TYPE I PLANETARY MIGRATION WITH MHD TURBULENCE

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ABSTRACT

This paper examines how type I planetary migration is affected by the presence of turbulent density fluctuations in the circumstellar disk. For type I migration, the planet does not clear a gap in the disk and its secular motion is driven by torques generated by the wakes it creates in the surrounding disk fluid. MHD turbulence creates additional density perturbations that gravitationally interact with the planet and can dominate the torques that stem from the protoplanetary wake. This paper shows that conventional type I migration can be readily overwhelmed by turbulent perturbations, and hence the usual description of type I migration should be modified in locations where the magnetorotational instability is active. In general, the migrating planet does not follow a smooth inward trend but rather exhibits a random walk through phase space. Our main conclusion is that MHD turbulence will alter the timescales for type I planetary migration and, because of chaos, requires the timescales to be described by a distribution of values.

Subject headings: MHD — planetary systems — planetary systems: formation — planets and satellites: formation — turbulence

On-line material: color figures

1. INTRODUCTION

Ever since their initial discovery (Mayor & Queloz 1995; Marcy & Butler 1996), extrasolar planets have been found in unexpectedly short-period orbits. Since theories of planet formation suggest that planets must form farther out in the disk (see, e.g., Lissauer 1993), these findings strongly indicate the necessity of planetary migration (e.g., Lin et al. 1996). A variety of migration scenarios have been suggested. Type I migration (Ward 1997) occurs when a planet (or protoplanet) is embedded in a circumstellar gaseous disk and the planet’s mass is too small to clear a gap in the disk. In this case, the planet drives wakes into the background fluid of the disk and these wakes in turn exert torques on the planet. These torques are generally greater in the inward direction than in the outward direction, and a net inward migration occurs.

These same disks are expected to contain magnetic fields that are subject to a robust instability (Ballbus & Hawley 1991), for regimes in which an ideal-MHD description is adequate. The nonlinear outcome of this magnetorotational instability is MHD-driven turbulence. Indeed, MHD-driven turbulence is commonly invoked as a primary source of disk viscosity that can drive mass accretion through the disk. The usual scenario for type I migration, however, implicitly assumes a smooth background, uninterrupted by turbulence (see, however, Goodman & Rafikov 2001). Given that such turbulence, along with associated stochastic density fluctuations in the disk gas, may often be present, we reexamine the type I migration scenario by considering the dynamical evolution of embedded protoplanets that are torqued by transient density perturbations arising from MHD turbulence.

Our analysis shows that gravitational torques arising from MHD turbulence will contribute a random walk component to the migratory evolution of the planet’s semimajor axis, and that this component is superposed on the underlying secular type I migration. The net radial change $\langle r \rangle$ in the planet’s location after $N$ orbits arising from the stochastic torques accumulates as $\langle r \rangle \propto AN^{1/2}$, whereas for standard type I migration $\langle r \rangle \propto A_{p}N$. Hence, in order for turbulent torques to have a decisive effect on the motion over a specified number of orbits $N$, the constant $A$ must satisfy the requirement

$$A \geq A_{c}N^{1/2},$$

where $A_{c}$ is a critical amplitude that we derive below.

The effects of large-scale turbulence on type II migration, where the planet clears a gap in the disk (e.g., Goldreich & Tremaine 1980; Lin & Papaloizou 1993), has been explored recently (Nelson & Papaloizou 2003; Papaloizou & Nelson 2003; Winters et al. 2003). In a related vein, Terquem (2003) has considered the possibility of modifying inward migration with a toroidal magnetic field. These previous papers show how turbulence and magnetic fields can produce important modifications to the standard migration picture. This present work carries out a complementary analysis for type I migration.

Our approach to the problem is as follows. We first compute a representative three-dimensional MHD simulation ($\S$ 2) in a circumstellar disk and use the results to quantify the spectrum of nonlinear density perturbations that are produced by MHD-driven turbulence. These density perturbations provide a stochastic time-varying gravitational torque on any embedded protoplanets that are orbiting in the disk. Our next step is to construct a parametric model that characterizes the essential aspects of the spectrum of turbulent density fluctuations that are observed in the three-dimensional simulation. This model is described in terms of a time-varying nonaxisymmetric potential function ($\S$ 3). The resulting prescription for the perturbations is then applied to two-dimensional hydrodynamic simulations of circumstellar disks to study type I protoplanetary migration ($\S$ 4). With this scheme, we can examine the effects of the turbulent perturbations on the migration of bodies that are not sufficiently massive to open gaps in a protostellar

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disk and we can obtain an overall initial exploration of the type I migration problem. Our approach allows for a rapid survey of the important regions of the parameter space, and is complementary to full-scale three-dimensional MHD simulations that trace the evolution of a planet in an ionized circumstellar disk. Because the saturation of the magnetorotational instability occurs at nonlinear amplitudes, MHD-driven turbulence naturally produces large density fluctuations, which in turn produce torques that generally dominate those produced by the planetary wakes of objects with \( M < 10 \, M_\odot \).

We quantify this threshold by deriving an analytic estimate of the fluctuation amplitudes that are required for type I migration to be highly distorted (§ 5) and then discuss the longer term behavior (§ 6).

2. MHD SIMULATIONS

Our first step is to conduct a representative three-dimensional simulation to evaluate the expected properties of MHD turbulence in a model circumstellar disk. We use the “Nirvana” MHD code (Ziegler & Rüdiger 2000) as modified by Steinacker & Papaloizou (2002). We work in cylindrical coordinates \((\xi, r, \phi)\), and use a system of units in which the gravitational constant \( G \) and the central stellar mass \( M \) are both unity. Our disk model has a nearly Keplerian radial domain that extends from an inner boundary at \( r = 1.5 \) out to \( r = 3.5 \) and subtends \( \pi/3 \) radians in azimuth. The equation of state is taken to be locally isothermal, with \( P(r) = \rho(r)\xi \rho^2(r) \) (see, e.g., Laughlin & Bodenheimer 1994). This approximation is reasonable for optically thick disks whose local radiative diffusion time is shorter than the local dynamical (orbital) timescale.

The gravitational potential from the central star is specified to depend only on the cylindrical radial coordinate \( \Phi = -GM_\star/r \). The model therefore does not consider the vertical density gradient that results from the \( z \)-component of the gravitational force. This approximation simplifies the physical situation, allowing the use of a modest number of zones in the vertical direction. An obvious future refinement will be to study vertically stratified models. The reference simulation described here uses 454 radial zones, 94 azimuthal zones, and 40 vertical zones.

In the nearly Keplerian region of our initial equilibrium model, the disk density varies inversely with radius, \( \rho(r) = \rho_0(r_0/r) \), the sound speed is given by \( a_s(r) = C_0(r_0/r)^{1/2} \), and the rotational velocity is \( v_\phi(r) = (GM_\star/r_0 - 2C_0^2 r_0/r)^{1/2} \). For numerical reasons, the disk also contains inner and outer radial boundary domains as described in Steinacker & Papaloizou (2002).

We adopt \( C_0 = 0.1 \), along with a subthermal poloidal seed field having zero net flux through the disk. With these starting parameters, the disk experiences the magnetorotational instability, progresses through the channel phase, and develops turbulence as described in detail by Steinacker & Papaloizou (2002). This configuration of initial conditions was found by Steinacker & Papaloizou (2002) and Hawley (2001), among others, to lead to the smallest overall amplitude in the turbulent density fluctuations. Initial magnetic configurations with net flux threading the disk can lead to turbulent fluctuations that are up to 2 orders of magnitude larger. This turbulence persists for the duration of the simulation, which was followed to 204 orbits at the outer disk edge.

Figure 1 shows a map of the surface density distribution \( \sigma(r, \phi) \) in the disk after the turbulence has entered its quasi-steady fully developed phase. The surface density fluctuations that are present in the disk provide stochastic gravitational torques that are capable of severely modifying the type I migration process. The corresponding global Fourier amplitudes,

\[
C_m = \frac{\int_0^{2\pi} \int_0^{\infty} \sigma(r, \phi) r \, dr \, d\phi \, \exp^{-im\phi}}{\int_0^{2\pi} \int_0^{\infty} \sigma(r, \phi) r \, dr \, d\phi},
\]

of the disk fluctuations shown in Figure 1 are plotted as filled black circles in Figure 2. The perturbation amplitude is strongest for the lowest azimuthal wavenumber \( (m = 6) \) that fits in the \( \pi/3 \) azimuthal domain. As the azimuthal mode number \( m = 6n \) is increased through \( n = 2, 3, 4, \) and \( 5 \), the Fourier amplitudes show a gradual decline. The concentration of power in lower order Fourier modes is a characteristic of MHD-generated turbulence and has been previously observed by other authors, including Armitage (1998) and Hawley et al. (1995).

3. PARAMETERIZATION OF THE TURBULENCE

In order to survey the effects of MHD turbulence on type I planetary migration, we have chosen to construct a simple heuristic model to describe the perturbations. By characterizing the turbulence arising in the full MHD treatment in a simple manner, we can use the resulting prescription to specify the development of time-varying disk fluctuations in two-dimensional planetary migration simulations. The alternative (and more rigorous) approach to the problem would involve running an expensive three-dimensional MHD simulation with every planetary migration calculation. Each long-duration run of the MHD code requires, for example, 30,000 CPU hours on an Origin 3800 parallel processing facility (Nelson &
MHD turbulence computed by the full three-dimensional code mimic the power spectrum of density fluctuations in the actual simulation, typically 512 here. We find that the choice of a lognormal distribution of $m$ gives the best approximation to the density fluctuation spectrum produced by the MHD simulation; this spectrum of $m$ provides power on all spatial scales. With $m$ specified, the mode extends for a distance $2\pi r_c/m$ along the azimuthal direction. The radial extent is then specified by choosing $\sigma = \pi r_c/4m$ so that the mode shapes have roughly a 4:1 aspect ratio (this statement is not precise because the profiles differ in the radial and azimuthal directions). Note also that the initial mode shapes are not tilted as would be expected from the usual spiral patterns seen in disks; here the durations of the modes are generally much longer than the time required for the Keplerian shear to adjust the orientations, so we can rely on the shear to take over.

The modes first appear at a time $t_0$, which is chosen so that the simulation contains 50 active modes at all times. The pattern speed $\Omega_c = \kappa_{\text{kepler}}/r_c$ in the time-dependent factor allows the mode center to travel along with the Keplerian shear. The duration of the mode $\Delta t$ is taken to be the sound crossing time of the mode along the angular direction, i.e., $\Delta t = 2\pi r_c/(\text{mas})$.

Finally, the strengths of all the modes are governed by an overall amplitude $A$, as well as individual amplitudes $\xi$. As written, the amplitude $A$ has units of $[A] = \rho^{1/2}l^{-2}$ (where $l$ is a length scale and $t$ is a timescale) so that the overall expression has units of potential. The dimensionless variable $\xi$ has a Gaussian distribution with unit width, whereas the overall amplitude $A$ has a fixed value for all of the modes. We can find the amplitude $A$ that reproduces the level of perturbations found in the MHD simulation. Alternately, we can treat the amplitude $A$ as an adjustable parameter and study the effects of these turbulent fluctuations on planetary migration as a function of the amplitude (see below).

Figure 2 shows the power spectra from both the full MHD treatment of the previous section and the parametric model developed in this section. We have explicitly chosen to make the amplitudes smaller for the two-dimensional simulations so that the turbulence does not completely overwhelm the type I migration effects. The slopes of the two spectra, however, are in reasonable agreement. As a result, we employ this approach to specify the perturbations for migration calculations, as presented in the following section.

4. MIGRATION WITH TURBULENT PERTURBATIONS

With the perturbations due to MHD turbulence specified (as described in §3), we have performed fluid dynamic simulations of the migration of a terrestrial planet (or the core of a giant planet) in the circumstellar disk. These simulations use a two-dimensional fluid dynamics code where we add the turbulent perturbations according to the prescription described above.

Our two-dimensional code (Laughlin et al. 1997) is based on second-order van Leer advection (e.g., Stone & Norman 1992). We generally use a grid of 256 evenly spaced radial zones and 512 azimuthal zones. The initial model has the same surface density and sound speed distributions and the same active radial domain as was employed in the three-dimensional simulation. We minimize reflection by implementing sponge boundary conditions at the radial edges of the computational domain. In the $n_{\text{rad}} = 12$ radial zones interior to...
5. THE CRITICAL AMPLITUDE

Equation (3) gives the potential produced by the turbulent fluctuations. The gradient of this potential provides a forcing term that acts on the gas in the disk (but does not act directly on the planet) and thereby produces corresponding fluctuations in the surface density of the disk. To obtain a rough understanding of the critical fluctuation amplitude $A_c$ required for turbulence to overwhelm the standard type I migration torques, we assume here that the gravitational potential of the perturbed gaseous disk (where this potential does act on the planet) has the same form as that of equation (3). However, as we show below the amplitude of the potential acting on the planet will in general be reduced from that acting on the gas by a factor $\Gamma \lesssim 1$. The torques exerted on the planet by the surface density perturbations in the disk will thus have the form

$$
\tau = -\frac{m\alpha \Gamma \mu M_p}{r^{1/2}} e^{-(r-r_0)/\sigma^2} \sin(m\theta - \phi) \sin(\pi t/\Delta t),
$$

where $M_p$ is the mass of the planet and all the other quantities are defined previously. At any given time, the disk will contain 50 modes that exert torques of this form. To evaluate the effectiveness of these torques, we need to define an average torque strength. First, we integrate over time to obtain the net change in angular momentum per mode, i.e.,

$$
\Delta J = \int_0^{\Delta t} dt \tau = \frac{m\alpha \Gamma \mu M_p}{r^{1/2}} e^{-(r-r_0)/\sigma^2} \frac{\Delta t}{\pi} \sin \left( \frac{a_0 \pi - \varphi_0 - \varphi}{a^2 - 1} \right),
$$

where $r_0$ is the radius of the planet and $\Delta t$ is the orbital period of the planet. The critical amplitude determined empirically from these two-dimensional disk simulations is $A_c \sim 1 \times 10^{-5}$. In the following section, we derive an analytic estimate for this critical value.
where the parameter $a$ is defined to be $a \equiv (m\Omega - \Omega_c)\Delta t/\pi$. The mean torque per mode $\tau_i$, averaged over the lifetime of the mode, is thus given by

$$\tau_i = \frac{\Delta J}{\Delta t} = \frac{A\Gamma M_p}{r^{1/2}} F,$$

(7)

where

$$F \equiv \frac{m \xi}{\pi} e^{-(r-r_0)^2/\sigma^2} \frac{\sin (a\pi - \varphi) - \sin \varphi}{a^2 - 1}$$

and the variables $r_0$, $\varphi$, $\xi$, and $m$ are chosen randomly as described in § 3.

At any given time, the disk contains 50 modes and hence 50 values of the function $F$. Since these torques can be either positive or negative, the 50 modes will tend to cancel each other out and the net forcing effect will be relatively small. We thus need to compute the forcing function $F_{50}$ defined by

$$F_{50} \equiv \sum_{j=1}^{50} F_j,$$

(8)

where the $F_j$ are sampled in the same way that the numerical simulations sample parameter space. The function $F_{50}$ should average to zero (or some low value) because the torques can be positive or negative. However, the rms value of $F_{50}$, denoted here as $\langle F_{50} \rangle$, should provide a good measure for the effective strength of this torque. Numerically sampling the function shows that $\langle F_{50} \rangle \approx 0.29$. Putting these results together, we find that the average torque $\tau_\gamma$ exerted on the planet through the action of turbulence is given by

$$\tau_\gamma = \langle F_{50} \rangle \frac{A\Gamma M_p}{r^{1/2}} \approx 0.29 \frac{A\Gamma M_p}{r^{1/2}}.$$

(9)

To estimate the response amplitude $\Gamma$ of the disk (due to the potential induced by turbulent fluctuations), we use a simple linearized WKB treatment of a two-dimensional disk (Shu 1992). However, unlike the usual spiral density wave theory, we include only the forcing due to the fluctuation potential $\Phi$ and neglect the self-gravity of the disk. In the WKB limit, we can solve for the radial part of the reduced surface density perturbation $S$ in terms of the forcing potential $\Phi$ to obtain

$$S = \frac{k^2 \sigma_d \Phi}{(\omega - m\Omega)^2 - \kappa^2 - k^2 \sigma_d^2},$$

(10)

where $\sigma_d$ is the surface local disk surface density, $\omega = m\Omega_c$, and the epicyclic frequency $\kappa = \Omega$ for a Keplerian disk. Next, we need to use the relationship between the surface density perturbation $S$ and the corresponding gravitational potential $V$. In the WKB limit, this relation has been calculated previously (e.g., Shu 1992) and can be written in the form

$$S \approx - \frac{|k|V}{2\pi G}.$$

(11)

Equating the expressions for the surface density response to the turbulent fluctuations (eq. [10]) and the corresponding perturbation in the gravitational potential (eq. [11]), we can solve for the response factor $\Gamma$, i.e.,

$$\Gamma \approx \frac{V}{\Phi} \approx \frac{2\pi G\sigma_d k}{(\omega - m\Omega)^2 - \Omega^2 - k^2 \sigma_d^2}.$$

(12)

To evaluate this expression analytically, we must make further simplifying assumptions. The turbulent fluctuations induce modes with relatively large wavenumbers $m = 2 - 32$, so we can approximate the denominator as $|D| \sim m^2\Omega^2 = m^2GM_c/r^3$. Given the radial dependence of the turbulent
fluctuations (eq. [3]), we can estimate the radial wavenumber \( k \) in the WKB limit:

\[
kr = (r/\Phi)(\partial \Phi/\partial r) \sim 2r^2/\sigma^2 \sim 32m^2/\pi^2.
\]

The expression for \( \Gamma \) thus becomes

\[
\Gamma = \frac{\pi \sigma_d r^2 64}{M_* \pi^2} \approx 0.06.
\] (13)

Next, we need to compare the average torque produced through turbulent fluctuations with that produced by the usual type I migration mechanism (Ward 1997). This torque \( \tau_I \) can be written

\[
\tau_I = \beta_I (M_p/M_*)^2 (\pi \sigma_d^2 r^2 \nu^2)^{1/2} r^2 (r/\nu)^{3/2},
\] (14)

where \( H \) is the disk scale height, \( \sigma_d \) is the disk surface density, and \( \beta_I \) is a dimensionless amplitude. Previous two-dimensional calculations (appropriate to the analysis here) suggest that \( \beta_I \sim 10^{-2} \) (Ward 1997). Recent three-dimensional calculations by Tanaka et al. (2002) suggest that \( \beta_I \) may be smaller, enough to increase type I migration timescales by a factor of 2–3.

By equating \( \tau_I \) and \( \tau_T \), we can solve for the critical amplitude \( A_C \) for which the torques due to turbulence become larger than those of standard type I migration. We find

\[
A_C = \frac{3.4 \pi^2}{64} \beta_I (M_p/M_*)^2 (r/H)^3 \approx 1 \times 10^{-5},
\] (15)

where the numerical value is obtained using the same parameters as in the numerical simulations presented here (for a 1 \( M_\odot \) planet). This analytic estimate for the critical amplitude is in good agreement with that observed in the numerical simulations themselves, which show an empirically determined value of \( A_C \sim 1 \times 10^{-5} \) (see § 4).

Before leaving this section, we note that the torque produced by type I migration (eq. [14]) and that produced indirectly through turbulence (eq. [9]) scale differently with the mass of the planet. Small planetesimals, with masses \( m \ll M_\text{E} \), will be tossed around violently because of MHD turbulence. On the other hand, sufficiently large planets can be impervious to its effects (for sufficiently small amplitudes \( A \)). If we assume that the maximum fluctuation amplitude \( A_{\text{max}} \approx r^{1/2}(r/\nu)^2 \), then we can define a critical mass scale \( M_{\text{CP}} \) such that larger masses will always be dominated by type I migration torques. This mass scale is given by

\[
M_{\text{CP}}/M_* = (64/\pi^2 \beta_I) (r/H)^3 \approx 0.04.
\]

A body of this mass far exceeds the \( M \sim 0.001M_\odot \) required to open up a gap in the disk and experience type II migration (Goldreich & Tremaine 1980; Lin & Papaloizou 1993; Ward 1997). As a result, there is no regime of parameter space for which the magnitude of the type I migration torques is necessarily larger than the torques produced by MHD turbulence.

6. LONG-TERM EVOLUTION

Given our analytic description of the effects of turbulence, we can understand the possible range of migration behavior over longer time intervals. This analytic approach is necessary, as it is difficult for numerical simulations to remain stable over millions of orbits, the dynamical time frames of the migration process. As we discuss below, several complications arise in the long term.

First, even if the amplitude of the turbulent fluctuations is that required for the resulting torques to have the same amplitude as those of type I migration (i.e., if \( A = A_C \)), the two sources of torque have different accumulative effects. The type I migration torque \( \tau_I \) is a secular torque that consistently moves the planet in one direction. The net turbulent torque \( \tau_T \) can be either positive or negative and changes its value over a sampling time that we denote here as \( t_{\text{smp}} \). The changes in angular momentum accumulate in random walk fashion. The change in angular momentum over the integrated time interval \( t = N t_{\text{smp}} \) for the two contributions can thus be written in the form

\[
\langle \Delta J \rangle_I = N \tau_I t_{\text{smp}},
\]

\[
\Delta J_I = N \tau_I t_{\text{smp}}.
\] (16)

The sampling time \( t_{\text{smp}} \) is the time required for the disk to produce an independent realization of the turbulent torques. The lifetime of a mode with wavenumber \( m \) is \( \Delta t = 2\pi r/(m \nu) \), which can be written in the form \( \Delta t = (2\pi/\Omega) (r/m H) \). The first factor is the orbital period \( P_{\text{orb}} \). The second factor is of order unity: the disk generally has \( r/H \approx 10 \) and the wavenumber \( m \) lies in the range \( 2 \leq m \leq 32 \). Since the wavenumbers \( m \) are logarithmically spaced and the lowest \( m \) modes live the longest (and are near the peak of the power spectrum; see Fig. 2), the appropriate value of \( m \) to use in estimating the sampling time lies near the low end of the range. As a result, we expect \( t_{\text{smp}} \approx P_{\text{orb}} \). In order for the turbulent fluctuations to dominate over a specified time interval \( t_\text{f} \), the amplitude of the turbulent fluctuations must be larger than the critical value \( A_C \) by an additional factor, i.e.,

\[
A \geq \left( \frac{t_\text{f}}{P_{\text{orb}}} \right)^{1/2} A_C.
\] (17)

A sufficiently large amplitude can keep the turbulent fluctuations dominant over the entire lifetime of the disk. The expected disk lifetime \( t_{\text{disk}} \) (specifically, the time over which planetary migration takes place) is of order 1–3 Myr (Haisch et al. 2001), while expected timescales for type I migration of individual (Earth-mass) planetesimals are considerably shorter, of order \( 10^5 \) yr (Ward 1997). The orbit time \( P_{\text{orb}} \) is of order 1–3 yr. So this problem contains another critical amplitude, namely that required for the turbulent fluctuations to overwhelm type I migration torques over a \( 10^5 \) yr secular migration timescale. This second critical amplitude \( A_2 \) is estimated to be

\[
A_2 \approx \left( \frac{t_{\text{disk}}}{P_{\text{orb}}} \right)^{1/2} A_C \approx 0.003.
\] (18)

Our two-dimensional simulations suggest that a turbulent amplitude \( A \approx A_2 \approx 3 \times 10^{-3} \) will result in density fluctuations
of order $\log_{10} C_m \sim -2.0$ for the strongest azimuthal wave-numbers. These amplitudes are comparable to or slightly smaller than the amplitudes observed in three-dimensional MHD simulations (see Fig. 2). We also note that at global Fourier amplitudes $\log_{10} C_m > -2.0$, nonlinear mode-mode coupling begins to become important (see Laughlin et al. 1997, 1998) and our linear treatment will start to fail.

A useful benchmark is the number of steps $N$ (or, equivalently, the timescale $N P_{\text{orb}}$) required for the random walk process to completely change the angular momentum. In other words, we set $(\Delta J)_T = (GM, r)^{1/2}$ in equation (17) and solve for $N$ to find

$$N = \left( \frac{A}{r^{5/2} \Omega} \right)^2 2\pi F_{50} \Gamma. \quad (19)$$

Even for fluctuations with the critical amplitude $A = A_C$, this number is rather large: $N \sim 3 \times 10^5$, with a corresponding timescale of 300 Myr. On the other hand, for $A = A_1$, the required number of steps falls to $N \sim 3 \times 10^3$, with a timescale of $\sim 3000$ yr.

Another complication is the radial dependence of the critical amplitude $A_C$. As shown by equation (15), this ratio scales as $A_C \propto r^{3-3\epsilon/2}$, where $q$ is the power-law index of the disk temperature profile. Although the temperature will not be purely a power law, $q$ is nonetheless expected to lie in the range $\frac{1}{2} \leq q \leq \frac{3}{2}$ so that the index $3 - 3q/2 = 15/8 - 9/4 \approx 2$. As a result, the amplitude of the turbulent modes required to dominate the type I migration torques is a rapidly increasing function of radius $r$ (decreasing as the planet moves in). As the planet migrates inward, $A_C \propto r^2$ decreases and turbulence tends to dominate to an ever-greater extent. On the other hand, if MHD turbulence shuts down in the inner disk because of magnetic decoupling (e.g., Fromang et al. 2002) then standard type I migration is resumed.

7. DISCUSSION AND CONCLUSION

In this paper, we have generalized the scenario of type I planetary migration to include the effects of fluctuations produced by MHD turbulence. To characterize the turbulence itself, we have run three-dimensional MHD simulations to study the onset and character of the turbulence and used the results to specify the power spectrum of turbulent fluctuations. These perturbations were then parameterized and used in a second set of two-dimensional numerical simulations that study the migration phase itself. Finally, we have developed an analytic description of the turbulent fluctuations, the torques they produce, and their long-term effects on the migration process. We now summarize our specific result.

MHD turbulence produces a full spectrum of fluctuations that peaks at relatively low azimuthal wavenumbers $m \sim 1 - 6$ (Figs. 1 and 2). These fluctuations can be characterized by potential fluctuations of the form given by equation (3). The formulation developed here may be useful in many other astronomical applications because it allows the fluctuations produced by turbulence to be effectively modeled with a minimum of computational effort, and even allows for many results to be determined analytically (see § 5). The effects of turbulence on type I migration depend sensitively on the amplitude of the turbulent potential perturbations. Furthermore, we have found the critical threshold $A_C$ for this amplitude, using both two-dimensional numerical simulations of the migration process (Figs. 3 and 4) and an analytic treatment (eq. [15]). For perturbation amplitudes below this threshold ($A < A_C$), type I migration proceeds smoothly inward, as envisioned in the original scenario. For small but finite amplitudes, the turbulence leads to random fluctuations superposed on the smooth inward migration, but the general trend remains. If the amplitude exceeds the threshold ($A > A_C$), the perturbations due to turbulence produce torques that are stronger than those induced by the planetary wake and the evolution changes dramatically. Instead of displaying a smooth inward progression, the planet exhibits random walk behavior. The planet can even move outward when the turbulence amplitude is large enough.

The critical amplitudes are $A_C \approx 1 \times 10^{-5}$ for a 1 $M_\oplus$ planet and $A_C \approx 1 \times 10^{-3}$ for a 10 $M_\oplus$ planet. These critical amplitudes are found both in the two-dimensional numerical simulations of type I migration and by the analytic treatment of § 5. For comparison, the expected amplitudes for the perturbations are $A \sim Z \approx 10^{-3}$, comfortably larger than the critical amplitudes needed to change the migration scenario. As a result, we expect MHD turbulence to change the character of the type I migration process under typical disk conditions where the requirements for ideal MHD are met.

The main conclusion of this paper is that the typical migration torques in a circumstellar disk will often be dominated by the perturbations due to MHD turbulence rather than the steady planetary wakes that drive type I migration. As a result, type I migration will often display far richer behavior than has been considered previously. When MHD instabilities are robust, migration will generally proceed in a highly chaotic fashion, with an element of a random walk behavior, although the underlying conventional type I secular torque acts to drive the planet inward over the long term. The complications introduced by turbulence have two important implications: (1) the timescale for planetary migration can be quite different, either longer or shorter, when turbulence is present; and (2) the result of any given migration episode in a circumstellar disk cannot be described by a single outcome; instead, because of chaos and extreme sensitivity to initial conditions, the result of a migration episode must be described in terms of a distribution of possible outcomes. In particular, the timescale required for planets to migrate inward will display a full distribution of values.

This chaotic element, including the necessity of finding the full distribution of possible outcomes, has recently been emphasized for migration scenarios that involve planet scattering (Adams & Laughlin 2003). Other recent work (Nelson & Papaloizou 2003; Papaloizou & Nelson 2003; Winters et al. 2003) outlines the effects of turbulence and chaos for type II migration. Since this paper shows that type I migration can often be dominated by turbulence/chaos as well, nearly all possible mechanisms for planetary migration are expected to display chaotic behavior. The ubiquity of chaotic processes during the formation and early evolution of planetary systems certainly contributes to the startling diversity that is observed among the known extrasolar planets.

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