Choice of Factors Modelling of Concrete

Varlamov A.A.1, Yakobchuk D. L.1, Lozhkin I. A.1

1 Magnitogorsk state technical University G. I. Nosov, Lenin Ave, 27, City of Magnitogorsk, 455000, Russian Federation
mgrp@mgn.ru

Abstract. The continued development of the theory of degradation. In this work, the experimental data were processed by the method of factor analysis. Conducted experimental research with the aim of charting the concrete for comparison with a new theory. The paper presents analysis of experimental data. Considered the factors which determine the behaviour diagram of concrete. Treated with own experimental data and data of other authors. Additionally, used the Varimax – method and "bootstrap analysis". The result was that two factors describe only 60% of the variance. The addition of the third factor allows us to describe the diagram of the concrete by 90%.

1. Introduction
To assess the condition of concrete structures and prediction of its behaviour is necessary to build a model of the behaviour of concrete [1-5]. Attempt to build a model of concrete based on the use of two variables – grade concrete and stress levels gave satisfactory results mainly in the central region loads [6-9]. At low and high loads the model using the two variables poorly described the behaviour of concrete [10-13]. Therefore it was decided at the first stage of modelling to solve the problem of allocating the factors, to build a model of the behaviour of concrete [14-16].

When conducting the factor analysis used the data obtained in our studies and the results of the tests given in other works [17-19].

2. Problem formulation and choice of model parameters
The purpose of the factor analysis was the isolation of m significant factors or component from the set N original attributes that characterize the phenomenon being studied. The original signs were replaced with fewer (m < N) standardized orthogonal factors or components.

General view of the model factor analysis is a system of equations that can be represented in matrix form:
\[ Z = W^*F + L, \]  
(1)

where \( Z = (Z_1, Z_2, ..., Z_N)^T \), \( F = (F_1, F_2, ..., F_m)^T \) – random vectors of size \( N \) and \( m \) respectively, and the random vector of errors (of specific characteristics) \( L = [\mu_1U_1, \mu_2U_2, ..., \mu_NU_N] \).

The main difficulty that arose in the analysis the lack of a unique solution \( W \) given here matrix equations. Indeed, in \( N \) – dimensional space reference system, i.e. the system of factor axes is not uniquely determined (follows from the uncertain decision system). And this clearly influenced the values of factor loadings, because of the projection factor – signs on the axis – factors depended on the provisions of the latter. Therefore, despite the uniqueness of the configuration of source characteristics, there was not a clear solution according to the factor loadings.

Thus, the set of solutions of the homogeneous system has led to the possibility to obtain the matrix of loads with almost "simple" structure.

Further solved the following problem: what considerations should guide the selection of common factors from an infinite number of such systems, that lie in the same linear subspace, and what computational methods should to implement this choice.

From the fundamental solution \( W \) is found from the correlation matrix \( R \) (or \( R_{\text{red}} \)) can be obtained the so-called “rotated” solution with the matrix \( A \):

\[ A = W^*L^T = L^*W, \]  
(2)

where \( L \) is the matrix of \( N \) – order arbitrary orthogonal transformation the result which the condition \( W^*W = R \), and consequently, \( A^T*A = R \).

The existence of an infinite number of equivalent structures \( (W, A', A'', A''', ...) \) due to an arbitrary orthogonal transformation \( L \) expresses the so-called "rotational" uncertainty component (factor) model that determines the opportunity again to expose the matrix \( A \) the rotational procedure. Naturally, when the number of components \( m < N \), the dispersion equation of the factor model for the \( j \) – sign \( S_j^2 = H_j^2 + V_j^2 \) has different residues \( V_j^2 \) for different values of \( m \). The uncertainty connected with the problem of finding of this assessment \( V_j^2 \), in which the model would be the most "economical", i.e. the number \( m \) of columns of the matrix \( W \) would be minimal.

Any two structures load of the order \( N \) of the form \( W = (W_{jk}) \) and \( A = (a_{jk}) \) are equivalent in the sense of rotational indeterminacy, if the matrix \( W \) and \( A \) obtained one from the other orthogonal transformations of the form \( L = (l_{jk}) \) of the same order. However, these structures can be really unequal (for practice), i.e. is not convenient in terms of interpretation or economically unprofitable to use in control tasks. That is why they should be subjected to rotation through any suitable operator \( L = (l_{jk}) \) in order to translate the original matrix \( W \) in the matrix \( A \) with the so-called “simple” structure.

The solution to the problem of rotation is still at the beginning of his development. To obtain meaningful information, the factor analysis used the most common method of rotation - is Varimax [20]. This method is easy to implement on a computer and allows you to rotate the principal components or factors in multivariate angle \( \phi \) giving the orthogonal matrix solutions to \( A = (a_{jk}) \), with an almost “simple” structure.

The solution to the problem of rotation is still at the beginning of his development. To obtain meaningful information, the factor analysis used the most common method of rotation is Varimax. For the implementation of factor analysis on the computer used the system STATISTIKA in a Windows environment (manufacturer StatSoft Inc., USA) where one of the modules of this system Factor Analysis.

3. Results of factor analysis

When conducting the factor analysis used the data obtained in our studies and the results of the tests given in other works. For the analysis of all research works under study in the variables (prism strength
(R_b), water / cement ratio (W/C), activity of cement (R_c), volume of rubble (V_G), the ratio by weight of sand and gravel (S/G), density (\(\rho\)), the amount of cement (V_C), type of cement, age of concrete, a breed of coarse aggregate, moisture content environment, the method of accelerating the hardening selected seven variables, the amount of cement, the fineness modulus of crushed stone and sand, the strength of the filler) was chosen seven. The principle of selection of variables was based on the possibility of determining them or their analogues in operated structures. For example, in more detail the results of the analysis of the data obtained in the work A. N. Bambur, V. J. Bachinsky [17]:

The source data

VAR1 – deformation (\(\varepsilon_b\)); VAR2 – prism strength (R_b); VAR3 – water-cement ratio (W/C); VAR4 – activity cement (R_c); VAR5 is the volume of gravel (V_G); VAR6 – the ratio by weight of sand and gravel (S/G); VAR7 – density (\(\rho\));

The main primary document for factor analysis is correlation matrix. In table 1 shows the correlation matrix obtained from the experimental data.

Table 1. Correlation matrix

|       | VAR1   | VAR2   | VAR3   | VAR4   | VAR5   | VAR6   | VAR7   |
|-------|--------|--------|--------|--------|--------|--------|--------|
| VAR1  | 1      | 0.8114 | -0.7821| -0.7085| 0.49388| -0.582 | 0.2727 |
| VAR2  | 0.81136| 1      | -0.9509| -0.453 | 0.1815 | -0.27  | 0.6969 |
| VAR3  | -0.7821| -0.951 | 1      | 0.5548 | -0.3286| 0.376  | -0.6184|
| VAR4  | -0.7085| -0.453 | 0.5548 | 1      | -0.9522| 0.9712 | 0.2941 |
| VAR5  | 0.49388| 0.1815 | -0.3286| -0.952 | 1      | -0.982 | -0.5347|
| VAR6  | -0.5817| -0.27  | 0.3760 | -0.9825| 1      | 0      | 0.4812 |
| VAR7  | 0.27277| 0.6969 | -0.6184| 0.2941 | -0.5347| 0.4812 | 1      |

Convert correlation matrix to diagonal form reveals its own values. In table 2, except calculated eigenvalues, shown percentage explain the General dispersion of signs, and accumulated (cumulative) the sum of the eigenvalues and the cumulative percentage of this amount.

Table 2. Cumulative sum of eigenvalues

| Room eigenvalues | Eigenvalues | Percentage of explained common variance of the signs | Of the accumulated (cumulative) sum of eigenvalues | cumulative percentage of this amount |
|------------------|-------------|-----------------------------------------------------|--------------------------------------------------|--------------------------------------|
| 1                | 4,16041     | 59,43455                                            | 4,160419                                         | 59,43455                            |
| 2                | 2,57277     | 36,75393                                            | 6,733194                                         | 96,18848                            |

For a visual comparison of the eigenvalues and adoption of decisions on the choice of the number of factors is a graph of all eigenvalues (figure 1).
Figure 1. Graph of eigenvalues of matrix R

To comparison, monitor and improve results in a new basis presents a number of paired graphs, with
the results of the factor analysis to rotation of the factors system and after the rotation.

Illustration of calculation of factor loadings in the plane of the factors F1 and F2 up to rotation of the
coordinate system shown in figure 2, and after rotation in figure 3 rotate the system of normalized or-
thogonal factors were Varimax-method.

To represent on the convergence process of factorization when using 2 factors in the table 3 shows
the matrix of correlations of residuals.

As follows from table 3, all residuals have the value > of 0.1, which confirms the adequacy of the
number of factors used in the analysis. If the number of factors will increase the convergence of the
original and the reproduced matrix will naturally increase (and even these matrices coincide when using
the model of 7 factors). However, the redistribution of loads in this case would not be in favor of their
significance, therefore, the matrix of weights $W = (W_{jk})$ is even more different from the matrix "simple
structure", which defeats the purpose of factor analysis.

Figure 2. Schedule loads for F1 and F2 up to rotation system
**Figure 3.** Schedule loads for F1 and F2 after rotation system

**Table 3.** Matrix residues correlations

| Variables | VAR1 | VAR2 | VAR3 | VAR4 | VAR5 | VAR6 | VAR7 |
|-----------|------|------|------|------|------|------|------|
| VAR1      | 0.15 | -0.01| 0.07 | 0.0195| -0.045| 0.017| -0.0371|
| VAR2      | -0.01| 0.02 | 0.01 | -0.002 | -0.0047| -0.006| -0.0022|
| VAR3      | 0.07 | 0.01 | 0.05 | 0.0062 | -0.0285| 0.0028| -0.0195|
| VAR4      | 0.02 | -0.00 | 0.01 | 0.0067 | -0.0048| -0.00 | -0.0053|
| VAR5      | -0.05| -0.00 | -0.03 | -0.0048 | 0.0199 | -0.00 | 0.0130|
| VAR6      | 0.02 | -0.01 | 0.00 | -0.0001 | -0.00 | 0.009 | -0.0033|
| VAR7      | -0.04| -0.00 | -0.02 | -0.0053 | 0.0130 | -0.0033| 0.0118|

Note. The main diagonal elements of the matrix residue correlations exceeding a given level of accuracy, not marked.

At the final stage the results of the analysis are also given in the table according to the factor inputs. In table 4 presents data on deposits of up to a factor rotation of the coordinate system as in table 5 shows the same data obtained after applying the Varimax – method to the normalized factors

**Table 4.** Coefficients of factor contribution for each characteristic prior to procedure rotation

| Variables | Factor 1 | Factor 2 |
|-----------|----------|----------|
| VAR1      | 0.210681 | -0.108129|
| VAR2      | 0.174114 | -0.262756|
| VAR3      | -0.190164| 0.222490|
| VAR4      | -0.225379| -0.131222|
| VAR5      | 0.192433 | 0.226345|
| VAR6      | -0.203924| -0.202341|
| VAR7      | 0.009204 | -0.386080|
Table 5. Coefficients of factor contribution for each characteristic after procedure of rotation

| Variables | Factor 1 | Factor 2 |
|-----------|----------|----------|
| VAR2      | 0.030822 | 0.313698 |
| VAR3      | 0.007044 | -0.2926  |
| VAR4      | 0.257634 | -0.04049 |
| VAR5      | -0.29213 | -0.05408 |
| VAR6      | 0.285886 | 0.028209 |
| VAR7      | 0.236571 | 0.305248 |

Table 6. Summarized data according to factor analysis of results of experiments obtained by A. N. Bambura, V. J. Bachinsky

| Factor | F1 | F2 |
|--------|----|----|
| Coefficient factor contribution to $\varepsilon$ | -0.095 | 0.217 |
| Coefficient factor contribution to $R$ | 0.031 | 0.314 |
| Variables | $R_e$ | $V_G$ | $S/G$ | $\varepsilon$ | $R_b$ | $W/C$ | $\rho$ |
| Factor load | 0.94 | -0.99 | 0.98 | 0.78 | 0.98 | -0.94 | 0.79 |
| Eigenvalues | 4.160 | 2.573 |

The following tables provide summary data received with our experiences and the experiences of other authors.

In the consolidated results of the factor analysis, the observed pattern of the distribution of variables between the factors. Variables that characterize the structural parameters of concrete are highlighted in a separate factor, and the variables characterizing the strength characteristics of mortar and concrete in the individual. However, in the results of data processing are given in table 6,9 structural parameters characterizing the composition concrete – $V_G$ and $S/G$, is clearly not isolated to a specific factor, but the results obtained cannot with certainty describe the experiences as the determinant of a matrix $detA$ small.

Table 7. Summarized data according to factor analysis of results of experiments obtained [18].

| Factor | F1 | F2 |
|--------|----|----|
| Coefficient factor contribution to $E$ | 0.326 | -0.030 |
| Coefficient factor contribution to $R$ | 0.326 | -0.030 |
| Variables | $E_b$ | $R_b$ | $\rho$ | $W/C$ | $R_c$ | $V_G$ | $S/G$ |
| Factor load | 0.99 | 0.99 | 0.99 | 0.78 | -0.95 | 0.91 | -0.78 |
| Eigenvalues | 3.509 | 2.565 |

Table 8. Summary data for factor analysis results of experiments obtained in work [19]:

| Factor | F1 | F2 | F3 |
|--------|----|----|----|
| Coefficient factor contribution to $E$ | -0.026 | 0.033 | 0.835 |
| Coefficient factor contribution to $R$ | 0.136 | 0.391 | -0.299 |
| Variables | $V_G$ | $S/G$ | $\rho$ | $R_c$ | $E_b$ |
| Factor load | -0.980 | 0.865 | -0.977 | 0.913 | 0.959 |
| Eigenvalues | 3.268 | 1.307 | 1.102 |
Table 9. Summarized data according to factor analysis, results of our experiments on fast impingement creep

| Variables | F1 | F2 |
|-----------|----|----|
| $\rho$    | 0.24 | 0.299 |

Table 10. Summarized data according to factor analysis of results of experiments obtained in our experiments on modulus of elasticity

| Variables | F1 | F2 |
|-----------|----|----|
| $\rho$    | 0.203 | -0.110 |

The reason for this may be the presence of several highly correlated features in the primary data array, which lead to a significant reduction in the level of information, while $\text{det } R \rightarrow 0$, and the phenomenon is called multicollinearity in the array.

More detailed analysis of the original data table 6 shows that in conducting the experiments is the composition of the concrete with high activity of cement and high value of $W/C$, and Vice versa. This gives rise to the phenomenon of multicollinearity and degeneration of the matrix. To avoid this was conducted an additional factor analysis of the original data with alternate exception described cases: bootstrap-analysis. Thus distinguished three factors from a set of source characteristics. Variables that explain these factors in different ways, crashed as follows:

| Option | $F_1$ | $F_2$ | $F_3$ |
|--------|-------|-------|-------|
| 1      | $W/C$ | $V_G, W/C$ | $E_b, \rho$ |
| 2      | $E_b, R_b$ | $W/C, R_c$ | $E_b, S/G$ |

A similar operation was conducted with the data table 9, and the final results are given below. Variables describing factors in different ways, crashed as follows:

| Option | $F_1$ | $F_2$ |
|--------|-------|-------|
| 1      | $W/C, R_b$ | $V_G, S/G, \rho$ |
| 2      | $W/C, R_b$ | $V_G, \rho$ |

In the results of structural variables $V_G, S/G, \rho$ clearly allocated to one factor and the variables determining the strength of concrete $W/C$ and $R_b, R_C$ to another.

4. Conclusions

1. The results showed a tendency of separation of the studied variables characterizing the properties of concrete, into two groups. While the strength and deformation characteristics of concrete with a high degree of reliability can be described in two factors that explain 80-95% of the study variance.

2. The first factor is mainly characterized by variables that depend on strength and explains about 60% of the variance, the second factor combines variables describing the structure of the material (variables $V_G, S/G, \rho$) and add it additionally contributes 20-35% of the variance.
References

[1] N. I. Karpenko, V. A. Eryshev, and V. I. Rimshin, The Limiting Values of Moments and Deformations Ratio in Strength Calculations Using Specified Material Diagrams, *IOP Conference Series: Materials Science and Engineering*, 2018.

[2] A. A. Varlamov, and Yu. M. Krutsilyak, Evaluation of variations of structural and deformation characteristics of concrete during its operation: *Beton i Zhelezobeton* (5), pp. 14-16, 2003.

[3] A. A. Varlamov, and Yu. M. Krutsilyak, Method of evaluating the stressed-strained state of operated reinforced concrete: *Beton i Zhelezobeton* (6), pp. 18-20, 2005.

[4] E. Kuzina, V. Rimshin, and V. Kurbatov, The Reliability of Building Structures Against Power and Environmental Degradation Effects, *IOP Conference Series: Materials Science and Engineering*, 2018.

[5] E. Kuzina, and V. Rimshin, Strengthening of Concrete Beams with the Use of Carbon Fiber, *Advances in Intelligent Systems and Computing*, 2019.

[6] A. A. Varlamov, V. I. Rimshin, and S. Y. Tverskoi, The modulus of elasticity in the theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 2018.

[7] A. A. Varlamov, V. I. Rimshin, and S. Y. Tverskoi, The General theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 2018.

[8] A. A. Varlamov, and S. Y. Tverskoi, Experimental selection of young's modulus according the structure of concrete, *IOP Conference Series: Materials Science and Engineering* 451(1), 2018.

[9] A. A. Varlamov, S. Y. Tverskoi, and V. B. Gavrilov, Charting standard concrete based on the theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 463(2), 2018.

[10] A. A. Varlamov, V. I. Rimshin, and S. Y. Tverskoi, Security and destruction of technical systems, *IFAC-PapersOnLine*, 1(30), pp. 808-811, 2018.

[11] A. A. Varlamov, V. I. Rimshin, and S. Y. Tverskoi, The modulus of elasticity in the theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 463(2), 2018.

[12] A. A. Varlamov, E. L. Shapovalov, V. B. Gavrilov, Estimating Durability of Reinforced Concrete, *IOP Conference Series: Materials Science and Engineering*, 262(1), 2017.

[13] A. A. Varlamov, S. Tverskoi, and V. Gavrilov, Samples of concrete of small sizes, *E3S Web of Conferences* 91, 2019.

[14] V. Rimshin, B. Labudin, V. Morozov, A. Kazarian and V. Kazaryan, Calculation of Shear Stability of Conjugation of the Main Pillars with the Foundation in Wooden Frame Buildings, *Advances in Intelligent Systems and Computing*, 2019.

[15] A. A. Varlamov, and V. I. Rimshin, Behaviors of concrete. The General theory of degradation, *IN-FRA-M*, 436p., 2019.

[16] N. V. Smirnov, I. V. Dunin-Barkovsky, Course of probability theory and mathematical statistics. Moscow, *Nauka*, - 512 p., 1969.

[17] A. N. Bambar, V. J. Bachinsky, Guidelines for refined analysis of reinforced concrete elements taking into account the full chart of the compression concrete, Kiev, 1987.

[18] E. A. Guzeev, S. N. Leonovich, A. F. Milovanov, K. A. Piradov, and L. A. Salanov, Destruction of concrete and its durability, Minsk: *Tizen*, 1997.

[19] E. N. Lviv, Experimental investigations of engineering structures, Leningrad, *Izd-vo Nauka*, 1973.

[20] L. D. Dewatchenko, L. E. Valeeva, and D. Kamardina, Matrix of multidimensional observations in linear regression analysis, *Magnitogorsk*, - 36p., 1988.