Quantum Corrections to Monopoles

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We summarize our recent work on low energy quantum corrections to monopoles in $N=2$ supersymmetric Yang-Mills theory. The details maybe found in [1].

Most of our understanding of solitons in quantum field theory comes from semiclassical expansion about classical solutions. However, it was realized long ago [2] that in principle there is an alternative: One can study “classical” solutions of the quantum effective action. Of course, in practice, one rarely has access to the full effective action. Nevertheless, one can study the solutions to various approximations and get approximate information. An example of this procedure is the Skyrme model where solitons of the QCD low energy effective action (depending on a mesonic field) are considered to be baryons. The key difference between that case and our problem is, as explained below, that we know the full non-perturbative as well as perturbative low-energy effective action; this should of course make our predictions more reliable.

Three years ago, Seiberg and Witten [3] started a revolution when they found the full nonperturbative low-energy effective action of the massless states in $N=2$ supersymmetric $SU(2)$ Yang-Mills theory. This effective action can be gauge covariantized with respect to the spontaneously broken $SU(2)$ gauge symmetry (as described below) to describe the massive charged vector multiplets of the theory. Evidence for the correctness of this procedure is provided by an interesting strong-coupling phenomenon: the massive charged vector multiplets destabilize for certain values of the Higgs vacuum expectation value, and precisely at these

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values the effective Hamiltonian changes sign \([4]\).

Using this low-energy nonperturbative approximation to the full effective action, we find that the resulting effective Hamiltonian admits BPS solutions that can be interpreted as quantum corrected monopoles and dyons.

In this report, we do not have space to review all of the results of Seiberg and Witten; a brief summary of the salient points will have to suffice. It is well known that \(N = 2\) Yang-Mills theory with gauge group \(SU(2)\) has a classical potential with valleys; different vacua are described by the VEV of the scalar fields in the theory, and as long as the VEV is nonzero, the gauge group is broken to \(U(1)\). Seiberg and Witten argued that nonperturbatively, the valleys remain, but that the vacuum with unbroken gauge symmetry is destabilized by quantum effects. They described the nonperturbative low-energy (up to quadratic order in gradients) effective action of the massless \(U(1)\) fields in terms of a single holomorphic function \(\mathcal{F}[5]\). They parametrized the space of vacua, or moduli space, in terms of a single complex variable \(u = \langle Tr \phi^2 \rangle\), where \(\phi\) is a complex scalar Higgs field in the adjoint representation of \(SU(2)\). The VEV \(a\) of the scalar field in the unbroken \(U(1)\) is given in terms of \(u\) as a certain integral, as is \(a_D \equiv \frac{\partial \mathcal{F}}{\partial u}\).

The effective coupling and \(\theta\)-angle are given by \(\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} \equiv \frac{\partial^2 \mathcal{F}}{(\partial a)^2}\).

As already observed by Seiberg and Witten, the low energy interactions of the massive charged supermultiplets in the broken part of the \(SU(2)\) gauge group can be described by simply covariantizing with respect to the gauge group: \(\mathcal{F}(a) \rightarrow \mathcal{F}(\sqrt{\phi \cdot \phi})\). If one regards this as a Wilsonian effective action, it ceases to become meaningful when the massive charged vector multiplets are no longer the lightest massive states in the theory; however, we see no compelling reason to take a Wilsonian view, and regard \(\mathcal{F}(\sqrt{\phi \cdot \phi})\) as the leading low-energy expansion of the 1-PI generating functional (to this level in the derivative expansion, the two appear to coincide \([6]\)). Evidence for this interpretation is provided by the observation that the effective action itself encodes the information about its range of validity: there is a disk bounded by a curve of marginal stability in the \(u\)-plane inside of which the charged supermultiplets become unstable and vanish from the spectrum. Precisely on this disk, the effective action for the charged vector multiplets breaks down by changing sign \([4]\).

The bosonic part of the low-energy effective Hamiltonian that we compute from this effective action has a bulk as well as a boundary contribution \(H = H_0 + H_{\text{top}}:\)

\[
H_0 = \frac{1}{8\pi} \, \Im \int d^3x \mathcal{F}_{AB} \left\{ (B_i^A + iE_i^A + \sqrt{2}e^{i\alpha} \nabla_i \phi^A) (B_i^A - iE_i^B + \sqrt{2}e^{-i\alpha} \nabla_i \phi^B) \right\}.
\]

Here \(\mathcal{F}_{AB} \equiv \frac{\partial^2 \mathcal{F}}{\partial \phi^A \partial \phi^B}\), and \(B\) and \(E\) are the nonabelian magnetic and electric fields. A constant phase \(e^{i\alpha}\) has been introduced and will be given explicitly below. It might appear that \(e^{i\alpha}\) could be absorbed into the complex field \(\phi\). However, the asymptotic phase of \(\phi\) has an independent
significance: it determines the \( \theta \)-angle of the vacuum. Note that \( H_0 \) is positive whenever \( \Im \mathcal{F}_{AB} \geq 0 \); hence the BPS equations for the general monopole and dyonic states is

\[
B^B_j + iE^B_j + e^{i\alpha} \sqrt{2} \nabla_j \phi^B = 0 .
\] (2)

These equations have the same form as in the classical Yang-Mills-Higgs action. Nevertheless, as we explain below, there are significant quantum corrections.

The energy associated with the boundary terms is

\[
H_{\text{top}} = -\sqrt{2} \int d\Sigma \cdot \left\{ \Pi_A \Im (e^{i\alpha} \phi^A) + \frac{1}{4\pi} \bar{B}^A \Im (e^{i\alpha} \mathcal{F}_A) \right\} .
\] (3)

Here \( \Pi \) is the momentum conjugate to the nonabelian vector potential, and generates gauge transformations. After some calculation, we find that for

\[
e^{i\alpha} = i \frac{\bar{Z}}{|Z|} ,
\] (4)

where \( Z \) is the central charge of the \( N = 2 \) superalgebra, we obtain the BPS bound for the total energy

\[
E \geq \sqrt{2}|Z| .
\] (5)

When the quantum corrected monopole/dyon solutions satisfy the \textit{usual} BPS equations, the bound is saturated.

We may next consider a radial ansatz, and study the structure of the solutions. Conceptually, we imagine that we are finding a solution \( u(r) \), where \( u \) is the modular parameter described above, and from it determine the \( \phi(r) \), \( B(r) \), and \( E(r) \). A key change from the classical solution arises when we consider the appropriate boundary conditions for the problem. In the classical theory, one imposes regularity everywhere in space, and consequently, the expectation value of the Higgs field vanishes at the origin. Seiberg’s and Witten’s representation of the VEV \( a(u) \) can \textit{never} vanish (we call this excluded region the hard core of the monopole). However, this does not lead to a contradiction: the low-energy approximation is expected to break down when derivatives become large, which happens at the core of the monopole; consequently, we cannot impose a boundary condition at the origin. This introduces an extra parameter into our solution which merely expresses our ignorance of what is happening inside the core of the soliton. Examination of how this parameter enters reveals that it has a simple physical significance: it determines the effective contribution of the massive charged vectors to the soliton.
A more fundamental change in the solution arises because the gauge theory that we are considering is asymptotically free, and undergoes dimensional transmutation: the coupling and $\theta$-angle are determined by the value of the Higgs field. Consequently, $\tau$ becomes effectively spatially dependent $\tau \rightarrow \tau(r)$, as does the central charge $Z \rightarrow Z(r)$. This has a dramatic effect: charge of the dyon depends of the local value of $\tau(r)$, and hence, the electric field has direct quantum corrections: it is no longer a simple duality rotation of the magnetic field. This is consistent with the breaking of the continuous duality group to a discrete subgroup.

The solution that we find is characterized by the condition that the phase of the local central charge $Z(r)$ is constant throughout space. We may follow lines of constant $\text{arg}(Z)$ in the $u$-plane or the $\tau$-plane; these are shown in Figure 1. A solution should be thought of starting at some point and following a line to the curve of marginal stability; beyond this curve, the solution cannot be trusted, as the massive vector multiplets are no longer in the spectrum; this defines a larger “soft” core for the monopole. We note that for approximately half the $u$-plane, the solutions all run to one fixed point $u = 1$. Seiberg and Witten argued that at this point the monopoles are becoming massless; notice that we cannot describe such monopoles as quantum corrected classical solutions, since massless monopoles have a core that fills all space.
Figure 1: (a) Some lines of constant $Z(r)$-phase in the $\tau$-plane. (b) Lines of constant $Z(r)$-phase in the $u$-plane are shown for $\pi/20$ increments.

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