Bounds on Transverse Spin Asymmetries for

Λ Baryon Production in pp Collisions at BNL RHIC

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Abstract

We study inclusive Λ hyperon production in pp collisions at BNL RHIC, with just one transversely polarized proton. We show that the measurement of the spin transfer between the initial proton and the produced Λ is sensitive to the proton transversity distributions and to the corresponding Λ transversity fragmentation functions. In view of our present ignorance of these distributions and fragmentation functions, we resort to positivity bounds for making some predictions for the corresponding spin transfer asymmetry.
From longitudinally polarized deep inelastic scattering (DIS) experiments, we only begin to gain some insight into the helicity parton densities \( \Delta_L f(x, Q^2) \), with \( f = q, \bar{q}, g \). Due to the scarcity of the data and their limited kinematical coverage in \( x \) and \( Q^2 \), many uncertainties remain though, in particular concerning the precise \( x \)-shape of the polarized gluon distribution and the flavour decomposition of the quark singlet combination. Hence the rôle played by (anti)quarks and gluons in the nucleon spin sum rule is still unsettled. To reduce our present ignorance, it is desirable to study polarization effects also in various other processes, rather than in fully inclusive DIS. This will be achieved by the vast spin physics programme which will be undertaken at the forthcoming RHIC collider at BNL \[1\]. Of particular interest here are also reactions with a detected (longitudinally) polarized hadron \( h \) in the final state which would allow us to study spin-dependent fragmentation functions \( \Delta_L D^h_f(x, Q^2) \), the even far less well-determined “time-like” counterparts of the parton density functions \( \Delta_L f(x, Q^2) \). In addition, such measurements could provide a deeper understanding of the hadronization mechanism for hadrons produced inclusively. So far, the only measured polarized hadron in the final state is the \( \Lambda \) hyperon. Its \( \Delta_L D^\Lambda_f(x, Q^2) \) are only poorly constrained by the existing \( e^+e^- \rightarrow \bar{\Lambda}X \) data \[2\], taken on the \( Z \)-resonance (henceforth, a horizontal (vertical) arrow will denote a longitudinally (transversely) polarized particle). However, it was shown in a recent work \[3\], that studying the helicity transfer between the initial proton and a high-P\( T \) \( \Lambda \) in \( \bar{p}p \rightarrow \bar{\Lambda}X \) would allow us to discriminate cleanly between various possible scenarios for \( \Delta_L D^\Lambda_f(x, Q^2) \) proposed in \[2\].

In the case of transverse polarization the situation is even worse. The transversity densities, denoted by \( \Delta_T q(x, Q^2) \) (or \( h_1^q(x, Q^2) \)), which are equally fundamental at leading twist \[4\] as the \( \Delta_L q(x, Q^2) \), are experimentally completely unknown for the time being. The chiral-odd \( \Delta_T q(x, Q^2) \) measure the difference of the probabilities to find a quark with its spin parallel to that of a transversely polarized nucleon and of finding it oppositely polarized. Unlike the case of unpolarized and longitudinally polarized densities, there is no gluon transversity distribution at leading twist due to angular momentum conservation \[4, 5\], and the \( \Delta_T q(x, Q^2) \) are not accessible in inclusive DIS measurements because of their chirality properties. Various ways to measure the \( \Delta_T q(x, Q^2) \) have been suggested, for
instance via the transversely polarized Drell-Yan process \[4, 6, 7\] at RHIC, but, as already mentioned, no data are available so far. In a similar way, one can define transversity fragmentation functions, denoted by $\Delta_T D^h_q(x, Q^2)$, to describe the fragmentation of a transversely polarized quark into a transversely polarized hadron. Needless to say that also in this case, we have no experimental information on these quantities. In view of the promising results obtained in \[3\] concerning a possible measurement of the $\Delta_L D^\Lambda_f(x, Q^2)$ in $\bar{p}p \rightarrow \Lambda X$, it seems worthwhile to study this reaction for the situation of transverse polarization at RHIC, i.e., for $p^\uparrow p \rightarrow \Lambda^\uparrow X$. This is the main purpose of this paper. In order to be able to make sensible predictions for the possible spin-transfer asymmetries for this process, we will exploit the positivity constraints derived in \[8\] to constrain the involved quantities $\Delta_T q(x, Q^2)$ and $\Delta_T D^h_q(x, Q^2)$ in a non-trivial way.

Let us first recall that a positivity constraint at the naive parton model level was obtained for the $\Delta_T q(x)$, which reads \[8\]

$$2|\Delta_T q(x)| \leq q(x) + \Delta_L q(x). \quad (1)$$

This result follows from the positivity properties of the forward quark-nucleon elastic amplitude, for which $\Delta_T q$ corresponds to

$$q_{h'}(q) + N_H(P) \rightarrow q_h(q) + N_{H'}(P), \quad (2)$$

where the helicities are such that $H = h = +1/2$ and $H' = h' = -1/2$. It was shown recently that Eq. (1) is preserved by the QCD $Q^2$ evolution, even to next-to-leading order (NLO) accuracy \[4, 8, 10\]. Eq. (2) as it stands only applies to the emission of a quark by a nucleon, but by using time reversal it is also related to the fragmentation of a quark into a nucleon. Here, keeping the same helicity labels as above, it corresponds to $\Delta_T D^h_q(x)$. Consequently an analogous positivity bound for the fragmentation functions of a quark $q$ into a hadron $h$ holds, namely

$$2|\Delta_T D^h_q(x)| \leq D^h_q(x) + \Delta_L D^h_q(x). \quad (3)$$

This new result is surely valid at the level of the naive parton model, and below, after specifying the densities on the r.h.s. of Eq. (3), we will show that it is also maintained.
by the QCD $Q^2$ evolution at leading order (LO). We will use these non-trivial bounds (1) and (3) to constrain the unmeasured transversity parton densities $\Delta_T q(x, Q^2)$ and fragmentation functions $\Delta_T D^h_q(x, Q^2)$ in our studies of the spin-transfer asymmetry for transversely polarized $\Lambda$ baryon production at RHIC below.

But before going into the details, we recall some general positivity constraints at the hadronic level that, even though not serving as a further constraint on our parton densities and fragmentation functions, will provide a consistency check on our calculation. A reaction of the type $pp \rightarrow \Lambda X$, where only one initial proton and the $\Lambda$ are polarized, can be described in terms of seven spin observables [11]. By studying the positivity domain of the reaction, one finds several model-independent constraints among these spin observables, which are valid at any kinematical point (total energy, transverse momentum, rapidity, etc.). If we restrict ourselves to the observables calculable in QCD at leading twist, only three of these spin transfer parameters survive, which can be chosen to be the spin transfer asymmetries $D_{LL}$, $D_{SS}$ and $D_{NN}$. Here “$L$” stands for longitudinal polarization of the proton and the $\Lambda$, whereas “$S$” and “$N$” denote transverse polarization, with the proton and the $\Lambda$ polarization vectors in, or normal to, the scattering plane, respectively. For all three cases one has the usual definition of a spin transfer asymmetry,

$$D_{PP} \equiv \frac{\sigma(s_p, s_\Lambda) - \sigma(s_p, -s_\Lambda)}{\sigma(s_p, s_\Lambda) + \sigma(s_p, -s_\Lambda)} , \quad (P = L, S, N) ,$$

(4)

where $s_p$, $s_\Lambda$ are the proton and $\Lambda$ spin vectors. In each of the cases $P = L, S, N$, the sum in the denominator of (4) corresponds to the usual unpolarized cross section for $\Lambda$ production in $pp$ scattering. As can be derived from [11], the $D_{PP}$ are subject to the following constraint:

$$|D_{LL} \pm D_{SS}| \leq 1 \pm D_{NN} .$$

(5)

For the process $p^\uparrow p \rightarrow \Lambda^\uparrow X$ considered here at LO QCD, it will actually turn out that $D_{SS} \equiv D_{NN}$ (see below). Eq. (3) therefore reduces to

$$|D_{LL} \pm D_{NN}| \leq 1 \pm D_{NN} .$$

(6)

Since we are left with essentially only one independent transverse-spin observable, we will
refer to it by the label “T” from now on and abbreviate the r.h.s. of Eq. (4) as
\[ D_{PP} \equiv \frac{\Delta P \sigma}{\sigma}, \]  
(7)
where \( P = L, T \). The case \( P = L \) was already studied in detail in [3]; as mentioned above, the present paper deals with \( P = T \). Here we will closely follow the procedure adopted in [3].

In various theoretical analyses of spin-transfer reactions it has turned out to be particularly useful to study distributions differential in the rapidity of a produced particle, see, e.g., [3], to which we therefore limit ourselves also in the present analysis. The rapidity differential polarized cross section in the numerator of (7) can be schematically written in a factorized form as
\[ \frac{d\Delta P \sigma_{pp \to \Lambda X}}{d\eta} = \int_{p_T^{\text{min}}} dp_T \sum_{ff' \to iX'} \int dx_1 dx_2 dz f^p(x_1, \mu^2) \times \Delta_P f^p(x_2, \mu^2) \times \Delta_P D_i^\Lambda(z, \mu^2) \times \frac{d\Delta_P \hat{\sigma}}{d\eta}, \]  
(8)
the sum running over all possible LO subprocesses \( ff' \to iX' \) (partons \( f' \) and \( i \) are polarized) with spin-transfer cross sections \( d\Delta_P \hat{\sigma}/d\eta \) defined in complete analogy with the numerator of Eq. (4). Note the appearance of the usual unpolarized parton densities \( f^p \) in (8), resulting from the fact that one initial proton is unpolarized. The expression for the unpolarized cross section \( d\sigma_{pp \to \Lambda X}/d\eta \), needed to calculate the spin-transfer asymmetries in (4), is similar to the one in (8), with all \( \Delta \)'s removed. In (8), we have integrated over the transverse momentum \( p_T \) of the \( \Lambda \), with \( p_T^{\text{min}} \) denoting some suitable lower cut-off to be specified below.

The spin-transfer cross sections for the subprocesses \( ff' \to iX' \) have been known for quite some time. They can be found in [12] for both polarization cases \( (P = L, T) \). The cross sections for the transversity case, \( P = T \), were presented in [12] in a form that also allows us to distinguish between the situations “S” and “N” introduced above, i.e., when the final-state particle “\( i \)”, and hence the \( \Lambda \), is transversely polarized in, or normal to, the scattering plane: writing the momentum of particle “\( i \)” in terms of its transverse momentum \( p_T^i \), its pseudorapidity \( \eta \) and its azimuthal angle \( \Phi \) as
\[ \vec{p}_i = p_T^i (\cos \Phi, \sin \Phi, \sinh \eta), \]  
(9)
one can parametrize the transverse spin vector of the $\Lambda$ by

$$\vec{s}_\Lambda(\beta) = (\sin \Phi \cos \beta + \tanh \eta \cos \Phi \sin \beta, -\cos \Phi \cos \beta + \tanh \eta \sin \Phi \sin \beta, -\sin \beta / \cosh \eta).$$

(10)

The angle $\beta$ in (10) is the rotational degree of freedom of the spin vector $\vec{s}_\Lambda$ around the momentum of the $\Lambda$ (or, equivalently, the momentum of the parton "i") while $\vec{s}_\Lambda \cdot \vec{p}_\Lambda = 0$. The values $\beta = 0$, $\Phi = \pi/2$ correspond to the proton and the $\Lambda$ being transversely polarized normal to the scattering plane, i.e., to calculating $D_{NN}$. Conversely, for $\beta = -\pi/2$, $\Phi = 0$ the proton and the $\Lambda$ are transversely polarized in the scattering plane; this we defined as $D_{SS}$. The analysis of [12] shows that the asymmetry for arbitrary values of $\beta$ and $\Phi$, i.e., general polarization, is proportional to $\sin(\Phi - \beta)$; we thus immediately arrive at

$$D_{NN} \equiv D_{SS}.$$

(11)

Interestingly, the inequality (6) is already satisfied on the partonic level: taking the spin-transfer subprocess cross sections for $ff' \rightarrow iX'$ of [12] and the unpolarized ones as compiled in [13], one easily verifies that for all $p_T$ and $\eta$

$$|d_{LL} \pm d_{NN}| \leq 1 \pm d_{NN},$$

(12)

where, in analogy with (4),

$$d_{PP} \equiv \frac{\hat{\sigma}(s_{f'}, s_i) - \hat{\sigma}(s_{f'}, -s_i)}{\hat{\sigma}(s_{f'}, s_i) + \hat{\sigma}(s_{f'}, -s_i)}, \quad (P = L, S, N).$$

(13)

It will therefore not come as a surprise that, at the hadron level, Eq. (8) is also satisfied in the framework of our calculation; see below.

Let us now turn to the phenomenological analysis. Before we can estimate the spin-transfer asymmetry $D_{NN}^\Lambda$ for transversely polarized $\Lambda$ baryon production in (8), we have to specify the various different parton distribution and fragmentation functions involved in this calculation. We will use the approach of saturating the positivity inequalities given in Eqs. (4) and (6) at some input resolution scale $Q_0$ to constrain the unknown transversity parton densities $\Delta_Tq(x, Q^2)$ and the $\Lambda$ fragmentation functions $\Delta_TD_q^\Lambda(x, Q^2)$, respectively. The QCD evolution then fully specifies both densities at all scales $Q \geq 5$. 

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$Q_0$. Such a framework is sufficient to derive a more or less rigorous estimate for an upper bound for the expected spin-transfer asymmetry $D_{NN}^\Lambda$ in (7). Since all relevant helicity transfer subprocess cross sections are available only at the Born level, we restrict ourselves also to LO for the $Q^2$ evolutions of the involved parton density and fragmentation functions. More precisely, for the $\Delta T q(x, Q^2)$ we follow the approach in [7] and use the unpolarized GRV [14] and the longitudinally polarized GRSV [15] LO parton densities $q$ and $\Delta_L q$, respectively, on the r.h.s. of Eq. (7). The unpolarized and the longitudinally polarized fragmentation functions $D_q^\Lambda$ and $\Delta_L D_q^\Lambda$ determined in [4] serve to constrain the transversity fragmentation functions $\Delta T D_q^\Lambda(x, Q^2)$ via the bound in (3). Since the available $e^+e^-$ data are not sufficient to constrain the $\Delta L D_q^\Lambda$ fully, three very distinct models for these densities were proposed in [2]. To take this uncertainty into account in the present analysis, we define in the same fashion also three different scenarios for the $\Delta T D_q^\Lambda(x, Q^2)$ by using all three $\Delta L D_q^\Lambda$ sets of [4] in (3). The idea behind these scenarios should be briefly recalled here (see [4] for more details):

**Scenario 1** is based on expectations from the non-relativistic naive quark model, where only strange quarks can contribute to the fragmentation processes that eventually yield a polarized $\Lambda$.

**Scenario 2** is inspired by estimates [16] for a fictitious DIS structure function $g_1^\Lambda$. Assuming the same features also for the $\Delta L D_q^\Lambda$, a sizeable negative contribution from $u$ and $d$ quarks to $\Lambda$ fragmentation is predicted here.

**Scenario 3** is the most extreme counterpart of scenario 1 since all the polarized quark fragmentation functions are assumed to be equal here.

In order to check numerically if the parton model bound [3] for the transversity fragmentation functions is respected also by the QCD $Q^2$ evolution at LO, Fig. 1(a) shows the ratio

$$R_q(z, Q^2) = \frac{2\Delta T D_q^\Lambda(z, Q^2)}{D_q^\Lambda(z, Q^2) + \Delta L D_q^\Lambda(z, Q^2)}$$

(14)

as a function of $z$, for several different $Q^2$ values, for scenario 3 (here $R_u = R_d = R_s$). Very similar results for $R_q$ are found within the other two scenarios. Clearly, (3) is
satisfied for all $Q^2$ values, and the bound remains saturated, i.e., $R_q(z, Q^2) = 1$, only for $z \to 1$, whereas at smaller $z$ it becomes more and more diluted with increasing $Q^2$. This finding is not really unexpected, since at the LO level all (polarized and unpolarized) QCD Altarelli–Parisi splitting functions for the fragmentation case are identical to those for the parton density case (see, e.g., [17]). The only difference between the evolutions of parton densities and fragmentation functions results from an interchange of the splitting functions for quark-to-gluon and gluon-to-quark transitions (which contribute to the evolution of unpolarized and longitudinally polarized parton densities and fragmentation functions but not to the transversity case).

Fig. 1(b) compares the $\Delta T^\Lambda D_q^\Lambda$ for the three different scenarios, by showing the partonic asymmetries $A_q^\Lambda(z, Q^2) \equiv \Delta T^\Lambda D_q^\Lambda(z, Q^2)/D_q^\Lambda(z, Q^2)$ at $Q^2 = 100 \text{ GeV}^2$. Actually the $Q^2$ dependence is rather weak in the $z$-range where fragmentation functions can be applied, i.e., for $z \gtrsim 0.05$. For smaller $z$ values finite-mass corrections to the cross section would become increasingly important, see, e.g., Ref. [2]. Furthermore, it was pointed out in [3] that small values of $z$ also have to be excluded in order to make sure that there are no unreasonably large NLO contributions induced by the extremely singular behaviour of the (unpolarized) NLO evolution kernels at small $z$. As can be inferred from Fig. 1(b), the differences between the scenarios are not very pronounced (especially between scenarios 1 and 2), in contrast to the corresponding results for the longitudinally polarized fragmentation functions, cf. Fig. 5 in [3]. This is readily explained by the fact that now the unpolarized $D_q^\Lambda$ play an important role in the construction of the $\Delta T^\Lambda D_q^\Lambda$ via Eq. (3) which dilutes the differences between the scenarios as implemented in the three $\Delta L^\Lambda D_q^\Lambda$ sets of [2].

Fig. 2 shows our predictions for the spin-transfer asymmetry $D_{NN}^\Lambda$ as a function of rapidity, calculated according to Eqs. (7) and (8) for $\sqrt{s} = 500 \text{ GeV}$ and $p_T^{\text{min}} = 13 \text{ GeV}$. Note that we have counted positive rapidity in the forward region of the polarized proton. We have used the three different scenarios for the $\Delta T^\Lambda D_q^\Lambda$ discussed above, employing the hard scale $\mu = p_T$. The possibility to have negative and positive asymmetries of the same size for each scenario reflects the freedom in the choice of the sign for the $\Delta T^\Lambda D_q^\Lambda$ and
the $\Delta_T q$ in Eqs. (3) and (4), respectively. It should be stressed that the $p_T$ cut we have introduced does not only guarantee the applicability of perturbative QCD (the hard scale $\mu$ in (8) should be $O(p_T)$), but also ensures that the fragmentation functions can be safely applied here, i.e., that $z \gtrsim 0.05$, as discussed above.

The “error bars” in Fig. 2 should give an impression of the achievable statistical accuracy for such a measurement at RHIC. They have been estimated via

$$\delta D_{NN}^\Lambda \simeq \frac{1}{P} \frac{1}{\sqrt{b_\Lambda \epsilon_\Lambda \mathcal{L} \sigma_{pp\to\Lambda X}}} ,$$

assuming a transverse polarization $P$ of the proton beam of about 70%, a branching ratio $b_\Lambda \equiv \text{BR}(\Lambda \to p\pi) \simeq 0.64$, a conservative value for the $\Lambda$ detection efficiency of $\epsilon_\Lambda = 0.1$, and an integrated luminosity of $\mathcal{L} = 800 \text{ pb}^{-1}$. The cross section $\sigma_{pp\to\Lambda X}$ in (15) is the unpolarized one, integrated over suitable bins of $\eta$. It should be mentioned that results almost identical to the ones in Fig. 2 can be obtained also for a lower c.m.s. energy of $\sqrt{s} = 200 \text{ GeV}$ and a correspondingly lowered $p_T^{\min}$ and luminosity of 8 GeV and 240 pb$^{-1}$, respectively. As expected (see Fig. 1(b)), the differences in $D_{NN}^\Lambda$ calculated within the three scenarios for the $\Delta_T D_q^\Lambda$ are not too pronounced, since the main contribution to the cross section comes from the region of rather small $z$ (see also Figs. 1 and 2 in [3] for the corresponding situation in the longitudinally polarized case). The $\eta$ dependence of the asymmetries in Fig. 2 is readily understood: at negative $\eta$ the parton densities of the transversely polarized proton are probed at small values of the momentum fraction $x_2$, where the ratio $\Delta_T q(x_2)/q(x_2)$ is also small [7]. On the contrary, at large positive $\eta$, the quarks are polarized much more strongly, resulting in an asymmetry that increases with $\eta$.

In Fig. 2 we have also studied the impact of one of the major theoretical uncertainties in a LO calculation of $D_{NN}^\Lambda$, the dependence on variations of the a priori unknown hard scale $\mu$ in (8). Luckily, it turns out that $D_{NN}^\Lambda$ depends only very weakly on the value of the hard scale in the range $\mu = p_T/2$ to $\mu = 2 p_T$, as is demonstrated for scenario 3 in Fig. 2 (very similar results hold for the other two scenarios).

The results shown in Fig. 2 clearly demonstrate the usefulness of studying also the
production of transversely polarized $\Lambda$ hyperons at RHIC. Of course, one should keep in mind that the asymmetries presented in Fig. 2 represent only a rough upper bound of what can be expected in an actual measurement. In order to arrive at this prediction we have saturated both positivity bounds to constrain the unknown transversity parton density and $\Lambda$ fragmentation functions in a non-trivial manner, which is, however, not very likely to be realized in nature. Hence the measured asymmetry will possibly be considerably smaller with respect to our prediction, but even when reduced by a factor of 2 or 4, a measurement of $D_{NN}^{\Lambda}$ would still remain feasible since the expected statistical errors are very small. To eventually disentangle the $\Delta_T q$ and the $\Delta_T D_q^\Lambda$ from a measurement of $D_{NN}^{\Lambda}$, one needs of course at least one other measurement in order to determine both unknown distributions. As already mentioned, the transversely polarized Drell-Yan process seems to be a realistic way to obtain first information on the $\Delta_T q$ at RHIC which could then be used to study the $\Delta_T D^\Lambda_q$ from a measurement of $D_{NN}^{\Lambda}$.

Let us finally return to the inequality (6) which relates the spin-transfer asymmetries for longitudinally and transversely polarized $\Lambda$ baryon production. We have already shown that this relation is fulfilled at the level of partonic cross sections (12), and in Fig. 3 we check whether (6) is also maintained. Taking our results for $D_{NN}^{\Lambda}$ shown in Fig. 2 and the corresponding ones for the longitudinally polarized case $D_{LL}^{\Lambda}$ as presented in Fig. 1 of [3] (note that in [3] we have denoted $D_{LL}^{\Lambda}$ by $A^{\Lambda}$), we present in Fig. 3 the ratio

$$R_D^\pm = \frac{|D_{LL}^{\Lambda} \pm D_{NN}^{\Lambda}|}{1 \pm D_{NN}^{\Lambda}}$$

(16)

as a function of the rapidity of the $\Lambda$ for both signs $\pm$ in (6) with all other parameters being the same as in Fig. 2 and in [3]. As expected, the inequality (6) holds. It should be stressed that this is not merely a result of our choice to use fully saturated transversity parton distributions and fragmentation functions, since with vanishing transversity densities, i.e., $D_{NN} = 0$, Eq. (6) reduces to the usual positivity limit $|D_{LL}| \leq 1$ for longitudinally polarized cross sections and is trivially fulfilled.
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References

[1] Proceedings of the RSC annual meeting, Marseille, CPT-96/P.3400 (1996);
    Proceedings of the workshop on RHIC Spin Physics, Riken-BNL Research Center, April 1998 (to appear).

[2] D. de Florian, M. Stratmann, and W. Vogelsang, Phys. Rev. D 57 (1998) 5811.

[3] D. de Florian, M. Stratmann, and W. Vogelsang, hep-ph/9802432, to appear in Phys. Rev. Lett.

[4] J.P. Ralston and D.E. Soper, Nucl. Phys. B152 (1979) 109;
    X. Artru and M. Mekhfi, Z. Phys. C45 (1990) 669;
    R.L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552, Nucl. Phys. B375 (1992) 527;
    J.L. Cortes, B. Pire, and J.P. Ralston, Z. Phys. C55 (1992) 409.

[5] R.L. Jaffe and A. Manohar, Phys. Lett. B223 (1989) 218;
    X. Ji, Phys. Lett. B289 (1992) 137.

[6] W. Vogelsang and A. Weber, Phys. Rev. D48 (1993) 2073;
    A.P. Contogouris, B. Kamal and Z. Merebashvili, Phys. Lett. B337 (1994) 169.

[7] O. Martin, A. Schäfer, M. Stratmann, and W. Vogelsang, Phys. Rev. D57 (1998) 3084.

[8] J. Soffer, Phys. Rev. Lett. 74 (1995) 1292.
[9] W. Vogelsang, Phys. Rev. D57 (1998) 1886.

[10] C. Bourrely, J. Soffer, and O.V. Teryaev, Phys. Lett. B420 (1998) 375.

[11] M.G. Doncel and A. Méndez, Phys. Lett. B41 (1972) 83.

[12] M. Stratmann and W. Vogelsang, Phys. Lett. B295 (1992) 277.

[13] R. Gastmans and T.T. Wu, “The Ubiquitous Photon” (Clarendon Press, Oxford, 1990).

[14] M. Gluck, E. Reya, and A. Vogt, Z. Phys. C67 (1995) 433.

[15] M. Gluck, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D53 (1996) 4775.

[16] M. Burkardt and R.L. Jaffe, Phys. Rev. Lett. 70 (1993) 2537.

[17] M. Stratmann and W. Vogelsang, Nucl. Phys. B160 (1997) 301.
Figure Captions

Fig. 1 (a) The ratio $R_q(z, Q^2)$ as defined in (14) for various different values of $Q^2$ using scenario 3 for the transversity fragmentation functions (here $R_u = R_d = R_s$). The results for the other two scenarios are very similar.
(b) The ratio $A^T_q \equiv \Delta_T D^\Lambda_q / D^\Lambda_q$ at $Q^2 = 100 \text{GeV}^2$ for the three different sets of transversity $\Lambda$ fragmentation functions $\Delta_T D^\Lambda_q$. The unpolarized $D^\Lambda_q$ in $A^T_q$ are taken from Ref. [2].

Fig. 2 Upper bounds for the spin-transfer asymmetry $D^\Lambda_{NN}$ according to Eqs. (7) and (8), as functions of the rapidity of the produced $\Lambda$ at RHIC energies, using saturated positivity bounds in (1) and (3) for the $\Delta_T q$ and for the three sets of transversity fragmentation functions $\Delta_T D^\Lambda_q$, respectively, as defined in the text. The “error bars” correspond to the expected statistical accuracy for such a measurement at RHIC and have been calculated according to (15) and as discussed in the text. For “scenario 3” we also illustrate the typical theoretical uncertainty induced by a variation of the hard scale $\mu$ in (8) in the range $p_T/2$ to $2p_T$.

Fig. 3 The ratio $R^\pm_D$ as defined in (16) for both signs $\pm$ and using the three different scenarios for the $\Delta_T D^\Lambda_q$. The other parameters are the same as in Fig. 2. The longitudinal spin-transfer asymmetry $D^\Lambda_{LL}$ is taken from [3].
Fig. 1

(a) $R_q(z,Q^2)$ (scen. 3)

- $Q^2 = Q_0^2$
- $Q^2 = 1$ GeV$^2$
- $Q^2 = 10$ GeV$^2$
- $Q^2 = 10^4$ GeV$^2$

(b) $A_{q}^{T}(z,Q^2)$

- $q = s$
- $q = u, d, s$
- $q = u, d$

scen. 1 scen. 2 scen. 3
$\sqrt{s} = 500$ GeV
$p_T > 13$ GeV

$D_{NN}^\Lambda$

Fig. 2
Fig. 3