B–PHYSICS IN HADRON COLLIDERS†

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The possibility of exploring the systematics of the spectroscopy, strong dynamics, and the weak and rare decay modes of b–quark systems at hadron colliders such as Fermilab, LHC and SSC, is discussed. A copious yield of $10^{10}$ detected $B$–mesons is readily accessible in a dedicated Fermilab program, and implies a vast array of accessible decay modes, including second order weak processes and $CP$–violation, which will be unavailable elsewhere until the commissioning of LHC or SSC. Kinematic and flavor tagging, utilizing the “daughter pions” from resonances, is expected to play a major role in semileptonic weak decay studies and the search for $CP$–violation.

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1. Introduction

1.1. Generalities

The $b$–quark offers a window on the standard model that is open to experimentalists at hadron colliders, where the largest yields of $b$–quarks occur. With existing facilities, such as CDF, it should be possible to achieve $\sim 10^9$ observable $B$–decays within the next few years. This entails evolution of the high resolution vertex detectors, e.g., CDF’s SVX, including full $r$-$\theta$-$z$ information, and especially generalized triggers, such as single lepton displaced vertices for semileptonic weak decay studies.$^{1,2}$ With a modest yet dedicated program, perhaps involving a new detector, $> 10^{10}$ observed $B$’s should be achievable at Tevatron to Main Injector luminosities within this decade. Such a program is essential to break the ground for future hadron–based $B$–physics programs at LHC and SSC. An ultimate hadron collider based program at Fermilab, LHC and SSC can look forward to recording the decays of $> 10^{12}$ produced $B$’s.

The present discussion is intended to be primarily a prospectus for such a program. We will, however, indulge in some speculations about tagging of flavors and the all–important kinematic reconstruction needed to do semileptonic weak studies. This reflects recent interest that has arisen in the possibility of “daughter pion” tagging, i.e., using the pions from the decays of parent resonances to tag the flavor.$^3$

The major advantages of the hadron based $B$–physics environment are the relatively large cross–section for $b$–quark production and the the “broad–band” nature of the beam. $b$–quark pairs are produced by (predominantly) gluon fusion$^4$ and arbitrarily massive states are available. Thus, all of the spectroscopy, including $B_c \sim \bar{b}c$ and the resonances, $B^{**}$ etc., are produced in hadronic collisions. This sharply contrasts the situation in $e^+e^-$ machines that make use of the $\Upsilon(4S)$ and $\Upsilon(5S)$ resonances in which only the low–lying $\bar{b}(u, d)$ combinations can be produced. Moreover, in $e^+e^-$ machines that operate in the continuum or on the $Z$–peak the cross–section for $b$ production is many orders of magnitude below that in the hadronic environment.

On the other hand these advantages imply major challenges as well.$^{1,2}$ The copious production at hadron machines implies that a substantial parsing of data must occur quickly on–line, i.e., a trigger that can keep interesting candidate events must be provided. To date in hadronic colliders the semileptonic decay modes have been largely discarded in favor of the much easier $\psi$ modes. A trigger capable of recovering the semileptonic decays is possible, and demonstrating its feasibility is of high priority for a number of reasons (conventional flavor tagging requires it).

Another issue is the extent to which decays involving missing mass, such as the semileptonic decays involving neutrinos, can be fully reconstructed. In $e^+e^-$ machines that make use of the $\Upsilon(4S)$ the $B$–mesons are produced with a known energy, the beam energy. In combination with the visible decay momentum, this completely determines the decay kinematics, e.g., the $Q^2$ of the lepton pair is determined even though the neutrino is never seen. In a hadronic mode we observe a $B$–meson flight direction and the visible momentum of the decay products, but this yields a two–
fold ambiguity in the $B$ energy. Thus, to make maximal use of a semileptonic decay sample it is imperative that efficient techniques evolve for resolving this ambiguity!

One technique would “bludgeon” the semileptonic processes with high statistics by insisting on keeping only those special kinematic configurations for which the ambiguity disappears. While inefficient, this technique is guaranteed to work. However, we will suggest another approach presently that is speculative, but may ultimately prove to be an efficient way of fully reconstructing $B$ processes with relatively high efficiency. It makes use of the fact that $B$–mesons will often be produced as decay fragments of a resonance as in $B^{∗∗} → B + π$. The $π$ meson here we will call a “daughter pion,” and it has previously been suggested as a flavor tagging mechanism for neutral $B$–mesons. The observation of daughter $π$–mesons from resonances is established by ARGUS, E-691 and CLEO, and E-687. However, we suggest here that it can potentially be used to resolve the two–fold kinematic ambiguity in the $B$–meson $4$–momentum. We describe this approach in Section 2.4 below. It may prove workable in some form as our understanding of $B$ production evolves.

The physics goals of a $> 10^{10}$ $B$–meson program are very rich and diverse. Heavy quark physics allows us to map out the CKM matrix of the standard model through the detailed studies of inclusive and exclusive decay modes. It will allow us to test the standard model beyond the leading order in radiative corrections, and through rare decay modes and mixing phenomena which are sensitive to $m_{top}$ and $V_{tq}$, etc. This will lead ultimately to experimental tests of the CKM theory of $CP$–violation, which is expected to manifest itself in many interesting new channels in the $b$–system. High statistics studies of the $b$–system will furthermore enable searches for exotic physics, signals of which might be expected to emerge in heavy quark processes.

We begin first with a brief overview of the physics considerations that are relevant to doing heavy quark physics in the hadronic collider environment.

1.2. Prima Facie Considerations of Hadronic $B$’s

$B$–physics at hadron colliders is often casually dismissed out–of–hand, preference given to $e^+ e^−$ production, because the hadronic environment is “too noisy.” It is important to realize that the “noise,” i.e., the background of high multiplicity, mostly low $p_T$ pions in a hadronic collision, is largely spread out over a large range of rapidity. The low–mass particle production follows an approximately constant distribution in the pseudo–rapidity:

$$\eta = -\ln[\tan(\theta/2)] \approx \tanh^{-1}(p_z/E)$$

Typically at Tevatron energies the number of pions per unit rapidity is given by:

$$\frac{dN_π}{d\eta} \approx 3.0 \text{ charged}; \quad \approx 1.5 \text{ neutral}$$

Thus, in a rapidity range of $|\eta| < 1$ we expect of order $π = 6$ charged pions, and $6$ $π^0$ gamma’s emanating from the beam collision spot.
Table I: Indicated yields of usable $B$–mesons running for a 3 year, 30% duty cycle, period for: (a) Tevatron at present attainable $\mathcal{L} = 10^{31}$ cm$^{-2}$ sec$^{-1}$ (b) Main Injector assuming $\mathcal{L} = 10^{32}$ cm$^{-2}$ sec$^{-1}$ (twice the design goal; multiply by 10 if the rapidity range is $|\eta| \leq 3$ and $p_{t} > 5$ GeV). (c) ABF – Asymmetric B-factory proposal at $\mathcal{L} = 10^{34}$ cm$^{-2}$ sec$^{-1}$ operating on the $\Upsilon(4S)$ (d) LEP at $Z^{0}$–pole with $\mathcal{L} = 2 \times 10^{31}$ cm$^{-2}$ sec$^{-1}$ (see M. Artuso in ref.[2]).

| Mode | Tevatron$^{(a)}$ | Main Injector$^{(b)}$ | ABF$^{(c)}$ | LEP II$^{(d)}$ |
|------|-----------------|---------------------|------------|--------------|
| $B_{u,d}$ | $6 \times 10^{9}$ | $6 \times 10^{10}$ | $3 \times 10^{8}$ | $4 \times 10^{6}$ |
| $B_{s}$ | $1.6 \times 10^{9}$ | $1.6 \times 10^{10}$ | none | $8 \times 10^{5}$ |
| $B_{c}$ | $10^4$ | $10^8$ | none | $4 \times 10^4$ |
| $\Lambda_{b}$ | $10^9$ | $10^{10}$ | none | $4 \times 10^4$ |

On the other hand, the finite and relatively large mass of the $b$–quark leads to a longitudinal momentum distribution that is centered on $\eta = 0$, and is fairly broad depending upon the $cm$ energy scale and the $p_{t}$ cut (see, e.g., Alan Sill in ref.[1]). In rapidity, the range of significant $b$–quark production with high $p_{t}$ is for the Tevatron $\sim \pm 3$; for the LHC $\sim \pm 4.5$; and for the SSC $\sim \pm 7$. Moreover, the transverse momentum distribution, $p_{t}$, of heavy quarks is set by the mass scale of the quark (generally, it requires a parton subprocess of larger $\hat{s}$ to make a heavier quark, hence larger values of $p_{t}$ become relatively more probable).

Moreover, $b$–hadrons have a fortuitously long life–time, and they therefore drift a resolvable distance away from the primary vertex before they decay. With high resolution vertex detectors it is easy to resolve the secondary vertex and isolate the heavy hadron decay. The typical displacement of a $b$–hadron secondary decay vertex is $\sim 400$ microns, while a resolution to better than $\sim 15$ microns is achieved with the SVX. With this secondary vertex separation there is only a very small combinatorial background to these displaced vertices coming from minimum bias physics. There remains, however, a significant background from charmed mesons which also have displaced secondary vertices. These can generally be controlled by demanding partial reconstruction of the heavy hadron decay with mass cuts, i.e., demand that the visible decay products have masses exceeding those of charmed particles, typically $\gtrsim 2.5$ GeV.

At the luminosity of $10^{31}$ cm$^{-2}$ sec$^{-1}$ in a $p\bar{p}$ collider, for which we assume $\sqrt{s} = 1.8$ GeV, $B$-meson pairs are produced in a rapidity range of $|\eta| \leq 1$ and $p_{t} > 10$ GeV, with a total cross-section of $\sim 10$ $\mu b$ or 100 Hz (ref.[1]; M. Artuso in ref.[2]). With the main injector, and the experience to date at the Tevatron, an ultimate luminosity of $10^{32}$ cm$^{-2}$ sec$^{-1}$ is thinkable (the present peak Tevatron luminosity is $\sim 0.8 \times 10^{31}$). Running at $10^{32}$ ($10^{31}$) for a total of 3 years, with a 33% duty factor yields $\sim 3 \times 10^{10}$ ($3 \times 10^{9}$) usable $B$–mesons. If we can triple the rapidity range to $|\eta| \leq 3$ and reduce the lower limit to $p_{t} > 5$ GeV the yields for useful $B$’s approach $\sim 3 \times 10^{11}$ ($3 \times 10^{10}$). Of this, the yield of $B_{s}$ is $\sim 18\%$, $\Lambda_{B}$ is $\sim 10\%$ and of $B_{c}$ is $\sim 0.1\%$. The yield of $b$–quark containing baryons is expected to
be of order 10%, though these are crude estimates at present, and should actually be measured at the end of run I. This compares with the idealized luminosity of $10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$ in an $e^+e^-$ storage ring, such as the proposed asymmetric $B$-factory (ABF) at SLAC or CESR (the present peak luminosity at CESR is $2.5 \times 10^{32}$). The cross-section for $B\bar{B}$ production on the $\Upsilon(4S)$ is $\sim 1 \text{ nb}$, which yields $B$ pairs on the $\Upsilon(4S)$ at a rate of $\sim 10 \text{ Hz}$. The yield for the same 3 year 30% duty cycle period is $\sim 3 \times 10^8 B$-mesons (note this is the proposed ultimate $300 \text{ fb}^{-1}$, lifetime $\int Ldt$ for the asymmetric $B$-factory). On the $Z^0$ pole the cross-section is $\sim 7.0 \text{ nb}$. Hence operating an $e^+e^-$ collider at the LEP luminosity of $2 \times 10^{31}$ on the $Z^0$ pole for the same 3 year continuous duty cycle period yields $\sim 3 \times 10^6 b$'s. For continuum $e^+e^-$ machines the cross-sections are $\sim 10^{-2}$ those on the $Z^0$ pole and we will not consider them for comparison.

We see from Table I that various new states and decay modes are available in the hadronic facility that are inaccessible, or of lower statistics in the $e^+e^-$ environment. Moreover, it appears that a reasonable goal for a dedicated hadron collider based program in the pre-SSC era is to produce a total of $> 10^{10}$ usable $b$ hadrons. In what follows we will take $10^{10} B$-mesons to be our standard reference normalization and give a preliminary consideration of what might be achieved in such a program.

2. Physical States and Leading Processes

2.1. Resonance Spectroscopy

The spectrum of resonances of the $B$-mesons imitates that of the charm system. We see this by comparison in Fig.(1), where the known and predicted resonances of $\ell = 0$ and $\ell = 1$ are indicated. The spectroscopy is actually reflecting a remarkable aspect of heavy quark symmetry, i.e., the heavy quark spin symmetry.\footnote{Put simply, heavy quark spin can be ignored in the dynamics, and acts effectively like a flavor symmetry. As a result, states which differ only by flipping the heavy quark spin will be degenerate (up to $O(1/M)$). It is convenient to describe this by classifying mesons as $(j_1, j_2)$, where $j_1$ is the spin of the heavy quark subsystem and $j_2$ is the spin of the remaining system. So, for a heavy–light meson $j_1 = \frac{1}{2}$, and the states of lowest mass will have $j_2 = \frac{1}{2}$ as well. Thus $(\frac{1}{2}, \frac{1}{2})$ describes the groundstate and this corresponds to total $J = 0$ or $J = 1$. Therefore, the groundstate consists of a degenerate $0^-$ and $1^-$ multiplet. We see the $D$ and $D^*$ are actually split by slightly more than a pion mass, while the splitting between the $B$ and $B^*$ decreases by $m_c/m_b$ in the $B$ system. It is important to note that $j_2$ is the quantum number of the “brown muck;” we cannot a priori separate the light quark and gluon degrees of freedom under rotations in QCD, though potential models do so (potential models refer to constituent quarks, and work remarkably well even in light heavy–light systems).\footnote{A fancier way of stating this is to note that spin is the classification of a state under the “little group;” the little group is the subgroup of the Lorentz group which commutes with the momentum of the state (i.e., it is just $O(3) = SU(2)$ for...}}
a massive particle, or \( O(2) = U(1) \) for a massless particle). Remarkably, we see that the little group of a heavy–light meson is enlarged to \( SU(2) \times SU(2) \), since we can rotate the heavy quark independently of the brown muck. The states for which \(|j_1 - j_2|\) is an integer are equivalent to representations of \( O(4) = SU(2) \times SU(2) \). Thus the groundstate is equivalent to a 4-plet under \( O(4) \), containing the 0\(^-\) and the 1\(^-\) mesons.

### Figure 1

The low–lying spectra of \( D \) and \( B \) states from EHQ.\(^8\) Solid lines are established, dashed lines are predictions (we omit the broad \((0^+, 1^+)\) p-waves\(^9\)).

The masses and decay widths of heavy–light resonances have been estimated recently by Eichten, Hill and Quigg (EHQ)\(^8\). The masses of these states seem to be well fit by using a Buchmüller–Tye potential for a static massive quark with a constituent light quark boundstate. Their decay widths were obtained by rescaling the known strange and charm widths with smearing. The spectra are presented in Fig.(1). There will generally occur a \((\frac{1}{2}, \frac{1}{2}) = 0^+ + 1^+\) parity partner of the groundstate (a \( p \)-wave in the constituent quark model) which has a very large \( \sim GeV \) width and will generally be unobservable.\(^9\) This state may be viewed as the “chiral partner” of the groundstate;\(^9\) if we imagine restoring the broken chiral symmetry the groundstate would have to linearly realize the chiral symmetry, thus becoming doubly degenerate (thus, the left–handed iso–doublet is \( 0^+ - 0^- \), while the right–handed iso–doublet is \( 0^+ + 0^- \) when the chiral symmetry is restored).

#### 2.2. Daughter Mesons

The resonances can be observed by studying the \( \pi \)'s and \( K \)'s produced in association with \( B \)-mesons. Some of the \( \pi \)-mesons will be decay relics from processes
like:
\[ p + \bar{p} \rightarrow X + (B^{**} \rightarrow B + (\pi, K)) \] (3)

The first objective is to establish the existence and masses of the resonant states and the fraction \( f = \sigma_{B^{**}}/\sigma_B \) by which a \( B \)-meson is produced through the decay of parent resonance. \( f \) is likely to be sensitive to the decay and production kinematics.

Experience in \( e^+e^- \) (ARGUS and CLEO) and charm photo-production experiments\(^6\) suggests \( f \sim 13\% \) for the fraction of \( D^* \) coming from the \( D^{**} \), and \( f \sim 7\% \) for the fraction of \( D \) coming from the \( D^{**} \). We note that photoproduction on a hadronic target (E-691, E-687 in ref.[6]) bears some formal resemblance to the gluon fusion process, and might be a good analogue process for calibrating our understanding of detailed production in \( \bar{p}p \) collisions. We would expect (heavy quark symmetry) that apart from normalization the charm production distributions can be taken over to \( B \)-physics directly. Thus, for tagging purposes an inclusive rate of \( f \sim 20\% \) of \( B \)'s coming from the \( B^{**} \rightarrow B^* \rightarrow B \) and \( B^{**} \rightarrow B \) chains might be expected. The experience in photoproduction suggests that the efficiency for finding the daughter pion is \( \sim 50\% \). We will therefore assume an overall tagging efficiency of \( \sim 10\% \) by daughter mesons is possible.

The production tagging efficiency is probably sensitive to \( p_T \) and to angular cuts (or rapidity cuts). The heavy quark limit ensures that the 4-velocity of the produced \( B^{**} \) is approximately equal to the 4-velocity of the \( B \) i.e., zero–recoil of the \( b \)-system is a good approximation. In hadronic collisions it is probably reasonable to assume that the \( B^{**} \) system at low \( p_T \) is produced in an unpolarized initial state and, thus, the distribution of decay pions in the process \( B^{**} \rightarrow B + \pi \) is spherical in the \( B^{**} \) rest frame. The (unit normalized) polar distribution of pions relative to the \( B \) flight direction is then obtained by boosting the spherical distribution:

\[
\frac{dN}{d\Omega} = \frac{1}{4\pi} \left[ \frac{\gamma(1 - \beta^2 \omega^2 - ((\beta^4 - 2\beta^2)\omega^2 + \beta^2) \cos^2 \theta) + 2A\beta\omega \cos \theta}{A\gamma^2(\beta^2 \cos^2 \theta - 1)^2} \right]
\] (4)

where:
\[
A = (1 - \beta^2 \omega^2 - \beta^2(1 - \omega^2) \cos^2 \theta)^{1/2}
\] (5)

and \( \omega = 1/\sqrt{1 - m_\pi^2/(\Delta M)^2} \approx 1.04 \). In the massless pion limit, \( \omega = 1 \) and this reduces to \( dN/d\Omega = 1/[4\pi\gamma^2(1 - \beta \cos \theta)^2] \) valid to order \( \frac{1}{2}O(m_\pi^2/(\Delta M)^2) \approx 4\% \). Note that 50\% of the pions will occur within a cone of opening angle \( \theta_{50\%} \) given by (for \( \omega = 1 \)):

\[
\tan \theta_{50\%} = \frac{1}{\gamma \beta} = \frac{1}{\sqrt{\gamma^2 - 1}}
\] (6)

For \( \gamma \approx 2 \) we see that \( \theta_{50\%} \approx 30^\circ \), and this defines a cone of small solid angle of \( 0.07 \times 4\pi \) steradians. The aligned daughter pions, coming from the primary vertex, are also expected be more energetic than typical minimum bias pions. Thus, the conical cut on pions with \( \theta < \theta_{50\%} \) should lead to a significant gain in signal to background for low–\( p_T \) (at high \( p_T \) the \( B \)-meson is enveloped in a jet with higher
\( \pi \) multiplicity within small conical angle). We do not consider the more general possibility of rapidity correlations here.\(^3\)

2.3. Semileptonic Weak Decays involving \( V_{cb} \)

High statistics measurements of exclusive semileptonic branching ratios such as \( B \rightarrow l + \nu + (D^{**}, D^*, D) \), etc., are possible at the level of \( \sim 10^9 \) decays. These are important processes for establishing the overall normalization of weak transitions in hadron colliders since the CLEO and ARGUS experiments are significantly improving the statistics of these processes. The key physics goal here is to obtain the highest precision determination of \( V_{cb} \) possible. This requires exploiting the heavy quark symmetry result, together with QCD and \( 1/M \) corrections, which fixes at special kinematic point \( w = v \cdot v' \rightarrow 1 \) the normalization of the Isgur–Wise function. The normalization of \( \xi(w \rightarrow 1) \) is known to a precision approaching \( 3\% \). Therefore, the goal of experiment should be to approach a \( 3\% \) determination of \( V_{cb} \). Much effort to date has gone into the measurements of semileptonic weak inclusive decays and exclusive decays of heavy mesons. In \( e^+e^- \) experiments such as CLEO or ARGUS, and as proposed for the asymmetric \( B \)–factory, one tunes the beam energy to produce the \( \Upsilon(4S) \) resonance, which decays to pairs of \( B^+B^- \) or \( B^0\bar{B}^0 \) mesons that are nearly at rest in their \( cm \) system. With tagging this can produce a clean sample of \( B \)'s for the exclusive decay modes. The \( B \)–mesons can then decay to a final state lepton either directly, semileptonically as \( B \rightarrow (l\nu)X \), or hadronically, cascading as \( B \rightarrow X \rightarrow (l\nu)X' \).

Various models,\(^1\) are used to fit the leptonic energy distribution to the various component subprocesses (see discussion of S. Stone in ref.[2]). The error in these results is dominated by the theoretical models used to fit the spectra, and is of order \( \sim 15\% \). At PEP, PETRA and the LEP experiments the semileptonic decays are studied at much higher energies. These results are consistent with the \( \Upsilon(4S) \) results to order \( \sim 15\% \).\(^2\) Alternatively, one can study exclusive modes using a tagged \( B \), and determine the missing \( M^2 \) distribution from the mass of the visible decay fragments of the other \( B \). The missing \( M^2 \) distribution will contain endpoint peaks from contributing subprocesses, such as \( B^0 \rightarrow l^-\nu(D^{*+}) \rightarrow \pi^+(D^0) \rightarrow K^-\pi^+ \). The subprocesses are then fit to the observed missing \( M^2 \) distribution. These methods, using different theoretical models, have broadly consistently yielded our first determinations of branching ratios and again yield results to order \( \sim 15\% \).\(^2,11\)

However, ultimately we want to minimize the sensitivity to theoretical models in extracting \( V_{cb}, V_{ub} \). Here we can use heavy quark symmetry in a model independent way,\(^1\) from the \( w \) distribution. The decay distribution in \( w \) for \( B \rightarrow \ell\nu D_i \) is:

\[
\frac{d\Gamma_{D_i}}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_B^2 m_D^2 (1 + w)^2 \sqrt{w^2 - 1} (F_{D_i}(r, w))
\]

where \( r = m_{D_i}/m_B \) and \( F_{D_i}(r, w) \) is a form factor.\(^1\) In the \( m_{B,D} \rightarrow \infty \) limit \( F \) is given in terms of the Isgur–Wise function \( \xi(w) \) and the known ratio \( r \). At the special “zero recoil” point \( \xi(1) = 1 + \epsilon \) where \( \epsilon \) is composed of (a) QCD corrections computed to NLLA order \( \pm 1\% \) and (b) \( 1/M \) effects that are dominant \( \pm 3\% \). Hence,
the strategy is to extract the functional dependence of $F(r, w)$, or $\xi(w)$ upon $w$ and extrapolate to $w = 1$ where theoretical corrections are under control. This implies that the experimental statistical uncertainties must become significantly smaller than $\sim 1\%$ and the limiting attainable precision of $V_{cb}$ is expected to be $\sim 3\%$, modulo improvements in the theoretical uncertainties.

Neubert\textsuperscript{10} has carried out this analysis with the existing CLEO and ARGUS data on the $q^2$ distributions, based upon $\sim (a few 100)$ events, to extract the model independent result $|V_{cb}| = 0.042 \pm 0.007$. This is indicative of the current statistical extrapolation errors attained with $\sim 300$ events, and this should improve in the near future. It would appear that with $10^4$ fully reconstructed events the statistical error in this approach will scale downward by a factor of 10. The key point here is that the theoretical modeling in the hadronic environment is now relegated to the corrections, and not to the result itself. The highest experimental statistics will drive the future determinations of $V_{cb}$.

The challenge for this approach in the hadronic experiments is the requirement to fully reconstruct the decaying $B$–meson, particularly with respect to kinematics. In $e^+ e^−$ experiments the beam energy, together with the flight direction of the $B$, supplies sufficient kinematic information to know the $B$ energy unambiguously. In the broad–band hadronic environment we are \textit{a priori} limited to knowing only the flight 3–vector of the $B$, and the visible 4–momenta; the unobserved neutrino momentum leads to the ambiguity.

Let us consider the semileptonic decay $B \rightarrow D + \ell^± + X$. Of course, $X$ contains the neutrino but may also contain missing neutrals as well. The first question is, can we select events in which $w \rightarrow 1$ using this information alone? If we consider events for which we hypothesize that the missing (mass)$^2$ is $M^2_X$, then the energy of the $B$ is determined up to a a two–fold ambiguity.

\begin{equation}
E_B = \frac{\Delta^2 E_{vis} \pm [\Delta^4 E_{vis}^2 - (E_{vis}^2 - \vec{p}_{vis}^2 \cos^2 \theta)(\Delta^4 + M_B^2 \vec{p}_{vis}^2 \cos^2 \theta)]^{1/2}}{E_{vis}^2 - \vec{p}_{vis}^2 \cos^2 \theta} \quad (8)
\end{equation}

where $(E_{vis}, \vec{p}_{vis}) = p^\mu_{vis} = p_{D}^\mu + p_{\ell}^\mu$ is the visible 4–momentum ($M^2_{vis} = p^2_{vis}$) and $\Delta^2 = \frac{1}{2}(M^2_B + M^2_{vis} - M^2_X)$.
\( \theta \) is the angle subtended by the flight vector of the \( B \) (primary to secondary vertex vector) and \( \vec{p}_{vis} \). Let us now further assume \( M_X = 0 \) (no missing neutrals, etc.). To observe \( w = v_D \cdot v_B = 1 \) we must have in the \( B \) rest–frame, \( M_B = M_D + 2E_\ell \), i.e., the massless leptons are back–to–back, whence

\[ M_{vis}^2 = (p_D + p_\ell)^2 = M_B^2 + 2M_D E_\ell = M_B M_D \]

The condition \( M_{vis}^2 = M_B M_D \), using,

\[ 0 = M_X^2 = (p_D + p_\ell - p_B)^2 \quad \text{implies} \quad x + y = \frac{1}{2}(1 + M_D/M_B), \]

which defines a line in the phase space of the decay Fig.(2) intersecting point (a). Unfortunately this line cuts across the physical region (interior to (abc)) and does not uniquely select \( w = 1 \), while \( M_{vis}^2 = M_D^2 \) and \( M_{vis}^2 = M_M^2 \) do uniquely select points (b) and (c). Thus, for \( w = 1 \):

\[ E_B = \frac{E_{vis}}{2} \left( \frac{M_B}{M_D} + 1 \right) \pm \frac{1}{2} \left( \frac{M_B}{M_D} - 1 \right) \]

Therefore, we see that we cannot uniquely reconstruct the Isgur–Wise point \( w = 1 \) from \( M_{vis}^2 \) alone. To uniquely reconstruct the kinematic point \( w = 1 \) using the information about the \( B \) decay alone we must have (i) \( M_X^2 = 0 \) (ii) \( M_{vis}^2 = M_B M_D \), (iii) and \( |\vec{p}_{vis}| = 0 \). Note that for \( |\vec{p}_{vis}| = 0 \) the \( B \)–energy is determined uniquely as \( E_B = \Delta^2/E_{vis} \).

P. Sphicas\(^5\) has examined by Monte Carlo the fraction of (hypothetical) events for which the two–fold energy ambiguity of the \( B \)–meson is less than 10%. For \( \sim 10^9 \) decays he finds (few)\( \times 10^3 \) decays in which \( \delta E_B/E_B < 10\% \). The slope of the Isgur–Wise function near \( v \cdot v' = 1 \) is \( \xi'/\xi \sim -0.4 \), thus a 10% precision in the \( B \) energy yields about an additional 4% uncertainty in the normalization, or about \( \sim 6\% \) overall. With \( 10^{11} \) \( B \)'s this would approach the desired limiting resolution.

How well does this do in excluding missing neutrals? If we allow \( M_X^2 = m_\pi^2 \), which occurs for a fast pion collinear to the neutrino, then one finds that the point (a) shifts by \( \delta x_a \sim \delta y_a \sim O(m_\pi^2/M_B^2) \) (the points (b) and (c) shift by \( O(m_\pi/M_B) \), which is easier to resolve). This is much less than the experimental momentum resolution, and is therefore problematic. However, the typical pion contribution is not collinear with the neutrino and \( M_X \sim m_\pi M_B \), whence \( \delta x_a \sim \delta y_a \sim O(m_\pi/M_B) \sim 3\% \), and is marginally resolvable.

2.4. Kinematic Tagging with Daughter Pions?

Let me indulge here in a speculative proposal. Clearly we can sacrifice the huge statistics available at the hadron machine to achieve reasonable kinematic
reconstruction for a (few)×10³ events. However, we would prefer a method which is efficient, covers all of phase space, not just \( \vec{p}_{\text{vis}} = 0 \), and ideally which offers greater leverage in momentum resolution.

Perhaps we can exploit the fact that a fraction \( f \sim 20\% \) of \( B \)–mesons will be produced as the daughters of the \( B^{**} \) resonance, together with the daughter pion. Thus, let us ask if we can select the \( B \)–meson energy in a typical process \( B \to D^{*} + \ell + \nu \), where the two hypothetical 4–momenta of the \( B \) are \( p^{(1)}_B, p^{(2)}_B \). We demand that we find a pion which matches a hypothetical solution for the \( B \)–meson 4–momentum, \( p_B \), satisfying either:

\[
(p_\pi + p^{(1)}_B)^2 = M^2_{B^{**}} \quad \text{or} \quad (p_\pi + p^{(2)}_B)^2 = (M_{B^{**}} + \delta M_{B^{**}})^2
\]

where \( \delta M_{B^{**}} \) is the width of the resonance parent. Then a difference between the hypothetical 4–momentum has a resolution given by the width:

\[
p_B^\mu(p^{(1)}_B - p^{(2)}_B) = M_{B^{**}}\delta M_{B^{**}} = E_\pi(\delta r E_B)(1 - (1 + \beta - \beta^2)\cos \theta)
\]

where \( \delta r E_B \) is the minimum resolvable \( B \)–energy. Hence, apparently we can directly reconstruct the \( B \) energy by this method to a limiting resolution of only:

\[
\frac{\delta r E_B}{M_{B^{**}}} \sim \frac{\delta M_{B^{**}}}{E_\pi} \gtrsim 5\%
\]

where we use \( E_\pi \sim 1 \) GeV, \( \delta M_{B^{**}} \sim 50 \) MeV, typically, and \( \theta \approx 90^\circ \). On the other hand, we see in eq.(11) that, using \( \vec{p}_{\text{vis}} \) the energy ambiguity is:

\[
\delta E_B = |\vec{p}_{\text{vis}}| \left( \frac{M_B}{M_D} - 1 \right)
\]

Note that \( |\vec{p}_{\text{vis}}| \) can be quite large; as we approach the Isgur–Wise point (a) in Fig.(2) and, taking for example the \( B \) rest–frame, we have \( |\vec{p}_{\text{vis}}| \sim \frac{1}{2}(M_B - M_D) \). The value \( \delta r E_B \) is then sufficiently small to allow a selection between the two solutions, since:

\[
\frac{\delta r E_B}{\delta E_B} \sim \frac{M_{B^{**}}\delta M_{B^{**}}}{E_\pi |\vec{p}_{\text{vis}}|} \left( \frac{M_B}{M_D} - 1 \right)^{-1} \sim 10\%
\]

using \( |\vec{p}_{\text{vis}}| \sim \frac{1}{2}(M_B - M_D) \). In other words, the energy ambiguity can be \( \sim 10\sigma \) of the minimum resolvable energy of the \( B \)–meson, using the daughter pion in combination.

Note that we are not then restricted to the special kinematic configurations \( |\vec{p}_{\text{vis}}| = 0 \); indeed, this approach would be complimentary to \( |\vec{p}_{\text{vis}}| = 0 \), and preferably requires that \( |\vec{p}_{\text{vis}}| \) be large. It does rely on being able to “cut hard” to reduce the background pions that fake a \( B^{**} \) daughter, and it is subject to background fakes that favor the wrong solution. This probably favors low \( p_T \) \( B \)’s with less of an enveloping jet structure, and then a \( < \theta_{50}\% \) cut. Again, this cannot resolve the missing collinear pion ambiguity, but it is potentially able to resolve the typical
missing neutral pion ambiguity. We have given here only a sketchy analysis of this. It requires serious study by Monte Carlo simulation, or direct application to the existing data of charm photoproduction experiments, and eventually in $B$ decays where the $B$–momentum is known (all decay products visible). With $f \sim 10\%$ we may hope to be able to select between kinematic options with efficiencies of order 1%, allowing $\sim 10^7$ fully reconstructed semileptonic weak decays.

2.5. Semileptonic Weak Decays involving $V_{ub}$

High statistics measurements of exclusive semileptonic branching ratios such as $B \to \ell + \nu + (\pi, (d, s))$, etc., are possible at the level of a (few)$\times 10^6$ decays. These are important processes to establish the general normalization of weak transitions involving $V_{ub}$. The statistical limitations together with theory imply better than a 3% determination of the quantity $f_B\sqrt{B}V_{ub}$ may be possible. The quantity $f_B\sqrt{B}$ is known poorly to about 20% precision, implying an overall determination of $V_{ub} \pm 20\%$.

The present determinations of $V_{ub}/V_{cb}$ are based upon the endpoint of the lepton spectrum for inclusive semileptonic decay rates (see the S. Stone review in ref.[2]). There have been searches for the exclusive decay mode $B \to \rho \ell \nu$. On the $\Upsilon(4S)$ the measurement of $E_\ell$ near the endpoint where the background from $b \to c\ell \nu$ and continuum $e^+e^-$ production becomes small in principle yields a determination of $V_{ub}/V_{cb}$, however it is subject to limitations from the knowledge of $m_b$ and $m_c$, and is highly model dependent. The extracted $V_{ub}/V_{cb}$ values range from $0.11 \pm 0.02$ for the ACM model to $0.17 \pm 0.03$ for the ISGW model. The statistical errors are large. The exclusive decay mode $B \to \rho \ell \nu$ has been studied, with greater model dependence, lower statistics < 100 events and a larger scatter of $0.1 < V_{ub}/V_{cb} < 0.3$.

| Mode | $B$ | $|V_{ub}/V_{cb}|$ | comment |
|------|-----|-----------------|---------|
| $B \to \rho \ell \nu \ (\rho \to \pi^\pm \pi^\mp)$ | $5.0 \times 10^{-5}$ | $1.5 \times 10^6$ | * lattice |
| $B \to X_{charmless} \ell \nu$ | $2.5 \times 10^{-4}$ | $1.7 \times 10^6$ | inclusive models |
| $B \to \omega \ell \nu \ (\omega \to \pi^+ \pi^-)$ | $1.0 \times 10^{-6}$ | $3.3 \times 10^3$ |
| $B \to \phi \ell \nu \ (\phi \to K^+ K^-)$ | $2.7 \times 10^{-7}$ | $8.3 \times 10^2$ |
| $B \to \pi \ell \nu$ | $3.0 \times 10^{-5}$ | $10^5$ | * chiral symmetry |
| $B \to \eta \ell \nu \ (\eta \to \pi^\pm \pi^\mp e^+ e^-)$ | $1.0 \times 10^{-7}$ | $10^4$ | chiral symmetry |
| $B \to D_s(\pi, \rho, \omega)$ | $10^{-4}$ | $10^6$ | Argus limit $\lesssim 1\%$ |
| $B_s \to K \ell \nu$ | $3 \times 10^{-5}$ | $6 \times 10^4$ | * yields $f_{B_s}/f_{B_{(u,d)}}$ |
| $B_s \to K^* \ell \nu$ | $5 \times 10^{-5}$ | $10^9$ | $\propto f_{B_s}$ |

Table II. Branching ratios estimated by rescaling charm analogues, assuming $|V_{ub}/V_{bc}| = 0.05$. The yields assume 33% $B^\pm$, 33% $B^0$, 18% $B_s$.

With a reasonable extrapolation to the SVX technology, and the copious yield of $B$'s we can imagine rather conservative cuts allowing the study of final states such $X = \rho$, $X = \pi$, $X = \omega$, $X = many \pi$'s, etc. In the decay $B^- \to \rho \ell^- \nu$ and the subsequent $\rho \to \pi^+ \pi^- \ (P = 0.5)$ we demand that the pions reconstruct to the
\( \rho \) mass, and connect to the lepton at the decay vertex of the \( B \). The estimated 
\[ \text{Br}(B^- \to \rho^- \nu) \sim (\text{Br}(B^- \to D^* \ell^- \nu) \sim 4\%) \times |V_{ub}/V_{cb}|^2 \times 1/2 \sim 5.0 \times 10^{-5}, \]
thus with \( 10^{10} \) produced \( B \)'s we will have \( \sim 1.5 \times 10^5 \) events. The problematic 
backgrounds are from \( B \to D \ell \nu \) and \( D \to 2\pi \) or \( D \to \rho \pi^0 \), with the \( \pi^0 \) undetected, 
\( B \to \rho D \) and \( D \to \ell \nu \). The \( \rho \) tends to be diluted by the pion background, which 
may require cutting on events in which the other \( B \) is seen in a semileptonic mode \( (\sim 10\%) \). The rejection of \( \gamma \)'s and the mass reconstruction of the \( \rho \), and a veto 
on more than 2 pions are important constraints to consider in fishing the \( \rho \) out of hadronic events.

Thus, a high statistics study of Cabibbo suppressed decay modes seems possible 
with \( 10^{10} \) \( B \)-mesons, but we are in a learning situation at present that must evolve 
considerably. This yields of order \( 10^5 \) decays. A form factor analysis may be possible 
for the \( \pi \ell \nu \) mode if daughter pion kinematic tagging is possible, yielding \( \sim 10^3 \) fully 
reconstructed decays. One can hope to exploit the fact that chiral symmetry fixes 
the normalization of this matrix element at \( w = 1 \). It should certainly be possible 
to achieve \( V_{ub} \) to better than \( \pm 20\% \) using models, and perhaps better precision by 
use of chiral symmetry. The quantity \( f_{B_s}/f_{Bu,d} \) would be probed to \( \pm 1\% \) precision.

### 2.6. \( B_s \) and \( B_c \)

The \( B_s = (\bar{b}s) \) has been seen at Aleph, Opal and CDF.\(^{12} \) CDF has observed 14 
fully reconstructed \( \psi \phi \) events, and reports a mass of \( M_{B_s} = 5383 \pm 7 \) MeV. With a 
yield of \( 10^{10} \) usable \( B \)'s there are expected to be produced \( 1.8 \times 10^9 \) \( B_s + \bar{B}_s \). This 
will allow survey of various decay modes, such as \( DK^*, D^* K, D_s^*D_s^*, D_s^*D^*, \) etc.
Also, of great interest will be the study of higher resonances producing daughter 
\( K^\pm \) mesons in association with the \( B_s \), e.g.,
\[ p\bar{p} \to B_s^{*+}(2^-, 3^-) \to K + B_s \]  
(17)
The prospects for the application of this to, e.g., flavor tagging for study of \( B_s\bar{B}_s \) 
mixing, is discussed below.

Table III. (a) Yields are for detectable decays and include the branching fractions \( \psi \to \mu^+ \mu^- \sim 7\% \) (b) includes \( (\psi \to \mu^+ \mu^-) \times (D_s^* \to \pi^+ (\phi \to K^+ K^-) \sim 2\%). \)

| Mode               | Br       | yield/10\(^{10}\) \( B \)'s | yield/100 \( pb^{-1} \) |
|--------------------|----------|-----------------------------|---------------------------|
| \( B_c \to \pi^+ \psi \) | \( 4.0 \times 10^{-3} \) | \( 2.8 \times 10^4 \) \(^{(a)} \)  | \( 276 \) \(^{(a)} \) |
| \( B_c \to D_s^* \psi \) | \( 5.0 \times 10^{-2} \) | \( 7.0 \times 10^2 \) \(^{(b)} \)  | few \(^{(b)} \) |
| \( B_c \to \psi \ell \nu \) | \( 10\% \)   | \( 7.0 \times 10^4 \) \(^{(a)} \)  |                           |

Perhaps the most interesting new mesonic system will be the \( B_c = (\bar{b}c) \). This is 
remarkable because we can say with certainty that non–relativistic potential models 
apply, and the spectrum is completely determined by those methods. Indeed, this is 
the true Hydrogen atom of QCD. Eichten and Quigg\(^{13} \) have estimated the spectrum 
and widths of the \( B_c \) system. They use the Buchmüller–Tye potential as fit to the \( \psi \) 
and \( Y \) systems (and use other potentials, e.g., the Cornell and Richardson potentials, 
for error estimation), finding:

\[ M_{B_c} = 6258 \pm 20 \text{ MeV} \quad M_{B_{cs}} - M_{B_c} = 73 \pm \text{ MeV} \]  
(18)
The prospects for observation of $B_c$ hinge upon the production cross-section. There is reasonable agreement amongst several groups\textsuperscript{14} that the ratio $\sigma(B_c)/\sigma(Bb) \sim 10^{-3}$. Thus, for $|\eta| \leq 1$ and $p_t > 10$ GeV/c we have $\sigma(B_c) \sim 10^{-2}$ mb, and a yield of $\sim 10^7$ $B_c$'s for $10^{10}$ $B$'s. Some of the principal detectable decay modes are listed in Table III.\textsuperscript{12}

Note that the decay mode $B_c \to \psi \ell \nu$ is the $B_c$ analogue of the $B_u \to D^* \ell \nu$ decay for which the Isgur–Wise function at $w = 1$ sets the normalization. Here the process is completely determined, and $w = 1$ involves only the overlap of the known $\psi$ and $B_c$ wave–functions. Thus, this is an interesting toy laboratory for the heavy quark symmetry methods where everything is perturbative. We should also mention that processes containing $CP$–violation, like $B_c \to D_s \phi$, involve both a direct short–distance penguin and interference terms with short–distance contributions to the imaginary parts. Here the factorization approximation is exact, and the short–distance imaginary parts are also in principle computable. Thus, $CP$–violation in the $B_c$ system may ultimately prove to be a fundamental issue in the $B$–physics program. The $B_c$ is a remarkable system in which much of the QCD dynamics is solvable by perturbative methods. It will thus provide a powerful laboratory for theorists and experimentalists, and possibly a probative system for new physics in the future.

2.7. Heavy Baryons

The spectroscopy and interactions of baryons consisting of two heavy quarks and one light quark simplify heavy quark mass limit, $m_Q \to \infty$. The heavy quarks are bound into a diquark whose radius $r_{QQ}$ is much smaller than the typical length scale $1/\Lambda$ of QCD. In the limit $r_{QQ} \ll 1/\Lambda$ the heavy diquark has interactions with the light quark and other light degrees of freedom which are identical to those of a heavy antiquark. Hence as far as these light degrees of freedom are concerned, the diquark is nothing more than the pointlike, static, color antitriplet source of the confining color field in which they are bound, i.e., these $QQq$ baryons are in a sense “dual” to heavy mesons $Q\bar{q}$.

The spectrum of $QQq$ baryons is thus related to the spectrum of mesons containing a single heavy antiquark. The groundstate is essentially a $(1, \frac{1}{2})$ or $(0, \frac{1}{2})$ heavy spin multiplet. The form factors describing the semileptonic decays of these objects may be directly related to the Isgur-Wise function, which arises in the semileptonic decay of heavy mesons. The production rates for baryons of the form $ccq$, $bbq$ and $bcq$ have been estimated in the approximation that the $QQ$ diquark is formed first by perturbative QCD interactions, and then this system fragments to form the baryon like a heavy meson.\textsuperscript{15} (In the $cc$ system the heavy diquarks are not particularly small relative to $1/\Lambda$, so there may be sizeable corrections to these results). Essentially the fragmentation of a heavy quark $Q$ into a $QQq$ (or $QQ'q$) baryon factorizes into short-distance and long-distance contributions. The heavy quark first fragmentation into a heavy diquark may be trivially related to the fragmentation of $Q$ into quarkonium $Q\bar{Q}$. This initial short distance fragmentation process is analogous to fragmentation into charmonium, $c \to \psi c$, which has been
analyzed recently by Braaten, et al., and others. The subsequent fragmentation of the diquark $QQ$ to a baryon is identical to the fragmentation of a $\bar{Q}$ to a meson $\bar{Q}q$. Experimental data on production of heavy mesons can be used here.

Table IV. Hadronically produced double heavy baryons for Tevatron ($3 \times 10^9 B_{u,d}$'s) and Main Injector ($3 \times 10^{10} B_{u,d}$'s).

| Mode         | Tevatron | Main Injector |
|--------------|----------|---------------|
| $\Sigma_{cc}$, $\Sigma_{cc}^*$ | $6 \times 10^4$ | $6 \times 10^5$ |
| $\Lambda_{bc}$ | $6 \times 10^4$ | $6 \times 10^4$ |
| $\Sigma_{bc}$, $\Sigma_{bc}^*$ | $\sim 10^5$ | $\sim 10^6$ |
| $\Sigma_{bb}$, $\Sigma_{bb}^*$ | $\sim 10^4$ | $\sim 10^4$ |
| $\Lambda_{bc}$ | $6 \times 10^2$ | $6 \times 10^2$ |
| $\Sigma_{bc}$, $\Sigma_{bc}^*$ | $6 \times 10^2$ | $6 \times 10^2$ |

The probability for $c \to \Sigma_{cc}$, $\Sigma_{cc}^*$ is estimated to be $\sim 2 \times 10^{-5}$, for $b \to \Lambda_{bc}$ to be $\sim 2 \times 10^{-5}$, and for $b \to \Sigma_{bc}, \Sigma_{bc}^*$ to be $\sim 3 \times 10^{-5}$. The probabilities for $b \to \Sigma_{bb}, \Sigma_{bb}^*$, $c \to \Lambda_{bc}$ and $c \to \Sigma_{bc}$, $\Sigma_{bc}^*$ are down by roughly $(m_c/m_b)^3$, or two orders of magnitude.

Detection of these objects is probably very difficult at best. Consider the $\Sigma_{bb}$ decay chain:

$$
\Sigma_{bb} \to D^* + X + (\Sigma_{bc})
$$

$$
\to D^* + X + (\Lambda_b)
$$

$$
\to D^* + X + (\Lambda_c)
$$

$$
\to K^* + X + \Lambda
$$

Each vertex above must be reconstructed, in spite of a high probability of missing neutrals, including the drift of $D^* \to D$'s away to branch vertices. A rough estimate is that a handful of such decay chains might be available in a $10^{10}$ program admitting reconstruction of the parent doubly–heavy baryon. However, there will come insights as to how to do this well as experience is gained.

3. Rare Processes

In this section we will briefly discuss some of the interesting “rare” processes that are the far–reaching goals of the initiatives of this decade. Much greater detail is afforded these topics in other talks in this conference, so we will focus only on issues that involve some of the aforementioned ideas. Clearly the ultimate structure of $CP$–violation is of great interest, but the first observation of $CP$–violation in the $B$-system will be an achievement of enormous importance. We will comment as to how this observation may be feasible in the hadronic collider mode by making use of daughter pion flavor tagging, in comparison to the conventional strategy. Indeed, many of the tools necessary to see the $CP$–asymmetry in $B \to \psi K_S$ are now in place at CDF, and this exciting observation may be only a few years away!
We describe the important observation of $B_s \bar{B}_s$ mixing. This process will be quite a bit more difficult to observe than $CP$–violation. This is likely, given that the large top mass implies a large $x_s$, and mandates very high statistics for flavor tagged, and kinematically tagged $B_s$ semileptonic decays. It may be a leap of faith to extrapolate to this process, given that there is limited experience with semileptonic decays of any $B$–system to date. In conjunction with flavor tagging, our experience here is $O(\epsilon^2)$ at present. We will also discuss the rare leptonic modes. Here we have made extensive use of a presentation by S. Willenbrock and G. Valencia from our in–house workshop. Thus, the last subsection is really their effort, more than mine.

3.1. $CP$ violation

There are well–known modes for the observation of $CP$–violation, such as $B^0 \rightarrow \psi K_S$, etc., and $B_s \rightarrow D_s^\pm K_s^\mp$, and self–tagging modes. To observe $CP$–violation we must tag the flavor of the initial state, which taxes the available statistics. $CP$–violation with self–tagging modes is experimentally attractive, but there exists no guarantee that observable $CP$–effects will be present in these modes. Since the volume of the Snowmass Proceedings is consumed with the intimate details of $CP$–violation in the $B$–system, we will simply focus on how one might use the conventional or daughter–meson tagging methods to observe the straightforward $B^0 \rightarrow \psi K_S CP$–asymmetry.

The decay mode $(B^0, \bar{B}^0) \rightarrow \psi K_S$ involves $CP$–violation. Thus the partial widths for $B^0$ and $\bar{B}^0$ to decay into the $\psi K_S$ final state differ, and the time integrated asymmetry is defined as:

$$a = \frac{\Gamma(B \rightarrow \psi K_S) - \Gamma(B \rightarrow \bar{\psi} K_S)}{\Gamma(B \rightarrow \psi K_S) + \Gamma(B \rightarrow \bar{\psi} K_S)} = \frac{x_d}{1 + x_d^2} \sin(2\beta) \sim 0.1 - 0.5$$

(20)

Note that the branching ratio for $B^0 \rightarrow K_S + (\psi \rightarrow \mu^+\mu^-)$ is $\sim 2 \times 10^{-5}$ (including the 7% dimuon mode of $\psi$).

To observe $a$ one must flavor–tag the neutral $B$–meson at production $t = 0$ to determine if it is a particle or anti–particle. Since $b$–quarks are produced in pairs, this is conventionally achieved by observing the semileptonic decay mode of the other $B$ in the event. For example, if the other meson is a $B^- (\bar{B}^+)$ it can decay semileptonically to a charge $-(+) \text{ lepton}$, with a $Br(B \rightarrow \ell\nu D) \sim 10\%$. This does not require full reconstruction of the semileptonic decay, so for $CP$–violation one is effectively measuring $\Gamma(\ell^+\psi K_S) - \Gamma(\ell^-\psi K_S)$ (Note that this does not require a new single lepton trigger since one can trigger on the $\psi$ dimuons). Including geometric efficiencies this conventional tagging efficiency is expected to be of order $\epsilon_1 \sim 10^{-2}$.

Gronau, Nippe and Rosner have pointed out that resonance daughter pions (as well as rapidity correlations associated with the jet fragmentation) are possible flavor–tags. A stunning implication of the daughter mesons from parent resonances is that all $CP$–violating processes in hadron machines are expected to be self–tagging! We should recognize that at low–$p_T$ the $b$–production mechanism is somewhat more akin to threshold production and the resonance mechanism may be favored.
Table V. Statistical significance $\sigma_i$ for tagging efficiencies $\epsilon_1, \epsilon_2$ and asymmetries $a$, for various integrated luminosities. We show the 100 pb$^{-1}$, i.e., prospects for run I(b) at Fermilab ($10^{10}$ $B$'s corresponds to $\int L dt = 10^3$ pb$^{-1}$).

| $a$ | $\epsilon_2 - \epsilon_1$ | $\int L dt$ | $\sigma_2 - \sigma_1$ |
|-----|----------------|-------------|----------------|
| 0.5 | 0.1 - 0.01 | 100 pb$^{-1}$ | 2.1 - 0.7 |
| 0.1 | 0.1 - 0.01 | 100 pb$^{-1}$ | 0.4 - 0.13 |
| 0.5 | 0.1 - 0.01 | 10$^4$ pb$^{-1}$ | 6.7 - 2.1 |
| 0.1 | 0.1 - 0.01 | 10$^3$ pb$^{-1}$ | 1.3 - 0.4 |
| 0.5 | 0.1 - 0.01 | 10$^4$ pb$^{-1}$ | 21.2 - 6.6 |
| 0.1 | 0.1 - 0.01 | 10$^4$ pb$^{-1}$ | 4.1 - 1.3 |

At higher $p_T$ the $b$-jet is forming and there would be more pions expected (a source of dilution), and perhaps the rapidity correlation idea is favored. This is not to advocate any theory of production, but rather to emphasize that the optimization may involve tuning of $p_T$, etc. For example, we may prefer operating at low $p_T$'s below the present cuts. While with optimization cuts it is possible that significant improvements in the tagging efficiency may occur, the charm photoproduction experiments suggest that a tagging efficiency of $\epsilon_2 \sim 10\%$ from daughter pions is possible. The flavor of a neutral $B^0 \sim b\bar{d}$ ($\bar{B}^0 \sim b\bar{d}$) is tagged by the presence of a $\pi^+$ ($\pi^-$) daughter, and the $CP$-asymmetry we measure in practice is effectively $\propto \Gamma(\pi^+\psi K_S) - \Gamma(\pi^-\psi K_S)$.

The overall efficiency for observing $B \rightarrow K_s(\psi \rightarrow \mu\mu)$ involves the physics branching ratio $\sim 2 \times 10^{-5}$ times the detection efficiency (including geometric efficiencies). The latter is $\sim 3\%$ at the CDF SVX at present, and we assume it in Table IV. Thus, the overall efficiency for $B \rightarrow K_s(\psi \rightarrow \mu\mu)$ is $\sim 6 \times 10^{-7}$, and, for 100 pb$^{-1}$, we expect $3 \times 10^8$ usable neutral $B$'s, therefore $\sim 180 \psi K_s$ events. Larger $\eta$ coverage, and other detector gains might boost this $\sim 5\times$.

The prospects for observing the CP-asymmetry at a statistical deviation $\sigma$ are indicated in Table V. Significant limits on $CP$–violation in the $B$ system will begin to be placed by end of run I. In the best case, $a = 0.5$ we can begin to see a signal with the conventional semileptonic tagging efficiency, $\epsilon = 0.01$, for $10^{10}$ produced $B$'s, or with the daughter pion tagging $\epsilon_2 = 0.1$ and the larger asymmetry a discovery is likely. Evidently a discovery is assured for $10^{11}$ $B$'s with daughter pion tagging.

### 3.2. $B_s\bar{B}_s$ Mixing

We have for the mixing parameter:

$$x_s = \frac{G_F m_{B_s} \tau_{B_s}}{6\pi^2} B_{s} f_{B_s}^2 \eta_B |V_{ts}^* V_{tb}|^2 m_t^2 F(m_t/M_W)$$

$$\approx \Delta M_{B_sB_s}/\Gamma \sim (14 \pm 6) (f_{B_s}/200 \text{ MeV})^2$$

where $F(z)$ is an Inami-Lim function. An expression for $x_d$ is gotten by replacing $s$
by $d$ everywhere. Note that:

$$\frac{x_s}{x_d} = \left| \frac{V_{ts}}{V_{td}} \right|^2 (1 + \delta) \quad \delta = \left( \frac{m_{B_s}f_{B_s}^2}{m_{B_d}f_{B_d}^2} - 1 \right) \sim 0.2 \quad (22)$$

$x_s$ is very sensitive to $m_{top}$ and we find:

$$x_s \sim 8.0 \leftrightarrow 18.0, \quad m_t = 140 GeV; \sqrt{Bf_B} = 200 MeV \quad (23)$$

$$x_s \sim 17.0 \leftrightarrow 40.0, \quad m_t = 200 GeV; \sqrt{Bf_B} = 220 MeV$$

and we must prepare ourselves for the possibility of large $x_s$, $8 \lesssim x_s \lesssim 40$. For large values the system oscillates many times per decay length ($x = \frac{1}{2}(\text{radians})/(\text{e-attenuation})$, thus $x = 10$ corresponds to 20 radians per decay length). This requires observing the time evolution of the system, which implies that fully reconstructed (energy and flavor), tagged $B_s$ decays are necessary. In contrast, $x_d = 0.66$ and is readily observed in time–integrated measurements. These requirements make the observation of $B_s\bar{B}_s$ mixing more challenging than the observation of $CP$–violation! However, it should be emphasized that this important phenomenon is likely to be the exclusive province of hadron collider experiments because of the large statistical requirements.

The key to observing oscillations is achieving the smallest proper time resolution, $\sigma_t$ (for a good schematic discussion of this see Mike Gold in ref. (1); we also thank John Skarha for discussions on this topic). $\sigma_t$ is composed of the beam-spot resolution $\delta L_{xy}/L_{xy}$ where $L_{xy}$ is the transverse path length (this is the dominant contribution), together with the momentum resolution $\delta p_T/p_T$ as:

$$\sigma_t = \left( \frac{\delta L_{xy}/L_{xy}}{2} + \frac{\delta p_T/p_T}{2} \right)^{\frac{1}{2}} \quad (24)$$

With $\delta_{xy} \sim 40 \mu m$, $L_{xy} \sim 600 \mu m$, we find $\sigma_t \sim 0.07$ characteristic of CDF-SVX.

The conventional triggers would use a produced $B_s \rightarrow l\nu(D_s \rightarrow \phi X)$ or $B_s \rightarrow \pi^+\pi^-\pi^+(D_s \rightarrow \phi X)$ and the opposite $B \rightarrow l\nu X$ for flavor tagging. By fully reconstructing the $B_s$ decay (requiring exclusively charged particles in $X$) and partially reconstructing the tagging decay, it has been estimated that one can reconstruct the oscillation in $\tau$ with $\sim 4000$ events.\(^1\) With the estimated efficiencies this requires about $3 \times 10^{10}$ to $10^{11}$ produced $B$’s. This appears to be a significant challenge!

Can we tag the $B_s$ flavor and kinematics by using the daughter $K$ mesons associated with it’s resonance production? For example, we expect the D-wave $B(2^-)$ and $B(3^-)$ to be above threshold for decay to $K^+ + B_s$ or $K^- + \bar{B}_s$. These resonances are estimated to be broad (250 to 400 MeV), but with a decay fraction to $B_s$ and $B_s^*$ of about 30%. Thus, with the favorable production and branching fractions we may have a flavor tag for $B_s$, but a kinematic tag seems less likely. The
charm system process \( D_s^{**} \to D^* K \) has been demonstrated, which is the opposite to \( D_s^{***} \to D_s K \), The higher resonances have not yet been seen.

### 3.3. Other Rare Modes

Length considerations preclude our giving any comprehensive discussion of the additional interesting rare modes in \( B \)-physics. We will, however, briefly mention a few of the leptonic modes. Rare \( B \) decays encompass such processes as:

\[
\begin{align*}
& (I) \quad B_{d,s} \to (e\bar{e}, \mu\bar{\mu}, \tau\bar{\tau}) \\
& (II) \quad B_{d,s} \to (e\bar{\mu}, \mu\bar{e}, e\tau, \tau\bar{e})
\end{align*}
\]

and additional hadrons in the final state may be included. We should remark that the \( \tau \) containing final states are unique to \( B \), never available in \( K \) decays, and at best phase space suppressed for \( D \)'s.

Such processes as (I) have low standard model rates and are probes of \( V_{td}, V_{ts}, \) and \( m_t \). Thus, they are good probes of the standard model if they are seen at the expected rates. Moreover, they are excellent probes of new physics, such as charged Higgs and flavor changing neutral Higgs couplings, which are generally \( \propto m \) mass. The conventional SM estimates are as follows:

| Mode | \( \tau\tau \) | \( \mu\bar{\mu} \) | \( e\bar{e} \) |
|------|----------------|----------------|----------------|
| \( B_s \to \) | \( 10^{-7} \) | \( 10^{-9} \) | \( 10^{-14} \) |
| \( B_d \to \) | \( 5.0 \times 10^{-9} \) | \( 5.0 \times 10^{-11} \) | \( 5.0 \times 10^{-16} \) |

A crude estimate of the background due to Valencia and Willenbrock is as follows. UA-1 has measured the continuum \( \mu \)-pair background cross-section near the \( B \)-mass, \( M_{\mu^+\mu^-}^2 = M_B^2 \) with momentum resolution \( \delta p \sim 100 \text{ MeV} \) to be \( \sigma(\mu^+\mu^-) \sim 10^{-5}\sigma_B \), where \( \sigma_B \) is the hadronic \( B \) cross-section. This can presumably be reduced to \( \sigma(\mu^+\mu^-) \sim 10^{-6}\sigma_B \) with improved momentum resolution from silicon vertex detectors. The probability that two stray muons make a vertex is geometrically \( \sim 10^{-2} \) and the probability that this yields a momentum vector pointing toward the primary vertex is \( \sim 10^{-2} \). Thus we have an overall background approaching \( 10^{-10}\sigma_B \) and a 3-\( \sigma \) \( B_s \)-peak is therefore possible. With a yield of \( 10^{11} \) \( B \)'s we expect therefore \( \sim 30 \) events from \( B_s \to \mu\bar{\mu} \). Since the signature is a clean displaced muon pair event with mass reconstruction, it is likely that this can be searched over a rapidity range of \( |\eta| \lesssim 3 \), and a \( p_t \) threshold of \( O(5) \) GeV/c.

Valencia and Willenbrock (VW) have given a nice characterization of the lepton-number violating processes (class II, above) which we describe here. First, note that \( (\tau e) \) and \( (\tau \mu) \) are unique to the \( B \)-system (not available in rare \( K \) decays). Since such processes can be generated in principle by Higgs-scalar exchange, which is a coupling constant \( \propto m \) mass, it is possible that the \( B \) system becomes sensitive to these processes at a level that is readily experimentally accessible, and complimentary to rare \( K \) decay searches, such as at KTEV.
VW begin by postulating general four–fermion interactions describing such processes as $B \rightarrow e\mu$ and $K \rightarrow e\mu$ as:

$$c_B(s\Gamma d \bar{p}\Gamma e) + c_K(b\Gamma s \bar{p}\Gamma e)$$

(26)

with arbitrary Dirac structures $\Gamma$. VW then consider the effects of different $\Gamma$'s and $c_X$'s on the ratio of branching ratios $R_1 = \text{Br}(B \rightarrow \mu e)/\text{Br}(K_L \rightarrow \mu e)$ and $R_2 = \text{Br}(B \rightarrow \mu e + h)/\text{Br}(K_L \rightarrow \mu e + h)$ (where $h$ is an extra hadron system, e.g., pions) as follows:

$$R_1 \approx \frac{c_B^2 f_B^2}{c_K^2 f_K^2} \left( \frac{m_B \tau_B}{m_K \tau_K} \right) \approx 10^{-4} \frac{c_B^2}{c_K^2} \quad \Gamma = (\gamma_\mu, \gamma_\mu \gamma^5)$$

$$R_1 \approx \frac{c_B^2 f_B^2}{c_K^2 f_K^2} \left( \frac{m_B^3 \tau_B}{m_K^3 \tau_K} \right) \approx 10^{-2} \frac{c_B^2}{c_K^2} \quad \Gamma = (1, \gamma^5)$$

$$R_2 \approx \frac{c_B^2 f_B^2}{c_K^2 f_K^2} \left( \frac{m_B^5 \tau_B}{m_K^5 \tau_K} \right) \approx \frac{c_B^2}{c_K^2}$$

(27)

Thus, to proceed we need input as to the magnitude of the ratio $c_B/c_K$. VW distinguish three cases: (i) (Current-like) $c_B/c_K \sim 1$ (ii) (Higgs-like) $c_B/c_K \sim m_B/m_K \sim 10^1$ (iii) (Box-like) $c_B/c_K \sim V_{tb}V_{ts}/V_{td}V_{ts} \sim 10^2$. The latter “box-like” result assumes that the process is induced via a top quark containing box diagram. Thus, the following table arises:
Table VII. Valencia and Willenbrock’s characterization of lepton–number violating modes of $B$ and $K$.

| $\frac{Br(B \to X)}{Br(K \to X)}$ | Current-like | Higgs-like | Box-like |
|-----------------------------------|--------------|------------|----------|
| $\Gamma = (\gamma_\mu, \gamma_\mu \gamma^5)$; $X = e\mu$ | $\sim 10^{-4}$ | $\sim 10^{-2}$ | $\sim 1$ |
| $\Gamma = (1, \gamma^5)$; $X = e\mu$ | $10^{-2}$ | 1 | $10^4$ |
| Any $\Gamma$; $X = e\mu + h$ | 1 | $10^2$ | $10^4$ |

Thus, in the “box–like” and “Higgs–like” limits the $B$ system maybe a better probe than the $K$ system for new physics. The VW characterization is general, and covers all possible models. It illustrates the possibility that $B$ decays are sensitive to new physics in a manner complimentary to rare $K$’s.

4. Summary

A program of producing $> 10^{10}$ detectable $B$’s is conservatively achievable within this decade. This offers an excellent conventional physics program of $\sim 10^9 B \to D^*\ell\nu$ decays and $\sim 10^5 B \to \rho\ell\nu$ decays, allowing a determination of $V_{cb} \pm 3\%$ and $V_{ub} \pm 20\%$. This also probes the quantities such as $\sqrt{Bf_B}$ and $f_{B_s}$ with high statistics.

The resonances and the prospects for flavor and kinematic tagging will emerge within the next few years. New states such as $B_c$ will be surveyed, and the list of $B_s$ and $B_c$ decay modes will grow. $CP$–violation with conventional or bachelor pion tagging may be first observed in the $\psi K_S$ asymmetry within such a $10^{10}$ program. $B_s\overline{B}_s$ mixing looks difficult, though $x_s \lesssim 20$ may be probed. Rare and radiative decays will be subject to their first probative examination.

In conclusion, we have seen that $B$–physics based in a hadron collider offers a rich and diverse, unique and powerful scientific program. It can peacefully coexist with a high–$p_T$ program and dominate the post–High–$p_T$ era at such facilities as Fermilab. Indeed, the prospects for observation of $CP$–violation in the $p\overline{p}$ collider environment are great. There are in fact advantages of the $p\overline{p}$ mode over $pp$ in the observation of $CP$–violation. A dedicated $B$–physics program at Fermilab is important to the evolution of the world–wide effort and a healthy base program for at least the next ten years and probably beyond.
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