The Physical Hilbert Space of SU(2) Lattice Gauge Theory

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Abstract

We solve the Gauss law of SU(2) lattice gauge theory using the harmonic oscillator prepotential formulation. We construct a generating function of a manifestly gauge invariant and orthonormal basis in the physical Hilbert space of (d+1) dimensional SU(2) lattice gauge theory. The resulting orthonormal physical states are given in closed form. The generalization to SU(N) gauge group is discussed.

1 Introduction

The physical states of gauge theories are gauge invariant. Therefore, an important problem is to label and construct a complete orthonormal basis in the physical (gauge invariant) Hilbert space of gauge theories. The motivation of the present work is to address this problem within the framework of lattice gauge theories using the recently proposed harmonic oscillator prepotential formalism. We will be working with the Hamiltonian formulation of (d+1) dimensional lattice gauge theories. Further, to keep the discussions simple, we will deal with SU(2) gauge groups and the generalization to SU(N) will be discussed at the end.

There have been two approaches to the above problem:

- The set of all Wilson loops \([1, 2]\) generates a basis in the physical Hilbert space which is manifestly gauge invariant. However, this basis is not orthogonal and it is overcomplete [3].

- Characterize the states associated with each lattice link by the eigenvalues of a complete set of commuting operators and then impose the Gauss law constraints on these states over the lattice [4]. Such solutions of Gauss law, have been written down in terms of Wigner D matrices in different spin representations [5]. Unlike the Wilson loop basis, they provide a complete orthonormal basis. However, these constructions lack the elegance of Wilson loop construction and are limited by the problem of rapid proliferation of the SU(2) group indices and the Clebsch Gordan coefficients.

Recently, we proposed a re-formulation of SU(2) lattice gauge theories in terms of harmonic oscillator prepotentials [6]. Two of the novel features of this formulation which are relevant for the present work are:

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1. In terms of harmonic oscillator prepotentials, the Hamiltonian has $SU(2) \otimes U(1)$ local gauge invariance.

2. The prepotentials with $U(1)$ charges transform as doublets under the $SU(2)$ gauge transformations. This enabled us to study the gauge invariance of the theory more critically.

Using this formulation, we now construct a basis in the physical Hilbert space which has the following desired properties:

- It is orthonormal and complete.
- Like Wilson loops, it is manifestly gauge invariant and is given in a closed form in terms of the harmonic oscillator prepotentials.
- In $d$ space dimensions, it is characterized by $3(d-1)$ quantum numbers per lattice site which is the number of physical transverse degrees of freedom of the $SU(2)$ gluons [4].

We should emphasize that this formulation completely bypasses the problem of proliferation of indices leading to cumbersome expressions faced by the earlier approaches. The reason for this simplification is the novel $SU(2) \otimes U(1)$ gauge group and the corresponding gauge transformation properties of the prepotentials (see section 3). These features of prepotential formulation of lattice gauge theories will be emphasized and compared with the standard formulation as we proceed.

The plan of the paper is as follows: In section (2), we start with a brief introduction to $SU(2)$ prepotential Hamiltonian formulation [6]. This section is included for the sake of completeness and for setting up the notations which are more suitable for the present work. The Section (3) is devoted to the study of physical Hilbert space in terms of prepotential operators. In this section, we explicitly construct a manifestly $SU(2)$ gauge invariant orthonormal basis in the physical Hilbert space and write down the corresponding generating function.

## 2 The Hamiltonian Prepotential Formulation

We start with $SU(2)$ lattice gauge theory in $(d+1)$ dimensions. The Hamiltonian is [2]:

$$
H = \sum_{n,i} tr E(n,i)^2 + K \sum_{\text{plaquettes}} \text{tr} \left( U_{\text{plaquette}} + U_{\text{plaquette}}^\dagger \right).
$$

where,

$$
U_{\text{plaquette}} = U(n, i)U(n + i, j)U^\dagger(n + j, i)U^\dagger(n, j); \quad E(n, i) \equiv E^a(n, i) \sigma^a 2.
$$

and $K$ is the coupling constant. The index $n$ labels the site of a $d$-dimensional spatial lattice and $i,j (=1,2,...,d)$ denote the direction of the links. Each link $(n,i)$ is associated
with a symmetric top, whose configuration (i.e the rotation matrix from space fixed to
body fixed frame) is given by the operator valued SU(2) matrix $U(n,i)$. The angular
momenta with respect to body and space fixed frames are given by $E^a(n, i)(a = 1, 2, 3)$
and $e^a(n + i, i)$ respectively. The quantization rules [2] are:

$$
\left[ E^a(n, i), E^b(n, i) \right] = i \epsilon^{abc} E^c(n, i), \quad \left[ e^a(n, i), e^b(n, i) \right] = i \epsilon^{abc} e^c(n, i).
$$

Thus, under SU(2) gauge transformations both a and b type potentials transform as

doublets. Therefore, the manifestly SU(2) gauge invariant operators are:

$$\text{E}^a(n, i) \text{ and } e^a(n + i, i) \text{ commute with each other and their magnitudes are same:}
$$

$$
\sum_{a=1}^{3} E^a(n_i) E^a(n_i) = \sum_{a=1}^{3} e^a(n + i, i) e^a(n + i, i).
$$

However, the axis of rotation can have arbitrary inclinations in the two frames. Therefore,
the complete set of commuting operators on any link $(ni)$ are: $\sum_{a=1}^{3} E^a(n, i) E^a(n, i) =
\sum_{a=1}^{3} e^a(n + i, i) e^a(n + i, i)$; $E^3(n, i); e^3(n + i, i)$. The corresponding eigenvectors will be
denoted by $|j(ni), \mathcal{m}(ni), \mathcal{m}(n+i, i)\rangle$. The Hamiltonian (1) and the quantization rules (2)
are invariant under:

$$
E(n_i) \rightarrow V(n) E(n_i) V^\dagger(n); \quad e(n_i) \rightarrow V(n) e(n_i) V^\dagger(n); \quad U(n_i) \rightarrow V(n) U(n_i) V^\dagger(n + i).
$$

The left and right gauge transformation on $U(ni)$ are generated by the $E^a(ni)$ and $e^a(n +
i, i)$ respectively. Therefore, we should think of $E^a(ni)$ and $e^a(n + i, i)$ as the operators
attached to the two ends $(n)$ and $(n+i)$ of the link $(ni)$ respectively. The SU(2) Gauss
law at every site $(n)$ is:

$$
\sum_{i=1}^{d} (E^a(n, i) + e^a(n, i)) = 0.
$$

Having fixed the notations, we now define the SU(2) prepotentials. To every link $(n.i)$, we
associate two doublets of harmonic oscillators $(a_\alpha(n, i), a_\alpha^\dagger(n, i))$ and $(b_a(n + i, i), b_\alpha^\dagger(n + i, i))$ attached to the two ends $(n)$ and $(n+i)$ respectively. They satisfy:

$$
[a_\alpha, a_\beta^\dagger] = \delta_{\alpha,\beta}; \quad [b_\alpha, b_\beta^\dagger] = \delta_{\alpha,\beta} \quad \alpha, \beta = 1, 2.
$$

Using the Jordan-Schwinger boson representation of SU(2) Lie algebra [7], we write:

$$
E^a(n, i) \equiv a^\dagger(n, i) \sigma^a a(n, i); \quad e^a(n, i) \equiv b^\dagger(n, i) \sigma^a b(n, i).
$$

The gauge transformation properties of electric fields (4) imply that

$$
a_\alpha^\dagger(n, i) \rightarrow V(n)_{\alpha\beta} a_\beta^\dagger(n, i), \quad b_\alpha^\dagger(n, i) \rightarrow V(n)_{\alpha\beta} b_\beta^\dagger(n, i).
$$

Thus, under SU(2) gauge transformations both a and b type prepotentials transform as
doublets. Therefore, the manifestly SU(2) gauge invariant operators are:

$$a^\dagger, b^\dagger \equiv \sum_{\alpha=1}^{2} a_\alpha^\dagger b_\alpha, \quad a^\dagger, b^\dagger \equiv \sum_{\alpha=1}^{2} \sum_{\beta=1}^{2} \epsilon_{\alpha\beta} a_\alpha^\dagger b_\beta^\dagger = a_1^\dagger b_2^\dagger - a_2^\dagger b_1^\dagger.
$$

Note that compared to [6], we have introduced a -ve sign in front of $E^a(n, i)$ for later convenience
The constraint (3) implies that the occupation numbers of the harmonic oscillator prepotentials located at (n) and (n+i) ends of the link (n,i) are equal.

$$a^\dagger(n, i).a(n, i) = b^\dagger(n + i, i).b(n + i, i) \equiv N(n, i); \quad i = 1, 2, \ldots, d.$$ (10)

Therefore, the Hilbert space of pure SU(2) lattice gauge theory is characterized by the following orthonormal state vectors at each link:

$$|j(n, i), m(n, i), \bar{m}(n + i, i)\rangle = \frac{1}{\sqrt{(j + m)!}} \frac{1}{\sqrt{(j - m)!}} \frac{1}{\sqrt{(j + \bar{m})!}} \frac{1}{\sqrt{(j - \bar{m})!}} \langle 0 | 0 0 0 \rangle. (11)$$

In (11) $a^\dagger$ and $b^\dagger$ (also $m$ and $\bar{m}$) are defined at the two ends n and n+i of the link (n,i) respectively. From now onwards we will denote the Hilbert space consisting of all possible state vectors (11) at all the links of the lattice as $\mathcal{H}$. Note that $\mathcal{H}$ defines the dual Hilbert space as it is characterized by the eigenvalues values of the angular momentum operators. The defining equations (7) for the SU(2) prepotentials and the Hilbert space in (11) are invariant under:

$$a^\dagger_\alpha(n, i) \rightarrow \exp i\theta(n, i) a^\dagger_\alpha(n, i); \quad b^\dagger_\alpha(n + i, i) \rightarrow \exp -i\theta(n, i) b^\dagger_\alpha(n + i, i). (12)$$

In (12), $\theta(n, i)$ is a phase angles at link (n,i). This is a novel U(1) local gauge invariance. Note that this is not an abelian subgroup of the SU(2) gauge group. The constraint (10) now becomes the Gauss law for this abelian gauge invariance. The $SU(2) \otimes U(1)$ gauge invariance imply [6]:

$$U(n, i)_{\alpha\beta} = F(n, i)(a^\dagger_\alpha(n, i)b^\dagger_\beta(n + i, i) + \tilde{a}_\alpha(n, i)b_\beta(n + i, i))F(n, i). (13)$$

In (13), $F(n, i) \equiv \frac{1}{\sqrt{N(n,i)+1}}$ with $N(n,i)$ defined in (10) is the normalization factor.

### 3 A Manifestly SU(2) Gauge Invariant Basis

In this section, we construct the generating function which produces a manifestly SU(2) gauge invariant orthonormal basis. The abelian gauge invariance (12) and the associated Gauss law (10) are simple and will be incorporated at the end. Before we start the construction of the gauge invariant basis, we would like to emphasize the following two features of the prepotential formulation which are relevant for the construction of generating function in this section.

1. In terms of the link variables $U(n,i)$, the state given in (11):

$$|j(n, i), m(n, i), \bar{m}(n, i)\rangle = \sum_{i_1, \ldots, i_{2j} \in S_{2j}} U_{m_1, m_1} U_{m_2, m_2} \ldots U_{m_{2j}, m_{2j}} |0\rangle. (14)$$

In (14), $(m_1, m_2, \ldots, m_{2j})$ and $(\bar{m}_1, \bar{m}_2, \ldots, \bar{m}_{2j})$ are the two sets of $\pm \frac{1}{2}$ with constraints: $m_1 + m_2 + \ldots m_{2j} = \bar{m}_1 + \bar{m}_2 + \ldots \bar{m}_{2j} = \bar{m}$ and $S_{2j}$ is the permutation group of...
order 2j. The construction (14) though the link operators (unlike (11)) becomes more and more complicated as j increases. Thus, the characterization of the Hilbert space, even before imposing the Gauss law constraints, through the prepotential formulation is much simpler than the standard formulation.

2. The dynamical variables at site n are $a^\dagger(n, i)$ and $b^\dagger(n, i)$, $i=1,2,...,d$. Under SU(2) gauge transformation at site n, they all transform as SU(2) doubles like matter fields. Therefore, the problem of constructing the SU(2) gauge invariant Hilbert space over the entire lattice using the link operators $U$ reduces to a (local) problem of constructing SU(2) singlets out of SU(2) harmonic oscillator doubles at one lattice site. The additional U(1) Gauss law then connects these local SU(2) gauge invariant Hilbert spaces at different lattice sites.

The above discussion suggests that we collect all the 2d prepotential doublets meeting at a particular lattice site n and label them$^3$ as $a_{\alpha}[i], a^\dagger_{\alpha}[i]$; $i=1,2,...,2d$. The corresponding angular momentum operators are $a^\dagger[i]a[i]$ and will be denoted by $J^a[i]$. The eigenvalues of $\sum_{a=1}^{3d} J^a[i].J^a[i]$ and $J^{(a=3)}[i]$ will be denoted by $j_i$ and $m_i$ respectively. Therefore, a generic state in the Hilbert space $\tilde{H}_n$ living at the site n is labelled by 4d quantum numbers:

$$ \prod_{i=1}^{2d} \otimes |j_i, m_i \rangle. $$

(15)

The complete Hilbert space $\tilde{H}$ consisting of state vectors in (11) is obtained by taking direct product:

$$ \tilde{H} = \prod_n' \tilde{H}_n. $$

(16)

In (16), the prime over the direct product over all lattice sites implies that the direct product is taken such that U(1) Gauss law (10) is satisfied.

The SU(2) Gauss law (5) at site n now takes a simpler form:

$$ J^a_{\text{total}} = J^a[1] + J^a[2] + \ldots + J^a[2d] = 0. $$

(17)

It simply states that the sum of all the 2d angular momenta meeting at a site (n) is zero. The new form of SU(2) Gauss law (17) immediately implies that the physical Hilbert space living at site n and denoted by $\tilde{H}_n^p$ can be trivially characterized as $^6$:

$$ |\vec{l} \rangle \equiv \begin{vmatrix} l_{12} & l_{13} & \ldots & l_{1 2d} \\ l_{23} & \ldots & \ldots & l_{2 2d} \\ \vdots & \ddots & \ddots & \ddots \\ l_{2d-1 2d} \\ l_{2d 2d-1} \\ \end{vmatrix} = \prod_{ij}^{i \neq j} (a^\dagger[i].a^\dagger[j])^{l_{ij}} |0 \rangle. $$

(18)

$^3$More explicitly, $a^\dagger_{\alpha}[i] \equiv a^\dagger_{\alpha}(n, i)$; and $a^\dagger_{\alpha}[d + i] \equiv b^\dagger_{\alpha}(n, i)$; $i=1,2,...,d$. From now on, our analysis will be only at a particular site n. Therefore, we will suppress the site index and show it explicitly only when required.
In (18), \( l_{ij} (\equiv l_{ji}) \) are \( N_d = d(2d - 1) \) +ve integers at site \( n \) which are invariant under the SU(2) gauge transformations. The states (18) \( \in \mathcal{H}_n^d \) with total angular momentum zero are also the eigenvectors of \( J_i J_i \), \( i = 1, 2,..,2d \) with eigenvalues \( j_i(j_i + 1) \) where \( 2j_i = (\sum_{j \neq i} l_{ij}) \). As the above 2d+2 mutually commuting operators do not form the complete set of commuting operators (except in the trivial case of \( d=1 \)), the SU(2) invariant basis (18), like the standard basis obtained by all Wilson loops, is obviously overcomplete and not orthonormal. Infact, (18) gives the dual description of the basis obtained by all possible Wilson loops [6].

To construct a complete orthonormal basis, we follow the following angular momentum addition scheme:

\[
\hat{J}[1] + \hat{J}[2] \rightarrow \hat{J}[12] \rightarrow \hat{J}[123] \rightarrow \ldots \hat{J}[1..(2d - 1)] + \hat{J}[2d] \rightarrow \hat{J}[12..(2d)] = 0. \tag{19}
\]

Note that (19) is a 2d-1 step process and the last step implies that the eigenvalues of \( \hat{J}[12..(2d - 1)] \) and \( \hat{J}[2d] \) are equal. What follows now is an appropriate generalization of the technique developed in [7]. To add the 2d angular momenta above, we consider a direct product of the generating functions of two SU(2) coherent states defined over the complex planes \( x (\equiv (x_1, x_2)) \) and \( y (\equiv (y_1, y_2)) \) respectively.

\[
|x\rangle \otimes |y\rangle \equiv \sum_{j_1 m_1, j_2 m_2} \phi_{j_1 m_1}(x)\phi_{j_2 m_2}(y)|j_1, m_1\rangle \otimes |j_2, m_2\rangle = \exp \left( x.a^\dagger[1] \oplus y.a^\dagger[2] \right) |0\rangle \tag{20}
\]

where,

\[
\phi_{jm}(x) = \frac{(x_1)^{j+m}(x_2)^{j-m}}{\sqrt{(j+m)!(j-m)!}}
\]

We apply the differential operator involving a triplet of complex parameters \((\delta_1, \delta_2, \delta_3)\) and a complex plane \(z (\equiv (z_1, z_2))\):

\[
\exp \left( \delta_3 (\partial_x \hat{\delta}_y) + \delta_1 (z.\partial_x) + \delta_2 (z.\partial_y) \right)
\]

on the both sides of (20) and put \( x = y = 0 \) to get [7]4:

\[
\sum_{j_1 j_2 j m} \phi_{j m}(z)\Phi_{j_1 j_2 j}(\hat{\delta})||j_1 j_2 j m\rangle = \exp \left( \delta_3 a^\dagger[1].\hat{a}^\dagger[2] + z.a^\dagger[12] \right)|0\rangle \tag{22}
\]

where,

\[
a^\dagger[12] \equiv \delta_1 a^\dagger[1] + \delta_2 a^\dagger[2] \tag{23}
\]

\[
\Phi_{j_1 j_2 j}(\hat{\delta}) \equiv \left(\frac{(j_1 + j_2 + j + 1)!}{(2j + 1)!}\right)^{\frac{1}{2}} \frac{(\delta_1)^{j_1-j_2+j}(\delta_2)^{j_1+j_2-j}}{[(j_1 - j_2 + j)!(j_1 + j_2 - j)!]^\frac{1}{2}}.
\]

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4Infact, the relation (22) is a simple consequence of the fact that it’s right hand side is a generator of SU(2) coherent states in the representation spaces of the total angular momentum operators.
For later convenience, $\Phi_{j_1j_2j}(\vec{\delta})$ will be called vertex factor as they are involved in adding the angular momenta $j_1$ and $j_2$ to give total angular momentum $j$ corresponding to the first step of the scheme (19). If we now put:

$$\vec{\delta} = (0, 0, \delta_3) \implies j_1 = j_2, \ j = 0.$$  

Therefore, in this (trivial) $d=1$ case, the manifestly SU(2) gauge invariant Hilbert space at a particular site is:

$$|j_1 = j, j_2 = j; j_{\text{total}} = 0⟩ = \frac{1}{\sqrt{(2j)!(2j+1)!}}(a_1^\dagger|12⟩.a_2^\dagger|2⟩)^{2j}|0⟩. \tag{24}$$

Note that the states in (24) form a complete set of orthonormal and manifestly gauge invariant basis and the U(1) Gauss law (10) makes $j$ site independent. These results are obvious to begin with and did not require any calculations. We now generalize this procedure to arbitrary $d$ dimensions. To do this, we define $(2d-1)$ triplets of complex parameters $\vec{\delta}[r]$ and the corresponding vertex factor $\Phi_{j_{12..r}j_{r+1}j_{12..(r+1)}}(\vec{\delta}[r])$ associated with adding the angular momenta $j_{12..r}$ to $j_{r+1}$ to give net angular momentum $j_{12..(r+1)}$. This is the $r_{th}$ step of the ladder (19). Therefore, iterating the above process sequentially at all the 2d-1 steps of the ladder (19), we get:

$$\sum_j \left[ \prod_{r=1}^{2d-1} \Phi_{j_{12..r}j_{r+1}j_{12..(r+1)}}(\vec{\delta}[r]) \right] \phi_{jm}(z)|j_{12..r}j_{r+1}j_{12..(r+1)}⟩; j ≡ j_{12..2d} m⟩ = \exp(z.a_1^\dagger|12..2d⟩.a_2^\dagger|2d⟩.a_3^\dagger|3⟩...|0⟩) \tag{25}$$

In (25), the summation over $\vec{\jmath}$ is the summation over all $(4d-1)$ angular momenta shown in (19) and the summation over $m$ is from $-j$ to $+j$. The generalization of (23) is:

$$a_{1α}|12..r⟩ ≡ δ_{1[r]}a_{1α}|12..r⟩ - 1⟩ + δ_{2[r]}a_{1α}|r⟩. \tag{26}$$

Further, like in $d=1$ case, to get the generating function of the gauge invariant basis we put:

$$\vec{\delta}[2d-1] = (0, 0, \delta_3[2d-1]) \implies j_{12..(2d-1)} = j_{2d}, \ j = j_{12..2d} = 0.$$  

This finally gives the generating function of a manifestly SU(2) gauge invariant basis:

$$\sum_{\vec{\jmath}} \left[ \prod_{r=1}^{2d-1} \Phi_{j_{12..r}j_{r+1}j_{12..(r+1)}}(\vec{\delta}[r]) \right] |j_{12..2d}⟩; j = 0⟩ = \exp(\sum_{r=1}^{2d-1} δ_3[r](a_1^\dagger|12..r⟩.a_2^\dagger|r+1⟩)|0⟩. \tag{27}$$

To solve (27) for the physical states, we note that $(j_{12..r} - j_{r+1} + j_{12..r+1}), (-j_{12..r} + j_{r+1} + j_{12..r+1}), (j_{12..r} + j_{r+1} - j_{12..r+1})$ are all non-negative integers and put

$$δ_1[r] = expiθ_1[r], \ δ_2[r] = expiθ_2[r] \ r = 1, 2, ..., 2d - 1.$$
This gives:

$$|j_1 j_2 j_{12} j_3 j_{123} \ldots ; j = 0 \rangle = N \left[ \prod_{r=1}^{2d-1} \prod_{\alpha=1}^{2} \frac{1}{2\pi} \int_{0}^{2\pi} d\theta_\alpha [r] \Phi_{j_{12..r} j_{r+1} j_{12..(r+1)}} \left( \delta^r [\alpha] \right) \right] |0\rangle. \quad (28)$$

In (28), $\delta_\alpha^r[r] = e^{i\theta_\alpha[r]}$, $\alpha = 1, 2$ and $\delta_3^r[2d-1]$ are the manifestly SU(2) gauge invariant operators. Further, $\delta_3^r[2d-1]$ is independent of $\delta_1[2d-1], \delta_2[2d-1]$ and therefore, the corresponding integrations project out the total angular momentum zero states. $N$ is the normalization constant at site n and is given by:

$$N \equiv \prod_{r=1}^{2d-1} \frac{(2j_{12..(r+1)} + 1)! (j_{12..r} - j_{r+1} + j_{12..(r+1)})! (-j_{12..r} + j_{r+1} + j_{12..(r+1)})!}{(j_{12..r} + j_{r+1} + j_{12..(r+1)} + 1)!}.$$

Therefore, at this stage the problem of solving the non-abelian Gauss law reduces to the problem of solving the abelian Gauss law (10). This abelian Gauss law simply states $j(n, i) = j(n + i, d + i)$ for $i = 1, 2, \ldots, d$. The complete $SU(2) \otimes U(1)$ invariant Hilbert space can be written as:

$$\tilde{H}^p = \prod_n \tilde{H}_n^p \quad (29)$$

In (29), the direct product is taken over all the lattice sites such that U(1) Gauss law (10) is satisfied. Note that 1) the states in (28) are the eigenstates of the complete set of commuting angular momentum operators shown in (19). Therefore, by construction they form orthonormal basis. 2) they are manifestly SU(2) gauge invariant. As is clear from (28), we require $(4d-3)$ quantum numbers at a given lattice site to characterize the SU(2) gauge invariant Hilbert space. Further, the U(1) Gauss law fixes $d$ of these quantum numbers in terms of the quantum numbers associated with the previous sites. Therefore, the $SU(2) \otimes U(1)$ gauge invariant Hilbert space is characterized by $3(d-1)$ quantum numbers per lattice site which is the number of physical transverse degrees of freedom of SU(2) gluons.

We now briefly discuss the SU(N) lattice gauge theories. The SU(2) Schwinger boson algebra was generalized to SU(N) group in [8]. This required introduction of (N-1) SU(N) fundamental multiplets of harmonic oscillators. Therefore, the prepotential formulation of SU(N) lattice gauge theory will be invariant under $SU(N) \otimes (U(1))^{N-1}$ gauge group. Like in the present case, this formulation is also useful in the construction of a manifestly gauge invariant orthonormal basis. The work in this direction is in progress and will be reported elsewhere [9].

### 4 Discussion and Summary

In this work, we have constructed an orthonormal basis in the physical Hilbert space of SU(2) lattice gauge theories in arbitrary dimension. This construction unlike the earlier approaches has both the desired features: a) it is manifestly gauge invariant like Wilson.
loops basis b) it is orthonormal and complete like basis given in terms of Wigner D matrices. This is the first application of the harmonic oscillator prepotential formulation of lattice gauge theories. The matter can be included as in [6]. Breaking the link operator U into left and right harmonic oscillator doublets was essential for this construction and it completely bypassed the permutation symmetry problems faced earlier. Using (13), it would be interesting to study the connection of (28) with the Wilson loop overcomplete basis as both are manifestly gauge invariant.

The investigation of lattice Hamiltonian eigenvalues and eigenvectors in the physical Hilbert space is an alternative to path integral formulation. It is generally the truncated basis with respect to the number of plaquettes which is included in such calculations. Therefore, the present work is going to be useful in this direction. The matrix elements of the Hamiltonian and the corresponding dynamical issues are under investigation and will be reported at a later stage.

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