Universal Quantum Computation using Exchange Interactions and Teleportation of Single-Qubit Operations

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We show how to construct a universal set of quantum logic gates using control over exchange interactions and single- and two-spin measurements only. Single-spin unitary operations are teleported instead of being executed directly, thus eliminating a major difficulty in the construction of several of the most promising proposals for solid-state quantum computation, such as spin-coupled quantum dots, donor-atom nuclear spins in silicon, and electrons on helium. Contrary to previous proposals dealing with this difficulty, our scheme requires no encoding redundancy. We also discuss an application to superconducting phase qubits.

Quantum computers (QCs) hold great promise for inherently faster computation than is possible on their classical counterparts, but so far progress in building a large-scale QC has been slow. An essential requirement is that a QC should be capable of performing “universal quantum computation” (UQC). I.e., it should be capable of computing, to arbitrary accuracy, any computable function, using a spatially local and polynomial set of logic gates. One of the chief obstacles in constructing large scale QCs is the seemingly innocuous, but in reality very daunting set of requirements that must be met for universality, according to the standard circuit model [1]: (1) preparation of a fiducial initial state (initialization), (2) a set of single and two-qubit unitary transformations generating the group of all unitary transformations on the Hilbert space of the QC (computation), and (3) single-qubit measurements (read-out). Since initialization can often be performed through measurements, requirements (1) and (3) do not necessarily imply different experimental procedures and constraints. Until recently it was thought that computation is irreducible to measurements, so that requirement (2), a set of unitary transformations, would appear to be an essential component of UQC. However, unitary transformations are sometimes very challenging to perform. Two important examples are the exceedingly small photon-photon interaction that was thought to preclude linear optics QCs, and the difficult to execute single-spin gates in certain solid state QC proposals, such as quantum dots [2, 3] and donor atom nuclear spins in silicon [4, 5]. The problem with single-spin unitary gates is that they impose difficult demands on g-factor engineering of heterostructure materials, and require strong and inhomogeneous magnetic fields or microwave manipulations of spins, that are often slow and may cause device heating. In the case of exchange Hamiltonians, a possible solution was recently proposed in terms of qubits that are encoded into the states of two or more spins, whence the exchange interaction alone is sufficient to construct a set of universal gates [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] (the “encoded universality” approach). In the linear optics case, it was shown that photon-photon interactions can be induced indirectly via gate teleportation [20]. This idea has its origins in earlier work on fault-tolerant constructions for quantum gates [21, 22, 23] (generalized in [24]) and stochastic programmable quantum gates [25, 26]. The same work inspired more recent results showing that, in fact, measurements and state preparation alone suffice for UQC [27, 28, 29, 30].

Experimentally, a minimalistic approach to constructing a QC seems appealing. In this sense, retaining only the absolutely essential ingredients needed to construct a universal QC may be an important simplification. Since read-out is necessary, measurements are inevitable. Here we propose a minimalistic approach for universal quantum computation that is particularly well suited to the important class of spin-based QC proposals governed by exchange interactions [2, 3, 4, 5, 31, 32], and other proposals governed by effective exchange interactions [33, 34, 35]. In particular, we show that UQC can be performed using only single- and two-qubit measurements and controlled exchange interactions, via gate teleportation. In our approach, which offers a new perspective on the requirements for UQC, the need to perform the aforementioned difficult single-spin unitary operations is obviated, and replaced by measurements, which are anyhow necessary. The tradeoff is that the implementation of gates becomes probabilistic (as in all gate-teleportation based approaches), but this probability can be boosted arbitrarily close to 1 exponentially fast in the number of measurements.

We begin our discussion with a relatively simple example of the utility of measurement-aided UQC. This example is not in the exchange-interaction category, but both serves to illustrate some of the more complex ideas needed below, and solves a problem of relevance to an important solid-state QC proposal. The proposal we have in mind is that using d-wave grain boundary (dGB) phase qubits [36, 37]. The system Hamiltonian is:

$$H_S = H_X + H_Z + H_{ZZ},$$

(1)

where $H_X = \sum_i \Delta_i X_i$ describes phase tunneling, $H_Z = \sum_i b_i Z_i$ is a bias, and $H_{ZZ} = \sum_{i,j} J_{ij} Z_i Z_j$ represents Josephson coupling of qubits; $X_i, Y_i, Z_i$ denote the Pauli matrices $\sigma^x, \sigma^y, \sigma^z$ acting on the $i^{th}$ qubit. It turns out that in this system only one of the terms $H_X, H_Z, H_{ZZ}$
universality framework. Since all terms in thus unifying all exchange-based proposals under a single be controllable. The method we present here works two-qubit evolution operator of the form

\[ H_{ij}^x(t) = J_{ij}^x(t)(X_iX_j + Y_iY_j) + J_{ij}^z(t)Z_iZ_j, \]  

(2)

where \( J_{ij}^\alpha(t) \) (\( \alpha = \perp, z \)) are controllable coupling constants. The XY (XXZ) model is the case when \( J_{ij}^z = 0 \) (\( \neq 0 \)). The Heisenberg interaction is the case when \( J_{ij}^\perp = J_{ij}^z \). See [12] for a classification of various QC models by the type of exchange interaction. In agreement with the QC proposals [2, 3, 4, 5, 31, 32, 33, 34, 35], we assume here that \( J_{ij}^\perp(t) \) is completely controllable and allow that the ratio between \( J_{ij}^\perp(t) \) and \( J_{ij}^z(t) \) may not be controllable. The method we present here works equally well for all three types of exchange interactions, thus unifying all exchange-based proposals under a single universality framework. Since all terms in \( H_{ex}(t) \) commute it is simple to show that it generates a unitary two-qubit evolution operator of the form \( U_{ij}(\varphi^\perp, \varphi^z) = \exp[-i \int^t dt' H_{ij}^x(t')] = e^{-i\varphi^z} e^{i\varphi^z \cos 2\varphi_{x,0} - i\varphi^z \sin 2\varphi_{x,0}} e^{-i\varphi^z \sin 2\varphi_{x,0}} e^{i\varphi^z \cos 2\varphi_{x,0}} e^{-i\varphi^z} \) (we use units where \( h = 1 \), where \( \varphi^z = \int^t dt' J^z(t') \), and we have suppressed the qubit indices for clarity. In preparation of our main result, we first prove:

**Proposition.** The set \( \mathcal{G} = \{ U_{ij}(\varphi^\perp, \varphi^z), R_{ij}^{\beta} \equiv \exp(i\frac{\pi}{4}\sigma_{ij}^\beta) \} \ (\beta = x, z) \) is universal for quantum computation.

**Proof:** A set of continuous one-qubit unitary gates and any two-body Hamiltonian entangling qubits are universal for quantum computation [39]. The exchange Hamiltonian \( H_{ex}^x \) clearly can generate entanglement, so it suffices to show that we can generate all single-qubit transformations using \( \mathcal{G} \). Two of the Pauli matrices are given simply by \( \sigma_j^x = -R_{ij}^z \). Now, let \( C_0^A \equiv \exp(i\varphi_B) \equiv \exp(-i\theta_A) \exp(+i\theta_A) \); two useful identities for anticommuting \( A, B \) with \( A^2 = I \) (the identity) are [16]:

\[ C_0^{\pi/4} \circ e^{-i\varphi_B} = e^{-i\varphi_B}, \quad C_0^{\pi/4} \circ e^{-i\varphi_B} = e^{i\varphi_AB}. \]

Using this, we first generate \( e^{-i\varphi_{X_1}X_2} = U_{12}(\varphi/2, \varphi^z)C_0^{\pi/4} \circ U_{12}(\varphi/2, \varphi^z) \), which takes six elementary steps (where an elementary step is defined as one of the operations \( U_{ij}(\varphi^\perp, \varphi^z), R_{ij}^{\beta} \)). Second, as we show below, our gate teleportation procedure can prepare \( R_{ij}^\beta \) just as efficiently as \( R_{ij}^\perp \) (also note that \( R_{ij}^\beta = -(R_{ij}^\perp)^3 \)), so that with two additional steps we have \( e^{-i\varphi_{Y_1}X_2} = C_{\pi/4} \circ e^{-i\varphi_{X_1}X_2} \). Finally, with a total of \( 8 + 6 + 8 = 22 \) elementary steps we have \( e^{-i\varphi_{Z_1}Z_2} = C_{\pi/4} \circ e^{-i\varphi_{X_1}X_2} \), where \( \varphi \) is arbitrary. Similarly, we can generate \( e^{-i\varphi_{Y_1}} \) in 22 steps using \( C_{\pi/4}^{\perp} \) instead of \( C_{\pi/4} \). Using a standard Euler angle construction we can generate arbitrary single-qubit operations by composing \( e^{-i\varphi_{Z_1}} \) and \( e^{-i\varphi_{Y_1}} \).

It is important to note that optimization of the number of steps given in the proof above may be possible. We now show that the single qubit gates \( R_{ij}^{\beta} \) can be implemented using cooling, weak spin measurements, and evolution under exchange Hamiltonians of the Heisenberg, XY, or XXZ type. Our method is inspired by the gate teleportation idea [20, 21, 22, 23, 24, 25, 26, 27, 28, 29], which we briefly review, along with state teleportation [40], in Fig. [1]. We proceed in two cycles. In Cycle (i), consider a spin (our “data qubit”) in an unknown \( \langle \psi \rangle = a |0\rangle + b |1\rangle \), and two additional (“ancilla”) spins, as shown in Fig. [2]. Our task is to apply the one-qubit operation \( R_{ij}^{\beta} \) to the data qubit. As in gate teleportation, we require an entangled pair of ancilla spins. However, it turns out that rather than one of the Bell states we need an entangled state that has a phase of \( i \) between its components. To obtain this state, we first turn on the exchange interaction \( H_{23}^x \) between the ancilla.
spins such that $J^z > 0$. The eigenvalues (eigenstates) are 
\{-2J^+ - J^z, 2J^+ - J^z, J^z, J^z\} \text{ and } \{S, \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle), \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)\}

The total state of the three spins then reads (neglecting an overall phase $e^{i\varphi}$):

$$|\psi\rangle_1 U_{23}(\pi/8, \varphi_0) |10\rangle_{23} = \frac{1}{\sqrt{2}} (a|001\rangle - ib|110\rangle)$$

$$+ \frac{1}{\sqrt{2}} (|T_0\rangle_{12} R_{3z} |\psi\rangle_3 - \frac{1}{\sqrt{2}} r^* |S\rangle_{12} R_{3z} |\psi\rangle_3)$$

(4)

where $r = \exp(-i\pi/4)$ and the subscripts denote the spin index.

At this point Alice makes a weak measurement of her spins [Fig. 2(b)]. Let $S_{ij} = \frac{1}{2}(\sigma_i + \sigma_j)$ be the total spin of qubits $i,j$; Alice measures $S_{12}$, with eigenvalues $S(S+1)$. Since only for the singlet state $|S\rangle_{12}$ do we have $S(S+1) = 0$, it follows that if the measurement yields $0$, then the state has collapsed to $|S\rangle_{12} R_{3z} |\psi\rangle_3$.

In this case, which occurs with probability $1/4$, Bob has $R_{3z} |\psi\rangle_3$, and we are done [Fig. 2(c), bottom]. If, on the other hand, Alice finds $S = 1$, then the normalized post-measurement state is

$$\frac{1}{\sqrt{3}} [r |T_0\rangle_{12} R_{3z} |\psi\rangle_3 + a\sqrt{2} (|001\rangle - ib|110\rangle)].$$

(5)

Similar to the gate teleportation protocol [27, 28, 29] shown in Fig. 2 (a), Alice and Bob now need to engage in a series of correction steps. In the next step Alice measures $S_2^z = \frac{1}{4} (\sigma_1^z + \sigma_2^z)^2 = \frac{1}{4} (I + \sigma_1^z \sigma_2^z)$ [Fig. 2(c), top]. Measurement of the observable $\sigma_1^z \sigma_2^z$ is discussed in [23]. If Alice finds $S_2^z = 0$ then with probability $1/3$ the state collapses to $|T_0\rangle_{12} R_{3z} |\psi\rangle_3$ and Bob ends up with the opposite of the desired operation, namely $R_{3z}^\dagger |\psi\rangle$ [Fig. 2(d), bottom]. We describe the required corrective action below, in Cycle (ii). If Alice finds $S_2^z = 1$, then the state is:

$$a|001\rangle - ib|110\rangle = \frac{1}{\sqrt{2}} (r^* R_{3z}^\dagger |\psi\rangle_3 |S\rangle_{23} + r R_{1z} |\psi\rangle_1 |T_0\rangle_{23}).$$

Bob now measures $S_{23}^x$. If he finds $S = 0$ then the state has collapsed to $R_{1z}^\dagger |\psi\rangle_1 |S\rangle_{23}$, while if $S = 1$ then the outcome is $R_{1z} |\psi\rangle_1 |T_0\rangle_{23}$, equiprobably. In the latter case Alice ends up with the desired operation [Fig. 2(c)].

In a similar manner one can generate $R_{k}$ or $R_{k}^\dagger$ acting on an arbitrary qubit state $|\psi\rangle$. Let $|\pm\rangle$ denote the $\pm 1$ eigenstates of the Pauli operator $\sigma^x$. As in the $R_z$ case above, first prepare a singlet state $|S\rangle = \frac{1}{\sqrt{2}} (-|+\rangle - |\pm\rangle)$ on the ancilla spins 2,3 by cooling. Then perform a

FIG. 1: Teleportation [11] is a method for transmitting an unknown quantum state $|\psi\rangle$ with the help of prior entanglement and classical communication. A state teleportation circuit is shown in (a), where time proceeds from left to right, and $\langle \psi |$ denotes the entangled (Bell) state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Alice has $|\psi\rangle_1$ and qubit 2 from the Bell state. Bob has qubit 3 from the Bell state. Alice measures $|\psi\rangle_1$ and qubit 2 in the Bell basis, obtaining one of 4 possible outcomes labeled $\alpha$. She communicates her result to Bob (double wires, who applies $\sigma^\alpha$ to his qubit, where $\sigma^\alpha$ are the four Pauli matrices $I, \sigma^x, \sigma^y, \sigma^z$. Bob then has $|\psi\rangle_3$. A gate teleportation circuit is shown in (b), following [22]. To teleport the single-qubit operation $U$, the state $|U\rangle_\beta = (I \otimes U \sigma^\beta) |0\rangle = |0\rangle + |1\rangle$ is prepared offline, by first preparing the state $|0\rangle$ and then measuring in the orthonormal basis of states $|U\rangle$. Alice and Bob now repeat the state teleportation protocol. With probability $1/4$ Alice finds $\alpha = \beta$, in which case Bob now has $U|\psi\rangle$. With probability $3/4$ she finds $\alpha \neq \beta$ and Bob needs to apply a correction $M_{\alpha\beta} = U \sigma^\alpha U^\dagger$ in order to end up with $U|\psi\rangle_3$. This is done by teleporting $M_{\alpha\beta}$, i.e., the procedure is repeated recursively. It succeeds on average after 4 trails.

FIG. 2: Gate teleportation of single-qubit operation $R_z$. Initially Alice has $|\psi\rangle_1$ and $|0\rangle$. Bob has $|1\rangle$. Time proceeds from left to right. Starting from the 3-qubit state $|\psi\rangle_1|0\rangle$, the task is to obtain $R_z |\psi\rangle$. The protocol shown succeeds with probability $1/2$. When it fails the operation $R_z^\dagger$ is applied instead. Fractions give the probability of a branch; 0 and 1 in a gray box are possible measurement outcomes of the observable in the preceeding gray box. See text for full details.
single-spin measurement of the observable $\sigma_x^j$ on each ancilla, which will yield either $|+\rangle$ or $|-\rangle$. For definiteness assume the outcome was $|+\rangle$. Observing that in the $\{|+\rangle, |−\rangle\}$ subspace, $H_{ij}^{\text{ex}} = -J_{ij}^x I + (J_{ij}^+ + J_{ij}^-)X$, where $X : |+\rangle \leftrightarrow |−\rangle$, it follows that $U(\pi/4 - \varphi_0, \varphi_0)|+\rangle = e^{-i\varphi_0/\sqrt{2}}( |+\rangle - i |−\rangle )$, so that we have a means of generating an entangled initial state. The unknown state $|\psi\rangle_1$ of the data qubit can be expressed as $|\psi\rangle = a_x |+\rangle + b_x |−\rangle$, where $a_x = (a + b)/\sqrt{2}$ and $b_x = (a - b)/\sqrt{2}$. Then (neglecting the overall phase $e^{-i\varphi_+}$):

$$\begin{align*}
|\psi\rangle_1 U_{23}(\pi/4 - \varphi_0^*, \varphi_0^*)|+\rangle_{23} &= \frac{1}{2} r^* |S\rangle_{12} R_{3x} |\psi\rangle_3 + \\
\frac{1}{2} r |T_{0}^x\rangle_{12} R_{3x}^* |\psi\rangle_3 + \frac{1}{\sqrt{2}}(a_x |+\rangle + b_x |−\rangle)\end{align*}$$

where $|T_0^x\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |−\rangle)$ is a triplet state, a zero eigenstate of the observable $\sigma_x + \sigma_y$. The gate teleportation procedure is now repeated to yield $R_x$ or $R_y$, respectively. First, Alice measures the total spin $S_{12}$. If she finds $S = 0$ (with probability $1/4$) Bob has spin $3$ in the desired state $R_{3x} |\psi\rangle_3$. If she finds $S = 1$ then she proceeds to measure the total length of the $x$ component $S_x = \frac{1}{4}(\sigma_x^1 + \sigma_x^2)^2$, yielding, provided she finds $S_x = 0$, the state $|T_0^x\rangle_{12} R_{3x}^* |\psi\rangle_3$ with probability $1/3$. If, on the other hand, she finds $S_x = 1$, i.e., the state is $a_x |+\rangle - b_x |−\rangle$, then by letting Bob measure $S_{23}^z$, the states $R_{12}^x |\psi\rangle_1 |S\rangle_{23}$ or $R_{1x} |\psi\rangle_1 |T_{0}^x\rangle_{23}$ are obtained, with equal probabilities.

Fig. 2 summarizes the protocol we have described thus far. The overall effect is to transform the input state $|\psi\rangle$ to either the output state $R_x |\psi\rangle$ or $R_y |\psi\rangle$, equiprobably.

We have now arrived at Cycle (ii), in which we must fix the erred state $R_{ij}^x |\psi\rangle_j$ ($j = 1$ or $3$). To do so we essentially repeat the procedure shown in Fig. 2. We explicitly discuss one example; all other cases are similar. Suppose that we obtain the erred state $R_{12}^x |\psi\rangle_1 |S\rangle_{23}$ [Fig. 3(e)]. It can be rewritten as

$$r R_{12}^x |\psi\rangle_1 |S\rangle_{23} = -\frac{i}{\sqrt{2}}(a |001\rangle - ib |110\rangle)$$

$$-\frac{1}{2} r^* |T_0\rangle_{12} R_{3z} |\psi\rangle_3 + \frac{1}{2} r^* |T_0\rangle_{12} R_{3z} |\psi\rangle_3,$$

which up to unimportant phases is identical to Eq. 4, except that the position of $R_{3z}^x$ and $R_{3z}$ has flipped. Correspondingly flipping the decision pathway in Fig. 3 will therefore lead to the correct action $R_{ij} |\psi\rangle$ with probability $1/2$, while the overall probability of obtaining the faulty outcome $R_{ij} |\psi\rangle$ after the second cycle of measurements is $1/4$. Clearly, after $n$ measurement cycles as shown in Fig. 3 the probability for the correct outcome is $1 - 2^{-n}$. The expected number of measurements per cycle is $\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} = 1$, and the expected number of measurement cycles needed is $\sum_{n=1}^{\infty} n 2^{-n} = 2$.

We note that in the case of the erred state $R_{ij}^x |\psi\rangle_j$ ($j = 1$ or $3$) there is an alternative that is potentially simpler than repeating the measurement scheme of Fig. 2. Provided the exchange Hamiltonian is of the XY type, or of the XXZ type with a tunable $J^z$ exchange parameter, one can simply apply the correction operator $U_{J^z}(\frac{\pi}{2}, 0) = Z_j Z_3$ to $R_{ij}^x |\psi\rangle_j$, yielding $R_{ij}^x |\psi\rangle_j$ as required. Finally, we note that Nielsen [27] has discussed the conditions for making a gate teleportation procedure of the type we have proposed here, fault tolerant.

To conclude, we have proposed a gate-teleportation method for universal quantum computation that is uniformly applicable to Heisenberg, XY and XXZ-type exchange interaction-based quantum computer (QC) proposals. Such exchange interactions characterize almost all solid-state QC proposals, as well as several quantum optics based proposals [12]. In a number of these QC proposals, e.g., quantum dots [3], exchange interactions are significantly easier to control than single-qubit operations [8, 12]. Therefore it is advantageous to replace, where possible, single-qubit operations by measurements. Moreover, spin measurements are necessary for state read-out, both at the end of a computation and at intermediate stages during an error-correction procedure, and often play an important role in initial-state preparation. Our method combines measurements of single- and two-spin observables, and a tunable exchange interaction. In a similar spirit we have shown how to replace with measurements certain difficult single-qubit operations in a QC-proposal involving superconducting phase qubits. We hope that the flexibility offered by this approach will provide a useful alternative route towards the realization of universal quantum computation.

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