A Spectrum Sensing Method based on Antieigenvalues and Stochastic Resonance

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Abstract - Cognitive radio is an influential technology to solve the issue of spectrum scarcity caused by the massive number of wireless mobile devices. Spectrum sensing can alleviate interference situations among cognitive radio devices, and efficiently utilize the available radio spectrum. This paper proposes a novel spectrum sensing approach based on antieigenvalue and stochastic resonance. Stochastic resonance is used to enforce the detecting signals from multiple antennas in low SNR condition. Then the sample covariance matrix and its antieigenvalues of the enforcing detection signal are computed for constructing test statistic. The simulation results demonstrate that the proposed detector is superior than the existing antieigenvalue based detector, and is robust in strong noise background.

Index Terms - spectrum sensing, stochastic resonance, multiple antenna, antieigenvalue.

1. Introduction

Accompanied with the popularization of 5G technology and wireless mobile applications, the available spectrum resources have been decreased largely. Cognitive radio (CR) has been considered as a prospective scheme to relieve the pressure of shortage of spectral resources, which allows secondary users (SUs) to utilize the authorized and long term idle frequency band [1]. As a basic technology of CR, spectrum sensing makes SUs probe such spectrum holes accurately and fast. So SUs can opportunistically access licensed frequency band and do not generate harmful collision to primary users (PUs).

To overcome the shortcoming of estimating various wireless background knowledge, a massive number of spectrum sensing methods based on multiple antennas and eigenvalue is proposed, e.g., the maximum eigenvalue (ME) [2], the eigenvalue weighting (EW) [3]. However, calculating all eigenvalues leads to large computation complexity in the process of eigenvalues decomposition particularly when more antennas are deployed. Thus, the mean to square extreme eigenvalue (MSEE) [4] detector use only maximum and minimum of eigenvalues to alleviate the computational complexity. Considering the rank of sample covariance matrix of PUs, The antieigenvalue (AE) detector [5] is proposed by employing multiple large and small eigenvalues with partial eigen decomposition.

In the circumstance of low signal to noise ratio (SNR), eigenvalue based detector could only increase the antenna number to compensate the deterioration of spectrum sensing performance. Therefore, the design cost and complexity of wireless mobile device will be augmented. Here, stochastic resonance (SR) is an available scheme, which is a nonlinear phenomenon extracting the weak signal character buried in intensive background noise [6]. SR has been extensively applied to spectrum sensing in weak signal condition, e.g., energy detection based on fixed parameter SR and
adaptive parameter SR [6], detector based on particle swarm algorithm and tri-stable SR [7], detector based on optimal dynamic overdamped bistable SR [8], detector based on polarization and SR [9]. However, the classical SR theories point out that the input signal of SR system could only worked in low frequency and small parameters [6]. However, actual wireless communications applications always work in carrier wave of high frequency. So frequency shifting technologies of SR are often applied to convert high frequency to low frequency equivalently.

This paper designed a detector based on antieigenvalue and SR technology. Firstly, multiple antenna signals are processed via SR system, in which normalized scale transformation (NST) frequency shifting scheme [10] is exploited. Secondly, the proposed detector employs multiple small antieigenvalues according to eigen decomposition. Numerical simulations are provided to demonstrate the superior performance of the proposed AE-SR detector compared to AE detectors.

2. System model

Primary user (PU) signal denotes the authorized and available signal. The PU signal is captured by the secondary user (SU), which should be equipped with multiple antennas \(M > 1\). The binary hypothesis model is:

\[
\begin{align*}
H_0: \mathbf{r}(t) &= \mathbf{n}(t) \\
H_1: \mathbf{r}(t) &= \mathbf{h}(t)\mathbf{s}(t) + \mathbf{n}(t)
\end{align*}
\]

where \(H_0\) and \(H_1\) represent that the frequency band of PU signal is vacant or busy respectively. The observed PU signal \(\mathbf{s}(t)\) is a discrete time baseband signal. \(\mathbf{h}(t)\) denotes complex channel matrix, which is expressed as \(\mathbf{h}(t) \in C^M \times 1\). Let \(\mathbf{s}(t) = \mathbf{h}(t)\mathbf{s}(t)\). \(\mathbf{s}(t)\) is assumed to have mean zero and covariance \(R_s\). Under the effection of channel matrix \(\mathbf{h}(t)\), \(\mathbf{s}(t)\) is often exhibit the change of amplitude, frequency and phrase. Therefore, the rank \(R_p\) of covariance matrix \(R_s\) often varies from 1 to \(M\). \(\mathbf{n}(t) = [n_1(t), n_2(t), \ldots, n_M(t)]^T\) denotes the additive white noise with circularly symmetric complex Gaussian (CSCG) distribution with mean zero and covariance \(\sigma^2\), i.e., \(\mathbf{n}(t) \sim CN(0, \sigma^2 \mathbf{I}_M)\). supposing that \(\mathbf{s}(t)\) and \(\mathbf{n}(t)\) are independent from each other. In actual wireless communication system, the covariance matrix \(R_s\) of PU signal has a bit number of larger eigenvalues [11]. for the signal vector \(\mathbf{r}(t) = [r_1(t), r_2(t), \ldots, r_M(t)]^T\), one antenna component is statistically independent from each other.

3. Proposed Method: AE-SR

To recover the original signal furthest from intensive noise, this section considers that the received signal in each antenna \(r_i\) will subsequently passes through the stochastic resonance (SR) system. The output signal is defined as a vector \(\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T = f(\mathbf{r}(t))\), where \(f(.)\) is a nonlinear function of SR, which can be defined as the following Langevin equation [6]:

\[
\frac{dx_i(t)}{dt} = -\frac{U(x_i)}{dx_i} + \bar{s}_i(t) + n_i(t)
\]

where \(a > 0\) and \(b > 0\); \(U(x_i)\) denotes the potential function: \(U(x_i) = -a \frac{1}{2} x_i^2 + \frac{b}{4} x_i^4\); \(\bar{s}_i(t) = h_i(t)A_1\cos(2\pi f_0 t)\); \(i\) denotes the \(i\)th antenna; \(A_1\) and \(f_c\) denotes amplitude and carrier frequency of weak signal. For high frequency signal, \(f_c \gg 1\) Hz, which is opposite with the classical adiabatic approximation and nonlinear response theory [6]. Thus, we exploit the normalized scale transformation (NST) method is to convert high frequency to low frequency. Firstly, the NST theory is reviewed and the variable substitutions are introduced as [10]

\[
\frac{dz_i(\tau)}{dt} = z_i - z_i^3 + h_i(\tau)A_0\cos(2\pi f_0 \tau) + n_0(\tau)
\]

where \(z = x\sqrt{a/b}\); \(\tau = at\); \(A_0 = \sqrt{b/a^2} A_1\) is the normalized amplitude; \(f_0 = f_c/a\) is the normalized frequency; \(n_0(\tau) = \sqrt{b/a^2} n_i(\tau)\) is the normalized noise with expectation 0 and variance \(\sigma_0^2 = \frac{b}{a^2} \sigma^2\). equation (3) is the standard normalized form of equation (2). They have the same dynamic
characteristics; however, the main significances and contributions lie in that equation (3) can satisfy the preconditions of the adiabatic approximation theory. Through the preset of \( f_0 \) and \( A_0 \), \( a \) and \( b \) could be obtained. In addition, adjusting \( A_0 \) based on input SNR will achieve the desired output state. In general, equation (3) is an expression form of one order ordinary differential equation, thus the exact solutions can not be obtained. However, it can be approximately solved by the fourth order Runge-Kutta (RK) as giving a numerical solution [6].

After the output signal of SR is achieved, the sample covariance matrix of the SR output signal will be calculated as [5]:

\[
\mathbf{R}_x = E[\mathbf{x} \mathbf{x}^T] = \frac{1}{N_s} \sum_{t=0}^{N_s-1} \mathbf{x}(t) \mathbf{x}(t)^T. 
\]

where \( N_s \) is the number of observed samples. The eigenvalues of \( \mathbf{R}_x \) are denoted as \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \). The antieigenvalue \( \nu_k \) is given by:

\[
\nu_k = 2\sqrt{\lambda_k \lambda_{M-k+1}} / (\lambda_k + \lambda_{M-k+1}), \quad k \leq M/2. 
\]

Supposing the smallest antieigenvalues number is \( K \), the test statistic is obtained as:

\[
T_{AE-SR} = \sum_{k=1}^{K} \nu_k. 
\]

4. Simulation Results

This section compared the performances of antieigenvalue and stochastic resonance detector (AE-SR) and antieigenvalue detector (AE). The PU signal is a sinusoidal wave and the signal amplitude is \( A_m = 3 \); The signal carrier frequency is \( f_c = 10 \) Hz; The normalized frequency is \( f_0 = 0.01 \) Hz; the sampling frequency is \( f_s = 5 \) MHz; The signal sample number is \( N_s = 1536 \); The normalized noise power is \( P_{n0} = 2 \); The false alarm probability is \( P_f = 0.1 \); The Monte Carlo simulation is 10000 times. The rank of sample covariance Matrix of PU signal is \( Rp = 5 \). The smallest antieigenvalue number is \( K = 5 \), the simulation result is shown as follows:

![Figure 1] The input and output signal of SR

![Figure 2] Detection probability versus SNR under various detectors and antenna number

![Figure 3] Detection probability versus \( Rp \) under various antieigenvalue number for AE-SR detector

![Figure 4] Detection probability versus \( P_{n0} \) under various smaller antieigenvalue number for AE detector
Figure 1 shows the input and output signal of SR, the weak signal is enforced obviously. Figure 2 shows the receiver operator character (ROC) courves of detection probability $P_d$ versus SNR ranged from $[-30, -13]$ dB. In which, different detector (AE, AE-SR) and different antenna number $M$ are choosen for testing. It can be found that the detection probability of the AE-SR detection approach is higher than those of the AE detector under low SNR circumstances. Meanwhile, accompany with the increase of antenna number, the detection probability is enhanced in AE and AE-SR. Figure 3 shows the AE-SR ROC courves of detection probability $P_d$ versus $R_p$ ranged from $[1, 10]$. In which, different antieigenvalue number $K$ ranged from $[1, 5]$ is choosen for testing; $SNR = -24$ dB and $M = 10$. It can be found that the detection probability of AE-SR is improved overall when $K$ is increased. Meanwhile, for constant value $K$, the increase of $R_p$ will give rise to The decreasedment of $P_d$ overall. It indicates that suitable parameter $K$ should be set to overcome the difficulties caused by $R_p$. That is: when $R_p \leq M/2, K = R_p$; when $M/2 < R_p \leq M, K = M/2$.

Figure 4 shows detection probability $P_d$ of AE-SR versus $P_{n_0}$ ranged from $[1, 10]$. In which, different $SNR$ varies from $[-30, -27, -24]$ dB are choosen for testing. It can be found that, more extensive noise power will deteriorate the detection performance of AE-SR for any $P_{n_0}$. In addition, when $P_{n_0} \leq 4$, the increase of $P_{n_0}$ will improve $P_d$. When $P_{n_0} > 4$, $P_d$ will converge to a stable value. Therefore, $P_{n_0} = 4$ is a suitable setting. extra large $P_{n_0}$ is nonsense because it will produce chaos in SR system.

5. Conclusion

This paper proposes a spectrum sensing method based on antieigenvalue and stochastic resonance (AE-SR). SR is adopted to achieve a better output signal for the received signals from multiple antennae. Normalized scale transformation technology is utilized to transform the high frequency application to low frequency. A test statistic is constructed according to the antieigenvalues of the covariance matrix. The simulation results validate the effectiveness of the proposed AE-SR detector in real applications, particularly under low SNR circumstances. This approach is significant for implementing and enlarging the application of CR networks.

Acknowledgments

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References

[1] Mitola, J., Iii and Maguire, G. Q., Jr. (1999) Cognitive radio: making software radios more personal. IEEE Pers Commun, vol. 6, pp. 13-18.
[2] Zan, L., Wang, D., Qi, P., and Hao, B. (2016) Maximum Eigenvalue Based Sensing and Power Recognition for Multi-Antenna Cognitive Radio System. IEEE Transactions on Vehicular Technology, vol. 65, pp. 8218-8229.
[3] Chang, L., Li, H., Jie, W., and Jin, M. (2017) Optimal Eigenvalue Weighting Detections for Multi-antenna Cognitive Radio Networks. IEEE Transactions on Wireless Communications, vol. 16, pp. 2083-2096.
[4] Bouallegue, K., Dayoub, I., Gharbi, M., and Hassan, K. (2017) Blind spectrum sensing using extreme eigenvalues for cognitive radio networks. IEEE Communications Letters, vol. 22, pp. 1386 - 1389.
[5] Guo, C., Jin, M., Guo, Q. H., and Li, Y. M. (2019) Antieigenvalue-Based Spectrum Sensing for Cognitive Radio. Ieee Wireless Communications Letters, vol. 8, pp. 544-547.
[6] Wang, J., Ren, X., Zhang, S. W., Zhang, D. M., Li, H. S., and Li, S. Q. (2014) Adaptive Bistable Stochastic Resonance Aided Spectrum Sensing. IEEE Transactions on Wireless Communications, vol. 13, pp. 4014-4024.
[7] Lu, J., Huang, M., and Yang, J. J. (2017) A Novel Spectrum Sensing Method Based on Tri-Stable Stochastic Resonance and Quantum Particle Swarm Optimization. Wireless Personal Communications, vol. 95, pp. 1-13.
[8] He, D., Chen, X., Pei, L., Jiang, L. G., and Yu, W. X. (2019) Improvement of Noise Uncertainty and Signal-To-Noise Ratio Wall in Spectrum Sensing Based on Optimal Stochastic Resonance. *Sensors*, vol. 19.

[9] Lu, J., Huang, M., and Yang, J. (2019) Study of Polarization Spectrum Sensing based on Stochastic Resonance in Partial Polarized Noise. *Wireless Networks*, vol. 25, pp. 4991-4999.

[10] Huang, D., Yang, J., Zhang, J., and Liu, H. (2018) An improved adaptive stochastic resonance with general scale transformation to extract high-frequency characteristics in strong noise. *International Journal of Modern Physics B*, vol. 32, pp. 185-205.