$S$-matrix description of the finite-temperature nonequilibrium media

J. Manjavidze

Abstract

The paper contains the real-time perturbation theory for description of a statistical system with the nonuniform temperature distribution. The formalism based on the Wigner-functions approach. The perturbation theory is formulated in terms of the local-temperature Green functions.
1 Introduction

The aim of this article is to construct the perturbation theory for generating functional of Wigner functions [1, 2, 3] for the case of nonuniform temperature distribution. As an example of interesting system one can have in mind the process of a very large number of hadrons creation at the high-energy collisions. The phenomenology of high-multiplicity processes was given in [4, 5].

In terms of QCD it is the rear process of the cold quark-gluon plasma formation. Qualitatively this is a process of total dissipation of high kinetic energy density initial state. One can consider this process also as the process of dissipation of a high-temperature local fluctuation in a low temperature equilibrium media. Generally speaking, the temperature freely evolves in this processes and is not distributed uniformly at least on the early stages. In this paper we will consider the general theory of such processes which can be used not only in the particles physics.

We should adopt the standard $S$-matrix formalism which is applicable to any nonequilibrium processes. In this microcanonical approach the temperature $T$ will be introduced as the Lagrange multiplier and the physical (measurable) value of $T$ will be defined by the equation of states at the very end of calculations. Using standard terminology [3], we will deal with the “mechanical” perturbations only [7] and it will not be necessary to divide the perturbations on “thermal” and “mechanical” ones [3] (see also Sec.5).

The usual Kubo-Martin-Schvinger (KMS) periodic boundary conditions [4, 10] can not be applied here since they are applicable for the equilibrium case only [11] (see also [12]). We will introduce the boundary conditions “by hands”, modeling the environment of the system [2]. Supposing that the system is in a vacuum we will have usual field-theoretical vacuum boundary condition (Sec.2). We will consider also the system in the background field of black-body radiation (Sec.4). Last one restores the theory with KMS boundary condition in the equilibrium limit.

Calculating the generating functional of Wigner functions the local temperature distribution will be introduced: $T(\vec{x}, t) = 1/\beta(\vec{x}, t)$ is the temperature in the measurement point $(\vec{x}, t)$ (Sec.3). In other words, we will divide the “measuring device” (but not the system as usually was done, [13]) on the cells the dimension of which tends to zero. The differential measure $D\beta(\vec{x}, t)$ will be defined taking into account the energy-momentum conservation law.

2 Vacuum boundary condition

The probability $r(P)$ of in- into out-states transition with fixed total 4-momentum $P$ can be calculated using the $n$- into $m$-particle transition amplitude $a_{n,m}$:

\[
r(P) = \sum_{n,m} \frac{1}{n! m!} \int d\omega_n(q) \omega_m(p) \times
\]

\[
\times \delta^{(4)}(P - \sum_{k=1}^{n} q_k) \delta^{(4)}(P - \sum_{k=1}^{m} p_k) |a_{n,m}|^2,
\]

(2.1)
\[ d\omega_n(q) = \prod_{k=1}^{n} d\omega(q_k) = \prod_{k=1}^{n} \frac{d^3q_k}{(2\pi)^3 2\epsilon(q_k)}, \quad \epsilon(q) = (q^2 + m^2)^{1/2}. \]  

Eq. (2.1) is the basic formula of our calculations. The microcanonical description was introduced in [12] considering the Fourier transformation of \( \delta \)-functions.

The amplitude \( a_{n,m} \) looks as follows [12]:

\[ a_{n,m}(q_n, p_m) = \prod_{k=1}^{n} \hat{\phi}(q_k) \prod_{k=1}^{m} \hat{\phi}^*(p_k) Z(\phi), \]  

where \( q_k(p_k) \) are the momentum of in(out)-going particles and the annihilation operator

\[ \hat{\phi}(q) = \int d^4x e^{-iqx} \hat{\phi}(x), \quad \hat{\phi} = \frac{\delta}{\delta \phi(x)}, \]  

was introduced. Correspondingly, \( \hat{\phi}^*(p) \) is the creation operator. One can put the auxiliary field \( \phi(x) \) equal to zero at the end of calculation. The vacuum into vacuum transition amplitude in presence of external field

\[ Z(\phi) = \int D\Phi e^{iS_{C_+}(\Phi) - iV_{C_+}(\Phi + \phi)} \]  

is defined on the Mills’ complex time contour \( C_+ \) [14], i.e. \( C_+ : t \to t + i\varepsilon, \varepsilon > 0 \). In eq. (2.3) \( S_{C_+} \) is the free part of the action and \( V_{C_+} \) describes the interactions.

In this section we will propose the vacuum boundary condition:

\[ \int_{\sigma_{\infty}} d\sigma_{\mu} \Phi \partial^\mu \Phi = 0 \]  

where \( \sigma_{\infty} \) is the infinitely far hypersurface.

We start consideration from the assumption that the temperature fluctuations are large scale. In a cell the dimension of which is much smaller then the fluctuation scale of temperature we can assume that the temperature is a “good” parameter. (The “good” parameter means that the corresponding fluctuations are Gaussian.)

Let us surround the interaction region, i.e. the system under consideration, by \( N \) cells with known space-time position and let us propose that we can measure the energy and momentum of groups of in- and out-going particles in each cell. The 4-dimension of cells can not be arbitrary small in this case because of the quantum uncertainty principle.

To describe this situation we decompose \( \delta \)-functions in (2.1) on the product of \((N + 1)\) \( \delta \)-functions:

\[ \delta^{(4)}(P - \sum_{k=1}^{n} q_k) = \int \prod_{\nu=1}^{N} dQ_\nu \delta(Q_\nu - \sum_{k=1}^{n_{\nu}} q_{k,\nu}) \delta^{(4)}(P - \sum_{\nu=1}^{N} Q_\nu), \]  

where \( q_{k,\nu} \) are the momentum of \( k \)-th in-going particle in the \( \nu \)-th cell and \( Q_\nu \) is the total 4-momenta of \( n_{\nu} \) in-going particles in this cell. The same decomposition will be used for
the second \(\delta\)-function in (2.11). Inserting this decompositions into (2.11) we must take into account the multinomial character of particles decomposition on \(N\) groups. This will give the coefficient:

\[
\frac{n!}{n_1! \cdots n_N!} \delta_K(n - \sum_{\nu=1}^N n_{\nu}) \frac{m!}{m_1! \cdots m_N!} \delta_K(m - \sum_{\nu=1}^N m_{\nu}),
\]

where \(\delta_K\) is the Kronecker’s \(\delta\)-function.

In result, the quantity

\[
r((Q)_N, (P)_N) = \sum_{(n,m)} \int |a_{(n,m)}|^2 \times
\]

\[
\times \prod_{\nu=1}^N \left\{ \prod_{k=1}^{n_{\nu}} \frac{d\omega(q_{k,\nu})}{n_{\nu}!} \delta^{(4)}(Q_{\nu} - \sum_{k=1}^{n_{\nu}} q_{k,\nu}) \prod_{k=1}^{m_{\nu}} \frac{d\omega(p_{k,\nu})}{m_{\nu}!} \delta^{(4)}(P_{\nu} - \sum_{k=1}^{m_{\nu}} p_{k,\nu}) \right\}
\]

(2.9)

describes the probability that in the \(\nu\)-th cell we measure the fluxes of in-going particles with total 4-momentum \(Q_{\nu}\) and of out-going particles with the total 4-momentum \(P_{\nu}\). The sequence of this two measurements is not fixed.

The Fourier transformation of \(\delta\)-functions in (2.4) gives the formula:

\[
r((Q)_N, (P)_N) = \int \prod_{k=1}^N \frac{d^4\alpha_{-\nu}}{(2\pi)^4} \frac{d^4\alpha_{+\nu}}{(2\pi)^4} e^{i \sum_{\nu=1}^N (q_{\nu} + k_{\nu}) \alpha_{-\nu} + (p_{\nu} - q_{\nu}) \alpha_{+\nu}} R((\alpha_-)_N, (\alpha_+)_N),
\]

(2.10)

where \(R((\alpha_-)_N, (\alpha_+)_N) = R(\alpha_{-,1}, \alpha_{-,2}, ..., \alpha_{-,N}; \alpha_{+,1}, \alpha_{+,2}, ..., \alpha_{+,N})\) has the form:

\[
R((\alpha_-)_N, (\alpha_+)_N) = \int \prod_{\nu=1}^N \left\{ \prod_{k=1}^{n_{\nu}} \frac{d\omega(q_{k,\nu})}{n_{\nu}!} e^{-i\alpha_{-\nu} \cdot q_{k,\nu}} \times \prod_{k=1}^{m_{\nu}} \frac{d\omega(p_{k,\nu})}{m_{\nu}!} e^{-i\alpha_{+\nu} \cdot p_{k,\nu}} \right\} |a_{(n,m)}|^2.
\]

(2.11)

Inserting (2.3) into (2.11) we find:

\[
R((\alpha_-)_N, (\alpha_+)_N) = \exp\{i \sum_{\nu=1}^N \int dx dx' [\hat{\phi}_+(x) D_{+-}(x - x'; \alpha_{+,\nu}) \hat{\phi}_-(x')] - \hat{\phi}_-(x) D_{-+}(x - x'; \alpha_{-,\nu}) \hat{\phi}_+(x')]\} Z(\phi_+) Z^*(\phi_-),
\]

(2.12)

where \(\phi_-\) is defined on the complex conjugate contour \(C_- : t \to t - i\varepsilon\) and

\[
D_{+-}(x - x'; \alpha) = -i \int d\omega(q) e^{iq(x-x')} e^{-i\alpha q},
\]

(2.13)

\[
D_{-+}(x - x'; \alpha) = i \int d\omega(q) e^{-iq(x-x')} e^{-i\alpha q}
\]

(2.14)

are the positive and negative frequency correlation functions correspondingly.
We must integrate over sets \((Q)_N\) and \((P)_N\) if the distribution of fluxes momenta over cells is not fixed. In result,

\[
r(P) = \int D^4\alpha_- (P) d^4\alpha_+ (P) R((\alpha_-)_N, (\alpha_+)_N),
\]

where the differential measure

\[
D^4\alpha (P) = \prod_{\nu=1}^{N} \frac{d^4\alpha_\nu}{(2\pi)^4} K(P, (\alpha)_N)
\]

takes into account the energy-momentum conservation laws:

\[
K(P, (\alpha)_N) = \int \prod_{\nu=1}^{N} d^4Q_\nu e^{\sum_{\nu=1}^{N} \alpha_\nu Q_\nu} \delta^{(4)}(P - \sum_{\nu=1}^{N} Q_\nu).
\]

The explicit integration gives that

\[
K(P, (\alpha)_N) \sim \prod_{\nu=1}^{N} \delta^{(3)}(\alpha - \alpha_\nu),
\]

where \(\vec{\alpha}\) is the center of mass (CM) 3-vector.

To simplify the consideration let us choose the CM frame and put \(\alpha = (-i\beta, \vec{0})\). In result,

\[
K(E, (\beta)_N) = \int_0^{\infty} \prod_{\nu=1}^{N} dE_\nu e^{\sum_{\nu=1}^{N} \beta_\nu E_\nu} \delta(E - \sum_{\nu=1}^{N} E_\nu)
\]

Correspondingly, in the CM frame,

\[
r(E) = \int D\beta_+(E) D\beta_-(E) R((\beta_+)_N, (\beta_-)_N),
\]

where

\[
D\beta(E) = \prod_{\nu=1}^{N} \frac{d\beta_\nu}{2\pi i} K(E, (\beta)_N)
\]

and \(R((\beta)_N)\) was defined in (2.12) with \(\alpha_{k,\nu} = (-i\beta_{k,\nu}, \vec{0})\), \(Re\beta_{k,\nu} > 0\), \(k = +, -\).

We will calculate integrals over \(\beta_k\) using the stationary phase method. The equations for mostly probable values of \(\beta_k\):

\[
- \frac{1}{K(E, (\beta)_N)} \frac{\partial}{\partial \beta_k} K(E, (\beta)_N) = \frac{1}{R((\beta)_N)} \frac{\partial}{\partial \beta_k} R((\beta)_N), \quad k = +, -
\]

always has the unique positive solutions \(\tilde{\beta}_{k,\nu}(E)\). We propose that the fluctuations of \(\beta_k\) near \(\tilde{\beta}_k\) are small, i.e. are Gaussian. This is the basis of the local-equilibrium hypothesis [13]. In this case \(1/\tilde{\beta}_-\) is the temperature in the initial state in the measurement cell \(\nu\) and \(1/\tilde{\beta}_+\) is the temperature of the final state in the \(\nu\)-th measurement cell.
The last formulation (2.13) imply that the 4-momenta \((Q)_N\) and \((P)_N\) can not be measured. It is possible to consider another formulation also. For instance, we can suppose that the initial set \((Q)_N\) is fixed (measured) but \((P)_N\) is not. In this case we will have mixed experiment: \(\tilde{\beta}_{-\nu}\) is defined by the equation:

\[
E_\nu = -\frac{1}{R} \frac{\partial}{\partial \beta_{-\nu}} R \tag{2.23}
\]

and \(\tilde{\beta}_{+\nu}\) is defined by second equation in (2.22).

Considering limit \(N \to \infty\) the dimension of cells tends to zero. In this case we are forced by quantum uncertainty principle to propose that the 4-momenta sets \((Q)\) and \((P)\) are not fixed. This formulation becomes pure thermodynamical: we must assume that \((\beta_-)\) and \((\beta_+)\) are measurable quantities. For instance, we can fix \((\beta_-)\) and try to find \((\beta_+)\) as the function of total energy \(E\) and the functional of \((\beta_-)\). In this case eqs. (2.22) become the functional equations.

In the considered microcanonical description the finiteness of temperature does not touch the quantization mechanism. Really, one can see from (2.12) that all thermodynamical information is confined in the operator exponent

\[
e^{N(\hat{\phi}_i\hat{\phi}_j)} = \prod_\nu \prod_{i \neq j} e^{\int d\omega(q) \hat{\phi}_i^*(q)e^{-\beta_{-\nu} \hat{\phi}_j(q)}z_{ij}(q)} \times
\]

\[
\times Z(\phi_+)Z^*(\phi_-). \tag{2.24}
\]

the expansion of which describes the environment, and the “mechanical” perturbations are described by the amplitude \(Z(\phi)\). This factorization was achieved by introduction of auxiliary field \(\phi\) and is independent from the choice of boundary conditions, i.e. from the choice of the considered systems environment.

3 The distribution functions

In the previous section the generating functional \(R((\beta)_N)\) was calculated by means of dividing the “measuring device” (calorimeter) on the \(N\) cells. It was assumed that the dimension of device cells tends to zero \((N \to \infty)\). Now we will specify the cells coordinates using the Wigner’s description [1, 2, 3].

Let us introduce the distribution function \(F_n\) which defines the probability to find \(n\) particles with definite momentum and with arbitrary coordinates. This probabilities (cross section) are usually measured in particle physics. The corresponding Fourier-transformed generating functional can be deduced from (2.12):

\[
F(z, (\beta)_N, (\beta)_N) = \prod_\nu \prod_{i \neq j} e^{\int d\omega(q) \hat{\phi}_i^*(q)e^{-\beta_{-\nu} \hat{\phi}_j(q)}z_{ij}(q)} \times
\]

\[
\times Z(\phi_+)Z^*(\phi_-). \tag{3.1}
\]

The variation of \(F\) over \(z_{ij}(q)\) generates corresponding distribution functions. One can interpret \(z_{ij}(q)\) as the local activity: the logarithm of \(z_{ij}(q)\) is conjugate to the particles
number in the cell $\nu$ with momentum $q$ for the initial ($ij = +$) or final ($ij = -$) states. Note that $z_{ij}(q)\hat{\phi}_i^*(q)\hat{\phi}_j(q)$ can be considered as the operator of activity.

The Boltzmann factor $e^{-\beta_i,\nu \epsilon(q)}$ can be interpreted as the probability to find a particle with the energy $\epsilon(q)$ in the final state ($i = +$) and in the initial state ($i = -$). The total probability, i.e. the process of creation and further absorption of $n$ particles, is defined by multiplication of this factors.

The generating functional (3.1) is normalized as follows:

$$F(z = 1, (\beta)) = R((\beta)), \quad F(z = 0, (\beta)) = |Z(0)|^2 = R_0(\phi_\pm)|_{\phi_\pm = 0}$$

(3.2)

Where

$$R_0(\phi_\pm) = Z(\phi_+)Z^*(\phi_-)$$

(3.3)

is the “probability” of the vacuum into vacuum transition in presence of auxiliary fields $\phi_\pm$. The one-particle distribution function

$$F_1((\beta_+)_N, (\beta_-)_N; q) = \frac{\delta}{\delta z_{ij}^\nu(q)}F|_{z=0} = \{\hat{\phi}_i^*(q)e^{-\beta_i^\nu \epsilon(q)/2}\} \{\hat{\phi}_j(q)e^{-\beta_j^\nu \epsilon(q)/2}\} R_0(\phi_\pm)$$

(3.4)

describes the probability to find one particle in the vacuum.

Using definition (2.4),

$$F_1((\beta_+)_N, (\beta_-)_N; q) = \int dx dx' e^{iq(x-x')} e^{-\beta_i,\nu \epsilon(q)} \hat{\phi}_i(x)\hat{\phi}_j(x') R_0(\phi_\pm) =$$

$$= \int dY \{dye^{iyq} e^{-\beta_i,\nu \epsilon(q)}\} \hat{\phi}_i(Y + y/2)\hat{\phi}_j(Y - y/2)R_0(\phi_\pm).$$

(3.5)

(3.6)

We introduce using this definition the one-particle Wigner function $W_1$ [2]:

$$F_1((\beta_+)_N, (\beta_-)_N; Y, q) = \int \int dY W_1((\beta_+)_N, (\beta_-)_N; Y, q).$$

(3.7)

So,

$$W_1((\beta_+)_N, (\beta_-)_N; Y, q) = \int dy e^{iyq} e^{-\beta_i,\nu \epsilon(q)} \hat{\phi}_i(Y + y/2)\hat{\phi}_j(Y - y/2)R_0(\phi_\pm).$$

(3.8)

This distribution function describes the probability to find in the vacuum particle with momentum $q$ at the point $Y$ in the cell $\nu$.

Since the choice of the device coordinates is in our hands it is natural to adjust the cell coordinate to the coordinate of measurement $Y$:

$$W_1((\beta_+)_N, (\beta_-)_N; Y, q) = \int dy e^{iyq} e^{-\beta_i(Y)\epsilon(q)} \hat{\phi}_i(Y + y/2)\hat{\phi}_j(Y - y/2)R_0(\phi_\pm).$$

(3.9)
This choice of the device coordinates lead to the following generating functional:

\[ F(z, \beta) = \exp\{ i \int dydY[\hat{\phi}_+(Y + y/2)D_{+\text{}}(y; \beta_+(Y), z)\hat{\phi}_-(Y - y/2) - \hat{\phi}_-(Y + y/2)D_{-\text{}}(y; \beta_-(Y), z)\hat{\phi}_+(Y - y/2)]\}R_0(\phi_\pm), \]  

(3.10)

where

\[ D_{+\text{}}(y; \beta_+(Y), z) = -i \int d\omega(q)z_{+\text{}}(Y, q)e^{iqy}e^{-\beta_+(Y)\epsilon(q)}, \]  

(3.11)

\[ D_{-\text{}}(y; \beta_+(Y), z) = i \int d\omega(q)z_{-\text{}}(Y, q)e^{-iqy}e^{-\beta_-(Y)\epsilon(q)} \]  

(3.12)

are the modified positive and negative correlation functions (2.13), (2.14).

The inclusive, partial, distribution functions are familiar in the particle physics. This functions describe the distributions in presence of arbitrary number of other particles. For instance, one-particle partial distribution function

\[ P_{ij}(Y, q; (\beta)) = \delta_{zij}(Y, q)F(z, (\beta))|_{z=1} = \frac{e^{-\beta(\epsilon(q))}}{(2\pi)^3\epsilon(q)} \int dy e^{iqy}\hat{\phi}_i(Y + y/2)\hat{\phi}_j(Y - y/2)R(\phi_\pm, (\beta)), \]  

(3.13)

where eq.(3.2) was used.

The mean multiplicity \( n_{ij}(Y, q) \) of particles in the infinitesimal cell \( Y \) with momentum \( q \) is

\[ n_{ij}(Y, q) = \int dq \frac{\delta}{\delta z_{ij}(Y, q)} \ln F(z, (\beta))|_{z=1}. \]  

(3.14)

If the interactions among fields are switched out we can find that (omitting indexes):

\[ n(Y, q_0) = \frac{1}{e^{\beta(q_0)0} - 1}, \quad q_0 = \epsilon(q) > 0. \]  

(3.15)

This is the mean multiplicity of black-body radiation.

4 The closed-path boundary condition

The developed in Sec.2 formalism allows to introduce the more general boundary conditions instead of (2.5). Considering the probability \( R \) which has the double path integral representation we will introduce integration over closed path. This allows to introduce the equality:

\[ \int_{\sigma_\infty} \sigma_\mu(\Phi_+ \partial^\mu \Phi_+ - \Phi_- \partial^\mu \Phi_-) = 0, \]  

(4.1)

as the boundary condition, where \( \sigma_\infty \) is the infinitely far hypersurface. The general solution of this equation is:

\[ \Phi_\pm(\sigma_\infty) = \Phi(\sigma_\infty) \]  

(4.2)
where $\Phi(\sigma_\infty)$ is the “turning-point” field. The result of this changing of boundary condition was analyzed in [12] for the case of uniform temperature distribution.

In terms of $S$-matrix the field $\Phi(\sigma_\infty)$ represent the background flow of mass-shell particles. We will propose that the probability to find a particle of the background flow is determined by the energy-momentum conservation law only. In another words, we will propose that the system under consideration is surrounded by the black-body radiation.

Presence of additional flow will reorganize the differential operator $\exp\{N(\hat{\phi}_i\hat{\phi}_j)\}$ only and new generating functional $R_{cp}$ has the form:

$$R_{cp}(\alpha_+, \alpha_-) = e^{N(\hat{\phi}_i\hat{\phi}_j)} R_0(\phi_{\pm}).$$

(4.3)

The calculation of operator $N(\hat{\phi}_i\hat{\phi}_j)$ is strictly the same as in [12]. Introducing the cells in the $Y$ space we will find that

$$N(\hat{\phi}_i\hat{\phi}_j) = \int dY dy \hat{\phi}_i(Y + y/2) n_{ij}(Y, y) \hat{\phi}_j(Y - y/2),$$

(4.4)

where the occupation number $n_{ij}$ carries the cells index $Y$:

$$n_{ij}(Y, y) = \int d\omega(q) e^{i\omega y} n_{ij}(Y, q)$$

(4.5)

and $(q_0 = \epsilon(q))$

$$n_{++}(Y, q_0) = n_{--}(Y, q_0) = \bar{n}(Y, (\beta_+ + \beta_-)|q_0|/2) = \frac{1}{e^{(\beta_+ + \beta_-)(Y)|q_0|/2} - 1},$$

(4.6)

$$n_{+ -}(Y, q_0) = \Theta(q_0)(1 + \bar{n}(Y, \beta_+ q_0)) + \Theta(-q_0) \bar{n}(Y, -\beta_- q_0),$$

(4.7)

$$n_{-+}(Y, q_0) = n_{+-}(Y, -q_0).$$

(4.8)

For simplicity the CM system was used.

Calculating $R_0$ perturbatively we will find that

$$R_{cp}(\beta) = \exp\{-iV(-i\hat{j}_+ + i\hat{j}_-\}) \times$$

\[ \times \exp\{i \int dY dy \hat{j}_i(Y + y/2) G_{ij}(y, (\beta(Y)) \hat{j}_j(Y - y/2) \} \]

(4.9)

where, using the matrix notations,

$$iG(q, (\beta(Y))) = \begin{pmatrix} \frac{i}{q^2 - m^2 + i\epsilon} & 0 \\ 0 & -\frac{i}{q^2 - m^2 - i\epsilon} \end{pmatrix} +$$

$$+ 2\pi\delta(q^2 - m^2) \begin{pmatrix} n(\frac{\beta_+ + \beta_-}{2})|q_0| & n(\beta_+(Y)|q_0|)a_+(\beta_+) \\ n(\beta_-(Y)|q_0|)a_-(&beta_-) & n(\frac{\beta_+ + \beta_-}{2})|q_0| \end{pmatrix},$$

(4.10)

and

$$g_{\pm}(\beta) = -e^{\beta(|q_0|\pm\phi_0)/2}.$$

(4.11)

Formally this Green functions obey the standard equations in the $y$ space:

$$(\partial^2 - m^2)_y G_{ii} = \delta(y),$$

$$((\partial^2 - m^2)_y G_{ij} = 0, \ i \neq j$$

(4.12)

since $\Phi(\sigma_\infty) \neq 0$ reflects the mass-shell particles. But the boundary conditions for this equations are not evident.
5 Concluding remarks

One can not expect the evident connection between the above considered and Zubarev’s approaches. The reason is as follows.

In Zubarev’s theory the “local-equilibrium” hypothesis was adopted as the boundary condition. It is assumed that in the suitably defined cells of a system at a given temperature distribution \( T(\vec{x}, t) = 1/\beta(\vec{x}, t) \) where \((\vec{x}, t)\) is the index of the cell, the entropy is maximum. The corresponding nonequilibrium statistical operator

\[ R_z \sim e^{-\int d^3x \beta T_{00}} \] (5.1)

describes evolution of a system. Here \( T_{\mu\nu} \) is the energy-momentum tensor. It is assumed that the system “follows” to \( \beta(\vec{x}, t) \) evolution and the local temperature \( T(\vec{x}, t) \) is defined as the external parameter which is the regulator of systems dynamics. For this purpose the special \( i\epsilon \)-prescription was introduced [13].

The KMS periodic boundary condition [6, 10] can not be applied [11, 12] and by this reason the decomposition:

\[ \beta(\vec{x}, t) = \beta_0 + \beta_1(\vec{x}, t) \] (5.2)

was offered in the paper [8]. Here \( \beta_0 \) is the constant and the inequality

\[ \beta_0 >> |\beta_1(\vec{x}, t)| \] (5.3)

is assumed. Then,

\[ R_z \sim e^{-\beta_0 (H_0 + V + B)} \] (5.4)

where \( H_0 \) is the free part of the Hamiltonian, \( V \) describes the interactions and the linear over \( \beta_1/\beta_0 \) term \( B \) is connected with the deviation of temperature from the “equilibrium” value \( 1/\beta_0 \). Considering \( V \) and \( B \) as the perturbations one can calculate the observables averaging over equilibrium states, i.e. adopting the KMS boundary condition. Using standard terminology [11] one can consider \( V \) as the “mechanical” and \( T \) as the “thermal” perturbations.

The quantization problem of operator (5.4) is connected with definition of the space-time sequence of mechanical (\( V \)) and thermal (\( B \)) excitations. It is necessary since the mechanical excitations give the influence on the thermal ones and vice versa. It was assumed in [8] that \( V \) and \( B \) are commuting operators, i.e. the sequence of \( V \)- and \( B \)-perturbations is not sufficient. This solution leads to the particles propagators renormalization by the interactions with the external field \( \beta(\vec{x}, t) \) even without interactions among fundamental fields. (Note absence of this renormalization in our formalism.)

In [14] the operators \( V \) and \( B \) are noncommuting ones and \( B \)-perturbations were switched on after \( V \)-perturbations. In this formulation the nondynamical renormalization are also present but it is not unlikely that they are canceled at the very end of calculations [15].

This formulation with \( \beta(\vec{x}, t) \) as the external field remained the old, firstly quantized, field theory in which matter is quantized but fields are not. It is known that consistent
quantum field theory requires the second quantization. Following to this analogy, if we want to take into account consistently the reciprocal influence of $V$- and $B$-perturbations the field $\beta(\vec{x}, t)$ must be fundamental, i.e. must be quantized (and the assumption of paper \[8\] becomes true). But it is evidently the wrong idea in the canonical Gibbs formalism. So, as in the firstly quantized theory, the theory with operator \(5.1\) must have the restricted range of validity \[13\].

Therefore, we must reduce our formalism just to the hydrodynamical accuracy to find the connection with Zubarev’s approach. There is the another side of this question. The offered formalism is able to describe an arbitrary nonequilibrium process since it based on the $S$-matrix, i.e. on the strict field-theoretical description. But the mechanism of irreversibility is not clearly seen: the generating functional $R_0(\phi_\pm)$ is described by the closed-path motion in the functional space, i.e. formally is the time-reversible quantity.

**Acknowledgement**

I would like to thank T.Bibilashvili for interesting discussions. This work was supported in part by the U.S. National Science Foundation.
References

[1] E. P. Wigner, *Phys. Rev.* **40** 749 (1932)

[2] E. Carruthers and F. Zachariasen, *Rev. Mod. Phys.* **55** 245 (1983)

[3] E. Calsetta and B. L. Hu, *Phys. Rev.* **D37** 2878 (1988)

[4] J. Manjavidze and A. Sissakyan, *JINR Rapid Comm.* **5[31]-88** 5 (1988)

[5] J. Manjavidze, *Particles & Nuclei (Sov.Phys.)* **16** 101 (1985)

[6] R. Kubo, *J. Phys. Soc. Japan* **12** 570 (1957)

[7] P. Kadanoff and P. C. Martin, *Ann. Phys. (NY)* **24** 419 (1963); M. Luttinger, *Phys. Rev.* **135A** 505 (1964); J. L. Jackson and P. Masur, *Physica* **30** 2295 (1964)

[8] T. Bibilashvili and I. Pasiashvili, *Ann. Phys. (NY)* **220** 134 (1992)

[9] T. Bibilashvili, *Phys. Lett.* **B313** 119 (1993)

[10] P. S. Martin and J. Schwinger, *Phys. Rev.* **115** 342 (1959)

[11] R. Haag, N. Hugenholtz and M. Winnink, *Comm. Math. Phys.* **5** 5 (1967)

[12] J. Manjavidze, *Preprint*, IP GAS-HE-5/95, [hep-ph/9506424](http://arxiv.org/abs/hep-ph/9506424) (submitted for *Ann.Phys.)*

[13] D. N. Zubarev, *Nonequilibrium Statistical Thermodynamics* (Consultants Bureau, NY, 1974)

[14] R. Mills, *Propagators for Many-Particles Systems* (Gordon and Breach, Science, 1970)

[15] T. Bibilashvili, *(privet comm.)*