THEORETICAL DEVELOPMENTS IN STANDARD-MODEL
HIGGS PHYSICS AT A FUTURE $e^+e^-$ LINEAR COLLIDER

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We review the decay properties and production mechanisms of the Standard-Model
Higgs boson at a future $e^+e^-$ linear collider with special emphasis on the influence
of quantum corrections. We also discuss how its quantum numbers and couplings
can be extracted from the study of appropriate final states.

1 Introduction
The Higgs boson is the missing link of the Standard Model (SM) of elementary
particle physics. At a future $e^+e^-$ linear collider (LC), an important experimental
task will be to determine the Higgs quantum numbers and couplings in
order to distinguish between the minimal SM and possible extensions. In particular, the measurement of the Higgs self-couplings will allow one to directly
test the Higgs mechanism.

In this presentation, we discuss the decay properties of the Higgs boson
and its main production mechanisms at a future $e^+e^-$ LC, emphasizing the
influence of radiative corrections, and explain how to extract its quantum numbers and couplings from the study of final states. For more details, we refer to
a recent review.

2 Decay Properties
At the tree level, the Higgs boson decays to pairs of massive fermions and
gauge bosons if $M_H > 2M_i$ ($i = f, V$). If $M_V < M_H < 2M_V$ ($M_H < M_V$),
then one of the (both) final-state gauge bosons are forced to be off shell, so
that one is dealing with three-particle (four-particle) decays. The Higgs boson also couples to photons (gluons), through loops involving charged (coloured)
massive particles, and one is led to consider the loop-induced decays $H \to Z\gamma$, $H \to \gamma\gamma$, $H \to gg$, etc.

In order to match the high experimental precision to be achieved with
a future $e^+e^-$ LC, it is indispensable to take radiative corrections into account. At one loop, the electroweak corrections to $\Gamma(H \to f\bar{f})$ and $\Gamma(H \to VV)$, and $\Gamma(H \to Zf\bar{f})$ and the QCD ones to $\Gamma(H \to q\bar{q})$ are well established, including the dependence on all particle masses. Beyond one loop,
only dominant classes of corrections were investigated, sometimes only in limiting cases. For a low-mass Higgs boson, these include corrections enhanced by the strong-coupling constant $\alpha_s$ and the top Yukawa coupling $g_{tH} = M_t/v$, where $v = 2^{-1/4}G_F^{-1/2} \approx 246$ GeV, with $G_F$ being Fermi’s constant. Specifically, the two-loop QCD corrections were found for $\Gamma(H \rightarrow l^+l^-)$, $\Gamma(H \rightarrow q\bar{q})$, and $\Gamma(H \rightarrow Z\gamma)$, and $\Gamma(H \rightarrow gg)$.

Even three-loop QCD corrections were calculated, namely for $\Gamma(H \rightarrow q\bar{q})$, and $\Gamma(H \rightarrow gg)$ in the last case, they are quite significant, the correction factor being $1 + 0.66 + 0.21$ for $M_H = 100$ GeV.

An efficient way of obtaining corrections leading in $X_i = g_{tH}^2/(4\pi)^2$ to processes involving low-mass Higgs bosons is to construct an effective Lagrangian by integrating out the top quark. This may be conveniently achieved by means of a low-energy theorem which relates the amplitudes of two processes which differ by the insertion of an external Higgs-boson line carrying zero four-momentum. In this way, the processes which differ by the insertion of an external Higgs-boson line carrying $O(\alpha)$ corrections to $\Gamma(H \rightarrow l^+l^-)$, $\Gamma(H \rightarrow Z\gamma)$, and $\Gamma(H \rightarrow gg)$.

The dominant mechanisms of Higgs-boson production in $e^+e^-$ collisions are Higgs-strahlung and $W^+W^-$ fusion. The cross section of $ZZ$ fusion is approximately one order of magnitude smaller than the one of $W^+W^-$ fusion, because of weaker couplings. Compact cross section formulas may be found in Ref. 27.

As for the Higgs-strahlung process, the electromagnetic and weak corrections are fully known at one loop. The electroweak corrections for $VV$ fusion, a $2 \rightarrow 3$ process, are not yet available. However, the leading effects can be conveniently included as follows. The bulk of the initial-state bremsstrahlung can be taken into account in the so-called leading logarithmic approximation provided by the structure-function method, by convoluting the tree-level cross

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section with a radiator function, which is known through $O(\alpha^2)$ and can be
further improved by soft-photon exponentiation. The residual dominant cor-
rections of fermionic origin can be incorporated in a systematic and convenient
fashion by invoking the so-called improved Born approximation.

4 Quantum Numbers and Couplings from Final States

The spin, parity, and charge-conjugation quantum numbers $J^{PC}$ of Higgs
bosons can be determined at a future $e^+e^-\text{LC}$ in a model-independent way by
analyzing the threshold behaviour and the angular dependence of the Higgs-
strahlung process. This process can also be employed to place limits on
anomalous $ZZH$ and $Z\gamma H$ couplings. The top Yukawa coupling $g_{ttH}$ and
the trilinear Higgs self-coupling $g_{HHH}$ can be probed by studying the processes
$e^+e^-\to t\bar{t}H$, $e^+e^-\to ZHH$, and $e^+e^-\to \nu_e\nu_eHH$.

5 Conclusions and Outlook

The theoretical predictions for the partial decay widths of the SM Higgs boson
and its production cross sections at a future $e^+e^-\text{LC}$ are generally in good
shape. The strategies for the determination of the Higgs profile are also well
elaborated. The list of urgent tasks left to be done includes the calculation of
the full $O(\alpha)$ corrections for important $2\to 3$ processes, such as $W^+W^-$ fusion,
$ZZ$ fusion, and $t\bar{t}H$ associated production, and the inclusion of background
processes and detector simulation.

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