Design of Feedback Matrix for Full-Order Flux Observer of Asynchronous Motor

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Abstract: The asynchronous motor full-order flux observer uses the motor itself as a reference model to observe the motor's flux and stator current by constructing a state equation. It takes the stator current output and introduces the error between the actual current and the observed current as feedback correction, and improves the performance of the full-order flux observer by adjusting the feedback gain of the correction item. Based on the in-depth analysis of the zero and pole images of the open-loop transfer function of the small signal linearized model, a feedback matrix design method is proposed to improve the smooth switching of the full-order flux observer and the dynamic performance of the full order flux observer speed identification. Simulation results verify the correctness of the analysis method and the feasibility of the proposed countermeasures.

1. Introduction

The research of sensor-less induction motor control technology has replaced the expensive, bulky and low reliability speed sensor, so it has been widely studied in the field of high-performance variable frequency speed regulation. At present, the rotor flux observation method of speed sensor-less vector control of induction motor is mainly based on model method. Compared with the voltage and current model and MRAS, the full-order flux observer has the advantages of high robustness to motor parameters and large stable speed estimation area, so it is widely used.

The design of error feedback matrix coefficients is a key step in full order flux observer. When the feedback matrix is zero, the convergence rate of the full order flux observer is not ideal, and the speed identification is stable in the electric condition, but unstable in the vulgar power generation condition. In reference [1], the feedback matrix is designed according to the pole multiple method. This method can make the observer converge faster, but there will be unstable region at low speed. According to the stability condition of the speed estimation system, the design method of the feedback matrix is obtained in reference [2]. This method makes the convergence and stability of the speed estimation of the observer better in a wide speed range, but the load carrying capacity is weak at low speed.

The current model flux observer is robust to stator resistance in low-speed region, while the voltage model flux observer is insensitive to parameters in high-speed region. Through the design of the feedback matrix, the full order flux observer can be equivalent to the voltage model flux observer and the current model flux observer respectively in the steady state. A composite model flux observer based on the two flux observers is proposed, and the smooth switching between the two models is adjusted by PI. This paper proposes a feedback matrix design method to improve the smooth switching of the full-order flux observer and the dynamic characteristics of the full-order flux observer's speed identification. The analysis and design methods are verified in simulation.
2. Mathematical model of asynchronous motor

According to the inverse-\( \Gamma \) steady-state equivalent circuit, the stator flux and rotor flux vector are selected as the state variables, and the state equations of asynchronous motor in the rotating coordinate system with arbitrary angular speed \( \omega_k \) can be established, as shown in equations (1) and (2).

\[
\dot{x} = \begin{bmatrix}
-\frac{1}{\tau_s'} - j\omega_k & \frac{1}{\tau_s'} \\
\frac{1-\sigma}{\tau_r'} & -\tau_r' - j(\omega_k - \omega_m)
\end{bmatrix} x + \begin{bmatrix}1 \end{bmatrix} u_s
\] (1)

\[
i_s = \begin{bmatrix} \frac{1}{L_s'} - \frac{1}{L_r'} \end{bmatrix} x
\] (2)

where: \( x = [\psi_s \ \psi_R] \), \( \sigma = \frac{L_s'}{L_M + L_s'} \), \( \tau_s' = \frac{L_s'}{R_s} \), \( \tau_r' = \frac{\sigma L_M}{R_R} \);

\( u_s \), \( i_s \), \( \psi_s \), \( \psi_R \) are the stator voltage vector, stator current vector, stator flux vector and rotor flux vector in turn; \( R_s \), \( R_R \), \( L_M \), \( L_s' \), \( \sigma \) are stator resistance, rotor resistance, mutual inductance, total leakage inductance and total leakage inductance coefficient in turn; \( \tau_s' \), \( \tau_r' \) are the instantaneous time constant of the stator and the instantaneous time constant of the rotor in turn. \( \omega_k \) is any angular velocity frequency; \( \omega_m \) is the rotor speed.

3. Design of full-order flux observer

The asynchronous motor full-order flux observer takes the motor itself as a reference model, observes the stator flux and rotor flux of the asynchronous motor according to the state estimation equation, and adds an error between the actual motor current and the observed current as a correction term in this equation. Improve the dynamic performance and steady-state performance of the observer through the matrix of correction terms, thereby improving the accuracy of estimation.

3.1 Stator flux and rotor flux as state variables

Refer to the asynchronous motor state equation, take the stator flux and the rotor flux as the state variables, and add the state feedback matrix to form a full-order observer model:

\[
\dot{\hat{x}} = \hat{A}\hat{x} + Bu_s + L(i_s - \hat{i}_s), \hat{i}_s = C\hat{x}
\] (3)

Where: \( \hat{x} = [\hat{\psi}_s, \hat{\psi}_R]^T \), \( \hat{A} = \begin{bmatrix}
-\frac{1}{\tau_s'} - j\omega_k & \frac{1}{\tau_s'} \\
\frac{1-\sigma}{\tau_r'} & -\tau_r' - j(\omega_k - \hat{\omega}_m)
\end{bmatrix} \), \( B = \begin{bmatrix}1 \end{bmatrix}, \hat{C} = \begin{bmatrix}1 & 0 \end{bmatrix} \)

\( L = [l_s \ l_r]^T \) is state feedback matrix, \( l_s = l_{sd} + jl_{sq}, \ l_r = l_{rd} + jl_{rq} \).

3.2 Stator current and rotor flux as state variables

When the stator current and rotor flux are selected as the state variables, the full-order observer model can also be expressed as:

\[
\dot{\hat{x}} = \hat{F}\hat{x} + \hat{K}u_s + G(i_s - \hat{i}_s), \hat{i}_s = H\hat{x}
\] (4)
Where: \[
\hat{F} = \begin{bmatrix}
\frac{1}{\tau_r} - j\omega_k & \frac{1}{L_s'} \left( \frac{1}{\tau_r} - j\hat{\omega}_m \right) \\
R_s & -\frac{1}{\tau_r} - j(\omega_k - \hat{\omega}_m)
\end{bmatrix}, \tau_r = \frac{L_s'}{R_s + R_s}, K = \begin{bmatrix}
1 \\
0
\end{bmatrix}, H = \begin{bmatrix}
1 & 0
\end{bmatrix};
\]

Refer to literature [3][4] to get the speed identification formula based on the full-order flux observer as shown in formula (5).

\[
\dot{\hat{\omega}}_m = -\left( k_{pos} \varepsilon + k_{io} \int \varepsilon dt \right)
\]

\[
e = \text{Im}\{e^*J R_R\}
\]

where: \(\varepsilon\), \(k_{pos}\), \(k_{io}\) are the speed adaptive signal, speed identification proportional coefficient, speed identification integral coefficient; \(J^*\) is the conjugate of \(J\), and \(\text{Im}\{\}\) is the imaginary part of the complex number.

4. Design of the feedback matrix of the full-order flux observer

In the synchronous coordinate system, the small-signal linearization model is used to obtain the steady-state operating point and the transfer function between the estimated speed and the actual speed is derived. By drawing the zero-pole distribution diagram of the transfer function, it is verified whether the designed feedback matrix meets the requirements (fastness, good damping characteristics).

4.1 Small signal linearization model

The error is calculated by the difference between formula (1) and formula (3) to get \(e = x - \hat{x}\):

\[
\dot{x} = (A - LC)x - (\hat{A} - LC)\hat{x} = (A - LC)e + \begin{bmatrix}
0 \\
0
\end{bmatrix}(\hat{\omega}_m - \hat{\omega}_m)
\]

In the estimated rotor flux reference system, the linearized model (6) becomes:

\[
\dot{e} = (A_0 - L_0 C)e + \begin{bmatrix}
0 \\
0
\end{bmatrix}(\omega_m - \hat{\omega}_m)
\]

Where: \(A_0 = \begin{bmatrix}
\frac{1}{\tau_r'} - j\omega_{s0} & \frac{1}{L_s'} \\
1 - \frac{1}{\tau_r'} & -\frac{1}{\tau_r'} - j\omega_{r0}
\end{bmatrix}, L_0 = \begin{bmatrix}
L_{r0} \\
L_{s0}
\end{bmatrix}, \omega_{r0} = \omega_0 - \omega_{m0}
\]

Where: \(\omega_{s0}\), \(\omega_{r0}\), \(\omega_{m0}\) Are slip, synchronous speed and rotor speed respectively.

From equation (7), the transfer function from speed error to current error can be derived as shown in equation (8). By taking the imaginary part of equation (8), the transfer function from speed error to q-axis current error can be derived as equation (9) Shown. In the synchronous rotating reference frame of the observation rotor flux orientation, \(\hat{\psi}_R = \hat{\psi}_R + j0\), the speed adaptation rate becomes as shown in equation (10), and equation (10) is changed to equation (11) through Laplace change.

\[
G(s) = C(sI - A_0 + L_0 C)^{-1}\begin{bmatrix}
0 \\
0
\end{bmatrix} = -\frac{\hat{\psi}_{r0}}{L_s'} \frac{s + j\omega_0}{A(s) + jB(s)}
\]
\[ G_q(s) = \text{Im}\{G(s)\} = -\frac{\psi_{r0} s A(s) + \omega_{r0} B(s)}{L'_r} \]  

\[ \dot{\omega}_m = -k_p \left( i_{sq} - \hat{i}_{sq} \right) \hat{\psi}_R - k_i \int \left( i_{sq} - \hat{i}_{sq} \right) \hat{\psi}_R dt \]  

\[ K(s) = \frac{\dot{\omega}_m}{i_{sq} - \hat{i}_{sq}} = -\left( k_p + \frac{k_i}{s} \right) \psi_{r0} \]  

According to equations (9) and (11), the transfer function from actual speed to estimated speed can be derived as shown in equation (12):

\[ G_{cl}(s) = \frac{\dot{\omega}_m(s)}{\omega_m(s)} = \frac{G_q(s) K(s)}{1 + G_q(s) K(s)} \]  

Where:

\[ A(s) = s^2 + \left( \frac{1}{\tau_s'} + \frac{1}{\tau_r'} + \frac{l_{ad0} - l_{rd0}}{L_s'} \right) s - \omega_{s0} \omega_r + \frac{\sigma}{\tau_s' \tau_r'} \omega_{ad'} \omega_{rd'} - \frac{\omega_{r0} l_{ad0}}{L_s'} + \frac{\sigma l_{ad0}}{\tau_s' L'_s} \]  

\[ B(s) = s \left( \omega_{s0} + \omega_{a0} + \frac{l_{ad0} - l_{rd0}}{L_s'} \right) + \frac{\omega_{a0} \tau_r' + \omega_{a0} \tau_r'}{\tau_s' \tau_r'} + \frac{\omega_{rd0} \tau_r' - \omega_{rd0} \tau_r'}{L_s'} + \frac{\sigma l_{ad0}}{\tau_s' L'_s} \]  

4.2 Design of feedback matrix

The feedback matrix given in [5] can be derived when the stator current and rotor flux are selected as the state variables, such as formula (13), which can promote the full-order flux observer in the current model and the voltage model flux observer Smooth switching between. However, this feedback matrix has the problem of pure integration of the stator flux and cannot provide sufficient damping when damping in the medium-high speed region.

\[ G = \begin{bmatrix} (-R_s - R_R) / L_s \\ R_R \end{bmatrix} \]  

In order to solve the problem of poor stability of open-loop transfer function of speed identification and easy oscillation of speed identification caused by poor damping of speed identification system and flux observer system in medium and high speed region, the proposed feedback matrix shown in formula (14) can provide sufficient damping for speed identification system, thus improving the stability of the system and effectively suppressing the occurrence of speed identification problem of oscillation.

\[ G = \begin{bmatrix} 2\lambda / \dot{L} \\ -1 + j \text{sign} \left( \omega_{a0} \right) \end{bmatrix} \]  

Where: \( \lambda \) is a nonlinear parameter related to speed.

Based on the above analysis, this paper proposes a new feedback matrix design method as shown in formula (15), which takes into account the design advantages of the above formulas (15) and (16). It makes the full-order flux observer switch smoothly between the current model and the voltage model flux observer and can provide sufficient damping for the speed identification system in the medium-high speed region. At the same time, the introduction of a 0-1 parameter \( k_g \) can effectively suppress the pure integral problem of the stator flux.

\[ G = \begin{bmatrix} (2\lambda - k_g R_s - R_R) / \dot{L} \\ R_R + \lambda \left( -1 + j \text{sign} \left( \omega_{a0} \right) \right) \end{bmatrix} \]  

Substituting the feedback matrix shown in formula (15) into formula (9) can obtain the zero and pole distribution diagram of the speed identification system, as shown in Figure2, and Figure1 is the...
zero and pole distribution diagram when the feedback matrix is zero. After using the feedback matrix coefficients shown in equation (15), the pole position of the observer is shifted to the left by a certain distance relative to the pole when the feedback matrix is zero, and the damping ratio is also improved. Because the pole position when the feedback matrix is zero is closer to the imaginary axis, the observer using the feedback matrix has a faster convergence rate, that is, the observer has a better dynamic response.

![Pole-Zero Map](image1.png)

**Figure 1.** The feedback matrix is zero

![Pole-Zero Map](image2.png)

**Figure 2.** Improved feedback matrix

## 5. System simulation and result analysis

Use the MATLAB/Simulink platform to build a vector control model of a sensor-less asynchronous motor based on a full-order flux observer. The motor parameters are shown in Table 1. The actual parameters of the motor used in the simulation are number of pole pairs 2, rated power 2.2kW, rated voltage 380V, rated frequency 50Hz, rated speed 1500r/min, stator resistance 2.804 Ω, rotor resistance 2.178 Ω, stator inductance 2.804H, rotor inductance 2.178H, stator and rotor mutual inductance 0.3303H, moment of inertia 0.03kg·m², damping coefficient 0kg·m²/s, rated torque 14.7N·m.

In the MATLAB/Simulink simulation, the m file is used to build the system, and the whole system is discretized. The standard unit method is used for discretization. The reference values of voltage, current and speed are 310V, 6.9A and 1500r/min respectively. During the simulation, the feedback matrix is set in sequence according to the method proposed in this paper. The set value of the rotor flux is 0.854Wb, and the speed command is given after 1s and with load. The observed flux of the full-order observer when the given speed is 1200r/min, the actual speed and the identification speed comparison chart are shown in Figure 3 to Figure 6.

When the motor is running in the medium and high-speed area, the estimated flux linkage and the identification speed under the full-order flux linkage observer have certain oscillations in the transient process. When the feedback matrix is zero, load is added after 1s. It can be found that the estimated flux oscillation is serious in the interval of 1~1.4s, so the speed identification error is also relatively large. After using the feedback matrix in this article, it can be seen from Figure 4 that the oscillation of the observed flux linkage has been improved. Therefore, it can be seen from Figure 6 that the accuracy of the speed identification has also been well improved, and the identification speed in the high-speed area is transient. The oscillation of the process basically disappeared. This is because after adding the feedback matrix, the full-order flux linkage observer has smoother switching between the voltage model and the current model, and the damping characteristics of the medium and high speeds have also been greatly improved.
6. Conclusion
This paper introduces in detail the method of designing the feedback matrix of the induction motor's full-order flux observer. The simulation results are analyzed under the condition of sudden torque change and high-speed switching between forward and reverse rotations. The simulation results show that this method is very good for asynchronous motors. The control function of the motor can achieve precise closed-loop control of the high-speed section of the motor, and the system has a faster response speed and dynamic performance.

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