Stiffness modelling of flexible support module for Large-aperture laser transmission unit

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Abstract. Large-aperture laser transmission unit (LLTU) is a device that focuses the laser beam to the center of the target, which are often designed as compliant mechanisms to achieve micro displacement adjustment. In the traditional mechanisms, they are designed as integrated micromanipulation systems, and driven by piezoelectric ceramics. However, most of these researches only focuses on motion accuracy, due to the lack of consideration of large load problems, the application is greatly limited. To this end, a flexible support module (FSM), as well as its stiffness model, was presented in this paper. Combined with finite element method (FEM) of FSM, structure size optimization was also completed, successfully solved the problems of stress concentration and load of FSM in engineering application. Moreover, a dual vision-based measurement method was introduced, to verify the stiffness model and analyze the repetitive error of FSM. From this result, the prototype enabled 5 mm and 0.007 rad of working area with average error of 0.3192 mm and -0.0036 rad. The repeatable error is within 7%, and will decreased to 4% with internal stress released in 5~15min.

1. Introduction
Large-aperture laser transmission unit (LLTU) is a device that focuses laser beams into the target centre with high precision, it is widely used in high-energy laser facilities, such as ShenGuang III (SG-III) [1] in China, National Ignition Facility (NIF) [2] in USA, and laser megajoule (LMJ) [3] in France. Generally, considering the requirements of high precision and high load, LLTU is designed as a compliant parallel mechanism (CPM). However, in most existing research cases, CPMs are designed as integrated mechanisms, and most of them only focus on motion accuracy, such as cell microinjector [4], fast steering mirror [5], micro positioning stage [6-8], which are difficult to transplant to high load compliant engineering.

Compared with integrated CPM, flexible support module (FSM) has more advantages in precision engineering applications, such as higher DOF and replaceability, which makes it more applicable for CPM research of LLTU. As the passive deformation unit of CPM, FSM usually provide the micro displacement via parallel installation. In the LLTU [9] designed by laser megajoule (LMJ) in France, the optic component was connected on both sides by elastic stems, and supported by stepping actuators. Different from LMJ, the optical frame of NIF [10] is constrained by three FSM, which were mounted on the back of the optical assembly, and axially point to the rotation centre of the optic component. The FSM developed in the NIF is a common structure, it can achieve high DOF by staggered series of multiple flexible hinges. This kind of structure has perpendicular DOF, so it is also

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called as flexure hook hinge. However, due to the need of confidentiality, these studies did not analyse the modelling and control process of the mechanism.

FSM were usually designed for higher degrees of freedom, such as U-shaped flexible joints [11], and spatial joint compliant module (SJCM) [12,13] developed by SIMTech, which are composed of several flexure hinges. According to different flexibility requirements of CPM, flexure hinges are designed into various shapes, including circular hinge [14], filled leaf hinge [4,15] and elliptical hinge [16]. To model these hinges, two most methods are widely used, namely finite element method (FEM) [17] and analytical modelling method [18-19], Pham et al. [20] presented an analytical stiffness modelling of CPM in six directions, and used the described model to establish the stiffness matrixes of a double linear spring and a three-degree-of freedom (DOF) translational CPM. Byung et al. [14] designed a planar 3-DOF parallel mechanism based on circular hinge, and analysed inverse kinematics, internal kinematics, and analytic stiffness modelling referenced to the task coordinate. Chen et al. [21] presented an analytical stiffness of CPM in six directions, and used the described model to establish the stiffness matrixes of a double linear spring and a three-degree-of freedom (DOF) translational CPM. Byung et al. [14] designed a planar 3-DOF parallel mechanism based on circular hinge, and analysed inverse kinematics, internal kinematics, and analytic stiffness modelling referenced to the task coordinate. However, the work mentioned above rarely optimizes the structural size parameters according to the engineering application requirements, which makes their non-working direction stiffness insufficient, and cannot be normally applied in heavy load precision engineering. In some follow-up studies, many novel flexure hinges have been proposed, such as bionic flexible joints [23,24] and corrugated flexure elements [25], which were originally designed to improve the working range of flexure hinges, and did not take the aforementioned issues into consideration.

2. Design of flexible support module
The traditional flexure hinge with the filled leaf can achieve the bending deformation to a certain extent, however, under the conditions with the encountered heavy load, the stress concentration will occur, and most of these stresses are distributed at both ends of the filled leaf, as shown in Fig.1 (a), the maximum concentrated stress is 3518.4MPa, which may cause the fatigue failure of the flexure hinge. For the case of stress concentration problem, an elliptical reinforced flexible hinge (ERFH) was designed, with the profile shown in Fig.4. The results of finite element analysis in Fig.1 (b) show that it can disperse the concentrated stress to the filled leaf, effectively reduce maximum stress to 1754.7MPa.

![Figure 1](image-url)  
Figure 1. (a) Finite element analysis of straight beam flexure hinge, and (b) elliptical reinforced flexure hinge

Flexible support module (FSM) is designed based on ERFH, as shown in Fig.2, it is composed of four groups of flexure hinges in staggered series. For each group of flexure hinges, two ERFHs was in parallel and in the same direction, which can transform the moment in X-direction into normal stress acted on each ERFH, to improve the stiffness in the non-working direction. Different from most existing integrated CPM, the design of FSM also needs to meet the requirements of load in some precision engineering. Therefore, factors such as working flexibility, non-working direction stiffness
and stress concentration will be fully considered in structural size parameters optimization process, corresponding detailed analysis will be given in Section 3

Figure 2. Model of flexible replaceable module optimized with ERFH

3. FSM stiffness analysis
Stiffness modelling is the basis of realizing the dynamic control of compliant mechanism, which gives the linear mapping between the driving force and small displacement, the flexible support module (FSM) is modelled analytically. Elliptical reinforced flexible hinge (ERFH), as the basic deformation unit of FSM, its compliance matrix is a basis to calculate structural stiffness, which will be analysed in detail in section 3.2.

3.1. FSM stiffness model
Assume a flexure hinge as a six degree-of-freedom beam with one end fixed, and subjected to bending, axial loading, shearing, and torsion. By defining the force actuating on the hinge by

$$ F = \begin{bmatrix} F_x & F_y & F_z & M_x & M_y & M_z \end{bmatrix}^T $$

and the corresponding deformations of the hinge by

$$ \Delta = \begin{bmatrix} \Delta_x & \Delta_y & \Delta_z & \alpha_x & \alpha_y & \alpha_z \end{bmatrix}^T $$

according to the Hooke's law, the relationship between the displacement and applied force of the flexure hinge can be expressed by

$$ F = K \cdot \Delta $$

where $K$ represents the stiffness matrix of compliant mechanism, and is the inverse of the compliance matrix, $C$.

As shown in Fig.2, the calculation of the compliance matrix of FSM can be regarded as four groups of flexure hinges in staggered series. In order to simplify the calculation process, each group of flexure hinges can be simplified and merged into one. Therefore, the overall compliance matrix of FSM can be calculated by series formula as [26]:

$$ C_e = \sum_{i=1}^{n} Ad_i \cdot C_i \cdot Ad_i $$

where $Ad_i \cdot C_i \cdot Ad_i^T$ is the adjoint transformation of compliance matrix, it represents the transformation of reference coordinate system. Assuming that the compliance matrix of the structural member in coordinate system $\{A\}$ is $C_A$, and the adjoint matrix from coordinate system $\{A\}$ to $\{B\}$ is $Ad_1$, then the compliance matrix $\{C\}$ in coordinate system $\{B\}$ can be defined as:

$$ C_s = Ad_i^T \cdot C_i \cdot Ad_i $$

with adjoint transformation and its transpose:

$$ Ad(\xi) = \begin{bmatrix} R & 0 \\ iR & R \end{bmatrix}, \quad Ad^T(\xi) = \begin{bmatrix} R^T & -R^T i \\ 0 & R^T \end{bmatrix} $$
where \( \mathbf{R} \) and \( \mathbf{t} \) represent the rotation and translation transformation from \( \{A\} \) to \( \{B\} \) respectively, the skew matrix of the translation vector, \( \hat{\mathbf{t}} \), defined as:

\[
\hat{\mathbf{t}} = \begin{bmatrix}
0 & -t_3 & t_2 \\
t_3 & 0 & -t_1 \\
-t_2 & t_1 & 0
\end{bmatrix}
\]

(7)

Therefore, for the FSM coordinate system shown in Fig.2, the kinematic relationship between them can be established as shown in Tab.1.

| Flexure hinge | Kinematic transformation | Parameter values (mm) |
|---------------|--------------------------|------------------------|
| 1             | \( R_1 = [0; 0; 0] \), \( P_1 = [p; 0; 0] \) | \( p = 88.18 \) |
| 2             | \( R_2 = [90^\circ; 0; 0] \), \( P_2 = [p; 0; 0] \) | \( p = 88.18 \) |
| 3             | \( R_3 = [0; 0; 0] \), \( P_3 = [q; 0; 0] \) | \( q = 36.82 \) |
| 4             | \( R_4 = [90^\circ; 0; 0] \), \( P_4 = [q; 0; 0] \) | \( q = 36.82 \) |

As aforementioned, each set of ERFH is considered as a combined flexure hinge in the simplified calculation. The combined flexure hinges will have the same structure. Accordingly, the compliance matrix of ERFH in their respective reference coordinate systems, \( \mathbf{C}_i \), will be the same, and have the following form [27]:

\[
\mathbf{C}_i = \begin{bmatrix}
\frac{\Delta_x}{F_x} & 0 & 0 & 0 & 0 \\
0 & \frac{\Delta_y}{F_y} & 0 & 0 & \frac{\Delta_y}{M_y} \\
0 & 0 & \frac{\Delta_y}{F_y} & 0 & \frac{\Delta_y}{M_y} \\
0 & 0 & 0 & \frac{\alpha_x}{M_x} & 0 \\
0 & 0 & \frac{\alpha_y}{F_y} & 0 & \frac{\alpha_y}{M_y} \\
0 & 0 & 0 & 0 & \frac{\alpha_y}{M_y}
\end{bmatrix}
\]

(8)

Each element in the compliance matrix is called a compliance factor, which represents the influence coefficient of displacement and angular deformation in different directions on force and moment.

3.2. Compliance Equations of ERFH

![Figure 3. Analysis of the elliptical reinforced flexible hinge profile](image)
The closed-form equation for compliance factors can be derived based on the Euler–Bernoulli beam theory. In order to deduce the compliance factor of the closed-form equation, the profile curve of ERFH should be formulated first, as shown in Fig.3, it can be described by piecewise equation,

\[
y(x) = \begin{cases} 
-\frac{t}{2} & (0 < x < b) \\
\sqrt{b^2 - \frac{b^2}{a^2}(x-a)^2} - \frac{b-t}{2} & (b \leq x < a+l) \\
\sqrt{b^2 - \frac{b^2}{a^2}(x-a-l)^2} - \frac{b-t}{2} & (a+l \leq x < 2a+l) 
\end{cases}
\]  

By parameterizing the independent variables, and applied Euler–Bernoulli beam theory, the closed-form equations of compliance factors can be derived, its corresponding detailed derivation process can be seen in the appendix website (http://jiabinpan.work/Publications/AnlsRFH.html), here, the results are given directly,

\[
\frac{\alpha_y}{M_y} = \frac{12l}{Ewt} - \frac{3a}{Ew} \cdot N_1 
\]

\[
\frac{\alpha_y}{F_y} = \frac{\Delta_y}{M_y} = \frac{6l(2a+l) - 3a(2a+l)}{Ewt} \cdot N_1 
\]

\[
\frac{\alpha_y}{F_y} = \frac{12l}{Ewt} - \frac{12a}{Ew} \cdot N_2 
\]  

\[
\frac{\alpha_x}{M_x} = \frac{\Delta_x}{M_x} = \frac{6l(2a+l)}{Ewt} \cdot N_1 + \frac{7a}{2Gw} \cdot N_2 + \frac{7l}{2Gwt} + \frac{7l}{2Gwt} \cdot N_4 
\]

\[
\frac{\Delta_y}{F_y} = \left[ \frac{2l}{Ewt} \frac{(2a+l)(3a+l)}{Gwl} \right] - \left[ \frac{6a}{Ew} \left( \frac{(2a+l)(3a+l)}{Gwl} \right) + \frac{ka}{Gw} \cdot N_1 + \frac{3a^2l}{Ew} \cdot N_2 + \frac{3a^2l}{Ew} \cdot N_3 - \frac{3a^3}{Ew} \cdot N_4 
\]

\[
\frac{\Delta_x}{F_y} = \frac{l}{Ewt} - \frac{a}{Ew} \cdot N_2 
\]  

where \(N_1\sim N_6\) is complex integral terms, which can be obtained via software, \textit{Mathematica},

\[
N_1 = \int_0^\frac{\pi}{b} \frac{\cos \theta}{(b \cos \theta - b - t/2)^4} \cos \theta d\theta = -8 \left( \frac{t^2 + 4bt + 6b^2}{(4b + t)^5} \cdot \frac{6b(2b + t)\arctan(\sqrt{4b/t} + 1)}{t^5 (4b + t)^6} \right) 
\]

\[
N_2 = \int_0^\frac{\pi}{b} \frac{\cos \theta}{b \cos \theta - b - t/2} \cos \theta d\theta = \frac{\pi}{2b} - \frac{2(2b + t)}{b^2 \sqrt{4b + t}} \arctan(\sqrt{4b/t} + 1) 
\]

\[
N_3 = \int_0^\frac{\pi}{b} \frac{\sin \theta}{b \cos \theta - b - t/2} \cos \theta d\theta = \frac{1}{b} - \frac{2b + t}{2b^2} \log \left( \frac{2b}{t} + 1 \right) 
\]

\[
N_4 = \int_0^\frac{\pi}{b} \frac{\cos \theta}{b \cos \theta - b - t/2} \cos \theta d\theta = \frac{3\pi + 4}{4b} + \frac{(\pi + 1)t}{2b^2} - \frac{6b^2}{8b^3} + \frac{(2b + t)^3}{2b^2 \sqrt{4b + t}} \arctan(\sqrt{4b/t} + 1) 
\]

\[
N_5 = \int_0^\frac{\pi}{b} \frac{\sin \theta}{b \cos \theta - b - t/2} \cos \theta d\theta = \frac{t - 2b}{b^2} - \frac{1}{b^3 (t + 2b)} 
\]
\[ N_e = \int_0^\pi \frac{\cos^2 \theta}{b \cos \theta - b - t/2} d\theta = \frac{\pi}{2b^2} \left[ \frac{2(2b+t)(6b^2-4bt-t^2)}{b^2 t^2 (4b+t)^2} \right] - \frac{2(2b+t)(24b^4-8b^3 t+14b^2 t^2+8bt^3+t^4)}{b^2 t^2 (4b+t)^{3/2}} \arctan \sqrt{4b/t+1} \]  

(23)

and \( E, G, \mu \) are the modulus of elasticity, the modulus of rigidity and Poisson ratio, respectively. The shear coefficient, \( k \), for the micro-beams with rectangular cross-section, is employed as [29]:

\[ k = \frac{12+11\mu}{10+10\mu} \]  

(24)

4. Optimal RFH structural parameters

There are two kinds of structural parameters for reinforced flexure hinges. The first is related to the elliptical profile, Category I \((a \text{ and } b)\), which is regarded as the parameters that determine the degree of stress concentration, and the second is main size parameters of flexure hinges, Category II \((l, w \text{ and } t)\), which is used to optimized to make the FSM meet the design goal, that is, when the compliance of the ERFH in the working direction meets the requirements, the compliance in the non-working direction need to be as large as possible. To optimized Category I and II, the parameters were limited in a certain range, as shown in Tab.2.

| Table 2. Range of structural parameters to be optimized |
|---|
| type | symbol | range |
| edge profile parameters (I) | \( a \) | 1 – 4 |
| | \( b \) | 1 – 3 |
| main size parameters (II) | \( l \) | 3 – 6 |
| | \( w \) | 8 – 12 |
| | \( t \) | 2 – 4 |

For parameters Category I, the optimization can be completed via finite element models (FEMs). ANSYS Workbench 19.2 is used to analyse the maximum stress for different flexure hinges, which are subjected to the external load \((Mx = 19.5 \text{ N/m}, My = 45 \text{ N/m})\), and have the same material properties \((\rho = 2770 \text{ kg/m}^3, E = 7.1e + 10, \mu = 0.33)\). The stress analysis results of ERFH are shown in Tab.3. The increase of parameters \( a \) and \( b \) will both reduce the concentrated stress. Compared with parameter \( b \), the effect of parameter \( a \) is more significant, especially when \( a=4 \), the influence of \( b \) on the maximum stress concentration is already very small. Therefore, \( a=4, b=2 \) (moderate value) are chosen as the parameters of the elliptical profile. At the same time, by observing the data in Tab.3, it is found that when \( a \geq 3 \), the maximum stress change is not so significant, which is due to the concentrated stress is dispersed to the straight beam section, and is distributed in strips (Fig.1 b).

| Table 3. The maximum stress (MPa) of flexure hinge with different ellipse parameters |
|---|
| \( a \) | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| \( b \) | 1 | 2111 | 1881.7 | 1786.5 | 1750.4 | 1735.6 | 1730.2 | 1728.6 |
| | 1.5 | 2088.2 | 1974.7 | 1847.3 | 1787.7 | 1760.0 | 1746.5 | 1741.6 |
| | 2 | 2187.1 | 2040.6 | 1906.3 | 1823.3 | 1784.6 | 1763.9 | 1754.7 |
| | 2.5 | 2293.5 | 2076.1 | 1947.4 | 1862.5 | 1808.7 | 1781.6 | 1768.2 |
| | 3 | 2365.9 | 2127.7 | 1991.8 | 1890.5 | 1837.4 | 1800.4 | 1780.2 |
Figure 4. Variation trend of compliance factors, $C_{y}^{F}$ and $C_{y}^{M}$, in terms of $t$, $w$ and $l$.

Figure 5. Compliance factors, $C_{x}^{F}$ and $C_{z}^{F}$, in terms of $w$ and $l$.

Among the Category II ($l$, $w$ and $t$), thickness $t$ has a much higher influence on the compliance of ERFH in working direction, as shown in Fig.4. In order to ensure the overall flexibility of FSM to meet the design requirements, the parameter $t$ needs to be determined first. In this process, the other two parameters $l$ and $w$ of ERFH are tentatively determined as the median value ($\beta = 0.5$) within the range, that is, $l = 4.5$ and $w = 10$. In the case of this paper, the designed FSM is needed to provide a displacement of 5mm at Y-direction under external load $M_x = 19.5$ N/m, $M_y = 45$ N/m, which requires that the compliance factor of each ERFH should meet $C_{y}^{M} = \frac{\Delta y}{M_{y}} \geq 7.39 \times 10^{-6}$, therefore, we determine $t = 3$ as the thickness of ERFH. Under this condition, parameters $l$ and $w$ should ensure that the stiffness of ERFH in the non-working direction is as high as possible. From the trend diagram of the compliance factor in the $x$ and $z$ directions with respect to $l$ and $w$, as shown in Fig.5, the values of $l$ and $w$ are selected as 3mm and 12mm respectively.

5. Experimental Verification

In order to further verify the stiffness model, and analyse the repetitive working error of FSM, an experimental platform based on the visual measurement was designed. As shown in Fig.6, an aluminium alloy plate, attached with 10mm×10mm chessboard, is fixed at the end of FSM, and its displacement is measured by two fixed 2D cameras. In terms of the external varied force and moment required by FSM, the force gauge is placed on the back of the aluminium alloy plate, and driven by a 3-DOF slider module. By adjusting the position of the driving point and the force of gauge, the equivalent moment acting on the centre of FSM will be changed. In this experiment, the force acting point is set at a distance of $D_y = -40$mm and $D_z = 160.75$mm from the centre of FSM, and the force gauge acts on the aluminium alloy plate with a driving force of 15N in steps to obtain chessboard images of FSM under different external forces.
As mentioned before, the displacement of FSM is measured by camera calibration [30], which has two advantages: one is to avoid the error caused by the installation of the measuring device; the other is that camera calibration is non-contact measurement, which can eliminate the influence of contact error. Fig.6 show the relationship between camera calibration and displacement measurement of compliant mechanism. The deformation displacement of FSM can be expressed by HTM between flange frames \( \{ F \} \),

\[
H_F = (A \cdot C)^{-1} \cdot (B \cdot C)
\]  

(25)

By transforming the \( H_F \) into vector form, the deformation displacement of FSM can be obtained. In these results, we mainly focus on the deformation in the direction of its working DOFs, Y and Z axis. For the direction of X axis, however, it is mainly caused by the parasitic motion rather than the deformation, which will not be considered when verifying the FSM flexibility matrix.

The experimental results, displacement of FSM under different driving forces, are shown in Fig.7, which indicate that FSM has good repetitive working accuracy in the linear direction of Y axis and angular direction of Z axis, its mean error is 0.0077 mm and -2.2067e-04 rad respectively, which is better than the linear direction of Z and angular direction of Y, 0.3192 mm and -0.0036 rad. For the deviation between the calculated and measured values, we can reflect it by the corresponding compliance factor. The average measured linear compliance in Y axis, \( \frac{\Delta y}{M_y} \), is 2.249694e-01, and -2.651412e-01 in Z axis, its calculated value are both (-) 2.748678e-01. Correspondingly, the angular compliance, \( \frac{a_y}{M_y} \) and \( \frac{a_z}{M_z} \), the average measured value are 3.708488e-03 and 4.124847e-03 respectly, calculated value is 4.133350e-03. Above results show that the compliance matrix of FSM is well in line with the stiffness test of FSM.

Why is the repeatability in the linear direction of Y axis and angular direction of Z axis much lower? It is not difficult to find that the moment \( My \) acting on FSM is much larger than \( Mz \), which leads to a greater stress in the corresponding ERFH. After the withdrawal of the external force, FSM is likely to cause errors due to the internal stress is not completely released. To verify this interpretation, we continue to measure the position and orientation variation of the FSM in these directions, as shown in Fig.10, \( t=0 \) represents the repetitive error of FSM final state without external load in Fig.8 experiment, that is, the linear displacement of Z-axis, -0.5707 mm, and the angular displacement of Y-axis, 0.0054 rad. After 5 min, the repeatability decreased rapidly, and its value reached to -0.1805 mm and 0.0025 rad at 15 min, which confirms the previous explanation of the results in Fig.9.
6. Conclusions
This paper presents a flexible support module (FSM) for compliant parallel mechanism, which can be used in precision engineering with high precision and large load, such as large aperture laser transmission unit. In this mechanism, the stiffness model of FSM is established based on the analysis of the elliptical enhanced flexure hinge, and its size parameters are optimized based on the finite element analysis. Moreover, to verify the stiffness model and analyse the repetitive error of FSM, a vision-based measurement method is introduced in this paper. The results show that the prototype can satisfy the application of the average error of 0.3192 mm and -0.0036 rad in the working space of 5mm and 0.07 rad, which is mainly caused by the lag of internal stress release, after the external force is removed, the repeatability error can be reduced to 4% rapidly in 5-15 minutes.

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