Wormhole Spaces: the Common Cause for the Black Hole Entropy-Area Law, the Holographic Principle and Quantum Entanglement

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Abstract
We present strong arguments that the deep structure of the quantum vacuum contains a web of microscopic wormholes or short-cuts. We develop the concept of wormhole spaces and show that this web of wormholes generate a peculiar array of long-range correlations in the patterns of vacuum fluctuations on the Planck scale. We conclude that this translocal structure represents the common cause for both the BH-entropy-area law, the more general holographic principle and the entanglement phenomena in quantum theory. In so far our approach exhibits a common structure which underlies both gravity and quantum theory on a microscopic scale. A central place in our analysis is occupied by a quantitative derivation of the distribution laws of microscopic wormholes in the quantum vacuum. This makes it possible to address a number of open questions and controversial topics in the field of quantum gravity.
1 Introduction

In the following we want to give a new explanation of the area law of black hole (BH) entropy and the more general and stronger holographic principle. Furthermore, we provide (in our view) convincing arguments that an important structural ingredient of the deep structure of our quantum vacuum is a network of microscopic wormholes. In contrast to e.g. string theory and loop quantum gravity (LQG), which both employ the quantum laws more or less unaltered all the way down to the remote Planck scale, we regard this as an at least debatable assumption. We rather view the holographic hypothesis as a means to understand how both quantum theory and gravitation do emerge as derived and secondary theories from a more fundamental theory living on a more microscopic scale. A central role in this enterprise is played by an analysis of the microscopic structure of the quantum vacuum which leads to the key concept of wormhole spaces.

This important conceptual structure makes it possible to understand the holographic aspects of quantum gravity, on the one hand, and the (non-local) entanglement phenomena pervading ordinary quantum physics, on the other hand, in a relatively natural way. Furthermore we think that there exist links to the old ideas of e.g. Sakharov and Zeldovich, dubbed induced gravity (see for example [1],[2],[3],[4]).

Some words are in order regarding the relation of our investigation to the analysis of BH entropy in, say, string theory. Three scenarios are in our view in principle possible. Either, both approaches address the same phenomena in different languages, or they deal with them on different scales of resolution of space-time. Be that as it may, we think that our observation that the true ground state of our quantum vacuum seems to be what we call a wormhole space (see section 3) is an aspect which is not apparent in the original string theory approach and may be helpful to fix the proper ground state in string theory.

In [5] Bekenstein remarked that the deeper meaning of black-hole entropy (BH-entropy) remains mysterious. He asks, is it similar to that of ordinary entropy, i.e. the log of a counting of internal BH-states, associated with a single BH-exterior? (6,7) or [8]). Or, similarly, is it the log of the number of ways, in which the BH might be formed. Or is it the log of the number of horizon quantum states? (9,10). Does it stand for information, lost in the transcendence of the hallowed principle of unitary evolution? ([11],[12]). He then claims that the usefulness of any proposed interpretation of BH-entropy depends on how well it relates to the original “statistical” aspect of entropy as a measure of disorder, missing information, multiplicity of microstates compatible with a given macrostate, etc.

Quite a few workers in the field argue that the peculiar dependence of BH-entropy on the area of the event horizon points to the fact that the degrees of freedom (DoF), responsible for BH-entropy, are situated near the event horizon. This seems to be further corroborated by the corresponding behavior of the so-called entanglement entropy, i.e. its (apparent) linear dependence on the area.
of the dividing surface (cf., just to mention a few sources, [13], [14], [15] or the lively debate in [16], concerning entanglement entropy in a more general setting, [17], [18]). This linear dependence does however not generally hold without further qualifications. It does in particular not hold for excited states (see [19])!

That is, while some particular sort of entanglement certainly plays an important role in this context, the real question is in our view the scale of resolution of space-time where this entanglement becomes effective and the nature of the quantum vacuum on this level of resolution.

Remark: We want to emphasize the in our view crucial (but frequently apparently not fully appreciated) point that the entropy content of a BH is maximal.

We think, the usual version of entanglement, we observe on the scales of ordinary quantum theory, is only an epiphenomenon, representing rather the coarse-grained effect of a hidden structure which lives on a much more microscopic scale. I.e., we are sceptical whether on such a microscopic scale the quantum vacuum can still be treated in the way of an ordinary quantum field theory vacuum as suggested in some of the papers cited above. We think, the maximum-entropy property of the BH-interior suggests another interpretation. We will come back to this point in more detail in section 4 (cf. also the sceptical remarks in some of the review papers by Wald, e.g. [20] (see in particular sect.6, Open Issues), [21] (see in particular sect.4, Some unresolved Issues and Puzzles), [22]).

As BH-entropy is widely regarded as an observational window into the more hidden and primordial quantum underground of space-time, it should be expected that it can be naturally explained within the frameworks of the leading candidates of such a theory, i.e., to mention the most prominent, string theory or LQG. For certain extreme situations string theory manages to give an explanation of the BH-entropy-area law. Whether the explanation is really natural is perhaps debatable (it relies in fact on a number of assumptions and correspondences as e.g. peculiar intersections of various classes of p-branes). In a sense, it is rather a correspondence between BH-behavior and the configurational entropy of certain string states. To mention some representative papers, [23], [24], [25], [26], [27], [28]. In LQG, on the other hand, it is assumed from the outset (at least as far as we can see) that the corresponding DoF are sitting at the BH-horizon. Therefore the observed area dependence of BH-entropy is perhaps not so surprising (cf. e.g. [29], [30]).

In the enumeration of the most promising candidates for a theory of quantum gravity one approach is usually left out which, we nevertheless think, has a certain potential. One may, for example, tentatively divide quantum gravity candidates into roughly three groups, the relativisation of quantum theory (with e.g. LQG and causal set theory as members), the quantisation of general relativity (string theory being a prominent candidate) or third, theories which underlie both general relativity and quantum theory but are in fact more fundamental and structurally different from both and contain these two pillars of modern physics as derived and perhaps merely effective sub-theories, living on coarser scales (cf. e.g. [31]). In the following we want to develop such a model
As far as we can see, such a philosophy is also shared by ‘t Hooft who emphasized this point in quite a few papers (see e.g. [32], [33], [34], [35]). We quote from [33]:

...it may still be possible that the quantum mechanical nature of the phenomenological laws of nature at the atomic scale can be attributed to an underlying law that is deterministic at the Planck scale but with chaotic effects at all larger scales...Since, according to our philosophy, quantum states are identified with equivalence classes...

Furthermore:

...It is the author’s suspicion however, that these hidden variable theories failed because they were based far too much upon notions from everyday life and ‘ordinary physics’ and in particular because general relativistic effects have not been taken into account properly.

While ‘t Hooft usually chooses his model theories from the cellular automaton (CA) class, we are adopting a point of view which is on the one hand more general and flexible but, on the other hand, technically more difficult and complex. Instead of a relatively rigid underlying geometric substratum in the case of CA (typically some fixed regular lattice) on which the CA are evolving according to a given fixed (typically local) CA-law, we are employing quite irregular, dynamic geometric structures called by us cellular networks, the main point being that connections (edges or links) between the respective nodes or cells can be created or annihilated according to a dynamical law which, in addition, determines the evolution of the local node- and edge-states.

To put it briefly, the ‘matter distribution’ (i.e. the global pattern of node-states) acts on the geometry of the network (the global pattern of active edges) and vice versa. Thus, as in general relativity, the network is supposed to find both its internal geometry and its matter-energy distribution with the help of a generalized dynamical law which intertwines the two aspects (cf. e.g. [36] or [37] and further references given there). Technically, the geometric substructure can be modelled by large, usually quite irregular (random) graphs.

To make our point clear, this approach should not be confused with e.g. the spin network approach in LQG or various forms of (dynamical) triangulations. Our networks are usually extremely irregular and wildly fluctuating on a microscopic scale, resembling rather Wheeler’s space-time foam, and smooth geometric structures (as e.g. dimensional notions) are hoped to emerge via some sort of a geometric renormalisation process (in fact a very particular organized form of coarse-graining steps). Some of the interesting deeper mathematical aspects can for example be looked up in [38].

In our dynamical network approach to quantum space-time physics the nodes are assumed to represent cells of some microscopic size (presumably Planck size), the internal details of which cannot be further resolved in principle or are ignored and averaged over for convenience and will be represented instead by a
simple ansatz for a local (node) state. It can perhaps be compared with the
many existing spin-models which are designed to implement certain characteristic features of complex solids. This is more or less the same philosophy as in the CA-framework. The elementary connections between the nodes (the edges in graph theory) are assumed to represent elementary interactions or information channels among the cells and also carry simple edge-states. We made a detailed numerical analysis of the behavior of such networks in \[39\].

Remark: We would like to emphasize however, that our approach does not really rely on this particular framework. It rather serves as a means to illustrate the various steps in our analysis within a concrete model theory.

The paper is organized as follows. In the next section we analyse the basic substratum, i.e. the microscopic patterns of vacuum fluctuations, in particular the negative energy fluctuations. In section \[3\] we describe the three different roads which lead (in our view: inevitably) to the concept of wormhole space. The preparatory sections 2 and 3 are then amalgamated in section 4 into a detailed analysis of the microscopic distribution pattern of short-cuts or wormholes and their consequences for the number of effective DoF in a volume of space. We introduce a new type of dimension, the so-called holographic dimension. Furthermore, we explain the microscopic basis of the holographic principle in general and the bulk-boundary correspondence between the DoF in the interior of e.g. a BH and the DoF on the boundary. Some apparent counter examples concerning the area-scaling property (see e.g. \[72\]; Wheeler’s ‘bag of gold’-spacetimes) are very briefly addressed. In the last section we briefly comment on a number of immediate applications of our microscopic holographic approach and (open) problems which can be settled with the help of our framework.

2 The Structure of the Vacuum Fluctuations on a Microscopic Scale

A characteristic feature of the dynamical network models we investigated is their undulatory character. As a consequence of the feedback structure of the coupling between node (cell) states and wiring diagram of edges (i.e. the pattern of momentary elementary connections or interactions) the network never settles in a static, frozen final state. The network may of course end up in some attracting subset of phase space but typical are wild fluctuations on a small (microscopic) scale with possibly some macroscopic patterns emerging on a coarser scale forming some kind of superstructure (see e.g. \[39\]).

It is in our view not sufficiently appreciated that, in contrast to most of the other systems being studied in physics, the quantum vacuum is in a state of eternal unrest on a microscopic scale, with, for all we know, short-lived excitations constantly popping up and being reabsorbed by the seething sea.

It therefore seems reasonable to regard our above network model (investigated in e.g. \[30\] to \[39\]) as a (toy) model of the quantum vacuum with the
energy-momentum fluctuations on short scales being associated with the fluctuations of the local node and edge states.

**Postulate 2.1** In the following we adopt the working hypothesis of a parallelism of network behavior and microscopic behavior of the quantum vacuum.

We now come to a detailed analysis of the microscopic pattern of vacuum fluctuations. In sect. 4 of [40] we made a calculation which shows that, given the huge number of roughly Planck-size grains in a macroscopic piece of space and assuming that the individual grains are allowed to fluctuate almost independently, more precisely, some grain variable like e.g. the local energy, the total fluctuations in a macroscopic or mesoscopic piece of space of typical physical quantities are still so large (i.e. macroscopic) that they should be observable. Note that with the number of nodes of roughly Planck-size, \( N_P \), in a macroscopic volume, \( V \), being gigantic, its square root is still very large (for the details of the argument see [40]). More precisely, with \( q_i \) some physical quantity belonging to a microscopic grain (e.g. energy, momentum, some charge etc. and taking for convenience \( \langle q_i \rangle = 0 \)) and \( Q_V := \sum_i q_i \) being the observable belonging to the volume \( V \), the fluctuation of the latter behaves under the above assumption as

\[
\langle Q_V Q_V \rangle^{1/2} \sim (V/l_p^3)^{1/2}
\]

with \( N_P \sim V/l_p^3 \) the number of grains in \( V \). This is a consequence of the central limit theorem. As such large integrated fluctuations in a macroscopic region of the physical vacuum are not observed (they are in fact microscopic on macroscopic scales), we conclude:

**Conclusion 2.2** The individual grains or supposed elementary DoF do not fluctuate approximately independently.

Remark: We note that this fact is also corroborated by other, independent observations.

We can refine the result further (cf. [40]) by assuming that the fluctuations in the individual grains are in fact not independent but correlated over a certain distance or, more precisely, are short-range correlated. In mathematical form this is expressed as integrable correlations. This allows that “positive” and “negative” deviations from the mean value can compensate each other more effectively. Letting e.g. \( q(x) \) be the density of a certain physical observable and \( Q_V := \int_V q(x) \, d^n x \) the integral over \( V \). In order that

\[
\langle Q_V Q_V \rangle^{1/2} \ll V^{1/2}
\]

we proved in [40] that it is necessary that

\[
\int_V d^n y \left\langle q(x)q(y) \right\rangle \approx 0
\]

We made a more detailed analysis in [40] under what physical conditions property 3 can be achieved, arriving at the result:
**Conclusion 2.3** Nearly vanishing fluctuations in a macroscopic volume, $V$, together with short-range correlations imply that the fluctuations in the individual grains are anticorrelated in a fine-tuned and non-trivial way, i.e. positive and negative fluctuations strongly compensate each other which technically is expressed by property (3).

Remark: In [41] we extended such a vacuum fluctuation analysis and applied it to measurement instruments, being designed to detect (possibly) microscopic fluctuations of distances due to passing gravitational waves.

We hence infer that the fluctuation pattern of e.g. energy-momentum has to be strongly **anticorrelated**. But under the above assumption it is possible that the underlying compensation mechanism which balances e.g. the positive and negative energy fluctuations is of short-range type, viz., individual grain-energies may still fluctuate almost independently if their spatial distance is sufficiently large. We show in the following that the true significance of the so-called **holographic principle** is it, to enforce a very rigid and **long-ranged** anticorrelated fluctuation pattern in the quantum vacuum. As we are at the moment only interested in matters of principle, we assume the simplest case to prevail, called the space-like holographic principle (holding in contexts like e.g. quasi-static backgrounds or asymptotic Minkowski-space; see e.g. the beautiful review [42]).

**Postulate 2.4** There exists a class of scenarios in which the maximal amount of information or entropy which can be stored in a spherical volume is proportional to the area of the bounding surface. The same holds then for the number of available DoF in $V$. This is the spacelike holographic principle.

In a series of papers Brustein et al. developed a point of view that relates typical fluctuation results of quantum mechanical observables in quantum field theory with the area-law-like behavior of entanglement entropy and BH-entropy (cf. e.g. [43]). We already made a brief remark to this approach in [40]. We note that we arrived at related results using different methods in another context (see e.g. [44] and [45]). As a more detailed comment would lead us too far astray, we plan to discuss this subject matter elsewhere.

What we are going to show in the following is that the mechanism leading to the strange area-behavior of the entropy of an enclosed volume, $V$, is considerably subtler as usually envisaged. On the one hand, we will show that the number of elementary DoF contained in $V$ is in principle proportional to the volume. On the other hand we infer from observations on the macroscopic or mesoscopic scale that the fluctuations of e.g. the energy are strongly anticorrelated. However, as long as this compensation mechanism is short-ranged, we would still have a number of nearly independently fluctuating clusters of elementary DoF which again happens to be proportional to the volume as the cluster size is roughly equal to the correlation length. So the conclusion seems to be inescapable that the patterns of vacuum fluctuations must actually be **long-range correlated** on a microscopic scale.
But we showed in [10] or [41] in quite some detail that even systems, displaying long-range correlations, will usually have an entropy which is proportional to the volume. A typical example is a (quantum) crystal ([10],[41]). It is certainly correct that below a phase transition point a system of particles in the crystal phase has a smaller entropy than in the liquid or gas phase, but still the entropy happens to be an extensive quantity. The reason is in our view that the system develops, as a result of the long-range correlations, new types of collective excitations (e.g. lattice phonons) which serve as new collective DoF. Approximately they may be treated as a gas of weakly interacting elementary modes with the usual extensive entropic behavior.

That is, the holographic principle entails that the elementary DoF have to be long-range anticorrelated (cf. also the remarks in [35] or sect.7 of [46]). But we see that this is only a necessary but not a sufficient property for an entropy-area law to hold. We hence arrive at the preliminary conclusion:

**Conclusion 2.5** From our preceding arguments and observations we conclude that the holographic principle implies that the fluctuation patterns in \( V \) are long-range anticorrelated in a fine-tuned way on a microscopic scale and are essentially fixed by the state of the fluctuations on the bounding surface. The dynamical mechanism, which generates these long-range correlations must however, by necessity, have quite unusual properties (cf. subsection 4.2).

Before we derive the wormhole structure of the quantum vacuum on a primordial scale in the next sections, we continue with the general analysis of the pattern of vacuum fluctuations and derive some useful properties of it.

A particular role is usually played by the energy and its fluctuations. Furthermore, vacuum fluctuations are frequently discussed together with the so-called zero-point energies. While they are not exactly the same, they are closely related. Both occur also in connection with the cosmological constant problem (to mention only a few sources see e.g. [47],[48],[49],[50],[51],[52],[53]).

In the simplest examples like e.g. the quantized harmonic oscillator or the electromagnetic field we have

\[
H = \frac{P^2}{2m} + \frac{m\omega}{2} \cdot Q^2
\]

(4)

and with

\[
\langle P \rangle_0 = \langle Q \rangle_0 = 0
\]

(5)

in the groundstate, \( \psi_0 \), we have

\[
\hbar \cdot \omega/2 = \langle H \rangle_0 = 1/2m \cdot (\langle (P - \langle P \rangle_0)^2 \rangle_0 + m\omega/2 \cdot (\langle (Q - \langle Q \rangle_0)^2 \rangle_0
\]

(6)

with

\[
\langle (P - \langle P \rangle_0)^2 \rangle_0 \cdot (\langle (Q - \langle Q \rangle_0)^2 \rangle_0 \geq h^2/4
\]

(7)

which follows from \([P,Q] = -i\hbar\). In the same way we have in (matter-free) QED:

\[
H = \text{const} \cdot (\mathbf{E}^2 + \mathbf{B}^2)
\]

(8)
with \[
\langle E \rangle_0 = \langle B \rangle_0 = 0 \tag{9}
\]
so that again \( \langle H \rangle_0 \) is a sum over pure vacuum fluctuations of the non-commuting quantities \( E \) and \( B \). One should however note that in the quantum field context products of fields at the same space-time point have to be Wick-ordered (in order to be well-defined). It is, on the other hand, frequently argued that with gravity entering the stage, these eliminated zero-point energy fluctuations have to be taken into account again. In our view, this problem is not really settled.

We now come to an important point. It is our impression that in some heuristic discussions (vacuum fluctuations as virtual particle-antiparticle pairs) the consequences of the fact that the vacuum state is an exact eigenstate of the Hamiltonian in a Hilbert space representation of some quantum field theory are not fully taken into account. I.e., we have

\[
H \Omega = 0 \tag{10}
\]
(provided the ground state energy is for convenience normalized to zero; note however that this may be problematical in a theory containing gravity). Eigenstates, however, have the peculiar property that the standard deviation is necessarily zero,

\[
\Delta_\Omega H = \langle (H - \langle H \rangle_\Omega)^2 \rangle_\Omega^{1/2} = \langle H^2 \rangle_\Omega^{1/2} = 0 \tag{11}
\]

According to the standard interpretation of quantum theory combined with spectral theory, \( H^2 \geq 0 \), this implies that in each individual observation process the total energy of the vacuum which is, according to conventional wisdom, the (hypothetical) sum or superposition of local (small scale) fluctuations, happens to be exactly zero. In other words, the elementary fluctuations have to exactly compensate each other in an apparently fine-tuned way. Put differently

**Observation 2.6** If there are positive local energy fluctuations, there have to be at the same time by necessity negative energy fluctuations of exactly the same order. That is, at each moment, the global pattern of energy fluctuations in the quantum vacuum is an array of rigidly correlated positive and negative local excitations.

Remark: Note the similarity of this independent observation to what we have said above in connection with the holographic hypothesis.

It should be mentioned that Hawking in [54] invoked exactly this picture of a particle pair excitation near the event horizon with the virtual particle, having negative energy, falling into the BH while the one with positive energy escapes to infinity.

It would be useful to get more quantitative information on the spectral properties of the local observables, in particular estimates on negative fluctuations. One could try to make an explicit spectral resolution of these quantities, e.g. of the energy, contained in a finite volume, \( V \), but this turns out to be difficult in general, even if one has given an explicit model theory in some Hilbert
space. As we prefer a more general, model independent approach (not necessarily based on Hilbert space mathematics), we proceed by using (similar to Bell in his papers) a general probabilistic approach which rather exploits the statistics of individual measurement results. Unfortunately, we found that the standard estimates, known to us in this context (e.g. the Markov-Chebyshev-inequality), always go in the wrong direction (see e.g. [55] or [56]). Therefore we present in the following our own estimate.

The strategy is the following. We take an observable, $E_V$, localized in $V$ with, for convenience, discrete spectral values, $E_i$, and corresponding probabilities denoted by $p_i > 0$. If we assume that the expectation of $E_V$ is zero (which can always be achieved by a simple shift) we have

$$\sum p_i = 1 \quad , \quad \sum p_i \cdot E_i = 0 \quad (12)$$

Furthermore, we assume its standard deviation in e.g. the vacuum, $\Omega$, to be finite (which is automatically the case for bounded operators, but we want to include also more general statistical variables)

$$\sum p_i \cdot E_i^2 = (\Delta \Omega E)^2 < \infty \quad (13)$$

In a first step we make the simplifying assumption (taking e.g. a bounded function of the energy)

**Assumption 2.7**

$$|E_i| \leq \Lambda \quad \text{for all} \quad E_i \quad (14)$$

We are interested in the amount of negative (e.g. energy) fluctuations we will observe in measurements. A reasonable quantitative measure of it is

$$\sum p_i^- \cdot |E_i^-| \quad (15)$$

with $E_i^-, p_i^-$ the negative spectral values and their corresponding probabilities.

We then have (with $|E_i^-|/\Lambda \leq 1$)

$$\sum p_i^- \cdot |E_i^-|/\Lambda \geq \sum p_i^- \cdot |E_i^-|^2/\Lambda^2 \quad (16)$$

For the lhs we have

$$\sum p_i^- \cdot |E_i^-|/\Lambda = \sum p_i^+ \cdot |E_i^+|/\Lambda \quad (17)$$

as the expectation of $E$ was assumed to be zero.

This yields

$$\sum p_i^- \cdot |E_i^-|/\Lambda = 1/2 \cdot \left( \sum p_i^- \cdot |E_i^-|/\Lambda + \sum p_i^+ \cdot |E_i^+|/\Lambda \right) \geq$$

$$1/2 \cdot \left( \sum p_i^- \cdot |E_i^-|^2/\Lambda^2 + \sum p_i^+ \cdot |E_i^+|^2/\Lambda^2 \right) \quad (18)$$
I.e.,
\[ \sum p_i^- \cdot |E_i^-| \geq 1/2 \cdot \sum p_i \cdot E_i^2 / \Lambda = 1/2 \Lambda \cdot (\Delta \Omega E)^2 \] (19)

On the other hand (Cauchy-Schwartz)
\[ \left( \sum p_i^- \cdot |E_i^-| \right)^2 = 1/4 \cdot \left( \sum p_i^- \cdot |E_i^-| + \sum p_i^+ \cdot |E_i^+| \right)^2 \leq 1/4 \cdot \sum p_i \cdot |E_i|^2 \] (20)

We hence arrive at the result

**Conclusion 2.8** If the observable, \( E \), is bounded, so that its spectral values fulfill \(|E_i| \leq \Lambda\), we have the estimate

\[ 1/2 \Lambda^{-1} (\Delta \Omega E)^2 \leq \left( \sum p_i^- \cdot |E_i^-| \right) \leq 1/2 (\Delta \Omega E) \] (21)

with \( p_i \) the probabilities that the negative spectral values \( E_i \) occur in an observation. That is, we manage to bound a quantity, which is difficult to measure directly, by quantities, which are usually more easily accessible.

We can generalize this result to situations where the \( E_i \) are not exactly bounded by some \( \Lambda \) but are bounded in at least an essential way. We assume that there exists some \( \Lambda \) so that

\[ \sum |E_i^-| > \Lambda \quad \Rightarrow \quad \sum p_i^- \cdot |E_i^-| / \Lambda \geq 1/2 \Lambda \cdot (\Delta \Omega E)^2 - \epsilon \Lambda / \Lambda^2 \] (22)

We then have

\[ \sum p_i^- \cdot |E_i^-| / \Lambda = 1/2 \left( \sum p_i^- \cdot |E_i^-| / \Lambda + \sum p_j^+ \cdot |E_j^+| / \Lambda \right) \geq \\
1/2 \left( \sum_{|E_i^-| \leq \Lambda} p_i^- \cdot |E_i^-| / \Lambda + \sum_{|E_j^+| \leq \Lambda} p_j^+ \cdot |E_j^+| / \Lambda \right) \geq \\
1/2 \left( \sum_{|E_i^-| \leq \Lambda} p_i^- \cdot |E_i^-|^2 / \Lambda^2 + \sum_{|E_j^+| \leq \Lambda} p_j^+ \cdot |E_j^+|^2 / \Lambda^2 \right) \geq \\
1/2 \left( \sum p_i \cdot |E_i|^2 / \Lambda^2 - \epsilon \Lambda / \Lambda^2 \right) \] (23)

**Corollary 2.9** Under the above assumption of an essentially bounded \( E \) we have

\[ \sum p_i^- \cdot |E_i^-| \geq 1/2 \Lambda \left( (\Delta \Omega E)^2 - \epsilon \Lambda \right) \] (24)

Another, rigorous, but not quantitative, argument can be derived from axiomatic quantum field theory (see e.g. [57]). It follows from the so-called *Reeh-Schlieder theorem* that there are no *local observables or fields* which can annihilate the vacuum (where by local we mean that the objects commute for space-like separation). I.e., we have for any local \( A \) (with \( A = A^* \))

\[ A \Omega \neq 0 \quad \Rightarrow \quad (A\Omega|A\Omega) = (\Omega|A^2 \Omega) \neq 0 \] (25)
We take now as local observable the energy density integrated over a certain spatial region, $V$,

$$H_V := \int_V h_{00}(\mathbf{x}, 0) \, d^3x$$  \hspace{1cm} (26)

One usually normalizes $h_{00}(x)$ to

$$\langle \Omega | h_{00}(x) \Omega \rangle = 0 \Rightarrow \langle \Omega | \int_V h_{00}(\mathbf{x}, 0) d^3x \Omega \rangle = 0$$  \hspace{1cm} (27)

The classical expression of the energy density, being derived in Lagrangian field theory, is positive. The corresponding quantized expression, after a necessary Wick-ordering (see e.g. [58] or [59]) is however no longer positive definite as an operator (density). This can be seen as follows. If the quantized energy density were still positive, one can take the square root (via the spectral theorem) of e.g. the positive operator $H_V$ and get:

$$0 = \langle \Omega | H_V \Omega \rangle = \langle H_V^{1/2} \Omega | H_V^{1/2} \Omega \rangle$$  \hspace{1cm} (28)

hence

$$H_V^{1/2} \Omega = 0$$  \hspace{1cm} (29)

As $H_V^{1/2}$ is also a local observable this is a contradiction due to the Reeh-Schlieder theorem.

**Conclusion 2.10** $H_V$ is not a positive operator, hence its spectrum contains negative spectral values. It is then easy to construct Hilbert-space vectors, $\psi$, so that the measurement of $H_V$ in $\psi$ yields negative values for the local energy.

Remark: We recently learned that this argument is originally attributed to Epstein (unpublished; [60] or see [61]), while the derivation which can be found in [62] is a completely different one.

The important message (in our view) of all this is that, perhaps in contrast to naive expectation, the quantum vacuum contains a lot of negative energy excitations which globally exactly balance the positive excitations. One may now speculate about the possibility of making use of this observation.

### 3 Wormhole Spaces

In this section we want to describe (very) briefly and sketchily the three different lines of reasoning which lead us to the concept of wormhole spaces. The first line originated from our investigation of the structure and dynamical behavior of the networks we described above. In e.g. [39] we analyzed in some quantitative detail the unfolding of the network structure and the various network epochs under the inscribed microscopic dynamical laws and developed the two-level concept of the network structure (or, rather, a multi-scale structure), which, under the right conditions, is relatively smooth on a sufficiently coarse-grained level (level
2) with, among other things, a distant measure (metric) of the more ordinary type and (hopefully) an integer-valued geometric dimension, while on a more microscopic scale (level 1) the network structure is expected to be very erratic with possibly a lot of links (elementary interactions or information channels) connecting regions which may be far apart with respect to the metric on level 2. The association of these links with microscopic wormholes thus suggests itself (cf. in particular observation 4.27 in [36]). Note furthermore that our network dynamics implies that these translocal connections are dynamically switched on or off. Compare this observation with the point of view expounded in e.g. [63].

...But if a wormhole can fluctuate out of existence when its entrances are far apart ... then, by the principle of microscopic reversibility, the fluctuation into existence of a wormhole having widely separated entrances ought to occur equally readily. This means that every region of space must, through the quantum principle, be potentially “close” to every other region, something that is certainly not obvious from the operator field equations which, like their classical counterparts, are strictly local. . . . It is difficult to imagine any way in which widely separated regions of space can be “potentially close” to each other unless space-time itself is embedded in a convoluted way in a higher-dimensional manifold. Additionally, a dynamical agency in that higher-dimensional manifold must exist which can transmit a sense of that closeness.

The quantitative network calculations in the mentioned papers have mainly been performed within the framework of random graphs. Important mathematical tools for the network analysis in the transition from microscopic, strongly fluctuating and geometrically irregular scales to coarse-grained and, by the same token, smoother scales have been the concepts of cliques of nodes, the clique-graph of a graph and an important network parameter which we dubbed intrinsic scaling dimension (we later learned, [38], that this concept plays also an important role in geometric group theory or Cayley-graphs where it is called the growth degree). To give a better feeling what is actually implied, we give the definitions of clique, clique-graph and internal scaling dimension (more about graph theory can e.g. be found in [64], notions and properties of graph dimension were studied in e.g. [65]).

**Definition 3.1** A simplex in a graph is a subset of vertices (nodes) with each pair of nodes in this subset being connected by an edge. In graph theory it is also called a complete subgraph. The maximal members in this class are called cliques.

**Definition 3.2** The clique graph, C(G), of a graph, G, is built in the following way. Its set of nodes is given by the cliques of G; an edge is drawn between two of its nodes if the respective cliques have a non-empty overlap with respect to their set of nodes.

Graphs carry a natural neighborhood structure and notion of distance. The neighborhood \( U_n(x) \) of a node \( x \) is the set of nodes \( y \) which can be reached,
starting at \( x \) in \( \leq n \) consecutive steps, i.e. there exists a path of \( \leq n \) consecutive edges connecting the nodes \( x \) and \( y \).

**Definition 3.3** The canonical network or graph metric is given by

\[
d(x, y) := \min_{\gamma} \{ l(\gamma) \mid \gamma \text{ a path connecting } x \text{ and } y \} \tag{30}
\]

Here \( l(\gamma) \) is the number of consecutive edges of the path. The above definition fulfills all properties of a metric. Thus graphs and networks are examples of metric spaces.

**Definition 3.4 (Internal Scaling Dimension)** Let \( x \) be an arbitrary node of \( G \). Let \( \#(U_n(x)) \) denote the number of nodes in \( U_n(x) \). We consider the sequence of real numbers \( D_n(x) := \frac{\ln(\#(U_n(x)))}{\ln(n)} \). We say \( D_S(x) := \liminf_{n \to \infty} D_n(x) \) is the lower and \( D_S(x) := \limsup_{n \to \infty} D_n(x) \) the upper internal scaling dimension of \( G \) starting from \( x \). If \( D_S(x) = D_S(x) =: D_S(x) \) we say \( G \) has internal scaling dimension \( D_S(x) \) starting from \( x \). Finally, if \( D_S(x) = D_S \) \( \forall x \), we simply say \( G \) has internal scaling dimension \( D_S \).

**Observation 3.5** We proved in [65] (among other things) that this quantity does not depend on the choice of the base point for most classes of graphs.

It turns out that this geometric notion is a very effective characteristic of the large-scale structure of graphs and networks. This topic was further studied in greater generality in e.g. [38].

In [37] we developed what we called the geometric renormalization group, to extract important geometric coarse grained, that is, large scale information from the microscopically quite chaotically looking network and its dynamics. The idea is, at least in principle, similar to the block spin transformation in statistical mechanics. That is, certain characteristic properties of the system are distilled from the microscopically wildly fluctuating statistical system by means of a series of algorithmic renormalization steps (i.e. coarse-graining plus purification). The central aim is it to arrive in the end at a system which resembles, on the surface, a classical space-time, or, on the other hand, to describe the criteria a network has to fulfill in order that it actually has such a classical fixed point.

In the course of this analysis we observed (cf. section VIII of [37]) that the so-called critical network geometries, i.e. the microscopic network geometries which are expected to play a relevant role in the analysis, are necessarily in a very specific way geometrically non-local, put differently, they have to contain a very peculiar structure of non-local links, or short-cuts, that is, in other words, the kind of wormhole structure, we already described above.

Relations to non-commutative geometry were established and studied in [66]. We mention in particular section 7.2 “Microscopic Wormholes and Wheeler’s Space-Time Foam” and section 8 “Quantum Entanglement and Quantum Non-Locality”. The possible relevance for quantum theory is in fact quite apparent.
(as has also been emphasized in the papers by 't Hooft), as these microscopic wormholes may be the origin of the ubiquitous entanglement phenomena in quantum theory. The following figures describe pictorially the nested structure of the cliques of nodes in consecutive renormalization steps and overlapping cliques of nodes, defining the local near-order of physical points together with shortcuts which connect distant parts of the coarse-grained surface structure.

![Figure 1: Nested Structure](image1)

Figure 1: Nested Structure; the (overlapping) cliques of a given level are represented as non-overlapping for reasons of pictorial clearness

![Figure 2: Translocal links](image2)

Figure 2: Translocal links, connecting some local clusters of nodes (grains)

A second complex of (related) phenomena emerges in the field of small world networks. This is a particular class of networks of apparently quite a universal character (described and reviewed in some detail, for the first time, in [67]) with applications in many fields of modern science. They consist essentially of an ordinary local network with its own local notion of distance superimposed by a typically very sparse network of so-called short-cuts living on the same set of nodes and playing a structural role similar to the microscopic wormholes described above. A typical example (with dimension of the underlying lattice $k = 1$) is given in the following figure. Some further (in fact very few) references, taken from quite diverse fields are e.g. [68], [69], [70].

**Observation 3.6** Its, in our view, crucial characteristic is the existence of two metrics over the same network or graph. The first, $d_1(x,y)$, is defined (cf. definition 3.3) by taking into account the full set of edges (i.e., including the
Figure 3: The smallworld model for $k = 1$. The number of nodes is $N = 30$. In this particular realisation we have inserted four additional shortcuts. The unfilled nodes are the vertices which can be reached by for example $\leq 3$ steps starting from node $x$. The black nodes are the vertices not reached after three steps.

short-cuts) and a second (local) metric, $d_2(x, y)$, taking into account only the edges of the underlying local network. It hence holds

$$d_1(x, y) \leq d_2(x, y)$$

Remark: The metric $d_2(x, y)$ may then be associated (after some renormalisation or coarse-graining steps) with an ordinary macroscopic metric defined on a smooth space (without wormholes) like our classical space-time. $d_1(x, y)$, on the other hand, should be regarded as a microscopic distance concept which employs the existence of wormholes.

While, on the surface, the origin of this concept of small-world networks seems to be quite independent of the wormholes in general relativity, it is the more surprising that on a conceptual meta level various subtle ties do emerge. To mention only one (in our view) important observation. In [71] it is for example shown, that a sparse network of shortcuts superimposed upon an underlying local network, has the propensity to stabilize the overall frequency pattern (phase locking) of so-called phase-oscillators which represent the nodes of the networks, the links representing the couplings. The oscillators are assumed to oscillate with (to a certain degree) independent frequencies. If we relate these local frequencies with some local notion of time (or clocks), we may infer that (microscopic) wormholes create or stabilize some global notion of time!

We now come to the third strand, viz. the real wormholes of general relativity or quantum gravity. We mainly concentrate on the wormholes in true, i.e. Lorentzian space-time. Euclidean wormholes also (may) play an important role and have been discussed extensively in the context of the (nearly) vanishing value of the cosmological constant (see e.g. [72], [73], [74], [75], [76]). Of particular relevance in the Lorentzian context are the so-called traversable wormholes. Their study started (as far as we know) with two seminal papers by Thorne and coworkers (see [78]). The geometric construction of such solutions
is in fact not so difficult if performed by the so-called \textit{g-method}. That is, one constructs a geometric wormhole, e.g. of the static type, and, in a second step, computes the energy-momentum tensor being consistent with this solution.

Giving a rough outline, this can be done in following way. Two open balls are removed from two different pieces of e.g. approximately flat 3-space. Their boundaries are glued together with the junction being smoothed. As a consequence of the smoothing process a tube emerges interpolating between the two spheres (see e.g. \cite{79}). It is a remarkable fact that in this process the \textit{weak energy condition} (WEC) has to be violated, the latter implying that

\begin{equation}
T_{00} \geq 0 \quad , \quad T_{00} + T_{ii} \geq 0 \quad \text{for} \quad i = 1, 2, 3
\end{equation}

that is, the matter-energy density is positive in any reference system. Put differently,

\begin{observation}
In order to get a traversable wormhole, one has to violate the WEC. The WEC is always satisfied by classical matter. Therefore quantum effects are needed. The kind of negative energy needed is also called exotic matter.
\end{observation}

We showed in quite some detail in the preceding section that the quantum vacuum abounds with negative energy fluctuations. Therefore the speculation in section H of the first paper in \cite{78} does not seem to be too far-fetched. In a next step one can study networks of such traversable wormholes. In \cite{80} it is speculated that such a network, existing in the early universe, may solve the \textit{horizon problem}. The same situation was discussed from the point of view of our network approach in section 4.1 (The Embryonic Epoch) of \cite{36}. All this comes already quite near the general picture we envoled in the beginning of this section. Furthermore one can envisage solutions combining black and white holes. This corresponds to some of our networks where the orientation (direction) of the links connecting two nodes can change under the dynamics. A review of Lorentzian wormholes can be found in the book by Visser \cite{81}. Some other references are e.g. \cite{82} and \cite{83}.

The above picture of a hypothetical network of wormholes sitting in the deep structure of the quantum vacuum is beautifully complemented by an approach (see e.g. \cite{84}, \cite{85}) which investigates within a (semi)classical approximation the energy of a quantum vacuum state containing such an array of wormholes (or, rather, a gas of such wormholes) and compare it with a vacuum state which in zeroth order is flat Minkowski space. It comes out (apparently being a kind of Casimir effect) that the quantum vacuum containing the wormhole gas has in this semiclassical approximation a lower energy compared to the state, being a perturbation of Minkowski space. One should note however that this is a first order quantum effect! Anyhow, this observation seems to corroborate the space-time foam picture of e.g. Wheeler and we conclude this section with

\begin{conclusion}
From our analysis in this and the preceding section emerges a model of the ground state of some preliminary version of quantum gravity which
\end{conclusion}
contains as an essential ingredient a network of microscopic wormholes. These wormholes can be created and annihilated and are in our picture the carriers of information between distant parts of classical space-time.

**Definition 3.9 (Wormhole Space)** We call such a physical structure a wormhole space and regard our cellular or small world networks, discussed above, as models, encoding and representing the typical characteristics of such systems. The typical characteristic is the existence of two types of distance, a microscopic one and an ordinary local one, being similar to ordinary macroscopic metrics on smooth spaces.

### 4 Wormhole Spaces as the Common Cause of the Holographic Principle and the Entropy-Area Law

We learned in the preceding sections that two (presumably crucial) properties govern the behavior of the quantum vacuum on a microscopic scale. First, the vacuum fluctuations are strongly long-range anticorrelated on a microscopic scale, i.e. there exists a fine-tuned pattern of positive and negative (energy) fluctuations. Second, a quantum mechanical stability analysis seems to show that the quantum vacuum is pervaded by a network of microscopic wormholes. We argued above that these two features are not independent phenomena but rather are the two sides of the same medal. Furthermore, the presumed wormhole structure has been supported by observations coming from other fields of research like e.g. cellular or small-world networks.

In this (central) section we will now combine these observations and show that they underlie (among other things) the holographic principle and the entropy-area law of BH-thermodynamics. In the following we will use (for convenience) the language of our networks with the nodes of the network representing microscopic grains of space (or space-time) of roughly Planck-size. Leaving out other details we treat our quantum vacuum as a wormhole space, i.e. as a (small world) network consisting of an ordinary local network structure being superimposed by a (presumably) sparse random network with edges consisting of short-cuts, i.e. links, connecting regions of space or space-time, which may be quite a distance apart with respect to the metric, belonging to the underlying local network. These short-cuts represent the wormholes of ordinary space-time.

The crucial characteristic, from which everything is expected to follow, is the pattern and distribution of these short-cuts being immersed in the underlying local network. That is, we randomly select a node $x$ in the network $G$ ($G$ standing for graph) and study the distribution of short-cuts connecting $x$ with nodes $y$ on spheres of radius $R$ around $x$ (measured with respect to some macroscopic metric or the natural metric of the underlying local network).

**Observation 4.1** We expect that the precise distribution law will depend on the concrete type of space-time we are dealing with. This holds in particular if
the space-time is not static. That is, our microscopic approach to holography makes it possible to understand how holography may depend on the concretely given type of space-time (cf. e.g. the covariant entropy bound of Bousso, [42]).

Remark: We emphasize that the network or the quantum vacuum it is representing, is basically a statistical system with all local DoF fluctuating. That means, most of our statements in the following are about mean values or averages over finer statistical details.

4.1 The Distribution of Short-Cuts or Wormholes

One can arrive at the law, describing the distribution of short-cuts or wormholes around some arbitrary but fixed generic node (viz. some fixed place in space-time) in roughly two ways. One can e.g. motivate the distribution law by appealing to certain fundamental principles like e.g. scale-freeness or absence of a particular and in some sense unnatural length scale on a fundamental level. Alternatively, one can show that a reasonable choice leads to far-reaching consequences and corroborates the findings and observations made on a more macroscopic level. To keep the discussion as briefly as possible we adopt in this section the second point of view. In the following we want to concentrate, for the sake of brevity, on a simple type of quantum vacuum, that is, the vacuum belonging to ordinary Minkowski space or a space-time which is asymptotically flat (e.g. a Schwarzschild space-time). We postpone the analysis of more general space-times as they occur in general relativity.

We make the following conjecture:

**Conjecture 4.2** On the average the number of short-cuts from a central node \( x \) to nodes \( y \), sitting on the sphere, \( S_R(x) \) about \( x \) is independent of \( R \). Denoting this number by \( N_{S_R(x)} \), we hence have

\[
N_{S_R(x)} = N_0
\]  

(33)

Remark: As this number is a statistical average, it need not be an integer.

The situation is depicted in the following picture.

**Observation 4.3** We will show in subsection 4.2 in a detailed quantitative analysis that this result approximately holds as well for nodes, not sitting exactly in the center of the spheres \( S_R \) (see the following picture).

**Definition 4.4** We denote the cluster of nodes in the ball \( B_R \) being connected to an \( x \) by short-cuts by \( C_{B_R}(x) \).

We previously introduced the internal scaling dimension of a network (see definition 3.4). It roughly describes how fast the network is growing with respect to some base node. As this growth degree is to a large degree independent of the base node (see e.g. [65]) it is a global characteristic of a given network, in fact of a whole class of similar networks ([38]). It is well known that the generalization of
the concept of dimension away from smooth geometric structures is not unique. The above type of dimension has the tendency to grow if additional short-cuts are inserted into a given network geometry. We now introduce another dimensional concept which catches other important network properties being more closely related to the phenomena we want to analyze in this paper. It uses in an essential way the two metrics, \( d_1, d_2 \), introduced above.

**Observation 4.5** From the above we infer that the number of nodes in the cluster \( C_{B_R}(x) \) is approximately equal to \( N_0 \cdot R \). Furthermore, if the network of short-cuts is very sparse, the clusters \( C_{B_R}(x_i), C_{B_R}(x_j) \) with \( x_i \neq x_j \) are essentially disjoint (the overlap is empty or very small). This is the phenomenon called spreading in the theory of random graphs.

Hence, the following concept is reasonable.

We define a **holographic dimension**, \( D_H \), of a network in the following way. We take some ball \( B_R \) with macroscopic radius \( R \) around some fixed but arbitrary node \( x \) with respect to the local metric \( d_2 \). We then form the \( U_1^{(1)}(y) \)-neighborhoods around the nodes \( y \in B_R \) with respect to the microscopic metric \( d_1 \). We construct a minimal cover of \( B_R \) by such \( U_1^{(1)}(y) \), i.e. a minimal
selection of such \( y_i \) s.t.

\[
\bigcup_i U_1^{(1)}(y_i) \supset B_R
\]

(34)

The cardinality of such a minimal set we denote by \( N_C(B_R) \). We take the limit \( R \) large or \( R \to \infty \) (in an infinite network) and define

**Definition 4.6** We call

\[
D_H := \lim_{R \to \infty} \frac{\ln N_C(B_R)}{\ln R}
\]

(35)

the holographic dimension of the graph (network), provided the limit exists. In the more general situation we can, as in definition 3.4, define upper and lower dimensions etc.

**Corollary 4.7** As for the previously defined graph dimension, the limit is independent of the selected base point, \( x \), if the network or graph is homogeneous on the average or in the large.

**Observation 4.8** Due to the sparseness of the embedded subgraph of short-cuts, which yields the spreading property mentioned above, the number \( N_C(B_R) \) scales for the wormhole spaces or small-world networks as

\[
N_C(B_R) \sim R^{n-1}
\]

(36)

with \( n \) the dimension of the local network or its coarse-grained continuum limit space.

Proof: The \( U_1^{(1)}(y) \)-neighborhoods consist of nodes lying in the neighborhoods with respect to the local metric, \( d_2, U_2^{(2)}(y) \), plus the vertices connected by short-cuts with \( y \). The cardinality of \( U_1^{(2)}(y) \) is independent of \( R \) and typically (at least in our models) a small number. For \( R \to \infty \) \( U_1^{(1)}(y) \cap B_R \) will therefore consist mainly of nodes connected to \( y \) by short-cuts. Sparseness of the short-cut graph and spreading yield the result. \( \square \)
Conclusion 4.9 For the type of wormhole spaces or small-world networks, defined above, we then have

\[ D_H = \lim_{R \to \infty} \frac{\ln(V(B_R)/R)}{\ln R} = n - 1 \]  

That is, in this case we have the important result

\[ D_H = \dim S_R = n - 1 \]  

We now come to the holographic principle and the BH-entropy area law. As already mentioned, we discuss in this paper only the example of 4-dim. asymptotically flat (Minkowski) space-time. In Planck units a macroscopic ball, \( B_R \), contains approximately

\[ |V(B_R)| := V(B_R)/l_p^3 \]  

DoF or grains of Planck size. The typical cluster size is

\[ |C_{B_R}(x_i)| \approx N_0 \cdot R/l_p \]  

Due to the mentioned spreading property the number of (effectively) independent cluster in the above minimal cover is approximately

\[ N_C(B_R) \approx \left( (4/3)\pi \cdot R^3/N_0 \cdot R \right) \cdot l_p^{-2} = (3N_0)^{-1} \cdot 4\pi R^2/l_p^2 = \\
(3N_0)^{-1} \cdot A(S_R)/l_p^2 =: (3N_0)^{-1} \cdot |A(S_R)| \]  

with \( A(S_R) \) denoting the area of \( S_R \).

Observation 4.10 The number of effectively independent clusters, \( C_{B_R}(x_i) \) in \( B_R \) is

\[ N_C(B_R) \approx (3N_0)^{-1} \cdot |A(S_R)| = (3N_0)^{-1} \cdot A(S_R)/l_p^2 \]  

with the typical cluster size

\[ |C_{B_R}(x_i)| \approx N_0 \cdot R/l_p \]  

To show now that the number of effective DoF in a generic volume (where by generic we mean a region in space with the diameter in all directions being roughly of the same order) is proportional to the surface area, \( A(V) \), of its boundary, we employ a general observation, made e.g. in statistical mechanics. An important tool for the analysis of systems in statistical mechanics are correlation functions. Correlations decay usually for large separation of the respective DoF, but what is on the other hand certainly the case is, that nearest neighbors are strongly correlated (near order versus far order).

Observation 4.11 We expect that the DoF in each of the \( U_1^{(1)}(x) \) are strongly correlated. We hence take it for granted, that they act effectively as a single collective DoF.
Remark: It may be possible, that this near order in the immediate neighborhood of the grains can be finally destroyed by the insertion of a huge amount of localized energy, but this does not seem possible with present means.

**Conclusion 4.12 (Area Law)** Due to the existence of wormholes or short-cuts, distributed in space-time, the number of effective DoF (affiliated with the respective clusters \( C \)) in e.g. a ball \( B_R \) equals \( N_C(B_R) \), that is

\[
\#(\text{DoF in } B_R) \approx (3N_0)^{-1} \cdot |A(S_R)| = (3N_0)^{-1} \cdot A(S_R)/l_p^2
\]

This is the area-law behavior of entropy or number of DoF in a volume of space found in e.g. BH-entropy. We note however, that this law, in our formulation, is essentially a statement about the collective behavior of the elementary DoF in (the interior of) a volume of space. I.e., the respective DoF are *not* really sitting on the boundary of \( V \). As to the details of the *bulk-boundary correspondence* see the following subsection.

If we adopt the entropy-area law of BH-thermodynamics, which is, expressed in Planck units,

\[
S = \frac{1}{4} \cdot |A|
\]

we have the possibility to fix our parameter \( N_0 \), which gives the number of wormholes connecting a central grain of space with the grains on a surrounding sphere \( S_R \) for any \( R \). However, entropy is not exactly identical to number of DoF. To relate the two, we have to make a simple model assumption. One frequently makes the assumption of *Boolean DoF*, i.e. the DoF on an elementary scale are two-valued.

**Observation 4.13** With this assumption we have the relation

\[
S = N \cdot \ln 2 \quad \text{i.e.} \quad N = |A|/4 \cdot \ln 2
\]

with \( S \) the entropy, \( N \) the number of DoF.

**Conclusion 4.14** With the help of this identification we get

\[
N_0 = 4/3 \cdot \ln 2
\]

which can in qualitative arguments be approximated by one!

That is, in Planck units, there exists roughly one short-cut between a central vertex and a surrounding sphere of radius \( R \). This shows that on an extremely microscopic scale, the network of short-cuts is indeed very sparse. However the picture changes considerably if we go over to more accessible length scales. If we use, for example an atomic length-scale of e.g. \( l_a := 10^{-10}m \), we have approximately

\[
(10^{-10})^3/(10^{-35})^3 = 10^{75}
\]

grains of Planck-size in a volume element of diameter \( l_a \). If we then choose, instead of a sphere \( S_R \), a spherical shell of radius \( R \) and thickness \( l_a \) we have approximately
Observation 4.15 The number of wormholes or short-cuts between a central volume element of size $l_a^3$ and a corresponding spherical shell of radius $R$ is approximately

$$\#(\text{short-cuts}) \approx 10^{75} \cdot 10^{25} = 10^{100}$$

which is quite a large number.

If we choose for example $R = 1m$, we see that roughly $10^{96}$ grains in the shell are the endpoints of about $10^{100}$ short-cuts coming from the central volume element of size $l_a^3$. If we replace $R$ by the approximate diameter of the universe, i.e. $R_0 \approx 10^{40}$ ly, we get (with $1\text{ ly} \approx 10^{17}m$):

$$R_0 \approx 10^{27} m$$

and for the number of Planck-size grains in a spherical shell of this radius:

$$\#(\text{grains in shell of radius } R_0) \approx 10^{149}$$

with still $10^{100}$ short-cuts ending there. That is, only one in $10^{49}$ grains is the endpoint of a respective short-cut. But if we select a volume element of size $l_a^3$ in this shell, we have still

Observation 4.16 The number of wormholes (short-cuts) between two volume elements of size $l_a^3$ being a distance $R_0$ apart, is still the large number

$$\#(\text{short-cuts}) \approx 10^{100} \cdot 10^{-149} \cdot 10^{75} = 10^{-49} \cdot 10^{75} = 10^{26}$$

that is, even over such a large distance there exist still a substantial number of wormholes connecting the two volume elements. But nevertheless, the network is sparse, viewed at Planck-scale resolution.

4.2 The Bulk-Boundary Correspondence

We now come to the last point of this section. From what we have learned above, it is intuitively clear, that the DoF sitting on the boundary $S_R$ of e.g. a ball $B_R$ should fix (or slave) the DoF in the interior. But we note that in order that this can hold, we have to verify our statement made in observation 4.3. Furthermore, it is of tantamount importance to understand in more quantitative detail the influence of different shapes of the region under discussion and the effect of different space-time geometries. The prerequisites for this enterprise will be derived in the following.

As an example we employ, as we already did above, the simple geometry of the spacelike holographic bound. For reasons of simplicity we place the center of the ball in the origin, i.e. $x_0 = 0$. It is of great help if we can transform the problem into a problem of ordinary continuous analysis. To this end we introduce the probability that a node in the interior of $B_R$ and an arbitrary node on the boundary $S_R$ are connected by a short-cut. With $y \in S_R$ and $x \in B_R$ there spatial euclidean distance in three dimensions is

$$|y - x| = \left(\sum_{i=1}^{3} (y_i - x_i)^2\right)^{1/2}$$

(53)
Observation 4.17 The edge probability is given by

\[ p(|y - x|) = \frac{N_0}{|A(S_{y-x})|} = \frac{N_0 \cdot l_p^2/4\pi \cdot |y - x|^{-2}}{} \]  \hspace{1cm} (54)

Here \( |A(S_{y-x})| \) is the number of nodes (or Planck-scale grains) on the sphere around \( x \) with radius \( |y - x| \). This follows directly from what we have learned in the previous sections.

What we are actually doing in the following is the calculation of the average number of short-cuts between an arbitrary node \( x \) in the interior of \( B_R \) and the nodes on the boundary \( S_R \). This will be done within the framework of random graphs. The above \( p \) is the so-called edge probability (for the technical details see [64] or [36],[37]). The sample space is the space of graphs with node set comprising the node in \( x \) and all the nodes sitting on the boundary \( S_R \) and edge set all possible different sets of short-cuts connecting \( x \) with the nodes on \( S_R \). The probability of each graph in the sample space is calculated with the help of the above elementary edge probability \( p \) and its dual \( q := 1 - p \).

We choose \( x \) arbitrary but fixed in \( B_R(0) \) and let \( y \) vary over the sphere \( S_R(0) \). The integral over \( S_R(0) \) will then give the mean number of short-cuts between \( x \) and the grains on \( S_R(0) \). The guiding idea is that the DoF in the interior are fixed by the DoF on the boundary if this integral is essentially \( \gtrsim 1 \), as according to our philosophy, developed previously, in that case every node in the interior has on average at least one partner on the boundary as nearest neighbor with respect to the microscopic metric \( d_1 \).

To make the integration easier we choose, without loss of generality,

\[ x = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}, \quad z := k \cdot R \]  \hspace{1cm} (55)

with \( 0 \leq k \leq 1 \). A straightforward calculation (using polar coordinates and appropriate variable transformations) yields for the average number of short-cuts, \( N_{S_R}(x) \),

\[ N_{S_R}(x) = \frac{N_0 \cdot l_p^2/4\pi}{|y - x|^{-2}} \int_{S_R} |y - x|^{-2} \, \, d\Phi = \]

\[ \left( \frac{N_0}{4\pi \cdot R^2} \cdot 2\pi R^{-2} \right) \int_{-1}^{+1} du \left( (1 + k^2) - 2ku \right)^{-1} = \]

\[ \frac{N_0}{2} \cdot \int_{-1}^{+1} du \left( (1 + k^2) - 2ku \right)^{-1} \]  \hspace{1cm} (56)

Observation 4.18 Note that the integrand \( ((1+k^2)-2ku)^{-1} \) is always positive. Furthermore, our choice of a Coulomb-like law (in three dimensions) for the distribution of short-cuts in the previous subsection, i.e. \( p \sim R^{-2} \), makes the above integral independent of \( R \).
We can find a closed expression for the definite integral, i.e.

\[ I := \int_{-1}^{+1} du ((1 + k^2) - 2ku)^{-1} = -1/2k \cdot \ln ((1 - k)^2/(1 + k)^2) > 0 \] (57)

Note that the position of the point \( x \) relative to the center and the boundary can be regulated by the value of the parameter \( 0 \leq k \leq 1 \). We have tabulated the integral for \( k \) from 0 to 0.9 in the following table.

| \( k \) | 0   | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( I_k \) | 2   | 2   | 2.04| 2.11| 2.19| 2.29| 2.45| 2.71| 3.23|

We see that the number of short-cuts is almost constant through the whole interior of \( B_R \) apart from a thin shell near the boundary. But this is not really surprising because there the main contribution comes from the near side of the boundary and is no longer of a true short-cut character. Taking into account the additional prefactor, \( N_0/2 \), in front of the integral which is \( \approx 1/2 \) we have

**Conclusion 4.19** The number of short-cuts from an arbitrary node \( x \) in \( B_R \) to the boundary \( S_R \) is approximately

\[ p(x) \gtrsim 1 \] (58)

for most of the nodes. Furthermore for our Coulomb-like distribution law it is independent of the radius of the sphere and is therefore consistent with the expected holographic behavior for this geometry.

It is instructive to evaluate the above formula for \( k > 1 \), i.e., the influence via short-cuts of the sphere \( S_R \) on a DoF in the exterior of \( S_R \). For \( k \) large, the integral is dominated by the first term in the integrand, viz. for \( k \) large we have

\[ I \approx \int_{-1}^{+1} du (1 + k^2)^{-1} \sim k^{-2} \] (59)

**Conclusion 4.20** For nodes, \( x \), lying outside of \( S_R \), the effect of the short-cut connections between \( x \) and \( S_R \) decays like a Coulomb-law. That is, the DoF in the exterior are no longer fixed by the DoF on \( S_R \). What remains instead is a statistical influence in form of a correlation which decays with increasing distance. By the same token, there cannot be an entropy-area law for the exterior of the sphere relative to its internal boundary. Anyhow, this example does not really contradict the correctness of the spatial holographic principle as being presented in this paper. It would be interesting to relate our findings to the covariant holographic principle of e.g. Bousso, [42]

This simple observation has an important consequence for arguments being sometimes invoked against the general nature of the spatial holographic principle (cf. e.g. [42]). While we do not intend to discuss the holographic principle for more general space-times in this paper, we mention one counter-example which one finds frequently in the literature, i.e. a universe containing a closed spatial slice, \( S \) with a small inner subregion, \( S_2 \) (see the following picture).
Observation 4.21 \textit{The area-law in the usual form applies for the subregion $S_2$ relative to its boundary. However, according to our (microscopic) version of spatial holography, the DoF on the inner boundary cannot slave the DoF in the large region $S_1$ if the inner boundary becomes too small. They only establish some kind of correlation in the exterior. The quantitative details are given by integrating our Coulomb-like influence law over the inner surface.}

Another, related, class of interesting (but perhaps pathological) apparent counter examples (which we plan to address in greater detail elsewhere) is discussed in e.g. [92], i.e. spacetimes which are called by Marolf 'bag-of-gold spacetimes'. An essential ingredient is some FRW-spacetime hidden in the interior of a region which resembles an ordinary BH. The inner FRW-universe has of course an entropy which is proportional to its volume while from the outside the whole configuration looks like a BH. This seeming contradiction can be easily understood with the help of our microscopic holographic law as the FRW-spacetime is actually only weakly coupled with the exterior of the BH via wormholes. The technical arguments are the same as above.

5 Commentary

In the preceding sections we developed only the groundwork of our approach. To keep the paper within reasonable size, we had to postpone a more detailed discussion of the many consequences and immediate applications. In this final section we at least undertake to briefly comment on a number of important points. It is however obvious that a more detailed discussion of each point would require a paper of its own.

i) The possible connections to the ubiquitous phenomenon of entanglement in ordinary quantum theory are obvious. Interesting in this respect is e.g. the well-known tension in quantum theory between the locality and causality principle of special relativity and the instantaneous state reduction, accompanying the measurement process (cf. the respective sections in e.g. [86]). We think, similar to e.g. 't Hooft, that (the microscopic form of) holography (we developed in this
paper) is the common basis which may unite quantum theory and gravitation.

ii) The consequences of the BH-entropy being maximal, which is quite uncharacteristic for the ground state entanglement entropy in say ordinary quantum theory, should be further analysed.

iii) The ADS-CFT-correspondence is regarded in string theory as the paradigm for bulk-boundary correspondence (we mention only the review [87] and the popular account [88]). In it two, at first glance, fundamentally different theories are related to each other, the one living in the bulk, the other living on the boundary at infinity. We must however say that the concrete physical epistemology of this latter notion is not entirely clear to us. The use of boundaries at infinity is wide spread in holography and is mathematically well-defined, in particular for certain well-adapted coordinate systems being in use in hyperbolic geometry. But in general it is rather an asymptotic property and not a concrete place. Note that in our approach full information about the interior of a (spatial) region is distributed essentially everywhere in the exterior of the region via wormholes, but usually not in the form of another field theory!

iv) A virulent problem (the unitarity problem) in BH-thermodynamics is the question whether a pure state goes over into a mixed state or not, that is, if the laws of ordinary quantum theory are possibly violated in BH-thermodynamics (instead of the many published papers we mention only the reviews by Wald, cited above). This is a quite intricate epistemological problem somewhat similar to the quantum measurement problem. We think, part of the problem is that frequently pure states and mixtures are regarded as complete opposites. But this is not really correct. It is here not the place to go into more details. But in some respect it lies rather in the eye of the beholder. That is, it is the problem of dealing with the complete microscopic information of a state, or rather with some coarse-grained form. Note that in our approach microscopic information is widely scattered via short-cuts or wormholes over essentially the whole space. I.e., it is not fully accessible to a local observer. We recommend the study of some older classics on the ergodic theorem in quantum statistical mechanics ([89], [90], [91]).

v) Our analysis should be extended to more general space-times where possibly different distribution laws may show up.

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