Analysis of the Installation Error of Full Accelerometers on the Projectile and Configuration Improvement

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Abstract. In order to improve the precision of the projectile’s angular velocity when there is a positional installation error of the accelerometer, an improved twelve-accelerometer mounting configuration is proposed in this paper. The improvement is to extend the installation distance of the accelerometers properly in the narrow space of the projectile. Integral method and logarithm method are used to solve the projectile angular velocity, and the error equations of projectile’s angular velocity are derived respectively in different installation configurations. On the basis of the air trajectory model, the projectile angular velocity errors are compared for the two configurations in the simulation, and it shows that increasing the mounting distance of the accelerometer can effectively improve the precision of the projectile’s angular velocity.

1. Introduction
The traditional strapdown inertial navigation system (SINS) uses three-axis accelerometer and gyroscope installed at the center of mass of the projectile to complete the navigation task, in which the accelerometer measures the linear motion information of the projectile while the gyroscope measures the angular motion information [1]. There are large linear acceleration and angular acceleration during the flight of the projectile, and the gyroscope is often damaged due to unbearable wide range of dynamic acceleration [2]. Compared with gyroscope, the accelerometer has the advantages of longer lifespan, higher reliability, lower cost, lower power consumption and faster dynamic response [3–4]. Therefore, the idea of using multiple accelerometers to replace gyroscopes has been increasingly attracted the researchers’ interest.

The full accelerometers navigation system which is also called gyro-free strapdown inertial navigation system (GFSINS) [5], relies on the special installation configuration of accelerometers on the projectile which includes the installation location and sensitive direction of the accelerometers. It has been proved that at least 6 accelerometers are required to accurately describe all motion information of the projectile [6]. Till now, the configuration with 6, 9, 10 and 12 accelerometers have been proposed and wildly studied by the researchers [7–8]. With the outstanding research of the installation configuration, many algorithms to calculate the angular velocity of the projectile using the measurements of the accelerometers have also been proposed. However, the accelerometers are directly mounted on the projectile, to calculate the specify force and angular velocity at the center of mass without gyroscope. When the accelerometer is installed with a little position error, there is a large measurement error of the accelerometer so that the whole system can’t complete navigation task as well as SINS. The installation errors of the accelerometer have been also studied by many researchers. Chin Woo Tan and Fanjun Yu have calibrated the accelerometer’s positional installation error and sensitive direction errors [3–9], Junwei Wu have used the parameter identification method to calibrate...
the accelerometer error [10]. In this paper, a novel installation configuration is proposed to improve the solution precision of the projectile’s angular velocity.

This paper will be structured as follows. Section II analyses the relationship between installation position error and accelerometer output error. Section III mainly proposes a novel configuration and deduces the relationship between the positional installation error and the solution error of the projectile’s angular velocity for different installation configurations. Section IV verifies the above analysis in simulation. The last section is the conclusions.

2. Location Installation Error Analysis

According to the non-centroidal specify force equation (1) [11], we can theoretically calculate the output of the accelerometer. In equation (1), $\theta^b$ is the accelerometer sensitive direction matrix composed by the sensitive direction vector of each accelerometer. $f_q^b$ is the specify force of the projectile at the center of mass on the projectile, $\Omega_{ib}$ is the anti-symmetric matrix of the projectile angular velocity, $r^b$ is the accelerometer position mounting matrix composed by position vector of every accelerometer, and $\dot{\omega}_{ib}^b$ is the angular acceleration of the projectile.

$$A = \left[(r^b \times \theta^b)^T \quad (\theta^b)^T \right] \left(\begin{array}{c} \omega_{ib}^b \\ f_q^b \end{array} \right) + \theta^b \cdot \Omega_{ib}^2 \cdot r^b$$

(1)

The relation between location installation error and measurement error of the accelerometer will be analyzed in the following. Assume that there is no error in the sensitive direction of full accelerometers, and the sensitive direction of the accelerometer is $\theta^b$. The ideal mounting position of the accelerometer is $l_i$, which corresponds to the output $A_i$ of the accelerometer. Similarly, under the existence of the location mounting error $l_e$, the actual installation location of the accelerometer is $l_t$, and the actual output of the accelerometer is $A_t$. As shown in figure 1, the figure shows that one accelerometer installs on the $Y$ axis with location installation error $l_e$ in the coordinate system, and the sensitive direction is $Z$ axis. So we can infer the mathematical relation between $l_i$, $l_e$ and $l_t$.

$$l_t = l_i + l_e$$

(2)

Figure 1. A accelerometer position installation error

When there is a positional installation error vector $l_e$ of all accelerometers on the projectile, according to the above derivation, it’s simply to infer the ideal accelerometer output equation $A_i$ and the actual accelerometer output equation $A_t$.

$$A_i = \left[(l_i \times \theta^b)^T \quad (\theta^b)^T \right] \left(\begin{array}{c} \omega_{ib}^b \\ f_q^b \end{array} \right) + \theta^b \cdot \Omega_{ib}^2 \cdot l_i$$

(3)
Based on equation (2), equation (3) and equation (4), the output error of twelve-accelerometer mounting configuration can be concluded as follows.

$$A_i = \left[ (l_i \times \theta^b)^T \cdot (\theta^b)^T \right] \cdot \left( \begin{array}{c} \dot{\omega}^b_{f_q} \\ \Omega_{ib}^2 \cdot l_i \end{array} \right)$$  \hspace{1cm} (4)$$

Define the accelerometer error item $A_{el}, i = 1, 2, \cdots, 12$ as the accelerometer error with positional mounting error $l_e$. And the derivation of $l_e$ in equation (5) is given

$$\frac{dA_{el}}{dl_e} = (\theta^b \times \dot{\omega}^b_{f_q})^T + \theta^b \Omega_{ib}^2$$  \hspace{1cm} (5)$$

From equation (5) and equation (6), it is clear that the output of the accelerometer is linear with the positional installation error of the accelerometer. Therefore, when the positional installation error of the accelerometer increases, the accelerometer output error also increases with respect to the previous.

3. The Improvement of Installation Configuration and Solution Error Analysis

In short-range projectile, high precision of the angular velocity can be obtained through the integral method and the logarithmic method [12]. In this section a novel configuration is proposed and the positional error is analyzed by the two methods mentioned above.

3.1. Traditional Installation Configuration and Improved Installation Configuration

By the knowledge of theoretical mechanics, in order to determine all the motion parameters of a rigid body in the three-dimensional space, it is necessary to know the information of six degree-of-freedom (DOF) about the rigid body, which includes three linear motion (the translational process of projectile) and three angular motion (the rotational process of the projectile). Therefore, in the full accelerometer strapdown inertial navigation system, six degree-of-freedom information will be obtained by properly configuring the accelerometers on the projectile. This part will propose a novel mounting configuration of 12 accelerometers. And a brief comparison of the proposed configuration and the traditional one is shown below.
Different from the traditional configuration in which the installation position of the three accelerometers in axis of $X$, $Y$ and $Z$ are the same, the proposed configuration enlarges the installation position of the accelerometers in axis $X$.

3.2. Projectile Angular Velocity Error Analysis with Positional Error of the Accelerometer

From figure 2 and figure 3 it is clearly to see that three accelerometers are installed at the same point of the projectile. Positional error of the accelerometers which is inevitable during the installation will lead to the angular velocity errors of projectile calculated by integral method and logarithmic method. In order to compare the magnitude of angular velocity error of the projectile, it is assumed that there is a $Y$-axis positional error of $A_{12}$ accelerometer in the integral method, while there is a $Y$-axis positional error of $A_{4}$ accelerometer in the logarithmic method. The positional error on the $Y$-axis is defined as $l_{e_y}$, and it is equal in the two configurations. So this section will analyze the two methods separately.

(1) Integral method

The accelerometer error item of $A_{12}$ is denoted as $A_{e_{12}}$ in the integral method referring to above content. According to equation (4), it can be derived that the $Y$-axis angular velocity of the projectile is largely affected when there is a mounting error of the $A_{12}$ accelerometer. Hence the $Y$-axis angular accelerations of the projectile in both mounting configurations are given.
\[ \dot{\omega}_{y_1} = \frac{1}{2l_1} (A_3 + A_9 - A_1 - (A_{12} + A_{11})) \]  
\[ \dot{\omega}_{y_2} = \frac{1}{2l_1 \cdot L_4} (l_4 \cdot A_9 - l_4 \cdot (A_{12} + A_{11}) - L_4 \cdot A_1 + L_4 \cdot A_3) \]  

where the subscript 1 in \( \dot{\omega}_{y_1} \) represents the traditional twelve-accelerometer mounting configuration, while the subscript 2 in \( \dot{\omega}_{y_2} \) defines the improved twelve-accelerometer mounting configuration. The angular velocity of the projectile can be obtained by integrating the angular accelerations in equation (7) and equation (8), and the angular velocity equation of the projectile is shown in equation (9).

\[ \omega_1(t + T) = \omega_1(t) + T \dot{\omega}_1(t) \]  

where \( T \) is the sample time, and \( \omega_1(t) \) is the angular velocity calculated at time \( t \). In order to simplify the analysis, the errors of \( A_{x12} \) in equation (7) and equation (8) are separately proposed as

\[ \Delta \dot{\omega}_{y_1} = -\frac{1}{2l_1} A_{x12} \]  
\[ \Delta \dot{\omega}_{y_2} = \frac{1}{2l_1 \cdot L_4} (-L_4 A_{x12}) = -\frac{1}{2L_4} A_{x12} \]  

As can be seen from figure 2 and figure 3, the installation distance of all accelerometers is \( l_4 = 0.07m \) in the traditional twelve-accelerometer mounting configuration, but in the improved twelve-accelerometer installation configuration, the installation distance of a three-accelerometer on \( Y \)-axis is \( L_4 = 0.5m \), while the installation distance of other accelerometers is \( l_4 = 0.07m \). So we have the following error relation.

\[ |\Delta \dot{\omega}_{y_1}| > |\Delta \dot{\omega}_{y_2}| \]  

Similarly, the relationship between the angular velocity of the \( Y \)-axis projectile in the two configurations is shown below.

\[ |\Delta \omega_{y_1}| > |\Delta \omega_{y_2}| \]  

It can be concluded that when the location mounting error of the accelerometer is the same, the precision of the angular velocity which solved by integral method can be improved by increasing the installation distance of the accelerometer on the projectile.

(2) Logarithm method

Logarithm method is the other method with high precision in the practical application, which can effectively suppress the divergence of the angular velocity error of the projectile. Similar to the integral method, the accelerometer error item of \( A_1 \) is denoted as \( A_{x4} \). According to the equation (4), the true angular velocity squared items are given as:

\[ \omega^2_{x1} = \frac{1}{2l_1} ((A_{x4} + A_{x4}) + A_5 + A_9 - A_1 - A_6 - A_{11}) \]  
\[ \omega^2_{x4} = \frac{1}{2l_1} (A_1 - (A_{x4} + A_{x4}) - A_5 + A_6 + A_9 - A_{11}) \]  
\[ \omega^2_{x1} = \frac{1}{2l_1} (A_1 - (A_{x4} + A_{x4}) + A_5 - A_6 - A_9 + A_{11}) \]
\[ \omega_{x2}^2 = -\frac{1}{2l_i} \left( l_i A_{i} - l_i \left( A_{i4} + A_{i4}\right) - L_3 A_{i5} + L_3 A_{i6} - L_3 A_{i9} + L_3 A_{i11} \right) \]

\[ \omega_{y2}^2 = \frac{1}{2l_i} \left( l_i A_{i} - l_i \left( A_{i4} + A_{i4}\right) - L_4 A_{i5} + L_4 A_{i6} + L_4 A_{i9} - L_4 A_{i11} \right) \]

\[ \omega_{z2}^2 = \frac{1}{2l_i} \left( l_i A_{i} - l_i \left( A_{i4} + A_{i4}\right) + L_5 A_{i5} - L_5 A_{i6} - L_5 A_{i9} + L_5 A_{i11} \right) \]  

(15)

The definition of relevant parameters in equation (14) and equation (15) are the same as the integral method. To simplify the problem, the accelerometer error terms in above equations are separated as

\[ \Delta \omega_{xy}^2 = \Delta \omega_{x2}^2 = \frac{1}{2l_i} A_{4} \]

\[ \Delta \omega_{yz}^2 = \Delta \omega_{z2}^2 = \frac{1}{2l_i} A_{4} \]  

(16)

According to the relationship between \( l_i \) and \( L_i \) from figure 3, the relation between the error equation (16) and equation (17) are shown in equation (18).

\[ \Delta \omega_{x1}^2 > \Delta \omega_{x2}^2, \Delta \omega_{y1}^2 > \Delta \omega_{y2}^2, \Delta \omega_{z1}^2 > \Delta \omega_{z2}^2 \]  

(18)

As with the integral method, it can also be concluded that with the same accelerometer position installation error, the precision of the improved twelve-accelerometer installation configuration is higher than the traditional twelve-accelerometer installation configuration when using the logarithm method.

4. Simulations and Results

The simulation is based on the air ballistic bomb model [13]. The initial velocity is 914m/s. The angle of shooting and the angle of emergence are 45°. The sampling period of the accelerometers is \( T = 0.01s \), and the accelerometer installation error on the Y-axis is \( l_{y1} = 0.01m \). In the following, integral method and logarithm method will be used to calculate the projectile’s angular velocity, and the corresponding error curves are given under different error conditions.

(1) Integral method

![Figure 4. Simulation results of integral method with positional error.](image)

There is a position installation error of the accelerometer \( A_{1,2} \), the angular velocity error curves of the projectile are shown in figure 4. From the above figures, it can be seen that there is an
accumulation of angular velocity error in integral method. In the traditional twelve-accelerometer mounting configuration, the Y-axis projectile angular velocity error can reach $-1.5 \times 10^{-7}$ rad/s, while the angular velocity error of the projectile only has $-8 \times 10^{-7}$ rad/s in the improved installation configuration, so the precision is greatly improved in improved installation configuration.

(2) Logarithm method

Similar to the integral method, there is a positional installation error of the $A_y$ accelerometer, the projectile angular velocity error curves are as follows.

![Figure 5. Simulation results of logarithm method with positional error.](image)

In the practical applications, the output error of the accelerometer contains not only the positional installation error, but also some measurement errors, such as accelerometer constant error, random walk and so on. In this simulation it is assumed that the constant error is $40 \times 10^{-3} \, g$, and the random walk coefficient is $80 \times 10^{-6} \, g/\sqrt{Hz}$. The angular velocity error curves of the projectile which calculated by logarithm method are shown below.

![Figure 6. Simulation results of logarithm method with positional and measured errors.](image)

From figure 4 and figure 5, it is well known that when there is a positional installation error of the accelerometer, the angular velocity of the projectile solved by logarithm method doesn’t appear an accumulation of errors compared with integral method. Reference to the analysis of equation (14), it can be seen that the angular velocity of the projectile is only related to the output value of the accelerometer at present time which has nothing to do with the sampling time $T$ in the logarithm...
method, so the calculated angular velocity of the projectile fluctuates around the real angular velocity. But in the integral method, the angular velocity error of the projectile will be a certain degree of accumulation with the sampling time $T$ according to equation (9). Therefore, the logarithm method is more effective than the integral method in solving the angular velocity of the projectile.

As can be seen from figure 5 and figure 6, when there are the same positional installation errors in the two configurations, the improved mounting configuration can achieve higher precision than the traditional installation configuration, which also verifies the equation (18). Moreover, there is a certain degree of divergence on the $Y$-axis angular velocity error of the projectile, but for the short-range projectile that the flight time is short; the error of $Y$-axis can satisfy the actual requirement.

5. Conclusion
In this paper, a novel mounting configuration to solve the problem of positional installation error in practical application of full accelerometers inertial navigation system is proposed. Compared to the traditional one, the improved installation configuration increases the installation distance of the accelerometer. The error equations of the projectile’s angular velocity are derived in the two mounting configurations by the integral method and logarithm method. From the simulation we can conclude that the precision of the projectile’s angular velocity in the improved mounting configuration is higher than the traditional installation configuration. Moreover, the simulation also shows that logarithm method can effectively suppress the dispersion of the projectile’s angular velocity.

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7. References
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