The Clique Problem in Ray Intersection Graphs

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Example
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Geometric Intersection Graphs
Cliqu es in Geometric Intersection Graphs

- Kratochv íl and Ne s t íl: “Independent set and clique problems in intersection defined classes of graphs”, 1990
  - Independent set is NP-hard in segment intersection graphs, even with a bounded number of distinct directions
  - Open question: is Clique NP-hard in segment intersection graphs?
Clique in Geometric Intersection Graphs

- Middendorf and Pfeiffer 1992: “The max clique problem in classes of string-graphs”
  - NP-hard for the intersection of 1-intersecting piecewise linear curves
- Ambühl and Wagner 2005: “The clique problem in intersection graphs of ellipses and triangles”
  - NP-hard for intersection graphs of ellipses of arbitrary, fixed, aspect ratio
  - breaks for the two limit cases of disks and segments!
Our Result

The Maximum Clique Problem is **NP-hard** in segment intersection graphs – even when a realization is given as input.
A Related Structural Question

- It is known that every planar graph is an intersection graph of segments (Chalopin and Gonçalves, 2009)
- Is every co-planar graph an intersection graph of segments?
It is known that every planar graph is an intersection graph of segments (Chalopin and Gonçalves, 2009).

Is every co-planar graph an intersection graph of segments?
We reduce from independent set in planar graphs

Even subdivisions increase the independence number by a fixed amount

Byproduct: every planar graph has an even subdivision whose complement is a segment intersection graph
Outline of the Construction

1. Segment realization of complements of trees
2. Segment realization of complements of extended trees: trees with short paths between leaves
3. Every planar graph has an even subdivision that is an extended tree

We check that the construction uses polynomial-size coordinates
Trees: the Snooker representation
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Extended Trees
From Planar Graphs to Extended Trees

(a) \( f \) (b) \( f^* \) (c) 

(d) 

(e) 

\( r \)
The segments we use can all be extended at infinity in one direction.

Hence the Clique problem is NP-hard in ray intersection graphs as well!
Open Problems

- Representability of \textit{co-planar graphs} as segment intersection graphs?
- \textbf{Approximability}?
- Is the Clique problem NP-hard for \textit{disk} intersection graphs?