Effects of Gauss-Bonnet term on final fate of gravitational collapse

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Abstract. We obtain a general spherically symmetric solution of a null dust fluid in \( n \geq 5 \)-dimensions in Gauss-Bonnet gravity. Using the solution for \( n \geq 5 \) with a specific form of the mass function, we present a model for a gravitational collapse in which a null dust fluid radially injects into an initially flat and empty region. It is found that a naked singularity is inevitably formed and its properties are quite different between \( n = 5 \) and \( n \geq 6 \). In the \( n \geq 6 \) case, a massless ingoing null naked singularity is formed, while in the \( n = 5 \) case, a massive timelike naked singularity is formed, which does not appear in the general relativistic case. The strength of the naked singularities is weaker than that in the general relativistic case. These naked singularities can be globally naked when the null dust fluid is turned off after a finite time and the field settles into the empty asymptotically flat spacetime.

1. Introduction
The final fate of gravitational collapse is one of the most important unsolved problems in gravitation physics. In this context, a cosmic censorship hypothesis (CCH) was proposed by Penrose, which asserts that singularities formed in generic gravitational collapse of physical matter cannot be observed; in other words, there are no naked singularities formed in physical gravitational collapse [1,2].

The effects of quantum gravity will be unavoidable in the very final stage of gravitational collapse. Superstring theory is the most promising candidate of quantum gravity, which predicts a higher-dimensional spacetime. If superstring theory is the correct quantum gravity, the effects of the extra dimensions will become significant near singularity formation. However, the non-perturbative aspects of superstring theory are so far not understood completely. Given the present circumstances, taking string effects perturbatively into classical gravity is one possible approach to study the quantum effects of gravity. The Gauss-Bonnet term in the Lagrangian is the higher curvature correction to general relativity and naturally arises as the next leading order of the \( \alpha' \)-expansion of heterotic superstring theory [3,4].

In this paper, considering the spherically symmetric gravitational collapse of a null dust fluid with the \( n(\geq 5) \)-dimensional action including the Gauss-Bonnet term, we investigate its effects on the final fate of gravitational collapse. This paper is based on [5]. Throughout this paper we use units such that \( c = 1 \). The Greek indices run \( \mu = 0, 1, \cdots, n - 1 \).
2. Model and solution

The $n$-dimensional ($n \geq 5$) gravitational equation for a null dust in Gauss-Bonnet gravity is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \alpha H_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_n \rho l_\mu l_\nu,$$

where

$$H_{\mu\nu} = 2 \left[ RR_{\mu\nu} - 2R_{\mu\sigma}R^\sigma_\nu - 2R^\sigma_\mu R_{\nu\alpha\beta\gamma} R^\alpha_\nu R^{\beta\gamma}_\mu \right] - \frac{1}{2}g_{\mu\nu}(R^2 - 4R_{\mu\nu}R^\mu^\nu + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}).$$

The Gauss-Bonnet term does not contribute to the field equations, i.e., $H_{\mu\nu} \equiv 0$ holds, for $n = 4$.

The general spherically symmetric solution in the single null coordinates is obtained by

$$ds^2 = - \left[ 1 + \frac{r^2}{2\tilde{\alpha}} \left( 1 + \sqrt{1 + 4\tilde{\alpha} \left( \frac{m(v)}{r^{n-1}} + \tilde{\Lambda} \right)} \right) \right] dv^2 + 2dvdr + r^2 d\Omega^2_{n-2},$$

where

$$\tilde{\alpha} \equiv (n - 3)/(n - 4) \alpha$$

and

$$\tilde{\Lambda} \equiv 2/[(n - 1)/(n - 2)]\Lambda.$$  

$m(v)$ is an arbitrary function of $v$ and $d\Omega^2_{n-2}$ is the line element of the $(n-2)$-dimensional unit sphere.

There are two families of solutions which correspond to the sign in front of the square root in Eq. (3). We call the family which has the minus (plus) sign the minus-branch (plus-branch) solution. In the general relativistic limit $\tilde{\alpha} \to 0$, the minus-branch is reduced to the $n$-dimensional Vaidya-(anti)de Sitter solution, while there is no such limit for the plus-branch.

3. Results

Applying the minus-branch solution to the situation in which a null dust fluid radially injects into an initially Minkowski region, we have investigated the effects of the Gauss-Bonnet term on the final fate of the gravitational collapse.

We have assumed that (i) the mass function has the form of $m(v) = m_0 v^{n-3}$, where $m_0$ is a positive constant, and (ii) the null geodesics obey a power law near the singularity. Then, it has been found that a naked singularity is inevitably formed. On the other hand, in the general relativistic case, a naked singularity is formed only when $m_0$ takes a sufficiently small value. This result implies that the effects of the Gauss-Bonnet term on gravity worsen the situation from the viewpoint of CCH rather than prevent naked singularity formation.

Furthermore, the Gauss-Bonnet term drastically changes the nature of the singularity and the whole picture of gravitational collapse. The picture of the gravitational collapse for $n = 5$ is quite different from that for $n \geq 6$. For $n \geq 6$, as well as the general relativistic case for $n \geq 4$, a massless ingoing null naked singularity is formed. On the other hand, for the special case $n = 5$, a massive timelike naked singularity is formed.

Although naked singularities are inevitably formed in Gauss-Bonnet gravity, the Gauss-Bonnet term makes the strength of the naked singularity weaker than that in the general relativistic case. In association with this, there does exist a possible formulation of CCH, which asserts that the formation of weak naked singularities need not be ruled out. In this sense, the Gauss-Bonnet term works well in the spirit of CCH.

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