Numerical Modeling the Effect of Static Indentation on the Rate and the Fatigue Crack Growth Trajectory

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Abstract. The results of numerical modeling (in ANSYS) are presented, concerning the effect of the local residual stress field in the vicinity of the crack tip on the fatigue crack growth rate and change of the crack trajectory. Static indentation by a spherical indenter is considered as a method for creating a residual stress field. In this case, the point (or points) of indentation can be located both symmetrically relative to the line of initial crack orientation and with an offset. It is shown that the fatigue crack can grow in the direction opposite to its initial orientation. This effect can be employed to control the fatigue crack growth trajectory.

1. Introduction
The indentation is widely used as a test method to determine the mechanical properties of structural materials [1-5]. At the same time, local indentation in the vicinity of the crack tip by means of a spherical indenter can lead to a significant (several times) decrease of the fatigue crack growth rate [6-8].

To estimate residual life of structural elements with cracks under fatigue loading, taking into account the influence of the residual stress field in the crack tip zone created by static or dynamic indentation, numerical modeling the fatigue crack propagation is very attractive. It is quite obvious that computational modeling is also the optimal way to solve the considered practical problem and determining the indentation parameters (the size and material of the indenter, localization of the indentation point and the load on the indenter).

Regarding the mentioned problem, numerical modeling the effect of the residual stress field in the vicinity of the crack tip, created by static indentation, on the fatigue crack growth is carried out. The results of the numerical analysis of series of model problems for various types of cracks can create the basis for proposals for increasing the residual life of structural elements with cracks.

2. Basic aspects of numerical modeling
A specialized algorithm has been developed in the ANSYS software. The algorithm uses the ANSYS Explicit STR solver, which is fully integrated into the unified calculation module Workbench Mechanical. The algorithm allows determining the field of residual stresses (RS) induced by indentation, used for the numerical modeling the fatigue crack growth. These two procedures are combined into one sharing common features like Mesh, Fracture, etc.

The Paris law is used for the numerical modeling of the fatigue crack propagation in the absence of trajectory changes:

\[
\frac{dl}{dN} = C(\Delta K)^m,
\]

where \(l\) is the crack length; \(N\) is the number of cycles; \(\Delta K\) is the stress intensity factor range; \(C\) and \(m\) are constants of the material. The stress intensity factor ratio is also introduced into consideration as

\[
R = \frac{K_{min}}{K_{max}},
\]

where \(K_{min}\) is the minimum value of the stress intensity factor; \(K_{max}\) is the maximum value of the stress intensity factor.
The equivalent stress intensity factor under mixed mode loading (modes I and II) equals

\[
\Delta K_{eq} = \frac{1}{2} \cos \left( \frac{\theta}{2} \right) \left[ \Delta K_I (1 + \cos \theta) - 3 \Delta K_{II} \sin \theta \right],
\]

where \(\Delta K_I = K_I^{max} - K_I^{min} = (1 - R) K_I^{max}\) and \(\Delta K_{II} = K_{II}^{max} - K_{II}^{min} = (1 - R) K_{II}^{max}\).

Here \(\theta\) is the angle between the crack growth direction and the original direction of the z-axis.

Calculation of the crack length increment \(ds\) is performed by calculating the lengths of the separated "open" finite elements (FE) describing the contour or the surface of crack edges in the vicinity of the crack tip (figure 1). The change of the crack length is the sum of the lengths of the open FE belonging to crack edges (figure 1a) for the plane problem. In the case of the three-dimensional problem, the change in the crack length is determined by the change in the position of each node of the crack front and it is the sum of the change in the FE lengths, which are located in the direction of crack growth \(\Delta S = \Sigma \Delta l\) (figure 1b). Thus, if the parameters are counted at 0% of the crack front length, then the increments at each step are taken from the specimen’s surface. If the parameters are counted at 50%, then the increments at each step are taken in the middle of the crack front.

The angle of fatigue crack propagation is determined by the following equation

\[
\theta = \cos^{-1} \left( \frac{3 \left( K_{II}^{max} \right)^2 + \left( K_I^{max} \right)^2 + 9 \left( K_{II}^{max} \right)^2}{\left( K_I^{max} \right)^2 + 9 \left( K_{II}^{max} \right)^2} \right).
\]

The stress intensity factors \(K_I\) and \(K_{II}\) are calculated through the interaction integral [9]:

\[
\left\{ q \left[ \sigma_{x_{II}} e_{x_{II}} \delta_x - \sigma_{y_{II}} e_{y_{II}} \delta_y - \sigma_{x_{III}} \delta_x - \sigma_{y_{III}} \delta_y - \sigma_{x_{IV}} \delta_x - \sigma_{y_{IV}} \delta_y \right] / \sqrt{\delta_{q}} \right\} dS,
\]

where \(\sigma_{ij}, e_{ij}, u_i\) are components of stresses, strains and displacements, respectively; \(\sigma_{ij}^{max}, e_{ij}^{max}, u_i^{max}\) are components of stresses, strains and displacements of the auxiliary field, respectively; \(q_i\) are increment vector components.

3. Fatigue crack growth with symmetric indentation

Two typical problems of fatigue crack growth are considered below, namely, a semi-elliptical crack in a plate under tension and a plane crack in a standard compact tension (CT) specimen.

3.1. Plate with a semi-elliptical crack

The scheme of a plate with a semi-elliptical crack and its finite element (FE) discretization are shown in figure 2 (\(l = 1.5b; h = 6b; L = 30b; B = 50b; b = 2mm\)). The material of the plate is AL 7075-T6. The cracked plate deforms according to the bilinear law. The mechanical properties of the material are given
in Ref. [2]. The parameters of the Paris law are the following: \( C = 2.7 \cdot 10^{-11}, \ m = 3.7 \) [10]. The crack growth rate is given in \( \text{mm/cycle} \) and the stress intensity factor is given in \( \text{MPa} \cdot \text{m}^{1/2} \). A cyclic stress \( \pm \sigma_z \) is applied to the end of plate \( z = \pm L/2 \).

**Figure 2.** Plate with a semi-elliptical crack: 
a) scheme of loading conditions; b) FE model

The **SMART Crack Growth** module in the Fracture section, which allows calculating the crack growth, need both free crack surfaces (which are built using the embedded **Semi-Elliptical Crack** function). Thus, it is not possible to use the symmetry of the problem.

A tetrahedral FE mesh is employed for calculating the fatigue crack growth. Taking into account the need to calculate the residual deformations in the indentation zone (the zone where the crack reaches the free surface \( x = 0 \)), the FE mesh has a local refinement. Total number of nodes equals 128129.

Calculation of the fatigue crack growth is performed for the case when a local RS field, obtained by indentation, takes place in the zones where the crack reaches the surface. Balls (structural steel [10]) with a diameter of 5 mm are considered as indenters. The case is investigated when the point of the first contact between the indenter and the plate is the point where the crack reaches the surface \( x = \pm a, y = 0 \) (figure 2a). It is assumed that there is no friction between surfaces of the indenter and the plate. The value of the static force on each indenter is taken as \( P = 860 \text{N} \). In this case, the maximum value of the displacement of the indenter is \( u^\text{max} = 60 \mu\text{m} \) and the maximum residual displacements \( u^\text{res} = 45\mu\text{m} \). Figure 3 shows the distribution of the residual equivalent stresses.

**Figure 3.** Distribution of the residual equivalent stresses in XOY plane.

The results of calculations with and without the indentation of the plate under study are presented in figure 4. Since calculation of the fatigue crack growth (and, accordingly, the **SMART Crack Growth** procedure used for the calculations) is applicable only for linear materials, it is assumed that hardening is absent.
The obtained results (figure 4a) show that a sufficiently small value of the force on the indenter can lead to a significant decrease of the crack growth rate. This concerns not only the effect of indentation for points where the crack reaches the surface, but also for the point (b, 0) farthest from the surface.

3.2. Compact tension specimen

The scheme of the CT specimen, as well as FE model, is shown in figure 5a (here the case is shown when the indentation points are displaced relative to the plane of symmetry YOZ). In order to increase the zone of influence of indentation on the fatigue crack growth rate, a small specimen thickness is considered: \( h = 2 \text{ mm} \). The material of the CT specimen is aluminum alloy 7075 T6. The geometric and mechanical parameters of the indenter are the same as in the above-mentioned problem. A schematic of the FE subdivision of the specimen surface is shown in figure 5b. The points in the vicinity of the crack tip on the specimen surface are subjected to indentation by two spherical indenters (diameter 5 mm) with a force \( P = 100 \text{ N} \). As a result, a residual stress field is created, and the maximum value of residual displacements is \( u_{\text{max}} = 8 \mu \text{m} \). A symmetric cycle \( F_{\text{max}} = 50 \text{ N} \) of loading is under consideration.

Figure 4. The results of calculation for a semi-elliptical crack:

- crack length increment dependencies \( l = \Delta l(N) \);
- at the point of indentation \( (0, \pm l) \);
- at the point farthest from the surface; b) distribution of \( K_1 \) along the crack front \( S \).

Figure 5. Scheme of the CT specimen (a), FE model of the specimen surface for a symmetric problem (b), FE model for the case of indenter displacement \( \Delta z = 2 \text{ mm} \) (c).
Figure 6 shows the calculated curves of the fatigue crack growth \( l = l(N) \) which are obtained with and without the indentation. The results show that indentation into the crack tip leads to the decrease of the crack growth rate on the basis \( N = 2.5 \times 10^5 \) cycles by more than 3 times even at very low residual stresses. Thus, the use of this technique is promising for extending the residual life of thin-walled structural elements with through-thickness cracks.

![Figure 6. Calculated curves of fatigue crack growth \( l = \Delta l(N) \)](image)

4. Indentation as a way of changing the crack growth trajectory

In this case, the indentation points \( k \) are displaced relative to the plane of symmetry of the specimen YOX by \( \Delta z \) and relative to the crack tip in the direction of the X axis by \( \Delta x \) (2 mm) (figure 5a).

It can be assumed that the influence of the indentation force \( P \) on the crack growth rate and changes of the crack trajectory will decrease with an increase of the tensile load \( F \). This is confirmed by the results of calculations (figure 7) which show the corresponding dependences for the case of indentation by force \( P = 450\,N \) to point \( k \) with \( \Delta x = 2 \, \text{mm} \) and \( \Delta z = 4 \, \text{mm} \) (figure 7a) and subsequent cyclic loading with \( F_1^{\max} = 50\,N \) and \( F_2^{\max} = 90\,N \).

![Figure 7. The results of evaluating the influence of the indentation force \( P \) on the growth rate (a) and the cyclic load \( F^{\max} \) on the crack propagation trajectory (b)](image)

It should be noted (figure 7a) that when the crack moves to the indentation point \( k \), the crack growth rate is slightly higher than the rate that occurs in the absence of indentation, and then it decreases. In this case, the trajectory of the crack comes out on a straight line parallel to the initial direction of the crack orientation (figure 7b).

The possibility of a significant change of the fatigue crack trajectory (trajectory control) based on indentation of the zones in the vicinity of crack tip is also considered. The effect of the value of the static force \( P \) (\( 1500\,N \leq P \leq 2000\,N \)), which is applied to the indenter, on the fatigue crack trajectory is...
analyzed for the case of the cyclic load $F^{\text{max}} = 50\,\text{N}$ and the constant position of the indentation point $k$ ($\Delta x = 2\,\text{mm}$, $\Delta z = 8\,\text{mm}$). The calculation results are given in figure 8.

It can be seen (Figure 8) that the increase of the indentation force $P$ leads to a significant deviation of the fatigue crack trajectory from the initial direction for the case of the same localization of the indentation point and the cyclic load. When the indentation force $P$ reaches a certain value $P^*$, the crack can propagate in the opposite direction to the original orientation of the crack. This effect can be successfully employed to control the fatigue crack growth trajectory.

![Figure 8: The effect of the indentation force $P$ on the trajectory of fatigue crack propagation](image)

**5. Conclusions.**
The method and the algorithm in the ANSYS software for numerical modeling of the fatigue crack growth under mixed mode loading have been developed taking into account the influence of the local residual stress field caused by static indentation in the vicinity of the crack tip.

It is shown that the indentation in the vicinity of the semi-elliptical crack tip, where the crack reaches the surface, can lead to the significant decrease of the fatigue crack growth rate not only in the vicinity of the indentation zone, but also in depth of the plate.

It is demonstrated for the CT specimen that when the indentation point is displaced relative to the initial crack propagation line, there is a significant deviation of the fatigue crack trajectory from the initial direction. Moreover, the fatigue crack can propagate in the opposite direction to the original orientation of the crack at certain parameters of the problem (the point of the indenter, the value of the indentation force, the value of the cyclic load).

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