Lepton flavor violating $^\!
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\beta \rightarrow \mu$ and $^\!
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\beta \rightarrow \tau$ conversion in unparticle physics

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Abstract

We have studied lepton flavor violation processes $^\!
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\beta \rightarrow \mu$ and $^\!
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\beta \rightarrow \tau$ conversion in nuclei induced by unparticle. Both $\text{Br}(^\!
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\beta \rightarrow \mu)$ and $^\!
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\beta \rightarrow \tau$ conversion rate $\text{CR}(^\!
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\beta \rightarrow \mu$;Nuclei) strongly depend on the scale dimension $d_U$ and the unparticle coupling $\xi_{K}^{0}$ ($K = V, A, S, P$). Present experimental upper bounds on $\text{Br}(^\!
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\beta \rightarrow \mu)$, $\text{CR}(^\!
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\beta \rightarrow \mu$;Ti) and $\text{CR}(^\!
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\beta \rightarrow \mu$;Au) put stringent constraints on the parameters of unparticle physics. The scale dimensions $d_U$ around 2 are favored for the unparticle scale $\mu$ of $O(10 \text{ TeV})$ and the unparticle coupling of $O(10^{-3})$. $\text{CR}(^\!
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\beta \rightarrow \mu$;Nuclei) is proportional to $Z_{\mu}^4 A^2 = Z$ for the pure vector and scalar couplings between unparticle and SM fermions, this peculiar atomic number dependence can be used to distinguish unparticle from other theoretical models.
I. INTRODUCTION

Scale invariance proves to be a very powerful concept in physics. At low energy, the scale invariance is explicitly broken by the masses of particles, and it is manifestly broken by the Higgs potential in the standard model (SM). However, there may exist a scale invariant sector at a much higher scale, e.g., above TeV scale. Motivated by the Banks-Zaks theory, recently Georgi suggested that a scale invariant sector with nontrivial infrared fixed-point may appear, which weakly couples to the SM. At a low energy scale, this sector is attached onto the so-called "unparticle" with non-integral scale dimension $d_U$. For simplicity, most literatures so far have assumed that scale invariance remains until the low energy scale.

Unparticle is very peculiar from the view of particle physics, it looks like a non-integral number $d_U$ of invisible massless particles, this leads to peculiar energy and momentum distributions, through which unparticle may be detected in high energy colliders. Unparticle doesn't have a definite invariant mass, instead a continuous mass spectrum, which can be represented by an infinite tower of massive particles from the perspective of particle physics. Moreover, the unparticle two-point correlation function has an unusual phase in the time-like region, which can produce interesting interference patterns between the amplitudes of S-channel unparticle exchange and that of SM processes.

Despite of the complexities of the scale invariant sector, we can use the effective theory to deal with its low energy behavior. The unparticle operator can have different Lorentz structures: scalar $O_{U}$, vector $O_{U}$, tensor $O_{U}$ or spinor. However, so far there is no principle to constrain the interactions between the SM fields and unparticle. The rich unparticle phenomenological implications have been extensively studied in particle physics, astrophysics, cosmology, gravity and so on.

In the minimal version of SM, lepton flavor violating (LFV) interactions are strictly forbidden. In the minimal extension of SM in order to accommodate the present data on neutrino masses and mixings, the LFV processes, such as $\nu_i \rightarrow \nu_j (i \neq j)$ and $\nu_e \rightarrow \nu_e$ are very strongly suppressed due to tiny neutrino masses and unitarity of the mixing matrix ($M_{\nu}$ matrix). In particular, the branching ratio for $\nu_e \rightarrow \nu_e$ amounts to at most $10^{-54}$, to be compared with the present experimental upper bound $12 \times 10^{-11}$. However, most extensions
of the SM predict LFV, and some of them predict LFV at much higher rates, which may be in conflict with the existing experimental bounds. LFV provides an unique insight into the nature of new physics beyond SM, and various LFV processes have been considered in many scenarios of new physics beyond SM, such as the see-saw model with or without GUT, supersymmetry, $Z'$ model, and so on. Three kinds of LFV processes are usually discussed: LFV radiative decays $\mu \rightarrow \tau \gamma$, $\mu \rightarrow e \gamma$, and $\mu \rightarrow e \nu \nu$ (i.e., $\mu \rightarrow e \nu$, or $\mu \rightarrow e \nu \nu$), 3 e-like processes and $\mu \rightarrow e$ conversion in nuclei. Unparticle induced $\mu \rightarrow e \nu$ and other cross symmetry related processes such as $\mu \rightarrow e \nu$, $\mu \rightarrow e^+\nu$ have been considered [11,14,18]. In this work, we will consider LFV radiative decay $\mu \rightarrow e \gamma$ and $\mu \rightarrow e$ conversion in nuclei.

Besides the great theoretical interests, there has been a lot of theoretical efforts in detecting LFV processes at CERN LEP and B-factories. The current experimental bound on the LFV radiative decay $\mu \rightarrow e \gamma$ is as follows [51]

$$\text{Br}(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11}; \text{CL} = 90\%$$

(1)

For $\mu \rightarrow e$ conversion in heavy nuclei, the most stringent constraints arise for Titanium and Gold, respectively, with $\text{CR}(\mu \rightarrow e \nu$, Ti) < $4.3 \times 10^{-12}$ [56] and $\text{CR}(\mu \rightarrow e \nu$, Au) < $7 \times 10^{-13}$ [57]. Several experiments have been designed to explore LFV with much higher sensitivity than presently available. In particular, the MEG experiment at PSI will detect $\mu \rightarrow e \gamma$ down to the $10^{-13}$ level in the very near future [58]. Concerning the challenging $\mu \rightarrow e \nu$ conversion in heavy nuclei, the J-PARC experiment PRIS/PRIME is expected to reach a sensitivity of $0 \times 10^{-18}$ [59], i.e., an improvement by six orders of magnitude relative to the present upper bound.

Motivated by the future considerable progress in experimental measurements, studying $\mu \rightarrow e \gamma$ and $\mu \rightarrow e$ conversion in unparticle physics are of great theoretical interests. The paper is organized as follows. In Section II, we review the basic aspects of unparticle physics. In Section III, we calculate the LFV radiative decays $\mu \rightarrow e \gamma$ and $\mu \rightarrow e \nu \nu$ conversion rates in nuclei are considered in Section IV. Finally we present our conclusions and some discussions.

II. THE MODEL

As was suggested by Georgi [2], we shall assume that at a very high energy scale, the world consists of the SM sector and the so-called Banks-Zaks (BZ) sector with non-trivial
infrared (IR) xed point, and the two sectors interact with each other via the exchange of particles with a large mass scale \( M_U \gg 1 \text{TeV} \). Below the scale \( M_U \), the interactions between these two sectors may be described by the effective non-renormalizable Lagrangian,

\[
\frac{1}{M_U^{d_{BZ} + d_{uW}}} O_{\text{SM}} O_{\text{BZ}}
\]

which is analogous to the four-fermion interactions in SM, where \( O_{\text{SM}} \) and \( O_{\text{BZ}} \) are respectively local operators constructed from the SM fields and the BZ fields. The BZ theory has IR xed point around an energy scale \( U \gg 1 \text{TeV} \), below this scale, the BZ sector undergoes dimensional transmutation and the scale invariant unparticle sector emerges. The BZ operator \( O_{BZ} \) matches onto the unparticle operator \( O_U \), and the interactions between the unparticle and the SM fields generally have the form

\[
L_{\text{eff}} = \frac{1}{M_U^{d_{BZ} + d_{uW}}} O_{\text{SM}} O_U
\]

where \( = C_U \left( \frac{m_{U}}{M_U} \right)^{d_{BZ} + d_{uW}} \) and \( C_U \) is the Wilson-like coefficient function. The lowest order effective interactions between the unparticle and the SM fermionic fields are as follows

\[
L_{\text{int}} = \frac{f f^0}{M_U^{d_{u} - d_{uW}}} f^0 O_U + \frac{f f^0}{M_U^{d_{u} - d_{uW}}} f^0 O_U + \frac{f f^0}{M_U^{d_{u} - d_{uW}}} f^0 O_U + \frac{f f^0}{M_U^{d_{u} - d_{uW}}} f^0 O_U
\]

Here \( f \) and \( f^0 \) denote SM fermions (leptons or quarks), and they should have the same electric charges. We note that both the third and the fourth term are absent, if we require that the effective Lagrangian \( L_{\text{int}} \) is consistent with the SM symmetry with unparticle being SM singlet. The unparticle operators have been set to be hermitian, and \( O_U \) is assumed to be transverse \( O_U = 0 \). The couplings between the SM fermionic fields and unparticle are quite arbitrary, it can be flavor conserving or changing. Moreover, there is no any correlation in the transitions among generations for flavor changing processes. In Ref. [8], the authors introduced BZ charges for the SM particles at very high energy scale, then tree level flavor changing neutral current (FCNC) can be induced by rediagonalizations of the SM fermion mass matrices. Under the Fritzsch ansatz of the mass matrices, the FCNC effects were found to be associated with the mass ratios \( m_i^2 \). Scale invariance fixes the two-point correlation function of unparticle, by dispersion relation, the two-point correlation function is determined to be

\[
Z^2 d^4 x e^{ip \cdot x} \langle 0 | T \left( O_U(x) O_U^\dagger(0) \right) | j 0 i = \frac{i A_{dU}}{2} \int_0^1 \frac{s^{d_U} 2}{P^2 s + 1} ds = \frac{i A_{dU}}{2 \sin(d_{uW})} (P^2 i)^{d_{uW}}
\]
where the normalization factor $A_{d_U}$ is chosen to be

$$A_{d_U} = \frac{16}{2\pi^2} \frac{5^{d_U}}{2^{d_U}} \frac{(d_U + \frac{1}{2})}{(d_U - 1)} \frac{1}{2^{d_U}(2d_U)}$$  \hspace{1cm} (6)$$

and the complex function $(P^2 i)^{d_U/2}$ is defined to be

$$\begin{align*}
(P^2 i)^{d_U/2} &= (P^2 i)^{d_U/2} ; P^2 < 0 \\
&= (P^2 i)^{d_U/2} e^{id_U} ; P^2 > 0
\end{align*}$$

In Eq. (5) the unparticle operator is scalar, it is straightforward to generalize to the vector unparticle operator $O_U$

$$d^4x e^{ipx} \Delta \Phi \left[ O^E_U(x)O^E_U(0) \right] 0 = iA_{d_U} \left( \frac{2}{2\sin(d_U)} (P^2 i)^{d_U/2} \right) \left( \gamma + P \gamma = P^2 \right)$$  \hspace{1cm} (8)$$

We note that the dispersion representation of the unparticle two-point correlation function is very useful, if unparticle appear in the loop, e.g. the unparticle induced lepton anomalous magnetic moment and LFV radiative decay $\tau \rightarrow e$ in the following.

III. LFV RADIATIVE DECAYS

![Feynman diagrams](image)

**FIG.1**: The Feynman diagrams contributing to $\tau \rightarrow e$

The diagrams for the LFV $\tau \rightarrow e$ are shown in Fig.1. Generally, the amplitude for $\tau \rightarrow e$ can be written as

$$M(\tau \rightarrow e) = \mathcal{U}_e(p_e)[\mathcal{I}q \left( A + B \right) + \left( C + D \right) + q \left( E + F \right)]\mathcal{U}_\tau(p)$$  \hspace{1cm} (9)$$

where $q$ and $\mathcal{I}$ are respectively the photon momentum and polarization, $A ; B ; \ldots ; F$ are invariant amplitudes. The electromagnetic gauge invariance requires the above amplitude is invariant under $\mathcal{I}q + q$, then we have

$$C = D = 0$$  \hspace{1cm} (10)$$
Since the photon is on shell $q^2 = 0$ and transverse $\mathbf{q} = 0, \mathbf{e}$ is a magnetic transition

$$M(\mathbf{e} \mathbf{e}^\ast) = \mathbf{\tau}_e(p_e)[i\gamma \cdot (A + B \Sigma)]u(p)$$

(11)

It is easy to calculate of the corresponding radiative decay width

$$\Gamma(\mathbf{e} 
\mathbf{e}^\ast) = \frac{m^3}{8}(\mathbf{A} \cdot \mathbf{f} + B \cdot \mathbf{f})$$

(12)

where we have neglected the final state electron mass. Using $(\mathbf{e}_e) = m^3 G_F^2 = 192$ cm, here $G_F$ is the Fermi constant, this can be converted into the branching ratio

$$\text{Br}(\mathbf{e} \mathbf{e}) = \frac{(\mathbf{e} \mathbf{e}^\ast)}{(\mathbf{e}_e)} = \frac{24^2}{m^2 G_F^2}(\mathbf{A} \cdot \mathbf{f} + B \cdot \mathbf{f})$$

(13)

We note that the couplings between unparticle and photon such as $O_\alpha O_\beta F$ also contribute to $\mathbf{e} \mathbf{e}$. However, its contribution is highly suppressed compared with those in Fig. 1. Using the dispersion representation of the unparticle two-point correlation function, it is straightforward, albeit somewhat lengthy, to work out these unparticle induced radiative decay $\mathbf{e} \mathbf{e}$ amplitude. In fact, we only need to consider Fig. 1(b), since the contribution of Fig. 1(a) and Fig. 1(c) are proportional to $\mathbf{\tau}_e(p_e) u(p)$ or $\mathbf{\tau}_e(p_e) s u(p)$.

$$A_S = \frac{e A_{\alpha}}{4 \sin(d_\alpha)} \int \frac{dx dy dz}{(4\pi)^2} \frac{Z_1}{e^a} \left[ \begin{array}{l} x^2 + y^2 + z^2 \\ x^2 y^2 + y^2 z^2 + z^2 x^2 \\ x^2 y^2 z^2 \end{array} \right]$$

(14)

$$A_P = \frac{e A_{\beta)}}{4 \sin(d_\beta)} \int \frac{dx dy dz}{(4\pi)^2} \frac{Z_1}{e^a} \left[ \begin{array}{l} x^2 + y^2 + z^2 \\ x^2 y^2 + y^2 z^2 + z^2 x^2 \\ x^2 y^2 z^2 \end{array} \right]$$

(15)

$$A_V = \frac{e A_{\gamma}}{4 \sin(d_\gamma)} \int \frac{dx dy dz}{(4\pi)^2} \frac{Z_1}{e^a} \left[ \begin{array}{l} x^2 + y^2 + z^2 \\ x^2 y^2 + y^2 z^2 + z^2 x^2 \\ x^2 y^2 z^2 \end{array} \right]$$

(16)
$$A_A = \frac{e A_{du}}{4 \sin (d_U)} \left( \frac{1}{4} \right)^2 \frac{x}{a_{e,\ell}} \left( \frac{2}{U_{du}} \right)^{1/2} \int_0^1 dx dy dz \ (x + y + z \ 1)f \left[ 4z(1 \ x) m_e \right.$$}

$$4z(1 \ y)m_e \ 8zm_a e^{|i(d_U \ 2)} |xzm_e^2 + yzm^2 \ (x + y)m_a^2 d_{du} \ z \ d_{du} \ +$$

$$2 \left[ 2y(m_a + m_e) \ 2x(m_a + m) \ (l + z)m_a + 2z(xm_e + ym) \ + z(1 \ x)m_e \right.$$}

$$+x(l \ \ y)m_e + zm_a \ 8zm_a e^{|i(d_U \ 2)} |xzm_e^2 + yzm^2 \ (x + y)m_a^2 d_{du} \ z \ d_{du} \ +$$

$$z^2 d_{du} = 2 \ d_{du} + \left[ 2y(m_a + m_e)(xzm_e^2 + (l \ y)(l \ z)m^2 \ ym_a m) \right.$$}

$$2x(m_a + m)(yzm^2 + (l \ x)(l \ \ y)m_e^2 \ xm_e m_a) + 2xy(m_a + m)(m_a + m_e)$$

$$+m_a m) e^{|i(d_U \ 3)} |xzm_e^2 + yzm^2 \ (x + y)m_a^2 d_{du} \ z \ d_{du} \ g \ (17)$$

FIG. 2: Variation of the branching ratios $\text{Br}( \mu \rightarrow e \gamma)$ with the scale dimension $d_U$. $V, A, S$ and $P$ respectively denote the branching ratios for the pure vector, axial vector, scalar and pseudoscalar couplings between unparticle and SM fermions. The horizontal line indicates the present experimental bounds for $\text{Br}( \mu \rightarrow e \gamma)$. We have taken $V = A = S = P = 0.001$, $= 3$ and $U = 10 \text{ TeV}$.

where the subscript denotes the contribution from the corresponding interactions between unparticle and the SM fermions, $B_S, B_P, B_V$ and $B_A$ equal zero. If both vector coupling and axial vector coupling between the unparticle and fermions (or scalar coupling and pseudoscalar coupling) are non-zero.
doscalar coupling) exist simultaneously, B would be non-zero. In Eq. (14)-Eq. (17), there is the factor \([xzm_e^2 + yzm^2 + (x+y)m_a^2]^{1/2}\) with \(a = e; \) (or \([xzm_e^2 + yzm^2 + (x+y)m_a^2]^{1/2}\)). It is well-defined if \(xzm_e^2 + yzm^2 + (x+y)m_a^2 > 0\), whereas \([xzm_e^2 + yzm^2 + (x+y)m_a^2]^{1/2} = \exp(i(d_\mu^2)/2)\) if \(xzm_e^2 + yzm^2 + (x+y)m_a^2 < 0\). Note that \(A_V\) and \(A_A\) are computed for a transverse vector unparticle operator \(O_u\), both \(g\) and \(P\) \(= P^2\) parts in the unparticle two-point correlation function contribute to the decay amplitude.

In Fig. 2 we present the variation of the branching ratio \(Br(\ ! e \ )\) as a function of the scale dimension \(d_u\) respectively for the pure vector, axial vector, scalar, pseudoscalar couplings between unparticle and the SM fermions. For simplicity, we assume that the unparticle couplings with the SM fermions are universal

\[
\frac{f_{u,\rho}}{K} = \frac{\delta}{K}; \ f = f^0
\]

(18)

where \(\delta > 1\) and \(K = V, A, S, P\) for vector, axial vector, scalar, pseudoscalar couplings respectively.

As we can see from Fig. 2, the branching ratio \(Br(\ ! e \ )\) decreases with \(d_u\) in the considered range, and it strongly depends on the scale dimension \(d_u\). There is little difference between \(Br(\ ! e \ )\) in the pure vector coupling case and that in the pure axial vector coupling case. The same is true for the pure scalar coupling and the pseudoscalar coupling. From Eq. (13) and Eq. (14)-Eq. (17), we can see \(Br(\ ! e \ )\) is proportional to \(1 = (\frac{2}{d_u})^{2d_u - 2}\). The \(d_u\) dependence of \(Br(\ ! e \ )\) for the pure vector coupling case is shown in Fig. 3.

From Fig. 2, we find that \(Br(\ ! e \ )\), for \(d_u = 1.1\) or 1.3 and other input parameters in that figure, is clearly above its present experimental upper bound. The important conclusion from Fig. 2 and Fig. 3 is that the present experimental data on \(Br(\ ! e \ )\) favors the scale dimension \(d_u\) close to 2 for \(\mu\) of \(O(10\ TeV)\) and the unparticle couplings of \(O(10^{-3})\).

IV. \(\text{EC}O\text{NS}V\text{ERTION IN NUCLEI}\)

The \(\text{e}C\text{ONV}E\text{RTION IN NUCLEI}\) is described by the Feynman diagram presented in Fig. 4, it means the following exotic process

\[
+ (A;Z) ! e + (A;Z)
\]

(19)
FIG. 3: $\text{Br}(\mu \rightarrow e)$ as a function of the unparticle scale $\nu$ for various scale dimension $d_0$ in the pure vector coupling case. The horizontal line indicates the present experimental bound for $\text{Br}(\mu \rightarrow e)$. We have taken $\nu = 0.001$, $\alpha = 3$.

FIG. 4: Feynman diagram for $e$ conversion in nuclei

It violates the conservation of lepton number $L_e$ and $L$ by one unit, but conserve the total lepton number $L$. The conversion rate is usually expressed by

$$\text{CR}(\mu; e; X) = \frac{+ X \rightarrow e + X}{+ X \rightarrow \text{capture}}$$

where $(+ X \rightarrow \text{capture})$ is the capture rate of the nucleus $X$. A very detailed calculation of the $e$ conversion rate in various nuclei has been performed in [60], using the methods...
developed by Czamecki et al. [61]. It has been emphasized in [60] that the atomic number dependence of the conversion rate can be used to distinguish between different theoretical models of LFV.

We will calculate the conversion rates in nuclei using the general model-independent formulae of both [60] and [61]. For the nucleon numbers relevant for the conversion experiments, the rate for the coherent process dominates over the incoherent excitations of the nuclear system, and the rate of the coherent conversion process over the incoherent ones is enhanced by a factor approximately equal to the number of nucleons in nucleus. Explicit calculations based on nuclear models [62] show that the ratio between the coherent rate and the total conversion rate for nuclei as $^{48}$Ti can be as large as 90%.

For coherent conversion, only vector coupling and scalar coupling between the quarks and unparticle do contribute, and the contributions of axial vector and pseudoscalar couplings are negligible. For the pure vector coupling between SM fermions and unparticle, the four fermion effective interaction, which describes coherent conversion, is given by

$$L^V_{e\text{ conv}} = e \bar{q} \gamma^\mu q \frac{A_d}{2 \sin(d_U)} \frac{1}{u} \left( \frac{q^2}{2} \right)^{d_U} 2 \bar{e} \gamma^\mu e$$

(21)

For the pure scalar coupling case,

$$L^S_{e\text{ conv}} = e \bar{q} \gamma^\mu q \frac{A_d}{2 \sin(d_U)} \frac{1}{u} \left( \frac{q^2}{2} \right)^{d_U} 2 \bar{e} \gamma^\mu e$$

(22)

In Eq. (21) and Eq. (22), $q^2$ is the momentum transfer in the conversion process ($q^2$, $m^2$), which is much smaller than the scale associated with the structure of the nucleon, and we can neglect the $q^2$ dependence in the nucleon form factors. The above effective Lagrangian at the quark level is then converted to the effective Lagrangian at the nucleon level, by means of the approximate nucleon form factors [52,60]. The matrix elements of the quark current for the nucleon $N = p,n$ can be written as,

$$h_p \gamma_k q^\mu p = G^{(u)p}_k$$

$$h_n \gamma_k q^\mu n = G^{(s)n}_k$$

(23)

where $k = 1; \ldots; 9$ respectively for $K = S,V$. The numerical values of the relevant $G_K$ are as follows [52]

$$G^{(u)p}_V = G^{(d)n}_V = 2; \quad G^{(d)p}_V = G^{(u)n}_V = 1; \quad G^{(s)p}_V = G^{(s)n}_V = 0$$

$$G^{(u)p}_S = G^{(d)n}_S = 5.1; \quad G^{(d)p}_S = G^{(u)n}_S = 4.3; \quad G^{(s)p}_S = G^{(s)n}_S = 2.5$$

(24)
Under the approximation of equal proton and neutron densities in the nucleus, and of non-relativistic muon wavefunction for the $1^s$ state, the general formula for the $e^-$ conversion rate for the pure vector coupling between SM fermions and unparticle, relative to the muon capture rate, is given by

$$\text{CR}(e;\text{Nucleus}) = \frac{p_e E_e m_e^3 Z^4}{2^2} \left[ e^e_v \frac{q q}{2} \frac{F_p}{2 \sin(d_\mu)} \right] \frac{1}{2^2} \left( \frac{m^2}{2} \right)^{\frac{1}{2}} \frac{d_\mu}{\sin(d_\mu)} 2 \int \Psi_\mu X \ G_\nu^{(qP)}$$

$$+ \frac{X}{q} G_\nu^{(qP)} \ j^\frac{1}{2} \ capt$$

(25)

For pure scalar coupling case, it is

$$\text{CR}(e;\text{Nucleus}) = \frac{p_e E_e m_e^3 Z^4}{2^2} \left[ e^e_s \frac{q q}{2} \frac{F_p}{2 \sin(d_\mu)} \right] \frac{1}{2^2} \left( \frac{m^2}{2} \right)^{\frac{1}{2}} \frac{d_\mu}{\sin(d_\mu)} 2 \int \Psi_\mu X \ G_S^{(qP)}$$

$$+ \frac{X}{q} G_S^{(qP)} \ j^\frac{1}{2} \ capt$$

(26)

where $Z$ and $N$ are the numbers of proton and neutron in nucleus, while $Z_e$ is an effective atomic charge, obtained by averaging the muon wavefunction over the nuclear density. $F_p$ is the nuclear matrix element and $capt$ denotes the total muon capture rate. $m$ is the muon mass, $p_e$ and $E_e$ is the momentum and energy of the electron. Since $P_q G^{(qP)}_\nu = P_q G^{(qP)}_\nu = 3$ and $P_q G^{(qP)}_\delta = P_q G^{(qP)}_\delta = 11.9$, the $e^-$ conversion rate is proportional to $Z^4 e^{-2A}$ with the atomic number $A = Z + N$, which can distinguish unparticle from other theoretical models.

In Fig. 5, we display the predicted $e^-$ conversion rates for Al, Ti, Sr, Sb, Au and Pb as a function of the scale dimension $d_\mu$ in the case of vector coupling between unparticle and SM fermions. The values of the relevant parameters for these nuclei, $Z_e$, $F_p$ and $capt$ have been collected in Table II. Here the universal couplings between unparticle and SM fermions are assumed as we have done in $e^\pm$. We clearly see that the $e^-$ conversion rates in nuclei $\text{CR}(e;\text{Nucleus})$ are sensitive to the scale dimension $d_\mu$, and they decrease with $d_\mu$ as well, which is obvious from Eq. (25) and Eq. (26), since $(m^2 = \frac{2}{d_\mu})^2$ dominates the $d_\mu$ dependence in the plot range, and $m^2 = \frac{2}{d_\mu}$ is a small quantity. Moreover, the present experimental bound on $\text{CR}(e;\text{Ti})$ and $\text{CR}(e;\text{Au})$ favor $d_\mu$ near 2 for the input parameters in this plot. The same conclusion has been found from LFV radiative decay $e^\pm$. 
FIG. 5: $e$ conversion rates for various nuclei as a function of the scale dimension $d_U$ for the vector coupling between unparticle and SM fermions. The horizontal lines denote the present experimental bounds for CR($e; Ti$) and CR($e; Au$). We have taken $V = 0.001$, $\alpha = 3$ and $U = 10$ TeV.

V. SUMMARY

Since LFV processes are sensitive probes to new physics beyond SM, we have explored the peculiar aspects of unparticle physics in $\pi e$ and $\mu e$ conversion in nuclei, where vector, axial vector, scalar, pseudoscalar couplings between unparticle and SM fermions are considered. The difference between the branching ratio $Br(\pi e)$ in the pure vector coupling case and that in the pure axial vector coupling case is small, the same is true for scalar coupling and pseudoscalar coupling. Only pure vector coupling and scalar coupling contribute to $\mu e$ conversion in nuclei, which is proportional to $Z_e^4 A^2 Z$, which can be used to distinguish unparticle from other theoretical models. Both $Br(\pi e)$ and CR($\mu e$; Nuclei) are sensitive to the scale dimension $d_U$ and the unparticle coupling $V^K_0$ ($K = V, A, S, P$), and
TABLE I: The value of $Z_e$, $F_p$ and $\text{capt}$ for various nuclei, which is taken from [60].

| Nucleus  | $Z_e$ | $F_p$ | $\text{capt}$ (G eV) |
|----------|-------|-------|----------------------|
| $^{28}_{13}$Al | 11.5  | 0.64  | $4.64079 \times 10^{19}$ |
| $^{48}_{22}$Ti | 17.6  | 0.54  | $1.70422 \times 10^{18}$ |
| $^{80}_{38}$Sr | 25.0  | 0.39  | $4.61842 \times 10^{18}$ |
| $^{121}_{51}$Sb | 29.0  | 0.32  | $6.71711 \times 10^{18}$ |
| $^{197}_{79}$Au | 33.5  | 0.16  | $8.59868 \times 10^{18}$ |
| $^{207}_{82}$Pb | 34.0  | 0.15  | $8.84868 \times 10^{18}$ |

The present data on $\text{Br}(\, \text{e}^+\!, \text{e}^-)$, $\text{CR}(\, \text{e}^+; \text{Ti})$ and $\text{CR}(\, \text{e}^+; \text{Au})$ put stringent constraints on the parameters of unparticle study. The scale dimensions $d_i$ near 2 are favored for the unparticle scale $v_0$ of $O(10 \text{ TeV})$ and the unparticle coupling of $O(10^{-3})$. The interactions between unparticle and SM fermions can also lead to LFV $\, \text{e}^-\text{e}^+\text{e}^+$ and cross symmetry related processes such as $\text{e}^+\text{e}^-\text{e}^+\text{e}^-$, detailed analyses of these processes have been performed [11,14,18]. Future dedicated LFV measurements MEG experiment and J-PARC experiment PRISM /PRIME would provide important clues to understanding the nature of unparticle.

Unparticle associated with conformal hidden sector may exist, and it has very distinctive phenomenologies. Unparticle may weakly couples to the SM field so that we are able to explore the peculiar properties of unparticle. However, whether observable effects can be produced strongly depends on how weakly the unparticle interacts with ordinary matter. So far there is no principle to constraint and organize the interactions between the SM particles and unparticles, therefore there are many freedom in the present phenomenological studies of unparticle. It would be enlightened and interesting to build an explicit model, where hidden sector with strict or broken scale invariance is realized and it connects to the SM fields via a connector sector. These issues lie outside the scope of the this work, and will be considered elsewhere [63].
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