Mathematical Modeling of a Virtual Platform for Training Specialists in the Oil and Gas Industry

N Chebotov¹, D Davydova¹, K Patutin¹, N Prokudina¹
¹Dept. of Automated Systems of Decision-Making Support, Tambov State Technical University, 392000 Tambov, Russia

E-mail: kirill-patutin@mail.ru

Abstract. A relevant task in the organization of comprehensive training of specialists in various fields is the development of the necessary theoretical and practical competencies. In the conditions of pandemic restrictions and the need to conduct classes in a distance mode, there is a need to develop new educational resources and means of representing knowledge, functioning on the basis of modern information technologies. One of the promising approaches is the transfer of laboratory and practical exercises to a virtual platform that allows you to master the necessary practical competencies on a computer. However, the functioning and visualization of physical or chemical processes in such virtual platforms must correspond to real processes. The article discusses the mathematical modeling of a virtual platform for training specialists in the oil and gas industry. An approach to the construction of a mathematical model based on the collected experimental data, the study of the laws of equilibrium of liquid and gas, and hydrostatic equations is considered. On the basis of the obtained model, an algorithm for the functioning of a virtual stand on the Unity platform is built. The presented approach can be used in the implementation of various virtual platforms for organizing the learning process in a distance form.

1. Introduction

The process of training specialists in the oil and gas industry includes both a theoretical course and a set of practical exercises [1-3]. To develop the required level of development of practical skills and competencies from a specialist in this area, it is necessary to conduct more laboratory classes and gain experience with the equipment. However, due to limitations caused by various reasons (pandemic, distance learning, lack of material opportunities for conducting experiments or their high danger for students and others), it is not always possible to organize practical exercises in full and at a sufficient level [4-6]. One of the options for solving this relevant problem is the use of virtual laboratories and platforms on the basis of which specialists are trained.

In the subject area of the oil and gas industry, when conducting practical exercises, it is necessary to study many complex chemical and physical processes and phenomena. In the course of the educational process, for a better understanding of the patterns occurring in this area, students get acquainted with the mathematical models of these processes. However, the complete transfer of such models to the area of virtual space often does not make sense because of the high complexity of mathematical equations and, consequently, the high cost and complexity of software implementation of such training systems [7].
Therefore, as a basis for the implementation of virtual platforms for laboratory and practical exercises, simplified mathematical models are used that provide the required level of realism and compliance with real processes [8-10].

The paper considers the solution to the problem of mathematical modeling in a virtual platform for training specialists in the oil and gas industry. The virtual platform is implemented on the example of a laboratory setup for the study of hydrodynamic processes. In the course of the study, the following problems were identified: the high complexity of modeling the hydrodynamic process, the high cost of real laboratory installations, the need for a large volume of premises for conducting group exercises [11]. To solve the indicated problems and implement a virtual platform for training specialists in the oil and gas industry, it is necessary: to develop a simplified mathematical model based on the approximation of experimental data, to implement it in a virtual platform, to develop a virtual platform environment, to test the platform in the educational process.

2. Analysis of physical processes in a real laboratory

At the first stage of solving the task of developing a virtual platform, it is necessary to analyze real physical processes [12]. The laboratory bench implements the hydrodynamic processes of fluid movement through pipes through pumps that are connected in series and in parallel. There is also a tank with a liquid, in which, under air pressure, the liquid occupies a certain level and, having occupied it, an equilibrium state of the system is observed.

The laboratory work consists of modeling two physical processes. The first process - the hydrodynamic process describes the Navier-Stokes equation [13, 14]. It is a system of partial differential equations describing the motion of a viscous Newtonian fluid. In vector form for a liquid, the equation is written as follows:

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho \eta} \nabla \Delta \mathbf{v}
\]

(1)

where \( \mathbf{v} \) – fluid particle velocity, \( t \) – time, \( F \) – external specific (per unit mass) force, \( \rho \) – density, \( p \) – pressure, \( \mu \) – kinematic viscosity coefficient (\( \mu \) – dynamic viscosity index), \( \nabla \) – Hamilton operator, \( \Delta \) – Laplace operator.

The viscosity coefficients depend on temperature and for a liquid, as a rule, are determined experimentally, and for a gas they are derived from the kinetic theory of gases.

It should be noted that without establishing the boundary and initial conditions, equation (1) has no solutions.

The second process is the equilibrium of liquid and gas in the tank. The balance of two forces in a vessel in mathematical form is represented as

\[
F_s + F_i = 0
\]

(2)

where \( F_s \) - gas force, \( F_i \) – fluid power

Pressure \( p \) there is power \( F \), per unit area \( S \), i.e. \( p = \frac{F}{S} \). Then formula (2) will take the form:

\[
p_s S + p_l S = 0
\]

(3)

or

\[
p_s + p_l = 0
\]

(4)
The gas pressure does not need to be simulated, since in a laboratory bench its value takes on a number set using a blower (pear) or other methods to pump gas.

The pressure in fluids at rest obeys Pascal's law: the pressure created by external forces that act on the surface of the fluid is transmitted unchanged to all points of the fluid. This law is equally true for gases.

As a result of the equilibrium condition and Pascal's law, the hydrostatic equation is obtained:

\[ p = p_0 + \rho gh, \quad (5) \]

where \( p_0 \) – pressure on the free surface of the liquid, and the product \( \rho gh \) – hydrostatic pressure.

Since our vessel is closed, then \( p_0 = 0 \). Substituting formula (5) into formula (4), we get:

\[ \rho gh + p_s = 0. \quad (6) \]

To solve the Navier-Stokes’s equation, it is required to allocate computing resources for the numerical solution of its equations (Ritz's method [15]) by specifying the initial and boundary conditions. Due to the complexity of solving this equation, it was decided to simplify the construction of the model and, based on the approximation of experimental data, to obtain the function necessary for the virtual platform [16,17].

3. Development of a simplified mathematical model of the laboratory

Due to the complexity of the implementation of the Navier-Stokes equations, it was decided to collect the data experimentally. The data obtained in the course of several attempts are averaged, and the patterns between the input and output parameters are recorded in tables. Consider a formalized mathematical model of the processes of a laboratory installation.

3.1. Model of the hydrodynamic process

The input parameters of this process include:

\[ X = \{x, E, m\}, \quad (7) \]

where \( x = \{x_1, x_2, x_3, x_4, x_5, x_6\} \) – the position of the valves, which take on the values 1 and 0, that is, whether the valve is open or closed. With a more accurate version of the stand and its large scale, the valves take the position as a percentage, that is, 0% - the valve is completely open, and 100% - it is completely closed;

\[ E = \{e_1, e_2, e_3\} \] - pumps, which can also take two values 0 and 1. All pumps can be operated in pairs, forming various combinations \( \{e_1, e_2\}, \{e_1, e_3\}, \{e_2, e_3\} \) and for this reason the input \( E \) takes the following form - \( E = \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}\} \). Then we highlight 7 subsets: \( E = \{E1, E2, E3, E4, E5, E6, E7\} \) or \( E = \{Ei | i = 1..7\} \);

\[ m = \{m_1, m_2, m_3\} \] – pump power and the higher it is, the better the pump pumps liquid through the pipes. The power of each pump is divided into three modes, that is, the power \( m \) comprises \( m_i = \{m_{11}, m_{12}, m_{13}\} \), power \( m_2 = \{m_{21}, m_{22}, m_{23}\} \), and the power \( m_3 = \{m_{31}, m_{32}, m_{33}\} \).

Output parameters include:

\[ Y = \{P, R\}, \quad (8) \]

where \( P \) – set of pressures removed by sensors from three places in the installation in kPa;

\( R \) – a variety of flow rates in liters / minute.
Next, a system of two functions is calculated \( S(X) \) and \( S2(X) \), which represent functions given in a tabular way, obtained by various approximation methods (polynomial, cubic spline, neural network, and so on):

\[
\begin{align*}
S(X) &= P \\
S2(X) &= R
\end{align*}
\]  

(9)

Based on functions (9), it is possible to implement a mathematical model of a laboratory installation for its implementation in the software environment of a virtual platform. Let's consider the procedure for obtaining each of the functions.

Function \( S(X) \) is the product of two functions \( \varphi(X) \) and \( F(X) \):

\[
S(X) = \varphi(X) F(E_i),
\]

(10)

where the function \( \varphi(X) \) is a logical error function, that is, if any of the inputs is closed, and it should not be so, then the function will return 0 and then the whole function \( S(X) \) will become equal to zero and the program warns us about the incorrect setting of the initial conditions. The function \( \varphi(X) \) as follows:

\[
\varphi(X) = (x_4 \oplus (x_5 \rightarrow x_i))^\wedge \wedge( (x_2 \wedge P_1 \wedge XOR(m_{i_1},m_{i_2},m_{i_3})) | (x_5 \wedge P_2 \wedge XOR(m_{i_2},m_{i_3},m_{i_3}))))
\]

\[
F(E_i) = p_{i,a,b,c},
\]

(11)

where \( p_{i,a,b,c} \in P \) - a function that selects pressure values from the experimental data table. Indexes \( i,a,b,c \) determine the coordinates of the cell in this multidimensional pressure matrix.

- \( i \) determines the operating mode of the pump;
- \( a \) - table index \( P \), taking a value from 1 to 3 in the case, if \( |E_i| = 1 \), otherwise \( a = 1 \);
- \( b \) - table index \( P \), taking a value from 1 to 3 in the case, if \( |E_i| = 2 \), otherwise \( b = 1 \);
- \( c \) - table index \( P \), taking a value from 1 to 3 in the case, if \( |E_i| = 3 \), otherwise \( c = 1 \).

Values accepted by indexes \( i,a,b,c \) depend on the indices of the elements of the set of pump capacities as follows:

\[
\begin{align*}
&\text{if } |E_i| = 1, \exists m_j, \text{ then } j \rightarrow a \\
&\text{if } |E_i| = 2, \exists m_{j_1},m_{j_2}, \text{ then } (j,l) \rightarrow b \\
&\text{if } |E_i| = 3, \exists m_{j_1},m_{j_2},m_{j_3}, \text{ then } (j,l,w) \rightarrow c
\end{align*}
\]

The output \( R \) is calculated similarly:

\[
S2(X) = \varphi(X) F2(X)
\]

(12)

\[
F2(X) = r_{a,b,c}
\]

(13)

The only difference is that a different table with a different dimension is used, where indexes \( a,b,c \) are used.
At the next stage of research, the simplified mathematical model was transformed into a graph model (Figure 1), which is already used for the software implementation of physical processes in a virtual platform.

![Figure 1. Graph model of the algorithm for the first pump.](image)

This graph model allows developers of a virtual platform to separate input and output variables, as well as form a number of approximating functions.

3.2. Model of the equilibrium state of the liquid tank

Next, consider a model of the equilibrium state of a tank with a liquid. In this case, the input parameters include:

\[ X = \{p_s, h, P\} \]

where \( p_s \) – gas pressure initially equal to 101 kPa, and which can be increased using a blower;

\( h \) – the height that the liquid occupies at the moment is in the initial state equal to 0;

\( P \) – fluid pressure that comes from \( Y \) out of the outlet first model;

Output parameters include:

\[ Y = \{h\} \]

where \( h \) – the height that the liquid should take.

Further, using formula (6), we understand that the main role here is played by the inputs \( p_s \) and \( h \).

For example, at the initial moment of time, the tank with the liquid is empty, that is \( h = 0 \), and \( p_s \) set at a value equal to 101 kPa, which means that the equation (6) the equation was correct \( h \) should be set to 10.1 m3. If you increase the gas pressure \( p_s \), let's say up to 120 kPa, then \( h \) needs to be reduced to 7.9 m3. If the gas pressure is reduced, the height of the liquid column will increase. The entrance \( P \) responsible for the rate at which the tank is filled with liquid. Within the framework of a simplified mathematical model for calculating \( h \) we will use the following formula:
4. Software implementation of the mathematical model
The collected experimental data and the dependences obtained on their basis in the form of approximating functions are used in the software implementation of the virtual stand based on the 3D model (Figure 2). This implementation is executed in the Unity software environment [18, 19]. It contains all the necessary functions that are required to develop applications based on 3D objects and the virtual world. The main advantages of the platform are cross-platform, visual development environment, modular system of components [20, 21]. This allows the developed project to be used on various platforms (personal computers, tablets, smartphones), which is especially important when organizing the educational process to expand the potential audience of users.

The algorithm for applying the functions represents the following set of conditions:
Collecting data from inputs X
- Position of valves. They can occupy only two positions - fully open or completely closed. For a more accurate representation, the gates can be presented in other positions, but this work does not require clarification of this aspect.
- The status of the pumps is on or off.
- Pump power. Each of them has three power modes of fluid flow, due to which the pressure in the pipes either increases or decreases.
- Establishing the value of the gas pressure in the tank.
Calculation of functions for each output Y:
- Calculation of functions S(X) and S2(X) for setting values on the scoreboard
- Calculating a function to fill to the required height
- Filling the required places on the stand screens with values from the outputs Y.
- Filling the liquid column to a certain level in the tank
These conditions are met due to the fact that the system is not closed. Otherwise, diffusion processes of dissolution or evaporation of gases begin to appear in the system.

5. Conclusion
Since the pandemic process negatively affected the educational process, many schools and universities have a need to conduct laboratory work without the use of training stands and the full-time presence of students. In this case, it is necessary to transfer these phenomena to the virtual world, where the user is only required to have a personal computer. The use of virtual platforms also addresses a number of
safety related issues, especially in a number of chemical and physical experiments. Also, the format of virtual platforms for laboratory and practical exercises allows you to carry out educational tasks in a distance format with the involvement of a larger number of students. However, the creation of such virtual platforms is associated with a number of non-trivial tasks, one of which is mathematical modeling of processes occurring in laboratory facilities.

The solution to this problem is the creation of a simplified mathematical model that reduces the time and complexity of the software implementation of a virtual installation, reduces the load on the system, allowing the software to free up resources for other tasks. The creation of such a mathematical model allows transferring various physical or chemical processes into a virtual environment.

The paper considers the solution to the problem of mathematical modeling of hydrodynamic processes in a laboratory setup and the implementation of a simplified mathematical model. The model is based on the approximation of the tabular values of the experimental data. An algorithm for using the obtained model in a virtual platform, software implementation of the platform for a laboratory installation for training specialists in the oil and gas industry is presented.

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