Safe Carrier Sensing Range in CSMA Network under Physical Interference Model

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Abstract—In this paper, we study the setting of carrier-sensing range in 802.11 networks under the (cumulative) physical interference model. Specifically, we identify a carrier-sensing range that will prevent collisions in 802.11 networks due to carrier-sensing failure under the physical interference model. We find that the carrier-sensing range required under the physical interference model must be larger than that required under the protocol (pairwise) interference model by a multiplicative factor. For example, if the SINR requirement is $10dB$ and the path-loss exponent is 4, the factor is 1.4. Furthermore, given a fixed path-loss exponent of 4, the factor increases as the SINR requirement increases. However, the limit of the factor is 1.84 as the SINR requirement goes to infinity.

Index Terms—carrier-sensing range, physical interference model, SINR constraints.

I. INTRODUCTION

In 802.11, carrier sensing is designed to prevent concurrent transmissions that can corrupt each other. The setting of the carrier-sensing range is crucial. Too large a carrier-sensing range will limit spatial reuse, while too small a carrier-sensing range will fail to prevent collisions.

To date, most studies on the proper setting of the carrier-sensing range [1]–[5] are based on the pairwise interference model (referred to as the protocol interference model in [6]). For a link under the pairwise interference model, the interferences from the other links are considered one by one. If the interference from each of the other links on the link concerned does not cause a collision, then it is assumed that there is no collision overall. In particular, the pairwise interference model does not take into account the cumulative effects of the interferences from the other links. The physical interference model [6], on the other hand, considers the cumulative interferences and thus is a more accurate model.

Under the pairwise interference model, hidden-node collision happens if the transmitters of two links do not carrier sense each other, but the two links are close enough to interfere with each other. Ref. [5] established the carrier-sensing range required to prevent hidden-node collisions in 802.11 networks under the pairwise interference model. This resulting carrier-sensing range is too optimistic when the more accurate physical interference model is considered instead. For a particular link, although the carrier-sensing range is set large enough with respect to the interference from each of the other links, the cumulative interference powers from all the other links may still corrupt the transmission on the link concerned. Since the collision is not due to a specific “hidden” node, the conventional term of “hidden node collision” does not quite apply to this situation that arises under the physical interference model. We define a new term “missed-carrier-sensing collision” to describe this phenomenon. Missed-carrier-sensing collisions occur when the transmitter does not sense the transmissions of the other links, but the cumulative interference power of all the other concurrent transmissions will interfere the transmission on this particular link. This paper is dedicated to the study of the required carrier-sensing range to prevent missed-carrier-sensing collisions in 802.11 networks under the physical interference model.

II. SYSTEM MODEL

A. Network and Physical Interference Model

We represent a wireless network by a set of directed links $\mathcal{L} = \{l_i, 1 \leq i \leq |\mathcal{L}|\}$. Let $T = \{T_i, 1 \leq i \leq |\mathcal{L}|\}$ and $\mathcal{R} = \{R_i, 1 \leq i \leq |\mathcal{L}|\}$ denote the set of transmitting nodes and the set of receiving nodes, respectively. A receiver decodes its signal successfully if and only if the signal-to-interference-plus-noise ratio (SINR) requirement at the receiver is above a certain threshold. We adopt the physical interference model, where the interference is the sum total of the powers the receiver receives from all transmitters except its own transmitter. We assume that radio signal propagation obeys the log-distance path model with path loss exponent $\alpha > 2$. The path gain $G(T_i, R_j)$ from transmitting node $T_i$ to receiving node $R_j$ follows a geometric model

$$G(T_i, R_j) = d(T_i, R_j)^{-\alpha},$$

where $d(T_i, R_j)$ is the Euclidean distance between nodes $T_i$ and $R_j$.

In 802.11, each data transfer on a link $l_i$ consists of a DATA frame in the forward direction (from transmitter $T_i$ to receiver $R_i$) followed by an ACK frame on the reverse direction (from $R_i$ to $T_i$). The data transfer on link $l_i$ is said to be successful if and only if both the DATA frame and the ACK frame are correctly received. So under the physical interference model,
the conditions for successful transmissions on link \( l_i \) are
\[
\frac{P \cdot G(T_i, R_i)}{N + \sum_{l_j \in L'} P \cdot G(S_j, R_i)} \geq \gamma_0, \quad \text{(DATA frame)}
\]
and
\[
\frac{P \cdot G(R_i, T_i)}{N + \sum_{l_j \in L''} P \cdot G(S_j, T_i)} \geq \gamma_0, \quad \text{(ACK frame)}
\]
where \( P \) is the transmission power level, \( N \) is the average noise power, and \( \gamma_0 \) is the SINR threshold for correct reception. We assume that all nodes use the same transmit power \( P \) and adopt the same SINR threshold \( \gamma_0 \). Let \( L' \) (\( L'' \)) denote the set of links that transmit concurrently with the DATA (ACK) frame on link \( l_i \). For a link \( l_j \) in \( L' \) or \( L'' \), the interference could be either from transmitter \( T_j \) or the receiver \( R_j \) of link \( l_j \) through the DATA or ACK transmission on link \( l_j \), respectively. So we use the notation \( S_j \) to denote the sender of link \( l_j \), which could be either \( T_j \) or \( R_j \).

B. Carrier Sensing in 802.11

Consider the wireless link set \( L \). If there exists a link \( l_i \in L \) such that not both (2) and (3) are satisfied, collision happens. In 802.11, carrier sensing can be used to prevent simultaneous transmissions that collide.

We assume carrier sensing by energy detection. That is, if the power received from another node is above a power threshold \( P_{cs} \), then a transmitter will not transmit and its backoff countdown process will be frozen. Given a carrier sensing power threshold \( P_{cs} \), it can be mapped to a carrier-sensing range \( CSRange \). Consider two links, \( l_i \) and \( l_j \). If the transmitters \( T_i \) and \( T_j \) can carrier-sense the frames transmitted by each other, simultaneous transmissions by them will be prevented. That is, if the distance between \( T_i \) and \( T_j \) satisfies
\[
d(T_i, T_j) < CSRange,
\]
the DATA frame transmissions on \( l_i \) and \( l_j \) are prevented beforehand.

Setting an appropriate \( CSRange \) is crucial to the performance of 802.11 network. If \( CSRange \) is too large, spatial reuse will be unnecessarily limited. If \( CSRange \) is not large enough, the missed-carrier-sensing collisions may occur. That is a number of transmitters may transmit simultaneously because condition (4) is not satisfied by all pairs of the transmitters. However, there may exist one link that not both conditions (2) and (3) are satisfied. In this case, collisions happen and the carrier sensing fails to prevent such collisions. We define a Safe \( CSRange \) that will prevent the missed-carrier-sensing collisions in 802.11 network under the physical interference model as follows:

**Definition 1 (Safe-CSRange):** Consider the wireless link set \( L \). Let \( L_{con} \subseteq L \) denote a subset of links that are allowed to transmit concurrently under a \( CSRange \) setting. Let \( L_C(\text{CSRange}) = \{ L_{con} \} \) denote all such subsets of links. A \( CSRange \) is said to be a Safe-CSRange if for any \( L_{con} \in L_C(\text{CSRange}) \) such that for any link \( l_i \in L_{con} \), both the conditions (2) and (3), with \( L' = L'' = L_{con} \setminus \{ l_i \} \), are satisfied.

In the following analysis, we assume that the background noise power \( N \) can be ignored.

III. SUFFICIENT CONDITION ON Safe-CSRange

In this section, we will derive a sufficient large value on the Safe-CSRange that will prevent the missed-carrier-sensing collisions in 802.11.

It is shown in [5] that although the \( CSRange \) is sufficient large for the SINR requirements of all nodes, transmission failures can still occur due to the “Receiver-Capture effect”. Consider two links \( l_i \) and \( l_j \) such that \( T_i \) and \( T_j \) are out of the \( CSRange \), but \( R_j \) is in the \( CSRange \) of \( T_i \). If \( T_i \) transmits first, then \( R_j \) will have sensed the signal of \( T_i \) and the default operation in most 802.11 products is that \( R_j \) will not attempt to receive the later signal from \( T_j \), even if the signal from \( T_j \) is stronger. This will cause the transmission on link \( l_j \) to fail. It is further shown in [5] that no matter how large the \( CSRange \) is, we can always come up with an example that give rise to transmission failures, if the “Receiver-Capture effect” is not dealt with properly. This kind of collisions can be solved with a receiver “RS(Re-Start) mode” which can be enabled in some 802.11 products (e.g., Atheros WiFi chips). With RS mode, a receiver will switch to receive the stronger packet as long as the SINR threshold \( \gamma_0 \) for the later link can be satisfied.

With the receiver’s RS mode, we can derive the Safe-CSRange that will prevent collisions in 802.11 network under the Physical Interference Model.

**Theorem 1:** Consider the wireless link set \( L \). The sufficient condition on the Safe-CSRange that will prevent collisions in 802.11 network under the Physical Interference Model is:
\[
\text{Safe-CSRange} = (K + 2)d_{\max},
\]
where \( d_{\max} = \max_{l_i \in L} d(T_i, R_i) \), which is the maximum link length in the network, and
\[
K = \left( 6\gamma_0 \left( 1 + \left( \frac{2}{\sqrt{3}} \right)^{\frac{\alpha}{\alpha - 2}} \right) \right)^{\frac{1}{\alpha}}.
\]

**Proof:** With the receiver’s RS mode, in order to prevent collisions in 802.11 networks, we only need to show that condition (5) is sufficient to satisfy both the SINR requirements (2) and (3) of all the concurrent transmission links. With the transmitter-side carrier sensing in 802.11, concurrent transmissions can only happen when the transmitters do not carrier sense each other. Let \( L_{con} \) denote the set of links which have concurrent transmissions. Consider any two links \( l_i \) and \( l_j \) in \( L_{con} \), we have the following inequality:
\[
d(T_j, T_i) \geq \text{Safe-CSRange} = (K + 2)d_{\max}.
\]
Because both the lengths of links \( l_i \) and \( l_j \) satisfy
\[
d(T_i, R_i) \leq d_{\max}, \quad d(T_j, R_j) \leq d_{\max},
\]

the conditions (2) and (3), with \( L' = L'' = L_{con} \setminus \{ l_i \} \), are satisfied.

\[
\]
using triangular inequality, we have
\[
d(T_j, R_i) \geq d(T_j, T_i) - d(T_i, R_i)
\geq (K + 2)d_{\text{max}} - d_{\text{max}} = (K + 1)d_{\text{max}},
\] (8)
\[
d(R_j, T_i) \geq d(T_i, T_j) - d(T_j, R_j)
\geq (K + 2)d_{\text{max}} - d_{\text{max}} = (K + 1)d_{\text{max}},
\] (9)
and also
\[
d(R_j, R_i) \geq d(R_i, T_j) - d(T_j, R_j)
\geq (K + 1)d_{\text{max}} - d_{\text{max}} = Kd_{\text{max}}.
\] (10)

We take the most conservative distance $Kd_{\text{max}}$ in our interference analysis (i.e., we will pack the interference links in a tightest manner given the $\text{CSRange}$ in (5)). Consider any two links $l_i$ and $l_j$ in $L_{\text{con}}$. The following four inequalities are satisfied:
\[
d(T_i, T_j) \geq Kd_{\text{max}},
\] (11)
\[
d(T_i, R_i) \geq Kd_{\text{max}},
\] (12)
\[
d(T_j, R_i) \geq Kd_{\text{max}},
\] (13)
\[
d(R_i, R_j) \geq Kd_{\text{max}}.
\] (14)

Consider any link $l_i$ in $L_{\text{con}}$. We will show that the SIR requirements for both the DATA frame and the ACK frame can be satisfied. We first consider the SIR requirement of the DATA frame. The SIR at $R_i$ is:
\[
SIR = \frac{\sum_{l_j \in L_{\text{con}}, j \neq i} P d^{-\alpha}(S_j, R_i)}{\sum_{l_j \in L_{\text{con}}, j \neq i} P d^{-\alpha}(T_i, R_i)}
\] (15)
For the received signal power we consider the worst case that $d(T_i, R_i) = d_{\text{max}}$. So we have
\[
P d^{-\alpha}(T_i, R_i) \geq P \cdot d_{\text{max}}^{-\alpha}.
\] (16)

To calculate the cumulative interference power, we consider the worst case that all the other concurrent transmission links have the densest packing, in which the link lengths of all the other concurrent transmission links are equal to zero. In this case, the links degenerate to nodes. The minimum distance between between any two links in $L_{\text{con}}$ is $Kd_{\text{max}}$. The densest packing of nodes with minimum distance requirement is the hexagon packing (as shown in Fig. 1).

If link $l_j$ is the first layer neighbor link of link $l_i$, we have $d(S_j, R_i) \geq \sqrt{3}Kd_{\text{max}}$. Thus we have
\[
P d^{-\alpha}(S_j, R_i) \leq P(Kd_{\text{max}})^{-\alpha} = \frac{1}{K^\alpha} P d_{\text{max}}^{-\alpha},
\] and there are at most 6 neighbor links in the first layer.

If link $l_j$ is the second layer neighbor link of link $l_i$, we have $d(S_j, R_i) \geq 3Kd_{\text{max}}$. Thus we have
\[
P d^{-\alpha}(S_j, R_i) \leq P(\sqrt{3}Kd_{\text{max}})^{-\alpha} = \frac{1}{(\sqrt{3}K)^\alpha} P d_{\text{max}}^{-\alpha},
\] and there are at most 12 neighbor links in the second layer.

If link $l_j$ is the $n$th layer neighbor link of link $l_i$, we have $d(S_j, R_i) \geq \sqrt{n^2Kd_{\text{max}}}$. Thus we have
\[
P d^{-\alpha}(S_j, R_i) \leq P(Kd_{\text{max}})^{-\alpha} = \frac{1}{K^\alpha} P d_{\text{max}}^{-\alpha},
\] and there are at most $6n$ neighbor links in the second layer.

So the cumulative interference power satisfies the following inequality:
\[
\sum_{l_j \in L_{\text{con}}, j \neq i} P d^{-\alpha}(S_j, R_i)
\leq 6 \cdot \left(\frac{1}{K}\right)^\alpha \left(1 + \sum_{n = 2}^{\infty} n \left(\frac{2}{\sqrt{3}nK}\right)^\alpha\right) \cdot P d_{\text{max}}^{-\alpha}
= 6 \cdot \left(\frac{1}{K}\right)^\alpha \left(1 + \sum_{n = 2}^{\infty} n \left(\frac{2}{\sqrt{3}n}\right)^\alpha\right) \cdot P d_{\text{max}}^{-\alpha}
= 6 \cdot \left(\frac{1}{K}\right)^\alpha \left(1 + \sum_{n = 2}^{\infty} \frac{1}{n^{\alpha - 1}}\right) \cdot P d_{\text{max}}^{-\alpha}
\leq 6 \cdot \left(\frac{1}{K}\right)^\alpha \left(1 + \frac{2}{\sqrt{3}} \cdot \frac{1}{\alpha - 2}\right) \cdot P d_{\text{max}}^{-\alpha}
\] (17)
\[
= \frac{P d_{\text{max}}^{-\alpha}}{\gamma_0},
\] (18)
where (17) follows from a bound on Riemann’s zeta function and (18) follows from the definition of $K$ in (6).

According to (16) and (18), we find that the SIR of the DATA frame of link $l_i$ at the receiver $R_i$ satisfies:
\[
SIR = \frac{\sum_{l_j \in L_{\text{con}}, j \neq i} P d^{-\alpha}(T_i, R_i)}{\sum_{l_j \in L_{\text{con}}, j \neq i} P d^{-\alpha}(S_j, R_i)} \geq \frac{P \cdot d_{\text{max}}}{P d_{\text{max}}^{-\alpha}} = \gamma_0.
\] (19)
This means the SIR requirement of the successful transmission of the DATA frame on link \( l_i \) can be satisfied.

The proofs that the SIR requirement of the ACK frame on link \( l_i \) can be satisfied follow the similar procedure as above. So for any link \( l_i \) in the concurrent transmission links \( L_{con} \), condition (5) is sufficient to satisfy the SIR requirements of the successful transmissions of both its DATA and ACK frames. This means that, together with the receiver’s RS mode, condition (5) is sufficient for preventing collisions in 802.11 networks under the physical interference model. □

Condition (5) provides a sufficiently large Safe-CSRange that prevents missed-carrier-sensing collisions in 802.11 networks. So there is no need to set a CSRange larger than (5) in order to prevent collisions. Setting a larger CSRange than (5) will only decrease spatial reuse.

IV. COMPARISON OF THE Safe-CSRange WITH THE PAIRWISE INTERFERENCE MODEL

This section compares the Safe-CSRange under the physical interference model to that under the pairwise interference model (derived in [5]). Let Safe-CSRange_{pairwise} denote the CSRange that prevents hidden-node collisions under the pairwise interference model. Let Safe-CSRange_{physical} denote the CSRange that prevents missed-carrier-sensing collisions under the physical interference model. The Safe-CSRange_{pairwise} and the Safe-CSRange_{physical} are:

\[
\text{Safe-CSRange}_{\text{pairwise}} = \left(2 + \gamma_0 \right) d_{\text{max}},
\]

\[
\text{Safe-CSRange}_{\text{physical}} = (K + 2) d_{\text{max}}.
\]

For example, if \( \gamma_0 = 10 \) and \( \alpha = 4 \), which are typical for wireless communications, we have

\[
\text{Safe-CSRange}_{\text{pairwise}} = 3.78 d_{\text{max}},
\]

\[
\text{Safe-CSRange}_{\text{physical}} = 5.27 d_{\text{max}}.
\]

The Safe-CSRange needs to be increased by a factor of 1.4 to ensure successful transmissions under the physical interference model.

Given a fixed path loss exponent \( \alpha \), both Safe-CSRange_{pairwise} and Safe-CSRange_{physical} will increase when the SIR requirement \( \gamma_0 \) increases. This is the case when the separation among links must be larger to meet the SIR targets. For example, if \( \alpha = 4 \), we have

\[
\text{Safe-CSRange}_{\text{pairwise}} = \left(2 + \gamma_0 \right) d_{\text{max}},
\]

\[
\text{Safe-CSRange}_{\text{physical}} = \left(2 + \left(\frac{34}{3} \gamma_0 \right)^{\frac{1}{4}} \right) d_{\text{max}}.
\]

The ratio of Safe-CSRange_{physical} to Safe-CSRange_{pairwise} is

\[
\frac{\text{Safe-CSRange}_{\text{physical}}}{\text{Safe-CSRange}_{\text{pairwise}}} = \frac{2 + \left(\frac{34}{3} \gamma_0 \right)^{\frac{1}{4}}}{2 + 4 \gamma_0^{\frac{1}{4}}},
\]

which is also an increasing function of \( \gamma_0 \). And the limit of the ratio of Safe-CSRange_{physical} to Safe-CSRange_{pairwise} as \( \gamma_0 \) goes to infinity is

\[
\lim_{\gamma_0 \to \infty} \frac{\text{Safe-CSRange}_{\text{physical}}}{\text{Safe-CSRange}_{\text{pairwise}}} = \lim_{\gamma_0 \to \infty} \frac{2 + \left(\frac{34}{3} \gamma_0 \right)^{\frac{1}{4}}}{2 + \gamma_0^{\frac{1}{4}}} = \left(\frac{34}{3} \right)^{\frac{1}{4}} \approx 1.8348
\]

Fig. 2 shows the ratio Safe-CSRange_{physical}/Safe-CSRange_{pairwise} as a function of the SIR requirements \( \gamma_0 \). Different curves represent different choices of path loss exponent \( \alpha \).

V. Conclusion

This paper studies the problem of setting the carrier-sensing range that will prevent missed-carrier-sensing collisions in 802.11 networks under the physical interference model. We establish a sufficient condition for the carrier sensing range. We call the minimum carrier sensing range that meets the condition Safe-CSRange and compare it with that established under the pairwise interference model [5]. We find that for the typical setting of path-loss exponent \( \alpha = 4 \) and SINR requirement \( \gamma_0 = 10 \), the Safe-CSRange needs to be increased by a factor of 1.4 under the physical interference model. We also find that, given a fixed path-loss exponent \( \alpha \), the factor increases when the SINR requirement \( \gamma_0 \) increases. And the factor tends to a constant as the SINR requirement \( \gamma_0 \) goes to infinity. For example, when the path-loss exponent \( \alpha = 4 \), the limit of the factor is 1.84 as the SINR requirement \( \gamma_0 \) goes to infinity.

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