Single Leptoquark Production at $e^+e^-$ and $\gamma\gamma$ Colliders

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ABSTRACT
We consider single production of leptoquarks (LQ’s) at $e^+e^-$ and $\gamma\gamma$ colliders, for two values of the centre-of-mass energy, $\sqrt{s} = 500$ GeV and 1 TeV. We find that LQ’s which couple within the first generation are observable for LQ masses almost up to the kinematic limit, both at $e^+e^-$ and $\gamma\gamma$ colliders, for the LQ coupling strength equal to $\alpha_{em}$. The cross sections for single production of $2^{nd}$- and $3^{rd}$-generation LQ’s at $e^+e^-$ colliders are too small to be observable. In $\gamma\gamma$ collisions, on the other hand, $2^{nd}$-generation LQ’s with masses much larger than $\sqrt{s}/2$ can be detected. However, $3^{rd}$-generation LQ’s can be seen at $\gamma\gamma$ colliders only for masses at most $\sim \sqrt{s}/2$, making their observation more probable via the pair production mechanism.
One of the more interesting environments in which to study physics beyond the standard model (SM) is at a high-energy linear $e^+e^-$ collider. Not only are $e^+e^-$ collisions clean, but it will likely be possible to adjust the centre-of-mass energy. Furthermore, it has been suggested that, by using backscattered laser beams, an $e^+e^-$ machine can be converted into an $e\gamma$ or $\gamma\gamma$ collider [1]. This is particularly exciting, since these different modes may be quite useful for looking for new physics.

Leptoquarks (LQ’s), which are absent in the SM but predicted by many of its extensions, are one example of the new physics which can be studied at such machines. These particles, which can have electromagnetic charge $Q_{em} = -1/3, -2/3, -4/3$ or $-5/3$, would decay into a lepton and a quark or antiquark, so the signal would be quite striking. In principle, LQ’s couple to fermions of either helicity. In general, leptoquarks can have spin 0 or 1, but here we concentrate only on scalar LQ’s.

Various processes constrain the strength and nature of the LQ couplings to fermions. For example, for LQ’s of charge $-1/3$ which couple to both $e^-u$ and $\nu_e d$, rare $\pi$ and $K$ decays constrain the couplings to be chiral [2]. That is, LQ’s must couple only to left-handed (LH) or right-handed (RH) quarks, but not both. For these same LQ’s, bounds from weak universality require that the LH couplings be at most about 10% of electromagnetic strength. However, these limits need not necessarily apply to leptoquarks of other charges.

One of the most stringent constraints on LQ couplings comes from the absence of low-energy flavour-changing neutral currents (FCNC’s). In order to avoid FCNC’s, one typically requires the LQ’s to couple within a single generation only. However, M. Leurer [3] has recently pointed out that this requirement is in fact impossible to meet in general. Due to Cabibbo-Kobayashi-Maskawa mixing in the left-handed quark sector, one cannot simultaneously diagonalize the couplings of the LQ in both the up-quark and down-quark sector. Thus, if one tries to evade constraints from FCNC’s in the down-quark sector, such as $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ mixing, by diagonalizing the LH leptoquark couplings, $D^0-\bar{D}^0$ mixing will then put very strong limits on the masses and couplings of left-handed LQ’s. There are no similar constraints for the right-handed LQ’s.

Of course, this should not discourage experimentalists from looking for left-handed LQ’s. After all, it is possible that there are other new particles whose effects in low-energy processes would cancel those due to leptoquarks. Thus, if a left-handed LQ were discovered, in fact two types of physics beyond the SM would have been found: the leptoquark itself, and the new physics responsible for the cancellations! This possibility is not totally fantastic, since models which include LQ’s will typically also contain other new particles (scalars, gauge bosons, etc.).

In a previous paper [4], two of us investigated the production of scalar leptoquarks
at $e\gamma$ colliders at two values of the centre-of-mass energy, $\sqrt{s} = 500$ GeV and 1 TeV. We showed that LQ’s with masses essentially up to the kinematic limit could be seen, even for couplings as weak as $O(10^{-3}) - O(10^{-2})\alpha_{em}$. In this paper we continue the investigation of single leptoquark production at both $e^+e^-$ and $\gamma\gamma$ colliders, again taking $\sqrt{s} = 500$ GeV and 1 TeV. The $e^+e^-$ case was studied some time ago by Hewett and Pakvasa [5], but only for charge $-1/3$ LQ’s. Here we do a more complete analysis. We do not, however, agree with their results.

The most general, model independent Lagrangian with $SU(3) \times SU(2) \times U(1)$ invariant couplings of the scalar leptoquarks and conservation of the baryon and lepton numbers [6] can be separated into two pieces:

$$
L_L = g_{1L} \bar{q}_L^c i\tau_2 l_L S_1 + g_{3L} \bar{q}_L^c i\tau_2 \tau^i l_L S^i_3 + h_{2L} \bar{q}_L^c i\tau_2 e_R R^c_2 , \\
L_R = g_{1R} \bar{u}_R^c e_R S_1^c + \tilde{g}_{1R} \bar{d}_R^c e_R \tilde{S}_1 + h_{2R} \bar{u}_R^c l_L R_2 + \tilde{h}_{2R} \bar{d}_R^c l_L \tilde{R}_2 .
$$

The LH quarks and leptons appear in the standard SU(2) doublets $q_L$ and $l_L$, and the superscript $c$ denotes charge conjugation. In the above equations, following Leurer [3], we have defined the ‘handedness’ of the leptoquarks according to the helicity of the quark/antiquark to which they couple*. That is, the LQ’s in $L_L$ and $L_R$ are left-handed and right-handed, respectively. From the above, we see that the LH leptoquarks transform as either a singlet, doublet or triplet of $SU(2)_W$, while those coupling to RH quarks are singlets or doublets. The $R$ and $S$ leptoquarks carry fermion number 0 and 2 respectively, with their subscript indicating the $SU(2)_W$ multiplet to which they belong.

Despite the rather complicated notation in Eq. 1, for our purposes the only important properties of the leptoquark are its charge and its handedness. It is straightforward to verify that, for each of the four possible electromagnetic charges, there exists a LH and a RH leptoquark. There is one other important point – in all processes of interest in this analysis, only the couplings of the LQ to the charged lepton will enter. We can therefore take the LQ couplings to be generation-diagonal. There is no conflict with Leurer’s result – there may indeed be a LQ-neutrino-quark/antiquark coupling which is not generation-diagonal, but this is unimportant here. In summary, then, the leptoquark is defined by its charge, by its handedness, and by the generation of the particles to which it couples. In this paper we will use the symbol $S$ to denote a leptoquark, while $q$ will refer to either a quark or an antiquark.

One advantage of the $\gamma\gamma \rightarrow \ell qS$ process is that it allows for the production of leptoquarks of each of the three generations [7]. As we will see, although $2^{nd}$- and $3^{rd}$-generation

* Note that this differs from the conventions of Ref. 4, in which the handedness of the LQ is defined by the helicity of the lepton.
LQ’s can indeed be produced in $e^+e^-$ collisions, the cross sections are much too small to be observed.

Let us first focus on single LQ production in $e^+e^-$ collisions. The diagrams which give rise to this are shown in Fig. 1. Although the large number of diagrams may seem daunting, most of these can be neglected. There is a relatively simple way to ascertain which are important and which are not. Consider the diagrams of Fig. 1a in which a photon is exchanged. When the virtual photon is aligned with the positron beam direction, the amplitude diverges. This divergence is regulated by the small mass of the positron, giving rise to a logarithmic enhancement of about 30 in the total cross section. In the following, we will refer to such enhancements as ‘large logs’. A quick way to spot diagrams which have large logs is to look for vertices involving three massless (or nearly massless) particles with at least one $t$- or $u$-channel propagator. Application of this rule reveals that none of the diagrams in Figs. 1b or 1c have large logs, so these diagrams can be neglected, and similarly for those diagrams in Fig. 1a in which a $Z$ is exchanged. In fact, with this rule one expects that the second diagram of Fig. 1a with photon exchange should have an additional large log due to the quark propagator. We will see below that this is indeed the case.

The presence of these divergences indicates that most of the cross section comes from a few directions in phase space. This makes it very difficult to use conventional Monte Carlo methods for computing phase space integrals. A much simpler way to evaluate the diagrams of Fig. 1a in which a photon is exchanged is to use the effective photon approximation [8]:

$$\sigma(s) = \int_{s_{th}/s}^{1} d\tau f_{\gamma}(\tau) \hat{\sigma}(\tau s),$$  \hspace{1cm} (2)

in which $\sigma(s)$ is the cross section for the process $e^+e^- \rightarrow e^+qS$ at centre-of-mass energy $s$, and $\hat{\sigma}(\tau s)$ is the cross section for the sub-process $\gamma e^- \rightarrow qS$ with centre-of-mass energy $\hat{s} = \tau s$. The minimum $\hat{s}$ required ($s_{th}$) is $(M_S + m_q)^2$. The photon distribution function $f_{\gamma}(\tau)$ is [8]:

$$f_{\gamma}(\tau) = \frac{\alpha}{2\pi} \left\{ \frac{[1 + (1 - \tau)^2]}{\tau} \ln \left[ \frac{s}{4m_e^2} \frac{(1 - 2\tau + \tau^2)}{(1 - \tau + \tau^2/4)} \right] + \tau \ln \left( \frac{2 - \tau}{\tau} \right) + \frac{2(\tau - 1)}{\tau} \right\}.$$  \hspace{1cm} (3)

As expected, $f_{\gamma}(\tau)$ contains a large log. Note that the more common form of this function,

$$f_{\gamma}(\tau) = \left( \frac{\alpha}{2\pi} \ln \frac{s}{4m_e^2} \right) \frac{1 + (1 - \tau)^2}{\tau},$$  \hspace{1cm} (4)

is quite adequate when $M_S$ is relatively small compared to $\sqrt{s}$. However, for large $M_S$ the full form (Eq. 3) must be used.
In order to use Eq. 2, we must evaluate the cross section for $\gamma e^- \rightarrow qS$. The diagrams describing this process are shown in Fig. 2. The key point which must be addressed is that, in the limit in which the quark mass is neglected, the second diagram diverges. This corresponds to the situation in which the photon and the quark are aligned. One way to deal with this is to impose a $p_T$ cut on the quark jet. This is the method used by Hewett and Pakvasa [5]. The problem with this solution is that, because the large logs are due to that region of phase space in which the entire event is collinear, one loses a significant fraction of the total cross section. An alternative procedure, which is the one we advocate, is to use the nonzero quark mass as a regulator. As we will see, this results in an enormous enhancement of the total cross section compared to the $p_T$ cut. Experimentally, the situation is that the entire event goes down the beam pipe. However, the leptoquark will then decay into a jet and a lepton, giving a signal in the detector which is unmistakable: $\gamma e^-$ (or $e^+e^-$) $\rightarrow e^- + jet$! For comparison, we will present both methods of regulating the divergence.

We first consider calculating the diagrams of Fig. 2 neglecting the lepton mass, but keeping a nonzero mass for the quark, $m_q$. For all leptoquarks we will use the generic Yukawa coupling constant $g$, with the understanding that the coupling could depend on the masses involved and might vary from one generation to the other. We parametrize the strength of the LQ coupling by comparing it to the electromagnetic interaction, i.e., $g^2 = 4\pi k\alpha_{em}$, and allowing $k$ to vary. Denoting the charge of the leptoquark by $Q_S$, the full expression for $\hat{\sigma}(\hat{s})$ is then found to be

\[
\hat{\sigma}(\hat{s}) = \frac{\pi k\alpha_{em}^2\beta}{2\hat{s}} \left[ \frac{\alpha}{2} + \frac{Q_S}{2} \left\{ 4 - 2\alpha + \frac{4}{\beta} \left( 1 - \alpha + \frac{m_q^2}{\hat{s}} \right) \ln \left[ \frac{2 - \alpha - \beta}{2 - \alpha + \beta} \right] \right\} 
+ \frac{Q_S^2}{2} \left\{ 12 - 10\alpha + \frac{4}{\beta} \left( (\alpha - 3)(\alpha - 1) + \frac{m_q^2}{\hat{s}} \right) \ln \left[ \frac{2 - \alpha - \beta}{2 - \alpha + \beta} \right] \right\} 
- \frac{Q_S + 1}{2} \left\{ 4 - 2\alpha + \frac{4 m_q^2}{\hat{s}} \ln \left[ \frac{\alpha + \beta}{\alpha - \beta} \right] \right\} 
+ \frac{(Q_S + 1)^2}{2} \left\{ 8 - 10\alpha + \frac{1}{\beta} \left( 2 - 4\alpha + 4\alpha^2 + \frac{4 m_q^2}{\hat{s}} \right) \ln \left[ \frac{\alpha + \beta}{\alpha - \beta} \right] \right\} 
+ Q_S(Q_S + 1) \left\{ -6 + 6\alpha + \frac{2 m_q^2}{\hat{s}} (1 - 2\alpha) \ln \left[ \frac{\alpha + \beta}{\alpha - \beta} \right] 
- \frac{2}{\beta} (3 - 2\alpha) \left( \frac{m_q^2}{\hat{s}} + 1 - \alpha \right) \ln \left[ \frac{2 - \alpha - \beta}{2 - \alpha + \beta} \right] \right\} \right),
\]

in which

\[
\alpha \equiv 1 - \frac{(M_S^2 - m_q^2)}{\hat{s}}.
\]
and
\[
\beta \equiv \left( 1 - 2\frac{(M^2_S + m_q^2)}{\hat{s}} + \frac{(M^2_S - m_q^2)^2}{\hat{s}^2} \right)^{\frac{1}{2}}. \tag{7}
\]

One important point to notice is that Eq. 5 is independent of the handedness of the LQ. In other words, the cross section for the subprocess $\gamma e^- \to qS$ is the same for both LH and RH leptoquarks of charge $Q_S$.

The cross section for single leptoquark production in the process $e^+e^- \to e^+qS$ can now be calculated using the effective photon approximation by substituting Eq. 5 into Eq. 2 and numerically computing the integral. Our results for $\sqrt{s} = 500$ GeV and 1 TeV appear in Figs. 3a and 3b, where we have taken the LQ coupling strength to be equal to that of the electromagnetic interaction, i.e. $k = 1$, and have set $m_q = 7$ MeV (this corresponds roughly to either a $d$-quark or a $u$-quark mass). Assuming the integrated luminosity at a high-energy $e^+e^-$ collider to be $10$ fb$^{-1}$ at 500 GeV, and $60$ fb$^{-1}$ at 1 TeV, and requiring 25 events for discovery, one can see that LQ’s almost up to the kinematic limit can be seen in $e^+e^-$ colliders. More precisely, LQ’s with $Q_S = -1/3$ and $-5/3$ are observable if $M_S \leq 475$ GeV (960 GeV) at $\sqrt{s} = 500$ GeV (1 TeV), and those with $Q_S = -2/3$ and $-4/3$ can be seen for $M_S \leq 420$ GeV (870 GeV) at $\sqrt{s} = 500$ GeV (1 TeV). These numbers are given explicitly in Table 1, where they can be compared with the prospects at $\gamma\gamma$ colliders, which we will discuss later in the paper. Since the cross section is linear in $k$, it is straightforward to scale the results shown in Fig. 3 to other values of $k$, if desired. It should also be stressed that, because we have used an approximation in the calculation, there is some uncertainty in the above numbers, perhaps as much as 5% [9].

As noted above, most of the cross section comes from that region of phase space in which the entire event goes down the beam pipe. Since the LQ decays to $\ell + jet$, there will be essentially no background from SM processes. Even for those events in which other particles are seen in the detector, there will be a sharp invariant mass peak in $M_{\ell+jet}$ at $M_S$, which is not present in SM decays.

One interesting feature of Fig. 3 is that the cross sections for the LQ’s with $Q_S = -1/3$ and $Q_S = -5/3$ are almost equal, and similarly for those LQ’s with $Q_S = -2/3$ and $-4/3$. This reflects the dominance of the second diagram in Fig. 2, since it has an extra large log compared to the other two. Since the amplitude for this diagram is proportional to the quark charge, $Q_q = -(Q_S + 1)$, the most important term in $\hat{\sigma}(\hat{s})$ is the one whose coefficient is $(Q_S + 1)^2$. From this it follows that, to a very good approximation, the cross sections for LQ’s with $Q_S = -1/3$ and $-5/3$ should be equal. Similarly, LQ’s with $Q_S = -2/3$ and $-4/3$ are expected to have equal cross sections, and these should be a factor of 4 smaller than the cross section for the LQ with $Q_S = -1/3$. These expectations are born out in Fig. 3.
We have also calculated the diagrams in Fig. 2 by imposing a $p_T$ cut on the quark jet. In this case, the cross section for $\gamma e^- \rightarrow qS$ takes the form

\[
\hat{\sigma}(\hat{s}) = \frac{\pi k\alpha_{em}^2}{2\hat{s}^2} \left[ -\frac{1}{2\hat{s}} (u_{\text{max}}^2 - u_{\text{min}}^2) + \frac{2(Q_S + 1)}{\hat{s}} \left\{ \frac{1}{2} (u_{\text{max}}^2 - u_{\text{min}}^2) - M_S^2 (u_{\text{max}} - u_{\text{min}}) \right\} \right.
\]

\[
- \frac{2Q_S}{\hat{s}} \left\{ \frac{1}{2} (u_{\text{max}}^2 - u_{\text{min}}^2) - M_S^2 (u_{\text{max}} - u_{\text{min}}) + \hat{s}M_S^2 \ln \left( \frac{\hat{s} + u_{\text{max}}}{\hat{s} + u_{\text{min}}} \right) \right\}
\]

\[
- (Q_S + 1)^2 \left\{ \frac{1}{\hat{s}} (u_{\text{max}}^2 - u_{\text{min}}^2) + \left( 2 - \frac{4M_S^2}{\hat{s}} \right) (u_{\text{max}} - u_{\text{min}}) \right. \right.
\]

\[
+ \left( -2M_S^2 + \hat{s} + \frac{2(M_S^2)^2}{\hat{s}} \right) \ln \left( \frac{u_{\text{max}}}{u_{\text{min}}} \right) \}
\]

\[
+ \frac{4Q_S(Q_S + 1)}{\hat{s}} \left\{ \frac{1}{2} (u_{\text{max}}^2 - u_{\text{min}}^2) + \left( -2M_S^2 + \frac{\hat{s}}{2} \right) (u_{\text{max}} - u_{\text{min}}) \right.
\]

\[
+ M_S^2 \left( M_S^2 + \frac{\hat{s}}{2} \right) \ln \left( \frac{\hat{s} + u_{\text{max}}}{\hat{s} + u_{\text{min}}} \right) \}
\]

\[
- \frac{2Q_S^2}{\hat{s}} \left\{ \frac{1}{2} (u_{\text{max}}^2 - u_{\text{min}}^2) - 2M_S^2 (u_{\text{max}} - u_{\text{min}}) \right.
\]

\[
+ \hat{s}(M_S^2)^2 \left\{ \frac{1}{\hat{s} + u_{\text{max}}} - \frac{1}{\hat{s} + u_{\text{min}}} \right\} + M_S^2 (M_S^2 + 2\hat{s}) \ln \left( \frac{\hat{s} + u_{\text{max}}}{\hat{s} + u_{\text{min}}} \right) \} \right] .
\]

Here,

\[
u_{\text{max}} = -\frac{1}{2} \hat{s} \beta' (1 - c_{\text{max}}) , \quad u_{\text{min}} = -\frac{1}{2} \hat{s} \beta' (1 + c_{\text{max}}) ,
\]

with

\[
\beta' = 1 - \frac{M_S^2}{\hat{s}} , \quad c_{\text{max}} = \sqrt{1 - \frac{2p_T\text{cut}}{\sqrt{\hat{s}} \beta'}} ,
\]

in which $p_T\text{cut}$ is the imposed $p_T$ cut which we take to be 10 GeV.

We now calculate as before the cross section for single leptoquark production in the process $e^+ e^- \rightarrow e^+ qS$ using the $\hat{\sigma}(\hat{s})$ given in Eq. 8. In order to compare with the results of Hewett and Pakvasa [5], we consider $Q_S = -1/3$ LQ’s only, use $\sqrt{\hat{s}} = 1$ TeV, and take $k = 2$ (due to a difference of $\sqrt{2}$ in the definition of the LQ coupling, this corresponds to $k = 1$ in the notation of Ref. 5). The result is shown in Fig. 4. Our results are significantly smaller than those obtained in Ref. 5, by roughly an order of magnitude for all values of $M_S$. The difference might perhaps be due to the form taken for the photon distribution function in the effective photon approximation (Eq. 3).

What is certain is that one gains a significant amount in the total cross section for single leptoquark production by not imposing a $p_T$ cut, but rather using the nonzero
quark mass to regulate the collinear divergence. For example, compare Fig. 3b with Fig. 4 (remembering to rescale the numbers one reads off of Fig. 4 by a factor of 2 in order to correspond to our $k = 1$). A LQ of mass 800 GeV has a cross section of about 4 fb if the quark mass method is used. On the other hand, using the $p_T$ cut, an 800 GeV LQ would have a cross section of 0.4 fb, which is considerably smaller. For the rest of the paper, we will restrict ourselves to evaluating the large logs using a nonzero quark mass.

Since we have assumed that each LQ couples generation-diagonally, the production cross sections shown in Fig. 3 hold only for first generation leptoquarks. The only way to produce single 2nd- or 3rd generation LQ’s in $e^+e^-$ collisions is through the graphs of Fig. 1c. Note, however, that there are no large logs in these diagrams, so we expect the cross sections to be smaller than those of Fig. 3 by at least two to three orders of magnitude. We have calculated these diagrams explicitly, and we find that indeed the single LQ production cross sections are typically $O(10^{-3})$ fb. Thus, there is no hope for seeing single 2nd- or 3rd generation LQ’s in $e^+e^-$ collisions*. It is, however, possible to observe such LQ’s in $\gamma\gamma$ collisions, and we now turn to a study of such processes.

For the process $\gamma\gamma \to \ell^+qS$, there are six diagrams, shown in Fig. 5. In fact, there are really twelve diagrams, since each final state must be symmetrized with respect to the initial photons. Again, it is not necessary to calculate all the graphs — some can be neglected. Using the large log counting rules introduced earlier, one finds that, for the case in which the quark mass is small, the diagram in Fig. 5a contains two large logs, the four diagrams of Figs. 5b and 5c each have one large log, while the diagram in Fig. 5d has none. Our experience with single LQ production in $e^+e^-$ collisions tells us that the graph in Fig. 5a will essentially completely dominate in this case. For 3rd-generation LQ’s, the mass of the top quark (\(\sim 150\) GeV) can no longer be considered small compared to the centre-of-mass energy. In this case, the large log counting changes — the graphs in Figs. 5a and 5b each have one large log, while those in Figs. 5c and 5d have none. Therefore, for LQ’s of all generations, it is an excellent approximation to keep only the diagrams in Figs. 5a and 5b in the calculation of the total cross section, and this is what we will do.

Note that, although we must include the interference among the three diagrams of Figs. 5a and 5b in our calculation, we may ignore the interference between this set of three graphs and the set which is symmetrized with respect to the initial photons. This can be seen intuitively as follows. The divergences in one set of graphs, which give rise to the large logs, occur when the virtual lepton goes in the direction of one of the initial photons, while the divergences of other set of graphs are present in that region of phase space in

* Note that 2nd- and 3rd-generation LQ’s could be seen if they were pair produced in $e^+e^-$ collisions via s-channel $\gamma$- or $Z$-exchange. Of course, this is only possible for $M_S \leq \sqrt{s}/2$. 

which the virtual lepton aligns itself with the other photon. Therefore, when these two
sets of graphs interfere, there are no divergences, and hence no large logs. We have verified
this intuitive picture analytically, and find that indeed there are no large logs coming from
the interference of the two sets of diagrams. Thus, it is only necessary to evaluate the
contribution of the graphs of Figs. 5a and 5b to the process $\gamma\gamma \rightarrow \ell^+q_S$, and then to
include a factor of two to take into account the symmetrized set of diagrams.

The easiest way to calculate the diagrams in Figs. 5a and 5b is to use a technique
similar to that used in the computation of $e^+e^- \rightarrow e^+q_S$, namely the effective fermion
approximation. That is,

$$\sigma(s) = \int_{s_{th}/s}^{1} d\tau f_\ell(\tau)\hat{\sigma}(\tau s), \quad (11)$$

in which $\sigma(s)$ is the cross section for $\gamma\gamma \rightarrow \ell^+q_S$ at centre-of-mass energy $s$, and $\hat{\sigma}(\tau s)$ is
the cross section for the sub-process $\gamma\ell^- \rightarrow q_S$ with centre-of-mass energy $\hat{s} = \tau s$, just as
in the effective photon approximation. The lepton distribution function is [9]

$$f_\ell(\tau) = \frac{\alpha}{2\pi} \left\{ [\tau^2 + (1-\tau)^2] \ln \left[ \frac{s}{4m_\ell^2}(1-\tau)^2 \right] + 2\tau(1-\tau) \right\}. \quad (12)$$

Again, as expected, this function contains a large log. Although $f_\ell(\tau)$ somewhat resembles
the photon distribution function $f_\gamma(\tau)$ (Eq. 3), there is one important difference. Due to
the absence of a factor of $\tau$ in the denominator, $f_\ell(\tau)$ is much smoother than $f_\gamma(\tau)$. Thus
we do not expect the cross section for $\gamma\gamma \rightarrow \ell^+q_S$ to be as strong a function of $M_S$ as we
found in $e^+e^- \rightarrow e^+q_S$.

The cross section for the subprocess $\gamma\ell^- \rightarrow q_S$ has already been calculated (Eqs. 5-7),
so we can simply perform the numerical integration of Eq. 11 for all three generations. We
have taken $m_e = 0.5$ MeV and $m_u = m_d = 7$ MeV for the first generation, $m_\mu = 100$
MeV, $m_s = 150$ MeV and $m_c = 1.5$ GeV for the second, and $m_\tau = 1.5$ GeV, $m_b = 5$
GeV and $m_t = 150$ GeV for the third. We remind the reader that the LQ’s of charge $-1/3$ and $-5/3$ couple to up-type quarks, while the $Q_{em} = -2/3$ and $-4/3$ LQ’s couple
to down-type quarks. In computing the cross section, we have included the factor two to
take into account the symmetrized set of diagrams.

The results are shown in Fig. 6 for the three generations, for $\sqrt{s} = 500$ GeV and
1 TeV, for $k = 1$. Before describing the results, let us note some general features. For
the $1^{st}$- and $2^{nd}$-generation LQ’s, the similarity of the curves for LQ’s of $Q_{em} = -1/3$
and $-5/3$, and for LQ’s with charges $-2/3$ and $-4/3$, again reflects the dominance of the
diagram in Fig. 5a (two large logs). Also, as expected, the cross sections are in general less
strongly dependent on the LQ mass than was found in $e^+e^-$ colliders. Finally, the cross
sections for $3^{rd}$-generation LQ’s are significantly smaller than for those coupling within
the 1\textsuperscript{st}- and the 2\textsuperscript{nd} generations. This reflects the fact that there is really one less large log in the cross section for 3\textsuperscript{rd}-generation LQ’s.

The figure of merit in Fig. 6 is the largest LQ mass observable for each of the three generations. The question is, which has the better prospects for LQ detection, the single LQ production mode, or the pair production mode (in which LQ’s of mass $M_s \leq \sqrt{s}/2$ can be seen)? Looking at Fig. 6, if LQ’s of mass greater than $\sqrt{s}/2$ can be seen, then it is better to try to detect leptoquarks in the single LQ production mode. However, if the maximum LQ mass which can be observed is less than $\sqrt{s}/2$, then pair production is more promising. In Table 1 we display ($M_s$)$_{max}$ for all four LQ charges and for all three generations.

From Figs. 6a and 6b and Table 1, we see that, as in $e^+e^-$ colliders, 1\textsuperscript{st}-generation LQ’s with masses almost up to the kinematic limit can be seen in $\gamma\gamma \rightarrow \ell^+qS$. As before, we have assumed an integrated luminosity of 10 fb\(^{-1}\) at 500 GeV, and 60 fb\(^{-1}\) at 1 TeV, and assumed a discovery signal of 25 events. And again, there is perhaps a 5% uncertainty in these numbers due to the approximations used [9].

The situation is similar, though not quite as promising, for 2\textsuperscript{nd}-generation LQ’s (Figs. 6c and 6d, Table 1). At $\sqrt{s} = 500$ GeV, the maximum mass allowed for observing a LQ is about 300-400 GeV, depending on the LQ charge, while at 1 TeV, it is 700-900 GeV. We remind the reader that we have used the c-quark mass in the cross sections for LQ’s with charge $-1/3$ and $-5/3$, and the s-quark mass for LQ’s with $Q_{em} = -2/3$ and $-4/3$.

Things are very different for 3\textsuperscript{rd}-generation LQ’s (Figs. 6e and 6f, Table 1). At $\sqrt{s} = 500$ GeV, the maximum mass is 100-200 GeV, except for the charge $-1/3$ LQ, which is not observable at all. At 1 TeV, ($M_s$)$_{max}$ is 180-540 GeV. For 3\textsuperscript{rd}-generation leptoquarks, then, it is almost always better to look for pair production in $e^+e^-$ or $\gamma\gamma$ collisions. It should be emphasized, however, that these cross sections have been calculated for LQ coupling strengths $k = 1$. If the LQ couplings were proportional to masses, then for those LQ’s which couple to the $t$-quark one might conceivably have $k$ larger than one, and the cross sections would increase accordingly. Of course, if $k$ were much larger than one, then at some point this perturbative analysis would break down.

An important point to remember is that, in $\gamma\gamma$ colliders created by the backscattering of laser light, the photon beams are not monochromatic. For a complete calculation it would be necessary to fold in the energy spectrum of the initial photons. Typically the highest energy photons would have about 80% of the energy of the parent electron machine so that the limits given in Table 1 would be scaled accordingly.

In conclusion, we have calculated the cross sections for single leptoquark production at high-energy $e^+e^-$ and $\gamma\gamma$ colliders of $\sqrt{s} = 500$ GeV and 1 TeV. For LQ’s coupling
within each of the 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} generations, we have considered the four LQ charges $Q_S = -1/3, -2/3, -4/3, -5/3$. Our results are independent of whether the LQ couples to left- or right-handed quarks/antiquarks.

For the process $e^+e^- \rightarrow e^+qS$ we have utilized the effective photon approximation, while for $\gamma\gamma \rightarrow \ell^+qS$ the effective fermion approximation was used. For each of these methods it was necessary to calculate the cross section for the subprocess $\gamma\ell^- \rightarrow qS$. We have shown that using a $p_T$ cut to regulate the collinear divergence in this process is in fact not a very good procedure – one loses too much of the total cross section. It is better to use the nonzero quark mass as a regulator. In this case the bulk of the cross section comes from that region of phase space in which the entire event goes down the beam pipe. When the LQ decays, this results in an unmistakable signal in the detector: $e^+e^-(or \gamma\gamma) \rightarrow \ell^- + jet$. This is virtually background-free.

We have found that 1\textsuperscript{st}-generation LQ’s of any charge can be observed almost up to the kinematic limit in both $e^+e^-$ and $\gamma\gamma$ colliders ($\sqrt{s} = 500$ GeV or 1 TeV), for LQ coupling strengths equal to that of the electromagnetic interaction. For 2\textsuperscript{nd}- and 3\textsuperscript{rd}-generation leptoquarks, the cross sections for single LQ production at $e^+e^-$ colliders are too small to be observable. These LQ’s can, however, be seen at $\gamma\gamma$ colliders. Depending on their charges, 2\textsuperscript{nd}-generation leptoquarks with masses between 700 and 900 GeV can be observed in $\gamma\gamma$ collisions at $\sqrt{s} = 1$ TeV, while at 500 GeV machines, LQ’s whose mass is between roughly 300 and 400 GeV are detectable. For 3\textsuperscript{rd}-generation leptoquarks, the situation is not nearly as promising. At $\sqrt{s} = 500$ GeV, only LQ’s with masses at most 190 GeV are observable, while at 1 TeV, it is possible to see LQ’s with $M_S$ up to just over 500 GeV. Thus, for 3\textsuperscript{rd}-generation leptoquarks, it seems that it is just as good, if not better, to look for signals from pair production. Of course, if the LQ coupling strength were significantly stronger than $\alpha_{em}$, as might be the case where the top quark is involved, then single 3\textsuperscript{rd}-generation LQ production would become more promising.

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| Process                        | Charge        | Generation | (a) $(M_S)_{max}$ | (b) $(M_S)_{max}$ |
|-------------------------------|---------------|------------|-------------------|-------------------|
| $e^+e^- \rightarrow e^+qS$:   | $Q_S$         | $1^{st}$   | 475               | 960               |
|                               |               | $2^{nd}$   | 420               | 870               |
|                               |               | $3^{rd}$   | 420               | 870               |
| $\gamma\gamma \rightarrow \ell^+qS$: |               | $1^{st}$   | 480               | 970               |
|                               |               | $2^{nd}$   | 420               | 920               |
|                               |               | $3^{rd}$   | 320               | 780               |

Table 1: The largest LQ mass (in GeV) observable, for each of the four LQ charges and for each of the three generations, in the processes $e^+e^- \rightarrow e^+qS$ and $\gamma\gamma \rightarrow \ell^+qS$ at (a) $\sqrt{s} = 500$ GeV and (b) $\sqrt{s} = 1$ TeV. $2^{nd}$- and $3^{rd}$-generation LQ’s cannot be seen at $e^+e^-$ colliders.
**Figure Captions**

1. The three sets of diagrams contributing to the process $e^+e^- \rightarrow e^+qS$. $q$ represents either a quark or an antiquark.

2. Diagrams contributing to the process $\gamma e^- \rightarrow qS$. $q$ represents either a quark or an antiquark.

3. Cross sections for single leptoquark production in $e^+e^-$ collisions at (a) $\sqrt{s} = 500$ GeV, (b) $\sqrt{s} = 1$ TeV, for the 4 possible LQ charges, $Q_S = -1/3, -2/3, -4/3, -5/3$. The results are given for $k = 1$. Here we have used the nonzero quark mass as a regulator (see text).

4. Cross section for single leptoquark production in $e^+e^-$ collisions at $\sqrt{s} = 1$ TeV, using $Q_S = -1/3$ and $k = 2$ (this corresponds to $k = 1$ in Ref. 5). Here we have used a 10 GeV $p_T$ cut as a regulator (see text).

5. The four sets of diagrams contributing to the process $\gamma\gamma \rightarrow \ell^+qS$. $q$ represents either a quark or an antiquark.

6. Cross sections for single leptoquark production in $\gamma\gamma$ collisions for the 4 possible LQ charges, $Q_S = -1/3, -2/3, -4/3, -5/3$, for (a) 1$^{st}$-generation LQ’s at $\sqrt{s} = 500$ GeV, (b) 1$^{st}$-generation LQ’s at $\sqrt{s} = 1$ TeV, (c) 2$^{nd}$-generation LQ’s at $\sqrt{s} = 500$ GeV, (d) 2$^{nd}$-generation LQ’s at $\sqrt{s} = 1$ TeV, (e) 3$^{rd}$-generation LQ’s at $\sqrt{s} = 500$ GeV, (f) 3$^{rd}$-generation LQ’s at $\sqrt{s} = 1$ TeV. The results are given for $k = 1$. 

