Measurement uncertainty evaluation of a hexapod-structured calibration device for multi-component force and moment sensors

J Nitsche\textsuperscript{1,2}, S Baumgarten\textsuperscript{2}, M Petz\textsuperscript{1}, D Röskes\textsuperscript{2}, R Kumme\textsuperscript{2} and R Tutsch\textsuperscript{1}

\textsuperscript{1} TU Braunschweig, Institut für Produktionsmesstechnik, Braunschweig, Germany
\textsuperscript{2} Physikalisch-Technische Bundesanstalt (PTB), Braunschweig, Germany

E-mail: jan.nitsche@tu-bs.de

Received 23 November 2016, revised 18 January 2017
Accepted for publication 23 January 2017
Published 15 February 2017

Abstract
As a reference measurement machine for multi-component force and moment sensors of up to six components, a hexapod-structured calibration device was developed at the Physikalisch-Technische Bundesanstalt in 2001. The machine can generate and measure forces of up to 10\,kN and moments of up to 1\,kN $\cdot$ m. In this paper, the measurement uncertainty budget of the machine is analyzed, beginning with an improved physical model and calculation of sensitivity coefficients using the implicit function theorem and the Monte Carlo method. The main influencing factors for the measurement uncertainty are discussed and suggestions for further reduction of the uncertainty are given.

Keywords: torque, force, multi-component, measurement uncertainty budget, hexapod, calibration, Stewart platform

(Some figures may appear in colour only in the online journal)

1. Introduction

Multi-component force and moment sensors are important measuring devices in applications where the direction of a force or moment vector is not known from the beginning or different components of forces or moments superimpose. Typical examples of such applications are robotics \cite{1}, wind tunnel balances \cite{2} or machine tool applications \cite{3, 4}, among others. Specific sensors for such demands are proposed, for example, in \cite{5–7}.

The calibration of such sensors is typically performed under uniaxial forces or moments, totally neglecting possible side-effects or parasitic influences. The behaviour under non-uniaxial forces or moments, however, reflects the typical use case of those devices which is not tested in calibration procedures. The influence of signal crosstalk for combined axial force and torque is investigated in \cite{8} and the results show that such effects cannot be neglected. In the following, if it is not specified if a force or a moment is adressed, the term load will be used.

Around the world, very few institutes have the equipment to calibrate multi-component sensors with superimposed loads. In 1993, a six-component calibration device was presented at INRIM in Torino, using crossed-flexure levers for axial forces up to 105\,kN, transverse forces up to 6\,kN and moments up to 2\,kN $\cdot$ m \cite{9}. In Korea, two devices have been installed generating forces in a range of 50–500\,N and moments of 5–50\,N $\cdot$ m \cite{10} and continuous forces of up to 2000\,N and moments of up to 400\,N $\cdot$ m \cite{11}. At the Physikalisch-Technische Bundesanstalt (PTB), the National Metrological Institute of Germany, a torque-generating device was installed in the 1\,MN force standard machine, capable of generating torque in a range of 20\,N $\cdot$ m–2\,kN $\cdot$ m \cite{8}. All of these machines use levers, mass stacks or pulleys to generate the required forces and moments.
At PTB, a calibration device was developed in 2001, which is capable of generating and measuring forces and moments with six degrees of freedom (DoF) [12, 13]. The machine is driven by six servo motors in a parallel kinematic structure and can generate forces of up to 10 kN and moments of up to 1 kN·m. This setup is unique worldwide as in theory any force-moment-combination in the mentioned range can be generated. From initial measurements of the machine, a relative uncertainty of $1 \cdot 10^{-3}$ was stated. The aim of this work is to perform a complete measurement uncertainty analysis of the system with the goal of reducing the measurement uncertainty budget.

2. The hexapod reference measurement system

The calibration device developed at PTB consists of two symmetrical hexapod structures. A CAD model of the machine is shown in figure 1; figure 2 shows a picture of the setup. The upper hexapod structure (1.a) is driven by six servo motors (1.3) which enable the driven platform (1.5) to be moved with six DoF. The lower hexapod structure (1.b) contains a single axis force sensor (1.8) in each of the six legs of the hexapod. A detailed picture of one leg of the lower hexapod structure is shown in figure 3. From the reading of the sensors and the geometry of the setup, the resulting force and moment vectors can be calculated using the physical model described in section 3. Those vectors serve as reference for the calibration of the specimen.

The multi-component sensor under test (1.6) is mounted between the driven platform (1.5) and the top platform (1.7) of the measuring hexapod (1.b). In order to be able to mount a sensor in the machine, the whole driven hexapod structure (1.a) can be moved vertically using a motor (1.1).

The links (1.10) between the bottom platform (1.9), the legs of the hexapod structures and the top platform (1.7) are implemented as elastic joints (see figure 3). The advantages of elastic joints over ball bearings in this specific application are discussed in [14].

Up to now, no complete analysis of the measurement uncertainty of the calibration device was performed. The aforementioned value of $1 \cdot 10^{-3}$ was based on the geometry of the setup derived from measurements of the individual machine parts in an unmounted state. Assembly uncertainties, deformation of the system due to thermal expansion and the limited stiffness of the mechanical structure were not included. In order to completely evaluate the measurement uncertainty, a more elaborate analysis needs to be performed.
3. Physical model

In order to calculate the reference vectors for the calibration, a physical model of the measuring hexapod structure is needed. The design model used in [12] is capable of calculating the force vector \( F = [F_x, F_y, F_z]^T \) and the moment vector \( M = [M_x, M_y, M_z]^T \) in an idealized, stiff model. To include production inaccuracies, deformation under load and thermal influences, the model needs to be extended. The driven hexapod has no influence on the measurement uncertainty, hence only the measuring hexapod will be examined in the following chapters.

In figure 4, the geometry of the measuring hexapod is shown, where figure 4(a) presents a top view of the measuring hexapod while figure 4(b) represents a side view towards leg 5. All geometric quantities of our analysis refer to these figures. The legs of the hexapod are identified using the index \( i = 1 \ldots 6 \), the bottom link of each leg is indexed \( b \) and the top link is indexed \( t \). The coordinate \( K_{bi} = [X_{bi}, Y_{bi}, Z_{bi}]^T \) describes the position of the bottom link of leg \( i \).

Based on the output data of a geometrical measurement, the physical model is modified to include link positions for each link individually in a Cartesian coordinate system. As the multi-component sensor under test will be mounted on the top platform, reference values need to be calculated according to the position of the top platform.

In order to calculate the load components of the applied force and moment vectors in a global Cartesian coordinate system, the force components of each leg of the hexapod need to be separated to their Cartesian components. To calculate the \( F_{ij} \)-component of leg \( L_i \), the force reading \( F_i \) and the inclination angle \( \alpha_i \), which describes the angle between the top platform and the leg \( L_i \), are needed:

\[
F_{ij} = \sin(\alpha_i)F_i
\]

For the calculation of the components \( F_{xi} \) and \( F_{yi} \), the angle \( \beta_i \), which describes the angle between the projection of leg \( i \) on the \( xy \)-plane and the \( x \)-axis of the top platform, is needed:

\[
\beta_i = \arctan\left(\frac{Y_{bi} - Y_{tbi}}{X_{bi} - X_{tbi}}\right) - \theta
\]

Both angles \( \alpha \) and \( \beta \) are calculated from the coordinates of the corresponding links according to (4) and (5):

\[
\alpha_i = \frac{\pi}{2} - \arccos\left(\frac{K_{bi} \cdot n}{|K_{bi}| \cdot |n|}\right)
\]

\[
\beta_i = \arctan\left(\frac{Y_{bi} - Y_{tbi}}{X_{bi} - X_{tbi}}\right) - \theta
\]
with the vector \( \mathbf{n} \) as the normal vector of the top platform, \( \mathbf{K}_{ini} \) as the directional vector of leg \( i \) and \( \theta \) as the rotation angle of the top platform around the \( z \)-axis.

Using (1)-(3), the force and moment vectors \( \mathbf{F} \) and \( \mathbf{M} \) can be calculated as a sum of the different components of each leg. The sign of each summand has to be identified according to the direction of the force \( F_i \) and the sing of \( \sin(\alpha_i) \):

\[
F_i = -\cos(\beta_i) \cos(\alpha_i)F_i + \sum_{i=2}^{5} \cos(\beta_i) \cos(\alpha_i)F_i
- \cos(\beta_0) \cos(\alpha_0)F_0
\]

\[
F_j = -\sin(\beta_i) \cos(\alpha_i)F_i + \sum_{i=2}^{5} \sin(\beta_i) \cos(\alpha_i)F_i
- \sin(\beta_0) \cos(\alpha_0)F_0
\]

\[
F_c = \sum_{i=1}^{6} \sin(\alpha_i)F_i
\]

(6)

(7)

(8)

For the bending moment calculation, the reference length \( l_{ref} \) between the top platform and the origin of the coordinate system of the sensor under test needs to be taken into account:

\[
M_x = \sum_{i=1}^{6} Y_i \sin(\alpha_i)F_i - F_y \cdot l_{ref}
\]

\[
M_y = \sum_{i=1}^{6} -X_i \sin(\alpha_i)F_i + F_x \cdot l_{ref}
\]

\[
M_z = [Y_i \cos(\beta_i) - X_i \sin(\beta_i)] \cos(\alpha_i)F_i
+ \sum_{i=2}^{5} [-Y_i \cos(\beta_i) + X_i \sin(\beta_i)] \cos(\alpha_i)F_i
+ [Y_0 \cos(\beta_0) - X_0 \sin(\beta_0)] \cos(\alpha_0)F_0
\]

\[
M_c = [X_i \sin(\beta_i) - Y_i \cos(\beta_i)] \cos(\alpha_i)F_i
\]

(9)

(10)

(11)

In this model, readings of the force sensors \( F_i \) are defined as positive for tension force while compression force is defined as negative.

### 4. Influencing parameters

From the physical model described in section 3, the influencing factors on the measurement uncertainty of the reference values for \( \mathbf{F} \) and \( \mathbf{M} \) are the coordinates \( K_i \) of the top and bottom links and the readings \( F_i \) of the force sensors in each leg of the hexapod. Changes of the link coordinates \( K_i \) due to thermal effects and deformation under load have an influence on the reference values and the measurement uncertainty.

To evaluate this influence, the actual change of the coordinates based on these effects is observed. Deformation under load is expected to have an effect on the force sensors and the elastic joints installed in the legs, resulting in a length change of the legs. All other components of the setup are expected to be stiff. Thermal expansion affects all parts of the machine which results in length changes for the different distances in the setup. All these changes can be estimated using thermal expansion models and deformation of rigid bodies. A problem results from the 6 DoF of the top platform of the parallel kinematic.

This problem constitutes the typical challenge of direct kinematics for parallel kinematic machines. Proceeding from a known set of lengths of the legs and geometrical boundary conditions in the top and bottom platforms, the position of the top platform needs to be calculated. The calculation of the geometry of parallel kinematics is discussed in detail in the literature of robotics and machine tool research [10, 15, 16]. A complete set of equations for parallel kinematics of 6 DoF does not have a unique solution [15]. Depending on the structure of the parallel kinematic setup, up to 16 different assembly modes can be found. As a result of the different solutions, the direct kinematic problem cannot be solved in a closed form without additional information [17] or linearization [18].

As an alternative to a closed form solution, the direct kinematic problem can be solved using an iterative optimization of the geometry. From the initial geometry \( K_{ini} \), the coordinates of the top links \( K_i \) are optimized iteratively until the set of equations is solved within a given threshold. In the case of the measurement hexapod, the geometrical changes are very small, so the risk of the iterative optimization converging to a wrong local minimum is small. In addition, the application is not time critical, hence an iterative optimization is an adequate and accurate option.

Input parameters for the optimization are the initial coordinates \( K_{ini} = [X_{ini}, Y_{ini}, Z_{ini}]^T \), which lead to initial leg lengths \( L_{ini} = [K_{ini} - K_{ini}] \) (see figure 4(b)), the stiffness \( s \) of the legs, the force readings \( F_i \) from the force sensors, the thermal expansion coefficient \( \alpha_T \) of the material of the components and the temperature difference \( \Delta T \) to normal temperature \( (T = 20 \, ^\circ C) \). In geometric measurements, 20 °C is the standard temperature for measurements to be performed and geometric characteristics to be referenced to. The result of the iterative optimization is the actual top link coordinates \( K_i = [X_i, Y_i, Z_i]^T \).

The parameters for the optimization algorithm contain an uncertainty which results in an uncertainty \( \Delta K_i = [\Delta X_i, \Delta Y_i, \Delta Z_i]^T \) for the resulting coordinates. To calculate the uncertainty \( \Delta K_i \), sensitivity coefficients \( c_i \), which describe the influence of the uncertainty of each input parameter on the resulting uncertainty, are needed. The Guide to the expression of uncertainty in measurement (GUM) [19] defines the sensitivity coefficients \( c_i \) as the partial derivative of the measurement result and the input parameter:

\[
c_i = \frac{\partial f}{\partial x_i}
\]

(12)

For the coordinates \( K_i \), no direct functional correlation in the form of

\[
K_i = f(x_i)
\]

(13)

exists due to the multiple solutions of the set of equations. The sensitivity coefficients cannot be calculated by calculating the partial derivative, and a measurement uncertainty calculation according to basic GUM is not possible for those coordinates.
5. Sensitivity coefficients and uncertainty for link coordinates

To calculate the sensitivity coefficients needed for calculating the uncertainty of the link coordinates, two approaches were used. In a first step, the coefficients are calculated using the implicit function theorem (IFT) [20]. Under certain conditions, the IFT provides a solution to calculate the derivative of a function without explicit knowledge of the function itself. In addition, a Monte Carlo method (MC) according to [21] is performed. MC is recommended by GUM, for example, when linearization results are inadequate for the given problem. Due to the large number of sensitivity coefficients needed to calculate the uncertainty $\Delta K_{ij}$ ($24 \cdot 42 = 1008$ sensitivity coefficients), MC is very time-consuming. For the automated calculation of sensitivity coefficients for each calibration run in a calibration device, MC is not suitable and is therefore used to confirm the results of the IFT. The much faster calculation using IFT can then be implemented in the calibration process.

5.1. Implicit function theorem

The implicit function theorem states that under certain regularity and continuity conditions an equation or a system of equations

$$I(x, y) = 0$$

(14)

implicitly defines a function

$$y = f(x)$$

(15)

in the neighbourhood of $x_0$ that satisfies

$$I(x, f(x)) = 0.$$ (16)

Continuity of the functions $f$ is needed to be able to calculate the partial derivatives of the functions; regularity ensures the invertibility of the partial derivative $\partial I/\partial y$. If the given conditions apply, the derivative $df/dx$ can be calculated without explicit knowledge of the function $y = f(x)$ using the following equation:

$$\frac{df}{dx}(x) = \left( \frac{\partial I}{\partial y}(x, f(x)) \right)^{-1} \cdot \frac{\partial I}{\partial x}(x, f(x)).$$ (17)

In the case of the hexapod setup, a system of 24 equations \( I(x, y) \) is used for the iterative optimization and the IFT. The parameter vector $x$ contains the 42 parameters leg length ($L_i$), boundary conditions in the top platform ($n_{i,i+1}$), boundary conditions for additional coordinates $j = 7 \ldots 8$ ($t_{ij}$) and coordinates of the bottom link ($K_{ib}$):

$$L_i = |K_{0ii} - K_{0bi}| \cdot (1 + \alpha T \Delta T) + F_i \cdot s$$ (18)

$$n_{i,i+1} = |K_{0ii} - K_{0,i+1}| \cdot (1 + \alpha T \Delta T)$$ (19)

$$t_{ij} = |K_{0ii} - K_{0ij}| \cdot (1 + \alpha T \Delta T)$$ (20)

$$K_{bi} = [X_{0bi}, Y_{0bi}, Z_{0bi}]^T \cdot (1 + \alpha T \Delta T)$$ (21)

The variable vector $y$ contains the 24 coordinates of the top links ($K_{0i}$) and the additional coordinates ($K_{bi}$). The system of equations $I$ as well as the vectors $x$ and $y$ are listed in appendix B.

To be able to use the IFT, regularity and continuity conditions need to apply. For continuity, the equations $I$ are analyzed. All of the equations are of the same type:

$$a^2 + b^2 + c^2 - l^2 = 0$$ (22)

Under the condition that the different summands $a^2, b^2, c^2, l^2$ are not equal to zero, the continuity condition is satisfied. All summands describe distances between two nonidentical links, so they can never become zero.

For regularity, the partial derivative $\partial I/\partial y$ is analyzed. The system of equations $I$ contains 24 equations; the variable vector $y$ contains 24 variables. The result of the partial derivative is a $24 \times 24$ matrix. The partial derivatives are calculated symbolically using python SymPy-Package. The columns of the matrix are linearly independent, so the matrix is invertible and the regularity condition is satisfied. From the given analysis, IFT can be used, resulting in a $24 \times 42$ matrix of sensitivity coefficients $c_j$.

5.2. Monte Carlo method

To verify the sensitivity coefficients, the Monte Carlo method was used. The iterative optimization was executed using a Levenberg–Marquardt algorithm [22] with the initial geometry of the hexapod as starting point for the optimization. Consecutively, to each of the 42 input parameters a normally distributed random number $d$ with $\mu(d) = 0$ and $\sigma(d) = 0.1$ was added over 10000 iterations and the coordinates $K_{ij}$ were calculated. The sensitivity coefficient is calculated from the ratio of the standard deviation of the resulting coordinates $\sigma(K_{ij})$ and the standard deviation of the used random input numbers $\sigma(d)$:

$$c_j = \frac{\sigma(K_{ij})}{\sigma(d)}.$$ (23)

The resulting sensitivity coefficients $c_j$ from MC match the results from IFT to the third decimal place. The maximum value for $c_j$ is 1.3157; the maximum deviation between IFT and MC is $1.9 \cdot 10^{-3}$.

5.3. Uncertainty of link coordinates

To calculate the uncertainty of the link coordinates $\Delta K_{ij}$, uncertainties $u_i(x)$ for the input parameters $x$ need to be known. As mentioned in section 5.1, the parameter vector $x$ contains 42 parameters which can be calculated from (18)–(21). The uncertainties $u_i(x)$ can be calculated by error propagation using uncertainties $\Delta p$ for parameters $p = [K_{0ij}, K_{0ji}, \alpha T, \Delta T, F_i, s]^T$ according to (24). The parameter vector $p$ contains 51 elements, 36 for $K_{0ij}, 6$ for $K_{0ji}, 6$ for $F_i$ and one each for $\alpha T, \Delta T, s$.

$$u_i(x) = \sqrt{\sum_{j=1}^{51} \left( \frac{\partial x}{\partial p_i} \cdot \Delta p_j \right)^2}.$$ (24)
The initial coordinates $K_0$ of the link positions were determined using optical measurement equipment [23]. From a combination of digitized reference points using a photogrammetric system GOM TRITOP and a fringe projection system GOM ATOS Core 200, each link is digitized in a global coordinate system. In figure 5 the resulting scan for one link is shown. Two cones (5.1, 5.5) are fitted to the surface points. The centre point (5.3) between their apexes (5.2, 5.4) is defined as the link point. The measurement is repeated ten times to estimate the uncertainty of the measured initial coordinates. Each measurement is performed independently which results in different global coordinate systems. To evaluate the measurement uncertainty, all measurements need to be aligned in one coordinate system using unique geometric features. For $x$- and $y$-alignment, a centre hole in the base plate and in the centre between $K_{01}$ and $K_{06}$ was used. $z$-alignment was reached from $Z_{65}$, $Z_{45}$ and $Z_{66}$ coordinates. From this alignment, the value of those three coordinates is zero for all measurements. The variance of the $z$-distribution, however, is transformed into the $Z$-coordinates of the remaining links, increasing the uncertainty $u(Z)$. The coordinates $K_{0i}$ are additional points for boundary conditions which do not represent a physical point in the setup and therefore do not contain any uncertainty.

For the thermal expansion coefficient $\alpha_T$, literature values of $1.2 \cdot 10^{-5}$ K$^{-1}$ were used for the alloy steel from construction data. The spring rigidity of the installed sensor is stated to be $350 \text{kN} \cdot \text{mm}^{-1}$. The stiffness $s$ of the legs, including the elastic joints and the installed force sensors, was determined experimentally to a value of $1.1 \cdot 10^{-5} \text{mm} \cdot \text{N}^{-1}$ or $91 \text{kN} \cdot \text{mm}^{-1}$. For the uncertainty of both values, an upper estimation of 10% was used. The hexapod setup has been installed in an air conditioned laboratory building with temperature readings averaged over a set of sensors distributed around the room. Temperature readings of the closest sensor are used for the uncertainty estimation. The sensor readings add a rectangular distributed uncertainty with a half-width of 0.1 °C.

The uniaxial force sensors installed in the legs of the measuring hexapod are classified as class 00 according to ISO 376 [24]. A measurement range of 25 kN for each sensor is given. The uncertainties are taken from calibration certificates and reach relative values of $u(F) \leq 0.037\%$, except for the tension force on leg 2 (see section 7). In [14], the design of the elastic joints connecting the legs of the hexapod and the platforms was performed in order to minimize the transmission of moments. From an FEM analysis during design, a maximum transmitted bending moment of $M_b \leq 0.5 \text{ N} \cdot \text{m}$ and torque of $M_t \leq 0.2 \text{ N} \cdot \text{m}$ was calculated. In the data sheet of the installed force sensors, a bending moment effect $u_b(F) < 0.003\% \ (\text{N} \cdot \text{m})^{-1}$ and a torque effect $u_t(F) = 0.0002\% \ (\text{N} \cdot \text{m})^{-1}$ is given. To consider the effects of bending moment and torque sensitivity in the uncertainty estimation, both values are included in the uncertainty of the force readings:

$$u_t(F) = \sqrt{(u(F))^2 + (u_b(F))^2 + (u_t(F))^2}. \quad (25)$$

Calculating the combined uncertainty $u_c(F)$ using the given values for $u(F) \leq 0.037\%$, $u_b$, $M_b$, $u_t$ and $M_t$ results in a value of $u_c(F) \leq 0.037\%$. Hence the moment sensitivity of the force sensors does not have a significant influence on the resulting uncertainty.

The influence of the parameters $p$ on the uncertainty $u_c(x)$ is listed exemplarily for $x(1) = l_1$ in table 1; values for $u_c(x)$ are listed in appendix C.

Using the sensitivity coefficients $c_i$ calculated from IFT in section 5.1 and the uncertainties $u_c(x)$ of the input values $x$, the uncertainties $u_c(y)$ are calculated:

$$u_c^2(y) = \sum_{i=1}^{42} (c_i^2) \cdot (u_c^2(x(i))). \quad (26)$$

Again exemplarily for $y(1) = X_{1b}$, the sensitivity coefficients $c_i = [c_1, \ldots, c_{42}]^T$ are listed in table 2. The maximum uncertainty for the link coordinates after iterative optimization is $\leq 0.014 \text{ mm}$. The values for $u_c(y)$ are listed in appendix C.

6. Measurement uncertainty budget (MUB)

With the resulting uncertainty for the link coordinates, the uncertainty for the reference force and moment components can be calculated based on the Guide to the expression of uncertainty in measurement (GUM) [19]. Values and uncertainties of the input parameters are used according to section 5. The uncertainties $u(Z_{65})$, $u(Z_{45})$ and $u(Z_{66})$ are zero, as these coordinates were used to align the repeated geometrical measurements for the estimation of the uncertainty of the initial geometry (see section 5.3).

A reference measurement of uniaxial force of $F_z = 5 \text{kN}$ was performed. This reference measurement serves as an example; any other combination of $\mathbf{F}$ and $\mathbf{M}$ within the specified range is possible. An analysis of the uncertainties for all six components for 10%, 50% and 100% of the nominal load based on theoretical force readings is performed at the end of this section.

Due to the control of the machine, strictly uniaxial loads cannot be generated ($F_x, F_y, M_x, M_y, M_z = 0$). Multi-component sensors can be very sensitive to combined force components.
Hence a set of all six force and moment components is necessary to describe the load on the specimen under test. The resulting force and moment components of the reference measurement including their expanded measurement uncertainty \((k = 2)\) are listed in table 3.

For \(F_\text{z} = 5\, \text{kN}\), the input parameters, uncertainties and sensitivity coefficients, as well as their contribution to the MUB are listed in table 4. The relative expanded measurement uncertainty \((k = 2)\) for \(F_z\) is \(1.5 \times 10^{-4}\). The analysis of the setup and the updated physical model improved the uncertainty in the range of one order of magnitude. For the components \(F_x, F_y, M_x, M_y,\) and \(M_z,\) the significant influencing factors (relative contribution to MUB \(\geq 0.5\%\)) for the aforementioned reference measurement of \(F_z = 5\, \text{kN}\) are listed in appendix D.

To complete the analysis of the setup, the uncertainty for all six force and moment components is calculated for a uniaxial load of \(\pm 10\%, \pm 50\%\) and \(\pm 100\%\) of the nominal load. For the calculation of the uncertainties, additional basic force from the top platform and from mounting adapters of \(F_z = 2817\, \text{N}\) was considered for the deformation and the readings of the force sensors in the hexapod legs. Table 5 shows the relative and absolute expanded uncertainties \((k = 2)\) for the different load levels.

7. Discussion and outlook

From the relative uncertainties listed in table 5 it is noticeable that the uncertainty for certain directions and load levels (e.g. \(F_z = 50\% \cdot F_{\text{max}}\)) differs significantly from others. Particularly the fact that the relative uncertainty for some load directions increases with higher load levels (e.g. \(F_z\)) is unusual. The difference of up to one order of magnitude can be explained by the quality of one single force sensor, installed in leg two of the hexapod. This sensor shows a strong deviation from linear behaviour for tension force, especially in the first calibration step of 0–4 kN. In this range, a relative uncertainty of \(5.14 \times 10^{-3}\) is given. The relative uncertainty of all other sensors is \(\leq 3.7 \times 10^{-4}\). The effect of different behaviour under compression force and tension force can be observed for all sensors installed, but only the sensor in leg two shows such distinctive characteristics. For any load combination which results in a tension force of 0–4 kN on leg two, this uncertainty has a major influence on the resulting uncertainty.

For load combinations which do not result in tension force on leg two (e.g. \(F_z = 50\% \cdot F_{\text{max}}\) table 4), the combined uncertainty contribution of the force readings \(F_i\) has a main influence of 68\% \((M_z = -10\% \cdot M_{\text{max}})\) to 98\% \((F_z = F_{\text{max}})\) on the MUB. The uncertainty of the geometry has a minor influence on the new model, which proves the used optical measurement setup to be appropriate for the described task. The information about the contribution of the different influence parameters to the MUB, which was not known before, is important for the understanding of the machine and shows a direction for possible improvements.

To reach an equal uncertainty distribution for all directions and load steps, the force sensor installed in leg 2 should be recalibrated or replaced. This would reduce the relative uncertainty for all considered loads to \(\leq 2.2 \times 10^{-4}\).
Further improvement of the measurement uncertainty can be obtained by installing force sensors of smaller individual uncertainty or more precise calibration of the sensors in all six legs. The sensors installed at the moment are already classified as class 00, which is the class of highest accuracy in this classification system, so the options for alternative sensors are limited.

The nominal force capacity of the sensors is $F = 25$ kN each, resulting in a maximum uniaxial force for the whole hexapod setup of $F_z = 110$ kN. Including the mass of the platforms, mounting devices, adapter plates and the maximum axial force of $F_z = 10$ kN, the sensors are not expected to reach their optimal operating range in the upper half of the nominal force capacity. The maximum force to be expected from uniaxial load on one single sensor, including the mass of the setup, is $<7500$ N. Installing force sensors with a nominal force of 10–15 kN can further improve the resulting measurement uncertainty.

All of the suggested improvements to the setup include recalibration or replacement of force sensors in the hexapod legs. It has to be pointed out that it is necessary to disassemble the whole measuring hexapod in order to dismount even one of the sensors. After reassembly, the whole geometry has to be analyzed again, which is very time-consuming.

The uncertainty of $\leq 2.2 \cdot 10^{-4}$ (with certain exceptions) reached by this analysis represents a big improvement of the setup in comparison to the previous uncertainty budget and links the MUB to the previously unknown influence of the geometry parameters.

The next steps to be performed will be a calibration of a commercially available multi-component sensor regarding crosstalk on multiple axes under non-uniaxial load.
8. Summary

A hexapod-structured calibration device for multi-component force and moment sensors, installed at PTB, was analyzed regarding its measurement uncertainty budget. An improved physical model of the measurement hexapod was described, based on the individual coordinates $K_i$ of the 12 links in the hexapod and six force readings $F_i$ of force sensors installed in the legs. The geometry of the setup and the uncertainties for the coordinates $K_i$ were determined from optical measurements; the uncertainty of the force readings $F_i$ was taken from calibration certificates. Two different approaches were used to calculate the sensitivity coefficients for the influencing parameters without direct functional correlation, resulting from an iterative optimization algorithm for the position estimation of the platform. From the results of that calculation, the measurement uncertainty budget for all six force and moment components of a reference measurement with a uniaxial force of $F_z = 5\text{kN}$ was calculated exemplarily. In addition, the MUB for all six components $\mathbf{F}$ and $\mathbf{M}$ was calculated for $\pm 10\%$, $\pm 50\%$ and $\pm 100\%$ of the nominal load capacity. Through this analysis, the relative expanded measurement uncertainty ($k = 2$) of the calibration device was improved from $1 \cdot 10^{-3}$ to $2.2 \cdot 10^{-4}$ with some exceptions. As the main influencing factors of the measurement uncertainty, the force readings $F_i$ were identified. This proves the optical measurement to be a good option for the determination of the hexapod geometry. Furthermore, the reason for the exceptions to the measurement uncertainty was identified and proposals for further reduction of the measurement uncertainty were given.

Acknowledgments

The author thanks Daniel Hutzschenreuter of PTB for discussion about and input in the implicit function theorem.

The authors gratefully acknowledge the funding of this work by the Deutsche Forschungsgemeinschaft (DFG) under grants Tu 135/24 and Ku 3367/1.

### Appendix A. Nomenclature

| $b$ | Bottom link |
|-----|-------------|
| $i$ | Top link |
| $i = 1, \ldots, 6$ | Leg number |
| $j = 7, \ldots, 8$ | Additional coordinates |
| $c_i$ | Matrix of sensitivity coefficients $\text{var}$ |
| $d$ | Normally distributed random number |
| $\mathbf{F}$ | Force vector $[F_x, F_y, F_z]^T$ N |
| $F_i$ | Force reading in leg $i$ N |
| $\mathbf{I}$ | Vector of equations |
| $K_i$ | Coordinate $[X_i, Y_i, Z_i]^T$ mm |
| $K_{ii}$ | Initial coordinate mm |
| $K_j$ | Calculated coordinate mm |
| $\Delta K$ | Uncertainty for coordinate $K$ mm |
| $K_{a0}$ | Directional vector from coordinate $K_{a0}$ to $K_i$ |
| $L_i$ | Leg length mm |
| $\mathbf{M}$ | Moment vector $[M_x, M_y, M_z]^T$ N m |
| $M_i$ | Bending moment N m |
| $M_t$ | Torque N m |
| $n_{i+j}$ | Dist. between links $i$ and $i+j$ mm |
| $s$ | Stiffness of legs mm kN$^{-1}$ |
| $T$ | Normal temperature of 20 °C °C |
| $T$ | Temperature difference to K |
| $t_{ij}$ | Dist. between link $i$ and point $j$ mm |
| $X$ | $x$-coordinate mm |
| $x$ | Vector of input parameters var |
| $Y$ | $y$-coordinate mm |
| $y$ | Vector of variables var |
| $Z$ | $z$-coordinate mm |
| $\alpha$ | Inclination angle rad |
| $\gamma$ | Thermal expansion coefficient K$^{-1}$ |
| $\beta$ | Angle between leg and $x$-axis rad |
| $\theta$ | Rotation angle of top platform rad |
| $\sigma$ | Standard deviation |

### Appendix B. Equations and parameters for implicit function theorem

$$\mathbf{I} = [f_1, \ldots, f_{24}]^T \quad (B.1)$$

\[ \text{Table 5. Uncertainties for } 10\%, 50\% \text{ and } 100\% \text{ of nominal load for } \mathbf{F} (10\text{kN}) \text{ and } \mathbf{M} (1 \text{ kN} \cdot \text{m}). \]

| Component | $u_i(10\%)$ (abs.) | $u_i(10\%)$ (rel.) | $u_i(50\%)$ (abs.) | $u_i(50\%)$ (rel.) | $u_i(100\%)$ (abs.) | $u_i(100\%)$ (rel.) |
|-----------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $F_x$     | 0.2 N              | $2.0 \cdot 10^{-4}$ | 0.97 N             | $1.9 \cdot 10^{-4}$ | 1.6 N              | $1.6 \cdot 10^{-4}$ |
| $-F_x$    | 0.2 N              | $2.0 \cdot 10^{-4}$ | 7.4 N              | $1.5 \cdot 10^{-3}$ | 2.8 N              | $2.8 \cdot 10^{-4}$ |
| $F_y$     | 0.21 N             | $2.1 \cdot 10^{-4}$ | 0.98 N             | $2.0 \cdot 10^{-4}$ | 2.9 N              | $2.9 \cdot 10^{-4}$ |
| $-F_y$    | 0.2 N              | $2.0 \cdot 10^{-4}$ | 1.0 N              | $2.0 \cdot 10^{-4}$ | 1.4 N              | $1.4 \cdot 10^{-4}$ |
| $F_z$     | 0.15 N             | $1.5 \cdot 10^{-4}$ | 0.73 N             | $1.5 \cdot 10^{-4}$ | 1.5 N              | $1.5 \cdot 10^{-4}$ |
| $-F_z$    | 0.15 N             | $1.5 \cdot 10^{-4}$ | 4.4 N              | $8.7 \cdot 10^{-4}$ | 8.7 N              | $8.7 \cdot 10^{-4}$ |
| $M_x$     | 0.021 N · m        | $2.1 \cdot 10^{-4}$ | 0.1 N · m          | $2.0 \cdot 10^{-4}$ | 0.2 N · m          | $2.0 \cdot 10^{-4}$ |
| $-M_x$    | 0.021 N · m        | $2.1 \cdot 10^{-4}$ | 0.74 N · m         | $1.5 \cdot 10^{-3}$ | 1.5 N · m          | $1.5 \cdot 10^{-3}$ |
| $M_y$     | 0.022 N · m        | $2.2 \cdot 10^{-4}$ | 0.17 N · m         | $3.3 \cdot 10^{-4}$ | 0.33 N · m         | $3.3 \cdot 10^{-4}$ |
| $-M_y$    | 0.022 N · m        | $2.2 \cdot 10^{-4}$ | 0.099 N · m        | $2.0 \cdot 10^{-4}$ | 0.2 N · m          | $2.0 \cdot 10^{-4}$ |
| $M_z$     | 0.017 N · m        | $1.7 \cdot 10^{-4}$ | 0.083 N · m        | $1.7 \cdot 10^{-4}$ | 0.87 N · m         | $8.7 \cdot 10^{-4}$ |
| $-M_z$    | 0.017 N · m        | $1.7 \cdot 10^{-4}$ | 0.080 N · m        | $1.6 \cdot 10^{-4}$ | 0.16 N · m         | $1.6 \cdot 10^{-4}$ |
\[ f_i = (X_{a1} - X_{a2})^2 + (Y_{b1} - Y_{b2})^2 + (Z_{c1} - Z_{c2})^2 - L_i^2 \]

\[ f_2 = (X_{a2} - X_{a3})^2 + (Y_{b2} - Y_{b3})^2 + (Z_{c2} - Z_{c3})^2 - L_2^2 \]

\[ f_3 = (X_{a3} - X_{a4})^2 + (Y_{b3} - Y_{b4})^2 + (Z_{c3} - Z_{c4})^2 - L_3^2 \]

\[ f_4 = (X_{a4} - X_{a5})^2 + (Y_{b4} - Y_{b5})^2 + (Z_{c4} - Z_{c5})^2 - L_4^2 \]

\[ f_5 = (X_{a5} - X_{a6})^2 + (Y_{b5} - Y_{b6})^2 + (Z_{c5} - Z_{c6})^2 - L_5^2 \]

\[ f_6 = (X_{a6} - X_{a7})^2 + (Y_{b6} - Y_{b7})^2 + (Z_{c6} - Z_{c7})^2 - L_6^2 \]

\[ f_7 = (X_{a7} - X_{a8})^2 + (Y_{b7} - Y_{b8})^2 + (Z_{c7} - Z_{c8})^2 - L_7^2 \]

\[ f_8 = (X_{a8} - X_{a9})^2 + (Y_{b8} - Y_{b9})^2 + (Z_{c8} - Z_{c9})^2 - L_8^2 \]

\[ f_9 = (X_{a9} - X_{a10})^2 + (Y_{b9} - Y_{b10})^2 + (Z_{c9} - Z_{c10})^2 - L_9^2 \]

\[ f_{10} = (X_{a10} - X_{a11})^2 + (Y_{b10} - Y_{b11})^2 + (Z_{c10} - Z_{c11})^2 - L_{10}^2 \]

\[ f_{11} = (X_{a11} - X_{a12})^2 + (Y_{b11} - Y_{b12})^2 + (Z_{c11} - Z_{c12})^2 - L_{11}^2 \]

\[ f_{12} = (X_{a12} - X_{a13})^2 + (Y_{b12} - Y_{b13})^2 + (Z_{c12} - Z_{c13})^2 - L_{12}^2 \]

\[ f_{13} = (X_{a13} - X_{a14})^2 + (Y_{b13} - Y_{b14})^2 + (Z_{c13} - Z_{c14})^2 - L_{13}^2 \]

\[ f_{14} = (X_{a14} - X_{a15})^2 + (Y_{b14} - Y_{b15})^2 + (Z_{c14} - Z_{c15})^2 - L_{14}^2 \]

\[ f_{15} = (X_{a15} - X_{a16})^2 + (Y_{b15} - Y_{b16})^2 + (Z_{c15} - Z_{c16})^2 - L_{15}^2 \]

\[ X_i = [x_1, \ldots, x_{42}]^T \]  \hspace{1cm} (B.2)

\[ y = [y_1, \ldots, y_{24}]^T \]  \hspace{1cm} (B.3)

\[ u_i(x) = [u_{i1}, \ldots, u_{i24}]^T \]  \hspace{1cm} (C.1)

\[ u_i(y) = [u_{i1}, \ldots, u_{i24}]^T \]  \hspace{1cm} (C.2)

\[ u_i(x) = [u_{i1}, \ldots, u_{i24}]^T \]  \hspace{1cm} (C.1)
Appendix D. Significant influencing factors and measurement uncertainty for $F_x$, $F_y$, $M_x$, $M_y$, $M_z$

| Quantity $X_i$ | Estimate $x_i$ | Standard uncertainty $u_i(x_i)$ | Distribution function | Sensitivity coefficient | Contribution to MuB (N) | Relative contribution to MUB (%) |
|---------------|----------------|-------------------------------|----------------------|-----------------------|------------------------|---------------------------------|
| $F_1$         | 1017.920 N     | 0.183 N                       | Normal               | −0.66 N/N             | −0.121                 | 22.8                            |
| $F_2$         | 1031.750 N     | 0.165 N                       | Normal               | 0.45 N/N              | 0.074                  | 8.6                             |
| $F_3$         | 1151.070 N     | 0.213 N                       | Normal               | 0.21 N/N              | 0.045                  | 3.1                             |
| $F_4$         | 1207.159 N     | 0.217 N                       | Normal               | 0.21 N/N              | 0.046                  | 3.2                             |
| $F_5$         | 1197.370 N     | 0.216 N                       | Normal               | 0.45 N/N              | 0.097                  | 14.7                            |
| $F_6$         | 1134.470 N     | 0.199 N                       | Normal               | −0.66 N/N             | −0.131                 | 26.9                            |
| $Z_1$         | 346.828 mm     | 0.012 mm                      | Normal               | 1.43 N mm$^{-1}$      | 0.019                  | 0.5                             |
| $X_3$         | −230.161 mm    | 0.014 mm                      | Normal               | 3.70 N mm$^{-1}$      | 0.044                  | 3.1                             |
| $Z_3$         | 347.405 mm     | 0.011 mm                      | Normal               | −3.40 N mm$^{-1}$     | −0.037                 | 2.2                             |
| $X_5$         | −230.102 mm    | 0.013 mm                      | Normal               | 3.03 N mm$^{-1}$      | 0.039                  | 2.4                             |
| $Y_4$         | −90.519 mm     | 0.013 mm                      | Normal               | −1.81 N mm$^{-1}$     | −0.023                 | 0.9                             |
| $Z_4$         | 347.785 mm     | 0.011 mm                      | Normal               | −3.60 N mm$^{-1}$     | 0.043                  | 2.9                             |
| $X_6$         | 192.112 mm     | 0.011 mm                      | Normal               | 2.36 N mm$^{-1}$      | 0.026                  | 1.1                             |
| $Z_6$         | 347.116 mm     | 0.010 mm                      | Normal               | 3.70 N mm$^{-1}$      | 0.037                  | 2.1                             |

Table D2. Uncertainty contribution of the most significant influencing factors on $F_y$ for a uniaxial force of $F_z = 5 \text{kN}$.

| Quantity $X_i$ | Estimate $x_i$ | Standard uncertainty $u_i(x_i)$ | Distribution function | Sensitivity coefficient | Contribution to MuB (N) | Relative contribution to MUB (%) |
|---------------|----------------|-------------------------------|----------------------|-----------------------|------------------------|---------------------------------|
| $F_1$         | 1017.920 N     | 0.183 N                       | Normal               | 0.14 N/N              | 0.026                  | 1.0                             |
| $F_2$         | 1031.750 N     | 0.165 N                       | Normal               | −0.50 N/N             | −0.083                 | 9.9                             |
| $F_3$         | 1151.070 N     | 0.213 N                       | Normal               | −0.64 N/N             | −0.136                 | 27.1                            |
| $F_4$         | 1207.159 N     | 0.217 N                       | Normal               | 0.64 N/N              | 0.139                  | 28.1                            |
| $F_5$         | 1197.370 N     | 0.216 N                       | Normal               | 0.50 N/N              | 0.108                  | 17.0                            |
| $F_6$         | 1134.470 N     | 0.199 N                       | Normal               | −0.14 N/N             | −0.028                 | 1.1                             |
| $Y_1$         | 153.104 mm     | 0.013 mm                      | Normal               | 2.10 N mm$^{-1}$      | 0.027                  | 1.1                             |
| $Z_2$         | 243.000 mm     | 0.012 mm                      | Normal               | 1.70 N mm$^{-1}$      | 0.020                  | 0.6                             |
| $Z_2$         | 346.961 mm     | 0.012 mm                      | Normal               | 4.30 N mm$^{-1}$      | 0.052                  | 3.9                             |
| $Y_5$         | 89.485 mm      | 0.013 mm                      | Normal               | 1.50 N mm$^{-1}$      | 0.020                  | 0.6                             |
| $Z_5$         | 347.405 mm     | 0.011 mm                      | Normal               | 2.30 N mm$^{-1}$      | 0.025                  | 0.9                             |
| $Y_5$         | −90.519 mm     | 0.013 mm                      | Normal               | 1.50 N mm$^{-1}$      | 0.020                  | 0.6                             |
| $Z_5$         | −243.853 mm    | 0.012 mm                      | Normal               | −2.60 N mm$^{-1}$     | −0.029                 | 1.2                             |
| $Z_6$         | 347.450 mm     | 0.009 mm                      | Normal               | −4.40 N mm$^{-1}$     | −0.040                 | 2.3                             |
| $Y_6$         | −154.023 mm    | 0.013 mm                      | Normal               | 2.40 N mm$^{-1}$      | 0.031                  | 1.4                             |
| $Z_6$         | 347.116 mm     | 0.010 mm                      | Normal               | −1.80 N mm$^{-1}$     | −0.018                 | 0.5                             |
| $F_y$         | 95.134 N       | 0.262 N                       |                       |                       |                        |                                 |
Table D3. Uncertainty contribution of the most significant influencing factors on $M_x$ for a uniaxial force of $F_z = 5\, \text{kN}$.

| Quantity $X_i$ | Estimate $x_i$ | Standard uncertainty $u_s(x_i)$ | Distribution function | Sensitivity coefficient | Contribution to MuB (N·m) | Relative contribution to MUB (%) |
|---------------|----------------|---------------------------------|-----------------------|------------------------|---------------------------|----------------------------------|
| $F_1$         | 1017.920 N     | 0.183 N                         | Normal                | $0.16 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | 0.029                     | 10.1                             |
| $F_2$         | 1151.070 N     | 0.213 N                         | Normal                | $-0.16 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | -0.034                    | 13.7                             |
| $F_3$         | 1207.159 N     | 0.217 N                         | Normal                | $0.16 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | 0.035                     | 14.2                             |
| $F_4$         | 1134.470 N     | 0.199 N                         | Normal                | $-0.16 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | -0.032                    | 11.9                             |
| $Y_1$         | 153.104 mm     | 0.013 mm                        | Normal                | $1.50 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.020                     | 4.5                              |
| $Z_1$         | 346.828 mm     | 0.012 mm                        | Normal                | $1.20 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.014                     | 2.4                              |
| $Y_2$         | 243.000 mm     | 0.012 mm                        | Normal                | $1.50 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.018                     | 3.8                              |
| $Z_2$         | 346.961 mm     | 0.012 mm                        | Normal                | $2.20 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.026                     | 8.2                              |
| $Y_3$         | 89.485 mm      | 0.013 mm                        | Normal                | $1.50 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.020                     | 4.5                              |
| $Z_3$         | 347.405 mm     | 0.011 mm                        | Normal                | $1.10 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.012                     | 1.7                              |
| $Y_4$         | $-90.519$ mm   | 0.013 mm                        | Normal                | $1.50 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.020                     | 4.5                              |
| $Z_4$         | 347.785 mm     | 0.011 mm                        | Normal                | $-1.20 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.013                    | 2.1                              |
| $Y_5$         | $-243.853$ mm  | 0.012 mm                        | Normal                | $1.80 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.022                     | 5.5                              |
| $Z_5$         | 347.450 mm     | 0.009 mm                        | Normal                | $-2.30 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.021                    | 5.0                              |
| $Y_6$         | $-154.023$ mm  | 0.013 mm                        | Normal                | $1.60 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.021                     | 5.1                              |
| $Z_6$         | 347.116 mm     | 0.010 mm                        | Normal                | $-1.10 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.011                    | 1.4                              |

$M_x$ $-17.788 \, \text{N} \cdot \text{m}$ $0.092 \, \text{N} \cdot \text{m}$

Table D4. Uncertainty contribution of the most significant influencing factors on $M_y$ for a uniaxial force of $F_z = 5\, \text{kN}$.

| Quantity $X_i$ | Estimate $x_i$ | Standard uncertainty $u_s(x_i)$ | Distribution function | Sensitivity coefficient | Contribution to MuB (N·m) | Relative contribution to MUB (%) |
|---------------|----------------|---------------------------------|-----------------------|------------------------|---------------------------|----------------------------------|
| $F_1$         | 1017.920 N     | 0.183 N                         | Normal                | $0.09 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | 0.016                     | 3.1                              |
| $F_2$         | 1031.750 N     | 0.165 N                         | Normal                | $-0.18 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | -0.030                    | 10.4                             |
| $F_3$         | 1151.070 N     | 0.213 N                         | Normal                | $0.10 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | 0.021                     | 5.0                              |
| $F_4$         | 1207.159 N     | 0.217 N                         | Normal                | $0.10 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | 0.021                     | 5.3                              |
| $F_5$         | 1197.370 N     | 0.216 N                         | Normal                | $-0.18 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | -0.039                    | 17.8                             |
| $F_6$         | 1134.470 N     | 0.199 N                         | Normal                | $0.09 \, \text{N} \cdot \text{m} \cdot \text{N}^{-1}$ | 0.018                     | 3.6                              |
| $X_{11}$      | 192.164 mm     | 0.013 mm                        | Normal                | $1.05 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.014                    | 2.2                              |
| $Z_{11}$      | 346.828 mm     | 0.012 mm                        | Normal                | $1.90 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.023                    | 6.1                              |
| $X_{12}$      | 36.875 mm      | 0.014 mm                        | Normal                | $-0.81 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.011                    | 1.5                              |
| $X_{13}$      | $-230.161$ mm  | 0.014 mm                        | Normal                | $-1.56 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.022                    | 5.6                              |
| $Y_{13}$      | 89.485 mm      | 0.013 mm                        | Normal                | $0.67 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.009                     | 0.9                              |
| $Z_{13}$      | 347.405 mm     | 0.011 mm                        | Normal                | $1.90 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.021                     | 5.2                              |
| $X_{14}$      | $-230.102$ mm  | 0.013 mm                        | Normal                | $-2.05 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.027                    | 8.4                              |
| $Z_{14}$      | 347.785 mm     | 0.011 mm                        | Normal                | $1.90 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | 0.021                     | 5.2                              |
| $X_{15}$      | 36.875 mm      | 0.012 mm                        | Normal                | $-2.12 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.025                    | 7.6                              |
| $X_{16}$      | 192.112 mm     | 0.011 mm                        | Normal                | $-1.89 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.021                    | 5.1                              |
| $Z_{16}$      | 347.116 mm     | 0.010 mm                        | Normal                | $-1.90 \, \text{N} \cdot \text{m} \cdot \text{mm}^{-1}$ | -0.019                    | 4.3                              |

$M_y$ $12.063 \, \text{N} \cdot \text{m}$ $0.092 \, \text{N} \cdot \text{m}$
Table D5. Uncertainty contribution of the most significant influencing factors on $M_c$ for a uniaxial force of $F_z = 5\, \text{kN}$.

| Quantity $X_i$ | Estimate $x_i$ | Standard uncertainty $u_i(x_i)$ | Distribution function | Sensitivity coefficient | Contribution to $\text{MuB} (\text{N} \cdot \text{m})$ | Relative contribution to $\text{MuB} (%)$ |
|---------------|----------------|---------------------------------|-----------------------|------------------------|-----------------------------------------------|-----------------------------------------------|
| $F_1$         | 1017.920 N     | 0.183 N                         | Normal                | 0.13 N · m $^{-1}$     | 0.024                                         | 10.2                                          |
| $F_2$         | 1031.750 N     | 0.165 N                         | Normal                | 0.13 N · m $^{-1}$     | -0.021                                        | 8.3                                           |
| $F_3$         | 1151.070 N     | 0.213 N                         | Normal                | 0.13 N · m $^{-1}$     | 0.028                                         | 13.9                                          |
| $F_4$         | 1207.159 N     | 0.217 N                         | Normal                | 0.13 N · m $^{-1}$     | -0.028                                        | 14.4                                          |
| $F_5$         | 1197.370 N     | 0.216 N                         | Normal                | 0.13 N · m $^{-1}$     | 0.028                                         | 14.3                                          |
| $F_6$         | 1134.470 N     | 0.199 N                         | Normal                | 0.13 N · m $^{-1}$     | -0.026                                        | 12.1                                          |
| $Y_1$         | 153.104 mm     | 0.013 mm                        | Normal                | 1.10 N · m $^{-1}$     | 0.014                                         | 3.7                                           |
| $X_2$         | 36.875 mm      | 0.014 mm                        | Normal                | -0.93 N · m $^{-1}$    | -0.013                                       | 3.1                                           |
| $Y_2$         | 243.000 mm     | 0.012 mm                        | Normal                | -0.52 N · m $^{-1}$    | -0.006                                       | 0.7                                           |
| $X_3$         | -230.161 mm    | 0.014 mm                        | Normal                | -1.00 N · m $^{-1}$    | -0.014                                       | 3.6                                           |
| $Y_3$         | 89.485 mm      | 0.013 mm                        | Normal                | -0.61 N · m $^{-1}$    | -0.008                                       | 1.1                                           |
| $X_4$         | -230.102 mm    | 0.013 mm                        | Normal                | 1.10 N · m $^{-1}$     | 0.014                                         | 3.7                                           |
| $Y_4$         | -90.519 mm     | 0.013 mm                        | Normal                | -0.64 N · m $^{-1}$    | -0.008                                       | 1.3                                           |
| $X_5$         | 36.873 mm      | 0.012 mm                        | Normal                | 1.10 N · m $^{-1}$     | 0.013                                         | 3.2                                           |
| $Y_5$         | -243.893 mm    | 0.012 mm                        | Normal                | -0.60 N · m $^{-1}$    | 0.007                                         | 0.9                                           |
| $Y_6$         | -154.023 mm    | 0.013 mm                        | Normal                | 1.20 N · m $^{-1}$     | 0.016                                         | 4.4                                           |
| $M_z$         | -1.15 N · m    | 0.074 N · m                    |                       |                        |                                               |                                               |

References

[1] Romiti A and Sorli M 1992 Force and moment measurement on a robotic assembly hand Sensors Actuators A 32 531–8
[2] Ewald B 2000 Multi-component force balances for conventional and cryogenic windtunnels Meas. Sci. Technol. 11 R81–94
[3] Mühring H-C, Litwinski K M and Günter O 2010 Process monitoring with sensory machine tool components CIRP Ann.—Manuf. Technol. 59 383–6
[4] Teti R, Jemiˇlniˇck K, O’Donnell G and Dornfeld D 2010 Advanced monitoring of machining operations CIRP Ann.—Manuf. Technol. 59 717–39
[5] Kim G-S, Shin H-J and Yoon J 2008 Development of 6-axis force/moment sensor for a humanoid robot’s intelligent foot Sensors Actuators A 141 276–81
[6] Genta G, Germak A, Barbato G and Levi R 2016 Metrological characterization of an hexapod-shaped multicomponent force transducer Measurement 78 202–6
[7] Ranganath R, Nair P S, Muthyuniyaya T S and Ghosal A 2004 A force-torque sensor based on a Stewart platform in a near-singular configuration Mech. Mach. Theory 39 971–98
[8] Baumgarten S, Kahrman H and Röske D 2016 Metrological characterization of a 2 kN · m torque standard machine for superposition with axial forces up to 1 MN Metrologia 53 1165–76
[9] Ferrero C, Zhong L Q, Marinari C and Martino E 1993 New automatic multicomponent calibration system with crossed-flexure levers Proc. of the 3rd Int. Symp. on Measurement and Control in Robotics pp Cm.1.31–Cm.1.39
[10] Kim G-S 2000 The development of a six-component force/moment sensor testing machine and evaluation of its uncertainty Meas. Sci. Technol. 11 1377–82
[11] Kim G-S and Yoon J 2007 Development of calibration system for multi-axis force/moment sensor and its uncertainty evaluation J. Korean Soc. Prec. Eng. 24 91–8
[12] Röske D, Peschel D and Adolff K 2001 The generation and measurement of arbitrarily directed forces and moments: the project of a multicomponent calibration device based on a hexapod structure Proc. of the 17th Int. Conf. of IMEKO TC5 pp 339–49
[13] Röske D 2003 Metrological characterization of a hexapod for a multi-component calibration device Proc. of the 17th IMEKO World Congress pp 347–51
[14] Röske D 2002 Investigation of different joint types for a multi-component calibration device based on a hexapod structure Proc. of the 18th IMEKO TC3 Conf.
[15] Merlet J-P 2006 Parallel Robots (Solid Mechanics and its Applications vol 74) 2nd edn (Dordrecht: Kluwer)
[16] Szatmári S 2007 Kinematic Calibration of Parallel Kinematic Machines on the Example of the Hexapod of Simple Design (Lehre, Forschung, Praxis) (Dresden: Inst. für Werkzeugmaschinen und Steuerungstechnik, Lehrstuhl für Werkzeugmaschinen)
[17] Parenti-Castelli V and Di Gregorio R 1996 Closed-form solution of the direct kinematics of the 6-3 type Stewart platform using one extra sensor Meccanica 31 705–14
[18] Kang C-G 2001 Closed-form force sensing of a 6-axis force transducer based on the Stewart platform Sensors Actuators A 90 31–7
[19] JCGM 2008 Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement JCGM 100:2008 (Paris: JCGM) (www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf)
[20] Krantz S G and Parks H R 2013 Implicit Function Theorem: History, Theory, and Applications (Modern Birkhäuser Classics) (New York: Birkhäuser)
[21] JCGM 2008 Evaluation of Measurement Data—Supplement 1 to the ‘Guide to the Expression of Uncertainty in Measurement’—Propagation of Distributions using a Monte Carlo Method JCGM 100:2008 (Paris: JCGM) (www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf)
[22] Marquardt D W 1963 An algorithm for least-squares estimation of nonlinear parameters J. Soc. Ind. Appl. Math. 11 431–41
[23] Nitsche J, Petz M, Röske D, Kummel R and Tutsch R 2016 Geometric characterization of a hexapod-structured calibration device for multi-component force and momentum transducers Int. Symp. on Optomechatronics Technology
[24] ISO 2011 International Organization for Standardization Metallic materials—calibration of force-proving instruments used for the verification of uniaxial testing machines ISO 376:2011 06.2011