that lead to the formation of concepts but, also, the formation of concepts modifies experience. This justifies the claim made above to the effect that the model being developed here is not dualistic. Indeed, it is more appropriately placed in the context of neutral monism or dual-aspect monism [28], and it also seems to be consistent with central tenets of buddhist philosophy [17], from which the expression “dependent origination” is borrowed.

Mathematically, Principle 2 implies that topologically equivalent qualia (i.e., qualia that belong exactly to the same open sets) have no physical grounds on which to be regarded as different subjective experiences, and thus should be identified. Hence, from here on \( \mathcal{Q} \) will be taken to be a \( T_0 \)-space.

### 2.4 The logic of concepts

The concept topology \( \Omega(\mathcal{Q}) \) can be regarded as being a logic of finitely observable properties, in a sense which arose in computer science [31]. In order to see this, let \( \alpha \in \mathcal{Q} \) be a quale, and let \( U, V \) be concepts such that \( \alpha \in U \cap V \). For instance, let \( \alpha \) correspond to the subjective experience that arises when Alice sees a certain flash of light. Let also \( U \) and \( V \) be Alice’s concepts of red and bright, respectively. Suppose that upon experiencing \( \alpha \) Alice concludes that she saw red light, and that upon experiencing \( \alpha \) a second time Alice realizes that the light is bright. The first realization establishes that \( \alpha \) “has” the property \( U \) (the light is red), and the second realization establishes the property \( V \) (the light is bright). Both realizations taken
together constitute a verification of the conjunction \( U \cap V \), which is done in finite time and consuming finite resources.

Similarly, the disjunction of \( U \) and \( V \) is conveyed by the union \( U \cup V \), and in order to verify the disjunction it suffices to verify one of the disjuncts: if \( \alpha \in U \) or \( \alpha \in V \) then \( \alpha \in U \cup V \). In fact, this applies to disjunctions of arbitrary arity, for in order to verify the inclusion \( \alpha \in \bigcup_i U_i \) it suffices to verify \( \alpha \in U_i \) for some \( i \in I \). So, provided the verification of each \( U_i \) consumes finite time and resources, so does the verification of \( \bigcup_{i \in I} U_i \).

This discussion shows that the topology \( \Omega(\mathcal{Q}) \) has a logical interpretation that is concretely grounded in the idea that concepts are partial aspects of qualia that can be apprehended by finite means, and that the logical connectives are interpreted as follows:

- The intersection \( U \cap V \) is the conjunction of the concepts \( U \) and \( V \);
- The disjunction \( \bigcup_{i \in I} U_i \) is the disjunction of the concepts \( U_i \) with \( i \in I \);
- \( \mathcal{Q} \) is the greatest concept, corresponding to the truth value “true”;
- \( \emptyset \) is the least concept, corresponding to the truth value “false” — it coincides with the empty disjunction \( \bigcup_{i \in \emptyset} U_i \).

Note that the conjunctions can be iterated but are always finitary, of the form

\[
(\cdots ((U_1 \cap U_2) \cap U_3) \cap \ldots) \cap U_k = U_1 \cap \ldots \cap U_k,
\]

where \( k \geq 0 \) — if \( k = 0 \) the conjunction coincides with \( \mathcal{Q} \), if \( k = 1 \) then the conjunction is just \( U_1 \). Therefore there is an asymmetry with respect to the disjunctions, which are of arbitrary arity.

As usual in propositional logics, models can be defined in terms of valuations, which consist of assignments of truth values to the “propositions” in a way that respects the logical connectives:

**Definition.** By a logical valuation of \( \Omega(\mathcal{Q}) \) is meant a mapping

\[
v : \Omega(\mathcal{Q}) \to \{0, 1\}
\]

such that for all concepts \( U, V, U_i, \ldots \) we have

- \( v(\mathcal{Q}) = 1 \),
- \( v(U \cap V) = 1 \) if and only if \( v(U) = 1 \) and \( v(V) = 1 \),
- \( v(\bigcup_{i \in I} U_i) = 1 \) if and only if \( v(U_i) = 1 \) for some \( i \in I \).

(Therefore \( v(\emptyset) = 0 \).)

**Remark.** In the language of locale theory, the topology \( \Omega(\mathcal{Q}) \) is a locale and the valuations are the points of the locale.

For each quale \( \alpha \) the mapping

\[
v_\alpha : \Omega(\mathcal{Q}) \to \{0, 1\}
\]
which is defined by

\[ v_\alpha(U) = 1 \iff \alpha \in U \]

is a valuation, and the Principle of Dependent Origination (i.e., \( Q \) is \( T_0 \)) is equivalent to the statement that the assignment \( \alpha \mapsto v_\alpha \) is injective:

\[ v_\alpha = v_\beta \Rightarrow \alpha = \beta. \]

This shows that there is a close relation between qualia and the logical valuations of \( \Omega(Q) \). Indeed, each quale \( \alpha \) can be identified with the set of concepts

\[ N_\alpha = \{ U \in \Omega(Q) \mid \alpha \in U \}, \]

so we can regard \( \alpha \) as being a kind of infinitary conjunction of concepts, namely those for which \( v_\alpha(U) \) is “true.” Hence, the concepts \( U \) such that \( \alpha \in U \) can be regarded as physically recordable snapshots of the quale \( \alpha \) but, equally, the whole collection of such concepts contains everything there is to say about \( \alpha \).

Therefore, the only way in which the set of valuations may differ from the set of qualia is that there may be valuations \( v \) which are not of the form \( v_\alpha \) for any \( \alpha \in Q \). How are we to interpret such valuations? According to the above discussion, they can be regarded as being “quasi-qualia” whose existence is derived from the logic of concepts (albeit in general transfinitely) by taking “conjunctions” of concepts in all the ways that are consistent with the logic. Therefore, if there is a valuation \( v \) which is not of the form \( v_\alpha \), the conclusion is that the logic of concepts is telling us a quale should exist where in fact no corresponding subjective experience can be found. This ultimately renders the logic irrational, which is something that should be avoided. So let us require every valuation \( v \) to be of the form \( v_\alpha \) for some quale \( \alpha \), and record the requirement as follows:

**PRINCIPLE 3** (Rationality). *Qualia arise in relation to concepts whenever this is logically consistent.*

Mathematically, together with Principle 2, which has already established that \( Q \) is a \( T_0 \)-space, Principle 3 asserts that \( Q \) is a sober topological space.

3 Operations on qualia

Mathematically, the content of the previous section can be summarized by saying that the space \( Q \) of qualia is a sober topological space whose open sets are the physically realizable concepts. This is perhaps best viewed as a mathematical description of proto-consciousness: it conveys a basic structural relation between conscious experience and physical phenomena but, in order to describe rich conscious experience of the kind usually associated to biological organisms such as human beings, additional mathematical structure is needed for reflecting the cognitive abilities which are involved in such rich experiences. There are various possibilities for this, and in this section we address two: *sequential compositions* of qualia, which account for the subjective experience of (linear) time; and *disjunctions* of qualia, which relate to the subjective experience associated to abstract logic, in particular to the experience of realizing that something is more general than something else. Both operations rely, each in its own way, on the assumption that qualia can be repeated. In addition to these also an *involution* will be described: this plays the role of a formal time reversal which, albeit being less intuitive from the point of view of qualia, is mathematically convenient and makes \( Q \) a space of measurements in the sense of [23].
3.1 Subjective time

Let us define algebraic structure on \( Q \) that concerns the subjective experience of time. Although there are several possibilities for this, let us consider only a binary operation of \textit{sequential composition} that expresses a subjective experience of \textit{linear time}.

As an example, let \textcolor{red}{red} and \textcolor{blue}{blue} be qualia that correspond to the experiences associated with observing pulses of red and blue light, respectively. For an organism possessing a minimal amount of cognitive ability, such as at least a tiny amount of memory, experiencing \textcolor{red}{red} followed by \textcolor{blue}{blue} may yield an AHA! moment associated to the realization that there were two things happening, one after the other. In other words, there may be a quale \textcolor{red}{red} and then \textcolor{blue}{blue}, which I shall denote by \textcolor{red}{red} & \textcolor{blue}{blue}:

**PRINCIPLE 4** (Experience of repetition). \textit{The experience of having two consecutive experiences exists.}

Regardless of what our favorite definition of objective time may be, the amount of it spent between experiences is deemed irrelevant: all that \textcolor{red}{red} & \textcolor{blue}{blue} stands for is the experience of \textcolor{red}{red} and \textcolor{blue}{blue} occurring one immediately after the other, where the sense in which “immediately” is being used is that nothing else is experienced between \textcolor{red}{red} and \textcolor{blue}{blue}. For instance, a man undergoing surgery may remember the last conscious moment before anesthesia as if it immediately preceded the slap on the face that is meant to wake him up, regardless of the fact that a couple of hours elapsed between those two events. The point is that no subjective time existed between both qualia. In other words, subjective time may be regarded as a measure of how many qualia are occurring. Hence, if \( \alpha, \beta, \) and \( \gamma \) are qualia, we make no distinction between \((\alpha \& \beta) \& \gamma \) and \( \alpha \& (\beta \& \gamma) \): both stand for the experience \( \alpha \) followed by \( \beta \) and then \( \gamma \).

The operation \((\alpha, \beta) \mapsto \alpha \& \beta\) will be called \textit{composition} or \textit{multiplication} of qualia. Of course there is in principle no reason to assume that every pair of qualia should be composable. For instance, if \( \alpha \) corresponds to the experience associated to a neural activation pattern in Alice’s brain, and \( \beta \) to a neural activation pattern in Bob’s brain, the experience that should correspond to \( \alpha \& \beta \) may simply not exist at all. This could be dealt with by making composition a partial operation, similar to the composition in a category \cite{EGA}, but instead we shall consider that the multiplication is a total operation and that, in addition, \( Q \) contains a special \textit{impossible quale} \( 0 \) such that the equation \( \alpha \& \beta = 0 \) means that the experience of \( \alpha \) and then \( \beta \) does not exist. Of course, the impossible quale does not represent an actual quale, so it should be regarded simply as a useful mathematical construct. Naturally, we shall assume that the absorption laws \( \alpha \& 0 = 0 \& \alpha = 0 \) hold, so \( Q \) is a \textit{semigroup with zero}.

It will be assumed that the multiplication of qualia is a continuous operation with respect to the topology \( \Omega(Q) \), so \( Q \) is a topological semigroup with zero. Noting that the largest open set, \( Q \), is the trivial concept that corresponds to no information at all, and since \( 0 \) is the impossible quale, it will also be imposed that \( Q \) is the only concept that contains \( 0 \).

In light of the above discussion, the multiplication of \( Q \) provides us with a very simple and abstract model of subjective time. I shall assume nothing concerning whether this should correspond to a fundamental structure in nature or instead just something that applies to a class, no matter how large, of sentient organisms. But it is worth remarking that at least our assumptions are minimal to the extent that a very large collection of organisms may be encompassed — certainly not just human beings.
3.2 Disjunctions of qualia

The type of operation that we shall study now is closely linked to the topological properties of $Q$ and to the idea that some qualia are more specific than others. For instance, the experience of just seeing a flash of light is less specific than the experience of seeing a flash of red light, or a flash of intense red light. This specificity relation can be identified with the specialization order of the topological space $Q$: we say that $\alpha$ is more specific than $\beta$, and write

$$\alpha \leq \beta,$$

if for all concepts $C$ the condition $\alpha \in C$ implies $\beta \in C$. In other words, every concept which can be triggered by $\alpha$ can also be triggered by $\beta$. The reason we consider $\alpha$ to be more specific is that fewer potential concepts can be associated with it, whereas $\beta$ has more associated concepts, and thus is less determined.

Letting $\alpha$ and $\beta$ be qualia, we denote by $\alpha \lor \beta$ a quale (should one exist) which is less specific than both $\alpha$ and $\beta$, but more specific than any other quale $\gamma$ which is less specific than both $\alpha$ and $\beta$ (this means that $\alpha \lor \beta$ is the supremum of $\alpha$ and $\beta$ in the specialization order of $Q$).

While it might not be immediately obvious what $\alpha \lor \beta$ means for a particular pair of qualia $\alpha$ and $\beta$, there are many situations where the meaning is clear. And there is a canonical interpretation of $\alpha \lor \beta$ in terms of the idea of repetition of qualia, provided a modicum of cognitive ability exists that enables an experiencer to compare different occurrences of the “same” situation. For instance let red and blue be qualia corresponding to the experience of watching a flash of red light and a flash of blue light, respectively, and let there be a flashlight that randomly produces red light or blue light each time we press a button. After pressing the button a few times, the realization that either red or blue will be seen is another AHA! moment that we interpret as a more abstract quale which can be read “either red or blue,” written red $\lor$ blue. Hence, we refer to $\alpha \lor \beta$ as the disjunction of $\alpha$ and $\beta$.

Remark. There is a subtlety here, namely that it may be impossible to produce an abstraction of, say, red and blue, which does not also include other colors, e.g., Alice’s experience of “either red or blue” could well coincide with her experience of “either as red or green”. Whether or not this is the case depends on the processing power that surrounds the occurrence of these qualia (e.g., the cognitive capabilities of a specific organism), but it may also depend on fundamental physical limitations, as explained in [23] in the context of physical measurements.

The canonical interpretation of disjunction as abstraction-after-repetition suggests that disjunctions should relate to the topology as follows: any concept $C$ such that $\alpha \lor \beta \in C$ can be obtained through repeated occurrences of either $\alpha$ or $\beta$ provided that there is an occurrence of $\alpha$ that triggers a concept $U$ and an occurrence of $\beta$ that triggers a concept $V$ such that the conjunction of concepts $U \cap V$ “implies” $C$; that is $U \cap V \subset C$.

From here on we shall assume that a disjunction $\alpha \lor \beta$ exists for any pair of qualia $\alpha$ and $\beta$:

**PRINCIPLE 5 (Disjunction of experiences).** The experience of having either of two other experiences exists.

This principle has two consequences. The first is that $\alpha \lor \beta$ is indeed the supremum in the topological order of $Q$, hence disjunctions are uniquely defined and they are associative, commutative and idempotent; that is, for all $\alpha, \beta, \gamma \in Q$ we have
• $(\alpha \lor \beta) \lor \gamma = \alpha \lor (\beta \lor \gamma)$,
• $\alpha \lor \beta = \beta \lor \alpha$ and
• $\alpha \lor \alpha = \alpha$.

The second consequence is that the operation $\lor : Q \times Q \rightarrow Q$ is continuous with respect to the topology $\Omega(Q)$. An explanation of this can be found in [23][24], in the context of physical measurements.

To conclude, the following algebraic laws will be assumed to hold:

• $\alpha \lor 0 = \alpha$ for all $\alpha \in Q$ (this means that the impossible quale 0 is the least element in the specialization order of Q);
• $\alpha \& (\beta \lor \gamma) = (\alpha \& \beta) \lor (\alpha \& \gamma)$ and $(\beta \lor \gamma) \& \alpha = (\beta \& \alpha) \lor (\gamma \& \alpha)$ for all $\alpha, \beta, \gamma \in Q$ (distributivity).

The first law is easy to understand, since the disjunction of $\alpha$ and something impossible can be nothing but $\alpha$ itself. As for distributivity, reading $\beta \lor \gamma$ as “$\beta$ or $\gamma$” makes it natural to identify $\alpha$ followed by $\beta \lor \gamma$ with the disjunction “$\alpha \& \beta$ or $\alpha \& \gamma$,” and similarly for multiplication by $\alpha$ on the right.

### 3.3 Involution

Let us write $\alpha^*$ for a time-reversed occurrence of $\alpha$; that is, $\alpha^*$ is, somehow, “similar” to $\alpha$ but with time flowing backwards. Then the following three laws should hold for all $\alpha, \beta \in Q$:

• $\alpha^{**} = \alpha$,
• $(\alpha \& \beta)^* = \beta^* \& \alpha^*$,
• $(\alpha \lor \beta)^* = \alpha^* \lor \beta^*$.

Such an operation is called an involution in semigroup theory. In addition to these algebraic laws we should require the involution to be compatible with the concept topology, so we assume that it is continuous.

It is less easy to justify the existence of an involution in terms of qualia than it is to justify the multiplication and the disjunction. For instance, a quale such as $\alpha \& \alpha^*$ is to be interpreted as $\alpha$ followed by its own time-reversal. For some qualia a possible interpretation of this could be that somehow $\alpha^*$ is able to “undo” $\alpha$. As an example, consider $\alpha$ to be a quale associated to an emotion for which there is an “antidote” $\alpha^*$ that neutralizes it — perhaps $\alpha$ could be anger and $\alpha^*$ could be love. This is of course a naive suggestion, and surely not every quale can be undone in such a way, but this operation will be added for mathematical convenience, and also in order to make Q a measurement space in the sense of [23].

A more precise account of when it is that $\alpha^*$ “undoes” $\alpha$ can then be formulated by imposing $\alpha \& \alpha^* \& \alpha = \alpha$ as a necessary condition. In order to avoid situations of “pseudo-reversibility” it will also be imposed that this equation must hold whenever the inequality $\alpha \& \alpha^* \& \alpha \leq \alpha$ holds.
3.4 Qualia as measurements

The structure and axioms which we have postulated so far can be summarized as follows:

**THESIS.** The space of qualia $Q$ is a measurement space in the sense of [23].

Therefore there is a mathematical analogy between physical measurements and qualia. But the aim of this paper is to convey that the analogy is deeper than just superficial mathematical pattern-matching. Indeed, in a measurement space the open sets are the observable physical properties, namely those which can be recorded in physical devices, and this is analogous to the interpretation of concepts in the present paper, since concepts are taken to be finite portions of classical information that are stored physically, for instance in brain synapses. And the role of measurements with respect to physical properties is entirely analogous to the role of qualia with respect to concepts. While qualia are the “generators” of concepts, measurements are the “generators” of physical properties. But, if indeed the physical character of a concept is the same as that of a physical property, it follows that qualia and measurements should also amount to the same thing. In other words, it is suggestive to think that, at a fundamental level, the distinctive feature of a procedure which performs a measurement of something is that ultimately qualia are involved, and, conversely, that the occurrence of qualia has the effect of a physical measurement on something:

**THESIS.** The physical processes that constitute measurements are precisely those in the course of which qualia occur, and, conversely, each quale is a measurement.

In this sense, no measurement exists unless it is subjectively experienced, albeit this is taken in a fundamental sense which does not rely on preexisting systems, observers, etc. Note also that the proposed identification of measurements with qualia is prior to any model of quantum theory and, in particular, prior to any notion of “collapse” of a wave function, so it is also prior to any discussion concerning whether or not wave functions are real entities, and prior to any arguments concerning whether consciousness causes collapse (as originally suggested by von Neumann [32]) or, conversely, collapse causes consciousness as in [10].

**Remark.** An interesting consequence of this discussion is that, whereas viewing physical measurements as fundamental may still lead one to thinking of measurements as being physical processes “out there” (as usually one does in physics), in the case of qualia one discusses the qualities of instances of subjective experience, which are usually conceived as being “internal.” However, the sense in which the word “internal” is to be taken presupposes the existence of some entity, which however is not postulated anywhere in the definition of $Q$. So, both in the theory of measurement spaces and the theory of qualia presented here, there is no separation between “internal” and “out there.”

3.5 Digression

There is more mathematical structure in $Q$ than meets the eye, as the topological properties together with those of the disjunction imply that the specialization order of $Q$ has (continuous) suprema of arbitrary subsets, and both the multiplication and the involution preserve arbitrary suprema in their variables. In other words, $Q$ is an involutive quantale in the sense of [13], in fact a stably Gelfand quantale due to the specific
conditions imposed on the involution \[22\]. This implies that, despite the seemingly minimal structure of \(Q\), quite a few interesting consequences follow from it, some of which directly relevant to the question of what an “observer” is. This will be hinted at again further below. For further mathematical details see \[23, 24\].

An additional type of structure that begs to be considered would take into account, again by repeating qualia \(\alpha\) and \(\beta\) multiple times, not just the subjective experience of logic abstraction conveyed by the disjunction of \(\alpha \lor \beta\), but also the experience of frequency of occurrence of \(\alpha\) relative to \(\beta\). This relates to the emergence of a subjective experience of probability rather than logic, but a mathematical treatment of this in the context of measurement spaces is yet to be developed.

Finally, note that also the concept of subjective time employed here is very simple, since it is purely sequential, but more complex approaches to time and causality have been proposed in \[23\], in particular with the aim of making measurement spaces compatible with relativity theory. However, it is interesting that much can be derived, in particular as regards observers, just from the minimal structure that we have placed on \(Q\), as we will see in the next section.

4 Observers and intersubjectivity

We have already insisted several times that the notion of qualia in this paper is independent of preexisting entities that “possess” the qualia. In this section we address how, on the contrary, entities such as observers can in fact emerge from the structure of \(Q\).

4.1 The emergence of space

In a pragmatic sense, one refers to Alice’s qualia as those which arise in relation to the functioning of her body, especially her nervous system. This distinguishes such qualia from those of Bob, which arise in relation to events in Bob’s body. So it is natural to regard “Alice’s consciousness” as the “sum” off all “her” qualia. How does one measure such consciousness? Integrated Information Theory, for instance, approaches this via suitable notions of complexity which are based on the structure of each entity whose consciousness we want to quantify \[20, 30\]. In doing so, there are no a priori restrictions regarding the type of physical structure that may be taken into account. In particular, such structure may depend on the notions of time and space that are available in current physics.

However, the perspective of this paper suggests that we try to use as a starting point for any model of a “conscious entity” the very same laws that govern qualia in general, by which we mean the mathematical properties of the space \(Q\), for these are meant to provide an already watered-down formulation of very general and abstract cognitive capabilities. Any convincing description of “entities” in these terms can be regarded as indirect evidence in favor of the mathematical approach to qualia via measurement spaces. In particular, noting that no preexisting notion of physical space has been assumed in the definition of \(Q\), but rather only the primitive notion of time given by multiplication, it is especially interesting to see how spaces that resemble physical space may actually emerge from the structure of \(Q\), so let us examine this.

Let \(\mathcal{A} \subset Q\) be the set of all of Alice’s qualia (in the pragmatic sense described above). Assuming that Alice’s cognitive capabilities are rich enough for her to be able
to experience multiplication and disjunctions of qualia, \( A \) should be closed under these operations; that is, for all \( \alpha, \beta \in A \) we should have \( \alpha \& \beta \in A \) and \( \alpha \vee \beta \in A \) (and \( 0 \in A \)), and we shall also take \( A \) to be closed under involution, so \( \alpha^* \in A \) for all \( \alpha \in A \). Adding to this, we want the principle of rationality to apply to Alice’s qualia, so \( A \), with the subspace topology inherited from \( Q \), should be a sober space. Then, from the properties of measurement spaces we obtain:

**Lemma.** \( A \) is a measurement space.

Now if Alice were a “sentient elementary particle” we would probably stop here, but entities such as Alice and Bob are macroscopic, and their subjective perceptions seem to be governed by the laws of classical logic and classical physics (so much so that it is hard for them to understand intuitively what quantum mechanics is really about). So let us make a crucial assumption: the qualia experienced by Alice due to interaction with the world, including those that are derived from the information processing done by her own brain, obey the laws of classical physics and classical computing. Concretely, we obtain the following conclusions, whose justification can be found in [23] for measurement spaces:

- There should be no quantum interference effects, which, it can be argued, translates to imposing that \( A \) is a distributive lattice — that is, for all \( \alpha, \beta, \gamma \in A \) we must have \( \alpha \land (\beta \lor \gamma) = (\alpha \land \beta) \lor (\alpha \land \gamma) \);
- \( A \) should have the topology that spaces of data types and computable functions have — that is, it should be a countably-based continuous lattice equipped with the Scott topology. See [29, Ch. VI].

The effect of these two conditions can be summarized as follows:

**Theorem.** \( A \) is a measurement space equipped with the Scott topology, and there is a (unique up to homeomorphism) second-countable locally compact sober topological space \( X \) whose topology \( \Omega(X) \) is order-isomorphic to \( A \).

This means that, even though no underlying notion of space has been assumed beforehand when defining \( Q \), from relatively simple logical and computational axioms it has followed that the qualia in \( A \) can be regarded as being open sets of a topological space \( X \) which, moreover, is fairly well behaved and (ignoring that it does not have to be Hausdorff) is close to the typical kind of spaces found in geometry and analysis. While in the context of measurement spaces this suggests an explanation of why physical systems look like they have “internal states,” in the context of qualia it is suggestive to think that such a structure of \( A \) is the reason why Alice has a geometric mental model of the world.

**Definition.** [23] Such a measurement space \( A \) (after we add a law stating that \( \alpha \leq \alpha \& \alpha^* \& \alpha \) for all \( \alpha \in A \)) is called a *classical measurement space*.

**Example.** Let us give a simple example. Suppose Alice looks at a wall. Any photon that is reflected by the wall and hits Alice’s retina carries information about a region of the wall, no matter how small, but never about precise points of the wall. So the visual qualia are more related to the topology of the wall than to the points of the wall. Nevertheless, when human beings look at a wall they seem to form a mental picture of a surface with points. It is reasonable to conclude that such a mental picture would
not exist were it not for processing done by the brain, and that the mental picture of
the wall is no more real than the space \( X \) in relation to \( A \) (but this issue will be
revisited in the next section). More elaborate examples can be obtained from locally
compact groupoids and their \( C^* \)-algebras \([24]\).

4.2 Experiencing concepts

The principles laid out in section 2 make qualia and concepts interdependent: on
one hand concepts are formed in response to qualia but, on the other hand, concepts
enable the existence of qualia. We can take the latter one step further and postulate
that concepts have qualia associated to them. Let us examine this idea.

Let \( A \subset Q \) be Alice’s space of qualia. The subspace topology \( \Omega(A) \) contains Alice’s
concepts (these are the intersections \( A \cap C \) for all concepts \( C \in \Omega(Q) \)), so let us
postulate that Alice’s subjective experience of her own concepts can be described by a
map

\[
\Phi : \Omega(A) \to A.
\]

It is natural to postulate that \( \Phi \) should respect the logic of concepts described in
section 2 by which is meant that \( \Phi \) should be a homomorphism of locales; that is,
for all \( U, V, U_i \in \Omega(A) \) we must have

- \( \Phi(A) = 1_A \),
- \( \Phi(U \cap V) = \Phi(U) \land \Phi(V) \),
- \( \Phi(\bigcup U_i) = \bigvee_i \Phi(U_i) \).

**Example.** A canonical definition of such a homomorphism \( \Phi \), which can be proved
to work because \( A \) is a locale and the open sets are upper-closed in the specialization
order of \( A \), is to define for each open set \( U \subset A \):

\[
\Phi(U) = \bigvee \{ \alpha \in A \mid \exists \beta \in U \alpha \land \beta = 0 \}.
\]

For the sake of the example, let us suppose that \( U \) is the concept Alice’s RED
of section 2.2. One way to interpret \( \Phi(U) \) is by noticing that the conditions \( \alpha \land \beta = 0 \)
and \( \beta \in U \) mean that \( \alpha \) is “completely disjoint” from a “red quale” \( \beta \). So \( \Phi(U) \) is a
disjunction whose associated subjective experience is that of experiencing something
which is distinguishable from at least one of Alice’s red experiences. It is the quality of
an AHA! moment in which Alice realizes there is a clear distinction between something
(corresponding to \( \alpha \)) and something else (corresponding to \( \beta \)) that she
knows is red (even if \( \alpha \) may happen to be a red quale, too). One way to phrase this is therefore to
say that \( \Phi \) has to do with knowledge: \( \Phi(U) \) is the most general subjective experience
that arises from confronting each new experience with Alice’s knowledge of the world
as expressed through the concept \( U \).

Let \( X \) be the sober space (unique up to homeomorphism) such that \( A \cong \Omega(X) \).
Locale theory tells us that one consequence of having a homomorphism of locales \( \Phi : \Omega(A) \to A \) is, since \( A \) is sober, that there exists a unique continuous map \( \phi : X \to A \)
such that the composition

\[
\Omega(A) \xrightarrow{\Phi} A \xrightarrow{\cong} \Omega(X)
\]
equals the inverse image homomorphism $\phi^{-1}$. This gives us a mathematical description of how, despite the fact that the points of $X$ seem to be nothing but a figment of Alice’s imagination, there are nevertheless qualia associated to them, given by the continuous map $\phi$.

Going back to the wall example at the end of section 4.1, where we discussed the qualia that correspond to the subjective experience of looking at a wall in terms of small regions of it (open sets), $\phi$ provides a possible description of subjective experience associated to the mental representation of the wall in terms of points. In hindsight the existence of this map is not surprising, for it accords with our pedestrian feeling that surfaces do have points, regardless of how little physical sense that makes.

### 4.3 Intersubjectivity

Let us look at intersubjectivity in the qualia space $Q$; that is, at mechanisms via which a shared reality among different observers may be formed. For this let again $A \subset Q$ be the qualia space associated to Alice, and let $B \subset Q$ be the qualia space associated to Bob.

Let us assume that Alice wants to know more about how Bob sees the world. As a first step, she will try to learn how Bob describes colors. She picks up a few flashlights of various colors, which she labels “red,” “blue,” etc., and points them at a white target near Bob’s location. When Bob sees each flash some color qualia occur, correlated with his brain activity. So for each of Alice’s color qualia she will activate the corresponding flashlight, following which Bob experiences color qualia himself. We can abstract away the details of this procedure simply saying that there is a function that translates Alice’s color qualia to Bob’s color qualia. As an idealization, let us assume that similar procedures can be carried out for all of Alice’s qualia, so we obtain a function $F : A \rightarrow B$.

But $F$ is just some physical channel through which Alice’s experiences are mapped to Bob’s experiences, hopefully in as accurate a manner as possible, and this in itself does not help Alice increase her knowledge about Bob’s view of the world. This is where concepts come into the picture, in accordance with Lewis’ notion of pure concept as “that meaning which must be common to two minds when they understand each other by the use of a substantive or its equivalent.” After Alice mapped her qualia to Bob’s via the function $F$, Bob replies telling Alice what concept each quale has triggered for him. For instance, upon experiencing a red quale, Bob, being belgian, will shout “rood!” to Alice, upon which she learns that “rood” may be the flemish word that corresponds to her concept “red.”

In general, the activation of each one of Bob’s concepts will, after being communicated to Alice, translate to one of Alice’s concepts, even if that concept merely corresponds to the knowledge “when I press this button Bob reacts in a particular way.” We can formalize this by imposing that the pre-image under $F$ of each concept $C \in \Omega(B)$ is a concept $F^{-1}(C) \in \Omega(A)$; that is, we impose that $F$ is continuous.

As generalization of this, we may consider an idealized situation in which there is a continuous map $F_B : Q \rightarrow B$ that translates every quale in $Q$ into Bob’s qualia. The existence of such a continuous map is at the basis of the definition of observer in measurement spaces [23], of which good examples based on groupoids and C*-algebras are described in [24]. Concretely, an observer context of $Q$ is defined to consist of a pair $(O, F_O)$ where $O \subset Q$ is a classical measurement space with the same operations of $Q$ and with the subspace topology, and $F_O : Q \rightarrow O$ is a continuous map that satisfies
the following properties:

- $F_O(\omega) = \omega$ for all $\omega \in O$,
- $F_O(\omega \& \alpha) = \omega \& F_O(\alpha)$ for all $\omega \in O$ and $\alpha \in Q$,
- $F_O(\alpha^*) = F_O(\alpha)^*$,
- $F_O(\alpha \lor \beta) = F_O(\alpha) \lor F_O(\beta)$.

The rationale behind these properties, in terms of measurements, is given in [23], and it is readily translated to the language of qualia.

Recall the spatial representation of $A$ and $B$ as topologies, letting $X$ and $Y$ be the unique sober spaces such that $A \cong \Omega(X)$ and $B \cong \Omega(Y)$. The restriction of $F_O$ to $A$ determines a suprema preserving map $F : A \to B$ which does not in general determine a continuous function $f : Y \to X$ such that $f^{-1} = F$ but rather a continuous function $f : Y \to C(X)$ (where $C(X)$ is the space of closed sets of $X$ with the lower Vietoris topology) [23]. Hence, each one of Bob’s points is represented by a set of Alice’s points.

Therefore Bob’s mental representation of $B$ in terms of the points of $Y$ does not translate precisely to Alice’s mental representation of $A$ in terms of the points of $X$ but rather to an “unfocused” representation. This phenomenon mimics the way in which each basis vector of a vector space is represented as a linear combination of basis vectors from a different basis, except that there are no numerical coefficients involved: each “vector” either belongs or it does not belong to a combination. The conclusion is that, solely due to the mathematical structure of $Q$, the viewpoints carried by different observers are not necessarily compatible. This phenomenon, albeit here in logical (rather than probabilistic) form, resembles the properties of bases of eigenvectors that correspond to pairs of noncommuting observables in quantum mechanics.

5 Discussion

The analogy with the measurement spaces of [23] suggests that qualia can be regarded as “seeds” of classical information: each quale is identified with the quality of an instance of production of classical information (cf. section 3.4). This concurs with the fundamental view of information as being the source of reality in the sense of Wheeler’s “it from bit” [33]. Therefore an essential question is: under what conditions is classical information produced? Or, in other words, what does it take to perform a measurement? If such information producing moments occur in insects or plants, or in artificial intelligence systems, then there will be a proto-consciousness associated to such entities in the same way that one exists for humans and other animals — the interesting issue is to identify where in the structure and behavior of each such entity the creation of information resides. In other words, the hard problem of consciousness can be identified with the physical problem of explaining the creation of classical information.

In terms of the standard formalism of quantum mechanics this brings us back to putative relations between consciousness and the collapse of the wave function, albeit not in the sense of von Neumann [32] but closer to the sense of Hameroff and Penrose [10], who see each collapse as the origination of a moment of consciousness. However, whereas the latter approach falls into the class of objective collapse models, which try to reduce collapse to more fundamental principles (see also [3]), and therefore would also reduce consciousness to more fundamental principles, measurement spaces
are agnostic with respect to the possibility of reductionism, instead taking measurements (and qualia) to be primitive processes whose mathematical structure needs to be investigated from scratch. The production of each new bit of information may even be regarded as an “elementary conscious choice” (between a 0 and a 1), and qualia may be closely related to free will in the sense of Conway and Kochen [8]. While this approach may eliminate conceptual difficulties of quantum mechanics, the extent to which such a view can provide a foundation for physics depends at least on whether standard quantum theory can be reconstructed in this way. For that it is necessary, among other things, to add measure-theoretic structure to the model, which in the qualia space \( Q \) amounts to taking into account subjective experience associated to probabilities (cf. section 3.5).

A second aspect worth discussing is that the type of structure which is carried by qualia spaces is considerably abstract, and certainly far-removed from biology and neuroscience. It is clear that such structure presupposes the existence of some information processing capabilities, for instance a modicum of memory. If we are to accept that a given biological organism, or an artificial intelligence, can have qualia associated to it, and if its information processing abilities can reproduce the type of mathematical structure described in this paper, then fairly rich conscious experiences will be associated to such entities insofar as such experiences can be explained entirely in terms of the structure of the space \( Q \).

Concretely, there are four directions which would benefit from being examined by neuroscientists, plant biologists, etc., regarding any entity, or group of entities, whose possibility of being conscious one would like to assess in terms of the mathematical properties of \( Q \): (i) Does the entity have the capability of experiencing repetition, so that the multiplication of qualia is defined and subjective time is experienced? (ii) Does the entity have the capability of experiencing logical abstraction and disjunctions of qualia? (iii) Is the entity able to sense its own concepts in the way illustrated in section 4.2? (iv) Does the entity, and others like it, possess the communication skills needed for a communication protocol as described in section 4.3, so that intersubjectivity may emerge?

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