Safety Criteria for Solid–Liquid Heterogeneous Systems in Semibatch Reactors
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ABSTRACT: An exothermic reaction in a semibatch reactor can potentially cause thermal runaway due to evolved energy accumulation or a secondary reaction. This research aims to propose safety criteria for solid–liquid reactions in semibatch reactors. Simulation modeling was carried out to build thermal runaway criteria for solid–liquid reactions in semibatch reactors. A new model for the energy and mass balance of solid–liquid reactions was successfully established. Criteria for the safety boundary diagram and the temperature diagram were ameliorated for solid–liquid reactions. The results showed that the dissolution heat has a great influence on the thermal behavior of the reaction. Experiments to neutralize citric acid and sodium hydroxide were carried out to determine the critical parameters for the neutralization reaction using the temperature diagram criterion. The proposed criteria would be reasonably expected to provide some guidance for chemical process optimization and safety design for engineering.

1. INTRODUCTION
Semibatch reactors (SBRs) used in the fine chemical and pharmaceutical industries has been studied extensively. Compared with a batch reactor (BR), a SBR can avoid extreme temperatures by controlling the dosing rate and the cooling temperature. However, thermal runaway accidents are not eliminated completely. Runaway occurs when the operation parameters are set inappropriately. To obtain safe operating conditions, a huge amount of work has been done on safety criteria. Semenov first presented the Semenov criterion in 1928. Considering an exothermal reaction with zero-order kinetics and neglecting the reactant consumption, Semenov provided the Semenov number to obtain the critical temperature. As for SBRs, in 1986, Hugo et al. found that the accumulation of reactants is the main reason for thermal runaway in SBRs and investigated the influence of the breakdown of cooling to the maximum process temperature. Steensma et al. introduced the target temperature \( T_{ta} \) to verify different thermal behaviors of reactions and proposed a boundary diagram for liquid–liquid homogeneous reactions in SBRs. Based on their work, Maestri et al. improved the boundary diagram for liquid–liquid heterogeneous reactions in SBRs and proposed an adiabatic temperature diagram by comparing the maximum reaction temperature and the maximum allowable temperature (MAT).

Recently, Ni et al. enhanced the boundary diagram by adding a secondary reaction temperature. Guo et al. defined \( T_{n} \) as a dimensionless constant temperature to identify the safe operating conditions and predict the maximum temperature of the synthesis reaction (MTSR) at the same time. Bai et al. identified four types of reactor thermal behaviors based on the profiles of MTSR, \( \theta_{MTSR} \), and the cooling temperature and proposed a new criterion for checking the safer operating conditions of homogeneous reactions in SBRs. After that, Han et al. extended the results of Bai’s research to heterogeneous reactions. Zhang et al. proposed a multifeature recognition criterion based on pattern recognition to develop a safety boundary diagram.

In addition, sensitive criteria and divergence criteria have been proposed to forecast thermal runaway. Casson et al. compared these works to investigate the hazard of acid-catalyzed esterification and pointed out that these criteria mainly aimed to detect runaway conditions without considering process optimization. Alex et al. summarized the criteria since 1928 and regarded the safety boundary diagram and the

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temperature diagram as easy and suitable for reactions in SBRs compared with other criteria.  

Previous safety boundary diagram- and temperature diagram-related works have generally been proposed for liquid–liquid reactions. Actually, a number of solid–liquid reactions can also have serious consequences due to the massive reaction heat and dissolution heat. The dynamics of solid–liquid reactions in SBRs was first presented by Samoilenko et al. In their study, they investigated the thermal explosion of solid–liquid reactions in a BR and a SBR. However, the mathematical model they proposed did not take into account the influence of dissolution, which can be an exothermic or endothermic process. Besides, the volume of the solid dissolved in the liquid was ignored.

In this paper, first, the dissolution of the solid and the mass and energy balance of solid–liquid reactions was studied in detail. Second, a safety boundary diagram and a temperature diagram were proposed for solid–liquid reactions. Besides, a series of calorimetric experiments were carefully carried out in a SBR to acquire the dissolution heat and the reaction heat. In addition, kinetic and thermodynamic parameters were obtained using Matlab, and the reliability of the theoretical simulation calculation results was demonstrated.

2. RESULTS AND DISCUSSION

2.1. Criteria for the Safety Boundary Diagram and the Temperature Diagram. The thermal safety of solid–liquid reactions in SBRs was assessed both qualitatively and quantitatively using criteria of the safety boundary and the temperature diagram. The necessary steps are listed in Figure 1.

2.1.1. Mathematical Model for Solid–Liquid Reactions in SBRs. In an indirect-cooling SBR, component A is fully dosed from the beginning, while component B is dosed at a constant rate. Here we consider A as the liquid phase and B as the solid phase. A reacts with B to form C and D as products. The reaction heat is removed by a flow of coolant through a jacket.

The mathematical model was formulated on the basis of the following assumptions:

1. The reaction mass is completely macromixed.
2. The volume change of the liquid phase is equal to the volume of the dissolved solid.
3. The heat effects are associated with the chemical reaction and the dissolution of the solid.
4. The physicochemical properties of all the components are constant during the reaction.
5. The reaction happens in the liquid phase only.
6. The heat exchange of the solid reactant completes immediately when dosed into the reactor.
7. The dosing temperature of the solid is equivalent to the temperature diagram as easy and suitable for reactions in SBRs compared with other criteria.  

The volume of solid B, in turn, that is affected by the dissolution and dosing can be written as shown in eq 1.

\[ \frac{dV_B}{dt} = -\delta S(eC_B - C_{B0}) + qC_B \] (1)

The interphase surface area of solid B is determined by the volume of solid B and the specific surface area per unit volume (S_sp).  

\[ S = S_{sp} \frac{V_B}{\mu} \] (2)

To determine the specific surface area, it must be calculated as an ensemble of particles (for example, equivalent to spherical particles) with an unknown size distribution. This characteristic of an ensemble of spherical particles is given by eq 3.

\[ S_{sp} = \frac{6\mu'}{\mu_3} = \frac{6}{\int_0^{d_{max}} x^2 f(x) \, dx} \int_0^{d_{max}} x^3 f(x) \, dx \] (3)

The value of S_sp is calculated by eq 4 based on a Pearson distribution of type I.

\[ f(x) = \frac{x^{m_1}(d_{max} - x)^{m_2}}{(d_{max})^{m_1} + m_1 + 1} = B(m_1 + 1, m_2 + 1) \] (4)

where \( 0 \leq x \leq d_{max} \), \( m_1 > 0 \), \( m_2 > 0 \), and \( B(m_1 + 1, m_2 + 1) \) is the special \( \beta \)-function. Thus, the second and third
Concentration of liquid A dosed at the beginning decreases as the dynamic behavior of the reaction system:

\[
\mu_2' = \frac{d_{\text{max}}^2}{(m_1 + 1)(m_1 + 2)(m_1 + m_2 + 3)}
\]

\[
\mu_3' = \frac{d_{\text{max}}^3}{(m_1 + 1)(m_1 + 2)(m_1 + m_2 + 3)(m_1 + m_2 + 4)}
\]

Thus, the specific surface area of solid B can be calculated according to eq 7.24

\[
S_p = \frac{6\mu_2'}{\mu_3'} = \frac{6(1 + \theta)}{\theta (m_1 + 1)} \quad \theta = \frac{(m_1 + 1)}{(m_1 + 3)} > 0
\]

The parameter \( \theta \) is a measure of the polydispersity of solid B. When \( \theta = 0 \), the distribution is monodisperse and \( S_p = (6/d_{\text{max}})V_B \). In this paper, \( \theta \) is assumed to be unity. Thus, the surface area is \( S = (12/d_{\text{max}})V_B \).

Before the dosing time \( t_d \), \( d_{\text{max}} \) is equal to the initial diameter of solid B. After \( t_d \), there is no more solid B dosed into the reactor, and \( d_{\text{max}} \) changes over time as shown in eq 8.24

\[
\frac{dd_{\text{max}}}{dt} = -2\left( e - \frac{C_{BA}}{C_B} \right)
\]

For the first stage, solid B is dosed into reactor from \( t_0 \) to \( t_d \). The following system of equations was established to describe the dynamic behavior of the reaction system:

Concentration of liquid A dosed at the beginning decreases as reactant B is dosed:

\[
\frac{dC_A}{dt} = -\nu_Ak_{\text{d}}\exp\left(-\frac{E_a}{RT}\right)C_A^nC_{BA}^m
\]

Concentration of reactant B dissolved in the liquid phase decreases while with reactant A:

\[
V_A\frac{dC_{BA}}{dt} = \delta d_{\text{max}}\frac{6V_B(1 + \theta)}{d_{\text{max}}} (\epsilon C_B - C_{BA})
\]

\[
- \nu_Ak_{\text{d}}\exp\left(-\frac{E_a}{RT}\right)C_A^nC_{BA}^m V_A
\]

Volume of solid B dosed constantly decreases as it dissolves in the liquid phase:

\[
C_B\frac{dV_B}{dt} = -\delta d_{\text{max}}\frac{6V_B(1 + \theta)}{d_{\text{max}}} (\epsilon C_B - C_{BA}) + qV_B
\]

With above equations, the energy balance equation can be written as follows:

\[
\epsilon \rho_1(V_A + q)\frac{dT}{dt} = \Delta H_v\nu_A k_{\text{d}}\exp\left(-\frac{E_a}{RT}\right)C_A^nC_{BA}^m V_A
\]

\[
- \alpha(S_0 + \Delta \varepsilon)(T - T_{\text{cool}})
\]

\[
+ \epsilon \rho_1 q(T_{\text{cool}} - T)
\]

\[
+ \left( C_B\frac{dV_B}{dt} - qC_B \right)\Delta H_{\text{dis}}
\]

This reveals that the temperature of the reaction system is influenced by four factors: the chemical reaction, the dissolution, the heat removal by the cooling jacket, and the dosing of solid B.

When \( t = 0 \), the conditions read as

\[
\begin{align*}
C_A &= C_{A0}, C_{BA} &= 0, V_B &= 0, T &= T_{\text{cool}}
\end{align*}
\]

The dosing is stopped at \( t_d \) when the ratio of moles solid B dosed into the reactor to the initial number of moles liquid A is equal to the stoichiometric ratio. After that, the maximum diameter should be calculated with eq 8. As no more reactant is dosed into the reactor, we need to adjust eqs 9–12 above for the second stage.

\[
\frac{dC_A}{dt} = -\nu_A k_{\text{d}}\exp\left(-\frac{E_a}{RT}\right)C_A^nC_{BA}^m
\]

\[
V_A\frac{dC_{BA}}{dt} = \delta d_{\text{max}}\frac{6V_B(1 + \theta)}{d_{\text{max}}} (\epsilon C_B - C_{BA}) - \nu_A k_{\text{d}}\exp\left(-\frac{E_a}{RT}\right)C_A^nC_{BA}^m V_A
\]

\[
C_B\frac{dV_B}{dt} = -\delta d_{\text{max}}\frac{6V_B(1 + \theta)}{d_{\text{max}}} (\epsilon C_B - C_{BA})
\]

\[
\epsilon \rho_1(V_A + q)\frac{dT}{dt} = \Delta H_v\nu_A k_{\text{d}}\exp\left(-\frac{E_a}{RT}\right)C_A^nC_{BA}^m V_A
\]

\[
- \alpha(S_0 + \Delta \varepsilon)(T - T_{\text{cool}}) + C_B\frac{dV_B}{dt}\Delta H_{\text{dis}}
\]

\[
\frac{d\Delta t}{dt} = -\Delta t \frac{\Delta t_{\text{dis}}}{1 + \rho P_2} - \alpha \Delta t \frac{\Delta t_{\text{diss}}}{1 + \rho P_2}
\]

\[
- R_H\frac{P_2}{1 + \rho P_2} \frac{\Delta t_{\text{dis}}}{1 + \rho P_2}
\]

where: \( \nu_A k_{\text{d}} = \nu_A k_{\text{d}} K_{\text{A}} C_{\text{A0}}, \gamma = E_a/(RT_0), P_1 = 6\delta t_d/d_{\text{max}}, P_2 = C_{\text{A0}}/C_{\text{B0}}, \Delta t_{\text{dis}} = -\Delta H_{\text{dis}}/\rho \Delta H_{\text{diss}}, (V_{\text{A0}}T_{\text{f}}/\rho \gamma), \alpha = 2\Delta t D/(\rho \gamma C_{\text{A0}}), \) and \( R_H = \epsilon \rho_1/(\epsilon \gamma) \).

At \( \theta = 0 \), the initial conditions are

\[
\eta_A = 1, \eta_B = 0, \eta_V = 0, \tau_0 = \tau
\]

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For the second stage ($\theta > 1$), the equations are nondimensionalized, as shown in eqs 25–29.

\[
\frac{dn_a}{d\theta} = -\nu_A D_a \exp\left[\left(1 - \frac{1}{\tau}\right)\eta_u \eta_a\right]
\]  

(25)

\[
\frac{dn^\Lambda}{d\theta} = \frac{dn_a}{d\theta} + \frac{R}{\eta_d} \frac{(1 + 8\varepsilon)}{(1 + 8\varepsilon)} \left(\frac{\varepsilon}{2} - \eta^\Lambda\right)
\]  

(26)

\[
\frac{dn}{d\theta} = -\frac{R}{\eta_d} \frac{(1 + 8\varepsilon)}{(1 + 8\varepsilon)} \left(\frac{\varepsilon}{2} - \eta^\Lambda\right) - \frac{dn}{d\theta} \frac{\Delta \tau_{\text{dis}}}{1 + 2\varepsilon}
\]  

(27)

\[
\frac{dn}{d\varepsilon} = \frac{R \eta_a (\eta - \varepsilon)}{3}
\]  

(29)

The initial conditions of eqs 25–29 can be found from the eqs 20–23, respectively, in the former stage. Additionally, $\eta_d = 1$ at $\theta = 1$.

Compared with the mathematical model proposed by Samoilenko,\textsuperscript{23} this new model takes into account the change of the volume of the liquid phase caused by the dissolved solid, which can influence the cooling effect of the cooling jacket. Moreover, the new model takes the heat effect of dissolution, which can be an exothermic or endothermic process, into consideration.

2.1.2. Safety Boundary Diagram. Liquid–liquid reactions and solid–liquid reactions share basically same equation for the target temperature, and the trend of each are the same. The temperature decreases as the dosing proceeds and become stable after the dosing is finished, as illustrated by eq 30.

\[
\begin{align*}
\tau_a &= \tau_0 + \frac{1.05(\Delta \tau_{\text{dis}} + \Delta \tau_{\text{sol}})}{P_2 R_{\text{ad}} + \alpha^* D_a (1 + 2\varepsilon)} \quad \theta \leq 1 \\
\tau_a &= \tau_0 + \frac{1.05(\Delta \tau_{\text{dis}} + \Delta \tau_{\text{sol}})}{P_2 R_{\text{ad}} + \alpha^* D_a (1 + 2\varepsilon)} \quad \theta > 1
\end{align*}
\]  

(30)

The reason for this downward trend is the change of the volume and the heat exchange area of the reaction system. In the case of a liquid–liquid reaction, the volume of the reaction system increases as the liquid is dosed into the reactor. The heat exchange area changes as well, thus the quantity of heat can be removed by the cooling jacket. The same as in liquid–liquid reactions, the change is caused by the dosing reactant in solid–liquid reactions. While the solid is dosed into the reactor, it also starts to dissolve, and the volume of the system increases steadily. As the reaction proceeds, the solid is dosed and dissolves into the liquid completely. Eventually, the entire volume of the reaction system is equal to its initial liquid volume plus the volume of the dosed solid, and the target temperature become the same as $\tau_a$.

Three characteristic types of thermal behaviors in a SBR were proposed by increasing the cooling temperature compared to $T_{\text{in}}$ as shown in Figure 2.\textsuperscript{25}

By changing the cooling temperature while keeping other conditions unchanged, it is possible to use eq 31 to find a range of cooling temperatures under which the reaction system can cause thermal runaway. Since the safety boundary diagram corresponds to the situation where the $\tau_{\text{max}}$ is equal to $\tau_{\text{in}}$, it is required to find the condition in compliance with the constraint

\[
\text{FF} = \tau_{\text{max}} - \tau_{\text{in}} \eta(\tau_{\text{max}})
\]  

(31)

As shown in Figure 3, the cooling temperature between the first two roots can lead to the thermal runaway of the system under a certain set of conditions. Thus, by changing the four parameters $D_u$, $\Delta \tau_{\text{d},0}$, $\gamma$, and $P_2$ and recording the corresponding roots of the eq 31, we can receive cooling
temperature intervals that can cause thermal runaway for different conditions.

Based on the research of Steensma and Westerterp, the reactivity factor ($F_R$), the exothermicity factor ($F_E$), the dissolution factor ($F_D$), the cooling factor ($F_{cool}$), the exothermicity number ($E_{ex}$), and the reactivity number ($R_y$) of solid−liquid reaction are as follows:

\[
F_R = v_A D_a \exp \left[ \gamma \left( 1 - \frac{1}{\tau_0} \right) \right]
\]

\[
F_E = \Delta \tau_{ad,0} \frac{\gamma}{\tau_0}
\]

\[
F_D = \Delta \tau_{dis}
\]

\[
F_{cool} = P_2 R_{H} + \alpha^+ D_1 (1 + P_2 \theta)
\]

\[
E_x = \frac{F_E + F_D}{F_{cool}} \bigg|_{\tau_0 = 0} = \frac{\Delta \tau_{ad,0} \frac{\gamma}{\tau_0} + \Delta \tau_{dis}}{P_2 R_{H} + \alpha^+ D_1 (1 + P_2 \theta)}
\]

\[
R_y = \frac{F_R}{F_{cool}} = \frac{v_A D_a \exp \left[ \gamma \left( 1 - \frac{1}{\tau_0} \right) \right]}{P_2 R_{H} + \alpha^+ D_1 (1 + P_2 \theta)}
\]

The boundary diagram can be established by putting the roots from eq 31 into eqs 36 and 37. Figure 4 is an example boundary diagram. Three thermal behaviors are shown, and an inherently safe region is divided by the dashed line.  

2.1.3. Temperature Diagram.

In addition to $T_{ad}$, maximum allowed temperature (MAT) is also used to define the thermal behavior. A reaction with a maximum temperature over the MAT is considered to have thermal runaway. Using the boundary diagram, we can judge whether the reaction is out of control because of its accumulation, but sometimes the reaction temperature can reach MAT without an obvious accumulation. To quantitatively describe the maximum reaction temperature under different conditions, a temperature diagram was proposed by Maestri and Rota.

**Figure 5a** displays temperature diagrams developed under certain conditions, where the lines are the fitting results. A line of the MAT, which is often chosen as the boiling point of
water, was drawn. As shown in Figure 5a, the reaction can reach the MAT at a low $E_d$ and a high $R_s$. This is mainly caused by a high cooling temperature. In addition, the curve of $R_s = 0.1$ exceeds the curve of $R_s = 1$ in a certain interval of $E_d$ because of its accumulation. For $R_s = 0.01−0.3$, there is the possibility that the maximum temperature can exceed the MAT, as shown in Figure 5b.

2.2. Influence of the Dissolution Heat. Calculating the heat of dissolution of various substances can be extremely different. For an electrolyte, the dissolution involves ionization and hydration processes, which are endothermic and exothermic, respectively. Thus, according to the intensity of ionization and hydration processes, which are endothermic and exothermic, respectively. Such as $\text{Al}_2(\text{SO}_4)_3$ with a dissolution heat of $-527$ kJ mol$^{-1}$, or positive, such as $\text{Na}_2\text{HPO}_4·12\text{H}_2\text{O}$ with a dissolution heat of 97 kJ mol$^{-1}$. As shown in Figure 6, the dissolution heat can be so intense, even more than the reaction heat sometimes, that is has a great influence on the boundary diagram. The minimum $R_s$ values of reactions with different dissolution heats had very little difference, but the maximum value of $R_s$ increases as $\Delta \tau_{dis}$ decreases. Futhermore, the minimum $E_d$ also became lower slightly as $\Delta \tau_{dis}$ decreased. This meant that the runaway region became larger when the reaction had an endothermic dissolution, and the reaction was more likely to accumulate and cause thermal runaway. Figure 7 shows profiles of temperature diagrams with different $\Delta \tau_{dis}$ values, and Figure 8 compares three $R_s$ lines under three values of $\Delta \tau_{dis}$. From the figures, we can know that the influence of $\Delta \tau_{dis}$ on the minimum $R_s$ in the boundary diagram is that the minimum $R_s$ decreases as $\Delta \tau_{dis}$ increases.

Combining the boundary diagram and the temperature diagram, we can draw the conclusion that endothermic dissolution is advantageous for thermal runaway caused by accumulation but is also able to lower the temperature for reactions that are not likely to accumulate. Conversely, exothermic dissolution is not only helpful for preventing the reaction from experiencing thermal runaway caused by accumulation but is also likely to heat the reaction to the MAT without accumulation.

A comparison of the elaborated safety diagram and temperature diagram to other criteria is shown in Table 1. The criteria of divergence and Morbidelli–Varma were compared with the safety boundary diagram by Westerterp et al. The result showed that the divergence criterion is too conservative, indicating a higher $T_{cool}$ as a critical temperature. The results of the Morbidelli–Varma criterion and the safety diagram were well-matched. However, it takes a tremendous amount of work to execute the numerical simulations required for the Morbidelli–Varma criterion. Thus, the safety boundary diagram is an efficient and robust method of obtaining safe operating conditions.

2.3. Experimental Validation. Figure 9 presents the temperature and the heat flow profile for the dissolution of NaOH and citric acid (CA). The $\Delta H_{dis}$ values of NaOH and CA calculated by integrating the heat flow were $-40.8$ and 21.1 kJ mol$^{-1}$, respectively. The temperature trend indicated that exothermic or endothermic dissolution can be serious and should not be ignored. The mass transfer coefficients ($\delta$) of NaOH and CA were calculated to be 0.00687 and 0.00532 cm$^{-1}$, respectively, using eq 11.

The calorimetry experiment was conducted to analyze thermal behaviors of the neutralization of CA and NaOH under various cooling temperatures and dosing rates. As shown in Figure 10, CA was dosed as the substrate, and the temperature increased as NaOH was dosed into the reactor, reaching the maximum temperature at $t_e$. The cooling temperature was set at 282.15, 287.15, 292.15, 297.15, or 302.15 K, but the maximum temperature had no notable difference and the temperature trends were also the same. This meant that the neutralization of CA and NaOH was not sensitive to the cooling temperature and that the reaction rate was too quick for NaOH to accumulate. Accordingly, its thermal behavior did not change much under different cooling temperatures. Therefore, changing the cooling temperature had little effect on the reaction process for the neutralization of CA and NaOH.

Figure 11 shows the thermal behavior of the neutralization of CA and NaOH at different dosing rates. The same as Figure 10, the temperature increased as NaOH was dosed into the reactor and reached the maximum temperature at $t_e$. However, the maximum temperature decreased when the dosing rate was set lower. Thus, it was possible to control the reaction process by lowering the dosing rate to avoid thermal runaway.

When the substrate changed from the 38.4% w/v CA solution to the 24% w/v NaOH solution with the same dosing time and cooling temperature, the maximum temperature was found to be much lower, as shown in Figure 12. The huge difference between the maximum temperatures of the experiments was caused by the dissolution of the solid. The $\Delta H_{dis}$ of NaOH measured previously is $-40.8$ kJ mol$^{-1}$, making it an exothermic process, while the $\Delta H_{dis}$ of CA measured previously is 21.1 kJ mol$^{-1}$, making it an endothermic process. As a result, the maximum temperature with the NaOH solution as the substrate is about 18.5 K lower than with the CA solution as the substrate due to the difference between the $\Delta H_{dis}$ values of CA and NaOH.

Using eqs 20−29 and the experimental data, $\Delta H_{\text{r,cal}}$ was calculated to be $-55.7$ kJ mol$^{-1}$ and $\Delta H_{\text{r,exp}}$ was found to be $-96.5$ kJ mol$^{-1}$, respectively. The kinetic parameters for the neutralization of CA and NaOH, as shown in Table 2, were calculated using Matlab and Origin. The simulation results are shown in Figure 13, where the solid lines represent...
experimental temperature trends and the dashed lines represent simulation temperature trends. It can be seen that the simulation temperature trends were close to the experimental temperature trends, indicating that the model proposed earlier has a sound goodness of fitting.

Some reaction parameters, such as the cooling coefficient were given by the calorimeter (RC1) directly, while the others were calculated earlier in this article using the experimental results. $E_v$ and $R_y$ can be calculated using eqs 36 and 37, respectively, after nondimensionalizing the reaction parameters. The calculated experimental results are listed in Table 3.

As there was no obvious accumulation during the reaction, a temperature diagram is more appropriate for the neutralization of CA and NaOH. The reaction parameters used to judge the thermal behaviors and the results can be seen in Figure 14. Using the temperature diagrams, the critical cooling temperature with a dosing rate of 30 g min$^{-1}$ was inferred to be 303.276 K, and the critical dosing rate with a cooling temperature of 292.15 K was inferred to be 37.585 g min$^{-1}$.

Figure 7. Temperature diagrams with different $\Delta \tau_{\text{dis}}$ values, where $0.025 < v_{0,\text{Da}} < 18, 0.1 < \Delta \tau_{\text{ad},0} < 1,32 < \gamma < 48, 0.1 < P_s < 1, \epsilon = 0.73, \theta = 1, \alpha^* Da = 10, R_H = 1, P_1 = 2, m = 1, and n = 1. (a) $\Delta \tau_{\text{dis}} = 0$, (b) $\Delta \tau_{\text{dis}} = -0.1$, (c) $\Delta \tau_{\text{dis}} = -0.5$, (d) $\Delta \tau_{\text{dis}} = 0.1$, and (e) $\Delta \tau_{\text{dis}} = 0.5$.  

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Critical thermal runaway condition is added to contain information on the dissolution heat. The exothermicity factor (∆T) should be applied to the operating conditions in a SBR, the temperature diagram should be used as a quantitative judgment for runaway caused by the accumulated reactant. Based on the studies by Maestri and Rota, the simulation results, the proposed criteria were shown to be in a good agreement with the practical situation. Using the temperature diagram criterion, the critical conditions were determined to be a cooling temperature of 303.28 K and a dosing rate of 37.59 g min⁻¹. This work would be reasonably expected to provide some guidance for process safety design and thermal runaway prevention in semibatch reactors.

3. CONCLUSIONS

In this study, safety criteria were proposed for solid–liquid heterogeneous systems in semibatch reactors. An energy and mass balance dimensionless mathematical model for solid–liquid reactions in SBRs was successfully established and used to ameliorate the boundary diagram and temperature diagram criteria. In addition to the reactive factor (Fₚ) and the exothermicity factor (∆T), the dissolution factor (Fₗ) was added to contain information on the dissolution heat. The critical thermal runaway condition is T₉ for the boundary diagram and the MAT for temperature diagram. Based on the result of the boundary diagram, it was found that the runaway region was smaller when the dissolution was exothermic, indicating that exothermic dissolution was advantageous to prevent runaway caused by the accumulated reactant. Based on the result of the temperature diagram, it was found that reactions were more likely to reach the MAT with endothermic dissolution when there was no obvious accumulation in the reactor. The two criteria are fairly related from the points of view of both calculation and application; to obtain the safe operating conditions in a SBR, the temperature diagram should be applied first to check the relationship between the reaction temperature and the MAT value. Then, the boundary diagram should be used as a quantitative judgment for runaway caused by the accumulated reactant.

Calorimetry experiments for the neutralization of CA and NaOH were carried out to obtain the thermodynamic and kinetic parameters. By comparing the experimental results and the simulation results, the proposed criteria were shown to be

![Graph showing the generalized temperature diagram with different ∆Tₙ values, where 0.025 < vₚDa < 18, 0.1 < ∆Tₙ < 1, 32 < γ < 48, 0.1 < P₂ < 1, ε = 0.73, f = 1, αₚDa = 10, Kₗ = 1, P₁ = 1, m = 1, and n = 1.]

4. MATERIALS AND METHODS

4.1. Establishment of Criteria. Before building the criteria, it is necessary to predict the thermal behavior of reactions with a mathematical model. In view of solid–liquid reactions, the mathematical model includes the dissolution of the solid and the mass and energy balance of the reaction. The equations in the model can be solved by ode23 (Bogacki–Shampine) in Matlab using Runge–Kutta methods. The algorithm is better than ode45 in terms of allowable error and the calculation of rigid problems.

Based on the studies by Maestri and Rota, T₉ was introduced as the target temperature. Based on a comparison of the maximum reaction temperature and T₉, the following three characteristic types are proposed for thermal behavior in a SBR:

1. No ignition: with a low cooling temperature, the reaction temperature is always lower than the corresponding target temperature. The reaction is too slow to cause thermal runaway in this condition, although the accumulation of the reactant can reach a high value.

2. Runaway region: as the cooling temperature increases further, the rate of the reaction is not low enough to avoid thermal runaway but is also not high enough to keep the accumulation at an acceptable value. Under this situation, thermal runaway of the reaction system may occur.

3. QFS region: The reaction rate is high enough to avoid thermal runaway by keeping the accumulation sufficiently low as an ideal condition. This situation is referred to as a quick onset, fair conversion, smooth temperature profile (QFS) situation because the reaction temperature swiftly approaches the target temperature but will not exceed it.

By changing the cooling temperature while keeping other conditions unchanged, it is possible to finding a range of cooling temperatures under which the reaction system can cause thermal runaway. When the maximum reaction temperature is equal to T₉, the corresponding cooling temperature is regarded as the critical temperature. To obtain the critical temperature under different conditions, the parameters νₚDa, ∆Tₙ, γ, and P₂ were changed randomly in a certain interval. The exothermicity number Eₜ and the reactivity number Rₜ were calculated, and the operating parameters and the boundary

| Table 1. Comparison of Several Criteria |
|----------------------------------------|
| criterion                            | advantages                                      | disadvantage             |
| divergence criterion                  | can be applied for several types of reactors    | too conservative         |
| Morbidelli–Varma criterion            | able to reflect the influence of different parameters on the reaction system | complicated calculation |
| Samoilenko’s work                     | able to determine critical conditions of thermal ignition for solid–liquid reactions in SBRs and BRs | ignores the influence of the dissolution process on the temperature and volume of the reaction system |
| safety boundary and temperature diagram criteria proposed in this study | convenient to use and the results are intuitive | cannot be applied for solid–liquid reactions |

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diagram can be built using $E_x$ and $R_y$ as coordinates, respectively.

To build a temperature diagram, the same dimensionless parameters are used (i.e., the exothermicity number $E_x$ and the reactivity number $R_y$). First, a set of parameters, namely $v_A D_A$, $\Delta r_{ad,0}$, $\Delta r_{dis}$, $P_1$, $P_2$, $\theta$, $\epsilon$, $\alpha^*$, $D_A$, $R_H$, $m$, and $n$, is assigned in compliance with an acceptable range. A certain value is chosen for $R_y$, and the cooling temperature and $E_x$ can be calculated.

Figure 9. Temperature and heat flow profiles for the dissolution of (a) NaOH and (b) CA in water.

Figure 10. Temperature vs time profiles with different cooling temperatures.

Figure 11. Temperature versus time profiles with different dosing rates.
using eqs 36 and 37, respectively. The parameters $v_A D_A \Delta r_{ad,0}$, $\gamma$, and $P_2$ were changed randomly in a certain interval while the remaining six parameters (i.e., $\Delta r_{ad,0} P_1 \theta_1 \epsilon$, $\alpha^\nu D_A$, and $R_H$) were kept constant, and the above procedure was repeated to find different cooling temperatures and values $E_x$ for a certain value of $R_y$. The temperature diagram can be built with $E_x$ and the maximum reaction temperature as coordinates.

### 4.2. Calorimetry Experiment

#### 4.2.1. Reaction and Materials

The acid–base neutralization reaction of CA and NaOH is widely used for sodium citrate production and is also a typical exothermic solid–liquid reaction.\(^{28}\) In this study, the neutralization reaction of CA and NaOH was chosen to prove the reliability of the proposed models. NaOH was purchased from Sinopharm Chemical Reagent Co. (A.R.), and CA was purchased from Macklin Reagent Co. (A.R.). The particle sizes were measured with micrometer to be 1.055 mm (NaOH) and 1.436 mm (CA).

#### 4.2.2. Apparatus and Calorimetric Tests

The integral dissolution heat of NaOH and CA was measured using a reaction calorimeter (RC1e, Mettler Toledo Ltd., Switzerland) with a Lambda Doser powder dosing instrument (Lambda Laboratory Instruments Ltd., Czech Republic). The experiment began with the addition of 1 L of water into the reactor. The stirring rate was set at 200 rpm, and the heat transfer coefficient and specific heat were determined by a thermal calibration procedure. Then, 6 mol NaOH was dosed at a rate of 30 g min\(^{-1}\) with a cooling temperature of 292.15 K. The integral dissolution heat of CA was also obtained with the same procedure. Roughly, 2 mol CA was dosed at a rate of 22.8 g min\(^{-1}\) with a cooling temperature of 292.15 K. The heat transfer coefficient and the specific heat were determined again.

### Table 2. Kinetic Parameters Calculated by Matlab for the Neutralization of CA and NaOH

| kinetic parameter | value         |
|-------------------|---------------|
| apparent activation energy ($E_a$) | 46.9 kJ mol\(^{-1}\) |
| pre-exponential factor ($A$) | $2.79 \times 10^{10}$ (cm\(^3\) mol\(^{-1}\) s\(^{-1}\) |
| reaction order (NaOH) | 3 |
| reaction order (CA) | 1 |

### Table 3. Dimensionless Experimental Results of R1-R8

| reaction | dimensionless maximum temperature | $E_x$ | $R_y$ |
|----------|----------------------------------|-------|-------|
| R1       | 1.159                            | 1.768 | 16.728|
| R2       | 1.172                            | 1.715 | 18.626|
| R3       | 1.192                            | 1.664 | 20.664|
| R4       | 1.217                            | 1.616 | 22.844|
| R5       | 1.236                            | 1.569 | 25.171|
| R6       | 1.073                            | 1.214 | 21.097|
| R7       | 1.124                            | 0.863 | 21.435|
| R8       | 1.173                            | 0.439 | 21.842|

#### Figure 12. Temperature versus time profiles with different substrates.

#### Figure 13. Comparison of fitting results and experimental data for different dosing rates.

#### Figure 14. Positions of experimental results in the temperature diagram. (a) Results of R1–R5 and (b) results of R3 and R6–R8.
when the dosing was finished. The experimental data were analyzed using eq 38.

\[
\Delta H_{\text{diss}} = \frac{\int_{t_0}^{t_e} q_{\text{hf}} \, dt}{n_s} \quad (38)
\]

The acid–base neutralization of CA and NaOH was performed in RC1 with the Lambda doser powder dosing instrument. The experimental conditions are summarized in Table 4. Approximately 1 L of the 38.4% w/v CA solution (2 mol) was prepared and dosed into the reactor first. The heat transfer coefficient and the specific heat were determined using a thermal calibration procedure with the stirring rate of 200 rpm. The reactor was cooled to the set temperature. After 10 min of stabilization, 6 mol NaOH was dosed under the set rate. The heat transfer coefficient and the specific heat were determined again when the reaction was finished. To observe the influence of the dosing reactants, we changed the substrate from 1 L of the 38.4% w/v CA solution to 1 L of the 24% w/v NaOH solution (6 mol), and the dosing rate was set at 23.6 g min\(^{-1}\) at 292.15 K. The stoichiometric scheme for the acid–base neutralization of CA and NaOH is given by eq 39.

\[
3\text{NaOH} + \text{C}_6\text{H}_5\text{O}_2 \rightarrow \text{C}_6\text{H}_5\text{Na}_2\text{O}_7 + 3\text{H}_2\text{O} \quad (39)
\]

The reaction enthalpy can be calculated by integrating the heat flow with eqs 40 and 41.

\[
\Delta H_{r,\text{exp}} = \frac{\int_{t_0}^{t_e} q_{\text{hf}} \, dt}{n_s} \quad (40)
\]

\[
\Delta H_{r,\text{exp}} = \Delta H_{r,\text{cal}} + \Delta H_{\text{diss}} \quad (41)
\]

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## Notes

The authors declare no competing financial interest.

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## NOMENCLATURE

**Latin Symbols**

- \(P\) concentration ratio of reactant A to reactant B
- \(n\) number of molar equivalents of reactant A (mol)
- \(n_0\) number of molar equivalents of reactant A (mol)
- \(P_1\) dimensionless parameter of the dissolution rate
- \(P_2\) concentration ratio of reactant A to reactant B
- \(q\) dosing rate (g min\(^{-1}\))
- \(q_{\text{hf}}\) exothermic rate (w)
- \(R\) ideal gas constant (8.314 J mol\(^{-1}\) K\(^{-1}\))
Greek Symbols

$\gamma$ dimensionless activation energy
$\nu$ distribution coefficient
$\theta$ dimensionless time
$\rho_{\text{L}}$ density of the liquid phase (mol m$^{-3}$)
$P_{\text{S}}$ density of the solid phase (mol m$^{-3}$)
$\tau$ dimensionless temperature
$\tau_0$ dimensionless initial temperature
$\tau_{\text{cool}}$ dimensionless cooling temperature
$\tau_{\text{max}}$ dimensionless maximum temperature
$\Delta T_{d,0}$ dimensionless adiabatic temperature rise
$\Delta T_{d,\alpha}$ dimensionless dissolution heat
$\mu_1$ second moments of distribution
$\mu_3$ third moments of distribution
$\delta$ polydispersity of the solid
$\delta$ mass transfer coefficients (cm s$^{-1}$)
$\eta_{\text{A}}$ dimensionless concentration of reactant A
$\eta_{\text{B}}$ dimensionless concentration of reactant B
$\eta_v$ dimensionless volume of solid reactant
$\eta_d$ dimensionless diameter
$\alpha^*$ Du dimensionless cooling number

Abbreviations

SBR semibatch reactor
BR batch reactor
MAT maximum allowable temperature
QFS quick onset, fair conversion, smooth temperature profile
CA citric acid

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