Economic order quantity (EOQ) by game theory approach in probabilistic supply chain system under service level constraint for items with imperfect quality

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Abstract. In this paper, Economic Order Quantity (EOQ) of probabilistic two-level supply – chain system for items with imperfect quality has been analyzed under service level constraint. A firm applies an active service level constraint to avoid unpredictable shortage terms in the objective function. Mathematical analysis of optimal result is delivered using two equilibrium scheme concept in game theory approach. Stackelberg’s equilibrium for cooperative strategy and Stackelberg’s Equilibrium for noncooperative strategy. This is a new approach to game theory result in inventory system whether service level constraint is applied by a firm in his moves.

1. Introduction
In recent of industrial competition, only thinking about the integrated scheme is no longer suitable for any case of supply chain management. Each member in supply chain system may play a different strategy rather than integrated scheme. That chosen one depends on the economic purposes of each member. They can choose the cooperative or non-cooperative type of strategy to each other. According to the current literature, games theory is the best mathematical tool for getting an analysis of this condition and also finding the optimal result. There are two types of games related to the strategy analysis in supply chain management that cooperative (centralized approach) and non-cooperative (decentralized approach) games. In cooperative games condition, there are no dominant players, the players choose their strategies simultaneously, for instances, an integrated scheme in the inventory system. While, in a non-cooperative game, this is not exactly true. There are some circumstances that each member in inventory system can dominate each other. It can be classified as leader and follower. There are two types of noncooperative games that are dynamics non-cooperative games and static non-cooperative games. According to the literature by Alaei et al [1], Stackelberg game is a non-cooperative game. Each firm makes an optimal decision given the behavior of the other firm, and therefore, none has an incentive to deviate unilaterally from the equilibrium. On the other hand, for the cooperative game, according to literature by Peters [2], the results of non-cooperative approaches are not Pareto efficient. In order to achieve this solution, which is Pareto-efficient, a cooperative game can be applied by Abad and Jaggi [3]. In this game, the weighted sum of the player’s objectives which agree on the result of the Nash equilibrium is optimized. There are many kinds of literature about the optimal analysis of supply chain model using game theory approach. For
example, Elyasi et al [4], Esmaeili et al [5], Setiawan and Triyanto [6], Setiawan [7]. But, according to current literature, all of them are still limited to the deterministic model and most of them did not consider the model with the probabilistic condition and imperfect process condition such as lead time demands, defective items, inspection error rate, shortage and partial back-ordering. Another literature about the application of game theory in order to strategic behavior modeling in the industrial and economic field had been written by Geckil and Anderson [8] and Esmaeili et al [5] have analyzed two level seller and buyer model, they use a basic deterministic model that proposed by Abad and Jaggi [3]. They use Stackelberg strategy solution concept to analyze the noncooperative game and Pareto optimal concept to analyze the cooperative game. But, this paper still meets with the assumption that shortage is not permitted and no back ordering or partial back ordering process allowed. Elyasi et al [8] study modified EOQ for deterministic two-echelon (a manufacturer and a supplier) model with imperfect quality items. They use just in time (JIT) concept (centralized model) in inventory problem to formulate total cost of inventory. In their paper, the optimal solution also analyzed by game theory approach: non-cooperative static game using and cooperative game that first introduced by Abad and Jaggi [3]. A cooperative game is applied in order to get Pareto – efficient solution which refers to literature by Peters [2]. In their result, they proved that the result of the cooperative model is the nearest one to the result of the centralized model. Wu et al. [9] consider a supply chain with one supplier and two competing retailers and develop six different scenarios of power imbalance that leads to different sequences of moving among the members of the supply chain. They implement both Nash and Stackelberg equations and compare the gained results.In this research, we consider with probabilistic inventory model. Information about lead time length is often limited only in first and two moments of order, so it's difficult to estimate lead time function precisely. Many researchers use leads free demand condition to anticipate those condition In practically, it's difficult to estimate the shortage cost, so a service level constraint criterion is easy to interpret in an objective function instead of shortage cost term. Ni and Xia [10] define the condition of SLC is condition for a given safety factor which satisfies the probability that lead demand at the buyer exceeds reorder point, the actual proportion of demands not met from stock should not exceed a specified value of α. In this research, we propose some new result in game theory approach in inventory system, that is result when service level constraint is applied to avoid shortage condition.

2. Methods

2.1. General assumptions

There are single-vendor and single-buyer. A single type of product is considered. Probabilistic conditions always exist in the inventory system, two of them are deteriorating items and uncertainty lead time demand. It’s assumed that defective items only occur in the production process. Follow to the random number of defective items in retailer arriving order lot. On the buyer’s side, the defective items exist in an arriving lot of size Q with defective percentage γ and probability function f(γ). Vendor’s production rate for non-defective items is greater than buyer’s demand rate. Shortage conditions are allowed in the relationship between the vendor and the buyer. Shortages are partially back ordered with a fraction of the demand β, β∈[0,1] during the stock out period that will be back ordered. We ignore the inspecting time on buyer's side so that defective items can be inspected immediately by a 100 % screening process of the lot. The Buyer will return all defective items to the manufacturer upon receipt of the next lot. The reorder point r = expected demand during lead time + safety stock (SS) and SS = kσ√L, where k is a safety factor. We refer to the assumption by Lin [11]. The lead time length L consists of m mutually independent components. The ith component has the normal duration ̃h_i, the minimum duration a_i, and the crushing cost per unit time c_i. Furthermore, this c_i are assumed to be arranged such that c_1 ≤ c_2 ≤ ⋯ ≤ c_m. The components of lead time are crashed one at a time starting with the component of least c_i. Let L_i be the length of lead time with components 1,2,..., i crashed to their minimum duration,
then \( L_{\min} = \sum_{i=1}^{m} a_i \leq L \leq \sum_{i=1}^{m} b_i = L_{\max} \), \( L_i = L_{\max} - \sum_{j=1}^{i} (b_j - a_j) \), and the lead time crashing cost per cycle \( C(L) \) for a given \( L \in (L_i, L_{i-1}) \) is given by \( C(L) = c_i (L_{i-1} - L) + \sum_{j=1}^{m} c_j (b_j - a_j) \).

2.2. Notations. In this paper, we use some following parameter with some specific notation

\( Q \): The size of the shipments from the vendor to the buyer
\( N \): The number of lots in which the product is delivered from the vendor to the buyer,
\( D \): Expected demand per unit time on the Buyer for deteriorating items
\( P \): The production rate at the vendor
\( F \): The freight (transportation) cost per shipment.
\( S_b \): Buyer’s ordering cost per order.
\( S_v \): Vendor's set up the cost per production.
\( h_{b1} \): Buyer’s holding cost for a defective item per unit time.
\( h_{b2} \): Buyer’s holding cost for a non-defective item per unit time.
\( h_v \): Vendor’s holding cost per unit time.
\( \pi \): Buyer’s shortage cost per unit short.
\( \gamma \): The probability that an item produced is defective.
\( \pi_0 \): Buyer’s marginal profit (cost of demand lost) per unit.
\( \beta \): The fraction of the demand during the stock-out period.
\( c_{wp} \): The vendor’s unit warranty cost per defective items.
\( c_{ib} \): Screening cost at the buyer.
\( r \): Reorder point for deteriorating items.
\( x \): Buyer’s unit screening cost

3. Result and Discussion

3.1. Buyer total cost

The buyer inventory cost per cycle per unit time is consists of components due to placing an order, transportation cost, screening cost, holding cost, expected shortage cost and lead time crashing cost. The mathematical formula of the expected average total cost per unit time for the buyer follows the following equation

\[
ETC_b(Q, k, n, L) = \frac{D(S_b + nF + c_{ib}Q + [\pi + \pi_0(1 - \beta)]E[(X - r)^+]) + C(L)}{(1 - \gamma)(1 - \theta)}
\]

\[
+ h_{b1} \left[ \frac{Q(1 - \gamma)(1 - \theta)}{2} + k\sigma\sqrt{L} + (1 - \beta)E[(X - r)^+] \right] + h_{b2}Q\gamma\theta
\]

\[
+ \frac{(h_{b1}D - h_{b2})Q\gamma\theta}{2x(1 - \gamma)(1 - \theta)}
\]

Where \( \pi + \pi_0(1 - \beta) = \bar{\pi} \). Then, the worst distribution of \( ETC_b(Q, k, L) \) is given by

\[
ETC_b(Q, k, n, L) = \frac{D\left(S_b + nF + c_{ib}Q + \frac{\bar{\pi}\sigma\sqrt{L}}{2}(\sqrt{1 + k^2} - k)\right) + C(L)}{(1 - \gamma)(1 - \theta)}
\]

\[
+ h_{b1} \left[ \frac{Q(1 - \gamma)(1 - \theta)}{2} + k\sigma\sqrt{L} + (1 - \beta)\frac{\sigma\sqrt{L}}{2}(\sqrt{1 + k^2} - k) \right]
\]

\[
+ h_{b2}Q\gamma\theta + \frac{(h_{b1}D - h_{b2})Q\gamma\theta}{2x(1 - \gamma)(1 - \theta)}
\]
3.1.1. Vendor total cost. The vendor’s inventory production per unit time can be obtained by subtracting the accumulated buyer’s inventory level from the accumulated vendor’s inventory level as follow:

\[
\left[ nQ\left(\frac{Q}{p} + (n - 1)\right) - \frac{nQ(nq)}{2p}\right] - T(Q + 2Q + \cdots + (n - 1)Q) \\
= \frac{nQ^2}{p} - \frac{n^2Q^2}{2p} + \frac{n(n - 1)Q^2(1 - \gamma)(1 - \theta)}{2D}
\]

(3)

Thereby, holding cost for the vendor is formulated by

\[
h_v\left(\frac{nQ^2}{p} - \frac{n^2Q^2}{2p} + \frac{n(n - 1)Q^2(1 - \gamma)(1 - \theta)}{2D}\right)
\]

(4)

\[
ETC_v(Q,n) = \frac{D}{nQ(1 - \gamma)(1 - \theta)}\left[S_v + c_{vw}Qq\theta \right] \\
+ h_v\left(\frac{nQ^2}{p} - \frac{n^2Q^2}{2p} + \frac{n(n - 1)Q^2(1 - \gamma)(1 - \theta)}{2D}\right)
\]

(5)

3.2. Economic order quantity under service level constraint

We will provide the result in economic order quantity when service level constraint is active. If we are talking about two level inventory model, it’s commonly on buyer’s side. In practically, it’s difficult to estimate the shortage cost, so a service level constraint criterion is easy to interpret in an objective function instead of shortage cost term. Ni and Xia [11] define the condition of SLC is condition for a given safety factor which satisfies the probability that leads demand at the buyer exceeds reorder point, the actual proportion of demands not met from stock should not exceed a specified value of \(\alpha\)

\[
\frac{Expected\ demand\ shortages\ at\ the\ end\ of\ cycle\ for\ given\ safety\ factor}{Quantity\ available\ for\ satisfying\ the\ demand\ per\ cycle} \leq \alpha
\]

(6)

Since \(r=DL+k\sigma\sqrt{L}\) and considering the safety factor \(k\) as decision variable instead of the reorder point \(r\), then for the fixed value of \(L\), we get the following inequality

\[
\frac{E[(X - r)^+]}{Q} \leq \frac{\sigma\sqrt{L}}{2Q} \left(\sqrt{1 + k^2} - k\right), L \in [L_{i-1}, L_i] \Leftrightarrow \frac{\sigma\sqrt{L}}{Q} \left(\sqrt{1 + k^2} - k\right)
\]

(7)

Hence, SLC on the buyer to prevent unacceptable stock outs and this cost. Furthermore, in this research, we consider the partial back ordering process with specified rate \(\beta\) in integrated inventory system which service level restriction was established.

3.2.1. Economic order quantity by non-cooperative decentralized model. The vendor as leader choose his move first, and then buyer as follower reacts by observing respective decision. The buyer gets his best own move consistent with available information. Then, the leader determines his optimal move as Stackelberg’s optimal move consider the optimal reaction from the follower. To avoid the unexpected shortage condition, we consider the situation that buyer prefers to apply service level constraint rather to shortage cost term in the objective function. We propose to take service level constraint term (7) in buyer's expected total cost while Stackelberg game is applied by vendor and buyer as a leader and its follower respectively. According to the best of author’s knowledge about the reference in this field, this is a new way to represent game theory result when one of the firms in inventory system is willing to use service level constraint. Using Karush-Kuhn-Tucker (KKT) conditions for optimum condition, we get EOQ of buyer as follower for fixed value of number of shipment \(n\) for buyer and fixed value of lead time \([L_{i-1}, L_i]\) as follows.
On the other hand, if we apply KKT condition to the vendor's expected average total cost, we get the optimal vendor move:

\[
Q^*_{fl} = \sqrt{\frac{S_v}{n_{fl} + \frac{\pi \sqrt{L}}{2} (n_{fl}^2 - 1)}} = \sqrt{\frac{2(S_v + c_{ewf} \theta Q^*_{fl})}{h_v \left( 1 + \frac{(1-\gamma)(1-\theta)}{\delta} \right)}}
\]

To get Stackelberg solution, vendor as the leader in that system observe the buyer’s independent optimal move. Roughly speaking, leader’s optimal move depends on follower’s optimal move. We only need to substitute Equation (8), Equation (9) and Equation (10) to Equation (11) to get Stackelberg solution for the system. In this research, we can only present the optimal solution in implicit equation

\[
\frac{D \left( S_b + nF + \frac{\pi \sqrt{L}}{2} \left( \sqrt{1 + k^*_{fl}^2} - k^*_{fl} \right) + C(L) \right)}{(1-\gamma)(1-\theta)} \left[ \frac{h_b(1-\gamma)(1-\theta)}{2} + h_{b2}\theta + \frac{(h_{b1}(1-\gamma)(1-\theta)) + \lambda^*}{2x(1-\gamma)(1-\theta)} \right] - \frac{2(S_v + c_{ewf} \theta Q^*_{fl})}{h_v \left( 1 + \frac{(1-\gamma)(1-\theta)}{\delta} \right)} = 0
\]

where \(Q^s\), \(k^s\) and \(\lambda^s\) is final EOQ of the inventory system, which is agreed to apply the noncooperative strategy.

### 3.2.2. Economic Order Quantity by Cooperative Decentralized Model

According to the Abad and Jaggi (2003) [3], the optimal result of the non-cooperative approaches is not Pareto – efficient. To get the Pareto-efficient solution, a cooperative game is applied. We use the cooperative game introduced by Abad and Jaggi [3] and also used by Elyasi et al [4]. The weighted sum of the players’ objectives is optimized under the assumption that all of the players (vendor and buyer) agree on the result of the Nash equilibrium. We take service level constraint term into that weighted sum of the players’
objectives to get new Lagrange function \( L^*_p(Q, k, L, \lambda) = \lambda_p(ETC_b) + (1 - \lambda_p)ETC_v + \lambda\left[\alpha - \sqrt[4]{L(\sqrt{1 + k^2} - k)}\right]\) where \(0 < \lambda_p < 1\). Using Karush-Kuhn-Tucker (KKT) conditions for optimum condition of \( L^*_p(Q, k, L, \lambda)\), we get Economic Order Quantity (EOQ) of buyer as follower for fixed value of lead time \( L \in [L_{i-1}, L_i]\) as follows

\[
Q^* = \frac{\alpha (2 + \alpha \sqrt{(1+k^2)\alpha})}{\sqrt{16 \pi^2 \sigma^2 + 4 \pi \sqrt{(1+k^2)\alpha} \sigma \sqrt{1+k^2}}}
\]

\[
n^* = \frac{\sqrt{\frac{n^*}{\pi^2}}}{\sigma (\sqrt{1+k^2} - k)} \left[ \frac{\sqrt{\frac{n^*}{\pi^2}}}{\sigma (\sqrt{1+k^2} - k)} \right] + \lambda_p \frac{\sigma (\sqrt{1+k^2} - k)}{\sqrt{2 \pi \sigma^2}} + \frac{\lambda_p \sigma (\sqrt{1+k^2} - k)}{\sqrt{2 \pi \sigma^2}}
\]

4. Conclusion
In this research, we propose the mathematical model of probabilistic inventory under probabilistic condition using game theory approach. We concern about service level constraint as a replacement part of an objective function to avoid unexpected shortage condition because of uncertainty lead-free time demand. In our model, service level constraint is depending on safety factor in lead time process. First one is a non-cooperative game using Stackelberg optimum, and then the third ones are cooperative schemes using Pareto Optimal scheme. While the vendor and the buyer agree to play cooperative strategy, then Pareto optimal scheme is used to get the optimal decision of each party. Based on the analytical result, it’s complicated to get an explicit form of an optimal solution (EOQ) each decision variable, so the analytical result is presented in implicit form only.

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