CLASSIFICATION PERFORMANCE METRIC FOR IMBALANCE DATA BASED ON RECALL AND SELECTIVITY NORMALIZED IN CLASS LABELS

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ABSTRACT

In the classification of a class imbalance dataset, the performance measure used for the model selection and comparison to competing methods is a major issue. In order to overcome this problem several performance measures are defined and analyzed in several perspectives regarding in particular the imbalance ratio. There is still no clear indication which metric is universal and can be used for any skewed data problem. In this paper we introduced a new performance measure based on the harmonic mean of Recall and Selectivity normalized in class labels. This paper shows that the proposed performance measure has the right properties for the imbalanced dataset. In particular, in the space defined by the majority class examples and imbalance ratio it is less sensitive to changes in the majority class and more sensitive to changes in the minority class compared with other existing single-value performance measures. Additionally, the identity of the other performance measures has been proven analytically.

Keywords Performance metrics · Imbalanced datasets · Model Selection · Imbalance ratio.

1 Introduction

One of the important threads in machine learning and data mining is the evaluation of the algorithm or classifier performance. In the case of a binary classification, classifier assessment methods use the data contained in the confusion matrix. Many metrics are considered to choose the right machine learning method for a specific life science problem. In bioinformatics, such problems are for example: the prediction of tissue condition based on gene expression, genome annotation or the gene classification. The performance measures, that have the intuitive visual interpretation are analyzed in [1], [2], [3]. Single-value metrics are also considered from different points of views [4], [5] [6].

In a wide range of scientific areas, including the life sciences, there is a class imbalance problem [7], [8]. The dataset is imbalanced when a difference in the numbers of positive and negative instances is significant. In bioinformatics unequal class distributions arise naturally [9], [10]. It is well-known that the most commonly used performance measure, the overall accuracy, could not be used to measure the true performance of an imbalanced dataset. For this reason the selection of an appropriate performance measure to skewed data is an important problem. The selection and analysis of properties of various performance measures is still a current research problem regarding the imbalance data. One of the problems considered is the impact of class imbalance on the classification performance measures [11]. The same problem with the imbalance factor is also considered for imbalanced and streaming data [12]. The work [13] presents another comprehensive study of the differences between various measures from several perspectives for imbalanced datasets. The problem of the classification difficulty and performance measures is considered in [14]. The generic performance measure for multiclass problems is instead introduced in [15].
This research is focused on the definition of a new classification performance metric which has the desired properties for imbalanced data. In particular, this applies to the fact that they are less sensitive to changes in the majority class and more sensitive to changes in the minority class compared with other existing single-value performance measures. The results presented in this article extend the discussion on the use of classification performance measures [5], [16], in particular for imbalanced data [11], [17].

The goal of this paper is to analyze the behavior of performance measures in the context of comparing two algorithms for imbalanced dataset. What is important in our analysis is the answer to the question: Which machine learning method is better for the classification of the majority or minority class? The absolute value of the two values of performance measures can be used for this purpose. In addition, our analysis concerns the comparison of the proposed in this paper performance measure with other metrics considered in the article.

The rest of the paper is organized as follows. Section 2 presents the definition of the existing performance measures based on the confusion matrix. A new performance measure is defined in Section 3. In Section 4 we demonstrate that the proposed measure can be successfully used for imbalanced datasets. In particular, compared to other metrics. The conclusions of presented results are addressed in Section 5.

2 Performance metrics

In binary classification when a classifier produces a real-valued output all possible combinations of actual and predicted class labels form a confusion matrix. The confusion matrix is $2 \times 2$ table containing a number of four types of an outcome:

- True Positive – $TP$,
- True Negative – $TN$,
- False Positive – $FP$,
- False Negative – $FN$.

$TP$ and $TN$ items denote the number of examples classified correctly by the classifier as positive and negative respectively, while $FN$ and $FP$ indicate the number of misclassified positive and negative examples respectively. Additionally, $TP + FN = P$, $TN + FP = N$, $TP + FP = \hat{P}$, $TN + FN = \hat{N}$, and $P + N = M$, where $M$ is a number of the test dataset examples, $P$ is a number of the positive and $N$ is a number of the negative test dataset examples, $\hat{P}$ and $\hat{N}$ are the predicted positive and negative number of examples respectively.

In this work, we consider the following classification performance metrics which are the functions based on values from the confusion matrix:

\[
Recall (REC) = \frac{TP}{P} \tag{1}
\]

\[
Precision (PRC) = \frac{TP}{\hat{P}} \tag{2}
\]

\[
Selectivity (SEL) = \frac{TN}{N} \tag{3}
\]

\[
Accuracy (ACC) = \frac{TP + TN}{M} \tag{4}
\]

\[
F_1 - score = 2 \frac{PRC * REC}{PRC + REC} \tag{5}
\]

\[
G - mean = \sqrt{REC * SEL} \tag{6}
\]

\[
Matthews cor. coef. (MCC) = \frac{TP * TN - FP * FN}{\sqrt{\hat{P} \cdot P \cdot N \cdot \hat{N}}} \tag{7}
\]

\[
Kappa stat. (Kappa) = \frac{ACC - \frac{1}{M} \left( \frac{P \cdot \hat{P}}{M} + \frac{N \cdot \hat{N}}{M} \right)}{1 - \frac{1}{M} \left( \frac{P \cdot \hat{P}}{M} + \frac{N \cdot \hat{N}}{M} \right)} \tag{8}
\]
The Receiver Operating Characteristic (ROC) analysis is widely used for classifiers with the soft type of output. As a performance measure the Area Under Curve (AUC) is commonly used. For the real-valued output of the classifier $AUC$ and $BACC$ are equal [18]. Therefore, in this paper, whenever $BACC$ is mentioned it could be also replaced by $AUC$.

Figure 1: Set of heat maps for $IR = 0.01$. 
2.1 Imbalance ratio

The class imbalance occurs when the difference between the number of positive and negative examples is high. In several articles there have been proposed different definitions of class imbalance [19], [11], [20]. In this paper we propose the imbalance ratio (IR) as

\[ IR = \frac{\min(P, N)}{\max(P, N)}. \]  

(9)

This IR is independent of the fact which class (positive or negative) is the majority or minority class.

3 A proposal of a new performance metric

Our motivation is to use the measures: REC and SEL taking into account the normalization of class labels. The normalization refers to the number of class labels \((P, N)\) in the number of all the objects \(M\) from the test dataset. The normalized in class labels performance metrics are therefore the following:

\[ \text{REC} = \frac{TP}{P + N}, \quad \text{SEL} = \frac{TN}{P + N}. \]

Given the above, we propose the following harmonic mean of \(\text{REC}\) and \(\text{SEL}\) normalized in the class labels \((HMNC)\) performance metric:

\[ HMNC = \frac{\text{HM}(\text{REC} \cdot \frac{P}{M}, \text{SEL} \cdot \frac{N}{M})}{\text{HM}(\frac{P}{M}, \frac{N}{M})}. \]

(10)

where \(\text{HM}\) is the harmonic mean.

Given the expression of the harmonic mean of two numbers \(\text{HM}(a, b) = \frac{2ab}{a+b}\) the proposed performance metric can be expressed as:

\[ HMNC = \frac{TP \cdot TN \cdot M}{(TP + TN) \cdot P \cdot N}. \]

(11)

The domain of HMNC measure is between 0 and 1, where 1 is the preferred value which means that all objects are classified correctly. If the value is 0, each object is incorrectly classified. The same properties have performance measures \((4)\)-(\(8\)) while the measures \((7)\) and \((8)\) return values between \(-1\) and \(+1\).

The proposed metric has also a property, which is common for the other metrics as shown below.

**Theorem** Performance measures \(HMNC\), \(ACC\), \(BACC\) and \(G - mean\) are equal if \(\frac{TP}{P} = \frac{TN}{N}\).

**Proof.** If the equation \(\frac{TP}{P} = \frac{TN}{N}\) occurs, then equations \(\frac{TP}{TN} = \frac{P}{N}\) and \(TP \cdot N = TN \cdot P\) are true. Therefore:

\[ HMNC = \frac{TP \cdot TN \cdot M}{(TP + TN) \cdot P \cdot N} = \frac{TP \cdot TN \cdot M}{TP \cdot P \cdot N + TP \cdot N \cdot N} = \frac{TN \cdot M}{TP \cdot N(P + N)} = \frac{TN}{N}. \]

\[ ACC = \frac{TP + TN}{M} = \frac{TN - P + TN}{P + N} = \frac{TN \left(\frac{P}{N} + 1\right)}{P + N} = \frac{TN}{N}, \]

\[ BACC = 0.5 \left(\frac{TP}{P} + \frac{TN}{N}\right) = 0.5 \left(\frac{TN}{N} + \frac{TN}{N}\right) = \frac{TN}{N}. \]

4
\[ G - \text{mean} = \sqrt{\frac{TP}{P} \frac{TN}{N}} = \sqrt{\frac{TN}{N} \frac{TN}{N}} = \frac{TN}{N} \]

In the case of fulfilled assumptions of the theorem all performance measures \( \text{HMNC}, \text{ACC}, \text{BACC} \) and \( G - \text{mean} \) are equal to \( \frac{TN}{N} \) (or equivalent \( \frac{TP}{P} \)) which completes the proof.

4 Analysis of HMNC performance metric

The measures defined by equations (1)-(8) were discussed and compared in various articles [4], [12], [18]. Additionally, visual-based analyses of performance measures are also presented, in particular ROC [1] and precision-recall curves [3] are analyzed.

Table 1: Values of performance metrics for four classifiers and theirs absolute value of the difference in measures – the case of IR=0.01, \( P=1000, N=10 \).

| Meth. | TP  | TN  | HMNC | ACC | BACC | MCC | \( F_1 \) | \( G - m \) | Kappa |
|-------|-----|-----|------|-----|------|-----|---------|---------|-------|
| 1     | 500 | 5   | 0.5  | 0.5 | 0.5  | 0   | 0.66    | 0.5     | 0     |
| 2     | 700 | 5   | 0.5  | 0.7 | 0.6  | 0.04| 0.82    | 0.59    | 0.01  |
| 3     | 700 | 7   | 0.7  | 0.7 | 0.7  | 0.09| 0.82    | 0.7     | 0.03  |
| 4     | 500 | 7   | 0.7  | 0.5 | 0.6  | 0.04| 0.67    | 0.59    | 0.01  |

| Absolute value of the difference in measures |
|------------------------------------------------|
| \( |\Psi_1 - \Psi_2| | 0.0 | 0.2 | 0.1 | 0.02 | 0.16 | 0.09 | 0.01 |
| \( |\Psi_1 - \Psi_3| | 0.2 | 0.2 | 0.2 | 0.04 | 0.16 | 0.09 | 0.01 |
| \( |\Psi_1 - \Psi_4| | 0.2 | 0.0 | 0.1 | 0.02 | 0.00 | 0.09 | 0.00 |
| \( |\Psi_2 - \Psi_3| | 0.2 | 0.0 | 0.1 | 0.02 | 0.00 | 0.11 | 0.01 |
| \( |\Psi_3 - \Psi_4| | 0.0 | 0.2 | 0.1 | 0.02 | 0.16 | 0.11 | 0.01 |

Table 2: Values of performance metrics for four classifiers and theirs absolute value of the difference in measures – the case of IR=0.1, \( P=1000, N=100 \).

| Meth. | TP  | TN  | HMNC | ACC | BACC | MCC | \( F_1 \) | \( G - m \) | Kappa |
|-------|-----|-----|------|-----|------|-----|---------|---------|-------|
| 1     | 500 | 50  | 0.5  | 0.5 | 0.5  | 0   | 0.65    | 0.5     | 0     |
| 2     | 700 | 50  | 0.51 | 0.68| 0.6  | 0.12| 0.8     | 0.59    | 0.09  |
| 3     | 700 | 70  | 0.7  | 0.7 | 0.7  | 0.24| 0.81    | 0.7     | 0.18  |
| 4     | 500 | 70  | 0.68 | 0.52| 0.6  | 0.12| 0.65    | 0.59    | 0.06  |

| Absolute value of the difference in measures |
|------------------------------------------------|
| \( |\Psi_1 - \Psi_2| | 0.01 | 0.18 | 0.1 | 0.06 | 0.15 | 0.09 | 0.05 |
| \( |\Psi_1 - \Psi_3| | 0.2  | 0.2  | 0.2 | 0.12 | 0.16 | 0.2   | 0.09 |
| \( |\Psi_1 - \Psi_4| | 0.18 | 0.02 | 0.1 | 0.06 | 0.01 | 0.09 | 0.03 |
| \( |\Psi_2 - \Psi_3| | 0.19 | 0.02 | 0.1 | 0.06 | 0.01 | 0.11 | 0.04 |
| \( |\Psi_3 - \Psi_4| | 0.02 | 0.18 | 0.1 | 0.06 | 0.16 | 0.11 | 0.06 |

Table 3: Values of performance metrics for four classifiers and theirs absolute value of the difference in measures – the case of IR=0.25, \( P=1000, N=250 \).

| Meth. | TP  | TN  | HMNC | ACC | BACC | MCC | \( F_1 \) | \( G - m \) | Kappa |
|-------|-----|-----|------|-----|------|-----|---------|---------|-------|
| \( \Psi_1 \) | 500 | 125 | 0.5  | 0.5 | 0.5  | 0   | 0.62    | 0.5     | 0     |
| \( \Psi_2 \) | 700 | 125 | 0.53 | 0.66| 0.6  | 0.17| 0.77    | 0.59    | 0.16  |
| \( \Psi_3 \) | 700 | 175 | 0.7  | 0.7 | 0.7  | 0.33| 0.79    | 0.7     | 0.3   |
| \( \Psi_4 \) | 500 | 175 | 0.65 | 0.54| 0.6  | 0.16| 0.63    | 0.59    | 0.12  |

| Absolute value of the difference in measures |
|------------------------------------------------|
| \( |\Psi_1 - \Psi_2| | 0.03 | 0.16 | 0.1 | 0.08 | 0.15 | 0.09 | 0.08 |
| \( |\Psi_1 - \Psi_3| | 0.2  | 0.2  | 0.2 | 0.16 | 0.17 | 0.2   | 0.15 |
| \( |\Psi_1 - \Psi_4| | 0.15 | 0.04 | 0.1 | 0.08 | 0.02 | 0.09 | 0.06 |
| \( |\Psi_2 - \Psi_3| | 0.17 | 0.04 | 0.1 | 0.08 | 0.02 | 0.11 | 0.07 |
| \( |\Psi_3 - \Psi_4| | 0.05 | 0.16 | 0.1 | 0.08 | 0.15 | 0.11 | 0.09 |

Tab. 1–3 show a comparison of the absolute values of the quality measures for the classification of two classifiers. The classification efficiency is given as \( TP \) and \( TN \) values. The tables relate to three different \( IR \) values (\( IR = 0.01, \) \( \) \( IR = 0.1, \) \( IR = 0.25 \)).
$IR = 0.1$ and $IR = 0.25$). It was assumed that the majority class is a positive class. So, for example, if $IR = 0.01$, $TP = 500$ and $TN = 5$ (see Tab.1) the method designated as $\Psi_1$ classifies 500 out of 1000 positive class examples correctly, as well as 5 out of 10 negative class examples. In each considered case of $IR$ the following relationships can be noted:

- The comparison of two classifiers in which the values of $TN$ do not change. This case means that the methods differ in the accuracy in relation to the majority class classification $P$. In Tab.2 and 3 these are the rows marked...
Figure 3: Set of heat maps for \( IR = 0.25 \).

\[
|\Psi_1 - \Psi_2| \text{ and } |\Psi_3 - \Psi_4| \text{. For these rows, the difference in the absolute value of the classification quality measure is the smallest for } HMNC \text{ measure compared to other measures.}
\]

- The comparison of two classifiers in which the values of \( TP \) do not change. This case means that the methods differ in the accuracy in relation to the minority class classification \( N \). In Tab.\[3\] these are the rows marked \( |\Psi_1 - \Psi_4| \text{ and } |\Psi_2 - \Psi_3| \). For these rows, the difference in the absolute value of the classification quality measure is the largest for \( HMNC \) measure compared to other measures.
Therefore, comparing the absolute values regarding the difference between two classification methods for different classification measures, one can indicate which of the two machine learning methods is more effective for a minority or majority class. The discussed results relate to the following values \( TP > 0.5 \times P \) and \( TN > 0.5 \times N \), which are presented in Tab. 1–3. Similar results apply to \( TP > 0.5 \times P \) and any \( TN \) value. This results from the fact of analyzing heat maps presented in Fig. 1–3. The heat maps indicate a change in the value of the relevant measure of classification quality in the space specified by \( TP \) (abscissa axis) and \( TN \) (ordinate axis).

In the case of the performance measure \( HMNC \), the changes are the smallest along the abscissa axis, i.e., for changing \( TP \) values, which were adopted at work as a majority class. Changes in the value of measure \( HMNC \) are the most significant along the ordinate axes, i.e., for changing \( TN \) values that were adopted at work as a minority class. The described changes occur for \( TP \) values, which are equal to approximately \( \frac{TP}{P} > IR \).

In the case of values for which \( \frac{TP}{P} < IR \) the heat maps for \( HMNC \) metric indicate an inverse trend. This means quick changes in the value of the measure value for small changes in \( TP \) and small changes in the value of the measure value for significant changes in \( TN \) value.

It should also be noted that the heat map of the proposed method \( HMNC \) is similar to the heat map of \( G – mean \) measure. The heat map curvature (change in the direction of the isoline from horizontal to vertical) for measure \( G – mean \) is independent of \( IR \) factor.

In the case of the heat map for the \( HMNC \) performance measure, the direction of isoline changes at \( \frac{TP}{P} < IR \). The proposed measure therefore reflects \( IR \) coefficients, and the analysis of the absolute values of the difference between the results of the two machine learning methods can clearly indicate which of the two methods analyzed is more effective for the minority or majority class.

Referring to the invariant properties of the classification performance metric described in [18], it can be stated that:

- For the correct classification of the majority classes larger than \( IR \) defined by equation (9) the proposed \( HMNC \) metric will be less sensitive to changes in the majority class and more sensitive to changes in the minority class,
- For the correct classification of the majority classes smaller than \( IR \) defined by equation (9) the proposed \( HMNC \) metric will be more sensitive to changes in the majority class and less sensitive to changes in the minority class.

Sensitivity to changes in classified objects is greater (in the case of the minority class) or lower (in the case of the minority class) than the change in sensitivity of other classification performance measures considered in the article, which was shown using heat maps (Fig. 1–3) and examples presented in Tab. 1–3.

5 Conclusion

In this study, we propose the new performance measure which has several properties required for the classification quality measures. The identity with other performance measures in the case of \( TP = TN \) has been proven analytically. The shape of the heat map isoline indicates that it depends on \( IR \) value. The proposed measure therefore has the desired properties for imbalanced data, as shown in the examples. The obtained results relate to the pairwise comparison of machine learning methods and their analysis in the context of other performance measures.

References

[1] Peter A Flach. Roc analysis. In Encyclopedia of Machine Learning and Data Mining, pages 1–8. Springer, 2016.
[2] Jan Grau, Ivo Grosse, and Jens Keilwagen. Prroc: computing and visualizing precision-recall and receiver operating characteristic curves in r. Bioinformatics, 31(15):2595–2597, 2015.
[3] Takaya Saito and Marc Rehmsmeier. Precrec: fast and accurate precision–recall and roc curve calculations in r. Bioinformatics, 33(1):145–147, 2017.
[4] Pierre Baldi, Søren Brunak, Yves Chauvin, Claus AF Andersen, and Henrik Nielsen. Assessing the accuracy of prediction algorithms for classification: an overview. Bioinformatics, 16(5):412–424, 2000.
[5] Davide Chicco and Giuseppe Jurman. The advantages of the matthews correlation coefficient (mcc) over f1 score and accuracy in binary classification evaluation. BMC genomics, 21(1):6, 2020.
[6] Nathalie Japkowicz and Mohak Shah. Evaluating learning algorithms: a classification perspective. Cambridge University Press, 2011.
[7] C Arun and C Lakshmi. Class imbalance in software fault prediction data set. In Artificial Intelligence and Evolutionary Computations in Engineering Systems, pages 745–757. Springer, 2020.

[8] Justin M Johnson and Taghi M Khoshgoftaar. Survey on deep learning with class imbalance. Journal of Big Data, 6(1):27, 2019.

[9] Barbara Pes. Learning from high-dimensional biomedical datasets: The issue of class imbalance. IEEE Access, 8:13527–13540, 2020.

[10] Randall Wald, Taghi M Khoshgoftaar, Alireza Fazelpour, and David J Dittman. Hidden dependencies between class imbalance and difficulty of learning for bioinformatics datasets. In 2013 IEEE 14th International Conference on Information Reuse & Integration (IRI), pages 232–238. IEEE, 2013.

[11] Amalia Luque, Alejandro Carrasco, Alejandro Martín, and Ana de las Heras. The impact of class imbalance in classification performance metrics based on the binary confusion matrix. Pattern Recognition, 91:216–231, 2019.

[12] Dariusz Brzezinski, Jerzy Stefanowski, Robert Susmaga, and Izabela Szczech. On the dynamics of classification measures for imbalanced and streaming data. IEEE Transactions on Neural Networks and Learning Systems, 2019.

[13] Takaya Saito and Marc Rehmsmeier. The precision-recall plot is more informative than the roc plot when evaluating binary classifiers on imbalanced datasets. PloS one, 10(3):e0118432, 2015.

[14] Xiaoli Zhang, Xiongfei Li, and Yuncong Feng. A classification performance measure considering the degree of classification difficulty. Neurocomputing, 193:81–91, 2016.

[15] Thomas Kautz, Bjoern M Eskofier, and Cristian F Pasluosta. Generic performance measure for multiclass-classifiers. Pattern Recognition, 68:111–125, 2017.

[16] Rosario Delgado and Xavier-Andoni Tibau. Why cohen’s kappa should be avoided as performance measure in classification. PloS one, 14(9), 2019.

[17] Sankha Subhra Mullick, Shounak Datta, Sourish Gunesh Dhekane, and Swagatam Das. Appropriateness of performance indices for imbalanced data classification: An analysis. Pattern Recognition, 102:107197, 2020.

[18] Marina Sokolova and Guy Lapalme. A systematic analysis of performance measures for classification tasks. Information Processing & Management, 45(4):427 – 437, 2009.

[19] László A Jeni, Jeffrey F Cohn, and Fernando De La Torre. Facing imbalanced data–recommendations for the use of performance metrics. In 2013 Humaine association conference on affective computing and intelligent interaction, pages 245–251. IEEE, 2013.

[20] Rui Zhu, Yiwen Guo, and Jing-Hao Xue. Adjusting the imbalance ratio by the dimensionality of imbalanced data. Pattern Recognition Letters, 133:217–223, 2020.