Quark Spin and Orbital Angular Momentum in the Baryon

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Abstract

The spin and orbital angular momentum carried by different quark flavors in the nucleon are calculated in the SU(3) chiral quark model with symmetry-breaking. The model is extended to all octet and decuplet baryons. In this model, the reduction of the quark spin, due to the spin dilution in the chiral splitting processes, is transferred into the orbital motion of quarks and antiquarks. The orbital angular momentum for each quark flavor in the proton as function of the partition factor $\kappa$ and the chiral splitting probability $a$ is shown. Although the total amount of the quark spin reduction is canceled by the equal amount increase of the quark orbital angular momentum, the cancellation does not apply to each quark flavor. Especially, the cancellation between the spin and orbital contributions in the baryon magnetic moment is discussed. Comparisons of our results with other models are also shown.

12.39.Fe, 13.40.Em, 13.88.+e, 14.20.-c
I. Introduction

One of the important tasks in hadron physics is to reveal the internal structure of the nucleon. This includes the study of flavor, spin and orbital angular momentum shared by the quarks and gluons in the nucleon. These structures determine the basic properties of the nucleon: spin, magnetic moment, axial coupling constant, elastic form factors, and the deep inelastic structure functions. The polarized deep-inelastic scattering (DIS) data \[1–3\] indicate that the quark spin only contributes about one third of the nucleon spin or even less. A natural and interesting question is: where is the missing spin? Intuitively, and also from quantum chromodynamics (QCD) \[4\], the nucleon spin can be decomposed into the quark and gluon contributions

\[
\frac{1}{2} = < J_z > q+\bar{q} + < J_z > G = \frac{1}{2} \Delta \Sigma + < L_z > q+\bar{q} + < J_z > G .
\]

Without loss of generality, in (1) the proton has been chosen to be longitudinal polarized in the z direction; it has helicity of \(+1/2\). The angular momentum \(< J_z > q+\bar{q}\) has been decomposed into the spin and orbital parts in (1). The total spin from quarks and antiquarks is \(\Delta \Sigma/2 = \sum (\Delta q + \Delta \bar{q})/2 = < s_z > q+\bar{q}\), where \(\Delta q \equiv q^\uparrow - q^\downarrow\), and \(\Delta \bar{q} \equiv \bar{q}^\uparrow - \bar{q}^\downarrow\), and \(q^\uparrow, \bar{q}^\uparrow\) are quark (antiquark) numbers with spin parallel and antiparallel to the nucleon spin, or more precisely, quark (antiquark) numbers of positive and negative helicities. \(< L_z > q+\bar{q}\) denotes the total orbital angular momentum carried by quarks and antiquarks, and \(< J_z > G\) is the gluon angular momentum. The smallness of \(\frac{1}{2} \Delta \Sigma\) implies that the missing part should be contributed either by the quark orbital motion or the gluon angular momentum. In the past decade, considerable experimental and theoretical progress has been made in determining the quark spin contribution in the nucleon \[4\]. There is no direct data on \(\Delta G\) except for a preliminary restriction on \(\Delta G(x)/G(x)\) given by experiment E581/704 \[3\], and another indirect result \(\Delta G \approx 0.5 - 1.5\) at \(Q^2 \approx 10\) GeV\(^2\) from the analysis of \(Q^2\) dependence of \(g_1(x,Q^2)\) \[3\]. Several experiments for measuring \(\Delta G\) \[7\] have been suggested. Most recently, it has been shown that \(< J_z > q+\bar{q}\) might be measured in the deep virtual Compton scattering (DVCS) \[8\], and one may then obtain the quark orbital angular momentum from the difference \(< L_z > - < s_z >\). Hence the study of the quark spin and orbital angular momentum are important and interesting both experimentally and theoretically.

Historically, when the quark model \[9\] was invented in 1960’s, all three quarks in the nucleon were assumed to be in S-states, so \(< L_z > q = 0\) and the nucleon spin arises entirely from the quark spin. On the other hand, in the simple parton model \[10\], all quarks, antiquarks and gluons are moving in the same direction, i.e. parallel to the proton momentum, there is no transverse momentum for the partons and thus \(< L_z > q+\bar{q} = 0\) and \(< L_z > G = 0\). This picture cannot be \(Q^2\) independent due to QCD evolution. In leading-log approximation, \(\Delta \Sigma\) is \(Q^2\) independent while the gluon helicity \(\Delta G\) increases with \(Q^2\). This increase should be compensated by the decrease of the orbital angular momentum carried by partons (see for instance Ref. \[11\] and later analysis \[12\]). Recently, the leading-log evolution of \(< L_z > q+\bar{q}\) and \(< L_z > G\); and an interesting asymptotic partition rule were obtained in \[13\]. The ‘initial’ value of the orbital angular momenta at renormalization scale \(\mu^2\) is determined by nonperturbative dynamics of the nucleon as a QCD bound state. Lattice QCD provides us with a nonperturbative tool to calculate the physical quantities of hadrons and has provided many interesting results \[14\]. Meantime, many QCD inspired nucleon models have
been developed to explain existing data and yield good physical insight into the nucleon. For instance, in the bag model \cite{15}, \( < s_z >_q \approx 0.39 \), and \( < L_z >_q \approx 0.11 \), while in the Skyrme model \cite{16,17}, \( \Delta G = \Delta \Sigma = 0 \), and \( < L_z > = 1/2 \), which implies that the nucleon spin arises only from orbital motion.

Phenomenologically, long before the EMC experimental data were published \cite{1}, using the Bjorken sum rule and low energy hyperon \( \beta \)-decay data (basically axial coupling constants), Sehgal \cite{18} showed that nearly 40% of the nucleon spin arises from the orbital motion of quarks; the remaining 60% is attributed to the spin of quarks and antiquarks. Most recently Casu and Sehgal \cite{19} show that to fit the baryon magnetic moments and polarized DIS data, a large collective orbital angular momentum \( < L_z > \), which contributes almost 80% of nucleon spin, is needed. Hence the question of how much of the nucleon spin is coming from the quark orbital motion remains. This paper will discuss this question within the chiral quark model given in \cite{20,21}. In section II, the basic assumption and formalism presented is briefly reviewed. A scheme for describing both spin and orbital angular momentum carried by quarks and antiquarks in the nucleon is discussed in section III. The important improvement is that the partition factor \( \kappa \) is no longer restricted to be 1/3, which was so called ‘equal sharing’ assumption in \cite{21}. In this paper, the \( \kappa \) can be any value in the range of \([0,1/2]\). Extension to the octet and decuplet baryons is given in section IV. The magnetic moments of the baryons are discussed in section V. A brief summary is given in section VI.

II. Chiral Quark Model

The chiral quark model was first employed by Eichten, Hinchliffe and Quigg \cite{22} to explain both the sea flavor asymmetry \( \bar{d} - \bar{u} > 0 \) \cite{23} and the smallness of \( \Delta \Sigma \) in the nucleon. The model was significantly improved by introducing U(1)-breaking \cite{24} and kaonic suppression \cite{25}. A description with both SU(3) and U(1)-breakings was developed in \cite{26–28}. Using the low energy hyperon decay data, the description given in \cite{26} was reformed to an one-parameter scheme in \cite{20} and the predictions are in good agreement with both spin and flavor observables. In this paper, we will use the notations given in \cite{20}.

The effective Lagrangian is

\[
L_I = g_8 \bar{q} \left( \begin{array}{c}
\frac{G_u^0}{\sqrt{\bar{c}K^-}} + \pi^+ \\
\frac{\pi^-}{\sqrt{\bar{c}K^0}} + \frac{\sqrt{\bar{c}K^+}}{\sqrt{\bar{c}K^0}}
\end{array} \right) q,
\]

(2a)

where \( G_{u(d)}^0 \) and \( GB_s^0 \) are defined as

\[
G_{u(d)}^0 = \frac{(+\sqrt{\frac{\bar{c}K^-}{2}} + \sqrt{\bar{c}K^0}}{\sqrt{\bar{c}K^0}} + \sqrt{\bar{c}K^0}, \quad G_{s}^0 = -\sqrt{\bar{c}K^-} \sqrt{\bar{c}K^-} + \sqrt{\bar{c}K^-} \sqrt{\bar{c}K^-}.
\]

(2b)

The breaking effects are explicitly included. \( a \equiv |g_8|^2 \) denotes the transition probability of chiral fluctuation or splitting \( u(d) \rightarrow d(u) + \pi^+(-) \), and \( \epsilon a \) denotes the probability of \( u(d) \rightarrow s + K^-(0) \). Similar definitions are used for \( \epsilon_o a \) and \( \zeta^2a \). If the breaking parameter is dominated by mass suppression effect, one reasonably expects \( 0 \leq \zeta^2a < \epsilon_o a \simeq \epsilon a \leq a \), then we have \( 0 \leq \zeta^2 \leq 1, 0 \leq \epsilon_o \leq 1, \) and \( 0 \leq \epsilon \leq 1 \). We note that in our formalism, only the integrated quark spin and flavor contents are discussed.

The basic assumptions of the chiral quark model are: (i) The quark flavor, spin and orbital contents of the nucleon are determined by its valence quark structure and all possible chiral fluctuations, and probabilities of these fluctuations depend on the interaction
The zeroth approximation, the SU(3) Lagrangian (2). (ii) The coupling between the quark and Goldstone boson is rather weak, one can treat the fluctuation \( q \to q' + \text{GB} \) as a small perturbation \( (a \sim 0.10 \sim 0.15) \) and the contributions from the higher order fluctuations can be neglected. (iii) Quark spin-flip interaction dominates the splitting process \( q \to q' + \text{GB} \). This can be related to the picture given by the instanton model \([29]\), hence the spin-nonflip interaction is suppressed.

Based upon the assumptions, the quark flips its spin and changes (or maintains) its flavor by emitting a charged (or neutral) Goldstone boson. The light quark sea asymmetry \( \bar{u} < \bar{d} \) is attributed to the existing flavor asymmetry of the valence quark numbers (two valence \( u \)-quarks and one valence \( d \)-quark) in the proton. On the other hand, the quark spin reduction is due to the spin flip in the chiral splitting processes. Furthermore, the quark spin component changes one unit of angular momentum, \( (s_z)_f - (s_z)_i = +1 \) or \(-1\), due to spin-flip in the fluctuation with GB emission. The angular momentum conservation requires the same amount change of the orbital angular momentum but with opposite sign, i.e. \((L_z)_f - (L_z)_i = -1 \) or \(+1\). This induced orbital motion is distributed among the quarks and antiquarks, and compensates the spin reduction in the chiral splitting. This is the starting point to calculate the orbital angular momenta carried by quarks and antiquarks in the chiral quark model. We note that the quark orbital angular momentum in the nucleon in the SU(3) symmetry chiral quark model was discussed in \([30]\).

For spin-up or spin-down valence \( u, d, \) and \( s \) quarks, up to the first order fluctuation, the allowed chiral processes are

\[
\begin{align*}
  u_{\uparrow,(i)} &\to d_{\uparrow,(i)} + \pi^+ , & u_{\uparrow,(i)} &\to s_{\uparrow,(i)} + K^+ , & u_{\uparrow,(i)} &\to u_{\Lambda,(i)} + G^0, & u_{\uparrow,(i)} &\to u_{\Lambda,(i)}. \\
  d_{\downarrow,(i)} &\to u_{\downarrow,(i)} + \pi^-, & d_{\downarrow,(i)} &\to s_{\downarrow,(i)} + K^0, & d_{\downarrow,(i)} &\to d_{\Lambda,(i)} + G^0, & d_{\downarrow,(i)} &\to d_{\Lambda,(i)}. \\
  s_{\downarrow,(i)} &\to u_{\Lambda,(i)} + K^-, & s_{\downarrow,(i)} &\to d_{\Lambda,(i)} + K^0, & s_{\downarrow,(i)} &\to s_{\Lambda,(i)} + G^0, & s_{\downarrow,(i)} &\to s_{\Lambda,(i)}. 
\end{align*}
\]

We note that the quark helicity flips in the chiral splitting processes \( q_{\uparrow,(i)} \to q_{\downarrow,(i)} + \text{GB} \), i.e. the first three processes in each of (3a), (3b), and (3c), but not for the last one. In the zeroth approximation, the SU(3)⊗SU(2) proton wave function gives

\[
n_p^{(0)}(u_\uparrow) = \frac{5}{3}, \quad n_p^{(0)}(u_\downarrow) = \frac{1}{3}, \quad n_p^{(0)}(d_\uparrow) = \frac{1}{3}, \quad n_p^{(0)}(d_\downarrow) = \frac{2}{3}.
\]

the spin-up and spin-down quark (or antiquark) contents, up to first order fluctuation, can be written as

\[
n_p(q_{\uparrow,\downarrow}^{\prime} , \text{or} \tilde{q}_{\uparrow,\downarrow}^{\prime}) = \sum_{q=u,d} \sum_{h=\uparrow,\downarrow} n_p^{(0)}(q_h) P_{q_h}(q_{\uparrow,\downarrow}^{\prime} , \text{or} \tilde{q}_{\uparrow,\downarrow}^{\prime}),
\]

where \( P_{q_h}(q_{\uparrow,\downarrow}^{\prime}) \) and \( P_{q_h}(\tilde{q}_{\uparrow,\downarrow}^{\prime}) \) are the probabilities of finding a quark \( q_{\uparrow,\downarrow}^{\prime} \) or an antiquark \( \tilde{q}_{\uparrow,\downarrow}^{\prime} \) arise from all chiral fluctuations of a valence quark \( q_{\uparrow,\downarrow} \). \( P_{q_{\uparrow,\downarrow}}(q_{\uparrow,\downarrow}^{\prime}) \) and \( P_{q_{\uparrow,\downarrow}}(\tilde{q}_{\uparrow,\downarrow}^{\prime}) \) can be obtained from the effective Lagrangian (2) and listed in Table I, where only \( P_{q_{\uparrow}}(q_{\uparrow,\downarrow}^{\prime}) \) and \( P_{q_{\downarrow}}(\tilde{q}_{\uparrow,\downarrow}^{\prime}) \) are shown. Those arise from \( q_{\downarrow} \) can be obtained by using the relations, \( P_{q_{\uparrow}}(q_{\uparrow,\downarrow}^{\prime}) = P_{q_{\downarrow}}(\tilde{q}_{\uparrow,\downarrow}^{\prime}) \), \( P_{q_{\downarrow}}(q_{\uparrow,\downarrow}^{\prime}) = P_{q_{\uparrow}}(\tilde{q}_{\uparrow,\downarrow}^{\prime}) \). The notations given in Table I are defined as

\[
f \equiv \frac{1}{2} + \frac{\epsilon_\eta}{6} + \frac{\zeta^2}{3}, \quad f_s \equiv \frac{2\epsilon_\eta}{3} + \frac{\zeta^2}{3},
\]

and

\[
A \equiv 1 - \zeta' + \frac{1 - \sqrt{\epsilon_\eta}}{2}, \quad B \equiv \zeta' - \sqrt{\epsilon_\eta}, \quad C \equiv \zeta' + 2\sqrt{\epsilon_\eta}.
\]
The special combinations $A$, $B$, and $C$ stem from the quark and antiquark contents in the octet and singlet neutral bosons $G^0_{u(d)}$ and $G^0_s$ [see (2b)] appeared in the effective chiral Lagrangian (2a), while $f$ and $f_s$ stand for the probabilities of the chiral splittings $u_\uparrow(d_\uparrow) \to u_\downarrow(d_\downarrow) + G^0_{u(d)}$ and $s_\uparrow \to s_\downarrow + G^0_s$ respectively. Although there is no valence $s$ quark in the proton and neutron, there are one or two valence $s$ quarks in $\Sigma$ or $\Xi$, or other strange decuplet baryons, and even three valence $s$ quarks in the $\Omega^-$. Hence for the purpose of later use we also give the probabilities arise from a valence $s$-quark splitting. In general, the suppression effects may be different for different baryons, hence the probabilities $P_{q\uparrow\downarrow}(q'\uparrow\downarrow)$ and $P_{q\uparrow\downarrow}(\bar{q}'\uparrow\downarrow)$ may vary with the baryons. But we will assume that they are universal for all baryons.

Using (4), (5) and the probabilities listed in Table I, the spin-up and spin-down quark and antiquark contents, and the spin average and spin weighted quark and antiquark contents in the proton were obtained in [26,20] and are now collected in Table II. For the purpose of later discussion, we write down the formula for the spin-weighted quark content

$$\Delta q' = \sum_q [n_B(0)(q\uparrow) - n_B(0)(q\downarrow)][P_{q\uparrow\downarrow}(q'\uparrow\downarrow) - P_{q\uparrow\downarrow}(q'\downarrow\downarrow)],$$

and the spin-weighted antiquark content is zero

$$\Delta \bar{q}' = 0.$$  

Hence one has $(\Delta q)_{sea} \neq \Delta \bar{q}$ in the chiral quark model. This is different from those models, in which the sea quark and antiquark with the same flavor are produced as a pair from the gluon (see discussion in [25]). The quark spin contents in the proton are

$$\Delta u^p = \frac{4}{5}\Delta_3 - a, \quad \Delta d^p = -\frac{1}{5}\Delta_3 - a, \quad \Delta s^p = -\epsilon a,$$

where $\Delta_3 = \frac{5}{3}[1 - a(\epsilon + 2f)]$. The total quark spin content in the proton is

$$\frac{1}{2}\Delta \Sigma^p = \frac{1}{2}(\Delta u^p + \Delta d^p + \Delta s^p) = \frac{1}{2} - a(1 + \epsilon + f) \equiv \frac{1}{2} - a\xi_1,$$

where the notation $\xi_1 \equiv 1 + \epsilon + f$ is used.

III. Quark Orbital Motion

(a) Quark orbital momentum in the nucleon

The quark orbital angular momentum can be discussed in a similar way. For instance, for a spin-up valence $u$-quark, only first three processes in (3a), i.e. quark fluctuations with GB emission, can induce a change of the orbital angular momentum. The last process in (3a), $u_\uparrow \to u_\uparrow$ means no chiral fluctuation and it makes no contribution to the orbital motion and will be disregarded. The orbital angular momentum produced in the splitting $q_\uparrow \to q'\uparrow + GB$ is shared by the recoil quark ($q'$) and the Goldstone boson (GB). If we define the fraction of the orbital angular momentum shared by the recoil quark is $1 - 2\kappa$, then the orbital angular momentum shared by the (GB) is $2\kappa$ which, we assume, equally shared by the quark and antiquark in the Goldstone boson. We call $\kappa$ the partition factor, which satisfies $0 < \kappa < 1/2$. For $\kappa = 1/3$, the three particles, the quark and antiquark in the
orbital motion of quarks and antiquarks. The amount of quark spin reduction
Therefore, in the chiral fluctuations, the missing part of the quark spin is
in (7d) is canceled by the equal amount increase of the quark orbital angular momentum in
the amount $\xi$, and the total angular momentum of nucleon is unchanged.

From the spin-up and spin-down valence quark fluctuations, it is easy to write down the total
orbital angular momentum carried by a specific quark flavor, for instance $u$-quark, in the proton

$$
< L_z >_{p_u} = \sum_{q=u,d} [n_p^{(0)}(q_\uparrow) - n_p^{(0)}(q_\downarrow)] < L_z >_{q/U},
$$

where $\sum$ summed over $u$ and $d$ valence quarks in the proton. $n_p^{(0)}(q_\uparrow)$ and $n_p^{(0)}(q_\downarrow)$ are given in (4). Similarly, one obtains the $< L_z >_{p_d}$, $< L_z >_{p_s}$, and corresponding quantities for the antiquarks. The numerical results are listed in Table IV. Note that different baryons have different valence quark structure and thus different $n_p^{(0)}(q_\uparrow)$ and $n_p^{(0)}(q_\downarrow)$.

Defining $< L_z >_{q} ( < L_z >_{q_\downarrow})$ as the total orbital angular momentum carried by all quarks (all antiquarks), we obtain

$$
< L_z >_{q} \equiv < L_z >_{q_{u+d+s}} = (2 + \delta)\kappa \xi_1 a, \quad (11a)
$$

$$
< L_z >_{q_\downarrow} \equiv < L_z >_{q_{u+d+s}} = \kappa \xi_1 a, \quad (11b)
$$

$$
< L_z >_{q_{u+d+s}} \equiv < L_z >_{q\downarrow} >_{q\downarrow} + < L_z >_{q\downarrow} \equiv \xi_1 a. \quad (11c)
$$

It means that the orbital angular momentum of each quark flavor may depend on the parton
factor $\kappa$, but the total orbital angular momentum (11c) is independent of $\kappa$. Furthermore,
the amount $\xi_1 a$ is exactly the same as the total spin reduction in (7d). The sum of (11c)
and (7d) gives

$$
< J_z >_{q_{u+d+s}} = < s_z >_{q_{u+d+s}} + < L_z >_{q_{u+d+s}} = < L_z >_{q_{u+d+s}} = \frac{1}{2}. \quad (11d)
$$

Therefore, in the chiral fluctuations, the missing part of the quark spin is transferred into the
orbital motion of quarks and antiquarks. The amount of quark spin reduction $a(1 + \epsilon + f)
$ in (7d) is canceled by the equal amount increase of the quark orbital angular momentum in
(11c), and the total angular momentum of nucleon is unchanged.

Two remarks should be made here. Although the orbital angular momentum carried by
quarks (or antiquarks) $< L_z >_{q}$ (or $< L_z >_{q_\downarrow}$) depends on the chiral parameters, $a$, $\epsilon$, $\epsilon_\eta$,
and ζ′, the ratio \( < L_z >^p_q / < L_z >^p_\bar{q} = 2 + \delta = (1 - \kappa)/\kappa \) is independent of the probabilities of chiral fluctuations. For \( \kappa = 1/3 \) (equal sharing), this ratio is 2:1. This is originated from the mechanism of the chiral fluctuation: there are two quarks and one antiquark in the final state. Secondly, the total loss of quark spin \( a(1 + \epsilon + f) \) appeared in Eq.(7d) is due to the fact that there are three splitting processes with quark spin-flip [see the first three processes in (3a) and (3b)], the probabilities of these spin-flip splittings are \( a, \epsilon a, \) and \( f a \) respectively. For the same reason, the total gain of the orbital angular momentum is \( a(1 + \epsilon + f) \).

The discussion can be easily extended to the neutron. Explicit calculation shows that

\[
\xi \equiv \frac{1}{2} (\epsilon a + \epsilon \bar{a} + f d + f \bar{d}) = \frac{1}{2} \left( \eta + \zeta^2 \right)
\]

\( < L_z >_{u,\bar{u}} = < L_z >_{d,\bar{d}} = < L_z >_{s,\bar{s}} = < L_z >_{p,\bar{p}} \). Using these relations, one can obtain the orbital angular momenta carried by quarks and antiquarks in the neutron. We have similar relations for \( \Delta q \) from the isospin symmetry, hence the main results (7d), (11a-d), and related conclusions hold for the neutron as well. Extension to other octet and decuplet baryons will be given in section IV.

(b) Numerical results

To determine model parameters, we use similar approach given in [20], where the chiral quark model with only three parameters gave a good description to most existing spin and flavor observables. The chiral parameters \( a, \epsilon \approx \epsilon_\eta, \) and \( \zeta' \) are determined by three inputs, \( \Delta u - \Delta d = 1.26, \Delta u + \Delta d - 2\Delta s = 0.60, \) and \( \bar{d} - \bar{u} = 0.143 \) (good agreement between the model prediction and spin-flavor data can be seen from Table XIII below). The three-parameter set is: \( a = 0.145, \epsilon = 0.46, \) and \( \zeta'^2 = 0.10 \). It gives

\[
\xi_1 \equiv 1 + \epsilon + f = 2.07.
\]

Comparing our choice of the parameters with two extreme cases,

\[
\begin{align*}
\xi_1 &\equiv 1 + \epsilon + f = 3.0, \quad \text{for } U(3) \text{ symmetry (} \epsilon = \epsilon_\eta = \zeta'^2 = 1) \quad (13a) \\
\xi_1 &\equiv 1 + \epsilon + f = 1.5, \quad \text{for } \text{ extreme breaking (} \epsilon = \epsilon_\eta = \zeta'^2 = 0) \quad (13b)
\end{align*}
\]

one can see that the value \( \xi_1 = 2.07 \) given in (12) is just between those given in (13a) and (13b).

Eq.(12) leads to

\[
< L_z >_{q,\bar{q}} \approx 0.30, \quad (14a)
\]

and

\[
\begin{align*}
< L_z >_u &\approx 0.225, \quad \text{(for } \kappa = 1/4), \quad (14b) \\
< L_z >_d &\approx 0.075, \quad \text{(for } \kappa = 1/3), \quad (14c) \\
< L_z >_s &\approx 0.200, \quad \text{(for } \kappa = 1/3), \quad (14d) \\
< L_z >_p &\approx 0.187, \quad \text{(for } \kappa = 3/8).
\end{align*}
\]

The orbital angular momenta shared by different quark flavors depend on the partition factor \( \kappa \) and they are listed in Table IV. We plot the orbital angular momenta carried by quarks and antiquarks in the proton as function of \( \kappa \) in Fig.1. Several comments should be made here. (1) Fig.1 shows that \( < L_z >_q \approx < L_z >_{\bar{q}} \) at \( \kappa = 1/3 \) and \( < L_z >_u = < L_z >_{\bar{u}} \) at \( \kappa \approx 0.28, \) but \( < L_z >_d = < L_z >_{\bar{d}} \) cannot be equal at any value of \( \kappa. \) (2) Although the orbital angular momentum carried by each flavor depends on \( \kappa, \) the total orbital angular momentum carried by the quarks and antiquarks does not, and is determined by the chiral parameters (see Table IV). (3) Using the parameter set given above, \( < L_z >_{q,\bar{q}} \approx 0.30, \) i.e.,
nearly 60% of the proton spin is coming from the orbital motion of quarks and antiquarks, and 40% is contributed by the quark and antiquark spins. Comparison of our result with other models is given in Table V and Fig. 2. (4) As indicated in Eq.(11d), the total quark spin reduction (with respect to the SU(6) value 1/2) is canceled by the equal amount increase of the quark orbital angular momentum. However, the cancellation does not apply to each quark flavor. For example, in the NQM, Δu_p(0) = 4/3, Δd_p(0) = −1/3, and Δs_p(0) = 0. However, from Table IV, we have

\[ \frac{1}{2}Δu^p + < L_z >_{u+\bar{u}}^p = 0.558 \neq \frac{1}{2}Δu^{(0)} = 0.667, \]

\[ \frac{1}{2}Δd^p + < L_z >_{d+\bar{d}}^p = -0.080 \neq \frac{1}{2}Δd^{(0)p} = -0.167, \]

\[ \frac{1}{2}Δs^p + < L_z >_{s+\bar{s}}^p = 0.022 \neq \frac{1}{2}Δs^{(0)p} = 0. \]

It is obvious that for the u-flavor, the orbital contribution is not big enough to compensate the quark spin reduction, while for the d-quark flavor, the orbital contribution is too big and the d-quark spin reduction is over compensated. However, taking the sum we have

\[ \frac{1}{2}Δq^p + < L_z >_{q+\bar{q}}^p = \frac{1}{2} = \sum_{q=u,d,s} \frac{1}{2}Δq^{(0)} \]  

(11d)

IV. Extension to other Baryons.

(a) Spin content in octet baryons

We take Σ^+(uuu) as an example, other octet baryons can be discussed in a similar manner. The valence quark structure of Σ^+ is the same as the proton with the replacement d → s. Hence one has

\[ n_{Σ^+}^{(0)}(u_1) = \frac{5}{3}, \quad n_{Σ^+}^{(0)}(u_4) = \frac{1}{3}, \quad n_{Σ^+}^{(0)}(s_1) = \frac{1}{3}, \quad n_{Σ^+}^{(0)}(s_4) = \frac{2}{3}. \]  

(15)

Using (5) (change p → Σ^+), (15), and Table I, we can obtain Δu_{Σ^+}, Δq_{Σ^+}, and Δs_{Σ^+}. Similarly, we can obtain the results for Σ^0, Λ^0, and Ξ^0. Those for Σ^−, and Ξ^−, can be obtained by using the isospin symmetry relations. All Δq^B are listed in Table VI.

In general, the total spin content of quarks and antiquarks in the octet baryons can be written as (see Table VI)

\[ < s_z >_q^B = \frac{1}{2} - \frac{a}{3}(c_1ξ_1 + c_2ξ_2), \]  

(16)

where c_1 and c_2 satisfy c_1 + c_2 = 3, and (c_1, c_2) = (3, 0), (4, −1), (0, 3), and (−1, 4) for B=N, Σ, Λ, and Ξ respectively. One can see that the spin reductions for all members in the same isospin multiplet are the same, but may be different for different isospin multiplets, except for the SU(3)-symmetry limit (ξ_1 = ξ_2 = 2 + f) and U(3)-symmetry limit (ξ_1 = ξ_2 = 3). In the U(3)-symmetry limit, < s_z >^N_{q+\bar{q}} = < s_z >^Σ_{q+\bar{q}} = < s_z >^Λ_{q+\bar{q}} = < s_z >^Ξ_{q+\bar{q}} = \frac{1}{2} - 3a. Using the parameters ξ_1 ≃ 2.07, and ξ_2 ≃ 1.27, we plot the quark and antiquark spin contents in different octet baryons as function of the parameter a in Fig.3. For a ≃ 0.145, one obtains

\[ < s_z >^N_{q+\bar{q}} ≃ 0.20, \quad < s_z >^Σ_{q+\bar{q}} ≃ 0.16, \quad < s_z >^Λ_{q+\bar{q}} ≃ 0.32, \quad < s_z >^Ξ_{q+\bar{q}} ≃ 0.35 \]  

(17)
(b) Orbital angular momentum in octet baryons

Similar to the nucleon case, the orbital angular momenta carried by quarks and antiquarks in other octet baryons can be calculated. The results for different isospin multiplets are listed in Table VI. The total orbital angular momentum carried by all quarks and antiquarks in the baryon $B$ is

$$<L_z>_q = \frac{a}{3}(c_1\xi_1 + c_2\xi_2).$$  \hspace{1cm} (18)

The sum of spin (16) and orbital angular momentum (18) gives

$$<J_z>_q = <s_z>_q + <L_z>_q = \frac{1}{2}, \quad (B = N, \Sigma^{\pm,0}, \Lambda^0, \Xi^{0,-})$$

Hence we obtain $<J_z>_q = 1/2$ for all octet baryons, i.e. the loss of the quark spin is compensated by the gain of the orbital motion of quarks and antiquarks. The results and conclusions obtained in section III for the nucleon hold for other octet baryons as well. The spin and orbital angular momentum for different quark flavors in the octet baryons are listed in Table VII. Using the isospin symmetry relations $<L_z>_{u,d}^\Sigma^- = <L_z>_{d,u}^\Sigma^+$, one can obtain the orbital angular momenta in $\Sigma^-$ and $\Xi^-$. Similar to the nucleon case, we have $<L_z>_d^\Xi^0 = <L_z>_d^\Xi^0$ at $\kappa = 1/3$ and $<L_z>_u^\Xi^0 = <L_z>_u^\Xi^0$ at $\kappa = 1/3$. This is because, for instance, the sea quark components of $\Sigma^+$ (uus) baryon are $d$ and $\bar{d}$, while those in the proton are $s$ and $\bar{s}$. The same is true for $\Xi^0$ (uss) baryon.

From Table VI, one has

$$\Delta u^B - \Delta d^B = c_B[1 - (\epsilon + 2f)],$$  \hspace{1cm} (19)

where $c_B = 5/3$, $4/3$, and $-1/3$ for $B = p$, $\Sigma^+$, and $\Xi^0$ respectively. Using the isospin symmetry relations, one obtains the following identity

$$\Delta u^p - \Delta u^n + \Delta u^{\Sigma^-} - \Delta u^{\Sigma^+} + \Delta u^{\Xi^0} - \Delta u^{\Xi^-} = 0.$$  \hspace{1cm} (20a)

The same relation holds for $d$–quark spin and $s$–quark spin as well.

One can show by explicit calculation that the orbital angular momentum $<L_z>_u^B$ in the octet baryons satisfy similar identity

$$<L_z>_u^p - <L_z>_u^n + <L_z>_u^{\Sigma^-} - <L_z>_u^{\Sigma^+} + <L_z>_u^{\Xi^0} - <L_z>_u^{\Xi^-} = 0.$$  \hspace{1cm} (20b)

The same relation holds for $<L_z>_d^B$, $<L_z>_s^B$, and $<L_z>_q^B$. Combining (20a) and (20b), one obtains the sum rule for the magnetic moments [see Eq.(24) below] of the octet baryons

$$\mu_p - \mu_n + \mu_{\Sigma^-} - \mu_{\Sigma^+} + \mu_{\Xi^0} - \mu_{\Xi^-} = 0.$$  \hspace{1cm} (21)

This sum rule was discussed in [31] without the orbital contributions. Our result shows that the sum rule (21) holds in the symmetry breaking chiral quark model even the orbital contributions are included. The quark spin contents, but not orbital angular momentum, in the octet baryons were discussed in [33,34].

(c) Decuplet baryons
The above discussion can be extended to the baryon decuplet. The quark spin and orbital angular momenta are listed in Table VIII (we note that the quark spin, without orbital contribution, in the decuplet baryons were discussed in [34]). Again, the explicit calculation shows that \((\Delta u)^{\Delta^+} = (\Delta d)^{\Delta^0}\), \((\Delta u)^{\Delta^0} = (\Delta d)^{\Delta^+}\), \((\Delta u)^{\Sigma^{*+}} = (\Delta d)^{\Sigma^{*0}}\), and \((\Delta u)^{\Xi^{*0}} = (\Delta d)^{\Xi^{*+}}\), which are due to the isospin symmetry of the decuplet baryon wave functions. In Table VIII, we only list the results for \(\Delta^{++}, \Delta^+, \Sigma^{*+}, \Sigma^*, \Xi^*, \Omega^-\).

It is interesting to see that there is an equal spacing rule for total quark spin in the decuplet baryons \(<s_z> = \frac{3}{2} - a[3\xi_1 + S(\xi_1 - \xi_2)], \tag{22a}\) where the \(S\) is the strangeness quantum number of the decuplet baryon \(B^*\). Hence we have \(<s_z> = \frac{3}{2} - a[3\xi_1 + S(\xi_1 - \xi_2)], \tag{22b}\) From (22a), for the strangeless \(\Delta\) multiplet, \(S = 0\), one obtains \(<s_z> = \frac{3}{2} - a(\xi_1 - \xi_2). \tag{22c}\) and \(<s_z> = 3 <s_z> = N <s_z> \), i.e. total spin content of \(\Delta\) baryon is three times that of the nucleon, which is a reasonable result. We note that the equal spacing rule (22b) was also discussed in [34].

For the orbital angular momentum (see Table VIII), we have \(<L_z> = a[3\xi_1 + S(\xi_1 - \xi_2)], \tag{23a}\) Similar equal spacing rule holds for the orbital angular momentum \(<L_z> = a(\xi_1 - \xi_2). \tag{23b}\) The sum of spin (22a) and orbital angular momentum (23a) gives \(<J_z> = \frac{3}{2}. \tag{23c}\) Once again, the spin reduction is compensated by the increase of orbital angular momentum and keep the total angular momentum of the baryon (now is 3/2 for the decuplet) unchanged.

V. Baryon Magnetic Moments.

The baryon magnetic moments depend on both spin and orbital motions of quarks and antiquarks. In the chiral quark model all antiquark sea polarizations are zero, the baryon magnetic moment can be written as \(\mu_B(B^*) = \sum_{q=u,d,s} \mu_q[(\Delta q)^{B(B^*)} + \Delta z_B(q) + \Delta z_q(B^*) - L_z>_{q}^{B(B^*)} \equiv \sum_{q=u,d,s} \mu_q C_q^B, \tag{24}\) where Eq.(7b) has been used. \(B(B^*)\) denote the octet (decuplet) baryons and \(\mu_q\)s are the magnetic moments of quarks. We have assumed that the magnetic moment of the baryon is the sum of spin and orbital magnetic moments of individual charged particles (quarks or
antiquarks). The assumption of additivity is commonly believed to be a good approximation for a loosely bound composite system, which is the basic description for the baryon in the effective chiral quark model. In addition, the baryon may contain other neutral particles, such as gluons (for example see discussion in [31]). Although the glouns do not make any contribution to the magnetic moment, the existence of intrinsic gluon would significantly change the valence quark structure of the baryon due to the spin and color couplings between the gluon and quarks. To calculate the baryon magnetic moments, we need to know the spin content $\Delta q$ (note that $\Delta \bar{q} = 0$ in the chiral quark model) and the difference between the orbital angular momentum carried by quark $q$ and that carried by corresponding antiquark $\bar{q}$, which is denoted by $< L_z >_B^{q-\bar{q}} \equiv < L_z >_B^q - < L_z >_B^{\bar{q}}$. For example, one has for $u$-quark

$$< L_z >_{u-\bar{u}} = \sum_q [n_B^{(0)}(q^\uparrow) - n_B^{(0)}(q^\downarrow)] [< L_z >_u - < L_z >_{\bar{u}}]$$  \hspace{1cm} (25)$$

similar equation holds for $d$-quark, $s$-quark, and corresponding antiquarks, where $\Sigma$ summed over all valence quarks in the baryon $B$. Having obtained $\Delta q^B$ from (7a), and $< L_z >_{q-\bar{q}}$ from (25), one obtains the baryon magnetic moments. Since the quantities $P_{qn}(q_{\uparrow, \downarrow}^q)$ are known (Tables I and II) and universal for all baryons, we only need to know the valence quark numbers $n_B^{(0)}(q_{\uparrow, \downarrow})$ in a specific baryon $B$, and these numbers depend on the model of the baryon. For our purpose of showing the effects of the orbital angular momentum, we only consider the valence quark structure SU(3)$_f \otimes$SU(2)$_s$ without gluon mixing. The hybrid quark-gluon mixing model - three valence quarks and a gluon [32] will be discussed elsewhere.

(a) Octet baryons

From (7a), (24) and (25), one can obtain the analytic expressions of the magnetic moments for the octet baryons. It is easy to verify that they satisfy the following sum rules

$$\mu_p - \mu_n = \mu_{\Sigma^+} - \mu_{\Sigma^-} - (\mu_{\Xi^0} - \mu_{\Xi^-})$$  \hspace{1cm} (4.22), \hspace{1cm} (26a)

$$-6 \mu_A = -2(\mu_p + \mu_n + \mu_{\Xi^0} + \mu_{\Xi^-}) + (\mu_{\Sigma^+} + \mu_{\Sigma^-})$$  \hspace{1cm} (3.34), \hspace{1cm} (26b)

$$\mu_p^2 - \mu_n^2 = (\mu_{\Sigma^+}^2 - \mu_{\Sigma^-}^2) - (\mu_{\Xi^0}^2 - \mu_{\Xi^-}^2)$$  \hspace{1cm} (3.56), \hspace{1cm} (26c)

$$\mu_p - \mu_{\Sigma^+} = \frac{3}{5}(\mu_{\Sigma^-} - \mu_{\Xi^-}) - (\mu_n - \mu_{\Xi^0})$$  \hspace{1cm} (0.31), \hspace{1cm} (26d)$$

where the values of the two sides taken from the data [35] are shown in parentheses. The relations (26a) [or (21)] and (26b) were first given by Franklin in [36]. The relations (26a), (26b), and nonlinear sum rule (26c) are not new and violated at about $10 - 15\%$ level. They have been discussed in many works, for instance [37, 39]. However, the new relation (26d) is rather well satisfied. Our result shows that if the $SU(3) \otimes SU(2)$ valence quark structure is used, chiral fluctuations cannot change these sum rules even the orbital contributions are included. Furthermore, we have shown in [39] that the sum rules (26a)-(26c) also hold for more general case.

The results of applying Eq.(24) to the magnetic moments of the baryon octet are shown in Table IX. We find that if we choose parameters $\mu_u$, $\mu_d$ and $\mu_s$ by fitting to the measured values of $\mu_p$, $\mu_n$ and $\mu_A$ as is also done in the simple SU(6) quark model (NQM) [35], the results for all different $\kappa$ values are completely identical with the NQM result. Several
Remarks should be made here. (1) According to the fitting procedure, for a given set of $\mu_{u,d,s}$ in the NQM, there is a corresponding set of $\mu_{u,d,s}$ which gives the same values of $\mu_\Lambda$ and $\mu_\Lambda$ in the chiral quark model. (2) Seven magnetic moments of the baryon octet satisfy four sum rules, Eqs.(26a-26d), which hold for both the chiral quark model and the NQM, hence as soon as three baryon magnetic moments are identical in both cases, the remaining four should be identical as well. (3) As we discussed in section III, the total amount of quark spin reduction is canceled by the equal amount increase of the quark orbital angular momentum, $<L_z>$, but the cancellation does not happen for each quark flavor. Now we turn into Eq.(24), the relevant term is $C_{q}^{B} \equiv \Delta q_{+}^{B} + <L_z>_{q+\bar{q}}$, which differs from $\Delta q_{+}^{B}/2+ <L_z>_{q+\bar{q}}$ in Eq.(11d), and we have

$$C_{u}^{(0)p} = 4/3, \quad C_{d}^{(0)p} = -1/3, \quad C_{s}^{(0)p} = 0, \quad (\text{NQM}, \quad <L_z>_{q+\bar{q}} = 0),$$

$$C_{u}^{(2)p} = 0.996, \quad C_{d}^{(2)p} = -0.403, \quad C_{s}^{(2)p} = -0.067, \quad (\text{with} \quad <L_z>_{q+\bar{q}}, \quad \kappa = 1/3),$$

$$C_{u}^{(1)p} = 0.863, \quad C_{d}^{(1)p} = -0.397, \quad C_{s}^{(1)p} = -0.067, \quad (\text{without} \quad <L_z>_{q+\bar{q}}).$$

The orbital contribution does move $C_{u}^{(1)p}$ up to $C_{u}^{(2)p}$, but it is still far below 4/3. For the $d$-quark flavor, the orbital contribution moves $C_{d}^{(1)p}$ down, and $C_{d}^{(2)p}$ is even more negative than -1/3. Hence in the magnetic moment, Eq.(24), the cancellation between the contributions of the quark spin reduction and the quark orbital angular momentum is more involved than in the total angular momentum case [see Eq.(11d)].

(b) Decuplet baryons

Similar to the octet baryons, the decuplet magnetic moments are calculated and listed in Table X. Here we use the same set of $\mu_{u}$, $\mu_{d}$ and $\mu_{s}$ as in the octet sector. It is easy to verify that the following equal spacing rules hold,

$$\mu_{\Delta^{++}} - \mu_{\Delta^{+}} = \mu_{\Delta^{+}} - \mu_{\Delta^{0}} = \mu_{\Delta^{0}} - \mu_{\Delta^{-}} = \mu_{\Sigma^{++}} - \mu_{\Sigma^{+}} = \mu_{\Sigma^{+}} - \mu_{\Sigma^{0}} = \mu_{\Sigma^{0}} - \mu_{\Sigma^{-}} = \mu_{\Xi^{0}} - \mu_{\Xi^{-}} \quad (27a)$$

$$\mu_{\Delta^{+}} - \mu_{\Sigma^{+}} = \mu_{\Delta^{0}} - \mu_{\Sigma^{0}} = \mu_{\Delta^{-}} - \mu_{\Sigma^{-}} = \mu_{\Sigma^{+}} - \mu_{\Xi^{0}} = \mu_{\Sigma^{0}} - \mu_{\Xi^{-}} = \mu_{\Xi^{0}} - \mu_{\Omega^{+}}. \quad (27b)$$

From Table X, the spacing is 2.82 n.m. in Eq.(27a), and -0.36 n.m. in Eq.(27b). Similar to the octet baryon, the decuplet magnetic moments with and without orbital contributions are approximately the same provided changing the quark magnetic moments accordingly. Hence the baryon magnetic moment is not a good nucleon property for revealing the quark orbital angular momentum, unless the quark magnetic moments are known.

V. Summary

An unified scheme for describing quark flavor, spin and orbital contents in the baryon in the chiral quark model is suggested. Contrary to the reduction effect on the quark spin component, the quark splitting mechanism produces a positive orbital angular momentum carried by the quarks and antiquarks. The results of the quark flavor and spin observables of the nucleon are listed in Table XI. They are in good agreement with the existing data.

We have calculated the orbital angular momentum carried by different quark flavors in the baryons. These orbital angular momenta might be determined indirectly in the DVCS or other processes. Attention has been paid to the orbital contributions on the baryon magnetic moments.
To summarize, the chiral quark model with only a few parameters can well explain many nucleon properties: (1) strong flavor asymmetry of light antiquark sea: $\bar{d} > \bar{u}$, (2) nonzero strange quark content, $<\bar{s}s> \neq 0$, (3) sum of quark spins is small, $<s_z>_{q+\bar{q}} \simeq 0.1-0.2$, (4) sea antiquarks are not polarized: $\Delta \bar{q} \simeq 0$ ($q = u, d, ...$), (5) polarizations of the sea quarks are nonzero and negative, $\Delta q_{\text{sea}} < 0$, and (6) the orbital angular momentum of the sea quark is parallel to the proton spin. (1)-(4) are consistent with data, and (5)-(6) could be tested by future experiments.

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TABLES

TABLE I. The probabilities $P_{q_i}(q_{i\uparrow}, q_{i\downarrow})$ and $P_{q_i}(q_{i\uparrow}, q_{i\downarrow})$

| $q'$ | $P_{u_+}(q_{i\uparrow})$ | $P_{d_+}(q_{i\downarrow})$ | $P_{s_+}(q_{i\downarrow})$ |
|------|-----------------|-----------------|-----------------|
| $u_\uparrow$ | $1 - (\frac{1}{2} + f) a + \frac{a}{18}(3 - A)^2$ | $\frac{1}{18} A^2$ | $\frac{1}{18} B^2$ |
| $u_\downarrow$ | $(\frac{1}{2} + f) a + \frac{a}{18}(3 - A)^2$ | $a + \frac{a}{18} A^2$ | $\epsilon a + \frac{a}{18} B^2$ |
| $d_+^\uparrow$ | $\frac{a}{18} A^2$ | $1 - (\frac{1}{2} + f) a + \frac{a}{18}(3 - A)^2$ | $\frac{a}{18} B^2$ |
| $d_\downarrow$ | $\frac{a}{18} A^2$ | $(\frac{1}{2} + f) a + \frac{a}{18}(3 - A)^2$ | $\epsilon a + \frac{a}{18} B^2$ |
| $s_+^\uparrow$ | $\epsilon a + \frac{a}{18} B^2$ | $\epsilon a + \frac{a}{18} B^2$ | $1 - (\epsilon + f_s) a + \frac{a}{18} C^2$ |
| $s_\downarrow$ | $\epsilon a + \frac{a}{18} B^2$ | $\epsilon a + \frac{a}{18} B^2$ | $(\epsilon + f_s) a + \frac{a}{18} C^2$ |

TABLE II. The spin-up, spin-down quark (antiquark), spin-average and spin-weighted quark (antiquark) contents in the proton. Where $U_1 = \frac{1}{3}[A^2 + 2(3 - A)^2]$, $D_1 = \frac{1}{3}[2A^2 + (3 - A)^2]$, and $U_2 = 4D_2 = 4(\epsilon + 2f - 1)$.

| $u_\uparrow$ | $\frac{a}{3} + \frac{a}{3}(3 + \frac{1}{3}U_1)$ | $d_1 = \frac{1}{3} + \frac{a}{3}(2 + \frac{D_1}{2} + \frac{D_2}{2})$ | $s_\uparrow = \epsilon a + \frac{a}{3} (\epsilon + f_s)$ |
| $u_\downarrow$ | $\frac{a}{3} + \frac{a}{3}(5 + \frac{1}{3}U_1)$ | $d_1 = \frac{2}{3} + \frac{a}{3}(4 + \frac{D_1}{2} - \frac{D_2}{2})$ | $s_\downarrow = \epsilon a + \frac{a}{3} (\epsilon + f_s)$ |
| $\bar{u}_\uparrow = \bar{u}_\downarrow$ | $\frac{a}{3} + \frac{a}{3}(\frac{1}{3})$ | $d_\uparrow = d_\downarrow = a + \frac{a}{3}(\frac{D_1}{2})$ | $\bar{s}_\uparrow = \bar{s}_\downarrow = \frac{3a}{2} + \frac{a}{3} (\epsilon + f_s)$ |
| $u = 2 + \frac{a}{3}(3 + U_1)$ | $d = 1 + \frac{a}{3}(6 + D_1)$ | $s = 3\epsilon a + \frac{a}{3} B^2$ |
| $\bar{u} = \frac{a}{3}(3 + U_1)$ | $\bar{d} = \frac{1}{3}(6 + D_1)$ | $\bar{s} = 3\epsilon a + \frac{a}{3} B^2$ |

$\Delta u = \frac{4}{3}[1 - a(\epsilon + 2f)] - a$ | $\Delta d = \frac{2}{3}[1 - a(\epsilon + 2f)] - a$ | $\Delta s = a(1 - \epsilon) - a$

| $\Delta \bar{u} = 0$ | $\Delta \bar{d} = 0$ | $\Delta \bar{s} = 0$ |

TABLE III. The orbital angular momentum carried by the quark $q'$ ($q''$), spin-up and spin-down are included, from a valence spin-up quark $q_\uparrow$ fluctuates into all allowed final states.

| $q'$ | $< L_z > q', q''/u_\uparrow$ | $< L_z > q', q''/d_\uparrow$ | $< L_z > q', q''/s_\uparrow$ |
|------|-----------------|-----------------|-----------------|
| $u$ | $\kappa a[\xi_1 + \delta f + (\frac{3-A)^2}{9}]$ | $\kappa a[1 + \delta + \frac{A^2}{9}]$ | $\kappa a[\epsilon(1 + \delta) + \frac{A^2}{9}]$ |
| $d$ | $\kappa a[1 + \delta + \frac{A^2}{9}]$ | $\kappa a[\xi_2 + f \delta + (\frac{3-A)^2}{9}]$ | $\kappa a[\epsilon(1 + \delta) + \frac{A^2}{9}]$ |
| $s$ | $\kappa a[\frac{1}{3} + \delta + \frac{A^2}{9}]$ | $\kappa a[\epsilon(1 + \delta) + \frac{A^2}{9}]$ | $\kappa a[\xi_2 + f \delta + (\frac{3-A)^2}{9}]$ |
| $\bar{u}$ | $\kappa a[\frac{3-A)^2}{9}]$ | $\kappa a[1 + \frac{A^2}{9}]$ | $\kappa a[\epsilon + \frac{B^2}{9}]$ |
| $\bar{d}$ | $\kappa a[1 + \frac{A^2}{9}]$ | $\kappa a[\xi_2 + f \delta + (\frac{3-A)^2}{9}]$ | $\kappa a[\epsilon + \frac{B^2}{9}]$ |
| $\bar{s}$ | $\kappa a[\epsilon + \frac{B^2}{9}]$ | $\kappa a[\xi_2 + f \delta + (\frac{3-A)^2}{9}]$ | $\kappa a[\epsilon + \frac{B^2}{9}]$ |
TABLE IV. Quark spin and orbital angular momentum in the proton in different models.

| Quantity | Data [3] | This paper | Sehgal [18] | NQM |
|----------|----------|------------|-------------|-----|
| $< L_z >_u^p$ | $\kappa = 1/4$ | $0.115$ | $0.130$ | $0.138$ | $-$ | $0$ |
| $< L_z >_d^p$ | $\kappa = 1/3$ | $0.073$ | $0.043$ | $0.027$ | $-$ | $0$ |
| $< L_z >_s^p$ | $\kappa = 3/8$ | $0.038$ | $0.028$ | $0.023$ | $-$ | $0$ |
| $< L_z >_{\bar{u}}^p$ | $-$ | $-0.003$ | $-0.003$ | $-0.004$ | $-$ | $0$ |
| $< L_z >_{\bar{d}}^p$ | $-$ | $0.057$ | $0.076$ | $0.086$ | $-$ | $0$ |
| $< L_z >_{\bar{s}}^p$ | $-$ | $0.021$ | $0.028$ | $0.031$ | $-$ | $0$ |
| $< L_z >_{q+\bar{q}}^p$ | $-$ | $0.30$ | $0.30$ | $0.30$ | $0.39$ | $0$ |
| $\Delta u^p$ | $0.85 \pm 0.05$ | $0.86$ | $0.78$ | $4/3$ |
| $\Delta d^p$ | $-0.41 \pm 0.05$ | $-0.40$ | $-0.34$ | $-1/3$ |
| $\Delta s^p$ | $-0.07 \pm 0.05$ | $-0.07$ | $-0.14$ | $0$ |
| $\frac{1}{2} \Delta \Sigma^p$ | $0.19 \pm 0.06$ | $0.20$ | $0.08$ | $1/2$ |

TABLE V. Quark spin and orbital angular momentum in different models.

| NQM | MIT bag | This paper | CS [19] | Skyrme |
|-----|---------|------------|---------|--------|
| $< s_z >_{q+\bar{q}}$ | $1/2$ | $0.32$ | $0.20$ | $0.08$ | $0$ |
| $< L_z >_{q+\bar{q}}$ | $0$ | $0.18$ | $0.30$ | $0.42$ | $1/2$ |

TABLE VI. The quark spin and orbital contents in the octet baryons.

| Baryon | $\Delta\kappa \xi_1$ | $\Delta\kappa \xi_2$ | $\Delta\kappa (4\xi_1 - \xi_2)$ | $\Delta\kappa (4\xi_2 - \xi_1)$ |
|--------|-----------------|-----------------|------------------|------------------|
| $p$    | $(2 + \delta)\kappa\xi_1$ | $\kappa\xi_1$ | $a\xi_1$ | $a\xi_1$ |
| $\Sigma^+$ | $(2 + \delta)\kappa\xi_1$ | $\kappa\xi_1$ | $a\xi_1$ | $\kappa\xi_1$ |
| $\Lambda^0$ | $(2 + \delta)\kappa\xi_2$ | $\kappa\xi_2$ | $\kappa\xi_2$ | $\kappa\xi_2$ |
| $\Xi^0$ | $(2 + \delta)\kappa\xi_2$ | $\kappa\xi_2$ | $\kappa\xi_2$ | $\kappa\xi_2$ |
TABLE VII. Quark spin and orbital angular momentum in other octet baryons.

| Baryon | $\Sigma^+$ | $\Lambda$ | $\Xi^0$ |
|--------|------------|-----------|---------|
|        | $\kappa = 1/4$ | $\kappa = 1/3$ | $\kappa = 3/8$ | $\kappa = 1/4$ | $\kappa = 1/3$ | $\kappa = 3/8$ | $\kappa = 1/4$ | $\kappa = 1/3$ | $\kappa = 3/8$ |
| $< L_z >_{\bar{\Sigma}^+}$ | 0.130 | 0.141 | 0.147 | 0.038 | 0.028 | 0.023 | 0.014 | 0.000 | −0.008 |
| $< L_z >_{\Lambda}$ | 0.096 | 0.071 | 0.058 | 0.038 | 0.028 | 0.023 | 0.023 | 0.017 | 0.014 |
| $< L_z >_{\bar{\Xi}}$ | 0.029 | 0.015 | 0.007 | 0.063 | 0.067 | 0.069 | 0.071 | 0.080 | 0.085 |
| $< L_z >_{\bar{\Xi}^0}$ | 0.005 | 0.007 | 0.008 | 0.021 | 0.028 | 0.031 | 0.025 | 0.033 | 0.037 |
| $< L_z >_{\bar{\Xi}^0}$ | 0.053 | 0.071 | 0.079 | 0.021 | 0.028 | 0.031 | 0.013 | 0.017 | 0.019 |
| $< L_z >_{\bar{\Xi}^0}$ | 0.026 | 0.035 | 0.039 | 0.004 | 0.006 | 0.007 | −0.001 | −0.001 | −0.002 |
| $< L_z >_{\bar{\Xi}^0}$ | 0.34 | 0.34 | 0.34 | 0.18 | 0.18 | 0.18 | 0.15 | 0.15 | 0.15 |

TABLE VIII. The quark spin and orbital contents in the decuplet baryons.

| Baryon | $\Delta u^{B^*}$ | $\Delta d^{B^*}$ | $\Delta s^{B^*}$ |
|--------|-----------------|-----------------|-----------------|
| $\Delta^{++}$ | $3 - 3a(2\xi_1 - \epsilon - 1)$ | $-3a$ | $-3a\epsilon$ |
| $\Delta^+$ | $2 - a(4\xi_1 - 2\epsilon - 1)$ | $1 - a(2\xi_1 - \epsilon + 1)$ | $-3a\epsilon$ |
| $\Sigma^+$ | $2 - a(4\xi_1 + \epsilon - 1)$ | $1 - 2a\xi_1$ | $1 - 2a\xi_2$ |
| $\Sigma^0$ | $1 - 2a\xi_1$ | $1 - 2a\xi_1$ | $-a(\epsilon + 2)$ |
| $\Xi^0$ | $-a(\epsilon + 2)$ | $-a(\epsilon + 1)$ | $2 - a(4\xi_2 - 3\epsilon)$ |
| $\Omega^-$ | $-3a\epsilon$ | $-3a\epsilon$ | $3 - 6a(\xi_2 - \epsilon)$ |

| $\frac{1}{2} \Delta \Sigma^{B^*}$ | $0.16$ | $0.32$ | $0.35$ |

| $\Delta$ | $(2 + \delta)\kappa a(3\xi_1)$ | $\kappa a(3\xi_1)$ | $a(3\xi_1)$ |
| $\Sigma$ | $(2 + \delta)\kappa a(2\xi_1 + \xi_2)$ | $\kappa a(2\xi_1 + \xi_2)$ | $a(2\xi_1 + \xi_2)$ |
| $\Xi$ | $(2 + \delta)\kappa a(\xi_1 + 2\xi_2)$ | $\kappa a(\xi_1 + 2\xi_2)$ | $a(\xi_1 + 2\xi_2)$ |
| $\Omega$ | $(2 + \delta)\kappa a(3\xi_2)$ | $\kappa a(3\xi_2)$ | $a(3\xi_2)$ |
TABLE IX. Comparison of our predictions with data for the octet baryon magnetic moments. The naive quark model (NQM) results are also listed. The quantity used as input is indicated by a star.

| Baryon | data   | $\kappa = 1/4$ | This paper $\kappa = 1/3$ | $\kappa = 3/8$ | NQM   |
|--------|--------|----------------|---------------------------|----------------|-------|
| p      | $2.79 \pm 0.00$ | $2.79^*$       | $2.79^*$                  | $2.79^*$       |       |
| n      | $-1.91 \pm 0.00$ | $-1.91^*$      | $-1.91^*$                 | $-1.91^*$      |       |
| $\Sigma^+$ | $2.46 \pm 0.01$ | 2.67          |                          | 2.67           |       |
| $\Sigma^-$ | $-1.16 \pm 0.03$ | $-1.09$       |                          | $-1.09^*$      |       |
| $\Lambda^0$ | $-0.61 \pm 0.00$ | $-0.61^*$     |                          | $-0.61^*$      |       |
| $\Xi^0$ | $-1.25 \pm 0.01$ | $-1.43$       |                          | $-1.43^*$      |       |
| $\Xi^-$ | $-0.65 \pm 0.00$ | $-0.49$       |                          | $-0.49^*$      |       |
| $\mu_u$ | 2.404  | 2.351         | 2.328                     | 1.85           |       |
| $\mu_d$ | $-1.047$ | $-0.944$     | $-0.892$                  | $-0.97^*$      |       |
| $\mu_s$ | $-0.657$ | $-0.623$     | $-0.606$                  | $-0.61^*$      |       |

TABLE X. Comparison of our predictions with data for the decuplet baryon magnetic moments. The naive quark model (NQM) results are also listed.

| Baryon | data   | $\kappa = 1/3$ | This paper $\kappa = 1/3$ | NQM   |
|--------|--------|----------------|---------------------------|-------|
| $\Delta^{++}$ | $4.52 \pm 0.50 \pm 0.45$ | $5.55$       | 5.58                      |       |
| $\Delta^+$ | $3.7 < \mu_{\Delta^{++}} < 7.5$ | $2.73$       | 2.79                      |       |
| $\Delta^0$ | $-0.09$ |               | 0.00                      |       |
| $\Delta^-$ | $-2.91$ |               | $-2.79$                   |       |
| $\Sigma^{++}$ | $3.09$ |               | 3.11                      |       |
| $\Sigma^{+0}$ | $0.27$ |               | 0.32                      |       |
| $\Sigma^{0-}$ | $-2.55$ |               | $-2.47$                   |       |
| $\Xi^{+0}$ | $0.63$ |               | 0.64                      |       |
| $\Xi^{*-}$ | $-2.19$ |               | $-2.15$                   |       |
| $\Omega^-$ | $-1.94 \pm 0.17 \pm 0.14$ | $-1.83$     | $-1.83$                   |       |
|            | $-2.024 \pm 0.056$ |               |                           |       |
| $\mu_u$ | 2.351  |               | 1.85                      |       |
| $\mu_d$ | $-0.944$ |               | $-0.97$                   |       |
| $\mu_s$ | $-0.623$ |               | $-0.61$                   |       |
TABLE XI. Quark spin and flavor observables in the proton. The quantity used as input is indicated by a star.

| Quantity | Data     | This paper | NQM |
|----------|----------|------------|-----|
| $\bar{d} - \bar{u}$ | $0.147 \pm 0.039$ | $0.143^*$ | 0   |
| $\bar{u}/\bar{d}$ | $0.100 \pm 0.018$ | $0.64$ | –   |
| $2\bar{s}/(\bar{u} + \bar{d})$ | $<\bar{s}/(\bar{u} + \bar{d})> = 0.477 \pm 0.051$ | $0.72$ | –   |
| $2\bar{s}/(u + d)$ | $<\bar{s}/(u + d)> = 0.099 \pm 0.009$ | $0.13$ | 0   |
| $\sum \bar{q}/\sum q$ | $<\bar{q}/q> = 0.245 \pm 0.005$ | $0.23$ | 0   |
| $f_s$ | $0.10 \pm 0.06$ | $0.10$ | 0   |
| $f_3/f_8$ | $0.21 \pm 0.05$ | $0.22$ | $1/3$ |
| $\Delta u$ | $0.85 \pm 0.05$ | $0.86$ | $4/3$ |
| $\Delta d$ | $-0.41 \pm 0.05$ | $-0.40$ | $-1/3$ |
| $\Delta s$ | $-0.07 \pm 0.05$ | $-0.07$ | 0   |
| $\Delta \bar{u}, \Delta \bar{d}$ | $-0.02 \pm 0.11$ | 0 | 0   |
| $\Delta_3$ | $2.17 \pm 0.10$ | $2.12$ | $5/3$ |
| $\Delta_8$ | $1.2601 \pm 0.0028$ | $1.26^*$ | $5/3$ |
| $\Delta_8$ | $0.579 \pm 0.025$ | $0.60^*$ | 1   |
FIG. 1. Quark or antiquark orbital angular momentum $< L_z >_{q,ar{q}}$ in the proton as function of the partition factor $\kappa$. 
FIG. 2. Quark spin and orbital angular momentum ($< s_z >_{q+q}$ versus $< L_z >_{q+q}$) in the nucleon in different models.
FIG. 3. Quark spin content ($< s_z >_{q+\bar{q}}$) in different octet baryons as function of $a$