Adjustment of the k-ω SST turbulence model for prediction of airfoil characteristics near stall

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Abstract. A version of k-ω SST turbulence model adjusted for flow around airfoils at high Reynolds numbers is presented. The modified version decreases eddy viscosity and significantly improves the accuracy of prediction of aerodynamic characteristics in a wide range of angles of attack. However, considered reduction of eddy viscosity destroys calibration of the model, which leads to decreasing accuracy of skin-friction coefficient prediction even for relatively simple wall-bounded turbulent flows. Therefore, the area of applicability of the suggested modification is limited to flows around airfoils.

1. Introduction
Prediction of airfoil characteristics in regimes with maximum lift and near stall is an important task for aviation and wind power, as well as for turbomachinery. Unfortunately, maximum lift coefficient and corresponding angle of attack are systematically overpredicted by the Reynolds Averaged Navier-Stokes (RANS) approach in combination with different semi-empirical turbulence models (see, for example, [1-3]). This disagreement is caused by delay of turbulent boundary layer separation under adverse pressure gradient on the suction side of the airfoil (Fig. 1). Since in the frame of RANS approach the separation position is controlled by the turbulence model, one of the ways to improve accuracy of the airfoil characteristics prediction is a special tuning of the models for such class of flows. This paper presents such adjustment for k-ω SST (2003) turbulence model [4].

![Figure 1. Scheme of flow around an airfoil at the near-stall regime.](image)

2. Model formulation
The equations of the turbulent kinetic energy (k) and specific dissipation rate (ω) for the SST turbulence model are obtained from a combination of k-ε and k-ω turbulence models and read as:
\[
\frac{D(\rho k)}{Dt} = \nabla \cdot \left[ (\mu + \sigma_k \mu_t) \nabla k \right] + \frac{P_t - \beta^* \rho \omega k}{\omega}
\]
\[
\frac{D(\rho \omega)}{Dt} = \nabla \cdot \left[ (\mu + \sigma_\omega \mu_t) \nabla \omega \right] + \gamma \frac{P_t - \beta^* \rho \omega^2 + 2(1-F_1)\rho \sigma_{a2}}{\omega} \left( \nabla k \cdot \nabla \omega \right)
\]

where \( P_t = \min(\mu, S^2, 10\beta^* \rho k \omega) \) is the production term,
\( d_w \) – wall distance,
\( \rho \) – density,
\( \mu \) and \( \nu \) – dynamic and kinematic viscosity correspondingly,
\( S \) – strain rate.

Functions and constants of the model are as follows:

\[
F_1 = \tanh\left( \arg_1^* \right) \quad \arg_1 = \min \left[ \max \left( \frac{k}{0.09 \omega d_w}, \frac{500 \omega}{d_w^2 \omega} \right), \frac{4 \rho \sigma_{a2} k}{CD_{\text{ref}} d_w^2} \right], \quad CD_{\text{ref}} = \max \left( 2 \rho \sigma_{a2} \frac{1}{\omega} \frac{\partial k}{\partial x} \frac{\partial \omega}{\partial x}, 10^{-10} \right)
\]
\[
F_2 = \tanh\left( \arg_2^* \right) \quad \arg_2 = \max \left( 2 \frac{k}{0.09 \omega d_w}, \frac{500 \omega}{d_w^2 \omega} \right)
\]
\[
\sigma_1 = F_1 \sigma_{a1} + (1 - F_1) \sigma_{a1}, \quad \sigma_a = F_1 \sigma_{a1} + (1 - F_1) \sigma_{a2}, \quad \beta = F_1 \beta_1 + (1 - F_1) \beta_2
\]
\[
\gamma = \frac{\beta^* - \sigma_a \kappa^*}{\sqrt{\beta^*}}
\]

Coefficients of the model read as:
\( \beta^* = 0.09, \quad \kappa^* = 0.41, \quad a_1 = 0.31, \quad \sigma_{a1} = 0.85, \quad \sigma_{a2} = 1.0, \quad \sigma_a = 0.5, \quad \sigma_{a2} = 0.856, \quad \beta_1 = 0.075, \quad \beta_2 = 0.0828. \)

Eddy viscosity definition \( \mu_t = \frac{\rho a_t k}{\max(a_t, \omega, SF_2^2)} \) includes the so called SST limiter, which is introduced into the model in order to prevent overprediction of the shear stress in the boundary layers under adverse pressure gradient. Therefore, behaviour of the model in such boundary layers as well as separation on the suction side of the airfoil is controlled by the \( a_1 \) constant. In the present work the SST model modification and adjustment for flows around airfoils is carried out by modification of \( a_1 \).

3. Problem definition

Four aerodynamic airfoils with different shapes and thicknesses (from 15% to 21%) were considered (Fig. 2). Experimental investigations [5 - 8] were carried out in low turbulence wind tunnels (1<1%), at relatively high Reynolds number (Re > 10^6) based on airfoils chord and freestream velocity.

A tunnel wall correction was applied to the airfoil characteristics and the angle of attack for comparison with freestream setup [9]. Since the experimental Mach number did not exceed 0.15, incompressible flow was considered.

![Figure 2. Considered airfoils.](image)

Numerical simulations of two-dimensional RANS equations in combination with the SST turbulence model were carried out using the double precision version of ANSYS Fluent 15.0. The pressure-based coupled solver was employed with the Second Order Upwind discretization scheme for the convective terms in all transport equations.

Fine structured C-type meshes were generated in 20Cx20C (C – airfoil chord) computational domain using ICEM CFD (Fig. 3). The meshes were refined normal to the wall in order to resolve the viscous sublayer (\( \Delta y^+ < 1 \)), near the leading edge in the streamwise direction for a proper resolution of thin
boundary layer, and near the trailing edge. This results in about 400 points along the airfoil and a total mesh size of about 40,000 cells.

A constant velocity is specified at the inlet section of the computational domain. Inlet turbulent kinetic energy corresponds to experimental turbulence intensity and the specific dissipation rate is specified as $\omega = 10 \cdot U_\infty / C$ [10]. No-slip conditions are used on the airfoil surface and constant pressure is specified on the outlet.

![Inlet and Outlet](image)

**Figure 3.** Computational domain, boundary conditions, and mesh.

4. Results

4.1. Model calibration

Preliminary results of the original SST model indicated that the size of the separation zone and position of the separation point on the suction side of the airfoil was substantially underestimated, especially for angles of attack near stall, which leads to an overestimation of the maximum lift coefficient. The calibration of $a_1$ was carried out for the S809 airfoil at $Re = 2 \cdot 10^6$ and angle of attack equals to $10^\circ$. One can see that decrease of $a_1$ leads to increase of the size of the recirculation zone and improves agreement with experimental data (Fig 4a, 5). The best agreement is achieved at $a_1 = 0.28$, so this value was used for further computations (Fig. 4b).

![Skin-friction and Pressure Coefficients](image)

**Figure 4.** Effect of $a_1$ constant on the prediction of the skin-friction (a) and pressure (b) coefficients for the S809 airfoil at $\alpha = 10^\circ$.
4.2. Prediction of flow around airfoils

Computations using the original and modified versions of the SST model show that both models predict virtually the same lift coefficient for low angles of attack when the flow is attached (Fig. 6). At higher angle of attack the modified SST model predicts lower value of the lift coefficient due to the larger size of the recirculation zone.

![Figure 5](image5.png)

**Figure 5.** Contours of streamwise velocity component and streamlines near the S809 airfoil at $\alpha = 10^\circ$.

![Figure 6](image6.png)

**Figure 6.** Comparison of experimental and computational lift coefficient at different angles of attack. Experiments were carried out for clean airfoil model (EXP: Clean) and model with rough leading edge (EXP: Tripped)

One can see (Fig. 7) that the same separation point is obtained at $4^\circ-5^\circ$ lower angles of attack. Lift coefficient distribution for modified version as well as pressure coefficient (Fig. 8) is in good agreement.
with the experimental data for all the considered airfoils over a wide range of angles of attack, while the original SST model strongly overpredicts lift value (error is about 25%). However even the modified model predicts somewhat higher deep stall angle. One of the possible reasons is elimination of laminar-turbulent transition effect [11].

Figure 7. Position of computational separation point on the suction side of the airfoils at different angles of attack.

|   | S805 | S809 | S827 |
|---|------|------|------|
|   | ![Graph](image1.png) | ![Graph](image2.png) | ![Graph](image3.png) |

Figure 8. Computational and experimental pressure coefficient for different airfoils at angle of attack near stall ($\alpha = 15^\circ$).

4.3. Basic turbulence flows.
The SST model was calibrated by the authors on a set of basic turbulent flows and decrease of the $a_1$ constant could destroy this calibration. The effect of reduction of $a_1$ was checked with prediction of flow on the flat plate [12] and in CS0 diffuser [13]. Results of computations show that the modified model systematically underpredicts the value of the skin-friction coefficient for even relatively simple wall-bounded turbulent flows. In particular, the error in prediction of skin friction coefficient on the flat plate reaches 25% (Fig. 9a), which confirms once again the imperfection of semi-empirical turbulence models for all types of flows.

Figure 9. Comparison of experimental and computational skin friction coefficient distribution for the flat plate (a) and CS0 diffuser (b).
5. Conclusion
Presented modification of the k-ω SST turbulence model with reduced $a_1$ constant ($a_1 = 0.28$) significantly improves the accuracy of prediction of aerodynamic characteristics over a wide range of angles of attack ($\alpha = 0^\circ \div 30^\circ$). Error in the prediction of maximal lift coefficient does not exceed 5% with the modified model (this error is about 25% for the original model). However, reduction of the $a_1$ constant destroys calibration of the model, which leads to decreased accuracy of the skin-friction coefficient even for relatively simple wall-bounded turbulent flows. In particular, the error of skin friction coefficient for the flat plate reaches 25%, which confirms once again the imperfection of semi-empirical turbulence models for all types of flows. As a result, the area of applicability of suggested modification is limited to flows around airfoils.

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