Nonequilibrium Quantum Fields in Cosmology: Comments on Selected Current Topics

B. L. Hu*

Department of Physics, University of Maryland, College Park, MD 20742, USA
Institute for Advanced Study, Princeton, New Jersey 08540, USA

September 24, 1994

(UMDPP-95-051,IASSNS-HEP-94/78)
Invited Talk given at the Second Journee Cosmologie, Observatorie de Paris, June 2-4, 1994

Abstract

Concepts of quantum open systems and ideas of correlation dynamics in nonequilibrium statistical mechanics, as well as methods of closed-time-path effective action and influence functional in quantum field theory can be usefully applied for the analysis of quantum statistical processes in gravitation and cosmology. We raise a few conceptual questions and suggest some new directions of research on selected current topics on the physics of the early universe, such as entropy generation in cosmological particle creation, quantum theory of galaxy formation, and phase transition in inflationary cosmology.

*Email: hu@umdhep.umd.edu, hu@sns.ias.edu
1 Introduction

The theme of this talk is to show how nonequilibrium statistical field theory is useful, and in some cases, essential to analyzing many problems in gravitation and cosmology, especially in black holes and the early universe, from the Planck to the Grand Unified epochs. There are a number of reviews I have written in the last two years on this general subject matter, to which the interested reader may refer. Each review discusses some general issues and then focuses on a specific topic. They are:

1. Waseda Conference [1]: on quantum statistical processes in the early universe, with emphasis on noise in quantum fields and gravitational entropy.
2. Belgium [2]: on the quantum origin of noise and fluctuations and the relevance of decoherence; presents an influence functional approach to the quantum theory of fluctuations and galaxy formation. For more recent work, see [3, 4].
3. Amsterdam [3]: critical dynamics in the early universe: ideas of scaling applied to the study of inflation; use of the quasilocal effective potential for the study of slow-roll inflation.
4. Los Alamos [3]: the nature and the roles of quantum noise in semiclassical gravity and cosmology; particle creation as parametric amplification of quantum noise; Hawking and Unruh radiation [7, 8] derived from a statistical field theory viewpoint. For details, see [9, 10].
5. Banff [11]: Nonequilibrium quantum field theory in gravitation; examples with interacting fields and semiclassical cosmology. For details, see [12, 13].

Let me mention some new developments in the last several months in these directions. I’ll comment on the nature and focuses of the problems without going into details, which can be found in the quoted papers.

2 Particle Creation: Quantum Noise and Thermal Radiation

Anglin [14], Hu and Matacz [9], Calzetta and Hu [3] and Hu and Sinha [12] recently showed how one can calculate particle creation in curved spacetimes via the influence functional method [15]. In particular, an expression for the influence functional in terms of the Bogolubov coefficients was derived. To place the old problem of particle creation in this new light enables one to address statistical mechanical issues related to these quantum processes, such as fluctuations and noise, decoherence, dissipation, backreaction, and entropy generation in a unified framework [16]. Currently, Anglin, Hu and Raval [17] are analyzing the two detector problem, trying to resolve the controversy raised by Grove, Raine and Sciama [18] on whether an inertial detector will detect radiation from an accelerated particle. They concentrate on the field correlation as modified by a moving particle. The influence functional approach affords one the freedom to examine the behavior of the particles or the field by coarse-graining the information carried by the other (particle or field). Since it includes the mutual effects of the particles on and by the field, the equations of motion of the subsystems (in the form of coupled Langevin equations) contain the backreaction self-consistently. New concepts are introduced, such as self- and mutual- noise and dissipation, to describe the
effective interaction of particles and field.

The conventional explanation of Unruh and Hawking radiation is that they are special consequences of the existence of event horizons in uniformly accelerating detectors or black holes. From physical considerations, cases with small departures from the uniform should also emit radiations, albeit with a non-Planckian spectrum. It is difficult to treat such cases from the geometric viewpoint in terms of event horizons. But it should be natural from the statistical field theory viewpoint, as I argued earlier [19, 20] (my emphasis is thus closer in spirit to the views of Bekenstein [22] and Unruh [23]). The influence functional formalism can treat any arbitrary state of motion. There the radiation is seen to originate from vacuum fluctuations excited by the motion of the particle, the limiting case of uniform acceleration yielding an exact thermal radiance. In statistical mechanics description, varying kinematical state of the detector can alter in varying degrees the appearance of vacuum fluctuations in Minkowsky space as a mixture of quantum and thermal noise in a different space (only in the exact uniform acceleration case would it be describable by the Rindler space). Currently Raval, Koks and Matacz [24] are working out some examples to illustrate these ideas I suggested earlier [19, 20, 21]. This should lead to a unified treatment and understanding of thermal radiance from cosmological and black hole spacetimes.

3 Entropy Generation in Cosmological Particle Creation

In 1984, in discussing the conceptual problems of entropy generation from particle creation I pointed out [25] that the usual simplistic identification of entropy with the number of created particles is valid only in the hydrodynamic (strongly interacting) regime. Theoretically, for a free field, particle pairs created from the vacuum will remain in a pure state and there should be no entropy generation. In 1986 Pavon and I [26] suggested that an intrinsic entropy of a (free) quantum field can be measured by the particle number (in a Fock space representation) or by the variance (in the coherent state representation). It was pointed out that the entropy of a (free) quantum field can change only if some information of the field is lost or excluded from consideration, either by choosing some special initial state or by introducing some measure of coarse-graining. For example, the predicted monotonic increase in the spontaneous creation of bosons is a consequence of the random phase initial condition implicitly assumed in most discussions of vacuum particle creation. For a recent discussion of spontaneous and stimulated cosmological particle creation in terms of squeezed states, see e.g., [31].

3.1 The entropy of quantum fields

Following these early discussions of the theoretical meaning of the entropy of quantum fields, a recent upsurge of interest on this issue was stimulated by the work of Brandenberger, Parker [27] first discussed the difference of spontaneous and stimulated creation of bosons versus fermions. Entropy generation in particle creation with interactions was discussed by Hu and Kandrup [29], the relation of random phase and particle creation was further elaborated by Kandrup [30].
Mukhanov and Prokopec (BMP) [32], Gasperini and Giovannini (GG) [33] and others on the entropy content of primordial graviton and other particles created. The language of squeezed states for the description of cosmological particle creation was introduced by Grishchuk and Sidorov [34]. Though the physics is the same [35, 31] as was originally described by Parker [27] and Zel’dovich [28] the language makes it easier to compare with similar problems in quantum optics, which has delved into many similar theoretical and practical issues. BMP suggested a coarse-graining of the field by integrating out the rotation angles, while GG used the dispersion in the superfluctuant operator. In the large squeezing limit (late times) they all get the same answer $2r$ for the entropy of spontaneously created particles, where $r$ is the squeezing parameter. Here, as in the original work of Hu and Pavon, the choice of representation of the state of the quantum field and the coarse-graining in the field are stipulated, not derived. The main point there is to show how the entropy of particle creation depends on the choice of a specific initial state and/or a particular way of coarse-graining. How natural or physical this particular choice is in a realistic cosmological context is not addressed. This includes conditions when, for example, the quantum field is at a finite temperature or is in disequilibrium, interacting with other fields, or that its vacuum state is dictated by the choice of boundary conditions in the earlier quantum cosmology regime (e.g., the Hartle-Hawking boundary condition leading to the Bunch-Davies vacuum in de Sitter spacetime), etc. To answer these questions, one needs to work with a larger theoretical framework, that of statistical field theory of quantum open systems, as I will now explain.

In the quantum Brownian motion (QBM) paradigm depiction of quantum field theory in curved spacetime studied in the series of papers by Hu, Paz and Zhang [36, 37], and Hu and Matacz [9], the system represented by the Brownian particle can act as a detector (as in the influence functional derivation of Unruh and Hawking radiation), a particular mode of a quantum field (such as the homogeneous inflaton field), or the scale factor of the background spacetime (as in minisuperspace quantum cosmology [38]), while the bath could be a set of coupled oscillators, a quantum field, or just the high frequency sector of the field, as in stochastic inflation [39]. The statistical properties of the system are depicted by the reduced density matrix formed by integrating out the details of the bath. Here one can use the reduced density matrix or the associated Wigner function to calculate the statistical averages of physical observables of the system, and, in particular, the uncertainty or the entropy functions. The time-dependence of the uncertainty function in a system interacting with a heat bath illustrates the relative importance of thermal and vacuum fluctuations and their roles in bringing about the decoherence of the system and the emergence of classical behavior [40, 41]. The entropy function for such open systems was used [42, 41] as a measure of how close the description in terms of different quantum states is to the classical dynamics. For example, in a quantum Brownian model Zurek, Habib and Paz [42] showed that the coherent state yields the minimal entropy, suggesting the special relation it bears with the classical depiction.

### 3.2 Entropy associated with the states of the system

Here, the entropy function constructed from the reduced density matrix (or the Wigner function) of a particular quantum state of the system measures the information loss of the system in that state to the environment (or, the ‘instability’ characterized by the loss
of predictive power relative to the classical description \[12\]). One can study the entropy increase for a specific state, or compare the entropy at each time for a variety of states characterized by the squeeze parameter. The uncertainty function plays a similar role \[10, 11\]. It measures the effects of vacuum and thermal fluctuations in the environment (at zero and finite temperature) on the observables of the system. The increase of their variances due to these fluctuations gives rise to the uncertainty and entropy increase.

Now what is the relation of this latter group of work to that of the former and how the issues addressed here can help to answer the questions I raised above?

The differences in appearance are obvious: The entropy of \[26, 32, 33\] and others refers to that of the field, and is obtained by coarse-graining some information of the field itself, such as making a random phase approximation, adopting the number basis, or integrating over the rotation angles. The entropy of the open system in \[40, 41, 42\] refers to that of the system and is obtained by coarse-graining the environment. Why is it that for certain generic models in some common limits (late time, high squeeze), both groups of work obtain the same result? For example, Matacz \[43\] considered a squeezed vacuum of a time-dependent harmonic oscillator system, and motivated by the special role of coherent states, modelled the effect of the environment by decohering the squeezed vacuum in the coherent state representation. He calculated the entropy function from this reduced density matrix, and found that it approaches $2\pi$ in the high squeezing limit. This calculation, though performed in the QBM open system framework, is in spirit closer to the former work of field entropy in that the bath only serves the token role for decohering the system and because the time-dependent oscillator system admits particle creation in the normal modes of the field. In a way, approaching this problem from the open system viewpoint improves on the earlier work in that one can perform different coarse-grainings of the system by changing its coupling to the environment. What is more important conceptually, it clarifies the relation between quantum and classical descriptions – it is through decoherence that the field quanta behave like classical particles (loosely speaking). Thus it is not too surprising that the two groups of inquiries lead to similar answers because the same questions are asked about the oscillator (with time-dependent frequency) as about the field.

Furthermore, an even closer relation exists between the entropy of a system and its environment theoretically. It can be shown that any two subsystems of a closed system in a pure state will have equal entropy. Indeed this is the reason why the derivation of black hole entropy can be obtained equivalently by computing the entropy of the radiation (e.g., \[14\]) emitted by the black hole, or by counting the internal states (if one knows how!) of the black hole (e.g., \[15\]). Here, if the open system and the environment together constitute a closed system and they interact coherently, one expects that they would have the same entropy \[16, 17\]. Physically one can view what happens to the particle as a probe into the state of the field. The application of open-system concepts to black hole entropy is a very fruitful avenue. (see the recent review of Bekenstein \[18\])

\[2\] When one says he or she wants to calculate the entropy content in classical matter or radiation observed today due to primordial particle creation, one needs to show the transition from a quantum field to a classical matter description. (This is similar to the questions on galaxy formation raised below, specifically, on the relation between quantum and classical fluctuations and how to get classical stochastic equations from quantum fields.)
3.3 Directions

Besides continuing the theoretical inquiries on uncertainty and entropy for open systems now pursued in earnest by Halliwell and coworkers \[49\], there are several directions one can advance on this issue of entropy associated with cosmological particle production. One currently undertaken by Koks and Matacz \[50\] is to analyze the time-dependent harmonic oscillator interacting with a bath of parametric oscillators. One expects to find the entropy function to depend nonlocally on the entire history of the squeezing parameter. This can be seen from the fact that the rate of particle creation varies in time and its effect is history dependent. Existing methods of calculating the entropy generation give results which only depend on the squeezing parameter at the time of coarse-graining. There, both the choice of coarse-graining and the time of its application are rather ad hoc and they affect the results obtained. The new method we are proposing has a more rigorous theoretical basis and should provide more accurate estimates of entropy generation from cosmological particle creation.

The other direction is to consider a bath of parametric oscillators mimicking the field which produces particles, and let the system be a particle detector probing the field. This picture shifts the focus back on the field entropy again. In the recent work of Calzetta, Hu and Matacz \[21, 9, 6\] a physical link between quantum noise and particle creation is established, and an expression for the influence functional in terms of the Bogolubov coefficients is derived. This formalism not only allows one to get everything familiar, such as particle creation and interaction depicted in the established theories (quantum fields in curved spacetimes) but also new statistical mechanics information from first principles, such as noise, as measured by the fluctuations in particle number, and entropy, as constructed from the reduced density matrix. One can also use the open system method to discuss entropy generation from particle creation in the reheating phase. I will comment on this at the end of my talk.

4 Galaxy Formation from Quantum Fluctuations

In \[2\], Hu, Paz and Zhang pointed out some basic deficiencies in the existing theoretical framework of structure formation from quantum fluctuations. One concerns the origin of noise, and the other concerns the use of a classical stochastic equation for the description of the dynamics of quantum fields. In that paper, they showed how to derive the characteristics of quantum noise and a classical stochastic equation from quantum field theory. They found that colored multiplicative noise arise naturally from fluctuations of interacting quantum fields, and that the justification of using a classical stochastic equation for the long wavelength modes depends on how successfully they decohere in the face of these noises. The recent work of Calzetta and Hu \[3\] raises yet a third important issue, questioning the validity of the conventional use of the classical correlation functions for quantum fields in these problems. Indeed because of the many ad hoc assumptions made in the conventional theory of structure formation involving quantum fluctuations, it is useful to reexamine the soundness of the whole theoretical foundation. Let me raise a few general questions in this regard.
4.1 Problems with the Existing Framework

In the classical gravitational instability theory of galaxy formation based on Lifshitz’s 1946 work, the wave equation for the linear perturbations of the background spacetime is driven by the first variations of the classical energy momentum tensor. The scalar part of the perturbation is related to the density contrast. It is often assumed that the initial source is a white noise. In the quantum theory of galaxy formation a quantum scalar field (inflaton) mediates inflation, and its quantum fluctuations seed the galaxies. Thus a) the vacuum energy density of the scalar field drives the background spacetime into inflationary expansion, while b) the fluctuations of the inflaton field produce the density contrasts. In stochastic inflation the long wavelength modes are driven by the short wavelength quantum fluctuations. The folklore is that the former behaves classically and the latter gives rise to a white noise. (We have remarked that neither of these assumptions have been justified satisfactorily \[21, 2\].)

Note first that level a) above invokes the semiclassical Einstein equation, where the source is given by the vacuum expectation value of the energy momentum tensor. In this case, it is the vacuum energy density of the symmetric state before the system undergoes phase transition. This is fine. At level b) there is confusion. Density contrast obtained from the first variation of the 00 component of the energy momentum tensor is a classical object, but fluctuations of the scalar field is quantum in nature. In relating the two, one has presumably made a tacit assumption, i.e., that it is the average value of the quantum fluctuations, or something which transforms them into classical fluctuations, that drives the scalar part of the gravitational perturbation related to the density contrast. What exactly turns the quantum fluctuations into classical fluctuations is never made clear. Indeed, most authors seem not to be bothered by this and just assume that the quantum fluctuations behave like a classical stochastic source. We feel rather uncomfortable about this state of affairs. In fact, in our investigations of noise and fluctuations in semiclassical gravity, Calzetta and I \[3\] pointed out that this is in general not the case. Luckily there is a way to make these concepts and procedures sound and rigorous via statistical field theory.

4.2 New Theoretical Framework and Potential Problems

The recent work of Calzetta, Hu, Matacz and Sinha \[3, 9, 10, 12\] showed that the semiclassical Einstein equation is only a mean-field theory. There is actually a stochastic source term arising from the quantum fluctuations of the matter field. It registers on the influence functional as the noise kernel, which balances the dissipation kernel responsible for the backreaction in the effective dynamics of the background geometry. The semiclassical equation which one customarily uses is the result of averaging over the noise distribution. It is by means of this stochastic source, and within this context, that the theory of galaxy formation based on quantum fluctuations of matter fields can be made precise. However, this new framework also raises a set of new issues. For example, it was shown that quantum fluctuations of the matter field in a dynamical spacetime are related to particle creation. When the excitation is below the particle creation threshold there is no stochastic source, but when it is above the threshold the stochastic source is related to the difference in the amount of particle creation in neighboring histories. Unlike classical fluctuations they don’t
come at all energy scales and span the full spectrum. Worse yet, for conformal fields in con-
formally static spacetimes, like photons in a Robertson-Walker universe, there is no particle
creation and there should not be any stochastic source due to quantum fluctuations. (Our
preliminary analysis indicates that vacuum polarization terms like the trace anomaly do not
contribute to the stochastic source.) For nonconformal fields, there is particle creation. But
during the slow-roll inflationary epoch particle creation from the slowly changing inflaton
field is small and concentrates in the low frequency modes. (By contrast, in the reheating
regime, the rapidly changing field induces abundant particle creation and entropy genera-
tion, but the spectrum will no longer be scale-invariant and there are other problems.) The
low production and weak fluctuations may not be so bad in comparison with the observed
inhomogeneity, but the existing theoretical basis could be at peril if this were true (we are
cautiously guarded from making such a claim yet). Further investigations on these issues
are in place.

4.3 Modelling and Analysis

Going from the basic theoretical issues to modelling, recall that in [2] a model of two interact-
ing fields was used for illustration. One can use more realistic models for a thermal bath (as
is in the follow-up work of these authors [51] in attempting to bridge their stochastic field the-
ory with the established thermal field theory), or try to do the high-low frequency mode split
(as originally intended in the stochastic inflation program [39]) to derive a set of stochastic
equations depicting more realistic conditions. As for analysis, the functional Langevin equa-
tion deduced by us can be simplified to an ordinary equation with some sampling functions
dictated by physical conditions. One can then be compare it with the stochastic equations
used for structure formation studies, and perform numerical analysis.

This program of study while filling a gap in the quantum theory of galaxy formation
also can lead to new elements of discovery, such as the effect of colored noise, non-Gaussian
galaxy distributions, and anomalous correlations. It is interesting to compare the predictions
from different theoretical models with the limits drawn from the COBE data.

5 Phase Transitions in the Early Universe

There are at least three aspects in this subject: a) the field theory aspect, which involves
an infrared analysis of the effective action; b) the spacetime aspect, where the effects of
spacetime curvature and topology enter; and c) the statistical mechanical aspect, dealing
with nonequilibrium quantum fields. (Only when they are in equilibrium can a finite tem-
perature field theory be useful). The field-theory aspect is at the heart of the matter. Many
techniques have been developed for treating infrared behavior, none could claim perfection
(or even accuracy). The approach of O’Connor and Stephens who have made significant
advances in the treatment of cross-over behaviors, seems to me to be the most hopeful [52].
The spacetime aspect was studied with techniques developed in quantum field theory in
curved spacetime. The first stage of work between 1980-86 was reviewed in my talk at the
Fourth Marcel Grossmann Meeting [53]. In the work of Hu and O’Connor, and O’Connor
et al [54, 55] the Coleman-Jackiw-Tomboulis method [56] was used to treat the infrared be-
behavior of quantum fields under rather general conditions. Effects of curvature and topology on phase transitions can be understood in terms of finite size effect. Their examples for symmetry breaking in product spacetimes contain many interesting subcases, such as the imaginary-time finite temperature theory, the Kaluza-Klein theory and Robertson-Walker universe. The idea proposed by Hu and Zhang to understand inflation as scaling [21, 5] can provide some deeper insight into the physics of inflation and black hole collapse. In the statistical mechanics aspect, the 1988 work of Calzetta and Hu [38] established a theoretical foundation for studying non-equilibrium quantum fields. The methods they proposed: the closed-time-path (CTP, or Schwinger-Keldysh) functional formalism [57], the n-particle irreducible effective action, and the Wigner function techniques, have since been used as essential ingredients for many statistical and kinetic field theory investigations into non-equilibrium quantum processes, including particle creation, heavy-ion collision, and quantum transport [59]. A quantum field theory of spinodal decomposition was first studied by Calzetta [60] using the CTP formalism. Recently Gleiser, Boyanovsky and others [61] have applied these concepts and techniques to study the nature of the electroweak phase transition (e.g., weakly first order), spinodal decomposition, tunneling and domain wall formation with interesting results. I refer to their talks in this conference. Another recent development is the use of influence functional formalism to describe noise and fluctuations. The recent work of Calzetta, Hu, Matacz, Paz, Sinha and Zhang mentioned above in applying the quantum Brownian model to field theory provides a theoretical framework to study the effects of quantum fluctuations and thermal noise on phase transition. I will find another occasion to discuss this aspect. Here, instead, I would like to describe some thoughts generated in my current work with Calzetta [63] on correlations and fluctuations in quantum field theory.

5.1 Correlation and Noise in Kinetic Theory

The relevant domain in the analysis of a system’s approach to a critical point is the infrared behavior of its fluctuations, and the important object which carries this information is the correlation function. A critical point is reached when the range of correlation becomes very large (approaching infinity in bulk samples, while limited in finite size systems). This was the starting point for our earlier investigations into the symmetry behavior of quantum fields in curved spacetimes (the work by Hu, O'Connor, Shen is summarized in [53]). In our current program of investigation [63], the objective is to extract the statistical information of interacting quantum fields. The infrared behavior of quantum fluctuations can be applied to phase transition studies, but the formalism itself is useful for a broader range of problems on relativistic kinetic processes.

The statistical mechanical paradigm used in our formalism is that of Boltzmann’s kinetic theory in the BBGKY hierarchy form. The key object is the n-point correlation function. This is different from the Brownian motion paradigm in the open system conceptual framework, which has recently been successfully applied to the discussion of decoherence, noise and dissipation problems in foundational aspects of quantum mechanics and semiclassical gravity. The philosophical difference between these two major paradigms of non-equilibrium statistical mechanics and their respective appropriateness in the application to different physical problems have been discussed in the Introduction of [64]. There we proposed the correlation history as a more natural way of coarse-graining in the decoherent history formulation of
quantum mechanics [13]. One of the lessons we learned in adopting the open system way of thinking is how to treat noise and fluctuations rigorously from first principles. In our present work we want to incorporate noise and fluctuations into dissipation, all described in terms of the hierarchy of correlation functions. Thus, concepts from both paradigms are utilized.

Many new results come from this line of inquiry. Let me just mention two, one concerns effective field theory, and the other concerns stochastic mechanics. The former addresses renormalization group theory and gauge hierarchy problems, and the latter addresses the source and nature of fluctuations in quantum fields, both are relevant to phase transition problems.

How does one define noise in the correlation framework? In an open system one coarsely-grains the environment and obtains noise. Here there is no clear separation of the system and the environment, as all particles in the kinetic theory are treated on equal footing. A separation is possible by the correlation order. If one keeps all orders one has complete information, just as the complete BBGKY hierarchy is equivalent to the full set of Newton’s equations for the n particles. In actual measurements, one does not have the ability to keep track of all orders, and usually settles with the one-particle distribution function and perhaps the two- or three-particle correlation functions. The description of the total system is therefore incomplete by choice. Recall that the BBGKY hierarchy has the dynamics of an n-th order correlation function driven by a source of n+1-th order functions, ad infinitum. Only when one assumes that the n+1-th order correlation function factorizes (into products of lower order correlation functions) will there be loss of information and the appearance of dissipation in the effective dynamics of the nth order correlation function. Physically, this is the molecular chaos assumption which Boltzmann used to explain dissipation in Hamiltonian dynamics and to postulate the H-theorem. One can define noise of the nth correlation order as the source term (formed by the n+1-th order correlation functions) in the nth order BBGKY equation. Usually this term is ignored when the hierarchy is simply truncated—like the collisionless Boltzmann equation. But the correlation noise has a place to itself, both in practical and theoretical terms. Practically, this is the stochastic source for the lower order equations, and theoretically, it is needed in the balance of information, or loss thereof. These two points are easier to see in the open system framework, the former via the Langevin equation and the latter via the fluctuation-dissipation relation. But they are also there in the kinetic theory framework.

5.2 Fluctuations and Correlation of Quantum Fields

In field theory language, the full set of nth order correlation functions enters into the n-particle irreducible (nPI) effective action. Complete information for the system is contained in the $\infty PI$, or what we call the master effective action. At any order determined by the level of accuracy of the experiment, the effective equation for the correlation function is driven by a noise term which contains the information of all the higher order correlations. Truncation at any level by factorization discards this information and introduces dissipation in the effective dynamics of that order. The ordinary effective action we encounter in field theory is a functional of the mean field. There is no room for the noise terms associated with quantum fluctuations. For phase transition studies one usually includes the 2-particle correlation function (there is of course no guarantee that this is sufficient to depict the
infrared behavior—usually they don’t—witness the frustration in the higher order calculations of the electroweak phase transition.) The relation between the mean field and the correlation function(s) was not clearly treated in the old way. But cast in this new statistical field theoretical framework, one can actually calculate the stochastic source term driving the correlation functions at each level, and verify their consistency by the fluctuation-dissipation relations at each level. In this light one can also understand why effective theories can be intrinsically dissipative in nature [20, 11, 13]. These stochastic equations can provide a new basis for the analysis of nucleation and spinodal decomposition processes, as the dynamics of fluctuations and correlations can be studied on the same footing. The implications of this new statistical field theory on renormalization group theory and phase transitions are under investigation [13, 63, 66].

5.3 Dynamics of the Inflaton Field

The dynamics of inflationary cosmology depends sensitively on how the order parameter field (inflaton) evolves, from the symmetric to the broken symmetry phase, i.e., to enter a vacuum-energy dominated phase (e.g., needs a metastable potential), to sustain the inflation (e.g., slow-roll) and to gracefully exit (reheating). The usual treatment is via a finite-temperature effective potential which a priori assumes an equilibrium condition. But the time-dependence of the inflaton in a dynamic spacetime really calls for a fully non-equilibrium treatment. As we pointed out before, this is one of the three major factors lacking in the usual treatment of phase transitions in the early universe [53] (i.e., field theory infrared behavior, geometric and topological effects, and statistical mechanical effects). With a stochastic field theory set up we can now improve upon this aspect of the problem. For example, in the study of inflaton dynamics by Guth and Pi [67] using a quantum mechanical inverted harmonic oscillator model, the transition to the classical regime was defined via a simple uncertainty principle argument. One can actually calculate the transition time by comparing the relative magnitude of the quantum and thermal fluctuations based on our recent result on decoherence [36] and the uncertainty principle [40]. This was done recently by Raval [68]. Cornwall and Bruinsma [69] first suggested using the influence functional method for this problem but oversimplified the problem in order to cater to the Calderia-Leggett model. One can provide a more realistic and in-depth depiction of the inflaton dynamics with the statistical field theory results [51].

5.4 Reheating

The standard picture of reheating is given qualitatively by Dolgov and Linde [70], Abbott, Farhi and Wise [71]. There is a recent revival of interest in the reheating phase of inflation stimulated by the work of Kofman, Linde and Starobinsky [72] and Shtanov, Traschen and Brandenberger [73]. I just want to call the attention to the fact that there is a rigorous way of calculating particle creation and interaction in the phase where the inflaton field undergoes rapid oscillation and damping. The method is the Schwinger-Keldysh (or closed-time-path) [57] effective action formalism, which has been applied to cosmological particle creation and backreaction problems before [74, 75]. Specifically for the reheating problem, Paz [76] and Stylianoupolis [77] have used this method to study particle creation and decay in field the-
ory models with quadratic and Yukawa type couplings. Their calculation also included the backreaction of created particles which is encapsuled in the viscosity function in the effective equation of motion for the inflaton field. Making use of the intimate relation of the CTP effective action with the influence functional formalisms [21, 12] one can also deduce the noise kernel associated with particle creation in the reheating phase and a fluctuation dissipation relation for such processes. This is a far more sophisticated and rigorous approach to this problem than the time-dependent perturbation theory used in the older works. With it one can address the issues raised in [72, 73] and discern the physical scenarios based on quantitative rather than descriptive results. The recent work of Boyanovsky, de Vega, Holman, Lee and Singh [78] seems to me to be the most advanced in the analysis of this problem.

Acknowledgement I thank Esteban Calzetta and Andrew Matacz for discussions on various topics touched upon in this talk. The kind hospitality of Professors de Vega and Sanchez made my short stay in Paris a very pleasant one. This work is supported in part by the National Science Foundation under grant PHYS91-19726, the General Research Board of the Graduate School of the University of Maryland and the Dyson Visiting Professor Fund at the Institute for Advanced Study, Princeton.

References

[1] B. L. Hu, “Quantum Statistical Processes in the Early Universe” in Quantum Physics and the Universe, Proc. Waseda Conference, Aug. 1992 ed. M. Namiki et al (Pergamon Press, Tokyo, 1993). Vistas in Astronomy 37, 391 (1993)

[2] B. L. Hu, J. P. Paz and Y. Zhang “Quantum Origin of Noise and Fluctuations in Cosmology”, in The Origin of Structure in the Universe, edited by E. Gunzig and P. Nardone (Kluwer, Dordrecht, 1993), p. 227.

[3] E. Calzetta and B. L. Hu,”Noise and Fluctuations in Semiclassical Gravity”, Phys. Rev. D49, 6636 (1994)

[4] E. Calzetta, B. L. Hu and A. Matacz, in preparation

[5] B. L. Hu, Class. Quan. Grav. 10, S93 (1993)

[6] B. L. Hu and A. Matacz, “Quantum Noise in Gravitation and Cosmology” Invited Talk at the Workshop on Fluctuations and Order, ed. M. Millonas (Springer Verlag, Berlin, 1994). Univ. Maryland preprint pp94-44 (1993) astro-ph/9312012

[7] S. W. Hawking, Comm. Math. Phys. 43, 199 (1975)

[8] W. G. Unruh, Phys. Rev. D14, 870 (1976)

[9] B. L. Hu and A. Matacz, “Quantum Brownian Motion in a Bath of Parametric Oscillators”, Phys. Rev. D49, 6612 (1994)
[10] B. L. Hu and A. Matacz, “Backreaction in Semiclassical Cosmology: the Einstein-Langevin Equation”, Univ. Maryland preprint 94-31 (1993). gr-qc/9403043

[11] B. L. Hu, “Quantum Statistical Fields in Gravitation and Cosmology” in Proc. Third International Workshop on Thermal Field Theory and Applications, eds. R. Kobes and G. Kunstatter (World Scientific, Singapore, 1994) gr-qc/9403061

[12] B. L. Hu and S. Sinha, “Fluctuation-Dissipation Relation in Cosmology”, Univ. Maryland preprint pp93-164 (1993) gr-qc/9403054

[13] E. Calzetta, B. L. Hu and Yuhong Zhang, “Dissipative Nature of Effective Field Theories” (in preparation)

[14] J. R. Anglin, Phys. Rev. D47, 4525 (1994)

[15] R. Feynman and F. Vernon, Ann. Phys. (NY) 24, 118 (1963). R. Feynman and A. Hibbs, Quantum Mechanics and Path Integrals, (McGraw - Hill, New York, 1965). A. O. Caldeira and A. J. Leggett, Physica 121A, 587 (1983); Ann. Phys. (NY) 149, 374 (1983). H. Grabert, P. Schramm and G. L. Ingold, Phys. Rep. 168, 115 (1988). B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D45, 2843 (1992); D47, 1576 (1993)

[16] B. L. Hu, in Thermal Field Theories, eds. H. Ezawa et al (North-Holland, Amsterdam, 1991) p.223

[17] A. Raval, B. L. Hu and J. R. Anglin , in preparation

[18] P. G. Grove, Class. Quan. Grav. 3, 801 (1986) D. J. Raine, D. W. Sciama, and P. G. Grove, Proc. Roy. Soc. Lond. A435, 205 (1991)

[19] B. L. Hu, in Proc. CAP-NSERC 1987 Summer Institute in Theoretical Physics eds G. Kunstatter et al, (World Scientific, Singapore, 1988) Vol 2, p. 252-276

[20] B. L. Hu, Physica A158, 399 (1989).

[21] B. L. Hu and Y. Zhang, “Coarse-Graining, Scaling, and Inflation” Univ. Maryland Preprint 90-186 (1990); B. L. Hu, in Relativity and Gravitation: Classical and Quantum Proc. SILARG VII, Cocoyoc, Mexico 1990. eds. J. C. D’ Olivo et al (World Scientific, Singapore 1991).

[22] J. D. Bekenstein, Phys. Rev. D12, 3077 (1975).

[23] W. G. Unruh, Phys. Rev. Lett. 46, 1351 (1981); “Dumb Holes”, gr-qc/9409008 (1994)

[24] B. L. Hu, A. Raval, D. Koks, A. Matacz, in preparation

[25] B. L. Hu, in Cosmology of the Early Universe eds. L. Z. Fang and R. Ruffini (World Scientific, Singapore, 1984)

[26] B. L. Hu and D. Pavon, Phys. Lett. B180, 329 (1986)
[27] L. Parker, Ph. D. Thesis, Harvard University, 1966; Phys. Rev. Lett. 21, 562 (1968); Phys. Rev. **183**, 1057 (1969); **D3**, 346 (1971)

[28] Ya. B. Zel’dovich, Pis’ma Zh. Eksp. Teor. Fiz, **12**, 443 (1970) [JETP Lett. **12**, 307(1970)];

[29] B. L. Hu and H. E. Kandrup, Phys. Rev. **D35**, 1776 (1987)

[30] H. E. Kandrup, Phys. Rev. **D37**, 3505 (1988)

[31] B. L. Hu, G. W. Kang and A. Matacz, Int. J. Mod. Phys. A9, 991 (1994)

[32] R. H. Brandenberger, V. Mukhanov and T. Prokopec, Phys. Rev. Lett. 69, 3606 (1992); Phys. Rev. D48, 2443 (1993)

[33] M. Gasperini and M. Giovanni, Phys. Lett. 301B, 334 (1993); M. Gasperini and M. Giovanni and Veneziano, Phys. Rev. D48, R439 (1993).

[34] L. Grishchuk and Y. V. Sidorov, Phys. Rev. D42, 3414 (1990)

[35] A. Albrecht, P. Ferreira, M. Joyce and T. Prokopec, Imperial College preprint TP/92-93/21, astro-ph/9303001

[36] B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. **D45**, 2843 (1992)

[37] B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. **D47**, 1576 (1993)

[38] Sukanya Sinha and B. L. Hu, Phys. Rev. D44, 1028 (1991)

[39] A. A. Starobinsky, in *Field Theory, Quantum Gravity and Strings*, ed. H. J. de Vega and N. Sanchez (Springer, Berlin 1986); J. M. Bardeen and G. J. Bublik, Class. Quan. Grav. **4**, 473 (1987).

[40] B. L. Hu and Yuhong Zhang, Mod. Phys. Lett. A8, 3575 (1993); Univ. Maryland preprint 93-162 (1993), gr-qc/9301034 B. L. Hu and Yuhong Zhang, in Proc. Third International Workshop on Quantum Nonintegrability, Drexel University, Philadelphia, May 1992, eds J. M. Yuan, D. H. Feng and G. M. Zaslavsky (Gordon and Breach, Langhorne, 1993).

[41] A. Anderson and J. J. Halliwell, Phys. Rev. **D48**, 2753 (1993).

[42] W. H. Zurek, S. Habib and J. P. Paz, Phys. Rev. Lett. 70, 1187 (1993)

[43] A. Matacz, Phys. Rev. D49, 788 (1994)

[44] V. Frolov and I. D. Novikov, Phys. Rev. D48, 4545 (1993)

[45] L. Bombelli, R. K. Koul, J. Lee and R. D. Sorkin, Phys. Rev. D34, 373 (1986)

[46] D. N. Page, Phys. Rev. Lett. 71, 1291 (1993)
[47] M. Srednicki, Phys. Rev. Lett. 71, 666 (1993)

[48] J. D. Bekenstein, Review Talk at MG7 (1994) gr-qc/9409015

[49] J. J. Halliwell, Phys. Rev. D48, 2739, 4785 (1993); C. Anastopoulos and J. J. Halliwell, Imperial College Preprint IC 93-94/53 (1994) gr-qc/9407039

[50] D. Koks, A. Matacz, B. L. Hu and A. Raval, in preparation

[51] B. L. Hu, J. P. Paz and Y. Zhang, “Stochastic Dynamics of Interacting Quantum Fields” in preparation (1994)

[52] Denjoe O’Connor and C. R. Stephens, “Environment-Friendly Renormalization Group Theory” Int. J. Mod. Phys. (1994)

[53] B. L. Hu, “Phase Transition in the Early Universe: Geometric Effects” in Recent Developments in General Relativity: Proc. 4th Marcel Grossmann Meeting, Rome, 1985 ed. R. Ruffini (North Holland, Amsterdam, 1986)

[54] B. L. Hu and D. J. O’Connor, Phys. Rev. D36, 1701 (1987).

[55] D. J. O’Connor, C. R. Stephens and B. L. Hu, Ann. Phys. (N.Y.) 190, 310 (1990).

[56] J. M. Cornwall, R. Jackiw and E. Tomboulis, Phys. Rev. D10, 2428 (1974)

[57] J. Schwinger, J. Math. Phys. 2 (1961) 407; P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. 4, 1 (1963), 4, 12 (1963). L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) [Engl. trans. Sov. Phys. JETP 20, 1018 (1965)]. G. Zhou, Z. Su, B. Hao and L. Yu, Phys. Rep. 118, 1 (1985); Z. Su, L. Y. Chen, X. Yu and K. Chou, Phys. Rev. B37, 9810 (1988). B. S. DeWitt, in Quantum Concepts in Space and Time, ed. R. Penrose and C. J. Isham (Claredon Press, Oxford, 1986); R. D. Jordan, Phys. Rev. D33, 44 (1986). E. Calzetta and B. L. Hu, Phys. Rev. D35, 495 (1987); D37, 2878 (1988); Phys. Rev. D40, 656 (1989).

[58] E. Calzetta and B. L. Hu, Phys. Rev. D37, 2878 (1988).

[59] See, e.g., F. Cooper, et al, hep-ph/9404357, Los Alamos Preprint LA-UR-94 783 (1994)

[60] E. Calzetta, Ann. Phys. (N.Y.) 190, 32 (1989)

[61] D. Boyanovsky and H. J. de Vega, Phys. Rev. D47, 2343 (1993); D. Boyanovsky, D. S. Lee and A. Singh, Phys. Rev. D48, 800 (1993); D. Boyanovsky, H. J. de Vega, and R. Holman, Univ. Pittsburg preprint 93-6; M. Gleiser, G. C. Marques and R. O. Ramos, Phys. Rev. D48, 1571 (1993); M. Gleiser and R. O. Ramos, Univ. Dartmouth preprint DART-HEP-93/06 (1993)

[62] W. Horsthemke and R. Lefever, Noise Induced Transitions (Springer, Berlin 1984)

[63] E. Calzetta and B. L. Hu, ”Correlation, Noise and Fluctuations in Interacting Quantum Fields” (1994)
[64] E. Calzetta and B. L. Hu, “Decoherence of Correlation Histories” in Directions in General Relativity, Vol II: Brill Festschrift, eds B. L. Hu and T. A. Jacobson (Cambridge University Press, Cambridge, 1993)

[65] R. Omnés, Rev. Mod. Phys. 64, 339 (1992); J. B. Hartle, “Quantum Mechanics of Closed Systems” in Directions in General Relativity, Vol. 1: Misner Festschrift, eds B. L. Hu, M. P. Ryan and C. V. Vishveswara (Cambridge Univ., Cambridge, 1993).

[66] E. Calzetta, B. L. Hu and J. P. Paz, in preparation

[67] A. H. Guth and S.-Y. Pi, Phys. Rev. D32, 1899 (1985)

[68] B. L. Hu and A. Raval, in preparation

[69] J. M. Cornwall and R. Bruinsma, Phys. Rev. D38, 3146 (1988)

[70] A. D. Dolgov and A. D. Linde, Phys. Lett. 116B, 329 (1982)

[71] L. Abbott, E. Farhi and M. Wise, Phys. Lett. 117B, 29 (1982)

[72] L. A. Kofman, A. D. Linde and A. A. Starobinsky, “Reheating After Inflation” hep-th/9405187 (1994)

[73] Y. Shtanov, J. Traschen and R. Brandenberger, “Universe Reheating After Inflation” Brown University preprint HET-957 (1994)

[74] E. Calzetta and B. L. Hu, Phys. Rev. D35, 495 (1987).

[75] E. Calzetta and B. L. Hu, Phys. Rev. D40, 656 (1989).

[76] J. P. Paz, Phys. Rev. D42, 529 (1990)

[77] A. Stylianopoulos, Ph. D. Thesis, University of Maryland (1991)

[78] D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee and A. Singh, “Dissipation via Particle Production in Scalar Field Theories” (1994) hep-ph/9408214