1. Introduction

In the pavement design methodology the typical approach is to apply the small deformation theory of elasticity or visco-elasticity (Akbulut, Aslantas 2005; Kim 2009). Road pavement model in the form of a multi-layered half-space with axi-symmetry assumption is solved using analytical, numerical or mixed (analytical-numerical) methods. Some selected components of the stress and strain fields at certain points are determined: for example, the value of the vertical stress component transmitted to the sub-base or the horizontal strain component in the bottom of asphalt layers, etc. The material parameters for this calculations should be adopted on the basis of experimental tests carried out in laboratory on samples prepared in laboratory or on in situ measurements made using e.g. falling weight deflectometer (FWD). The obtained results together with the design assumptions for traffic categories provide a starting point for estimating the fatigue life of the road structure. Durability is estimated based on empirical equations (Kim 2009) and the results of fatigue tests (Chkir et al. 2009; Mangiafico et al. 2015; Matallah, La Borderie 2009). Fatigue tests are generally carried out on four-point bending beams in a displacement control mode specifying function of stiffness modulus as a function of number of cycles (Ning et al. 2013; Pronk 2012). These functions are determined for a few values of displacements (strain in the most distant fibers from the axis of the beam) to form the so-called full fatigue characteristic – in case of symmetric with respect to zero sinusoidal displacement function. Sometimes the term degradation is reserved for elasticity properties (Lemaitre, Chaboche 2002), and damage is used for plasticity properties (Lubliner 1990). In this paper for both the "degradation" term is used. This paper is an extension of the idea shown in conference paper (Ga
djewski, Jemioło 2010) and presents the influence of degradation not only in the properties of elasticity but also plasticity on the nature of the failure mechanism of road structure. Degradation of the elastic properties was taken into account by the Young's modulus, and degradation of the plastic properties of the material by a cohesion present in the formulation of elasto-plasticity with the Coulomb-Mohr yield condition. This paper presents the study of the influence of parameters, such as Young's modulus and cohesion, on structural damage development in the pavement. However, it should be noted that the application of the limit analysis in the designing of the road structure (that is, in fact, taking into account the plastic or visco-plastic material properties) seems reasonable, especially in relation to the structure composed with the materials degraded with respect to their elastic and plastic properties.
Of course, currently in many finite element method (FEM) programs constitutive models taking into account the degradation (damage) are available (in most cases degradation is described with scalar parameter (Carol et al. 2001; Lemaitre, Chaboche 2002; Sullivan 2008), however, due to the fact that the road structures are subjected to a large number of cycles, it is impossible to carry out numerical calculations taking into account the entire history of the loading (e.g. because of the numerical computation cost, overlapping numerical error, etc.). However, this type of calculation can be performed for a limited number of cycles to determine the qualitative conclusions in relation to the distribution of degradation/damage parameter in different pavement layers, i.e. to determine which of them are the most degraded, and which actually are not subjected to degradation. In this work, for simplicity in the following calculations it was assumed that all pavement layers have been degraded to the same extent. Another issue that should be taken into account in the calculations together with cyclical nature of the pavement loading is the phenomenon of material (structure) adapting to repeated load. This type of analysis in relation to the road structures has been carried out in (Chazallon et al. 2009) and can be also studied in (Lemaitre, Chaboche 2002).

2. Constitutive relationships for materials modelling

The paper deals with modelling of vehicle wheel interaction with pavement of layered construction without taking into account a typical contact formulation. The layered structure of the pavement is modelled as axi-symmetrical and load from vehicle is modelled through stress type boundary conditions. The problem was formulated with application of the large deformation plasticity theory in which a non-associated flow rule is assumed (Lubliner 1990; Crisfield 1997; Gajewski, Jemioło 2010). The finite element software ABAQUS was used as a tool for solution of the formulated task. In applied theory the additive decomposition of total strain rate \( \dot{d} \) into elastic part \( \dot{d}_e \) and plastic (permanent) part \( \dot{d}_p \) was used by analogy to classical small deformation theory of elastic-plastic materials. However, differently from classical theory, the strain in current configuration of the body is described with logarithm of left stretch tensor \( V \). Tensor \( V \) results from polar decomposition of \( F \) which is a deformation gradient tensor with positive determinant (Lubliner 1990). The invariant \( J = \det F < 0 \) locally characterizes material volume changes. Because for considered boundary value problem (BVP) strains and local rotations are relatively small than the logarithm from left stretch tensor was chosen as a strain measure. Thus, the following decomposition is assumed:

\[
\dot{d} = \dot{d}(\ln V_e) = \dot{d}(\ln V_e) + \dot{d}(\ln V_p) = \dot{d}_e + \dot{d}_p, \quad (1)
\]

where \( V \) – left Cauchy’s stretch tensor; \( V_e \) – elastic part of the left Cauchy’s stretch tensor; \( V_p \) – plastic part of the left Cauchy’s stretch tensor.

The elasticity constitutive relationship of linear isotropic materials written in the following form:

\[
\sigma = \lambda(\ln J_e) I + 2\mu \ln V_e, \quad (2)
\]

where \( \lambda(N), \mu(N) \) – Lame’s constants, as functions of number of cycles \( N; J_e \) – determinant of tensor \( V_e, \sigma \) – Cauchy’s stress tensor (symmetrical and called “true stress”).

In case of the materials like mineral-asphalt mixes, cement concrete and most of the soils the Coulomb-Mohr yield condition (CM in Fig. 1) can be assumed valid in most cases. In analysed problem also the CM yield condition with hemitropic hardening evolution depending on equivalent plastic strains is used (Lubliner 1990). It can be written in the following form:

\[
\bar{F}(\rho, q, \theta) - c(\bar{\pi}^{pl}, N) = 0, \quad (3)
\]

where \( (\rho, q, \theta) \) – three “true stress” tensor invariants expressed by

\[
\rho = \frac{1}{3} \text{tr} \sigma = \frac{1}{3} \bar{\pi}, \quad q = \sqrt{\frac{3}{2} \text{tr} s^2} = \sqrt{\frac{3}{2} \|s\| \cos(3\theta)} = \frac{\text{tr} s^3}{q^3}, \quad (4)
\]

and \( c(\bar{\pi}^{pl}, N) \) – a cohesion as a function of number of cycles and \( \bar{\pi}^{pl} \) stands for equivalent plastic strains \( \bar{\pi}^{pl} = \bar{\pi}_p + \int_0^t \sqrt{\dot{\pi}_p} \dot{\pi}_p dt \). In Eq (4) \( s = \sigma - \rho I \) is deviatoric part of \( \sigma \) stress tensor and \( \|s\| \) is a norm of the deviatoric part of this tensor. Finally the CM yield condition in an invariant form can be written as follows:

\[
F(p, q, \theta) = q R_{mc}(\theta, \phi) + p \tan \phi, \quad (5)
\]
where yield condition meridian cross-section depends on shear angle \( \phi \) and third stress invariant \( \theta \):

\[
R_{mc}(\theta, \phi) = \frac{1}{\sqrt{3 \cos \phi}} \sin \left( \theta + \frac{\pi}{3} \right) + \frac{1}{3} \cos \left( \theta + \frac{\pi}{3} \right) \tan \phi.
\]  

(6)

The plasticity flow function \( G(\sigma) \) (denoted as MW in Fig. 1) is a smooth approximation of CM yield condition in the form given by (Menétrey, William 1995):

\[
G(p, q, \theta) = \left( \frac{\varepsilon (\varepsilon^p \theta) \tan \psi}{3} \right)^2 + \left( R_{mw}(\theta, e) q \right)^2 + \frac{p \tan \psi}{3} = 0.
\]  

(7)

The plasticity flow function \( G(\sigma) \) is a smooth approximation of CM yield condition in the form given by (Menétrey, William 1995):

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\]  

(7)

The dependence between \( R_{mw}(\theta, e) \) and \( R_{mc}(\pi/3, \phi) \) thus between meridians of approximation and CM functions written in the following form:

\[
R_{mw}(\theta, e) = \frac{4 \left( 1 - e^2 \right) \cos^2(\theta) + (2e - 1)^2}{2 \left( 1 - e^2 \right) \cos(\theta) + (2e - 1) \sqrt{4 \left( 1 - e^2 \right) \cos^2(\theta) + 5e^2 - 4e}} R_{mc}(\pi/3, \phi).
\]  

(8)

where \( e \) – function shape parameter with interpretation shown in Fig. 1; \( e \) – parameter defining of approximation smoothness; \( \psi \) – dilatation angle. The parameter \( e \) is assumed as equal to: \( e = \frac{3 - \sin \phi}{3 + \sin \phi} \). Due to the convexity assumption of the set bounded with yield condition, the following constraints (Menétrey, William 1995): \( 0.5 < e \leq 1 \) for parameter \( e \) have to be satisfied.

Of course there is possibility to use more sophisticated constitutive models specially developed for road materials (Kim 2009) but in some cases it is difficult to obtain parameters for them. Model presented here needs two parameters for elasticity and three \( (c_0, \phi, \psi) \) for plasticity. Still there is need for Young modulus and cohesion as functions of the number of cycles. The first function obtained from the test which in the road engineering industry is a standard one (Ning et al. 2013; Pronk 2012) (4PB, 3PB etc.) but the other is not.

3. Boundary value problem formulation

In the analysed BVP the interaction of wheel with typical pavement structure is dealt with (Gajewski et al. 2007; Gajewski, Jemioło 2010). The load from the wheel is modelled through stress boundary conditions, and the pavement is analysed as an axi-symmetrical layered half space (Fig. 2).

In the analysed problem the wheel loading is applied through increasing pressure \( p_0 \) evenly distributed over circular sub-region (with the 10 cm radius). In Fig. 2 the constant pressure modelling loading is shown on ED boundary segment. The proper density of the mesh for (FEM) solutions was determined on the basis of elasticity task correctness of displacement estimation, and also the modelled region of 2.5×2.5 m dimensions seems enough (also having in mind that infinite finite elements are going to be applied). The mesh in sub-region close to load application is much finer than around boundaries where displacement type boundary conditions are assumed (AB and BC) and also the thicknesses of each layer has influenced somehow mesh density. Finally, FEM mesh build from three and four node elements (with linear shape functions) consists of 6248 elements and 6283 nodes. In BVP domain modelling there is essential need for usage of infinite finite elements at the boundaries (AB and BC) to model real infiniteness of the analysed region (Zienkiewicz et al. 1983). The formulated FEM task was solved in two steps. In first step (step I) load linearly increases to its extreme value \( p_0 \), while in the following step (step II) the pavement is unloaded.

For each pavement layer the material parameters’ values were assumed on the basis of (Gajewska et al. 2012; Gajewski, Jemioło 2010; Raad, Minassian 2005) and are shown in Table 1. In constitutive Eq (2) the Lame’s constants appear. For convenience instead of them, having in mind the following relationships:

\[
\lambda = \frac{\nu E}{(1-2\nu)(1+\nu)}, \quad \mu = \frac{E}{2(1+\nu)},
\]  

(9)

Table 1. Values of materials parameters for all pavement layers in its original state (before degradation)

| Thickness and material parameters for each layer | \( h \), cm | \( E_0 \), GPa | \( \nu \) | \( c_0(*) \), kPa | \( \phi \), deg | \( \psi \), deg |
|-----------------------------------------------|------------|---------------|------|----------------|---------|---------|
| (W1) Asphalt concrete wearing layer           | 5          | 19.3          | 0.25 | 5000           | 20      | 15      |
| (W2) Concrete base                            | 7          | 18.1          | 0.25 | 5000           | 20      | 15      |
| (W3) Sub-base                                 | 20         | 0.4           | 0.3  | 60             | 45      | 22.5    |
| (W4) Subgrade improved with cement            | 215        | 0.12          | 0.3  | 80             | 15      | 5       |

Note: * – strain hardening was taken into account assuming that cohesion value has increased by 10% for equivalent plastic strains increasing to 100%.
the technical constants $E$ and $\nu$ are used.

In Table 1, $h$ stands for the layer thickness, $E_0$ for Young modulus before degradation (material in an original state) and $\nu$ for Poisson ratio.

The BVP was solved with FEM application for several sets of material parameters. Because of the lack of experimental data for multi-axial stress states which are crucial to obtain parameters for presented constitutive models, some rational assumptions regarding their values have been made. In case of elasticity the Poisson ratio was assumed as constant and in case of Young modulus for all layers 50% decrease of its value is assumed. Nevertheless real

![Fig. 3. Normalized Young modulus and normalized cohesion as a functions of cycle number in 4PB bending test with sinusoidal excitation with constant strain amplitude](image)

![Fig. 4. Typical results of the 4PB fatigue results—Young modulus norm as a function of cycle number with constant amplitude: A – porous asphalt with 50/70 binder 15 Hz in 10 °C, micro-strain = 350; B – HMA 10Hz in 10 °C, micro-strain = 130](image)

![Fig. 5. Contour graph of plastic strain norm $|e_p|$ at the end of step I for load equal to 6.5 MPa (materials without the influence of the degradation)](image) behaviour of materials is very close to the one presented in Fig. 3, thus it is possible to formulate reliable qualitative conclusions on that base. Typically for example for asphalt mixes the fatigue develops in three phases (Fig. 3) (Chkir et al. 2009; Ning et al. 2013). The results of test confirming this fact in the case of complex modulus norm is given in Fig. 4 for two typical mixtures and in (Chkir et al. 2009; Kim 2009). As there is no results available of the cohesion as a function of the number of cycles it assumed that this function is analogous to the modulus function.

### 4. Analysis of the FEM results

For better understanding how layered pavement structure works in the elastic-plastic state a contour maps of stress, strain and displacement fields are further presented. In Fig. 5 the contour graph of plastic strain norm at the end of step I for a task with the data as in Table 1, which corresponds to non-degraded materials, is presented. Similar problem was analysed in case of non-degraded materials in work (Gajewski et al. 2007), the load value of 6.5 MPa was assumed, which is around ten times more than typical pavement load (assumed for designing purposes). The same value was used, and it naturally means that application of real load on non-degraded pavement structure will not result in its failure.

Figure 6 shows various stages of plastic zone development for linearly increasing load to a value $p_0$ at the end of Step I, and then reducing the load to zero at the end of step II. Plastic zone begins to develop at the bottom of the layer W3 under the stress boundary condition. Plastic zone gradually increases (Fig. 6b) but still remains only in the layer of W3. Only when the load achieves the value approximately 0.6075 $p_0$ (Fig. 6c) plastic zone extends to the lower part of the layer W2. This process proceeds until a load equal to about 0.9075 $p_0$ (Fig. 6d) and in the final phase the plastic strain appears in W4 layer.

Further, the several boundary value problems were solved in which it was assumed that the initial Young's modulus decreased by half, and in turn cohesion was equal to: 0.15, 0.25, 0.35, 0.45, 0.55, 0.65 and 0.75 of the initial cohesion $c_0$. Selected results are presented in graphs in Figs 7–13.

A graph of a minimum (maximum of an absolute value) of $u_2$ displacement component in point $E$ as a function of applied load in steps I (loading) and II (unloading) is shown in Fig. 7. A significant increase in irreversible deformation with decreasing cohesion of materials is observed. The relationship is strongly non-linear (Mangiafico et al. 2015). In order to better understand this mechanism an extreme vertical displacement at point $E$ was plotted as a function of cohesion of the material at a constant value of $E = 0.5E_0$. From this graph results that the difference in displacement at the end of step II and I is almost constant as a function of cohesion. However, the permanent deformation increases rapidly when $c < 0.7c_0$ and for $c = 0.2c_0$ are several times higher than for non-degraded material.

In order to evaluate the qualitative effect of the degradation on the layered structure behavior in Fig. 9, the same way as in Fig. 6, the development of the plastic zone
for \( E = 0.5E_0 \) and \( c = 0.45c_0 \) is shown. At the beginning of the step I the development of plastic zone is similar in both cases, compare Figs 6a–6c and Figs 9a–9c and differences are apparent at the end of step II (Figs 6d and 9d). In the first case \((c = c_0)\) no plastic deformation occurred in the layer W1 and in W4 layer only to a small extent. In the second case \((c = 0.45c_0)\) at the end of step I, the plastic deformation appeared in the layer W1 and in W4 layer the plastic zone is heavily developed. After unloading of the structure, i.e. at the end of Step II in the second case the W1 layer is also affected by permanent deformation.

It can be concluded, that depending on the degree of degradation of the material (a change in the cohesion of materials for different layers) qualitatively different failure modes are observed. It corresponds also with totally different permanent deformation (the norm of plastic strain tensor) contour graphs, as shown in Figs 5, 10–11. It is worth noting that the 55% reduction in material cohesion causes an extreme increase in permanent strain norm by almost 500%. However, the displacement vector norm increases by an order of magnitude only (Fig. 12). Analysing the Mises equivalent stresses contour graph presented in Fig. 13, it is clear that residual stresses accumulated after loading beyond the plasticity limit in case of more degraded material have more substantial influence on overall capacity and initiated the deterioration not only the W2 layer but also W1 layer.

5. Limit analysis – parametric study

In case of the analysed BVP, the parametric study of the influence of degradation ratio of elastic (Young modulus) and plastic (cohesion) material properties on the bearing

![Fig. 6. The development of plastic zone for increasing load (non-degraded material): a – 0.2075p_0; b – 0.4075p_0; c – 0.6075p_0; d – p_0; e – unloaded](image)

![Fig. 7. Displacement component \( u_2 \) (in m) in point E as a function of pressure \( p \) at ED boundary](image)

![Fig. 8. The value of the vertical displacement \( u_2 \) (in m) in point E for fully loaded/unloaded pavement as a function of cohesion](image)
The development of plastic zone for increasing load (degraded material i.e. $E = 0.5E_0$ and cohesion $c = 0.45c_0$): a – 0.2075$p_0$; b – 0.4075$p_0$; c – 0.6075$p_0$; d – $p_0$; e – unloaded

capacity (load limit value $p_{max}^L$) of pavement structure was carried out. For this purpose in the constitutive model the strain hardening in plastic range is neglected, the assumed model is called elastic-perfectly plastic. This means that the limit state problem is analysed, i.e., the load at which the pavement is in failure mode is determined, and hence the stress and displacement velocity fields satisfy limit-analysis theorems (Lubliner 1990). In the present case, this leads to creation of a kind of wedge in the loaded region, which try to cut layered half space. With such formulation of the BVP using the FEM, only the starting point of this process specified, as in the following increments in the solution algorithm, even the smallest increase in load causes an imbalance of the system. In addition, the problem is the

Fig. 10. The contour graph of plastic strain tensor norm at the end of step II in case of material with $E = 0.5E_0$ according to Table 1 and cohesions $c = c_0$

Fig. 11. The contour graph of plastic strain tensor norm at the end of step II in case of material with $E = 0.5E_0$ according to Table 1 and cohesions $c = 0.45c_0$

Fig. 12. Contour graph of the displacement vector norm (in m) at the end of the step II, i.e., after unloading of the material for $E = 0.5E_0$ according to Table 1 and cohesions $a – c = c_0$, $b – c = 0.45c_0$
excessive deformation of FEM mesh which also leads to deterioration of the numerical solution.

However, several tasks have been solved with different values of parameters $c$ and $E$ ($c/c_0 \in [0.15, 1.00]$ and $E/E_0 \in [0.25, 1.00]$). The results are presented in Table 2 and in Fig. 14. The maximum value of limit load for the structure ($p_{max}^L = 24.3$ MPa) in the assumed range of parameters was received for $c/c_0 = 1$ and $E/E_0 = 0.5$. The minimum limit load value was obtained for $c/c_0 = 0.15$ and $E/E_0 = 1.00$.

Analysing the results presented in Table 2 and in the form of a graph in Fig. 14 seen that the value of Young’s modulus is of secondary importance in relation to cohesion in determining the capacity of the layered structure. For example, change the cohesion from 0.15$c_0$ to $c_0$ results in more than six-fold increase in capacity. Changes in Young’s modulus at a fixed value of cohesion generate only a few percent difference in the estimates of the bearing capacity of the structure. Interestingly the received load function is not uniformly monotonic relative to both arguments. There appear situations in which reduction in Young’s modulus while maintaining the cohesion of the structure gives higher limit load. The fact that the task is highly non-linear in this case is clearly revealed. Note, however, that these estimates of limit loads for layered structure were obtained using the same finite element mesh and the same control parameters for incremental algorithm. It is known that in the case of plasticity boundary value problems the influence of the mesh quality on the estimation of limit load is very significant (difference can be as high as several percent). Not without significance is the fact of maintaining the same incremental algorithm parameters for all tasks.

6. Discussion and final conclusions

Problems connected with the issue of modeling and rational design of road structures are still actual. The fact was shown in a number of publications (Loizos et al. 2007; Scarpas, Shourkry 2002). Separate layers of the road have significantly different rheological properties which, to different extent, decide how the structure “works” as a whole, (Gajewska et al. 2012; Gajewski et al. 2015). In road designing the solutions of boundary problems of continuum mechanics are used more often. Due to using them it is possible to explain many effects and phenomena developing locally in layers of the road. It was shown in (Loizos et al. 2007; Scarpas, Shourkry 2002; Zbiciak et al. 2016) that there is need to consider layered road structure (deformable pavement) in a frame of large deformation continuum mechanics, but still describing road materials in frame of small deformation theory is very complicated task (Chazzallon et al. 2009; Kim 2009; Sullivan 2008). Here the theory of large deformation with constitutive equations taking into account elastic-plastic properties and degradation caused by cyclic loading was used. On the basis of analysis of the issues presented in this paper it is possible to draw the following conclusions:

### Table 2. Influence of the degree of degradation of the elasticity and plasticity properties on the bearing capacity of the road structure (limit load value)

| $E/E_0$ | 0.15 | 0.25 | 0.35 | 0.45 | 0.55 | 0.65 | 0.75 | 1.00 |
|---------|------|------|------|------|------|------|------|------|
| $c/c_0$ |      |      |      |      |      |      |      |      |
| 0.25    | 0.17 | 0.30 | 0.42 | 0.40 | 0.59 | 0.68 | 0.75 | 0.94 |
| 0.50    | 0.16 | 0.25 | 0.40 | 0.52 | 0.48 | 0.70 | 0.90 | 1.00 |
| 0.75    | 0.20 | 0.25 | 0.37 | 0.64 | 0.60 | 0.76 | 0.93 | 0.98 |
| 1.00    | 0.14 | 0.29 | 0.41 | 0.44 | 0.55 | 0.70 | 0.75 | 0.95 |

Figure 13. Contour graph of the equivalent Mises stresses (in Pa) at the end of the step II, i.e. after unloading of the material for $E = 0.5E_0$ according to Table 1 and cohesions a – $c = c_0$, b – $c = 0.45c_0$

Figure 14. Influence of the degree of degradation of the elasticity and plasticity properties on the bearing capacity of the road structure (limit load value)
1. The proposed method of analysing the layered road pavement structure in frame of plasticity theory taking into account the degradation of the elasticity and plasticity properties is so simple that it could successfully be used in pavement design.

2. For standard arrangement of layers, always extreme permanent deformation is localized in the layers of sub-base or sub-soil, and not in the upper layers of asphalt concrete. Only for significant loads the destruction of the surface layer can be observed. It should be emphasized, however, that with each successive cycle of loading and unloading, beyond the yield condition of the individual layers, the enlargement of permanent deformation zone is observed.

3. Taking into account the degradation of elasticity and plasticity properties has not only the influence on reducing limit load, but also changes the nature of the structure behavior for next loading. In the case of structures with non-degraded materials the plastic zone starts to develop in the sub-base, and never reach the surface layer, and in the case of materials for which the degradation is taken into account also surface layers are accumulating permanent deformation.

4. In this article it was shown that taking into account, even in the simplest manner, the information about the degradation of the material in a quantitative and qualitative manner changes the nature of structural damage of the road. The aim of further work should be experimental studies performed on asphalt mixtures for multi-cyclic compressive and tensile loads, which can determine the cohesion of the material as a function of the number of cycles, see Figure 3. It should be noted that currently only test in which the complex modulus norm is determined as a function of the loading cycles is a standard one which is not sufficient to determine the parameters for materials plastic behaviour.

References

Akbulut, H.; Aslantas, K. 2005. Finite Element Analysis of Stress Distribution on Bituminous Pavement and Failure Mechanism, Materials and Design 26(4): 383–387. 
http://dx.doi.org/10.1016/j.matdes.2004.05.017

Carol, I.; Rizzi, E.; Willam, K. 2001. On the Formulation of Anisotropic Elastic Degradation. I. Theory Based on a Pseudo-Logarithmic Damage Tensor Rate, International Journal of Solids and Structures 38(4): 491–518. 
http://dx.doi.org/10.1016/S0020-7683(00)0030-5

Chazzallon, C.; Allou, F.; Hornych, P.; Mouhoubi, S. 2009. Finite Elements Modelling of the Long Term Behaviour of a Full Scale Flexible Pavement with the Shakedown Theory, International Journal for Numerical and Analytical Methods in Geomechanics 33(1): 45–70. 
http://dx.doi.org/10.1002/nag.702

Chkir, R.; Bodin, D.; Piaudier-Cabot, G.; Gauthier, G.; Gallet, T. 2009. An Inverse Analysis Approach to Determine Fatigue Performance of Bituminous Mixes, Mechanics of Time-Dependent Materials 13: 357–373. 
http://dx.doi.org/10.1007/s11043-009-9096-7

Crisfield, M. A. 1997. Non-Linear Finite Element Analysis of Solids and Structures. Vol. 2: Advanced Topics. Chichester-Singapore, John Wiley and Sons. 509 p.

Gajewska, B.; Kraszewski, C.; Gajewski, M.; Rafalski, L. 2012. Investigation of Resilient Moduli of Selected Hydraulically Bound Mixtures (HBM) Under Cyclic Load, Road and Bridges – Drogi i Mosty 11(4): 269–280. 
http://dx.doi.org/10.7409/rabdim.012.001

Gajewski, M.; Jemioło, S. 2010. Modelling of Pavement Failure Taking into Account Material Degradation Caused by Cyclic Loading, in Proc. of XIX - Slovak-Polish-Russian Seminar “Theoretical Foundation of Civil Engineering”, 12–16 September 2010, Žilina, Slovakia. Moscow: ACB, 103–110.

Gajewski, M.; Sybilski, D.; Bankowski, W. 2015. The Influence of Binder Rheological Properties on Asphalt Mixture Permanent Deformation, The Baltic Journal of Road and Bridge Engineering 10(1): 54–60. 
http://dx.doi.org/10.3846/bjrbe.2015.07

Gajewski, M.; Jemioło, S.; Maliszewski, M.; Mularzuk, R.; Sybilski, D. 2007. Modeling of Permanent Deformation Development in Multilayered Flexible Pavement, in Proc. of the International Conference on Advanced Characterization of Pavement and Soil Engineering Materials, vol. 1, Ed. by Loizos, A; Scarpas, T; Al-Quadi, I. 20–22 June, 2007, Athens, Greece. Taylor & Francis. 381–388.

Kim, Y. R. 2009. Modeling of Asphalt Concrete. ASCE Press, Mc Grow Hill. 459 p.

Lemaitre, J. L.; Chaboche, J. 2002. Mechanics of Solid Materials. Cambridge University Press, United Kingdom. 559 p.

Loizos, A.; Scarpas, T; Al-Quadi, I. (eds). 2007. Advanced Characterisation of Pavement and Soil Engineering Materials, Proceedings of the International Conference on Advanced Characterization of Pavement and Soil Engineering Materials. Taylor & Francis. 1858 p.

Lubliner, J. 1990. Plasticity Theory. Macmillan Publishing Company, New York. 544 p.

Mangiatico, S.; Sauzeat, C.; Benedetto, H.; Planque, L. 2015. Quantification of Biasing Effects During Fatigue Tests on Asphalt Mixes: Non-Linearity, Self-Heating and Thixotropy, Road Materials and Pavement Design 16(2): 73–99. 
http://dx.doi.org/10.1080/14680629.2015.1077000

Matallah, M.; La Borderie, C. 2009. Inelasticity-Damage-Based Model for Numerical Modeling of Concrete Cracking, Engineering Fracture Mechanics 76(8): 1087–1108. 
http://dx.doi.org/10.1016/j.engfracmech.2009.01.020

Menétrey, Ph.; William, K. J. 1995. Triaxial Failure Criterion for Concrete and Its Generalization, ACI Structural Journal 92: 311–318.

Ning, L.; Pronk, A. C.; Molenaar, A. A. A.; van de Ven, M. F. C.; Wu, S. 2013. Comparison of Uniaxial and Four-Point Bending Fatigue Tests for Asphalt Mixture, Transportation Research Records 2373: 44–53.

Pronk, A. C. 2012. Description of a Procedure for Using the Modified Partial Healing Model (MPH) in 4PB Test in Order to Determine Material Parameters, in Proc. of the 3rd 4PB Workshop, Davis, California, USA.
Raad, L.; Minassian, G. 2005. The Influence of Granular Base Characteristics on Upper Bound Shakedown of Pavement Structures, *International Journal of Road Materials and Pavement Design* 6(1): 53–79. http://dx.doi.org/10.3166/rmpd.6.53-79

Scarpas, A.; Shourkry, S. N. (Eds). 2002. 3D Finite Element Modelling of Pavement Structures, in *Proc. of the 3rd International Symposium on 3D Finite Element for Pavement Analysis, Design and Research*. 2–5 April, 2002, Amsterdam, Holland, UB/TIB Hannover. 496 p.

Sullivan, R. W. 2008. Development of Viscoelastic Continuum Damage Model for Cyclic Loading, *Mechanics of Time-Dependent Materials* 12: 329–342. http://dx.doi.org/10.1007/s11043-008-9069-2

Zienkiewicz, O. C.; Emson, C.; Bettess, P. 1983. A Novel Boundary Infinite Element, *International Journal for Numerical Methods in Engineering* 19: 393–404. http://dx.doi.org/10.1002/nme.1620190307

Zbiciak, A.; Michalczyk, R.; Brzeziński, K. 2016. Evaluation of Fatigue Strength of Pavement Structure Considering the Effects of Load Velocity and Temperature Variations, *Procedia Engineering* 153: 895–902. http://dx.doi.org/10.1016/j.proeng.2016.08.222

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