Hair distributions in noncommutative Einstein-Born-Infeld black holes

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Abstract

We study hair mass distributions in noncommutative Einstein-Born-Infeld hairy black holes with non-zero cosmological constants. We find that the larger noncommutative parameter makes the hair easier to condense in the near horizon area. We also show that the Hod’s lower bound can be invaded in the noncommutative gravity. However, for large black holes with a non-negative cosmological constant, the Hod’s lower hair mass bound almost holds in a sense that nearly half of the hair lays above the photonsphere.

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I. INTRODUCTION

The famous black hole no hair theorem introduced by Wheeler was motivated by the uniqueness theorem that a Einstein-Maxwell black hole can be determined only by the three conserved global charges associated with Gauss laws as ADM mass $M$, electric charge $Q$ and angular momentum $J$. In accordance with this no hair theorem, stationary black holes indeed cannot support the existence of scalar fields, massive vector fields and spinor fields in the exterior spacetime, for references see

However, nowadays we are faced with the surprising discovery of various types of hairy black holes in theories like Einstein-Yang-Mills, Einstein-Skyrme, Einstein-non-Abelian-Proca, Einstein-Yang-Mills-Higgs, Einstein-Yang-Mills-Dilaton and non-static spin gravies, which cannot be unique described by the three conserved charges $M$, $Q$ and $J$, for references please refer to and reviews can be found in. Recently, a no short hair theorem was proposed as an alternative to the classical no hair theorem based on the fact that the hair satisfying the weak energy condition and the energy-momentum tensor dominant condition must extend above the photonsphere. And it was found that no short scalar hair behaviors also exist in non-spherically symmetric non-static Kerr black holes. It also provided a nice heuristic picture that the formation of hair is due to the self-interaction which can bind together the hair below the photonsphere and hair above the photonsphere relatively distant from the horizon. It should be emphasized that for various types of black hole hair, these two conditions are indeed satisfied.

Along this line, it is interesting to study the hair distribution outside the black hole horizon. For the limit case of the linear Maxwell field, Hod showed that the region above the photonsphere contains at least half of the total mass of Maxwell fields and also found that this lower bound holds for various genuine hairy black holes in Einstein-Yang-Mills, Einstein-Skyrme, Einstein-non-Abelian-Proca, Einstein-Yang-Mills-Higgs and Einstein-Yang-Mills-Dilaton systems. And Hod further conjectured that the hair mass lower bound exists in all hairy black holes. In fact, it was found that the non-linear Einstein-Born-Infeld black holes also satisfy this lower bound that half of the Born-Infeld hair is above the photonsphere. As a further step, we showed that the Hod’s lower bound holds in asymptotically dS Einstein-Born-Infeld hairy black holes.

The known results imply that Hod’s lower bound of hair mass ratio may be a general property in the hairy black hole background. However, all of these calculations were based on a commutative spacetime. Recently, noncommutative black holes have been studied on the motivation that noncommutativity is expected to be
relevant at the Planck scale where it is known that usual semiclassical considerations break down. For example, modifications to the semiclassical area law in the noncommutative (NC) spacetime have been obtained [47–52]. Another important motivation to study noncommutative theories is due to its natural emergence in string theory and some surprising consequences [53–58]. In this work, we plan to extend the discussion of hair distributions to noncommutative spacetimes and also examine whether the Hod’s lower hair mass bound holds in noncommutative hairy black holes.

In the following, we introduce noncommutative Einstein-Born-Infeld black holes and disclose effects of parameters on hair distributions. We also examine whether the Hod’s lower bound holds in this noncommutative model. And we will summarize our main results at the last section.

II. HAIR MASS BOUNDS IN NONCOMMUTATIVE HAIRY BLACK HOLES

In this paper, we choose the background of noncommutative Einstein-Born-Infeld hairy black holes and the corresponding Lagrangian density with non-zero cosmological constant $\Lambda$ is [59–62]

$$L = R - 2\Lambda + 4b^2(1 - \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{2b^2}}). \quad (1)$$

Here $R$ is the scalar curvature, $b$ is the Born-Infeld factor parameter and the limit of $b \to \infty$ corresponds to the Maxwell field case.

Now we introduce the line element of Einstein-Born-Infeld black holes with noncommutative mass deformation as follows [51, 63]

$$ds^2 = -f_{EBI}(r)dt^2 + f(r)^{-1}_{EBI}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2)$$

The metric function is $f_{EBI}(r) = 1 - \frac{4M}{\sqrt{\pi}r} \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) - \Lambda r^2 + \frac{2b^2}{3}r^2\left(1 - \sqrt{1 + \frac{Q^2}{b^2r^4}}\right) + 4\frac{Q^2}{b^2r^4} F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{b^2r^4}\right)$, where $Q$ is the charge, $M$ is the ADM mass, $F$ is the hypergeometric function satisfying $\left(\frac{1}{4} F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{b^2r^4}\right)\right)' = -\frac{1}{\sqrt{r^4 + \frac{Q^2}{b^2}}}$. and $\gamma$ is the incomplete gamma function defined as $\gamma(n, z) = \int_0^z t^{n-1}e^{-t}dt$. We also label $\theta$ as the noncommutative parameter and the model goes back to the commutative case in the limit of $\theta \to 0$.

The mass in a sphere of radius $r$ is

$$m(r) = M - \frac{\sqrt{\pi}}{\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)} \frac{b^2r^3}{3} \left(1 - \sqrt{1 + \frac{Q^2}{b^2r^4}}\right) + \frac{2Q^2}{3r} F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{b^2r^4}\right). \quad (3)$$

It was found that the black hole horizon $r_H$ and the photonsphere $r_\delta$ can be conveniently used to describe spatial distribution of the matter field outside the horizon [42, 44]. And the spatial distribution of the hair is
characterized by the dimensionless hair mass ratio $\frac{m_{\text{hair}}^+}{m_{\text{hair}}^-}$, where

$$m_{\text{hair}}^+ = M - m(r_\delta)$$  \hspace{1cm} (4)$$

is the hair mass above the photonsphere and

$$m_{\text{hair}}^- = m(r_\delta) - m(r_H)$$  \hspace{1cm} (5)$$

is the hair mass between the horizon and the photonsphere. Here, the black hole horizon $r_H$ is defined by $f_{EBI}(r_H) = 0$. According to the approach in [44], the radius $r_\delta$ of the null circular geodesic (photonsphere) is determined by the relation

$$2f_{EBI}(r_\delta) - r_\delta f'_{EBI}(r_\delta) = 0.$$  \hspace{1cm} (6)$$

And the hair mass ratio can be expressed as

$$\frac{m_{\text{hair}}^+}{m_{\text{hair}}^-} = \frac{M - m(r_\delta)}{m(r_\delta) - m(r_H)} \approx \frac{1}{\gamma(\frac{3}{2}, \frac{r_\delta^2}{2})[b^2 r_\delta^3(1 - \sqrt{1 + \frac{Q^2}{b^2 r_\delta^2}}) + \frac{2Q^2}{r_\delta} F(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{r_\delta^2})]}.$$  \hspace{1cm} (7)$$

It was found that the Hod’s lower hair mass bound $\frac{m_{\text{hair}}^+}{m_{\text{hair}}^-} \geq 1$ holds for various asymptotically flat static hairy black holes in Einstein-Yang-Mills, Einstein-Skyrme, Einstein-non Abelian-Proca, Einstein-Yang-Mills-Higgs, Einstein-Yang-Mills-Dilaton and Einstein-Born-Infeld systems [44, 45]. In fact, this lower bound also exists in asymptotically dS static Einstein-Born-Infeld hairy black holes [46]. In the following, we extend the discussion to the case of noncommutative static Einstein-Born-Infeld hairy black holes.

Case I: $\Lambda = 0$

We calculate the hair mass ratio in the noncommutative Einstein-Born-Infeld asymptotically flat black holes. We firstly show effects of noncommutative parameter on the hair mass ratio. With $M = 1.0$, $Q = 0.8$, $\Lambda = 0$, $b = 2$ and various $\theta$, we obtain smaller ratio with larger $\theta$ as

$$\frac{m_{\text{EBI}}^+}{m_{\text{EBI}}^-} \approx 1.779 \text{ for } \theta = 0.08;$$  \hspace{1cm} (8)$$

$$\frac{m_{\text{EBI}}^+}{m_{\text{EBI}}^-} \approx 1.740 \text{ for } \theta = 0.10;$$  \hspace{1cm} (9)$$

$$\frac{m_{\text{EBI}}^+}{m_{\text{EBI}}^-} \approx 1.588 \text{ for } \theta = 0.12.$$  \hspace{1cm} (10)$$

We further show $\frac{m_{\text{EBI}}^+}{m_{\text{EBI}}^-}$ as a function of $\theta$ in the left panel of Fig. 1. From the left panel, it can be easily seen that $\frac{m_{\text{EBI}}^+}{m_{\text{EBI}}^-}$ decreases with respect to $\theta$. Then, whether very large $\theta$ can invade the Hod’s lower bound is
FIG. 1: (Color online) In the left panel, we show the hair mass ratio \( \frac{m_{EBI}^+}{m_{EBI}^-} \) as a function of the noncommutative parameter \( \theta \) with \( M = 1.0, Q = 0.8, \Lambda = 0 \) and \( b = 2 \) and in the right panel, we show the metric function \( f_{EBI} \) with \( M = 1.0, Q = 0.8, \Lambda = 0, b = 2 \) and various \( \theta \). The three curves in the right panel from top to bottom correspond to \( \theta = 0.1600, \theta = 0.1395 \) and \( \theta = 0.1200 \).

A question to be answered. We numerically show that \( \theta \) cannot be arbitrarily large and there exists a critical noncommutative parameter \( \theta_0 \), above which the black hole horizon cannot form as can be seen in the right panel of Fig. 1. When fixing \( M = 1.0, Q = 0.8, \Lambda = 0 \) and \( b = 2 \), the lowest ratio can be reached at \( \theta_0 \) as

\[
\frac{m_{EBI}^+(\theta_0)}{m_{EBI}^-(\theta_0)} \approx 1.158 \quad \text{with} \quad \theta_0 \approx 0.1395.
\]  

That is to say the noncommunity cannot invade the Hod’s lower bound in this specific case of \( M = 1.0, Q = 0.8, \Lambda = 0 \) and \( b = 2 \). Now we study the ratio with various \( Q \) and \( \theta \). With \( M = 1.0, \Lambda = 0, b = 2 \) and various \( Q \), we can search for \( \theta_0 \) and calculate the minimum ratios at \( \theta_0 \) as

\[
\frac{m_{EBI}^+(\theta_0)}{m_{EBI}^-(\theta_0)} \approx 1.134 \quad \text{for} \quad \theta_0 \approx 0.1705 \quad \text{and} \quad Q = 0.7;
\]

\[
\frac{m_{EBI}^+(\theta_0)}{m_{EBI}^-(\theta_0)} \approx 1.076 \quad \text{for} \quad \theta_0 \approx 0.1974 \quad \text{and} \quad Q = 0.6;
\]

\[
\frac{m_{EBI}^+(\theta_0)}{m_{EBI}^-(\theta_0)} \approx 0.822 \quad \text{for} \quad \theta_0 \approx 0.2221 \quad \text{and} \quad Q = 0.5;
\]

\[
\frac{m_{EBI}^+(\theta_0)}{m_{EBI}^-(\theta_0)} \approx 0.721 \quad \text{for} \quad \theta_0 \approx 0.2403 \quad \text{and} \quad Q = 0.4.
\]

In Fig. 2, we plot the minimum ratio \( \frac{m_{EBI}^+(\theta_0)}{m_{EBI}^-(\theta_0)} \) as a function of \( Q \) with \( M = 1.0, \Lambda = 0 \) and \( b = 2 \) in the left panel and \( M = 1.5, \Lambda = 0 \) and \( b = 2 \) in the right panel. Here, we numerically find that the Hod’s lower bound can be invaded in the noncommutative static asymptotically flat Einstein-Born-Infeld black hole model. We also mention that the Hod’s lower bound is more likely to be invaded in the small charge region. In contrast, it should be emphasized that the Hod’s lower bound always holds in the commutative static EBI black holes with non-negative cosmological constants \[45,46\].
Case II: $\Lambda > 0$

Setting $M = 1.0, Q = 0.4, b = 2$ and $\Lambda = 0.0001$ in the asymptotically dS black holes, we get the ratio at $\theta = 0.2403$ as

$$\frac{m^+_{EBI}}{m_{EBI}} \approx 0.933 < 1.$$  \hspace{1cm} (16)

So the Hod’s lower bound can be invaded in the noncommutative static asymptotically dS hairy black hole. Setting $M = 1.0, Q = 0.4, b = 2$ and $\Lambda = 0.01$, we get the ratio at $\theta = 0.2403$ as

$$\frac{m^+_{EBI}}{m_{EBI}} \approx 1.425 > 1.$$  \hspace{1cm} (17)

With the relations (15), (16) and (17), we see that large cosmological constants lead to a mass ratio above the Hod’s lower mass bound. With detailed calculation, we find that $\frac{m^+_{EBI}}{m_{EBI}}$ increases with respect to $\Lambda$.

Case III: $\Lambda < 0$

Now we show that the Hod’s bound can also be invaded in the AdS noncommutative background. With $M = 1.0, Q = 0.6, b = 2, \theta = 0.18$ and various $\Lambda$, we find

$$\frac{m^+_{EBI}}{m_{EBI}} \approx 1.448 > 1 \text{ for } \Lambda = 0;$$  \hspace{1cm} (18)

$$\frac{m^+_{EBI}}{m_{EBI}} \approx 0.726 < 1 \text{ for } \Lambda = -0.05.$$  \hspace{1cm} (19)

It means that the Hod’s bound can be invaded by imposing an AdS boundary. Due to the confinement of the AdS boundary, this result is natural and effects of negative cosmological constants on hair distribution should be qualitatively the same for other types of black hole hairs.

In the front analysis, we find that the Hod’s lower bound can be widely invaded in the noncommutative gravity and the Hod’s bound is not such a general property as cases in the commutative case. However, we
will show in the following that the Hod’s bound almost holds in noncommutative black holes of large size with non-negative cosmological constants. Our numerical data shows that the following relation exactly holds with non-negative cosmological constants and various other parameters

\[
1 \leq \frac{r_H^3(1 - \sqrt{1 + \frac{Q^2}{r_H^2 b^2}}) + \frac{2Q^2}{r_H} F\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{Q^2}{r_H^2 b^2}\right)}{r_H^3(1 - \sqrt{1 + \frac{Q^2}{r_H^2 b^2}}) + \frac{2Q^2}{r_H} F\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{Q^2}{r_H^2 b^2}\right)} \leq 2. \tag{20}
\]

In the large black hole limit or \(r_H \gg r_f \gg \theta\), the model goes back to the commutative case and the relation (20) is equivalent to Hod’s lower bound in the commutative black hole. Here, we further find that (20) holds beyond the large black hole limit. According to (20) and the fact that \( \gamma(\frac{5}{2}, \frac{7}{4}) \approx 1 \), the ratio (7) can be expressed as

\[
\frac{m_{\text{hair}}^+}{m_{\text{hair}}} = \frac{1}{\gamma(\frac{5}{2}, \frac{7}{4})^2 r_H^2 (1 - \sqrt{1 + \frac{Q^2}{r_H^2 b^2}}) + \frac{2Q^2}{r_H} F\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{Q^2}{r_H^2 b^2}\right) - 1} \geq \frac{1}{2 \gamma(\frac{5}{2}, \frac{7}{4})} - 1.
\]

Considering that \( \gamma(\frac{5}{2}, \frac{7}{4}) \) increases as a function of \( r \) with values of \( \gamma(\frac{5}{2}, \frac{7}{4}) \) in the range \([0, \sqrt{\frac{3}{2}}]\), we obtain lower bound of the ratio in the noncommutative gravity as

\[
\frac{m_{\text{hair}}^+}{m_{\text{hair}}} \geq \frac{1}{2 \gamma(\frac{5}{2}, \frac{7}{4}) - 1} \geq \frac{1}{\sqrt{\pi} - \gamma(\frac{5}{2}, \frac{7}{4})} \approx 1. \tag{22}
\]

In the limit of large black hole or \( r_H \gg \theta \), there is

\[
\frac{m_{\text{hair}}^+}{m_{\text{hair}}} \geq \frac{\gamma(\frac{5}{2}, \frac{7}{4})}{\sqrt{\pi} - \gamma(\frac{5}{2}, \frac{7}{4})} \approx 1. \tag{23}
\]

According to results in [46], the ratio of (20) is equal to 2 in the limit of large \( b \) and small \( Q \). So the lower bound (22) should be also the approximate formula of the hair mass ratio in the case of large \( b \) and small \( Q \). That is to say

\[
\frac{m_{\text{hair}}^+}{m_{\text{hair}}} \approx \frac{\gamma(\frac{5}{2}, \frac{7}{4})}{\sqrt{\pi} - \gamma(\frac{5}{2}, \frac{7}{4})}
\]

in the nearly neutral black hole with large \( b \), which is also well
supported by our numerical data. For example, in the case of \( M = 1.0, \ Q = 0.1, \ b = 2 \) and \( \theta = 0.275 \), we have \( r_H \approx 1.590 \) and

\[
\frac{m_{\text{hair}}^+}{m_{\text{hair}}} \approx 0.743;
\]

\[
\frac{\gamma\left(\frac{3}{2}, \frac{r_H^2}{4\theta}\right)}{\sqrt{\pi} - \gamma\left(\frac{3}{2}, \frac{r_H^2}{4\theta}\right)} \approx 0.733.
\]

(26)

(27)

In summary, we show that the Hod’s bound can be invaded in the noncommutative Einstein-Born-Infeld black holes. We obtain a lower bound (22) expressed with black hole horizon and noncommutative parameters. And (22) shows that the Hod’s bound almost holds for large black holes in flat or dS backgrounds. Since there is also no scalar hair theorem in regular neutral reflecting stars and static scalar fields can condense around charged reflecting stars, it is also very interesting to extend the discussion to the horizonless reflecting star background.

III. CONCLUSIONS

We studied hair distributions of the static spherically symmetric Einstein-Born-Infeld black hole in the noncommutative geometry. We used the photonsphere to divide the matter into two parts and obtained lower bounds of the mass ratio. We found that the noncommutative parameter makes the hair easier to condense in the near horizon area. We further showed that the Hod’s bound can be invaded in the noncommutative hairy black holes and the Hod’s bound is not such a general property as cases in the commutative case. We also mentioned that the Hod’s lower bound is more likely to be invaded in the small charge region. However, for large black holes with a non-negative cosmological constant, the Hod’s lower hair mass bound almost holds in a sense that nearly half of the hair lays above the photonsphere.

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[1] R. Ruffini and J. A. Wheeler, Introducing the black hole, Phys. Today 24, 30 (1971).
[2] J. D. Bekenstein, Black holes and entropy, Phys. Rev. D 7, 2333 (1973).
[3] Panofsky, W.K.H, Needs Versus Means In High-energy Physics, Phys. Today 33, 24 (1980).
[4] W. Israel, Event horizons in static vacuum space-times, Phys. Rev. 164, 1776 (1967).
[5] W. Israel, Event horizons in static electrovac space-times, Commun. Math. Phys. 8, 245 (1968).
[6] B. Carter, Axisymmetric Black Hole Has Only Two Degrees of Freedom, Phys. Rev. Lett. 26, 331 (1971).
[7] S. W. Hawking, Black holes in general relativity, Commun. Math. Phys. 25, 152 (1972).
[8] D. C. Robinson, Classification of black holes with electromagnetic fields, Phys. Rev. D 10, 458 (1974).
[9] D. C. Robinson, Uniqueness of the Kerr black hole, Phys. Rev. Lett. 34, 905 (1975).
[10] J. Isenberg, Electromagnetic Test Fields Around a Kerr-Metric Black Hole, Phys. Rev. Lett. 27, 529 (1971).
[11] J. E. Chaise, Commun. Math. Phys. 19, 276 (1970).
[12] J. D. Bekenstein, Transcendence of the law of baryon-number conservation in black hole physics, Phys. Rev. Lett. 28, 452 (1972).
[13] C. Teitelboim, Nonmeasurability of the baryon number of a black hole, Lett. Nuovo Cimento 3, 326 (1972).
[14] J. D. Bekenstein, Nonexistence of baryon number for static black holes, Phys. Rev. D 5, 1239 (1972).
[15] M. Heusler, A No hair theorem for self-gravitating nonlinear sigma models, J. Math. Phys. 33, 3497 (1992).
[16] D. Sudarsky, A Simple proof of a no hair theorem in Einstein Higgs theory, Class. Quantum Grav. 12, 579 (1995).
[17] J. Hartle, Long-range neutrino forces exerted by kerr black holes, Phys. Rev. D 3, 2938 (1971).
[18] C. Teitelboim, Nonmeasurability of the lepton number of a black hole, Lett. Nuovo Cimento 3, 397 (1972).
[19] P. Bizoń, Colored black holes, Phys. Rev. Lett. 64, 2844 (1990).
[20] M. S. Volkov and D. V. Gal’tsov, Sov. J. Nucl. Phys. 51, 1171 (1990).
[21] H. P. Kuenzle and A. K. M. Masood-ul-Alam, Spherically symmetric static SU(2) Einstein Yang-Mills fields, J. Math. Phys. 31, 928 (1990).
[22] G. Lavrelashvili and D. Maison, Regular and black hole solutions of Einstein Yang-Mills Dilaton theory, Nucl. Phys. B 410, 407 (1993).
[23] P. Bizoń and T. Chamj, Gravitating skyrmions, Phys. Lett. B 297, 55 (1992).
[24] Serge Droz, Markus Heusler, Norbert Straumann, New black hole solutions with hair, Phys. Lett. B 268, 371 (1991).
[25] M. Heusler, S. Droz, and N. Straumann, Stability analysis of self-gravitating skyrmions, Phys. Lett. B 271, 61 (1991).
[26] M. Heusler, S. Droz, and N. Straumann, Linear stability of Einstein Skyrme black holes, Phys. Lett. B 258, 21 (1992).
[27] B. R. Greene, S. D. Mathur, and C. O’Neill, Eluding the no hair conjecture: Black holes in spontaneously broken gauge theories, Phys. Rev. D 47, 2242 (1993).
[28] T. Torii, K. Maeda, and T. Tachizawa, Non-Abelian black holes and catastrophe theory. 1. Neutral type, Phys. Rev. D 51, 1510 (1995).
[29] N. Straumann and Z.-H. Zhou, Instability of a colored black hole solution, Phys. Lett. B 243, 33 (1990).
[30] N. E. Mavromatos and E. Winstanley, Aspects of hairy black holes in spontaneously broken Einstein-Yang-Mills systems: Stability analysis and entropy considerations, Phys. Rev. D 53, 3190 (1996).
[31] M. S. Volkov and D. V. Gal’tsov, Gravitating non-Abelian solitons and black holes with Yang-Mills fields, Phys. Rept. 319, 1 (1999).
[32] P. Bizoń and T. Chmaj, Remark on formation of colored black holes via fine tuning, Phys. Rev. D 61, 067501 (2000).
[33] G. V. Lavrelashvili and D. Maison, A Remark on the instability of the Bartnik-McKinnon solutions, Phys. Lett. B 343, 214 (1995).
[34] Yves Brihaye, Carlos Herdeiro, Eugen Radu, D. H. Tchrakian, Skyrmions, Skyrme stars and black holes with Skyrme hair in five spacetime dimension, JHEP 11 (2017) 037.
[35] Shahar Hod, Stationary Scalar Clouds Around Rotating Black Holes, PRD 86 (2012) 104026.
[36] C. A. R. Herdeiro and E. Radu, Kerr black holes with scalar hair, Phys. Rev. Lett. 112, 221101 (2014).
[37] C. Herdeiro, E. Radu, and H. Runarsson, Non-linear QQ-clouds around Kerr black holes, Phys. Lett. B 739, 302 (2014).
[38] P. V. P. Cunha, C. A. R. Herdeiro, E. Radu, and H. F. Runarsson, Shadows of Kerr black holes with scalar hair, Phys. Rev. Lett. 115, 211102 (2015).
[39] Y. Brihaye, C. Herdeiro, and E. Radu, Inside black holes with synchronized hair, Phys. Lett. B 760, 279 (2016).
[40] J. D. Bekenstein, Black hole hair: twenty-five years after, arXiv:gr-qc/9605059.
[41] Carlos A. R. Herdeiro, Eugen Radu, Asymptotically flat black holes with scalar hair: A review, Int. J. Mod. Phys. D 24 (2015) 09152014.
[42] D. Núñez, H. Quevedo, and D. Sudarsky, Black Holes Have No Short Hair, Phys. Rev. Lett. 76, 571 (1996).
[43] Shahar Hod, A no-short scalar hair theorem for rotating Kerr black holes, Class. Quant. Grav. 33 (2016) 114001.
[44] S. Hod, Hairy Black Holes and Null Circular Geodesics, Phys. Rev. D 84, 124030 (2011) arXiv:1112.3286 [gr-qc].
[45] Yun Soo Myung, Taeyoon Moon, Hairy mass bound in the Einstein-Born-Infeld black hole, Phys. Rev. D 86 (2012) 084047.
[46] Yan Peng, Hairy mass bound in the black hole with non-zero cosmological constants, arXiv:1807.06257.
[47] For a review and complete list of papers on logarithmic corrections, see D.N. Page, New J. Phys. 7 203 (2005); hep-th/0409024.
[48] R. Banerjee, B.R. Majhi, S.K. Modak, Class. Quant. Grav., 26, 085010 (2009). [arXiv:0802.2176].
[49] R. Casadio, P. Nicolini, JHEP 0811, 072 (2008).
[50] R. Banerjee, B. Chakraborty, S. Ghosh, P. Mukherjee, S. Samanta, Found. Phys. 39, 1297 (2009). arXiv:0909.1000 [hep-th].
[51] P. Nicolini, A. Smailagic and E. Spallucci, Phys. Lett. B 632 547 (2006) arXiv:gr-qc/0510112.
[52] R. Banerjee, S. Gangopadhyay, S.K. Modak, Phys. Lett B 686 181 (2010).
[53] E. Witten, Nucl. Phys. B 268, 253 (1986).
[54] N. Seiberg and E. Witten, JHEP 9909, 032 (1999).
[55] S. Minwalla, M. Van Raamsdonk and N. Seiberg, JHEP 0002, 020 (2000).
[56] J. Gomis and T. Mehen, Nucl. Phys. B 591, 265 (2000).
[57] D. Bahns, S. Doplicher, K. Fredenhagen and G. Piacitelli, Phys. Lett. B 533, 178 (2002).
[58] Kai Ma, Ya-Jie Ren, Ya-Hui Wang, Kai-Qiang Liu, Probing Noncommutativities of Phase Space by Using Persistent Charged Current and Its Asymmetry, Phys. Rev. D 97 (2018) no.11, 115011.
[59] N. Breton, Phys. Rev. D 67 (2003) 124004.
[60] T. K. Dey, Phys. Lett. B 595, 484 (2004) [arXiv:hep-th/0406169].
[61] S. Fernando, Phys. Rev. D 74 (2006) 104032 [arXiv:hep-th/0608040].
[62] Y. S. Myung, Y. W. Kim and Y. J. Park, Phys. Rev. D 78 (2008) 084002 [arXiv:0805.0187 [gr-qc]].
[63] Angélica González, Román Linares, Marco Maceda, Oscar Sánchez-Santos, Thermodynamics of a Higher Dimensional Noncommutative Inspired Anti-de Sitter-Einstein-Born-Infeld Black Hole, Int. J. Theor. Phys. 57 (2018) no.7, 2041-2063.
[64] S. Hod, Physics Letters B 718, 1489 (2013); Srijit Bhattacharjee, Sudipta Sarkar, Phys. Rev. D 95 (2017) 084027.
[65] S. Hod, Physics Letters B 763, 275 (2016); S. Hod, Physics Letters B 768 (2017) 97-102; Shahar Hod, European Physical Journal C 78, 173 (2017); Elisa Maggio, Paolo Pani, Valeria Ferrari, Phys. Rev. D 96 (2017) 104047; Yan Peng, Physics Letters B 780 (2018) 144-148; Yan Peng, [arXiv:1803.09148] Yan Peng, Physics Letters B 782 (2018) 717-722; Yan Peng, Nuclear Physics B 934 (2018) 459-465.