Numerical simulation of adiabatic shear bands formation processes on two-dimensional eulerian meshes

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Abstract. In this work we suggest a mathematical model of motion of the elasto-plastic materials with nonlinear plasticity constitutive law; we also propose an effective numerical method for numerical simulations of such tasks on two-dimensional eulerian meshes. Based on the method, we research formation of multiple adiabatic shear bands (ASB) at high-speed shear deformations. We test our approach on two-dimensional problem where the initial heterogeneity of temperature leads to formation of adiabatic shear band.

1. Introduction
The study of the strength properties of different technologically important materials subjected to shear deformations is one of the most important sections of mechanics [1, 2]. In addition to real experiments, numerical simulation is also a very useful tool for investigation of the strength properties of different technologically important materials [3, 4, 5]. Most often, the finite element method is used in the numerical simulation of solid materials, but it has some drawbacks. Disadvantages such as mesh entanglement, as well as a decrease in the accuracy of calculations with significant distortion of elements, often arise when high-speed deformations are simulated. To avoid this problem we propose a finite volume scheme for numerical simulation of ASB formation on fixed meshes.

2. Mathematical model of ASB formation
We consider a large deformations of an elasto-plastic material. A motion of continuum substance is described by a classical continuity equations which have following form. A conservation of mass (1), the Cauchy momentum equation (2), a conservation of energy (3).

\[
\frac{\partial \rho}{\partial t} + \text{div} (\rho \vec{v}) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t} (\rho \vec{v}) + \text{div} (\rho \vec{v} \times \vec{v} - \sigma) = 0. \tag{2}
\]

\[
\frac{\partial}{\partial t} (\rho E) + \text{div} (\rho E \vec{v} - \sigma \vec{v} - k \text{ grad } T) = 0, \tag{3}
\]

\[E = e(T) + \frac{1}{2} \vec{v}^2, \quad e(T) = C \nu T,\]
where \( \rho \) is density, \( \vec{v} \) is velocity, \( T \) is temperature, \( e \) and \( E \) are internal and total energies respectively, \( \sigma \) is Cauchy stress tensor.

Then we split a total deformations into elastic and plastic parts \( \varepsilon = \varepsilon^e + \varepsilon^p \). We use Hooke’s law for elastic part and associative flow rule for plastic part of deformation. As a result, we obtain a following equations

\[
\overset{\circ}{\sigma} = 2 \mu (\overset{\circ}{\varepsilon} - \overset{\circ}{\varepsilon}^p), \quad \overset{\circ}{\varepsilon}^p = \frac{3s}{2\sigma}, \quad \overset{\circ}{\tau} = \sigma_Y^{-1}(\sigma, T),
\]

where \( s \) is a deviatoric stress tensor, \( \overset{\circ}{\varepsilon} \) is a deviatoric strain-rate tensor (index \( p \) for plastic part), \( \overset{\circ}{\sigma} \) is an objective stress rate (we use Jaumann derivative), \( \overset{\circ}{\tau} \) and \( \sigma \) are effective strain-rate and effective stress respectively.

The function \( \sigma_Y(\overset{\circ}{\tau}, T) \) is a plasticity flow law which is written in the form of Litonsky law [REF]:

\[
\sigma_Y^{-1}(\sigma, T) = \begin{cases} \dot{\varepsilon}_y \left( \left( \frac{\sigma}{\sigma_0 g(T)} \right)^{\frac{1}{\gamma}} - 1 \right), & \sigma > \sigma_0 g(T), \\ 0, & \text{otherwise,} \end{cases}
\]

where \( g(T) = 1 - aT \) is temperature softening factor and \( g(T) \) is equal to zero at melting temperature.

Let us consider two dimensional case. We have twelve equations and the same amount of dependent variables: \( \rho, u, v, p, e, T, s_1, s_2, \tau, \dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\gamma} \). We redefine some variables for simplicity. Variables \( u \) and \( v \) are components of velocity, \( p \) is the pressure, \( s_1 \) and \( s_2 \) are diagonal components of deviatoric stress tensor, \( \tau \) is shear component of deviatoric stress tensor, \( \dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\gamma} \) are components of plastic part of the deviatoric strain-rate tensor (we reduce index \( p \)).

A system of equations in two-dimensional case:

\[
\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + p - s_1) + \frac{\partial}{\partial y}(\rho uv - \tau) = 0,
\]

\[
\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho uv - \tau) + \frac{\partial}{\partial y}(\rho v^2 + p - s_2) = 0,
\]

\[
\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x}(u(\rho E + p - s_1) - \tau v) + \frac{\partial}{\partial y}(v(\rho E + p - s_2) - \tau u) = k \Delta T.
\]

The stress tensor evolution:

\[
s_{1,t} + us_{1,x} + vs_{1,y} + \frac{2\mu}{3} (v_y - 2u_x) = \tau(u_y - v_x) - 2\mu \dot{\varepsilon}_1,
\]

\[
s_{2,t} + us_{2,x} + vs_{2,y} + \frac{2\mu}{3} (u_x - 2v_y) = \tau(v_x - u_y) - 2\mu \dot{\varepsilon}_2,
\]

\[
\tau_t + u \tau_x + v \tau_y - \mu (v_x + u_y) = \frac{1}{2} (s_1 - s_2) (v_x - u_y) - 2\mu \dot{\gamma},
\]

\[
\dot{\varepsilon}_1 = \frac{3s_1}{2\sigma} \sigma_Y^{-1}(\sigma, \theta), \quad \dot{\varepsilon}_2 = \frac{3s_2}{2\sigma} \sigma_Y^{-1}(\sigma, \theta), \quad \dot{\gamma} = \frac{3\tau}{2\sigma} \sigma_Y^{-1}(\sigma, \theta).
\]

An equation of state in form \( p(\rho, e) \) close the system of equations. We use equation of state in form of Tillotson

\[
p(\rho, e) = B \left( \frac{\rho}{\rho_0} - 1 \right).
\]
Table 1. Thermophysical parameters of steel.

| $C_v$, J/(kg °C) | $k$, W/(m °C) | $\rho_0$, kg/m³ | $B$, GPa | $\mu$, GPa | $\sigma_0$, MPa | $a$, 1/°C | $\dot{\varepsilon}_y$, 1/s | $m$ |
|------------------|--------------|----------------|----------|---------|-------------|----------|----------|-----|
| 473              | 49.22        | 7800           | 128      | 80      | 333         | 0.0222   | 0.1      | 0.025 |

3. Numerical approximation

The left-hand side of the equations (6–10) constitute a hyperbolic subsystem of equations, so to approximate this part we use a scheme of the Godunov type. Let’s mark vector of variables as $\vec{q} = (\rho, \rho u, \rho v, \rho E, s_1, s_2, \tau)^T$, then finite volume scheme will take the following form

$$\vec{q}^{n+1}_{\alpha} = \vec{q}^n_{\alpha} - \frac{\Delta t}{V_{\alpha}} \sum_{\beta \in V(\alpha)} R^{-1}_{\alpha\beta} \vec{F}^*_{\alpha\beta} S_{\alpha\beta} + \Delta t \vec{H}^n_{\alpha}, \quad (13)$$

where $\alpha, \beta$ are indexes of cells, $V(\alpha)$ are indexes of cells adjacent to $\alpha$, $V_{\alpha}$ is volume of cell, $S_{\alpha\beta}$ is area of face between cells $\alpha\beta$, $\vec{n}_{\alpha\beta}$ is outward normal to face (from cell $\alpha$ to cell $\beta$). The vector $\vec{H}$ includes the right-hand side of equations (6–10). The matrix $R$ is rotation matrix to local basis related to outward normal, so vector $\vec{q}$ in local basis is defined as $\vec{Q} = R \vec{q}$.

The vector $\vec{F}^*_{\alpha\beta}$ is the numerical flux between cells for locally one-dimensional Riemann problem that is non-conservative:

$$\frac{\partial \vec{Q}}{\partial t} + B \frac{\partial \vec{Q}}{\partial x} + \frac{\partial \vec{F}^*}{\partial x} = 0.$$

To compute numerical flux we use Courant-Isaacson-Rees scheme, and we get following formula

$$\vec{F}^*_{\alpha\beta} = \frac{1}{2} B_{\alpha\beta} \cdot (\vec{Q}_{\beta} - \vec{Q}_{\alpha}) + \frac{1}{2} \vec{F}_{\alpha\beta} - \frac{1}{2} |A_{\alpha\beta}| \left( \vec{Q}_{\beta} - \vec{Q}_{\alpha} \right), \quad (14)$$

$$|A| = \Omega_R |A| \Omega_L, \quad A = \Omega_R A \Omega_L = B + \frac{\partial \vec{F}^*}{\partial \vec{Q}}.$$

Here $A = \Omega_R A \Omega_L$ is the eigendecomposition of the Jacobian matrix of the hyperbolic system. The eigenvalues of the system is a real numbers and

$$\Lambda = \text{diag}(u - c_p, u - c_s, u, u, u, u + c_s, u + c_p), \quad (15)$$

$$c_s^2 = \frac{\mu}{\rho}, \quad c_p^2 = \frac{B}{\rho_0} + \frac{4}{3} c_s^2. \quad (16)$$

where $c_p$ and $c_s$ are speed of primary and secondary waves respectively.

4. Benchmark problem

We test our approach on the plane problem was investigated in the work [6] by Batra. We study formation of adiabatic shear band in an infinite steel sample during shear deformation. The computational domain is $(x, y) \in [-2H, 2H] \times [-H, H]$, where $H$ equals to 5 mm. Top and bottom boundaries are insulated and shifted in opposite directions so that an overall applied strain-rate $\dot{\varepsilon}_0$ is $5 \cdot 10^3$ 1/s.

$$v(x, \pm H) = \pm \dot{\varepsilon}_0 H, \quad T_y(x, \pm H) = 0. \quad (17)$$
a) Temperature ($T$) at $\varepsilon_{\text{nom}} = 0.18$

b) Temperature ($T$) at $\varepsilon_{\text{nom}} = 0.23$

c) Velocity ($u$) at $\varepsilon_{\text{nom}} = 0.18$

d) Velocity ($u$) at $\varepsilon_{\text{nom}} = 0.23$

Figure 1. Temperature and velocity in the sample at different time moments.

We use periodic boundary conditions along $y$ axis.

The velocity $u$ at initial moment has a linear distribution and the temperature has a perturbation at center of the slab that leads to ASB formation. For the other variables zero initial conditions are used.

$$u(x, y) = \dot{\varepsilon}_0 y, \quad T(x, y) = 18(1 - r^2)^9 \exp(-5r^2), \quad r^2 = \left(x^2 + y^2\right)/H^2.$$ (18)

The thermophysical parameters of the steel can be found in table 1.

The initial heteroehenity of temperature results in the formation of single ASB in the center of the sample. Stages of ASB formation are shown in figure 1. All parameters in table 1 equals to the parameters in the article [6]. These parameters correspond to high tensile steel HY-100, except for the softening coefficient $a$. That value is increased to reduce the computational time required to solve problem, and melting temperature in this case is about 45°C. As in the article [6], we introduce dimensionless variable $\varepsilon_{\text{nom}} = \dot{\varepsilon}_0 t$. At $\varepsilon_{\text{nom}} = 0.23$ temperature at the center of the sample reaches a melting point (Fig. 1b), and the discontinuity appears on graph of the velocity (Fig. 1d). The results obtained are consistent with the results obtained by Batra.

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