MATTER CONTRIBUTIONS TO THE EXPANSION RATE OF THE UNIVERSE

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ABSTRACT

We consider the effect of various particles on the cosmic expansion rate relative to that of the graviton. Effectively massless fermions, gauge bosons and conformally coupled scalars make only minuscule contributions due to local conformal invariance. Minimally coupled scalars can give much stronger contributions, but they are still sub-dominant to those of gravitons on account of global conformal invariance. Unless effectively massless scalar particles with very particular couplings exist, the leading effect on the expansion rate is furnished solely by the graviton. An upper bound on the mass of such scalar particles is obtained.

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1. Introduction

In this note, we analyze the contribution of the matter sector to the physical rate of expansion of the universe. The pure gravitational contribution has been considered elsewhere [1]. There, we calculated the expectation value of the invariant element in the presence of a homogeneous and isotropic state which is initially free de Sitter vacuum:

\[ \langle 0 | g_{\mu\nu}(t, \vec{x}) \, dx^\mu \, dx^\nu | 0 \rangle = -dt^2 + a^2(t) \, d\vec{x} \cdot d\vec{x}. \]  

(1)

We worked on the manifold \( T^3 \times \mathbb{R} \), using zero temperature quantum field theory based on the two-parameter Lagrangian:

\[ \mathcal{L}_{GR} = \frac{1}{16\pi G} \left( R - 2\Lambda \right) \sqrt{-g}. \]  

(2)

We inferred the physical rate of expansion from the effective Hubble parameter:

\[ H_{\text{eff}}(t) \equiv \frac{d}{dt} \ln(a), \]  

(3)

under the assumption that the scale of inflation \( M \sim (\Lambda/G)^{\frac{1}{4}} \) is adequately below the Planck mass \( M_{\text{Pl}} \sim G^{-\frac{1}{2}} \):

\[ M \lesssim 10^{-3} \, M_{\text{Pl}}. \]  

(4)

And we discovered – at two loops – a decrease in this rate by an amount which becomes non-perturbatively large at late times: *

\[ H_{\text{eff}}(t) = H \left\{ \frac{1}{2} \left[ \frac{1}{6} (Ht)^2 + \text{(subdominant)} \right] + O(\kappa^6) \right\}, \]  

(5)

This may mean that the actual cosmological constant is not unnaturally small and that our current expansion is the result of a screening effect whose slow onset allows a long period of inflation without the need for new particles or severe fine tuning.

* Throughout this note, \( a(t) \) is the scale factor of a homogeneous and isotropic background geometry, \( \kappa^2 = 16\pi G \) is the loop counting parameter of quantum gravity and \( 3H^2 \equiv \Lambda \) gives the bare Hubble constant.
Result (5) was obtained by taking the onset of inflation to be at $t = 0$ and by using perturbation theory around the classical background:

$$a_{\text{class}}(t) = \exp(Ht) \, .$$  \hspace{1cm} (6)

It turns out that the two-loop effect drives a crucial denominator to zero and extinguishes inflation at a time:

$$Ht \sim \left( \frac{M_{\text{Pl}}}{M} \right)^{\frac{8}{3}} \gtrsim 10^8 \, .$$  \hspace{1cm} (7)

when all higher gravitational corrections are insignificant [2].

It is natural to wonder how the back-reaction induced by matter compares with the graviton result (5). Since the basic mechanism for any secular effect in vacuum must be infrared, we need consider only quanta which are effectively massless. This is because infrared effects influence local observables through the coherent superposition of distant interactions in the past lightcone of the observer. Massive propagators oscillate inside the lightcone and this leads to destructive interference.

The phrase “effectively massless” means that the particle’s mass $m$ results in only small distortions of its free mode functions over the relaxation time (7). When $m/H \ll 1$ it turns out that these distortions give a multiplicative factor of $a(t)$ raised to the $-m^2/3H^2$ power. Requiring this multiplicative factor to be of order one gives the following bound:

$$m \lesssim M \left( \frac{M}{M_{\text{Pl}}} \right)^{\frac{7}{3}} \, .$$  \hspace{1cm} (8)

A high inflation scale, say $M \sim 10^{16}$ GeV, means that effectively massless particles must be less than about $10^9$ GeV. Lower inflationary scales provide a more stringent bound. For instance, if $M$ descends all the way to $10^3$ GeV, then $m \lesssim 10^{-34}$ GeV.

If the massless particle has locally conformally invariant interactions, then any local quantity – when written in conformal coordinates – is the same as in flat space. However,
the global effect is nill because the conformal coordinate volume is only $H^{-4}$. * Physically, such a massless particle is unaware of the inflating spacetime. All the observed effectively massless fermions and gauge bosons are eliminated as dominant contributors to $H_{\text{eff}}(t)$ because of the aforementioned argument. The same is true for conformally coupled scalars.

Minimally coupled scalar particles could have a leading influence on $H_{\text{eff}}(t)$ and it is the purpose of this note to evaluate their effect. The first non-trivial diagrams occur at two loops – see, for instance, Fig. 1a. Although they should naively contribute as strongly as the graviton, explicit calculation shows that they are in fact subleading (Section 2). Section 3 explains this as a consequence of \textit{global} conformal invariance. In addition to its phenomenological significance, this result provides a severe test on the master integration programs used in the graviton and scalar cases.

\begin{center}
\begin{tikzpicture}
\draw (0,0) circle (1);\draw[dashed] (0,0) circle (0.5);
\draw (3,0) circle (1);\draw (3,0) circle (0.5);
\end{tikzpicture}
\end{center}

\textbf{Fig. 1:} Various contributions to $H_{\text{eff}}(t)$. Gravitons reside on segmented lines, scalar fields on solid lines.

The only exception is effectively massless scalars with non-derivative self-interactions. (See, for example, Fig. 1b.) These are considered in Section 4. Although they can contribute more strongly than gravitons, the effect is always to slow inflation.

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*(The relevant conformal parameter range is: $x^i \in [-\frac{1}{2}H^{-1}, \frac{1}{2}H^{-1})$, $u \in [H^{-1}, 0)$.*
2. The Result

The complete Lagrangian \( \mathcal{L} \) to be studied consists of the gravitational part \( \mathcal{L}_{GR} \) – given by (2) – and the matter part \( \mathcal{L}_{SC} \) representing a massless minimally coupled scalar field:

\[
\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_{SC}, \tag{9a}
\]

\[
\mathcal{L}_{SC} = -\frac{1}{2} \sqrt{-g} \, g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \tag{9b}
\]

As in the purely gravitational case, it is most convenient to employ the “open-conformal” set of coordinates:

\[
-dt^2 + a_{\text{class}}^2(t) \, d\vec{x} \cdot d\vec{x} = \Omega^2 \left(-du^2 + d\vec{x} \cdot d\vec{x}\right), \tag{10a}
\]

\[
\Omega \equiv (Hu)^{-1} = \exp(Ht), \tag{10b}
\]

and to organize perturbation theory in terms of the quantum fields \( \phi \) and \( \psi_{\mu\nu} \):

\[
g_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu} \equiv \Omega^2 \left(\eta_{\mu\nu} + \kappa \psi_{\mu\nu}\right). \tag{11}
\]

Note that the conformal time \( u \) is inverted with respect to the co-moving time \( t \). The onset of inflation at \( t = 0 \) corresponds to \( u = H^{-1} \) while the infinite future at \( t = +\infty \) corresponds to \( u = 0 \).

We shall compute the scalar contributions to the amputated expectation value of the pseudo-graviton field in the presence of free de Sitter vacuum. By using the manifest homogeneity and isotropy of the theory and the initial state, we have expressed the expectation value in terms of two functions \( a(u) \) and \( c(u) \): *

\[
D_{\mu\nu}^{\rho\sigma} \left\langle 0 \right| \kappa \psi_{\rho\sigma}(x) \left| 0 \right\rangle = a(u) \, \eta_{\mu\nu} + c(u) \, \delta_{\mu}^0 \delta_{\nu}^0. \tag{12}
\]

* The gauged-fixed kinetic operator \( D_{\mu\nu}^{\rho\sigma} \) is most conveniently expressed in terms of the kinetic operator \( D_A \equiv \Omega (\partial^2 + \frac{\dot{a}^2}{a^2}) \Omega \) for a massless, minimally coupled scalar and the kinetic operator \( D_B = D_C \equiv \Omega \partial^2 \Omega \) for a conformally coupled scalar:

\[
D_{\mu\nu}^{\rho\sigma} \equiv \left[ \sqrt{\frac{\partial}{\partial t}} (\rho) \delta_{\nu}^\sigma - \frac{1}{2} \eta_{\mu\nu} \eta^{\rho\sigma} - \frac{1}{2} \delta_{\nu}^0 \delta_0^\rho \delta_0^\sigma \right] D_A + \delta_{(\mu}^0 \delta_{\nu)}^{\rho} \delta_{0}^{\sigma} D_{B} + \delta_{\mu}^0 \delta_{\nu}^0 \delta_0^\rho \delta_0^\sigma D_{C},
\]

where the bar above a symbol means that its zero component is projected out.
Although (12) is not gauge invariant, it is quite simple to obtain from it the effective Hubble constant \( H_{\text{eff}}(t) \) which is a genuine observable and the quantity of physical interest [1,3].

The complete set of Feynman rules for the purely gravitational part can be found elsewhere [1] and will not be presented here. The scalar propagator consists of a “normal” and a “logarithmic” part:

\[
i\Delta(x; x') \approx \frac{H^2}{8\pi^2} \left\{ \frac{2u'u}{\Delta x^2 - \Delta u^2 + 2i\epsilon|\Delta u| + \epsilon^2} - \ln \left[ H^2 \left( \Delta x^2 - \Delta u^2 + 2i\epsilon|\Delta u| + \epsilon^2 \right) \right] \right\},
\]

where \( \Delta x \equiv \|\vec{x}' - \vec{x}\| \) and \( \Delta u \equiv u' - u \). * To regulate the ultraviolet sector, the mode cutoff \( \epsilon \) which appears in the propagators has been used throughout [1,3]. The relevant graviton-scalar interactions are:

\[
L_{\text{INT}}^{(1,2)} = \kappa \Omega^2 \left\{ -\frac{1}{4} \eta^{\mu\nu} \psi,\mu \phi,\nu + \frac{1}{2} \psi^{\mu\nu} \phi,\mu \phi,\nu \right\}, \quad (14a)
\]

\[
L_{\text{INT}}^{(2,2)} = \kappa^2 \Omega^2 \left\{ \frac{1}{4} \psi \psi^{\mu\nu} \phi,\mu \phi,\nu - \frac{1}{2} \psi^{\mu} \phi,\mu \psi^{\nu} \phi,\nu - \frac{1}{16} \eta^{\mu\nu} \psi^2 \phi,\mu \phi,\nu \\
+ \frac{1}{8} \eta^{\mu\nu} \psi_{\alpha} \phi,\alpha \psi_{\beta} \phi,\beta \right\}. \quad (14b)
\]

These interactions have the same generic form as those of pure gravity – two derivatives with a factor of \( \Omega^2 \). Since the scalar propagator also has the same form as that of the graviton, one might expect the back-reaction from the scalar to be of the same strength as that of the graviton.

If quantum corrections are to exceed the classical result for \( H_{\text{eff}}(t) \) at late times, \( a(u) \) or \( c(u) \) must grow faster than \( u^{-4} \) as \( u \to 0^+ \) [3]. With these interactions all diagrams in perturbation theory that contribute to (12) are subject to a maximum growth of \( u^{-4} \) times powers of \( \ln(Hu) \) [3,4]. If the effect is to be interesting, such logarithmic terms must

* This form of the propagator is obtained by turning the mode sum on \( T^3 \) into an integral – an excellent approximation since the propagator is only needed for small conformal coordinate separations [3].
be present since pure $u^{-4}$ behavior simply renormalizes $\Lambda$. The two sources of logarithmic terms are:

(i) The integration over the interaction vertices of the theory. This is the classic source of an infrared effect – access to an arbitrarily large invariant volume in the past lightcone of the observation.

(ii) The “logarithmic” piece of a propagator. This source is particular to de Sitter spacetime and reflects the increasing correlation of the vacuum at constant invariant separation.

Both sources are absent in the first scalar diagram that contributes to (12) beyond tree order (see Fig. 2). Amputation fixes the single interaction vertex at $(u, \vec{x})$ and one obtains only the coincidence limit of derivatives of the scalar propagator at this point. In fact, the maximum growth we can obtain is $u^{-2}$. Therefore, we must go to the two-loop diagrams of Fig. 3 to identify the first potentially relevant graphs in the infrared. Starting in reverse order, diagrams (3d-e) are entirely canceled by the counterterms needed to renormalize their coincident lower loops.* Of the remaining graphs, one free interaction vertex exists in (3c) and two in (3a-b), whereas all three graphs can have up to one undifferentiated graviton propagator.

Our counting indicates – in precise analogy with the pure graviton diagrams – that the graphs (3a-c) should contribute terms of order $u^{-4} \ln^2(Hu)$ at late physical times.

* Not shown is the ultra-local tadpole formed from the $\psi^3 \partial \phi \partial \phi$ vertex, which must also be renormalized away.
These terms would lead to contributions to $H_{\text{eff}}(t)$ that are equal in strength to the pure gravitational ones (5) and, consequently, would modify the expansion rate of the universe.

![Diagrams](a)(b)(c)(d)(e)

**Fig. 3:** Two-loop scalar contributions to $H_{\text{eff}}(t)$.

The actual calculation was performed using the same techniques as with the more complicated pure gravitational case. The symbolic manipulation program Mathematica [5] was used throughout. * Acting the derivatives from interactions on the propagators produces many terms. Some of these integrate to contribute to the coefficient functions $a(u)$ and $c(u)$ at order $u^{-4}\ln^2(Hu)$ — the same as gravitons. However, the net result is sub-dominant to that of pure gravitation, *for each of the three diagrams separately.* In fact, this cancellation was found to occur whenever one sums over all the pure graviton interactions and the various terms which come from *any combination* of the six scalar-graviton interactions given in (14a) and (14b).

### 3. The Justification

The sub-dominance of diagrams involving any combination of scalar-graviton interactions suggests that the cause is a symmetry of the scalar action which is not shared by the gravitational action. The natural candidate is global conformal invariance, whose action

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*Copies of the computer programs employed and the intermediate expressions they generated can be made available. Both the tensor algebra and the loop integrations used programs identical to those in the pure graviton case.
on the fields can be given in terms of a spacetime constant parameter $s$:

$$g'_{\mu\nu}(x) = s^2 g_{\mu\nu}(x), \quad \phi'(x) = s^{-1} \phi(x). \quad (15)$$

The scalar Lagrangian is invariant under (15) while the gravitational Lagrangian is not. We will see how this, combined with dimensional analysis and a few pieces of field theory lore suffice to explain the cancellation.

We seek to understand why the amputated 1-point function acquires no terms of the form $u^{-4}\ln^2(u)$ from diagrams which involve a scalar loop. Since $u$ is dimensionful, each factor of $\ln(u)$ must be paired with the logarithm of some other dimensionful parameter. Before renormalization there are only two such parameters: the Hubble constant $H$ and the ultraviolet regularization parameter $\epsilon$. Factors of $\ln(H)$ enter from undifferentiated propagators (13), which must be graviton lines since all scalar fields in $L_{SC}$ carry derivatives. * Two-loop diagrams which contribute to the amputated 1-point function can have at most one such undifferentiated propagator [1]. Factors of $\ln(\epsilon)$ come from logarithmic ultraviolet divergences, of which there can be at most two at two loops. The two possibilities are therefore a double logarithmic ultraviolet divergence or else a single logarithmic ultraviolet divergence combined with an undifferentiated graviton propagator.

![Fig. 4:](image) Upon replacing the logarithmic piece of the undifferentiated propagator – in this case $i\Delta(x'';x)$ – with a constant, the two-loop diagram becomes one-loop.

In either of the aforementioned possibilities, the scalar loop must contribute a factor

* There can be no factors of $\ln(H)$ from the $H^{-1}$ limit on the conformal time integrations because power counting shows that the integrands converge for large conformal time [1].
of $\ln(\epsilon)$. This is obvious for the double logarithm since then each loop must be logarithmically divergent. The statement is also true when an undifferentiated graviton propagator contributes a factor of $\ln(H)$ because this removes any spacetime dependence from the associated graviton line, effectively cutting it — see Fig. 4. But then the factor of $\ln(\epsilon)$ must come from the only intact loop: that of the scalar.

\[ \ldots \]

**Fig. 5:** The diagrammatic expansion of the effective action $\Gamma_{SC}$.

The natural vehicle for analyzing such issues is the gravitational effective action induced by the scalar, $\Gamma_{SC}[g]$ — see Fig. 5. The amputated 1-point function can be expressed in terms of this quantity by performing the Gaussian functional integration over the scalar field:

\[
D_{\mu\nu}^{\rho\sigma} \left\langle 0 \mid \kappa \psi_{\rho\sigma}(x) \mid 0 \right\rangle = \int [d\psi][d\phi] D_{\mu\nu}^{\rho\sigma} \kappa \psi_{\rho\sigma}(x) \exp\left\{ iS_{GR} + iS_{SC} \right\}, \quad (16a)
\]

\[
= \int [d\psi] D_{\mu\nu}^{\rho\sigma} \kappa \psi_{\rho\sigma}(x) \exp\left\{ iS_{GR} \right\} \exp\left\{ i\Gamma_{SC} \right\}. \quad (16b)
\]

The resulting formalism is that of a purely gravitational theory whose action is that of classical gravity, $S_{GR}$, plus $\Gamma_{SC}$. Two-loop contributions involving the scalar consist of one-loop diagrams in this purely gravitational theory, where a single interaction comes from $\Gamma_{SC}$ and the rest from $S_{GR}$ — see Fig. 6.

* In the interest of clarity we have suppressed the gauge fixing paraphernalia, as well as the forward and backwards evolving fields needed to give an expectation value rather than an “in”-“out” amplitude.
The ultraviolet divergences of $\Gamma_{SC}$ multiply local invariant functions of the metric, as usual. Although our mode cut off $\epsilon$ breaks global conformal invariance, the result of a global conformal transformation is simply to rescale $\epsilon$. It follows that the local invariants which multiply $\ln(\epsilon)$ must possess global conformal invariance. Only two such terms exist in four dimensions:

$$C^2 \sqrt{-g}, \quad R^2 \sqrt{-g}.$$ (17)

The first of these is actually invariant under local conformal transformations, so it has no dependence upon the conformal factor $\Omega = (Hu)^{-1}$. We also know that the expansion of its integral in powers of the pseudo-graviton field begins at quadratic order. The second term in (17) has neither of these properties: it depends upon $\Omega$ and the expansion of its integral begins at linear order in $\psi$. One consequence of this is that $\Gamma_{SC}$ cannot actually have a term involving $\ln(\epsilon)$ times this second term. If it did, the one-loop diagram of Fig. 2 would possess a logarithmic divergence, which it does not.

Now consider one-loop contributions to the amputated 1-point function which come from a single insertion of an interaction from $C^2 \sqrt{-g}$, plus the possibility of a single interaction vertex from $L_{GR}$ at the fixed observation point $x^\mu$. These diagrams can indeed give a factor of either $\ln(\epsilon)$ or $\ln(H)$, but without the essential multiplicative factor of $u^{-4}$. The $C^2 \sqrt{-g}$ vertex is entirely independent of the conformal time, and the vertices from $L_{GR}$ can contribute at most a factor of $u^{-3}$. Propagators give only positive powers of conformal time. We can get higher inverse powers from lower ones by decomposing by
partial fractions terms which produce ultraviolet divergences:

\[
\frac{1}{u'(u' - u - i\epsilon)^2} = \frac{1}{u(u' - u - i\epsilon)^2} - \frac{1}{u^2(u' - u - i\epsilon)} + \frac{1}{u^2 u'},
\]

but only if the integrand already contains at least one inverse power of the conformal time which is being integrated. It cannot get any powers from the insertion, propagators give only positive powers, and the possible vertex from \( \mathcal{L}_{GR} \) must be external.

We can therefore exclude the possibility of two-loop scalar contributions of strength \( u^{-4} \ln^2(Hu) \). In addition to understanding why these terms can not come from free scalars we have uncovered the reason why they can come from gravitons: \( \mathcal{L}_{GR} \) is not invariant under global conformal transformations. We should also note that there are presumably non-zero scalar contributions at order \( u^{-4} \ln(Hu) \) although we did not compute them.

### 4. The Scalar Self-Interactions

For completeness we consider the case of a self-interacting scalar which somehow avoids developing an unacceptably large mass. Suppose we add an \( N \)-point self-interaction:

\[
-\frac{1}{N!} \lambda \sqrt{-g} \phi^N,
\]

to the scalar Lagrangian (9b). There are two basic diagrams that must be considered at the first non-trivial order, both of which involve the vertices:

\[
\mathcal{L}_{INT}^{(0,N)} = -\frac{1}{N!} \lambda \Omega^4 \phi^N ; \quad \mathcal{L}_{INT}^{(1,N)} = -\frac{1}{2N!} \kappa \lambda \Omega^4 \psi \phi^N.
\]

The two diagrams can be seen in Fig. 7 and their respective contributions to the amputated expectation value (12) are:

\[
I_{(a)}^{\mu \nu} = \frac{i}{2(N-1)!} \kappa^2 \lambda^2 \Omega^2 \int d^4x' \Omega^4 \int d^4x'' \Omega^{4'} \left( -\frac{1}{4} \eta^{\rho \sigma} \eta^{\mu \nu} + \frac{1}{2} \eta^{\mu \rho} \eta^{\nu \sigma} \right)
\left[ \partial_{\rho} i\Delta(x'; x') \right] \left[ \partial_{\sigma} i\Delta(x, x'') \right] \left[ i\Delta(x'; x'') \right]^{N-1},
\]

\[
I_{(b)}^{\mu \nu} = -\frac{i}{2N!} \kappa^2 \lambda^2 \Omega^4 \int d^4x' \Omega^4 \eta^{\mu \nu} \left[ i\Delta(x; x') \right]^N,
\]

\[12\]
where $x$ is the observation event while $x'$ and $x''$ are the locations of the interaction vertices that must be integrated.

\begin{center}
\begin{tikzpicture}
  \node at (0,0) {\includegraphics[width=0.3\textwidth]{diagram}};
\end{tikzpicture}
\end{center}

**Fig. 7:** The first basic scalar contributions to $H_{\text{eff}}(t)$ due to an $N$-point scalar self-interaction.

The leading contribution for diagram 7b comes entirely from the logarithmic part of the propagator. It is easily computed to be:

$$I_{(b)}^{\mu\nu} = -\frac{1}{6} \frac{\kappa^2 \lambda^2 H^{2N-8} \eta_{\mu\nu}}{(2\pi)^{2N-2} N!} \left[ \frac{1}{u^4} \ln(Hu) \right] (\text{subdominant}) .$$  \hfill (22)

Diagram 7a is sub-dominant; it can give at most $(N - 1)$ logarithms. The effect on the expansion rate is:

$$H_{\text{eff}}^S(t) = H \left\{ 1 - \frac{\kappa^2 \lambda^2 H^{2N-6}}{(2\pi)^{2N-2} N!} \left[ \frac{1}{u^4} (Ht)^N + (\text{subdominant}) \right] + \ldots \right\} ,$$  \hfill (23)

Note that the effect is to slow inflation for all values of $N$. However unlikely it is to have light, self-interacting scalars, we need not worry that they prevent screening. They can only add to the effect already provided by gravitons.

Note also that we get a non-zero effect even for $N = 4$, when the scalar action has global conformal invariance. The argument of the previous section does not apply because non-derivative interactions allow the survival of many more factors of $\ln(H)$ from undifferentiated propagators. The integration over $u'$ also diverges for large conformal times, which gives another factor of $\ln(H)$. 
5. Epilogue

The special thing about gravitons is their combination of masslessness with an intrinsic scale which breaks conformal invariance. This feature is what allows them to screen the cosmological constant. We have shown that even global conformal invariance results in the absence of leading order contributions from minimally coupled scalars which lack self-interactions. The addition of non-derivative self-interactions allows minimally coupled scalars to slow inflation as much or more than gravitons, but only if they can somehow satisfy our bound (8) for remaining effectively massless.

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