Abstract: The consideration of the N-body gravitational problem equations can give to us some class of boundary-value problems defined on the "beem’s" construction. One can considere it as weak or so-called finite element method’s approximation with the linear function’s basis. So there are time and coordinate dependent beems as distances between points and unknown coordinate functions which are defined on the beems. The possible sample from this class is simple Lalplas equation where potential energy are maximal. Some computer simulations can give to us symmetries on spherical coordinates (similar for well known mathematical structures - configurations like Pascal or Desargus configurations).

But in other hand, the consideration of the natural mass distribution can give to us evidence of the physical reality of the similar structures on the star sphere. Moreover, there are coupled invariancy for geometry and arithmetics.

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Key words: N-body problem, finite elements, natural symmetries, configurations

Introduction.

So, it is well known that there is a structure of young O, B and A0 stars. Herschel (1847) was the first who noted and statistically convincing arguments were given by Guld (1879), that there is so called Guld zone. There was discussion of physical validity and stability of problem of so called O, B associations and superassociations [1], [2], [3]. At the last time appeared the results on the periodicity for the clusters of the galaxies [4], [5]. Possibly there is mathematical unity for some astrophemonema or in other words automodility for mass distribution so J.Einasto wrote: ’ If this reflects the distribution of all matter (luminous and dark), then there must exists some hitherto unknown process that produces regular structure on large scales’ [5]. But word ‘unknown’ does not mean existence of really new physical law which take place only for large scales. We hope that there are mathematical methods yet and order for distances smaller than cosmological sizes. We start our investigation from famous and old N-body problem [6], then for assemblies of bodies with low velocities we can find coherent structures.
But there is similar visual coherency, moreover automodel symmetry for some scales on natural coordinate systems (spherical systems of coordinates related with momentum).

**N-body equations for** $\ddot{x}, \ddot{y}, \ddot{z} = 0$.

Let us consider $N$-body equations for $N$ points with masses $M_j$ on OXYZ coordinate system [7]:

$$\ddot{u}_i = \sum_{j=1}^{j=N'} M_j (u_j - u_i) / r_{ij}^3, u_i = x_i, y_i, z_i$$ \hspace{1cm} (1)

where $r_{ij}$ is distance between points $i$ and $j$, sign ' noted so there is no case $i=j$.

After decomposition we can write:

$$\ddot{u}_i = -u_i (\sum_{j=1}^{j=N'} M_j / r_{ij}^3) + (\sum_{j=1}^{j=N'} u_j M_j / r_{ij}^3), u_i = x_i, y_i, z_i$$ \hspace{1cm} (2)

Note, that right parts of (2) are equal to product of the positive matrix $A(x, y, z)$ (by weak diagonal predominance) on the vector $w = (x_1, x_2, ... x_N, y_1, ... y_N, ... z_N)$.

Then let us consider (2) as finite approximation of some boundary-value problem on the "beem’s" mesh where lengthess of beems or in other words distances between points must be solution depended.

We’ll try find form from our finite-difference equations or 'revers motion' on well known transformations:

$$\rho_t + (\rho \phi_x)_x + (\rho \phi_y)_y = 0, \phi_n = 0$$ \hspace{1cm} (3)

$$v(x, y)(\rho_t + (\rho \phi_x)_x + (\rho \phi_y)_y) = 0,$$ \hspace{1cm} (4)

$$\int_\Omega (v \rho_t - \rho \phi_x v_x - \rho \phi_y v_y) d\omega = 0$$ \hspace{1cm} (5)

For right part we can find function $f(l)$ from:

$$\text{RightPart} = \int_L f(l) u_l v_l dl$$ \hspace{1cm} (6)

where $L$ is beem in the distinguished moment of the time and integration are going along directions between points, $v(l)$ is test function.

Then we can take finite basis of functions $v_i(l)$ (Figure 1) which related with points and which have finite region where $v(l)$ is not equal to zero (so-called
finite elements [8]). In (3) we have:

\[ u(l) = \sum_{k=1}^{k=N} \alpha_k v_k(l) \]  

(7)

After that we have that \( f(l) = M_j/R^2 \) constant on every beam where \( R = r_{ij} \).

For left part we have:

\[ \text{LeftPart} = 2/K \left( \int_L v(l) u_{tt}/R - R/6u_{ttv}dl \right) \]  

(8)

where \( K \) is the number of the nodes which significant connected with the given one.

In differential form and for the equal masses \( M \) we have:

\[ M u_{tt}/R + (M R/6u_{tt})_t = (M^2/R^2 u_t)_t \]  

(9)

where we take \( K = 2 \) without less of generality with respect to physical case.

We can consider the row of equations where one of them is (6) and others are:

\[ M/R u_{tt} = (M^2/R^2 u_t)_t \]  

(10)

\[ (M/R u_t)_t = 0 \]  

(11)

where \( R \) is equal to small number when we take by unit size of our constellation and gravitational interaction between points defined on finite distance which are less than \( MR \) for some natural \( M \).

Let us consider (11). We can see so there are spherical symmetry for form (11) and hence there are some hope for symmetries on spherical representations on 'natural' (that it to say spherical) coordinates.

There are easy way for computer simulation. Let us consider volume \( G \) and points with equal masses where some points (set \( A \)) are into the volume \( G \) and other points (set \( B \)) are outside the volume \( G \). The every point is surrounded by an \( P \) envelope of some (set \( P \) is not equal \( A+B \)) points. Note, near \( G \) boundary \( P_z \) have points from \( A \) and \( B \) too.

Let’s define iterative motion for every point \( z \) from volume \( G \) into mass center of \( P_z \). Then after some steps we will have solution for (11).

We can see some examples of simulation on Figures 2, 3, 4 where \( P \) defined by choosen distance.

By the way — it is simple and fast algorithm for Steiner problem [10] for two or three dimensional cases — Figure 5.

Application
Well known so equation (11) seems as fully unacceptable for real masses and formal solutions of (11) seems as very inconstant by time.

But let us consider nearest and well known light galaxies — M31, M81, M82, M83, M101, NGC 5128, Dwingeloo1 (data from nedwww.ipac.caltech.edu).

We can see pictures — Figures 6,7,8,9 which are similar for our computer simulation.

At next let us consider "histogram" for natural stars as distribution on the star's sphere or in other words as the distribution on the one of the planes: (l, b), (α, δ) or (λ, β) on the one or two coupled rectangles of size $2\pi \times \pi$ (there are periodicity). Also there is physical value for star which can be correlated with possible order. It is so-called visual brightness or in the order words order which was related with some degree of the $M^2/R_{0.6}$ where $R$ is distance between star and view point [11]. Below are given the enumeration for 50 lightest stars [11]:

1 αCma 2 αCar 3 αBoo 4 αLyr 5 αCen 6 αAur 7 βOri 8 αCmi 9 αOri 10 αEri 11 βCen 12 αAql 13 αCru 14 αTau 15 αSco 16 αVir 17 βGem 18 αPsa 19 βCru 20 αCyg 21 αLeo 22 εCma 23 αGem 24 λSco 25 γOri 26 γCru 27 βTau 28 βCar 29 εOri 30 αGru 31 εMa 32 ζOri 33 αUma 34 αPer 35 γVel 36 εSgr 37 δCMa 38 ηUma 39 εCar 40 δSco 41 βAur 42 γGem 43 αTra 44 δVel 45 αPav 46 βCMa 47 αHy 48 αCet 49 αAri 50 αUMi

As an example, let us consider the stars with numbers 5 αCen, 6 αAur, 8 αCmi, 12 αAql, 14 αTau, 15 αSco on (λ, β) coordinate plane (or on cilinder) — Figure 10.

It is an approximate example of graph of a new kind, similar for graceful graphs [12] and harmonious graphs [13], where arithmetic, geometry and topology are tightly coupled.

We can give definition:

each point $i$ belongs to a straight line drawn through other two points $j,k$ from the list, the sum of $j$ and $k$ being equal to 20.

where is exception to the rule for point 5.

Also we have on the (l, b) and (α, δ) planes, Figures 11, 12 —

for every points $i,k,j$ which belong to a one of the straight lines there is other straight line with points $i$, $j$, $k$, or $i$, $j$, 20-$k$ or $i,j-20$,k-20 (on (l, b)).

Note that these are really the nearest extremally massive objects (with exception of point 15). Moreover, there are the correlations with velocity vectors.
for presented configurations (or quasi-configurations) [14] in the all planes by time.

Also we can construct similar figures for other viewpoints - view from $\alpha Cmi$ give to us Figure 13, view from $\alpha Cen$ Figure 14, view from $\alpha Aql$ Figure 15. We can see so there are same numbers— 5, 6, 8, 12, 14, 15.

Let us consider the stars with the numbers $5k$ and stars nearest by same order. We have paradoxical meshes of the parallel lines for these stars— Fig. 16 ($\lambda, \beta$) plane, Figure 17 ($\alpha, \delta$) plane, and Figure 18— for ($l, b$) planes (with parallelism on projective sense). Note that these are really extremely massive objects— more than 5 Sol’s mass.

With rare exception, most part of $O, B$ and $A0$ stars (brighter than 6.0$^m$) is concentrated near the straight lines drawn through pairs of the stars with numbers 2, 3, 5, 6, 7, 10, 11, 15— Figure 19; that is true for ($l, b$), ($\alpha, \delta$), ($\lambda, \beta$) coordinate planes.

There is a substructure of stars of $\beta Per$ (EA) and $\beta Lyr$ (EB) types, which concentrate only to some of the lines in all the coordinate systems. It is an independent test, the brightness of these stars is $8 - 9^m$ on the average. $K, M$ giants also gravitate towards these lines, but their positions are more smeared than those of $O, B$ stars.

**Conclusion**

Perhaps phenomenon does not completely visual and there are order on native coordinate representation for equations (1) and (11). Maybe gravitational interactions for far and large masses are masked by heat motions of small stars and interstellar media.

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1. $v_i = v_i(l)$, 
$v_{il} = 1/R_{ij}$

Fig. 1 Example of nodes related linear function
Fig. 2. Simulation, 4 points, spherical plane, one on the viewpoint.
Fig. 3. Simulations result, 5 points, one on the viewpoint.
Fig. 4. Simulation result, some points.
Fig. 5 Circles - 'boundary' points, filled circles-points in the 'volume', 2 and 3 D Steiner configurations,
Fig. 6. Light and nearest galaxies, lighter than $8.5^m$ -(l,b) representation.
Fig. 7. Light and nearest galaxies, lighter than 8.5$^m$, $(\lambda, \beta)$ representation.
Fig. 8. \((\alpha, \delta)\)-representation.
Fig. 9. Supergalactic plane representation.
Fig 10. 5- αCen, 6-αAur, 8- alphaCmi, 12 -αAql, 14- αTau, 15- αSco (λ, β) plane.
Fig. 11 (l,b) plane.
Fig 12 ($\alpha, \delta$) plane.
Fig 13. Viewpoint is $\alpha Cmi$, 5- $\alpha Boo$, 6- $\beta Gem$, 8- $\alpha Tau$, 12- $\alpha Cru$, 14- $\alpha Gem$, 15- $\alpha Sco$
Fig. 14 Viewpoint is $\alpha$Cen, 5-$\alpha$Lyr, 6-$\alpha$Aur, 8-$\alpha$Ori,

12-$\alpha$Cru, 14-$\alpha$Sco, 15-$\alpha$Vir
Fig. 15. Viewpoint $\alpha Aql$
Fig 16 ($\lambda$, $\beta$) "multiplied by 5".
Fig 17 (\(\alpha, \delta\)) "multiplied by 5".
Fig. 18 For first 50 stars on (l,b).
Fig 19 For first light 500 stars on $\alpha, \beta$.

- EA, EB stars