Decoherence and purity of a driven solid-state qubit in Ohmic bath

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March 31, 2008

Abstract
In this paper we study the decoherence and purity of a driven solid-state qubit in the Ohmic bath by using the method based on the master equation. At first, instead of solving the master equation we investigate the coefficients of the equation which describe the shift in frequency, diffusive, decoherence, and so on. It is shown that one of the coefficients (we called it decoherence coefficient) is crucial to the decoherence of the qubit in the model. Then we investigate the evolution of the purity of the state in the model. From the analysis of the purity we see that the decoherence time of the qubit decrease with the increase of the amplitude of the driven fields and it is increase with the increase of the frequency of the driven fields.

Keywords: Solid-state qubit; Decoherence; Master equation; Driven field.
PACS number: 03.67.HK; 42.50.CT; 03.65.YZ

1 Introduction
Solid-state qubits are considered to be promising candidates for realizing building blocks of quantum information processors. In past few years, many kinds of these qubit models are introduced and many efforts not only theoretical but also experimental have been performed on investigating the decoherence, relaxation and manipulation for these qubit models. Most of the investigations for these models are considered them as a spin-boson model which is modeled with a two-level system coupled to a bath and the bath always constructed with a set of harmonic oscillators. The qubits can be manipulated with a driven field [1, 2, 3, 4, 5], so a more realistic description requires the inclusion of an external control field in the qubit-bath model. The quantum tunneling or other quantum properties of the driven two-level system have been investigated at dept in last

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years \[6, 7\]. Recently, the driven spin-boson model attracting a lot of attention because of its interest in connection with quantum computing with solid state devices. The control of the coherence dynamics of a two-level atom placed in a lossy cavity with an external periodic driving field \[24\], the dynamics of a XOR gate operation with an external source \[25\], and the consequence of driving in terms of multi-phoyo transitions experimentally \[26\] are investigated. In this paper, we shall analyze the influence of the driven control field on the decoherence of the qubits by using the master equation method \[12, 13, 19, 18\]. At first we shall derive out a master equation from the Hamiltonian of the driven qubit in the Ohmic bath. Then we analyze the decoherence and other decay coefficients based on the master equations. Finally, a rate of purity decay for this model is obtained and discussed.

2 Models and Master Equation

In this paper we concentrate on the case of a persistent current qubit based on Josephson junction \[9, 8, 10, 11\]. It is driven with the magnetic flux through the loop and damped predominantly by flux noise with Gaussian statistics. This setup is accurately expressed by the driven spin-boson (DSB) model \[14, 15, 16, 6\]

\[
H = H_S + H_B + H_I, \tag{1}
\]

where

\[
H_S = \frac{1}{2} \varepsilon(t) \sigma_z - \frac{1}{2} \Delta \sigma_x,
\]

\[
H_B = \sum_n (\frac{p_n^2}{2m_n} + m_n \omega_n^2 x_n^2),
\]

\[
H_I = \sigma_z \sum_n \lambda_n x_n. \tag{2}
\]

Here, \(\sigma_i (i = x, y, z)\) are Pauli matrices. The quantum environment is modeled with an infinite set of harmonic oscillators of mass \(m_n\), angular frequency \(\omega_n\), momentum \(p_n\) and position coordinate \(x_n\) which are coupled independently to the spin \(\sigma_z\) with strength measured by the set \(\{ \lambda_n \}\) and \(\varepsilon(t) = \varepsilon_0 + s \cos(\Omega t)\) is the external, time-dependent control field with the static basis \(\varepsilon_0\). For \(s = 0\), the system degenerates to be a common two-level one.

The evolution of the total density matrix for the system, in the interaction picture, reads

\[
i \hbar \dot{\rho}_T = [\tilde{H}_I, \rho_T], \tag{3}
\]

where the interaction representation of the operators are given by

\[
\dot{\tilde{\rho}}_T(t) = \exp(\frac{iH_0 t}{\hbar}) \rho_T \exp(-\frac{iH_0 t}{\hbar}), \tilde{\sigma}
\]

\[
\dot{\tilde{H}}_I(t) = \exp(\frac{iH_0 t}{\hbar}) H_I \exp(-\frac{iH_0 t}{\hbar}). \tag{4}
\]

Here \(\rho_T = \rho_T(0)\) and \(H_I = H_I(0)\), where \(H_I(0)\) is the Hamiltonian of the system in the Schrödinger picture. The perturbative expression of the Eq. (1)
in the interaction picture is given to second order of $H_I$ by

$$\tilde{\rho}_T(t) = \tilde{\rho}_T(0) + \frac{1}{i\hbar} \int_0^t dt_1 [\tilde{H}_I(t_1), \tilde{\rho}_T(0)] + (\frac{1}{i\hbar})^2 \int_0^t dt_1 \int_0^{t_1} dt_2 [\tilde{H}_I(t_1), [\tilde{H}_I(t_2), \tilde{\rho}_T(0)]] . \tag{5}$$

The reduced density operator for the system is defined by $\tilde{\rho}(t) = \text{Tr}_B(\tilde{\rho}_T(t))$, where $\text{Tr}_B$ indicates a trace over environment variables, then we assume that initially the system and environment are uncorrelated that the total density matrix is a tensor product of the form $\tilde{\rho}_T(t) = \tilde{\rho}(0) \otimes \tilde{\rho}_B(0)$. So we get

$$\tilde{\rho}(t) = \tilde{\rho}(0) + \frac{1}{i\hbar} \int_0^t dt_1 \text{Tr}[\tilde{H}_I(t_1), \tilde{\rho}(0) \otimes \tilde{\rho}_B(0)] + (\frac{1}{i\hbar})^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \text{Tr}[\tilde{H}_I(t_1), [\tilde{H}_I(t_2), \tilde{\rho}(0) \otimes \tilde{\rho}_B(0)]] . \tag{6}$$

Next, we make a rather trivial operation that enables us to finish the derivation in a simple way, we can express the initial state $\tilde{\rho}(0)$ in terms of $\tilde{\rho}(t)$ using the same perturbative expansion and rewrite the Eq. (3) while the initial state $\tilde{\rho}(0)$ appears in the right-hand side only, then inserting this expression into Eq. (4) and making the derivation [20], we can obtain

$$\tilde{\rho}(t) = \frac{1}{i\hbar} \int_0^t dt_1 \text{Tr}[\tilde{H}_I(t_1), \tilde{\rho} \otimes \tilde{\rho}_B] - \frac{1}{\hbar^2} \int_0^t dt_1 \text{Tr}[\tilde{H}_I(t), [\tilde{H}_I(t_1), \tilde{\rho} \otimes \tilde{\rho}_B]] + \frac{1}{\hbar^2} \int_0^t dt_1 \text{Tr}[\tilde{H}_I(t), \text{Tr}[\tilde{H}_I(t_1), \tilde{\rho} \otimes \tilde{\rho}_B] \otimes \tilde{\rho}_B] . \tag{7}$$

The Eq. (7) is in the interaction picture. By virtue of the evolution operator $U_s = \exp[-\frac{i}{\hbar} \int_0^t dt_1 H_s(t_1)]$ we rewrite the equation in the Schrödinger picture, and considering the coupling term in the DSB with $H_I = \sigma_z \sum_n \lambda_n x_n$, then we get the master equation of DSB in the Schrödinger picture is

$$\dot{\rho} = \frac{1}{i\hbar} [H_{\text{eff}}, \rho] + \frac{1}{i\hbar} \sum_n [\lambda_n \langle x_n | \sigma_z | \rho] \rho] - \frac{1}{\hbar^2} \sum_n \int_0^t dt_1 (k_n^{(1)} |\sigma_z, [\sigma_z(t_1 - t), \rho] + k_n^{(2)} |\sigma_z, \{\sigma_z(t_1 - t), \rho]\}) , \tag{8}$$

with

$$k_n^{(1)} = \frac{1}{2} \lambda_n^2 \langle \{x_n(t), x_n(t_1)\} \rangle - \lambda_n^2 \langle x_n | x_n \rangle ,$$

$$k_n^{(2)} = \frac{1}{2} \lambda_n^2 \langle \{x_n(t), x_n(t_1)\} \rangle , \tag{9}$$

$$H_{\text{eff}} = a_1 \hat{\sigma}_x + b_1 \hat{\sigma}_y + c_1 \hat{\sigma}_z ,$$
where,

\[
a_1 = \frac{\Delta}{2} \cos\left(\frac{\varepsilon_0 t + s}{\hbar} \sin(\Omega t)\right) + \frac{1}{4\hbar} \left[ \Delta \left(\frac{\varepsilon_0 t + s}{\Omega} \sin(\Omega t)\right) + \Delta t (\varepsilon_0 + s \cos(\Omega t)) \right] \cos\left(\frac{\varepsilon_0 t + s}{\hbar} \sin(\Omega t)\right),
\]

\[
b_1 = \frac{\Delta}{2} \sin\left(\frac{\varepsilon_0 t + s}{\hbar} \sin(\Omega t)\right) - \frac{1}{4\hbar} \left[ \Delta \left(\frac{\varepsilon_0 t + s}{\Omega} \sin(\Omega t)\right) + \Delta t (\varepsilon_0 + s \cos(\Omega t)) \right] \cos\left(\frac{\varepsilon_0 t + s}{\hbar} \sin(\Omega t)\right),
\]

\[
c_1 = -\frac{\Delta}{2} (\varepsilon_0 + s \cos(\Omega t)) + \frac{1}{4\hbar} \left[ \Delta \left(\frac{\varepsilon_0 t + s}{\Omega} \sin(\Omega t)\right) + \Delta t (\varepsilon_0 + s \cos(\Omega t)) \right] \sin\left(\frac{\Delta t}{\hbar}\right).
\]

(10)

Here, the notation \([\cdot,\cdot],\{\cdot,\cdot\}\) denote the commutators and anticommutators, and \(\langle \cdot \rangle = \text{Tr}(\cdot \rho)\) are reserved for quantum expectation values. So far, we only make two important assumptions: first, we used a perturbative expansion up to second order, which is accurate enough as the interaction small enough comparing to the Hamiltonians of system and bath; second, we assumed that the initial state is not entangled. Next we consider another important approximation, which is usually considered in the master equation, the Markov approximation. Assuming the kernel \(k^{(i)}\) are strongly peaked at the point \(t = t_1\), with slowly varying functions, then we can transform the temporal integrals over the variable \(\tau = t - t_1\). And assumed that the initial state of the environment to be thermal equilibrium at temperature \(T = 1/k_B\beta\), thus the first order term in the master equation of DSB disappears because \(\langle x_n \rangle = 0\). Therefore, the master equation becomes

\[
\dot{\rho} = \frac{1}{i\hbar} [H_{\text{eff}}, \rho] - \frac{1}{\hbar} \int_0^t dt_1 (\nu(t_1) [\sigma_z, [\sigma_z (-t_1), \rho]] - i\eta(t_1) [\sigma_z, \{\sigma_z (-t_1), \rho\}]),
\]

(11)

with the two kernels are the dissipation and noise kernels respectively and are defined as

\[
\nu(t) = \frac{1}{2\hbar} \sum_n \lambda_n^2 \langle \{x_n(t), x_n(0)\} \rangle = \int_0^\infty d\omega J(\omega) \cos(\omega t) \coth\left(\frac{\beta \hbar \omega}{2}\right),
\]

(12)

\[
\eta(t) = \frac{1}{2\hbar} \sum_n \lambda_n^2 \langle [x_n(t), x_n(0)] \rangle = \int_0^\infty d\omega J(\omega) \sin(\omega t),
\]

(13)

where \(J(\omega)\) is the bath spectral density function defined by

\[
J(\omega) = \frac{\pi}{2} \sum_n \frac{\lambda_n^2}{m_n \omega_n^2} \delta(\omega - \omega_n).
\]

(14)
We consider that the Hamiltonian of the spin system $H_s = \frac{1}{2} \varepsilon(t) \hat{\sigma}_z - \frac{1}{2} \Delta \hat{\sigma}_x$, the evolution operator has the form

$$U_s = \exp[-i \int_0^t dt_1 H_s(t_1)]$$

$$= \exp\{ -\frac{i}{2\hbar} [\varepsilon_0 t + \frac{s}{\Omega} \sin(\Omega t)] \hat{\sigma}_z \} \exp\{ \frac{i}{2\hbar} \Delta t \hat{\sigma}_x \} \times \exp\{ -\frac{i\Delta}{(2\hbar)^2} [\varepsilon_0 t^2 + t \frac{s}{\Omega} \sin(\Omega t)] \hat{\sigma}_y + o(t^3) \}, \quad (15)$$

where we use the formula $e^{A+B} = e^A e^B \exp \sum_j \frac{1}{j!} \frac{1}{2} [B, A^{(j)}]$. If we set the time $t$ very small (namely, $t$ is smaller than the characteristic time of the qubit), we can ignore the term of $o(t^3)$. So we can solve the Heisenberg equations for the system and determine the operator $\sigma_z(t)$ to be

$$\sigma_z(t) = -\sigma_x \cos\left( \frac{\Delta t}{\hbar} \right) \sin\left( \frac{\Delta t}{2\hbar^2} [\varepsilon_0 t + \frac{s}{\Omega} \sin(\Omega t)] \right) - \sigma_y \sin\left( \frac{\Delta t}{\hbar} \right)$$

$$+ \sigma_z \cos\left( \frac{\Delta t}{\hbar} \right) \cos\left( \frac{\Delta t}{2\hbar^2} [\varepsilon_0 t + \frac{s}{\Omega} \sin(\Omega t)] \right). \quad (16)$$

So the results of this paper are all the short-time results. Substituting this equation into Eq. (11), we obtain the final expression for the master equation of DSB. It is

$$\dot{\rho} = \frac{1}{i\hbar} [H_{eff} + \hat{\Omega}(t), \rho] - D(t)[\sigma_z, [\sigma_z, \rho]] - G(t)[\sigma_z, [\sigma_z, \rho]]$$

$$- f(t)[\sigma_z, [\sigma_y, \rho]] + ir_1(t)[\sigma_z, \{\sigma_x, [\sigma_z, \rho]\}] + ir_2(t)[\sigma_z, \{\sigma_y, \rho\}] \quad (17)$$

where

$$D(t) = \int_0^t dt_1 \nu(t_1) \left( \cos\left( \frac{\Delta t_1}{\hbar} \right) \cos\left( \frac{\Delta t_1}{2\hbar^2} [\varepsilon_0 t_1 + \frac{s}{\Omega} \sin(\Omega t_1)] \right) \right),$$

$$f(t) = \int_0^t dt_1 \nu(t_1) (-\sin\left( \frac{\Delta t_1}{\hbar} \right)),$$

$$G(t) = \int_0^t dt_1 \nu(t_1) (-\cos\left( \frac{\Delta t_1}{\hbar} \right) \sin\left( \frac{\Delta t_1}{2\hbar^2} [\varepsilon_0 t_1 + \frac{s}{\Omega} \sin(\Omega t_1)] \right)), $$

$$\hat{\Omega}(t) = \int_0^t dt_1 \eta(t_1) \left( \cos\left( \frac{\Delta t_1}{\hbar} \right) \cos\left( \frac{\Delta t_1}{2\hbar^2} [\varepsilon_0 t_1 + \frac{s}{\Omega} \sin(\Omega t_1)] \right) \right), \quad (18)$$

$$r_1(t) = \int_0^t dt_1 \eta(t_1) (-\cos\left( \frac{\Delta t_1}{\hbar} \right) \sin\left( \frac{\Delta t_1}{2\hbar^2} [\varepsilon_0 t_1 + \frac{s}{\Omega} \sin(\Omega t_1)] \right)), $$

$$r_2(t) = \int_0^t dt_1 \eta(t_1) (-\sin\left( \frac{\Delta t_1}{\hbar} \right)).$$
3 Decoherence and purity decay

Having obtained a master equation describing the evolution of a controllable solid state qubit system coupled to an Ohmic bath, we now proceed to examine the consequences of that evolution. In the following we shall analyze the problem with two different methods. At first, we use the coefficients in the master equation to detect the instantaneous effects of the environment. Then we can investigate the time evolution of the purity. For the first method, all the effects including controlled external field and uncontrolled environment are considered in the above coefficients. These coefficients are to renormalize the frequency as well as to introduce the decay of the system. From these equations, it is possible to have a qualitative idea of the effects for the environment producing on the system. First, we can observe that the effective Hamiltonian $H_{\text{eff}}$ is evolved with time-dependence. The system Hamiltonian $H_S$ and the coupling Hamiltonian $H_I$ do not commute with each other and the evolution of $H_{\text{eff}}$ is periodic because the driven controlled field is periodic. The term $\tilde{\Omega}(t)$ is the shift in frequency which produces the renormalized frequency. This term does not affect the unitarity of the evolution. The terms $D(t)$, $f(t)$, $G(t)$, $r_1(t)$ and $r_2(t)$ are diffusive terms and bring about non-unitary effects. The terms of $r_1(t)$ and $r_2(t)$ are the dissipation coefficients related to the dissipation kernel $\nu(t)$ defined already, which play the role of a time-dependent relaxation rate, are independent of temperature. And $D(t)$, $f(t)$ and $G(t)$ are the diffusion coefficients, which produce the decoherence effects, they are not only time-dependent but depend on temperature. Of cause, the explicit time dependence of the coefficients can only be computed once we specify the spectral density of the environment. To illustrate their qualitative behavior, we focus on the case that the environment is the Ohmic bath in the following, whose spectral density of the Ohmic bath can be expressed as

$$J(\omega) = 2\pi \alpha \omega \exp\left(-\frac{\omega^2}{\Lambda}\right),$$

where $\Lambda$ is the physical high-frequency cutoff, which represents the highest frequency presented in the environment and the parameter $\alpha$ is dimensionless parameter reflecting the strength of dissipation. We set $\alpha = 0.01$ as in [23]. We can plot the diffusive terms $D(t)$, $f(t)$ and $G(t)$ as Fig.1.

Fig.1

It is shown that all these plots appear to be a periodically diverging function, which stems not only from the periodic control driven field but also from the bath, and the peaks of $D(t)$ is much higher than that of $f(t)$ and $G(t)$. It means that the term proportional to $D(t)$ in Eq. (18) plays the main role to decoherence. The form of the decoherence coefficient $D(t)$ [20] is not necessarily intuitive, so we evaluate it numerically as Fig.2.

Fig.2

One of the parameters ($s$, $T$, $\Omega$) is varied in Figs.2 (a, b, c). All of the plots demonstrate the same basic behavior: $D(t)$ decreases with periodic oscillations.
It is shown that the influence of the heat bath to the decoherence of the qubit can be described by these coefficients of the master equation. It can also be described by a computationally convenient way, namely the linear entropy \( \zeta = tr(\rho - \rho^2) \) which is another measure of the purity of a quantum state. For the pure state it is approximative equals to \( \zeta = 1 - tr(\rho^2) \) so we shall study the evolution of the purity of the system as measured by \( \xi = tr(\rho^2) \) for simplicity [20] [17] [21]. It is equal to one for a pure state and decreases when the state of the system gets mixed because the destruction of quantum coherence is generated by evolution. By virtue of the master equation, we can easily obtain an evolution for the purity \( \xi \) and the equation we obtained is:

\[
\dot{\xi} = -4D(t)Tr(\rho^2 \sigma_z^2 - \rho \sigma_z \rho \sigma_z) - 4G(t)Tr(\rho \sigma_x \rho \sigma_z) - f(t)Tr(\rho \sigma_y \rho \sigma_z) - 4r_1(t)Tr(\rho^2 \sigma_y) + 4r_1(t)Tr(\rho^2 \sigma_x).
\]

(20)

Setting the initial state is pure then \( \rho^2 = \rho \). Similar to Ref. [22] we can obtain

\[
\dot{\xi}(t) = -\frac{4}{3}D(t).
\]

(21)

It is interesting that the Eq. (21) only correspond to the decoherence coefficient \( D(t) \). It is shown that the purity decay rate is proportional to the decoherence coefficient \( D(t) \), and do not include any other coefficients. It confirms that \( D(t) \) plays the main role in decoherence which is analyzed in the above section in detail. From Eq. (21) we can also see that the larger the rate of the purity is, the smaller the decoherence becomes. In Fig.3, we plot the evolutions of the state purity with time \( t \).

It is show that the purity of qubit decreases fast with the increasing of the amplitude and decreasing of the frequency of the driven control field. This is agree with the result in Ref. [22] that the low frequency driven field is destructive to the coherence of the qubit.

4 Conclusions

In this paper we have investigated the effects of the environment for a solid state qubit coupled with a driven control field. At first, instead of solving the master equation we investigated the coefficients in the equation which express the shift in frequency, diffusive, decoherence and so on. Here we suppose the environment is modeled with a Ohmic bath and it is coupled with the qubit linearly. It shows that the term proportional to the coefficient \( D(t) \) in master equation plays the main role to the decoherence. Then, we concretely investigated the decay of purity. It is shown that the decay rate of purity behaves fast as the amplitude increasing and the frequency decreasing of the driven field. It is suggested that if the driven field is necessary for some qubit system a higher frequency may help to decrease the decoherence.
Acknowledgement 1 This project was sponsored by National Natural Science Foundation of China (Grant No. 10675066) and K. C. Wong Magna Fundation in Ningbo University.

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Figures Captions

Fig. 1: Coefficients in master equation versus time $t$. (a) $D(t)$; (b) $f(t)$; (c) $h(t)$. Here, the parameters are $\varepsilon_0 = \Delta$, $s = \Delta$, $\Omega = \Delta$, $\Lambda = 36\Delta$, $\Delta = 5$, $T = 30$ mK, here and in the following figures the times are expressed in unit of $5/\Delta$.

Fig. 2: Dependence of $D(t)$ on coefficients of the driven field and temperature $T$. The parameters $s$, $\Omega$, $\beta$, and $\Delta$ are same as Fig. 1. (a) $D(t)$ versus $s$; (b) $D(t)$ versus $\Omega$; (c) $D(t)$ versus $\beta = 1/kT$.

Fig. 3: Dependence of purity of the system on coefficients of the driven field. The parameters $s$, $\Omega$, $\beta$, and $\Delta$ are same as Fig. 1. (a) $\xi$ versus $s$; (b) $\xi$ versus $\Omega$. 

5 Figures Captions
fig. 1a
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1c.png}
\caption{A graph showing the function $g(t)$ versus $t (5/\Delta)$, with a distinct oscillatory pattern.}
\end{figure}

\textbf{fig. 1c}
fig. 3b

(b)