Acceleration of particles near the inner black hole horizon

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We study the possibility of obtaining unbound energy $E_{c.m.}$ in the centre of mass frame when two particles collide near the inner black hole horizon. We consider two different cases - when both particles move (i) in the same direction or (ii) in the opposite ones. We also discuss two different versions of the effect - whether an infinite energy can be released in the collision (strong version) or the energy $E_{c.m.}$ is finite but can be made as large as one likes (weak version). We demonstrate that the strong version of the effect is impossible in both cases (i) and (ii). In case (i) this is due to the fact that in the situation when $E_{c.m.}$ formally diverges on the horizon, one of particles passes through the bifurcation point where two horizons meet while the second particle does not, so collision does not occur. In case (ii), both particles hit different branches of the horizon. The weak version is possible in both cases, provided at least one of particles starts its motion inside the horizon along the direction of spatial symmetry from infinity.

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I. INTRODUCTION

In recent years, an interesting effect was discovered: when two particles collide near the event horizon of a black hole, their energy $E_{c.m.}$ in the centre of mass frame can grow unbound (so-called the BSW effect [1]). This provoked a series of papers in which properties of such collisions were investigated in detail (see, e.g., the recent works [2], [3] and references therein). Meanwhile, for collisions near the inner horizon the situation turned out to be

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contradictory. At first, the possibility of the BSW effect on the inner horizons of the Kerr and BTZ black holes was claimed in [4]. Later, in a brief note [5], K. Lake claimed that although the formulas for the energy in [4] are formally correct they are physically irrelevant since actual collision between particles which would lead to infinite $E_{\text{c.m.}}$ cannot occur. A similar result was obtained in [6], [7] for the Kerr metric. However, in a recent work [8], the kinematics of collisions inside black holes was discussed again with the conclusion that the divergencies do occur on the inner horizon (provided the energy and angular momentum of one particle are fine-tuned properly). The same conclusion (but without consideration of the kinematics of collisions) is made in [9] for a cosmological horizon. Moreover, the results of [9] would have implied that an infinite energy can be achieved during a finite interval of time that looks unphysical.

The aim of the present paper is to give general explanation of the situation with collisions near the inner black hole horizon valid for generic rotating black holes and investigate the similar issue for charged nonrotating black holes. (The same reasonings apply also to the nonextremal cosmological horizons, so for definiteness we restrict ourselves by the inner black holes ones). As was noticed in [10], the counterpart of the BSW effect for rotating black holes reveals itself for charged nonrotating ones. The latter case captures main features of the phenomenon but consideration is simpler, so at the beginning we discuss the motion of particles in the Reissner-Nordström metric. We show that the same conclusions apply to any nonextremal spherically symmetric black holes having the inner horizon.

We also consider generic rotating nonextremal black holes (cf. [11]). We show that the arguments of [6], [7] were incomplete but the result is correct in agreement with [5], so there is no BSW effect with infinite energy on the inner or cosmological horizon in contrast to the claims made in [9].

Apart from the BSW effect, we also consider another type of the effect connected with the near-horizon collisions - the Piran and Shanam (PS) one [12]. The difference between both effects consists in that in the BSW case both particles move in the same directions whereas in the PS case they do it in opposite ones (see [13] for details).

In what follows, we need to distinguish two different versions of the effects under discussion. I call it the ”strong one” if the value of the energy in the centre of mass frame is divergent in the point of collision. And, it is called ”weak version” if the energy is finite but can be made as large as one likes. In the pioneering paper [1] where the BSW effect was dis-
covered, the energy in the centre of mass frame was found to be infinite in the horizon limit, if special relationship between the energy and momentum of one particle holds. However, later it was observed [14], [7], [15] that corresponding collision requires an infinite proper time, so physically the infinite energy cannot be realized. This observation was extended to the generic case in [11]. Apart from this, it is pointed out in [14], [11], [7] that the BSW effect is valid for nonextremal horizons, provided multiple scattering occurs. In each collision the energy $E_{\text{c.m.}}$ is finite but it can be made as large as one likes if the radial coordinate of collision becomes closer and closer to the horizon radius. (Additionally, it requires some special relationship between the energy and angular momentum or the energy and electric charge for one of colliding particles.) Thus for both types of the event horizons (extremal and nonextremal) only the weak version of the BSW effect can be realized.

In this terminology, when it applies to inner horizons, refutations made in [5] and [6], [7] concern the strong version of the BSW effect only but they say nothing about the possibility of the weak version. This will be done below. The claim of [8] can be reformulated by saying that the reason why the strong version of the BSW effect is not realized is due to an infinite time required for the critical particle to reach the horizon. We will see that this is incorrect since the corresponding time is finite and the reason why the strong version of the BSW effect does not happen is quite different. The result of the paper [9] for the cosmological horizon is even more unphysical since it would have meant that the strong version of the effect of collision does occur.

It is worth noting that there are possible limitations on the BSW effect due to backreaction and gravitational radiation near the event horizon [15], [16]. Similar factors can come into play near the inner horizon as well. At present, their role is not fully understood and we refrain from discussing this important physical issue that needs separate investigation.

II. MOTION INSIDE THE REISSNER-NORDSTRÖM BLACK HOLE

At first, let us consider the metric of the Reissner-Nordström black hole

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2.$$  \hspace{1cm} (1)

Here $d\omega^2 = \sin^2 \theta d\phi^2 + d\theta^2$, $f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$, $M$ is the black hole mass, $Q$ is its charge, $M > Q$. We use the geometrical system of units with $c = G = 1$ ($G$ is the gravitational
constant, \( c \) is the speed of light). The function \( f(r) = 0 \) at \( r_- = M - \sqrt{M^2 - Q^2} \) (the inner horizon) and \( r_+ = M + \sqrt{M^2 - Q^2} \) (the event horizon).

We are interested in the region \( r_- \leq r < r_+ \) where \( f = -g \leq 0 \). In this region, the coordinate \( r \equiv T \) has timelike character and \( t \equiv y \) is spacelike. Then, the metric takes the form

\[
ds^2 = -\frac{dT^2}{g(T)} + g(T)dy^2 + T^2d\omega^2.
\]

Let a particle have the charge \( q \) and the rest mass \( m \). For simplicity, we consider its motion along the \( y \)-direction. As the metric does not depend on \( y \), the momentum \( P_y \equiv P \) is conserved. We assume that \( r = T \) decreases, so the region under discussion is the \( T_- \) region in the Novikov’s terminology \[17\]. The equations of motion read

\[
m\dot{y} = \frac{X}{g}, \quad X = (P + \frac{gQ}{T}),
\]

\[
\dot{T} = -\sqrt{g + \frac{X^2}{m^2}}
\]

where dot denotes differentiation with respect to the proper time \( \tau \). It follows from (4) that

\[
\tau = m \int_{r}^{r_1} \frac{dr}{\sqrt{m^2g + X^2}}
\]

where we an initial value moment of time \( r_1 \) such that \( r_- < r \leq r_1 < r_+ \).

The choice of signs in (3) takes into account that \( P = -E \) where \( E \) has the meaning of the conserved energy in \( R \) region. It follows from (3), (4) that

\[
\frac{dy}{dT} = -\frac{1}{g} \frac{X}{\sqrt{m^2g + X^2}}.
\]

It follows from (3) that

\[
y = \int_{r}^{r_1} \frac{Xdr}{g\sqrt{m^2g + X^2}} + y(r_1).
\]

If there are two particles with charges \( q_1 \) and \( q_2 \) and masses \( m \), the energy in the centre of mass frame is equal to (see \[10\], \[11\] and references therein)

\[
\frac{E_{cm}^2}{2m^2} = 1 + \frac{Z_1Z_2 - X_1X_2}{gm^2}.
\]

Here, \( i = 1, 2 \) enumerates particles,

\[
Z_i = \sqrt{X_i^2 + m^2g},
\]
$X_1X_2 > 0$ if particles move in the same direction and $X_1X_2 < 0$ if the particles move in the opposite ones. If the charges $q_1$ and $q_2$ have the same sign, Coloumb repulsion somewhat complicates the picture of collision. To avoid such unnecessary details, we assume that, say, $q_2 = 0$. The potential divergencies can occur in the limit $g \to 0$ only, i.e. near the inner horizon where $r \to r_-$.

Now, we examine the possibility of two effects separately.

**III. BSW EFFECT ($X_1X_2 > 0$).**

**A. Energy of collision**

Here, one should distinguish two types of particles. By analogy with previously used terminology [11], we call a particle usual if $X_i(r_-) \neq 0$ and critical if $X_i(r_-) = 0$ ($i = 1, 2$). In the latter case

$$X(r) = -\frac{q_i Q}{rr_-}(r - r_-).$$  \hfill (10)

If both particles are usual, near the horizon $Z_i \approx |X_i| + \frac{m^2 g}{2|X_i|}$ and, according to [8], $E_{cm}^2$ remains finite, the effect is absent. If both particles are critical, near the horizon $X_i \sim r - r_- \sim g$, $Z_i \approx m\sqrt{g}$, and the effect is also absent. The only potential case of interest arises when the particles belong to different types. Say, particle 1 is critical and particle 2 is usual. Then,

$$\frac{E_{cm}^2}{2m^2} \approx |X_2(r_-)| \frac{1}{\sqrt{g}m} \to \infty$$  \hfill (11)

when $r \to r_-$. However, the crucial point consists in that one should check whether a collision as such occurs near the inner horizon.

**B. Trajectories in original coordinates**

For what follows, we need explicit asymptotic behavior of space-time trajectories near the horizon. For a usual particle, one can easily obtain from [3] - [9] that

$$t = y \approx C - \frac{\text{sign}X(r_-)}{2\kappa_-} \ln(r - r_-),$$  \hfill (12)
\[ r = T \approx r_+ + \frac{|X(r_-)|}{m}(\tau_- - \tau). \]  

(13)

Here, \( \kappa_- = \frac{1}{2} \left( \frac{d \omega}{d \rho} \right)_{\rho=r_-} \) has a meaning of the surface gravity of the inner horizon, \( C \) and \( \tau_- \) are constants.

For the critical particle,

\[ y - y_- \approx A\sqrt{r - r_-}, \]

(14)

\[ \tau - \tau_- \approx -B\sqrt{r - r_-}, \]

(15)

where \( y_-, A, B \) are constants,

\[ A = \frac{-qQ}{\sqrt{2m\kappa_-^{3/2}r_-^2}}, \]

(16)

\[ B = \frac{\sqrt{2}}{\kappa_-^{1/2}}. \]

(17)

It is seen from (12), (13) that the proper time is finite both for usual and critical particles (in contrast to the results described in the end of Sec. III of Ref. [8]). For usual particles, \( \frac{dt}{d\tau} \to \infty \) or \( \frac{dt}{d\tau} \to -\infty \), depending on the sign of the momentum \( X \) in full analogy with the situation for the Kerr metric [6], [7]. Meanwhile, for critical particles, \( \frac{dt}{d\tau} \) remains finite.

It was concluded in [6], [7] that the collision between the critical particle and a usual one (which is necessary for divergences of \( E_{cm}^2 \)) cannot occur since the difference in the variable \( t \) is infinite for them. Such a conclusion looks plausible but not quite rigorous since the coordinate system used in the analysis becomes degenerate near the horizon, so the behavior of coordinates gives essentially incomplete information about the process. Apart from this, we want to examine not only the strong version of the BSW effect but also the weak one. To give full self-consistent picture, one should reformulate the metric in well-behaved coordinates.

C. Trajectories in Kruskal coordinates

In original coordinates (1), the metric coefficients become ill-behaved near the horizon. To remedy this drawback, one is led to using coordinates in which the metric coefficients are analytical near the horizon. We take advantage of the Kruskal-like coordinates which were introduced in [18] for the Schwarzschild metric. Now, we apply corresponding formulas to the region inside the horizon where the metric reads
\[ ds^2 = -F dU dV + r^2 d\omega^2, \]  
(18)

and

\[ U = \exp[-\kappa_-(t - r^*)], \]
(19)

\[ V = \exp[\kappa_-(t + r^*)], \]
(20)

the tortoise coordinate

\[ r^* = \int \frac{dr}{g}. \]
(21)

\[ F = g\kappa_+^2 \exp(-2\kappa_- r^*). \]
(22)

Near the horizon, the tortoise coordinate diverges,

\[ r^* \approx \frac{1}{2\kappa_-} \ln(r - r_-) + r_0^* \]
(23)

where \( r_0^* \) is a constant.

The coordinates \( U \) and \( V \) take finite values near the horizon. The surface \( U = 0 \) corresponds, say, to the left inner horizon in the standard Carter-Penrose diagram while \( V = 0 \) corresponds to the right one. Near any of two horizons,

\[ g \approx 2\kappa(r - r_-), \]
(24)

\( F \) is finite. As a result, the zeros in the numerator and denominator in (22) compensate each other, so the metric coefficient \( F \) is finite and nonzero on the horizon.

Then, it follows from formulas (19) - (23) that

\[ \frac{U}{V} = \exp(-2\kappa_- t) \]
(25)

and, near the horizon

\[ UV \approx \text{const}(r - r_-). \]
(26)

For usual particles near, say, the left horizon the value of coordinate \( V \) is finite, \( V \neq 0 \), \( t \rightarrow \infty, r - r_- \sim \exp(-2\kappa t) \rightarrow 0, U \rightarrow 0 \). For critical particles, it is seen from (14) that \( t \) is finite, \( V \sim U \sim \sqrt{t_- - \tau} \sim \sqrt{\tau_- - \tau} \rightarrow 0 \). Thus, critical and usual particles have at the horizon different values of \( V \) and, therefore, cannot collide there. This confirms the statements of \( [5] \) and \( [6], [7] \).
Here, we comment shortly on the corresponding claims made in [8]. These authors rely on the properties of the critical particle and claim that it only asymptotically spirals onto the horizon for an infinite proper time similarly to the situation analyzed in [15], [6]. Then, according to Ref. [8], collision with an infinite energy would occur at an arbitrary point of the inner horizon, and only an infinite proper time prevents it from being actual event. Meanwhile, the corresponding observations in Refs. [15], [6] refer to the situation in $R-$region outside the extremal event horizon. They do not apply to particles’ motion near the inner horizon which is nonextremal. Mathematically, the integral (5) converges even in the critical case since the function $g$ has the simple zero. Therefore, the proper time for the critical particle to reach the horizon is finite in contrast to the claim made in Ref. [8]. Moreover, for the Reissner-Nordström (or Reissner-Nordström - de Sitter one like in Ref. [9]) the effect reveals itself even for a zero angular momentum, so there is no any spiraling at all. Actually, the mechanism preventing the strong version of the effect is completely different and this will be shown below.

D. Critical particle and bifurcation point

Now, it is worth noting that motion of the critical particle admits simple geometrical interpretation. It follows from (4) that the critical particle reaches the horizon in finite proper time (in contrast to the situation with the event horizon [6], [10], [11], [15]). It means that a particle should cross the horizon. However, in the $R$ region, where $f > 0$, such a particle cannot be situated in the immediate vicinity of the horizon. Indeed, in that region, eq. (27) turns to

$$\dot{r}^2 = \frac{X^2}{m^2} - f. \tag{27}$$

Near the horizon, for $r < r_-$, $f \sim r_- - r$. For the critical particle, $X \sim r_- - r$. Thus, near $r_-$ for $r < r_-$, the right hand side of (27) becomes negative and motion is impossible. The same conclusion is valid for rotating nonextremal black holes [11], [8]. Thus, the critical particle cannot find itself in the $R$ region. The only possibility that remains for it is to enter $T$ region again. But for the Reissner-Nordström nonextremal metric the only possible path for it passes through the bifurcation point where two horizons meet. Meanwhile, a usual particle reaches the horizon somewhere in an intermediate point, so these points
are separated geometrically. The corresponding situation is represented at the part of the Carter-Penrose diagram in Fig. 1 where B is the bifurcation point.

It is instructive to remind that if both particles collide approaching the extremal horizon from the $R$ region (outside the event horizon)\cite{1}, the critical particle plays a crucial role in the BSW effect but in such a case there is no bifurcation point at all.

\section*{E. Critical particle and speed of motion}

In addition to geometrical properties of the trajectory of the critical particle near the horizon, it is instructive to look at the kinematic ones. One can define the velocity with respect to an observer who remains at rest: $v = \frac{dl}{d\tau} = dy \sqrt{g}, d\tau = \frac{dT}{\sqrt{g}}$, so $v = g \frac{dy}{dT}$. Then, after simple manipulations, one obtains from (3), (4) that

$$X_m = \pm \sqrt{\frac{v}{1 - v^2}}. \quad (28)$$

For usual particles, $X(r_-) \neq 0$, so near the horizon $v \approx 1 - \kappa_- \frac{m^2}{X^2(r_-)} (r - r_-)$ where we took into account (24). For the critical particle, $X(r) \sim r - r_-$ near the horizon, so $v \sim \frac{X}{\sqrt{g}} \sim \sqrt{r - r_-} \rightarrow 0$. Thus, the critical particle not only has the velocity different from that of light - moreover, it approaches the horizon with almost vanishing velocity.

For particles which collide near the horizon from the outside, the BSW effect received a simple explanation based on kinematic properties\cite{20}. Namely, in the static frame a usual particle have the velocity $v \rightarrow 1$ near the horizon, whereas for critical ones $v \neq 1$ near the horizon. Then, the relative velocity tends to the speed of light, the Lorentz factor grows unbound and the energy in the comoving frame tends to infinity. Roughly speaking, a quick particle overtakes a slow one that results in the almost infinite energy of collision in the centre of mass frame. However, collision with the literally infinite energy in the centre of the type represented in Fig. 1 is impossible.

\section*{IV. ARE NEAR-HORIZON COLLISIONS WITH FINITE BUT UNBOUND ENERGY POSSIBLE?}

Thus we saw that the collisions with the infinite energies cannot occur. In other words, the strong version of the BSW effect cannot be realized physically. Meanwhile, one can
ask, whether we can at least arrange collisions not exactly on the horizon but somewhere in its vicinity with the energy which would grow while approaching the horizon (the weak version of the BSW effect). It is instructive to remind that for collisions which occur from the outside the horizon, such situation is indeed possible not only for extremal black holes but also for nonextremal ones \[14\], \[7\], \[11\].

Let particle 1 be the critical one, as before. For a generic particle 2 that would hit the horizon with generic value of coordinate \(V\) both particles are still separated in agreement with Fig. 1. Meanwhile, we can arrange collision not exactly on the horizon but somewhere in its vicinity. We can assume that an usual particle which without collision would hit the right horizon at point A with \(V \to 0\), now collides with particle 1 at point C - see Fig. 2.

It means that collision is adjusted in such a way that points A, B and C are close to each other and the value of \(V\) in point \(C\) is small. Correspondingly, the value \(r_0\) in the point of collision is close to \(r_-\). Then, according to eq. (11), the energy becomes arbitrarily large.

Meanwhile, there is a kinematic issue to be clarified. The situation under discussion implies that particle 2 possesses two important properties: (i) it is usual, so \(X_2(r_-) \neq 0\),(ii) in the absence of collision it would hit the horizon with arbitrarily small \(V\). Are properties (i) and (ii) mutually consistent? Now we will show that the answer is "yes".

It follows from eqs. \[12\] (with \(X_2 > 0\) for definiteness), \[20\] and \[23\] that near the horizon \(V_2 \approx e^{\kappa-C}\) where for simplicity we put \(r_0^* = 0\) for the constant in \[23\]. Therefore, for generic finite \(C\) particle 2 cannot have small \(V\) near the horizon. However, this becomes possible if the constant \(C\) is chosen to be (in modulus) large and negative that implies that the constant \(y(r_1)\) in \[7\] is also large and negative. A typical trajectory of such a kind is given in Fig. 2. Particle 2 passes nearly to the left corner of the Carter-Penrose diagram, keeps moving closely to the left horizon and would hit the right horizon in point A in the absence of collision.

Taking into account eqs. \[12\] - \[13\] and \[19\], \[20\], one can write for coordinates of both particles (critical and usual) near the horizon

\[
U_1 \approx U_{10}\sqrt{r_0-r_-}, \quad V_1 \approx V_{10}\sqrt{r_0-r_-},
\]

\[
U_2 \approx U_{20}(r_0-r_-), \quad V_2 \approx V_{20}
\]

where

\[
U_{10} = e^{-\kappa-y_-}, \quad V_{10} = e^{\kappa-y_-}, \quad U_{20} = e^{-\kappa-C}, \quad V_{20} = e^{\kappa-C}.
\]
It is seen that the condition of collision \( U_1 = U_2, V_1 = V_2 \) has one solution for which

\[
U_{10} = U_{20} \sqrt{r_0 - r_\ominus}, V_{20} = V_{10} \sqrt{r_0 - r_\ominus}, e^{\kappa(C-y\cdot)} = \sqrt{r_0 - r_\ominus}
\]  

(32)

where \( r_0 \) is a point of collision and eq. (14) was taken into account. This is achieved at the expense of large and negative \( C \). It is obvious that one can deform slightly the mutual disposition of particles in Fig. 2 in such a way that particle 1 ceases to be critical but remains near-critical that does not change our main conclusions.

Then, the collision occurs near the bifurcation point with small (although nonzero) coordinates \( U \) and \( V \) and with the large (although finite) energy \( E_{\text{cm}} \). Thus the BSW effect is present in the weak version under discussion. However, this imposes conditions not only on particle 1 which must be critical or near-critical but requires also that an usual particle belong to a special class of trajectories. More precisely, in eq. (7), (12) constants \( y(r_1) \) and \( C \) should be large and negative. This means that a usual particle 2 starts its motion from "almost" infinity in the sense that \( |y| \) is large. It starts from large and negative \( y \) and moves from the left to the right if \( X_2 > 0 \) (so \( \dot{y} > 0 \)) and vice versa.

It is instructive to compare the situation of infinitely growing energies for collisions on the inner end event horizons. In both cases, one of particle should be critical or near-critical. In doing so, the velocity of a usual particle with respect to \((r, t)\) coordinates approaches the speed of light whereas the velocity of the critical one does not. Meanwhile, there is difference in geometric properties between both cases. Near the event horizon, the four-velocity of a critical particle has the component along one horizon generator much larger than along the other one [13]. For the inner horizon the situation is opposite as (without mathematical rigor) can be seen from Fig. 2 clearly.

V. PS EFFECT \((X_1X_2 < 0)\)

Now, let us discuss the case when both colliding particle move in the opposite directions (the PS effect). We would also like to emphasize the difference between the PS effect on the event horizon [12] and on the inner one. In the first case, one has to combine the metric of a black hole with the state of a particle moving away from the horizon instead of a usual picture of a particle approaching the horizon in the course of gravitational collapse. This requires the choice of very special initial conditions - say, such a particle should acquire its
momentum (or be created) in some other precedent process. By contrary, inside the event horizon motion in both directions are physically equivalent, so in this region the PS effect is more natural than outside.

Hereafter, we assume that $X_1 > 0$ (so $\dot{y} > 0$) and $X_2 < 0$ (so $\dot{y} < 0$). In contrast to the collisions in $R$ region, now particles move along the legs of a cylinder and for both of them $\dot{r} < 0$. It is seen from [8] that if both particles are usual, $E_{cm}^2 \sim g^{-1} \to \infty$ near the inner horizon. If both particles are critical, $X_i \sim g$, $Z_i \sim \sqrt{g}$, and $E_{cm}^2$ is finite, so this case is not interesting. If, say, particle 1 is critical and particle 2 is usual, $X_1 \sim g$, $X_2(r_-) \neq 0$, $Z_1 \sim \sqrt{g}$, $Z_2(r_-) \neq 0$, $E_{cm}^2 \sim g^{-1/2} \to \infty$, so the energy diverges, although more slowly than in the case when both particles are usual. Thus, it is interesting that, in contrast to the BSW effect, the most rapid grow of the energy $E_{c.m.}$ occurs when none of particles is critical.

Now, we must analyze the possibility of collision from the kinematic viewpoint.

A. Impossibility of strong version

First of all, we must check that the strong version of the effect is impossible. Let us consider different cases separately.

1. Particle 1 is critical, particle 2 is usual.

Then, the situation is presented at Fig. 3 which is similar to Fig. 1, so collision with the infinite energy does not occur.

2. Both particles are usual.

Using the asymptotic form of trajectories (12), (13) and eqs. (19), (20) we find that

$$U_1 \approx U_{10}(r - r_-), \ V_1 \approx V_{10},$$

$$U_{10} = e^{-\kappa-C_1}, \ V_{10} = e^{\kappa-C_1}. \quad (33)$$

For particle 2,

$$U_2 \approx U_{20}, \ V_2 \approx V_{20}(r - r_-). \quad (35)$$
\[ U_{20} = e^{-\kappa-C_2}, \quad V_{20} = e^{\kappa-C_2}. \] (36)

If constants \( C_1, C_2 \) (hence, also \( U_{10}, U_{20}, V_{10}, V_{20} \)) are all finite, it is seen that in the horizon limit when \( r \to r_-, U_1 \to 0, V_1 \neq 0 \) and \( V_2 \to 0, U_1 \neq 0 \). This means that particle 1 hits the left horizon whereas particle 2 hits the right one, so again collision with the infinite energy does not happen. The situation is represented at Fig. 4.

**B. Weak version**

Obviously, if the particles start having some separation and move along the \( y \) axis to meet each other, head-on collision is inevitable and occurs in some finite proper time. In doing so, the energy is finite. However, it will be seen now that the energy can be made as large as one wants if one makes the point of collision closer and closer to the horizon. Again, we analyze different cases separately.

1. **Particle 1 is critical, particle 2 is usual.**

Then, eqs. (29) - (32) are valid and the corresponding analysis applies. The situation is represented at Fig. 5 which is similar to Fig. 2.

2. **Both particles are usual.**

Then, near the horizon eqs. (33) - (36) are valid. One can arrange collision \( (U_1 = U_2, V_1 = V_2) \) if one chooses \( U_{20} = U_{10}(r_0 - r_-), V_{10} = V_{20}(r_0 - r_-) \) or, equivalently, \( e^{\kappa-C_1} = e^{\kappa-C_2}(r_0 - r) \) where \( r_0 \) corresponds to the point of collision. This can be also obtained in original coordinates (2) directly, taking into account the asymptotic form of trajectories from (3), (4). We can rewrite the relation between the constants as

\[ C_2 \approx C_1 - \frac{1}{\kappa_-} \ln(r_0 - r_-). \] (37)

For any \( r_0 \neq r_- \), the energy \( E_{\text{cm}} \) is finite. However, choosing \( r_0 - r_- \ll r_- \) and simultaneously taking trajectories with larger and larger \( C_2 \), we can indeed achieve collision with \( E_{\text{c.m.}} \) as large as we want.
The situation is represented at Fig. 6 (collision near the generic point of the horizon, $C_1 \to -\infty$, $C_2$ is finite)

and Fig. 7 (collision near the bifurcation point, $C_1 \to -\infty$, $C_2 \to \infty$). It is worth reminding that large and negative (positive) $C$ corresponds to particles which start from the left (right) infinity in terms of the coordinate $y$.

VI. INTERMEDIATE CASE ($X_1X_2 = 0$)

Now, we will discuss the case intermediate between BSW and PS effects - namely, when one of $X_i$ vanishes (say, $X_1 = 0$). This means that $y_1 = const$, the particle remains at rest with respect to this coordinate system. (Meanwhile, the geometry evolves since the region is nonstationary, $\dot{r} < 0$.) Then, collision between particles can be thought of as a counterpart of that in the Kerr background when one particle is on the circular orbit.

It follows from (3) that the condition $X_1 = 0$ for all $r$ is possible only for a special case $P_1 = 0 = q_1$. Actually, particle 1 is formally critical since $X_1(r_-) = 0$ (although now this holds not only for $r_-$ but for all $r$). Therefore, the above analysis applies. In the present case, $Z_1 = m\sqrt{g}$,

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{Z_2}{m\sqrt{g}} = 1 + \sqrt{1 + \frac{X_2^2}{m^2g}}. \quad (38)$$

If particle 2 is critical, $E_{cm}^2$ remains finite in accordance with what is said above. If particle 2 is usual, $X_2(r_-) \neq 0$, $\frac{r_{c2}^2}{2m^2} \to \infty$ when the moment of collision $r_0 \to r_-$. For any finite $y_1$, collision occurs outside $r_-$. However, taking $r_0 - r_- \to 0$ and, correspondingly, $|y_1| \to \infty$, one can achieve the collision near the horizon with $E_{cm}^2$ as large as one wishes, so again we have the weak version of the effect.

VII. GENERALIZATION, EXTENSION TO GENERIC ROTATING BLACK HOLES

We can consider more general spherically symmetric black hole metrics of the form

$$ds^2 = -dt^2 f_1 + \frac{dr^2}{f_2} + r^2 d\omega^2. \quad (39)$$
Let us suppose that such a black hole is nonextremal and near the inner horizon \( f_1 \sim f_2 \sim r_- - r \). Then, all above consideration applies since it was based on the asymptotic properties of the metric coefficients, while the explicit form of the Reissner-Nordström metric was irrelevant. Repeating all reasonings step by step we arrive at the conclusion that the strong version of the BSW and PS effects is forbidden whereas the weak one is allowed.

The situation with rotating black holes is somewhat more complicated. The metric reads

\[
\begin{align*}
    ds^2 &= -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dt^2 + B dz^2. \\
\end{align*}
\]

(40)

We assume that all metric coefficients do not depend on \( t \) and \( \phi \). We also assume that \( \theta = z (z = 0) \) is the symmetry plane and restrict ourselves by motion in this plane. Then, the equations of motion read \[11\]

\[
\begin{align*}
    \dot{t} &= \frac{E - \omega L}{N^2}, \\
    \dot{\phi} &= \frac{L}{g_{\phi\phi}} + \frac{\omega (E - \omega L)}{N^2}, \\
    \dot{t}^2 &= \frac{(E - \omega L)^2}{N^2} - 1 - \frac{L^2}{g_{\phi\phi}}
\end{align*}
\]

(41, 42, 43)

where \( L \) is the angular momentum, \( E \) is the energy and the value \( \theta = \frac{\pi}{2} \) is put in all metric coefficients. Inside the horizon we must replace \( N^2 \) by \(-g < 0\) and the proper distance by the proper time \( \tau \). Now, the metric takes the form

\[
\begin{align*}
    ds^2 &= -d\tau^2 + g dy^2 + g_{\phi\phi}(d\phi - \omega dy)^2 + B dz^2.
\end{align*}
\]

(44)

where \( t = y \) is a spatial coordinate.

Then, near the inner horizon,

\[
\begin{align*}
    \sqrt{g} &\approx \kappa_- \tau + O(\tau^3), \\
    \omega &\approx \omega_i + A(z) \tau^2.
\end{align*}
\]

(45, 46)

where \( \kappa_- \) and \( \omega_i \) are constants. The quantity \( \kappa_- \) has the meaning of the surface gravity of the inner horizon. Derivation of (45), (46) can be found in \[21\] with obvious replacement \( l \to \tau \).

It is also convenient to introduce the coordinates \( x = \frac{1}{4} \tau^2 \) and \( \tilde{\phi} = \phi - \omega_i y, \ y = \frac{\tilde{y}}{2\kappa_-} \). Then,

\[
\begin{align*}
    ds^2 &\approx -\frac{dx^2}{x} + d\tilde{y}^2 x + g_{\tilde{\phi}\tilde{\phi}}(d\tilde{\phi} - \tilde{A}x d\tilde{y})^2 + B^{-1} dz^2,
\end{align*}
\]

(47)
\[ g_{\phi\phi}^- = g_{\phi\phi}|_{x=0}, \quad B^- = B|_{x=0}, \quad \tilde{A} = \frac{2A|_{x=0}}{\kappa_-}. \] (48)

Near the horizon, one can obtain that the asymptotic behavior of space-time trajectories (12) - (15) is still valid.

Further, let us introduce

\[ x^* = \ln x, \quad u = x^* - \tilde{y}, \quad v = \tilde{y} + x^*, \] (49)

\[ U = \exp\left(\frac{u}{2}\right), \quad V = \exp\left(\frac{v}{2}\right). \] (50)

Then, it is seen that the metric becomes analytical near the horizon where

\[ ds^2 \approx -4dU dV + g_{\phi\phi}^- [d\tilde{\phi} - \tilde{A}(U dV - V dU)]^2 + B^- dz^2 \] (51)

Once this fact is established, we can repeat the same reasonings as in the case of charged nonrotating black holes, and arrive at the same conclusions. Namely, on the horizon (say, \( U = 0 \)) a usual particle has \( V \neq 0 \) whereas the critical one has \( V = 0 \) (that corresponds to the bifurcation point), so the collision does not occur. For other pairs of particles (two critical or two usual ones) collision can occur but with the finite energy \( E_{\text{c.m.}} \). This generalizes observations made for the particular case of the Kerr metric in [3] and [6]. This means that the strong version of the BSW effect is not possible. However, the weak version is admissible.

The same conclusions apply to the PS effect.

**VIII. HOW NATURE ESCAPES INFINITE ENERGIES: STRONG VERSION OF EFFECT AND KINEMATIC CENSORSHIP**

Obviously, in any physical process an infinite energy cannot be released, so some mechanism should act that prevents the strong version of the effect and excludes the events in which such an energy could be otherwise realized. For collisions on the event horizon, such mechanism consists in impossibility to reach the extremal horizon within a finite interval of the proper time. In the nonextremal case, the proper time is always finite but the critical particle cannot reach the horizon because of the potential barrier. A near-critical particle can do it but the corresponding energy \( E_{\text{c.m.}} \) is finite although can become larger and larger as the state of a particle comes closer and closer to the critical one. (See [6], [11], [7] for details.).
For the inner horizon, neither of two aforementioned mechanisms which were valid for the event horizon, now applies. The inner horizon is nonextremal by its very meaning, so the proper time needed to reach it, is finite. Apart from this, there is no potential barrier between a near-critical particle and the horizon. Instead, now quite different reason makes collision with an infinite energy $E_{c.m.}$ impossible. As far as the BSW effect is concerned, an infinite energy $E_{c.m.}$ requires that one of two particle be critical while the other one must be usual. Then, it turns out that such particles cannot meet each other in the same point of space-time. This is because the first particle passes through the bifurcation point where horizons intersect whereas the other one does not. This means that some kind of censorship (let us call it "kinematic censorship") indeed acts in these processes forbidding infinite energies in any collision but its manifestation is quite different for the event horizons and the inner ones. It is worth noting that the claim of [9] are in contradiction with kinematic censorship. As regards, the PS effect, the requirement of an infinite energy $E_{c.m.}$ enforces colliding particles to hit the different branches of the horizon. Thus both in the BSW and PS effects the particles turned out to be separated in space-time although the details of such a separation are different.

IX. CONCLUSION

The situation with collisions on the inner horizon proved to be more diverse than simply impossibility of the BSW effect. It required careful distinction between two different effects (the BSW and PS ones) and examining two versions of each of them (strong and weak). We checked that the strong version is impossible and interpreted it as a particular realization of "kinematic censorship" which has different manifestations on the event and inner horizons. The weak version of both effects can be realized on the inner horizon. It is worth paying attention that in the BSW effect one of particles should be critical or near-critical while the other one should be usual. In this respect, the situation is quite similar to what happens for the high-energy collisions near the event horizon. Meanwhile, geometrically, the critical and usual particles change their mutual role with respect to the local light cone as compared to the BSW effect near the event horizon. There is one more difference. For the BSW effect to occur near the nonextremal event horizon, multiple scattering is required in the course of which a particle changes its angular momentum, overcomes the potential barrier and gets a
critical value in the vicinity of the horizon. Meanwhile, the inner horizon is not surrounded by a potential barrier, so multiple scattering is not necessary for the BSW effect to occur there.

Also, we showed that inside the event horizon the PS effect becomes as physically relevant as the BSW one. For the PS effect, fine-tuning the parameters of one particle to the critical values is not necessary.

The results obtained in the present paper are valid not only for the Kerr metric but also for generic spherically symmetric black holes as well as generic rotating ones.

It is known that there are instabilities inherent to inner horizons (see, e.g., the reviews [22], [23] and references therein). One can ask whether ultra-high energy collisions (even with finite but growing energy) expand the list of these instabilities.

X. ACKNOWLEDGEMENT

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Figure 1. Impossibility of the strong version of the BSW effect. The critical particle 1 passes through the bifurcation point whereas a usual one 2 hits the left horizon.

Figure 2. The weak version of the BSW effect. Near-horizon collision between critical particle 1 and usual one 2.

Figure 3. Impossibility of the strong version of the PS effect. The critical particle 1 passes through the bifurcation point whereas a usual one 2 hits the left horizon.

Figure 4. Impossibility of the strong version of the PS effect. Two usual particles hit different branches of the horizon.

Figure 5. The weak version of the PS effect. Near-horizon collision between critical particle 1 and usual one 2.

Figure 6. The weak version of the PS effect. Collision between two usual particles near the left horizon.

Figure 7. The weak version of the PS effect. Collision between two usual particles near the bifurcation point.
