THE IMPLICATIONS FOR HELIOSEISMOLOGY OF
EXPERIMENTAL UNCERTAINTIES IN NEWTON’S CONSTANT

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ABSTRACT

The experimental uncertainties in G between different experiments have important implications for helioseismology. We show that these uncertainties for the standard solar model lead to a range in the value of the square of the sound speed in the nuclear region that is as much as 0.15\% higher than the inverted helioseismic sound speed. While a lower value of G is preferred for the standard model, any definite prediction is masked by the uncertainties in the solar models available in the literature. However future refinements of helioseismology with an accuracy of the order of $10^{-3}$ to $10^{-4}$ in the square of the sound speed, especially in combination with precision measurements of the $^8B$ solar neutrino flux should be capable of independently testing these experimental values of G.

\textit{Subject headings:} Key words: stars: oscillations - stars: interiors - Sun: oscillations - Sun: interior: cosmology - dark matter

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Newton’s gravitational constant, \( G \), stands apart from all the other constants of physics in that the accepted uncertainty of a few per thousand for \( G \) is several orders of magnitude larger than for other fundamental constants (1998 CODATA report; Mohr & Taylor 1999). An accurate determination of \( G \) is important for many fields of modern physics, and in particular for the new alternative theories to general relativity that have started to emerge (Forgács & Horváth 1979; Albrecht & Magueijo 1999; Barrow 1999; Barrow & Magueijo 1998; Avelino, Martins, Rocha 2000; Mbelek & Lachièze-Rey 2001). A common consequence of these unified theories, applied to cosmology, is that they allow a space and time dependence of the coupling constants, such as the speed of light and Newton’s gravitational constants. Therefore an accurate determination of \( G \) in the laboratory is essential for testing these new theories.

On the experimental side, the current interest in measuring \( G \) was stimulated by a publication in 1996 by Michaelis, Haars & Augustin of a value of \( G \) that differed by 0.7% from the accepted value given in the previous 1986 CODATA report (see table 1; for recent reviews, see Quinn 2001, Mohr & Taylor 1999). To take this difference into account, the 1998 CODATA report recommends a value of \( G \) of \( 6.673 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \) with an uncertainty of 0.15%, some ten times worse than that in 1986. Whereas the other fundamental constants are more accurately known than in 1986, the uncertainty in \( G \) has increased drastically. In an attempt to improve the measurement of \( G \), several groups around the world have made new measurements using a range of different experimental methods (see table 1). The experimental targets represent an accuracy of between 0.01% and 0.001%. New results have recently been published, but in spite of the improved accuracy obtained by the recent experiments, the disagreement between the different measurements is still quite large (see table 1). In particular, we refer to the result of Luo et al. (1999), which determines a value of \( G \) that is 0.0026 smaller that the adopted value of CODATA in 1986. More recently, Gundlach & Merkowitz (2000) determined a value of \( G \) that is 0.001215 above the 1998 CODATA value. Using two independent methods Quinn et al. (2001) found a value of \( G \) 0.0026 above the 1998 CODATA value. Even if the two more recent experiments lead to a value above the 1998 CODATA report, both the trend of other experiments (Mohr & Taylor 1999), and the values of Gundlach and Quinn et al. (2001) lead to a value above the 1998 CODATA value.
We use the solar seismic data and the Super-Kamiokande (SK; Fukuda flux as probes. Furthermore, we confront these results with the recent solar neutrino measurements of Super-Kamiokande (SK; Fukuda et al. 2001) and the Sudbury Neutrino Observatory (SNO; Ahmad et al. 2002). Table 1: Summary of the recent experimental values of the Newton’s constant with their relative standard uncertainties. For reference, the values adopted by the CODATA report in 1986 and 1998 are also represented. Note - An extensive list of experiments is present in the CODATA report, so we selected a subset of these which represents the current determination of G. Details on the experimental determination of G and on the different experiments can be found in the Mohr & Taylor (1999) report.

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| Source                   | $G/(10^{-8} \text{cm}^3 \text{g}^{-1} \text{s}^{-2})$ | Rel. Stand. Uncert. |
|--------------------------|-----------------------------------------------|---------------------|
| Luo et al. (1999)        | 6.6699(7)                                    | $1.0 \times 10^{-4}$ |
| CODATA (1986)            | 6.67239(85)                                 | $1.3 \times 10^{-4}$ |
| CODATA (1998)            | 6.673(10)                                   | $1.5 \times 10^{-3}$ |
| Kleinevoss et al. (1999) | 6.6735(29)                                  | $4.3 \times 10^{-4}$ |
| Gundlach & Merkowitz (2000) | 6.674215                                   | $1.0 \times 10^{-5}$ |
| Quinn et al. (2001)      | 6.67559(27)                                 | $4.1 \times 10^{-5}$ |
| Richman et al. (1999)    | 6.68311                                     | $1.7 \times 10^{-3}$ |
| Schwarz et al. (1999)    | 6.6873(94)                                  | $1.4 \times 10^{-3}$ |
| Michaelis et al. (1996)  | 6.71540(56)                                 | $8.3 \times 10^{-5}$ |

& Merkowitz (2000) and Quinn et al. (2001), do not seem to agree (see table 1). The accuracy of the experiments has improved by as much as 1 $10^{-5}$, but the disagreement between the different results is of the order of 1.4 $10^{-3}$, which is quite striking. In general the experimental values of G varies between $6.669 \times 10^{-8} \text{cm}^3 \text{g}^{-1} \text{s}^{-2}$ and $6.715 \times 10^{-8} \text{cm}^3 \text{g}^{-1} \text{s}^{-2}$.

In this Letter we study the consequences of these new measurements of G for the evolution of the Sun. We use the solar seismic data and the $^8B$ neutrino flux as probes. Furthermore, we confront these results with the recent solar neutrino measurements of Super-Kamiokande (SK; Fukuda et al. 2001) and the Sudbury Neutrino Observatory (SNO; Ahmad et al. 2001).

The idea of using stellar evolution to constrain the possible value of G was originally proposed by Teller (1948), who stressed that the evolution of a star was strongly dependent on G. The luminosity of a main sequence star can be expressed as a function of Newton’s gravitational constant and its mass by using homology relations (Teller 1948, Gamow 1967, Kipperhahn & Weigert 1994). In the particular case that the opacity is dominated by free-free transitions, Gamow (1967) found that the luminosity of the star is given approximately by $L \approx G^{7.8} M^{5.5}$. In the case of the Sun, this would mean that for higher values of G, the burning of hydrogen will be more efficient and the star evolves more rapidly, therefore we need to increase the initial content of hydrogen to obtain the present observed Sun. In a numerical test of the previous expression, Delg’Innocenti et al. (1996) found that low-mass stars evolving from the Zero Age Main Sequence to the red giant branch satisfy $L \propto G^{5.6} M^{4.7}$, which agrees to within 10% of the numerical results, following the idea that Thomson scattering contributes significantly to the opacity inside such stars. Indeed, in the case of the opacity being dominated by pure Thomson scattering, the luminosity of the star is given by $L \approx G^4 M^3$. It follows from the previous analysis that the evolution of the star on the main sequence is highly sensitive to the value of G. Following this idea, several attempts to directly check the sensitivity of G to stellar evolution, and in particular its temporal variation, have been previously performed. The effect of a possible time-dependence of G on luminosity has been studied in the case of globular cluster H-R diagrams but has not yielded any stronger constraints than those relying on celestial mechanics (Will 1993, reference therein). In 1998, Guenther and collaborators used solar acoustic oscillation spectra available at that time to constrain the time variation of G, setting an upper limit on the variation of G that was $1.6 \times 10^{-12} \text{yr}^{-1}$, almost one order of magnitude smaller than the constraints obtained by binary pulsar timing measurements (Will 1993). In this context, the evolution of main sequence stars like the Sun presents an excellent probe for discussing new experimental values of G. This argument is validated by a strong constraint that can be used to diagnose the internal structure of our star, namely in the nuclear region, through the new results of helioseismology. A larger value of Newton’s gravitational constant increases the gravitational force, which, for stars on the main sequence in hydrostatic equilibrium,
is compensated by an increase in the rate of thermonuclear reactions. This leads to an increase of the central temperature and has two main consequences: since central pressure support must be maintained, the central density is increased in the solar models with $G$ larger than the reference model (in our case CODATA 1986), and since more hydrogen is burnt at the centre of the Sun, the central helium abundance and the central molecular weight are larger than in standard solar models.

The Sun is a unique star for research because its proximity allows a superb quality of solar data, enabling precision measures of its luminosity, mass, radius and chemical composition. Therefore it naturally becomes a privileged tool to be used as a laboratory for physics. In recent years, different groups around the world have produced solar models in the framework of classical stellar evolution, taking into account the best known physics as well as all the available observational seismic data. This has led to the determination of a well-established model for the Sun, the so-called standard solar model (Turck-Chièze & Lopes 1993; Christensen-Dalsgaard et al. 1996; Brun, Turck-Chièze & Zahn 1999; Provost, Berthomieu & Morel 2000; Turck-Chièze, Nghiêm, Couvidat & Turcotte 2001, Turck-Chièze S. et al. 2001a; Bahcall, Pinsonneault, Basu 2001), for which the acoustic modes are in very good agreement with observation. Furthermore, this model has established considerable consensus among the different research groups, concerning the predictions of the solar neutrino fluxes, and has unambiguously helped define the difference between the theoretical predictions and the experimental results. In this context, we use the standard solar model as a reference to test the new experimental measurements of $G$ (see Table 1). We have pro-
duced different solar models which are distinguished from the solar standard model by adopting different values of $G$. As usual, the model starts to evolve from a standard primordial chemical composition star to reach the present Sun with the observed luminosity and radius at its present age 4.6 Gyr, by readjusting the initial helium abundance and the mixing length parameter (Turck-Chièze et al. 2001; Lopes, Silk and Hansen 2001; Lopes, Bertone & Silk 2001). There has been considerable discussion recently regarding the precise value of the solar radius and solar luminosity, therefore in order to obtain the present solar models for different values of $G$ we have performed a calibration by choosing the value of mixing-length and the initial content of helium in such a way that the present luminosity and the present solar radius are reproduced with an accuracy better than $10^{-7}$ (Lopes & Silk 2002, in preparation). This calibration precision is much higher than the precision of the global quantities which is of the order of a few thousand. At this stage, it is worth noticing that the standard evolution of the Sun is independent of the total mass of the star. The observational determination of the product $GM_\odot$, i.e., the product of Newton’s constant with the total mass of the Sun, the so-called Gaussian constant, is known with an accuracy better than $10^{-7}$, therefore the $GM_\odot$ product can be used to explicitly write the equations of stellar structure as a function of Newton’s constant. Actually, it is this fact that provides us with a possible means of probing the value of $G$ by using the evolution of the Sun and the highly accurate results of helioseismology and solar neutrinos (Lopes & Silk 2002, in preparation).

In Fig. 1, we compare the square of the sound speed for different solar models with the new experimental values of $G$ and the solar standard model. In the same figure, we show the square of the sound-speed as inferred for the present Sun by using the data from Global Oscillations at Low Frequency (GOLF; Gabriel et al. 1995) and Michelson Doppler Imager (MDI; Scherrer et al. 1995) experiments. It follows from our analysis that the new experimental values of $G$ determined by Michaelis et al. (1996), Schwarz et al. (1999), Richman et al. (1999), produce changes in the profile of the sound speed, compared with the inverted sound speed, that are larger than the differences currently obtained with the solar standard model. Conversely, models computed with lower values of $G$, of the order of $6.63 \times 10^{-8} \text{cm}^3\text{g}^{-1}\text{s}^{-2}$ reproduce the differences observed between the solar standard model and the inverted sound speed from the more recent seismic data. Naturally, the inversion of the sound speed in the center is not totally reliable but we infer that the seismology seems to favor a lower value of $G$. If we consider that among the present values of $G$ the more reliable are those of Quinn et al. (2001) and Gundlach & Merkowitz (2000), than we anticipate that in the coming years solar physics and seismology can progress together to a level such that the accuracy of the square of the sound speed can be obtained with an accuracy of 1 part in $10^4$. In this case it follows follows that the standard solar model will be capable of distinguishing between the more likely values of $G$ (see Fig. 2).

Indeed, it is important to remark that the inversion of the sound speed is still uncertain in the central region due to the lack of seismic data, mainly due to the small number of acoustic modes that reach the nuclear-burning region. The inversions are not very reliable at the surface, above 98% of the solar radius, due to a poor description of the interaction of acoustic waves with the radiation field and turbulent convection, namely, in the superadiabatic region (Lopes & Gough 2001). In spite of these uncertainties, it is not possible to explain the large differences in the nuclear region obtained by these experimental values of $G$ in the solar standard model. Indeed, a positive difference of as larger as 0.3% cannot be accommodated by our present understanding of the internal structure of the Sun. However, a lower value of $G$, typically of the order of $6.63 \times 10^{-8} \text{cm}^3\text{g}^{-1}\text{s}^{-2}$, produces changes in the structure comparable to the helioseismic sound speed and accurately reproduces its shape in the nuclear region.

If we believe in the diagnostic capability of the seismic techniques presented here, a lower value of $G$ is better accommodated in the present picture of the evolution of the Sun than the experimental values of $G$ measured by Quinn et al. (2001) and Gundlach & Merkowitz (2000), among others. However, we stress that in order to determine with certainty the impact of the new values of $G$ on the evolution of the Sun, a more careful analysis of this problem must be made.

The SNO collaboration have published their result for the $^8B$ flux measured by neutrino-electron scattering reactions and reported a lower $^8B$ flux as compared to the theoretical predictions of the standard solar model (Bahcall, Pinsonneault and Basu 2001; Turck-Chièze, Nghiem, Couvidat & Turcotte 2001). Furthermore, this result is in agreement with
the $^{8}B$ flux measured by Super-Kamiokande detector through the same reaction. Therefore, the experiment confirms the deficit of solar neutrino fluxes of the Chlorine experiment of Davis et al. (1998) and subsequently confirmed by Kamiokande, and by the Gallium experiments SAGE (Abdurashitov et al. 1996), GALLEX (Kirsten et al. 2000) and GNO (Belloti et al. 2000).

The production of $^{8}B$ takes place in the inner 2% of the solar mass core. The $^{8}B$ decay reaction presents the strongest dependence on the temperature: the $^{8}B$ neutrino production is maximum at quite small radii, 5% of the solar radius, and its generation is confined to the region between 2% and 7% of the solar radius. Consequently, this flux of neutrinos becomes the best signature of the temperature at the center of the Sun. Indeed, if the SNO measurement of the $^{8}B$ neutrino fluxes is correct, the central temperature of the standard solar model is within less than 0.5% of the temperature deduced from the measured $^{8}B$ neutrino flux (Bahcall 2001, Turck-Chièze 2001). It follows that the high constraint on the solar central temperature imposed by the SNO results can be used to constrain some physical processes occurring in the center of the Sun, or even test some SUSY dark matter particles (Lopes & Silk 2002; Lopes Hansen & Silk 2002; Lopes, Bertone & Silk 2002).

The evolution of the Sun on the main sequence occurs under hydrostatic equilibrium, and accordingly the kinetic energy of electrons and nuclei in the Sun is proportional to the gravitational potential. Therefore the central temperature can be used to measure G. In Fig. 2 we present a solar model for different values of G. At present the error bar of SNO does not allow us to identify which is the best value of G. Nevertheless, SNO is expected to attain an accuracy of 10% or even 5% in future years (Wark 2001, private communication). With this precision it is not possible to unequivocally determine the best value of G, based solely upon the constraint imposed by SNO neutrinos. However these results, combined with other neutrino experiments and with a better understanding of the mechanisms of neutrino oscillations, will enable us to open a new means of constraining the value of G in the solar interior. The information provided by the neutrino experiments is quite significant because it constitutes an independent test of G complementary to the one provided by helioseismology.

It has been known for the last two hundred years that Newton’s constant is very difficult to measure accurately. Simply stated, G is determined by measuring the gravitational attractive force between two masses at a known distance apart. The problem is that the gravitational attraction between two laboratory-sized masses is simply too small. However, using a very large body like the Sun and the solar acoustic spectrum, it is possible to constrain the gravitational self-attraction and, in so doing, test the new experimental values of $G$. It follows from our analysis that the low values of G seem to be favoured. However, the solar standard model is quite complex, and the accurate determination of G can be masked by other uncertainties in the solar model. Only a systematic study of these uncertainties can lead to an accurate determination of G. However, the fact that the neutrino flux measured by SNO (and possibly other neutrino experiments) can be combined with a better model for the oscillation properties of neutrinos will provide a promising means of potentially determining G with improved accuracy, specially if current experimental error bars are significantly reduced.

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