Application of modified temperature-moisture relationships in the calculations of drying kinetics

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Abstract. The paper presents a description of the modified temperature-moisture relationships, selected on the basis of the analysis of the kinetics of drying of liquid dispersed products on substrates and the methods of engineering calculation of the kinetics of drying with their application. Classification of the types of kinetic curves characterizing the change in the type of thermogram, such as a change in the angle of inclination, deformation and degeneration of the wet-bulb thermometer and boiling plateaux, allows obtaining a modified type of temperature-moisture relationships for their use in calculations of the drying kinetics. The choice of the type of temperature-moisture relationship is carried out on the basis of the solution developed by the authors of the forecast model of the kinetic curve type. The engineering method for calculating the kinetics of drying liquid dispersed products on substrates is based on solving the basic equation of drying kinetics, choosing the type of temperature-moisture relationship, calculating the coordinates of the reference points of temperature-moisture relationship, determined on the basis of the drying mode and the properties of the dried material.

1. Introduction
Traditionally, the approximations of the drying kinetics by the drying speed curves \( N(U) \) and temperature-moisture relationships \( T(U) \) are used for engineering calculations of the drying kinetics [1]. The approximations of the drying kinetics by drying speed curves \( N(U) \) were developed by A.V. Lykov on the basis of studies of the drying kinetics of wet materials with a solid skeleton [1]. It is necessary to have an explicit form of the approximation relationship of the drying speed on the current moisture content for the calculation using \( N(U) \). Such relationships, as a rule, can only be obtained empirically [2]. This leads to the fact that the solution will be valid only for the experimentally researched product. Features of obtaining differential drying speed curves lead to the fact that they have significant errors, e.g. a significant error was noted when modeling the evaporation of suspended droplets of water-ethanol solution [3]. In [4] this is shown for an investigation on transport of methylhydroxyethylcellulose (MHEC) during drying of a model porous material. In [5], it is shown that a similar effect is observed in several approaches to modeling the heating of evaporating droplets and predicting various heating and evaporation characteristics. Using differential drying speed curves to calculate the evaporation kinetics in the presence of non-spherical droplets [6] and in the model for the convective drying of a kaolin spherical sample [7] also leads to significant errors. Accordingly, they can be used for analysis and calculation only in cases when the dried material forms the classical drying curves and thermograms [8].
As the experience of research of drying kinetics shows, thermograms $T(\tau)$ are the most informative, in terms of analysis and calculation [1, 2]. Therefore, experimentally obtained kinetic curves (thermograms and weight loss curves (drying curves)) are most widely applied in drying studies. Accordingly, the most appropriate as basic dependencies for the analysis and description of the kinetics of non-isothermal drying processes are temperature-moisture curves $T(U)$ [1, 8]. Due to the simultaneous use of data on moisture content and temperature of the material on such curves, certain areas are visually identified on them [9, 10].

The appearance of such areas is especially typically for high-temperature and high-intensity modes, for drying processes accompanied by complicating phenomena, as well as for a number of other specific processes in which the mechanism of heat and moisture transfer is significantly different at different stages [5, 11-13]. At the same time, the initial temperature-moisture curves are not overly sensitive to the thin physical details of the process and experimental errors, like the differential curves $dU/d\tau$, $dT/d\tau$, $dT/dU$.

In the works of the school of prof. V I Konovalov the temperature-moisture relationships are taken as the “base” and are used as the main tool for drying and thermal research [1]. The feature of temperature-moisture relationships is that their character is directly related to chemical, physical-chemical and structural-deformation developments, such as structure formation, shrinkage, etc. At the same time, the presence of so-called thermal sites on temperature and temperature-moisture curves is especially reliable and informative for analysis.

The basic models of temperature-moisture relationships, used in the classical engineering method for calculating the drying kinetics, are based on the so-called linear-beam scheme, when certain reference points of temperature-moisture relationships can be obtained by intersection of lines drawn through reference points of temperature-moisture relationships. In this case, the thermograms in zones of the wet-bulb thermometer $T_{wb}$ and boiling $T_{boil}$ are horizontal.

Using the basic models of temperature-moisture relationships, an engineering method was developed for calculating the drying kinetics [1]. We call this technique classic. In order to calculate the drying kinetics, a number of experimental studies are carried out in the classical method, on the basis of which the type of temperature-moisture relationship is obtained and its relationship to the base model. The classical method recommends using the universal radial temperature-moisture relationship [1] for products, for which the study of the drying kinetics was not carried out or is connected with certain difficulties.

2. Methods and materials

However, the application of universal temperature-moisture relationship gives an acceptable result only in the case when the product forms the classical kinetic curves during the drying process. In the course of research in the field of drying liquid dispersed products on substrates, a strong influence of surface structure formation processes on the character of kinetic curves, in particular, on the type of thermogram, was found. The authors proposed a classification of the types of temperature-moisture curves of drying, based on the presence and type of temperature sites on the drying thermograms. The introduction of such classification is related to the fact that the nature of the development of surface phenomena in various drying modes, associated with a corresponding change in the transfer mechanisms, strongly influences the form of kinetic curves [1].

The development of the introduced classification of types of kinetic curves is the selection of certain subtypes of kinetic curves that take into account the characteristic features of thermograms observed in the process of drying liquid dispersed products, inclined to structure formation. Accordingly, the introduction of 7 subtypes characterizing the change in the type of thermogram (in particular, the change in the angle of inclination, deformation and degeneration of the wet-bulb thermometer and boiling area) was proposed. Such a classification allows obtaining a modified type of temperature-moisture relationships for application in the drying kinetics calculations.

In order to use the modified temperature-moisture relationships in engineering calculations of the kinetics of convective drying of liquid dispersed products on substrates, it is necessary to separately
distinguish the dependence type for soft (drying agent temperature $T_a$ less than 100 °C) and hard drying modes (drying agent temperature $T_a$ more than 100 °C).

For soft drying modes, depending on the observed features of the thermogram, 5 types of wet-bulb thermometer area $T_{wb}$ are determined. Accordingly, temperature-moisture relationships for soft regimes can be divided into 3 sections. Sector I – warming up to the site $T_{wb}$. Sector II – area site $T_{wb}$. Sector III – post-drying – the end of the process.

The diagram of temperature-moisture relationships for soft regimes of drying liquid disperse products on substrates is shown in figure 1.

In some methods of calculating the kinetics of drying, the sector of heating the material is excluded. This is due to the fact that the size of the sector I is negligible. However, in the case of drying liquid dispersed products, it is impossible to neglect the warm-up area. This sector is recommended to be modeled by linear dependence on the base points.

As the analysis of experimental data shows, the form of the post-drying sector III is non-linear in nature and does not change for all the selected types of kinetic curves. It is recommended to mathematically approximate the curve in sector III using the dependence of the hyperbolic type or power-law dependence.

The sector in the area of the temperature of the wet-bulb thermometer $T_{wb}$ is the most important from the point of view of obtaining the correct result of the calculation of the kinetics of drying using temperature-moisture relationships. The most correct approximation of the temperature-moisture relationship in this area is linear with a certain angle of inclination.

As can be seen from figure 1, an increase in the speed of the process of surface structure formation leads to deformation of the $T_{wb}$ site. Accordingly, the sector II of temperature-moisture relationship increases the angle of inclination. Table 1 are presented the coordinates of the reference points of the temperature-moisture relationships for soft modes, depending on the type of thermogram.

Table 1. The reference points of temperature-moisture relationships for soft drying modes.

| Point | Temperature $T$ | Moisture $U$ |
|-------|-----------------|--------------|
| O     | $T_0$           | $U_0$        |
| A     | $T_{wb}$        | $U_0$        |
| B     | $T_{wb}$        | $U_1$        |
| C     | $T_{wb} - \Delta T_1$ | $U_1$        |
| D     | $T_{wb}$        | $U_{cr}$     |
| E     | $T_{wb} + \Delta T_2$ | $U_{cr}$     |
| F     | $T_a$           | $U_{eq}$     |
The calculation of the coordinates of the reference points is reduced to setting the drying mode, calculating the value of the wet-bulb thermometer temperature $T_{wb}$ and the critical moisture content $U_{cr}$. The values of $T_{wb}$ and $U_{cr}$ depend on the drying mode and material properties. The recommended dependences for $\Delta T_i$ are obtained by the authors as a result of processing experimental data.

In the classification of the types of kinetic curves for hard drying regimes, in addition to the 5 types of the wet-bulb thermometer $T_{wb}$ area, there are 3 types of the boiling point $T_{boil}$ area. The selected types of sites determine the type and characteristics of kinetic drying curves. Figure 2 presents the general scheme of temperature-moisture relationships for hard drying modes.

![Figure 2. Diagram of temperature-moisture relationships for hard drying modes. I - material heating zone, II - wet-bulb thermometer zone, III - the heating zone to the boiling point, IV - boiling zone, V - final zone.](image)

The temperature-moisture relationships for hard modes should be divided into 2 zones: $Z_1$ - area of the wet-bulb thermometer $T_{wb}$, $Z_2$ - area of the boiling point $T_{boil}$. In each zone there are specific sectors. For zone $Z_1$, this is sector I: warming up to the corresponding site of a wet-bulb thermometer $T_{wb}$, sector II: area of the site of a wet-bulb thermometer $T_{wb}$, sector III: warming up to the start of zone $Z_2$ - boiling point site $T_{boil}$. For zone $Z_2$, this is sector IV: area of the boiling site of $T_{boil}$, sector V: the end of the process is warming up to the final temperature $T_a$.

Table 2 are presented the coordinates of the reference points of the temperature-moisture relationships for hard modes, depending on the type of thermogram. The coordinates of the reference points are obtained from the calculations, similarly to the soft modes.

Table 2. The reference points of temperature-moisture relationships for hard drying modes

| Point | Temperature $T$ | Moisture $U$ |
|-------|-----------------|--------------|
| O     | $T_0$           | $U_0$        |
| A     | $T_{wb}$        | $U_0$        |
| B     | $T_{wb}$        | $U_1$        |
| C     | $T_{wb} + \Delta T_1$ | $U_1$   |
| D     | $T_{wb}$        | $U_{cr1}$    |
| E     | $T_{wb} + \Delta T_2$ | $U_{cr1}$ |
| G     | $T_{boil} + \Delta T_3$ | $U_0$   |
| H     | $T_{boil}$      | $U_0$        |
| K     | $T_{boil}$      | $U_{cr2}$    |
| L     | $T_{boil} + \Delta T_4$ | $U_{cr2}$ |
| F     | $T_a$           | $U_{eq}$     |
3. Results and discussion

The engineering method for calculating the kinetics of drying liquid dispersed products on substrates, using modified temperature-moisture relationships, is based on solving the basic equation of the drying kinetics [1, 3, 8]:

\[-M \cdot r \cdot \frac{dU}{d\tau} + M \cdot c \cdot \frac{dT}{d\tau} = Q,\]

where \(T\) – temperature, K; \(U\) – moisture content, kg kg\(^{-1}\); \(M\) – mass, kg; \(Q\) – heat flow, J s\(^{-1}\); \(c\) – heat capacity, J kg\(^{-1}\) K\(^{-1}\); \(r\) – latent heat, J kg\(^{-1}\); \(\tau\) – time, s.

Mathematically, the temperature-moisture relationship must be specified as a function \(T(U)\) of a certain type. The values of the coordinates of the reference points of the temperature-moisture relationship are determined on the basis of the drying mode and the properties of the dried material. According to the calculations, we obtain the functionally specified form of the temperature-moisture relationship \(T(U)\). For a given form of the function \(T(U)\), equation (1) is converted to the form:

\[\tau = \int_{U_s}^{U_f} f(U)dU,\]

where \(U_s\) and \(U_f\) are the boundaries of the integration of temperature-moisture relationship (determined by the coordinates of the reference points). Equation (2) can be solved analytically or numerically, depending on the form of the function \(T(U)\). We solve equation (2) to obtain numerical dependencies of the form \(T(\tau)\) and \(U(\tau)\), which determine the kinetics of drying.

As an example of the use of modified temperature-moisture relationships, we consider the results of the calculation of the kinetics of convective drying under soft and hard modes. The dried product is a liquid polymethylene-B-naphthalene sulfonates. Some results of experimental studies of the drying kinetics of this product are presented in [1, 10].

Calculation No. 1. Drying mode - convective, \(T_a = 80\) °C, \(w = 7\) m/s, substrate - fluoroplastic. \(T_0= 20\) °C, \(U_0= 8.6\) kg/kg, \(T_a= 80\) °C.

\(U_{eq}= 0.086\) kg/kg, \(T_{wb}=40\) °C, \(U_{cr}= 2.58\) kg/kg, \(\Delta T_1=1.3\) °C, \(\Delta T_2= 1.6\) °C (calculated).

The calculation of \(T_{wb}\), \(U_{eq}\), \(U_{cr}\) and external heat exchange conditions was carried out according to the method [1]. The reference points of temperature-moisture relationship is O-C-E-F. The results of the calculation of the drying kinetics are presented in figure 3, where \(\tau\) is the time; \(T_e\), \(U_e\) are the experimental temperature and moisture content; \(T_c\), \(U_c\) are the calculated temperature and moisture content. The maximum discrepancy between experimental and calculated temperature data was 2.3 °C (4 %). The maximum discrepancy between experimental and calculated data on moisture content was 0.5 kg hum/ kg abs dry (6 %). The discrepancy between the calculated and experimental total drying time was 133 s (9%).

![Figure 3](image-url)
Calculation No. 2. Drying mode - convective, $T_c = 120 \, ^\circ C$, $w = 2 \, m/s$, substrate - fluoroplastic. $T_e = 20 \, ^\circ C$, $U_e = 8.6 \, kg/kg$, $T_r = 120 \, ^\circ C$.

$U_{eq} = 0.086 \, kg/kg$, $T_{wb} = 45 \, ^\circ C$, $T_{boil} = 100 \, ^\circ C$, $U_{cr} = 2.6 \, kg/kg$, $\Delta T_1 = 2.5 \, ^\circ C$, $\Delta T_2 = 4.3 \, ^\circ C$ (calculated).

The calculation of $T_{wb}$, $T_{boil}$, $U_{eq}$, $U_{cr}$ and external heat exchange conditions was carried out according to the method [1].

The reference points of temperature-moisture relationship is O-C-E-H-K-L. The calculation results are presented in figure 4, where $\tau$ is the time; $T_e$, $U_e$ are the experimental temperature and moisture content; $T_c$, $U_c$ are the calculated temperature and moisture content. The maximum discrepancy between experimental and calculated data on temperature was 9 °C (7%). The maximum discrepancy between the experimental and calculated data on moisture content was 0.44 kg/kg (5%).

The discrepancy between the calculated and experimental total drying time was 203 s (10%).

![Graph](image)

**Figure 4.** The results of calculation and experiment for drying a liquid polymethylene-B-naphthalene sulfonates on hard drying mode.

4. Conclusion

The presented modified temperature-moisture relationships, in their physical sense, are some integral characteristics of the drying process, taking into account external and internal heat and mass transfer and the phenomena of structure formation. The separation of the drying process into zones (sectors) in which temperature-moisture relationships have a certain appearance allows us to take into account certain phenomena limiting the drying process. For example, for soft and hard regimes in the first period, the type of temperature-moisture relationships takes into account surface evaporation, complicated by the formation of a membrane, which reduces the intensity of evaporation compared to the free surface.

Accordingly, the degree of reflection of these phenomena (in particular, surface and bulk structure formation) on the temperature-moisture relationships corresponds to the identity of the calculated and experimental kinetic curves. The proposed method for calculating the kinetics of drying liquid dispersed products gives satisfactory results on the evaluation of the main parameters of the process, the duration of individual sections of kinetic curves and the entire process as a whole.

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