Thermal relic abundance of the lightest Kaluza-Klein particle in phenomenological universal extra dimension models

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Abstract

Universal extra dimension models with Kaluza-Klein parity provide us excellent candidates for dark matter. We consider phenomenological universal extra dimension models where the Kaluza-Klein (KK) mass spectrum is different from that of the minimal universal extra dimension model, and compute the thermal relic abundance of the first KK mode of the photon taking into account the production of second KK particles. It is pointed out that its thermal relic abundance depends significantly on the mass degeneracy between the KK-photon and other KK particles because of considerable coannihilation effects. The cosmologically favored compactification scale is shown to range from around 1 TeV to a few TeV even in the cases where one of the first KK particles is tightly degenerate with the first KK photon in mass.

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The discovery of the Higgs boson with a mass of 125 GeV at the CERN Large Hadron Collider (LHC), which was announced on July 4th, 2012 [1, 2], is one of the most notable breakthroughs in the last decades. It has been clearly established that elementary particles in the Standard Model (SM) acquire mass through the Higgs mechanism. Meanwhile, no clear collider signatures that demand extensions of the SM have been found at the LHC. On the other hand, cosmological and astrophysical observations have accumulated evidence of new physics beyond the SM (BSM). Such BSM phenomena include the existence of dark matter (DM), the baryon asymmetry of the Universe, the cosmic inflation as well as neutrino oscillations. Revealing these phenomenological problems by extending the SM is a matter of primary importance.

As for issues concerning DM, one of the most promising candidates for DM is a Weakly Interacting Massive Particle (WIMP). It is remarkable that the existence of a terascale new physics is inferred from the condition that the thermal WIMP relic abundance coincides with the DM abundance $\Omega h^2 = 0.1$ determined most notably by the WMAP [3] and Planck [4] observations. This energy scale is now explored by the ongoing LHC Run-II and will be scrutinized by the future electron-positron colliders, such as International Linear Collider (ILC) [5–8], the Compact LInear Collider (CLIC) [9] and the Future Circular Collider of electrons and positrons (FCC-ee) [10], as well as DM direct and indirect detection experiments. Therefore, it is of particular importance to narrow down the energy scale of new physics models through the precise computation of the WIMP abundance for such experiments.

In this Letter, as a new paradigm realized at the terascale, we address models with Universal Extra Dimensions (UEDs) where all the SM particles freely propagate in the bulk of extra dimensions [11]. For reviews, see, for example Refs. [12–14]. It is intriguing that in UED models equipped with Kaluza-Klein (KK) parity, the Lightest KK Particle (LKP) is stabilized and serves as a candidate for WIMP DM [15]. One of the simplest UED models is the so-called the minimal UED (mUED) model, which assumes the existence of a flat fifth dimension compactified on an $S^1/Z_2$ orbifold with a compactification radius $R$ in order to obtain chiral fermions. In the mUED model, it is also hypothesized that brane localized operators are absent at the cutoff scale $\Lambda$, and that mass splitting among KK particles is attributed to radiative corrections [16]. Therefore, physical observables in the mUED model are controlled solely by the compactification scale $1/R$ and the cutoff scale $\Lambda$. The LKP in the mUED model is the first KK mode of the photon $\gamma^{(1)}$, which is the lighter eigenstate of
the admixtures of the first KK modes of the $U(1)_Y$ $B$-boson and the neutral $SU(2)_L W^3$-boson. Since KK particles are almost degenerate in mass at each KK level in the mUED model, a large number of coannihilation processes must be taken into account in evaluating the relic abundance of $\gamma^{(1)}$. The complicated computation of the relic abundance of $\gamma^{(1)}$ has been developed in Refs. [17–23]. In particular, it has been pointed out that the production of second KK particles gives the dominant contribution to the effective annihilation cross section [23]. The phenomenology of the mUED model has been extensively investigated from many aspects. Recent related works include Higgs phenomenology [24, 25], DM direct detection [23, 26], DM indirect detection [27] and collider tests [28–30]. The strongest experimental bound on the mUED parameter space is obtained through the LHC Run-II $3.2\text{ fb}^{-1}$ multijets plus missing transverse energy searches as $R^{-1} > 1100 \text{ GeV}$ insensitive to $\Lambda R$ [30]. On the other hand, it should be noticed that the mUED model is constructed based merely on minimality. Even in the framework of five-dimensional (5D) space-time, many extended UED models have been proposed including non-minimal UED models [31] and split UED models [32]. In addition, the form of renormalization group equations and resulting KK mass spectra are easily affected by introducing new multiplets below the cutoff scale $\Lambda$. The phenomenological and cosmological consequences of such extended UED models are quite different from those of the mUED model.

The goal of this paper is to investigate the thermal relic abundance of the $\gamma^{(1)}$ LKP in 5D phenomenological UED (pUED) models where the masses of KK particles are arbitrary. Particular attention is paid to the cases where the above-mentioned second KK particle production is significant. We show that the compactification scale consistent with the measured dark matter abundance is considerably dependent on the mass differences among KK particles.

We first briefly review the framework of 5D UEDs with the spatial extra dimension compactified on an $S^1/Z_2$ orbifold whose compactification radius is $R$, and resulting phenomenological consequences. The range of the fifth coordinate $y$ is $0 \leq y < \pi R$ because $y$ and $-y$ are identified by orbifolding. All the SM fields mediate in the bulk of the flat extra dimension, resulting in KK tours truncated at the cutoff scale $\Lambda$ in the four-dimensional (4D) viewpoint. In the mUED, the conservation of local 5D Lorentz symmetry and the absence of brane-localized operators are assumed at the cutoff scale $\Lambda$, where $n$th KK particles have a mass of $n/R$ up to contributions from the Higgs vacuum expectation value.
\( v = 246 \text{ GeV} \). However, note that the \( S^1 \) compactification violates the 5D Lorentz invariance, and that the \( Z_2 \) orbifolding does momentum conservation along the fifth dimension. Therefore, radiative corrections generate bulk and brane-localized operators that give rise to mass splitting among KK particles at the same KK level. Although the KK number conservation is violated, the KK parity defined by \( P = (-1)^n \) is still a conserved quantum number. Consequently, the LKP is stabilized and can become a candidate for DM. In the mUED, the LKP is the first KK photon \( \gamma^{(1)} \), which is the lighter mass eigenstate of the first \( U(1)_Y \) gauge boson, \( B^{(1)} \), and the neutral component of the first \( SU(2) \) gauge bosons, \( W^{3(1)} \). In the mass matrix in the \((B^{(1)}, W^{3(1)})\) basis, the diagonal components are substantially larger than the off-diagonal ones, which are in proportion to \( v^2 \), because \( 1/R \gg v \). In practice, the weak mixing angle of the first KK gauge bosons is small enough to take \( \gamma^{(1)} \simeq B^{(1)} \). It has been shown that the effective annihilation cross section of the \( \gamma^{(1)} \) LKP is significantly enhanced by the production processes of second KK particles and that the mass of the \( \gamma^{(1)} \) LKP consistent with the DM abundance is pushed above 1 TeV in the mUED model [23]. This is because the second KK production processes occur near a pole as \( m^{(2)} \simeq 2m^{(1)} \), and some of the produced second KK particles decay dominantly into a pair of SM particles.

Fig. 1 shows the thermal relic abundance of the first KK photon \( \Omega h^2 \) as a function of the compactification scale \( 1/R \) in the mUED model. The cases with \( \Lambda R = 5 \) (red line), 20 (orange) and 50 (blue) are plotted from the top. The green band shows the DM abundance at the 2\( \sigma \) level determined by Planck [4]. Here, we include all the coannihilation processes taking the production of second KK particles into account. In computing the KK mass spectrum and the thermal relic abundance of \( \gamma^{(1)} \), we use the same model files as Ref. [23] employs. Namely, these model files are generated by LanHEP [33] and implemented into CalcHEP [34] and micrOMEGAs4.3 [35]. For the details of the computations, see Ref. [23]. The mass of the Higgs boson is set at \( m_h = 125 \text{ GeV} \). From this figure, the cosmologically favored compactification scale, which is roughly the mass of the first KK photon \( \gamma^{(1)} \), is shown to range typically from around 1300 GeV to 1500 GeV in the mUED model.

We analyze the thermal relic density of the \( \gamma^{(1)} \) LKP in pUED models. We emphasize that the mUED model presented above is just a benchmark model of the framework of 5D UEDs on an \( S^1/Z_2 \). Namely, the mUED is the UED counterpart of the constrained Minimal Supersymmetric Standard Model (cMSSM) in that the number of free parameters is minimized. In general, UED models offer us a variety of phenomenological consequences.
Baring this situation in mind, we consider pUED models that allow arbitrary KK masses retaining the feature that the mass of the \( n \)th KK particle is \( n \) times that of the first KK particle in the same 5D field. To this end, we introduce new field strength parameters \( Z_A \), \( Z_\psi \), and \( Z_\Phi \) in the kinetic terms of the gauge bosons, fermions and Higgs boson in the 5D Lagrangian,

\[
\mathcal{L}_{(5D)}^{(A)} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2} Z_A F_{\mu5}^a F^{a\mu5},
\]
\[
\mathcal{L}_{(5D)}^{(\psi)} = \bar{\psi}_i \gamma^\mu D_\mu \psi_i - Z_\psi \bar{\psi}_i \gamma^5 D_5 \psi_i,
\]
\[
\mathcal{L}_{(5D)}^{(\Phi)} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - Z_\Phi (D_5 \Phi)^\dagger (D_5 \Phi) - \mu^2 \Phi^\dagger \Phi,
\]

respectively. Before the electroweak symmetry breaking, the (squared) masses of \( n \)th KK particles of the gauge bosons, fermions and Higgs boson are given by \( m_{A(n)}^2 = Z_A n^2 / R^2 \), \( m_{\psi(n)} = Z_\psi n / R \), and \( m_{\Phi(n)}^2 = Z_\Phi n^2 / R^2 + \mu^2 \), respectively. If we fix the values of these field strength parameters to those determined solely by the radiative corrections from the SM particles and their KK modes, we recover the mUED results. Instead of taking the mUED
values, we regard the field strength parameters as arbitrary and investigate to what extent the thermal relic density of the first KK photon is changed. It has been known that in UED models the inclusion of coannihilation processes can significantly affect the resultant relic density \[17–22\]. In discussing the coannihilation effects, it is convenient to introduce the following mass degeneracy parameter:

\[
\Delta X = \frac{m_{X(1)} - m_{\gamma(1)}}{m_{\gamma(1)}}. 
\]

We focus on the cases where only the mass of one of the first KK particles is taken arbitrary and set the others to the mUED values. Since the radiative corrections to the $B^{(1)}$-boson are negligible in the mUED, we obtain $Z_B \simeq 1$. Therefore, the mass degeneracy parameter for $X$ is roughly rewritten as $\Delta X \simeq Z_X - 1$. As in the mUED model, we set the bulk Higgs mass parameter at $\mu = 0$.

Fig. 2 shows the contours of the thermal relic abundance of the first KK photon satisfying $\Omega h^2 = 0.12$ in the $(1/R, \Delta X)$ plane. We investigate the degenerate cases where the first KK particle of $X$ is the first KK gluon $g^{(1)}$ (cyan line), the first KK $W$-bosons $W^{(1)}$ (light green), the first KK left-handed top quark $T^{(1)}$ (dark green), the first KK right-handed top quark $t^{(1)}$ (blue), the three generations of the first KK right-handed down-type quarks $3d^{(1)}$ (purple), the three generations of the first KK left-handed leptons $3E^{(1)}$ (pink), the three generations of the first KK right-handed leptons $3e^{(1)}$ (red), and the first KK Higgs bosons $H^{(1)}$ (orange). The other mass parameters than $X$ are set to the mUED values with $\Lambda R = 5$. As in the mUED case, we take into account all the contributions from the coannihilation modes with the other 1st KK particles including the production of 2nd KK particles in the final state. The forms of the KK-number violating vertices, which make the second KK particles decay, are set to those in the mUED model for simplicity. The nontrivial behavior for the $Z^{(1)}$ line stems from whether or not the mass of the second KK charged $W$-bosons, $W^{(2)\pm}$, is larger than the sum of those of $Z^{(1)}$ and the first KK charged Higgs bosons, $a^{(1)\pm}$. For comparison, the mUED prediction (black) is also shown. From this figure, it clear that in general the predicted compactification scale varies significantly depending on the mass degeneracy parameters. Even in these examples, the allowed compactification scale ranges from around 1 TeV to a few TeV.

In conclusion, we have evaluated the thermal relic abundance of the first KK photon $\gamma^{(1)}$
FIG. 2: The contours of the thermal relic abundance of the first KK photon satisfying $\Omega h^2 = 0.12$ in the $(1/R, \Delta_X)$ plane. We show the degenerate cases where the first KK particle of $X$ is the first KK gluon $g^{(1)}$ (cyan line), the first KK $W$-bosons $W^{(1)}$ (light green), the first KK left-handed top quark $T^{(1)}$ (dark green), the first KK right-handed top quark $t^{(1)}$ (blue), the three generations of the first KK right-handed down-type quarks $3d^{(1)}$ (purple), the three generations of the first KK left-handed leptons $3E^{(1)}$ (pink), the three generations of the first KK right-handed leptons $3e^{(1)}$ (red), and the first KK Higgs bosons $H^{(1)}$ (orange). The other mass parameters than $X$ are set to the mUED values with $\Lambda R = 5$. For comparison, the mUED prediction (black) is also shown.

including the production of second KK particles in pUED models where KK mass shifts are taken arbitrary. We have shown that the thermal relic abundance of the $\gamma^{(1)}$ LKP depends crucially on the mass degeneracy between $\gamma^{(1)}$ and other KK particles as coannihilation effects are significant. The range of the compactification scale consistent with the observed DM abundance has been shown to be from around 1 TeV to a few TeV if the mass of one of the first KK particles is tightly degenerate with that of the $\gamma^{(1)}$ LKP.

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