Secrecy Rate of Resource-Constrained Mobile Relay Model under Two-Way Wiretap Channel

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Abstract: Relay communication is emerging as a promising solution to improving the reliability of long-distance communication systems. However, transmitting data in a secure way is challenging due to the possibility of eavesdroppers wiretapping such systems. To address the challenge, this paper proposes a joint secure transmission and graph mobility model. With the proposed model, the secrecy rate of the resource-constrained two-way wiretap channel mobile relay system is formulated as a mixed integer nonlinear programming (MINLP) problem. Furthermore, efficient algorithms that achieve a local optimal solution are derived. Numerical results are provided to validate the performance of the proposed algorithms.

Keywords: secrecy rate; mobile relay; two-way wiretap channel; power control; path planning

1. Introduction

It is well known that secure information transmission is an essential communication requirement. Resource-constrained two-way communication occurs in a four-node network, where two users exchange information through a relay station under the condition of an eavesdropper wiretapping the channels. This four-node relay-eavesdropper network has been widely studied [1–5], and with the help of relay, the network is able to adjust its uplink and downlink channels and thus improves the system security. With the rapid development of Internet of Things (IoT), huge numbers of long-distance sensors need to interact with each other to jointly complete tasks [6]. Resources such as time and energy in this type of cooperative sensor network are important factors that need to be carefully considered. Therefore, the resource-constrained relay-eavesdropper network is becoming a novel interesting research topic and is attracting considerable attention.

Currently, most existing works focus on two-way communication systems with fixed relays [7–10]. In particular, relay cooperative jamming has been studied extensively [7–10] to protect confidential message from wiretapping. This type of method, although effectively improving the secrecy rate, may lead to harmful interference and additional energy consumption [7,8]. Thus, to eliminate interference, interference cancellation techniques have to be taken into account [10]. On the other hand, in order to improve energy utilization efficiency, energy harvesting [7] or wireless energy transfer [8] is adopted to provide the relay and users with sufficient energy. Nowadays, there is a growing interest in deploying unmanned aerial vehicles (UAVs) in two-way relay systems [11–13]. UAV schemes...
indeed provide flexible mobile relays to implement air-to-ground or air-to-air wireless communication. However, for ground secure transmission, especially indoor, complicated pitches (e.g., warehouses), this beyond the reach of UAV-based relay system. Moreover, when an UAV serves as a flying relay, it requires large propulsion energy to keep itself aloft, which leads to a short life flight. This feature prevents the UAV from participating in many applications. Therefore, two-way secure relay system needs further investigation, and this paper presents an investigation of the secrecy rate region of a ground mobile vehicle-based relay system, which remains an open problem.

To quantify the performance gain brought by our scheme, this paper proposes a two-way secure transmission and a graph mobility model, as well as formulating a secrecy rate maximization optimization problem with resource allocations incorporating path planning constraints, which represents a mixed integer nonlinear programming (MINLP) problem. The major challenges in solving this discrete problem are twofold. First, the path planning refers to a considerable number of discrete binary variables, and the solution space grows exponentially with the number of anchor points. This characteristic results in a substantial difference from the design of UAVs’ trajectories, which only involve continues variables. Second, when the relay moves along an unknown path, finding the achievable secrecy rate becomes very challenging. This is because two major reasons. One is that the transmission time at different location point needs to be determined. The other is that the powers at relay and users at each location need to be jointly optimized. As a result, in contrast to the fixed relay case, the dimension of variables significantly increase, resulting in additional nonlinear coupling between time allocation and uplink–downlink power allocation. In order to overcome the above challenges, auxiliary and relaxation methods are adopted to transform the non-convex problem into a convex problem, and a local optimal solution is derived. Finally, numerical results show that our mobile relay scheme outperforms the fixed relay scheme, and the relay trajectory is highly dependent on noise power.

Notation: Italic letters, simple bold letters, and capital bold letters represent scalars, vectors, and matrices, respectively. \((\cdot)^T, (\cdot)^{-1}\text{vec}(\cdot)\) denote the transpose, inverse and, vectorization operators of a matrix, respectively. \(E(\cdot)\) takes the expectation of a random variable, and \(|\cdot|\) is the modulus of the complex-valued number. The notation \([a]^+\) is used as \(\max(a,0)\); symbol \(I_N\) represents the \(N \times N\) identity matrix. Curlicue letters denote sets; \((s_1, s_2, \cdots)\) represents a sequence; and \([s_1, s_2, \cdots]^T\) is a column vector.

2. Materials and Methods
2.1. System Model
2.1.1. Mobility Model

The two-way wireless secure transmission mobile relay system shown in Figure 1 was considered, and it consists of two IoT users (Alice and Bob), an eavesdropper (Eve), and a relay. The relay is equipped with \(N\) antennas and mounted on a ground mobile vehicle. Alice, Bob, and Eve are configured with a single antenna. Alice and Bob exchange data through the mobile relay under the condition of Eve eavesdropping the whole process. The scenario is described as a directed graph \((\nu, \varepsilon)\) as shown in Figure 2, where \(\nu\) is the set of \(M\) vertices representing the possible stopping points, and \(\varepsilon\) is the set of directed edges representing the allowed movement paths [14,15]. We let \(D = (D_{m,j}), 1 \leq m \leq j \leq M\) be an asymmetric \(M \times M\) non-negative real distance matrix associated with \(\varepsilon\), and we assume that the following three conditions hold:

1. The mobile relay returns to the starting point.
2. The mobile relay visits all important positions.
3. There is no subtour in the planned path.
Accordingly, we define a selection path variable $s = [s_1, \cdots, s_M]^T$, where $s_m = 1$ if the vertex $m$, $1 \leq m \leq M$, appears in the visiting path and $s_m = 0$ otherwise. In addition, we introduce an $M \times M$ matrix $Z$ with variables $Z_{m,j}$ being set to 1 or 0, $1 \leq m \leq j \leq M$ and interpret the value 1 (0 resp.) to mean “in path” (“not in path” resp). Then, motion distance $\Theta(Z)$ is given by

$$\Theta(Z) = \min \left\{ \sum_{m=1}^{M} \sum_{j=1}^{M} Z_{m,j} D_{m,j} : Z \in \mathcal{A}(s,Z) \right\}$$

(1)
and we define set $\mathcal{A}(s,Z)$ as the visiting path, which can be expressed as [14]:

$$\mathcal{A}(s,Z) = \left\{ (s,Z) : \sum_{j=1}^{M} Z_{m,j} = s_m, \sum_{j=1}^{M} Z_{j,m} = s_m, \forall m = 1, \ldots, M \right\} \quad \text{(2a)}$$

$$\lambda_m - \lambda_j + (\sum_{i=1}^{M} s_l - 1)Z_{m,j} + (\sum_{i=1}^{M} s_l - 3)Z_{j,m} \leq \sum_{l=1}^{M} s_l - 2 + j(2 - s_m - s_j), \forall m, j \geq 2, m \neq j \quad \text{(2b)}$$

$$s_m \leq \lambda_m \leq (\sum_{i=1}^{M} s_l - 1)s_m, \forall m \geq 2 \quad \text{(2c)}$$

$$Z_{m,j} \in \{0,1\}, \forall m, j, Z_{m,m} = 0, \forall m \quad \text{(2d)}$$

$$s_1 = 1, s_m \in \{0,1\}, \forall m \geq 2 \quad \text{(2e)}$$

Constraint (2a) guarantees that vertex $m$ being visited only once at the most, while (2b) and (2c) are sub-tour elimination constraints, which ensure that the visiting path is connected. (2d) represents whether edge $(m,j)$ belongs to the path and also ensures that no self-loop appears in the visiting path. (2e) stands for the first vertex is the starting point. Note that $\lambda_m$ represents slack variables to guarantee a connected trajectory, and $f$ is a constant set to $f = 10^6$.

With the moving time from vertex $m$ to vertex $j$ being $D_{m,j}/a$, where $a$ is the average velocity, the total moving time along a selected visiting path is

$$\frac{\Theta(Z)}{a} = \frac{1}{a} \sum_{m=1}^{M} \sum_{j=1}^{M} Z_{m,j} D_{m,j} \quad \text{(3)}$$

Furthermore, the total motion energy $E_G$, along with the visiting path, is proportional to the total moving time [16]; it can be expressed as

$$E_G = \zeta(a) \sum_{m=1}^{M} \sum_{j=1}^{M} Z_{m,j} D_{m,j} \quad \text{(4)}$$

where $\zeta(a)$ is a simple parameter function of velocity $a$ of the model. It can be described as $\zeta(a) = (\tau_1 + \tau_2/a)$, where $(\tau_1, \tau_2)$ are parameters related to the weight of ground mobile vehicle (e.g., for the considered Pioneer 3DX robot, we have $\tau_1 = 7.4$ and $\tau_2 = 0.29$) [6,16,17].

2.1.2. Uplink Signal Model

For uplink transmission, Alice transmits vector $x_m \in \mathbb{C}^{L_m \times 1}$ to relay with transmit power $q_m = \frac{1}{L_m} \mathbb{E}[\|x_m\|^2]$, where $L_m$ is the number of symbols and $t_m$ is the transmission time assigned to the mobile relay at the stopping point $m$. Then, the received signal $Y_m \in \mathbb{C}^{N \times L_m}$ at relay is given by $Y_m = h_m x_m^T + N_m$, where $h_m \in \mathbb{C}^{N \times 1}$ denotes the uplink channel vector of Alice-to-relay, and $N_m \in \mathbb{C}^{N \times L_m}$ is the Gaussian noise with $\mathbb{E}[\text{vec}(N_m)\text{vec}(N_m)^H] = \sigma_e^2 I_{NL_m}$. Given perfect channel state information (CSI) at the receiver, and applying a receive beamformer $w_m^H$ (with $\|w_m\|^2 = 1, w_m = h_m/\|h_m\|$) to $Y_m$, the uplink signal-to-noise ratio (SNR) of Alice-to-relay at the relay is $q_m \|w_m^H h_m\|^2/\sigma_e^2 = q_m \|h_m\|^2/\sigma_e^2$. On the other hand, as the computational capability of the eavesdropper is not known, we focus on the worst case scenario for facilitating secrecy supply. In particular, we assume that the eavesdropper is equipped with a noise-free receiver and able to remove other interference. In this case, the received signal $y_e^{UP} \in \mathbb{C}^{1 \times L_m}$ at Eve is given by $y_e^{UP} = h_e x_m^T + n_e$, where $h_e \in \mathbb{C}^{1 \times 1}$ denotes the uplink wiretap channel scalar of
Alice-to-Eve, and \( n_e \in \mathbb{C}^{1 \times L_m} \) is the Gaussian noise with \( \mathbb{E}[n_e n_e^H] = \sigma_e^2 I_{L_m} \). Applying a receive beamformer \( w_e \) (with \( w_e = h_e n_e \)) to \( y_e^{UP} \), the uplink wiretap SNR of Alice-to-Eve at Eve is 

\[
q_m |w_e^H h_e|^2 / \sigma_e^2 = q_m |h_e|^2 / \sigma_e^2.
\]

Therefore, the uplink achievable secrecy rate \( R_m^{UP - Sec} \) at the \( m \)th stopping point from Alice to relay is given by

\[
R_m^{UP - Sec} = \left[ \log_2 \left( 1 + \frac{q_m |h_m|^2}{\sigma_f^2} \right) - \log_2 \left( 1 + \frac{q_m |h_e|^2}{\sigma_f^2} \right) \right]^+. \tag{5}
\]

2.1.3. Downlink Signal Model

For downlink transmission, the relay uses \( w^D_m \) to generate a symbol \( b_m \in \mathbb{C}^{L_m \times 1} \) with power \( p_m = \frac{1}{\|b_m\|^2} \). Then, the relay transmits \( b_m \) to Bob with the transmit beamforming vector \( v_m \in \mathbb{C}^{N \times 1} \), which is given by \( v_m = \frac{g_m}{\|g_m\|}, \quad \|v_m\| = 1 \). Therefore, the received signal \( r_m^T \in \mathbb{C}^{1 \times L_m} \) at Bob is 

\[
r_m^T = g_m^H v_m b_m^T + n_m^T \quad \text{with} \quad g_m^H \in \mathbb{C}^{1 \times N} \text{ is the downlink channel vector of relay-to-Bob, and} \quad n_m^T = n_m^T \in \mathbb{C}^{1 \times L_m} \text{ is the Gaussian noise at Bob with} \quad \mathbb{E}[n_m n_m^H] = \sigma_e^2 I_{L_m}. \]

The received signal \( y_e^{DL} \in \mathbb{C}^{N \times L_m} \) at Eve is given by 

\[
y_e^{DL} = g_e m \sigma e m^T + n_e,
\]

where \( g_e m \) is the Gaussian noise at Bob with \( \mathbb{E}[vec(n_e)vec(n_e)] = \sigma_e^2 I_{N \times L_m} \). Applying a receive beamformer \( w'_c \) (with \( \|w'_c\| = 1 \), \( w'_c = \frac{g_{ec}}{\|g_{ec}\|} \)) to \( y_e^{DL} \), the downlink wiretap SNR of relay-to-Eve at Eve is 

\[
q_m |w'_c^H g_{ec}|^2 / \sigma_e^2 = q_m |g_{ec}|^2 / \sigma_e^2.
\]

Therefore, the downlink achievable secrecy rate \( R_m^{DL - Sec} \) at the \( m \)th stopping point from relay to Bob is given by

\[
R_m^{DL - Sec} = \left[ \log_2 \left( 1 + \frac{p_m |g_{ec}|^2}{\sigma_e^2} \right) - \log_2 \left( 1 + \frac{p_m |g_{ec}|^2}{\sigma_e^2} \right) \right]^+. \tag{6}
\]

2.2. Achievable Secrecy Rate Problem Formulation

The achievable secrecy rate region of the resource-constrained two-way mobile relay system is the average of all combinations of the worst case uplink–downlink achievable secrecy rate; therefore, the achievable secrecy rate \( R^{Sec} \) requirement that the system needs to meet is

\[
R^{Sec} \leq \frac{1}{T} \sum_{m=1}^{M} t_m \min \{ R_m^{UP - Sec}, R_m^{DL - Sec} \}, \tag{7}
\]

where \( T \) is the time duration, consisting of all transmission time \( t_m \) and total moving time. Moreover, the transmit power and energy requirements must also satisfy

\[
\sum_{m=1}^{M} q_m t_m \leq Q, \quad q_m \geq 0 \tag{8}
\]

\[
\sum_{m=1}^{M} p_m t_m \leq (E - E_G), \quad p_m \geq 0 \tag{9}
\]

where \( q_m \) and \( Q \) are transmit power and initial available energy at Alice, respectively. In addition, \( p_m, E, \) and \( E_G \) are the transmit power, initial available energy, and motion energy of the relay, respectively. If the relay is fixed, \( E_G = 0 \); otherwise, \( E_G \neq 0 \), which is determined by a selected visiting path. Having the power allocation and graph mobility constraints satisfied, the achievable secrecy rate problem is formulated as P1.
where the constraints mean:

1. (10a) and (10b) are for secure transmission.
2. (10c) ensures that the mobile ground vehicle moving and data transmission are completed in T seconds.
3. (10g) is the constraint that the stopping time and power allocation are zeros provided that the vertex is not visited.

However, problem P1 is a MINLP problem due to the discontinuity of integer constraints of (10d), which is NP hard and nontrivial to solve [18]. Moreover, the secrecy rate depends on the transmit power \( \{p_m, q_m\} \) and transmission time \( \{t_m\} \), which present unknown and nonlinear coupling with each other.

### 2.3. Local Optimal Solution to P1

To solve P1, we first utilize auxiliary and relaxation methods to transform the non-convex constraints of (10a) and (10b) into convex. Then, by fixing \( s \), an optimization problem P2 is derived, which is equivalent to P1. Through further transformation, P1 is converted to P3, which only involves variable \( s \). Lastly, we exploit the neighborhood search method (NSM) [19–21] and develop an iterative algorithm to obtain the local optimal solution.

#### 2.3.1. Convex Approximation via Taylor Expansion

We first consider the constraints of (10a) and (10b) because of the nonlinear coupling between \( \{t_m\} \) and \( \{p_m, q_m\} \). We replace \( \{q_m\} \) with a new variable \( \{Q_m\} \) such that \( Q_m := q_m t_m \). Similarly, \( \{p_m\} \) is replaced with a new variable \( \{P_m\} \) satisfying \( P_m := p_m t_m \). Therefore, the above constraints of (10a), (10b), and (10e)–(10g) of P1 are equal to the following:
Theorem 1. Perspective function has convexity.

Proof of Property 2. Follows a similar proof to [14] (Property 1).

Due to the concavity and monotonically increasing property, we can take derivatives of \( f_2(t_m, Q_m) \) and \( g_2(t_m, P_m) \) and have the following equivalent inequalities of constraints of (10a) and (10b) by applying the first-order Taylor expansion at the given points \( (t_m^0, Q_m^0) \) and \( (t_m^0, P_m^0) \),
\[
\begin{align*}
\text{TR}^\text{Sec} = & \sum_{m=1}^{M} f_1(t_m, Q_m) + \sum_{m=1}^{M} \left[ f_2(t_m^0, Q_m^0) + \frac{\partial f_2(t_m, Q_m^0)}{\partial t_m} \right] |_{t_m=0} (t_m - t_m^0) + \frac{\partial f_2(t_m, Q_m^0)}{\partial Q_m} \left|_{Q_m=Q_m^0} (Q_m - Q_m^0) \right| \leq 0 \\
\text{TR}^\text{Sec} = & \sum_{m=1}^{M} g_1(t_m, P_m) + \sum_{m=1}^{M} \left[ g_2(t_m^0, P_m^0) + \frac{\partial g_2(t_m, P_m^0)}{\partial t_m} \right] |_{t_m=0} (t_m - t_m^0) + \frac{\partial g_2(t_m, P_m^0)}{\partial P_m} \left|_{P_m=0} (P_m - P_m^0) \right| \leq 0
\end{align*}
\]

2.3.2. Optimal Solution of \((Z)\) (namely \(\{Z_{mj}\}\)) and \(\{t_m, P_m, q_m\}\)

Given \(s = \tilde{s}\), where \(\tilde{s}\) is any feasible solution to \(P_1\), the constraint \((2e)\) of \(A(s,Z)\) can be discarded since it only involves \(s\). Combined with Equation (1) and \((10c)\), it is proved in Appendix A that we can obtain the following equation:

\[
\sum_{m=1}^{M} \sum_{j=1}^{M} Z_{mj} D_{mj} = a(T - \sum_{m=1}^{M} t_m)
\]

Putting the above Equation (22) into Equation (13), we can obtain a new equal constraint of \((10e)\),

\[
\sum_{m=1}^{M} P_m + \zeta(a)(T - \sum_{m=1}^{M} t_m) \leq E
\]

Based on \((13)\)–\((15)\), \((20)\)–\((23)\), it is proved in Appendix B that \(P_1\) is equivalently transformed into the two-layer optimization problem shown below:

\[
P_2 : \max_{R_1^\text{Sec}, \{t_m, Q_m, P_m\}} R_1^\text{Sec}
\]

\[
\text{s.t. } \begin{align*}
& \text{TR}^\text{Sec} = \sum_{m=1}^{M} f_1(t_m, Q_m) + \sum_{m=1}^{M} \left[ f_2(t_m^0, Q_m^0) + \frac{\partial f_2(t_m, Q_m^0)}{\partial t_m} \right] |_{t_m=0} (t_m - t_m^0) + \frac{\partial f_2(t_m, Q_m^0)}{\partial Q_m} \left|_{Q_m=Q_m^0} (Q_m - Q_m^0) \right| \leq 0 \\
& \text{TR}^\text{Sec} = \sum_{m=1}^{M} g_1(t_m, P_m) + \sum_{m=1}^{M} \left[ g_2(t_m^0, P_m^0) + \frac{\partial g_2(t_m, P_m^0)}{\partial t_m} \right] |_{t_m=0} (t_m - t_m^0) + \frac{\partial g_2(t_m, P_m^0)}{\partial P_m} \left|_{P_m=0} (P_m - P_m^0) \right| \leq 0
\end{align*}
\]

\[
\sum_{m=1}^{M} t_m = \max_{\{Z_{mj}, \lambda_m\}} \left\{ T - \frac{1}{a} \sum_{m=1}^{M} \sum_{j=1}^{M} Z_{mj} D_{mj} : A(\tilde{s}, Z) \right\}
\]

\[
(1 - \tilde{s}_m)(t_m^2 + P_m + Q_m) = 0,
\]

\[
t_m \geq 0, P_m \geq 0, Q_m \geq 0 \quad \forall m = 1, \ldots, M
\]

\[
\sum_{m=1}^{M} P_m + \zeta(a)(T - \sum_{m=1}^{M} t_m) \leq E
\]

\[
\sum_{m=1}^{M} Q_m \leq Q
\]

In \(P_2\), the right hand of \((24c)\) is the crucial issue of the traveling salesman problem as shown below:

\[
\max_{\{Z_{mj}, \lambda_m\}} T - \frac{1}{a} \sum_{m=1}^{M} \sum_{j=1}^{M} Z_{mj} D_{mj} \quad \text{s.t. } A(\tilde{s}, Z)
\]

Problem \((25)\) can be optimally solved via the software Mosek \([23,24]\). \(\{\tilde{Z}_{mj}, \tilde{\lambda}_m\}\) are supposed to be the optimal solution to the problem \((25)\), and the optimal objective value of problem \((25)\) is given by \(T - (\sum_{m=1}^{M} \sum_{j=1}^{M} \tilde{Z}_{mj} D_{mj}) / a\). Then, the \((24c)\) constraint of \(P_2\)
can be re-written as $\sum_{m=1}^{M} l_m = T - (\sum_{m=1}^{M} \sum_{j=1}^{M} \tilde{z}_{mj} d_{mj}) / a$. Since other constraints of P2 are convex, P2 can be optimally solved by CVX. Denoting its solution as $\{\hat{R}_{\text{Sec}}, \hat{\imath}_m, \hat{\tilde{Q}}_m, \hat{\tilde{P}}_m\}$, the optimal $\{\hat{q}_m, \hat{\tilde{p}}_m\}$ with fixed $s = \hat{s}$ can be obtained from $\hat{q}_m = \hat{Q}_m / \hat{\imath}_m$ and $\hat{\tilde{p}}_m = \hat{\tilde{P}}_m / \hat{\imath}_m$.

2.3.3. Local Optimal Solution of $s$

With $\hat{R}_{\text{Sec}}, \{\hat{z}_{mj}\}, \{\hat{\imath}_m, \hat{\tilde{Q}}_m, \hat{\tilde{P}}_m, \hat{\tilde{Q}}_m\}$, which were derived in a previous section, the secrecy rate, the left-hand sides of Equations (13) and (14) with $s = \hat{s}$, can be written as

$$\Phi(s) = \hat{R}_{\text{Sec}}$$

$$\Xi(s) = \zeta(a)(\sum_{m=1}^{M} \sum_{j=1}^{M} \hat{z}_{mj} d_{mj}) + \sum_{m=1}^{M} \hat{\tilde{P}}_m$$

$$\Psi(s) = \sum_{m=1}^{M} \hat{\tilde{Q}}_m$$

Therefore, problem P1 is re-written as

$$\text{P3} : \max_s \Phi(s)$$

s.t. $\Xi(s) \leq E$ \hspace{1cm} (27a)

$\Psi(s) \leq Q$ \hspace{1cm} (27b)

$s_1 = 1, \ s_m \in \{0, 1\}, \ \forall m \geq 2$ \hspace{1cm} (27c)

To solve P3, an effective way is to apply the neighborhood search method (NSM) [19–21], which greatly reduces the amount of computation compared to an exhaustive search. The critical point of NSM is the idea of the neighborhood $\mathcal{N}(-)$. Taking $s^{(0)} = [1, 0, \cdots, 0]^T$, for example, the neighborhood $\mathcal{N}(s^{(0)})$ is

$$\mathcal{N}(s^{(0)}) = \{s \in \{0, 1\}^M : ||s - s^{(0)}||_0 \leq L, s_1 = 1\}$$ \hspace{1cm} (28)

where $L \geq 1$ is the size of neighborhood [21], and for arbitrary $s^{(n)}$, the neighborhood $\mathcal{N}(s^{(n)})$ is obtained in Algorithm 1.

**Algorithm 1 Computing the neighborhood $\mathcal{N}(s^{(n)})$**

1. **Initialize** $s^{(n)}$ (i.e., for $n = 0, s^{(0)} = [1, 0, \cdots, 0]^T$) and a proper $L$.
2. Flip the binary values of $\{s_m\}$ according to (28).
3. **Output** $\mathcal{N}(s^{(n)})$.

2.3.4. Summary of Algorithm

At the beginning, we start from a feasible solution of $s = s^{(0)}$ and compute $\Phi(s^{(0)})$ by solving P2 with $s = s^{(0)}$. Then, we randomly select a candidate solution $s' \in \mathcal{N}(s^{(0)})$ and compute $\Phi(s')$. Next, we compare $\Phi(s')$ with $\Phi(s^{(0)})$. If $\Phi(s') \geq \Phi(s^{(0)})$, we update $s^{(1)}$ with $s^{(1)} = s'$ and view $s^{(1)}$ as a new feasible solution to construct the next neighborhood. Otherwise, we check the $\mathcal{N}(s^{(0)})$ to determine whether there are points not yet being sampled. If yes, we find another point within the neighborhood $\mathcal{N}(s^{(0)})$ and proceed to the above steps; otherwise, we terminate the procedure.

To obtain the local optimal solution of $s$, we repeat the above process and obtain a sequence of $\{s^{(0)}, s^{(1)}, s^{(2)}, \cdots \}$. The convergence point is guaranteed to be a local optimal solution to P1 [14]. To summarize the process given above, an algorithm for obtaining the local optimal solution to P3 (equal to P1) is proposed as Algorithm 2.

In terms of computational complexity, the computing problem (25) requires $O((M - 1)^2 \cdot 2^{M-1})$ in the worst case [14,25]. Solving P2 by CVX requires a complexity of $O((3M +
due to the fact that $P2$ has $3M + 1$ variables. Therefore, the complexity order of Algorithm 2 is dominated by $O(X \cdot [(M - 1)^2 \cdot 2^{M-1} + (3M + 1)^{3.5}])$, where $X$ is the number of iterations.

**Algorithm 2** Proposed local optimal solution to $P1$

1: Repeat
2: Select a candidate $s'$ from $\mathcal{N}(s^{(n)})$, where $\mathcal{N}(s^{(n)})$ is obtained through Algorithm 1.
3: Set $s = s'$, and solve $P2$ to obtain $\Phi(s')$ (namely $R^{Sec}$).
4: If $\Phi(s') \geq \Phi(s^{(n)})$, update $s^{(n+1)} \leftarrow s'$ and $n \leftarrow n + 1$.
5: Else go to step 2.
6: Until Convergence.

### 3. Results

Numerical results are provided in this section to evaluate the performance of the resource-constrained two-way relay network. In particular, the distance-dependent path-loss model $\varrho_{i,m} = \varrho_0 \cdot (\frac{d_{i,m}}{d_0})^{-\alpha}$ is adopted, where $d_{i,m}$ is the distance from user 1 (Alice: user 1; Bob: user 2) to the $m$th stopping point of the relay; $\varrho_0 = 10^{-3}$ is the path loss at distance $d_0 = 1$ m; and $\alpha$ is the path-loss exponent set to be 2.5 [6,27]. Based on the path-loss model, channels $h_m$ and $g_m$ are generated according to $CN(0, \varrho_{i,m} I_N)$. Similarly, the wiretap channels $h_{e,m}$ and $g_{e,m}$ are generated according to $CN(0, \varrho_{e,m} I_N)$ and $CN(0, \varrho_{e,m} I_N)$, respectively, where $\varrho_{e}$ and $\varrho_{e,m}$ are the distance-dependent path loss of uplink and downlink wiretap channels, respectively. The receiver noise powers are set to $\sigma_r^2 = \sigma_u^2 = -70$ dBm and $\sigma_e^2 = -60$ dBm. Furthermore, within the duration of $T = 50$ s, the energies available at Alice and relay are $Q = 300$ mJ and $E = 1000$ J, respectively. Each point in the figures is obtained by averaging over 100 simulation runs.

Based on the above settings, we simulate the secrecy rate map in a 20 m × 20 m = 400 m$^2$ square area [14,27], which is a typical size of a smart warehouse. Inside this map, 2 IoT users, 1 eavesdropper, 1 relay with 8 ($N = 8$) antennas, and $M = 15$ vertices representing stopping points are randomly scattered, with the eavesdropper being near to the starting point of the relay.

To verify the convergence of Algorithm 2 in the previous section, Figure 3 shows the relationship between the secrecy rate and the number of iterations. It can be seen that with $L = 3$, a stable result is obtained. This verifies the convergence of our proposed algorithm with a moderate number of iterations.

![Figure 3. Secrecy rate vs. iterations.](image-url)
In order to illustrate the proposed scheme’s performance gains, Figure 4 shows a comparison between the fixed relay case and mobile relay scheme. The secrecy rate under the mobile relay scheme is much higher than that of the fixed case, which is almost zero due to the relay being near to Eve. Moreover, the performance gap concisely quantifies the secrecy rate gain brought by mobile relay, which demonstrates the advantage of employing movement of the relay.

Figure 4. Secrecy rate comparison between fixed relay case and moving relay scheme.

To gain more insight into the mobile relay scheme, Figure 5 can be observed, from which it can be observed that Algorithm 2 can automatically determine where to move and how far to move. The relay can smartly avoid the dangerous zones that are close to the eavesdropper.

Figure 5. path for mobile relay with M = 15 when noise power $\sigma^2_e = -60$ dBm.

Furthermore, to show how the mobile relay scheme works effectively, we set $\sigma^2_e = -90$ dBm and obtained Figure 6. Compared with Figure 5 ($\sigma^2_e = -60$ dBm), it
can be seen in Figure 6 that the visiting path includes more far-away stopping points. This is because when the eavesdropper’s noise power is decreased, denoting that Eve has a stronger wiretap ability, the mobile relay has to move far way to guarantee secure transmission.

Figure 6. Selected path for mobile relay with M = 15 when noise power \( \sigma_e^2 = -90 \text{ dBm} \).

4. Discussion

In this section, we compare our proposed scheme and algorithms with state-of-the-art methods from the literature.

Unlike [1–5], which illustrate a four-terminal relay-eavesdropper network, there is no direct link between the source (Alice) and legitimate destination (Bob) in our system model due to the long distance between them and limited transmit power at Alice; therefore, the channel model and subsequent signal analysis are quite different from those in [1–5]. Moreover, in contrast to [2–4] with the relay’s location variation restricted in a line, the relay in our scheme moves around in a two-dimensional (2D) space, which is more complicated. In particular, unlike [3], where the relay assists the wire tapper rather than the destination, the mobile relay in our scheme helps the destination and suppresses the eavesdropper.

Our scheme presents moving trajectories, as shown in Figures 5 and 6, which are not present in [7–10]. This is because [7–10] focus on fixed relay scenarios, in which relays are not able to move. Additionally, in contrast to [7], we obtain a competitive secrecy rate versus number of iterations. However, unlike [7], the secrecy rate of the fixed relay case in our scheme is near 0, which is depicted in Figure 4. This is because the eavesdropper in our paper is very close to the relay.

Ref. [11–13] aim to develop unmanned aerial vehicle (UAV)-based relay systems. They indeed provide flexible moving relays to implement air-to-ground or air-to-air wireless communication, while our ground mobile vehicle-based relay system is designed to solve the ground secure transmission problem, especially the indoor complicated pitch case. However, there are some identical issues both in UAV and in our ground mobile vehicle scheme, such as joint trajectory and communication design, which is generally formulated as mixed integer non-convex optimization problem, and this kind of problem is usually nontrivial to solve due to the integer constraints and other nonlinear parts. Contrary to expectations, we all utilize successive convex optimization techniques to solve the intractable optimization problem. Nevertheless, there are other different aspects among [11–13] and our scheme. Firstly, there is no eavesdropper in [11,13]; thus, it is not the maximum secrecy rate but the minimum communication delay in [11] and the maximum throughput in [13].
Secondly, multi-user communication scheduling and association in [13] are regarded as the integer constraints, while in our scheme, the integer constraints are the graph mobility model. Thirdly, Ref. [12] introduces an additional UAV-based jammer to ensure secure communication against eavesdropping. In comparison with [12], no jamming is added in our scheme, which leads to a notably different system model from that in [12]. Finally, unlike [11–13] employing the block coordinate descent method, we propose an iterative algorithm described as Algorithm 2 with Algorithm 1 embed in it, which is capable of obtaining a local optimal solution with much lower complexity.

A future research goal is to derive the secrecy rate bounds of the resource-constrained two-way mobile relay system. Additionally, finding the global optimal solution is a consequent work plan, and the secrecy rate with cache-aided at the mobile relay may also be worth investigation.

5. Conclusions

In the current study, the secrecy rate of a ground mobile vehicle-based relay system was investigated. Firstly, an integrated graph mobility model and a secrecy rate of the mobile relay system model were proposed. The joint secure transmission and graph mobility management problem was formulated to achieve secrecy rate maximization, subject to resource constraints and mobility graph structure constraints. Then, algorithms that achieve a local optimal solution were proposed. Simulation results were presented to demonstrate that our scheme outperforms the fixed relay scheme and that the mobile relay trajectory highly depends on noise power.

Author Contributions: Conceptualization and methodology: H.W.; software: H.W. and Y.W.; validation, formal analysis, investigation, resources, visualization, and writing—original draft preparation: H.W.; writing—review and editing: H.W. and Y.W.; supervision and project administration: M.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Overseas Study Fund of Jiangsu University of Science and Technology grant number just-2018-05.

Acknowledgments: The authors are grateful to Yik-Chung Wu at the University of Hong Kong for providing the visiting researcher project; thank Shuai Wang (Southern University of Science and Technology) technique help; and acknowledge the funding support from Jiangsu University of Science and Technology.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

- MINLP Mixed integer nonlinear programming
- UAV Unmanned aerial vehicles
- CSI Channel state information
- SNR Signal-to-noise ratio
- NSM Neighborhood search method

Appendix A

$(Z)\ast$ and $t\ast_m$ are supposed to be optimal for P1; then, $(Z)\ast$ and $t\ast_m$ must activate the constraint $(10c)$, i.e., $\frac{1}{n} \sum_{m=1}^{M} \sum_{j=1}^{M} Z_{m,j} D_{m,j} + \sum_{m=1}^{M} t\ast_m = T$. Otherwise, we can increase the value of $l_m$ such that the left-hand sides of (11) and (12) are increased. This means that the objective value of P1 can also be increased, which contradicts $t\ast_m$ being the optimal. Therefore, the constraint (10c) can be restricted to an equality $\frac{1}{n} \sum_{m=1}^{M} \sum_{j=1}^{M} Z_{m,j} D_{m,j} + \sum_{m=1}^{M} t_m = T$. 
Appendix B

Transformation from P1 to P2 with fixed \( s = \tilde{s} \).

Based on P1’s constraints (10a) and (10b), we can obtain the following:

\[
R_{Sec}^{S} \leq \frac{1}{T} \sum_{m=1}^{M} t_m \left[ \log_2(1 + \frac{q_m h_m^2}{\sigma^2}) - \log_2(1 + \frac{q_m h_e^2}{\sigma^2}) \right] \tag{A1a}
\]

\[
R_{Sec}^{S} \leq \frac{1}{T} \sum_{m=1}^{M} t_m \left[ \log_2(1 + \frac{p_m g_m^2}{\sigma_g^2}) - \log_2(1 + \frac{p_m g_{e,m}^2}{\sigma_g^2}) \right] \tag{A1b}
\]

With the fixed \( s = \tilde{s} \), an optimal solution \( \{ R_{Sec}^{S}, (Z)^* \}, \{ \lambda^*_m \}, \{ t^*_m \} \) to P1 activates (10a), (A1a), and (A1b), and the following holds:

\[
R_{Sec}^{S} \min \left\{ \frac{1}{T} \sum_{m=1}^{M} t_m \left[ \log_2(1 + \frac{q_m h_m^2}{\sigma^2}) - \log_2(1 + \frac{q_m h_e^2}{\sigma^2}) \right], \frac{1}{T} \sum_{m=1}^{M} t_m \left[ \log_2(1 + \frac{p_m g_m^2}{\sigma_g^2}) - \log_2(1 + \frac{p_m g_{e,m}^2}{\sigma_g^2}) \right] \right\} \tag{A2}
\]

(A2) shows that the objective function \( R_{Sec}^{S} \) of P1 is a monotonically increasing function of \( t_m \); the maximum value of \( R_{Sec}^{S} \) in (A2) is obtained when \( \{ t_m \} \) are maximized. Since (22) holds, \( \sum_{m=1}^{M} t_m = T - \frac{1}{2} \Sigma_{m=1}^{M} \Sigma_{j=1}^{M} Z_{m,j}D_{m,j} \) can be obtained, and maximizing \( \{ t_m \} \) is equal to obtaining the maximum value of \( T - \frac{1}{2} \Sigma_{m=1}^{M} \Sigma_{j=1}^{M} Z_{m,j}D_{m,j} \); therefore, problem P1 is equivalently transformed into P2.

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