Triangular and $Y$-shaped hadrons in QCD

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Abstract

Gauge invariant extended configurations are considered for the three fundamental (quarks) or adjoint (gluons) particles. For quarks it is shown that the $Y$-shaped configuration is the only possible. For adjoint sources both the $Y$-shaped and triangular configurations may realize. The corresponding static potentials are calculated in the Method of Field Correlators and in the case of baryon shown to be consistent with the lattice simulations. For adjoint sources the potentials of $Y$-shaped and $\Delta$-shaped configurations turn out to be close to each other, which leads to almost degenerate masses of $3^{--}g$ glueballs and odderon trajectories.

1. To make conclusions on the structure of gluonic fluxes confining color charges in physical states, one has to start from the considering of the space-extended gauge invariant wave function of the hadron. It is easy then to show that in the case of the static charges the Green function of the hadron reduces to the Wilson loop, which in the case of the baryon has the $Y$-type shape and consists from the three contours formed by the quark trajectories and joined at the point of the string junction [1, 2]. The Wilson loop of the tree adjoint sources is less known than the 3q one. In the paper we will show that it can have both $Y$-type and $\Delta$-type shape.

Using the formalism of the Method of the Field Correlators (MFC), we compute the static potentials, corresponding to the Wilson loops of the hadrons. In the case of the baryon the static potential was used long ago in many dynamical calculations [3, 4]. Recently it has been computed in lattice gauge theory in a number of papers [5, 6, 7]. We show in the paper that our potential is in a full agreement with the lattice studies.

In the case of adjoint sources we find that the $Y$-type and $\Delta$-type potentials remain near each other at the characteristic hadronic size. Using that we estimate masses of lowest $3g$ glueballs, lying on the corresponding odderon trajectories, and show that they are close to each other, implying that there are two possible odderon trajectories with not much different Regge slopes. A short discussion of physical implications of these results concludes the paper.

To avoid confusion, we should stress that the term ”$\Delta$ configuration” used in [8, 5, 6, 7] in the context of the static baryon potential, refers to the perimeter behaviour of the potential and not to the gauge invariant configurations as well as to the structures of fluxes discussed in the present paper.

2. Hadron building in SU(3) starts with listing elementary building blocks: quarks $q^{a}$, $\alpha = 1, 2, 3$, gluons (or adjoint static sources) $g^{a}$, $a = 1, \ldots, 8$, parallel transporters (PT) in fundamental representation $\Phi_{\alpha}^{a}(x, y) = (P \exp ig \int A_{\mu}(z)dz_{\mu})_{\alpha}^{a}$, adjoint parallel transporters...
Φ_{ab}(x, y), generators \( t^{(a)\beta}_\alpha \) symmetric symbols \( \delta^\beta_\alpha \), \( \delta_{ab} \), \( d^{abc} \), and antisymmetric ones, \( e_{\alpha\beta\gamma} \) and \( f^{abc} \). Note that we always use Greek indices for fundamental representation and Latin ones for the adjoint.

To construct a real extended (not point-like) hadron one uses all listed elements, PT included, and forms a white (gauge-invariant) combination. It is convenient to form an extended quark (antiquark) operator

\[
q^\alpha(x, Y) \equiv q^\beta(x)\Phi^\alpha_\beta(x, Y);
\]

\[
\bar{q}_\alpha(x, Y) = \bar{q}_\beta(x)\Phi^\beta_\alpha(x, Y).
\] (1)

In this way one has for the Y-shaped baryon:

\[
B_Y(x, y, z, Y) = e_{\alpha\beta\gamma}q^\alpha(x, Y)q^\beta(y, Y)q^\gamma(z, Y).
\] (2)

One can also define quark operator with two lower indices: \( e_{\alpha\beta\gamma}q^\alpha(x) \equiv q_{\beta\gamma}(x) \). However an attempt to create a gauge-invariant combination from 3 operators \( q_{\beta\gamma}(x) \) and 3 PT to construct a ∆-type configuration fails: the structure

\[
B_\Delta(x, y, z) = q_{\alpha\beta}(x)\Phi^\alpha_\beta(x, y)q_{\gamma\delta}(y, y)\Phi^\gamma_\delta(y, z)q_{\epsilon\rho}(z, z)\Phi^\epsilon_\rho(z, x)
\] (3)

is not gauge invariant, that can be checked directly, substituting in (3) \( q^\alpha(x) \to U^\alpha_\beta(x)q^\beta(x) \). One can try all combinations, but it is impossible to form a continuous chain of indices to represent the ∆-type structure using as operators \( q_\alpha \) as \( q_{\alpha\beta} \). Thus one can conclude that the Y-shaped configuration is the only possible gauge-invariant configuration of wave function for baryons.

One may wonder, what is the relation between the spatial structures of wave function of hadrons and their gluonic flux? The answer from the flux-tube models is that these structures coincide. In realistic lattice calculations one is to use the Wilson loop, which describes the gauge invariant state of hadron generated at some initial and annihilated at final moment of time. As is well known, for static charges the Wilson loop consists of two wave functions considered above, joined by parallel transporters. Let us imagine now some evolution of the fluxes in baryon which would lead to the emerging of the ∆-type configuration of fluxes in some intermediate time. First of all we should note that the cross-section of the Wilson loop by the time-like hypersurface will recover a gauge-invariant \( 3q \) state, i.e. the Y-type configuration. To have a ∆-shape for fluxes one should admit that the fluxes have no relation with the wave function, which is improbable.

Consider now the adjoint source \( g^\alpha(x)t^{(a)\beta}_\alpha \equiv G^\beta_\alpha(x) \). We do not specify here the Lorentz structure of \( g^\alpha(x) \), but only impose condition that it should gauge transform homogeneously, \( G^\beta_\alpha \to U^+_{\beta\gamma}G^\gamma_\alpha U^\alpha_\beta \). Therefore \( g^\alpha(x) \) can be either the field strength \( F^\alpha_{\mu\nu}(x) \), or valence gluon field \( a^\alpha_\alpha(x) \) in the background-field perturbation theory \([9]\). It is easy to construct a ∆-type configuration for 3 such sources;

\[
G_\Delta(x, y, z) = G^\beta_\alpha(x)\Phi^\gamma_\beta(x, y)\Phi^\delta_\gamma(y, z)G^\epsilon_\delta(z, x)\Phi^\alpha_\epsilon(z, x).
\] (4)

It is clear that in (4) all repeated indices form gauge-invariant combinations, and \( G_\Delta(x, y, z) \) is a gauge-invariant ∆-type configuration, which was used previously for the 3g glueball in \([10]\).
But one can persuade oneself that (4) is not the only 3g gauge-invariant configuration. Consider adjoint sources and adjoint PT (here distinguishing upper and lower indices is not necessary) and form as in (3) an extended gluon operator:

$$g_a(x, Y) \equiv g^b(x)\Phi_{ab}(x, Y)$$  \hspace{1cm} (5)

and an Y-shaped configuration

$$G_Y^{(f)}(x, y, z, Y) = f^{abc}g_a(x, Y)g_b(y, Y)g_c(z, Y).$$  \hspace{1cm} (6)

In the same way one constructs $G_Y^{(d)}$ replacing $f$ by $d$ in (6). It is clear that $G_Y$ is gauge-invariant and should be considered on the same grounds as $G_\Delta$.

At this point it is necessary to clarify how (2), (3) generate Green’s functions and Wilson loops.

To this end consider initial and final states made of (2), (4), (5) and for simplicity of arguments take all fundamental and adjoint sources to be static, i.e. propagating only in Euclidean time.

Then the Green’s function for the object will be

$$G_i(X, X) = \langle \Psi_i^\dagger(X)\Psi_i(X) \rangle$$  \hspace{1cm} (7)

where $\Psi_i = G_\Delta, G_Y, B_Y; X = x, y, z$ for $G_\Delta$ and $x, y, z, Y$ otherwise. Now it is important that vacuum average in (7) produces a product of Green’s functions for quarks or for valence gluons in the external vacuum gluonic field, which is proportional to the corresponding PT, fundamental – for quarks and adjoint – for gluons. Namely,

$$\langle \bar{q}_\beta(x)q^\alpha(x) \rangle \sim \Phi^\alpha_\beta(\bar{x}, x),$$  \hspace{1cm} (8a)

$$\langle g^a(\bar{x})g^b(x) \rangle \sim \Phi_{ab}(\bar{x}, x).$$  \hspace{1cm} (8b)
Figure 2: θ-shaped Wilson loop

(This statement is well known for static sources, for relativistic quarks and gluons this follows directly from the exact Fock-Feynman-Schwinger representation (FFSR), see \[11, 12\] and for a review \[13\]).

As a result one obtains a gauge-invariant Wilson-loop combination for each Green’s function (7). In particular for $B_Y$ (2) one has a familiar 3-lobe Wilson loop $W_Y$:

$$W_Y(\xi, \bar{\xi}) = \text{tr}_Y \prod_{i=1}^{3} W_i(C_i),$$  \hspace{1cm} (9)

where $\text{tr}_Y = \frac{1}{6} e_{\alpha\beta\gamma} e_{\alpha'\beta'\gamma'}$, and the contour $C_i$ in the open loop $W_i$ passes from $Y$ to $\bar{Y}$ through points $x, \bar{x}$ $(i = 1)$, $y, \bar{y}$ $(i = 2)$, or $z, \bar{z}$ $(i = 3)$, as shown in Fig.1.

This situation is well-known and was exploited in numerous applications. Relatively less known are the Wilson-loop configurations for $G_Y$ and $G_{\Delta}$. In the first case the structure is the same with the replacement of fundamental lines and symbols by the adjoint ones: $e_{\alpha\beta\gamma} \rightarrow f^{abc}$ or $e_{\alpha\beta\gamma} \rightarrow d^{abc}$, $\Phi_{\alpha} \rightarrow \Phi_{ab}$, so that the whole structure in (9) is the same with this replacement. Contrary to the baryon case, we can contract adjoint indices in two ways, using antisymmetric symbol $f^{abc}$ or symmetric one $d^{abc}$. The proper choice is related to the Bose-statistics of the gluon system which ensures the full coordinate-spin function to be symmetric.

In the case of $G_{\Delta}$ using (4) and (8) one can write the resulting structure symbolically as follows

$$G_{\Delta}(\bar{X}, X) = \Delta_{a'b'c'}(\bar{x}, \bar{y}, \bar{z}) \Phi_{a'a'}(\bar{x}, x) \Phi_{b'b}(\bar{y}, y) \Phi_{c'c}(\bar{z}, z) \Delta_{abc}(x, y, z)$$  \hspace{1cm} (10)

where we have denoted

$$\Delta_{abc}(x, y, z) = t^{(a)\beta}_{\alpha} \Phi_{\beta}^{\gamma}(x, y) t^{(b)\delta}_{\gamma} \Phi_{\delta}^{\epsilon}(y, z) t^{(c)\rho}_{\epsilon} \Phi_{\rho}^{\alpha}(z, x).$$  \hspace{1cm} (11)

To understand better the structure of (10), one can use the large $N_c$ approximation, in which case one has

$$\Psi_{\alpha\alpha'}^{\beta\beta'} \equiv t^{(a')\beta'}_{\alpha'} \Phi_{a'a'}(\bar{x}, x) t^{(a)\beta}_{\alpha} \approx \frac{1}{2} \Phi_{\alpha}^{\beta}(x, \bar{x}) \Phi_{\alpha}^{\beta}(\bar{x}, x).$$  \hspace{1cm} (12)
As a result in this approximation $G_\Delta$ appears to be a product of 3 fundamental closed loops, properly oriented with respect to each other

$$G_\Delta(X, X) \sim W(\bar{x}, \bar{y}|x, y)W(\bar{y}, \bar{z}|y, z)W(\bar{z}, \bar{x}|z, x) \equiv W_\Delta(X, X),$$

(13)

it is displayed in Fig. 2.

3. Static potentials for configurations (2), (4), (6) can be computed using Field Correlator Method (FCM)\[14\], through the equation

$$V = -\lim_{T \to \infty} \frac{1}{T} \ln \langle W \rangle,$$

(14)

where $T$ is the time extension of the Wilson loop.

For the baryon in the case of 3 quarks at the vertices of an equilateral triangle, at the distance $R$ from the string junction Y, the static baryon potential reads as \[15\]

$$V^{(B)}(R) = 3V^{(M)}(R) + V^{(nd)}(R),$$

(15)

where

$$V^{(M)}(R) = \frac{2\sigma}{\pi} \left\{ R \int_{0}^{R/T_g} dx x K_1(x) - T_g \left( 2 - \frac{R^2}{T_g^2} K_2 \left( \frac{R}{T_g} \right) \right) \right\}$$

(16)

is the mesonic confining potential with the asymptotic slope $\sigma \approx 0.18$ GeV$^2$ and the gluonic correlation length $T_g = 0.12 \div 0.2$ fm \[16\], and the nondiagonal part of the potential,

$$V^{(nd)}(R) = \frac{2}{\sqrt{3}} \sigma T_g - \frac{3\sqrt{3} \sigma R^2}{2\pi} \int_{-\pi}^{\pi} d\varphi \frac{1}{\cos \varphi} K_2 \left( \frac{\sqrt{3}R}{2T_g \cos \varphi} \right),$$

(17)

appears due to the interference of the gluonic fields on different lobes of the Wilson loop. Note the difference in the overall factor $-1/2$ with the previous calculations \[17\], where it was
Figure 4: The lattice baryon potential in the equilateral triangle with quark separations \( r \) from \([6]\) (points) at \( \beta = 5.8 \) and the MFC potential \( V^{(B)} + V_{\text{pert}}^{(\text{fund})} \) (solid line) at \( \alpha_s = 0.18, \sigma = 0.18 \text{ GeV}^2, \) and \( T_g = 0.12 \text{ fm}. \)

erroneously omitted. Let us denote \( L \equiv 3R \) the total length of the string. In Fig. 3 from \([15]\) the dependence of lattice nonperturbative baryon potential from \([4]\) on \( L \) along with the MFC potential \((15)-(17)\) is shown. One can see that our potential is in the complete agreement with the lattice results. In the asymptotic region \( L \gtrsim 1.5 \text{ fm} \) the potential has a linear form

\[
V^{(B)}(R) \approx \sigma L + \left( \frac{2}{\sqrt{3}} - \frac{12}{\pi} \right) \sigma T_g. \tag{18}
\]

The dotted tangent in Fig. 3 demonstrates that in the range \( 0.3 \text{ fm} \lesssim L \lesssim 1.5 \text{ fm} \) the lattice data can be described by the linear potential with the slope some 10\% less than \( \sigma. \)

The potential written so far contains only the nonperturbative confining part. To obtain the total potential we should add to it the perturbative color-coulombic potential

\[
V_{\text{pert}}^{(\text{fund})}(r) = -\frac{3}{2} \frac{C_2(\text{fund})}{r} \alpha_s, \tag{19}
\]

where \( r = \sqrt{3}R \) is the interquark distance in the equilateral triangle and \( C_2(\text{fund}) = 4/3. \)

In Fig. 4 lattice data from the last ref. of \([3]\) and the potential \( V_B(r) + V_{\text{pert}}^{(\text{fund})}(r) \) are shown. One can see that our results are in the complete agreement with this independent set of the lattice data as well.

In a similar way one can write the static potential for the adjoint sources, neglecting the nondiagonal term, which is different in symmetric and antisymmetric states:

\[
V_Y^{(G)}(R) = \frac{C_2(\text{adj})}{C_2(\text{fund})} V_Y^{(B)}(R) = \frac{9}{4} V_Y^{(B)}(R). \tag{20}
\]

Consider the \( \Delta \)-configuration in the approximation \([12]\). In this case \( V_\Delta^{(G)}(R) \) reduces to the sum of the mesonic potentials corresponding to area laws for all three loops minus the nondiagonal interference term, and one obtains just as in \([17]\)

\[
V_\Delta^{(G)}(r) = 3V^{(M)}(r) + V^{(\text{nd})}(r). \tag{21}
\]
Along with the adjoint perturbative potential

\[ V_{\text{adj}}^\text{pert}(r) = -\frac{3}{2} C_2(\text{adj}) \alpha_s, \]

where \( C_2(\text{adj}) = 3 \), we plot both \( V_Y^{(G)} \) and \( V_\Delta^{(G)} \) in Fig. 5 without the interference terms. We see from the figure that the curves intersect at \( r \approx 0.5 \) fm, and are very close to each other.

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4. To summarize our results, we have considered possible gauge-invariant configurations of 3 fundamental or adjoint sources and corresponding Wilson loops, which have \( Y \)-type shape for fundamental charges and may have both \( Y \)-type and \( \Delta \)-type shapes for the adjoint ones. We have shown that the static baryon potential obtained in the MFC is in the complete agreement with the lattice data.

For adjoint sources it was demonstrated that two possible configurations yield static potentials differing only a little. This in turn implies that 3-gluon glueballs [10] may be of two distinct types, with no direct transitions between them (quark-containing hadrons must be involved as intermediate states). The mass of the \( \Delta \)-shaped \( 3^{--} \) glueball was found in [10] to be \( m^{(3g)}_\Delta = 3.51 \) GeV for \( \sigma_f = 0.18 \) GeV\(^2\) (or 4.03 GeV for \( \sigma_f = 0.238 \) GeV\(^2\)) to be compared with lattice one calculated in [19] 4.13±0.29 GeV. The mass of the \( Y \)-shaped glueball can easily be computed from the baryon mass calculated in [20], multiplying it by \( \sqrt{9/4} = 3/2 \). In this way one obtains \( m_Y^{(3g)} = 3.47 \) GeV (\( \sigma_f = 0.18 \) GeV\(^2\)). The slope of the corresponding odderon trajectory is almost the same and corresponds to \( g - gg \)-configuration. Thus one obtains the \( \Delta \)-odderon (slope)\(^{-1} \) to be twice the standard Regge slope, while for \( Y \)-odderon it is \( 9/4 \) of the standard slope. In both cases the intercept comes out as in [10] to be rather low (\( -1.8 \) for the \( Y \)-shape and \( -2.4 \) for the \( \Delta \)-shape) implying very small odderon contribution to reactions under investigation [21] in agreement with measurements. We plan to perform more accurate calculations of glueball potentials and spectra taking into account the string-string interference in subsequent publications.

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