Effect of Seabed Instability on Pile Soil Pressure

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Abstract. In a narrow sense, the seabed instability means that the shear failure, liquefaction or scour occurs in the seabed; in a broad sense, the seabed instability means that the soil displacement occurs in the seabed. The seabed instability widely exists in the ocean environment, however, it has not been considered in the current design and calculating methods of the marine pile foundation, consequently, the accuracy and reliability of design results can not be guaranteed, which may induce that the ocean buildings are not safe enough. Therefore, it is necessary to found one new method to calculate the pile soil pressure with the influence of seabed instability, before which, one approximate mathematical model shall be established. Thus, based on Biot consolidation theory and fictitious pile technique, this paper firstly establishes a mathematical model describing pile-soil interaction considering the seabed instability, the soil pressure obtained through which can cover such factors as ocean environment, seabed soil condition and pile condition etc. And then, this paper introduces one mathematical solver (FlexPDE) to get the numerical solutions of the mathematical model presented. Finally, based on the numerical solutions, this paper discusses the effects of pile deflection, wave height, water depth, soil modulus, pile length and diameter on soil pressure, and some conclusions are obtained, which can be referenced in the design of ocean pile foundation.

1. Introduction
Over the years, geotechnical engineers and structural engineers have put much attention to the investigation on pile-soil interaction because it is important to the design safety of buildings. It is worth mentioning that the pile-soil interaction issue in ocean shall consider two aspects, namely, the wave-seabed interaction and the seabed-pile interaction. The former can be classified into the wave-induced seabed instability. When the wave propagates over the seabed, the dynamical pressure would impose on the seabed, the pore water pressure within the seabed will change, and the effective stress will also change. And the excess pore pressure will increase, at the same time the vertical effective stress will decrease, which might cause the local seabed instability, behaving as that the soil displacement occurs inside the seabed. The seabed instability occurs in a wide variety of offshore regions, such as near-shore zones, continental slopes, shallow water, even the deep ocean floors. It has been well known that, the seabed instability induced by wave is one substantial factor to make the offshore structures damaged or destructed. Some case histories related to the failure or damage of offshore structures due to hurricane-triggered submarine slides that occurred in the Mississippi River Delta [1-4]. Sterling and Strohbeck discussed the reasons of failure of the South Pass 70 "B" Platform in Hurricane "Camille" occurred in 1969, which was turned over and slid up to 30 m downward along the slope [4]. They concluded that the failure of platform was not induced by the simple overload, but primarily due to a major submarine slide which extended to a considerable depth. They
disclosed that a "B" structure could withstand a soil slide of 9-12m, but the structure could not withstand the these slide forces. The wave-induced seabed instability was first investigated in 1940s, and many research scholars, such as Yamamoto (1978,1981,1983), Gade (1958), Hsu JRC and Jeng, D S (1993, 1994, 2006) etc., have presented several analytical and numerical solutions [5-11]. Regarding the seabed-pile interaction, Matlock (1970), Poulos (1973, 1991, 2001), Reese (1974), Mostafa YE and El Naggar MH (2002, 2006) etc., have put forward several analysis methods [12-19], such as analytical method, winkler foundation beam method, boundary element method and finite element method etc. It is worth stressing that the researchers concerning pile-seabed interaction only used the single phase medium to describe the seabed behavior. However, the seabed is a porous medium filled with seawater. Furthermore, the evaluation of seabed soil condition is crucial for the bearing capacity of the pile. Therefore, the above two issues should be integrated together when investigating the pile-soil interaction in ocean, that is to say, the effect of wave-induced seabed instability should be considered when calculating the pile soil pressure, which may leave hidden trouble for the safety of ocean buildings in the design process. This paper firstly founds one new mathematical model describing the pile-soil interaction in ocean based on Biot consolidation theory and fictitious pile technique, and then conducts several numerical examples by employing FlexPDE. Based on the numerical solutions, the effects of pile deflection, wave height, water depth, compression modulus of soil, pile length and diameter on soil pressure are discussed, and several conclusions are obtained, which can be referenced in the design of ocean pile foundation, and used to develop one new method for calculating the pile soil pressure in ocean.

2. Pile-soil Interaction Model Considering Seabed Instability

2.1. Governing Equations

When the wave propagates over the seabed, the pore water would flow along the horizontal and vertical directions inside the seabed, and the pore water pressure and the effective stresses would change. Assuming that the seabed is a hydraulically isotropic poro-elastic medium and the pore water is compressible; the flow of pore water within the seabed obeys the law of conservation of mass and the Darcy's law. The Cartesian coordinate system for the wave-seabed interaction is founded as shown in Figure 1, the wave crests are assumed to propagate in the positive x-direction, while the z-direction is upward from the seabed surface, $d$ is the depth of seabed and $h$ is the water depth, $L$ is the wave length and $H$ is the wave height. Based on the above assumptions, the seepage continuous equation of pore water can be expressed as

$$ k_x \frac{\partial^2 p}{\partial x^2} + k_z \frac{\partial^2 p}{\partial z^2} - \gamma_w n \beta \frac{\partial p}{\partial t} = \gamma_w \frac{\partial}{\partial t} (\epsilon_v) \tag{1} $$

where $p$ is the excess pore water pressure; $k_x$ is the permeability of seabed in x-direction; $k_z$ is the permeability of seabed; $k_x = k_z$ for the isotropic seabed; $n$ is the soil porosity; $\gamma_w$ is the unit weight of pore water; $\beta$ is the compressibility of pore water, which is defined as

![Figure 1. Wave-seabed interaction sketch.](image-url)
\[ \beta = \frac{1}{K} + \frac{1 - S_r}{p_{w0}} \]  

(2)

where \( K \) is the true bulk modulus of pore water; \( S_r \) is the degree of saturation; \( p_{w0} \) is the absolute pore water pressure. \( \varepsilon_v \) is the volume strain of the porous medium, which is defined as

\[ \varepsilon_v = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \]  

(3)

where \( u \) and \( w \) are respectively soil displacements in \( x \) and \( z \) directions.

According to Biot consolidation theory [20], the two-dimensional seabed soil satisfies the following equilibrium equations:

\[ G\nabla^2 u + \frac{G}{1-2\mu} \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial p}{\partial x} \]  

(4)

\[ G\nabla^2 w + \frac{G}{1-2\mu} \frac{\partial \varepsilon_v}{\partial z} = \frac{\partial p}{\partial z} \]  

(5)

where \( G \) is the shear modulus of soil; and \( \mu \) is the Poisson ratio of soil. Three partial differential equations, Equations (1), (4) and (5), form the governing equations of the two-dimensional Biot consolidation model which uses three unknown variables, namely, \( u \), \( w \) and \( p \), which can be solved in particular boundary conditions.

The constitutive equations of isotropic soil are given as

\[ \sigma_x = 2G\left[\frac{\partial u}{\partial x} + \frac{\mu}{1-2\mu} \varepsilon_v \right] \]  

(6)

\[ \sigma_z = 2G\left[\frac{\partial w}{\partial z} + \frac{\mu}{1-2\mu} \varepsilon_v \right] \]  

(7)

\[ \tau = 2G\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right] \]  

(8)

where \( \sigma_x \) and \( \sigma_z \) are the effective stresses in the \( x \) and \( z \) directions, respectively; and \( \tau \) is the shear stress. Thus, if \( u \) and \( w \) are obtained, the stress condition of soil can be determined according to Equations (6) through (8).

2.2. Boundary Conditions

Generally, the marine single pile foundation is the steel pipe pile with large diameter, as shown in Figure 2. Such a structural system can be reduced mathematically to the system shown in Figure 3, where the dotted lines denote the pile, and the region surrounded by the solid lines denotes the seabed soil, the response of which under wave action can be described by the above governing equations, yet, several boundary conditions shall be introduced in order to get the unique solutions of governing equations.

For the system shown in Figure 3, the boundary conditions at the seabed surface can be expressed as

\[ \begin{align*}
\sigma_z &= 0 \\
\tau &= 0 \\
p &= P_z
\end{align*} \]

(9)

At \( z=0 \).
where $P_z$ is the wave pressure at the seabed surface, can be given by

$$P_z = \frac{\gamma_w H}{2\cosh(kh)} \cos(kx - \omega t)$$  \hspace{1cm} (10)

where $k$ is the wave number, $k = 2\pi / L$; $\omega$ is the angular frequency of wave motion; and $t$ is the time.

For the system shown in Fig.3, the boundary conditions at the seabed bottom can be expressed as

$$u = 0$$

$$w = 0$$

$$\frac{\partial p}{\partial z} = 0$$

$$z = -d$$  \hspace{1cm} (11)

The interface of seabed and pile is impermeable. Assuming that the soil and the pile never separate, the lateral deflection profile is linear, the deflection at the pile toe is equal to zero, and the pile deflection time-history is also sinusoidal as the soil displacement time-history under the linear wave action is sinusoidal, the boundary conditions of the left side interface of soil and pile can be expressed as

$$u = y_0 (1 + z/l) \cos(kx - \omega t)$$

$$\frac{\partial p}{\partial x} = 0$$

$$z = -d$$  \hspace{1cm} (12)

where $y_0$ is the pile deflection at the top; and $l$ is the pile length.

Then, the boundary conditions of the right side interface of soil and pile can be expressed as

$$u = y_0 (1 + z/l) \cos(kx - \omega t - kb)$$

$$\frac{\partial p}{\partial x} = 0$$

$$z = l$$  \hspace{1cm} (13)

where $b$ is the pile diameter.

The boundary conditions at both sides of seabed soil can be expressed as

$$\sigma_x = \sigma_0$$

$$\frac{\partial p}{\partial x} = 0$$

$$z = 0$$  \hspace{1cm} (14)

where $\sigma_0$ is the confining pressure of soil. In ocean, the pile-soil interaction system can be described
by the governing equations formed by Equations (1), (4) and (5) and the boundary conditions formed by Equations (9) through (14), while the wave-seabed interaction system can be described by Equations (1), (4) and (5) and the boundary conditions formed by Equations (9)-(11) and (14). After the lateral stresses of soil at both sides of pile are obtained, the pile soil pressure can be obtained through solving their differences.

3. Discussion on Soil Pressure of Pile

3.1. Numerical Method

The numerical solutions of pile soil pressure can be obtained through solving the mathematical model described in Section 1 by employing FlexPDE, which is a solver for finding numerical solutions to systems of linear or non-linear partial differential equations. The basic principle of FlexPDE is that it turns the partial differential equations into finite element model using Galerkin method, the quantity and density of meshes could be adjusted automatically, and the modified Newton-Raphson iteration method is used to solve the nonlinear partial differential equation [21]. FlexPDE is quite applicable to solve multi-field coupling problem. One example is given to verify its validity to solve the above mathematical model, and the solutions will be compared with the analytical solutions [22]. Taking two kinds of homogeneous saturate single-layer soil as the investigated subject, in this section we solve the wave-seabed interaction problem. The parameters used in the seabed seepage calculation are given in Table 1, and the results are given in Figure 4. The real lines represent the analytical solutions presented by Yongli Zhang and Jie Li, and the dotted lines represent the numerical solutions by use of FlexPDE. Figure 4 clearly shows that, the numerical solutions agree well with the analytical solutions, Therefore, FlexPDE is applicable to solve the above mathematical model.

Table 1. Seabed seepage calculation parameters.

| Item                  | Value     |
|-----------------------|-----------|
|                       | Case1     | Case2     |
| Coefficient of permeability/(m/s) | 1E-3      | 1E-4      |
| Porosity              | 0.35      | 0.35      |
| Poisson ratio         | 0.3       | 0.3       |
| Elastic modulus /Pa   | 2E7       | 4E7       |
| Water depth/m         | 10        | 20        |
| Wave length/m         | 100       | 50        |
| Wave height/m         | 5         | 8         |
| Wave period/s         | 6         | 10        |
| Saturation level/%    | 100       | 100       |
| Seabed thickness/m    | 80        | 50        |

Figure 4. Verification of FlexPDE’s applicability.
From the above mathematical model, it can be seen that the pile soil pressure obtained integrates the effects of ocean environment, geotechnical conditions and pile conditions. In order to evaluate their influence, several factors significant to the wave-induced seabed instability are discussed as below.

3.2. Effect of Pile Deflection

The ocean environment parameters and geotechnical parameters are given in Table 2. Suppose the pile length is 40 m, and the pile diameter is 3 m. This section discusses the effect of pile deflection on soil pressure, taking four pile top deflections, namely, 10 mm, 20 mm, 30 mm and 40 mm. Based on these parameters, the mathematical model given in Section 1 can be solved in FlexPDE through programming, the calculation region is taken as \(-200 \leq x \leq 200\) m and \(-100 \leq z \leq 0\). Figure 5 shows the temporal and spatial distribution of pile soil pressure when the pile top deflection is 10 mm, and it demonstrates that the variation amplitude of soil pressure at the two ends of pile is bigger than that in the middle; there exists the lagging phase of soil pressure along the depth, which discloses that unlike the pile foundation on shore, the soil surrounding the ocean pile can not exert its capability to restrain the pile displacement due to wave action, even it would enlarge the pile displacement. The traditional calculation methods of pile soil pressure, such as m method, p-y curve method and NL method etc., can not represent this characteristic owing to without considering the wave-induced seabed instability, thus, they may leave hidden trouble for safety of ocean buildings in the design. Figure 6 shows the amplitude envelops of pile soil pressure for four different pile deflections, which demonstrates that the soil pressure firstly increases along with the depth, reaches the most at the depth of 0.2 L, then decreases along with the depth; without considering the plasticity of soil, the pile soil pressure increases as the pile deflection increases in whole, and the more the soil pressure, the more the increasing amplitude.

| Item                          | Value       |
|-------------------------------|-------------|
| Poisson ratio                 | 0.3         |
| Elastic modulus /Pa           | 2.76E6      |
| Water depth/m                 | 12          |
| Compressibility of pore water | 5.26E-10    |
| Wave length/m                 | 100         |
| Wave period/s                 | 8           |
| Wave height/m                 | 6           |
| Wave period/s                 | 8           |
| Coefficient of permeability/(m/s) | 3E-9     |
| Porosity                      | 0.35        |
| Saturation level/%            | 100         |

Table 2. Seabed seepage calculation parameters.

Figure 5. Temporal and spatial distribution of pile soil pressure.

Figure 6. Effect of pile deflection on amplitude of soil pressure.
3.3. Effect of Wave Height
The wave height is one of important ocean environment parameters, shall be considered carefully in the design of marine buildings, and its effect on pile soil pressure is discussed in this section. Four wave heights, namely, 6 m, 8 m, 10 m and 12 m, are taken for investigation, the pile top deflection is 10 mm, and the other parameters are the same as Table 2. Figure 7 shows the amplitude envelops of pile soil pressure for four different wave heights. It is indicated that the pile soil pressure increases as the wave height increases without considering the plasticity of soil, the increasing amplitude at under part is bigger than that at upper part in whole, and the most increasing value occurs near the pile toe while there is almost no change near the pile top; and if the wave height increases continuously, it will dramatically change the distribution of soil pressure along the pile shaft.

3.4. Effect of Water Depth
This section discusses the effect of water depth on pile soil pressure. Four water depths, namely, 8 m, 10 m, 12 m and 14 m, are taken for investigation, the pile top deflection is 10 mm, and the other parameters are the same as Table 2. Figure 8 shows the amplitude envelops of pile soil pressure for four different water depths. It is indicated that the pile soil pressure decreases as the water depth increases without considering the plasticity of soil, the decreasing amplitude at under part is bigger than that at upper part in whole, and the most decreasing value occurs near the pile toe while there is almost no change near the pile top.

3.5. Effect of Compression Modulus of Soil
Obviously, the compression modulus of soil is a key factor to decide the degree of seabed instability, and its effect on pile soil pressure is discussed in this section. Four soil moduluses, namely, 1.38 \times 10^6 \text{ Pa}, 2.76 \times 10^6 \text{ Pa}, 1.38 \times 10^7 \text{ Pa} and 2.76 \times 10^7 \text{ Pa}, are taken for investigation, the pile top deflection is 10 mm, and the other parameters are the same as Table 2. Figure 9 shows the amplitude envelops of pile soil pressure for four different soil moduluses. It is indicated that the pile soil pressure increases as the soil modulus increases without considering the plasticity of soil and the more the soil pressure, the more the increasing amplitude.

3.6. Effect of Pile Length
In the design of pile foundation, the effective embedded length of pile shall be selected approximately, and it is closely related with the soil pressure distribution. This section discusses the effect of pile length on soil pressure. Four pile lengths, namely, 30 m, 40 m, 50 m and 60 m, are taken for investigation, the pile top deflection is 10 mm, and the other parameters are the same as Table 2. Figure 10 shows the amplitude envelops of pile soil pressure for four different pile lengths. It is indicated that the pile soil pressure in the middle, whereas, the soil pressure near the pile toe decreases as the pile length increases without considering the plasticity of soil, that is to say, the constraint force
of soil to the pile gets more and more weak as the pile length increases, and it will become useless to add the pile length beyond a certain degree.

3.7. Effect of Pile Diameter
This section discusses the effect of pile diameter on soil pressure. Four pile lengths, namely, 30 m, 40 m, 50 m and 60 m, are taken for investigation, the pile top deflection is 10 mm, and the other parameters are the same as Table 2. Figure 11 shows the amplitude envelops of pile soil pressure for four different pile diameters. It demonstrates that the pile diameter has little influence on soil pressure without considering the plasticity of soil, which may be resulted from that the pile diameters are quite small compared to the wave length. In practice, the pile diameter is usually quite small compared to the design wave length, thus, the effect of pile diameter can be neglected in determining the pile soil pressure distribution if the plasticity of soil is not accounted for.

![Figure 9. Effect of soil modulus on amplitude of soil pressure.](image1)

![Figure 10. Effect of pile length on amplitude of soil pressure.](image2)

From the above discussion it is illustrated that the seabed instability has big influence on the amplitude and distribution of pile soil pressure, and it is necessary to found a new method for calculating the pile soil pressure with the influence of seabed instability, to ensure the safety of ocean pile foundation and upper structure.

![Figure 11. Effect of pile diameter on amplitude of soil pressure.](image3)

4. Conclusions
Based on Biot consolidation theory and fictitious pile technique, the mathematical model of the pile-soil interaction considering the seabed instability is founded, and solved by use of FlexPDE. Based on the numerical solutions, the effects of pile top deflection, wave height, water depth, soil
modulus, pile length and diameter on soil pressure are discussed, and the main conclusions obtained are as following

(1) If the seabed instability is considered, the pile soil pressure will firstly increase then decrease along the depth, and the maximum value occurs at the depth of 0.2 L.

(2) There exists the phase lag for the soil pressure along the depth, which means that the soil pressure along the pile can not reach the maximum value at the same time, thus it is necessary to consider the effect of phase lag of soil pressure in the pile-soil interaction analysis.

(3) The pile soil pressure increases as the pile deflection or the soil modulus increases, and the more the soil pressure, the more the increasing amplitude.

(4) The pile soil pressure increases as the wave height increases or the water depth decreases, and the increasing amplitude at under part is bigger than that at upper part; the wave height has big influence on the distribution of pile soil pressure.

(5) If the plasticity of soil is not regarded, the pile diameter has little influence on the soil pressure, and can be neglected in determining the distribution of the pile soil pressure.

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References
[1] Bea RG. How sea floor slides affect offshore structures. Oil Gas. 1971, 29: 88-92.
[2] McClelland B, Cox WR. Performance of pile foundations for fixed offshore structures. Proceedings, BOSS’76, international conference on behavior of off-shore structures, vol. 2. University of Trondheim, Norway; 1976. p. 528-44.
[3] Bea RG, Audibert JME. Offshore platforms and pipelines in Mississippi River Delta. J Geotechn Eng Div, ASCE 1980, 106 (GT8):853-69.
[4] Sterling GH, Strobeck EE. The failure of the South Pass 70 “B” Platform in Hurricane Camille. Proceedings of the 5th offshore technology conference, vol. 2, paper OTC 1898; 1973. p. 719–30.
[5] Yamamoto, T. et al. On the response of a poro-elastic bed to water waves. Journal of Fluid Mechanics. 1978, 87(1):193-206.
[6] Yamamoto T. Wave-induced pore pressures and effective stresses in inhomogeneous seabed foundations. Ocean Engineering, 1981, 8(1):1-16.
[7] Yamamoto T. On the response of a Coulomb-damped poro-plastic bed to water waves. Marine Geotechnology, 1983, 5(2): 93-130.
[8] Gade H G. Effects of a non-rigid impermeable bottom on plane surface waves in shallow water. J. Mar. Res. 1958,16: 61-82.
[9] Hsu JRC, Jeng, DS and Tasi CP. Short-crested wave-induced soil response in a porous seabed of finite thickness. Int. J. Numer. Analyt. Meth. Geomech. 1993,17(8): 553-576
[10] Hsu JRC and Jeng DS. Wave-induced soil response in an unsaturated anisotropic seabed of finite thickness, Int. J. Numer. Analyt. Meth. Geomech. 1994, 18(11): 785-807
[11] Liu H and Jeng DS. Response of a porous seabed under random wave loading. In Proceedings of the 25th International Conference on Offshore Mechanics and Arctic Engineering, Humberg, Germany, 2006, 9p
[12] Poulos H G. Analysis of piles in soil undergoing lateral movement .JSMFD, ASCE, 1973, 99(SM5):391-406.
[13] Stewart D P, Jewell R J, Randolph M F. Design of piled bridge abutments on soft clay for loading from lateral soil movements. Geotechnique, 1994, 44(2): 277-296.
[14] Matlock, H. Correlations for design of laterally-loaded piles in soft clay[C]. Proc. 2nd Annual Offshore Tech. Conf., Vol.1, Houston, TX,1970,577-594.
[15] Reese. L.C., W. R. Cox, et al. Analysis of laterally loaded in sand. Proc. Fifth Offshore Tech. Conf, Houston, TX, 1974.
[16] Lee CY, Poulos HG, Hull TS. Effect of seabed instability on offshore pile foundations. Can Geotechn J, 1991, 28: 729-37.
[17] Xu K J, Poulos H G. 3-D elastic analysis of vertical piles subjected to “passive” loadings. Computers and Geotechnics, 2001, Vol. 28, pp. 349-375.

[18] Mostafa YE, El Naggar MH. Dynamic analysis of laterally loaded pile groups in sand and clay. Can Geotech J 2002, 39 (6):1358-83.

[19] Mostafa YE, El Naggar MH. Effect of seabed instability on fixed offshore platforms. Soil Dynamic and Earthquake Engineering, 2006, vol.26:1127-1142.

[20] Biot, M.A. General theory of three-dimensional consolidation. Journal of Applied Physics, 1941, Vol. 12:155-164.

[21] FlexPDE 6 Help, Version 6.06. PDE Solutions Inc. 2009.

[22] YongLi Zhang and Jie Li. Analytical solution for wave-induced response of isotropic poro-elastic seabed. SCIENCE CHINA-TECHNOLOGICAL SCIENCES, 2010, 53(10):2619-29.