The flavor distribution of Cosmic Neutrinos†.

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Abstract

Simple approximate expressions for the relative flavor fluxes of energetic cosmic neutrinos detected on Earth, are presented in terms of their initial fluxes at the surface of the producing cosmic sites, assuming the neutrino mixing angles to lie in the experimentally favored region. These expressions highlight the main sensitivities to the initial production fluxes as well as to small variations of the mixing angles within the experimentally preferred region, thus providing simple methods to disentangle these physical quantities from cosmic neutrino data. This is more so, due to some striking mathematical properties of the relative neutrino flavors at \((s_{23} = 1/\sqrt{2}, \ s_{13} = 0)\), which somehow characterize the whole experimentally preferred region. We also assess the quality of our approximate expressions through a numerical comparison with the exact results.

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The relative flavors of energetic Neutrinos reaching the Earth, after they have been emitted at various cosmic sites, provide useful information on the physical conditions there. Such very energetic neutrinos, approaching the $10^3$ TeV \[1\], or even the $10^6$ TeV scale \[2, 3, 4, 5\], may be generated in extra-galactic sites like Gamma Ray Bursts and Active Galactic Nuclei (AGN). Galactic candidates that may emit neutrinos of up to 100 TeV have also been identified at distances of at least 2.6 kpc \[6, 5\]. [Exploding galactic Supernovae may also induce observable neutrino fluxes with energies in the few MeV range \[7\].]

It is commonly believed that these neutrinos are produced mainly through the decay of high energy $\pi^\pm$ and K (and may be also some D) mesons, which implies that the initial relative neutrino flavors at the cosmic sites satisfy

$$F^0_e = 1/3 , \ F^0_\mu = 2/3 , \ F^0_\tau = 0 ,$$

\[3, 4, 2, 8\]. This case is referred to below, as the **canonical case**. It may be useful to remember though, that our present understanding of the mechanisms for generating high energy neutrinos is rather primitive, and sites may exist in the Universe where the produced neutrinos have a different initial structure \[2, 9\]. Therefore, the measurement of the relative intensities of the various neutrino flavors on Earth, may provide useful direct information on the mechanism responsible for their generation in the Cosmos and may possibly lead to the discovery of some kind of New Physics \[10\].

Once the various neutrino flavors produced inside some cosmic object reach its surface, they propagate oscillating through space, as dictated by the vacuum oscillation formalism\(^1\). Some of the intrinsic properties of the flavor oscillations are, however, not very easy to readout from the general vacuum oscillation formulae, especially when relying only on numerical scans. These properties can be of importance in the perspective of disentangling, through the measurement of relative fluxes on earth, the astrophysical uncertainties encoded in the initial flavor fluxes on the cosmic sites, from the particle physics features in the neutrino sector. It will therefore be useful to have simple formulae giving the observable relative neutrino flavors $F_e, F_\mu, F_\tau$ on Earth, in terms of the initially produced ones $F^0_e, F^0_\mu, F^0_\tau$ at the surface of the cosmic object.

The aim of the present paper is to give such simple analytical expressions, assuming only three active neutrino flavors which propagate oscillating among themselves \[11\]; (in particular no cosmic neutrino decay is assumed \[12\]). These expressions have the advantage of identifying specific sensitivities and degeneracies in the mixing angles, and of encoding some essential qualitative and quantitative properties of the full formalism. Some of these properties turn out to be highly non-generic consequences of the concomittance of the physically favored values for the mixing angles $s_{23}, s_{13}$, and the initial fluxes.

To derive the aforementioned formulae, we take into account the basic experimental characteristics of the neutrino masses and mixings. These are summarized as follows: The recent SNO \[13\] data combined with those of Super-Kamiokande \[14\] strongly favor the LMA MSW \[15\] solar solution with three active neutrinos and $\theta_{12} \simeq \pi/5.1$ and

\[^1\text{For a review see e.g. 11.}\]
\[|m_3^2 - m_1^2| \approx 5 \times 10^{-5} \text{eV}^2 \] \cite{16}. The atmospheric neutrino \cite{17} data imply \(\theta_{23} \approx \pi/4\) and \(|m_3^2 - m_2^2| \approx 2.5 \times 10^{-3} \text{eV}^2\); while the CHOOZ experiment constrains \(\theta_{13} \lesssim 0.1\), \cite{18,11}.

Defining \(s_{ij} \equiv \sin \theta_{ij}\) and choosing the “central values”

\[
s^c_{12} = \frac{1}{\sqrt{3}} \quad , \quad s^c_{23} = \frac{1}{\sqrt{2}} \quad , \quad s^c_{13} = 0
\] \hspace{1cm} (2)

one has

\[
s_{12} \equiv s^c_{12} + \delta s_{12} \quad , \quad s_{23} \equiv s^c_{23} + \delta s_{23} \quad , \quad s_{13} \equiv s^c_{13} + \delta s_{13}
\] \hspace{1cm} (3)

where \cite{16,17,11,19},

\[
-0.11 \lesssim \delta s_{12} \lesssim 0.04 \quad , \quad -0.12 \lesssim \delta s_{23} \lesssim 0.10 \quad , \quad 0 \leq \delta s_{13} \lesssim 0.1
\] \hspace{1cm} (4)

For realistic neutrino mass differences, and neutrino energies in the range \(E \lesssim 10^6 \text{TeV}\), the vacuum oscillation lengths \(\lambda_{ij} = 4\pi E / |m_i^2 - m_j^2|\), always satisfy \(\lambda_{ij} \lesssim 1 \text{pc}\), which is much smaller than the distances to all cosmic neutrino emitting sites, beyond our solar system \cite{6}. Consequently, the number of oscillations performed by the cosmic neutrinos before arriving at the Earth, is so large, that \(\sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right)\) averages to \(1/2\), and the CP-violating contributions vanish.

Studying the properties of the relative neutrino fluxes in the \(s_{23}s_{13}\)-plane, for any fixed values of \(s_{12}\) and \(\cos \delta\), we remark that the point \((s^c_{23}, s^c_{13}) = (1/\sqrt{2}, 0)\), (lying within the experimental range of Eqs.\((4, 5)\)), presents the striking property of being the unique point of this \(s_{23}s_{13}\)-plane\(^2\), where a common \((s_{12}-\text{ and } \cos \delta\)-dependent) direction exists, along which all three fluxes \(F_e, F_\mu, F_\tau\) are stationary.

As an example, we expand the standard vacuum oscillation formulae to first order in \(s_{13}\) and \((\delta s_{12}, \delta s_{23})\) defined in \cite{4}, getting the relative neutrino fluxes on Earth,

\[
F_e = \frac{1}{3} + \left( \frac{1 - 2\sqrt{3} \delta s_{12}}{9} \right) (3F_e^0 - 1) + \delta s_{12} (F_\tau^0 - F_\mu^0) ,
\]

\[
F_\mu = \frac{1}{3} + \left( \frac{2\sqrt{3} \delta s_{12} - 1}{18} \right) (3F_e^0 - 1) + \delta s_{12} (F_\mu^0 - F_e^0) ,
\]

\[
F_\tau = \frac{1}{3} + \left( \frac{2\sqrt{3} \delta s_{12} - 1}{18} \right) (3F_e^0 - 1) + \delta s_{12} (F_\tau^0 - F_e^0) ,
\] \hspace{1cm} (6)

where

\[
\delta s_{123} \equiv \frac{\sqrt{2}}{9} \left( \kappa \delta s_{23} - \delta s_{13} \cos \delta \right) ,
\] \hspace{1cm} (7)

with \(\kappa = 4\), and \((F_e^0, F_\mu^0, F_\tau^0)\) being the initial neutrino relative flavors at the cosmic site.

The experimental constraints \cite{11,13} then imply

\[
-0.09 \lesssim \delta s_{123} \lesssim +0.07 .
\] \hspace{1cm} (8)

\(^2\)At least for \(s_{23} < 0.5\).
In writing Eq. (6) we took into account the unitarity relation

\[ F_e + F_\mu + F_\tau = F_e^0 + F_\mu^0 + F_\tau^0 = 1 \]

where the right hand side is just a normalization.

The set of equations (6) clearly indicates that for \( s_{12}^c \equiv 1/\sqrt{3} \), all three relative fluxes on Earth are stationary along the direction \( \kappa = \frac{1}{\sqrt{2}} \left( s_{23} - 1/\sqrt{2} \right) - s_{13} \cos \delta = 0 \) passing through the point \((s_{23}, s_{13}) = (1/\sqrt{2}, 0)\) of the \( s_{23}s_{13}\)-plane\(^3\). An alternative way of expressing this "stationary along a direction" property is to observe that \( \delta s_{23} \) and \( s_{13} \) enter Eqs. (6) only through one specific combination, like \( \delta s_{123} \) of (7). This signals an approximate degeneracy in the sensitivity to these mixing angles, which would affect their reconstruction from experimental data.

If second order effects are retained, then dependencies on all four mixing angle combinations \( \delta s_{12}, \delta s_{23}, \delta s_{123} \) and \( s_{13} \) appear, leading to

\[
F_e = \frac{1}{3} + (1 - 2\sqrt{3}\delta s_{12} + 9\delta s_{12}^2 - 5s_{13}^2) \left( \frac{3F_e^0 - 1}{9} \right) \\
+ \left[ (1 - \frac{7}{2}\sqrt{3}\delta s_{12})\delta s_{123} + \delta s_{123}(2\sqrt{6}\delta s_{12} + \frac{4}{9}\delta s_{23}) \right](F_\mu^0 - F_\tau^0),
\]

\[
F_\mu = \frac{1}{2}(1 + \Delta F_{\mu\tau} - F_e),
\]

\[
F_\tau = \frac{1}{2}(1 - \Delta F_{\mu\tau} - F_e),
\]

\[
\Delta F_{\mu\tau} \equiv F_\mu - F_\tau = \\
\left[ 12\delta s_{123}^2 + \delta s_{23}(\frac{68}{9}\delta s_{23} - 2\sqrt{6}\delta s_{12}) + \delta s_{123}(\frac{7}{2}\sqrt{3}\delta s_{12} - 12\sqrt{2}\delta s_{23} - 1) \right](3F_e^0 - 1) \\
+ 8(3\delta s_{123}^2 - 3\sqrt{2}\delta s_{123}\delta s_{23} + 2\delta s_{23}^2)(3F_\mu^0 - 1).
\]

We next turn to the discussion of three interesting specific cases.

**Equipartition condition for fluxes on Earth.** There is in fact still another reason that makes the specific point \((s_{23}, s_{13}) = (1/\sqrt{2}, 0)\) unique in the \( s_{23}s_{13}\)-plane. To explain this, we study in general terms the conditions required for the initial fluxes and the mixing angles, in order that flavor fluxes on Earth become equal; i.e. \( F_e = F_\mu = F_\tau \), a property referred to as flux equipartition. It is indeed noteworthy that, besides the Supernovae case where flux equipartition on Earth is an immediate consequence of the (approximate) initial flux equipartition\(^4\), the **canonical** case also leads to equal neutrino fluxes on Earth, provided the neutrino mixing angles are appropriate.

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\(^3\)For a given central value \( s_{12}^c \), the stationary direction is determined by \( \kappa = \frac{2\sqrt{2}s_{12}^c\sqrt{1 - (s_{12}^c)^2}}{(1 - 2(s_{12}^c)^2)} \), which indeed leads to \( \kappa = 4 \) for \( s_{12}^c = 1/\sqrt{3} \).

\(^4\)This case is discussed in more detail below.
Using the exact results for large distance oscillations, one finds that
\[ s_{23}^2 = \frac{1 - F_\mu^0 - 2F_\tau^0}{F_\mu^0 - F_\tau^0} + \frac{2/3 - (F_\mu^0 + F_\tau^0)}{(F_\mu^0 - F_\tau^0)(1 - s_{13}^2)} , \]

is a necessary condition for equipartition of the neutrino fluxes on Earth. This condition shows clearly that equipartition requires a specific correlation between two sets of physically independent quantities, namely the mixing angles \( s_{23}, s_{13} \) and the initial flavor fluxes. It is noteworthy that since Eq. (11) is independent of \( s_{12} \), the latter does not have any influence on equipartition, in agreement with the findings of [20] for the canonical case.

If \( F_\mu^0 + F_\tau^0 = 2/3 \) is now assumed, then Eq. (11) implies \( s_{23}^2 = (1 - F_\mu^0 - 2F_\tau^0)/(F_\mu^0 - F_\tau^0) = 1/2 \), which has been first observed by [21]. If, on top of this, we impose the canonical initial fluxes satisfying Eqs. (1), then \( s_{13} = 0 \) is also uniquely required for equipartition. Thus, the requirement of exact equipartition of the fluxes on Earth, combined with the assumption that the initial fluxes are canonical, uniquely determines \( s_{23} = 1/\sqrt{2} \) and \( s_{13} = 0 \).

The fact that the initial fluxes resulting from the astrophysical mechanism, and the experimentally favored neutrino mixing angles fall in the vicinity of the unique equipartition solution for the fluxes on Earth, is a striking coincidence! This is even more so, if we also remember the ”stationary” property of these fluxes described above.

We next turn to the fluxes as determined by the actual ranges of the mixing angles appearing in [4, 5, 8]. Using the first order expression (6), together with (1), we get
\[ F_e = \frac{1}{3}(1 - 2\delta s_{123}) , \]
\[ F_\mu = F_\tau = \frac{1}{3}(1 + \delta s_{123}) , \]
which agrees with the conclusion of [4, 22] that for bimaximal neutrino mixing with very small \( s_{13} \), the relative neutrino flavor fluxes are \( F_e \simeq F_\mu \simeq F_\tau \simeq 1/3 \).

Our formalism goes beyond this though, since it also considers in detail the small deviations from the mixing angle-values (\( \theta_{12} \simeq \pi/5.1, \theta_{23} \simeq \pi/4, \theta_{13} \simeq 0 \)). To linear order in these deviations, we find that the arriving neutrino fluxes are independent of \( \delta s_{12} \), and only depend on the specific combination of \( \delta s_{23} \) and \( s_{13} \cos \delta \) given in Eq. (7). Moreover, \( F_\mu \) and \( F_\tau \) are always equal and described by the ”green” line along the diagonal of the triangle in Fig.1.

As seen from Eqs. (10) though, to second order in \( (\delta s_{12}, \delta s_{23}, s_{13}) \), a weak intrinsic dependence on all four parameters \( (\delta s_{12}, \delta s_{23}, s_{13}) \) and \( \cos \delta \) appears. To study this in somewhat more detail, we have reproduced in Fig.1, the implications of the constraints [4, 5]. The second order formulae in Eq. (10) describes these constraints by the ”blue-plus-red” region within the triangle in Fig.1, which almost completely overlaps with the ”red”

\[ 5 \text{Barring some special cases which we have also identified, where } (s_{12} = 0, 1/\sqrt{2}, 1) \text{ or } (s_{23} = 0, 1) \text{ or } (s_{13} = 1). \text{ Such cases are clearly excluded by experiment. In particular, the case } s_{12} = 1/\sqrt{2} \text{ studied in this equipartition context in [21], has been recently reported to be excluded at the } 5 \sigma \text{ level [19].} \]
region resulted from the numerical analysis of the exact expressions \([4, 22]\). As seen there, \(F_{\mu} \geq F_{\tau}\) in the whole allowed region. This property can actually be shown analytically, both from the exact formalism or from the approximation Eq.\((10)\) which leads to

\[
F_{\mu} - F_{\tau} = \frac{8}{27} (13\delta_{23}^2 + s_{13}^2 \cos^2 \delta + (\delta_{23} + s_{13} \cos \delta)^2) \geq 0 .
\]  

(13)

On the basis of this analysis, we find that in the canonical case

\[
0.28 \lesssim F_{e} \lesssim 0.39 , \\
0.30 \lesssim F_{\mu} \lesssim 0.36 , \\
0 < \Delta F_{\mu\tau} \lesssim 0.073 ,
\]  

(14)

and

\[
0.73 \lesssim \frac{F_{\mu}}{F_{e}} \simeq (1 + 3 \delta_{123}) \lesssim 1.21 ,
\]  

(15)

which, as already said, are consistent with the results of \([4, 22]\).

A virtue of the present derivation, is that the effect of a future reduction of the experimental uncertainties on the mixing angles, can be straightforwardly read analytically from Eq.\((10)\) or even Eq.\((6)\). As an example we note that if it turns out that e.g. \(\delta_{123} = 0.1\) (compare Eq.\((8)\)) and that \(F_{\mu} \simeq F_{\tau}\), then the linear formulae \((12)\) should be adequate, leading to \(F_{e} \simeq 0.27\) and \(F_{\mu} \simeq F_{\tau} \simeq 0.37\). In a future sufficiently large neutrino detector such as IceCube \([23]\), it might be possible to discriminate this case from the ideal prediction \(F_{e} \simeq F_{\mu} = F_{\tau} \simeq 1/3\), in the TeV-PeV energy range.

Equal initial neutrino fluxes. Because of unitarity, if the initial relative flavors satisfy \(F_{e}^{0} = F_{\mu}^{0} = F_{\tau}^{0}\), then the final ones also obey \(F_{e} = F_{\mu} = F_{\tau} = 1/3\), irrespective of the neutrino mixing angles. This is e.g. the situation roughly half a second or so after a supernova collapse and explosion, as suggested by typical simulations \([7]\). However, since such configurations can be simulation dependent, and keeping in mind the uncertainties related to supernova physics\([7]\), we may parameterize a deviation in the electron flavor from the initial flux equipartition in the form

\[
F_{e}^{0} = \frac{1 - 2\epsilon}{3} \\
F_{\mu}^{0} = F_{\tau}^{0} = \frac{1 + \epsilon}{3}
\]  

(16)

The expected flux differences on Earth are then easily obtained from Eq.\((6)\) as

\[
F_{\tau} - F_{e} = (\frac{7}{16} - 3\frac{1}{2} \delta_{12} - \delta_{123}) \epsilon \\
F_{\mu} - F_{e} = (\frac{7}{16} - 3\frac{1}{2} \delta_{12} + \delta_{123}) \epsilon \\
F_{\mu} - F_{\tau} = 2\delta_{123} \epsilon
\]  

(17)
We call Eqs. (16, 17) the "Supernova-type case", allowing it to cover also the possibility of TeV neutrino sources, which somehow produce roughly equal neutrino fluxes for all neutrino and antineutrino flavors.

It is interesting to note that even in the linear approximation in this case, an initial $\mu/\tau$ flavor equipartition tends to be removed by neutrino oscillation effects, as can be seen from the third equation in Eq. (17). Moreover, a simultaneous measurement of the three flux differences in Eq. (17), provided it is sufficiently accurate, would allow the reconstruction of the three quantities $\epsilon, \delta s_{123}$ and $\delta s_{12}$. It should be clear, though, that our analysis is valid only if matter effects on the observed fluxes can be neglected. Obviously, this would not be the case for neutrinos which cross the Earth before detection[24]. Furthermore, star matter effects can be important in some regions of the supernova [25].

$F_0^e = 1$ case. As a last illustration we consider the rather exotic case where $F_0^e = 1$, $F_0^\mu = F_0^\tau = 0$. In the context of the linear approximation formulae Eqs. (16), we get in this case

\[
F_e = \frac{5}{8} - \delta s_{12}, \\
F_\mu = \frac{3}{16} + \frac{\delta s_{12}}{2} - \delta s_{123}, \\
F_\tau = \frac{3}{16} + \frac{\delta s_{12}}{2} + \delta s_{123},
\]

(18)

where, in contrast to the previous situation, the relative neutrino fluxes have some sensitivity to $\delta s_{12}$ also. Using Eqs. (4, 8) we then find

\[
0.44 \lesssim F_e \lesssim 0.67, \\
-0.24 \lesssim F_\mu - F_\tau \lesssim 0.24,
\]

(19)

where the uncertainties induced by $\delta s_{12}$ and $\delta s_{123}$ are separated.

To summarize, we have studied in this paper some useful analytical properties of the neutrino flavor fluxes, assuming just three active neutrino flavors propagating in the vacuum space (from the surface of the cosmic sites where they are produced, to Earth) oscillating among themselves with no neutrino decay processes\(^6\). We have first emphasized that for any given $s_{12}$ and $\cos \delta$, the point ($s_{23} = 1/\sqrt{2}$, $s_{13} = 0$) is essentially the unique point where (a) all three relative fluxes ($F_e$, $F_\mu$, $F_\tau$) on Earth are equal, when the initial fluxes are canonical, and (b) a common ($s_{12}$- and $\cos \delta$-dependent) direction exists in the $s_{23}s_{13}$-plane, [irrespectively of the values of the initial fluxes], along which all three relative fluxes ($F_e$, $F_\mu$, $F_\tau$) on Earth are stationary. These features have immediate consequences on the sensitivity to the mixing angles and flavor fluxes. Assuming then that the deviations of the neutrino mixing angles from their favored experimental values $s_{12} = 1/\sqrt{3}$, $s_{23} = 1/\sqrt{2}$ and $s_{13} = 0$ are small, we have expressed the observable neutrino fluxes on Earth

\(^6\)The above formulae can of course be straightforwardly extended to cases including sterile neutrinos.
in terms of the original ones at the cosmic sites, keeping either linear or quadratic terms in the above angle-deviations, and assessed the validity of this approximation through a numerical comparison with the exact formalism. These expressions allow to extract information from Neutrino Astronomy data in a fairly simple way.

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Figure 1: The relative neutrino flavor fluxes on Earth $F_e$, $F_\mu$, $F_\tau$ are shown in the canonical case $F^0_e = 1/3$, $F^0_\mu = 2/3$, $F^0_\tau = 0$. The colored regions correspond to mixing angle variations in the intervals $[4, 5]$ using green: for the linear approximation (6); red: for the exact formalism result; blue plus red: for the second order expression (10). The $F_e$, $F_\tau$, $F_\mu$ coordinates of a specific point inside the triangle are obtained by reading the intersections with the appropriate axes, of lines emanating from this specific point along the indicated arrows; e.g., the illustrated point corresponds to $(F_e, F_\mu, F_\tau) = (1/5, 1/5, 3/5)$. 
References

[1] E. Waxman, arXiv:astro-ph/0211358

[2] F. Halzen and D. Hooper, Rept. Prog. Phys. 65: 1025 (2002), arXiv:astro-ph/0204527

[3] H. Athar, arXiv:hep-ph/0001128, hep-ph/0004083, A. Yu Neronov and D.V. Semikoz, hep-ph/0208248

[4] O. Yasuda, arXiv:hep-ph/0005135

[5] B. Zhang, Z. G. Dai, P. Mészáros and E. Waxman, arXiv:astro-ph/0210382

[6] D. Guetta, C. Distefano, A. Levinson and W. Waxman, arXiv:astro-ph/0207359, L.A. Anchordoqui, D.F. Torres, T.P. McGauley and G.E. Romero, arXiv:hep-ph/0211231

[7] G.G. Raffelt, Nucl.Phys.Proc.Suppl. 110: 254 (2002), arXiv:hep-ph/0201099, arXiv:hep-ph/0208024

[8] H. Athar, arXiv:hep-ph/0209130, G. Sigl, Annals Phys. 301: 53 (2002), arXiv:hep-ph/0205051, hep-ph/0109202, J. H. MacGibbon and U. Wichoski, Nucl.Phys.Proc.Suppl. 110: 528 (2002), arXiv:hep-ph/0111436, A.Yu. Neronov and D.V. Semikoz, arXiv:hep-ph/0208248, J. Granot, D. Guetta, arXiv:astro-ph/0211433, D. Fargion, arXiv:hep-ph/0211153

[9] S.F. King, arXiv:hep-ph/0210089

[10] H. Shahid and D. W. McKay, hep-ph/0310091

[11] M Apollonio et.al., arXiv:hep-ph/0210192

[12] J.F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, T.J. Weiler, arXiv:hep-ph/0211305

[13] Q.R. Ahmad et.al., SNO Collaboration, Phys. Rev. Lett. 89: 011301 (2002), arXiv:nucl-ex/0204081, Phys. Rev. Lett. 89: 011302 (2002), arXiv:nucl-ex/0204009, S.N. Ahmed et.al., nucl-ex/0309004

[14] S. Fukuda et.al., SuperKamiokande Collaboration, Phys. Rev. Lett. 86: 5656 (2001), arXiv:hep-ex/0103033

[15] L. Wolfenstein, Phys. Rev. D17: 2369 (1978), ibid Phys. Rev. D20: 2634 (1979); S.P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. 42:1441 (1985) [ Sov. J. Nucl. Phys. 42: 913 (1985)].
[16] V. Barger, D. Marfatia, K. Whisnant and B.P. Wood, Phys. Lett. B537: 179 (2002), arXiv:hep-ph/0204253; A. Bandyopadhyay, S. Choubey, S. Goswami, D.P. Roy Phys. Lett. B540: 14 (2002), arXiv:hep-ph/0204286; J.N. Bahcall, M.C. Gonzalez-Garcia, C. Pena-Garay, JHEP 0207: 054 (2002), arXiv:hep-ph/0204314; A. Yu. Smirnov, arXiv:hep-ph/0209131.

[17] Y. Fukuda et al. (Superkamiokande Collaboration), Phys. Rev. Lett. 81: 1562 (1998), arXiv:hep-ex/9807003; M.C. Gonzalez-Garcia, "Theory of neutrino masses and mixings", Plenary report at the 2003 International Conference on High Energy Physics, Amsterdam, hep-ex/0210359.

[18] CHOOZ Collaboration (M. Apollonio et al.), Phys. Lett. B466: 415 (1999), arXiv:hep-ex/9907037, arXiv:hep-ex/0301017.

[19] S.N. Ahmed et al. arXiv:nucl-ex/0309004; P. Migliozi, hep-ph/0311269; M. Maltoni, T. Schwetz, M.A. Tórtola and J.W.F. Valle, hep-ph/0309130.

[20] G. Barenboim and C. Quigg Phys. Rev. D67: 073024 (2003), arXiv: hep-ph/0301220.

[21] D.V. Ahluwalia, Mod. Phys. Lett. A16: 917 (2001).

[22] J.G. Leanard and S. Pakvasa, Astropart. Phys. 3: 267 (1995); O. Yasuda, Acta Phys. Pol. B30: 3089 (1999), arXiv:hep-ph/9910428; H. Athar, M. Jezabek, O. Yasuda, Phys. Rev. D62: 103007 (2000).

[23] F. Halzen, Int. J. Mod. Phys. A17: 3432 (2002), arXiv:hep-ph/0111059.

[24] A. S. Dighe and A. Yu. Smirnov, Phys. Rev. D62:033007, 2000.

[25] S. Hannestad, H.-T. Janka, G. G. Raffelt, G. Sigl, Phys. Rev. D62:093021, 2000, arXiv:astro-ph/9912242.