The New Symmetries Beyond the Standard Model
(The Body-centred Cubic Periodic Symmetries
in Particle Physics)

Jiao-Lin Xu

The Center for Simulational Physics,
The Department of Physics and Astronomy
University of Georgia,
Athens, GA 30602, USA
jxu@hal.physast.uga.edu

Institute of Theoretical Physics,
Chinese Academy of Sciences, P. O. Bax 2735, Beijing 100080, China.

December 10, 2009

Abstract

This paper proposes new symmetries (the body-centred cubic periodic symmetries) beyond the standard model. Using a free particle expanded Schrödinger equation with the body-centred cubic periodic symmetry condition, the paper deduces a full baryon spectrum (including mass $M$, $I$, $S$, $C$, $B$, $Q$, $J$ and $P$) of all 116 observed baryons. All quantum numbers of all deduced baryons are completely consistent with the corresponding experimental results. The deduced masses of all 116 baryons agree with (more than average 98 percent) the experimental baryon masses using only four constant parameters. The body-centred cubic periodic symmetries with a periodic constant “a” about $10^{-23}$m play a crucial role. The results strongly suggest that the new symmetries really exist. This paper predicts some kind of “Zeeman effect” of baryons, for example: one experimental baryon $N(1720)\frac{3}{2}^+$ with $\Gamma = 200$ Mev is composed of two $N$ baryons $[(N(1659)\frac{3}{2}^+ + N(1839)\frac{3}{2}^+)] = N(1749)\frac{3}{2}^+$ with $\Gamma = 1839-1659 = 180$ Mev.

PACS numbers 11.30.-j 12.40.Yx
key words: new symmetry, beyond standard model, baryon spectrum, phenomenology.

1 Introduction

The standard model of particle physics has been enormously successful in predicting a wide range of phenomena. M. K. Gaillard, P. D. Grannis and F. J. Sciulli [1] point out. “And, just as
ordinary quantum mechanics fails in the relativistic limit, we do not expect the standard model to be valid at arbitrarily short distances. However, its remarkable success strongly suggests that the standard model will remain an excellent approximation to nature at distance scales as small as $10^{-18}$ m. How about the nature at distance scales smaller than $10^{-18}$ m?

*The Particle Physics Roadmap [2]*, in the “Beyond the Standard Model” section, proposes nine terms “as a mission statement for the field. 1) Are there undiscovered principles of nature: new symmetries, new physical laws? 2) How can we solve the mystery of dark energy? 3) Are there extra dimensions of space? . . . 9) What happened to the antimatter?” The first mission of all is to discover new symmetries.

Where can we find the undiscovered symmetries? How do we find the new symmetries? Professor Wilczek [3] points out: “that empty space—the vacuum—is in reality a richly structured, highly symmetrical, medium. . . . Because the vacuum is a complicated material governed by locality and symmetry, one can learn how to analyze it by studying other such materials—that is, condensed matter.” Wilczek not only pointed out where we can find the symmetries—“that empty space—the vacuum”, but also provided a very practical and efficient way for the study—“learning from studying condensed matter.”

Since the simplest and common symmetries of condensed matter is the body centred cubic periodic symmetries, this paper suggests that there are new symmetries beyond the standard model: the local body-centred cubic periodic symmetries with a periodic constant $a \sim 10^{-23}$ m around baryons. Using the symmetries and a free particle expanded Schrodinger equation, the paper can deduce the full baryon spectrum (including mass M, isospin I, strange number S, charmed number C, bottom number B, electric charge Q, total angular momentum J and parity P) with only four constant parameters. The deduced quantum numbers (I, S, C, B, Q, J and P) of all baryons (116 baryons) are completely the same as the 116 corresponding experimental baryons. The deduced masses of all 116 baryons agree with the corresponding experimental baryons with more than average 98 percent accuracy. The symmetries play a crucial role in the deduction. Thus the paper will show that the body-centred cubic periodic symmetries really exist in nature at distance scales smaller as $10^{-23}$ m by deduction of the full baryon spectrum that agrees with the experimental results.

The great success and exploration of the standard model, has prepared the conditions for deduction of the full baryon spectrum. Many excellent experimental physicists have searched for free quarks with as high as possible energies (accelerators and cosmic ray) to “unglue” quarks for more than 40 years, yet no one individually free quark has been found [4]. This has been a great exploration. The result strongly shows that the binding energy of the quarks in the baryons is extremely high. From that fact, we infer that the low mass spectrum of the baryons might not have originated from the internal quark motions inside the baryons since the energy levels of the internal quark motions might be much higher than the observable mass spectrum of the baryons. Such as the energy levels of nuclei with much high binding energy are much higher than the energy levels of molecules with much lower binding energy. The fact is really so, “the experimentally observed excitation spectrum of the nucleon ($N$ and $\Delta$ resonances) is compared to the results of a typical quark model calculation. Many more states are predicted than observed but on the other hand states with certain quantum numbers appear in the spectrum at excitation energies much lower than predicted.” [5] Many outstanding theoretical physicists have deduced the mass spectrum of the baryons using the internal motions of the quarks inside baryons for more than 40 years, but a completely satisfying united theoretical mass spectrum has still not arisen [6]. The quark model
does not give out the precise quark masses in theory and the quark masses cannot be measured directly by experiments, so we do not know the precise quark masses. **Trying to deduce the baryon masses from the unknown precise masses of the inside quarks is very difficult work.** Therefore, we try to deduce the mass spectrum of the baryons from outside symmetries. Then, using the qqq baryon model, we deduce the quark masses from the deduced ($\simeq$ measured) masses of the most important baryons.

In the low-energy phenomena, the baryons usually act as single particles. They are just like the atoms in chemical reactions in that the atoms look like elementary particles without internal structure. All baryons automatically decay into the ground baryon (proton and neutron) in a very short time [7]. **These facts make us think that the baryons might be different excited states of only one kind of particles. The simplest model that really works is a single free particle expanded Schrodinger equation with the body centered cubic periodic (BCCP) condition.**

In order to deduce the full spectrum of baryons using the BCCP symmetries, we will propose three postulates to set up a single free particle expanded Schrodinger equation and a body-centred cubic periodic (BCCP) condition in Section 2. We will deduce energy bands from the expanded Schrodinger equation with BCCP condition in Section 3. We will find phenomenological formulae about quantum numbers of the energy bands in Section 4. We will deduce the baryons using the phenomenological formulae and the energy bands in Section 5. We will compare the deduced baryons with the experimental results in Section 6. We will deduce the masses and the mass parameters of the five quarks from the masses of the most important deduced baryons in Section 7. Finally we will discuss some important problems (in Section 8) and will give the conclusions (in Section 9) as well as will give some predictions (in Section 10).

### 2 The Expanded Schrodinger Equation and the body-centred cubic Periodic Condition

All the experiments of particle physics show that (1) the baryons always are born and act as whole particles (not individual quarks), and (2) the baryons always automatically decay into the ground state baryons (protons and neutrons) in a very short time. From these very important experiments, we infer a single free particle approximation of the baryons (neglecting the internal structure and motions) to deduce the masses and the quantum numbers (I, S, C, B, L, J and P) of the baryons.

#### 2.1 The first postulate (perfect SU(N) symmetry groups)

The quark model [8] assumes five quarks (u, d, s, c and b) with special quantum numbers (see Table 14.1 of [9]) as the components of baryons and mesons to explain the observable quantum numbers of baryons and mesons with sum laws. The five quarks satisfy SU(5)$_f$ symmetries. The existence of baryons with t-quark is very unlikely due to the short lifetime of the top quark. In order to explain the mass differences of baryons, it also assumes that the five (four or three) quarks satisfy SU(5)$_f$ (SU(4)$_f$ or SU(3)$_f$) approximate symmetries and the symmetries have been broken by the mass differences of the quarks.
Since the quark model was born more than forty years ago, many new particles have been
discovered. Especially the surprising experimental facts—the quark confinement \[^4\] was discovered
which is entirely unexpected. These new experimental discoveries should be adopted to estimate
the quark masses, and to further check whether the \( SU(5)_f \) symmetry would be broken due to the
differences in quark masses.

The experimental facts of the quark confinement and the stability of proton (proton \( p \) with
lifetime \( \tau \) about \( 10^{30} \) years) inevitably result in huge binding energies of the three quarks in baryons.
According to the \( E = MC^2 \) formula, the huge binding energies and the small experimental masses
of baryons inevitably lead to huge quark masses. From \(qqq\) baryon model of the quark model, for
the five most important baryons (proton, neutron, \( \Lambda(1116) \), \( \Lambda_c(2286) \) and \( \Lambda_b(5624) \)), there are five
equations:

\[
\begin{align*}
    m_u + m_u + m_d - |E_{\text{bin}}(uud)| &= 938(\text{Mev}), \\
    m_u + m_d + m_d - |E_{\text{bin}}(udd)| &= 940(\text{Mev}), \\
    m_u + m_d + m_s - |E_{\text{bin}}(uds)| &= 1116(\text{Mev}), \\
    m_u + m_d + m_c - |E_{\text{bin}}(udc)| &= 2286(\text{Mev}), \\
    m_u + m_d + m_b - |E_{\text{bin}}(ubd)| &= 5624(\text{Mev}).
\end{align*}
\]

In order to find the maximum possible values of the quark mass differences, we assume that the
mass difference of the five baryons are completely from the mass differences of the five quarks, the
binding energy are completely the same,

\[
|E_{\text{bin}}(uud)| = |E_{\text{bin}}(udd)| = |E_{\text{bin}}(uds)| = |E_{\text{bin}}(udc)| = |E_{\text{bin}}(udd)| = |E_{\text{Bind}}|.
\]

From (6), (1) and (2) we have
\[
m_d - m_u = 2.
\]

Putting (7) into (1) and (2), from (6), we get

\[
\begin{align*}
    m_u &= \frac{1}{3}[|E_{\text{Bind}}|] + 312, \\
    m_d &= \frac{1}{3}[|E_{\text{Bind}}|] + 314.
\end{align*}
\]

Similarly, we can find:

\[
\begin{align*}
    m_s &= \frac{1}{3}[|E_{\text{Bind}}|] + 490, \\
    m_c &= \frac{1}{3}[|E_{\text{Bind}}|] + 1660, \\
    m_b &= \frac{1}{3}[|E_{\text{Bind}}|] + 4998.
\end{align*}
\]

Thus, from (8), (9), (10), (11) and (12), we have the masses of the quarks for the binding
energy = zero, 300 Gev, 3000 Gev and 30000 Gev as shown in Table 11:
Table 1. The masses of the quarks

| $E_{\text{bind}}(uud)$ | $m_u$ | $m_d$ | $m_s$ | $m_c$ | $m_b$ |
|------------------------|-------|-------|-------|-------|-------|
| 0 Gev                  | 312   | 314   | 490   | 1660  | 4998  |
| 300 Gev                | 100312| 100314| 100490| 101660| 104998|
| 3000 Gev               | 1000312| 1000314| 1000490| 1001660| 1004998|
| 30000 Gev              | 10000312| 10000314| 10000490| 10001660| 10004998|

The huge quark masses and the very small differences of the baryon masses naturally bring about that the very small mass differences of the quarks can be neglected as shown in Table 1. And the $SU(3)_f$, $SU(4)_f$ and $SU(5)_f$ symmetries give out completely correct quantum numbers of all baryons and all mesons. These facts force us to think that the $SU(3)_f$, $SU(4)_f$ and $SU(5)_f$ symmetries not only really exists, but also are perfect symmetries. The observable mass differences of baryons are not from the mass difference of the quarks (while they are from outside symmetry as shown in the following sections). Thus the experimental facts now have already shown that the $SU(N)$ ($N = 3, 4$ and $5$) symmetry groups are perfect since the five quark masses are the same essentially. Therefore we propose the first postulate:

Postulate I: **All five quarks have the same bare mass $m_q$ in the vacuum (but different physical masses in the observable baryons) and they satisfy perfect $SU(3)_f$ (for $u, d$ and $s$), $SU(4)_f$ (for $u, d, s$ and $c$) and $SU(5)_f$ (for $u, d, s, c$ and $b$) symmetries. In other words, the flavor $SU(N)_f$ ($N = 3, 4$ and $5$) symmetry groups are not broken.**

According to Postulate I, the five quarks have the same unknown large bare masses. Since $SU(5)_f$ symmetry group and $SU(3)_c$ symmetry group are both perfect, the interaction energies of the three quarks inside various baryons are the same. Thus we have a corollary:

Corollary: **The baryons composed of any three quarks will have the same unknown large bare mass $M_b$ in the vacuum.**

The observable physical masses of the various baryons with the same bare mass $M_b$ but different quantum numbers, however, are different since they are in different energy excited states (from the vacuum).

The experimentally observed baryons almost all belong to $SU(4)$ multiplets of baryons made of $u, d, s$ and $c$. The quark model has already given the quantum numbers of the baryons of the 20-plet with an $SU(3)$ Octet as appearing in Table 2 [10]:

Table 2. The quantum numbers of baryons with $s = 1/2$

| Baryons | $I$ | $I_Z$ | S   | C   | Q   | Bare M | Color |
|---------|-----|-------|-----|-----|-----|--------|-------|
| $N$     | $\frac{1}{2}$ | $\frac{1}{2}, -\frac{1}{2}$ | 0   | 0   | +1, 0 | $M_b$  | 0     |
| $\Sigma$ | 1 | 1, 0, -1 | -1  | 0   | 1, 0, -1 | $M_b$ | 0     |
| $\Lambda$ | 0 | 0       | -1  | 0   | 0    | $M_b$  | 0     |
| $\Xi$   | $\frac{1}{2}$ | $\frac{1}{2}, -\frac{1}{2}$ | -2  | 0   | 0, -1 | $M_b$  | 0     |
| $\Sigma_c$ | 1 | 1, 0, -1 | 0   | +1  | +2, +1, 0 | $M_b$ | 0     |
| $\Lambda_c$ | 0 | 0       | 0   | +1  | +1   | $M_b$  | 0     |
| $\Xi_c$ | $1/2$ | $\frac{1}{2}, -\frac{1}{2}$ | -1  | +1  | 1, 0  | $M_b$  | 0     |
| $\Xi_c'$ | $1/2$ | $\frac{1}{2}, -\frac{1}{2}$ | -1  | +1  | 1, 0  | $M_b$  | 0     |
| $\Omega$ | 0 | 0       | -2  | +1  | 0    | $M_b$  | 0     |
| $\Xi_{cc'}$ | $1/2$ | $\frac{1}{2}, -\frac{1}{2}$ | 0   | +2  | +2, +1 | $M_b$ | 0     |
| $\Omega_{cc'}$ | 0 | 0       | -1  | +2  | +1   | $M_b$  | 0     |
The quark model has already given the quantum numbers of baryons in the 20-plet of SU(4) with an SU(3) Decuplet as appearing in Table 3 [10]:

| Baryons | I | I_z | S | C | Q | Bare M | Color |
|---------|---|-----|---|---|---|--------|-------|
| Δ       | 3/2 | 3/2, -1/2 | 0 | 0 | 2, 1, 0, -1 | M_b | 0 |
| Σ       | 1   | 1, 0, -1  | -1 | 0 | 1, 0, -1 | M_b | 0 |
| Ξ       | 1/2 | 1/2, -1/2 | -2 | 0 | 0, -1 | M_b | 0 |
| Ω       | 0   | 0       | -3 | 0 | -1 | M_b | 0 |
| Σ_c     | 1   | 1, 0, -1  | 0 | +1 | 2, 1, 0 | M_b | 0 |
| Ξ_c     | 1/2 | 1/2, -1/2 | -1 | +1 | 1, 0 | M_b | 0 |
| Ω_c     | 0   | 0       | -2 | +1 | 0 | M_b | 0 |
| Ξ_{cc}  | 1/2 | 1/2, -1/2 | 0 | +2 | +2, +1 | M_b | 0 |
| Ω_{cc}  | 0   | 0       | -1 | +2 | +1 | M_b | 0 |
| Ω_{ccc} | 0   | 0       | 0  | +3 | +2 | M_b | 0 |

According to particle and anti-particle symmetry, there always exists an anti-quark for each quark of the quarks u, d, s, c and b in the vacuum, and there always exists an anti-baryon for each baryon of Table 2 or Table 3 in the vacuum. We can easily find the quantum numbers of the anti-quarks from the quarks in Table 14.1 of [9] and anti-baryons from Table 2 and Table 3. Thus we do not list the quantum numbers of the anti-quarks and the anti-baryons.

For convenience, we make an idea baryon with a unknown large bare mass $M_b$ and colorless. We call the idea particle b-particle. In other words, for all the baryons, we give the common name “b-particle.” A b-particle represents any baryon with the same large bare mass $M_b$ and other common properties (colorless) in the vacuum.

### 2.2 The second postulate (the free b-particle expanded Schrodinger equation and the BCCP symmetry condition)

A big macroscopic body can affect the space around it. For example, it is well known that under the effect of the sun, the space around the sun is curved. The small microscopic particles look to affect the space around it also. The micro-particles have wave-particle duality. The wave-particle duality might hint that there are some periodic symmetries in the space. The periodic symmetries cause the wave-particle duality. The symmetry of the apparent empty space is a very complex problem. Frank Wilczek [3] point out: “that empty space—the vacuum—is in reality a richly structured, highly symmetrical, medium. . . , one can learn how to analyze it by studying other such materials—that is, condensed matter.” The most natural symmetry of condensed matter with the highest density and the simplest structure is the body-centred cubic periodic symmetries. After many years research, we have discovered that a body-centred cubic periodic condition is the best physical and mathematical simplification for the problem and this condition will gives a full baryon spectrum that consistent with experimental results. Thus we propose the second postulate:

Postulate II: After a pair of b-particle and anti b-particle is excited from the vacuum, the excited b-particle gets a baryon number $B = +1$, the excited anti b-particle gets a baryon number $B = -1$. They all get at least an energy $M_bC^2$. At the same time, as a
single particle, the b-particle is freely moving within a local weak body-centred cubic periodic (BCCP) symmetry space around it. The b-particle with large bare mass \(M_b\) obeys a free particle expanded Schrödinger equation with a body center cubic periodic condition. The periodic constant “\(a\)” ∼ 10\(^{-23}\) m.

The following sections will show that it can give the correct baryon spectrum. According to Postulate II, when a b-particle (or an anti b-particle) is in the vacuum state, its baryon number \(B = 0\). If, however, a pair of b-particle and anti b-particle is excited out from the vacuum state, they become an observable baryon and an anti-baryon with the baryon numbers

\[
B = +1, \text{ for } b - \text{particle} ; \quad B = -1, \text{ for anti } b - \text{particle.} \tag{13}
\]

The anti b-particle well annihilate with a existing b-particle imminently.

The excited b-particle will obey a free b-particle expanded Schrödinger equation

\[
\frac{\hbar^2}{2M_b} \nabla^2 \Psi + \varepsilon \Psi = 0, \tag{14}
\]

where \(M_b\) is the unknown very large bare mass of the b-particle (and anti b-baryon) with baryon number \(B = +1\) (or an anti-baryon number \(B = -1\)). This Schrödinger equation is an expanded Schrödinger equation beyond the standard model. It is different from the ordinary Schrödinger equation. (1) In the ordinary equation the mass of the particle is an observable mass of the particle, but in this equation the mass is the bare mass of the b-particle. (2) The ordinary equation is working in smooth and straight space, but this equation is working in a BCCP symmetry space. (3) This equation is available to the distance scale ∼ 10\(^{-23}\)m beyond the standard model. It calculates the effect of the vacuum material around the excited b-particle. We simplify the effect into a body-centred cubic periodic condition and a bare mass in the expanded Schrödinger equation phenomenologically. (4) This Schrodinger equation can replace the Dirac equation, since the bare mass [11] of the b-particle (or anti b-particle) \(M_b\) is much larger than the observable masses of the baryons. The ultimate test of the validity of any physical theory must be whether it is consistent with observations and measurements of physical phenomena. Thus the ultimate test of these approximations is the experiments. The following sections will show that the expansion and the approximations are very good.

The body-centred cubic periodic symmetry (BCCP) condition is

\[
\Psi(\vec{r} + \vec{S}) = \Psi(\vec{r}), \tag{15}
\]

where the \(\vec{S}\) is

\[
\vec{S} = A\mathbf{j} \tag{16}
\]

the matrix \(A\) is the \(A\) matrix of a body-centred cubic lattice. It is

\[
A = \begin{bmatrix}
-a/2 & a/2 & a/2 \\
 a/2 & -a/2 & a/2 \\
a/2 & a/2 & -a/2
\end{bmatrix}.
\]
The constant “a” is the periodic constant of the body-centred cubic periodic symmetry; \( a \sim 10^{-23} \) m. The vector \( \vec{j} \) is

\[
\vec{j} = \begin{bmatrix}
  j_1 \\
  j_2 \\
  j_3
\end{bmatrix}
\]

where \( j_1, j_2 \) and \( j_3 \) are positive or negative integers or zero.

From (14) and (15), we can see that the equation and the condition are independent from spin, isospin, strange number, charm number, bottom number, electric charge and color that the b-particle might have. They are dependent only on the bare mass \( M_b \) of the b-particle. From the Corollary of Postulate I, all baryons have the same bare mass \( M_B \). Thus the equation and condition describe all baryons. In mathematics this equation and its condition are the same with the free electron limit in a body center cubic lattice of the energy band theory. In the free electron limit case [12], the periodic potential of the lattice becomes arbitrarily weak while the symmetry properties (the BCC lattice) of the wave functions are preserved. The mathematical form of the free limit case is that a free particle Schrodinger equation \( (V(\vec{r}) = 0) \) and the wave functions satisfy the symmetry of the body-centred cubic lattice. According to the energy band theory [13], an excited (from the vacuum) b-particle in the BCCP symmetry space will be in an energy band with BCCP symmetry. In order to find the observable baryons, we propose the third postulate.

### 2.3 The third postulate (the observable baryons)

According to the energy band theory [13], with the expanded Schrodinger equation and BCCP condition, we can find energy bands. Then using phenomenological formula, we can find the intrinsic quantum numbers \( (I, S, C, B \) and \( Q \) of an energy band from its degeneracy, the rotational fold of symmetry axis and the index number \( \vec{n} \) of the energy band that are deduced from the expanded Schrodinger equation with BCCP boundary condition. In order to recognize the baryons from the energy bands, we propose the third postulate:

Postulate III: The intrinsic quantum numbers \( (I, S, C, B \) and \( Q \) of baryons are given by the perfect SU(N) symmetry in the vacuum. The intrinsic quantum numbers of the energy bands are deduced from the expanded Schrodinger equation and the BCCP condition as well as phenomenological formulae of the quantum numbers. The baryons with certain quantum numbers (as appearing in Table 2 and Table 3) will be excited only into the energy bands with the same quantum numbers as the baryon with in the vacuum. The lowest energy (the excited energy from vacuum and the excited energy in a band) of this energy band is the observable mass of the observable baryon.

Thus the intrinsic quantum numbers \( (I, S, C, B \) and \( Q \) of the observable baryons are mainly determined by the SU(N) symmetries. We will deduce the energy bands from the expanded Schrodinger equation and the BCCP condition in Section 3.

### 3 The Energy Bands and Wave Functions of the B-particle

The expanded Schrodinger equation of the b-particle [14] describes a b-particle with unknown very large bare mass \( M_b \) motion inside the vacuum material; the wave functions satisfy the body-centred cubic periodic symmetries of the local space around the b-particle. In mathematical terms,
this equation and its condition are the same as the free electron limit in a body center cubic lattice of the energy band theory \[12\]. For the equation and condition, there are already solutions in energy band theory \[13\]. We will use the mathematical results. This, however, is not a solid state problem. The periodic constant \(a\) is about \(10^{-23}\) m. It is much smaller than the standard atomic lattice constant (about 2-6 \(\times\) 10\(^{-10}\) m). This is a particle work–deducing the baryon spectrum using free particle limit approximation methods of energy band theory.

### 3.1 The solutions of the free b-particle expanded Schrodinger equation with the BCCP condition

According to the energy band theory, the solution of the equation (14) and the BCCP condition (15) is the Bloch wave function \[14\]:

\[
\Psi_{\vec{k}}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u(\vec{r}),
\]

where \(\vec{k}\) is a wave vector and \(u(\vec{r})\) is a periodic function with body-centred cubic periodic symmetries

\[
u(\vec{r} + \vec{S}) = u(\vec{r}),
\]

where the \(\vec{S}\) is shown in formula (16). If we put the Bloch function (17) into the Schrodinger equation (14), we get an equation of the \(u(\vec{r})\):

\[
\nabla^2 u + 2i\vec{k} \cdot \nabla u + \frac{2M_b}{\hbar^2} (\varepsilon - \frac{\hbar^2 k^2}{2M_b}) u = 0.
\]

A periodic solution [(3.91) of [13]] of the above equation is

\[
u_{\vec{l}}(\vec{r}) = e^{-i\vec{l} \cdot \vec{B} \cdot \vec{r}}.
\]

The corresponding energy value [(3.92) of [13]] is

\[
\varepsilon_{\vec{k}} = \frac{\hbar^2}{2M_b} (\vec{k} - \vec{l} \cdot \vec{B})^2
\]

where the vector \(\vec{l}\) is

\[
\vec{l} = (l_1, l_2, l_3);
\]

\(l_1, l_2\) and \(l_3\) are positive or negative integers or zero; and the matrix \(\vec{B}\) is the \(\vec{B}\) matrix of the body centred cubic lattice

\[
\vec{B} = \begin{bmatrix}
0 & \frac{2\pi}{a} & \frac{2\pi}{a} \\
\frac{2\pi}{a} & 0 & \frac{2\pi}{a} \\
\frac{2\pi}{a} & \frac{2\pi}{a} & 0
\end{bmatrix},
\]

where a (\(\sim 10^{-23}\) m) is the periodic constant of the body center cubic lattice.

Since the wave function satisfies the body-centred cubic periodic condition, the wave vector \(\vec{k}\) satisfies the reciprocal (lattice) periodic symmetry of the body-centred cubic (lattice). If we take

\[
\vec{k} = \frac{2\pi}{a} (\xi, \eta, \zeta),
\]
using $\vec{l}$ value and B-matrix value, we can get
\[
(\vec{k} - \vec{l} \cdot \vec{B})^2 = \left(\frac{2\pi}{a}\right)^2 [\xi - n_1]^2 + (\eta - n_2)^2 + (\zeta - n_3)^2],
\] (24)
where
\[
\begin{align*}
n_1 &= l_2 + l_3, \\
n_2 &= l_3 + l_1, \\
n_3 &= l_1 + l_2;
\end{align*}
\] (25)
and
\[
\begin{align*}
l_1 &= 1/2(-n_1 + n_2 + n_3), \\
l_2 &= 1/2(+n_1 - n_2 + n_3), \\
l_3 &= 1/2(+n_1 + n_2 - n_3).
\end{align*}
\] (26)
Condition (26) implies that the vector $\vec{n} = (n_1, n_2, n_3)$ can only take certain values that make the $\vec{l}$ to be an integral vector ($l_1, l_2, l_3$ are positive or negative integers or zero). For example, the $\vec{n}$ cannot take (0,0,1) or (-1,1,-1), but can take (0,0,2) and (-1,1,2) since the wave functions must satisfy the symmetries of the BCCP symmetry. From (21) and (24), we get
\[
\varepsilon_k = \frac{\hbar^2}{2M_b a^2} [\xi - n_1]^2 + (\eta - n_2)^2 + (\zeta - n_3)^2].
\] (27)
If we assume
\[
\alpha = \frac{\hbar^2}{2M_b a^2}
\] (28)
and
\[
E(\vec{k}, \vec{n}) = (n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2
\] (29)
we get the energy formula
\[
\varepsilon(\vec{k}, \vec{n}) = \alpha E(\vec{k}, \vec{n}).
\] (30)
From (20), (17), (23), (22), (25) and the expression of the B matrix, we can find the total wave function
\[
\Psi_{\vec{k}}(\vec{r}) = \exp(-\frac{i2\pi}{a})[(n_1 - \xi)x + (n_2 - \eta)y + (n_3 - \zeta)z].
\] (31)

3.2 The energy bands and wave function of the b-particle

According to the energy band theory, the first Brillouin zone of the body-centred cubic lattice appears in Fig. 1. [depicted from (Fig. 1) of [15] and (FIGURE 8.10) of [16] as well as Fig. 42 of [13]:

![Image of the first Brillouin zone of the body-centred cubic lattice.](image-url)
Fig. 1. The first Brillouin zone of the body-centred cubic lattice. The symmetry points and symmetry axes are showing. The Δ-axis is a four-fold rotary axis, the strange number $S = 0$, the baryon family $\Delta (\Delta^+, \Delta^-, \Delta^0, \Delta^-)$ and $N (N^+, N^0)$ will be born on the axis. The Λ-axes and F-axis are three-fold rotary axes, the strange number $S = -1$, the baryon family $\Sigma (\Sigma^+, \Sigma^0, \Sigma^-)$ and $\Lambda$ will be born on the axes. The $\Sigma$-axes and G-axis are two-fold rotary axes, the strange number $S = -2$, the baryon family $\Xi (\Xi^0, \Xi^-)$ will appear on the axes. The D-axis is parallel to the axis $\Delta$, $S = 0$. The D-axis is a two-fold rotary axis, the baryon family $N (N^+, N^0)$ will appear on the axis.

The energy band theory [13] has already given out the symmetry groups of the symmetry points ($\Gamma$, H, M, P and N) and the symmetry axes ($\Delta$ ($\Gamma$-H), $\Lambda$ ($\Gamma$-P), $\Sigma$ ($\Gamma$-N), D (P-N), F (P-H) and G (M-n)].

At the points $\Gamma$, H and M, the wave functions have the symmetries of the $O_h$ group. The irre-
ducible representations and basis function (or symmetry type) of the $O_h$ group appear in Table A1.

At the point $P$, the wave functions have the symmetries of $T_d$ group. The irreducible representations, characters and basis function (or symmetry type) of the group appear in the Table A2.

At the points $N$, the group of the wave vector $N$ is of order 8. Each operation forms a class by itself, so that there are eight irreducible representations, and all the states are not degenerate. The wave functions have the symmetry of $D_{2h}$ group. The irreducible representations, characters and basis function (or symmetry type) of the group appear in Table A3.

On the $\Delta$-axis, the wave vectors obey the symmetry group $C_{4v}$. The irreducible representations, characters and basis function (or symmetry type) of the group $C_{4v}$ appear in Table A4.

On the $\Lambda$-axis, the wave vectors obey the symmetry group $C_{3v}$. The irreducible representations, characters and basis function (or symmetry type) of the group $C_{3v}$ appear in Table A5.

On the $\Sigma$-axis, $D$-axis and $G$-axis, the wave vectors obey the symmetry group $C_{2v}$. The irreducible representations, characters and basis function (or symmetry type) of the group $C_{2v}$ appear in Table A6.

The coordinates of the symmetry points ($\Gamma$, $H$, $P$, $N$ and $M$) are:

\[
\Gamma = (0, 0, 0), \quad H = (0, 0, 1), \quad P = (1/2, 1/2, 1/2), \quad N = (1/2, 1/2, 0), \quad M = (1, 0, 0). \quad (32)
\]

The coordinates ($\xi$, $\eta$, $\zeta$) of the symmetry axes are:

\[
\Delta = (0, 0, \zeta), \quad 0 < \zeta < 1; \quad D = (1/2, 1/2, \xi), \quad 0 < \xi < 1/2;
\]
\[
\Lambda = (\xi, \xi, \xi), \quad 0 < \xi < 1/2; \quad F = (\xi, \xi, 1 - \xi), \quad 0 < \xi < 1/2;
\]
\[
\Sigma = (\xi, \xi, 0), \quad 0 < \xi < 1/2; \quad G = (\xi, 1 - \xi, 0), \quad 1/2 < \xi < 1. \quad (33)
\]

For any valid value of the vector $\vec{n}$, substituting the ($\xi$, $\eta$, $\zeta$) coordinates of the symmetry points or the symmetry axes into Eq.(29) and Eq.(31), we can get the $E(\vec{k}, \vec{n})$ values and the wave functions at the symmetry points and on the symmetry axes. In order to show how to calculate the energy bands, we give the calculation of some low energy bands in the symmetry axis $\Delta$ as an example (the results appear in Fig. 2).

First, from (29) and (31), we find the formulae for the $E(\vec{k}, \vec{n})$ values and the wave functions at the end points $\Gamma$ and $H$ of the symmetry axis $\Delta$, as well as on the symmetry axis $\Delta$ itself:

\[
E_\Gamma = n_1^2 + n_2^2 + n_3^2, \quad (34)
\]
\[ \Psi_\Gamma = \exp(-\frac{i2\pi}{a}[n_1x + n_2y + n_3z]); \]  
\[ E_H = n_1^2 + n_2^2 + (n_3 - 1)^2; \]  
\[ \Psi_H = \exp(-\frac{i2\pi}{a}[n_1x + n_2y + (n_3 - 1)z]); \]  
\[ E_\Delta = n_1^2 + n_2^2 + (n_3 - \zeta)^2; \]  
\[ \Psi_\Delta = \exp(-\frac{i2\pi}{a}[n_1x + n_2y + (n_3 - \zeta)z]). \]

Then, using (34)–(39), and beginning from the lowest possible energy, we can obtain the corresponding integer vectors \( \vec{n} = (n_1, n_2, n_3) \) (satisfying condition (26)) and the wave functions.

The lowest \( E(\vec{k}, \vec{n}) \) is at \( (\xi, \eta, \zeta) = 0 \) (the point \( \Gamma \)) and with only one value of \( \vec{n} = (0,0,0) \) (see (34) and (35)):

\[ \vec{n} = (0,0,0), \quad E_\Gamma = 0, \quad \Gamma_1 : \quad \Psi_\Gamma = 1. \]  

This wave function is a \( \Gamma_1 \) symmetry type from Table A1.

Starting from \( E_\Gamma = 0 \), along the axis \( \Delta \), there is one energy band (the lowest energy band) \( E_\Delta = \zeta^2 \), with \( n_1 = n_2 = n_3 = 0 \) (see (38) and (39)) ending at the point \( E_H = 1 \):

\[ \vec{n} = (0,0,0), \quad E_\Gamma = 0, \quad E_\Delta = \zeta^2, \quad E_H = 1, \quad \Delta_1 : \quad \Psi_\Delta = \exp[i\frac{2\pi}{a}\zeta z]. \]

This wave function is a \( \Delta_1 \) symmetry type from Table A4.

At the point \( H \) of the energy band \( E_\Gamma = 0 \rightarrow E_\Delta = \zeta^2 \rightarrow E_H = 1 \) (the end point). Also at point \( H, E_H = 1 \) when \( \vec{n} = (\pm 1, 0, 1), (0, \pm 1, 1) \) and \( (0,0,2) \) (see (36) and (37)):

\[ E_H = 1, \quad \Psi_H = e^{i\frac{2\pi}{a}(\pm x)}, e^{i\frac{2\pi}{a}(\pm y)}, e^{i\frac{2\pi}{a}(\pm z)}. \]

From Table A1, the symmetrized wave functions are

\[ H_1 : \Psi = \cos \frac{2\pi}{a}x + \cos \frac{2\pi}{a}y + \cos \frac{2\pi}{a}z, \]

\[ H_{12} : \Psi = \cos \frac{2\pi}{a}z - \frac{1}{2}(\cos \frac{2\pi}{a}x + \cos \frac{2\pi}{a}y), \cos \frac{2\pi}{a}x - \cos \frac{2\pi}{a}y \]

\[ H_{15} : \Psi = (\sin \frac{2\pi}{a}x, \sin \frac{2\pi}{a}y, \cos \frac{2\pi}{a}z). \]

Starting from \( E_H = 1 \), along the axis \( \Delta \), there are three energy bands ending at the points \( E_\Gamma = 0, E_\Gamma = 2 \) and \( E_\Gamma = 4 \), respectively:

\[ \vec{n} = (0,0,0), E_H = 1 \rightarrow E_\Delta = \zeta^2 \rightarrow E_\Gamma = 0, \quad \Delta_1 : \quad \Psi_\Delta = \exp[i\frac{2\pi}{a}(\zeta z)]. \]
This is a $\Delta_1$ symmetry type function from Table A4 on the $\Delta$ axis.

\[
\tilde{n} = (0, 0, 2), \quad E_H = 1 \rightarrow E_\Delta = (2 - \zeta)^2 \rightarrow E_\Gamma = 4, \Psi_\Delta = exp\left[i\frac{2\pi}{a}(2 - \zeta)z\right]. \quad (47)
\]

This is a $\Delta_1$ symmetry type function from Table A4 on the $\Delta$ axis.

\[
\tilde{n} = (\pm 1, 0, 1)(0, \pm 1, 1), \quad E_H = 1 \rightarrow E_\Delta = 1 + (1 - \zeta)^2 \rightarrow E_\Gamma = 2,
\]

\[
\Psi_\Delta = e^{(-i\frac{2\pi}{a})[\pm x + (1-\zeta)z]}, e^{(-i\frac{2\pi}{a})[\pm y + (1-\zeta)z]}.
\]

The energy bands with four sets of values $\tilde{n}$ ($\tilde{n} = (\pm 1, 0, 1), (0, \pm 1, 1)$) are ending at $E_\Gamma = 2$. The four wave functions can be composed of $\Delta_1, \Delta_2'$ and $\Delta_5$ symmetry type functions:

\[
\Delta_1 : e^{i\frac{2\pi}{a}(\zeta-1)z}(\cos\frac{2\pi}{a}x + \cos\frac{2\pi}{a}y),
\]

\[
\Delta_2' : e^{i\frac{2\pi}{a}(\zeta-1)z}(\cos\frac{2\pi}{a}x - \cos\frac{2\pi}{a}y),
\]

\[
\Delta_5 : [e^{i\frac{2\pi}{a}(\zeta-1)z}\sin\frac{2\pi}{a}x, e^{i\frac{2\pi}{a}(\zeta-1)z}\sin\frac{2\pi}{a}y].
\]

From (34), $E_\Gamma = 2$ also when $\tilde{n}$ takes another eight sets of values: $\tilde{n} = (1, \pm 1, 0), (-1, \pm 1, 0)$, and $(\pm 1, 0, -1), (0, \pm 1, -1)$. Putting the 12 sets of $\tilde{n}$ values into Eq. (35), we can obtain 12 plane wave functions:

\[
\Psi_\Gamma = exp(-\frac{i2\pi}{a}(\pm x + z)), \quad \Psi_\Gamma = exp(-\frac{i2\pi}{a}(\pm y + z)),
\]

\[
\Psi_\Gamma = exp(-\frac{i2\pi}{a}[x \pm y]), \quad \Psi_\Gamma = exp(-\frac{i2\pi}{a}(-x \pm y)),
\]

\[
\Psi_\Gamma = exp(-\frac{i2\pi}{a}(\pm x - z)), \quad \Psi_\Gamma = exp(-\frac{i2\pi}{a}(\pm y - z)).
\]

Using the 12 plane wave functions, we can compose $\Gamma_1, \Gamma_{12}, \Gamma'_{25}, \Gamma_{15}$ and $\Gamma_{25}$ symmetry types at the point $E_\Gamma = 2$. Since this paper does not need the complex wave functions, we omit them.

Starting from $E_\Gamma = 2$, along the axis $\Delta$, there are three energy bands ended at the points $E_H = 1$, $E_H = 3$ and $E_H = 5$, respectively:

\[
\tilde{n} = (\pm 1, 0, 1), (0, \pm 1, 1); \quad E_\Gamma = 2 \rightarrow E_\Delta = 1 + (1 - \zeta)^2 \rightarrow E_H = 1,
\]

\[
\tilde{n} = (1, \pm 1, 0), (-1, \pm 1, 0); \quad E_\Gamma = 2 \rightarrow E_\Delta = 2 + \zeta^2 \rightarrow E_H = 3,
\]

\[
\tilde{n} = (\pm 1, 0, -1), (0, \pm 1, -1); \quad E_\Gamma = 2 \rightarrow E_\Delta = 1 + (\zeta + 1)^2 \rightarrow E_H = 5.
\]

Continuing the process, we can find all low energy bands and the corresponding wave functions as well as the irreducible representations with symmetry types. The wave functions are not necessary for deduction of quantum numbers and masses of baryons, so we only show the energy bands with their index numbers $\tilde{n}$ values and the irreducible representations at symmetry points and on symmetry axes in Fig.2–Fig.8. There are six figures (Fig.2–Fig.7); each of them shows the energy bands for one of the six axes in Fig. 1. For convenience, we only deduce the $E(\vec{k}, \tilde{n})$ values in (29).

Using (30), the energy of the energy band $\varepsilon(\vec{k}, \tilde{n}) = \alpha E(\vec{k}, \tilde{n})$. 

14
According to Postulate II, when a b-particle excites from the vacuum, it will get an energy \( M_0 C^2 \) at least; at the same time it will go into an energy band with energy \( \epsilon \). From Postulate III, the observable mass \( M_f \) of the baryons is

\[
M_{F\text{ig}} = M_0 + \text{Minimum}(\epsilon).
\]  (59)

From (59), (29) and (30), the masses \( M_{F\text{ig}} \) of the baryons (appearing in the Fig.2–Fig.7) can change to

\[
M_{F\text{ig}} = M_0 + \text{minimum}(\alpha E(\vec{k}, \vec{n})).
\]  (60)

From (60) and the experimental mass spectrum of baryons [7], we find that the lowest mass of all baryons is \([\text{minimum}(\alpha \ E(\vec{k}, \vec{n})) = 0] \)

\[
M_{F\text{ig}} = M_0 = (1/2)(M_p + M_n) = 939 \ (\text{Mev}).
\]  (61)

Comparing the mass spectrum of the baryons [7] and the mass of the energy bands of the b-particle (60), we find

\[
\alpha = 360 \ (\text{Mev}).
\]  (62)

From (61), (62), (60) and (29), we get the mass formula of the baryons that is

\[
M_{F\text{ig}} = 939 + 360 \times \text{minimum}[(n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2].
\]  (63)

Using (63), we can find the masses of the energy band shown in Fig.2–Fig.8. These figures are schematic ones where the straight lines of the energy bands should be parabolic curves. The energy band indexes \([\vec{n} = (n_1, n_2, n_3), \vec{n}' = (n'_1, n'_2, n'_3), \ldots] \) appear above the lines of the energy bands; the irreducible representations at symmetry points appear near the points; the lowest energies (baryon masses) and highest energy of the energy bands are marked near by the two end points of the energy bands; and the irreducible representations on symmetry axes appear above (or under) the lines of the energy band also. The index numbers of energy bands appearing in the figures [such as \((101,011,-101,0-11)\)] are the simplified index numbers of \((1, 0, 1; 0, 1, 1; -1, 0, 1; 0, -1, 1)\).

Both end points of an energy band (the intersections of the energy band line and the two vertical lines) show the highest and lowest energies (in Mev) of the energy bands in Figure 2–8. We can easily find the lowest energy of the energy bands from Fig.2–Fig.8. It is the mass of the baryon as shown in Fig.2–Fig.8.

Fig.2. The energy bands on the \( \Delta \)–axis. The energy 939+360\( E_\Gamma \) (Mev) is the energy at \( \Gamma \) point (one end) of the energy band, and the energy 939+360\( E_H \) (Mev) is the energy at \( H \) point (another end) of the energy band.

Fig.3. The energy bands on the \( \Lambda \)–axis. The energy 939+360\( E_\Gamma \) (Mev) is the energy at \( \Gamma \) point (one end) of the energy band, and the energy 939+360\( E_P \) (Mev) is the energy at \( P \) point (another end) of the energy band.

Fig.4. The energy bands on the \( \Sigma \)–axis. The energy 939+360\( E_\Gamma \) (Mev) is the energy at \( \Gamma \) point (one end) of the energy band, and the energy 939+360\( E_N \) (Mev) is the energy at \( N \) point (another end) of the energy band.
Fig. 5. The energy bands on the D-axis. The energy $939 + 360E_P$ (Mev) is the energy at $P$ point (one end) of the energy band, and the energy $939 + 360E_N$ (Mev) is the energy at $N$ point (another end) of the energy band.

Fig. 6. The energy bands on the F-axis. The energy $939 + 360E_P$ (Mev) is the energy at $P$ point (one end) of the energy band, and the energy $939 + 360E_H$ (Mev) is the energy at $H$ point (another end) of the energy band.

Fig. 7. The energy bands on the G-axis. The energy $939 + 360E_N$ is the energy at $N$ point (one end) of the energy band, and the energy $939 + 360E_H$ is the energy at $H$ point (another end) of the energy band.

Fig. 8. The single energy bands on the $\Delta$-axis. The energy $939 + 360E_\Gamma$ (Mev) is the energy at $\Gamma$ point (one end) of the energy band, and the energy $939 + 360E_H$ (Mev) is the energy at $H$ point (another end) of the energy band.
4 The Phenomenological Formulae for the Quantum Numbers of the Energy bands

We have deduced the energy bands of the b-particle in the above section (Fig.2–Fig.8). From postulates II and III, these band exited states of the b-particle are baryons. Their masses $M_{fig}$ have already appeared in Fig.2–Fig.8. In order to recognize the baryons, we need phenomenological formulae of the baryon quantum numbers ($I$, $S$, $C$, $B$, $Q$, $J$ and $P$). Using these formulae, we can find the quantum numbers for each energy band. Then comparing the quantum numbers of the energy bands with the quantum numbers of the experimental baryons [7] and quark model in Table 2 and Table 3, we can recognize the baryons of the energy bands. The formulae follow:

1. Isospin number $I$: $I$ results from the energy band degeneracy $d$ using a formula

$$d + \delta(|n|) = 2I_m + 1,$$

where

$$|n| = |n_1| + |n_2| + |n_3|,$$

and $\delta(|n|)$ is a Dirac function; for $|n| = 0$, $\delta(|n|) = 1$; for $|n| \neq 0$, $\delta(|n|) = 0$. If the $d$ is larger than the rotary fold $R$ of the symmetry axis,

$$d > R,$$

the $d$ will be divided $\gamma$ sub-degeneracies first,

$$\gamma = \frac{d}{R};$$

then, using (64), we can find the isospin values for each sub-energy band. Using the $I_m$ value, we can find the $2I_m + 1$ components: $I_z = I_m$, $I_m - 1$, $I_m - 2$, ..., $-I_m$. At the same time, there is another $I$ value for $I_m = 1$ and $\frac{3}{2}$:

$$I = I_m - 1.$$ (68)

Thus for $I_m = 1$ ($\Sigma$ baryons with $I = 1$ and $S = -1$) there is another baryon ($\Lambda$ baryons with $I = 0$ and $S = -1$); and also for $I_m = \frac{3}{2}$ ($\Delta$ baryons with $I = \frac{3}{2}$ and $S = 0$), there is another baryon ($N$-baryons with $I = \frac{1}{2}$ and $S = 0$).

2. Strange number $S$: For an energy band of a symmetry axis with a symmetry rotary fold $R$, its strange number $S$ results from the rotary fold $R$ of the symmetry axis with

$$S_{axis} = B(1 - \delta(|n|))(R - 4),$$

where $B = 1$ for baryons (or $= -1$ for anti baryons) from Postulate II and the number 4 is the highest possible rotary fold number of the symmetry axes in the body-centred cubic periodic symmetry. The $|n| = |n_1| + |n_2| + |n_3|$, $\delta(|n|)$ is a Dirac function: if $|n| = 0$ $\delta(|n|) = 1$, if $|n| \neq 0$ $\delta(|n|) = 0$. From (69), for the ground energy band with $\vec{n} = (0, 0, 0),$

$$the \quad S_{ground} = 0.$$ (70)
Using (69), we can find that the strange numbers of the bands on the $\Delta$-axis ($R = 4$) $S_\Delta = 0$, the strange number of the energy bands on the $\Lambda$-axis ($R = 3$) $S_\Lambda = -1$, and the strange number of the energy bands on the $\Sigma$-axis ($R = 2$) $S_\Sigma = -2$:

$$S_\Delta = 0, \quad S_\Lambda = -1 \quad \text{and} \quad S_\Sigma = -2.$$  \hfill (71)

Using (69) we can also find the opposite strange numbers for the anti-baryon.

The formula (69) is working well for the three axes (the $\Delta$-axis, the $\Lambda$-axis and the $\Sigma$-axis) inside the first Brillouin zone. How to find the strange numbers for the three surface axes though? We think that the strange numbers of the surface axes are determined by an inside axis having the same direction with it. The $D$-axis being parallel to the $\Delta$-axis, from (71), the strange number of the bands on the $D$-axis is

$$S_D = S_\Delta = 0.$$ \hfill (72)

The $F$-axis being parallel to the $\Lambda$-equivalent-axis and $R = 3$, from (71), the strange number of the bands on the $F$-axis is

$$S_F = S_\Lambda = -1.$$ \hfill (73)

The $G$-axis being parallel to the $\Sigma$-equivalent-axis and $R = 2$, from (71), the strange number of the bands on the $G$-axis is

$$S_G = S_\Sigma = -2.$$ \hfill (74)

We have deduced all strange numbers of all energy bands with $d \geq R$ on all symmetry axes using the formula (69). For the energy bands with $d \geq R$, the formula (69) is working well. For the energy bands with $d$ less than $R$, the formula might need to adjust since these energy bands have not the $R$-fold rotary symmetries of the symmetry axis.

While to $R - d = 2$ cases, for first case $R = 4$ and $d = 2$, $R - d = 2$. The $R = 4 \rightarrow \Delta$-axis with $S_\Delta = 0$ from ($\Delta$-baryon), $d = 2 \rightarrow I = \frac{1}{2}$ from (64). The band with $I = \frac{1}{2}$ and $S_\Delta = 0$ might be a $N$-baryon, in this case the formula might not need to adjust. The second case $R = 3$ and $d = 1$ $R - d = 2$. The $R = 3 \rightarrow \Lambda$-axis with $S_\Lambda = -1$ and $I = 1$ the $\Sigma$-baryon. The $d = 1$ $I = 0$ from (64). The band with $S = -1$ and $I = 0$ might be a $\Lambda$-baryon. In this case the formula might be not need to adjust. Thus for

$$R - d = 2$$ \hfill (75)

the formula (69) does not need to adjust, the energy band has $S = S_{ax}$. For other cases,

$$d < R \quad \text{and} \quad R - d \neq 2,$$ \hfill (76)

we have to find a revised formula of the strange number.

3. Electric charge $Q$: Using the baryon number $B$, the isospin $I$ and the strange number $S$ ($C$ and $b$) of the energy band, from the Gell-Mann–Nishijima relation [17], we can deduce the electric charge $Q$ of the energy band:

$$Q = I_z + \frac{1}{2}(B + S + C + B).$$ \hfill (77)

We must emphasize that (77) can give the correct electric charges based on the quark model. According to Postulate III, the charges of baryons really come from the charges of the quarks.
Without the quarks, we cannot have solid physical foundation to deduce the correct charges of the baryons with this formula.

In the above paragraphs (about strange number paragraphs), we stated that “for an energy band with the degeneracy \(d\) less than \(R\) and \(R-d\neq 2\), the formula (69) need to adjust. For example, a single band on \(\Delta\) axis (see Fig. 2), \(S=0\) from (69), \(I=0\) from (64). Using Gell-Mann–Nishijima relation (77), we get a fractional charge:

\[
Q = 0 + \frac{1}{2}(1 + 0 + 0 + 0) = \frac{1}{2}.
\]  

(78)

The experiments show that the charges of the baryons are always integers or zero and never have fractional charge. Thus there is a mistake. We believe that the Gell-Mann–Nishijima relation, the baryon number \((\mathcal{B}=1)\) and the isospin \((I=0)\) are all correct, but the strange number \((S=R-4=0)\) is not correct since the energy band (with \(d\) less than \(R\) and \(R-d\neq 2\)) has already broken the \(R\) fold rotation symmetry of the symmetry axis.

4. In order to find the correct formula of the strange number for the energy bands with \(d<R\) and \(R-d\neq 2\), we compare the important single baryons \((I=0)\) with the possible corresponding single bands. For the \(\Sigma\)-axis, from Fig. 4, we have:

Table 4. The \(S\) of baryons and the n-value of the single bands of \(\Sigma\)-axis

| Baryon or Band | Isospin | M   | \(S\) or \(S_\Sigma\) | \(\Delta S\) | n-value | \(S\) |
|---------------|---------|-----|----------------------|-------------|---------|------|
| \(\Lambda(1116)\) | 0       | 1116| \(S=-1\)            | \(\Delta S=+1\) | (1,1,0) | -1   |
| \(E_N=1/2\)   | 0       | 1119| \(S_\Sigma=-2\)     | \(\Delta S=+1\) | (1,1,0) | -1   |
| \(\Omega(1672)\) | 0       | 1672| \(S=-3\)            | \(\Delta S=-1\) | (-1,-1,0) | -3   |
| \(E_T=2\)     | 0       | 1659| \(S_\Sigma=-2\)     | \(\Delta S=-1\) | (-1,-1,0) | -3   |

In Table 4, the I, M and S of \(\Lambda(1116)\) and \(\Omega(1672)\) are the experimental values [7]; the I values of the two bands are from (64), the M values of the two bands are from Fig. 4 and the \(S_{ax}\) values of the two bands are from (69).

Since the two bands are born on the \(\Sigma\)-axis, the strange number of the two bands will near the \(S_\Sigma=-2\). From Table 4, we can see that if we assume the strange number equals \(S_\Sigma\) plus a \(\Delta S\)

\[
S = S_\Sigma + \Delta S
\]

(79)

and

\[
\Delta S = \mathcal{B} \text{Sign}(n_1 + n_2 + n_3)
\]

(80)

we will get correct strange numbers. The sign function

\[
\text{Sign}(n_1 + n_2 + n_3) = \begin{cases}  +1, & \text{for } (n_1 + n_2 + n_3) > 0, \\  -1, & \text{for } (n_1 + n_2 + n_3) < 0, \\  0, & \text{for } (n_1 + n_2 + n_3) = 0. \end{cases}
\]

(81) (82) (83)
For single bands \( (I = 0) \) of the \( \Delta \)-axis, from Fig. 2, we have:

| Baryon or Band | \( I \) | \( M \) | \( S \) or \( S_\Delta \) | \( \Delta S \) | n-value | \( S \) or \( C \) |
|----------------|--------|------|----------------|--------|--------|--------|
| \( \Lambda(1405) \) | 0      | 1405 | \( S = -1 \)  |        | -1     |        |
| \( E_G=1 \)  | 0      | 1299 | \( S_\Delta = 0 \) | \( \Delta S = -1 \) | \( 0,0,2 \) | -1     |
| \( \Lambda_c(2286) \) | 0      | 2286 | \( C = +1 \)  | \( \Delta S = +1 \) | \( 0,0,-2 \) | +1     |
| \( E_T=4 \)  | 0      | 2379 | \( S_\Delta = 0 \) | \( \Delta S = +1 \) | \( 0,0,-2 \) | +1     |

From Table 5 we can see that the formula \( \text{(80)} \) is not working simply for the single bands on the \( \Delta \)-axis. For the \( \Delta \)-axis, the band with \( \vec{n} = (0, 0, 2) \), \( \Delta S = +1 \) from \( \text{(80)} \), but it needs \( \Delta S = -1 \) as appearing in Table 5. Thus the correct \( \Delta S \) formulae, for the energy bands with \( d \) less than \( R \) and \( R - d \neq 2 \), is

\[
\Delta S = \mathcal{B}[(1 - 2\delta(S_{axis}))\text{Sign}(n_1 + n_2 + n_3)],
\]

\[
S = S_{axis} + \Delta S.
\]

From Table 4 and Table 5, we can see that the correct strange numbers of the bands with \( d \) less than \( R \) and \( R - d \neq 2 \) are \( \text{(85)} \). The strange numbers oscillate around the strange number value of the axis \( S_{axis} \). This situation shows that the fluctuation exists also in the particle world.

5. Charmed number \( C \) and bottom number \( B \): the strange number of the single bands on the \( \Delta \)-axis, when \( \vec{n} = (0, 0, -2) \), \( S = S_\Delta + \Delta S = +1 \). In order to compare it with the experimental results, we would like to give the baryon of the energy band a new name. The new name will be the charmed baryon \( \Lambda_c \) with charmed number \( C = +1 \). If an energy band (baryon) with a \( \Delta S = +1 \) \( \text{(84)} \) and its mass \( \geq \) mass of \( \Lambda_c(2286) \), then we call it the charmed baryon with a charmed number

\[
C = +1.
\]

If an energy band with a \( \Delta S = +1 \) \( \text{(84)} \) and \( S = -1 \) as well as its energy being near even large the mass of the \( \Lambda_b = 5624 \) Mev, we call it (the baryon of the energy band) the bottom baryon with

\[
B = -1.
\]

6. The spins of the baryons: The spins of the baryons are determined by the quark model. Such as in Table 2 (baryons with \( s = \frac{1}{2} \)) and Table 3 (baryons with \( s = \frac{3}{2} \)). Since the quarks all have spin \( s = \frac{1}{2} \) and the baryons all composed of three quarks, there are only two possible spin values: \( s = \frac{1}{2} \) and \( s = \frac{3}{2} \). For the lowest energy band at each symmetry points (\( \Gamma \), \( H \), \( M \), \( N \) and \( P \)) and the lowest mass baryons of each kind of baryons (\( \Sigma \), \( \Xi \), \( \Sigma_c \), \( \Xi_c \), \( \Omega_c \), \( \Xi_{cc} \), \( \Omega_{cc} \) and \( \Lambda_b \)) see Table 2, the spin \( s = \frac{1}{2} \) since the energies are not enough to excited the spin to higher value \( s = \frac{3}{2} \); otherwise the \( s = \frac{3}{2} \). Thus the corresponding energy bands of the varied ground baryons \([N(939), \Lambda(1116), \Sigma(1193), \Xi(1318), \Lambda_c(2286), \Sigma_c(2455), \Xi_c(2470), \Xi_{cc}(2577), \Omega_c(2698), \text{and } \Lambda_b(5624)]\) have the spin

\[
s = \frac{1}{2}.
\]
The other energy bands (baryons) have spin $s$

$$s = \frac{1}{2} \text{ and } \frac{3}{2}. \quad (89)$$

7. The orbit angular momentum $L$: the wave functions of the energy bands are not eigenfunctions of the orbit angular momenta since they satisfy the body-centred cubic periodic symmetries. Thus the energy bands of the b-particle have short lifetimes and might be “strange.” There are, however, equivalent orbit angular momenta. The equivalent orbit angular momenta are determined by the symmetry types of the irreducible representations. The lowest equivalent orbit angular momentums ($L$) of the irreducible representations have appeared in Table A1–Table A6.

The single energy bands of the b-particle are usually longer lifetime baryons ($p$, $n$, $\Lambda$, $\Omega$, $\Lambda_c$ and $\Lambda_b$). Their orbit angular momenta are determined by the irreducible representations of the symmetry group of the symmetry axis. Such as for $\Delta_1$ of the $\Delta$-axis, $\Lambda_1$ of the $\Lambda$-axis, $\Sigma_1$ of the $\Sigma$-axis, the orbit angular momenta $L_{axis}$ are

$$L_{axis} = 0. \quad (90)$$

The energy band theory [15] points out that the basis functions make possible an approximate correspondence (see Table A1), for examples, $\Gamma_1 \rightarrow s$ wave ($L = 0$); $\Gamma_{15} \rightarrow p$ wave ($L = 1$); $\Gamma_{12}, \Gamma_{25} \rightarrow d$ wave ($L = 2$); $\Gamma'_2, \Gamma_{25}, \rightarrow f$ wave ($L = 3$), ... (see Table A1).

The energy band theory [15] further more points out that these correspondences are only approximate in that if a wave function, belonging to $\Gamma'_{25}$, for instance, is expanded in spherical harmonics (the eigenfunctions of angular momentum), terms involving $L = 4$ and $L = 6$, etc., might be present in addition to those of $L = 2$. Thus, when the energy higher the possible $L$ values of a representation are higher as following:

$$L = L_{low} + 2 + 2 \ldots, \quad (91)$$

does the $L_{low}$ is the lowest possible $L$ value which shown in Table A1–A6 for each representation. The highest $L$ value is given in formula (93).

The case can been seen from Table 6:

Table 6A. Full rotation group compatibility table for the group $O$

| $D^\pm_0$  | $\Gamma_1$ |
|------------|------------|
| $D^\pm_1$  | $\Gamma_4$ |
| $D^\pm_2$  | $\Gamma_3 + \Gamma_5$ |
| $D^\pm_3$  | $\Gamma_2 + \Gamma_4 + \Gamma_5$ |
| $D^\pm_4$  | $\Gamma_1 + \Gamma_3 + \Gamma_4 + \Gamma_5$ |
| $D^\pm_5$  | $\Gamma_2 + 2\Gamma_4 + \Gamma_5$ |
| $D^\pm_6$  | $\Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + 2\Gamma_5$ |
| ...        | ...        |

The point group $O_h = O \otimes C_1$. The $O_h$ group is the point group of the points $\Gamma$, point $H$ and point $M$.  

24
8. The total angular momenta of the energy bands (baryons) are found by
\[ \vec{J} = \vec{s} + \vec{L}. \] (92)

Generally a particle with higher angular momenta has higher energy. The experimental baryon spectrum [7] shows that if the masses of the baryons are higher, the angular momenta of the baryons are higher also. In order to simplify, we make a phenomenological formulae for the highest total angular momentum of a baryon. The maximum \( J_{\text{max}} \) values of a baryon will reach but do not exceed \( J_{\text{max}} \):
\[ J_{\text{max}} \leq \frac{M - 939}{360} + \frac{1}{2} \left[ 1 + \delta(S) \frac{M - 939}{939} \right], \] (93)
where \( M \) is the mass of the baryon and the \( S \) is the strange number of the baryon. The \( \delta(S) \) is a Dirac function: if \( S = 0 \), \( \delta(S) = 1 \); if \( S \neq 0 \), \( \delta(S) = 0 \). Since the values of \( J \) are quantized: \( \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \), if (93) gives out a value that is different from these quantized values, for example \( \frac{13}{2} \), the \( J_{\text{max}} \) will take the nearest quantized value \( \frac{7}{2} \). If (93) gives values that have two nearest quantized \( J \) values, for examples \( \frac{4}{2}, \frac{6}{2}, \frac{8}{2}, \ldots \), the \( J_{\text{max}} \) will take the larger ones:
\[ \frac{4}{2} \rightarrow \frac{5}{2}, \frac{6}{2} \rightarrow \frac{7}{2}, \frac{8}{2} \rightarrow \frac{9}{2}, \ldots \] (94)

Using (93) and (94), we can find the \( J_{\text{Max}} \) of baryons as appearing in Table 6B:

| \( M \) | 1299 | 1479 | 1659 | 1814 | 1839 | 1929 | 1945 | 2019 |
|--------|------|------|------|------|------|------|------|------|
| \( J_{\text{Max}} \) of \( \Lambda \) or \( \Sigma \) or \( \Xi \) | \( \frac{2}{2} \) | \( \frac{2}{2} \) | \( \frac{5}{2} \) | \( \frac{5}{2} \) | \( \frac{7}{2} \) | \( \frac{7}{2} \) | \( \frac{9}{2} \) | \( \frac{9}{2} \) |
| \( J_{\text{Max}} \) of \( \Delta \) or \( N \) | \( \frac{2}{2} \) | \( \frac{2}{2} \) | \( \frac{5}{2} \) | \( \frac{5}{2} \) | \( \frac{7}{2} \) | \( \frac{7}{2} \) | \( \frac{9}{2} \) | \( \frac{9}{2} \) |
| \( M \) | 2199 | 2379 | 2444 | 2559 | 2649 | 2739 | 2919 | 3099 |
| \( J_{\text{Max}} \) of \( \Lambda \) or \( \Sigma \) or \( \Xi \) | \( \frac{5}{2} \) | \( \frac{5}{2} \) | \( \frac{5}{2} \) | \( \frac{11}{2} \) | \( \frac{11}{2} \) | \( \frac{13}{2} \) | \( \frac{13}{2} \) | \( \frac{13}{2} \) |
| \( J_{\text{Max}} \) of \( \Delta \) or \( N \) | \( \frac{9}{2} \) | \( \frac{9}{2} \) | \( \frac{9}{2} \) | \( \frac{11}{2} \) | \( \frac{11}{2} \) | \( \frac{13}{2} \) | \( \frac{13}{2} \) | \( \frac{13}{2} \) |

Table A1–Table A6 give out the lowest possible \( L \) value of the representations. For higher mass baryons, the formulae (93) and (94) and Table 6B will give out correct \( J \) values of baryons.

9. Parity \( P \): If the orbit angular momentum of the baryon is \( L \), for a ground state of a kind of baryons, the parity \( P \) is
\[ P = (-1)^{L_{\text{axis}}}, \] (95)
where the \( L_{\text{axis}} \) is the orbit angular momentum from symmetry axes (90).

For an excited baryon, the parity \( P \) is
\[ P = -(-1)^{L_{\text{point}}}, \] (96)
where \( L_{\text{point}} \) is the orbit angular momentum from Table A1–A6.

10. An adjusted mass for some special baryons with \( \Delta S \neq 0 \): For the energy bands with \( d \) less than \( R \) and \( d \neq R - 2 \), there is a \( \Delta S \neq 0 \). At the same time, there will be an adjusted energy (mass) \( de \). Thus for these baryons, the total mass will be \( M_{\text{Fig}} \) (60) plus the adjusted mass \( de \):
\[ M = M_{\text{Fig}} + de. \] (97)
For simplification we assume that the adjusted energy $de$ is:

$$de = -\beta \delta(S_{ax} + f)(C - S) + B + fS - n_c f]\Delta S,$$

(98)

where $\beta = 115$ (Mev); $\delta(S_{ax} + f)$ is a Dirac function, for $S_{ax} + f = 0$, it = 1; otherwise it = 0. The $S_{ax}$ is the strange number of the symmetry axis. The $S$ is strange number of the baryon. The $C$ is the charmed number of the baryon. For the three inside axes ($\Delta$-axis, $\Lambda$-axis and $\Sigma$-axis), $f = 0$; while for the three surface axes (the $d$-axis, the $F$-axis and the $G$-axis), $f = 1$. The $n_c = 0, 1, 2, \ldots$, is the order number of the charmed baryons. For one kind of charmed baryon in a symmetrical axis, to the energy band (charmed baryon) with the same energy and the same parity, the order number $n_c$ from the lowest to higher $J$ values, the order number $n_c = 0, n_c = 1, n_c = 2, \ldots$. The adjusted energy formula $de$ can be simplified to each symmetrical axes as appearing in Table 7.

Table 7. The adjusted energy formulae of the symmetry axes

| Axis   | $S_{ax}$ | $f$ | $de$                                      |
|--------|----------|-----|------------------------------------------|
| $\Delta$-axis | 0        | 0   | $-115(C - S)\times\Delta S$              |
| $\Lambda$-axis | -1       | 0   | $0$                                      |
| $\Sigma$-axis | -2       | 0   | $-115B\times\Delta S$                   |
| $D$-axis   | 0        | 1   | $115(n_c - S)\times\Delta S, n_c = 0, 1, 2, \ldots$ |
| $F$-axis   | -1       | 1   | $115(n_c - C)\times\Delta S, n_c = 0, 1, 2, \ldots$ |
| $G$-axis   | -2       | 1   | $115(n_c - S)\times\Delta S, n_c = 0, 1, 2, \ldots$ |

5 Deduction of the Baryons Spectrum (Including M, I, S, C, B, Q, J and P of Baryons)

The energy bands of the b-particle have been deduced in section 3 (as appearing in Fig.2–Fig.8). Using above phenomenological formulae (64)–(98), we can deduce the quantum numbers of the energy bands from Fig.2–Fig.8. According to Postulate III, for each energy band of the b-particle, there is only one kind of baryons that has the same quantum numbers with the energy band. This baryon can been excited (from the vacuum) into the energy band to form an observable physical baryon. The mass of the baryon is the lowest energy (of the energy band) that can be deduced using (97) and Fig.2–Fig. 8 ($M_{Figg}$) as well as Table 7 ($de$).

From Fig.2–Fig.8 and Table 7, we can get the quantum numbers and masses of the energy bands of the b-particle. We can get a baryon number $B = 1$ (-1) for each energy band of the b-particle (or anti b-particle) from Postulate II. Thus each energy band always has a baryon number $B = +1$. We will directly cite this result (each band has $B = +1$) without deduction again.

First we deduce the ground baryons with the lowest mass of all baryons.

5.1 The ground state baryons

From Fig.2, Fig.3 and Fig.4, we can see there are three lowest energy bands with $\vec{n} = (0, 0, 0)$ and $d = 1$ on the $\Delta$-axis, $\Lambda$-axis and $\Sigma$-axis. From the Postulate II and (13), for each band of the three bands, we get

$$B = 1.$$

(99)
Since $\vec{n} = (0, 0, 0)$ and $d = 1$, from (64), we get
\[
I = \frac{1}{2}.
\] (100)

Since $\vec{n} = (0, 0, 0)$, from (69), (85), (84), (86) and (87) we get
\[
S = C = B = 0.
\] (101)

The three energy bands with $\vec{n} = (0, 0, 0)$ [in fact they are a single band in three dimensional reciprocal lattice space] have the $I = \frac{1}{2}$, $S = C = B = 0$. For $I = \frac{1}{2}$ band, there are two members with $I_z = +\frac{1}{2}$ and $-\frac{1}{2}$. Using Gell-Mann–Nishijima relation (77), we can deduce their electric charge: for $I_z = +\frac{1}{2}$, $Q = I_z + 1/2(B + S + C + B) = +1$; for $I_z = -\frac{1}{2}$, $Q = I_z + 1/2(B + S + C + B) = 0$. Thus the three bands (one band) have $I = \frac{1}{2}$, $S = C = B = 0$ and $Q = +1, 0$. Comparing the quantum numbers (I, S, C, B and Q) of the bands with the quantum numbers of the baryons in Table 2 and in [7], we find there is only the N-baryon having the same quantum numbers as the band. According to Postulate III, the N-baryon will excite from the vacuum into the bands to form the observable N-baryons.

From Fig. 2, Fig. 3 and Fig. 4, we get $M_{Fig} = 939$ (Mev), and from (98) and (101), $de = 0$. From (97), the mass $M$ of the band is
\[
M = M_{Fig} + de = 939 \text{ (Mev)}.
\] (102)

From Fig. 2, the energy band with $\vec{n} = (0, 0, 0)$ has $\Delta_1$ type. Then from Table A4, $L_{axis} = 0$ and $P = +1$ from (95). From Fig. 3, the energy band with $\vec{n} = (0, 0, 0)$ has $\Lambda_1$ type. Then from Table A5, $L_{axis} = 0$ and $P = +1$ from (95). From Fig. 4, the energy band with $\vec{n} = (0, 0, 0)$ has $\Sigma_1$ type. Then from Table A6, $L_{ax} = 0$ and $P = +1$ from (95). Thus the three bands have $L = 0$, $P = +$ and $s = \frac{1}{2}$ from (88). From (92), thus $J^P = \frac{1}{2}^+$. Therefore the three single energy bands with $\vec{n} = (0, 0, 0)$ of the b-particle are the ground baryons $N(939)$ (proton and neutron) with $B = +1$; $I = \frac{1}{2}$; $S = C = B = 0$ and $Q = +1, 0$; $J^P = \frac{1}{2}^+$ and $M = 939$ (Mev):
\[
\vec{n} = (0, 0, 0), \ N(939) \frac{1}{2}^+.
\] (103)

In Fig. 4 and Fig. 8, at $M_{Fig} = 1119$, the single band with representation $\Sigma_1$ and $\vec{n} = (1, 1, 0)$ on the $\Sigma$-axis , has $B = +1$, $I = 0$ from (64). Its $S_{\Sigma} = -2$ from (69) and $\Delta S = +1$ from (84), thus the band has $S = S_{\Sigma} + \Delta S = -1$. It also has $Q = 0$ from (77). In total the band has $I = 0$, $S = -1$ and $Q = 0$. Comparing the quantum numbers of the band with the quantum numbers of the baryons in Table 2 and in [7], we find there is only the $\Lambda$-baryon having the same quantum numbers as the band. According to Postulate III, the $\Lambda$-baryon will excite from the vacuum into the band to form an observable $\Lambda(1119)$-baryon. For the single band with $\vec{n} = (1, 1, 0)$ and $\Sigma_1$, from (90), (92) and (95), we have
\[
\vec{n} = (1, 1, 0) \ \Lambda(1119) \ \frac{1}{2}^+.
\] (104)
The band with $\vec{n} = (1,1,0)$ of the two-fold band with $M_{F_{ig}} = 1119$ and $\vec{n} = (1,1,0; 0,0,0)$ in Fig.5 and Fig.7 has the $B = +1$. The two-fold asymmetry band with $(1,1,0; 0,0,0)$ will divide into two single bands. The band with $(0,0,0)$ forms N(939) from \(^{(103)}\). The another band with $\vec{n} = (1,1,0)$ forms the $\Lambda(1119)$-baryon similarly to the one mentioned above.

In Fig.6 at $M_{F_{ig}} = 1209$, the single band with $\vec{n} = 1,1,0$ might form a $\Lambda(1209)$-baryon as mentioned above. Since these bands with $\vec{n} = (1,1,0)$ in various axes are the same band in the receptive three dimension space, the lowest energy of the three dimensional band is 1119 (Mev). Thus this band forms a $\Lambda(1119)$-baryon.

Similarly to above, the band with $\vec{n} = (1,1,0)$ (of the three-fold band with $M_{F_{ig}} = 1209$ and $\vec{n} = (1,1,0;1,0,1;0,1,1)$ in Fig.3) form a $\Lambda(1119)$ also. Therefore the energy bands with $\vec{n} = (1,1,0)$ and $M_{F_{ig}} = 1119$ or 1209 will form a $\Lambda(1119)$-baryon as

\[ \text{the band with (110) and } M_{F_{ig}} = 1119 \text{ or } 1209 \rightarrow \Lambda(1119). \] \(^{(105)}\)

5.2 The baryons on the $\Delta$-axis

The $\Delta$-axis is a four-fold rotary axis (see Fig.1), $R = 4$. From Postulate II and \(^{(69)}\), we have

\[ B = 1 \text{ and } S_{\Delta} = 0. \] \(^{(106)}\)

Thus all energy bands of the $\Delta$-axis have $B = 1$ and $S_{\Delta} = 0$.

For low energy levels, there are eight-fold degenerate and four-fold degenerate energy bands as well as single bands on the axis in Fig.2. The eight-fold degenerate band will divide into two four-fold degenerate bands from \(^{(67)}\).

For four-fold degenerate bands, using \(^{(64)}\), we get $I_m = 3/2$, and $I_z = 3/2, 1/2, -1/2, -3/2$. From \(^{(77)}\), the four-fold energy bands have $Q = 2, 1, 0, -1$. Thus, each four-fold degenerate band of the b-particle has

\[ B = 1, S = 0, I_m = 3/2, Q = 2, 1, 0, -1. \] \(^{(107)}\)

Comparing the quantum numbers of the four-fold energy bands with the quantum numbers of the baryons in Table 3 and the experimental baryons in \([7]\), we find that there are only the $\Delta$-baryon having the same quantum numbers as the energy band. Thus, from Postulate III, the $\Delta$-baryons will excite from the vacuum into the energy bands to form the observable $\Delta$-baryons. From Fig. 2, we can find the mass $M_{F_{ig}}$ of the $\Delta$-baryon corresponding to the energy band as appearing in Table 8. Since $C = S = 0$ of the $\Delta$-baryons, $d_e = 0$ from Table 7. Thus the $\Delta$-baryons have $M = M_{F_{ig}}$ from \(^{(97)}\).

At the same time, for each four-fold degenerate band, there is always at least a two-fold band [sometimes, there are two two-fold bands] with $S = 0$, $I = \frac{1}{2}$ from \(^{(68)}\), $I_z = \frac{1}{2}, -\frac{1}{2}$ and $Q = +1$ and 0 from \(^{(77)}\). Comparing the quantum numbers of the bands with the quantum numbers of baryons deduced by the quark model in Table 2 and the experimental baryons \([7]\), we find there is only the N-baryons having the same quantum numbers as the band. According to Postulate III, the N-baryons will excited from the vacuum into the bands to form the observable N(M)-baryons. Similarly to the $\Delta$-baryons, we can find the mass of the N-baryons corresponding to the energy bands as appearing in Table 8.
At $M_f = 1659$, $\vec{n} = (110,1-10,-110,-1-10)$, there are two representations $\Gamma_2^{25}$ and $\frac{1}{2}\Gamma_{12}$ that they have the same $L = 2$. Thus there are two $N(1659)$ with $L = 2$. Similarly, at $M_f = 2739$ and $\vec{n} = (211,2-11,-211,-2-11)$, there are two $N(2739)$ also shown in Table 8.

For the single bands, $d = 1$ $I = 0$ from (64), $B = 1$ and $S_\Delta = 0$.

At $M_{Fig} = 1299$, the single band with $\vec{n} = (0,0,2)$ in Fig. 2, from (85) and (84), has the strange number

\[ S = S_{axis} + \Delta S = 0 - 1 = -1. \]  

(108)

The charge $Q = 0$ from (77). The mass $= M_{Fig} + de = 1299 +115 = 1414$ from (91) and Table 7. Thus the single band has $B = 1$, $I = 0$, $S = -1$, $Q = 0$. Comparing the quantum numbers with the quantum numbers of the baryons in Table 2, we find there will be a $\Lambda(1414)$-baryon in the band.

At $M_{Fig} = 2379$, the band with $\vec{n} = (0,0,-2)$, has $S = +1 = C$ from (85) and (86):

\[ S = S_{axis} + \Delta S = 0 + 1 = +1 \rightarrow C. \]  

(109)

Thus the band has $B = 1$, $I = 0$, $C = +1$ and $Q = +1$ from (77). Comparing the quantum numbers of the band with the quantum numbers of the baryons in Table 2 and in [7], we find there is only the $\Lambda_c$-baryon having the same quantum numbers as the band. According to Postulate III, the $\Lambda_c$-baryon will excite from the vacuum into the band to form the observable $\Lambda_c(M)$-baryon. Using Table 7, $de = -115$ (Mev) from (109) and Table 7. Thus the baryon mass $M = M_{Fig} + de = 2379 -115 = 2264$ (Mev) from (97). For the baryon $\Lambda_c(2264)$, $J^P = \frac{1}{2}^+$, from (90), (92) and (95).

It is very important to pay attention to the charmed baryon $\Lambda_c(2264)$ born on the single energy band of the $\Delta$-axis from $\Delta S = +1$ as appearing in Table 8.

Table B1 gives the possible $J^P$ values of the deduced baryons; Table 6B gives the $J_{Max}$ values for the deduced baryon. The correct $J^P$ appear in Table 8.
Table 8. The baryons on the $\Delta$-axis

| $M_{Fg}$ | $(n_1, n_2, n_3)$ | S | C | de | Baryon($\bar{M}$) | J(1/2) | J(3/2) | P |
|----------|--------------------|---|---|----|------------------|------|------|---|
| 939      | (0, 0, 0)          | 0 | 0 | 0  | N(939)           | $\frac{1}{2}$ |          | + |
| 1299     | (101,-101, 011,0-11) | 0 | 0 | 0  | $\Delta(1299)$  | $\frac{5}{2}$ | $\frac{3}{2}$ | + |
| 1299     | (0,0,2)            | -1 | 0 | 115 | $\Lambda(1414)$ | $\frac{1}{2}$ |          | - |
| 1659     | (110,1-10, -110,-1-10) | 0 | 0 | 0  | $\Delta(1659)$  | $\frac{5}{2}$ | $\frac{3}{2}$ | - |
| 1659     | (10-1,-10-1, 01-1,0-1-1) | 0 | 0 | 0  | $\Delta(1659)$  | $\frac{5}{2}$ | $\frac{3}{2}$ | + |
| 2019     | (112,1-12, -112,-1-12) | 0 | 0 | 0  | $\Delta(2019)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | + |
| 2379     | (200,-200, 020,0-20) | 0 | 0 | 0  | $\Delta(2379)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | - |
| 2379     | (0, 0,-2)          | 0 | 1 | -115| $\Lambda_c(2264)$ | $\frac{1}{2}$ |          | + |
| 2739     | (121,1-21, -121,-1-21, 211,2-11, -211,-2-11) | 0 | 0 | 0  |              |              |          |   |
| 2739     | (121,1-21, -121,-1-21,) | 0 | 0 | 0  | $\Delta(2739)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | + |
| 2739     | (211,2-11, -211,-2-11) | 0 | 0 | 0  | $\Delta(2739)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | - |
| 2739     | (202,-202, 022,0-22) | 0 | 0 | 0  | $\Delta(2739)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | + |
| 2739     | (013,0-13, 103,-103) | 0 | 0 | 0  | $\Delta(2739)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | - |
| 3099     | (12,1-1-2, -12,1,-1-2-1) | 0 | 0 | 0  | $\Delta(3099)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | + |
| 3099     | (21,1-2,1-1, -21,1,-2-11) | 0 | 0 | 0  | $\Delta(3099)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | - |
| 3099     | (11-2,1-1-2, -11-2,-1-1-2) | 0 | 0 | 0  | $\Delta(3099)$  | $\frac{3}{2}$ | $\frac{1}{2}$ | + |
5.3 The baryons on the Λ-axis

The Λ-axis is a three-fold rotary axis (see Fig. 1), R = 3, from (69) and Postulate II, any energy band has

\[ B = 1 \quad \text{and} \quad S_\Lambda = -1. \]  \hspace{1cm} (110)

Thus all energy bands of the Λ-axis have \( B = 1 \) and \( S_\Lambda = -1 \).

From Fig. 3, we see that there is a single energy band with \( \vec{n} = (0, 0, 0) \); it is the baryon N(939) from (103). All other bands are three-fold or six-fold energy bands. From (67), the six-fold energy bands will divide into two three-fold energy bands. Thus we only need to discuss the three-fold energy bands.

For a three-fold energy band, \( I = 1 \) from (64); we have already found the strange number of all energy bands \( S = -1 \) and the baryon number \( B = +1 \) in (110). Thus from (77), we have \( Q = 1, 0, -1 \). Comparing the quantum numbers of the three-fold energy bands with the quantum numbers of the baryons in Table 2 and the experimental baryon spectrum in [7], we find there is only the Σ-baryon having the same quantum numbers as the band. According to Postulate III, the Σ-baryons will excite from the vacuum into the three-fold energy bands to form the observable Σ(M)-baryons (Σ⁺, Σ⁰, Σ⁻). Since \( de = 0 \) from Table 7, the mass \( M = M_{Fig} \) (from Fig. 3) as appearing in Table 9.

At the same time, from (68), there is another baryon with \( I = 1 - 1 = 0 \) for each three-fold energy band. The baryon has \( B = 1, S = -1, I = 0 \) and \( Q = 0 \) from (77). Comparing the quantum numbers of the baryon with the quantum of the baryons in Table 2 and the experimental baryon spectrum in [7], we find that the baryon is Λ(M)-baryon. The mass \( M = M_{Fig} \) as appearing in Table 9. Thus there is a Σ-baryon and a Λ-baryon in each three-fold energy band as appearing in Table 9.

On the Λ-axis, there is a Σ(1209) and a Λ(1209) in the lowest three-fold band with \( \vec{n} = (110,101,011) \) and \( M_f = 1209 \). The Λ(1209) corresponds to the band with (1,1,0). From (105), the band with (1,1,0) forms a Λ(1119)-baryon as appearing in Table 9 also.

Table A2 gives the possible \( J\P \) values of the deduced Σ-baryons and Λ-baryons; and Table 6B limits out the correct \( J\P \) values as appearing in Table 9:

At \( M_{Fig} = 1659 \), a six fold band with \( \vec{n} = (1-10,-110,011) \) divided into two three-fold bands. The first three-fold band with \( \vec{n} = (1-10,-110,011) \) has \( I = 1 \), \( S = -1 \) and \( Q = 1, 0, -1 \) from (77). This is a Σ(1659). At same time, there is a Λ(1659) also. From Table B2, corresponding to this band the representation is a three fold \( \Gamma_{25} \) \((L = 2)\). Thus \( J\P = \frac{3}{2}, \frac{5}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \). The second three-fold sub-band with \( \vec{n} = (0-11,01-1) \), in Table B2, spin \( s = \frac{1}{2} \) and \( \frac{3}{2} \), representation = \( \Gamma_{12} \) \((L = 2)\) and \( \Gamma_{11} \), \((L = 0)\). Thus for three-fold Σ(1659), \( J\P \) will take the common values \( \frac{1}{2}, \frac{3}{2} \). For single Λ(1659), \( J\P \) will take \( \frac{1}{2}, \frac{3}{2} \) for \( L = 0 \) and \( \frac{3}{2}, \frac{5}{2}; \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \) for \( L = 2 \). The Σ(1659), \( J\P \) values and Λ(1659), \( J\P \) values will appear in Table 9.
Table 9. The baryons on the \( \Lambda \)-axis

| \( M_{Fig} \) (\( n_1, n_2, n_3 \)) | I | S | \( \Sigma(M) \) or \( \Lambda(M) \) | \( J(1/2) \) | \( J(3/2) \) | P |
|---|---|---|---|---|---|---|
| 1209 (110,101,011) | 1 | -1 | \( \Sigma(1209) \) | \( \frac{1}{2}, \frac{3}{2} \) | - | + |
| 1209 (1, 1, 0) | 0 | -1 | \( \Lambda(1119) \) | \( \frac{1}{2} \) | - | + |
| 1659 (1-10,-110,1-101) | -1 | \( \Sigma(1659) \) or \( \Lambda(1659) \) | \( \frac{3}{2}, \frac{5}{2} \) | \( \frac{3}{2}, \frac{5}{2} \) | - |
| 1659 (1-10,-110,1-101) | 1 or 0 | -1 | \( \Sigma(1659) \) or \( \Lambda(1659) \) | \( \frac{3}{2} \) | - |
| 1929 (020,002,200) | 1 or 0 | -1 | \( \Sigma(1929) \) or \( \Lambda(1929) \) | \( \frac{3}{2}, \frac{5}{2} \) | - |
| 2379 (00-2,0-20,-200) | 1 or 0 | -1 | \( \Sigma(2379) \) or \( \Lambda(2379) \) | \( \frac{3}{2}, \frac{5}{2} \) | + |
| 2649 (12-1,1-12,21-1 2-11,-121,-112) | -1 | \( \Sigma(2649) \) or \( \Lambda(2649) \) | \( \frac{3}{2}, \frac{5}{2} \) | \( \frac{3}{2}, \frac{5}{2} \) | + |
| 2649 (12-1,1-12,21-1) | 1 or 0 | -1 | \( \Sigma(2649) \) or \( \Lambda(2649) \) | \( \frac{3}{2}, \frac{5}{2} \) | - |
| 2649 (2-11,-121,-112) | 1 or 0 | -1 | \( \Sigma(2649) \) or \( \Lambda(2649) \) | \( \frac{3}{2}, \frac{5}{2} \) | + |
| 2649 (202,220,022) | 1 or 0 | -1 | \( \Sigma(2649) \) or \( \Lambda(2649) \) | \( \frac{3}{2}, \frac{5}{2} \) | + |

5.4 The baryons on the \( \Sigma \)-axis

The \( \Sigma \)-axis is a two-fold rotary axis, \( R = 2 \), \( S_\Sigma = -2 \) from (69). Thus all energy bands of the \( \Sigma \)-axis have \( S_\Sigma = -2 \) and \( B = 1 \) from Postulate II.

\[ S_\Sigma = -2 \text{ and } B = 1. \]  \hspace{1cm} (111)

For low energy levels, there are two-fold degenerate energy bands, four-fold degenerate energy bands and single energy bands on the axis (see Fig. 4). From (67), each four degenerate energy band on the symmetry \( \Sigma \)-axis will divide into two two-fold degenerate bands. Thus we only need to discuss the two-fold degenerate bands for four-fold and two-fold degenerate bands.
For the two-fold degenerate energy bands, I = 1/2 from (64); S = -2 from (69); Q = 0, -1 from (77). Comparing the quantum numbers of the energy bands with the quantum numbers of the baryons in Table 2 and the experimental baryon spectrum in \[7\], we find there are only the \(\Xi\)-baryons having the same quantum numbers as the energy band. According to Postulate III, the \(\Xi\)-baryons will excite from the vacuum into the two-fold energy bands to form the observable \(\Xi(M)\)-baryons. The masses \(M = M_{Fg} + de = M_{Fg}\) since \(de = 0\) from Table 7 (\(\Delta S = 0\)).

Table B3 gives the possible J and parity P for each energy band and Table 6B limits off extra J values for each deduced \(\Xi\)-baryon. Fig. 4 gives the masses of the \(\Xi(M)\). Thus we deduce the \(\Xi(M)\)-baryons with correct \(J^P\) values as appearing in Table 10:

| \(M_{Fg}\)  | \(n_1, n_2, n_3\) | I  | S  | Baryon(M) | \(J(1/2)\) | \(J(3/2)\) | P  |
|----------|-------------------|----|----|-----------|-----------|-----------|----|
| 1479     | (101, 101, 01-1)  | 1/2| -2 | \(\Xi(1479)\) | 1/2        | 3/2       | -  |
| 1479     | (1.0, 0, 1.1)     | 1/2| -2 | \(\Xi(1479)\) | 1/2        | 3/2       | +  |
| 1659     | (-1, 0, 1.1)      | 1/2| -2 | \(\Xi(1659)\) | 1/2        | 3/2       | +  |
| 1659     | (-101, 10, 1-1)   | 1/2| -2 | \(\Xi(1659)\) | 1/2        | 3/2       | -  |
| 1839     | (2, 0, 0); (0, 2, 0)| 1/2| -2 | \(\Xi(1839)\) | 1/2        | 3/2       | -  |
| 2199     | (121, 121, 12-1)  | 1/2| -2 | \(\Xi(2199)\) | 1/2        | 3/2       | -  |
| 2199     | (1.2, 1; 2, 1, 1)| 1/2| -2 | \(\Xi(2199)\) | 1/2        | 3/2       | -  |
| 2379     | (0, 0, 2; 0, 0, -2)| 1/2| -2 | \(\Xi(2379)\) | 1/2        | 3/2       | -  |
| 2559     | (112, 11-2)       | 1/2| -2 | \(\Xi(2559)\) | 1/2        | 3/2       | -  |

5.5 The baryons of the single energy bands on the \(\Sigma\)-axis

The single energy bands of the \(\Sigma\)-axis appear in Fig. 8. For the single energy bands, \(d = 1\), \(I = 0\) from (64); for the \(\Sigma\)-axis, the strange number \(S_\Sigma = -2\) from (69) (see Fig. 1) and \(L_{axis} = 0\) from (90) and Fig. 8. All single energy bands of the \(\Sigma\)-axis have

\[
B = 1, \quad I = 0, \quad S_\Sigma = -2, \quad \text{and} \quad L = 0.
\]  

(112)

In Fig. 8, at \(M_{Fg} = 1119\) Mev, the energy band with \(\vec{n} = (1, 1, 0)\) is the \(\Lambda(1119)\) from (105)

\(\Lambda(1119)\) with \(I = 0, S = -1, Q = 0\) and \(M = 1119\).

(113)
It is very important to pay attention to the strange baryon $\Lambda(1119)\frac{1}{2}^+$ born on the single energy band of the $\Sigma$-axis from $\Delta S = +1$.

At $M_{\text{Fig}} = 1659$ Mev in Fig. 8, the band with $\vec{n} = (-1, -1, 0)$ has $\Delta S = -1$ from (83). Thus $S = S_\Sigma + \Delta S = -3$, $I = 0$, $Q = -1$ from (77). Comparing the quantum numbers of the energy band with the quantum numbers of the baryons in Table 3 and in [7], we find there is only the $\Omega$-baryon having the same quantum numbers as the energy band. According to Postulate III, the $\Omega$-baryon will excite from the vacuum into the energy band to form a $\Omega(1659)$-baryon. From Table 3, its spin $s = \frac{3}{2}$; $L_{\text{axis}} = 0$ from Fig. 8 and (90). Thus $J^P = \frac{3}{2}^+$ from (92).

At $M_{\text{Fig}} = 2559$ Mev, the band with $\vec{n} = (2, 2, 0)$ has $\Delta S = +1$ from (84), $S = -2 + 1 = -1$ and $Q = 0$. Similarly to $M_{\text{Fig}} = 1119$ Mev, there is a $\Lambda(2559)$-baryon. $J^P = \frac{1}{2}^+$, from Table 2 (spin $s = 1/2$), (90) ($L_{\text{axis}} = 0$), (92) and (95).

At $M_{\text{Fig}} = 1659$ Mev, at $M_{\text{Fig}} = 3819$, the band with $\vec{n} = (2, 2, 0)$ has $\Delta S = -1$ from (84), $S = -2 + 1 = -1$ and $Q = 0$. Similarly to $M_{\text{Fig}} = 1659$ Mev, we find there is only the $\Lambda_b$-baryon having the same quantum numbers as the energy band. Thus the $\Lambda_b$-baryon will be excited from the vacuum into the energy band to form a $\Lambda_b(5554)$-baryon since $M = M_{\text{Fig}} + de = 5439 + 115 = 5554$ Mev from Table 7. Its $J^P = \frac{1}{2}^+$, from (88), (90) ($L_{\text{axis}} = 0$), (92) and (95). Using Fig. 8, we have the baryon as appearing in Table 11:

| $M_{\text{Fig}}$ | $\vec{n}$ | $S_{\text{axis}}$ | $\Delta S$ | $S$ | $B$ | Baryon(M) | $L$ | $J$ | $P$ |
|---|---|---|---|---|---|---|---|---|---|
| 1119 | (1, 1, 0) | -2 | +1 | -1 | 0 | $\Lambda(1119)$ | 0 | $\frac{3}{2}$ | + |
| 1659 | (-1, -1, 0) | -2 | -1 | -3 | 0 | $\Omega(1659)$ | 0 | $\frac{3}{2}$ | + |
| 2559 | (2, 2, 0) | -2 | +1 | -1 | 0 | $\Lambda(2559)$ | 0 | $\frac{1}{2}$ | + |
| 3819 | (-2, -2, 0) | -2 | -1 | -3 | 0 | $\Omega(3819)$ | 0 | $\frac{1}{2}$ | + |
| 5439 | (3, 3, 0) | -2 | +1 | 0 | -1 | $\Lambda_b(5554)$ | 0 | $\frac{1}{2}$ | + |

It is very important to pay attention to the bottom baryon $\Lambda_b(5554)$ born on the single energy band with $\vec{n} = (3, 3, 0)$ and $M_\Sigma = 5439$ Mev from the $\Delta S = +1$ and $de = 115$.

5.6 Three axes on the surfaces of the first Brillouin zone

For the three symmetry axes that are on the surface of the first Brillouin zone (the D-axis (P-N), the F-axis (P-H) and the G-axis (M-N), see Fig.1), the energy bands with the same energy might have asymmetric $\vec{n}$ values (see Fig. 5, Fig. 6 and Fig. 7). For symmetrical $\vec{n}$ values we give a definition: for two $\vec{n}$ values [such as $\vec{n} = (n_1, n_2, n_3)$ and $\vec{n}' = (n'_1, n'_2, n'_3)$], if using the permutation of the three components and changing the signs of the components (one, two, or three), they can be interchanged each other, then we call they are symmetrical $\vec{n}$ values. For example, the following eight $\vec{n}$ values are all symmetric: $(1,1,0), (1,-1,0), (-1,1,0), (-1,-1,0),(1,0,-1), (-1,0,-1), (0,1,-1), (0,-1,-1)$. While $(1,-1,1)$ and $(1,1,0)$ are asymmetric. $(0,0,2)$ and $(1,1,2)$ are asymmetric also.

For an energy band with asymmetric $\vec{n}$ values, for example $(10,-110,020,200)$, we cannot use (61) to find $I = \frac{3}{2}$. We have to divide it into two symmetric values $(10,-110)$ and $(020,200)$ first.
Then, using (64), we can get two sub-bands with $I = \frac{1}{2}$.

For this division ($\gamma = \frac{d}{R}$), if the R-fold “sub-degeneracy” bands have R-fold symmetry $\vec{n}$ values, the strange number of the sub-energy band will not change ($\Delta S = 0, \rightarrow S_{sub} = S_{ax}$) since the sub-band has the same R-fold symmetries as the axis has.

\[ If \ a \ sub-band \ has \ R-fold \ symmetry \ \vec{n} \ values, \ \Delta S = 0. \] (114)

If the R-fold “sub-degeneracy” bands have asymmetric $\vec{n}$ values, however, the strange number of the sub-energy band will change ($S_{sub} \neq S_{ax}$) since the sub-band has not the R-fold symmetries. $\Delta S$ can be found using the below formula (115).

Sometimes, after dividing, although the $\vec{n} \rightarrow \text{Sign}(n_1 + n_2 + n_3) = 0 \rightarrow \Delta S = 0$ from (84), this is not correct. For example, on the F-axis ($S_{F} = -1$), the energy band $M_{Fig} = 1929$ with $\vec{n} = (0,0,2;0,1,1;0,-1,0)$. This three-fold band with three asymmetric $\vec{n}$ values divide into two bands with $(2,0,0)$ and $(-1,0,0;-1,1)$. The energy band with two symmetric $\vec{n}$ values $(-1,0,1;0,-1,1) \rightarrow \text{Sign}(n_1 + n_2 + n_3) = 0 \rightarrow \Delta S = 0$ from (84). This result is not correct. The F-axis $S_{F} = -1$, the two-fold degeneracy band $I = 1/2$ from (84), $Q = \frac{1}{2}, \frac{1}{2}$ from (77). Since this division has broken the three-fold symmetry of the F-axis, the strange number $S_{F} = -1$ has to change. We have to change $\text{Sign}(n_1 + n_2 + n_3)$ into $\text{Sign}(n_1 + n_2 + n_3 + f)$. When the sub-band with $d \neq R$ and $R-d \neq 2$ as well as $\text{Sign}(n_1 + n_2 + n_3) = 0$, the formula (84) is changed into

\[ \Delta S = [1 - 2\delta(S_{ax})] \text{Sing}(n_1 + n_2 + n_3 + f), \] (115)

where $f = 1$ for the surface axes (the D-axis, the F-axis and the G-axis), and $f = 0$ for inside three axes (the $\Delta$-axis, the $\Lambda$-axis and the $\Sigma$-axis) as appearing in Table 7.

Generally, if an energy band has $d$ asymmetric $\vec{n}$ values, the energy band will divide into subgroups of energy bands with $d_1$, $d_2$, ..., symmetric (or single) $\vec{n}$ values. For each subgroup of energy bands, using (85) and (84) as well as (115), we can find the strange numbers of the subbands. Finally we recognize the baryons using the quantum numbers (I, S and Q). Generally, the two (or three or four) sub-bands have different S (or C or B) → different baryons. If the two (or three or four) sub-energy bands with $d_1$ and $d_2$ (or $d_3$ or $d_4$) get the same S (or C or B), they might combine back to the energy band with $d = d_1 + d_2$:

\[ d = d_1 + d_2 + \ldots. \] (116)

5.7 The baryons on the D-axis (P-N)

The energy bands of the D-axis appear in Fig. 5. The strange number of the energy bands on the D-axis has $S_D = 0$ from (72). All energy bands have

\[ B = 1, \ and \ S_D = 0. \] (117)

For low energy levels, there are four-fold energy bands and two-fold energy bands on the axis (see Fig. 5). In Fig. 5, the lowest energy band has two fold asymmetric $\vec{n} = (000, 110)$. The two-fold band will divide into two single bands. The band with $\vec{n} = (0, 0, 0)$ is N(939)$^{1+}$ from (103). The band with $\vec{n} = (1,1,0)$ is the $\Lambda(1119)^{1+}$ from (104). In Fig. 5, the second lowest band with two fold $\vec{n} = (011,101)$ and $M_{Fig} = 1209$, but uncertain L, J and P from Table B4. It cannot form a two-fold N(1209)-baryon. They might be regarded as $\Delta(1232)^{1+}$ and $\Delta(1232)^{0}$. 

35
From Fig. 5, we can see that each four-fold energy band has \( \vec{n} \) as symmetry. They all can be divided into two groups. Each of them has two symmetric \( \vec{n} \) values. For these two-fold energy bands with symmetry \( \vec{n} \) values, \( I = \frac{1}{2} \) from (64), \( I_Z = 1/2, -1/2; Q = 1, 0 \) from (77) and \( S_D = 0 \). Comparing the quantum numbers (\( I, S \) and \( Q \)) of the two-fold energy bands with the quantum numbers of the baryons in Table 2 and in [7], we find only the N-baryons with completely the same quantum numbers as the two-fold energy bands. According to Postulate III, the N-baryons will excite from the vacuum into the two-fold energy bands to form the \( N(M_{Fig}) \)-baryon. Fig. 5 gives the mass \( M = M_{Fig} \) of the N-baryons as appearing in Table 12.

There is a four-fold energy band with \( M_{Fig} = 1839 \) and \( \vec{n} = (1-10, -110, 020, 200) \), as well as the lowest energy at the point N. Since the four-fold \( \vec{n} \) values are asymmetrical, it divide into two two-fold sub-bands with symmetrical \( \vec{n} \) values (1-10, -110) and (020, 200). Since the two sub-bands have \( d = R \) and two symmetrical \( \vec{n} \) values, the \( \Delta S = 0 \) from the formula (114). The two sub-bands have the same masses \( M = 1839 \) and same \( S = 0 \), from (116), they should combine back to form a four-fold \( \Delta \)-baryon. There, however, is not any four-fold single and double representation [18] in the N-group Table A3. Thus the two sub-bands only form two \( N(1839) \) as appearing in Table 12.

There is another four-fold energy band with \( M_{Fig} = 1929 \) and \( \vec{n} = (-101, 0-11, 211, 121) \) as well as the lowest energy at the point P (a special case). The band with four asymmetry \( \vec{n} \) should divide into two bands with two-fold symmetry \( n \)-values, the two two-fold bands still have the same strange number \( S = 0 \) since the two sub-bands have \( d = R \) \( \Delta S = 0 \) from (114). From (116), the two two-fold bands combine back to form a four-fold band with \( S = 0, \) \( M =1929, \) \( I = \frac{3}{2} \) and \( Q = +2, +1, 0, -1 \) since there are four-fold double representations in the P-group [18]. This is a \( \Delta(1929) \)-baryon. At the same time, there is a \( N(1929) \) also from (68). The \( J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^- \) and \( \frac{2}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+ \) and \( \frac{7}{2}^+ \) as appearing in Table 12.

At \( E = 1929 \), there is a two-fold band with asymmetry (002,112). It will divide into two single bands with (0,0,2) and (1,1,2). From (115), the band with (0,0,2) and \( M_{Fig} = 1929 \) has \( S = S_D + \Delta S = 0 + (-1) = -1, I = 0 \) from (64) and \( Q = 0 \) from (77). Similarly to the band with (1,1,2) and \( M_{Fig} = 1929 \), the band with (2,2,0) and \( M_{Fig} = 2559 \) and the band with (1,1,-2) and \( M_{Fig} = 2559 \), they all have \( S = -1, I = 0 \) and \( Q = 0 \). Comparing the quantum numbers of these energy bands with the quantum numbers of the baryons in Table 2 and in [7], we find there are only the \( \Lambda \)-baryons having completely the same quantum numbers as these energy bands. According to Postulate III, the \( \Lambda \)-baryons will excite into these energy bands to form observable \( \Lambda \)-baryons. There is an adjust energy \( de = - 115 \) \( S \times \Delta S = -115 \) (Mev) for each \( \Lambda \)-baryon from Table 7. The masses of the baryons are \( M = M_{Fig} + de = M_{Fig} - 115 \). So that the baryons are \( \Lambda(1814) \)–the band with (002), \( \Lambda(1814) \)–the band with (112), \( \Lambda(2444) \)–the band with (220), \( \Lambda(2444) \)–the band with (11-2) as appearing in Table 12.

At \( E_N = 2559 \), there are two two-folds bands with two asymmetric (220,-1-10) and (11-2,00-2). The band with (2,2,0,-1,-1,0) divides to two bands with (2,2,0) and (-1,-1,0). The band with (2,2,0) has \( I = 0 \) from (64), \( \Delta S = -1 \) from (64), \( Q = 0 \) from (77) and \( de = -115 \) from Table 7. The band \( \rightarrow \Lambda(2444) \) as appears above. The band with (-1,-1,0), \( \Delta S = +1 \) from (64). Thus the band has \( I = 0 \) from (64), \( C = +1 \) from (86) and \( Q = +1 \) from (77). Comparing the quantum numbers of the band with the quantum numbers of the baryons in Table 2 and in [7], we find there are the \( \Lambda_c \)-baryons having completely the same quantum numbers as the band. According to Postulate III, the \( \Lambda_c \)-baryons will excite from the vacuum into the band to form three \( \Lambda(M)_c \)s. The mass
Table 12. The Baryons of the Energy Bands on the D-axis (P-N)

| $M_{Fig}$ | $(n_1, n_2, n_3)$ | I | $\Delta S$ | S | C | de | Baryon(M) | J | P |
|-----------|------------------|---|-----------|---|---|----|----------|---|---|
| 1479      | (10-1,01-1)      | $\frac{1}{2}$ | 0         | 0 | 0 | 0  | N(1479) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 1839      | (1-10,-110, 020,200) | 0            | 0         | 0 | 0 | 0  | N(1839) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 1839      | (1-10,-110)      | $\frac{1}{2}$ | 0         | 0 | 0 | 0  | N(1839) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 1929      | (-101,0-11, 211,121) | $\frac{3}{2}$ | 0         | 0 | 0 | 0  | $\Delta$1929 | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 1929      | (002,112)        | 0            | 0         | 0 | 0 | 0  | N(1929) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 1929      | (0,0,2)          | 0            | -1        | -1 | 0 | -115 | $\Lambda$(1814) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 1929      | (1,1,2)          | 0            | -1        | -1 | 0 | -115 | $\Lambda$(1814) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2199      | (12-1,21-1)      | $\frac{1}{2}$ | 0         | 0 | 0 | 0  | N(2199) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 2199      | (-10-1,0-1-1)    | $\frac{1}{2}$ | 0         | 0 | 0 | 0  | N(2199) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2559      | (220,-1-10)      | 0            | -1        | -1 | 0 | -115 | $\Lambda$(2444) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 2559      | (-1,-1,0)        | 0            | +1        | 0   | 0 | +1  | $\Lambda_c$(2559) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 2559      | (-1,-1,0)        | 0            | +1        | 0   | 0 | +1  | $\Lambda_c$(2674) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 2559      | (-1,-1,0)        | 0            | +1        | 0   | 0 | +1  | $\Lambda_c$(2789) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
| 2559      | (11-2,00-2)      | 0            | -1        | -1 | 0 | -115 | $\Lambda$(2444) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2559      | (1,1,-2)         | 0            | -1        | -1 | 0 | -115 | $\Lambda$(2444) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2559      | (0,0,-2)         | 0            | +1        | 0   | 0 | +1  | $\Lambda_c$(2904) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2559      | (0,0,-2)         | 0            | +1        | 0   | 0 | +1  | $\Lambda_c$(3019) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2559      | (0,0,-2)         | 0            | +1        | 0   | 0 | +1  | $\Lambda_c$(3134) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2649      | (-121,2-11)      | $\frac{1}{2}$ | 0         | 0   | 0 | 0   | N(2649) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2649      | (-112,1-12, 202,022) | $\frac{3}{2}$ | 0         | 0   | 0 | 0   | $\Delta$(2649) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | + |
| 2919      | (2-1,-12-1)      | $\frac{1}{2}$ | 0         | 0   | 0 | 0   | N(2919) | $\frac{3}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{7}{2}$ | - |
At $M_f = 2649$, a band with $\vec{n} = (-121,2-11)$ has $I = \frac{1}{2}$ from (64), $S = 0$ from (117), $Q = 1, 0$ from (77). Similar to above, it is a $N(2649)$.

At $M_f = 2649$, a band with $\vec{n} = (-112,1-12,202,022)$ is similar to the four-fold band with $M_{F_{\text{ig}}} = 1929$ and $\vec{n} = (-101,0-11,211,121)$. The four-fold band forms a $\Delta(2649)$ and $N(2649)$ as appearing in Table 12.

Table B4 gives the possible $J^P$ values of each energy band, Table 6B limits extra $J^P$ values off. The deduced baryons with correct $J^P$ values appear in Table 12.

5.8 The baryons on the F-Axis

The F-axis is a three-fold symmetric axis, from (73), $S_F = -1$. All energy bands of the F-axis have

$$B = 1, \text{ and } S_F = -1.$$

(118)

There are six-fold energy bands, three-fold energy bands and single energy bands on the axis (see Fig. 6).

For the single bands, $S_F = -1$, the isospin $I = 0$ from (64) and $Q = 0$ from (77). Comparing the quantum numbers of the energy bands with the quantum numbers of the baryons in Table 2 and in [7], we find there is only the $\Lambda$-baryon having the same quantum numbers as the single bands. According to Postulate III, the $\Lambda$-baryons will excite from the vacuum into the single bands to form the $\Lambda$-baryons. The single band with $\vec{n} = (1,1,0)$ forms the strange $\Lambda(1119)$-baryon from (105). The single band with $\vec{n} = (1,-1,2)$ forms a strange baryon $\Lambda(2019)$.

In Fig. 6, the three-fold energy bands all have the three asymmetry $\vec{n}$ values. Each of the three-fold asymmetry $\vec{n}$ values composes of a 2-fold symmetry $\vec{n}$ values and a single $\vec{n}$ value. The single energy bands, similarly to the above paragraph, are the $\Lambda(M)$-baryons with $S = -1, I = 0$ and $Q = 0$ as well as $M = M_{F_{\text{ig}}}$ as appearing in Table 13. Using (84) or (115), if the two-fold symmetry bands have $\Delta S = +1$, the two-fold energy bands form $N$-baryons; otherwise, if the two-fold symmetry bands have $\Delta S = -1$, the two-fold energy bands form $\Xi$-baryons as appearing in Table 13.

The six-fold bands all have six asymmetry $\vec{n}$-values. Each of them composes of three two-fold symmetry $\vec{n}$-values. There are always two $(n_1 + n_2 + n_3)$ values larger than 0 and two $(n_1 + n_2 + n_3)$ values smaller than 0. It will divide into two three fold bands since the rotary fold $R = 3$ of the F-axis and (67). The one contains the two $(n_1 + n_2 + n_3)$ larger than 0, while the another contains the two $(n_1 + n_2 + n_3)$ smaller or equal 0.

For example, at $E_P = 2649$, a six fold-band with $\vec{n} = (202,022,-121,2-11,0-1-1,-10-1)$ will divided into two three-fold sub-bands from (67). The first sub-band with $\vec{n} = (202,022,-121)$ has two n-values larger than zero; the second sub-band with $\vec{n} = (2-11,0-1-1,-10-1)$ has two n-values smaller than zero.

For first division, using (84), the first sub-band with an asymmetry $\vec{n} = (202,022,-121)$ will get a strange number $\Delta S = +1$ for each of the three values (202,022,-121). Then they become a three-fold band with $S = S_F + \Delta S = -1 + 1 = 0$. Since the three $\vec{n}$ values are asymmetric, the band will divide again. For the second time division, using (84), we get $\Delta S = +1 = C$ from (86) for each band of the three bands. Although the three bands with asymmetry $\vec{n}$ values have
been divided into a single band and a two-fold band, the single band and the two-fold band have the same quantum numbers S and C as well as the same mass. From (116), the single band and the two-fold bands will combine back to form a three-fold band with \( S = 0 \), \( C = +1 \) and \( I = 1 \) from (117). Comparing the quantum numbers of the three-fold band with the quantum numbers of the baryons in Table 2 and in [7], we find there is only the \( \Sigma_c \)-baryon with the same quantum numbers as the three-fold band. According to Postulate III the \( \Sigma_c \)-baryons will excite from the vacuum into the band to form \( \Sigma_c(M) \)-baryons. The \( M = M_{fig} + de = 2649 -115 (n_c = 0, 2534); 2649 +0 (n_c = 1, 2649); 2649 + 115 (n_c = 2, 2764) \) from Table 7 appear in Table 13.

For the second sub-bands with asymmetry \( \vec{n} = (2-11,0-1,-10,-1) \), using (84), it will divide into a single band and a two-fold band with different \( \Delta S \) from (115). Thus the single band and the two-fold bands cannot combine back into a three-fold band with the same S, C and mass as the first sub-bands. The second sub-bands will divide into two bands: a two-fold band with symmetry \( \vec{n} \) values \((0,-1,-1;0,-1)\) and a single band with \((2,-1,1)\). Using (84) for the two-fold band, we get \( \Delta S = -1 \). Since the \( S_F = -1 \), the strange number of the band \( S = S_F + \Delta S = -1 + (-1) = -2 \). The band has \( S = -2 \), \( I = \frac{1}{2} \) from (64), \( Q = 0 \) and \(-1 \) from (77). Comparing the quantum numbers of the band with the quantum numbers of the baryons in Table 2 and in [7], we find there is only the \( \Xi \)-baryon having the same quantum numbers as the band. According to Postulate III, the \( \Xi \) baryon will excite from the vacuum into the energy band to form a \( \Xi(M) \) baryon. The mass of the baryon \( M = M_{fig} + de = 2649 + de \) since \( de = 0 \) from Table 7. The single band with \((2, -1, 0)\) and \(d = 1, R - d = 3-1 = 2 \) from (75), the strange number \( S = S_F = -1 \) and \( Q = 0 \) from (77) as well as \( I = 0 \) from (64). The band with \( I = 0 \), \( S = -1 \) and \( Q = 0 \) leads to a \( \Lambda(2649) \)-baryon as appearing in Table 13.

At \( M_{fig} = 2649 \), the three-fold band with \( \vec{n} = (21-1,1-12,2-12,0) \) will divide into a two-fold symmetry band with \( \vec{n} = (21-1,12-1) \) and a single band with \( \vec{n} = (2, 2, 0) \). The single band, similarly to the above single bands, is a \( \Lambda(2649) \). The two-fold band has \( I = \frac{1}{2} \) from (64), \( S = S_F = -1 \), \( \Delta S = +1 = C \) from (84) and (86) and \( Q = +1, 0 \) from (77). Comparing the quantum numbers of the band with the quantum numbers of the baryons in Table 2 and in [7], we find there is only a \( \Xi_c \)-baryon with the same quantum numbers as the band. According to Postulate III, the \( \Xi_c \)-baryon will excite from the vacuum into the energy band to form a \( \Xi_c(M) \) baryon. The \( M = M_{fig} + de = 2739 -115 (n_c = 0, 2534); 2739 + 0 (n_c = 1, 2649); 2739 + 115 (n_c = 2, 2764) \) from (97) and Table 7 as appearing in Table 13. Since there are \( \Xi_c \) in Table 2, there are two \( \Xi_c(2534) \).

There is second six-fold energy bands with \( M_{fig} = 2739 \) and \( \vec{n} = (1-21,013,103;0-20,-200,-211) \). From (67), the six fold band divides into two three sub-bands with \( \vec{n} = (1-21,013,103) \) and (0-20,-200,-211) respectively. From (84), the first sub-band with \((1-21,013,103)\) gives a sub-sub-bands with \( \vec{n} = (013,103) \) and \( \Delta S = +1 = C \) from (86) and another sub-sub-band with \((1,-2,1)\) and \( \Delta S = 0 \). The sub-sub-band with \((013,103)\) has \( I = \frac{1}{2} \) from (64), \( S = S_F = -1 \), \( C = +1 \) from (86) and \( Q = +1, 0 \). These are \( \Xi_c(M) \) baryons with \( M = M_f + de = 2739-115 (2624 n_c = 0); 2739 + 0 (2739 n_c = 1); 2739+115 (2875 n_c = 2) \) from Table 7. The second sub-band with \( \vec{n} = (0-20,-200,-211) \) divide to a sub-sub-band with \( \vec{n} = (0-20,-200) \) and a sub-sub-band with \( \vec{n} = (-2, 1, 1) \). The sub-sub-band with \( \vec{n} = (-2, 1, 1) \) has \( I = 0 \) from (64), \( S = S_F = -1 \) from (75) and \( Q = 0 \) from (77). It is a \( \Lambda(2739) \). The sub-sub-band with \( \vec{n} = (0-20,-200) \) has \( I = \frac{1}{2} \) from (64), \( S = -2 \) from (84) and \( Q = 0, -1 \) from (77). This is a \( \Xi(2739) \).
Table 13a. The baryons on the F-axis (P-H)

| $M_{Fig}$ | $(n_1, n_2, n_3)$ | I | $\Delta S$ | S | C | de | baryon | J  | P  |
|----------|------------------|---|------------|---|---|----|--------|----|----|
| 1209     | (0,1;1,0,1)      | 1/2 | +1         | 0 | 0 |    | N(1209) | 1/2 | +  |
| 1229     | (002,-101,0-11)  | 0  | 0          | -1| 0 | 0   | $\Lambda$(1229) | 1/2 | -  |
| 1229     | (0, 0, 2)        | 0  | 0          | -1| 0 | 0   | $\Lambda$(1299) | 1/2 | -  |
| 1299     | (-1,0,1;0,-1,1)  | 1/2 | +1         | 0 | 0 | 0   | N(1299) | 1/2 | -  |
| 1299     | (112,-110,1-10)  | 0  | 0          | -1| 0 | 0   | $\Lambda$(1299) | 1/2 | -  |
| 1299     | (-1,1,0;1,-1,0)  | 1/2 | +1         | 0 | 0 | 0   | N(1299) | 1/2 | -  |
| 1229     | (121,211,200     | 0  | 0          | -1| 0 | 0   | $\Lambda$(1229) | 1/2 | -  |
| 1229     | (020,01-1,10-1)  | 0  | 0          | -1| 0 | 0   | $\Lambda$(1299) | 1/2 | -  |
| 1229     | (0, 1, 0)        | 0  | 0          | -1| 0 | 0   | $\Lambda$(1299) | 1/2 | -  |
| 1299     | (01-1,10-1)      | 1/2 | +1         | 0 | 0 | 0   | N(1299) | 1/2 | -  |
| 1299     | (2, 0, 0)        | 0  | 0          | -1| 0 | 0   | $\Lambda$(1299) | 1/2 | -  |
| 1299     | (1,2,1;2,1,1)    | 1/2 | +1         | 0 | 0 | 0   | N(1299) | 1/2 | -  |
| 1229     | (-1-10,-112,1-12)| 0  | 0          | -1| 0 | 0   | $\Lambda$(1229) | 1/2 | -  |
| 1229     | (-1,-1,0)        | 0  | 0          | -1| 0 | 0   | $\Lambda$(1229) | 1/2 | -  |
| 1229     | (-1,1,2;1,-1,2)  | 1/2 | +1         | 0 | 0 | 0   | N(1299) | 1/2 | -  |
| 1229     | (-1,-1,2)        | 0  | 0          | -1| 0 | 0   | $\Lambda$(1229) | 1/2 | -  |
| 2649     | (220,022,-121,   | 1  | +1         | 0 | 0 | +1    | -15 | $\Sigma_c$(2534) | 1/2 | +  |
| 2649     | (2-11,0-1,1-10-1)| 1  | +1         | 0 | 0 | +1    | -115 | $\Sigma_c$(2649) | 1/2 | +  |
| 2649     | (0-1,-10-1)      | 1  | +1         | 0 | 0 | +1    | -115 | $\Sigma_c$(2764) | 1/2 | +  |
| 2649     | (21,-1,12,220)   | 1  | +1         | 0 | 0 | +1    | -115 | $\Sigma_c$(2964) | 1/2 | +  |
| 2649     | (21,-1,12,-1)    | 1  | +1         | 0 | 0 | -1    | -115 | $\Sigma_c$(2964) | 1/2 | +  |
| 2649     | (21,-1,12,-1)    | 1  | +1         | 0 | 0 | -1    | -115 | $\Sigma_c$(2964) | 1/2 | +  |
Table 13b: The baryons on the F-axis (P-H)

| $M_{\text{Fig}}$ | $(n_1, n_2, n_3)$ | I | $\Delta S$ | S | C | de | baryon | J | P |
|------------------|------------------|---|-----------|---|---|----|--------|---|---|
| 2739             | (013,103,-21,   |   |           |   |   |    | $\Xi_{c}(2739)$ | $\frac{1}{2}$ | - |
|                  | -211,0-20,-200) |   |           |   |   |    |        |   |   |
| 2739             | (013,103,-21)   | $\frac{1}{2}$ | +1 | -1      | +1 | 0   | $\Xi_{c}(2739)$ | $\frac{1}{2}$ | - |
| 2739             | (013,103,-21)   | $\frac{1}{2}$ | +1 | -1      | +1 | 115 | $\Xi_{c}(2854)$ | $2\frac{3}{4}$ | - |
| 2739             | (013,103,-21)   | $\frac{1}{2}$ | +1 | -1      | +1 | 230 | $\Xi_{c}(2969)$ | $2\frac{3}{4}$ | - |
| 2739             | (-2,1,1)        | 0   | 0 | -1      | 0  | 0   | $\Lambda(2739)$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 2739             | (0-20,-200)     | $\frac{1}{2}$ | -1 | -2      | 0  | 0   | $\Xi(2739)$      | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 2739             | (0-13,-103,0-22,|   |   |           |   |   |    | $\Xi_{c}(3084)$ | $\frac{1}{2}$ | + |
|                  | -202,-211,1-21) |   |   |           |   |   |    |        |   |   |
| 2739             | (0-13,-103,0-22,|   |   |           |   |   |    | $\Xi(3199)$      | $\frac{1}{2}$ | + |
|                  | -202,-211,1-21) |   |   |           |   |   |    |        |   |   |
| 2739             | (0-13,-103)     | $\frac{1}{2}$ | +1 | -1      | +1 | 345 | $\Xi_{c}(3319)$ | $\frac{1}{2}$ | + |
| 2739             | (0-13,-103)     | $\frac{1}{2}$ | +1 | -1      | +1 | 575 | $\Xi_{c}(3429)$ | $\frac{1}{2}$ | + |
| 2739             | (-2,11,-1-21)   | $\frac{1}{2}$ | -1 | -2      | 0  | 0   | $\Xi(379)$      | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 2739             | (-2,0,2)        | 0   | 0 | -1      | 0  | 0   | $\Lambda(2739)$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

Similarly to above paragraph, the third six-fold band with $M - \text{Fig} = 2739$ and $\vec{n} = (-202,0,13,-103;2-10,-1-21,0-22)$ leads to a $\Xi(M) = M_{\text{Fig}} + \text{de} = 2739 + 230 (2969, n_c = 3); 2739 + 345 (3314, n_c = 4); 2739 + 460 (3199, n_c = 5)$ from Table 7. There are $\Xi(2739)$ and two $\Lambda(2739)$. They appear in Table 13b.

Table B5 gives the J and P of the energy bands (the baryons) as appearing in Table 13. Using Table 6B, we can find the $J_{\text{Max}}$ for each baryon. Thus we omit the J larger than the $J_{\text{Max}}$ from Table B5 as appearing in Table 13.

5.9 The baryons on the G-axis (M-N)

The G-axis is a two-fold symmetric axis, $S_G = -2$ from (114) and $B = 1$ from Postulate II. All energy bands have a baryon number $B = 1$ and $S_G = -2$:

$$B = 1 \text{ and } S_G = -2.$$  (119)

There are two-fold, four-fold and six-fold energy bands on the axis (see Fig. 7). At $M_{\text{Fig}} = 1119$, there is a two-fold energy band with asymmetry $\vec{n} = (0,0,0,1,1,0)$. The band with $\vec{n} = (0,0,0)$ is
the N(939)-baryon from\cite{103}, and the band with $\vec{n} = (1, 1, 0)$ is the baryon $\Lambda(1119)$ from \cite{105}.

The four-fold bands with asymmetric $\vec{n}$ values divide into two two-fold bands with symmetric $\vec{n}$ values from \cite{67}. Thus, we only need to discuss the two-fold bands.

The two-fold bands with symmetric $\vec{n}$ values have $S = -2$ from \cite{74}, $I = \frac{1}{2}$ from \cite{64}, $Q = 0$ and -1 from \cite{77}. Comparing the $I$, $S$ and $Q$ of the bands with the $I$, $S$ and $Q$ of the baryons in Table 2 and in \cite{7}, we find there are only the $\Xi$-baryons with the same quantum numbers as the band. According to Postulate III, the $\Xi$-baryons will excite from the vacuum into the two-fold energy bands to form $\Xi(M_{Figg})$-baryons as appearing in Table 14.

There are two two-fold bands with asymmetric $\vec{n}$ values: at $E_M = 1299$, a band with $\vec{n} = (200, 1-10)$; at $E_N = 1839$, a band with $(020, -110)$. The two two-fold bands will divide into four single bands. Each single band has $S = S_F + \Delta S = -2 + 1 = -1$ from \cite{115} and \cite{85}, $I = 0$ from \cite{64} and $Q = 0$ from \cite{77}. Comparing the quantum numbers of the single bands with the quantum numbers of the baryons in Table 2 and in \cite{7}, we find there is only the $\Lambda$-baryons with the same quantum numbers as the single bands. According to Postulate III, the four $\Lambda$-baryons will excite from the vacuum into the four single bands to form four $\Lambda(M)$-baryons. The mass $M = M_{Figg} + de = 1299 + 115 = 1414$ for the two bands with $(200)$ and $(1-10)$; the $M = 1839 + 115 = 1954$ for the two bands with $(020)$ and $(-110)$ as appearing in Table 14.

At $M_{Figg} = 2559$, there is a six-fold band with asymmetry $\vec{n} = (112, 11-2, 002, 00-2, 220, -1-10)$. From \cite{67}, the band is divided into two two-fold bands with symmetric $\vec{n}$ values $(112, 11-2)$ and $(002, 00-2)$ as well as one two-fold band with asymmetric $\vec{n} = (220, -1-10)$. The two two-fold bands with symmetric $\vec{n}$ values have $I = 1/2$, $S = -2$, $Q = 1$, 0. They are two $\Xi(2559)$-baryons as mentioned above. The third two-fold sub-band with asymmetric $\vec{n} = (220, -1-10)$ divide into a single sub-sub-band with $\vec{n} = (2, 2, 0)$ and another single sub-sub-band with $\vec{n} = (-1, -1, 0)$. The single sub-band with $(2, 2, 0)$ has $I = 0$, $S = -2$, $\Delta S = +1 = C$ from \cite{86} and $Q = 0$ from \cite{77}. Comparing the quantum numbers of the sub-sub-band with the quantum numbers of the baryons in Table 2 and in \cite{7}, we find there is only the $\Omega_c$-baryons with the same quantum numbers as the sub-sub-band. According to Postulate III, the $\Omega_c$-baryons will excite from the vacuum into the sub-band to form $\Omega_c(M)$-baryons. The $M = M_{Figg} + de = 2559 + 230 (n_c = 0, 2789); 2559 + 3450 (n_c = 1, 2904); 2559 + 230 (n_c = 0.30199)$ from Fig. 7 and Table 7 (de = $115(n_c - S)\Delta S$). The $\Omega_c$-baryons are shown in Table 14. The sub-sub-band with $(-1, -1, 0)$ has $I = 0$, $S = S_F + \Delta S = -2 + (-1) = -3$ from \cite{115} and $Q = -1$ from \cite{77}. Comparing the quantum numbers of the band with the quantum numbers of the baryons, we find there is only an $\Omega$-baryon with the same quantum numbers as the band with. According to Postulate III, The $\Omega$-baryon will be excited from the vacuum into the sub-sub-band to form an $\Omega(M)$-baryon. The $M = M_{Figg} + de = 2559 + (-345) = 2214$ from Figure 7 and Table 7. The $\Omega(2214)$ ($\xi n_c = 0$) appear in Table 14.

At $M_f = 2739$ there is another six-fold energy band with $(310, 0-20, 202, 0-2, 220, -1-2)$. Similarly to the six-fold band at $M_f = 2559$ as mentioned above, we can deduce the baryons of the six-fold band. The six-fold band divide into two two-fold symmetry sub-bands with $\vec{n} = (202, 20-2)$ and $(1-12, 1-1-2)$ as well as an asymmetry two-fold sub-band with $(310, 0-20)$. The two two-fold sub-bands lead to two $\Xi(2739)$-baryons. The asymmetry two-fold sub-band $(310, 0-20)$ divides into two single sub-sub-bands with $\vec{n} = (3, 1, 0)$ and $(0, -2, 0)$. The single sub-sub-band with $\vec{n} = (3, 1, 0)$ leads to $\Omega_c(M)$-baryons with $M = M_{Figg} + de = 2739 + 230$ ($\xi n_c = 0$, 2969); 2739 + 345 ($\xi n_c = 1$, 3084); 2739 + 460 ($\xi n_c = 2$, 2199) from de = $115(n_c - S)\Delta S$ in Table 7. The other single sub-sub-band with $\vec{n} = (0, -2, 0)$ leads to an $\Omega(2394)$-baryon (de = -345). These baryons appear in Table 14.
Table 14a. The Baryons on the G-axis

| $M_{F_{1g}}$ | ($n_1,n_2,n_3$) | I | $\Delta S$ | S | C | de | baryon | J | P |
|-------------|----------------|----|-----------|---|---|----|--------|----|---|
| 1299        | (1,0,1;0,1,-1) | $\frac{1}{2}$ | 0 | -2 | 0 | 0 | $\Xi(1299)$ | $\frac{1}{2}$ | + |
| 1299        | (2,0,0;1,-1,0) | $\frac{1}{2}$ | 0 | +1 | -1 | 0 | 115 $\Lambda(1414)$ | $\frac{2}{3}$ | + |
| 1299        | (2,0,0)       | 0   | +1 | -1 | 0 | 115 $\Lambda(1414)$ | $\frac{2}{3}$ | + |
| 1479        | (0,1,1;0,1,-1) | $\frac{1}{2}$ | 0 | -2 | 0 | 0 | $\Xi(1479)$ | $\frac{1}{2}$ | + |
| 1839        | (0,2,0;1,1,0) | 0   | +1 | -1 | 0 | 115 $\Lambda(1954)$ | $\frac{2}{3}$ | - |
| 1839        | (0,2,0,0)     | 0   | +1 | -1 | 0 | 115 $\Lambda(1954)$ | $\frac{2}{3}$ | - |
| 2019        | (0,1,1;0,1,1) | 0   | -1 | 0  | 0 | $\Xi(2019)$ | $\frac{2}{3}$ | - |
| 2019        | (2,1,1;2,1,1) | $\frac{1}{2}$ | 0 | -2 | 0 | $\Xi(2019)$ | $\frac{3}{2}$ | - |
| 2019        | (2,1,1;2,1,1) | $\frac{1}{2}$ | 0 | -2 | 0 | $\Xi(2019)$ | $\frac{3}{2}$ | - |
| 2559        | (121,12-1)    | $\frac{1}{2}$ | 0 | -2 | 0 | $\Xi(2559)$ | $\frac{3}{2}$ | - |
| 2559        | (002,00-2)    | $\frac{1}{2}$ | 0 | -2 | 0 | $\Xi(2559)$ | $\frac{3}{2}$ | - |
| 2739        | (310,0-20,202 | $\frac{1}{2}$ | 0 | -2 | 0 | $\Xi(2739)$ | $\frac{3}{2}$ | - |
| 2739        | (3,1,0;0,-2,0)| $\frac{1}{2}$ | 0 | +1 | -2 | +1 | $+345$ $\Omega_{c}(2904)$ | $\frac{3}{2}$ | - |
| 2739        | (3,1,0)       | $\frac{1}{2}$ | 0 | +1 | -2 | +1 | $+345$ $\Omega_{c}(3199)$ | $\frac{3}{2}$ | - |
| 2739        | (0,-2,0)      | $\frac{1}{2}$ | 0 | -1 | -3 | 0  | $-345$ $\Omega_{c}(2394)$ | $\frac{3}{2}$ | - |
| 2739        | (202,20-2)    | $\frac{1}{2}$ | 0 | -2 | 0 | $\Xi(2739)$ | $\frac{3}{2}$ | + |
| 2739        | (1-12,1-1-2)  | $\frac{1}{2}$ | 0 | -2 | 0 | $\Xi(2739)$ | $\frac{3}{2}$ | + |
Table B6 gives the J and P values for these baryons of the G-axis. The formula \[ (92) \] and Table 6B give the maximum \( J_{\text{Max}} \) for each baryon. Using it we can take off the extra J values (larger than \( J_{\text{Max}} \)) as appearing in Table 14.

### 6 Comparison of the Deduced Baryons with the Experimental Baryons

We have deduced the baryons appearing in \((103), (104)\) and Table 8–Table 14 in Section 5. Using Table 15–Table 22, we will compare the deduced baryons with the experimental results \([7]\). In the comparisons, we use the baryon name to represent the intrinsic quantum numbers (I, S, C, B and Q) of baryons as appearing in Table 2 and Table 3 as well as in \([7]\). If the name is the same between theory and experiment, this means that the intrinsic quantum numbers (I, S, C, B and Q) of the deduced and experimental baryons are completely the same. In the comparison, the same kind of baryons (with same I, S, C, B and Q) will be divided into groups with the same J and P as well as similar masses (about inside the experimental width). For each group, we deduce the average mass and use it to compare with the experimental mass of the corresponding baryon.

#### 6.1 The most important baryons

First we compare the most important baryons. From \((103), (104)\) and Table 8–Table 14, we get the most important deduced baryons \( N(939), \Lambda(1119), \Sigma(1209), \Xi(1318), \Omega(1659), \Lambda_c(2264) \) and \( \Lambda_b(5554) \) as appearing in Table 15. The corresponding experimental results are from \([7]\).

| Table 15. The comparison of the most important baryons |
|------------------------------------------------------|
| Experiment   | \( N(939) \frac{1}{2}^+ \) | \( \Lambda(1116) \frac{1}{2}^+ \) | \( \Sigma(1193) \frac{1}{2}^+ \) | \( \Xi(1318) \frac{1}{2}^+ \) |
| Theory       | \( N(939) \frac{7}{2}^+ \) | \( \Lambda(1119) \frac{7}{2}^+ \) | \( \Sigma(1209) \frac{7}{2}^+ \) | \( \Xi(1299) \frac{7}{2}^+ \) |
| Where        | Table 11                   | Table 9                        | Table 14                      |
| \( \Delta M/M_{\text{Exp.}} \)   | 0.0                        | 0.3                            | 1.3                           | 1.4                           |

| Experiment   | \( \Omega(1672) \frac{3}{2}^+ \) | \( \Lambda_c(2286) \frac{3}{2}^+ \) | \( \Lambda_b(5624) \frac{1}{2}^+ \) |
| Theory       | \( \Omega(1659) \frac{3}{2}^+ \) | \( \Lambda_c(2264) \frac{3}{2}^+ \) | \( \Lambda_b(5554) \frac{1}{2}^+ \) |
| Where        | Table 11                   | Table 8                        | Table 11                      |
| \( \Delta M/M_{\text{Exp.}} \)   | 0.8                        | 1.0                            | 1.2                           |

Table 15 shows that the deduced quantum numbers (I, S, C, B, Q) are exactly the same as the experimental results of the seven most important baryons. Their masses are consistent with experimental results more than 98.5 percent using only three constant parameters (\( M_0 = 939 \) Mev, \( \alpha = 360 \) Mev and \( \beta = 115 \) Mev).

#### 6.2 The comparison between the deduced and experimental \( \Delta \)-baryons

The deduced \( \Delta \)-baryons are from Table 8 and Table 12 (\( \Delta(1929) \) and \( \Delta(2649) \)). The experimental \( \Delta \)-baryons are from \([7]\). We compare the deduced \( \Delta \)-baryons with the experimental
The comparison between the deduced and experimental $\Sigma$-baryons

The deduced $\Sigma$-baryons are from Table 9. The experimental $\Sigma$-baryons are from [7]. The comparison appears in Table 17:
Table 17. Comparing theory Σs with the experimental results

| Order, Compose                  | Theory $J^P$      | Experiment $J^P$, $\Gamma$   | $\Delta M / M$ |
|--------------------------------|-------------------|-------------------------------|----------------|
| 1. $\Sigma(1209)$, $(\frac{1}{2}^+)$ | $\Sigma(1209)$, $(\frac{1}{2}^+)$ | $\Sigma(1193)$, $(\frac{1}{2}^+)$, **** | 1.4 |
| 2. $\frac{1}{2}[\Sigma(1209) + \Sigma(1659)]$, $(\frac{3}{2}^-)$ | $\Sigma(1434)$, $(\frac{3}{2}^-)$ | $\Sigma(1385)$, $(\frac{3}{2}^-)$, .36**** | 3.5 |
| 3. $\Sigma(1659)$, $(\frac{1}{2}^-)$ | $\Sigma(1659)$, $(\frac{1}{2}^-)$ | $\Sigma(1620)$, $(\frac{1}{2}^-)$, .43** | 2.4 |
| 4. $2\Sigma(1659)$, $(\frac{1}{2}^-)$ | $\Sigma(1659)$, $(\frac{1}{2}^-)$ | $\Sigma(1580)$, $(\frac{1}{2}^-)$, 15* | 5.0 |
| 5. $\Sigma(1659)$, $(\frac{1}{2}^+)$ | $\Sigma(1659)$, $(\frac{1}{2}^+)$ | $\Sigma(1660)$, $(\frac{1}{2}^+)$, 100*** | .06 |
| 6. $2\Sigma(1659)$, $(\frac{1}{2}^-)$ | $\Sigma(1659)$, $(\frac{1}{2}^-)$ | $\Sigma(1670)$, $(\frac{1}{2}^-)$, .60**** | 0.7 |
| 7. $\Sigma(1659)$, $(\frac{3}{2}^-)$ | $\Sigma(1659)$, $(\frac{3}{2}^-)$ | $\Sigma(1690)$, $(\frac{3}{2}^-)$, 130** | 1.8 |
| 8. $\frac{1}{2}[\Sigma(1659) + \Sigma(1929)]$, $(\frac{1}{2}^-)$ | $\Sigma(1794)$, $(\frac{1}{2}^-)$ | $\Sigma(1750)$, $(\frac{1}{2}^-)$, .90*** | 2.5 |
| 9. $\frac{1}{8}[4\Sigma(1659)+4\Sigma(1929)]$, $(\frac{3}{2}^-)$ | $\Sigma(1794)$, $(\frac{3}{2}^-)$ | $\Sigma(1775)$, $(\frac{3}{2}^-)$, 120**** | 1.1 |
| 10. $4\Sigma(1929)$, $(\frac{3}{2}^-)$ | $\Sigma(1794)$, $(\frac{3}{2}^-)$ | $\Sigma(1794)$, $(\frac{3}{2}^-)$, 120**** | 1.4 |
| 11. $\frac{1}{2}[\Sigma(1659)+\Sigma(1929)]$, $(\frac{3}{2}^+)$ | $\Sigma(1794)$, $(\frac{3}{2}^+)$ | $\Sigma(1770)$, $(\frac{3}{2}^+)$, 100* | .65 |
| 12. $\frac{1}{2}[\Sigma(1659)+2\Sigma(1929)]$, $(\frac{3}{2}^+)$ | $\Sigma(1839)$, $(\frac{3}{2}^+)$ | $\Sigma(1840)$, $(\frac{3}{2}^+)$, 100* | .05 |
| 13. $\Sigma(1929)$, $(\frac{1}{2}^-)$ | $\Sigma(1929)$, $(\frac{1}{2}^-)$ | $\Sigma(1915)$, $(\frac{1}{2}^-)$, 120**** | 0.7 |
| 14. $2\Sigma(1929)$, $(\frac{1}{2}^-)$ | $\Sigma(1929)$, $(\frac{1}{2}^-)$ | $\Sigma(2100)$, $(\frac{1}{2}^-)$, 100* | 8.1 |
| 15. $2\Sigma(1929)$, $(\frac{1}{2}^+)$ | $\Sigma(1929)$, $(\frac{1}{2}^+)$ | $\Sigma(1880)$, $(\frac{1}{2}^+)$, 100** | 2.6 |
| 16. $\Sigma(1929)$, $(\frac{3}{2}^-)$ | $\Sigma(1929)$, $(\frac{3}{2}^-)$ | $\Sigma(2000)$, $(\frac{3}{2}^-)$, 200* | 3.6 |
| 17. $\frac{1}{2}[\Sigma(1929) + 2\Sigma(2379)]$, $(\frac{3}{2}^+)$ | $\Sigma(2079)$, $(\frac{3}{2}^+)$ | $\Sigma(2080)$, $(\frac{3}{2}^+)$, 200** | .04 |
| 18. $\frac{1}{8}[4\Sigma(1929)+2\Sigma(2379)]$, $(\frac{3}{2}^+)$ | $\Sigma(2079)$, $(\frac{3}{2}^+)$ | $\Sigma(2070)$, $(\frac{3}{2}^+)$, 300* | 0.4 |
| 19. $\frac{1}{2}[\Sigma(1929) + \Sigma(2379)]$, $(\frac{3}{2}^+)$ | $\Sigma(2079)$, $(\frac{3}{2}^+)$ | $\Sigma(2030)$, $(\frac{3}{2}^+)$, 180**** | 0.5 |
| 20. $\frac{1}{2}[\Sigma(1929)+3\Sigma(2379)]$, $\frac{3}{2}^+\frac{3}{2}^+\frac{3}{2}^+ \frac{3}{2}^+ \frac{3}{2}^+$ | $\Sigma(2267)$, $(?)^*$ | $\Sigma(2250)$, $(?)^*$, 100** | 0.8 |
| 21. $\frac{1}{2}[\Sigma(2379)+\Sigma(2649)]$, $(\frac{9}{2}^-)$ | $\Sigma(2514)$, $(\frac{9}{2}^-)$ | $\Sigma(2455)$ Bumps, $(\frac{3}{2}^-)$, 120** | 2.4 |
| 22. $\Sigma(2649)$, $(\frac{9}{2}^-)$ | $\Sigma(2649)$, $(\frac{9}{2}^-)$ | $\Sigma(2620)$ Bumps (?), 200** | 1.1 |
| 23. $2\Sigma(2649)$, $\frac{9}{2}^+_\frac{9}{2}^+_\frac{9}{2}^+_\frac{9}{2}^+$ | $\Sigma(2649)$, $(?)^*$ | $\Sigma(2620)$ Bumps (?), 200** | ? |

The quantum numbers (I, S, C, B, Q, J and P) of all 22 deduced Σ-baryons are completely the same as the experimental results. The masses of the all 22 observed Σ-baryons agree more than 98.2 percent with the experimental results. The largest error of the nine established Σ-baryons is only 3.5 percent.

### 6.4 The comparison of the N-baryons

The deduced N-baryons are from Table 8, Table 12 and Table 13; and the experimental N-baryons are from [7]. The comparison between the deduced N-baryons with the experimental N-baryons appears in Table 18.
The all quantum numbers (I, S, C, B, Q, J and P) of all deduced 23 N-baryons are completely the same as the experimental results. The masses of 23 observed N-baryons agree more than average 98 percent with the experimental results. The largest error of 15 established N-baryons is only 3.7 percent.
6.5 The comparison of the $\Lambda$-baryons

The deduced $\Lambda$-baryons are from Table 9, Table 12, Table 13 and Table 14. The experimental $\Lambda$-baryons are from [7]. The comparison between the deduced and the experimental $\Lambda$-baryons appears in Table 19.

| Order, $J^P$, Compose | Theory       | Exper.$\Lambda(M)$, $\Gamma$ | $\Delta M/M$ |
|-----------------------|--------------|------------------------------|--------------|
| 1. $1/2^+$, $\Lambda(1119)$ | $\Lambda(1119)$ | $\Lambda(1116)$ | 0.3 |
| 2. $3/2^-$, $\frac{1}{2}\{\Lambda(1299)+2\Lambda(1414)\}$ | $\Lambda(1376)$ | $\Lambda(1405)$, 50$^{****}$ | 2.1 |
| 3. $3/2^-$, $\frac{1}{2}\{\Lambda(1414)+\Lambda(1659)\}$ | $\Lambda(1537)$ | $\Lambda(1520)$, 16$^{****}$ | 1.1 |
| 4. $1/2^+$, $\Lambda(1659)$ | $\Lambda(1659)$ | $\Lambda(1670)$, 35$^{****}$ | 0.7 |
| 5. $3/2^-$, $4\Lambda(1659)$ | $\Lambda(1659)$ | $\Lambda(1690)$, 60$^{****}$ | 1.8 |
| 6. $1/2^+$, $\frac{1}{2}\{\Lambda(1414)+2\Lambda(1659)\}$ | $\Lambda(1577)$ | $\Lambda(1600)$, 150$^{***}$ | 1.4 |
| 7. $1/2^+$, $2\Lambda(1814)$ | $\Lambda(1814)$ | $\Lambda(1810)$, 150$^{***}$ | 0.2 |
| 8. $3^+$, $\frac{1}{15}\{\Lambda(1414)+2\Lambda(1659)+2\Lambda(1814)+9\Lambda(1929)+\Lambda(2019)\}$ | $\Lambda(1849)$ | $\Lambda(1890)$, 100$^{****}$ | 2.2 |
| 9. $5^+$, $\frac{1}{7}\{\Lambda(1659)+\Lambda(1814)+2\Lambda(1929)\}$ | $\Lambda(1833)$ | $\Lambda(1820)$, 80$^{****}$ | 0.7 |
| 10. $5^-$, $\frac{1}{7}[2\Lambda(1659)+2\Lambda(1814)+3\Lambda(1929)]$ | $\Lambda(1819)$ | $\Lambda(1800)$, 300$^{***}$ | 1.1 |
| 11. $3^-$, $\frac{1}{18}[4\Lambda(1814)+7\Lambda(1929)+5\Lambda(1954)+2\Lambda(2019)]$ | $\Lambda(1920)$ | $\Lambda(2000)$ (7), 125$^*$ | 4.0 |
| 12. $5^-$, $\frac{1}{15}[4\Lambda(1659)+4\Lambda(1814)+6\Lambda(1929)+4\Lambda(1954)]$ | $\Lambda(1849)$ | $\Lambda(1830)$, 95$^{****}$ | 1.0 |
| 13. $7^-$, $\frac{1}{17}[3\Lambda(1929)+2\Lambda(1954)+3\Lambda(2019)+3\Lambda(2444)]$ | $\Lambda(2099)$ | $\Lambda(2100)$, 200$^{***}$ | 0.0 |
| 14. $3^+$, $3\Lambda(2379)$ | $\Lambda(2379)$ | ?? | ?? |
| 15. $5^+$, $\frac{1}{12}[7\Lambda(1929)+2\Lambda(2019)+3\Lambda(2379)]$ | $\Lambda(2056)$ | $\Lambda(2110)$, 200$^{***}$ | 2.6 |
| 16. $7^+$, $\frac{1}{10}[6\Lambda(1929)+2\Lambda(2019)+2\Lambda(2379)]$ | $\Lambda(2037)$ | $\Lambda(2020)$, 140$^*$ | 0.8 |
| 17. $9^+$, $\frac{1}{7}[3\Lambda(2379)+\Lambda(2444)]$ | $\Lambda(2412)$ | $\Lambda(2350)$, 150$^{***}$ | 2.6 |
| 18. $3^+$, $2\Lambda(2444)$ | $\Lambda(2444)$ | $\Lambda(2325)$, 169$^*$ | 5.1 |
| 19. $5^+$, $\frac{1}{7}[2\Lambda(2444)+4\Lambda(2649)]$ | $\Lambda(2581)$ | $\Lambda(2585)$, 225$^{**}$ | 0.2 |
| 20. $5^+$, $\frac{1}{7}[2\Lambda(2444)+2\Lambda(2649)]$ | $\Lambda(2547)$ | ??, ?? | ?? |

The quantum numbers (I, S, C, B, Q, J and P) of all deduced $\Lambda$-baryons are completely the same as the experimental results. The masses of the 14 established (four or three stars) $\Lambda$-baryons agree more than 97.8 percent with the experimental results.

6.6 The comparison of the $\Xi$-baryons

From Table 10, Table 13 and Table 14, we can find the deduced $\Xi(M)$-baryons and from Table B3, Table B5 and Table B6, we can find $J^P$ value for each $\Xi(M)$. Using [7], we can find the experimental $\Xi(M)J^P$-baryons. Then we comparing the deduced $\Xi J^P$-baryons with the
experimental $\Xi(M)$ $J^P$-baryons [7] as appearing in Table 20.

For many experimental $\Xi(M)$-baryons, the $J^P$ are uncertainly and the widths are very small. There are three possibility: The first possible, there is a main $\Xi(m)$ with the largest number of the same mass $\Xi(M)$-baryons; for example, $\frac{1}{7}[\Xi(1299)+4\Xi(1479)+2\Xi(1659)]^{3+}$, the $4\Xi(1479)^{3+}$ is the main $\Xi(m)$ with the largest number 4 of the same mass $\Xi(M)$-baryons. The second possible, there are two nearest different mass kinds of $\Xi(M)$-baryons with many different $J^P$ values; for examples, $\frac{1}{15}[8\Xi(1479)+10\Xi(1659)]$, $\frac{1}{9}[3\Xi(1659)+2\Xi(1839)]$ and $\frac{1}{11}[6\Xi(2199)+8\Xi(2019)]$. Third possible, there are many energy bands with the same mass but many different $J^P$ values; for example $\frac{1}{34}[34\Xi(2199)]$ and $\frac{1}{23}[23\Xi(2379)]$ and $\frac{1}{23}[23\Xi(2559)]$.

Table 20. The comparison of the $\Xi(M)$-baryons

| Order, Compose | Deduced $\Xi(M)$ | Experiment $\Xi(M)$, $J^P$, $\Gamma$ | $\Delta M$/$M$ |
|----------------|------------------|--------------------------------------|--------------|
| 1. $\Xi(1299)^{1/2}+$ | $\Xi(1299)^{1/2}$ | $\Xi(1318)^{1/2}$ ** ** | 1.5 |
| 2. $\frac{1}{7}[\Xi(1299)+4\Xi(1479)+2\Xi(1659)]^{3+}$ | $\Xi(1505)^{3/2}$ | $\Xi(1532)^{3/2}$, 9.1 ** ** | 1.8 |
| 3. $\frac{1}{9}[3\Xi(1659)+2\Xi(1839)]$ | $\Xi(1731)$ | $\Xi(1620)$, (?), 27* | 2.5 |
| 4. $2\Xi(1839)^{3/2}$ | $\Xi(1839)^{3/2}$ | $\Xi(1820)^{3/2}$, 24** ** | 2.4 |
| 5. $\frac{1}{11}[6\Xi(2199)+8\Xi(2019)]$ | $\Xi(1947)$ | $\Xi(1950)$, (?), 60** | 1.0 |
| 6. $\frac{1}{34}[34\Xi(2199)]$ | $\Xi(2019)$ | $\Xi(2030)$, (?), 55** | 0.2 |
| 7. $\frac{1}{23}[23\Xi(2379)]$ | $\Xi(2199)$ | $\Xi(2250)$, (?), 46** | 0.8 |
| 8. $\frac{1}{15}[6\Xi(2199)+8\Xi(2019)]$ | $\Xi(2214)$ | $\Xi(2394)$, (?), 9** | 0.5 |
| 9. $\frac{1}{23}[23\Xi(2379)]$ | $\Xi(2250)$ | $\Xi(2250)$, (?), 46** | 2.3 |
| 10. $\frac{1}{23}[23\Xi(2559)]$ | $\Xi(2394)$ | $\Xi(2394)$, (?), 80** | 0.4 |
| 11. $\frac{1}{23}[23\Xi(2559)]$ | $\Xi(2394)$ | $\Xi(2394)$, (?), 80** | 0.4 |
| 12. $\frac{1}{23}[6\Xi(2649)]$ | $\Xi(2559)$ | $\Xi(2500)$, (?), 105* | 2.3 |

The all quantum numbers (I, S, C, B and Q) of all deduced $\Xi$-baryons are completely the same as the experimental results. The all deduced masses agree more than average 98.5 percent with the experimental results. The largest error of the established is only 2.4 percent.

### 6.7 The comparison of $\Omega$-baryons

Table 11 and Table 14b have given the deduced $\Omega$-baryons, and [7] has already shown the experimental $\Omega$-baryons. We compare the deduced and experimental $\Omega$-baryons in Table 21:

Table 21. The comparison of $\Omega$-baryons

| Theory | Experiment |
|--------|------------|
| $\Omega(1659)^{3/2}+$ | $\Omega(1672)^{3/2}$, ** ** | 0.8 |
| $\Omega(2214)^{3/2}+$ | $\Omega(2250)$, 55 ** | 1.6 |
| $\Omega(2394)^{3/2}+$ | $\Omega(2380)$, ? ** | 1.6 |
| $\Omega(3819)^{3/2}+$ | $\Omega(2470)$, 26 ** | 0.6 |

The deduced quantum numbers (I, S, C, B and Q) of the four $\Omega$-baryons are completely the same with the corresponding experimental results. The deduced masses of the four $\Omega$-baryons agree with average more than 98.4 percent experimental masses.
6.8 The comparison of charmed-baryons

The \( \Lambda_c(2264) \frac{1}{2}^- \) is from Table 8. In Table 12, the band with \( M_{F \bar{g}} = 2559 \) and \( \vec{n} = (-1,-1,0) \) has a \( \Lambda_c \) with \( \frac{1}{2}^-, \frac{3}{2}^+ \) and \( \frac{5}{2}^- \). The band with \( M_{F \bar{g}} = 2559 \) and \( \vec{n} = (0,0,-2) \) has a \( \Lambda_c \) with \( 2\frac{1}{2}^+, 2\frac{3}{2}^+, \frac{5}{2}^+ \) from Table 7, they form \( \Lambda_c(2559) \frac{1}{2}^-; \Lambda_c(2674), \frac{3}{2}^-; \Lambda_c(2789), \frac{5}{2}^- \) and \( \Lambda_c(2904), 2\frac{1}{2}^+, 2\frac{3}{2}^+, \frac{5}{2}^+ \) as appearing in Table 22.

In Table 13, the band with \( M_{F \bar{g}} = 2649 \) and \( \vec{n} = (21-1,12-1) \) has \( \Xi_c \) with \( \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+ \). Using adjusted energy \( \Delta \bar{s} \) in Table 7, we get \( \Xi_c(2534), \frac{1}{2}^+; \Xi_c(2534), \frac{3}{2}^+; \Xi_c(2649), \frac{5}{2}^+; \Xi_c(2649), \frac{5}{2}^+ \). The band with \( M_{F \bar{g}} = 2739 \) and \( \vec{n} = (013,103) \) has \( \Xi_c(2739), \frac{1}{2}^-; \Xi_c(2854), \frac{3}{2}^-; \Xi_c(2969), \frac{5}{2}^- \) from \( \Delta \bar{s} \) of the F-axis in Table 7. The band with \( M_{F \bar{g}} = 2739 \) and \( \vec{n} = (0-13,-103) \) has \( \Xi_c \) with \( \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+ \). From \( \Delta \bar{s} \) in Table 7, we get \( \Xi(3084), \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+ \). The deduced \( \Xi_c \)-baryons appears in Table 22.

In Table 13, at \( M_{F \bar{g}} = 2649 \), the sub-band with \( \vec{n} = (202,022,-121) \) has \( \Xi_c \) with \( \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+ \). From \( \Delta \bar{s} \) in Table 7, we get \( \Xi_c(2534), \frac{1}{2}^+; \Xi_c(2649), \frac{3}{2}^+; \Xi_c(2649), \frac{5}{2}^+ \) as appearing in Table 22.

In Table 14, at \( M_{F \bar{g}} = 2559 \) the band with \( \vec{n} = (2,2,0) \) has \( \Omega_c(2789), 2\frac{1}{2}^+, \Omega_c(2904), 2\frac{3}{2}^+, \Omega_c(3019), \frac{5}{2}^+ \) from \( \Delta \bar{s} = 115(n_c-S)\Delta S \) in Table 7. The \( \Omega_c \)-baryons appear in Table 22.

| \( M_{F \bar{g}}, n_c \) | Theory | Experiment | \( \Delta M \) |
|----------------|--------|------------|------------|
| 2379, 0        | \( \Lambda_c(2264), \frac{1}{2}^- \) | \( \Lambda_c(2286), \frac{1}{2}^+ \) | 1.0        |
| 2559, 0        | \( \Lambda_c(2559), \frac{1}{2}^- \) | \( \Lambda_c(2593), \frac{1}{2}^-, 3.6, ** \) | 1.3        |
| 2559, 1        | \( \Lambda_c(2674), \frac{3}{2}^- \) | \( \Lambda_c(2625), \frac{3}{2}^-, 1.9, *** \) | 1.8        |
| 2559, 2        | \( \Lambda_c(2789), \frac{5}{2}^+ \) | \( \Lambda_c(2765), ?, 50, \) | 0.9        |
| 2559, 3        | \( \Lambda_c(2904), \frac{7}{2}^+ \) | \( \Lambda_c(2880), ? \) | 0.8        |
| 2559, 4        | \( \Lambda_c(3019), \frac{9}{2}^+ \) | \( ? \), \( ? \) | \( ? \)        |
| 2649, 0        | \( \Xi_c(2534), \frac{1}{2}^+ \) | \( \Xi_c(2470), \frac{1}{2}^+ \) | 2.6        |
| 2649, 0        | \( \Xi_c(2534), \frac{3}{2}^- \) | \( \Xi_c(2577), \frac{3}{2}^+ \) | 1.7        |
| 2649, 1        | \( \Xi_c(2649), \frac{5}{2}^- \) | \( \Xi_c(2645), \frac{5}{2}^+ \), \( \leq 4.6 \) | 0.2        |
| 2649, 2        | \( \Xi_c(2764), \frac{7}{2}^+ \) | \( ? \), \( ? \) | \( ? \)        |
| 2739, 1        | \( \Xi_c(2739), \frac{3}{2}^- \) | \( \Xi_c(2700), \frac{3}{2}^- \), \( \leq 14 \) | 1.8        |
| 2739, 2        | \( \Xi_c(2854), \frac{5}{2}^- \) | \( \Xi_c(2815), \frac{5}{2}^- \), \( \leq 5 \) | 1.4        |
| 2739, 3        | \( \Xi_c(2969), \frac{7}{2}^- \) | \( ? \), \( ? \) | \( ? \)        |
| 2739, 4        | \( \Xi_c(3084), \frac{9}{2}^- \) | \( ? \), \( ? \) | \( ? \)        |
| 2649, 0        | \( \Sigma_c(2534), \frac{1}{2}^+ \) | \( \Sigma_c(2455), \frac{1}{2}^+, \sim 2.5 \) | 3.2        |
| 2649, 1        | \( \Sigma_c(2649), \frac{3}{2}^- \) | \( \Sigma_c(2520), \frac{3}{2}^- \), \( \sim 16 \) | 5.1        |
| 2649, 2        | \( \Sigma_c(2764), \frac{5}{2}^- \) | \( \Sigma_c(2800), \sim 66 \) | 1.3        |
| \( \Omega_c \)  | \( \Omega_c(2789), \frac{5}{2}^- \) | \( \Omega_c(2698), \frac{5}{2}^- \) | 3.4        |
| \( \Omega_c \)  | \( \Omega_c(2904), \frac{7}{2}^- \) | \( ? \), \( ? \) | \( ? \)        |
| \( \Omega_c \)  | \( \Omega_c(3019), \frac{9}{2}^- \) | \( ? \), \( ? \) | \( ? \)        |
The $\Sigma_c$-baryons are from Table 13 also. The $\Omega_c(2789)$-baryons are from Table 14. The experimental $\Lambda_c$-baryons, $\Xi_c$-baryons, $\Sigma_c$-baryons, $\Omega_c$-baryons and $\Lambda_b$-baryon are all found from [7]. We show the comparisons of these deduced baryons with the experimental baryons in Table 22.

The $J^P$ values of the charmed-baryons have not been measured by experiment, we will show the $J^P$ values of the quark model in Table 22.

Table 22 shows that the all quantum numbers (I, S, C, Q) of all 14 deduced baryons (5 $\Lambda_c$, 5 $\Xi_c$, 3 $\Sigma_c$ and one $\Omega_c$) are completely the same with the corresponding experimental results. The deduced masses of the $\Lambda_c$-baryons agree average more then 98.8 percent. The deduced masses of the $\Xi_c$-baryons consistent with the corresponding experimental results more then average 98.4 percent. The deduced masses of the $\Sigma_c$-baryons agree average more then 96.8 percent the experimental masses. The deduced mass of the $\Omega_c$-baryon consists with the experimental result 96.6 percent.

### 6.9 In Summary

Table 15 shows that the all deduced quantum numbers (I, S, C, B, Q) of the seven most important baryons (N(939), $\Lambda$(1119), $\Sigma$(1209), $\Xi$(1299), $\Omega$(1659), $\Lambda_c$(2264) and $\Lambda_b$(5554)) are exactly the same as the experimental results. Their masses are consistent with experimental results more than 98.5 percent using only three constant parameters ($M_0 =939$ Mev, $\alpha = 360$ Mev and $\beta = 115$ Mev ).

The all deduced quantum numbers (I, S, C, B, Q, J and P) of all observed 116 baryons are completely the same as corresponding experimental result. The masses of the 68 established baryons (10 $\Delta$-baryons, nine $\Sigma$-baryons, 15 N-baryons, 14 $\Lambda$-baryons, five $\Xi$-baryons, two $\Omega$-baryons, three $\Lambda_c$-baryons, three $\Sigma_c$-baryons, five $\Xi_c$-baryons, one $\Omega_c$-baryon and one $\Lambda_b$-baryon) agree with more than the average 98.3 percent with the experimental results as appearing in Table 23. The masses of all observed 116 baryons agree with more than the average 98 percent with the experimental results as appearing in Table 23.

| Baryon | Established | Average of $\Delta M$ | Max. $\Delta M$ | Total number | Total Average |
|--------|-------------|------------------------|-----------------|--------------|---------------|
| $\Delta(M)$ | 10 | 2.19 | 5.4 | 22 | 2.44 |
| $\Sigma(M)$ | 9 | 1.23 | 3.5 | 23 | 1.77 |
| $N(M)$ | 15 | 1.87 | 3.7 | 23 | 1.81 |
| $\Lambda(M)$ | 14 | 1.27 | 2.6 | 18 | 1.55 |
| $\Xi(M)$ | 5 | 1.44 | 2.4 | 11 | 1.43 |
| $\Omega(M)$ | 2 | 1.20 | 1.6 | 4 | 1.15 |
| $\Lambda_c(M)$ | 3 | 1.37 | 1.8 | 5 | 1.16 |
| $\Xi_c(M)$ | 5 | 1.54 | 2.6 | 5 | 1.54 |
| $\Sigma_c(M)$ | 3 | 3.2 | 5.1 | 3 | 3.2 |
| $\Omega_c(M)$ | 1 | 3.4 | 3.4 | 1 | 3.4 |
| $\Lambda_b(M)$ | 1 | 1.21 | 1.21 | 1 | 1.21 |
| All Baryons | 68 | 1.68 | 5.4 | 116 | 1.82 |

Inside Table 23, the term “Established” means the numbers of the experimental established
baryons. The term “Max. ∆M” is the largest ∆M of all established baryons belong to one kind. The term “Total Number” is the numbers of the all observed baryons. The term “Total average” is the average of $\frac{\Delta M}{M}$ of all observed baryons.

All 116 observed baryons (22 Δ-baryons, 23 Σ-baryons, 23 N-baryons, 18 Λ-baryons, 11 Ξ-baryons, four Ω-baryons, five Λc-baryons, three Σc-baryons, five Ξc-baryons, one Ωc-baryon and one Λb-baryon) have been deduced. All deduced baryons with $M \leq 2019$ Mev have been found in experiments (see Table 15-22) except the Λ-baryons with $J = \frac{1}{2}$. All deduced 116 baryons have completely the same quantum numbers (I, S, C, B, Q, J and P) as the corresponding experimental baryons. The deduced masses of the 116 baryons agree average more than 98 percent with experimental result. These results are the results of the expanded Schrodinger equation and BCCP condition using only four constant parameters ($\alpha = 360$ Mev, $M_0 = 939$ Mev, $\beta = 115$ Mev and $f = 0$ for inside axis or $f = 1$ for the axes on the surface of the first Brillouin zone Fig. 1). There is not any established baryon outside the deduced baryons. Considering the large widths of the baryons, we have to say the deduced baryon masses consistent well with the experimental results.

7 Deducing the Masses of the Quarks from the Masses of the Most Important Deduced Baryons

In the first postulate, we assumed that the five quarks have the same unknown large bare mass $m_q$. When a baryon excite from the vacuum state into an energy band, it becomes an observable baryon with the same quantum numbers as in the vacuum state as appearing in Table 2 or 3, but different physical masses from its bare mass. The three quarks inside the baryons are excited from vacuum state into the observable physical state with the same quantum numbers as in the vacuum (Table 14.1 of [9]), but different observable masses from the bare mass $m_q$. The mass of a quark inside a observable hadron relating to the laboratory reference system is the “quark mass.” The “quark masses” are different values for different quarks since they are inside different baryons with different energies.

Estimating the “quark masses” is a difficult job since all quarks are confined inside hadrons. We can estimate the quark masses from the masses of the baryons composed of the quarks. In order to distinguish the masses of u-quark with d-quark, we use the measured masses of proton and neutron. For the five most important baryons (proton, neutron, Λ(1119), Λc(2264) and Λb(5554)), there are five quark mass equations:

\begin{align}
    m_u + m_d + m_u + E_{bin}(uud) &= 938, \\
    m_u + m_d + m_d + E_{bin}(udd) &= 940, \\
    m_u + m_d + m_s + E_{bin}(uds) &= 1119, \\
    m_u + m_d + m_c + E_{bin}(udc) &= 2264, \\
    m_u + m_d + m_b + E_{bin}(udb) &= 5554.
\end{align}
Where $E_{\text{bin}}(uud)$ is the binding energy of three quarks inside the proton, $E_{\text{bin}}(udd)$ is the binding energy of three quarks inside the neutron, $E_{\text{bin}}(uds)$ is the binding energy of three quarks inside $\Lambda(1119)$, . . . Using the perfect $SU(5)_f$ symmetry, we can have

$$E_{\text{bin}}(uud) = E_{\text{bin}}(udd) = E_{\text{bin}}(uds) = E_{\text{bin}}(udc) = E_{\text{bin}}(udb).$$  \hspace{1cm} (125)

From the above six equations \((120)-(125)\), we can get

$$m_d - m_u = 2, m_s - m_u = 181, m_c - m_u = 1326, m_b - m_u = 4616. $$ \hspace{1cm} (126)

Assuming $E_{\text{bin}}(uud) = E_{\text{bin}}(udd) = E_{\text{bin}}(uds) = \ldots = 0$ (it is impossible!), we can obtain the masses of the quarks:

$$m_u = 312, \ m_d = 314, \ m_s = 493, \ m_c = 1638, \ m_b = 4928. $$ \hspace{1cm} (127)

Since the baryons have never been divided into free quarks, the binding energies $E_{\text{bin}}(uud)$, $E_{\text{bin}}(udd)$, $E_{\text{bin}}(uds)$, $E_{\text{bin}}(udc)$ and $E_{\text{bin}}(udb)$ shall be very big. If making strong stable baryons, the masses of quarks will be much larger than the values appearing in Table 24:

**Table 24. The quark masses for various possible binding energies**

| $E_{\text{bind}}$ (Mev) | $m_u$ | $m_d$ | $m_s$ | $m_c$ | $m_b$ |
|--------------------------|-------|-------|-------|-------|-------|
| 0                        | 312   | 314   | 493   | 1638  | 4928  |
| 30000                    | 10312 | 10314 | 10493 | 11638 | 14928 |
| 300000                   | 100312| 100314| 100493| 101638| 104928|
| 3000000                  | 1000312| 1000314| 1000493| 1001638| 1004928|
| 30000000                 | 10000312| 10000314| 10000493| 10001638| 10004928|

Historically, the first determinations of quark masses were performed using quark models. The resulting masses only make sense in the limited context of a particular quark model, and cannot be related to the quark mass parameters of the standard Model. Similarly, these masses of the quarks are deduced using the quark model also. They are in the limited context of the quark model. For the quark model with binding energy $E_{\text{bind}} = 300000$ Mev, $m_d = 100312$, $m_u = 100314$, $m_s = 100493$, $m_c = 101653$ and $m_b = 104913$. The deduced quark masses are really different as formula \((126)\). The mass differences of the five quarks are independent from the binding energies. They are very small constants. Comparing with the huge quark masses, they can neglect. The masses of the quarks are almost the same for the five quarks (see Table 24) since the binding energies very big to confine the quarks. Thus the SU(3), SU(4) and SU(5) are really perfect symmetry groups.

Using the deduced masses in Table 23 (laboratory system) of the quarks, we can deduced the quark mass parameters of the standard model\[19\]. The mass parameters describe the scattering and the decays of the quarks. Since they are measured in mass center inside hadrons, if subtracting the binding energies $E_{\text{bind}}$ and the energy differences between the mass center system and the laboratory system from the quark masses, we will get the mass parameters of the quarks:

$$m(q)_{\text{par.}} = m_q - E_{\text{bind}} - DE$$ \hspace{1cm} (128)

where $DE$ phenomenologically $= \frac{1}{3} M_p - 2.7 + 100|S + C + B| = 310 + 100|S + C + B|$. Using \((128)\), we find
\[ m(u)_{\text{par.}} = 2, \ m(d)_{\text{par.}} = 4, \ m(c)_{\text{par.}} = 83, \ m(c)_{\text{par.}} = 1233, \ m(b)_{\text{par.}} = 4503. \quad (129) \]

Comparing the deduced mass parameters in \( (129) \) with experimental results in \( [19] \), we find the deduced mass parameters of the standard model agree well with experimental results:

| Table 25. The comparison of the quark mass parameters |
|-----------------------------------------------------|
| Mass Parameter | \( m(u)_P \) | \( m(d)_P \) | \( m(s)_P \) | \( m(c)_P \) | \( m(b)_P \) |
| Deduced M-Par. | 2 | 4 | 83 | 1233 | 4503 |
| Measured M-Par. | 1.5 to 3.0 | 3 to 7 | 95±25 | 1250 ± 90 | 4200 ± 70 |

Table 25 and formula \( (129) \) not only use new method to deduce the mass parameters, but also give the relationship between the masses and the mass parameters of the quarks. Using the relationship, we can deduce the masses of quarks from the corresponding mass parameters and we can deduce the quark mass parameters from the corresponding quark masses.

8 Discussion

1. Although the bare mass \( M_b \) of the b-particle is unknown, we have the constant \( \alpha = \frac{h^2}{2\lambda a^2} = 360 \text{ Mev} \) from \( (28) \) and \( (62) \). Thus we can get

\[ M_b = \frac{h^2}{2\alpha a^2}. \quad (130) \]

Here the periodic constant \( a \) of the body-centred cubic periodic symmetry condition, \( a \sim 10^{-23} \text{ m} \). The bare mass of the b-particle will be

\[ M_b \sim \frac{h^2}{2\alpha a^2} \sim 2.14 \times 10^{19} \text{ (Mev)} \gg M_p = 938 \text{ (Mev)}. \quad (131) \]

The estimated bare masses \( M_b \sim 2.14 \times 10^{19} \text{ (Mev)} \gg M_p \). Thus we can use the Schrodinger equation \( (14) \) instead of the Dirac equation to deduce the baryon spectrum. In fact the results of this paper in Table 15-Table 22 also show that this is a very good approximation.

2. The experimental facts of the quark confinement and the stability of the proton (proton \( p \) with \( \tau \) about \( 10^{30} \text{ years} \) inevitably result in huge binding energies of the three quarks in baryons. According to the \( E = MC^2 \), the huge binding energies and the small masses of baryons inevitably lead to huge quark masses. The huge masses of quarks and small mass differences of baryons inevitably bring about that differences of the quark masses can be neglected as appearing in Table 24. Thus, the new experimental facts have already shown clearly that the \( SU(N) \) \( (N = 3, 4 \text{ and } 5) \) symmetry groups are perfect since the five quark masses are essentially the same.

3. In the deduction of quantum numbers, this paper does not make any approximation. The degeneracy \( d \) of the energy bands, the rotary fold number \( R \) of the symmetry axis and the index number \( \vec{n} \) of the energy bands are exactly the results of the Schrodinger equation with BCCP boundary condition. The isospins of the baryons are from the energy band degeneracy \( d \) \( (64) \); the
strange numbers of the baryons are from the R of the symmetry axes \((69)\); the charmed number and bottom number are from the fold numbers R and the energy band index number \(\vec{n} (86) and (87)\). Thus the strange number, the charmed number and the bottom number of the energy bands are the products of the BCCP symmetry and the expanded Schrodinger equation. In fact, the quantum numbers originate from the body-centred cubic periodic symmetries.

4. Although there are infinite high energy bands in theory, this case means there will be infinite high baryons in theory. In fact, however, since the density of the energy bands is higher and higher (the distances between energy bands is smaller and smaller) as the energy increases and the experimental width of the baryons are larger and larger as the energy grows, so that many energy bands will mix together to form the physical back ground in the higher energy case. For example, for N-baryons, at \(M = 2200\) Mev, \(N(2190) \Gamma = 500\) Mev, \(N(2220) \Gamma = 400\) Mev and \(N(2250) \Gamma = 500\) Mev; but the distances between the energy bands are about 180–360 Mev in the theory. Thus there are the upper limits of various observable baryons that are dependent on the experimental technique and equipments. As improvements of the experimental technique and equipments occur, many new baryons might be discovered in the future.

5. This paper can help us to understand the instability of the baryons since all energy bands (baryons) are not the eigenstates of angular momentum and all baryons are the excited states of the same b-particle. The higher excited states decay into lower energy states and the energy excited states decay into the ground state is a natural law in physics. Thus the baryons do so.

6. How about the nature at distance scale larger than \(10^{-18} m\)? Since the distance scale is larger than the periodic constant “a” \((\sim 10^{-23})\) by more than \(10^5\) times, physicists using the nature at distance scale larger than \(10^{-18} m\) will not see the body-centred cubic symmetries of the local spaces around baryons. Thus there is not the BCCP symmetry in their physical theory–the standard model.

9 Conclusions

1. There is a new symmetry beyond the standard model-the body-centred cubic periodic symmetry (BCCP) with a periodic constant “a” \(\sim 10^{-23} m\) in the local space around baryons. The masses and the space quantum numbers (L, J and P) of baryons result from the BCCP symmetries.

2. The \(SU(N)_f\) (\(N = 3, 4\) and 5) symmetry groups not only really exist, but also they are perfect. The \(SU(N)_f\) groups give the intrinsic quantum numbers (I, S, C, B, Q and spin s) of the baryons.

3. The free b-particle expanded Schrodinger equation with large bare mass \(M_b\) is a new motion equation beyond the standard model. It is available to the distance scale \(\sim 10^{-23} m\) and BCCP condition. Using it and the BCCP symmetry condition, this paper deduces the full (including M, I, S, C, B, Q, J and P) baryon spectrum (116 observed baryons, 68 established baryons) with only four constant parameters \(M_0 = 939\) Mev, \(\alpha = 360\) Mev, \(\beta = 115\) Mev and \(f = 1\) or 0). The deduced quantum numbers are consistent with experimental results completely; the deduced masses agree with the experimental results average more than average 98 percent the experimental results.

4. This paper has deduced the masses of the five quarks (u, d, s, c and b), from the masses of the most important deduced baryons, in terms of the qqq baryon model of the quark model.
The deduced quark masses are huge. The huge masses not only provide a necessary condition for the quark confinement, but also provide a solid physical foundation of perfect $SU_f(N)$ symmetry. The huge mass quarks stabilize the proton with the lifetime $10^{30}$ years. This paper also deduces the mass parameters (of the standard model) which agree with the measured results as appearing in Table 25.

5. The experimental large width of baryons (special N-baryons) result from the several same kind of baryons with the same I, S, C, B, Q, J and P as well as similar masses (see Table 18).

10 Prediction—“Zeeman Effect” of Baryons

There are many baryons with large widths ($\Gamma$) as 300 Mev, 400 Mev, 500 Mev and 650 Mev. Table 18 shows that the big widths of the N-baryons result from the composition of several nearby energy bands with the same I, S, C, B, Q, J and P. There are several baryons that are the composition baryons such as $\Delta$-baryons in Table 16, $\Sigma$-baryons in Table 17, N-baryons in Table 18 and $\Lambda$-baryons in Table 19. This phenomena we call the “Particle Zeeman Effect.” For example, the baryon $N(2190)_{\frac{7}{2}^-}$ with width $\Gamma = 500$(Mev) is composed of $[N(1839) + 4N(1929) + 3N(2019) + 3N(2199) + 3N(2379) + 2N(2649)]_{\frac{7}{2}^-} \rightarrow N(2165)_{\frac{7}{2}^-}$ with $\Gamma \geq [(2379-1929)] = 450$ Mev. There are 16 energy bands with fluctuation in a region about 500 Mev. They are difficultly separated by experiments. For other example, $N(1720)_{\frac{3}{2}^+}$ with $\Gamma = 200$ Mev and $\frac{3}{2}^+$, however, composed of only two energy bans: $\frac{1}{2}[N(1659)+N(1839)]_{\frac{3}{2}^+} = N(1749)_{\frac{3}{2}^+}$ with $\Gamma = 1839 - 1659 = 180$ (Mev). Thus we suggest that using experiments to separate the $N(1720)$-baryon $\frac{3}{2}^+$, we will observe one $N(1659)_{\frac{3}{2}^+}$ and one $N(1839)_{\frac{3}{2}^+}$. If we actually get the two N-baryons with $JP = \frac{3}{2}^+$, this will show that the BCCP symmetry really exists.

Acknowledgments

I sincerely thank Professor Robert L. Anderson for his valuable advice. I acknowledge my indebtedness to Professor D. P. Landau for his help also. I would like to express my heartfelt gratitude to Dr. Xin Yu. I sincerely thank Professor Kang-Jie Shi for his important advice. I wholeheartedly thank Professor Han-yin Guo, Professor Zhan Xu, Professor Xing-chang Song for their useful advice.
A The Irreducible Representations and Their Compatibility Relations

Table A1. The irreducible representations and basis functions of $O_h$

| $O_h$     | Basis function                                      | Deg. | L  |
|-----------|-----------------------------------------------------|------|----|
| $\Gamma_1, H_1, M_1$ | $1$                                                   | 1    | 0  |
| $\Gamma_2, H_2, M_2$ | $x^4(y^2 - z^2) + y^4(z^2 - x^2) + z^4(x^2 - y^2)$ | 1    | 6  |
| $\Gamma_{12}, H_{12}, M_{12}$ | $z^2 - \frac{1}{2}(x^2 + y^2), (x^2 - y^2)$ | 2    | 2  |
| $\Gamma_{15}, H_{15}, M_{15}$ | $(x, y, z)$                                             | 3    | 1  |
| $\Gamma_{25}, H_{25}, M_{25}$ | $x(y^2 - z^2), y(z^2 - x^2), z(x^2 - y^2)$ | 3    | 3  |
| $\Gamma_1', H_1', M_1'$ | $xyz[x^4(y^2 - z^2) + y^4(z^2 - x^2) + z^4(x^2 - y^2)]$ | 1    | 9  |
| $\Gamma_2', H_2', M_2'$ | $xyz$                                                  | 1    | 3  |
| $\Gamma_{12}', H_{12}'', M_{12}'$ | $(xyz[z^2 - \frac{1}{2}(x^2 + y^2)], xyz(x^2 - y^2))$ | 2    | 5  |
| $\Gamma_{15}', H_{15}'', M_{15}'$ | $xy(x^2 - y^2), yz(y^2 - z^2), zx(z^2 - x^2)$ | 3    | 4  |
| $\Gamma_{25}', H_{25}'', M_{25}'$ | $(xy, yz, zx)$                                         | 3    | 2  |

Table A1 is copied from Table IV of [15].

The wave functions at the point P have the group p symmetry. Irreducible representations and basis functions appear in Table A2. The group of P is the tetrahedral group. It is interesting to see the way in which the representations at $\Gamma$ combine to give those at P: $P_1$ contains functions belonging to $\Gamma_1$ and $\Gamma_2'$; $P_2$ contains those belonging to $\Gamma_2$ and $\Gamma_1'$; $P_3$ contains $\Gamma_{12}$ and $\Gamma_{12}'$; $P_4$ contains $\Gamma_{15}$ and $\Gamma_{25}'$; $P_5$ contains $\Gamma_{25}$ and $\Gamma_{15}'$ [15].

Table A2. The irreducible representations and basis Functions of the Group P

| $I_d$ | Basis function                                      | Deg. | L  |
|-------|-----------------------------------------------------|------|----|
| $P_1$ | 1, xyz                                              | 1    | 0  |
| $P_2$ | $x^4(y^2 - z^2) + y^4(z^2 - x^2) + z^4(x^2 - y^2)$ | 1    | 6  |
| $P_3$ | $z^2 - \frac{1}{2}(x^2 + y^2), (x^2 - y^2); (xyz[z^2 - \frac{1}{2}(x^2 + y^2)], xyz(x^2 - y^2))$ | 2    | 5  |
| $P_4$ | x, y, z; xy, yz, zx                                 | 3    | 1, 2 |
| $P_5$ | $x(y^2 - z^2), y(z^2 - x^2), z(x^2 - y^2)$          | 3    | 3  |

Table A2 is copied from Table V of [15].

The wave functions at the point N have the symmetries of the group N.
Table A3. The irreducible representations and basis functions of the Group N

| $D_{2h}$ | $E$ | $C_4$ | $C_1$ | $C_2$ | $JC_4$ | $JC_1$ | $JC_2$ | B. function | d | L  |
|----------|-----|-------|-------|-------|--------|--------|--------|-------------|---|----|
| $N_1$    | 1   | 1     | 1     | 1     | 1      | 1      | 1      | 1, xy      | 1 | 0,2 |
| $N_2$    | 1   | -1    | 1     | -1    | 1      | -1     | 1      | z(x-y)     | 1 | 2  |
| $N_3$    | 1   | -1    | 1     | 1     | -1     | -1     | 1      | z(x+y)     | 1 | 2  |
| $N_4$    | 1   | 1     | -1    | -1    | 1      | 1      | -1     | $x^2 - y^2$| 1 | 2  |
| $N'_1$   | 1   | 1     | -1    | 1     | 1      | 1      | -1     | x+y        | 1 | 1  |
| $N'_2$   | 1   | 1     | -1    | -1    | 1      | 1      | 1      | $z(x^2 - y^2)$ | 1 | 2 |
| $N'_3$   | 1   | 1     | -1    | -1    | -1     | 1      | 1      | z          | 1 | 1  |
| $N'_4$   | 1   | -1    | -1    | 1     | 1      | 1      | 1      | x-y        | 1 | 1  |

Table A3 is copied from Table VI of [15] and Table 29 of [13].

Table A4. The irreducible representations and basis functions of the group $\Delta$

| $C_4$ | $E$ | $C_2^4$ | $C_4$ | $JC_4^2$ | $JC_2$ | B. function | Deg. | L |
|-------|----|---------|-------|----------|--------|-------------|------|---|
| $\Delta_1$ | 1 | 1     | 1     | 1       | 1      | 1           | 1    | 0 |
| $\Delta_2$ | 1 | 1     | -1    | 1       | -1     | $(x^2 - y^2)$ | 1    | 2 |
| $\Delta'_2$ | 1 | 1     | -1    | -1     | 1      | xy         | 1    | 2 |
| $\Delta'_1$ | 1 | 1     | 1     | -1     | -1     | xy(x^2 - y^2) | 1    | 4 |
| $\Delta_5$ | 2 | -2    | 0     | 0       | 0      | (x, y)     | 1    | 1 |

Table A4 is copied from Table 22 of [13].

Table A5. The irreducible representations and basis functions of the Group $\Lambda$

| $C_3v$ | $E$ | 2$C_3$ | $3\sigma_v$ | B. function | Deg. | L |
|--------|----|--------|-------------|-------------|------|---|
| $\Lambda_1 or F_1$ | 1 | 1     | 1            | 1           | 1    | 0 |
| $\Lambda_2 or F_2$ | 1 | 1     | -1           | xy(x-y), yz(y-z), zx(z-x) | 1 | 4 |
| $\Lambda_3 or F_3$ | 2 | -1    | 0            | [(x - z), (y-z)] | 1    | 1 |

Table A5 is copied from Table 23 of [13] and Table 7.7 of [16].

Table A6. The Irreducible Representations and Basis Functions of the Groups $\Sigma$, D and G

| $C_{2v}$ | $E$ | $C_2$ | $JC_4^2$ | $JC_2$ | B. function | Deg. | L |
|----------|----|-------|----------|--------|-------------|------|---|
| $\Sigma_1, D_1, G_1$ | 1 | 1     | 1        | 1      | 1           | 1    | 0 |
| $\Sigma_2, D_2, G_2$ | 1 | 1     | -1       | -1     | z(x-y)     | 1    | 2 |
| $\Sigma_3, D_3, G_3$ | 1 | -1    | -1       | 1      | z          | 1    | 1 |
| $\Sigma_4, D_4, G_4$ | 1 | -1    | 1        | -1     | x-y        | 1    | 1 |

Table A6 is copied from Table 24 of [13] and Table 7.7 of [16].

The compatibility relations between states along symmetry axes and the states at the end-points appear in the following tables:
Table A7. The compatibility relations of the $\Gamma$-$\Delta$, $\Gamma$-$\Lambda$ and $\Gamma$-$\Sigma$ axis

| $\Gamma$-Point($O_h$) | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_{12}$ | $\Gamma_{15}$ | $\Gamma_{25}$ |
|-----------------------|------------|------------|--------------|--------------|--------------|
| $\Delta$ - Axis($C_{4v}$) | $\Delta_1$ | $\Delta_2$ | $\Delta_1 \Delta_2$ | $\Delta_1 \Delta_5$ | $\Delta_2 \Delta_5$ |
| $\Lambda$ - Axis($C_{3v}$) | $\Lambda_1$ | $\Lambda_2$ | $\Lambda_1 \Lambda_3$ | $\Lambda_2 \Lambda_3$ | $\Lambda_1 \Lambda_3$ |
| $\Sigma$ - Axis($C_{2v}$) | $\Sigma_1$ | $\Sigma_4$ | $\Sigma_1 \Sigma_4$ | $\Sigma_2 \Sigma_3 \Sigma_4$ | $\Sigma_1 \Sigma_2 \Sigma_3$ |

Table A7 is copied from Table 26 of [13] and Table 8.5 of [16].

Table A8. The compatibility relations of the $\Gamma$-$\Delta$, and $H$–$F$-axis

| $\Gamma$-Point($O_h$) | $\Gamma'_1$ | $\Gamma'_2$ | $\Gamma'_{12}$ | $\Gamma'_{15}$ | $\Gamma'_{25}$ |
|-----------------------|------------|------------|--------------|--------------|--------------|
| $\Delta$ - Axis($C_{4v}$) | $\Delta_1$ | $\Delta_2$ | $\Delta_1 \Delta_2$ | $\Delta_1 \Delta_5$ | $\Delta_2 \Delta_5$ |
| $\Lambda$ - Axis($C_{3v}$) | $\Lambda_1$ | $\Lambda_2$ | $\Lambda_1 \Lambda_3$ | $\Lambda_2 \Lambda_3$ | $\Lambda_1 \Lambda_3$ |
| $\Sigma$ - Axis($C_{2v}$) | $\Sigma_2$ | $\Sigma_3$ | $\Sigma_1 \Sigma_3 \Sigma_4$ | $\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4$ | $\Sigma_1 \Sigma_2 \Sigma_3$ |

Table A8 is copied from Table 8.5 of [16].

Table A9. The compatibility relations of the point $M$–the G-axis

| M-Point($O_h$) | $M_1$ | $M_2$ | $M_{12}$ | $M_{15}$ | $M_{25}$ |
|----------------|------|------|--------|--------|--------|
| $G$ - axis($C_{2v}$) | $G_1$ | $G_4$ | $G_1 G_4$ | $G_2 G_3 G_4$ | $G_1 G_2 G_3$ |

Table A9 is copied from Table 8.5 of [16].

Table A10. The compatibility relations of P-D, P-F and P-$\Lambda$

| $T_d$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ |
|-------|------|------|------|------|------|
| $C_{3v}$ | $A_1$ | $A_2$ | $A_3$ | $A_1 A_3$ | $A_2 A_3$ |
| $C_{3v}$ | $F_1$ | $F_2$ | $F_3$ | $F_1 F_3$ | $F_2 F_3$ |
| $C_{2v}$ | $D_1$ | $D_2$ | $D_1 D_2$ | $D_1 D_3 D_4$ | $D_2 D_3 D_4$ |

Table A10 is copied from Table 8.5 of [16].

59
Table A11. The compatibility relations of N-D, N-G and N-Σ

|       | N_1 | N_2 | N_3 | N_4 | N'_1 | N'_2 | N'_3 | N'_4 |
|-------|-----|-----|-----|-----|------|------|------|------|
| D_{2h} | Σ_1 | Σ_2 | Σ_3 | Σ_4 | Σ_1 | Σ_2 | Σ_3 | Σ_4 |
| C_{2v} | D_1 | D_3 | D_4 | D_4 | D_2 | D_4 | D_1 | D_3 |
| C_{2v} | G_1 | G_3 | G_1 | G_4 | G_2 | G_4 | G_3 | G_1 |

Table A11 is copied from Table 8.5 of [16].

B The Angular Momenta J and Parities P of the Energy Bands

From Fig. 2–Fig. 8 and Table A1–Table A11 as well as the formulae [88]–[96], we can find the orbit angular momenta L, the total angular momenta J and the parities P of the energy bands on the Δ-axis, the Λ-axis, the Σ-axis, the D-axis, the F-axis and the G-axis as appearing in Table B1–Table B6. Since the compatibility relations (in Table 7, Table 8 and Table 9) cannot completely determine the representations at a symmetry point correspondent to an energy band, generally, we choose the representation with lower L value for the lower energy band; and we choose the representation with higher L value for higher energy band.

choosing the representation with lower L, for lower energy band.  \hfill (132)

This is a useful formula for choosing a correct representation.

B.1 The J and P of the energy bands on Δ-axis

There are many four-fold energy bands in Fig. 2. For a four-fold energy band, there are always two irreducible representations in the same band since the highest fold of the representations is only three.

For these complex cases, in order to get a definite result, we assume that if the two representations have different orbit angular momenta (L) and parities (P), the parity (P) of the energy band is determined by the representations with higher fold (most three-fold). The total angular momentum J is the common J values of the two representations in ground state. For example, the energy band with M = 1299 and \(\vec{n} = (101,011,-101,0-11)\), has \(H_{15} (3 \text{ fold}) + \frac{1}{2}H_{12} (1 \text{ fold})\). The P = "+" is determined by \(H_{15}\) from [90]. \(H_{15}\) has \(s = \frac{1}{2}\) from [88] and \(L = 1\) from Table A1. Thus \(J = \frac{1}{2}, \frac{3}{2}\). Similarly, \(\frac{1}{2}H_{12}\) has \(s = \frac{1}{2}\) and \(J = \frac{3}{2}, \frac{5}{2}\). Therefore the two representations have a common \(J^p = \frac{3}{2}^+\). In exited state, however, the angular momentum L is determined by the representations with higher fold (most three-fold). Since a three-fold single group representation correspond to a four-fold double representation and a two-fold double representation, excited band may excite the four-fold double representation [18]. Such as, the band with M = 2019 and \(\vec{n} = (112,1-12,-112,-1-12)\), has the representation \(H_{15}\) and \(H'_2\). The L value and P are determined by \(H_{15}\) as shown in Table B1. Similarly, we can find the J and P for the energy bands with M = 1659 and \(\vec{n} = (10-1,-1-10,01-1,0-1-1)\); M = 1659 and \(\vec{n} = (10-1,-10-1,01-1,0-1-1)\). The results appearing in Table B1.
For the band with $M = 2379$ and $\vec{n} = (200, -200, 020, 0, -20)$, there are two representations $\Gamma_{12}$ and $\frac{2}{3}\Gamma_{15}$. The two-fold single representation $\Gamma_{12}$ corresponding to a four-fold double representation, and the $\frac{2}{3}\Gamma_{15}$ corresponding to a four-fold double representation also [18]. From Table A1 and (93), for $\frac{2}{3}\Gamma_{15}$, $J^P = \frac{5}{2}^+, \frac{7}{2}^+, \frac{9}{2}^+, \frac{11}{2}^+$ from (96); for $\Gamma_{12}$, $J^P = \frac{5}{2}^-, \frac{7}{2}^-, \frac{9}{2}^-, \frac{11}{2}^-$ from (96).

There are three single bands: $M = 939$, $\vec{n} = (0, 0, 0) \rightarrow N(939)$-ground baryon from (103); $M = 1299$, $\vec{n} = (0, 0, 2) \Lambda(1299)$ is not ground baryons, the representation is $H_1$, $L = 0$ from Table A1, $J = \frac{1}{2}$, $P = "-"$ from (96); $M = 2379$, $\vec{n} = (0, 0, -2)$, $A_c$ is a ground baryon, the representation $\Delta_1 \rightarrow J^P = \frac{1}{2}^+$ from (95).

From Fig.2, Table A1, Table A4, Table A7 and Table A8, we can deduce the total angular momenta $J$ and parities $P$ of the energy bands on the $\Delta$-axis as appearing in Table B1.
Table B1. The J and P of the energy bands on the \( \Delta \)-axis

| M    | \((n_1, n_2, n_3)\) | s | Repres. | L | J(1/2) | J(3/2) | P  |
|------|---------------------|---|---------|---|---------|---------|----|
| 939  | (0, 0, 0)           | \(\frac{1}{2}\) | \(\Delta_1\) | 0* | \(\frac{1}{2}\) | \(\frac{1}{2}\) | +  |
| 1299 | (101,-101, 011,0-11)| \(\frac{1}{2}\) | \(H_{15}\) | 1  | \(\frac{3}{2}\) |          | +  |
| 1299 | (0, 0, 2)           | \(\frac{1}{2}\) | \(H_1\)  | 0  | \(\frac{3}{2}\) | \(\frac{5}{2}\) | -  |
| 1659 | (110,1-10, -110,-1-10)| \(\frac{1}{2}, \frac{3}{2}\) | \(\Gamma_{25}\) | \(\frac{1}{2}\) | \(\Gamma_{12}\) | 2  | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | -  |
| 1659 | (10-1,-10-1, 01-1,0-1-1)| \(\frac{1}{2}, \frac{3}{2}\) | \(\Gamma_{15}\) | \(\frac{1}{2}\) | \(\Gamma_{12}\) | 1  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | +  |
| 2019 | (112,1-12, -112,-1-12)| \(\frac{1}{2}, \frac{3}{2}\) | \(H_{15}\) | \(\frac{1}{2}\) | \(H_2\)  | 1  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | +  |
| 2379 | (200,-200, 020,0-20)| \(\frac{1}{2}, \frac{3}{2}\) | \(\Gamma_{12}\) | \(\frac{1}{2}\) | \(\Gamma_{15}\) | 2  | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | -  |
| 2379 | (0, 0, -2)          | \(\frac{1}{2}\) | \(\Delta_1\) | 0* | \(\frac{1}{2}\) | \(\frac{1}{2}\) | +  |
| 2739 | (121,1-21, -121,-1-21, 211,2-11, -211,-2-11)| \(\frac{1}{2}, \frac{3}{2}\) | \(H_{15}\) | \(\frac{1}{2}\) | \(H_{12}\) | 1  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | +  |
| 2739 | (121,1-21, -121,-1-21)| \(\frac{1}{2}, \frac{3}{2}\) | \(H_{15}\) | \(\frac{1}{2}\) | \(H_{12}\) | 2  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | +  |
| 2739 | (211,2-11, -211,-2-11)| \(\frac{1}{2}, \frac{3}{2}\) | \(H'_{25}\) | \(\frac{1}{2}\) | \(H_{12}\) | 3  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | -  |
| 2739 | (202,-202, 022,0-22)| \(\frac{1}{2}, \frac{3}{2}\) | \(H_{25}\) | \(\frac{1}{2}\) | \(H_{12}\) | 4  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | +  |
| 2739 | (013,0-13, 103,-103)| \(\frac{1}{2}, \frac{3}{2}\) | \(H_{15}\) | \(\frac{1}{2}\) | \(H_{12}\) | 5  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | -  |
| 3099 | (12,-1,2-1, -12,-1,1-2-1)| \(\frac{1}{2}, \frac{3}{2}\) | \(\Gamma'_{25}\) | \(\frac{1}{2}\) | \(\Gamma_{12}\) | 2  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | -  |
| 3099 | (21,1,-2-1, -21,-1,2-11)| \(\frac{1}{2}, \frac{3}{2}\) | \(\Gamma'_{15}\) | \(\frac{1}{2}\) | \(\Gamma_{12}\) | 4  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | -  |
| 3099 | (11,-2,1-1-2, -11,-2,-1-1-2)| \(\frac{1}{2}, \frac{3}{2}\) | \(\Gamma_{25}\) | \(\frac{1}{2}\) | \(\Gamma_{2}\)  | 3  | \(\frac{1}{2}, \frac{3}{2}\) | \(\frac{3}{2}, \frac{5}{2}\) | \(\frac{7}{2}\) | \(\frac{7}{2}\) | +  |

0* means L = 0 from Table A4.
### B.2 The J and P of the energy bands on Λ-axis

From Fig.3, Table A1, Table A2, Table A7 and Table 10, we can get the irreducible representations of the energy bands at the lowest energy point. Using Table A1, Table A2 and (92)–(96), we can find the J and P of the energy bands on Λ-axis as appearing in Table B2. At $M_{Fig} = 1929$ and 2649, the band with P-representations of the P-group (Table A2). $P_4$ contains functions belonging to $\Gamma_{15}$ and $\Gamma_{25}'$, $P_5$ functions belonging to $\Gamma_{25}$ and $\Gamma_{15}'$ [15].

| M (Mev)  | $(n_1, n_2, n_3)$ | $s$ | Repres. | L | J(1/2) | J(3/2) | P  |
|----------|------------------|-----|---------|---|--------|--------|----|
| 939      | (0, 0, 0)        | $\frac{1}{2}$ | $\Lambda_1$ | 0** | $\frac{1}{2}$ | $\frac{3}{2}$ | +  |
| 1209     | (011,101,110)    | $\frac{1}{2}$ | $P_4$ | 1 | $\frac{3}{2}$ | $\frac{5}{2}$ | +  |
| 1659     | (1-10,-110,01-1, 0-11,10-1,-101) | $\frac{1}{2}$ | $\frac{3}{2}$ | $\Gamma_1\Gamma_{12}\Gamma_{25'}$ | +  |
| 1659     | (1-10,-110,01-1) | $\frac{1}{2}$ | $\frac{3}{2}$ | $\Gamma_{25'}$ | 2 | $\frac{5}{2}$ | $\frac{7}{2}$ | -  |
| 1659     | (0-11,10-1,-101) | $\frac{1}{2}$ | $\frac{3}{2}$ | $\Gamma_{12}\Gamma_1$ | 2 | $\frac{5}{2}$ | $\frac{7}{2}$ | -  |
| 1659     | (-1-10,-10-1,0-1-1) | $\frac{1}{2}$ | $\frac{3}{2}$ | $\Gamma_{15}$ | 1 | $\frac{5}{2}$ | $\frac{7}{2}$ | +  |
| 1929     | (020,002,200)    | $\frac{1}{2}$ | $\frac{3}{2}$ | $P_4$ | 1 | $\frac{5}{2}$ | $\frac{7}{2}$ | +  |
| 1929     | (211,121,112)   | $\frac{1}{2}$ | $\frac{3}{2}$ | $P_4$ | 1 | $\frac{5}{2}$ | $\frac{7}{2}$ | +  |
| 2379     | (00-2,0-20,00-2) | $\frac{1}{2}$ | $\frac{3}{2}$ | $\Gamma_{15}$ | 1 | $\frac{5}{2}$ | $\frac{7}{2}$ | +  |
| 2649     | (12-1,1-12,21-1 2-11,-121,-112) | $\frac{1}{2}$ | $\frac{3}{2}$ | $P_4P_4$ | +  |
| 2649     | (12-1,1-12,21-1) | $\frac{1}{2}$ | $\frac{3}{2}$ | $P_4$ | 1 | $\frac{5}{2}$ | $\frac{7}{2}$ | +  |
| 2649     | (2-11,-121,-112) | $\frac{1}{2}$ | $\frac{3}{2}$ | $P_4$ | 1 | $\frac{5}{2}$ | $\frac{7}{2}$ | +  |
| 2649     | (202,220,022)   | $\frac{1}{2}$ | $\frac{3}{2}$ | $P_5$ | 3 | $\frac{9}{2}$ | $\frac{11}{2}$ | +  |
| 2649     | (202,220,022)   | $\frac{1}{2}$ | $\frac{3}{2}$ | $P_5$ | 4 | $\frac{9}{2}$ | $\frac{11}{2}$ | +  |

0* means $L = 0$ from Table A4
B.3 The J and P of the energy bands on Σ-axis

Using Fig. 4, Table A1, Table A3, Table A7 and (92)–(96), we can deduce the J and P of the energy bands on the Σ-axis as appearing in Table B3:

Table B3. The J and P of energy bands on the Σ-axis

| M (Mev) | (n₁, n₂, n₃) | s   | Repres. | L  | J(1/2) | J(3/2) | P |
|---------|---------------|-----|---------|----|--------|--------|---|
| 939     | (0, 0, 0)     | 1   | Σ₁      | 0* | 0      | +      |
| 1119    | (1, 1, 0)     | 1   | Σ₁      | 0* | 0      | +      |
| 1479    | (101,101)     | 1/2 | Σ₁      | 0.2| 1/2    | 3/2    | -  |
| 1659    | (1-10,-101)   | 1/2 | Σ₁      | 1  | 1      | 3/2    | +  |
| 1839    | (200,020)     | 1   | Σ₁      | 0* | 0      | +      |
| 2199    | (121,211)     | 1/2 | Σ₁      | 2  | 2/3    | 1/2    | -  |
| 2379    | (002,002)     | 1/2 | Σ₁      | 1  | 1/3    | 1/3    | +  |
| 2379    | (002,002)     | 1/2 | Σ₁      | 2  | 2/3    | 1/2    | -  |
| 2559    | (112,112)     | 1/2 | Σ₁      | 2  | 2/3    | 1/2    | -  |

0* means L = 0 from Table A4

B.4 The J and P of the energy bands on D-axis

Using Fig. 5, Table A2, Table A3, Table A6, Table A8, Table A9 and (92)–(96), we can deduce the J and P of the energy bands on the D-axis as appearing in Table B4. At the lowest energy Mᵢ = 1209, a band with (011,101) will take the representations with lower L = 1/3P₄ + P₁ from P - 1 and P₁ from (132). From (93), J_max ≤ 3/2, it cannot get certain L, J and P:
Table B4. The L, J and P of the energy bands on the D-axis

| $M_{MeV}$ | $(n_1, n_2, n_3)$ | s | Repres. | L | J(1/2) | J(3/2) | P |
|-----------|------------------|---|---------|---|---------|---------|---|
| 1209      | (0,1;1,0;1)      | $\frac{1}{7}$ | $\frac{1}{7}P_4 + P_1$ | ? | ?       | ?       | ? |
| 1479      | (10-1,0-1)       | $\frac{5}{2} \cdot \frac{3}{2}$ | $N_1, N_2$ | 2 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 1389      | (1-10,-110, 020,200) | $\frac{5}{2} \cdot \frac{3}{2}$ | $N_1, N_4$ | 2 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 1929      | (002,112)        | $\frac{5}{2} \cdot \frac{3}{2}$ | $P_3$ | 2 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 1929      | (112)            | $\frac{5}{2} \cdot \frac{3}{2}$ | $P_4$ | 1 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2199      | (12-1,21-1, -10-1,0-1) | $\frac{5}{2} \cdot \frac{3}{2}$ | $N_3, N_4$ | 2 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2199      | (1,2,-1:2,1,-1)  | $\frac{5}{2} \cdot \frac{3}{2}$ | $P_3, P_4$ | 1 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2559      | (220,-10-1)      | $\frac{5}{2} \cdot \frac{3}{2}$ | $N_1$ | 0 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2559      | (-1,-1,0)        | $\frac{5}{2} \cdot \frac{3}{2}$ | $N_2$ | 2 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2559      | (11-2,00-2)      | $\frac{5}{2} \cdot \frac{3}{2}$ | $N_3$ | 1 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2649      | (-121,2-11)      | $\frac{5}{2} \cdot \frac{3}{2}$ | $P_3$ | 1 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2649      | (-112,1-12, 202,022) | $\frac{5}{2} \cdot \frac{3}{2}$ | $P_5$ | 3 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
| 2919      | (2-1,-1-12)      | $\frac{5}{2} \cdot \frac{3}{2}$ | $N_1, N_2$ | 2 | $\frac{5}{2}, \frac{3}{2}$ | $\frac{3}{2}, \frac{3}{2}$ | - |
B.5  The J and P of the energy bands on F-axis

Table B5a. The J and P of the energy bands on F-axis

| $M_{ev}$  | $(n_1, n_2, n_3)$ | s | Repr. | L | J(1/2) | J(3/2) | P |
|-----------|------------------|---|-------|---|---------|---------|---|
| 1209      | (101,011)        | $^{3/2}$ | $P_4$ | 1 | $^{3/2}$ | $^{3/2}$ | + |
| 1299      | (002,-101,0-11)  | $^{1/2}$ | $H_{12}$ | 0 | $^{1/2}$ | $^{1/2}$ | - |
| 1929      | (112,-110,1-10)  | $^{3/2}$ | $P_3$ | 2 | $^{3/2}$ | $^{3/2}$ | - |
| 1929      | (01-1,10-1,020   | $^{1/2}$ | $P_4$ | 1 | $^{1/2}$ | $^{1/2}$ | + |
| 1929      | (01-1,10-1,020   | $^{1/2}$ | $P_4$ | 2 | $^{1/2}$ | $^{1/2}$ | + |
| 1929      | (01-1,10-1,020   | $^{1/2}$ | $P_5$ | 3 | $^{1/2}$ | $^{1/2}$ | + |
| 1929      | (01-1,10-1,020   | $^{1/2}$ | $P_5$ | 4 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (202,022,121, 2-11,0-1-1,-10-1) | $^{1/2}$ | $P_4$ | 1 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (202,022,121, 2-11,0-1-1,-10-1) | $^{1/2}$ | $P_4$ | 1 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (202,022,121, 2-11,0-1-1,-10-1) | $^{1/2}$ | $P_5$ | 3 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (2-1,-1)         | $^{1/2}$ | $P_5$ | 3 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (2-1,1,220)      | $^{1/2}$ | $P_4$ | 1 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (2-1,12-1,-220)  | $^{1/2}$ | $P_4$ | 1 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (2-1,12-1)       | $^{1/2}$ | $P_4$ | 2 | $^{3/2}$ | $^{3/2}$ | - |
| 2649      | (2-1,12-1)       | $^{1/2}$ | $P_4$ | 2 | $^{3/2}$ | $^{3/2}$ | - |
| 2649      | (2-1,12-1)       | $^{1/2}$ | $P_4$ | 2 | $^{3/2}$ | $^{3/2}$ | - |
| 2649      | (22,0)           | $^{1/2}$ | $P_4$ | 1 | $^{1/2}$ | $^{1/2}$ | + |
| 2649      | (22,0)           | $^{1/2}$ | $P_4$ | 2 | $^{3/2}$ | $^{3/2}$ | - |
Table B5b. The J and P of the energy bands on F-axis (continued from B5a)

| $M_{MeV}$ | $(n_1, n_2, n_3)$ | s | Repr. | L | J(1/2) | J(3/2) | P |
|-----------|------------------|---|-------|---|--------|--------|---|
| 2739      | (013,103,1-21, -211,0-20,-200) | $\frac{1}{2}, \frac{3}{2}$ | $H_{15}, H_{25'}$ |   |        |        |   |
| 2739      | (013,103,1-21) | $\frac{1}{2}, \frac{3}{2}$ | $H_{15}$ | 1, 3 | $\frac{3}{2}$ | $\frac{3}{2}$ | 6/22, 9/212/2 |
| 2739      | (013,103) | $\frac{1}{2}, \frac{3}{2}$ | $H_{15}$ | 1, 3 | $\frac{3}{2}$ | $\frac{3}{2}$ | 6/22, 9/212/2 |
| 2739      | (1-21) | $\frac{1}{2}, \frac{3}{2}$ | $\frac{1}{3} H_{15}$ | 1, 3 | $\frac{3}{2}$ | $\frac{3}{2}$ | 6/22, 9/212/2 |
| 2739      | (-211,0-20,-200) | $\frac{1}{2}, \frac{3}{2}$ | $H_{25'}$ |   |        |        |   |
| 2739      | (0-20,-200) | $\frac{1}{2}, \frac{3}{2}$ | $\frac{2}{3} H_{25'}$ | 2, 4 | $\frac{4}{3}$ | $\frac{7}{3}$ | 6/22, 9/212/2 |
| 2739      | (-2,1,1) | $\frac{1}{2}, \frac{3}{2}$ | $\frac{4}{3} H_{25'}$ | 2, 4 | $\frac{4}{3}$ | $\frac{7}{3}$ | 6/22, 9/212/2 |
| 2739      | (0-13,-103,0-22, -202,0-2-1,-1-21) | $\frac{1}{2}, \frac{3}{2}$ | $H_{25}H_{15'}$ |   |        |        |   |
| 2739      | (0-13,-103,0-22) | $\frac{1}{2}, \frac{3}{2}$ | $H_{25}$ | 3, 5 | $\frac{3}{2}$ | $\frac{7}{2}$ | 6/22, 9/212/2 |
| 2739      | (0-13,-103) | $\frac{1}{2}, \frac{3}{2}$ | $\frac{2}{3} H_{25}$ | 3, 5 | $\frac{3}{2}$ | $\frac{7}{2}$ | 6/22, 9/212/2 |
| 2739      | (1-21) | $\frac{1}{2}, \frac{3}{2}$ | $H_{25}$ | 3, 5 | $\frac{3}{2}$ | $\frac{7}{2}$ | 6/22, 9/212/2 |
| 2739      | (-202,-2-11,-1-21) | $\frac{1}{2}, \frac{3}{2}$ | $H_{15'}$ | 4 |        |        |   |
| 2739      | (-2-11,-1-21) | $\frac{1}{2}, \frac{3}{2}$ | $\frac{4}{3} H_{15'}$ | 4 | $\frac{4}{3}$ | $\frac{7}{3}$ | 6/22, 9/212/2 |
| 2739      | (-2,0,2) | $\frac{1}{2}, \frac{3}{2}$ | $H_{15'}$ | 4 | $\frac{7}{2}$ | $\frac{7}{2}$ | 6/22, 9/212/2 |
### B.6 The J and P of the energy bands on G-axis

Table B6. The J and P of the energy bands on the G-axis

| M (MeV) | (n₁, n₂, n₃) | s | Repres. | L | J(1/2) | J(3/2) | P |
|---------|------------|---|---------|---|--------|--------|---|
| 1299    | (1,0,1;0,1,-1) | ½ | ½M₁₁₁₁ | 1 | ½ | ½ | + |
| 1299    | (2,0,0;1,-1,0) | ½ | M₁₁₁₁ | 1 | ½ | ½ | + |
| 1299    | (2,0,0,) | ½ | ½M₁₁₁₁ | 1 | ½ | ½ | + |
| 1299    | (1,-1,0) | ½ | M₁ | 0 | ½ | ½ | - |
| 1479    | (0,1,1;0,1,-1) | ½ | N₁₁₁₁ | 1 | ½ | ½ | + |
| 1839    | (0,2,0;-1,1,0) | ½ | N₁₁₁₁ | 1 | ½ | ½ | + |
| 1839    | (0,2,0,) | ½ | N₁ | 0 | ½ | ½ | - |
| 1839    | (-1,1,0) | ½ | N₁ | 2 | ½ | ½ | - |
| 2019    | (0-11,0-1-1,121,121) | ½ | M₁₁₁₁ | 1 | ½ | ½ | - |
| 2019    | (0-11,0-1-1) | ½ | M₁₁₁₁ | 2,1 | ½ | ½ | ? |
| 2019    | (21-1,121) | ½ | M₁₁₁₁ | 2 | ½ | ½ | - |
| 2019    | (2,-1,1;2,-1,1) | ½ | M₁₁₁₁ | 2 | ½ | ½ | - |
| 2199    | (-101,-10-1,121,121-1) | ½ | N₁₁₁₁ | 1 | ½ | ½ | + |
| 2199    | (-101,-10-1) | ½ | N₁₁₁₁ | 1 | ½ | ½ | + |
| 2199    | (121,12-1) | ½ | N₁₁₁₁ | 2 | ½ | ½ | - |
| 2559    | (002,00-2) | ½ | N₁₁₁₁ | 2 | ½ | ½ | - |
| 2559    | (112,11-2) | ½ | N₁₁₁₁ | 1 | ½ | ½ | + |
| 2559    | (220,-1-10) | ½ | N₁₁₁₁ | 1 | ½ | ½ | + |
| 2559    | (2,2,0) | ½ | N₁₁₁₁ | 1 | ½ | ½ | + |
| 2559    | (-1,-1,0) | ½ | N₁₁₁₁ | 0 | ½ | ½ | - |
| 2739    | (202,20-2) | ½ | M₁₁₁₁ | 1 | ½ | ½ | + |
| 2739    | (1-12,1-1-2) | ½ | M₁₁₁₁ | 3 | ½ | ½ | + |
| 2739    | (3, 1, 0) | ½ | M₁₁₁₁ | 1 | ½ | ½ | + |
| 2739    | (0,-2,0) | ½ | M₁₁₁₁ | 3 | ½ | ½ | + |
| 2739    | (301,30-1) | ½ | M₁₁₁₁ | 2 | ½ | ½ | - |
| 2739    | (1-21,1-2-1) | ½ | M₁₁₁₁ | 2 | ½ | ½ | - |
| 2739    | (3,-1,0) | ½ | M₁₁₁₁ | 2 | ½ | ½ | - |
| 2739    | (2,-2,0) | ½ | M₁₁₁₁ | 2 | ½ | ½ | - |
References

[1] M. K. Gaillard, P. D. Grannis, F. J. Sciulli, Rev. Mod. Phys., 71 No 2, Centenary S96 (1999).

[2] P5 Report: The Particle Physics Roadmap,
www.science.doe.gov/hep/P5RoadmapfinalOctober2006.pdf.

[3] F. Wilczek, Phys. Tod. Jan. 11 (1998).

[4] M. L. Perl, E. R. Lee and D. Lomba, Mod. Phys. Lett, A19, 2595 (2004).

[5] W. M. Yao et al. (Particle Data Group), J. Phys. G33, 170 (2006).

[6] R. H. Dalitz and L. J. Reinders, in Hadron Structure as known from Electromagnetic and Strong Interactions, Proceedings of the Hadron ’77 Conference (veda,1979), P. 11; N. Isgur and G. Karl, Phys. Rev. D18, 4187 (1978); ibid. D19, 2653 (1979); ibid. D20, 1191(1979); K.-T. Chao, N. Isgur and G. Karl, Phys. Rev. D23, 155 (1981).

[7] W. M. Yao et al. (Particle Data Group), J. Phys. G33, 955(2006).

[8] M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig, CERN Preprint CERN-Th-401, CERN-Th-412 (1964).

[9] W. M. Yao et al. (Particle Data Group), J. Phys. G33, 165 (2006).

[10] W. M. Yao et al. (Particle Data Group), J. Phys. G33, 169 (2006).

[11] S. Weinberg, The Quantum Theory of Fields, (Cambridge , New York) 499(1995).

[12] J. Callaway, Energy Band theory (Academic Press, New York ), 9 (1964); P. T. Landsberg, Solid State Theory Methods and Applications (Wiley-Interscience, New York), 222 (1969).

[13] H. Jones, The Theory of Brillouin Zones and Electronic States in Crystals,(North-Holand/American Elsevier, Amsterdam, New York) 117 (1975).

[14] F. Bloch, Zeit. F. Phys. 52, 555 (1928); H. Jones, The Theory of Brillouin Zones and Electronic States in Crystals,(North-Holand/American Elsevier, Amsterdam, New York) 34 (1975).

[15] J. Callaway, Energy Band theory (Academic Press, New York and London), 24 (1964).

[16] A. W. Joshi, Elements of Group Theory for Physicists ( Wiley Eastern Limited, New Delhi), 294(1982).

[17] M. Gell-Mann, Phys. Rew. 92, 833 (1953) ; K. Nishijima, Prog. Theor. Phys. 12, 107 (1954).

[18] H. Jones, The Theory of Brillouin Zones and Electronic States in Crystals,(North-Holand/American Elsevier, Amsterdam, New York) 256 (1975).

[19] W. M. Yao et al. (Particle Data Group), J. Phys. G33, 36 (2006).