Analysis of applicability of SWRC equation to different types of soils and its prediction method considering initial dry density

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ABSTRACT

This paper is centered on the assessment of 3 soil-water retention curves (SWRC) equations for unsaturated soils, for its predictive capability with respect to different soil types. The fitting effects of van Genuchten equation, Fredlund and Xing equation and Brooks and Corey equation on clay, silt and sand are studied. The influence of different soils with different initial dry density on parameters in different equations is analyzed. A new SWRC prediction method considering the influence of initial void ratio is proposed, which assumes a linear relationship between the initial void ratio and the saturation under the same suction. By analyzing the test data of sand, the predicted results are compared well with test results.

Keywords: unsaturated soil, soil-water retention curve, initial dry density, suction

1 INTRODUCTION

The soil water retention curve (SWRC) is the basis of studying the strength, permeability and constitutive theory of unsaturated soil (Fredlund and Rahardjo, 1993). It can be described by the relationship between saturation, $S_r$ (or the volumetric water content, $\theta$) and the matric suction, $s$ (or soil-water potential, $\psi$), which represents the water holding capacity of unsaturated soil. There are many factors affecting the soil water retention curve, such as soil type, mineral composition, pore structure, stress state, pore ratio, salt content and temperature, etc (Miller and Yesiller et al., 2002; Yang and Rahardjo et al., 2004; Vanapalli and Fredlund et al., 1999; Marinho, 2005; Ng and Pang, 2000; Sun and You et al., 2016; Romero and Gens, 2001; Uchaipichat and Khalili, 2009; Zhou and Sheng et al., 2014). In the natural environment, the dry density of the soil increases with the increase of the depth. The soil with different initial dry density is the main factor affecting the water retention ability, so it is of great significance to study the influence of the initial void ratio or dry density on the soil water retention curve.

Recent years, the problem of unsaturated soil has often been encountered in engineering. Among them, engineering accidents related to unsaturated soil have also appeared in highway and railway projects, causing economic losses and casualties. Rainfall in nature, seepage through the surface, and changes in groundwater levels can all contribute to changes in the volume content of unsaturated soils. The suction of unsaturated soils has a significant contribution to its strength, changes of the water content of the soil can cause changes in the suction of the matrix, resulting in changes in the strength of the slope soil. If the stress of the unsaturated soil is not fully considered in the actual engineering process, the risk of the construction process will increase, posing a major threat to the entire project. On December 20th, 2015, a landslide accident occurred in the red slag soil of Guangming New District in Shenzhen, causing economic losses of 881 million RMB. The main reason for the landslide is that the soil is saturated and a weak sliding zone is formed at the bottom, which leads to the instability and sliding out of the residual soil. The accident shows that the rainfall changes the saturation of unsaturated soils, resulting in a decrease in the matrix suction of the soil, which reduces the strength of the soil. The shear strength of unsaturated soil is related to the suction.

The study of unsaturated soil began in the 1930s. At that time, the theory was still immature. It is not realistic to study the strength of soil in the three-phase state, and it is hard to measure the relevant parameters. In...
addition, the theoretical foundation needed is lacking, so the flow of capillary water becomes the focus of research (Xie, 2005). SWRC can express many properties of unsaturated soil, such as permeability, strength, stress state, etc (Fredlund and Xing et al., 1994). At the same time, as a basic constitutive relationship to explain the behavior of unsaturated soils, the soil-water retention curve is one of the important relationships that combine theory, experiment and prediction methods (Barbour, 1998; Leogn and Rahardjo, 1997). Salager and Nuth (2013) used pressure plate and salt solution experiments to draw soil-water retention curves with different initial pore ratios in a wide suction range. Sheng and Zhou (2011, 2012) proposed an incremental relationship between the degree of saturation (Sr) and the void ratio (e) by realising that the SWRC is obtained under constant stress instead of constant volume.

This paper presents a systematical study to analyses the applicability of typical SWRC equations to various soils by collecting SWRC test data of various types of soils, and discusses the variation of equation parameters with initial dry density. At the same time, a new SWRC prediction method considering the influence of initial dry density is proposed.

### 2 SWRC EQUATION APPLICABILITY ANALYSIS

Analysis of a typical SWRC equation includes: van Genuchten (vG) equation (van Genuchten, 1980), Fredlund-Xing (FX) equation (Fredlund and Xing, 1994) and Brooks-Corey (BC) equation(Brooks and Corey, 1964). The specific equation form is as follows:

\[
S = \left[1 + \left(\frac{\psi}{a}\right)^n\right]^{-\frac{1}{m}}
\]

\[
S = \left[\ln\left(\frac{\psi}{e + \left(\frac{\psi}{a'}\right)^n}\right)\right]^{-\frac{1}{m'}}
\]

\[
S = \left(\frac{\psi_{awc}}{\psi}\right)^{\lambda}
\]

Where \(a\), \(n\) and \(m\) is vG equation fitting parameters; \(a'\), \(n'\) and \(m'\) is FX equation fitting parameters. Since the FX equation is based on mathematical fitting, each parameter has no clear physical meaning; \(\lambda\) and \(\psi_{awc}\) is BC equation fitting parameters, which \(\lambda\) is for the pore distribution coefficient and \(\psi_{awc}\) is for the air entry value. Van Genuchten (1985) pointed out that when the value of \(m/n\) is small, the parameter \(a\) in the vG equation is equal to the reciprocal of the air entry value, and when the value of \(m/n\) is large, the parameter \(a\) is approximately equal to the reciprocal of the suction at the inflection point. For convenience, the parameters in each equation are referred to as parameter 1, parameter
2, and parameter 3, respectively. To discuss the applicability of each equation for different soil types, the SWRC test results of four types of soils are selected for fitting. The properties and components of soil samples are listed in Table 1. The fitting results are shown in Fig. 1 and Table 2.

Table 1. Selection of test data for soil samples.

| Type              | Composition                | GI  |
|-------------------|----------------------------|-----|
| Expansive rock soil | Expansive rock soil        | --  |
| Remolding clay    | Clay                       | 2.74|
| Touchet Silty sand| 32% sand, 53% silt, 15% clay | 2.60|
| Columbia sand     | 54% sand, 35% silt, 11% clay | 2.67|

By the fitting results, the following conclusions can be drawn: (1) The vG equation has a good fitting effect on expansive soil and remolded clay. For sand, it is difficult to show the relationship between suction and saturation. (2) The FX equation has a good fitting effect on four types of soil, and the correlation coefficient is as high as possible as 0.99 or more, and the fitting effect on the boundary effect region, the transition region and the unsaturated residual region curve is better. (3) The BC equation has a poor effect on the fitting of clay and expansive soil test data, and the correlation coefficient is lower than other equations. However, the fitting effect on sand and silty soil is better, and the correlation coefficient is above 0.99. Since the two parameters of the BC equation are relatively clear, the air entry value and the pore distribution coefficient can be directly determined, so it is the first choice for the practical application of silt and sand.

Table 2. Fitting results of SWRC test data.

| Type              | Equation | Parameter 1 | Parameter 2 | Parameter 3 | $R^2$  |
|-------------------|----------|-------------|-------------|-------------|-------|
| Expansive rock soil | vG       | 0.0081      | 0.213       | 1.279       | 0.9987|
|                   | FX       | 130.3       | 1.211       | 0.505       | 0.9988|
|                   | BC       | 0.155       | 39.88       | --          | 0.9821|
| Remolding clay    | vG       | 0.0233      | 0.412       | 2.126       | 0.9983|
|                   | FX       | 53.33       | 1.851       | 1.406       | 0.9978|
|                   | BC       | 0.603       | 24.45       | --          | 0.9846|
| Touchet Silty sand| vG       | 0.100       | 3.010       | 0.952       | 0.9775|
|                   | FX       | 7.126       | 5.563       | 1.000       | 0.9966|
|                   | BC       | 1.427       | 5.731       | --          | 0.9991|
| Columbia sand     | vG       | 0.100       | 2.356       | 3.320       | 0.9590|
|                   | FX       | 3.579       | 6.155       | 0.969       | 0.9974|
|                   | BC       | 1.429       | 2.849       | --          | 0.9956|

3 EFFECT OF DRY DENSITY ON PARAMETERS OF SWRC EQUATION

ZHOU’s studies have shown that the pore ratio, hydraulic path and other factors affect the soil-water retention of the soil. This section uses the vG equation, FX equation, and BC equation to fit the SWRC test results of remolded clay and Columbia sand at different initial dry densities to analyze the effect of initial dry density on parameters of SWRC equations.

3.1 Remolded clay

The fitting results of the experiment data of the remolded clays of different initial dry densities of each equation are shown in Fig. 2 and Table 3. The reciprocal of the vG equation parameter a is 38.3, 46.8, and 43.6. The fitting results show that the initial dry density is approximately proportional to the air entry value. However, dry density is not the only factor affecting the air entry value, and the pore distribution also affects the air entry value. The parameter n represents the pore size distribution in the soil. In the fitting result, the parame-
ter $n$ decreases with the increase of the dry density, which indicates that the large pores are gradually compressed into small pores as the density increases. The parameters $a'$ and $n'$ of the FX equation increase as the dry density increases, and the parameter $m'$ decreases as the dry density increases.

According to the meaning of the parameters of the BC equation, as the dry density of the soil sample increases, the drainage of the soil deteriorates and the air entry value increases. The pore distribution coefficient decreases as the dry density increases, and the pore distribution changes.

Table 3. Fitting results of SWRC test data.

| Equation | Initial dry density (g/cm$^3$) | Parameter 1 | Parameter 2 | Parameter 3 | $R^2$  |
|----------|---------------------------------|-------------|-------------|-------------|-------|
| vG       | 1.68                            | 0.0261      | 1.097       | 0.771       | 0.9961|
|          | 1.72                            | 0.0214      | 1.233       | 0.750       | 0.9972|
|          | 1.74                            | 0.0229      | 2.097       | 0.422       | 0.9981|
| FX       | 1.68                            | 51.30       | 1.097       | 0.771       | 0.9959|
|          | 1.72                            | 55.83       | 1.135       | 2.206       | 0.9969|
|          | 1.74                            | 54.19       | 1.827       | 1.432       | 0.9975|
| BC       | 1.68                            | 0.490       | 11.43       | --          | 0.9916|
|          | 1.72                            | 0.495       | 13.47       | --          | 0.9856|
|          | 1.74                            | 0.603       | 24.46       | --          | 0.9842|

3.2 Columbia sand

The fitting results of the columbia sand with different void ratio are shown in Fig. 3 and Table 4. The results show that the fitting correlation coefficient of the vG equation is only 0.97-0.98, which is inferior to the fitting effect of the other two equation.

The first parameter of the fitting equations is 0.1, which means the air entry value of the soil samples are all around 10kPa. This shows that the vG equation cannot accurately reflect the air entry value of sand. The FX equation and the BC equation fit well to the test data, and the correlation coefficients are all above 0.99. The parameter $a$ in the BC equation is a good representation of the law that the air entry value of the soil sample gradually increases with the decrease of the initial void ratio.

Table 4. Fitting results of SWRC test data.

| Equation | Void ratio | Parameter 1 | Parameter 2 | Parameter 3 | $R^2$  |
|----------|------------|-------------|-------------|-------------|-------|
| vG       | 1.193      | 0.100       | 2.35579     | 3.31977     | 0.95825|
|          | 1.075      | 0.100       | 2.39483     | 2.89382     | 0.98569|
|          | 0.984      | 0.100       | 2.77451     | 2.05787     | 0.96977|
| FX       | 1.193      | 3.57867     | 6.15504     | 0.96911     | 0.99742|
|          | 1.075      | 4.085       | 5.98146     | 1.01111     | 0.99500|
|          | 0.984      | 4.97568     | 6.57004     | 0.96714     | 0.99598|
| BC       | 1.193      | 1.42492     | 2.84921     | --          | 0.99561|
|          | 1.075      | 1.56128     | 3.38232     | --          | 0.99908|
|          | 0.984      | 1.61767     | 4.14527     | --          | 0.99826|

4 SWRC PREDICTION METHOD

Zhou (2012) proposed a quantitative description of the effect of initial void ratio on unsaturated soil-water retention curves based on the coordination constraints of solid, liquid and gas three-phase deformation and the difference between macro and micro pore water. The method gives the evolution equation of saturation with void ratio:
where $S_e$ is the saturation degree, $e$ is the void ratio, and $\zeta$ is a fitting parameter. Under the equal stress condition, the void ratio $e$ is only related to the initial state and can be degenerated to the initial void ratio $e_0$. Parameter $\zeta$ can control the moving range of SWRC, which affects the shape of SWRC as a scaling factor. For the same type of soil, even if the hydraulic path is the same as the suction, the saturation of the soil will increase as the void ratio decreases, as shown in Fig. 4. The accuracy of this method is good, but it is more complicated to calculate. This paper attempts to propose a simple prediction method based on the SWRC curve with known initial dry density.

Fig. 4. Diagram of SWRC with different void ratios.

In Fig. 4, the saturations on the $a$, $b$, and $c$ point curves corresponding to the same suction are $S_{rai}$, $S_{rbi}$, and $S_{rci}$, respectively. It is assumed that there is a linear correspondence between the initial void ratio and the saturation under the same suction, so the relationship is satisfied:

$$\frac{S_{rai} - S_{rbi}}{S_{rbi} - S_{rci}} = \frac{e_0 - e_c}{e_a - e_b}$$

(5)

Based on this formula, the SWRC curve of any initial void ratio to soil sample can be predicted based on the known fitting equation of the soil water retention test data of the initial void ratio to soil sample. The calculation process is as follows:

$$S_e = \frac{(e_a - e_s)S_{rai} - (e_a - e_c)S_{rci}}{e_a - e_c}$$ when $e_s > e_a > e_c$

$$S_e = \frac{(e_a - e_s)S_{rai} + (e_b - e_c)S_{rbi}}{e_b - e_c}$$ when $e_s > e_b > e_a$ (6)

$$S_e = \frac{(e_a - e_s)S_{rai} - (e_a - e_c)S_{rci}}{e_a - e_c}$$ when $e_s > e_a > e_c$

(1) According to the experimental data of two sets of different initial pore ratios ($e_1$ and $e_2$), two sets of parameters are obtained by fitting the same soil-water retention curve equation. (2) Calculate the saturation sum corresponding to the target suction using the two sets of equations obtained by fitting, using formula 6. (3) For any initial porosity ratio $e_s$ soil sample, the formula for calculating the saturation under the target suction is:

In order to verify the effectiveness of the method, the BC equation was used to verify the results of the Touchet silt test at different initial void ratios. The BC equation parameters were obtained by fitting the soil sample test results with initial void ratios of 0.972 and 0.754 and were (4.898, 1.322) and (6.592, 1.471). Based on the parameters obtained by fitting, the results of soil samples with an initial void ratio of 0.862 were predicted. The relevant experiments, fitting, and prediction results are shown in Fig. 5. It can be seen from the prediction graph and test data that the curves predicted by the method compare very well with the test data for this soil.

5 CONCLUSIONS

The soil-water retention curve, SWRC, plays an important role in calculating soil-water problems. Research shows that this curve is not a characteristic, but is influenced by the initial dry density or void ratio. Several conclusions can be drawn based on this study:

(1) The applicability of three typical SWRC equations was analyzed by fitting the results of SWRC test data for four different types of soil samples. The vG equation has a good fitting effect only on clay and expansive soil; the fitting accuracy of the FX equation on various soil sample test results is high, but the meaning of each parameter in the equation is not clear enough. The BC equation only fits well to the experimental data of sand and silty soil.

(2) As the initial dry density of the soil sample increases, the air entry value will gradually increase. The reciprocal of the parameter $a$ of the vG equation directly characterizes the magnitude of the air entry value. The fitting results show that the parameter $a$ can reflect the variation of the air entry value with the dry density. However, the sand does not show a similar law. On the contrary, the parameters of the BC equation show the above regularity for all types of soil.
(3) A SWRC prediction method considering the influence of the initial void ratio is proposed. The method assumes a linear corresponding relationship between initial void ratio and saturation under the same suction. The comparison between the experimental data and predictions is of acceptable accuracy.

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