Actuarial Study on Equal Progressive and Equal Ratio Progressive Repayment Model

Yongkuo Liu*

School of Finance, Shanxi University of Finance and Economics, Taiyuan, Shanxi, 030000

*Corresponding author: liuyongkuo@sxufe.edu.cn

Abstract. Equal progressive repayment method and equal ratio progressive repayment method are two kinds of repayment modes which are widely used in countries and regions where individual housing loan businesses flourish internationally. Firstly, starting from cash flow generated during repayment process, modeling loan repayment process, obtains relations between payment, principal and interest in general repayment mode; secondly, uses relational formula to generate equal progressive (withdrawal) repayment method and equal ratio progressive (withdrawal) repayment method next payment, principal and interest mathematical expression; Finally through case analysis give two kinds of repayment methods applicable to housing groups provide reference for financial institutions to introduce diversified repayment methods. When buyers handle mortgage loans, financial institutions generally offer two kinds of repayment methods: equal principal repayment method and equal principal repayment method. Equal principal repayment means equal amounts of loans within repayment periods, repayment of principal amount per month (month) and interest generated during that period, i.e. fixed assets per month (month). Equal interest payment method is to add principal sum of mortgage loan sum interest sum then share each repayment period (month) namely each period of period for fixed. With rapid economic development of our country, residents’ consumption demand has changed greatly, and the concept of advanced consumption gradually accepted by people. Financial institutions have created many new types of loan repayment methods to satisfy people's consumption demand such as single debt service payment maturity repayment method equal progressive repayment method mobile combination repayment method check-in payment method double week supply relay loan circulation loan balloon loan hole loan etc. Most of these innovative repayment methods are used in short-term consumer loans while residents mortgage loans are long term loans and belong to banks’ high quality assets. Shao Lei and Mr Chen Yonghong pointed out that from America Japan and Hong Kong China such as private housing mortgage lending business booming countries and regions actual situation such as progressive repayment method and equal progressive repayment method is one of two most popular repayment modes internationally especially welcomed by newly married couples and young entrepreneurs. However, these two kinds of repayment methods are complicated, coupled with computer settings and other reasons, currently in China temporarily less used by banks. In our country loan consumption mode gradually accepted by ordinary people. In our country common loan repayment mode includes one-time repayment method equal principal interest repayment method equal principal repayment method. Besides three common repayment modes mentioned above, there are other kinds of progressive repayment methods and equal progressive repayment methods which are commonly used internationally, although these two kinds of repayment methods are less used by banks in China than in China. However, in countries where individual loan business flourish, two kinds of consumer credit repayment modes are very popular. This
paper establishes mathematical models of equal ratio progressive repayment method and equal progressive repayment method by using annuity theory in financial mathematics, and analyzes the suitable crowd of these two kinds of repayment methods.

**Keywords:** Equal Progressive repayment method; equal ratio Progressive repayment method; actuarial model; diversified repayment mode.

1. **Introduction**

The law of reimbursement of principal of equal amount [1] is to point to be in reimbursement period loan total amount is equal cent, every period (month) the interest that the principal of reimbursement of equal amount and rest loan produce inside this period, namely every period (month) place is returned principal is fixed. Equivalent principal and interest repayment method [2] is the total amount of the mortgage loan principal and interest sum, and then the average allocation to each repayment period (month), that is, the period of each period fixed. Zheng Zexing and Tang Zhenrong [3] pointed out that these two repayment modes require the repayment principal and interest to be evenly distributed during the repayment period. Although it is convenient for Banks to determine their own future cash flow, it does not give borrowers more room to choose the repayment mode according to their income distribution. In fact, different borrowers have different repayment demands: young people who have just come out to work are in the beginning stage of their careers, lack working experience and corresponding working skills, have low salary in the early stage, and have to consider various issues such as house decoration after paying down payment. As a result, such borrowers are particularly keen to pay less upfront and more later. For retiring old buyers, their life after the early stage of the struggle, has accumulated a certain amount of money and the corresponding work skills, early work much higher income, they will consider is retired mortgage paid off so that they can be abetted, therefore, hope to take early has this kind of borrowers, the late reimbursement means less also.

From the perspective of the original intention of financial institutions' design of mortgage loan, its purpose is to hope that the borrower can repay the loan principal and the corresponding interest in full and on time within the stipulated loan term in accordance with the repayment method agreed in advance. Based on the reimbursement process and generate cash flow as a starting point, the loan repayment process modeling, obtained the general reimbursement means next to, the relationship between the principal and interest, and by using this formula has been further period for (1) is a progressive (back), waiting for the forehead geometric progressive trend periods (back), the mathematic expression of the principal and interest. In the end, through case analysis, this paper gives the suitable borrowers of these two repayment methods, and provides some references for financial institutions to launch diversified repayment methods.

2. **Reimbursement model**

Suppose the loan amount is M yuan, the loan period is N years, the annual repayment in period, the loan interest rate is \( \alpha \) (annualized), the term interest rate \( r = \frac{\alpha}{\beta} \). If the principal payable in the N period is \( A(n) \), the interest payable is \( B(n) \), and the principal payable is \( C(n) \), then \( A(n) + B(n) = C(n) \).

2.1. **General repayment model**

Let the repayment amount flow be \( C(1), C(2), \ldots, C(\beta N) \), since the interest is generated from the principal of the loan,

\[
\begin{align*}
B(1) &= \frac{M\alpha}{\beta}, A(1) = C(1) - B(1) \\
B(2) &= \left(\frac{M-A(1)\alpha}{\beta}\right), A(2) = C(2) - B(2)
\end{align*}
\]
Here \( A(I) > 0, B > 0, (I) I = 1, 2, ..., K, k \) is the last payment.

If \( K < \beta N \), the borrower prepayments, the loan contract automatically terminated; If \( k = \beta N \), the borrower normally pays off the loan on schedule, and the remaining principal and the corresponding interest will be paid off in the last installment.

In the process of the entire loan repayment due (left) the principal is not 0, so the issue also interest must be fully, namely must have \( A B(I) > 0 \), but the period for the principal part of the can is 0, that is \( A(I) \leq 0 \).

In order to obtain the relationship among the interim supply, principal and interest of the general repayment model, formula (2) is further obtained.

\[
B(n + 1) = \frac{(M - \sum_{i=1}^{n} A(I)) \cdot a}{\beta} = A(n + 1) = C(n + 1) - B(n + 1)
\]

(4)

\[
B(n) - B(n + 1) = A(n) \frac{a}{\beta}
\]

(5)

\[
C(n) - C(n + 1) + A(n + 1) = A(n)(1 + \frac{a}{\beta})
\]

(6)

The above gives a more general expression of the relationship between principal and interest, principal and maturity in the loan repayment process. Moreover, Formula (5) has a strong intuitive meaning, that is, the interest will gradually decrease with the reduction of the loan principal.

2.2. A progressive (regressive) repayment model of equal amount

Equal progressive (refund) repayment method [4] is a repayment method that divides the whole repayment period according to a certain period of time, in which each period is more (less) than the previous period and a fixed amount is agreed, and the principal and interest of the loan should be repaid every month with the same amount of repayment within each period. Without loss of generality, suppose

\[
C(n) = C(1) + (n - 1)d, n = 1, 2, ..., \beta N
\]

(7)

The size of \( D \) is agreed between the borrower and the bank.

Obviously \( d > 0 \), is equal amount progressive repayment method; When \( d < 0 \), is the equal amount regressive repayment method; When \( D = 0 \), is the repayment method of principal and interest of the same amount. And \( d \) must be such that \( A(I), B(I), I = 1, 2, ..., N \) satisfies the constraint equation (3).

According to Equation (6), \( A(n + 1) - d = A(n)(1 + r) \) Let's add to both sides:

\[
A(n + 1) + \frac{d}{r} = [A(n) + \frac{d}{r}] \cdot (1 + r)
\]

\[
A(n) = [A(1) + \frac{d}{r}] \cdot (1 + r)^{n-1} - \frac{d}{r}
\]

(8)

The constraints must be satisfied \( \sum_{n=1}^{\beta N} A(n) = M \)

\[
\sum_{n=1}^{\beta N} \left( [A(1) + \frac{d}{r}] \cdot (1 + r)^{n-1} - \frac{d}{r} \right) = M
\]
2.3. Equal-ratio progressive (regressive) repayment model

Equal-ratio progressive (refundable) repayment method [5-7] is a repayment method that divides the whole repayment period according to a certain period of time, in which each period is more (less) than the previous period and a fixed proportion is agreed, and the principal and interest of the loan should be repaid every month with the same amount of repayment within each period. Without loss of generality, let

\[ C(n) = C(1)q^{n-1} \]

Obviously, when \( q > 1 \), it is equal ratio progressive repayment method; When \( 0 < q < 1 \), it is equal ratio regressive repayment method. When \( Q = 1 \), it is the repayment method of principal and interest of the same amount.

According to the discounted cash flow method, there are:

\[ C(1) \left( \frac{1}{1+r} \right) + C(2) \left( \frac{1}{1+r} \right)^2 + \cdots + C(\beta N) \left( \frac{1}{1+r} \right)^{\beta N} = M \]

Plug in: \( C(n) = C(1)q^{n-1} \)

\[ \frac{C(1)}{1+r} + \frac{C(1)q}{(1+r)^2} + \cdots + \frac{C(1)q^{\beta N-1}}{(1+r)^{\beta N}} \cdot M \]

When \( q = 1 + r \), it can be calculated from equation (10) \( C(1) = \frac{M(1+r)}{\beta N}, C(n) = \frac{M}{\beta N} \cdot (1 + r)^n \). When \( q \neq 1 + r \), it can be calculated by Equation (10), \( C(1) = \frac{M(1+r-q)}{1-(\frac{1}{1+r})^{\beta N}} \) and \( C(n) = \frac{M(1+r-q)}{1-(\frac{1}{1+r})^{\beta N}} \cdot q^{n-1} \).

Take A(n) and B(n).

\[ C(n) = \frac{M}{\beta N} (1 + r)^n \]

\[ \frac{M}{\beta N} (1 + r)^n - \frac{M}{\beta N} (1 + r)^{n+1} + A(n + 1) = A(n)(1 + r) \]

Divide both sides by \((1 + r)^{n+1}\)

\[ \frac{M}{\beta N(1 + r)} - \frac{M}{\beta N} + \frac{A(n + 1)}{(1 + r)^{n+1}} = \frac{A(n)}{(1 + r)^n} \]

Thus, \( \{\frac{A(n)}{(1 + r)^n}\} \) is \( \frac{A(1)}{1 + r} \) the first item, and
\[ A(n) = \left[ A(1) + (n - 1) \frac{Mr}{\beta N} \right] \cdot (1 + r)^{n-1} \quad (11) \]

\[ B(n) = C(n) - A(n) = \frac{M}{\beta N}(1 + r)^n - \left[ A(1) + (n - 1) \frac{Mr}{\beta N} \right] \cdot (1 + r)^{n-1} = \frac{M}{\beta N} \cdot [1 + (2 - n) \cdot r] - A(1) \cdot (1 + r)^{n-1} \quad (12) \]

According to Formula (1), there are initial conditions:

\[ A(1) = C(1) - B(1) = \frac{M(1 + r)}{\beta N} - Mr = \frac{M}{\beta N}(1 + r - \beta Nr) \]

And according to formula (11),

\[
\sum_{n=1}^{\beta N} A(n) = \sum_{n=1}^{\beta N} \left\{ \left[ A(1) + (n - 1) \frac{Mr}{\beta N} \right] \cdot (1 + r)^{n-1} \right\} = \sum_{n=1}^{\beta N} \left[ A(1) \cdot (1 + r)^{n-1} \right] \\
+ \sum_{n=1}^{\beta N} \left[ \frac{Mr}{\beta N} \cdot (n - 1)(1 + r)^{n-1} \right] = M
\]

This law is applicable; otherwise, it will be inconsistent with reality.

When \( C(n) = \frac{M(1+r-q)}{1-(\frac{r}{1+r})^{\beta N}} \cdot q^{n-1} \),

\[
C(1)q^{n-1} - C(1)q^n + A(n + 1) = (1 + r)A(n)
\]

Divide both sides by \( q^n \),

\[
\frac{C(1)}{q} - C(1) + q A(n+1) = (1 + r) \frac{A(n)}{q^n}.
\]

Make \( h(n) = \frac{A(n)}{q^n} \)

\[
C(1)\left(\frac{1}{q} - 1\right) + qh(n + 1) = (1 + r)h(n)
\]

\[
\frac{M(1+r-q)}{1-(\frac{r}{1+r})^{\beta N}} \cdot \frac{1-q}{q} + qh(n + 1) = (1 + r)h(n)
\]

\[
\frac{M(1-q)}{q(1-(\frac{r}{1+r})^{\beta N})} = D_0 \quad \text{the} \quad \frac{C(1)}{1+r-q} = \frac{D_0q}{1-q}.
\]

When \( 0 < q < 1 \) or \( q > 1 + r \), \( D_0 > 0 \); When \( 1 \) is less than \( q \) is less than \( 1 \) plus \( r \), \( D_0 \) is less than \( 0 \).

Thus \( D_0(1 + r - q) + qh(n + 1) = (1 + r)h(n) \).

\[
q(h(n + 1) - D_0) = (1 + r)(h(n) - D_0)
\]

\[
h(n) = (h(1) - D_0)\left(\frac{1 + r}{q}\right)^{n-1} + D_0
\]

In turn, \( A(n) = (A(1) - D_0q)(1 + r)^{n-1} + D_0q^n \)

\[
B(n) = C(n) - A(n) = \frac{D_0}{1-q} \cdot (1 + r - q)q^n - (A(1) - D_0q)(1 + r)^{n-1} - D_0q^n
\]

\[
= \frac{D_0r}{1-q} \cdot q^n - (A(1) - D_0q) \cdot (1 + r)^{n-1}
\]
According to 1

\[ B(1) = Mr, A(1) = C(1) - B(1) = \frac{M(1 + r - q)}{1 - (\frac{q}{1 + r})^\beta N} - Mr \]

For A (1), which requires A (1) ≥ 0

\[
\sum_{n=1}^{\beta N} A(n) = \sum_{n=1}^{\beta N} ((A(1) - D_0q) \cdot (1 + r)^{n-1} + D_0q^n) = \\
\sum_{n=1}^{\beta N} ((A(1) - D_0q) \cdot (1 + r)^{n-1}) + \sum_{n=1}^{\beta N} (D_0 \cdot q^n) = \\
(A(1) - D_0q) \cdot \frac{(1 + r)^{\beta N - 1}}{r} + D_0q \cdot \frac{1 - q^{\beta N}}{1 - q} \\
\]

### 3. The empirical analysis

A buyer is going to purchase a house with a bank loan of 1 million yuan. The loan term is 30 years, and the monthly repayment is on time. The annual interest rate of the loan is 5.88%.

(1) Equal amount progressive (refund) repayment method.

For the equal amount progressive (back) repayment method, d=±5 can be taken here. (The value of D needs to make the principal and interest of each period meet the constraint equation (3).) According to Equations (7), (8) and (9), we can find:

Fig 1. Equal progressive repayment
It can be seen from Figure 1 and Figure 2 that the equal amount progressive (back) repayment method shows a linear monotone increase (decrease) of the term supply, but the principal repaid in each period shows a nonlinear monotone increase trend, and the interest decreases gradually with the decrease of the principal, which is consistent with the reality.

(2) Equal-ratio progressive (refund) repayment method

For the equal-ratio progressive (refund) repayment method, three situations should be considered: $0 < q < 1$, $q = 1 + r$, $q > 1$ and $q \neq 1 + r$

Let's say that $q$ is equal to 0.999, $q = 1.001$ (The value of Q should make the principal and interest of each period meet the conditions (3)). According to equations (13) and (14),

Fig 2. Equal amount of regressive repayment

![Fig 2](image1)

Fig.3 Equal ratio regressive repayment

![Fig 3](image2)
It can be seen from Figure 3 that when $Q = 0.999$, the picture of "term supply" is almost a monotonically decreasing line, similar to the method of equal amount regressive repayment. Similarly, in FIG. 4, when $Q = 1.001$, the maturity profile is almost a monotonically increasing line, which is similar to the equal-amount progressive repayment method. The values of $Q$ above are all close to 1, which makes the future supply image appear to be a straight line. In order to verify this model and illustrate the importance of condition 3, a positive example and a negative example are given below. $Q = 0.98$ and $Q = 1 + r$ are taken respectively, and the results are shown in Figure 5 and Figure 6 below.
It can be seen from FIG.5 that, when Q = 0.98, before about 50 terms, the monthly payment of the borrower was above 10,000 yuan, and the initial payment was closer to 25,000 yuan. However, after the 200 period, the images of monthly payment, principal and interest almost coincide, indicating that the monthly payment is almost 0, in line with the requirements of early payment more, later payment less.

Figure 6 shows a counter example. The monthly payment is less in the early stage and more in the later stage, showing an increasing trend of equal ratio. However, before the period of 150, the principal is negative, and the trend of interest is to increase first and then decrease.

Therefore, for the isometric regressive repayment method, as long as 0 < q < 1, no matter what value q takes, it does not violate the reality. At the same time, the closer Q is to 0, the greater the repayment pressure in the early stage, the closer Q is to 1, and the flatter the image of the term supply is. When the value of Q is close to 1, the equal ratio progressive (back) repayment method can be replaced by the equal amount progressive (back) repayment method. When the value of Q is greater than 1 and deviates from 1 for a long time, it can be seen from constraint condition 3 and FIG. 6 that the initial principal part will be negative and the interest will increase first and then decrease, which is contrary to the reality. As for this problem, financial institutions can try to apply to borrowers that they only need to repay the interest in the early stage and begin to repay the principal and interest in a later period, or adopt another algorithm, assuming that the principal repaid in each period increases proportionally, and use Equations (5) and (6) to recalculate the mathematical expressions of the period supply and interest.

To sum up, equal amount progressive (refund) repayment method and equal ratio progressive (refund) repayment method are mainly applicable to the future income expectation has the big change (or gradually increase or gradually reduce) the population. For borrowers who wish to pay back less in the first few years and more in the later years, the equal amount progressive repayment method of d > 0 or the equal proportion progressive repayment method of q b> 1 can be adopted. For borrowers who wish to repay more in the early stage and less in the later stage, equal amount regressive repayment method with D < 0, equal amount regressive repayment method with 0 < q < 1 or traditional repayment method with equal amount of principal. Of course, the value of D or Q should be in line with the actual situation and take into account the future cash flow of the payer.

Geometric progressive repayment method and equal a progressive payment method due to less common in the loan business in our country, these two kinds of payment method is mainly used in the expected future income has great changes (or increase gradually, or gradually reduce) populations, such as the couple of young people and starting a business is expected future income has increased greatly, can use q > 1 geometric progressive payment method or d > 0 matching a progressive payment method; For example, if older people expect to have less income in the future, they can use the equal progressive repayment method with q < 1 or d < 0. The value of Q or D can be set according to personal circumstances.
The different value of the repayment period L, Q or D directly affects the monthly repayment amount within each period.

4. Conclusion
It is the most powerful competitive means for financial institutions to take the customer as the center and provide diversified products and services for customers. In the progressive repayment method, the principal is recovered less in the early stage, and financial institutions have more interest-bearing assets. In the early stage of the regressive repayment method, the principal is recovered more, the loan risk is small, and the capital utilization rate of financial institutions is high. However, when financial institutions launch new products, they must also bear certain expenses and bear the risks of the products themselves. The mathematical expressions of period, principal and interest obtained by using the annuity theory in this paper can provide some references for financial institutions to create new products.

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