FARIMA MODELING OF SOLAR FLARE ACTIVITY FROM EMPIRICAL TIME SERIES OF SOFT X-RAY SOLAR EMISSION

A. A. Stanislavsky1, K. Burnecki2, M. Magdziarz2, A. Weron3, and K. Weron3

1 Institute of Radio Astronomy, National Academy of Sciences of Ukraine, 4 Chervonopraporna St., Kharkov 61002, Ukraine; alexstan@ri.kharkov.ua
2 Hugo Steinhaus Center, Institute of Mathematics and Computer Science, Wrocław University of Technology, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland
3 Institute of Physics, Wrocław University of Technology, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland

Received 2008 May 7; accepted 2008 December 9; published 2009 March 10

ABSTRACT

A time series of soft X-ray emission observed by the Geostationary Operational Environmental Satellites from 1974 to 2007 is analyzed. We show that in the solar-maximum periods the energy distribution of soft X-ray solar flares for C, M, and X classes is well described by a fractional autoregressive integrated moving average model with Pareto noise. The model incorporates two effects detected in our empirical studies. One effect is a long-term dependence (long-term memory), and another corresponds to heavy-tailed distributions. The parameters of the model: self-similarity exponent $H$, tail index $\alpha$, and memory parameter $d$ are statistically stable enough during the periods 1977–1981, 1988–1992, 1999–2003. However, when the solar activity tends to minimum, the parameters vary. We discuss the possible causes of this evolution and suggest a statistically justified model for predicting the solar flare activity.

Key words: methods: data analysis – methods: statistical – Sun: activity – Sun: flares – Sun: X-rays, gamma-rays

Online-only material: color figures

1. INTRODUCTION

Observations of solar flare phenomena in X-rays became possible in the 1960s with the availability of space-borne instrumentation. Since 1974, broadband soft X-ray emission of the Sun has been measured almost continuously by the meteorology satellites operated by the US National Oceanic and Atmospheric Administration (NOAA) as well as the Synchronous Meteorological Satellite (SMS) and the Geostationary Operational Environmental Satellite (GOES). The solar soft X-ray flare data are widely available from the NOAA Space Environment Center site (http://goes.ngdc.noaa.gov/data/avg/).

At present, the accurate solar data are available for the most recent three cycles only. Individual solar cycles are different in form, amplitude, and length. Hence, understanding the long-term solar variability and predicting the solar activity is an actual problem for solar physics. It is associated with a variety of space weather effects (Mann et al. 1998; Svensmark & Friis-Christensen 1997). Solar activity is known to correlate with flare activity, and a variety of flare properties (time and strength) could be incorporated into predictions. Moreover, these disturbances pose serious threats to man-made spacecraft, disrupt electronic communication channels, and can even set up huge electrical currents in power grids (Clark 2006).

Within the last few years, theoretical models of solar dynamo have been developed to explain various aspects of the solar activity. Dikpati & Gilman (2006) made the first attempt at using a theoretical dynamo model to predict the strength of the upcoming Cycle 24. They have shown that this cycle will be the strongest in 50 years. But later Choudhuri et al. (2007) pointed out that some assumptions in the Dikpati–Gilman model are unjustified. In contrast, their model, based on the earlier work of Nandy & Choudhuri (2002), predicts that the 24th cycle will be weaker than the 23rd. The key problem here is the following: the dominant processes such as the magnetic field advection and toroidal field generation by differential rotation are fairly regular during the rising phase of a cycle from a minimum to a maximum, and hence, good knowledge of magnetic configurations during a minimum would enable a good theoretical model to predict the next maximum reliably. However, the dominant process in the declining phase of a cycle contains the poloidal field generation by the Babcock–Leighton mechanism which involves randomness (primary cause of solar cycle fluctuations) and cannot be predicted in advance by any deterministic model. Thus, the prediction methods of flare activity should be based on statistical analysis of temporal and spatial features of the experimental evidence. For example, Li et al. (1998) studied the distribution of the X-ray flares ($M \geq 1$) from 1987 to 1992 with respect to helio longitude. They have shown that the flares were not uniformly distributed in longitude. The analysis of temporal features of the X-flare statistics basically has been related to the waiting-time distributions (see, for example, Boffetta et al. 1999; Moon et al. 2001; Lepreti et al. 2001; Wheatland 2002; Veronig et al. 2002).

Based on a Bayesian approach, the statistical solar flare forecast method has been developed by Wheatland (2005). His method is less accurate at predicting mean numbers of M-X events than the NOAA method (developed by McIntosh 1990 and adopted by the NOAA; see Gallagher et al. 2002), but more accurate at predicting X events, which are important contributors to space weather (see also Wheatland 2000, 2001, 2004, 2008).

As a candidate for extensive statistical studies of the soft X-ray solar emission time series we consider the fractional autoregressive integrated moving average (FARIMA) model (Granger & Joyeux 1980; see also Stoëv et al. 2002 and Stoëv & Taqqu 2004, and the references therein). The FARIMA model is a discrete-time analog of the fractional Langevin equation (Magdziarz & Weron 2007) that takes into account the non-Gaussian statistics and the long-term dependence (long-term memory), i.e., the events that are arbitrarily distant still influence each other exceptionally strong. Briefly, the physical arguments for our subsequent consideration are the following.

Processes resulting in solar flares are associated with a complicated motion of charged particles (plasma) in magnetic and electric fields. It generates the fields, accumulates their energy, and transforms into the energy of flares. The classical problem for a motion of the charged Brownian particle into magnetic...
and electric fields can be described in the framework of the Langevin equations. However, in the strongly nonequilibrium plasmas, where turbulence is a prevailing phenomenon, the non-Gaussian (Lévy) statistics of random force becomes dominant (Chechkin et al. 2002). As a consequence, the solar data provide information on anomalous (non-Gaussian) diffusion and non-Maxwell stationary states. For example, the random motion of bright points (BP), associated with magnetic fields at the solar photosphere, with times less than 20 minutes has a subdiffusive character (Cadavid et al. 1999). Probably, this behavior is connected with the appearance of traps due to the existence of closed magnetic surfaces. As it has been shown by Stanislavsky & Weron (2007), the anomalous diffusion of BPs can be described by the fractional Fokker–Planck equation generalizing the Leighton model of diffusion (Leighton 1964).

A significant difference between Gaussian and Lévy stable distributions is that the latter have heavy tails. This means that much larger jumps or flights are possible for Lévy stable distributions as compared to Gaussian ones. Hence, in the case of heavy-tailed distributions, many more rare events are observed than in the case of Gaussian distributions. For example, the random motion of solar flares shows that much larger jumps or flights are possible for Lévy stable distributions (Chechkin et al. 2002). As a consequence, the solar data provide information on anomalous (non-Gaussian) diffusion and non-Maxwell stationary states. For example, the random motion of solar flares shows that much larger jumps or flights are possible for Lévy stable distributions (Chechkin et al. 2002).

The process \( X(n) = \sum_{j=0}^{\infty} c_j \epsilon_{n-j} \),

where \( c_j \)'s are defined by the equation

\[
\frac{\Theta_p(z)(1-z)^{-d}}{\Phi_p(z)} = \sum_{j=0}^{\infty} c_j z^j, \quad |z| < 1
\]

(3)

for details see Taqqu & Teverovsky (1998). The variable \( \epsilon_j \) may be either Gaussian, non-Gaussian with finite variance, or may have infinite variance. For infinite variance variables one may consider, for example, symmetric and skewed stable distributions, as well as Pareto distributions. Both are characterized by the index \( \alpha \) and their tails \( P(\epsilon > x) \) satisfy

\[
P(\epsilon > x) = 1 - F(x) \sim x^{-\alpha}, \quad x \to \infty,
\]

(4)

where \( F(x) \) denotes the corresponding distribution function and \( \sim \) denotes that the ratio of the left to the right-hand side tends to 1, as \( x \to \infty \). It should be noted that for the Lévy-stable distributions we have the following restriction: \( 0 < \alpha < 2 \), whereas for the Pareto distribution the index \( \alpha \) is only greater than zero. The resulting process \( X(n) \) will be Lévy-stable if the variables are Lévy-stable, and asymptotically will be in the domain of attraction of a Lévy-stable distribution if the variables are Pareto with \( \alpha \) in the range \( 0, 2 \) (see Samorodnitsky & Taqqu 1994). Moreover, such FARIMA processes are asymptotically self-similar with parameter \( d - 1/\alpha \).

The process \( X(n) \) is a stationary moving average and the necessary condition for the series (2) to converge a.s. is \(-\infty < d < 1 - 1/\alpha \). In the Gaussian case, i.e., when \( \alpha = 2 \), the rate of decay of the covariance function \( \text{Cov}(n) = E[X(n)X(0)] - E[X(n)]E[X(0)] \) for the FARIMA model is \( t^{2d-1} \), which shows that for \( d \geq 0 \) we have \( \sum_{n=0}^{\infty} |\text{Cov}(n)| = \infty \) and \( X(n) \) is a process with long-term dependence. Additionally, the spectral density \( f(\omega) \) (Fourier transform of \( \text{Cov}(n) \)) satisfies \( f(\omega) \sim c|\omega|^{-2d} \) as \( \omega \to 0 \). For \( \alpha < 2 \) the covariance does not exist and one has to replace it, e.g., with the codifference. It is defined in the following way (see Samorodnitsky & Taqqu 1994): the codifference \( \tau_{X,Y} \) of two jointly \( \alpha \)-stable random variables \( X \) and \( Y \) equals

\[
\tau_{X,Y} = \ln E e^{i(X-Y)} - \ln E e^{iX} - \ln E e^{-iY}.
\]

(5)

The codifference \( \tau(n) \) of the FARIMA process was studied in Kokoszka & Taqqu (1995), where it was proved that for \( d > 1 - 2/\alpha \) FARIMA possesses long-term dependence in the above sense.

The Lévy-stable distributions are conveniently described by their characteristic function \( \phi(\theta) \)—the inverse Fourier transform of the probability density function (pdf). The most popular form of the characteristic function of a Lévy-stable random variable

\[
\Phi(\theta) = \Theta(\theta)^{d} X(n) = \Theta(\theta)^{d} X(n-1),
\]

where \( B \) is the shift operator \( BX(n) = X(n-1) \), and \( \Delta \) is the difference operator, i.e., \( \Delta X(n) = X(n) - X(n-1) \). The fractional difference operator \( \Delta^d = (1 - B)^d \) is defined by means of a binomial expansion, namely \( \Delta^d = (1 - B)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-B)^j = \sum_{j=0}^{\infty} \pi_j B^j \), where \( \pi_j = \frac{\Gamma((j+d)/\alpha)}{\Gamma(d/\alpha)} \), and \( \Gamma \) is the Gamma function. Here, \( \Phi \) and \( \Theta \) are the polynomials of degree \( p \) and \( q \), respectively, \( d < 1 - 1/\alpha \) takes fractional values, either positive or negative, and \( \epsilon_j \), representing the noise in the model—are independent and identically distributed (i.i.d.) random variables. The polynomials \( \Phi(\theta) = 1 - a_1 B - a_2 B^2 - \cdots - a_p B^p \) and \( \Theta(\theta) = 1 - b_1 B - b_2 B^2 - \cdots - b_q B^q \) have no roots in common and neither have roots in the closed unit disk. They correspond to autoregressive (AR) and moving average (MA) parts, respectively.

The linear solution of the above equation (known as the FARIMA process) takes the form of the following time series:

\[
X(n) = \sum_{j=0}^{\infty} c_j \epsilon_{n-j} ,
\]

(2)
is given by the expression

\[
\log \phi(\theta) = \begin{cases} 
-\alpha^n|\theta|^n [1 - i \beta \text{sign} (\theta) \tan (\pi \alpha / 2)] + i \mu \theta, & \alpha \neq 1, \\
-\alpha |\theta| [1 + 2i \beta \text{sign} (\theta) \log |\theta| / \pi] + i \mu \theta, & \alpha = 1,
\end{cases}
\]

where \(0 < \alpha \leq 2, -1 < \beta < 1, \sigma > 0, \) and \(\mu \in \mathbb{R}\) are parameters of this distribution (Samorodnitsky & Taqqu 1994). The Pareto distribution is represented by the power-law probability density \(f(x) = \frac{\alpha \lambda ^\alpha}{x^{\alpha+1}},\) where \(\lambda\) and \(\alpha\) are positive constants (Burnecki et al. 2005).

The power-law behavior of the tails implies that the variance is infinite if \(\alpha < 2\). The tail index \(\alpha\) controls the rate of decay of the tail of the distribution function \(F(x)\). Modeling with FARIMA time series with infinite variance allows one to take into account heavy tails. The FARIMA processes offer also a lot of flexibility in modeling long- and short-range dependences by choosing the memory parameter \(d\) and appropriate AR and MA coefficients in expression (2).

Finally, the linear predictor for the FARIMA process based on the finite past \(X_n, \ldots, X_0\) takes the form

\[
\hat{X}_{n+h} = \sum_{j=0}^{n} a_j X_{n-j},
\]

where the sequence \(\{a_0, a_1, \ldots, a_n\}\) is given by

\[
a_j = -\sum_{\ell=0}^{j-1} c_\ell h_{j-k},
\]

the \(c_j\)'s are defined by Equation (3) and \(h_j\)'s are given by

\[
\Phi_\theta(z)(1 - z)^d \Theta_\theta(z) = \sum_{j=0}^{\infty} h_j z^j, \quad |z| < 1,
\]

see Kokoszka (1995) for the discussion of the prediction problem in the infinite variance FARIMA case.

3. SOLAR FLARE DATA ANALYSIS

In analysis presented below, we restrict our attention to such time intervals in which the solar activity is strong, in particular, to the intervals: 1978–1981, 1988–1992, and 1999–2003. We use X-ray flare data from the GOES satellite that contain information about the time of appearance and energy of solar flares. The solar soft X-ray flares are classified according to peak X-ray fluxes. At low peak fluxes there is a departure from ideal case and the max-spectrum plot is almost perfectly linear with the number of blocks, but most real data sets deviate from the ideal case and the max-spectrum becomes linear only over a range of relatively large scales. On the other hand, the solar data are power-law distributed for large peak X-ray fluxes. At low peak fluxes there is a departure from power-law behavior, which is due to the problem of identifying small events against the time varying soft X-ray background. Using a range of relatively large scales in the max self-similarity approach gives a necessary background subtraction.

Using the max self-similarity estimator and considering the data in year intervals, we have analyzed how the solar cycle influences the tail index. A clear correlation between the tail index \(\alpha\) and solar activities is shown in Figure 1. When the solar activity is around maxima, the tail index is larger than 1, whereas in minima it tends to fall down less than 1. The index value in the period of solar maximum almost coincides with the result of Weron et al. (2005). Their data insert X-ray solar flares of C, M, and X classes, and the value \(\alpha\) was estimated to be 1.2674. The present analysis extends the tail index analysis on some cycles and speaks surely that the index tendency observed earlier is kept at least during the recent three solar cycles. McCulloch's method (McCulloch 1986) gives similar results. It confirms that the tail index \(\alpha\) depends on the solar activity value during the three solar cycles. For the last solar-maximum period, when

\[\text{ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_FLARES/}
\]

\[\text{XRAY_FLARES}.\]

et al. 1998; Park & Willinger 2000). However, the availability of huge amount of various data poses a set of new challenges for the problem of estimating the tail index. The point is that the data can be contaminated by extraneous oscillations, different noises with finite variance, etc. This makes the analysis of heavy-tailed data more complicated (Janicki & Weron 1994; Weron 2001; Lynch et al. 2005). The time series of soft X-ray solar emission relates to such a problematic data (Baiesti et al. 2006). Therefore, for reliability we have estimated the tail index by different statistical procedures. One of them is based on the asymptotic max self-similarity properties of heavy-tailed maxima (Stoev & Michailidis 2006). In this test the maximum values of data are calculated over blocks of size \(m\), scaled at rate of \(m^{1/\alpha}\). By examining a sequence of growing block sizes \(m = 2^j, 1 < j < \log_2 N, j \in \mathbb{N}\), and subsequently estimating the mean of logarithms of block-maxima one obtains an estimation of the tail index \(\alpha\). The slope of the plot for large block-sizes yields just an estimate of \(1/\alpha\). In the ideal case, the max-spectrum plot is almost perfectly linear with the number of blocks, but most real data sets deviate from the ideal case and the max-spectrum becomes linear only over a range of relatively large scales. On the other hand, the solar data are power-law distributed for large peak X-ray fluxes. At low peak fluxes there is a departure from power-law behavior, which is due to the problem of identifying small events against the time varying soft X-ray background. Using a range of relatively large scales in the max self-similarity approach gives a necessary background subtraction.

Using the max self-similarity estimator and considering our data in year intervals, we have analyzed how the solar cycle influences the tail index. A clear correlation between the tail index \(\alpha\) and solar activities is shown in Figure 1. When the solar activity is around maxima, the tail index is larger than 1, whereas in minima it tends to fall down less than 1. The index value in the period of solar maximum almost coincides with the result of Weron et al. (2005). Their data insert X-ray solar flares of C, M, and X classes, and the value \(\alpha\) was estimated to be 1.2674. The present analysis extends the tail index analysis on some cycles and speaks surely that the index tendency observed earlier is kept at least during the recent three solar cycles. McCulloch's method (McCulloch 1986) gives similar results. It confirms that the tail index \(\alpha\) depends on the solar activity value during the three solar cycles. For the last solar-maximum period, when
energy was transmitted by X-rays emitted during blasts on a solar surface from 2000 January 1 to 2002 December 31 (see the time series presented in Figure 2) the McCulloch’s method yields $\alpha = 1.213$.

The second aim of this section is to find the memory parameter $d$ which in the FARIMA model is related to the tail index $\alpha$ and self-similarity exponent $H$ by the relationship: $d = H - 1/\alpha$. The relationship implies that assessing $\alpha$ and $H$ is equivalent to estimating $d$. This also allows to double-check the results via different methods of estimation of $\alpha$, $d$, and $H$ parameters.

The first estimation procedure of the self-similarity exponent $H$ is the so-called finite impulse response transformation (FIRT). The FIRT estimator involves an array of coefficients. The array is made out of finite impulse response coefficients. The estimator $H_{\text{FIRT}}$ is obtained by performing a log-linear regression on the coefficients and measuring the slope (Stoev et al. 2002). It is important to note that the estimator $H_{\text{FIRT}}$ is unbiased for all $\alpha$ falling in the range $(0, 2)$. For the last solar-maximum period (see Figure 2), we have found that $H_{\text{FIRT}} = 1.1424$.

An alternative method of testing scaling and correlation properties of a time series is the variance of residuals method (McCulloch 1986). First, the series is divided into blocks. Then, within each block, the partial sums are calculated. A least-squares line is fitted to the partial sums and the values of the statistics over the blocks are calculated and a least squares line is fitted to the mean for different lengths of the blocks. The slope should be equal to $d + 1/2$. We have found that $d_{\text{RS}} = 0.2408$.

Figure 2. Energy–time series of solar flares from 2000 January 1 to 2002 December 31. Peaks and high variability of the data suggest that underlying distribution is heavy-tailed. Moreover, the data do not resemble white (pure random) noise. The clustering in the data is visible. This indicates the presence of self-similarity.

(A color version of this figure is available in the online journal.)

Our analysis of the data has shown that the tail of the underlying distribution of the energy of solar flares conforms to the power law. Hence, we suggest to describe the data by a FARIMA model with Pareto variables since the analyzed data are numerically positive. As the power-law distributions belong to the domain of attraction of stable law (see, e.g., Janicki & Weron 1994), the resulting distribution of the FARIMA process should be close to the stable one. We applied the McCulloch quantile fit to obtain the parameters of the distribution (McCulloch 1986). The value of $\alpha$ was estimated to be 1.213. One may check that the estimated value of $\alpha$ for simulated FARIMA times series with Pareto variables is usually underestimated. Therefore, we have assumed that the variables in our model follow the Pareto law with $\alpha = 1.25$.

According to Weron et al. (2005), using an arbitrary estimator of $H$, in order to recover both the self-similarity exponent $H$ and the tail index $\alpha$ (hence, the memory parameter $d$) we can apply a procedure which is based on the concept of surrogate data. In our case the surrogate data have been obtained by random shuffling of the original data positions. We have studied the surrogate data of the empirical time series recorded from the system describing the energy of solar flares (Figure 2) taking into account the following

1. If the process is FARIMA with finite variance (e.g., Gaussian) noise, then the values of the estimator should change to $1/2$ for the surrogate data independently on the initial values.

2. If the process is FARIMA with $\alpha$-stable or Pareto noise for $\alpha < 2$, then the values of the estimator should change to $1/\alpha$ for the surrogate data independently on the initial values.

The values of the parameters are listed in Table 1. From the results for the surrogate data, the corresponding estimates for the parameter $1/\alpha$ are: $1/\alpha_{\text{FIRT}} = H_{\text{FIRT}} = 0.8452$ and $1/\alpha_{\text{VR}} = H_{\text{VR}} = 0.7722$. We observe that the estimators are close to the one assumed in our model: $1/\alpha_{\text{MC}} = 0.8$. Moreover, we choose $d = 0.19$ as the highest admissible value of $d < 1/\alpha$ for the FARIMA model which is close to the one obtained via the R/S method for the original data.

One may note that the estimators of $H$ obtained via FIRT and VR methods (see Table 1) are greater than theoretically admissible in the FARIMA model, i.e., they exceed 1. As stated in Burnecki et al. (2008), this can be justified by performing

| Data Set            | $H_{\text{FIRT}}$ | $H_{\text{VR}}$ | $d_{\text{RS}}$ |
|---------------------|-------------------|-----------------|-----------------|
| Original Time Series|                   |                 |                 |
| Solar Flares        | 1.1424            | 1.0665          | 0.2408          |
| Surrogate Data      | 0.8452            | 0.7722          | 0.0507          |

4. CALIBRATION OF FARIMA MODEL

The obtained values of the parameters are listed in Table 1. From the results for the surrogate data, the corresponding estimates for the parameter $1/\alpha$ are: $1/\alpha_{\text{FIRT}} = H_{\text{FIRT}} = 0.8452$ and $1/\alpha_{\text{VR}} = H_{\text{VR}} = 0.7722$. We observe that the estimators are close to the one assumed in our model: $1/\alpha_{\text{MC}} = 0.8$. Moreover, we choose $d = 0.19$ as the highest admissible value of $d < 1/\alpha$ for the FARIMA model which is close to the one obtained via the R/S method for the original data.

One may note that the estimators of $H$ obtained via FIRT and VR methods (see Table 1) are greater than theoretically admissible in the FARIMA model, i.e., they exceed 1. As stated in Burnecki et al. (2008), this can be justified by performing
Figure 3. Combined MSE of the calculated FIRT, VR, and R/S estimators for the simulated FARIMA \((2, 0.19, 0)\) time series with respect to the ones calculated for the solar flare data for different linear and quadratic coefficients of the AR(2) part. The index \(\alpha\) is equal to 1.25. One may see that the error decreases slightly as \(a_1\) and \(a_2\) grow, it reaches a minimum around the values: \(a_1 = 0.02\) and \(a_2 = 0.03\), and then quickly increases when \(a_1\) or \(a_2\) get large.

Figure 4. Solar flare data (circles) and 1 day ahead prediction in the FARIMA \((2, 0.19, 0)\) model (asterisks). The prediction applies the linear predictor based on the previous observations. The error of the prediction is not low but we have to take into account that there is no strong deterministic component in the data, only stochastic one with strongly persistent noise.

Finally, we have calculated the 1 day ahead prediction for the FARIMA \((2, 0.19, 0)\) time series applying formula (7) and setting \(h = 1\). The results are depicted in Figure 4.

5. CONCLUDING REMARKS

In this paper, we have suggested the FARIMA \((2, d, 0)\) model with Pareto noise for predicting solar flare appearance in the period of solar maxima. It should be noted that the FARIMA model is a discrete-time analog of the fractional Langevin equation describing a motion of charged particles of strongly nonequilibrium plasmas into magnetic and electric fields. The motion may be characterized by three parameters: self-similarity exponent \(H\), tail index \(a\), and memory parameter \(d\). They obey the law \(d = H - 1/\alpha\). This law holds when the solar activity is strong and the solar plasma is very perturbed in active regions around sunspots, otherwise, the model is unacceptable.

The procedure is applied in Section 3 for the solar X-ray flares from 2000 January 1 to 2002 December 31. Comparing the values of the different estimators of the self-similarity exponent \(H\) for the original data series and for the surrogate data, we have found the values of the memory parameter \((d = 0.19)\) and the tail index \((\alpha = 1.25)\), see Table 1. These results allow, in principle, to build a proper physical model for analyzing the solar activity.

The analysis of soft X-ray emission observations shows that this series is enough complicated in nature. It contains both long-term dependence and heavy-tailed effects. The first creates a random number of strong flares on a background, and the second implies their persistence between each other. The most convenient model for their joint description is the FARIMA time series. The model permits one to predict a time series of soft X-ray solar flares, when the solar activity will be again near its maximum in 2010–2014 years. This approach provides additional information in comparison with other attempts like NOAA or Wheatland’s. None of them is preferable up to now because each of them is restricted by the model itself.
A.A.S. is grateful to the Institute of Physics and the Hugo Steinhaus Center for pleasant hospitality during his visit to Wrocław University of Technology. The $\textit{GOES}$ X-ray light curve was made available courtesy of the NOAA Space Environment Center, Boulder, CO.

REFERENCES

Adler, R., Feldman, R., & Taqqu, M. S., ed. 1998, A Practical Guide to Heavy Tails: Statistical Techniques and Application (Boston, MA: Birkhauser)

Baiesi, M., Paczuski, M., & Stella, A. L. 2006, \textit{Phys. Rev. Lett.}, 96, 051103

Beran, J. 1994, Statistics for Long-Memory Processes (New York: Chapman & Hall)

Bertacca, M., Berizzi, F., & Mese, E. D. 2005, \textit{IEEE Trans. Geosci. Remote Sens.}, 43, 2484

Boffetta, G., Carbone, V., Giuliani, P., Veltri, P., & Vulpiiani, A. 1999, \textit{Phys. Rev. Lett.}, 83, 4662

Brandimarte, L., Montanari, A., Briaud, J.-L., & D’Odorico, P. 2006, \textit{J. Hydr. Eng.}, 132, 493

Burnecki, K., Klafter, J., Magdziarz, M., & Weron, A. 2008, \textit{Physica A}, 387, 1077

Burnecki, K., Misiolek, A., & Weron, R. 2005, in Statistical Tools for Finance and Insurance, ed. P. Čižek, W. Härdle, & R. Weron (Berlin: Springer), 289

Cadavid, A. C., Lawrence, J. K., & Ruzmaikin, A. A. 1999, \textit{ApJ}, 521, 844

Chechkin, A. V., Gonchar, V. Yu., & Szydlowsky, M. 2002, \textit{Phys. Plasmas}, 9, 78

Choudhuri, A. R., Chatterjee, P., & Jiang, J. 2007, \textit{Phys. Rev. Lett.}, 98, 131103

Clark, S. 2006, \textit{Nature}, 441, 402

Dikpati, M., & Gilman, P. A. 2006, \textit{ApJ}, 649, 498

Gallagher, P. T., Moon, Y.-J., & Wang, H. 2002, \textit{Solar Phys.}, 209, 171

Granger, C. W. J., & Joyeux, R. 1980, \textit{J. Time Ser. Anal.}, 1, 15

Hurst, H. E. 1951, \textit{Trans. Am. Soc. Civ. Eng.}, 116, 770

Janicki, A., & Weron, A. 1994, A Simulation and Chaotic Behavior of $\alpha$-Stable Stochastic Processes (New York: Dekker)

Kokoszka, P. S. 1995, \textit{Probab. Math. Stat.}, 16, 83

Kokoszka, P. S., & Taqqu, M. S. 1995, \textit{Stoch. Proc. Appl.}, 60, 19

Leighton, R. B. 1964, \textit{ApJ}, 140, 1547

Lepreti, F., Carbone, C., & Veltri, P. 2001, \textit{ApJ}, 555, L133

Li, K.-J., Schmieder, B., & Li, Q.-Sh. 1998, \textit{ApJS}, 131, 99

Lynch, V. E., Carreras, B. A., Sanchez, R., LaBombard, B., van Milligen, B. Ph., & Newman, D. E. 2005, \textit{Phys. Plasmas}, 12, 052304

Magdziarz, M., & Weron, A. 2007, \textit{ Stud. Math.}, 181, 47

Mandelbrot, B. B. 1960, \textit{Int. Econ. Rev.}, 1, 79

Mandelbrot, B. B., & Wallis, J. R. 1969, \textit{Water Resour. Res.}, 5, 228

Mann, M. E., Bradley, R. S., & Hughes, M. K. 1998, \textit{Nature}, 392, 779

McCulloch, J. H. 1986, \textit{Comm. Statist. Simulation Comput.}, 15, 1109

McIntosh, P. S. 1990, \textit{Solar Phys.}, 125, 251

Montanari, A., Rosso, R., & Taqqu, M. S. 1997, \textit{Water Resources Res.}, 33, 1055

Moon, Y.-J., Choe, G. S., Yun, H. S., & Park, Y. D. 2001, \textit{J. Geophys. Res.}, 106, A12, 29951

Nandy, D., & Choudhuri, A. R. 2002, \textit{Science}, 296, 1671

Peng, C.-K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., & Goldberger, A. L. 1994, \textit{Phys. Rev. E}, 49, 1685

Samorodnitsky, G., & Taqqu, M. S. 1994, \textit{Stable Non-Gaussian Random Processes} (New York: Chapman & Hall)

Scafetta, N., & West, B. J. 2003, \textit{Phys. Rev. Lett.}, 90, 248701

Stanislavsky, A. A., & Weron, K. 2007, \textit{Astrophys. Space Sci.}, 312, 343

Stoev, S., & Michailidis, G. 2006, Technical Report 447 Department of Statistics, Univ. Michigan, \url{http://www.stat.lsa.umich.edu/~sstoev/max-spectrum-dep.pdf}

Stoev, S., Pipiras, V., & Taqqu, M. S. 2002, \textit{Signal Process.}, 82, 1873

Stoev, S., & Taqqu, M. Q. 2004, \textit{Fractals}, 12, 95

Svensmark, H., & Friis-Christensen, E. 1997, \textit{Solar-Terr. Phys.}, 59, 1225

Taqqu, M. S., & Teverovsky, V. 1998, in A Practical Guide to Heavy Tails: Statistical Techniques and Applications, ed. R. Adler, R. Feldman, & M. S. Taqqu (Boston, MA: Birkhuser), 177

Veronig, A., Temmer, M., Hanslmeier, A., Otruba, W., & Messerotti, M. 2002, \textit{A&A}, 382, 1078

Weron, R. 2001, \textit{Int. J. Mod. Phys. C}, 12, 209

Weron, R. 2002, \textit{Physica A}, 312, 285

Weron, A., Burnecki, K., Mercik, Sz., & Weron, K. 2005, \textit{Phys. Rev. E}, 71, 016113

Wheatland, M. S. 2000, \textit{ApJ}, 536, L109

Wheatland, M. S. 2001, \textit{Sol. Phys.}, 203, 87

Wheatland, M. S. 2002, \textit{Sol. Phys.}, 208, 33

Wheatland, M. S. 2004, \textit{ApJ}, 609, 1134

Wheatland, M. S. 2005, \textit{Space Weather}, 3, S07003

Wheatland, M. S. 2008, \textit{ApJ}, 679, 1621

Zipf, G. 1932, Selective Studies and Principle of Relative Frequency in Language (Cambridge, MA: Harvard Univ. Press)