Gravitational Waves: A Test for Modified Gravity

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In a modified gravity theory, the propagation equation of gravitational waves will be in a non standard way. Therefore, it provides a crucial test on deviation of general relativity. In this paper, we propose a parameterized modification to the propagation of gravitational waves and study its effects on the tensor modes of cosmic microwave polarization B modes and its degeneracy to the tensor mode power spectrum index $n_t$ and its running $\alpha_t$. At last, we reported the current status on the detection to deviation of general relativity through the currently available data sets. Our results showed no significant deviation of general relativity was probed.

I. Introduction

A modified gravity (MG) at large scales and an addition of dark energy (DE) in general relativity (GR) both can explain the late time accelerated expansion of our Universe. However, they are totally different in nature. The previous one means the discovery of a new gravity theory, and the late one implies the existence of new energy component which is still unknown now. Therefore, the current task is how to distinguish MG theory from DE model through cosmic observations, in other words, is how to probe any deviation to GR or break the degeneracy between MG and DE in GR. Due to the great diversity of MG theories and DE models, it is almost impossible to test each of them one by one. On the other side, one needs a general formalism which grasps the main properties of MG theory and DE. Then the deviation to GR and properties of DE can be determined by the cosmic observations. Therefore, a parameterized MG theory and DE model were proposed. However, to find a character which can distinguish MG from DE efficiently is really complicated. In the literature, the parameterized modification to GR in the scalar mode perturbation was studied, see [1, 2] for examples, and modification to the tensor mode perturbation was studied recently in Refs. [3–7] where the speed of gravitational waves can deviate from the speed of light \textsuperscript{1}. If it is true, this difference of speeds could produce interesting phenomena and difficulties, for example the causality problem. It means that one can feel the gravitational force firstly, then sees the object in sequence, when the speed of gravitational waves is larger than the speed of light. Again, if the speed of gravitational waves is smaller than the speed of light, to understand the concept of space-time is a challenge based on the relation between space-time and the distribution of energy-momentum. Actually, it also commits the causality problem. Due to these difficulties, in this paper, we will not consider this kind of modified gravity theory any more. Therefore, we propose a general parameterized modification to the propagation of gravitational waves in the following form

\[ \ddot{h} + 3\xi(k, a)H\dot{h} + \frac{k}{a^2}h = 8\pi G\mu(k, a)\Pi, \]

where $\xi(k, a)$ parameterizes the deviation to GR coming from a MG theory in tensor mode perturbation. The $\mu(k, a)$ term comes form the well-known effective time and scale variable Newtonian gravitational constant. To study its effect on the tensor modes of cosmic microwave polarization B modes on different time ($a$) and space scale ($k$), we propose an explicit function of $a$ and $k$ in the following form, as an example,

\[ \xi(k, a) = \frac{1 + \beta_1 a^2}{1 + \lambda_1 a^2}, \]

which is inspired by the forms proposed by Bertschinger and Zukin [1]. The modification to the scalar mode perturbation was taken in the following form [1]

\[ \mu(k, a) = \frac{1 + \beta_1 a^2}{1 + \lambda_1 a^2}, \quad \gamma(k, a) = \frac{1 + \beta_2 a^2}{1 + \lambda_2 a^2}. \]

It has already been pointed out that it may not be so easy to distinguish MG and DE in the scalar perturbation mode due to the facts that the perturbation and anisotropy stress of DE can introduce an effective time and scale variable Newtonian gravitational constant [9]. It was also reported that the anisotropic stress of dark energy can modify the propagation of gravitational waves [4]. However, we assume the anisotropic stress modification is stored in the $\mu(k, a)$ term. It is reasonable because it modifies the potential slip in the form of [4]

\[ \Phi - \Psi = \sigma(k, a)\Pi. \]

It implies the $\xi(k, a)$ term describes the pure MG effects which cannot be mimicked by DE in GR. Therefore it provides the character which can distinguish MG and DE in principle.

Recently, the Background Imaging of Cosmic Extragalactic Polarization (BICEP2) experiment [10, 11] has detected the B-modes of polarization in the cosmic microwave background, where the tensor-to-scalar ratio $r = 0.20^{+0.07}_{-0.05}$ with $r = 0$ disfavored at $7.0\sigma$ of the lensed-ΛCDM model was found. Although, combining with WMAP9 polarization data,

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\textsuperscript{1} For the measurement of the speed of cosmological gravitational waves, one can see in Ref. [8]
ACT and SPT, Planck group reported a much smaller tensor-to-scalar ratio compared to that from BICEP2 $r < 0.11$ at 95% C.L. in the $\Lambda$CDM+$r$ model [12]. However, it was debated that what BICEP2 detected is not the signal of the primordial gravitational waves but is the contamination coming from the foreground [13–15]. Recently, Planck group measured the dust angular power spectrum of $D_{l}^{XX} = k(l + 1)c_{l}^{XX}/(2\pi)$ ($XX$ denotes EE or BB) over the multipole range $40 < l < 600$ at intermediate and high Galactic latitudes form 100GHz to 353GHz. Extrapolation of the Planck 353GHz data to 150GHz gives almost the same magnitude as that of BICEP2 signal [16] in the range $40 < l < 120$. Huang, et al reanalyzed the simple $\Lambda$CDM+$r$ model with the addition of Planck dust data and didn’t find any significant evidence of the primordial gravitational waves [17]. Considering the error bars of the dust polarization parameters $D_{l}^{BB/EE}$, $A_{TT/EE}$ and $a^{XX}$, Xu reinvestigated the $\Lambda$CDM+$r$ model and found very weak primordial gravitational waves signals at the cost of decreasing the value of $D_{l=40}^{BB}$ to $0.67^{+0.25}_{-0.25}$ with the running of the scalar spectral tilt [18]. Before it is done and dusted, more data are required. Anyway, the gravitational waves allow us to study the possible deviation to GR in tensor perturbation case.

This paper is structured as follows. At first, in Section II we show the effects to the CMB TT and BB power spectrum from the possible deviation to GR in the proposed forms (1) and (2). To confirm the effects coming from MG purely, we should also test the effects of the spectrum index $n$ and its running $\alpha = dn_{i}/d \ln k$ to the CMB TT and BB power spectrum. Therefore we also discuss the degeneracies between MG and inflation model. We reported the current status of the discrimination of MG from GR by the cosmic observations and performing Markov chain Monte Carlo (MCMC) analysis in Section III. Section IV is the conclusion.

II. Effects to the CMB TT and BB Power Spectrum

The degeneracy between a dark energy model and a modified gravity theory at the background level is well-known. Therefore, in this work, the $\Lambda$CDM model is respected at the background. It means that the deviation to GR occurs at the first order perturbation level. Although for a concrete modified gravity theory, the model parameter space is a little worse constrained due to the lack of geometrical information at the background level, it is already useful to study the possible deviation to GR in cosmological scales.

To study the effects to the CMB TT and BB power spectrum, we modified the MGCAMB code [2] to include the modification to the tensor perturbation equation as shown in Eq. (2). The relevant cosmological parameters are fixed to their mean values obtained by Planck group [19] and BICEP2 group [11], but the values of MG parameters run freely. We show the effects to the CMB TT and BB power spectrum in Figure 1 and Figure 2 due to the modification to GR from $\mu(k,a)$ and $\xi(k,a)$ respectively.

FIG. 1. The effects to CMB TT and BB power spectrum with the variation of $\beta$, $\lambda^{2}$ and $s$, where the relevant cosmological parameters are fixed to their mean values obtained by Planck group [19] and BICEP2 group [11]. The other relevant MG parameters are fixed as follows: $\beta = 0.78$, $\lambda^{2} = 1.0E4$, $\beta_{s} = 1.125$ and $\lambda^{2} = 0.67E4$. 
In Figure 1, the modification to GR comes from an effective time and scale variable redefinition of the Newtonian constant \( \mu(k, a) \). This modification will change the potential along the line of sight, therefore it has effects to the integrated Sachs-Wolfe effect as shown at \( l < 100 \) in the CMB TT power spectrum. And this modification has also lensing effects on the CMB BB power spectrum as shown at \( l > 100 \) where the lensing effects are dominated. This modification keeps the CMB BB power spectrum almost untouched at low multipole \( l < 10 \).

In Figure 2, the effects to CMB TT and BB power spectrum due to the modification of \( \xi(k, a) \) term is shown. This term changes the expansion rate of B mode propagation. As a result, it has effects on the CMB TT and BB power spectrum at low multipole \( l < 10 \), i.e. the large scale.

Comparing the CMB BB power spectrum in the Figure 1 and the Figure 2, one can discriminate the MG theory due to modification of B mode propagation equation (expansion rate) from that of the effective Newtonian gravitational constant. As is well known, the anisotropic stress of dark energy will also modify the gravitational potential slip. As a result, this modification introduce an effective Newtonian gravitational constant. Therefore, the CMB BB power spectrum provides a crucial tool to distinguish dark energy model with anisotropic stress and MG theory. Furthermore, the CMB BB power spectrum is crucial to detect the deviation to GR. But it depends on the measurement at low multipole \( l < 10 \), i.e. at the large scales.

However, all the above observations are based on the facts that the tensor-to-scalar ratio \( r \), the tensor mode power spectrum index \( n_t \) and its running \( \alpha_t \) are fixed. Therefore, one should worry about the degeneracies between MG and inflation models in GR. Here, we show the effects to the CMB TT and BB power spectrum due to different values of \( r, n_t \) and \( \alpha_t \) in Figure 3. We only focus on the CMB BB power spectrum. One can see \( r \) will change the total amplitude at \( l < 100 \) but keep the shapes. The \( n_t \) changes the amplitude and shape simultaneously. And the running \( \alpha_t \) changes the shape at \( l < 10 \) and has little effects in the range \( l \in (10, 100) \). Therefore, \( \alpha_t \) mainly has the same effects to the CMB BB power spectrum as that of MG theory. Thus, to confirm the MG or dark energy, one still needs to understand the inflation very well. So in this work, we assume the inflation model is parameterized by \( r \) only.

III. Data Set and Results

In this section, we show the current status of the discrimination of MG from GR, under the assumption that the inflation model is well understood and is parameterized by \( r \) only, through the cosmic observations which include:

(i) The newly released BICEP2 CMB B-mode data [10, 11]. It will be denoted by BICEP2.

(ii) The full information of CMB which include the recently released Planck data sets which include the high-l TT likelihood (CAMSpec) up to a maximum multipole number of \( l_{\text{max}} = 2500 \) from \( l = 50 \), the low-l TT likelihood (lowl) up to \( l = 49 \) and the low-l TE, EE, BB likelihood up to \( l = 32 \) from WMAP9, the data sets are available on line [20]. This data set combination will be denoted by P+W.

(iii) For the BAO data points as ‘standard ruler’, we use the measured ratio of \( D_V/r_s \), where \( r_s \) is the co-moving sound horizon scale at the recombination epoch, \( D_V \) is the ‘volume
distance’ which is defined as

\[ D_v(z) = [(1 + z)^2 D_A^2(z)cz/H(z)]^{1/3}, \]

where \( D_A \) is the angular diameter distance. The BAO data include \( D_v(0.106) = 456 \pm 27 \) [Mpc] from 6dF Galaxy Redshift Survey [21]; \( D_v(0.35)/r_s = 8.88 \pm 0.17 \) from SDSS DR7 data [22]; \( D_v(0.57)/r_s = 13.62 \pm 0.22 \) from BOSS DR9 data [23]. This data set combination will be denoted by BAO.

(iv) The present Hubble parameter \( H_0 = 73.8 \pm 2.4 \) [km \( s^{-1} \)Mpc\(^{-1}\)] from HST [32] is used.

(v) The ten \( f\sigma_8(z) \) data points from the redshift space distortion (RSD) are used, they are summarized as in Table I.

We perform a global fitting on the Computing Cluster for Cosmos by using the publicly available package CosmoMC.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\# & \( z \) & \( f\sigma_8(z) \) & Survey and Refs \\
\hline
1 & 0.067 & 0.42 \pm 0.06 & 6dFGRS (2012) [25] \\
2 & 0.17 & 0.51 \pm 0.06 & 2dFGRS (2004) [26] \\
3 & 0.22 & 0.42 \pm 0.07 & WiggleZ (2011) [27] \\
4 & 0.25 & 0.39 \pm 0.05 & SDSS LRG (2011) [28] \\
5 & 0.37 & 0.43 \pm 0.04 & SDSS LRG (2011) [28] \\
6 & 0.41 & 0.45 \pm 0.04 & WiggleZ (2011) [27] \\
7 & 0.57 & 0.43 \pm 0.03 & BOSS CMASS (2012) [29] \\
8 & 0.60 & 0.43 \pm 0.04 & WiggleZ (2011) [27] \\
9 & 0.78 & 0.38 \pm 0.04 & WiggleZ (2011) [27] \\
10 & 0.80 & 0.47 \pm 0.08 & VIPERS (2013) [30] \\
\hline
\end{tabular}
\caption{The data points of \( f\sigma_8(z) \) measured from RSD with the survey references.}
\end{table}

For the growth function, the values of \( f = d \ln \Delta_m/d \ln a \) at different values \( a \) and \( k \) are stored in a two dimensional table, where \( \Delta_m = \delta_m + 3H(1 + \omega_m)\theta_m/k^2 \). Then their values can be used freely.

(vi) The consistence of \( \Omega_m \) between Ia supernovae and Planck 2013 was shown by SDSS-II/SNLS3 joint light-curve analysis, for the details please see [31].

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Parameters} & \textbf{Prior} & \textbf{Mean with errors} & \textbf{Best fit} \\
\hline
\( \Omega_{\Lambda}h^2 \) & [0.005, 0.1] & 0.0217 \pm 0.0024 & 0.02209 \\
\( \Omega_b h^2 \) & [0.005, 0.99] & 0.1164 \pm 0.0116 & 0.1162 \\
\( 100\theta_{MC} \) & [5, 10] & 1.04161 \pm 0.01056 & 1.04177 \\
\( \tau \) & [0.01, 0.81] & 0.0888 \pm 0.0050 & 0.105 \\
\ln(10^{10}A_s) & [27, 4] & 3.077 \pm 0.0921 & 3.111 \\
n_s & [0.9, 1.1] & 0.9691 \pm 0.0167 & 0.9713 \\
r & [0.1] & 0.2074 \pm 0.0072 & 0.246 \\
\beta_1 & [0.5, 1.5] & 0.947 \pm 0.0119 & 0.974 \\
\sigma_8 & [0.1, 0.7E8] & 2.92E8 & 2.92E8 \\
\beta_1 & [0.4] & 1.25 \pm 0.125 & 1.10 \\
\sigma_8 & [0.5E1, 0.6E8] & 5.2E8 \pm 0.5E8 & 5.2E8 \\
\beta_1 & [0.23] & 11.8 \pm 6 & 3.9 \\
\beta_1 & [1E5, 1E7] & 0.5E7 & 0.5E7 \\
\delta & [0.1E2, 0.7E8] & 6.66E8 & 6.66E8 \\
\hline
\end{tabular}
\caption{The mean values with 1\( \sigma \) errors and the best fit values of the model parameters and the derived cosmological parameters, where the Planck 2013, WMAP9, BAO, BICEP2, JLA, HST and RSD data sets were used. ‘~’ denotes the one which is not well constrained.}
\end{table}

\[ P = \{ \Omega_b h^2, \Omega_{\Lambda} h^2, 100\theta_{MC}, \tau, \ln(10^{10}A_s), n_s, r, \beta_1, \beta_2, \lambda_2, \beta_3, \lambda_3, s \}, \]

their priors are shown in the second column of Table II. The obtained results are shown in Table II for the data combinations: Planck 2013, WMAP9, BAO, BICEP2, JLA, HST and RSD. The obtained contour plots for the interested model parameters are shown in Figure 4.
One can clearly see that no significant deviation from GR was detected for the scalar perturbations even in 1σ regions. For the tensor perturbations, as shown in Figure 2, one cannot obtain any tight constraint to the model parameters $\lambda_2^3$ and $\beta_3$ due to the lack of data points below $l < 10$. However, to detect the possible deviation from GR in this region is a tough task due to the domination by the cosmic variance.

IV. Conclusion

In this paper, we proposed a general parameterized modification to the propagation of gravitational waves inspired by the forms presented in Ref. [1]. In this general form, we showed the effects to the CMB TT and BB power spectrum due to this kind of modification to GR by adopting different values of the model parameters. We also showed the possible degeneracy to the tensor mode power spectrum index $n_t$ and
its running $\alpha_t$. Our analysis reveals that the modification to GR has effects on the CMB TT and BB power spectrum at low multipole $l < 10$, i.e. the large scale and keeps the shape of the CMB BB power spectrum. And the tensor mode power spectrum index $n_t$ and its running $\alpha_t$ have effects to CMB BB power spectrum in the range $l \in (1, 100)$ and change the shape of the CMB BB power spectrum. It implies a precise data points below $l \sim 10$ can break this degeneracy between them. However it is a tough task due to the domination by the cosmic variance in this region.

We also used the currently available cosmic observational data sets, which include Planck 2013, WMAP9, BAO, BICEP2, JLA, HST and RSD, to detect the possible deviation to GR. The results were gathered in Table II and Figure 4. For the scalar perturbation part, the model parameters $\lambda^2_1$ and $\lambda^2_2$ vary in large ranges and were not well constrained. Due to the large values of $\lambda^2_1$ and $\lambda^2_2$, the values of $\beta_1 = 0.947^{+0.077}_{-0.051}$ and $\beta_2 = 1.25^{+0.125}_{-0.30}$ imply no significant deviation to GR even in $1\sigma$ region. For the tensor perturbation part, the model parameter $\beta_3$ and $\lambda^2_3$ were not well constrained due to the lack of data points. Therefore, one still cannot read any valuable information in this polarization data sets now.

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