On fractional neutral Volterra-Fredholm integro-differential systems with non-instantaneous impulses in Banach space

M. Mallika Arjunan

Abstract
The primary objective of this paper is to analyze the existence of $PC$-mild solution of fractional neutral Volterra-Fredholm integro-differential systems with non-instantaneous impulses in Banach spaces. Based on the Banach contraction principle, we develop the main results. An example is given to support the validation of the theoretical results achieved.

Keywords
Fractional neutral equations, mild solution, non-instantaneous impulses, fixed point theorem.

AMS Subject Classification
34K30, 35R12, 26A33.

1 Introduction

In the past decades, research on fractional differential equations has been published. There has been a considerable improvement and new applications have been proposed since then. For example, in [3], where models of viscoplasticy are discussed, some examples of applications of fractional order equations can be identified. Due to its broad applications in physics, economics, population dynamics, control theory, etc., the study of differential equations with sudden and instantaneous impulses seems to be of great importance. The behavior of instantaneous impulses, however, does not characterize some physical processes. For this reason, Hernandez et al. [2] introduced the concept of non-instantaneous impulses.

Motivated by [1, 2, 4], in this paper, we consider a class of fractional neutral Volterra-Fredholm integro-differential systems with non-instantaneous impulses of the form

$$\begin{align*}
{}^cD_{0+}^\alpha [x(t) + h(t,x(t))] \\
= A[x(t) + h(t,x(t))] + f \left( t,x(t), \int_0^t k(t,s,x(s))ds, \int_0^T \tilde{k}(t,s,x(s))ds \right), \\
t \in (s_i,t_{i+1}], i = 0,1,2,\ldots,m \\
x(t) = h_i(x(t_i)) + g_i(t,x(t)), \quad t \in (t_i,t_i], i = 1,2,\ldots,m \\
x(0) = x_0,
\end{align*}$$

where $^cD_{0+}^\alpha$ is the Caputo fractional derivative of order $\alpha \in (0,1)$ with the lower limit zero, $A : D(A) \subset X \to X$ is the generator of a $C_0$-semigroup of bounded linear operators \{\{T(t)\}_{t \geq 0}\} on a Banach space \{X, \| \cdot \|\}, x_0 \in X, 0 = t_0 = s_0 < t_1 < t_2 < s_2 < \cdots < s_m < t_m + 1 = T$ are fixed numbers, $h : [0,T] \times X \to X; g_i \in C(\{t_i,s_i\} \times X; X); I_i : X \to X$ for $i = 1,2,\ldots,m, f : [0,T] \times X^3 \to X$ is a nonlinear function and $k, \tilde{k} : \Delta \times X \to X$, where $\Delta = \{(x,s) : 0 \leq s \leq x \leq \tau \}$ are given functions which satisfies assumptions to be specified later on.

For our convenience, we denote $E_1 x(t) = \int_0^t k(t,s,x(s))ds$ and $E_2 x(t) = \int_0^T \tilde{k}(t,s,x(s))ds$.

The impulses in problem (1.1) start abruptly at the points $t_i$ and their action continues on the interval $[t_i,s_i]$. To be...
precisely, the function \( x \) takes an abrupt impulse at \( t_i \) and follows different rules in the two subintervals \([t_i, t_{i+1}]\) and \([t_{i+1}, t_{i+2}]\) of the interval \([t_i, t_{i+2}]\). At the point \( s_i \), the function \( x \) is continuous. The term \( I_i(x(t_i)) \) means that the impulses are also related to the value of \( x(t_i) = x(t_i^-) \).

We remark that if \( t_i = s_i \) and the second equation of (1) takes the form of \( \Delta x(t_i) = I_i(x(t_i)) = x(t_i^+) - x(t_i^-) \) with \( x(t_i^+) = \lim_{t \to 0^+} x(t_i + \varepsilon), \) \( x(t_i^-) = \lim_{t \to 0^-} x(t_i + \varepsilon) \) representing the right and left limits of \( x(t) \) at \( t = t_i \).

We also study the nonlocal Cauchy problems for impulsive fractional evolution equations

\[
\begin{align*}
\alpha D^q x(t) + h(t, x(t)) &= A x(t) + g(t, x(t)), \\
\int_0^t \kappa(t, s, x(s)) ds &= \int_0^t \bar{k}(t, s, x(s)) ds, \\
& \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \ldots, m, \\
& x(t) = \ell(x(t_i)) + g(t, x(t)), \quad t \in (t_i, s_i], i = 1, 2, \ldots, m, \\
x(0) &= x_0 + b(x),
\end{align*}
\]

where \( A, f, h, I, g, k, \ell, \bar{k} \) are the same as above, \( b \) is a given function; this constitutes a nonlocal Cauchy problem. It is well known that the nonlocal condition has a better effect on the solution and is more precise for physical measurements than the classical initial condition alone.

The rest of the paper is organized as follows. In Section 2, we present the notations, definitions and preliminary results needed in the following sections. In Section 3 is concerned with the existence results of problems (1.1) and (1.2). An example is given in Section 4 to illustrate the results.

## 2. Preliminaries

Let us set \( J = [0, T], J_0 = [0, t_1], J_1 = (t_1, t_2], \ldots, J_{m-1} = (t_{m-1}, t_m], J_m = (t_m, t_{m+1}] \) and introduce the space \( PC(J, X) := \{ u : J \to X \mid u \in C(J_0, X), k = 0, 1, 2, \ldots, m, \text{ and there exist } u(t_i^+), u(t_i^-), k = 1, 2, \ldots, m, \text{ with } u(t_i^-) = u(t_i^+) \} \). It is clear that \( PC(J, X) \) is a Banach space with the norm \( \| u \|_{PC} = \sup \{ ||u(t)|| : t \in J \} \).

**Definition 2.1.** [3] The Riemann-Liouville fractional integral of order \( q \) with the lower limit zero for a function \( f \) is defined as

\[
I^q f(t) = \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} f(s) ds, \quad q > 0
\]

provided the integral exists, where \( \Gamma(\cdot) \) is the gamma function.

**Definition 2.2.** [3] The Riemann-Liouville derivative of order \( q \) with the lower limit zero for a function \( f : [0, \infty) \to \mathbb{R} \) can be written as

\[
D^q f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t (t-s)^{n-q-1} f(s) ds, \quad n-1 < q < n, t > 0
\]

Let us recall the following definition of mild solutions for fractional evolution equations involving the Caputo fractional derivative.

**Definition 2.3.** [5, 6] A function \( x \in C(J, X) \) is said to be a mild solution of the following problem:

\[
\begin{align*}
\alpha D^q x(t) &= Ax(t) + y(t), \quad t \in (0, T] \\
x(0) &= x_0
\end{align*}
\]

if it satisfies the integral equation

\[
x(t) = \int_0^t (t-s)^{q-1} Q(t-s)y(s) ds.
\]

Here

\[
P_\alpha(t) = \int_0^t \xi_\alpha(t)Q(t,s) d\theta, \quad Q(t,s) = \alpha \int_0^t \Gamma(n\alpha + 1) \frac{sin(n\pi \alpha)}{n!} \theta^n d\theta,
\]

and \( \xi_\alpha \) is a probability density function defined on \((0, \infty)\), that is,

\[
\int_0^\infty \xi_\alpha(\theta) d\theta = 1
\]

It is not difficult to verify that

\[
\int_0^\infty \theta^n \xi_\alpha(\theta) d\theta = \frac{1}{\Gamma(1+\alpha)}.
\]

We make the following assumption on \( A \) in the whole paper.

\( H(A) \) : The operator \( A \) generates a strongly continuous semigroup \( \{ T(t) : t \geq 0 \} \) in \( X \), and there is a constant \( M_A \geq 1 \) such that \( \sup_{t \in [0, \infty)} \| T(t) \|_{L(X)} \leq M_A \). For any \( t > 0, T(t) \) is compact.

**Lemma 2.4.** [5, 6] Let \( H(A) \) hold, then the operators \( P_\alpha \) and \( Q_\alpha \) have the following properties:

1. For any fixed \( t \geq 0, P_\alpha(t) \) and \( Q_\alpha(t) \) are linear and bounded operators, and for any \( x \in X \),

\[
\| P_\alpha(t)x \| \leq M_A \| x \|, \quad \| Q_\alpha(t)x \| \leq \frac{\alpha M_A}{\Gamma(1+\alpha)} \| x \|
\]

2. \( \{ P_\alpha(t), t \geq 0 \} \) and \( \{ Q_\alpha(t), t \geq 0 \} \) are strongly continuous;

3. for every \( t > 0, P_\alpha(t) \) and \( Q_\alpha(t) \) are compact operators.

Next, by using the concept discussed in [1], we define the following definition of the mild solution for problem (1.1).

**Definition 2.5.** A function \( x \in PC(J, X) \) is said to be a PC-mild solution of problem (1.1) if it satisfies the following
relation:

\[
x(t) = \begin{cases}
  P_\alpha(t)[x_0 + h(0,x(0)) - h(t,x(t))] + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\
  f(s,x(s),E_1x(s),E_2x(s))ds, t \in [0,t_1] \\
  I_1(x(t_i)) + g_1(t,x(t)), t \in [t_i,s_i] \\
  P_\alpha(t-s_i)d_1 - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\
  f(s,x(s),E_1x(s),E_2x(s))ds, t \in [s_i,t_i] \\
  \vdots \\
  I_1(x(t_i)) + g_1(t,x(t)), t \in [t_i,s_i], i = 1,2,\ldots,m, \\
  P_\alpha(t-s)d_i - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\
  f(s,x(s),E_1x(s),E_2x(s))ds, t \in [s_i,t_i+1]
\end{cases}
\]

where, for \( i = 1,2,\ldots,m \)

\[
d_i = I_1(x(t_i)) + g_1(s_i,x(s_i)) + h(s_i,x(s_i)) \\
- \int_{t_i}^{s_i} (s_i-s)^{\alpha-1} Q_\alpha(s_i-s)f(s,x(s),E_1x(s),E_2x(s))ds.
\]

(3.1)

3. Existence Results

In this section, we present and prove the existence and uniqueness of the system (1.1) under Banach contraction principle fixed point theorem.

From Definition 3.1, we define an operator \( S : PC(J,X) \to PC(J,X) \) as

\[
(Sx)(t) = \begin{cases}
  P_\alpha(t)[x_0 + h(0,x(0)) - h(t,x(t))] + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\
  f(s,x(s),E_1x(s),E_2x(s))ds, t \in [0,t_1] \\
  I_1(x(t_i)) + g_1(t,x(t)), t \in [t_i,s_i] \\
  P_\alpha(t-s)d_1 - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\
  f(s,x(s),E_1x(s),E_2x(s))ds, t \in [s_i,t_i+1]
\end{cases}
\]

with \( d_i, i = 1,2,\ldots,m \), defined by (2.1).

To prove our first existence result we introduce the following assumptions:

(H(f)) The function \( f \in C((J \times X^3) ; X) \) and there exists \( L_f \in L^1(J,\mathbb{R}^+) \) with \( \alpha \in (0,\alpha) \) such that

\[
\| f(t,x_1,x_2,x_3) - f(t,y_1,y_2,y_3) \| \\
\leq L_f(t) \| x_1 - y_1 \| + \| x_2 - y_2 \| + \| x_3 - y_3 \|
\]

for all \((x_1,x_2,x_3),(y_1,y_2,y_3)\) \( \in \) \( X \) and every \( t \in J \).

(H(h)) The function \( h \in C([0,T] \times X;X) \) and there exists a positive constant \( L_h > 0 \) in a way that

\[
\| h(t,x) - h(t,y) \| \leq L_h \| x - y \|, \quad x,y \in X and \ t \in J.
\]

(H(\( k,\overline{k} \))) The functions \( k, \overline{k} : \Delta \times X \to X \) are continuous and there exist constants \( L_k, L_{\overline{k}} > 0 \) such that

\[
\left\| \int_0^t [k(t,s,x(s)) - k(t,s,y(s))]ds \right\| \leq L_k \| x - y \|
\]

for all \( x,y \in X \); and

\[
\left\| \int_0^T \overline{k}(t,s,x(s)) - \overline{k}(t,s,y(s))ds \right\| \leq L_{\overline{k}} \| x - y \|
\]

for all \( x,y \in X \).

(H(I)) For \( i = 1,2,\ldots,m, I_i \in C(X,X) \) and there is a constant \( L_I > 0 \) such that \( \| I_i(x) - I_i(y) \| \leq L_I \| x - y \| \) for all \( x,y \in X \).

(H(g)) For \( i = 1,2,\ldots,m, \) the functions \( g_i \in C([t_i,s_i] \times X;X) \) and there exists \( L_g \in C (J,\mathbb{R}^+) \) such that

\[
\| g_i(t,x) - g_i(t,y) \| \leq L_g(t) \| x - y \|
\]

for all \( x,y \in X \) and \( t \in [t_i,s_i] \).

Theorem 3.1. Assume \( H(f), H(h), \) \( H(I), H(k,\overline{k}) \) and \( H(g) \) are satisfied and

\[
M_\alpha \left( L_f + \| L_g \|_{C(J)} \right) + (1+M_\alpha) \left( L_h + \frac{\alpha M_\lambda}{\Gamma(\alpha+1)} \left( \frac{1}{\alpha} \right)^{1-\tau} \right)<0.
\]

Then there exists a unique \( PC \)-mild solution of problem (1.1).

Proof. From the assumptions it is easy to show that the operator \( S \) is well defined on \( PC(J,X) \). Let \( x,y \in PC(J,X) \). For \( t \in [0,t_1] \), from Lemma 2.1, we have

\[
\| (Sx)(t) - (Sy)(t) \|
\leq \| h(t,x(t)) - h(t,y(t)) \| + \int_0^t (t-s)^{\alpha-1} \| Q_\alpha(t-s) \|
\| f(s,x(s),E_1x(s),E_2x(s)) - f(s,y(s),E_1y(s),E_2y(s)) \| ds
\leq L_h + \frac{\alpha M_\lambda}{\Gamma(\alpha+1)} \left( \frac{1}{\alpha} \right)^{1-\tau} \int_0^t (t-s)^{\alpha-1} \left( \int_0^t (t-s)^{\alpha-1} \| Q_\alpha(t-s) \| \right)ds
\| x-y \|_{PC}.
\]

Similarly, we have, for \( t \in [t_i,s_i], i = 1,2,\ldots,m \)

\[
\| (Sx)(t) - (Sy)(t) \|
\leq \| I_i(x(t_i)) - I_i(y(t_i)) \|
\| g_i(t,x(t)) - g_i(t,y(t)) \|
\leq \left( L_I + \| L_g \|_{C(J)} \right) \| x-y \|_{PC}
\]

and, for \( t \in [s_i,t_i+1], i = 1,2,\ldots,m \)

\[
\| (Sx)(t) - (Sy)(t) \|
\leq \left( L_I + \| L_g \|_{C(J)} \right) \| x-y \|_{PC}.
\]
\[ \left\| (Sx)(t) - (Sy)(t) \right\| \]
\[ \leq \left\| P_{\alpha}(t - s) \left[ L_{1}(x(t)) - L_{1}(y(t)) + g_{i}(s_{i}, x(s_{i})) - g_{i}(s_{i}, y(s_{i})) \right] \right. \]
\[ - f(s, y(s), E_{1}y(s), E_{2}y(s))ds \right\| \]
\[ + \left\| h(t, x(t)) - h(t, y(t)) \right\| + \int_{0}^{t}(t - s)^{\alpha - 1} Q_{\alpha}(s - t) f(s, x(s), E_{1}x(s), E_{2}x(s))ds \right\| \]
\[ + \left\| Q_{\alpha}(t - s)(f(s, x(s), E_{1}x(s), E_{2}x(s)) - f(s, y(s), E_{1}y(s), E_{2}y(s)))ds \right\| \]
\[ \leq M_{A} \left( L_{f} + \left\| L_{g}\right\|_{C(J)} + L_{h} + \frac{\alpha M_{A}}{\Gamma(\alpha + 1)} \left( \frac{1 - \tau}{\alpha - \tau} \right)^{1 - \tau} \right) \]
\[ s_{i}^{\alpha - \tau} \left\| L_{f}\right\|_{L^{1}_{+}(0, s_{i})} \left[ 1 + L_{h} + L_{g} \right] \|x - y\|_{PC} \]
\[ + L_{h} + \frac{\alpha M_{A}}{\Gamma(\alpha + 1)} \left( \frac{1 - \tau}{\alpha - \tau} \right)^{1 - \tau} \int_{0}^{t+1} \left\| L_{f}\right\|_{L^{1}_{+}(0, x_{i+1})} \left[ 1 + L_{h} + L_{g} \right] \|x - y\|_{PC}. \]

From the above we can deduce that (since \( M_{A} \geq 1 \)) \( \left\| (Sx)(t) - (Sy)(t) \right\|_{PC} \leq M_{A} \left( L_{f} + \left\| L_{g}\right\|_{C(J)} + L_{h} + \frac{\alpha M_{A}}{\Gamma(\alpha + 1)} \left( \frac{1 - \tau}{\alpha - \tau} \right)^{1 - \tau} \right) \)
\[ s_{i}^{\alpha - \tau} \left\| L_{f}\right\|_{L^{1}_{+}(0, s_{i})} \left[ 1 + L_{h} + L_{g} \right] \|x - y\|_{PC} \]
\[ + L_{h} + \frac{\alpha M_{A}}{\Gamma(\alpha + 1)} \left( \frac{1 - \tau}{\alpha - \tau} \right)^{1 - \tau} \int_{0}^{t+1} \left\| L_{f}\right\|_{L^{1}_{+}(0, x_{i+1})} \left[ 1 + L_{h} + L_{g} \right] \|x - y\|_{PC}. \]

Then it follows from condition (3.1) that \( S \) is a contraction on the space \( PC(J, X) \). Hence by the Banach contraction mapping principle, \( S \) has a unique fixed point \( x \in PC(J, X) \) which is just the unique \( PC \)-mild solution of problem (1.1). The proof is now complete. \( \square \)

**Definition 3.2.** A function \( x \in PC(J, X) \) is said to be a \( PC \)-mild solution of problem (1.2) if it satisfies the following relation:
\[ x(t) = \begin{cases} \left[ \begin{array}{l} P_{\alpha}(t) \left( x_{0} + b(x) + h(0, x(0)) \right) - h(t, x(t)) \\ + \int_{0}^{t}(t - s)^{\alpha - 1} Q_{\alpha}(t - s) f(s, x(s), E_{1}x(s), E_{2}x(s))ds, \\
L_{i}(x(t)) + g_{i}(s_{i}, x(t)), t \in \left( t_{i}, t_{i+1} \right], i = 1, \ldots, m \end{array} \right] \\
\left[ \begin{array}{l} P_{\alpha}(t - s_{i})d_{i} + \int_{0}^{t}(t - s)^{\alpha - 1} Q_{\alpha}(t - s) f(s, x(s), E_{1}x(s), E_{2}x(s))ds, t \in \left[ s_{i}, t_{i+1} \right[ \\
d_{i}, i = 1, \ldots, m, \end{array} \right] \end{cases} \]
with \( d_{i}, i = 1, \ldots, m, \) defined by (2.1).

**Theorem 3.3.** Assume \( H(f), H(h), H(I), H(k, \bar{k}) \) and \( H(g) \) are satisfied and
\[ \left[ M_{A} \left( L_{f} + \left\| L_{g}\right\|_{C(J)} + L_{h} \right) + \left( 1 + M_{A} \right) \left( \frac{\alpha M_{A}}{\Gamma(\alpha + 1)} \left( \frac{1 - \tau}{\alpha - \tau} \right)^{1 - \tau} \right) \right] \left\| L_{f}\right\|_{L^{1}_{+}(J)} \left[ 1 + L_{h} + L_{g} \right] \]
\[ < 1. \]

Then there exists a unique \( PC \)-mild solution of problem (1.2).

**Proof.** The proof of this theorem is very similar to Theorem 3.1, so we omit it. \( \square \)

### 4. Application

A simple example is given in this section to illustrate the result.

Let \( X = L^{2}([0, \pi]) \). Define an operator \( A : D(A) \subseteq X \rightarrow X \) by \( Ax = x'' \) with \( D(A) = \{ x \in X : x'' \in X, x(0) = x(\pi) = 0 \} \).
It is well known that \( A \) is the infinitesimal generator of a strongly continuous semigroup \( \{ T(t) : t \geq 0 \} \) in \( X \). Moreover, \( T(t) \) is compact for \( t > 0 \) and \( \| T(t) \|_{(X)} \leq e^{-\tau} \leq 1 = M_{A}, t \geq 0 \).

Consider the following impulsive problem:
\[ c D_{T}^{\alpha} [u(t,y) + h(t, u(t,y))] = \frac{\partial^{2}}{\partial y^{2}} [u(t,y) + h(t, u(t,y))]
+ f(t, u(t,y), E_{1}u(t,y), E_{2}u(t,y)), \]
\[ t \in \left[ 0, \frac{1}{2} \cup \left[ \frac{2}{3}, 1 \right], y \in [0, \pi] \right), \]
\[ u(t, y) = I \left( u \left( \frac{1}{2}, y \right) \right) + g(t, u(t,y)), \]
\[ t \in \left[ 1, \frac{2}{3} \right], y \in [0, \pi] \]
\[ u(t,0) = u_{0}(y), \quad y \in [0, \pi] \]
\[ u(t, \pi) = u(t, \pi) = 0, \quad t \in [0, 1] \]

(4.1)

Here \( c D_{T}^{\alpha} \) means that the Caputo fractional derivative is taken for the time variable \( t \) with the lower limit zero.

Let
\[ f(t, u(t,y), E_{1}u(t,y), E_{2}u(t,y)) = \frac{e^{-\frac{t}{2}} u(t,y)}{(9 + e^{x})(1 + [u(t,y)])} + \frac{1}{10} \int_{0}^{t} e^{-\frac{t}{2}} u(s,y)ds \]
\[ + \frac{1}{10} \int_{0}^{t} e^{-\frac{t}{2}} u(s,y)ds \]
\[ h(t, u(t,y)) = \frac{1}{10} u(t,y), \quad I(u(t,y)) = \frac{[u(t,y)]}{9 + [u(t,y)]}, \]
\[ g(t, u(t,y)) = \frac{1}{16} \sin u(t,y) + e^{t} \]
Define \( x(t)(y) = u(t,y), (t,y) \in [0,1] \times [0,\pi] \). Then \( f, I, \) and \( g \) can be rewritten as

\[
\begin{align*}
f(t,x(t),E_1x(t),E_2x(t)) &= \frac{e^{-t}x(t)}{(9 + e^t)(1 + |x(t)|)} + \frac{1}{10} \int_0^t e^{-\frac{1}{2}s}ds \\
&+ \frac{1}{10} \int_0^t e^{-\frac{1}{4}s}ds \\
h(t,x(t)) &= \frac{1}{10} x(t), \quad I(x(t)) = \frac{|x(t)|}{9 + |x(t)|}, \\
g(t,x(t)) &= \frac{1}{16} \sin x(t) + e^t
\end{align*}
\]

We can verify that \( H(f), H(h), H(I), H(k, \tilde{k}) \) and \( H(g) \) hold by putting \( L_f(t) = \frac{1}{10}, L_h = \frac{1}{10}, L_I = \frac{1}{2}, L_k = \frac{1}{4}, L_{\tilde{k}} = \frac{1}{8} \) and \( L_g(t) = \frac{1}{16} \).

Moreover, since \( \alpha = \frac{3}{4} \), let \( \tau = \frac{1}{2} \), we have

\[
\begin{align*}
M_A \left( L_f \| L_e \|_{C(J)} \right) + (1 + M_A) \left( L_h + \frac{\alpha M_A}{1 + (1 - \frac{\alpha}{\tau})^\tau} \right) \\
\leq \frac{1}{9} \left( 1 + \frac{1}{16} + 2 \left( 0.1 + 0.8160 \times 1.4142 \right) \right) + \frac{1}{10} \left( 1 + \frac{1}{4} + \frac{1}{8} \right) = 0.52 < 1.
\end{align*}
\]

Therefore by Theorem 3.1, we deduce that problem (4.1) has a unique \( PC \)-mild solution on \([0,1]\).

### References

[1] X. Fu, X. Liu and B. Lu, On a new class of impulsive fractional evolution equations, *Advances in Difference Equations*, (2015) 2015:227.

[2] E. Hernández and D. O’Regan, On a new class of abstract impulsive differential equations, *Proc.Amer. Math. Soc.*, 141 (2013), 1641–1649.

[3] A.A. Kilbas, H.M. Srivastava and J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, vol. 204. Elsevier, Amsterdam, 2006.

[4] M. Mallika Arjunan, Existence results for nonlocal fractional mixed type integro-differential equations with non-instantaneous impulses in Banach space, *Malaya Journal of Matematik*, 7(4)(2019), 837–840.

[5] Y. Zhou and F. Jiao, Nonlocal Cauchy problem for fractional evolution equations, *Nonlinear Anal., Real World Appl.* 11(2010), 4465–4475.

[6] Y. Zhou and F. Jiao, Existence of mild solutions for fractional neutral evolution equations, *Comput. Math. Appl.*, 59(2010), 1063–1077.