SELF-CONSISTENT EVOLUTION OF GAS AND COSMIC RAYS IN CYGNUS A AND SIMILAR FR II CLASSICAL DOUBLE RADIO SOURCES

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ABSTRACT

In Cygnus A and other classical FR II double radio sources, powerful opposing jets from the cores of halo-centered galaxies drive out into the surrounding cluster gas, forming hotspots of shocked and compressed cluster gas at the jet extremities. The moving hotspots are sandwiched between two shocks. An inner-facing shock receives momentum and cosmic rays from the jet and creates additional cosmic rays that form a radio lobe elongated along the jet axis. An outer-facing bow shock moves directly into the undisturbed group or cluster gas, creating a cocoon of shocked gas enclosing the radio lobe. We describe computations that follow the self-consistent dynamical evolution of the shocked cluster gas and the relativistic synchrotron-emitting gas inside the lobes. Relativistic and non-relativistic components exchange momentum by interacting with small magnetic fields having dynamically negligible energy densities. The evolution of Cygnus A is governed almost entirely by cosmic ray energy flowing from the hotspots. Mass flowing into hotspots from the jets is assumed to be small, greatly reducing the mass of gas flowing back along the jet, common in previous calculations, that would disrupt the spatial segregation of synchrotron-loss ages observed inside FR II radio lobes. We compute the evolution of the cocoon when the velocity and cosmic ray luminosity of the hotspots are constant and when they vary with time. If cosmic rays mix with cluster gas in hotspots before flowing into the radio lobe, the thermal gas is heated to mildly relativistic temperatures, producing an unobserved pressure inside the lobe.

Key words: galaxies: clusters: intracluster medium – galaxies: individual (Cygnus A) – hydrodynamics – radio continuum: galaxies – X-rays: galaxies: clusters – X-rays: individual (Cygnus A)

Online-only material: color figures

1. INTRODUCTION

Figure 1 shows a ~200 ks Chandra image of the archetypical twin-jet FR II source Cygnus A and a second X-ray image with radio contours at 5 GHz superimposed (Wilson et al. 2006). The bright core of Cygnus A is coincident with the nucleus of a massive galaxy at the center of a cluster with mass exceeding ~3 × 10^14 M⊙ (Smith et al. 2002). We adopt a distance ~230 Mpc to Cygnus A at which 1″ corresponds to 1 kpc.

As first discussed by Blandford & Rees (1974) and Scheuer (1974), Cygnus A and other FR II radio sources are formed by two essentially identical oppositely directed jets created near cluster-centered black holes that penetrate the cluster gas at mildly relativistic speeds ≈0.01–0.1c. The jets impact the cluster gas in strong (reverse) shocks at the inner boundaries of bright kpc-sized “hotspots” visible in Figure 1 at the tips of the radio and X-ray images about 60 kpc from the center. The supersonic motion of these relatively dense hotspots through the ambient cluster gas forms symmetric outward-facing bow shocks that propagate directly into undisturbed cluster gas. The football-shaped bow shock forms a cocoon of shocked cluster gas that encloses the radio jets and lobes. The age of Cygnus A, about 10^7 yr, can be estimated from the observed age of the oldest radio-synchrotron electrons (e.g., Machalski et al. 2007). From this age and the hotspot–center distance, the average velocity of the hotspots is about v_{hs} ~ 6000 km s^{-1} which should also be consistent with the strength of the cocoon bow shock. The total power being delivered to the hot gas is ~10^{46} erg s^{-1}, as suggested by Wilson et al. (2006).

We describe here approximate computations of the evolution of Cygnus A—and FR II sources in general—that include for the first time dynamical interactions between the hot cluster gas and the relativistic fluid confined inside the radio lobes. In view of the unknown nature and contents of the Cygnus A jets and the very small volume they occupy, we regard the hotspot as the principal source of energy. As a hotspot moves at some assumed velocity away from the cluster center, gas inside the hotspot receives a local compression from the jet impact. Moreover, the jet–hotspot shock transmits cosmic rays (CRs) from the jet and accelerates additional CRs associated with the shock compression. By regarding the hotspots as sources of dynamic and relativistic energy we avoid a detailed computation of the flow of energy and relativistic particles along the jet about which rather little is known from observation.

2. THE RADIO LOBES

The relativistic electrons responsible for the non-thermal radio-synchrotron emission in Figure 1 are assumed to be transported to the hotspot in the jets or produced locally in the strong jet–hotspot shock, or both. In a weakly magnetized plasma electrons and protons in the high-energy tail of the Maxwellian thermal distribution can be accelerated to relativistic energies by multiple scatterings across strong shock waves (Blandford & Eichler 1987). CR acceleration increases with the shock duration and Mach number. For this reason we see 5 GHz synchrotron emission from relativistic electrons streaming away from...
powerful jet-driven shocks in the luminous hotspots in Figure 1, but not from the much weaker bow shocks (Mach \( \sim 1.3–2.5 \)) that enclose the cocoon. Evidently hotspots receive a huge momentum kick and compression at their inner edges where the jet impact occurs. As a result, gas flows away from the hotspots at a high, nearly sonic velocity in all directions roughly perpendicular to the jet axis, carrying gas and newly accelerated relativistic electrons with it. This powerful transverse flow contributes to the width of the cocoon-enclosing shock perpendicular to the jet axis and the width of the radio-emitting lobes.

The double-lobe radio image at 5 GHz in Figure 1 does not show the full extent of the radio lobes in Cygnus A. Non-thermal emission at lower radio frequencies extends all the way to the center where the two lobes merge, maintaining an approximately uniform width (e.g., Steenbrugge et al. 2010). While FR II sources like Cygnus A are often referred to as classical radio double sources, many authors use the term “bridge” if the low-frequency radio emission extends continuously to the cluster center. The 5 GHz lobes in Figure 1 appear truncated at their inner boundaries because the electrons that radiate most efficiently at this (relatively high) frequency are no longer present closer to the center of Cygnus A, reflecting a loss of electron energy and possible variations in the magnetic field. In fields of a few \( \mu \)G electrons of energy \( \gamma = E/m_e c^2 \sim 10^{10}–10^5 \) emit radio-synchrotron radiation concentrated near frequency \( \nu_c = \sqrt{\frac{2U}{\gamma_0}} \) and have lifetimes \( t_{\text{age}} \approx (3m_e c^2/4\pi\gamma^3)/\gamma u \), where \( u = (B^2 + B_{\text{cmb}}^2)/8\pi \) is the energy density and \( B_{\text{cmb}} = 3 \mu \)G is the equivalent magnetic field for cosmic microwave background inverse Compton (IC-CMB) losses. The entire region between opposing hotspots is filled with radio emission having progressively steeper radio spectra near the center where older electrons, ejected from the hotspot at earlier times, radiate at lower \( \nu \). This radio-wake provides essential information about the past velocity of the reverse shock and hotspot and the local magnetic field affecting \( t_{\text{age}} \).

Placing a transparent ruler on the Chandra image in Figure 1 reveals that (1) the brightest hotspots on the east and west sides are exactly collinear with the central galactic source and (2) the smaller hotspot in the west is aligned with faint, broad X-ray “jets” on both sides. Furthermore, the current direction of the very thin one-sided western (Doppler-boosted approaching) jet, visible in the radio image, is terminated by none of the hotspots visible on the west side. Evidently the jet direction varies with time.

Observations of IC-CMB X-ray emission arising from interactions of radio-synchrotron electrons with the CMB, when combined with low-frequency radio-synchrotron observations from CR electrons having the same energy, allow a direct determination of the magnetic field strength. Moreover, if the radio lobes in Cygnus A are assumed to be in approximate pressure equilibrium with their hot cocoon gas environment of known or estimated pressure, it is possible to estimate the energy density of the relativistic particles in the radio lobes. By this means it is found that the relativistic electron energy density greatly exceeds the magnetic field energy density by factors of 10–600 (Hardcastle & Croston 2010; Yajie et al. 2010). It is remarkable that the X-ray image in Figure 1 shows no clear evidence of a cavity associated with the 5 GHz radio emission, although the crowding of radio contours at the lobe boundaries indicates a sharp contact discontinuity between CRs and the thermal gas. Evidently the X-ray cavity is filled in with IC-CMB (and possibly also synchrotron self-Compton (SSC)) emission that fortuituously matches the thermal X-ray emission just outside the lobes so there is no easily visible X-ray cavity. Normally, one expects some diffusion of relativistic CR electrons, but the sharp radio lobe boundary indicates that very little diffusion occurs at this interface with the thermal gas. In addition, the stratification of radio-synchrotron electrons of different ages and energies along the Cygnus A radio cavity of age \( 10^7 \) yr implies an upper limit on the CR diffusion coefficient inside the radio cavity, \( \kappa < (60 \text{kpc})^2/(10^7 \text{yr}) \approx 10^{22} \text{cm}^2 \text{s}^{-1} \), which seems easy to satisfy.

3. THE PHANTOM HOTSPOT

The evolution of FR II radio sources similar to those in Cygnus A has been the subject of many detailed computations (e.g., Carvalho et al. 2005 or O’Neill & Jones 2010 and references therein). Nevertheless, there is little consensus about the internal nature or contents of the jets—electron pairs, CRs, magnetic field, thermal gas, etc. Perhaps the jets are initially purely electromagnetic but entrain some cluster gas and inertia as they progress outward. In addition, internal shocks perhaps arising from perturbations or changes in the shape of the jet wall geometry can rejuvenate and accelerate new relativistic particles inside jets. Nevertheless, for computational expediency in most or all previous computations of FR II jets the jet content is assumed to be thermal gas, sometimes with adiabatic index...
\( \gamma = 4/3 \) rather than 5/3. However, a significant mass of thermal gas flowing out in the jet invariably results in fast “backflows,” a contrary flow just outside the jet boundary that flows anti-parallel to the outgoing jet. After passing through the reverse shock in the hotspot, mass-carrying jets encounter high-pressure cluster gas locally compressed near the moving hotspot. This produces a positive pressure gradient in the gas just outside the jet (opposite to the negative pressure gradient in the undisturbed cluster gas) that drives backflows of shocked gas back toward the center of the cluster. Such computed backflows can generate strong Kelvin–Helmholtz (KH) instabilities and general turmoil which is not observed in FR II radio cocoons.

Thermal gas backflows are described in some detail by Krause (2005) who considered so-called “light” jets with internal densities that are \( \sim 10^{-4} \) below that of the initial central cluster gas. The (negative) backflow velocity is fast, typically exceeding in magnitude the outward velocity of the hotspot—by nearly 10 in the FR II calculations of Reynolds et al. (2002). However, strong backflows appear to be inconsistent with radio observations. In particular, computed backflowing gas moving through the radio cavity region becomes highly disordered by shear-driven KH instabilities. Such backflows would advect and upset the radially ordered age-related energy distributions of radio-synchrotron electrons observed along the radio lobes by Carilli et al. (1991) and illustrated by Steenbrugge et al. (2010).

Moreover, the sharply defined outer boundaries of the Cygnus A radio lobes in Figure 1 would not in general be possible. Finally, the faint irregular X-ray jet-like features visible in Figure 1, whatever their uncertain origin, and also the very narrow and sharply defined radio jet appear to be completely undisturbed by turbulent backflow activity similar to that predicted by most, possibly all, previous FR II calculations.

To avoid this undesirable outcome, we consider here jets that carry a very small amount of non-relativistic gas and which occupy such a small volume, as in the radio image in Figure 1, that they can be ignored altogether, i.e., our computation is driven not by the jet but by hotspots energized by the jet. Although our understanding of the physical environment within hotspots is still very incomplete, because of their relatively high luminosity and resolvable structure, more is known about the internal physics of hotspots (e.g., Stawarz et al. 2007) than the jets themselves.

The FR II hotspot energy source in our computations is represented with a “phantom” hotspot compression that moves out in the computational grid in the jet direction and is also the source of CR energy. When the hotspot compression moves supersonically relative to the cluster gas, bow shocks form, producing cocoons of shocked gas as observed in Cygnus A. While there is no compelling observational evidence of deceleration in FR II sources (O’Dea et al. 2009) we nevertheless entertain this possibility. The phantom hotspot velocity is parameterized as follows:

\[
v_{hs} = (v_0 - v_\infty)e^{-t/\tau} + v_\infty, \tag{1}\]

where

\[
\tau = t/t_a \tag{2}\]

is the time normalized by the current age \( t_a \) of the FR II structure and \( v_0 \) is the initial velocity. The time parameter \( t_0 = \tau_0/t_a \) defines the deceleration epoch of the hotspot compression which moves with uniform velocity \( v_0 \) if \( \tau_0 = t_0/t_a \) is assumed to be very large. The average velocity at any time is

\[
\langle v_{hs} \rangle = \frac{\tau_0}{\tau}(v_0 - v_\infty)(1 - e^{-t/\tau}) + v_\infty \tag{3}\]

and \( r_{hs} = (v_{hs})t \) is the radial position of the hotspot at any time. The final velocity \( v_\infty \) is not a free parameter but instead ensures that the mean velocity is \( \langle v_{hs} \rangle = r_{hs}/t_a \) at time \( t_a \) for any choice of \( v_0 \) and \( t_0 \), i.e.,

\[
v_\infty = \frac{(v_{hs})_a - v_0\tau_0(1 - e^{t/\tau_0})}{1 - \tau_0(1 - e^{t/\tau_0})}. \tag{4}\]

We assume that the volume of the phantom hotspot \( V_{hs} \) remains constant during its evolution. When the CR power density \( S_{cr} \) injected into the moving hotspot is uniform in space and time, then the rate of change of the energy density of CRs in the hotspot \( e_{cr} \) is also constant:

\[
\dot{S}_{hs} = \frac{dE_{e,hs}}{dt} = \frac{L_{cr}}{V_{hs}} \text{ erg cm}^{-3} \text{ s}^{-1}, \tag{5}\]

where \( L_{cr} \) is the mean CR luminosity of the hotspot over time \( t_a \). However, when the hotspot has a non-uniform velocity, the phantom hotspot receives the same total energy in CRs over time \( t_a \), but at a variable rate,

\[
\dot{S}_{hs} = \left(\frac{(v_{hs})_a}{v_{hs}(\tau)} \frac{L_{cr}}{V_{hs}} \text{ erg cm}^{-3} \text{ s}^{-1}.\right. \tag{6}\]

While the calculations we describe here are not intended to exactly reproduce the observed radio and X-ray properties of Cygnus A, we choose parameters that approximate those of this relatively nearby and well-observed FR II source. The current age of Cygnus A, \( t_a = 10^5 \) yr, is taken from a recent detailed analysis of the radio spectrum distribution along the radio lobe axes (Machalski et al. 2007). The current distance of the Cygnus A hotspot from the galactic core, \( r_s \approx 60 \) kpc, is based on the assumption that the source is oriented in the plane of the sky although the higher visibility of radio jets on the western side suggests that this side may be aligned toward the center of the galaxy. With these parameters the mean velocity of the hotspot, \( \langle v_{hs} \rangle = r_s/t_a \approx 5870 \text{ km s}^{-1} \), is well in excess of the sound speed in the hot cluster gas, \( c_s \approx 1100(T/4.6 \text{ keV})^{1/2} \text{ km s}^{-1}. \)

Also of interest is the possibility that the CR energy provided to the Cygnus A hotspots by the jets and reverse shock can vary with time in a manner that is independent of the hotspot velocity. FR II observational data suggest that neither the jet power nor the hotspot velocity varies with time (O’Dea et al. 2009), but these statistical arguments are based on a sample of 31 FR II sources that contains very little information about hotspots at small distances, \( \lesssim 50 \) kpc, that are most relevant to the past history of Cygnus A of current size \( r_s \approx 60 \) kpc. The size of FR II hotspots is observed to increase with the distance of the hotspot from the core of the host galaxies (Jeyakumar & Saikia 2000; Peruchi & Marti 2003; Kawakatu et al. 2008), where it is assumed that FR II sources evolve from the class of compact and medium-sized symmetric radio sources.

If the CR power density received by the hotspot has an intrinsic time variation, this can be expressed with a properly normalized dimensionless function \( \sigma_{hs}(\tau) \) of \( \tau = t/t_a \),

\[
\dot{S}_{hs} = \left. \frac{dE_{e,hs}}{dt} = \frac{L_{cr}}{V_{hs}} \sigma_{hs}(\tau) \text{ erg cm}^{-3} \text{ s}^{-1}. \right. \tag{6}\]

The dimensionless time variation is assumed to be

\[
\sigma_{hs}(\tau) = \frac{(1 + \eta) + (1 - \eta) \tanb[(\tau - \tau_0)/(\Delta \tau)]}{(1 + \eta) + (1 - \eta) \Delta \tau \ln \left(\frac{\cosh[(1 - \tau)/\Delta \tau]}{\cosh[-(1 - \tau)/\Delta \tau]}\right)}, \tag{7}\]
where $\eta$, $\Delta t$, and $\tau_c$ are adjustable parameters. If $\eta = 1$ then $\sigma_{hs} = 1$ and the jet power of the hotspot does not vary; if $\eta < 1$, the numerator in the expression for $\sigma_{hs}$ increases smoothly from $(1+\eta)+(1-\eta)\tanh[-\tau_c/(\Delta t)](1+\eta)+(1-\eta)\tanh[(1-\tau_c)/(\Delta t)]$ near time $\tau_c$, during a time interval parameterized with $\Delta t$. The denominator in Equation (6) (which does not vary with $\tau$) normalizes $\sigma_{hs}(\tau)$ so that
\[
\int_0^1 \sigma_{hs}(\tau)d\tau = 1, \quad (8)
\]
equating that the mean rate of CR injection into the hotspot at time $\tau_0$ is $L_{CR}/V_{hs}$ for any choice of parameters $\eta$, $\Delta t$, and $\tau_c$.

In Section 5, we describe how the gas and CR dynamics of FR II sources like Cygnus A can be completely determined by the assumed hotspot evolution in a given cluster environment.

4. COMPUTATIONAL PROCEDURE

4.1. Equations of Cosmic Ray and Gas Dynamics

Magnetic fields of strength $0.3-10$ $\mu$G are ubiquitous in cluster gas (Govoni & Feretti 2004) and are generally stronger in the enhanced feedback environment of cool-core clusters (Feretti et al. 2009). The origin of these fields is controversial but only their presence concerns us here. Fields at this level, with energy densities $B^2/8\pi \approx 10^{-13}(B/4 \mu G)^2$ erg cm$^{-3}$, cannot significantly alter the dynamics of cluster gas with thermal energy density $3P/2 = 5 \times 10^{-11}(n_e/0.01 \text{ cm}^{-3})(T/\text{keV})$ erg cm$^{-3}$. Since $B^2$ and $P$ both decrease with cluster radius in about the same way, $B^2/8\pi \ll 3P/2$ holds at every radius. Magnetic fields are almost always dynamically subordinate in the cluster gas.

CRs are spatially confined by their relativistic Larmor radii and are assumed to be confined eddy current times and to be gaseous fluid. Typical radio-synchrotron CR electrons with energies $\gamma \sim 10^3-10^5$ and have small gyroradii, $r_e = \gamma mc^2/eb \approx 4 \times 10^{12}(\gamma/10^4)(B/4 \mu G)^{-1}$ cm. Pressure gradients in the CR fluid communicate momentum to fields and also to the cluster gas since fields and gas are frozen together. Most importantly, CRs and thermal gas can exchange momentum even when the energy density of the communicating field is very small, having no dynamical consequence of its own.

The dynamical interaction of CR with hot gas in groups/ clusters can be studied by solving the following equations:
\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0 \quad (9)
\]
\[
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) = -\nabla(P + P_e) - \rho \mathbf{g} + \rho \mathbf{a}_{hs} \quad (10)
\]
\[
\frac{\partial e}{\partial t} + \mathbf{u} \cdot \nabla e = -P(\nabla \cdot \mathbf{u}) \quad (11)
\]
\[
\frac{\partial e_c}{\partial t} + \mathbf{u} \cdot \nabla e_c = -P_e(\nabla \cdot \mathbf{u}) + \nabla \cdot (\kappa \nabla e_c) + S_{hs} \quad (12)
\]
where artificial viscosity terms are suppressed. Pressures and thermal energy densities in the plasma and CRs are related, respectively, by $P = (\gamma - 1)e$ and $P_e = (\gamma_e - 1)e_c$. The dynamics of the CRs are described by $e_c$, the integrated energy density over the CR energy, or momentum distribution, $e_c \propto \int E \mathrm{d}E \propto \int p^4 f(p)(1 + p^2)^{-1} dp$. If desired, $e_c$ can refer to the relativistic CR energy density for any combination of CR electrons or protons; we do not make this distinction in our models of Cygnus A, but synchrotron and IC radiation from relativistic electrons are obviously required. Because of their negligible collective rest mass, a mass conservation equation for the CRs is unnecessary. Although CRs and hot plasma are coupled by mutual interactions with a small magnetic field, no magnetic terms need explicitly appear in the equations provided magnetic stresses are small, $B^2/8\pi \ll P + P_e$, and the (Alfvén) velocity of the magnetic scatterers is small relative to the thermal gas (e.g., Drury & Falle 1986); these reasonable assumptions are adopted here. We consider only non-relativistic bulk velocities $u \lesssim 0.1c$, appropriate for our hotspot-driven flows. The hotspot acceleration $\sigma_{hs}$ and CR luminosity density $S_{hs} = d\varepsilon_{cr,inh}/dt$ are included as source terms in the equations above.

These equations are solved here using our own 2D axisymmetric ZEUS-like hydrocode in cylindrical coordinates. This code has been rather extensively checked in particular by exactly duplicating the shock structure of Jones & Kang (1990) which is modulated by diffusing CRs. In our code the CR diffusion term $\nabla \cdot (\kappa \nabla e_c)$ is solved using operator-splitting and fully implicit Crank–Nicolson differencing. As in previous recent papers (Mathews & Brighenti 2008; Mathews 2009), we employ a CR diffusion coefficient that varies inversely with the local gas density, crudely assuming that the magnetic field also scales with density. The diffusion coefficient depends on a single density parameter $n_{e,0}$:
\[
\kappa = \begin{cases} 10^{30} \text{ cm}^2 \text{s}^{-1} : & n_e \lesssim n_{e,0} \text{ cm}^{-3} \\ 10^{30}(n_{e,0}/n_e) \text{ cm}^2 \text{s}^{-1} : & n_e > n_{e,0} \text{ cm}^{-3}. \end{cases}
\]

However, as discussed earlier, it is very unlikely that the relativistic component in Cygnus A diffuses significantly during its short age $t_s = 10^7$ yr. For this reason we consider only a small density parameter $n_{e,0} = 6 \times 10^{-6}$ cm$^{-3}$ that effectively suppresses the role of CR diffusion in the solutions.

In the calculations described here, the CR component is assumed not to lose energy by synchrotron or inverse Compton emission. This is probably a reasonable assumption because, during the relatively short lifetime of CRs in Cygnus A, the total CR energy is not expected to be greatly degraded by this emission. Radiative losses depend on the square of the particle energy, while, for the original particle energy spectrum in Cygnus A, $N(\gamma) \propto \gamma^{-p}$ with $p = 2.2$ (Machalski et al. 2007), most of the total CR energy $e_c$ is contained in low-energy particles which emit very little radiation. However, the purely adiabatic evolution of the relativistic energy density $e_c$ does not mean that the total CR energy integrated over volume $E_c = \int e_c dV$ is conserved during the cocoon evolution. Mechanical $PdV$ work done by (or on) the CR fluid as it interacts with the cluster gas can alter its total energy.

4.2. The Cluster Surrounding Cygnus A

X-ray observations of the cluster gas surrounding Cygnus A are discussed in Smith et al. (2002) and subsequently modified by Wilson et al. (2006). The density and temperature profiles of this cluster are closely matched by those of Abell 478 which has been extensively observed by Vikhlinin et al. (2006). Consequently we use the temperature fitting functions for Abell 478 provided by Vikhlinin et al. but reduce all temperatures by a factor of 0.915 to exactly duplicate the innermost temperature measured near Cygnus A by Wilson et al., $kT = 4.60$ keV, at radius 32 kpc. We assume that the cluster potential is described with an NFW potential for virial mass $1.25 \times 10^{15} M_\odot$. and
We adopt a 2D cylindrical computational grid with 150 uniform zones \( \Delta r = \Delta z = 0.5 \) kpc extending out to 75 kpc in both directions, well beyond the Cygnus A cocoon at the present time. Beyond 75 kpc, 50 additional zones having geometrically increasing sizes extend the grid to 0.8 Mpc in both directions. The phantom hotspot moves out along the symmetry \( z \)-axis at velocity \( v_{\text{hs}}(t) \). In keeping with the kpc sizes of observed hotspots, we consider phantom hotspots that extend 1 kpc in the \( r \)-direction (two \( j \)-zones) and 0.5 kpc in the \( z \)-direction (one \( i \)-zone). When the instantaneous phantom hotspot radius satisfies \( z_{i-1} < r_{\text{hs}}(t) < z_{i+1} \), the innermost two \( j \)-zones at the \( i \)-th grid along the \( z \)-axis are identified as the hotspot. In updating the gas velocity during each computational time step \( \Delta t \), the hotspot zones receive an additional acceleration in the \( z \)-direction

\[
a_{\text{hs};i,j} = \frac{\rho_{i,j} v_{\text{hs}}(t)^2 A_{i,j}}{\rho_{i,j} A_{i,j} \Delta z} = \frac{v_{\text{hs}}(t)^2}{\Delta z},
\]

where \( A_{i,j} = \pi (r_{j+1}^2 - r_{j}^2) \) is the area of the \( j \)-th hotspot zone and \( \Delta z = z_{i-1} - z_{i} \). Time steps are chosen so that the phantom hotspot computation moves slowly along the \( z \)-axis, spending many time steps in each hotspot zone as it accelerates to \( v_{\text{hs}}(t) \). Consequently, each hotspot zone undergoes an additional operator-splitting step,

\[
u_{i,j}^{n+1} = \min \left[ u_{i,j}^n + v_{\text{hs}}^2 \Delta t/\Delta z, v_{\text{hs}}^n \right].
\]

During each time step \( \Delta t = t^{n+1} - t^n \) hotspot zones also receive an additional increment of CRs:

\[
e_{c_{i,j}}^{n+1} = e_{c_{i,j}}^n + \hat{S}_{\text{hs}} \Delta t
\]

and \( \hat{S}_{\text{hs}} \propto L_{\text{CR}}/V_{\text{hs}} \) (Equations (4), (5), or (6)), where \( V_{\text{hs}} \) is the total volume of all hotspot zones, assumed to be constant during the calculation.

5. COMPUTED COCOONS FOR CYGNUS A

We compare the distribution of gas and CRs in three dynamical models, all at time \( t_a = 10^7 \) yr when the hotspot has moved 60 kpc from the galaxy/cluster center. The total CR energy injected into the hotspot, \( E_{0,\text{CR}} = t_a L_{\text{CR}} \approx 3.15 \times 10^{60} \) erg, is the same in all models, but the mechanical work done by hotspot compression varies with the choice of \( v_{\text{hs}}(t) \).

5.1. Cocoon with Uniform \( v_{\text{hs}} \) and \( \sigma_{\text{hs}} \)

Figure 3 shows the FR II cocoon at time \( t_a = 10^7 \) yr for model 1 in which \( \eta = \sigma_{\text{hs}} = 1 \) (see Equation (5)) so that both the hotspot velocity \( v_{\text{hs}} = r_a/t_a = 5870 \) km s\(^{-1}\) and CR production are constant along the hotspot trajectory (horizontal or \( z \)-axis). The upper panel shows in cross section white contours for the CR energy density \( \rho_e(z, r) \) superimposed on a logarithmically scaled image of the gas density \( \rho(z, r) \). The very low density dark core of the cocoon is completely filled with CRs that define the radio lobe region. As in Figure 1, the CR contours in Figure 3 are closely spaced around the perimeter of the radio cavity where the relativistic and thermal gases meet in a contact discontinuity. The hotspot is visible at the far right as a small completely white region \( 60 \) kpc along the \( z \)-axis. The cavity surface is disturbed near \( (z, r) = (25, 7.5) \) kpc, but very little gas flows across the radio lobe boundary.

The lower panel in Figure 3 shows an image of the bolometric thermal X-ray surface brightness \( \Sigma_x = \int (\rho/m)^2 \alpha d\ell \int \ell \) integrated along the line of sight \( \ell \) assumed to be perpendicular to the \( z \)-axis. Here the bolometric emissivity \( (\rho/m)^2 \alpha \) erg cm\(^{-3}\) s\(^{-1}\) is evaluated assuming solar abundance. The contours in this panel outline the projected energy density \( \int e_v d\ell \), giving a rough idea of the limits and appearance of the radio emission in projection if the cavity magnetic field were uniform.

Figure 4 shows profiles of \( \rho(z, 0) \) (solid line) and \( e_v(z, 0) \) (dashed line) along the jet axis \( (r = 0) \) at time \( t_a = 10^7 \) yr. The dramatic concentration of CRs in the hotspot qualitatively resembles the Cygnus A X-ray and radio images (Figure 1), but the computed hotspot is not bright in X-rays in Figure 3 since we do not include SSC or IC emission from the CMB. As expected,
the gas density is compressed just ahead of the hotspot forming the apex of the cocoon bow shock.

The image in Figure 5 shows the temperature distribution $T(z,r)$ of the extremely hot, very low density thermal gas within the radio cavity. The thermal gas inside the radio lobe is evidently heated by multiple shocks produced by rapidly propagating waves in the relativistic gas trapped within the radio cavity walls. These low-amplitude waves are visible in Figure 4 and in the cavity contours in Figure 3. The temperature of essentially all of the heated thermal gas inside the X-ray cavity is relativistic, i.e., $T \gtrsim 5 \times 10^8$ K, so its thermal properties and total energy are not accurately computed with the non-relativistic gas equations used here. The acceleration of cluster gas to mildly relativistic velocities inside the radio cavity is almost certainly a collisionless process because of the very low particle density.

After $10^7$ yr the total mass of thermal gas with $T > 10^8$ K, all of it inside the radio cavity, is $1.7 \times 10^8 M_\odot$. Most of this gas flows directly from the hotspot. For example, the mass of a cylindrical core through the undisturbed cluster gas having the same 1 kpc radius as the hotspot and length equal to $r_h = 60$ kpc is $1.7 \times 10^8 M_\odot$, essentially identical to the mass of ultra-hot cluster gas. The average density of gas with $T > 10^9$ K inside the radio lobe is $(ne)_{\text{lobe}} = 1.8 \times 10^{-3}$ cm$^{-3}$, which is slightly less than 1% of the density in the original cluster gas in the same region. We suspect that the mass of ultra-hot cavity gas with $T > 10^9$ K depends on detailed grid-level assumptions about the exact spatial distribution of CRs inside the hotspot. In our calculations, we assume that CRs are completely mixed with the thermal gas in the hotspot grid zones, but if they are not so efficiently mixed, the outflow of thermal gas from the hotspot and the mass of ultra-hot cavity gas might be lowered.

In Figure 6 we show cross-section profiles of the gas pressure $P$, relativistic pressure $P_{\text{r}}$, and the gas density $n_e$ plotted perpendicular to the jet (or z) axis at $z = 10$ and 45 kpc, both at time $t_a = 10^7$ yr. The total pressure $P + P_{\text{r}}$ is shown with a dotted line, indicating a cocoon overpressure several times larger than that in the original cluster gas at both locations along the jet axis. The total pressure in the radio lobe, $P + P_{\text{r}} \approx 10^{10} \text{ dyn cm}^{-2}$, is essentially constant along the jet axis (as in Figure 4), consistent with an absence of significant mass backflow in the cavity region. In the lower panel of Figure 6 (at $z = 45$ kpc), note that the dash-dotted profile for the thermal gas density $n_e$ becomes very small within $r = 6$ kpc where the gas pressure is still dominant and appreciable; this explains the dark, low-density transition region in Figure 3 that extends a little beyond the region of CR confinement, most visible at $z \gtrsim 30$ kpc.

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**Figure 3.** Model 1 at time $t_a = 10^7$ yr. Top: contours of $e_c(z,r)$ superimposed on the image of $\log \rho(z,r)$ shown with values indicated in the color bar. Bottom: image of the projected bolometric X-ray and emission contours of the projected CR energy density $f_e d\ell$. The horizontal $z$-axis and the vertical $r$-axis are marked in (unlabeled) ticks every 2 kpc with larger ticks every 10 kpc; both panels are $70 \times 35$ kpc. The relativistic CRs are confined in an elongated cavity in the shocked hot gas which is enclosed within the cocoon bow shock. (A color version of this figure is available in the online journal.)

**Figure 4.** Profiles of $10^9 \rho(z,0)$ g cm$^{-3}$ (solid line) and $10^9 e_c(z,0)$ erg cm$^{-3}$ (dashed line) along the jet axis ($r = 0$) for model 1 at time $t_a = 10^7$ yr.

**Figure 5.** Image of log $T$ (K) of the very hot (thermal) gas within the X-ray cavity on the same $70 \times 35$ kpc spatial scale as in Figure 3. The hottest gas log $T \sim 11.5–12$ is in the wake just behind the hotspot and the temperature decreases to log $T \sim 9.5–10.5$ closer to the cluster center where it is being mixed with thermal gas flowing into the cavity near $(z, r) = (25, 7.5)$ kpc. (A color version of this figure is available in the online journal.)
The final drop in $P$ and $n_e$ in the panels of Figure 6 occurs at the bow shock, broadened by artificial viscosity. In blast waves of this type it is possible to determine the Mach number $M$ from the Rankine-Hugoniot shock jump transition $P_2/P_1$ or $\rho_2/\rho_1$. The gas pressure and density shock transitions in Figure 6 indicate $M \approx 1.8$ at both $z = 10$ and 45 kpc. This Mach number, $M = v_{sh}/c_s \approx 2.0$ is confirmed by direct measurement of the velocity $v_{sh}$ of the profiles during times near $t_a$ and the local cluster gas sound speed. Wilson et al. (2006) estimate a somewhat smaller Mach number, 1.3, from X-ray observations of the cocoon shock in Cygnus A.

While the cocoon in Figure 3 has many features in common with Cygnus A, the overall shape of the computed cocoon shock and radio cavity is different. The cocoon shock in Figure 1 is uniformly convex everywhere, unlike the mildly concave shock surface in Figure 3 with a slope change near $(z, r) = (10.7, 25)$ kpc. In addition, the radio “bridge” region that connects opposing hotspots in Cygnus A (e.g., Steenbrugge et al. 2010) is more nearly cylindrical, unlike the quasi-conical radio cavity region shown in Figure 3. Evidently, too many CRs are produced in model 1 near the center at early times in the evolution.

Our computed cocoon receives some energy from hotspot acceleration and compression but the CR energy, $E_{cr} = L_{cr} t_a = 3.1 \times 10^{60}$ erg, dominates. The allocations of cocoon energy into kinetic, thermal, and CRs at time $t_a = 10^7$ yr are listed in Table 1. The change in potential energy at time $t_a$ is negligibly small, but is expected to become larger at later times (Mathews & Brighenti 2008). It is seen that the relativistic CR energy has dropped to about a third of its original value because of $PdV$ work done on the surrounding cluster gas. To explore the energy introduced by hotspot acceleration, we compute “model 1nocr” for the cocoon evolution with hotspot acceleration as in model 1 but with no contribution from CRs, $L_{cr} = 0$. Figure 7, showing the cocoon density pattern $\rho(z, r)$ for model 1nocr at time $t_a$, is strikingly different from the top panel in Figure 3. The dark wake behind the moving hotspot along the $z$-axis is a region of only moderately higher temperature, $T \sim 15 \times 10^7$ K, about three times hotter than the original cluster gas. The shock wave is outwardly convex everywhere in Figure 7 and encloses a much narrower cocoon along the (vertical) $r$-axis than in Figure 3. The cocoon shock Mach numbers are also lower at corresponding places, $M = 1.17$ at $z = 10$ kpc and $M = 1.27$ at $z = 45$ kpc. The energetics for model 1nocr listed in Table 1 reveal that the cocoon kinetic and thermal energies are both very much less than in model 1. The kinetic energy generated by hotspot acceleration and compression in model 1nocr can be estimated by assuming that the total mass of initial cluster gas contained in a cylinder of hotspot area out to the current hotspot radius $r_a = 60$ kpc, $M_{bcyl} = 1.7 \times 10^8 M_\odot$, is accelerated to velocity $v_{hs} = 5800$ km s$^{-1}$. The resulting kinetic energy, $0.5 M_{bcyl} v_{hs}^2 = 0.057 \times 10^{60}$ erg, agrees very well with the relatively small total energy for model 1nocr in the final column of Table 1. This confirms the critical importance of CR energy to the overall cocoon energetics in model 1 and implies that the width and shape of the radio cavity are sensitive to the local rate that CRs are processed in the moving hotspot. The final column of Table 1 is the sum of the previous three columns, $\sum E_i = E_{kin} + \Delta E_{th} + E_{cr}$, which for model 1 is very close to the total CR energy injected, $E_{cr,\text{inj}} = L_{cr} t_a = 3.1 \times 10^{60}$ erg. This serves as a global check on the accuracy of the calculations and the energy check itself. Finally, we estimate the total energy inside the X-ray cavity in Figure 3; the sum of the internal energy $PV/(\gamma_c - 1)$ and the work done to generate the cavity.

### Table 1

| Model | $v_{hs}$ (km s$^{-1}$) | $\sigma$ | $E_{kin}$ (10^{60} erg) | $\Delta E_{th}$ (10^{60} erg) | $E_{cr}$ (10^{60} erg) | $\sum E_i$ (10^{60} erg) |
|-------|-------------------|-------|----------------|-----------------|----------------|----------------|
| 1     | 5866              | 1.0   | 0.608          | 1.482           | 0.893          | 2.98           |
| 1nocr | 5866              | 1.0   | 0.015          | 0.042           | 0.893          | 2.98           |
| 2     | 5866              | 0.609 | 1.497          | 1.022           | 3.13           |
| 3     | $v_{hs}(t)$      | 1.0   | 0.543          | 1.308           | 1.269          | 3.15           |

**Notes.**

* a Hotspot velocity.
* b Time variation of hotspot CR source.
* c Cocoon energies at time $t_a$: $E_{kin}$, kinetic; $\Delta E_{th}$, thermal (with original cluster energy subtracted); $E_{cr}$, CR. Final column is the sum of all three energies. All energies refer to one hemisphere.
* d Model 1 with no hotspot CRs.
* e Variable hotspot CR source: $\eta = 0.1$, $\tau_c = 0.3333$, and $\Delta \tau = 0.15$.
* f Variable hotspot velocity: $v_h = 2 \times 10^7$ km s$^{-1}$ and $v_0 = 1.5 \times 10^6$ yr.

Figure 7. Image of log $\rho(z, r)$ at time $t_a = 10^7$ yr for model 1nocr which is identical to model 1 but with no CRs produced in the hotspot. The image is 70 × 35 kpc.

(A color version of this figure is available in the online journal.)
The less conical shape of the radio cavity and the overall convex bow shock profile that are more in agreement with those of Figure 8.

Variation of the intrinsic hotspot CR luminosity factor \( \sigma(\tau) \) with parameters \( \eta = 0.1, \tau_e = 0.3333, \) and \( \Delta \tau = 0.1. \)

\( \dot{P} V \approx 4 \dot{P} V \approx 3.1 \times 10^{60} \) erg, which is very similar to the total CR energy injected, \( E_{\text{cr.inj}}. \) Here, we use the mean cluster gas pressure \( P = 1.1 \times 10^{-9} \) dy cm\(^{-2} \) within \( r_a \) and the volume of the computed cavity \( V = 2.4 \times 10^4 \) kpc\(^3 \).

5.2. Cocoon Models with Varying Hotspot Speed or Cosmic-ray Luminosity

The preceding discussion suggests that the shapes of the cocoon shock and radio cavity could be improved if the hotspot source produced fewer CRs during its early evolution. Such a reduction could be achieved if the CR luminosity of the hotspot were lower or if the hotspot velocity \( v_{hs} \) were larger at early times. In this section we adjust the hotspot parameters to explore these two limiting cases in more detail.

First consider cocoon model 2 having a non-uniform hotspot luminosity \( \sigma(\tau) \) with parameters \( \eta = 0.1, \tau_e = 1/3, \Delta \tau = 0.1, \) and source term \( S_{hs} \) as in Equation (6). Recall that \( \tau = t/t_a \) and that \( \sigma(\tau) \) is normalized so that the total CR energy injected in the hotspot \( E_{\text{cr.inj}} = L_{cr} t_a \) remains unchanged at time \( t = t_a \) but is distributed differently during the hotspot evolution. The \( \sigma(\tau) \) corresponding to these parameters is plotted in Figure 8. Figure 9 illustrates the density \( \rho(z, r) \) image and CR energy density \( e_{cr}(z, r) \) contours for model 2 and their projected counterparts at time \( t_a. \) As expected, this solution has a radio cavity boundary and bow shock profile that are more in agreement with those of Cygnus A in Figure 1.

However, the increasing hotspot luminosity we consider in model 2 is opposite to the decreasing luminosity observed in the combined sample of compact symmetric and FR II radio sources discussed by Jeyakumar & Saikia (2000). Consequently, it is of interest to consider model 3 in which the hotspot CR luminosity remains constant during its short lifetime \( t_e, \) but its velocity \( v_{hs}(\tau) \) decelerates with time. Using Equation (1) for \( v_{hs}(\tau), \) the decelerating hotspot reaches \( r = r_a = 60 \) kpc at time \( t_a \) while producing the same total CR energy \( E_{\text{cr.inj}} = L_{cr} t_a. \) For model 3 we explore the decelerating hotspot velocity \( v_{hs}(\tau) \) illustrated in Figure 10 which is characterized with parameters \( v_0 = 20,000 \) km s\(^{-1} \) and \( \tau_0 = 0.15 \) and source term \( S_{hs} \) as in Equation (5). The resulting gas density \( \rho(z, r) \) and relativistic energy density \( e_{cr}(z, r) \) plus projections are shown in Figure 11. The less conical shape of the radio cavity and the overall convex shape of the cocoon shock in model 3 are closer to those observed in Cygnus A (top panel of Figure 1). The evolution of FR II sources from more compact sources has been discussed by Kawakatu et al. (2008) who suggest hotspot deceleration in various evolutionary scenarios.

Finally, we note that the mass of ultra-hot gas \( (T > 10^9 \) K) inside the radio cavities in models 2 and 3, \( 1.5 \times 10^8 \) and \( 1.4 \times 10^8 M_{\odot}, \) respectively, is comparable with that found in model 1 and is nearly equal to the total gas mass in a column of original cluster gas containing all hotspot zones within the current hotspot–center distance, \( r_a = 60 \) kpc. The volumes of the radio cavities in models 2 and 3, \( 2.2 \times 10^4 \) and \( 2.8 \times 10^4 \) kpc\(^3 \), are also similar to that in model 1, \( 2.4 \times 10^4 \) kpc\(^3 \).
6. DISCUSSION

6.1. Orientation of Cygnus A

In this paper, we assume with Smith et al. (2002) that the image of Cygnus A in Figure 1 lies entirely in the plane of the sky. However, the western radio jet appears more luminous than its eastern counterpart (lower panel of Figure 1); the radio flux density ratio of the W/E jets on kpc scales, $R = 2.6 \pm 1$ (Carilli et al. 1996), is uncertain but consistent with some Doppler boosting. It is likely therefore that the image in Figure 1 with major to minor axis ratio of the cocoon shock $R_{\text{maj/max}} = 2.2$ (Wilson et al. 2006) is somewhat foreshortened. If so, the current distance of the hotspot from the galaxy center exceeds the apparent distance $r_a = 60$ kpc and the ratios of the major to minor axis for the cocoon shock that we calculate ($R_{\text{maj/max}} = 2.1, 3.0, \text{ and } 2.4$ in Figures 3, 9, and 11, respectively) are systematically too small. In Figure 7, for model 1 no. the aspect ratio $R_{\text{maj/max}} = 3.6$ is very large, indicating that the non-uniform production of CR energy by the hotspot must be adjusted for any assumed foreshortening correction until $R_{\text{maj/max}}$ as well as the shape of the FR II X-ray cocoon shock and its radio lobe all come into agreement.

6.2. Unexplained Features in the X-ray Image

In addition to the purely thermal X-ray emission in our models, the X-ray image of Cygnus A shown in Figure 1 also includes SSC X-ray emission from the hotspots and from the X-ray cavity (radio lobe region). To access the visibility of the Cygnus A cavity from thermal X-ray emission alone, we show in Figure 12 X-ray surface brightness scans perpendicular to the jet axis for our model 1. Evidently the X-ray cavity visible in Figure 12 is filled with SSC emission at a level that by coincidence approximately matches the maximum thermal X-ray surface brightness just outside the radio lobe boundary, concealing the expected cavity in thermal X-rays. For example, the sharp outer edges of the radio lobes in the lower panel of Figure 1 do not appear to correspond to changes in the X-ray surface brightness in the upper panel. (The shape of the leading edge of the radio lobe in Figure 1 is broader than in our calculations presumably because the jet direction in Cygnus A changes rather abruptly, depositing CRs over a broader region.)

We also note the bright, asymmetric thermal X-ray emission visible in Figure 1 distributed roughly perpendicular to the jet direction and extending $\sim 20$ kpc from the center of Cygnus A mostly toward the east. This prominent X-ray feature is unrelated to the cocoon evolution from cluster gas that we study here. Instead, this irregular X-ray emission may be the expanded remnant of denser gas formerly located near the center of Cygnus A that was shocked and heated by AGN energy during the early stages of FR II jet development. There is considerable observational evidence at other wavelengths for cooler, high-density gas extending several kpc from the center of Cygnus A. Optical emission lines observed in Cygnus A are characteristic of warm gas at temperature $T \sim 10^4$ K that may be photoionized and heated by a hard UV to X-ray spectrum from the central AGN (Osterbrock & Miller 1975). The total mass of warm gas is $\sim 10^7 M_\odot$. Line emission from the nuclear regions is observed to be significantly reddened by dust intrinsic to Cygnus A (Taylor et al. 2003). Near-infrared observations by Wilman et al. (2000) detect rovibrational emission lines of H$_2$ as well as [Fe II] and H recombination lines. These lines are spatially extended by a few kpc and appear to come from different regions with complex velocity profiles having widths up to nearly $500$ km s$^{-1}$ (Bellamy & Tadhunter 2004). Wilman et al. estimate the mass of molecular gas to be $10^8-10^{10} M_\odot$. Soft X-ray absorption columns of $N \sim 10^{21}$ cm$^{-2}$ are commonly observed in FR II sources with accompanying fluorescent Fe Kα emission from cooler gas (Evans et al. 2006), so Cygnus A

Figure 11. Model 3 at time $t_s = 10^7$ yr. Top: contours of $\rho(z, r)$ shown with values indicated in the color bar. Bottom: image of the projected bolometric X-ray emission and contours of the projected CR energy density $\int \rho(z, r) dt$. Both panels have dimensions $70 \times 35$ kpc.

(A color version of this figure is available in the online journal.)

Figure 12. Eight scans of the bolometric X-ray surface brightness $\Sigma_X$ perpendicular to the jet direction for model 1 (bottom panel of Figure 3). From top to bottom the scans are at $z = 10, 15, 20, 25, 30, 35, 40,$ and $45$ kpc from the center of Cygnus A.
is not unusual in having massive central reservoirs of colder gas. If the asymmetric display of thermal X-ray emission in Figure 1 results from colder gas that was shock-heated near the center of Cygnus A during the early development of its FR II event, as we suggest here, it is possible that the energy absorbed in heating and expanding this gas to its present position would reduce the width of the cocoon shock near the center computed in our model 1, agreeing better with the observed cocoon shape and preserving the assumption of spatially uniform CR luminosity in the hotspot.

Finally, we draw attention again to the faint but clearly visible X-ray “jets” in Figure 1, several kpc in width, extending along the major axis of the cocoon both east and west from the center of Cygnus A. This surface brightness of this quasi-linear feature is irregular but has a cylindrical appearance overall. It is difficult to understand how such a feature could be created by a radio jet. Fluorescent AGN X-ray emission from a radiating bi-cone along these opposing directions also fails unless the gas temperature in this region is very much less than a few keV.

6.3. Components in the Radio Lobe Pressure

The initial undisturbed cluster gas pressure varies across the Cygnus A cocoon from $20 \times 10^{-10}$ dyn cm$^{-2}$ at the cluster center to $1.9 \times 10^{-10}$ dyn cm$^{-2}$ at the hotspot radius $r_a = 60$ kpc. The initial pressure scale height in the cluster gas is approximately $r_P \propto P/\rho g \propto T/g$. By comparison, inside the radio cavity the relativistic temperature $T_{\text{lobe}}$ and pressure scale height greatly exceed those in the cluster gas, $r_{P\text{lobe}} \propto T_{\text{lobe}}/g \gg r_P$, explaining why the (wave-averaged) pressure inside the radio cavity $P_{\text{lobe}}$ is nearly constant with cluster radius. The total cavity pressure in models 1–3 discussed above is $P_{\text{lobe}} = 10.5, 13.0$, and 8.5, respectively, in units of $10^{-10}$ dyn cm$^{-2}$, all approximately equal to the average pressure of the initial undisturbed cluster gas within the hotspot radius.

Recently Hardcastle & Croston (2010) and Yaji et al. (2010) detected IC X-ray emission from Cygnus A. In many FR II sources, the Compton emission results from electron interactions with cosmic background radiation (IC-CMB). However, since the synchrotron photon density in Cygnus A exceeds that of the CMB, SSC X-ray emission dominates. The additional information provided by Compton emission allows independent estimates of the energy density in the magnetic field $u_B = B^2/8\pi$ and in synchrotron-emitting electrons $u_e = m_e c^2 \int \gamma n(\gamma) d\gamma$. The non-thermal synchrotron spectrum, $F_\nu \propto \nu^{-\alpha}$, indicates that the electron energy distribution is also a power law $n(\gamma) \propto \gamma^{-p}$. The exponent $p \approx 2$ is expected from traditional shock-accelerated CRs and is related to the spectral slope by $p = 2\alpha + 1$, suggesting $\alpha = 0.6$–0.7 for electrons when they are first injected into the radio cavity. Much of the non-thermal cavity energy is contained in low-energy electrons, and it often assumed that the integral for $u_e$ extends to some $\gamma_{\text{min}}$ far less than values $\gamma \sim 10^3$–$10^4$ that contribute to the observed radio spectrum. If the total pressure in the radio lobe $P_{\text{lobe}}$ is known from X-ray observations or from dynamical models such as those presented here, it is possible to determine an energy density of non-thermal particles that matches the radio spectrum and, in combination with magnetic energy density (from IC), provides a pressure that must not exceed $P_{\text{lobe}}$. The parameters that define possible solutions for the particle energy densities are $\rho_e, \gamma_{\text{min}}$, and $\kappa$, the ratio of the energy density in non-radiating particles to radiating electrons. Both Hardcastle & Croston (2010) and Yaji et al. (2010) find that the particle energy density in Cygnus A dominates over that of the magnetic field, $u_e \gg u_B$, so the interesting question is whether the observed synchrotron-radiating electrons provide enough pressure to support the cavity. The lobe pressure adopted by Yaji et al., $P_{\text{lobe}} \approx 20–40 \times 10^{-10}$ dyn cm$^{-2}$, significantly exceeds those in our models while the value assumed by Hardcastle & Croston, $P_{\text{lobe}} \approx 6 \times 10^{-10}$ dyn cm$^{-2}$, is in better agreement.

With parameters $(p, \gamma_{\text{min}}, \kappa) = (2, 1, 0)$ Hardcastle & Croston find that the electron energy density necessary to match $P_{\text{lobe}}$ in Cygnus A produces IC emission that exceeds X-ray observations by factors of 2–5, where the range reflects the rather large uncertainty in the X-ray Compton flux. Solutions (also with $u_e \gg u_B$) that match the Compton X-ray emission are possible if $(p, \gamma_{\text{min}}, \kappa) = (2, 1, 1–4)$ or $(p, \gamma_{\text{min}}, \kappa) = (2.3, 1.0)$. Hardcastle & Croston argue that $p = 2$ is more physically acceptable for shock acceleration, and that a pressure component from additional non-radiating particles ($\kappa > 0$) contributes to the Cygnus A energy density, as in other FR II sources. However, as discussed by Stawarz et al. (2007), the physical conditions inside the Cygnus A hotspots are not those of diffusive acceleration in a single traditional non-relativistic shock. Their infrared observations are consistent with a very flat low-energy particle spectrum in the hotspot, $p \sim 1.5$, and large fields, $\sim 200–300 \mu G$, possibly indicating non-linear turbulent field amplification in the relativistic reverse shock. Above $\nu \sim 1$ GHz, this flat hotspot spectrum steepens to $\alpha \gtrsim 1$, suggesting to Stawarz et al. (2007) that two different accelerating mechanisms are involved and that only electrons emitting above $\sim 1$ GHz may be accelerated by traditional diffusive-shock Fermi processes.

Notwithstanding all these complications, the conclusion of Hardcastle & Croston that some additional non-radiating pressure is required to support the Cygnus A cavity may well be correct. Adding relativistic shock-accelerated protons gains a factor of 2 but even this may not be sufficient. Another possibility is that there is an additional weakly relativistic electron population with energies $1 \lesssim \gamma \lesssim 10^3$ (or $10^4 \lesssim K \lesssim 10^2$ K) too low to contribute to currently available radio observations and therefore qualify as “non-radiating.” Note that these are similar to the temperatures of wave-shocked thermal gas inside our cavities, as in Figure 5. Unlike diffusive shock acceleration where only about 10% of the thermal gas becomes relativistic, in our models the entire (small) mass of thermal gas inside the cavities is heated to relativistic temperatures presumably in a series of wave-driven shocks. Since our computation of the dynamics of thermal gas is completely non-relativistic, we expect that the total pressure $P + P_c$ inside the cocoon cavity is correctly computed, but the pressure ratio $P_c/P$ inside the cavities is incorrect by a factor of order unity. In a more realistic calculation that accurately describes the shock-heating transition of thermal gas to relativistic temperatures, we expect that some memory of the non-relativistic Maxwellian peak would be retained with a spectrum $n_{\text{max}}(\gamma)$ quite unlike the power-law spectrum resulting from traditional diffusive shock acceleration. The collisionless nature of cavity shocks introduces additional complications, but it is possible that shock-heated thermal gas provides an additional radio lobe pressure that is not apparent in the observed radio spectrum, even at the lowest currently feasible radio frequencies. As discussed earlier, the presence of a small mass of ultra-hot thermal gas in our computed cavities may be a model-dependent result from our assumptions that the FR II jets carry very few non-relativistic particles and CRs and
thermal gas fully mix before leaving the hotspot. In any case, it is essential that wave activity or coherent fluid motions within the cavity do not disrupt the gradient of radio-synchrotron spectral steepening and electrons ages observed along the jet axis in the radio lobes of many FR II sources including Cygnus A; some collisionless wave-damping mechanism may be required to accomplish this.

7. CONCLUSIONS

We describe computations of the gas-dynamical evolution of FR II radio cocoons in galaxy clusters and, evidently for the first time, also include the self-consistent dynamical evolution of the relativistic synchrotron-emitting plasma. Cluster and FR II parameters are chosen to approximately match those of Cygnus A, the most intensively observed and studied FR II source. Our calculation is based on the hypothesis that the dynamics and energy content of FR II cocoons and non-thermal radio sources have their origin in the hotspots, where FR II jets first encounter the cluster gas. Our hotspot-oriented computations allow us to avoid detailed calculations of the dynamical structure and particle content of the FR II jets about which very little is known (e.g., Kataoka et al. 2008). The strong reverse shock that forms at the inner hotspot boundary as a result of the jet impact imparts a momentum that drives the bow shock surrounding a cocoon of shocked cluster gas. To simulate this momentum transfer, we consider a phantom hotspot that moves out in some prescribed fashion into the computational grid along the jet axis, marking the grid zone which is compressed and accelerated in the reverse shock. However, the total thermal and kinetic power delivered by the hotspot is only a few percent of the power $\sim 10^{46}$ erg s$^{-1}$ flowing from the jet in relativistic CR particles including the electrons that emit the observed radio radiation. The CR energy introduced inside the hotspot is a combination of relativistic particles arriving in the jet at the reverse shock and additional CRs created in this shock by the diffusive Fermi mechanism.

In view of the domination of relativistic energy in forming the Cygnus A cocoon, we do not explicitly consider a non-relativistic mass flux from the jet into the hotspot. As a result, in our calculations no mass accumulates in the high-pressure region near the hotspot which in many previous FR II calculations drives a turbulent backflow near the jet in the opposite direction. Such backflows would disrupt radial gradients of radio-synchrotron ages and spectral steepening observed in FR II radio lobes and could generate irregularities in the radio cavity boundaries that are also not observed.

For computational simplicity we represent the CR energy only with its relativistic energy density $e_r$, erg cm$^{-3}$ and the corresponding pressure $P_r = (\gamma_r - 1)e_r$ with $\gamma_r = 4/3$. We do not explicitly consider the energy spectrum of CR particles or the physical nature of these particles which could be any combination of electrons and protons. In keeping with the small observed magnetic fields in Cygnus A and other similar FR II sources, we assume that the magnetic energy density is everywhere considerably less than the sum of the local thermal and relativistic energy densities. However, a modest magnetic field that is frozen into the thermal cluster gas is essential for thermal and relativistic fluids to exchange momentum and respond to the total pressure gradient. With these simplifying assumptions, it is not possible to compute the synchrotron emission and radio spectrum since the energy spectrum of the electrons and the field strength are unspecified, but the radio lobe region is clearly defined by cavities in the hot gas displaced by the relativistic fluid component. Finally, we assume that the relativistic and thermal fluids interact and share energy adiabatically with no radiative losses due to thermal X-ray or synchrotron radio emission. This assumption is reasonable because of the short age of Cygnus A.

We regard the approximate synchrotron decay age observed in the radio lobes of Cygnus A, $t_d = 10^7$ yr, as the dynamical age of the entire cocoon. The current projected distance of the hotspots from the center of Cygnus A, $r_s = 60$ kpc, is assumed to be identical to the actual physical distance, i.e., we imagine that the Cygnus A cocoon is essentially in the plane of the sky.

An initial calculation in which the hotspot velocity and its CR luminosity $L_{cr} = 10^{46}$ erg s$^{-1}$ are assumed to be constant with time reproduces all the essential features of Cygnus A and those of FR II sources in general. The radio lobe filled with CRs is extended along the jet direction and (for Cygnus A) merges at the origin with an identical mirror-imaged counterlobe, forming a single extended "bridge" of radio-emitting CRs along the jet–counterjet axis. The bow shock produced by the kpc-sized phantom hotspot encloses a cocoon of shocked gas that is comparable in size and aspect ratio to that in the X-ray image of Cygnus A. The extended radio lobe confines the relativistic particles within the cocoon and at the same time displaces the shocked cluster gas in a sharp contact discontinuity similar to that observed in Cygnus A at radio frequencies.

The energy budget is dominated by the internal energy of the relativistic component. When an otherwise identical calculation is performed without CR injection in the hotspot, the total kinetic and thermal energy received by the cluster gas is only a few percent of $t_d L_{cr}$.

The high pressure of CRs received and generated by the jet at the hotspot causes both CRs and thermal gas to flow roughly transverse to the jet, helping to widen the cocoon and strengthen the bow shock that encloses the cocoon. As observed, the relativistic CRs are confined inside elongated radio lobes. However, we assume that the CRs and thermal gas inside the hotspots are mixed and flow together into the apex of the radio cavity which contains a small mass of thermal gas. The small mass of thermal gas that enters the radio lobe from the hotspot is shocked by high-velocity waves to pressures that are comparable with that in the relativistic gas. This results in extremely high relativistic temperatures in the thermal gas that are not accurately calculated with our current non-relativistic hydrocode. Nevertheless, the radially uniform pressure inside the Cygnus A radio lobe, about 2–3 times larger than the pressure at mid-lobe in the initial undisturbed cluster gas, should be accurately computed. The total energy in our computations is conserved to a few percent.

We describe two additional computations in which fewer CRs are introduced at early times in the Cygnus A evolution, while keeping the total CR energy ejected by the hotspot and its average velocity unchanged. In these computations the shapes of the radio lobe and cocoon shock after $t_s = 10^7$ yr are significantly improved. We considered two ways of achieving this early CR reduction: first by simply adjusting downward the CR hotspot luminosity at early times and second by assuming that the hotspot luminosity is constant but the hotspot velocity decelerates with time. The improved results of these two calculations are similar. However, a uniform production of CRs in the moving hotspot may still be possible if a significant fraction of the energy released by the hotspot during its early evolution is absorbed in heating and accelerating cold gas away from the center of Cygnus A, producing the asymmetric emission observed in thermal X-rays.
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