Twist-three analysis of photon electroproduction off pion.

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Abstract

We study twist-three effects in spin, charge, and azimuthal asymmetries in deeply virtual Compton scattering on a spin-zero target. Contributions which are power suppressed in $1/Q$ generate a new azimuthal angle dependence of the cross section which is not present in the leading twist results. On the other hand the leading twist terms are not modified by the twist three contributions. They may get corrected at twist four level, however. In the Wandzura-Wilczek approximation these new terms in the Fourier expansion with respect to the azimuthal angle are entirely determined by the twist-two skewed parton distributions. We also discuss more general issues like the general form of the angular dependence of the differential cross section, validity of factorization at twist-three level, and a relation of skewed parton distributions to spectral functions.

Keywords: deeply virtual Compton scattering, twist-three contributions, asymmetries, skewed parton distribution
1 Introduction.

Hard electroproduction of a photon, i.e. deeply virtual Compton scattering (DVCS) \[1, 2, 3\], measures non-forward matrix elements \(\langle P_2 | O_i | P_1 \rangle\) of quark and/or gluon non-local composite operators \(O_i\). The former possesses a more diverse structure than processes with forward kinematics e.g. arising in conventional inclusive reactions. Fourier transforms of \(\langle P_2 | O_i | P_1 \rangle\) define a new type of hadron characteristics — generalized parton distributions (GPDs). Real photon electroproduction on nucleon targets has already been studied by DESY experiments: for moderate Bjorken variable \(x_B\) by HERMES \[4\] and for small \(x_B\) by ZEUS and H1 collaborations \[5\]. Since these new functions are poorly known, our primary goal is to constrain diverse models from experimental data. To extract GPDs from future experiments \[6\] one has to address the problem of appropriate observables. The four-fold differential cross section for \(e(k)h(P_1) \rightarrow e(k')h(P_2)\gamma(q_2)\)

\[
\frac{d\sigma}{dx_Bdyd|\Delta^2|d\phi} = \frac{\alpha^3 x_B y}{8\pi Q^2} \left(1 + \frac{4M^2 x_B^2}{Q^2}\right)^{-1/2} \left|\mathcal{T}\right|^2
\]

depends on the Bjorken variable \(x_B = -q_1^2/(2P_1 \cdot q_1)\) (with \(q_1 = k - k'\) and \(q_1^2 = -Q^2\)), the momentum transfer \(\Delta^2 = (P_2 - P_1)^2\), the fraction of the lepton energy loss \(y = P_1 \cdot q_1/P_1 \cdot k\) and the azimuthal angle \(\phi\) between lepton and hadron scattering planes. The amplitude \(\mathcal{T}\) is a sum of the virtual Compton scattering (VCS) amplitude, \(\mathcal{T}_{VCS}\), and the Bethe-Heitler (BH) amplitude, \(\mathcal{T}_{BH}\), since they have the same initial and final states. Fortunately, the interference term, \(\mathcal{I} \equiv \mathcal{T}_{VCS}\mathcal{T}_{BH}^* + \mathcal{T}_{VCS}^*\mathcal{T}_{BH}\), in \(|\mathcal{T}|^2\) may be isolated by measuring various asymmetries like the lepton charge asymmetry, the hadron/lepton spin asymmetry and the azimuthal angle asymmetry \[2, 7, 8\]. Since \(\mathcal{I}\) is linear in the DVCS amplitude, one can extract its real or imaginary part from an observable defined in an appropriate manner. Thus, one directly accesses a specific linear combination of GPDs convoluted with the real or imaginary part of a hard scattering amplitude.

The theoretical description of DVCS amplitudes is simplified for small values of \(x_B\). This region is relevant for the HERA collider experiments. For small \(x_B\) one naively assumes that GPDs are essentially determined by the forward parton densities \(1\) multiplied by partonic form factors. The momentum transfer dependence of GPDs may be parametrized by a hadron form factor \[9\] or for larger values it can be computed within QCD \[10\]. For larger values of \(x_B\) the ‘exclusive’ domain of GPDs becomes important. Then their functional form cannot be approximated by conventional densities. Rather one has to rely on models or plausible parametrizations \[8\]. Such predictions can be unstable under QCD radiative- and/or higher twists corrections. Therefore both issues deserve a detailed investigation. Although the evolution effects are quite moderate (of order \(10-15\%\) \[11\]), the next-to-leading order corrections in the coefficient functions have a significant impact on the

\[1\] The only free parameter is the renormalization group scale where this identification is done.
handbag approximation and can be of order 50% in the valence region \[12\]. On the other hand the exploration of power suppressed contributions has just begun. Recent studies explored twist-three GPDs in the DVCS amplitudes \[13\]-\[17\] and relations of twist-three GPDs to already discussed leading twist functions \[15\]-\[17\] and interaction dependent antiquark-gluon-quark correlations \[15\]. A natural next step is to study power suppressed contributions in the coefficients of the Fourier expansion of the differential cross section. For the sake of simplicity and clarity we consider a pion target, \( h = \pi \). While barely a target for an experiment, (pseudo) scalar particles are instructive for theoretical reasons. They avoid difficulties accompanying particles with non-zero spin. Our goal is thus to gain insights into gross features of twist-three effects in physical observables which are expected to hold true also for DVCS on higher spin targets. Presently, we discuss issues of the azimuthal angle dependence of the differential cross section \( \langle 1 \rangle \) with contributions from twist-three GPDs and qualitative estimates of higher twist effects.

The paper is organized as follows. In Section 2 we calculate the azimuthal angle dependence of the differential cross section in terms of the hadronic amplitudes, which appear in the DVCS tensor for a spin-zero target. We derive constraints between the angular moments of the amplitude squared. In Section 3 we evaluate the twist-three predictions based on perturbative leading order calculations. In Section 4 we give a consistent definition of spectral functions, so-called double distributions, and derive their relationship to GPDs. After giving a few numerical estimates in Section 5 we summarize.

2 Angular dependence of the cross section.

As we observed before, the total electroproduction amplitude of a real photon consists of two terms, \( T_{\text{VCS}} \), we are interested in, and the contaminating \( T_{\text{BH}} \). \( T_{\text{BH}} \) is purely real and arises from a contraction of the leptonic tensor, \( L_{\mu\nu} = \bar{u}(k',\lambda')[\gamma_\mu(k'-\Delta)^{-1}\gamma_\nu + \gamma_\nu(k'+\Delta)^{-1}\gamma_\mu]u(k,\lambda) \), with a hadronic electromagnetic current, which for the pion target reads \( J_\mu = P_\mu F(\Delta^2) \). It is parametrized by the electromagnetic form factor \( F(\Delta^2) \) described by a simple monopole form \((1 - \Delta^2/m_V^2)^{-1} \) with \( m_V \approx 0.46 \text{ GeV}^2 \). Similarly, the DVCS amplitude is expressed by \( T_{\text{VCS}} = \pm \frac{e^2}{Q^2} T_{\mu\nu} T_{\mu\nu}^* (q_2) \bar{u}(k')\gamma_\nu u(k) \), where the sign depends on the beam charge, \( \pm e \pm \). The hadronic tensor is given by

\[
T_{\mu\nu}(q, P, \Delta) = i \int dxe^{ix\cdot q} \langle P_2 | T j_\mu(x/2) j_\nu(-x/2) | P_1 \rangle.
\]

It depends on the momenta \( q = (q_1 + q_2)/2, P = P_1 + P_2 \) and \( \Delta = P_2 - P_1 \) which can be reexpressed in terms of the Lorentz invariants \( \xi = Q^2/P \cdot q, \eta = \Delta \cdot q/P \cdot q, Q^2 = -q^2 = -\frac{1}{4}(q_1 + q_2)^2 \) and \( \Delta^2 \). For the DVCS kinematics, the first two scaling variables are related to each other by
\[ \eta = -\xi \left(1 - \frac{\Delta^2}{Q^2}\right) \approx -\xi. \]  

The ‘experimental’ variables referred to in the introduction are expressed by the present ones to twist-four accuracy via, see e.g. [5], \( Q^2 \approx \frac{1}{2} Q^2, \xi \approx x_B/(2-x_B) \). There are five independent kinematical structures in Eq. (3) for a general two photon process on a (pseudo) scalar target:

\[
T_{\mu\nu}(q, P, \Delta) = -\mathcal{P}_{\mu\sigma} g^{\sigma\tau} \mathcal{P}_{\tau\nu} F_1 + \frac{\mathcal{P}_{\mu\sigma} P^\sigma P^\tau \mathcal{P}_{\tau\nu}}{2P \cdot q} F_2 + \frac{\mathcal{P}_{\mu\sigma} (P^\sigma \Delta^\tau + \Delta^\sigma P^\tau) \mathcal{P}_{\tau\nu} F_3}{2P \cdot q} \\
+ \frac{\mathcal{P}_{\mu\sigma} (P^\sigma \Delta^\tau - \Delta^\sigma P^\tau) \mathcal{P}_{\tau\nu} F_4}{2P \cdot q} + \frac{\mathcal{P}_{\mu\sigma} \Delta^\sigma \Delta^\tau \mathcal{P}_{\tau\nu} F_5}{M^2}. \tag{3}
\]

Here current conservation is ensured by means of the projector \( \mathcal{P}_{\mu\nu} = g_{\mu\nu} - q_{\mu} q_{\nu}/q \cdot q \). The transverse component of the momentum transfer is \( \Delta^\perp \equiv \Delta - \eta P_{\mu} \). We scaled the kinematical factors in such a way, that all five dimensionless scalar amplitudes change only logarithmically if quark masses are set to zero. Contracting the hadronic tensor with the leptonic current we can conclude that the amplitudes \( F_1, F_2 \) and \( F_5 \) are the leading contributions in \( Q \), while \( F_3 \) and \( F_4 \) are suppressed by the powers \( 1/Q \) and \( 1/Q^2 \), respectively. Note also that the Bose symmetry requires \( F_1, F_2, F_4, F_5 \) to be even and \( F_3 \) to be odd in \( \eta \). For the case at hand the outgoing photon is real. Then the on-shell condition \( q_0^2 = 0 \) forces \( \mathcal{P}_{\mu\sigma} \Delta^\sigma \) to be proportional to \( q_{2\mu} \). When this term is contracted with the leptonic part its contribution vanishes. Only three contributions are left then

\[
T_{\mu\nu}(q, P, \Delta)|_{q^2=0} = -\mathcal{P}_{\mu\sigma} g_{\sigma\tau} \mathcal{P}_{\tau\nu} \left( T_1 + \frac{(1-\xi^2)(\Delta^2 - \Delta_{\min}^2)}{4\xi M^2} T_3 \right) \\
+ \frac{\mathcal{P}_{\mu\sigma} P^\sigma P^\tau \mathcal{P}_{\tau\nu}}{2P \cdot q} \left( T_2 + \frac{\Delta^2}{4M^2} T_3 \right) + \frac{\mathcal{P}_{\mu\sigma} \Delta^\sigma \Delta^\tau \mathcal{P}_{\tau\nu} T_3}{2M^2}. \tag{4}
\]

The new form factors are parametrized in such a way that gluonic transversity only contributes to \( T_3 \) at \( \mathcal{O}(\alpha_s) \). We also dropped \( T_3/Q^2 \) contributions in the first Lorentz structure. The new amplitudes are parametrized by Compton form factors, which are related to the old ones by the following set of equations

\[
T_1 = \left( F_1 - \frac{(1-\xi^2)(\Delta^2 - \Delta_{\min}^2)}{2M^2} F_5 \right)|_{\eta=-\xi}, \quad T_2 = \left( F_2 + \xi F_3 - \xi F_4 - \frac{\xi \Delta^2}{2M^2} F_5 \right)|_{\eta=-\xi}, \\
T_3 = \frac{2\xi F_5}{\eta=-\xi}.
\]

We again neglected twist four terms. Due to current conservation, the substitution \( P_{\sigma} \to \Delta^\perp/\xi \) may be performed. It is obvious then that the second term on the r.h.s. of Eq. (4) is suppressed by at least one power in \( 1/Q \) w.r.t. the leading terms which contain \( T_1 \) and \( T_3 \). Once more we emphasize that \( T_3 \) is absent at the Born level.

Now we are in a position to discuss the azimuthal angle dependence of the DVCS and BH amplitudes squared as well as of the interference term. To ensure that the Fourier series contains no terms which are artificially generated by kinematical subtleties, we choose a frame
rotated w.r.t. the laboratory one\textsuperscript{3}. In our frame the virtual photon has no transverse components and a negative $z$ component $q_1 = (q_1^z, 0, -|q_1^z|)$. The $x$ component of the incoming positron is positive $k = (E, E \sin \theta_e, 0, E \cos \theta_e)$. Other momenta read $P_1 = (M, 0, 0, 0)$ and $P_2 = (E, |P_2| \cos \phi \sin \theta_H, |P_2| \sin \phi \sin \theta_H, |P_2| \cos \theta_H)$. $\phi$ is the azimuthal angle between the lepton and hadron scattering planes.

The calculation of the squared amplitudes is straightforward and yields (the electric charge is set equal to one):

- Bethe-Heitler squared term.

The amplitude squared for a spin-zero target does not depend on the lepton polarization. It reads

$$|T_{BH}|^2 = \frac{8F^2}{\Delta^2} \left\{ \frac{4M^2 - \Delta^2}{2\Delta^2} \left( 1 + \frac{Q^4 + \Delta^4}{2(2k \cdot \Delta - \Delta^2)(Q^2 + 2k \cdot \Delta)} \right) + \frac{((q - k) \cdot P)^2 + (k \cdot P)^2}{(2k \cdot \Delta - \Delta^2)(Q^2 + 2k \cdot \Delta)} \right\}. \quad (5)$$

The denominators of both lepton propagators $(2k \cdot \Delta - \Delta^2)$ and $(2k \cdot \Delta + Q^2)$ depend on the azimuthal angle $\phi$. Moreover, the Taylor expansion of $(Q^2 + 2k \cdot \Delta) = (k - q_2)^2 = -\frac{1}{y}Q^2(1 + \mathcal{O}(1/Q))$ induces a pole for $y \to 1$. We introduce as a new notation $P_1$ and $P_2$ for the dimensionless lepton propagators:

$$Q^2P_1 \equiv (k - q_2)^2 = Q^2 + 2k \cdot \Delta, \quad Q^2P_2 \equiv (k - \Delta)^2 = -2k \cdot \Delta + \Delta^2. \quad (6)$$

Their $\phi$ dependence is contained in

$$k \cdot \Delta = -\frac{Q^2}{2y(1 + \epsilon^2)} \left\{ 1 + 2K \cos \phi - \frac{\Delta^2}{Q^2} \left( 1 - x_B(2 - y) + \frac{y \epsilon^2}{2} \right) + \frac{y \epsilon^2}{2} \right\}, \quad (7)$$

where $\epsilon \equiv 2x_B M/Q$. The kinematical factor

$$K = \left[ -\frac{\Delta^2}{Q^2} (1 - x_B) \left( 1 - y - \frac{y \epsilon^2}{4} \right) \right] \left( 1 - \frac{\Delta^2_{\min}}{\Delta^2} \right) \left\{ \sqrt{1 + \epsilon^2} - \frac{4x_B(1 - x_B) + \epsilon^2 \Delta^2_{\min} - \Delta^2}{4(1 - x_B)} \right\}^{\frac{1}{2}}, \quad (8)$$

is $1/Q$ power suppressed. It vanishes at the kinematical boundary $\Delta^2 = \Delta^2_{\min}$ determined by

$$-\Delta^2_{\min} = Q^2 \frac{2(1 - x_B) \left( 1 - \sqrt{1 + \epsilon^2} \right) + \epsilon^2}{4x_B(1 - x_B) + \epsilon^2} = \frac{M^2x_B^2}{1 - x_B + x_MB^2/Q^2} \left\{ 1 + \mathcal{O} \left( M^2/Q^2 \right) \right\}. \quad (9)$$

Let us take a closer look at equation (5). In the second numerator, \((q - k) \cdot P)^2 + (k \cdot P)^2 = (q \cdot P - k \cdot \Delta - 2k \cdot P_1)^2 + (k \cdot \Delta + 2k \cdot P_1)^2\), only $k \cdot \Delta$ depends on $\phi$. Therefore equation (5) contains terms proportional to $\cos^0 \phi$, $\cos^1 \phi$ and $\cos^2 \phi$. In other words (5) may be written as

$$|T_{BH}|^2 = -\frac{F^2(\Delta^2)}{x_B^2y^2(1 + \epsilon^2)\Delta^2 P_1 P_2} \sum_{m=0}^2 c_m^{BH} K_m \cos (m\phi), \quad (10)$$

\textsuperscript{2}This reference frame is related to the centre-of-mass system in Ref. \cite{2} by a boost of the hadron in the $z$-direction.
where the expansion coefficients are given by
\[
c^0_{\text{BH}} = \left\{ (2 - y)^2 + y^2(1 + \epsilon^2) \right\} \left\{ 4x_B^2 \frac{M^2}{\Delta^2} + 4(1 - x_B) + (4x_B + \epsilon^2) \frac{\Delta^2}{Q^2} \right\}
+ 32x_B^2K^2 \frac{M^2}{\Delta^2} + 2\epsilon^2 \left\{ 4(1 - y)(3 + 2\epsilon^2) + y^2(2 - \epsilon^2) \right\} - 4x_B^2(2 - y)^2(2 + \epsilon^2) \frac{\Delta^2}{Q^2},
\]
\[
c^1_{\text{BH}} = -8(2 - y) \left\{ 2x_B + \epsilon^2 - 4x_B^2 \frac{M^2}{\Delta^2} \right\},
\]
\[
c^2_{\text{BH}} = 32x_B^2 \frac{M^2}{\Delta^2}.
\]
(11)

It should be noted, that all coefficients are bounded in the whole kinematical region, since \( \frac{M^2}{\Delta^2} \) is always multiplied by \( x_B^2 \) and \( \Delta^2_{\text{min}} \propto M^2 x_B^2 \). The \( K \) term in the Fourier coefficients \( c_0, c_1 \) and \( c_2 \) suppresses the angular moments by \( \int d\phi \cos(m\phi) |\mathcal{T}_{\text{BH}}|^2 \propto \left\{ - \frac{\Delta^2}{Q^2} \left( 1 - \frac{4\Delta^2}{\Delta^2_{\text{min}}} \right) \right\}^{m/2}. \) To prevent induced azimuthal angle dependence due to the expansion of lepton propagator in \( 1/Q \), angular moments with an additional weight of the two propagators \( P_1 P_2 \) can be calculated. Then the averaged BH squared term reads for large \( Q^2 \)
\[
\int_0^{2\pi} \frac{d\phi}{2\pi} P_1(\phi) P_2(\phi) |\mathcal{T}_{\text{BH}}|^2 = - \frac{F^2(\Delta^2)}{x_B^2 y^2 \Delta^2} \left\{ 8(2 - 2y + y^2)(1 - x_B) \left( 1 - \frac{\Delta^2_{\text{min}}}{\Delta^2} \right) + \mathcal{O} \left( \frac{1}{Q^2} \right) \right\}.
\]
(12)

- Bethe-Heitler–DVCS interference term.

The interference term can be treated in the same way.
\[
\mathcal{I} = - \frac{F(\Delta^2)}{x_B^2 y^2 \Delta^2} \left\{ \frac{\Delta^2}{Q^2} c_0^I \sum_{m=1}^{2} K^m \left[ c_m^I \cos(m\phi) + \lambda s_m^I \sin(m\phi) \right] + \frac{Q^2}{M^2} x_B^2 \frac{M^2}{\Delta^2} \right\}
\]
(13)

As we see the Fourier sums terminate with \( \cos(3\phi) \) and \( \sin(2\phi) \) terms for unpolarized and polarized scattering, respectively. The information about GPDs is contained in the dimensionless coefficient functions \( c_m^I \) and \( s_m^I \). They are linear combinations of the amplitudes \( T_i \) introduced in [9]. The exact results are rather lengthy. Therefore we neglect terms which are power suppressed w.r.t. the leading contribution:
\[
c^0_I = -8x_B(2 - y) \left\{ (2 - x_B)(1 - y) - (1 - x_B)(2 - y)^2 \left( 1 - \frac{\Delta^2_{\text{min}}}{\Delta^2} \right) \right\} \text{Re} \ T_1,
\]
\[
c^1_I = -8x_B(2 - 2y + y^2) \text{Re} \ T_1,
\]
\[
c^2_I = -16(2 - y) \text{Re} \left\{ x_B T_1 - T_2 + \frac{(2 - x_B)\Delta^2 - 6(1 - x_B)(\Delta^2 - \Delta^2_{\text{min}})}{2(2 - x_B)M^2} T_3 \right\},
\]
\[
c^3_I = -\frac{16}{2 - x_B} \text{Re} \ T_3,
\]
\[
s_1^I = 8x_B y(2 - y) \text{Im} \ T_1,
\]
\[
s_2^I = 16y \text{Im} \left\{ x_B T_1 - T_2 - \frac{\Delta^2}{2M^2} T_3 \right\}.
\]
(14)
As we can see, the interference term falls off like \(1/\sqrt{-\Delta^2 Q^2}\) for large \(Q\). Its average over \(\phi\) is suppressed by \(1/Q^2\). Note that the coefficient \(c_3^T\) arises at twist-two level due to the gluon transversity in \(T_3\).

- DVCS squared term.

Finally, the DVCS squared term reads

\[
|T_{\text{DVCS}}|^2 = \frac{1}{x_By^2Q^2} \left\{ c_0^{\text{DVCS}} + K \left[ c_1^{\text{DVCS}} \cos(\phi) + \lambda s_1^{\text{DVCS}} \sin(\phi) \right] + \frac{Q^2}{M^2} K^2 c_2^{\text{DVCS}} \cos(2\phi) \right\}. \tag{15}
\]

The Fourier expansion terminates with \(\cos(2\phi)\) and has only a \(\sin(\phi)\) term. The coefficients are quadratic in the Compton form factors \(T_i\) and to leading order in \(1/Q\) they are given by

\[
c_0^{\text{DVCS}} = 2(2 - 2y + y^2) \left\{ x_B T_1 T_3^* + \frac{(1 - x_B)^2}{x_B(2 - x_B)} \frac{\left( \Delta^2 - \Delta_{\text{min}}^2 \right)^2}{M^4} T_3 T_3^* \right\}, \tag{16}
\]

\[
c_1^{\text{DVCS}} = 8(2 - y) \text{Re} \left\{ \left( T_1 - \frac{1 - x_B}{2 - x_B} \Delta^2 - \Delta_{\text{min}}^2 \right) \left( x_B T_1 - T_2 \right) - \frac{1 - x_B}{2 - x_B} \frac{\Delta^2 - \Delta_{\text{min}}^2}{M^2} T_3 T_3^* \right\},
\]

\[
c_2^{\text{DVCS}} = \frac{8}{2 - x_B} \text{Re} T_1 T_3^*,
\]

\[
s_1^{\text{DVCS}} = -8y \text{Im} \left\{ \left( T_1 - \frac{1 - x_B}{2 - x_B} \Delta^2 - \Delta_{\text{min}}^2 \right) \left( x_B T_1 - T_2 \right) - \frac{1 - x_B}{2 - x_B} \frac{\Delta^2 - \Delta_{\text{min}}^2}{M^2} T_3 T_3^* \right\}.
\]

The Compton form factor \(T_3\) contributes to \(c_0^{\text{DVCS}}\) and \(c_2^{\text{DVCS}}\) at leading twist. In the remaining two coefficients the interference of \(T_1\) with \(T_3\) (as well as in \(c_2^T\) and \(s_2^T\)) requires a twist-three analysis at next-to-leading order. Note that all angular coefficients are determined by geometrical (i.e. dimension minus spin) twist-two (for \(c_0^T, c_1^T, s_1^T, c_3^T, c_0^{\text{DVCS}}\) and \(c_2^{\text{DVCS}}\)) or by geometrical twist-three contributions. However, \(c_3^T\) and \(c_2^{\text{DVCS}}\) may be stronger contaminated by power suppressed contributions than the other coefficients, since gluon transversity is perturbatively suppressed.

It is important to note, that some of the coefficients are related to each other. We recall that there are only three Compton form factors in equation (4). Therefore, the four coefficients \(c_1^T\) are reduced to three independent ones. To leading order approximation in \(1/Q\) this relation looks like:

\[
c_0^{\text{DVCS}} = 2 - y \left\{ (2 - x_B)(1 - y) - (1 - x_B)(2 - y)^2 \left( 1 - \frac{\Delta_{\text{min}}^2}{\Delta^2} \right) \right\} c_1^T + \mathcal{O} \left( 1/Q^2 \right). \tag{17}
\]

Measuring charge asymmetries yields information about the real part of all three amplitudes and the imaginary part of two of them (see discussion below). If we assume for a moment that the \(T_3\) contribution is negligibly small, we find the following relations

\[
c_0^{\text{DVCS}} = \frac{2 - 2y + y^2}{32x_B} \left\{ \frac{\left( c_1^T \right)^2}{(2 - 2y + y^2)^2} + \frac{\left( s_1^T \right)^2}{y^2(2 - y)^2} \right\} + \mathcal{O} \left( 1/Q^2 \right),
\]

\[
\]
These constraints can be used to test $Q$ scaling of the Compton form factors. They can be generalized to the case of non-zero $T_3$. The real part of $T_3$ enters in $c_3^D$, while the imaginary part of $T_3$ can be extracted from $c_2^D$ (assuming we know $T_1$). In this case we obtain the same set of equations (18), but the coefficients $c_0^D$, $c_1^D$ and $s_1^D$ then depend on all angular moments of the interference term as well as on $c_2^D$.

Finally, let us give a few remarks concerning the actual measurements of the angular coefficients. Of course, experiments with pions are improbable at present but the remarks also apply to the analogous case of lepton-nucleon scattering. For optimal experiments one needs both polarized electron and positron beams. Since the amplitudes squared are charge-even, while the interference term is charge-odd, this allows to access all moments of the interference term. The $\cos (\phi)$ and $\sin (\phi)$ dependence should dominate away from the kinematical boundary, because other contributions are either kinematically or perturbatively suppressed. After subtraction of the BH squared contribution measurements of the charge-even part of the cross section for unpolarized lepton beams allow to access all azimuthal angular moments of the DVCS squared term. Here the constant term is dominant, because $K$ suppresses the other terms kinematically and because of the expected smallness of transversity. With a polarized beam the $\sin (\phi)$ term can be directly accessed, because the BH process is independent of the beam polarization. Following the outlined schedule one should, thus, be able to test the three equalities (18).

Let us now comment on experiments where a polarized lepton beam is only available for one charge. Then single spin asymmetries may be used to yield information about the angular coefficients by weighting the differential cross section with $P_1 P_2$. Unfortunately the DVCS squared term will inevitably contribute to the azimuthal asymmetries then. Although its contribution is suppressed by $y \Delta^2 / Q^2$ [compare Eq. (13) with Eq. (15)] compared to the interference term it is also affected by the ratios $\text{Re} T_i / F$. For the unpolarized case the following observations are relevant. Near the kinematical boundary $\Delta^2 = \Delta_{\text{min}}^2$ all terms of the amplitude squared tend to be of the same order starting with $1 / Q^2$. Away from $\Delta^2 = \Delta_{\text{min}}^2$ one still has a relative kinematical suppression of $|T_{\text{DVCS}}|^2$ by $y \Delta^2 / Q^2$. However, $y \Delta^2 / Q^2$ is now multiplied by terms containing $\text{Im} T_i / F$. Of course, one has to subtract the BH processes to get the desired information. Fortunately, for the $\cos (\phi)$ and $\cos (2\phi)$ moments the BH term enters with the same power $1 / Q$ as the interference term [compare Eq. (10) with Eq. (13)]. The latter again includes the ratios $\text{Re} T_i / F$. 

\[
\begin{align*}
c_1^{\text{DVCS}} &= \frac{1}{16x_B} \left\{ \frac{c_1^T c_2^T}{2 - 2y + y^2} + \frac{s_1^T s_2^T}{y^2} \right\} + \mathcal{O} \left( 1/Q^2 \right), \\
s_1^{\text{DVCS}} &= \frac{1}{16x_B} \left\{ \frac{s_1^T c_2^T}{(2 - y)^2} - \frac{c_1^T s_2^T}{2 - 2y + y^2} \right\} + \mathcal{O} \left( 1/Q^2 \right).
\end{align*}
\]
3 Twist-three GPDs.

In this Section we derive the explicit expression for the Fourier coefficients in terms of GPDs from the hadronic tensor, which is known to twist-three accuracy in the Born approximation. We will discuss different aspects of the result. Since factorization has not yet been proven for $1/Q$ suppressed contributions it might happen (in contrast to the leading twist situation \[3, 20, 21\]) that the results are plagued by singularities. Universality of higher twist distributions could not be guaranteed then. A key role in factorization plays the property that non-perturbative functions do have specific analytical properties. For instance, in the case of the pion transition form factor measured in $\gamma^* \gamma^* \rightarrow \pi^0$, it requires that the meson distribution should vanish at the end-points and this can indeed be proven \[22\]. In the case of DVCS an analogous requirement is the continuity of skewed parton distributions at $|x| = \eta$. The tree level analysis has been recently done \[13\]-\[17\]. For transverse polarization of the initial photon a divergent amplitude is encountered. However, this divergence does not show up in the cross section because of the final state $\gamma^* \cdot q_2 = 0$ \[17\]. Note that this cancelation of singularities has not yet been proven for twist contributions higher than twist-three.

For any target the hadronic tensor reads ($\epsilon^{0123} = 1$) in the considered order \[15\]:

$$T_{\mu\nu}(q, P, \Delta) = -P_{\mu\sigma} g_{\sigma\tau} P_{\tau\nu} \frac{q \cdot V_1}{P \cdot q} + (P_{\mu\sigma} P_{\rho\nu} + P_{\mu\rho} P_{\sigma\nu}) \frac{V_{2\rho}}{P \cdot q} - P_{\mu\sigma} i \epsilon_{\sigma\tau\rho\nu} P_{\tau\nu} A_{1\rho} + \text{twist-four.} \tag{19}$$

$V_{i\rho}$ and $A_{1\rho}$ are given as matrix elements of vector and axial-vector light-ray operators, respectively. We should note that the gauge invariance beyond the twist-three accuracy has been restored by hands. Furthermore, in our approximation the vector $V_{2\rho}$ depends on the other ones by the relation:

$$V_{2\rho} = \xi V_{1\rho} - \frac{\xi \cdot P_{\rho}}{2 P \cdot q} q \cdot V_1 + \frac{i \epsilon_{\rho\sigma\Delta q}}{2 P \cdot q} A_{1\sigma} + \text{twist-four.} \tag{20}$$

The Lorentz structure of the $A_{1\sigma}$ term in (20) when combined with the last term in (19) changes the gauge invariant $P_{\mu\sigma} \epsilon_{\sigma\tau\rho\nu} P_{\tau\nu}$ projector into $(g_{\mu\sigma} - P_{\mu} q_{2\sigma}/P \cdot q_2) \epsilon_{\sigma\tau\rho\nu} (g_{\nu\tau} - P_{\nu} q_{1\tau}/P \cdot q_1)$ \[13\]. The result \[13\]-\[20\] contains relations that are well known from deep-inelastic scattering. Obviously, in the forward case, where $V_{i\rho} \propto P_{\rho} F_i$ and $\xi = x_B$, we rediscover the Callan-Gross relation. Besides the generalization of this relation, there are other ones at twist-three level. They will be discussed below. In our case of a scalar target, the matrix element of the axial-vector operator has a non-zero form factor at twist-three level, while the vector operator already has a non-zero form factor at leading twist level:

$$V_{1\rho} = P_{\rho} \mathcal{H} + (\Delta_{\rho} - \eta P_{\rho}) \mathcal{H}^{\text{tw3}} + \text{twist-four,} \quad A_{1\rho} = \frac{i \epsilon_{\rho\Delta P q}}{P \cdot q} \tilde{\mathcal{H}}^{\text{tw3}}. \tag{21}$$
The (twist-two and three) form factors $H$ and $\tilde{H}$ are given as convolutions in momentum fraction $x$ of perturbatively calculable hard scattering parts with GPDs ($\otimes \equiv \int dx$):

$$\left\{ \frac{H}{\tilde{H}} \right\} (\xi, \eta, \Delta^2 | Q^2) = \sum_{i=u,d,..} \left\{ \begin{array}{cc} C_{i}^{(-)} \end{array} \right\} (\xi, x | Q^2 / \mu^2) \otimes \left\{ \begin{array}{cc} H_{i} \end{array} \right\} (x, \eta, \Delta^2 | \mu^2). \quad (22)$$

The summation runs over different parton species and $\mu^2$ is the factorization scale. To simplify notations we have kept only scaling variables as an argument of the functions. For a $Q_1$-charged quark the leading order coefficient function reads $\xi C_{i}^{(\mp)} (\xi, x) = Q_{i}^2 (1 - x / \xi - i \epsilon)^{-1} \mp (x \rightarrow -x)$ with $- (+)$ standing for parity even (odd).

The complete twist decomposition of the vector and axial-vector operator is given in \cite{15} and our parameterization \cite{21} reads for the twist-two GPDs

$$(H, \tilde{H}) (x, \eta) = \int \frac{dk}{2\pi} e^{ikP x} \langle P_2 | ^{V,A} O_{+} (\kappa, -\kappa) | P_1 \rangle, \quad ^{V,A} O_{+} (\kappa, -\kappa) = \bar{\psi} (-\kappa n) (1, \gamma_3) \psi (\kappa n). \quad (23)$$

We omitted gauge links here. Note that these functions can be related to other fundamental non-perturbative amplitudes, namely hadronic wave functions, via overlap integrals \cite{18}. The twist-three functions contain two pieces, the so-called Wandzura-Wilczek (WW) piece expressed in terms of the twist-two function and a correlation function, $G$, of antiquark-gluon-quark operators \cite{15}:

$$H^{tw3} (x, \eta) = \int_{1}^{2} dy \frac{dy}{|\eta|} W_{+} \left( \frac{x}{\eta}, \frac{y}{\eta} \right) \left( \frac{\vec{\partial}}{\partial y} - \frac{\vec{\partial}}{\partial \eta} \right) H (y, \eta) + \left( \frac{1}{\eta} \right) H (x, \eta) - G^{+} (x, \eta),$$

$$\tilde{H}^{tw3} (x, \eta) = \int_{1}^{2} dy \frac{dy}{|\eta|} W_{-} \left( \frac{x}{\eta}, \frac{y}{\eta} \right) \left( \frac{\vec{\partial}}{\partial y} - \frac{\vec{\partial}}{\partial \eta} \right) H (y, \eta) - G^{-} (x, \eta), \quad (24)$$

Here $W_{\pm} (x, y) = \frac{1}{2} \{ W (x, y) \pm W (-x, -y) \}$ and the $W$ kernel reads $W (x, y) = \Theta_{11} (1 + x - y)$ with \cite{19} $\Theta_{11} (x, y) = \int_{0}^{1} d\alpha \delta (x\alpha + y\alpha) = [\theta (x) - \theta (y)] / (x - y)$. Note that the first moment with respect to $x$ vanishes. Moreover, the time reversal invariance together with hermiticity tells us that the twist-three GPDs are real valued functions with the symmetry properties $H^{tw3} (-\eta) = -H^{tw3} (\eta)$, $\tilde{H}^{tw3} (-\eta) = \tilde{H}^{tw3} (\eta)$. The only new dynamical information is contained in the antiquark-gluon-quark GPDs \cite{15}:

$$G^{\pm} (x, \eta) = \int_{1}^{2} dy \int_{1}^{2} du \left\{ \frac{1 - u}{2} W'' \left( \frac{x}{\eta}, \frac{y}{\eta} \right) S^{+} (y, u, \eta) \pm \frac{1 + u}{2} W'' \left( -\frac{x}{\eta}, -\frac{y}{\eta} \right) S^{-} (y, u, \eta) \right\} \quad (25)$$

We used the convention $W'' \left( \pm \frac{x}{\eta}, \pm \frac{y}{\eta} \right) \equiv \frac{\partial^2}{\partial y^2} W \left( \pm \frac{x}{\eta}, \pm \frac{y}{\eta} \right)$. Due to this second derivative the antiquark-gluon-quark operators do not contribute to the second moment of twist-three GPDs. Since the operators defined in the parity even and odd sectors can be expressed through each other,
only two independent functions \( \langle P_2 | S^\pm_\rho (\kappa, u\kappa, -\kappa) | P_1 \rangle = P_\rho^2 \Delta_\rho^\pm \int dx \exp(-i\kappa x P_+) S^\pm(x, u, \eta) \) appear, where the operators are \( S^\pm_\rho (\kappa_1, \kappa_2, \kappa_3) = ig\bar{\psi}(\kappa_3 n) \left[ \gamma_+ G_{+\rho}(\kappa_2 n) \pm i\gamma_5 \tilde{G}_{+\rho}(\kappa_2 n) \right] \psi(\kappa_1 n) \). Note that the twist-three form-factors possess discontinuities [16, 17] at the points \( x = |\eta| \), for instance:

\[
H^{tw3}(\eta + 0, \eta) - H^{tw3}(\eta - 0, \eta) = \frac{1}{2} \text{PV} \int_{-1}^{1} dy \frac{1}{y - \eta} \left\{ \left( \frac{\partial}{\partial \eta} + \frac{\partial}{\partial y} \right) H(y, \eta) - 2 \frac{\partial^2}{\partial y^2} \int_{-1}^{1} du (1 + u) S^-(y, u, \eta) \right\},
\]

where we assumed that the GPDs vanish at the boundary \( x = \pm 1 \). The appearence of discontinuities is a general artefact of the procedure to separate twist-two and -three contributions and is not solely due to the WW-approximation.

There is a number of relations between the amplitudes \( F_i \) in [3] at the Born level. The first one is a generalization of the twist-two Callan-Gross relation:

\[
F_1(\xi, \eta) = \frac{1}{2\xi} F_2(\xi, \eta) = \sum_{i=u,d,...} \int_{-1}^{1} dx \ C_i^{(-)}(x, \xi) H_i(x, \eta).
\]

This equation can be easily deconvoluted by \( \sum_i [H_i(x, \eta) - H_i(-x, \eta)] = \text{Im} F_1(x, \eta)/\pi \). Moreover, by means of antisymmetry we can continue \( F_1(x, \eta) \) to negative values of \( x \). Thus, we obtain a dispersion relation, \( \pi \text{Re} F_1(\xi, \eta) = \text{PV} \int_{-1}^{1} \frac{dx}{\xi y} \text{Im} F_1(x, \eta) \), which tests the dominance of perturbative leading order predictions. In the WW approximation for twist-three functions two further relations hold, which express the amplitudes \( F_3 \) and \( F_4 \) in terms of twist-two GPDs:

\[
F_3(\xi, \eta) = \xi \sum_{i=u,d,...} Q_i^2 \int_{-1}^{1} dx \left\{ \left( \frac{\text{sign}(\eta)}{\eta + x} \ln \frac{\xi + \eta - i0}{\xi - x - i0} + (\eta \to -\eta) \right) \left( \frac{\partial}{\partial \eta} - \frac{x}{\eta} \frac{\partial}{\partial x} \right) \\
+ \frac{2}{\eta(\xi - x - i0)} (x \to -x) \right\} H_i(x, \eta),
\]

\[
F_4(\xi, \eta) = \xi \sum_{i=u,d,...} Q_i^2 \int_{-1}^{1} dx \left\{ \left( \frac{\text{sign}(\eta)}{\eta + x} \ln \frac{\xi + \eta - i0}{\xi - x - i0} - (\eta \to -\eta) \right) \left( \frac{\partial}{\partial \eta} - \frac{x}{\eta} \frac{\partial}{\partial x} \right) \\
- (x \to -x) \right\} H_i(x, \eta).
\]

As we can see, in the kinematics of a two-photon process the WW approximation exist, although, the \( \ln(\xi \pm \eta) \) terms could provide a numerical enhancement. In the case of an outgoing on-shell photon, i.e. \( \xi = -\eta + O(\Delta^2/Q^2) \), one naively expects a logarithmic enhancement (stemming from (23)) due to \( \ln(\xi + \eta) \to \ln(\Delta^2/Q^2) \), which would imply difficulties with factorization. However, it has been shown that such contributions cancel in physical amplitudes [17, 18], and thus legitimize the WW approximation. It is interesting to note that the same situation appears also in the antiquark-gluon-quark sector.
The DVCS \((\eta = -\xi)\) amplitudes in the tensor decomposition (4) are given by

\[
T_1 = \mathcal{H}, \quad T_2 = 2\xi \mathcal{H} + 2\xi^2 \left\{ \mathcal{H}^{tw3} - \tilde{\mathcal{H}}^{tw3} \right\},
\]

(29)

and the form factor \(T_3\) is determined at twist-two level by the gluon transversity. All Compton form factors also contain twist-four contributions, which have not been computed yet. Obviously, the generalized Callan-Gross relation can not be tested in the DVCS process, since it is modified by twist-three contributions, see (29). The modification is given as a difference of \(\mathcal{H}^{tw3}\) and \(\tilde{\mathcal{H}}^{tw3}\) and, thus, ensures the cancellation of the \(\ln(\xi + \eta)\) term for the WW and the antiquark-gluon-quark contributions:

\[
\left\{ \mathcal{H}^{tw3} - \tilde{\mathcal{H}}^{tw3} \right\}(\xi, -\xi) = -\frac{1}{\xi} \mathcal{H}(\xi, -\xi) - \sum_{i=u,d,...} \int_{-1}^{1} dx \frac{Q_i^2}{\xi + x} \ln \frac{2\xi}{\xi - x - i0} \times \left\{ \left( \frac{\partial}{\partial \xi} - \frac{x}{\xi} \frac{\partial}{\partial x} \right) \{ H_i(x, \xi) - H_i(-x, \xi) \} - \frac{\partial^2}{\partial x^2} S_i(x, \xi) \right\},
\]

(30)

where \(S_i(x, \xi) = \int_{-1}^{1} du (1 + u) \left[ S^+_i(-x, -u, -\xi) - S^-_i(x, u, -\xi) \right]\). Note that this special combination (30) of twist-three amplitudes can be represented as \(\mathcal{H}^{tw3} - \tilde{\mathcal{H}}^{tw3} = \int \frac{dx}{x} C(-)(x, \xi) \delta H^{tw3}(x, -\xi)\).

It is free from singularities, since the integrand \(\delta H^{tw3}(x, -\xi) = x H^{tw3} - \xi \tilde{H}^{tw3}\) is a continuous function in the momentum fraction \(x\). It remains an open problem if this cancellation of \(\ln(\Delta^2/Q^2)\) terms is ensured by general principles, or if it is only valid in leading order for the Wilson coefficients and breaks down once radiative corrections are accounted for. Unfortunately, contrary to the case of general kinematics, in the WW approximation for DVCS we can not express \(T_2\) in terms of \(T_1\). To test the WW approximation one has to use models for GPDs in order to study their influence on \(T_1\) and \(T_2\).

The tree-level result for the angular coefficients up to twist-three accuracy in leading order of the coupling constant\(^3\) reads for

- the interference term (14)

\[
\begin{align*}
 c_0^{\bar{T}} &= -8(2 - y) x_B \Delta^2/Q^2 \left\{ (2 - x_B)(1 - y) - (1 - x_B)(2 - y)^2 \left( 1 - \frac{\Delta_{\min}^2}{\Delta^2} \right) \right\} \text{Re } \mathcal{H}, \\
 c_1^{\bar{T}} &= -8(2 - 2y + y^2) x_B \text{Re } \mathcal{H}, \quad s_1^{\bar{T}} = 8y(2 - y) x_B \text{Im } \mathcal{H}, \\
 c_2^{\bar{T}} &= -16 \frac{(2 - y) x_B}{2 - x_B} \text{Re } \mathcal{H}^{\text{eff}}, \quad s_2^{\bar{T}} = 16 \frac{y x_B}{2 - x_B} \text{Im } \mathcal{H}^{\text{eff}},
\end{align*}
\]

(31)

\(^3\)The twist two contribution including gluonic transversity is known in next-to-leading order.
where we defined the ‘effective’ twist-three function
\[ H_{\text{eff}} = -2\xi \left( \frac{1}{1+\xi} H + H_{\text{tw3}} - \tilde{H}_{\text{tw3}} \right) \]

in such a way, that the remaining \( x_B \) dependence of the twist-three angular coefficients is the same as for twist-two. The coefficients \( c_3^T \) and \( c_{2\text{DVCS}}^T \) contain, besides the gluonic twist-two transversity, also geometrical twist-four contributions. In leading order of perturbation theory the latter can be evaluated from handbag diagrams with additional transversal gluons and, so-called, cat-ear diagrams. As we observe the twist-three contribution induces a new \( \cos(2\phi) / \sin(2\phi) \) angular dependence of charge asymmetries as well as a new \( \cos(\phi) / \sin(\phi) \) angular dependence in the DVCS squared term with coefficients proportional to a ‘universal’ combination of GPDs given in Eq. (33). In contrast, the angular independent part of the interference term, i.e. \( c_0^T \), arises from a pure kinematical twist-three effect.

4 Double and skewed parton distributions.

Before we give examples for the shape of twist-three GPDs in the next section let us discuss the relation between skewed and double distributions (DDs). The double distributions were originally defined by
\[ \langle P_2 | \mathcal{O}_+(\kappa, -\kappa) | P_1 \rangle = P_+ \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz e^{-i\kappa P_+ y - i\kappa \Delta_2 z} F(y, z). \]

By means of the so-called \( \alpha \)-representation it has been shown that \( F(y, z) \) is a generalized function defined in the region \( |y| + |z| \leq 1 \). This original definition, also introduced in [3], leads to a violation of the polynomiality condition for GPDs [2]. The reasoning goes as follows. The \( j \)th-moment of an GPD is given by the expectation value of local twist-two operators with spin \( j + 1 \), i.e. \( \mathcal{R}_{\rho_1...\mu_j}^2 = S_{\rho_1...\mu_j} \bar{\psi} \gamma_\rho i \overset{\leftarrow}{D}_\mu_1 ... i \overset{\leftarrow}{D}_\mu_j \psi \), which are completely symmetrized and traceless, and where \( \overset{\leftarrow}{D}_\mu = \overset{\rightarrow}{D}_\mu - \overset{\rightarrow}{D}_\mu \) and \( \overset{\rightarrow}{D}_\mu = \partial_\mu - igB_\mu \). Thus, they are polynomials of order \( j + 1 \) in \( \eta \):
\[ \int_{-1}^{1} dx x^j H(x, \eta) = n_{\rho_1...\mu_j} \langle P_2 | \mathcal{R}_{\rho_1...\mu_j}^2 | P_1 \rangle = P_+^{j+1} \sum_{k=0}^{j+1} \eta^k H_{j+1,j+1-k}. \]

On the other hand
\[ H(x, \eta) = \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz \delta(y + \eta z - x) F(y, z) = \int_{-1}^{1} \frac{dy}{\eta} \Xi(y|x, \eta) F \left( y, \frac{x-y}{\eta} \right), \]
with

$$\Xi(y|x, \eta) = \theta(x > \eta)\theta\left(\frac{x+y}{1+\eta} > y > \frac{x-y}{1-\eta}\right) + \theta(-\eta > x)\theta\left(\frac{x+y}{1-\eta} > y > \frac{x-y}{1+\eta}\right) + \theta(\eta > x > -\eta)\theta\left(\frac{x+y}{1+\eta} > y > \frac{x-y}{1+\eta}\right).$$

The $j$th-moment of $H(x, \eta)$ generates only a polynomial of order $j$ in $\eta$: $\int dx x^j H(x, \eta) = \int dy \int dz (y + \eta z)^j F(y, z)$. As suggested in Ref. [23] one can cure this problem by adding in the definition (34) of DDs an extra independent term concentrated in $y = 0$ and proportional to $\Delta_+$:

$$H(x, \eta) = \int dy \int dz \delta(y + \eta z - x) \{F(y, z) + \eta \delta(y) D(z)\}. \quad (36)$$

Since $D(z)$ has the support $|z| \leq 1$, it induces a term in the GPD that is entirely concentrated in the ‘exclusive’ region. Indeed this choice is unique and the $D$-term can be extracted from a given distribution by $D(x) = \lim_{\eta \rightarrow \infty} H(\eta x, \eta)$. The first term in the bracket of Eq. (36) is understood as a representation of a subtracted GPD, namely, $H(x, \eta) - \text{sign}(\eta) D(x/\eta)$. Note that $D(z)$ is an antisymmetric function in $\eta$ since $H(x, -\eta) = H(x, \eta)$.

In the following we give an alternative solution of the problem stated above, in which the spectral function does not depend on the skewedness parameter $\eta$. The new representation will enable us to derive an inverse transformation in a simple way, since it does not explicitly depend on $\eta$. It is instructive to start with local operators. The parameterization for the matrix elements of the completely symmetrized and traceless local vector operators sandwiched between spin-zero states reads:

$$\langle P_2 | \mathcal{R}^2_{\rho;\mu_1 \ldots \mu_j} | P_1 \rangle = \mathcal{S} \left\{ P_\rho \ldots P_{\mu_j} H_{j+1,j+1} + \Delta_\rho P_{\mu_1} \ldots P_{\mu_j} H_{j+1,j} + \ldots + \Delta_\rho \ldots \Delta_{\mu_j} H_{j+1,0} \right\}. \quad (37)$$

Next we introduce a generating function for the coefficients $H_{jk}$. It is easy to check that after contraction with the light-cone vector $n_\mu$ and summation over the local operators, i.e. $\mathcal{O}_+ (\kappa, -\kappa) = n_\rho \mathcal{R}^2_\rho (\kappa, -\kappa)$, $\mathcal{R}^2_\rho (\kappa, -\kappa) = \sum_{j=0}^{\infty} \frac{(-i\kappa)^j}{j!} n_{\mu_1} \ldots n_{\mu_j} \mathcal{R}^2_{\rho;\mu_1 \ldots \mu_j}$, the following definition of $H(x, \eta)$

$$H_{j,j-k} = \frac{1}{k!} \frac{d^k}{d\eta^k} \int_{-1}^{1} dx x^{j-1} H(x, \eta)|_{\eta=0}, \quad \text{where} \quad 0 \leq k \leq j, \quad 1 \leq j,$$

is equivalent to that one in Eq. (23). Let us remark that the term proportional to $\Delta_\rho$ in the matrix elements of the twist-two light-ray operator $\mathcal{R}^2_\rho (\kappa, -\kappa)$ arises from combinatorics and is fixed by a WW type relation [13]

$$\langle P_2 | \mathcal{R}^2_\rho (\kappa, -\kappa) | P_1 \rangle = \int_{-1}^{1} dx e^{-ikP_\rho x} \left( P_\rho H(x, \eta) + \Delta_\rho \int_{-1}^{1} dy W_2(x, y) \frac{d}{d\eta} H(y, \eta) \right), \quad (37)$$

where the kernel reads $W_2(x, y) = \Theta^0_{11}(x, x - y)$. 

13
Analogous to the definition of the GPD as a generating function of moments $H_j(\eta)$, we now introduce a double distribution $f(y, z)$ with the moments $H_{j,j-k} \equiv \binom{j}{k} \int dy \int dz \ y^{j-k} z^k f(y, z)$ where $0 \leq k \leq j$, $1 \leq j$. Summing the series of moments up again we find

$$H(x, \eta) = \int dy \int dz \ x \delta(y + \eta z - x) f(y, z).$$

(38)

so that the transformation (33) is modified in a minimal way by an additional factor of $x$. The matrix elements of twist-two light-ray operators are expressed in terms of the new double distribution by

$$\langle P_2| R^2_\rho(\kappa, -\kappa)|P_1 \rangle = \int dy \int dz f(y, z) \ (y P_\rho + z \Delta_\rho) \ e^{-i\kappa P_1 y - i\eta \Delta_\rho z}.$$  

(39)

Now we are able to derive an inverse transformation for equation (38). In the first step we project the Lorentz index $\rho$ in Eq. (39) onto the transverse plane and subsequently perform Fourier transformations w.r.t. $\Delta_\rho$ and $\kappa$. Employing the representation (37) for the matrix elements in terms of GPDs, we immediately obtain the desired transformation:

$$zf(y, z) = -\frac{1}{2\pi^2} \int dx \int d\eta \ PV(y + \eta z - x)^{-2} \int dx'W_2(x, x') \frac{d}{d\eta} H(x', \eta).$$

Here we used the following Fourier representation for $|\kappa P_+|$: $|a| = -\frac{1}{\pi} \int d\lambda \ \exp(i\lambda a) \ PV \frac{1}{\lambda^2}$, where we understand the PV prescription as $PV \frac{1}{\lambda^2} \equiv \frac{1}{2} \left\{ \frac{1}{(\lambda+i0)^2} + \frac{1}{(\lambda-i0)^2} \right\}$. The final steps are to perform the $x$ integration and a partial integration w.r.t. $\eta$. In the last step we dropped a surface term. We find

$$f(y, z) = \frac{1}{2\pi^2} \int d\eta \int \frac{dx}{x} PV \ (y + z \eta)^{-2} - (y + z \eta - x)^{-2} \} \ H(x, \eta).$$

(40)

Finally, we express the $F$- and $D$-functions of the representation (39) in terms of the $f$-function. The $D$-term is easily extracted by taking the limit $\eta, x \to \infty$ with $x/\eta$ fixed:

$$D(z) = z \int dy \ f(y, z).$$

(41)

Consequently, the term $\int dy \int dz \ x \ {\delta(y + \eta z - x) - \delta(\eta z - x)} f(y, z)$ corresponding to the first one on the r.h.s. of Eq. (36) can be cast in the form (36) by means of

$$x \ {\delta(y + \eta z - x) - \delta(\eta z - x)} = y \delta(y + \eta z - x) + z \frac{d}{dz} \ \int dy' \delta(y' + \eta z - x) W_2(y', y) y.$$

It leads to

$$F(y, z) = y f(y, z) - \frac{d}{dz} z \int dy' W_2(y, y') y f(y', z).$$

(42)
Thus, both $F$ and $D$ functions in (36) turn out to be different projections of the same function $f$. Note that the projection (42) together with the inversion formula (40) give us the function $F(y, z)$ in terms of the GPD:

$$F(y, z) = -\frac{1}{2\pi^2} \int d\eta \int dx PV \left\{ (y + z\eta - x)^{-2} \right\} \left[ H(x, \eta) - \text{sign}(\eta) D(x/\eta) \right].$$ (43)

Here the $D(x/\eta)$ term appears from a surface term that arose from the convolution with the $W_2$ kernel [see r.h.s. of Eq. (42)].

5 Features of WW approximation.

In this Section, which aims to serve illustrative purposes only, we will give qualitative estimates of the twist-three contributions. An analysis and numerical estimates for targets with spin will be given elsewhere. In the following we assume that the GPD factorizes in a partonic form factor $F(\Delta^2)$ and a function depending only on the remaining three variables $H(x, \eta, \Delta^2|Q^2) = F(\Delta^2)H(x, \eta|Q^2)$. The non-trivial polynomiality requirement for the moments of $H(x, \eta)$ is obeyed by using the transformation formula discussed in the previous Section. To ensure the reduction of the GPD to the parton densities in the forward case, we assume that the double distribution can be modeled, following the proposal in Ref. [24], as a product of the quark densities with a profile function that has certain properties. In the case of the transformation (36) the ansatz for the quark DD is $(\bar{y} \equiv 1 - y)$

$$F_i(y, z) = f_i(y)\pi(|y|, z), \quad \pi(y, z) = \frac{3y^2 - z^2}{4y^3}, \quad f_i(y) = q_i(y)\theta(y) - \bar{q}_i(-y)\theta(-y).$$ (44)

In the following we limit ourselves to the valence quark approximation, thus, we set $\bar{q} = 0$ and $q = q_{\text{val}}$ which is, for the sake of simplicity, chosen to be determined by Regge and quark counting rules $q_{\text{val}}(y) = q_u + q_d = \frac{3}{2}y^{-1/2}(1 - y)$.

In Fig. 1 (a) the shape of the GPD that results from the transformation (36) with $D = 0$ is shown (dashed). Taking a simple ansatz for the $D$-term, i.e. $D(z) = \theta(1 - |z|)2z(1 - z^2)$, we see that such a term induces a complex shape (dash-dotted line). We also plotted the so-called forward parton distribution (FPD) model, where the GPD is equated to the quark density, i.e. $\pi(y, z) = \delta(z)$. As we already know, the WW approximation for the twist-three GPDs induces jumps at point $|x| = \xi$. However, they cancel in the combination $\delta H^{\text{tw3}}$, see Fig. 1 (b).

In Fig. 2 (a,b) we show the DVCS amplitude $H$ as defined in Eq. (22) for the same twist-two models as introduced above. Here we observe that the DD-distribution model (36) without $D$-term and the FPD model give almost the same DVCS amplitude. We also see that a $D$-term...
Figure 1: Twist-two GPDs $H(x, \eta) \eta = \frac{1}{2}$ are shown in (a) for the transformation (36) without (dashed) and with (dash-dotted) $D$ term and the FPD model (dotted). In (b) we plot the corresponding WW approximation for $\delta H^{tw3}(x, \eta)$ for $\eta = \frac{1}{2}$. Changing $\eta$ to $-\xi$ leaves (a) intact and reflects (b) w.r.t. the horizontal axis.

can change the real part drastically, while it drops out in the imaginary part of the amplitude. It is worth mentioning that quite different shapes of the GPD do not change the simple convex (concave) form of the real (imaginary) part of the DVCS amplitude.

Now let us address the WW approximation of the ‘universal’ effective combination (33) that determines the normalization of the twist-three angular coefficients (31), (32). We decompose it in WW and antiquark-gluon-quark pieces $H^{eff}(\xi) = H^{eff-WW}(\xi) + H^{\bar{q}Gq}(\xi)$. In the definition (30) we rewrite the derivatives\(^5\) and find

$$H^{eff-WW} = \frac{2}{1+\xi}H + 2\xi \frac{\partial}{\partial \xi} \sum_{i=u,d,...} \int_{-1}^1 dx \frac{Q_i^2}{\xi + x} \ln \frac{2\xi}{\xi - x - i0} \{H_i(x, \xi) - H_i(-x, \xi)\}. \quad (45)$$

Since the integrand only has a logarithmic singularity at $x = \xi$, the integration will result into a smooth $\xi$ dependence, which, however, will be partly removed by the differentiation w.r.t. $\xi$. As we can see in Fig. 2 (c,d) for the real part the normalization and shape of $H^{eff-WW}$ differs only slightly from that of $H$, while the differences in the normalization for the imaginary part are caused by the behaviour of the twist-two GPD at the point $x = \xi$. In any case we observe no numerical enhancement of the twist-three contributions in the WW approximation. The curves suggest a direct connection of the WW approximation with the twist-two GPD in the small $\xi$ region. For the FPD model we can even analytically perform the integration and the amplitude is entirely determined by the small $x$ behaviour of the parton density. In the case $q(x) = Ax^{-a}$ with $a > 0$ for $x \to 0$, we immediately obtain the imaginary and real parts:

$$\text{Im} H = A\pi \xi^{-a}, \quad \text{Im} H^{eff-WW} = 2A\pi \xi^{-a} \left[1 - \frac{a}{2} \psi \left(\frac{1+a}{2}\right) + \frac{a}{2} \psi \left(\frac{a}{2}\right)\right],$$

\(^5\)Obviously, for any smooth function $\tau(z)$, the following identity $\frac{\partial}{\partial z} \frac{\tau(z)}{z} = -\frac{\tau(z)}{z^2}$ holds true.
Figure 2: The twist-two predictions (a,b) and the WW approximation (c,d) of $H^{\text{eff-\ WW}}$, for the convolution of the hard scattering amplitude with the GPDs specified in Fig. (1) are shown for the real (a,c) and imaginary part (b,d), respectively.

\[
\text{Re} H = - A \pi \xi^{-a} \cot \left( \frac{a \pi}{2} \right), \quad \text{Re} H^{\text{eff-\ WW}} = - 2 A \xi^{-a} \left[ \pi \cot \left( \frac{a \pi}{2} \right) - a C(a) \right],
\]

(46)

where $C(a) = \int_0^\infty dx \, x^{-a} \left[ \ln \left( \frac{1-x}{2} \right) / (1+x) - \ln \left( \frac{1+x}{2} \right) / (1-x) \right]$. It is important to note that a different ansatz can give quite different amplitudes in the small $\xi$ region. This fact may be helpful in order to distinguish GPD models by comparison with small $x_B$ data from HERA experiments.

Finally, we take a look beyond the WW approximation. Both the measurement of the polarized structure functions in deep inelastic scattering as well as lattice data suggest that in the forward limit the WW approximation is quantitatively valid. For the DVCS process we have to compare the size of the effective twist-three contribution (45) in the WW approximation with that one coming from the antiquark-gluon-quark correlation functions:

\[
H^{qGq}(\xi) = - 2 \xi \sum_{i=u,d,\ldots} \int_{-1}^{1} dx \frac{Q_i^2}{\xi + x} \frac{2 \xi}{\xi - x - i0^+} \frac{\partial^2}{\partial x^2} S_i(x, \xi).
\]

Since the coefficient function contains now a double pole at $x = \xi$, one has in general a numerical enhancement of $S_i(x, \xi)$. Thus, the antiquark-gluon-quark contribution would have to be much
smaller than $H_i(x,\xi)$ to favour the WW approximation. This can presumably be tested by extracting different azimuthal angular moments from experimental data as discussed in the previous Sections.

6 Conclusions.

In the present contribution we have studied DVCS on a scalar target to twist-three accuracy. Let us summarize the lessons we have learnt from this exercise:

- The interference term is dominated by $\cos \phi/\sin \phi$ dependence away from the kinematical boundary. $Q$ scaling of the Compton form factors can also be tested by a set of relations among the azimuthal angular coefficients.

- Twist-three effects do not show up in the coefficients of the azimuthal angle dependence already present in the leading twist approximation but rather they induce new Fourier components. Thus, we expect at most $O(\Delta^2/Q^2)$ and $O(M^2/Q^2)$ corrections to the leading twist angular dependence.

- Twist-three functions contribute in a singularity free combination to the physical cross sections and there is no violation of factorization.

- GPDs are related to a single spectral density, or double distribution.

- Rather different shapes of GPD models result in quite similar shapes for the predicted DVCS amplitudes manifesting low sensitivity to the former. However, the overall normalization differs significantly.

A similar analysis for the DVCS cross section on the nucleon will be given elsewhere.

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Note added: Recently it has been pointed out in [25] that the inversion of Eq. (38) for the function $H(x,\eta)/x$ is known as the inverse Radon transformation [26]. Our result (40) differs from the standard formula by the first term in the integral on the r.h.s. of Eq. (40) [and a surface term]. Obviously, our integral exist also in the case if $H(x,\eta)/x$ has a non-integrable singularity at $x = 0$. In the case when $H(x,\eta)/x$ is integrable, the first term in the integrand can be safely neglected, being proportional to $\delta(y)\delta(z)$, and we obtain the inverse Radon transformation for the function $H(x,\eta)/x$. Note that the PV prescription can be rewritten as $\text{PV} \int_{-\infty}^{\infty} dz \tau(z)/z^2 = \int_{0}^{\infty} dz \{\tau(z) + \tau(-z) - 2\tau(0)\}/z^2$ [27].
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