PID Controller Design with Cuckoo Search Algorithm for Stable and Unstable SOPDT Processes

Durga Vivek¹, Dr.P.Bharath kumar²
¹ PG student, Dept. Of EEE, JNTUA, Ananthapuram, AP, India
² Assistant Professor, Dept. Of EEE, JNTUA, Ananthapuram, AP, India

¹E-mail:-durga.vivek10@gmail.com
²E-mail:- pbk.eee@jntua.ac.in

Abstract. The design of Proportional–Integral–Derivative (PID) controller for the various types of stable and unstable processes with the dynamic model having Second-Order Plus-Dead-Time (SOPDT) is focused in this work. This work initiated with the rejection of load disturbance by connecting PID controller to the system through feedback and then a set-point filter is connected in forward path to achieve proper tracking of reference. The direct synthesis method is applied to get the ranges of PID parameter values and the sharp optimum parameter values which suits for the process can be achieved by introducing Cuckoo Search algorithm which is taken from a class of meta-heuristic algorithms.

KEY WORDS: SOPDT, PID Controller, Direct Synthesis, Cuckoo Search Algorithm.

1. Introduction
In process industries, the equipment like heating boilers, liquid storage tanks, liquid level system and chemical reactors having some transportation delays, analysis loops for composition and recycle loops, etc. due to this, the equipment dynamic response become slow with large time constant [1]. To control such type of systems (FOPDT, SOPDT and integrating systems), some authors proposed variety of design techniques of controllers [2-5].

Using generalized IMC–PID method, Lee and Shamsuzzoha in [6], designed a PID controller to achieve better load disturbance rejection and by using set-point filter reference tracking was achieved. In [1], S. R. Rao and A. S. Rao designed direct synthesis-based controller for integrating process with time delay. Tao and Zhang in [a] implemented two-degree-of freedom (2-DoF) control for open loop unstable process and they tried to improve both set-point filter performances as well as load disturbance reduction. Initially they utilized PD controller and they got optimized values simultaneously and separately. Panda and vijayan in [7] implemented double feedback method to draw into stable operation and to achieve better performance of system. A PID controller which works based on IMC (internal model controller) is implemented in the outer loop to made set-point tracking is satisfactory and the inner loop is tuned with the auto tuning Ziegler-Nichols method to stabilize the system. Vilanova and Alfaro in [9] presented 2DoF PI controller for robust tuning and they used model reference optimization procedure with servo and regulatory closed-loop transfer functions targets. It is concluded from their work that there may be always a tradeoff between performance and robustness [8]. Robustness can be maintained with the factor of maximum sensitivity Ms, for desirable robustness of ISOPDT models and IPDT models, the maximum sensitivity range is 1.2≤Ms≤2.0 for stable processes and it should be Ms ≥2.0 for unstable process. In [10], Ajmeri M and Ali A altered Parallel Control Structure (PCS) which was derived from 2-DoF Control Structure for some class of unstable processes in isothermal chemical reactors that reflects multi steady state solutions. They proposed new tuning methods using direct synthesis (DS) method.
In [11], Kasi, V. R. and Majhi used DS method to find the approximate ranges of parameters and they implemented a new nature inspired optimization technique with the name called Particle Swarm Optimization algorithm. In that work the authors achieved satisfactory performance during load disturbance rejection and set-point tracking.

Most of the tuning methods described above concentrate on either load disturbance rejection or monitoring of set-points. It can be achieved by selecting proper objective function during the design. Always a tradeoff takes place between these two (controller and set-point filter) performances. So, the objective here is to design a PID controller which can reduce the load disturbance and to design a set-point filter for satisfactory reference tracking. Initially the approximate ranges of PID controller parameters will be obtained by using Direct Synthesis method and the values of parameters are captured from Cuckoo Search Algorithm.

The proposed approach is contrasted with the current algorithm for Particle Swarm Optimization (PSO) and evaluated under the uncertainty parameter.

2. Mathematical relations

The structure of PID controller is taken in the parallel form as

\[ G_c(s) = K_p(1 + \frac{1}{T_i} + T_ds) \]  

(1)

Where \( K_p \), \( T_i \) and \( T_d \) are parameters of the controller to be optimized. The transfer function of set-point filter is given in (2) and \( \beta \) is set-point coefficient.

\[ G(s) = \frac{K e^{-\beta s}}{(sT_1 \pm 1)(sT_2 + 1)} \]  

(2)

Now let \( G(s) \) be a transfer function of an SOPDT stable/unstable process by including steady state gain \( K \) and time delay \( \theta \), then

\[ G(s) = \frac{s^2T_1^2T_2^2}{(sT_1T_2 + 1)(sT_1 + 1)(sT_2 + 1)} \]  

(3)

where \( T_1 \) and \( T_2 \) \((T_1 > T_2)\) are time constants.

The PID controller is primarily designed for the rejection of load disturbance by implementing the direct synthesis process. Then, for reference tracking, a set point filter is designed. In terms of load disturbance, the desired tf can be taken as

\[ \frac{Y}{D} = \frac{\frac{T_p e^{-\beta s}}{(sT_1 + 1)}}{K_p} \]  

(4)

Where \( T \) is the time constant and \( \theta \) is the effective time delay of the SOPDT process model. Here \( T_{c1} \) is the desired tuning parameter.

An expression shown below comes out by including transfer functions \( G \) and \( G_c \) and from the comparison of the denominator parts

\[ s^3(T_1^2T_2 - T_1T_2^2) + s^2(T_1(T_1 \pm T_2) + (T_1T_2 - \theta T_2)) + s(T_1 - \theta + \frac{T_1}{K_p}) + 1 = 0 \]  

\[ T_c^3s^3 + 3T_c^2s^2 + 3T_c + 1 = 0 \]  

(5)

For stable system, the controller parameters from above expression (5) are given in (6), (7) and (8)

\[ K_p = \frac{-T_c^3\theta^2 - 3T_c^2T_2^2 + T_c(3T_cT_2^2 + 3T_1T_2^2\theta + 3T_1^2T_2^2\theta)}{\theta^2(T_1^2T_2 + T_2^3)^3} \]  

(6)

\[ T_d = \frac{\theta(T_1 T_2^3 + 3T_cT_2^2 + 3T_cT_1^2 - T_2^2(T_1 \theta + T_2 \theta + \theta^2))}{-T_c^3\theta^2 - 3T_c^2T_2^2 + T_c(3T_cT_2^2 + 3T_1T_2^2\theta + 3T_1^2T_2^2\theta) + T_1^2T_2^3 + T_2^3T_1^3 + T_2^3\theta + T_1^3\theta^2} \]  

(7)

\[ T_i = \frac{\theta(-T_c^3\theta^2 - 3T_c^2T_2^2 + T_c(3T_cT_2^2 + 3T_1T_2^2\theta + 3T_1^2T_2^2\theta) + T_1^2T_2^3 + T_2^3T_1^3 + T_2^3\theta + T_1^3\theta^2)}{\theta^2(T_1 + T_2 \theta + \theta^2 + T_1T_2)^3} \]  

(8)

Similarly, for unstable system the controller parameters from (5) are given in (9), (10) and (11)

\[ K_p = \frac{T_c^3\theta^2 + 3T_c^2T_2^2 + T_c(3T_cT_2^2 + 3T_1T_2^2\theta - 3T_cT_1^2\theta)}{\theta^2(T_1^2T_2 + T_2^3)^3} \]  

(9)
Now to track the reference input, the filter coefficient $\beta$ (0 to 1) used and below expression is the desired transfer function of closed loop system.

$$T_d = \frac{\theta (T_1 T_2 T^3 + 3 T_1 T_2 T^2 T_{c1} + 3 T_1 T_2 T^2 T_{c2} + T_{c2}^3 (-T_1 \theta + T_2 \theta + \theta^2))}{T_{c2}^3 \theta^2 + 3 T_{c2} T_1 T_2 T^2 + 3 T_{c2} T_1 T^2 \theta - 3 T_1 T_{c2} T^2 \theta + T_1 T_2 T^3 + T_1 T^3 \theta - T_2 T^3 \theta}$$  \hfill (10)

$$T_i = \frac{\theta (T_{c2} T^2 + 3 T_{c2} T^2 \theta + T_{c1} (3 T_1 T_2 T^2 + 3 T_1 T^2 \theta - 3 T_1 T_{c2} T^2 \theta) + T_1 T_2 T^3 + T_1 T^3 \theta - T_2 T^3 \theta)}{(T_1 \theta - T_2 \theta - \theta^2 + T_1 T_2) T^2}$$  \hfill (11)

By rearranging the function in terms of desired transfer function

Substituting the desired transfer function in above expression and by comparing can get

$$\beta = \frac{\theta T_{c1}}{T T_i}$$  \hfill (16)

After verifying so many outcomes it is observed that during set-point response, $\beta$ in (16) causes more peak overshoot. So it is desired to adopt the following set-point filter coefficient.

$$\beta = 1 - \frac{\theta T_{c1}}{T T_i}$$  \hfill (17)

Where $T_{c1}$ is the desired time constant and can be obtained by applying Cuckoo search algorithm. The goal here is to reduce Integral Square Error (ISE).[18].

The objective function $ISE$ is

$$ISE = \int_0^\infty e(t)^2 dt$$  \hfill (18)

where, $e(t)$ is the error in system.

3. The cuckoo search algorithm

The Cuckoo search algorithm is an algorithm inspired by nature that belongs to the meta-heuristic community that imitates the behaviour of a family of birds called cuckoo brood parasitism.[12]. These bird types used to lay their own eggs in other host bird nests by raising the chance of hatching by choosing newly spawned nests and eliminating established eggs. By assuming that the eggs are their own, the host birds will take care of those eggs. Some of the host birds might, however, notice that the eggs are not their own and then they can either create new nests in some other new places through the eggs or through them. The similar analogy is used to solve several optimization problems. The steps related to the algorithm are as it follows:

Step1: Specify the parameters for Cuckoo hunt such as number of nests (n), step size ($\alpha$), discovering probability ($p_a$) and maximum number of generations for conclusion.

Step2: Generate the initial locations of nests (Eggs of host birds) by assigning set of random values to each variable as shown below:

$$\text{nest}_{i,j}^0 = \text{Round}(x_{j,\text{min}} + \text{rand}[(x_{j,\text{max}} - x_{j,\text{min}})])$$  \hfill (19)
Where $n_{est}^{(0)}_{i,j}$ is the initial value of the $j_{th}$ variable for the $i_{th}$ nest; and $x_{j} \text{min}$, $x_{j} \text{max}$ are the minimum and the maximum permissible values for the $j_{th}$ variable; $\text{rand}$ is a random number in the interval $[0, 1]$. Because of the discrete nature of the problem, the round function is carried out.

**Step 3:** Based on the quality of new cuckoo eggs produced with Levy flights from their positions, all nests except for the best one is replaced as:

$$n_{est}^{(t+1)}_{i,j} = n_{est}^{(t)}_{i,j} + \alpha S(n_{est}^{(t)}_{best} - n_{est}^{(t)}_{i,j}) \text{rand}$$

(20)

**Step 4:** The probability matrix used for each solution to discover the aliened eggs given as

$$P_{ij} = \begin{cases} 1 & ; \text{rand} < P_a \\ 0 & ; \text{rand} \geq P_a \\ \end{cases}$$

(24)
Existing eggs will be replaced by random phase walks with a good quality of new produced eggs from their current positions as follows:

\[ S = \text{rand.}(\text{nests(randperm1}(n), :)) - \text{nests(randperm2}(n), :)) \]

And \( n_{\text{est}}^{(t+1)} = n_{\text{est}}^{(t)} + S \cdot P \)  \[ (25) \]

Where \text{randperm1} and \text{randperm2} are random permutation functions used for various permutation rows added to the matrix of nests, \( P \) is the matrix of probability.

**Step 5: Termination Criterion**
Steps are repeated until a termination condition is reached by producing new cuckoos and finding alien eggs. [14].

4. Results and discussions

**Example 1:** Consider a stable SOPDT process \( G_1(s) = \frac{2e^{-5}}{(10s+1)(5s+1)} \)

The parameter values of controller which suit for the process \( G_1(s) \) are mentioned in Table 1. For this process \( M_0 \) value is given as 2.0. The output characteristics of system with the PID-PSO controller [7] along with proposed controller are plotted in Fig 2. Here the load disturbance is given in the form of a step function with the magnitude of \( -0.5 \) exactly at time \( t = 40 \text{ sec} \). The algorithm given by Venkata Ramana gives a better set-point output, but settling time is very high and so system is slow and during disturbance also system takes more time to reach steady state. The suggested method has strong rejection of load disturbance and also takes less rise time and settles time during rejection of load disturbance compared to the PID-PSO method. This method is also undergone parameters variation with the time constants \( T_1 \) and \( T_2 \) as \( -10\% \) and \( K \) and \( \theta \) as \( +10\% \) and the corresponding output is shown in Fig 3.

![Figure 2. Output characteristics of \( G_1(s) \)](image)

![Figure 3. Output characteristics of \( G_1(s) \) with perturbation](image)

**Example 2:** Consider an unstable SOPDT process \( G_2(s) = \frac{e^{-0.5s}}{(2s-1)(0.5s+1)} \)

The parameter values of controller which suit for the process \( G_2(s) \) are mentioned in Table 1. For this process \( M_0 \) value is given as 2.98. The output characteristics of system with the PID-PSO controller [7] along with proposed PID-CSA controller are plotted in Fig 4. Here the load disturbance is given in the form of a step function with the magnitude of \( -0.5 \) exactly at time \( t = 40 \text{ sec} \). The algorithm given by Venkata Ramana gives a better set-point output, but settling time is very high and so system is slow and during disturbance also system takes more time to reach steady state. The proposed method has better load disturbance rejection and also requires less rise time and settling time during load disturbance rejection when compared with the PID-PSO method. This method is also undergone
parameters variation with the time constants \((T_1 \text{ and } T_2)\) as \(-10\%\) and \(K\) and \(\theta\) as \(+10\%\) and the corresponding output is shown in Fig 5.

\[ G_2(s) = \frac{1}{(5s-1)(2.07s+1)} \]

**Example 3:** Consider another unstable SOPDT process \(G_3(s) = \frac{e^{-0.939s}}{(5s-1)(2.07s+1)}\).

The parameter values of controller which suit for the process \(G_3(s)\) are mentioned in Table 1. For this process \(M_s\) value is given as 2.95. The output characteristics of system with the PID-PSO controller [7] along with proposed PID-CSA controller are plotted in Fig 6. Here the load disturbance is given in the form of a step function with the magnitude of \(-0.5\) exactly at time \(t = 40\text{sec}\). The algorithm given by Venkata Ramana gives less settling time and less rise time, but the proposed method reduces peak overshoots and also reduces oscillations of response while reaching steady state when comparing with the PID-PSO method. This method is also undergone parameters variation with the time constants \((T_1 \text{ and } T_2)\) as \(-10\%\) and \(K\) and \(\theta\) as \(+10\%\) and the corresponding output is shown in Fig 7.
5. Conclusion
In this work the design of PID controller to control the stable and unstable SOPDT processes is performed. The ranges of controller parameters are obtained by applying Direct Synthesis method. The narrowed values of parameters obtained by applying Cuckoo Search optimization algorithm. The proposed algorithm performance compared with the existed PSO algorithm and the algorithm generated optimal values to fulfill the control objective. The Set-Point philtre architecture, however, does not require any additional parameters.

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Table 1: PID controller parameters for stable and unstable process

| System/process | Algorithm | $K_p$  | $K_i$  | $K_d$  |
|----------------|-----------|--------|--------|--------|
| $G_1(s) = \frac{2e^{-3}}{(10s + 1)(5s + 1)}$ | PSO | 7.7213 | 1.1962 | 15.0023 |
| $G_2(s) = \frac{e^{-0.65}}{(2s − 1)(0.5s + 1)}$ | CSA | 7.3978 | 1.8434 | 15.9410 |
| $G_3(s) = \frac{e^{-0.9395s}}{(5s − 1)(2.07s + 1)}$ | PSO | 5.1481 | 0.7305 | 7.0039 |

5. Conclusion
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