Spin injection into a metal from a topological insulator

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We study a junction of a topological insulator with a thin two-dimensional (2D) non-magnetic or partially polarized ferromagnetic metallic film deposited on a 3D insulator. We show that such a junction leads to a finite spin current injection into the film whose magnitude can be controlled by tuning a voltage V applied across the junction. For ferromagnetic films, the direction of the component of the spin current along the film magnetization can also be tuned by tuning the barrier potential V0 at the junction. We point out the role of the chiral spin-momentum locking of the Dirac electrons behind this phenomenon and suggest experiments to test our theory.

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Topological insulators (TI), a class of three-dimensional (3D) insulators with strong spin-orbit coupling, are known to possess gapless Dirac-like quasiparticles on their surfaces whose existence originates from the special topological properties of their bulk bands [1–9]. Such insulators, which are essentially 3D generalizations of their 2D counterparts which exhibit the quantum spin-Hall effect, have attracted a lot of theoretical and experimental attention in recent years. These TIs can be classified as strong or weak depending on their sensitivity to time reversal symmetric perturbations. The surfaces of the strong TIs have an odd number of Dirac cones; the number and positions of these cones depend on the nature of the surface concerned [1, 2, 4]. The odd number of the Dirac cones ensures that any surface impurity which conserves time reversal symmetry does not destroy the low-energy Dirac properties of the quasiparticles on the surface. For several compounds such as Bi2Te3 and Bi2Se3, specific surfaces have been found with a single Dirac cone near the Γ point of the 2D surface Brillouin zone [2, 7, 9].

A Dirac cone on the surface of a TI is described by the Hamiltonian

\[ H_{\tilde{n}}[v_F] = \int \frac{dk_xdk_y}{(2\pi)^2} \psi^{\dagger}_k (\hbar v_F \tilde{n} \cdot \mathbf{k} - \mu I) \psi_k, \]

where \( \tilde{n}(I) \) denotes the Pauli (identity) matrices in spin space, \( \tilde{n} \) denotes the unit vector normal to the TI surface which hosts electrons with momentum components \( k_i \) and \( k_j \), \( v_F \) is the Fermi velocity, \( \mu \) is the chemical potential, and \( \psi = (\psi_\uparrow, \psi_\downarrow)^T \) is the annihilation operator for the Dirac spinor [10]. In this notation, \( \uparrow (\downarrow) \) denotes components of the quasiparticle spin along (opposite to) \( \tilde{n} \). Recently, several novel features of these surface Dirac electrons have been studied. These include the existence of Majorana fermions in the presence of a magnet-superconductor interface on the surface [11], generation of a state resembling a \( p_x + ip_y \)-wave superconductor but with time reversal symmetry via proximity to a \( s \)-wave superconductor [10], anomalous magnetoresistance of ferromagnet-ferromagnet junctions [12], realization of a magnetic switch in junctions of these materials [13], spin textures with chiral properties [9], control of spin transport and polarization using gate voltages and electric fields [14, 15], realization of a Lifshitz transition in thin TI films [16], and spin-polarized STM spectra [17]. However, junctions of such TIs with conventional metals and ferromagnets have not been studied so far.

In this Letter, we study the transport properties of a junction of a TI with a conventional insulator with a non-magnetic or partially polarized ferromagnetic metallic film deposited on it as schematically shown in Fig. 1. The regions I and II in Fig. 1 refer to the two surfaces of the TI which host chiral Dirac quasiparticles, while region III consists of conventional electrons obeying the Schrödinger equation. We show that due to the chiral spin-momentum locking of the Dirac electrons on the surfaces of the TI, the transport through such a junction can lead to a finite spin current in the film without any skew scattering which is necessary in conventional spin-Hall materials to generate such currents. The magnitude of the spin current generated can be controlled by a voltage \( V \) applied across the junction without the in-
volvement of any external magnetic field. We also study
the dependence of this spin current on the barrier poten-
tial $V_0$ at the junction and show that it displays an os-
cillatory feature with a decaying envelope as a function of
$V_0$. This behavior draws from both the chiral nature of
the Dirac quasiparticles in regions I and II which leads
to the oscillatory nature of the spin current $[15]$, and the
presence of the conventional Schrödinger electrons in re-

gion III which leads to an exponential decay of the spin
current with increasing $V_0$. Finally, we demonstrate that
for ferromagnetic films, the direction of the component of
the spin current along the film magnetization ($J_s$) can be
controlled by tuning $V_0$. To the best of our knowledge,
the generation of spin current using the chirality of the
Dirac quasiparticles on the surface of the TIs whose di-
rection and magnitude can both be controlled electrically
has not been proposed before; we therefore expect our
proposal to generate significant interest in the field of
spintronics.

We begin with the analysis of the junction in Fig. 1
when $V_0 = 0$. In region I, the Hamiltonian for the Dirac
quasiparticles is given by Eq. (11) with $\hat{n} = \hat{x}$. The wave
function for these quasiparticles with a transverse mo-
mentum $k_x$ and energy $\epsilon = \hbar v$ moving along $\hat{z}$ can be ob-
tained by solving the Dirac equation $H_\parallel |\psi_1\rangle = \epsilon |\psi_1\rangle$
and is given by $\psi_\pm = [1, -i \exp(\pm i t/k)]^T \exp[\pm k_x x \hat{z} (k_y y)] / \sqrt{2}$,
where $\theta_k = \arccos[h v_1 k_x / (\epsilon + \mu)]$, $\epsilon = -\mu + h v_1 \sqrt{k_x^2 + k_y^2}$,
and $v_1$ is the Fermi velocity of the quasiparticles in the
region I. The wave function of the Dirac quasiparticles in
region I can thus be written as
\[
\psi_I = \psi_+ + r \psi_-,
\]
where $r$ is the reflection amplitude. We note that for
any incident angle $\theta$, $\psi_+$, and $\psi_-$ have
$\langle \psi_+ | \sigma_z | \psi_- \rangle = \sin(\theta_k) = -\langle \psi_- | \sigma_z | \psi_+ \rangle$. Thus the reflected Dirac
quasiparticle has the opposite spin orientation along $\hat{x}$
compared to its incident counterpart. This phenomenon is
reminiscent of Andreev reflection from a superconducting
junction where the charge and the transverse momenta of
the reflected quasiparticle change sign. In contrast,
for the junctions considered here, the transverse in-plane
component of the quasiparticle spin changes sign upon
reflection. In what follows, we shall show that this spin
reversal is at the heart of the generation of a finite spin
current in region III.

In region II, $\hat{n} = \hat{y}$, and the wave function of the Dirac
quasiparticles moving along $\hat{z}$ with transverse momenta
$k_z$ and energy $\epsilon$ can be obtained by solving $H_y |\psi_2\rangle = \epsilon |\psi_2\rangle$ and is given by
\[
\psi_{II} = t_1 |\psi_2\rangle, \quad |\psi_2\rangle = [u_k, v_k]^T \exp(-i k_z z + k_x x) / \sqrt{2},
\]
where $u_k/v_k = \sqrt{1 + [-1 \cos(\theta_k)] / \sqrt{2}}$, $\phi_k = \arccos[h v_2 k_x / (\epsilon + \mu)]$, $\epsilon = -\mu + h v_2 \sqrt{k_x^2 + k_y^2}$, $v_2 = \beta^2 v_1$
is the Fermi velocity, and $t_1$ denotes the transmission
probability of the Dirac quasiparticles in region II. In
the rest of this work, we shall choose $\beta = \sqrt{v_2/v_1} \leq 1$.

In the metallic film (region III), the Hamiltonian
for the electrons can be written as $H_{III} = h^2 (k_x^2 + k_y^2) / (2m) - \mu - A \sigma_z$, where $\mu$ and $m$ are the chemical
potential and mass of the electrons in the film, and $A$
is proportional to the magnetization of the electrons. For
a non-magnetic film $A = 0$, while for a fully polarized
ferromagnetic film $A \to \infty$. In what follows, we first
consider a non-magnetic film for which $A = 0$. The wave
function of the electrons in region III is then given by
\[
\psi_{III} = [t_2, t_3]^T e^{i(k_x x + k_y y) / \sqrt{2}},
\]
where $\epsilon = -\mu + h^2 (k_x^2 + k_y^2) / (2m)$, and $t_2$ and $t_3$
denote the transmission amplitudes of spin-up and spin-down
electrons in region III.

The boundary condition on these wave functions in-
volves continuity of current through the junction which
yields
\[
v_1 \psi_+^\dagger \sigma_x \psi_I - v_2 \psi_+^\dagger \sigma_x \psi_{II} = \frac{\hbar}{m} I m(\psi_{III}^\dagger \partial_y \psi_{III}),
\]
where it is understood that all fields are evaluated at
the junction line $y = 0$ (for $\psi_I$ and $\psi_{III}$) and $z = 0$
(for $\psi_{II}$). We note that the unusual boundary condition
(Eq. 5) in which a current without derivatives in regions
I and II (Dirac equation) has to be matched with a cur-
rent involving a first derivative in region III (Schrödinger
equation) is generic for any junction involving a TI and
a conventional material. (The situation here is different
from a junction of ordinary materials with spin-orbit cou-
ping where the current involves a first derivative on both
sides $[19]$). Below, we present a general solution to this
problem with (discussed in Eq. 11) below) and without
a barrier potential $[21]$.

In the absence of any barriers at the junction, the gen-
teral solution of Eq. 5 is given by two linear conditions
on the wave functions $[21]$.
\[
\psi_{III} = c (\psi_I + \beta \psi_{II}), \quad \frac{\hbar}{m} \partial_y \psi_{III} = \frac{i}{c} \psi_{II}^\dagger \sigma_z [\psi_I - \beta \psi_{II}],
\]
where $c$ is an arbitrary real constant; we will set $c = 1$
for simplicity. Note that the metal/ferromagnet decouples
from the TIs for $c \to 0$ or $\infty$. Substituting Eqs. 3
in Eq. 6 we obtain the following relations between $r, t_1, t_2,$
and $t_3$,
\[
1 + r + (-) \beta u_k t_1 = t_2 (\alpha t_3),
\]
\[
e^{i \theta_k} r + r e^{-i \theta_k} (-) \beta v_k t_1 = i t_3 (\alpha t_2),
\]
where $\alpha = h k y / (m v_1)$. Solving for $r, t_{1,2,3}$ from Eq. 7,
we get
\[
N = D / (2(1 - \alpha^2) \sin(\theta_k)/(\beta D)),
\]
\[
t_1 = \frac{2}{D} \sin(\theta_k) (u_k + \alpha v_k),
\]
\[
t_2 = \frac{4}{D} \sin(\theta_k) (u_k + \alpha v_k),
\]
\[
t_3 = \frac{4}{D} \sin(\theta_k) (v_k + \alpha u_k),
\]
where $N = -i u_k \exp(i \theta_k)(1 + \omega^2) - 2i \alpha - v_k[(1 + \omega^2) + 2i \exp(i \theta_k)],$ and $D = [i u_k(1 + \omega^2) + 2i \alpha v_k \exp(-i \theta_k)] + v_k(1 + \omega^2) + 2i \alpha v_k.$

Eq. (S) predicts a net spin current along $\hat{x}$ in the metallic region: $J_x = (hv_1/2) \sum_{k_y} \langle \psi_{II} | \alpha \sigma_x \psi_{III} \rangle = (hv_1/2) \sum_{k_z} \alpha (t^I_3 t^I_3 + \text{h.c.}).$ This can be written as

$$J_x = \frac{8J_0}{\pi} \int_{E/(2E_0)}^E \int_{E/(2E_0)}^E dx \frac{\alpha [(1 + \omega^2) \sin(\phi_k) + \alpha] \sin^2(\theta_k)}{|D|^2},$$

where $J_0 = h v_1 k_0, x = k_y/k_0, k_0 = m v_1/h, E = eV + \mu,$ and $E_0 = h v_3 k_0/2.$ A plot of $J_x/J_0$ as a function of the applied voltage $eV/E_0,$ shown in the bottom panel of Fig. 2, confirms that the net spin current is finite and its amplitude depends on $V.$ The inset shows the dependence of the spin current on $\beta$ for a fixed $V$ and confirms the presence of a finite $J_x$ for the entire range of $0 < \beta \leq 1.$ The charge current is given by $I_c = e v_1 \sum_{k_z} \alpha (|t^I_2|^2 + |t^I_3|^2)$ which can be written as

$$I_c = \frac{4I_0}{\pi} \int_{-E/(2E_0)}^{E/(2E_0)} dx \frac{\alpha [1 + \omega^2 + 2 \alpha \sin(\phi_k)] \sin^2(\theta_k)}{|D|^2},$$

where $I_0 = e v_1 k_0.$ The top panel of Fig. 2 shows the variation of $I_c/I_0$ with the applied voltage $V,$ while the inset depicts its variation with $\beta.$ We note that the charge current displays a qualitatively similar behavior as the spin current along $\hat{x}.$ The net spin current along $y$ and $z$ vanishes. For $J_x$ this can be seen by noting that $u_k \to v_k$ under $k_y \to -k_y.$ Consequently $t_2 \to t_3$ under this transformation which leads to a zero net value for $J_z = \sum_{k_z} \alpha (|t^I_2|^2 - |t^I_3|^2).$ Also, using the fact that $t_2$ and $t_3$ are both purely imaginary (Eq. (S)), it can be easily shown that $J_y = 0.$

Next, we address the behavior of the spin and charge currents for a ferromagnetic film where $A \neq 0.$ In this case, the wave function in region III is given by

$$\psi_{III}^\text{FM} = [t_2 e^{i k_y y}, t_3 e^{i k_y y}]^T e^{i k_0 x}/\sqrt{2},$$

where $k_y^{(1)(2)} = [2m(\epsilon + \mu + [-]A)/h^2 - k_z^2]^{1/2}.$ The boundary condition on the wave function is given by Eq. (i) and yields

$$t_2[0] = -4i e^{i \theta_k} \sin(\theta_k)(u_k[\alpha_2 u_k] + \alpha_1 v_k[\alpha_1 v_k]) / D_4,$$

where $\alpha_1 \alpha_2 = \alpha_2 \alpha_3 = \alpha_1 \alpha_3 = \alpha_1 \alpha_2; \text{ in this limit Eq. (12) matches with Eq. (S).} Using this wave function, it is straightforward to compute the expression for $I_c$ following the method outlined earlier which shows qualitatively similar behavior as in metallic films.

The key difference between the ferromagnetic and the non-magnetic films which we now focus on is that the former films allow a non-zero $J_z.$ This is easily seen from Eq. (12) by noting that $t_2(k_x) \neq t_3(-k_x)$ due to the difference of velocities of the up and the down spin quasiparticles; this leads to a finite $J_z$ in region III given by

$$J_z = \frac{J_0}{2\pi} \int_{-E/(2E_0)}^{E/(2E_0)} dx (\alpha_1 |t^I_2|^2 - \alpha_2 |t^I_3|^2).$$

In what follows, we set $\mu = A$ or $\mu = -A$ so that the spin-up or spin-down Fermi surface is aligned with the Fermi surface in region I. Increasing $A$ therefore pushes...
the other Fermi surface away from the Fermi surface of region I and hence increases the effective spin polarization of the film in region III. The behavior of $J_z$, computed by substituting Eq. (13) in Eq. (12), is shown in Fig. 4.

The top panel of the figure shows the variation of $J_z$ as a function of $eV/E_0$ for $A/E_0 = 3$, while the bottom panel shows the dependence of $J_z$ on $A$ for fixed $eV/E_0 = 3$ and $\beta = 1$. These plots demonstrate the presence of a finite $J_z$ for a large range of $V$; the sign of $J_z$ depends on the magnetization of the film while its magnitude can be controlled by the applied voltage $V$.

Next, we consider the effect of a finite barrier potential $V_0$ applied over a region $d$ at the junction. In what follows we will consider the thin-barrier limit where $V_0 \to \infty$ and $d \to 0$ keeping $\chi = V_0d/(\hbar v_1)$ finite. In this limit, the boundary condition to be imposed on the wave functions reads [21,22]

$$\psi_{III} = e^{-i\chi \sigma_z} \psi_I + \beta e^{i\chi \sigma_z/\beta^2} \psi_{III}, \quad (14)$$

Substituting Eqs. (2), (3), and (11) in Eq. (14), we again obtain a set of four equations for $r$ and $t_{1,2,3}$ which reads

$$t_{2[3]} = \frac{1}{\sqrt{2}} (a_1^* b_2 - a_2^* b_1), \quad (\alpha_1^2 + 2i\chi) t_{3[2]} = \frac{1}{\sqrt{2}} (a_1^* b_3 - a_3^* b_1 - \beta t_2 b_1),$$

where $a_1 = \cos(\chi) - \sin(\chi) \exp(i\theta_k), \quad a_2 = \sin(\chi) + \cos(\chi) \exp(i\theta_k)$, and $b_{1[2]} = u_k |v_k| \cos(\chi/\beta^2) + i u_k v_k \sin(\chi/\beta^2)$. The solution of these equations yields

$$t_{2[3]} = [-2(b_{1[2]} + b_{2[1]}(\alpha_1^2 + 2i\chi))(a_1^* o_2 - a_2^* o_1)]/D_2,$$

(15)

where $D_2 = -2b_1(2\chi + i\alpha_1) + b_2 [1 + \chi + \alpha_2](\alpha_2 - 2i\chi)]a_1 + [b_1(1 + (\alpha_1 + 2\chi)(\alpha_2 + 2i\chi) + \alpha_2 b_2(2\chi)/\alpha_2^2$ for magnetic films. The corresponding expressions for the non-magnetic films can be obtained by putting $\alpha_1 = \alpha_2$ in Eq. (15). We note that Eq. (15) reproduces Eq. (12) for $\chi = 0$.

From Eq. (15), we find that the barrier potential $\chi$ enters the transmission amplitudes $t_{2[3]}$ both through the $\cos(\chi)$ and $\sin(\chi)$ factors in $a_{1(2)}$ and $b_{1(2)}$ leading to an oscillatory $\chi$ dependence of $t_{2[3]}$ and through the appearance of $\chi$ in the numerator and denominator of Eq. (15), which, in the limit of large $\chi$, leads to a decay of $t_{2[3]}$ with increasing $\chi$. The former behavior arises from the Dirac nature of the electrons in regions I and II, while the latter is a consequence of the conventional Schrödinger nature of the electrons in region III. Consequently, we expect $t_{2[3]}$ to have an oscillatory dependence on $\chi$ along with a decaying envelope. We note that such a behavior is different from what is found in analogous junctions involving solely Dirac or solely conventional materials.

To compute the spin and charge currents, we substitute the values of $t_{2[3]}$ from Eq. (15) in the expressions $J_z = \hbar v_1 \sum_k (\alpha_1 |t_2|^2 - \alpha_2 |t_3|^2)$ for ferromagnetic films, and $I_c = e v_1 \sum_k \alpha_1 |t_2|^2 + |t_3|^2$ and $J_s = \hbar v_1 \sum_k \alpha (t_2^* t_2 + h.c.)$ for non-magnetic films. The resulting dependence of $J_z$ and $I_c$ on $\chi$ is shown in Fig. 3 for $\beta = 1$. The top panel shows the behavior of $J_z$ and $I_c$ for non-magnetic films for $eV/E_0 = 3$. We find that both $J_z$ and $I_c$ display small oscillatory features with an overall monotonic decay as a function of $\chi$. We have found qualitatively similar behavior of $J_z$ and $I_c$ in magnetic films. In contrast, $J_z$ for magnetic films, shown in the left (right) bottom panels of Fig. 4 for $\mu = A (\mu = -A)$, displays a non-monotonic behavior with increasing $\chi$. In particular, these plots demonstrate that the sign of the spin currents gets reversed with increasing $\chi$, which allows for the possibility of tuning the sign of $J_z$ by tuning $V_0$. This leads to electrical control of both the magnitude and the direction of $J_z$ in magnetic films via tuning externally applied voltages $V$ and $V_0$.

To experimentally verify our theory, we propose measuring the current in region III for magnetic films using a ferromagnetic contact. When the direction of magnetization of the contact is along $\hat{z}$, it will measure only the current due to the spin-up electrons: $I_{e} = e v_1 \sum_k \alpha_1 |t_2|^2 = [I_{c} + e e J_z/(2\hbar)]/2$. Similarly a contact with magnetization along $-\hat{z}$ will record $I_s = [I_{c} - e e J_z/(2\hbar)]/2$. Thus the difference between these two currents will provide a measure of the spin current through the film as a function of the applied voltage $V$ and the potential barrier $V_0$: $J_z = 2h(I_{e} - I_{c})/e$. Thus such an experiment can verify the predicted dependence of $J_z$ on $V$ and $V_0$ [21].

In conclusion, we have studied a junction of a TI with
a thin metallic/partially polarized ferromagnetic film deposited over an ordinary insulator. For ferromagnetic films, we have shown that such a junction can be used to generate a finite spin current along $\hat{z}$ whose magnitude and direction can both be controlled by externally applied voltages without the presence of any external magnetic field. Our work shows that the chiral spin-momentum locking of the Dirac quasiparticles on the surfaces of the TI is at the heart of this phenomenon. Finally, we have suggested a simple experiment to test our theory.

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[21] See Supplemental Material for a derivation of the boundary conditions at the junction and for numerical estimates of the currents to be measured.
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APPENDIX A

In this Section, we provide some supplemental material to the main text related to the derivation of the boundary condition Eq. [14] and numerical estimate of the measured current in suggested experiment.

Current conserving boundary conditions at a junction

In this section, we will find the general time reversal invariant boundary condition which satisfies the current conservation relation at the junction discussed in our paper. We begin with the Hamiltonians in the three regions. Region I of the topological insulator (TI) is defined by the half-plane $z = 0$ and $y < 0$, and has the Dirac Hamiltonian

$$H_I = i\hbar v_1 [ -\sigma_x \partial_y + \sigma_y \partial_x ] .$$

Region II of the TI, given by the half-plane $y = 0$ and $z < 0$, has the Hamiltonian

$$H_{II} = i\hbar v_2 [ -\sigma_z \partial_x + \sigma_x \partial_z ] .$$

Region III of the non-magnetic metal/ferromagnet thin film is defined by the half-plane $z = 0$ and $y > 0$, and has the Schrödinger Hamiltonian

$$H_{III} = -\frac{\hbar^2}{2m} (\partial_x^2 + \partial_y^2) - \mu - A\sigma_z .$$

The time evolution equations $i\hbar \partial \psi_a / \partial t = H_a \psi_a$ ($a = I, II, III$) are invariant under time reversal ($t \rightarrow -t$ and...
complex conjugation of all numbers) if $\psi_i \rightarrow \sigma_y \psi_i^*$ and $A = 0$.

The junction of the three regions is given by the line $y = z = 0$. The conservation of the total current coming into the junction from the three regions is given by Eq. (5) of our paper, namely,

$$v_1(\psi_I^\dagger \sigma_x \psi_I)_y=0- - v_2(\psi_{II}^\dagger \sigma_x \psi_{II})_z=0- = \frac{\hbar}{m} Im(\psi_{III}^\dagger \partial_y \psi_{III})_{y=0+}. \hspace{1cm} (19)$$

In order to satisfy this equation, let us assume linear relations between the wave functions at the junction of the form

$$(\psi_{III})_{y=0+} = A_1(\psi_I)_{y=0-} + A_2(\psi_{II})_{z=0-},$$

$$\frac{\hbar}{m} (\partial_y \psi_{III})_{y=0+} = i\sigma_x [A_3(\psi_I)_{y=0-} + A_4(\psi_{II})_{z=0-}], \hspace{1cm} (20)$$

where the $A_i$ are four parameters. The relations in Eq. (20) will be time reversal invariant if the $A_i$ are real. We can now check that Eq. (19) will be satisfied if $A_1A_3 = v_1$, $A_2A_4 = -v_2$ and $A_3A_4 + A_2A_4 = 0$. This implies that the $A_i$ can be written in terms of a single real parameter $c$ as $A_1 = c$, $A_2 = c\beta$, $A_3 = v_1/c$ and $A_4 = -v_1\beta/c$, where $\beta = \sqrt{v_2/v_1}$; this gives

$$\frac{\hbar}{m} (\partial_y \psi_{III})_{y=0+} = \frac{iv_1\sigma_x}{c} [(\psi_I)_{y=0-} - \beta(\psi_{II})_{z=0-}], \hspace{1cm} (21)$$

which is Eq. (6) of our paper. In the limits $c \rightarrow 0$ (or $\infty$), we obtain $\psi_{III} = 0$ (or $\partial_y \psi_{III} = 0$); in either case, the current into the junction from region III vanishes, so that the metal/ferromagnet gets decoupled from the two TI regions. The value of $c$ in a given system will depend on its microscopic details such as an underlying lattice model. For the numerical calculations in our paper, we have simply set $c = 1$.

The above analysis assumed that there is no barrier present at the junction. A realistic system may be expected to have some potential barriers present at the junction in all three regions. Let us assume thin bars-

riers of the form $V(y) = V_0$ for $-d < y < 0$ in region I, $V(z) = V_0$ for $-d < z < 0$ in region II, and $V(y) = V_0$ for $0 < y < d$ in region III. For simplicity, we have assumed the barrier width ($d$) and height ($V_0$) to be the same in all three regions; we will eventually be interested in the $\delta$-function limit $d \rightarrow 0$ and $V_0 \rightarrow \infty$ keeping $dV_0/(\hbar v_1) = \chi$ constant. In Ref. 7, it has been shown that $\delta$-function barrier in a Dirac Hamiltonian produces a discontinuity in the wave function of the form

$$(\psi_{I})_{y=0-} = e^{-i\chi \sigma_z} (\psi_I)_{y=-d},$$

$$(\psi_{II})_{z=0-} = e^{i\chi (v_1/v_2) \sigma_x} (\psi_{II})_{z=-d}. \hspace{1cm} (22)$$

A $\delta$-function barrier in a Schrödinger Hamiltonian produces no discontinuity in the wave function (i.e., $(\psi_{III})_{y=d} = (\psi_{III})_{y=0+}$), but there is a discontinuity in the first derivative of the form

$$\frac{\hbar}{m} [(\partial_y \psi_{III})_{y=d} - (\partial_y \psi_{III})_{y=0+}] = 2\chi v_1(\psi_{III})_{y=d}. \hspace{1cm} (23)$$

Substituting Eqs. (22,23) in Eq. (21), and setting $c = 1$, we obtain

$$\frac{\hbar}{m} (\partial_y \psi_{III})_{y=d} = e^{-i\chi \sigma_z} (\psi_I)_{y=-d} + \beta e^{i(\chi/\beta^2) \sigma_x} (\psi_{II})_{z=-d},$$

$$\frac{\hbar}{m} (\partial_y \psi_{III})_{y=d} = \frac{iv_1\sigma_x}{c} [e^{-i\chi \sigma_z} (\psi_I)_{y=-d} - \beta e^{i(\chi/\beta^2) \sigma_x} (\psi_{II})_{z=-d}]. \hspace{1cm} (24)$$

In the limit $d \rightarrow 0$, this gives Eq. (14) of our paper.

**Numerical estimates of measured currents**

In this section we provide estimates for $I_\uparrow$ and $I_\downarrow$ which are to be measured in the proposed experiment. For a typical TI surface, the group velocity of the Dirac electrons turns out to be $v_1 \approx 10^6$ m/s. This gives us an estimate of $k_0 = mv_1/\hbar \approx 8.63 \times 10^{-5}$ m$^{-1}$. Using this, one finds $I_0 = e v_1 k_0 \approx 13.83$ mA and $E_0 = m v_1^2/2 \approx 2.89$ eV. Since $I_\uparrow$ and $I_\downarrow$ can be written as

$$I_{\uparrow(\downarrow)} = \frac{4I_0}{\pi} \int_{-E/(2E_0)}^{E/(2E_0)} dx \, \alpha_1 |t_2|^2 (\alpha_2 |t_3|^2), \hspace{1cm} (25)$$

we find numerical values of $I_\uparrow = 0.113$ mA and $I_\downarrow = 0.0906$ mA for $\chi = 0$ and $\mu = -A = 3 E_0$. This indicates that the visibility $V$ defined by

$$V = \left| \frac{I_\uparrow - I_\downarrow}{I_\uparrow + I_\downarrow} \right| \hspace{1cm} (26)$$

is close to 0.1 which means that such current measurements are well within the reach of current experimental standards. For finite and large barrier strength $\chi = 5$, the corresponding numbers are $I_\uparrow = 0.0068$ mA and $I_\downarrow = 0.012$ mA which leads to $V \approx 0.28$. 