An Optimization Model of Reverse Logistics Network Design

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Abstract: With the increasing development of China’s economy, product and materials of reverse logistics is also increasing. Reverse logistics network design is of great significance to reduce cost of reverse logistics enterprises and society. In this paper, 0-1 mixed integer programming is used to establish a reverse logistics network optimization model which takes into account establishment of fixed facilities and flow between different logistics nodes. The objective function is profit maximization, and the constraints contain the processing capacity of facilities vehicle loading capacity. Lingo 14.0 is used to solve the model. The results show that the application of the model can improve the profits of reverse logistics enterprises on the basis of optimizing logistics facility allocation and material flow.

1. Introduction

China's output of industrial products ranks first in the world. While producing a large number of industrial products, more and more waste products are produced. Therefore, design and planning of waste product logistics network is important in current logistics research. In addition, due to requirements of "carbon neutralization", reverse logistics enterprises should not only reduce logistics costs, but also reduce carbon emissions. In reverse logistics network design, Waste fire extinguisher (WFE) plays important role.

Many scholars study reverse logistics network design. Mutha A and pokharel¹ established a mixed integer programming model for dealing with the recovery and return of products with different structures. Das K and Chowdhury A H² studied the modular structure of the product to simplify the recycling process. Qiang³ built a production planning model to evaluate the benefits of re-manufacturing in closed-loop supply chain. Liao T Y⁴ established a reverse logistics network...
model with mixed integer programming considering the return quality in the model, and solved the model with hybrid genetic algorithm. Kara and Onut\[5\] studied the logistics network planning combining forward and reverse logistics in the field of paper industry. Alshamsi A and Diabat A\[6\] established a reverse supply chain network considering different return quality.

Figure 1 depicts the networks of WFE supply chain.

![Diagram of Reverse Logistics Network of Used Battery](image)

So far, the research on reverse logistics mainly aims at cost minimization and less focuses on profit maximization. In addition, vehicle scheduling problem is rarely considered in the design of reverse logistics network. This paper aims to make up for the shortcomings of the current research and establish reverse logistics network model with the goal of profit maximization, and vehicle scheduling problems between nodes are embedded in the model. The example analysis shows that this model can effectively improve the efficiency of reverse logistics enterprises through reasonable network node setting and vehicle scheduling between nodes.

2. Mathematical Model

2.1. Parameters

\(E\): Suppliers of WFE.  
\(F\): Collection points of WEF.  
\(G\): Processing center of WFE.  
\(H\): Destruction center of WFE.  
\(H\): Secondary markets of WFE.  
\(I\): Recycling centers of WFE.  
\(J\): vehicles available.  
\(S_e\): Number of WFE provided by supplier \(e (e \in E)\).  
\(P_j\): Cost of buying vehicle \(j (j \in J)\).  
\(VC_j\): Cost of driving one kilometer for vehicle \(j (j \in J)\).  
\(SP_{fg}\): Actual kilometers from collection point \(f\) to processing center \(g (f \in F, g \in G)\).  
\(SP_{gh}\): Actual kilometers from processing center \(g\) to destruction center \(h (g \in G, h \in H)\).  
\(SP_{gi}\): Actual kilometers from processing center \(g\) to recycling center \(i (g \in G, i \in I)\).  
\(SP_{gk}\): Actual kilometers from processing center \(g\) to secondary market \(k (g \in G, k \in K)\).  
\(FIDX_{C_j}\): Cost of establishing and putting collecting point \(f\) into operation, that is, fixed cost of collecting point \(f\).  
\(VAC_{fj}\): Cost of handling a WFE in collecting point \(f\), that is, variable cost of collecting point \(f\).  
\(L\): Classes of components that a product can be decomposed into.
$O_l$: Number of components(type l) that can be taken part from a WFE. ($l \in L$)

$\theta_g$: Proportion of WFE flowing into secondary markets from processing center $g$, which have not been taken apart.

$\varphi_g$: Proportion of WFE flowing into recycling centers from processing center $g$, which have not been taken apart.

$CO2_j$: Grams of carbon dioxide emitted by vehicle $j$ driving one kilometer.

$FIXC_g$: Cost of establishing and putting processing center $g$ into operation, that is, fixed cost of processing center $g$.

$VAC_g$: Cost of handling a WFE in processing center $g$, that is, variable cost of processing center $g$.

$CP_j$: Load limit of vehicle $j$.

$UC_f$: Handling capacity of collecting point $f$.

$UC_g$: Handling capacity of processing center $g$.

$W$: A positive number big enough.

### 2.2. Decision Variables

$z_a$: If facility $a$ is built and operated, then $z_a = 1$, or else $z_a = 0$. ($a = f, g$)

$z_{ef}$: If supplier $e$ offer WFE to collecting point $f$, then $z_{ef} = 1$, or else $z_{ef} = 0$.

$y_j$: If vehicle $j$ is put in transportation, then $y_j = 1$, or else $y_j = 0$.

$y_{fg}^i$: If vehicle $j$ is used for transportation from collecting point $f$ to processing center $g$, then $y_{fg}^i = 1$, otherwise, $y_{fg}^i = 0$.

$y_{gh}^i$: If vehicle $j$ is used for transportation from processing center $g$ to destruction center $h$, then $y_{gh}^i = 1$, otherwise, $y_{gh}^i = 0$.

$y_{gk}^i$: If vehicle $j$ is used for transportation from processing center $g$ to secondary market $k$, then $y_{gk}^i = 1$, otherwise, $y_{gk}^i = 0$.

$x_{fs}^j$: Number of WFE from collecting point $f$ to processing center $g$ transported by vehicle $j$.

$x_{gi}^j$: Number of WFE from processing center $g$ to secondary market $i$ transported by vehicle $j$.

$x_{gkl}^j$: Number of components(type l) transported by vehicle $j$ from processing center $g$ to recycling center $k$.

$x_{gkl}^j$: Number of components(type l) transported by vehicle $j$ from processing center $g$ to disposal center $h$.

### 2.3. Objective Function

$$MinZ = \sum_{f} FIXC_f z_f + \sum_{e} \sum_{f} VAC_f S_e z_{ef} + \sum_{g} FIXC_g z_g + \sum_{j} \sum_{f} \sum_{g} VAC_j x_{fs}^j$$
\[ + \sum_{j} \sum_{f} \sum_{g} SP_{jg} VC_{jg} x_{jg}^{f} \] 
\[ + \sum_{j} \sum_{g} \sum_{i} SP_{gi} VC_{jg} x_{gi}^{f} \] 
\[ + \sum_{j} \sum_{g} \sum_{k} \sum_{p} SP_{kg} VC_{jgk} x_{jgkp}^{f} \] 
\[ + \sum_{j} \sum_{g} \sum_{k} \sum_{p} SP_{kg} VC_{jgk} x_{jgkp}^{f} \] 
\[ + \sum_{j} y_{j} \] 
\[ MinZ_{2} = \sum_{j} \sum_{f} \sum_{g} SP_{jg} CO_{2j} y_{jg}^{f} \] 
\[ + \sum_{j} \sum_{g} \sum_{h} \sum_{k} SP_{ghk} CO_{2j} y_{gk}^{f} \] 
\[ + \sum_{j} \sum_{g} \sum_{k} SP_{gk} CO_{2j} y_{gk}^{f} \] 

2.4. Constraints

\[ \sum_{f} z_{f} \geq 1 \] 

Constraint (3) indicates requirement of number of collecting points established and put into operation.

\[ \sum_{g} z_{g} \geq 1 \]

Constraint (4) indicates requirement of number of processing centers established and put into operation.

\[ \sum_{f} z_{ef} = 1 \quad \forall e \in E \]

Constraint (5) indicates for a supplier, it has to supply WFE to one and only one collecting point.

\[ \sum_{c} z_{cf} \leq V z_{f} \quad \forall f \in F \]

Constraint (6) indicates when and only when a processing point operates, suppliers can provide WFE to it.

\[ \sum_{c} S_{c} z_{ef} = \sum_{j} \sum_{g} x_{jg}^{f} \quad \forall f \in F \]

Constraint (7) indicates that for a collecting point, quantity of incoming products is equal to those outgoing.

\[ \sum_{c} S_{c} z_{cf} \leq UC_{j} z_{jf} \quad \forall f \in F \]

Constraint (8) indicates that quantity of WFE flowing into a collecting point cannot exceed its handling capacity

\[ x_{jg}^{f} \leq y_{j} \quad \forall j \in J, f \in F, g \in G \]

Constraint (9) indicates that only vehicle \( j \) used in the whole logistics process, it can be used in the transportation process from collecting point \( f \) to processing center \( g \).
\[ x_{jfg}^j \leq CP_j \times y_{jfg}^j \quad \forall j \in J, f \in F, g \in G \]  
(10)

Constraint (10) indicates loading capacity for vehicle \( j \) from collecting point \( f \) to processing center \( g \).

\[ \sum_j \sum_f x_{jfg}^j \leq UC_{g} \quad \forall f \in F \]  
(11)

Constraint (11) indicates limit for number of WFE processed by processing center \( g \).

\[ \psi \sum_j \sum_f x_{jfg}^j = \sum_j \sum_i \sum_g x_{gji}^j \quad \forall g \in G \]  
(12)

Constraint (12) indicates flow of WFE into processing center \( g \) and out to secondary market \( i \).

\[ \sum_i O_i \cdot (\sum_j \sum_f x_{jfg}^j) = \sum_i \sum_k \sum_h \sum_l x_{hkl}^j \quad \forall g \in G \]  
(13)

Constraint (13) indicates flow of WFE/its components into processing center \( g \) and out to a recycling center \( k \).

\[ y_{ghi}^j + y_{gk}^j + x_{gji}^j \leq y_{ji} \quad \forall i \in I, j \in J, h \in H, g \in G, k \in K \]  
(15)

Constraint (15) indicates that only vehicle \( j \) used in the whole logistics process, it can be used in the transportation process from processing center \( g \) to secondary market \( i \)/recycling center \( k \)/destruction center \( h \).

\[ \sum_j \sum_{ghi} x_{ghi}^j \leq CP_j \cdot y_{ghi}^j \quad \forall j \in J, h \in H, g \in G \]  
(16)

Constraint (16) indicates that only vehicle \( j \) used in the whole logistics process, it can be used in the transportation process from processing center \( g \) to destruction center \( h \).

\[ \sum_j \sum_{gjl} x_{gjl}^j \leq CP_j \cdot y_{gjl}^j \quad \forall j \in J, k \in K, g \in G \]  
(17)

Constraint (17) indicates that only vehicle \( j \) used in the whole logistics process, it can be used in the transportation process from processing center \( g \) to recycling center \( k \).

\[ x_{jgi}^j \leq CP_j \times y_{gji}^j \quad \forall j \in J, g \in G, i \in I \]  
(18)

Constraint (18) indicates that only vehicle \( j \) used in the whole logistics process, it can be used in the transportation process from processing center \( g \) to secondary market \( i \).

\[ x_{jfg}, x_{gji}, x_{gjl}, x_{ghi} \geq 0 \quad \forall j \in J, f \in F, h \in H, g \in G, l \in L \]  
(19)

Constraint (19) indicates non-negative conditions.
3. Solution

Multi-objective programming is changed into single objective programming.

\[
\text{Min} Z = u_1 Z_1 + u_2 Z_2 \quad (\lambda_1, \lambda_2 \geq 0, \lambda_1 + \lambda_2 = 1)
\]  

Constraints (3)-(19)

4. Numerical Examples

Zhejiang Qianjin logistics company is business with reverse logistics services of WFE. This paper analyzes a practical case of the company. Up to now, the company has 10 suppliers, 6 collecting points, 2 processing centers and 2 secondary markets, 2 destruction centers and 2 recycling centers are served by it. Cost of buying one vehicle is 23 yuan and cost of driving a kilometer per vehicle is 5 yuan. Cost of building a collecting center and putting it into operation is 150 yuan. Cost per ton of WFE processed by ta collecting point is 15 yuan. Cost of building a processing center and put it into operation is 125 yuan and cost of handling WFE per ton is 22 yuan. Loading capacity of a vehicle is 8 tons. Amounts of WFE given by a supplier is 70 tons, handling capacity of a collecting point/processing center is 400/500 tons. Selling price of a WFE in secondary market is 180 yuan. Distance from a collecting point to a processing center is illustrated in Table 1.

Table1: Distance from collecting points to processing centers(Unit: Kilometers)

| Collecting point1 | Collecting point2 | Collecting point3 | Collecting point4 | Collecting point5 | Collecting point6 |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Processing center 1 | 110               | 108               | 107               | 110               | 107               | 110               |
| Processing center 2 | 101               | 99                | 96                | 98                | 96                | 101               |

The following Table2 shows distances from a processing center to a secondary market/recycling center/destruction center are distributed.

Table2: Distances from a processing center to a secondary market/recycling center/destruction center (Unit:Kilometers)

| Destruction center1 | Destruction center2 | Secondary market1 | Secondary market 2 | Recycling center1 | Recycling center2 |
|---------------------|---------------------|-------------------|-------------------|------------------|------------------|
| Processing center 1 | 125                 | 109               | 99                | 90               | 103              | 108              |
| Processing center 2 | 119                 | 99                | 98                | 94               | 101              | 109              |

Classes and their quantities of components from a WFE is distributed in the following Table 3.

Table 3: Type/quantity of parts with selling price

| components     | 11 | 12 | 13 | 14 | 15 | 16 |
|----------------|----|----|----|----|----|----|
| Selling price  | 71  | 70  | 58  | 48  | 47  | 45  |

The solution results are as follows (\( \mu_1 = \mu_2 = 0.5 \)). Objective function value \( z_1 = 59678 \) and \( z_2 = 21795 \). Because the objective function is to minimize value of the objective function, we always selects the route with the lowest transportation cost to reduce transportation cost in the transportation plan. For example, for WFEs’ transportation from collection point 2 to processing centers, transportation to processing center 2 is given priority, and for WFEs’ transportation from
processing center 1 to secondary markets, transportation to secondary market 2 is given priority, and for WFEs’ transportation from processing center 1 to secondary markets. Objective function value is minimized by selecting the route with lower transportation cost.

If \( \mu_1 \) and \( \mu_2 \) are given different values, logistics cost \( (z_1) \) and carbon emission \( (z_2) \) are illustrated in the following Table 4.

Table 4: Value of objective function when \( \mu_1 \) and \( \mu_2 \) are changed (Carbon emission unit: gram; Cost unit: Yuan)

|          | \( \mu_1=0.65, \ \mu_2=0.35 \) | \( \mu_1=0.6, \ \mu_2=0.4 \) | \( \mu_1=0.45, \ \mu_2=0.55 \) |
|----------|--------------------------------|-------------------------------|-------------------------------|
| \( z_1 \) | 58756                          | 57003                         | 60035                         |
| \( z_2 \) | 22957                          | 21976                         | 20776                         |

As can be seen from Table 4, with the continuous increase of \( \lambda_2 \), carbon emissions are decreasing, while logistics costs are increasing. Reverse logistics enterprises should timely adjust the value of carbon emission according to the requirements of carbon emission, so as to reduce the total logistics cost under the condition of ensuring that the carbon emission does not exceed the standard.

5. Conclusions

Due to huge number of waste fire extinguishers, design of waste fire extinguisher reverse logistics network is one of the main contents of reverse logistics research. In this paper, a 0-1 mixed integer multi-objective programming method is used to establish the reverse logistics network model of waste fire extinguishers. The objective functions are to minimize total logistics cost and carbon emission respectively. The constraints consider the processing capacity of nodes and transportation capacity in reverse logistics network. The example shows that application of this model can effectively reduce logistics cost for WFE reverse logistics enterprises under the premise carbon emission does not exceed the standard.

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