An Experimental and Computer Simulation Study of Tailing Flow Attributed to A Dam Breach

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Abstract. The tailing flow generated after dam failure will evolve into Geo-hazards like debris flows. Therefore, it is of great significance to study the evolution and influence range of tailing flow for disaster prevention and mitigation. Physical model experiments and numerical calculation are two major methods in Dam-failure research. These two methods can verify and complement each other, because the former can modify parameters of the latter and the latter can expand the research scope. In this study, a proposed tailing dam was simulated and calculated by using physical model experiment and simulation (Ramms). The erosion process of the dam body of the tailing flow was monitored by the model experiments. The numerical calculation parameters were adjusted to make the results closer to the actual conditions. This study aim to verify the reliability and effectiveness of the debris flow Ramms based on Voellmy model in dam failure simulation. The experiment showed that the velocity (V_{max}=38.3 \text{ m/s}), submerge area (S_{max}=0.558 \text{ km}^2) and the deposition depth (H_{max}=36 \text{ m}) of the tailing flow. It reflected that the Voellmy model can predict the flow state of tailing flow in the physical model experiments. So it can provide theoretical support and technical reference for the design and management of tailing dam.

Keywords. Tailing flow, Ramms, simulation, model experiment.

1. Introduction
Tailings dam is a store facility to store byproducts of metal and minerals extracted from mined ore [1]. There are an estimated 3,500 active tailing dams worldwide, some of which have been breached by DAMS, threatening the safety of downstream residents and facilities, and causing environmental disasters [2-3]. For instance, a catastrophic tailings breach accident occurred on September 8, 2008, Xinta Mining co., LTD., Xiangfen County, China, killing 270 people and causing devastated environmental disaster [4]. A catastrophic tailing dam collapsed suddenly on January 25, 2019 in Brumadinho, Brazil, with over 300 people missing [5]. Accordingly, many countries have adopted legislation to regulate the construction and management of modern tailings ponds; it is stipulated that in the process of design, construction, operation and monitoring of tailings dams, strict evaluation must be made to prevent the environmental consequences of dam break of tailings pond [6-7].

To reduce the huge losses caused by tailings dam break, many scholars have conducted statistical analysis and tests to quantitatively assess the consequences of dam failure. There are three main research methods for dam breake: theoretical analysis, numerical simulation and model test [8]. Rico [9] analyzed 147 tailings dam failures worldwide; they proposed that the major causes of the instability of the dam include the high-steep dam (10-45 m), unusual rainfall and seismic liquefaction. Two hundred
eighteen tailings dam failures have been recorded, among which extreme rainfall was the dominant cause; nearly 76% of the tailings dam accidents occurred in the center-line tailings dam [10].

To facilitate prevention and mitigation of tailings dam failure, the formation mechanism of tailings dam failure was studied systematically in worldwide since the 1960s [11-12]. Numerical simulation has been used for prediction tailing dam environment evolution and safety; the research showed that the outflow distances, impact pressure and sediment depth were the key factors in the prediction of tailings dam failure [13-15]. The physical tailing dam failure models often act as methods to predict the three key factors. However, different numerical simulation methods adopt different mathematical models, with different calculation formulas and basic theories. For instance, different numerical methods and constitutive models were used to analyze the flow characteristics and discontinuities of complex tailings dam failure [16].

In order to clarify the movement law of dam-breaking debris flow and predict the movement distance and influence range of debris flow, many researchers have used numerical models from soft tools such as Ramms, Massflow, Dan3D, SPH and CFD to estimate its movement parameters [17-18]. The Voellmy friction model can simulate the remote movement of debris flow more accurately [19]. For the Voellmy model in RAMMS software to be calibrated, well-documented historical cases should be referenced to determine the best fitting set of parameters applicable to subsequent analysis. Besides, Ramms can also retrieve the results of DEM or DSM and modify terrain data, increasing the amount of additional parameters, and facilitating the prediction of the debris flow [20]. This article is to predict the movement law of the tailings flow in the dam-break of the tailing reservoir. First, a physical model was built for dam break model test to study the situation of tailings dam break, its influence range, movement speed of tailings flow and deposition thickness. Second, RAMMS software based on Voellmy friction model is used to numerically calculate the potential hazard area and the range affected by the disaster of the tailings dam's instability. The third is the comparative analysis of the experimental results. Finally, the critical parameters of tailings pond dam break and the dynamic and motion characteristics of tailings flow are presented.

2. Materials and Methods

2.1. DEM Data

DEM file is the most important input data in the pre-processing of numerical simulation, capable of precisely reflecting the noticeable terrain features of the tailing reservoir area. Digital elevation model (DEM) data applied here were extracted and converted from the survey geomorphologic data of tailing reservoir area using AUTOCAD software to yield a contour topographic map. Subsequently, CASS software and ArcGIS software were adopted to generate topographic files, triangular network files and raster files in the dam-break research area in sequence. Lastly, raster files underwent ASCII extraction to generate DEM files of the study area of tailings pond dam break under sophisticated topographic conditions. To integrate with DEM data, the three-dimensional geometric shape of tailings pond was built according to height, volume and surrounding topography of tailings dam (figure 1).

![Figure 1. Acquisition process of DEM data from the field investigation.](image-url)
2.2. Experimental Facilities

Physical model test has been extensively used in tailings dam break to simulate specific geomorphologic characteristics in microfilm chamber. In the present study, a 1:150 scale experimental model was built, with tailings similar to the original sand as the model test material. This material exhibits a small cohesion between the particles, making the model tailings pond destroyed by saturation quickly. The height variation of infiltration line in the tailing pond was monitored and regulated by the arrangement of plum flower-shaped seepage points. Since the study aimed to analyze and predict the damage consequences of the tailings dam, the water saturation time and the height of the infiltration line were tested to reach the flow of the dam break of the tailings pond. Using the experimental model of infiltration instability, a real demonstration is achieved for the occurrence, development and inundation range in the process of the dam failure of tailings dam transforming into mud and stone. Previous work has suggested that the wetting dam-break test exhibits a scale property similar to that of tailings reservoir catchment [21-22].

2.2.1 Tailing Prototype. The proposed tailings pond is located in Fujian Province of China. The designed maximum capacity of the tailings pond is $9291.4 \times 10^4$ m$^3$. Since this research area is close to the east China sea, Typhoons and rainstorms frequently occur from June to September. Thus, before the construction of tailings dam, the downstream loss caused by tailings dam break under extreme rainfall conditions must be fully considered, and the risk assessment of possible damage consequences should be prioritized.

The topography of the tailings area includes low hill and hills. The overall topography is high in the west and low in the east. The tailings area is surrounded by mountains with slopes of 25-45 degrees on both sides, the trending of valleys and ridges is from northwest to south-east, the elevation of tailing ditch ranges between 555 m and 855 m, and The elevation ranges of the watershed basin between 657 m and 1,100.7 m. The study site is characterized by sandy shale, slate and granite. As shown in figure 2, Shangdi village (0.7 km from the tailings dam), Shizhou Village (2.7 km), Laotulou village (3.35 km), Menkouan village (3.54 km), Shizhou Reservoir (5.8 km), Shizhou Hydropower Station (7.12 km), Beikeng village (7.98 km), Tulou Village (9.62 km) and Songxi Town (11 km) are located downstream of the tailings dam in sequence. A RAMMS numerical simulating method was referenced to verify the accuracy of the physical model test result.

![Figure 2](image-url)  
*Figure 2. The topography of tailings pond and downstream villages (Google Earth).*

2.2.2. Model Building. A physical model of tailings reservoir and the watershed basin was established at 1:150 scale. According to field investigation and DEM data, the model was set as six parts (mountain terrain, downstream terrain, water injection system, infiltration line control and measurement system, water and sand recovery system and monitoring system). Figure 3 suggests that the model was set as 35 m$\times$21 m$\times$3.3 m (length $\times$ width $\times$ height), representing the actual runout range from tailing pond to Shizhou village. The infiltration line control and measurement system was developed for flowing...
towards the downstream flume shown in figure 3 via a water injection system. Some systems of water pumps and piezometers were set up at the bottom of the tailings pond to measure the water infiltration pressure. Mud-flow waveform and velocity data were collected using a range of real-time monitor equipment and analysis software.

Figure 3. Physical model setup and dimensions

The regulation of saturated line variations was observed by installed the infiltration level monitor and flow monitor spots in the physical model of the tailings pond (figure 3(b)-3(c)). By RIEGL type 3D Laser Scanning (3DLS), Spark type Unmanned Aerial Vehicle (UAV) and 5KF20 type High-Definition Cameras System(HDCS), the tailing dam failure displacement was monitored; by the VDMS-LAB-04C type Surface Flow Field Real-Time Measurement System (SFFRTM), breach-flow changes and flow peak process were detected. The layout of the tailing dam failure monitor system is shown in figure 4.

Figure 4. Layout of the flow field monitor system.

2.3. RAMMS Method

The Ramms method has the core of an effective second order numerical solution of the depth motion equation of debris flow. The height and velocity of water flow are fully calculated on the 3D digital topography model, and block-release areas can be specified using the GIS mapping tool. Moreover, another approach is to use the input line to specify the flow as a function of time to provide the release area, debris-flow or stopping behavior for the users. Topographic maps and satellite cloud images can be superimposed on physical models to calibrate the runout properties (e.g., flow velocities, sediment depth, and flow distance). The physical model of RAMMS-DBF can be split into the friction resistance into a dry-Columb type friction (coefficient $\mu$) and viscous-turbulent friction (coefficient $\xi$). The frictional resistance $S$ is expressed as:

$$S = \frac{P_g U^2}{\xi} + \mu N$$

(1)
where \( \rho, g \) denote the density and gravitation acceleration, respectively; \( N=\rho g h \cos(\phi) \) is the normal stress on the running surface, \( h \) is the flow-height, the vector \( U=(U_x,U_y) \), which is composing of the flow-velocity in the \( x \) and \( y \) directions [23]. The magnitude of velocity is defined as:

\[
\|U\| = \sqrt{U_x^2 + U_y^2} \tag{2}
\]

Equation (2) can be expressed in an Voellmy-fluid friction model by a mass balance equation as:

\[
Q(x,y,t) = \partial_x (H U_x) + \partial_y (H U_y) \tag{3}
\]

where \( H \) denotes the flow height; \( Q(x,y,t) \) is the mass source term. If \( Q>0 \), it's termed as entrainment rate. when \( Q=0 \), there is no sediment erosion or deposition; when \( Q<0 \), it represents the deposition rate [24]. The average depth equilibrium equation of the fluid is written as:

\[
\begin{align*}
S_{gx} - S_{fx} &= \partial_x (C_x H U_x^2 + g \frac{k_{ap} H^2}{2}) + \partial_y (H U_x) + \partial_y (H U_y) \\
S_{gy} - S_{fy} &= \partial_y (C_y H U_y^2 + g \frac{k_{ap} H^2}{2}) + \partial_y (H U_x) + \partial_y (H U_y)
\end{align*} \tag{4}
\]

where \( C_x \) and \( C_y \) denote the profile coefficient; \( g \) is the gravity acceleration vertically; \( S_{gx}=g_x H \), \( S_{gy}=g_y H \) is the meaning of driving. The acceleration of gravity in the \( x \) and \( y \) directions, respectively; \( S_{fx}, S_{fy} \) are the friction of driving, gravitation acceleration in \( x \) and \( y \) direction, respectively, as expressed below:

\[
S_{gx} = n_{ux} \left[ \mu g_x + g \frac{\|U\|^2}{\xi} \right] \quad \text{and} \quad S_{fy} = n_{uy} \left[ \mu g_y + g \frac{\|U\|^2}{\xi} \right] \tag{6}
\]

In the Voellmy model, the contact relationship in the vertical direction can be defined as the heterogeneous Mohr-Coulomb relationship [24], where \( n_{ux} \), \( n_{uy} \) denotes the unit vector of velocity in the \( x \) direction, \( n_{ux} \), \( n_{uy} \) is the unit vector of the velocity is in \( y \) direction, and \( k_{ap} \) is the coefficient of soil pressure, as expressed below:

\[
k_{ap} = \tan^2 \left[ 45^\circ \pm \frac{\phi}{2} \right] \tag{7}
\]

where \( \phi \) stands for Angle of internal friction. \( \kappa/\rho \) is the active/passive earth pressure. Active area refers to the expansion flow area \( \nabla \cdot U \geq 0 \), the passive area is the compression area \( \nabla \cdot U < 0 \) [24]. Moreover, In addition to the Rankine theory, other methods of Savage and Hart were used to estimate the coefficient of earth pressure [25]. By using the above method the Voellmy-fluid formula can be obtained:

\[
\frac{d(U_h)}{dt} = (z \cdot n) h n - [(z \cdot n) h \mu + \frac{U^2}{\xi}] s - k(\nabla h) h \tag{8}
\]

where the variables are measured by length \( L \), velocity \((gL)/2 \) and time \((L/g)/2 \) to achieve a unified Froude value; \( k, \mu, s \) represents the ratios of anisotropic lateral earth pressure, effective coefficient of dynamic friction and direction of movement, respectively; Gravity vector is \( z=(0,0,-1) \), and the disturbance factor is defined as \( \xi=\xi/g \), which is dimensionless.

By sorting out equation (8) for gravity flow, the formula for calculating the fluid internal resistance in the viscous flow of gravity rock-soil mass can be yielded as:
\[ S = (1 - \mu)N_0 + \mu N + \frac{\rho g U^2}{\xi} - (1 - \mu)e^{\frac{-N}{\gamma}} \]  

(9)

where \( N_0 \) denotes the yield stress of the flowing tailings; \( \mu \) is a “hardening” parameter. Different from the standard Mohr-Coulomb type relation, this formula ensures that \( S \rightarrow 0 \) as \( N \rightarrow 0 \) and \( U \rightarrow 0 \). It increases the shear stress, the value of \( N_0 \) causes the tailing-flow to stop earlier. The fluid properties and 3d geometry Settings reconstructed for Ramms represent the results of physical experiments. The viscous/turbulent flow type parameter (\( \xi \)) and dry friction parameter (\( \mu \)) were set to 0.07 and 1500, according to the computing efficiency and memory size of the voellmy method. Finally, 1334514 computer flow cells and 1338190 fluid nodes were generated.

3. Results

3.1. The Deformation Development of the Experimental Tailing Dam Failure

Water is injected into the reservoir area to simulate overflow dam break, which occurs when 0.313 m³ of water is injected. The dam-breaking process results of the physical tailings model are shown in figure 5. As the water level was improved, the wide and the depth of the dam breach were up-regulated gradually, resulting in considerable debris flow, and eventually the downstream channel’s and part of the villages were submerged. According to similarity theory, the dam failure stopped at 12:12’12” on August 23 2018, it is suggested that the tailing flow has lasted for 20 min and 10 sec (4.1 h prototype). The width×depth of the model dam breach was about 1.5 m×1.4 m, indicating that the eventual deformation of width×depth of prototype tailing dam will be 225 m×210 m. The gully length and widest in the model tailing ponds area were nearly 2.8 m and 0.56 m, respectively, revealing that gully length and widest in prototype tailing ponds are nearly 420 m and 84 m.

![Figure 5. Dam failure process result of physical tailing model on 23 August 2018.](image)

From the UAV photo and 3D Laser Scan results, the physical tailings dam model failure is characterized by significant drop and high potential energy to produced artificial debris flow. After the dam break, considerable tailings were carried by the rushed flood from the tailing pond. When the artificial flow stopped, the total area of the submerged range was nearly 24.8 m², equivalent to 0.558 km² in the prototype (figure 6(a)). The total amount of tailings collapse was about 3.85 m³, nearly 1300×10⁴ m³ in the prototype, taking up 14% of the total tailing storage capacity (figure 6(b)). According to the calculation results of dam break by 3DLS, the tailing flow were primarily deposited in the dam foot area (Area 1), taking up about 36% of the total flow volume. The second sedimentary tailing flow was at the Shizhou village (Area 2), river gully (Area 3), Shangdi village (Area 4), and it finally flew into the downstream of the Shizhou village (Area 5) with the volume about 9.1% of the total flow material (figure 6(c)).
3.2. Sedimentary Thickness at Key Locations by Dem Data and 3Dls

According to the DEM data and 3DLS results of physical tailings dam failure experiment, the dam failure produced tailing flow, passing over 3 km through Shangdi village and Shizhou village. Figure 7 showed that the farther the distance from the dam foot, the smaller the siltation depth will be. To detect the siltation thickness along the gully, six key locations were taken from the cross section to measure and compare the flood inundation depth and tailings siltation depth. After the experiment ended, the inundation and deposition thickness of each section were measured (figure 8).

Figure 7. Profile of siltation depth downstream deduced by the model experiment measurement result.

Figure 8. Inundation and siltation thickness maps for each cross section.

Figure 8(a) shows that section 1# is located at the foot of the dam, and the maximum siltation depth thickness and siltation elevation reached 36 m and 600 m, respectively. Shangdi village is in the...
middle of section 2# (figure 8(b)) and section 3# (figure 8(c)), with siltation thickness of 18.9 m and 5.4 m, respectively. Compared with the elevation of houses in Shangdi village, all the houses would be submerged by the tailing flow. Sections 4#, 5# and 6# are located in the upper, middle and lower reaches of Shizhou village, respectively, suggesting that the houses at Shizhou village below the elevation 461 m would be submerged by the tailing flow (figures 8(d)-8(f)).

3.3. Ramms Simulation and Comparison with the Experiments
In the RAMMS-DBF simulation, as with the experimental landscape, topographic input data was quickly captured using the Cartesian coordinate system as the Digital Elevation Model (figure 9(a)). Based on the Voellmy-fluid approach, the unchanneled debris flow was used for tailing flow, the depth, elevation and average slope angle of tailings pond was defined as initial reference information of the RAMMS release area [26] According to figure 9(b), the gray “L” shaped area represents the calculation area containing critical topographic information, and the green area is the release area of the dam break movement, with a height of 54 m and a volume of $1.74 \times 10^7$ m$^3$. The simulation stopped when the momentum of all node elements was below 5% of the maximum value. According to a flume test, the numerical simulation parameters of the original tailing material were verified as listed table 1.

![Figure 9.](image)

**Figure 9.** (a) Topography map with elevation information from DEM data; (b) A comparison of experimental results with physical models.

| Input parameters of the RAMMS model | Simulation results by RAMMS |
|-------------------------------------|-----------------------------|
| Release depth 54.00 m              | Actual simulation end time 14616.00 s |
| Average slope angle 33.72°         | Release volume 17453124 m$^3$ |
| Elevation 695.57 m                 | Total maximum velocity 38,3477 m/s |
| Simulation end time set 1000 s     | Total maximum deposit height 126,363 m |
| Numerical solver second order accuracy | Total maximum Head pressure 2573.46 kPa |
| Curvature 1                        | Number of cells/nodes 1334514 / |
| Simulation grid resolution 2 m     | 1338190                      |
| GIF-Animation Interval 5 s         |                             |
| Simulation stop set Low flux equilibrium |                           |

Using the described method, the run-out slurry routing was modelled and predicted from the potential failure of a proposed ‘overhead tailing pond’ at the research area. The RAMMS numerical results in figure 10 reveal that the slurry flow velocity increased rapidly from the dam toe downstream in the first 30 s. Its peak value reached 28.47 m/s due to the steep slope under the dam toe. At t=100 s, the run-out slurry submerged the Shangdi village buildings and the river side farmland at a peak velocity 27.07 m/s, and it continued to rapidly flow downstream, passing through the Shizhou village at t=400 s. At t=1000 s, the slurry front flow moved from Shizhou village to the topography junction at the upstream of Shizhou reservoir, and the flow velocity of the debris flow was down-regulated. However, the velocity of the slurry flow remained high, and the slurry flowed downstream with a peak value 26.13 m/s.
To predict the inundation depth of dam break flood along the downstream, the submerged depth, velocity at the six key position in the study area were compared with the physical model experiment result. Such comparison revealed that the slurry sedimentation thickness deduced by the physical mode experiment based on the 1:150 scale was less deep than the simulation results in the upstream of the section 4#, whereas it was deeper in the downstream of the section 4#. For the safety of downstream communities and residents, the maximum value between the two was taken as the recommended value for the safety evaluation of the proposed tailing pond (figure 11).

The maximum velocity distribution of tailings in the study area is illustrated in figure 12. In the low-lying of the tailings pond, the velocity of tailings in the blue area was very slow, and the velocity of tailings in the relatively high lying area on both sides was very high with $V_{\text{max}} = 38.3$ m/s, indicating that the movement of tailing was accelerated to the low-lying area in the middle of gully at the early stage of the dam failure. The high velocity appeared at the toe of tailing dam, whereas the high velocity area was significantly down-regulated compared with the upstream area of the tailing pond. The velocity of slurry in Shangdi village decreased to 20.2 m/s compared with the toe of the tailing dam, indicating that the kinetic energy of slurry dissipated obviously. After passing several “gourd-shaped” topographical junctions downstream of Shangdi village, the area with a large flow velocity was further narrowed, the velocity decreased to 13.3 m/s, and the velocity of tailings near Shizhou village was below $V \leq 8$ m/s.
4. Conclusions
In this study, a physical model experiment based on DEM data and a 3D field-scale numerical modelling method in accordance with Voellmy theory and the advanced rapid mass movements simulation (Ramms) method was performed to study the tailing slurry flows through the downstream terrain, for the purpose of determining the impact range of tailings failures. Compared with result from physical experimental test and numerical simulation, a recommended value for the safety evaluation of the tailing pond was proposed.

1) The physical modeling experiment of the proposed tailing pond was rebuilt and verified using the Ramms method. The results showed that this method can effectively solve the problem of sediment crossing the actual terrain of tailings dam break. Parameters (e.g., flow velocities, move distance, sediment depths, as well as impact area) should be obtained for further analysis.

2) Modeling of tailing pond in mountain area suggested that if tailings pond dam break when the reservoir is full, the time of tailings flow bursting and releasing would be extremely short. The numerical simulation results of different debris flow locations in tailings showed that the value of pore model parameters has a significant influence on the simulation results. Further analysis and assessments should be adopted before the building and the management of these “overhead tailings ponds”.

3) It is considered that the velocities of the slurry induced by the tailings dam failure under the heavy rainfall are relatively high. Drainage measures were adopted to reduce the effect of water pressure on stability of tailings pond. If water level is allowed to rise uncontrollably over a tailings dam, the stored tailings can be penetrated by gulling process in a very short period of time.

4) The results could evidence safety evaluation and mitigation engineering design before the tailing pond construction, emergency management plan. Ramms is being developed, so it can better model the 3D topography tailings dam failure. The laboratory-scale physical model experiment and computer modelling also allow geological information development to be examined over time, which is difficult in the field.

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