Unifying the flavor origin of dark matter with leptonic nonzero $\theta_{13}$

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We propose a flavor symmetric approach to unify the origin of dark matter (DM) with the non-zero $\theta_{13}$ in the lepton sector. In this framework, the breaking of a $U(1)$ flavor symmetry to a remnant $Z_2$ ensures the stability of the DM and gives rise to a modification to the existing $A_4$-based tri-bimaximal neutrino mixing to attain the required non-zero values of $\sin \theta_{13}$. This results in a range of Higgs portal coupling of the DM which can be potentially accessible at various ongoing and future direct and collider search experiments.

Flavor symmetries play important roles in understanding many issues in particle physics including quark and lepton mixing as well as mass hierarchies. Historically a global $U(1)$ flavor symmetry was proposed to explain the quark mass hierarchy and Cabibbo mixing angle \[1\] which was extended to explain neutrino masses and mixing later. Among others, a tri-bimaximal (TBM) lepton mixing generated from a discrete flavor symmetry such as $A_4$ gets particular attention \[2, 3\] due to its simplicity and predictive nature. However, the TBM mixing primarily is associated with a vanishing reactor mixing angle $\theta_{13}$ which is against the recent robust observation of non-zero $\theta_{13} \approx 9^\circ$ \[4–6\] by DOUBLE CHOOZ \[7\], Daya Bay \[8\], RENO \[9\] and T2K \[10\] experiments. Hence, an alteration to TBM structure has been under scanner.

Understanding the nature of dark matter (DM) is another outstanding problem in particle physics today. Although astrophysical evidences, such as rotation curves of galaxies, gravitational lensing and large scale structure of the Universe supports the existence of DM \[11\], a discovery in laboratory is still awaited. The relic abundance of DM has been measured by WMAP \[12\] and PLANCK \[13\] satellite experiments to be about 26.8% of the total energy budget of the Universe. Although this hints towards a broad classification of DM scenarios, its properties apart from gravitational interactions, are not known yet.

In this paper we propose a $U(1)$ flavor extension of the Standard Model (SM) to unify the origin of DM with the simultaneous realization of non-zero $\sin \theta_{13}$ in the lepton sector. For this purpose, we presume the existence of a TBM neutrino mixing pattern (in a basis where charged leptons are diagonal) and a dark sector consisting of vector like leptons. We will argue that this serves as a minimal extension of the SM to accommodate DM and non-zero $\sin \theta_{13}$. A pictorial presentation of the model is shown in Fig.1. Here $f$ represents the flavon fields charged under $A_4$, the vacuum expectation values (vevs) of which $(\langle f \rangle)$ would break the $A_4$ and generate the flavor structure of the lepton sector. The flavon field $(\phi)$ charged under the $U(1)$ plays the role of a messenger between the dark sector and SM particles including left-handed neutrinos. The $U(1)$ symmetry, once allowed to be broken by the vev of $\phi$, generates a non-zero $\sin \theta_{13}$ and a Higgs portal coupling to the vector-like leptonic DM in the effective field theory. We will show that the non-zero values of $\sin \theta_{13}$ are correlated to the Higgs portal coupling of the DM which yields the correct relic density measured by WMAP \[12\] and PLANCK \[13\]. Future direct search experiments, such as Xenon1T \[14\] and the Large Hadron Collider (LHC) \[15\] can establish a bridge between the two invisible sectors by measuring the Higgs portal coupling of DM.

\[\text{FIG. 1: Non-zero values of } \sin \theta_{13} \text{ predict Higgs portal couplings of DM via a } U(1) \text{ flavour symmetry: a schematic presentation.}\]

We consider an effective field theory approach for the demonstration purpose and begin by assuming a typical well known structure of the neutrino mass matrix \[2,3\], $(m_\nu)_0$, given by

\[
(m_\nu)_0 = \begin{pmatrix}
2b/3 & b/3 & b/3 \\
-2b/3 & a+b/3 & b/3 \\
b/3 & a+b/3 & -2b/3
\end{pmatrix},
\]

which results in a TBM neutrino mixing pattern while the charged lepton mass matrix is diagonal. The TBM mixing matrix \[17\] can be represented by:

\[
U_{\text{TBM}} = \begin{pmatrix}
\sqrt{2}/\sqrt{3} & \frac{1}{\sqrt{6}} & 0 \\
-\frac{\sqrt{2}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{\sqrt{2}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix},
\]
implying $\sin \theta_{13} = 0$, $\sin^2 \theta_{12} = 1/3$ and $\sin^2 \theta_{23} = 1/2$. The above structure of $(m_{\nu})_0$ can be obtained in a $A_1$ based set-up either in a type-I or II see-saw framework [18-20] or through higher dimensional lepton number violating operators. For example, we can have Altarelli-Feruglio (AF) model [8], where the SM doublet leptons ($\ell$) are transforming as triplet under the $A_4$ while the singlet charged leptons $\epsilon_L, \mu_R$ and $\tau_R$ transform as $1, 1'$ and $1$ respectively. Then a higher dimensional operator of the form, $(\ell H \ell H)(\xi - y \phi_S)/\Lambda^2$ can be considered, where $\xi$ and $\phi_S$ are singlet and triplet flavon fields (they are SM singlet and transform under $A_4$) respectively. $\Lambda$ is the cut off scale of the theory and $y$ represents the relative strength between the two couplings involved. Once these flavons get vevs, a flavon field can be generated after electroweak symmetry breaking with $a = (v^2/\Lambda)e$ and $b = y(v^2/\Lambda)e$ where $e = (\xi)/\Lambda = (\phi_S)/\Lambda$. With a judicious choice of additional discrete symmetries like $Z_3$ or more, one can ensure that no other terms involving these flavons and SM fields are allowed at $1/\Lambda^2$ order or below so as to keep the structure of $(m_{\nu})_0$ intact as in Eq. (1). In what follows, we introduce an additional global $U(1)$ flavor symmetry, which will be broken into a remnant $Z_2$ and an additional contribution to $(m_{\nu})_0$ becomes functional. None of the fields in the above dimension-6 operators, responsible for TBM mixing, would carry any $U(1)$ charge in order to generate non-zero $\sin \theta_{13}$ and establish a connection to the dark sector.

Simplest way to connect non-zero $\sin \theta_{13}$ and dark sector is achieved by assuming a minimal fermionic DM framework consisted of a vector-like $SU(2)_L$ doublet fermion $\psi^T = (\psi^0, \psi^-)$ and a vector-like neutral singlet fermion $\chi^0$ [14]. These fermions are charged under the additional $U(1)$ flavor symmetry, but neutral under the existing symmetry in the neutrino sector (say the non-abelian $A_4$ and additional discrete symmetries required). We also introduce two other SM singlet flavon fields $\phi$ and $\eta$ which carry equal and opposite charges under the $U(1)$ symmetry but transform as $1$ and $1'$ under $A_4$. Note that the SM fields are neutral under this additional $U(1)$ symmetry. The effective Lagrangian, invariant under the symmetries considered, describing the interaction between the dark and the SM sector is then given by:

$$L_{int} = \left(\frac{\phi}{\Lambda}\right)^n \bar{\psi} H \chi^0 + \left(\frac{H^H \phi \eta}{\Lambda^3}\right).$$  

We keep $n$ as a free parameter at present. The first term is allowed since the $U(1)$ charge of $\phi^n$ is compensated by $\psi$ and $\chi^0$, while the second term is allowed since the $U(1)$ charges of $\phi$ and $\eta$ cancel with each other. This also ensures that $\phi$ and $\eta$ do not take part in $(m_{\nu})_0$. The detailed structure of the scenario is left for a future work [21]. The idea of introducing a vector like fermion in the dark sector is also motivated by the fact that we expect a replication of the SM Yukawa type interaction to be present in the dark sector as well. Here the field plays the role of the messenger field similar to the one considered in [22]. See [23], for some earlier efforts to relate $A_1$ flavor symmetry to DM.

When $\phi$ and $\eta$ acquire vevs, the $U(1)$ symmetry breaks into a remnant $Z_2$ symmetry under which the vector-like fermions $\psi$ and $\chi^0$ are odd. Consequently the DM emerges as an admixture of the neutral component of the vector-like fermions $\psi$ and $\chi^0$ and yields a larger region of allowed parameter space as we will shortly demonstrate. The interaction strength of the DM with the SM Higgs is then given by $(\langle \phi \rangle / \Lambda) \equiv \epsilon^\phi$. Similarly the second term in Eq. (3) provides an additional contribution to the light neutrino mass matrix as follows:

$$\delta m_{\nu} = \begin{pmatrix} 0 & 0 & d \\ 0 & 0 & 0 \\ d & 0 & 0 \end{pmatrix},$$  

where $d = (v^2/\Lambda)e^2$ with $e = \langle \phi \rangle / \Lambda \equiv \langle \eta \rangle / \Lambda$. This typical flavor structure follows from the involvement of $\eta$ field, which transforms as $1'$ under $A_4$ [24].

From Eqs. (1) and (4), we get the light neutrino mass matrix as $m_{\nu} = (m_{\nu})_0 + \delta m_{\nu}$. We have already seen that the $(m_{\nu})_0$ can be diagonalized by $U_{TBM}$ alone, so an additional rotation ($U_1$) is required to diagonalize $m_{\nu}$:

$$U_1 = \begin{pmatrix} \cos \theta_\nu & 0 & \sin \theta_\nu \\ 0 & 1 & 0 \\ -\sin \theta_\nu & 0 & \cos \theta_\nu \end{pmatrix}. $$  

Here we assume all parameters $a, b, d$ are real for simplicity. We therefore obtain [18]

$$\tan 2\theta_\nu = \sqrt{3d} = \frac{\sqrt{3e}}{\epsilon - 2}. $$  

Then comparing the standard $U_{PMNS}$ parametrization and neutrino mixing matrix $U_e(= U_{TBM}U_1)$ we get $\sin \theta_{13} = \sqrt{\frac{2}{3}} |\sin \theta_\nu|, \sin^2 \theta_{12} = \frac{1}{3(1 - \sin^2 \theta_{13})}, \sin^2 \theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{3}} \sin \theta_{13}, \delta = \arg([U_{e13}].) = 0. $$

Clearly sin $\theta_{13}$ depends only on $\epsilon$ as shown in Fig. 2. The horizontal patch in Fig. 2 denotes the allowed $3\sigma$ range of sin $\theta_{13}$ ($\equiv 0.1330-0.1715$) [6] which is in turn restrict the range of $\epsilon$ parameter denoted by the vertical patch on the figure. Note that the interaction strength of DM with the SM particles depend on $\epsilon^n$. Therefore we find that the size of sin $\theta_{13}$ is intimately related with the Higgs portal coupling of DM. This is an important observation in this letter and is demonstrated in the rest of the paper. The two other mixing angles $\theta_{12}$ and $\theta_{23}$ fall in the right ballpark while light neutrino mass satisfy the $\Delta m^2_{23} = 7.60 \times 10^{-5} eV^2$ and $|\Delta m^2_{atm}| = 2.48 \times 10^{-3} eV^2$ [4, 8].

Now we focus on the 1st term of Eq. (3) to estimate the relic density of dark matter as a function of $\epsilon$. Note, both flavour and the dark sector constrains the ratio $\epsilon$, instead of the new physics scale $\Lambda$ in the effective operator formalism considered here. Since $\psi$ and $\chi^0$ are vector-like fermions, they can have bare masses, $M_\psi \bar{\psi} \psi$ and $M_\chi \bar{\chi} \chi$, which are not protected by the SM sym-
There are two additional terms, \( \chi^0 H^1 H / \Lambda \) and \( \bar{\psi} H^1 H / \Lambda \), which are also allowed by the symmetry considered. However, their contribution (\( \sim v^2 / \Lambda \)) to the mass matrix \( M \) is negligibly small compared to the bare masses and \( Y v \). They also have negligible impact on the DM annihilation processes as the DM-DM-\( \psi \) vertex would be suppressed by \( v / \Lambda \).

by three parameters \( \sin \theta_d, M_1, M_2 \). In the following we use MicrOmega [29] to find the allowed region of correct relic abundance for \( \psi_1 \) DM satisfying WMAP [12] constraint

\[
0.094 \leq \Omega_{DM} h^2 \leq 0.130.
\]
The most stringent constraint on the Higgs portal coupling $Y \simeq \sin \theta_q \Delta M/(2v)$ comes from the direct search of DM at Xenon-100 [30], LUX [31] as demonstrated in Fig. 5. We see that the bound from LUX, constrains the coupling: $Y \sim 0.05$ for DM masses $\gtrsim 800$ GeV (Green points). The Yukawa coupling needs to be even smaller for $M_1 \simeq 100$ GeV. Though large couplings are allowed by correct relic density, but they are highly disfavored by the direct DM search at terrestrial experiments. Note that these constraints are less dependent on $\Delta M$ as to the mixing angle, which plays otherwise a crucial role in the relic abundance of DM.

We can now combine the outcome of the two sectors into Fig. 6. The allowed range of $Y$-values can be translated in terms of $\epsilon - n$ as shown here. Correct $\sin \theta_{13}$ allowed $\epsilon$ within $0.328 - 0.4125$ (see Fig. 2). Therefore, the Higgs portal couplings: $Y \lesssim 0.05$, allowed by correct relic density and direct search of DM can be satisfied with $n = 3$ or more.

![FIG. 5: Allowed values of the Higgs portal coupling $Y$ by the direct search experiments, Xenon100, LUX and Xenon-IT: $Y : \{0.001 - 0.05\}$ (Green), $Y : \{0.05 - 0.1\}$ (Purple), $Y : \{0.1 - 0.15\}$ (Red). $\Delta M = 100$ GeV is used for the scan.]

![FIG. 6: $n$ vs $\epsilon$ to generate different values of $Y = \epsilon^n$.]

The $U(1)$ symmetry of the model is broken by the vev of a flavon field to a remnant $Z_2$, whereas the breaking of $A_4$ (and additional discrete symmetry) is responsible for producing the flavor structure of neutrino mass matrix. The details of symmetry breaking pattern and charge assignment of the flavon fields is worthy of attention. Non-zero $\sin \theta_{13}$ appeals for finite values of phases and hence CP-violations, which have been ignored in this letter. They will be discussed together in a future publication [21].

In summary, the observed value of non-zero $\sin \theta_{13}$ and its link to Higgs portal coupling of a vector-like fermionic DM was obtained in a further $U(1)$ flavor extension of the SM. We showed that the non-zero values of $\sin \theta_{13}$ fixes a range of Higgs portal coupling $Y = \epsilon^n$, $n \gtrsim 3$ which can be probed at the future direct DM search experiments such as Xenon-1T. Also note that the next to lightest stable particle (NLSP) could be a charged fermion which can be searched at the LHC [10]. In the limit of small $\sin \theta_{13}$, the NLSP can give rise to a displaced vertex at LHC, a rather unique signature for the model discussed [15].

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