Bounding CKM Mixing with a Fourth Family

Michael S. Chanowitz

Theoretical Physics Group
Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

Abstract

CKM mixing between third family quarks and a possible fourth family is constrained by global fits to the precision electroweak data. The dominant constraint is from nondecoupling oblique corrections rather than the vertex correction to $Z \rightarrow b\bar{b}$ used in previous analyses. The possibility of large mixing suggested by some recent analyses of FCNC processes is excluded, but 3-4 mixing of the same order as the Cabbibo mixing of the first two families is allowed.
A fourth family of quarks and leptons is an obvious extension of the Standard Model (SM) that will be investigated at the LHC. If a fourth family is discovered, it is likely to have consequences at least as profound as those that have emerged from the discovery of the third family. The necessarily heavy neutrino mass, $m_{\nu_4} > m_Z/2$, would be surprising, but without a theory of neutrino masses we are not really in a position to judge; if a fourth family were discovered, it would instantaneously refocus the effort to understand neutrino masses (see for instance \cite{2, 3}). A fourth family is consistent with precision electroweak (EW) data,\cite{4, 5, 6} and can remove\cite{5} the tension between the SM fit and the LEP II lower bound on $m_H$ that arises if the $3.2\sigma$ discrepancy between hadronic and leptonic determinations of $\sin^2 \theta_W^{\text{eff}}$ turns out to be the result of underestimated systematic error.\cite{7} The SM augmented with a fourth family is consistent with $SU(5)$ gauge coupling unification without supersymmetry.\cite{8} Electroweak baryogenesis might be viable with four families,\cite{9} although it is not in the SM with only three. Since the plausible parameter space includes the strong coupling region determined by perturbative unitarity,\cite{10} $m_{Q_4} \gtrsim 550$ GeV, a heavy fourth family could naturally play a role in the dynamical breaking of electroweak symmetry.\cite{2, 11} Even if fourth family quarks are very heavy, e.g., $m_Q \gtrsim 1$ TeV, and difficult or impossible to observe directly, they will give rise to a large nonresonant signal for production of longitudinally polarized $Z$ boson pairs from $gg \to ZZ$, that could be seen at the LHC with $5\sigma$ significance over backgrounds with only $O(10)$ fb$^{-1}$ of integrated luminosity.\cite{12}

Motivated initially by an interesting study of the FCNC constraints on a unitary $4 \times 4$ CKM matrix by Bobrowski \textit{et al.}\cite{13} we have studied the constraint on 3-4 family CKM mixing that can be obtained from precision electroweak data. They found, in addition to the expected small angle solutions, that surprisingly large mixing between the third and fourth family quarks is also allowed. They exhibit fits with $|V_{tb}|/|V_{tb}^{\text{SM}}|$ as small as 0.73, corresponding to $|V_{tb}|$ as large as $\simeq 0.63$, just at the edge of the 95% allowed region for $|V_{tb}|$ determined from single top production.\cite{14} We find however that these fits are decisively excluded by the precision EW data and present the EW constraints on 3-4 family CKM mixing for a range of fourth family masses favored by the EW data. Although the large-mixing FCNC fits are excluded, the EW constraints do allow 3-4 CKM mixing of the same order as the Cabbibo mixing of the first two families. Our results are also inconsistent with large 3-4 mixing parameters obtained in another recent study\cite{15} and constrain proposals to explain the CP anomalies suggested by B meson data.\cite{16, 17}

In the presence of 3-4 CKM mixing there are two nondecoupling radiative corrections to the precision EW observables with quadratic sensitivity to heavy fourth-family fermion masses: the $\rho$ parameter correction,\cite{18, 10} (AKA the oblique parameter $\alpha T$\cite{19}) and the $Zb\bar{b}$ vertex correction.\cite{20} In the SM both are proportional to $G_F m_t^2$ at one loop order. In the
four-family model they give rise to corrections proportional to $|V_{tb}^\prime|^2 m_t^2$ and, in the case of the oblique corrections, there are also large corrections proportional to $|V_{tb}|^2 m_b^2$ if $m_b^2 \gg m_t^2$.

Previous consideration of the precision EW constraint on 3-4 CKM mixing focused on the effect of the vertex correction on $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$, [21, 22] which was used as a constraint in subsequent FCNC studies (e.g., [16, 17]). However the oblique corrections are of precisely the same order and actually provide the most important constraint. To obtain a valid bound it is essential to reevaluate the global EW fit in the new physics model, since the not infrequently followed practice of using just the magnitude of shifts from the values in the SM fit does not take into account the possibility that a global fit incorporating the new physics may have its $\chi^2$ minimum at significantly different values of the SM parameters, e.g., $m_t$ and, especially, $m_H$.\textsuperscript{1} This in fact occurs in the results presented below. In the fits that establish the 95% CL limits (e.g., tables 2 and 5) $R_b$ is only 1$\sigma$ from its experimental value and does not contribute to the constraint.\textsuperscript{2} In addition, global fits are less susceptible to statistical fluctuation or systematic uncertainty than a constraint based on a single observable. For instance, the 2$\sigma$ upper limit on 3-4 mixing obtained from the (negative) shift in $R_b$ from its SM value would have been significantly weaker if the SM value for $R_b$ were 0.7$\sigma$ above the experimental value rather than 0.7$\sigma$ below.

In the following sections we briefly review the nondecoupling corrections and present the bounds on the mixing angle that follow from the global fits. Because the heaviest quark masses considered are at the threshold of the strong coupling region, we use the two loop correction to the $\rho$ parameter as a guide to the applicability of perturbation theory and the accuracy of the results.

**Nondecoupling corrections**

For vanishing CKM angles, the one loop correction to the $\rho$ parameter from a heavy fermion doublet ($f_1$, $f_2$) is [18, 10]

$$\delta \rho = N_C \frac{\alpha}{8\pi x_W (1 - x_W)} F_{12}$$

(1)

where $N_C = 1, 3$ for leptons and quarks respectively, $x_W = \sin^2 \theta_W$, and $F_{12}$ is

$$F_{12} = \frac{x_1 + x_2}{2} - \frac{x_1 x_2}{x_1 - x_2} \ln \frac{x_1}{x_2}$$

(2)

with $x_i = m_i^2/m_Z^2$. To study the EW constraints on the large-mixing parameter sets of

\textsuperscript{1}Alwall et al. consider the oblique constraints and remark that they are sensitive to $m_H$. They do not perform a global fit, which would incorporate the $m_H$ dependence, and instead rely on $R_b$ for their strongest precision EW constraint.

\textsuperscript{2}The shift in $R_b$ is dominated by the nondecoupling vertex correction. The oblique corrections to $\Gamma(Z \rightarrow \text{hadrons})$ and $\Gamma(Z \rightarrow b\bar{b})$ are significantly larger but cancel in the ratio. $\Gamma(Z \rightarrow \text{hadrons})$ and $\Gamma(Z \rightarrow b\bar{b})$ are not included in the fits but can be derived from combinations of observables that are.
it suffices to assume a block-diagonal form for the 3-4 CKM submatrix,\(^3\) which is then characterized by a single angle, \(\theta_{34}\), with \(|V_{tb}| = |V_{t'\nu'}| = c_{34}\) and \(|V_{t\nu}| = |V_{t'\nu'}| = s_{34}\), where \(c_{34} \equiv \cos \theta_{34}\) and \(s_{34} \equiv \sin \theta_{34}\). The oblique correction \(T\) from the fourth family is then

\[
T_4 = \frac{1}{8\pi x_W(1 - x_W)} \left\{ 3 \left[ F_{t'\nu'} + s_{34}^2(F_{t\nu} + F_{t'\nu'} - F_{t\nu} - F_{t'\nu'}) \right] + F_{\nu\nu}\right\}.
\]

(3)

The term \(-s_{34}^2 F_{t\nu}\) is the decrease from the three-family SM \(t\bar{b}\) contribution to the \(W\) boson vacuum polarization. \(F_{t\nu}\) is the largest term and puts the strongest constraint on \(\theta_{34}\).

Equation (3) is easily obtained following the derivation for \(V_{tb} = 1\) in the second paper cited in [10]; for \(V_{tb} \neq 1\) the GIM mechanism and the custodial \(SU(2)\) together ensure that the divergences cancel between the \(W\) and \(Z\) vacuum polarization terms, leaving the finite correction in (3).

We also need the oblique correction \(S\) for the fourth family quark doublet, which to a very good approximation[4] is given by

\[
S_4 = \frac{N_C}{6\pi} \left( 1 - \frac{1}{3} \ln \frac{x_1}{x_2} \right).
\]

(4)

For the lepton masses considered below, the leptonic contribution to \(S\) is negligible.

Finally there is the nondecoupling correction to the \(Zb\bar{b}\) interaction of the left handed \(b\) quark, which arises in the SM from one loop vertex corrections containing the \(t\) quark and the \(W\) boson. In our notation the interaction Lagrangian is

\[
\mathcal{L} = \frac{g}{\cos \theta_W} g_{bL} \bar{b}_L Z b_L
\]

(5)

where in the SM \(g_{bL} = -\frac{1}{2} + \frac{1}{3} x_W\). In the SM the one loop, nondecoupling vertex correction from the \(t + W\) loop graphs is[20]

\[
\delta^V g_{bL}^{SM} = \frac{\alpha}{16\pi x_W(1 - x_W)} \frac{m_t^2}{m_Z^2}
\]

(6)

The one loop correction from 3-4 CKM mixing is then

\[
\delta^V g_{bL}^{3-4} = s_{34}^2 \frac{\alpha}{16\pi x_W(1 - x_W)} \left( \frac{m_t^2}{m_Z^2} - \frac{m_t^2}{m_Z^2} \right)
\]

(7)

where again the last term accounts for the decrease of the SM top quark correction.

Bounds on \(\theta_{34}\)

Like Bobrowski et al.[13] we focus on parameters for the fourth family shown by Kribs et al.[6] to be favored by the precision EW data. In addition to the three parameter sets

\(^3\)We have verified explicitly for the large-mixing parameter sets of [13] that this approximation is valid to better than 1% for the diagonal matrix elements and to better than 3% for the off-diagonal ones.
Table 1: The three large-mixing parameter sets of [13] with the corresponding values of $T_4$ from equation (3) and the $\chi^2$ for 12 degrees of freedom from the global fits. The pulls of four sensitive observables are compared with the pull of $R_b$.

|  | $m_{t'}$ | $|s_{34}|$ | $T_4$ | $\chi^2$ | $m_W$ | $\Gamma_Z$ | $A_{FB}^b$ | $A_{LR}$ | $R_b$ |
|---|---|---|---|---|---|---|---|---|---|
| I | 326 | 0.51 | 1.09 | 188 | 9 | 5 | 5 | 0.8 | 3 |
| II | 654 | 0.37 | 3.58 | 5750 | 53 | 30 | 27 | 24 | 7 |
| III | 389 | 0.63 | 2.59 | 2530 | 35 | 19 | 18 | 15 | 6 |

identified by Bobrowski et al., shown in table 1, we survey other $t'$ masses between 300 and 600 GeV. Following [5] and [6] we confirm for $s_{34} = 0$ that $\chi^2$ is minimized for $|m_{t'} - m_{b'}| \sim 45 - 75$ GeV and set the $b'$ mass to

$$m_{b'} = m_{t'} - 55\text{ GeV}.$$  

(8)

As discussed below, the limits on $s_{34}$ do not depend sensitively on this choice. The lepton masses, which have relatively little effect on the limit on $s_{34}$, are chosen as $m_{\nu_4} = 100$ GeV and $m_{l_4} = 145$ GeV.\textsuperscript{4}

Table 1 shows that the three large-mixing parameter sets of [13] are excluded “with extreme prejudice” by the EW data. The large contributions to $T_4$ are responsible for the huge $\chi^2$ values. In these fits the dominant contributors to $\chi^2$ are the $W$ boson mass, the $Z$ boson width, and one or both of $A_{FB}^b$ and $A_{LR}$. Since the FCNC constraints for these three parameter sets have been carefully considered in [13], we include them in the 95% CL limit fits presented below.

Following the procedure of the EWWG,[23] the global fits are obtained by minimizing $\chi^2$ while varying four SM parameters: $m_t$, $\Delta \alpha_5$, $\alpha_S$, and $m_H$, with $m_H$ allowed to vary freely between 10 GeV and 1 TeV. Like the EWWG we leave $\alpha_S$ unconstrained and determine it from the fits; for all the fits at or within the 95% CL limit for $|s_{34}|$, the values of $\alpha_S$ are in reasonable agreement with other determinations. The global constraints on $\theta_{34}$ are fairly valued because they allow for the possibility of $\chi^2$ minima at points in the parameter space that are quite different than the location of the SM minimum, and they are efficient because they aggregate the effect of the new physics on all of the relevant observables.

Radiative corrections for the global fits are computed with ZFITTER[24], including the two loop corrections to $\sin^2 \theta_W^{\text{eff}}$ [25] and $m_W$.[26] Our fits include the largest experimental correlations, taken from the EWWG. When we use the same measurement set, our SM fit

\textsuperscript{4}We assume a Dirac mass for $\nu_4$. A dynamically generated Majorana mass for the fourth neutrino which made a negative contribution to $T$ could weaken the constraints for given $m_{\nu}$.\textsuperscript{3}
agrees closely with the EWWG. Unlike typical fits with the oblique parameters $S$ and $T$, which are performed with respect to fixed reference values of the SM parameters, we use the complete set of radiative corrections from ZFITTER to compute the dependence of $\chi^2$ on the SM parameters $m_H$, $m_t$, $\Delta\alpha_5$, and $\alpha_s$, for each $(S_4, T_4)$ pair, which represent only the fourth family corrections. This procedure is then more accurate, however the resulting $S$, $T$ values are not directly comparable to $S$ and $T$ from typical fits with fixed “reference” values of the SM parameters.

In this work we focus on the EWWG set of observables. We also briefly describe the results for the data set without the three hadronic front-back asymmetry measurements.\textsuperscript{5} The fits are performed for four-family models with $m_t'$ between 300 GeV and 1 TeV, including the three parameter sets of \cite{13}, with $b'$ and lepton masses as described above. In all cases the $\chi^2$ minimum occurs at $\theta_{34} = 0$. The 95\% CL upper limit is obtained by increasing $\theta_{34}$ until $\chi^2$ increases by 3.84 units, corresponding to CL($\Delta\chi^2$, 1) = 0.95, like the procedure commonly used to obtain the 95\% upper limit on $m_H$ in the SM fits.\textsuperscript{6} The fits at $m_t' = 1$ TeV are intended primarily to probe the range of applicability of perturbation theory.

The results for $m_t' = 500$ GeV are illustrated in figure 1 and table 2. Table 2 displays the SM fit, the four-family fit for $m_t' = 500$ GeV and $\theta_{34} = 0$ and also for $\theta_{34}$ at its (one loop) 95\% upper limit, $|s_{34}| = 0.15$. $\chi^2$ as a function of $|s_{34}|$ is shown in figure 1. The $\chi^2$ of the four family model with $\theta_{34} = 0$ is little changed from the SM but the central value of $m_H$ is increased, from 85 to 139 GeV, and the 3.2\sigma discrepancy between $A_{LR}$ and $A_{FB}^b$ is more equally shared between the two, in contrast to the SM fit in which $A_{FB}^b$ is the outlier.\textsuperscript{7} In the fit at the edge of the 95\% confidence level, $m_H$ increases to the strong coupling regime near 1 TeV because of the increased value of $T_4$, and $A_{LR}$ has become the outlier, while the pull of $R_b$ is only 1.2. The central value of $m_H$ as a function of $|\sin\theta_{34}|$ is shown in figure 2.

In table 3 we summarize the limits on $\theta_{34}$ for seven values of $m_t'$ between 300 to 650 GeV, including the three parameter sets from reference \cite{13}, and in addition at $m_t' = 1$ TeV. In all cases the fits at $\theta_{34} = 0$ are nearly identical to the fit shown in table 2 for $m_t' = 500$ GeV. As we would expect from equations (3) and (7), the limit on $|\sin\theta_{34}|$ becomes proportional to $1/m_{t'}$ for $m_{t'} \gg m_t$. For these fits at the 95\% confidence limit, the Higgs boson mass is at $m_H = 790 \pm 30$ GeV and $T_4 = 0.47 \pm 0.01$. In all cases we have $|V_{tb}| \simeq |V_{t'b'}| \simeq |\cos\theta_{34}| \geq 0.94$ and for $m_{t'} \geq 500$ GeV we have $|\cos\theta_{34}| \geq 0.99$.

\textsuperscript{5}Unlike the EWWG we do not include the $W$ boson width, which with a 2\% error is not a precision measurement in the sense of the other measurements that typically have part per mil precision. In any case, $\Gamma_W$ has a negligible effect on the fits.

\textsuperscript{6}For the SM fits the limit is at $\Delta\chi^2 = 2.71$ corresponding to the 90\% symmetric confidence interval and the 95\% upper limit. Because the bound on $|\theta_{34}|$ is one-sided, the 95\% limit is at $\Delta\chi^2 = 3.84$.

\textsuperscript{7}We assign 12 degrees of freedom to each four-family fit, assuming discovery at the LHC as a prior. This convention has no effect on the 95\% CL limit.
Figure 1: $\chi^2$ distribution as a function of $|\sin \theta_{34}|$ for the global fit to the four family model with $m_{t'} = 500$ GeV. The horizontal line indicates the 95% confidence interval.
|                  | Experiment | SM Pull | SM$_4$ Pull | $s_{34}[95\%]$ Pull |
|------------------|------------|---------|-------------|---------------------|
| $A_{LR}$         | 0.1513 (21)| 0.1480  | 1.6         | 0.1466 2.2          |
| $A'_{FB}$        | 0.01714 (95)| 0.01642 | 0.8         | 0.01592 1.3         |
| $A_{e,\tau}$    | 0.1465 (32)| 0.1480  | -0.5        | 0.1466 -0.03        |
| $A''_{FB}$       | 0.0992 (16)| 0.1037  | -2.8        | 0.1028 -2.2         |
| $A_{FB}$         | 0.0707 (35)| 0.0741  | -1.0        | 0.0734 -0.8         |
| $Q_{FB}$         | 0.23240 (120)| 0.23140 | -0.8        | 0.23158 -0.7        |
| $m_W$            | 80.398 (25)| 80.374  | 0.9         | 80.398 0.0          |
| $\Gamma_Z$      | 2495.2 (23)| 2495.9  | 0.3         | 2498.1 -1.3         |
| $R_\ell$        | 20.767 (25)| 20.744  | 0.9         | 20.733 1.4          |
| $\sigma_h$      | 41.540 (37)| 41.477  | 1.7         | 41.484 1.5          |
| $R_b$            | 0.21629 (66)| 0.21586 | 0.7         | 0.21587 0.6         |
| $R_c$            | 0.1721 (30)| 0.1722  | -0.04       | 0.1722 -0.03        |
| $A_b$            | 0.923 (20) | 0.935   | -0.6        | 0.935 -0.6          |
| $A_c$            | 0.670 (27) | 0.668   | 0.07        | 0.668 0.09          |
| $m_t$            | 172.6 (1.4)| 172.3   | 0.2         | 172.3 0.2           |
| $\Delta \alpha_5(m_Z)$ | 0.02758 (35)| 0.02768 | -0.3        | 0.02747 0.3         |
| $\alpha_S(m_Z)$ | 0.02758 (35)| 0.02768 | -0.3        | 0.02747 0.3         |

|                  |           |         |             |                     |
|------------------|------------|---------|-------------|---------------------|
| $m_{\nu}$        |            | 500     | 500         |                     |
| $s_{34}$          |            | 0.0     | 0.15        |                     |
| $T_4$             |            | 0.20    | 0.48        |                     |
| $S_4$             |            | 0.15    | 0.15        |                     |
| $x_{\nu}$        |            | 0.0     | 0.00052     |                     |
| $m_H$             |            | 85      | 139         | 810                 |
| CL($m_H > 114$)   |            | 0.26    | 0.67        | 1.00                |
| $m_H(95\%)$       |            | 148     | 235         | > 1000              |
| $\chi^2$/dof     |            | 17.3/12 | 17.0/12     | 20.9/12             |
| CL($\chi^2$)     |            | 0.14    | 0.15        | 0.05                |

Table 2: Global fits: the SM, the 4 family SM with $m_{\nu} = 500$ GeV and $s_{34} = 0$ and again with $s_{34}$ at the 95% confidence level.
Figure 2: Higgs boson mass as a function of $|\sin \theta_{34}|$ for the global fit to the four family model with $m_{t'} = 500$ GeV. The horizontal line indicates the 95% confidence interval for $|\sin \theta_{34}|$.

| $m_{t'}$ | $T_4$ | $m_H$ (GeV) | $|s_{34}^{(1)}|$ | $|s_{34}^{(2)}|$ | $\pm \Delta \theta_{t'}^{(2)}$ | $|c_{34}^{(2)}|$ |
|---|---|---|---|---|---|---|
| 300 | 0.46 | 760 | 0.32 | 0.35 ± 0.001 | 0.94 |
| 326 | 0.47 | 760 | 0.28 | 0.30 ± 0.002 | 0.95 |
| 389 | 0.48 | 760 | 0.21 | 0.23 ± 0.004 | 0.97 |
| 400 | 0.47 | 800 | 0.20 | 0.22 ± 0.005 | 0.98 |
| 500 | 0.48 | 810 | 0.15 | 0.17 ± 0.007 | 0.99 |
| 600 | 0.48 | 800 | 0.12 | 0.14 ± 0.010 | 0.99 |
| 654 | 0.48 | 820 | 0.11 | 0.13 ± 0.013 | 0.99 |
| 1000 | 0.49 | 820 | 0.07 | 0.11 ± 0.010 | 0.99 |

Table 3: 95% CL upper limits on $|s_{34}|$ at one and two loops from global fits to the EWWG data set. $T_4$ and $m_H$ from the 95% CL fits are also shown.
To gauge the range of applicability of perturbation theory for large quark masses, the limits in table 3 have been obtained at both one and two loop order in the leading, non-decoupling electroweak corrections to the $\rho$ parameter.\(^{8}\) The leading two loop corrections for large quark mass are known,\(^{27, 28, 29}\) but in no case with both the Higgs boson mass dependence, which is large, and the dependence on the masses of both quarks in the doublet. The two loop corrections computed by Barbieri et al.,\(^{28}\) are best suited for our purpose: they include the full $m_H$ dependence but for only one heavy quark in the doublet, i.e., for $m_{Q_1} \gg m_{Q_2} \approx 0$. This captures the contribution that is most important for the bound on the mixing angle, because the $\sin \theta_{34}$ dependent term in $T_4$, equation (3), is dominated by $F_{tb'}$, for which it is always an excellent approximation.

We do not need to consider the two loop correction to $F_{tb'}$, because even the one loop term has a negligible effect on the $\sin \theta_{34}$ dependence of $T_4$. The $F_{tb'}$ term is however somewhat problematic. The two loop correction, $F_{tb'}^{(2)}$, can safely be neglected for the smallest masses we consider, say $m_\nu \lesssim 400$ GeV, for which the one loop term $F_{tb'}^{(1)}$ is not very important, but for $500$ GeV $\lesssim m_\nu \lesssim 1$ TeV, $F_{tb'}^{(1)}$ is not negligible and approximating $m_\nu = 0$ in the two loop correction $F_{tb'}^{(2)}$ may be a poor approximation. To give a conservative indication of the possible error, the effect on the bounds of a $\pm 100\%$ variation in the value of $F_{tb'}^{(2)}$ is shown as $\pm \Delta_{tb'}^{(2)}$ in table 3. The conclusion is that the bounds on $\sin \theta_{34}$ are probably good to a few percent at $m_\nu = 300$ GeV, while at $m_\nu = 650$ GeV they are probably valid to from 10 to 20%. At $m_\nu = 1$ TeV the order one variation from $\Delta_{tb'}^{(2)}$ significantly overestimates the uncertainty from $F_{tb'}^{(2)}$, but the order 50% shift from the first to second order limit on $|s_{34}|$, from 0.07 to 0.11, is a signal that perturbation theory has become unreliable.

The upper limits do not depend sensitively on the choice of $m_\nu - m_\nu'$ in equation (8). For larger mass differences, e.g., $m_\nu - m_\nu' \gtrsim 100$ GeV, there are no acceptable fits: at $\theta_{34} = 0$ the fits are poor, with $\text{CL}(\chi^2) < 0.03$, and quickly become much poorer as $|\theta_{34}|$ increases. For smaller mass splitting, e.g., at the extreme, $m_\nu = m_\nu'$, the $\chi^2$ CL's are acceptable for $\theta_{34} = 0$, with $\text{CL}(\chi^2) = 0.18$, but the predictions for the Higgs mass are unacceptable, with $m_H = 35$ GeV and $\text{CL}(m_H > 114) = 0.0016$. In this case marginally acceptable fits can be found by increasing $|s_{34}|$, which raises the Higgs mass toward the allowed region while maintaining acceptable $\chi^2$ CL's. For instance, with $m_\nu = m_\nu' = 500$ GeV and $s_{34} = 0.07$, the fit is just consistent at 5% CL with the 114 GeV lower limit on $m_H$, with $m_H = 58$ GeV, $\text{CL}(m_H > 114) = 0.05$ and $\text{CL}(\chi^2) \simeq 0.18$. Taking $\Delta \chi^2 = 3.84$ from either this fit or from the fit at $\theta_{34} = 0$, we find the one loop 95% upper limit at $|s_{34}| < 0.155$, little changed from the limit that we obtained using $m_\nu - m_\nu' = 55$ GeV. There is a simple reason for the insensitivity of the limit to the mass splitting: as $|s_{34}|$ increases, the terms in equation (3)

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\(^{8}\)We have not included the two loop correction to the $Z\bar{t}bb$ vertex, since the nondecoupling vertex correction does not have an important effect on the global fits.
Figure 3: $\chi^2$ distribution as a function of $|\sin \theta_{34}|$ of the global fit to the four-family model with $m_{t'} = 500$ GeV, for the data set without the hadronic asymmetry measurements. The horizontal line indicates the 95% confidence interval.
Table 4: 95% CL upper limits on $|s_{34}|$ at one and two loops from global fits to the EWWG data set without the hadronic asymmetry measurements. $T_4$ and $m_H$ from the 95% CL fits are also shown.

$|s_{34}|$, $|s_{34}| \pm \Delta_{1b'}^{(2)}$, $|\epsilon_{34}|$

| $m_{t'}$ | $T_4$ | $m_H$(GeV) | $|s_{34}|^{(1)}$ | $|s_{34}|^{(2)} \pm \Delta_{1b'}^{(2)}$ | $|\epsilon_{34}|^{(2)}$ |
|-------|-------|------------|----------------|------------------|----------------|
| 300   | 0.35  | 300        | 0.25           | 0.26 $\pm$ 0.0008 | 0.97            |
| 326   | 0.35  | 280        | 0.21           | 0.22 $\pm$ 0.0010 | 0.98            |
| 389   | 0.35  | 270        | 0.16           | 0.17 $\pm$ 0.0016 | 0.99            |
| 400   | 0.35  | 290        | 0.15           | 0.16 $\pm$ 0.0016 | 0.99            |
| 500   | 0.35  | 270        | 0.11           | 0.12 $\pm$ 0.0027 | 0.99            |
| 600   | 0.35  | 290        | 0.087          | 0.095 $\pm$ 0.0033 | 0.995           |
| 654   | 0.35  | 280        | 0.078          | 0.086 $\pm$ 0.0035 | 0.996           |
| 1000  | 0.35  | 270        | 0.048          | 0.059 $\pm$ 0.007  | 0.998           |

It is interesting to consider the data set with hadronic asymmetries excluded, motivated by the possibility that underestimated systematic error might contribute to the 3.2\sigma discrepancy in the SM determination of $\sin^2 \theta_W^{\text{eff}}$ from the hadronic and leptonic asymmetry measurements.[7] The hadronic asymmetry measurements are more challenging, both experimentally and, especially, theoretically, because of the difficulty of extracting quark asymmetries from the actual measurements of hadronic final states. The QCD corrections are large, three times the experimental error, and the Monte Carlo calculations needed to merge the large QCD corrections with the experimental cuts give rise to a systematic error that is difficult to quantify. Without the three hadronic measurements, $A_{FB}^b$, $A_{FB}^c$, and $Q_{FB}$, the confidence level of the SM fit improves dramatically but the fit predicts a very light Higgs boson, $m_H = 50$ GeV, with 95% upper limit at 105 GeV, in conflict at 97% CL with the 114 GeV LEP II lower limit. Results are shown in figure 3 and tables 4 and 5. As found by Novikov et al.[5], the four-family model removes the conflict with the LEP II lower limit for this data set. The limits in table 4 are \(\simeq 40\%\) stronger than for the full data set in table 3. At the 95% limit for $\theta_{34}$ the Higgs boson masses range from 275 to 300 GeV with $T_4 = 0.35$ in all cases. Because $T_4$ and $m_H$ are smaller for this data set, both the one loop and two loop corrections are smaller, and perturbation theory appears to be under better control.

Discussion

While this work was initially motivated by the paper of Bobrowski et al., several other studies have considered the possible role of a fourth generation on FCNC phenomena and
|                          | Experiment | SM   | Pull | SM₄ | Pull | s₃₄[95%] | Pull |
|--------------------------|------------|------|------|-----|------|----------|------|
| A₉LR                    | 0.1513 (21)| 0.1503 | 0.5  | 0.1483 | 1.4  | 0.1474  | 1.8  |
| A₁₉FB                   | 0.01714 (95)| 0.01694 | 0.2  | 0.1649 | 0.7  | 0.01630 | 0.9  |
| Aₑ,τ                    | 0.1465 (32)  | 0.1503 | -1.2 | 0.1483 | -0.6 | 0.1474  | -0.3 |
| mₜ₆                     | 80.398 (25)  | 80.403 | 0.03 | 80.423 | -1.0 | 80.425  | -1.1 |
| Γ₉                       | 2495.2 (23)  | 2496.0 | -0.3 | 2498.5 | -1.4 | 2499.2  | -1.7 |
| Rₑ                       | 20.767 (25)  | 20.741 | 1.0  | 20.729 | 1.5  | 20.725  | 1.7  |
| σ₉ₜ                     | 41.540 (37)  | 41.482 | 1.6  | 41.489 | 1.4  | 41.491  | 1.3  |
| R₉ₚ                     | 0.21629 (66) | 0.21584 | 0.7  | 0.21586 | 0.6  | 0.2157  | 1.0  |
| R₉ₚ                     | 0.1721 (30)  | 0.1722 | -0.04 | 0.1722 | -0.03 | 0.1722  | -0.05 |
| Aₑ                       | 0.923 (20)   | 0.935   | -0.6 | 0.935  | -0.6 | 0.935   | -0.6 |
| Aₑ                       | 0.670 (27)   | 0.669   | 0.03 | 0.668  | 0.06 | 0.668   | 0.08 |
| mₜₚ                     | 172.6 (1.4)  | 172.3   | 0.2  | 172.3  | 0.2  | 172.3   | 0.2  |
| Δα₅(m₉)                 | 0.02758 (35) | 0.02754 | 0.1  | 0.02747 | 0.3  | 0.2732  | 0.7  |
| αₛ(m₉)                  | 0.1174       | 0.1162  | 0.1168 |

|                          |            |      |      |      |      |      |
|--------------------------|------------|------|------|------|------|------|
| mₜₚ                     |            | 500  | 500  |
| s₃₄                      |            | 0.0  |      | 0.11 |
| T₄                       |            | 0.20 |      | 0.35 |
| S₄                       |            | 0.15 |      | 0.15 |
| x₅ₚ                     |            | 0.0  |      | 0.0028 |
| mₜₚ                     |            | 500  |      |      |
| CL(mₜₚ > 114)            |            | 0.03 |      | 1.0  |
| mₜₚ(95%)                 |            | 105  | 174  | 480  |
| χ²/dof                   |            | 5.6/9 | 9.8/9 | 13.7/9 |
| CL(χ²)                   |            | 0.78 | 0.36 | 0.13 |

Table 5: Global fits for the data set without the hadronic asymmetry measurements: the SM, the 4 family SM with mₜₚ = 500 GeV and s₃₄ = 0, and again with s₃₄ at the 95% confidence level.
the CKM matrix. The fits of Yanir\cite{21} for $m_{t'} = 500$ GeV require $|s_{34}| \lesssim 0.14$ and therefore fall within the 95\% CL limit of the fit to the complete data set, table 3, and just beyond the limit from the fit with hadronic asymmetries excluded, table 4. The 95\% CL limit quoted by Alwall et al., $|c_{34}| > 0.93$\cite{22} is weaker than the global fit limits in tables 3 and 4, especially for the larger values of $m_{t'}$. Herrera et al.\cite{15} have studied the FCNC constraints in texture models of the $4 \times 4$ quark mass matrices. They remark on an isolated solution with $|V_{tb}| \sim 0.88$, implying $|s_{34}| \sim 0.47$, which is decisively excluded by the EW fits, like the parameter sets in table 1. Most of their fits lie within $0.90 \leq |V_{tb}| \leq 0.94$, which is excluded at 95\% CL in all cases considered here except the fit to the complete data set with $m_{t'} = 300$ GeV shown in table 3.

Hou et al.\cite{16} and Soni et al.\cite{17} have identified regions in the CKM$_4$ parameter space that could explain possible anomalies in B meson CP measurements, requiring large mixing between not only the third and fourth families but also between the second and the fourth. For instance, Hou et al. consider the four family model with $m_{t'} = 300$ GeV, $s_{34} = 0.22$, and with 2-4 mixing given by $|V_{ts}| = 0.114$ and $|V_{cb'}| = 0.116$. There are then nonnegligible contributions to $T_4$ from the 2-4 family mixing and additional terms must be added to equation (3), which becomes

$$T_4 = \frac{1}{8\pi x_W (1 - x_W)} \left\{ 3 \left[ F_{s3} + s_{34}^2 (F_{sb} + F_{bb} - F_{tb} - F_{t'b'}) \right] + F_{t'4} \right\}$$

$$+ \frac{3}{8\pi x_W (1 - x_W)} \left[ |V_{ts'}|^2 F_{s's} + |V_{cb'}|^2 F_{cb'} \right].$$

Using the above mixing angles and $m_{t'} = 300$ GeV the fit to the full data set yields $\Delta \chi^2 = 2.44$, which falls within the 95\% CL limit at $\Delta \chi^2 = 3.84$. For the fit with hadronic asymmetries excluded we find $\Delta \chi^2 = 4.64$, just beyond the 95\% CL limit.

Soni et al.\cite{17} quote a range of values for the product $\lambda_{t'} = |V_{ts'} V_{tb'}|$, for values of $m_{t'}$ between 400 and 700 GeV. The limit on $\lambda_{t'}$ then depends on the hierarchy between the CKM matrix elements,

$$r = \frac{|V_{ts'}|}{|V_{tb'}|}.$$ 

Neglecting a possible contribution which might be expected from $F_{cb'}$ but is not specified in \cite{17}, the limits quoted in tables 3 and 4 for $s_{34}^2$ now apply instead to the combination $(1 + r^2)s_{34}^2$. The corresponding upper limit on $\lambda_{t'} = rs_{34}^2$ is then a function of $r$,

$$\lambda_{t'} < \frac{r}{1 + r^2} X_{95}$$

where $X_{95}$ is the square of the 95\% CL upper limits on $s_{34}$ given in the tables. The limit is maximal for $r = 1$, corresponding to $\lambda_{t'} < X_{95}/2$; in this case the full range of preferred
values in [17] is allowed for the fit to the full data set and only slightly restricted in the fit with hadronic asymmetries excluded. If we assume a hierarchy of order the Cabibbo angle, \( r \sim 0.2 \), then the bound is tighter, \( \lambda_s ^{\prime} \lesssim X_{95}/5 \); in this case a portion of the preferred range in [17] is excluded at 95\% CL for both data sets, but a significant portion continues to be allowed.

If a fourth family were discovered at the LHC, the subsequent study of its properties would be a major undertaking, with many profound implications. The elucidation of the four family CKM matrix would be important to understand the on-shell measurements at the LHC as well as the virtual implications for flavor physics and CP violation. Electroweak precision data, perhaps eventually augmented by a second generation \( Z \) boson factory, should continue to play an important role, by constraining and vetting the emerging picture and even by indicating a mismatch with direct high energy measurements that could be a signal for still unobserved new physics.

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