Networked Control System in Quadrotor Altitude Control with Time Delay Compensation

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Abstract. Networked control system (NCS), which has the advantage of control signal efficiency, suits quadrotor control scheme. However, the application of NCS in quadrotor may suffer from time delay. Thus, a strategy for compensating time delay is needed. In this paper, we used Smith predictor to compensate delay in quadrotor altitude control. Simulation experiment with quadrotor model indicates that the addition of Smith predictor successfully tracked reference signal with better performance compared to the conventional method in various time delay, even under simulated wind disturbance.

1. Introduction
Networked control system (NCS) is an attention drawing research topic in recent years as communication technology rises. It is a closed loop feedback control that relies on networks for exchanging signal data as figured in Fig. 1. One of strongest advantages of NCS application is efficiency. It makes the use of multiple signaling line among electronic components in a plant can be reduced in number, thus makes the wiring or signal line far more simple. This signal line number reduction is important as the space constraints are usually inevitable in real physical plant, such as in multi axis actuator or complex machine. Yet, the maintenance cost can also be reduced significantly.

Networked control system has wide range of application ranging from industrial automation, medics, military to autonomous vehicle. One of autonomous vehicle that gained huge popularity over recent decade is quadrotor. Quadrotor is a helicopter like vehicle with four rotors. These rotors are placed in square formation with same distance to each other, making the center of square as center of mass. This configuration allows quadrotor to perform versatile maneuver such as vertical take off and vertical landing where the stability and altering can be controlled by adjusting rotor speed [1]. This means NCS is suited to be applied to quadrotor system. Even though NCS application suits quadrotor control architecture, however, it may suffer from time delay.

A networked control system usually involves time delay, which is a result of time taken of data sequence from sensors to controller and controller to actuators or vice versa. The time delay may lead into stability problem, thus it needs to be compensated with particular control technique. Researchers have been using NCS architecture to cope with different plants with
addition of time delay compensation, such as [2] which tried to compensate time delay with future states prediction, while also eliminating disturbance and uncertainty. Research [3] used $H_{\infty}$ fault detection which considers packet loss and random delay. Research [4] implemented dynamic output feedback controller on aperiodic sampling and time varying delay for DC motor plant which the research topic has established already. Research [5] which uses observer based controller with anticipative and nonanticipative methods then compared them to pure plant mathematical modeling, and [6] with state feedback control of network. Research [7] tried to make a control that is able to overcome data loss due to jitter and data drop in Pressure Heavy Water Reactor (PHWR) plant, while [8] implemented event triggered controller to reduce workload in NCS with on an inverted pendulum plant. However, none of them implemented NCS architecture on quadrotor altitude control with time delay compensation.

Our goal is to minimize error at tracking the reference signal, so that the system has better response. In this paper, a networked control system in quadrotor altitude control with time delay compensation is presented. Our approach is to use Smith predictor to compensate time delay in quadrotor NCS scheme.

This paper is structured as follows: section 2 describes mathematical modeling of quadrotor and conventional PID control, section 3 introduces our control architecture approach, section 4 presents numerical simulation with the proposed method, and finally, section 5 summarizes the results.

2. The Quadrotor Model and PID Control
Before we design a compensation of time delay caused from NCS, we need to define the mathematical model of quadrotor and PID control. In this paper, we use the plant model from [1].

2.1. Quadrotor Model
Quadrotor moves in translation and rotation. The translation movement is position in three dimensional space namely $x$, $y$, and $z$ axis, while quadrotor rotation is movement in roll, pitch, and yaw. The state space of a quadrotor can be defined as (1)

$$x = [\phi \; \dot{\phi} \; \theta \; \dot{\theta} \; \psi \; \dot{\psi} \; x \; \dot{x} \; y \; \dot{y}]^T$$

where $[x \; y \; z]$ represents quadrotor position with $[\phi \; \theta \; \psi]$ angle position and $[\dot{\phi} \; \dot{\theta} \; \dot{\psi}]$ angular velocity.

The input signal of a quadrotor can be defined as (2)
\[ u = [u_1 \ u_2 \ u_3 \ u_4]^T \]  

(2) 

where (3) 

\[ u_1 = K_f(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4) \]
\[ u_2 = K_f(-\Omega_2 + \Omega_4) \]
\[ u_3 = K_f(\Omega_1 - \Omega_3) \]
\[ u_4 = K_M(\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4) \]  

(3) 

that yields (4) 

\[
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
  u_4
\end{bmatrix} =
\begin{bmatrix}
  K_f & K_f & K_f & K_f \\
  0 & -K_f & 0 & K_f \\
  K_f & 0 & -K_f & 0 \\
  K_M & -K_M & K_M & -K_M
\end{bmatrix}
\]  

(4) 

with \( \Omega_n \) is angular velocity of rotor \( n \), \( K_f \) is aerodynamic force constant, and \( K_M \) is moment constant. 

2.2. Quadrotor Altitude Model 

Physical modeling is needed to achieve simpler model but still maintain minimal error if compared to the real quadrotor.

From Newton’s second law, it is known that (5) 

\[ \Sigma F = ma \]  

(5) 

with \( \Sigma F = F_r - mg \) it yields \( F_r - mg = ma \) so that (6) 

\[ a = \frac{FT - mg}{m}. \]  

(6) 

As acceleration \( a \) is a second derivative of position, it can be Laplace transformed by assuming initial condition \( h(0) = 0 \) and \( \dot{h}(0) = 0 \), then the equation becomes (7) 

\[ s^2h = \frac{FT - mg}{m}. \]  

(7) 

And by defining \( F_M = F_r - mg \), (7) becomes (8) 

\[ h = \frac{1}{ms^2}F_M(s). \]  

(8) 

We need to define input, output, and the states (9) 

Input: 
\[ u = F_M \]  

State: 
\[ x_1 = h \ \text{(altitude)} \]  
\[ x_2 = v \ \text{(velocity)} \]  

Output: 
\[ y = x_1. \]
With those definitions, we can make transfer function into state space (10)

\[
\begin{align*}
    u &= F_M \\
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= a = \frac{F_M}{m} = \frac{u}{m} \\
    \begin{pmatrix}
        \dot{x}_1 \\
        \dot{x}_2
    \end{pmatrix} &= \begin{pmatrix}
        0 & 1 \\
        0 & 0
    \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix}
        0 \\
        \frac{1}{m}
    \end{pmatrix} u \\
    y &= (1 \ 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}
\end{align*}
\]

With \( \phi = 0 \) and \( \theta = 0 \), the quadrotor is assumed to only move along \( z \)-axis, then the \( \ddot{z} \) is (12)

\[
\ddot{z} = g - \frac{-K_f(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4)}{m}
\]

and the Laplace transform of it yields (13)

\[
z(s) = \frac{-K_f(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4) - mg}{mS^2}
\]

which means quadrotor movement along \( z \)-axis depends on all four rotors.

The dynamic model of quadrotor altitude can be defined as (14)

\[
m\ddot{z} = 4F\cos(\phi)\cos(\theta) - mg
\]

where \( F \) is every rotor thrust force, \( m \) is total quadrotor mass, \( z \) is \( \phi \) height, and \( \theta \) is roll and pitch angle. From the equation (14), we can divide both sides with \( m \) resulting (15)

\[
\ddot{z} = \frac{4F\cos(\phi)\cos(\theta)}{m} - g.
\]

Finally, we can find quadrotor altitude at \( z \)-axis by integrating equation (15) two times becomes (16)

\[
z = \int \int \frac{4F\cos(\phi)\cos(\theta)}{m} - g.
\]

2.3. Plant with PID Control

The form of PID control for the system assuming no delay with control signals is \( u(t) \) as defined in equation (17). The difference between the reference value and the current value is defined as error and denoted as \( e(t) \).

\[
U(t) = K_pe(t) + Ki \int_0^t e(t)dt + Kd \frac{de(t)}{dt}
\]

The above equation is in the time domain, if it is changed to the frequency domain with Laplace transform it will be as follows (18)

\[
U(s) = K_p e(s) + K_i \frac{1}{s} e(s) + K_d s e(s) = (K_p + \frac{1}{s}K_i + sK_d)e(s).
\]
By aligning with the system to be controlled with delay, the equation in the time domain becomes (19)

\[ U(t-t_0) = Kpe(t-t_0) + Ki \int_0^t e(t-t_0) \, dt + Kd \frac{de(t-t_0)}{dt} \] (19)

The above equation is still in the time domain, if it is changed to the frequency domain with Laplace transform it will become (20)

\[ U(s) = Kpe^{-t_0}e^{-t_0} + Kie^{-t_0}e^{-t_0} + Kde^{-t_0} = (Kp + \frac{1}{s}Ki + sKd)e^{-t_0}. \] (20)

3. Time Delay Compensation Design in NCS

Fig. 2 shows the diagram of networked control system architecture we use in this paper with quadrotor as the plant. We assume there is no delay transmission via network from controller to actuator. We assume delay only occurs in network transmission from sensor to controller. Quadrotor plant model with transfer function of \( \frac{1}{s^2} \) is used and the delay is assumed to occur when sending data from the plant to the controller, which is defined as \( t_0 \). The value of parameter \( \phi \) and \( \theta \) are assumed to be 0 and \( m \) is assumed to be 1.

The usual control used in quadrotor control is PID algorithm which can be seen at 3. The input signal is sent to the plant and the output generates feedback back to the controller. However, the time delay from networked control system scheme may lead the control to fail at tracking reference signal. Thus, a time delay compensation is needed.

In Fig. 4, we modified the controller by adding plant model and time delay model. The signal from plant model is sent as feedback to controller first but it suffers from modeled time delay. Output from plant, which also suffers from time delay, is combined with modeled time delay output before fed to the controller again together with plant model output signal.

We use Smith predictor to compensate the delay. In using Smith Predictor, it can be assumed that \( P^*(z)z_s^{-\alpha} = P(z)z_s^{-\alpha} \) where \( P^*(z) \) is the plant model controller and \( z_s^{-\alpha} \) is the delay on the sensor with the delay value \( \alpha \) during sampling time \( k \).

Then the Smith predictor as inspired by [9] is (21)
Figure 3. Plant with delay and PD controller.

Figure 4. Plant with delay compensation using Smith predictor.

\[ e(k) = r(k) - (P^\ast(z)z_a^{-\alpha} - P(z)z_a^{-\alpha}(u(k) - P^\ast(z)u(k)) \]  

While \( P^\ast(z)z_a^{-\alpha} \) and \( P(z)z_a^{-\alpha} \) has the same value, then the equation becomes (22)

\[ e(k) = r(k) - (P^\ast(z)z_a^{-\alpha} - P(z)z_a^{-\alpha}(u(k) - P^\ast(z)u(k)) \]  

The random delay value used in the method is Gaussian distributed with a mean value of 0 and a standard deviation of 1. Because we need positive value delay, then absolute value is used.

4. Numerical Simulation Experiment

In order to show the effectiveness of our method, we performed simulation on the networked control system in quadrotor model then compared the plant performance between the use of PD controller alone and the integration of Smith predictor and PD controller. The comparison results of both methods are evaluated in terms of tracking performance and error with integral absolute error (IAE). Integral absolute error explains the sum of errors of \( M + 1 \) data points. It is defined by equation (23)

\[ \text{IAE} = \sum_{k=0}^{M} |r(k) - y(k)| \]
where \( r(k) \) represent reference signal at time sampling \( k \), \( y(k) \) represent the output signal at time sampling \( k \). The ideal best IAE value is 0 which means that there is no error from output signal compared to reference signal. Closer-to-zero IAE means that the control signal is better at following reference signal.

In this experiment, the Proportional-Derivative (PD) controller is used because the plant is in the form of a double integrator. The plant has to be good at tracking reference signal, thus, intuitively, we need to differentiate it with derivative control to eliminate its integral properties. There are four scenarios on the experiment:

4.1. **PD Controller versus PD and Smith predictor under small time delay**

In first scenario, both method are tested to track reference signal with small constant time delay of 0.1 seconds.

![Comparison of both methods on small time delay in the network.](image)

**Figure 5.** Comparison of both methods on small time delay in the network.

The result can be seen at Fig. 5. The system experiences a time delay on the sensor to the controller with a constant value of 0.1 seconds. It can be observed that the control waveform between the PD Controller and Smith predictor with PD is almost similar, but the signal from Smith predictor and PD method is slightly better in terms of tracking performance. This observation is supported by the fact that IAE value for PD Controller alone, which is 365.9507, is slightly bigger than the use of Smith predictor and PD at 346.4633. From this scenario, we can conclude that the use of both methods has no significant difference when the short time delay occurs in the network.

4.2. **PD Controller versus PD and Smith predictor under large time delay**

In second scenario, both methods are tested with constant time delay of 0.9 seconds. This scenario is to test the methods performance against large time delay which may occur in the network.

The IAE value for PD Controller is 1,274.1, while IAE for Smith predictor and PD is 355.6337. With a time delay of 0.9 seconds, it appears in Fig. 6 that the control waveform between PD Controller and Smith predictor and PD is completely different. The implementation of Smith predictor and PD produces much better performance at tracking reference signal than the use of PD, as well as much smaller IAE which means the Smith predictor and PD is significantly better in large time delay compared to the previous scenario which uses small time delay.
Figure 6. Comparison of both methods on large time delay in the network.

4.3. PD Controller versus PD and Smith predictor under random time delay
The constant value of time delay in previous scenarios may not represent real application as time delay fluctuation usually occurs, thus a scenario to test the system performance with random time delay is needed.

In Fig. 7, the system experiences a time delay on the sensor to the controller with a normal distribution that is absolute. IAE value for PD controller is 457.8097, while IAE for Smith Predictor and PD is 261.2406. It can be seen that the control waveform between PD Controller and Smith Predictor and PD differ greatly with the Smith Predictor and PD shows much better performance at tracking reference signal in network with random time delay, even better IAE than the previous scenario which has large time delay.

Figure 7. Comparison of both methods on random time delay in the network.

4.4. PD Controller versus PD and Smith predictor under random time delay and external disturbances
In this scenario, the quadrotor is simulated with close-to-real condition by considering random time delay and simulated wind as external disturbance. The random time delay is normally distributed and the wind is assumed as uniformly distributed noise.

The result of both methods under such condition is shown at Fig. 8. Again, we can observe that the control waveform between the PD controller and Smith predictor and PD also differ
greatly in terms of tracking reference signal and IAE, identical with previous scenario. But, with the addition of simulated wind, we can see that both method produces noticeable ripple. However, still, the integration of Smith predictor and PD does significantly better than PD controller alone at tracking reference signal. This observation is also verified by IAE which resulted 388.9889 value for Smith Predictor and PD which is far less than the use of PD controller at 799.5430.

Overall, the experiments from four scenarios above shows that the application of Smith predictor and PD, compared to PD, is able to greatly reduce the IAE when large and random time delay in the network occurs. This means that the time delay model used in the Smith predictor loop cancels the simulated large and random time delay better than small time delay. However, the delay compensation performance at small time delay can still be improved in future research.

5. Conclusion
The implementation of Smith Predictor and PD controller on the quadrotor altitude control with networked control system shows better performance on tracking reference signal than the use of PD controller alone in various time delay, as well as in the presence of simulated wind as external disturbance, significantly when the value of time delay is large and random. This research can still be improved a lot, such as adding time delay on the actuator side which connects to the plant.

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