Power-suppressed effects in heavy quark fragmentation functions

Matteo Cacciari
LPTHE, Université P. et M. Curie - Paris 6, France
Paolo Nason
INFN, Sezione di Milano, Piazza della Scienza 3, 20126 Milan, Italy
Carlo Oleari
Università di Milano-Bicocca, Piazza della Scienza 3, 20126 Milan, Italy

This talk summarizes the results of a phenomenological analysis of heavy quark fragmentation data published by the CLEO and BELLE collaborations at $\sqrt{s} = 10.6$ GeV and by the LEP collaborations at $\sqrt{s} = 91.2$ GeV. Several theoretical ingredients are employed: next-to-leading order initial conditions, evolution and coefficient functions; soft-gluon resummation to next-to-leading-log accuracy; a next-to-leading order matching condition for the crossing of the bottom threshold in the evolution. Important initial-state electromagnetic radiation effects in the CLEO and BELLE data are also accounted for. We find that with reasonably simple choices of a non-perturbative correction to the fixed-order initial condition for the evolution, the data from CLEO and BELLE can be fitted with remarkable accuracy. The fitted fragmentation function, when evolved to LEP energies, does not however represent fairly the $D^*$ fragmentation spectrum measured by ALEPH. Large non-perturbative corrections to the coefficient functions of the meson spectrum are needed in order to reconcile CLEO/BELLE and ALEPH results.

1. Introduction

The CLEO [1] and the BELLE [2] Collaborations have recently published high-statistics and high-accuracy data for various charmed mesons fragmentation in $e^+e^-$ collisions at a centre-of-mass energy of 10.6 GeV. These data are the first ones able to rival in quality (and, in fact, to best) similar ones published a few years ago by the ALEPH Collaboration [3] at 91.2 GeV.

Taken together these sets of data allow for a phenomenological analysis [4] which spans a fairly large energy gap. It is therefore possible not only to test the ability of the theoretical framework to describe the data well, but also to perform the evolution from one energy to the other, and look for evidence (or absence) of power suppressed effects.

The theoretical framework employed is based on a next-to-leading order QCD description of the fragmentation process [5], including a next-to-leading log accurate resummation of collinear and soft gluons [6].

A few more details complete the theoretical picture. They are briefly summarized below, and they are outlined in detail in [4].

First, and most important, the soft-gluon resummation needs to be artificially regularized at large $x$ (or, equivalently, large $N$ in moment space). The choice of the regularization defines the perturbative distribution and therefore directly influences the non-perturbative one extracted by fitting the experimental data. In this work we have decided to use a prescription which, while being as simple as possible, possesses the following desirable features:

(i) it is consistent with all known perturbative results,

(ii) it yields physically acceptable results,

(iii) it does not introduce power corrections larger than generally expected for the processes in question, i.e. $N\Lambda/m$ for the initial condition [9–12] and $N\Lambda^2/q^2$ for the coefficient functions [13], where $\Lambda$ is a typical hadronic scale

---

*Talk given by M. Cacciari at FRIF workshop on first principles non-perturbative QCD of hadron jets, LPTHE, Paris, France, 12-14 Jan 2006

1It is worth noting that very recently next-to-next-to leading order results for the time-like non-singlet splitting function have become available [7]. Together with the previously available $O(\alpha_s^2)$ initial conditions [8] they make possible to repeat at least part of this analysis with higher accuracy.
of a few hundreds MeV.

In practical terms, our choice will yield a fragmentation function which does not become negative in the large-\(X\) region. This will allow for a good description of the data up to the the \(x = 1\) endpoint.

Second, while evolving the charm fragmentation function through the bottom threshold, one needs in principle to modify the evolution equations and properly match at this threshold. Moreover, production of charm via gluon splitting should be allowed, since it represents a non entirely negligible source at LEP energies. Both these features have been implemented in [4], thus departing from the simple non-singlet only description previously usually employed.

Finally, the experimental data as measured by CLEO and BELLE still contain the effect of electromagnetic initial state radiation. We have estimated that this effect is not negligible in this case (as it is, instead, at LEP, due to the physical cutoff provided by the \(Z^0\) resonance peak). We have therefore proceeded to simulate it and to deconvolute it from the data before fitting them with a pure QCD description of the fragmentation process. Again, details can be found in [4].

2. Non-perturbative fragmentation function

In the heavy-quark fragmentation-function formalism, the largest non-perturbative effects come from the initial condition, since one expects power corrections of the form \(\Lambda/m\). We assume that all these effects can be described by a non-perturbative fragmentation function \(D_{NP}\), that takes into account all low-energy effects, including the process of the heavy quark turning into a heavy-flavoured hadron, that has to be convoluted with the perturbative cross section. Thus, the Mellin transform of the full resummed cross section, including non-perturbative corrections, is

\[
\sigma_{H}(N, q^2) = \sigma_{Q}(N, q^2, m^2) D_{NP}(N) .
\]

We have attempted to fit CLEO and BELLE \(D^*\) data using several forms for \(D_{NP}\). We found that the best fits are obtained with the two-component form

\[
D_{NP}(x) = \text{Norm.} \times \frac{1}{1 + e \left[ \delta(1 - x) + cN_{a,b}^{-1}(1 - x)^a x^b \right]} ,
\]

where

\[
N_{a,b} = \int_0^1 (1 - x)^a x^b .
\]

This form is a superposition of a maximally hard component (i.e. the delta function) and the form proposed in Ref. [14]. It can be given a simple phenomenological interpretation, the hard term corresponding in some sense to the direct exclusive production of the \(D^*\), and the Colangelo-Nason form accounting for \(D^*\)'s produced in the decay chain of higher resonances.

Following the approach of Ref. [15], we assume that the \(D\) meson non-perturbative fragmentation function is the sum of a direct component, which is isospin invariant, plus the component arising from the \(D^*\) decay. The decay \(D^* \rightarrow D \pi\) is very close to threshold, so that the \(D\) has the same velocity of the \(D^*\), and their momenta are thus proportional to their masses. Under these circumstances, the component of the \(D\) fragmentation function arising from \(D^* \rightarrow D \pi\) decays is given by

\[
B(D^* \rightarrow D \pi) \tilde{D}_{\pi}^{D}(x) ,
\]

where we have defined

\[
\tilde{D}_{\pi}^{D}(x) = D_{NP}^{D^*} \left( \frac{m_{D^*}}{m_D} \right)^{m_{D^*}/m_D} \theta \left( 1 - \frac{m_{D^*}}{m_D} \right) ,
\]

E005
and $B(D^* \rightarrow D\pi)$ is the branching ratio of $D^* \rightarrow D\pi$. Observe that $\tilde{D}_\pi^D$ has been defined so as to have the same normalization as $D_{NP}^{D^*}$. In $N$ space we obtain immediately

$$\tilde{D}_\pi^D(N) = D_{NP}^{D^*}(N) \left[ \frac{m_D}{m_{D^*}} \right]^{N-1}.$$  

For the $D^* \rightarrow D\gamma$ decay, in the $D^*$ frame, the $D$ has non-negligible velocity, but it is non-relativistic, its momentum being given by

$$p_D = \frac{m_{D^*} - m_D}{2m_{D^*}}.$$  

It can easily be shown [4] that, in moment space, we can write

$$\tilde{D}_\gamma^D(N) = D_{NP}^{D^*}(N) \frac{m_{D^*}}{2p_D} \frac{(m_D + p_D)^N - (m_D - p_D)^N}{N m_D^N}.$$  

We thus describe $D^{+/0}$ production as the sum of a primary (i.e. not coming from $D^*$ decays) component, plus the contributions coming from $D^*$ decays

$$D_{NP}^{D^+}(x) = D_{NP}^{D^+,p}(x) + B(D^{++} \rightarrow D^+\pi^0)\tilde{D}_\pi^{D^+}(x) + B(D^{+\gamma} \rightarrow D^+\gamma)\tilde{D}_\gamma^{D^+}(x),$$

$$D_{NP}^{D^0}(x) = D_{NP}^{D^0,p}(x) + [B(D^{++} \rightarrow D^0\pi^+) + B(D^{+\gamma} \rightarrow D^0\gamma)]\tilde{D}_\pi^{D^0}(x) + B(D^{*0} \rightarrow D^0\gamma)\tilde{D}_\gamma^{D^0}(x).$$

### 3. $D$ mesons data fits near the $\Upsilon(4S)$

Several parameters enter our calculations. First of all, at all matching points, there are scale choices that could be varied, to yield a perturbative uncertainty in our result. Those are the initial evolution scale $\mu_0$, the matching scale for the crossing of the $b$ threshold $\mu_{thr}$, and the final evolution scale $\mu$. In the present work we fix

$$\mu_0 = m, \quad \mu = \sqrt{q^2} \equiv \sqrt{s}, \quad \mu_{thr} = m_{thr} = m_b.$$  

These scales could, in principle, be varied by a factor of order two around the values listed above, yielding a sensibly different result. However, in general, the scale variation will simply result in different values for the fitted parameters of the non-perturbative form. When computing cross sections for different processes, one should then use the parametrization appropriate for the scale choice that has been made in the fit, hence compensating for the change. In the present work we will not pursue this issue further, since our aim is simply to show that a fit within QCD is possible. A similar remark applies to the value of $\Lambda_{QCD}$ and the quark masses, that we will fix at

$$\Lambda_{QCD}^{(5)} = 0.226 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_b = 4.75 \text{ GeV}.$$  

The result of the fit is reported in Table I, and in Figs. 1, 2 we show some of the data and the corresponding fitted curve, both in $x$ and moment space.
4. $D$ mesons data fits on the $Z^0$

Fig. 3 shows a similar fit to ALEPH $D^{*+}$ data [3]. We fit the data in the region $x \in [0.4, 1]$ using the non-singlet component only, since a subtraction of the gluon-splitting contributions was performed by ALEPH. Observe that, in this calculation, the bottom-threshold crossing has to be dealt with. We also show, for comparison, the full evolution result (dashed line), using the same parameters obtained in the non-singlet fit. As we can see, the difference is only
visible at small $x$. The result of the fit for the non-perturbative parameters is

$$
a = 2.4 \pm 1.2, \quad b = 13.9 \pm 5.7, \quad c = 5.9 \pm 1.7, \quad (13)
$$

with a $\chi^2 = 4.2$ for 13 fitted points. These results are not really consistent with those for the $\Upsilon(4S)$ data in Tab. I.

In order to better quantify the discrepancy between Eq. (13) and Tab. I we use the parametrization of CLEO and BELLE data to predict the $D^*$ fragmentation function at LEP energies. The LEP prediction, using the parametrization of Table I, is reported in Fig. 4 together with the ALEPH data. We find a $\chi^2 = 60.1$ (for 13 fitted points) for this parametrization. Thus, the description is not satisfactory, especially in the large-$x$ (large-$N$) region.

In Fig. 5 we show the ratio of the moments of ALEPH $D^{*+}$ data over our prediction. We observe that the $N$ dependence of the ratio is well described by the functional form

$$
\frac{1}{1 + 0.044 (N - 1)}, \quad (14)
$$

where, since the first moment of the non-singlet distribution should be exactly given by the theory (because of charge conservation), we normalize to one the extrapolation of the data to $N = 1$.

We can only speculate about the possible origin of the discrepancy and the form of the coefficient of $(N - 1)$ in Eq. (14). Assuming that we are dealing with a non-perturbative correction to the coefficient function of the form

$$
1 + \frac{C(N - 1)}{q^2}, \quad (15)
$$

this would lead to the extra factor

$$
\frac{1 + \frac{C(N - 1)}{M_Z^2}}{1 + \frac{C(N - 1)}{M_\Upsilon^2}}, \quad (16)
$$

(where $M_Z$ and $M_\Upsilon$ are the $Z^0$ and $\Upsilon(4S)$ mass) to be applied to our prediction for the ALEPH data. For $C = 5$ GeV$^2$ we reproduce the behaviour of Eq. (14). In Ref. [13], on the basis of a calculation of infra-red renormalon effects,
a $1/q^2$ power correction is found, with an $N$ dependence marginally compatible with (15). No $1/\sqrt{q^2}$ correction is found. Ref. [16] also predicts a leading $1/q^2$ power correction. However, the $C \approx 5$ GeV$^2$ coefficient would appear to be somewhat too large$^2$. Alternatively, if we admitted the existence of corrections to the coefficient functions of the form

$$
1 + \frac{C(N-1)}{\sqrt{q^2}} .
$$

then we would find $C \approx 0.52$ GeV, a much more acceptable value. We observe that a form

$$
\left(1 + \frac{C}{\sqrt{q^2}}\right)^{N-2} \approx 1 + \frac{C(N-2)}{\sqrt{q^2}}
$$

was required in Ref. [17] to fit light-hadron fragmentation data.

Demonstrating the absence (or the existence) of $1/\sqrt{q^2}$ corrections in fragmentation functions would be a very interesting result, since it would help to validate or disprove renormalon-based predictions. Unfortunately, the low precision of the available data does not allow, for the time being, to resolve this issue.

We would like to remark that the discrepancy between the CLEO/BELLE and ALEPH data exclusively depends upon the evolution between the $\Upsilon(4S)$ and $Z^0$ energies. The method we used to describe the CLEO/BELLE data (i.e. the perturbative calculation of the fragmentation function, the Sudakov effects in the initial conditions and the parametrization of the non-perturbative part) does not affect the conclusions of the present section. In fact, we can simply compute the ratio of the moments of the inclusive $D^{*+}$ (ISR corrected) distribution at CLEO/BELLE and ALEPH, and compare it to the theoretical prediction. The result of this comparison (where we have used, for simplicity, BELLE data only) is displayed in Fig. 6. The curves are given by

$^2$If we believe that it is the maximum meson energy, not $\sqrt{q^2}$, that controls power effects, than we would have $C \approx 1$ GeV$^2$, a more acceptable value.
Figure 5: ALEPH $D^{*+}$ data, compared to the QCD prediction.

\[
\frac{\sigma_Q(N, M_{Z}^2, m^2)}{\sigma_Q(N, M_{\Upsilon}^2, m^2)} = \frac{\bar{a}_q(N, M_{Z}^2, \mu_{Z}^2)}{1 + \alpha_s(\mu_{Z}^2)/\pi} E(N, \mu_{Z}^2, \mu_{\Upsilon}^2) \frac{1 + \alpha_s(\mu_{\Upsilon}^2)/\pi}{\bar{a}_q(N, M_{\Upsilon}^2, \mu_{\Upsilon}^2)} \tag{19}
\]

where $\mu_{Z}$ and $\mu_{\Upsilon}$ are the factorization scales and the evolution factor $E$ is given by the solution of the Altarelli-Parisi evolution equation. Notice that low-scale effects, both at the heavy quark mass scale and at the non-perturbative level, cancel completely in this ratio, making its prediction entirely perturbative. As we can see from the figure, the rather large scale uncertainty displayed by the NLO result is much reduced when Sudakov effects are included. In both cases, however, the data clearly undershoot the pure QCD prediction, being instead compatible with the inclusion of the correction factor (14) (dotted lines).

One can legitimately wonder whether some of the theoretical ingredients might hide further uncertainties. We have checked that the regularization procedure needed to deal with the Landau pole in the soft-gluon resummation has very little impact on our curves. Using the very large value $\Lambda_{\text{QCD}}^{(5)} = 0.3$ GeV would lower the theoretical predictions by no more than 11% for $N \leq 20$, very far from explaining the observed effect. The deconvolution of ISR effects, that hardens the $\Upsilon(4S)$ data, but is insignificant on the $Z^0$, widens the discrepancy, which would however still be partially visible even if the data were not corrected for e.m. radiation.

Because of the relatively low energy of the data on the $\Upsilon(4S)$, it is also legitimate to wonder whether charm-mass effects could be responsible for the discrepancy between LEP and $\Upsilon(4S)$ data. We have not included mass effects in the present calculation. However, in Ref. [18], mass effects in charm production on the $\Upsilon(4S)$ where computed at order $\alpha_s^2$, and found to be small. In fact, they amount to a correction of the order of 1% at $N = 5$, 4% at $N = 11$ and 7% at $N = 20$, very far from being able to explain our observation. We thus believe that it is unlikely that mass effects could play an important role in explaining this discrepancy.

\[\text{Solid line: } 1/(1 + 0.044 (N-1))\]

\[\text{in Refs. [10] and [9], on the basis of the analogy with the spacelike case, corrections of the form } \Lambda m/q^2 \text{ are introduced. The importance}\]
5. Conclusions

This phenomenological analysis [4] of heavy quark fragmentation in $e^+e^-$ collisions shows that it is possible to perform excellent fits of $D^*/D$ meson fragmentation spectra in perturbative QCD, using all known results on the perturbative heavy-quark fragmentation function, and compounding them with a simple parametrization of non-perturbative effects.

A second striking result is the evidence of large non-perturbative effects, visible in the relation between the $D^*$ fragmentation function at the $\Upsilon(4S)$ and $Z^0$ energies. It would be interesting to understand the power law of these contributions. Their magnitude would suggest a $1/\sqrt{q^2}$ scaling law. Theoretical arguments based upon infrared renormalons would favour, instead, a $1/q^2$ behaviour. Because of the lack of precise $D^*$ production data in the intermediate region, it is difficult, at this point, to discriminate between the two possibilities. We point out, however, that, if these non-perturbative corrections involve the coefficient functions, they may be present also in light-hadron production, where data at intermediate energy are available. It is thus possible that fits to the light-hadron fragmentation functions from $\Upsilon(4S)$ up to $Z^0$ energies may clarify this issue.

References

[1] M. Artuso et al. Charm meson spectra in $e^+e^-$ annihilation at 10.5-gev-cms. Phys. Rev., D70:112001, 2004.

of these corrections in the present framework would require further investigation.
[2] R. Seuster et al. Charm hadrons from fragmentation and b decays in e+ e- annihilation at s**(1/2) = 10.6-gev. 
*Phys. Rev.*, D73:032002, 2006.

[3] R. Barate et al. Study of charm production in z decays. *Eur. Phys. J.*, C16:597–611, 2000.

[4] Matteo Cacciari, Paolo Nason, and Carlo Oleari. A study of heavy flavoured meson fragmentation functions in 
e+ e- annihilation. *JHEP*, 04:006, 2006.

[5] B. Mele and P. Nason. The fragmentation function for heavy quarks in qcd. *Nucl. Phys.*, B361:626–644, 1991.

[6] Matteo Cacciari and Stefano Catani. Soft-gluon resummation for the fragmentation of light and heavy quarks 
at large x. *Nucl. Phys.*, B617:253–290, 2001.

[7] A. Mitov, S. Moch, and A. Vogt. Next-to-next-to-leading order evolution of non-singlet fragmentation functions. 
2006. hep-ph/0604053.

[8] Kirill Melnikov and Alexander Mitov. Perturbative heavy quark fragmentation function through o(alpha(s)**2). 
*Phys. Rev.*, D70:034027, 2004.

[9] P. Nason and B. R. Webber. Non-perturbative corrections to heavy quark fragmentation in e+ e- annihilation. 
*Phys. Lett.*, B395:355–363, 1997.

[10] R. L. Jaffe and Lisa Randall. Heavy quark fragmentation into heavy mesons. *Nucl. Phys.*, B412:79–105, 1994.

[11] Lisa Randall and N. Rius. Using heavy quark fragmentation into heavy hadrons to determine qcd parameters 
and test heavy quark symmetry. *Nucl. Phys.*, B441:167–196, 1995.

[12] Matteo Cacciari and Einan Gardi. Heavy-quark fragmentation. *Nucl. Phys.*, B664:299–340, 2003.

[13] M. Dasgupta and B. R. Webber. Power corrections and renormalons in fragmentation functions. *Nucl. Phys.*, 
B484:247–264, 1997.

[14] G. Colangelo and P. Nason. A theoretical study of the c and b fragmentation function from e+ e- annihilation. 
*Phys. Lett.*, B285:167–171, 1992.

[15] Matteo Cacciari and Paolo Nason. Charm cross sections for the tevatron run ii. *JHEP*, 09:006, 2003.

[16] M. Beneke, Vladimir M. Braun, and Lorenzo Magnea. Phenomenology of power corrections in fragmentation 
processes in e+ e- annihilation. *Nucl. Phys.*, B497:297–333, 1997.

[17] P. Nason and B. R. Webber. Scaling violation in e+ e- fragmentation functions: Qcd evolution, hadronization 
and heavy quark mass effects. *Nucl. Phys.*, B421:473–517, 1994. Erratum-ibid.B480:755,1996.

[18] Paolo Nason and Carlo Oleari. A phenomenological study of heavy-quark fragmentation functions in e+ e- 
annihilation. *Nucl. Phys.*, B565:245–266, 2000.