Predictions of striking energy and angular dependence in $pp \rightarrow (pp)_{S\text{-wave}}\pi^0$ production

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Abstract

A phenomenological calculation from threshold to 800 MeV of the initial proton beam energy is presented to describe recent data on the reaction $pp \rightarrow (pp)_{S\text{-wave}}\pi^0$ with a low energy cut on the final state diproton excitation energy. A strong forward dip is obtained in the differential cross section as in the data from COSY at 800 MeV, although the absolute value of the forward cross section is too low. Earlier low energy data from CELSIUS are reasonably well reproduced. In the unexplored energy interval between these two experiments the model predicts a spectacular energy dependence both in the forward direction and in the angle-integrated cross section.

PACS: 13.75.Cs, 25.40.Qa

1 Introduction

Pion production cross sections in two-nucleon collisions have in a broad sense existed for a long time (for a historical reference see Ref. [1], a modern review close to threshold Ref. [2]). However, only quite recently have experiments on $NN \rightarrow NN\pi$ reactions with a cut-off on the final $NN$ excitation energy opened a new chapter in comparison with theory. Restriction to only one $NN$ partial wave ($S$ wave) simplifies the comparison tremendously to be basically similar to the simple $NN \rightarrow d\pi$. It is clear that in this kind of experiments good resolution of momenta is essential and cooled beams give an obvious advantage, although such experiments were initiated at TRIUMF with measurements of the differential cross section and analyzing power $A_y$ in quasifree $pn \rightarrow (pp)_{S\text{-wave}}\pi^- [3,4,5]$. In these experiments a cut-off of $\approx 1.5$ MeV was applied on the final diproton energy ($37.5$ MeV/c on the canonical c.m. momentum [6]). The data agreed reasonably well with the predictions of Refs. [7,8] for the inverse quasifree absorption of negative pions on the $^1S_0$ $pp$ pair in $^3$He.
Later differential cross sections between 310 and 425 MeV for $pp \rightarrow (pp)\pi^0$ have been obtained at CELSIUS both integrated over the final momentum magnitudes (and the nucleon relative angle) and also applying a diproton energy cut of 3 MeV (53 MeV/c momentum) [9]. In the latter case the final diproton should be rather purely $S$ wave. An interesting feature was that above 350 MeV the slope of the angular distribution applying the energy cut was opposite as compared with the case without the cut. Normally the cross sections tend to find a maximum in the forward direction. However, the $pp \rightarrow (pp)_{S\text{-wave}}\pi^0$ cross section decreases for the decreasing reaction angle. This is in agreement with the predictions given already in Ref. [7] for pion absorption on a $pp$ pair in the corresponding isospin situation. A similar behaviour is also seen in a very recent measurement by the ANKE collaboration at COSY of this reaction at 800 MeV very near the forward direction [10].

2 Model

The basically phenomenological model has been presented in the past in some detail for mechanisms in Refs. [7,8] (albeit for pion absorption on a bound diproton) and for the treatment of the long range free nucleon wave functions and the Coulomb interaction in Ref. [11]. The mechanisms in the production operator involve first the direct production from each nucleon with distorted initial and final $pp$ states. This is Galilean invariant with the axial current part $\propto \mathbf{q} \cdot \mathbf{\sigma}$ and the corresponding recoil term (axial charge) $\omega_q (\mathbf{p} + \mathbf{p'}) \cdot \mathbf{\sigma} / 2M$. In pion reactions the all important $\Delta(1232)$ resonance is treated as excitation of a $\Delta N$ intermediate state by coupled channels with a transition potential including pion and $\rho$ meson exchange. This covers pion rescattering in the $\pi N \, p_{3/2}$ partial wave. Following Ref. [8] pion $s$-wave rescattering from the second nucleon is parameterized as occurring on the energy shell with the corresponding propagator for a pion emitted by a $\Delta$ taken also on shell and off shell as in e.g. Ref. [12] if emitted by a nucleon. A monopole form factor with the cut-off mass 550 MeV is also included in the exchange. Further, the heavy meson exchange effect suggested in Refs. [14,15] to account for the missing $pp \rightarrow (pp)\pi^0$ threshold strength [16] is used. The latter is implemented as in Ref. [8] fitting the data at a single energy 290 MeV and the agreement with data is very good up to $\eta = q_{\text{max}} / m_{\pi} \approx 0.6$, i.e. in the range where the $Ss$-wave production is by far dominant [8].

The interactions thus fixed will be applied at higher energies with a constraint

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1 On chiral perturbation arguments Ref. [13] suggests putting the pion on shell also in this case.
2 As another possibility to account for this at least partially off-shell pion rescattering has been proposed [17,18].
of the two final state protons being in the $^1S_0$ state. Having only one $NN$ final state simplifies the theoretical treatment significantly to resemble the reaction $pp \rightarrow d\pi^+$, although the long-range behaviour of the free protons requires some extra care [11]. When the laboratory energy increases above, say, 350 MeV, the final state nucleons will not remain in the $S$ wave, if the whole phase space is included, and the results of the present calculation involving only that should fall below any such experiment as seen e.g. in Ref. [8]. With a cut on the final diproton excitation energy the validity range is increased and only the limitations of the model itself will eventually make it fail. In experiments this cut is the way to single out $S$-wave final nucleons.

One specific but relevant feature concerning the role of the ∆ should still be mentioned. The reactions $pp \rightarrow d\pi^+$ and $pp \rightarrow np\pi^+$ are dominated by the ∆ causing a wide peak at the nominal mass of the ∆ excitation around 600 MeV (lab). However, in the present case with a final $^1S_0$ nucleon pair the initial state $^1D_2$ coupled to an $S$-wave $\Delta N$ is not possible, but the ∆ is excited at least in a $p$ wave. Because of the centrifugal energy the ∆ excitation should be somewhat suppressed and its position displaced by about 80 MeV in the c.m.s. to appear most prominently at about 800 MeV laboratory energy [19]. This is just the energy of the recent COSY/ANKE experiment [10]. However, actually no peaking is is seen at this energy but rather close to the nominal ∆ mass of $\sqrt{s} \approx 1310$ MeV.
3 Results

Figure 1 shows the calculation compared with a representative selection (four of six energies with the best error limits) of the Celsius differential cross section data [9], which are constrained to the final $^1S_0$ pp state by a cut off $E_{pp} \leq 3$ MeV on the diproton energy. The trends are very similar and also the agreement in the absolute magnitude is good in this energy range, although the highest maximum final pion momentum is even twice that of Ref. [16]. In particular the cross section distinctly gets a minimum in the forward direction. This is in contrast to most other situations, e.g. the same cross sections without the cut [9].

In the recent data [10] from COSY measured at 800 MeV in the near-forward direction it was found an extraordinarily strong angular dependence with the cross section dropping down by 30% in the interval where $\cos \theta$ changed only from 0.97 to 1. This steep dip is rather unexpected even in the light of the previous CELSIUS results showing the minimum in the forward direction. As seen in Fig. 2 such a very steep descent is also obtained by the present model, although the absolute scale is too low. However, since the cross section drops by an order of magnitude, it is clear that there is extremely strong cancellation in the forward direction between different amplitude components (three important ones expressed earlier in the Introduction) and so a relatively minor change in a single partial wave may cause a large change in the cross section. Also another minimum is predicted at $90^\circ$ and a maximum at about $50^\circ$.

With such a strong variation and interference it is also relevant to divide the cross section into partial waves to find the important ones. As can be seen in Fig. 3 all partial wave amplitudes $s_{01}$, $d_{21}$ and $d_{23}$ (in the notation $l_{\pi, JL}$) are about equally important at 800 MeV. In this figure the contribution of each partial wave to the angle integrated cross section is presented as a function of the incident laboratory energy. The $g$-wave pions contribute negligibly. The low-energy Celsius data [9] are reasonably well reproduced. Unfortunately there are no comparable data at other energies.

Further, a drastic energy dependence between 500 and 800 MeV is found. This may be related to the constraint of small energy of the final state diproton. Namely in that case the phase space integral is very limited so that the angle integrated cross section is nearly just a sum of the squared reaction matrix elements taken with the maximal pion momentum. In this way the role of the $d$-wave pions is emphasized. Also the contribution of a single partial wave can reach even zero as seems to happen for the $s_{01}$ pions. In an incoherent sum this could not happen. Similarly a phase space integral over a wider range of momenta would probably have a moderating effect in sharp changes and
oscillations.

For orientation of the most imminent further experiments at COSY [20], Fig. 4 presents the energy dependence of the forward cross section and its slope. The theoretical slope is defined as the difference of the cross section at $\cos^2 \theta = 0.9$ and $\cos^2 \theta = 1$ divided by 0.1 (i.e. approximately the derivative with respect to $\cos^2 \theta$ but plotted as positive). For the COSY data [10] the extremes of $\cos^2 \theta$ were used and for the CELSIUS the fits published in Ref. [9]. If possible, the energy dependence is even more dramatic in this quantity just in the experimentally uncharted region.

Here actual interference of all different partial waves is possible and apparently at about 550 and 700 MeV the forward amplitude changes its sign producing the small minima. Also, because the forward cross section may be an order of magnitude smaller than the bulk of the cross section as in Fig. 2, due to
Fig. 3. The cumulative sum of the contributions to the overall cross section from different partial wave amplitudes: solid $s_{01}$, dashed $d_{21}$, dotted $d_{23}$. The solid curve denoting the total integrated cross section including also the $g$-wave pions is indistinguishable from the dotted one. The data are obtained by integrating the fits of Ref. [9].

destructive interferences the absolute detailed prediction may not be exactly correct, but still violent energy and angular variations are expected. Certainly the expected behaviour of the cross section is sharper than that in the widely studied $pp \rightarrow d\pi^+$ both as a function of energy and angle.

4 Conclusion

In summary, a phenomenological model calculation is performed for the reaction $pp \rightarrow (pp)_{S\text{-wave}}\pi^0$. Partly the aim has been to provide some predictions in anticipation of experiments at COSY. However, the finding of extreme energy and angular dependencies may have also wider interest and applications in other similar reactions in attempts to extract information on reaction matrix elements. The constraint of a small relative momentum for the final state protons seems to favour this strong variation of the cross section in particular in the forward direction but also in the angle-integrated cross section. Obviously this cut also tends to stress higher pion waves than cross sections integrated over all possible momenta. Such a strict constraint may be a way to get hands on the squared reaction matrix elements at (nearly) a single momentum choice.

Already the experimental finding of opposite slopes of $d\sigma/d\Omega$ with and without the cut on the final $pp$ excitation energy is suggestive of physical differences. Calculations of total production cross sections (integrated over all
Fig. 4. Predictions for the forward cross section and its slope defined as in the text. The data are from the fits of Ref. [9] and from Ref. [10] (800 MeV). The forward cross section at 800 MeV would be outside the figure at 700 nb/sr.

phase space) show that already at 400 MeV the amplitudes $^1D_2 \rightarrow ^3P_2 s$, $^3P_1 \rightarrow ^3P_0 p$ and $^3F_3 \rightarrow ^3P_2 p$ are each as large as $^3P_0 \rightarrow ^1S_0 p$ with also sizable contributions from $^3P_2 \rightarrow ^3P_1 p$, $^3P_2 \rightarrow ^3P_2 p$ and $^3F_2 \rightarrow ^3P_2 p$. The $^1S_0s$ part of the cross section would then be only about one sixth of the total in line with the phenomenological fits of Ref. [9]. Apparently this complexity makes a detailed partial wave analysis improbable. However, with a cut, due to the additional simplicity of the spin structure of the final $^1S_0$ state, such an analysis is amenable with measurements of also spin observables. This, in turn, would act as a strong constraint on any modelling of the reaction over the whole phase space range.

The discrepancy with the COSY data in the forward direction may be due to overly delicate destructive interference, which a minor change in just one of the amplitudes might moderate. By some exploratory model variations it was not possible to significantly improve the situation. Making pion $s$-wave rescattering somewhat stronger actually decreased the forward cross section.
Change of the size of the cut-off within the experimental precision has no significant effect. An intriguing possibility could be a need for explicit pion $d$-wave rescattering possibly involving the $N(1520)\frac{1}{2}^-$ resonance.

It would certainly be interesting to extend experiments both to larger angles to check whether the model gives the total normalization reasonably and also to the unexplored energies accessible at COSY, where extreme energy dependence shown in Figs. 3 and 4 is predicted. Further details with model dependence and spin observables will be studied in a forthcoming paper [21].

Acknowledgements

I thank C. Hanhart, Y. Uzikov and in particular C. Wilkin for numerous useful discussions and suggestions for this work and J. Zlomanczuk for providing the data of Ref. [9]. This work was supported by the DAAD and Academy of Finland exchange programme projects DB000379 (Germany) and 211592 (Finland). I also thank the Magnus Ehrnrooth Foundation for partial support and IKP of Forschungszentrum Jülich for hospitality.

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