Kähler Potential of Moduli Space

of Calabi-Yau $d$-fold

embedded in $\mathbb{CP}^{d+1}$

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ABSTRACT

We study a Kähler potential $K$ of a one parameter family of Calabi-Yau $d$-fold embedded in $\mathbb{CP}^{d+1}$. By comparing results of the topological B-model and the data of the CFT calculation at Gepner point, the $K$ is determined unambiguously. It has a moduli parameter $\psi$ that describes a deformation of the CFT by a marginal operator. Also the metric, curvature and hermitian two-point functions in the neighborhood of the Gepner point are analyzed. We use a recipe of $tt^*$ fusion and develop a method to determine the $K$ from the point of view of topological sigma model. It is not restricted to this specific model and can be applied to other Calabi-Yau cases.

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1 Introduction

There are great progresses in studying worldsheet instanton corrections by the discovery of the mirror symmetry \[1\],\[2\]. When we study topological sigma model \[3\], physical observables are constructed by combining couplings, correlation functions and metrics of operators. Typical constituent blocks are three-point and two-point functions (metrics). Their behaviors are controlled by moduli parameters and these observables contain information about moduli space of the sigma model \[4\]-\[10\]. In this paper, we investigate a Kähler potential of the B-model moduli space from the point of view of the topological sigma model together with results by CFT at Gepner point.

2 Calabi-Yau \(d\)-fold

We concentrate on a one-parameter family of Calabi-Yau \(d\)-fold \(M\) realized as a zero locus of a hypersurface in a projective space \(CP^{d+1}\)

\[
W; \{ p = 0 \}/\mathbb{Z}_N^{(N-1)},
\]

\[
p = X_1^N + X_2^N + \cdots + X_N^N - N\psi X_1 X_2 \cdots X_N = 0,
\]

where we introduce a number \(N = d + 2\). Hodge numbers \(h^{p,q}\) of this \(d\) fold are calculated as \[11\]

\[
h^{p,q} = \delta_{p+q,d}, \quad (0 \leq p \leq d, 0 \leq q \leq d, p \neq q),
\]

\[
h^{p,p} = \delta_{2p,d} + \sum_{m=0}^{p} (-1)^m \binom{d+2}{m} \binom{(p+1-m)(d+1)+p}{d+1},
\]

and deformation of complex structure is parametrized by a complex parameter \(\psi\).

In order to describe the complex structure moduli space near Gepner point (\(\psi \sim 0\)), we uses a basis of periods

\[
\tilde{\omega}_k(\psi) = \left[ \frac{k}{N} \right]^N \frac{(N\psi)^k}{\Gamma(k)} \cdot G_k(\psi),
\]

\[
G_k := 1 + \sum_{n=1}^{\infty} \left[ \frac{\Gamma\left(\frac{k}{N} + n\right)}{\Gamma\left(\frac{k}{N}\right)} \right]^N \frac{\Gamma(k)}{\Gamma(Nn+k)} (N\psi)^{Nn}.
\]

They behave around \(\psi \sim 0\)

\[
\tilde{\omega}_k(\psi) = \left[ \frac{k}{N} \right]^N \frac{(N\psi)^k}{\Gamma(k)} \cdot [1 + \mathcal{O}(\psi^N)].
\]
This set is a projective coordinate of the B-model moduli space. Also we use a canonical basis \( \{ \omega_k^{(0)} \} \) of the coordinates at the Gepner point as
\[
\tilde{\omega}_k = \omega_k^{(0)} \cdot \left[ \Gamma \left( \frac{k}{N} \right) \right]^N,
\]
and take ratios to construct normalized coordinates
\[
\omega_{k-1} := \frac{\omega_k^{(0)}}{\omega_1^{(0)}} = \frac{(N\psi)^{k-1}}{(k-1)!} \cdot \frac{G_k}{G_1} \quad (k = 1, 2, \cdots, N - 1).
\]
Some part of the cohomology classes of \( H^d(W) \) is associated with the complex structure deformation with Hodge numbers \( h_\ell,d - \ell = 1 \) \( (\ell = 0, 1, 2, \cdots, d) \). Then associated operators are represented as
\[
O^{(k)} := (X_1X_2\cdots X_N)^k \quad (k = 0, 1, 2, \cdots, N - 2),
\]
and each \( O^{(k)} \) is related to a canonical period \( \omega_k \). Also a coupling \( \tilde{\psi} := N\psi \) is associated with an operator \( O^{(1)} = X_1X_2\cdots X_N \). When we choose a canonical set of periods, these operators are normalized appropriately and fusion couplings \( \kappa_\ell \) are evaluated at \( \psi = 0 \)
\[
\tilde{\omega} := \left( \begin{array}{cccc}
\omega_0 & \omega_1 & \cdots & \omega_{N-2}
\end{array} \right),
\]
\[
K := \left( \begin{array}{cccc}
0 & K_0 & & \\
0 & 0 & K_1 & \\
& 0 & 0 & K_2 \\
& & \ddots & \ddots \\
& & & 0 & K_{d-1}
\end{array} \right)
\]
\[
\partial_{\tilde{\psi}} \tilde{\omega} = K \tilde{\omega},
\]
\[
\leftrightarrow O^{(1)} \cdot O^{(\ell)} = K_\ell O^{(\ell+1)} \quad (\ell = 0, 1, \cdots, d - 1).
\]
\[
K_0 = 1, \quad K_1 = \partial \omega_1 = 1 + O(\psi^N),
\]
\[
K_\ell = \frac{1}{K_{\ell-1}} \frac{1}{K_{\ell-2}} \cdots \frac{1}{K_0} \partial \omega_\ell = 1 + O(\psi^N), \quad (\ell \geq 2).
\]
In this canonical set of periods, the three-point couplings are normalized to units up to terms with order \( O(\psi^N) \).

### 3 Kähler potential

Next let us consider a Kähler potential \( K \) of the moduli space. We can construct \( K \) as a quadratic form of periods
\[
e^{-K} = i^d \int_M \Omega \wedge \bar{\Omega} = \sum_{m,n=1}^{N-1} I_{m,n} \tilde{\omega}_m^\dagger \tilde{\omega}_n.
\]
The matrix $I = \{I_{m,n}\}$ is determined by properties of intersection numbers of homology cycles and does not change under any infinitesimal deformation of continuous moduli parameters. That is, the $I_{m,n}$s cannot depend on any moduli parameters and turns out to be constant numbers.

On the other hand, global structures of the moduli space are encoded in monodromies and it is important to see property of $K$ under a global monodromy transformation. Our basis $\{\tilde{\omega}_k\}$ diagonalizes a cyclic $\mathbb{Z}_N$ monodromy $\psi \rightarrow \alpha \psi$ ($\alpha = e^{2\pi i/N}$) at $\psi = 0$

$$\tilde{\omega}_k(\alpha \psi) = \alpha^k \tilde{\omega}_k(\psi) \quad (k = 1, 2, \cdots, N - 1).$$

$$\alpha = e^{2\pi i/N}.$$ But this transformation induces a change of the $I_{m,n}$

$$I_{m,n} \rightarrow I_{m,n} \cdot \alpha^{-m+n} \quad (m, n = 1, 2, \cdots, N - 1).$$

Because the Kähler potential is physical quantity and should not depend on monodromies, it is invariant under the transformation. Only invariant parts of this transformation are diagonal ones $I_{m,m} (m = 1, 2, \cdots, N - 1)$ and we find that the matrix $I$ is diagonal one:

$$e^{-K} = \sum_{k=1}^{N-1} I_k \tilde{\omega}_k^\dagger \tilde{\omega}_k. \quad (1)$$

4 Determination of $I_k$

Next, in order to determine the Kähler potential, all we have to do is to fix the diagonal matrix. We use a method of the $tt^*$ fusion in fixing the $I_k \quad (k = 1, 2, \cdots, N - 1)$. First note that the Kähler potential is related to a moduli space metric $g_{\tilde{\psi}\tilde{\psi}}$

$$\partial \bar{\partial} K = g_{\tilde{\psi}\tilde{\psi}}, \quad \partial = \frac{\partial}{\partial \tilde{\psi}}, \quad \bar{\partial} = \frac{\partial}{\partial \bar{\tilde{\psi}}}. \quad (2)$$

When we introduce a set of hermitian two-point functions [1]

$$\langle \mathcal{O}(\ell) | \mathcal{O}(\ell') \rangle = e^{q_{\ell}} (0 \leq \ell \leq N - 2), \quad (2)$$

the Kähler potential and Zamolodchikov metric $g_{\tilde{\psi}\tilde{\psi}}$ are expressed as

$$e^{q_0} = e^{-K}, \quad g_{\tilde{\psi}\tilde{\psi}} = e^{q_{1-q_0}},$$

$$-\partial \bar{\partial} q_0 = e^{q_{1-q_0}}. \quad (3)$$

[1] These two-point functions are different from topological metrics $\eta_{\ell m} = \langle \mathcal{O}(\ell) \mathcal{O}(m) \rangle = N \delta_{\ell+m,d}$. 

3
The equation Eq.(3) is a part of the $tt^*$-fusion equation [12, 9] of the Calabi-Yau $d$-fold with fusion couplings $\kappa_n$

\[
\begin{align*}
\partial\bar{\partial}q_0 + |\kappa_0|^2 e^{q_1-q_0} &= 0 , \\
\partial\bar{\partial}q_\ell + |\kappa_\ell|^2 e^{q_{\ell+1}-q_\ell} - |\kappa_{\ell-1}|^2 e^{q_{\ell}-q_{\ell-1}} &= 0 \ (1 \leq \ell \leq d - 1) , \\
\partial\bar{\partial}q_d - |\kappa_{d-1}|^2 e^{q_{d}-q_{d-1}} &= 0 .
\end{align*}
\]

(4)

This set of equations Eq.(4) represents an $A_d$ type Toda system. By introducing new variables

\[
\begin{align*}
\tilde{q}_0 &= q_0 , \\
\tilde{q}_\ell &= q_\ell + \sum_{n=0}^{\ell-1} \log |\kappa_n|^2 \ (\ell \geq 1) ,
\end{align*}
\]

and noting relations $\partial\bar{\partial}\tilde{q}_\ell = \partial\bar{\partial}q_\ell$, we reexpress the Toda system into a formula

\[
U_\ell = (\ell + 1)U_0 + \sum_{n=0}^{\ell-1} (\ell - n)D \log(-U_n) \ (1 \leq \ell \leq d - 1) ,
\]

\[
U_0 = Dq_0 = D \log \left[ 1 + \sum_{\ell=2}^{N-1} \frac{I_\ell}{I_1} \left| \frac{\tilde{\omega}_\ell}{\omega_1} \right|^2 \right] ,
\]

\[
U_d = 0 ,
\]

\[
U_n = \sum_{m=0}^{n} Dq_m \ (1 \leq n \leq d) ,
\]

\[
D := \partial\bar{\partial} ,
\]

\[
|\kappa_n|^2 \cdot e^{q_{n+1}-q_{n}} = -U_n \ (0 \leq n \leq d - 1)
\]

(5)

The number of diagonal components $I_k$ in the Kähler potential is $(d+1)$ and the above Toda system with $(d+1)$ equations gives us consistency conditions of the $I_k$s. Now recall that the components of the matrix $I$ are numerical constants. We may truncate terms of the periods $\tilde{\omega}_k$s in the series expansions up to order $O(\psi^N)$ for the purpose of determination of $I$

\[
\tilde{\omega}_k(\psi) = f_k \cdot (N\psi)^k \left[ 1 + O(\psi^N) \right] \ (k = 1, 2, \cdots, N - 1) ,
\]

\[
U_0 = D \log \left[ 1 + \sum_{\ell=2}^{N-1} a_\ell (\tilde{\omega}_\ell/\omega_1)^{\ell-1} + \cdots \right] ,
\]

\[
f_k := \left[ \Gamma \left( \frac{k}{N} \right) \right]^N \frac{1}{\Gamma(k)} , \ a_n := \frac{I_n}{I_1} \left| \frac{f_n}{f_1} \right|^2 .
\]

(6)

We calculate the $U_n$ concretely for $n \leq 10$, and propose a conjecture for the $\{U_n\}$s at the Gepner point $\psi = 0$

\[
\begin{align*}
U_0 &= a_2 , \ U_d = 0 , \\
U_n &= (n + 1)^2 \frac{a_{n+2}}{a_{n+1}} \ (1 \leq n \leq d - 1).
\end{align*}
\]
When we use definitions of $a_n$ and $f_n$, the $U_n$s are expressed as

$$U_0 = a_2 = \frac{I_2}{I_1} \left[ \frac{\Gamma\left(\frac{2}{N}\right)}{\Gamma\left(\frac{1}{N}\right)} \right]^{2N},$$

$$U_n = (n + 1)^2 \frac{a_{n+2}}{a_{n+1}} \left( \frac{\Gamma\left(\frac{n+2}{N}\right)}{\Gamma\left(\frac{n+1}{N}\right)} \right)^{2N} \quad (1 \leq n \leq d - 1).$$

Also we can obtain expressions of hermitian two-point functions by remarking that the three-point couplings become constants $\kappa_n = 1$ ($n = 0, 1, \cdots, d$) at the $\psi = 0$

$$e^{q_{n+1} - q_n} = -\frac{I_{n+2}}{I_{n+1}} \left[ \frac{\Gamma\left(\frac{n+2}{N}\right)}{\Gamma\left(\frac{n+1}{N}\right)} \right]^{2N} \quad (0 \leq n \leq d - 1).$$

Equivalently, normalized two-point functions are represented as

$$\frac{\langle \bar{O}^{(m)}|O^{(m)} \rangle}{\langle \bar{O}^{(0)}|O^{(0)} \rangle} = e^{q_m - q_0} = (-1)^m \cdot \frac{I_{m+1}}{I_1} \left[ \frac{\Gamma\left(\frac{m+1}{N}\right)}{\Gamma\left(\frac{1}{N}\right)} \right]^{2N} \quad (0 \leq m \leq d). \quad (7)$$

Now let us compare these results with those of CFT calculations [13, 12] for minimal models associated with the $d$-fold $M$

$$e^{q_n} = \frac{1}{N^N} \left[ \frac{\Gamma\left(\frac{n+1}{N}\right)}{\Gamma\left(1 - \frac{n+1}{N}\right)} \right]^N \quad (n = 0, 1, \cdots, N - 2),$$

$$\rightarrow e^{q_{n+1} - q_0} = \left[ \frac{\Gamma\left(\frac{n+1}{N}\right)}{\Gamma\left(\frac{1}{N}\right)} \Gamma\left(1 - \frac{1}{N}\right) \right]^N \quad (8)$$

By comparing two results Eq.(7) and Eq.(8), we can obtain the components of the matrix $I$ up to one numerical constant $c$

$$I_m = c \cdot (-1)^m \left( \sin \frac{\pi m}{N} \right)^N \quad (m = 1, 2, \cdots, N - 1).$$

In this case, the $e^{-K}$ in Eq.(1) is given as

$$e^{-K} = \sum_{m=1}^{N-1} c \cdot (-1)^m \alpha^N \left[ \frac{\Gamma\left(\frac{m}{N}\right)}{\Gamma\left(1 - \frac{m}{N}\right)} \right]^N \times \left( \frac{N^2 \psi \bar{\psi}}{[\Gamma(m)]^2} \right)^m \cdot |G_m(\psi)|^2.$$ 

Because the Kähler potential is not a function but a section of a line bundle, there is an arbitrariness of multiplication of arbitrary (anti-)holomorphic functions. We choose the normalization factor as

$$c = \frac{-1}{\pi N \cdot N^{N+2}}.$$
then the Kähler potential is written as

\[ e^{-K} = (\bar{\psi}\psi) \cdot \sum_{m=1}^{N-1} (-1)^{m-1} \left[ \frac{1}{N} \frac{\Gamma\left(\frac{m}{N}\right)}{\Gamma\left(1 - \frac{m}{N}\right)} \right]^{N} \frac{(N^2\bar{\psi}\psi)^{m-1}}{[\Gamma(m)]^2} |G_m(\psi)|^2. \] (9)

In order to confirm the validity of this convention, we restrict ourselves to the 3-fold \( (N = 5) \) case and consider a 3-point function \( \kappa \) of the operator \( \mathcal{O}(1) \) in the B-model

\[ \kappa := \kappa_{\bar{\psi}\psi\psi} = \frac{1}{5^3} \cdot \frac{5\psi^2}{1 - \psi^5}. \] (10)

(In considering the \( tt^* \) equation, we use normalized periods \( \omega_k \) by taking ratios. In that case, the \( e^{-K} \) in Eq.(9) and the three-point function Eq.(10) are divided by a factor \( \sim |\psi|^2. \) )

This coupling is a section of holomorphic line bundle and changes with a normalization of the Kähler potential. But there is an invariant 3-point coupling

\[ (g_{\bar{\psi}\psi})^{-3/2} \cdot e^K \cdot |\kappa|, \]

and its value is evaluated at the \( \psi = 0 \) in our normalization

\[ (g_{\bar{\psi}\psi})^{-3/2} \cdot e^K \cdot |\kappa| = \left[ \frac{\Gamma\left(\frac{2}{5}\right)}{\Gamma\left(\frac{4}{5}\right)} \right]^{15/2} \cdot \left[ \frac{\Gamma\left(\frac{1}{5}\right)}{\Gamma\left(\frac{1}{5}\right)} \right]^{5/2} = 1.5553189899632389725 \cdots. \]

This result coincides with that of the calculation of CFT [13, 2].

5 Metric and Curvature

Now we have an exact formula of the \( K \) with a moduli parameter \( \psi \) and can evaluate the corrections in the physical observables by the marginal deformation of the CFT. For simplicity, we will consider leading corrections of the Kähler potential, metric and scalar curvature in the moduli space. When we define a function \( A_m \)

\[ A_m = (-N^2)^{m-1} \cdot \frac{1}{[\Gamma(m)]^2} \left[ \frac{1}{N} \frac{\Gamma\left(\frac{m}{N}\right)}{\Gamma\left(1 - \frac{m}{N}\right)} \right]^{N} \quad (1 \leq m \leq N - 1), \]

we can obtain these formulae

\[ \frac{A_m}{A_n} = (-N^2)^{m-n} \left( \frac{\Gamma(n)}{\Gamma(m)} \right)^2 \left[ \frac{\Gamma\left(\frac{m}{N}\right)}{\Gamma\left(1 - \frac{m}{N}\right)} \right]^{N} \quad (m, n \leq N - 1), \]

\[ e^{-K} = \left[ \frac{1}{N} \frac{\Gamma\left(\frac{1}{N}\right)}{\Gamma\left(1 - \frac{1}{N}\right)} \right]^{N} \cdot (\bar{\psi}\psi) \times \left[ 1 - N^2 \cdot \left[ \frac{\Gamma\left(\frac{2}{5}\right)}{\Gamma\left(1 - \frac{2}{5}\right)} \right]^{N} \right]^{N} \quad (N \geq 3), \]
\[ g_{\psi \bar{\psi}} = N^2 \cdot \left[ \frac{\Gamma\left(\frac{2}{N}\right) \Gamma\left(1 - \frac{1}{N}\right)}{\Gamma\left(1 - \frac{3}{N}\right) \Gamma\left(\frac{1}{N}\right)} \right]^N + (\psi \bar{\psi}) \left[ 2 \cdot \left( \frac{A_2}{A_1} \right)^2 - 4 \left( \frac{A_3}{A_1} \right) \right] + \cdots \quad (N \geq 4), \]

\[ R = -4 + 2 \cdot \left[ \frac{\Gamma\left(\frac{2}{N}\right) \Gamma\left(3 - \frac{3}{N}\right)}{\Gamma\left(1 - \frac{1}{N}\right) \Gamma\left(1 - \frac{3}{N}\right)} \right]^N \left[ \frac{\Gamma\left(1 - \frac{2}{N}\right)}{\Gamma\left(\frac{2}{N}\right)} \right]^{2N} \]

\[ + (\psi \bar{\psi}) \left[ 24 \left( \frac{A_3}{A_2} \right) - 96 \left( \frac{A_1}{A_2} \right)^2 + 72 \left( \frac{A_1}{A_2} \right) \left( \frac{A_3}{A_2} \right) \right] + \cdots \quad (N \geq 5). \]

The Ricci tensor of the moduli space is represented as \( R_{\psi \bar{\psi}} = \frac{1}{2} g_{\psi \bar{\psi}} R \). For the torus \((N = 3)\) case, its metric is a standard one of the upper-half plane and the scalar curvature is constant \( R = -4 \) except for points \( \psi = e^{2\pi i \ell/3} \) \((\ell = 0, 1, 2)\). Also the K3 \((N = 4)\) metric is given by the above formula and related scalar curvature is a negative constant number \( R = -2 \) except for \( \psi = e^{\pi i \ell/2} \) \((\ell = 0, 1, 2, 3)\).

We plot this scalar curvature at the \( \psi = 0 \) in Fig.1. It increases monotonically with the dimension \( d \). But the derivative with respect to \( \partial_{\psi} \partial_{\bar{\psi}} \) is negative for \( N \geq 11 \) cases at \( \psi = 0 \) as shown in Fig.2.

![Figure 1: Leading part \( R_0 \) of Scalar Curvature \( R \). This part depends on the dimension \( N = d + 2 \) of the Calabi-Yau \( d \)-fold. It increases monotonically with the \( d \). For the torus \((N = 3)\) and K3 \((N = 4)\) cases, associated curvatures are negative. But the other cases have positive curvatures at \( \psi = 0 \).](image)

Let us return to the hermitian two-point functions. They are represented as some combinations of these \( K, g_{\psi \bar{\psi}} \) and \( R \)

\[ e^{\tilde{q}_0} = e^{-K}, \quad e^{\tilde{q}_1 - \tilde{q}_0} = \frac{1}{N^2} g_{\psi \bar{\psi}}, \quad e^{\tilde{q}_2 - \tilde{q}_1} = \frac{1}{N^2} g_{\psi \bar{\psi}} \left( \frac{R}{2} + 2 \right), \]

\[ e^{\tilde{q}_3 - \tilde{q}_2} = \frac{1}{N^2} g_{\psi \bar{\psi}} \left[ 3 \left( \frac{R}{2} + 1 \right) - g_{\psi \bar{\psi}} \partial_{\psi} \partial_{\bar{\psi}} \log \left( \frac{R}{2} + 2 \right) \right], \]
Figure 2: Subleading part $R_1$ of Scalar Curvature $R$. This part is a coefficient of $\psi \bar{\psi}$ in the $R$ and depends on the dimension of the CY $d$-fold. For the $5 \leq N \leq 10$ cases, its value is positive. But the $R_1$ is negative for the other cases $N \geq 11$.

\[
e^{\tilde{q}_4-\tilde{q}_3} = \frac{1}{N^2} g_{\psi \bar{\psi}} \left[ 4 + 3R - 2g^{\psi \bar{\psi}} \partial_\psi \bar{\partial}_\psi \log \left( \frac{R}{2} + 2 \right) 
- g^{\psi \bar{\psi}} \partial_\psi \bar{\partial}_\psi \log \left[ 3 \left( \frac{R}{2} + 1 \right) - g^{\psi \bar{\psi}} \partial_\psi \bar{\partial}_\psi \log \left( \frac{R}{2} + 2 \right) \right] \right],
\]

Now we know the formula of the $K$, $R$, and $g_{\psi \bar{\psi}}$ and evaluate moduli dependences of these correlators.

In this paper we develop a method to determine the Kähler potential unambiguously by comparing the result of CFT with that of topological sigma model. We calculate the metric and curvature in the neighborhood of the Gepner point, which have dependences of moduli parameter $\psi$. The result represents a marginal deformation of the CFT. But the formula of the $K$ is exact and we can study it at all points in the moduli space by analytic continuation. The method we developed here is not restricted to the specific model and can be applied to any other Calabi-Yau spaces.

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