A Spatially Weighted Degree Model for Network Vulnerability Analysis

WAN Neng¹, ZHAN F. Benjamin¹,², CAI Zhongliang²
1. School Texas Center for Geographic Information Science, Department of Geography, Texas State University, San Marcos, TX 78666, USA
2. School of Resource and Environmental Science, Wuhan University, 129 Luoyu Road, Wuhan 430079, China

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Abstract Using degree distribution to assess network vulnerability represents a promising direction of network analysis. However, the traditional degree distribution model is inadequate for analyzing the vulnerability of spatial networks because it does not take into consideration the geographical aspects of spatial networks. This paper proposes a spatially weighted degree model in which both the functional class and the length of network links are considered to be important factors for determining the node degrees of spatial networks. A weight coefficient is used in this new model to account for the contribution of each factor to the node degree. The proposed model is compared with the traditional degree model and an accessibility-based vulnerability model in the vulnerability analysis of a highway network. Experiment results indicate that, although node degrees of spatial networks derived from the traditional degree model follow a random distribution, node degrees determined by the spatially weighted model exhibit a scale-free distribution, which is a common characteristic of robust networks. Compared to the accessibility-based model, the proposed model has similar performance in identifying critical nodes but with higher computational efficiency and better ability to reveal the overall vulnerability of a spatial network.

Keywords GIS; network analysis; spatial analysis; vulnerability analysis

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Introduction

Spatial networks are networks whose nodes or links are imbedded in a physical space or have specific spatial positions.¹ Examples of spatial networks in geographic space include road networks, airline networks, power grids, and facility networks. Most of these networks are indispensable for today’s society because they provide critical functions, such as power transmission and transportation. The malfunction or destruction of these spatial networks may bring devastating consequences. For example, interruption in a power transmission network may bring great economic loss, especially in metropolitan areas; the long time blockage of a highway may seriously impact local transportation. Effective planning and management of spatial networks require appropriate analysis of their vulnerability to a variety of natural or human-induced disasters.²

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WAN Neng was a Ph.D. student at the Texas Center for Geographic Information Science, Department of Geography, Texas State University-San Marcos when submitting this paper. His research interests include vulnerability analysis, health services research, health disparity, and environmental injustice.

E-mail: wanneng99@gmail.com
This paper defines the vulnerability of a spatial network as the extent to which its proposed function can be affected by the removal or malfunction of its basic elements. This concept considers serviceability, topological integrity, and geography as the most important factors. The corresponding measure of vulnerability, therefore, should not only be able to identify critical elements but also be able to reflect how the entire network is resilient to element destructions.

The past decade witnessed an increasing interest on vulnerability analysis of spatial networks. Researchers tried to analyze the vulnerability of a network either from the perspective of network accessibility or from node degree distribution. Although the degree model based on node degree distribution outperforms accessibility-based measures in revealing the overall vulnerability of a network, it does not work well for spatial networks because it does not adequately account for the geographic aspects of spatial networks.

The goal of this paper is to analyze the vulnerability of spatial networks by integrating geographic knowledge into network topology. We argue that the functional class and length of network links are important factors that affect the node degrees of spatial networks. Based on this observation, we propose a spatially weighted degree model for analyzing the vulnerability of a spatial network. As a validation, we applied the proposed model, the traditional degree model, and an accessibility-based model to a highway network and compared their performances in revealing the overall vulnerability of the network as well as in identifying critical nodes of the network.

1 Related work

1.1 Vulnerability analysis of spatial networks

Vulnerability is also referred to as “robustness” [7] or “resilience” [8] without any apparent distinction. Although the word vulnerability has been defined as the “sensitivity to attack or injury,” [5] a one-for-all definition for the vulnerability of spatial networks has not been proposed. [9] Generally, researchers tend to define the vulnerability of spatial networks based on their own research purposes and the functions of the networks in question. For example, Berdica [3] defines the vulnerability of a road network as susceptibility to unexpected accidents that can result in significant degradation in road serviceability, in which the serviceability of a network element refers to the probability of being in use; Holmgren [4] places emphasis on the collection of properties of an infrastructure network to perform its intentional functions or services when it suffers threats or hazards; and Willis [10] focuses on the ability of a network to resist an attack and suggests vulnerability as the probability of damage from an attack, if it happens. Although their definitions vary, researchers agree that vulnerability is closely dependent on the topological integrity of a network, which can be disrupted by internal or external disturbances.

Generally, researchers evaluate the vulnerability of a spatial network by identifying critical locations or elements within the network, with the emphasis that the malfunction or removal of these locations or elements may decrease the transmission efficiency or increase the total travel cost to a much worse extent than the removal of other elements. For example, Taylor et al. [9] considered a node to be vulnerable if the absence of a small portion of links significantly diminishes the accessibility of that node. Demšar et al. [2] utilized a more complicated mathematical model, which combines a dual graph method with connectivity analysis and topological measures, to identify critical locations in a street network. They concluded that cut nodes and betweenness are the two useful measures for identifying critical nodes for a spatial network. Scott et al. [6] integrated traffic flow and link capacity into a robustness index to identify critical links of transportation networks. These models are generally considered accessibility-based models because they determine critical network elements based on local accessibility within the network. Although these methods have been proven useful in identifying critical locations of a network, none of them was designed to assess the overall vulnerability of a network. In other words, these methods cannot provide a quantitative measure indicating how vulnerable a network is. This study attempts to fill this void.

1.2 Degree distribution

Node degree is the number of links connected to a network node. Degree distribution, or the frequency
of node degrees of a network, is an effective measure of network vulnerability based on the assumption that nodes with higher degrees are more important than those with lower degrees.\cite{8,11} A robust degree distribution is characterized by the fact that the random removal of a small portion of nodes will not affect the stability and functionality of the entire network.\cite{7}

Based on the distribution of node degrees, networks can be categorized into two types: random networks (or homogeneous networks)\cite{12,13} and scale-free networks.\cite{7,14,15} For random networks, each node has approximately the same number of links. For scale-free networks (or inhomogeneous networks), the probability of a node having \(k\) links \((P(k))\) follows a power law:

\[
P(k) \sim k^{-\gamma}
\]

where \(\gamma\) is a skew coefficient.

For a scale-free network, a small portion of nodes have relatively higher degrees, while the others have lower degrees. These highly connected nodes act as “hubs” and play a critical role in maintaining the topological integrity and the basic function of the network. An obvious characteristic of scale-free networks is that they are resilient to random removal of nodes, which means that random attacks on the network will not hinder it from functioning well. This resilience enables networks to work in a steady status. However, scale-free networks are sensitive to attacks targeted at hubs whose malfunction may bring disastrous outcomes for the entire network. The skew coefficient \(\gamma\), which can reflect the proportion of hubs among all nodes, indicates the extent to which the whole network is resilient to random or targeted attacks. Compared to accessibility-based methods, degree distribution not only identifies critical locations, or hubs, but also provides an overall measure of vulnerability of the entire network.

Although some spatial networks, such as the WWW and power grids, have been proved scale-free,\cite{14,16} it is inappropriate to study their vulnerability directly from degree distribution. The main reason is that node degree is purely a topological concept, but the robustness of spatial networks does not rely solely on network topology. The geographical information that comes with the spatial locations of network elements is an important factor for spatial network analysis and should not be omitted.\cite{17} However, geographical information has seldom been incorporated into node degree models for the purpose of vulnerability analysis.

### 2 A spatially weighted degree model

The importance of geographical information can be further illustrated in a highway network in Fig.1, in which nodes 1 and 2 have the same degree value of three based on the traditional definition of node degree. However, the importance of the two nodes may differ since node 2 is only connected by two local highways, while node 1 is the intersection of two interstate highways. Also, longer links can make the node more critical because their blockage may impact the transportation of more people along the highway. The removal of node 1 may bring worse consequences than the removal of node 2 if the differences in functional class and length of their links are considered.

![Fig. 1 Difference of nodes with the same degree value](image)

Since the traditional definition of a node degree is insufficient to differentiate the links of spatial networks, we introduce the concept of spatially weighted node degree in this paper. As an effective measure to distinguish critical nodes, this spatially weighted node degree should not only be able to reflect the number of links, as the traditional degree model does, but also be capable of incorporating geographic knowledge, which varies among different links. We consider two geographic factors that have a direct effect on node degree: functional class and length of links. Functional class categorizes links according to their functional importance. For example, in a highway network, the important road that links from the view of socio-economic effect will be sections of the main roads (interstate highway) going through populated areas. Although link length does not contribute to spatial degree in all cases, it is an important measure for some networks, such as road networks be-
cause longer links would certainly lead to higher travel cost or longer transition time.

Based on the discussions given above, the spatially weighted degree model can be expressed as

\[
N_{sd}^i = \sum_{j=1}^{ec_i} c_i (1 + \omega \frac{l_j - l_{min}}{l_{max} - l_{min}})
\]

(2)

where \(N_{sd}^i\) is the spatially weighted degree of node \(i\), \(ec_i\) is the number of links connected to node \(i\), \(c_j\) and \(l_j\) are the functional class and the length of the \(j^{th}\) link, \(l_{min}\) and \(l_{max}\) are the minimum and maximum link length of the entire network under consideration, and \(\omega\) is a weight coefficient specifying the importance of link length in the specific domain.

The physical implication of the model is that the number of links connected to a node, the functional class, and the length of each link are factors that should be accounted for in calculating node degrees of a network, and the relative importance of each factor to the node degree in question can be reflected in the weight coefficient \(\omega\). A small value of \(\omega\) means that the link length does not contribute much to the node degree, and a larger \(\omega\) value indicates otherwise. The existence of \(\omega\) is necessary because the contribution of link length varies according to the specific type of a spatial network. For example, link length has less impact on the WWW than on ground transportation networks because of the fast speed of data transmission through wires.

3 Experimental analysis

We tested the proposed model using the highway network of Texas, USA. The network data was derived from the U.S. National Highway Planning Network (NHPN) database,[18] which represents approximately 400000 miles of federal-aided highways in the 50 states and Puerto Rico. The Texas part of NHPN is composed of 6687 links and 4099 nodes (Fig.2).

This study used the Texas portion of the NHPN highway network, and as such, the following experiments and analysis were restricted to it as the sole road network for road transportation. This is an approximate representation of networks in the real world because some local roads or streets are neglected. However, it suffices to use the network to demonstrate the usefulness of the model for network vulnerability analysis.

Fig. 2 The highway network of Texas derived from NHPN. Texas county map (gray line) is used as the base map.

The functional class for highway roads (or links) was set based on the classification of the 2002 Highway Performance Monitoring System (HPMS), which distinguished roads according to their geographic locations (Rural or Urban), physical class (Interstate, Arterial, Collector, or Local), and socio-economic importance (Principle, Major, or Minor). Each class was assigned a value to denote its significance (Table 1). The classification scheme is considered reasonable for the analysis because it accounts for all the factors mentioned in Section 2. The weight coefficient \((\omega)\) was set to 1 since it was assumed that link length is important for the analysis.

We conducted the experiment analysis using the ArcGIS software version 9.3 (http://www.esri.com). The mathematical models mentioned in this paper were developed with Visual Basic for Applications (VBA) that comes with ArcGIS 9.3.

Table 1 Class definition of US highways according to HPMS and the functional class value of each class
(A collector is a road whose service level is between arterial road and local road)

| Class definition | Rural interstate | Rural principal arterial | Rural minor arterial | Rural major collector | Rural minor collector | Rural local |
|------------------|------------------|-------------------------|---------------------|----------------------|----------------------|------------|
| Class value      | 6                | 5                       | 4                   | 3                    | 2                    | 1          |
| Class definition | Urban interstate | Urban freeway           | Urban principal arterial | Urban minor arterial | Urban collector | Urban local |
| Class value      | 12               | 10                      | 8                   | 6                    | 4                    | 2          |
4 Results

4.1 Degree distribution

The degree distributions of the traditional degree model and the spatially weighted degree model are shown in Fig. 3. To further illustrate how network vulnerability is reflected by degree distribution, we proposed a simple classification method to categorize nodes based on the degree value. The method divides degree values into four subranges: Small Values (SVs), Medium Values (MVs), Large Values (LVs), and Extra Large Values (ELVs). The numbers of nodes whose degree values fall into the four subranges are recorded for further analysis. The classification scheme is applied to degrees calculated from both models for comparison.

As shown in Table 2, node degrees calculated by the traditional model mainly focus on SVs and MVs, with few cases of LVs and ELVs. This means nodes have approximately the same number of degrees as determined by the traditional model, thus indicating a homogeneous topology of the network. The result is in accordance with the work of Barabasi and Bonabeau,[16] which categorized highway networks into random networks and showed that their degrees follow a bell-shape distribution.

Table 2 Numbers of nodes whose values fall into the subranges for the two degree models

| Subrange          | Frequency(%) | Subrange          | Frequency(%) |
|-------------------|--------------|-------------------|--------------|
| Small values      | 3            | [24.0, 35.2)      | 43.5         |
| Medium values     | 4            | [35.2, 46.4)      | 28.2         |
| Large values      | 5            | [46.4, 57.6)      | 26.1         |
| Extra large values| [6,7]        | [57.6, 80.0]      | 2.2          |

Note: The interval between subranges is 1/5 of the total range.

Node degrees of the network as determined by the spatially weighted model follow a scale-free distribution with an approximate $\gamma$ value of 1.4 (Fig. 3(b)). As determined by $\gamma$, the spatially weighted model produces fewer SVs and MVs and more LVs and ELVs than the traditional model. This means that node degrees determined by the spatially weighted model are more diversely distributed. The physical implications of this structure can be understood in two ways: (1) the highway network is resistant to random errors or perturbations, because most of the nodes are with small or medium degree values, and (2) the existence of a certain amount of large degree and extra large degree nodes makes the highway network vulnerable to targeted attacks on these nodes. However, since we analyze the vulnerability of the network from the socio-economic perspective instead of homeland security or military purposes, the possibility of targeted attack on hubs of the network can be mostly neglected. From this point of view, the results suggest that the Texas highway network is robust to node destruction.

Another characteristic of the spatially weighted model is that its degree values have a wider range. As shown in Fig. 3, the proposed model yields more degree values than the traditional model. In fact, the range could be wider by adjusting the parameters of the graphing software. On the other hand, the traditional model produces only a limited number of degrees (in this paper, the degrees are 3, 4, 5, 6, and 7). This advantage makes the spatially weighted model more suitable for research purposes because it can provide a much smaller sampling distance for the relatively larger degree range.
4.2 Critical nodes

In this section, the ability of the proposed model to identify critical nodes is assessed. The traditional degree model and an accessibility-based vulnerability measure, the Hansen Index, are also implemented for comparison.

The Hansen Index, proposed by Taylor et al., can be expressed as

\[
A_i = \frac{\sum_j B_j f(c_{ij})}{\sum_j B_j} \tag{3}
\]

where \(A_i\) is the Hansen Index of the \(i\)th node, \(B_j\) is the attractiveness (e.g., population or economic status) of the \(j\)th node, and the impedance function \(f(c_{ij})\) represents the separation between nodes \(i\) and \(j\). The reason why this model is chosen is because, compared to other topology-based methods, Hansen Index reveals network accessibility from the perspective of socio-economic impact (local attractiveness), which is exactly what the spatially weighted model is proposed for. The Hansen Index has a better capability in identifying critical network elements than other topology-based measures.

Local population and the reciprocal of shortest paths between nodes are used as the attractiveness and the impedance function, as suggested by Taylor et al. The population density of network nodes is estimated from the 2000 census block population data (http://www.census.gov/main/www/cen2000.html).

Nodes with LVs and ELVs of the proposed model are selected as critical nodes (Fig.4(b)). The same number of critical nodes for the Hansen Index model and the traditional degree model are also selected for a comparison (Fig.4). As shown in the figures, there is no global clustering for any model but local clusters for all of the three models. For the spatially weighted model and Hansen Index, critical nodes are distributed in a similar pattern: nodes are prone to cluster in urban areas. The Dallas-Fort Worth metropolitan area and the Houston metropolitan area have the highest density of critical nodes. The Austin-San Antonio corridor area and the El Paso metropolitan area also have high densities. However, for the traditional degree model, critical nodes are more sparsely distributed: although urban-clusters can be found in the Dallas-Fort Worth area and the San Antonio area, more critical nodes are located in nonmetropolitan regions. Differences among different models can be reflected in overlap rates of their critical nodes. The overlap rate between the proposed model and Hansen Index is 84.1%, as compared to 56.4% between the traditional model and Hansen Index. This result indicates that the proposed model has similar performance in identifying critical elements as Hansen Index than the traditional model.

![Fig. 4 Geographical distribution of critical nodes](image)

Critical nodes are determined as the first 1072 nodes with high strength calculated by the respective model. Critical nodes are shown as black dots in (b), (c), and (d).
To gain a further view of the local distribution of these critical nodes, we examined their quantity and density distribution among the 254 counties of Texas. The top 10 counties with most quantity and high density of critical nodes as determined by both models are listed in Table 3.

Table 3 Top 10 counties with most critical nodes and The highest density of critical nodes

| Ranking | County name | No. of critical nodes | County name | No. of critical nodes (per square miles) |
|---------|-------------|-----------------------|-------------|------------------------------------------|
| 1       | Dallas      | 166                   | Dallas      | 0.185                                    |
| 2       | Tarrant     | 156                   | Tarrant     | 0.173                                    |
| 3       | Harris      | 155                   | Harris      | 0.089                                    |
| 4       | Bexar       | 79                    | Bexar       | 0.063                                    |
| 5       | Travis      | 66                    | Travis      | 0.063                                    |
| 6       | El Paso     | 41                    | Lubbock     | 0.045                                    |
| 7       | Lubbock     | 41                    | El Paso     | 0.041                                    |
| 8       | Nueces      | 22                    | Gregg       | 0.035                                    |
| 9       | Smith       | 21                    | Nueces      | 0.026                                    |
| 10      | Ector       | 20                    | Galveston   | 0.024                                    |

As is shown in Table 3, Dallas county, Tarrant county, and Harris county have the highest density and most quantity of critical nodes. It can also be observed that most of these top ten counties are in the eastern part of the state. The reason may lie in the high-density highway network or high level of economic development in these counties. This result is useful for policy makers for socio-economic development decision-making since the quantity and density of critical nodes can directly or indirectly reflect the regional vulnerability of the highway network.

4.3 Computational efficiency of the models

Computational efficiency is another important criterion for algorithm evaluation. In this part, the efficiencies of the three models are evaluated. The models were implemented in a PC equipped with an Intel Core2 2.00 GHz CPU with 2.00 GB Memory. The running time of the models for processing the network data is listed in Traditional degree model, which is 0.50 s. Spatially weighted degree model is 0.59 s. Hansen index is 401.45 s.

As can be seen from above, it took 0.50 s, 0.59 s, and 401.45 s for the three methods to process the entire highway network used in this study. The Hansen Index took much more time than the other two. Considering the time spent on selecting shortest paths between nodes (about 25 hours under the same PC environment), the calculation time could be even longer for the Hansen Index. Thus, it can be concluded that the proposed model and the traditional model have better computational efficiency than Hansen Index.

5 Conclusion

This paper presents a spatially weighted degree model for analyzing the vulnerability of spatial networks. Instead of focusing only on the topology of network, the proposed model takes into account of geographic knowledge, such as functional class and length of network links when calculating node degrees. Experimental results using the Texas highway network indicate that the proposed spatially weighted degree model is more suitable in vulnerability analysis of spatial networks than the traditional degree model. The new model also has advantages in computational efficiency and in revealing overall vulnerability of the entire network when compared with an accessibility-based method, the Hansen Index.

This spatially weighted model may not be a good choice for vulnerability analysis of networks whose function depends solely on topology. However, it is applicable to network analysis, which considers not only network topology but also other characteristics of a network. The functional class in the model can be substituted by other factors, depending on the specific aims of the network analysis. This flexibility makes the model applicable to different kinds of spatial networks and a wide variety of applications involving spatial networks.

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**Notes to Contributors**

Contributions are welcomed on one of the following subjects or in related areas:

- ★ GIS
- ★ GPS
- ★ RS
- ★ Cartology
- ★ Geodynamic
- ★ Geo-surveying
- ★ Photogrammetry
- ★ Graphics
- ★ Physical geo-surveying
- ★ Engineering surveying
- ★ Mapping apparatus

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Feel free to contact us.

E-mail: gsis@whu.edu.cn
          sjjs.2008@gmail.com
Tel:+86-27-68778045