The Hubble parameters in the D-brane models

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Abstract

We consider the DBI action for the D-branes with the dynamic em-beddings in the background produced by p-branes. For the D-brane with
the special topology we obtain two Hubble parameters on this brane. The
condition for the equality of these parameters is analyzed. In the special
case a mass and a charge of the background p-branes are derived from
this condition.

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1 Introduction

The motion of the D-branes in the diversity backgrounds has been considered
in the variety of papers e.g. [1-5]. The applications D-branes to the cosmology
and gravity are also widely discussed e.g. [2-4]. In these approaches there
are considered small size and big size compactified directions. The D-brane
is considered as an embedded submanifold of the ten-dimensional spacetime
with nontrivial backgrounds fields. The effective theory is described by the
Dirac-Born-Infeld (DBI) action. This action also determine the motion in the
spacetime. From the D-brane point of view this motion is interpreted as the
evolution of the world-volume of the brane. In the D-brane models of the
universe all fields and particles of the Standard Model (SM) are fixed to the
world-volume. Thus evolution of D-brane corresponds to cosmological evolution
for the observer fixed to the world-volume.

In this paper we consider the cosmological evolution of a probe Dk-brane
with the DBI action in the backgrounds of p-branes. The probe Dk-brane
means that the backreactions are neglected. We assume that the Dk-brane
has the topology of the direct product of a compact space and an non-compact
space. In this case we will obtain the Hubble parameters for these spaces. These
parameters are depend on the tangent and the normal directions to Dk-brane.
The condition for the equality of these parameters is given.
In the section 2 we recall the DBI action for a Dk-brane in the backgrounds produced by p-branes and derive it equation of motion in given embedding. In the section 3 we derive a ratio of the Hubble parameters for k=3. They are obtained from the metric on the world-volume of D3-brane. This case can be considered as a toy cosmological model corresponding to 3+1 spacetime. In the general relativity models with several Hubble parameters were considered long time ago e.g.[6]. In these models space is anisotropic and rate of expansion depends on directions. From other side cosmological models derived from M-theory admit warped factors which depend on time [7]. These factors on IIB string theory side correspond to different rates expansion of the tangent directions to a D3-brane. The section 4 is devoted to conclusions.

2 D-brane evolution

In this section we consider the motion of a Dk-brane. The Dk-brane action is described by the DBI action:

$$ S = -T_k \int d^{k+1} \xi e^{-\phi} \sqrt{-\det(\gamma_{\mu\nu} + 2\pi \alpha' F_{\mu\nu} + B_{\mu\nu}) + T_k \int \sum_{i} \tilde{A}_{(i)} \wedge e^{2\pi \alpha' F + B}}, $$

where $\gamma_{\mu\nu}$ is the pull back of the background metric, $B_{\mu\nu}$ is the pull back of the background the NS 2-form, $F_{\mu\nu}$ is the strength of the abelian gauge field on the worldvolume and $\tilde{A}_{(i)}$ are pull-back of the background i-forms $A_{(i)}$ with odd (even) degrees: $i = 1, 3, 5, 7$ ($i = 0, 2, 4, 6, 8$) in the Type IIA (IIB) theory.

We consider the background solutions with the symmetry group $\mathbb{R}^1 \times E_{(6-p)} \times SO(p+3)$, where $E_{(6-p)}$ is the Euclidean group. They are given by [8,9,10,11]:

- the metric:

$$ ds^2 = -\Delta_+ \Delta_- \frac{r_p}{r} dt^2 + \Delta_+^{1/2} \Delta_-^{1/2} \Omega_{p+2}^2 + \Delta_+^{(3-p)/2} dr^2 + r^2 \Delta_+^{(3-p)/2} d\Omega_{p+2}^2, \tag{2.2} $$

where:

$$ \Delta_\pm (r) = 1 - \left( \frac{r_\pm}{r} \right)^{p+1}, $$

- the gauge strength $F = dA_{(p+1)}$:

$$ F = (p+1) \left( r_+ r_- \right)^{(p+1)/2} \varepsilon_{p+2}, \tag{2.3} $$

$\varepsilon_{p+2}$ is the volume form on $S^{p+2}$.
• the dilaton field:

\[ e^{-2\phi} = \Delta^{2}, \quad (2.4) \]

where \( a^2 = (3 - p)^2 / 4. \)

The topological charge \( g_{7-p} \) and the mass \( m_{7-p} \) of the background are expressed by \( r_{+}, r_{-} : \)

\[ g_{7-p} = vol \left( S^{p+2} \right) \frac{(p + 1)(r_{+}r_{-})^{(p+1)/2}}{\sqrt{2\kappa}}, \quad (2.5) \]

\[ m_{7-p} = \frac{vol \left( S^{p+2} \right)}{2\kappa^2} \left( (p + 2) r_{+}^{p+1} - r_{-}^{p+1} \right). \quad (2.6) \]

The above solution becomes the BPS state for \( r_{+} = r_{-} = R \) with the metric:

\[ ds^2 = \Delta^{\frac{p+1}{2}} \left( -dt^2 + dX_i dX^i \right) + \Delta^{2}\frac{r^2}{p+2} \left( d\rho^2 + \rho^2 d\Omega_{p+2}^2 \right), \quad (2.7) \]

where \( \rho \) is related to \( r \) as follows: \( \rho^{p+1} = r^{p+1} - R^{p+1} \) and \( \Delta = 1 + (R/\rho)^{p+1}. \)

We have considered this background solution since they are general and for \( p = 3 \) the last metric has the form used in the warp compactification.

In the general case the Dk-brane and D(6-p)-brane do not intersect if their spatial dimensions obey the relation:

\[ 6 \geq k + 6 - p. \]

We denote the background coordinates as follows:

\[ X^M = (t, X^1, ..., X^{6-p}, r, \varphi^1, ..., \varphi^{p+2}), \]

where \( \varphi^1, ..., \varphi^{p+2} \) are coordinates on the sphere \( S^{p+2} \), so \( r \) and \( \varphi^1, ..., \varphi^{p+2} \) span the transverse directions to the (6-p)-brane. The Dk-brane propagating in this background has \( n \)-directions parallel to (6-p)-brane and \( k - n \) directions perpendicular to (6-p)-brane where the number \( n \) is given by [1]:

\[ n \leq n_0 = \min(k, 6 - p). \quad (2.8) \]

We will consider free falling Dk-brane in its rest frame with the proper time \( \tau \). We assume that \( r \) is always transverse to Dk-brane and Dk-brane has the topology of the direct product:

\[ V_n \times S^{k-n}, \]

where \( V_n \) is some n-dimensional space. Thus the embedding field has the form:

\[ X^M (\tau) = \left( t (\tau), \xi^1, ..., \xi^n, X^{n+1}, ..., X^{6-p}, r (\tau), \theta^1, ..., \theta^{k-n}, \varphi^{k-n+1} (\tau), ..., \varphi^{p+2} (\tau) \right), \quad (2.9) \]
where $\xi^1, \ldots, \xi^n$ are coordinates on $V_n$ and $\theta^1, \ldots, \theta^{k-n}$ are coordinates on $S^{k-n}$. The induced metric $\gamma_{\mu\nu}$ on the world-volume by the embedding (2.9) has the form:

$$
\gamma_{00} = -\Delta_+ \Delta_- \frac{r^2}{r^2 + \Delta_- (\frac{3-p}{r^2})^2} + r^2 h_{\bar{r}\bar{s}} \tilde{\gamma} \cdot \tilde{\gamma},
$$

(2.10)

$$
\gamma_{ab} = \Delta_- \delta_{ab},
$$

(2.11)

$$
\gamma_{\tilde{a}\tilde{b}} = \tilde{r}^2 h_{\tilde{a}\tilde{b}},
$$

(2.12)

where $\tilde{r}, \tilde{s} = k-n+1, \ldots, p+2$, $a, b = 1, \ldots, n$, and $\tilde{a}, \tilde{b} = 1, \ldots, k-n$. The metrics $h_{\tilde{a}\tilde{b}}$ and $h_{\tilde{r}\tilde{s}}$ are expressed by the metric $h_{rs}$ on the sphere $S^{p+2}$:

$$
(h_{rs}) = \left( \begin{array}{cc} h_{\tilde{a}\tilde{b}} & 0 \\ 0 & h_{\tilde{r}\tilde{s}} \end{array} \right).
$$

The dot over coordinates means the derivative with the respect to the proper time $\tau$. In the case when the background NS form $B$ is zero and the abelian gauge field on the world-volume vanishes the WZ term in (2.1) takes the form:

$$
\int \tilde{A}_{(k+1)},
$$

where the form $\tilde{A}_{(k+1)}$ is given by the pull back of background form $A_{(k+1)}$. In the considered background the only non-vanishing form is $A_{(p+1)} = A_{M_0 \ldots M_p} dX^{M_0} \wedge \ldots dX^{M_p}$ such that $dA_{(p+1)}$ is given by (2.3). Thus WZ term does not vanish if $k=p$ and the DBI action takes the form:

$$
S = -T_k \int d\tau d^n\xi d^{k-n} \theta \left( e^{-\phi} \sqrt{-\det (\gamma_{\mu\nu}) - \delta_{k,p} A} \right)
$$

(2.13)

and $A = A_{0 \ldots p}$. Since the terms in (2.13) do not depend on coordinates $\xi$ we get:

$$
S = -T_k \text{vol} (V_n) \int d\tau d^{k-n} \theta \left( e^{-\phi} \sqrt{-\det (\gamma_{\mu\nu}) - \delta_{k,p} A} \right).
$$

In the considered background we obtain:

$$
e^{-\phi} \sqrt{-\det (\gamma_{\mu\nu})} = \left( t^2 - \Delta_+^2 \Delta_-^{-2} \frac{r^2}{r^2 + \Delta_- (\frac{3-p}{r^2})^2} - \Delta_-^{-1} \Delta_+ \tilde{r}^2 h_{\bar{r}\bar{s}} \tilde{\gamma} \cdot \tilde{\gamma} \right)^{1/2} \times \sqrt{r^{k-n} \Delta_+^{1/2} \Delta_-^{\frac{n}{2}}} / \sqrt{\text{vol} (V_n)}.
$$

(2.14)

In non-rotating case $\tilde{\varphi} = 0$ the action simplifies and takes the form:

$$
S = -T_k \text{vol} (V_n) \int d\tau L,
$$

(2.15)

where the Lagrangian $L$ has the form:

$$
L = \left[ \text{vol} (S^{k-n}) \left( t^2 - \Delta_+^2 \Delta_-^{-2} \frac{r^2}{r^2 + \Delta_- (\frac{3-p}{r^2})^2} \right)^{1/2} r^{k-n} \Delta_+^{1/2} \Delta_-^{\frac{n}{2}} \text{vol} (V_n) \right] - \delta_{k,p} A \text{vol} (V_n).
$$

(2.16)
and \( w = \int d^{k-n} \theta \). Variation \( L \) with respect to \( t \) gives:

\[
\text{vol} (S^{k-n}) \frac{tr^{k-n} \Delta_+^{1/2} \Delta_-^{[5(1-p)+n(1+p)]/16}}{\sqrt{t - \Delta_-^2 \Delta_+^{1+p}}} - \delta_{k,p} Aw = E, \tag{2.17}
\]

where \( E \) is a constant of motion. Thus:

\[
\left( \frac{dr}{dt} \right)^2 = \left[ 1 - \frac{r^{2(k-n)} \Delta_+^{1/2} \Delta_-^{[5(1-p)+n(1+p)]/16}}{(E + \delta_{k,p} Aw)^2} - \text{vol}^2 (S^{k-n}) \right] \Delta_+^2 \Delta_-^{1+p}.
\tag{2.18}
\]

The proper time \( \tau \) of the \( Dk \)-brane is expressed by:

\[
d\tau^2 = \gamma_{\mu \nu} d\xi^\mu d\xi^\nu = g_{MN} \partial_\mu X^M \partial_\nu X^N d\xi^\mu d\xi^\nu.
\]

In the rest frame of the \( Dk \)-brane and for the considering embedding this proper time has the form:

\[
d\tau^2 = - \left( g_{00} + g_{rr} \dot{r}^2 + r^2 h_{\varphi \varphi} \dot{\varphi}^2 \right) dt^2,
\tag{2.19}
\]

where:

\[
g_{00} = -\Delta_+ \Delta_-^{-\frac{1}{1+p}}, \quad g_{rr} = \Delta_+ \Delta_-^{-(\frac{3-p}{2})}.
\]

In the non-rotating case \( \dot{\varphi} = 0 \) the derivatives with the respect to the proper time \( \tau \) and coordinate time \( t \) are related:

\[
\left( \frac{dr}{dt} \right)^2 = \left( \frac{dr}{d\tau} \right)^2 \left( \frac{d\tau}{dt} \right)^2,
\]

so:

\[
\left( \frac{dr}{dt} \right)^2 = - \frac{g_{00}}{1 + g_{rr} \left( \frac{dr}{d\tau} \right)^2} \left( \frac{dr}{d\tau} \right)^2.
\]

From (2.18) and (2.19) we obtain relation between the radial position and the proper time:

\[
\left( \frac{dr}{d\tau} \right)^2 = \frac{(E - \delta_{k,p} Aw)^2 - r^{k-n} \Delta_+^{1/2} \Delta_-^2 \text{vol}^2 (S^{k-n})}{(E - \delta_{k,p} Aw)^2 - \Delta_-^2 + r^{k-n} \Delta_+^{3/2} \Delta_-^2 \text{vol}^2 (S^{k-n})} \Delta_+^2 \Delta_-^\alpha, \tag{2.20}
\]

where the exponents are:

\[
\alpha = \frac{-1 + 14p - p^2}{8 (1 + p)}, \quad \beta = \frac{5 (1 - p) + n (1 + p)}{16},
\]
\[
\gamma = \frac{3 (p^2 - 10 p + 9)}{8 (1 + p)}, \quad \delta = \frac{p^2 - 60 p + 61 + n (1 + p)^2}{16 (1 + p)}.
\]

In the coordinate time \( t \) the induced metric on the Dk-brane by the embedding (2.9) has the form:

\[
ds^2 = -\left( \Delta_+ \Delta_-^{-2/3} - \Delta_+^{-1} \Delta_-^{-2/3 p/3 - 1/3} r^2 - r^2 h_{\tau \varphi} \varphi^{-3} \right) \, dt^2 + \Delta_+ \Delta_-^{-2/3} d\xi_a d\xi^a + r^2 h_{\tilde{a} \tilde{b}} d\tilde{\theta}^\tilde{a} d\tilde{\theta}^\tilde{b}.
\]

Using (2.19) we get:

\[
ds^2 = -d\tau^2 + \Delta_+ \Delta_-^{-2/3} d\xi_a d\xi^a + r^2 h_{\tilde{a} \tilde{b}} d\tilde{\theta}^\tilde{a} d\tilde{\theta}^\tilde{b},
\]

where \( r (\tau) \) is the solution of the (2.20). This metric has the form of the FRW-like metric with two scale factors namely \( \Delta_+ \Delta_-^{-2/3} \) and \( r^2 \). Assuming that the evolution of the world-volume seeing by the observer fixed to the brane, is determined by the gravity produced by (2.22) we get following equation of motion:

\[
n (n - 1) \lambda^2 + 2 m n \lambda \beta + m (m - 1) \beta^2 + e^{-2 \beta} \tilde{R} = 16 \pi G \rho,
\]

where \( m = k - n \), \( \exp 2 \lambda = \Delta_+ \Delta_-^{-2/3} \), \( \exp 2 \beta = r^2 \), the scalar curvature \( \tilde{R} \) is obtained from the metric \( h_{\tilde{a} \tilde{b}} \) and \( \rho \) is the energy density on the world-volume. The dot means: \( \cdot = d \lambda / d r \) and so on. Hence from (2.22) follows that the evolution in the non-compact directions \( \xi \) is given by \( \Delta_+ \Delta_-^{-2/3} \) and the second factor \( r^2 h_{\tilde{a} \tilde{b}} \) concerns the evolution in compact directions corresponding to the sphere.

### 3 Hubble parameters

We relate to the metric (2.22) two Hubble parameters:

\[
H_n = \frac{1}{\Delta_+ \Delta_-^{-2/3}} \frac{d}{d\tau} \left( \Delta_+ \Delta_-^{-2/3} \right),
\]

\[
H_c = \frac{1}{r} \frac{dr}{d\tau},
\]

where in \( H_c \) is assumed isotropic evolution, it means that \( d \left( h_{\tilde{a} \tilde{b}} \right) / d\tau = 0 \). The eq. (3.1) takes the form:

\[
H_n = \frac{(p + 1)^2}{16} \frac{r r^{p+1}_{p+1}}{r^{p+1}_{p+1} - r^{p+1}_{p+1}} \frac{dr}{d\tau},
\]

where \( dr/d\tau \) is given by (2.20). The ratio of these Hubble parameters is given by the relation:

\[
\frac{H_n}{H_c} = \frac{(p + 1)^2}{16} \frac{r^2 r^{p+1}_{p+1}}{r^{p+1}_{p+1} - r^{p+1}_{p+1}} \equiv \eta (r).
\]
It depends on the position $r$ of the D$k$-brane and $r$ is given by the solution of the equation (2.20).

We investigate the ratio (3.4) as a function of $t$ in the case when $r_+ = r_- = R$. Thus the eq. (2.18) takes the form:

$$\left(\frac{dr}{dt}\right)^2 = \left[1 - \frac{r^{2(k-1)}\Delta^{1/2+[(1-p)+(1+p)]/16}}{(E + \delta_{k,p}Aw)^2} \right] vol^2 \left(S^{k-n}\right) \Delta^{1+3p}_{1+p} \quad (3.5)$$

and the metric on the world-volume has the form:

$$ds^2 = -d\tau^2 + \Delta^{1+3p} d\xi_a d\xi_a + r^2 h_{\hat{a}\hat{b}} d\theta^\hat{a} d\theta^\hat{b}.$$ 

The ratio of the Hubble parameters in this case is given by:

$$\frac{H_n}{H_c} = (p+1)^2 \cdot \frac{r^{2p+1}}{r^{p+1} - R^{p+1}}. \quad (3.6)$$

We restrict ourselves to the case when $k = 3$ which corresponds to D3-brane. Thus $a, b = 1, \ldots, n$, and $\hat{a}, \hat{b} = 1, \ldots, 3 - n$. Then the eq.(3.5) takes the form:

$$\left(\frac{dr}{dt}\right)^2 = \left[1 - \frac{r^{2(3-n)}\Delta^{1/2+[(1-p)+(1+p)]/16}}{(E + \delta_{3,p}Aw)^2} \right] vol^2 \left(S^{3-n}\right) \Delta^{1+3p}_{1+p}. \quad (3.7)$$

In order to get how change (3.6) in time we need find solutions of (3.7). These solutions among other depends on the dimension $p$ of the background branes. Thus we have to consider each dimension $p$ separately. The solutions of (3.7) for different $p$ are given below where the number of the non-compact dimensions $n$ is given by the condition (2.8).

For $p = 0$ (D-particle) the eq. (3.7) gives following result:

$$\int \frac{\sqrt{r} dr}{\sqrt{r - R} \sqrt{1 - r^{2(\alpha - \beta_0)} (r - R)^{2\beta_0} \sigma_n^2}} = t + t_0, \quad (3.8)$$

where

$$\sigma_n^2 = \left(\frac{vol \left(S^{3-n}\right)}{E}\right)^2$$

and $\alpha = 3 - n$, $2\beta_0 = (13 + n)/16$. The number $n$ of the non-compact dimensions is:

$$n = 0, 1, 2, 3.$$ 

The cases $n = 0$ and $n = 3$ correspond to the only one Hubble parameter.

For $p = 1$ (D-string) we get:

$$\int \frac{r^2 dr}{(r^2 - R^2) \sqrt{1 - r^{2(\alpha - \beta_1)} (r^2 - R^2)^{2\beta_1} \sigma_n^2}} = t + t_0, \quad (3.9)$$
where $2\beta_1 = (9 + 2n)/16$.

For $p = 3$:

$$\int \frac{r^5 dr}{(r^4 - R^4)^{5/4} \sqrt{1 - r^{2(n-\beta_3)} (r^4 - R^4)^{2\beta_3} (E + Aw)^2 (S^3 - n)}} = t + t_0, \quad (3.10)$$

where $2\beta_3 = (2n - 1)/8$.

In the both above cases the number $n$ of the non-compact dimensions is equal to:

$$n = 0, 1, 2, 3.$$  

The first and the last cases correspond to the only one Hubble parameter. For $p = 5$:

$$\int \frac{r^8 dr}{(r^6 - R^6)^{4/3} \sqrt{1 - r^{2(n-\beta_5)} (r^6 - R^6)^{2\beta_5} \sigma_n^2}} = t + t_0, \quad (3.11)$$

where $2\beta_5 = 3(n - 2)/8$. The number $n$ of the non-compact dimensions is equal to: $n = 0, 1$.

The above integrals are complicated. One can evaluated them in the limit when the parameter $E$ goes to infinity ($E \to \infty$). In this case all the above integrals have simply asymptotes: $r \sim t$. It means that the D3-brane and background $p$-branes does not form bounded system. Thus one can notice from (3.6) that:

$$\frac{H_n}{H_c} = \eta \to \frac{\infty}{R^2/4} \quad p = 0 \quad \frac{\infty}{0} \quad p = 1 \quad \frac{0}{p > 1} \quad (3.12)$$

and $\eta$ has singularity for all $p$ in $r = R$. As was mentioned above the considered background solutions are right for $r > R$. So one can conclude that background produced by D1-branes (D-strings) gives flat Minkowski 4-dimensional space-time with the equal Hubble parameters if $R = 2$. This condition put constraint on a topological charge $g_6$ and a mass $m_6$ of a dual D6-branes to the background D1-branes, because $R$ is related to these parameters by (2.5) and (2.6). For $r_+ = r_- = R$ these relations takes the form:

$$g_6 = \frac{3\text{vol} (S^3)}{\sqrt{2}\kappa} R^3, \quad (3.13)$$

$$m_6 = \frac{3\text{vol} (S^3)}{2\kappa^2} R^3. \quad (3.14)$$

Thus we get following values of $g_6$ and $m_6$:

$$g_6 = 24\sqrt{2}\pi^2/\kappa, \quad (3.15)$$

$$m_6 = 24\pi^2/\kappa^2. \quad (3.16)$$
In the background produced by D1-branes the condition of the isotropic expansion leads to the (3.15) and (3.16).

In the case other backgrounds branes one can see from (3.12) that for \( p = 0 \) the expansion of the non-compact dimensions is much faster than compact dimensions or \( H_c = 0 \) which corresponds to the static compact space. For \( p > 2 \) the result is that compact dimensions expand faster than non-compact or \( H_n = 0 \) which gives static non-compact space.

4 Conclusions

In this paper we have obtained Hubble parameters for Dk-brane embedded in the backgrounds produced by the black p-branes. These parameters are related to the topology of the Dk-brane: the Dk-brane is represented as the Cartesian product of the \( n \)-dimensional non-compact space and some \( (n - k) \)-dimensional compact space (in our case this space is sphere). In general case these parameters have different values. It means that evolution from the point of view an observer fixed to the Dk-brane in the compact and non-compact directions is different. The ratio of these parameters has been obtained in explicit form for big values of \( r \). This ratio is equal to one only in one case for \( p = 1 \) (D-strings) and for special value of \( R = 2 \). It means that in asymptotic region \( (r \to \infty) \) and for \( R = 2 \) expansion is the same in all directions. In this case the mass and the topological charge are given by eqs. (3.15-3.16 ). The above results are valid if D3-brane and background branes does not form bounded system. It is true for sufficient big parameter \( E \). In general case the ratio \( \eta \) (eq. (3.4)) depends on the position of the D3-brane.

The considered model is an example of a toy cosmological model. The observed isotropic expansion of our universe is realized in this model as the condition on equality of Hubble parameters. This condition puts constraint on the allowed masses and charges of the background D6-branes which are dual to D1-branes.

Form the other side one can consider this world-volume expansion as driven by some fictitious fields (mirage cosmology [2]) and postulate that their energy densities obey the Friedmann equations:

\[
\begin{align*}
H_n^2 &= \rho_n(\Phi_n), \\
H_c^2 &= \rho_c(\Phi_c),
\end{align*}
\]

where \( \rho_n(\Phi_n) \) and \( \rho_c(\Phi_c) \) are the density energy produced by the fictitious scalar fields \( \Phi_n \) and \( \Phi_c \) respectively. These fields determinate cosmological evolution of the world-volume of D3-brane in non-compact and compact directions.

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