Center Vortices at Strong Couplings and All Couplings

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Abstract

Motivations for the center vortex theory of confinement are discussed. In particular, it is noted that the abelian dual Meissner effect, which is the signature of dual superconductivity, cannot adequately describe the confining force at large distance scales. A long-range effective action is derived from strong-coupling lattice gauge theory in D=3 dimensions, and it is shown that center vortices emerge as the stable saddlepoints of this action. Thus, in the case of strong couplings, the vortex picture is arrived at analytically. I also respond briefly to a recent criticism regarding maximal center gauge.

In this talk I would like to present some recent work, done in collaboration with Manfried Faber and Štěfan Olejník, concerning center vortices in strong coupling lattice gauge theory. I will also touch on results obtained in collaboration with Jan Ambjørn and Joel Giedt, and with Faber, Olejník, and Roman Bertle.

1 Why Center Vortices?

We begin with a simple question: If confinement is defined by the Wilson area-law criterion, then what charge is actually confined in an SU(N) gauge theory? The first answer that comes to mind is that confined charge is just the SU(N) color charge. But this answer can’t be quite right, at least according to the Wilson criterion, because not all color charges are confined in this sense. For example, due to color screening, there exists no asymptotic linear potential between heavy charges in the adjoint representation. A second possible answer, motivated by dual-superconductor models, is that abelian electric charge (identified by abelian projection $SU(N) \rightarrow U(1)^{N-1}$) is the charge that is confined. But this doesn’t work either, since not all electric charges are confined. In, e.g., SU(2) lattice gauge theory, there is no asymptotic linear potential between charges of $q = \pm 2$ multiples of the elementary charge. Finally, consider N-ality, i.e. the charge associated with the $Z_N$ subgroup of SU(N). It is well known that in SU(N) gauge theory, only color charges with non-zero N-ality are confined, and the asymptotic string tension depends only on the N-ality of

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the color charge representation. Thus we conclude that it is really $Z_N$ charge which is confined in non-abelian gauge theories.

This simple fact is interesting, because the type of charge confined is an indication of the type of field configuration which does the confining. Of course, the $N$-ality dependence of the QCD string tension is no mystery; it is simply due to color screening of higher charge representations by gluons. We have an intuitive picture of one or more gluons bound to a static charge. On the other hand, Wilson loops can also be interpreted as a probe of vacuum fluctuations in the absence of external sources (think of evaluating spacelike loops in the Hamiltonian formulation), and it is generally assumed that loop observables are disordered by certain large-scale topological configurations. If that is the case, then such configurations must have the very non-trivial property that $N$-ality $= 0$ loops are somehow not disordered, and that the induced string tension in general depends only on $N$-ality. The only known gluonic field configurations with this property are the center vortices.

The center group is also singled out by the deconfinement phase transition, which involves the breaking of a global $Z_N$ symmetry, and certain features found in the deconfined phase are elegantly explained in terms of the vortex picture. Note that only global symmetries can actually break spontaneously. In the absence of gauge fixing, the VEV of a Higgs field in any gauge theory, dual or otherwise, is zero in any phase. This is due to the Elitzur theorem, and also to the status of local gauge symmetry as a genuine redundancy in field variables.

Let us consider the non-confinement of abelian electric charge in more detail (cf. Ambjørn et al. for an extended discussion). Fixing to maximal abelian gauge, we write the link variables in the usual form $U_\mu = W_\mu A_\mu$ where $A_\mu$ is the abelian (diagonal) link variable. Imagine integrating out the $W$ and ghost fields, to obtain an effective abelian action

$$\exp[S_{\text{eff}}[A]] = \int DW_\mu D(\text{ghosts}) e^{S_{\text{gf}} + S_{\text{eff}}[A]}$$  \hspace{1cm} (1)

The reduction to $U(1)^{N-1}$ degrees of freedom is not particularly significant in itself; this procedure could be carried out for any subgroup of SU(N), including the $Z_N$ center. Moreover, such effective actions are likely to be extremely complicated and non-local. The reduction only becomes physically interesting if $S_{\text{eff}}[A]$ takes on some particularly simple form at large scales, e.g. if $S_{\text{eff}}[A]$ describes a dual superconductor and/or monopole Coulomb gas of some kind. There exist, in fact, some very concrete proposals along these lines. One feature which is always found in such proposals is that all multiples of abelian electric charge are confined by the dual Meissner effect. If that is really so,
then an immediate consequence is that abelian Polyakov lines corresponding to any integer multiple of the electric charge ought to vanish in the confined phase. In SU(2) lattice gauge theory, this means that

\[ P_q = \langle \text{Tr}[(A_0 A_0 \ldots A_0)^q] \rangle_{S_{\text{eff}}} = 0 \quad \text{all } q \quad (2) \]

In addition, if \( P_{Mq} \) denotes the “monopole dominance” approximation to \( P_q \), and if \( S_{\text{eff}}[A] \) is well described by a monopole Coulomb gas, we would also expect

\[ P_{Mq} \approx P_q \quad (3) \]

The fact is, however, that while both of the above relations hold at \( q = 1 \) (\( Z_2 \) charged), neither relation holds at \( q = 2 \) (\( Z_2 \) neutral). The relevant \( q = 2 \) Polyakov lines in the confined phase (for \( T = 4 \) lattice spacings in the time direction), are shown in fig. 1 below. From the data showing \( P_2 \neq 0 \), we conclude that there is no confinement for \( q = 2 \). From \( |P_2| \gg |P_{M2}| \), it seems there is no monopole dominance either. And from the fact that \( P_2 < 0 \) we find that positivity is broken as well. Actually, positivity can be restored by going to spacelike maximal abelian gauge, but then 90° rotation symmetry is lost, and \( q = 2 \) string-breaking still occurs.

\[ \text{Figure 1: Double-charge abelian Polyakov lines} \]

Non-confinement of \( q = 2 \) charge, and the breakdown of monopole dominance, pose severe difficulties for dual-superconductor/dual abelian Higgs/monopole Coulomb gas models based on the abelian projection. All of these pictures predict confinement of all \( q \); therefore none of them is a good description of \( S_{\text{eff}}[A] \).
A similar objection can be raised to the monopole confinement picture in the D=3 Georgi-Glashow model. Although the monopole Coulomb gas picture, developed by Polyakov, is certainly valid for some intermediate range of distances, this picture must break down asymptotically. The reason is that in a monopole Coulomb gas we find string tensions $\sigma_q \propto q$ between objects with $q$ units of U(1) charge. But the Georgi-Glashow model has W-bosons with $q = 2$. Taking charge screening by these fields into account, we must get eventually

$$\sigma_q = \begin{cases} 
\sigma_1 \text{ odd } q \\
0 \text{ even } q 
\end{cases}$$

(4)

which contradicts the Coulomb gas picture. There is a moral here: In a confining theory, massive charged fields are relevant to far-infrared vacuum structure, and cannot be ignored.

These comments apply also to the Seiberg-Witten model. The low-energy effective action, derived in this model, again explicitly neglects the massive W-particles, and therefore misses the screening effects due to those particles. For this reason, the Seiberg-Witten effective action (which assumes locality) is not really the same thing as a Wilsonian effective action obtained from integrating out massive charged fields, and does not adequately describe physics beyond the double-charge screening scale.

In pure SU(2) gauge theory in a physical abelian gauge, non-confinement of $q = \text{ even}$ charge can likewise be deduced from the inevitable electric charge screening by off-diagonal gluons (the W-fields of eq. (1)). The effect is no mystery, but the consequences are important. While non-confinement ($\sigma_q = 0$) for $q = \text{ even}$ must be a property of the true effective action $S_{eff}[A]$ for the abelian field, a very different $q$-dependence ($\sigma_q \sim q$) is found in dual superconductor and monopole gas models. This indicates that the latter are either incorrect or, at best, incomplete in some way. In contrast, the correct $q$-dependence of the abelian string tension is quite natural in the framework of the vortex theory (cf. Ambjørn et al.).

2 Center Vortices at Strong Couplings

In strong-coupling lattice gauge theory in $D > 2$ dimensions, we have both confinement for N-ality $\neq 0$ charges, and color screening for N-ality $= 0$ charges. These facts suggest the existence of a vortex mechanism. On the other hand, there is a bit of folklore about strong coupling, namely, that confinement in $D > 2$ dimensions is just due to plaquette disorder, as in $D = 2$ dimensions. If so, vortices (and any other topological objects), have nothing to do with confinement at strong coupling. This folklore, however, is misleading.
Consider SU(2) lattice gauge theory at strong-coupling, and denote by $U(C)$ the product of link variables around loop $C$. Let the minimal area of a planar loop be decomposed into a set of smaller areas, bounded by loops $\{C_i\}$. We ask: Do the $\{U(C_i)\}$ fluctuate (nearly) independently, for large areas and small $\beta$? The test is whether

$$< \prod_i F[U(C_i)]> \approx \prod_i <F[U(C_i)]>$$

for any class function

$$F[g] = \sum_{j \neq 0} f_j \chi_j[g]$$

In fact, in $D = 2$ dimensions, it is easy to show that this equality is satisfied exactly. However, for dimensions $D > 2$, evaluating the left- and right-hand sides of (5) we find for the exponential falloff on each side

$$e^{-4\sigma P(C)} \prod_i \frac{1}{3} f_1 \gg \prod_i f_1 e^{-4\sigma P(C_i)}$$

where the inequality holds for perimeters $P(C) \ll \sum_i P(C_i)$. The conclusion is that the holonomies $U(C_i)$ do not fluctuate independently, even at strong-coupling, for $D > 2$. Where, then, does the area-law falloff come from?

The question is resolved by extracting a center element from the holonomies

$$z[U(C)] = \text{signTr}[U(C)] \in \mathbb{Z}_2$$

and asking if the center elements fluctuate independently; i.e

$$< \prod_i z[U(C_i)]> \approx \prod_i <z[U(C_i)]>$$

In fact, it is easy to show that they do:

$$e^{-\sigma A(C)} \prod_i \frac{3}{4\pi} = \prod_i \frac{3}{4\pi} e^{-\sigma A(C_i)}$$

Thus, confining disorder is center disorder, at least at strong couplings. Confining configurations must disorder the center elements $z$, but not the coset elements, of SU(2) holonomies $U(C_i)$. Again, the only configurations known to have this property are center vortices.

If center vortices are, in fact, the confining configurations of strong-coupling lattice gauge theory, then it would be interesting if this fact could be demonstrated analytically. A reasonable conjecture is that if the Wilsonian effective
action could be computed at a scale beyond the vortex thickness (4-5 lattice spacings at strong couplings), then at this scale “thin” center vortices will be stable saddlepoints of the action.

Suppose we define an effective long-range action $S_{eff}$ by, e.g.

$$\exp [S_{eff}[V]] = \int DU \prod_{\nu} \delta \left[ V_{\nu}^I(UUU)_{\nu} - I \right] e^{S_W[U]} \quad (11)$$

where the V-lattice spacing is $L$ U-lattice spacings. In D=2 dimensions

$$\exp [S_{eff}[V]] = \mathcal{N} \exp \left[ \sum_{P'} \log \left( 1 + \sum_{j=\frac{1}{2}, \frac{3}{2}}^{} \left( 2j + 1 \right) \left( \frac{I_{2j+1}(\beta)}{I_1(\beta)} \right)^{L^2} \chi_j[V(P')] \right) \right] \quad (12)$$

But this must be wrong for $D > 2$ dimensions, because it leads to a perimeter-law falloff

$$\langle \chi_1[V(C)] \rangle \sim \exp[-\mu \mathcal{P}(C)] \quad (13)$$

with an $L-$dependent “gluelump” mass

$$\mu = 4L \log \left( \frac{\beta}{4} \right) \quad \text{(wrong)} \quad (14)$$

The correct coefficient is

$$\mu = 4 \log \left( \frac{\beta}{4} \right) \quad (15)$$

In fact, $S_{eff}$ in $D > 2$ dimensions is non-local. It contains loops in all $j =$ integer representations, with perimeter-law weightings, derived from diagrams in which plaquettes on the U-lattice form a “tube” around a contour $C$ on the V-lattice. These diagrams lead to non-local contributions to $S_{eff}$ such as

$$S_{eff}[V] \supset \left( \frac{\beta}{4} \right)^{4(\mathcal{P}(C)-4)} \chi_1[V(C)] \quad (16)$$

We would like to derive a local effective action which would produce at least the leading contribution to any Wilson loop on the V-lattice. To achieve this, we integrate over all links on the U-lattice except on 2-cubes surrounding
V-lattice sites, as shown in Fig. 2. This defines an effective action \( \tilde{S}_L \)

\[
Z = \int DV \int \prod_{l \in 2\text{-cubes}} d\tilde{U}_l \left\{ \int \prod_{l' \in 2\text{-cubes}} dU_{l'} \prod_{l'} \delta \left[ V_{l'}^\dagger (UU..U)_{l'} - I \right] \right\} e^{S_{W[U]}}
\]

\[
= \int DV \int \prod_{l \in 2\text{-cubes}} d\tilde{U}_l \exp \left[ \tilde{S}_L[V, \tilde{U}] \right]
\]

Introduce group-valued plaquette variables in the 2-cubes \( h, g \), where \( h \) variables run around plaquettes on the surface of the 2-cube, and \( g \) variables run around plaquettes in the interior. Both sets of contours begin and end at the center of the 2-cube. After a number of manipulations, which include changing variables from \( \tilde{U} \) to \( h, g \) and integrating over the \( g \) variables, we obtain

\[
Z \approx \int DV D_h \prod_{2\text{-cubes}} K \left\{ 1 + 2 \left( \frac{\beta}{4} \right)^3 \sum_{c \in K} \chi_{\frac{1}{2}}[(hhh)_{c}] 
+ 2 \left( \frac{\beta}{4} \right)^4 \sum_{c_1, c_2 \in K} \chi_{\frac{3}{2}}[(hhh)_{c_1}(hhh)_{c_2}] + \ldots \right\}
\]

\[
\times \exp \left[ \frac{\beta}{2} \sum \text{Tr}[h] + 2 \left( \frac{\beta}{4} \right)^{4(L-2)} \sum_{l'} f_{l'}^{ijkl} \text{Tr}[h_{l'}^\dagger V_l h_{l'}^\dagger V_l^\dagger] 
+ 2 \left( \frac{\beta}{4} \right)^{L^2} \sum_{l'} \text{Tr}[VVV^\dagger V^\dagger] \right]
\]

(17)

\textit{Note:} We work in D=3 dimensions. The extension to D=4 should be straightforward.
where $f^{ijkl}_{i'} = 1$ if two plaquettes on neighboring 2-cubes can be joined by a cylinder of plaquettes on the U-lattice adjacent to V-link $l'$; $f^{ijkl}_{i'} = 0$ otherwise. This resembles an adjoint-Higgs theory, with an SU(2) gauge field $V_\mu$ coupled to 24 “matter” fields $h$ in the adjoint representation. Note that for large $L$, the “Higgs” potential term is much larger than the “kinetic” (V-link) and pure-gauge (V-plaquette) terms, so the $h$-fields fluctuate almost independent of $V_\mu$.

We then do a unitary gauge-fix of the $h$-fields (which leaves a remnant $Z_2$ symmetry), and integrate out the remaining $h$ d.o.f. to obtain

$$S_{\text{eff}}[V] \approx S_{\text{link}}[V, \langle h \rangle] + S_{\text{plaq}}[V]$$

$$= 2 \left( \frac{\beta}{4} \right)^{4(L-2)} \sum_{i'} f^{ijkl}_{i'} \text{Tr} \left[ (h^\dagger_{ij})_h V_\nu (h^\dagger_{kl})_h V^\dagger_{i'} \right]$$

$$+ 2 \left( \frac{\beta}{4} \right) L^2 \sum_{\nu'} \text{Tr} [VV^\dagger V^\dagger]$$

(18)

Now look for saddlepoints. We find that $S_{\text{link}}$ is maximized at

$$V_\mu (\vec{n}) = Z_\mu (\vec{n}) \times g(\vec{n}) g^\dagger (\vec{n} + \vec{\mu})$$

$$Z_\mu = \pm 1$$

where $g(\vec{n}) g^\dagger (\vec{n} + \vec{\mu})$ is fixed by the particular unitary gauge choice, while $S_{\text{plaq}}$ is maximized if $ZZZZZ = +1$. This is the unitary gauge ground state. Create a thin center vortex on this state by a discontinuous gauge transformation, e.g.

$$Z_y (\vec{n}) = \begin{cases} -1 & n_1 \geq 2, \ n_2 = 1 \\ +1 & \text{otherwise} \end{cases}$$

$$Z_z (\vec{n}) = Z_z (\vec{n}) = 1$$

This configuration is stationary: $S_{\text{link}}[V]$ is still a maximum, and $S_{\text{plaq}}$ is extremal (max or min) on all plaquettes. Stability depends on the eigenvalues of

$$\frac{\delta^2 S_{\text{eff}}}{\delta V_\mu (n_1) \delta V_\nu (n_2)} = \frac{\delta^2 S_{\text{link}}}{\delta V_\mu (n_1) \delta V_\nu (n_2)} + \frac{\delta^2 S_{\text{plaq}}}{\delta V_\mu (n_1) \delta V_\nu (n_2)}$$

(19)

and we find

$$\frac{\delta^2 S_{\text{link}}}{\delta V_\mu (n_1) \delta V_\nu (n_2)} \sim \left( \frac{\beta}{4} \right)^{4(L-2)+12}$$

$$\frac{\delta^2 S_{\text{plaq}}}{\delta V_\mu (n_1) \delta V_\nu (n_2)} \sim \left( \frac{\beta}{4} \right) L^2$$

(20)
The crucial observation is that for $\beta/4 \ll 1$ and
\[
4(L - 2) + 12 < L^2 \quad \Rightarrow \quad L \geq 5 \quad (21)
\]
the contribution of $\delta^2 S_{\text{plaq}} / \delta V \delta V$ to the stability matrix (and therefore to the eigenvalues of the stability matrix) is negligible compared to $\delta^2 S_{\text{link}} / \delta V \delta V$, which has only stable modes. This implies:

1. **Vortex Stability**: The thin vortex is a stable saddlepoint of the full effective action $S_{\text{eff}}$ at $L \geq 5$.

2. **Vortex Thickness**: A “thin” vortex on the V-lattice means thickness $< L$ on the U-lattice. This means that stable center vortices are $\approx 4 - 5$ lattice spacings thick. For the strong coupling Wilson action, this is the distance where the adjoint string breaks! The correspondence between the adjoint string-breaking length, and the thickness of center vortices, has been emphasized by our group in connection with Casimir scaling (see also Cornwall).

3. **Percolation**: From $S_{\text{eff}}$, we see that center vortices in D=3 cost an action $8(\beta/4)L^2$/unit length, while the entropy is $O(1)$/unit length. Since entropy $\gg$ action, this implies that vortices percolate through the lattice, and confine $N$-ality $\neq 0$ charge.

### 3 P-Vortices, Gauge Copies, and Lattice Size

The original calculations of center-projected Creutz ratios $\chi_{cp}(I, I)$, in direct maximal center gauge, used 3 gauge copies for gauge-fixing each lattice (picking the best of the three). Very recently Bornyakov et al. have claimed that $\chi_{cp}(I, I)$ varies with the number of gauge copies used, and disagrees, in the large copy number limit, with the unprojected string tension by as much as 30%.

In our opinion, the reported strong disagreement between projected and unprojected string tensions is due to finite-size effects. Lattice sizes used by Bornyakov et al. were $12^4$ at $\beta = 2.3, 2.4$, and $16^4$ at $\beta = 2.5$, while our published results were obtained on $16^4$ lattices at $\beta = 2.3, 2.4$ and $22^4$ lattices at $\beta = 2.5$. Projected lattices are more sensitive to finite-size effects than unprojected lattices, and this is probably due to the fact that center projection has difficulty finding vortices, when most of the lattice volume is taken up by the vortex cores. There are now good estimates for vortex thickness, coming from three sources: First, from the ratio of “vortex-limited” Wilson loops. Second, from the adjoint string-breaking distance, measured by de Forcrand...
and Philipsen. Third, the vortex thickness is found in a very interesting calculation, reported here by Terry Tomboulis, of the vortex free energy vs. lattice size. All three estimates are in rough agreement, and give a vortex thickness of a little over one fermi. This means that center vortices are $\approx 12-14$ lattice spacings thick at $\beta = 2.5$; a $16^4$ lattice may just be too small on this scale.

We have therefore repeated the calculation of Bornyakov et al. on a variety of lattice sizes. Some typical results at $\beta = 2.5$ are shown in fig. 3, where the finite size dependence is clearly seen. When the lattice volume is large enough, increasing the number of copies does not seem to make any substantial difference to our previously reported results for projected Creutz ratios and vortex densities. The details will be presented in a separate publication.

Finally, some questions about propagating ghosts in center gauges were raised at this meeting in the summary talk by Schierholz. Center gauges, like Landau gauge, are not ghost-free, and this would be a real problem if the aim of center gauge-fixing were to eliminate all unphysical modes in the Lagrangian. But the issue has little relevance, in our opinion, to the actual purpose of center gauge fixing, which is used in conjunction with center projection as a vortex finder. The rationale underlying this procedure and its empirical success in finding vortices on thermalized lattices have been discussed at length elsewhere. The same speaker questions whether vortex physics will be found to be consistent with instanton physics. We see no evidence of a problem in this area; in fact there are some suggestive findings to the effect that removing vortices from a lattice configuration also removes the topological charge.
In any case, the study of instanton physics in relation to center vortices has only just begun, and there is no reasonable basis, at this stage, for strong conclusions.

In this talk I have indicated how the center vortex picture of confinement can be derived at strong lattice couplings, and why this picture is attractive at any gauge coupling. The strong-coupling analysis shows that center vortices are stabilized by color-screening terms in the long-range effective action, and that screening terms dominate the action at the adjoint string-breaking scale. We expect that these qualitative features of the strongly coupled gauge theory are also found in the continuum $\beta \to \infty$ limit.

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