Baryogenesis via leptogenesis in adjoint $SU(5)$

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Abstract. The possibility of explaining the baryon asymmetry in the Universe
through the leptogenesis mechanism in the context of adjoint $SU(5)$ is
investigated. In this model neutrino masses are generated through the type
I and type III seesaw mechanisms, and the field responsible for the type III
seesaw, called $\rho_3$, generates the $B-L$ asymmetry needed to satisfy the observed
value of the baryon asymmetry in the Universe. We find that the $CP$ asymmetry
originates only from the vertex correction, since the self-energy contribution is not
present. When neutrino masses have a normal hierarchy, successful leptogenesis
is possible for $10^{11}$ GeV $\lesssim M_{\rho_3}^{NH} \lesssim 4 \times 10^{14}$ GeV. When the neutrino hierarchy is
inverted, the allowed mass range changes to $2 \times 10^{11}$ GeV $\lesssim M_{\rho_3}^{IH} \lesssim 5 \times 10^{11}$ GeV.
These constraints make it possible to rule out a large part of the parameter space
in the theory which was allowed by the unification of gauge interactions and the
constraints coming from proton decay.

Keywords: cosmological neutrinos, baryon asymmetry, physics of the early
universe

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1. Introduction

The origin of the baryon asymmetry in the Universe is one of the most interesting issues in modern cosmology. One of the simplest explanations is provided by the baryogenesis via leptogenesis scenario [1], where the lepton asymmetry generated in the out-of-equilibrium decays of the fields responsible for the seesaw mechanism of neutrino masses [2] is converted into a baryon asymmetry by sphaleron transitions [3]. This idea is very appealing due to the strong connection with the origin of neutrino masses.

Grand unified theories (GUTs) [4] are among the most appealing extensions of the standard model (SM), explaining e.g. the origin of the SM gauge interactions and neutrino masses. In particular, these theories provide a natural framework for the implementation of the baryogenesis via leptogenesis mechanism mentioned above. In this paper we study the leptogenesis mechanism and its possible predictions in the context of the adjoint \( SU(5) \) [5]. In this theory the Higgs sector is composed of \( 5_H, 24_H \) and \( 45_H \) Higgses and the matter lives in the \( \bar{5}, 10, \) and \( 24 \) representations. All fermion masses are generated at the renormalizable level, and neutrino masses are generated through the type I and type III [6] seesaw mechanisms. The predictions coming from the unification of gauge interactions and proton decay were studied in great detail in [7], where the authors concluded that the lightest fermionic field living in the adjoint representation has to be the field responsible for the type III seesaw mechanism. This result strongly motivates us to study the leptogenesis mechanism in the context of adjoint \( SU(5) \) since in this case there is no ambiguity about which field is responsible for leptogenesis.

Our first finding is that the \( CP \) asymmetry is generated only by the vertex correction since the self-energy contribution vanishes. When the neutrino mass hierarchy is normal, successful leptogenesis is possible in a large region of the parameter space. In contrast, when the spectrum for neutrinos is inverted, we find consistent solutions for a very restricted mass range. Finally, we show that imposing the constraints coming from leptogenesis one can rule out a large region in the parameter space of the theory which was
allowed by the unification of gauge interactions and the constraints coming from proton decay.

The paper is organized as follows. In section 2 we discuss the theory of adjoint $SU(5)$ and its predictions for neutrino masses. In section 3 we present our computation of the baryon asymmetry through leptogenesis, and derive bounds on the mass of the field responsible for the type III seesaw. In section 4 we discuss the possible constraints on the spectrum of the theory from successful leptogenesis. In the last section we summarize our main results.

2. Adjoint $SU(5)$ unification and neutrino masses

In the context of adjoint $SU(5)$ [5] neutrino masses are generated through the type I [2] and type III [6] seesaw mechanisms. In this theory the Higgs sector is composed of $5_H$, $24_H$, and $45_H$ and the matter fields live in $5 = (d^C, l)_L$, $10 = (u^C, Q, e^C)_L$ and $24 = (\rho_3, \rho_0)_L$ \(= (8, 1, 0) \oplus (1, 3, 0) \oplus (3, 2, -5/6) \oplus (3, 2, 5/6) \oplus (1, 1, 0)\). In our notation $\rho_3$ and $\rho_0$ are the $SU(2)_L$ triplet responsible for type III seesaw and the singlet responsible for type I seesaw, respectively. See reference [8] for the supersymmetric version of the theory.

The relevant interactions for neutrino masses in this context are given by

\[
V = c_\alpha \bar{5}_\alpha 245_H + p_\alpha \bar{5}_\alpha 2445_H + M_{\bar{\nu}_\alpha} 24^2 + \lambda \text{Tr}(24^2 24_H) + \text{h.c.}
\]

\[
= -c_\alpha \ell^T_\alpha \sigma_2 \rho_3 H_1 + 3p_\alpha \ell^T_\alpha \sigma_2 \rho_3 H_2 + \frac{3c_\alpha}{2\sqrt{15}} \ell^T_\alpha \sigma_2 \rho_0 H_2 + \frac{\sqrt{15}}{2} p_\alpha \ell^T_\alpha \sigma_2 \rho_0 H_2 + \cdots,
\]

where $\alpha = 1, 2, 3$, and $H_1$ and $H_2$ are the Higgs doublets living in $5_H$ and $45_H$, respectively. Once $24_H$ gets the expectation value $\langle 24_H \rangle = v \text{ diag}(2, 2, 2, -3, -3)/\sqrt{30}$, the masses of the fields responsible for seesaw living in $24$ are given by

\[
M_{\rho_0} = \left| m - \frac{\tilde{\lambda} M_{\text{GUT}}}{\sqrt{\alpha_{\text{GUT}}}} \right|, \quad \text{and} \quad M_{\rho_3} = \left| m - \frac{3\tilde{\lambda} M_{\text{GUT}}}{\sqrt{\alpha_{\text{GUT}}}} \right|,
\]

where we have used the relations $M_V = v\sqrt{5\pi \alpha_{\text{GUT}}}/3$, $\tilde{\lambda} = \lambda/\sqrt{50\pi}$ and chose $M_V$ as the unification scale. The predictions coming from the unification of gauge interactions and proton stability were studied in detail in [7]. In this recent study the authors concluded that in order to satisfy the unification and proton decay constraints the field $\rho_3$ has to be the lightest field in the $24$ representation. Therefore, the theory [5] is a good theory for leptogenesis since one can predict which field generates the lepton asymmetry.

Once the GUT symmetry is broken by the vev of $24_H$, all elements of the mass term for the Higgs doublets are large, of order the GUT scale. Diagonalizing the mass matrix, one obtains one light eigenstate, to be identified with the SM Higgs $H$, and one heavy eigenstate $H'$ with a mass at the GUT scale. In particular, it is relevant for our study that $\rho_3$ is only kinematically allowed to decay into the SM Higgs $H$.

Writing $H_1 = \cos \alpha H - \sin \alpha H'$ and $H_2 = \sin \alpha H + \cos \alpha H'$, and since only $H$ gets a vev $\langle H \rangle = v_0/\sqrt{2}$, we have that $\cos \alpha = v_5/v_0$ and $\sin \alpha = v_{145}/v_0$. The relevant terms in
equation (1) with the addition of the mass terms are then given by
\[ V_\nu = h_{\alpha 1}^T \sigma_2 C \rho_3 H + h_{\alpha 2}^T \sigma_2 C \rho_0 H + M_{\nu 3}^T C \rho_3 + \frac{1}{2} M_{\rho 3}^T C \rho_0 + h.c., \] (3)

where
\[ h_{\alpha 1} = \frac{1}{2 \sqrt{2 v_0}} (c_{\alpha} v_5 - 3 p_{\alpha} v_{45}), \quad \text{and} \quad h_{\alpha 2} = \frac{\sqrt{15}}{2 \sqrt{2 v_0}} \left( \frac{c_{\alpha} v_5}{5} + p_{\alpha} v_{45} \right), \] (4)

with \( v_0 = 174 \text{ GeV} \). In the above equations \( v_5/\sqrt{2} = \langle 5_H \rangle, \ v_{45}/\sqrt{2} = \langle 45_H \rangle^{15} = \langle 45_H \rangle^{25} = \langle 45_H \rangle^{35} \) and the matrix representation for \( \rho_3 \) is given by
\[ \rho_3 = \frac{1}{2} \left( \begin{array}{cc} T^0 & \sqrt{2}T^+ \\ \sqrt{2}T^- & -T^0 \end{array} \right). \] (5)

Integrating out the fields responsible for the seesaw mechanism, the mass matrix for neutrinos reads as
\[ M_{\alpha \beta}^\nu = \left( \frac{h_{\alpha 1} h_{\beta 1}}{M_{\rho 3}} + \frac{h_{\alpha 2} h_{\beta 2}}{M_{\rho 0}} \right) \varepsilon_0^2. \] (6)

The theory [5] predicts one massless neutrino at tree level. Therefore, we could have either a normal neutrino mass hierarchy: \( m_1 = 0, \ m_2 = \sqrt{\Delta m^2_{\text{sol}}} \) and \( m_3 = \sqrt{\Delta m^2_{\text{atm}}} \), or the inverted neutrino mass hierarchy: \( m_1 = 0, \ m_2 = \sqrt{\Delta m^2_{\text{atm}}} \) and \( m_3 = \sqrt{\Delta m^2_{\text{sol}}} \). \( \Delta m^2_{\text{sol}} \approx 8 \times 10^{-5} \text{ eV}^2 \) and \( \Delta m^2_{\text{atm}} \approx 2.5 \times 10^{-3} \text{ eV}^2 \) are the mass-squared differences of solar and atmospheric neutrino oscillations, respectively [9].

Finally, the kinetic terms for the fields responsible for seesaw read as
\[ L_{\text{kin}} = i \text{Tr} \bar{\rho}_3 \gamma^\mu D_\mu \rho_3 + i \bar{\rho}_0 \gamma^\mu \partial_\mu \rho_0, \] (7)

where \( D_\mu \rho_3 = \partial_\mu \rho_3 + ig_2 [W_\mu, \rho_3] \) and
\[ W_\mu = \frac{1}{2} \left( \begin{array}{cc} W^3_\mu & \sqrt{2} W^+_\mu \\ \sqrt{2} W^-_\mu & -W^3_\mu \end{array} \right). \] (8)

The gauge scattering term coming from the kinetic term will be crucial in the leptogenesis analysis, which we will be dealing with in section 3. A convenient parametrization of the \( 3 \times 2 \) Yukawa coupling matrix \( h \) is the so-called Casas–Ibarra [10]
\[ h = UD_u^{1/2} \Omega D^1_{\rho} / v_0, \] (9)

where \( U \) is the PMNS lepton mixing matrix, \( D_\nu = \text{diag}(m_1, m_2, m_3) \) and \( D_\rho = \text{diag}(M_{\rho 3}, M_{\rho 0}) \). The \( \Omega \) matrix takes here the well-known form corresponding to the type I seesaw case with two right-handed neutrinos [11]:
\[ \Omega^{\text{NH}} = \begin{pmatrix} 0 & 0 \\ \pm \sqrt{1 - \omega^2} \xi \omega & \pm \xi \sqrt{1 - \omega^2} \end{pmatrix}, \quad \Omega^{\text{IH}} = \begin{pmatrix} \pm \sqrt{1 - \omega^2} & -\omega \\ \xi \omega & \pm \xi \sqrt{1 - \omega^2} \end{pmatrix}, \] (10)

\(^3\) Compared to [5] our treatment uses the more convenient definition in the context of leptogenesis \( a_\alpha \equiv h_{\alpha 1} \) and \( b_\alpha \equiv h_{\alpha 2} \), which makes apparent the similarity between the model under consideration and the type I seesaw model with two right-handed neutrinos.
in the normal and inverted hierarchy, respectively, and where \( \omega \) is a complex parameter. \( \xi = \pm 1 \) is a discrete parameter that accounts for a discrete indeterminacy in \( \Omega \). For the PMNS matrix \( U \), we adopt the usual parametrization [12]

\[
U = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
  -s_{12} c_{23} + c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
  s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix} \times \text{diag}(1, e^{i \Phi / 2}, 1),
\]

(11)

where \( s_{ij} \equiv \sin \theta_{ij} \), \( c_{ij} \equiv \cos \theta_{ij} \), \( \delta \) is the Dirac CP-violating phase, \( \Phi \) is a Majorana CP-violating phase. Neglecting statistical errors, we will use throughout our study the values \( \theta_{12} = \pi / 5 \) and \( \theta_{23} = \pi / 4 \), compatible with the results from neutrino oscillation experiments. Moreover, we will adopt the 3\( \sigma \) range \( s_{13} = 0 \)\( -20 \).

3. Baryogenesis via leptogenesis: \( M_\nu \) and lower bound on \( M_{\rho_3} \)

The leptogenesis mechanism was investigated in the context of the type III seesaw [6] mechanism in [13]. However, the case of a hybrid seesaw, type I plus type III, has not been investigated before, and it is the main focus of our work. As emphasized in section 2, the model predicts \( \rho_3 \) lighter than \( \rho_0 \), so, in a first approximation, we will focus on the decay of \( \rho_3 \) for generating the observed baryon asymmetry of the Universe via leptogenesis.

One of Sakharov’s necessary conditions [14] to satisfy in order to produce a baryon asymmetry in the early Universe is CP violation, which naturally occurs in our model in the decays of \( \rho_3 \). We define the CP asymmetry parameter as

\[
\varepsilon_{\rho_3, \alpha} = \frac{\Gamma(\rho_3 \to \ell_\alpha H^\dagger) - \Gamma(\rho_3 \to \bar{\ell}_\alpha H)}{\sum_\alpha \Gamma(\rho_3 \to \ell_\alpha H^\dagger) + \Gamma(\rho_3 \to \bar{\ell}_\alpha H)}.
\]

(12)

In the pure type III case, the CP asymmetry was computed in [13], and was found to differ from that for the pure type I case by a factor 1/3 in the very hierarchical limit. Here, the computation is slightly different from that of [13] because we have one singlet running in the loops of the self-energy and the vertex corrections. The group theory product is therefore different. Interestingly, the self-energy contribution vanishes in our case. Hence, the only non-vanishing contribution is the vertex correction, which turns out to have the same magnitude and sign as for the type I seesaw. The CP asymmetry in our model is therefore given by [15]

\[
\varepsilon_{\rho_3, \alpha} = \frac{1}{8 \pi (h^\dagger h)_{11}} \text{Im} [h^*_{\alpha 1} h_{\alpha 2} (h^\dagger h)_{12}] f(M_{\rho_0}^2 / M_{\rho_3}^2),
\]

(13)

where

\[
f(x) = \sqrt{x} \left[ (1 + x) \ln \left( \frac{1 + x}{x} \right) - 1 \right] \xrightarrow{x \gg 1} \frac{1}{2 \sqrt{x}},
\]

(14)

which is a factor 3 smaller than in the type I case in the hierarchical limit \( M_{\rho_3} \ll M_{\rho_0} \). It should be noted that, even though the CP asymmetry in our model has the same magnitude in the hierarchical limit as in the type III seesaw, this is for a completely different reason. In the latter case, the factor 1/3 shows up because the vertex correction,
which takes a negative sign, partially cancels the self-energy contribution, which does not vanish in that case [13].

Note that the loop factor $f(x)$ in the CP asymmetry from the singlet $\rho_0$ with the triplet $\rho_3$ running in the loop is suppressed by a factor $M_{\rho_3}/M_{\rho_0}$ [16], which is required by the unification constraints to be less than $1/40$ [7]. Therefore, without even considering washout aspects, the contribution to leptogenesis from the singlet can be safely neglected.

The third Sakharov condition, namely departure from thermal equilibrium, can be conveniently described by the so-called decay parameter $K \equiv \Gamma/H_{T=M_{\rho_3}}$, given by the ratio of the decay widths to the expansion rate when $\rho_3$ starts to become non-relativistic at $T = M_{\rho_3}$. In terms of Yukawa couplings, the decay parameters can be written respectively as

$$K_\alpha = \frac{v_0^2}{m_* M_{\rho_3}} |h_{a1}|^2,$$

and

$$\sum_\alpha K_\alpha = K = \frac{v_0^2}{m_* M_{\rho_3}} (h^1 h)^{11},$$

where $m_*$ is the equilibrium neutrino mass [17] given by

$$m_* \simeq 1.08 \times 10^{-5} \text{ eV}.$$  \hspace{1cm} (16)

Let us now express the quantities in equations (13) and (15) in terms of the convenient Casas–Ibarra parametrization introduced above, equation (9). In the case of a normal hierarchy, the flavored decay parameters and their sum can be written respectively as

$$K_\alpha = \frac{m_2 |U_{a2}|^2}{m_*} (1 - \omega^2) + \frac{m_3 |U_{a3}|^2}{m_*} |\omega|^2 \pm \frac{2\sqrt{m_2 m_3}}{m_*} \text{Re} \left( U_{a2} U_{a3}^* \sqrt{1 - \omega^2} \omega^* \right),$$

$$K = \frac{m_2}{m_*} (1 - \omega^2) + \frac{m_3}{m_*} |\omega|^2,$$  \hspace{1cm} (17)

and since the mass splitting in the model is quite large, as pointed out above, the flavored CP asymmetries read as

$$\varepsilon_{\rho_3,\alpha} \simeq \frac{1}{8\pi v_0^2} \frac{M_{\rho_\alpha}}{m_2 |1 - \omega^2| + m_3 |\omega|^2} \left[ (m_3^2 |U_{a3}|^2 - m_2^2 |U_{a2}|^2) \text{Im}(\omega^2) \right.$$

$$\left. \pm \xi \sqrt{m_2 m_3} (m_3 + m_2) \text{Re}(U_{a2}^* U_{a3}) \text{Im}(\omega \sqrt{1 - \omega^2}) \right.$$ \hspace{1cm} (18)

$$\left. \pm \xi \sqrt{m_2 m_3} (m_3 - m_2) \text{Im}(U_{a2}^* U_{a3}) \text{Re}(\omega \sqrt{1 - \omega^2}) \right].$$

The total CP asymmetry $\varepsilon_{\rho_3} = \sum_\alpha \varepsilon_{\rho_3,\alpha}$ can be readily obtained from the latter expression:

$$\varepsilon_{\rho_3} \simeq - \frac{1}{8\pi v_0^2} \frac{M_{\rho_3}}{m_2 |1 - \omega^2| + m_3 |\omega|^2} \left( m_3^2 - m_2^2 \right) \text{Im}(\omega^2).$$ \hspace{1cm} (19)

Note that the case of inverted hierarchy in equations (17)–(20) is obtained by changing the indices 3 → 2 and 2 → 1. It can be noticed from the two above expressions that the factor $\xi$ does not open any new region in the parameter space since it always multiplies ±1. We will therefore assume $\xi = 1$ in the following.

Now that we have defined the essential quantities for leptogenesis, we can turn to the Boltzmann equations, which will have to be written in two different regimes, the two-flavor regime and the unflavored regime. As a matter of fact, when the mass range for $\rho_3$ is between $10^9$ GeV and $5 \times 10^{11}$ GeV, flavor effects cannot be neglected, and the so-called two-flavor regime applies, with flavors denoted as $\alpha = e, \mu, \tau$ [18]–[20]. For the range of masses $5 \times 10^{11}$–$10^{15}$ GeV we will use unflavored Boltzmann equations.
3.1. Flavored regime

Let us first discuss the two-flavor regime, which will give the lowest bound on $M_{\rho_3}$. The relevant Boltzmann equations, taking into account decays and inverse decays with proper subtraction of the resonant contribution from $\Delta L = 2$ [21] and $\Delta L = 0$ [18, 22] processes as well as $\Delta L = 1$ scatterings [23, 24], are given by

$$\frac{dN_{\rho_3}}{dz} = -(D + S)(N_{\rho_3} - N_{\rho_3}^{\text{eq}}) - 2S_g(N_{\rho_3} - (N_{\rho_3}^{\text{eq}})^2), \quad (21)$$

$$\frac{dN_{\Delta\alpha}}{dz} = \varepsilon_{\rho_3,\alpha}(D + S)(N_{\rho_3} - N_{\rho_3}^{\text{eq}}) - W^{\text{ID}}_{\alpha} \sum_{\beta} C_{\alpha\beta}N_{\Delta\beta} - W^{\Delta L=1}_{\alpha} \sum_{\beta} C'_{\alpha\beta}N_{\Delta\beta}, \quad (22)$$

where $z \equiv M_{\rho_3}/T$, $\Delta_{\alpha} = B/3 - L_{\alpha}$, and we indicated with $N_X$ any particle number or asymmetry $X$ calculated in a portion of co-moving volume containing one $\rho_3$ (i.e. three components) in ultra-relativistic thermal equilibrium, so that $N_{\rho_3}^{\text{eq}}(T \gg M_{\rho_3}) = 1$. The decay factor is given by

$$D \equiv \frac{\Gamma_D}{H z} = 3K z \left( \frac{1}{\gamma} \right), \quad (23)$$

where $H$ is the expansion rate and the factor 3 comes again from the three components of $\rho_3$. The total decay rate, $\Gamma_D \equiv \Gamma + \bar{\Gamma}$, is the product of the decay width and the thermally averaged dilation factor $(1/\gamma)$, given by the ratio $K_1(z)/K_2(z)$ of the modified Bessel functions. A simple analytic approximation for the sum $D + S$, where $S$ is the contribution from the Higgs-mediated scattering processes, is given by [17]

$$D + S \simeq 3 \times 0.1 K \left[ 1 + \ln \left( \frac{M_{\rho_3}}{M_h} \right) z^2 \ln \left( 1 + \frac{a}{z} \right) \right], \quad (24)$$

where $M_h$ is the Higgs mass and

$$a = \frac{8\pi^2}{9 \ln(M_{\rho_3}/M_h)}. \quad (25)$$

The equilibrium abundance and its rate are also expressed through the modified Bessel functions,

$$N_{\rho_3}^{\text{eq}}(z) = \frac{1}{2} z^2 K_2(z), \quad \text{and} \quad \frac{dN_{\rho_3}^{\text{eq}}}{dz} = -\frac{1}{2} z^2 K_1(z). \quad (26)$$

The inverse decay washout term, with the resonant $\Delta L = 2$ contribution properly subtracted [21], is given by

$$W^{\text{ID}}_{\alpha}(z) = \frac{3}{4} K_{\alpha} K_1(z) z^3. \quad (27)$$

It was shown in [17] that the complete washout term can be conveniently expressed as

$$W^{\Delta L=1}_{\alpha}(z) = j(z) W^{\text{ID}}_{\alpha}(z), \quad (28)$$

where

$$j(z) = 0.1 \left( 1 + \frac{15}{8z} \right) \left[ z \ln \left( \frac{M_{\rho_3}}{M_h} \right) \ln \left( 1 + \frac{a}{z} \right) + \frac{1}{z} \right] \quad (29)$$

in the strong washout regime, which will be the relevant one in the subsequent discussion.
The effects of the so-called ‘spectator processes’ [25, 26], which translate into a non-trivial relation between the asymmetries stored in the lepton doublets $\ell_\alpha$ and the asymmetries $\Delta_\alpha$ [27] as well as into an additional washout due to the asymmetry in the Higgs field [26], are accounted for by the matrices $C$ [26, 28] and $C'$, whose components are given by

$$C = \begin{pmatrix} 1.11 & 0.25 \\ 0.21 & 1.01 \end{pmatrix}, \quad \text{and} \quad C' = \begin{pmatrix} 0.98 & 0.08 \\ 0.08 & 0.84 \end{pmatrix}. \quad (30)$$

The matrix $C'$ is different from $C$ because the Higgs asymmetry contribution is divided by 2 in the $\Delta L = 1$ scattering case [26].

Compared to the ‘usual’ computation with singlet neutrinos, there is a new term in the equation for the abundance of $\rho_3$, as pointed out in [13]. It originates from scatterings allowed by the interaction equation (7). From the calculation in [13] we found the useful fit (within 30% accuracy in the relevant range $0.1 \lesssim z \lesssim 10$)

$$S_g \simeq 10^{-3} \frac{M_{Pl}}{M_{\rho_3}} \sqrt{\frac{1 + (\pi/2)\rho_3}{15/8 + z}} e^{-0.3z}. \quad (31)$$

The small uncertainty introduced by using this fit will translate into less than 10% effects on the final baryon asymmetry.

In writing equations (21), (22) we are neglecting the non-resonant $\Delta L = 2$ process contribution and $\Delta L = 0$ processes, a good approximation for $M_1 \ll 10^{14}$ GeV ($m_{\text{atm}}^2 / \sum_i m_i^2$), certainly satisfied in the flavored regime. We are also neglecting thermal corrections [21], which are expected to be small in the strong washout regime.

Solving the Boltzmann equations (21), (22), one obtains $N^{f}_{\Delta_\alpha} = N^{f}_{\Delta_\alpha}(z \to \infty)$, and hence the baryon-to-photon ratio predicted is

$$\eta_B \simeq 3 \times 0.96 \times 10^{-2} \left( N^{f}_{\Delta\nu} + N^{f}_{\Delta \tau} \right), \quad (32)$$

where the factor 3 comes from the three degrees of freedom in the fermionic triplet $\rho_3$. This prediction must then be compared with the observed value [29]

$$\eta_B^{\text{CMB}} = (6.2 \pm 0.15) \times 10^{-10}. \quad (33)$$

### 3.2. Unflavored regime

Let us now discuss the unflavored regime, for $5 \times 10^{11}$ GeV $< M_{\rho_3} < 10^{15}$ GeV. The relevant Boltzmann equations are

$$\frac{dN_{\rho_3}}{dz} = -(D + S) (N_{\rho_3} - N_{\rho_3}^{\text{eq}}) - 2 S_g (N_{\rho_3}^2 - (N_{\rho_3}^{\text{eq}})^2), \quad (34)$$

$$\frac{dN_{B-L}}{dz} = \varepsilon_{\rho_3} (D + S) (N_{\rho_3} - N_{\rho_3}^{\text{eq}}) - W N_{B-L}, \quad (35)$$

where $\varepsilon_{\rho_3} = \sum_\alpha \varepsilon_{\rho_3, \alpha}$ and $W(z) = j(z) \sum_\alpha W^{ID}_{\alpha}(z) + \Delta W(z)$. The contribution to the washout by the non-resonant $\Delta L = 2$ processes, $\Delta W(z)$, is given by [17]

$$\Delta W(z) \simeq 3 \times 10^{-3} \frac{0.186}{z^2} \left( \frac{M_{\rho_3}}{10^{10} \text{ GeV}} \right) \left( \frac{\overline{m}^2}{\text{eV}^2} \right), \quad (36)$$

with $\overline{m}^2 \equiv m_1^2 + m_2^2 + m_3^2 = 2.7 (4.9) \text{ eV}^2$ for normal (inverted) hierarchy.
After solving equations (34), (35), one obtains

\[ N_{B-L}^f = N_{B-L}(z \to \infty) \]

from which the final baryon asymmetry

\[ \eta_B \simeq 3 \times 0.96 \times 10^{-2} N_{B-L}^f \]  

is derived, to be compared with the measured value, equation (33).

### 3.3. Numerical results

In order to obtain the region in the parameter space \((K, M_{\rho_3})\) that is allowed by successful leptogenesis, one needs to solve the Boltzmann equations in the two-flavor regime, equations (21), (22), and then to maximize the asymmetry over all unknown parameters \((\theta_{13}, \delta, \Phi, \omega)\) at every given value of \(K\). In the unflavored regime, one needs to solve equations (34), (35) and maximize over \(\omega\) at every given value of \(K\). The result is shown in figure 1 for a normal hierarchy of light neutrinos and in figure 2 for an inverted hierarchy.
Let us explain the origin of the shaded areas in the plots. Imposing that the Yukawa couplings $h_{\alpha,1,2}$ remain perturbative, i.e. $h_{\alpha,1,2} < 2\sqrt{\pi}$, implies

$$M_{\rho_3} < \frac{4 \times 10^{17} \text{ GeV}}{K},$$

excluding the triangle shaded area in the figures. Additionally, the atmospheric scale must be accounted for, i.e. $h_{\alpha,1,2}^2 v_0^2 / M_{\rho_3} \gtrsim m_{\text{atm}}$, implying

$$M_{\rho_3} < 8 \times 10^{15} \text{ GeV},$$

which excludes the horizontally shaded area in the figures.

Turning to figure 1, one can clearly see the transition from the two-flavor to the unflavored regime, when the mass of $\rho_3$ goes over $5 \times 10^{11} \text{ GeV}$. Flavor effects would introduce a relaxation of the lowest bound by roughly an order of magnitude if the gauge scattering term $S_g$ was not present, but accounting for this, the relaxation is only by a factor 2–3. We therefore confirm that the gauge scattering term induces a reduction of the maximal efficiency factor, as pointed out in [13] and [30]. Furthermore, we would like to point out that spectator processes, whose effects are accounted for in $C$ and $C'$ in equation (22), induce a reduction of the allowed region by about 30% in the flavored regime. As for the $\Delta L = 1$ scatterings, their inclusion changes the final asymmetry only very marginally, confirming what was found in [31].

A nice feature of the computation is that the final asymmetry is insensitive to the initial number of $\rho_3$. The reason is that one has a regime of strong washout, also when flavor effects are included. The gauge scattering term, which quickly thermalizes the abundance of $\rho_3$, also contributes to the independence of the initial number of $\rho_3$. The strong washout is ensured by the fact that $K \geq K_{\text{sol}} \equiv 8.2 \gg 1$ in the case of normal hierarchy and $K \geq K_{\text{atm}} \equiv 46 \gg 1$ in the case of inverted hierarchy.

In figure 2, where the case of inverted hierarchy is displayed, it is apparent that the allowed region is very small. Actually, only in the flavored regime below $5 \times 10^{11} \text{ GeV}$ is there an allowed region. In the unflavored regime the usual suppression of the $CP$ asymmetry for the case of two right-handed neutrinos in the inverted hierarchy [32], combined with the washout from the non-resonant $\Delta L = 2$ processes, leaves no allowed region. On the other hand, when flavor effects are included, the $CP$ asymmetry is not suppressed, and the final asymmetry can be orders of magnitude higher than what would be predicted with an unflavored calculation. This was already noticed in the very similar case of two singlet neutrinos in [24] and confirmed recently in [28]. It is important to say that there is a big sensitivity to the ‘low-energy’ $CP$-violating phases in the PMNS matrix in that case which will be studied in a future publication. This behavior in our model is not surprising if one remembers the similarity with the model with two right-handed neutrinos, where the crucial role played by the $CP$-violating phases in the PMNS matrix in the case of inverted hierarchy was recently emphasized in [28] (see the right panel of figure 5 there).

### 4. Constraints from leptogenesis and the spectrum of the theory

In the previous section we have obtained the allowed region by leptogenesis for each neutrino mass spectrum. In the case of the normal hierarchy we find

$$10^{11.1} \text{ GeV} \lesssim M_{\rho_3}^{\text{NH}} \lesssim 10^{14.5} \text{ GeV},$$

in agreement with recent bounds from the spectrum of the theory.
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Figure 3. Constraints coming from unification when $M_{\Phi_1} = 200$ GeV, including the bound from leptogenesis.

while for inverted hierarchy we find

$$10^{11.3} \text{ GeV} \lesssim M_{\rho_3}^{IH} \lesssim 10^{11.7} \text{ GeV}.$$  

(41)

In reference [7] the solutions for the spectrum in the theory which are consistent with the unification of gauge interactions and proton decay were shown. Let us investigate the role of the above constraints coming from leptogenesis. The relation between the masses of the fermionic fields living in the adjoint representation is given by

$$M_{\rho_0} = \frac{1}{\hat{m}} (3 + 2 \hat{m}) M_{\rho_3}, \quad M_{\rho_8} = \hat{m} M_{\rho_3}, \quad \text{and} \quad M_{\rho(3,2)} = M_{\rho(3,2)} = \frac{1}{2} (1 + \hat{m}) M_{\rho_3}. $$  

(42)

Since the mass of all these fields should be below the GUT scale, one can use these relations as well as the bounds on $M_{\rho_3}$ coming from leptogenesis in order to constrain the spectrum. The most relevant constraint comes from the relation between $M_{\rho_8}$ and $M_{\rho_3}$, from which we find

$$10^{11.1} \text{ GeV} \hat{m} \lesssim M_{\text{GUT}} \lesssim 10^{15.90} \text{ GeV},$$  

(43)

where the upper bound is coming from the unification and proton decay constraints [7] and the lower bound is due to the minimal allowed value for $M_{\rho_3}$. We recall that $\hat{m}$ is the mass splitting. From equation (43) one readily obtains an upper bound on the mass splitting between the fields in the adjoint representation:

$$10^2 \lesssim \hat{m} \lesssim 10^{1.8},$$  

(44)

where the lower bound comes from the unification constraints [7]. Therefore, one excludes a large part of the allowed parameter space shown in [7]. In order to show the importance of these bounds we present in figure 3 the parameter space allowed by unification when $M_{\Phi_1} = 200$ GeV. The fields $\Phi_1 \sim (8, 2, 1/2)$ and $\Phi_3 \sim (3, 3, -1/3)$ live in $45_H$ while $\Sigma_3 \sim (1, 3, 0)$ is in $24_H$. See [7] for details. Notice that once we include the leptogenesis constraints a large part of the parameter space is excluded. Now, since $M_{\Phi_3}$ has to
be larger than $10^{12}$ GeV in order to satisfy the constraints coming from proton decay, the only allowed region in figure 3 is the area bounded by the lines $M_{\Sigma_3} = 200$ GeV, $M_{\phi_3} = 10^{12}$ GeV and $M_{\text{GUT}} = 10^{11}$ GeV. This means that the model is quite predictive.

5. Summary

We have presented a detailed study of the baryogenesis via leptogenesis mechanism in the context of adjoint $SU(5)$ where neutrino masses are generated by the type I and type III seesaw mechanisms. Through the decays of the field responsible for the type III seesaw, $\rho_3$, a lepton asymmetry is produced and later converted in a baryon asymmetry by sphalerons. In our model, the $CP$ asymmetry is generated by the vertex correction since the self-energy contribution vanishes. Imposing successful leptogenesis, we found that the case of normal hierarchy for the neutrinos is possible for a large range of $\rho_3$ masses (see figure 1). On the other hand, when the spectrum is inverted, the allowed region is very small (see figure 2). Finally, we have shown that, imposing successful leptogenesis, one rules out a large region in the parameter space allowed by the unification of gauge interactions and the constraints coming from proton decay (see figure 3).

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