Abstract. Multi-attributed relational structures (MARSs) have been proposed as a formal data model for generalized property graphs, along with multi-attributed rule-based predicate logic (MARPL) as a useful rule-based logic in which to write inference rules over property graphs. Wikidata can be modelled in an extended MARS that adds the (imprecise) datatypes of Wikidata. The rules of inference for the Wikidata ontology can be modelled as a MARPL ontology, with extensions to handle the Wikidata datatypes and functions over these datatypes. Because many Wikidata qualifiers should participate in most inference rules in Wikidata a method of implicitly handling qualifier values on a per-qualifier basis is needed to make this modelling useful. The meaning of Wikidata is then the extended MARS that is the closure of running these rules on the Wikidata data model. Wikidata constraints can be modelled as multi-attributed predicate logic (MAPL) formulae, again extended with datatypes, that are evaluated over this extended MARS. The result models Wikidata in a way that fixes several of its major problems.

1 Introduction

Wikidata is already an extremely useful source of information about notable entities in the world, containing high-quality information about over 87 million items. However, Wikidata must be used with care as it does not present a cohesive view of the world. To pick a simple example, Wikidata has a property for \texttt{spouse} that Wikidata also states that \texttt{spouse} is its own inverse and has symmetric constraints among its property constraints. Either of these alone should be adequate to allow users of Wikidata to assume that to find the spouse of someone it suffices to query that someone’s value for \texttt{spouse}. But this is not necessarily so—as of mid-June 2020 there were over thirty-eight hundred non-symmetric spousal relationships in Wikidata, for items ranging from Roman gods to politicians. So any attempt to accurately determine the spouse of someone in Wikidata needs to perform two queries. Making this fix for \texttt{spouse} is relatively easy, but in general it is impossible for an application to automatically determine what to do to access all the information that it needs that is inherent in Wikidata.

\footnote{This is an extended version of a paper that appears in the 2020 Description Logic Workshop.}

\footnote{For readability we use the English label (with underscores) to identify Wikidata items, here \url{https://www.wikidata.org/wiki/Property:P26}, even though that might not uniquely identify an item.}

\footnote{The number of violations is helpfully provided on a database report page that collects symmetry violations for \texttt{spouse}.}
Wikidata has a large, but simple, ontology, containing classes such as human and person and a subclass of relationship from human to person. Retrieving all the people in Wikidata should thus be a simple matter of finding all the items that have an instance of link to person. But as of mid-June 2020 there are only about 30 items that have an instance of link to person—far fewer than the over 7.9 million humans in Wikidata. To find all instances of person in Wikidata requires asking for instances of the transitive closure of subclass of under person. Again, it is not hard to remedy this particular problem, and it is so important that most accessors of Wikidata make the extra effort to look through subclasses. On the other hand, based on our personal experiences the analogous workaround for subproperties is often not done.

As yet another illustrative example, what if an application wants to know what information is current, i.e., true at the present time. Wikidata has quite a number of constructs that specify temporal extents for pieces of information, including end time. How can the application know what it needs to do to filter out statements that are not valid at the current time?

Providing an interface to Wikidata that presented all and only the valid information would make it much easier to write applications that just want to know what’s true based on what is in Wikidata, as opposed to how the information is said or who said it.

Maybe this effort is doomed to be fruitless. Maybe Wikidata is just data and there is no way to provide a decent logical view of the data in Wikidata. This would indeed be a shame, as Wikidata is the best example of an open Knowledge Graph, and so it should be possible to treat the Wikidata data logically.

The need for well-founded reasoning capabilities in Wikidata has been noted and lightly documented in a WikiProject\(^5\), but to date this project has not produced any concrete results. The current work may be viewed as a proposal to satisfy a superset of the requirements documented there.

### 1.1 Areas To Be Addressed

Wikidata is data, but many portions of Wikidata that are vital to Wikidata’s utility and success are conceived, discussed, and documented as if Wikidata is more than just data, as illustrated by spouse and instance of above. To reliably provide construction of the meanings implicit in the descriptions of even the central properties of Wikidata requires a formal theory for the properties and an implementation of this theory. Without such, different users and tool builders end up determining and implementing the meaning on their own, and this often produces different, non-compatible results.

One way of providing a common meaning is via a set of axioms in a logic and a service that implements reasoning in the logic, either in general or for the specific axioms. The service then provides a common resource that serves up the meaning of the various pieces of Wikidata. This is particularly vital for the central, ontological portion of Wikidata, including instance of as well as symmetric property.

Other parts of Wikidata are also currently under-specified and differentially implemented and can be improved by a logical formulation. Wikidata constraint\(^6\) express regularities that are supposed to hold in general, but which may have exceptions. These

\(^5\) https://www.wikidata.org/wiki/Wikidata:WikiProject_Reasoning
\(^6\) https://www.wikidata.org/wiki/Help:Property_constraints_portal
constraints are not used to draw inferences but instead to point out potential problems to interested contributors who can then either fix the problem or indicate that the particular anomaly is acceptable.

Wikidata constraints, albeit useful, are represented and processed in an incomplete, ad hoc fashion. Although in most cases they are declared and documented reasonably clearly, the declarations do not fully express their meaning, and thus do not provide a precise, unambiguous basis, or a logical foundation, for constraint-checking implementations. Further, building a constraint checker for a new constraint may be laborious, idiosyncratic, and error-prone. A logical formulation and implementation of constraints would permit constraints to be quickly specified and eliminate the implementation burden for each new type of constraint.

As noted in [9], Wikidata’s custom data model supports attributed statements (with the attributes referred to as qualifiers), and allows attributes with multiple values. [9] refers to this sort of generalized Property Graph model as a multi-attributed graph, and observes that “In spite of the huge practical significance of these data models ..., there is practically no support for using such data in knowledge representation.” This creates not only the challenge of constructing a logic in which qualifiers are first-class citizens, but also the challenge of creating a reasoning framework in which the behavior of qualifiers in axioms can be specified in a practical fashion.

Wikidata has many classes and properties, such as female, human, that are missing many of their expected instances or relationships. Axioms that give a logical formulation for these classes and properties would add back their missing elements and eliminate the incorrect information that currently comes from accessing them in Wikidata.

Wikidata has several ways of providing something other than a particular value for a property. The some-value snak represents some “unknown” value and the no-value snak says that there is no value. However the precise meaning of both of these is unclear, and a logical formulation for them would provide a better meaning for these parts of Wikidata.

1.2 Contributions

We provide a logical foundation for Wikidata, building on previous work on multi-attributed predicate logic (MAPL), including multi-attributed rule-based predicate logic (MARPL) and multi-attributed relational structures (MARSs) [9]. Our work 1) supports the explicit expression of Wikidata’s ontological axioms, and their use in accessing Wikidata; 2) supports the explicit expression of other axioms that can benefit other areas of Wikidata; 3) supports the specification of qualifiers as first-class citizens in axioms and constraints, and provides a means for effectively handling them; and 4) supports the expression of nearly all current Wikidata property constraints, plus a variety of other constraints and the identification of constraint violations, all in one logical framework.

2 Development

Wikidata’s custom data model goes beyond the Property Graph data model, which associates sets of attribute-value pairs with the nodes and edges of a directed graph, by
allowing for attributes with multiple values. Marx et al. refer to such generalized Property Graphs as *multi-attributed graphs*, and observe that “In spite of the huge practical significance of these data models ..., there is practically no support for using such data in knowledge representation”. Given that motivation, Marx et al. introduce the *multi-attributed relational structure* (MARS) to provide a formal data model for generalized Property Graphs, and *multi-attributed predicate logic* (MAPL) for modeling knowledge over such structures. MARS and MAPL may be viewed as extensions of FOL to support the use of attributes (with multiple values).

The essential new elements over FOL are these:

- a *set term* is either a set variable or a set of attribute-value pairs \( \{ a_1 : b_1, ..., a_n : b_n \} \), where \( a_i, b_i \) can be *object terms*. Object terms are the usual basic terms of FOL, and can be either constants or *object variables*.
- a *relational atom* is an expression \( p(t_1, ... t_n) \)\( \subseteq S \), where \( p \) is an n-ary predicate, \( t_1, ... t_n \) are object terms and \( S \) is a set term.
- a *set atom* is an expression \( (a : b) \in S \), where \( a, b \) are object terms and \( S \) a set term.

These elements are best illustrated with a simple example (taken directly from [9]):

\[
\forall x, y, z_1, z_2, z_3. \text{spouse}(x, y) \subseteq \{ \text{start} : z_1, \text{loc} : z_2, \text{end} : z_3 \} \\
\rightarrow \text{spouse}(y, x) \subseteq \{ \text{start} : z_1, \text{loc} : z_2, \text{end} : z_3 \}
\]

This MAPL formula states that spouse is a symmetric relation, where the inverse statement has the same start and end dates, and location. The entire second line of the formula is a relational atom, which includes the set term \( \{ \text{start} : z_1, \text{loc} : z_2, \text{end} : z_3 \} \). If that set term were represented by a set variable \( U \), then one could make an assertion about its membership using the set atom \( (\text{start} : z_1) \in U \).

In Wikidata terms, this particular relational atom (once \( x \) and \( y \) have been instantiated to specific Wikidata items) corresponds to a statement, and each attribute-value pair (once the \( z_i \) variable has been instantiated to a specific value), corresponds to a qualifier of the statement. \( x \), of course, is called the subject of the statement, and \( y \) the value or object of the statement. While MAPL allows for predicates of arbitrary arity, in Wikidata all statements are triples; i.e. Wikidata properties have arity 2.

Marx et al. go on to introduce multi-attributed rule-based predicate logic (MARPL), roughly the Horn-clause fragment of MARS plus functions that compute the attributes of atoms in the consequent of rules. MARPL is decidable for fact entailment, but still provides a high level of expressivity. Note that Formula 1 falls within the MARPL fragment.

2.1 eMARS

MARPL is close to what is needed as a basis for Wikidata but MARPL is missing datatypes and each rule in MARPL has to specifically account for attributes (essentially the qualifiers of Wikidata). Datatypes play a large role in Wikidata and handling Wikidata qualifiers correctly requires accounting for many qualifiers in each of many rules, which is infeasible from a practical perspective with MARPL. Using an extension of
MARPL means that we have the chance of being able to relatively easily compute the consequences of Wikidata. We also use MAPL formulae in constraints, so we also need to extend MAPL.

As MARPL and MAPL are based on MARSs we start by adding datatypes to MARSs. As Wikidata, like RDF [2], has a single domain for everything, including predicates, we will be extending MARS in this direction, but in a way that does not permit some of the strange situations possible in RDF. Because of space limitations we write our definitions in a somewhat informal manner, but the details can be filled out as is usual.

**Definition 1.** A datatype theory, $D$, consists of a finite set of named datatypes, $D$, each of which has a finite or infinite set of data values; a finite set of named and typed datatype relations, $R$, over $D$; and a finite set of named and typed datatype functions, $F$, over $D$. The relations are closed under negation.

So datatype theory for the rationals and integers would have as data values all the rational numbers (with the integers as a subset). The datatype functions and relations could include the comparison relations (both within each datatype and between the two datatypes) and arithmetic functions.

**Definition 2.** An extended MARS (eMARS), $M$, is a MARS extended with a datatype theory, $D$. All datatypes, datatype relations, and datatype functions of $D$ as well as all predicates are distinct elements of the domain of $M$, $\delta^M$. All data values in $D$ are also elements of $\delta^M$. Each datatype is a unary predicate of $M$ which is true on the data values of the datatype.

The domain elements for datatypes, datatype relations, datatype functions, and predicates are all distinct, thus eliminating several unusual situations that can occur in RDF and can be forced in extensions of RDF.

**Wikidata datatypes.** The defined datatypes of Wikidata[7] are IriValue; StringValue; MonolingualTextValue, strings with language tags; MultilingualTextValue, strings in multiple languages; QuantityValue, with an associated unit; GeoCoordinatesValue, in some coordinate system; and TimeValue, including a timezone. The last three of these are unusual in that they have imprecise values, containing a main value and some notion of precision (not necessarily symmetric), indicating an interval or range of possible values.

In our datatype theory for Wikidata all datatypes have equality and inequality relations that take into account all aspects of the data values. The precise datatypes have the usual set of relations and functions that extract their pieces. IriValue has no extra relations or functions. StringValue has as well the usual lexicographic relations and pattern matching relations. The relations for StringValue are extended to MonolingualTextValue (the languages have to be the same for the relation to hold). There are functions to extract the string and language tag from MonolingualTextValue. The relations for StringValue are extended to MonolingualTextValue by requiring that the strings for each languages be in the relation. There is a function to extract the string for a language

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[7] Most information shown here about the Wikidata data model is taken from [https://www.mediawiki.org/wiki/Wikibase/DataModel](https://www.mediawiki.org/wiki/Wikibase/DataModel).
tag, which returns the empty string if there is no string with that language tag in the data value.

Imprecise datatypes have as well equality and inequality for their main values, a predicate for whether a value is precise, and overlaps and disjoint and emptiness. They also have intersection functions that result in a smallest (in imprecision) value that includes the intersection of the two values, if the two values intersect, otherwise an empty value (not currently in Wikidata) results; and a kind of union function that produces a smallest (again in imprecision) value that includes both the values. (Some imprecise Wikidata datatypes have multiple values that are equivalent. Some imprecise Wikidata datatypes are not closed under intersection.)

QuantityValue has ordering relations of three flavours, one for the main value, e.g., the main value of the first argument is less than that of the second; one for necessary ordering, e.g., all possible values of first argument are less than those of the second; and one for possible ordering, e.g., some possible values of first argument is less than one of the second. GeoCoordinatesValue has various direction relations, such as north_of, must_be_north_of and can_be_north_of, the latter two taking into account imprecision.

TimeValue has ordering relations, as for QuantityValue, although the base names are before and after. There are also functions to return the part of one time that must (or can) be before (or after) the second. As well, there is a first function whose returned TimeValue has a main time, first possible time, and last possible time that are the earlier ones from either of its two (TimeValue) arguments, and an analogous last function.

This is just an initial set of relations and functions, and likely needs expansion. There are currently no construction functions, for example, to construct a MultilingualTextValue from several MonolingualTextValues, pending an analysis of which construction functions do not cause computational difficulties.

2.2 eMAPL

Extended MAPL (eMAPL) is then MAPL based on eMARS. Datatype predicates and functions are allowed in eMAPL, of course. To support the representation of constraints, we add equality and, as syntactic sugar, counting quantifiers.

Definition 3. eMAPL terms are MAPL terms with the addition of datatype function applications. eMAPL formulae are MAPL formulae using eMAPL terms plus the addition of datatype relations as predicates, an equality predicate, and counting quantifiers. Atoms with datatype relations cannot have non-empty attribute sets.

Definition 4. An eMAPL interpretation is a MAPL interpretation with the following additions and modifications:

- An eMAPL interpretation is into an eMARS.
- The interpretations of datatypes, predicate and datatype relations, and functions are themselves.
- The interpretation of a datatype function term is the application of the (fixed) function to its arguments.
- The interpretation of datatype relations use the (fixed) datatype relation itself.
– The interpretation of the equality predicate is the identity relation.
– Counting quantifiers have the obvious interpretation.

2.3 eMAPL rules

Because we are interested in effective reasoning, we restrict axioms to rules. We make the usual additions and allowances for datatype predicates and functions.

**Definition 5.** An eMAPL rule is a MAPL rule modified as in eMAPL. (Note that the functions in MAPL rules are different from datatype functions.) A variable is relational if it occurs in the rule body as the argument of a predicate that is not a datatype relation (including the datatype equality relations). Terms in rule bodies can use datatype functions, but the variables in these terms must be relational. Atoms in rules can use datatype relations but variables in these atoms must be relational.

Remember that in MARPL ontologies attributes are processed using rules that take attributes and their values from instantiations of body atoms and determine how they augment the head instantiation that is the consequent of the rule. We want to be able to handle attributes (Wikidata qualifiers) in a uniform manner so that each rule does not need to say, for example, how the Wikidata temporal qualifiers are processed.

We do this by characterizing each Wikidata qualifier, providing instructions on how it is to be processed in each rule that does not say anything about how the attribute’s values are added to the consequences of the rule.

**Definition 6.** An attribute is characterized in one of several ways.

– No values for the attribute are to be added to the consequents of rules. We expect that most qualifiers will fit into this ignore characterization, so it is the default.
– The values of the attribute in the facts matched by body atoms are each added to the consequents of rules, in an additive fashion.
– The consequents of rules are given a single value for the attribute, formed by combining all the values for the attribute in matched body atoms by a datatype function that maps pairs of values in one datatype into a value in the same datatype. If the result fails to satisfy a unary predicate then the rule produces no result.
– The most sophisticated characterization involves starting by combining values for several attributes as above and then blending the resultant values using a different function, provided that the resultant values satisfy a datatype relation.

Combination is only suitable for an attribute whose value has the datatype of the function. The datatype function should be commutative and associative, but other functions could be used if there is only a single value for the attribute, with the resultant value being the left reduction of the values taken in the order of occurrence in the body of the rule.

**Wikidata qualifiers.** Wikidata has the qualifier point in time, which is used to state when some statements are valid. Combining statements with this qualifier should take into account whether the time intervals overlap, so this qualifier would be characterized as a combining qualifier with the intersection function and the emptiness predicate.
Other Wikidata statements use the qualifiers start time and end time to indicate an extended period when the statement is valid. To produce the value for start time when making inferences from statements with these qualifiers requires first finding the last possible start time and the first possible end time and then taking the part of the combined start times that can be before the combined end times.

So the Wikidata start qualifier could be characterized as a blending qualifier by first combining start and end qualifiers using the last and first functions, respectively, giving start and end times for the result, and then using the could be before function to cut out start times that cannot be before the end time. If there is no start time before the end time (not could be before is true between the combined start and end times), then the inference produces no result.

**Definition 7.** An eMARPL ontology consists of a finite set of eMAPL rules, a finite set of function definitions, and a finite set of attribute characterizations as above. The function definitions are as in MARPL ontologies except that they can have datatype function terms in their consequences (i.e., in the consequences of their conditionals).

These attribute characterizations are used as macros, modifying the functions and rules of an eMARPL ontology.

**Definition 8.** The expansion of an eMARPL ontology uses the attribute characterization to modify its rules as follows:

- First give each rule its own function by making copies of functions or creating a new function for the rule. Ensure that each atom in the rule body has its own set variable, adjusting the function as needed to access this set variable.

- For each attribute whose characterization is addition, for each rule where the function does not mention the attribute in its consequences, augment the function to copy over all the attribute's values.

- For each attribute whose characterization is combine, for each rule where the function does not mention the attribute in its consequences, augment the function to add a single attribute value that is the combination of all the values in the body atoms. Also add a new body atom to check whether the combined value satisfies the combination predicate. This will require adding multiple clauses to the function as well as splitting the rule to take into account the presence or absence of the attribute in body atoms.

- For each attribute whose characterization is blend, for each rule where the function does not mention the attribute in its consequences, augment the function to compute the combination values and add the blend result. Also add a new body atom to check whether the blended value satisfies the blend predicate. This will cause similar but more pronounced increase in the number of clauses than combination attributes.

The meaning of an expanded eMARPL ontology on an eMARS is just the inferential closure of the rules on the eMARS.

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[8] In actuality, all three of these qualifiers should take into account all of them, so a better characterization would have a more complex blending specification.
2.4 Complexity

An expansion can be exponentially larger than the original ontology in the number of attributes with combination or blending characterizations. Implementations would not actually construct the expansion, instead gathering up values internally and applying the functions and predicates only to the existing values. So the size of the expansion is not a real complexity problem, by itself.

Datatype functions and relations are limited so that rules cannot add new datatype values to predicate extensions, eliminating this particular cause of intractability or undecidability in rules.

Even so, reasoning in MARPL, let alone eMARPL, is intractable because the number of attribute values can grow large. We view this as unavoidable, but we do not expect this worst-case intractability to be much of a problem in practice.

3 Wikidata on eMARS

Now that eMARPL ontologies have been defined we can show how we determine the meaning of Wikidata as an eMARS. Following that, we will show how the eMARS can be used to check Wikidata’s constraints.

Objects in Wikidata are items, which include predicates (properties). Facts in Wikidata are statements, consisting of a subject (an item) and a main snak. Snaks are predicate-object pairs, or some-value snaks, or no-value snaks. Statements also have associated qualifiers, which are also snaks. Statements have a rank, which is regular, preferred, or deprecated. Wikidata also provides optional typing information for the values of properties. We also use a characterization for each property used in a qualifier in Wikidata; a set of ontological rules for Wikidata; and a set of constraints.

First we turn Wikidata itself into an eMARS. The domain elements of the eMARS are all the items in Wikidata, with additions as below. The predicate extensions in the eMARS are pairs of subjects and objects from statements with the predicate as property. Statements whose main snak is a some-value snak are modelled by adding a fresh element to the domain. (This treats a some-value snak as stating that its value is a distinct domain element, which might not be quite the best treatment, but avoids computational problems having to do with determining which regular domain element the value is equal to.) Each qualifier snak becomes an attribute of the statement, in the obvious manner. For statement ranks we add a special rank attribute with value preferred or normal. We ignore reference records for now.

Statements whose main snak is a no-value snak are ignored here, and left for future work, because the meaning of no-value snaks is uncertain in Wikidata. We think the best treatment of a no-value snak is as a constraint but it is unclear whether a no-value snak means no value at all, no value with the same qualifiers, or something in between. These options can be modelled as eMAPL constraint formulae.

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9 One way to handle reference records is to add new objects to the domain for reference records and make these objects be values of a special qualifier, such as reference_record.
Property typing is modelled as rules requiring that the values of the property belong to the datatype. So the datatype for date\(_\text{of\,}\text{birth}\) is modelled by the rule:

\[
date\text{of\,}birth(s, o) \rightarrow \text{Point\,in\,time}(o)
\] (2)

### 3.1 Wikidata Ontology Rules

Some parts of formalizing the Wikidata ontology have already been described above as examples of quantifier characterizations. Arguably, the most important part of this formalization, though, is formalizing the ontological rules of Wikidata.

The ontology problems with Wikidata can be solved by treating Wikidata as an eMARPL ontology, transforming Wikidata into an eMARS and adding a few ontology rules, such as those shown in (3). Each atom on the right-hand side of these rules is implicitly associated with an attribute set that is constructed by a rule function.

\[
\begin{align*}
\text{subclass}\text{of}(c, d) \land \text{subclass}\text{of}(d, e) & \rightarrow \text{subclass}\text{of}(c, e) \\
\text{instance}\text{of}(y, c) \land \text{subclass}\text{of}(c, d) & \rightarrow \text{instance}\text{of}(y, d) \\
\text{subproperty}\text{of}(c, d) \land \text{subproperty}\text{of}(d, e) & \rightarrow \text{subproperty}\text{of}(c, e) \\
\text{subproperty}\text{of}(p, q) \land p(x, y) & \rightarrow q(x, y) \\
\text{instance}\text{of}(p, \text{symmetric}\,\text{property}) \land p(y, x) & \rightarrow p(x, y) \\
\text{instance}\text{of}(p, \text{transitive}\,\text{property}) \land p(x, y) \land p(y, z) & \rightarrow p(x, z)
\end{align*}
\] (3)

These rules look higher order, but they are not. The quantification is only over Wikidata properties, so the fourth rule, for example, is more properly written as

\[
\text{instance}\text{of}(p, \text{Wikidata}\,\text{property}) \land \text{subproperty}\text{of}(p, q) \land p(x, y) \rightarrow q(x, y)
\]

Even the appearance of being higher order can be removed by changing to a triple-based formulation, as is done in RDF, where the rule would be written something like:

\[
T(p, \text{instance}\text{of}, \text{Wikidata}\,\text{property}) \land T(p, \text{subproperty}\text{of}, q) \land T(x, p, y) \rightarrow T(x, q, y)
\]

So far, these ontology rules are just MARPL, and even just regular Horn rules. But Wikidata qualifiers need to be taken into account, even if qualifiers are forbidden in ontology atoms (i.e., those with predicates instance\text{of}, subclass\text{of}, subproperty\text{of}), because of occurrences of non-ontology atoms such as \(p(x, y)\) in some rules. In MARPL, such rules would either have to take into account every possible Wikidata qualifier, or different rules would have to be written for each Wikidata property, but even these rules would have to take into account all the qualifiers that are present on Wikidata statements for the property. In eMARPL a single rule can be written, without regard to qualifiers, and the qualifier characterizations are used to handle the qualifiers that are present.

### 3.2 Wikidata Qualifier Characterization

The most important, and most involved, Wikidata qualifier characterizations are probably the ones for temporal qualifiers, as they impact very many statements in Wikidata.
and missing temporal qualifiers can lead to considering statements that are not relevant to the user’s context. The characterizations of temporal qualifiers above as blending attributes are, we feel, typical and show off the capabilities of eMARPL.

Many Wikidata qualifiers have little logical import or are specific to a statement, such as the measurement method used to determine a statement or the location of an event. The former can be simply ignored for most uses of Wikidata and the latter, although of interest, should not be carried along in most inferences, so they would be classified in the default classification. Other Wikidata qualifiers have logical import and thus should be carried along, but are multivalued and do not in general interact with other qualifiers. These can be categorized in the “additive” categorization.

3.3 The Meaning of Wikidata

The meaning of Wikidata is then the inferential closure of the eMARS above under this eMARPL ontology. It is this eMARS that is to be used when querying or otherwise requesting what is true in Wikidata, or checking constraints.

3.4 Constraints

Wikidata has constraints that do not add directly to the meaning of Wikidata but instead provide signals that there is something questionable in Wikidata, consistent with the view taken by other work on constraints for knowledge-graph-like systems [3,12]. We model Wikidata constraints as eMAPL formulae that are evaluated over the eMARS that is the meaning of an eMARPL ontology. We note that attribute sets and functions for checking set membership, which are useful for expressing constraints involving qualifiers, are already included in the definition of MAPL in [9], and we have also included equality and counting quantifiers in eMAPL. For space considerations:

Constraints can either be given a positive formulation, which indicates a pattern of data elements that conform to the constraint, or a negative formulation, which indicates a pattern of data elements that violate the constraint. In our view, it is most natural to first write the positive formulation, and then from that derive the negative formulation, which can then be used as a query. Instantiations of these constraints that are false are to be reported as violations of the constraint.

For example, the distinct values constraint in Wikidata is supposed to say that a given property should have different values for different items. Constraints like this one are currently implemented (if at all) by special-purpose code. The following eMAPL formula embodies this constraint

\[
\text{property\_constraint}(p, \text{distinct values constraint}) \land \\
p(s_1, o_1) \land p(s_2, o_2) \land s_1 \neq s_2 \rightarrow o_1 \neq o_2
\]  

(4)

Formula (4) directly expresses the meaning of the constraint in the usual fashion of first-order logic. When this formula is satisfied for all bindings of the free variables the constraint holds. Bindings of the free variables that falsify the formula are violations of the constraint. The query formulation of this constraint is just its negation:

\[
\text{property\_constraint}(p, \text{distinct values constraint}) \land \\
p(s_1, o_1) \land p(s_2, o_2) \land s_1 \neq s_2 \land o_1 = o_2
\]  

(5)
Bindings of the free variables that make Formula 5 true, i.e., the results of the query, are violations of the constraint.

In a separate paper [7] we show that with the proposed formalism nearly all of Wikidata’s existing property constraints can be given a complete characterization in an economical, natural, and relatively easy-to-understand fashion. Such characterizations, unlike documentation in natural language, provide an unambiguous basis for understanding and for implementing constraint checkers. Constraint violations can be found by an expression-evaluator for the formalism, with no additional engineering effort. In other words, all that is required is to obtain the constraint’s negative formulation (which, as mentioned earlier, can be automatically derived from its positive formulation) and submit it to the evaluator. Moreover, we show that additional constraints can easily be added, so long as they are expressible in eMAPL. eMAPL allows for representing and handling a broad range of constraints, which goes beyond property constraints, in the same formalism.

3.5 Rules for Recognizing Class Instances

There are many classes in the Wikidata ontology that have very few instances, even when considering instances of subclasses. For example, as of mid-June 2020 in Wikidata female:human has only 5 instances, even including instances of subclasses, as opposed to the several million expected. The problem is that most female humans are stated as belonging to human and having sex:or:gender female. There is nothing in Wikidata to suggest, however, that asking for instances of female:human is unreasonable.

Adding a rule of the form

\[ \text{instance of}(\text{human}, h) \land \text{sex:or:gender}(h, \text{female}) \rightarrow \text{female:human}(h) \]  

results in female:human having the correct instances.

Many other notable under-populated classes can be handled in the same way.

4 Related Work

We highlight relevant work from several slices of logical foundations for knowledge bases.

*Logical foundations for Wikidata.* SQID [8] is a browser and editor for Wikidata, which draws inferences from a collection of MARPL rules. Our work was informed by SQID’s embodiment of MARPL-based reasoning, and motivated in part by the desire to expand the expressiveness of MARPL rules, as illustrated by the SQID rule set (particularly limitations related to attribute sets) to provide a more complete reasoning framework, and to accommodate Wikidata constraints. [11] also formalizes a model of Wikidata based on MARS, but with a different objective: the application of “Formal Concept Analysis to efficiently identify comprehensible implications that are implicitly present in the data”. [11] is thus nicely complementary with [9] and with our work.

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10 SQID’s rule set may be viewed at https://tools.wmflabs.org/sqid/#/rules/browse.
in that it provides a basis for discovering, rather than hand-authoring, new (e)MARPL rules.

Logical foundations for annotated KBs. Annotated RDFS [13] extends RDFS and RDFS semantics to support annotations of triples. A deductive system is provided, and extensions to the SPARQL query language that enable querying of annotated graphs. While this approach could provide a useful target formalism for Wikidata’s RDF dumps, we have chosen instead to represent Wikidata’s data model as directly as possible, and thus we deliberately avoid the use of the RDF dumps, and the complexities that could arise from adopting RDF as the modeling framework.

Adding attributes to logics. Just as MARPL was developed to provide a (rule-based, Datalog-like) decidable fragment of MAPL, Krötzsch, Ozaki, et al. have also explored description logics as a basis for other decidable fragments of MAPL, and have analyzed the resulting family of attributed DLs in [4,5,6,10,11]. We believe that MARPL provides the best available starting point for modeling Wikidata, but we also agree that this ongoing thread of research will lead to attributed DLs with the right level of expressivity for other sorts of applications.

5 Conclusion and Future Work

We have described eMARS, eMAPL, and eMARPL (extensions of previous work of Marx et al. [9]), which can provide a practically usable foundation for a logic of Wikidata. These extensions are centered around the introduction of datatypes and the specification of the behavior of qualifiers in connection with axioms. We have outlined a reasoning framework for using these formalisms with Wikidata, and have indicated how its use could add substantial value to Wikidata and in particular bring benefits from well-founded reasoning involving axioms, constraints, qualifiers, and special values. The adoption of this framework could support more complete, meaningful, and consistent querying of Wikidata, as well as better facilities for implementing KB completion and other reasoning capabilities.

Our plans for future work include the discussion of other areas of Wikidata that can benefit from its use and the creation of a prototype implementation and then a detailed design for a scalable deployment.

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