Measurement of the absolute branching fraction of $D^+ \to \bar{K}^0 e^+ \nu_e$ via $\bar{K}^0 \to \pi^0 \pi^0$
I. INTRODUCTION

The study of semileptonic decays of $D$ mesons can shed light on the strong and weak effects in charmed meson decays. The absolute branching fraction $B$ of the semileptonic decay $D^+ \rightarrow K^0 e^+ \nu_e$ can be used to extract the form factor $f_K^+(0)$ of the hadronic weak current or the quark mixing matrix element $|V_{cs}|$ [1], which are important to calibrate the lattice quantum chromodynamics calculation on $f_K^+(0)$ and to test the unitarity of the quark mixing matrix. In addition, the measured $B(D^+ \rightarrow K^0 e^+ \nu_e)$ can also be used to test isospin symmetry in the $D^+ \rightarrow K^0 e^+ \nu_e$ and $D^0 \rightarrow K^- e^+ \nu_e$ decays [2,3,4]. Therefore, improving the measurement precision of $B(D^+ \rightarrow K^0 e^+ \nu_e)$ will be helpful to better understand the $D$ decay mechanisms.

Measurements of $B(D^+ \rightarrow K^0 e^+ \nu_e)$ via $K^0 \rightarrow K^0_S \rightarrow \pi^+ \pi^-$ have been performed by the MARKIII, BES, CLEO and BESIII Collaborations [2,4,8]. Recently, a measurement of $B(D^+ \rightarrow K^0_S e^+ \nu_e)$ has been carried out by the BESIII Collaboration [8]. However, no measurement of $B(D^+ \rightarrow K^0 e^+ \nu_e)$ using $K^0 \rightarrow K^0_S \rightarrow \pi^0 \pi^0$ has been reported so far. As a first step, we present in this paper a measurement of $B(D^+ \rightarrow K^0 e^+ \nu_e)$ using $K^0 \rightarrow K^0_S \rightarrow \pi^0 \pi^0$, based on an analysis of 2.93 fb$^{-1}$ of $e^+e^-$ collision data [7] accumulated at the center-of-mass energy $\sqrt{s} = 3.773$ GeV with the BESIII detector [8]. Since there is currently no room to improve our measurement of $f_K^+(0)|V_{cs}|$ [8], we only aim to measure the $B(D^+ \rightarrow K^0 e^+ \nu_e)$ in this work.

II. BESIII DETECTOR AND MONTE CARLO

The BESIII detector is a cylindrical detector with solid-angle 93% of 4$\pi$ that operates at the BEPCII collider. It consists of several main components. A 43-layer main drift chamber (MDC) surrounding the beam pipe performs precise determinations of charged particle trajectories and provides ionization energy loss ($dE/dx$) measurements that are used for charged particle identification (PID). An array of time-of-flight counters (TOF) is located radially outside the MDC and provides additional charged particle identification information. A CsI(Tl) electromagnetic calorimeter (EMC) surrounds the TOF and is used to measure the energies of photons and electrons. A solenoidal superconducting magnet located outside the EMC provides a 1 T magnetic field in the central tracking region of the detector. The iron flux return of the magnet is instrumented with about 1272 m$^2$ of resistive plate muon counters (MUC) arranged in nine layers in the barrel and eight layers in the endcaps that are used to identify muons with momentum greater than 0.5 GeV/$c$. More details about the BESIII detector are described in Ref. [8].

A GEANT4-based [10] Monte Carlo (MC) simulation software, which includes the geometric description and a simulation of the response of the detector, is used to determine the detection efficiency and to estimate the potential backgrounds. An inclusive MC sample, which includes generic $\psi(3770)$ decays, initial state radiation (ISR) production of $\psi(3686)$ and $J/\psi$, QED ($e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$) and $q\bar{q}$ ($q = u, d, s$) continuum processes, is produced at $\sqrt{s} = 3.773$ GeV. The MC events of $\psi(3770)$ decays are produced by a combination...
of the MC generators KKMC \cite{11} and PHOTOS \cite{12}, in which the effects of ISR \cite{13} and Final State Radiation (FSR) are considered. The known decay modes of charmonium states are generated using EvtGen \cite{14} with the branching fractions taken from the Particle Data Group (PDG) \cite{15}, and the remaining events are generated using LundCharm \cite{16}. The $D^+ \to K^0 e^+ \nu_e$ signal is modeled by the modified pole model \cite{17}.

III. MEASUREMENT

A. Single tag $D^-$ mesons

With a mass of 3.773 GeV just above the open charm threshold, the $\phi(3770)$ resonance decays predominately into $D^0 D^0$ or $D^+ D^-$ meson pairs. In each event, if a $D^-$ meson can be fully reconstructed via its decay into hadrons (in the following called the single tag (ST) $D^-$), there must be a recoiling $D^+$ meson. Using a double tag technique which was first employed by the MARKIII Collaboration \cite{2}, we can measure the absolute branching fraction of the $D^+ \to K^0 e^+ \nu_e$ decay. Throughout the paper, charge conjugation is implied.

The ST $D^-$ mesons are reconstructed using six hadronic decay modes: $K^+ \pi^- \pi^-$, $K_S^0 \pi^-$, $K^+ \pi^- \pi^- \pi^0$, $K_S^0 \pi^- \pi^-$, $K_S^0 \pi^+ \pi^- \pi^-$ and $K^+ K^- \pi^-$. The daughter particles $K_S^0$ and $\pi^0$ are reconstructed via $K_S^0 \to \pi^+ \pi^-$ and $\pi^0 \to \gamma \gamma$, respectively.

All charged tracks are required to be reconstructed within the good MDC acceptance $|\cos \theta| < 0.93$, where $\theta$ is the polar angle of the track with respect to the beam direction. All tracks except those from $K_S^0$ decays are required to originate from the interaction region defined as $V_{xy} < 1.0$ cm and $|V_z| < 10.0$ cm. Here, $V_{xy}$ and $|V_z|$ are the distances of closest approach to the Interaction Point (IP) of the reconstructed track in the plane transverse to and along the beam direction, respectively. For PID of charged particles, we combine the $dE/dx$ and TOF information to calculate Confidence Levels for the pion and kaon hypotheses ($CL_\pi$ and $CL_K$). A charged track is taken as kaon (pion) if has $CL_K > CL_\pi$ ($CL_\pi > CL_K$).

The charged tracks from $K_S^0$ decays are required to satisfy $|V_z| < 20.0$ cm. The two oppositely charged tracks, which are assumed as $\pi^+ \pi^-$ without PID, are constrained to originate from common vertex. A $\pi^+ \pi^-$ combination is considered as a $K_S^0$ candidate if its invariant mass lies in the mass window $|M_{\pi^+ \pi^-} - M_{K_S^0}| < 12$ MeV/$c^2$, where $M_{K_S^0}$ is the nominal $K_S^0$ mass \cite{18}. The $\pi^+ \pi^-$ combinations with $L/\sigma_L > 2$ are retained, where $\sigma_L$ is the uncertainty of the $K_S^0$ reconstructed decay length $L$.

Photon candidates are selected by using the EMC information. The shower time is required to be within 700 ns of the event start time. The shower energy is required to be greater than 25 (50) MeV in the barrel (endcap) region. The opening angle between the candidate shower and the closest charged track is required to be greater than 10°. A $\gamma \gamma$ combination is considered as a $\pi^0$ candidate if its invariant mass falls in $(0.115, 0.150)$ GeV/$c^2$. To obtain better mass resolution for the $D^-$ candidates, the $\gamma \gamma$ invariant mass is constrained to the $\pi^0$ nominal mass via a kinematic fit.

To suppress combinatorial backgrounds, we define the variable $\Delta E = E_{mK\pi} - E_{beam}$, which is the difference between the measured energy of the $mK\pi$ ($m = 1, 2, n = 1, 2, 3$) combination ($E_{mK\pi}$) and the beam energy ($E_{beam}$). For each ST mode, if there is more than one $mK\pi$ combination satisfying the above selection criteria, only the one with the minimum $|\Delta E|$ is kept. The $\Delta E$ is required to be within $(-25, +25)$ MeV for the $K^+ \pi^- \pi^-$, $K_S^0 \pi^-$, $K_S^0 \pi^+ \pi^- \pi^-$ and $K^+ K^- \pi^-$ decay modes, and be within $(-55, +40)$ MeV for the $K^+ \pi^- \pi^- \pi^0$ and $K_S^0 \pi^- \pi^- \pi^0$ combinations.

To measure the yield of ST $D^-$ mesons, we fit the spectra of the beam energy constrained masses $M_{BC} = \sqrt{E_{beam}^2 - |p_{mK\pi}|^2}$ of the accepted $mK\pi$ combinations, as shown in Fig. 1 Here, $p_{mK\pi}$ is the measured momentum of the $mK\pi$ combination. In the fits, the $D^-$ signal is modeled by the MC simulated $M_{BC}$ distribution convoluted with a double Gaussian function, and the combinatorial background is described by an ARGUS function \cite{19}. The candidates in the ST $D^-$ signal region defined as (1.863, 1.877) GeV/$c^2$ are kept for further analysis. Single-tag reconstruction efficiencies $\epsilon_{ST}$ are estimated by analyzing the inclusive MC sample. The ST yields $N_{ST}$ and the ST efficiencies are summarized in Table III The total ST yield is $N_{ST}^{tot} = 1522474 \pm 2215$.

![FIG. 1. Fits to the $M_{BC}$ spectra of the (a) $K^+ \pi^- \pi^-$, (b) $K_S^0 \pi^-$, (c) $K^+ \pi^- \pi^- \pi^0$, (d) $K_S^0 \pi^+ \pi^- \pi^-$, (e) $K_S^0 \pi^+ \pi^- \pi^-$ and (f) $K^+ K^- \pi^-$ combinations. The dots with error bars are data, the blue solid curves are the fit results, the red dashed curves are the fitted backgrounds and the pair of red arrows in each sub-figure denote the ST $D^-$ signal region.](image-url)
TABLE I. Summary of the ST yields ($N_{\text{ST}}^i$), the ST and DT efficiencies ($\epsilon_{\text{ST}}^i$ and $\epsilon_{\text{DT}}^i$), and the reconstruction efficiencies of $D^+ \rightarrow K^0_{S} \pi^0 \nu_e$ ($\epsilon_{D^+\rightarrow K^0_{S} \pi^0 \nu_e}^i$). The efficiencies do not include the branching fractions for $K^0_{S} \rightarrow \pi^+\pi^-$ (used in the reconstruction of ST $D^-$ mesons), $K^0 \rightarrow \pi^0 \pi^0$ and $\pi^0 \rightarrow \gamma \gamma$. The uncertainties are statistical only. The index $i$ represents the $i$th ST mode.

| ST mode $i$ | $N_{\text{ST}}^i$ | $\epsilon_{\text{ST}}^i$ (%) | $\epsilon_{\text{DT}}^i$ (%) | $\epsilon_{D^+\rightarrow K^0_{S} \pi^0 \nu_e}^i$ (%) |
|-------------|-----------------|-----------------|-----------------|-----------------|
| $D^- \rightarrow K^+ \pi^- \pi^-$ | 782669 ± 990 | 50.61 ± 0.06 | 13.39 ± 0.07 | 26.45 ± 0.14 |
| $D^- \rightarrow K^0_{S} \pi^-$ | 91345 ± 320 | 50.41 ± 0.17 | 13.81 ± 0.22 | 27.40 ± 0.44 |
| $D^- \rightarrow K^+ \pi^- \pi^- \pi^0$ | 251008 ± 1135 | 26.74 ± 0.09 | 6.23 ± 0.06 | 23.29 ± 0.25 |
| $D^- \rightarrow K^0_{S} \pi^- \pi^0$ | 215364 ± 1238 | 27.29 ± 0.07 | 6.88 ± 0.07 | 25.21 ± 0.28 |
| $D^- \rightarrow K^0_{S} \pi^- \pi^- \pi^0$ | 113054 ± 889 | 28.31 ± 0.12 | 6.74 ± 0.10 | 23.79 ± 0.37 |
| $D^- \rightarrow K^+ K^- \pi^-$ | 69034 ± 460 | 40.83 ± 0.24 | 10.54 ± 0.20 | 25.81 ± 0.50 |

B. Double tag events

In the system recoiling against the ST $D^-$ mesons, the $D^+ \rightarrow K^0_{S} e^+ \nu_e$ candidates, called the double tag (DT) events, are selected via $K^0_{S} \rightarrow K^0_{S} \rightarrow \pi^0 \pi^0$. It is required that there be at least four good photons and only one good charged track that have not been used in the ST selection. The good charged track, photons and $\pi^0 \pi^0$ mesons are selected using the same criteria as those used in the ST selection. If there are multiple $\pi^0 \pi^0$ combinations satisfying these selection criteria, only the combination with the minimum value of $\chi^2_1(\pi^0 \rightarrow \gamma \gamma) + \chi^2_2(\pi^0 \rightarrow \gamma \gamma)$ is retained, where $\chi^2_1$ and $\chi^2_2$ are the chi-squares of the mass constrained fits on $\pi^0 \rightarrow \gamma \gamma$. A $\pi^0 \pi^0$ combination is considered as a $K^0_{S}$ candidate if its invariant mass falls in $(0.45, 0.51)$ GeV/$c^2$. For electron PID, we combine the $dE/dx$, TOF and EMC information to calculate Confidence Levels for the electron, pion and kaon hypotheses ($CL_e$, $CL_\pi$ and $CL_K$), respectively. The electron candidate is required to have $CL_e > 0.001$ and $CL_e/(CL_e + CL_\pi + CL_K) > 0.8$, and to have a charge opposite to the ST $D^-$ meson. To partially recover the effects of FSR and bremsstrahlung, the four-momenta of photon(s) within 5° of the initial electron direction are added to the electron four-momentum. To suppress the backgrounds associated with fake photon(s), we require that the maximum energy ($E_{\text{extra}}^{\gamma}$) of any of the extra photons, which have not been used in the DT selection, be less than 300 MeV.

In order to obtain the information of the missing neutrino, we define the kinematic quantity

$$U_{\text{miss}} \equiv E_{\text{miss}} - |\vec{p}_{\text{miss}}|,$$

where $E_{\text{miss}}$ and $|\vec{p}_{\text{miss}}|$ are the total energy and momentum of the missing particle in the event, respectively. $E_{\text{miss}}$ is calculated by

$$E_{\text{miss}} = E_{\text{beam}} - E_{K^0} - E_{e^+},$$

where $E_{K^0}$ and $E_{e^+}$ are the energies carried by $K^0$ and $e^+$, respectively. $|\vec{p}_{\text{miss}}|$ is calculated by

$$|\vec{p}_{\text{miss}}| = |\vec{p}_{D^+} - \vec{p}_{K^0} - \vec{p}_{e^+}|,$$

where $\vec{p}_{D^+}$, $\vec{p}_{K^0}$ and $\vec{p}_{e^+}$ are the momenta of $D^+$, $K^0$ and $e^+$, respectively. To obtain better $U_{\text{miss}}$ resolution, $\vec{p}_{D^+}$ is constrained by

$$\vec{p}_{D^+} = -\vec{p}_{D^+} \sqrt{E_{\text{beam}}^2 - m_{D^+}^2},$$

where $\vec{p}_{D^+}$ is the momentum direction of the ST $D^-$ meson and $m_{D^+}$ is the $D^+$ nominal mass [18].

To determine the number of DT events, we apply a fit to the $U_{\text{miss}}$ distribution of the accepted DT candidates, as shown in Fig. 2. In the fit, the DT signal and the combinatorial background are modeled by the MC simulated $U_{\text{miss}}$ shapes, respectively. From the fit, we obtain the DT yield in data as

$$N_{\text{DT}} = 5013 \pm 78.$$

C. Branching fraction

The efficiency of reconstructing the DT events, called the DT efficiency $\epsilon_{\text{DT}}$, is determined by analyzing the signal MC events. Dividing $\epsilon_{\text{DT}}$ by $\epsilon_{\text{ST}}$, we obtain the reconstruction efficiency for $D^+ \rightarrow K^0_{S} e^+ \nu_e$ in each ST mode, $\epsilon_{D^+\rightarrow K^0_{S} e^+ \nu_e}^i$, as summarized in Table I. Weighting

![Fig. 2](image_url)
them by the ST yields observed in data, we obtain the averaged reconstruction efficiency of $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$
\[
\epsilon_{D^+ \rightarrow \bar{K}^0 e^+ \nu_e} = (25.58 \pm 0.11)\%	ext{,}
\]
which does not include the branching fractions of $\bar{K}^0 \rightarrow \pi^0 \pi^0$ and $\pi^0 \rightarrow \gamma \gamma$.

The branching fraction of $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ is determined by
\[
B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = \frac{N_{\text{DT}}}{N_{\text{ST}}^\text{tot}} \epsilon_{D^+ \rightarrow \bar{K}^0 e^+ \nu_e} B(\bar{K}^0 \rightarrow \pi^0 \pi^0) B^2(\pi^0 \rightarrow \gamma \gamma),
\]
where $N_{\text{DT}}$ is the DT yield, $N_{\text{ST}}^\text{tot}$ is the total ST yield, $\epsilon_{D^+ \rightarrow \bar{K}^0 e^+ \nu_e}$ is the averaged reconstruction efficiency of $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, $B(\bar{K}^0 \rightarrow \pi^0 \pi^0)$ and $B(\pi^0 \rightarrow \gamma \gamma)$ are the branching fractions of $\bar{K}^0 \rightarrow \pi^0 \pi^0$ and $\pi^0 \rightarrow \gamma \gamma$, respectively. Here, we assume that $K_S^0$ constitutes half the decays of the neutral kaons.

Inserting the numbers of $N_{\text{DT}}$, $N_{\text{ST}}^\text{tot}$, $\epsilon_{D^+ \rightarrow \bar{K}^0 e^+ \nu_e}$, $B(\bar{K}^0 \rightarrow \pi^0 \pi^0)$ and $B(\pi^0 \rightarrow \gamma \gamma)$ in Eq. (7), we obtain
\[
B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = (8.59 \pm 0.14)\%	ext{,}
\]
where the uncertainty is statistical only.

### D. Systematic uncertainty

In the measurement of the branching fraction, the systematic uncertainty arises from the uncertainties in the fits to the $M_{\text{BC}}$ spectra of the ST candidates, the $\Delta E$, $M_{\text{BC}}$ and $\bar{K}^0(\pi^0 \pi^0)$ mass requirements, the $\pi^0$ reconstruction, the $e^\pm$ tracking, the $e^\pm$ PID, the $E_{\text{extra}}$ requirement, the $U_{\text{miss}}$ fit, the $\chi_1^2$ and $\chi_2^2$ selection method, the MC statistics and the quoted branching fractions.

The uncertainty in the fits to the $M_{\text{BC}}$ spectra of the ST candidates is estimated to be 0.5% by observing the relative change of the ST yields of data and MC when varying the fit range, the combinatorial background shape or the endpoint of the ARGUS function. To estimate the uncertainties in the $\Delta E$, $M_{\text{BC}}$ and $\bar{K}^0(\pi^0 \pi^0)$ mass requirements, we examine the change in branching fractions when enlarging the $\Delta E$ selection window by 5 or 10 MeV; varying the $M_{\text{BC}}$ selection window by $\pm 1$ MeV and using alternative $\bar{K}^0(\pi^0 \pi^0)$ mass windows (0.460, 0.505), (0.470, 0.500), (0.480, 0.500) GeV/$c^2$, respectively. The maximum changes in the branching fractions, 0.3%, 0.2%, and 0.9%, are assigned as the systematic uncertainties. The $\pi^0$ reconstruction efficiency is examined by analyzing the DT hadronic decays $D^0 \rightarrow K^- \pi^+$ and $K^- \pi^+ \pi^- \pi^0$ versus $D^0 \rightarrow K^- \pi^+ \pi^0$ and $K_S^0(\pi^+ \pi^- \pi^0)$. The difference of the $\pi^0$ reconstruction efficiencies between data and MC is found to be $(-1.0 \pm 1.0)$% on $\pi^0$. The systematic uncertainty in $\pi^0$ reconstruction is taken to be 1.0% for each $\pi^0$ after correcting the MC efficiency of $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ to data. The uncertainty in the tracking or PID for $e^\pm$ is estimated by analyzing $e^+e^- \rightarrow \gamma e^+e^-$ events. It is assigned to be 0.5%, which is the re-weighted difference of the $e^\pm$ tracking (or PID) efficiencies between data and MC. The uncertainty in the $E_{\text{extra}}$ requirement is estimated to be 0.1% by analyzing the DT hadronic $D \bar{D}$ decays. The uncertainty in the $U_{\text{miss}}$ fit is assigned to be 0.5%, which is obtained by comparing with the nominal value of the branching fraction measured with an alternative signal shape obtained with different requirements on the MC-truth matched signal shape, an alternative background shape after changing the relative ratios of the dominant backgrounds (doubling each of the simulated backgrounds for $D^0 \bar{D}^0$, $D^+D^-$ and $q\bar{q}$ continuum processes), and alternative fit range ($\pm 50$ MeV). The difference of 0.3% in the $\pi^0 \pi^0$ acceptance efficiencies between data and MC, which is estimated by the DT hadronic decays $D^0 \rightarrow K^- \pi^+ \pi^0$ versus $D^0 \rightarrow K^- \pi^- \pi^0$, is assigned as a systematic uncertainty due to the $\chi_1^2$ and $\chi_2^2$ selection method. In this analysis, the $K^0 \rightarrow \bar{K}^0(\pi^0 \pi^0)$ meson from the signal side is formed with photon candidates reconstructed under the assumption that they originate at the IP. We examine the DT efficiencies of the signal MC events in which the lifetimes of $K_S^0$ meson from the signal side are set to the nominal value and 0, respectively. The difference of these two DT efficiencies, which is less than 0.2%, is taken as the systematic uncertainty of the $K_S^0(\pi^0 \pi^0)$ reconstruction. The uncertainties in the MC statistics and the $B(\bar{K}^0 \rightarrow \pi^0 \pi^0)$ are 0.5% and 0.2% [13], respectively. In our previous work, the uncertainty in the signal MC generator is estimated to be 0.1%, which is obtained by comparing the DT efficiencies before and after re-weighting the $q^2 (= (p_D - p_K)^2)$ distribution of the signal MC events of $D^0 \rightarrow K^-e^+\nu_e$ to the distribution found in data [13], where the $p_D$ and $p_K$ are the four-momenta of the $D$ and $K$ mesons. The systematic uncertainties are summarized in Table [I]. Adding all uncertainties in quadrature, we obtain the total systematic uncertainty to be 2.5%.

### E. Validation

The analysis procedure is examined by an input and output check using an inclusive MC sample equivalent to a luminosity of 3.26 fb$^{-1}$. Using the same selection criteria as those used in data analysis, we obtain the ST yield, the DT yield and the weighted reconstruction efficiency of $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$, to be 1683631 $\pm$ 1768, 5802 $\pm$ 85 and (26.07 $\pm$ 0.11)%), where no efficiency correction has been performed. Based on these numbers, we determine the branching fraction $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = (8.82 \pm 0.13)\%$, where the uncertainty is statistical only. The measured branching fraction is in excellent agreement with the input value of 8.83%.

To validate the reliability of the MC simulation, we examine the cos $\theta$ and momentum distributions of $\bar{K}^0$ and $e^+$ of the $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ candidates, as shown in Fig. [I]. We can see that the consistency between simulation and
TABLE II. Relative systematic uncertainties (in %) in the measurement of $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$.

| Source               | Uncertainty |
|----------------------|-------------|
| $M_{BC}$ fit         | 0.5         |
| $\Delta E$ requirement | 0.3        |
| $M_{BC} \in (1.863, 1.877)$ GeV/c² | 0.2         |
| $M_{\pi\pi\pi}$ | 0.9         |
| $\pi^0$ reconstruction | 2.0        |
| Tracking for $e^\pm$ | 0.5         |
| PID for $e^\pm$      | 0.5         |
| $E_{\text{extra}}^\gamma < 0.3$ GeV | 0.1        |
| $U_{\text{miss}}$ fit | 0.5        |
| $\chi^2_1 + \chi^2_2$ selection method | 0.3        |
| $K^0_S(\pi^0\pi^0)$ reconstruction | 0.2        |
| MC statistics        | 0.5         |
| $B(\bar{K}^0 \rightarrow \pi^0\pi^0)$ | 0.2        |
| MC generator         | 0.1         |
| **Total**            | **2.5**     |

...data is very good.

IV. SUMMARY AND DISCUSSION

Based on the analysis of 2.93 fb⁻¹ data collected at $\sqrt{s} = 3.773$ GeV with the BESIII detector, we measure the absolute branching fraction $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = (8.59 \pm 0.14 \pm 0.21)%$, using $\bar{K}^0 \rightarrow K^0 \rightarrow \pi^0\pi^0$. Figure 4 presents a comparison of $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$ measured in this work with the results obtained by other experiments. Our result is well consistent with the other measurements within uncertainties and has a precision comparable to the PDG value [18]. Our measurement will be helpful to improve the precision of the world average value of $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$.

![Figure 4](image_url)

FIG. 4. Comparison of the $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$ measured in this work with those measured by other experiments, where the slash band is the world averaged branching fraction with uncertainty. For the BESIII measurement using $\bar{K}^0 \rightarrow K^0_L$, we take $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e) = 2B(D^+ \rightarrow K^0_L e^+ \nu_e)$.

Combining the PDG values for $B(D^0 \rightarrow K^- e^+ \nu_e)$, $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$ [18], and the lifetimes of $D^0$ and $D^+$ mesons ($\tau_{D^0}$ and $\tau_{D^+}$) [18] with the value of $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$ measured in this work, we determine

$$\begin{align*}
\Gamma(D^0 \rightarrow K^- e^+ \nu_e) &= B(D^0 \rightarrow K^- e^+ \nu_e) \times \tau_{D^0} \\
\Gamma(D^+ \rightarrow K^0 e^+ \nu_e) &= B(D^+ \rightarrow K^0 e^+ \nu_e) \times \tau_{D^+} \\
&= 0.969 \pm 0.025, \quad \text{(8)}
\end{align*}$$

where $B(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)$ is the the averaged branching fraction based on the PDG value and the one measured in this work. This gives a more stringent test on isospin symmetry in the $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ and $D^0 \rightarrow K^- e^+ \nu_e$ decays.

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