Explosive Dark Matter Annihilation

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If the Dark Matter (DM) in the Universe has interactions with the standard-model particle, the pair annihilation may give the imprints in the cosmic ray. In this paper we study the pair annihilation processes of the DM, which is neutral, however has the electroweak (EW) gauge non-singlet. In this estimation the non-relativistic (NR) effective theory in the EW sector is a suitable technique. We find that if the DM mass is larger than about 1 TeV, the attractive Yukawa potentials induced by the EW gauge bosons have significant effects on the DM annihilation processes, and the cross sections may be enhanced by several orders of magnitude, due to the zero energy resonance under the potentials. Especially, the annihilation to two γ’s might have a comparable cross section to other tree-level processes, while the cross section under the conventional calculation is suppressed by a loop factor. We also discuss future sensitivities to the γ ray from the galactic center by the GLAST satellite detector and the Air Cerenkov Telescope (ACT) arrays.

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Nature of the Dark Matter (DM) in the Universe is an important problem in both particle physics and cosmology. The Weakly-Interacting Massive Particle (WIMP), χ0, is a good candidate for the DM. It works as the cold dark matter in the structure formation in the Universe. High resolution N-body simulations show that the cold dark matter hypothesis explains well the structure larger than about 1 Mpc [1]. Also, the WMAP measured the cosmological abundance precisely as ΩDM = 0.27 ± 0.04 [2]. Now we know the gravitational property of the DM in the structure formation and the abundance and distribution in the cosmological scale. The next questions are the constituent of the DM and the distribution in the galactic scale.

If the DM is SU(2)L non-singlet, a pair of the DM could annihilate into the standard-model (SM) particles with significant cross sections [3]. We call such DM’s as electroweak-interacting massive particle (EWIMP) DM in this paper. The detection of exotic cosmic ray fluxes, such as positron, anti-proton and γ ray, may be a feasible technique to search for the DM’s. Since some DM candidates in the supersymmetric (SUSY) models have interactions with the SM particles, these annihilation processes are extensively studied. Especially, excess of monochromatic γ ray due to the pair annihilation is a robust signal if observed, because the diffused γ-ray background must have a continuous energy spectrum [4]. Searches for the exotic γ ray from the galactic center, the galactic halo, or even from extra galaxies are ones of the projects in the GLAST satellite detector and the big Air Cerenkov Telescope (ACT) arrays such as CANGAROO III, HESS, MAGIC and VERITAS.

In the previous estimates, the cross sections for the EWIMP are evaluated at the leading order in the perturbation. However, the DM is non-relativistic (NR) in the current Universe. In this case, if the EWIMP mass m is much heavier than EW scale, the EWIMP wave function may be deformed under the Yukawa potentials induced by the EW gauge boson exchanges and it may give a non-negligible effect in the annihilation processes. Furthermore, the neutral EWIMP should has a charged SU(2)L partner, χ±. When the EWIMP is heavier than EW scale, their masses are almost degenerate, and the unsuppressed transition between the two-body states of 2χ0 and χ−χ+ may play an important role in the 2χ0 pair annihilation.

In this letter we reevaluate the pair annihilation cross sections of the EWIMP’s, for the two cases that the DM is a component of two SU(2)L-doublet fermions or of an SU(2)L-triplet fermion. These correspond to the Higgsino-like and Wino-like DM’s in the SUSY models, respectively. Most interesting fact we find is that the annihilation cross sections to the gauge boson pairs for the SU(2)L-doublet (triplet) DM suffer from a zero energy resonance around m ∼ 6(2) TeV, whose binding energy is zero [5] under the potential. Therefore, the cross sections would be enhanced significantly compared with ones in the perturbative estimations for m > 1(0.5) TeV. Furthermore, it is found that the cross section for 2χ0 → γγ, which is usually suppressed by a one-loop factor, becomes comparable to the other tree-level processes, such as 2χ0 → W+W−, around the resonance. This means that the mixing between the two-body states of χ−χ+ and 2χ0 is maximal under the potential. Due to the explosive enhancement of the cross sections, the SU(2)L-triplet DM is already partially constrained by the EGRET observation of the γ ray from the galactic center, and the future γ ray searches may have sensitivity to the heavier EWIMP DM.

First, we summarize properties of the EWIMP DM’s. If the DM has a vector coupling to the Z boson, the current bound from the direct DM searches through the spin-independent interaction [6] is stringent. This means that the EWIMP DM should be a Majorana fermion or a real scalar if it is relatively light. Here we consider a former case for simplicity.
A simple example for the EWIMP DM's is a neutral component of an SU(2)\(_L\)-triplet fermion (T) whose hypercharge is zero. This corresponds to the Wino-like LSP in the SUSY models. It is accompanied with the a charged fermion, \(\chi^{\pm}\). While \(\chi^0\) and \(\chi^{\pm}\) are almost degenerate in mass in the SU(2)\(_L\) symmetric limit, the EW symmetry breaking by the Higgs field, \(h\), generates the mass splitting, \(\delta m\). If \(\delta m\) comes from the radiative correction, \(\delta m \simeq 1/2\alpha_2(m_W - c_W^2m_Z) \sim 0.18\) MeV for \(m \gg m_W\) and \(m_Z\). Here, \(m_W\) and \(m_Z\) are the \(W\) and \(Z\) boson masses, respectively, and \(c_W(\equiv \cos\theta_W)\) is for the Weinberg angle. Effective higher-dimensional operators, such as \(h^3T^2/\Lambda^3\), also generate \(\delta m\), however they are suppressed by the new particle mass scale \(\Lambda\). The thermal relic density of the DM with mass around 1.7 TeV is consistent to the WMAP data.

Another example for the EWIMP DM's is a neutral component of a pair of SU(2)\(_L\)-doublet fermions (\(D\) and \(D'\)) with the hypercharges \(\pm 1/2\). This corresponds to the Higgsino-like LSP in the SUSY models. The \(\chi^0\) is accompanied with a neutral Majorana fermion, \(\chi^0\), as well as a charged Dirac fermion, \(\chi^{\pm}\). They are again degenerate in mass in the SU(2)\(_L\) symmetric limit. The mass difference is generated by the effective operators, such as \(h^3D^2/\Lambda\), via the EW symmetry breaking. The thermal relic density of the DM explains the WMAP data when the mass is around 0.6 TeV.

In the current Universe the DM is expected to be highly non-relativistic as mentioned before. In this case, the perturbative pair annihilation cross sections of the EWIMP DM may have bad behaviors if the DM mass is heavier than the weak scale. One of the example is the annihilation cross section to 2\(\gamma\) at the leading order. The process is induced at one-loop level, and the cross section is \(4(1/4)\pi\alpha_2^2c_2^2/m_W^2\) for the SU(2)\(_L\)-triplet (doublet) DM in the SU(2)\(_L\) symmetric limit. The cross section is not suppressed by \(1/m^2\), and the perturbative unitarity is violated when \(m\) is heavy enough.

The NR effective theory is useful to evaluate the cross sections in the NR limit. In Ref. [7] we studied the NR effective theory for the EWIMP in a perturbative way and found that the trouble in the cross section to 2\(\gamma\) is related to the threshold singularity. In order to evaluate the cross section quantitatively, we have to calculate the cross section non-perturbatively using the NR effective theory [12].

For evaluation of the annihilation cross sections for heavy EWIMP, we need to solve the EWIMP wave function under the EW potential. In this paper, we show the formulae for evaluating the cross sections in the SU(2)\(_L\)-triplet DM case. Those for the SU(2)\(_L\)-doublet case will be shown in the further publications [8].

The NR effective Lagrangian for two-body states, \(\phi_N(r)(\simeq 1/2\chi^0\chi^0)\) and \(\phi_C(r)(\simeq \chi^-\chi^+\), is given as

\[
\mathcal{L} = \frac{1}{2} \Phi^T(r) \left( \left( E + \frac{\nabla^2}{m} \right) 1 - \mathbf{V}(r) + 2i\Gamma\delta^3(r) \right) \Phi(r),
\]

where \(\Phi(r) = (\phi_C(r), \phi_N(r))\), \(r\) is the relative coordinate \((r = |r|)\), and \(E\) is the internal energy of the two-body state. The EW potential \(\mathbf{V}(r)\) is

\[
\mathbf{V}(r) = \left( \begin{array}{cc} 2\delta m - \frac{\alpha}{r} - \alpha_2^2c_2^2 \frac{e^{-m_Zr}}{r} - \sqrt{2}\alpha_2 \frac{e^{-m_Wr}}{r} & -\sqrt{2}\alpha_2 \frac{e^{-m_Wr}}{r} \\ -\sqrt{2}\alpha_2 \frac{e^{-m_Wr}}{r} & 0 \end{array} \right).
\]

(2)

In this equation we keep only \(2\delta m\) in (1,1) components in order to calculate the DM annihilation rate up to \(O(\sqrt{\delta m/m})\) [7]. \(\Gamma\) is the absorptive part of the two-point functions. Note that a factor of 1/2 (1/\(\sqrt{2}\)) is multiplied for \(V_{22}\) and \(\Gamma_{22}\) (\(V_{12}\) and \(\Gamma_{12}\)) since \(\phi_N\) is a two-body state of identical particles. Thus, \(\Gamma_{22}\) (\(\Gamma_{11}\)) is the tree-level annihilation cross section multiplied by the relative velocity \(v\) and 1/2(1). Since the SU(2)\(_L\)-triplet DM is assumed to be a Majorana fermion, the \(1S\)-wave gauge contribution to \(\Gamma\) is relevant to the NR annihilation, and then,

\[
\Gamma = \frac{\pi\alpha_2^2}{m^2} \left( \frac{3}{2\sqrt{2}} \frac{1}{2\sqrt{2}} \frac{1}{1} \right).
\]

The annihilation cross section of \(\chi^-\chi^+\) or \(2\chi^0\) to the EW gauge boson pair can be expressed using the two-by-two Green function, \(G(r, r')\), which is given by

\[
\left( \left( E + \frac{\nabla^2}{m} \right) 1 - \mathbf{V}(r) + 2i\Gamma\delta^3(r) \right) G(r, r') = \delta^3(r - r')1.
\]

(4)

Due to the optical theorem, the long-distance (wave function) and the short-distance (annihilation) effects can be factorized [9]. The annihilation cross sections to \(VV'\) (\(V, V' = W, Z, \gamma\)) are written as

\[
(\sigma\nu)_{VV'} = c_i \sum_{ab} \langle ab\rangle \nu_{VV'} \times A_a A_b^*\,
\]

where \(i\) represents the initial state (\(i = 0\) and \(\pm 2\chi^0\) and \(\chi^-\chi^+\) pair annihilation, respectively) and \(A_a = \int d^3r e^{-i\mathbf{k}\cdot\mathbf{r}} (E + \nabla^2/M) G_{ai}(r, 0)\) with \(\mathbf{k} = \sqrt{mE = mv/2}\). Here \(c_0 = 2\) and \(c_{\pm} = 1\), where \(c_0\) is a factor needed to compensate the symmetric factor for \(\Gamma\) and \(\mathbf{V}\). \(\langle ab\rangle_{VV'}\) is the contribution to \(\Gamma_{ab}\) from the final states \(VV'\). It is clear that if the long-distance effect is negligible, \(\langle ab\rangle_{VV'} = c_i \Gamma_{ai}\).

The S-wave annihilation is dominant in the NR annihilation. Thus, the Green function is reduced to \(G(r, r') = g(r, r')/rr'\). Similar to the case in one-flavor system, we find that \(g(r, r')/rr'\) is expressed by the independent solutions of the homogeneous part of the Eq. (4), \(g_{>}(r)/r\) and \(g_{<}(r)/r\), as

\[
g(r, r') = \frac{m}{4\pi} g_{>}(r) g_{>}(r') \theta(r - r')
\]

\[
+ \frac{m}{4\pi} g_{<}(r) g_{<}(r') \theta(r' - r).
\]

(6)

The solutions \(g_{>}(r)\) and \(g_{<}(r)\) are also two-by-two matrices since \(\Phi(r)\) has two degrees of freedom. The boundary
conditions at \( r = 0 \) are \( g_\lambda(r)|_{r=0} = 0 \), \( g'_\lambda(r)|_{r=0} = 1 \), and \( g''_\lambda(r)|_{r=0} = 1 \). In the following, we assume \( E < 2\delta m \) so that a pair annihilation of \( \chi^0 \) does not produce on-shell \( \chi^-\chi^+ \). As the result,

\[
g_\lambda(r)|_{r=\infty} = \begin{pmatrix} 0 & 0 \\ d_1 e^{ikr} & d_2 e^{ikr} \end{pmatrix}.
\]  

In this case, the \( \chi^0 \)-pair annihilation cross sections are \( (\sigma v) \propto V^2 \propto (1 + \sum_{ij} |V_{ij}|^2) \), as expected. It is enough to calculate \( d \) in order to evaluate the cross sections.

In Fig. (1) we show the annihilation cross sections of the SU(2)\(_L\)-triplet DM pair to \( 2\gamma \) and \( W^+W^- \) as functions of \( m \). We evaluated the cross section numerically. Here, we take \( v/c = 10^{-3} \), which is the typical averaged velocity of the DM in our galaxy, and \( \delta m = 0.1 \) GeV and 1 GeV. The perturbative cross sections are also plotted. Large \( \delta m \) leads to unreliable numerical calculation for large \( m \), and then some curves are terminated at some points. However, \( \delta m \) should be suppressed around the regions.

When \( m \) is around 100 GeV, the cross sections to \( 2\gamma \) and \( W^+W^- \) are almost the same as the perturbative ones. The cross section to \( 2\gamma \) is suppressed by a loop factor there. However, when \( m \gtrsim 0.5 \) TeV, the cross sections are significantly enhanced and have the resonance structure. Especially, the cross section to \( 2\gamma \) becomes comparable to that to \( W^+W^- \) around the resonance. This suggests that the \( 2\chi^0 \) state is strongly mixed with \( \chi^-\chi^+ \). The cross section (9) is reduced to \( 4\pi\alpha^2/\sqrt{\tilde{m}_W^2} \) for \( \alpha_2 m \lesssim m_W \). On the other hand, it is not suppressed by a one-loop factor for \( \alpha_2 m \gtrsim m_W \) and has a correct behavior as \( \sim 1/m^2 \) in a heavy \( m \) limit. When \( k_\perp R = (2n-1)\pi/2 \) \((n = 1, 2, \cdots)\), the zero energy resonance, whose binding energy is zero, appears and the cross section is enhanced significantly. In Fig. (1), the \( n \)-th zero energy resonance appears at \( m = m_{(n)} \sim n^2 \times m_{(1)} \), while the well potential \( V = V_W \) and the unitarity is not broken.

We also show the annihilation cross sections for the SU(2)\(_L\)-doublet DM in Fig. (1). The SU(2)\(_L\)-doublet DM has the smaller gauge charges compared with the SU(2)\(_L\)-triplet DM. As the result, the cross section is smaller, and the first zero energy resonance appears at 5 TeV. The enhancement for the DM annihilation rates gives significant impacts on the indirect searches for the DM.

![FIG. 1: The \( \chi^0 \)-pair annihilation cross sections to \( 2\gamma \) and \( W^+W^- \) when \( \delta m = 0.1 \) GeV (slid lines) and 1 GeV (dashed lines). \( \chi^0 \) is the SU(2)\(_L\)-triplet or doublet DM. Here, \( v/c = 10^{-3} \). The leading-order cross sections in the perturbation are also shown for \( \delta m = 0 \) (dotted lines).](image-url)
The line $\gamma$ from the pair annihilation to $2\gamma$ or $Z\gamma$ at the galactic center is a robust signal for the DM. Also, $W^-W^+$ and $2Z^0$ final states produce the continuum $\gamma$ spectrum through $\pi^0 \to 2\gamma$, and the observation may constrain the EWIMP DM. The $\gamma$ flux, $\Psi_\gamma(E)$, is given as

$$\frac{d\Psi_\gamma(E)}{dE} = 9.3 \times 10^{-12} \text{cm}^{-2} \text{sec}^{-1} \text{GeV}^{-1} \times \bar{J} \Delta \Omega$$

$$\times \left(\frac{100 \text{GeV}}{m}\right)^2 \sum_{VV'} dN^{VV'}/dE \left(\frac{\langle \sigma v \rangle_{VV'}}{10^{-27} \text{cm}^3 \text{sec}^{-1}}\right). \quad (10)$$

where $N^{VV'}$ is the number of photons from the final state.

In Fig. (2a) we show the line $\gamma$ flux from the galactic center in the cases of the SU(2)$_L$-triplet and doublet DM’s. Here, we take $\delta m = 0.1, 1, 10 \text{ GeV}$. We also show the flux obtained by the leading-order calculation for comparison. The ACT detectors have high sensitivity for the TeV-scale $\gamma$ ray. MAGIC and VERITAS in the north hemisphere might reach to $10^{-14} \text{cm}^{-2} \text{sec}^{-1}$ at the TeV scale $V V'$ and $\langle \sigma v \rangle$ is the averaged cross section by the velocity distribution function. Note that the angular acceptance of the detector, $\Delta \Omega$, is $10^{-3}$ typically for the ACT detectors. The flux depends on the halo DM density profile $\rho$ through

$$\bar{J} \Delta \Omega = \frac{1}{8.5 \text{kpc}} \int_{1.0 \text{s}} d\Omega dl \left(\frac{\rho}{0.3 \text{ GeVcm}^{-3}}\right)^2, \quad (11)$$

where the integral is along the line of sight. $\bar{J}$ is studied for various halo models, and $3 \lesssim \bar{J} \lesssim 10^5$ [4]. The cuspy structures in the halo density profiles, which are suggested by the $N$-body simulations, tend to give larger numbers to $\bar{J}$. In the following we take a moderate value as $\bar{J} = 500$, which is typical for the NFW profile [10], while CANGAROO III and HESS in the south hemisphere to $10^{-13} \text{cm}^{-2} \text{sec}^{-1}$ [4]. These ACT detectors are expected to cover the broad region.

In Fig. (2b), the contour plot of the continuum $\gamma$ flux from the galactic center is presented. For $dN^{VV'}/dE$ we use the fitting functions given in [4]. Shaded regions correspond to $S/B > 1$. In order to evaluate the back-
ground $B$, we assume a power low fall-off in the energy for the diffused $\gamma$ ray flux $\Psi_{BG}(E)$ as $d\Psi_{BG}(E)/dE = 9.1 \times 10^{-5} \text{cm}^{-2} \text{sec}^{-1} \text{GeV}^{-1} \times (E/1 \text{ GeV})^{-2.7} \Delta \Omega$ [4]. The EGRET experiment has observed the diffused $\gamma$ ray emission from the galactic center up to about 10 GeV [11]. Even the small regions around the resonances in addition to the triplet DM with $m_\chi = 100$ GeV are already constrained by the EGRET observation. The GLAST satellite detector, which will detect $\gamma$ ray with $1 \text{ GeV} < E < 300$ GeV, have more sensitivity to the around the region.

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