Recently, critical phenomena in the vicinity of the magnetic quantum critical point (QCP) have attracted much interest in strongly correlated metals. Experimentally, the outer magnetic field is frequently used to change the distance from the QCP. As for the antiferromagnetic (AFM) QCP, the magnetic field is believed to increase the distance to the QCP in general. Spin fluctuation theories such as the SCR theory [1] and the fluctuation-exchange (FLEX) approximation [2], have succeeded in describing various critical phenomena in metals close to the AFM-QCP, such as the non-Fermi liquid-like behaviors of various transport coefficients [3,4]. However, previous studies on the effect of the magnetic field based on the spin fluctuation theory were not comprehensive [5,6].

CeMIn$_5$ (M=Rh, Co, or Ir) is a well-known quasi-two-dimensional heavy fermion compound, where single conductive CeIn layers stack perpendicular to the c-axis. CeCoIn$_5$ is a superconductor with $T_c = 2.3$ K at ambient pressure [7]. In CeRhIn$_5$, the AFM order emerges at $T_N = 3.8$ K at ambient pressure, and the superconductivity emerges at $T_c \approx 2K$ below $P = 1.6$ GPa [8,9]. Recent experiments reveals that the $T_N$ increases under the magnetic field along the a(b)-axis. When $B = 9$ T, the increment in $T_N$ is approximately 0.15 K. A small increment in $T_N$ is also observed in Ce$_2$RhIn$_8$ which is composed of double CeIn layers. However, there has been no theoretical explanation for this phenomenon.

In the present study, we investigate the two-dimensional Hubbard model under the uniform magnetic field $B$ along the x-axis, based on the FLEX approximation. Recently, experiments reveals that the ductivity emerges at the magnetic field along the $a$-axis. When $T = 9$ T, the increment in $T_N$ is approximately 0.15 K. A small increment in $T_N$ is also observed in Ce$_2$RhIn$_8$ which is composed of double CeIn layers. However, there has been no theoretical explanation for this phenomenon.

We propose the mechanism for the magnetic-field-induced antiferromagnetic (AFM) state in a two-dimensional Hubbard model in the vicinity of the AFM quantum critical point (QCP), using the fluctuation-exchange (FLEX) approximation by taking the Zeeman energy due to the magnetic field into account. In the vicinity of the QCP, we find that the AFM correlation perpendicular to $B$ is enhanced, whereas that parallel to $B$ is reduced. This fact means that the finite magnetic field increases $T_N$, with the AFM order perpendicular to $B$. The increment in $T_N$ can be understood in terms of the reduction of both quantum and thermal fluctuations due to the magnetic field, which is caused by the self-energy effect within the FLEX approximation.

We analyze the following two-dimensional Hubbard model:

$$H = \sum_{k\sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{kk'q} \epsilon_{k+q\sigma}^q c_{k-q\sigma}^\dagger c_{k'\sigma}^\dagger,$$

where $\sigma = 1(-1)$ corresponds to the $\uparrow$ (down) spin state and $\epsilon_{k\sigma} = \epsilon_k + \sigma B$, where the factor $\sigma B$ represents the Zeeman energy. The spin quantization axis is the $x$-axis.

We study the square lattice tight-binding model with nearest neighbor hopping ($t$) and next-nearest neighbor hopping ($t'$). The dispersion of the electron is given by $\epsilon_k = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y$. We study the case of $(t,t') = (1,-0.25)$ with the electron density $n = 0.90$ ($n = 1.20$) per site, which corresponds to a hole-doped (electron-doped) high-$T_c$ cuprates. In the case of $n = 0.90$, the Fermi surface(FS) is very close to the van-Hove singular point (at $(\pi,0)$ in this case; see Fig. 3), and it is similar to the largest (main) cylindrical FS in CeMIn$_5$ (M=Co,Ir,Rh) [10]. Assuming a similar single cylindrical FS, many aspects of CeMIn$_5$, particularly the $d_{x^2-y^2}$-wave superconductivity, can be reproduced by the perturbation study [11,12].

In the presence of the magnetic field along the $x$-axis, the dynamical spin susceptibilities within the FLEX approximation (or random-phase approximation (RPA)), $\chi_\sigma^{\pm}(q)$ and $\chi_\sigma^{0}(z)(q)$, are given by

$$\chi_{\sigma}(q) = \chi_{\sigma}(q) = (\chi_{\uparrow\uparrow}(q) + \chi_{\downarrow\downarrow}(q)) / 4$$

$$\chi_{\sigma}(q) = [\chi_{\uparrow\uparrow}(q) + \chi_{\downarrow\downarrow}(q)] / 4 + U\chi_{\uparrow\downarrow}(q)\chi_{\downarrow\uparrow}(q) / 2,$$

$$\chi_{\sigma}^{0}(q) = \chi_{\sigma}^{0}(q) = \frac{\chi_{\sigma}^{0}(q)}{1 - U^2\chi_{\sigma}^{0}(q)\chi_{\sigma}^{0}(q)}, \quad \chi_{\sigma}^{0}(q) = \frac{\chi_{\sigma}^{0}(q)}{1 - U^2\chi_{\sigma}^{0}(q)\chi_{\sigma}^{0}(q)},$$

as a function of the magnetic field.
\[ \chi_{\sigma\sigma}(q) = -T \sum_k G_\sigma(k+q)G_\sigma(k). \]  

Note that \( \chi_{\uparrow\downarrow}(q) = \{ \chi_{\downarrow\uparrow}(q) \}^* \). Here and hereafter, we promise that \( q = (q_x, i\omega_n) = (q_x, 2\pi i n T) \) and \( k = (k_x, i\epsilon_n) = (k_x, \pi(2n+1) T) \). Apparently, both \( \chi_x(q) \) and \( \chi_{y(z)}(q) \) are even functions of \( B \), reflecting the reflectional symmetry in spin space. Apparently, \( \chi_x(q) = \chi_y(q) \) when \( B = 0 \).

The self-energy in the FLEX approximation is given by

\[
\Sigma_\sigma(k) = U^2 T \sum_q \left[ G_\sigma(k-q)\left(\chi_{-\sigma,-\sigma}(q) - \chi_{\sigma\sigma}(0)\right) + G_{-\sigma}(k-q)\chi_{\sigma,-\sigma}(q) \right] + U n_{-\sigma},
\]

where \( n_\sigma = T \sum_k \text{Im} \chi_{\sigma\sigma}(k) \) is the density of electrons with \( \sigma \)-spin. Here, we solve the Eqs. (2)-(7) together with the Dyson equation \( G^{-1}_\sigma(k) = i\epsilon_n + \mu - \epsilon_k - \sigma B - \Sigma_\sigma(k) \) numerically, by adjusting the chemical potential \( \mu \) so that \( n = n_\sigma \).

Here, we discuss the numerical results obtained by the FLEX approximation. We use \( 64 \times 64 \times 1028 \) Matsubara frequencies in the present numerical study by FLEX approximation. Figure 1 shows the obtained static staggered spin susceptibilities: \( \chi_{\alpha}^{\text{max}} \equiv \max_q \chi_{\alpha}(q,0) \), where \( \alpha = x, y, z \). \( \chi_y^{\text{max}} \) is the Stoner factor without \( B \). In the FLEX approximation, \( \alpha_S < 1 \) is always satisfied at finite \( T \) in two-dimensional systems, so the Marmin-Wagner-Hohenberg theorem is satisfied [13,14]. The momentum dependence of \( \chi_{\alpha}(q,0) \) \( (\alpha = x, z) \) and the splitting of the FS under the magnetic field are given in figs.2 and 3, respectively, in the case of \( n = 0.90 \).

In Fig. 1, \( \chi_y^{\text{max}} \) decreases whereas \( \chi_x^{\text{max}} \) increases with \( B \parallel \hat{x} \) in both cases of \( n = 0.90 \) and \( n = 1.20 \) by FLEX approximation. Their field dependence becomes more prominent as \( U \) increases, that is, as \( \alpha_S \) approaches unity. These results indicate that the distance to the AFM-QCP decreases owing to the uniform magnetic field. In the FLEX approximation, the field dependence of the susceptibility is caused by (i) the change in the nesting conditions owing to the Zeeman splitting of the FS, and (ii) the self-energy effect (or mode-mode coupling effect) which represents the reduction in \( \chi_{\text{tot}}^{\text{max}} \) and its Curie-Weiss-like temperature dependence owing to the spin-fluctuations. In the FLEX approximation, a large \( \text{Im} \Sigma(k, -i\delta) \) caused by spin fluctuations reduces the density of states (DOS) at \( \mu \), which makes \( \chi_{\text{FLEX}} \gg \chi_{\text{RPA}} \). Below, we will discuss that the effect (ii), which is absent in the RPA is important to explain why \( \chi_y^{\text{max}} \) is enhanced under the magnetic field parallel to the \( x \)-axis.

We discuss the physical reason for the field enhancement of the AFM correlation: First, the uniform magnetization induced by \( B \parallel \hat{x} \) will reduce the AFM correlation along the \( x \)-direction. This leads to the enhancement of \( \chi_y^{\text{max}} \) by contraries, as a result of solving the conflict between spin-fluctuations with different components. The increase in \( \chi_y^{\text{max}} \) will be more prominent in lower dimensional systems because the reduction of \( T_N \) due to fluctuations is large in general. Note that the reduction of the staggered moment at \( T = 0 \) owing to the quantum fluctuations is approximately 40%-50% in two (three) dimensional \( S = 1/2 \) Heisenberg model without a magnetic field.

Consistently with the above discussion, \( \chi_x^{\text{max}} \) increases whereas \( \chi_y^{\text{max}} \) decreases under \( B \parallel \hat{x} \) in the present model by the FLEX approximation. We have checked that this is a universal behavior in two-dimensional systems close to the AFM-QCP, by studying various types of Hubbard model. Here, we briefly discuss the self-energy effect for susceptibilities: When \( \hat{B} = 0 \), \( \chi_{\alpha}^{0}(q,0) \) by the FLEX approximation is reduced from the RPA’s value because of the reduction of the DOS, which is caused by the large \( \text{Im} \Sigma \) under strong spin-fluctuations. Considering that \( \Sigma(k) \approx U^2 T \sum_q \chi_{\alpha}(q) G_\alpha(k+q) \), the change in \( \chi_{\alpha}^{0}(q,0) \) within the lowest order with respect to the self-energy is given by

\[
\delta\chi_{\alpha}^{0}(q,0) \approx -T^2 \sum_{k,q} \chi_{\alpha}(k) G_\alpha(k+q) G_\alpha(k+q') \times 2U^2 (\chi_x(q') + 2\chi_y(q')).
\]
AFM metals in two dimensions, which has been pointed out in the present work for the first time. In Fig. 4, $T_N$ starts to increase in proportion to $B^2$, and it almost saturates at approximately $B^* \sim 0.3$ when $n = 0.90$. This result also means that the system approaches the AFM-QCP by applying a magnetic field. The increment in $T_N$ is larger when $n = 0.90$, reflecting the closeness to the van-Hove singularity.

Here, it is notable that in the antiferromagnetic isotropic Heisenberg chain under the magnetic field along the $x$-axis, $\langle S_i^x S_j^x \rangle - M^2 \propto (-1)^{i-j} |i-j|^{-1/2} \cos(2\pi M (i-j))$ and $\langle S_i^y S_j^y \rangle \propto (-1)^{i-j} |i-j|^{-\eta}$, where $\eta$ decreases from unity with the magnetic field [15]. Their field dependencies are consistent with the present study of a two-dimensional Hubbard model. In the XXZ-Heisenberg chain, an infinitely small magnetic field along the $x$-axis induces the staggered magnetization of the $y$-component in the case of $J_z < J_x$ [16,17]. In the opposite case, $J_z > J_x$, the staggered magnetization along the $z$-axis, which exists without the field, is enhanced by $B \parallel \hat{x}$ [18]. We also point out that Fukusima and Kuramoto studied a localized electron model with interactions between quadrupole moments by a local approximation, and found the field enhancement in $T_Q$ due to the suppression of fluctuations [19].

Note that the field-induced SDW is realized in the quasi-one dimensional metal, TMTSF, owing to the orbital motion of electrons, free from the Zeeman effect [20]. However, various characteristics of the FI-AFM in CeRhIn$_5$ do not coincide with that observed in TMTSF. In fact, CeRhIn$_5$ possesses both cylindrical and spherical FS's. They are naturally explained in terms of the Zeeman effect as discussed in the present study.

Now we discuss the experimental results of CeMIn$_5$ in the present study. The bandwidth of the present model is $\sim 10$. If we estimate the renormalized quasiparticle bandwidth of CeMn$_5$ to be $\sim 1000$ K [21], the temperature $T = 0.02$ corresponds to $\sim 2$ K, which is close to $T_c$ in CeCoIn$_5$ [22]. The magnetic field $B = 0.1$ in the present work corresponds to $\sim 5$ T for the $M = \pm 5/2$ Kramers doublet (KD), because the Zeeman energy for Ce$^{3+}$ is $(6/7)\mu_B M H$ ($6/7$ is the g-value of Ce$^{3+}$). Note that the renormalization factor averaged over the FS is 0.217 in the present FLEX approximation for $U = 5$ at $T = 0.02$. $T_N$ in CeRhIn$_5$ continues to increase with the magnetic field parallel to the $a$-$b$-plane, at least below 9 T; $T_N = 3.8$K at 0 T, and $T_N(9T) - T_N(0T) \approx 0.15$ K [8,9]. Whereas $T_N$ decreases monotonically when $B \parallel \hat{c}$, as is observed in usual 3D heavy Fermion systems. This is naturally understood because the orbital motion of electrons, which is absent in the present study where $B$ is parallel to the 2D system, will destroy the AFM state to obtain the energy due to the Landau diamagnetism.

Furthermore, we discuss the anisotropy of $\chi(q)$ in CeRhIn$_5$: The lowest KD of Ce$^{3+}$-ion in CeRhIn$_5$ is $J_{1,2}^0 = \frac{1}{4}$; $|z;\pm\rangle = \pm(M_z = \pm 3/2 - \alpha |M_z = \mp 3/2| [23,24], which is approximately 70 K lower than the second lowest KD. If we put $(x,\beta) \approx (0.44,0.9)$ [24], $\langle z;\pm|J_z z;\pm\rangle = \pm(2.5 \times M_z = \pm 0.885$, where $|x;\pm\rangle = (|z;\pm\rangle \mp |z;\mp\rangle)/\sqrt{2}$. Then, the anisotropy of the susceptibility of a single Ce$^{3+}$-ion is $\chi_a/\chi_c \approx 1.74/0.885 = 1.97$, which is similar to the experimental ratio. On the other hand, several neutron experiments on CeRhIn$_5$ revealed that the magnetic moments on Ce sites lie on the $ab$-plane below $T_N$, whose effective moment is $\mu_{\text{eff}} = 0.264\mu_B$ [23,24]. This suggests that the antiferromagnetic RKKY interaction between nearest neighbor Ce sites is XY-like; $J_{a,b} > J_c$ [25]. Then, the magnetic field along the $a$-axis will enhance the AFM correlation along the $b$-axis as a result of the reduction of fluctuations, which is similar to the behavior of the XXZ-Heisenberg chain under $B$ [16,17]. In fact, $\mu_{\text{eff}}$ is much smaller than $(6/7)\mu_B (x;+|J_z|x;+) \approx 0.76\mu_B$, which suggests that the quantum fluctuations are strong in CeRhIn$_5$, reflecting its two-dimensionality. As a result, the field-enhancement in $T_N$ observed in CeRhIn$_5$ is well understood in terms of the reduction of spin-fluctuations by a magnetic field. It is a future research problem to take the Kondo effect into account beyond the FLEX approximation.

In summary, on the basis of the FLEX approximation, we found the field-induced antiferromagnetism in a two-dimensional Hubbard model, as a result of solving the conflict between fluctuation with different directions by a magnetic field. This phenomenon is expected to be prominent and universal in the vicinity of the AFM-QCP in lower dimensional systems, irrespective of the fact that the field-induced uniform magnetization tends to decrease the AFM moments. The induced AFM moments are almost on the plane perpendicular to the applied magnetic field, to earn the Zeeman energy by canting. Experimentally, the field-induced increment in $T_N$ will be more prominent when $B$ is parallel to the 2D system, because the reduction of $T_N$ caused by the orbital motion effect (Landau quantization) is absent.

As for two-dimensional organic metals, the field-induced transition from the paramagnetic metal to the AF insulator is found in $\kappa$-(ET)$_2$Cu[N(CN)$_2$]Cl under a pressure, below $T_{\text{co}} = 13$ K and above $H_{c2}$ [26]. Also, field-induced SDW due to the Zeeman effect is expected to be realized in $\tau$-phase organic metals [27]. These phenomena will be explained by the present mechanism [28].

Finally, we note that the present results by the FLEX approximation seems reasonable in terms of the fluctuation-dissipation theorem; $\langle S_\alpha^2 \rangle = \sum_q J_0^\alpha \frac{\alpha}{2\pi} \text{Im} \chi_\alpha(q) \coth(q\omega_n)$ ($\alpha = x,y,z$), and $\sum_{x,y,z} \langle S_\alpha^2 \rangle$ grows, $\langle S_\alpha^2 \rangle$ will increase (especially in 2D systems). Thus, $\chi_\alpha(q) = \langle S_\alpha (q) \rangle$ will increase when $\chi_x(q)$ decreases by $B \parallel \hat{x}$ [28].

We are grateful to K. Yamada, D.S. Hirashima, K. Miyake, Y. Kuramoto, M. Tsuchizhu and Y. Matsuda for useful discussions.
[1] T. Moriya and K. Ueda: Adv. Physics 49 (2000) 555.
[2] N. E. Bickers and S. R. White: Phys. Rev. B 43 (1991) 8044.
[3] H. Kontani, K. Kanki and K. Ueda: Phys. Rev. B 59 (1999) 14723.
[4] H. Kontani: Phys. Rev. Lett. 89 (2003) 237003.
[5] Effect of the magnetic field on a model close to the ferromagnetic CQP was studied in S. Shioda, Y. Takahashi and T. Moriya: J. Phys. Soc. Jpn. 57 (1988) 3146.
[6] Hatatani and K. Miyake: thesis (2001).
[7] C.Petrovic et al.: J. Phys.: Condens. Matter 13 (2001) L337.
[8] A.L. Cornelius, P.G. Pagliuso, M.F. Hundley, and J.L. Sarrao: Phys. Rev. B 64 (2001) 144411.
[9] B.E. Light, R.S. Kumar, A.L. Cornelius, P.G. Pagliuso, and J.L. Sarrao: Phys. Rev. B 69 (2004) 024419.
[10] T. Maehira, T. Hotta, K. Ueda and A. Hasegawa: J. Phys. Soc. Jpn. 72 (2003) 854.
[11] Y. Nisikawa, H. Ikeda and K. Yamada: J. Phys. Soc. Jpn. 71 (2002) 1140.
[12] Y. Yamase, T. Jujo, T. Nomura, H. Ikeda, T. Hotta and K. Yamada: Phys. Rep. 387 (2003) 1481.
[13] T. Hikihara and A. Furusaki: Phys. Rev. B 69 (2004) 064427.
[14] J.S. Caux, F.H.L. Essler and U. Low: Phys. Rev. B 68 (2003) 134431.
[15] D.V. Dmitriev and V.Y. Krivnov: cond-mat/0403035.
[16] W. Bao, P.G. Pagliuso, J.L. Sarrao, J.D. Thompson, Z. Fisk, J.W. Lynn and R.W. Erwin: Phys. Rev. B 62 (2000) R14621.
[17] A.D. Christianson, J.M. Lawrence, P.G. Pagliuso, N.O. Moreno, J.L. Sarrao, J.D. Thompson, P.S. Riseborough, S. Kern, E.A. Goremychkin, and A.H. Lacerda: Phys. Rev. B 66 (2002) 193102.
[18] The second lowest KD or the $l$-$s$ coupling term in $p$-orbital may be indispensable because $J_a = J_b = J_c$ in the present crystal structure within the single KD; see

FIG. 1. Field dependences of $\chi_x$ and $\chi_y$ by the FLEX approximation (RPA), under conditions (a) $n = 0.90$, $U = 5$ ($U = 2.10$) and (b) $n = 1.20$, $U = 8$ ($U = 2.96$), at $T = 0.02$. In the numerical study by RPA, $256 \times 256$ k-meshes and 256 Matsubara frequencies are used.
FIG. 2. $\chi_x(\mathbf{q}, 0)$ and $\chi_y(\mathbf{q}, 0)$ under finite $B$ for $n = 0.90$ at $T = 0.02$.

FIG. 3. FS's for $\uparrow$- and $\downarrow$-electrons under various magnetic fields for $n = 0.90$ and 1.20 at $T = 0.02$.

FIG. 4. Obtained phase diagram for $T_N$ vs $B$ for various $\alpha_0^S$'s.