A priori mixed hadrons, weak radiative and non-leptonic decays of hyperons

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Abstract

A priori mixings of eigenstates in physical states are quantum mechanical effects well known in several realms of physics. The possibility that such effects are also present in particle physics, in the form of flavor and parity mixings, is studied. Applications to weak radiative and non-leptonic decays of hyperons are discussed.

I. INTRODUCTION

Because parity and strong flavors (strangeness, charm, etc.) are violated in nature, the physical (mass eigenstates) hadrons cannot be either parity or flavor eigenstates, i.e., the former must be admixtures of the latter. It is generally believed that the breaking of flavor global groups is caused by the mass differences of hadrons, but in such a way that parity and all flavors are conserved, i.e., the mass operator of hadrons giving rise to such breakings does not contain a piece that violates parity and flavor. The flavor and parity mixings in physical hadrons are attributed to the perturbative intervention of $W^\pm_\mu$ and $Z^0_\mu$ (parity mixing only).

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Precisely because such intervention is perturbative, such mixings can appear only in higher orders of perturbation theory; thus, such mixings appear, so to speak, a posteriori.

However, the possibility that the mass operator of hadrons does contain a (necessarily) very small piece that is flavor and parity violating is not excluded by any fundamental principle. If such a piece does exist, then, the parity and flavor admixtures in hadrons must come a priori, in a non-perturbative way. It is not idle to emphasize that such a piece could not be attributed to the $W^\pm$ and $Z^0$.

Our purposes in this paper are (i) to explore the possibility that the mass operator of hadrons contain flavor and parity violating pieces leading to a priori mixings, (ii) to study how to implement the a priori mixings in hadrons, and (iii) to illustrate the potential usefulness such mixings might have. Accordingly, in Sec. II we discuss how a priori mixings may be introduced at the hadron level via an ansatz, and in Secs. II and IV we apply a priori mixings to weak radiative and non-leptonic decays of hyperons in order to show how the framework we introduced can be used. We reserve the last section to discuss the potential implications of a priori mixings in particle physics.

To close this section, let us remark that a priori mixings are quantum mechanical effects well known in other realms of physics, e.g., atomic physics. Thus, another way to put the aims of this paper is to explore the questions whether a priori mixings are also present in particle physics and what consequences this could have.

II. AN ANSÄTZ

The implementation of a priori mixings for practical applications cannot, as of today, be achieved from first principles, i.e., by starting from a model at the quark level and then performing the QCD calculations to obtain the physical hadrons and their couplings. In order to proceed we must elaborate an ansatz. We shall do this in a series of steps (or working hypothesis) and we shall restrict what follows to spin-1/2 baryons and spin-0 mesons.

Our ansatz consists of the following steps:

S1. In addition to ordinary $s$-baryons and $p$-mesons there exist $p$-baryons and $s$-mesons. Let us assume that the $s$-baryons and $p$-mesons have intrinsic parity opposite to the one of the $p$-baryons and $s$-mesons, respectively. This is a crucial assumption in our approach. The indices $s$ and $p$ will refer to this, $s$ means positive intrinsic parity and $p$ means negative intrinsic parity. Both sets have the same strong-flavor assignment and belong to two different 20 and 16 representations of $SU_4$.

S2. There exist very small flavor and parity violating pieces in the mass operators for such hadrons and the passage to the physical hadrons is performed by the final rotations $R = (r_{ij})$ and $R^M = (r^M_{ij})$ that diagonalize the mass operators. $R$ and $R^M$ will be considered real for simplicity and since we are not taking into account the $CP$-violation problem in baryon decays. This leads to a priori flavor and parity admixtures in the physical (mass eigenstates) baryons and mesons, for example, like $\Lambda_{ph} = \Lambda_s + \alpha n_s + \alpha' n_p + \beta \Xi^0_s + \beta' \Xi^0_p + \cdots$. We do not know how to fix the matrix elements of $R$ and $R^M$, but on experimental grounds we can advance that the mixing angles are very small, so that, $r_{ij} = \delta_{ij} + \epsilon_{ij}$, with $\epsilon_{ji} = -\epsilon_{ij}$ and $i, j = 1, \ldots, 40$, and similarly for $r^M_{ij}$.
S3. The small mixing parameters ($\alpha$, $\alpha'$, $\beta$, etc.) are determined by assigning strong-flavor group properties to the transformation matrices $R$ and $R^M$. For example, for $SU_3$ octets of baryons and mesons:

$$R = 1 + a(U_+ + U_-) + c(O_+ + \hat{O}_-) + a'(\hat{U}_+ + \hat{U}_-) + c'(\hat{O}_+ + O_-)$$

and

$$R^M = 1 + a(U_+ + U_-) + c(O_+^M + \hat{O}_-) + a'(\hat{U}_+ + \hat{U}_-) + c'(\hat{O}_+^M + O_-^M),$$

where $U_\pm$, $\hat{U}_\pm$, $O_\pm$, $O_\pm^M$, $\hat{O}_\pm$, and $\hat{O}_\pm^M$ are all $U$-spin type ladder operators (charge conserving), with $U_\pm$, $O_\pm$, and $O_\pm^M$ acting on ordinary hadrons ($s$-baryons and $p$-mesons) and with $\hat{U}_\pm$, $\hat{O}_\pm$, and $\hat{O}_\pm^M$ acting on mirror hadrons ($p$-baryons and $s$-mesons). The $U_\pm$ and $\hat{U}_\pm$ operators only connect hadrons in the same representation, so that, they are generators, but $O_\pm$, $O_\pm^M$, $\hat{O}_\pm$, and $\hat{O}_\pm^M$ are not, of necessity, because they only connect hadrons that belong to different representations. With the properties $RR^\dagger = R^\dagger R = R^M R^M = I$ and choosing the symmetric $D$-type couplings of $O_\pm$ and $\hat{O}_\pm$ equal to zero, then the a priori flavor and parity mixings for $SU_3$ octets can be described in terms of only four independent mixing angles named: $\sigma$, $\delta$, $\delta'$, and $\hat{\sigma}$. The appropriate identifications are:

$$\sigma = a(- \frac{F}{\sqrt{6}}), \quad \hat{\sigma} = a'(- \frac{\hat{F}}{\sqrt{6}}),$$

$$\delta = c'(- \frac{\hat{F}_0}{\sqrt{6}}) = c'\left(\sqrt{\frac{3}{10}} \hat{D}_0^M - \frac{\hat{F}_0^M}{\sqrt{6}}\right) = c\left(\sqrt{\frac{3}{10}} D_0^M + \frac{F_0^M}{\sqrt{6}}\right),$$

$$\delta' = c\left(\frac{F_0}{\sqrt{6}}\right) = c'\left(- \sqrt{\frac{3}{10}} \hat{D}_0^M - \frac{\hat{F}_0^M}{\sqrt{6}}\right) = c\left(- \sqrt{\frac{3}{10}} D_0^M + \frac{F_0^M}{\sqrt{6}}\right),$$

where $F$, $\hat{F}$, $\hat{F}_0$, and $F_0$ are the $F$-type couplings of $U_\pm$, $\hat{U}_\pm$, $\hat{O}_\pm$ ($= O_\pm^L$), and $O_+$ ($= \hat{O}_\pm^L$), respectively; while $\hat{D}_0^M$ and $F_0^M$ and $D_0^M$ and $F_0^M$ are the $D$ and $F$-type couplings of $\hat{O}_\pm^M$ ($= O_\pm^M$) and $O_\pm^M$ ($= \hat{O}_\pm^M$), respectively. We must point out that the previous rules in this step have a parallelism at the quark level so that they should be necessary to develop a formulation at that level. This matter will not be tried here.

Step S3 leads to:

$$p_{ph} = p_s - \sigma \Sigma_s^+ - \delta \Sigma_p^+ + \cdots$$

$$\Sigma_{ph}^+ = \Sigma_s^+ + \sigma p_s - \delta' p_p + \cdots$$

$$\Sigma_{ph}^- = \Sigma_s^- + \sigma \Xi_s^- + \delta \Xi_p^- + \cdots$$

$$\Xi_{ph}^- = \Xi_s^- - \sigma \Sigma_s^- + \delta' \Sigma_p^- + \cdots$$
\[ n_{ph} = n_s + \sigma \left( \frac{1}{\sqrt{2}} \Sigma^0_s + \sqrt{\frac{3}{2}} \Lambda_s \right) + \delta \left( \frac{1}{\sqrt{2}} \Sigma^0_p + \sqrt{\frac{3}{2}} \Lambda_p \right) + \cdots \]

\[ \Lambda_{ph} = \Lambda_s + \sigma \sqrt{\frac{3}{2}} \left( \Xi^0_s - n_s \right) + \delta \sqrt{\frac{3}{2}} \Xi^0_p + \delta' \sqrt{\frac{3}{2}} n_p + \cdots \]

\[ \Sigma^0_{ph} = \Sigma^0_s + \sigma \left( \frac{1}{\sqrt{2}} \Sigma^0_s + \sqrt{\frac{3}{2}} \Lambda_s \right) + \delta \left( \frac{1}{\sqrt{2}} \Sigma^0_p + \sqrt{\frac{3}{2}} \Lambda_p \right) + \cdots \]

\[ \Xi^0_{ph} = \Xi^0_s - \sigma \left( \frac{1}{\sqrt{2}} \Sigma^0_s + \sqrt{\frac{3}{2}} \Lambda_s \right) + \delta' \left( \frac{1}{\sqrt{2}} \Sigma^0_p + \sqrt{\frac{3}{2}} \Lambda_p \right) + \cdots \]

\[ K^+_{ph} = K^+_p - \sigma \pi^+_p - \delta' \pi^+_s + \cdots \]

\[ K^0_{ph} = K^0_p + \sigma \frac{1}{\sqrt{2}} \pi^0_p + \delta' \frac{1}{\sqrt{2}} \pi^0_s + \cdots \]

\[ \pi^+_p = \pi^+_p + \sigma K^+_p - \delta K^+_s + \cdots \]

\[ \pi^0_p = \pi^0_p - \sigma \frac{1}{\sqrt{2}} (K^0_p + \bar{K}^0_p) + \delta \frac{1}{\sqrt{2}} (K^0_s - \bar{K}^0_s) + \cdots \]

\[ \pi^-_{ph} = \pi^-_p + \sigma K^-_p + \delta K^-_s + \cdots \]

\[ \bar{K}^0_{ph} = \bar{K}^0_p + \sigma \frac{1}{\sqrt{2}} \pi^0_p - \delta' \frac{1}{\sqrt{2}} \pi^0_s + \cdots \]

\[ K^-_{ph} = K^-_p - \sigma \pi^-_p + \delta' \pi^-_s + \cdots \]  \hspace{1cm} (3)

Notice that the physical mesons are \( CP \)-eigenstates, e.g., \( CP K^+_p = -K^-_{ph} \), etc., because we have assumed \( CP \)-invariance. We have displayed only the predominantly ordinary matter physical hadrons in terms of hadrons that correspond to \( SU_3 \) octets, so that only three independent mixing angles \( \sigma, \delta, \) and \( \delta' \) survive in this calculation. The mixings with the other hadrons corresponding to the 20 and 16 representations of \( SU_4 \) are similar to the above ones. In Eqs. (3) the dots stand for the latter flavor and parity mixings.

We have in mind an application to the observed weak radiative decays of hyperons. In this respect we introduce two more steps.

S4. The e.m. current operator \( J^\text{em}_\mu \) for baryons is a flavor conserving Lorentz proper vector.
S5. The leading form factors $f_1$ in the matrix elements of $J_{em}^\mu$ between $s$ and $s$, $s$ and $p$, and $p$ and $p$ baryons are governed by the e.m. charge operator and the induced form factors $f_2$ are independent of the $s$ and $p$ indices (because of hermiticity, the sign of $f_2$ in the matrix elements between $p$ and $s$ baryons must be reversed w.r.t. the sign of $f_2$ in the matrix elements between $s$ and $p$ baryons).

We wish to caution the reader that in assumption S5 the subindices $s$ and $p$ in the form factors $f_2$ should not be confused and taken to mean that they correspond to transition matrix elements between predominantly ordinary matter baryons and predominantly mirror matter baryons. This is important because the dimensionful magnetic-type $f_2$ depend on a mass scale determined by the masses of the physical baryons used. In Eqs. (3) the masses are of the order of 1 GeV and the pieces of the matrix elements of $J_{em}^\mu$ between these baryons that carry the indices $s$ and $p$ have a mass scale of this 1 GeV order. If one were to compute transitions between a predominantly ordinary matter baryon and a predominantly mirror matter baryon then, of course, the mass scale would be dominated by the mass of the latter baryon, a scale which is unknown and by necessity must be very large. In the next section we shall be concerned with transitions between predominantly ordinary matter baryons exclusively.

III. APPLICATION TO WEAK RADIATIVE DECAYS

Our paper would not be complete if we did not attempt an application of the physical hadrons with the non-perturbative a priori mixings of flavor and parity eigenstates. A most direct application we may have is the weak radiative decays of hyperons, although admittedly these may not necessarily be the easiest physical processes to understand.

The important point to remark is that, in contrast to $W^\pm$ mediated weak radiative decays, a priori mixed baryons can produce weak radiative decays via the ordinary electromagnetic interaction hamiltonian $H_{int}^{em} = eJ_{em}^\mu A^\mu$, where $J_{em}^\mu$ is the familiar e.m. current operator which is a flavor conserving Lorentz proper four-vector. That is, a priori mixings in baryons lead to weak radiative decays that in reality are ordinary parity and flavor conserving radiative decays, whose transition amplitudes are non-zero only because physical baryons are not flavor and parity eigenstates.

The radiative decay amplitudes we want are given by the usual matrix elements $\langle \gamma, B_{ph} | H_{int}^{em} | A_{ph} \rangle$, where $A_{ph}$ and $B_{ph}$ stand for hyperons. A very simple calculation leads to the following hadronic matrix elements

$$\langle p_{ph} | J_{em}^\mu | \Sigma^+_{ph} \rangle = \bar{u}_p \sigma (f_2^+ - f_2^p) + (\delta^p f_2^p - \delta f_2^+ \Sigma^+ \gamma^5) i\sigma^{\mu\nu} q_\nu u_{\Sigma^+}$$

$$\langle \Sigma^-_{ph} | J_{em}^\mu | \Xi^-_{ph} \rangle = \bar{u}_{\Sigma^-} \sigma (f_2^- - f_2^-) + (\delta f_2^\Sigma^- - \delta^\Sigma^- f_2^- \gamma^5) i\sigma^{\mu\nu} q_\nu u_{\Xi^-}$$

$$\langle n_{ph} | J_{em}^\mu | \Lambda_{ph} \rangle = \bar{u}_n \left\{ \sigma \left( \frac{\sqrt{2}}{2} (f_2^\Lambda - f_2^n) + \frac{1}{\sqrt{2}} f_2^{\Sigma^0\Lambda} \right) \right\} i\sigma^{\mu\nu} q_\nu u_{\Lambda}$$
\[
\langle \Lambda_{ph} | J_{em}^\mu | \Xi_{ph}^0 \rangle = \bar{u}_\Lambda \left\{ \sigma \left[ \sqrt{\frac{3}{2}} (f_2^\Sigma^0 - f_2^\Sigma^0) - \frac{1}{\sqrt{2}} f_2^{\Sigma^0 A} \right] + \left[ \sqrt{\frac{3}{2}} (\delta' f_2^\Sigma^0 - \delta f_2^{\Sigma^0}) + \delta' \frac{1}{\sqrt{2}} f_2^{\Sigma^0 A} \right] \right\} \gamma^5 i\sigma^{\mu\nu} q_\nu u_{\Xi^0}
\]

\[
\langle \Xi_{ph}^0 | J_{em}^\mu | \Lambda_{ph} \rangle = \bar{u}_{\Xi^0} \left\{ \sigma \left[ \frac{1}{\sqrt{2}} (f_2^{\Lambda^0} - f_2^{\Xi^0}) - \sqrt{\frac{3}{2}} f_2^{\Sigma^0 A} \right] + \left[ \frac{1}{\sqrt{2}} (\delta f_2^{\Xi^0} - \delta f_2^{\Xi^0}) + \delta' \sqrt{\frac{3}{2}} f_2^{\Sigma^0 A} \right] \right\} \gamma^5 i\sigma^{\mu\nu} q_\nu u_{\Xi^0}
\] (4)

The spinors \( u_A, A = p, \Sigma^+, \text{etc.} \) are ordinary four-component Dirac spinors and \( q = p_B - p_A \). In accordance with S5, in Eqs. (4) we have used the generator properties of the electric charge, which require \( f_{1s}^p = f_{1s}^{\Sigma^+} = 1 \), etc. and also, since \( s \) and \( p \) baryons belong to different irreducible representations, \( f_{1sp}^p = f_{1sp}^{\Sigma^+} = 0 \), etc. In addition, we have dropped the indices \( s \) and \( p \) in the \( f_2 \), so that \( f_{2s}^p = f_{2sp}^p = f_{2s}^{\Sigma^+} = f_{2sp}^{\Sigma^+} \), etc. All the matrix elements are of the form \( \bar{u}_B (C + D\gamma^5) i\sigma^{\mu\nu} q_\nu u_A \), where \( C \) and \( D \) would, respectively, correspond to the parity conserving and parity violating amplitudes of the \( W_\mu^\pm \) mediated decays, although in our case both amplitudes are indeed parity conserving. Notice that Eqs. (4) comply with e.m. gauge invariance.

We shall compare Eqs. (4) with experiment, ignoring the contributions of \( W_\mu^\pm \) amplitudes. We shall do this in order to be able to appreciate to what extent a priori mixings provide on their own right a framework to describe weak radiative decays.

To be able to proceed, we must decide what are the \( f_2 \) form factors in Eqs. (4). They are anomalous magnetic moment transition form factors, because, for example, \( f_2^{\Sigma^+} \) corresponds to a form factor between \( \Sigma^+ \) flavor eigenstates present in the incoming physical \( \Sigma^+ \) with mass \( m_{\Sigma^+} \) and in the outgoing physical \( p \) with mass \( m_p \). The \( f_2 \) form factors are affected by the masses of physical states. However, we shall assume that as a first approximation such mass dependence may be ignored and we will make a detailed analysis of this point in a separate point [2]. In this case, the \( f_2 \) in Eqs. (4) may be identified with the measured anomalous magnetic moments of the hyperons, i.e., \( f_2^\Lambda = \mu_A^{\text{exp}} - e_A/e_p \) (in nuclear magnetons). Only \( f_2^{\Sigma^0} \) is not measured [3], we shall use its \( SU_3 \) estimate, \( \mu_{\Sigma^0} = (\mu_{\Sigma^+} + \mu_{\Sigma^-})/2 \), as its central value with a 10% error bar. Also, we allow a 6% theoretical error in all the others.

The unknown quantities in Eqs. (4) are \( \sigma, \delta, \text{and} \delta' \). We have no theoretical argument available to try to fix their values. We must leave them as free parameters and extract their values from experiment. For this purpose amplitudes (4) should be plugged into the usual formulas for the decay rates and angular asymmetries. These formulas and the experimental data can be found in Ref. [3]. The results are displayed in Table [4]. The values obtained for the a priori mixing angles are

\[
\sigma = (1.4 \pm 0.3) \times 10^{-6}
\]

\[
\delta = (-0.35 \pm 0.13) \times 10^{-6}
\]
\[
\delta' = (-0.22 \pm 0.13) \times 10^{-6}
\]  

From Table I one can see that, given its simplicity, the above weakly mixed baryon scheme provides a qualitative reasonable description of weak radiative decays of hyperons. For completeness, our results may be compared with those obtained when the \( W \)-boson is responsible for these decays. This path has been extensively discussed, very recent reviews are found in Ref. [4]. All the models considered so far contain three or more free parameters, most of them are fixed with non-leptonic hyperon decays data. The main conclusion of Ref. [4] is that we still do not have a satisfactory theoretical explanation of weak radiative decays of hyperons. In this respect, it is important to remark that following our approach the calculations are appreciably simpler.

Nevertheless, it must be stressed that these results must be taken only as qualitative and not as quantitative. Given the simplicity of the above approach we find them encouraging enough as to take the a priori mixings in hadrons as a serious possibility.

IV. APPLICATION TO NON-LEPTONIC DECAYS

The possibility that strong-flavor and parity violating pieces in the mass operator of hadrons exist does not violate any known fundamental principle of physics. If they do exist they would lead to non-perturbative a priori mixings of flavor and parity eigenstates in physical (mass eigenstates) hadrons. Then, two paths for weak decays of hadrons to occur would be open: the ordinary one mediated by \( W^\pm (Z_\mu) \) and a new one via the strong-flavor and parity conserving interaction hamiltonians. The enhancement phenomenon observed in non-leptonic decays of hyperons (NLDH) could then be attributed to this new mechanism. However, for this to be the case it will be necessary that a priori mixings produce the well established predictions of the \(|\Delta I| = 1/2\) rule [5,6].

The a priori mixed hadrons will lead to NLDH via the parity and flavor conserving strong interaction (Yukawa) hamiltonian \( H_Y \). The transition amplitudes will be given by the matrix elements \( \langle B_{ph} M_{ph} | H_Y | A_{ph} \rangle \), where \( A_{ph} \) and \( B_{ph} \) are the initial and final hyperons and \( M_{ph} \) is the emitted meson. Using the above mixings, Eqs. (3), these amplitudes will have the form \( \bar{u}_B(A - B \gamma_5)u_A \), where \( u_A \) and \( u_B \) are four-component Dirac spinors and the amplitudes \( A \) and \( B \) correspond to the parity violating and the parity conserving amplitudes of the \( W^\pm \) mediated NLDH, although with a priori mixings these amplitudes are both actually parity and flavor conserving. As a first approximation we shall neglect isospin violations, i.e., we shall assume that \( H_Y \) is an \( SU_2 \) scalar. However, we shall not neglect \( SU_3 \) breaking. One obtains for \( A \) and \( B \) the results:

\[
A_1 = \delta' \sqrt{3} g^{p,sp}_{p,pp0} + \delta((g^{s,ss}_{\Lambda,pK^0} - g^{s,pp}_{\Lambda,\Sigma^+e^-}),
\]

\[
A_2 = -\frac{1}{\sqrt{2}}[\delta' \sqrt{3} g^{p,sp}_{p,pp0} + \delta((g^{s,ss}_{\Lambda,pK^0} - g^{s,pp}_{\Lambda,\Sigma^+e^-})],
\]

\[
A_3 = \delta(\sqrt{2} g^{s,ss}_{\Sigma^+,pK^0} + \sqrt{\frac{3}{2}} g^{s,pp}_{\Sigma^+,\Lambda\Sigma^+} + \frac{1}{\sqrt{2}} g^{s,pp}_{\Sigma^+,\Sigma^+e^0}),
\]
\[ A_4 = -\delta' \sqrt{2} g_{p, \pi^0}^{s, pl} + \delta \left( \sqrt{\frac{3}{2}} g_{\Sigma^+, \Lambda^+}^{s, pl} - \frac{1}{\sqrt{2}} g_{\Sigma^+, \Sigma^+}^{s, pl} \right), \]

\[ A_5 = -\delta' g_{p, n}^{s, pl} - \delta \left( g_{\Sigma^0, p}^{s, pl} + g_{\Sigma^+, \Sigma^+}^{s, pl} \right), \]

\[ A_6 = \delta' g_{\Sigma^+, \Lambda^+}^{s, pl} + \delta \left( g_{\Sigma^-, \Lambda^-}^{s, pl} + \sqrt{3} g_{\Xi^0, \Xi^0}^{s, pl} \right), \]

\[ A_7 = \frac{1}{\sqrt{2}} \left[ \delta' g_{\Sigma^+, \Lambda^+}^{s, pl} + \delta \left( g_{\Sigma^-, \Lambda^-}^{s, pl} + \sqrt{3} g_{\Xi^0, \Xi^0}^{s, pl} \right) \right], \tag{6} \]

and

\[ B_1 = \sigma \left( -\sqrt{3} g_{p, \pi^0}^{s, pl} + g_{\Lambda, \pi^0} + g_{\Lambda, \pi^+} \right), \]

\[ B_2 = -\frac{1}{\sqrt{2}} \sigma \left( -\sqrt{3} g_{p, \pi^0}^{s, pl} + g_{\Lambda, \pi^-} - g_{\Lambda, \pi^-} \right), \]

\[ B_3 = \sigma \left( \sqrt{2} g_{p, \pi^0}^{s, pl} + \sqrt{\frac{3}{2}} g_{\Sigma^+, \Lambda^+}^{s, pl} + \frac{1}{\sqrt{2}} g_{\Sigma^+, \Sigma^+}^{s, pl} \right), \]

\[ B_4 = \sigma \left( \sqrt{2} g_{p, \pi^0}^{s, pl} + \sqrt{\frac{3}{2}} g_{\Sigma^+, \Lambda^+}^{s, pl} - \frac{1}{\sqrt{2}} g_{\Sigma^+, \Sigma^+}^{s, pl} \right), \]

\[ B_5 = \sigma \left( g_{p, \pi^0}^{s, pl} - g_{\Sigma^0, p}^{s, pl} - g_{\Sigma^+, \Sigma^+}^{s, pl} \right), \]

\[ B_6 = \sigma \left( -g_{\Sigma^+, \Lambda^+}^{s, pl} + g_{\Xi^-, \Lambda^-}^{s, pl} + \sqrt{3} g_{\Xi^0, \Xi^0}^{s, pl} \right), \]

\[ B_7 = \frac{1}{\sqrt{2}} \sigma \left( -g_{\Sigma^+, \Lambda^+}^{s, pl} + g_{\Xi^-, \Lambda^-}^{s, pl} + \sqrt{3} g_{\Xi^0, \Xi^0}^{s, pl} \right). \tag{7} \]

The subindices 1, \ldots, 7 correspond to \( \Lambda \rightarrow p \pi^-, \Sigma \rightarrow n \pi^0, \Sigma^- \rightarrow n \pi^-, \Sigma^+ \rightarrow n \pi^+, \Sigma^+ \rightarrow p \pi^0, \Xi^- \rightarrow \Lambda \pi^-, \) and \( \Xi^0 \rightarrow \Lambda \pi^0, \) respectively. The \( g \)-constants in these equations are Yukawa coupling constants (YCC) defined by the matrix elements of \( H_{\gamma} \) between flavor and parity eigenstates, for example, by \( \langle B_{\Lambda} M_{\Lambda} | H_{\gamma} | A_{\gamma} \rangle = g_{A, B, M}^{s, pl} \). We have omitted the upper indices in the \( g \) ’s of the \( B \) amplitudes because the states involved carry the normal intrinsic parities of hadrons. In Eqs. (6) we have used the \( SU_2 \) relations \( g_{p, \pi^0}^{s, pl} = -g_{n, n^0, n^0}^{s, pl} = \left. g_{p, n^+}^{s, pl} \right| \sqrt{2}, \)

\( g_{\Sigma^+, \Lambda^+}^{s, pl} = \sqrt{3} g_{\Sigma^0, \Lambda^+}^{s, pl} = g_{\Xi^0, \Lambda^+}^{s, pl} = -g_{\Lambda, \Sigma^0, \Lambda^+}^{s, pl} = g_{\Xi^-, \Lambda^+}^{s, pl} = -g_{\Sigma^+, \Sigma^+}^{s, pl} = g_{\Xi^0, \Sigma^+}^{s, pl} = -g_{\Sigma^-, \Xi^0}^{s, pl} = g_{\Xi^-, \Sigma^+}^{s, pl} = -g_{\Xi^-, \Xi^0}^{s, pl} = g_{\Xi^-}^{s, pl} \). Similar relations are valid within each set of upper indices, e.g.,
$g_{p, p}^{p, s} = -g_{n, n}^{p, s}$, etc.; the reason for this is, as we discussed in Ref. [7], mirror hadrons may be expected to have the same strong-flavor assignments as ordinary hadrons. Thus, for example, $\pi_+^s$, $\pi_0^s$, and $\pi_-^s$ form an isospin triplet, although a different one from the ordinary $\pi_+^p$, $\pi_0^p$, and $\pi_-^p$ isospin triplet. These latter relations have been used in Eqs. (6).

From the above results one readily obtains the equalities:

$$A_2 = \frac{1}{\sqrt{2}} A_1, \quad A_5 = \frac{1}{\sqrt{2}} (A_4 - A_3), \quad A_7 = \frac{1}{\sqrt{2}} A_6,$$

$$B_2 = -\frac{1}{\sqrt{2}} B_1, \quad B_5 = \frac{1}{\sqrt{2}} (B_4 - B_3), \quad B_7 = \frac{1}{\sqrt{2}} B_6.$$

(8)

These are the predictions of the $|\Delta I| = 1/2$ rule. That is, a priori mixings in hadrons as introduced above lead to the predictions of the $|\Delta I| = 1/2$ rule, but notice that they do not lead to the $|\Delta I| = 1/2$ rule itself. This rule originally refers to the isospin covariance properties of the effective non-leptonic interaction hamiltonian to be sandwiched between strong-flavor and parity eigenstates. The $I = 1/2$ part of this hamiltonian is enhanced over the $I = 3/2$ part. In contrast, in the case of a priori mixings $H_Y$ has been assumed to be isospin invariant, i.e., in this case the rule should be called a $\Delta I = 0$ rule.

It must be stressed that the results (8) and (9) are very general: (i) the predictions of the $|\Delta I| = 1/2$ rule are obtained simultaneously for the $A$ and $B$ amplitudes, (ii) they are independent of the mixing angles $\sigma$, $\delta$, and $\delta'$, and (iii) they are also independent of particular values of the YCC. They will be violated by isospin breaking corrections. So, they should be quite accurate, as is experimentally the case.

Although a priori mixings do not violate any fundamental principle, the reader may wonder if they do not violate some important theorem, specifically the Feinberg–Kabir–Weinberg theorem [8]. They do not. This theorem is useful for defining conserved quantum numbers after rotations that diagonalize the kinetic and mass terms of particles. It presupposes on mass-shell particles and interactions that can be diagonalized simultaneously with those terms. This last is sometimes not clearly stated, but it is an obvious requirement. Quarks inside hadrons are off mass-shell; so the theorem cannot eliminate the non-diagonal $d-s$ terms which lead to non-diagonal terms in hadrons. It has not yet been proved for hadrons, but one can speculate: what if it had? Hadrons are on mass-shell, but they show many more interactions than quarks, albeit, effective ones. The Yukawa interaction cannot be diagonalized along with the kinetic and mass terms, as can be seen through the YCC of the amplitudes above. Therefore, this theorem would not apply to the last rotation leading to a priori mixings in hadrons. Another example is weak radiative decays, it is interesting because it is a mixed one. The charge form factors can be diagonalized while anomalous magnetic ones cannot. The theorem would apply to the former but not to the latter.

A detailed comparison with all the experimental data available in these decays requires more space and will be presented separately [9]. Nevertheless, we shall briefly mention a few very important results.

First, the experimental $B$ amplitudes (displayed in Table II) are reproduced within a few percent by accepting that the YCC are given by the ones observed in strong interactions [10], an assumption which cannot be avoided in this approach. The best predictions for these amplitudes are $B_1 = 22.11 \times 10^{-7}$, $B_2 = -15.63 \times 10^{-7}$, $B_3 = 1.39 \times 10^{-7}$, $B_4 = -42.03 \times 10^{-7}$,
\[ B_5 = -30.67 \times 10^{-7}, \quad B_6 = 17.45 \times 10^{-7}, \quad \text{and} \quad B_7 = 12.34 \times 10^{-7}. \] The only unknown parameter \( \sigma \) is determined at \((3.9 \pm 1.3) \times 10^{-6}\). We quote the experimental values of the \( B \) amplitudes in the natural scale of \( 10^{-7} \), see Ref. [3]. Their signs are free to choose; actually, the comparison with theoretical predictions is only meaningful for their magnitudes. The signs we display are for convenience only. This is not the case for the signs in the \( A \) amplitudes.

Second, although the \( A \) amplitudes involve new YCC, an important prediction is already made in Eqs. [3]. Once the signs of the \( B \) amplitudes are fixed, one is free to fix the signs of four \( A \) amplitudes — say, \( A_1 > 0, \ A_3 < 0, \ A_4 < 0, \ A_6 < 0 \) — to match the signs of the corresponding experimental \( \alpha \) asymmetries, namely, \( \alpha_1 > 0, \ \alpha_3 < 0, \ \alpha_4 > 0, \ \alpha_6 < 0 \). Then the signs of \( A_2 < 0, \ A_5 > 0, \ A_7 < 0 \) are fixed by Eqs. (6) and the fact that \( |A_4| \ll |A_3| \). In turn the signs of the corresponding \( \alpha \)'s are fixed. These three signs agree with the experimentally observed ones, namely, \( \alpha_2 > 0, \ \alpha_5 < 0, \ \alpha_7 < 0 \).

The above predictions are quite general because only assumptions already implied in the ansatz for the application of a priori mixings have been used. A detailed comparison of the \( A \) amplitudes with experiment is limited by our current inability to compute well with QCD. However, one may try simple and argumentable new assumptions to make predictions for such amplitudes. Since QCD has been assumed to be common to both ordinary and mirror quarks, it is not unreasonable to expect that the magnitudes of the YCC in the \( A \) amplitudes have the same magnitudes as their corresponding counterparts in the ordinary YCC of the \( B \) amplitudes. The relative signs may differ, however. Introducing this assumption we obtain the predictions for the \( A \) amplitudes displayed in Table I. The predictions for the \( B \) amplitudes must also be redone, because determining the \( A \) amplitudes alone may introduce small variations in the YCC that affect importantly the \( B \) amplitudes, i.e., both the \( A \) and \( B \) amplitudes must be simultaneously determined, the \( B \)'s act then as extra constraints on the determination of the \( A \)'s. The new predictions for the \( B \)'s are also displayed in Table I. In obtaining Table I we have actually used the experimental decay rates \( \Gamma \) and \( \alpha \) and \( \gamma \) asymmetries [3], but we only display the experimental and theoretical amplitudes.

The predictions for the \( A \)'s agree very well with experiment to within a few percent, while the predictions for the \( B \)'s remain as before. The a priori mixing angles are determined to be \( |\delta| = (0.23 \pm 0.07) \times 10^{-6}, \ |\delta'| = (0.26 \pm 0.07) \times 10^{-6} \), and \( \sigma = (4.9 \pm 1.5) \times 10^{-6} \). This last value of \( \sigma \) is consistent with the previous one. The overall sign of the new YCC can be reversed and the new overall sign can be absorbed into \( \delta \) and \( \delta' \). This can be done partially in the group of such constants that accompanies \( \delta \) or in the group that accompanies \( \delta' \) or in both. Because of this, we have determined only the absolute values of \( \delta \) and \( \delta' \). In order to emphasize this fact we have inserted absolute value bars on \( \delta \) and \( \delta' \). The more detailed analysis of the comparison of the \( A \)'s and \( B \)'s with experiment is presented in Ref. [3].

The above results, especially those of Eqs. (5) and (6) and the determination of the amplitudes, satisfy some of the most important requirements that a priori mixings must meet in order to be taken seriously as an alternative to describe the enhancement phenomenon observed in non-leptonic decays of hadrons. This means then that another source of flavor and parity violation may exist, other than that of \( W_\mu^\pm \) and \( Z_\mu \).
In the previous sections we have explored the possibility that flavor and parity violating pieces in the mass operator of hadrons may exist. In this case, physical hadrons would show non-perturbative mixings of flavor and parity eigenstates, i.e., right from the start. These we have called a priori mixings to distinguish them from the mixings originated by the intervention of the $W^±_\mu$ and $Z^0_\mu$ bosons, which are perturbative and lead to such mixings in hadrons, but in an a posteriori fashion.

If a priori mixings are present, then weak decays may go via the flavor and parity conserving hamiltonians of strong and electromagnetic interactions. That is, with these mixings there would exist another mechanism to produce weak radiative, non-leptonic, and rare mode decays of hadrons, in addition to the already existing mechanisms provided by the $W^±_\mu$ and $Z^0_\mu$ bosons. It is worthwhile to point out that the calculation of decays and reactions through the $W/Z$ exchange mechanisms is obtained in the present scheme in the usual way. The weak hamiltonian is, so to speak, sandwiched between a priori mixed hadrons; to lowest order only the parity and flavor eigenstates survive, the mixed eigenstates contribute negligible corrections. Thus, beta and semileptonic decay remain practically unchanged, while nonleptonic kaon decays, hypernuclear decays, and others in which the enhancement phenomenon could be present should be recalculated. One is immediately led to several questions: if a priori mixings in hadrons do exist in nature, how do their contributions compare to those of the $W^±_\mu$?, can their contributions be relevant?, and if so, would they improve our understanding of weak decays of hadrons?

Before discussing these questions one must first be able to calculate such contributions. This is not an easy task; however, one can introduce working hypotheses, based on educated guesses as much as possible. This we have done in Sec. II for spin-1/2 baryons and spin-0 mesons. This collection of working hypotheses or ansatz enabled us to perform some calculations. As an illustration, we made an application to weak radiative and non-leptonic decays of hyperons, in Secs. III and IV. In order to keep things still at a relatively simple level, we introduced some approximations and, because of this, the results obtained should be judged as qualitative only. We find them to be encouraging enough as to conclude that a priori mixings in hadrons should be taken seriously, as a novel possibility in Particle Physics.

Let us retake the above questions. As we mentioned in Sec. II, we lack any theoretical argument to roughly estimate the size of the a priori mixing angles. Clearly, it could well be the case that they are non-zero, so that this new effect does exist in Particle Physics as it does in other realms of physics, but they are extremely small. This would mean that with even very precise data a priori mixings would go undetected. In other words, the effect might exist but it would be a theoretical curiosity, irrelevant for practical purposes. The next possibility would be that the mixing angles be such that they lead to observable weak decays comparable to those mediated by $W^±_\mu$. In this case, one would have to face the complicated situation of disentangling what belongs to what in describing experimental data. The last possibility is that the a priori mixing angles be such that they lead to contributions appreciably larger than the corresponding ones of $W^±_\mu$. In-as-much as a priori mixings are concerned, this is the really interesting situation. Their experimental predictions could then be subject to conclusive tests. Therefore, it is this last possibility we shall concentrate upon.

In the understanding of non-leptonic, weak radiative, and rare mode decays of hadrons...
a long-standing problem still remains an open challenge. This is the enhancement phenomenon. An impressive amount of effort has been invested in trying to demonstrate that the strong interactions that dress the hadron weak decays mediated by $W^\pm_\mu$ are responsible for such enhancement. The results so far are disappointing. It is commonly believed that the reason for this failure is our inability to compute with QCD, but once we can calculate better this problem will be solved favorably. Along this line of reasoning, the situation envisaged is that the intermediation of $W^\pm_\mu$ will saturate all measurements on flavor changing decays of hadrons and if any other mechanism exists it will necessarily be negligible small, e.g., a priori mixings could not go beyond the theoretical curiosity level we just mentioned. However, it may happen that — once we can calculate better with QCD and contrary to expectations — it is demonstrated that enhancement cannot be produced by strong interactions. In this situation a new mechanism would be required.

This last comment provides the means to subject a priori mixings to critical tests. One of these is that, if they are to be an interesting effect in hadron weak decays, they should produce the observed enhancement phenomenon. Another very important one is that one should expect that the a priori mixing angles show a universality-like property, i.e., that their values appear reasonably stable in different types of weak decays. However the judgment of how these tests and others are passed or failed will also be limited in the near future by our inability to calculate better with QCD. Accordingly, one should first expect to obtain relevant qualitative results and afterwards quantitative results based on educated guesses and simple models as we have illustrated in Secs. I, II, and IV. Clearly, it is along these lines that efforts of future research in this subject should be addressed. Also, the contributions of $W^\pm_\mu$ should be included at some point at a, for consistency, small level, say, by assuming that $|\Delta I| = 1/2$ amplitudes are of the same order of magnitude as the $|\Delta I| = 3/2$ amplitudes.

To close this paper and in the light of this discussion, we must stress that our applications to weak radiative and non-leptonic decays of hyperons should be taken more than anything else just as an exercise to learn to use a priori mixings of baryons. A more detailed analysis of these decays should be retaken later on. Nevertheless, for the time being we may point out that the lesson in Secs. II and IV is encouraging enough so as to take with seriousness the possibility of the existence of this effect in Particle Physics.

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\[
\begin{align*}
g_{p,p\pi^0} &= 1.001 = -g_{p,p\pi^0}^{s,ss}, \\
g_{\Sigma^+,\Sigma^+} &= 0.987 = g_{p,p\pi^0}^{s,ss}, \\
g_{\Sigma^+,\Sigma^0} &= 0.936 = g_{p,p\pi^0}^{s,ss}, \\
g_{\Sigma^0,\pi^+} &= 0.251 = g_{p,p\pi^0}^{s,ss}, \\
g_{\Lambda,\Lambda^+} &= -0.897 = g_{p,p\pi^0}^{s,ss}, \\
g_{\Lambda,\Sigma^+} &= -0.812 = g_{p,p\pi^0}^{s,ss}, \\
g_{\Lambda,\Sigma^+} &= -0.360 = g_{p,p\pi^0}^{s,ss}, \\
g_{\Sigma^0,\Lambda^-} &= -0.303 = g_{p,p\pi^0}^{s,ss},
\end{align*}
\]

These values are in reasonable agreement with the experimental counterparts. We use a chi-square method in which the experimental YCC are added as constraints.
### TABLE I.

Predictions for the asymmetries and branching fractions (in units of $10^{-3}$) of the weak radiative decays considered, along with the eight experimental measurements from Ref. [3].

| Decay          | $\alpha_{\text{th}}$ | $\alpha_{\text{exp}}$ | Fraction ($\Gamma_i/\Gamma$)$_{\text{th}}$ | Fraction ($\Gamma_i/\Gamma$)$_{\text{exp}}$ |
|----------------|-----------------------|------------------------|---------------------------------------------|---------------------------------------------|
| $\Sigma^+ \rightarrow p\gamma$ | -0.75                 | -0.76 ± 0.08           | 1.3                                         | 1.25 ± 0.07                                 |
| $\Xi^- \rightarrow \Sigma^- \gamma$ | 0.57                   | —                      | 0.14                                        | 0.127 ± 0.023                               |
| $\Lambda \rightarrow n\gamma$ | -0.85                 | —                      | 1.8                                         | 1.75 ± 0.15                                 |
| $\Xi^0 \rightarrow \Lambda \gamma$ | -0.23                 | 0.4 ± 0.4              | 1.1                                         | 1.06 ± 0.16                                 |
| $\Xi^0 \rightarrow \Sigma^0 \gamma$ | -0.03                 | 0.20 ± 0.32            | 3.2                                         | 3.5 ± 0.4                                   |

### TABLE II.

Predictions for the $A$ amplitudes, along with the accompanying predictions for the $B$ amplitudes, obtained by assuming that the magnitudes of the YCC of Eqs. (6) match their corresponding counterparts in Eqs. (7). The values of the YCC are listed in Ref. [10]. All amplitudes are given in units of $10^{-7}$.

| Decay          | $B_{\text{exp}}$ | $B_{\text{th}}$ | $A_{\text{exp}}$ | $A_{\text{th}}$ |
|----------------|------------------|-----------------|------------------|-----------------|
| $\Lambda \rightarrow p\pi^-$ | -22.09 ± 0.44    | -22.38          | -3.231 ± 0.020   | -3.262          |
| $\Lambda \rightarrow n\pi^0$ | 15.89 ± 1.01    | 15.83           | 2.374 ± 0.027    | 2.307           |
| $\Sigma^- \rightarrow n\pi^-$ | 1.43 ± 0.17     | 1.34            | -4.269 ± 0.014   | -4.264          |
| $\Sigma^+ \rightarrow n\pi^0$ | -42.17 ± 0.18   | -42.09          | -0.140 ± 0.027   | -0.152          |
| $\Sigma^+ \rightarrow p\pi^0$ | -26.86 ± 1.10   | -30.72          | 3.247 ± 0.089    | 2.907           |
| $\Xi^- \rightarrow \Lambda \pi^-$ | -17.47 ± 0.50   | -17.27          | 4.497 ± 0.020    | 4.521           |
| $\Xi^0 \rightarrow \Lambda \pi^0$ | -12.29 ± 0.70   | -12.21          | 3.431 ± 0.055    | 3.197           |