Time-resolved measurement of Landau-Zener tunneling in periodic potentials

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We report time-resolved measurements of Landau-Zener tunneling of Bose-Einstein condensates in accelerated optical lattices, clearly resolving the step-like time dependence of the band populations. Using different experimental protocols we were able to measure the tunneling probability both in the adiabatic and in the diabatic bases of the system. We also experimentally determine the contribution of the momentum width of the Bose condensates to the width of the tunneling steps and discuss the implications for measuring the jump time in the Landau-Zener problem.

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Tunneling is one of the most striking manifestations of quantum behaviour and has been the subject of intense research both in fundamental and applied physics \cite{1}. While tunneling probabilities can be calculated accurately even for complex quantum systems and have an intuitive interpretation as statistical mean values of experimental outcomes, the concept of tunneling time and its computation are still the subject of debate even for simple systems \cite{2}. How much time a quantum system spends in a classically forbidden area, e.g., inside a barrier separating two potential wells, has been theoretically calculated for a wide range of physical systems \cite{3,4} and measured in recent experiments \cite{5}. A well-known example of a simple tunneling problem is Landau-Zener (LZ) tunneling, in which a quantum system tunnels across an energy gap at an avoided crossing of the system’s energy levels. In recent years, LZ tunneling has been used as a paradigm for studies of the quantum Zeno effect (controlling the decay by repeated measurements of the systems \cite{6}), and more generally in the observation of deviations from a pure exponential decay of a quantum system consisting of just a few energy levels. An example of the latter was investigated by Raizen and co-workers \cite{7,8,9}, where the decay in a Wannier-Stark system was shown to deviate from an exponential law at short times. Moreover, LZ tunneling provides a building block for the quantum control \cite{10} of complex many-body systems \cite{11}.

According to the adiabatic theorem an infinitely slow change in the Hamiltonian of a quantum system leads to an adiabatic following of the instantaneous eigenstate, whereas a controlled time dependence can cause transitions to other eigenstates. In the case of a linear time dependence in a Hamiltonian describing two crossing eigenstates that are repelled by a coupling energy, the probability of a transition between these eigenstates is described by the celebrated Landau-Zener model \cite{12,13}. While in most treatments of LZ-tunneling only the (asymptotic) tunneling probability is considered, the full time dependence of the LZ dynamics as determined by the time evolution of the quantum-mechanical wavefunction also plays an important role, especially if the asymptotic limit assumed in LZ theory is not reached in a single transition. The characteristic time required for the tunneling between initial and final quantum states to be completed \cite{14,15} is sometimes referred to as the "jump time".

In this Letter, we present experimental results, backed up by numerical simulations, on the LZ dynamics and on the jump time for a Wannier-Stark system realized with ultracold atoms forming a Bose-Einstein condensate (BEC) inside an optical lattice \cite{16,17}. This system gives us an optimal control of single as well as sequences \cite{18} of LZ processes. In contrast to the above mentioned experiments with cold atoms \cite{7,8,9} our BEC has an initial width in momentum space that is much smaller than the characteristic momentum scale of the problem given by the momentum width $p_B = 2p_{rec} = 2\pi\hbar/d_L$ of the first Brillouin zone of a periodic potential with lattice constant $d_L$. This possibility of engineering the initial state for the LZ process in momentum space enables us to observe the time dependent dynamics with a non-exponential decay in the survival probability for single or multiple LZ crossings \cite{19} as well as oscillations in the transition probability after the system has passed the avoided crossing (as predicted for LZ tunneling in atomic Rydberg states \cite{20} and experimentally observed in a wave-optical two-level system \cite{21}), the only limitation being the initial momentum width of the condensates and nonlinear effects. This possibility of engineering the initial state for the LZ process in momentum space enables us to observe the time dependent dynamics with a non-exponential decay in the survival probability for single or multiple LZ crossings \cite{19} as well as oscillations in the transition probability after the system has passed the avoided crossing, the only limitation being the initial momentum width of the condensates and nonlinear effects. Such oscillations were predicted for LZ tunneling in atomic Rydberg states \cite{20} and experimentally observed in a wave-optical two-level system \cite{21}). Our experiments are similar to recent studies of LZ transitions at avoided crossings in the energy levels of a solid-state artificial atom \cite{22}, but the high level of control over the light-induced periodic potential...
Experimental protocol (shown in the band-structure representation of energy $E(q)$ versus lattice quasimomentum $q$). Left: The lattice is accelerated, (partial) tunneling occurs. Right: The acceleration is then suddenly reduced and the lattice depth increased so as to ‘freeze’ the instantaneous populations in the lowest two bands; finally, further acceleration is used to separate these populations in momentum space. (b) Experimental results for $V = 1 E_{\text{rec}}$ and $F_0 = 0.383$, giving $T_B = 0.826$ ms. The solid and dashed lines are a numerical simulation of our experimental protocol and an exponential decay curve for our system’s parameters, respectively.

also allowed us to measure the tunneling dynamics in different eigenbases of the system’s Hamiltonian.

In our experiments we created Bose-Einstein condensates of $5 \times 10^4$ rubidium-87 atoms inside an optical dipole trap (mean trap frequency around 80 Hz). A one-dimensional optical lattice created by two counter-propagating, linearly polarized gaussian beams was then superposed on the BEC by ramping up the power in the lattice beams in 100 ms. The wavelength of the lattice beams was $\lambda = 842$ nm, leading to a sinusoidal potential with lattice constant $d_L = \lambda/2 = 421$ nm. A small frequency offset between the two beams could be introduced through the acousto-optic modulators in the setup, allowing us to accelerate the lattice in a controlled fashion.

The time-resolved measurement of LZ tunneling was then effected as follows [see Fig. 1 (a)]. After loading the BEC into the ground state energy band of an optical lattice of depth $V_0$ as described above, the lattice was accelerated with acceleration $a_{\text{LZ}}$ for a time $t_{\text{LZ}}$ by chirping the frequency offset between the lattice beams, resulting in a corresponding force $F_{\text{LZ}}$ on the atoms in the rest frame of the lattice. The lattice thus acquired a final velocity $v = a_{\text{LZ}} t_{\text{LZ}}$. During $t_{\text{LZ}}$ the quasimomentum of the BEC swept the Brillouin zone, and at multiples of half the Bloch time $T_B = 2\pi \hbar (M a_{\text{LZ}} d_L)^{-1}$ (where $M$ is the atomic mass) i.e. at times $t = (n+1/2)T_B$ ($n = 0, 1, 2, \ldots$) when the system was close to the Brillouin zone edge, tunneling to the upper band became increasingly likely. At time $t = t_{\text{LZ}}$ the acceleration was abruptly reduced to a smaller value $a_{\text{sep}}$ and the lattice depth was increased to $V_{\text{sep}}$ in a time $t_{\text{sep}} \ll T_B$. These values were chosen in such a way that at time $t = t_{\text{LZ}}$ the probability for Landau-Zener tunneling from the lowest to the first excited energy band dropped from between $\approx 0.1 - 0.9$ (depending on the initial parameters chosen) to less than $\approx 0.01$, while the tunneling probability from the first excited to the second excited band remained high at about 0.95. This meant that at $t = t_{\text{LZ}}$ the tunneling process was effectively interrupted and for $t > t_{\text{LZ}}$ the measured survival probability $P(t) = N_0/N_{\text{tot}}$ (calculated from the number of atoms $N_0$ in the lowest band and the total number of atoms in the condensate $N_{\text{tot}}$) reflected the instantaneous value $P(t = t_{\text{LZ}})$.

The lattice was then further accelerated for a time $t_{\text{sep}}$ such that $a_{\text{sep}} t_{\text{sep}} \approx 2 n p_{\text{rec}} / M$ (where typically $n = 2$ or 3). In this way, atoms in the lowest band were accelerated to a final velocity $v \approx 2 n p_{\text{rec}} / M$, while atoms that had undergone tunneling to the first excited band before $t = t_{\text{LZ}}$ underwent further tunneling to higher bands with a probability $> 0.95$ and were, therefore, no longer accelerated. At time $t_{\text{sep}}$ the lattice and dipole trap beams were suddenly switched off and the expanded atomic cloud was imaged after 23 ms. In these time-offlight images the two velocity classes 0 and $2 n p_{\text{rec}} / M$ were well separated and the atom numbers $N_0$ and $N_{\text{tot}}$ could be measured directly. Since the populations were “frozen” inside the energy bands of the lattice, which represent the adiabatic eigenstates of the total Hamiltonian of the system, this experiment effectively measured the time dependence of the LZ survival probability $P_1$ in the adiabatic basis. The result of a typical measurement is shown in Fig. 1 (b). One clearly sees two “steps” at times $t = 0.5 T_B$ and $t = 1.5 T_B$, which correspond to the instants at which the atoms cross the Brillouin zone edges, where the lowest and first excited energy bands exhibit avoided crossings. For comparison, the result of a numerical simulation (integrating the linear Schrödinger equation for the experimental protocol) as well as an exponential decay as predicted by LZ theory are also shown.

The LZ tunneling probability can be calculated by considering a two-level system with the adiabatic Hamiltonian

$$
H_a = H_d + V = a t \sigma_y + \frac{\Delta E}{2} \sigma_z,
$$

where $\sigma_i$ are the Pauli matrices. The eigenstates of the diabatic Hamiltonian $H_a$, whose eigenenergies vary linearly in time, are mixed by the potential $V$ characterized
FIG. 2: (a) LZ survival probability in the adiabatic basis for a fixed force \(F_0 = 1.197\) and different lattice depths (filled squares: \(V_0 = 2.3\ E_{\text{rec}}\); open circles: \(V_0 = 1.8\ E_{\text{rec}}\); open squares: \(V_0 = 1\ E_{\text{rec}}\); filled circles: \(V_0 = 0.6\ E_{\text{rec}}\)). The dashed lines are sigmoid fits to the experimental data. Inset: Survival probability in both the adiabatic (open squares) and diabatic (filled triangles) bases for \(V_0 = 1\ E_{\text{rec}}\) and \(F_0 = 1.197\). (b) Step height \(h\) as a function of the inverse adiabaticity parameter \(1/\gamma\) for varying lattice depth and \(F_0 = 1.197\) (open symbols), and for varying force with fixed \(V_0 = 1.8\ E_{\text{rec}}\) (filled symbols). The dashed line is the prediction of Eq. (3) for the LZ tunneling probability.

by the energy gap \(\Delta E\). Applying the Zener model [13] to our case of a BEC crossing the Brillouin zone edge leads to a band gap \(\Delta E = V_0/2\) and to \(\alpha = 2v_{\text{rec}}M a_{LZ} = 2F_0E_{\text{rec}}/\hbar\) for \(E_{\text{rec}} = \hbar^2\pi^2/(2M d_B^2)\) the recoil energy and \(F_0 = M a_{LZ} d_B/E_{\text{rec}}\) the dimensionless force. The limiting value of the adiabatic and diabatic Landau-Zener survival probabilities (for \(t\) going from \(-\infty\) to \(+\infty\)) in the eigenstates of \(H_a\) and \(H_d\), respectively, is

\[
P_a(t \to +\infty) = 1 - P_d(t \to +\infty) = 1 - P_{LZ}, \tag{2}
\]

where

\[
P_{LZ} = e^{-\frac{\gamma}{2}} \tag{3}
\]

with the adiabaticity parameter \(\gamma = 4\hbar\alpha(\Delta E)^{-2}\) is the standard LZ tunneling probability [24].

In order to test whether this tunneling probability correctly predicts the height of the step corresponding to a single LZ tunneling event, we performed the experiment described above for a variety of values of \(V_0\) and \(F\). Figure 2(a) shows the first tunneling step for different lattice depths \(V_0\), measured in units of \(E_{\text{rec}}\), at a given LZ acceleration determined by \(F_0\). The steps can be well fitted with a sigmoid function

\[
P_a(t) = 1 - \frac{1}{1 + \exp((t_0 - t)/\Delta t_{LZ})}, \tag{4}
\]

where \(t_0\) is the position of the step (which may deviate slightly from the expected value of \(0.5\ T_B\), e.g. due to a non-zero initial momentum of the condensate), \(h\) is the step height given by Eq. (3) and \(\Delta t_{LZ}\) represents the width of the step. As expected, the step height \(h\) follows the LZ result for the tunneling probability (see Fig. 2(b)).

We note here that while the experimental protocol described above measures the LZ tunneling probability in the adiabatic basis, it is also possible to make analogous measurements in the diabatic basis of the unperturbed free-particle wavefunctions (plane waves with a quadratic energy-momentum dispersion relation) by abruptly switching off the lattice and the dipole trap after the first acceleration step. In this case, after a time-of-flight the number of atoms in the \(v = 0\) and \(v = 2p_{\text{rec}}/M\) momentum classes are measured and from these the survival probability (corresponding to the atoms remaining in the \(v = 0\) velocity class relative to the total atom number) is calculated. The inset of Fig. 2(a) (filled triangles) shows such a measurement. Again, a step around \(t = 0.5\ T_B\) is clearly seen, as well as strong oscillations for \(t > 0.5\ T_B\). While (weaker) oscillations can also be measured in the adiabatic basis (see the results for \(V_0 = 2.3\ E_{\text{rec}}\) in Fig. 2(a)), they are much stronger and visible for a wider range of parameters in the diabatic basis, as expected from theoretical calculations [15].

The widths \(\Delta t_{LZ}\) corresponding to the steps shown in Fig. 2(a) should, according to our interpretation, reflect the "tunneling time" or "jump time" for LZ tunneling \(\Delta t_{LZ} = \Delta v_{LZ}/\alpha_{LZ}\) during which the probability of finding the atoms in the lowest energy band goes from \(P_a(t = 0) = 1\) to its asymptotic LZ value \(P_{LZ}\). In the theoretical literature this tunneling time has been calculated by several authors [4, 14, 15]. Vitanov [15] defines the jump time in the adiabatic basis as

\[
\tau_a^{\text{jump}} = \frac{P_a(t = +\infty)}{P_a(t = t_0)}, \tag{5}
\]

where \(P_a(t = t_0)\) denotes the time derivative of the tunneling probability \(P_a(t)\) evaluated at the crossing point of \(H_a\). A sigmoidal function for \(P_a(t)\) leads to \(\tau_a^{\text{jump}} = 4\Delta t_{LZ}\). For large values of \(\gamma\), which is the regime of our experiments, the theoretical jump time is given by \(\tau_a^{\text{jump}}/T_B \approx (V_0/E_{\text{rec}})/8\). From our sigmoidal fits we retrieve \(\tau_a^{\text{jump}}/T_B \approx 0.15 - 0.35\) (corresponding to absolute jump times between 50 \(\mu\)s and 200 \(\mu\)s), whereas the theoretical values for our experimental parameters are in the region of \(0.1 - 0.15\). We interpret this discrepancy...
as being due the fact that in our experiment the condensate does not occupy one single quasimomentum but is represented by a momentum distribution of a finite width $\Delta p / p_B \gtrsim 0.1$ due to the finite number of lattice sites (around 50) it occupies and the effects of atom-atom interactions. Also, during the acceleration process dynamical instabilities can further broaden the momentum distribution.

In order to test the dependence of the measured step width on the width of the momentum distribution of the condensate, we created initial distributions of different widths using a dynamical instability in a controlled way [26]. The condensate was loaded into a lattice moving at a finite velocity corresponding to quasimomentum $q = -0.3 p_B$ and held there for a time up to 3 ms. During this time the dynamical instability associated with the negative effective mass at $q = -0.3 p_B$ led to an increase in the momentum width of the condensate. After this preparatory stage, the LZ dynamics was measured as described above and the width of the tunneling step was measured [see Fig. 3(a)]. As expected, the larger the initial momentum width of the condensate, the larger the step width [Fig. 3(b)]. This was also confirmed by a numerical integration of the Schrödinger equation in which the momentum width of the condensate was varied by changing the initial trap frequency. The simulation also showed that for a vanishing momentum width, the step width still remains finite and in that limit directly reflects the jump time given by Eq. (3).

In summary, we have measured the full dynamics of a single LZ transition of matter waves in an accelerated optical lattice. In both the adiabatic and diabatic bases the step-like behaviour as well as oscillations of the survival probability were clearly seen and agree with theoretical predictions. While there is quantitative agreement between the measured and theoretically expected step heights, the jump times calculated from the step widths are larger by $\approx 50 - 100\%$ than the theoretical results because of the finite initial momentum widths of the condensates in our experiments.

In future investigations one could reduce the initial momentum width by using, e.g., appropriate trap geometries or by controlling the nonlinearity through Feshbach resonances. This would allow one to obtain a more quantitative measurement of the jump time and enable a comparison with theoretical results related to the minimum time for a single LZ crossing limited by fundamental quantum (or wave, see [22]) mechanical properties [27]. Also, clearer observations of the short-time oscillations (happening on even shorter time scales), whose signatures can be seen in Fig. 2(a) should be possible in this way. Our method can also be used to study multiple LZ crossings, e.g., in order to observe Stückelberg oscillations.

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![FIG. 3: LZ transition for different momentum widths of the condensate. (a) Survival probability for $\Delta p / 2p_{rec} = 0.2$ (filled squares) and $\Delta p / 2p_{rec} = 0.6$ (open squares). The solid and dashed lines are the results of a numerical simulation and of a sigmoid fit, respectively. (b) Step width $\Delta t_{LZ}$ of the LZ transition as a function of the momentum width of the condensate. The solid line is the result of a numerical simulation.](image-url)
[9] M. C. Fischer, B. Gutiérrez-Medina, and M. G. Raizen, Phys. Rev. Lett. 7, 040402 (1999).
[10] J.M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. 73, 565 (2001).
[11] G.E. Santoro, R. Martoňák, E. Tosatti, and R. Car, Science 295, 2427 (2002).
[12] L.D. Landau, Phys. Z. Sowjetunion, 2, 46 (1932).
[13] C. Zener, Proc. R. Soc. London, Ser. A 137, 696 (1932).
[14] K. Mullen, E. Ben-Jacob, Y. Gefen, and Z. Schuss, Phys. Rev. Lett. 62, 2543 (1989).
[15] N.V. Vitanov, Phys. Rev. A 59, 988 (1999).
[16] B.P. Anderson and M.A. Kasevich, Science 27, 282 (1998).
[17] O. Morsch, J.H. Müller, M. Cristiani, D. Ciampini, and E. Arimondo, Phys. Rev. Lett. 87, 140402 (2001).
[18] C. Sias et al., Phys. Rev. Lett. 98, 120403 (2007).
[19] Ref. [20] studied the influence of atomic interactions in BEC samples on the non-exponential decay in LZ sequences.
[20] S. Wimberger et al., Phys. Rev. A 72, 063610 (2005).
[21] J.R. Rubmark, M.M. Kash, M.G. Littman, and D. Kleppner, Phys. Rev. A 23, 3107 (1981).
[22] D. Bouwmeester et al., Phys. Rev. A 51, 646 (1995).
[23] D.M. Berns et al., Nature (London) 455, 51 (2008).
[24] Ref. [23] derived a more refined Zener model for ultracold atoms in an optical lattice.
[25] M. Holthaus, J. Opt. B:Quantum Semiclass Opt. 2, 589 (2000).
[26] M. Cristiani et al., Opt. Express 12, 4 (2004).
[27] Y. Makhlin, and P. Hakonen, Phys. Rev. Lett. 96, 187002 (2006); T. Caneva et al., [arXiv:0902.4193]