Collision-dominated dust sheaths and voids—observations in micro-gravity experiments and numerical investigation of the force balance relations

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Abstract. Numerical solutions of stationary force balance equations are used to investigate the possible dust configurations (dust structures) in complex plasmas between two floating potential plane electrodes. The distance between electrodes is assumed to be larger than the ion–neutral mean free path and the hydrodynamic description is used. It includes the known forces operating in this limit, the ionization source and the dust charge variations. The stationary balance equations are solved both in the case of the presence of one-size dust grains and for the case of a mixture of grains with two different sizes. Recent micro-gravity experiments with single-size dust grains and two-different-size dust grains show the formation of a system of dust sheaths and dust voids between the two plane electrodes. The observed configurations of dust structures depend strongly on the gas pressure and the degree of ionization used. The numerical investigations are able to show the necessary conditions for the types of structure to be created and give their size. The size of the structures observed is larger than the ion–neutral mean free path and is of the order of magnitude of that obtained numerically. The numerical investigations give details of the spatial distributions, the dust particles, the electron/ion densities, the ion drift velocity and dust charges inside and outside different dust structures. These details have not yet been investigated experimentally and can indicate directions for further experimental work to be performed. The single-dust-sheath structure with single-size dust particles surrounded by dust free regions (dust wall-voids) and floating potential electrodes...
is computed. Such a structure was observed recently and the computational results are in agreement with observations. It is shown that more often a dust void in the centre is observed. It is found that a dust void in the centre region between two electrodes is formed if the ionization rate is larger than the critical ionization rate and that in the presence of the floating potential walls the central void should be surrounded by two dust sheaths. The necessary condition for this dust structure to be formed is found to be that between the sheaths and the walls there are formed two other wall-void regions. The size of the central void and the distributions of the structure parameters in the two sheaths and in the three voids are computed. The qualitative features of the structure obtained in the numerical computations correspond to those observed. The distributions of the structure parameters in the case of the two dust sheaths are quite different from that for the case of a single central sheath. The possible structures between the electrodes for the case of the presence of dust particles of two different sizes are analysed numerically. It is shown that dust particles with different sizes cannot coexist in equilibrium at the same position and that the regions with different size dust particles must be separated in space. This conclusion is in agreement with most observations performed so far. It is illustrated numerically that for the case where the central void is present the dust particles of larger size form a separate dust sheath which should be located at larger distances from the centre than that for the smaller dust particles. This result also coincides qualitatively with the observations. Computations for the distributions of the parameters in the larger size dust sheath were performed both in the case where the central part is occupied by a dust sheath with smaller size dust particles and for the case where in the central part there exists a dust void surrounded by dust sheaths with smaller size dust particles. The size of the dust void between the sheaths with different size dust particles is calculated and shown to be small as compared to the sheath thickness. In the sheath with larger size dust particles the distribution of dust and plasma parameters differs qualitatively from that of the first dust sheath with smaller size dust particles. The stability of the stationary structures both with respect to excitation of dust convection cells and with respect to oscillations of dust void size is discussed.
Preface

This article was begun in 2001 when the first experimental indications came that the dust distributions in structures with two-different-size grains can be very unusual with spatial separation of different-size grains. Additional computations in July 2002 were made when the results of the first micro-gravity experiments on the Space Station were available. The recent experimental results showing the possibility of closing the central void in the structure observed in the Space Station experiment require a more detailed investigation of the necessary conditions for existence of a collision-dominated dust sheath in the absence of the central void. These numerical results were obtained at the end of 2002 and the beginning of 2003 and are added to the present final version of the paper.

1. Introduction

Theoretically it is expected that dusty plasmas have universal properties to form structures [1, 2], with dust clumps separated by dust-free regions (dust voids). This structuring of dusty plasmas has been observed in most macro-gravity and laboratory experiments performed so far. The term ‘dust structure’ is used below for the different configuration of mixtures of the dust voids and the dust clumps (which correspond to dust sheaths in plane geometry). The formation of the dust structures is simpler in the micro-gravity experiments since the gravity force is absent, but still the finite geometry of the experiments introduces a certain asymmetry which seems to be an important factor in formation of the dust structures. In the presence of the walls, which is inevitable in any real experiments, there appears an ion flux toward the walls (which are usually charged negatively if they are at the floating potential conditions). This effect changes the structure formation.

The aim of the present investigations is to obtain the form of the stationary dust structures in plane geometry of finite size where the dusty plasma exists between the two floating potential electrodes. From the previous investigations of the nonlinear dusty structures [3]–[6] it is known that the structures can be either collision-less with size much less than the ion–neutral mean free
path or collision dominated with the size much larger than the ion–neutral mean free path. In numerical analysis in the present paper we will assume that the distance between the electrodes $L$ is much larger than the ion–neutral mean free path $\lambda_{in}$ and moreover that it is larger than or of the order of the $\lambda_{st} = \lambda_{in} T_e/T_i = \lambda_{in}/\tau$ where $T_e$ and $T_i$ are the electron and ion temperatures respectively. In the existing experiments $T_i/T_e \equiv \tau \ll 1$, therefore $\lambda_{st} \gg \lambda_{in}$ and the inequality $L \geq \lambda_{st}$ is well fulfilled. The reason for introducing the distance $\lambda_{st}$ is that it is expected to be of the order of the structure size (of the order of either the void or the sheath size). The numerical investigation previously performed for the collision-dominated dust voids [6] shows that the size of the void calculated was of the order of $\lambda_{st}$. We will prove that the characteristic size is indeed also of the order of $\lambda_{st}$ for most of the collision-dominated structures for finite geometry between two floating potential electrodes. This is the case for the structures calculated numerically such as plane sheaths, central voids surrounded by sheaths and further by wall-voids between the sheaths and the walls. Therefore, $\lambda_{st}$ can be named the characteristic size of structures, and we have therefore denoted it with the subscript $st$.

Having defined the numerical problem of computations we should demonstrate some recent observations of dust structures in micro-gravity conditions observed between two plane electrodes. These electrodes naturally have a finite size in the plane of the electrodes. The 1D computations which are performed here with infinite size plane electrodes can describe only approximately these experiments for those parts of the structures which are approximately flat. In experiments such parts of the structures really exist although all structures are more complicated due to the edge effects (see figure 1—the void and convection structures obtained in the first micro-gravity parabolic flight experiment [7]—and figure 2—the big central void structure surrounded by convection vortices obtained in the first Space Station micro-gravity experiment [8]). The recent efforts in extending the PKE–Nefedov experiments [8] to remove the central void by changing the external parameters succeeded in obtaining a dust sheath at the centre with only wall-voids present (figure 3). The edge effect is observed in all examples shown. They can cause the formation of vortices (dust convection cells). These cells can also be created by the low frequency instabilities of the structures. The term low frequency is used in the sense that the time of cell formation is much longer that the time of the quasi-stationary structure formation, which satisfies the condition of the stationary balance of forces. In the computations we will assume that such nonlinear stationary structures can be reached in the time evolution of the structuring instability leaving for the future the investigation of their stability, which can cause not only the formation of dust convective cells but also the ‘breathing’ of the structures. Such breathing (a slow periodic change of dust structure size) is well known in nonlinear physics. The breathing structures were also observed in micro-gravity experiments and called the ‘heartbeat modes’ (in 3D geometry the void size changes in time resembling the beating of the heart). We will not consider the problems of the stability of the structures obtained here, investigating only the conditions of the stationary balance of forces together with dust charging. From the approach used here we will be able to obtain several important features of the observed dust structures. Note that the convection instability was previously considered theoretically in [9], but its modification for the finite geometry which is treated here remains a problem for future consideration. The method used in [6] for the movement of the void boundary can also be used in future investigations of the stability problems of the plane sheath structures considered here.

The observations shown in figures 1–3 indicate that under micro-gravity conditions different types of dust structure can be created including different dust sheaths and different dust voids if the plasma is surrounded by plane or other shape floating potential electrodes. The results
of another important experiment [8] are shown in figure 4. These experiments were performed for two different-size dust particles injected in the space between electrodes. They show an interesting phenomenon of separation of dust particles with different size and formation of more complicated sequences of dust sheaths and dust void structures. The aim of the present simplified plane geometry investigation is to show that such a phenomenon of separation of different-size dust particles and formation of a separate dust sheaths for different-size grains is a natural phenomenon and is indeed obtained in numerical computations. In particular, for the computed void structures, the dust particles with larger size are located at larger distances from the centre as really observed.
Figure 3. The single dust sheath observed in PKE–Nefedov experiments [8]. The scales are the same as in figure 2 and only half of the sheath is shown.

Figure 4. The two-size dust particle micro-gravity experiments [8]. The void and the two sheaths with different-size dust particles are separated by the interface, which appears as a thin dust void. The vortices at the edges between the plane electrodes are apparently interacting with other observed structures pressing them to the central region.

Recently it was found experimentally [8] that in micro-gravity conditions one can tune the parameters determining the structure to observe the existence of a single dust sheath in the centre of the region between the two plane electrodes. This type of structure is shown to exist in the numerical computation model. The necessary physical requirements for such a structure to be created are found. In the present paper the 1D geometry is used in numerical computations and
Figure 5. Sketch of a dust sheath between the electrodes. The arrows indicate the direction of the ion flow velocity to the surfaces of the voids. The voids at the wall always contain the plane where the direction of the ion flow changes its sign.

This configuration is shown schematically in figure 5. The figures show only a part of the plane structure between the electrodes; the plane considered theoretically is infinite and the edge effect is not taken into account.

More often in the experiments in the central region between electrodes a dust void (figures 1, 2, 4) is created. In the plane geometry used in the present computations this configuration is sketched in figure 6. We find the necessary physical requirements for void appearance for the central void creation and demonstrate that the central void should be surrounded by the two dust sheaths and further by two wall-voids between the sheaths and the electrodes. For the case where the experiments are performed with two-size dust particles [8] the location of particles with different size was different and the sheaths of different-size dust were separated by dust voids. In particular for void structure the dust particles with a smaller size were located at larger distances from the centre.

In the present paper we will deal only with the case when the role of gravity forces is negligible and will only analyse numerically the nonlinear stationary balance equations, thus assuming that such stationary balance is achieved. The stability of the stationary structures is still not analysed. The structures obtained numerically have many features resembling the observed structures. This includes the conversion of a single-sheath structure to a void structure surrounded by dust sheaths with an increase of the ionization rate, separation of dust sheaths of different-size dust particles etc. Therefore there is a possibility to use the analyses of the nonlinear balance equation to predict the configuration of other dust structures which may be obtained in future micro-gravity experiments. The model based on the balance equation is shown to explain qualitatively the observations. It can also predict more complex structures to be created as well as predicting the condition for their experimental detection. We will consider both the
case of one-size dust particles and two-size dust particles and will explain in physical terms why the dust particles of different sizes will be located at different positions for the force balance conditions to be satisfied. The system is assumed to be symmetric to the position called the centre of the system which is the plane in the middle between the electrodes. Only the floating potential boundary conditions will be considered at the electrodes. The aim of the consideration of the force balance equations for different-size dust particles is also to find the critical ionization rate where at the centre of the system the dust sheath will not satisfy the force balance conditions and therefore at the centre of the system a dust void region is expected to be formed.

2. Basic concept of nonlinear force balance and global balance condition

We will analyse only the balance equations in the collision dominated regime where the characteristic size of the structure is much larger than the mean free path for ion–neutral atom collisions and we will assume that the dust size is much less than both the Debye length and the ion–neutral atom collision mean free path. The corresponding system of nonlinear equations for plane geometry is given in the appendix. It takes into account the pressure and the electric field force in the balance equation for electrons, the drag and the electric field force in the balance equation for dust of different sizes, the momentum transfer force in the ion–dust collisions,

**Figure 6.** Sketch of a central void surrounded by two dust sheaths and the voids at the wall. The arrows indicate the direction of the ion flow to the surfaces of the sheaths.
the nonlinear friction force due to ion–neutral collisions, the electric field force in the balance
equation for ions, the diffusion of ions due to collisions with neutral atoms and the ionization
in the ion continuity equation, the change in the dust charges due to the ion and electron density
changes in the dust charging equation and deviations from the complete local quasi-neutrality
in the Poisson equation. The corresponding equations are given in the appendix both for the
dust regions and dust-void regions. In the latter case only the ion and electron equation together
with the Poisson equation are operating. Thus the balance equations are quite general for the
case where the ion–neutral collisions are dominating and for the case where the forces are
described using their hydrodynamic expressions (without a more detailed kinetic description). It
is therefore assumed that the ions are described by drifting thermal distributions and the change
of the ion drift velocity distribution is also calculated for all structures. The limit
\[ \tau = T_i / T_e \ll 1 \]
is used in the expressions for the dependence of the drag and charging coefficients on the ion drift
velocity. The drag coefficient is determined in this limit only by Coulomb ion–dust collisions
(taking into account the collective dust effects and large angle scattering in the expression for
the Coulomb logarithm). The condition \( \tau \ll 1 \) is fulfilled in most experiments. The ion drift
velocity \( u_i \) will be normalized to the ion thermal velocity \( v_{Ti} = \sqrt{T_i/m_i} \) (\( m_i \) is the ion mass) by
introducing the dimensionless ion drift velocity \( u \)
\[ u = \frac{u_i}{\sqrt{2v_{Ti}}} . \]

We will demonstrate later for all the structures considered here that in the collision-
dominated regime the value of \( u \) can reach unity but does not much exceed it. Nevertheless
we will use an exact expression for the ion drag force and the dust charging coefficient as
functions of arbitrary \( u \). We denote the drag coefficient as \( \alpha_{dr}(u) \) and the charging coefficient
as \( \alpha_{ch}(u) \). They depend only on the ion drift velocity. An explicit analytic expression for them
(through the error function) in the limit \( \tau \ll 1 \) is used through all calculations. These expressions
are given in the appendix. The dependence of these coefficients on ion drift velocity is important
for \( u \) reaching a value close to or larger than unity. Numerical calculations show that the latter
happens for all types of structure considered here and the existing experiments also indicate that
\( u \) is of the order of or somewhat larger than unity. At the electrodes due to the floating boundary
conditions (Bohm criteria) \( u \gg 1 \), but as we show the dust is absent close to the wall and in the
description of dust free regions (voids) the drag and the charging coefficients do not enter the
balance equations. We keep in the calculations the general expressions for the dependence of
the drag coefficient and the charging coefficient on the ion drift velocity.

For the multi-sheath structures the nonlinear balance equations are solved numerically
separately in the dust regions and in the void regions and are joined at the void boundaries using
the boundary conditions of continuity of the electron and the ion densities, continuity of the
ion drift velocity and continuity of the electric fields. Finally the boundary floating potential
conditions are satisfied at the wall boundaries (electrodes). The balance equations start to be
solved from \( x = 0 \), the centre of the distance between the electrodes, and the spatial dependences
of plasma and dust characteristics, such as ion density \( n_i \), electron density \( n_e \), dust density \( n_d \),
dust charge \( Z_d \), ion drift velocity \( u \) and electrostatic field strength \( E \), are examined as functions
of the distance \( x \) from the centre. The dimensionless variables
\[ n \equiv \frac{n_i}{n_{cr}} , \quad n_e \equiv \frac{n_e}{n_{cr}} ; \quad P \equiv \frac{n_d Z_d}{n_{cr}} ; \quad E \equiv \frac{Ee}{T_i} ; \quad z \equiv \frac{Z_de^2}{aT_e} . \]
are used, where \( \lambda_{\text{in}} \) is the mean free path for ion–neutral collisions and

\[
    n_{\text{cr}} = \frac{\tau^2 T_e}{4\pi e^2 \lambda_{\text{in}} a}.
\]

The value of \( n_{\text{cr}} \) is of the order of \((10^8–10^9)\) cm\(^{-3}\) for the existing experiments and therefore the normalized densities \( n, n_e \) should be of the order of unity.

The distance will be normalized to \( \lambda_{\text{in}}/\tau = \lambda_{\text{st}} \)

\[
    x \equiv \frac{x}{\lambda_{\text{in}}/\tau}.
\]

In laboratory experiments \( \tau \approx 0.02, 0.04 \) and the characteristic distances \( \lambda_{\text{in}}/\tau \) are much larger then the ion–neutral mean free path. It is of the order of the structure size observed in the experiments. This coincidence is not occasional. It follows from the balance equation which needs such normalization (this can be easily found by writing the balance equations in the dimensionless form—see the appendix). The numerical computation of the dimensionless balance equations supports the statement that the normalized distance is of the order of unity (sometimes less than unity). The analysis will show the dependences of dust structures on three main dimensionless parameters:

1. \( d = \sqrt{\frac{a}{\lambda_{\text{in}}}} \)

   where \( a \) is the dust diameter and \( \lambda_{\text{in}} \) is the mean free path for ion–neutral atom collisions;

2. \( x_i = \frac{\lambda_{\text{ion}} \tau}{\lambda_{\text{in}}} \)

   where \( 1/\lambda_{\text{ion}} \) is the ionization length and \( 1/x_i \) is the number of electron–ion pairs created by the ionization source per length \( \lambda_{\text{i,n}}/\tau, \tau = T_i/T_e. \)

3. The central ion density \( n(0) \). The central electron density \( n_e(0) \) is also another parameter. The constraint of the global balance limits the possible range of the parameters and specially the value of \( n_e(0) \). The change in \( x_i \) converts the type of structure with a dust sheath in the centre to the type of structure with a dust void in the centre.

An important aspect of the problem is the applicability of the collision-dominated limit which is determined by the condition that the characteristic size of the structure is larger than the ion–neutral mean free path. The hydrodynamic approach used here for the description of the collision-dominated regime can have intrinsic properties related to an increase of the spatial derivatives reaching the point where it start to be not valid (the characteristic size becomes of the order of the mean free path). We stop the numerical calculations at the point where the collision-dominated regime starts to be not valid and will discuss different possibilities for further investigation. Thus we will be able to compare with experiments the obtained single-dust-sheath structures, the single-central-void structure and the separation of two dust sheaths with different-size dust particles but not the thickness of the second sheath.

The intrinsic nonlinear properties of the balance equation leading at a certain point to violation of the hydrodynamic approach are a new type of hydrodynamic singularity related to the dust charging process and indicating the start of dust convection. Thus we give here a
qualitative description and a definition of the singular points. The ion drift velocity is regulated by the dust drag and neutral friction. Most important is the dependence of the drag force on the ion drift velocity and the dependence of the charging coefficient on the ion drift velocity. These forces are described by the drag coefficient $\alpha_{dr}(u)$ and $\alpha_{ch}(u)$, for which an explicit expression is given in the appendix in the limit $\tau = T_i/T_i \ll 1$. The force balance changes with the distance and the ion drift velocity $u$ changes with the distance. A change of the ion drift will create both changes in the drag and charging processes. The drag is regulating the ion drift velocity through the ion balance condition. The space derivative of the dust charge $z$, $dz/dx$, will contain a term proportional to the space derivative of the ion drift velocity, $du/dx$, and also the derivative of the ion drift velocity will contain a term proportional to the derivative of the dust charge. The system of equations for these derivatives can be resolved only if the determinant for this system is not zero. The expression for the latter $R(u, z)$

$$R(u, v) = \frac{u}{\alpha_{dr}(u)} \frac{d\alpha_{dr}(u)}{du} \left( \frac{1}{z + 1} \frac{u}{\alpha_{ch}(u)} \frac{d\alpha_{ch}(u)}{du} \right)$$

is derived in the appendix. The singular point corresponds to $R = 0$. For distances close to this point, say $x_0$ where $u = u_0$, $z = z_0$, we will have $(u - u_0)^2$, $(z - z_0)^2 \propto (x - x_0)$ and if, on one side of this point, say $x < x_0$, $(u - u_0)^2$, $(z - z_0)^2$ is positive, then on the other side of the point, $x > x_0$, $(u - u_0)^2$, $(z - z_0)^2$ is negative and the balance for $x > x_0$ cannot be satisfied. The absence of zeros of $R$ can be checked numerically for any stationary structure found by the solution of the nonlinear balance equations for different phase space domains of the parameters (1)–(3). If the zero exists presumably an instability will not allow us to reach the balance from the initial state. It is also possible that for $x > x_0$ the balance will be determined by the collision-less relations (with respect to ion–neutral collisions but not with respect to the ion–dust collisions). But the most important possibility is that at the points close to $R = 0$ the convection of dust converts the 1D structure to a 2D structure. The dissipative processes related to diffusion of ions due to ion–neutral collisions are carried out through all balance relations and it is shown that they do not avoid the appearance of the singularity. The dissipative process related to delay in charging is operating in the collision-less domain. In the present investigation we can answer only the questions of existence or non-existence of a stationary force balance state in the collision-dominated domain.

An important parameter is the total number of dust particles per unit surface area confined in nonlinear structures $N_d$. The units of distance in calculation of the area are the dimensionless length introduced above and the dust dimensionless density is $n_d = P/z$. $N_d$ is defined by the expression

$$N_d = \int \frac{P}{z} \, dx.$$  

The value of $N_d$ is calculated for all types of structure. If the dust surface density per cm$^2$ is denoted as $n_d^s$, then $n_d^s = \tau N/4\pi a^2$ and for given $N$ depends on the dust size $a$ and $\tau$.

### 3. Structures with a dust sheath at the centre

#### 3.1. One-size dust particles forming a single dust sheath

We start by showing the results of the calculations demonstrating the possibility of the existence of a dust sheath at the centre of the region separated by two plane electrodes with floating potential
conditions (zero net current). These calculations are performed to illustrate the experimentally found structure with a sheath at the centre of the system. The calculations are started from the centre \((x = 0)\), assuming a symmetry in \(x\), which allows us to consider half of the space between the electrodes \((x > 0)\). In the numerical calculations the central ion density can be used as a free parameter which finally determines the distance from the centre up to the electrode (in units of ion–neutral atom mean free path divided by \(\tau\)). The distance from the centre to the electrode is fixed in experiments and the central ion density should be determined by the distance between the electrodes. Here we invert the problem, fix in the calculations the value of the ion central density and find from the balance equations the distance between the electrodes. Since this distance is dependent on the ion–neutral atom mean free path (or in other words on gas pressure) one can say that the results given below illustrate the case of a certain gas pressure where the position of the electrode coincides with that numerically calculated. We take as an example the normalized value of ion density to be \(n(0) = 2.5, \tau = 0.05\). The central electron density was chosen from the condition that the centre the total flux coincides with the convection flux (when the drag force directed to the centre is largest): \(n_e(0) = 1.024\). The ion drift velocity at the centre \(u \rightarrow u_0, x \rightarrow 0\) is negative with \(u_0 = -0.219\). Then at the centre the value of the dimensionless dust charge is \(z(0) = 2.317\) and the parameter \(P\) at the centre is \(P_0 = n(0) - 2/\alpha \mu(0) z(0) = 1.503\). The ionization rate we chose to be \(x_i = 2\) and the parameter \(d = 0.15\). These parameters satisfy all necessary conditions for not having a dust void at the centre of the structure and the conditions for optimization of the structure [10] (see the relations (45)–(47) of [10]—they are also valid for the 1D case, not relations (40), (41) of [10] where for the 1D case a factor of three should be added in the right-hand side). Namely, the requirements are
\[
x_i > x_i^\text{cr} = 2\sqrt{\pi} \frac{n_e(0)}{n(0)(n(0) - n_e^\text{cr})}; \quad n_e > n_e^\text{cr} = \frac{3\sqrt{\pi}}{z(0) \ln \Lambda}
\]
(10)

where \(\ln \Lambda\) is the Coulomb logarithm which takes into account both the collective effects and large angle scattering. For the parameters we used above \(x_i^\text{cr} = 0.963 < x_i = 2, n_e^\text{cr} = 0.997 < n_e(0) = 1.024\).

The first result of numerical calculations is that the structure found numerically is qualitatively the same as that observed in experiments [8]. It can be found from the balance equations and satisfies all necessary boundary conditions. The calculations in the dust region joined by the void-wall region show that the floating conditions at the electrode can be satisfied only for the size of the dust sheath less than \(x_{vw}\) (the subscript \(vw\) is used to indicate the position of the wall-void to start). The position of the dust-void from the centre is found to be \(x_{vw} = 0.561\) and the position of the plane electrode from the centre is found to be \(x_w = 0.8529\). Also \(x_v = 0.6\) was found which corresponds in the dust sheath to the point where \(P = 0\) is reached. \(x_{vw} = 0.561 < x_v = 0.6\) but the difference between \(x_v\) and \(x_{vw}\) is small, 0.039, which indicates that the dust sheath occupies almost the whole maximum possible space allowed for dust to occupy by the balance equation. At \(x = x_{vw}\) the parameter \(P\) jumps from its value \(P_{vw} = 0.68\) for \(x < x_{vw}\) to \(P = 0\) for \(x > x_{vw}\). Thus the dust wall-void has a sharp boundary and the dust sheath occupies 66.1% of the distance between the electrodes. The dimensionless number of dust particles per unit square surface is \(N_d = 0.2973\). This value corresponds to several thousand dust grains in the sheath with the surface of several cm\(^2\). The number of dust grains per cm\(^2\) varies mainly with dust size \((\alpha 1//\alpha^2)\) but the dimensionless \(N\) also vary with gas pressure (ion–neutral atom mean free path), with ionization rate and with \(\tau\) until these parameters are

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in the range where such a type of structure (dust sheath without a void in the centre) can exist. In the central part of the dust sheath the ion drift velocity is small and \( u \ll 1 \). In the outer part of the dust sheath it can reach a value of the order of unity. Figure 7(a) shows the ion drift velocity and the total flux (convective and diffusion) in the central inner part of the dust sheath at \( x < 0.3 \) where \( u \ll 1 \) and figure 7(b) shows the parameter \( P \) in the central part of the structure. The separation of the central part of the sheath from the outer part is dictated only by numerical computations which allows us to use in the central part asymptotic expressions for \( \alpha_{dr} \) and \( \alpha_{ch} \) for \( u \to 0 \)—the exact analytical expressions for dependence of these coefficients on \( u \) cannot be used since for \( u \to 0 \) these expressions need a numerical evaluation of \((\to 0/\to 0)^3\). One can see from figures 7(a) and (b) that the ion drift velocity increases in absolute value, being always directed to the centre of the structure. The variation of the parameter \( P \) is not large in absolute value although these changes become more pronounced in the other computations where the initial starting parameters are different (particularly for larger central ion density).

The distributions of the parameters \( P \), the ion density \( n \), the dust charge \( z \), the dust density \( n_d = P/z \), the total flux \( \Phi \) and the ion drift velocity \( u \) in the dust sheath are shown in figures 7(c) and (d) for all distances both larger than \( x = 0.3 \) (outer part of the sheath) and less than \( x = 0.3 \) (inner part of the sheath). In the outer part \( x \) varies from 0.3 up to the point where the dust density \( n_d \to 0 \) and \( P = 0 \) or until \( x = x_c \). These computations for the outer part are joined with the distributions for \( x < 0.3 \) from the previous computations (shown in figures 7(a) and (b)). It can be seen that at the edge of the sheath the ion drift velocity reaches a value close to \(-1\) and that in the inner part the total flux \( \Phi \) practically coincides with the convection flux \( nu \), while at the edge of the sheath the contribution of the diffusion flux becomes substantial, which is obviously related to an increase of the ion density derivative (see figure 7(d)). Since this derivative is negative the diffusion flux is oriented outwards and partially compensates the convective flux \( nu \), still leaving the total flux as negative, i.e. directed inwards to the sheath centre. The dust charge \( z \) and the electron density \( n_e \) increase toward the edge of the dust sheath. Notice that \( x_{vw} = 0.561 \) is less than \( x_c = 0.6 \) corresponding to \( P = 0 \) and therefore the last part \( x_{vw} < x < x_w \) shown in figures 7(a)–(d) can correspond to the case where at the wall the boundary conditions are not the floating potential conditions. For floating potential conditions at the wall the dust sheath is terminated at \( x = x_{vw} = 0.561 \).

Figure 7(e) contains the information of possible crystallization of dust in the sheath. This can occur if the parameter proportional to the coupling constant \( \Gamma \) (being inversely proportional to the dust temperature) becomes larger than a certain critical value \( \Gamma_{cr} \). We plot the dimensionless parameter \( \Gamma = z^{5/3} P^{1/3} \) which is the only one varying inside the sheath (the coefficient relating this \( \Gamma \) to a conventional one depends on the dust size, on dust temperature and on \( n_d \)). We demonstrate by the results shown in figure 7(e) that the dimensionless \( \Gamma \) is almost constant in the sheath (at least at \( x < x_{vw} \)) and therefore we can conclude that the crystallization can occur (when the dust temperature is decreased) almost simultaneously in the whole sheath. The results presented in figure 7(e) also indicate that, if the crystallization occurs, it occupies almost the whole sheath leaving only a thin outer part of the dust layer, probably in a dust liquid state.

Figure 7(f) gives the change of parameters \( nu \) (the convective flux), \( u \) (ion drift velocity), \( \Phi \) (the total flux) and \( \Phi_{th} = \Phi - nu \) (the diffusion flux) in the wall-void region from \( x > x_{vw} \) up to the electrode \( x = x_w \) where the floating boundary conditions are satisfied. It illustrated that at certain point in the void region the flux and the drift velocity change their sign and start to be directed to the wall (electrode). These dependences are obtained for the set of initial parameters chosen above and illustrate the possibility of existence of a smooth dust sheath with maximum of
the ion and the dust densities located at the centre of the system. The number of dust grains \( N_d \) given above corresponds to the maximum possible dust grains which can be confined in the dust sheath (if one injects more dust grains these will be not ‘accepted’ by the sheath structure and will fall probably on the electrodes). If \( x_{vw} \) was chosen to be less than the number found above, \( 0.561 \)—then \( N_d \) and \( x_w \) are decreasing (for example for \( x_{vw} = 0.54, N_d = 0.290, x_w = 0.687; \) since \( x \) is measured in terms of the ion–neutral mean free path this decrease of \( N_d \) corresponds to an increase of the gas pressure).

An investigation was performed to find the range of parameters where such smooth dust sheaths can exist. The calculations were performed for changes of the parameters \( d \) and \( x_i \). The ion density at the centre was unchanged and was the same as in the previous calculations \( n(0) = 2.5 \) to find explicitly the dependence of the structures on the two other parameters \( d_j \) and \( x_i \).

Figures 8(a)–(c) show the dependences of initial sheath parameters (at the centre of the sheath) on the \( x_i \) and the \( d \) obtained from the condition that the flux close to the centre of the sheath is convective (see [10]) and from the condition that the drift velocity in the region close to the centre of the sheath is directed to the centre of the sheath (of course both of them vanish at the centre).

The change of the sheath structures with change of the parameter \( d \) are found to be the following. A large increase of the parameter \( d \) is not possible due to the starting assumption \( d \ll 1 \). An increase of the parameter \( d \) up to 0.5 (which is the limit of possible increase of the parameter \( d \)), with the parameter \( x_i \) unchanged, increases the size of the dust region up to \( x_{vw} = 0.712 \) and the wall size up to \( x_w = 0.993 \) with the dust occupying 71.7% of the space between the electrodes. The dimensionless number of dust particles per unit square area then increases up to \( N_d = 0.378 \). The parameter \( d \) is proportional to the square root of the gas pressure. Therefore, an increase of the volume occupied by dust corresponds to an increase of the gas pressure keeping the normalized central ion density unchanged. If, on the other hand, we start to decrease the parameter \( d \) without changing the parameter \( x_i \) we find that for \( d = 0.03 \) the dust sheath will still exist and has the parameters \( x_{vw} = 0.6931, x_w = 0.7692 \); the dust structure thus occupies 90% of the space between the electrodes, while \( N_d = 0.5938 \).

A further decrease of the parameter \( d \) to \( d = 0.025 \) will lead to a qualitative change of the dependence of the dust density on the distance from the centre. The dust density starts to increase from the centre and reaches a rather large value at a certain point. In these conditions the void region can be joined to the dust region at any point in the dust region but limitations appear due to a change in the dependence of the ion drift velocity from the centre of the sheath—it starts at a certain distance to decrease in absolute value, which decreases the drag force which confines the dust. This determines the maximum possible size of the dust sheath, found to be 0.723, with the size of the wall 0.734 corresponding to the case of maximum possible space occupied by dust between the electrodes, namely 95.5%. The maximum dimensionless number of dust particles per unit square area is then found to be \( N_d = 0.90 \). When the number of dust particles injected is less than this maximum possible number the dust sheath occupies smaller space than that corresponding to the maximum number of dust particles. The threshold for \( d \) when the qualitative change in the dust and the ion drift velocity starts is determined by the condition of local quasi-neutrality of the dust sheath which corresponds to \( d_{cr} = 0.025 \) for the parameters considered in these computations.

The physical reason for the absence of dust sheath structure for \( d < d_{cr} \) is the following. To form a sheath the absolute value of the ion drift velocity should increase to the periphery
Figure 7. (a) The ion drift velocity and the total flux (the convective flux plus the diffusion flux) in the central part of the single dust sheath at $x < 0.3$. (b) The parameter $P$ in the central part of the single-sheath structure. (c) The distributions of the parameter $P$, the dust density $P/z$, the total flux $\Phi$ and the ion drift velocity $u$ for distances larger than $x = 0.3$ (outer part of the structure); $x$ varies from 0.3 up to the point where the dust density becomes zero, $P = 0$. The distance $x_{wv}$ is less than that corresponding to $P = 0$. (d) The distributions of the ion density $n$, the electron density $n_e$ and the dust charge $z$ in the same region as in (c). (e) The crystallization of dust in the sheath can occur if the parameter proportional to the coupling constant $\Gamma$ becomes larger than a certain critical value $\Gamma_{cr}$. The computations shown in the figure indicate that inside the structure the parameter $\Gamma$ is approximately constant. (f) The change of the parameters $nu$ (the convective flux), $u$ (the ion drift velocity), $\Phi$ (the total flux) and $\Phi_{th} = \Phi - nu$ (the diffusion flux) in the dust void region surrounding a single dust sheath up to the electrodes where the floating boundary conditions were satisfied.
Figure 8. Dependences of initial sheath parameters (at the centre of the sheath) on the $x_i$ and $d$ obtained from the conditions that the flux close to the centre of the sheath is convective.

of the sheath since the larger the distance from the centre of the sheath the larger the number of dust particles that should be confined by the ion drag. Therefore the absolute value of the ion drift velocity should also decrease toward the centre because the ions are transferring their momentum to dust particles and lose their kinetic energy. Apart from losing their momentum in ion–dust collisions the ions also lose their momentum in ion–neutral collisions. If the latter dominates the ion drag will not increase to the periphery of the sheath, then fewer dust particles
will be compressed toward the centre and the dust density starts to increase to the periphery which, in turn, decreases the absolute value of the ion drift velocity. This determines the critical value $d_c$ when the dust sheath can be formed.

We then describe qualitatively the dependence of the dust sheath structure on the ionization rate $x_i$ as compared to the value $x_i = 2$ used in the described example of the numerical computations. An increase of $x_i$ by a factor of ten (a tenfold decrease of the ionization rate) decreases slightly the number of dust particles in the sheath $N_d$ and its size $x_{wv}$ to 0.3317 and 0.561 respectively. The decrease of $x_i$ to 0.5 leads to a violation of the condition of existence of a sheath at the centre since both the drift velocity and the flux change sign at the centre. Thus one

Figure 9. (a) Distribution of the parameter $R$ in the second dust sheath with larger dust grains of size $2a$ located at distances larger than the first dust sheath with smaller dust particles of size $a$. The point at which the parameter $R$ becomes zero is that at which the motion becomes 2D and it is most probable that dust convection is created. (b) Distribution of other parameters in the dust sheath with size $2a$ dust particles for the conditions shown in (a). (c) Distribution of the parameters in the first sheath with dust size $a$ for lower rate of ionization $x_i = 1$ as compared to that used in the results of figure 7 ($x_i = 2$) and for larger central ion density $n(0) = 4$ as compared to that used in results shown in figure 7 ($n(0) = 2.5$). (d) Distribution of some parameters in the second sheath with the dust size $2a$ for the other parameters the same as in (c).
finds the critical ionization rate $1/x_{i,cr}$ such that for the ionization rates larger than the critical one the dust sheath at the centre cannot be created. The calculations show that there exists another critical ionization rate $1/x'_{i,cr}$ larger than $1/x_{i,cr}$ ($x'_{i,cr} > x_{i,cr}$) when a dust void is created at the centre. For $x_{i,cr} < x_i < x'_{i,cr}$ no stationary structures (dust voids or dust sheaths) can be created and presumably such a state can correspond to a non-stationary ‘breathing’ shear and can apparently explain the observed phenomenon of the ‘heart-beat’ mode. This state should be investigated in more detail to answer whether the latter hypothesis is correct or not. The problem of void formation for $x_{i} > x'_{i,cr}$ is discussed in the next section.

The absence of singular points in the balance equations was checked for the single dust sheaths by substituting in expression (3) the dependences on $x$ of $z$ and $u$ obtained in the sheath in numeric computations. In all cases the expression $R$ does not reach the zero point which shows the absence of singularities in the treatment of the single-sheath structure.

This investigation shows qualitatively the range of the parameters of the phase space {d, $x_i$} where one dust sheath with one-size dust particles can be formed. This phase space is substantially large for experimental detection of such structures. The parameters of such structures observed experimentally correspond to this range. The mean free path for ion–neutral atom collisions was in [7] of the order of 0.1 cm and the size of the dust sheath was $\approx 2$ cm which corresponds to our dimensionless size 0.5 ($0.5/\tau 0.1$ cm $\approx 2$ cm for $\tau = T_i/T_e = 0.05$). The critical density $n_*$ for the size of dust particles ($\approx 3$ $\mu$m for experiments [7]) is $2 \times 10^8$ and the dimensionless value $n = 2$ corresponds to the ion density of the order of $4 \times 10^8$ cm$^{-3}$ which is approximately the value corresponding to that in the experiment [7]. A more detailed comparison can be made if one measures the actual value of the ionization rate for different RF powers and one measures the actual ion density and drift velocity distributions.

3.2. Dust sheath structure with two-size dust particles

One easily finds that the equilibrium position of two dust particles with different sizes should be different if the important forces are only the drag force and the electric field force. Indeed the drag force is proportional to $a^2$ while the electric field force is proportional to $a$. Thus for example for particles with the size of that previously considered (with a radius $a/2$) at the same space position as the particles with radius $a$ the balance is possible if $1 = 1/2$ (in other words it is not possible). The drag acting on larger size particles is larger. The drag due to the collective flux created by all particles of the sheath confines them and creates the sheath as described in the previous section. The equilibrium positions of dust grain of different sizes are different and the grains with size $a/2$ should have equilibrium positions different from those for grains with size $a$.

We start the calculations using the parameters we considered in the previous section, namely $n_0 = 2.5$, $x_i = 2$ and $d = 0.15$ for particles of size $a$, and then we will try to find at which position the sheath with particles of size either $2a$ or $a/2$ can start. The value given above $x_{wv}$ of the distance from the centre to where the void starts $x_{wv} = 0.561$ corresponds to the maximum possible size of the dust sheath. The distance of the wall for this case was $x_w = 0.8529$.

There are two equations to be solved simultaneously to find the possible position where the second dust layer with particles of different size can start: the electric field continuity equation at the surface of the second layer.

$$E(x) = sn(x)a_{dr}(u(x))u(x)z(x);$$

(11)
and the charging equation at the surface of the second layer
\[ \exp(-z(x)) = \frac{2\sqrt{\pi}}{\sqrt{\mu\tau} \sqrt{n_e(x) n_{e\alpha ch}(u(x))}} \]
with \( s = 2, 1/2 \ldots \) for different-size particles. Equations (11) and (12) should be solved outside the central sheath region, but the dust-free region outside the sheath can start at any \( x_{vw} < x_{vw,\text{max}} = 0.561 \). Equations (11) and (12) were solved for \( \tau = 0.05 \) and \( \mu = m_i/m_e \) corresponding to singly ionized ions in argon gas (as for all previous results given above). The numerical solution of equations (11) and (12) for \( x_{vw} = x_v = 0.6 \) or \( x_{vw} = x_{vw,\text{max}} = 0.561 \) (determined by the floating boundary conditions at the wall for a single dust particle sheath) shows that although (11) and (12) can be satisfied at a certain position outside the first sheath the subsequent solutions of equations for double-size or half-size particles give the unphysical result \( P < 0 \) for the second sheath.

If the total number of particles of the first sheath is decreased (is less than its maximum value and \( x_{vw} < x_{vw,\text{max}} \)) there exists a solution with \( P > 0 \) for the second sheath. For example, for \( x_{vw} = 0.324 \) we find that the second sheath with double-size particles (\( s = 2 \)) starts at \( x = 0.389 \), i.e. the void between the two sheaths is rather thin, \( \delta x = 0.055 \). The increase of \( P \) in the second sheath is rapid and at \( x = 0.412 R \) in the second sheath reaches zero (figures 9(a) and (b)). The convective flux and the drift velocity in the second sheath becomes positive (directed to the wall), but the total flux is negative which indicates that the diffusion flux plays an important role in this case. For \( s = 1/2 \) the solutions are not physical, \( P < 0 \), which only indicates that the balance conditions in the second sheath are not satisfied.

Thus we conclude that the second sheath with double-size grains can exist only if in the first sheath the number of grains is substantially less than the maximum possible, if the two sheath are separated by a thin void and if in the second sheath the convection can be generated rather close to its beginning. When increasing the central density in the first sheath and increasing the degree of ionization (i.e. decreasing \( x_i \)) we can satisfy (11) and (12) for \( s = 2 \) (but not \( s = 1/2 \)) even for \( x > x_v \), i.e. for a dust sheath with \( P = 0 \) at its edge. For example, for \( x_i = 1, n(0) = 4 \) and \( n_e(0) = 1.28 > n_{e\text{cr}} = 1.19 \) (for the flux to be convective at the centre) we find \( x_v = 0.425 \) and the start of the second sheath at \( x = 0.446 \) \( 27 \), with \( R \to 0 \) (start of convection) at \( x = 0.447 \) 38 (figures 9(c) and (d)). One can see a decrease of \( P \) in the central part of the first sheath which precedes the void formation in the centre.

Thus the procedure for calculating the sheaths with different size particles was established but in certain cases no solutions were found in the case where at the centre region the void is absent. The definite conclusion from the calculations performed so far is that the structures with two sheaths of different-size particles could be separated by a thin void and that the creation of convection in the second sheath is rather probable. The solutions also exist for larger size particles in the second sheath. In the case where the second sheath consists of smaller particles the balance equations in the second sheath are rarely satisfied.

### 4. Structures with a dust void in the centre

#### 4.1. Dust void surrounded by dust sheaths with equal size dust particles

The dust void at the centre is created in the case where the ionization rate exceeds its critical value and the ion drift velocity at the centre start to be directed from the centre. In this case
the drag forces act to create a void at the centre until the balance of forces is reached at certain distance from the centre which determines the size of the void.

The theory of the collision-dominated voids was developed in [6], but this theory deals only with the problem of the existence of the void boundary. It was shown in [6] that such a boundary can exist and the dependence of the position of that boundary on different parameters such as ionization rate, ion density at the centre etc was investigated. The theory proceeded to find the distributions of the dust and plasma parameters in the dust region. Thus one can call the surface found in [6] a virtual surface, i.e. the surface with a property that in the case where even a single dust particle is injected at this surface the balance of forces and the charges obtained from the plasma flux on the dust particle will allow this particle to stay at this ‘virtual’ surface. This means that this surface can operate as a virtual void surface without many dust particles being present at the distances larger than the virtual surface or as a void without a dust cloud present.

If the number of dust grains injected into the system is increased the stationary dust distributions at the dust size of the mentioned surface will obey the force balance conditions at the positions further out from the void surface and will then finally form a dust void. The question is: what will be the distribution of dust grains at the dust part of the surface and will it finally form dust sheaths from both sides of the central void?

This problem is investigated in the present paper for the case of the presence of walls at remote distances from the centre defined as surfaces satisfying floating boundary conditions. It is found that in these conditions a dust structure can indeed be formed such that the central void is surrounded by two dust sheaths which in turn are surrounded by two dust wall-void regions between the sheaths and the walls. In the central void region the void surface is formed when the balance of forces is reached. The ionization source in the central region produces sufficient electron–ion pairs to form a plasma flux which evacuates dust from the void region toward the dust region. The electrons move with almost thermal electron velocity while the ions, being attracted to the negative charged dust in the dust region, are accelerated towards the dust region and their drift velocity is soon larger than their thermal velocity.

The numerical computations show that at the void surface the ion drift velocity reaches a value several times larger than the thermal velocity but in the collision-dominated regime it is still of the order of the ion thermal velocity and is not much larger than the latter. In these conditions in the void region the electron density should be smaller than the ion density and the quasi-neutrality is violated. The difference between the ion and the electron densities in the void is determined by the boundary conditions at the virtual surface and the virtual dust charge which in turn depends on the size of the dust grains in the dust region.

In the computations we fix at the centre the ratio of electron to ion densities and find the form of dust sheaths created. Thus we again invert the problem. We give an example of numerical computations for $n_e(0)/n_i(0) = 0.25$, $n_i(0) = 0.8$, $d = 0.7$, $x_i = 0.5$ for argon gas. Not all values of parameters at the centre lead to formation of a void with a dust sheath at larger distances, because the dust sheath itself and its structure as well as the void structure determines these parameters. An example of the parameters was found to satisfy all necessary conditions for a void–sheath system to be formed and these conditions were found by a trial and error procedure to satisfy the boundary conditions at the dust sheath. The whole system should have a proper asymptotic behaviour at the centre. The latter can be found from the void equations given in the
appendix. These asymptotic expressions are for $x \to 0$

\[
\begin{align*}
    u & \to \frac{n(0) - n_e(0)}{2d^2} x; \\
    n & \to n(0) + \frac{x^2}{2\tau} \left( \frac{(n(0) - n_e(0)n(0))}{2d^2} - \frac{n_e(0)}{x_i} \right), \\
    n_e & \to n_e(0); \quad \Phi \to \frac{n_e(0)}{x_i} x
\end{align*}
\]

(13)

The calculations in the void region are started using these asymptotic expressions at $x = 0.001$. The results are presented in figure 10(a), illustrating an increase of the ion drift velocity, the total flux and the ion density from the centre. The size of the void is determined by simultaneous solution of equations (11) and (12) for $s = 1$ (all dust particles have the same size $a$). The position of the virtual void surface we denote here as $x_v$. The result of computations gives the size of the void (the position where the dust sheath starts), $x_v = 0.672$, and the dust charges at this surface, $z_v = 0.933$. All other parameters at this surface are the ion drift velocity $u_v = 0.56$, the ion density $n_v = 3.82$, the electron density $n_{e,v} = 0.142$ and the total (convection plus diffusion) flux $\Phi_v = 0.244$.

These parameters are sufficient to solve the nonlinear balance equations for the structure of the dust sheath at $x > x_v$. The results are given in figure 10(b) for the dust density $P/z$, for the parameter $P$, for the total flux $\Phi$, for the electron density $n_e$, for the ion density $n$ and for the ion drift velocity $u$. As can be seen the dust sheath has a very regular structure with a maximum of the dust density shifted towards larger distances from the centre. The maximum distance from the centre for the dust particles in the sheath (the point where $P = 0$ is vanishing a second time) is 0.799, thus the thickness of the sheath found is 0.127. The change of the parameter $R$ in this region is shown in figure 10(c)—it is always positive and the singular point is not located in the sheath region (convection is not created).

Starting with the same parameters we can consider the location and the dust distribution in the sheath consisting only of dust particles with a double size $2a$. In this case the solution of equations (11) and (12) should be found for $s = 2$. We find $x_v = 0.825$ and $z_v = 0.50$, showing that the dust sheath is then located farther from the centre and that the charge on each dust particle is smaller. The distribution of the parameters in the sheath with size $2a$ dust particles is shown in figure 10(d) and the parameter $R$ is shown in figure 10(e). In this case the maximum size of the sheath is determined by the point $R = 0; P$ in the sheath increases continually. There exist two possibilities:

1. the convection is started before the large gradients appear;
2. when the large gradients of all parameters are met the validity of the hydrodynamic approach is lost; the solution should be joined with the one where the role of ion–neutral collisions is negligible. Close to the singular point the derivative of $P$ increases and the dust charge decreases but still does not reach the zero value (figure 10(f)). Thus the sheath with double-size dust particles can have a different structure, but this statement is true for the same central parameters as for the sheath of dust size $a$. If we choose the central $n(0)$ to be $2n(0)$ we obviously get the same kind of distribution for the double-size $2a$ dust particles as we get for the size $a$ particles.
Figure 10. (a) Increase of the ion drift velocity, of the total flux and of the ion density with the distance from the centre in the central void region surrounded by a dust sheath with dust grains of the same size. (b) Distributions of the dust density $P/\Phi$, the parameter $P$, the total flux $\Phi$, the electron density $n_e$, the ion density $n$ and the ion drift velocity $u$ in the dust sheath with the dust particles of the same size (equal to $a$) surrounding the central void region. (c) Parameter $R$ in the dust sheath of the single-size $a$ dust particles; it is always positive and the singular point is not located in the sheath. (d) The distribution of the parameters in the sheath with single-size $2a$ dust particles. (e) The distribution of the parameter $R$ for the sheath shown in figure 10(d). (f) Distribution of $z$ close to the singular point where the derivative of $P$ increases and the dust charge decreases (but does not reach the zero value).
4.2. Dust sheaths of different-size dust particles surrounding the void region

As shown in the previous section two types of sheath are possible (say, with convection and without convection). If there exist simultaneously dust particles of both sizes they can create such conditions at the centre that either both sheaths will be of the same type or they are of different types. Let us then return to the case of a dust sheath with dust particles of size $a$ for the central parameters used in the previous section where we calculated the structure of the central void and the sheath with size $a$ dust particles. In the case where there are simultaneously present dust particles with size $2a$ their position cannot coincide with the position of the dust particles with size $a$ as emphasized above. The distribution of the parameters of the first sheath with size $a$ dust particles is the same as that of the previous section (see figures 10(a)–(c)). Thus we solve then the equations for the void region between the two sheaths for the distances larger than that for the first sheath with particles of size $a$ by using as starting parameters the values at the right-hand surface of the first sheath. Than we solve the system of equations (11) and (12) to find the position of the second sheath. Such a solution exists; the starting position of the second sheath is $x_{2v} = 0.834$ and the dust charge is $z_{2v} = 0.149$. The distance between the two sheaths (the size of the void separating the two sheaths) is rather small, 0.055, which seems to be in accordance with observations [8] (see figure 4). The position of the second sheath is somewhat farther from the centre than in the case where the sheath with particles of size $a$ was absent ($x_v = 0.825$) and the dust charge is smaller. The numerical calculation of the structure of the second sheath shows that it is of the second type approaching a point at the right-hand boundary of the second sheath with $R = 0$ (figures 11(a)–(c)). At this point it is most probable that the 1D approach is lost and the convection can start. At the right-hand side of the second sheath the wall-void region appears with a jump of the parameter $P$ at the right-hand side of the second sheath. This jump is determined by the floating boundary conditions at the wall. For the distributions shown in figure 11(c) the boundary conditions at the wall can always be fulfilled and determine the size of the second sheath for a given distance between the electrodes. In fact we are then joining the void-wall solution at the given point of the right-hand part of the second sheath to calculate the dimensionless distance to the wall. In experiments the distance between the electrodes is fixed. We are able to find the point in the second sheath where the wall-void solution can be joined. Than the conclusion of the presence of convection in the second sheath has no basis since the wall-void starts before the point $R = 0$ is reached. If the distance between the electrodes is larger than that which allows us to join the wall-void solution at the edge of the second sheath, the creation of dust convective cells is very probable. Apparently the observations shown in figure 4 correspond to this possibility. The convection cannot be investigated in the frame of the 1D model used in the present paper but the threshold for convection is related to the violation of the 1D approach which appears in calculations particularly when $R \to 0$.

Thus we found that for a central void structure with two-size dust grains the sheath has the following features:

1. the sheath with smaller grains is located at smaller distances as also observed (see figure 4);
2. the interface between the two sheaths is small (a thin void is formed) as also observed (see figure 4);
3. the dust convection seems to be a natural effect in such systems as observed (see figures 1–4);
4. the dust convection can be absent for small inter-electrode distances;
5. the computed characteristic size of the structure is of the order of $\lambda_{in}/\tau$ and is of the order of characteristic sizes of the structures in the micro-gravity experiment [8] (figures 2–4);
Figure 11. Distributions of the parameters in the second dust sheath, with the dust size equal to $2a$, for the case where the size of the grains in the first sheath is $a$ (see figures 10(a)–(c)); the distributions show that in the second sheath the singular point can be approached at the right-hand boundary of the sheath.

(6) the wall-void dust distribution can have rather sharp edges with a jump of the dust density and

(7) the position of the wall-void surface is determined by the distance to the electrode with floating potential boundary conditions. The calculations also indicate that by choosing different parameters at the centre one can get the first sheath to be with sharp edges and the second sheath smooth. This also depends on the inter-electrode distance.

5. Discussion and conclusions

The aim of the present paper was to give examples of the sheath and the void distributions in the plane geometry with two electrodes at floating potential at the edge of the system. From a mathematical point of view it was simple to start with some parameters at the centre and to calculate the structures of the sheaths and void satisfying the floating potential boundary...
conditions at the electrodes. Such calculations finally determine the distance between the electrodes in terms of $\lambda_{Di}/\tau$. In real experiments the distance between the electrodes is fixed and the central parameters are determined by this distance, by the gas pressure and by the ionization rate. To convert these calculations by fixing the distance between the electrodes is not a difficult problem. At least using our approach we were able to demonstrate that all qualitative features of the sheath–void structures observed so far satisfy the force balance conditions and qualitatively explain them by using the self-consistent treatment of the balance equations. Our investigations show that a totally self-consistent treatment of a completely nonlinear system is one of the important aspects in any more sophisticated approaches which may be used in the future. We use here the simplest one related only to the force balance relations and dust charge variations. The new result found from this investigation is the possibility to find the physical requirements for the parameters for the different types of structure to be created.

The singular points found in calculation can have several reasons for appearing. They are certainly not computational singularities. One of the most probable reasons is that at these points the dust convection is started. We collect here the arguments for the latter statement.

1. The singular points in computations can appear due to an inappropriate choice of the central parameters of the system in numerical computations. In fact these points can be avoided if one starts the calculations from their vicinity requiring that the numerator of the nonlinear responses in the set of nonlinear equations also vanish at those points. This determines at those points the relation between different parameters such as drift velocity, density, dust charge and others. The computations started at those points can be performed up to the centre and determine the relation between the parameters at the structures which do not have the singularities. These structures will be of special type and require an adjustment of parameters in the structures. Probably these structures will be rarely found in the experiments. An investigation of these structures is left for future research.

2. The appearance of singularities is not related to the presence of two-size dust particles; they can exist even for a single sheath with single-size dust particles. Thus it is a general phenomenon depending on the choice of the parameters of the system.

3. There exists an answer on the question of whether the dissipative process can smooth the singularity. The diffusion due to collisions with neutral atoms is already included in the consideration and the dissipation in the charging process has scales much less than the ion–neutral mean free path. The singularities can be made smooth by the delay in the charging process but still they remain sharp on the scales of the ion–neutral mean free path. These dissipative processes cannot smooth the singularity.

4. The theoretical investigations were based on the assumption of stationarity of all distributions. The time-dependent 1D investigation should include the movement of the time dependent boundaries which will open to investigate the nonlinear 'breathing' structures. Such ‘breathing’ boundaries will include the dust movement.

5. The dust 1D movement can be also stationary (time independent where the dust velocity depends only on distance). The mathematical description of such structures will include the dust–neutral friction and the change in the charging equation with the additional term $v_d dz/dx$. One can easily find that in this case the $v_d$ should be a periodical function with a point where $v_d = 0$, thus causing a new singularity in $dz/dx$ at $v_d = 0$. Thus we can see that a stationary dust movement in the 1D case does not remove the singularities.
(6) This discussion suggests that only 2D dust movements can remove these singularities. Indeed in the 2D case the charging equation will contain a term \( v_d \cdot \frac{d}{dx} \). It can be not-vanishing at any point only for circular dust motion and this means the creation of dust convection cells. This is an important conclusion which shows the possibility that the singular points found above can be the point where the dust convection starts. The theoretical investigation of this problem needs a 2D numerical consideration, but the existence of dust vortices in experiments (see figures 1–4) is of no doubt. In figure 4 one can see that even the vortices in the second layer start at rather short distances from its boundary as the singular point is in the computations. The 2D convection can start as a convective instability, proposed in [9] (the latter is due to perturbations in which the electric field force and the drag force are not aligned). One can expect that the structures with singular points are convectively unstable.

For existing and future experiments the obtained results indicate the following.

(1) The size of the single dust sheath in the collision dominated range of parameters is of the order of \( \lambda_{st} \); the pressure and temperature dependence of the latter is known and can be checked with the observed structures in future experiments.

(2) The predicted separation of the sheaths of different size particles coincides with first observations in micro-gravity conditions.

(3) The theoretical predictions for the distributions of the dust and the other parameters in a single sheath and in the more complicated sheath–void structures with single-sized dust grains and with different-size dust grains is one possible test of the theory.

(4) In future experiments it is particularly necessary to measure the distribution of the ion drift velocity which seems to be one of the decisive parameters determining the type of structure formed.

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Appendix

(1) Balance equation for electrons

\[
E = -\frac{1}{n_e} \frac{dn_e}{dx}. \tag{A.1}
\]

(2) Balance equation for ions

\[
E = u(2 + |u|) + P \nu z \alpha_{dr}(u). \tag{A.2}
\]

(3) Balance equation for dust

\[
E = \alpha_{dr}(u)nu_z. \tag{A.3}
\]
(4) Definition of total flux $\Phi$ as sum of convection flux and diffusion flux

$$\Phi = nu - \tau \frac{dn}{dx}.$$  \hfill (A.4)

(5) Equation for total flux

$$\frac{d\Phi}{dx} = \frac{n_e}{x_i} - \alpha_{ch}(u) P_n.$$  \hfill (A.5)

(6) Poisson equation

$$\frac{dE}{dx} = \frac{1}{d^2} (n - n_e - P).$$  \hfill (A.6)

(7) Dust charging equation

$$\exp(-z) = \frac{2\sqrt{\pi}}{\sqrt{\tau \mu}} \frac{n}{n_e} \frac{n - ne}{P n}; \quad \mu = \frac{m_i}{m_e}.$$  \hfill (A.7)

(8) Analytic expressions for the drag and charging coefficients in the limit $\tau ll 1$

$$\alpha_{dr}(u) = \left[ \frac{\text{erf}(u)}{2u^3} - \frac{\exp(-u^2)}{\sqrt{\pi} u^3} \right] \ln \left( \frac{d_i}{a} \right)$$  \hfill (A.8)

$$\alpha_{ch}(u) = \frac{4}{u} \text{erf}(u).$$  \hfill (A.9)

(9) System of nonlinear balance equations in the dust region

$$\begin{align*}
\frac{du}{dx} &= R_1; \quad \frac{dn}{dx} = \frac{1}{\tau} (nu - \Phi); \quad \frac{dn_e}{dx} = -n_e n z u \alpha_{dr}; \quad \frac{dz}{dx} = R_2 \\
\frac{d\Phi}{dx} &= \frac{n_e}{x_i} - \alpha_{ch}(u) n \left( n - 2 + |u| \right) \left( \alpha_{dr}(u) n \right)
\end{align*}$$  \hfill (A.10)

$$R(u, z) = 1 + \frac{u}{\alpha_{dr}(u)} \frac{d\alpha_{dr}(u)}{du} \left( \frac{u}{\alpha_{dr}(u)} \right)$$

$$R_1(u, n, n_e, \Phi, z) = \frac{\alpha_{dr} u u^2 n z}{z + 1} + \frac{zu(\Phi - nu)}{(z + 1)n \tau} + \frac{1}{d^2 n \alpha_{dr}(u)} \left( \frac{2 + |u|}{\alpha_{dr}(u) z} - n_e \right)$$

$$R_2(u, n, n_e, \Phi, z) + \frac{z}{z + 1} \left[ \frac{1}{\alpha_{ch}(u)} \frac{d\alpha_{ch}(u)}{du} \frac{R_1(u, n, n_e, \Phi, z)}{R(u, z)} - \frac{\Phi - nu}{\tau n} + \alpha_{dr}(u) z n \right].$$  \hfill (A.11)

(10) System of nonlinear balance equations in the void region

$$\begin{align*}
\frac{dn}{dx} &= \frac{1}{\tau} (nu - \Phi); \quad \frac{dn_e}{dx} = -n_e u (2 + |u|); \quad \frac{d\Phi}{dx} = \frac{n_e}{x_i}; \quad \frac{du}{dx} = \frac{n - n_e}{d^2 (1 + |u|)}.
\end{align*}$$  \hfill (A.12)

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