Vibration analysis on compact car shock absorber

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Abstract. Shock absorber is a part of the suspension system which provides comfort experience while driving. Resonance, a phenomenon where forced frequency is coinciding with the natural frequency has significant effect on the shock absorber itself. Thus, in this study, natural frequencies of the shock absorber in a 2 degree-of-freedom system were investigated using Wolfram Mathematica 11, CATIA, and ANSYS. Both theoretical and simulation study how will the resonance affect the car shock absorber. The parametric study on the performance of shock absorber also had been conducted. It is found that the failure tends to occur on coil sprung of the shock absorber before the body of the shock absorber is fail. From mathematical modelling, it can also be seen that higher vibration level occurred on un-sprung mass compare to spring mass. This is due to the weight of sprung mass which could stabilize as compared with the weight of un-sprung mass. Besides that, two natural frequencies had been obtained which are 1.0 Hz and 9.1 Hz for sprung mass and un-sprung mass respectively where the acceleration is recorded as maximum. In conclusion, ANSYS can be used to validate with theoretical results with complete model in order to match with mathematical modelling.

1. Introduction

Car suspension system is known to absorb vibration from the vehicle itself. Suspension system consists of two main components, which are springs and shock absorber. This allows the vehicle to absorb vibration or to minimize the vibration level to provide a comfort driving experience.

In the existing suspension system, different types of spring were used, such as leaf spring and coil spring. Leaf spring is arc-shaped spring steels which cascade together into few layers. It is normally locate at the axle of the wheels. While coil spring, unlike leaf spring consists of a few layers of spring steel, it is a helical-shaped spring. Coil spring mounted on both rear wheels and front wheels [1]. It is normally installed in light vehicles unlike leaf spring which is commonly used in heavy type vehicles such as trucks [2]. Kong et al. mentioned that the ease of maintenance and low cost of leaf spring are
the reasons why leaf spring is commonly used in heavy vehicles [3]. Car suspension system can be considered as spring-mass-damper system in other words the system consists of spring, mass and damper.

For a car suspension system, shock absorber can have a huge impact on the comfort level of driving experience. According ISO 2631-1, ride comfort can be affected between 0.5 Hz and 80 Hz [4]. In order to analyse the suspension system, mathematical model can be used such as quarter-car, half-car and full-car models. The models can be generally defined with the governing equation.

Mathematical modelling can be used to analyse the car suspension system. A car suspension system can be expressed as spring-damper-mass system which means the system consists of these three main components: spring, damper and mass. Different car model can be used for the analysis such as quarter-car model, half-car model and full-car model. Quarter-car model offers simplicity by defining the system solely in vertical motion. Bhargav et al. also stated that quarter-car model is easily understandable as it only described the suspension system in vertical motion [5]. Half-car model, unlike quarter-car model, it takes into account rotational motion which is one of the important element that could has direct impact on the ride comfort [6]. Another study, Gang Wang et al. stated that half-car model is normally used due to its symmetry which analysis of both front wheel and rear wheel can be viewed [7]. Full-car model is the most complex model among the other two models which showed the suspension system for both front wheels and rear wheels. Full car model can either be seen as four quarter-car models which could be considered an approach to analyse or direct analysis on the model [8]. Full-car model has better accuracy than the other two models which are quarter-car model and half-car model because full-car model can analyse if in a case where asymmetry exist such as uncertainties of road.

Vibration control can be classified into three types and they are passive, semi-active, and active. Passive suspension system has the advantages of low cost and zero energy consumption and is commonly used in suspension system. Passive suspension system can be understand as a system consists of energy storing element and dampers according to Syed et al [9]. Javad et al. supported the usage of passive suspension system as this is an approach to obtain appropriate spring stiffness and damping coefficient with the lowest possible cost [10]. However, passive suspension system’s performance can be limited because of unable to dissipate or add energy in a control manner like active suspension system.

Syed et al. also mentioned that semi-active suspension system can dissipate energy in a control manner but energy cannot be build up to the system. In recent researches, Magneto-rheological (MR) damper is mostly used in developing semi-active suspension system [9]. Low power consumption and safety provided by MR encouraged the usage of it in the suspension system [11].

Active suspension system performs better than both passive and semi-active suspension system. It provides better ride comfort due to its ability to attenuate vibration amplitude with the use of actuator [12]. Active suspension system is still not commonly used as the technology on active suspension system is not that advance enough and costly compare to passive and semi-active suspension system although it has advantages over passive and semi-active suspension system.

In this study, it aims to investigate and conduct analysis on car’s suspension system. Another purpose is to investigate the resonance frequency that occurs on the suspension system. Resonance is a phenomenon when natural frequency is same with the forcing frequency. Excessive deflection will happened if suspension system is exposed to resonance which will eventually leads to system failure.

2. Methodology
In this study, there are two phases of analysis which are mathematical modelling and vibration analysis. Mathematical modelling is used to define the suspension system. Mathematical modelling used in this study is quarter-car model which represent one-quarter of the suspension system. Derivations of the equations are developed in phase 1 to obtain the theoretical result. Phase 2, the simulation, by using finite element analysis (FEA) based software – ANSYS to conduct the modal analysis on the suspension system. To import 3D model to ANSYS, a model is created using CATIA.
In ANSYS, it is aimed to study the characteristic of the shock absorber when exposed to the natural frequency.

2.1 Mathematical modelling

Mathematical modelling on suspension system is vastly used and quarter-car model is the model most commonly used in the activities of research due to its simplicity. Based on the several studies from literature review [12 - 14], quarter-car model has been used to analyse the vehicle suspension system. In this study, quarter-car model is developed according to a passive suspension system which consists of a few important elements which are sprung mass, un-sprung mass, spring and damper. Figure 1 shows a drawn 2 degree-of-freedom (DOF) quarter-car model of suspension system.

![Figure 1. Two degree-of-freedom quarter-car model of shock absorber](image)

By applying Newton’s second law of motion to both sprung mass \( m_1 \) and un-sprung mass \( m_2 \), equations of motion can be represented in the model in Figure 1 as below:

\[
\begin{align*}
\quad m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 - c_1 \dot{x}_2 - k_1 x_2 &= 0 \\
\quad m_2 \ddot{x}_2 - c_1 (\dot{x}_1 - \dot{x}_2) - k_1 (x_1 - x_2) &= k_2 y 
\end{align*}
\]

In Figure 1, \( c_1 \) represents suspension damping coefficient while \( k_1 \) represents suspension stiffness and \( k_2 \) represents tire stiffness, \( m_1 \) and \( m_2 \) represent sprung mass and un-sprung mass respectively. Road, \( y \) acts as base excitation of this system and assuming it is sinusoidal. Equations (1) and (2) are equations of motion based on the quarter-car model.

\[
\begin{align*}
\quad x_1(t) &= X_1 e^{i\omega t} \\
\quad x_2(t) &= X_2 e^{i\omega t} \\
\quad y(t) &= Y e^{i\omega t}
\end{align*}
\]

\( x_1(t) \), \( x_2(t) \) and \( y(t) \) can be represent as in Equation (3), Equation (4) and Equation (5) respectively and \( X_1 \), \( X_2 \) and \( Y \) represent the amplitude for each equation. After that, Equation (3), Equation (4) and Equation (5) are to substitute into equation (1) and (2) and transform into matrix form

\[
\begin{align*}
\quad Z &= \begin{bmatrix}
\quad i\omega c_1 + k_1 - \omega^2 m_1 & -i\omega c_1 - k_1 \\
\quad -i\omega c_1 - k_1 & i\omega c_1 + k_1 + k_2
\end{bmatrix} \begin{bmatrix}
\quad X_1 \\
\quad X_2
\end{bmatrix} = \begin{bmatrix}
\quad 0 \\
\quad k_2 Y
\end{bmatrix} \\
\quad F_0 &= \begin{bmatrix}
\quad 0 \\
\quad k_2 Y
\end{bmatrix}
\end{align*}
\]
Equation (6) can be obtained after the substitution of the equations (3), (4) and (5) into equation (1) and equation (2). Then, $Z$ represents the mechanical impedance which is also the first term of the matrix, Equation (6) while $F_0$ represents external force exerted on the system.

$$
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} = Z^{-1}F_0
$$

(9)

$X_1$ and $X_2$ can be obtained by using Equation (9) where $Z^{-1}$ is the inverse matrix of $Z$. Input of the parameters refer to previous study and is showed in Table 1.

| Parameters                              | Value     |
|-----------------------------------------|-----------|
| Sprung mass, $m_1$                      | 290 kg    |
| Un-sprung mass, $m_2$                   | 59 kg     |
| Shock absorber spring stiffness, $k_1$ | 16812 N/m |
| Tire stiffness, $k_2$                   | 190000 N/m|
| Shock absorber damping coefficient, $c_1$ | 1000 N.s/m |

### 2.2 Phase 2: Simulation

Phase 2 consists of two main steps. The first step is to create a 3D model in CATIA which will be imported to ANSYS. While second step is to convert the CAD file into a compatible file type and import to ANSYS, then modal analysis will be conducted.

### 3. Results and Discussions

In analytical analysis, acceleration against frequency graphs were plotted after solving by using Wolfram Mathematica 11. Equation of motions which represent the 2 DOF system were used for the calculation and further solving using Wolfram Mathematica.

![Figure 2. Graph of acceleration against frequency of both sprung mass and un-sprung mass at t=1s](image)

In figure 2, it shows the acceleration against frequency for both sprung mass and un-sprung mass. At 6.3856 rad/s or 1.0 Hz the acceleration for sprung mass is 0.1779 m/s$^2$ while at 57.1314 rad/s, acceleration of un-sprung mass is 2.8562 m/s$^2$. The magnitude of acceleration of un-sprung mass is comparatively higher than that of sprung mass. The frequencies mentioned are the resonant frequency...
for sprung mass and un-sprung mass respectively where the maximum acceleration occurred. It could be explained that the resonance has greater effect on sprung mass than un-sprung mass.

3.1 Parametric Studies
There are three parametric study cases were investigated by changing one of the parameters by two times. The changing variables are $k_1$, $k_2$ and $c_1$ while the remaining variables are remained constant.

3.1.1 Case 1: $k_1 = 33624$ N/m

In figure 3, maximum acceleration $0.9133$ m/s$^2$ occurs at frequency $10.0245$ rad/s while in figure 4, maximum acceleration $2.6125$ m/s$^2$ occurs at frequency $60.4339$ rad/s. The natural frequency of sprung shifted to right compare to original due to the linear relationship of stiffness to natural frequency and the increase in magnitude of acceleration of sprung mass due to the greater stiffness greater potential energy which able to absorb as well as able to release more energy compare to original parameters. However, negligible change in un-sprung mass as no change in stiffness for un-sprung mass.

3.1.2 Case 2: $k_2 = 380000$ N/m
In figure 5, at maximum acceleration \(0.2505 \text{ m/s}^2\) occurs at frequency \(7.2777 \text{ rad/s}\). While in figure 6, at maximum acceleration \(-0.84645 \text{ m/s}^2\) occurs at frequency \(80.9205 \text{ rad/s}\). In this case, negligible change happened in sprung mass while significant change as there is only increase of stiffness of un-sprung mass but not sprung mass.

### 3.1.3 Case 3: \(c_1 = 2000 \text{ N.s/m}\)

![Figure 7. Graph of acceleration against frequency for sprung mass when \(c_1 = 2000 \text{ N.s/m}\)](image)

![Figure 8. Graph of acceleration against frequency for un-sprung mass when \(c_1 = 2000 \text{ N.s/m}\)](image)

In figure 7, at maximum acceleration \(-0.0988 \text{ m/s}^2\) occurs at frequency \(8.0861 \text{ rad/s}\). While in figure 8, at maximum acceleration \(-0.7686 \text{ m/s}^2\) occurs at frequency \(55.7153 \text{ rad/s}\). The increase in damping coefficient also increase the energy loss when the car suspension system vibrate and affect both sprung mass and un-sprung mass with decrease in magnitude of acceleration.

### 3.1.4 Analysis

| Case no. | Sprung mass \(x_1(t), \text{m/s}^2\) | \(\omega_1, \text{rad/s}\) | Un-sprung mass \(x_2(t), \text{m/s}^2\) | \(\omega_2, \text{rad/s}\) |
|----------|-------------------------------|-----------------|-------------------------------|-----------------|
| 1. \(k_1 = 33624 \text{ N/m}\) | 0.9133 | 10.0245 | 2.6125 | 60.4339 |
| 2. \(k_2 = 380000 \text{ N/m}\) | 0.2505 | 7.2777 | -8.4645 | 80.9205 |
| 3. \(c_1 = 2000 \text{ N.s/m}\) | -0.0988 | 8.0861 | -0.7686 | 55.7153 |

In table 2, it concludes the results from figure 3 to figure 8, shows that when increased the spring stiffness either of sprung or of un-sprung mass, the acceleration of sprung mass or un-sprung mass increased significantly, respectively, but with its natural frequency shifted away when compared to original parameters for both Case 1 and Case 2. While in Case 3, the change in damping coefficient has almost negligible effect on the natural frequency of both sprung mass and un-sprung mass but greatly reduced the magnitude of acceleration of both sprung mass and un-sprung mass. It can be explained that the natural frequency can be affected by the spring stiffness but not the damping coefficient and it is directly proportional to the spring stiffness.
3.2 Analysis Simulation

![Figure 9. Total deformation at 35.612 Hz](image)

![Figure 10. Total deformation at 39.464 Hz](image)

In figure 9, it shows the first mode shape of the shock absorber at 35.612 Hz and in figure 10, shows the first mode shape of the shock absorber at 39.464 Hz. From the mode shapes, the coil spring vibrates the most at natural frequencies. It can be explained that the coil spring will most likely to fail first before the shock absorber itself failed. Once the coil spring failed, without the spring, the shock absorber can hardly return to original position since the function of coil spring is to absorb the potential energy and helps the shock absorber to return to original position so that the wheel can always keep intact with the road.

4. Conclusion

According to this study, natural frequencies of the car shock absorber can be obtained using both calculations and simulation. In calculation, it investigated the quarter-car model of the suspension system and obtained frequency response of the model to observe how the natural frequencies effect on the system. Based on the results, resonance has more significant effect on the un-sprung mass which is also known as the shock absorber compare to the sprung mass.

In simulation, it is further studies the effect of resonance on shock absorber only. The result explains that the coil spring of the shock absorber will fail first before the tubes of shock absorber fail. Thus, both theoretical and simulation results can be validated as the resonance has significant effect on the shock absorber.

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