Exploring new Boundary Conditions for $\mathcal{N} = (1, 1)$ Extended Higher Spin $AdS_3$ Supergravity

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In this paper, we present a candidate for $\mathcal{N} = (1, 1)$ extended higher spin $AdS_3$ supergravity with the most general boundary conditions discussed by Grumiller and Riegler recently. We show that the asymptotic symmetry algebra consists of two copies of the $\mathfrak{osp}(3|2)_k$ affine algebra in the presence of the most general boundary conditions. Furthermore, we impose some certain restrictions to gauge fields on the most general boundary conditions and that leads us to the supersymmetric extension of the Brown-Henneaux boundary conditions. We eventually see that the asymptotic symmetry algebra reduces to two copies of the $\mathfrak{SW}(3,2)$ algebra for $\mathcal{N} = (1, 1)$ extended higher spin supergravity.

Keywords: $AdS_3/CFT_2$ Correspondance, Asymptotic Symmetry Algebra, Higher Spin Supergravity, Chern-Simons Gauge Theory, Affine algebras.

I. INTRODUCTION

The most common asymptotic symmetries of $AdS_3$ gravity with a negative cosmological constant in 3D are known as two copies of the Virasoro algebras and this has been written first by Brown and Henneaux in their seminal paper. So this work is well known as both a pioneer of $AdS_3/CFT_2$ correspondence and also a realization of Holographic Principle. Clearly, one of the biggest breakthroughs in theoretical physics in the past few decades is undoubtedly the discovery of the $AdS_3/CFT_2$ correspondence describing an equivalence between the Einstein gravity and a large $N$ gauge field theory.

The pure Einstein gravity in this context is simply a Chern-Simons gauge theory, that is, it is rewritten as a gauge field theory, in such a way that the structure simplify substantially. This recalls us that there is no local propagating degrees of freedom in the theory, and hence no graviton in three-dimensions. Therefore, this gauge theory is said purely topological and only global effects are of physical relevance. Finally, one must emphasize here that the dynamics of the theory is controlled entirely by the boundary conditions, because its dynamical content is far from insignificant due to the existence of boundary conditions. This fact was first discovered by Achucarro and Townsend, and subsequently developed by Witten. The things found here is that the gravity action in three-dimensions and equations of motions are in the same class with a Chern-Simons theory for a suitable gauge group. Under an convenient choice of boundary conditions, there is actually an infinite number of degrees of freedom living on the boundary. These boundary conditions are required, but these conditions are not unique in the selections. In fact, the dynamical properties of the theory are also highly sensitive to the selection of these boundary conditions. Thus, the situations of asymptotic symmetries as the remnant global gauge symmetries occur.

In the case of $AdS_3$ gravity above with a negative cosmological constant in 3D, the most famous of these boundary conditions is worked in the same paper. These boundary conditions also contain BTZ
black holes. Besides, the Chern-Simons higher spin theories are purely bosonic theories as versions of Vasiliev higher spin theories with higher spin fields of integer spin, they are based on the \( \mathfrak{sl}(N, \mathbb{R}) \) algebras and higher spin algebras \( \mathfrak{hs}(\lambda) \) respectively. The Chern-Simons higher spin theories can also realize the \( W_N \) algebras as the asymptotic symmetries of the two dimensional CFT's.\(^{1,15}\) The validity of these results can be extended to supergravity theory,\(^{1,16}\) as well as higher spin theory.\(^{6,20}\) Beyond that a supersymmetric generalization of these bosonic theories can be achieved by considering Chern-Simons theories based on superalgebras such as \( \mathfrak{sl}(N|N-1) \), see e.g.\(^{14,16-20}\), or \( \mathfrak{osp}(N|N-1) \), which can be obtained by truncating out all the odd spin generators and one copy of the fermionic operators in \( \mathfrak{sl}(N|N-1) \).

Our motivation is to construct a candidate solution for the most general \( \mathcal{N} = (1,1) \) extended higher spin supergravity theory in \( AdS_3 \). Our theory falls under the same metric class as the recently constructed most general \( AdS_3 \) boundary condition by Grumiller and Riegler.\(^{22}\) This method recently has also been applied to flat-space\(^{21}\) and chiral higher spin gravity\(^{24}\) which showed a new class of boundary conditions for higher spin theories in \( AdS_3 \). This is an alternative to the non-chiral Drinfeld-Sokolov type boundary conditions. In particular we first focus on the simplest example, \( \mathcal{N} = (1,1) \) Chern-Simons theory based on the superalgebra \( \mathfrak{osp}(2|1) \). It is clear that the related asymptotic symmetry algebra is two copies of the \( \mathfrak{osp}(2|1)_k \) affine algebra. Then one can extend this study to the \( \mathcal{N} = (1,1) \) Chern-Simons theory based on the superalgebra \( \mathfrak{osp}(3|2) \) and the symmetry algebras are given by two copies of the \( \mathfrak{osp}(3|2) \) affine algebra. Furthermore, we also impose certain restrictions to the gauge fields on the most general boundary conditions and that lead us to the supersymmetric extensions of the Brown-Henneaux boundary conditions. From these restrictions we also impose how the asymptotic symmetry algebras reduce to two copies of the \( \mathcal{SW}(\frac{3}{2},2) \) algebra for the most general \( \mathcal{N} = (1,1) \) extended higher spin supergravity theory in \( AdS_3 \). So, one can think that this method provides a good laboratory for investigating the rich asymptotic structure of extended supergravity.

The outline of the paper is as follows. In the next section, we give briefly a fundamental formulation of \( \mathcal{N} = (1,1) \) supergravity in the perspective of \( \mathfrak{osp}(2|1) \oplus \mathfrak{osp}(2|1) \) Chern-Simons gauge theory for both affine and superconformal boundaries respectively in three-dimensions. In section 3, we carry out our calculations to extend the theory to \( \mathfrak{osp}(3|2) \oplus \mathfrak{osp}(3|2) \) higher spin Chern-Simons supergravity in the presence of both affine and superconformal boundaries, in which we showed explicitly principal embedding of \( \mathfrak{osp}(2|1) \oplus \mathfrak{osp}(2|1) \) and also demonstrated how asymptotic symmetry and higher spin Ward identities arise from the bulk equations of motion coupled to spin \( s = \frac{3}{2}, 2, \frac{5}{2} \) currents. Finally, the last section is devoted to the case of classical two copies of the \( \mathfrak{osp}(3|2)_k \) affine algebra on the affine boundary and \( \mathcal{SW}(\frac{3}{2},2) \) symmetry algebra on the superconformal boundary as asymptotic symmetry algebras, and also the chemical potentials related to source fields appearing through the temporal components of the connection are obtained. The final section contains our summary and conclusion.

II. SUPERGRAVITY IN THREE-DIMENSIONS, A REVIEW:

In this section we review the Chern-Simons formalism for higher spin supergravity. In particular, we use this formalism to study supergravity in the \( \mathfrak{osp}(2|1) \) superalgebra basis under the same metric class as the recently constructed most general \( AdS_3 \) boundary condition by Grumiller and Riegler.

A. Connection to Chern-Simons Theory

The three-dimensional Einstein-Hilbert action for \( \mathcal{N} = (1,1) \) supergravity with negative cosmological constant is classically equivalent to the Chern-Simons action, as it was first proposed by Achucarro and Townsend in\(^{13}\) and developed by Witten in\(^{2}\). One can start by defining 1-forms \( (\Gamma, \tilde{\Gamma}) \) taking values in the gauge group’s \( \mathfrak{osp}(2|1) \) superalgebra, and also the supertrace \( \mathfrak{str} \) is taken over the superalgebra...
generators. The Chern-Simons action can be written in the form,

\[ S = S_{CS}[\Gamma] - S_{CS}[\bar{\Gamma}] \]  

(1)

where

\[ S_{CS}[\Gamma] = \frac{k}{4\pi} \int \text{str} \left( \Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right). \]  

(2)

Here \( k = \frac{\ell^2}{4G_{\text{str}}(L_0^2)} \) is the level of the Chern-Simons theory depending on the \( \text{AdS} \) radius \( \ell \) and the Newton’s constant \( G \) with the related central charge \( c \) of the superconformal field theory. Nevertheless, supertrace shows a metric on the \( \text{osp}(2|1) \) superalgebra. If \( L_i, (i = \pm 1, 0) \) and \( G_p, (p = \pm \frac{1}{2}) \) are the generators of \( \text{osp}(2|1) \) superalgebra. We have expressed \( \text{osp}(2|1) \) superalgebra such that

\[ [L_i, L_j] = (i - j)L_{i+j}, \quad [L_i, G_p] = (\frac{i}{2} - p)G_{i+p}, \quad \{G_p, G_q\} = -2L_{p+q}. \]  

(3)

The equations of motion for the Chern-Simons gauge theory give the flatness condition \( F = \bar{F} = 0 \) where

\[ F = d\Gamma + \Gamma \wedge \Gamma = 0 \]  

(4)

is the same as the Einstein’s equation. \( \Gamma \) and \( \bar{\Gamma} \) are related to the metric \( g_{\mu\nu} \) through the veilbein

\[ g_{\mu\nu} = \frac{1}{2} \text{str}(e_{\mu}e_{\nu}). \]  

(5)

One can choose a radial gauge of the form

\[ \Gamma = b^{-1}a(t, \phi) b + b^{-1}db, \quad \bar{\Gamma} = b\bar{a}(t, \phi) b^{-1} + bdb^{-1} \]  

(6)

with state-independent group element as

\[ b(\rho) = e^{\rho_{\text{Lo}}}. \]  

(7)

which manifests all the \( \text{osp}(2|1) \) charges and chemical potentials and also the choice of \( b \) is irrelevant in the case of asymptotic symmetry, as long as \( db = 0 \). Moreover, \( a(t, \phi) \) and \( \bar{a}(t, \phi) \) in the radial gauge are the \( \text{osp}(2|1) \) superalgebra valued fields, which are independent from the radial coordinate, \( \rho \) as

\[ a(t, \phi) = a_t(t, \phi) dt + a_\phi(t, \phi) d\phi. \]  

(8)

B. \( \text{osp}(2|1) \oplus \text{osp}(2|1) \) Chern-Simons \( \mathcal{N} = (1, 1) \) Supergravity for Affine Boundary

Affine case is given by reviewing asymptotically \( \text{AdS}_3 \) boundary conditions for a \( \text{osp}(2|1) \oplus \text{osp}(2|1) \) Chern-Simons theory, and how to determine the asymptotic symmetry algebra using the method described in\(^{22} \). Thus the most general solution of the Einstein’s equation that is asymptotically \( \text{AdS}_3 \), as a generalization of Fefferman-Graham method is given by with a flat boundary metric

\[ ds^2 = d\rho^2 + 2 \left[ e^{\rho}N^{(0)}_i + N^{(1)}_i + e^{-\rho}N^{(2)}_i + \mathcal{O}(e^{-2\rho}) \right] d\rho dx^i \\
+ \left[ e^{2\rho}g^{(0)}_{ij} + e^\rho g^{(1)}_{ij} + g^{(2)}_{ij} + \mathcal{O}(e^{-\rho}) \right] dx^i dx^j. \]  

(9)
We need to choose the most general boundary conditions for $\mathcal{N} = (1, 1)$ supergravity such that they maintain this metric form. In the following we only focus on the $\Gamma$-sector. Therefore, one can propose to write the components of the $\mathfrak{osp}(2|1)$ superalgebra valued connection in the form,

$$
a_{\varphi} (t, \varphi) = \sum_{i=-1}^{+1} \alpha_i L^i (t, \varphi) L_i + \sum_{p=-1/2}^{+1/2} \beta_p G^p (t, \varphi) G_p
$$

where $(\alpha_i, \beta_p)$'s are some scaling parameters to be determined later and we have five functions: three bosonic $L^i$ and two fermionic $G^p$. They are usually called \textit{charges} and also the time component of the connection $a_t (t, \varphi)$

$$
a_t (t, \varphi) = \sum_{i=-1}^{+1} \mu^i (t, \varphi) L_i + \sum_{p=-1/2}^{+1/2} \nu^p (t, \varphi) G_p.
$$

Here, the time component have in total five independent functions $(\mu^i, \nu^p)$. They are usually called \textit{chemical potentials}. But, they are not allowed to vary

$$
\delta a_{\varphi} = \sum_{i=-1}^{+1} \alpha_i \delta L^i L_i + \sum_{p=-1/2}^{+1/2} \beta_p \delta G^p G_p,
\delta a_t = 0.
$$

The flat connection conditions \cite{16} for fixed chemical potentials impose the following additional conditions as the temporal evolution of the five independent source fields $(L^i, G^p)$ as

$$
\alpha_{\pm 1} \partial_t L^{\pm 1} = \partial_{\varphi} \mu^{\pm 1} \mp \alpha_0 L^0 \mu^{\pm 1} \pm \alpha_{\pm 1} L^{\pm 1} \mu^0 - 2\beta_{\pm 1/2} G^{\mp \beta} \nu_{\pm 1/2},
$$

$$
\frac{1}{2} \alpha_0 \partial_t L^0 = \frac{1}{2} \partial_{\varphi} \mu^0 + \alpha_{+1} L^{+1} \mu^{-1} - \alpha_{-1} L^{-1} \mu^{-1} - \beta_{+1/2} G^{+ \beta} \nu^{-1/2} - \beta_{-1} G^{- \beta} \nu^{+1/2},
$$

$$
\beta_{\pm 1/2} \partial_t G^{\pm 1/2} = \partial_{\varphi} \nu^{\pm 1/2} \pm \alpha_{\pm 1} L^{\pm 1} \nu^{\mp 1/2} + \frac{1}{2} \alpha_0 L^0 \nu^{\mp 1/2} \mp \beta_{+1/2} \mu^{\pm 1/2} \nu^{\mp 1/2} \pm \frac{1}{2} \beta_{-1/2} \mu^0 G^{\mp 1/2}.
$$

After the temporal evolution of the source fields, one can start to compute the gauge transformations for asymptotic symmetry algebra by considering all transformations

$$
\delta \lambda \Gamma = d\lambda + [\Gamma, \lambda] = \mathcal{O} (\delta \Gamma)
$$

that preserve the boundary conditions with the gauge parameter in the $\mathfrak{osp}(2|1)$ superalgebra

$$
\lambda = b^{-1} \left[ \sum_{i=-1}^{+1} \epsilon^i (t, \varphi) L_i + \sum_{p=-1/2}^{+1/2} \zeta^p (t, \varphi) G_p \right] b.
$$

Here, the gauge parameter have in total five arbitrary functions : bosonic $\epsilon^i$ and fermionic $\zeta^p$ on the boundary. The condition \cite{17} impose that transformations on the gauge are given by

$$
\alpha_{\pm 1} \partial_\lambda L^{\pm 1} = \partial_\varphi \epsilon^{\pm 1} \mp \alpha_0 L^0 \epsilon^{\pm 1} \pm \alpha_{\pm 1} L^{\pm 1} \epsilon^0 - 2\beta_{\pm 1/2} G^{\mp \beta} \zeta_{\pm 1/2},
$$

$$
\frac{1}{2} \alpha_0 \partial_\lambda L^0 = \frac{1}{2} \partial_\varphi \epsilon^0 + \alpha_{+1} L^{+1} \epsilon^{-1} - \alpha_{-1} L^{-1} \epsilon^{-1} - \beta_{+1/2} G^{+ \beta} \zeta^{-1/2} - \beta_{-1} G^{- \beta} \zeta^{+1/2},
$$

$$
\beta_{\pm 1/2} \partial_\lambda G^{\pm 1/2} = \partial_\varphi \zeta^{\pm 1/2} \pm \alpha_{\pm 1} L^{\pm 1} \zeta^{\mp 1/2} + \frac{1}{2} \alpha_0 L^0 \zeta^{\mp 1/2} \mp \beta_{+1/2} \epsilon^{\pm 1/2} G^{\mp 1/2} \pm \frac{1}{2} \beta_{-1/2} \epsilon^0 G^{\pm 1/2}.
$$
Since the chemical potentials \((\mu^m, \nu^p)\) are fixed, they obey the following transformations

\[
\alpha_{\pm 1} \partial_\lambda \mu^{\pm 1} = \partial_\epsilon^{\pm 1} = \alpha_0 \mu^{\epsilon} + \alpha_{\pm 1} \mu^{\pm 1} \epsilon - 2 \beta_{\pm 1} \nu^{\pm 1} \zeta^{\pm 1},
\]

\[
\frac{1}{2} \alpha_0 \partial_\lambda \mu^0 = \frac{1}{2} \partial_\epsilon^0 + \alpha_{\pm 1} \mu^{\epsilon + 1} \epsilon - \alpha - \mu^{\epsilon - 1} - \beta_{\pm 1} \nu^{\pm 1} \zeta^{\pm 1} - \beta_{\pm 1} \nu^{\pm 1} \zeta^{\pm 1},
\]

\[
\beta_{\pm 1} \partial_\lambda \nu^{\pm 1} = \partial_\zeta^{\pm 1} = \alpha_{\pm 1} \mu^{\zeta} + \frac{1}{2} \alpha_0 \mu^0 \zeta^{\pm 1} + \beta_{\pm 1} \nu^{\pm 1} \zeta^{\pm 1} + \frac{1}{2} \beta_{\pm 1} \nu^0 \zeta^{\pm 1}.
\]

As a final step, one now has to determine the canonical boundary charge \(Q[\lambda]\) that generates the transformations \([19]-[21]\). Therefore, the corresponding variation of the boundary charge \(Q[\lambda]\) in order to show the asymptotic symmetry algebra, is given by

\[
\delta \lambda Q = \frac{k}{2\pi} \int d\varphi \, \text{str} (\lambda \delta \Gamma_\varphi).
\]

The canonical boundary charge \(Q[\lambda]\) can be integrated which reads

\[
Q[\lambda] = \int d\varphi \left[ L^0 \epsilon^0 + L^{\pm 1} \epsilon^{-1} - L^{-1} \epsilon^{+1} - G^{+1} \zeta^{-1} + G^{-1} \zeta^{+1} \right] .
\]

We now prefer to work in complex coordinates for affine boundary, \(z(\bar{z}) \equiv \varphi \pm i \bar{\varphi}\). After both the infinitesimal transformations and the canonical boundary charge have been determined, one can yields the Poisson bracket algebra by using the method \([20]\) with

\[
\delta \lambda F = \{ F, Q[\lambda] \}
\]

for any phase space functional \(F\):

\[
\{ L^i(z_1), L^j(z_2) \}_{F, \theta} = (i - j) L^{i+j}(z_2) \delta(z_1 - z_2) - \frac{1}{\alpha_{\pm 1}} \eta^{ij} \partial_\varphi \delta(z_1 - z_2),
\]

\[
\{ L^i(z_1), G^p(z_2) \}_{F, \theta} = \left( \frac{i}{2} - p \right) G^{i+p}(z_2) \delta(z_1 - z_2),
\]

\[
\{ G^p(z_1), G^q(z_2) \}_{F, \theta} = -2L^{p+q}(z_2) \delta(z_1 - z_2) - \frac{1}{\alpha_{\pm 1}} \eta^{pq} \partial_\varphi \delta(z_1 - z_2)
\]

where \(\alpha_{\pm 1} = \frac{-2\pi}{k}\) for the convention in the literature and \(\eta^{ij}\) is the bilinear form in the fundamental representation of \(osp(2|1)\) superalgebra. One can also expand \(L(z)\) and \(G(z)\) charges into Fourier modes \(L^i(z) = \frac{1}{2\pi} \sum_n L^i_n \zeta^{-n-1}\), and \(G^p(z) = \frac{1}{2\pi} \sum_q G^p_q \zeta^{-p-q}\), and also replacing \(i \{ \cdot, \cdot \}_{F, \theta} \rightarrow [ \cdot, \cdot ]\). A mode algebra can then be defined as:

\[
\left[ L^i_n, L^j_n \right] = (i - j) L^{i+j}_{m+n} + k n \eta^{ij} \delta_{m+n, 0},
\]

\[
\left[ L^i_n, G^p_r \right] = \left( \frac{i}{2} - p \right) G^{i+p}_{m+r},
\]

\[
\{ G^p_r, G^q_s \} = -2L^{p+q}_{r+s} + k s n \eta^{pq} \delta_{r+s, 0}.
\]

Besides, this mode algebra in these space is equivalent to operator product algebra,

\[
L^i(z_1) L^j(z_2) \sim \frac{k \eta^{ij}}{z_{12}^2} + \frac{(i - j)}{z_{12}} L^{i+j}(z_2),
\]

\[
L^i(z_1) G^p(z_2) \sim \frac{(\frac{i}{2} - p)}{z_{12}} G^{i+p}(z_2),
\]

\[
G^p(z_1) G^q(z_2) \sim \frac{k \eta^{pq}}{z_{12}^2} + \frac{2}{z_{12}} L^{p+q}(z_2)
\]
where $z_{12} = z_1 - z_2$, or in the more compact form,

$$J^A(z_1)J^B(z_2) \sim \frac{k \eta^{AB}}{z_{12}} + \frac{f^{AB} C^C(z_2)}{z_{12}}. \quad (37)$$

Here, $\eta^{AB}$ is the supertrace matrix and $f^{AB}$'s are the structure constants of the related algebra with $(A, B = 0, \pm 1, \pm \frac{1}{2})$, i.e. $\eta^{p+} = 0$ and $f^{ij}_{i+j} = (i - j)$. After repeating the same algebra for $\Gamma$-sector, one can say that the asymptotic symmetry algebra for the most general boundary conditions of $\mathcal{N} = (1, 1)$ supergravity is two copies of the affine $\mathfrak{osp}(2|1)_k$ algebra as in Ref.30.

**C. $\mathfrak{osp}(2|1) \oplus \mathfrak{osp}(2|1)$ Chern - Simons $\mathcal{N} = (1, 1)$ Supergravity for Superconformal boundary**

Under the following restrictions as the Drinfeld - Sokolov highest weight gauge condition,

$$L^0 = G^+ = 0, \quad L^{-1} = L, \quad G^- = G, \quad \alpha_+ L^+ = 1 \quad (38)$$
on on the boundary conditions with the $\mathfrak{osp}(2|1)$ superalgebra valued connection\[^{10}\], one can get the superconformal boundary conditions as the supersymmetric extension of the Brown-Henneaux boundary conditions proposed in\[^{14}\] for $\text{AdS}_3$ supergravity. Therefore we have the supersymmetric connection as,

$$a_\phi = L_1 + \alpha_- L_{-1} + \beta_{-\frac{1}{2}} G_{-\frac{3}{2}}, \quad (39)$$

$$a_\epsilon = \mu L_1 + \sum_{i=1}^{0} \mu^i L_i + \nu G_{\frac{1}{2}} + \nu^{-\frac{1}{2}} G_{-\frac{1}{2}} \quad (40)$$

where $\mu \equiv \mu^+ 1, \nu \equiv \nu^{+\frac{1}{2}}$ can be interpreted as the independent chemical potentials. This means that we assume the chemical potential to be fixed at infinity, i.e. $\delta \mu = 0$. The functions $\mu^0, \mu^{-1}$ and $\nu^{-\frac{1}{2}}$ are fixed by the flatness condition \[^{11}\] as

$$\mu^0 = -\mu', \quad (41)$$

$$\mu^{-1} = \frac{1}{2} \mu'' + \alpha_- L \mu + \beta_{-\frac{1}{2}} G \nu, \quad (42)$$

$$\nu^{-\frac{1}{2}} = -\nu' + \beta_{-\frac{1}{2}} G \nu. \quad (43)$$

For the fixed chemical potentials $\mu$ and $\nu$, the time evolution of canonical boundary charges $L$ and $G$ can be written as

$$\partial_\epsilon L = -\frac{\mu''}{2\alpha_-} + 2 L \mu' + L' \mu + \frac{\beta_{-\frac{1}{2}}}{\alpha_-} \left( 3 G \nu' + G' \nu \right), \quad (44)$$

$$\partial_\epsilon G = -\frac{\nu''}{\alpha_-} - \frac{3}{2} L \nu + \frac{1}{\beta_{-\frac{1}{2}}} \left( 3 G \nu' + G' \nu \right) \quad (45)$$

where $\alpha_- 1$ and $\beta_{-\frac{1}{2}}$ are some scaling parameters to be determined later. Now, we are in a position to work the superconformal asymptotic symmetry algebra under the Drinfeld - Sokolov reduction. This reduction imply that the only independent parameters $\epsilon \equiv \epsilon^{+1}$, and $\zeta \equiv \zeta^{+\frac{1}{2}}$. One can start to compute the gauge transformations for asymptotic symmetry algebra by considering all transformations \[^{17}\] that preserve the boundary conditions, with the gauge parameter in the $\mathfrak{osp}(2|1)$ superalgebra \[^{18}\] as

$$\lambda = b^{-1} \left[ \alpha L_1 - \epsilon' L_0 + \frac{1}{2} \epsilon'' + \alpha_- L \epsilon + \beta_{-\frac{1}{2}} G \zeta \right] L_{-1} + \zeta G_{-\frac{1}{2}} + \left( \beta_{-\frac{1}{2}} G \epsilon - \zeta' \right) G_{-\frac{1}{2}}. \quad (46)$$
The condition (17) impose that transformations on the gauge with \( \alpha_{-1} = \frac{6}{7} \) and \( \beta_{-1} = -\frac{3}{7} \) are given by

\[
\partial_{\lambda} \mathcal{L} = \frac{c}{12} \epsilon'' + 2 \mathcal{L} \epsilon' + \mathcal{L}' \mathcal{L} - \frac{1}{2} \left( 3 \mathcal{L} \zeta' + \mathcal{G}' \zeta \right),
\]

\[
\partial_{\lambda} \mathcal{G} = \frac{c}{3} \zeta'' + 2 \mathcal{L} \zeta + \frac{3}{2} \epsilon' \mathcal{G} + \epsilon \mathcal{G}'.
\]

The variation of canonical boundary charge \( \mathcal{Q}[\lambda] \) can be integrated which reads

\[
\mathcal{Q}[\lambda] = \int d\phi \left[ \mathcal{L} \epsilon + \mathcal{G} \zeta \right].
\]

This leads to operator product expansions in the complex coordinates by using (27)

\[
\mathcal{L}(z_1) \mathcal{L}(z_2) \sim \frac{z_2}{z_1^2} + \frac{c L}{z_1^2} + \frac{c L'}{z_1^2},
\]

\[
\mathcal{L}(z_1) \mathcal{G}(z_2) \sim \frac{2 G}{z_1^2} + \frac{G'}{z_1^2}, \quad \mathcal{G}(z_1) \mathcal{G}(z_2) \sim \frac{G}{z_1^2} + \frac{2 C}{z_1^2}.
\]

After repeating the same algebra for \( \bar{\Gamma} \)-sector, one can say that the asymptotic symmetry algebra for a set of boundary conditions of \( \mathcal{N} = (1,1) \) supergravity is two copies of the super-Virasoro algebra with central charge \( c = 6k \).

III. \( \mathcal{N} = (1,1) \) \( \text{osp}(3|2) \oplus \text{osp}(3|2) \) HIGHER SPIN CHERN - SIMONS SUPERGRAVITY

A. For Affine Boundary

In this section, an extension of the \( \mathcal{N} = (1,1) \) higher spin supergravity theory constructed here as \( \text{osp}(3|2) \oplus \text{osp}(3|2) \) is defined by the \( \text{osp}(2|1) \oplus \text{osp}(2|1) \) algebra that is present as a sub-superalgebra as the ordinary case of \( \text{osp}(N|N-1) \) gauge algebra. If \( L_i (i = \pm 1, 0), G_p (p = \pm \frac{1}{2}), A_i (i = \pm 1, 0), \) and \( S_p (p = \pm \frac{1}{2}, \pm \frac{3}{2}) \) are the generators of \( \text{osp}(3|2) \) superalgebra, we have expressed \( \text{osp}(3|2) \) superalgebra such that

\[
[L_i, L_j] = (i - j) L_{i+j}, \quad [L_i, G_p] = \left( \frac{i}{2} - p \right) G_{i+p},
\]

\[
\{G_p, G_q\} = \sigma_1 A_{p+q} + \sigma_2 L_{p+q}, \quad \{G_p, S_q\} = \left( \frac{3p}{2} - q \right) \left( \sigma_3 A_{p+q} + \sigma_4 L_{p+q} \right),
\]

\[
[L_i, A_j] = \left( \frac{i}{2} - j \right) A_{i+j}, \quad [L_i, S_p] = \left( \frac{3i}{2} - p \right) S_{i+p},
\]

\[
[A_i, A_j] = (i - j) (\sigma_5 L_{i+j} + \sigma_6 A_{i+j}), \quad [A_i, G_p] = \sigma_7 S_{i+p} + \sigma_8 \left( \frac{i}{2} - p \right) G_{i+p},
\]

\[
[A_i, S_p] = \sigma_9 \left( \frac{3i}{2} - p \right) S_{i+p} + \sigma_{10} \left( 3i^2 - 2ip + p^2 - \frac{9}{4} \right) G_{i+p},
\]

\[
[S_p, S_q] = \left( \frac{3p^2 - 4pq + 3q^2 - \frac{9}{2}}{2} \right) \left( \sigma_1 A_{p+q} + \sigma_2 L_{p+q} \right).
\]

The super Jacobi identities give us the nontrivial relations for some constants \( \sigma_i' \), \( i = 1, 2, \ldots, 12 \) appearing on the RHS of eqs. (52) - (57) as,

\[
\sigma_1 = 0, \quad \sigma_2 = -\sigma_3 \sigma_7, \quad \sigma_4 = -\sigma_3 \sigma_8, \quad \sigma_5 = \frac{1}{2} (\sigma_8 - 2 \sigma_9) (\sigma_8 + \sigma_9), \quad \sigma_6 = \frac{1}{2} (5 \sigma_9 - \sigma_8), \quad \sigma_{10} = \frac{(\sigma_8 - \sigma_9)^2}{4 \sigma_7}, \quad \sigma_{11} = \frac{\sigma_3 (\sigma_8 - \sigma_9)}{4 \sigma_7}, \quad \sigma_{12} = \frac{\sigma_3 \sigma_9 (\sigma_8 - \sigma_9)}{4 \sigma_7}.
\]
For the corresponding affine algebra, the resulting boundary relations are as \( \sigma_3 = 2, \ \sigma_7 = -1, \ \sigma_8 = 0, \ \sigma_9 = 1 \). We are now ready to formulate most general boundary conditions for asymptotically \( AdS_3 \) spacetimes:

\[
\begin{align*}
\mathcal{A}_\varphi &= \alpha_i \mathcal{L}_i + \gamma_i \mathcal{A}_i + \beta_p \mathcal{G}_p + \delta_p \mathcal{S}_p, \\
\mathcal{A}_i &= \mu^i \mathcal{L}_i + \chi^i \mathcal{A}_i + f^i \mathcal{G}_p + \nu^i \mathcal{S}_p
\end{align*}
\]

(60)

(61)

where \((\alpha_i, \gamma_i, \beta_p, \delta_p)\)'s are some scaling parameters to be determined later and we have twelve functions: six bosonic \((\mathcal{L}^i, \mathcal{A}^i)\) and six fermionic \((\mathcal{G}^p, \mathcal{S}^p)\) as the *charges* and also here, the time component have in total twelve independent functions \((\mu^i, \chi^i, f^i, \nu^i)\) as the *chemical potentials*. The flat connection conditions \([\ref{11}]\) for fixed chemical potentials impose the following additional conditions as the temporal evolution of the twelve independent source fields, \((\mathcal{L}^i, \mathcal{A}^i, \mathcal{G}^p, \mathcal{S}^p)\) as

\[
\begin{align*}
\alpha_{\pm 1} \partial_t \mathcal{L}^{\pm 1} &= \partial_{\varphi} \mu^{\pm 1} + \alpha_{\pm 1} \mu^{0} \mathcal{L}^{\pm 1} + \alpha_{0} \mu^{\pm 1} \mathcal{L}^{0} + 2 \beta_{\pm 1} \mathcal{G}^{\pm \frac{1}{2}} \nu^{\pm \frac{1}{2}} + \mathcal{A}^{\pm 1} \gamma_{\pm 1} \chi^{0} + \mathcal{A}^{0} \gamma_{0} \chi^{1} \\
&+ \frac{3 \beta_{\pm 1} f^{\pm \frac{1}{2}} \delta_{\pm 1} \mathcal{S}^{\pm \frac{1}{2}}}{2} - 2 \delta_{\pm 1} f^{\frac{1}{2}} \mathcal{S}^{\pm \frac{1}{2}} + \frac{3 \beta_{\pm 1} f^{\frac{1}{2}} S^{\pm \frac{1}{2}}}{2},
\end{align*}
\]

(62)

\[
\frac{1}{2} \alpha_{0} \partial_t \mathcal{L}^{0} = \frac{1}{2} \partial_{\varphi} \mu^{0} + \alpha_{0} \mu^{0} \mathcal{L}^{0} - \alpha_{1} \mu^{1} \mathcal{L}^{0} - \beta_{1} \mathcal{G}^{0} \mathcal{S}^{0} - \delta_{1} \mathcal{G}^{0} \mathcal{S}^{0}.
\]

(63)

\[
\begin{align*}
\alpha_{\pm 1} \partial_t \mathcal{G}^{\pm \frac{1}{2}} &= \partial_{\varphi} \nu^{\pm \frac{1}{2}} + \frac{3}{2} \alpha_{0} \nu^{\pm \frac{1}{2}} \mathcal{L}^{\pm 1} + \beta_{1} \mathcal{G}^{0} \mathcal{S}^{1} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \frac{1}{2} \beta_{1} \mathcal{G}^{1} \mathcal{S}^{0} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \frac{1}{2} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0}.
\end{align*}
\]

(64)

\[
\frac{1}{2} \gamma_{0} \partial_t \mathcal{A}^{0} = \frac{1}{2} \partial_{\varphi} \chi^{0} + \mathcal{A}^{1} \gamma_{1} \mathcal{L}^{1} - \mathcal{A}^{0} \gamma_{1} \mathcal{L}^{0} + 2 \beta_{1} \mathcal{G}^{0} \mathcal{S}^{1} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + 2 \beta_{1} \mathcal{G}^{1} \mathcal{S}^{0} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \frac{1}{2} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0}.
\]

(65)

\[
\begin{align*}
\gamma_{\pm 1} \partial_t \mathcal{A}^{\pm 1} &= \partial_{\varphi} \chi^{\pm 1} + \mathcal{A}^{1} \gamma_{1} \mathcal{L}^{1} + \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \beta_{1} \mathcal{G}^{1} \mathcal{S}^{0} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \beta_{1} \mathcal{G}^{0} \mathcal{S}^{1} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \beta_{1} \mathcal{G}^{1} \mathcal{S}^{0} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \frac{1}{2} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0}.
\end{align*}
\]

(66)

\[
\begin{align*}
\delta_{\pm 2} \partial_t \mathcal{S}^{\pm \frac{1}{2}} &= \partial_{\varphi} f^{\pm \frac{1}{2}} - \mathcal{A}^{1} \gamma_{1} \mathcal{L}^{1} + \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \beta_{1} \mathcal{G}^{0} \mathcal{S}^{1} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \beta_{1} \mathcal{G}^{0} \mathcal{S}^{1} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0} + \frac{1}{2} \mathcal{A}^{0} \gamma_{0} \mathcal{L}^{0}.
\end{align*}
\]

(67)

After the temporal evolution of the source fields, one can start to compute the gauge transformations for asymptotic symmetry algebra by considering all transformations \([\ref{17}]\) that preserve the boundary
conditions, with the gauge parameter $\lambda$ in the $\mathfrak{osp}(3|2)$ superalgebra as,

$$\lambda = b^{-1} \left[ e^i L_i + \kappa^i A_i + \zeta^\rho G_\rho + \varrho^\rho S_\rho \right] b.$$  \hfill (69)

Here, the gauge parameter have in total twelve arbitrary functions six bosonic ($e^i, \kappa^i$) and six fermionic ($\zeta^\rho, \varrho^\rho$) on the boundary. The condition (17) impose that transformations on the gauge are given by

$$\alpha_{\pm 1} \partial_\lambda L^{\pm 1} = \partial_\lambda (e^{\pm 1} \pm \alpha_{\pm 1} e^0 L^{\pm 1} + \alpha_0 e^{\pm 1} L^0 + 2 \beta_{\pm 4} G^{\pm 4} g^{-\frac{1}{2}} + \mp A^{\pm 1} \gamma_{\pm 1} \kappa^0 \pm A^0 \gamma_0 \kappa^0 \pm 3 \delta_{\pm 4} \zeta^{2 \pm 4} S^{\mp 2} - 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{\mp 2} + 3 \delta_{\pm 4} \zeta^{2 \pm 4} S^{\mp 2},$$  \hfill (70)

$$\frac{1}{2} \alpha_0 \partial_\lambda L^0 = \frac{1}{2} \partial_\lambda e^0 + \alpha_{\pm 1} e^{\pm 1} L^0 - \alpha_{\pm 1} e^0 L^{\pm 1} + \beta_{\pm 4} G^{\pm 4} g^{-\frac{1}{2}} + \beta_{\pm 4} G^{\pm 4} g^{-\frac{1}{2}} + \beta_{\pm 4} G^{\pm 4} g^{-\frac{1}{2}}$$  \hfill (71)

$$\frac{1}{2} \alpha_0 \partial_\lambda L^{0} = \frac{1}{2} \partial_\lambda e^0 + \alpha_{\pm 1} e^{\pm 1} L^0 + \beta_{\pm 4} G^{\pm 4} g^{-\frac{1}{2}} e^{\pm 1} + \frac{1}{2} \beta_{\pm 4} G^{\pm 4} e^0 + \frac{3}{2} \partial_{\pm 4} \gamma_{\pm 1} \kappa^0 + \frac{1}{2} \alpha_{\pm 1} e^{\pm 1} L^0 - \frac{1}{2} \beta_{\pm 4} G^{\pm 4} g^{-\frac{1}{2}} e^0$$  \hfill (72)

$$\frac{1}{2} \gamma_0 \partial_\lambda A^0 = \frac{1}{2} \partial_\lambda e^0 + A^{\pm 1} \gamma_{\pm 1} e^{-1} - A^{-1} \gamma_{-1} e^{1} + \frac{5}{2} \alpha_{\pm 1} e^{\pm 1} L^0 - \frac{5}{2} A^{-1} \gamma_{-1} e^{1} + \beta_{\pm 4} G^{\pm 4} S^{-\frac{3}{2}} + \beta_{\pm 4} G^{\pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} - 9 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}}$$  \hfill (73)

$$\gamma_{\pm 1} \partial_\lambda A^{\pm 1} = \partial_\lambda e^{\pm 1} \pm A^{\pm 1} \gamma_{\pm 1} e^{-1} + 2 A^{\pm 1} \gamma_{\pm 1} e^{-1} + 2 A^{\pm 1} \gamma_{\pm 1} e^{1} - 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} - 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} - 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \alpha_{\pm 1} \kappa^0 L^0 - \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}}$$  \hfill (74)

$$\delta_{\pm 4} \partial_\lambda S^{\pm 1/2} = \partial_\lambda e^{\pm 1} - A^{\pm 1} \gamma_{\pm 1} e^{-1} - A^{-1} \gamma_{-1} e^{1} + 3 A^{\pm 1} \gamma_{\pm 1} e^{1} + \frac{1}{2} e^0 \gamma_0 e^{\pm 1} + 2 A^{1} \gamma_{1} e^{1} + 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} - 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \alpha_{\pm 1} \kappa^0 L^0 - \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}}$$  \hfill (75)

$$\delta_{\pm 4} \partial_\lambda S^{\pm 1/2} = \partial_\lambda e^{\pm 1} - A^{\pm 1} \gamma_{\pm 1} e^{-1} - A^{-1} \gamma_{-1} e^{1} + 3 A^{\pm 1} \gamma_{\pm 1} e^{1} + \frac{1}{2} e^0 \gamma_0 e^{\pm 1} + 2 A^{1} \gamma_{1} e^{1} + 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} - 2 \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \alpha_{\pm 1} \kappa^0 L^0 - \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}} + \delta_{\pm 4} \zeta^{2 \pm 4} S^{-\frac{3}{2}}$$  \hfill (76)

Since the chemical potentials ($\mu^i, \chi^i, f^p, \nu^p$) are fixed, they obey the following transformations

$$\alpha_{\pm 1} \partial_\mu \mu^{\pm 1} = \partial_\lambda e^{\pm 1} + \alpha_{\pm 1} e^{\pm 1} \mu^{\pm 1} + \alpha_0 e^{\pm 1} \mu^0 + 2 \beta_{\pm 4} f^{\pm 4} g^{-\frac{1}{2}} + \chi^{\pm 1} \gamma_{\pm 1} \kappa^0 + \chi^{\pm 1} \gamma_{\pm 1} \kappa^0 + \frac{3}{2} \alpha_0 \zeta^{2 \pm 4} L^0 + \alpha_{\pm 1} \zeta^{2 \pm 4} L^0$$  \hfill (77)
As in the $\text{osp}(2|1) \oplus \text{osp}(2|1)$ case one can now determine the canonical boundary charges $Q[\lambda]$ that generates the transformations (70)-(76). Therefore, the corresponding variation of the boundary charge $Q[\lambda]$, in order to show the asymptotic symmetry algebra, is given by (25). The canonical boundary charge $Q[\lambda]$ can be integrated which reads

$$ Q[\lambda] = \int d\varphi \left[ \mathcal{L}^i \epsilon^{-i} + \mathcal{A}^i \zeta^{-i} + G^p \zeta^{-p} + S^p \varphi^{-p} \right]. $$

Having determined both the infinitesimal transformations and the canonical boundary charges as the generators of the asymptotic symmetry algebra one can also get the Poisson bracket algebra by using the methods with (27) again for any phase space functional $F$, the operator product algebra can then be defined as,

$$ \mathcal{L}^i(z_1) \mathcal{L}^j(z_2) \sim \frac{k_{ij}}{z_{12}^2} + \frac{(i-j)}{z_{12}} \mathcal{L}^{i+j}, $$

$$ \mathcal{L}^i(z_1) G^p(z_2) \sim \frac{(i-p)}{z_{12}} G^{i+p}, $$
flatness condition (4). For the fixed chemical potentials, the time e volution of canonical boundary charges proposed in perconformal boundary conditions as the supersymmetric exten sion of the Brown - Henneaux boundary identities of the osp

where $z_{12} = z_1 - z_2$, or in the more compact form,

$$3^A(z_1)3^B(z_2) \sim \frac{k \eta^{AB}}{z_{12}^2} + \frac{f^{ABC} \gamma^C(z_2)}{z_{12}^2}. \tag{94}$$

Here, $\eta^{AB}$ is the supertrace matrix and $f^{ABC}$’s are the structure constants of the related algebra with $(A, B = 0, \pm 1, \pm \frac{1}{2})$, i.e, $\eta^{ip} = 0$ and $f^{ij}_{i+j} = (i-j)$. After repeating the same algebra for $\Gamma$ - sector, one can say that the asymptotic symmetry algebra for the most general boundary conditions of $\mathcal{N} = (1, 1)$ supergravity is two copies of the affine $\mathfrak{osp}(3|2)_k$ algebra.

B. For Superconformal Boundary

Under the following restrictions as the Drinfeld - Sokolov highest weight gauge condition,

$$\mathcal{L}^0 = A^0 = A^+ = \mathcal{G}^+ = \mathcal{S}^+ = 0, \quad \mathcal{L}^{-1} = \mathcal{L}, \quad A^{-1} = A, \quad \mathcal{G}^- = \mathcal{G}, \quad \mathcal{S}^- = \mathcal{S}, \quad \alpha_{-1} \mathcal{L}^+ = 1 \tag{95}$$

on the boundary conditions with the $\mathfrak{osp}(3|2)$ superalgebra valued connection[10], one can get the superconformal boundary conditions as the supersymmetric extension of the Brown - Henneaux boundary conditions proposed in[1] for $\text{AdS}_3$ supergravity. Therefore we have the supersymmetric connection as,

$$a_p = L_1 + \alpha_{-1} A L_{-1} + \gamma_{-1} A L_{-1} + \beta_{-1} \mathcal{G}_{-1} + \delta_{-1} \mathcal{S}_{-1}, \tag{96}$$

$$a_t = \mu L_1 + \alpha A_1 + f \mathcal{G}^+ + \nu \mathcal{S}^+ + \sum_{i=-1}^{0} \mu^i L_i + \sum_{i=-1}^{0} \chi^i A_i + f^+ \mathcal{G}^- + \sum_{p=-\frac{1}{2}}^{0} \nu^p S_p \tag{97}$$

where $\mu \equiv \mu^+, \quad \chi \equiv \chi^+, \quad \nu \equiv \nu^+, \quad f \equiv f^+$, and $f^+ \mathcal{G}^- + \sum_{p=-\frac{1}{2}}^{0} \nu^p S_p$ can be interpreted as the independent chemical potentials. One can choose for the corresponding asymptotic symmetry algebra, the resulting relations as $\sigma_3 = \frac{1}{\sqrt{3}}, \quad \sigma_7 = i \sqrt{3}, \quad \sigma_8 = 0, \quad \sigma_9 = 1$, appearing on the RHS of eq. (52) - (57) in the super Jacobi idenities of the $\mathfrak{osp}(3|2)$ superalgebra. The functions, except the chemical potentials are fixed by the flatness condition [3]. For the fixed chemical potentials, the time evolution of canonical boundary charges
where $\alpha_{-1}, \gamma_{-1}, \beta_{-\frac{1}{2}}$ and $\delta_{-\frac{3}{2}}$ are some scaling parameters to be determined later. Now, we are in a position to work the superconformal asymptotic symmetry algebra under the Drinfeld-Sokolov reduction. This reduction imply that the only independent parameters $\epsilon \equiv \epsilon^{+1}$, $\kappa \equiv \kappa^{+}$, $\zeta \equiv \zeta^{+}$ and $\varphi \equiv \varphi^{+}$. One can start to compute the gauge transformations for asymptotic symmetry algebra by considering all transformations (17) that preserve the boundary conditions with the gauge parameter $\lambda$ in the $\mathfrak{osp}(3|2)$ superalgebra (62) - (57) as

$$
\lambda = b^{-1} \left[ e_{L_{1}} - \epsilon^{L_{0}} + \left( A_{1-1} - \beta_{-\frac{1}{2}} \zeta G - \frac{9}{10} \delta_{-\frac{3}{2}} \rho S + \alpha_{-1} L_{0} \epsilon^{+1} + \frac{\epsilon^{''}}{2} \right) L_{-1} 
+ \zeta G_{+\frac{1}{2}} + \left( \frac{3i \gamma_{-1} \varphi}{2} - \frac{3i \gamma_{-1} \varphi}{2} \right) G_{-\frac{1}{2}} + \kappa A_{+1} + \left( -\kappa' + \frac{3i \beta_{-\frac{1}{2}} \varphi G'}{\sqrt{5}} \right) A_{0} 
+ \left( -\frac{5}{2} i A_{1-1} \epsilon - \frac{3i \beta_{-\frac{1}{2}} \varphi G'}{2 \sqrt{5}} - \frac{1}{2} \sqrt{5} \beta_{-\frac{1}{2}} \zeta G' + \frac{\kappa''}{2} + \frac{9}{10} \delta_{-\frac{3}{2}} \rho S + \alpha_{-1} \kappa L \right) A_{-1} 
+ \frac{\varphi S}{2} - \varphi' S_{+\frac{1}{2}} + \left( \frac{3i A_{1-1} \rho}{2} + \frac{\varphi''}{2} + \frac{3}{2} \alpha_{-1} \varphi L \right) S_{-\frac{1}{2}} 
+ \left( \frac{1}{2} i A_{1-1} A^{'} - \frac{1}{2} i \sqrt{5} A_{1-1} \zeta + \frac{7}{6} i A_{1-1} \epsilon^{+1} + \frac{\varphi^{2}}{2} - G^{2} \rho - \frac{1}{6} i \sqrt{5} \beta_{-\frac{1}{2}} \kappa G' 
- \frac{1}{2} \sqrt{5} \beta_{-\frac{1}{2}} \zeta G' - \frac{\varphi''}{6} - \delta_{-\frac{3}{2}} \kappa S + \frac{\delta_{-\frac{3}{2}}}{4} \varphi S \epsilon - \frac{1}{2} \alpha_{-1} \varphi L' - \frac{7}{6} \alpha_{-1} \varphi L' \right) S_{-\frac{1}{2}} \right] b. \tag{102a}
$$

The condition (17) impose that transformations on the gauge with $\alpha_{-1} = \gamma_{-1} = \frac{d}{c}, \beta_{-\frac{1}{2}} = -\frac{d}{c}$ and $\delta_{-\frac{3}{2}} = -\frac{10}{c}$ are given by

$$
\partial_{\lambda} L = \frac{e^{''}}{12} + e^{L'} + 2 e^{L'} + \kappa A^{'} + 2 A \kappa' + \frac{G'}{2} + \frac{3 \rho S'}{2} + \frac{5 \rho S'}{2}, \tag{103}
$$

$$
\partial_{\lambda} G = \frac{3i \rho A'}{\sqrt{5}} - \frac{4i \varphi A'}{\sqrt{5}} + \frac{e^{''}}{3} + e^{G'} + \frac{3 G'}{2} + i \sqrt{5} \kappa S + 2 \zeta L, \tag{104}
$$
\[ \partial A = -\frac{5}{2} \alpha A + \alpha A' - 5 i A \kappa' + 2 A' + \frac{27 \alpha G \rho}{\sqrt{5c}} + \frac{27 i G \rho L}{2 \sqrt{5c}} + \kappa''' \]
\[ + \frac{3 i \rho G''}{4 \sqrt{5}} + \frac{2 G' \rho}{\sqrt{5}} + \frac{3 i G \rho'}{2 \sqrt{5}} - \frac{3}{2 i} \rho \sigma - i \sqrt{5} i S - \frac{5}{2 i} i \rho' + \kappa' + 2 L, \text{(105)} \]
\[ \partial S = -\frac{3}{10} \rho A'' + \frac{i c A'}{\sqrt{5}} - i A' \rho' + \frac{4 i A c'}{\sqrt{5}} - i A' \rho'' - \frac{27 A G \kappa}{\sqrt{5c}} - \frac{54 i A \rho L}{5c} + \frac{63 G L \rho'}{20c} + \frac{27 i G \kappa L}{2 \sqrt{5c}} + \frac{c p'''}{60} \]
\[ + \frac{27 \rho L^2}{5c} - \frac{i c G''}{4 \sqrt{5}} - \frac{i G c' L}{2 \sqrt{5}} - \frac{3 i G \rho'''}{5 c} + i c S' - \frac{5}{2} i S \kappa' + \frac{5 S \kappa'}{2} + \frac{3 \rho L'''}{10} + \rho' \kappa' + \kappa'' L. \text{(106)} \]

The variation of canonical boundary charge \( \Omega[\rho] \text{(25)} \) can be integrated which reads
\[ \Omega[\rho] = \int d \varphi \ [L \rho + A \kappa + G \zeta + S_0]. \text{(107)} \]

This leads to operator product expansions in the complex coordinates by using \( \text{(27)} \)

\[ \mathcal{L}(z_1) \mathcal{L}(z_2) \sim \frac{\mathcal{L}}{z_{12}^2} + \frac{2 \mathcal{L}}{z_{12}} + \frac{\mathcal{L}'}{z_{12}}, \text{(108a)} \]
\[ \mathcal{G}(z_1) \mathcal{G}(z_2) \sim \frac{\mathcal{G}}{z_{12}^2} + \frac{\mathcal{G}'}{z_{12}} + \mathcal{G}(z_1) \mathcal{G}(z_2) \sim \frac{\mathcal{G}}{z_{12}} + \frac{2 \mathcal{L}}{z_{12}}, \text{(108b)} \]
\[ \mathcal{G}(z_1) A(z_2) \sim \frac{2 A}{z_{12}^2} + \frac{A'}{z_{12}}, \mathcal{L}(z_1) S(z_2) \sim \frac{2 S}{z_{12}} + \frac{S'}{z_{12}}, \text{(108c)} \]
\[ \mathcal{G}(z_1) A(z_2) \sim -\frac{i \sqrt{5} A}{z_{12}}, \mathcal{G}(z_1) S(z_2) \sim \frac{i \sqrt{5} A}{z_{12}} - \frac{i \sqrt{5} A'}{z_{12}}, \text{(108d)} \]
\[ \mathcal{A}(z_1) A(z_2) \sim \frac{\mathcal{A}}{z_{12}^2} + \frac{2 \mathcal{L} - 5 i A}{z_{12}} + \frac{\mathcal{L} - \frac{5 i}{2} A}{z_{12}}, \text{(108e)} \]
\[ \mathcal{A}(z_1) S(z_2) \sim \frac{3 i G}{z_{12}^2} - \frac{i G'}{z_{12}} + \frac{5 i S}{2 z_{12}^2} - \frac{1}{z_{12}} \left( \frac{27 A G}{\sqrt{5c}} + \frac{27 i G L}{2 \sqrt{5c}} + i S' + \frac{i G''}{4 \sqrt{5}} \right), \text{(108f)} \]
\[ S(z_1) S(z_2) \sim \frac{2 \mathcal{L} - i A}{z_{12}} + \frac{\mathcal{L}'}{z_{12}} - \frac{i A'}{z_{12}} + \frac{1}{z_{12}} \left( \frac{27 \mathcal{L}^2}{5c} + \frac{54 i A \mathcal{L}}{5c} + \frac{63 G G'}{20c} - \frac{3 i A''}{10} + \frac{3 \mathcal{L}''}{10} \right). \text{(108g)} \]

After repeating the same algebra for \( \Gamma \)-sector, one can say that the asymptotic symmetry algebra for a set of boundary conditions of \( N = (1, 1) \) supergravity is two copies of the \( SW(\frac{3}{2}, 2) \) algebra with central charge \( c = 6 k \). Finally, one can also take into account normal ordering (quantum) effects of this algebra as in Refs. \[31,32].

### IV. SUMMARY AND CONCLUSION

In this work, a relation between \( AdS_3 \) and \( \mathfrak{osp}(2|1) \oplus \mathfrak{osp}(2|1) \) Chern-Simons theory was first reviewed. The Chern-Simons formulation of \( AdS_3 \) allows for a straightforward generalization to a higher spin theory as in the bosonic cases. The higher spin gauge fields have no propagating degrees of freedom, but we noted that there can be a large class of interesting non-trivial solutions. Specifically, \( AdS_3 \) in the presence of a tower of higher spin fields up to spin \( \frac{3}{2} \) is obtained by enlarging \( \mathfrak{osp}(2|1) \oplus \mathfrak{osp}(2|1) \) to \( \mathfrak{osp}(3|2) \oplus \mathfrak{osp}(3|2) \). Finally, classical two copies of the \( \mathfrak{osp}(3|2)_k \) affine algebra on the affine boundary and two copies of \( \mathcal{SW}(\frac{3}{2}, 2) \) symmetry algebra on the superconformal boundary as asymptotic symmetry algebras, and also the chemical potentials related to source fields appearing through the temporal components of the connection are obtained. Therefore, we think that this method provides a good laboratory for investigating the rich asymptotic structure of extended supergravity.
Our results presented in this paper can be extended in various ways. One possible extension is by enlarging $\mathfrak{sl}(2|1) \oplus \mathfrak{sl}(2|1)$ to $\mathfrak{sl}(3|2) \oplus \mathfrak{sl}(3|2)$ as $\mathcal{N} = (2, 2)$ supergravity. This possible extension will be to look for in our forthcoming paper.

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VI. REFERENCES

1. J. D. Brown and M. Henneaux, Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity, Commun. Math. Phys. 104 (1986) 207.
2. J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. 38 (1999) 1113 [Adv. Theor. Math. Phys. 2 (1998) 231].
3. L. Susskind, The World as a hologram, J. Math. Phys. 36, 6377 (1995).
4. A. Achucarro and P. K. Townsend, A Chern-Simons Action for Three-Dimensional Anti-De Sitter Supergravity Theories, Phys. Lett. B 180 (1986) 89.
5. E. Witten, (2+1)-Dimensional Gravity as an Exactly Soluble System, Nucl. Phys. B 311 (1988) 46.
6. M. Banados, C. Teitelboim and J. Zanelli, The Black hole in three-dimensional space-time, Phys. Rev. Lett. 69, 1849 (1992).
7. M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, Geometry of the (2+1) black hole, Phys. Rev. D 48, 1506 (1993) Erratum: [Phys. Rev. D 88, 069902 (2013)].
8. E. Bergshoeff, M. P. Blencowe and K. S. Stelle, Area Preserving Diffeomorphisms and Higher Spin Algebra, Commun. Math. Phys. 128, 213 (1990).
9. M. P. Blencowe, A Consistent Interacting Massless Higher Spin Field Theory in D = (2+1), Class. Quant. Grav. 6, 443 (1989).
10. M. A. Vasiliev, Higher spin gauge theories: Star product and AdS space, In *Shifman, M.A. (ed.): The many faces of the superworld* 533-610.
11. M. A. Vasiliev, Higher spin symmetries, star product and relativistic equations in AdS space, [hep-th/0002183].
12. M. Henneaux and S. J. Rey, Nonlinear $W_{\infty}$ as Asymptotic Symmetry of Three-Dimensional Higher Spin Anti-de Sitter Gravity, JHEP 1012, 007 (2010).
13. A. Campoleoni, D. Francia and C. Heissenberg, Asymptotic symmetries and charges at null infinity: from low to high spins, EPJ Web Conf. 191, 06011 (2018).
14. H. S. Tan, Exploring Three-dimensional Higher Spin Supergravity based on sl(n, n−1) Chern-Simons theories, JHEP 1211, 063 (2012).
15. H. T. Özer and A. Filiz, On the explicit asymptotic $W_3$ symmetry of 3D Chern-Simons higher spin AdS3 gravity, J. Math. Phys. 59, no. 8, 083504 (2018).
16. M. Banados, K. Bautier, O. Coussaert, M. Henneaux and M. Ortiz, Anti-de Sitter / CFT correspondence in three-dimensional supergravity, Phys. Rev. D 58, 085020 (1998).
17. C. Candu and M. R. Gaberdiel, Supersymmetric holography on AdS3, JHEP 1309, 071 (2013).
18. M. Henneaux, G. Lucena Gmez, J. Park and S. J. Rey, Super- $W(\infty)$ Asymptotic Symmetry of Higher Spin AdS3 Supergravity, JHEP 1206, 037 (2012).
19. C. Peng, Dualities from higher-spin supergravity, JHEP 1303, 054 (2013).
20. K. Hanaki and C. Peng, Symmetries of Holographic Super-Minimal Models, JHEP 1308, 030 (2013).
21. B. Chen, J. Long and Y. N. Wang, Conical Defects, Black Holes and Higher Spin (Super-)Symmetry, JHEP 1306, 025 (2013).
22. D. Grumiller and M. Riegler, Most general AdS5 boundary conditions, JHEP 1610, 023 (2016).
23. D. Grumiller, W. Merbis and M. Riegler, Most general flat space boundary conditions in three-dimensional Einstein gravity, Class. Quant. Grav. 34, no. 18, 184001 (2017).
24. C. Krishnan and A. Raju, Chiral Higher Spin Gravity, Phys. Rev. D 95, no. 12, 126004 (2017).
25. M. Banados, Three-dimensional quantum geometry and black holes, AIP Conf. Proc. 484, no. 1, 147 (1999).
26. S. Carlip, Conformal field theory, (2+1)-dimensional gravity, and the BTZ black hole, Class. Quant. Grav. 22, R85 (2005).
27. M. Banados, Global charges in Chern-Simons field theory and the (2+1) black hole, Phys. Rev. D 52, 5816 (1996).
28. M. Banados, T. Broz and M. E. Ortiz, Boundary dynamics and the statistical mechanics of the (2+1)-dimensional black hole, Nucl. Phys. B 545, 340 (1999).
29. M. Blagojevic, Gravitation and gauge symmetries, IOP publishing, U.K., (2002).
30. C. E. Vakárcel, New boundary conditions for (extended) AdS3 supergravity, Class. Quant. Grav. 36, 065002 (2019).
31. J. M. Figueroa-O’Farrill and S. Schraun, Extended superconformal algebras, Phys. Lett. B 257, 69 (1991).
32. J. M. Figueroa-O’Farrill, A Note on the extended superconformal algebras associated with manifolds of exceptional holonomy, Phys. Lett. B 392, 77 (1997).