Do we observe Lévy flights in cosmic rays?

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We argue that the so called long flying component (LFC) observed in some cosmic ray experiments are yet another manifestation of Lévy distributions (with index $q = 1.3$), this time of the distribution observation probability of the depths of starting points of cascades. It means that LFC is governed by the so called long-tail Lévy-like anomalous superdiffusion, a phenomenon frequently encountered in Nature. Its connection with the so called Tsallis’s statistics is also briefly discussed.

1. INTRODUCTION

Cosmic ray experiments report for some time the existence of the apparent long flying component (LFC) phenomenon in the propagation of the initial flux of incoming nucleons. We shall concentrate here on the example of the distribution of cascade starting points in the extra thick lead chamber of the Pamir experiment (cf. \cite{1} for detailed references concerning the whole subject considered here). What is regarded as peculiar or even unexpected is the fact that instead of the normally expected exponential fall off of the depth distribution of the starting points of cascades

$$\frac{dN(T)}{dT} = \text{const} \cdot \exp \left(-\frac{T}{\lambda}\right)$$

one gets power law behaviour of the type (cf. Fig. 1):

$$\frac{dN(T)}{dT} = \text{const} \cdot \left[1 - \frac{(1 - q)T}{\lambda}\right]^{-\frac{1}{q}}$$

with parameter $q$ (to be specified below) equal $q = 1.3$ and with the absorption length (in c.u.) for the interaction of hadrons in the lead chamber equal to $\lambda = 18.85 \pm 6.62$. The immediate question one is faced with is: have somebody seen something like this somewhere?. The answer to it is positive and we shall concentrate on it in what follows.

2. LÉVY DISTRIBUTIONS

In many places (albeit in different circumstances than encountered here like, for example, distributions of heartbeats, travel patterns of albatrosses, behaviour of stock market, time dependence of the leaky faucet and numerous other) one discovers that the respective distributions are of the type

$$P(x) \sim \frac{1}{x^{1+\gamma}} \quad \text{for large } x.$$  \hspace{1cm} (3)

They represent the so called Lévy distributions $L_\gamma(x)$ the characteristic feature of which is that they allow for much longer jumps (called Lévy flights) than in normal diffusion \cite{2}. Whereas in normal diffusion emerging from the Brownian motion one encounters chaotic (usually small) jumps governed by Gaussian distribution with finite variance, in Lévy flights long jumps appear intermittently with short ones and variance is divergent. As a result one gets a fractal structure of the distribution of points visited by a random walker as demonstrated in \cite{3}.
Figure 1. Depth distribution of the starting points of cascades in Pamir lead chamber. Notice the non-exponential behaviour of data points (for their origin cf. [1]).

3. SIMPLE-MINDED DERIVATION

The only possible way to (formaly) reconcile the power-like behaviour of eq.(2) with the exponential eq.(1) is to make substitution $\lambda \rightarrow \lambda(T) = \lambda - (1 - q)T$ in the later. Notice that our previous explanation of the LFC [1] which was based on the idea of fluctuating cross sections leads effectively to such a replacement. Performing now such replacement in the derivation of eq.(2) one gets

$$\frac{dN}{N} = \frac{-dx}{\lambda} \Rightarrow -\frac{dx}{\lambda [1 - (1 - q)\frac{x}{\lambda}]}.$$ (4)

Changing now variables to $z = 1 - (1 - q)\frac{x}{\lambda}$ one gets

$$\frac{dN}{N} = \frac{1}{1 - q} \frac{dz}{z} \Rightarrow N = N_0 \frac{1}{z^{1 - q}},$$ (5)

i.e., equation (2).

4. TSALLIS’S STATISTICS

The whole above discussion can be put on more formal level by introducing the concept of information entropy

$$S = -\sum_i p_i \ln p_i$$ (6)

for a given probability distribution $p_i$. It can be shown (cf. [1] for relevant references and discussion for production processes) that the most plausible or least biased model independent $p_i$’s for a given process can be obtain by maximalizing $S$ under constraints:

$$\sum_i p_i = 1 \quad \text{and} \quad \sum_l R_i^{(l)} p_i = <\hat{R}_i^{(l)}>, \quad (7)$$

where first one assures proper normalization of the probability distribution $p_i$ whereas the rest $l = 1, 2, \ldots$ of them represent our experimental knowledge on the problem under consideration [1]. In this way $p_i$ tells us the truth, the whole truth about our experiment, i.e., reproduces known information but also it tells us nothing but the truth, i.e., it conveys the least information. The Gaussian diffusion can be then derived straightforwardly [1] by demanding finitness of the second moment of $p(x)$. Hower, this formalism fails to describe random walks with more complex jump probabilities (like Lévy distributions). In general it fails for all physical systems involving either long range correlations or long-range memory effects or fractal space-time structure. To overcome this problem Tsallis proposed generalised statistics leading to generalised entropy $S_q$:

$$S_q = -k \frac{1 - \sum p_i^q}{1 - q}$$ (8)

which in the limit when new parametr $q$ approaches unity, $q \rightarrow 1$, goes into (2) ($k$ is constant, which in eq.(2) was put equal unity).
entropy can be now used in the same way as mentioned above with the \( l = 1, 2, \ldots \) constraints representing measured observables changed to

\[
\sum R_q^{(l)} p_q^l = < \hat{R}_q^{(l)} >. \tag{9}
\]

In this manner we get instead the Gaussian distribution, as before, the following power-like distribution:

\[
p_q(x) = \frac{1}{Z_q} [1 - \beta (1 - q) x^2]^{-1}, \tag{10}
\]

where

\[
Z_q = \int dx \left[ 1 - \beta (1 - q) x^2 \right]^{1/(1-q)}. \tag{11}
\]

Notice that for large \( x \) distribution (10) with \( q = (3 + \gamma)/(1 + \gamma) \) coincides with distribution given by eq. (8) representing Lévy flights.

The new parameter \( q \) introduced in (8) describes in a simple way possible correlations present in the system in the sense that, as is obvious from the new form of the constraint equations, it either enhances frequent events (for \( q > 1 \)) or enhances the rare ones (for \( q < 1 \)). The best way to demonstrate this property is to look at the entropy of the system composed with two subsystems \( A \) and \( B \):

\[
S_q(A \cup B) = S_q(A) + S_q(B) + (1 - q) S_q(A) \cdot S_q(B). \tag{12}
\]

It is in obvious way non-additive now, the normal additivity is recovered only in the usual case of \( q = 1 \) (leading to eq. (3)). It means that we are facing a nonextensivity here: subadditivity for \( q > 1 \) and superadditivity for \( q < 1 \).

5. SUMMARY AND CONCLUSIONS

We have proposed a novel approach to the (regarded as "strange" or "suspicious") power-like behaviour of some observables measured in cosmic ray experiments. The novelty of this approach is in the fact that even without detail dynamical knowledge of the origin of such effects (as provided, for example, in [6]) it is still worth to parametrize them with a single new parameter \( q \). This parameter expresses summarily some new, so far undiscovered, dynamics by showing that there are some correlations present in these particular measurements. The final deciphering of its meaning is out of the scope of this presentation and will be attempted elsewhere. Here we would only like to bring ones attention to the fact that in essentially the same way one can generalize the usual information entropy approach used to describe single particle distributions in multiparticle production processes (as provided for example, in [6]) to the case of more general Tsallis’s entropy \( S_q \). The ultimate goal in this case would be the unique description of leading particle spectra in terms of a single physical parameter \( q \). This would be especially suited for description of a great amount of cosmic ray data. Such project is at present under investigation. Finally, let us mention that Tsallis’s entropy can still be generalized even more (cf. [6]) but physical meaning of this generalization is, at least from the point of view of cosmic ray applications, not yet clear.

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