Finite element study of tunable cantilever plate structure using position change

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Abstract. In this paper, a tunable cantilever plate structure is presented, in which a box is attached to each corner of its free sides and a ball is placed in each box. Change of the balls position makes natural frequency of the structure variable, which expands working bandwidth of the structure to accommodate the complex ambient vibration. The modal analysis of the tunable cantilever plate is carried out by the finite element software ABAQUS. Changing the balls position, it is found that there are maximum and minimum values in the natural frequency of the self-tuning structure, which forms a resonance bandwidth. Moreover, there are multiple position combinations of two balls at the same natural frequency in the range of resonance band. The paper studies the influence of mass and geometrical dimensions of boxes and balls on the natural frequency and bandwidth. While ensuring strength and stiffness of the structure, optimal material and size parameters are selected to improve the resonant bandwidth. This provides a theoretical basis for applying the tunable structure to piezoelectric energy harvester.

1. Introduction
Vibration energy widely exists in daily life and practical engineering. Vibration energy harvester is an effective way to convert vibration energy of surrounding environment into electrical energy [1, 2]. The advantages of the piezoelectric energy harvester are high conversion efficiency, simple structure, no additional power supply, environment-friendly, etc., and its application prospect is broad [3-7]. Generally, working bandwidth of piezoelectric energy harvester is relatively narrow, but power of the energy harvester can be maximized when external excitation frequency is consistent with natural frequency of the system. Because of low frequency and wide frequency band of the external vibration, Natural frequency of the piezoelectric energy harvester cannot match the external excitation frequency, which leads to a great reduction of the output power of the harvester [8, 9].

In order to improve efficiency of piezoelectric energy harvester, it is necessary to automatically adjust natural frequency of the device according to the frequency of external environment vibration to achieve resonance. The earliest tunable energy harvester relied on manually varying position of additional mass [10] to adjust natural frequency. After that, some tunable devices using not manual operation but additional energy [11], such as electromagnetic [12] to change the stiffness of the system, so the actual capture efficiency is low. In addition, there was a reference that combined several cantilever beams with different natural frequencies to a complex structure, but it occupies a large space [13]. The devices in the reference [14, 15] have achieved self-tuning, but the frequency is higher and the tunable
range is narrower. Reference [16] designed a self-tuning device by attaching a box with two cylinders based on cantilever beam structure, and did not further study the relationship between position and frequency/bandwidth. Based on cantilever plate, this paper proposes a tunable cantilever plate structure with additional balls and boxes, no electricity recovery part. By changing position of the balls, tunable function in broad-frequency vibration environment is realized.

2. Finite element modeling

The research is based on a cantilever plate structure with one end fixed and the other end free. A box is attached to each corner of free end of cantilever plate, and a small ball is placed in each box. The model structure is shown in figure 1. Natural frequency of the structure is adjusted by changing position of balls within boxes.

![Figure 1. Sketch of the cantilever plate structure.](image)

In order to study the influence of the balls position, size parameters and mass of boxes and the mass of balls on the natural frequency, modal analysis is carried out by using ABAQUS—a common finite element software. ABAQUS has 433 types of units, which provides designers with more choices and can reflect the subtle structural phenomena and differences between phenomena. It is easy to simulate the mechanical behavior of the actual structure. In view of the universality of contact problems in practical engineering, ABAQUS has separately set up an interaction module, which can accurately simulate various contact problems existing in actual engineering. In this paper, cantilever plate and boxes are built with S4R shell elements, and the balls are built with C3D10M solid elements. Boxes and plate are linked by the TIE in ABAQUS. Size of cantilever plate is 120mm×60mm×0.06mm, and diameter of each sphere is 16 mm. Model of finite element simulation is shown in figure 2.

![Figure 2. Finite element simulation of the structure.](image)

3. Result and analysis

3.1. Influence of ball location

The purpose is to obtain the influence regulation of the ball position on the natural frequency, first-order natural frequency of the structure is obtained by modal analysis using ABAQUS, and three special change cases of sphere position are studied:
1. Initial positions of both balls are in the left-most side of box near the fixed end of the cantilever plate. Changing the positions of the balls two balls sites equidistantly toward free end of the plate until the balls reach the right-most side of the boxes, modal analysis is carried out at each new situation of two balls.

2. Initial positions of two balls are the same as in case 1. One ball is fixed and position of another ball is changed equidistantly to the right end of box. Modal analysis is performed for each change of the balls' position.

3. Initially, two balls are on the left and right side of box respectively. The sphere at the free end is fixed, and position of the left ball is changed equidistantly to the right until the ball reaches the free end. Modal analysis is carried out each time the ball position is changed.

Figure 3. Change curves of natural frequency in three cases.

Figure 4. Position distribution of balls at different frequencies.

The curves of the first-order natural frequency following balls position in three cases are shown in figure 3 from ABAQUS. As position of balls changes from left to right (from 0 to 0.04m), the natural frequency gradually decreases. When two balls are at the left-most side of boxes, the value of the natural frequency is the highest, named \( f_{\text{max}} \). When the balls are simultaneously at the right-most side near the free end, the value is the lowest and is named \( f_{\text{min}} \). In other positions, the natural frequency is in the range of \( f_{\text{max}} \) and \( f_{\text{min}} \), so the difference between \( f_{\text{max}} \) and \( f_{\text{min}} \) can be determined as bandwidth,
called $f_b = f_{max} - f_{min}$. As the site of two balls changes, the natural frequency varies within the bandwidth, so that tunable function of the structure is achieved.

In the above three cases, the ball position is changed in a special way, that is, either the two balls are in the same situation (case 1), or one ball is fixed at an end, and site of another ball is changed (case 2, 3). In general, during the vibration of the board, two balls can roll freely in the box, and the position is not limited. Taking three frequencies in the bandwidth as examples, they are defined as $f_r = (f_{min} + f_b/4)$, $f_{mid} = (f_{max} + f_{min})/2$ and $f_l = (f_{max} - f_b/4)$ respectively, and the positions distribution of two balls at a natural frequency is analyzed. As shown in figure 4a-c.

It is found that within the bandwidth range, a natural frequency corresponds to multiple groups of two balls’ positions, and each group is represented by a line between two balls in figure 4. All of the lines corresponding to a certain natural frequency constitute an X-shaped region. When the magnitude of natural frequency approach es the midpoint of bandwidth ($f_{mid}$), the range in which the balls can roll is maximized, and the area of the region is the largest. Therefore, as the frequency changes from $f_{min}$ to $f_{max}$, the X-region moves from right to left, and its area increases first and then decreases.

3.2. Effect of ball mass
In order to enlarge the bandwidth, the influence of ball’ mass on natural frequency and bandwidth of the system is investigated. Respectively, steel and aluminium are selected as balls material. Fixed the balls at the right or left end of the boxes, modal analysis is performed to obtain the minimum and maximum natural frequencies ($f_{min}$ and $f_{max}$) respectively. As shown in table 1, the results of $f_{min}$ are very similar, but the $f_{max}$ of steel balls is larger than that of aluminium balls. Therefore, the bandwidth of the structure with steel balls is superior, and finally steel is selected as the material of the balls.

| Material  | $f_{min}$ | $f_{max}$ | Bandwidth |
|-----------|-----------|-----------|-----------|
| Steel     | 16.421    | 22.707    | 6.286     |
| Aluminum  | 16.206    | 17.634    | 1.428     |

3.3. Effect of box
In order to widen the frequency band further, the influence of box mass on the natural frequency is investigated. Steel, aluminum and acrylic commonly used in engineering are selected as box materials respectively to calculate the maximum and minimum natural frequencies. As shown in table 2, the natural frequencies of structures with steel or aluminum boxes are lower. In contrast, although the natural frequency of the structure with acrylic box is higher, its bandwidth is wider. For the purpose of increasing the resonant frequency band, acrylic is chosen as the material of two boxes.

| Material    | $f_{min}$ | $f_{max}$ | Bandwidth |
|-------------|-----------|-----------|-----------|
| Stainless steel | 13.965    | 16.655    | 2.690     |
| Aluminum     | 14.702    | 18.166    | 3.464     |
| Acrylic      | 16.421    | 22.707    | 6.286     |

On the basis of previous studies, the influence of box length on the natural frequencies is studied. The width, height and wall thickness of the box are 20mm, 20mm and 1mm respectively. The maximum and minimum natural frequencies of the structures with box length of 30mm, 60mm and 90mm are calculated respectively, and the results are shown in table 3. Increase in the length of boxes causes the balls’ travels to become larger in the box, so the bandwidth gradually increases. Considering that the rigidity of the boxes is much larger than the stiffness of board, the stress concentration near the fixed
end of the cantilever plate will be too serious, which may cause the destruction of the cantilever plate, if the box is too long. Therefore, 60 mm box is optimum for the structure.

**Table 3.** Influence of length on natural frequency and bandwidth (Hz).

| Lengths | \( f_{\text{min}} \)  | \( f_{\text{max}} \)  | Bandwidth |
|---------|-----------------|-----------------|------------|
| 30mm    | 15.971          | 17.996          | 2.025      |
| 60mm    | 16.421          | 22.707          | 6.286      |
| 90mm    | 20.612          | 31.341          | 10.729     |

4. Conclusions

In this paper, a tunable cantilever plate structure is designed and studied by using the finite element software—ABAQUS. Natural frequency of the structure is changed through the variation of balls’ position, so that there is a resonance bandwidth formed by the maximum and minimum natural frequencies. By altering the relative position of the balls, any natural frequency in the bandwidth range corresponds to multiple position combinations of the two spheres, which establish an X-shaped region. Therefore the tunable ability of the structure is excellent. The influence of balls and boxes on the natural frequency is studied, and it is found that in order to widen the bandwidth under the premise of ensuring the structural strength and rigidity, it is necessary to elevate the mass ratio of ball to box, and raise the boxes’ lengths. Results of the study contribute to the structural optimization of energy harvester to increase its operating bandwidth.

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