Student's understanding of numbers through the number sense strategy

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Abstract. Many students have difficulties to do arithmetic operations with fraction. One way to overcome student difficulties is to implement number sense strategy. Therefore, it is necessary to develop a learning instrument that consists of the lesson plan, worksheet, and test. This study is qualitative research that only discusses student understanding after implementation of number sense strategy. The participants are 19 students of year 7 in one of the secondary schools in Meurah Mulia, North Aceh, Indonesia. The test used in this study are number sense problems and unstructured interview guidelines. The tests were conducted before and after the lesson. The result of the research shows that the students' understanding of the numbers has increased.

1. Introduction
Understanding numbers is not just about knowing arithmetic, but it is beyond that, namely having a good intuition about numbers, understanding well the properties of numbers, and knowing the relations between numbers. Students have started studying the numbers since the beginning of school, and fraction began to be taught in Year 3. Unfortunately, many students still have difficulties to do the arithmetic operation. This is according to Sutrisno that states that students still do not understand the standard procedures of the fractional operation [1].

![Figure 1. Example of student answer.](image)

This student made a mistake in adding fractions, he changed $\frac{9}{20}$ to $\frac{9}{5}$ and he sums up both the numerator with the numerator and the denominator with the denominator, so the result is wrong. Some previous research also mentions that students still make mistakes in calculating operations, such errors including procedural errors, concept errors, misinterpretation, error due to carelessness, application error, and learning errors [2, 3]. Witri, Putra, and Nurhanida [4] mentioned that students had more difficulty to solve tasks on fractions than integers. Usher [5] says that if someone has a problem to
understand the fractions and we are not doing anything, then he will get lost. Therefore, efforts are needed to help students improve their understanding.

One effort that can be conducted to help improve students' understanding is to introduce students to number sense strategies. Yang [6] states that the number sense refers to one's understanding of numbers and the ability to handle daily problems associated with numbers. Number sense is not a method of calculation; number sense is just a kind of sensitivity or awareness to numbers. A person with a good number sense will have the ability to process numbers. This ability is not only used in mathematical calculations but also in daily life. The importance of studying the number sense was expressed in several studies [6-11].

Number sense can be well taught if supported by appropriate learning materials. Therefore, the development of learning materials and tests based on the number sense strategies is needed to improve students' understanding on fraction. This paper only examines the development of students' understanding of the numbers using learning instruments, so that the problem in this research is how does students' understanding about fraction develop after learning based on the number sense strategy?

2. Method
This study is a qualitative research that aims to describe the students' understanding of numbers through the number sense strategies after learning processes of remedial I. The participants were 19 students from year 7 of junior high school in Meurah Mulia, North Aceh, Indonesia. This study used learning materials and tests that have been developed in the previous study. The test was designed to meet the number sense indicators: 1) to understand the density of numbers, 2) to use the benchmark, 3) to estimate and assess the reasonability of the calculation, 4) to know an operation effect, 5) to recompose/reconstruct the number [9]. The test given to the students consisted of nine questions: five multiple choice and four essay questions (Examples of question can be seen in student interviews on results and discussion). Students given the test were also interviewed to see which strategies they used.

The materials had been assessed and validated by four validators: a learning expert, a mathematics teacher, a teacher who has researched number sense and a friend who have attended research and learning mathematics workshops, so the developed materials have “worth using with revision” criteria. Validation results are revised based on validator suggestions and comments provided so that the device is eligible for use. In addition, the test used in the study also met valid and reliable criteria [12].

The data of this research were student answer sheets and interview results. Data collection was conducted in two stages. The first stage was conducted before the learning process took place to determine the initial ability of students and the second stage was implemented after learning processes to know the development of students' abilities. The learning conducted in this research was remedial learning on fraction. The learning processes were conducted for two cycles, namely remedial I and remedial II. Each remedial was designed for two meetings. The first meeting, students were taught about the density of numbers, comparing numbers, addition and subtraction of fractions including decimal fractions, and on second meeting students were taught about the multiplication of fraction. After following the learning processes, students were given a test to see their understanding and progress. Students who did not pass in Remedial I must follow Remedial II which was also designed for two meetings with different worksheet and test that are given at remedial I.

3. Result and discussion
Pre-test and interview results show that no students used number sense strategy to answer the question and the highest score obtained by students is 42.5%. Many students made a lot of mistakes when trying to do an appropriate calculation, for instance, on the task about (adding fractions). Figure 2 is an example of students’ mistake.
\[
\frac{2}{3} + \frac{14}{15} = \frac{16}{18}
\]

Figure 2. Student’s written answer on adding fraction.

This student sums up the fractions without equating the denominator, but he instantly sums up both the numerator with the numerator and the denominator with the denominator, which is wrong. The students were focused only on solving the given problem by using a standard procedure, so students did not understand the meaning underlying the calculation or operations [4, 13].

Students are given a test after remedial I to see their number sense development. Here are some examples of interviews with students:

3.1 Multiple choice test result
Each multiple choice item contains an indicator of estimating the result of a calculation operation. However, each question could be conducted using another indicator. Problem 1 and 5 were given to test students’ ability to estimate multiplication results. The following is an example of an interview with a student.

S06 : By using the estimation, the location of the decimal point in the following multiplication is … 4.346 x 3.125 = 1358125
P : The answer?
S06 : B. 13.58125. Because the initial numbers lie in the multiplication is 4 and 3, if they are multiplied, they will produce a two-digit number that is 12, but if all numbers are multiplied, they will be equal to the result above.

The student rounded each number (4.346 and 3.125) into one digit decimals (4 and 3), and then he tried to estimate the correct answer by looking the number closed to 12. So, he used one of number senses strategies called recomposing numbers. The similar idea was also presented by eight students.

Problems 2 and 3 are designed to train students’ ability to estimate the sum. Many students answered this question using a number sense strategy correctly. They used benchmark \( \frac{1}{2} \) to answer this question. An excerpt of an interview with one of the students is showed as follows.

S01 : Without doing a calculation, the following fractions which result in more than 1 are ...

a. \( \frac{5}{11} + \frac{3}{7} \)  
   b. \( \frac{7}{15} + \frac{5}{13} \)  
   c. \( \frac{1}{2} + \frac{4}{9} \)  
   d. \( \frac{5}{9} + \frac{8}{15} \)

P : Why did you choose d?
S01 : It is because \( \frac{5}{9} \) is greater than \( \frac{1}{2} \), \( \frac{8}{15} \) also greater than \( \frac{1}{2} \). So, greater than \( \frac{1}{2} \) + greater than \( \frac{1}{2} \) equal greater than 1

P : What about option a? Why don’t you choose a. \( \frac{5}{11} + \frac{3}{7} \)?
S01 : It is less than 1

P : less than 1? Why is it less than 1?
S01 : Because \( \frac{5}{11} \) is less than \( \frac{1}{2} \), \( \frac{3}{7} \) is less than \( \frac{1}{2} \).

P : So?
S01 : It is less than 1
Alasan:
\[
\frac{5}{9} \quad \text{lebih dari setengah tambah lebih setengah adalah lebih dari 1.}
\]

**Figure 3.** S01's written answer.

This student knows that \(\frac{5}{9}\) and \(\frac{8}{15}\) is a fraction of which each value is greater than \(\frac{1}{2}\), so if they are summed up \((\frac{5}{9} + \frac{8}{15}) = \text{greater than} \ \frac{1}{2} + \text{greater than} \ \frac{1}{2} = \text{greater than} \ 1\). She used one of number senses strategies called benchmark. The benchmark used in solving this problem is \(\frac{1}{2}\). This similar idea also presented by 11 students.

Question number 4 was given to know the student's ability to estimate the result of fraction subtraction. Six students answered this question correctly, but only one student answered with number sense.

**3.2 Essay test result**

The essay test consisted of four questions. The first question was given to test the students' understanding of the density between the two decimal numbers. The following dialogue is an excerpt of interviews with students who understood the density of numbers.

P : How many decimal numbers are between 1.21 and 1.22? Please, Explain clearly!
S02 : Too much
P : Why is so?
S02 : It is because between two numbers there are infinitely decimal numbers.
P : Please, Explain your reason!
S02 : It is because there are 1.211; 1.2111; etc. Also 1.12; 1.1211; etc. Regardless we mention the numbers, they remain less than 1.22

Students who answered the questions and gave reasons above understood the density of numbers and knew that between two numbers there are infinite numbers. The correct answer was also given by 11 students.

Essay question number 2 and number 3 were given to know the ability of students in comparing two numbers. Eighteen students answered this question correctly. The following is the excerpt from the interview.

P : \(A = \frac{5}{8}\) \(B = \frac{8}{9}\) which one is greater?
S06 : \(\frac{8}{9}\)
P : Why?
S06 : Because it is closer to 1, so \(\frac{8}{9}\) is greater than \(\frac{5}{8}\). Because b also has fewer remaining namely \(\frac{1}{9}\) so b is the answer.

This student used 1 as a mental reference and decided that \(\frac{8}{9}\) is closer with 1 so \(\frac{8}{9}\) is greater of both. In addition, these students also used residual theorems, in which a fraction that has less residue is larger than another one.

Essay question number 4 was given to know the ability of students to understand the effect of the multiplication operation. Eighteen students answered this question correctly (nine of them using the
number sense strategy), there were also students who did the formal calculation or short multiplication method to answer the question. An interview excerpt is showed as follows.

S01: Consider the following multiplication!
   3 x 0.2
   3 x 0.5
   Without doing the calculations, which one of the multiplications produces a greater result?
P: What do you think? Which one is greater?
S01: 3 x 0.5
P: Why is 3 x 0.5 greater?
S01: Because they are multiplied by a larger number.
   Here, there is 3 in both questions, so 0.5 is greater than 0.2.
P: So?
S01: 3 x 0.5 is greater than the two of them.

This student knew that a multiplication operation would produce a larger number when it was multiplied by larger numbers. The student was aware that the two given operations contain three, so the student must determine a larger multiplier, 0.5 or 0.2. Then he determined whether a larger or smaller multiplier had a larger result.

The problem with the right answer at least for the right reasons was essays question number 1, 2, and 3 about the density of numbers and comparing fractions. However, the students' understanding of numbers has shown progress in determining the right calculation strategies and also students have begun to use number sense strategies to solve the given problems. Number sense strategies used include re-composing numbers, using benchmarks, understanding the value of the place (number magnitude), assessing the reasonableness of the results, and knowing the effects of the operation. In addition, the highest score that students got after learning is 90%. This finding was in accordance with the study [14] which stated that number sense learning could provide positive results on student learning outcomes. These results also corresponded to those found in previous studies [6, 15] which suggested that number sense could be a starting point for students who wanted to understand the concept deeply.

Based on the result above, it can be seen that students have experienced the development of understanding. However, it was also found that most students had difficulty in subtracting and comparing fractions. During the learning process, these difficulties could be overcome by guiding students when working on material or answering questions. In addition, the need for improvement is to add a special task for students who have difficulties with certain materials so that they can continue learning. Student's worksheet and other assessment instruments have reached the desired indicator so that it can be used without revision.

4. Conclusion
The test results and student interviews showed progress toward students' understanding of numbers. The results of the first test showed that most students could not determine the appropriate calculation strategy to answer the question and no one student used number sense strategy. However, after the learning was completed, students began to show progress where students have been able to determine the appropriate algorithm to answer the problem and also already used the number sense. The number sense indicators used were re-composition of numbers, using benchmarks, understanding the value of the place (number magnitude), assessing the reasonableness of the results, and knowing the effects of the operation.

Learning materials developed were good in terms of preparing the learning steps and division of the group, but need a little revision on the part of the special assignment for students who experienced
obstacles which were expected to help students could continue the learning process. In addition, student worksheets and other assessment instruments can be used without revision.

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