Black holes and gravitational waves: simultaneous discovery by initial laser interferometers

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Study of gravitational-radiation induced merging rates of relativistic binary stars (double neutron stars; neutron star + black hole; double black holes) shows that the first-generation gravitational wave interferometers with an rms-sensitivity of $10^{-21}$ at frequency 100 Hz can detect $10^{-700}$ black hole and only $\sim 1$ neutron star coalescences in a 1-year integration time in a wide range of stellar evolution parameters. It is notable that modern concepts of stellar evolution predict that the first detection of gravitational wave will independently discover black holes.

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The final merging stage of binary relativistic star evolution containing two compact stars (neutron stars (NS) or black holes (BH)) that merge due to gravitational-radiation induced orbit decay on a time-scale shorter than the Hubble time ($1.5 \times 10^{10}$ years) are among the primary targets for gravitational wave (GW) interferometers currently under construction (LIGO, VIRGO, GEO-600) [1]. Reality of such events in the Universe is confirmed by binary pulsar observations [3]. To date, 5 binary NS are known in our Galaxy; 3 of them should coalesce within the Hubble time [3]. Based on the properties of these binary pulsars, one may estimate the total number of such systems and hence the merging rate of binary NS in the Galaxy, and then extrapolate this rate to the volume of the Universe where GW-detectors will be able to detect GW-signal at a given signal-to-noise level. Such estimations made in the last five years [4] yield a “realistic” galactic rate of double NS mergings of $10^{-5} - 10^{-6}$ per year.

Theoretical estimate of double NS merging rate may be deduced from stellar evolution [5] and give $\approx 10^{-4}$ NS-NS coalescences per year per Galaxy. It can be shown that these two estimates do not contradict to each other if one takes into account NS+NS binaries which contain no pulsar (ejecting NS) and thus are unobservable by traditional radioastronomical means [6].

The stellar evolution theory also predicts that neutron star – black hole (NS+BH) and black hole – black hole (BH+BH) binaries should be present in the Galaxy. So far the indirect existence of BH in stellar binary systems has been argumented by a high mass ($> 3M_\odot$) of the unseen companion in 11 close X-ray binary systems [6]. No PSR+BH binary system has yet been discovered in spite of optimistic theoretical expectations [6] of $\sim 1$ per 1000 single pulsars. Theoretical estimates of NS+BH/BH+BH merging rate in the Galaxy are 1-2 orders of magnitude lower than NS+NS merging rate. The difficulty in doing such estimates is that additional parameters of BH formation appears: the critical mass $M_{cr}$
beyond which the star collapses into BH and the mass of BH formed.

From the point of view of GW detection, NS+BH and BH+BH binary systems are advantageous in having higher "chirp mass" \( \mathcal{M} = M(\mu/M)^{3/5} \), \( M = M_1 + M_2 \), \( \mu = M_1 M_2/M \) that determines the amplitude of the dimensionless metric strain produced by a binary system:

\[
h_{\text{amp}} \sim \mathcal{M}^{5/3} f^{2/3} / r,
\]

where \( f \) is the GW frequency and \( r \) is the distance to the binary. In fact, matched filtering technique of data analysis \(^9\) allows enhancement of the signal-to-noise ratio from coalescing binaries \( S/N \propto \sqrt{n} \)

\[
\text{where } n = f^2 / \dot{f}
\]

is the number of cycles of the signal passing through the detector frequency band \( \Delta f \sim f \), so one usually considers the characteristic strain amplitude

\[
h_c = h_{\text{amp}} \sqrt{n} \sim \mathcal{M}^{5/6} f^{-1/6} / r.
\]

This means that at a given \( S/N \) the detector will be sensitive to a distance \( r \propto \mathcal{M}^{5/6} \). For example, if we consider a NS+NS binary consisting of two identical NS \( M_1 = M_2 = 1.4 M_\odot \) \( (M_{\text{ns}} \approx 1.2 M_\odot) \) and a BH+BH binary consisting of two identical BH \( M_1 = M_2 = 10 M_\odot \) \( (M_{\text{bh}} \approx 8.7 M_\odot) \), the limiting distance \( r_{\text{bh}} \approx 5.2 r_{\text{ns}} \), or the volume available to search for such BH+BH systems is 140 times as large as the volume in which NS+NS binaries can be detected. Assuming homogeneous matter distribution, the detection rate in a 1-year integration time with a given signal-to-noise ratio is

\[
N_{\text{bh}}/N_{\text{ns}} = (R_{\text{bh}}/R_{\text{ns}}) \times (M_{\text{bh}}/M_{\text{ns}})^{15/6}
\]

where \( R_{\text{bh}} \) and \( R_{\text{ns}} \) are galactic merging rates for BH and NS binaries, respectively. Clearly, to answer the question what type of binary systems will be more numerous during 1-year operation time of a GW-interferometer one should study in detail different types of binary mergings in the Galaxy.

In this paper we present calculations of NS+NS, NS+BH and BH+BH galactic merging rates in a wide range of main parameters of stellar evolution. We address the question how many binary mergings can one expect to detect with the rms-sensitivity \( h_{\text{rms}} \simeq 10^{-21} \) at \( f_c = 100 \) Hz corresponding to \( S/N = 1 \) for GEO-600 and \( S/N = 3 \) for LIGO/VIRGO-I \(^1\). We find that the prospect to discover BH-events (binary BH merging or BH+NS merging) is at least 10 times better than for NS+NS mergings. Moreover, for currently popular high recoil velocities of young pulsars \( \sim 400 \) km/s \(^1\) only BH+BH binaries may be detected by the first-order laser interferometers.

Qualitatively, a crude estimate of the ratio of BH-containing binary merging rate to NS binary merging rate may be done as follows. BH is thought to result from the core collapse of massive stars. Single massive stars lose roughly half their mass through intensive stellar wind; when in a close binary system, the mass may be transferred onto the secondary companion. An approximate relation between the initial mass of the star and its core is \( M_{\text{core}} \simeq 0.1 M_{\text{ns},1.4} \). Let us assume the mass of BH progenitor just before the
collapse be $35M_\odot$, which would correspond roughly to $M_{\text{ms}} \sim 60M_\odot$ on the initial main sequence. On the other hand, any star with $M_{\text{ms}} \geq 10M_\odot$ evolves to form a NS. Using the Salpeter mass function for star formation ($dN/dM \approx (M/M_\odot)^{-2.35}$ star per year), we obtain that BH formation rate relates to NS formation rate as $(60/10)^{-1.35} \approx 0.09$. Extrapolating this logic to binary BH/NS systems, we might expect $R_{\text{bh}}/R_{\text{ns}} \sim 1/10$, to a half-order accuracy. Actually, the situation is complicated by several factors: an asymmetry of the supernova explosion which may act more efficiently in the case of NS formation; mass exchange between the components; distribution by mass ratio, etc. All these factors will be accounted for in our calculations.

To perform evolutionary calculations, we employ Monte-Carlo method for binary stellar evolution studies developed by us over last ten years (the Scenario Machine code); we refer to [12] for a detailed description of the method and evolutionary scenarios used. The basic idea is to calculate the evolution of a representative number of binary stars (typically $10^6$) whose orbital and physical parameters are distributed corresponding to the observational data when available (e.g. initial masses, semimajor axes, recoil velocities of newborn neutron stars), or are chosen according to some model laws (e.g. initial rotation periods and magnetic fields of neutron stars). The calculated number of mergers is then scaled to the total stellar mass of the Galaxy (taken $10^{11}M_\odot$) and after dividing by the age of the Galaxy ($1.5 \times 10^{10}$ years) yields the galactic merging rate $R$. Note (see below) that when reducing to the total mass of the Galaxy we must take into account the fraction of stars entering binary systems. This fraction is at least 50%.

A BH is known to be fully described by three parameters: its mass $M_{\text{bh}}$, angular momentum, and electric charge. For our purposes, however, only mass is important as it determines the orbital evolution when the BH resides in a close binary system. At present, stellar-mass BH are thought to result from the core collapse of high-mass stars with $M > M_*$ where $M_*$ is the pre-supernova mass (see [13] for recent discussion). It seems reasonable to assume that the mass of BH is proportional to $M_*$, i.e. $M_{\text{bh}} = k_{\text{bh}}M_*$, $0 < k_{\text{bh}} \leq 1$. This mass in turn is calculated from modern stellar evolution theory. Analysis of observational data on BH-candidates in binary systems [8] shows that the most plausible BH-formation parameters are $k_{\text{bh}} = 0.25 - 0.5$, $M_*= 25-50$. Some restriction to these parameters are discussed below.

When in binaries, another important parameter appears: the additional velocity $w_{\text{bh}}$ imparted to BH during the anisotropic collapse. Here several different possibilities are feasible: (1) $w_{\text{bh}}$ is proportional to the mass ejected during the collapse $M_{\text{ej}} = M_*(1 - k_{\text{bh}})$; (2) the momentum $M_{\text{bh}}w_{\text{bh}}$ is proportional to the ejected mass $M_{\text{ej}}$; (3) $M_{\text{bh}}w_{\text{bh}}$ is proportional to the ejected momentum.

We shall assume a universal mechanism giving anisotropic velocity for both NS and BH. In the case (1)
we have:

\[ w_{bh} = w_{ns}(1 - k_{bh})(1 - M_{OV}/M_*)^{-1} \]  

(1)

where \( M_{OV} = 2.5 M_\odot \) is the Oppenheimer-Volkoff limit for NS mass. This law is chosen assuming boundary conditions \( w_{bh} = 0 \) at \( k_{bh} = 1 \) (i.e. when the total mass of the collapsing star goes into a BH) and \( w_{bh} = w_{ns} \) once \( M_{bh} = M_{OV} \). Clearly, the case (2) is similar to the case (1) but produces lower BH-velocities while the case (3), instead, assumes \( w_{bh} = w_{ns} \) irrespective of BH mass and thus produces higher BH velocities. We found, however, that all three cases yield BH+BH/BH+NS merging rates practically coinciding with each other at high kick velocities, so here we represent the results for the case (1) only.

The three-dimensional NS kick velocity is assumed to be arbitrarily directed in space and to be distributed so as to reproduce the observed pulsars’ transverse (two-dimensional) velocities \cite{10} (see \cite{12} for more detail):

\[ f_{LL}(x)dx \propto x^{0.19}(1 + x^{6.72})^{-1/2}dx \]  

(2)

where \( x = w/w_0 \), \( w_0 \) is the characteristic velocity; the observed Lyne-Lorimer 2D-distribution is obtained at \( w_0 = 400 \) km/s. The mean 3-D kick velocity differ from \( w_0 \) by \( \approx 10\% \). Our analysis of modern evolutionary scenarios \cite{12} shows that the most probable kick velocities lie in the range 200-400 km/s.

In Fig. 1 we plot the relativistic compact binaries’ merging rates as a function of the mean kick velocity assuming Lyne-Lorimer distribution \cite{4}. BH-formation parameters were taken within the limits \( M_* = 15 - 50 M_\odot \), \( k_{bh} = 0.25 \). Different scenarios for stellar evolution were considered (see ?? for more detail about evolutionary scenarios). From Fig. 1 we see that the theoretical expectation for the NS+NS merging rate in a model spiral galaxy with typical mass of \( 0.5 \times 10^{11} M_\odot \) in binary stars lie within the range from \( \sim 3 \times 10^{-4} \) yr\(^{-1} \) to \( \sim 3 \times 10^{-5} \) yr\(^{-1} \), depending on the assumed mean kick velocity and the shape of its distribution. Note that for spherically symmetric collapse the calculated rates coincide well with those found by Tutukov & Yungelson \cite{5} who used another method of calculations. For Lyne-Lorimer law with the mean value of 400 km/s, we obtain \( R_{NS+NS} \approx 5 \times 10^{-5} \) yr\(^{-1} \). BH+BH and BH+NS merging rates are typically 1-2 orders of magnitude smaller.
FIG. 1. The merging rate of NS+NS, NS+BH, and BH+BH binaries for Lyne-Lorimer kick velocity distribution as a function of $w_0$ assuming BH formation parameters $M_\ast = 15-50 \, M_\odot$, $k_{bh} = 0.25$, for different scenarios of binary star evolution in a model spiral galaxy with the total stellar mass $10^{11} \, M_\odot$ and 50% baryon fraction in binary stars.

Two details in Fig. 1 are worth noting: 1) the binding effect at small kick velocities and 2) the smaller effect of high kicks on the BH+BH/BH+NS rate. The first fact is qualitatively clear: a high kick leads to the system disruption; however, if the system has survived the explosion, its orbit would have a periastron always smaller than in the case without kick. During the subsequent tidal circularization a closer binary system will form which will spend less time prior to the merging. The binding effect at small recoil velocities is more pronounced in the case of binary BH. At higher kicks their merging rate decreases slower than that of binary NS due to higher masses of the components.

Having found galactic merging rates $R$, we can calculate them within a volume accessible for GW-observations of each type of binaries (which from the point of view of GW-signal differ, to the first approximation, in having different chirp masses). Here we may either use normalization to the IR-luminosity per cubic megaparsec (as in Phinney’s paper [4]), or scale the galactic rates per baryonic density in stars and then extrapolate it to the volume desired. In the latter case we obtain the formula

$$R = 0.0063 \, R (\epsilon_d/0.5) (\Omega_b/0.0046) \, h_{75}^2 \, \text{Mpc}^{-3}$$

where $\Omega_b$ is the baryon density (in units of critical density to close the Universe), $h_{75} = H_0/75 \, \text{km/s/Mpc}$ is the present value of the Hubble constant, $\epsilon_d$ is the fraction of baryons contained in binary stars. Typically $0.25 < \epsilon_d < 0.75$. This normalization coincides with Phinney’s one assuming $\epsilon_d \Omega_b = 0.0046$. According
to \[14\], \(\Omega_b = 0.0015\) in stars within galactic disks and \(\Omega_b = 0.003\) in spherical bulges of spiral galaxies and in elliptical galaxies. Calculations of NS+NS merging rate in elliptical galaxies show \[15\] that although a strong decrease in binary merging rate occurs, it tends to a nearly constant value several times lower than the binary merging rate in a spiral galaxy with constant star formation. So taking \(\epsilon_d \Omega_b = 0.5 \times 0.0046\) would not be too far off and we use it to evaluate binary merging rates in a given volume of the Universe. Note, however, that in view of unknown exact value of \(\epsilon_d\) all absolute rates we obtain may be uncertain to within a factor of 2.

Consider now whether it is possible to restrict BH formation parameters from the existing astronomical observations. From known 11 BH candidates in binary systems the evolutionary status of Cyg X-1 is mostly well understood (a massive X-ray binary containing a 10 \(M_\odot\) BH and a 30 \(M_\odot\) OB-supergiant). The evolution of 6 X-ray Novae – BH candidates is much more controversial. Only one CYg X-1-like source is observed in the Galaxy, so for the number of such system we can adopt the lower limit \(N(\text{Cyg X-1}) \geq 1\). On the other hand, as the analysis of stellar evolution shows \[8\], 1 binary PSR with BH should exist per 1000 single radiopulsars in the Galaxy. So far no such systems have been discovered, i.e. there is an upper limit \(N(\text{PSR+BH})/N(\text{BPSR}) < 1/700\). Binary systems like Cyg X-1 and PSR+BH are evolutionary related \[8\] and hence put bounds on BH formation parameters as shown in Fig. \[8\]: \(k_{bh} > 0.5\), \(M_* > 18M_\odot\) (or equivalently, \(M_{cr} > 35M_\odot\)).
Fig. 2. The galactic number of Cyg X-1-like systems (upper curves) and the fraction of binary PSR with BH among the total number of single PSR in the Galaxy (lower curves) as a function of BH formation parameters. The upper horizontal scale is in presupernova masses $M_*$, the lower scale in the critical main sequence mass $M_{cr}$ (both in solar masses). The fraction of presupernova mass to collapse into BH $k_{bh}$ is along the vertical scale. The shaded region shows the most plausible BH formation parameters.

Fig. 3 shows the expected total (NS+NS, NS+BH, BH+BH) and NS+NS detection rates at $S/N = 1$ level in 1-year integration on a GW-detector with the initial laser interferometers rms-sensitivity $h_{\text{rms}} = 10^{-21}$. All possible combinations of BH formation parameters $M_*$, $k_{bh}$ with Lyne-Lorimer kick velocity law with $w_0 = 400$ km/s were calculated for different scenarios of stellar evolution. It is seen from Fig. 3 that NS+NS detection rate is $\approx 0.3 - 0.7$ per year whereas BH+BH/BH+NS coalescences are parameters-dependent and can be much more numerous. The filled "Loch-Ness monster"-headed region corresponds to the “most realistic” BH-formation parameters ($M_* > 18$, $k_{bh} > 0.5$, $w_0 = 200 - 400$ km/s and the “low mass-loss” scenario for evolution of single stars (see [?]) for more detail). Within this region we may expect from $\sim 10$ to $\sim 700$ events per year, mostly (more than 80%) BH+BH coalescences.

![Plot of total merging rate of NS+NS, NS+BH, BH+BH binaries as would be detected by a laser interferometer with $h_{\text{rms}} = 10^{-21}$ for Lyne-Lorimer kick velocity distribution with $w_0 = 200 - 400$ km/s and BH progenitor’s masses $M_* = 15 - 50 M_\odot$, for different scenarios of binary star evolution as a function of $k_{bh}$. NS+NS mergings are shown separately. In all cases BH+BH mergings contribute more than 80% to the total rate. The filled “Loch-Ness monster-head”-like region correspond to BH formation parameters $M_* > 18 M_\odot$ and $k_{bh} = 0.5$.](image-url)
For a coalescing binary, the total mass $M = M_1 + M_2$ and the chirp mass $\mathcal{M}$ determine the final merging frequency (through the 3d Kepler’s law) and the GW-waveform amplitude, respectively. These mass distributions for binary BH mergings in the scenario with $M_* = 35 \, M_\odot$, $k_{bh} = 0.3$ and $w_0 = 400 \, \text{km/s}$ are shown in Fig. 4. The distributions are normalized so as to give $\int (dN/dM)dM = 1$.

The results presented in this paper were obtained on the base of astronomically well-grounded binary stellar system evolutionary scenario. We then may ask the question: do there exist some parameters of the modern evolutionary scenario with which less than 1 coalescing relativistic binary should be observed with our detector in 1-year integration time? The answer is “yes” only assuming $k_{bh} < 0.1$. Then low-mass BH are formed which are subjected to the disruptive action of both the high mass-loss during the collapse and the presumed collapse anisotropy. These small $k_{bh}$ seem to be unrealistic considering observed masses of BH-candidates $\sim 10 M_\odot$ [7] (then the progenitor’s mass would be about $100 \, M_\odot$ which corresponds to implausibly high mass of $160 \, M_\odot$ on the main sequence).

We conclude that for a wide range of BH-formation parameters ($M_* \approx 18 – 80 M_\odot$, $k_{bh} > 0.5$) and a high birth velocity of newborn collapsed stars (300–400 km/s) we always expect $\sim 10 – 700$ binary merging events (mostly BH+BH pairs) to have the signal-to-noise ratio equal unity during 1-year integration by the first-order GW-detectors. Irrespective of the absolute number of merging events (which are subjected to at least a factor of 2 uncertainty due to unknown fraction of baryons in binary stars), the relative number of detected BH mergings at any detector should typically be $\sim 10$ times higher than
NS+NS coalescences.

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