Topology bifurcation of a magnetic flux surface in magnetized plasmas

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Abstract. Bifurcation phenomena in the radial propagation of a heat pulse driven by modulated electron cyclotron heating are observed in the Large Helical Device. There are two patterns of heat pulse propagation observed in the flat temperature region. One is a bi-directional slow heat pulse propagation observed in the fast magnetic shear drop and the other is a fast heat pulse propagation observed in the slow magnetic shear drop. This bifurcation in the heat pulse propagation suggests the topology bifurcation of a magnetic flux surface in the plasma.

The transition from regular to chaotic motion is very often represented as a topological bifurcation of a torus in phase space \([1]\). Islands are formed in nested tori by resonant perturbations, and if global stochasticity sets in, turbulence-driven transport (such as super-diffusion) can occur \([2]\). This topological bifurcation has been considered to happen in high-temperature toroidal plasmas, e.g. during disruption in tokamaks \([3, 4]\), by a large deformation such as a spontaneous transition from the stochastic state to helical equilibrium in reversed-field pinch (RFP) plasmas \([5–7]\), or at the plasma edge as seen in the edge-localized-modes mitigation

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shows the time evolution of the toroidal plasma current, 

\[ I_p(t) = \frac{r B_0}{r B_0} \]

and magnetic shear (faster drop and slower drop) are plotted in figure 0.16 s in this experiment. Two cases of the discharge with slight differences in plasma current \[ \times 3.6 \text{ m} \] and the plasma density is 1

\[ B \]

in this experiment, the magnetic field, in which 99% of the plasma kinetic energy is confined and is 0.63 m in this discharge. In this

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This is because of the complex interplay between transport and sheared radial electric field, which contributes to the reduction of turbulence-driven transport in the proximity of rational surfaces [13].

More recently, fine structures of internal magnetic field and current density have been measured with the Faraday effect [14, 15], and direct measurement of the internal magnetic field structure associated with a three-dimensional (3D) helical equilibrium generated spontaneously in the core of an axisymmetric toroidal plasma clearly demonstrated the bifurcation from a stochastic magnetic field to a helical flux surface [16]. Alternatively, the change of magnetic topology, for example the change from a nested magnetic flux surface to a magnetic island, can be identified by heat pulse propagation [17, 18]. These experiments suggest that the stochasticization of magnetic flux surfaces can also be identified by the characteristics of heat pulse propagation. In this paper, the topology bifurcation of magnetic topology (e.g. magnetic flux with normal helical equilibrium, magnetic island, and stochastic magnetic field region) in the core plasma in the Large Helical Device (LHD) [19], identified by a significant change in the heat pulse propagation properties, is described.

In the LHD, radial profiles of the rotational transform, \( t = \frac{r B_0}{r B_0} \), are derived from the polarization angle measured with motional Stark effect (MSE) spectroscopy [20].

Figure 1 shows the time evolution of the toroidal plasma current, \( I_p \), and magnetic shear \( s \) \[ = \frac{d}{d r} \] and temperature gradient at a rotational transform of \( t = 0.5 \), where the \[ m/n = 2/1 \] magnetic island is expected to appear. Here \( m \) is the poloidal mode number and \( n \) is the toroidal mode number. The co-injection (parallel to the equivalent plasma current) neutral beam (NB) is injected for \( t = 0.3–4.3 \text{ s} \) and one of the counter (ctr)-injection (anti-parallel to the equivalent plasma current) NB is injected for \( t = 3.3–7.3 \text{ s} \) for MSE spectroscopy and another ctr-injection NB is injected for \( t = 4.3–7.3 \text{ s} \) to reduce the magnetic shear further and modulated electron cyclotron heating (MECH) is applied for \( t = 4.2–5.2 \text{ s} \). When the NB direction is reversed from co-injection to ctr-injection at \( t = 4.3 \text{ s} \) in the middle of the discharge, the magnetic shear at \( t = 0.5 \ \left( r_{\text{eff}}/a_0 \sim 0.4 \right) \) gradually decreases from 0.9 to 0.2–0.3 in time. This is because the edge rotational transform decreases due to the non-inductive current driven by the NB, while the central rotational transform even increases due to the return current. Here, \( r_{\text{eff}} \) is the averaged minor radius on a magnetic flux surface and \( a_0 \) is the effective minor radius in which 99% of the plasma kinetic energy is confined and is 0.63 m in this discharge. In this experiment, the magnetic field, \( B \), is 2.705 T at the magnetic axis in the vacuum field, \( R_{\text{ax}} \), is 3.6 m and the plasma density is \( 1 \times 10^{19} \text{ m}^{-3} \). The slowing down time of fast ions of the NB is 0.16 s in this experiment. Two cases of the discharge with slight differences in plasma current change and magnetic shear (faster drop and slower drop) are plotted in figure 1, which are due to slight differences in plasma density and therefore the decay of the magnetic shear, \( s \), can be controlled with high reproducibility. In these discharges, there are no external perturbation fields applied and the magnetic island formation is due to the growth of a tearing mode at the rational surface. When the magnetic shear drops to \( \sim 0.5 \), the flattening of the temperature profile at the rational surface \( (t = 0.5) \) takes place in both cases due to the change of magnetic topology as seen in figures 1(b) and (d).
Figure 1. Time evolution of (a) the toroidal plasma current, (b) magnetic shear \( s = (r/r) \frac{dr}{dr} \) evaluated from the gradient of the rotational transform measured with MSE spectroscopy in the range of \( \iota = 0.5 \pm 0.05 \) as indicated in the rotational transform profiles and electron temperature gradient evaluated from the electron temperature profiles within \( \pm 0.1 \) m at \( \iota = 0.5 \) measured with a neodymium-doped yttrium aluminum garnet (YAG) Thomson scattering system in the discharges with (c) a slower magnetic shear drop and (d) a faster magnetic shear drop.

In the Heliotron configuration, neoclassical tearing modes are usually stable because of the negative magnetic shear and positive bootstrap current. In this experiment, magnetohydrodynamics activities (interchange modes \([21]\)) with the amplitude of temperature fluctuation >1% are observed in electron cyclotron emission (ECE) signals in the frequency range of 0.4–1.4 kHz at half of the plasma minor radius \( r_{\text{eff}}/a_{99} \sim 0.5 \) later in the discharge (>5.9 s). However, there is no significant activity observed before or during the phase where the electron temperature gradients drop significantly at \( t = 4.8 \) s as indicated in figures 1(c) and (d). This fact suggests that the flattening of the electron temperature is due to the formation of a magnetic island or to their overlapping rather than the growth of the interchange modes.

The speed of the heat pulse propagation in the flat temperature region is measured to investigate the topology of the magnetic flux surface. In order to investigate heat pulse propagation, the modulation of ECH starts at \( t = 4.3 \) s with a frequency of 39 Hz (there is no modulation for 0.1 s at the beginning of the MECH pulse) in this experiment as seen in the time evolution of the electron temperature measured with ECE at the plasma center and off-axis in

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Figure 2. Time evolution of (a) the electron temperature at the plasma center ($r_{\text{eff}}/a_{99} = 0.09$) and the off-axis ($r_{\text{eff}}/a_{99} = 0.24$) measured with a 27-channel ECE radiometer during the MECH phase and (b) the square of the modulation amplitude of the temperature at the modulation frequency of MECH in the discharges with a slower magnetic shear drop. Radial profiles of electron density measured with YAG Thomson scattering just before ($t = 4.8$ s) and after ($t = 5.1$ s) the flattening of the electron temperature and time sliced electron temperature profiles measured with ECE during the flattening ($t = 4.81 - 5.11$ s) are also plotted.

The drop in the central electron temperature and temperature gradients starts at $t = 4.8$ s and ends at $t = 5.1$ s. As seen in the time evolution of the electron temperature profiles during this drop, the decrease of the electron temperature gradient extends to the magnetic axis. The electron density profile is flat even before the flattening of the temperature ($t = 4.8$ s) and there is no change in the density gradient during this phase. A wavelet technique is used to derive the modulation amplitude and phase of the temperature in order to analyze the modulation and phase change in time [22]. Just before the flattening of the electron temperature profile ($t < 4.8$ s), the modulation amplitude is peaked at the plasma center, and decreases toward the plasma edge and the modulation amplitude at half of the plasma minor radius ($r_{\text{eff}}/a_{99} = 0.54$) is smaller than that at the plasma center by one order of magnitude because of the radial heat transport. The peaked amplitude profile shows that the heat deposition of the centrally focused MECH is well localized at the magnetic axis within $r_{\text{eff}}/a_{99} < 0.1$, while the heat deposition of neutral beam injection (NBI) is on board. The square of the modulation amplitude near the plasma center drops by an order of magnitude during the flattening of the temperature profile, while that at the half minor radius of the plasma ($r_{\text{eff}}/a_{99} = 0.54$) even increases by an order of magnitude. After the flattening of the temperature ($t > 4.0$ s), the radial profile of the modulation amplitude also becomes flat, which suggests fast heat pulse propagation in the radial direction along the magnetic field of line.

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Figure 3. Radial profiles of the delay time of a heat pulse in the plasma with (a) a fast shear drop and (b) a slow shear drop during the flattening of the electron temperature profiles. Radial profiles of the deposition power density of MECH and NBI are also plotted.

Figure 4. Time evolution of the normalized $m/n = 2/1$ perturbation field, $B_n[= (RB_r)/(r_sB_\phi)]$, estimated from the saddle loop measurements. The poloidal distribution of the perturbation field and the fitted curve with $m = 1$ and 2 Fourier component at $t = 5.1$ s in the discharge with a slow shear drop and a fast shear drop are also plotted.

As seen in figures 3 and 4, clear differences in heat pulse propagation and the perturbation magnetic field with the lowest mode number of $m/n = 2/1$ are observed between the plasma with fast and slow magnetic shear drop. It should be noted that the slight difference in magnetic shear causes a bifurcation of the characteristics of heat pulse propagation and the perturbation magnetic field. There are two patterns of heat pulse propagation observed in the flat temperature
Figure 5. Poincaré map of the magnetic field of line calculated by the 3D equilibrium code in the plasmas with (a) an \(m/n = 2/1\) magnetic island and (b) a stochastic region with \(m/n = 2/1, 4/2, 6/3\) and \(8/4\) perturbations of toroidal current using the iota profile consistent with the measurements.

flat region; one is simultaneous propagation at two separate points (figure 3(a)) and the other is very fast propagation (figure 3(b)). The former is consistent with a magnetic flux surface with a magnetic island \([17, 18]\). The perturbation is felt simultaneously at two points separated in radius, which might be interpreted as a surface equilibration of the magnetic surfaces that form the island via fast parallel transport. The result is consistent with the observation of a fast growth of the \(m/n = 2/1\) perturbation magnetic field measured with the saddle loop in the discharge with the fast shear drop in figure 4. In contrast, in the latter case, the delay time is uniform throughout that radial region, and the \(m/n = 2/1\) perturbation magnetic field is weak as seen in the discharge with a slow shear drop in figure 4, which implies the disappearance of the magnetic island with the lowest mode number of \(m/n = 2/1\) and a stochastic magnetic field due to the overlapping of magnetic islands with higher order at the rational surface of \(\iota = 0.5\).

Based on the differences in the heat pulse propagation characteristics, there is a possibility of a topology bifurcation between the two magnetic flux surfaces; one has a magnetic island and the other has a stochastic magnetic field due to the overlapping of magnetic islands with higher order. Since there is no magnetic fluctuation observed at the onset (or just before) the flattening of the electron temperature profiles in both cases, the flattening of temperature and delay time profiles are not due to the fluctuation, which is in contrast to the RFP plasmas, where the magnetic fluctuation plays an important role in transport \([15]\). In order to investigate the possibility of stochastization due to the steady-state perturbation magnetic field, two 3D equilibria with \(m/n = 2/1\) and with \(m/n = 2/1, 4/2, 6/3\) and \(8/4\) perturbations, respectively, of toroidal current are calculated. Figure 5 shows the Poincaré map of magnetic field of lines in the plasmas with a magnetic island and stochastic region calculated by an equilibrium code (HINT \([23]\)) and a magnetic field tracing code (MGTRC \([24]\)) with a perturbation magnetic
field consistent with the measured iota profile and poloidal distribution of the perturbed radial magnetic field. Both the plasma current and beam pressure have a significant contribution to the equilibrium especially the magnetic axis shift in this plasma. The calculations of NBI-driven current and non-thermal pressure profiles have a relatively large uncertainty in the low-density plasma in this experiment. Therefore the pressure and current profiles, which give a consistent magnetic axis shift evaluated from the radial profile of electron temperature measured with YAG Thomson scattering [25] and rotational transform measured with MSE, are used in the 3D equilibrium calculation. When only the toroidal current perturbation with the lowest mode number of \( m/n = 2/1 \) exists in the plasma, a clear magnetic island appears at the rational surface \( (\iota = 0.5) \) as seen in figure 5(a). In contrast, as the higher harmonic perturbation toroidal currents with the mode numbers of \( m/n = 4/2, 6/3 \) and \( 8/4 \) become dominant, the magnetic flux surfaces become stochastic due to the overlapping of side band magnetic islands as seen in figure 5(b), which is consistent with the fact that the intermediate magnetic shear is necessary for the transition to stochastization.

In this experiment, the ratio of parallel to perpendicular transport is assumed to be significantly large because of the low collisionality, which is in contrast to the experiment in high collisionality where the finite gradient of temperature exists even inside the magnetic island [26]. Therefore the region where the heat pulse propagates bi-directionally and with a speed comparable to that in the helical equilibrium region is inferred to have a magnetic island, while the region where the heat pulse propagates fast with the thermal diffusivity of \( 10^2 \text{m}^2 \text{s}^{-1} \) is inferred to be a stochastic region. The thermal diffusivity in the stochastic region can be evaluated as \( \chi_{st} = D_{FL} v_e \), where \( v_e \) is the thermal velocity of electrons and \( D_{FL} \) is the diffusion of the field line defined by \( 2\pi (r_s^2/R) B_s^2 \), \( r_s \) is the radius of the resonant surface and \( B_n \) \( = (R B_t)/(r_s B_s) \) is the normalized perturbation field [27, 28]. The normalized perturbation field, \( B_n \), evaluated from the island size is \( \sim 0.01 \) (0.06% of \( B_p \)) and is consistent with that estimated from the saddle loop measurements within a factor of 2. The thermal diffusivity in the stochastic region, \( \chi_{st} \), is \( 2-3 \times 10^2 \text{m}^2 \text{s}^{-1} \). The lower limit of the thermal diffusivity experimentally determined by the width of the stochastic region, \( W \), and the minimum detectable delay time \( (\tau \sim 50 \mu\text{s}) \) using a simple formula of \( \chi_{st} = (W/2)/\tau \) is \( 10^2 \text{m}^2 \text{s}^{-1} \). In the flat temperature region, the radial electric field becomes zero in LHD [29] and large thermal diffusivity is expected in the flat temperature region. However, the thermal diffusivity evaluated here is much larger than the thermal diffusivity due to the transport perpendicular to the magnetic field even with zero radial electric field \( (< 10 \text{m}^2 \text{s}^{-1}) \) [30].

Figure 6 shows the relation between the normalized width \( W_0 \) of the magnetic island or stochastic region and magnetic shear at the rational surface of \( m/n = 2/1 \). When the magnetic shear exceeds the critical value \( \sim 0.6 \), there is no magnetic island and no stochastization at the rational surface. As the magnetic shear decreases, the formation of a magnetic island or stochastic region starts to develop and their size increases as the magnetic shear decreases. Since the stochastization needs the overlapping of a secondary magnetic island with a different mode number, only the formation of the magnetic island is expected at lower magnetic shear, which is consistent with the observation. When the magnetic shear is medium, the flat delay time region can be localized near the rational surface (inferred to be partially stochastized) or it can extend up to the magnetic axis (inferred to be fully stochastized). Because of the limit of spatial resolution of the ECE measurements, the possible existence of a nested magnetic island inside the stochastic region cannot be excluded when the stochastic region is small \( (W_0 \sim 0.2) \) in partial stochastization. In the plasma with a magnetic island and partial stochastization, the
Figure 6. Normalized island width \( W_n = \frac{r_{\text{in}} - r_{\text{out}}}{r_{\text{in}} + r_{\text{out}}} \), where \( r_{\text{in}} \) and \( r_{\text{out}} \) are the minor radii of the inner and outer boundaries, respectively, of the magnetic island or stochastic region as a function of magnetic shear and radial profiles of the delay time of the heat pulse in the plasma.

Flattening of the electron temperature does not extend to the magnetic axis and slight peaking of the electron temperature is observed near the plasma center (\( r_{\text{eff}}/a_{99} < 0.2 \) for the magnetic island and \( r_{\text{eff}}/a_{99} = 0.26 \) for the partial stochasticization), while the flattening extends to the magnetic axis for the full stochasticization.

In this experiment, the control parameter for the bifurcation is the drop rate of magnetic shear at the rational surface. The magnetic shear drops because the edge rotational transform decreases due to the non-inductive current driven by the NB, while the central rotational transform even increases due to the return current. As the electron density is increased, both non-inductive and return currents decrease and the drop rate of magnetic shear also decreases. Therefore the density is just an indirect control parameter for bifurcation. The direct control parameter is the drop rate of magnetic shear in the magnetic shear range of 0.4–0.6 as seen in figure 6. It is an interesting issue how the plasma potential and radial electric field change at the onset of stochasticization, because the change of topology should have a strong impact on zonal flow, mean flow and turbulence-driven transport [13]. In this experiment, the central plasma potential measured with a heavy ion beam probe drops by 0.4 kV associated with the transition from nested flux surfaces to a stochastic magnetic field, which suggests the disappearance of the positive radial electric field in the stochastic region. The investigation of the detailed structure of the plasma flow and the radial electric field in the stochastic region is left to a future study.

The present experiment implies that there are three states of the magnetic flux surface (nested magnetic island and partial and full stochasticization) besides normal nested flux surfaces as predicted by a theoretical model [28]. This observation also demonstrates that the heat pulse propagation experiment is a useful tool for identifying the stochasticization of the magnetic field, which is expected (or calculated) but not experimentally confirmed in the plasma periphery with a resonance magnetic field perturbation.
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