Color Superconductivity with Non-Degenerate Quarks

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Abstract

In dense quark matter, the response of the color superconducting gaps to a small variation, $\delta \mu$, in the chemical potential of the strange quark was studied. The approximation of three massless flavors of quarks and a general ansatz for the color flavor structure of the gap matrix was used. The general pole structure of the quasi-particle propagator in this ansatz is presented. The gap equation was solved using both an NJL interaction model and perturbative single gluon exchange at moderate densities and results are presented for varying values of $\delta \mu$. Quantitative and qualitative differences in the dependence of the gaps on $\delta \mu$ were found.
1 Introduction

The physics of strongly interacting matter at high densities and low temperatures has been the subject of much research in recent years. It has long been known\textsuperscript{[1]} that at sufficiently high densities a system of quarks should form a condensate of Cooper pairs which breaks the $SU_C(3)$ symmetry and becomes a color superconductor. The formation of the condensates leads to gaps in the quasi-particle mass spectrum. The authors of \textsuperscript{[1]} estimated that the gaps were of the order of $\Delta \sim 10^{-3} \mu$ where $\mu$ is the quark chemical potential. Recently it was shown at realistic values of $\mu$ in an instanton induced NJL model that gaps of the order of $\mu$ could be obtained\textsuperscript{[2]}. This stimulated a great deal of research\textsuperscript{[1]} in the ensuing years and has resulted in a proliferation of predicted superconducting ground states. These states may be realized in the cores of neutron stars and lead to observable effects\textsuperscript{[4, 5]}.

It is widely accepted\textsuperscript{[6, 7]} that the color superconducting ground state at asymptotic densities is the Color Flavor Locked (CFL) state\textsuperscript{[6]}:

$$\langle q^i_\alpha C \gamma^5 q^j_\beta \rangle = \Delta_{33}(\delta^i_\alpha \delta^j_\beta - \delta^i_\beta \delta^j_\alpha) + \Delta_{66}(\delta^i_\alpha \delta^j_\beta + \delta^i_\beta \delta^j_\alpha), \quad (1)$$

where the Greek indices are color indices, the Latin indices are flavor indices and the 3 and 6 subscripts refer to anti-triplet or sextet configurations in color and flavor spaces respectively. At lower densities ($\mu \sim m_s^2/4\Delta$), it is likely that the ground state is a superconducting state involving the condensation of Cooper pairs in the $u-d$ sector only (2SC). Finally at still lower densities the favored ground state will be ordinary hadronic matter. Two new phases have recently been predicted: Crystalline Color Superconductivity\textsuperscript{[8]} and CFL with meson condensation\textsuperscript{[9]}. These predictions, while not necessarily at odds with one another, indicate that the transition region between the CFL state and hadronic matter is not completely understood.

Including the strange quark mass in the gap equation with no approximations introduces two sets of complications: 1) massive quarks means that there are 4 new types of Dirac structures allowed for the condensates, and coupling between the corresponding gaps; 2) the fact that the strange quark is different from the other quarks means that gaps involving the strange quark should be different from those with zero strangeness.

In order to understand the implications of these two complications it is useful to separate them and understand them individually before tackling

\textsuperscript{1}For extensive references see the review articles \textsuperscript{[3].}
the full problem. Therefore, in this paper, all three quarks are treated as massless and the strange quark is distinguished by giving it a different chemical potential ($\mu_s = \mu - \delta \mu$) than the $u$ and $d$ quarks. This is motivated by observing\[10\] that for fixed Fermi energy, the Fermi momentum of a massive particle is less than the Fermi momentum of a massless particle:

\[ E_F = p_{F0} = \mu_0 \quad \text{for massless particles}, \]
\[ E_F = \sqrt{p_{Fm}^2 + m^2} \approx p_{Fm} + \frac{1}{2} \frac{m^2}{p_{Fm}} = \mu_0 \quad \text{for massive particles}. \]

This approximately corresponds to a shift in the Fermi momentum or the chemical potential:

\[ \mu_m = p_{Fm} \approx \mu_0 - \frac{1}{2} \frac{m^2}{p_{Fm}} \equiv \mu_0 - \delta \mu. \]

This approach is similar to that taken in [11] where this approach was studied using a four fermion interaction (NJL) model. Qualitatively similar results to those of [11] were obtained in [10] using an NJL interaction and a different prescription for inclusion of strange quark mass effects. In this paper an NJL model is considered in Section 3 for comparison to previous results and to illustrate the contrast with the results of the subsequent section. In section 4, perturbative single gluon exchange is considered for the interaction of the quarks. This approach is valid in the high density regime and gives an approximate model in the moderate density regime which is distinct from and can be compared to the NJL interaction model.

Understanding the effects of a shift in the chemical potential of one of three quark flavors is another motivation for this research. The papers [8, 12, 13, 14] also deal with shifts in the chemical potential in an NJL model, mostly in the two flavor case and find some interesting effects. The three flavor case was discussed in [8] and [13] but was not analyzed in detail. In [13] it was predicted that the number densities of the 3 quark flavors are constrained to be equal up to $\delta \mu = \Delta_{3\bar{3}}/\sqrt{2}$, at which point there is a first order transition to some less symmetric phase of quark matter. It would be interesting to test this prediction using perturbation theory. Finally in [15] first order perturbation theory is used to study the non-degenerate two flavor case. In this work, first order perturbation theory is applied in the non-degenerate three flavor case.
Perturbation theory applied to Color Superconductivity has been the subject of much important research\textsuperscript{2}. It has not previously been applied to the case of color superconductivity with three flavors of non-degenerate quarks.

Recently numerical results were obtained\textsuperscript{13} in the 2 flavor case in perturbation theory with single gluon exchange and massless and degenerate quarks. The $\delta \mu = 0$ results presented in this paper for perturbative single gluon exchange with three flavors are qualitatively similar and can be considered as independent confirmation of \textsuperscript{13} as well as an extension of them to three flavors.

The fact that the three flavor case is different from the two flavor case for non-degenerate flavors should be emphasized at this point. This is clear from the NJL analyses of the two different cases. In \textsuperscript{3,14} the gap is shown to be independent of $\delta \mu$ up to some critical value. In \textsuperscript{10,11}, $\delta \mu$ has a significant effect on the pattern of condensation even at small values of $\delta \mu$. This difference is simply a result of the fact that the algebra of the generators of $SU(2)$ is much simpler than the algebra of the generators of $SU(3)$. The important point that this illustrates is that results from the two flavor case may not tell the full story in the three flavor case.

The goals of this work are threefold. The first goal is to understand the effects of the non-degeneracy of the strange quark in a simplified model of QCD as a first step toward understanding the effect of the strange quark mass in a more complete way. The second goal of this work is to understand the effects of a small shift in the chemical potential on solutions of the gap equation for comparison with results in other approaches to the problem. The third goal is to determine how first order perturbative results compare to results obtained using the NJL interaction model.

Results presented in this paper are the poles of the quasiparticle propagator in this approach in a more general ansatz than in \textsuperscript{11} and in a different model than \textsuperscript{10}. Numerical solutions for the gaps at moderate densities using both an NJL interaction model and perturbative single gluon exchange are presented to illustrate the effect of $\delta \mu$ and the qualitative and quantitative differences in the results.

\textsuperscript{2}See for example \textsuperscript{10,17,18} and references in \textsuperscript{3}. 

3
2 Gap Equation

The gaps resulting from the formation of a color superconducting ground state can be determined by solving the mean field gap equation [20]:

$$\Delta(k) = -ig^2 \int \frac{d^4q}{(2\pi)^4} \sum_{A,B} D_{AB}^{\mu\nu}(k-q)\gamma^{\mu}(\frac{\lambda^A_c}{2})^T G^{-}_0(q)\Delta(q)G^{+}(q)\gamma^\nu\frac{\lambda^B_c}{2}$$

(5)

which is specialized to an interaction with the color structure of single gluon exchange. $D_{AB}^{\mu\nu}(k-q)$ is the gluon propagator, $\lambda^A_c$ are the Gell-Mann matrices, $G^{-}_0(q)$ is the bare antiquark propagator, $G^{+}(q)$ is the full quasiparticle propagator and $\Delta(k)$ is the gap matrix.

The gap matrix is a matrix in color, flavor and spinor space that contains all the specific gaps. Restricting consideration to spin zero gaps in the massless case the Dirac structure of the gap matrix can be written in a basis of four projectors [20]:

$$P^e_h(\vec{k}) = \Lambda^e(\vec{k}) P_h(\vec{k}) = \left( \frac{1 + e \gamma_0 \vec{\gamma} \cdot \vec{k}}{2} \right) \left( \frac{1 + e \gamma_5 \gamma_0 \vec{\gamma} \cdot \vec{k}}{2} \right), \quad e, h = \pm 1$$

(6)

which are products of projectors onto positive and negative energy states and positive and negative helicity states.

The color flavor structure of all objects will be given as $9 \times 9$ matrices consisting of a $3 \times 3$ matrix in flavor space whose components are $3 \times 3$ matrices in color space:

$$i = u \quad j = u, d, s$$

$$i = d \quad \quad \alpha, \beta = r, g, b$$

$$i = s$$

The most general gap matrix can be written as:

$$\Delta(\vec{k}) = \sum_{e,h=\pm 1} \left( \Delta_{ij}^{\alpha\beta} \right)_h^e P^e_h(\vec{k})$$

(8)
The gluon propagator, $D_{AB}^{\mu\nu}$, is the object that differs between the NJL model and perturbation theory. In the NJL model the interaction is modeled by a 4 fermion interaction and the gluon propagator is replaced by an identity matrix. This constitutes a low energy effective model of the interaction. In perturbative single gluon exchange $D_{AB}^{\mu\nu}$ is the bare gluon propagator. Perturbation theory is definitely valid at high energies and densities. At moderate densities perturbation theory can be considered a model for the quark interactions.

The bare propagator for the a quark of flavor $f$ is given by:

$$
(G_0^+(k))_f = (\gamma^\nu k_\nu \pm \mu_f \gamma_0)^{-1} = \left((k_0 \pm \mu_f)\gamma_0 - \bar{\gamma} \cdot \vec{k}\right)^{-1}
$$

(9)

If all $\mu_f$ are equal the particles are degenerate and the bare propagator is the identity on the color-flavor space. In the case considered here the bare antiquark propagator can be written in the form:

$$
G_0^- = \left((k_0 - \mu - \delta \mu P_f^{(s)})\gamma_0 - |k| \bar{\gamma} \cdot \vec{k}\right)^{-1}
$$

\begin{align}
&= \gamma_0 \sum_{e,h} \frac{P_f^{(u)} + P_f^{(d)}}{(k_0 - \mu) + e |k|} P_e(k) + \gamma_0 \sum_{e,h} \frac{P_f^{(s)} P_h(k)}{(k_0 - \mu - \delta \mu \delta_3) + e |k|}
\end{align}

(10)

where the $P^{(i)}$ are projectors onto a specific flavor sub-space, ie:

$$
P_f^{(u)} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

(11)

where bold font indicates $3 \times 3$ matrices in color space, and similarly for $P_f^{(d)}$ and $P_f^{(s)}$.

The quasiparticle (quasi-antiparticle) propagator is determined from the bare particle and antiparticle propagators and the gap matrix by the relation:

$$
G^\pm \equiv \left\{ [G_0^\pm]^{-1} - \Delta^\mp G_0^\pm \Delta^\pm \right\}^{-1}.
$$

(12)

where $\Delta^+ \equiv \Delta$ and $\Delta^- \equiv \gamma_0 \Delta^+ \gamma_0$.

The ansatz for the color-flavor structure of the gap matrix used in this
research is:

$$\Delta_{ij} = \begin{pmatrix}
\frac{a+h+e}{2} & 0 & 0 & 0 & \frac{a+h-e}{2} & 0 & 0 & 0 & c \\
0 & 0 & 0 & e & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & f & 0 & 0 & 0 \\
0 & e & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{a+h-e}{2} & 0 & 0 & 0 & \frac{a+h+e}{2} & 0 & 0 & 0 & c \\
0 & 0 & 0 & 0 & 0 & 0 & f & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & f & 0 & 0 & 0 & 0 & 0 \\
c & 0 & 0 & 0 & c & 0 & 0 & 0 & a-h \\
\end{pmatrix}$$

(13)

where:

$$\frac{a+h-e}{2} = \Delta_{66}(ud) + \Delta_{33}(ud),$$

(14)

$$e = \Delta_{66}(ud) - \Delta_{33}(ud),$$

(15)

$$c = \Delta_{66}(us) + \Delta_{33}(us),$$

(16)

$$f = \Delta_{66}(us) - \Delta_{33}(us),$$

(17)

$$a - h = \Delta_{66}(ss)$$

(18)

The form of this ansatz is essentially the same as in [10] with minor differences to simplify the poles of the quasi-particle propagator. The correspondence between the different conventions is:

$$b = \frac{a+h-e}{2} \quad d = a-h$$

(19)

and $e,c,f$ unchanged.

In the case of degenerate color-flavor locking:

$$c = \frac{a+h-e}{2} \quad e = f \quad a-h = \frac{a+h+e}{2}$$

(20)

In order to simplify the calculation, following the example of [10], the ansatz (13) is rotated to a different basis. The columns (rows) are referred to by the combination of color and flavor indices, $(i, \alpha)$.\footnote{Therefore for instance the 2nd column is the $(i, \alpha) = (u, g)$ column.} The $(u, r)$ and $(d, g)$ basis vectors are rotated to new basis vectors $1/\sqrt{2}((u, r) \mp (d, g))$ and all
the other basis vectors are left unchanged[^4]. Note that this change of basis leaves \( P_f^{(u)} + P_f^{(d)} \) and \( P_f^{(s)} \) operators unchanged which means that the bare propagator has the same form in this new basis. In this basis the gap matrix now has the form:

\[
\Delta_{ij}^{\alpha\beta} = \begin{pmatrix}
e & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & e & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & f & 0 & 0 \\
0 & e & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a + h & 0 & 0 & \sqrt{2}c \\
0 & 0 & 0 & 0 & 0 & 0 & f & 0 \\
0 & 0 & f & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & f & 0 & 0 & 0 \\
0 & 0 & 0 & \sqrt{2}c & 0 & 0 & a - h \\
\end{pmatrix}
\]

(21)

Of course all other objects with color or flavor structure must be transformed to the same basis. These details are only included in order that the quasi-particle propagator can be presented below in a fairly concise way.

Up until this point no assumptions about the Dirac structure of the gap matrix has been made. For ease of calculation in the perturbative analysis the gap matrix is assumed to have the usual form:

\[
\Delta(k) = \langle q^i_\alpha(-k) C \gamma_5 q^j_\beta(k) \rangle = \Delta_{ij}^{\alpha\beta} \sum_{e,h=\pm1} e h P_h^e(k)
\]

(22)

Substituting (22) and (21) into (12), the quasiparticle propagator is:

\[
G^+ = \begin{pmatrix}
E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & E & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & F_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & E & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A & 0 & 0 & C \\
0 & 0 & 0 & 0 & 0 & F_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & F_2 & 0 \\
0 & 0 & 0 & 0 & C & 0 & 0 & B \\
\end{pmatrix},
\]

(23)

[^4]: This means for example that the new first column is \( \frac{1}{\sqrt{2}} ((u, r) - (d, g)) \) and similarly for the fifth column.
where:

\[
E = \frac{q_0 + l}{(q_0 - l)(q_0 + l) - e^2} \Lambda^+ + [l \rightarrow -l'] \Lambda^- \tag{24}
\]

\[
F_1 = \frac{q_0 + l - \delta \mu}{(q_0 - l)(q_0 + l - \delta \mu) - f^2} \Lambda^+ + [l \rightarrow -l'] \Lambda^- \tag{25}
\]

\[
F_2 = \frac{q_0 + l}{(q_0 - l + \delta \mu)(q_0 + l) - f^2} \Lambda^+ + [l \rightarrow -l'] \Lambda^- \tag{26}
\]

\[
A = -\left[ \frac{(a - h)^2(l + q_0) + (2c^2 + (l + q_0)(l - q_0 - \delta \mu))(l + q_0 - \delta \mu))}{(q_0^2 - \epsilon_+(a, h, c, l, \delta \mu)^2)(q_0^2 - \epsilon_-(a, h, c, l, \delta \mu)^2)} \right] \Lambda^+ - [l \rightarrow -l'] \Lambda^- \tag{27}
\]

\[
B = -\left[ \frac{(a + h)^2(l + q_0 - \delta \mu) + (2c^2 + (l - q_0)(l + q_0 - \delta \mu))(l + q_0))}{(q_0^2 - \epsilon_+(a, h, c, l, \mu, \delta \mu)^2)(q_0^2 - \epsilon_-(a, h, c, l, \mu, \delta \mu)^2)} \right] \Lambda^+ - [l \rightarrow -l'] \Lambda^- \tag{28}
\]

\[
C = \left[ \frac{\sqrt{2c}(a(2l + 2q_0 - \delta \mu) - h \delta \mu)}{(q_0^2 - \epsilon_+(a, h, c, l, \mu, \delta \mu)^2)(q_0^2 - \epsilon_-(a, h, c, l, \mu, \delta \mu)^2)} \right] \Lambda^+ + [l \rightarrow -l'] \Lambda^- \tag{29}
\]

and:

\[
\epsilon_\pm(a, h, c, l, \delta \mu)^2 = (l - \delta \mu + \delta \mu^2 + a^2 + 2c^2 + h^2) = \pm \sqrt{\frac{4a^2(2c^2 + h^2) + 8a(2l - \delta \mu)h + 1}{4} \delta \mu^2(8c^2 + (2l - \delta \mu)^2)} ,
\]

using the definitions:

\[
l = (|q| - \mu), \tag{31}
\]

\[
l' = (|q| + \mu), \tag{32}
\]

which are the only ways that the functions depend on \(|q|\) and \(\mu\).

The form \((23)\) of the propagator is consistent with expectations. The strange quark has been distinguished from the up and down quarks. The ansatz still has flavor locked to color and therefore the blue quark has been distinguished from the red and green quarks. The \((u, g)\) and \((d, r)\) quarks propagators are unchanged. The \((u, b)\) and \((d, b)\) quarks propagators are affected in exactly the same way. The \((s, r)\) and \((s, g)\) propagators are also affected in exactly the same way. If one rotated back to the original basis one would
find the \((u, r)\) and \((d, g)\) propagators are also affected in exactly the same way. The propagator could be completely diagonalized to separate the poles which would simplify the calculation in some ways but would complicate it in others, so this form is used in this work.

The poles of the quasi-particle propagator are therefore:

\[
q_0 = \pm \sqrt{l^2 + e^2} \quad \text{degeneracy } = 3, \quad (33)
\]

\[
q_0 = \frac{\delta \mu}{2} \pm \sqrt{\left(l - \frac{\delta \mu}{2}\right)^2 + f^2} \quad \text{degeneracy } = 2, \quad (34)
\]

\[
q_0 = -\frac{\delta \mu}{2} \pm \sqrt{\left(l - \frac{\delta \mu}{2}\right)^2 + f^2} \quad \text{degeneracy } = 2, \quad (35)
\]

\[
q_0 = \pm \epsilon_-(a, h, c, l, \delta \mu) \quad \text{degeneracy } = 1, \quad (36)
\]

\[
q_0 = \pm \epsilon_+(a, h, c, l, \delta \mu) \quad \text{degeneracy } = 1. \quad (37)
\]

It can be shown that in the case where the sextet gaps\(^5\) are neglected, the poles of this propagator reduce to those given in [11]. Further the first eight of the poles reduce to the octet poles and the last pole reduces to the singlet pole at \(\delta \mu = 0\). The poles obtained in this model are based on a different model than [10].

Substituting the ansatz [21], the solution for \(G^+([23])\), and the bare propagator into the gap equation [5] one obtains a matrix gap equation.

\[
\Delta(k) = -ig^2 \int \frac{d^4 q}{(2\pi)^4} D^{\mu \nu}(k - q) \gamma_\mu \gamma^0 \gamma^5 \gamma^0 \gamma \cdot \hat{k} \quad (38)
\]

\[
\left(\Lambda^+(q) M(a, h, c, l, q_0, \delta \mu) - \Lambda^-(q) M(a, h, c, -l', q_0, \delta \mu)\right) \gamma^0 \gamma_\nu
\]

\[
= -ig^2 \int \frac{d^4 q}{(2\pi)^4} D^{\mu \nu}(k - q) \gamma^5 \gamma_\mu \gamma \cdot \hat{k} \gamma^0 \quad (39)
\]

\[
\left(\Lambda^-(q) M(a, h, c, l, q_0, \delta \mu) - \Lambda^+(q) M(a, b, a_{12}, -l', q_0, \delta \mu)\right) \gamma_\nu
\]

\(^5\)The sextet gaps are much smaller than the anti-triplet gaps.
where $D^\mu _{AB}(k - q) = \delta _{AB}D^\mu (k - q)$ has been used, the index $A$ has been summed over and the $\lambda$ matrices have been multiplied through.

The matrix $M(a, h, c, l, q_0, \delta \mu)$ has the form:

$$M = \begin{pmatrix}
M_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & M_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & M_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & M_{55} & 0 & 0 & 0 & M_{59} \\
0 & 0 & 0 & 0 & 0 & M_{55} & 0 & 0 & M_{37} \\
0 & 0 & M_{73} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M_{73} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M_{95} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M_{95} & 0 & 0 & M_{99}
\end{pmatrix} \tag{40}$$

which should be compared to the form of the ansatz (24). In general $M_{73} \neq M_{37}$ and $M_{95} \neq M_{59}$ which can be shown to be inevitable if one examines the product $G_0^- \Delta^+ G^+$ carefully. However, the terms which spoil the symmetry of these components can be shown to vanish under the $q_0$ integration over the interval $(−\infty, \infty)$. This is immediately clear in the NJL model and follows with a little more care in perturbation theory. Therefore the matrix gap equation does close under this ansatz.

Everything discussed so far follows without specification of the interaction model. In the following section the gap equations in the NJL interaction model will be given for the ansatze above. Solutions for the gaps will be presented in order to compare to previous results and to highlight the different behavior of the gaps using perturbation theory.

### 3 NJL Interaction Model

In an NJL model the fermion interaction is taken to be:

$$g^2 D^\mu _{AB} \rightarrow Gg^\mu \delta _{AB} \tag{41}$$

which is the same form as the interaction model of [10] and the gap equation becomes:

$$\Delta(k) = -iG \int \frac{d^4 q}{(2\pi)^4} \gamma ^5 \gamma ^\nu \gamma \cdot \hat{k} \gamma ^0 \left( \Lambda ^-(q)M(a, h, c, l, q_0, \delta \mu) - \Lambda ^+(q)M(a, h, c, -l', q_0, \delta \mu) \right) \gamma _\nu \tag{42}$$
Dirac indices one obtains:

\[ h = 2(a - h) c^2 - (a + h) ((a - h)^2 + (l - \delta \mu)^2 - q_0^2) \]
\[ e = 2(a - h) c^2 - (a + h) ((a - h)^2 + (l - \delta \mu)^2 - q_0^2) \]
\[ f = 4iG \int \frac{d^4q}{(2\pi)^4} \left( \frac{c(a^2 - 2c^2 - h^2 - l^2 + q0^2 + l\delta\mu)}{2(q_0^2 - \epsilon_+^2)(q_0^2 - \epsilon_-^2)} \right) + [l \rightarrow -l'] \]
\[ a = 4iG \int \frac{d^4q}{(2\pi)^4} \left( \frac{c(a^2 - 2c^2 - h^2 - l^2 + q0^2 + l\delta\mu)}{2(q_0^2 - \epsilon_+^2)(q_0^2 - \epsilon_-^2)} \right) + [l \rightarrow -l'] \]
\[ h = -4iG \int \frac{d^4q}{(2\pi)^4} \left( \frac{3e}{8(e^2 + l^2 - q_0^2)} + \frac{2(3a + 5h) c^2 - (a + h)(2l - \delta \mu)\delta \mu - (3a - 5h)(l^2 - q_0^2) - (a - h)(a + h)(3a + 5h)}{24(q_0^2 - \epsilon_+^2)(q_0^2 - \epsilon_-^2)} \right) + [l \rightarrow -l'] \]
\[ c = -4iG \int \frac{d^4q}{(2\pi)^4} \left( \frac{c(a^2 - 2c^2 - h^2 - l^2 + q0^2 + l\delta\mu)}{6(q_0^2 - \epsilon_+^2)(q_0^2 - \epsilon_-^2)} \right) + [l \rightarrow -l'] \]

These gap equations include the terms \( l \rightarrow -l' \) which correspond to the contribution of the anti-particle gap and do not lead to a gap at the Fermi
The anti-particle gap is not necessarily small and in fact is equal to the particle gap in the NJL model. These terms are suppressed, however, by at least $1/\mu^2$ and are usually neglected as is done here. This is crucial because the antiparticle gap is gauge dependent (see for example [11]).

Evaluation of the integrals on the right hand side of the gap equations is facilitated by the analytic continuation, $q_0 \to -i q_4$. The $q_4$ integral is done by contour integration closing the contour in the upper half plane and picking up the poles, (33)-(37), which lie in the upper half plane after analytic continuation. The angular integrals can be done trivially reducing the right hand side of the gap equation to an integration over $q(\equiv |\vec{q}|)$ which can be done numerically. This approach has the advantage that the quasi-particles are automatically on the mass shell.

The range of integration for $q$ is not infinite since the NJL model is a four-fermion interaction model and must have an UV cutoff. In this work the integrals are simply regularized by the factor:

$$R(q) = \frac{\Lambda^4}{(q^2 + \Lambda^2)^2}.$$ (50)

with $\Lambda = 1000$ MeV chosen so that the magnitude of the results in the NJL model are comparable to the perturbative results.

The coupled gap equations were solved iteratively for $\mu = 500$ MeV using numerical integration and the results are shown in Figure 1, after conversion to the convention of [11] for comparison. The effect of a shift in the strange quark chemical potential is a disruption to the pairing of the heavy quark with the light quarks in the dominant channel and a slightly smaller enhancement of the pairing of the light quarks. The results shown in Fig. 1 exhibit the same qualitative behavior as shown in Figure 6 of [10]. Note that the range of $\delta\mu$ shown in Fig. 1 does not go all the way up to the phase transition illustrated in [10] and the shifts seen in the results of this paper are larger than in [10].

The results are shown in terms of gaps with specific color-flavor symmetry in Fig. 2. These results show that the $\Delta_{66}$ gaps are non-zero but small. As well, the effect described above is seen more clearly: significant disruption to the pairing of the heavy quark with the light quarks in the dominant channel and a slightly smaller enhancement of the pairing of the light quarks. These results qualitatively agree with the results of [11] as far as the increase in $\Delta_{33}(ud)$ and decrease of in $\Delta_{33}(us)$. The quantitative agreement appears at first sight to be rather worse than above. The differences between the
results of [11] and [10], however, are mainly the result of different parameter choices [21]. Therefore the results presented here for the NJL model are consistent with previous analyses.

The mathematical reason for this phenomenon can be seen by examining the gap equations for the individual $\Delta_{33}$ gaps, but neglecting the $\Delta_{66}$ gaps and the difference between the $\Delta_{33}$ gaps on the right hand side of the gap equation. Expanding the integrands on the right hand side of the gap equations one obtains:

$$\Delta_{33}(ud) = 4G \int \frac{dq}{2\pi^2} q^2 \mathcal{R}(q) \left[ \frac{2\Delta_{33}}{9(l^2 + \Delta_{33}^2)^{1/2}} + \frac{\Delta_{33}}{9(l^2 + 4\Delta_{33}^2)^{1/2}} \right.$$ (51)

$$+ \left( \frac{l(2l^2 + 5\Delta_{33}^2)}{162\Delta_{33}(l^2 + \Delta_{33}^2)^{3/2}} - \frac{l(2l^2 + 14\Delta_{33}^2)}{162\Delta_{33}(l^2 + 4\Delta_{33}^2)^{3/2}} \right) \delta \mu$$

$$+ \left( \frac{8l^6 + 28l^4\Delta_{33}^2 + 29l^2\Delta_{33}^4 + 18\Delta_{33}^6}{486\Delta_{33}^3(l^2 + \Delta_{33}^2)^{3/2}} - \frac{8l^6 + 88l^4\Delta_{33}^2 + 314l^2\Delta_{33}^4 + 396\Delta_{33}^6}{486\Delta_{33}^3(l^2 + 4\Delta_{33}^2)^{3/2}} \right) \delta \mu^2$$

$$+ \mathcal{O}(\delta \mu^3)$$
Figure 2: Numerical solutions for gaps as a function of $\delta \mu$ with specific color flavor symmetries.

\[
\Delta_{33}(us) = 4G \int \frac{dq}{2\pi^2} q^2 \mathcal{R}(q) \left[ \frac{2\Delta_{33}}{9(l^2 + \Delta_{33}^2)^{1/2}} + \frac{\Delta_{43}}{9(l^2 + 4\Delta_{33}^2)^{1/2}} + \frac{l(l^2 - 2\Delta_{33}^2)}{12(l^2 + \Delta_{33}^2)^{3/2}} \right] \delta \mu
\]

One expects these equations to be dominated by the region near the Fermi surface, $l = 0$ ($q = \mu$). Expanding about $l = 0$ one obtains an approximation for the integrand in this region of the integration:

\[
\Delta_{33}(ud) : q^2 \mathcal{R}(q) \left[ \frac{5}{18} + \frac{13}{648} \frac{l \delta \mu}{\Delta_{33}} + \frac{5}{432} \frac{l \delta \mu^2}{\Delta_{33}^2} \right] + O(l/\Delta_{33}) \tag{53}
\]

\[
\Delta_{33}(us) : q^2 \mathcal{R}(q) \left[ \frac{5}{18} - \frac{83}{648} \frac{l \delta \mu}{\Delta_{33}} - \frac{1}{27} \frac{l \delta \mu^2}{\Delta_{33}^2} \right] + O(l/\Delta_{33}). \tag{54}
\]
The $l > 0$ part of this approximation is dominant because of the factor, $q^2 = (l+\mu)^2$. Therefore, if this region of the integration is dominant, $\Delta_{33}(ud)$ should increase linearly with $\delta \mu$ and a quadratic component for larger values of $\delta \mu$. Similarly $\Delta_{33}(us)$ should decrease linearly with $\delta \mu$ and a quadratic component for larger values of $\delta \mu$. The decrease in $\Delta_{33}(us)$ should be greater than the increase in $\Delta_{33}(ud)$. This is exactly the type of behavior exhibited by the solutions as shown in Fig. 2.

As was mentioned above, this behavior was already seen in a different model in [10]. One reason for presenting these results is as a check of the method to show that the model used here gives results that are consistent with previous analyses. The second and more important reason is to contrast with the results in the next section using perturbation theory and to explain the differences.

4 Perturbative Single Gluon Exchange

In the perturbative analysis, following the analysis of [13], the gluon propagator:

$$D_{\mu \nu}(q) = \frac{P^L_{\mu \nu}}{q^2 - \Pi_L} + \frac{P^T_{\mu \nu}}{q^2 - \Pi_T(q)} - \xi \frac{q_{\mu} q_{\nu}}{q^2},$$

is used in the weak coupling limit where:

$$P^L_{\mu \nu} \approx \delta_{\mu 0} \delta_{\nu 0}, \quad \frac{q_{\mu} q_{\nu}}{q^2} \approx \hat{q}_i \hat{q}_j.$$  \hspace{1cm} (56)

and using:

$$\Pi_L = m_D^2, \quad \Pi_T = \frac{\pi}{4} m_D^2 \frac{|m|}{|q|}$$ \hspace{1cm} (57)

with:

$$m_D^2 = \frac{N_f g^2 \mu^2}{2 \pi^2}$$ \hspace{1cm} (58)

$$g^2 = 4\pi \left( \frac{12\pi}{(11N_c - 2N_f) \log \left[ \mu^2/\Lambda_Q^2 \right]} \right).$$ \hspace{1cm} (59)

Evaluating the right hand side of the gap equations is more complicated in this case. The gluon propagator depends on the angle, $\theta$, between the momenta of the scattered fermions and this integral must be done numerically.
There is still one trivial angular integral that can be done immediately. As in the previous section, the analytic continuation \( q_0 \to -i q_4 \) is done and the \( q_4 \) integration is done by picking up the poles (33)-(37) of the quasiparticle propagator in the upper half plane. The right hand side of the gap equations is then reduced to integration over \( q(\equiv |q|) \) and one angular integral, \( \int d\cos \theta \), which can be done numerically. This approach has the advantage that the quasi-particles are automatically on the mass shell.

The gluon propagator is not the same for each term since it must be evaluated at the poles of the quasiparticle propagator. The poles of the gluon propagator are ignored in this calculation.

As was done in the previous section and in [16] the contribution of the antiparticle gaps is ignored. This is because the antiparticle gap does not lead to a gap at the Fermi surface and it’s contribution in the gap equation is suppressed by at least a factor of \( 1/\mu^2 \). As well, in a recent paper [22] it was shown that the anti-particle gap is further suppressed by an extra power of the coupling in perturbation theory if the quasiparticles are on the mass shell.

In order to make a numerical solution of this problem feasible we assume that the dominant contribution to the gap equation comes from the mean value of the \( ud \) and \( us \) gaps which are antisymmetric in color and flavor:

\[
\bar{\Delta}_{33} = \frac{\Delta_{33}(ud) + \Delta_{33}(us)}{2} = \frac{1}{8}(a + h - 3e) + \frac{1}{4}(c - f) .
\]  

The other gaps and the splitting between \( \Delta_{33}(ud) \) and \( \Delta_{33}(us) \) were evaluated after solving to check that they were significantly smaller than \( \bar{\Delta}_{33} \) and could safely be neglected.

The gap equation in this case is:

\[
\bar{\Delta}_{33}(p) = \frac{g^2}{12\pi^2} \int_0^\infty dq \int_{-1}^1 dx \quad \text{Residue} \quad q_4 \to q_i \quad F(q, \bar{\Delta}_{33}(q), \delta \mu)
\]

\[
\left[ \frac{3/2 - 1/2x}{p^2 + q^2 - 2pqx + [\Pi_T - (p_4 - q_4)^2]} + \frac{1/2 + 1/2x}{p^2 + q^2 - 2pqx + [\Pi_L - (p_4 - q_4)^2]} \right]
\]

where \( q_i \) are the poles of the quasiparticle propagator, (33)-(37) and \( x \equiv \cos \theta \).
\[
F(q, \Delta_{33}(q), \delta \mu) = 2\pi \left[ \frac{\Delta_{33}}{8\sqrt{l^2 + \Delta_{33}^2}} + \frac{\Delta_{33}}{12\sqrt{(l - \delta \mu/2)^2 + \Delta_{33}^2}} \right]
+ \frac{\Delta_{33} \left( (2l + \delta \mu)\delta \mu + 7\Delta_{33}^2 + D(q, \Delta_{33}(q), \delta \mu) \right)}{24\sqrt{2}D(q, \Delta_{33}(q), \delta \mu) \left( 2l^2 - 2l\delta \mu + \delta \mu^2 + 5\Delta_{33}^2 + D(q, \Delta_{33}(q), \delta \mu) \right)}
- \frac{\Delta_{33} \left( (2l + \delta \mu)\delta \mu + 7\Delta_{33}^2 - D(q, \Delta_{33}(q), \delta \mu) \right)}{24\sqrt{2}D(q, \Delta_{33}(q), \delta \mu) \left( 2l^2 - 2l\delta \mu + \delta \mu^2 + 5\Delta_{33}^2 - D(q, \Delta_{33}(q), \delta \mu) \right)}
\]

\[
D(q, \Delta_{33}(q), \delta \mu) = \sqrt{9\Delta_{33}^4 + 2\Delta_{33}^2\delta \mu(2l + 3\delta \mu) + \delta \mu^2(2l - \delta \mu)}
\]

One can solve the gap equation (61) iteratively using numerical integration with some care. In particular the collinear singularity at \(|p| = |q|\) and \(x = 1\) in the gluon propagator must be handled. In addition there are other features of the integrand near \(x = 1\) which are nonsingular but must be handled with a little care.

Perturbative analysis is certainly reliable at asymptotic densities where the Fermi momentum is very high. If color superconducting quark matter is actually observed, however, it will be at moderate densities of 4-5 times nuclear matter density in the core of neutron stars. At \(\mu = 500\) MeV where the results below are obtained, the perturbative single gluon exchange interaction should be considered a model which is distinct from and can be compared to the NJL model which is a low energy model.

Using this procedure a solution of (61) for \(\Delta_{33}(p)\) was obtained for different values of \(\delta \mu\). These results were then used to calculate estimates for the individual gaps. The results are shown for \(\delta \mu = 0\) MeV and \(\delta \mu = 45\) MeV in Figure 3. The error in the solution for \(\Delta_{33}\) is estimated to be less than 0.01% and the error in the values for the individual gaps are probably less than 0.25%.

Comparing the \(\delta \mu = 0\) results to previous analyses one sees that the peak value of \(\Delta_{33}(p)\) is of the same order of magnitude as predicted by the analysis of [16] where \(\Delta_{33}(p_0)\) is determined for the case when both magnetic and electric gluon exchanges are taken into account. The form of the \(\delta \mu = 0\) results is qualitatively similar to results found in [19] for \(\Delta_{33}(p)\) in the two flavor case at a similar density.
Figure 3: Numerical solutions for the gaps as a function of \( p \) for \( \delta \mu = 0 \) MeV and \( \delta \mu = 45 \) MeV.

The agreement with [19] includes a small cusp feature seen most clearly in the \( \delta \mu = 0 \) results in Figure 3. This feature arises because the magnetic gluon propagator is non-analytic at \( p_0 = q_0 \) due to the form of \( \Pi_T(p-q) \sim |p_0 - q_0| \) in (57). This leads to a non-analyticity of the gap function as a function of \( p \). This feature arises in the results of [19] as seen in their Fig. 6(a). It is a small effect, but the fact that it occurs in both analyses supports the conclusion that the independent analyses are consistent.

The neglected \( \Delta_{66} \) gaps are less than 5\% of the \( \Delta_{33} \) gaps, and \( \Delta_{33}(ud) \) and \( \Delta_{33}(us) \) are within 5\% of their mean value, \( \Delta_{33} \), at the maximum values of \( \delta \mu \) investigated. This indicates that the estimates obtained by solving for \( \Delta_{33} \) and then calculating the individual gaps should be fairly reliable.

The most important result of this analysis is that both \( \Delta_{33}(ud) \) and \( \Delta_{33}(us) \) decrease with \( \delta \mu \). This is in contrast to the results obtained in the NJL model where only the \( \Delta_{33}(us) \) gap decreases and the \( \Delta_{33}(ud) \) gap increases. The peak value for these gaps is shown in Figure 3 as a function of \( \delta \mu \) with the NJL model results from the previous section included for com-
Figure 4: Comparison of the gaps as a function of $\delta \mu$ for the NJL model and single gluon exchange.

The results are also shown at 3 other fixed values of $p$ in Figures 5, 6 and 7. These results show that both gaps consistently decrease with increasing $\delta \mu$ over the whole range of momenta and the decrease is basically linear in $\delta \mu$ with a small quadratic component.

If one takes the objects in square brackets from the equations for $\Delta_{33}(ud)$ and $\Delta_{33}(us)$ (51 and 52) in the NJL model, substitutes into the perturbative equation and examines the region away from the peak at $l = 0$ ($l >> \bar{\Delta}_{33}$ but not asymptotic values of $l$), these terms are:

$$\Delta_{33}(ud)(p) : \left[ \frac{5}{18} - \frac{\bar{\Delta}_{33} \delta \mu}{18 l^2} + \mathcal{O}(\delta \mu^2) \right]$$

$$\Delta_{33}(us)(p) : \left[ \frac{5}{18} - \frac{5\bar{\Delta}_{33} \delta \mu}{36 l^2} + \mathcal{O}(\delta \mu^2) \right]$$

Thus the effect of this region of the integration is to cause both gaps to decrease linearly in $\delta \mu$ which is seen above.

The other effect that one observes from Figure 4 is that the changes in the gaps in the NJL model are much more drastic than in perturbation theory. This is likely a result of the fact that, with the cut-off in momentum space
in the NJL model, the region of the integration near the Fermi momentum forms a much larger part of the complete integral, and so the relative effect is larger. In perturbation theory where one integrates over a larger momentum region within which different effects play a role, the relative change in the integral is much smaller.

Summarizing the results, at $\mu = 500$ MeV the dependence of $\Delta_{33} (ud)$ and $\Delta_{33} (ud)$ on $\delta \mu$ in first order perturbation theory is much weaker than in the NJL interaction model. More importantly, the dependence of $\Delta_{33} (ud)$ on $\delta \mu$ actually changes sign.

The implications of this change in sign could be significant. As $\delta \mu$ increases, flavor is still locked to color except that $s$ flavor and $b$ color are distinguished from the other flavors and colors. Above $\delta \mu = 2(\Delta_{33} \delta \mu = 0)$ the two flavor color superconducting phase is favored and the gap in this case is larger than the $ud$ gap in the CFL phase. If there is no intervening phase between the three flavor superconducting phase and the two flavor color superconducting phase, the fact that the $\Delta_{33} (ud)$ gap decreases suggests that the phase transition from the three to two flavor phases is first order. This analysis should be extended all the way to the three to two flavor phase transition point in order to verify this. As well, at this density, the inclusion of higher orders of perturbation theory must be examined to determine how they affect the results. Finally this reasoning ignores the possibility of intervening phases such as a three flavor equivalent of crystalline color super-

Figure 5: Gaps as a function of $\delta \mu$ in perturbation theory at $p = 0$. 

\[ \text{Figure 5: Gaps as a function of $\delta \mu$ in perturbation theory at $p = 0$.} \]
conductivity, which has been predicted in the two flavor case\cite{8}, or CFL with meson condensation predicted in \cite{9} in an effective Lagrangian model. It would be very interesting to study both of these phenomena in the present framework.

The results presented in this section were obtained using first order perturbation theory at a moderate density where it should be considered a model and may not be accurate. One could speculate that at much higher values of the chemical potential where perturbation theory is definitely valid and the solution for $\Delta_3$ is more strongly peaked at $l = 0$ \cite{19}, the differences between the perturbative results and the NJL model may not be as large as here. However, these results indicate that the dependence on $\delta \mu$ should be weaker in perturbation theory than in the NJL model.

5 Conclusion

In this paper the general poles of the quasiparticle propagator in the case where the $s$ quark has a different chemical potential than the other two quarks has been presented. These are a generalization of the poles given in \cite{11}. These poles are valid in a different model than \cite{10} and the relationship between the two models is not clear.

These poles were used in numerical solutions of the gap equations in an
NJL model to demonstrate that they produce results for the gaps which are qualitatively consistent with [10, 11].

The poles were also used in a numerical solution of the gap equation for $\Delta_{33}(p)$ using perturbative single gluon exchange and to obtain fairly reliable estimates of all the gaps. The results obtained for $\delta\mu = 0$ are qualitatively and quantitatively similar to those in [19], which can be seen as support for both results. The results of [19] are obtained in the two flavor case while the results presented here are obtained in the three flavor case.

Finally, numerical results for the gaps were found as a function of $\delta\mu$ out to $\delta\mu = 45\text{MeV} \approx (\Delta_{33})_{\delta\mu=0}$. The main conclusion is that perturbation theory at $\mu = 500\text{MeV}$ predicts a decrease in $\Delta_{33}(ud)$ as a function of $\delta\mu$ where the NJL model predicts an increase in $\Delta_{33}(ud)$. This observation suggests that the phase transition from three to two flavor color superconductivity is first order. The second conclusion is that the dependence of both $\Delta_{33}(ud)$ and $\Delta_{33}(us)$ on $\delta\mu$ is much weaker than in the NJL model for $\mu = 500\text{MeV}$. Therefore, there are both qualitative and quantitative differences between the NJL model results and the leading perturbative results at moderate density. It is not clear which of these results should be closest to the real world.

The weaker dependence of the gaps on $\delta\mu$ is likely to be true even at much higher densities, but how much different perturbative results might be from NJL results in this case remains to be seen. It is not clear if the increase of $\Delta_{33}(ud)$ with $\delta\mu$ will occur at higher $\mu$ and this determination will be left for

Figure 7: Gaps as a function of $\delta\mu$ in perturbation theory at $p = 1500$. 
Analyzing the effect of shifting the chemical potential of the strange quark by $\delta \mu$ is a first approximation to including the effect of a strange quark mass. This method has the benefit of separating out the difficulties introduced by fully including a strange quark mass in the gap equation. The knowledge gained in this research will be useful in an analysis where the $s$ quark is given a non-zero mass at the level of the bare quark propagator which is the ultimate goal of this line of research.

As well, the dependence of the gaps on $\delta \mu$ presented in this paper may be interesting when compared with other papers that consider this effect. In [14] it was shown in the two flavor case that up to $\delta \mu = \Delta_0/\sqrt{2}$, $\delta \mu$ will have no affect on the gaps. In [13] it is argued that in the three flavor case, $\delta \mu$ will affect the values of the gaps and the number densities of the quarks but the densities of the different flavors of quarks will remain equal. This situation would mean that the superconducting ground state could remain electrically neutral without the addition of electrons which would drastically change the properties of this type of matter. This conclusion could presumably be tested using the results of this paper.

This work can be extended in a number of ways. The analysis should be extended to $\delta \mu = 2(\Delta_{33})_{\delta \mu=0}$ corresponding to the phase transition to two flavors. The analysis should also be done at higher densities to determine how the results depend on $\mu$. Determine of higher order corrections to these results are necessary to determine the validity of the perturbative approach at this density. As well, the analysis of this paper could be extended is to non-zero temperature following [17].

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6Recent papers [23] have discussed progress in this direction.
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