The low-energy effective action
for perturbative heterotic strings on $K_3 \times T^2$
and the d=4 N=2 vector-tensor multiplet

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Abstract

We consider $d = 4$ $N = 2$ supergravity theories which serve as low-energy effective actions for heterotic strings on $K_3 \times T^2$. At the perturbative level we construct a new version of the heterotic effective action in which the axion has been traded for an antisymmetric tensor field. In the string frame the antisymmetric tensor doesn’t transform under Poincaré supersymmetry into the dilaton-dilatini system. This indicates that in this frame the antisymmetric tensor field and the dilaton are not contained in an $N = 2$ vector-tensor multiplet. Instead, we find that the heterotic dilaton is part of a compensating hypermultiplet, whereas the antisymmetric tensor is part of the gravitational multiplet. In order to obtain our results we use superconformal techniques. This enables us to comment on the range of applicability of this particular framework.
1 Introduction

During the last few years much progress has been made in the study of supersymmetric field theories and string theories in various dimensions thanks to the discovery of a whole set of duality symmetries. Four dimensional models with $N = 2$ supersymmetry have proven to be particularly interesting because of the rich physical phenomena that appear in this context. Moreover these phenomena can be studied in great detail thanks to the specific structure imposed by the $N = 2$ supersymmetry. A by-now famous example concerns the duality that relates heterotic strings on $K_3 \times T^2$ to type IIA strings on $K_3$-fibered Calabi-Yau manifolds [1, 2, 3, 4]. This duality has very powerful consequences. Suppose for instance that one is describing the low-energy dynamics of these strings in terms of an effective $N = 2$ supergravity action with vector and hypermultiplet couplings. The vector multiplet sector of such an effective action can in principle be determined exactly by performing a tree level computation in the type II picture. Thanks to the heterotic-type II duality one can reinterpret this exact result in a heterotic context. This automatically solves a strong coupling problem because in terms of the heterotic variables the vector multiplet action receives tree level, one-loop and non-perturbative contributions.

The interpretation of the type II result in a heterotic language is not so obvious however, because the variables that are naturally inherited from the type II side turn out to be complicated functions of the “natural” heterotic variables. The latter transform in a simple way under the $SO(2, n)$ T-duality group, whereas the type II variables in general don’t. Amongst other things this implies that one needs to perform a whole set of field redefinitions before the effective action can really be compared to direct perturbative heterotic string computations. Consider for instance the dilaton-axion-like scalar field $S = \phi - ia$ which on the type II side is defined as the complexified Kähler modulus of the $\mathbb{P}^1$ base of the $K_3$-fibration. This field $S$ is an $N = 2$ “special coordinate” and it is shifted by a purely imaginary constant under a (quantised) Peccei-Quinn symmetry. It is well-known [5, 6] that $S$ is not invariant under $SO(2, n)$ transformations once loop and non-perturbative contributions are taken into account. This indicates that $\phi$, which coincides with the true heterotic dilaton at the string tree level, starts to differ from it at the one-loop level or non-perturbatively. Therefore it is necessary to express the field $\phi$ as a function of the true dilaton $\phi_{\text{inv}}$ and the other moduli, before one can properly separate the non-perturbative effects from the perturbative ones. It is also known [7] that one has to perform a change of variables at the level of the vector fields. The reason is that the $SO(2, n)$ transformations mix the field strengths for the type II inherited vectors with their duals, which implies that these vectors themselves transform in a non-local way. In order to avoid this, one can perform an electro-magnetic duality transformation on one of the vectors, such that one ends up with a new set of “stringy” vectors which transform just linearly under $SO(2, n)$.

In addition to the above-mentioned field redefinitions there is a last change of variables which so far has been less well under control, and which we intend to study in the course of the present work. What we have in mind here is the duality transformation which trades the axion $a$ for an antisymmetric tensor field $\tilde{B}_{\mu \nu}$. Of course this transformation can only be implemented after going to the perturbative region of the vector multiplet moduli space, where the Peccei-Quinn symmetry is continuous instead of being quantised. The axion can then be identified with the zero form gauge potential associated to this continuous symmetry and as such it can be dualised into an antisymmetric tensor field. In [3] it was conjectured that in the $N = 2$ context this duality transformation would replace the vector multiplet which originally contained the scalar $S$ by a so-called vector-tensor multiplet [3, 8] in which $\phi_{\text{inv}}$ and $\tilde{B}_{\mu \nu}$ would find their

\footnote{Incidently we will refer to these vector fields as the STU vectors, because they lead to a vector multiplet action which is characterised by a prepotential $F(X)$ of the form $STU + \text{more}$. Here $T$ and $U$ stand for the moduli of the heterotic $T^2$.}
natural place. In order to test this conjecture a systematic study of vector-tensor supergravities was undertaken in \[9\]. An interesting class of interacting vector-tensor theories came out of this analysis, but quite surprisingly the sought-after heterotic vector-tensor theory was not found. In this article we further investigate the issue of a possible vector-tensor structure in the antisymmetric tensor effective action for heterotic strings. We do this by explicitly constructing this antisymmetric tensor effective action, together with its associated $N = 2$ supersymmetry transformation rules. The outcome is first of all that we have to be careful before drawing rigorous conclusions about the possible (non)existence of the heterotic vector-tensor multiplet because the $N = 2$ supersymmetry of our final model is only realised on-shell. This means that one first has to decide on which variables one uses as the fundamental ones, before one can identify the kind of multiplets these variables belong to. When the string metric and the corresponding string gravitinos are used as fundamental variables, the antisymmetric tensor $\tilde{B}_{\mu\nu}$ is not linked to dilaton-dilatini system by supersymmetry. As a result there is no vector-tensor multiplet in the string frame theory.

In this article we also wish to comment on other issues which are of interest from a purely supergravity point of view. Since it is not always possible (nor desirable) to keep the heterotic string and supergravity ideas completely separated throughout the main text, we summarise the different lines of thought already here, for the sake of clarity.

1.1 Construction of the antisymmetric tensor effective action for perturbative heterotic strings with $N=2$ supersymmetry

Our strategy for obtaining the antisymmetric tensor effective action for heterotic strings consists of performing a sequence of duality transformations. As a starting point we take a conventional $N = 2$ supergravity theory coupled to a set of vector and hypermultiplets, and we use the well-known vector multiplet prepotential for type II strings on $K3$-fibered Calabi-Yau manifolds. This yields what we call the STU version of the heterotic effective action. We review the basic properties of this model including its Peccei-Quinn and $SO(2,n)$ symmetries. We briefly discuss how the $SO(2,n)$ symmetry can be made manifest at the lagrangian level by implementing the higher-mentioned duality transformation on the vector gauge fields \[7\].

Before we dualise the axion we broaden our point of view and study generic vector multiplet theories containing a Peccei-Quinn symmetry. We show that the class of Peccei-Quinn invariant models is in fact quite restricted and precisely comprises the cases discussed in \[9\] plus the case which is relevant for perturbative heterotic strings. The various Peccei-Quinn invariant models can be dualised in a unified way and this explains why the resulting antisymmetric tensor theories share a similar gauge structure. Most notably there exists a particular $U(1)$ gauge symmetry which acts as a shift symmetry on the antisymmetric tensor. For those cases that allow for an off-shell treatment along the lines of \[9\] this $U(1)$ transformation is nothing but the central charge transformation. We show that in the cases of \[9\] the central charge-like structure is really indispensable (even on-shell!) whereas this is not so for the heterotic antisymmetric tensor theory. In the latter case the central charge-like structure can be completely removed, and one ends up with a set of vector fields which all appear on an equal footing. This of course reflects the underlying $SO(2,n)$ invariance of the heterotic model.

It is worth mentioning that very little string information is used in our construction of the “heterotic” antisymmetric tensor theory. In fact we just start from the most general Peccei-Quinn invariant vector multiplet theory and then select the case which contains an $SO(2,n)$ symmetry. At the end of the whole dualisation procedure we find that several known string theoretical properties are manifestly realised in our model, which shows that in fact they can

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\[2\] More recently superspace descriptions of the vector-tensor multiplet have appeared in \[10\].
be thought off as being a consequence of $N = 2$ supersymmetry alone, rather then being of an intrinsic stringy nature. First of all we nicely reproduce the specific couplings of the dilaton and the antisymmetric tensor to the other moduli. Furthermore we see that the supersymmetry transformation rules for all the fields are (almost) completely independent of the one-loop part of the theory. In this respect the antisymmetric tensor formulation is simpler then the other possible formulations of the heterotic effective action in terms of vector and hypermultiplets only.

1.2 N=2 superconformal supergravity and the perturbative heterotic string

It so happens that the low-energy effective action for perturbative heterotic strings is a perfect laboratory to address some interesting supergravity issues. These supergravity issues form a second main ingredient of this paper. In particular we focus on the superconformal framework for $d = 4$ $N = 2$ supergravity \[11, 12, 13\] which is very well suited for our present purposes.

The general philosophy of the superconformal approach to $N = 2$ supergravity is the following. Although the ultimate goal of the superconformal techniques is to construct super Poincaré theories describing the on-shell interactions of a certain set of physical fields, one starts off with various multiplets that form a representation of the \textit{off-shell superconformal algebra} (which is considerably larger then the super Poincaré algebra). Due to this high degree of symmetry several expressions —like the supersymmetry transformation laws for the fields— have a relatively simple form. In a second step one reduces the symmetry algebra to the super Poincaré algebra by implementing a partial gauge choice. Moreover one eliminates several auxiliary fields. In doing so the supersymmetry transformations in general acquire a more complicated form, but in any case they can be obtained by a number of well-defined algorithmical steps.

What makes the perturbative heterotic string so interesting from an $N = 2$ supergravity point of view is that it gives us two different examples of theories where the direct application of the superconformal ideas is problematic. The first example arises when one goes to the stringy vector formulation for which a prepotential — which is a crucial ingredient in any off-shell superconformal vector multiplet theory— doesn’t exist \[7\]. The second example concerns the heterotic antisymmetric tensor theory, which, as we already said, escaped any direct superconformal treatment along the lines of \[9\].

Although we know in advance that somehow the off-shell superconformal framework must break down when we dualise towards these “problematic” theories, we perform our computations in a superconformal setting. First of all this enables us to verify which ingredient of the standard superconformal framework is incompatible with the dualisation procedure. We find that the duality transformations that are used to obtain the two problematic theories both affect the Weyl multiplet, such that the latter is no longer realised off-shell\[4\]. In this sense the nonexistence of a conventional superconformal description for both the stringy vector multiplet version and the antisymmetric tensor version of the heterotic effective action has a common origin. There is no problem in preserving the various superconformal symmetries during the duality transformations. This is important from a technical point of view, because it implies that we can determine the supersymmetry transformation laws for the stringy vectors and antisymmetric tensor field before having to impose the conventional superconformal gauge choices and even before eliminating the auxiliary fields. In this way the necessary computations can be kept relatively simple.

\[3\] This case can be treated in an elegant way if one goes on-shell, where it is possible to write down an action \[14\] in terms of symplectic sections rather then the prepotential itself. In the stringy basis these symplectic sections remain well-defined even though they are no longer based on an underlying prepotential.

\[4\] Note that this departure from off-shellness does not occur in the cases described in \[9\] because there the Weyl multiplet is never touched.
One of the beautiful aspects of the superconformal setup is that it allows for some flexibility in the Poincaré reduction. We can benefit from this flexibility by choosing a dilatation gauge which immediately leads to a string frame Poincaré action. It is satisfactory to see that from a purely supergravity point of view the string frame is selected as a very natural one. It can be defined as the only dilatation gauge which makes the supersymmetry transformations of the stringy vector fields dilaton independent. Once this gauge has been chosen one may verify that the antisymmetric tensor $\tilde{B}_{\mu\nu}$ does not transform into the dilaton or dilatini, which means that in the string frame these fields don’t combine into a vector-tensor supermultiplet. What happens instead is that the dilaton and the dilatini effectively become part of the compensating hypermultiplet, whereas the antisymmetric tensor becomes part of the gravitational multiplet. Of course one can go to the Einstein frame, by applying an interpolating dilatation and $S$-supersymmetry transformation. The supersymmetry variation of $\tilde{B}_{\mu\nu}$ is not altered by this step because $\tilde{B}_{\mu\nu}$ is inert under the interpolating transformations. On the other hand the Einstein gravitinos are dilatino dependent functions of the original string gravitinos, so when $\delta \tilde{B}_{\mu\nu}$ is written in terms of the Einstein frame variables a (spurious) dilatino dependence gets induced. One might interpret the resulting Einstein frame configuration as an on-shell heterotic vector-tensor multiplet, but this configuration is clearly not selected by string theory.

2 \ N=2 superconformal supergravity coupled to vector and hypermultiplets

In this section we discuss some basic facts concerning $d = 4 \ N = 2$ superconformal supergravity and its couplings to vector and hypermultiplets. Our discussion will be rather brief, as we intend to use the standard superconformal vector and hypermultiplet theories merely as a starting point for our construction of the antisymmetric tensor version of the low-energy effective action for heterotic strings. Symplectic transformations are discussed in some more detail though, because as far as we know they never received a full treatment in the superconformal framework. The interested reader can find more details about the superconformal approach to $N = 2$ supergravity in the original articles [11, 12, 13]. For more recent texts we refer to [15] and also to the last article of [9] (which contains a comprehensive list of conventions).

The Weyl multiplet is a central object in the superconformal multiplet calculus as it contains the gravitational degrees of freedom. It consists of the vierbeins $e_\mu^a$, the gravitinos $\psi_\mu^i$, the gauge fields $b_\mu$, $A_\mu$, $V_{\mu i}^j$ — which gauge dilatations, chiral $U(1)$ and $SU(2)$ transformations — and a set of matter fields: a selfdual tensor $T_{\mu\nu ij}^+$ which is antisymmetric in its $SU(2)$ indices, a real scalar $D$ and a doublet of chiral fermions $\chi^i$. As such the Weyl multiplet forms the basic representation of (a deformed version of) the superconformal algebra, which closes off-shell. This algebra consists of general coordinate, local Lorentz, chiral $U(1)$ and $SU(2)$ transformations, dilatations, special conformal and $Q$- and $S$-supersymmetry transformations. For future reference we list the transformation laws for the vierbeins and the gravitinos

$$
\begin{align*}
\delta e_\mu^a &= \left( \bar{\epsilon}^a \gamma_\mu \psi_\mu^i + h.c. \right) - \Lambda_D e_\mu^a, \\
\delta \psi_\mu^i &= 2D_\mu \epsilon^i - \frac{1}{4} \sigma \cdot T^{- ij} \gamma_\mu \epsilon_j - \gamma_\mu \eta^i - \left( \frac{1}{2} \Lambda_D + \frac{i}{2} \Lambda_{U(1)} \right) \psi_\mu^i.
\end{align*}
$$

Here $\epsilon^i, \eta^i, \Lambda_D$ and $\Lambda_{U(1)}$ are parameters for $Q$- and $S$-supersymmetry transformations, dilatations and chiral $U(1)$ transformations respectively. General coordinate, local Lorentz and $SU(2)$ transformations (with parameter $\Lambda_{ij}$) are not explicitly given because these can automatically be inferred from the index structure of the fields that are being transformed. Here and in what follows a derivative $D_\mu$ stands for a covariant derivative with respect to local Lorentz, dilatation,
chiral \(U(1), SU(2)\) and gauge transformations. Since the parameter \(\epsilon^i\) carries non-trivial Weyl and chiral \(U(1)\) weights one has that
\[
D_\mu \epsilon^i = \left( \partial_\mu - \frac{1}{2} \omega_\mu^{ab} \sigma_{ab} + \frac{1}{2} b_\mu + \frac{i}{2} A_\mu \right) \epsilon^i + \frac{1}{2} V_\mu^i \epsilon^j .
\] (2.2)

Here \(\omega_\mu^{ab}\) is the (dependent) gauge field for local Lorentz transformations.

Next one introduces a set of vector multiplets (labeled by an index \(I\)) which can be consistently coupled to the Weyl multiplet. Each vector multiplet contains a complex scalar \(X^I\), a vector gauge field \(W^I_\mu\), a real \(SU(2)\) triplet of scalars \(Y^I_{ij}\) and a doublet of chiral gauginos \(\Omega^I_i\). The lagrangian describing the most general couplings of vector multiplets to superconformal supergravity was given in \cite{13} in formula (3.9). We will use this lagrangian several times in what follows and denote it by \(e^{-1} \mathcal{L}_{\text{vector}}\). In order to clarify our notations we list the bosonic terms:
\[
e^{-1} \mathcal{L}_{\text{vector}} = \ XNX \left( \frac{1}{6} R + D \right) + N_{IJ}D_\mu X^I D^\mu \tilde{X}^J - \frac{1}{8} N_{IJ} Y^I_{ij} Y^{ijj} - \frac{i}{8} \tilde{F}_{I}^{\mu} F^{\mu + \mu J} - \frac{i}{8} X N_{IJ} F^{\mu + \mu I} T^{\mu + \mu I} + \frac{1}{64} X N X \left( T^{\mu + \mu ij} \epsilon^{ij} \right)^2 + h.c.
\] (2.3)

This lagrangian depends on a holomorphic prepotential \(F(X)\) which is homogeneous of second degree in the \(X^I\)s.\footnote{Note that compared to \cite{13} we have rescaled the prepotential \(F(X) \rightarrow 2i F(X)\) as is common practice in the recent literature. We also changed the sign of the Ricci scalar \(R\) such that under finite Weyl rescalings} Derivatives of the prepotential with respect to the scalars \(X^I\) are given by
\[
F^I_{\mu \nu} = 2 \partial_\mu W^I_\nu \quad \tilde{F}^{\mu \nu I} = \frac{1}{2} \epsilon_{\mu \nu \lambda \sigma} F^I_{\lambda \sigma} \quad F^{\pm \mu I} = \frac{1}{2} \left( F^{\mu I} \pm \tilde{F}^{\mu I} \right).
\] (2.4)

We take \(\epsilon^{0123} = i\) such that complex conjugation interchanges \(F^{+ \mu I}\) with \(F^{- \mu I}\). The vector multiplet fields transform as follows:
\[
\delta X^I = - \bar{\epsilon}^i \Omega^I_i + \left( \Lambda_D - i \Lambda_{U(1)} \right) X^I \\
\delta Y^I_{ij} = 2 \partial_i X^I \bar{\epsilon}_j + \bar{\epsilon}_j \sigma_{\mu \nu} \epsilon^i \left( \tilde{F}^{\mu \nu I} - \frac{1}{4} X^I T^{\mu \nu \eta \xi} \epsilon_{\eta \xi} \right) + Y^I_{ij} \eta^j + 2 X^I \eta_j + \left( \frac{2}{3} \Lambda_D - \frac{i}{2} \Lambda_{U(1)} \right) \Omega^I_i \\
\delta W^I_\mu = \left( \bar{\epsilon}_j \gamma_\mu \Omega^I_j \bar{\epsilon}^j + 2 \bar{X}^I \bar{\epsilon} \psi^j_\mu \bar{\epsilon}^j + \text{h.c.} \right) + \partial_\mu \theta^I \\
\delta Y^I_{ij} = 2 \epsilon_{(i} \partial_\eta \Omega^I_{j)} + 2 \epsilon_{ik} \epsilon_{ij} \left( k \partial \Omega^I \right) + 2 \Lambda_D Y^I_{ij} .
\] (2.5)

The derivatives \(D_\mu\) are covariant with respect to all the superconformal (and possibly also gauge) symmetries. The covariant field strengths for the vectors are given by
\[
\tilde{F}^{\mu \nu I} = 2 \partial_\mu W^I_\nu + \left( \tilde{\Omega}^{I \mu \nu} \gamma_\mu \psi^j_\nu \bar{\epsilon}^j - \tilde{X}^I \psi^j_\mu \psi^j_\nu \bar{\epsilon}^j + \text{h.c.} \right).
\] (2.6)

Finally one adds a set of hypermultiplets consisting of \(r\) quaternions \(A^I_\alpha\) and \(2r\) chiral fermions \(\zeta^\alpha\), where \(\alpha = 1, \cdots, 2r\). The scalar fields \(A^I_\alpha\) satisfy a reality constraint given in \cite{13} which
implies that they describe 4r real degrees of freedom. In order to obtain a fully off-shell description for the hypermultiplets, one must add a set of 4r auxiliary scalars. These auxiliary degrees of freedom completely decouple from the other fields, so we choose to integrate them out right from the start. Having done so we get the following transformation rules for the hypermultiplets

\[
\begin{align*}
\delta A_i^\alpha &= 2\zeta^\alpha \epsilon_i + 2\rho^{\alpha\beta} \epsilon_{ij} \zeta^\beta \epsilon^j + \Lambda_D A_i^\alpha \\
\delta \zeta^\alpha &= \mathcal{D} A_i^\alpha \epsilon_i + A_i^{\alpha\eta} \eta^i + \left( \frac{3}{2} \Lambda_D - \frac{i}{2} \Lambda_U(1) \right) \zeta^\alpha.
\end{align*}
\]  

(2.7)

The superconformal action describing the hypermultiplet couplings to the Weyl multiplet is given in [13] in formula (3.29). We list its bosonic part:

\[
e^{-1} L_{\text{hyper}} = \frac{1}{2} A_i^\alpha A_i^\beta d_{\alpha\beta} \left( -\frac{1}{3} R + D \right) - D_{\mu} A_i^\alpha D^\mu A_i^\beta d_{\alpha\beta} + \text{fermionic terms}.
\]

(2.8)

For conventions concerning the constant matrices \(\rho^{\alpha\beta}\) and \(d_{\alpha\beta}\) we refer to [13].

### 2.1 Symplectic transformations

The superconformal vector multiplet theories we just described can be acted upon with symplectic transformations. In general these transformations relate a given theory (characterised by a prepotential \(F(X)\)) to other vector multiplet theories which are classically equivalent to the first one, in the sense that their equations of motion and Bianchi identities are transformed into eachother.

Symplectic transformations can be introduced in the following way. First of all one requires that they act linearly on the equations of motion and the Bianchi identities that are satisfied by the field strengths for the vectors [16]:

\[
\begin{align*}
\partial^\mu (F^-_{\mu\nu} - F^+_{\mu\nu}) &= 0 \quad \text{Bianchi identities} \\
\partial^\mu (G^-_{\mu\nu} - G^+_{\mu\nu}) &= 0 \quad \text{Equations of motion} \\
G^-_{\mu\nu} &= -4i \frac{\delta e^{-1} L_{\text{vector}}}{\delta F^\mu_{\nu}^-}.
\end{align*}
\]

(2.9)

This can be accomplished by putting \(F^-_{\mu\nu}\) and \(G^-_{\mu\nu}\) into a vector transforming as:

\[
\begin{pmatrix} F^-_{\mu\nu} \\ G^-_{\mu\nu} \end{pmatrix} \rightarrow \begin{pmatrix} F^+_{\mu\nu} \\ G^+_{\mu\nu} \end{pmatrix} = \begin{pmatrix} U^I_J & Z^{IJ} \\ W_{IJ} & V^J_I \end{pmatrix} \begin{pmatrix} F^-_{\mu\nu} \\ G^-_{\mu\nu} \end{pmatrix},
\]

(2.10)

where \(U^I_J, Z^{IJ}, W_{IJ}, V^J_I\) are constant real matrices. These matrices cannot be chosen arbitrarily because one must ensure that the \(G^+_{\mu\nu}\) result from varying the new vector multiplet theory with respect to the new field strengths \(F^+_{\mu\nu}\). This implies [12] that the transformation must be symplectic, i.e.

\[
\begin{pmatrix} U^T & W^T \\ Z^T & V^T \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} U & Z \\ W & V \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
\]

(2.11)

and that the scalars \(X^I\) and the \(F_I\) must form a symplectic vector too:

\[
\begin{pmatrix} X^I \\ F_I \end{pmatrix} \rightarrow \begin{pmatrix} X'^I \\ F'_I \end{pmatrix} = \begin{pmatrix} U^I_J & Z^{IJ} \\ W_{IJ} & V^J_I \end{pmatrix} \begin{pmatrix} X^I \\ F_I \end{pmatrix}.
\]

(2.12)

In the present superconformal setup one has to impose an additional restriction on the concept of symplectic transformations. The transformed theory only makes sense as a superconformal
theory when it is based on a new prepotential, say $F'(X')$. But this presupposes that all the transformed scalars $X'^I$ are independent variables, which excludes some a priori interesting symplectic transformations. The symplectic transformation which transforms the perturbative heterotic STU theory into its stringy analog is a particular example of a transformation which leads to new scalars which are not all independent. In section 3.2 we verify in which sense this forces us to exceed the off-shell superconformal framework.

Note that only a subset of the full symplectic group maps the original theory onto itself. These so-called duality invariances \[12\] are characterised by the fact that $F'(X') = F(X')$. This implies that the $F'_I = W_{IJ}X^J + V_I^J F_J(X)$ are simply given by $F_I(X')$. Sometimes the duality invariances are also called proper symmetries, while the other symplectic transformations which really change the form of the prepotential are called symplectic reparametrisations \[17\] or pseudo-symmetries \[18\].

Let us now proceed with an explicit description of the symplectic properties of the lagrangian \[2.3\] including the fermionic terms and the Weyl multiplet auxiliaries. Of course many of these symplectic properties overlap with those that have been studied previously in the Poincaré case \[12\], so we will concentrate on the ingredients that are specific to the superconformal setup. See also \[21\] for a (purely bosonic) treatment of symplectic transformations in the presence of a Weyl multiplet background.

The symplectic vectors that are relevant for \[2.3\] can be derived starting from the basic symplectic vector $(X^I, F_I)$ by requiring consistency with supersymmetry. If one computes successive supersymmetry variations of the basic symplectic vector, one gets the following results:\[14\]

$$\delta Q \left( \begin{array}{c} X^I \\ F_I \\ \Omega^I \\ F_I \Omega^J \\ \eta_I \\ \eta_J \end{array} \right) = \epsilon^I \left( \begin{array}{c} \Omega^I \\ (F_I \Omega^J) \\ \Omega^I \\ (F_I \Omega^J) \\ \eta_I \\ \eta_J \end{array} \right) + \epsilon_{ij} \sigma^{\mu \nu} \epsilon^3 \left( \begin{array}{c} \hat{F}^{-I}_{\nu \mu} \\ \hat{G}^{-I}_{\mu \nu} \end{array} \right) D_\mu \left( \begin{array}{c} X^I \\ F_I \\ \Omega^I \\ F_I \Omega^J \end{array} \right) + \epsilon_{ij} \eta\sigma^{\mu \nu} \epsilon^3 \left( \begin{array}{c} \hat{F}^{-I}_{\nu \mu} \\ \hat{G}^{-I}_{\mu \nu} \end{array} \right) \right)$$

$$\hat{G}^{-I}_{\mu \nu} \equiv F_{IJ} \hat{F}^{-J}_{\mu \nu} + i \frac{1}{2} \hat{F}^{-I}_{\mu \nu} \hat{F}^{-I}_{\nu \mu} - \frac{1}{4} F_{IKJ} \hat{\eta}_I^{\mu} \hat{\eta}_J^{\nu} \epsilon_{ij} - \frac{1}{4} F_{IKJ} \hat{\eta}_I^{\mu} \hat{\eta}_J^{\nu} \epsilon_{ij}$$

$$Z_{ij \ell} \equiv F_{IJ} \eta^{IJ} - \frac{1}{2} F_{IKJ} \hat{\eta}_I^{\mu} \hat{\eta}_J^{\nu} \epsilon_{ij} \ell.$$

The expressions $(\Omega^I, F_I, \Omega^J)$ and $(\hat{F}^{-I}_{\mu \nu}, \hat{G}^{-I}_{\mu \nu})$ automatically define new consistent symplectic vectors. Note that $(\hat{F}^{-I}_{\mu \nu}, \hat{G}^{-I}_{\mu \nu})$ is equivalent to the vector $(F^{-I}_{\mu \nu}, G^{-I}_{\mu \nu})$ we introduced in \[2.3\] and \[2.10\], because the difference between the two is itself symplectic. However, the would-be vector $(Y_{ij}^I, Z_{ij \ell})$ is not automatically consistent. The reason is that the auxiliary fields $Y_{ij}^I$ satisfy a reality condition, viz. $Y_{ij}^I \epsilon^{jk} = \epsilon_{ij} Y^{jkl}$, whereas the $Z_{ij \ell}$ apparently don’t. But one can verify that the $Z_{ij \ell}$ do satisfy a similar reality condition if one imposes the $Y_{ij}^I$ equations of motion:

$$N_{IJ} Y_{ij}^J - \frac{4}{3} F_{IKJ} \hat{\eta}_I^{\mu} \hat{\eta}_J^{\nu} \epsilon_{ij} \ell = 0.$$  

\textsuperscript{6}It is well-known that one can circumvent this last restriction if one goes on-shell. The point is that the Poincaré theories for $N = 2$ vector multiplets can be formulated in terms of symplectic sections $(X^I, F_I)$ \[14\], which need not be derived from a prepotential. The restriction on the admissible symplectic transformations then evaporates accordingly \[5\].

\textsuperscript{7}In fact we will show that one can stay quite close to the original superconformal setup. It suffices to go partly on-shell by eliminating the $T^+_{\mu \nu \nu}$ auxiliary field.

\textsuperscript{8}Remark that $F(X)$ is not an invariant function because $F'(X') \neq F(X)$.

\textsuperscript{9}For a related discussion, see \[21\] in which symplectic transformations are defined on entire $N = 2$ chiral superfields.
We henceforth assume that these equations of motion have been enforced. The \( Y_{ij} I \)'s must then be viewed as particular dependent expressions quadratic in the gauginos \( \Omega^I_\mu \) or their complex conjugates. Nevertheless we will find it useful to keep on using the shorthand \( Y_{ij} I \) in what follows.

It is clear from equation (2.13) that the symplectic vectors \( (X^I, F_I) \) and \( (\Omega^I_\mu, F_{I\mu} \Omega^J_\nu) \) transform into other symplectic vectors under supersymmetry. This fact holds in general although one has to be careful. One may verify that the symplectic vector \( (F_{\mu\nu}^+, \hat{G}_{\mu\nu}^- I) \) only transforms into other symplectic vectors provided one uses the equations of motion for the gauginos \( \Omega^I_\mu \).

At this point, however, we prefer not to impose the latter equations of motion, because they are not needed in order to guarantee the consistency of the symplectic vectors we just defined. Therefore we simply use the symplectic vectors of (2.13) as they stand. The resulting symplectic transformation rules for the vector multiplet fields make sense at a lagrangian level, and the lagrangian (2.3) — with dependent \( Y_{ij} I \), but all the other Weyl multiplet auxiliaries still present as independent degrees of freedom — turns out to be symplectically invariant, up to a familiar \( \text{Im}(F_{\mu\nu}^+ G_{\mu\nu}^+) \) term.

When checking the behaviour of the lagrangian (2.3) under symplectic transformations it suffices to concentrate on those terms that would vanish if the auxiliary fields were eliminated and the superconformal gauge choices were imposed. The symplectic properties of the other terms are known in advance as they are not changed by the transition to the Poincaré theory for which a complete treatment has already appeared elsewhere \[7, 12\]. One finds that the vector multiplet lagrangian can be written as
\[
e^{-1} \mathcal{L}_{\text{vector}} = -\frac{i}{8} F_{\mu\nu}^+ G_{\mu\nu}^+ + \text{h.c.} + \text{invariant terms},
\]
which is completely analogous to the Poincaré result. In principle there could have been additional non-invariant terms, which then would have to vanish in the Poincaré limit, but such terms do not appear. As an illustration, we treat the \( T_{\mu\nu ij} \) terms explicitly and show that they can be nicely absorbed into symplectic invariant combinations. We start with the invariant
\[
\begin{align*}
\frac{i}{32} \left\{ X^I \hat{G}^+_{\mu\nu} I - F_I \hat{F}^+_{\mu\nu} \right\} T_{\mu\nu ij} + \text{h.c.} \\
= \frac{1}{64} \left\{ X N I X^T_{\mu k l} \varepsilon_{k l} - 4 X N I \hat{F}^+_{\mu\nu} - \frac{i}{2} X^I \hat{F}^*_{\mu\nu} \hat{F}^+_{\mu\nu} \right\} T_{\mu\nu ij} + \text{h.c.}
\end{align*}
\]
(2.16)
which contains all the \( \mathcal{O}(T_{\mu\nu ij}^+)^2 \) terms of (2.3). After taking into account the terms linear in \( T_{\mu\nu ij}^+ \) which appear in (2.14) and in the \( F_{\mu\nu}^+ G_{\mu\nu}^+ \) term of (2.15) one is left over with
\[
\begin{align*}
\frac{1}{24} \left\{ X N I \hat{G}^+_{\mu\nu} I \gamma_{\mu} \bar{\psi}^j_{\nu} \bar{\psi}^j_{\nu} \right\} T_{\mu\nu ij} + \text{h.c.} \\
- \frac{1}{16} X N I \left\{ \hat{G}^+_{\mu\nu} \bar{\psi}^j_{\nu} \bar{\psi}^j_{\nu} - \bar{X}^I \bar{\psi}^j_{\mu} \bar{\psi}^j_{\nu} \varepsilon_{ij} \right\} T_{\mu\nu ij} + \text{h.c.} \\
- \frac{i}{64} X^I \hat{F}^*_{\mu\nu} \hat{G}^+_{\mu\nu} I \gamma_{\mu} \bar{\Omega}^J_\nu \bar{\psi}^j_{\nu} \varepsilon_{ij} + \text{h.c.}
\end{align*}
\]
(2.17)
The first line of (2.17) is invariant by itself. The second line conspires with suitable \( T_{\mu\nu ij}^+ \) independent terms in (2.3) to form the invariant
\[
-\frac{i}{8} \left\{ G_{\mu\nu}^+ \left( \hat{G}^+_{\mu\nu} I \gamma_{\mu} \bar{\psi}^j_{\nu} \varepsilon_{ij} - \bar{X}^I \bar{\psi}^j_{\mu} \bar{\psi}^j_{\nu} \varepsilon_{ij} \right) - F_{\mu\nu}^+ \left( \hat{F}^+_{\mu\nu} I \gamma_{\mu} \bar{\psi}^j_{\nu} \varepsilon_{ij} - \hat{F}^*_{\mu\nu} \bar{\psi}^j_{\mu} \bar{\psi}^j_{\nu} \varepsilon_{ij} \right) \right\} + \text{h.c.}
\]
(2.18)
One can check that (2.14), (2.18) and the \( F_{\mu\nu}^+ G_{\mu\nu}^+ \) term of (2.17) already contain all the \( F_{\mu\nu}^+ \) terms of the lagrangian apart from
\[
\frac{i}{32} F_{\mu\nu}^+ \hat{F}^*_{\mu\nu} \hat{G}^+_{\mu\nu} I \gamma_{\mu} \bar{\Omega}^J_\nu \bar{\psi}^j_{\nu} \varepsilon_{ij} + \text{h.c.}
\]
(2.19)
But the sum of (2.19) and the last line of (2.17) transforms into itself up to 4-fermi terms (which can be further analysed in the same way as in the Poincaré case).

To finish our discussion of symplectic transformations we introduce a useful piece of terminology $\tilde{\Omega}$. A given symplectic transformation is said to be of the “semi-classical” type when $Z^{IJ} = 0$ (in a well chosen symplectic basis). In that case the full set of Bianchi identities is left invariant, which implies that the vectors transform linearly into themselves. For $Z^{IJ} = 0$ one makes a further distinction between the $W_{IJ} = 0$ case which is called “classical”, and the genuine “semi-classical” case $W_{IJ} \neq 0$. It follows from (2.10), (2.11) and (2.15) that the lagrangian (2.3) is left invariant under semi-classical transformations up to a topological term:

$$e^{-1} \mathcal{L}_{\text{vector semi-class.}} = e^{-1} \mathcal{L}_{\text{vector}} - \frac{i}{8} F_{\mu\nu}[U^T W]_{IJ} \tilde{F}^{\mu\nu \cdot J}. \quad (2.20)$$

### 2.2 The transition to the Poincaré theory

The off-shell theories for $N = 2$ vector and hypermultiplets coupled to superconformal supergravity contain some redundant variables which do not describe true physical degrees of freedom. These redundant variables can be eliminated by going on-shell and by reducing the superconformal symmetry algebra to the super Poincaré algebra. As has been explained in [13] there exists a well-defined procedure to perform this step, and we now recall its most important ingredients.

As a starting point one considers the sum of the vector multiplet action (2.3) and the hypermultiplet action (2.8). The total number of hypermultiplets one introduces is equal to $r = (N_h + 1)$, where $N_h$ denotes the number of physical hypermultiplets one wants to obtain in the final theory. The extra hypermultiplet is a so-called compensating multiplet which plays an important rôle in the whole Poincaré reduction. In a moment we will sketch how the degrees of freedom of this compensating hypermultiplet can be eliminated. In the vector multiplet sector there are some compensating degrees of freedom too: one complex scalar and its fermionic partner are unphysical and as such they will be removed in the process of going to the Poincaré theory. This implies that the number $I$ always runs over $(N_v + 1)$ values, where $N_v$ counts the (complex) dimension of the vector multiplet moduli space.

The Weyl multiplet fields $D, \chi_i, A_\mu, \nu^{ij}_\mu$, and $T^{+}_{\mu \nu ij}$ are auxiliary fields. $D$ and $\chi_i$ are Lagrange multipliers which enforce the following constraints on the matter sector of the theory:

$$\begin{align*}
X N X + \frac{1}{2} A_i^\alpha A^\beta_i d_{\alpha \beta} &= 0 \\
\bar{X} N_I \Omega^I_j + 2 A_i^\alpha \zeta_\beta d_{\alpha \beta} &= 0.
\end{align*} \quad (2.21)$$

One can use these constraints to eliminate several component fields of the compensating hypermultiplet, i.e. one real component of the scalar and all the components of the associated fermion. The auxiliary fields $A_\mu, \nu^{ij}_\mu$, and $T^{+}_{\mu \nu ij}$ appear quadratically in the action so they can be solved for by imposing their own equations of motion. In the sequel we will explicitly need the $A_\mu, \nu^{ij}_\mu$ and $T^{+}_{\mu \nu ij}$ equations of motion which are given by

$$\begin{align*}
X N X \ Partial derivative of the second line of (2.21) is $$-rac{i}{8} N_{IJ} \tilde{\Omega}^{ij}_{\mu} \gamma_\mu \gamma_i \gamma_j d_{\alpha \beta}$$ which we used in the expression for $X N X A_\mu$. The expression $-\frac{i}{8} N_{IJ} \gamma_\mu \gamma_i \gamma_j d_{\alpha \beta}$ was used in the expression for $X N X \nu^{ij}_\mu$.

In order to derive the equation (2.22) we used the constraints (2.21). We also neglected any $b_\mu$ dependence, because we will put $b_\mu = 0$ in a moment. By $\partial_\mu$ we mean a derivative which
is covariant with respect to local Lorentz, $Q$-supersymmetry and gauge transformations. One easily verifies that the equations (2.22) are symplectically invariant, see for instance (2.16).

The superconformal algebra can be broken down to the super Poincaré algebra by imposing several gauge choices. First one breaks the special conformal symmetry by putting the dilatation gauge field $b_\mu = 0$. Next one uses the dilatation symmetry to bring the $R$ term in the action into a conventional form. It is common practice in the $N = 2$ supergravity literature to go to the Einstein frame for the metric, i.e. $e^{-1/4}L = \frac{1}{2}R + \text{more}$. Later on in the heterotic string case (section 5.4) we will not follow this common practice, but rather choose an alternative gauge which immediately leads to the string frame. Nevertheless we present the standard Einstein frame results here, so that the reader may compare to them in section 5.4. Given the fact that the $D$ equation of motion (2.21) has already been imposed, one finds that the dilatation gauge choice leading to the Einstein frame reads

$$XN\bar{X} = 1.$$  

The corresponding standard $S$-supersymmetry gauge choice reads

$$XN_\Omega^{ij} = 0.$$  

This $S$-gauge choice is chosen such that the dilatation gauge choice (2.23) is invariant under $Q$-supersymmetry transformations. Moreover one may verify that (2.24) removes many of the mixed gravitino-gaugino propagators. It is important to realise that the condition (2.24) itself is not $Q$-supersymmetric invariant. This shows that the Poincaré supersymmetry transformations—which by definition leave the various gauge choices invariant—are in fact composed out of a $Q$-supersymmetry part and a compensating $S$-supersymmetry part:

$$\delta(e) = \delta_Q(e) + \delta_S(\eta(e))$$
$$\eta_i(e) = \gamma^i_\mu \epsilon_j \left\{ \frac{1}{4} N_{IJ} \left( \bar{\Omega}^I \gamma_\mu \Omega^J - \frac{1}{2} \delta^I_j \bar{\Omega}^k \gamma_I \gamma_\mu \Omega_k^J \right) \right\} - \varepsilon_{ij} \sigma^{\mu
u} \epsilon^\mu d_\alpha^\beta \rho^{\alpha\gamma} \bar{\zeta}_\beta \sigma_{\mu\nu} \zeta_\gamma.$$  

(2.25)

In order to finish the whole Poincaré reduction one has to fix the chiral $U(1)$ and $SU(2)$ symmetries. The $U(1)$ gauge freedom can be used to further restrict the scalar fields $X^I$. If one introduces special coordinates

$$Z^I \equiv -i \frac{X^I}{X^0}$$  

(2.26)

one derives from eq.(2.23) that

$$|X^0|^2 = (ZN\bar{Z})^{-1},$$  

(2.27)

so in this case the dilatation gauge choice has effectively fixed the length of the scalar $X^0$. One then uses the $U(1)$ symmetry to fix the phase of $X^0$ as well. A convenient choice is

$$X^I = (ZN\bar{Z})^{-\frac{1}{2}} Z^I.$$  

(2.28)

This expresses the (dependent) scalars $X^I$ in terms of the (independent) $Z^I$. It should be noted that the previous formula and the subsequent ones are also valid when the $Z^I$ are not just special coordinates, but rather holomorphic sections $Z^I(z^A)$ of a symplectic bundle over the special Kähler manifold $SK$ which is defined by the vector multiplet theory. The $z^A$ with $A = (1, \cdots, N_v)$ are arbitrary coordinates on $SK$. The relation (2.28) is not $Q$-supersymmetry

---

10Our convention for $\hat{\partial}_\mu$ differs from the one given in [13] in that our $\hat{\partial}_\mu$ is covariant with respect to $Q$-supersymmetry as well.
invariant, which implies that a compensating $U(1)$ transformation must be included in order to obtain the correct Poincaré supersymmetry transformation rules. The precise form of this compensating transformation can be deduced from the fact that

$$\delta(c)X^I = \bar{\epsilon}^i \Omega^I_i - i \Lambda_{U(1)} X^I$$

$$= (ZN\bar{Z})^{-\frac{1}{2}} \bar{\epsilon}^i \Lambda^I_i - \frac{1}{2} X^I \left( \bar{Z} N \frac{1}{ZN} \bar{Z} \Lambda^I_i + h.c. \right),$$

where the fermions $\Lambda^I_i$ are defined by

$$\delta(c)Z^I = \bar{\epsilon}^i \Lambda^I_i.$$  \hfill (2.30)

From (2.29) one immediately reads off that

$$\Lambda^I_{U(1)}(c) = \frac{i}{2} \bar{Z} N \Lambda^I_i + h.c.$$  \hfill (2.31)

Remark that the $Z^I$ (and thus any Kähler coordinates $z^A$) are defined to be chiral fields, in the sense that they only transform into fermions of a definite handedness under Poincaré supersymmetry.  \hfill (2.11)

The scalars $X^I$ are not chiral under Poincaré supersymmetry because of the compensating $U(1)$ transformation. The $SU(2)$ symmetry, finally, can be used to remove the last 3 degrees of freedom of the compensating hypermultiplet scalar. A similar reasoning as the one followed for the chiral $U(1)$ gauge fixing leads to

$$A_i^\alpha = \sqrt{-\frac{2}{C(B)}} \delta_i^s B_s^\alpha, \quad s = 1, 2 \quad C(B) = B_s^\alpha B_{s*}^\beta d^\alpha_{\beta}$$

$$\zeta^\alpha = \sqrt{-\frac{2}{C(B)}} \left( \zeta^\alpha - B_s^\alpha B_{s*}^\beta d^\beta_{\gamma} \zeta^\gamma \right) / C(B)$$

$$\Lambda^\gamma_i = 4 \bar{\epsilon}^i \delta_s^B B_s^\alpha d^\alpha_{\beta} \xi^\beta \left( C(B) \right)^{-1} - h.c.; \text{ traceless}$$

where the $\xi^\alpha$ are defined by

$$\delta B_s^\alpha = \left( 2 \zeta^\alpha \epsilon_i + 2 \rho^\alpha_{\beta} \epsilon_i \xi^j \xi^j \right) \delta_s^i.$$  \hfill (2.33)

3 The vector multiplet effective action for perturbative heterotic strings on $K_3 \times T^2$

In this section we concentrate on a particular subset of all possible $N = 2$ vector multiplet theories, namely those that arise from type II strings on $K_3$-fibered Calabi-Yau manifolds, or from heterotic strings on $K_3 \times T^2$. We begin with type II strings because the vector multiplet sector of their low-energy effective action is relatively easy to analyse. This is so because the type II dilaton is contained in the hypermultiplet sector of the theory. \hfill (2.22)

As usual we use Weyl fermions for which the position of the $SU(2)$ index also indicates the chirality.

11To clarify our notation $A^i_j - h.c.; \text{ traceless} = A^i_j - A^j_i - \frac{1}{2} \delta^i_j (A^k_j - A^j_k)$ and $A^j_i = (A^i_j)^*$.  \hfill (2.22)

12One must be careful with this statement, though. As has been explained in \hfill (2.22) the type II dilaton is naturally described by the real part of the sum of a compensating $N = 2$ vector and tensor multiplet. Only after imposing an Einstein frame dilatation gauge choice the type II dilaton appears to be sitting in a physical tensor multiplet which can be converted into a hypermultiplet for practical reasons. So it is only in an Einstein frame that the type II dilaton can be identified with one of the hypermoduli.

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moduli (3.2) and the corresponding scalars \( X \) be identified with the standard moduli of the T space d mirror manifold ˜φ invariant dilaton Y with the complexified K"ahler moduli of

Here the index \( A \) is a dilaton-like field which is closely related to the heterotic string loop counting parameters, while \( a \) is an axion-like field. The heterotic prepotential thus satisfies the following

\[
F_{\text{type} II}(X^0, X^A) = i(X^0)^2 f_{\text{type} II}(Z^A)
\]

\[
f_{\text{type} II}(Z^A) = \frac{1}{6} d_{A B C} Z^A Z^B Z^C - \frac{c(3)}{16 \pi^3} \chi(Y) + \frac{1}{8 \pi^3} \sum_{d_1, \ldots, d_{n+1}} n_{d_1, \ldots, d_{n+1}} \text{Li}_3\left[e^{-2\pi d_A Z^A}\right]
\]

Here the index \( A \) runs from 1, ..., \( N_v = n + 1 \) (\( n \geq 1 \)). The special coordinates \( Z^A \) are identified with the complexified K"ahler moduli of \( Y \), the \( d_{A B C} \) are the Calabi-Yau triple intersection numbers, \( \chi(Y) \) is the Euler characteristic and the rational instanton numbers \( n_{d_1, \ldots, d_{n+1}} \) count the number of rational curves of multi degree \( (d_1, \ldots, d_{n+1}) \) on \( Y \). When \( Y \) is a \( K_3 \)-fibration over \( \mathbb{P}^1 \), there is one distinguished modulus, the K"ahler modulus of the \( \mathbb{P}^1 \) base, which we call \( S \). We may choose a basis such that

\[
Z^A = (S, Z^A) \quad A = 2, \ldots, n + 1.
\]  

The intersection numbers of the \( K_3 \)-fibered Calabi-Yau manifold then satisfy

\[
d_{111} = 0 \quad d_{11A} = 0 \quad d_{1AB} = \eta_{AB},
\]

where the \( n \times n \) “metric” \( \eta_{AB} \) is of signature \((1, n-1)\). Notice that the index \( I \) — which runs over all the vector multiplets — takes the values 0, 1, \( A \). As it will be relevant further on, we introduce a \((n + 2) \times (n + 2)\) metric \( \eta_{IJ} \) of signature \((2, n)\), which is defined as

\[
\eta_{IJ} = \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & \eta_{AB}
\end{pmatrix}.
\]

### 3.1 Identification of the dilaton-axion complex

Thanks to the type II-heterotic duality hypothesis — which has been tested very succesfully in many circumstances, see for instance \([3, 4, 25]\), the nice review \([25]\) and references therein — one can view the prepotential implied by the relations (3.1), (3.2), (3.3) as a prepotential describing heterotic strings on \( K_3 \times T^2 \). Of course it is crucial to make contact with the results which have been obtained in the past by direct computations in the heterotic string picture \([27, 28]\). In order to do so, one identifies the modulus \( S \) with the heterotic dilaton-axion complex:

\[
S = -i \frac{X^1}{X^0} \overset{\text{def}}{=} \phi - i a.
\]

Here \( \phi \) is a dilaton-like field which is closely related to the heterotic string loop counting parameter, while \( a \) is an axion-like field. The heterotic prepotential thus satisfies the following

\[\text{(3.5)}\]

\[\text{(3.1)}\]

\[\text{(3.3)}\]

\[\text{(3.4)}\]

\[\text{(3.5)}\]
expansion in the string coupling constant $S$:

$$ F_{\text{het}}(X) = -\frac{1}{2} \frac{X^1}{X^0} \eta_{AB} X^A X^B + F^{(0)}(X^0, X^A) + \sum_{k=1}^{\infty} F^{(k)}(X^0, X^A) e^{-2\pi k S}. \quad (3.6) $$

At the tree level one recovers the well-known prepotential $-\frac{1}{2} \frac{X^1}{X^0} \eta_{AB} X^A X^B$ corresponding to the special Kähler manifold

$$ SU(1,1) \otimes \frac{SO(2,n)}{SO(2) \times SO(n)} \quad (3.7) $$

which describes the classical vector moduli space for heterotic strings. This manifold is the only special Kähler manifold having a direct product structure [29]. This reflects the fact that (in the Einstein frame) the heterotic dilaton has no tree-level couplings to the other vector moduli. When $n \geq 2$ the tree-level prepotential can be brought into a standard form by performing some linear redefinitions of the special coordinates $Z^A$ to find

$$ F_{\text{tree}} = -\frac{1}{2} \frac{X^1}{X^0} \eta_{AB} X^A X^B = i (X^0)^2 \left\{ STU - S \sum_{i=4}^{n+1} \phi^i \phi^i \right\}. \quad (3.8) $$

The fields $T$ and $U$ are the moduli of the $T^2$ and the $\phi^i$ are possible Wilson line moduli. The $n = 1$ case can be viewed as degenerate case for which the difference $T - U$ has been frozen to a zero value. Due to the result (3.8) we say that the prepotential (3.6) leads to the heterotic vector multiplet effective action in the $STU$ basis.

The other terms in (3.6) can be “explained” by noting that they are the only possible ones allowed by the quantised Peccei-Quinn symmetry under which the heterotic theory is invariant. This symmetry maps

$$ S \xrightarrow{P,Q} S - ic \quad c \in \mathbb{Z}, \quad (3.9) $$

and leaves the other moduli untouched. In order to fully understand the consequences of this particular symmetry one needs some extra knowledge which will be provided in section 4.1 where we study generic Peccei-Quinn invariant models. At present it suffices to mention that in the case at hand the prepotential has to transform as follows [30,31].

$$ F_{\text{het}}(X) \xrightarrow{P,Q} F_{\text{het}}(X) = -\frac{1}{2} c \eta_{AB} X^A X^B. \quad (3.10) $$

Since the tree level part of the prepotential already saturates the latter equation all the other terms must be Peccei-Quinn invariant by themselves which directly leads to (3.10). In this way one has proven a powerful non-renormalisation theorem which states that perturbatively there is just the tree-level contribution and the one-loop term $F^{(0)}(X^0, X^A)$. The Peccei-Quinn symmetry is continuous at the perturbative level. This indicates that the axion $a$ describes the same physical degrees of freedom as the dual antisymmetric tensor $B_{\mu\nu}$ which is familiar from the the standard world-sheet formulation of heterotic strings. As usual this continuous symmetry is broken to its discrete subgroup at the full quantum level due to space-time instanton effects. These give rise to the non-perturbative $F^{(k)}(X^0, X^A) \exp[-2\pi kS]$ terms.

The heterotic string is invariant under a $SO(2,n)$ T-duality group [31], which is expected to survive at the non-perturbative level because it can be viewed as a discrete gauge symmetry [31]. The $SO(2,n)$ transformations can be embedded into the symplectic group $Sp(2(n+2);\mathbb{Z})$.

\[13\] This follows from the equations (4.3) and (4.11) (appropriately applied to the heterotic string example) and the fact that $F(X) = \frac{1}{2} X^i F_i$. At first sight the reader might be surprised by the fact that the prepotential is not invariant under the Peccei-Quinn symmetry even though this symmetry corresponds to a duality invariance. The point is that $F'(X') = F(X')$ but $\neq F(X)$. 

---

[13]
and thus naturally act on the vector multiplets contained in the low-energy effective action. These $SO(2, n)$ symplectic transformations are most easily written down in a symplectic basis which is different from the $STU$ basis. This new basis is completely specified by the “stringy” symplectic vector $(\tilde{X}^I, F_I)$ which is related to the $STU$ symplectic vector $(X^I, F_I \overset{\text{def}}{=} \frac{\delta F_{\text{het}}(X)}{\delta X^I})$ in the following way [7]:

\[
\begin{pmatrix}
\tilde{X}^0 \\
\tilde{X}^1 \\
\tilde{X}^A \\
\tilde{F}_0 \\
\tilde{F}_1 \\
\tilde{F}_A \\
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & \delta^A_B & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \delta_A^B \\
\end{pmatrix}
\begin{pmatrix}
X^0 \\
X^1 \\
X^A \\
F_0 \\
F_1 \\
F_A \\
\end{pmatrix},
\tag{3.11}
\]

In the stringy basis the $SO(2, n)$ transformations are of the semi-classical type, and are given by

\[
\begin{pmatrix}
\tilde{X}^I \\
\tilde{F}_I \\
\end{pmatrix} \overset{\text{SO}(2, n)}{\longrightarrow} \begin{pmatrix}
U^I_J \\
(U^{-1})^T_{IJ} \Lambda_{KJ} (U^{-1})^T_{IJ} \\
\end{pmatrix}
\begin{pmatrix}
\tilde{X}^J \\
\tilde{F}_J \\
\end{pmatrix},
\tag{3.12}
\]

where

\[
[U^T \eta U]_{IJ} = \eta_{IJ} \\
\Lambda_{IJ} = \text{symmetric, real}.
\tag{3.13}
\]

The matrices $\Lambda_{IJ}$ are absent at the string tree-level. They must be introduced at the one-loop level and also non-perturbatively to accommodate for the monodromies generated by encircling the singularities in the quantum moduli space. See e.g. [27, 6] for more details concerning this point.

The relations (3.11) - (3.13) have some important consequences. First of all one verifies that

\[
\tilde{F}_I = -i S \tilde{X} \eta_I + \sum_{k=0}^{\infty} e^{-2\pi k S} \partial_I F^{(k)}(X^0, X^A) \tag{3.14}
\]

where the $\partial_I$ stand for partial functional derivatives with respect to the original scalars $X^I$. Note that we have conveniently included the one-loop term ($k = 0$) into the instanton sum. In the $I = 1$ version of the equation (3.14) the one-loop and non-perturbative contributions clearly vanish, so one can derive from the variation of $\tilde{F}_1$ how the dilaton-axion field $S$ behaves under the T-dualities. This yields

\[
S \overset{\text{SO}(2, n)}{\longrightarrow} S + \sum_{k=0}^{\infty} i \frac{\partial_I F^{(k)}(U^{-1})^I_1}{[X \eta U^{-1}]_1} e^{-2\pi k S} + i \frac{[\tilde{X} \Lambda U^{-1}]_1}{[X \eta U^{-1}]_1} \tag{3.15}
\]

In other words, the $N = 2$ special coordinate $S$ is only invariant under $SO(2, n)$ transformations in the classical limit, when the $F^{(0)}(X^0, X^A)$, $F^{(k>0)}(X^0, X^A)$ and $\Lambda_{IJ}$ dependent terms vanish. From the moment on that one-loop and non-perturbative effects are taken into account $S$ is no longer inert under $SO(2, n)$ transformations, and even ceases to be single-valued due to the non-trivial monodromies. Therefore it is best to perform a change of coordinates on the Kähler manifold which amongst others trades the special coordinate $S$ for a single-valued and $SO(2, n)$ invariant alternative, which we call $S_{\text{hol}}$. This field $S_{\text{hol}}$ was introduced in [6] at the perturbative level and in [7] non-perturbatively, and it turns out to be a complicated holomorphic function of the moduli $S$ and $Z^A$. Therefore $S_{\text{hol}}$ is not a $N = 2$ special coordinate. In the non-perturbative case one defines

\[
S_{\text{hol}} = \frac{i}{(n + 2)} \left\{ \eta_{IJ} \tilde{F}_{IJ} + L \right\}. \tag{3.16}
\]
In order to properly understand this last formula a few more remarks must be added. Equation (3.11) implies that

\[ \tilde{X}^I = (X^0, \tilde{X}^1, X^A) \]

\[ \dot{\tilde{X}}^1 = -\frac{1}{2} \eta_{AB} X^A X^B X^0 + \sum_{k=1}^{\infty} \frac{2\pi i k}{X^0} F^{(k)}(X^0, X^A) e^{-2\pi k S} . \]  

(3.17)

As has been emphasized in [6] the \( \tilde{X}^I \) in general define a set of independent variables. However, this is no longer true in the perturbative regime, when the instanton terms are suppressed. In that case \( \tilde{X}^1 \) is a function of \( (X^0, X^A) \) only, which is reflected in the constraint

\[ \dot{\tilde{X}} \eta \tilde{X} = \sum_{k=1}^{\infty} 4\pi i k F^{(k)}(X^0, X^A) e^{-2\pi k S} \rightarrow 0 . \]  

(3.18)

So it is only for finite \( S \) that the stringy scalars \( \tilde{X}^I \) are really independent, which implies that only in that case one can define a stringy prepotential and derivatives thereof

\[ \dot{\tilde{F}}(\tilde{X}) = \frac{1}{2} \tilde{X}^I \dot{\tilde{F}}_I \quad \dot{\tilde{F}}_{IJ} = \tilde{\partial}_I \tilde{\partial}_J \tilde{F} . \]  

(3.19)

In the perturbative case a prepotential doesn’t exist, so the abstract expression (3.16) doesn’t make sense there. Nevertheless one can work out (3.16) in terms of the functions \( F^{(k)}(X^0, X^A) \) and then take the perturbative limit. This procedure leads to the perturbative expression for \( S_{\text{hol}} \) as it was given in [5]:

\[ S_{\text{hol}} \quad \rightarrow \quad S + \frac{i}{(n + 2)} \left\{ \eta^{AB} F^{(0)}_{AB} + L^{(0)} \right\} . \]  

(3.20)

The function \( L \) and its semi-classical limit \( L^{(0)} \) are necessary to cancel the infinities contained in \( \eta^{IJ} \dot{F}_{IJ} \) and \( \eta^{AB} F^{(0)}_{AB} \) respectively. Moreover, one must impose \( L \rightarrow L - \eta^{IJ} \Lambda_{IJ} \) in order to keep \( S_{\text{hol}} \) invariant under the monodromies.

Although \( S_{\text{hol}} \) clearly is a natural coordinate on the special Kähler manifold, it is still not describing the true heterotic dilaton \( \phi_{\text{inv}} \). Perturbatively one finds [2] that

\[ \phi_{\text{inv}}^{\text{perturb}} = \phi + \frac{i \tilde{X}^I F^{(0)}_I (X) + h.c.}{2 \tilde{X} \eta \tilde{X}} , \]  

(3.21)

where the second term is the so-called Green-Schwarz term. At this point one may want to introduce yet another complex scalar, called \( S_{\text{inv}} \), which contains the invariant dilaton \( \phi_{\text{inv}} \) as its real part. We propose the following definition, which makes sense at the full non-perturbative level and reduces to (3.21) semi-classically:

\[ S_{\text{inv}} = i \frac{\dot{\tilde{F}}_I \tilde{X}^I}{\tilde{X} \eta \tilde{X}} - i M = \phi_{\text{inv}} - i a_{\text{inv}} . \]  

(3.22)

Here \( M \) is a real function transforming as

\[ M \rightarrow M + \frac{\tilde{X} \Lambda \tilde{X}}{\tilde{X} \eta \tilde{X}} , \]  

(3.23)

which ensures that \( S_{\text{inv}} \) is monodromy invariant. Note that \( S_{\text{inv}} \) is a non-holomorphic function of the special coordinates \( S, Z^A \) so one cannot use it as a preferred Kähler coordinate. However, all the physical couplings in the effective action should be expanded in terms of the true dilaton-axion complex \( S_{\text{inv}} \) in order to properly identify the perturbative and non-perturbative contributions.
3.2 The “stringy” vector fields

In (3.11) we introduced the stringy symplectic vector \((\vec{X}^I, \vec{F}^I)\) in terms of which the T-duality transformations acquire a simple form. Of course one can also write the T-dualities in terms of the \(STU\) symplectic vector \((X^I, F_I)\) in which case — in the notation of (2.12) — a \(Z^{IJ} \neq 0\) term is generated. This means that some of the field strengths \(F^I_{\mu\nu}\) transform into their “duals” \(G_{\mu\nuI}\) under the T-dualities, which implies that the \(STU\) vectors \(W^I_\mu\) themselves transform in a non-local way. This state of affairs indicates that the \(STU\) vectors \(W^I_\mu\) are not the most natural objects to work with. As was explained in [7] one may proceed by trading the vector \(W^I_\mu\) for a new vector \(\vec{W}^I_\mu\) via a duality transformation. In this way one gets the so-called stringy vector fields

\[
\vec{W}^I_\mu = (W^0_\mu, \vec{W}^1_\mu, W^A_\mu) .
\]

By construction these satisfy

\[
2\partial_{[\mu}W^I_{\nu]} = F^I_{\mu\nu} = (F^0_{\mu\nu}, G_{\mu\nu1}, F^{A}_{\mu\nu}) ,
\]

from which one easily derives that the stringy vectors \(\vec{W}^I_\mu\) transform just linearly into each other under the T-dualities.

Having established that the stringy variables are the most natural ones to work with, we must still explain how one constructs the stringy version of the vector multiplet lagrangian. At the non-perturbative level everything is straightforward because one may effectively implement the duality transformation on the vector \(W^I_\mu\) by applying the symplectic transformation (3.11). This leads to the prepotential \(\vec{F}(\vec{X})\) of (3.19) which can be inserted into the general superconformal action (2.3). In the perturbative case a sensible stringy prepotential doesn’t exist because the \(\vec{X}^I\) are not independent. As a result the superconformal action formula (2.3) cannot be used as it stands.

In what follows we will explore in which sense the superconformal approach fails to deal with the perturbative effective action in the stringy basis. In particular we will show that one can actually stay pretty close to the familiar superconformal ideas and still obtain the desired theory. We take the superconformal action in the \(STU\) basis as a starting point and explicitly dualise the vector \(W^I_\mu\). In doing so we will clearly see which ingredient of superconformal supergravity is incompatible with the dualisation procedure. The crucial point is that one is forced to eliminate the auxiliary field \(T^+_{\mu\nu ij}\) contained in the Weyl multiplet in course of the dualisation. As a result the Weyl multiplet is no longer realised off-shell. Below we exhibit the results of the necessary computations in rather detail, amongst others because it sets the stage for the other duality transformation we intend to perform, namely the dualisation of the heterotic axion \(a\). Both dualisations share a lot of common features, as will become clear in section 5.

We consider the lagrangian (2.3) with as prepotential the perturbative expression

\[
F(X^I) = -\frac{1}{2} \frac{X^1}{X^0} \eta_{AB} X^A X^B + F^{(0)}(X^0, X^A) .
\]

From now on we will always work in the perturbative regime, which from a supergravity point of view is the most interesting one. The stringy variables \(\vec{X}^I\) and \(\vec{\Omega}^I_i\) are then given by

\[
\vec{X}^I = (X^0, \vec{X}^1, X^A) \\
\vec{\Omega}^I_i = (\Omega^0_i, \vec{\Omega}^1_i, \Omega^A_i) \\
\vec{X}^1 \overset{\text{def}}{=} F_1 = -\frac{1}{2} \frac{\eta_{AB} X^A X^B}{X^0} \\
\vec{\Omega}^1_i \overset{\text{def}}{=} F_{1I} \Omega^I_i = -\frac{\vec{X}^1}{X^0} \Omega^0_i - \frac{\eta_{AB}}{X^0} \vec{X}^A \Omega^B_i .
\]
The vector $W^{1}_{\mu}$ can be dualised by treating the field strength $F^{1}_{\mu\nu}$ as an independent variable on which a Bianchi identity has been imposed by means of a lagrange multiplier $\bar{W}^{1}_{\mu}$. This leads to the lagrangian

$$e^{-1}L_{\text{vector}} + \frac{i}{4}e^{-1}e^{\mu\nu\lambda\sigma}F^{1}_{\mu\nu}\partial_{\lambda}\bar{W}^{1}_{\sigma}.$$  \hspace{1cm} \text{(3.28)}

For future use we list the $F^{1}_{\mu\nu}$ dependent terms in the lagrangian:

$$e^{-1}L_{\text{vector}} = e^{-1}L_{\text{vector}} |_{F^{1}_{\mu\nu}=0} - \frac{i}{4}F^{1}_{\mu\nu} \left( F^{1}_{1}F^{\mu\nu+I} - \frac{1}{2}X_{\mu}T_{\mu
u+I} \delta^{ij} - \frac{i}{4}F^{1}_{IJ}\tilde{\Omega}^{I\mu\nu}\Omega^{J\nu}\delta_{ij} \right) + \text{h.c.}$$

$$+ \frac{i}{8}e^{-1}e^{\mu\nu\lambda\sigma}F^{1}_{\mu\nu} \left( \tilde{\Omega}^{I}_{\mu}\gamma_{\lambda}\psi_{\sigma j}\delta^{ij} - \bar{X}^{1}\gamma_{\mu}\bar{\psi}_{\sigma j}\delta^{ij} \right) + \text{h.c.} \hspace{1cm} \text{(3.29)}$$

Note that (3.29) depends at most linearly on $F^{1}_{\mu\nu}$. This is a direct consequence of the fact that the prepotential (3.26) is at most linear in $X^{1}$. The action (3.28) —which still contains all the auxiliary fields of the Weyl multiplet— is most suitable for determining the supersymmetry transformation of the Lagrange multiplier $\bar{W}^{1}_{\mu}$. The supersymmetry transformation rules for the original STU vector multiplet fields are as in (2.5), except for $\delta W^{1}_{\mu}$ which is replaced by

$$\delta F^{1}_{\mu\nu} = 2\partial_{[\mu}\left\{ \bar{e}_{i}\gamma_{[\nu]}\Omega^{I}_{\mu}\delta^{ij} + 2\bar{X}^{1}\bar{e}^{i}\psi_{\nu j}\delta^{ij} + \text{h.c.} \right\}.$$ \hspace{1cm} \text{(3.30)}

The variation of the $\bar{W}^{1}_{\mu}$-independent part of the lagrangian is necessarily of the following form

$$\delta L_{\text{vector}} = -\frac{i}{4}e^{\mu\nu\lambda\sigma}O_{\mu} \partial_{\nu}F^{1}_{\lambda\sigma} + \text{total derivatives},$$ \hspace{1cm} \text{(3.31)}

otherwise one would not regain invariance if the Lagrange multiplier would be eliminated again. Here $O_{\mu}$ stands for an a priori unknown expression whose precise form can be determined by direct computation. Transforming $F^{1}_{\mu\nu}$ in the Lagrange multiplier term of (3.28) generates a total derivative, so if one lets the variation of $\bar{W}^{1}_{\mu}$ be equal to $O_{\mu}$ one obtains invariance. This reasoning yields

$$\delta \bar{W}^{1}_{\mu} = \bar{e}_{i}\gamma_{\mu}\tilde{\Omega}^{I}_{\mu}\delta^{ij} + 2\bar{X}^{1}\bar{e}^{i}\psi_{\nu j}\delta^{ij} + \text{h.c.},$$ \hspace{1cm} \text{(3.32)}

Note that $\delta W^{0}_{\mu}, \delta W^{A}_{\mu}$ and $\delta W^{1}_{\mu}$ nicely rotate into eachother under $SO(2, n)$ transformations, as expected.

In order to finish the dualisation we eliminate the auxiliary field $F^{1}_{\mu\nu}$. From (3.29) one derives that the equation of motion enforced by $F^{1}_{\mu\nu}$ reads

$$4\bar{X}^{1}\eta I\tilde{F}^{I}_{\mu\nu} - \bar{X}\eta I\bar{T}^{+\mu\nu}_{ij}\delta^{ij} - \eta IJ\tilde{\Omega}^{I\mu\nu}\tilde{\Omega}^{J\nu}\delta_{ij} = 0,$$ \hspace{1cm} \text{(3.33)}

where the covariant field strength $\tilde{F}^{+1}_{\mu\nu}$ is defined in the same spirit as in (2.6). Remark that (3.33) is saturated at the string tree level in the sense that it doesn’t receive any loop corrections. Moreover it is manifestly $SO(2, n)$ invariant. However, the most important property of the $F^{1}_{\mu\nu}$ equation of motion is that it doesn’t determine the value of $F^{1}_{\mu\nu}$ itself, at least as long as one treats $T^{+\mu\nu}_{ij}$ as an independent auxiliary field. But in fact $T^{+\mu\nu}_{ij}$ cannot be kept as an independent variable, because (3.33) must be viewed as a constraint determining $T^{+\mu\nu}_{ij}$ as a function of the stringy field strengths, scalars and gauginos.\hspace{0.5cm}[17]

As a result the $T^{+\mu\nu}_{ij}$ equation of motion (2.22) must be imposed as well. In the present case this equation of motion can be worked out, yielding

$$\bar{Z}\eta\tilde{Z}\left( F^{+1}_{\mu\nu} + i\Delta F^{+0}_{\mu\nu} - i\phi X^{0}T^{+\mu\nu}_{ij}\delta^{ij} \right)$$

$$= 4\phi \tilde{F}^{+1}_{\mu\nu}\eta IJ \text{Re} \tilde{Z}^{J} + \text{fermions}^{2} + F^{(0)} \text{terms} + \text{hypermultiplet terms},$$ \hspace{1cm} \text{(3.34)}

\hspace{2cm} \text{[17]The factor $\bar{X}\eta\tilde{X}$ can be inverted, otherwise the $R$ term in the action would vanish.}
where \( \tilde{Z}^I = -i \frac{\bar{X}^I}{X^I} \). In this way \( F^{I}_{\mu \nu} \) finally becomes a dependent expression of the remaining physical fields. Remark that the actual expression for \( F^{I}_{\mu \nu} \) receives loop and hypermultiplet dependent contributions.

To finish our construction of the vector multiplet theory in the stringy basis we discuss its symplectic properties. We start from the lagrangian (3.28), impose the two equations of motion we just mentioned and call the resulting lagrangian \( e^{-1}L_{\text{stringy}} \). In fact the expression (3.33) is nothing but a nice way of writing that \( \tilde{F}^{\mu \nu+1} = G^{1 \mu \nu+} \). Moreover we define that \( \tilde{F}^{\mu \nu+1} = F^{I}_{\mu \nu} \). The symplectic transformations of \( \tilde{F}^{\mu \nu+1} \) and \( \tilde{G}^{1 \mu \nu+} \) are by definition given by the transformations of \( G^{1 \mu \nu+} \) and \( -F^{\mu \nu+1} \) respectively. One may verify that with these definitions the stringy action is symplectically invariant apart from the non-invariant term

\[
-\frac{i}{8} F^{I+1 \mu \nu} + \frac{i}{4} F^{I+1 \mu \nu} + h.c. = -\frac{i}{8} F^{I\mu \nu} G^{1 \mu \nu} + h.c. .
\]

In particular this implies that the stringy lagrangian transforms as follows under \( SO(2,n) \) transformations:

\[
e^{-1}L_{\text{stringy}} \xrightarrow{SO(2,n)} e^{-1}L_{\text{stringy}} - \frac{i}{8} A_{IJ} \tilde{F}^{I\mu \nu} \tilde{F}^{J\mu \nu}.
\]

### 4 Construction of generic antisymmetric tensor theories

In this section we take a general point of view and study arbitrary vector multiplet theories containing a Peccei-Quinn symmetry. We impose just one restriction on this class of theories, namely we demand that there exists a set of special coordinates on which the Peccei-Quinn transformation acts in a standard way. By this we mean that there exists one distinguished special coordinate, henceforth called \( S \), which shifts under the Peccei-Quinn symmetry by an imaginary constant, whereas the other special coordinates \( Z^A \) are invariant under it. Given this ansatz we are able to completely characterise the Peccei-Quinn invariant vector multiplet theories and we show that they precisely comprise the cases discussed in [9] plus the perturbative heterotic string case which was still missing there.

For all these theories we can obtain an antisymmetric tensor version by dualising the axion \( a = -\text{Im}S \). This dualisation can be performed in a way which is to a large extent model independent and as such it explains why the resulting antisymmetric tensor theories show some universal behaviour. We will see for instance that they are all characterised by a similar gauge structure. They contain a particular \( U(1) \) gauge symmetry, with parameter \( z \), under which the antisymmetric tensor field \( B_{\mu \nu} \) and an appropriately defined vector gauge field \( V_\mu \) transform in the following way:

\[
V_\mu \rightarrow zV_\mu^{(z)} \quad B_{\mu \nu} \rightarrow zB_{\mu \nu}^{(z)} .
\]

Here the fields \( V_\mu^{(z)} \) and \( B_{\mu \nu}^{(z)} \) stand for some (complicated) functions of the independent fields in our superconformal theory which will be specified below. In the treatment of [9] this \( z \) gauge symmetry coincides with the central charge transformation which is necessary for off-shell closure of the supersymmetry algebra.

At a later stage we verify whether or not one may dualise the vector \( V_\mu \) and we find that the different antisymmetric tensor theories behave differently in this respect. The so-called

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18 Notice that \( F^{\mu \nu+1} \) as a dependent field still transforms in the naive way due to the fact that the \( T^{\mu \nu+1}_{\mu \nu+1} \) equation of motion is symplectically invariant.

19 The fields \( S, Z^A \) are the obvious generalisations of the variables we encounter in the description of perturbative heterotic strings. The interpretation of \( S \) as a string coupling constant is of course not valid in the general case.
non-linear vector-tensor multiplet theory of\[9\] doesn’t allow for this dualisation. The linear vector-tensor theory\[9\] can be dualised. This last theory is known to contain two abelian background vector fields, one of which gauges the central charge transformation. Under the duality the role of both background vectors gets interchanged. In the heterotic case, finally, the dualisation removes the complete central charge-like structure. We will see later that in this last case the dual of $V_\mu$ is nothing but the stringy vector $\tilde{W}_1^\mu$ we encountered before. So the stringy vectors play at least two important roles. We saw in section 3.2 that they make the $SO(2,n)$ invariance of the heterotic vector multiplet theory as manifest as possible, and at present we find that they also considerably simplify the gauge algebra acting on the heterotic vector and tensor gauge fields.

4.1 Peccei-Quinn invariant vector multiplet theories

The aim of this subsection is to characterise the set of Peccei-Quinn invariant $N = 2$ vector multiplet theories. We demand that any theory in this set contains some special coordinate moduli $(S, Z^A)$ transforming in the following way under a continuous Peccei-Quinn symmetry:

\[
S \overset{P,Q}{\rightarrow} S - ic,
\]

\[
Z^A \overset{P,Q}{\rightarrow} Z^A, \quad (4.2)
\]

Here $c$ is an arbitrary real constant. In order to understand how the Peccei-Quinn symmetry acts on a the full vector multiplet theory (so not only on the scalars) we embed the Peccei-Quinn transformation into the symplectic group $Sp(2(n + 2), \mathbb{R})$

\[
\begin{pmatrix}
U^I_J & Z^{IJ} \\
W_{IJ} & V^I_J
\end{pmatrix}_{P,Q} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
c & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & \delta^A_B & 0 & 0 & 0 \\
W_{00} & -\frac{c}{2}W_{11} + W_1 & -\frac{c}{2}W_{1B} + W_B & 1 & -c & 0 \\
\frac{c}{2}W_{11} + W_1 & W_{11} & W_{1B} & 0 & 1 & 0 \\
\frac{c}{2}W_{1A} + W_A & W_{1A} & W_{1B} & 0 & 0 & \delta^B_A
\end{pmatrix}. \quad (4.3)
\]

This particular symplectic transformation is a crucial object in the construction of antisymmetric tensor theories, so we will comment on some of its characteristic features. First we note that $Z^{IJ} = 0$, so the symplectic transformation is of the semi-classical type. In particular this means that the Peccei-Quinn symmetry transforms the vectors $W^I_\mu$ just linearly into eachother. Note that the submatrices $U^I_J$ and $V^I_J = (U^{-1})^{T_J}_I$ are completely fixed by the transformations (4.2) of the special coordinate moduli and by the defining symplectic relation (2.11), so they are valid for all cases we are interested in. The submatrix $W_{IJ}$ is model dependent. In equation (4.3) the most general matrix $W_{IJ}$ has been given which is consistent with (2.11) (provided $W_{AB}$ is taken to be symmetric), but in fact there are additional restrictions. In any case the various entries of $W_{IJ}$ must be real constants. They may depend on the parameter $c$ but not for instance on any of the moduli fields. The additional restrictions we just mentioned stem from the fact that the symplectic transformation (4.3) must generate a duality invariance of the theory and not just a symplectic reparametrisation. This implies that

\[
F_I(X^0, X^1, X^A) \overset{P,Q}{\rightarrow} F_I(X^0, X^1 + cX^0, X^A), \quad (4.4)
\]

so when a suitable prepotential $F(X)$ is given the explicit form of $W_{IJ}$ can be read off easily. Nicely enough the reverse procedure is possible as well. As we are going to explain now, one
can selfconsistently solve for the most general matrix \( W_{IJ} \) leading to a duality invariance. The most general prepotential \( F(X) \) can then be reconstructed from the knowledge of \( W_{IJ} \).

We recall that the lagrangian (4.3) is Peccei-Quinn invariant up to a topological term

\[
e^{-1} \mathcal{L}_{\text{vector}} = e^{-1} \int_{\mathbb{R}^4} \frac{P}{Q} e^{-1} \mathcal{L}_{\text{vector}} - \frac{i}{8} (U^T W)_{IJ} F_{\mu \nu}^I \tilde{F}^{\mu \nu J}, \tag{4.5}
\]

thanks to the fact that the transformation (4.3) is semi-classical. In terms of the standard vector multiplet variables the non-invariance of the lagrangian may have different sources, because not only the axion \( a \) shifts under Peccei-Quinn transformations, but also \( W_\mu^I, \Omega_i^I, \Omega_{0i}^I, Y_{ij}^I, \) and \( Y_{0i}^I \) transform due to the equation (4.3). The situation becomes much more transparent though, if one performs a change of variables and rather works with a set of variables which — apart from the axion itself— are all invariant under Peccei-Quinn. Given equation (4.3) one easily verifies that the following variables do the job:

\[
a, \phi = \text{Re} S, X^0, X^A \\
\lambda_i = -i \frac{2 X^0}{X^0} \left\{ \Omega_1^i - \frac{X^1}{X^0} \Omega_0^i \right\}, \Omega_0^i, \Omega_1^i \\
V_\mu = W_\mu^I - a W_\mu^0, W_\mu^0, W_\mu^A \\
Z_{ij} = Y_{ij}^1 - a Y_{ij}^0, Y_{ij}^0, Y_{ij}^A 	ag{4.6}
\]

When the lagrangian is expressed in terms of (4.6) there is only one source for the non-invariance term in (4.3): it must come from terms in the lagrangian in which the axion \( a \) appears undifferentiated. As an example we concentrate on the term

\[
- \frac{i}{8} (U^T W)_{IJ} F_{\mu \nu}^I \tilde{F}^{\mu \nu J} = - \frac{i}{8} W_{AB}(c) F_{\mu \nu}^A \tilde{F}^{\mu \nu B} + \cdots \tag{4.7}
\]

This term should arise upon varying

\[
e^{-1} \mathcal{L}_{\text{vector}} = - \frac{i}{8} \text{Re} F_{AB} F_{\mu \nu}^A \tilde{F}^{\mu \nu B} + \cdots \tag{4.8}
\]

from which we derive that the following relation must be valid:

\[
\text{Re} F_{AB}(a \rightarrow a + c) = \text{Re} F_{AB}(a) + W_{AB}(c) \tag{4.9}
\]

It is then clear that

\[
W_{AB}(c) = -c \eta_{AB} \\
\text{Re} F_{AB}(a) = -a \eta_{AB} + a \text{ independent terms}, \tag{4.10}
\]

where \( \eta_{AB} \) is a real symmetric constant not depending on the parameter \( c \). In the same spirit one can determine the other entries in \( W_{IJ} \) by writing (4.5) in terms of the Peccei-Quinn invariant field strengths \( F_{\mu \nu}^0, 2 \partial_\mu V_\nu, F_{\mu \nu}^A \) and by reconstructing the undifferentiated \( a \) terms in the lagrangian which lead to (4.5). The result is

\[
W_{IJ} = -c \left( \begin{array}{cccc}
\eta_{00} - \frac{2}{5} \eta_{11} & \eta_{01} - \frac{4}{5} \eta_{11} & \eta_{0B} - \frac{4}{5} \eta_{1B} \\
\eta_{01} + \frac{4}{5} \eta_{11} & \eta_{11} & \frac{2}{5} \eta_{1B} \\
\eta_{0A} + \frac{4}{5} \eta_{1A} & \frac{2}{5} \eta_{1A} & \eta_{AB} \\
\end{array} \right), \tag{4.11}
\]

where all \( \eta_{IJ} \) are constants. This result can be integrated back to yield the prepotential

\[
F(X) = - \frac{1}{6} (X^1)^3 - \frac{1}{4} (X^0)^2 \eta_{11} \eta_{1A} X^A \ tag{4.12}
\]

\[
- \frac{1}{2} \left\{ (X^1)^2 \eta_{01} + X^1 X^0 \eta_{10} + 2 X^1 X^A \eta_{0A} \right\}
\]

\[\text{In order to make contact to the perturbative heterotic prepotential given in (3.20) all entries of } \eta_{IJ} \text{ apart from } \eta_{AB} \text{ must be put equal to zero. Therefore the } \eta_{IJ} \text{ which is used throughout this section doesn’t coincide with the } SO(2, n) \text{ metric defined in (4.3). Note that all } \eta_{ij} \text{’s occurring outside section 4 refer to the } SO(2, n) \text{ metric and not to the matrix } \eta_{IJ} \text{ we just defined.} \]
Note that the $\eta_I$ terms in the prepotential are quadratic polynomials with real coefficients. These just add total divergence terms to the action \[13\] so from now on we put $\eta_I = 0$ for simplicity. As such we precisely obtain the theories discussed in \[1\] and on top of that the perturbative heterotic string case $\eta_1 = 0$, $\eta_A = 0$ which was excluded in a fully off-shell superconformal context. We see no sign of any other vector multiplet theories which might have a dual antisymmetric tensor description. Remark that the prepotential (4.12) has a natural type II interpretation. For type II strings on generic Calabi-Yau manifolds there is a Peccei-Quinn symmetry for every modulus $Z_A$ \[33, 19\]. One may pick any value for $A$ and identify the corresponding modulus with $S$. In the large $S$ limit the prepotential (3.1) coincides with the prepotential we just found, provided we take $d_{111} = \eta_{11}$, $d_{11A} = 1/2 \eta_{1A}$, $d_{1AB} = \eta_{AB}$.

4.2 Dualising the axion

All the Peccei-Quinn invariant theories we just specified can be dualised into antisymmetric tensor theories. In order to check that this is indeed possible, it suffices to show that the vector multiplet lagrangians for these theories can be brought into a form such that the axion $a$ appears only via its “field strength” $\partial_\mu a$ (up to total derivative terms in the lagrangian). The existence of the variables (4.6) is very important in this respect. When the lagrangian (2.3) is written in terms of the standard vector multiplet variables one sees undifferentiated $a$ dependencies occurring at various places. By going to the new variables one effectively absorbs most of these unwanted $a$ terms. Only in the gauge sector some undifferentiated $a$ terms remain. As we remarked before these left-over $a$ terms can be reconstructed on the base of equation (4.5). They read

$$
\frac{i}{8} \left\{ a \left( \eta_{11} F_{\mu\nu}(V) \tilde{F}^{\mu\nu}(V) + \eta_{1A} F_{\mu\nu}(V) \tilde{F}^{\mu\nu A} + \eta_{AB} F_{\mu\nu}^A \tilde{F}^{\mu\nu B} \right) \\
+ a^2 \left( \eta_{11} F_{\mu\nu}(V) \tilde{F}^{\mu\nu 0} + \frac{1}{2} \eta_{1A} F_{0\mu}^0 \tilde{F}^{\mu\nu A} \right) + \frac{1}{3} a^3 \eta_{11} F_{\mu\nu}^0 \tilde{F}^{\mu\nu 0} \right\},
$$

(4.13)

where the gauge covariant field strength $F_{\mu\nu}(V)$ is defined as

$$
F_{\mu\nu}(V) = 2 \partial_\mu V_\nu - 2 W^0_\mu \partial_\nu a.
$$

(4.14)

Equation (4.13) can be rewritten as

$$
- \frac{i}{4} e^{-1} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu a \left( \eta_{11} V_\nu \partial_\lambda V_\sigma + \eta_{1A} W^A_\nu \partial_\lambda V_\sigma + \eta_{AB} W^A_\nu \partial_\lambda W^B_\sigma \right) + e^{-1} \left( \text{total derivative} \right).
$$

(4.15)

After dropping the total derivative we may dualise the axion by replacing the “field strength” $\partial_\mu a$ everywhere in the lagrangian by an auxiliary vector $V^{(z)}_\mu$ and by adding a Lagrange multiplier term

$$
\frac{i}{4} e^{-1} \epsilon^{\mu\nu\lambda\sigma} V^{(z)}_\mu \partial_\nu B_{\lambda\sigma}.
$$

(4.16)

In principle the auxiliary field $V^{(z)}_\mu$ can then be eliminated (possibly together with some other auxiliaries) in order to obtain an antisymmetric tensor theory. This turns $V^{(z)}_\mu$ into a dependent expression of the physical fields. In section 5.3 we will explicitly discuss the elimination of $V^{(z)}_\mu$ in the perturbative heterotic context.

4.3 The gauge structure of the antisymmetric tensor theories

It is interesting to see how a central-charge-like gauge structure, which is a crucial ingredient in the off-shell construction of \[9\], arises in the present context. The gauge transformations of
the vector fields are easily determined starting from \( \delta_{\text{gauge}} W^I_\mu = \partial_\mu \theta^I \) and the redefinition (4.6). Defining \( z = \theta^0, \ y = \theta^1 - \alpha \theta^0 \) one gets:

\[
\delta_{\text{gauge}} V_\mu = \partial_\mu y + z V^{(z)}_\mu \quad \delta_{\text{gauge}} W^0_\mu = \partial_\mu z \quad \delta_{\text{gauge}} W^A_\mu = \partial_\mu \theta^A . \tag{4.17}
\]

Notice that the \( z \)-gauge transformation maps the vector \( V_\mu \) into \( V^{(z)}_\mu \) which explains why \( V^{(z)}_\mu \) appears in the gauge covariant field strength \( F_{\mu \nu} (V) \). The vector field \( W^0_\mu \) is a distinguished one in that it gauges this \( z \)-gauge transformation. The gauge transformations of the antisymmetric tensor field \( B_{\mu \nu} \) can most easily be determined from the lagrangian

\[
e^{-1} L_{\text{vector}} + \frac{i}{4} e^{-1} \epsilon^{\mu \nu \lambda \sigma} V^{(z)}_\mu \partial_\nu B_{\lambda \sigma} ,
\]

in which \( V^{(z)}_\mu \) and the other auxiliaries are kept as independent non-propagating degrees of freedom. The \( B_{\mu \nu} \) independent part of this lagrangian is not invariant under gauge transformations. First there are the \( F_{\mu \nu} (V) \) dependent terms. Due to the fact that \( \delta_{\text{gauge}} F_{\mu \nu} (V) = 2 z \partial_\mu V^{(z)}_\nu \) they give a non-zero contribution. Secondly there is the non-total-derivative part of (4.17) (with \( \partial_\mu a \) replaced by \( V^{(z)}_\mu \)) which contains explicit gauge fields. Note that all these non-invariances are proportional to \( \partial_\mu V^{(z)}_\nu \), so they can be cancelled by a suitable choice of the gauge variation of the antisymmetric tensor field. In this way one finds that

\[
\delta_{\text{gauge}} B_{\mu \nu} = 2 \partial_\mu A_\nu + \eta_{11} y \partial_\mu V_\nu + \eta_{1A} \theta^A \partial_\mu V_\nu + \eta_{AB} \theta^A \partial_\mu W^B_\nu + z B^{(z)}_{\mu \nu} \\
B^{(z)}_{\mu \nu} = 4 i \dot{A}_{\mu \nu} - \eta_{11} V^{(z)}_\mu V^{(z)}_\nu - \eta_{1A} W^{(z)}_mü V^{(z)}_\nu \\
A_{\mu \nu} = \frac{\delta e^{-1} L_{\text{vector}}}{\delta F^{\mu \nu} I} | F^{\mu \nu} \rightarrow F_{\mu \nu} (V), F_{I J} \rightarrow F_{I J} (a = 0) ,
\]

where \( \Lambda_\mu \) is a parameter for tensor gauge transformations. Under the \( z \)-gauge transformation the antisymmetric tensor is mapped into \( B^{(z)}_{\mu \nu} \). This is a complicated function which does not only contain the field strengths but also scalars, gauginos, gravitinos etcetera. The gauge covariant field strength for \( B_{\mu \nu} \) reads

\[
H_{\mu \nu \lambda} = \partial_\mu B_{\nu \lambda} - \eta_{11} V_{[\mu} \partial_\nu V_{\lambda]} - \eta_{1A} W^{A}_{[\mu} \partial_\nu V_{\lambda]} - \eta_{AB} W^{A}_{[\mu} \partial_\nu W^{B}_{\lambda]} - W^{0}_{[\mu} B^{(z)}_{\nu \lambda]} . \tag{4.20}
\]

The constants \( \eta_{IJ} \) which were previously introduced in order to specify the prepotential (4.12) turn out to be directly related to the various Chern-Simons couplings of the antisymmetric tensor. It was already remarked in [4] that a cubic in \( S \) prepotential leads to a Chern-Simons coupling which is quadratic in the vector \( V_\mu \) and that the quadratic in \( S \) prepotential leads to a Chern-Simons coupling which is linear in \( V_\mu \). Here we see that the Chern-Simons couplings depend on the background vector multiplet fields only, when the prepotential is just linear in \( S \).

There is a last point concerning the gauge structure of the antisymmetric tensor theories which deserves to be investigated. When we specialise to the heterotic string case we see that we have constructed an antisymmetric tensor theory containing the vector \( V_\mu \) which is directly related to the \( STU \) vector \( W^1_\mu \). We already discussed in section 3.2 that in the vector multiplet version of the perturbative heterotic theory one may benefit from trading \( W^1_\mu \) for its dual, the stringy vector \( W^\perp_\mu \). One may ask a similar question in the present context, and verify what are the effects of dualising the vector \( V_\mu \). First of all we note that we have to exclude the \( \eta_{11} \neq 0 \) case, because otherwise the theory contains explicit \( V_\mu \) terms (see for instance (4.13)), which prevent us from performing the dualisation we have in mind. In the other cases there is no obstruction, so one may replace the field strength \( 2 \partial_\mu V_\nu \) by an auxiliary field \( C_{\mu \nu} \) and add the Lagrange multiplier term

\[
\frac{i}{4} e^{-1} \epsilon^{\mu \nu \lambda \sigma} C_{\mu \nu} \partial_\lambda V^d_\sigma , \tag{4.21}
\]

22
The gauge transformation of \( V^d_\mu \) can be fixed in the standard way by adopting \( \delta_{\text{gauge}} C_{\mu \nu} = 2 \partial_\mu \{ z V_\nu \} \) and checking the gauge invariance of the theory. One finds that

\[
\delta_{\text{gauge}} V^d_\mu = \partial_\mu y^d - \frac{1}{2} \eta_{IA} \theta^A C^{(z)}_{\mu \nu}.
\] (4.22)

But now we have a extra possibility which crucially hinges on the fact that \( C_{\mu \nu} \) is not a total derivative. We can introduce an extra gauge transformation \( \delta_{\text{extra}} C_{\mu \nu} = -2 z \partial_\mu V_\nu \) and still maintain the gauge invariance of the full theory provided we add a compensating \( \delta_{\text{extra}} B_{\mu \nu} \). This yields a modified

\[
\delta_{\text{gauge}} B_{\mu \nu} = 2 \partial_\mu \Lambda_\nu + 2 z \partial_\mu V_\nu^{(z)} + \eta_{AB} \theta^A \partial_\mu W^{B}_\nu + \frac{1}{2} \eta_{IA} \theta^A C_{\mu \nu}.
\] (4.23)

What have we gained by this whole operation? First we look at the \( \eta_{IA} \neq 0 \) case. The vector \( V^d_\mu \) transforms under a central charge-like transformation gauged by the vector \(-\frac{1}{2} \eta_{IA} W^{A}_\mu \). In going from (4.19) to (4.23) the complicated expression \( B_{\mu \nu}^{(z)} \) has been replaced by the field strength \( 2 \partial_\mu V_\nu^{(z)} \) and conversely the field strength \( 2 \partial_\mu V_\nu \) has been replaced by \( C_{\mu \nu} \). After imposing the equation of motion for \( C_{\mu \nu} \), this field becomes a dependent expression of the physical fields and as such it can be viewed as a dual \( B_{\mu \nu}^{(\eta_{IA} \theta^A)} \). So under the duality transformation the theory has been mapped onto a similar one in which the role of the vectors \( W_\mu^A \) and \(-\frac{1}{2} \eta_{IA} W^{A}_\mu \) has been interchanged\[21]. In the \( \eta_{IA} = 0 \) case the whole central charge-like structure disappears and the Chern-Simons couplings are of a completely conventional form. We will see later that in that case \( V_\mu \) can be identified with the stringy vector \( \tilde{W}^1_\mu \). The vector fields \( W_\mu^0, V_\mu^d, W_\mu^A \) then appear on an equal footing, reflecting the underlying \( SO(2, n) \) invariance of the heterotic string theory. The resulting antisymmetric tensor theory will be presented in the next section.

## 5 The antisymmetric tensor effective action for heterotic strings

### 5.1 The \( SO(2, n) \) invariant antisymmetric tensor theory

The results of the preceding section for the particular case of perturbative strings can be summarised as follows. One starts from the vector multiplet lagrangian (2.3) in the \( STU \) basis and introduces the new Peccei-Quinn invariant variables (4.6). Then one subtracts the total derivative term of (4.15) and adds the \( B_{\mu \nu} \) and \( V^d_\mu \) Lagrange multiplier terms given in (4.16) and (4.21) respectively. The order in which the various dualisations are carried out should not matter so one can equally start from the stringy vector multiplet theory described in section 3.2 in which the vector \( W^1_\mu \) has already been traded for its dual \( \tilde{W}^1_\mu \) and afterwards dualise the axion. Let us sketch the different steps one takes in this second scenario and verify that it indeed leads to the same theory as in section 4.3. The advantage of first dualising the vector

\[21\] As an example we may take \( A = 2, \eta_{12} = 2 \). The vector-tensor theory is then really mapped onto itself under the duality. This can be checked by going back to vector multiplet language and verifying that the symplectic transformation

\[
\begin{pmatrix}
U_{IJ} & Z_{IJ} \\
W_{IJ} & V_{IJ}
\end{pmatrix}
\] _dual

\[
= \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0
\end{pmatrix}
\]

(4.24)

corresponds to a duality invariance.

\[23\]
$W^1_\mu$ is that one can more easily keep track of the $SO(2, n)$ invariance of the final antisymmetric tensor theory.

Our starting point is the lagrangian (3.22), or rather $e^{-1} \mathcal{L}_{\text{stringy}}$ which is obtained from it by eliminating the auxiliaries $F^{-1}_\mu$ and $T_{\mu\nu ij}$. This lagrangian transforms in a simple way under $SO(2, n)$ transformations, see equation (3.36). As before one introduces the Peccei-Quinn invariant variables $\phi, \lambda_i, Z_{ij}$ given in (4.4), as well as a new field $C'_{\mu\nu}$ which is defined as the Peccei-Quinn invariant part contained in the dependent field $F^1_{\mu\nu}$:

$$C'_{\mu\nu} \equiv F^1_{\mu\nu} - aF^0_{\mu\nu}. \quad (5.1)$$

The stringy vectors $\tilde{W}^I_\mu$ are automatically Peccei-Quinn invariant as can be verified by rewriting the symplectic transformation (4.3) in the stringy basis. Therefore the stringy vectors need not be redefined and as a result no central charge-like gauge transformations appear. The Peccei-Quinn argument can be repeated, mutatis mutandis, in order to compute the explicit axion dependence of the lagrangian. One finds that the only undifferentiated axion term reads

$$\frac{i}{8} a_{\text{inv}} \eta_{IJ} \tilde{F}^I_{\mu\nu} \tilde{F}^{\mu\nu J}, \quad (5.2)$$

which can be rewritten as

$$-\frac{i}{4} e^{-1} \epsilon_{\mu\nu\lambda\sigma} \partial_{\mu} a_{\text{inv}} \eta_{IJ} \tilde{W}^I_\nu \partial_{\lambda} W^J_\sigma + e^{-1} \text{total derivative}. \quad (5.3)$$

Note that in these formulae we introduced the $SO(2, n)$ invariant axion $a_{\text{inv}}$ of (3.22) which is related to $a$ by

$$a_{\text{inv}} = a - \frac{1}{2} \frac{F^{(0)}_I \tilde{X}^I + h.c.}{\tilde{X} \eta \tilde{X}} + M^{(0)} \quad (5.4)$$

The use of $a_{\text{inv}}$ is just a matter of convenience. It guarantees that the total derivative term in (5.3) is $SO(2, n)$ invariant. At this point we drop the total derivative, replace $\partial_{\mu} a_{\text{inv}}$ by $V^{(z)}_\mu a_{\text{inv}}$ and add the $SO(2, n)$ invariant Lagrange multiplier $\tilde{B}_{\mu\nu}$. This leads to our final antisymmetric tensor lagrangian

$$e^{-1} \mathcal{L}_{\text{tensor}} = e^{-1} \left\{ \mathcal{L}_{\text{stringy}} - \text{total derivative of (5.3)} + \frac{i}{4} \epsilon_{\mu\nu\lambda\sigma} V^{(z)}_{\mu\nu} \partial_{\lambda} B_{\sigma}\right\}. \quad (5.5)$$

This lagrangian clearly transforms under the T-dualities as

$$e^{-1} \mathcal{L}_{\text{tensor}} \xrightarrow{SO(2,n)} e^{-1} \mathcal{L}_{\text{tensor}} - i \frac{1}{8} \Lambda_{IJ} \tilde{F}^I_{\mu\nu} \tilde{F}^{\mu\nu J}. \quad (5.6)$$

One can verify that the lagrangian we just specified equals the one discussed in section 4.3 provided we identify

$$\tilde{W}^I_\mu \equiv V^d_\mu \quad \tilde{B}_{\mu\nu} \equiv B_{\mu\nu} + W^0_{[\mu} \tilde{W}_{\nu]} \quad (5.8)$$

Note that the gauge transformation of $V^d_\mu$ as given in (4.22) is consistent with the transformation of $\tilde{W}^I_\mu$. Taking into account the result (4.28) one finds the following gauge transformations for the antisymmetric tensor theory:

$$\delta_{\text{gauge}} \tilde{W}^I_\mu = \partial_{\mu} \tilde{t}^I \quad \delta_{\text{gauge}} \tilde{B}_{\mu\nu} = 2 \partial_{[\mu} \tilde{A}_{\nu]} + \eta_{IJ} \partial_{[\mu} \tilde{W}_{\nu]} \tilde{t}^I. \quad (5.9)$$

To be completely precise: both lagrangians differ at the one-loop level by the gauge invariant total derivative term

$$-\frac{i}{4} e^{-1} \epsilon_{\mu\nu\lambda\sigma} \partial_{\mu} \left\{ \frac{1}{2} \frac{F^{(0)}_I \tilde{X}^I + h.c.}{\tilde{X} \eta \tilde{X}} - M^{(0)} \right\} \left( \partial_{\nu} B_{\lambda\sigma} - \eta_{IJ} \partial_{[\nu} \tilde{W}_{\sigma]} \tilde{t}^I \right). \quad (5.7)$$
5.2 The supersymmetry transformation of the antisymmetric tensor

The supersymmetry transformation of the antisymmetric tensor field \( \hat{B}_{\mu\nu} \) can be determined from the lagrangian (5.5). As input one takes the supersymmetry variations of the background vector multiplet fields as given in (2.5) and the variation of \( \tilde{W}_\mu^I \) in (3.32). Taking into account the redefinitions (4.6) one finds that

\[
\begin{align*}
\delta \phi & = e^i \lambda_i + h.c. \\
\delta V^{(z)}_\mu & = i \partial_\mu \{ e^i \lambda_i \} + h.c. \\
\delta \lambda_i & = \left( \partial_\mu - \frac{i}{2} X^0 \epsilon_{ij} \sigma^{\mu\nu} \epsilon^j \right) \left( \hat{C}^{(z)}_{\mu\nu} - i \phi \hat{F}_\mu^0 + \frac{i}{2} \phi \hat{X}_0 T^{kl} \epsilon_{kl} \right) \\
& - \frac{1}{2X^0} \left( i Z_{ij} + \phi Y^{(0)}_{ij} \right) \epsilon^j - \frac{1}{X^0} \left( \lambda_i \epsilon^j \Omega^{(0)}_{ij} + \Omega^0_{ij} \epsilon^j \lambda_j \right)
\end{align*}
\]

with

\[
\begin{align*}
\hat{C}^{(z)}_{\mu\nu} & = C^{(z)}_{\mu\nu} - \left( i \tilde{\psi}_{[\mu} \gamma_{\nu]} (2X^0 \lambda_j + \phi \Omega^{(0)}_{ij} \epsilon^j + i \phi X^0 \tilde{\psi}_{[\mu} \gamma_{\nu]} \epsilon^j + h.c. \right) \\
\hat{V}^{(z)}_\mu & = V^{(z)}_\mu - \left( \frac{i}{2} \psi^{(z)}_\mu \lambda_i + h.c. \right).
\end{align*}
\]

In order to find the supersymmetry variation of \( \hat{B}_{\mu\nu} \) it suffices to check only those terms in the variation of the lagrangian (5.5) which would vanish if the Bianchi identity

\[
\epsilon^{\mu\nu\lambda\sigma} \partial_\mu V^{(z)}_\mu = 0
\]

would be imposed. There are two different ways to obtain \( V^{(z)}_\mu \) terms in the variation of the lagrangian. In the first place there is the \( V^{(z)}_\mu \) dependence of the lagrangian itself

\[
\begin{align*}
e^{-1} L_{\text{tensor}} & = e^{-1} L_{\text{tensor}} \bigg|_{V^{(z)}_\mu = 0} + \frac{i}{4} e^{-1} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu V^{(z)}_\mu \left( \partial_\nu \hat{B}_{\lambda\sigma} - \eta_{IJ} \tilde{W}^I_\nu \partial_\lambda \tilde{W}^J_\sigma \right) \\
& - \frac{i}{2} V^{(z)}_\mu X_{\eta I} D^\mu \hat{X}^I - \frac{i}{8} V^{(z)}_\mu \eta_{IJ} \hat{\Omega}^I_\gamma \hat{\Omega}^J_\gamma + h.c. \\
& + \frac{i}{4} V^{(z)}_\mu X_{\eta I} \hat{\Omega}^I_\gamma \gamma^\mu \psi_{\eta i} + \frac{i}{8} e^{-1} \epsilon^{\mu\nu\lambda\sigma} \partial_\nu V^{(z)}_\mu \hat{X}_{\eta I} \hat{\psi}_{\eta I J} \gamma^\mu \psi_{\sigma i} + h.c.,
\end{align*}
\]

from which the relevant contributions to \( \delta e^{-1} L_{\text{tensor}} \) can be derived in a straightforward way. Secondly there are the \( \lambda_i \) terms which vary into \( V^{(z)}_\mu \) due to \( \delta \lambda_i = -i \tilde{V}^{(z)}_\mu \epsilon_i + \cdots \). Taking everything together one finds that the antisymmetric tensor action is invariant under supersymmetry if one defines

\[
\delta \hat{B}_{\mu\nu} = -2 \epsilon_i \sigma_{\mu\nu} \hat{X}_{\eta I} \hat{\Omega}^{I \eta} - \eta_{IJ} \hat{W}^I_\nu \left( \epsilon_{\gamma I J} \hat{\Omega}^{J \gamma} + 2 \hat{X}^I \epsilon_{\gamma I J} \epsilon_{\gamma J} \right) - 2 \hat{X}_{\eta I} \hat{X}^\gamma \gamma_{[\mu} \psi_{\nu \sigma]} + h.c.
\]

The \( S \)-supersymmetry invariance of the theory is automatically guaranteed so \( \hat{B}_{\mu\nu} \) doesn’t transform under \( S \)-supersymmetry. The covariant field strength for \( \hat{B}_{\mu\nu} \) can be determined from the equations (5.9) and (5.14):

\[
\hat{H}_{\nu \lambda \sigma} = \partial_\nu \hat{B}_{\lambda\sigma} - \eta_{IJ} \hat{W}^I_\nu \partial_\lambda \hat{W}^J_\sigma - \left( \hat{X}_{\eta I} \hat{\Omega}^{I \eta} - \sigma_{\nu \lambda} \psi_{\sigma I} - \frac{1}{2} \hat{X}_{\eta I} \hat{\psi}_{\nu \sigma I} + h.c. \right),
\]

while the dual of \( \hat{H}_{\mu \nu \lambda} \) is defined as

\[
\hat{H}^\mu \overset{\text{def}}{=} \frac{i}{2} e^{-1} \epsilon^{\mu\nu\lambda\sigma} \hat{H}_{\nu \lambda \sigma}.
\]

The bosonic part of these field strengths is denoted by dropping the “hat”.

25
It is important to notice that $B_{\mu\nu}$ transforms only into background Weyl multiplet and vector multiplet fields, and not for instance into the dilatchi $\lambda_i$. One can easily understand why this is the case. At this particular stage of our computation the fields $\phi, V_{\mu}^{(z)}$ and $\lambda_i$ are the only remnants of the original vector multiplet based on the scalar $X^1$. They appear at most linearly in the lagrangian $e^{-1}L_{\text{tensor}}$, which immediately follows from the linear $X^1$ dependence of the prepotential (3.26). The fields $\phi, V_{\mu}^{(z)}$ and $\lambda_i$ also transform at most linearly into each other, so $e^{-1}L_{\text{tensor}}$ can only transform into

$$
\frac{i}{4} e^{-1} \epsilon^{\mu\nu\lambda\sigma} V_{\mu}^{(z)} \partial_\nu \left( \text{background fields} \right) \lambda_\sigma . \tag{5.17}
$$

For the other antisymmetric tensor theories, namely the $\eta_{11} \neq 0$ or $\eta_{1A} \neq 0$ cases described in section 4 the situation is quite different. The supersymmetry variation of the tensor $B_{\mu\nu}$ as it was defined in section 4.2 can be computed much in the same way as $\delta B_{\mu\nu}$, and one finds that

$$
\delta B_{\mu\nu} = -4 \left( 2\eta_{11}\phi + \eta_{1A} \text{Re} Z^A \right) \left| X^0 \right|^2 e^i \sigma_{\mu\nu} \lambda_i + \text{h.c.} + \cdots . \tag{5.18}
$$

The latter transformation law automatically coincides with what was found in [9]. It then suffices to add an independent auxiliary scalar $\phi^{(z)}$ to recover the off-shell vector-tensor multiplets of [4].

### 5.3 Eliminating the auxiliary field $V_{\mu}^{(z)}$

Having determined the $Q$-supersymmetry transformation rule for $B_{\mu\nu}$ we proceed by eliminating the auxiliary field $V_{\mu}^{(z)}$. Before actually doing so we briefly recall what happens in the vector-tensor multiplet cases of [8]. There one can eliminate $V_{\mu}^{(z)}$ by imposing its own equation of motion, which is of the following type:

$$
V_{\mu}^{(z)} \sim H_{\mu} + \cdots . \tag{5.19}
$$

A relation like (5.19) is also what we expect to find in the present case because it expresses what duality is all about: $V_{\mu}^{(z)}$ was first introduced as the field strength for the axion and it should become equal to the dual of the field strength for $B_{\mu\nu}$ after the dualisation. However, in the heterotic theory a relation like (5.19) cannot be recovered in one go. Given (5.13) one readily verifies that the equation of motion enforced by the auxiliary $V_{\mu}^{(z)}$ reads

$$
\hat{H}_{\mu} = i\hat{X} \eta_I D_{\mu} \tilde{X}^I + i \frac{i}{4} \eta_{IJ}\check{\Omega}^{IF} \gamma_\mu \check{\Omega}^I + \text{h.c.}.
$$

$$
= \tilde{X} \eta_I \check{X} A_\mu + i\tilde{X} \eta_I \check{\partial}_\mu \tilde{X}^I + i \frac{i}{4} \eta_{IJ} \check{\Omega}^{IF} \gamma_\mu \check{\Omega}^I + \text{h.c.} , \tag{5.20}
$$

which cannot be used to solve for $V_{\mu}^{(z)}$ itself. Instead the rôle of (5.20) is to constrain the auxiliary Weyl multiplet field $A_\mu$, such that it becomes a dependent expression involving the dual field strength $H_\mu$. Of course, the $A_\mu$ equation of motion (2.22) must be imposed as well

$$
\phi \tilde{X} \eta_I \check{X} A_\mu = \frac{1}{2} \tilde{X} \eta_I \check{X} V_{\mu}^{(z)} - \left( \frac{i}{4} \phi \tilde{X} \eta_I \left( \partial_\mu - b_\mu \right) \tilde{X}^I + \text{h.c.} \right) + \text{fermions}^2 + F^{(0)} \text{terms} + \text{hypermultiplet terms} , \tag{5.21}
$$

and this relation finally determines the value of $V_{\mu}^{(z)}$ as a function of $A_\mu$ such that altogether equation (5.19) is indeed fulfilled. The fact that we have to eliminate $V_{\mu}^{(z)}$ and $A_\mu$ both at the same time is of course not completely innocent. During the axion dualisation process we loose (another) part of the Weyl multiplet. As we already said, this is different in the $\eta_{11} \neq 0$ or $\eta_{1A} \neq 0$ cases of section 4 where the Weyl multiplet is not affected by the dualisation. This
seems to be a crucial property which guarantees the existence of the off-shell vector-tensor multiplets of $\mathcal{M}$. We also note the different character of the relations (5.20) and (5.21). The former just depends on the tree-level part of the theory, whereas the latter receives one-loop and hypermultiplet dependent corrections. Both relations are $SO(2, n)$ invariant.

We would like to stress that the duality transformations we have encountered so far, i.e. the one yielding the stringy vectors on the one hand and the one yielding the heterotic antisymmetric tensor field on the other hand, are very similar in nature. In both cases the Weyl multiplet cannot be preserved during the dualisation process such that the final theories are far from being realised off-shell. This is in line with the observations in [7] and [9] that the off-shell superconformal constructions that are known today are not applicable in case of the stringy vectors or the heterotic antisymmetric tensor theory. Of course it cannot really be excluded that there might exist alternative yet unexplored off-shell superconformal theories, which would reduce —after a straightforward elimination (in the sense of not involving any duality transformations) of some well-chosen auxiliary fields— to the on-shell stringy vector or antisymmetric tensor theories we have described so far. We don’t know what such off-shell theories would have to look like, if they exist at all.

5.4 Implementing the superconformal gauge choices

So far we described the various ingredients of the heterotic antisymmetric tensor theory. According to the equations (5.14) and (3.32) the tensor $B_{\mu\nu}$ and the vectors $W^I_{\mu}$ do not transform under supersymmetry into the scalar $\phi$ and the fermions $\lambda_i$ but rather into the background fields $X^I$ and $\Omega^I_i$. In the present on-shell situation it is in fact quite artificial to call $X^I$ and $\Omega^I_i$ “background” fields because due to the relations (3.33) and (5.20) the gravitational, vector and antisymmetric tensor variables interfere with each other such that one can no longer tell which multiplet serves as a background for the others. Due to the on-shellness there is also no more reason to keep on using all the variables $\phi$, $X^I$, $\lambda_i$ and $\Omega^I_i$ as independent degrees of freedom. In order to reduce the number of matter degrees of freedom then, we go to the Poincaré version of the heterotic antisymmetric tensor theory. The general strategy for going from a superconformal to a Poincaré theory has been discussed in section 2.2, so we can simply apply the general rules to the case at hand.

As before the lagrange multipliers $D$ and $\chi_i$ enforce the constraints (2.21) which can be used to restrict the hypermultiplet variables. When these constraints are taken into account the Einstein term in the action reads

$$
\frac{1}{2}X N \tilde{X} R = \frac{1}{2} \left\{ \phi \tilde{X} \eta \tilde{X} + \frac{i}{2} \left( F^{(0)}_I \tilde{X}^I - h.c. \right) \right\} R = \frac{1}{2} \phi_{\text{inv}} \tilde{X} \eta \tilde{X} R
$$

(5.22)

Remark that this term does not depend on the one-loop part of the theory. It is the relation between the true dilaton $\phi_{\text{inv}}$ and the field $\phi = \Re S$ which is subject to loop-corrections and one may view the appearance of $F^{(0)}_I$ in (5.22) as (yet another) indication that $\phi$ should not be identified with the true string loop counting parameter. In fact one might use the result (2.23) as a purely supergravity definition of what the true dilaton field is: the field $\phi_{\text{inv}}$ is the only scalar field which 1) makes the Einstein term (and hence any sensible dilatation gauge choice) loop independent, and 2) reduces to $\phi$ when loop-effects are neglected. Note that the requirement of $SO(2, n)$ invariance is not strong enough as a criterion to select the true dilaton because it would yield both $\phi_{\text{inv}}$ and $\phi_{\text{hol}} = \Re S_{\text{hol}}$ as possible candidates.

If we would impose the standard Einstein-frame dilatation gauge at this point (as well as the usual $U(1)$ gauge) we would find that the $\tilde{X}^I$ fields would become loop-independent functions of the special coordinates $\tilde{Z}^I$ multiplied by a common dilaton factor. But it is straightforward to

23 with $\tilde{Z}^I = (-i, -\frac{1}{2} i Z^A \eta_{AB} Z^B, Z^A)$
see that this dilaton dependence can be completely removed by choosing an alternative dilatation
gauge, i.e.

$$\dot{X} \eta \ddot{X} = 1$$

(5.23)

Nicely enough this last gauge choice directly leads to the heterotic effective action in the string
frame, because one may identify

$$\phi_{\text{inv}} = e^{-2\varphi}$$

(5.24)

where the expectation value of $e^{\varphi}$ equals the string coupling constant $g_s$.24 The $S$-supersymmetry
choice which keeps equation (5.23) invariant under $Q$-supersymmetry reads

$$\ddot{X} \eta_0 \Omega_i^I = 0.$$

(5.25)

As usual the $S$-supersymmetry gauge choice itself is not $Q$-supersymmetric invariant. As a first
step in the computation of the compensating $S$-transformation we note that

$$\delta \Omega_i^I = 2 \dot{\eta}_I \eta_i + \varepsilon_{i j} \sigma^{\mu \nu} e^I (\dot{F}_{\mu \nu} - \frac{1}{4} \dot{X} T_{\mu \nu} \epsilon_{k l}) + \dot{Y}_{i j} e^I \epsilon_{k l}$$

(5.26)

where by definition $\dot{Y}_{i j}^0 = Y_{i j}^0$ and $\dot{Y}_{i j}^A = Y_{i j}^A$ while $\dot{Y}_{i j}^1$ is fixed by the constraint

$$\ddot{X} \eta_1 \dot{Y}_{i j} = \frac{1}{2} \eta_{I J} \dot{\Omega}^{I J} \epsilon_{i k} \epsilon_{j l}.$$  
(5.27)

Using this result the compensating $S$-supersymmetry transformation can be determined to be

$$\eta_i(\epsilon) = -i \bar{H} \epsilon_i - \frac{1}{2} \epsilon_{i j} \sigma^{\mu \nu} e^I \dot{X} \eta_I F_{\mu \nu}^I - \frac{1}{4} \epsilon_{I J} \eta_I \dot{\Omega}^{I J} \epsilon_{i k} \epsilon_{j l}$$

$$+ \frac{1}{4} \gamma^j e^I \left\{ \eta_I \dot{\Omega}^{I J} (\dot{\Omega}_j^I - \delta_j^I \Omega_i^J + \delta^I j \Omega_j^I \Omega_i^J \right\}.$$  
(5.28)

In order to obtain the equations (5.26) and (5.28) the dependent expressions for $T_{\mu \nu}^+ (3.33)$ and
$A_\mu (5.20)$ were freely used. As we already discussed it is only thanks to the fact that we dualised
the $STU$ vector $W_\mu^I$ and the axion $a$ that we could make the dependent expressions for $T_{\mu \nu}^+ (3.33)$
and $A_\mu (5.20)$ — and hence also $\eta_i(\epsilon)$ — completely loop and hypermultiplet independent. Note that
$\eta_i(\epsilon)$ depends on the gauge fields, so the $\dot{F}_{\mu \nu}^I$ and $H_\mu$ dependence of various supersymmetry
transformation laws changes by going to the Poincaré theory.

Fixing the $U(1)$ gauge leads to the following expressions for the dependent scalars $X^I$, the
gauginos $\Omega_i^I$ and the compensating $U(1)$ transformation respectively

$$X^I = (\bar{Z} \eta \bar{Z})^{-\frac{1}{2}} \bar{Z}^I$$

$$\hat{\Omega}_i^I = (\bar{Z} \eta \bar{Z})^{-\frac{1}{2}} \left( \hat{\Lambda}_i^I - \bar{Z}^I \ddot{Z} \eta \ddot{Z} \hat{\Lambda}_i^I \right)$$

$$\Lambda_{U(1)}(\epsilon) = \frac{i}{2} \bar{Z} \eta_1 \bar{Z}^I \hat{\Lambda}_i^I + \text{h.c.},$$

(5.29)

where as usual the $\hat{\Lambda}_i^I$ are defined as the fermionic partners of the special coordinates $\bar{Z}^I$.

Now we concentrate on the hypermultiplet variables. Given the conditions (2.21), (5.23) and
(5.25) we have that

$$\frac{1}{2} A_\alpha^I A_I^\beta d_\alpha^\beta = -e^{-2\varphi}$$

$$A_\alpha^I \zeta_\beta d_\alpha^\beta = -e^{-2\varphi} \varphi_i.$$  
(5.30)

24Later on we will see that any one-loop correction to our theory will be weighted by a relative $e^{2\varphi}$ factor which
proves that the latter identification is indeed the correct one.
where the $SO(2, n)$ invariant dilatini $\vartheta_i$ are defined as

$$\vartheta_i \equiv -\frac{1}{2} e^{2\varphi} \lambda_i - \frac{1}{4} \tilde{\Omega}_i^j \tilde{Z} \tilde{Z} - \frac{1}{2} e^{2\varphi} \left\{ \tilde{Z}^j \left( F_{ij}^{(0)} - h.c. \right) - \frac{\hat{Z} \eta_j}{\tilde{Z} \eta \tilde{Z}} \left( \tilde{Z}^K F_{iK}^{(0)} (Z) - h.c. \right) \right\}. \quad (5.31)$$

The string-frame dilatation gauge choice is directly responsible for the dilaton-dilatini dependence at the right hand side of $(5.30)$. The $\varphi$ and $\vartheta_i$ dependence would have been moved from the hypermultiplet sector to the vector multiplet sector if we would have imposed the alternative Einstein frame dilatation gauge. In the present string frame we may split the hypervariables into a dilaton-dilatini part and the rest:

$$A_i^\alpha = e^{-\varphi} A_i'^\alpha \quad \zeta^\alpha = e^{-\varphi} \left( \zeta^\alpha - A_i'^\alpha \vartheta^i \right) \quad (5.32)$$

Fixing the SU(2) gauge then leads to

$$A_i'^\alpha = \sqrt{-\frac{2}{C(B)}} \delta^s_B B_s^\alpha \quad s = 1, 2 \quad C(B) = B_{s}^\alpha B_{s}^\beta d_{\alpha \beta}$$

$$\zeta^\mu = \sqrt{-\frac{2}{C(B)}} \left( \zeta^\alpha - B_{s}^\alpha 2 B_{s}^\beta d_{\alpha \beta} \xi^\beta \right) \quad \Lambda_i^j = 2\epsilon_i \partial^j + 4\epsilon_i \delta^j B_{s}^\alpha d_{\alpha \beta} \xi^\beta C(B)^{-1} - h.c.; \ \text{traceless} \quad (5.33)$$

where $B_s^\alpha$ and $\xi^\alpha$ are the physical hypervariables. Special coordinates on the quaternionic manifold are defined by splitting the index $\alpha$ into the values 1, 2 and the rest, and by putting

$$B_s^\alpha = 1, 2 \quad \delta^s_1, 2 \quad \delta^s_1, 2 = 0 \quad (5.34)$$

In that case

$$\sqrt{\frac{C(B)}{-2}} A_{i}^{1, 2} = e^{-\varphi} \delta_{i}^{1, 2} \quad \zeta^{1, 2} = -e^{-\varphi} \vartheta^{1, 2} - 2e^{-\varphi} \delta_{s}^{1, 2} B_{s}^\alpha d_{\alpha \beta} \xi^\beta C(B). \quad (5.35)$$

which indicates that after the Poincaré reduction to the string frame the compensating hypermultiplet $(A_{i}^{1, 2}, \zeta^{1, 2})$ describes the dilaton and dilatini degrees of freedom. This is the $N = 2$ analog of the $N = 1$ statement [23, 34] that in $N = 1$ $d = 4$ heterotic string effective actions the dilaton can be viewed as sitting in the real part of a compensating chiral + antichiral multiplet.

It remains to integrate out the auxiliary field $V_{\mu,i}$. When writing the second equation of $(2.22)$ in a manifestly $SO(2, n)$ invariant way, and taking into account the various constraints which have been imposed on the vector and hypermultiplet variables we get that

$$V_{\mu,i} = -\left( A_i'^\alpha \delta_{\mu} A_i'^j d_{\alpha \beta} - A_i'^j \delta_{\mu} A_i'^\alpha d_{\alpha \beta} \right)$$

$$-\frac{1}{2} \left( \eta_{ij} - e^{2\varphi} \text{Im} F_{ij}^{(0)} \right) \left( \tilde{\Omega}_{i}^j \gamma_{\mu} \tilde{\Omega}_{j}^i - \frac{1}{2} \delta_{\mu} \tilde{\Omega}_{i}^j \tilde{\Omega}_{j}^i \right) \quad (5.36)$$

Remark that together with the $Y_{i}^j$ equation of motion $(2.14)$ this equation is the only one —out of all the equations of motion or gauge choices we have imposed so far on the antisymmetric
tensor theory— which does involve the one-loop part of the theory. In (5.36) the following 
covariantised second derivative matrix appears

\[
F^{(0)\text{cov}}_{IJ} = F^{(0)}_{IJ} - \eta_{IJ} \frac{F^{(0)}_K \bar{X}^K}{X \eta \bar{X}} - 2 \frac{\bar{X} \eta (F^{(0)}_{JK} - h.c.) \bar{X}^K}{X \eta \bar{X}} + \bar{X}^K (F^{(0)}_{KL} - h.c.) \bar{X}^L \frac{\bar{X} \eta (I \bar{X} \eta) (F^{(0)}_K - h.c.)}{(X \eta \bar{X})^2} + 2 \frac{\bar{X} \eta (I \bar{X} \eta) (F^{(0)}_K - h.c.)}{(X \eta \bar{X})^2}.
\]

This (non-holomorphic!) function \( F^{(0)\text{cov}}_{IJ} \) is a natural object in terms of which many one-
loop properties of the heterotic effective action can be expressed.23 As can be verified by a 
straightforward but tedious computation \( F^{(0)\text{cov}}_{IJ} \) transforms as a tensor under the \( SO(2, n) \) 
transformations (apart from a non-trivial monodromy transformation):

\[
F^{(0)\text{cov}}_{IJ} \xrightarrow{SO(2,n)} (F^{(0)\text{cov}}_{KL} + \Lambda_{KL}) (\mathcal{U}^{-1})^K_I (\mathcal{U}^{-1})^L_J - \eta_{IJ} \frac{\bar{X} \Lambda \bar{X}}{X \eta \bar{X}}.
\]

At this point we have finished the construction of the Poincaré version of the antisymmetric 
version of the heterotic effective action and we may now present the supersymmetry transformation 
rules for all the fields, as well as the (bosonic part) of the action. The supersymmetry 
transformation rules read

\[
\delta e_\mu^a = \dot{e}^i \gamma^a \psi_{\mu i} + h.c.
\]

\[
\delta \psi_{\mu i} = 2 \dot{\psi}_{\mu i} \dot{e}^i + \gamma_{\mu j} \rho_{ij} \bar{X} \eta \tilde{\delta}_{\mu} \bar{X}^I \dot{e}^i - i \sigma_{\mu \nu} \tilde{F}_{\nu} \dot{e}^i + \dot{\gamma}^{ij} \gamma^a \epsilon_j \bar{X} \eta \tilde{F}_{\mu} \dot{e}^I + \ldots
\]

\[
\delta \tilde{B}_{\mu \nu} = - \left( 2 \dot{e}^i \gamma_{\mu \nu} \psi_{ij} + \eta_{IJ} \tilde{W}_\mu^I \left( \epsilon_{ij} \Omega_j^I + 2 \tilde{X} \hat{e} \psi_{ij} \right) \dot{e}^i + h.c. \right) + 2 \delta_{\mu \nu} \tilde{W}_\mu^I
\]

\[
\delta \tilde{Z}^I = \dot{e}^i \tilde{A}_I^I
\]

\[
\delta \tilde{W}^I_\mu = \left( \epsilon_{ij} \Omega_j^I \dot{e}^i + 2 \tilde{X} \hat{e} \psi_{ij} + h.c. \right) + \partial_\mu \tilde{Z}^I
\]

\[
\delta \tilde{\Omega}_j^I = 2 \left( \partial_\mu \bar{X}^I - X^I \bar{X} \eta \tilde{\delta}_{\mu} \bar{X}^J \right) \epsilon_i + \epsilon_{ij} \sigma_{\mu \nu} \epsilon^j \left( \tilde{F}_{\mu \nu}^I - 2 \Re (X^I \bar{X} \eta) \tilde{F}^I_{\mu \nu} \right) + \tilde{Y}_j^I \dot{e}^i + \ldots
\]

\[
\delta \varphi = \dot{e}^i \varphi_i + h.c.
\]

\[
\delta \varphi = \dot{\varphi} \epsilon_i + 2 \gamma_{\mu \nu} \gamma_{\mu \nu} \tilde{F}_{\mu} \dot{e}^i + \ldots
\]

\[
\delta B_{\alpha \gamma} = \left( 2 \dot{\zeta}^\alpha \epsilon_i + 2 \dot{\gamma}^{\alpha \beta} \epsilon_{ij} \tilde{\xi}_{ij} \dot{e}^i \right) \delta_{\gamma}^s
\]

\[
\delta \zeta_{\alpha} = \left( \dot{\zeta}^\alpha \epsilon_i + A^I_{\alpha} \dot{A}_{\mu}^I \gamma^\beta \delta_{\beta \gamma} + \frac{1}{2} \epsilon_{ij} \sigma_{\mu \nu} \epsilon^j \bar{X} \eta \tilde{F}_{\mu} \dot{e}^i + \ldots
\]

Let us discuss a few aspects of the equation (5.39). Almost all transformation laws are completely 
saturated at the string tree level. The only exceptions are \( \delta \psi_{\mu i}^I \) and \( \delta \hat{\varphi}_i \), because these 
contain \( \gamma_{\mu j} \) and \( \tilde{Y}_j^I \) terms. As we already said the latter generate an \( F^{(0)} \) dependence which 
is of higher order in the fermions. The \( \ldots \) stand for other higher-order fermion terms which 
are not affected by string loop effects. These \( \ldots \) terms are not particularly interesting, and

\[25\] The following identities are handy in explicit computations:

\[
X^{I} F^{(0)\text{cov}}_{IJ} \bar{X}^J = F^{(0)} - \bar{X} \eta \frac{F^{(0)}_K \bar{X}^K}{X \eta \bar{X}}
\]

\[
\bar{X}^I F^{(0)\text{cov}}_{IJ} \bar{X}^J = F^{(0)}
\]

\[
\bar{X}^{\dot{I}} F^{(0)\text{cov}}_{IJ} \bar{X}^J = 0.
\]
in any case they can be reconstructed on the base of the results we presented before. It is particularly interesting to see that the antisymmetric tensor $\tilde{B}_{\mu\nu}$ is still completely decoupled from the dilaton-part of the theory. Rather then transforming into $\vartheta_i$ the antisymmetric tensor transforms into the gravitinos $\psi_{\mu}^i$, so in the string frame $\tilde{B}_{\mu\nu}$ has become part of the gravitational multiplet. The gravitinos themselves transform back into the antisymmetric tensor, thanks to the compensating $\eta_i(\epsilon)$ transformation. The $\delta Q_i^i$ are $H_\mu$ independent because there is a cancellation between two contributions coming from the $Q$- and the $S$-supersymmetry sectors respectively. The variation of the dilatini $\vartheta_i$ is most easily obtained by varying the left hand side of the second line of (5.30), although it can also be computed directly from (5.11) and (5.31). Note that the complete $V_{\mu j}^i$ and $\eta_i(\epsilon)$ dependence of $\delta \zeta^\alpha$ feeds into $\delta \vartheta_i$ and not into $\delta \zeta_i$. As a result the latter is $H_\mu$, $F_{\mu\nu}^I$ and loop-independent as expected.

The antisymmetric tensor lagrangian reads

$$e^{-1}L_{\text{tensor}} = e^{-2\varphi}\left\{\frac{1}{2}R + 2\partial^\mu \varphi \partial_\mu \varphi + \frac{1}{4}H^\mu H_\mu + G_{IJ}^i \partial^\mu \tilde{Z}^I \partial_\mu \tilde{Z}^J + \Delta_{\alpha}^\beta \partial^\mu B^{\alpha}_s \partial_\mu B^{\beta}_s \right\}$$

$$+ e^{-2\varphi}\left\{\frac{1}{8}N_{IJ}^i \tilde{F}_{\mu\nu}^{i+I} \tilde{F}^{\mu\nu+J} + h.c.\right\}$$

$$- \frac{1}{2}H^\mu \partial_\mu \tilde{Z}^I \frac{F_{IJ}^{(0)\text{cov}} \tilde{Z}^J}{\tilde{Z} \eta \tilde{Z}} + \frac{1}{4}H^\mu \partial_\mu M^{(0)} + h.c. \right\}$$

(5.40)

with

$$G_{IJ} = \frac{\eta_{IJ}}{\tilde{Z} \eta \tilde{Z}} - \frac{\tilde{Z} \eta \tilde{Z}}{(\tilde{Z} \eta \tilde{Z})^2} + i \frac{F_{IJ}^{(0)\text{cov}}}{\tilde{Z} \eta \tilde{Z}} - h.c. \right\}$$

$$\Delta_{\alpha}^\beta = \frac{2}{C(B)}d_{\alpha}^\beta - \frac{4}{C(B)^2}(B^{\gamma R}d^\gamma_{\alpha})(B^{\delta R}d^\delta_{\beta})$$

$$N_{IJ} = \left(\eta_{IJ} - 4 \frac{\tilde{Z} \eta \tilde{Z}}{\tilde{Z} \eta \tilde{Z}}\right) - i \left(\tilde{F}_{IJ}^{(0)\text{cov}} + M^{(0)} \eta_{IJ}\right) \right\}$$

(5.41)

This lagrangian is manifestly $SO(2, n)$ invariant except for the $N_{IJ}$ term which generates a shift in the $\theta$-angles according to (5.4). The dilaton almost completely decouples from the other fields except from the familiar $e^{-2\varphi}$ prefactor appearing at the string tree level. The antisymmetric tensor field also largely decouples from the rest, although it starts to interact with the vector multiplet moduli $\tilde{Z}^I$ at one loop. Of course these facts were already known from string theory, but here we see that we can reproduce them on the base of $N = 2$ supersymmetry only. We want to draw attention to the fact that the dilaton kinetic term (with the characteristic $+2$ prefactor) is entirely generated by the compensating hypermultiplet contribution to the $-D_\mu A_i^\alpha D^\mu A^i_\beta d_{\alpha}^\beta$ term in the original superconformal action. Had we imposed the Einstein frame dilatation gauge, then the dilaton dynamics (with a -1 prefactor) would have come from the compensating vector multiplet part of $N_{IJ} \tilde{D}_\mu X^I D^\mu \tilde{X}^J$.

In this article we have chosen a string frame dilatation gauge because then the metric $g_{\mu\nu}$ automatically coincides with the string metric. Given the string frame results (5.33) and (5.40) one can of course immediately read off what the Einstein-frame theory would look like. It suffices to perform an interpolating dilatation and $S$-supersymmetry transformation on the various fields, which implies that

$$e_{\mu}^a = e^{\varphi} e_{\mu}^{a(E)}$$

$$\psi^i_{\mu} = e^{\frac{\varphi}{2}} \left(\psi^i_{\mu}^{(E)} + \gamma_{\mu}^{(E)} \dot{q}^i(E)\right)$$

$$\tilde{\Lambda}^I_i = e^{-\frac{\varphi}{2}} \tilde{\Lambda}_i^{I(E)}$$

$$\partial_{\dot{i}} = e^{-\frac{\varphi}{2}} \partial_{\dot{i}}^{(E)}$$

$$\xi^\alpha = e^{-\frac{\varphi}{2}} \xi^\alpha(E)$$

$$\epsilon_{\dot{i}} = e^{\frac{\varphi}{2}} \epsilon_{\dot{i}}^{(E)}$$

(5.42)
while $\bar{B}_{\mu \nu}, \bar{Z}^I, W^I_{\mu}, \varphi, B_\alpha^s$ are left invariant. In this way one finds that

$$
\delta \bar{B}_{\mu \nu} = -e^{2\varphi} \left( 4\bar{i} \sigma_{\mu \nu} \partial_i - 2\bar{i} \gamma_{[\mu} \psi_{\nu]} \right)^{(E)} - \eta_{I J} \bar{W}^I_{[\mu} \left( \bar{\epsilon}_i \gamma_{\nu]} \bar{\Omega}^J_{ij} + 2 \bar{X}^I \bar{\epsilon}_i \bar{\psi}^J_{ij} \right)^{(E)} \bar{e}^{ij} + \text{h.c.}
$$

$$
+ 2\partial_{[\mu} \bar{A}_{\nu]} + \eta_{I J} \bar{\theta}^I \partial_{[\mu} \bar{W}^J_{\nu]} \tag{5.43}
$$

where

$$
\bar{X}^{I(E)} = e^{\varphi} \left( \bar{Z} \eta \bar{Z} \right)^{-\frac{1}{2}} \bar{Z}^I
$$

$$
\bar{\Omega}^{I(E)} = e^{\varphi} \left( \bar{Z} \eta \bar{Z} \right)^{-\frac{1}{2}} \left( \bar{A}^{I(E)} - \bar{Z} \eta \bar{\Lambda}^{I(E)} \eta \bar{Z} + 2 \bar{Z}^I \partial_{i}^{(E)} \right) \tag{5.44}
$$

So in the Einstein-frame the antisymmetric tensor field finally does transform into the dilatini, and one ultimately recovers what might be called the (on-shell) heterotic vector-tensor multiplet. Of course one is really looking at is a spurious dilatino dependence which has to correct for the fact that one is not working with the true string gravitinos.

### 6 Conclusions

In this paper we presented the antisymmetric tensor version of the low-energy effective action for perturbative heterotic strings on $K_3 \times T^2$. In fact we took a slightly broader point of view and showed that the complete set of antisymmetric tensor theories is quite restricted, and that in the dual vector multiplet picture they all lead to a theory characterised by one of the standard prepotentials given in [12]. Contact between the vector multiplet and the antisymmetric tensor multiplet pictures can be made by performing the change of variables [6], which immediately leads to the appearance of (central charge-like) shift symmetries. Out of the general class of antisymmetric tensor theories the one relevant for heterotic strings is then singled out as the only one for which these shift symmetries appear to be inessential. In fact one can obtain a completely conventional gauge structure for the heterotic effective action by using the so-called stringy vector fields instead of the usual $STU$ vectors. As such the stringy vectors play a double rôle because they were already known to make the $SO(2, n)$ invariance manifest at a lagrangian level.

Another remarkable feature of the heterotic antisymmetric tensor theory is that it seems to resist an off-shell description. Within the superconformal setup which we used, this on-shell character comes about when one is forced to eliminate the auxiliary fields $T_{\mu \nu}^{ij}$ and $A_\mu$ contained in the Weyl multiplet. On the other hand we have seen that the dependent expressions for $T_{\mu \nu}^{ij}$ and $A_\mu$ we obtain, are relatively simple and in particular do not depend on the one-loop part of the heterotic theory. We want to emphasize that the loop-independent expressions for the auxiliary fields only arise when one uses the stringy vectors $\bar{W}^I_{\mu}$ and the antisymmetric tensor field $\bar{B}_{\mu \nu}$ as the fundamental variables. This at the same time explains why the final Poincaré supersymmetric effective action is considerably simplified by going to the $\bar{W}^I_{\mu}$ and $\bar{B}_{\mu \nu}$ formulation.

Several times in this article we found examples where $N = 2$ supersymmetry considerations are sufficient to unique select the most natural variables for the heterotic effective action. The heterotic dilaton $\phi_{\text{inv}}$ for instance arises as the only $SO(2, n)$ invariant generalisation of $\phi = \Re S$ which makes the Einstein term in the superconformal theory loop independent. Choosing a string frame formulation for the Poincaré theory also has a very clear supergravity interpretation: it corresponds to imposing a dilatation gauge which makes the stringy scalars $X^I$ dilaton independent. As such one forces the dilaton and dilatini to sit in a compensating hypermultiplet, which —after three bosonic degrees of freedom have been removed by an $SU(2)$ gauge choice—
indeed describes one bosonic and 8 fermionic degrees of freedom. When using the natural hetero-
erotic variables we discussed before one gets a final theory in which 1) the $SO(2, n)$ symmetry  
is manifestly realised, 2) the supersymmetry transformation laws are loop independent apart  
from some higher order fermionic corrections and 3) the specific couplings of the dilaton  
and the antisymmetric tensor (as we know them from string theory) are easily reproduced.

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