Turbulent luminance in impassioned van Gogh paintings

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Everything in the last period of Vincent van Gogh paintings seems to be moving; this dynamical style served to transmit his own feelings about a figure or a landscape. Since the early impressionism, artists empirically discovered that an adequate use of luminance could generate the sensation of motion. This sensation was more complex in the case of van Gogh paintings of the last period: turbulence is the main adjective used to describe these paintings. It has been specifically mentioned, for instance, that the famous painting *Starry Night*, vividly transmits the sense of turbulence and was compared with a picture of a distant star from the NASA/ESA Hubble Space Telescope, where eddies probably caused by dust and gas turbulence are clearly seen [1]. It is the purpose of this paper to show that the probability distribution function (PDF) of luminance fluctuations in some impassioned van Gogh paintings, painted at times close to periods of prolonged psychotic agitation of this artist, compares notable well with the PDF of the velocity differences in a turbulent flow as predicted by the statistical theory of Kolmogorov. This is not the first time that this analogy with hydrodynamic turbulence is reported in a field far different from fluid mechanics; it has been also observed in fluctuations of the foreign exchange markets time series [2].

Luminance is a measure of the luminous intensity per unit area. It describes the amount of light that passes through or is emitted from a particular area, and falls within a given solid angle [4]. Its psychological effect is bright and thus is an indicator of how bright a surface will appear. Luminance has been used by artists to produce certain effects. For instance, the technique of equiluminance has been used since the early impressionism to transmit the sensation of motion in a painting. Notably Claude Monet in his famous painting *Impression, Sunrise* used regions with the same luminance, but constrasting colors, to make his sunset twinkle. The biological basis behind this effect is that color and luminance are analyzed by different parts of the visual system; shape is registerd by the region that process color information but motion is registered by the colorblind part. Thus, albeit equiluminant regions can be differentiated by color contrast, they have poorly defined positions and it may seem to vibrate [3]. It seems likely that van Gogh dominated this technique but some of the paintings of his last period produce a more disturbing feeling: they transmit the sense of turbulence. By assuming that luminance is the property that van Gogh used to transmit this feeling (without being aware of it), we will quantify the turbulence of some impassioned paintings by means of a statistical analysis of luminance, similar to the statistical approach that Andrei Kolmogorov used to study fluid turbulence.

We mainly study van Gogh’s *Starry Night* (June 1889), which undoubtedly transmits the feeling of turbulence. Also, as samples of another turbulent pictures, we analyze *Wheat Field with Crows* (July, 1890, just before van Gogh shot himself), *Road with Cypress and Star* (May, 1890) and *Self-portrait with pipe and bandaged ear* (January, 1889). By considering the analogy with the Kolmogorov turbulence theory, from our results we can conclude that the turbulence in luminance of the studied van Gogh paintings is like real turbulence. Thus, the distribution of luminance on these paintings exhibit the same characteristif features of a turbulent fluid. This result may
Vincent van Gogh’s *Starry Night*

www.moma.org/collection/printable_view.php?object_id=79802

FIG. 1: Vincent van Gogh’s *Starry Night.*

offer a clue on why van Gogh was capable of transmitting the feeling of turbulence with high realism. Our results also reinforce the idea that scientific objectivity may help to determine the fundamental content of artistic paintings, as was already done with Jackson Pollock’s fractal paintings [3, 4]. Along this same ideas, it also worthy to mention that another notable ability of van Gogh was recently remarked with an experiment with bumblebees that had never seen natural flowers; insects were more attracted by van Gogh’s *Sunflowers* than by other paintings containing flowers [7]. From this observation, Chitka and Walker suggest that van Gogh’s flower paintings have captured the essence of floral features from a bee’s point of view.

The statistical model of Kolmogorov [8, 9] is a foundation for modern turbulence theory. The main idea is that at very large Reynolds numbers, between the large scale of energy input (L) and the dissipation scale (η), at which viscous frictions become dominant, there is a myriad of small scales where turbulence displays universal properties independent of initial and boundary conditions. In particular, in the inertial range Kolmogorov predicts a famous scaling property of the second order structure function, $S_2(R) = (\langle \delta v_R \rangle)^2$, where $\delta v_R = v(r + R) - v(r)$ is the velocity increment between two points separated by a distance $R$ and $v$ is the component of the velocity in the direction of $R$. In his first 1941 paper Kolmogorov postulates two hypotheses of similarity that led to the prediction that $S_2(R)$ scales as $(\varepsilon R)^{2/3}$, where $R = \|R\|$ and $\varepsilon$ is the mean energy dissipation rate per unit mass. Under the same assumptions, in his second 1941 turbulence paper Kolmogorov found an exact expression for the third moment, $\langle (\delta v_R)^3 \rangle$, which is given by $S_3(R) = -\frac{2}{3} \varepsilon R$. And even more, he hypothesized that this scaling results generalizes to structure functions of any order, i.e., $S_n(R) = \langle (\delta v_R)^n \rangle \propto R^{\xi_n}$, where $\xi_n = n/3$. Experimental measurements show that Kolmogorov was remarkably close to the truth in the sense that statistical quantities depend on the length scale $R$ as a power law. The intermittent nature of turbulence causes, however, that the numerical values of $\xi_n$ deviate progressively from $n/3$ when $n$ increases, following a concave curve below the $n/3$ line [10]. In 1962, Kolmogorov [11] and Obukhov [12] recognize that turbulence is too intermittent to be described by simple power laws and propose a refinement that yields a log-normal form of the probability density of $\varepsilon$.

An important function to characterize turbulence is the PDF of velocity differences $\delta v_R$, and different models have been proposed to describe the shape of this function at different scales $R$. We adopt here the approach by Castaign et al. [13] that, supported by experimental results, follows the idea of the log-normal form of $\varepsilon$. By superimposing several Gaussians at different scales, it is inferred that the shape of the PDF goes from nearly Gaussian at large scales $R$ to nearly exponential at small scales. The number of superimposed Gaussians is controlled by a parameter, $\lambda$, which is the only parameter that must be fitted to the data. A large value of $\lambda$ means that many scales contribute to the results, and thus the PDF develops tails that decay much slower than a pure Gaussian correlation.

*Starry Night*, painted during his one year period in the Saint Paul de Mausole Asylum at Saint-Rémy-de-Provence, is undoubtedly one of the best known and most reproduced paintings by van Gogh (Fig. 1). The composition describes an imaginary sky in a twilight state, transfigured by a vigorous circular brushwork. To perform the luminance statistics of *Starry Night*, we start from a digitized, 300dpi, $2750 \times 3542$ image obtained from The Museum of Modern Art in New York (where the original paint lies), provided by Art Resource, Inc. In a digital image, the luminance of a pixel is obtained from its RBG (red, green and blue) components as $0.299R + 0.587G + 0.114B$. This approximate formulae takes into account the fact that the human eye is more sensitive to green, then red and lastly blue. Thus, we calculate the PDF of pixel luminance...
fluctuations by building up a matrix whose rows contain difference in luminance $\delta u$ and columns contain separation between pixels $R$. From this matrix, we determine the normalized PDF of the luminance differences $P_R(\delta u_R) = \delta u_R/\langle (\delta u_R)^2 \rangle^{1/2}$. In Figure 2, we show this function for six pixel separations, $R = 60, 240, 400, 600, 800, 1200$. In order to rule out scaling artifacts, we have systematically recalculate the PDF function to images with lower resolutions (with an adequate rescaling of the pixel separations $R$). No significative differences appear up to images with resolutions lower that $150 \times 117$ pixels, where the details of the brushwork are lost.

We can take the analogy with fluid turbulence further. By considering the large length scale as $L = 2000$ pixels, which is size of the largest eddy observed in the Starry Night, in Fig. 3a, we show a log-log plot of the statistical moments with $n = 1, 2, 3, 4, 5$ (from bottom to top), that show power-law regimes. In each case straight line indicates the least-squares fit to the range of scales limited by the two dashed lines in the plot. In Fig. 3b, the scaling exponent $\xi_n$, of the first ten statistical moments are shown as a function of $n$. Albeit data point can be fitted with great accuracy to a straight line (implying a simple scaling consistent with a self-similar picture of turbulence but no with intermittence), scaling exponents show deviations from the self-similar values as indicated by the error bars. To determine scaling exponents and error bars, we follow the method proposed in Ref. [15], based on local slopes.

The PDF of luminance, for a given $R$, shown in Fig 2 were fitted according to the model by Castaign et al. [13] yielding a notably good fit. Results are shown in the same figure with full lines; parameter values are $\lambda = 0.2, 0.15, 0.12, 0.11, 0.09, 0.0009$ (from bottom to top).

Finally, Figure 3c shows the dependence of $\lambda^2$ on $\ln(R)$ for the first six moments. As expected, the parameter $\lambda^2$, which measures the variance of the log-normal distribution [13], decreases linearly with $\ln(R)$.
For comparison purposes, in Fig. 6 we show van Gogh’s Self-portrait with pipe and bandaged ear and its PDF for six pixel separations. In a well known episode of his life, on 23 December 1888, Vincent van Gogh mutilated the lower portion of his left ear. He was hospitalized at the Hôtel-Dieu hospital in Arles and prescribed potassium bromide [16]. After some weeks, van Gogh recovered from the psychotic state and, in a stage of absolute calm (as himself described in a letter to his brother Theo and sister Wilhemina [17]), he painted the self-portrait with pipe. As it can be seen in Fig. 6, the PDF of this paint departs from what is expected in Kolmogorov’s model of turbulence. We analyzed a 300dpi, 605 × 732 image obtained from The Vincent van Gogh Gallery webpage.

In summary, our results show that Starry Night, and other impassioned van Gogh paintings, painted during periods of prolonged psychotic agitation transmitted the essence of turbulence with high realism. We use Kolmogorov’s model of turbulence to determine the degree of “realism” contained in the paintings. We are also suggesting new tools and approaches that open the possibility of quantitative objective research for art representation.

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[1] NASA Press Release, March 4, 2004: [http://hubblesite.org/newscenter/newsdesk/archive/releases/2004/10](http://hubblesite.org/newscenter/newsdesk/archive/releases/2004/10)
[2] S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner and Y. Dodge, *Nature* (London) 381, 767 (1996).
[3] M.S. Livingstone, *Vision and Art: The Biology of Seeing*, Harry N Abrams (New York, 2002).
[4] Falta ...
[5] R.P. Taylor, A.P. Micolich and D. Jonas, *Nature* (London) 399, 422 (1999).
[6] J.R. Mureika, C.C. Dyer and Y. Dodge, *Nature* (London) 399, 422 (1999).
[7] L. Chittka, and L. Walker, *Opt. Laser Technol.* 38, 323-328 (2006).
[8] A.N. Kolmogorov, *Dokl. Akad. Nauk SSSR* 30, 299-303 (1941) (reprinted in *Proc. R. Soc. Lond. A* 434, 9-13 (1991)).
[9] A.N. Kolmogorov, *Dokl. Akad. Nauk SSSR* 32, 16-18 (1941) (reprinted in *Proc. R. Soc. Lond. A* 434, 15-17 (1991)).
[10] Z. Warhaft, *Annu. Rev. Fluid Mech.* 32, 203-240 (2000).
[11] A. Kolmogorov, *J. Fluid. Mech.* 13, 82-85 (1962).
[12] A. Obukhov, *J. Fluid. Mech.* 13, 77 (1962).
[13] B. Castaing, Y. Gagne and E.J. Hopfinger, *Physica D* 46, 177-200 (1990).
[14] R.C. González, R.E. Woods and S.L. Eddins, *Digital Image Processing Using MATLAB*, Prentice Hall (New Jersey USA, 2003).
[15] D. Mitra, J. Bec, R. Pandit and U. Frisch, *Phys. Rev. Lett.* 94, Article No. 194501 (2005).
[16] D. Blumer, *Am. J. Psychiatry* 159, 519-527 (2002).
[17] The Complete Letters of Vincent van Gogh, Bullfinch Press (Minnetonka, MN, 2000).