The photon and a preferred frame scenario

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Abstract

Structure of the space of photonic states is discussed in the context of a working hypothesis of existence of a preferred frame for photons. Two polarisation experiments are proposed to test the preferred frame scenario.

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I. INTRODUCTION

One of the most interesting questions in the present-day physics concerns fundamental theory of space-time in the context of quantum gravity and a need of an extension of the Standard Model. A possible implication of contemporary approaches to quantisation of gravity is breaking of the Lorentz symmetry in the boost sector \[1, 2\]. In consequence, this leads to existence of a preferred frame (PF) of reference. A possibility that Nature might exhibit a preferred foliation of space-time at its most fundamental level has attracted a serious attention since the last two decades. One can mention Lorentz-violating extensions of the Standard Model \[3–6\] as well as new models of classical and quantum gravity e.g. Einstein-aether \[7\] and Hořava–Lifshitz theories of gravity \[8\] (including vacuum solutions in this model \[9\]). Also worth mentioning are the so called doubly-special relativity (DSR) theories \[10\] which are characterised by modified dispersion relations for Lorentz violating models. Almost all of the above theories predict new effects, however suppressed by a power of the Planck scale. In particular, low energy signatures of Lorentz symmetry breaking in the photon sector include vacuum birefringence. This is a consequence of asymmetry of the modified, helicity dependent, dispersion relations for the photon. As a result, rotation of the polarization plane is predicted, depending on the distance between the source and the detector. Moreover, this effect also depends on a specific mechanism of Lorentz symmetry breaking by higher order differential operators \[11\].

In this paper, motivated by the preferred frame scenario, we consider the problem of quantum description of the photon and its polarisation under a minimal number of assumptions and from completely different perspective than in the above mentioned dynamical theories. It is shown that the presence of a PF of reference could results in some polarisation phenomena caused by a specific structure of the Hilbert space of photonic states.

II. THE RELATIVISTIC APPROACH TO PHOTONIC STATES

In the standard description of photonic states one uses Hilbert space \(\mathcal{H}\), which is a carrier space of a unitary, irreducible representation of the inhomogeneous Lorentz group. The action of the Lorentz group in \(\mathcal{H}\) is obtained by the Wigner-Mackey induction procedure \[12, 13\]. It can be realised on the eigenvectors of the four-momentum operator and next
extended by linearity to the entire space. As a result one obtains:

\[ U(\Lambda)|k, \lambda\rangle = e^{i\phi(\Lambda,k)}|\Lambda k, \lambda\rangle, \]  

(1)

where \( k = [k^\mu] \) is the photon four-momentum satisfying the dispersion relation \( k^2 = k_\mu k^\mu = 0 \), \( \Lambda \) is an arbitrary element of the homogenous Lorentz group and the photon helicity \( \lambda = \pm 1 \). Hereafter we will use the natural units with \( c = 1, \hbar = 1 \). The inhomogeneous part of the Lorentz group is represented by \( e^{ik_\mu\hat{P}_\mu} \), where \( \hat{P}_\mu \) is the self-adjoint four-momentum operator. The phase factor \( e^{i\lambda\phi(\Lambda,k)} \), representing the Wigner little group element \( L_{-1}^\lambda \Lambda L_k \) belonging to the Euclidean group \( E(2) \), realises its homomorphic unitary irreducible representation which is isomorphic to the \( SO(2) \) subgroup of \( E(2) \) (an explicit form of the phase \( \phi(\Lambda,k) \) can be found elsewhere [14]). Here \( L_kq = k, q^T = \kappa(1; 0, 0, 1) \) and \( \kappa > 0 \). The Lorentz-invariantly normalised states \( |k, \lambda\rangle, \langle k, \lambda|p, \sigma\rangle = 2k^0\delta^3(k-p)\delta_{\lambda,\sigma} \), where \( k^0 = |k| \), are identified with monochromatic, circularly polarised. Therefore, the corresponding linearly polarised photonic states have the form:

\[ |\theta, k\rangle = \frac{1}{\sqrt{2}}(e^{i\theta}|k, 1\rangle + e^{-i\theta}|k, -1\rangle), \]  

(2)

where \( \theta \) is the polarisation angle. Consequently, under Lorentz transformations (1) the states (2) transform as:

\[ U(\Lambda)|\theta, k\rangle = |\theta + \phi(\Lambda,k), \Lambda k\rangle, \]  

(3)

which means that linearly polarised states are transformed into linearly polarized states related to a new phase.

III. THE PREFERRED FRAME APPROACH TO PHOTONIC STATES

Our aim is to apply the Wigner-Mackey construction to the case when one inertial frame is physically distinguished. We will assume the Lorentz covariance under transformations between inertial frames. Obviously, this assumption does not exclude the case when the Lorentz symmetry is broken because we deal with passive space-time transformations. From the point of view of an inertial observer, the preferred frame has a time-like four-velocity \( u^\mu \) i.e. \( u^0 - u^2 = 1 \). The observer’s frame will be denoted as \( \Sigma_u \) while the PF corresponds to \( u_{PF}^T = (1; 0, 0, 0) \). It can be seen that the working hypothesis that Nature distinguishes a
preferred inertial frame of reference is nontrivial in the photonic sector only if the monochromatic photonic states are frame-dependent i.e. they depend not only on $k^\mu$ but also on $u^\mu$. Hereafter we will denote them as $|k, u, \lambda\rangle$. The Hilbert space of the observer in $\Sigma_u$ will be denoted by $\mathcal{H}_u$. The family of Hilbert spaces $\mathcal{H}_u$ form a fiber bundle corresponding to the bundle of inertial frames $\Sigma_u$ with the quotient manifold $SO(1, 3)/SO(3) \sim \mathbb{R}^3$ as the base space. As in the standard case, the Hilbert space $\mathcal{H}_u$ is spanned by eigenvectors of the four-momentum operator but these base vectors are $u^\mu$-dependent. To apply the Wigner-Mackey construction to this case we should relate each pair of four-vectors $(k, u)$ with the "standard" pair $(q, u_{PF})$ and determine the stabiliser of the standard pair. It is obvious that the little group of the pair $(q, u_{PF})$ is $O(2) \sim E(2) \cap O(3)$. Moreover, the pair $(k, u)$ can be obtained from the standard pair $(q, u_{PF})$ by the sequence of Lorentz transformations $L_u R_n$, where $L_u$ is the Lorentz boost transforming $u_{PF}$ into $u$ and $R_n$ is the rotation of $q$ into four-vector $\kappa(1; n)$, provided the unit vector $n$ is equal to:

$$n = n(k, u) = \frac{1}{uk} \left( k - \frac{|k| + uk}{1 + u^0} \right),$$

where $uk = u^\mu k_\mu = \kappa$. Applying the Wigner-Mackey procedure to the base vectors $|k, u, \lambda\rangle$ in the manifold of Hilbert spaces $\mathcal{H}_u$ we obtain the unitary action of the Lorentz group of the form:

$$U(\Lambda)|k, u, \lambda\rangle = e^{i\varphi(\Lambda, k, u)}|\Lambda k, \Lambda u, \lambda\rangle,$$

where $e^{i\varphi(\Lambda, k, u)}$ is the phase representing the Wigner rotation:

$$W(k, u, \Lambda) = (L_{\Lambda u} R_n(k, u_{\Lambda}))^{-1} \Lambda L_u R_n(k, u);$$

belonging to the subgroup $SO(2)$. The Wigner rotations satisfy the group composition law of the form:

$$W(k, u, \Lambda_2 \Lambda_1) = W(\Lambda_1 k, \Lambda_1 u, \Lambda_2) W(k, u, \Lambda_1).$$

Now, by means of (5) the linearly polarised states:

$$|\theta, k, u\rangle = \frac{1}{\sqrt{2}} (e^{i\theta}|k, u, 1\rangle + e^{-i\theta}|k, u, -1\rangle)$$

transform under the Lorentz group action unitarily from $\mathcal{H}_u$ into $\mathcal{H}_{\Lambda u}$ according to the transformation law:

$$U(\Lambda)|\theta, k, u\rangle = |\theta + \phi(\Lambda, k, u), \Lambda k, \Lambda u\rangle.$$
Furthermore, the ideal polariser, regarded as a quantum observable in $\mathcal{H}_u$, can be defined as the projector:

$$\Pi_{\Omega(n)}^{u,\Theta} = \int_{\mathbb{R}^+ \times \Omega} \frac{d^3p}{2|p|} |\Theta, p, u\rangle \langle \Theta, p, u| \frac{1}{2} \int_0^\infty |p| d|p| \int_{\Omega(n)} d\Omega |\Theta, p, u\rangle \langle \Theta, p, u|,$$

(10)

where $\frac{d^3p}{2|p|}$ is the Lorentz invariant measure, the polarisation angle $\theta$ is fixed, while $\Omega(n)$ is a solid angle around a fixed direction $n$. A photon, in order to be detected, should have his momentum direction in the solid angle $\Omega(n)$; otherwise it cannot pass through the polariser. Indeed, applying $\Pi_{\Omega(n)}^{u,\Theta}$ to a linearly polarised state $|\theta, k, u\rangle$ we find that:

$$\Pi_{\Omega(n)}^{u,\Theta}|\theta, k, u\rangle = \begin{cases} 
\cos(\Theta - \theta)|\Theta, k, u\rangle & \text{if } k \in \Omega(n), \\
0 & \text{if } k \notin \Omega(n).
\end{cases}$$

(11)

It is evident that this observable satisfies the quantum Malus law [15]. Indeed, the probability $p(\theta, \Theta)$ of finding a linearly polarised photon in the polarised state determined by the polarisation angle $\Theta$ has the form:

$$p(\theta, \Theta) = \cos^2(\Theta - \theta).$$

(12)

It follows from the definitions of the phase factors in eqs. (1) and (5) that the change of the polarisation angle is different in the presence or absence of a PF. The difference is expressed by a nontrivial phase shift $\Delta \phi = \phi(\Lambda, k, u) - \phi(\Lambda, k)$. In principle, this "geometric" phase shift can be explicitly calculated by means of (6) as well as measured.

IV. TWO DIRECT EXPERIMENTS

In order to illustrate the above result, let us analyse two possible experiments. Firstly, let us imagine two observers in inertial frames $\Sigma_u$ and $\Sigma_{u'}$ related by the Lorentz transformation $\Lambda(V)$ determined by the velocity $V$ along the photon momentum direction i.e. $k \parallel V \parallel k'$ as shown in Fig. 1. Linearly polarised photons are send by one observer and detected by the other. In that case the standard phase shift is equal to zero [14]. On the other hand, the phase shift $\phi(V, \vartheta, \chi)$ in this configuration, calculated from (6), is given by:

$$\phi(V, \vartheta, \chi) = \arcsin \frac{V \vartheta \sin \chi}{\sqrt{2(1 + \sqrt{1 - V^2})(1 + \sqrt{1 - \vartheta^2})(V \vartheta \cos \chi + \sqrt{1 - V^2} \sqrt{1 - \vartheta^2} + 1)}}.$$

(13)
FIG. 1. A schematic presentation of the polarisation experiment for inertial observers in relative motion.

Here the velocities \( V = \pm |V| \) and \( \vartheta = |\vartheta| \) are expressed in the units of \( c \) and \( \chi \) is the angle between \( k \) and the preferred frame velocity \( \vartheta = \frac{\chi}{\mu} \), as seen by the observer in the frame \( \Sigma_u \). In this case the phase difference in general does not vanish, \( \Delta \phi = \phi(V, k, u) \neq 0 \). One can see that for \( \vartheta \) parallel to the photon momentum (\( \chi = 0 \)), the phase shift is zero whereas for \( \vartheta \) perpendicular to the photon momentum (\( \chi = \frac{\pi}{2} \)) it reaches the maximal value. The dependence of \( \phi(V, k, u) \) as a function of the relative velocity \( V \) with the choice of the CMBR frame as the PF for \( \chi = \frac{\pi}{2} \) is presented in Fig. 2. In this case the observer in \( \Sigma_u' \) measures the rotation of the polarisation plane of the photon in comparison to the polarisation plane of the initial light beam.

Now, let us consider another possible consequence of the influence of the PF on the photon polarisation. If we apply the formula (6) to the case of a rotation \( R(\delta) \) around the photon momentum in a fixed frame we obtain:

\[
\varphi[R(\delta), k, u] = 2 \arctan \frac{\sqrt{1 - \vartheta^2} + [(1 - \sqrt{1 - \vartheta^2}) \cos \chi - \vartheta] \cos \chi}{(1 - \vartheta \cos \chi) \cot \frac{\delta}{2} + [(1 - \sqrt{1 - \vartheta^2}) \cos \chi - \vartheta] \sin \chi}.
\]

(14)

Taking into account that the corresponding phase shift for the standard case is exactly \( \delta \)
FIG. 2. The phase $\phi$ as a function of the boost velocity $V$ for PF identified with the CBMR frame. The photon momentum $k$ is chosen as perpendicular to the preferred frame velocity $\vartheta$ ($\chi = \frac{\pi}{2}$).

[13], the phase shift difference is of the form $\Delta \phi = \delta - \varphi[R(\delta), k, u]$. For $\vartheta \ll 1$ we obtain:

$$\Delta \phi \simeq 2 \vartheta \frac{\sin \chi \tan^2 \frac{\delta}{2}}{1 + \tan^2 \frac{\delta}{2}},$$

(15)

which is depicted in Fig. 3, where the preferred frame is identified with the CMBR frame. In this case an anomalous correction to the classical Malus law is present.

CONCLUSIONS

We have analysed possible consequences of existence of a preferred frame for the photon assuming the standard dispersion relation and the Lorentz covariance realised via passive transformations. The crucial assumption is that the quantum state of the photon, as seen by an inertial observer, depends on the observed velocity of the PF. Under these assumptions, by means of the Wigner- Mackey method of the induced representations, we have constructed the space of single photon states. We also have defined an ideal polariser, regarded as a quantum observable in the Hilbert space of states, satisfying the quantum Malus law. For linearly polarised states we have obtained a difference between the Wigner phases for the standard case (absence of PF) and for the theory with a PF. This difference manifests as an additional rotation of the polarisation plane of linearly polarised photons. Such optical effect has rather geometrical than dynamical nature and is different from the vacuum birefringence.
FIG. 3. The phase shift difference $\Delta \varphi$ as a function of the rotation angle $\delta$ around of the photon velocity and of the angle between preferred frame velocity and the photon velocity for PF identified with the CBMR frame.

which appears in models with modified energy-momentum dispersion relations of the photon. The effect is independent of the photon energy (frequency) and of the distance between the source and observer but instead depends on relative velocity of the reference frames, velocity of PF and on relative configuration of these velocities. Two direct experiments were proposed to test the PF hypothesis. In one of them the predicted effect, if exists, can be observed as a deviation from the classical Malus law.

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[1] S. Liberati and L. Maccione, Ann. Rev. Nuc. Part. Sci. 59, 245-267 (2009).
[2] S. Liberati, Class. Quant. Grav. 30, 133001 (2013).
[3] D. Colloday and V. A. Kostelecky, Phys. Rev. D 55, 6760 (1997).
[4] D. Colloday, V. A. Kostelecky, Phys. Rev. D 58, 116002 (1998).
[5] S. R. Coleman and S. L. Glashow, Phys. Rev. D 59, 116008 (1999).
[6] A. G. Cohen and S. L. Glashow, Phys. Rev. Lett. 97, 021601 (2006).
[7] T. Jacobson, Einstein-aether gravity: A Status report. PoS, QG-PH:020 (2007).
[8] P. Horava, Phys. Rev. D 79, 084008 (2009).
[9] J. Rembieliński, Physics Letters B 730, 67 (2014).
[10] G. Amelino-Camelia, Living Rev. Rel. 16, 5 (2013).
[11] V. A. Kostelecky, M. Mewes, Astrophys. J. 689, L1-L4 (2008).
[12] G. Mackey, *Induced representations of groups and quantum mechanics* (W. A. Benjamin, University of Michigan, 1968).
[13] A.O. Barut and R. Rączka, *Theory of Group Representations and Applications* (PWN, Warsaw, 1977).
[14] P. Caban, J. Rembieliński, Phys. Rev. A 68, 042107 (2003).
[15] K. Wódkiewicz, Phys. Rev. A 51, 2785 (1995).