Superluminal neutrinos at the OPERA?

Robert B. Mann\textsuperscript{1} and Utpal Sarkar\textsuperscript{2,3}

\textsuperscript{1} Department of Physics \& Astronomy, University of Waterloo, Waterloo, ON N2L 3G1, Canada
\textsuperscript{2} Physical Research Laboratory, Ahmedabad 380009, India
\textsuperscript{3} McDonnell Center for the Space Sciences, Washington University in St. Louis, MO 63112, USA

We argue that the recent measurement of the neutrino velocity to be higher than the velocity of light could be due to one or more of several mechanisms: violation of Lorentz invariance, violation of the equivalence principle in the neutrino sector, or a form of dark energy originating from neutrino condensates. This result need not undermine special-relativistic foundational notions of causality. We suggest different possibilities for understanding the phenomenology of neutrino oscillations, Z-strahlung, pion decay kinematics and the consistency of this result with supernova 1987A data.

\textbf{Introduction}

Lorentz invariance has been foundational in formulating both the Standard Model of particle physics and general relativity. Remarkably precise experimental tests have been carried out to check its validity. One generally quantifies the accuracy of such tests by adding small Lorentz-invariance-violating terms to a conventional Lagrangian, with data from experiments subsequently used to set upper bounds on the coefficients of these terms. For example, modifying the square of the magnetic field yields a photon velocity $c_{\gamma}$ that differs from the maximum attainable velocity of a material body $c_{M}$\textsuperscript{1}. This perturbation breaks Lorentz invariance, preserving rotational and translational invariance in one frame (the preferred frame) but not in any other frame. It is common to interpret the preferred frame as that for which the cosmic microwave background is isotropic, in which case small anisotropies appear in laboratory experiments. High-precision spectroscopic experiments have set a bound of $|1 - c_{\gamma}^2/c_{M}^2| < 6 \times 10^{-22}$\textsuperscript{2}.

A Lorentz transformation of the coordinates for an observer moving at velocity $\vec{v}$ yields

$$\vec{p} = \gamma m \vec{v}; \quad \text{and} \quad E = \gamma m c^2,$$

for the energy and momentum of a body, where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor. The invariant momentum is defined as

$$p^2 = p_{\mu}p^{\mu} = \frac{E^2}{c^2} - |\vec{p}|^2 = m^2c^2,$$

so that

$$E^2 = |\vec{p}|^2c^2 + m^2c^4.$$  \hfill (3)

For any massive particle we thus require $v < c$, otherwise the Lorentz factor $\gamma$ becomes imaginary and the total energy cannot be positive definite. This contradicts the recent measurement of the muon-neutrino velocity with the OPERA detector at Gran Sasso of the CNGS $\nu_{\mu}$ beam from CERN\textsuperscript{3}. Furthermore, it is straightforward to show (using equation \cite{1}) that for a muon neutrino with mass around $m \sim 1$ eV and energy $E \sim 17$ GeV that $(c - v)/c \sim 10^{-20}$.

Should this result be confirmed by subsequent experiments, it will require a significant revision of the Standard Model. Constructing such a revision will be a major challenge, since any new model must be consistent with astrophysical bounds (from SN1987A\textsuperscript{4}) and neutrino oscillation experiments, as well as explain the OPERA data without undermining basic notions of cause and effect.

\textbf{Violation of Lorentz Invariance}

We argue here that the present measurement of the neutrino velocity could be consistent with causality if Lorentz invariance is violated for the interactions of the muon neutrinos. We regard this as the most conservative interpretation of the OPERA data in conjunction with other experiments. It retains the essential foundations of relativity, namely that the velocity of light (photons) is constant and the causal structure of spacetime is preserved. The validity of Lorentz invariance beyond the first generation of the Standard Model has never before been directly tested, and so its violation is not empirically ruled out\textsuperscript{5,6}. Violation of Lorentz invariance (VLI) has also been considered to evade the GZK cosmic ray cut off and explain the ultra high energy cosmic rays\textsuperscript{8}, taking the strong constraints\textsuperscript{9,10} on the parameters coming from studies of cosmic rays into consideration. Lorentz invariance violation has also been studied extensively in extensions of the standard model and constraints on various model parameters have been reviewed\textsuperscript{11}.

We follow the analysis of Coleman and Glashow\textsuperscript{9}, where they made a general construction of Lorentz non-invariant interactions of particles, keeping the gauge invariance intact. We introduce a Lorentz non-invariant interaction term for the muon neutrinos

$$i \frac{u^\dagger}{\sqrt{2}} \left[ D_0 - i \left( 1 - \frac{1}{2} \epsilon \right) \overline{\sigma} \cdot \vec{D} \right] u. \quad (4)$$

The parameter $\epsilon$ is a measure of violation of Lorentz invariance. This will modify the high energy behaviour of the muon neutrinos and modify its energy momentum...
relation. The renormalized Lorentz invariant propagator

$$i \, S_F(p) = \frac{i}{(p^2 - m^2 + m^2)A(p^2)},$$

(5)

for some function $A(p^2)$ normalized as $A(m^2) = 1$, will be modified to

$$i \, S_F(p) = \frac{i}{(p^2 - m^2 + m^2) + \frac{1}{2} \gamma \cdot \not{p} B(p^2)},$$

(6)

where we normalize the Lorentz non-invariant interaction as $B(m^2) = 1$.

The lowest order shift in the poles of the propagator then gives

$$p^2 = E^2 - |\vec{p}|^2 = m^2 + \epsilon |\vec{p}|^2.$$  

(7)

This implies that the Lorentz non-invariant contribution added a shift in the momentum, which will result in a shift in the maximum attainable velocity of the particle from the velocity of light $c$ (for our choice of units $c = 1$, which remains the maximum attainable velocity for all other particles except the muon neutrino) to a new value

$$c^2_{\nu_\mu} = (1 + \epsilon)c^2 = (1 + \epsilon)$$

(8)

The energy momentum relation then becomes

$$E^2 = |\vec{p}|^2 c^2_{\nu_\mu} + m^2 c^2_{\nu_\mu}.$$  

(9)

The muon neutrino mass has now shifted by a factor, $m_{\nu_\mu} = m/(1 + \epsilon)$, so that we still have $m_{\nu_\mu} c^2_{\nu_\mu} = m$ to be the rest mass of the muon neutrino. The Lorentz factor will also be shifted because of this Lorentz non-invariance, and the new Lorentz factor is given by

$$\gamma_{\nu_\mu} = \frac{1}{\sqrt{1 - v^2 / c^2_{\nu_\mu}}}.$$  

(10)

so that the mass energy relation becomes

$$E = \gamma_{\nu_\mu} m_{\nu_\mu} c^2_{\nu_\mu} = \gamma_{\nu_\mu} m$$  

(11)

and

$$\frac{(c_{\nu_\mu} - v)}{c_{\nu_\mu}} \approx \frac{m^2}{2E^2}.$$  

(12)

for very large $v$.

These modifications will now allow us to interpret the recent measurement of the neutrino velocity $v$ from the OPERA detector. The present result from OPERA gives

$$\frac{v - c}{c} = v - 1 = (2.48 \pm 0.28 \pm 0.3) \times 10^{-5}.$$  

(13)

However, in the presence of the Lorentz invariance violating interactions, the Lorentz factor that the muon neutrinos experience contains the factor

$$\frac{(c_{\nu_\mu} - v)}{c_{\nu_\mu}} \approx \frac{m^2}{2E^2} \approx 10^{-20}.$$  

(14)

This determines the amount of violation of Lorentz invariance, given by

$$\epsilon = \frac{c_{\nu_\mu}^2}{c^2} - 1 = \left[ \frac{(c_{\nu_\mu} - v)}{c} + \frac{(v - c)}{c} \right] \left( \frac{c_{\nu_\mu}}{c} + 1 \right) \approx 2 \left[ \frac{(c_{\nu_\mu} - v)}{c_{\nu_\mu}} + v - 1 \right]$$  

(15)

Since $v < c_{\nu_\mu}$, the present measurement of the muon neutrino velocity $v$ (with $v > c$) becomes consistent with the modified Lorentz factor and the muon neutrino remains time-like in the same context.

**Models for VLI**

In a recent article it has been pointed out that the models of VLI suffer from a serious constraint coming from the pion lifetime kinematics [13]. The amount of VLI required to explain the OPERA result is in contradiction with present data from accelerators and cosmic rays. We thus propose here a couple of scenarios that may give rise to an effective VLI for muon neutrinos that does not affect the pion lifetime. These models are based on the fact that the effective VLI originates from an interaction of the propagating neutrinos with the environment, so during a pion decay there is no VLI.

In the first solution we consider the possibility that the neutrinos interact with the background dark energy, which gives rise to an effective VLI. We consider the scenarios, in which neutrinos form condensates after they acquire masses and that explains the observed dark energy at present times [14]. This background neutrino condensate dark energy can, in principle, affect the dynamics of the neutrinos, compared to other particles. For example, a $\nu_\mu$ with momentum $p$ can collide with a condensate $\overline{\nu}_\mu - \nu_\mu$ pair and bind with the $\overline{\nu}_\mu$. The liberated $\nu_\mu$, located at a distance $x$ away from its condensate partner, will continue with momentum $p$ due to momentum conservation. As this process is repeated, the net effect is that the $\nu_\mu$ "hops" through the condensate at an effective speed greater than unity, resulting in a different maximum attainable velocity for the muon neutrinos. Only the maximum attainable velocity of the neutrinos can be affected by this mechanism. This yields an interesting solution to the OPERA result, because the velocity of the muon neutrino becomes larger than the velocity of light when the muon neutrinos interact with the background dark energy. It evades constraints due to $Z$-strahlung radiation ($\nu_\mu \rightarrow \nu_\mu + Z \rightarrow \nu_\mu + e^+ + e^-$) [12] since the $\nu_\mu$ does not actually travel faster than the speed of light as it moves through the condensate. The strong constraints coming from observed pion decays [13] and cosmic rays are also not applicable, whereas these constraints can cripple many of the conventional models of VLI.

We now turn to the second solution for an effective VLI, which is through a violation of the equivalence principle (VEP), considered previously as a mechanism for
neutrino oscillations [13]. An equivalence of this type of VEP and an effective VLI at the phenomenological level has already been demonstrated [16], although their origin is completely different. The violation of equivalence principle will introduce a shift in the momentum in a constant gravitational potential if the neutrinos couple to gravity with a different coupling, which can be treated as equivalent to a shift in the maximum attainable velocity of the neutrinos. This mechanism satisfies the $Z$-strahlung constraints [12] because neutrinos effectively follow subluminal geodesics of a different metric than other particles due to VEP, and so do not undergo the intense Z-strahlung effect (see figure 1). This proposal is also unaffected by the pion decay constraint, because the neutrino velocity becomes large only when it propagates in a background gravitational field.

Without going into the details of the mechanism that gives us the violation of equivalence principle, we just assume that a background gravitational potential $\phi$ changes the metric $g_{44} = (1+\alpha\phi)$, which in turn gives a correction to the energy of the test neutrinos

$$E_0 = p + \frac{m^2}{2p} - \alpha\phi p,$$

where $\alpha$ could be different for different materials, in violation of the weak equivalence principle, and is a measure of the amount of VEP. From this correction it is clear that $\alpha\phi c$ may be considered as the change in the maximum attainable velocity of the neutrinos giving rise to an effective VLI.

Neutrino Phenomenology

Turning to the phenomenology of our proposal, the simplest scenario to explain neutrino oscillation data with an effective Lorentz invariance violation (EVLI) is to assume that all the three neutrinos have the same maximum attainable velocity, $c_{\nu_1} = c_{\nu_2} = c_{\nu_3} = c$, and there is no energy dependence. These assumptions are consistent with MINOS [17], the short baseline experiments [18], and also the present observation at OPERA in the two energy bins of 13 GeV and 42.9 GeV, but fail to explain the supernovae SN1987A bound [4] of $|1 - c_{\nu_1}/c| < 10^{-9}$, where $\bar{\nu}_e$ was observed. If we further assume that the there is no EVLI for any other particles, there are no other phenomenological constraints. We shall now consider a few possibilities to explain the supernovae bound along with the neutrino oscillation constraints.

- We consider that the neutrino condensates changes the maximum attainable velocities of all the three neutrinos by same amount, so there is no constraint from neutrino oscillation experiments. However, during their propagation through intergalactic medium, they experience a new source of VEP, which slows them down and they satisfy the supernovae bound.

- The VEP in the dark matter background changes because they interact in a different manner compared to interaction of VEP with ordinary matter. During the propagation of the neutrinos on Earth, they experience the effect of VEP in the background of ordinary matter, which gives an EVLI that can explain the OPERA result. But during the propagation of the neutrinos from supernovae, they experience VEP in the dark matter background, which makes the neutrinos slower explaining the supernovae bound.

- This solution is similar to that of reference [19], where a light sterile neutrino is introduced, but the model is similar to that of reference [14]. We assume the existence of four neutrino states, $\nu_1, \ldots, \nu_4$, three of which have the same maximum attainable velocity similar to other particles $c_{\nu_1} = c_{\nu_2} = c_{\nu_3} = c = 1$, with the fourth having $c_{\nu_4} \approx (1 + 2.5 \times 10^{-5})c$, because the $\nu_4$ condensates explains the dark energy. This fourth neutrino is a combination of $\nu_\mu$ and a sterile neutrino $\nu_4$: $\nu_4 = \nu_\mu + \nu_\mu^\ast$; and has negligible mixing with $\nu_e$. After production, muon neutrinos will propagate partly as the fourth physical state $\nu_4$ because its wave function has a large component of $\nu_4$. As a result the velocity of the muon neutrino will appear to be similar to $c_{\nu_4}$, explaining the observation at OPERA and MINOS. However neutrinos emitted from supernova SN1987A travel mostly as $\nu_1$, with only a small fraction traveling as $\nu_4$. Consequently the required constraints from the SN1987A data are respected.

Comments

It is crucial to repeat the OPERA experiment. Should its findings be confirmed its implications for the foundation of physics will be profound. We have proposed what is perhaps the minimal change in this regard: VLI or VEP implemented in such a way that all neutrino species have the same effective maximal attainable velocity. From this perspective the photon moves at a speed...
slower than this maximum. However this cannot be due to a slightly massive photon. Current upper bounds on the photon mass range from $10^{-14}$ eV to $10^{-24}$ eV \cite{2}. Taking the more stringent bound of $m_\gamma \sim 10^{-24}$, then from $E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$, we have from $E \sim 17$ GeV, $v_\gamma - c_{\text{max}} \sim 10^{-68}$, so $c_\mu < c_{\text{max}}$ cannot explain the OPERA result. The slower photon speed could be due to an aether whose motion is so tightly correlated with that of the earth that the stringent anisotropy constraints of $c_\phi/c < 10^{-15}$ are evaded \cite{21}. This is not unreasonable if the aether is the condensate dark energy mentioned above, since (apart from neutrinos) the earth is therefore composed of particles that respond to the aether exactly as the photon does. Yet another possibility is that the universe is governed by a bi-metric theory, with all particles coupling to one metric except for neutrinos, that only neutrinos can approach. The models we have proposed satisfy constraints from neutrino oscillation, SN1987A, and pion decay kinematics, Z-Strahlung, and are consistent with OPERA/MINOS data. Whether or not this class of models survives future empirical scrutiny remains to be seen.

**Summary**

To summarize, we have pointed out that the OPERA data can be consistent with all other experiments if there is a maximal speed larger than the speed of light that only neutrinos can approach. The models we have proposed satisfy constraints from neutrino oscillation, SN1987A, and pion decay kinematics, Z-Strahlung, and are consistent with OPERA/MINOS data. Whether or not this class of models survives future empirical scrutiny remains to be seen.

**Note Added** Many interesting works followed the announcement of the OPERA result, we mention here a few of them \cite{22}.

**Acknowledgements** This work was supported in part by the Natural Sciences and Engineering Research Council of Canada. U. Sarkar would like to thank Prof. R. Cowsik, Director, McDonnell Center for the Space Sciences, Washington University in St. Louis, for arranging his visit as the Clark Way Harrison visiting professor.

[1] M.P. Hagen and C.M. Will, Phys. Today 40, 69 (1987); Y. Grossman, C. Kilic, J. Thaler, and D.G.E. Walker, Phys. Rev. D 72, 125001 (2005).
[2] S. K. Lamoreaux, J. P. Jacobs, B. R. Heckel, F. J. Raab, and E. N. Fortson, Phys. Rev. Lett. 57, 3125 (1986); C.J. Berghold, L.R. Hunter, D. Krause, Jr., E.O. Frigge, M.S. Ronfeldt, and S.K. Lamoreaux, Phys. Rev. Lett. 75, 1879 (1995).
[3] OPERA Collaboration: T. Adam, et al, arXiv:1109.4897v1 [hep-ex].
[4] K. Hirata, et al, Phys. Rev. Lett. 58, 1490 (1987); R.M. Bionta, et al, Phys. Rev. Lett. 58, 1494 (1987); M.J. Longo, Phys. Rev. D 36, 3276 (1987).
[5] T. Hambye, R. B. Mann, U. Sarkar, Phys. Rev. D58, 025003 (1998). hep-ph/9804307.
[6] T. Hambye, R. B. Mann, U. Sarkar, Phys. Lett. B421, 105 (1998). hep-ph/9709350.
[7] V.A. Kostelecky, Phys. Rev. Lett. 80, 1818 (1998); Phys. Rev. D 61, 016002 (1999); R. Bluhm, V.A. Kostelecky, and C.D. Lane, Phys. Rev. Lett. 84, 1098 (2000).
[8] S. Coleman and S.L. Glashow, hep-ph/9808446 (1998); F.W. Stecker and S.T. Scully, Astroparticle Physics 23, 203 (2005); L. Maccione, A.M. Taylor, D.M. Mattingly, and S. Liberati, JCAP 0904, 022 (2009).
[9] S. Coleman and S.L. Glashow, Phys. Rev. D 59, 116008 (1999).
[10] S. Coleman and S.L. Glashow, Phys. Lett. B 405, 249 (1997); R. Cowsik and B.V. Sreekantan, Phys. Lett. B 449, 219 (1999); X.-J. Bi, P.-F. Yin, Z.-H. Yu, and Q. Yuan, arXiv:1109.6661 [hep-ph] (2011).
[11] V.A. Kostelecky and N. Russel, Rev. Mod. Phys. 83, 11 (2011).
[12] A. Cohen and S. Glashow, arXiv: 1109.6562 [hep-ph] (2011).
[13] R. Cowsik, S. Nussinov and U. Sarkar, arXiv:1110.0241 [hep-ph] (2011).
[14] J.R. Bhatt, B. Desai, E. Ma, G. Rajasekaran and U. Sarkar, Phys. Lett. B 687, 75 (2010).
[15] M. Gasperini, Phys. Rev. D 38, 2635 (1988); M. Gasperini, Phys. Rev. D 39, 3606 (1989); J.T. Pantaleone, A. Halprin, and C.N. Leung, Phys. Rev. D 47, 4199 (1993). [hep-ph/9211214]; J.R. Mureika and R.B. Mann, Phys.Rev. D54, 2761 (1996); R. B. Mann, U. Sarkar, Phys. Rev. Lett. 76, 865 (1996). hep-ph/9505253.
[16] A. Halprin and H.B. Kim, Phys. Lett. B 469, 78 (1999).
[17] MINOS Collaboration: P. Adamson, et al, Phys. Rev. D 76, 072005 (2007).
[18] G.R. Kalbfleisch, N. Baggett, E.C. Fowler, J. Alspector, Phys. Rev. Lett. 43, 1361 (1979); J. Alspector, et al, Phys. Rev. Lett. 36, 837 (1976).
[19] S. Hannestad and M.S. Sloth, arXiv:1009.6282 [hep-ph] (2011); I.Ya. Aref’eva, and I.V. Volovich, arXiv:1110.0456 [hep-ph] (2011); H. Pas, S. Pakvasa, T.J. Weiler, Phys. Rev. D 72, 095017 (2005).
[20] A. S. Goldhaber, M. M. Nieto, Rev. Mod. Phys. 82, 939 (2010). arXiv:0809.1003 [hep-ph].
[21] H. Muller, S. Herrmann, C. Braxmaier, M. Schiller, and A. Peters, Phys. Rev. Lett. 91, 020401 (2003).
[22] J. Alexandre, J. Ellis and N.E. Mavromatos, arXiv:1109.6296 [hep-ph] (2011); G.F. Giudice, S. Sibiryakov, and A. Strumia, arXiv:1109.5682 [hep-ph] (2011); R. A. Konoplya, arXiv:1109.6215 [hep-th] (2011); C. Pfeifer, M. N. R. Wohlfarth, arXiv:1109.6005 [gr-qc] (2011); N.D. Haridass, arXiv:1110.0351 [hep-ph] (2011); T. Li and D.V. Nanopoulos, arXiv:1110.0451 [hep-ph] (2011); H. Gilles, arXiv:1110.0239 [hep-ph] (2011); G. Amelino-Camelia, G. Gubitosi, N. Loret, F. Mercati, G. Rosati, and P. Lipari, arXiv:1109.5172 [hep-ph] (2011); P. Wang, H. Wu, and H. Yang, arXiv:1110.0449 [hep-ph] (2011); arXiv:1109.6930 [hep-ph] (2011).