Higher dimensional slowly rotating dilaton black holes in AdS spacetime

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In this paper, with an appropriate combination of three Liouville-type dilaton potentials, we obtain the higher dimensional charged slowly rotating dilaton black hole solution for asymptotically anti-de Sitter spacetime. The angular momentum and the gyromagnetic ratio of such a black hole are determined for the arbitrary values of the dilaton coupling constant. It is shown that the dilaton field modifies the gyromagnetic ratio of the rotating dilaton black holes.

I. INTRODUCTION

Over the past years there has been a growing interest for studying the rotating black hole solutions in the background of anti-de Sitter (AdS) spacetimes. This interest is motivated by the correspondence between the gravitating fields in an AdS spacetime and conformal field theory on the boundary of the AdS spacetime \cite{1}. It was argued that the thermodynamics of black holes in AdS spaces can be identified with that of a certain dual conformal field theory (CFT) in the high temperature limit \cite{2}. In the AdS/CFT correspondence, the rotating black holes in AdS space are dual to certain CFTs in a rotating space \cite{3}, while charged ones are dual to CFTs with chemical potential \cite{4}. The most general higher dimensional uncharged rotating black holes in AdS space have been recently found \cite{3, 5}. As far as we know, rotating black holes for the Maxwell field minimally coupled to Einstein gravity in higher dimensions, do not exist in a closed form and one has to rely on perturbative or numerical methods to construct them in the background of asymptotically flat \cite{6, 7} and AdS \cite{8} spacetimes. There has also been recent interest in constructing the analogous charged rotating solutions in the framework of gauged supergravity in various dimensions \cite{9, 10}.

On another front, scalar coupled black hole solutions with different asymptotic spacetime structure is a subject of interest for a long time. There has been a renewed interest in such studies ever since new black hole solutions have been found in the context of string theory. The low energy effective action of string theory contains two massless scalars namely dilaton and axion. The dilat-
ton field couples in a nontrivial way to other fields such as gauge fields and results into interesting solutions for the background spacetime \cite{11,12}. These scalar coupled black hole solutions \cite{11,12}, however, are all asymptotically flat. It was argued that with the exception of a pure cosmological constant, no dilaton-de Sitter or anti-de Sitter black hole solution exists with the presence of only one Liouville-type dilaton potential \cite{13}. In the presence of one or two Liouville-type potentials, black hole spacetimes which are neither asymptotically flat nor (A)dS have been explored by many authors (see e.g. \cite{14,15,16,17}). Recently, the “cosmological constant term” in the dilaton gravity has been found by Gao and Zhang \cite{18,19}. With an appropriate combination of three Liouville-type dilaton potentials, they obtained the static dilaton black hole solutions which are asymptotically (A)dS in four and higher dimensions. The motivations for studying such dilaton black holes with nonvanishing cosmological constant originate from supergravity theory. Gauged supergravity theories in various dimensions are obtained with negative cosmological constant in a supersymmetric theory. In such a scenario AdS spacetime constitutes the vacuum state and the black hole solution in such a spacetime becomes an important area to study \cite{1}.

In the light of all mentioned above, it becomes obvious that further study on the rotating black hole solutions in a spacetime with nonzero cosmological constant in the presence of dilaton-electromagnetic coupling is of great importance. The properties of charged rotating dilaton black holes, for an arbitrary dilaton coupling constant, in the small angular momentum limit in four \cite{20,21} and higher dimensions have been studied \cite{22,23}. Recently, in the presence of a Liouville-type dilaton potential, one of us has constructed a class of charged slowly rotating dilaton black hole solutions in arbitrary dimensions \cite{24}. Unfortunately, these solutions \cite{24} are neither asymptotically flat nor (A)dS. Besides, they are ill-defined for the string case where $\alpha = 1$. More recently, a class of slowly rotating charged dilaton black hole solutions in four-dimensional anti-de Sitter spacetime has been found \cite{25}. Until now, higher dimensional charged rotating dilaton black hole solutions for an arbitrary dilaton-electromagnetic coupling constant in the background of anti-de Sitter spacetime have not been constructed. Our aim in this paper is to construct a higher dimensional charged rotating dilaton black hole solution for asymptotically AdS spacetime in the small angular momentum limit with an appropriate combination of three Liouville-type dilaton potentials. We then determine the angular momentum and the gyromagnetic ratio of such a black hole for the arbitrary values of the dilaton coupling constant. We will restrict ourselves to the rotation in one plane, so our black hole has only one angular momentum parameter.
II. FIELD EQUATIONS AND SOLUTIONS

We consider the $n$-dimensional ($n \geq 4$) theory in which gravity is coupled to the dilaton and Maxwell field with an action

$$S = -\frac{1}{16\pi} \int_{\mathcal{M}} d^n x \sqrt{-g} \left( R - \frac{4}{n-2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - e^{-4\alpha \Phi/(n-2)} F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^{n-1} x \sqrt{-h} \Theta(h), \quad (1)$$

where $R$ is the scalar curvature, $\Phi$ is the dilaton field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and $A_\mu$ is the electromagnetic potential. $\alpha$ is an arbitrary constant governing the strength of the coupling between the dilaton and the Maxwell field. The last term in Eq. (1) is the Gibbons-Hawking surface term. It is required for the variational principle to be well defined. The factor $\Theta$ represents the trace of the extrinsic curvature for the boundary $\partial \mathcal{M}$ and $h$ is the induced metric on the boundary. While $\alpha = 0$ corresponds to the usual Einstein-Maxwell-scalar theory, $\alpha = 1$ indicates the dilaton-electromagnetic coupling that appears in the low energy string action in Einstein’s frame. For an arbitrary value of $\alpha$ in AdS space the form of the dilaton potential in arbitrary dimensions is chosen as \[19\]

$$V(\Phi) = \frac{\Lambda}{3(n-3+\alpha^2)^2} \left[ -\alpha^2(n-2)(n^2-n\alpha^2-6n+\alpha^2+9) e^{[-4(\alpha^2+\alpha^2)\Phi/(n-2)\alpha]} + (n-2)(n-3)^2(n-1-\alpha^2)e^{4\alpha \Phi/(n-2)} + 4\alpha^2(n-3)(n-2)^2 e^{[-2\Phi/(n-3-\alpha^2)/(n-2)\alpha]} \right]. \quad (2)$$

Here $\Lambda$ is the cosmological constant. It is clear the cosmological constant is coupled to the dilaton in a very nontrivial way. This type of dilaton potential can be obtained when a higher dimensional theory is compactified to four dimensions, including various supergravity models \[26\]. In the absence of the dilaton field the action (1) reduces to the action of Einstein-Maxwell gravity with cosmological constant. Varying the action (1) with respect to the gravitational field $g_{\mu\nu}$, the dilaton field $\Phi$ and the gauge field $A_\mu$, yields

$$R_{\mu\nu} = \frac{4}{n-2} \left( \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{4} g_{\mu\nu} V(\Phi) \right) + 2e^{-4\alpha \Phi/(n-2)} \left( F_{\mu\eta} F^\eta_\nu - \frac{1}{2(n-2)} g_{\mu\nu} F_{\lambda\eta} F^{\lambda\eta} \right), \quad (3)$$

$$\nabla^2 \Phi = \frac{n-2}{8} \frac{\partial V}{\partial \Phi} - \frac{\alpha}{2} e^{-4\alpha \Phi/(n-2)} F_{\lambda\eta} F^{\lambda\eta}, \quad (4)$$

$$\partial_\mu \left( \sqrt{-g} e^{-4\alpha \Phi/(n-2)} F^{\mu\nu} \right) = 0. \quad (5)$$

We would like to find $n$-dimensional rotating solutions of the above field equations. For small rotation, we can solve Eqs. (3)-(5) to first order in the angular momentum parameter $a$. Inspection
of the $n$-dimensional Kerr solutions shows that the only term in the metric that changes to the first order of the angular momentum parameter $a$ is $g_{t\phi}$. Similarly, the dilaton field does not change to $O(a)$ and $A_\phi$ is the only component of the vector potential that changes. Therefore, for infinitesimal angular momentum we assume the metric being of the following form

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{W(r)} - 2af(r)\sin^2 \theta dt d\phi$$

$$+ r^2R^2(r) \left( d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega^2_{n-4}\right),$$

where $d\Omega^2_{n-4}$ denotes the metric of a unit $(n - 4)$-sphere. The functions $U(r)$, $W(r)$, $R(r)$ and $f(r)$ should be determined. In the particular case $a = 0$, this metric reduces to the static and spherically symmetric cases. For small $a$, we can expect to have solutions with $U(r)$ and $W(r)$ still functions of $r$ alone. The $t$ component of the Maxwell equations can be integrated immediately to give

$$F_{tr} = \sqrt{U(r)} \frac{Q e^{4a\Phi/(n-2)}}{(rR)^{n-2}}.$$ 

where $Q$, an integration constant, is the electric charge of the black hole. In general, in the presence of rotation, there is also a vector potential in the form

$$A_\phi = -aQC(r)\sin^2 \theta.$$ 

The asymptotically (A)dS static ($a = 0$) black hole solution of the above field equations was found in [19]. Here we are looking for the asymptotically (A)dS solution in the case $a \neq 0$. Our strategy for obtaining the solution is the perturbative method suggested by Horne and Horowitz [20]. Inserting the metric (6), the Maxwell fields (7) and (8) into the field equations (3)-(5), one can show that the static part of the metric leads to the following solutions [19]:

$$U(r) = \left[ 1 - \left(\frac{r}{r_+}\right)^{n-3} \right] \left[ 1 - \left(\frac{r}{r_-}\right)^{n-3} \right]^{1 - \gamma(n-3)} - \frac{1}{3}Ar^2 \left[ 1 - \left(\frac{r}{r_-}\right)^{n-3} \right]^{\gamma},$$

$$W(r) = \left\{ \left[ 1 - \left(\frac{r}{r_+}\right)^{n-3} \right] \left[ 1 - \left(\frac{r}{r_-}\right)^{n-3} \right]^{1 - \gamma(n-3)} - \frac{1}{3}Ar^2 \left[ 1 - \left(\frac{r}{r_-}\right)^{n-3} \right]^{\gamma} \right\}$$

$$\times \left[ 1 - \left(\frac{r}{r_-}\right)^{n-3} \right]^{\gamma(n-4)},$$

$$\Phi(r) = \frac{n-2}{4} \sqrt{\gamma(2 + 3\gamma - n\gamma)} \ln \left[ 1 - \left(\frac{r}{r_-}\right)^{n-3} \right]^{\gamma},$$

$$R(r) = \left[ 1 - \left(\frac{r}{r_-}\right)^{n-3} \right]^{\gamma/2},$$
while we obtain the following solution for the rotating part of the metric

\[
f(r) = \frac{2\Lambda r^2}{(n-1)(n-2)} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^\gamma + (n-3) \left( \frac{r_+}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{-\alpha^2/n+\alpha^2} \\
+ \frac{(\alpha^2 - n + 1)(n-3)^2}{\alpha^2 + n-3} r_-^{n-3} r \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^\gamma \times \int \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{(2-n)} \frac{dr}{r^n},
\]

(13)

\[
C(r) = \frac{1}{r^{n-3}}.
\]

(14)

We can also perform the integration and express the solution in terms of the hypergeometric function

\[
f(r) = \frac{2\Lambda r^2}{(n-1)(n-2)} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^\gamma + (n-3) \left( \frac{r_+}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{-\alpha^2/n+\alpha^2} \\
+ \frac{(\alpha^2 - n + 1)(n-3)^2}{(1-n)(\alpha^2 + n-3)} \left( \frac{r_-}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^\gamma \\
\times \_2F_1 \left( \left[ (n-2)\gamma, \frac{n-1}{n-3} \right], \left[ \frac{2n-4}{n-3} \right], \frac{r_-}{r} \right)^{n-3}.
\]

(15)

Here \( r_+ \) and \( r_- \) are, respectively, the event horizon and Cauchy horizon of the black hole, and the constant \( \gamma \) is

\[
\gamma = \frac{2\alpha^2}{(n-3)(n-3 + \alpha^2)}.
\]

(16)

The charge \( Q \) is related to \( r_+ \) and \( r_- \) by

\[
Q^2 = \frac{(n-2)(n-3)^2}{2(n-3 + \alpha^2)} r_+^{n-3} r_-^{n-3},
\]

(17)

and the physical mass of the black hole is obtained as follows [27]

\[
M = \frac{\Omega_{n-2}}{16\pi} \left[ (n-2)r_+^{n-3} + \frac{n-2 - p(n-4)}{p+1} r_-^{n-3} \right],
\]

(18)

where we have ignored the term of the order of \( a^2 \) in the mass expression for the anti-de Sitter dilatonic black hole [25, 28]. Here \( \Omega_{n-2} \) denotes the area of the unit \( (n-2) \)-sphere and the constant \( p \) is

\[
p = \frac{(2-n)\gamma}{(n-2)\gamma - 2}.
\]

(19)

It is apparent that the metric corresponding to (9)-(15) is asymptotically (A)dS. For \( \Lambda = 0 \), the above solutions recover our previous results for asymptotically flat rotating dilaton black holes [28].

In the special case \( n = 4 \), the static part of our solution reduces to

\[
U(r) = W(r) = \left( 1 - \frac{r_+}{r} \right) \left( 1 - \frac{r_-}{r} \right)^{(1-\alpha^2)/(1+\alpha^2)} - \frac{1}{3} \Lambda r^2 \left( 1 - \frac{r_-}{r} \right)^{2\alpha^2/(1+\alpha^2)},
\]

(20)
\[ \Phi (r) = \frac{\alpha}{\alpha^2 + 1} \ln \left( 1 - \frac{r_-}{r} \right), \]  
\[ (21) \]

\[ R (r) = \left( 1 - \frac{r_-}{r} \right)^{\alpha^2/(1+\alpha^2)}, \]  
\[ (22) \]

while the rotating part reduces to

\[ f (r) = - \left( 1 - \frac{r_-}{r} \right)^{(1-\alpha^2)/(1+\alpha^2)} \left( 1 + \frac{(1+\alpha^2)^2}{(1-\alpha^2)(1-3\alpha^2)r_+^2} + \frac{(1+\alpha^2)r_- - r_+}{r} \right) \]
\[ + \frac{r^2(1+\alpha^2)^2}{(1-\alpha^2)(1-3\alpha^2)r_+^2} \left( 1 - \frac{r_-}{r} \right)^{2\alpha^2/(1+\alpha^2)} + \frac{1}{3} \Lambda r^2 \left( 1 - \frac{r_-}{r} \right)^{2\alpha^2/(1+\alpha^2)}, \]  
\[ (23) \]

which is the four-dimensional asymptotically (A)dS charged slowly rotating dilaton black hole solution presented in [25]. One may also note that in the absence of a nontrivial dilaton \( \alpha = 0 = \gamma \), our solutions reduce to

\[ U (r) = W (r) = \left[ 1 - \left( \frac{r_+}{r} \right)^{n-3} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right] - \frac{1}{3} \Lambda r^2, \]  
\[ (24) \]

\[ f (r) = (n-3) \left[ \frac{r_-^{n-3} + r_+^{n-3}}{r^{n-3}} - \left( \frac{r_+ r_-}{r^2} \right)^{n-3} \right] + \frac{2 \Lambda r^2}{(n-1)(n-2)}, \]  
\[ (25) \]

which describe the \( n \)-dimensional charged Kerr-(A)dS black hole in the limit of slow rotation.

Next, we calculate the angular momentum and the gyromagnetic ratio of these rotating dilaton black holes which appear in the limit of slow rotation parameter. The angular momentum of the dilaton black hole can be calculated through the use of the quasilocal formalism of Brown and York [29]. According to the quasilocal formalism, the quantities can be constructed from the information that exists on the boundary of a gravitating system alone. Such quasilocal quantities will represent information about the spacetime contained within the system boundary, just like the Gauss’s law. In our case the finite stress-energy tensor can be written as

\[ T^{ab} = \frac{1}{8\pi} \left( \Theta^{ab} - \Theta h^{ab} \right), \]  
\[ (26) \]

which is obtained by variation of the action \[I\] with respect to the boundary metric \( h_{ab} \). To compute the angular momentum of the spacetime, one should choose a spacelike surface \( \mathcal{B} \) in \( \partial \mathcal{M} \) with metric \( \sigma_{ij} \), and write the boundary metric in ADM form

\[ \gamma_{ab} dx^a dx^b = -N^2 dt^2 + \sigma_{ij} \left( d\phi^i + V^i dt \right) \left( d\phi^j + V^j dt \right), \]

where the coordinates \( \phi^i \) are the angular variables parametrizing the hypersurface of constant \( r \) around the origin, and \( N \) and \( V^i \) are the lapse and shift functions, respectively. When there is a
Killing vector field $\xi$ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (26) can be written as

$$Q(\xi) = \int_B d^{n-2} \varphi \sqrt{\sigma} T_{ab} n^a \xi^b,$$

(27)

where $\sigma$ is the determinant of the metric $\sigma_{ij}$, $\xi$ and $n^a$ are, respectively, the Killing vector field and the unit normal vector on the boundary $B$. For boundaries with rotational ($\varsigma = \partial / \partial \varphi$) Killing vector field, we can write the corresponding quasilocal angular momentum as follows

$$J = \int_B d^{n-2} \varphi \sqrt{\sigma} T_{ab} n^a \varsigma^b,$$

(28)

provided the surface $B$ contains the orbits of $\varsigma$. Finally, the angular momentum of the black holes can be calculated by using Eq. (28). We find

$$J = \frac{a \Omega_{n-2}}{8\pi} \left( r_+^{n-3} + \frac{(n-3)(n-1-\alpha^2)r_-^{n-3}}{(n-3+\alpha^2)(n-1)} \right).$$

(29)

For $a = 0$, the angular momentum vanishes, and therefore $a$ is the rotational parameter of the dilaton black hole. For $n = 4$, the angular momentum reduces to

$$J = \frac{a}{2} \left( r_+ + \frac{3 - \alpha^2}{3(1 + \alpha^2)} r_- \right),$$

(30)

which restores the angular momentum of the four-dimensional Horne and Horowitz solution [20].

Finally, we calculate the gyromagnetic ratio of this rotating dilaton black hole. As we know, the gyromagnetic ratio is an important characteristic of the Kerr-Newman-AdS black hole. Indeed, one of the remarkable facts about a Kerr-Newman black hole in asymptotically flat spacetime is that it can be assigned a gyromagnetic ratio $g = 2$, just as an electron in the Dirac theory. It should be noted that, unlike four dimensions, the value of the gyromagnetic ratio is not universal in higher dimensions [28]. Besides, scalar fields such as the dilaton, modify the value of the gyromagnetic ratio of the black hole and consequently it does not possess the gyromagnetic ratio $g = 2$ of the Kerr-Newman black hole [20]. Here, we wish to calculate the value of the gyromagnetic ratio when the dilatonic black hole has an asymptotic AdS behavior. The magnetic dipole moment for this asymptotically AdS slowly rotating dilaton black hole can be defined as

$$\mu = Qa.$$

(31)

The gyromagnetic ratio is defined as a constant of proportionality in the equation for the magnetic dipole moment

$$\mu = g \frac{QJ}{2M}.$$

(32)
Substituting $M$ and $J$ from Eqs. (18) and (29), the gyromagnetic ratio $g$ can be obtained as

$$g = \frac{(n-1)(n-2)[(n-3+\alpha^2)r_-^{n-3} + (n-3-\alpha^2)r_-^{n-3}]}{(n-1)(n-3+\alpha^2)r_+^{n-3} + (n-3)(n-1-\alpha^2)r_+^{n-3}}. \quad (33)$$

One can see that in the linear approximation in the rotation parameter $\alpha$, the above expression for $g$ turns out to be same as that found in [23] for the asymptotically flat slowly rotating dilaton black hole. This means that the dilaton potential (cosmological constant term) does not change the gyromagnetic ratio of the rotating (A)dS dilaton black holes, as discussed in [25]. However, the dilaton field modifies the value of the gyromagnetic ratio $g$ through the coupling parameter $\alpha$ which measures the strength of the dilaton-electromagnetic coupling. This is in agreement with the arguments in [20]. We have shown the behavior of the gyromagnetic ratio $g$ of the dilatonic black hole versus $\alpha$ in Fig. 1. From this figure we find out that the gyromagnetic ratio decreases with increasing $\alpha$ in any dimension. In the absence of a nontrivial dilaton ($\alpha = 0 = \gamma$), the gyromagnetic ratio reduces to

$$g = n-2, \quad (34)$$

which is the gyromagnetic ratio of the $n$-dimensional Kerr-Newman black hole with a single angular momentum in the limit of slow rotation [7]. When $n = 4$, it reduces to

$$g = 2 - \frac{4\alpha^2 r_-}{(3-\alpha^2)r_- + 3(1+\alpha^2)r_+}, \quad (35)$$

which is the gyromagnetic ratio of the four-dimensional slowly rotating dilaton black hole [20, 25].
III. SUMMARY AND CONCLUSION

It is well known that in the presence of one Liouville-type dilaton potential, no de Sitter or anti-de Sitter dilaton black hole exists even in the absence of rotation \[13\]. In this paper, with an appropriate combination of three Liouville-type dilaton potential proposed in \[19\], we showed that such potential leads to higher dimensional slowly rotating charged dilaton black holes solutions in an anti-de Sitter spacetime. The presence of such an AdS dilatonic charged rotating black hole is inevitably associated with an accompanying scalar field with appropriate Liouville-type potential. Their study therefore may lead to a better understanding of the origin of the dark matter in the universe. We started from the nonrotating charged dilaton black hole solutions in anti-de Sitter spacetime \[19\] and then successfully obtained the solution for the rotating charged dilaton black hole in higher dimensions by introducing a small angular momentum and solving the equations of motion up to the linear order of the angular momentum parameter. We discarded any terms involving \(a^2\) or higher powers in \(a\) where \(a\) is the rotation parameter. For small rotation, the only term in the metric which changes is \(g_{t\phi}\). The vector potential is chosen to have a nonradial component \(A_\phi = -aQC(r)\sin^2 \theta\) to represent the magnetic field due to the rotation of the black hole. As expected, our solution \(f(r)\) reduces to the Ghosh and SenGupta solution for \(n = 4\), while in the absence of the dilaton field \((\alpha = 0 = \gamma)\), it reduces to the \(n\)-dimensional slowly rotating Kerr-Newman-AdS black hole. We calculated the angular momentum \(J\) and the gyromagnetic ratio \(g\) which appear up to the linear order of the angular momentum parameter \(a\). Interestingly enough, we found that the dilaton field modifies the value of the gyromagnetic ratio \(g\) through the coupling parameter \(\alpha\) which measures the strength of the dilaton-electromagnetic coupling. This is in agreement with the arguments in \[20\].

Finally, we would like to mention that in this paper we only considered the higher dimensional charged slowly rotating black hole solutions with a single rotation parameter in the background of AdS spacetime. In general, in more than three spatial dimensions, black holes can rotate in different orthogonal planes, so the general solution has several angular momentum parameters. Indeed, an \(n\)-dimensional black hole can have \(N = [(n - 1)/2]\) independent rotation parameters, associated with \(N\) orthogonal planes of rotation where \(\lfloor x \rfloor\) denotes the integer part of \(x\). The generalization of the present work to the case with more than one rotation parameter and arbitrary dilaton coupling constant is now under investigation and will be addressed elsewhere.
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