Multi-rate co-simulation of power system transients using dynamic phasor and EMT solvers

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Abstract: This study presents a novel multi-rate algorithm for the co-simulation of power system transients using base-frequency dynamic phasor solver for frequency adaptive simulation of transient (BFAST) and electromagnetic transient (EMT) solvers. The BFAST solver alters its solution technique from dynamic phasors to EMT based upon the frequency contents of the waveforms being simulated. A changeover algorithm between the two solvers is also presented. The BFAST solver is then integrated with an industrial-grade EMT solver to develop a BFAST–EMT multi-rate co-simulator. The interface between the two solvers is established using transmission lines at the partitioning locations. The co-simulator combines the benefits of dynamic phasors, frequency adaptive simulation of transients, parallel processing, and multi-rate simulation. The study describes the several solution modes of the proposed co-simulator. Illustrative examples are included to demonstrate the accuracy and computational benefits of the proposed co-simulator.

1 Introduction

Studies of transients in power systems are usually carried out with simulation tools such as transient stability (TS) and electromagnetic transient (EMT) solvers. These simulators use various numerical techniques and solution algorithms to model and solve the dynamic behaviour of a power system. The level of accuracy provided by each simulator generally has a commensurate effect on its computational complexity. TS simulations [1] focus only on slow electromechanical transients using quasi-steady-state assumption; TS simulators allow usage of large simulation time-steps of the order of milliseconds but face limitations in simulating fast-acting systems such as high-voltage DC converters and converter-tied distributed generators at high frequencies. EMT simulators [2, 3], on the other hand, represent fast transients over a large frequency range. However, given the small time-step size employed, their computational burden rapidly increases with the network size and complexity. This is not a limitation merely during transients; e.g. power electronic converters require small simulation time-steps even during normal operation while the rest of the system may require small time-steps only during transients. With conventional single-rate EMT algorithms, the entire network needs to be simulated with a small time-step even if high-frequency components and events are confined to small areas of the network.

To improve the computational efficiency of EMT solvers, several methods are proposed. Examples include parallel processing [4, 5] and network equivalents [6, 7]. Parallel processing requires specialised and expensive hardware, and network equivalents are often cumbersome to derive and need re-derivation if the network undergoes a change. In [8], a concept referred to as frequency-adaptive simulation of transient (FAST) was proposed for simulation in both time and frequency domains within a single solver. In this method, the simulator switches between the detailed EMT solution with a small time-step and the phasor solution with a large time-step by changing a simulation parameter, known as the shift frequency. This method is unique for its ability to capture fast transients and to accelerate the simulation. The challenge in this context is the representation of fast phenomena such as those in power electronic converters in power systems. The transition between EMT and phasor solutions is realised through the application of switching functions [9, 10]. A problem still to be addressed is the integration of power electronic converter models that accurately represent devices inside the converter.

Co-simulation (or hybrid simulation) [11, 12] combines two or more solvers to perform simulations of a single network by segmenting it into several subsystems and assigning them to distinct solvers based on the transient properties of each subsystem. This method marries the accuracy of an EMT solver with the computational efficiency of a less detailed solver (e.g. a TS solver) and enables simulation of large power systems. Co-simulators may use parallel processing and multi-rate techniques [13] to accelerate their computations. Recent developments in co-simulation are reported in EMT–TS co-simulation [14–18], EMT–dynamic phasor (DP) co-simulation [19–21], and EMT–DP–TS co-simulation [22]. Common problems with these models and algorithms include, but are not limited to, (i) reduced accuracy of the overall simulation when disturbances occur near the interface, (ii) possibility of one time-step delay between decoupled systems, (iii) inaccuracies of interaction in the presence of high-frequency waveforms, and (iv) lack of control on the simulation's computational burden.

This study proposes a new co-simulation platform, which combines the benefits of FAST, multi-rate simulation, parallel processing, DPs, and EMT for detailed and accelerated transient simulations of large power systems. Firstly, the study develops a base-frequency DP solver for frequency adaptive simulation of transient (BFAST) that is built upon the FAST method and takes advantage of the frequency shifting property of DPs. The study then shows the development of a novel BFAST–EMT co-simulator in which the external system is simulated with two time-steps in the context of the BFAST solver while the detailed system is simulated in the EMT solver. A novel changeover algorithm is developed to ensure an efficient and smooth transition between EMT and DP solutions. This co-simulator allows the accurate study of both external and detailed systems' dynamics with adjustable computational efficiency.

The proposed co-simulator is implemented using an industrial-grade EMT simulator (PSCAD/EMTDC), which enables direct usage of the already existing EMT models of complex components and systems. This feature is particularly appealing as it enables tapping into the vast libraries of models for sophisticated components and systems that are developed over several decades in
commercial EMT simulators, thereby facilitating the development of simulation cases of large and complex systems.

2 Brief overview of DPs

DP modelling provides efficiency and selectivity by decomposing the response waveforms into their harmonic components. Moreover, it enables the usage of large time-steps in frequency-domain simulation, which is computationally beneficial in large systems. Consider a time-domain signal, \( x(t) \), over the time window \((t - T, t)\). The Fourier series of \( x(t) \) over this time window is given in (1) [23]

\[
x(t - T + s) = \sum_{k=-\infty}^{\infty} \langle x \rangle_k e^{jk\omega_0(t - T + s)}
\]

where \( s \in (0, T) \), \( k \) is the harmonic order, and \( \omega_0 = 2\pi/T \). The \( k \)th Fourier coefficient of \( x(t) \) is

\[
\langle x \rangle_k(t) = \frac{1}{T} \int_0^T x(t - s) e^{-jk\omega_0(t - T + s)} ds
\]

The coefficient \( \langle x \rangle_k(t) \) is time-dependent and referred to as the DP of the \( k \)th harmonic of \( x(t) \). In practice, the infinite series in (1) is truncated to a finite number of terms depending on the accuracy required.

Equation (2) provides a simple way of extracting DPs from a time-domain signal. However, each harmonic component must be modelled separately using (2), thus creating a computational burden when multiple harmonics are present. A computationally efficient method that projects all the frequency contents including the dc component to the frame of the fundamental-frequency component is introduced in [19]. For this, (1) is re-written in the following form:

\[
x(t - T + s) = \langle x \rangle_0(t) + \text{Re} \left( 2 \sum_{k=1}^{\infty} \langle x \rangle_k(t) e^{jk\omega_0(t - T + s)} \right)
\]

\[
= \text{Re}(X_0(t)) e^{jk\omega_0(t - T + s)} + 2 \sum_{k=1}^{\infty} \langle x \rangle_k(t) e^{jk\omega_0(t - T + s)}
\]

is termed as the base-frequency DP of \( x(t) \). Note that there is only one frequency in the last term in (3), which includes all the harmonics of the signal. However, direct extraction of the base-frequency DP is still expensive as it requires computing each Fourier coefficient separately before combining them as shown. For this reason, an efficient method is used to extract the base frequency DP from a time-domain signal in accordance with [19].

Note that the Fourier series in (3) may be written as

\[
x(t - T + s) = \text{Re}(2\langle x \rangle_0(t)) e^{jk\omega_0(t - T + s)} + \sum_{k=\pm 1}^{\infty} \langle x \rangle_k(t) e^{jk\omega_0(t - T + s)}
\]

\[
= \text{Re}(2\langle x \rangle_0(t)) e^{jk\omega_0(t - T + s)} + X_k(t) e^{jk\omega_0(t - T + s)}
\]

where

\[
X_k(t) = \sum_{k=\pm 1}^{\infty} \langle x \rangle_k(t) e^{jk\omega_0(t - T + s)}
\]

\[
(x)_{0h}(t) = 2\langle x \rangle_0(t) + X_0(t)
\]

\[
= 2\langle x \rangle_0(t) + (x)_{0h}(t)
\]

\[
(x)_{0v}(t) = 2\langle x \rangle_0(t) - X_0(t)
\]

is a composite complex signal, which comprises all harmonic contents of \( x(t) \) except the fundamental. Equation (5) can be used to extract \( \langle X \rangle_{0h}(t) \) by first calculating the fundamental Fourier coefficient, \( \langle x \rangle_0(t) \), using (2) with \( k = 1 \). This is used to form the first term on the right-hand side of (5), which is then subtracted from \( x(t) \) to obtain the second term. From the second term \( X_0(t) \) can be readily obtained. Finally, the base-frequency DP is computed as

\[
\langle X \rangle_{0h}(t) = 2\langle x \rangle_0(t) + X_0(t)
\]

This averts computing individual coefficients and is efficient when \( x(t) \) has many harmonics. Additionally, it accurately captures all the frequency deviations of the time-domain signal (e.g. a 59 Hz signal in a 60 Hz system). In this study, all DP quantities shown are base-frequency DPs.

3 Novel solver for base-frequency phasor adaptive simulation of transients

Transients are normally confined to small-time intervals in a simulation. Before and after a transient, signals settle into periodic regimes. A conventional DP [24] is well suited to describe a sinusoidal function with an essentially constant amplitude and phase angle or slowly-varying ones but fails when fast and harmonic-rich changes are present. As opposed to the formulation in (2) that may need to rely on a significant number of coefficients, the base-frequency DP catches both steady-state and high-frequency transient regimes by preserving the entire harmonic spectrum. The BFAST solver presented in this section exploits this feature and adapts its solution method and time-step according to the frequency contents of the waveforms being simulated. During transients, it uses a detailed EMT-type solution with a small time-step and during steady-state reverts to a phasor-based solution with a large time-step. Note again that all DP quantities in this study are base-frequency DPs as defined in the previous section.

3.1 Frequency-dependent network modelling with DPs

The concept of changing the time-step during a simulation is built upon the ability of the solver to switch between the DP and the EMT solutions. For this purpose, discretised frequency-dependent equivalents of network components are developed using base-frequency DPs and the trapezoidal integration rule for discretisation as shown below for basic circuit elements. More details may be found in [19, 25–27].

3.1.1 Resistor:

\[
\langle i \rangle_{Rh}(t) = R\langle i \rangle_{R}(t)
\]

3.1.2 Inductor:

\[
\langle i \rangle_{Lh}(t) = \gamma_L\langle i \rangle_{L}(t) + I_{Lh}(t)
\]

where

\[
\gamma_L = \frac{\Delta t}{2\pi(1 + j\omega_0\Delta t/2)}
\]

\[
I_{Lh}(t) = \frac{1 - j\omega_0\Delta t/2}{1 + j\omega_0\Delta t/2}\langle i \rangle_{L}(t - \Delta t)
\]

\[
-\gamma_L\langle i \rangle_{L}(t - \Delta t)
\]

3.1.3 Capacitor:

\[
\langle c \rangle_{Ch}(t) = \gamma_C\langle c \rangle_{C}(t) + I_{C,Ch}(t)
\]

where
\[
\chi_c = \frac{2C}{\Delta t} \left( 1 + \frac{io\omega\Delta t}{2} \right)
\]
(13)

\[
I_{C_U}(t) = -\left( i\chi_c \omega t - \chi_c \omega t \right) - \chi_c \omega \chi_c \omega(t - \Delta t)
\]
(14)

Using these, the network admittance matrix becomes a function of the time-step, \(\Delta t\), and the base frequency term, \(\omega_0\), which is also referred to as the shift frequency. Examination of (8)-(14) reveals that for \(\omega_0 = 0\) (i.e. no shift of the harmonic spectrum) these models reduce to Dommel's EMT companion models [2]. Setting \(\omega_0\) to the fundamental frequency of the system yields DPs at that frequency; however, since base-frequency DPs are used, no harmonic information will be lost, because all harmonics have been transferred into the frame of the fundamental frequency component. The ability of the developed models to be either DP or EMT models enables solving the network with either approach in a single solver only by setting the shift frequency \(\omega_0\) and using a suitable time-step, which provides a way to adapt the solver, i.e. DP versus EMT, to the nature of the waveforms being simulated. Earlier work in this area [8] has identified this dual-approach possibility albeit in the context of shifted-frequency analysis.

3.2 Proposed algorithm for solver changeover

To switch between DP and EMT solvers requires determining the correct instant to change \(\omega_0\) and calculating history current sources in the proper domain. The algorithm proposed in [28] uses analytic signals together with adaptive frequency shifting for a smooth changeover; the algorithm presented hereinafter processes the base-frequency DP. Assume that the DP and EMT solutions are obtained with a large time-step, \(\Delta t_{DP}\), and a small time-step, \(\Delta t_{EMT}\), respectively, and that the solver switches from DP to EMT at time \(t_s\) and back to DP at time \(t_f\).

3.2.1 Changeover from DP to EMT solution: At time \(t = t_s + \Delta t_{EMT}\) (see Fig. 1a), the solution requires all the sources that represent previous time-step values calculated in time-domain. For that, all the variables at \(t = t_s\), which are already calculated in the phasor-domain, must be converted to time-domain before the solution at subsequent time-step is obtained. This can be readily done using (3).

The need for switching to the EMT solution arises only when there is a disturbance in the system. The changeover instant from DP to EMT can be determined either by monitoring the admittance matrix or simply by means of pre-specified instants of network disturbances that are set by the user. The latter is particularly practical when the eventual objective of the development of the BFAST solver is to co-simulate with an industrial-grade EMT solver. If the inception point of the disturbance and DP solution does not coincide on the discretised time axis, the solution can be interpolated to the actual inception point before the changeover or the changeover point can be set to the time-step just before the disturbance.

3.2.2 Determining the end of the transient: Once the solution method is changed to EMT (i.e. with \(\omega_0 = 0\)) reverting to the DP solution is done only after the transient has settled into a steady state. This needs a criterion to automatically determine the end of the transient, which is shown in Fig. 1b. As soon as DP to EMT changeover occurs, the solver begins to calculate the Fourier coefficients of the signals that are affected by the disturbance. These signals are normally confined to small areas of the network where the impact of the transient is most severe. For example, in the event of a fault, the voltages and currents of the buses in the electrical vicinity of the fault's location are often sufficient for this purpose. To avoid calculations at each time-step and to maintain low computational burden, Fourier coefficients are only calculated once per fundamental-frequency cycle. As a result, the computational burden of this task is negligible compared with the computations of the actual network solution. If the calculated coefficients match the corresponding coefficients in the previous cycle within a small pre-specified tolerance, then the solver decides that the transient has ended. The number of Fourier coefficients compared is based upon the harmonics present during normal operation of the system; this implies that a comparison of the fundamental coefficient is sufficient for most systems as they are expected to be operating at the fundamental frequency in steady state.

3.2.3 Changeover from EMT solution to DP solution: Once the simulator detects that the transient has settled, it begins to extract the DPs of each variable from time-domain instantaneous data (note that the solver is still operating with EMT solution). This can be readily achieved using the method in Section 2, for that the operation has to be carried over an exact period. Therefore, changeover from EMT to DP solution is set at \(t = t_s\), which is exactly one cycle from the end of transient detection as illustrated in Fig. 1c.

3.3 Simulation results

The circuit in Fig. 2 is simulated using the BFAST solver with \(\Delta t_{DP} = 1000\) and \(\Delta t_{EMT} = 10\) ms. Simulation results of transients due to opening the circuit breaker at \(t = 0.3\) s and closing it at \(t = 0.6\) s are depicted in Fig. 3. Waveforms obtained by simulating the same circuit entirely in PSCAD/EMTDC are used as a benchmark. The BFAST results include the envelopes of the waveforms (magnitude of the DPs) when the DP solution is obtained and the detailed waveform for the parts when the EMT solution is found. Fig. 3 readily shows that the BFAST uses the EMT solution \((\omega_0 = 0)\) during the start-up transient and the transients due to circuit breaker operation. At the end of each transient (detected automatically by the simulator), the simulator switches to the DP solution by changing the shift frequency to \(\omega_0 = 2\pi\times60\) rad/s and uses a large time-step to accelerate the simulation. In Figs. 3a and b, the BFAST solver switches from the EMT to the DP solution.
after the transients in the waveforms have settled sufficiently, i.e. around \( t = 0.1 \) and 0.7 s, when the decaying dc component is negligibly small. Fig. 3c shows the impact of solver changeover from EMT to DP well before \( t = 0.1 \) and 0.7 s, when the decaying dc component is still present. As seen, the solver is still able to capture the envelope of the settling waveform; however, small oscillations in the captured amplitude are observed that are due to the shifting of the dc component to the frame of the fundamental component.

4 Novel BFAST–EMT co-simulator

Although the BFAST simulator can solve a network as a DP and an EMT solver, it requires both EMT and DP models. This limits the applicability of the solver as a stand-alone platform. DP models of many components such as static converters may not be readily available or may need to be developed for specialised operating conditions such as commutation failure or imbalances. It is, therefore, logical and practical to envision a co-simulation platform wherein an existing EMT solver and BFAST are used. Such an environment lends itself to several simulation modes, as depicted in Fig. 4.

(I) EMT–EMT multi-rate co-simulator (mode-1)

In this mode, both external and detailed subsystems are simulated with the EMT method. The two sides may use different time-steps (larger in the external subsystem). This mode is useful when the external subsystem consists of fast—but comparatively slower—dynamics compared to the detailed subsystem. For example, with a switching converter, only the electrical vicinity of the converter may need a small time-step.

(II) DP–EMT multi-rate co-simulator (mode-2)

In this mode, the external subsystem is simulated using DPs with a large time-step and the detailed subsystem is simulated with the EMT solution using a small time-step. This mode can be used if the external subsystem dynamics are negligible or very slow.

(III) BFAST–EMT multi-rate co-simulator (mode-3)

In this mode, the solution method and time-step in the external system switch between DP and EMT. The detailed subsystem is simulated with the EMT solution using a small time step. Note that this mode is a superset of the other two. Thus, in this study, only this mode is explained. Regardless of the co-simulation mode, the BFAST and EMT solvers need to be interfaced and a signal conversion mechanism is required to exchange data between them. These are explained next. Before proceeding, it must be noted that the solver changeover from EMT to DP follows the same logic outlined in Section 3 and is based upon a calculation of the Fourier coefficients of signals. In the BFAST–EMT co-simulator, the variables selected for harmonic assessment are the interface voltages and currents. These are only a small number of variables and do not introduce any considerable computational burden. In the BFAST–EMT co-simulator, EMT to DP changeover is only performed when the dc and harmonic components of the waveforms are fully diminished.

4.1 Partitioning at a transmission line

Co-simulation needs partitioning the network into subsystems and forming interfaces between them. Network partitioning for explicit coupling introduces one time-step delay for data exchange between solvers. Applications proposed in [10, 18] ignore this delay when interfacing with external systems. However, if not properly compensated, this delay may cause phase errors and numerical instability, particularly when one side uses a large time-step.

There are several delay compensation methods in the literature. The method proposed in [29] uses extrapolation, and its accuracy drops when there is a discontinuity in signals or when a large solution time-step is used. The interface used in [30] adopts multi-area Thevenin equivalent technique to partition the network. This method avoids the partitioning delay but at the expense of added computational burden. The transmission line interface based on the Bergeron model [3] is widely used to partition a network using the natural wave propagation time through the line [15, 19]. The main restraint of this interface is the maximum time delay that can be compensated, which is equal to the wave travel time through the line and is limited by its length. The transmission line method is used to form the interface considering its simplicity and robustness. Other forms of interface may also be used with minimal implications. Fig. 5 shows the model of a lossless line between nodes K and M, where, \( Z_c = \sqrt{L/C} \) is the surge impedance of the line. If the wave propagation delay of the line is \( \tau \), then current injections at each node are

\[
h_d(t) = \frac{2v_d(t-\tau)}{Z_c} - h_k(t-\tau) \tag{15}\n\]

\[
h_k(t) = \frac{2v_k(t-\tau)}{Z_c} - h_d(t-\tau) \tag{16}\n\]
Assume that the time-step of the EMT solver is $\Delta_{\text{EMT}}$, then one can choose BFAST solver's time-steps as

$$\Delta_{\text{DP}} = N_e \Delta_{\text{EMT}}$$

where $N_e$ and $N_i$ are integers. $N_i$ determines the time-step ratio when both solvers operate with the EMT method, i.e. mode-1. $N_e$ can be set based upon the maximum time-step allowed for the DP solution, which is equal to $\tau$ (the wave propagation delay through the transmission line interface). The interaction between the BFAST and the EMT solvers for $N_i = 1$ ($\Delta_{\text{DP}} = \Delta_{\text{EMT}}$) and $N_i = 4$ is shown in Fig. 6.

When the BFAST solver operates with the EMT method, the current and voltage values at the interface buses of each side are used to update the current sources of the other side of the interface after a delay of $\tau$ using (15) and (16) as shown on the left side of Fig. 6. When $N_i$ is larger than one, the intermediate values of the BFAST solver can be interpolated to balance the granularity of data samples.

Once the BFAST solver switches to the DP solution with a large time-step, a sophisticated algorithm is required to (i) convert data between time-domain and frequency-domain signals and (ii) balance the granularity of data samples. It is important to note that the interface current source of the BFAST side at node-K must now be updated with DP values. For that, (16) needs to be transformed to the DP domain as follows:

$$\langle h_k \rangle_{\text{DP}}(t) = \frac{2\langle v_M \rangle_{\text{DP}}(t - \tau) - \langle h_k \rangle_{\text{DP}}(t - \tau)}{Z_c} e^{j\omega \tau} - \langle h_k \rangle_{\text{DP}}(t - \tau) e^{j\omega \tau}$$

Equation (18) requires the extraction of DPs of both the current and voltage from the other side of the interface, which increases the number of computations as well as conversion errors. This can be readily avoided if (18) is rewritten in the following recursive form by eliminating current samples:

$$\langle h_k \rangle_{\text{DP}}(t) = \frac{2\langle v_M \rangle_{\text{DP}}(t - \Delta_{\text{DP}}) - \langle h_k \rangle_{\text{DP}}(t - \Delta_{\text{DP}})}{Z_c} - \frac{2\langle v_M \rangle_{\text{DP}}(t - 2\Delta_{\text{DP}}) - \langle h_k \rangle_{\text{DP}}(t - 2\Delta_{\text{DP}})}{Z_c} e^{j\omega \Delta_{\text{DP}}}$$

Calculation of the interface current source on the EMT side, i.e. $h_0(t)$, can be done using (15) once the DP quantities are converted to natural time-domain values. Extracting $\langle v_M \rangle_{\text{DP}}(t - \Delta_{\text{DP}})$ required for (19) and converting to time-domain can be done as explained in Section 2. Once data conversion is established, the process of data communication between the solvers commences. Consider the case shown in the right-hand side of Fig. 6, where $\Delta_{\text{DP}}$ is chosen to be the same as the travel time of the interfacing transmission line, $\tau$, and solutions corresponding to $t = t_{k-1}$ are known for both solvers. The following interactions take place:

(i) BFAST interface is updated as in (19) using history values and the EMT samples at $t = t_{k-1}$, which reach the BFAST solver after a delay of $\tau$.
(ii) The solution for the BFAST solver at $t = t_k$ is obtained.
(iii) Intermediate values of interface currents and voltages between $t_{k-1}$ and $t_k$ of the BFAST solver are linearly interpolated.
(iv) Saved and interpolated interface values are converted and used to update the EMT solver after a delay of $\tau$.

It is apparent from the above sequence that the solution of one solver between $t_{k-1}$ and $t_k$ is not affected by the operations in the other solver during the same period. This implies that the constituent simulators can operate in parallel as long as $\Delta_{\text{DP}}$ is kept $\leq \tau$. Fig. 7 shows the flowchart of the co-simulation algorithm.
BFAST–EMT co-simulator with $\Delta t_T = 20$, $\Delta t_{DP} = 100$, and $\Delta t_{EMT} = 20 \mu s$, and its dynamic performance is examined by applying a line-to-ground fault to bus-6, which is inside the EMT solver, at $t = 0.5 \text{s}$ for a duration of six cycles. To validate the accuracy of co-simulation results, the entire system is also simulated in PSCAD/EMTDC with a 20 $\mu$s time-step, and its results serve as the baseline for comparison. In the shown waveforms, portions where the DP solution is obtained are presented with the envelope of the waveform and the detailed instantaneous waveforms are displayed for the parts where the EMT solution is found.

**Table 1** Co-simulation test system specifications

| $S_1$, $S_2$, $S_3$ | 1.01∠11.5°, 1.02∠21.5°, 1.00∠0.0° |
|---------------------|--------------------------------------|
| $\pi_1$, $\pi_5$, $\pi_6$ | $R = 0.00168 \text{ pu}$; $X = 0.01333 \text{ pu}$; $B = 0.02770 \text{ pu}$ |
| $\pi_2$, $\pi_4$ | $R = 0.00336 \text{ pu}$; $X = 0.03306 \text{ pu}$; $B = 0.05550 \text{ pu}$ |
| $\pi_3$ | $R = 0.00115 \text{ pu}$; $X = 0.00911 \text{ pu}$; $B = 0.01830 \text{ pu}$ |
| $Q_C$ | 0.6 pu |
| load$_1$, load$_2$ | $1 + j0.25 \text{ pu}$, $5 + j2.5 \text{ pu}$ |
| system base | 100 MVA, 230 kV |

Co-simulation results of two systems, a 12-bus system [31] and the IEEE’s 118-bus system [32], using the proposed BFAST–EMT platform are shown in this section. These examples serve to show the accuracy and computational efficiency of the co-simulator.

### 5.1 Segmentation of the IEEE 12-bus system

The single-line diagram of the segmented system is shown in Fig. 12. Transmissions lines are modelled as lumped parameter $\pi$-sections. The network is partitioned at the 100 km line connecting buses 1 and 2 and the 300 km line connecting buses 4 and 5. Each interface line, i.e. B5m–B5k and B1m–B1k, is 60 km long. The remaining 40 km portion of the original B1–B2 line is modelled in the EMT subsystem and the remaining 240 km portion is modelled in the EMT subsystem and the remaining 240 km portion
of the original B4–B5 line is in the BFAST subsystem. The characteristic impedance \(Z_c\) and wave propagation delay \(\tau\) of the interface lines are computed as 368.5 \(\Omega\) and 200 \(\mu s\), respectively. The EMT subsystem in Fig. 12 is implemented in PSCAD/EMTDC using standard library components. The external system of the BFAST solver is implemented in an external integrated development environment using C++. The interaction between EMTDC and the external system is via an inbuilt co-simulation component in PSCAD/EMTDC. The entire system is also modelled in PSCAD/EMTDC and simulated with a small time-step; the results of this simulation are used to validate co-simulation results.

### 5.2 Simulation setup and results

Transients in the systems are generated by applying disturbances in both the EMT and BFAST subsystems. Solution time-steps are chosen as \(\Delta t = 10, \Delta t_{\text{DP}} = 200,\) and \(\Delta t_{\text{EMT}} = 10 \mu s\).

#### 5.2.1 Disturbance in the EMT subsystem: A solid line-to-ground fault is applied to bus-2 at \(t = 2.0 s\). The fault is intentionally cleared at a non-zero crossing point of the fault current after six cycles (0.1 s) to assess the co-simulator's performance under high-frequency transients. The transient behaviour of interface transmission line currents from both sides are shown in Fig. 13; BFAST and EMT subsystem waveforms during the fault are shown in Fig. 14. The comparison shows that the co-simulation waveforms are essentially identical to those from standalone PSCAD/EMTDC (i.e. the benchmark); most notably, the co-simulator is able to capture high-frequency contents of waveforms after the fault is cleared without losing accuracy. The entire frequency band of the transient is conveyed to the external system via the interface. This is a major gain compared to other co-simulation models available in the literature and is achieved due to the harmonic-rich base-frequency DP modelling and also adaptive usage of both time-step and shift-frequency in the BFAST solver.

#### 5.2.2 Disturbance in the BFAST subsystem: Similarly, a solid line-to-ground fault is applied to bus-1 in the BFAST subsystem for assessing the impact of the external system dynamics on the EMT subsystem. It is observed from Fig. 15 that the co-simulator produces results that are in good agreement with EMTDC simulation waveforms. The transient details of waveforms are accurately conveyed to the EMT subsystem through the interface.

#### 5.2.3 Computational gain comparisons: A comparison of computational gains is done to validate the efficiency of the new co-simulation algorithm against standalone EMT simulation. Simulations are performed for 10 s duration for the 12-bus system on a computer with a 3.20 GHz, Intel Core i7-8700M processor and 16 GB RAM. The PSCAD/EMTDC (i.e. the standalone EMT solver) took 81 s to complete the simulation with a 10 \(\mu s\) time-step while the co-simulation shows noticeable computational gain as given in Table 2, for different time-step ratios and different modes of operations. The results show a nearly two-fold acceleration. It is important to note that the 12-bus system is a small network, and co-simulation is not expected to provide massive gains here as the EMT subsystem constitutes a noticeable portion of the overall system.

### 5.3 Co-simulation of the IEEE 118-bus system

IEEE's 118-bus system [32] is a larger system that is modelled using the developed BFAST–EMT co-simulator. It is also modelled in PSCAD/EMTDC in its entirety and used for comparative assessment. In this example, all the transmissions lines are modelled as lumped parameter \(\pi\)-sections, and all the generators are modelled as voltage sources. The system data for the test system are based on [30]. Bus numbers 98–112, the chosen detailed part of the system, are implemented in detail in PSCAD/EMTDC using standard library components. The remaining buses are included in the BFAST solver. As in the previous example, the interface between detailed subsystem and the external subsystem is formed assuming 60 km length of interfaced transmission lines (see Table 3).

### 5.4 Simulation setup

Transients of the co-simulated network are analysed by applying disturbances in both the EMT and BFAST subsystems. Solution
time-steps are chosen as $\Delta t_T = 50$, $\Delta t_{DP} = 200$, and $\Delta t_{EMT} = 10 \mu s$. The maximum time-step in this case is restricted to 200 $\mu s$ due to the length of the interface transmission lines. Methods to relieve this restriction are available but are outside the scope of this study.

Several tests similar to the previous example are performed on the system including the application of faults in the EMT and DP subsystems. The results (not shown for brevity) confirm the accuracy of the developed co-simulator in response to disturbances in both subsystems.

### 5.4.1 Computational gain comparisons:
Simulations of the 118-bus system for a 10 s duration. The standalone EMT simulator (PSCAD/EMTDC) took 740 s to complete the simulation with a 10 $\mu s$ time-step while the co-simulation shows noticeable computational gain as given in Table 4, for different time-step ratios and different modes of operations. It is clear that the computational gain in this example is much larger than what was observed in the 12-bus system. This is due to the fact that in this co-simulation example, the EMT subsystem is a smaller portion of the overall system. Relieving the rest of the system from having to use a small time-step contributes to a large reduction in the computing time needed for the whole simulation. Larger computational gains are expected from co-simulation of larger systems, in which the EMT subsystems are a small proportion of the entire system.

### 6 Conclusion
An advanced adaptive multi-rate co-simulation platform was developed and validated in this study for accurate and accelerated simulation of power system transients. For that, a base frequency dynamic phasor-based transient simulator (BFAST), which can adapt its simulation time-step and the solution method between EMT and DP, was established. To ensure a smooth and accurate transition between two solution techniques, a novel algorithm was developed and validated.

The BFAST–EMT co-simulator enables parallel processing and the ability to use large time-step ratios between the two solvers when the system is in normal operation adds considerable computational advantage. It allows us to operate different EMT segments at different time-steps, which is a major benefit compared to the standalone EMT solver. Implementation of the co-simulator using a commercial-grade EMT solver interfaced with the BFAST algorithm readily enables usage of standard library components as well as custom models developed for sophisticated components such as converters.

The co-simulation of a power system model by integrating the BFAST solver to an industrial-grade EMT simulator (PSCAD/EMTDC) was successful. Simulation results were observed to be giving essentially identical results for both the EMT and BFAST systems compared to those of standalone PSCAD/EMTDC simulation. The example systems studied showed that for larger networks wherein the EMT subsystem is relatively small, large computational gains are to be expected from the developed co-simulator.
7 References

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