IS THERMAL INSTABILITY SIGNIFICANT IN TURBULENT GALACTIC GAS?

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ABSTRACT

We investigate numerically the role of thermal instability (TI) as a generator of density structures in the interstellar medium (ISM), both by itself and in the context of a globally turbulent medium. We consider three sets of numerical simulations: (1) flows in the presence of the instability only; (2) flows in the presence of the instability and various types of turbulent energy injection (forcing), and (3) models of the ISM including the magnetic field, the Coriolis force, self-gravity and stellar energy injection. Simulations in the first group show that the condensation process that forms a dense phase ("clouds") is highly dynamical and that the boundaries of the clouds are accretion shocks, rather than static density discontinuities. The density histograms (probability density functions [PDFs]) of these runs exhibit either bimodal shapes or a single peak at low densities plus a slope change at high densities. Final static situations may be established, but the equilibrium is very fragile: small density fluctuations in the warm phase require large variations in that of the cold phase, probably inducing shocks in the clouds. Combined with the likely disruption of the clouds by Kelvin-Helmholtz instability, this result suggests that such configurations are highly unlikely. Simulations in the second group show that large-scale turbulent forcing is incapable of erasing the signature of TI in the density PDFs, but small-scale, stellar-like forcing causes the PDFs to transit from bimodal to a single-slope power law, erasing the signature of the instability. However, these simulations do not reach stationary regimes, with TI driving an ever-increasing star formation rate. Simulations in the third group show no significant difference between the PDFs of stable and unstable cases and reach stationary regimes, suggesting that the combination of the stellar forcing and the extra effective pressure provided by the magnetic field and the Coriolis force overwhelm TI as a density-structure generator in the ISM, with TI becoming a second-order effect. We emphasize that a multimodal temperature PDF is not necessarily an indication of a multiphase medium, which must contain clearly distinct thermal equilibrium phases, and that this "multiphase" terminology is often inappropriately used.

Subject headings: instabilities — ISM: structure — turbulence

1. INTRODUCTION

The idea that thermal instability (TI) plays a fundamental role in controlling important aspects of the interstellar gas and star formation in galaxies has been tremendously influential. As a model for the formation of cool dense clouds in pressure equilibrium with a warm rarefied intercloud medium, TI has been invoked to explain the existence of diffuse interstellar clouds (Field, Goldsmith, & Habing 1969, hereafter FGH), as a means of regulating the mass flow between different components of the ISM and the star formation rate (Chieze 1987; Parravano 1989), as a major factor allowing star formation to occur at an appreciable rate in galaxies (Norman & Spaans 1997; Spaans & Norman 1999), to explain the fragmentation of protoglobular clusters (Murray & Lin 1996), and to explain cooling flows around luminous cluster elliptical galaxies and more general situations (for recent discussions, see Nulsen & Fabian 1997; Allen & Fabian 1998; Mathews & Brighenti 1999).

The linear stability analysis for TI was first worked out in detail by Field (1965), who clearly delineated the isobaric, isochoric, and isentropic modes and examined effects such as magnetic fields and conduction. Subsequent work generalized the analysis to include chemical reactions (Yoneyama 1973) and refined the calculation of relevant heating and cooling rates (see Wolfire et al. 1995 for a recent discussion). In its simplest form, the isobaric mode of TI occurs when a compression leads to such enhanced cooling that pressure in the compressed region decreases. This leads to a picture of dense clouds in pressure equilibrium with their surroundings, the basis for the "two-phase" ISM model of FGH. This conceptual framework was extended by Cox & Smith (1974) and McKee & Ostriker (1977) to include a third, hot, phase representing the interiors of expanding supernova remnants, but it still assumes that clouds form by TI. Examples of the interesting physical phenomena that might occur because of phase transitions in a two-phase medium can be seen in the recent work of Kamaya (1997a, 1997b; 1998, 1999), Kamaya & Shchekinov (1997a, 1997b) and Burkert & Lin (2000). We are interested in finding whether this sort of two- or three-phase model is relevant, even as a zeroth-order approximation, in a turbulent ISM, and, if not, what sorts of residual effects TI might have.

The idea that clouds form by TI in the ISM is often justified in terms of the near constancy of the thermal pressure observed over a range of densities (0.1–100 cm⁻³; e.g., Myers 1978). However, closer inspection of Figure 1 of Myers suggests that instead \( P \sim \rho^{1.4} \) in that density range and that, really, the effective exponent of this relation is a function of the density over the whole range of densities.
condensation might avoid disruptive turbulent interactions.

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lence is supersonic, the incipient condensation should be
is the characteristic turbulent speed at scale
forming condensations on a crossing timescale
while the turbulence shears (or expands, or compresses) the
vective motions could suppress TI (e.g., Balbus & Soker 1989).
Numerical studies of the nonlinear evolution of TI in the solar
chromosphere transition region and corona (Dahlberg et al. 1987; Karpen, Picone, & Dahlburg 1998) and in galaxy
cluster cooling flows (Lowenstein 1989; Malagoli, Rosner, & Fryxell 1990; Hattori & Habe 1990; Yoshida, Habe, & Hattori 1991) have generally confirmed this suspicion. The motion of the incipient condensations leads to vortices, or to complete disruption by Kelvin-Helmholtz and Rayleigh-Taylor instabilities, although a magnetic field can diminish the effect in these convective cases. Moreover, Murray et al. (1993) have shown in some detail how the motion of a condensation initially in a two-phase medium in pressure equilibrium can easily lead to disruption of the condensation by dynamical instabilities, similar to the results quoted above. Strongly self-gravitating condensations can avoid this fate, but there is no reason to assume that such condensations were formed by thermal instability; supersonic turbulence interactions are sufficient to produce such structures (e.g., Vázquez-Semadeni, Passot, & Pouquet 1996).

What about thermal instability in a supersonic turbulent medium, like the gas in galaxies? (By “turbulent” we mean a disordered but not completely random velocity [and density] field covering a large range of scales.) In the ISM of the Milky Way there is certainly strong evidence for supersonic motions at all scales above at least 0.1 pc, in every sort of environment. Supersonic spectral line widths are observed even in regions with no detectable internal star formation (e.g., the Madalena molecular cloud complex; see Williams & Blitz 1998) and in diffuse mostly-H ii clouds in which self-gravity is unimportant (known since the 1950s; see Heithausen 1996 for a recent study of a subclass of such clouds), as well as in the more intensively studied star-forming molecular cloud structures.

A simple argument might suggest that thermal instability may be inoperative, or at least less efficient, as a cloud formation process in a supersonically turbulent ISM. The isobaric mode of TI in a region of size L condenses on a characteristic timescale L/c, where c is the sound speed, while the turbulence shears (or expands, or compresses) the forming condensations on a crossing timescale L/v, where v is the characteristic turbulent speed at scale L. If the turbulence is supersonic, the incipient condensation should be disrupted faster than it can condense. However this argument applies only in an average sense; a region of incipient condensation might avoid disruptive turbulent interactions for an unusually long time. More importantly, as we show in § 3.2, when one considers that the instability growth time decreases with scale size while the crossing time increases with scale, the above argument can apply only at sufficiently large scales, and below some size scale the instability could still operate. Moreover, Hennebelle & Pérault (1999) have recently shown that TI can be triggered by a dynamical compression in an originally stable flow.

A quantitative result suggesting that the idea of pressure-confined clouds, central to models of the ISM that rely on thermal instability, as well as other quasi-static conceptions, is not viable in a supersonically turbulent medium was given, using numerical simulations, by BVS99. They showed, partly through the evaluation of volume and surface terms in the virial theorem, that the importance of the kinetic energy surface terms implies that clouds cannot be considered as quasi-permanent entities with real “boundaries” but are instead continually changing, forming and dissolving. Not only is thermal pressure incapable of confining turbulent density fluctuations, but the idea of external turbulent-pressure confinement seems self-defeating, since the external turbulent stresses mostly serve to distort and disrupt clouds.

There are additional problems with the two- or three-phase ISM models. One involves timescales. The timescale on which a given parcel of the ISM is subjected to impulsive perturbations, whether by heating or by shock interactions, may be smaller than the timescale for TI and smaller than the time for readjustment to pressure equilibrium. That clouds must be subjected to frequent stochastic heating events was the basis for the early Kahn (1955) cloud-cloud collision model for the temperature distribution of clouds, and the “time-dependent ISM” models proposed by Bottcher et al. (1970) and examined in detail by Gerola, Kafatos, & McCray (1974). The latter work included only the influence of stochastic heating and ionization sources (no hydrodynamics), but the resulting statistical description of the diffuse ISM (Gerola et al. 1974) was in better agreement with available observations than the static two-phase models. A recent summary of (mostly observational) arguments against the specific three-phase McKee & Ostriker model is given by Elmegreen (1997). Moreover, the conclusion that pressure equilibrium is unlikely because of the frequency of either cloud collisions or shocks from supernovae or cluster superbubbles has been found independent-ly several times in the literature (e.g., Stone 1970; Heathcote & Brand 1984; Bowyer et al. 1995; Berghöfer et al. 1998; Kornreich & Scalo 2000).

Another important consideration is that TI is not the only physical ingredient influencing the dynamics of the ISM, other equally important ones being self-gravity, the magnetic field, shear and the Coriolis force due to Galactic rotation and, of course, turbulence (operationally, the advection [u · ∇u] term in the momentum equation). Elmegreen (1991, 1994; see also Passot, Vázquez-Semadeni, & Pouquet 1995 for the two-dimensional case) has investigated the linear stability criterion for all these processes combined (except turbulence, which is an intrinsically non-linear phenomenon), finding that the magnetic field and the Coriolis force may counteract the combined action of cooling and self-gravity. In any case, it is important to note that the combined instability behaves very differently from TI alone, with a different dispersion relation, unstable wave-number range and growth rates than any of the individual
instabilities considered (see also § 2.1), so the pure TI may represent an incomplete dynamical scenario even in the purely linear case.4

Given all the above results, it is important to examine whether the traditional picture of a multiphase ISM structured by TI should be retained as a useful model. By “multiphase” we are referring to a medium containing various different thermodynamic equilibrium regimes, some of them possibly stable and others unstable, with the requirement that a multiphase medium involves fluid parcels undergoing a “phase transition” when transiting between these equilibria. The structuring of the density field by the multiphase nature of the medium should be reflected in multimodal density and temperature probability density functions (PDFs), with gas accumulating in the stable “phases.”

However, note that the temperature field is expected to show a bimodal PDF even in the absence of TI, simply because of the functional shape of the interstellar cooling curve, which has the form of two plateaus separated by a sharp discontinuity at \( \sim 10^4 \) K (see Dalgarno & McCray 1972, Fig. 11). For example, the simulations of Korpi et al. (1999) do not contain real phases, since no background heating capable of balancing the cooling is included; yet, the temperature still exhibits a bimodal distribution in those simulations. But for a TI-structured multiphase ISM, the gas must be concentrated into two (or more) density components: dense clouds and a rarified intercloud medium. Two-dimensional simulations of the ISM including a typical cooling function, but containing no TI (Scalo et al. 1998) showed no indication of a bimodal density PDF. For this reason in this paper we consider a bimodal density distribution as the signature of a TI-structured ISM. Actually, our experiments have shown that, under certain conditions, the PDFs of unstable flows may be unimodal, but with an added slope change where the second peak should be located. Thus, we consider either of these PDF shapes as a signature of density structuring by TI.

It is also of great interest to investigate whether TI can significantly enhance the formation of clouds by turbulent interactions. As shown in Vázquez-Semadeni, Ballesteros-Paredes, & Rodríguez (1997), BVS99 and references therein, “clouds” naturally form in a supersonic turbulent medium, without any need for instabilities, as turbulent velocity streams interact to form compressed layers, as suggested by Elmegreen (1993).

The present work attempts to investigate these ideas using numerical simulations in two dimensions. Two-dimensional simulations of the development of the pure instability in a nearly isobaric and quasi-static regime have been presented by Shaviv & Regev (1994), but here we analyze a number of highly dynamical cases, from the pure-instability case to full models of the ISM, in order to investigate the relative importance of TI on the process of structure formation in each case. First we simulate the evolution of an initially static gas whose cooling and heating functions should guarantee TI and show that, as expected, the density distribution shows the signature of the instability. Second, we perform simulations of a randomly forced turbulent flow in the presence of the instability, showing that large-scale forcing does not seem to be able to prevent the development of the instability, but small-scale forcing does.

Finally, a third series of simulations considers a supersonic turbulent gas, including self-gravity, the magnetic field, the Coriolis force, and energy input due to star formation, in stable, marginal and unstable cases. We demonstrate that in the latter case the density distribution does not show the signature of the instability. The temperature and density are still anticorrelated, as expected from the cooling function, but a two-phase density field is not realized. This is ultimately due to the fact that, when the cooling timescale is much smaller than the hydrodynamic timescale, the density is “slaved” to the dynamics, not the temperature (cf. Vázquez-Semadeni, Passot, & Pouquet 1995a); another way of expressing this is that the density field is controlled by the ram pressure/advection, not by the thermal pressure.

2. THE MODEL

We use the numerical ISM model presented by Passot et al. (1995), which uses a single-fluid approach to describe the ISM on a 1 kpc² plane on the Galactic disk, centered at the solar Galactocentric distance. In this model, various physical ingredients can be included at will, such as self-gravity, the magnetic field, disk rotation, as well as model terms for the radiative cooling (\( \Lambda \)), a diffuse background radiation (\( \Gamma_s \)), and energy input due to star formation (\( \Gamma_s \)). In this paper, we solve the equations in logarithmic form for the density and temperature, as done in Gazol-Patiño & Passot (1999). This is because the shocks induced by the development of the instability are quite strong, and the logarithmic approach avoids the appearance of negative values of the density and temperature due to the Gibbs phenomenon in the vicinity of discontinuities, giving somewhat more robustness to the code. The equations solved read

\[
\frac{\partial \ln \rho}{\partial t} + \mathbf{V} \cdot (\ln \rho \mathbf{u}) + (1 - \ln \rho) \mathbf{V} \cdot \mathbf{u} = \mu [\nabla^2 \ln \rho + (\nabla \ln \rho)^2],
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P \rho - \left( \frac{J}{M} \right)^2 \nabla \phi - v_s \nabla^2 \mathbf{u} + v_s \left( \nabla^2 \mathbf{u} + \frac{1}{3} \nabla \nabla \cdot \mathbf{u} \right) + \frac{1}{\rho} (\nabla \times \mathbf{B}) \times \mathbf{B} - 2 \Omega \times \mathbf{u}, \tag{2}
\]

\[
\frac{\partial \ln e}{\partial t} + \mathbf{u} \cdot \nabla \ln e = -(\gamma - 1) \nabla \cdot \mathbf{u} + \frac{k_T}{\rho} [\nabla^2 \ln e + (\nabla \ln e)^2] + \Gamma_s + \Gamma_s + \rho \Lambda e, \tag{3}
\]

4 Note that, strictly speaking, the calculations of Elmegreen (1991, 1994) refer to a different system, a so-called “cloud fluid,” in which the colliding clouds replace atoms or molecules (see also Struck-Marcell & Scalo 1984). In this case the instability is not due to the interstellar (atomic) cooling curve, but to the assumed forms of the stellar heating rate and cloud dissipation rate, and the “sound speed” is taken as the cloud velocity dispersion. However, the formalism used by Elmegreen can be applied to the perfect-gas case with atomic and molecular cooling as well (Passot et al. 1995).
\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nu_\mathbf{s} \nabla^2 \mathbf{B} + \eta \nabla^2 \mathbf{B}
\]  
(4)

\[
\nabla^2 \varphi = \rho - 1 ,
\]  
(5)

\[P = (\gamma - 1) \rho e ,
\]  
(6)

where \(\rho\) is the fluid density, \(\mathbf{u}\) the velocity, \(P\) the thermal pressure, \(\varphi\) the gravitational potential, \(\Omega\) the angular velocity due to Galactic rotation, \(\mathbf{B}\) the magnetic field, and \(e\) the specific internal energy, related to the temperature \(T\) by \(e = c_s^2 T\), \(c_s\) being the specific heat at constant pressure. In equation (2), \(J \equiv L_0/L_2\) is the Jeans number, measuring the angular momentum of the velocity unit \(u_o\) with respect to the temperature unit, given by its corresponding speed of sound \(c_s\). In equation (3), \(\gamma\) is the ratio of specific heats at constant pressure and volume, and \(\kappa_T\) is the thermal diffusivity.

Momentum and magnetic field dissipation are included by a combination of a quadrilaplacian “hyperviscosity” operator, which confines the viscous effects to the smallest scales, and a second-order operator, which filters out possible oscillations in the vicinity of strong shocks. The use of the hyperviscosity operator allows the use of much smaller (typically by a factor of 10) values of the second-order diffusivities than otherwise. An artificial mass diffusion term in Fourier space, which is added in order to smooth out the density gradients, allowing the code to handle the continuity equation has been added to spread out and somewhat compress density peaks. The effect of this term has been quantified by Vázquez-Semadeni, Ballesteros-Paredes, & Rodríguez (1997) and BVS99.

The physical units are \(L_0 = 1\) kpc, \(\rho_0 = 1\) \(m_H\) cm\(^{-3}\), \(u_0 = 11.7\) km s\(^{-1}\), \(t_0 = (L_0/2\pi u_0)^{-1} = 1.3 \times 10^7\) yr, \(T_0 = 10^4\) K, and \(B_0 = 5\) \(\mu\)G. Throughout the paper, we omit \(m_H\), the proton mass, in the specification of densities.

The above equations are solved in two dimensions at a resolution of 128\(^2\) grid points (implying a spatial resolution of 7.8 pc) by means of a pseudospectral method, which imposes periodic boundary conditions. The temporal scheme is a third-order Runge-Kutta for the nonlinear terms, combined with a Crank-Nicholson for the linear terms. The initial conditions are set up in Fourier space, uncorrelated in all variables, and characterized by a power spectrum of the form \(k^4 \exp \left[ -2(k/k_0)^2 \right] \), with random phases. In particular, the size scale associated to wavenumber \(k_0\) determines the size of the initial density fluctuations.

In the case of the magnetic field, the initial condition involves a uniform component of strength \(B_z = 1.5\) \(\mu\)G in the \(x\)-direction mimicking the azimuthal component of the field on the Galactic plane, upon which a fluctuating field of rms strength \(5\mu\)G is added.

An important note is that, in spite of the usage of logarithmic variables and hyperviscosity, our simulations cannot handle very strong shocks because of the spectral scheme used (the Gibbs phenomenon produces artificial oscillations near discontinuities), and therefore they must be stopped when the gradients of the physical variables become too steep. However, they are able to reach advanced enough stages of the development of the instability that its fully dynamical character is seen, and the structures arising are well defined.

We describe below only those model terms that are directly related with simulations presented in the following sections. Further details concerning the choice of the other terms and parameters can be found in Passot et al. (1995) and Vázquez-Semadeni et al. (1996). Table 1 presents the relevant parameters for the runs in this paper.

### 2.1. Radiative Cooling and Diffuse Heating

The heating (\(\Gamma\)) and cooling (\(\rho\Lambda\)) rates are per unit mass and thus differ somewhat from the standard definitions, which are per unit volume. In equation (3), the term \(\Gamma_d\) models the heating of the gas due to photoelectrons ejected from dust grains because of the background UV radiation field, or heating by low-energy cosmic rays and is taken as a

#### Table 1

| Run Number | Label  | \(\Gamma_0\) | \(\beta_{12}\) | \(\beta_{23}\) | \(\beta_{34}\) | \(c_{12}\) | \(c_{23,34}\) | \(T_{2g}\) | \(\rho_2\) | \(\rho_3\) | \(T_{eq}\) | \(\gamma_{12,3}\) | \(\gamma_{23}\) | Forcing\(^m\) | ISM\(^a\) |
|------------|--------|-------------|---------------|---------------|---------------|---------|----------------|---------|-----------|--------|----------|---------------|-------------|----------------|---------|---|
| 1          | PT5    | 5           | 1             | 0.25          | 4.18          | 80.62   | 7.184          | 0.0398  | 1.56      | 0.696  | 0.2347   | 0.0           | 3           | ...             | ...     |
| 2          | PT3    | 3           | 1.5           | 0.25          | 4.18          | 170     | 3.022          | 0.0398  | 2.22      | 0.993  | 0.9711   | 0.33          | ...         | ...             | ...     |
| 3          | PT2.5  | 0.25        | 1.5           | 0.25          | 4.18          | 200     | 0.6325         | 0.01    | 1.25      | 0.395  | 0.0244   | 0.33          | ...         | ...             | ...     |
| 4          | HPS5   | 5           | 1             | 0.25          | 4.18          | 80.62   | 7.184          | 0.0398  | 1.56      | 0.696  | 0.2347   | 0.0           | -3          | ...             | ...     |
| 5          | HPS1   | 5           | 1             | 0.25          | 4.18          | 80.62   | 7.184          | 0.0398  | 1.56      | 0.696  | 0.2347   | 0.0           | -3          | ps = 0.5        | ...     |
| 6          | HSF    | 5           | 1             | 0.25          | 4.18          | 80.62   | 7.184          | 0.0398  | 1.56      | 0.696  | 0.2347   | 0.0           | -3          | ps = 1          | SF      |
| 7          | ISMS   | 3           | 1.5           | 1.5           | 3.0          | 150     | 150            | 0.0398  | 2.52      | 0.020 | 0.0737   | 0.33          | 0.33        | SF              | On      |
| 8          | ISMM   | 3           | 1.5           | 1.0           | 3.0          | 160     | 31.91          | 0.0398  | 2.36      | 0.094 | 0.0940   | 0.33          | 0.0         | SF              | On      |
| 9          | ISMU   | 3           | 1.5           | 0.25          | 4.18          | 170     | 3.022          | 0.0398  | 2.22      | 0.993  | 0.9711   | 0.33          | -3         | SF              | On      |

- **\(\Gamma_0\)**: Diffuse heating rate.
- **\(\beta_{12}\)**: Cooling law exponent between 10 K and \(T_2\).
- **\(\beta_{23}\)**: Cooling law exponent between \(T_2\) and \(10^5\) K.
- **\(\beta_{34}\)**: Cooling law exponent between \(10^5\) K and \(10^8\) K.
- **\(c_{12}\)**: Cooling law coefficient between 10 K and \(T_2\).
- **\(c_{23,34}\)**: Cooling law coefficients between 10 K and \(10^5\) K.
- **\(T_{2g}\)**: Transition temperature in units of \(10^4\) K.
- **\(\rho_2\)**: Transition density at \(T = T_2\).
- **\(\rho_3\)**: Transition density at \(T = 10^4\) K.
- **\(T_{eq}\)**: Equilibrium temperature at \(\langle \rho \rangle\) in units of \(10^4\) K.
- **\(\gamma_{12,3}\)**: Effective polytropic index between 10 K and \(T_2\).
- **\(\gamma_{23}\)**: Effective polytropic index between \(T_2\) and \(10^4\) K.
- **Forcing\(^m\)**: Type of forcing.
- **ISM\(^a\)**: ISM additional physics (magnetic field, self-gravity and rotation).
constant \( \Gamma_0 = \Gamma_0 \) everywhere and throughout the duration of the simulations.

The simulations use a cooling function with piecewise power-law dependence on the temperature of the form

\[
\Lambda = C_i l_{i+1} T^{\beta_i + 1} \quad \text{for} \quad T_i \leq T < T_{i+1}.
\]

The various runs have different values of \( \beta_i l_{i+1} \), given in Table 1. One set of values of these exponents, namely, \( \beta_{12} = 1 \), \( \beta_{23} = 0.25 \), \( \beta_{34} = 4.18 \), and \( \beta_{65} = -0.53 \), was chosen as a reasonable piecewise power-law fit to the radiative cooling curve of Dalgarno & McCray (1972) for a fractional ionization of \( 10^{-4} \) at \( T < 10^4 \) K (see also Spitzer 1978). For this fit, we have chosen the transition temperatures \( T_i \) as \( T_2 = 10^3 \) K, \( T_3 = 398 \) K, \( T_4 = 10^4 \) K, \( T_5 = 10^5 \) K, and \( T_6 = 10^7 \) K. As discussed below, this particular combination of values of \( \beta_i l_{i+1} \) and \( T_i \) gives a thermally unstable range (isobaric mode) between \( T_2 \) and \( T_3 \). Other runs have variations of this cooling function in order to obtain thermal stability or marginal instability in the range \( 398 - 10^4 \) K without a significant increase of the cooling rate above \( 10^4 \) K. For reasons of continuity, the values of coefficients \( C_i l_{i+1} \) must also be changed and are also listed in Table 1. Finally, all runs share the same values of \( \beta_{45} \) and of \( T_i, T_3, T_5, \) and \( T_6 \) and are therefore not listed in Table 1.

As discussed in previous papers (e.g., Vázquez-Semadeni et al. 1995a, 1996), the thermal timescales are typically much shorter than the dynamical ones, implying that the turbulent motions are quasi-static compared to the background heating and cooling rates, and allowing for the establishment of thermal equilibrium. With the adopted power-law behavior for these processes, the equilibrium temperature and pressure are

\[
T_{eq} = \left( \frac{\Gamma_0}{C_i l_{i+1} \rho} \right)^{\frac{1}{\beta_i l_{i+1}}},
\]

\[
P_{eq} = \frac{\rho T_{eq}^{\gamma_i l_{i+1} - \beta_i l_{i+1}}}{\gamma_i l_{i+1}} \left( \frac{\Gamma_0}{C_i l_{i+1}} \right)^{\frac{1}{\beta_i l_{i+1}}},
\]

where the (piecewise) effective polytropic index is given by \( \gamma_i l_{i+1} = 1 - 1/\beta_i l_{i+1} \). The isobaric mode of the TI, characterized by a decrease in the pressure as the density increases, develops when \( \gamma_i l_{i+1} < 0 \), corresponding in this case to \( \beta_i l_{i+1} < 1 \). The lower pressure causes an effective "suctioning" action by the high-density regions. The values of the resulting \( \gamma_{12} \) and \( \gamma_{23} \) are also given in Table 1. The value of \( \gamma_{54} \) is always taken as \( \gamma_{54} = 0.76 \) and therefore is not listed in Table 1. Note that the flow behaves as \( P \propto \rho^{\gamma_i l_{i+1}} \) in the density interval \( \rho_1 < \rho < \rho_{l+1} \), where the "transition" densities \( \rho_i \) are defined as the values whose corresponding equilibrium temperatures (eq. [8]) coincide with \( T_i \). Note also that some of the runs, in addition to having a thermally unstable range with \( \gamma_{23} < 0 \), have a marginally stable regime at low temperatures, with \( \gamma_{12} = 0 \). This is the case in particular for our fit of the Dalgarno & McCray (1972) cooling curves, and its consequences are discussed in § 3.1.1.

The dispersion relations for the combined thermal and gravitational instability in the presence of a uniform field \( B_1 \)

have been derived for the two-dimensional case by Passot et al. (1995) (for the general three-dimensional case, see Elmegreen 1994) and read

\[
\omega_i^2 = J^2 - k^2 \left( \frac{\gamma_i l_{i+1}}{\gamma} T_{eq} + B_1^2 \right)^{-1} \nu^2,
\]

\[
\omega_i^2 = J^2 - k^2 \frac{\gamma_i l_{i+1}}{\gamma} T_{eq} - \frac{k^2 \omega_i^2}{\omega_i^2 + k^2 B_1^2},
\]

where \( \omega_i \) and \( \omega_0 \) denote the growth rates for perturbations in the longitudinal and transverse directions, respectively, with respect to \( B_1 \). \( k \) denotes the wavenumber, \( J \) is the Jeans number defined above, and \( \kappa = 20[1 + (1/2)(\Omega d\omega/d\Omega)]^{1/2} \) is the epicyclic frequency. Unstable wavenumbers are those for which \( \omega_i^2 > 0 \). For the values of the parameters used in the unstable runs, typical unstable ranges are \( k \approx 1 \), implying that essentially all scales included in the simulations are unstable. Note that the thermal instability (\( \gamma_i < 0 \)) has the effect of reversing the instability criterion with respect to the purely gravitational case, in the sense that now it is scales smaller than some threshold value that are unstable.

Table 1 also contains information about the thermal equilibrium regime for the heating and cooling functions used here, in particular \( \rho_2 \) and \( \rho_3 \), and the equilibrium temperature \( T(\rho) \) at the mean density, \( \langle \rho \rangle = 1 \) cm\(^{-3}\). Note that the value of \( \Gamma_0 \) used here is in general not the same as that used by Vázquez-Semadeni et al. (1995a). We have chosen \( \Gamma_0 \) in order to ensure that \( T(\rho) \) lies within the unstable range.

2.2. Forcing

We consider two kinds of forcing (i.e., energy injection) for the turbulent runs. The first is a large-scale forcing, accomplished through an additional acceleration term in the momentum equation (eq. [2]), taken as a random vector field with a spectrum proportional to \( k^4 \) below the forcing wavenumber \( k_{for} = 4 \), and \( k^{-4} \) above \( k_{for} \). We consider two subcases of this forcing, one with 100% solenoidal (rotational) components and the other with 50% solenoidal and 50% compressible components.

The second type of forcing is applied at small scales, and consists of local, discrete heating sources turned on at a grid point \( x \) whenever \( \rho(x) > \rho_{SP} \) and \( \mathbf{V} \cdot \mathbf{u}(x) < 0 \), where \( \rho_{SP} \) is a free parameter, but taken equal to \( 6\langle \rho \rangle \) always. The sources stay on for a time interval \( \Delta t = 6 \times 10^6 \) yr. This forcing mimics the effect of ionization heating from massive stars, and acts on the scale of \( \sim 5 \) pixels (Vázquez-Semadeni et al. 1995a; Passot et al. 1995).

2.3. The Simulations

In order to investigate the interplay between TI and the turbulent dynamics, we have performed three sets of three simulations each. First, we have considered the development of the instability alone (runs labeled "P" for "pure" in Table 1), by simulating a purely hydrodynamic flow without the magnetic field, rotation, or self-gravity. Only the appropriate cooling and background heating functions necessary for the existence of a thermally unstable range are included. The three runs differ in details of the cooling and background heating functions, chosen to show the variety of structures that develop and the effects on them of the closeness of the transition densities to the mean density. The label "P" is thus followed by a mnemonic indicating the value of \( \Gamma_0 \) used in each run.
The second set of simulations (referred to as the “hydrodynamic” runs, or “H” in Table 1) are similar to the P runs, except that they include a variety of forcing schemes, as described in § 2.2. These are designed to test the effect of the scale and the amount of compressible content of the energy injection on the suppression of the instability by the turbulence. As indicated in § 2.2, the small-scale forcing is stellar-like, i.e., in the form of localized sources of heating, which create expanding bubbles of hot gas, while the large-scale forcing is random, with varying ratios of compressible-to-solenoidal content.

Finally, we have performed three ISM-like simulations (labeled “ISM” in Table 1), including the magnetic field, rotation and self-gravity, as well as the stellar-like forcing. The runs differ from each other in whether the cooling/heating combinations produce a stable, marginal or unstable regime and are designed to test the effects of the presence of the TI in an ISM-like medium.

For the “pure” and “hydro” runs we have used initial fluctuations in the density and temperature of rms amplitude equal to 0.1 times their mean values (=1), with zero initial velocity fluctuations. For the full ISM-like runs, we have used initial fluctuations with rms amplitudes equal to the mean values of the variables, except for the magnetic field, whose fluctuations are taken as described in § 2.

3. RESULTS

3.1. Development of the Pure Instability

3.1.1. Morphology and Dynamics

The density fields for runs P5 and P3 at an advanced stage of the instability development are shown in the left and right panels, respectively, of Figure 1. In run P5 (left panel), the gas is seen to evolve into a “bee’s-nest” structure formed by a network of dense (~10 cm⁻³), cold (~50 K) filaments enclosing warm (~10⁴ K) and rarefied (~0.3 cm⁻³) cells. The cells’ shape and size are determined by the initial conditions. In this run we have taken $k_0 = 2$, implying that the characteristic scale of the initial fluctuations is one-half of the box size. Note that the TI leads to the collapse of regions containing a density excess, whose characteristic size is one-fourth of the integration domain.

The roundish blobs formed at intersections between filaments originate from density maxima, while the dense filaments can be traced back to “saddle points” located between two density maxima and two minima in the initial conditions. The evolution of this simulation is reminiscent of “pancake” formation due to the gravitational instability in cosmological simulations of large-scale structure formation (e.g., Padmanabhan 1993). However, this bee’s-nest structure is transient, as we discuss below.

The condensation process in principle stops after the pressures in the warm and the dense (cold) phases equalize and any remaining kinetic energy due to the inertia of the gas is dissipated. This implies that the mass redistribution induced by the instability does not end when the density has “crossed” the transition values, but at more disparate values of the density such that the pressures in the two phases are equal. For example, Figure 2 shows the thermal equilibrium pressure, as given by equation (9), for runs P5 and P3. It can be seen that in order for the pressure in the warm phase to equal that of the dense phase, a density of roughly 0.03 cm⁻³ is required in the former. (Note that, since run P5 has $\gamma_{12} = 0$, dynamical [pressure] equilibrium can be reached only by decreasing the warm phase’s density.) However, at the final time reached by run P5, the pressure in the warm phase is still much larger than in the dense phase, as can be seen in Figure 3. That is, this run stopped only because the shocks became too strong for the code to handle, but it was still in a highly dynamical contracting stage.

The highly dynamical nature of the condensation process is clearly seen in Figure 4, which shows horizontal cuts at $y = 42$ through the (log of the) density (dotted line), (log of the) pressure (dashed line), and the x-velocity (solid line) fields of run P5 (Fig. 1, left panel). The cuts go through two blobs and one filament. A double-shock structure is observed in the blobs, which are the two-dimensional equivalent of one-dimensional shock-bounded slabs. This should

6 The initial conditions for the two-dimensional density field are mosaics of small-amplitude, alternating maxima and minima. Typically, imaginary lines joining two maxima and lines joining two minima intersect, forming an “X” pattern, at the center of which there is a saddle point.
also be the case of the filament, but it is too thin for the code to resolve the double-shock structure, since shocks in the code are spread over roughly five grid points by viscosity.

In order to see the subsequent evolution, we thus performed another run, PT.25, with slower heating and cooling rates (by decreasing the coefficients) but with the same temperature dependence as run PT.3 (except for a slight change in $T_0$) and which therefore evolved more slowly, developing milder shocks, and allowing us to reach more advanced stages of the instability development. A temporal sequence in the evolution of this run is shown in Figure 5. It is seen that this run also develops the bee’s-nest structure, but later the filaments disrupt and are accreted into the actual peaks. Moreover, some of the cells “implode,” and their surrounding filaments and peaks collapse into the void, because of the small pressure differences between cells, traceable to the randomness of the initial conditions. The behavior thus continues to resemble that induced by gravitational instability in cosmological simulations, as the pancakes later collapse into isolated structures (e.g., Padmanabhan 1993).

Run PT.3 (Fig. 1, right panel), on the other hand, does not show the bee’s-nest structure of run PT.5 but shows roundish blobs within elongated, isolated structures, and only one truly filamentary structure. More importantly, run PT.3 never develops the thin, dense filaments seen in run PT.5. Instead, only thick, moderate-density bands appear between the peaks, and then disappear. We attribute this difference in part to the fact that in run PT.3, $\rho_3$ is very close to the mean density (see Table 1). This causes any initial underdensity to reach the stable range quickly, implying that a relatively large amount of mass remains in the warm phase (note that the buildup of an underdensity entails the evacuation of that region).

However, the main reason for the nonappearance of the filaments seems to be again that the densities of the warm and cold phases required for equilibrium are significantly more distant from the mean density than $\rho_3$ and $\rho_1$. This implies that, during the development of the instability, the pressure maxima do not occur at the centers of the voids, but in the boundaries between the voids and the high density regions (Fig. 6). In the particular case of run PT.3, $\rho_3$ is so close to $\langle \rho \rangle$ that the voids reach smaller pressures earlier than the unstable filaments. The latter then reverse...
their condensation process, loosing their mass to both the voids and the peaks, and ultimately merging with the inter-cloud medium. The peaks do not suffer this "rebound" because they reach lower pressures faster than the voids because of their larger initial densities.

Even though the above effect is particularly noticeable in run PF3, we expect it to apply in general. Therefore, unless the density field is perfectly uniform within the phases, and the interface between them is a perfect discontinuity, a larger pressure is expected to be always present near the interface between the two phases than in either one of them. This suggests that a steady state with constant pressure everywhere is difficult to attain.

3.1.2. On the Final State of the Instability

Another consequence of the above discussion is that, since the gas is a fluid, the boundary of the "clouds" that make up the dense phase are accretion shocks, as discussed by BVS99, rather than the quiescent density jump at constant pressure usually assumed in the literature. Of course, quiescent pressure equilibrium may be established after the pressures have equalized, and the boundary shocks subside, but this may require very long times, and besides, the balance is very delicate. In general, there is a continuum of possible equilibria between the dense and rared phase, obtained by equating their pressures, as given by equation (9), and solving for \( \rho_d \), the density in the dense phase, as a function of \( \rho_c \), the density in the rared (warm) phase. We obtain

\[
\rho_d = \left[ \left( \frac{C_{12}}{\Gamma_0} \right)^{1/\beta_{12}} \left( \frac{\Gamma_0}{C_{34}} \right)^{1/\beta_{34}} \rho_c^{34} \right]^{1/12}. \tag{12}
\]

The dependence of \( \rho_d \) on \( \rho_c \) is thus determined essentially by the ratio \( \gamma_{34}/\gamma_{12} \). As an example, plugging in the corresponding parameters for run PF3, which has \( \gamma_{12} = \frac{1}{3} \), we find that \( \rho_d = 3.2 \times 10^3 \rho_c^{2.28} \). Thus, in this case, the equilibrium density of the dense phase depends sensitively on the density of the warm phase, minute fluctuations in \( \rho_c \) requiring large changes in \( \rho_d \). For example, at \( \rho_c = 0.1 \), \( \rho_d = 16.8 \), while at \( \rho_c = 0.3 \), \( \rho_d = 205 \) (all in units of the mean density). Such large density variations for the dense phase’s density will require highly dynamical adjustments that will most likely involve new shocks, especially since the sound speed in the warm phase is much larger than in the dense medium. This, together with the result by Murray et al. (1993) that the condensed clouds are likely to be set in motion relative to the diffuse phase by buoyancy or other effects, and then easily disrupted by Kelvin-Helmholtz-type instabilities, strongly suggests that the static configuration is highly improbable.\(^7\)

In cases where equation (12) does not apply, and the pressure of the dense phase becomes independent of its density. Thus, pressure equalization can be accomplished only through evacuation of the warm phase until its pressure drops to the value within the dense gas. In the mean-

\(^7\) It is possible that the K-H mode of cloud erosion could be stabilized by heat conduction at cloud boundaries if a two-phase system with discrete clouds in a hot intercloud medium did form, as has been speculated for proto-globular clusters (Kamaya 1998). However for the interstellar medium of disk galaxies that we are investigating here, the intercloud medium, if it could exist as a stable phase, would have a temperature of only around \( 10^4 \) K, and so conduction should exert a negligible stabilizing effect.
time, the density of the dense gas may increase by very large amounts, most likely involving violent compressions. We conclude that the static configuration is very hard to realize, in agreement with the conclusions of BVS99.

In any case, the formation of static configurations is possible in principle, although we have seen here that this is a direct consequence of the change in the effective polytropic exponent $\gamma^e$ from one phase to the other, as discussed in detail by BVS99. This suggests that cores within molecular clouds, which have essentially the same value of $\gamma^e$ as their parent clouds, should not be expected to reach hydrostatic equilibrium (BVS99).

3.1.3. PDFs

The density PDF has been the subject of much recent work. Vázquez-Semadeni (1994) and Padoan, Nordlund, & Jones (1997) have reported lognormal PDFs for two- and three-dimensional isothermal flows, respectively. Passot & Vázquez-Semadeni (1998) discovered that the functional form of the PDF for polytropic ($P \propto \rho^{\gamma^e}$) one-dimensional flows depends on the effective polytropic exponent $\gamma^e$, developing a power-law tail at high densities for $0 < \gamma^e < 1$ and vice versa, with a lognormal indeed developing when $\gamma^e = 1$ (see also Nordlund & Padoan 1999). Scalo et al. (1998) found that the simulations of the ISM by Passot et al. (1995), as well as two-dimensional Burgers-like (without thermal pressure) runs developed power-law PDFs analogous to the $0 < \gamma^e < 1$ cases of Passot & Vázquez-Semadeni (1998).

In the thermally unstable case, the instability causes the evacuation of material from the unstable to the stable regions, producing a two-phase density field characterized in principle by a bimodal PDF. In the remainder of this paper we use the density PDF as a diagnostic of the relative importance of the instability in the dynamics of the various simulations.

The density PDFs for runs PT5 and PT3 corresponding to the fields shown in Figure 1 are presented in Figure 7. The vertical lines denote the transition densities $\rho_2$ and $\rho_3$ for the two runs. The morphological and dynamical features described in the last section manifest themselves in the PDFs as well. In both cases, a peak in the PDF is observed just below the lower transition density, $\rho_2$. However, in the case of run PT5, $\rho_2$ is not too similar to the mean density, and this implies that a substantial amount of mass has to be transferred to the dense phase. As a consequence, a noticeable peak appears in the PDF large and of the upper transition density, $\rho_3$. Instead, for run PT3, much of the mass remains in the low-density phase, and there is no clear peak PDF.
above \( \rho_2 \). Only a change in the slope of the PDF is seen there.

Furthermore, the PDF for run PT3 seems to extend to much higher densities than that of run PT5. We interpret this as a consequence that run PT3 reached a more advanced stage in the development of the instability than run PT5, so that densities much higher than the transition value are reached as the densities in each phase approach the equilibrium values. In fact, this phenomenon should have also been observed in run PT5, had it reached more evolved stages. Indeed, in Figure 8 we show the density PDF at three different times for run PT.25, which, as mentioned above, is similar to run PT5 but evolves more slowly, producing weaker shocks and therefore reaching more advanced stages before the code cannot handle the shocks anymore. In this figure it is clearly seen that the PDF is similar to that of run PT5 at first, but then overshoots and becomes more similar to that of run PT3, the peak above the upper transition density disappearing altogether.

We conclude from this section that both a bimodal PDF and a unimodal one with a slope change above \( \rho_2 \) may be considered the signatures of the TI.

3.2. Effects of Turbulent Forcing

We now discuss the effects of adding turbulent forcing to purely hydrodynamic simulations in the presence of the TI. Our main objective is to determine whether a turbulent regime can prevent the development of the instability, in the sense of destroying the bimodality (or slope change; cf. § 3.1.3) of the density PDF, so that the two-phase nature of the medium disappears. In particular, we also wish to determine whether different types of forcing have different effects on the development of the instability.

To this end we consider run PT5, which is subject only to the TI, and three forced runs with exactly the same heating and cooling functions, but subject also to various types of forcing. As described in § 2.3, we refer to the forced runs generically by “H,” and then add a mnemonic for the type of forcing used. Run HPS.5 uses large-scale random forcing with 50% solenoidal (rotational) content (“PS” = “percent solenoidal”). Run HPS1 uses the same type of forcing, except with 100% solenoidal content. Finally, run HSF (“star formation”) uses small-scale (a few pixels) stellar-like forcing, which is 100% compressible.

In Figure 9 we compare the density PDFs for the four runs. Interestingly, the two runs with large-scale forcing are not able to counteract the development of the instability, as indicated by the fact that their PDFs still show a significant slope change above the upper transition density \( \rho_2 \), similar to that of run PT3 (§ 3.1.3). In fact, in these cases the turbulence appears to reinforce the instability, since both runs stopped because of the large gradients at earlier times than the pure-instability run PT5. Note, however, that a peak above \( \rho_2 \) is not present for either run, apparently because the turbulence contributes to the overshooting effect, which spreads the mass in the dense phase over a larger range of densities than in run PT5. The inability of the turbulence to counteract the collapse of the filaments, since the stellar
heating reverts the compression of the dense gas by the warm phase. This effect causes the PDF for this run (dotted-dashed line) to exhibit the interesting feature that the high-density peak spreads out, and in particular invades the unstable density region line in Figure 9, ultimately erasing the signature of the TI. Indeed, in this case, the shoving of the gas by expanding shells continually redistributes it among the various thermal regimes. This erasure effect is clearly shown in Figure 10, in which the run’s PDF is seen to transit from clear bimodality to essentially a power-law tail at densities above the mean. However, this run is “unstable” in the sense that it displays a runaway star formation rate (recall star formation in the simulations is triggered by large densities). This suggests that TI in this run still has the effect of promoting excessive star formation.

The inability of large-scale forcing to prevent the development of the instability may be understood in terms of the characteristic timescale for the growth of fluctuations under the effect of the instability \( \tau_{\text{TI}} \) increases linearly with wavenumber. This implies that the nonlinear crossing time at scale \( l \) varies as \( \sim (\beta + 1)/2 \), with \( \beta = \frac{3}{4} \) for a Kolmogorov flow and \( \beta = \frac{1}{2} \) for a shock-dominated flow. In either case, it is seen that, for sufficiently small scales, \( \tau_{\text{TI}} < \tau_{\text{NL}} \). Thus, a turbulent flow forced at large scales and cascading its energy to small scales cannot prevent the development of the instability at sufficiently small scales.

Note that the above argument assumes that the effect of turbulence is to always fight the instability. However, for compressible flows, it is in fact possible that in a fraction of the volume the turbulent transfer from large to small scales occurs via the formation of shock-bounded slabs (e.g., Elmegreen 1993; Vázquez-Semadeni et al. 1995a; BVS99), in which case the transfer from large scales promotes compression at small scales, thus helping the instability rather than opposing it. In fact, Hennebelle & Pérault (1999) have recently shown the triggering of TI by compressive motions in originally stable flows. Furthermore, the slabs formed either by turbulent compressions or by TI alone may themselves become unstable by either dynamical or gravitational instabilities (e.g., Hunter et al. 1986; Nishi 1992; Stevens, Blondin, & Pollack 1992; Vishniac 1994; Anninos, Norman, & Anninos 1995; Klein & Woods 1998).

In either case, it is seen that in a turbulent flow forced at large scales, the turbulent transfer is incapable of preventing the development of the instability in general. It is worth noting that these results are qualitatively similar to the instability of large-scale turbulent forcing to suppress the gravitational instability (Léorat, Passot, & Pouquet 1990; Klessen, Heitsch, & Mac Low 2000). On the other hand, as we have seen, stellar-like small-scale forcing, applied at the sites of maximum density, is capable of erasing the signature of the instability.

### 3.3. ISM-like Simulations

We now consider a set of three ISM-like turbulent simulations including self-gravity, the magnetic field, large-scale shear, the Coriolis force, and energy input due to star formation, but differing in the cooling functions in order to make them either thermally stable, marginal or unstable. These runs are respectively labeled “ISMS,” “ISMM,” and “ISMU.” The initial conditions include also velocity and magnetic field fluctuations. In particular, run ISMU has identical heating and cooling functions as run PF3.

It should be pointed out that we have tested numerically that run ISMU is also unstable with respect to the combined effect of all physical agents included, but without star formation. As discussed by Elmegreen (1994) and Passot et al. (1995), in the presence of shear, equations (10) and (11) are valid only near \( t = 0 \). At later times, the development of the perturbations has to be investigated numerically. We also tested that, in the absence of shear, run ISMU is unstable as well, as indicated by the dispersion relations.

Figure 11 shows the density PDFs for these three runs. These runs reach a stationary regime, and therefore we have integrated the PDFs over several timesteps to improve the statistics. The most important difference in the PDFs actually occurs at low densities. This seems to be a consequence that the lower transition density for run ISMU is very close to the mean density, as was the case for run PF3, and thus the warm phase is not strongly evacuated. Instead, for the marginal and stable runs, the lower transition densities are quite small (see Table 1). Although at first one might think the transition densities should not matter in unstable cases, they actually do. This is because the density range between \( \rho_{2} \) and \( \rho_{3} \) continues to be “softer” (i.e., more compressible) than densities below \( \rho_{3} \). Thus, once
Although this slope variation is minimal compared to the unstable run above its upper transition density. The main difference between the PDFs is at low densities, because its lower transition density is higher than those of the other two runs. At high densities, virtually no difference is seen between the PDFs.

the density drops to these values, the thermal pressure acts more strongly against further evacuation of these regions by the turbulence (cf. Passot & Vázquez-Semadeni 1998).

At large densities, the three curves are remarkably similar, with only a trace of a slope change in the PDF of the unstable run above its upper transition density. However, this slope variation is minimal compared to the dramatic one seen in the PDF of run PF3 (Fig. 7). This shows that the effect of TI on the density distribution has been erased by the small-scale energy injection from the stellar sources.

4. DISCUSSION

4.1. Limitations of the Simulations and Effects of Parameter Choice

4.1.1. Effects of the Two-Dimensionality

For economy, we have restricted our simulations to relatively low resolution and two dimensions. Thus, it is important to discuss the limitations and possible consequences of this setup. Extensive discussions on the effects of dimensionality have been presented in previous papers (Vázquez-Semadeni 1994; Passot et al. 1995; BVS99), so here we focus exclusively on its effects in the modeling of TI. The pure linear thermal instability analysis is independent of dimensionality, since, for example, the isobaric mode, which is the one we have focused on, requires only an inverse dependence of thermal pressure with density. In fact, numerous studies in less than three dimensions exist in the literature (see, e.g., Shaviv & Regev 1994; Wada & Norman 1999; Hennebelle & Pérault 2000 for two-dimensional studies, and Elphick, Regev, & Spiegel 1991; Elphick, Regev, & Shaviv 1992; Hennebelle & Pérault 1999 for one-dimensional studies). For the combined instability, as mentioned in § 2.1, the instability criterion does depend on dimensionality (Passot et al. 1995), but we have used the two-dimensional criterion in order to be fully consistent. Furthermore, the two-dimensional criterion does not differ too much from the three-dimensional one (Passot et al. 1995).

The type of structures that develop in the pure-TI case also are not expected to change with dimensionality, as the interfaces between the cold and warm gas are shocks, which form independently of dimensionality, and simply are hypersurfaces of one dimension less than the dimension of the simulation. In summary, we are confident that all our results are applicable to the real ISM, regardless of the two-dimensionality of the simulations.

The relatively low resolution we have used clearly prevents resolving further levels of substructure that may develop inside the clouds, but that is not the scope of this paper. Instead, we have been interested in whether cloud formation in the ISM is dominated by TI or by turbulence. We have based our diagnostic on the density PDFs, and the resolution has been sufficient to distinguish between the two scenarios using this tool.

4.1.2. Choice of Initial Conditions

For the pure-TI simulations we have chosen small amplitudes of the initial density fluctuations, in order to allow the instability to develop starting from the linear phase, in agreement with the linear analysis. Also, as discussed in § 3.1.1, the choice of the power spectrum of the initial fluctuations affects only the size of the condensations that form. Therefore, given the low resolution we have used, we have naturally chosen a spectrum containing most of the fluctuating power at large scales, so that the clouds are well resolved. Of course, as the clouds condense and contract, they decrease their characteristic sizes.

In the case of the H runs forced at large scales (HPS.5 and HPS1), the size of the forming clouds is also determined by the size of the initial density fluctuations, although this does not qualitatively affect our results. On the other hand, for all simulations forced by small-scale stellar sources, the memory of the initial conditions is lost as soon as the first stars begin to form.

4.1.3. Hyperviscosity and Shock Resolution

The use of hyperviscosity in the simulations is also not expected to have any effect on our results. Although it has been shown (Passot & Pouquet 1988) that hyperviscosity may cause artificial oscillations in the vicinity of shocks, we add small amounts of regular second-order viscosity in order to smooth those out, while still being able to use much smaller amounts of the latter than would be possible without the hyperviscosity. In effect, this increases the effective resolution of the simulations and allows the code to handle steeper gradients than otherwise possible.

Anyway, as we have mentioned in § 2, some simulations may eventually reach gradients too steep to handle, and must be stopped. Therefore we cannot see the evolution after the formation of the strongest shocks. This problem occurs in the pure-TI (P) simulations and the HPS runs, which develop the strongest shocks, because of the highly dynamical and continuously accelerating nature of the condensation process. However, as mentioned in § 2, concern-

![PDF graph](image.png)
ing the P runs, we have been interested mostly in showing
the highly dynamical character of the condensation process,
the appearance of accretion shocks, and the fragility of the
static equilibrium state. The first two goals are clearly
accomplished within the regimes accessible to the simula-
tions, while the latter was shown on the basis of a simple
calculation, independently of the simulations.

Instead, run HSF and the ISM simulations develop
milder shocks because the stars revert the condensation
process, and, in the case of the ISM runs, the magnetic field
and rotation weaken the shock strength. Indeed, as men-
tioned in §3.3, the ISM runs survive indefinitely, again
showing the relatively minor importance of TI in the evolu-
tion of the simulations most representative of the actual
regime in the ISM.

4.2. Comparison with Previous Work

Shaviv & Regev (1994) have investigated, both analyti-
cally and numerically, aspects of the pure-instability de-
velopment in a quasi-static regime, such as the growth of the
domain ("cloud") size in time, and the kinetics of the inter-
faces between the cold and warm phases. They showed that
clouds merge to form larger, less numerous structures, in
such a way that the correlation length in the simulations
increases as a power law in time, with the exponent depend-
ing on the value of the initial pressure compared to the
critical value at which both phases are energetically sym-
metric.

Shaviv & Regev's two-dimensional simulations used a
more idealized cooling function than the one used here.
More importantly, in order to match the assumptions made
in their analytical work, these authors considered shorter
dynamical times than the cooling ones. Such assumption is
exactly opposite to the situation in the warm and cold ISM
(e.g., Spitzer & Savedoff 1950; Elmegreen 1993; Vázquez-
Semadeni et al. 1995a). Because of that property, the simu-
lations by those authors are nearly spatially isobaric. In
turn this implies that no large velocities driven by pressure
gradients appear in their simulations. Under these condi-
tions their simulations cannot exhibit the highly dynamical
regimes we encounter in the pure-TI simulations. However,
their restrictive assumptions are made in order to match
those needed for their analytical work, rather than to model
realistic conditions in the ISM. Thus, our P simulations,
even though not very realistic compared to the ISM runs,
are expected to be better models of how TI would develop
in the ISM (if it could be isolated from other processes) than
the simulations by Shaviv & Regev.

More recently, Hennebelle & Péraúlt (1999, 2000) have
presented one-dimensional and two-dimensional numerical
and analytical studies in the nonmagnetic and magnetic
cases of the triggering of the instability by dynamical com-
pressions in an initially stable phase, finding that this is
indeed possible in both the HD case, and also in the MHD
case if the magnetic field and the compressive motions are
oblique. Their HD conclusion is consistent with our result
that the HP runs, forced at large scales, actually seem to
promote the development of the instability, since those
simulations reach comparable density contrasts to those of
the pure runs in shorter times. Also, although Hennebelle &
Péraúlt (1999) do not mention it explicitly, the accretion
shocks we have described here are also present in their
simulations. Thus, we feel their HD studies and our P and
H simulations are entirely consistent. On the other hand, we
have not performed simulations of the pure-TI plus the
magnetic field suitable of comparison with their MHD
study. However, we reiterate our feeling that studies of TI
alone do not represent the full regime existing in the actual
ISM, and in fact, that TI alone becomes a second-order
effect compared to the role of the small-scale–forced MHD
turbulence that prevails.

A model including a wider variety of physical agents has
been advanced by Wada & Norman (1999), who have pre-
pared high-resolution two-dimensional simulations of a
differentially rotating disk, including self-gravity and
detailed heating and cooling, and using a numerical method
that allows a very large dynamic range in the density. The
"turbulent" forcing in their calculation is due to shear from
the imposed differential rotation and from gravitational
fluctuations, and does not include star formation. Still, even
though their initial conditions are gravitationally and ther-
mailly unstable and they see a trace of TI activity in the
pressure-density diagram, their results are completely con-
sistent with ours in that there is no tendency for distinct
phases. In particular, even though they refer to their system
as "multiphase," they recognize that "...the system cannot
be described as a simple two- or three-phase medium with
pressure equilibrium between the phases" (p. L15) and
"The gas is in so many and such various phases represented
by a wide range of density and temperature that ‘multiphase’
‘is probably an inadequate description. The gas has
properties more like a phase continuum.” (p. L16). We
agree, except that we see no need to use the term "phase":
the turbulence smears out the thermal phases, creating a
continuous distribution of the physical properties.

Almost simultaneously to the acceptance of the present
paper, we learned about the acceptance of a paper by Wada,
Spaans, & Kim (2000) presenting simulations of an entire
dwarf irregular LMC-type galaxy, including the gravita-
tional and thermal instabilities, and stellar energy injection,
but without the magnetic field. Their numerical scheme
allows them to reach very large density contrasts, and to
proceed to the fully nonlinear stage of the instability de-
velopment. These authors then show that models without
stellar energy injection can develop large "cavities" filled
with warm gas ($T \sim 10^4$ K), without the need for stellar
energy, the latter being responsible for the formation of
medium-to-large cavities filled with hot gas instead. A
similar behavior is seen in the kiloparsec-sized simulations
by Gazol-Patino & Passot (1999) including the magnetic
field and the gravitational instability, but not the isobaric
TI. Interestingly, the "clouds" formed by the combined
instability in the simulations by Wada et al. (2000) are con-
tinually distorted and disrupted by the turbulent flow,
which in turn is driven by the instabilities themselves.8 This
result is thus consistent with our discussion in §3.1.2 on the
"fragility" and unlikelihood of the static regime.

5. CONCLUSIONS

5.1. Summary

In this paper we have discussed the development of the
thermal instability (TI), both by itself and in the presence of
turbulent forcing. We have also examined its role in deter-
mining the statistical distribution of the density field, as measured by its probability density function (PDF), in an ISM-like regime with magnetic fields, self-gravity, stellar-like energy injection, and rotation. We used two-dimensional numerical simulations on the plane of the Galactic disk at moderate resolutions, allowing us to perform a reasonable coverage of parameter space. Our simulations are not able to reach the stage at which pressure equilibrium is established because of the strong shocks involved, but stop instead at some earlier time.

The simulations of the development of the TI alone, starting from small perturbations, showed that the morphologies that arise depend quite sensitively on the proximity of the transition densities \( \rho_1 \) and \( \rho_2 \) (the values of the density bounding the unstable regime from above and below, respectively) to the mean density \( \langle \rho \rangle \) of the medium. If the lower transition density \( \rho_1 \) is very close to the mean density, a large fraction of the mass remains in the warm (low-density) phase, and there is little mass available for forming the dense phase ("clouds"), which consists essentially of isolated, roundish structures. Conversely, simulations in which \( \rho_2 \) is significantly smaller than \( \langle \rho \rangle \) evacuate more mass from the warm phase, and develop a transient filament network before the filaments are accreted into the actual peaks, similarly to pancake formation in cosmological large-scale structure formation simulations. The PDFs of advanced stages of the TI are either bimodal with peaks above and below the upper and lower transition densities, respectively, or else show the low-density peak plus a slope change above \( \rho_2 \).

The condensation process that originates the "clouds" was shown to be highly dynamical, with their boundaries being accretion shocks rather than static density discontinuities. Although in principle the formation of the latter is possible, it was shown that, for realistic cooling functions, the equilibrium density in the cold phase depends sensitively on that of the warm gas, so that small changes in the latter require large changes in the former, which most likely would imply the formation of shocks. Furthermore, if the density is continuous from one phase to the other, then the two phases must be mediated by larger pressure regions, which are expected to induce further motions. The static configuration is possible in principle, but appears highly unlikely.

The inclusion of energy-injection processes ("forcing") has very different effects depending on the nature of the forcing. Random, large-scale forcing is not able to inhibit the development of the instability, and only distorts the structures it forms on timescales longer than the growth rates of the instability. In fact, large-scale turbulent modes naturally generate compressions, so the large-scale forcing actually aids the instability. Instead, small-scale, stellar-like forcing systematically injects energy within the structures formed by the instability and is capable of destroying the signature of the instability in the PDF, pushing gas from the dense phase back into the unstable regime. However, the flow continues to be extremely compressible, as indicated by an ever-growing star formation rate and the inability to develop a stationary regime.

Finally, ISM-like simulations with and without TI show little difference in their PDFs, suggesting that the combined effect of the stellar-like forcing, the magnetic pressure and the Coriolis force overwhelm the thermal pressure deficit in the unstable cases. Furthermore, all of the ISM cases reach stationary regimes, indicating that the presence of a TI in the medium is of relatively minor importance in the overall cycle of such a regime. However, since the turbulent forcing originates from the stellar activity, the importance of the TI in the formation of the first generation of stars in a galaxy cannot be ruled out, although, on the other hand, the weaker cooling in those epochs due to the absence of heavier elements may have prevented the appearance of the instability altogether. The likely disruption of clouds by Kelvin-Helmholtz instability (Murray et al. 1993) suggests that TI, even if it can occur in primordial galaxies, will be unable to form stable clouds.

We conclude that TI alone should not be expected to lead to static regimes (as also suggested by the recent simulations by Wada et al. 2000), and that, in the presence of turbulence with small-scale driving, the magnetic field and rotation, the resulting density distribution is substantially different from that due to TI alone. However, the largest scales (both over- and underdensities of sizes over 1 kpc) may indeed be formed preferentially by the combined thermal + gravitational + magnetic instability, as suggested analytically by Elmegreen (1991, 1994) and numerically by Vázquez-Semadeni, Passot, & Pouquet (1995b), and confirmed in the nonmagnetic case by Wada et al. (2000).

5.2. On the Meaning of a Multiphase Medium

We wish to emphasize that a bimodal temperature distribution is not necessarily evidence of a multiphase medium but only of a nonlinear dependence of the equilibrium temperature on density. A true multiphase medium must involve clearly distinct phases, which, in the case of TI, means different thermal equilibria, some of which may be stable and others unstable. For example, Korpi et al. (1999) have also investigated the temperature and density PDFs in realistic (though not self-gravitating and not including TI) three-dimensional ISM simulations with stellar energy injection. However, in their model there is no background heating, so in effect their simulations are always in a cooling regime (albeit possibly very slowly at low densities), except in the vicinity of stellar thermal energy sources, and thus there are no stable "phases." Yet, those authors still interpret the resulting bimodal temperature distribution as an indication that the medium is "multiphase," in spite of the absence of stable phases in their model, and the facts that pressure varies by over one order of magnitude and that the density PDF shows no sign of bimodality. A bimodal temperature distribution is a simple consequence of the form of the cooling law and will occur in any model of the ISM in which gas can attain large temperatures and cool, even in a nondynamical time-dependent model with no isobaric instabilities (see Gerola et al. 1974). However, in general, this does not imply the existence of stable or unstable phases in the medium. Thus, we feel that the concept of a multiphase medium has sometimes been inappropriately used in the ISM literature. For example, we see no rationale for referring to the correlation-function turbulence model of Norman & Ferrara (1996) as a "continuum of phases," when in actuality it simply corresponds to a nonsothermal density continuum. The same comment applies to the density structure in the simulations by Wada & Norman (1999). These authors do consider a thermally unstable regime, but the turbulence in their simulations "pushes" the gas out of the equilibrium phases, creating again a continuum in which the vast majority of the density states are...
not true phases, since they do not correspond to equilibrium states of the unperturbed system. We feel that the use of such terminology unjustifiably perpetuates the idea that the ISM is controlled by phase transitions.

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