A counterexample to the Nelson-Seiberg theorem

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ABSTRACT: We present a counterexample to the Nelson-Seiberg theorem and its extensions. The model has 4 chiral fields, including one R-charge 2 field and no R-charge 0 filed. Giving generic values of coefficients in the renormalizable superpotential, there is a supersymmetric vacuum with one complex dimensional degeneracy. The superpotential equals zero and the R-symmetry is broken everywhere on the degenerated vacuum. The existence of such a vacuum disagrees with both the original Nelson-Seiberg theorem and its extensions, and can be viewed as the consequence of a non-generic R-charge assignment. Such counterexamples may introduce error to the field counting method for surveying the string landscape, and are worth further investigations.

KEYWORDS: Global Symmetries, Supersymmetry Breaking, Supersymmetric Effective Theories

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1 Introduction

In 4-dimensional $N = 1$ supersymmetry (SUSY) theories [1, 2], the relation between SUSY breaking and R-symmetries in Wess-Zumino models are described by the Nelson-Seiberg theorem and its extensions [3-6]. The original Nelson-Seiberg theorem [3] states that for SUSY breaking at a stable vacuum in a generic model, a necessary condition is to have an R-symmetric superpotential, and a sufficient condition is to have the R-symmetry spontaneously broken at the vacuum. The statement can be extended to metastable SUSY breaking models with approximate R-symmetries [7-9]. A revised theorem [5] claims that with a polynomial superpotential, the necessary and sufficient condition for SUSY breaking is to have an R-symmetric superpotential and more R-charge 2 fields than R-charge 0 fields. The revised theorem can be generalized to include models with non-polynomial superpotentials [6], where the condition for SUSY breaking is the same as before for superpotentials smooth at the origin of the field space, and a singularity at the origin implies both R-symmetry breaking and supersymmetry breaking. Half of the revised theorem has even stronger claims [4]: if we have an R-symmetric superpotential and less or equal R-charge 2 fields than R-charge 0 fields, the SUSY vacua claimed by the revised theorem have an additional property that the superpotential equals zero at the vacuum, and this part of claim is also true for non-$Z_2$ discrete R-symmetries or non-Abelian discrete R-symmetries [10, 11].

The Nelson-Seiberg theorem and its extensions provides various tools for new physics model building. In SUSY phenomenology beyond the Standard Model, the SUSY breaking effect is mediated to the SUSY Standard Model sector through a messenger sector. So constructing the SUSY breaking sector with R-symmetries makes the first step towards a full model. In string phenomenology, flux compactification of type IIB string theory [12-15] gives low energy effective theories formulated as a supergravity (SUGRA) version of Wess-Zumino models, and discrete R-symmetries come from geometrical symmetries of the Calabi-Yau manifold used for compactification. The previous condition for SUSY vacua with zero superpotentials gives again SUSY vacua with zero vacuum energy in SUGRA. SUSY breaking and the vacuum energy are then non-perturbatively generated at lower scales, notably through the racetrack mechanism [16]. Such vacua contribute to the third branch of the string landscape which prefers low scale SUSY breaking [17-19]. And the de
Sitter swampland conjecture [20–23] may hopefully be evaded by these vacua. Following the revised Nelson-Seiberg theorem, whether a SUSY vacuum exists or not can be generically determined by counting R-charge 2 and R-charge 0 fields, and the explicit vacuum solution is not needed. Such field counting method makes it possible to do a fast survey of the string landscape.

Both the original Nelson-Seiberg theorem and its extensions require genericness assumptions. The usual concept of genericness means that parameters take generic values. It is related to naturalness, fine-tuning or hierarchy problems [24–27]. This type of non-generic models only compose a null set in the parameter space, and can be neglected in both phenomenology or string phenomenology studies. Another lesser-known concept of genericness is about R-charges. With some special R-charge assignment which determines a renormalizable superpotential respecting the R-symmetry, the explicit vacuum solution disagrees with what the previous theorems predict. These models still have generic parameters, and can be viewed as having a non-generic R-charge assignment. This work is to present the first known model of this type as a constructive proof. If such non-genericness occurs often, it may introduce non-neglectable error to the field counting method from the revised Nelson-Seiberg theorem.

The rest of this paper is organized as follows. Section 2 presents the model and its vacuum structure, showing that it is a counterexample to both the original Nelson-Seiberg theorem and the revised theorem, with non-generic R-charges. Section 3 discusses properties of the SUSY vacuum and implications for both phenomenology and string phenomenology studies.

2 The counterexample

The model presented here has four chiral fields \( \{z_1, z_2, z_3, z_4\} \) with the R-charge assignment

\[
R(z_1) = 2, \quad R(z_2) = -2, \quad R(z_3) = 6, \quad R(z_4) = -6. \tag{2.1}
\]

In the following context, the same set of symbols are used for chiral fields and their scalar components which determine vacuum solutions. A renormalizable R-symmetric superpotential has the form

\[
W = az_1 + bz_1^2 z_2 + cz_2^2 z_3 + dz_1 z_3 z_4, \tag{2.2}
\]

where the coefficients \( a, b, c \) and \( d \) take generic complex values. All R-charges of fields are uniquely fixed by requiring \( W \) to have R-charge 2, and all R-charge 2 monomial terms up to cubic are included in \( W \). Following the field counting method used in [4, 5], the number of R-charge 2 fields \( N_X = 1 \) is greater than the number of R-charge 0 fields \( N_Y = 0 \). The revised Nelson-Seiberg theorem [5] claims that there is no SUSY vacuum. But solving the SUSY equations

\[
\partial_1 W = a + 2bz_1 z_2 + dz_3 z_4 = 0, \tag{2.3}
\]
\[
\partial_2 W = bz_1^2 + 2cz_2 z_3 = 0, \tag{2.4}
\]
\[
\partial_3 W = cz_2^2 + dz_1 z_4 = 0, \tag{2.5}
\]
\[
\partial_4 W = dz_1 z_3 = 0 \tag{2.6}
\]
gives a SUSY vacuum at
\[ z_1 = z_2 = 0, \quad z_3 z_4 = -\frac{a}{d}, \]  
which also satisfies an additional equation \( W = 0 \). The vacuum has one complex dimensional degeneracy on the \( z_3 \)-\( z_4 \) space for non-zero \( a \) and \( d \). Since \( z_3 \) and \( z_4 \) has non-zero R-charges, the R-symmetry is spontaneous broken everywhere on the degenerated vacuum, which should lead to SUSY breaking according to the original Nelson-Seiberg theorem [3].

The existence of the SUSY vacuum means that this model is a counterexample to both the original Nelson-Seiberg theorem and the revised one. The SUSY solution is a result of the particular form of the renormalizable superpotential (2.2), which is constructed from fields with the particular R-charges (2.1). Thus the counterexample presented here can be viewed as having a non-generic R-charge assignment.

The full vacuum structure of the model may be obtained from the scalar potential
\[ V = (\partial_i W)^* \partial_i W, \]  
where a minimal Kähler potential is assumed, and the Einstein summation convention for field indices is adopted. Stationary points are found by solving the zero points of the first derivatives of \( V \)
\[ \partial_i V = (\partial_j W)^* \partial_j \partial_i W = 0. \]  
Whether a stationary point is a minimum, maximum or saddle point can be determined by checking the eigenvalues of the second derivative matrix of \( V \)
\[ \partial^2 V = \begin{pmatrix} \partial_i \partial_j V & \partial_i \partial_j V \\ \partial_i \partial_j V & \partial_i \partial_j V \end{pmatrix} = \begin{pmatrix} (\partial_i \partial_k W)^* \partial_j \partial_k W & (\partial_i \partial_j W)^* \partial_k W \\ (\partial_i \partial_j W)^* \partial_i \partial_k W & (\partial_i \partial_j W)^* \partial_k W \end{pmatrix}. \]  
There is a saddle point at the origin of the field space
\[ z_1 = z_2 = z_3 = z_4 = 0. \]  
Using the complexified symmetry technique developed in [28–31], SUSY runaways are found along the direction
\[ z_1 = \frac{c}{d} u v, \quad z_2 = \frac{u}{v}, \quad z_3 = \left( \frac{2bc}{d^2} v^2 - \frac{a}{d} \right) \frac{u^3}{v}, \quad z_4 = \frac{v}{u^3}, \]  
\[ v = \sqrt{\frac{ad(10 + 2|u|^4)}{bc(25 + 4|u|^4)}} \]  
\[ u \to 0. \]  
Further analytical search for stationary point easily exhausts our available computational resource. Numerical calculation with typical coefficient values shows that several degenerated saddle points exist in addition to the one at the origin. But no local minimum other than the SUSY one has been found so far after extensive search. The SUSY vacuum (2.7) is most likely the only metastable (and stable) minimum in our model.
3 Discussions

Considering non-renormalizable superpotentials, there are simpler models than ours. For example, the quartic superpotential

\[ W = az_1 + bz_1^3z_3 + cz_1z_2z_3 \]  

(3.1)

with the R-charge assignment

\[ R(z_1) = 2, \quad R(z_2) = 4, \quad R(z_3) = -4, \]  

(3.2)

gives a similar vacuum structure with a SUSY vacuum at

\[ z_1 = 0, \quad z_2z_3 = -\frac{a}{c}. \]  

(3.3)

This model is also a counterexample to both the original Nelson-Seiberg theorem and the revised one. The quartic term has to be included to uniquely fix all R-charges. If the superpotential is restricted to cubic to be renormalizable, at least 4 fields are needed to construct a counterexample and our model is the simplest one.

It is worth to note that a different type of R-symmetry breaking SUSY vacua already exist in some previously known examples with non-generic superpotentials [32], or generic models with non-renormalizable superpotentials [33]. Their degenerated SUSY vacua contain the origin of the field space which preserves the R-symmetry. Since the Nelson-Seiberg theorem gives no information on the existence of a vacuum at the origin, those models do not really contradict with the theorem. Moreover, the models in [33] have less or equal R-charge 2 fields than R-charge 0 fields. So the field counting method gives correct prediction on the SUSY vacua. On the contrary, our model has R-symmetry breaking everywhere on the degenerated SUSY vacuum (2.7), and more R-charge 2 fields than R-charge 0 fields. Thus it is the first known counterexample to both the original Nelson-Seiberg theorem and the revised one, with a non-generic R-charge assignment and a generic renormalizable superpotential respecting the R-symmetry.

Because of the R-symmetry breaking feature of the vacuum, our model can serve as a tree-level R-symmetry breaking sector separate from a SUSY breaking sector. It is distinct from previous tree-level R-symmetry breaking models where both SUSY breaking and R-symmetry breaking happen in the same sector [34–38]. The separation of SUSY breaking and R-symmetry breaking may gives the possibility to generate the gaugino mass from tree-level R-symmetry breaking in gauge mediation models [39].

The superpotential (2.2) vanishes at the SUSY vacuum (2.7). Similarly to the case with less or equal R-charge 2 fields than R-charge 0 fields [4], the SUGRA version of the model gives again a SUSY vacuum with zero vacuum energy, and contributes to the third branch of the string landscape. Since the model has more R-charge 2 fields than R-charge 0 fields, the revised Nelson-Seiberg theorem does not predict a SUSY vacuum. But when realized in flux compactification of type IIB string theory [12–15], the R-symmetry breaking feature of the vacuum means that the expectation values of moduli has sent the Calabi-Yau manifold away from the R-symmetric point of its moduli space. It is then unnatural to turn on
only R-symmetric fluxes and obtain an R-symmetric effective superpotential. Our model does not affect the accuracy of the field counting method if we only consider R-symmetric SUSY vacua in the third branch, or string vacua with enhanced symmetries [40–44]. A recent exploration of new counterexamples [45] shows that such R-symmetry breaking feature of SUSY vacua is common in all currently known constructions. Whether other types of counterexamples exist is worth further investigations for an accurate survey of the string landscape.

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