New Insights into Uncertainties in the Relic Neutrino Background and Effects from the Nuclear Equation of State

G. J. Mathews and L. Boccioli
Center for Astrophysics, Department of Physics, University of Notre Dame, Notre Dame, IN 46556, USA
gmathews@nd.edu, lbocciol@nd.edu

J. Hidaka
Mechanical Engineering Department, Meisei University, Tokyo 191-8506, Japan
National Astronomical Observatory of Japan Tokyo, 181-8588, Japan
jun.hidaka@meisei-u.ac.jp

T. Kajino
National Astronomical Observatory of Japan Tokyo, 181-8588, Japan
International Research Center for Big-Bang Cosmology and Element Genesis, and School of Physics, Beihang University, Beijing 100083, Peoples Republic of China
Graduate School of Science, The University of Tokyo, Tokyo, 113-0033, Japan
kajino@buaa.edu.cn, kajino@nao.ac.jp

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We review the computation of and associated uncertainties in the current understanding of the relic neutrino background due to core-collapse supernovae, black hole formation and neutron-star merger events. We consider the current status of uncertainties due to the nuclear equation of state (EoS), the progenitor masses, the source supernova neutrino spectrum, the cosmological star formation rate, the stellar initial mass function, neutrino oscillations, and neutrino self-interactions. We summarize the current viability of future neutrino detectors to distinguish the nuclear EoS and the temperature of supernova neutrinos via the detected relic supernova neutrino spectrum.

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1. Introduction

For a number of years there has been interest\textsuperscript{1,20} in the possibility of detecting the cosmic background due to supernova relic neutrinos (SRN). Massive stars ($M \geq 8 M_\odot$) culminate their evolution as either a core collapse supernovae (CCSNe) or as a failed supernova (fSNe) leading to a black-hole remnant. In either case, such explosive events are expected to produce an intense flux of neutrinos with total
energy of order $\sim 10^{53}$ ergs emitted over an interval of $\sim 10$ s for CCSNe or $\sim 1$ s for fSNe. There are also contributions to the SNR background from neutrinos produced in neutron star merger events (NSMs) associated with neutron-star + neutron-star binaries or black-hole + neutron star binaries.

Neutrinos are weakly interacting elementary particles, and therefore can emerge from within the interior of CCSNe, fSNe, and NSMs. As such, neutrinos can provide information regarding the physical processes that take place inside these explosive environments. The detection of this diffuse relic neutrino background and the associated energy spectra could provide information on neutrino properties such as flavor oscillations and/or the neutrino temperatures produced in supernova explosions. One may even be able to discern the shock revival time from the detected spectrum of relic neutrinos. Moreover, these neutrino emission processes have continued over the history of the Galaxy. Hence, a detection of the SRN background will not only probe the physics of CCSNe, fSNe, and NSMs, but also the history of explosive events in the Galaxy.

The prospect for detecting the SRN background in the near future seems possible due to plans for a next generation Hyper-Kamiokande detector with a mega-ton of pure water. Moreover, results from the Super-Kamiokande collaboration already place some marginal limits on the total neutrino energies and temperatures from the background spectrum of supernova relic neutrinos. The possibility of a liquid scintillation detector has also been discussed.

However, as analyzed in Refs. theoretical predictions of the detection rate of supernova relic neutrinos are still subject to a number of uncertainties. These uncertainties include the time dependence of the stellar initial mass function, the star formation rate and/or supernova rate, the roles of neutrino oscillations and/or neutrino self interaction, and the spectral energy distributions of the three flavors of supernova neutrinos. These in turn depend upon the explosion mechanism, whether the final remnant is a neutron star or black hole, the equation of state (EoS) for proto-neutron stars, and the possibility of neutrino oscillations and/or self interactions, etc. In this review we summarize the current status of some of these uncertainties.

2. The Relic Neutrino Background

The presently arriving cosmic diffuse neutrino flux spectrum $dN_\nu/dE_\nu$ can be derived from an integral over the cosmic redshift $z$ of the neutrinos emitted per event (SN, or NSM) and the cosmic event rate per redshift per comoving volume ($R_{SN}(z)$ and $R_{NSM}(z)$):

$$
\frac{dN_\nu}{dE_\nu} = \frac{c}{H_0} \int_0^{z_{\text{max}}} \left[ R_{SN}(z) \frac{dN^{SN}_\nu(E'_\nu)}{dE'_\nu} + R_{NSM}(z) \frac{dN^{NSM}_\nu(E'_\nu)}{dE'_\nu} \right] \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}},
$$

(1)
where \( z_{\text{max}} \approx 5 \) is the redshift at which star formation begins. The various contributions to \( R_{\text{SN}} \) are described below, while \( R_{\text{NSM}} \) is the corresponding number of NSM events. The quantities \( dN_{\nu}^{\text{SN}}(E'_{\nu})/dE'_{\nu} \) and \( dN_{\nu}^{\text{NSM}}(E'_{\nu})/dE'_{\nu} \) are the emitted neutrino spectra at the various sources, where the energy \( E'_{\nu} = (1 + z)E_{\nu} \) is the energy at emission, while \( E_{\nu} \) is the redshifted energy to be observed in the detector.

In addition to integrating over the range of progenitor models to determine \( R_{\text{SN}} \) for normal CCSNe, one should add the neutrino emission from ONeMg electron-capture supernovae, the neutrino emission from the collapse of massive stars that form black holes, and the contribution from NSMs. The contribution from NSMs is small but potentially interesting. The reader is referred to Refs. [21, 22] for discussions of this possibility. For the remainder of this review we will only consider contributions from various classes of supernovae or black-hole formation.

Figure 1 from Ref. [15] illustrates the arriving neutrino flux from different supernova sources as labelled. This figure is for the case of no neutrino oscillations (Case III as defined below). We next review the various factors in Eq. (1) in detail.

### 2.1. Core-Collapse Supernova Rate \( R_{\text{SN}} \)

The core-collapse supernova rate \( R_{\text{SN}} \) can be determined from the total star formation rate (SFR) [i.e. total mass in stars formed per year per comoving volume] if one determines the fraction of total stars that produce each observable class of supernovae or other sources. The general form for the supernova rate at a given redshift \( z \) is related to the star formation rate \( \psi_s(z) \) and the initial mass function.
(IMF) $\phi(M)$.

$$R_{SN}(z) = \psi(z) \times \frac{\int_{M_{min}}^{M_{max}} dM \phi(M)}{\int_{M_{min}}^{M_{max}} dM M \phi(M)},$$

(2)

where $\text{min}$ and $\text{max}$ denote the range of progenitors for supernovae. As described below this may be a sum of segments that lead to CCSNe or fSNE separately. The quantities $M_{min}$ and $M_{max}$ refer to the range over all stars formed.

It is often assumed\textsuperscript{28} that all massive progenitors from 8 to 40 $M_\odot$ lead to visible SNe. The higher end of this mass range corresponds to SNIb,c, while the lower end corresponds to normal SNII.

In Ref.\textsuperscript{15}, however, it was pointed out contributions from both normal core-collapse SNII and SNIb,c should be considered separately, along with the possibility of fSNe forming a black hole and dim NeOMg supernovae in the low-mass end. The SNIb,c events accounts for about $25 \pm 10\%$ of observed core-collapse supernova rate at redshift near $z = 0$.\textsuperscript{29} Hence, for the total observed core-collapse SN rate one can write:

$$R_{SN}(z) = R_{SNII}(z) + R_{SNb,c}(z)$$

(3)

Nevertheless, there is large uncertainty in the mass ranges of CCSNe and the degree to which progenitors have experienced mass loss and the true initial progenitor mass could be greater.

Observational evidence for the mass of SN progenitors has been obtained\textsuperscript{29,30} by comparing sky images of nearby SNe before and after explosion. This suggests that the maximum mass of the red supergiant progenitors of core-collapse SNe may be as small as 16.5 $M_\odot$. This is the so-called ”red supergiant (RSG) problem”\textsuperscript{29,30} A subsequent study\textsuperscript{31} however, suggested that this value could be as much as 18 $M_\odot$. A similar range was deduced in Ref.\textsuperscript{32} who argued that this limit is closely related to the transition from convective to radiative transfer during the central carbon-burning phase. Hence, 16.5 to 18 $M_\odot$ was adopted in Ref.\textsuperscript{16} as a reasonable range of the uncertainty in the lower mass limit for fSNe.

It is also believed\textsuperscript{33} that initial progenitor stars in the mass range of 8-10 $M_\odot$ collapse as an electron degenerate ONeMg core and do not produce bright supernovae. Hence, one should consider a lower limit to the progenitor of normal CCSNe of 10 $M_\odot$ when allowing for the possibility of ONeMg SNe.

Regarding SNIb,c, although it is known\textsuperscript{29} that they are associated with massive star forming regions, there is some ambiguity as to their progenitors. There are two theoretical possibilities. Refs.\textsuperscript{33,34} have studied the possibility that massive stars ($M \sim 25 – 100 M_\odot$) can shed their outer envelope by radiative driven winds leading to bright SNIb,c supernovae. This source for SNIb,c was adopted in\textsuperscript{25} by assuming that all stars with ($M \sim (25 – 40) M_\odot$) explode as SNIb,c, while Ref.\textsuperscript{23} adopted a range of (25-50) $M_\odot$ for SNIb,c. Changing the upper mass limit from 40 $M_\odot$ to 50 $M_\odot$, however, makes little difference (1%) in the inferred total core-collapse supernova rate.\textsuperscript{15}
Theoretically, however, this single massive-star paradigm does not occur as visible SNIb,c supernovae until well above solar metallicity. At lower metallicity most massive stars with $M > 25 M_\odot$ collapse as failed supernovae with black hole remnants. Hence, this mechanism is not likely to contribute to the observed SNIb,c rate at higher redshifts. On the other hand, it has been estimated that 15 to 30% of massive stars up to $\sim 40 M_\odot$ are members of interacting binaries that can shed their outer envelopes by Roche-lobe overflow leading to bright SNIb,c supernovae. That hypothesis was considered in Ref. [15]. That is, a fraction $f_b$ of all massive stars in the range of $(8$ to $40) M_\odot$ could result in bright SNIb,c via binary interaction.

2.2. Failed Supernovae

It is usually presumed that all single massive stars with progenitor masses with $M > 25 M_\odot$ end their lives as failed supernovae fSNe. However, this cutoff is uncertain. For single stars above this cutoff mass the whole star collapses leading to a black hole and no detectable supernovae. Allowing for a fraction $f_b \sim 25\%$ of stars with $M = (25 - 40) M_\odot$ to form SNIb,c by binary interaction, then the rate for failed supernovae is given by:

$$R(\text{fSN}) = (1 - f_b) \psi_*(z) \times \frac{\int_{M_{\min}}^{M_{\max}} dM \phi(M)}{\int_{M_{\min}}^{M_{\max}} dMM \phi(M)} + \psi_*(z) \times \frac{\int_{M_{\min}}^{125 M_\odot} dM \phi(M)}{\int_{M_{\min}}^{M_{\max}} dMM \phi(M)}.$$  (4)

There appears to be a non-monotonic dependence of the success of the explosion on progenitor mass. These numerical investigations suggest that the compactness of the progenitor core just prior to collapse is a good indicator of the success of the explosion, although the onset of supernova turbulence may be a better indicator. A correct estimate of the relic neutrino background should include segmented ranges for failed supernovae and successful supernovae in the range from 10 to 25 $M_\odot$.

2.3. ONeMg Supernovae

ONeMg (or electron-capture) supernovae involve explosion energies and luminosities that are an order of magnitude less than normal core collapse supernovae. Hence, they may go undetected. ONeMg supernovae are believed to arise from progenitor masses in the range of $8 M_\odot \leq M \leq 10 M_\odot$. Such supernovae occur when an electron-degenerate ONeMg core reaches a critical density (at $M_{\text{core}} \sim 1.4 M_\odot$). In this case the electron Fermi energy exceeds the threshold for electron capture on $^{24}\text{Mg}$. These electron captures cause a loss of hydrostatic support from the electrons and also a heating of the material. Ultimately, this leads to a dynamical collapse and the ignition of an O-Ne-Mg burning front that consumes the star.
2.4. Star formation rate

The cosmic star formation rate $\psi^*(z)$ is a key component of theoretical estimates of the SRN background. Determining the SFR is an involved procedure based upon different sources, e.g., UV light from galaxies and the far infrared (FIR) luminosity. It also depends upon modeling fits to the spectral energy distribution (SED). Uncertainties from this procedure were analyzed in Refs. [15, 17]. The star formation rate also depends upon the choice of the stellar initial mass function (IMF).

It was noted in Ref. [28] that the measured core-collapse supernova rate $R_{SN(Obs)}$ in the redshift range $0 \leq z \leq 1$ appears to be about a factor of two smaller than the core-collapse supernova rate deduced from the measured cosmic massive-star formation rate (SFR). This is the so-called supernova rate problem. One possible solution to this involves uncertainty in the inferred supernova rate out to redshift $z \sim 1$. Another possible explanation is from the uncertainty in the lower mass limit for the progenitors of CCSNe.

A commonly adopted form for a fit to the SFR is the Madau formula given by:

$$\psi^*(z) = \frac{a(1+z)^b}{1 + [(1+z)/d]c} M_\odot \text{year}^{-1} \text{Mpc}^{-3},$$

(5)

where $a = 0.015$, $b = 2.7$, $c = 5.6$, and $d = 2.9$.

Another convenient functional form for the SFR is conveniently parameterized as a piecewise linear fit to the observed star formation rate vs. redshift:

$$\psi^*(z) = \dot{\rho}_0 [(1+z)^{\alpha \eta} + \frac{1+z}{B}^{\beta \eta} + \frac{1+z}{C}^{\gamma \eta}]^{\frac{1}{\eta}}.$$

(6)

Parameters adopted in Ref. [27] are $\dot{\rho}_0 = 0.017 \, h_{73}^3 \, M_\odot \, \text{Mpc}^{-3} \, \text{yr}^{-1}$ for the cosmic SFR at $z = 0$, as well as the parametrization $a = 3.4$, $b = -0.3$, $c = -3.5$, $z_1 = 1$, $z_2 = 4$, and $\eta = -10$. A somewhat different parameterization was deduced in [15] based upon different data at low redshift. The piecewise fit from [15] is illustrated in Figure 2. The fit from [27] is shown by the dotted line.

Most evaluations of the SFR are based upon the UV luminosity that is mainly due to OB stars. The dust surrounding galaxies, however, always complicates the evaluation of the UV. There is a claim that the SFR based upon UV light is underestimated especially for starburst galaxies. A method based upon SED modeling has been proposed that takes into account the star formation embedded in dense molecular clouds. This study indicated that the SFR for $z > 3.5$ is higher by a factor of 2 to 3 than the estimate from UV light. Effects of this were analyzed in [17].

In Ref. [15] three different versions of the star formation rates were studied as a means to estimate the overall uncertainty in the SFR dependence. One was a revised SFR based upon a piecewise linear fit to the observed star formation rate vs. redshift. They also considered SFR models with and without corrections for dust extinction.
Fig. 2. Piecewise linear star formation rate function from a fit to the subset of observed dust-corrected data. Red, blue, magenta, light blue, and green points show the observed data in IR, optical, UV, X-ray / γ-ray, and radio bands, respectively. Red lines show the SFR as function of redshift $z$ deduced from $\chi^2$ fitting, along with the ±1σ upper and lower limits to the SFR. The reduced $\chi^2$ for the fit is 2.3. The black dotted line shows the previous SFR.

2.5. Initial mass function $\phi(M)$

A convincing theory for the universal IMF has not yet been established. A remaining uncertainty, for example, is the metallicity dependence of the IMF. In Ref. [17], the dependence of the detected neutrino spectrum on a metallicity dependent IMF and other changes in the IMF were analyzed. A broken power-law Salpeter A IMF (Sal-A) is often assumed,

$$\phi(M) = M^{-\zeta},$$

with $\zeta = 2.35$ for stars with $M \geq 0.5 \, M_\odot$ and $\zeta = 1.5$ for $0.1 \, M_\odot < M < 0.5 \, M_\odot$.

Most of the popular IMFs including the Sal-A are observationally deduced for stars in the Milky Way. There are also theoretical derivations of the power-law index for the IMF. For example, a power law IMF is explained by the turbulent motion of molecular clouds in a magnetic field.

The relationship between metallicity and the IMF has also been considered in [17]. The IMF in metal poor environments could favor higher mass stars because molecular clouds may not sufficiently cool to allow gravitational collapse on small scales. Ref. [17] explored the IMF-metallicity relationship by adopting a cosmological metallicity evolution. This was based upon observational evidence that IMF
evolution occurs in low-z galaxies for which the IMF-Metallicity relationship obeys
\[ \zeta = 2.2(\pm 0.1) + 3.1(\pm 0.5) \times [M/H] \]  
\[ \quad \text{(8)} \]

Metallicity evolution is closely related with galaxy evolution. However, both the stellar and gas phase metallicity must be accurately considered. Ref. [49] studied the galaxy mass-metallicity relation based upon cosmological zoom-in simulations that include gas inflow and outflow to estimate the metallicity evolution accurately. They found \[ [M/H] = 0.4[\log(M_*/M_\odot) - 10] + 0.67\exp(-0.50z) - 1.04 \]
and \[ [M/H] = 0.35[\log(M_*/M_\odot) - 10] + 0.93\exp(-0.43z) - 1.05 \]. In addition, galactic mass evolution is expected, and the average of \( M_\star \) depends upon the redshift \( z \). The galaxy cosmological mass function can be expressed as \[ \langle M_\star \rangle = 10^{11}(1 + z)^{-0.58}[M_\odot] \]. These two studies have been used to deduce the cosmic metallicity evolution and deduce a \( z \)-dependent IMF along to evaluate the relic SN neutrino spectrum.

3. Source Neutrino Spectrum \( (dN^{SN}_\nu(E'_\nu)/dE'_\nu) \)

The uncertainty due to variations of estimates of the emergent neutrino spectra from core collapse supernovae was analyzed in [15]. It is usually assumed that the dependence on progenitor mass is small compared to the uncertainty in the neutrino temperatures themselves. The reason for this is that the mass of most observed neutron-star supernova remnants is rather narrowly constrained to be \( \sim 1.4M_\odot \). This narrow range of observed remnant neutron star masses suggests that the associated neutrino temperatures are also tightly constrained. Hence, it is reasonable to adopt a SN 1987A model (i.e. progenitor mass \( \sim 16.2M_\odot \), remnant mass \( \sim 1.4M_\odot \), liberated binding energy \( \sim 3.0 \times 10^{53}\text{ erg} \) [32]). Nevertheless, an evaluation of the validity of the assumption that this SN1987A model is representative of every core-collapse SN with progenitor masses in the range of 8 to 25 \( M_\odot \) and also every SNIb,c over the mass range of 8-40 \( M_\odot \) in any of the possible paradigms needs to be done.

The liberated binding energy is divided roughly equally among the 6 neutrino species (3 flavors and their anti-particles). Even in the context of a single progenitor model, however, there has been a range of predicted neutrino temperatures and chemical potentials in various supernova core-collapse simulations in the literature. These have been analyzed as a means to estimate the uncertainty in the neutrino detection rate due to the source neutrino spectra.

In Ref. [15] it was concluded that the uncertainty (\( \sim \pm 50\% \)) in the neutrino temperatures constitutes one of the largest uncertainties in the expected relic neutrino detection rate. However, this conclusion was based upon a sampling of many older calculations in the literature. The most modern supernova collapse simulations, however, all tend to predict lower uniform neutrino temperatures. Figure 3 shows a comparison of a benchmark study with a number of modern spherical 1D
Fig. 3. Comparison of neutrino luminosity light curves predicted by various CCSN codes in the literature. There is remarkable agreement among most codes except the NDL prediction. Presumably this relates to the use of more modern neutrino interaction rates in the newer codes.

SN codes plus our own results based upon the older general relativistic Wilson (NDL) code.

In previous works, the neutrino spectra were presumed to obey a Fermi-Dirac spectrum with temperatures determined from fits to various models in the literature, including models with high temperature such as the NDL EoS. Figure shows, however, that there is now remarkable similarity among the neutrino spectra for all of the modern codes except for the older CCSN models like the NDL. This difference can be traced to the use of better neutrino interaction rates in the newer codes and suggests that there is less temperature uncertainty in the SRN spectrum than was deduced in Ref. [15].

4. SRN Detection Rate

A Hyper-Kamiokande next-generation Čerenkov detector has been proposed consisting of a mega-ton of pure water laden with 0.2% GdCl₃ to reduce the background. The detected SRN energy spectrum in such a detector can be written as:

\[ \frac{dN_{\text{event}}}{dE_{e^+}} = N_{\text{target}} \cdot \varepsilon(E_\nu) \cdot \frac{1}{c} \cdot \frac{dF_\nu}{dE_\nu} \cdot \sigma(E_\nu) \cdot \frac{dE_\nu}{dE_{e^+}} \]  

(9)

where \( N_{\text{target}} \) is the number of target particles in the water Čerenkov detector, \( \varepsilon(E_\nu) \) is the efficiency for neutrino detection, \( dF_\nu/dE_\nu \) is the incident flux of SRNs, and \( \sigma(E_\nu) \) is the cross section for neutrino absorption: \( \bar{\nu}_e + p \rightarrow e^+ + n \), and \( E_\nu = E_{e^+} + 1.3 \text{ MeV} \). For simplicity one can set \( \varepsilon(E_\nu) = 1.0 \), and we use the cross sections given in [57] when calculating the reaction rate of the SRN with detector target material.

One only needs to consider the detection of \( \bar{\nu}_e \), because the cross section in a water Čerenkov detector for the \( \bar{\nu}_e + p \rightarrow e^+ + n \) reaction is about \( 10^2 \) times larger.
Fig. 4. Predicted $e^+$ energy spectra and uncertainties\cite{10} in the total SRN detections for the case of no neutrino oscillations. The uncertainty bands are based upon the observed cosmic SFR and the detector statistics. The results for two different SNe models based upon a stiff Shen\cite{59} EoS and the soft LS EoS\cite{58} are included. The grey shaded energy range below 10 MeV indicates the region where the background noise due to reactor $\bar{\nu}_e$ may dominate. The shaded energy range that intersects the spectrum at $\sim$ 30 to 46 MeV indicates the region where the background may be dominated by noise from atmospheric neutrinos. Rectangular regions represent the peak of the spectrum and the locations where the neutrino signals overwhelm the atmospheric neutrino noise.

than that for $\nu_e$ detection via $\nu_e + n \rightarrow e^- + p$. The threshold detection energy for SRN is $\sim$ 10 MeV due to the existence of background $\bar{\nu}_e$ emitted from terrestrial nuclear reactors. An upper detection limit is $\sim$ 30 to 40 MeV due to the existence of noise from the atmospheric $\bar{\nu}_e$ background.

An illustration of the detector SRN spectrum from Ref.\cite{17} is shown in Figure 4. This shows the anticipated number of events after 10 years run time. The vertical shaded region below 10 MeV illustrates the background due to terrestrial reactors. The lower shaded region indicates the expected background noise due to atmospheric neutrinos. The shaded bands indicate the uncertainty in the predicted neutrino signal from the uncertainty in the SFR and the detector statistics. It was suggested in Ref.\cite{17} that the largest uncertainty may be due to different models for the equation of state.

4.1. New insights into the Nuclear EoS Uncertainty

The results shown in Figure 4 and the EoS uncertainty deduced in Refs.\cite{15,16,17} are based upon the effects of two extreme equations of state. One is the very soft Thomas-Fermi plus Liquid Drop Model EoS of Lattimer and Swesty\cite{58} (for a
nuclear compression modulus of \( K = 180 \). The other is the rather stiff Relativistic Mean Field (RMF) 1998 model Shen et al.\(^{59}\) These were chosen because there were available calculations of the neutrino spectra from both CCSNe and SNe.\(^{60,61}\)

It should be noted, however, that there are now more stringent observational limits on the EoS than when those two formulations were developed. For one, there are now firm determinations\(^{62,63}\) of binary neutron star systems for which a neutron star mass as large as \( 2.01 \pm 0.04 \, M_\odot \) is obtained.\(^{64}\) More recently, the mass of a neutron star in the MSP J0740+6620 system was measured\(^{65}\) to be \( 2.17^{+0.11}_{-0.10} \, M_\odot \). Such large neutron star masses cannot be achieved with as EoS as soft as that of the Lattimer & Swesty (K=180) formulation.

Moreover, another important development is the observation by the LIGO and VIRGO Scientific collaboration of gravity waves from the binary neutron-star merger GW170817.\(^{66}\) Subsequent analysis\(^{67}\) in which both stars are required to have the same EoS places strong limits on the tidal polarizability which in turn depends upon the neutron star radius to the 5th power. Indeed, only an EoS for which the neutron-star radius of a 1.4 \( M_\odot \) is less than 13.3 km is required at the 90% confidence limit. A separate analysis\(^{68}\) of neutron star atmospheres concludes that the neutron star radius must be less than about 12.5 km. Such compact stars are incompatible with the stiff 1998 RMF Shen EoS.

Indeed, the set of equations of state that can satisfy both the neutron-star maximum mass constraint and the neutron-star radius constraint is rather limited.\(^{69}\) This implies that there is probably less EoS uncertainty on the relic neutrino spectrum than that deduced in Refs.\(^{15,16,17}\). Indeed, a new analysis of the uncertainties in the observed relic neutrino spectrum is warranted based upon these constraints. As an illustration, Figure 5 shows a new series of calculations\(^{69}\) of the emergent electron neutrino spectrum from a set of EoSs\(^{58,69–71}\) that are mostly consistent with the neutron star maximum mass requirement. Calculations were run with the modern fully general-relativistic spherically-symmetric open-source code GR1D.\(^{72}\) Neutrino luminosities were computed using the open-source library NuLib.\(^{73}\) The progenitor is the 20 \( M_\odot \) star calculated by Farmer et al. 2016.\(^{76}\) Note that HShen refers to the 2011 version of the RMF EoS\(^{69}\) not the 1998 version used in Refs.\(^{15,16,17}\). Similarly, LS220 denotes the compression modulus \( K = 220 \) version of the Lattimer and Swesty 1991 EoS, not the \( K = 180 \) version used previously\(^{15,17}\). Here one can see that there is less uncertainty the source luminosity from the newer equations of state that satisfy the neutron star maximum mass constraint. However, the LS220 and HShen equations of state keep the neutrino luminosity high for longer times. A similar conclusion has been obtained in Ref.\(^{74}\) for the same progenitor and GR1D CCSN model, but for 97 new Skyrme parameterized EoSs all of which satisfy observational constraints, yet show very little dispersion in the neutrino spectra.
4.2. Dependence on Neutrino Oscillations

It is known that the flavor eigenstates for the neutrino are not identical to the mass eigenstates. The flavor eigenstates $\nu_e$, $\nu_\mu$, $\nu_\tau$ are related to the mass eigenstates by a unitary matrix $U$:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

(10)

where the matrix $U$ can be decomposed as:

$$
U =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
$$

(11)
Here, $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, and $\theta_{ij}$ is the mixing angle between neutrinos with mass eigenstates $i$ and $j$, and $\delta$ represents the CP phase.

For the mixing of mass eigenstates one must consider two cases: 1) the normal mass hierarchy, $m_1 < m_2 < m_3$; and 2) the inverted mass hierarchy, $m_3 < m_1 < m_2$. Both models have two resonance mass regions. The resonance at higher density is called the H-resonance, and the one at lower density is called the L-resonance. In the normal mass hierarchy, both resonances are in the neutrino sector. In the inverted mass hierarchy, however, one resonance is in the neutrino sector while the other is in the anti-neutrino sector.

The prospects for detecting effects of neutrino flavor oscillations in the spectrum of detected SRNs has been discussed using a parameterized form for the emitted neutrino spectrum from Ref. [77], and also in Ref. [12] using the supernova simulations of the Basel group. In Refs. [15, 16, 17] the limit of 3 neutrino oscillation paradigms were considered according to Ref. [79].

If one assumes an efficient conversion probability $P_H$ of $\bar{\nu}_3 \leftrightarrow \bar{\nu}_1$ at the H-resonance in the inverted hierarchy, then the $\bar{\nu}_e$ flux emitted from CCSNe becomes:

$$\phi_{\bar{\nu}_e} = |U_{e1}|^2 \phi_{\bar{\nu}_1} + |U_{e2}|^2 \phi_{\bar{\nu}_2} + |U_{e3}|^2 \phi_{\bar{\nu}_3}
= |U_{e1}|^2 \{(1 - P_H)\phi_{\bar{\nu}_1}(0) + P_H \phi_{\bar{\nu}_1}(0)\}
+ |U_{e2}|^2 \phi_{\bar{\nu}_2}(0) + |U_{e3}|^2 \{P_H \phi_{\bar{\nu}_1}(0) + (1 - P_H) \phi_{\bar{\nu}_3}(0)\}.$$  \hfill (13)

From this, one can deduce the survival probability $\bar{P}$ for $\bar{\nu}_e$:

$$\bar{P} = |U_{e1}|^2 P_H + |U_{e3}|^2 (1 - P_H) \approx 0.7 P_H.$$  \hfill (14)

The same result occurs for the normal mass hierarchy. Hence, one can define this case of complete non-adiabatic mixing as:

$$\bar{\nu}_e(0) \rightarrow 0.7 \times \bar{\nu}_e^0 + 0.3 \times \nu_x^0 \quad \text{(Case I)}.$$  \hfill (15)

It is now known that the $\theta_{13}$ mixing is relatively large [$\sin^2 (\theta_{13}) = 0.092 \pm 0.0016 \text{(stat)} \pm 0.005 \text{(syst)}$]. Thus, if there is no supernova shock, then the survival probability can be small ($P_H \rightarrow 0$) so that the conversion of $\bar{\nu}_e$ into $\nu_x$ is efficient. One can define this situation of complete adiabatic mixing in the inverted neutrino mass hierarchy as:

$$\bar{\nu}_e(0) \rightarrow \nu_x^0 \quad \text{(Case II)}.$$  \hfill (16)

The case with no neutrino oscillations is adopted as Case III in Refs. [15, 16, 17].
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Figure 6 illustrates the calculated dependence of detection event rates on the three oscillation scenarios. These results are similar to those of other studies. Shaded bands show uncertainties from the SFR and detector statistics.

Fig. 6. Effects of neutrino oscillations on the predicted $e^+$ energy spectra as a function of $e^+$ energy for a $10^6$ ton water Čerenkov detector with 10 years of run time. Red, green, and blue lines show the SRN detection rate in the case of non-adiabatic (Case I), adiabatic (Case II), or the no neutrino oscillations (Case III), respectively for the case of a stiff RMF EoS.

The calculated positron detection rate in Figure 6 shows the effect of the three different neutrino oscillation scenarios for a stiff RMF EoS. Here, one can see that the main effect of the neutrino oscillations is on the amplitude of the detected signal. This may be difficult to distinguish from the similar effect from the EoS and/or uncertainty in the SFR as shown in Figure 4.

4.3. Dependence of SRN detection on the neutrino self interaction effect

In the case of an inverted mass hierarchy, a “self interaction effect” among neutrinos might cause an additional difference between the detected energy spectrum of supernova neutrinos. The possible importance of this effect was calculated in Ref. for the simplest case with a neutrino self-interaction, i.e. a single angle interaction.

A single angle interaction causes the energy spectra of $\bar{\nu}_e$ and $\nu_x$ ($\bar{\nu}_\mu$ and $\bar{\nu}_\tau$) to exchange with each other at about 4.0 MeV. This means that the energy spectra of
\( \bar{\nu}_e \) and \( \nu_x \) are perfectly exchanged in the detectable energy range of water Čerenkov detectors. Hence, a neutrino self interaction could change the SRN energy spectrum in oscillation Case II into that of Case III.

Figure 7 from Ref. [15] shows that for the most part the correction for neutrino self interaction produces a slight increase in the high energy end of the detected neutrino spectrum. More recently, it has been demonstrated\(^{22} \) that the combined effects of neutrino oscillations and neutrino self interaction can have a dramatic effect on the emergent neutrino spectrum. Such effects may be detectable in the detected SNR spectrum and warrant further study.

![Graph](image)

**Fig. 7.** Calculation\(^{15} \) of the detected SRN energy spectrum cases with (dashed line) or without (solid line) a single-angle neutrino self interaction. Shaded regions are the backgrounds as defined in Fig. [4].

### 5. Conclusions

The essential results of the past studies in Refs. [15, 16, 17] of the uncertainties in the predicted SRN spectrum are summarized on Figures [8] and [9]. These works have considered a wide variety of astrophysical scenarios and investigated the EoS dependence of the SRN spectrum for each case with and without the occurrence of neutrino oscillations. It was consistently found that the EoS dependence of the SRN spectrum manifests prominently in the high energy tail in any scenario considered. The robustness of this EoS dependence can be understood in that the SRN spectrum is determined mostly by the SFR for \( z < 2 \), so that cosmological effects

and metallicity-dependent effects at high redshift have a negligible effect. However, as noted above, this EoS dependence needs to be re-evaluated in the context of the current more stringent constraints on the possible equations of state consistent with observations.

Figure 8 shows rectangles indicating the location of the peak and tail of the detected SRN for a variety of astrophysical conditions and equations of state. In that paper it was concluded that the EoS sensitivity is strong enough to differentiate the EoS during CCSNe and fSNe. However, as noted above new constraints on the EoS may preclude this sensitivity.

Figure 9 illustrates how the neutrino temperature $T_\nu$ in CCSNe explosions can affect the detected spectra. In particular, the neutrino temperature influences the value of the positron energy that gives the maximum event rate, i.e. $E_{\text{peak}}$. A measurement of the ratio of the number of events at $E_{\text{peak}}$ compared to the events at higher energy (say 25 MeV) might be used to directly infer the neutrino temperature emerging from supernovae.
Fig. 9. Sensitivity or the ratio of events at the observed positron peak to events with a positron energy of 25 MeV, corresponding to 7 fiducial SN collapse models and different neutrino oscillation conditions. Circles and dashed line are for the case of neutrino oscillations with complete non-adiabatic mixing. Squares and dot-dashed line are for the case of adiabatic mixing in the an inverted mass hierarchy. Triangles and the solid line are for the case of no neutrino oscillations. These points illustrates how the observed positron spectrum might be used to directly infer the supernova electron-neutrino temperature. Error bars drawn indicate the adopted 10% uncertainty in model neutrino temperatures. The vertical scatter in the points indicates the uncertainty due to neutrino oscillation scenarios.

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