Tight Bayesian Ambiguity Sets for Robust MDPs

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Why Robustness in Reinforcement Learning

- **Batch RL**: Learn from logged data
- Limited data leads to uncertain transition probabilities
- Brittle policies fail when deployed
- Unacceptable **risk** in high-stakes domains: medicine, industry, ...
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- **Batch RL**: Learn from logged data
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- Compute **robust** policies without being too **conservative**?
  - Optimize size and **location** of ambiguity sets in robust MDPs using (hierarchical) Bayesian models
Robust Reinforcement Learning

- Batch of domain samples (log data, no simulator):
  \[ s_1, a_1, r_1, s_2, a_2, r_2, \ldots, s_n, a_n, r_n \]

- **Robust policy** \( \pi \): Guarantee lower bound on **true** return \( \rho_{\text{true}}(\pi) \) when deployed
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  \( s_1, a_1, r_1, s_2, a_2, r_2, \ldots, s_n, a_n, r_n \)

- **Robust policy** \( \pi \): Guarantee lower bound on true return \( \rho_{true}(\pi) \) when deployed

- **Approach**: Estimate return \( \rho_{estim}(\pi) \) of \( \pi \) such that:
  1. Lower bound: \( \rho_{estim}(\pi) \leq \rho_{true}(\pi) \)
  2. Tractable: \( \max_{\pi} \rho_{estim}(\pi) \)

- Solve \( \max_{\pi} \rho_{estim}(\pi) \)
Robust Estimate of Policy Return

- Use **rectangular robust MDPs** \( \rho_{\text{estim}}(\pi) = p_0^T v_\pi^R \):
  \[
  v^R(s) = \max_a \min_{p_{s,a} \in \mathcal{P}_{s,a}} \left( r_{s,a} + \gamma \cdot p_{s,a}^T v^R \right)
  \]

- Ambiguity set: \( \mathcal{P}_{s,a} = \{ p \in \Delta^s : \| p - \bar{p}_{s,a} \|_1 \leq \psi_{s,a} \} \)

- \( \approx \) principled regularization

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**MDP**

\[ p_{s,a} = [0.4, 0.2, 0.2] \]

**Robust MDP**

\[ \bar{p}_{s,a} = [0.4, 0.2, 0.2], \psi_{s,a} = 0.4 \]
Research Challenge: Data-driven Ambiguity Sets

- Too small: not robust, too large: very conservative
- **Standard approach**: Concentration inequality around the \( \max \) likelihood estimate (UCRL, \ldots )

\[
\text{Guarantee } \rho_{\text{estim}}(\pi) \leq \rho_{\text{true}}(\pi) \text{ with }
\]

### 30% confidence

### 90% confidence
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Robust but too conservative to be practical!
Getting Robustness Right: Main Insights

1. Capture prior knowledge using (hierarchical) Bayesian models
2. Optimize size and location of ambiguity sets
3. Ambiguity set need not be a confidence interval (similar to Gupta [2018])
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Guarantee $\rho_{\text{estim}}(\pi) \leq \rho_{\text{true}}(\pi)$ with 90% confidence
RSVF: Optimizing Bayesian Ambiguity Sets

- Fixed value function $v^R$: Guarantee $\rho_{\text{estim}}(\pi) \leq \rho_{\text{true}}(\pi)$ if ambiguity sets intersects a hyperplane
- RSVF: Incrementally grow a set of plausible $v^R$ values

1. Guess $v^R$

1. $v^R = [0, 0, 1]$

2. $v^R = [0, 0, 1]$ or $[2, 1, 0]$

3. $v^R = [0, 0, 1]$ or $[2, 1, 0]$ or $[3, 1, 0]$
Uninformative Dirichlet Prior (95% confidence)

Smaller error means less conservative solution
Informative Hierarchical Prior (95% confidence)

![Graph showing calculated return error vs. number of samples]

Smaller error means less conservative solution
Conclusion

- Data-driven construction of robust ambiguity sets
  1. Capture prior knowledge using (hierarchical) Bayesian models
  2. Optimize size and **location** of ambiguity sets
  3. Ambiguity set need **not** be a **confidence interval**

- Pros:
  1. Robust but not too much
  2. Finite-sample guarantees
  3. Easy to define prior knowledge (e.g. Stan, PyMC)

- Cons:
  1. Increased computational complexity
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Thank you