Three-Form Flux with $\mathcal{N} = 2$ Supersymmetry on $\text{AdS}_5 \times S^5$

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Abstract

In the context of the AdS/CFT correspondence the general form of a three-form flux perturbation to the $\text{AdS}_5 \times S^5$ solution in the type IIB supergravity which preserves $\mathcal{N} = 2$ supersymmetry is obtained. The arbitrary holomorphic function appearing in the $\mathcal{N} = 1$ case studied by Graña and Polchinski is restricted to a quadratic function of the coordinates transverse to the D3-branes.

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1. Introduction

It was proposed that the type IIB string theory compactified on AdS$_5 \times S^5$ has a dual description by the $\mathcal{N} = 4$ super Yang-Mills theory in the large $N$ limit [1, 2, 3]. This conjecture of the AdS/CFT correspondence has been supported by comparison of spectra, correlation functions and anomalies calculated in both of the supergravity and the Yang-Mills theory. (For a review, see ref. [4].) The AdS/CFT correspondence was also studied in various other spacetime dimensions. At first the correspondence was studied for theories with high supersymmetries such as $\mathcal{N} = 4$. To apply it to more realistic models one has to consider theories with lower supersymmetries.

One of the ways to obtain the AdS/CFT correspondence for lower supersymmetric cases is to modify supergravity solutions by adding a perturbation. In ref. [5] a perturbation of the three-form flux was added to the AdS$_5 \times S^5$, which breaks $\mathcal{N} = 4$ to $\mathcal{N} = 1$. This perturbation corresponds to fermion mass terms of the three $\mathcal{N} = 1$ chiral multiplets in the $\mathcal{N} = 4$ super Yang-Mills theory and polarizes D3 branes into 5-branes [6, 7]. Similar constructions of the AdS/CFT correspondence with lower supersymmetries were discussed in refs. [8, 9, 10, 11].

The general form of a three-form flux perturbation to the AdS$_5 \times S^5$ solution which preserves $\mathcal{N} = 1$ supersymmetry and satisfies the Bianchi identity and the linearized field equation was obtained in ref. [12]. It contains an arbitrary holomorphic function and an arbitrary harmonic function of the coordinates for the directions transverse to the D3-branes. It was argued that the holomorphic function corresponds to a superpotential in the dual super Yang-Mills theory. When the holomorphic function is quadratic in the transverse coordinates, the three-form flux coincides with that of ref. [5].

The purpose of the present paper is to obtain the general form of a three-form flux perturbation to the AdS$_5 \times S^5$ solution which preserves $\mathcal{N} = 2$ supersymmetry. We use the result of the $\mathcal{N} = 1$ case [12] and require further that the second supersymmetry is preserved. We find that the arbitrary holomorphic function in the $\mathcal{N} = 1$ case is restricted to a quadratic function of the transverse coordinates. This is a special form of the perturbation studied in ref. [5], which has one vanishing mass. It would be interesting to study a relation of our result to other works on soft breaking of $\mathcal{N} = 4$ to $\mathcal{N} = 2$ in the Coulomb branch [13, 14, 15]. In order to discuss the corresponding dual field theory and its RG flows we need to find...
out an exact solution with non-vanishing three-form flux. In addition, it would be also interesting to discuss the brane representations and massive vacua using S-dual transformations.

2. Unperturbed solution

The field content of the type IIB supergravity in ten dimensions \[16, 17\] is a metric \(g_{MN}\), a complex Rarita-Schwinger field \(\psi_M\), a real fourth-rank antisymmetric tensor field with a self-dual field strength \(F_{MNPQR}\), a complex second-rank antisymmetric tensor field with a field strength \(G_{MNP}\), a complex spinor field \(\lambda\) and a complex scalar field \(\tau = C + ie^{-\Phi}\). We denote ten-dimensional world indices as \(M, N, \cdots = 0, 1, \cdots, 9\) and local Lorentz indices as \(A, B, \cdots = 0, 1, \cdots, 9\). The fermionic fields satisfy chirality conditions \(\bar{\Gamma}^{10} D \psi_M = \psi_M\), \(\bar{\Gamma}^{10} D \lambda = -\lambda\), where \(\bar{\Gamma}^{10} D = \Gamma^0 \Gamma^1 \cdots \Gamma^9\) is the ten-dimensional chirality matrix. We choose the ten-dimensional gamma matrices \(\Gamma^A\) to have real components.

The field equations of this theory have a solution with the AdS\(_5\) \(\times\) S\(_5\) metric \[18, 19\]

\[
g_{MN} dx^M dx^N = Z^{-\frac{4}{5}} \eta_{\mu\nu} dx^\mu dx^\nu + Z^\frac{2}{5} \delta_{mn} dx^m dx^n, \tag{1}
\]

where \(M = (\mu, m)\) (\(\mu = 0, 1, 2, 3\); \(m = 4, 5, \cdots, 9\)), \(Z = \frac{R^4}{r^4}\) and \(r^2 = x^m x^n \delta_{mn}\). The constant \(R\) is a radius of AdS\(_5\) and S\(_5\). The fifth-rank field strength has non-vanishing components

\[
F_{\mu\nu\rho\sigma m} = \frac{1}{\kappa Z^2} \epsilon_{\mu\nu\rho\sigma} \partial_m Z, \\
F_{mnprq} = -\frac{Z^\frac{2}{5}}{\kappa} \epsilon_{mnprq} \partial^s Z, \tag{2}
\]

where \(\kappa\) is a coupling constant. This solution represents a supergravity configuration produced by D3-branes located at \(x^m = 0\). More generally, the warp factor \(Z\) can be an arbitrary function of \(x^m\) which is harmonic except at points where D3-branes exist. We will consider the general \(Z\) but assume that the density of D3-branes vanishes for \(r \to \infty\) and therefore \(Z \to \frac{R^4}{r^4}\) for \(r \to \infty\).

We are interested in how many supersymmetries are preserved by this solution and by a solution with a perturbation of \(G_{MNP}\) discussed later. They are found by
studying vanishing of local supertransformations of the fermionic fields $\psi_M$ and $\lambda$.

The supertransformations of the fermionic fields [16, 17] in these backgrounds are

$$\delta\psi_M = \frac{1}{\kappa} \mathcal{D}_M \epsilon + \frac{1}{16 \cdot 5!} i F_{P_1 \cdots P_5} \Gamma^{P_1 \cdots P_5} \Gamma_M \epsilon - \frac{1}{96} G_{NPQ} \left( \Gamma_M^{NPQ} - 9 \delta_M^{NPQ} \rho^{PQ} \right) \epsilon^*, $$

$$\delta\lambda = \frac{1}{24} G_{MNP} \Gamma^{MNP} \epsilon, $$

where the transformation parameter $\epsilon$ is a complex spinor satisfying the chirality condition $\bar{\Gamma}^{10D} \epsilon = \epsilon$. To study the supertransformations for the above backgrounds it is convenient to represent the ten-dimensional gamma matrices as

$$\Gamma^{\mu} = \gamma^{\mu} \otimes 1,$$

$$\Gamma^{m} = \bar{\gamma}^{4D} \otimes \gamma^{m},$$

where $\gamma^{\mu}$ and $\gamma^{m}$ are the SO(3,1) and SO(6) gamma matrices respectively. The chirality matrices are defined as

$$\bar{\gamma}^{4D} = i \gamma^{0} \gamma^{1} \gamma^{2} \gamma^{3}, \quad \bar{\gamma}^{6D} = i \gamma^{4} \gamma^{5} \gamma^{6} \gamma^{7} \gamma^{8} \gamma^{9},$$

which are related to the ten-dimensional one as $\bar{\Gamma}^{10D} = -\bar{\gamma}^{4D} \bar{\gamma}^{6D}.$

The above solution (1), (2) without a perturbation has 32 supersymmetries [18, 19]. This can be seen as follows. The supertransformation $\delta\lambda$ automatically vanishes, while the vanishing of $\delta\psi_M$ requires

$$\tilde{D}_M \epsilon = 0,$$

where we have defined

$$\tilde{D}^{\mu} = \partial^{\mu} - \frac{1}{8Z} \partial_{m} Z \gamma^{m} \gamma^{\mu}(1 + \bar{\gamma}^{4D}),$$

$$\tilde{D}^{m} = \partial^{m} - \frac{1}{8Z} \partial_{n} Z \left( \delta^{m}_{n} \bar{\gamma}^{4D} - \gamma^{m} \gamma^{n}(1 + \bar{\gamma}^{4D}) \right).$$

For solutions of eq. (6) to exist the integrability condition

$$[\tilde{D}_M, \tilde{D}_N] \epsilon = 0$$

must be satisfied. Using the expression (7) it is easy to show that eq. (8) is satisfied for an arbitrary $\epsilon$. Therefore, all of 32 supersymmetries are preserved [18, 19]. From the four-dimensional field theoretical point of view in the AdS/CFT correspondence
16 of them are Poincaré supersymmetries while other 16 are conformal supersymmetries. Thus, we have $\mathcal{N} = 4$ supersymmetry in four dimensions. More explicitly, the solutions of eq. (6) with the chirality $\bar{\gamma}_{4D} = -1$ have a form

$$\epsilon = Z^{-\frac{1}{8}}\eta,$$

(9)

where $\eta$ is an arbitrary constant spinor with the chirality $\bar{\gamma}_{4D} = -1$. These solutions correspond to Poincaré supersymmetries. The solutions with the chirality $\bar{\gamma}_{4D} = +1$ depend on $x^\mu$ and correspond to conformal supersymmetries.

3. Three-form flux with $\mathcal{N} = 2$ supersymmetry

By introducing a perturbation of the three-form flux $G_{mnp}$ the $\mathcal{N} = 4$ supersymmetry of the unperturbed supergravity background is broken to lower $\mathcal{N}$. In ref. [12] the conditions on $G_{mnp}$ for unbroken $\mathcal{N} = 1$ supersymmetry were studied. The supersymmetry parameter is expanded as $\epsilon = \epsilon_0 + \epsilon_1 + \cdots$, where $\epsilon_0$ is a solution of eq. (6) for the unperturbed background and $\epsilon_1$ is the first order correction due to the perturbation. Substituting it into eq. (6) $\epsilon_1$ is determined by $\epsilon_0$. To proceed it is convenient to define complex coordinates $z^i (i = 1, 2, 3)$ from $x^m$

$$z^1 = \frac{1}{\sqrt{2}}(x^4 + ix^7), \quad z^2 = \frac{1}{\sqrt{2}}(x^5 + ix^8), \quad z^3 = \frac{1}{\sqrt{2}}(x^6 + ix^9).$$

(10)

It was required in ref. [12] that one of the four Poincaré supersymmetries $\epsilon_0 = Z^{-\frac{1}{4}}\eta$, where $\eta$ is a constant spinor satisfying

$$\gamma^1 \eta = \gamma^2 \eta = \gamma^3 \eta = 0,$$

(11)

is preserved. Here, $\bar{i}$ denote indices of $\bar{z}^i$, while $i$ denote those of $z^i$. Using the expression $\bar{\gamma}_{6D} = (1 - \gamma^1 \gamma^1)(1 - \gamma^2 \gamma^2)(1 - \gamma^3 \gamma^3)$ it is easy to see that this $\epsilon_0$ has the chirality $\bar{\gamma}_{4D} = -\bar{\gamma}_{6D} = -1$ appropriate for the Poincaré supersymmetry. Then, this $\mathcal{N} = 1$ supersymmetry restricts the form of $G_{mnp}$ as [12]

$$G_{ijk} = 0,$$

$$G_{ijk} = \frac{2}{3} \hat{\epsilon}^{[p} \partial^{-2} \partial_p \partial_{i[} \partial_{j]} \partial_{[q} Z + \hat{\epsilon}_{[i} \partial_k \partial_{l]} \psi,$$

$$G_{ijk} = \frac{1}{12} \hat{\epsilon}_{[i} \delta_{jl]} \left( 2 \partial_i \partial_l \phi Z - \alpha \hat{\epsilon}_{il} \partial_k Z - 4 \partial_{i[} \phi \partial_{l]} Z \right),$$

$$G_{ijk} = \frac{1}{6} \hat{\epsilon}_{ijk} \delta_{il} \partial_i \phi \partial_l Z,$$

(12)
where $\phi(z^1, z^2, z^3)$ is an arbitrary holomorphic function, $\alpha$ is an arbitrary constant and $\psi$ is an arbitrary harmonic function.* In eq. (12) $\hat{\epsilon}_{ij}^k$ and $\hat{\epsilon}_{ij}^k$ are totally antisymmetric in their indices and take constant values 0, $\pm 1$ regardless of index positions, and $\partial^2 = 2\delta^a_i \partial_a \partial_i$ is the Laplacian. The three-form flux (12) also satisfies the Bianchi identity as well as the linearized field equation.

We shall obtain conditions on $G_{mnp}$ for unbroken $\mathcal{N} = 2$ supersymmetry. We require that in addition to $\epsilon_0 = Z^{-\frac{1}{2}}\eta$ the second supersymmetry with the parameter

$$\epsilon_0 = Z^{-\frac{1}{4}} \gamma^1 \gamma^2 \eta$$

is also preserved. This $\epsilon_0$ satisfies

$$\gamma^1 \epsilon_0 = \gamma^2 \epsilon_0 = \gamma^3 \epsilon_0 = 0$$

and has the chirality $\bar{\gamma}_{AD} = -1$. Comparing eqs. (11) and (14) it is easy to see that the conditions for the second supersymmetry are obtained from eq. (12) by the replacements

$$1 \leftrightarrow \bar{1}, \quad 2 \leftrightarrow \bar{2}, \quad \alpha \rightarrow \alpha', \quad \phi(z^1, z^2, z^3) \rightarrow \phi'(\bar{z}^1, \bar{z}^2, z^3), \quad \psi \rightarrow \psi'$$

for new $\alpha'$, $\phi'$ and $\psi'$.

We now require that the expression (12) and that with the replacements (15) are compatible each other. Let us first consider $G_{123}$. From the expression (12) we have $G_{123} = 0$. From the other expression we have $G_{123} = \frac{1}{6} \partial^2_3 \phi' Z$, which is obtained from $G_{123}$ in eq. (12) by the replacements (15). Thus we obtain a condition

$$G_{123} : \quad \partial^2_3 \phi' = 0.$$  

(16)

Similarly, we obtain conditions

$$G_{221} + G_{331} : \quad \partial_2 \partial_3 \phi' = 0,$$

$$G_{112} + G_{332} : \quad \partial_1 \partial_3 \phi' = 0,$$

$$G_{123} : \quad \partial^2_3 \phi = 0,$$

$$G_{221} + G_{331} : \quad \partial_2 \partial_3 \phi = 0,$$

$$G_{112} + G_{332} : \quad \partial_1 \partial_3 \phi = 0,$$

$$G_{123} : \quad \partial^2_3 \phi = \partial^2_3 \phi',$$

$$G_{123} : \quad \partial^2_3 \phi = \partial^2_3 \phi'.$$

(17)

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*In ref. [12] the constant $\alpha$ is required to vanish by the Bianchi identity. However, we do not agree with this result and leave $\alpha$ non-vanishing.
The component $G_{113} + G_{223}$ vanishes in both of the two expressions and gives no condition. These conditions fix the forms of $\phi$ and $\phi'$ as

$$
\phi = m_1 (z^1)^2 + m_2 (z^2)^2 + 2a z^1 z^2 + b_1 z^1 + b_2 z^2 + b_3 z^3,
$$

$$
\phi' = m_2 (z^1)^2 + m_1 (z^2)^2 + 2a' z^1 z^2 + b_1' z^1 + b_2' z^2 + b_3' z^3,
$$

where $m_1, m_2, a, a', b_i$ and $b_i'$ are arbitrary constants. We further obtain conditions

$$
G_{123} : \quad \partial_1^2 \psi = \partial_2^2 \psi', \\
G_{123} : \quad \partial_2^2 \psi = \partial_1^2 \psi', \\
G_{311} : \quad \partial_1 \partial_2 \psi = -\partial_1 \partial_2 \psi', \quad a = -a'.
$$

(19)

By a linear transformation $z^i \rightarrow U_{ij} z^j (i, j = 1, 2)$ with a unitary matrix $U$ we can set $a = -a' = 0$.

So far we have not used a particular form of $Z$. We now examine the remaining conditions first by using the asymptotic form $Z \sim R^4/r^4$ for $r \to \infty$ to fix the coefficients in eq. (18) and $\alpha, \alpha'$. We then check that the conditions are satisfied also for $r < \infty$.

From the equation for $G_{113}$ we obtain

$$
G_{113} : \quad -\frac{1}{6} \partial_1 \partial_2 \phi Z + \frac{1}{12} (\alpha \partial_3 Z + 2 \partial_1 \phi \partial_2 Z - 2 \partial_2 \phi \partial_1 Z)
$$

$$
= \frac{1}{6} \partial_1 \partial_2 \phi' Z - \frac{1}{12} (\alpha' \partial_3 Z + 2 \partial_1 \phi' \partial_2 Z - 2 \partial_2 \phi' \partial_1 Z).
$$

(20)

The equation for $G_{223}$ gives the same condition. Substituting the asymptotic form $Z \sim R^4/r^4$ and eq. (18) into eq. (20) we find $\alpha' = -\alpha$ and $b_1 = b_2 = b_1' = b_2' = 0$. The remaining conditions become

$$
G_{112} : \quad \partial_1 \partial_3 \psi' = \frac{1}{12} (\alpha \partial_2 + 2b_3 \partial_1) Z,
$$

$$
G_{331} : \quad \partial_2 \partial_3 \psi' = -\frac{1}{12} (\alpha \partial_1 - 2b_3 \partial_2) Z,
$$

$$
G_{123} : \quad \partial_3^2 \psi' = \frac{1}{6} b_3 \partial_3 Z,
$$

$$
G_{233} : \quad \partial_1 \partial_3 \psi = -\frac{1}{12} (\alpha \partial_2 - 2b'_3 \partial_1) Z,
$$

$$
G_{122} : \quad \partial_2 \partial_3 \psi = \frac{1}{12} (\alpha \partial_1 + 2b'_3 \partial_2) Z,
$$

$$
G_{123} : \quad \partial_3^2 \psi = \frac{1}{6} b'_3 \partial_3 Z.
$$

(21)
Comparing the equation obtained by applying $\partial_3$ to the first equation in eq. (21) and that obtained by applying $\partial_1$ to the third equation we find $\alpha = 0$. Then, eq. (21) determines $\psi, \psi'$ as

$$\partial_3\psi = \frac{1}{6}b'_3Z + f(z^1, z^2, \bar{z}^3),$$
$$\partial_3\psi' = \frac{1}{6}b_3Z + f'(\bar{z}^1, \bar{z}^2, z^3),$$

(22)

where $f$ and $f'$ are arbitrary functions of each variables. Substituting eq. (22) into the $\bar{z}^3$ derivative of eq. (19) and using the asymptotic form $Z \sim \frac{R^4}{r^4}$ we obtain $b_3 = b'_3 = 0$.

As a result of these analyses at asymptotic region $r \sim \infty$ we obtain

$$\phi = m_1(z^1)^2 + m_2(z^2)^2,$$
$$\phi' = m_2(z^1)^2 + m_1(z^2)^2.$$  

(23)

We have to check that eqs. (19), (20) and (21) are satisfied even for $r < \infty$. Substituting eq. (23) into eq. (21) we find that their right-hand sides vanish. The general solution of these equations are

$$\psi = f(z^1, z^2, \bar{z}^3)\bar{z}^3 + g(z^1, \bar{z}^1, z^2, \bar{z}^2, \bar{z}^3),$$
$$\psi' = f'(\bar{z}^1, z^2, \bar{z}^3)\bar{z}^3 + g'(z^1, z^1, z^2, \bar{z}^2, \bar{z}^3),$$

(24)

where $f, f', g$ and $g'$ are arbitrary functions of each variables. The conditions in eq. (19) then require

$$\partial_1^2 g = \partial_1^2 g', \quad \partial_2^2 g = \partial_2^2 g', \quad \partial_1 \partial_2 g = -\partial_1 \partial_2 g'.$$  

(25)

The conditions that $\psi$ and $\psi'$ in eq. (24) are harmonic are

$$\partial^2 g(z^1, \bar{z}^1, z^2, \bar{z}^2, \bar{z}^3) = -\partial_3 f(z^1, z^2, z^3),$$
$$\partial^2 g'(z^1, \bar{z}^1, z^2, \bar{z}^2, \bar{z}^3) = -\partial_3 f'(\bar{z}^1, z^2, z^3).$$

(26)

The functions $f$ and $f'$ do not appear in $G_{mnp}$ as one can see by substituting eq. (24) into eq. (12). We only need to consider $g$ and $g'$. Eq. (26) means that $\partial^2 g$ and $\partial^2 g'$ are independent of $\bar{z}^1, \bar{z}^2$ and $z^1, z^2$ respectively. These conditions are automatically satisfied when $g$ and $g'$ satisfy eq. (25). The functions $g$ and $g'$ need not be harmonic. Finally, we have to consider eq. (20). Substituting eq. (23) into eq. (20) we obtain

$$\left(m_1 z^1 \partial_2 - m_2 z^2 \partial_1 + m_2 \bar{z}^1 \partial_2 - m_1 \bar{z}^2 \partial_1\right)Z = 0.$$  

(27)
This means that $Z$ is invariant under SO(2) rotation of $(\sqrt{m_1} z^1, \sqrt{m_2} z^2)$ and $(\sqrt{m_2} \bar{z}^1, \sqrt{m_1} \bar{z}^2)$. Therefore, $Z$ must be a function of SO(2) invariant variables $r^2 = 2(z^1 \bar{z}^1 + z^2 \bar{z}^2)$, $m_1(z^1)^2 + m_2(z^2)^2$, $m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2$ and $m_1 z^1 \bar{z}^2 - m_2 z^2 \bar{z}^1$.

Let us summarize the result. The general form of the three-form flux $G_{mnp}$ which preserves the $\mathcal{N} = 2$ supersymmetry at the first order of the perturbation is given by eq. (12) with $\alpha = 0$, $\phi$ in eq. (23) and $\psi$ replaced by $g(z^1, \bar{z}^1, z^2, \bar{z}^2)$ satisfying eq. (25) for some function $g'(z^1, \bar{z}^1, z^2, \bar{z}^2)$. Thus, $\phi$, which is an arbitrary holomorphic function in the $\mathcal{N} = 1$ case [12], is severely restricted to a quadratic function in the $\mathcal{N} = 2$ case. Such $\mathcal{N} = 2$ preserving perturbation is possible only when the warp factor $Z$ satisfies eq. (27).

In our analysis at the first order of the perturbation we did not need the condition $m_1 = m_2$ to obtain the $\mathcal{N} = 2$ supersymmetry. At higher orders [20] we would need the condition $m_1 = m_2$ since these parameters correspond to masses of two $\mathcal{N} = 1$ chiral multiplets, which should be combined into an $\mathcal{N} = 2$ hypermultiplet. This is indeed the case in the field theory side. To see this let us consider two $\mathcal{N} = 1$ chiral supermultiplets $(A_1, \psi_1)$ and $(A_2, \psi_2)$, where $A_1$, $A_2$ are complex scalar fields and $\psi_1$, $\psi_2$ are Weyl spinor fields, with the action

\[
S = \int d^4x \left[ -\partial_\mu A_1^* \partial^\mu A_1 - \partial_\mu A_2^* \partial^\mu A_2 - i \psi_1 \sigma^\mu \partial_\mu \psi_1 - i \psi_2 \sigma^\mu \partial_\mu \bar{\psi}_2 \\
- m_1^2 A_1^* A_1 - m_2^2 A_2^* A_2 - \frac{1}{2} m_1 \left( \psi_1 \psi_1 + \bar{\psi}_1 \bar{\psi}_1 \right) - \frac{1}{2} m_2 \left( \psi_2 \psi_2 + \bar{\psi}_2 \bar{\psi}_2 \right) \right].
\]

(28)

Here we have used the two-component spinor notation in ref. [21]. $S$ is invariant under the $\mathcal{N} = 1$ supertransformation

\[
\delta A_i = \sqrt{2} \epsilon \psi_i, \quad \delta \psi_i = \sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_i - \sqrt{2} m_i \epsilon A_i^* \quad (i = 1, 2).
\]

(29)

The exact $N = 2$ supersymmetry of course requires $m_1 = m_2$. However, even for $m_1 \neq m_2$, it is also invariant under the second supertransformation

\[
\delta A_1 = \sqrt{2} \epsilon \psi_2, \quad \delta \psi_1 = \sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_2 - \sqrt{2} m_1 \epsilon A_2^*, \\
\delta A_2 = -\sqrt{2} \epsilon \psi_1, \quad \delta \psi_2 = -\sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_1 + \sqrt{2} m_2 \epsilon A_1^*
\]

(30)

at the first order in $m_1$, $m_2$. Thus, the condition $m_1 = m_2$ is needed only in quadratic and higher order terms for the $\mathcal{N} = 2$ supersymmetry.
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