Crystallographic features of magnetostatic waves spectra in ferrite films

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Abstract. Influence of crystallographic magnetic anisotropy on dispersion laws of magnetostatic spin waves is discussed. Classification of waves spectra in single crystal films of ferrites is given. The scale of magnetic anisotropy influence on spectra of volume and surface types of waves is investigated. The film model with cubic magnetic anisotropy is constructed and expressions for components of a tensor of the effective demagnetization anisotropy factors are deduced. Results of spectra calculation of magnetostatic waves in a film of yttrium iron garnet are given.

1. Introduction
One of the perspective directions of functional electronics is development of devices, in which transformation of signals is carried out by means of magnetostatic spin waves (MSWs). The greatest interest is represented by MSWs extending in films of ferrite [1, 2]. Their phase speed is some orders less than velocity of light and variations of fields in space than in time are more essential to them. The theory of such waves can be constructed on the basis of the equation of the movement of magnetization and Maxwell's equations taken in magnetostatic approach.

Important feature of films is their perfect crystal structure and as a result films possess anisotropy of magnetic properties. Crystallographic magnetic anisotropy belongs to the factors defining dispersive characteristics of MSWs. Radiation effects and changes of temperature lead to changes of anisotropy parameters and it influences on dispersion waves laws. To the present time many aspects of such an influence are studied. Private results relating to the films of the cubic ferrites are together with common approaches to a description of the MSWs spectra in the films with arbitrary magnetic anisotropy type were obtained [3 – 8]. The orientation of the investigations just to such ferrites is caused by wide use in the spin-wave electronics of the yttrium iron garnet films (YIG, $Y_3Fe_5O_{12}$), which are grown up on the single crystal gadolinium gallium garnet (Gd$_3$Ga$_5$O$_{12}$) substrates. In the present work the model, in which crystallographic orientation of a cubic ferrite film is the varied parameter, is built and investigated. Thus the considered earlier separate orientations become only special cases of the developed model.

2. General ratios
The studied model is presented in figure 1. The ferrite film thickness of $d$ and the unlimited sizes in the plane is magnetized before saturation by an external constant magnetic field. The vector of magnetization $M_0$ forms an angle $\theta$ with the film plane. Two systems of coordinates are entered into consideration. In $xyz$ system the $y$ axis is parallel to the film plane and the $z$ axis is parallel to the $M_0$
vector, \( z \parallel \mathbf{M}_0 \). In \( \xi\eta\zeta \) system the \( \xi \) axis is directed along a normal to a film \( n \) and the \( \eta \) axis along a wave vector of MSW \( k \). The \( \phi \) angle defines a direction of the \( \eta \) axis regarding the \( xz \) plane. The dispersive equations (DEs) of the magnetostatic waves are deduced by joint integration of the linearized equation of the magnetization movement without an exchange and losses and Maxwell's equations taken in magnetostatic approach and with the corresponding electrodynamic boundary conditions. As a result the DEs can be presented in the form of the equations expressed through the components of the magnetic permeability tensor of the film material taken in \( \xi\eta\zeta \) system

\[
\tan \left[ kd \sqrt{\left( \mu_{\xi\eta} + \mu_{\eta\zeta} \right)^2 - 4\mu_{\xi\zeta}\mu_{\eta\eta}} \right] = \sqrt{\left( \mu_{\xi\eta} + \mu_{\eta\zeta} \right)^2 - 4\mu_{\xi\zeta}\mu_{\eta\eta}} \over 2\mu_{\xi\zeta} \] (1)

if \( \left( \mu_{\xi\eta} + \mu_{\eta\zeta} \right)^2 \geq 4\mu_{\xi\zeta}\mu_{\eta\eta} \) and

\[
\exp \left[ kd \sqrt{4\mu_{\xi\zeta}\mu_{\eta\eta} - \left( \mu_{\xi\eta} + \mu_{\eta\zeta} \right)^2} \over \mu_{\xi\zeta} \right] = \left[ \sqrt{4\mu_{\xi\zeta}\mu_{\eta\eta} - \left( \mu_{\xi\eta} + \mu_{\eta\zeta} \right)^2 - 2} \right]^2 + \left( \mu_{\xi\eta} - \mu_{\eta\zeta} \right)^2 \over \left[ \sqrt{4\mu_{\xi\zeta}\mu_{\eta\eta} - \left( \mu_{\xi\eta} + \mu_{\eta\zeta} \right)^2 + 2} \right]^2 + \left( \mu_{\xi\eta} - \mu_{\eta\zeta} \right)^2 \] (2)

at a reverse inequality.

Figure 1. The coordinates systems used at a conclusion of the dispersion equations of the magnetostatic waves.

In the \( xyz \) coordinates system the tensor of magnetic permeability has a view

\[
\tilde{\mu} = \begin{pmatrix} \mu_x & \mu_s + i\mu_a & 0 \\ \mu_s - i\mu_a & \mu_y & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
\mu_x = 1 + \frac{4\pi M_0 \omega_x}{\Omega}, \quad \mu_y = 1 + \frac{4\pi M_0 \omega_y}{\Omega}, \quad \mu_s = -\frac{4\pi M_0 \omega_x}{\Omega}, \quad \mu_a = \frac{4\pi M_0}{\Omega} \begin{pmatrix} f \\ g \end{pmatrix},
\]

\[
\omega_x = H_{in} + M_0 \left( N_{yy}^a - N_{zz}^a \right), \quad \omega_y = H_{in} + M_0 \left( N_{xx}^a - N_{zz}^a \right), \quad \omega_s = M_0 N_{xy}^a,
\]

2
Here \( g = 2.8 \text{ MHz/Oe} \) is the gyromagnetic ratio; \( H_e \) is the external magnetizing field strength; \( N^n_i \) are the tensor components of effective demagnetizing anisotropy factors [9]; \( f \) is the wave frequency. In passing to the \( \zeta \eta \zeta \) coordinate system, the components of tensor \( \mu \) are transformed as follows

\[
\begin{align*}
\mu_{\zeta\zeta} &= \mu_x \cos^2 \theta + \sin^2 \theta, \\
\mu_{\eta\eta} &= \left(1 - \mu_x\right) \sin \theta \cos \varphi + (\mu_x^* + i \mu_y) \sin \varphi \cos \theta, \\
\mu_{\zeta\eta} &= \left(1 - \mu_x\right) \sin \theta \cos \varphi + (\mu_x^* - i \mu_y) \sin \varphi \cos \theta, \\
\mu_{\zeta\eta} &= \left(\mu_x \sin^2 \theta + \cos^2 \theta\right) \cos^2 \varphi - \mu_x \sin 2\varphi \sin \theta + \mu_y \sin^2 \varphi.
\end{align*}
\] (3)

The analysis of the spectra of the main types MSWs is based on the equations (1), (2).

2.1. The perpendicularly magnetized layer when \( \varphi = \pi/2 \), and it is also possible to combine \( y \) and \( \eta \) axes and therefore equality \( \varphi = \pi/2 \) is carried out.

Then \( \mu_{\zeta\zeta} = 1, \mu_{\eta\eta} = \mu_{\zeta\eta} = 0 \) and DE has a view

\[
\tan(kd\sqrt{-\mu_{\eta\eta}}) = -\frac{2\sqrt{1 + \mu_{\eta\eta}}}{\mu_{\eta\eta}},
\] (4)

The equation (4) describes modes of direct volume MSW. \( H_{ec} \) is a projection of the vector of the external magnetizing field strength to a magnetization vector.

2.2. Tangentially magnetized layer when \( \varphi = 0 \) and \( k \parallel M_0 \), thus equality \( \varphi = 0 \) is carried out.

Then \( \mu_{\eta\eta} = 1, \mu_{\zeta\zeta} = \mu_{\zeta\eta} = 0 \) and DE has a view

\[
\tan \frac{kd}{\sqrt{-\mu_{\zeta\zeta}}} = \frac{2\sqrt{1 + \mu_{\zeta\zeta}}}{\mu_{\zeta\zeta}},
\] (5)

The equation (5) describes modes of the backward volume waves.

2.3. Tangentially magnetized layer when \( \theta = 0 \) and \( k \perp M_0 \), \( \varphi = \pi/2 \).

The equations (1), (2) after all substitutions and algebraic transformations assume the following view

\[
\tan \frac{kd\sqrt{(f_1^2 - f_2^2)(f_2^2 - f_3^2)}}{f_2^2 - f_0^2} = \frac{2\sqrt{(f_1^2 - f_2^2)(f_2^2 - f_3^2)}}{(4\pi M_0 g)^2 + f_1^2 + f_2^2 - 2f_2^2},
\] (6)
The equation (6) corresponds to bulk wave and (7) to surface wave (or mixed at $N_{xy}^a \neq 0$). Here

$$\left( f_0 / g \right)^2 = H_{cc}^2 + H_{cc}M_0 \left( N_{xx}^a + N_{yy}^a - 2N_{zz}^a + 4\pi \right) + M_0^2 \left[ \left( N_{xx}^a - N_{yy}^a + 4\pi \right) \left( N_{yy}^a - N_{zz}^a \right) - \left( N_{xy}^a \right)^2 \right].$$

$$f_{1,2}^2 = \frac{f_0^2}{2} + 2\pi M_0^2 g^2 \left[ \left( N_{xx}^a - N_{yy}^a \right) \pm \sqrt{\left( N_{xx}^a - N_{yy}^a \right)^2 + 4\left( N_{xy}^a \right)^2} \right].$$

Just surface MSW represents the greatest practical interest.

In experiment orientation of the external magnetizing field but not orientation of magnetization is usually controlled. In the anisotropic and uniformly magnetized film at $H_{\parallel} \perp n$ and $k \perp H_{\parallel}$ the angles $\theta$ and $\varphi$ defining the $M_0$ orientation can differ from values $\theta = 0$ and $\varphi = \pi/2$. It is physically clear that this difference of subjects is less than intensity of the magnetizing field in comparison with the magnetic anisotropy effective fields is more. The detailed consideration of a given question is a separate task and goes out beyond the frames of carried out investigation of dispersion laws. It is possible to note however that on the formulas given above can be calculated angular derivative of frequency and at $\theta = 0$, $\varphi = \pi/2$ they become zero: $df / d\theta = 0$ and $df / d\varphi = 0$. So the corrections of dispersion laws connected with consideration of small deflections of the angles from the considered values will have the second order of smallness.

The example of the spectrum calculated on formulas (6) and (7) for values $(f_0 / 4\pi M_0 g)^2 = 1$; $(f_1 / f_0)^2 = 1.1$; $(f_2 / f_0)^2 = 0.95$ is given in figure 2. SMSW, FVMSWs and BVMSWs are respectively surface, forward volume and backward volume MSWs.

Figure 2. Spectrum of magnetostatic waves in the tangentially magnetized film and with a wave vector perpendicular to a magnetization vector.
Unlike a surface wave both types of volume waves are multimode. In figure 2 are represented some modes with initial numbers “$n$”. Spectrum of the forward volume waves is situated in interval $f_0 < f < f_1$, and in the interval $f_2 < f < f_0$ there is a spectrum of the backward volume waves. The surface MSW spectrum occupies frequency interval

$$f_1 < f < f_\infty = gH_{ec} + 2\pi M_0 g + 0.5gM_0 \left(N_{xx}^a + N_{yy}^a - 2N_{zz}^a \right)$$

and the wave vectors are restricted to the values

$$kd > k_{sv}d = \frac{2(f_1^2 - f_0^2)}{(4\pi M_0 g)^2 - (f_1^2 - f_0^2)}.$$  

Dispersive dependence of the main mode of a forward volume wave smoothly passes into dispersive dependence of a surface wave at $f = f_1$ and $k = k_{sv}$.

The comparative estimation of intervals of the frequencies occupied by volume waves and a surface wave looks as follows

$$\frac{f_1^2 - f_2^2}{f_0^2 - f_0^2} = \frac{16\pi \sqrt{(N_{xx}^a - N_{yy}^a)^2 + (2N_{xy}^a)^2}}{(4\pi + N_{xx}^a - N_{yy}^a)^2 + (2N_{xy}^a)^2}.$$  

It follows from this expression that in the tangentially magnetized films of the weakly anisotropic ferrites (when $|N_{ij}^a| < 1$) the large part of MSW spectrum with wave vectors perpendicular to the magnetization will occupy by a surface type wave. Besides spectra of the volume waves arise in the considered conditions only due to crystallographic magnetic anisotropy and in isotropic model these spectra aren’t present.

3. Anisotropy of the surface wave spectrum in a cubic ferrite film

In connection with the considered task we will note that YIG films with crystallographic orientation of \{111\} type are most widely applied now in spin-wave devices. At the same time some characteristics of devices can be improved when using films of other orientations. In particular the thermostability of SMSW spectra in the YIG films with orientations of \{110\} and \{100\} type is higher than in the \{111\} films.

We shall build a model, allowing executing of calculations for the films oriented along by any of the crystallographic planes passing across an axis of <110> type. Then, for example, all three mentioned orientations \{111\}, \{110\}, \{100\} become special cases of such model. Geometrical aspects of placed task are represented in figure 3.
The crystallographic orientation of the magnetization vector is defined by two angles. The angle $\psi$ defines the direction in the film plane concerning $<110>$ axis and the $\delta$ angle sets an inclination of the plane $\{100\}$ passing through the considered $<110>$ axis to the film plane.

Magnetic anisotropy of ferrite is taken into account with the help of $M_0 N_{ij}^{(l)}$ components. Expressions for them can be deduced from expression for energy of cubic magnetic anisotropy [9]. As a rule it is enough to take only one constant into account in this energy (two constants are necessary for $\{111\}$ orientation and this aspect is discussed below).

\[
M_0 N_{xx}^{(l)} = -\frac{K_1^c}{M_0} \left\{ 1 - \frac{1 + 3 \cos 2\delta}{16} \left[ (1 + 3 \cos 2\delta) + 3(1 - \cos 2\delta) \cos 2\psi \right] \right\},
\]
\[
M_0 N_{yy}^{(l)} = -\frac{K_1^c}{M_0} \left\{ 1 + \frac{1 + 3 \cos 2\delta}{32} \left[ (1 + 3 \cos 2\delta) + 3(5 - \cos 2\delta) \cos 4\psi \right] \right\},
\]
\[
M_0 N_{zz}^{(l)} = -\frac{K_1^c}{M_0} \left\{ 1 + \frac{1 + 3 \cos 2\delta}{32} \left[ (1 + 3 \cos 2\delta) + 4(1 - \cos 2\delta) \cos 2\psi - (5 - \cos 2\delta) \cos 4\psi \right] \right\},
\]
\[
M_0 N_{xy}^{(l)} = \frac{3}{2} \frac{K_1^c}{M_0} \sin 2\delta \left[ \cos 3\psi + \frac{1 + 3 \cos 2\delta}{8} (\cos \psi - \cos 3\psi) \right].
\]

$K_1^c$ is the first constant of cubic magnetic anisotropy. In figure 4a examples of angular dependences of SMSW frequencies calculated on DE (7) with substitutions of (8), (9) and $M_0 N_{ij}^{(l)}$ expressions are given. The crystallographic orientation of a film was defined by the values of the $\delta$ angle. Calculations correspond to the following values of magnetic parameters of YIG crystals (at 293 K): $4\pi M_0 = 1750$ G, $K_1^c / M_0 = -43$ Oe. It was supposed that $H_{cz} / 4\pi M_0 = 1/3$ and $kd = 1$.

![Figure 4](image_url)

**Figure 4.** Angular dependences of SMSW frequencies in films with various crystallographic orientations.

Films with $\{111\}$ orientation possess the following feature. Equality of $(1 + 3 \cos 2\delta) = 0$ is carried for them and as appears from the $M_0 N_{ij}^{(l)}$ expressions dependence on a $\psi$ angle remains only at $M_0 N_{xy}^{(l)}$. And unlike diagonal components, the parameter $M_0 N_{xy}^{(l)}$ is included into DE in the quadratic kind and therefore angular dependence of the frequencies in the $\{111\}$ plane will be the weakest. In this case the account not only the first, but the second constant of magnetic cubic anisotropy is necessary [5]. Calculation of the effective demagnetizing anisotropy factors corresponding to the accounting of the second constant ($K_2^c$) leads to the following formulas
\[ M_0 N_{x_1}^{a(2)} = \frac{K_2^\zeta}{6 M_0}, \quad M_0 N_{x_y}^{a(2)} = \frac{K_2^\zeta}{18 M_0} (1 + 5 \cos 6\psi), \quad M_0 N_{x_z}^{a(2)} = \frac{K_2^\zeta}{18 M_0} (1 - \cos 6\psi), \]
\[ M_0 N_{x_y}^{a(2)} = \frac{\sqrt{2}}{6} \frac{K_2^\zeta}{M_0} \cos 3\psi. \]

The results shown in figure 4b visually demonstrate effect of the second constant on the angular dependence of the SMSW frequency in the \{111\} film. Curve 1 take into account only the first anisotropy constant while curve 2 account for the first and the second anisotropy constant. The latter was taken as \( K_2^\zeta / M_0 = -2 \text{Oe} \). Values of other parameters didn't change.

4. Conclusions
The MSW dispersion laws in ferrite films are deduced and analysed. It is shown that the accounting of crystallographic magnetic anisotropy of ferrite leads to emergence in spectra of MSW of the additional dispersive dependences which are absent in isotropic models. The surface MSW spectrum in the tangentially magnetized films was in details investigated. It is established that the greatest anisotropy of spectrum is inherent in the films with \{100\} and \{110\} orientations and smallest with \{111\} type.

The results can be used for developing spin-wave devices. In particular change of characteristics of these devices at radiation and thermal effects will be defined by changes of the magnetic parameters of ferrite of such as saturation magnetization and a magnetic crystallographic anisotropy field. The obtained analytical dependences allow both forecasting of these changes and minimize them promoting development of devices with thermostable and resistant to radiation characteristics.

5. References
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