Evolution of jets driven by relativistic radiation hydrodynamics as Long and Low Luminosity GRBs

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ABSTRACT

We present the three-dimensional numerical simulation of jets modeled with Relativistic Radiation Hydrodynamics (RRH), that evolve across two environments: i) a stratified surrounding medium and ii) a 16TI progenitor model. We consider opacities consistent with various processes of interaction between the fluid and radiation, specifically, free-free, bound-free, bound-bound and electron scattering. We explore various initial conditions, with different radiation energy densities of the beam in hydrodynamical and radiation pressure dominated scenarios, considering only ultra-relativistic jets. In order to investigate the impact of the radiation field on the evolution of the jets, we compare our results with purely hydrodynamical jets. Comparing among jets driven by RRH, we find that radiation pressure dominated jets propagate slightly faster than gas pressure dominated ones. Finally, we construct the luminosity Light Curves (LCs) associated with all these cases. The construction of LCs uses the fluxes of the radiation field which is fully coupled to the hydrodynamics equations during the evolution. The main properties of the jets propagating on the stratified surrounding medium are that the LCs show the same order of magnitude as the gamma-ray luminosity of typical Long Gamma-Ray Bursts $10^{50} - 10^{54}$ erg/s and the difference between the radiation and gas temperatures is of nearly one order of magnitude. The properties of jets breaking out from the progenitor star model are that the LCs are of the order of magnitude of low-luminosity GRBs $10^{46} - 10^{49}$ erg/s, and in this scenario the difference between the gas and radiation temperature is of four orders of magnitude, which is a case far from thermal equilibrium.

Key words: gamma-rays: general - methods: numerical - opacity - radiative transfer.

1 INTRODUCTION

There is observational evidence that long gamma ray bursts (LGRBs) are produced after the death of massive stars (Woosley 1993; Galama et al. 1998; Stanek et al. 2003; Hjorth et al. 2003; Woosley & Bloom 2006), whose spectrum agrees with those of type Ic supernovae (SNe). Although the LGRBs have been identified spectroscopically with SNe, many of these events show smaller luminosity than those of standard LGRBs. These events are called low-luminosity GRBs (LLGRBs). The emission mechanisms and the surrounding medium density profiles that make the difference between LGRBs and LLGRBs are still a matter of debate. As a consequence of this, several authors have developed numerical models for the jet propagation in a stratified surrounding medium applied to LGRBs and massive star models applied to LLGRBs.

In this context, several jet numerical studies have been done. For instance, the evolution of jets within a surrounding medium using different approximations has been studied. In a first approximation, in (MacFadyen & Woosley 1999; Aloy et al. 2000; Zhang et al. 2003; Zhang et al. 2004; Mizuta et al. 2006 & 2013; Morsony et al. 2007; Lazzati et al. 2013; De Colle et al. 2012; Lopez-Camara et al. 2013; De Colle et al. 2017; Obergaulinger & Aloy 2017). However, in some previous works the radiation is not coupled with hydrodynamics during the evolution of the system, and
the radiation is attached to the system by post-processing the numerical simulations of hydrodynamics.

From a theoretical point of view, in the RRH simulations there are two key variables at modeling the system, these are: the rest-mass density of the fluid and the radiation energy density of the radiation field, which in turn has an impact on the optical depth of the fluid and the radiation pressure (Mihalas & Mihalas 1984). In accordance with the later, the radiative transport does not significantly affect the fluid dynamics as long as the optical depth is in the optically thin regime or the fluid pressure dominates. In this case, the radiation attached to the system in a post-processing of the numerical simulations is a good approach because emitted photons can be assumed to escape with no further interactions. Otherwise, in the optically thick regime or in the radiation pressure domination scenario, the effect of radiative transport cannot be negligible because the photons carry significant momentum and energy that affect the dynamics of the fluid around.

In this case, the radiation and fluid feed back each other’s dynamics and we need to solve the equations of radiative transport during the evolution at the same time as Euler’s equations. In view of the above, and under the assumption that the radiation carried by jets goes from an optically thick region to an optically thin region, coupling the radiation field and the fluid during the evolution, is expected to be a better approximation than post-processing or pure hydrodynamics. Therefore, in this paper we assume a model of the jet where the fluid is coupled to the radiation field and the evolution is dictated by the RRH equations. In our numerical simulations we assume, initially, that the radiation field and matter are in local thermal equilibrium and evolves according to the RRH equations. However, after initial time, the system loses the local thermal equilibrium. We explore various initial conditions for the jet, with different radiation energy density and a fixed but high Lorentz factor.

In order to describe GRB events in a more realistic astrophysical scenario, we study the dynamics of the jet when it propagates in two different environments. In the first one, the jets propagate in a stratified external medium with rest mass toy density profile $\rho \sim r^{-2}$ which is associated to LGBRs (Mignone et al. 2005). In the second one, the jet propagates within its progenitor star. For the later, we consider a pre-supernova 16T1 model as the progenitor star which is considered to be a progenitor of LLGBRs. In each one of the models we construct the LCs associated with particular processes of interaction between the fluid and radiation, specifically free-free, bound-bound and electron scattering opacities.

The paper is organized as follows. In section 2 we describe the system of the RRH equations. In section 3 we describe the initial conditions for the parameters of our simulations. In section 4 we study the evolution of jets on a stratified medium and illustrate how the LCs can be associated to LGBRs. Also we discuss the implications of the radiation field in the evolution of the system. In section 5 we present the evolution of jets starting from inside a progenitor and their relation to LLGBRs. Finally, in section 6 we discuss our results.

2 EQUATIONS OF EVOLUTION AND NUMERICAL METHODS

As mentioned before, the model assumed for the jet corresponds to a fluid interacting with a radiation field. This implies the need to solve the equations coupling such a system in order to capture the back-reaction of one of the components onto the other. One important advantage of considering the radiation field is that the construction of LCs is natural because one is constantly calculating the variables of the radiation field. The RRH equations governing the evolution of this system are (Farris et al. 2008; Fragile et al. 2012):

\[ \nabla_a (\rho u^a) = 0, \]
\[ \nabla_a T_{m}^{\alpha \beta} = G_r^\beta, \]
\[ \nabla_a T_{r}^{\alpha \beta} = -G_r^\beta, \]

where $T_{m}^{\alpha \beta}$ is the stress-energy tensor of a perfect fluid

\[ T_{m}^{\alpha \beta} = \rho u^\alpha u^\beta + P g^{\alpha \beta}, \]

where $g^{\alpha \beta}$ is the metric of the spacetime, $u^\alpha$ is the four-velocity of fluid elements, $\rho$, $h = 1 + \epsilon + P/\rho$, $\epsilon$ and $P$ are the rest-mass density, specific enthalpy, specific internal energy and the thermal pressure, respectively. The thermal pressure is related to $\rho$ and $\epsilon$ through a gamma-law equation of state $P = \rho (\Gamma - 1)$, where $\Gamma$ is the adiabatic index of the fluid. Here $T_r^{\alpha \beta}$ is the stress-energy tensor that describes the radiation field and is given by

\[ T_r^{\alpha \beta} = (E_r + P_r) u^\alpha u^\beta + F_r^\alpha u^\beta + u^\alpha F_r^\beta + P_r g^{\alpha \beta}, \]

where $E_r$, $F_r^\alpha$, and $P_r$ are the radiated energy density, radiated flux, and radiation pressure, respectively, measured in the comoving reference frame. The source term $G_r^\alpha$ is the radiation four-force that describes the interaction between the fluid and the radiation field. Among the various regimes of coupling between radiation and fluid, we choose the “gray-body” approximation, which technically means that the radiation field variables do not depend on its frequency (Mihalas & Mihalas 1984). In this case the radiative four-force is given by (Farris et al. 2008)

\[ G_r^\alpha = \chi^\alpha (E_r - 4\pi B) u^\alpha + (\chi^1 + \chi^\gamma) F_r^\alpha, \]

with $\chi^\gamma$ and $\chi^\gamma$ the coefficients of thermal and scattering opacities, respectively. Finally $B = \frac{1}{2\pi} a_r T_{\text{fluid}}$ is the Planck function, $T_{\text{fluid}}$ the temperature of the fluid and $a_r$ the radiation constant.

The above set of equations is completed with a closure relation that identifies the second moment of radiation with one of the lower order moments. The simplest approach is the Eddington approximation, which assumes a nearly isotropic radiation field and in the fluid frame shows a pressure tensor with the form $P_r^{ij} = \frac{1}{2} E_r \delta^{ij}$ (Mihalas & Mihalas 1984). This assumption is valid only in the optically thick regime within the diffusion limit. The radiation field in the optically thin regime requires a more general assumption. A scheme that allows a description of the radiation field in both optically thick and thin regimes is the M1 (Levermore 1984)
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where $f_i^r = F_i^r/cE_r$ is the reduced radiative flux and $\zeta = \frac{3f_i^f/f_i^r}{5+2\sqrt{4-3f_i^r}}$ is the Eddington factor (Levermore 1984). This closure relation recovers the two regimes of radiative transport, and which corresponds to Eddington’s approximation. On the other hand, in the optically thin regime $f_i^r = cE_r$, $f_i^f = 1$, and $\zeta = 1$ which corresponds to the free-streaming limit.

The fluid temperature is estimated taking into account contributions of both, baryons and radiation pressure. An approximate expression for the total pressure is written as (Cuesta-Martínez et al. 2015)

$$P_i = \frac{k_B}{\mu m_p}\rho T_{\text{fluid}} + (1-e^{-\tau})\zeta(T)\alpha_r T_{\text{rad}}^4,$$

where $k_B$ is the Boltzmann constant, $\mu$ the mean molecular weight, and $m_p$ the mass of the proton, $\tau = \int(\rho\chi^f + \rho\chi^r)ds$ is the total optical depth. Finally, $T_{\text{rad}} = (E_r/a_r)^{1/4}$, is the temperature of radiation. Here $\tau$ depends on the temperature only if any of the opacity coefficients does. The path to integrate $\tau$ in our simulations are straight lines parallel to the $z$ axis. In general the Eddington factor depends on the temperature $\zeta = \zeta(T_{\text{fluid}})$. When the fluid and radiation are in local thermal equilibrium (LTE), that is $T_{\text{fluid}} = T_{\text{rad}}$, the temperature approximately obeys a fourth degree equation similar to (3) (see Cuesta-Martínez et al. 2013).

We programmed a code that solves the three dimensional RRH equations above, together with the M1 closure relation, using the following numerical methods. First the RRH equations are written in flux balance form $\partial_t \mathbf{U} + \partial_x \mathbf{F} = \mathbf{S}$, where $\mathbf{U}$ is the vector of conserved variables, $\mathbf{F}$ the fluxes and $\mathbf{S}$ the sources (Zanotti et al. 2011). Based on this structure of the equations, we apply high-resolution shock capturing (HRSC) methods, that use a finite volume discretization, the HLLE flux formula and the minmod slope limiter. For the evolution we use the Method of Lines, with an explicit-implicit Runge-Kutta (IMEX RK) time integrator with second order accuracy, as done in (Rivera-Paleo & Guzmán 2016). In order to benefit from efficient parallelization and standard I/O, we mounted our code on the Cactus frame (CactusCode), using the Carpet driver (Schnetter et al. 2004) and in all the cases we use an unigrid discretization. In the appendices we present canonical tests showing that our implementation works properly.

3 OPACITIES

An essential ingredient in our analysis, is the use of appropriate opacities, because their values determine the radiative processes associated with the GRB emission. When temperature is of the order of $\sim 10^8K$, the energy of photons becomes an appreciable fraction of the electron rest mass, photons may be scattered only on some electrons, and the electron scattering opacity is given by (Buchler & Yuen 1976)

$$\chi^e = 0.2(1+X)\left[1 + 2.7 \times 10^{11}\frac{\rho}{T^{7/2}}\right]^{-1}\left[1 + \left(\frac{T}{4.5 \times 10^8}\right)^{0.86}\right]^{-1}.$$  

Moreover, at this high-temperatures ($\sim 10^8K$) and low densities, a primary source of opacity comes from the creation of pairs. On the other hand, at intermediate temperatures ($< 10^8K$) and low densities ($\sim 1\text{gr/cm}^3$), bound-free opacity may be dominant. Finally, at sufficiently low temperatures and densities, bound-bound absorption in the UV and far UV dominate the opacity. This effect is relevant in the low density regions, both in the stratified model and the progenitor model used later, specially in the wind-like structure surrounding the progenitor star. The opacity due to free-free, bound-free and bound-bound emission, can be approximated with the so called Kramers formula (Rybicky & Lightman 2004)

$$\kappa^{ff} = 10^{23}Z^2\rho T^{-7/2}\text{cm}^2/\text{gr},$$
$$\kappa^{bf} = 10^{23}Z(1+X)\rho T^{-7/2}\text{cm}^2/\text{gr},$$
$$\kappa^{bb} = 10^{23}Z\rho T^{-7/2}\text{cm}^2/\text{gr}.$$  

The total coefficients of thermal opacity may be approximated over a wide range of temperatures, $10^6 \leq T \leq 10^9K$ with the sum of free-free, bound-free, and bound-bound coefficients ($\chi^t = \kappa^{ff} + \kappa^{bf} + \kappa^{bb}$). In all opacities $X$ and $Z$ represent the mass fractions of hydrogen and heavy elements, respectively.

4 SIMULATIONS SETUP

The GRB jet model that we study here, is produced by the injection of a relativistic beam evolving through a fluid at rest, starting from a nozzle with radius $r_0$ and velocity $v_0$. The process is characterized by the ratio between the density of the beam (subindex $b$) and that of the medium (subindex $m$) $\eta = \rho_b/\rho_m$, as well as by the ratio between their pressures $K = P_b/P_m$. The relativistic Mach number of the beam is $M_b = M_b W_b\sqrt{1 - c_s^2}$, where $M_b$, $W_b$, and $c_s^2$ are the Newtonian Mach number, Lorentz factor, and speed of sound, respectively. Outflow boundary conditions are used at the boundaries, except inside the nozzle radius, where the values of the variables are kept constant in time during the time window in which we inject the beam.

In order to study the jet interaction with the surrounding medium, we assume the radiation field of the beam stays in an optically thick region and propagates towards an optically thin region (Pe’er & Ryde 2011). This fact allows one to assume local thermal equilibrium between the fluid and the radiation field initially. This assumption, is satisfied by the external medium density $\rho_m$ and pressure profiles $P_m$ that decrease with distance. We use a density and pressure profiles described by the following power law (Mignone et al. 2005; De Colle et al. 2012).
\[ \rho_m = \rho_0 \left( \frac{v_b}{c} \right)^2, \quad P_m = P_0 \left( \frac{v_b}{c} \right)^2, \]

where the parameters of the surrounding medium are \( \rho_0 = 10^5 \text{g/cm}^3 \) and \( P_0 = 10^{22} \text{g/cm}^3 \text{s}^2 \). This is a simplified model for the propagation of a relativistic jet through a collapsing, non-rotating massive star with 0.1 times solar metallicity [Mignone et al. 2005].

Other interesting properties of GRBs are the luminosity and total injected energy \( L_j \) and \( E_j \), respectively. The injected jet luminosity is given by the flux of the momentum density equation times the surface of injection, \( A_b \), which in the optically tick regime is

\[ L_j \simeq \left[ (\rho_b h_b + 4/3 E_{r,b}) W^2_b v_{z,b} + F^j_b W_b (v_{z,b} + 1) \right] A_b, \]

where \( h_b, E_{r,b}, W_b \), and \( v_{z,b} \) are the specific enthalpy, radiated energy density, Lorentz factor and velocity of the beam. Finally, the total injected energy is approximately

\[ E_j \simeq L_j t_{inj}, \]

where \( t_{inj} \) is the injection time.

We explore various initial conditions for an ultra-relativistic jet, with different radiation energy densities, in inertia of the radiation field \( (4 \rho) \). This jet is radiation or matter dominated to determine whether a jet is radiation or matter dominated initially, we measure the ratio between the effective inertia of the radiation field \( (4/3 E_{r,b} W^2_b) \) and the effective inertia in the purely hydrodynamical case \( (\rho_b h_b W^2_b) \), as well as the ratio between radiation and gas pressures. These are, respectively

\[ g_{1,b} = \frac{4/3 E_{r,b}}{\rho_b h_b}, \]
\[ g_{2,b} = \frac{1/3 E_{r,b}}{P_b}. \]

A very important part of our simulations’ diagnostics is the LC curve that we calculate by directly integrating the radiation fluxes in the laboratory-frame \( \hat{F}^r \), on a given surface

\[ L = \int \hat{F}^r \cdot \hat{n} \, dA. \]

The relation between laboratory and comoving frames of the radiation moments, is given by a Lorentz transformation [Mihalas & Mihalas 1984] [Myeong-Gu 2006].

\[ E_r' = W^2 \left( E_r + 2v_i F^r_{iv} + v_i v_j P^{(ij)}_r \right), \]
\[ F^r_{vi} = W^2 v^i E_r + \left( \delta^i_k + \frac{W - 1}{v^i v^j} W^j \right) F^r_{vj} + W v_j \left( \delta^i_k + \frac{W - 1}{v^i v^k} v^j v_k \right) P^{hk}_r, \]
\[ P^{(ij)}_r = W^2 v^i v^j E_r + W \left( v^i \delta^j_k + v^j \delta^i_k - \frac{W - 1}{v^i v^j} v^k \right) F^r_{kj} + \left( \delta^j_k + \frac{W - 1}{v^j} v^k \right) \left( \delta^i_k + \frac{W - 1}{v^i} v^j \right) P^{ki}_r, \]

where the primed variables are those measured by an observer in the laboratory frame. As a detecting surface in the laboratory frame, we choose \( A \) to be a plane included in the numerical domain. The surface is located in a region where the optically thin regime holds. We calculate this luminosity in two different planes, the first one perpendicular to \( \hat{z} \), whereas the second one is a plane perpendicular to \( (\hat{x} + \hat{z})/\sqrt{2} \).

In order to evaluate the radiation LC seen by a distant observer, we need to compute quantities in an observer frame that takes into account the cosmological effects induced by the redshift at which the source is located. We consider a distant observer whose line of sight makes an angle \( \theta \) with respect to the jet axis. We define the time at which the observer sees the radiation coming from a fluid element located at a distance \( z_i \) at time \( t \) (both measured in a lab-frame) as \( t_{\text{det}} = t - z_i \cos \theta/c \). Assuming that the emitting source is located at redshift \( z \), the time measured in the observer’s frame is \( t_{\text{obs}} = t_{\text{det}}(1 + z) \) [Cuesta-Martinez et al. 2015] [Rueda-Beccerril et al. 2017]. On the other hand, the total luminosity in the observer’s frame is given by \( I_{\text{obs}}(t_{\text{obs}}) = (1 + z)D^2L \), where \( D = [W(1 - v_z \cos \theta/c)]^{-1} \) is the Doppler factor [Dermer 2004]. In our calculations we assume a generic \( z = 1 \) and \( \theta = 0 \) for the perpendicular detector \( \hat{n} = \hat{z} \) and \( \theta = \pi/4 \) for the inclined detector \( \hat{n} = (\hat{x} + \hat{z})/\sqrt{2} \).

5 DIFFERENT JET MODELS

The aim of this paper is to study the evolution of jets and their LCs across interesting surrounding media. The first case assumes the medium density has the toy density profile [10] and we analyze various scenarios. The parameters of the cases studied are summarized in Table 1. In these models, we choose the density and pressure ratios to be initially \( q = 0.01 \) and \( K = 0.01 \), a nozzle radius of \( r_0 = 8 \times 10^3 \text{cm} \) and a beam Lorentz factor \( W_b = 10 \). We inject the jet during a finite time \( t_{inj} = 12\text{s} \), which is a lapse consistent with the amount of total energy of a generic GRB.

In all these simulations, the jet propagates along the \( z \)-direction and the numerical parameters have been standardized such that we use a resolution of \( \Delta x = \Delta y = \Delta z = 1 \times 10^6 \text{cm} \), in a numerical domain of \([-1, 1] \times [-1, 1] \times [0, 5] \times 10^{10} \text{cm} \). This resolution is enough to contain 8 zones per beam radius, which is a recommended resolution to properly resolve the internal structure and the jet/external medium interaction [Xloy et al. 2000].

Model 1 corresponds to a purely hydrodynamical jet and will serve as reference to learn how much the radiation field affects the dynamics of the evolution. This is important, because in the ideal case one would like to avoid the use of post-processing to include the radiation effects that do not back-react into the hydrodynamics, and these hydrodynamical jets show how much of the dynamics one loses when the radiation is not considered during the evolution. Even though the jets propagate through a stratified surrounding medium, they are collimated along their entire length and present a morphology similar to that of jets propagating on a constant surrounding medium [Martí et al. 1994] [Martí et al. 1995] [Belan et al. 2013] [Hughes et al. 2002]. The properties are pretty much the same among models 1, 2 and 3, namely the...
beam, bow shock, contact discontinuity, cocoon, back-flow and internal shocks. However, quantitatively there are differences among these models. In Fig. 1 we show the rest-mass density profile along the z-axis of the purely hydrodynamical jet (model 1), the gas pressure dominated jet (model 2) and the radiation pressure dominated jet (model 3) at time \( t = 11 \) s in the laboratory frame. From Fig. 1 we can distinguish a shocked region, which is in front of the jet’s head, the profile is nearly steady and the radiation field does not contribute significantly because in the three cases the profile is similar. Behind the jet’s head, the radiation field plays a more important role, because there the rest mass density is higher for the radiation dominated case by a 10%.

In the following subsections, we will analyze in more detail the implications of the interaction of the radiation field with the fluid. We explore two cases in which the radiation photon field is coupled with the fluid.

### 5.1 Gas-pressure dominated case

This is model 2 of Table 1 and in Fig. 2 we show the LCs measured in the two different planes mentioned above for comparison. In the top right panel of Fig. 2 we indicate the surface of the two detectors with white lines, used to calculate the LC using (18). From now on we will refer to these planes as perpendicular and inclined planes where the LC is measured.

### 5.2 Radiation-pressure dominated case

This is model 3 in Table 1. In Fig. 3 we show the LCs for this model in the two detectors. The difference with respect

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**Table 1.** Parameters of the jet models. In all cases we use the opacities that emulate the free-free, bound-free, bound-bound and electron scattering precesses, adiabatic index \( \Gamma = 4/3 \) and Lorentz factor \( W_0 = 10\).
properties of the jet, these are a collimation sock in the beam, a bow shock, a contact discontinuity, back-flow, the reverse shock that is produced by the interaction between the jet/external medium and the formation of a cocoon. At this time, the Lorentz factor of the jet beam remains similar to its initial value, whereas the Lorentz factor of the head and cocoon is smaller. Later on, the jet starts to propagate in a very dilute external medium, and consequently the pressure of the cocoon drops and it begins to expand laterally into the circumstellar matter. We can see a snapshot of this behavior at \( t \sim 9s \). Also at this time, the beam Lorentz factor grows. When the head of the jet reaches the boundary in the \( z \) axis at \( t \sim 13.12s \), a Kelvin-Helmholtz instability is formed behind the head of the jet, as well as, the Lorentz factor of the beam starts to decrease because the jet is not being injected anymore.

Comparing this case with its counterpart models, gas-pressure dominated and purely hydrodynamics, in Fig. 1 we can see a difference along the polar axis of the jet. In this model, we see that the rest-mass density of the jet is bigger than that of models 1 and 2. Quantitatively, at time \( t = 11s \), the difference between the maximum rest mass density of model 3 with respect to its hydrodynamical version is of the order of \( \sim 10\% \).

In order to learn the effect of radiation pressure in the case when the radiation-pressure is dominant, we compare the Lorentz factor profile of model 3 with the gas-pressure dominated (model 2) and the purely hydrodynamical (model 1) cases, at the same time \( t = 11s \). This is shown in Fig. 4 where we can see that the radiation-pressure dominated jet propagates slightly faster than the gas-pressure dominated and the purely hydrodynamic models. This illustrates the influence of radiation pressure in the dynamics of the jet.

To finalize this section, in Fig. 6 we show the \( z \) component of the radiative flux during the whole evolution. Because we are assuming that the jet starts in an optically thick regime, the radiative flux in comoving frame at \( t = 0 \) is equal to zero. At \( t \sim 5s \) we can see that the highest radiative flux takes place in the head of the jet, collimation, oblique, and blow shock. At \( t \sim 9s \), the radiative flux decreases in the blow shock. Finally, at \( t \sim 13s \) we can see how the radiative flux in the jet is dissipating because the matter and radiation was switched off at time \( t = 12s \).

6 APPLICATION TO MODELS OF LOW LUMINOSITY GRBS

Now we study the scenario of jet propagation within a progenitor star, which has been applied to model Low luminosity GRBs [Bronberg et al. 2011 & 2016] Mizuta et al. 2006 & 2013 De Cole et al. 2017 Senno et al. 2016 & Geng et al. 2016]. In order to study the LLGRBs, we evolve a jet propagating through its progenitor star assuming the 16TI progenitor density profile (Woosley & Heger 2006). This model consists of a pre-supernova star with radius of \( R = 4 \times 10^{10} \) cm, 13.95 solar masses, and 1% of solar metallicity. From \( 10^9 \) cm to \( 6 \times 10^9 \) cm the density falls quickly as a power-law \( \sim r^{-1.5} \), from this point to a radius of \( 4 \times 10^{10} \) cm it decays exponentially. Finally, the surrounding medium from the surface to \( 1.8 \times 10^{11} \) cm, the density falls off like \( \sim r^{-2} \).
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Figure 5. Model 3. We show the rest mass density (top) and Lorentz factor (bottom) at different times. The jet was injected during 12 s with a Lorentz factor $W_b = 10$. The white lines in the top right panel indicates the position of both detectors.

In Fig. we show the initial rest mass density profile of this progenitor model. The rest mass density and pressure of the beam are $10^3$ g cm$^{-3}$ and $10^{19}$ g cm$^{-1}$s$^{-2}$ respectively. The radius of the jet is $r_b = 1 \times 10^9$ cm and the ratio between their pressures is $K = 0.01$. In our evolution we do not consider the gravitational field of the progenitor because the jet is moving with high enough velocity such that the dominant effects are due to the interaction of the jet with the matter of the star.

**Set up of jets.** Initially, we launched the jet at a distance of $10^9$ cm from the center of star in the z direction, as done in [Mizuta et al. 2006 & 2013]. Unlike in the previous scenario, where the jet was injected during 12 s, in this case the jet is injected during $t_{inj} = 20$ s.

We carried out simulations with a the Lorentz factor $W_b = 10$ and consider the case where the gas and radiation pressure are dominant. The specific values for these parameters are in Table 2. These models have been standardized with a resolution of $1.25 \times 10^8$ cm for the numerical domain.

In the dynamical evolution of model 4, there are two important phases shown in Fig. In the first phase, when the jet is propagating through the star, the Lorentz factor of the beam around the nozzle begins to grow up as expected, because the nozzle is continuously injecting energy to this region, whereas the head of the jet propagates with smaller velocity due to the interaction with the stellar envelope. Also, as a consequence of the interaction between the jet and stellar envelope a reverse shock is formed, which
interacts with the jet and when the beam crosses the reverse shock a cocoon is created and the beam is deflected sideways. We show a snapshot of the rest mass density and Lorentz factor in Fig. 8 at \( t = 3 \) s in the laboratory frame. At this time the jet is confined by the pressure in the cocoon.

The second phase of the evolution starts when the jet breaks out the progenitor star. In Fig. 8 we show the rest mass density and Lorentz factor at various times. In particular at time \( t \sim 7 \) s after the breakout, the jet expands into a rarefied medium, the gas pressure is smaller than inside of the star, as a result of this, the cocoon starts to expand laterally. Later on at \( t = 13 \) s, we can see a stratified cocoon close to the jet head, which allows the advance of the head. At this time, the jet propagates with high velocity whereas the cocoon expands with a slower velocity. Likewise, we can see that after crossing the collimation shock, the jet is not confined enough as to keep the cylindrical radius fixed and it starts a lateral expansion. Finally, at \( t \sim 18 \) s the jet continues to expand laterally whereas the collimation shock is larger than at previous times.

The LC and the difference of the gas and radiation temperatures for this model 4 are shown in Fig. 10. The first peak in the luminosity measured by a perpendicular plane is produced by the breakout of the jet from the stellar surface. The main peak is due to the one produced by the material near the working surface when it crosses the detector location. The main characteristic of this luminosity curve is that the amplitude of the LC lies within the LLGRBs range \((10^{46} - 10^{49} \text{ erg s}^{-1})\). The radiation and fluid temperatures have a behavior similar to those measured in the previous section for LGRBs, that is, the fluid temperature is higher than the radiation temperature. Nevertheless, the fluid temperature is of the order of \(10^8\) K, whereas the radiation temperature is of the order of \(10^4\) K. Unlike in the jets evolving on the stratified medium from the previous section, in this case the gas and radiation are not as close to thermal equilibrium, showing a difference temperature including four orders of magnitude. It is worth noticing that the opacities used in our simulations are appropriate for this range of temperature.

The dynamical evolution of model 5 is similar to that of model 4. The results are shown in Fig. 9 for model 5 at the

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**Table 2.** Parameters of the jets that we evolve on the progenitor model. We use the opacities that emulate the free-free, bound-free, bound-bound and electron scattering opacities with adiabatic index \( \Gamma = 4/3 \).

| Model | \( L_j \) (erg/s) | \( E_j \) (erg) | \( E_{r,b} \left( \frac{\text{erg cm}}{\text{s}} \right) \) | \( g_{1,b} \) (\( g_{2,b} \)) |
|-------|----------------|--------------|----------------|-------------|
| 4     | \( 4.97 \times 10^{50} \) | \( 9.94 \times 10^{51} \) | \( 1 \times 10^{19} \) | 0.33 (0.33) |
| 5     | \( 1.28 \times 10^{52} \) | \( 2.58 \times 10^{53} \) | \( 1 \times 10^{21} \) | 33.3 (33.3) |

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same times of model 4. At $t \sim 3s$, before the jet reaches the surface of the progenitor star, the reverse shock produced by the jet/stellar envelope interaction forms a cocoon and the pressure in the cocoon confines the jet. At this time, the only difference with respect to the jet in the gas-pressure dominated scenario is that in this case the jet is propagating faster.

At $t = 7.5\,s$, we can see a collimated beam after the jet breaks out the progenitor star, and at this time there are two important differences with respect to the model 4. The first one is that the collimation shock appears at $\sim 2 \times 10^{10}\,\text{cm}$ further ahead on the beam jet. The second one is related to the jet opening angle, for model 5 the opening angle is slightly bigger than for model 4. This shows that the jet opening angle not only depends of the initial Lorentz factor (Mizuta & Ioka 2013), but on other variables as well, like the radiative energy density. At $t = 13.12\,s$ a collimated jet continues propagating through a stratified external medium. In this case, the pressure of the cocoon is enough to keep the structure collimated. As time goes on, $t = 18.7\,s$, the matter continues flowing along the forward direction and the jet remains collimated.

In the bottom of Fig. 8 we show the evolution of the Lorentz factor along the progenitor and external medium. By $t = 3.75\,s$ the jet is still within the star, and the jet Lorentz factor near the zone of injection grows, whereas the Lorentz factor in the head of the jet decreases. When the jet breaks out the surface of the star, the jet not only continues...
propagating faster compared with the jet of model 4 but with a Lorentz factor slightly bigger.

In Fig. 11 we show the LCs for model 5. The initial injected energy arrives to the detector with sufficient energy as to obtain a LCs of the order of $10^{48}$ ergs/s, two orders of magnitude bigger than the LCs of model 4. The bottom of this figure shows the maximum temperatures, in this case the fluid temperature is about $\sim 10^8$ K, whereas the radiation temperature is around $\sim 10^4$ K.

7 DISCUSSION AND CONCLUSIONS

We implemented a relativistic radiation hydrodynamics code in 3D, with the main objective of constructing the LCs produced by jets. The LC is calculated by the integration of the radiation flux, which is fully coupled to the hydrodynamics during the simulations. We present essential tests that validate our code and applications to the jet propagation in stratified toy media and onto a progenitor star model.

As a first application of our code, we considered a model of LGRBs jet evolving through a stratified surrounding medium. For the surrounding medium, we use a density and pressure profiles that decrease as a power of law $\sim r^{-2}$. The model assumes local thermal equilibrium between the...
Jets driven by relativistic radiation hydrodynamics

Fluid and the radiation field initially. For the definition of the jet one needs nine parameters: three components for the velocity, the rest mass density and pressure of matter for hydrodynamics, three components for the radiative flux and the radiated energy density for the radiation. In particular, we explored the regime in which the jets goes along $z$-axis with an ultra-relativistic velocity. We have combined each one of the jets with values of radiated energy density that created scenarios where the radiation-pressure or gas-pressure are dominant, one at a time. Our model is also restricted to a single frequency and opacities associated to free-free, bound-free, bound-bound and electron scattering processes, but can be extended to other scenarios.

Under these conditions, we have compared the evolution of jet models with and without the coupling of hydrodynamics to the radiation field and have also shown the effects of gas and radiation pressure domination. We have found that when gas-pressure dominates, the dynamics of the jet is pretty similar to its purely hydrodynamical counterpart. On the other hand, when the radiation-pressure is dominant, the effect of radiation is noticeable in comparison to gas-pressure and purely hydrodynamics versions, because the radiation field act as a boost accelerating the material density around. This effect is more important in the case of jets propagating across the progenitor density model.

Regarding the luminosity LCs, depending on the combination of radiated energy density, the maximum amplitude of the luminosity lies within the range $\sim [10^{40} - 10^{52}]$ erg s$^{-1}$ for the jets propagating along the stratified medium. The scenario with the smallest amplitude is the gas-pressure dominated, whereas the biggest amplitude is achieved when the jet is dominated by radiation-pressure. This is physically consistent with the fact that the energy injected in the radiation-pressure dominated scenario is bigger by two orders of magnitude than in the scenario where the gas-pressure dominates.

Additionally, we compute the maximum of the radiation and fluid temperatures for each of the jets, found to be right behind the working surface. For the gas-pressure dominated scenario the fluid temperature is of $\sim 10^8$ K and the radiation temperature is around one order of magnitude smaller. In the case where the radiation-pressure dominates, the gas temperature is of order of $10^9$ K and the radiation temperature is $\sim 10^8$ K. An important point here to highlight is that during a large part of the evolution, the fluid temperature is bigger than the radiation temperature, which is consistent with the notion that radiation carries energy and acts as a fluid cooling mechanism.

We also applied our code to evolve jets triggered inside a progenitor. We verified that the jets propagate inside of the progenitor and successfully break out the surface. For our initial conditions, the LCs peak are of order of $\sim 10^{48}$ erg s$^{-1}$ and $\sim 10^{46}$ erg s$^{-1}$ for the radiation pressure and gas pressure domination scenarios, respectively. This is
comparable to the luminosity of LLGRBs. Similarly to the previous scenarios, the fluid temperature is bigger than radiation temperature, however in this case the difference is of four orders of magnitude. Even though initially the gas and radiation are assumed in thermal equilibrium, during the evolution the temperature difference between the two reaches four orders of magnitude, which indicates how far from thermal equilibrium these components are.

ACKNOWLEDGMENTS
We appreciate the comments and recommendations from some members of the Valencia Group and from an anonymous Referee. This research is supported by grants CIC-UAM-UNAM-4.9 and CONACyT 258726 (Fondo Sectorial de Investigación para la Educación). Most of the simulations were carried out in the computer farm funded by CONACyT 106466 and the Big Mamma machine of the Laboratory of Artificial Intelligence at the IFM. The authors also acknowledge the computer resources, technical expertise and support provided by the Laboratorio Nacional de Supercómputo del Sureste de México, CONACyT network of national laboratories. We also thank ABACUS Laboratorio de Matemáticas Aplicadas y Cómputo de Alto Rendimiento del CINVESTAV-IPN, grant CONACT-EDOMEX-2011-C01-165873, for providing computer resources.

APPENDIX A: BASIC TESTS
We only present the standard tests in two dimensions, because the standard 1D tests of our code were presented in (Rivera-Paleo & Guzmán 2016). These tests show the performance of our code in both, optically thick and optically thin regimes. The initial conditions for the tests are the following:

(i) Single beam test. This test is intended to verify that our code can work properly in optically thin media, where gas and radiation are decoupled ($\kappa_a = \kappa_{\text{total}} = 0$). This test consists in injecting a simple beam of radiation and checking that the beam does not present any rupture during its evolution. The test was solved in the plane $z = 0$ on a $31 \times 31$ grid. The boundary conditions for all borders are outflow, except in the given region delimited by $y \in [0.4, 0.6]$, where the beam is injected with energy density $100$ times larger than that of the environment. The value of the $\alpha_r$ and adiabatic index of the gas are $1.118 \times 10^{17}$, code units, and $4/3$ respectively.

In Fig. A1 we can see, at $t = 10$ that radiation beam through the whole domain without presenting any rupture. The standard snapshot can be compared with (Sadkowski et al. 2013 & 2014).

(ii) Shadow. In order to verify that the M1 approximation works properly, and to illustrate the difference between the M1 and Eddington approximations we solve this problem. The test consists of an optically thick gas lego circle, immersed in an optically thin environment. We solve the test on the plane $z = 0$ on a $100 \times 50$ grid, with a fixed mass density within a lego circle given by

$$\rho_0 = \rho_a + (\rho_b - \rho_a) e^{-\sqrt{x^2+y^2+z^2}/\omega^2}, \quad (A1)$$

where $\rho_a = 10^{-4}$, $\rho_b = 10^3$, and $\omega = 0.22$. Initially the system is in thermal equilibrium, and has velocities and radiative fluxes equal to zero. The boundary conditions are: inflow at the left border and outflow at the right border. On all other borders we use periodic boundary conditions. At the border where we imposed the inflow. The values for radiated energy density, radiated flux and gas temperature are $E_L = a_r T_{g,L}^4$, $F^2 = 0.99999 E_L$, $y T_{g,L} = 100 T_a$ respectively. The value of $a_r = 351.37$ and $\kappa_a = \kappa_{\text{total}} = \rho_0$.

The gas temperature is given such that the pressure is constant throughout the domain,

$$T_g = T_a \frac{\rho_a}{\rho_0}, \quad (A2)$$

with an adiabatic index $\Gamma = 1.4$.

In Fig. A2 We show the results when the incoming radiation beam passes through the entire domain and reaches a steady state ($t \sim 10$). In the upper panel we show the solution obtained with the Eddington approximation. This approximation treats the radiation field isotropically, as consequence the radiation diffuses rapidly behind the sphere and a shadow cannot be formed. In the lower panel we show the solution with the M1 approximation, contrary to the Eddington approximation, here we can see that a shadow is formed behind the circle because it is designed to keep moving the flow parallel to itself in optically thin regions for $F_r \approx F_\gamma$. The standard snapshot can be compared with (Sadkowski et al. 2013 & 2014).

(iii) Double shadow. To test the performance and efficiency of our code with multiple sources of light, whose radiative flux is not parallel to the direction of propagation, we implement the double shadow problem described in (Sadkowski et al. 2013 & 2014). In this test a beam of light is injected into a static environment, where the photons move in different directions than the direction of propagation of

![Figure A1. Radiation energy density at $t = 10$. The radiation beam is located at the left boundary.](image-url)
Figure A2. Radiation energy density at $t = 10$. The source of radiation is located at the left boundary. Top: result corresponding to the Eddington approximation. Bottom: result corresponding to the M1 approximation.

Figure A3. Radiation energy density at $t = 10$. The source of radiation is located at the left boundary. This result corresponds to the M1 closure approximation.

The fact that the incident beam is inclined has an effect on the shadow produced behind the lego sphere. On the one hand we have regions of partial shadow (penumbra) that result from perpendicular photons, while the region of total shadow (umbra) is limited by the edges of the penumbra. This test shows the limits of the M1 approximation which, in principle, does not limit the specific intensity of radiation to a particular direction. But in the case of multiple light sources it should be used with caution, as seen in Fig. A3, the M1 approximation produces an extra horizontal shadow along the x-axis where the penumbra overlaps. In this region it is expected to be uniform and without shadow (Jiang et al. 2012). The standard snapshot, Fig. A3 can be compared with (Sadowski et al. 2013 & 2014).

**APPENDIX B: CONVERGENCE TEST**

In order to test the convergence of the evolution of the jets and select an adequate mesh resolution, we performed a convergence test. For this we use three different resolutions: low resolution which uses 6 zones per beam radius ($\Delta x_1 = 1.333 \times 10^8 \mathrm{cm}$), medium resolution that uses 8 zones per beam radius ($\Delta x_2 = 3 \Delta x_1 / 4$), employed in all the simulations listed in Tables 1 and 2 and high resolution with 10 zones per beam radius ($\Delta x_3 = 3 \Delta x_1 / 5$), the high resolution. These resolutions were chosen so that the simulations could be done during the time it takes the jet to travel through the domain.

Fig. B1 shows the morphology of the rest mass density for model 3 at $t \sim 9.37s$. With the three resolutions the morphology is consistent in the sense that the jet head has reached the same position in the z-axis in all the cases. Also, the transverse expansion of the jet is consistent. However, as expected, higher resolution reveals smaller structures and the exact morphology of the turbulent internal part of the jet is not exactly the same for the different resolutions. The exact details of that region are not expected to contribute to the thermal emission, which is dominated by the jet/external interaction, nevertheless they have an effect on the LCs which are different for different resolutions, however their time series should converge. This is the reason why we practice a self-convergence test on the final result of the simulations, namely the LC. Using the three resolutions mentioned $\Delta x_1 < \Delta x_2 < \Delta x_3$ we calculate the respective LCs $L_1$, $L_2$, $L_3$ and perform a self convergence test by comparing the differences among them. The convergence factor is given by

$$CF(L_1, L_2, L_3) = \frac{L_1 - L_2}{L_2 - L_3} \simeq \frac{(\Delta x_1)^Q - (\Delta x_2)^Q}{(\Delta x_2)^Q - (\Delta x_3)^Q}$$

where $Q$ is the accuracy order of the methods. In our case, the numerical methods used, that is, the piecewise linear reconstructor of variables, the HLLE flux formula and the IMEX integrator, all combined in our simulations, in the presence of shocks are expected to converge within first and second order. The convergence factor for for $Q = 1$ is $CF_{Q=1} = 5/3$, whereas for $Q = 2$ is $CF_{Q=2} = 175/81 \simeq 2.16$. We show in Fig. B2 that the Convergence Factor has a value between these two, which shows our results self-converge with the expected accuracy and thus also shows that our simulations use a resolution within a convergence regime.
Figure B1. Morphology of the rest mass density for Model 3 using three different resolutions. From left to right: lower, standard and high resolution.

Figure B2. Order of self-convergence of the light curve measured with a perpendicular detector for model 3. First order of accuracy is expected for systems with shocks during the evolution and our sample simulations shows a convergence slightly better than first order. The two constant lines indicate the value of $CF$ for first and second order convergence for our resolutions, respectively.

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