We discuss a trajectory of penguins’ faeces after the powerful shooting due to their strong rectal pressure. Practically, it is important to see how far faeceses reach when penguins expel them from higher places. Such information is useful for keepers to avoid the direct hitting of faeceses. We estimate the upper bound for the maximum flight distance by solving the Newton’s equation of motion. Our results indicate that the safety zone should be 1.34 meters away from a penguin trying to poop in typical environments. In the presence of the viscous resistance, the grounding time and the flying distance of faeces can be expressed in terms of Lambert $W$ function. Furthermore, we address the penguin’s rectal pressure within the hydrodynamical approximation combining Bernoulli’s theorem and Hagen-Poiseuille equation for viscosity corrections. We found that the calculated rectal pressure is larger than the estimation in the previous work.

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I. INTRODUCTION

Penguins, which are aquatic birds living mostly in the Southern Hemisphere [1], strongly shoot their faeceses towards their rear side [2]. It is believed that this is because penguins avoid getting the faeces on themselves as well as the nest. Although such a tendency is not limited to penguins and can be found in the case of other birds, these bombings sometimes embarrass keepers under breeding environments like an aquarium. It is practically important to know how far their faeceses reach from the origin. Such information would save keepers from the crisis. It would also be helpful for a newcomer guidance for keepers to avoid such an incident.

The flying distance of penguin’s faeces reaches about 0.4 m even on the ground. Since a typical height of a Humboldt penguin is given by 0.4 m, this distance corresponds to the situation that if a human being whose height is 1.7 m tries to evacuate his/her bowels, the object could fly to 1.7 m away. Therefore, one can immediately understand that penguin’s rectal pressure is relatively much strong compared to that of a human kind. In the pioneering work of Ref. [2], it is reported that this actual pressure could range from 10 kPa to 60 kPa for relevant values of the faeces viscosity and the radius of the bottom hole.

However, in Ref. [2], the projectile trajectory of the faeces is not taken into account but the horizontal distance is employed to estimate the fluid volume. In general, the actual trajectory would be longer than the horizontal distance during the projectile motion as shown in Fig. 1. In addition, since the ejection angle is not always horizontal and penguins sometimes shoot them out from higher place under the breeding environment, it is important to consider such a motion of faeces in a more general manner. The rectal pressure is regarded as an impulse force to accelerate the faeces up to the initial velocity in Ref. [2]. For non-viscous fluids, we can use Bernoulli’s theorem which is related to the energy conservation. To estimate the viscosity effects on the rectal pressure, Ref. [2] assumed the Hagen-Poiseille flow in the air, which is laminar flow of Newtonian liquid in a cylindrical pipe geometry. A similar approach for uniaxial urinary flow was employed to estimate the duration of urination of various animals in Refs. [3, 4].

In this work, we calculate the maximum flying distance of penguin’s faeces from a high place, which is relevant for several breeding environments. Such a projectile trajectory is described by Newton’s equation of motion. We assume that the upper bound for the flying distance can be obtained by the equation of motion in the absence of the air resistance. Moreover, we revisit the rectal pressure by using Bernoulli’s theorem and the Hagen-Poiseuille equation [5] to estimate the mechanical contributions of non-viscous flow and the viscosity correction during the flow in the stomach and in the air, respectively.

FIG. 1: Configuration for a penguin trying to defecate towards his/her rear side. A penguin stands on the rock with the height $h$ from the ground. We parameterize the ejection angle $\theta$ with the initial velocity $v = (v_{0x}, v_{0y})$. We estimate a flying distance $d$ of the faeces from the origin $O$. 

This paper is organized as follows. In Sec. III we explain our setup for the projectile motion of penguin’s faeces and the evacuation of them from their intestines. The latter is used for the estimation of penguin’s rectal pressure. In Sec. IV we show Newton’s equation of motion for the faeces after the shoot. We show the maximum flying distance at arbitrary angle and height. In Sec. V we discuss effects of the viscous air resistance during the projectile motion. In Sec. VI we calculate the rectal pressure under the assumption that the faeces liquid can approximately regarded as an ideal fluid. In Sec. VII we estimate the additional contribution to the rectal pressure due to the viscosity correction within the Hagen-Poiseuille equation. Finally, we summarize this paper in Sec. VII.

II. SETUP

Before moving to the calculation, we explain the configurations and the parameters we consider in this work. Figure 1 shows the situation where a Humboldt penguin shoots the faeces out from the higher place with the height $h$. Here, we employ an initial velocity $|v_0| \equiv v_0 = 2.0 \, \text{m/s}$ reported in Ref. [2] as a typical value. The ejection angle is denoted by $\theta$, and the gravitational constant is fixed at $g = 9.8 \, \text{m/s}^2$.

Figure 2 shows the effective model of penguin’s stomach. We employ properties of the faeces used in Ref. [2], where the mass density $\rho = 1141 \, \text{kg/m}^3$, the viscosity $\eta = 0.02 \sim 0.08 \, \text{Pa\cdot s}$ are employed. In our model the radius of bottom hole is given by $r = 0.004 \, \text{m}$. Although it is known that penguin’s rectum has a form of a straight tube [3], in this work we approximate it as a cylindrical tank with the radius $R$ for simplicity. The pressure in the stomach is given by $P = P_0 + P_t$ where $P_0 = 1013 \, \text{hPa}$ and $P_t$ are the atmospheric pressure and the rectal pressure, respectively.

III. NEWTON’S EQUATION OF MOTION WITHOUT AIR RESISTANCES

We consider Newton’s equations of motion
\[
m \frac{d^2x}{dt^2} = 0, \tag{3.1}
\]
\[
m \frac{d^2y}{dt^2} = -mg, \tag{3.2}
\]
where $m$ is the total mass of penguin’s faeces. By solving them, one can obtain textbook results
\[
v_x = \frac{dx}{dt} = v_0 \cos \theta, \tag{3.3}
\]
\[
v_y = \frac{dy}{dt} = v_0 \sin \theta - gt, \tag{3.4}
\]
and
\[
x(t) = v_0 t \cos \theta, \tag{3.5}
\]
\[
y(t) = h + v_0 t \sin \theta - \frac{1}{2}gt^2. \tag{3.6}
\]

We obtain the grounding time $t_g$ from $y(t_g) = 0$, which reads
\[
\frac{1}{2}gt_g^2 - v_0 y_g - h = 0. \tag{3.7}
\]

Thus, one can obtain
\[
t_g = \frac{v_0 \sin \theta}{g} + \sqrt{\frac{v_0^2 \sin^2 \theta}{g^2} + \frac{2h}{g}}. \tag{3.8}
\]

Since the flying distance is given by $d = v_0 t_g \cos \theta$, we obtain
\[
d = v_0 \cos \theta \left( \frac{v_0 \sin \theta}{g} + \sqrt{\frac{v_0^2 \sin^2 \theta}{g^2} + \frac{2h}{g}} \right). \tag{3.9}
\]

In Fig. 3 we plot $d$ as a function of $\theta$ at a typical height $h = 1.1 \, \text{m}$. The angle $\theta_{\max}$ where $d$ becomes maximum is given by the condition
\[
\left. \frac{\partial d}{\partial \theta} \right|_{\theta=\theta_{\max}} = -v_0 \sin \theta \left( \frac{v_0 \sin \theta}{g} + \sqrt{\frac{v_0^2 \sin^2 \theta}{g^2} + \frac{2h}{g}} \right) + v_0 \cos \theta \left( \frac{v_0 \cos \theta}{g} + \sqrt{\frac{v_0^2 \sin^2 \theta \cos \theta}{g^2}} + \frac{2h}{g} \right) = 0. \tag{3.10}
\]
IV. NEWTON’S EQUATION OF MOTION WITH VISCOUS RESISTANCE

Here, we discuss effects of the air resistance. For simplicity, we consider the case with viscous resistance which is proportional to \( v \). The equations of motion are given by

\[
\frac{md^2x}{dt^2} = -kv_x, \quad (4.1)
\]

\[
\frac{md^2y}{dt^2} = -mg - kv_y. \quad (4.2)
\]

Resulting velocities and distances are given by

\[
v_x(t) = v_{0x}e^{-\frac{kt}{m}}, \quad (4.3)
\]

\[
v_y(t) = v_{0y}e^{-\frac{kt}{m}} + \frac{mg}{k}(e^{-\frac{kt}{m}} - 1), \quad (4.4)
\]

\[
x(t) = \frac{mv_{0x}}{k}(1 - e^{-\frac{kt}{m}}), \quad (4.5)
\]

and

\[
y(t) = h + \frac{m}{k} \left( v_{0y} + \frac{mg}{k} \right) (1 - e^{-\frac{kt}{m}}) - \frac{mg}{k} t. \quad (4.6)
\]

We note that by expanding them with respect to \( k \), one can reproduce the results in the absence of the resistance. Since \( v_x(t) \) exponentially decreases due to the nonzero \( k \), one can confirm that \( d_{\text{max}} \) with \( k = 0 \) gives an upper bound for the distance. We note that in this case the grounding time is given by

\[
t_g = \frac{m}{k} W_0 \left( 1 + \frac{KV_{0x}}{mg} e^{-(1 + \frac{KV_{0y}}{mg} + \frac{h}{mg})} \right)
\]

\[
+ \frac{m}{k} + \frac{hk}{mg} + \frac{v_{0y}}{g}, \quad (4.7)
\]

FIG. 3: The flying distance \( d \) as a function of \( \theta \) at \( h = 1.1 \) m.

From Eq. (3.10) we obtain

\[
\theta_{\text{max}} = \sin^{-1} \left( \frac{1}{\sqrt{2 \left( 1 + \frac{h}{v_0^2} \right)}} \right). \quad (3.11)
\]

In this regard, in terms of \( h \) and \( v_0 \), the maximum flying distance \( d_{\text{max}} \) is given by

\[
d_{\text{max}} = v_0 \cos \left[ \sin^{-1} \left( \frac{1}{\sqrt{2 \left( 1 + \frac{h}{v_0^2} \right)}} \right) \right]
\times \left[ \frac{v_0}{g} \frac{1}{\sqrt{2 \left( 1 + \frac{h}{v_0^2} \right)}} + \frac{\sqrt{v_0^2 - 2h \frac{v_0}{g} \left( 1 + \frac{h}{v_0^2} \right)}}{\sqrt{2 \left( 1 + \frac{h}{v_0^2} \right)}} \right].
\]

(3.12)

Figure 4 shows \( h \)-dependence of \( d_{\text{max}} \) at \( \theta = \theta_{\text{max}} \). The inset shows \( \theta_{\text{max}} \) as a function of \( h \). In the case of \( h = 1.1 \) m which is typical height of rocks in the penguin’s area of the Katsurahama aquarium, we obtain \( d_{\text{max}} = 1.03 \) m at \( \theta_{\text{max}} = 21.6^\circ \). It is slightly longer the case of the horizontal ejection \( d(\theta = 0) = 0.95 \) m. We note that the maximal height of rocks is \( h = 2.0 \) m in the Katsurahama aquarium. In this case, we obtain \( d_{\text{max}} = 1.34 \) m at \( \theta_{\text{max}} = 16.9^\circ \) and \( d(\theta = 0) = 1.24 \) m. Since the air resistance in principle shortens \( d \) as we discuss in Sec. IV this value can be regarded as an upper bound for \( d_{\text{max}} \). Therefore, we found that penguin keepers should keep the distance being longer than 1.34 m from penguins trying to eject faeces in the Katsurahama aquarium.

FIG. 4: The maximum flying distance \( d_{\text{max}} \) as a function of \( h \). The inset shows the corresponding angle \( \theta_{\text{max}} \).
where $W_0(x)$ is the main branch of the Lambert $W$ function. The distance can be obtained by $d = x(t_g)$.

We also note that the resistance given by $k|v|v$ is more realistic in the air [2]. In such a case, an approximated solution for a low-angle trajectory has also been obtained by using the Lambert $W$ function [8, 9]. We note that an analytic solution in this case can be obtained by the homotopy analysis method [10]. In addition, the viscosity of faeces itself may shorten the flying distance due to the energy dissipation originating from the internal friction. More sophisticated treatments are required to take such an effect into account. However, our estimation for the upper bound on $d_{\text{max}}$ is robust against these resistance effects.

V. BERNOULLI’S THEOREM AND ABDOMINAL PRESSURE OF PENGUINS

Bernoulli’s theorem for non-viscous faeces liquids is given by

$$\frac{1}{2} \rho v^2 + P = \text{const.}$$

where $z$ is the initial height of liquids in a penguin as shown in Fig. 2. While the faeces are immobile under the abdominal pressure $P_a$ initially, they are released with $v_0$ in the air. Such an assumption gives

$$\rho g z + P_a + P_0 = \frac{1}{2} \rho v_0^2 + P_0,$$ (5.2)

where $P_0$ is the atmospheric pressure. At this stage, we do not consider the viscosity correction. Thus we obtain

$$P_a = \frac{1}{2} \rho v_0^2 - \rho g z.$$ (5.3)

The unknown parameter $z$ can be estimated from

$$z = \frac{V}{\pi R^2},$$ (5.4)

where $V$ is the fluid volume after the ejection. $R$ is the radius of stomach. For simplicity, we use $R = 0.1$ m. Here, one may notice that the system is quite similar to the so-called tank orifice with a vena contracta [12]. In such a case, the cross-section area of non-viscous flow after the ejection is approximately given by $C \pi r^2$, where $C \approx 0.611$ is a typical value of the coefficient of contraction [12]. In this regard, we obtain

$$V = C \pi r^2 \ell$$

$$= C \pi r^2 \int_0^{t_g} \sqrt{(v_{0y} - (v_{0y} - gt_g)(v_{0y} - gt_g))^2 + v_{0x}^2} dt + v_{0x}^2 \ln \left( \frac{(v_{0y} - (v_{0y} - gt_g))^2 + v_{0x}^2}{v_0 + v_{0y}} \right),$$ (5.5)

where $\ell$ is the total path of the faeces. In particular, in the case of $\theta = 0$ (namely, $v_{0y} = 0$ and $v_{0x} = v_0$), we obtain

$$V = \frac{C \pi r^2}{2g} \left[ gt_g \sqrt{g^2 t_g^2 + v_0^2} - v_0^2 \ln \left( \frac{\sqrt{g^2 t_g^2 + v_0^2} - gt_g}{v_0 + v_{0y}} \right) \right].$$ (5.6)

By substituting Eq. (5.5) to

$$P_a = \frac{1}{2} \rho v_0^2 - \frac{\rho g}{\pi R^2} V,$$ (5.7)

we can obtain $P_a \approx 2.3$ kPa at $\theta = 0$ and $h = 0.2$ m following the previous work [2]. It is smaller than the estimated pressure 4.6 kPa in Ref. [2], where the initial pressure for the acceleration was obtained by $P_a = \rho v_0^2$. However, $P_a$ are small compared to the viscosity effect which is addressed in Sec. VI.

VI. HAGEN-POISEUILLE EQUATION AND VISCOSITY EFFECT

The Hagen-Poiseuille equation gives a relation between an additional pressure and fluid flow rate $Q$ for laminar flow though a cylindrical pipe [11] as

$$Q = \frac{\pi r^4 \alpha}{8 \eta},$$ (6.1)

where $\alpha = \nabla P_b$ and $Q$ can approximately be given by $\alpha \approx \frac{P_a}{\ell}$ and $Q \approx \frac{V}{\ell}$, respectively. Therefore, we obtain

$$P_b = \frac{8V \eta}{t_g \pi R^4},$$ (6.2)

which is consistent with Ref. [2]. Furthermore, we estimate the additional pressure contributions for the rectal flow as

$$P_c = \frac{8V \eta}{t_d \pi R^4} \approx \frac{4V v_{0y} \eta}{\pi R^4},$$ (6.3)

where $t_d$ is the flowing time inside the intestine. We have estimated it as $t_d \approx \frac{3}{10} \eta \approx 0.45$ ms. Even the maximum duration $t_g + t_d \approx 0.20$ s is quite short compared to the urination time 8.2 (M/kg) 0.13 s obtained in Ref. [3] (where $M$ is the mass of an animal). We note that a typical value of $M$ for a Humboldt penguin is about 4 kg. In this way, the total rectal pressure is given by $P_t = P_a + P_b + P_c$.

Figure 5 shows the viscosity dependence of $P_t$ with $r = 0.004$ m. For comparison, we plot the previous result in Ref. [2] and our result of $P_t$ without $C$. While the previous work gives $P_t = 8.56 \sim 20.6$ kPa for $\eta = 0.02 \sim 0.08$ Pa-s (noting that $\eta$ may change due to the physical
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