LETTER

Absorbing phase transition in a four-state predator–prey model in one dimension

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Abstract. The model of competition between densities of two different species, called predator and prey, is studied on a one-dimensional periodic lattice, where each site can be in one of the four states, say, empty, or occupied by a single predator, or occupied by a single prey, or by both. Along with the pairwise death of predators and growth of prey, we introduce an interaction where the predators can eat one of the neighboring prey and reproduce a new predator there instantly. The model shows a non-equilibrium phase transition into an unusual absorbing state where predators are absent and the lattice is fully occupied by prey. The critical exponents of the system are found to be different from those of the directed percolation universality class and they are robust against addition of explicit diffusion.

Keywords: percolation problems (theory), phase transitions into absorbing states (theory)
Absorbing configurations do not have any outgoing rates [1]. Once they have been reached, the system cannot escape from these configurations. The presence of absorbing configurations in a phase space raises a possibility that the concerned system may undergo a non-equilibrium phase transition into absorbing states. The critical behavior of these absorbing state phase transitions (APTs) [2] depends on the conservation in the dynamics and the symmetry between absorbing states. It has been conjectured [3] that in the absence of any special symmetry the APTs belong to the directed percolation (DP) universality class as long as the system has a single absorbing state. Additional symmetries, like particle–hole symmetry [4], conservation of parity [5], and symmetry between different absorbing states [6] lead to different universalities. A spreading process with spatially quenched randomness [7] or with long-term memory [8] is known to destroy the critical behavior completely, whereas the long-range interaction leads to continuous variation [9] of the critical exponents. The presence of infinitely many absorbing states may [10] or may not [11] belong to the DP universality class. Again a different critical behavior is observed when the activity field does not have any special symmetry, but it is coupled to a conserved density [12]. Recent studies have indicated that DP critical behavior is possible, even in the presence of an additional conserved field [13]. It is not quite clear what microscopic ingredients can make an APT belong to the DP-class.

The model of directed percolation has been extended to more than one species [14]. Along with the simple DP behavior, a line of first order transition [15] has been observed in 1 + 1 dimensions when two species compete for survival. Janssen [16] studied coupled DP processes with bilinear and bidirectional interspecies couplings in the framework of bosonic field theory, where no other critical phenomena were found other than the DP. The hierarchy of unidirectionally coupled DP processes with many species shows multicritical behavior [17]. Coupled percolation processes have been also studied [18], where the absorbing phase become unstable with respect to an arbitrarily small branching rate even in one dimension.

Predator–prey cellular automaton models [19] in two dimensions show DP universality class. Coupled directed percolation (DP) processes with more than two species of particles (in one dimension) with different kinds of interspecies coupling have shown DP-type [20] transitions. Lotka–Volterra like models in one dimension always show co-existence [21], either in the form of well mixed states or as irregular bursts of the predator and prey populations. A four-state predator–prey model [22] in one dimension with a restriction that a site can have at best one particle of each kind, shows an APT to an absorbing (extinct) state which belongs to the DP-class.

In this paper we study a model of two species, say $A$ (prey) and $B$ (predator), on a (1 + 1)-dimensional lattice. Each lattice site is either vacant $\emptyset$ or occupied by at best one particle of each kind. The prey grow independently as $A\emptyset \rightarrow AA$ and the predators die as $BB \rightarrow \emptyset\emptyset$, whereas they interact through a process $BA \rightarrow BB$, where birth of a new predator occurs instantly along with the death of the prey. The system shows a line of continuous absorbing state transition different from DP as the rates of these processes are tuned. Unlike other multispecies models, in the absorbing state both prey and predators do not separately extinct, rather predators become extinct and prey proliferate to fill up the whole prey branch. The model is defined on a one-dimensional periodic lattice with lattice sites labeled by $i = 1, 2, \ldots, L$. Each site $i$ can be vacant, occupied by a single particle $A$ (prey), occupied by a single particle $B$ (predator) or occupied by both particles.

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(co-existing \(A\) and \(B\)), thus the model can be treated as a four-state predator–prey (4SPP) model. More than one particle of any kind is not allowed. These hardcore restrictions on individual particles, where co-existence is allowed, can be realized alternatively by considering two separate branches, one for \(A\) and the other for \(B\) particles. Particles living in one branch cannot move to the other branch, as schematically shown in figure 1. Correspondingly, each site \(i\) is associated with four states: \(s_i^A = 0, 1\) and \(s_i^B = 0, 1\), where 1 (0) denotes the presence (absence) of a particle at site \(i\).

On a periodic lattice, these particles interact following a random sequential dynamics given below.

The prey (\(A\)) can grow on their own branch with rate \(p\) independent of the predators (\(B\)). Again, two neighboring predators (\(B\)) die simultaneously with rate \(q\) independent of \(A\) due to their own crowding or overpopulation. Here \(X\) in the \(A\)-branch (\(B\)-branch) corresponds to an arbitrary state of \(A\) (\(B\)) particles, i.e. presence or absence. These two species interact with rate \(r\) as follows: when a predator \(B\) at site \(i\) meets a prey \(A\) as the right neighbor \(i + 1\), and site \(i + 1\) is not already occupied by a predator \(B\), then the predator \(B\) eats the prey \(A\) and simultaneously reproduces another predator \(B\). Note that the dynamics is spatially asymmetric as both species grow only in one direction (here, right), and it does not include explicit diffusion of species. The effects of addition of symmetry and diffusion are briefly discussed toward the end of this letter.

Let the steady state density of \(A\) and \(B\) particles be \(\rho_A^*\) and \(\rho_B^*\) respectively. Clearly in the absence of predators (\(\rho_B^*\)) the prey density \(\rho_A^* = 1\) as the prey do not have an independent death process. Again, note that the dynamics does not allow \(A\) particles to be eaten when they co-exist with \(B\) on the same site. This indicates that these \(A\)s can only die after their co-existing \(B\)s die, which can occur with rate \(q\) if there is a \(B\) particle present or created at the immediate left neighbor. Thus, for a reasonably small
death rate of predators, $\rho_A^s$ is expected to have a small value ($\rho_A^s < 1$) when $\rho_B^s \simeq 0$ and then it increases along with $\rho_B^s$; so, the prey density $\rho_A^s$ can never vanish. The predator density $\rho_B^s$ can, however, become zero by repetitive death processes. The isolated $B$s wait until the prey invades their neighboring site and then they subsequently eat and reproduce with rate $r$ and die with rate $q$. So, along with the co-existing phase\(^1\) where both $\rho_A^s$ and $\rho_B^s$ are nonzero, we have another phase where $\rho_A^s = 1$ and $\rho_B^s = 0$. Clearly, the latter phase ($\rho_A^s = 1, \rho_B^s = 0$) is absorbing as even once all the predators have died, a single surviving prey can lead to proliferation of the prey population in the whole lattice. Of course, as argued earlier, $\rho_A^s$ cannot vanish, and the other possible absorbing state ($\rho_A^s = 0, \rho_B^s = 0$), which requires simultaneous death of all predators and prey, is not dynamically accessible. Thus the 4SPP model can undergo an absorbing state phase transition by tuning the different rates with $\rho_B^s$ as the order parameter. Our aim here is to study this critical behavior in detail.

We have used the standard Monte Carlo methods to study the critical behavior of this model. From an initial arbitrary configuration, where each branch $A$ and $B$ is filled by an arbitrary number of respective particles, the system is allowed to evolve according to the random sequential update following the Monte Carlo dynamics of the 4SPP model. We have studied the system with different values of the rates $p$, $q$ and $r$ with system size $L = 10^3$. For illustration, we have fixed two of the reaction rates, say $q = 0.02$, $r = 0.9$, and varied $p$ as the control parameter in the following simulations.

For $p < p_c$ the average $B$ density $\rho_B^s$ decreases continuously until the system reaches a state with no predators ($\rho_B^s = 0$) and ultimately the whole prey lattice branch is filled with prey. Once it has reached this absorbing state ($\rho_A^s = 1, \rho_B^s = 0$), the system remains there forever. However, for $p > p_c$ the average density of $B$ particles $\rho_B^s$ saturates to a nonzero value along with $\rho_A^s$ which also takes a value smaller than unity. In figure 2(a) we have plotted $\rho_A^s$ and $\rho_B^s$ as a function of $p$ for a system size $L = 10^3$. As expected, the density of the prey $\rho_A^s$ never vanishes and they proliferate in the whole lattice (with $\rho_A^s = 1$) in the absorbing state where predators are absent. Thus the system undergoes an absorbing state phase transition as the birth rate of prey $p$ crosses a critical threshold $p_c = 0.148(4)$.

For $p > p_c$, the order parameter $\rho_B^s$ shows power law behavior with the distance from criticality,

$$\rho_B^s \sim (p - p_c)^\beta,$$

\(^1\) It has been predicted earlier [22] that, in the absence of site restriction, the predator and prey system in 1D always remains in the co-existing phase.
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Figure 2. (a) The average steady state density $\rho s A, \rho s B$ is plotted against the control parameter $p$ for fixed values of $q = 0.02, r = 0.9$ and $L = 10^3$; the critical point is indicated at $p_c = 0.148(4)$ showing absorbing to active phase transition. Both densities are again plotted against $q$ keeping $p = 0.55, r = 0.9$ fixed; the critical point is $q_c = 0.095(6)$ showing active to absorbing phase transition (see inset). (b) Phase diagram in the $(p,q)$ plane for $r = 0.9$, showing the active and absorbing phases.

Figure 3. (a) The average predator density ($\rho_B^s$) is plotted with fixed values of $q = 0.02, r = 0.9$, with different choices of $p_c = 0.153, 0.151, 0.148, 0.146, 0.144$ shown from top to bottom. The correct choice of $p_c = 0.148(4)$ and the corresponding slope in the logarithmic plot give the critical exponent $\beta = 0.367(7)$. (b) Time evolution of $\rho_B$ with $q = 0.02, r = 0.9$. Below the critical point for $p = 0.142, 0.144, 0.146$ the density $\rho_B$ eventually becomes extinct and above the critical point $p = 0.150, 0.154, 0.160$ the density $\rho_B$ saturates. At the critical point $p = 0.148(4)$, $\rho_B(t) \sim t^{-\alpha}$ gives the critical exponent $\alpha = 0.194(4)$ and the data collapse gives $\nu = 1.8(1)$.

density in the active phase approaches the steady state value $\rho_B^s$ in the $t \to \infty$ limit. So $\rho_B$ must scale as

$$\rho_B(t, p) = t^{-\alpha} F(t|p - p_c|^{\frac{1}{\nu}}).$$

(2)

Thus, one expects that $\rho_B(t)$ for different values of $p$ (shown in figure 3(b)) collapse into a single scaling function $F$ when $\rho_B t^{\alpha}$ is plotted against $t|p - p_c|^{\frac{1}{\nu}}$. The main figure here
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Figure 4. (a) The finite size scaling function $\rho_B t^\alpha$ plotted against the scaled variable $tL^{-z}$ for $L = 50, 100, 200, 300, 400, 500$ (bottom to top in the figures) with $p = 0.148(4)$ estimates $z = 1.52(0)$. (b) The same data could be collapsed according to equation (4) by choosing $\beta/\nu_\perp = 0.30$, which gives an estimate $\nu_\perp = 1.2(2)$. 

shows the data collapse when we choose $\alpha = 0.194(4)$ and $\nu_\parallel = 1.8(1)$. Since at the critical point $\rho_B(t, p_c) = t^{-\alpha} F(0)$, one can obtain both $p_c$ and $\alpha$ directly from the log scale plot of $\rho_B$ versus $t$ which is linear (as shown in the inset of figure 3(b)). The resulting $p_c$ and $\alpha$ are consistent with those obtained from the data collapse. Again, in the upper critical regime, $\rho_B^p$ vanishes as $|p - p_c|^\beta$ in the $t \to \infty$ limit. This can happen only when the off-critical scaling function $F(x) \sim x^{\beta/\nu_\parallel}$; thus

$$\alpha = \beta/\nu_\parallel.$$

Since all three exponents $\beta$, $\alpha$ and $\nu_\parallel$ are calculated independently, one can check whether the above scaling relation holds. In this case it holds to a great accuracy for the values of $\beta$, $\alpha$ and $\nu_\parallel$ calculated here.

Now we turn our attention to the finite size scaling of $\rho_B$ at the critical point. Again, the system of length $L$ with a high density of predators $\rho_B(t, L)$ decays as $t^{-\alpha}$, indicating a scaling form

$$\rho_B(t, L) = t^{-\alpha} G(t/L^z),$$

where $z$ is the dynamic critical exponent. Thus, one expects $\rho_B$ for different values of $L$ to be collapsed to a single function when plotted against $t/L^z$. This is described in figure 4(a). The inset there shows variation of $\rho_B(t)$ for different system sizes $L = 50, 100, 200, 300, 400, 500$, which were made to collapse to a single function using $\alpha = 0.194(4)$ and $z = 1.52(0)$. From the scaling relation

$$z = \nu_\parallel/\nu_\perp$$

one expects that $\nu_\perp = 1.19$. This can be verified from the modified scaling relation of equation (3). Since $z\alpha = \beta/\nu_\parallel$, we have

$$\rho_B(t, L) = L^{-\beta/\nu_\parallel} \tilde{G}(t/L^z),$$

where $\tilde{G}(x) = x G(x)$. In figure 4(b) we have plotted $\rho_B L^{\beta/\nu_\parallel}$ as a function of $t/L^z$ and found that the data for system sizes $L = 50, 100, 200, 300, 400$, and 500 could be collapsed

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Figure 5. Time evolution of the 4SPP model for $10^3$ time steps with prey $A$ (green) and predator $B$ (blue) with system size $L = 10^3$ for the regime $p < p_c$ and at $p = p_c$: (a) with asymmetric rules (no diffusion), (b) with asymmetric rules and explicit diffusion.

Table 1. Comparison of the critical exponents between the 4SPP model and the DP universality class.

|     | $\beta$ | $\alpha$ | $\nu_\alpha$ | $z$     | $\nu_\perp$ |
|-----|---------|---------|------------|--------|-----------|
| DP  | 0.276   | 0.159   | 1.733      | 1.580  | 1.096     |
| 4SPP | 0.367(7) | 0.194(4)| 1.8(1)     | 1.52(0) | 1.2(2)    |

into a single curve by choosing $\beta / \nu_\perp = 0.30$. This gives us $\nu_\perp = 1.2(2)$, which is consistent with the earlier estimation.

The critical exponents of the model are summarized in table 1 along with the critical exponents of the directed percolation universality class. Clearly the exponents, particularly $\beta$ and $\alpha$, are very different from those of the DP-class, indicating that the 4SPP model belongs to a different universality class. We have studied the model by varying other rates also.

For example, one can take $q$ as the control parameter, keeping $p$ and $r$ fixed. The inset of figure 2(a) shows the variation of $\rho_A^*$ and $\rho_B^*$ as a function of $q$ for fixed $p = 0.55$ and $r = 0.9$. Evidently, the order parameter $\rho_B^*$ vanishes continuously as $q$ crosses the threshold value $q_c = 0.095(6)$. The critical exponents obtained in this case were found to be consistent with table 1. Thus, for any fixed values of $r$, one expects a line of criticality in the $(p, q)$-plane, which is shown in figure 2(b).

That the critical behavior of the absorbing phase transition observed in the 4SPP model is different from DP can be visualized from the growth of clusters. The space–time diagram, starting from an arbitrary initial configuration, is shown in figure 5(a), where the occupancies of species $A$ and $B$ are represented separately in the upper and lower parts,
Figure 6. (a) Average density of predators ($\rho_s^B$) with $q = 0.2$, $r = 0.9$ with explicit diffusion: absorbing to active state phase transition at $p = 0.323(5)$ and the critical exponent $\beta = 0.370(9)$ (see inset). (b) Above the critical point for $p = 0.330, 0.335, 0.345$ the density $\rho_s^B$ saturates and below the critical point for $p = 0.305, 0.310, 0.315$ the density $\rho_s^B$ eventually becomes extinct. At the critical point $p = 0.323(5)$, $\rho_B(t) \sim t^{-\alpha}$ gives the critical exponent $\alpha = 0.190(5)$ and $\nu = 1.7(5)$.

respectively. The evolution of clusters is visibly different from that of the 1+1-dimensional directed percolation model. Since the prey species can grow independently, the space is always filled locally by prey where predators are absent. Again, regular striped structures appear in these figures as the species do not diffuse. A natural question would be whether diffusion can drive the system to have an absorbing state transition belonging to the DP universality class or not.

In the following we introduce diffusion of both the species explicitly in this model. Along with the usual dynamics of the 4SPP model described earlier, both $A$ and $B$ particles are now allowed to move to the neighboring available vacant spaces in their respective branches. In this case, it turns out that the clusters evolve more like the DP model (see the space–time diagram in figure 5(b)). However, a detailed study of the absorbing phase transition reveals that the critical exponents are the same as given in table 1.

In the presence of explicit diffusion, we choose to study the system with fixed rates $q = 0.2$, $r = 0.9$. Monte Carlo simulations show that the predator density $\rho_B^s$ vanishes continuously as $p$ is decreases below a critical threshold $p_c = 0.323(5)$. As described in figure 6(a), near the critical point, $\rho_B^s \sim (p - p_c)^\beta$ with $\beta = 0.370(9)$. Again, starting from a large number of predators, the density $\rho_B(t)$ decays to its stationary value $\rho_B^*$ which is nonzero only in the upper critical region $p > p_c$. We find that $\rho_B(t)$ for different values of $p$ could be merged to a unique scaling function which satisfies equation (2) by choosing $\alpha = 0.190(5)$ and $\nu = 1.7(5)$. This data collapse is described in figure 6(b). The critical exponents $\beta, \alpha$ and $\nu$ are more or less consistent with those listed in table 1. It is not surprising that the addition of explicit diffusion did not alter the universal behavior. In fact, although slow, effective diffusion of predators was already occurring in the 4SPP model through the rates $q$ and $r$.

Some comments are in order here. The fact that the absorbing state phase transition in the 4SPP model is different from that of DP can be seen from the dynamical mean-field
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analysis (ignoring spatial density correlations). The mean-field densities $\rho_A = \langle s_A \rangle$ and $\rho_B = \langle s_B \rangle$, in the continuum limit, evolve as

$$\frac{\partial \rho_A}{\partial t} = p \rho_A (1 - \rho_A) - r \rho_A \rho_B - v_A \frac{\partial \rho_A}{\partial x} + D_A \frac{\partial^2 \rho_A}{\partial x^2},$$

$$\frac{\partial \rho_B}{\partial t} = -2q \rho_B^2 + r \rho_A \rho_B - v_B \frac{\partial \rho_B}{\partial x} + D_B \frac{\partial^2 \rho_B}{\partial x^2},$$

where $v_{A,B}$ denote the velocities of the respective species which appear due to asymmetric dynamics, and $D_{A,B}$ are the coefficients of diffusion. The first term in equation (5) captures the growth of a prey restricted by hardcore interactions and the second term there corresponds to the interaction between the two species. Again, the first term in equation (6) represents the simultaneous death of two predators. Evidently, these mean-field equations have two fixed points: the unstable one ($\rho_A^* = 0, \rho_B = 0$) and the stable one ($\rho_A^* = (2pq/2pq + r^2), \rho_B^* = (pr/2pq + r^2)$). However, when $\rho_B \rightarrow 0$ faster than $\rho_A$, one can get another fixed point ($\rho_A^* = 1, \rho_B^* = 0$) from equation (5). This unusual absorbing state ($\rho_A^* = 1, \rho_B^* = 0$), as discussed earlier, raises a possibility that the corresponding absorbing state phase transition can be different from the usual APT to ($\rho_A^* = 0, \rho_B^* = 0$). Clearly, for $\rho_A^* = 1$, the mean-field equation for the predator density $\rho_B$ (from equation (6)) is identical to that of the DP. Thus, in higher dimension (larger than the critical dimension) one expects that the 4SPP model results in the same mean-field critical exponents ($\beta_{MF} = 1 = \nu_{MF}^{MF}, z_{MF}^{MF} = 2$) as those of DP. Another possible reason for the new universality class is the asymmetric dynamics, which generates density dependent velocity terms $v_{A,B}$. Note that the absorbing state phase transition in the asymmetric contact process [23] belongs to the DP-class, whereas asymmetric updating is a relevant perturbation to models with extremal dynamics [24]. Detailed study of the 4SPP model with symmetric dynamics will be reported elsewhere.

In summary, we introduce a two-species (predator $B$ and prey $A$) model in one dimension where each lattice site is vacant or occupied by a single predator, a single prey or both. More than one predator or prey are not allowed at any site. The prey are allowed to grow independently of the predators, whereas two predators, if present at neighboring sites, die simultaneously. The species interact through a dynamics where the predator produces an offspring by eating a prey from its neighbor. When the predator density $\rho_B^* = 0$, even a single prey can invade the whole lattice by its independent birth process. Thus, the system has an unusual absorbing state ($\rho_A^* = 1, \rho_B^* = 0$); the other absorbing state ($\rho_A^* = 0, \rho_B^* = 0$) is not dynamically accessible. Using dynamical Monte Carlo simulation we show that the system shows an absorbing state phase transition, as the birth rate of prey $p$ is increased beyond a critical value $p_c$, keeping the death rate of predators $q$ fixed. For a fixed $r$, the line of criticality ($p_c$ as a function of $q$) is governed by a universality class different from the most generic one, namely directed percolation. This critical behavior is found to be robust against the addition of explicit diffusion. Note that the dynamical rules of the model are different from other four-species predator–prey models studied earlier [22] in that the predators in the 4SPP model cannot eat prey at the same site. We believe that non-equilibrium phase transition to an unusual and unique absorbing state may result in a different universality class. Further study in this direction could clarify this issue.

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