Projectile deformation effects in the breakup of $^{37}$Mg

SHUBHCHINTAK$^1$, R. CHATTERJEE$^2$ and R. SHYAM$^{3,2}$

$^1$Department of Physics and Astronomy, Texas A&M University
Commerce, TX-75428, USA
$^2$Department of Physics, Indian Institute of Technology
Roorkee, 247667, India
$^3$Theory Group, Saha Institute of Nuclear Physics
1/AF Bidhannagar, Kolkata 700064, India

Abstract

We study the breakup of $^{37}$Mg on Pb at 244MeV/u with the recently developed extended theory of Coulomb breakup within the post-form finite range distorted wave Born approximation that includes deformation of the projectile. Comparing our calculated cross section with the available Coulomb breakup data we determine the possible ground state configuration of $^{37}$Mg.

1 Introduction and formalism

Coulomb breakup of nuclei away from the valley of stability has been one of the most successful probes to unravel their structure. However, it is only recently that one is venturing into medium mass nuclei like $^{23}$O [1] and $^{31}$Ne [2], especially in and around the so called “island of inversion”. This is a very new and exciting development which has expanded the field of light exotic nuclei to the deformed medium mass region.

We consider the elastic breakup of a two body composite ‘deformed’ projectile $a$ in the Coulomb field of a target $t$, $a + t \rightarrow b + c + t$, where
projectile a breaks up into fragments b (charged) and c (uncharged). The reduced transition amplitude, $\beta_{\ell m}$, for this reaction is given by [3]

$$
\beta_{\ell m} = \langle e^{i(\gamma q_e - \alpha K) \cdot r_1} | V_{bc}(r_1) | \phi_a^{\ell m}(r_1) \rangle \times \langle \chi_b^{(-)}(q_b, r_1) e^{i\delta q_e \cdot r_1} | \chi_a^{(+)}(q_a, r_1) \rangle.
$$

The ground state wave function of the projectile $\phi_a^{\ell m}(r_1)$ appears in the first term (vertex function), while the second term that describes the dynamics of the reaction, contains the Coulomb distorted waves $\chi^{(\pm)}$. This can be expressed in terms of the bremsstrahlung integral. $\alpha$, $\gamma$ and $\delta$ are the mass factors pertaining to the three-body Jacobi coordinate system (see Fig. 1 of Ref. [2]). In Eq. 1, $K$ is an effective local momentum appropriate to the core-target relative system and $q_i$’s ($i = a, b, c$) are the Jacobi wave vectors of the respective particles.

$V_{bc}(r_1)$ [in Eq. (1)] is the interaction between b and c, in the initial channel. We introduce an axially symmetric quadrupole-deformed potential, as

$$
V_{bc}(r_1) = \frac{V_{ws}}{1 + \exp\left(\frac{r_1 - R}{a}\right)} - \beta_2 R V_{ws} \frac{d f(r_1)}{d r_1} Y_0^2(\hat{r}_1),
$$

where $V_{ws}$ is the depth of the spherical Woods-Saxon potential, $\beta_2$ is the quadrupole deformation parameter. The first part of the Eq. (2) is the spherical Woods-Saxon potential $V_s(r_1)$ with radius $R = r_0 A^{1/3}$. $r_0$ and $a$ being the radius and diffuseness parameters, respectively. To preserve the analyticity of our method, we calculate the radial part of the ground state wave function of the projectile using the undeformed Woods-Saxon potential (radius and diffuseness parameters taken as 1.24 fm and 0.62 fm respectively, which reproduce the ground state binding energy). We emphasize that the deformation parameter ($\beta_2$) has already entered into the theory via $V_{bc}$ in Eq. (1). For more details on the formalism we refer to Ref. [4].

### 2 Results and discussions

The nucleus $^{37}\text{Mg}$ has a large uncertainty in its one-neutron separation energy ($0.162 \pm 0.086$ MeV [7]) and has controversies regarding its ground state spin-parity. Recently measured large breakup cross section [5] and reaction cross section [6] seems to suggest a halo structure in $^{37}\text{Mg}$. 
Figure 1: (a) Pure Coulomb total one-neutron removal cross section, $\sigma_{-1n}$, in the breakup reaction of $^{37}$Mg on a Pb target at 244 MeV/nucleon beam energy as a function of one-neutron separation energy $S_{-1n}$ obtained with configurations $^{36}$Mg($0^+ \otimes 2p_{3/2}^\nu$, $C^2S = 0.31$, $^{36}$Mg($0^+ \otimes 2s_{1/2}^\nu$, $C^2S = 1.00$) and $^{36}$Mg($0^+ \otimes 1f_{7/2}^\nu$, $C^2S = 0.001$) for $^{37}$Mg$_{gs}$ using the shell model spectroscopic factors ($C^2S$) as indicated in each case. The experimental cross section (taken from Ref. [5]) is shown by the shaded band. (b) Same reaction as in (a) for the $^{36}$Mg($0^+ \otimes 2s_{1/2}^\nu$ configuration using the $C^2S$ value of 0.40 as deduced in Ref. [5].

The nuclei in island of inversion are expected to have significant components of $2p - 2h [\nu(sd)^{-2}(fp)^2]$ neutron intruder configurations. Indeed, in Ref. [5], it has been argued that the valence neutron in $^{37}$Mg$_{gs}$ is most likely to have a spin parity ($J^\pi$) of $3/2^-$ that corresponds to the $2p_{3/2}$ orbital. For the sake of completeness, in this work we have considered neutron removal from $2p_{3/2}$, $2s_{1/2}$ and $1f_{7/2}$ orbitals.

In Fig. 1(a), we show the results of our calculations for the pure Coulomb $\sigma_{-1n}$ in the breakup reaction of $^{37}$Mg on a Pb target at the beam energy of 244 MeV/nucleon as a function of $S_{-1n}$ corresponding to one-neutron removal from the $2p_{3/2}$, $2s_{1/2}$ and $1f_{7/2}$ orbitals. For $C^2S$ we have used the shell model values as given in Ref. [5], which are 0.31 and 0.001 for the $^{36}$Mg($0^+ \otimes 2p_{3/2}^\nu$, and $^{36}$Mg($0^+ \otimes 2s_{1/2}^\nu$ configurations, respectively. However, for the $^{36}$Mg($0^+ \otimes 1f_{7/2}^\nu$ configuration the SM $C^2S$ is not given.
in this reference. Therefore, we have assumed a $C^2S$ of 1.0 for this case. The shaded band in this figure shows the corresponding measured cross section taken from Ref. [5] with its width representing the experimental uncertainty. We note that calculated cross sections obtained with the $^{36}\text{Mg}(0^+) \otimes 2p_{3/2}\nu$ configuration (solid line in Fig. 1), overlap with the experimental band in the $S_{-1n}$ region of $0.10 \pm 0.02$. Theoretical cross sections for the $2p_{1/2}$ case are almost identical to those of the $2p_{3/2}$ case. On the other hand, for the $^{36}\text{Mg}(0^+) \otimes 2s_{1/2}\nu$ and $^{36}\text{Mg}(0^+) \otimes 1f_{7/2}\nu$ configurations there is no overlap between calculated cross sections and the data band. Therefore, our results support a $J^\pi = 3/2^-$ ground state for $^{37}\text{Mg}$ with a one-neutron separation energy of $0.10 \pm 0.02$. The $S_{-1n}$ deduced in our work is closer to the evaluated value of $0.16 \pm 0.06$ [7], with lesser uncertainty.

Nevertheless, with $C^2S$ for the $^{36}\text{Mg}(0^+) \otimes 2s_{1/2}\nu$ configuration as extracted in Ref. [5] (0.40), the cross section curve for this case will also overlap with the experimental data band as shown in Fig. 1(b). However, the extracted $S_{-1n}$ in our case is $0.26 \pm 0.04$ MeV, instead of $0.40^{+0.19}_{-0.13}$ MeV deduced in Ref. [5]. In fact, with the combination of the mean values of $S_{-1n}$ and $C^2S$ (0.40 MeV and 0.40) for this configuration deduced in Ref. [5], there would be no overlap between the theoretical cross section and the experimental data band in our calculations.

In Fig. 2(a), we show our results for $\sigma_{-1n}$ as a function of $\beta_2$ for the $^{36}\text{Mg}(0^+) \otimes 2p_{3/2}\nu$ configuration of $^{36}\text{Mg}_{gs}$ with $C^2S$ values of 0.42 and 0.31 and taking a $S_{-1n}$ of 0.22 MeV (as in Ref. [5]) in both the cases. For $\beta_2 = 0$, the $\sigma_{-1n}$ in each case is the same as that shown in Fig. 1, which is below the experimental data. With increasing $\beta_2$, the cross sections in both the cases increase, and the overlaps between calculations and the data take place in ranges $0.35 < \beta_2 < 0.68$ and $0.62 < \beta_2 < 0.94$ for $C^2S$ of 0.42 and 0.31, respectively. We add that if for $C^2S = 0.31$ the $S_{-1n}$ as deduced in Fig. 1 is taken then calculated cross section would overlap with the data band at much lower values of $\beta_2$.

In Fig. 2(b) we show the same results for the $^{36}\text{Mg}(0^+) \otimes 2s_{1/2}\nu$ configuration with $C^2S$ and $S_{-1n}$ values of 0.40 and 0.40 MeV, respectively, (which is the mean value of these quantities as deduced in Ref. [5]). In this case, we see that in contrast to the results in Fig. 2(a), there is no overlap between calculated cross sections and the data band for any value of $\beta_2$ in the range of $0-1.0$. We have checked that the situation remains the same for $\beta_2 > 1.0$. Moreover, the contribution of the deformation term to the cross section is substantially low for the $s$-wave configuration, which results in almost constant $\sigma_{-1n}$ as a function of $\beta_2$ as seen in Fig. 2(b). Therefore, in our calculation, even with deformation a $s$-wave configuration is ruled out.
Figure 2: (a) $\sigma_{-1n}$ as a function of the deformation parameter $\beta_2$ in the Coulomb breakup of $^{37}$Mg on a Pb target at the beam energy of 244 MeV/nucleon with the configuration $^{36}$Mg$(0^+) \otimes 2p_{3/2}^\nu$ for $^{37}$Mg gs. The $S_{-1n}$ is taken to be 0.22 MeV with $C^2S$ values being 0.42 (solid line) and 0.31 (dashed line). (b) Same as in Fig. 2(a) for $^{36}$Mg$(0^+) \otimes 2s_{1/2}^\nu$ configuration with $C^2S$ and $S_{-1n}$ of 0.40 and 0.40 MeV, respectively. In both (a) and (b) the experimental data (shown by the shaded region) are taken from Ref. [5].

for $^{37}$Mg gs for the $C^2S$ and $S_{-1n}$ combination deduced in Ref. [5].

3 Conclusion

In this paper we have studied the Coulomb breakup reaction $^{37}$Mg + Pb → $^{36}$Mg + n + Pb at the beam energy of 244 MeV/nucleon, within the framework of the post form finite range distorted wave Born approximation theory that is extended to include the projectile deformation effects. In this formalism the transition amplitude is factorized into two parts - one containing the dynamics of the reaction and the another the projectile structure informations such as the fragment-fragment interaction and the corresponding wave function in its ground state. Analytic expressions can be written for both parts. This formalism opens up a route to perform realistic quan-
tum mechanical calculations for the breakup of neutron-drip line nuclei in the medium mass region that can be deformed.

We calculated the total one-neutron removal cross sections ($\sigma_{-1n}$) in this reaction and compared our results with the corresponding data reported in a recent publication [5]. Our calculations seem to favor a $J^\pi = 3/2^-$ spin assignment to the $^{37}\text{Mg}$ ground state with one-neutron separation energy ($S_{-1n}$) of $0.10 \pm 0.02$ MeV, if the spectroscopic factor ($C^2S$) for this state is taken to be the corresponding shell model value of 0.31. However, the deduced $S_{-1n}$ depends on the chosen value of $C^2S$. Our study shows that $S_{-1n}$ rises steadily with increasing $C^2S$. Indeed, due to the uncertainty in the $C^2S$ value for the $^{36}\text{Mg}(0^+ \otimes 2s_{1/2}\nu)$ configuration for the $^{37}\text{Mg}$ ground state, the $J^\pi = 1/2^+$ spin assignment to it can not be fully excluded based on the present data.

In order to gain more insight in the ground state structure of $^{37}\text{Mg}$, we studied the effect of the projectile deformation on $\sigma_{-1n}$. We find that for the configuration $^{36}\text{Mg}(0^+ \otimes 2p_{3/2}\nu)$ for the $^{37}\text{Mg}$ ground state, the calculated $\sigma_{-1n}$ overlaps with the experimental data band in certain range (that depends on the value of $C^2S$) of the quadrupole deformation parameter ($\beta_2$). However, with the $^{36}\text{Mg}(0^+ \otimes 2s_{1/2}\nu)$ configuration, the overlap does not occur between calculated and measured $\sigma_{-1n}$ for any reasonable combination of $\beta_2$ and $C^2S$ values. This supports the $J^\pi = 3/2^-$ spin assignment for the $^{37}\text{Mg}$ ground state.

However, for unambiguous confirmation one also needs to calculate [4] more exclusive observables such as the core-valence neutron relative energy spectra, the energy-angle and the angular distributions of the emitted neutron and the parallel momentum distribution of the core fragment. The position of the peak as well as the magnitude of the cross section near the peak of the core-valence neutron relative energy spectra would to be dependent on the configuration of the projectile ground state as well as on its deformation.

Our study is expected to provide motivation for future experiments on breakup reactions of the neutron rich medium mass nuclei.

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