Deterministic Rendering of BB84 for Practical Quantum Cryptography

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We describe how to modify the BB84 protocol for quantum cryptography in order to make it deterministic. We study both theoretical and experimental aspects of this issue, showing that the new scheme is as secure as the old one, more efficient on small-scale distances, and within the range of current technology.

PACS numbers: 03.67.Dd, 03.67.Hk

Introduction

Classically it is possible to convey information from one user (Alice) to another distant user (Bob) with perfect fidelity. This reliable transmission of information is usually obtained by means of redundancy, i.e. by repeating the main signal as many times as necessary for compensating the noise and the losses of the channel connecting the users. However things turn difficult if Alice and Bob wish to communicate privately, i.e. keeping any third party (usually called “Eve”) ignorant about the information exchanged over the channel. In this situation redundancy plays against privacy and is not at all simple to guarantee the confidentiality of the communication.

Quantum mechanics provide a solution to this issue: by using quantum signals a secret random key can be distributed at distant places and eventually used for a private communication. Several protocols for quantum key distribution (QKD) have been suggested so far [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Among these the BB84 [11] is arguably the most popular, and has been implemented both in free-space and in optical fiber setups even beyond the limit of 100 km [11, 12, 13]. Its easiness of implementation also triggered the commercialization of prototypes by a number of brands [14].

The BB84, as the vast majority of the QKD schemes, is a non-deterministic protocol. A protocol is defined “deterministic” when Alice can in-principle transmit a predetermined sequence of bits to Bob in a reliable way. The “in-principle” conditions basically mean “under ideal working conditions”, e.g. with a noiseless and lossless channel connecting Alice and Bob, or with a Bob endowed with perfect detectors. The simplest example of a deterministic protocol is any protocol working with classical signals (which is reliable but not secure, as explained above), like a standard fax transmission. On the contrary it is easy to see why the standard BB84 is non-deterministic. In the BB84 Alice (Bob) prepares (measures) the quantum signals using one out of two non-commuting observables, or bases. Only when the bases chosen by Alice and Bob coincide the two users get correlated results. In all the other instances they

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must discard their bits. This happens with probability $1/2$, and leads to the waste of half of the acquired bits, on the average. Then Alice can not, even in principle, transmit a predetermined sequence of bits to Bob, because the final sequence will depend also on Bob’s choice of the basis, about which Alice has no control.

Quite recently several deterministic protocols have been proposed \cite{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}. All of them satisfy the requisite of an in-principle reliable communication, but none of them comes with a rigorous proof of its unconditional security. The reason is that the newly proposed deterministic schemes are usually quite different from the existing ones, thus preventing a straightforward application of the mathematical tools developed for the standard protocols in the last two decades. The lack of a security proof in turn prevents a direct comparison between a deterministic and a non-deterministic protocol.

The deterministic rendering of BB84 (Det-BB84 hereafter) fills this gap, and lets a first precise quantification of the advantages coming from determinism. The point is that the security of Det-BB84 does not need to be demonstrated \textit{ab initio}, because it follows from the standard BB84’s one. This allows us to directly compare the two protocols, and establish that determinism increases the rate of secure transmission when Alice and Bob are separated by a small-scale distance.

The paper is organized as follows. In Section \textbf{II} we review what is known on the possibility of a deterministic rendering of BB84. In Section \textbf{III} we give our protocol and discuss its security. Finally in Section \textbf{IV} we propose a practical implementation of our protocol and contrast it with the traditional BB84.

\section{Deterministic BB84}

The possibility of a Det-BB84 is a common knowledge \cite{26}. The very first formulation of BB84 foresees that Bob is endowed with a quantum memory to store the quantum systems (qubits) received by Alice until Alice’s public disclosure of the bases. After knowing the bases of all the qubits Bob can measure them in a deterministic way, i.e. without risk of choosing a wrong basis that makes his measure’s outcome random.

However a simple-minded Det-BB84 appears to be totally unfeasible because of the presence of a quantum memory and of the demanding security requirements pertaining to QKD. For this reason it has been used so far only as a mathematical tool apt to simplify the analysis of the security. Even recently it has been adopted in \cite{27} to provide a stronger security for a QKD performed with BB84.

To better explain the current impracticality of the deterministic scheme we start our discussion with a basic version of Det-BB84, reported below, which exploits a quantum memory for the storage of the qubits. This version of Det-BB84 follows the steps of the BB84 described in \cite{27}, to which we refer for a detailed description, and of that reported in \cite{28}. We also refer to \cite{29} and \cite{30} for QKD-related technical details.

\textbf{DET-BB84 (basic version)}

(1) \textbf{Data-bit choice.} Alice chooses a random $W$-bit string $d$ (data string), where $W = (4 + \eta_c + \eta_m)N$. The factor $\eta_c$ accounts for the losses of the channel while $\eta_m$ accounts for the losses of Bob’s imperfect quantum memory.

(2) \textbf{Basis-bit choice and encoding.} Alice chooses a random $W$-bit string $b$ (basis string). She encodes each bit of $d$ on the qubits as $\{|0\rangle, |1\rangle\}$ if the corresponding bit of $b$ is 0 ($Z$ basis) or $\{|+, |-\rangle\}$ if the corresponding bit of $b$ is 1 ($X$ basis). Alice sends the resulting states to Bob.

(3) \textbf{Storage.} Bob receives on average $(4 + \eta_m)N$ qubits and stores them in a (imperfect) quantum memory.

(4) \textbf{Receipt.} He announces the completion of step 2 on the (authenticated or unjammable) classical channel.

(5) \textbf{Basis revelation.} Alice announces $b$.

(6) \textbf{Deterministic measurement.} Bob retrieves on average $4N$ qubits from the memory and measures each of them in the $X$ or $Z$ basis according to the disclosed value of $b$. In this way the outcome of his measure is deterministic and Alice and Bob do not discard any bits. After the public announcement by Bob of the addresses of the lost qubits, Alice and Bob will get on the average $4N$ pairs of correlated bit, a fraction of which contains errors due to the possible noise on the channel.

(7) Alice selects a subset of $2N$ bits that will serve as a check on Eve’s interference, and tells Bob which bits she selected.

(8) Alice and Bob announce and compare the values of the $2N$ check bits. If more than an acceptable number disagree, they abort the transmission.

(9) Alice and Bob perform error correction and privacy amplification on the remaining $2N$ bits to obtain $2M$ private key bits ($M \leq N$).

Points 2, 4 and 9 of the above protocol makes it deterministic, because they let Bob always measure in the right basis. In case of a noiseless and lossless channel between the users, the newly acquired determinism would enable the possibility of a \textit{direct communication} \cite{15, 19} via BB84. However, even for an imperfect channel and quantum memory, one can notice that the coefficient in front of $M$, the final number of distilled bits, is 2. This should be
compared with the coefficient 1 pertaining to BB84. It is apparent that this corresponds to a doubling of the theoretical final secure bit-rate.

A crucial point that makes Det-BB84 as secure as the original BB84 is the 4, which represents the receipt by Bob of the qubits sent by Alice. Without it there’s a risk that Eve delays the qubits until the public disclosure of the basis, thus gaining for herself the possibility of a deterministic measurement. In such a case Eve would go entirely undetected.

This security issue apart, point 4 represents the main obstacle toward a practical implementation of Det-BB84. In fact, to send a receipt, Bob must acknowledge that a given number of signals (for example photons) entered his station. The only way to do that without altering the information carried by the photons is represented by an ideal quantum nondemolition measurement (QND), which is still a demanding technology ([31], and references therein).

Point 4 also implies that Bob must store the qubit until Alice’s basis revelation (point 3). If we follow the qubit in its travel we see that the minimum storage time for a Det-BB84 with a receipt’s transmission is 2τ, where τ is the time for a signal to cover the distance between Alice and Bob: one τ is to let Bob’s receipt reach Alice, and one τ is to let Alice transmit the basis to Bob (we assume for simplicity that Alice and Bob use the same channel, hence the two times are equal in both directions).

The simplest example of a quantum memory is an optical fiber loop of length L that allows to store a photon for a time nL/c, with n the refractive index of the fiber, c the velocity of light in vacuum. At least in this simple case it is plain that the longer the photons are stored, the lower the probability to recover. Then, it would be necessary to keep the storage time as lower as possible.

II. PRACTICALITY AND SECURITY OF DET-BB84

The considerations of Section II suggest that the impracticality of Det-BB84 is mainly related to its point 4 (receipt of the qubits). In the following we show how to remove this point from the protocol without affecting its security. Despite some steps might result unusual (clock synchronization, initial measurement of the time delay), they have already been considered elsewhere and belong to the standard implicit structure of any QKD.

DET-BB84 (practical version)

(1) Preliminaries. Alice and Bob measure the time τ that a classical pulse (e.g. an intense laser pulse) employs to cover the distance between them. Then they use the (authenticated or unjammable) classical channel to (i) publicly declare the measured time τ (ii) establish the value of a positive security parameter, ∆, used later for the security analysis.

(2) Data-bit choice. Alice chooses a random W-bit string d (data string), with W = (4 + ηc + ηm)N. We indicate with di (i = 1, ..., W) the i-th bit of the string d.

(3) Basis-bit choice. Alice chooses a random W-bit string b (basis string). We indicate with bi (i = 1, ..., W) the i-th bit of the string b.

(4) Encoding and transmission of quantum information. Beginning with i = 1 Alice encodes the data di into the qubit qi. She encodes each bit of d as {0, 1} if the corresponding bit of b is 0 (X basis) or {+1, −1} if the corresponding bit of b is 1 (Z basis). At time τi Alice starts the transmission of the qubits to Bob. At the generic time τi Alice will send out the qubit qi. We note that the times τi (included the initial time τ1) need not to follow any particular prescription, and are simply related to Alice’s source’s repetition rate.

(5) Transmission of classical information. At time τi = (τi + τ + ∆), without waiting for Bob’s receipt, Alice starts the transmission of the basis bits bi using the classical channel. At the generic time τi = τi + τ + ∆ she will send out the bit bi. We note that τ and ∆ have been declared on the authenticated channel during step 1. We also note that the classical channel can be thought for simplicity and without loss of generality as the same channel used for the qubits, for example an optical fiber, but with an intense signal traveling in it. This entails that the bit bi employs an additional time τ to reach Bob. In any case the traveling time on the classical channel is measured and declared during step 1.

(6) Acquisition of classical information. At certain times Ti Bob acquires the W basis bits bi, and labels them as Bi. This step is very similar to Bob receiving a normal telephone call: he records both the values of the Bi’s and their times of arrival Ti. For what said at point 5, the expected times of arrival are Ti = (τi + 2τ + ∆ + δ), where δ > 0 is a certain unavoidable temporal delay due to the electronics of Bob’s apparatus.

(7) Deterministic measurement. As soon as the values Bi are available to Bob, he uses them to perform a deterministic measure of the qubits. The timing of this new measure is given by (Ti + δ') ± ε, where δ' ≥ δ is another temporal delay, known to Bob, due to the imperfectness of his apparatus and ε ≪ ∆ is the temporal acquisition window of his detectors. Bob labels the outcomes of this measure as Di and builds up the data string D.
(8) After Bob’s public announcement of the losses the users should share on average $4N$ pairs of correlated bits (if it is not so they abort the transmission). Alice selects a subset of $2N$ bits of $d$ and $2N$ bits of $b$ that will serve as a check of Eve’s interference, and tells Bob the addresses of the selected bits. Bob selects the same addresses from the strings $B$ and $D$.
(9) Alice and Bob announce on the classical channel (i) the values of the selected $2N$ basis bits $B_i$ and $B_t$. If any of them does not coincide they abort the whole transmission. (ii) The times of arrival $t^b_i$ and $T_i$ corresponding to the selected $2N$ pairs of bits from $b$ and $B$. If any of them does not fulfill the relation $T_i = t^b_i + 2\tau + \Delta + \delta$ within the experimental error they abort the transmission (note that for this step the clocks of the users are assumed to be synchronized). (iii) The values of the selected $2N$ pairs of check bits from $d$ and $D$. If more than an acceptable number of these values disagree, they abort the transmission.
(10) Alice and Bob perform error correction and privacy amplification on the remaining $2N$ bits to obtain $2M$ private key bits ($M \leq N$).

This version of Det-BB84, with the crucial point 5, removes the problem of Bob’s receipt, relying more on the classical communication. The main ingredient is a kind of “postselected” receipt by Bob: Alice transmits the information about the basis without waiting for Bob’s receipt, and Bob does not send the receipt in the very moment he receives the photon. Yet his final measurement will reveal whether the photon was there at the expected time or not. Thus the main problem of a QND measurement is removed at the roots. Another advantage of the above protocol is that the storage time at Bob’s site is reduced from $2\tau$ (as discussed in Section I) to $\tau + \Delta$. This reduces considerably the losses due to the storage in Bob’s quantum memory.

Security of the scheme

Apart from the removal of Bob’s receipt of the qubits, Det-BB84 is entirely equivalent to the protocol we described in Section I, which, in turn, has been shown to be secure and equivalent to the original BB84 in [27]. Hence, our security analysis aims at showing the security of Det-BB84 against attacks based on the potential weakness created by the Bob’s receipt removal. It can also be seen as a new security argument in the frame of “sequential” protocols for QKD [32, 33].

For the moment we consider Bob’s measuring apparatus as ideal, and we do not include in the proof the experimental parameters $\delta$, $\delta'$ and $\epsilon$ introduced above. The attackable point of our protocol is the lack of a qubit receipt from Bob to Alice. The risk is that Eve uses the disclosed bases to measure the qubits without perturbing them. Any other kind of eavesdropping is tantamount to Eve attacking a qubit just as she would do against a normal BB84 system. In order to exploit the basis information Eve must delay the qubit until the basis is disclosed. But any variation of the basis time of arrival respect to what declared in point 1 is detected during the check performed at point (ii) of 9, and any variation of the values of the bases decided by Alice, e.g. through an impersonation attack [34], is detected during the check of point (i) of 9.

Then assume that Eve controls opportunely the length of the channel between Alice and Bob in order to intercept the qubit $q_i$, wait for the basis information $b_i$, measure the qubit without perturbing it, and forward it to Bob without being detected. Let us examine the timing of the protocol: Alice transmits the qubit $q_i$ at time $t^q_i$, and the basis information at time $t^b_i = t^q_i + \tau + \Delta$.

Bob waits for the basis and deterministically measures the qubit at $T_i = t^q_i + 2\tau + \Delta$ (if this last relation is not a posteriori satisfied the protocol is aborted, according to point (ii) of 9). It is easy to see that Eve would go undetected only if she is able to do her attack without changing the time signature represented by $T_i$. Therefore, since the storage time of Bob’s quantum memory is $\tau + \Delta$, Eve must let the qubit enter Bob’s station at time

$$T_i - (\tau + \Delta) = t^q_i + 2\tau + \Delta - (\tau + \Delta) = t^q_i + \tau$$

(2)

to go undetected. But this time is always less than that at which the basis is revealed (Eq.1), as long as $\Delta > 0$. In other words when Eve knows the basis from Alice she does not get the qubit anymore.

Now let us discuss the experimental delays $\delta$, $\delta'$ and $\epsilon$ of Bob’s apparatus in relation to the security issue. The crucial quantity is the parameter $\Delta$: how big should it be to maintain the security of the protocol? The quantity $\epsilon$ represents a kind of experimental error in determining the exact time of arrival of the photons at Bob’s site. For example when the BB84 is implemented using weak pulses as a photon source $\epsilon$ is the time window of Bob’s “gated mode” detectors (i.e. detectors which are open only when a photon is expected to be there): otherwise, when the photons are generated through the spontaneous parametric down conversion, $\epsilon$ is the time window of the coincidence...
counts. In both cases typical values of $\varepsilon$ are less than 10 ns. In order to maintain the security of our protocol, at point 7 of Det-BB84 we required that $\varepsilon \ll \Delta$. But it descends from our security argument that actually the condition $\varepsilon < \Delta$ is sufficient to guarantee the security of the protocol. Hence, for all practical purposes, we can set $\Delta = 10\varepsilon \approx 100$ ns.

III. EXPERIMENTAL ISSUES

In this section we discuss the feasibility of Det-BB84. We consider a fiber-based configuration with weak pulses as a photon source. For the only purpose of comparison we make our proposal very similar to the one-way scheme recently reported on in [12]. However it is straightforward to work out a different setup, for example using the two-way plug-and-play configuration [35, 36].

The experimental Det-BB84 requires a fast and precise synchronization: fast enough to reduce Bob’s storage time, and precise enough to fulfill the security criteria. Specifically points 4-7 require a precise synchronization between the line carrying the quantum information (the qubit) and the line carrying the classical information (the basis).

All the QKD realizations known so far use three lines for communication: the quantum channel, the timing channel, or trigger, and the classical channel. In [12] the quantum channel is a pulsed attenuated laser at the wavelength of 1550 nm, the trigger is a pulsed bright laser at the wavelength of 1300 nm, which is used to synchronize the whole apparatus, and the classical channel is the Internet, which is employed to transfer the information about the bases and about error correction and privacy amplification. Now it is plain that to obtain the synchronization between the qubit and the basis mentioned above one can not rely on the Internet, because it can be unpredictably slow and random in the delivery of the TCP/IP packets. The solution is to use the trigger pulse, already synchronized with the quantum line, to convey also the basis information. To do that one can for example modulate the intensity of the trigger pulse: ‘low-pulse’ can represent a ‘0’, while ‘high-pulse’ can represent a ‘1’. Or it is possible to adequately digitalize the signal.

Our scheme is sketched in Figure 1. The start pulse from the computer drives the two laser sources (L1 @1550 nm, the quantum signal, and L2 @1300 nm, the trigger) and the phase modulator which encodes the information in the
relative phase of the two pulses generated by L1 and Alice’s interferometer. We drew the random number generator (RNG) as detached from the computer for simplicity. The phase encoded on the pulses is determined by the sum of the values of the basis (0 or \(\pi/2\)) and those of the state (0 or \(\pi\)). The important feature is that the basis is also written on the bright pulse @1300 nm, which now has a twofold role: time reference for Bob and carrier of the basis information. Along the bright pulse path there is a delay line, represented by a number of fiber loops, of length \(L + \Delta\). To use the parameters given in Det-BB84 we set \(L = \tau c/n\), and \(\Delta = \Delta c/n\), with \(n\) the refractive index of the fiber and \(c\) the speed of light in vacuum.

At Bob’s site the WDM selects the bright pulse, which is directed at a PIN photodiode detector. This acts as a trigger for the gate of the avalanche photodiode detectors APD1 and APD2. Moreover the value read by the detector (i.e. the basis used by Alice) acts as an input to the phase modulator represented by \(\varphi_B\) in the figure, thus allowing the deterministic measurement by Bob. On the other hand, the path followed by the quantum carrier ( photon from laser L1) is the same as in [12]. The only difference is the delay on Bob’s site, which is equal to the one at Alice’s. This delay represents the simplest quantum memory and allows Bob to wait for the information about the basis before his final, deterministic measurement. So in the whole, with respect to the usual BB84, no additional material other than some software and electronics is required for the implementation of Det-BB84.

Comparison with the BB84

In this section we compare our proposal for a practical Det-BB84 with the BB84 of [12] in terms of the rate of secure bits, \(R_{\text{sec}}\), introduced by N. Lutkenhaus in [37]. \(R_{\text{sec}}\) is a pure number and represents the fraction of distilled secure bits after the procedures of error correction [38, 39, 40] and privacy amplification [41]. It must be multiplied by the effective repetition rate in order to obtain the total secure rate of the considered setup. The \(R_{\text{sec}}\) for a BB84 implemented with weak pulses is defined as [37]:

\[
R_{\text{BB84}}^{\text{sec}} = \frac{1}{2} p_{\text{exp}} \{ \beta [1 - \tau (e/\beta)] - f_{\text{casc}} h(e) \}.
\] (3)

The coefficient 1/2 comes from the basis reconciliation procedure, in which the users’ bases coincide with an average probability of 1/2 [12]; \(p_{\text{exp}}\) is the signal of the experiment, which is given by the formula:

\[
p_{\text{exp}} = p_{\text{exp}}^{\text{signal}} + p_{\text{exp}}^{\text{dark}} - p_{\text{exp}}^{\text{dark}} p_{\text{exp}}^{\text{signal}};
\] (4)

\(p_{\text{exp}}^{\text{dark}}\) is the probability Bob gets a dark count in his detectors, while \(p_{\text{exp}}^{\text{signal}}\) is the probability that Bob’s detector fires because of a photon emitted by Alice’s source. This probability decreases with the distance between the users according to the expression:

\[
p_{\text{exp}}^{\text{signal}} = 1 - \exp(-\eta_B \eta_T \mu),
\] (5)

where \(\eta_B\) is the quantum efficiency of Bob’s detectors, \(\mu\) is the average number of photons per pulse, and \(\eta_T\) is the transmission probability of the channel, given by:

\[
\eta_T = 10^{-\left(\alpha L + L_c\right)/10}.
\] (6)

\(\alpha\) is the absorption coefficient of the fiber, \(L_c\) is the loss rate at receiver’s station and \(L\) is the distance between the users, as reported in Figure 1. Furthermore in Eq. (3) \(\beta\) is defined as:

\[
\frac{p_{\text{exp}} - S_m}{p_{\text{exp}}},
\] (7)

with \(S_m\) the probability that Alice photon source emits more than a single photon per pulse. \(\beta\) is a sort of security parameter; until it is positive the protocol is secure against the so-called PNS attacks [37, 43, 44]. \(f_{\text{casc}}\) is a function defined in [37] that takes into account the imperfect (although efficient) error correction procedure performed with the Cascade protocol. For simplicity we set it equal to 1 in our simulations. \(h(e)\) is the Shannon entropy pertaining to a given QBER \(e\). Finally \(\tau\) is the fraction of the error-corrected key which has to be discarded during privacy amplification when only single-photon pulses are taken into account [45]; it is a function of the QBER and amounts to: \(\tau(e) = \log_2(1 + 4e - 4e^2)\) for \(0 \leq e \leq 1/2\) and \(\tau(e) = 1\) for \(1/2 < e \leq 1\).

Analogously we define the secure rate for Det-BB84 as:

\[
P_{\text{secure}}^{\text{Det-BB84}} = p_{\text{exp}} \{ \beta [1 - \tau (e/\beta)] - f_{\text{casc}} h(e) \}.
\] (8)
Notice that the coefficient 1/2 is replaced by 1 in the above equation, due to Bob’s deterministic measurement. Furthermore the transmission probability $\eta_T$ is different from the one in Eq. (6). In fact in our scheme the photon is stored in the fiber loops at Bob’s site, whose length is $L + \Lambda$. Then the transmission probability becomes:

$$\eta'_T = 10^{-\alpha(2L+\Lambda)+L_c}/10.$$  \hspace{1cm} (9)

This entails that Det-BB84 is more affected by losses than BB84. However when the distance between Alice and Bob is small enough, the loss-rate is low, and the determinism still provides a nontrivial increase of the secure bit-rate.

For every fixed distance $L$ between the users the secure rate has a different maximum in the average photon number $\mu$ [37]. In our numerical simulation we chose the value of $\mu$ as such as to independently maximize the secure rate of BB84 and Det-BB84 at given lengths $L$. These values are reported in Table I.

| distance (km) | $\mu_{\text{Det-BB84}}$ | $\mu_{\text{BB84}}$ | $R_{\text{Det-BB84}}$ | $R_{\text{BB84}}$ | $R_{\text{Det-BB84}}/R_{\text{BB84}}$ |
|--------------|------------------|-----------------|------------------|------------------|------------------|
| 2            | 0.03820          | 0.04200         | 6.9145×10^{-4}   | 4.1686×10^{-4}   | 1.6587           |
| 4            | 0.03155          | 0.03818         | 4.7567×10^{-4}   | 3.4572×10^{-4}   | 1.3759           |
| 8            | 0.02162          | 0.03156         | 2.2498×10^{-4}   | 2.3783×10^{-4}   | 0.9460           |
| 16           | 0.01025          | 0.02165         | 4.9456×10^{-5}   | 1.1249×10^{-4}   | 0.4396           |

TABLE I: Values used for the numerical calculation of the secure key rates of BB84 and Det-BB84 protocols. For each distance and for each protocol the average photon number $\mu$ has been optimized to maximize the rate. Experimental parameters taken from [12].

In Figure 2 the secure rate is plotted for BB84 (Eq. 3) and Det-BB84 (Eq. 8) as a function of $L$. Only the average photon number $\mu$ is different, according to what just explained. The diagrams (a), (b), (c) and (d) have been obtained by fixing four values of $L$ and finding the values $\mu_i$ that maximize $R_{\text{sec}}(\mu_i|L_j)$ separately for BB84 and Det-BB84. Vertical lines have been drawn at the crucial distances $L_j$.

FIG. 2: Secure rate of Det-BB84 and BB84 optimized for distances between Alice and Bob of 2, 4, 8 and 16 km. Experimental parameters taken from [12].
It can be seen that in the plots (a) and (b) the secure rate provided by the Det-BB84 is higher than that pertaining to BB84. After that, in plot (c), the rates provided by the two protocols are almost the same. Finally in plot (d), the standard BB84 provides a higher rate. In other words, for distances up to about 8 km the Det-BB84 provides a better rate than the non-deterministic BB84. For distances of less than 2 km the improvement factor is more than 1.65, nearing the final value of 2 for very short distances and for a lossless setup. We remember that the maximum secure distance achievable with the BB84 setup described in [12] is about 60 km.

It should be noted that the secure rate is a figure of merit of a QKD setup, and is not a trivial task to increase it. The rate of transmission in any fiber-based setup is currently limited by detectors’ minimum dead times, which are of the order of microseconds for a standard InGaAs Avalanche Photodiode Detector (APD). This is a technological limitation that can be surpassed only by improving the detection mechanism. All the same, in the setups exploiting the spontaneous parametric down-conversion as a single-photon source is not possible to increase the signal on demand. The improvement brought about by Det-BB84 works in both the situations as it concerns the protocol itself, not the way it is implemented. In this respect the plots in Figure 2 are “universal”, i.e. independent of the particular technology employed in the experiments. For example it is possible to simply change the scaling factor of the plots drawn above to know the performances of a Det-BB84 realized in free space at the wavelength of 800 nm. It is worthwhile to mention that a low-loss high-rate QKD on very short distances has attracted recently renewed interest because of its closeness to the credit-card security issue [46]. The rate of this kind of transmissions can be almost doubled using Det-BB84.

We also remark that the performances of the practical Det-BB84 studied here are not the best possible. For instance we assumed a poor quantum memory for the storage of the photons at Bob’s site, i.e. an optical-fiber loop with the same transmission as the one used to connect the users. The maximum distance over which the Det-BB84 outperform the BB84 directly depends on this storage mechanism: the better it is the longer the distance. Technological progresses concerning quantum memories are continuously reported, and values of storage time up to microseconds have been recently achieved [47].

IV. CONCLUSION

In this paper we have provided a first evidence that a deterministic rendering of the BB84 protocol is not only conceivable in theory, but also realizable in practice with current technology. We have introduced the new protocol Det-BB84 which is as secure as the standard BB84 and does not need a Bob’s receipt for the qubits, thus paving the way to a feasible setup. In fact we have proposed an implementation of Det-BB84 and compared it with the BB84 reported in [12] in terms of the secure rate of distilled bits, finding a nontrivial increase of the bit-rate at small distances between Alice and Bob. This increase can become even more relevant as long as technology provides better memories for the storage of the qubits.

V. ACKNOWLEDGEMENTS

The core of this work has been prepared in the labs of MIMOS Berhad, Kuala Lumpur. For its completion one of the authors (M. L.) acknowledges the European Commission through the Integrated Project Qubit APPlications (QAP), Contract No. 015848, funded by the IST directorate.
