Three-Neutrino Oscillations of Atmospheric Neutrinos, \(\theta_{13}\), Neutrino Mass Hierarchy and Iron Magnetized Detectors

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Abstract

We derive predictions for the Nadir angle (\(\theta_n\)) dependence of the ratio \(N_{\mu^-}/N_{\mu^+}\) of the rates of the \(\mu^-\) and \(\mu^+\) multi-GeV events, and for the \(\mu^- - \mu^+\) event rate asymmetry, \(A_{\mu^- - \mu^+} = [N(\mu^-) - N(\mu^+)]/[N(\mu^-) + N(\mu^+)]\), in iron-magnetized calorimeter detectors (MINOS, INO, etc.) in the case of 3-neutrino oscillations of the atmospheric \(\nu_\mu\) and \(\bar{\nu}_\mu\), driven by one neutrino mass squared difference, \(|\Delta m^2_{31}| \sim (2.0 - 3.0) \times 10^{-3}\text{ eV}^2 \gg \Delta m^2_{21}\). The asymmetry \(A_{\mu^- - \mu^+}\) (the ratio \(N_{\mu^-}/N_{\mu^+}\)) is shown to be particularly sensitive to the Earth matter effects in the atmospheric neutrino oscillations, and thus to the values of \(\sin^2 \theta_{13}\) and \(\sin^2 \theta_{23}\), \(\theta_{13}\) and \(\theta_{23}\) being the neutrino mixing angle limited by the CHOOZ and Palo Verde experiments and that responsible for the dominant atmospheric \(\nu_\mu \rightarrow \nu_\tau\) (\(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau\)) oscillations. It is also very sensitive to the type of neutrino mass spectrum which can be with normal (\(\Delta m^2_{31} > 0\)) or with inverted (\(\Delta m^2_{31} < 0\)) hierarchy. We find that for \(\sin^2 \theta_{23} \gtrsim 0.50\), \(\sin^2 2\theta_{13} \gtrsim 0.06\) and \(|\Delta m^2_{31}| = (2 - 3) \times 10^{-3}\text{ eV}^2\), the Earth matter effects produce a relative difference between the integrated asymmetries \(\bar{A}_{\mu^- - \mu^+}\) and \(\bar{A}_{\mu^- - \mu^+}^{2\nu}\) in the mantle (\(\cos \theta_n = 0.30 - 0.84\)) and core (\(\cos \theta_n = 0.84 - 1.0\)) bins, which is bigger in absolute value than approximately \(\sim 15\%\), can reach the values of (30 – 35)\%, and thus can be sufficiently large to be observable. The sign of the indicated asymmetry difference is anticorrelated with the sign of \(\Delta m^2_{31}\). An observation of the Earth matter effects in the Nadir angle distribution of the asymmetry \(A_{\mu^- - \mu^+}\) (ratio \(N_{\mu^-}/N_{\mu^+}\)) would clearly indicate that \(\sin^2 2\theta_{13} \gtrsim 0.06\) and \(\sin^2 \theta_{23} \gtrsim 0.50\), and would lead to the determination of the sign of \(\Delta m^2_{31}\).

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1 Introduction

There has been a remarkable progress in the studies of neutrino oscillations in the last several years. The experiments with solar, atmospheric and reactor neutrinos \[1, 2, 3, 4, 5, 6, 7\] have provided compelling evidences for the existence of neutrino oscillations driven by nonzero neutrino masses and neutrino mixing. Evidences for oscillations of neutrinos were obtained also in the first long baseline accelerator neutrino experiment K2K \[8\]. It was predicted already in 1967 \[9\] that the existence of solar neutrino oscillations would cause a deficit of solar neutrinos detected on Earth. The hypothesis of solar neutrino oscillations, which in one variety or another were considered starting from the late 60's as the most natural explanation of the observed \[1, 2\] solar neutrino deficit (see, e.g., refs. \[9, 10, 11, 12\]), has received a convincing confirmation from the measurement of the solar neutrino flux through the neutral current reaction on deuterium by the SNO experiment \[4, 5\], and by the first results of the KamLAND experiment \[7\]. The analysis of the solar neutrino data obtained by Homestake, SAGE, GALLEX/GNO, Super-Kamiokande and SNO experiments showed that the data favor the Large Mixing Angle (LMA) MSW solution of the solar neutrino problem (see, e.g., ref. \[4\]). The first results of the KamLAND reactor experiment \[7\] have confirmed (under the very plausible assumption of CPT-invariance) the LMA MSW solution, establishing it essentially as a unique solution of the solar neutrino problem.

The latest addition to this magnificent effort is the evidence presented recently by the Super-Kamiokande (SK) collaboration for an “oscillation dip” in the \(L/E\) dependence, of the (essentially multi-GeV) \(\mu\)-like atmospheric neutrino events \[1, 2\], \(L\) and \(E\) being the distance traveled by neutrinos and the neutrino energy. As is well known, the SK atmospheric neutrino data is best described in terms of dominant 2-neutrino \(\nu_\mu \to \nu_\tau (\bar{\nu}_\mu \to \bar{\nu}_\tau)\) vacuum oscillations with maximal mixing, \(\sin^2 2\theta_{23} \approx 1\). The observed dip is predicted due to the oscillatory dependence of the \(\nu_\mu \to \nu_\tau\) and \(\bar{\nu}_\mu \to \bar{\nu}_\tau\) oscillation probabilities, \(P(\nu_\mu \to \nu_\tau) \approx P(\bar{\nu}_\mu \to \bar{\nu}_\tau)\), on \(L/E\). The dip in the observed \(L/E\) distribution corresponds to the first oscillation minimum of the \(\nu_\mu\) (\(\bar{\nu}_\mu\)) survival probability, \(P(\nu_\mu \to \nu_\mu) = 1 - P(\nu_\mu \to \nu_\tau)\), as \(L/E\) increases starting from values for which \(\Delta m^2_{31} L/(2E) \ll 1\) and \(P(\nu_\mu \to \nu_\mu) \approx 1\), \(\Delta m^2_{31}\) being the neutrino mass squared difference responsible for the atmospheric \(\nu_\mu\) and \(\bar{\nu}_\mu\) oscillations. This beautiful result represents the first ever observation of a direct effect of the oscillatory dependence on \(L/E\) of the probability of neutrino oscillations in vacuum.

The interpretation of the solar and atmospheric neutrino, and of KamLAND data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current (see, e.g., ref. \[14\]):

\[
u_{lL} = \sum_{j=1}^{3} U_{lj} \nu_{jL}.
\]

Here \(\nu_{lL}, l = e, \mu, \tau\), are the three left-handed flavor neutrino fields, \(\nu_{jL}\) is the left-handed field of the neutrino \(\nu_j\) having a mass \(m_j\) and \(U\) is the Pontecorvo-Maki-Nakagawa-Sakata
The solar neutrino, CHOOZ and KamLAND data showed [21] that for solar oscillations, the data suggest that with the neutrino mass squared differences which drive the solar and atmospheric neutrino oscillations, the indicated data in terms of \( \nu \) somewhat larger values of \( \sin^2 \theta_{13} \).

Earlier combined analyzes of the SK atmospheric neutrino and the K2K data produced where we have used a standard parametrization of \( U \) with the usual notations, \( s_{ij} \equiv \sin \theta_{ij} \), \( c_{ij} \equiv \cos \theta_{ij} \), and \( \delta \) is the Dirac CP-violation phase \(^2\). If one identifies \( \Delta m^2_{21} > 0 \) and \( \Delta m^2_{31} \) with the neutrino mass squared differences which drive the solar and atmospheric neutrino oscillations, the data suggest that \( |\Delta m^2_{31}| \gg \Delta m^2_{21} \). In this case \( \theta_{12} \) and \( \theta_{23} \), represent the neutrino mixing angles responsible for the solar and the dominant atmospheric neutrino oscillations, \( \theta_{12}, \theta_{23} \), while \( \theta_{13} \) is the angle limited by the data from the CHOOZ and Palo Verde experiments [19, 20].

The 3-neutrino oscillations of the solar \( \nu_e \) depend in the case of interest, \( |\Delta m^2_{31}| \gg \Delta m^2_{21} \), not only on \( \Delta m^2_{21} \) and \( \theta_{12} \), but also on \( \theta_{13} \). A combined 3-neutrino oscillation analysis of the solar neutrino, CHOOZ and KamLAND data showed [21] that for \( \sin^2 \theta_{13} \lesssim 0.05 \) the allowed ranges of the solar neutrino oscillation parameters do not differ substantially from those derived in the 2-neutrino oscillation analyzes (see, e.g., refs. [5, 22]). A description of the indicated data in terms of \( \nu_e \to \nu_{\mu,\tau} \) and \( \bar{\nu}_e \to \bar{\nu}_{\mu,\tau} \) oscillations is possible (at 99.73% C.L.) for \( \sin^2 \theta_{13} \lesssim 0.075 \). The data favor the LMA-I MSW solution (see, e.g., refs. [21, 22]) with \( \Delta m^2_{21} \approx 7.2 \times 10^{-5} \text{ eV}^2 \) and \( \sin^2 \theta_{12} \approx 0.30 \). The LMA-II solution, corresponding to \( \Delta m^2_{21} \approx 1.5 \times 10^{-4} \text{ eV}^2 \) and approximately the same value of \( \sin^2 \theta_{12} \), is severely constrained by the fact [23] that the rate of the CC and NC reactions on deuterium, measured with a relatively high precision in SNO during the salt phase of the experiment [5], turned out to be definitely smaller than 0.50. This solution is currently allowed by the data only at 99.13% C.L. [21].

The preliminary results of the most recent improved analysis of the SK atmospheric neutrino data, performed by the SK collaboration, gave at 90% C.L. [24]

\[
1.3 \times 10^{-3} \text{ eV}^2 \leq |\Delta m^2_{31}| \leq 3.1 \times 10^{-3} \text{ eV}^2, \quad 0.90 \leq \sin^2 2\theta_{23} \leq 1.0,
\]

with best fit values \( |\Delta m^2_{31}| = 2.0 \times 10^{-3} \text{ eV}^2 \) and \( \sin^2 2\theta_A = 1.0 \). Adding the K2K data [8], the authors [25] find the same results for \( \sin^2 2\theta_A \), the same \( |\Delta m^2_{31}| \) best fit value and

\[
1.55 \times 10^{-3} \text{ eV}^2 \lesssim |\Delta m^2_{31}| \lesssim 2.60 \times 10^{-3} \text{ eV}^2, \quad 90\% \text{ C.L.}
\]

Earlier combined analyzes of the SK atmospheric neutrino and the K2K data produced somewhat larger values of \( |\Delta m^2_{31}| \): the best fit value and the 90% C.L. allowed interval of values, found, e.g., in ref. [20] read \( |\Delta m^2_{31}| = 2.6 \times 10^{-3} \text{ eV} \) and \( 2.0 \times 10^{-3} \text{ eV}^2 \lesssim |\Delta m^2_{31}| \lesssim 3.2 \times 10^{-3} \text{ eV}^2 \). Finally, the values of \( |\Delta m^2_{31}| \) and \( \sin^2 2\theta_{23} \), deduced from the SK analysis of the \( L/E \) dependence of the observed \( \mu \)-like atmospheric neutrino events [13], are comfortably compatible with the values obtained in the other analyzes: \( |\Delta m^2_{31}| = 2.4 \times 10^{-3} \text{ eV}, \sin^2 2\theta_{23} = 1 \) (best fit), and \( 1.9 \times 10^{-3} \text{ eV}^2 \lesssim |\Delta m^2_{31}| \lesssim 3.0 \times 10^{-3} \text{ eV}^2, \) \( 0.90 \leq \sin^2 2\theta_{23} \leq 1.0 \) (90% C.L.). We will use in our further discussion as illustrative the values \( |\Delta m^2_{31}| = (2.0; 3.0) \times 10^{-3} \text{ eV}^2 \) and \( \sin^2 2\theta_{23} = 0.92; 1.0 \).

\(^2\)We have not written explicitly the two possible Majorana CP-violation phases [16, 17], which do not enter into the expressions for the oscillation probabilities of interest [16, 18].
Let us note that the atmospheric neutrino and K2K data do not allow one to determine the signs of $\Delta m^2_{31}$, and of $\cos 2\theta_{23}$ if $\sin^2 2\theta_{23} \neq 1.0$. This implies that in the case of 3-neutrino mixing one can have $\Delta m^2_{31} > 0$ or $\Delta m^2_{31} < 0$. The two possibilities correspond to two different types of neutrino mass spectrum: with normal hierarchy (NH), $m_1 < m_2 < m_3$, and with inverted hierarchy (IH), $m_3 < m_1 < m_2$. The fact that the sign of $\cos 2\theta_{23}$ is not determined when $\sin^2 2\theta_{23} \neq 1.0$ implies that when, e.g., $\sin^2 2\theta_{23} = 0.92$, two values of $\sin^2 \theta_{23}$ are possible, $\sin^2 \theta_{23} \simeq 0.64$ or 0.36.

The precise limit on the angle $\theta_{13}$ from the CHOOZ and Palo Verde data is $\Delta m^2_{31}$-dependent (see, e.g., ref. [27]). Using the 99.73% allowed range of $\Delta m^2_{31} = (1.1 - 3.2) \times 10^{-3} \, \text{eV}^2$, from ref. [25], one gets from a combined 3-neutrino oscillation analysis of the solar neutrino, CHOOZ and KamLAND data [21]:

$$\sin^2 \theta_{13} < 0.047 \ (0.074), \quad 90\% \ (99.73\%) \ \text{C.L.} \quad (5)$$

The global analysis of the solar, atmospheric and reactor neutrino data performed in ref. [28] gives $\sin^2 \theta_{13} < 0.054$ at 99.73% C.L.

It is difficult to overestimate the importance of getting more precise information about the value of the mixing angle $\theta_{13}$, of determining the sign of $\Delta m^2_{31}$, or the type of the neutrino mass spectrum (with normal or inverted hierarchy), and of measuring the value of $\sin^2 \theta_{23}$ with a higher precision, for the future progress in the studies of neutrino mixing. Although this has been widely recognized, let us repeat the arguments on which the statement is based.

The mixing angle $\theta_{13}$, or the absolute value of the element $U_{e3}$ of the PMNS matrix, $|U_{e3}| = \sin \theta_{13}$, plays a very important role in the phenomenology of the 3-neutrino oscillations. It drives the sub-dominant $\nu_\mu \leftrightarrow \nu_e$ ($\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$) oscillations of the atmospheric $\nu_\mu$ ($\bar{\nu}_\mu$) and $\nu_e$ ($\bar{\nu}_e$) [29, 30]. The value of $\theta_{13}$ controls also the $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ transitions in the long baseline neutrino oscillation experiments (MINOS, CNGS), and in the widely discussed very long baseline neutrino oscillation experiments at neutrino factories (see, e.g., refs. [31, 32, 33]). The magnitude of the T-violating and CP-violating terms in neutrino oscillations probabilities is directly proportional to $\sin \theta_{13}$ (see, e.g., refs. [34, 35]).

If the neutrinos with definite mass are Majorana particles (see, e.g., ref. [11]), the predicted value of the effective Majorana mass parameter in neutrinoless double $\beta$-decay depends strongly in the case of normal hierarchical or partially hierarchical neutrino mass spectrum on the value of $\sin^2 \theta_{13}$ (see, e.g., ref. [36]).

The sign of $\Delta m^2_{31}$ determines, for instance, which of the transitions (e.g., of atmospheric neutrinos) $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$, or $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$, can be enhanced by the Earth matter effects [37, 38, 39]. The predictions for the neutrino effective Majorana mass in neutrinoless double $\beta$-decays depend critically on the type of the neutrino mass spectrum (normal or inverted hierarchical) [36, 40]. The knowledge of the value of $\theta_{13}$ and of the sign of $\Delta m^2_{31}$ is crucial for the searches for the correct theory of neutrino masses and mixing as well.

Somewhat better limits on $\sin^2 \theta_{13}$ than the existing one can be obtained in the MINOS, OPERA and ICARUS experiments [41, 42]. Various options are being currently discussed (experiments with off-axis neutrino beams, more precise reactor antineutrino and long baseline experiments, etc., see, e.g., ref. [43]) of how to improve by at least an order of magnitude, i.e., to values of $\sim 0.005$ or smaller, the sensitivity to $\sin^2 \theta_{13}$. The sign of $\Delta m^2_{31}$ can be determined in very long baseline neutrino oscillation experiments at neutrino factories (see,
e.g., refs. [31, 32], and, e.g., using combined data from long baseline oscillation experiments at the JHF facility and with off-axis neutrino beams [43]. If the neutrinos with definite mass are Majorana particles, it can be determined by measuring the effective neutrino Majorana mass in neutrinoless double $\beta$-decay experiments [36, 40]. Under certain rather special conditions it might be determined also in experiments with reactor $\bar{\nu}_e$ [45].

In the present article we study possibilities to obtain information on the value of $\sin^2 \theta_{13}$ and on the sign of $\Delta m^2_{31}$ using the data on atmospheric neutrinos, which can be obtained in experiments with detectors able to measure the charge of the muon produced in the charged current (CC) reaction by atmospheric $\nu_\mu$ or $\bar{\nu}_\mu$. It is a natural continuation of our similar study for water-Čerenkov detectors [40]. In the experiments with muon charge identification it will be possible to distinguish between the $\nu_\mu$ and $\bar{\nu}_\mu$ induced events. As is well known, the water-Čerenkov detectors do not have such a capability. Among the operating detectors, MINOS has muon charge identification capabilities for multi-GeV muons [41]. The MINOS experiment is currently collecting atmospheric neutrino data. The detector has relatively small mass, but after 5 years of data-taking it is expected to collect about 440 atmospheric $\nu_\mu$ and about 260 atmospheric $\bar{\nu}_\mu$ multi-GeV events (having the interaction vertex inside the detector). There are also plans to build a 30-50 kton magnetized tracking iron calorimeter detector in India within the India-based Neutrino Observatory (INO) project [47]. The INO detector will be based on MONOLITH design [48]. The primary goal is to study the oscillations of the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$. This detector is planned to have efficient muon charge identification, high muon energy resolution ($\sim 5\%$) and muon energy threshold of about 2 GeV. It will accumulate sufficiently high statistics of atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ induced events in several years, which would permit to search for effects of the subdominant $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu$) and $\bar{\nu}_\mu \to \bar{\nu}_e$ ($\bar{\nu}_e \to \bar{\nu}_\mu$) transitions.

If $\nu_\mu$ and $\bar{\nu}_\mu$ with energies $E_{\nu,\bar{\nu}} \gtrsim 2$ GeV take part in 2-neutrino $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ oscillations, one would have $P(\nu_\mu \to \nu_\tau) = P(\bar{\nu}_\mu \to \bar{\nu}_\tau)$. If a given detector collects a sample of atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ induced CC events with $\mu^\pm$ having energies exceeding a few GeV, the difference between the $\mu^-$ and $\mu^+$ event rates in magnetized iron detectors of interest (MINOS, INO), apart from detector effects, would essentially be due to the difference between the $\nu_\mu$ and $\bar{\nu}_\mu$ charged current (CC) deep inelastic scattering cross sections. In the case of 3-neutrino oscillations of the atmospheric $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$ and $\bar{\nu}_e$, the Earth matter effects can generate a difference between the $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu$) and $\bar{\nu}_\mu \to \bar{\nu}_e$ ($\bar{\nu}_e \to \bar{\nu}_\mu$) transitions. For $\Delta m^2_{21} \ll |\Delta m^2_{31}|$, which is implied by the current solar and atmospheric neutrino data, and if $\sin^2 \theta_{13} \neq 0$, the Earth matter effects can resonantly enhance either the $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\mu$, or the $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ transitions, depending on the sign of $\Delta m^2_{31}$. The effects of the enhancement can be substantial for $\sin^2 \theta_{13} \approx 0.01$. Under the condition $\Delta m^2_{21} \ll |\Delta m^2_{31}|$, the expressions for the probabilities of $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu$) and $\bar{\nu}_\mu \to \bar{\nu}_e$ ($\bar{\nu}_e \to \bar{\nu}_\mu$) transitions contain also $\sin^2 \theta_{23}$ as a factor which determines their maximal values. Since the fluxes of multi-GeV $\nu_\mu$ and $\nu_e$ ($\bar{\nu}_\mu$ and $\bar{\nu}_e$) differ considerably (by a factor $\sim (2.6 - 4.5)$ for $E_{\nu,\bar{\nu}} \sim (2 - 10)$ GeV), the Earth matter effects can create a substantial difference between the $\mu^-$ and $\mu^+$ rates of events, produced by the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ in MINOS, INO, or any other detector of the same type. This difference can be relatively large and observable in the samples of the multi-GeV $\mu^\pm$ events ($E_{\mu} \sim (2 - 10)$ GeV), in which the muons are produced by atmospheric neutrinos with relatively large path length in the Earth, i.e., by neutrinos crossing deeply the Earth mantle [32, 33] or the mantle.
and the core [29, 50, 51, 52]. These are the neutrinos for which $46 \cos \theta_n \gtrsim (0.3 - 0.4)$, $\theta_n$ being the Nadir angle characterizing the (atmospheric) neutrino trajectory in the Earth.

For $\Delta m^2_{31} > 0$, the $\nu_\mu \rightarrow \nu_e (\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ and $\nu_e \rightarrow \nu_\mu (\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ transitions in the Earth lead to a reduction of the rate of the multi-GeV $\mu^-$ events observable in MINOS, INO, etc., with respect to the case of absence of these transitions (see, e.g., refs. [29, 49, 50, 51, 53]). If $\Delta m^2_{31} < 0$, the $\mu^+$ event rate will be reduced. Correspondingly, as observable which is sensitive to the Earth matter effects, and thus to the value of $\sin^2 \theta_{13}$ and the sign of $\Delta m^2_{31}$, as well as to $\sin^2 \theta_{23}$, we can consider the Nadir-angle distribution of the ratio $N(\nu^-)/N(\nu^+)$ of the multi-GeV $\mu^-$ and $\mu^+$ event rates, or, equivalently, of the $\mu^- - \mu^+$ event rate asymmetry $A_{\mu^- - \mu^+} = [N(\nu^-) - N(\nu^+)]/[N(\nu^-) + N(\nu^+)]$. The systematic uncertainty, in particular, in the Nadir angle dependence of $N(\nu^-)/N(\nu^+)$, and correspondingly in the asymmetry $A_{\mu^- - \mu^+}$, can be smaller than those on the measured Nadir angle distributions of the rates of $\mu^-$ and $\mu^+$ events, $N(\nu^-)$ and $N(\nu^+)$. We have obtained predictions for the Nadir-angle distribution of $A_{\mu^- - \mu^+}$ in the case of 3-neutrino oscillations of the atmospheric $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$ and $\bar{\nu}_e$, both for neutrino mass spectra with normal ($\Delta m^2_{31} > 0$) and inverted ($\Delta m^2_{31} < 0$) hierarchy, $(A^{\nu_\mu}_{\mu^- - \mu^+})_{\text{NH}}$, $(A^{\nu_{\bar{\mu}}}_{\mu^- - \mu^+})_{\text{IH}}$, and for $\sin^2 \theta_{23} = 0.64; 0.50; 0.36$. These are compared with the predicted Nadir-angle distributions of the same asymmetry in the case of 2-neutrino $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations of the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ (i.e., for $\sin^2 \theta_{13} = 0$), $A^{\nu_{\tau}}_{\mu^- - \mu^+}$. Predictions for the three types of asymmetries indicated above of the suitably integrated Nadir angle distributions of the $\mu^-$ and $\mu^+$ multi-GeV event rates are also given. Our results show, in particular, that for $\sin^2 \theta_{23} \gtrsim 0.50$ and $\sin^2 2\theta_{13} \gtrsim 0.06$ the effects of the Earth matter enhanced subdominant transitions of the atmospheric neutrinos, $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\mu$, or $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$, can be sufficiently large to be observable with INO and possibly with MINOS detectors. Conversely, if the indicated effects are observed in the MINOS and/or INO experiments, that would imply that $\sin^2 \theta_{13} \gtrsim 0.05$, $\sin^2 \theta_{23} \gtrsim 0.50$ and at the same time would permit to determine the sign of $\Delta m^2_{31}$ and thus to answer the fundamental question about the type of hierarchy - normal or inverted, the neutrino mass spectrum has.

Let us note that the Earth matter effects in atmospheric neutrino oscillations have been widely studied (for recent detailed analyzes see, e.g., refs. [52, 40], which contain also a rather complete list of references to earlier work on the subject). A rather detailed analysis for the MONOLITH detector has been performed in ref. [53]. A large number of studies have been done for the Super-Kamiokande detector, or more generally, for water-Čerenkov detectors. In ref. [40], in particular, the magnitude of the Earth matter effects in the Nadir angle distribution of the ratio of the multi-GeV $\mu^-$–like and $e^-$–like events, measured in water-Čerenkov detectors, $N_\mu/N_e$, has been investigated. This Nadir angle distribution is the observable most sensitive to the matter effects of interest. It was concluded that for $\sin^2 \theta_{23} \gtrsim 0.50$, $\sin^2 \theta_{13} \gtrsim 0.01$ and $\Delta m^2_{31} > 0$, the effects of the Earth matter enhanced $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions of the atmospheric $\nu_\mu$ and $\nu_e$, might be observable with the Super-Kamiokande detector. However, determining the sign of $\Delta m^2_{31}$ would be quite challenging in this experiment (or its bigger version - Hyper-Kamiokande [54]). In general, the matter effects in the Nadir angle distribution of the ratio of the multi-GeV $\mu^-$–like and $e^-$–like events, $N_\mu/N_e$, which can be measured in the Super-Kamiokande or other water-Čerenkov detectors, are smaller than the matter effects in the Nadir angle distribution of the ratio of the multi-GeV $\mu^-$ and $\mu^+$ events, $N(\nu^-)/N(\nu^+)$, which can be measured in MINOS,
INO, or any other atmospheric neutrino experiment with sufficiently good muon charge identification. The reason is that in the case of water-Čerenkov detectors, approximately 2/3 of the rate of the multi-GeV \( \mu \)-like events is due to \( \nu_\mu \), and \( \sim 1/3 \) is due to \( \bar{\nu}_\mu \); similar partition is valid for the multi-GeV \( e \)-like events. Depending on the sign of \( \Delta m^2_{31} \), the matter effects enhance either the neutrino transitions, \( \nu_\mu \rightarrow \nu_e \) and and \( \nu_e \rightarrow \nu_\mu \), or the antineutrino transitions, \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) and \( \bar{\nu}_e \rightarrow \bar{\nu}_\mu \), but not both types of transitions. Correspondingly, because the \( \nu_{\mu,e} \)- and \( \bar{\nu}_{\mu,e} \)-induced events are indistinguishable in water-Čerenkov detectors, only \( \sim 2/3 \) or \( \sim 1/3 \) of the events in the multi-GeV \( \mu \)-like and \( e \)-like samples collected in these detectors are due to neutrinos whose transitions can be enhanced by matter effects. This effectively reduces the magnitude of the matter effects in the samples of multi-GeV \( \mu \)-like and \( e \)-like events. Obviously, such a “dilution” of the magnitude of the matter effects does not take place in the samples of the multi-GeV \( \mu^- \) and \( \mu^+ \) events, which can be collected in MINOS and INO experiments, i.e., in the experiments with muon charge identification.

2 Subdominant 3-\( \nu \) Oscillations of Multi-GeV Atmospheric Neutrinos in the Earth

In the present Section we review the physics of the subdominant 3-neutrino oscillations of the multi-GeV atmospheric neutrinos in the Earth (see, e.g., ref. [46]).

The subdominant \( \nu_\mu \rightarrow \nu_e \) (\( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)) and \( \nu_e \rightarrow \nu_\mu(\tau) \) (\( \bar{\nu}_e \rightarrow \bar{\nu}_\mu(\tau) \)) oscillations of the multi-GeV atmospheric neutrinos of interest should exist and their effects could be observable if three-flavor-neutrino mixing takes place in vacuum, i.e., if \( \sin^2 2\theta_{13} \neq 0 \), and if \( \sin^2 2\theta_{13} \) is sufficiently large [29, 51, 52]. These transitions are driven by \( \Delta m^2_{31} \). The probabilities of the transitions contain \( \sin^2 \theta_{23} \) as factor which determines their maximal value (see further).

For \( \Delta m^2_{31} > 0 \), the \( \nu_\mu \rightarrow \nu_e \) (\( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)) and \( \nu_e \rightarrow \nu_\mu(\tau) \) (\( \bar{\nu}_e \rightarrow \bar{\nu}_\mu(\tau) \)) transitions of the multi-GeV atmospheric neutrinos, are amplified (suppressed) by the Earth matter effects; if \( \Delta m^2_{31} < 0 \), the transitions of neutrinos are suppressed and those of antineutrinos are enhanced. Therefore for a given sign of \( \Delta m^2_{31} \), the Earth matter affects differently the transitions of neutrinos and antineutrinos. Thus, the study of the subdominant atmospheric neutrino oscillations can provide information, in particular, about the type of neutrino mass hierarchy (or sign of \( \Delta m^2_{31} \)), the magnitude of \( \sin^2 \theta_{13} \) and the octant where \( \theta_{23} \) lies.

Under the condition \( |\Delta m^2_{31}| > \Delta m^2_{21} \), which the neutrino mass squared differences determined from the existing atmospheric and solar neutrino and KamLAND data satisfy, the relevant 3-neutrino \( \nu_\mu \rightarrow \nu_e \) (\( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)) and \( \nu_e \rightarrow \nu_\mu(\tau) \) (\( \bar{\nu}_e \rightarrow \bar{\nu}_\mu(\tau) \)) transition probabilities reduce effectively to a 2-neutrino transition probability [53] with \( \Delta m^2_{31} \) and \( \theta_{13} \) playing the role of the relevant 2-neutrino oscillation parameters.

The 3-neutrino oscillation probabilities of interest for atmospheric \( \nu_{e,\mu} \) having energy \( E \) and crossing the Earth along a trajectory characterized by a Nadir angle \( \theta_n \), have the following form [55]:

\[
P_{3\nu}(\nu_e \rightarrow \nu_e) \cong 1 - P_{2\nu}, \tag{6}
\]
\[
P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s^2_{23} P_{2\nu}, \tag{7}
\]
\[
P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c^2_{23} P_{2\nu}, \tag{8}
\]
oscillation probabilities are analogous to those for neutrinos: they can be obtained formally from eqs. (6) - (13) by replacing the neutrino related quantities - probabilities, $\nu$ where $\Phi$, trajectories for which $0$.

Here $P_{2\nu} \equiv P_{2\nu}(\Delta m^2_{31}; \theta_{13}; E, \theta_n)$ is the probability of 2-neutrino $\nu_e \rightarrow \nu^\prime$ oscillations in the Earth, where $\nu^\prime_\tau = s_{23}\nu_\mu + c_{23}\nu_\tau$ [55], and $\kappa$ and $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ are known phase and 2-neutrino transition probability amplitude $29, 30, 46$.

The fluxes of atmospheric $\nu_{e,\mu}$ of energy $E$, which reach the detector after crossing the Earth along a given trajectory specified by the value of $\theta_n$, $\Phi_{\nu_{e,\mu}}(E, \theta_n)$, are given by the following expressions in the case of the 3-neutrino oscillations under discussion $30$:

\[
\Phi_{\nu_e}(E, \theta_n) \cong \Phi_{\nu_e}^0 \left( 1 + [s^2_{23} r - 1] P_{2\nu} \right),
\]

\[
\Phi_{\nu_\mu}(E, \theta_n) \cong \Phi_{\nu_\mu}^0 \left( 1 + s^4_{23} \left( [s^2 r]^{-1} - 1 \right) P_{2\nu} - 2 s^2_{23} [1 - Re (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))] \right),
\]

where $\Phi_{\nu_{e,\mu}}^0 = \Phi_{\nu_{e,\mu}}^0(E, \theta_n)$ is the $\nu_{e,\mu}$ flux in the absence of neutrino oscillations and

\[
\rho \equiv r(E, \theta_n) \cong \frac{\Phi_{\nu_\mu}(E, \theta_z)}{\Phi_{\nu_e}(E, \theta_z)} .
\]

The interpretation of the SK atmospheric neutrino data in terms of $\nu_\mu \rightarrow \nu_\tau$ oscillations requires the parameter $s^2_{23}$ to lie approximately in the interval (0.30 - 0.70), with 0.5 being the statistically preferred value. For the predicted ratio $r(E, \theta_n)$ of the atmospheric $\nu_\mu$ and $\nu_e$ fluxes for i) the Earth core crossing and ii) only mantle crossing neutrinos, having trajectories for which $0.3 \lesssim \cos \theta_n \lesssim 1.0$, one has $50, 51, 58$: $r(E, \theta_z) \cong (2.0 - 2.5)$ for the neutrinos giving contribution to the sub-GeV samples of Super-Kamiokande events, and $r(E, \theta_n) \cong (2.6 - 4.5)$ for those giving the main contribution to the multi-GeV samples. If $s^2_{23} = 0.5$ and $r(E, \theta_z) \cong 2.0$, we have $(s^2_{23} r(E, \theta_z) - 1) \cong 0$, and the possible effects of the $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions on the $\nu_e$ and $\nu_\mu$ fluxes, and correspondingly in the sub-GeV $e$-like and $\mu$-like samples of events, would be strongly suppressed independently of the values of the corresponding transition probabilities. For the multi-GeV neutrinos one finds $s^2_{23}[1 - (s^2_{23} r(E, \theta_z))^{-1}] \cong 0.06 - 0.14 (0.16 - 0.27)$ and $(s^2_{23} r(E, \theta_z) - 1) \cong 0.3 - 1.3 (0.66 - 1.9)$ for $s^2_{23} = 0.5 (0.64)$. Obviously, the effects of interest are much larger for the multi-GeV neutrinos than for the sub-GeV neutrinos. They are also predicted to be larger for the flux of (and event rate due to) multi-GeV atmospheric $\nu_e$ than for the flux of (and event rate due to) multi-GeV atmospheric $\nu_\mu$.

The same conclusions are valid for the effects of oscillations on the fluxes of, and event rates due to, atmospheric antineutrinos $\bar{\nu}_e$ and $\bar{\nu}_\mu$. The formulae for anti-neutrino fluxes and oscillation probabilities are analogous to those for neutrinos: they can be obtained formally from eqs. (6) - (13) by replacing the neutrino related quantities - probabilities, $\kappa$, $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau)$ and fluxes, with the corresponding quantities for antineutrinos: $P_{2\nu}(\Delta m^2_{31}; \theta_{13}; E, \theta_n) \rightarrow \bar{P}_{2\nu}(\Delta m^2_{31}; \theta_{13}; E, \theta_n)$, $\kappa \rightarrow -\bar{\kappa}$, $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \rightarrow A_{2\nu}(\bar{\nu}_\tau \rightarrow \bar{\nu}_\tau)$ $\Rightarrow \bar{A}_{2\nu}$, $P_{3\nu}(\nu_\mu \rightarrow \nu_\tau) \rightarrow P_{3\nu}(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$, $\Phi_{\nu_{e,\mu}}(E, \theta_n) \rightarrow \Phi_{\bar{\nu}_{e,\mu}}(E, \theta_n)$ and $r(E, \theta_n) \rightarrow \bar{r}(E, \theta_n)$ (see refs. $30, 46$).

Equations (6) - (9), (11) - (12) and the similar equations for antineutrinos imply that in the case under study the effects of the $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, and $\nu_e \rightarrow \nu_{\mu(\tau)}$, $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu(\tau)}$, oscillations i) increase with the increase of $s^2_{23}$ and are maximal for the largest allowed value of $s^2_{23}$, ii) should be substantially larger in the multi-GeV samples of events than in the
sub-GeV samples, and iii) in the case of the multi-GeV samples, for \( \Delta m_{31}^2 > 0 \) they lead to a decrease of the \( \mu^- \) event rate, while if \( \Delta m_{31}^2 < 0 \), the \( \mu^+ \) event rate will decrease. The last point follows from the fact that the magnitude of the effects we are interested in depends also on the 2-neutrino oscillation probabilities, \( P_{2\nu} \) and \( \bar{P}_{2\nu} \), and that \( P_{2\nu} \) or \( \bar{P}_{2\nu} \) (but not both probabilities) can be strongly enhanced by the Earth matter effects. In the case of oscillations in vacuum we have \( P_{2\nu} = \bar{P}_{2\nu} \sim \sin^2 2\theta_{13} \). Given the existing limits on \( \sin^2 2\theta_{13} \), the probabilities \( P_{2\nu} \) and \( \bar{P}_{2\nu} \) cannot be large if the oscillations take place in vacuum.

If \( \sin^2 \theta_{13} \neq 0 \), the Earth matter effects can resonantly enhance either the \( \nu_\mu \rightarrow \nu_e \) and \( \nu_e \rightarrow \nu_\mu \), or the \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) and \( \bar{\nu}_e \rightarrow \bar{\nu}_\mu \) transitions, depending on the sign of \( \Delta m_{31}^2 \). The enhancement mechanisms are discussed briefly in the next subsection.

### 2.1 Enhancing Mechanisms

As is well-known, the Earth density distribution in the existing Earth models is assumed to be spherically symmetric \(^3\) and there are two major density structures - the core and the mantle, and a certain number of substructures (shells or layers). The core radius and the depth of the mantle are known with a rather good precision and these data are incorporated in the Earth models. According to the Stacey 1977 and the more recent PREM models \(^{[59, 60]}\), which are widely used in the calculations of the probabilities of neutrino oscillations in the Earth, the core has a radius \( R_c = 3485.7 \) km, the Earth mantle depth is approximately \( R_{\text{man}} = 2885.3 \) km, and the Earth radius is \( R_\oplus = 6371 \) km. The mean values of the matter densities and the electron fraction numbers in the mantle and in the core read, respectively: \( \bar{\rho}_{\text{man}} \cong 4.5 \text{ g/cm}^3 \), \( \bar{\rho}_c \cong 11.5 \text{ g/cm}^3 \), and \(^{[61]}\) \( Y_{e}^{\text{man}} = 0.49 \), \( Y_e^c = 0.467 \).

The corresponding mean electron number densities in the mantle and in the core read: \( \bar{N}_{e}^{\text{man}} = \bar{\rho}_{\text{man}} Y_{e}^{\text{man}} / m_N \cong 2.2 N_A \text{cm}^{-3} \), \( \bar{N}_e^c = \bar{\rho}_c Y_e^c / m_N \cong 5.4 N_A \text{cm}^{-3} \), \( m_N \) and \( N_A \) being the nucleon mass and Avogadro number.

Numerical calculations show \(^{[29, 34]}\) that, e.g., the \( \nu_e \rightarrow \nu_\mu \) oscillation probability of interest, calculated within the two-layer model of the Earth with \( \bar{\rho}_{\text{man}} \) (or \( \bar{N}_{e}^{\text{man}} \)) and \( \bar{\rho}_c \) (or \( \bar{N}_e^c \)) for a given neutrino trajectory determined using the PREM (or the Stacey) model, reproduces with a remarkably high precision the corresponding probability, calculated by solving numerically the relevant system of evolution equations with the much more sophisticated Earth density profile of the PREM (or Stacey) model.

In the two-layer model, the oscillations of atmospheric neutrinos crossing only the Earth mantle (but not the Earth core), correspond to oscillations in matter with constant density. The relevant expressions for \( P_{2\nu}, \kappa \) and \( A_{2\nu} (\nu_\tau \rightarrow \nu_\tau) \) are given by (see, e.g., ref. \(^{[46]}\)):

\[
P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n) = \sin^2 \left( \frac{\Delta M^2 L}{4E} \right) \sin^2 2\theta_m', \tag{14}
\]

\[
\kappa \approx \frac{1}{2} \left[ \frac{\Delta m_{31}^2}{2E} L + \sqrt{2} G_F \bar{N}_e^\text{man} L \right], \tag{15}
\]

\[
A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) = 1 + \left( e^{-\frac{i \Delta m_{31}^2}{2E} L} - 1 \right) \cos^2 \theta_m'. \tag{16}
\]

\(^3\)Let us note that because of the approximate spherical symmetry of the Earth, a given neutrino trajectory through the Earth is completely specified by its Nadir angle.
Here
\[
\Delta M^2 = \Delta m_{31}^2 \sqrt{(1 - \frac{\bar{\rho}_{\text{man}}}{\rho_{\text{res}}})^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13}},
\]  
(17)
is the mass difference between the two mass-eigenstate neutrinos in the mantle, \(\theta'_m\) is the mixing angle in the mantle,

\[
\sin^2 2\theta'_m = \frac{\sin^2 2\theta_{13}}{(1 - \frac{\bar{\rho}_{\text{man}}}{\rho_{\text{res}}})^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13}},
\]  
(18)

\(L\) is the distance the neutrino travels in the mantle, \(\bar{\rho}_{\text{man}}\) and \(\rho_{\text{res}}\) are the mean density along the neutrino trajectory and the resonance density in the mantle,

\[
\rho_{\text{res}} = \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2E\sqrt{2G_F Y_e} m_N}.
\]  
(19)

For a neutrino trajectory which is specified by a given Nadir angle \(\theta_n\) we have:

\[
L = 2R_{\oplus} \cos \theta_n
\]  
(20)

where \(R_{\oplus} = 6371\) km is the Earth radius (in the PREM \[60\] and Stacey \[59\] models) 4.

Consider for definiteness the case of \(\Delta m_{31}^2 > 0\). It follows from eqs. (11) and (12) that the oscillation effects of interest will be maximal if \(P_{2\nu} \approx 1\). The latter is possible provided i) the well-known resonance condition \[38, 39\], leading to \(\sin^2 2\theta_{m} \sim 1\), is fulfilled, and ii) \(\cos (\frac{\Delta M^2 L}{2E}) \sim -1\). Given the values of \(\bar{\rho}_{\text{man}}\) and \(Y_e^{\text{man}}\), or the value of \(N_e^{\text{man}}\), the first condition determines the neutrino energy at which \(P_{2\nu}\) can be enhanced:

\[
E_{\text{res}} \approx 6.6 \left(\frac{\Delta m_{31}^2}{10^{-3} \text{ eV}^2}\right) \left(\frac{N_A \text{cm}^{-3}}{N_e^{\text{man}}}\right) \cos 2\theta_{13} \text{ GeV}.
\]  
(21)

If the first condition is satisfied, the second determines the length of the path of the neutrinos in the mantle for which one can have \(P_{2\nu} \approx 1\):

\[
\left(\frac{\Delta M^2 L}{2E}\right)_{\text{res}} \approx 1.2\pi \tan 2\theta_{13} \left(\frac{N_e^{\text{man}}}{N_A \text{cm}^{-3}}\right) \left(\frac{L}{10^4 \text{ km}}\right) = \pi,
\]  
(22)

Taking \(\Delta m_{31}^2 \approx (2.0 - 3.0) \times 10^{-3} \text{ eV}^2\), \(N_e^{\text{man}} \approx 2 N_A \text{cm}^{-3}\) and \(\cos 2\theta_{13} \approx 1\) one finds from eq. (22): \(E_{\text{res}} \approx (6.6 - 10.0) \text{ GeV}\). The width of the resonance in \(E, 2\delta E\), is determined, as is well-known, by \(\tan 2\theta_{13}\): \(\delta E/E_{\text{res}} \sim \tan 2\theta_{13}\). For \(\sin^2 2\theta_{13} \sim (0.01 - 0.05)\), the resonance is relatively wide in the neutrino energy: \(\delta E/E_{\text{res}} \approx (0.27 - 0.40)\). Equation (22) implies that for \(\sin^2 2\theta_{13} = 0.05 (0.025)\) and \(N_e^{\text{man}} \approx 2.2 N_A \text{cm}^{-3}\), one can have \(P_{2\nu} \approx 1\) only if \(L \approx 8000 (10000)\) km.

It follows from the above simple analysis \[33\] that the Earth matter effects can amplify \(P_{2\nu}\) significantly when the neutrinos cross only the mantle i) for \(E \sim (6 - 11) \text{ GeV}\), i.e., in the multi-GeV range of neutrino energies, and ii) only for sufficiently long neutrino paths in

4Neutrinos cross only the Earth mantle on the way to the detector if \(\theta_n \gtrsim 33.17^\circ\).
In the case of atmospheric neutrinos crossing the Earth core, new resonant effects become apparent. For \( \sin^2 \theta_{13} < 0.05 \) and \( \Delta m^2_{31} > 0 \), we can have \( P_{2\nu} \equiv 1 \) only due to the effect of maximal constructive interference between the amplitudes of the \( \nu_e \rightarrow \nu_\tau \) transitions in the Earth mantle and in the Earth core [29]. The effect differs from the MSW one [29] and the enhancement happens in the case of interest at a value of the energy between the resonance energies corresponding to the density in the mantle and that of the core. The \textit{mantle-core enhancement effect} is caused by the existence (for a given neutrino trajectory through the Earth core) of \textit{points of resonance-like total neutrino conversion}, \( P_{2\nu} = 1 \), in the corresponding space of neutrino oscillation parameters [49, 50]. The points where \( P_{2\nu} = 1 \) are determined by the conditions [49, 50]:

\[
\tan \phi' \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos (2\theta''_m - 4\theta''_m)}}, \quad \tan \phi'' = \pm \frac{\cos 2\theta''_m}{\sqrt{-\cos 2\theta''_m \cos (2\theta''_m - 4\theta''_m)}},
\]

where the signs are correlated and \( \cos 2\theta''_m \cos (2\theta''_m - 4\theta''_m) \leq 0 \). In eq. (23) \( 2\phi' \) and \( 2\phi'' \) are the oscillation phases (phase differences) accumulated by the (two) neutrino states after crossing respectively the first mantle layer and the core, and \( \theta''_m \) is the neutrino mixing angle in the core. A rather complete set of values of \( \Delta m^2_{31}/E \) and \( \sin^2 2\theta_{13} \) for which both conditions in eq. (23) hold and \( P_{2\nu} = 1 \) for the Earth core-crossing atmospheric \( \nu_\mu \) and \( \nu_e \) having trajectories with Nadir angle \( \theta_n = 0; \) 13°; 23°; 30° was found in ref. [50]. The location of these points determines the regions where \( P_{2\nu} \) is large, \( P_{2\nu} \gtrsim 0.5 \). These regions vary slowly with the Nadir angle, they are remarkably wide in the Nadir angle and are rather wide in the neutrino energy [50], so that the transitions of interest produce noticeable effects: we have \( \delta E/E \approx 0.3 \) for the values of \( \sin^2 \theta_{13} \) of interest [50, 50].

For \( \sin^2 \theta_{13} < 0.05 \), there are two sets of values of \( \Delta m^2_{31} \) and \( \sin^2 \theta_{13} \) for which eq. (23) is fulfilled and \( P_{2\nu} = 1 \). These two solutions of eq. (23) occur for, e.g., \( \theta_n = 0; \) 13°; 23°, at 1) \( \sin^2 2\theta_{13} = 0.034; \) 0.039; 0.051, \( \Delta m^2_{31}/E = 7.2; \) 7.0; \( 6.5 \times 10^{-7} \text{ eV}^2/\text{MeV} \), and at 2) \( \sin^2 2\theta_{13} = 0.15; \) 0.17; 0.22, \( \Delta m^2_{31}/E = 4.8; \) 4.5; \( 3.8 \times 10^{-7} \text{ eV}^2/\text{MeV} \) (see Table 2 in ref. [30]). The first solution corresponds to \( \cos 2\phi' \approx -1 \), \( \cos 2\phi'' \approx -1 \) and \( \sin^2 (2\theta''_m - 4\theta''_m) = 1 \). For \( \Delta m^2_{31} = 2.0 \) (3.0) \( \times 10^{-7} \text{ eV}^2 \), the total neutrino conversion occurs in the case of the first solution at \( E \approx (2.8 - 3.1) \text{ GeV} \) (\( E \approx (4.2 - 4.7) \text{ GeV} \)). The values of \( \sin^2 2\theta_{13} \) at which the second solution takes place are marginally allowed. If, e.g., \( \Delta m^2_{31} = 2.5 \times 10^{-3} \text{ eV}^2 \), one has \( P_{2\nu} = 1 \) for this solution for a given \( \theta_n \) in the interval \( 0 \leq \theta_n \leq 23° \) at \( E \) lying in the interval \( E \approx (5.3 - 6.7) \text{ GeV} \).

The effects of the mantle-core enhancement of \( P_{2\nu} \) (or \( \tilde{P}_{2\nu} \)) increase rapidly with \( \sin^2 2\theta_{13} \) as long as \( \sin^2 2\theta_{13} \lesssim 0.06 \), and should exhibit a rather weak dependence on \( \sin^2 2\theta_{13} \) for \( 0.06 \lesssim \sin^2 2\theta_{13} < 0.19 \). If 3-neutrino oscillations of atmospheric neutrinos take place, the magnitude of the matter effects in the multi-GeV \( \mu \)–like and \( e \)–like event samples, produced by neutrinos crossing the Earth core, should be larger than in the event samples due to neutrinos crossing only the Earth mantle (but not the core). This is a consequence of the fact that in the energy range of interest the atmospheric neutrino fluxes decrease rather

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5 The term \textit{“neutrino oscillation length resonance” (NOLR)} was used in ref. [29] to denote the mantle-core enhancement effect in this case.
rapidly with energy - approximately as $E^{-2.7}$, while the neutrino interaction cross section rises only linearly with $E$, and that the maximum of $P_{2\nu}$ (or $\bar{P}_{2\nu}$) due to the resonance-like mantle-core interference effect takes place at approximately two times smaller energies than that due to the MSW effect for neutrinos crossing only the Earth mantle (e.g., at $E \approx (3.5 - 3.9)$ GeV and $E \approx 8.3$ GeV, respectively, for $\Delta m_{31}^2 = 2.5 \times 10^{-3}$ eV$^2$).

The same results, eqs. (21) and (22), and conclusions are valid for the antineutrino oscillation probability $\bar{P}_{2\nu}$ in the case of $\Delta m_{31}^2 < 0$. As a consequence, a preferable detector for distinguishing the type of mass hierarchy would be the one with muon charge discrimination, such that neutrino interactions can be distinguished from those due to antineutrinos.

### 3 Results

It follows from the preceding analysis that in the case of detectors with muon charge identification, as observable which is most sensitive to the Earth matter effects, and thus to the value of $\sin^2 \theta_{13}$ and the sign of $\Delta m_{31}^2$, as well as to $\sin^2 \theta_{23}$, we can consider the Nadir-angle ($\theta_n$) distribution of the ratio $N(\mu^-)/N(\mu^+)$ of the multi-GeV $\mu^-$ and $\mu^+$ event rates, or equivalently the Nadir-angle distribution of the $\mu^−-\mu^+$ event rate asymmetry

$$A_{\mu^−-\mu^+} = \frac{N(\mu^-) - N(\mu^+)}{N(\mu^-) + N(\mu^+)}.$$  \hspace{1cm} (24)

We have obtained predictions for the $\cos \theta_n$ distribution of the ratio $N(\mu^-)/N(\mu^+)$ and the asymmetry $A_{\mu^−-\mu^+}$ in the case of 3-neutrino oscillations of the atmospheric $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$ and $\bar{\nu}_e$, both for neutrino mass spectra with normal ($\Delta m_{31}^2 > 0$) and inverted ($\Delta m_{31}^2 < 0$) hierarchy, and for $\sin^2 \theta_{23} = 0.64$; 0.50; 0.36, and $\sin^2 2\theta_{13} = 0.05$; 0.10. These are compared with the predicted Nadir-angle distributions of the same ratio and asymmetry in the case of 2-neutrino ($\sin^2 \theta_{13} = 0$) vacuum $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations of the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$, $A_{2\nu}^{\mu^−-\mu^+}$.

In the calculations we have used the predictions for the Nadir angle and energy distributions of the atmospheric neutrino fluxes given in ref. [58]. The interactions of the atmospheric neutrinos are described by taking into account only the $\nu_\mu$ and $\bar{\nu}_\mu$ deep inelastic scattering (DIS) cross sections. The latter are calculated using the GRV94 parton distributions given in ref. [62]. We present here results for the asymmetry $A_{\mu^−-\mu^+}$ \hspace{1cm} (21). They are shown graphically in Figs. 1 - 4. The figures correspond to three different intervals of integration over the energies of the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$, and of the $\mu^−$ and $\mu^+$ they produce in the detector, $E = [2, 10]$, [2, 20], [5, 20] GeV, and thus to three different possible event samples. Figures 1 - 4 show the the asymmetries $A_{\mu^−-\mu^+}$ and $A_{2\nu}^{\mu^−-\mu^+}$ as functions of $\cos \theta_n$ for two “reference” values of $|\Delta m_{31}^2|$, $\Delta m_{31}^2 = \pm 2 \times 10^{-3}$ eV$^2$ and $\Delta m_{31}^2 = \pm 3 \times 10^{-3}$ eV$^2$, while in

\hspace{1cm} (21) It is interesting to note that the ratio $N(\mu^-)/N(\mu^+)$ exhibits essentially the same dependence on $\cos \theta_n$ as the asymmetry $A_{\mu^−-\mu^+}$. This is a consequence of the special form of the dependence of $A_{\mu^−-\mu^+}$ on $N(\mu^-)/N(\mu^+)$ and of the fact that typically one finds $N(\mu^-)/N(\mu^+) \sim (1.5 - 2.4)$ for the ranges of the values of the parameters of interest. Correspondingly, the following approximate relation holds (within \hspace{1cm} (22) $\sim 20\%$ and typically with much higher precision) for the range of values of the parameters of interest: $N(\mu^-)/N(\mu^+) \approx 6 A_{\mu^−-\mu^+}$.

\hspace{1cm} (23) The iron-magnetized calorimeter detectors allow to reconstruct with a certain precision the initial neutrino energy as well, see refs. [11] [17] [18].
Figs. 5-9 we present results for the asymmetries in the rates of the multi-GeV $\mu^-$ and $\mu^+$ events, integrated over $\cos \theta_\nu$ in the intervals $[0.30,0.84]$ (mantle bin) and $[0.84,1.0]$ (core bin), $\bar{A}_{\mu^-\mu^+}$ and $\bar{A}_{\mu^-\mu^+}^{2\nu}$. The dependence of the latter on $\sin^2 2\theta_{13}$ for $\Delta m_{31}^2 = \pm 2 \times 10^{-3}$ eV$^2$, and on $\Delta m_{31}^2$ for $\sin^2 2\theta_{13} = 0.10$, is shown for three values of $\sin^2 \theta_{23} = 0.36; 0.50; 0.64$.

As Figs. 1-9 indicate, the Earth matter effects can produce a noticeable deviations of $A_{\mu^-\mu^+}$ from the 2-neutrino vacuum oscillation asymmetry $A_{\mu^-\mu^+}^{2\nu}$ at $\cos \theta_\nu \gtrsim 0.3$. As a quantitative measure of the magnitude of the matter effects one can use the deviation of the asymmetry $A_{\mu^-\mu^+}$ in the case of 3-neutrino oscillations, $\sin^2 2\theta_{13} \neq 0$, $\sin^2 2\theta_{13} \gtrsim 0.04$, from the asymmetry, $A_{\mu^-\mu^+}^{2\nu}$, predicted in the case of 2-neutrino oscillations, i.e., for $\sin^2 2\theta_{13} = 0$, or the relative difference between the two asymmetries,

$$\Delta = \frac{A_{\mu^-\mu^+} - A_{\mu^-\mu^+}^{2\nu}}{A_{\mu^-\mu^+}^{2\nu}} \ .$$  (25)

The magnitude of the matter effects, or the relative difference $\Delta$, depends critically on the value of $\sin^2 \theta_{23}$: $|\Delta|$ increases rapidly with the increasing of $\sin^2 \theta_{23}$. This is clearly seen in Figs. 1-9. The matter effects in $A_{\mu^-\mu^+}$ are hardly observable for $\sin^2 \theta_{23} \lesssim 0.30$. For $\cos \theta_\nu \leq 0.84$, i.e., in the mantle bin, the asymmetry difference $|\Delta|$ increases practically linearly with $\sin^2 2\theta_{13}$. In the case of $0.84 \leq \cos \theta_\nu \leq 1.0$, i.e., in the core bin, $|\Delta|$ increases rapidly with $\sin^2 2\theta_{13}$ until the latter reaches the value of $\sin^2 2\theta_{13} \approx 0.06$. For values of $\sin^2 2\theta_{13} \approx (0.06 - 0.15)$, $\Delta$ is essentially independent of $\sin^2 2\theta_{13}$ and is given by its value at $\sin^2 2\theta_{13} \approx 0.06$ (Figs. 5-9). The magnitude of the asymmetry difference $\Delta$ depends weakly on $\Delta m_{31}^2$ taking values in the interval $\sim (2 - 3) \times 10^{-3}$ eV$^2$, as long as the energy integration interval is sufficiently wide to include the energy regions where the Earth matter effects enhance strongly the subdominant transition probabilities. If this is not the case, a noticeable dependence on $\Delta m_{31}^2$ can be present. This is illustrated e.g., in Fig. 5 which corresponds to $E = [2, 10]$ GeV. The asymmetry difference in the mantle bin diminishes monotonically as $|\Delta m_{31}^2|$ increases starting from the value of $\sim 1.3 \times 10^{-3}$ eV$^2$ and becomes rather small at $|\Delta m_{31}^2| \gtrsim 3 \times 10^{-3}$ eV$^2$. This behavior can be easily understood: for $|\Delta m_{31}^2| \approx 2 \times 10^{-3}$ eV$^2$, the region of enhancement of the subdominant neutrino oscillations lies in the region of energy integration, while for $|\Delta m_{31}^2| > 3 \times 10^{-3}$ eV$^2$ the enhancement region is practically outside the region of integration over the neutrino energy.

For the ranges considered of the three oscillation parameters, $\sin^2 \theta_{23}$, $\sin^2 2\theta_{13}$ and $|\Delta m_{31}^2|$, the magnitude of the asymmetry difference $|\Delta|$ depends weakly on the maximal neutrino (and muon) energy, $E_{\text{max}}$, for the chosen event sample as long as $E_{\text{max}} \gtrsim 10$ GeV. By increasing the minimal energy of the neutrinos contributing to the event sample, $E_{\text{min}}$, from 2 GeV to, e.g., 5 GeV, one could diminish the asymmetry in the core bin substantially (Fig. 1). In that case, a large fraction of the region of enhancement is not included within the interval of integration.

For $\sin^2 \theta_{23} \gtrsim 0.50$, $\sin^2 2\theta_{13} \gtrsim 0.06$ and $|\Delta m_{31}^2| = (2 - 3) \times 10^{-3}$ eV$^2$, the Earth matter effects produce an integrated asymmetry difference $|\Delta|$ which is bigger than approximately $\sim 15\%$, can reach the values of $(30 - 35)\%$ (Figs. 4-9), and thus can be sufficiently large to be observable. As Figs. 5-9 clearly show, the sign of the relative difference of the integrated asymmetries is anticorrelated with the sign of $\Delta m_{31}^2$: for $\Delta m_{31}^2 > 0$ we have $A_{\mu^-\mu^+} < A_{\mu^-\mu^+}^{2\nu}$,
while for $\Delta m^2_{31} < 0$, the inequality $A_{\mu^- \mu^+} > A^{2\nu}_{\mu^- \mu^+}$ holds. Therefore the measurement of $A_{\mu^- \mu^+}$ can provide a direct information on the sign of $\Delta m^2_{31}$, i.e., on the neutrino mass hierarchy.

It follows from Figs. 1-4 that the deviations of the asymmetry $A_{\mu^- \mu^+}$ from the 2-neutrino oscillation one, $A^{2\nu}_{\mu^- \mu^+}$, are maximal typically i) in the “core bin”, $\cos \theta_n = [0.84 - 1.0]$, and ii) at $\cos \theta_n \sim 0.50$ in the “mantle bin”, $\cos \theta_n \leq 0.84$. For $\Delta m^2_{31} = 2 \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{23} = 0.50$ (0.64) and $E = [2, 10]$ GeV (Fig. 4), we have at $\cos \theta_n \sim 0.50$ and $\sin^2 2\theta_{13} = 0.05$: $A^{2\nu}_{\mu^- \mu^+} \approx 0.29$, while the matter effects lead to $A_{\mu^- \mu^+} \approx 0.25 (0.24)$. This corresponds to a negative relative difference between $A(\mu^- \mu^+) \text{ and } A^{2\nu}(\mu^- \mu^+)$, $\Delta < 0$, and $|\Delta| \approx 14\% (17\%)$. For $\sin^2 2\theta_{13} = 0.10$, one finds $\Delta \sim -28\% (34\%)$. The relative difference $\Delta$ in the core bin is also negative and $|\Delta|$ has similar or larger values. For $E$ in the interval $E = [5, 20]$ GeV (Fig. 4), we get at $\cos \theta_n \sim 0.50$ for $\Delta m^2_{31} = 3 \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{13} = 0.10$ and $\sin^2 2\theta_{23} = 0.64$: $A^{2\nu}_{\mu^- \mu^+} \approx 0.25$, $A_{\mu^- \mu^+} \approx 0.075$, and a relative difference between the two asymmetries $\Delta \approx -70\%$.

Our results show that the Earth matter effects in the Nadir-angle distribution of the ratio $N(\mu^-)/N(\mu^+)$ of the rates of multi-GeV $\mu^-$ and $\mu^+$ events, or equivalently in the Nadir-angle distribution of the $\mu^- - \mu^+$ event rate asymmetry $A_{\mu^- \mu^+}$, eq. (24), can be sufficiently large to be observable in the current and planned experiments with iron magnetized calorimeter detectors which have muon charge identification capabilities (MINOS, INO, etc.).

## 4 Conclusions

We have studied the possibilities to obtain information on the values of $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$, and on the sign of $\Delta m^2_{31}$ using the data on atmospheric neutrinos, which can be obtained in experiments with detectors able to measure the charge of the muon produced in the charged current (CC) reaction by atmospheric $\nu_\mu$ or $\bar{\nu}_\mu$ (MINOS, INO, etc). The indicated oscillation parameters control the magnitude of the Earth matter effects in the subdominant oscillations, $\nu_\mu \rightarrow \nu_e \ (\nu_e \rightarrow \nu_\mu)$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e \ (\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$, of the multi-GeV ($E \sim (2 - 10)$ GeV) atmospheric neutrinos. As observable which is most sensitive to the Earth matter effects, and thus to the value of $\sin^2 \theta_{13}$ and and the sign of $\Delta m^2_{31}$, as well as to $\sin^2 \theta_{23}$, we have considered the Nadir-angle ($\theta_n$) distribution of the ratio $N(\mu^-)/N(\mu^+)$ of the multi-GeV $\mu^-$ and $\mu^+$ event rates, and the corresponding $\mu^- - \mu^+$ event rate asymmetry $A_{\mu^- \mu^+}$, eq. (24). The systematic uncertainty, in particular, in the Nadir angle dependence of $N(\mu^-)/N(\mu^+)$ and of the asymmetry $A_{\mu^- \mu^+}$, can be smaller than those on the measured Nadir angle distributions of the rates of $\mu^-$ and $\mu^+$ events, $N(\mu^-)$ and $N(\mu^+)$. We have obtained predictions for the $\cos \theta_n$ distribution of the asymmetry $A_{\mu^- \mu^+}$ (and of the ratio $N(\mu^-)/N(\mu^+)$) in the case of 3-neutrino oscillations of the atmospheric $\nu_\mu$, $\nu_\tau$, $\nu_e$ and $\bar{\nu}_e$, both for neutrino mass spectra with normal ($\Delta m^2_{31} > 0$) and inverted ($\Delta m^2_{31} < 0$) hierarchy, and for $\sin^2 \theta_{23} = 0.64$; 0.50; 0.36, and $\sin^2 2\theta_{13} = 0.05; 0.10$. These are compared with the predicted Nadir-angle distribution of the same ratio and asymmetry in the case of 2-neutrino ($\sin^2 \theta_{13} = 0$) vacuum $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations of the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$, $A^{2\nu}_{\mu^- \mu^+}$.

\footnote{Note that $A^{2\nu}_{\mu^- \mu^+} > 0$. This is a consequence of the fact that the $\nu_\mu$ DIS cross section is approximately by a factor 2 bigger than the $\bar{\nu}_\mu$ DIS cross section and that the fluxes of atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$, ($\nu_e$ and $\bar{\nu}_e$) do not differ considerably.}
Our results are summarized in Figs. 1 - 9. Figures 1 - 4 show the dependence of the asymmetries \(A_{\mu^-\mu^+}\) and \(A_{\mu^-\mu^+}^{2\nu}\) on \(\cos\theta_n\) for two “reference” values of \(|\Delta m^2_{31}|\), \(\Delta m^2_{31} = \pm 2 \times 10^{-3}\) eV\(^2\) and \(\Delta m^2_{31} = \pm 3 \times 10^{-3}\) eV\(^2\), and for three possible ranges of energies of the atmospheric neutrinos, contributing to the event rates of interest, \(E = [2,10], [2,20], [5,20]\) GeV. In Figs. 5 - 9 we present results for the asymmetries in the rates of the multi-GeV \(\mu^-\) and \(\mu^+\) events, integrated over \(\cos\theta_n\) in the intervals \([0.30,0.84]\) (mantle bin) and \([0.84,10]\) (core bin). We find that for \(\sin^2\theta_{23} \gtrsim 0.50, \sin^22\theta_{13} \gtrsim 0.06\) and \(|\Delta m^2_{31}| = (2-3) \times 10^{-3}\) eV\(^2\), the Earth matter effects produce a relative difference between the integrated asymmetries \(\bar{A}_{\mu^-\mu^+}\) and \(\bar{A}_{\mu^-\mu^+}^{2\nu}\) which is bigger in absolute value than approximately \(\sim 15\%\), can reach the values of \((30-35)\%\) (Figs. 5 - 9), and thus can be sufficiently large to be observable. As our results show (Figs. 5 - 9), the sign of the indicated asymmetry difference, \((\bar{A}_{\mu^-\mu^+} - \bar{A}_{\mu^-\mu^+}^{2\nu})\), is directly related to the sign of \(\Delta m^2_{31}\): for \(\Delta m^2_{31} > 0\) we have \((\bar{A}_{\mu^-\mu^+} - \bar{A}_{\mu^-\mu^+}^{2\nu}) < 0\), while if \(\Delta m^2_{31} < 0\) then \((\bar{A}_{\mu^-\mu^+} - \bar{A}_{\mu^-\mu^+}^{2\nu}) > 0\). Therefore the measurement of the Nadir angle dependence of \(A_{\mu^-\mu^+}\), or of the value of \(\bar{A}_{\mu^-\mu^+}\) in the mantle and/or in the core bins, can provide a direct information on the sign of \(\Delta m^2_{31}\), i.e., on the neutrino mass hierarchy.

To summarize, the studies of the oscillations of the multi-GeV atmospheric \(\nu_{\mu}\) and \(\bar{\nu}_{\mu}\) in experiments with detectors having good muon charge identification capabilities (MINOS, INO, etc.), can provide fundamental information on the values of \(\sin^2\theta_{13}\) and \(\sin^2\theta_{23}\), and on the sign of \(\Delta m^2_{31}\), i.e., on the neutrino mass hierarchy.

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Figure 1: The Nadir angle distribution of the charge asymmetry, $A_{\mu^-\mu^+}$, eq. (24), of the multi-GeV $\mu^-$ and $\mu^+$ event rates, integrated over the neutrino (and muon) energy in the interval $E = (2.0 - 10.0)$ GeV, in the cases i) of 2-neutrino $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations in vacuum of the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ and no $\nu_e$ and $\bar{\nu}_e$ oscillations, $A_{\mu^-\mu^+}^{2\nu}$ (solid lines), ii) 3-neutrino oscillations of $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$ and $\bar{\nu}_e$ in the Earth and neutrino mass spectrum with normal hierarchy ($A_{\mu^-\mu^+}^{3\nu}$)$_{NH}$ (dashed lines), or with inverted hierarchy, ($A_{\mu^-\mu^+}^{3\nu}$)$_{IH}$ (dotted lines). The results shown are for $|\Delta m^2_{31}| = 2 \times 10^{-3}$ eV$^2$, $\sin^2\theta_{23} = 0.36$ (upper panels); 0.50 (middle panels); 0.64 (lower panels), and $\sin^2 2\theta_{13} = 0.05$ (left panels); 0.10 (right panels).
Figure 2: The same as in Fig. 1 but for $\mu^−$ and $\mu^+$ event rates integrated over the neutrino (and muon) energy in the interval $E = (2.0 – 20.0)$ GeV.
Figure 3: The same as in Fig. 1 but for $\mu^-$ and $\mu^+$ event rates integrated over the neutrino (and muon) energy in the interval $E = (2.0 - 20.0)$ GeV and $|\Delta m^2_{31}| = 3 \times 10^{-3}$ eV$^2$. 
Figure 4: The same as in Fig. 1 but for $\mu^-$ and $\mu^+$ event rates integrated over the neutrino (and muon) energy in the interval $E = (5.0 - 20.0)$ GeV and for $|\Delta m^2_{31}| = 3 \times 10^{-3}$ eV$^2$. 
Figure 5: The charge asymmetry $A_{\mu^-\mu^+}$ of the multi-GeV $\mu^-$ and $\mu^+$ event rates, integrated over the neutrino (and muon) energy in the interval $E = (2.0 - 10.0)$ GeV and over the Nadir angle in the interval corresponding to $0.30 \leq \cos \theta_n \leq 0.84$ (mantle bin), as function i) of $\sin^2 2\theta_{13}$ for $|\Delta m^2_{31}| = 2 \times 10^{-3}$ eV$^2$ (left panels), and ii) of $|\Delta m^2_{31}|$ for $\sin^2 2\theta_{13} = 0.10$ (right panels): $A_{\mu^-\mu^+}^{2\nu}$ (solid lines), $A_{\mu^-\mu^+}^{3\nu}$ (dashed lines) and $A_{\mu^-\mu^+}^{3\nu}$ (dotted lines). The results shown are obtained for $\sin^2 \theta_{23} = 0.36$ (upper panels); 0.50 (middle panels); 0.64 (lower panels).
Figure 6: The same as in Fig. 5 but for $\mu^{-}$ and $\mu^{+}$ event rates integrated over the Nadir angle in the interval corresponding to $0.84 \leq \cos \theta_n \leq 1.00$ (core bin).
Figure 7: The same as in Fig. 5 but for $|\Delta m^2_{31}| = 3 \times 10^{-3} \text{ eV}^2$ and $\mu^-$ and $\mu^+$ event rates, integrated over the neutrino (and muon) energy in the interval $E = (2.0 - 20.0) \text{ GeV}$. 
Figure 8: The same as in Fig. 5 but for $|\Delta m^2_{31}| = 3 \times 10^{-3}$ eV$^2$ and $\mu^-$ and $\mu^+$ event rates, integrated over the neutrino (and muon) energy in the interval $E = (2.0 - 20.0)$ GeV and over the Nadir angle in the interval corresponding to $0.84 \leq \cos \theta_n \leq 1.00$ (core bin).
Figure 9: The same as in Fig. 5, but for $\mu^-$ and $\mu^+$ event rates integrated over the neutrino (and muon) energy in the interval $E = (5.0 - 20.0)$ GeV, and for $|\Delta m_{31}^2| = 3 \times 10^{-3}$ eV$^2$. 