Standard Model Light-By-Light Scattering in SANC:
Analytic and Numeric Evaluation*

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Abstract—The implementation of the Standard Model process $\gamma\gamma \rightarrow \gamma\gamma$ through a fermion and boson loop into the framework of SANC system and additional precomputation modules used for calculation of massive box diagrams are described. The computation of this process takes into account nonzero mass of loop particles. The covariant and helicity amplitudes for this process, some particular cases of $D_0$ and $C_0$ Passarino–Veltman functions, and also numerical results of corresponding SANC module evaluation are presented. Whenever possible, the results are compared with those existing in the literature.

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1. INTRODUCTION

SANC is a computer system for semi-automatic calculation of realistic and pseudo-observables for various processes of elementary particle interactions in Standard Model (SM) “from the SM Lagrangian to event distributions” at the one-loop precision level for the present and future colliders—TEVATRON, LHC, electron Linear Colliders (ILC, CLIC), muon factories, and others. To learn more about available processes in SANC see the description in [1, 2] and look at our home pages at JINR and CERN [3].

Light-by-light scattering is one of the most fundamental processes. It proceeds via one-loop box diagrams containing charged particles. The first results for the quantum electrodynamics (QED) low-energy limit of this process were obtained by Euler [4]. Then Karplus and Neuman [5] found a solution for QED in a general but complicated way. The QED cross sections in the high-energy limit were calculated by Ahiezer [6]. Nowadays there are computations for $\gamma\gamma \rightarrow \gamma\gamma$ process in the electroweak (EW) SM [7–10] and even for two-loop quantum chromodynamics (QCD) and QED corrections [11].

In this paper we describe the implementation of the SM process $\gamma\gamma \rightarrow \gamma\gamma$ through fermion [12] and boson loops [13] and corresponding precomputation modules into the framework of SANC system. The computations of this process take into account nonzero mass of the loop particles.

One should emphasize that the obtained building blocks and procedures of precomputation for box diagrams in QED and EW (as in $\gamma\gamma \rightarrow \gamma\gamma$) is the first step in the creation of environment for implementation of the similar four-boson processes in the SM (like $\gamma\gamma \rightarrow ZH$, $\gamma\gamma \rightarrow ZZ$ [10]) and in QCD (like $gg \rightarrow \gamma\gamma$, $gg \rightarrow ZZ$, $gg \rightarrow W^+W^-$, etc.).

In Section 2 we discuss some notation and common expression for cross section, diagrams for $\gamma\gamma \rightarrow \gamma\gamma$ process and covariant amplitude tensor structure, then the helicity amplitudes approach [1, 14] and their expressions for light-by-light scattering in general (massive) and in limiting (massless) cases are listed.

In Section 3 we briefly describe precomputation strategy, the place of this process on the SANC process tree, and the implementation of analytical results and the SANC modules concept. At last, one can find the numerical result and comparison with those existing in the literature.

Additionally, in Appendix we present strings and basis for covariant amplitude and list answers for particular cases of $D_0$, $C_0$, and $B_0$ Passarino–Veltman (PV) functions [15] (see also [16]), which are needed for calculation of light-by-light scattering through massive and massless loop particles.

2. LIGHT-BY-LIGHT SCATTERING PROCESS

2.1. Notation, Cross Section

The 4-momenta of incoming photons are denoted by $p_1$ and $p_2$, of the outgoing ones—by $p_3$ and $p_4$. 
The amplitudes are calculated for scattering of real photons, that is:
\[
p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = 0.
\]
The 4-momentum conservation law reads:
\[
p_1 + p_2 - p_3 - p_4 = 0.
\]
The Mandelstam variables are \(1\):
\[
s = -(p_1 + p_2)^2 = -2p_1 \cdot p_2,
\]
\[
t = -(p_1 - p_3)^2 = 2p_1 \cdot p_3,
\]
\[
u = -(p_1 - p_4)^2 = 2p_1 \cdot p_4, \quad s + t + u = 0.
\]

For the \(2 \rightarrow 2 \gamma \gamma \rightarrow \gamma \gamma\) process the cross section has the form
\[
d\sigma_{\gamma \gamma \rightarrow \gamma \gamma} = \frac{1}{j} |A_{\gamma \gamma \rightarrow \gamma \gamma}|^2 d\Phi^{(2)},
\]
where \(j = 4\sqrt{(p_1 p_2)^2}\) is the flux, \(A_{\gamma \gamma \rightarrow \gamma \gamma}\) is the covariant amplitude of the process, and \(d\Phi^{(2)}\) is the two-body phase space:
\[
d\Phi^{(2)} = (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) \frac{d^4p_3(d^4p_3^2) d^4p_4(d^4p_4^2)}{(2\pi)^3 (2\pi)^3}.
\]

For the differential cross section one gets:
\[
d\sigma_{\gamma \gamma \rightarrow \gamma \gamma} = \frac{1}{128\pi s} |A_{\gamma \gamma \rightarrow \gamma \gamma}|^2 d\cos\theta,
\]
where \(s = 4\omega^2\), \(\omega\) is the photon energy and \(\theta\) is the scattering angle in the center-of-mass system (c.m.s.).

2.2. Covariant Amplitude

The covariant one-loop amplitude (CA) corresponds to a result of the straightforward standard calculation of all diagrams contributing to a given process at Born (tree) and one-loop levels.

The CA is being represented in a certain basis, made of strings of Dirac matrices and/or external momenta (structures), contracted with polarization vectors of vector bosons, \(\epsilon(p)\), if any. The amplitude also contains kinematical factors and coupling constants and is parameterized by a certain number of Form Factors (FFs), which are denoted by \(F_i\), in general with the index \(i\), labeling the corresponding structure. The number of \(F_i\) is equal to the number of independent structures.

The \(\gamma \gamma \rightarrow \gamma \gamma\) process in quantum field theory appears due to nonlinear effects of interaction with vacuum, so this process has no Born or tree level. Corresponding diagrams start from the one-loop level and in QED there are box diagrams with four internal fermions of equal mass. The number of not identical diagrams (or topologies) is equal to six. But three of them differ from others only by the orientation of the internal fermionic loop, giving the same contribution or a factor two to the amplitude. So, only three topologies \((st, su, and ut\) channels\) remain which are related by simple permutations of external photons in the diagrams shown in Fig. 1: \(su\) channel is obtained from \(st\) channel by \(p_3 \leftrightarrow p_1\) rotation and \(ut\) channel—by \(p_2 \leftrightarrow p_3\). The sum of these fermionic diagrams is a gauge invariant in each generation of particles.

In the EW boson sector we have three types of diagrams to classify: box topologies, pinch topologies, and fish topologies (shown in Fig. 2). There are three channels of each topology \((st, su, and ut\) channels as in QED\) and we have \(W^+, W^-, \phi^+, \phi^-, and \(X^+, X^-\) (bosons and ghosts) as internal particles in \(R_\xi\) gauge theory.

As in fermionic part we can choose only positive or negative charged bosons and \(X^+, X^-\) ghosts to appear as loop particles and multiply the result by factor two to dismiss the double counting diagrams, which differ from others only by the orientation of the loop charge flow.

So, we have three structures (three channels, combinatorical factor is 1) in box type of diagrams, twelve structures (three channels by four corresponding pinches, combinatorical factor is \(1/2\)) in pinch type, and six structures (three channels by two corresponding combinations of propagators—direct and crossed, combinatorical factor is \(1/4\)) in fish type of diagrams—each of them is a sum of the appropriate sets of loop particle diagrams.

The full CA of given process for off-shell photons \((p_i \epsilon_i \neq 0)\) with corresponding combinatorial factors can be written as a sum of bosonic part minus fermionic and ghost parts.

In terms of Lorentz structures we have:
\[
A_{\gamma \gamma \rightarrow \gamma \gamma} = \sum_{i=1}^{43} \left[ F_i^{\text{bosons}} (s, t, u) + F_i^{\text{fermions}} (s, t, u) \right] T_i^{\alpha \beta \mu \nu}.
\]

The \(F_i\) are normalized by corresponding factors for fermion and boson parts:
\[
C_i^{\text{fermions}} = 8\alpha^2 Q_i^2 N_c, \quad C_i^{\text{bosons}} = 12\alpha^2,
\]
where \(\alpha\) is the fine structure constant, \(Q_i\) is the fraction of charge of loop fermion in units of electron charge \(e\), \(N_c\) is the number of colors for given fermion, \(T_i^{\alpha \beta \mu \nu}\) are tensors defined with the aid of auxiliary strings \(\tau_j\) presented in the Appendix. The off-shell