Off–shell pion electromagnetic form factor from a
gauge–invariant Nambu–Jona-Lasinio model

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Abstract

The off–shell electromagnetic vertex function of pions and kaons is studied in a
bosonized Nambu–Jona-Lasinio model with a gauge–invariant proper–time cutoff.
The slope of the pion form factor with respect to the pion 4–momentum is equal
to the on–shell pion charge radius in the chiral limit. The off–shell slope of the $K^0$
form factor is zero, that of the $K^\pm$ about 15% smaller than that of the pion. We
compare with results of a recent calculation in chiral perturbation theory.

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Electromagnetic interactions of pions play an important role in low-energy hadronic and nuclear physics. The on-shell electromagnetic form factor of the charged pion has been measured rather accurately in $\pi e^–$-scattering and $e^+e^–$-annihilation [1]. This function has also been calculated in a variety of dynamical models of the pion, such as the vector dominance model [2], chiral perturbation theory [3], or models of the Nambu–Jona–Lasinio–type, which describe the dynamical breaking of chiral symmetry at quark level [4]. However, in more complicated processes the electromagnetically interacting pion is in general not on its mass shell. For example, pion electroproduction involves the half-off-shell pion vertex function [5], while in nuclear electromagnetic processes with pion exchange currents generally both legs of the pion vertex function are off-shell [6]. The analysis of such processes requires knowledge of the electromagnetic vertex function for arbitrary pion 4-momenta.

Recently, Rudy et al. have calculated the off-shell electromagnetic form factor of the pion in chiral perturbation theory [7]. For off-shell pion momenta the vertex function receives a contribution from a tree-level term of order $O(p^4)$, which vanishes on-shell by virtue of the equation of motion of the pion field. This term brings in an unknown parameter, unrelated to the on-shell pion charge radius, $L_9$, which leaves the off-shell behavior of the vertex function undetermined at this level. In contrast, a picture in which the pion is considered as a bound state in a chirally invariant quark model assigns a definite off-shell behavior to the pion vertex function.

In this letter, we study the off-shell pseudoscalar meson electromagnetic vertex function in a bosonized Nambu–Jona–Lasinio (NJL) model [8, 9], defined with a gauge invariant proper-time cutoff [10]. This finite, chirally and gauge-invariant model action provides an ideal framework for the study of pion electromagnetic properties. We consider the full momentum dependence of the meson propagator and vertex function and do not restrict ourselves to a gradient expansion of the quark loop. Our method is an extension of the approach recently employed to study diquark electromagnetic form factors inside baryons [11]. A general expression for the vertex function is obtained, which is then analyzed by expanding around the on-shell point and studying the slopes of the form factors as functions of the meson mass. The general structure of our results is in agreement with the one found in chiral perturbation theory [7]. Moreover, we obtain a specific prediction for the off-shell behavior of the pion form factor. In particular, in the chiral limit, the off-shell slope of the form factor is found to be equal to the on-shell pion charge radius. Finally, we discuss the role of $SU(3)$-symmetry breaking in the slopes of the $K^±$- and $K^0$-form factors.

It is well-known in field theory that the off-shell behavior of correlation functions depends on the choice of interpolating field. Only matrix elements between asymptotic states are protected by the equivalence theorem [12]. Thus, individual results for off-shell correlation functions should be regarded as building blocks in describing more complicated processes like e.g. pion Compton scattering or electroproduction. Nevertheless, it is instructive to compare the predictions for the off-shell behavior of the pion vertex function in different but related approaches. Furthermore, considering the difficulty of a unified field-theoretic description of nuclear processes like electroproduction, a realistic approach is likely to be patchwork of different phenomenological descriptions.
We start from the action of the bosonized NJL model in the form [3, 13]

\[ S = -i \text{Tr}_A \log G^{-1} - \frac{N_C}{4g} \int d^4x \phi_\alpha \phi^\alpha. \]  

(1)

Here, \( g \) is the NJL coupling constant, \( G^{-1} = i\partial - m_0 - \phi_\alpha \Lambda^\alpha \) the inverse quark propagator, with \( m_0 = \text{diag}(m_u, m_d, m_s) \) the current quark mass matrix, and \( \phi_\alpha \) the meson field. The vertices, \( \Lambda^\alpha \), are matrices in color, flavor and Dirac spinor space, \( \Lambda^\alpha = 1_C \left( \frac{1}{2} \lambda^\alpha \right)_F O^a \), \( \alpha = (a, a) \), where \( O^a = \{ 1, i\gamma_5, i\gamma^\mu/\sqrt{2}, i\gamma^\mu\gamma_5/\sqrt{2} \} \), corresponding to scalar, pseudoscalar, vector and axial vector mesons. The vacuum value of the scalar meson field defines the constituent quark mass, \( M = \text{diag}(M_u, M_d, M_s) \) [3]. To eq. (1) we couple an electromagnetic field by way of minimal substitution, \( i\partial \to i\partial - cQ A_\mu \gamma^\mu \), where \( Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) \) is the quark charge matrix, and expand the effective action in the fluctuating pseudoscalar meson and the electromagnetic field,

\[ S = S_0 + \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \phi_\alpha(-p) (D^{-1})^{\alpha\beta}(p) \phi_\beta(p) \]

\[ + \int \frac{d^4p_t}{(2\pi)^4} \int \frac{d^4p_i}{(2\pi)^4} \phi_\alpha(-p_i) \phi_\beta(p_i) A_\mu(p_i - p_t) F^{\alpha\beta\mu}(p_i, p_t) + \ldots \]

(2)

Up to field renormalization, \( D \) is the meson propagator and \( F \) the (irreducible) electromagnetic vertex function. For pseudoscalar mesons (\( O^a = i\gamma_5 \)) they are of the form

\[(D^{-1})^{\alpha\beta}(p) = N_C \sum_{ij} \left( \frac{\lambda^\alpha}{2} \right)_{ij} \left( \frac{\lambda^\beta}{2} \right)_{ji} \left( -\frac{1}{2g} + I_{ij}(p^2) \right), \]

(3)

\[F^{\alpha\beta\mu}(p_i, p_t) = eN_C \sum_{ij} \left( \frac{\lambda^\alpha}{2} \right)_{ij} \left( \frac{\lambda^\beta}{2} \right)_{ji} \left( F_{ij}(p_i, p_t, q^2)(p_i + p_t)^\mu + G_{ij}(p_i, p_t, q^2)q^\mu \right), \]

(4)

where \( q = p_t - p_i \). In the absence of flavor mixing, the invariant functions \( I_{ij}, F_{ij}, G_{ij} \) directly correspond to a physical meson channel if the meson is labeled by its quark flavor content, i.e., \( F_{ud} \) is the \( \pi^+ \)–vertex function, \( F_{us} \) the \( K^+ \)–vertex function, etc..

We define the quark loop in eq. (1) using a gauge–invariant proper–time regularization,

\[ \text{Re Tr}_A \log G_E^{-1} = \frac{1}{2} \int_{\Lambda^{-2}}^{\infty} \frac{ds}{s} \text{Tr} \log (-s G_E^{-1} G_E^{-1}). \]

(5)

Here, \( G_E^{-1} \) is the quark propagator after continuation to euclidean space [3, 10]. The functions \( I_{ij}, F_{ij}, G_{ij} \) are obtained by expanding the proper–time integral, eq. (3), in the fluctuating meson and electromagnetic field [10, 11]. The resulting expressions are then continued back to Minkowski space. In this way one finds for the meson propagator

\[ I_{ij}(p^2) = (p^2 - (M_i - M_j)^2)A_{ij}(p^2) + M_i^2 B_i + M_j^2 B_j \]

(6)

\[ A_{ij}(p^2) = \frac{1}{16\pi^2} \int_0^1 d\alpha \Gamma \left( 0, \frac{\alpha M_i^2 + (1-\alpha)M_j^2 - \alpha(1-\alpha)p^2}{\Lambda^2} \right), \]

(7)

\[ B_i = \frac{1}{16\pi^2} \Gamma \left( -1, \frac{M_i^2}{\Lambda^2} \right). \]
For the vertex function, eq. (4), we obtain the general result

\[ F_{ij}(p_1^2, p_1^2, q^2) = (Q_i - Q_j) \frac{1}{2} \left[ A_{ij}(p_1^2) + A_{ji}(p_1^2) + p_1^2 C_{1,ij}^+ + q^2 C_{2,ij}^+ \right] \quad (8) \]

\[ + (Q_i + Q_j) \frac{1}{2} \left[ p_1^2 C_{1,ij}^- + q^2 C_{2,ij}^- \right], \]

\[ G_{ij}(p_1^2, p_1^2, q^2) = (Q_i - Q_j) \frac{1}{2} \left[ -A_{ij}(p_1^2) + A_{ji}(p_1^2) + p_1^2 C_{3,ij}^+ + p_1^2 C_{2,ij}^+ \right] \quad (9) \]

\[ + (Q_i + Q_j) \frac{1}{2} \left[ p_1^2 C_{3,ij}^- + p_1^2 C_{2,ij}^- \right], \]

\[ p_1^2 = p_1^2 + p_2^2 - q^2 - 2(M_i - M_j)^2, \quad p_2^2 = p_1^2 - p_2^2, \]

\[ C_{k,ij}^\pm \equiv C_{k,ij}^\pm(p_1^2, p_1^2, q^2) = \frac{1}{2} \left( C_{k,ij}(p_1^2, p_1^2, q^2) \pm C_{k,ij}(p_1^2, p_1^2, q^2) \right), \quad (10) \]

\[ C_{k,ij}(p_1^2, p_1^2, q^2) = \frac{1}{16\pi^2} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta X_k \frac{\exp(-Y^2/\Lambda^2)}{Y^2}, \]

\[ Y^2 = \alpha M_1^2 + (1 - \alpha) M_2^2 - \alpha(1 - \alpha - \beta)p_1^2 - \alpha \beta p_1^2 - \beta(1 - \alpha - \beta)q^2, \]

\[ X_1 = \alpha, \quad X_2 = 1, \quad X_3 = 1 - \alpha - 2\beta. \]

In arriving at eqs. (8, 9) we have made use of the symmetry properties \( C_{k,ij}(p_1^2, p_1^2, q^2) = \varepsilon_k C_{k,ij}(p_1^2, p_1^2, q^2), \varepsilon_k = \{+, +, -\}. \) As a consequence, \( F_{ij}(p_1^2, p_1^2, q^2) = F_{ij}(p_1^2, p_1^2, q^2) \) and \( G_{ij}(p_1^2, p_1^2, q^2) = -G_{ij}(p_1^2, p_1^2, q^2), \) as required by time reversal invariance. Note that, for a charged meson \((\pi^\pm, K^\pm), Q_i - Q_j = \pm 1, \) while for a neutral meson \((\pi^0, K^0, K^0), Q_i - Q_j = 0, \) \( Q_i + Q_j \neq 0. \) In the limit of flavor symmetry, \( M_i = M_j, \) the part of the vertex function proportional to \( Q_i + Q_j \) vanishes identically. We consider here the isospin limit, \( M_u = M_d \neq M_s, \) Thus, the \( \pi^0 \) vertex function is identically zero, but \( K^0, K^0 \) have vertex functions proportional to \( \pm (Q_d + Q_s). \)

We note that, as a result of the gauge–invariant definition of the effective action, eq. (3), the meson vertex function and propagator satisfy the Ward identity

\[ q_u \left( F_{ij}(p_1^2, p_1^2, q^2)(p_1^2 + p_1^2) + G_{ij}(p_1^2, p_1^2, q^2)q_4 \right) = (Q_i - Q_j) \left( I_{ij}(p_1^2) - I_{ij}(p_1^2) \right). \quad (11) \]

On the mass shell, \( p_1^2 = p_1^2 = m^2, \) one has \( C_{3,ij}(m^2, m^2, q^2) \equiv 0, \) and the longitudinal part of the vertex function vanishes, \( G_{ij}(m^2, m^2, q^2) \equiv 0. \) The mass of the meson bound states of flavor \( ij \) is determined as the zero of the inverse propagator, \( I_{ij}(m^2) - \frac{1}{2}g^{-1} = 0. \) We normalize the meson field to unit residue of the propagator at \( p^2 = m^2. \) For a given flavor channel this means dividing \( F_{ij}(p_1^2, p_1^2, q^2) \) and \( G_{ij}(p_1^2, p_1^2, q^2) \) of eqs. (3, 3) by

\[ Z_{ij}(m^2) = \frac{\partial}{\partial p^2} I_{ij}(p^2)|_{p^2=m^2}. \quad (12) \]

The field renormalization factor, \( Z_{ij}(m^2), \) depends on the meson mass but is otherwise independent of the field momentum. We discuss in the following the off–shell behavior of the vertex function subject to this definition of the meson field. This redefinition corresponds to a (momentum–dependent) finite multiplicative renormalization of the meson source in the original generating functional of the NJL model1. The normalized on–shell meson charge form factor is then given by \( F_{ij}(m^2, m^2, q^2)/Z_{ij}(m^2). \)

1For the bosonization of the NJL model including meson source terms, see [4].
In order to exhibit the off–shell behavior of the vertex function, and to make contact with the results of chiral perturbation theory, we expand eqs.(8, 9) simultaneously in the momentum transfer, $q^2$, and the off–shellness, $p_i^2 - m^2$ and $p_f^2 - m^2$. In practice, this is an excellent approximation, as the full expressions eqs.(8, 9) are almost linear in these variables for momenta up to $\sim 0.25\text{GeV}^2$. The expansion is of the form

$$Z_{ij}^{-1}(m^2)F_{ij}(p_i^2, p_f^2, q^2) = (Q_i - Q_j) \left[ 1 + r_{ij}^+(m^2)q^2 + s_{ij}^+(m^2)(p_i^2 + p_f^2 - 2m^2) + \ldots \right]$$

$$+ (Q_i + Q_j) \left[ r_{ij}^-(m^2)q^2 + s_{ij}^-(m^2)(p_i^2 + p_f^2 - 2m^2) + \ldots \right], \quad \text{eq.(13)}$$

$$Z_{ij}^{-1}(m^2)G_{ij}(p_i^2, p_f^2, q^2) = (Q_i - Q_j) \left[ t_{ij}^+(m^2)(p_i^2 - p_f^2) + \ldots \right]$$

$$+ (Q_i + Q_j) \left[ t_{ij}^-(m^2)(p_i^2 - p_f^2) + \ldots \right]. \quad \text{eq.(14)}$$

The slopes, $r_{ij}^\pm(m^2)$, $s_{ij}^\pm(m^2)$ and $t_{ij}^\pm(m^2)$, which are functions of the meson mass, are easily obtained as derivatives of eqs.(8, 9). Here, $r_{ij}^\pm(m^2)$ are related to the (on-shell) meson r.m.s. charge radius, while $s_{ij}^\pm(m^2)$ are the off–shell slopes of the form factor. From the Ward identity, eq.(11), it follows that $t_{ij}^\pm(m^2) \equiv r_{ij}^\pm(m^2)$. Furthermore, using the symmetry properties of the Feynman parameter integrals, eq.(10), one finds $s_{ij}^-(m^2) \equiv 0$ for any flavors $i, j$ and arbitrary $m^2$.

It is worthwhile to investigate the slopes of the meson form factor in dependence of the meson mass, i.e., on the symmetry–breaking current quark masses. For the charged pion in the isospin limit, $M_u = M_d$, the slopes of the meson form factor are $r_{\pi^+}(m_\pi^2) = r_{ud}^+(m_\pi^2)$ and $s_{\pi^+}(m_\pi^2) = s_{ud}^+(m_\pi^2)$. In particular, in the chiral limit, $m_\pi^2 = 0$, the Feynman parameter integrals determining $r_{\pi^+}(0)$ and $s_{\pi^+}(0)$ can be performed trivially, and one obtains

$$r_{\pi^+}(0) = s_{\pi^+}(0) = \frac{Z_{ud}^{-1}(0)}{96\pi^2 M_u^2} \exp(-M_u^2/\Lambda^2) = \frac{N_C}{24\pi^2 f_\pi^2} \exp(-M_u^2/\Lambda^2). \quad \text{eq.(15)}$$

Thus, in the chiral limit the off–shell slope of the pion form factor is equal to the $q^2$–slope, i.e., the charge radius. (In the last equation we have made use of the fact that, in the chiral limit, $Z_{ud}(0) = A_{ud}(0) = \frac{1}{4}N_C^{-1}M^2 \pi^2 f_\pi^2$.)

In chiral perturbation theory, the off–shell slope of the form factor, $s_{\pi^+}(m_\pi^2)$, introduces a new phenomenological parameter unrelated to the meson charge radius at tree level, $L_\theta$ [7]. In the notation of ref. [7], $s_{\pi^+}(0) = 16\beta_1 f_\pi^2$, where $\beta_1$ is the parameter multiplying an $O(p^4)$–tree level term, which vanishes if the equation of motion for the pion field holds. From eq.(15) we find $\beta_1 = (N_C/384\pi^2) \exp(-M_u^2/\Lambda^2)$. In the limit $\Lambda \to \infty$, keeping $f_\pi$ fixed, this agrees with the value quoted by Rudy et al. [7], which they infer by rewriting the lagrangian obtained from a gradient expansion of the quark determinant [7] in a form which exhibits the off–shell term determining $s_{\pi^+}(0)$. In our treatment this coefficient is obtained directly, with no need to “undo” the equation of motion for the pion field. Moreover, the direct relation between the off–shell slope and the charge radius in the chiral limit, eq.(15), is lost in the other approach. Crudely speaking, eq.(15) means that a massless pion and a photon with $q^2 = 0$ “see” the quark loop in the same way, which seems intuitively plausible.

The coefficients $r_{\pi^+}(m_\pi^2)$ and $s_{\pi^+}(m_\pi^2)$ are shown in fig.1 as functions of the pion mass, which is generated by chiral symmetry breaking of the form $m_u = m_d \neq 0$. The common
parameters are $M_u = M_d = m_\rho/\sqrt{6} = 315$ MeV, as suggested by the KSFR relation, and $\Lambda = 660$ MeV determined from fitting $f_\pi = 93$ MeV at the physical pion mass. (Here, when changing $m_u = m_d$, we keep $M_{u,d}$ and $\Lambda$ fixed and adjust the NJL coupling, $g$, according to the gap equation.) With these parameters the charge radius in the chiral limit is $\langle r^2 \rangle_{\pi^+} = (0.52 \text{ fm})^2$. If $\pi$–$A_1$–mixing is taken into account in fixing the cutoff, significantly larger values for $\Lambda$ are obtained, leading to larger values of the pion charge radius; in the limit $\Lambda \to \infty$ one finds $\langle r^2 \rangle_{\pi^+} = (0.58 \text{ fm})^2$, cf. [13]. Fig.1 shows that $r_{\pi^+}(m^2_\pi)$ and $s_{\pi^+}(m^2_\pi)$ vary only little from the chiral limit up to the physical pion mass, so that relation eq. (13) is well satisfied for physical pions.

Eq. (2) takes into account the direct coupling of the photon to the pion through the quark loop. This is not necessarily in contradiction to the vector dominance picture, as the mass of the $\bar{q}q$–intermediate state is approximately equal to $m_\rho$. Moreover, the quark core radius, eq. (12), is reasonably close to the experimental value, $\langle r^2 \rangle_{\pi^+} = (0.66 \text{ fm})^2$. We therefore have reason to believe that the quark loop also accounts for most of the off–shell behavior of the form factor. Our intention here is to compare with chiral perturbation theory, which does not include resonance contributions. The coupling of the photon through the vector mode of eq. (1) should, however, be included when studying the pion form factor in the timelike region.

For completeness we note that the off–shell behavior of the pion propagator is governed by the same parameter as that of the vertex function, i.e., one has $I_{ij}(p^2) = Z_{ij}(m^2)(p^2 - m^2)[1 + s_{ij}(m^2)(p^2 - m^2) + \ldots]$ near $p^2 = m^2$. This result is required in constructing the reducible 3–point Green’s function from the vertex function.

In the neutral kaon vertex function only the part proportional to $(Q_d + Q_s)$ contributes. Since $s_{ds}(m^2_K) \equiv 0$, the $K^0$ form factor does not show any off–shell dependence to order $(p_1^2 - p_2^2 - m^2_K)$, in agreement with the chiral perturbation theory calculation [7]. However, this result is completely independent of the quark masses and the kaon mass and thus not obviously related to chiral symmetry, as suggested in [7].

In evaluating the charged kaon form factors and the $q^2$–slope of the neutral kaon form factor, one has to take into account effects of $SU(3)$–symmetry breaking. We consider symmetry breaking of the form $M_s \neq M_u = M_d$, corresponding to $m_s \neq m_u = m_d = 0$. In doing so we have to keep in mind that $m^2_K$ depends on $m_s$. Fig.1 shows the $K^+$ charge radius and off–shell slope, $r_{K^+}(m^2_K) = (Q_u - Q_s)r_{us}^+(m^2_K) + (Q_u + Q_s)r_{us}^-(m^2_K)$, and $s_{K^+}(m^2_K) = (Q_u - Q_s)s_{us}^+(m^2_K) + (Q_u + Q_s)s_{us}^-(m^2_K)$, as well as the $K^0$ charge radius, $r_{K^0}(m^2_K) = (Q_d + Q_s)r_{ds}^-(m^2_K)$, as a function of $m^2_K$. (Here, $M_{u,d}$ and $\Lambda$ are as above, $g$ is kept fixed, while $M_s$ and $m_K$ vary in dependence on $m_s$.) At the physical kaon mass, the $K^+$ charge radius is roughly 10%, the off–shell slope 15% smaller than the value for the pion, while the $K^0$ charge radius is around 20% of the pion charge radius. These ratios are rather insensitive to the cutoff and the constituent quark mass. One may also evaluate $r_{K^+}(m^2_K)$, $s_{K^+}(m^2_K)$ and $r_{K^0}(m^2_K)$ by expanding to leading order in $M_s - M_u$. The slopes change of order $M_s - M_u \sim m^2_K$ when going away from the chiral limit. However, the first–order approximation to $SU(3)$–breaking does not provide a reliable quantitative estimate of the kaon radii, since the non–linear terms are large in this model, cf. fig.1. We note that the present approach takes into account $SU(3)$–symmetry breaking effects.\footnote{The contributions of the quark core and of composite pion loops to the pion charge radius have recently been estimated in the framework of an effective quark theory [10].}
to all orders. This is different from chiral perturbation theory, where symmetry breaking is governed by terms of order \( \log m_0 \) originating from pion and kaon loops. The relations 
\[
r_{K^+} = r_{\pi^+} + r_{K^0} \quad \text{and} \quad s_{K^+} = s_{\pi^+} \]
of [3] and [7] for physical pion and kaon masses only hold up to “ordinary” chiral symmetry breaking effects of order \( m_s \), cf. the comments in [4]. As fig.1 shows, they are satisfied at the 10% –level in the NJL model.

In summary, we have discussed the off–shell electromagnetic vertex function for pions and kaons in a gauge–invariant version of the bosonized NJL model. The off–shell slope of the form factor coincides with the charge radius in the chiral limit. The pion vertex function thus exhibits a high degree of symmetry; it is essentially characterized by one parameter — the pion charge radius. Modifications for finite pion masses are small. We note that the smooth nature of the chiral limit in this approach is essential in obtaining that result. It would be interesting to see if relation eq.(15) is realized also in other dynamical models of the pion, i.e., to what extent it depends on the particular choice of interpolating pion field. The general pattern of the off–shell behavior of the form factors agrees with the one obtained in chiral perturbation theory at the level to be expected.

The effective meson action derived from the NJL model, eq.(1), has been generalized to include baryon fields as composites of diquark and quark fields [13, 14]. The resulting meson–baryon theory allows in principle to calculate also the pion–nucleon part of the pion electroproduction amplitude, based on the same quark dynamics. In this framework the off–shell effects in the pion form factor, the pion–nucleon vertex and the pion propagator could be incorporated consistently, i.e., with the same interpolating pion field. Such an approach, which requires at least approximate knowledge of the baryon diquark–quark wave function, remains an interesting possibility.
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Figure caption

Fig.1: The $q^2$–slope and the off–shell slope of the electromagnetic form factor of pions and kaons, as a function of the meson mass. All quantities are expressed in units of the slope of the pion form factor in the chiral limit, $r_{\pi^+}(0)$, cf. eq. (13). Common parameters are $M_u = M_d = 315$ MeV and $\Lambda = 660$ MeV. Thin solid and dashed line: $r_{\pi^+}(m_{\pi}^2)/r_{\pi^+}(0)$ and $s_{\pi^+}(m_{\pi}^2)/r_{\pi^+}(0)$, with $m_{\pi}$ generated by isospin–symmetric chiral symmetry breaking of the form $m_u = m_d \neq m_s = 0$. Fat solid and dashed line: $r_{K^+}(m_{K}^2)/r_{\pi^+}(0)$ and $s_{K^+}(m_{K}^2)/r_{\pi^+}(0)$; fat dot–dashed line: $r_{K^0}(m_{K}^2)/r_{\pi^+}(0)+1$ (note the shift by +1; $r_{K^0}(0) = 0$). Here, $m_K$ and $M_s$ are driven by $SU(3)$–symmetry breaking of the form $m_s \neq m_u = m_d = 0$. Note that the pion and kaon masses in this plot are determined by different types of explicit chiral symmetry breaking. The physical pion and kaon masses are indicated by arrows.
This figure "fig1-1.png" is available in "png" format from:

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