Laser noise control in the optoacoustical gravitational antenna

V A Krysanov
Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia
E-mail: kv@sai.msu.ru

Abstract. It is shown that the metric sensitivity theory of the OGRAN project contains description of the Fabry-Perot cavity displacement readout scheme proposed by Braginsky. In theories a quality factor defines laser noise. As an alternative, two phenomenological intrinsic controlled parameters of laser noise are proposed to analyze readout resolution: spectral densities of frequency deviations and power random modulation $P(t) = P_0[1 + \xi(t)]$. The formula for vibration noise is derived. The new theory of instrumental sensitivity is applied to explain the effect of readout noise rise while increasing digitization time. To reduce the readout threshold signal from real value of $2 \cdot 10^{-15}$ cm/Hz$^{1/2}$ to required value of $3 \cdot 10^{-16}$ cm/Hz$^{1/2}$ there are proposed actions: 1) to limit temperature drift difference between laser and bar cavity frequencies by value of 250 Hz; 2) to suppress by circuitry means vibration disturbance voltage at the discriminator output while increasing by 20 dB width of feedback frequency range and feedback gain in this range.

1. Readout systems for Weber-type antennas

The gravitational wave antenna OGRAN [1] is installed in the underground laboratory of the Baksan Neutrino Observatory of the Institute for Nuclear Research and is currently in the stage of adjusting and testing [2]. It is a Weber-type the antenna as a massive solid-state test body of aluminum in the form of a cylinder (bar) [3]. It transforms metric variations into resonant acoustic oscillations; bar length variations are registered by the readout system. The Fabry-Perot (FP) interferometer mirrors are fastened rigidly at top ends of the bar. Eigenfrequency variations of the cavity locked fundamental mode are registered by the laser optoelectronic scheme.

Braginsky proposed to use a FP interferometer as readout scheme for mechanical displacements [4]. This method is continuation and extension of RF capacitive meter technique, in which eigen frequency of LC tank is changed [3,5]. A capacitive meter replaced the Weber’s passive piezoelectric transducer readout and added tank thermal noise. Also generator “driving generator noise” had been noticed, and the RF generator it presumably is a source of flicker-noise as jumps of amplitude [6].

The intention had arised to remove bias generator and enter into LC tank negative resistance to convert it in auto generator [7], where generation frequency deviates. In this article frequency stochastic deviations are analyzed both in created RF generator and in optical quantum generator simultaneously. In the OGRAN project, signal and noise are encoded in laser frequency deviations too.
In a capacitance sensor the additional noise source takes place: thermal noise of electrical tank causes the back force fluctuation influence on a bar, since a multiplicative effect with bias RF voltage arises on the quadratic electrostatic force. This effect increases noise temperature of a bar. To control this influence special theory, circuitry and efforts have been applied [8]. And in the OGRAN readout this back effect is absent [9]. So, in the article [1] we can read: there is “the possibility of attaining a sensitivity that is limited only by the thermal noise of the detector body . . . owing to the small fluctuation effect of the optical degree of freedom”.

Since room-temperature RF meters are well studied with there significant drawbacks, return to passive Weber-type readout has been made and the antenna ”Ulitka” on modern technological basis has been created. Its achieved readout resolution 10$^{-14}$ cm/Hz$^{1/2}$ [10] became a starting point for the OGRAN project. So, for the pilot model, the design value is of 10$^{-16}$cm/Hz$^{1/2}$ [11]. For the available test body the readout displacement resolution is a single registered instrumental parameter. Antennae’s metric characteristics are recalculated; either signal astrophysical forecast is used [10,11] or total noise is represented by spectral density of metric [12].

The aim of the long-time investigations was to reduce a threshold signal of displacement readout system. In a capacitive readout thermal noise dominates. In paper [13] appropriate steps and actions to increasing sensitivity of this scheme had been represented and described; it had marked the beginning of cryogenic direction in MSU. As the result of development, the measurer with a record resolution of 6 · 10$^{-17}$ cm/Hz$^{1/2}$ had been created [14]. The special low-noise bias generator had been created and used. The paper [13] is notable also for the fact it contains only projects, suggestions, proposals, recommendations and evaluations; and the presented message is made in this format. While deep cooling, superconductivity effect became appreciable. This effect has made promising alternative inductive meter with quantum interferometer devices (SQUIDs) and specific circuitry [15-17].

Optical scheme has unique high potential sensitivity even at room temperature. Its realization is prevented by laser noise. Intention to develop cryogenic version of OGRAN project to reduce test body thermal noise had been announced still in report [18]. Now the revival of cryogenic technology has begun [2,19].

2. Braginsky’s optical readout scheme

A conceptual readout scheme using a Fabry-Perot resonator had been proposed by Braginsky [4]. According to the principle of operation, it is similar to a capacitive sensor. Variations in the length of the test body cause variations in the capacity gap and LC tank eigenfrequency. Oscillations are generated in a LC tank or in a FP resonator by means of external generator; its frequency is tuned to the steep part of the slope of resonance curve.

In optical readout operating mode has been established: “In order that one obtain the greatest sensitivity, resonator eigenfrequency must be somewhat out of tune with the laser frequency $\nu_L$: $\Delta \nu_d = \nu_L / 2Q_{opt}$, where $Q_{opt}$ is the quality”. As to noise and readout resolution, we read: “If we use the known expressions for the spectral density of the deviation of the frequency ..., we may obtain a simple analytical expression for the smallest displacement”:

$$x_{MIN} = \frac{(1 - R)c}{2\pi} \sqrt{\frac{2h\Delta f}{N_0\nu_L}A} = \frac{\lambda \sqrt{h\nu_L\Delta f}}{F} \frac{A}{2N_0}.$$  (1)

Here $N_0$ is “power of the light current”, $\Delta f$ is a frequency band, and a photodetector quantum yield $\eta$ is close to unity. In the right formula, the modern parameter finess is applied [20]:

$$F_G = \pi(1 - R)^{-1}.$$  (2)

The evaluative parameter $A$ had been presented as “a dimensionless factor which takes account of the rigidity of the limit cycle of the nonlinear characteristics of the active medium
in optical generator” [4]. Additionally, the author indicates that the quantity $A$ characterizes “the “quality” of the optical source” and for “helium-neon lasers $A \approx 10^2 \div 10^3$”. When testing readout schemes, the formula (1) may be used as a theoretical forecast.

Information about the readout scheme is contained in OGRAN project theory. It defines threshold signal of antennae for gravity metric variations. The formula for the antennae’s noise factor contains an expression for certain spectral density [1,2,9,11,18]:

$$G_V(\omega) = B\omega^2_p \left( \frac{2h\nu}{\eta P} \right) \left( \frac{\lambda}{2\pi N} \right)^2,$$

where “laser power” $P$ and quantum yield $\eta$ are presented. In article [1] the factor $B$ is represented: “$B = 1–10^3$ is the phenomenological factor that indicates by how many times laser noise exceeds the Poisson level”. Also here “$N$ is the number of reflections of FP resonator of the detecting system” [2].

To extract formula for readout resolution from the OGRAN metric theory we compel to return to general description of Weber-type antennae, where an equivalent oscillator represents a bar; and its stochastic motion is determined by the equation [3]

$$M\ddot{x} + H\mu \dot{x} + k\mu x = F_N.$$  

Here $H\mu = M\omega_\mu/Q$ is a viscosity factor, $k\mu$ is a stiffness, and $F_N$ is a fictitious stochastic force in Langevin’s performance. In formulas of the paper [9] one can find the expression for energy spectral density $G_F(\omega)$ of generalized adducted force:

$$G_F(\omega) = G_B + G_f + |Z\mu(j\omega)|^2 G_V(\omega).$$

Here $G_B(\omega) = 2kT M\omega_\mu/\pi Q$ is Nyquist force spectral density ($G_B(f) = 4kTH\mu$), $G_f$ represents the stochastic component of light pressure, $Z\mu = M[j\omega + 2\delta\mu + \omega_\mu^2/(j\omega)]$ is the acoustic impedance of the oscillator and $\delta\mu = \omega_\mu/2Q$. There has been shown $G_f \ll G_B$ [9].

The special spectrum solution of the equation (4) allows us to find the expression for spectral density of stochastic oscillator displacements: $G_X = G_F/\omega^2 |Z\mu|^2$,

$$G_X = G_B/\omega^2 |Z\mu|^2 + \omega^{-2} G_V \equiv G_{XB}(\omega) + G_S(\omega),$$

where spectral density $G_{XB}$ characterizes the thermal Brownian motion of the acoustic resonator-bar; $G_S(\omega)$ is spectral density of noise of the readout (sensor) scheme defining its resolution (sensitivity, threshold signal) by the relation:

$$G_S(\omega) = \omega^{-2} G_V.$$

As the acoustic impedance $Z\mu$ is resonant, here is relevant to take into account only range near the frequency $\omega_\mu$. From expressions (3), (6) and (7) we find

$$G_X(f) = \frac{4kTQ}{M\omega_\mu^3} \frac{1}{1 + (\Delta \omega/\delta\mu)^2} + B \frac{h\nu}{\eta P_0} \left( \frac{\lambda}{\pi N} \right)^2.$$
Here $\Delta \omega = 2\pi f - \omega_\mu$. It is an expression for the spectral density of total noise, observed in test at a spectrogram, in particular, at the figure 7 in [1]. The first term describes quantitatively the resonant thermal noise peak of the bar. The second term is a forecast for readout displacement noise in the OGRAN set (“the optical degree of freedom” [1]); it is the “pedestrian” at the resonant thermal noise peak of the bar. The second term is a forecast for readout displacement at a spectrogram, in particular, at the figure 7 in [1]. The first term describes quantitatively the potential and instrumental sensitivity. Until, we assume the factor $B$ unattainable, although when reducing values of finesse and power, it is possible to pull together the resultant value of $2 \cdot 10^{-15}$ cm/Hz$^{1/2}$ has reported. Also for full-scale installation the resultant value of $2 \cdot 10^{-15}$ cm/Hz$^{1/2}$ has reported. Finally, in article [1] we see the value of $3 \cdot 10^{-16}$ cm/Hz$^{1/2}$.

Derivation of the formula (1) is also relevant because it contains two expressions. The one defines signal conversion and the other defines noise; while the threshold condition is $S/N = 1$. These expressions should be divided, since the adjustment of a readout scheme envisages a priori numerous calculation of signal and noise values at each control point with proper accuracy. In addition, it should be clarified what kind of laser power is fixed in the formula. This is relevant because in the OGRAN optical setup, from the beam splitter almost 1 W of radiation power enters each channel, but power only about $P_{PH} = 50$ mW reaches each photoreceiver [1]. Latter value is in priority to be taken into account as it forms shot noise of photoelectrons. In [4] the incident laser beam power $P_{IN}$ is denoted. And another value is relevant; it is power $P_0$ leaving the cavity in resonance. Since a FP resonator has energy losses, a relation $P_0 < P_{IN}$ is valid, whereas in ref. [3,4,9,18] the condition $P_0 = P_{IN}$ is denoted.

To clarify radiation power loose effect and to present “resonance curve form” mentioned in [3], we derive in notations of article [20] the expression defining electric field $E_{OUT}$ in passing beam:

$$E_{OUT} = t^2 E_{IN} \left[1 - r^2 \exp \left(\frac{j\omega}{\Delta \nu_{fsr}}\right)\right]^{-1}.$$  

Here $E_{IN} = E_0 e^{jat}$ is magnitude of incident beam electric field, $t$ and $r$ are amplitude transmission and reflection coefficients of mirrors, $\Delta \nu_{fsr} = c/2L$ is a “free spectral range” of a FP cavity.

Near resonance we can put $\omega = 2\pi n \Delta \nu_{fsr} + \delta \omega$; where $n$ “is an integer and $\delta \omega$ is deviation of laser frequency from resonance” [20]. When expanding in the small parameter $\delta \omega/2\pi \Delta \nu_{fsr}$ we can get

$$E_{OUT} = \frac{t^2 E_{IN}}{1 - r^2(1 + j\delta \omega/\Delta \nu_{fsr})}, \quad P_{OUT} = \frac{t^4 P_{IN}}{(1 - r^2)^2 + (\delta \omega/\Delta \nu_{fsr})^2}.$$
Under condition \((1-r^2)^2 = (\delta \omega / \Delta \nu_{_{fsr}})^2\) we have found standard bandwidth \(\Delta \nu_{_{res}}\) (at level -3 dB): \(\Delta \nu_{_{res}} = \Delta \nu_{_{fsr}}(1 - r^2) / \pi\). Incidentally, we have defined finesse: \(F_{_{G}} = \Delta \nu_{_{fsr}} / \Delta \nu_{_{res}} = \pi / (1 - R)\) \([20], (2)\). When \(r^2 + t^2 = 1\), in resonance (\(\delta \omega = 0\)) we find:

\[ P_{_{OUT}} = P_0 = P_{_{IN}}. \]

While deriving the formula (1), we base only on the expression for signal transmission \(M_x\) \([3]\):

\[ M_x = 2 \pi (1 - R) - \frac{x}{\lambda}. \]

It is “the relative change in light flux power at a photodetector”.

To determine the permanent power component we must refer to the established operating mode. One can read: “resonator must be somewhat out of tune with the frequency \(\nu_{_{L}}\): by \(\Delta \nu_{d} = \nu_{R} / 2Q_{_{opt}}\), where \(Q_{_{opt}}\) is the quality” \([4]\). The meaning of factor \(Q_{_{opt}}\) is defined in monograph \([3]\): \(Q_{_{opt}} = \nu_{R} / \Delta \nu_{_{res}}\). Then we find \(\Delta \nu_{d} = \Delta \nu_{_{res}} / 2\). It means \(\Delta P_{_{PH}} = P_0 / 2\) and \(M_x = \delta P_{_{PHS}} / (P_0 / 2)\), where \(\delta P_{_{PHS}}\) is variable signal component. We find the transmission expression: \(\delta P_{_{PHS}} = F_{_{G}}P_0 \lambda^{-1}x\).

It should be noted that at first variations of bar length \(x\) are converted into variations \(\delta \nu_{_{RS}}\) of a FP resonator eigenfrequency \(\nu_{R}\) \([3,12]\):

\[ \frac{\delta \nu_{_{RS}}}{\nu_{R}} = x / L_{_{G}}. \]  

Further variations \(\delta \nu_{_{RS}}\) are registered by means of conversion into light power variations \(\delta P_{_{PHS}}\) on a slope of resonance curve: \(\delta P_{_{PHS}} = D_{_{P}}\delta \nu_{_{RS}}\), where \(D_{_{P}}\) is a “decrement”:

\[ D_{_{P}} = \frac{F_{_{G}}P_0L_{_{G}}}{c} = \frac{F_{_{G}}P_{_{PH}}}{\Delta \nu_{_{fsr}}} = \frac{P_{_{PH}}}{\Delta \nu_{_{res}}}. \]  

In the GaAs photodiode \([1]\) signal current variations are determined by a conversion factor:

\[ \alpha \equiv \frac{\delta I_{_{S}}}{x} = \frac{F_{_{G}}P_0 \lambda^{-1} \eta e}{\hbar \nu}. \]  

As to noise, we can write out the expression for shot noise spectral density of photocurrent:

\[ S_{_{IE}}(f) = 2eI_0 = \eta e^2 P_0 / h \nu. \]

This noise determines the limit, potential sensitivity of the scheme. A threshold signal \(x_{_{N0}}\) is determined by addition of this noise to the displacement meter input:

\[ G_{_{S}}(f) = \frac{S_{_{IE}}}{\alpha^2}, \]

\[ x_{_{N0}} = \frac{\lambda}{F_{_{G}}} \sqrt{\frac{\hbar \nu}{\eta P_0}} \Delta f. \]  

Thus, we have executed derivation of the formula (1) for an ideal light source. The numerical estimation is relevant; for the OGRAN project values \(F_{_{G}} = 3000, \eta = 0.7, \lambda = 1.06 \ \mu m \ [1]\) and supposing \(P_{_{PH}} = 40 \ mW\), we find

\[ x_{_{N0}} = 6.5 \cdot 10^{-17} \ cm/Hz^{1/2}. \]  

When using the value of \(L_{_{G}} = 2 \ m\) the important transmission relation (9) gains certainty

\[ \frac{x}{\delta \nu_{_{RS}}} = 0.7 \cdot 10^{-12} \ cm/Hz. \]  

The paper \([23]\) represents creation of OGRAN pilot model bar cavity with finesse value of 50 000; this increases signal transmission and reduces a potential threshold signal by an order of magnitude. Besides, the paper \([18]\) represents “the designed OGRAN optical parameter
\[ W = (1 \div 3) W' \]. This parameter \( W \) is presented in the formula (3) and it means \( W = P_L \approx P_0 \). Moreover, in [18] we read: “In the OGRAN setup the beam was splitted with the beam-splitters to 16 equal parts and 16 photodetectors are used in both channels. It gives additional profit of technical signal-to-noise ratio increasing by factor 4”. Indeed, as a result of improvement of laser [24], the power of its radiation had been increased by ILF from value of 50 mW to value of 2 W. However, in article [1] there has been pointed the phenomenon of wastage more than 90% of optical power on the way to the photodetector; and 4-16 photodiodes only mentioned. Apparently, in test [11] such loss had not been noticed.

We can calculate extremely high potential resolution; it is higher than that of cryogenic sensors. There is a reason to present the OGRAN installation as “Room-temperature gravitational bar-detector with cryogenic level of sensitivity” [9]. Laser “quality factor” \( B \) is a powerful source of promise. So, the perspectives of applying values of \( B = 3 \div 10 \) in new forecasts have been shown in article [2].

Achieved readout resolution should be compared with theoretical forecast. Article [21] provides an example of forecast and reality comparison; also laser optical readout scheme is represented. Theory of the OGRAN project is based on this fundamental article, as pointed in [1]. It represents the expression for metric threshold signal \( h_{\text{min}} \) of “a free-mass gravitational antenna with a laser-interferometric readout system. The system registers relative displacements of two remote probe bodies; its resolution \( \Delta L_{\text{min}} \) is determined by the simple relation: \( \Delta L_{\text{min}} = L h_{\text{min}} \), where \( L \) is distance between bodies. Using it we find the formula for displacement resolution:

\[
\Delta L_{\text{min}} = \frac{\lambda}{2\pi N} \left( A \frac{2\hbar \omega_0}{\eta W_0 \tau} \right)^{1/2} = \frac{\lambda}{F_G} \left( A \frac{\hbar \omega_0}{2\eta W_0} \Delta f \right)^{1/2}.
\]

Here \( N = (1-R)^{-1} \), \( W_0 \) is “incident laser light power” and “measurement time \( \tau \) is equal to the time the detector is affected by an external signal” [21]. In right part relation (2) is used, \( N = F_G/\pi \). Replacement \( \Delta f \approx 1/\tau \) is standard [1,2]; it takes into account approximate width of pulse spectrum. We have obtained just the formula (1). It has presented without reference to paper [4], as well known.

In article [21] meaning of factor \( A \) had been interpreted more definitely: “The factor \( A \) determines the reduction in sensitivity due to a variety of noise sources, relative to the photon noise level”. The same meaning follows from the expression (8). Accordingly, a method for determining the factor \( A \) has specified. So, parameters of the readout circuit and the result of its test should be substituted into the formula (16); and parameter \( A \) is considered as unknown value. Thus, it provides connection of potential sensitivity with instrumental sensitivity, which is determined by technical noise sources.

As result of tests of the 40-meter GW antenna constructed at the California Institute of Technology (Caltech) the resolution value had been published. While applying the presented method, the value of \( A = 10^4 \) had been defined by calculation [21]. Without repeating this calculation and using the idea and methodology, we can obtain own updated estimation. For this, the real resolution of the readout system should be revealed.

3. Noise and sensitivity of the registration scheme

Published results [1] need comments and interpretations. There are four values that pretend to be the main result of the Moscow period of the antenna’s creation and development. The optoelectronic readout system has been created and adjusted by ILF of RAS [2]. However, the resultant total noise spectrograms had been obtained when registering thermal noise of the test body as a narrow peak. In the text [1] in connection to spectrogram at the figure 7, we read: “This is the thermal-noise peak of the mode, which is seen above the “pedestal” of the detection
noise that limits the actual sensitivity of the facility”. This “pedestal” is readout displacement noise. And we can see its level as $4.5 \cdot 10^{-3} \text{ Hz/Hz}^{1/2}$, which corresponds to $3 \cdot 10^{-15} \text{ cm/Hz}^{1/2}$ at the right scale or when using the ratio (15).

The second result located at the end of article [1] is declared as “a frequency band of $\sim 4 \text{ Hz}$”. This is a new format of result performance; it cannot be compared with anything. In [25] this performance has clarified and the digit has been shown to be determined by value of $3 \cdot 10^{-15} \text{ cm/Hz}^{1/2}$ at the figure 7.

The third result has been reported in connection with the figure 6 in [1]: “The measured level of the spectral density of the total antenna noise (background above which a thermal peak dominates) in the operating antenna range is $\sim 0.003 \text{ Hz/Hz}^{1/2}$. This view is almost a novel too. Once it was applied in article [26], which is remarkable for description the previous successful step on the way of increasing readout sensitivity and for the first observation of test body thermal peak in the OGRAN project. The representation of test result in dimension “Hz/Hz$^{1/2}$” is exclusive, except that noise on spectrograms at left scales on figures 6 and 7 is represented in this dimension. In fact, a readout circuit registers namely the frequency variations of a FP resonator.

Noise recalculation is implemented by means of relation (15). It can be noted that amplitudes of oscillations of cavity mirror and cylinder end are not equal. The text indicates the difference of 1.7 times [1]. The experimental method of this digit defining has not pointed out, as and its accuracy. The figure 1.7 has not taken into account further in the article. So, the resultant figure (0.003 Hz/Hz$^{1/2}$) characterizes resolution of measurer of cavity eigenfrequency and length variations namely. Right scales of spectrograms are represented in displacement dimension “cm/Hz$^{1/2}$” using relation (15).

In article [1], standard adducted noise recalculation has not been performed; that has created some uncertainty. If we do it; we get of $2 \cdot 10^{-15} \text{ cm/Hz}^{1/2}$. Just the same value had been published recently in the message [22]; and the spectrum on figure 6 illustrates how namely this result had been obtained. This result seems quite reliable, since the thermal peak at 1.3 kHz on this spectrogram of low-resolution has the value of $0.3 \text{ Hz/Hz}^{1/2}$, that is, $2 \cdot 10^{-13} \text{ cm/Hz}^{1/2}$. And this peak exceeds the noise level of the displacement meter by 40 dB. This peak value is under control as it can be calculated by the formula (8). So, substituting values $Q = 10^5$ and $M = 10^6 \text{ g}$ we get the value of $[G_{XB}(f)]^{1/2} = 1.7 \cdot 10^{-13} \text{ cm/Hz}^{1/2}$.

It should be recalled for comparison that the “Ulitka” detector readout has adducted resolution of $10^{-14} \text{ cm/Hz}^{1/2}$ with the peak height of 20 dB [10].

It is possible to understand for what purpose standard recalculation (9) has not executed with the third result. It has been done to avoid presence of two numerical values in the dimension “cm/Hz$^{1/2}$” in the text. So, the value of $3 \cdot 10^{-16} \text{ cm/Hz}^{1/2}$ already takes place. It is presented in connection with the figure 7 [1]. In standard dimension it can pretend to be considered as the fourth, the most significant result of Moscow period of OGRAN project development. This number in the middle of the text has the convenient form for citations and references out of context. It looks to be ready and suitable for comparison with the published designed resolution value of $10^{-16} \text{ cm/Hz}^{1/2}$ for the OGRAN pilot model [11]. Unlike the real result it should be considered rather as a desired value. This article is devoted to propose actions and operations to achieve this designed value in practice, ”in hard”.

For the full-scale antenna a designed value has not published; this is consequence of pilot model tests, where thermal peak has not been detected due to powerful excess technical noise [18,22].

The two resultant digits following from low-resolution and high-resolution spectrograms differ. It must be noted that values of thermal peak tops differ twice, 6 dB. As a result the second peak height is 30 dB. Top difference is a result of error in “optical calibration”. If we try to correct this mistake in order of interpretation, the resolution achieved becomes of $6 \cdot 10^{-15} \text{ cm/Hz}^{1/2}$.
This correction does not change the article final result of 4 Hz; the effect of noise increasing is explained quantitatively below.

Origin of resultant band of 4 Hz we can explain briefly. So, the width of the thermal peak, as well as bandwidth of acoustic resonator at level -3 dB, is 0.013 Hz. For the level -30 dB the width is 0.4 Hz in accordance with the analytical dependence (8). Within this band, the spectral density of metric threshold signal is \( \sim 10^{-20} \text{ Hz}^{-1/2} \) according to approximate estimate of article [1]. This calculated range of 0.4 Hz corresponds to lowest part of frequency dependence presented in figure 8. This noise spectrum is result of recalculation of spectrum at the figure 7 [25]. At to spectrum of figure 8, the interval between two points with ordinate value of 10 Hz\(^{-1/2}\) is 4 Hz, which is 10 times more than that of at the lowest part. Ttis representation of antenna’s general characteristic is available in paper [12].

If the 40 dB peak were used to calculate the reception band, it would be of 12 Hz instead of 4 Hz.

The revealed value of actually achieved resolution of OGRAN readout of \( 2 \times 10^{-15} \text{ cm/Hz}^{1/2} \) can be compared with the value of potential resolution (14). According to methodology of article [21] mentioned above we have find own “laser quality factor” estimation of \( B \approx 2 \times 10^3 \). Thus, we have considered laser noise in the theory accompanying OGRAN project development.

The representation of laser noise by factor \( B \) introduces uncertainty into calculations and makes it difficult to struggle with this noise source. When perceiving, some structure in noise can be found.

Two phenomenological components can be distinguished in noise of a bias source. So, in the cryogenic capacitive meter there have been represented energy spectrum densities of amplitude and frequency fluctuations of the special pumping generator; corresponding numerical values have been presented [14]. In a capacitive meter these noise components differ on a slope of a resonance curve.

In whole, there is a general representation of generator output signal; it involves the radiophysical determination of a narrow-band stochastic process as a harmonic base with random amplitude and stochastically varying phase; while frequency is a derivative of phase.

Powerful technical non-Poisson intensity fluctuations of laser light are evident by photoreceiver. They are presented quantitatively, in particular, in article [12], where they determine a noticeable stochastic component in pressure of light on an auxiliary intermediate body.

In the theory of heterodyne detection of optical signals, there is applied the representation of phenomenological intrinsic laser “power noise”: \( P(t) = P_0[1 + \xi(t)] \), where \( \xi(t) \) is a dimensionless modulating process having the spectral density \( m_L(f) \) [25].

There are also references to intrinsic laser “frequency noise” as a chaotic process \( \nu_N(t) \) of frequency deviations with spectral density \( S_\nu(f) \) [3,7,12]. If in article [4] Braginsky only mentions frequency noise of a laser, in monograph [3] he considers it separately and specially.

We can consider power noise in Braginsky’s optical scheme. Stochastic power variation \( \delta P_N \) arrive at a photodiode: \( \delta P_N = \delta P_{FH}\xi \). In current spectral density, the factor \( B \) is determined by the expression: \( S_I(f) = m_L I_0^2 + 2e I_0 = 2e I_0 B \), and we obtain relation, where the factor \( B \) attains significant certainty: \( B = 1 + m_L I_0/2e \). Applying the formula (12) we obtain the expression

\[
x_{N_m} = \frac{\lambda}{2F_G} \sqrt{m_L \Delta f}.
\]

Thus, in this laser noise performance an increase in power does not produce an effect of lowering a threshold signal. Although increase in radiation power provides proportional increase in signal transmission (11), it should be taken into account that laser is a source of noise itself.

For the laser used in the installation [24], there has been measured value of \( m_L \approx 10^{-12} \text{ Hz}^{-1} \) [25]. Using it and above values we get estimations \( B = 7.5 \times 10^4 \) and \( x_{N1} \approx 2 \times 10^{-14} \text{ cm/Hz}^{1/2} \).
this corresponds to resolution values of the “Ulitka” detector [10] or the pilot model [18,22].

Meanwhile, we know that the resolution value achieved in the OGRAN installation is an order of magnitude higher, and it should be increased by another order of magnitude. This has been ensured by the application of Pound-Drever-Hall (PDH) technique in which low-frequency (LF) power noise is replaced by noise on certain radio frequency (RF).

Consider frequency fluctuations $\nu_N(t)$ in the scheme. They are converted to power fluctuations on a slope: $\delta P_N = D_P \nu_N(t)$ and $S_1(f) = (\eta_e/\hbar \nu)^2 S_\nu D_P^2$. Using the expression (12), we find $G_S(f) = (L_G \lambda/c)^2 S_\nu$. Assuming for example $(S_\nu)^{1/2} = 3 \times 10^{-3}$ Hz/Hz$^{1/2}$ (stabilized laser), we find $(G_S)^{1/2} = 2 \times 10^{-15}$ cm/Hz$^{1/2}$. Laser frequency noise has significant but controlled effect on resolution of this readout scheme.

Suppression of laser frequency noise in the OGRAN readout is considered in article [25]:

$$S_{\nu d} = \frac{S_\nu}{K_0^2}.$$  

Here $S_\nu$ is intrinsic laser spectral density (LSD), $S_{\nu d}$ is LSD in licked scheme, $K_0$ is feedback gain.

We have $S_{\nu}^{1/2} \approx 3$ Hz/Hz$^{1/2}$, $K_0 = 10^3$ [1], $S_{\nu d}^{1/2} = 3 \times 10^{-3}$ Hz/Hz$^{1/2}$ and $G_S^{1/2} = 2 \times 10^{-15}$ cm/Hz$^{1/2}$. In this way there is implemented a function of stabilizing laser frequency by means of a FP interferometer.

Since the task is to reduce contribution of frequency noise to the value of $2 \times 10^{-16}$ cm/Hz$^{1/2}$, this noise should be suppressed additionally by 20 dB, the feedback gain should be increased by 20 dB, that is, to value of $10^3$. It may be raised to $3 \times 10^3$ and the compensation method may be applied, since at output of the synchronous detector of sensor channel frequency noise is present in pure form.

4. **Pound-Drever-Hall technique**

In the OGRAN GW antenna laser optical readout system published in paper [12] is implemented. It is an automatic frequency control scheme which provides fast and accurate tracking of laser frequency deviations $\delta \nu_{RS}$ for signal eigenfrequency deviations $\delta \nu_{RS}$ of the bar cavity. PDH technique [20] is used; laser light is modulated with a Pockels cell (EOM) driven by a local RF oscillator. Electric field of a beam becomes phase modulated:

$$E_{INC} = E_0 \exp[j(\omega_L t + \beta \sin \Omega_{RFt})],$$

where $E_{INC}$ marks beam incident on a cavity, $\omega_L = 2\pi \nu_L$, $\Omega_{RF}$ is modulating radio frequency and $\beta$ is a modulation depth. The beam contains a carrier power ($P_C$) component and two sidebands ($P_S$); there is a relation $P_S \approx 0.4P_C$ for the optimum value of the parameter $\beta$; $P_{INC} \approx P_C + 2P_S$.

Power of a beam reflected from a cavity $P_{REF}$ is defined by article [20]:

$$P_{REF} \approx P_C \left( \frac{\Delta \nu_{RL}}{\Delta \nu_{R0}} \right)^2 + 2P_S + D_{PG} \Delta \nu_{RL} \cos \Omega_{RFt}. \quad (18)$$

Here $\Delta \nu_{R0}$ is a cavity’s linewidth ($\Delta \nu_{R0} \equiv \Delta \nu_{res}$), $\Delta \nu_{RL} = \nu_R - \nu_L$ and $D_{PG} = 8(P_C P_S)^{1/2}/\Delta \nu_{R0}$ is the bar cavity power decrement.

The decrement $D_{PG}$ defines signal transmission. It is an analog of the decrement in expression (10). For another reason, we return to power value that arrives at a photodiode. In the absence of energy losses in a cavity we have: $P_{PH} = 2P_S$, $P_C \approx 1.25P_{PH}$ and $D_{PG} \approx 5P_{PH}/\Delta \nu_{R0}$. The presence of energy losses significantly reduces signal conversion and ads shot noise. So, loss of energy determines the parameter “contrast” $K_G = 0.2$ [1]; it reduces the transmission by 10 times, and another reflection expression takes place $P_{PH-} = 2P_S + (1 - K_G)P_C$ [25].

Reflected beam power (18) generate current in a diode, and AC component after amplification and synchronous demodulation forms the sign alternating discriminator characteristic.
5. Temperature drifts power noise

An explanation version of excess noise source has been formed \cite{25}. So, laser is known for producing powerful LF intensity noise. But it was believed that internal modulation removes this noise without remainder. Mechanism of these fluctuation penetrations has revealed. Representation has used: $P_{IN} = P_{IN0}[1 + \xi(t)]$, and the expression (19) takes the form:

$$U_{SD,OUT} = K_{SD}K_{F}R_{A} \left[8 \left(\frac{h\nu}{\eta e}\right) \sqrt{\frac{P_{CA}P_{S0}}{\Delta\nu_{RG0}}}(\Delta\nu_{RLT} + \delta\nu_{R})[1 + \xi(t)]\right],$$

where value $\Delta\nu_{RLT}$ is a quasi-static frequency mismatch caused by thermal drift.

The expression contains the product $(\Delta\nu_{RLT} + \delta\nu_{R})(1 + \xi) \approx \Delta\nu_{RLT} + \Delta\nu_{RLT}\xi + \delta\nu_{R}$. The second term here $\Delta\nu_{RLT}\xi$ represents new chaotic process (noise source $N$) and the third term represents signal $S$. Their equality $(S/N = 1)$ determines corresponding contribution to threshold signal in frequency deviations: $\delta\nu_{NT} = \Delta\nu_{RLT}\xi$. With recalculation (9) we have displacement noise: $x_{N} = (L_{G}/\nu_{L})\Delta\nu_{RLT}\xi$,

$$G_{S\Delta T}^{1/2} = (L_{G}/\nu_{L})\Delta\nu_{RLT}m_{L}^{1/2}$$

This main formula shows LF laser power noise penetration. For value $\Delta\nu_{RLT} = 250$ Hz, we obtain estimate $(G_{S\Delta T})^{1/2} = 2 \cdot 10^{-16}$ cm/Hz$^{1/2}$.

For next calculations consideration of residual mismatch value $\Delta\nu_{RLT}$ is required. Tracking system performs the second function: laser frequency follows bar cavity eigenfrequency drift caused by aluminum temperature expansion. Immediately after tracking capture, difference $\Delta\nu_{RLT}$ equal to zero, and afterwards cavity eigenfrequency $\Delta\nu_{RT}(t)$ changes. Error signal (output voltage of the synchronous detector) is formed by PDH technique: $\Delta U_{SD} = D_{GSD}(\Delta\nu_{RT} - \Delta\nu_{LT})$, where $D_{GSD} = K_{SD}R_{A}(\eta e/h\nu)D_{PG}$. Then laser frequency deviation $\Delta\nu_{LT}$ is formed: $\Delta\nu_{LT} = \beta_{AG}K_{S}\Delta U_{SD}$. Here $\beta_{AG}$ is a laser actuator constant, $K_{S}$ is the gain of the servo amplifier. This relation locks the feedback circuit and forms the equation. Its
solution has the form: \( \Delta \nu_{LT} = [K_0/(1 + K_0)] \Delta \nu_{RT} \), where \( K_0 = K_S \beta AG D_{GSD} \) is a total feedback gain on quasistatics. Substituting \( \Delta \nu_{RLT} = \Delta \nu_{RT} - \Delta \nu_{LT} \) we find

\[
\Delta \nu_{RLT} = \Delta \nu_{RT}/(1 + K_0). \tag{21}
\]

Therefore, to combat temperature drift noise (20) the gain \( K_0 \) should be increased.

The DC servo amplifier has complex frequency dependence; it defines the relationship

\[
K_0/K_0 \gg 1.
\]

Laser noise determines instrumental sensitivity of the OGRAN readout. Theory of instrumental sensitivity has based on technology of suppressing intrinsic fluctuations of laser frequency. Then, in article [25] two new noise sources are introduced, which are manifestations of laser power noise. This theory is independent of potential sensitivity theory; this is theory of readout designing and adjusting.

The new theory of instrumental sensitivity can be applied to explain the effect of increase in noise of the OGRAN installation while increasing digitization time. Certain difficulty is that the general feedback gain \( K_0 \) has not reported. Also there should be evaluated digitization time of the “high-resolution” spectrum at figure 7 [1] and temperature drift value in the boathouse of SAI MSU.

It seems possible to estimate those parameters approximately and simultaneously explain numerically the effect of decreasing the thermal peak value from 40 dB at the spectrogram on figure 6 to 30 dB at the spectrogram on figure 7 in article [1].

To estimate the minimum feedback gain \( K_0 \) of the main PDH channel we take into account the feedback gain of \( K_0 = 10^3 \) at operating frequency of 1.3 kHz [1]. Special methodology dealing with phase shifts is used to avoid self-excitation in locked circuit. According to this method the amplitude-frequency dependence of gain should fall moderately rapidly with frequency increasing. The total drop should be either 12 dB or 18 dB per octave. Two or three consecutive cascades provide this decay; each is a RC low pass filter with decay of 6 dB. For effectiveness of overall decline, we suppose that each cascade should have slope steepness close to the maximum already at operating frequency. This means the cutoff frequency of each pass filter should be three times less minimum than operating frequency (<450 Hz). Accordingly, gain of each corrective cascade at zero frequency should be three times more than that of at operating frequency. If there are three correcting cascades, the total gain addition will be minimum 30. Thus, we have got the estimate of \( K_0 \approx 3 \cdot 10^3 \).

As the next assumption, thermal noise peak on figure 7 [1] can be estimated to have digitization time of 10 minutes or less. This is experience of observing thermal peak of the “Ulitka” detector [10].

As the third assumption, we believe temperature drift in the SAI boathouse of 0.3 degrees per hour when industrial seismic disturbances are in minimum.

While calculating, within 10 minutes we get temperature variation of 0.05 degrees. Then for the values of aluminum linear expansion of 2.3 \( \cdot \) \( 10^{-5} \) K\(^{-1}\) and the bar length of 200 cm, we obtain variation of the FP resonator length of 2.3 \( \cdot \) \( 10^{-4} \) cm and eigenfrequency variation of \( \Delta \nu_{RT} \approx 330 \) MHz. For the above value \( K_0 \) using the relation (21) we find \( \Delta \nu_{RLT} \approx 11 \) KHz.

Substituting this frequency variation into the formula (20), we find the value of excess noise in the tenth minute of 8 \( \cdot \) \( 10^{-15} \) cm/Hz\(^{1/2}\). This estimation is quit close to real displacement noise 6 \( \cdot \) \( 10^{-15} \) cm/Hz\(^{1/2}\) in figure 7. So, by manipulation with parameters we have explained of observed effect.

At the same time, by this calculation we confirm indirectly the assumption that gain value is in the region \( K_0 = 10^4 \div 10^5 \). Taking into account the rigid requirement \(|\Delta \nu_{RLT}| \leq 250 \) Hz, it can be recommended to increase total feedback gain value to \( K_0 \approx 10^7 \) in order to use whole servo amplifier and “slow” PZT actuator retuning range of \( \pm (2.5 - 3) \) GHz, which corresponds to designed temperature operating range of \( \pm 0.5 \) K.
If there is not possible to increase the value of $K_{00}$, it is suggested to create a method of bar heating and cooling and introduce it into an active system of temperature auto-control using voltage at input of the laser tuning piezoelectric element (output of DC servo amplifier) as a temperature sensor in zero-indicator operating mode. As to laser cavity, its temperature is well stabilized [24].

6. Discriminator power vibration noise

Another way of LF laser power fluctuation penetration has revealed in the discriminator channel [25]. The eigenfrequency of the discriminator reference FP resonator is retuned by a piezoelectric element, to which one mirror is attached. On the slope of PDH discriminatory curve laser frequency deviations are converted into synchronous detector output voltage variations: $U_{SDD} = D_D U_{PH}$ (error signal); here $D_D$ is a voltage decrement.

The second function of this system is to track quasi-static thermal drift of laser and bar resonator frequencies. This implies range of reference cavity eigenfrequency tuning of $\sim \pm 3$GHz.

Also a phenomenon is observed: industrial and acoustic vibration disturbances (interference) with infra low sound frequency range of $\sim (5 \div 100)$ Hz is available at output. Usually it takes up entire bandwidth of the reference resonator $\Delta \nu_{RD0} = 28$ kHz [25] or more. It has been observed also that background of permanent noise spectrogram increases with increasing this disturbance. Therefore, tests were carried out in the SAI MSU boathouse at night and in calm weather. Thus, excess noise was reduced to an acceptable level corresponding to assessment of laser frequency noise contribution.

The third function of the discriminator channel is some suppression of vibration interference. So, in article [1] we read: “The feedback in the second arm is realized only at low frequencies ($< 100$ Hz); at higher frequencies, a feedback is absent”. To evaluate interference suppression we consider all functions of the tracking system simultaneously. The functional diagram of this system is represented at the Figure 1.

![Figure 1. Discriminator tracking system. 1 - FP cavity, 2 - photoreceiver, 3 - RF preamplifier, 4 - synchronous detector, 5 - servo amplifier.](image)

We write down relations that determine operation of the locked PDH scheme [12]:

$$\delta \nu_D = K_L \beta_D U_{SDD}, \quad U_{PH} = D_D (\delta \nu_L - \delta \nu_D), \quad U_{SDD} = K_D (U_{PH} + U_N).$$

Here $\delta \nu_D$ is tracking retune of cavity eigenfrequency, $\beta_D$ is the PZT driver retune parameter of the discriminator cavity, $K_L$ is a gain of servo amplifier 5, $D_D$ is photodetector output voltage...
decrement, $K_D$ is a gain of RF amplifier and synchronous detector (RF noise transformation is strictly considered in article [25]); process $U_N(t)$ represent shot noise.

The solution gives the total output voltage expression:

$$U_{SDD} = K_D(D_D\delta\nu_L + U_N)(1 + K_D\beta_D D_DU)\]

Here $K_0D(\omega) = 1 + K_L(\omega)\beta_D D_DU$ is an attenuation factor, $D_DU = K_D D_D$.

There is not much choice in frequency correction circuits, and we believe that there are two stages of correction: $K_L = K_{L0}/(1 + j\omega\tau)^2$. For $\omega = 0$ we have $K_0D(0) \approx K_{L0}\beta_D D_DU$. To evaluate latest feedback parameters assume that laser frequency retune of 3 GHz leads to deviation from cavity resonance of 12 kHz ($\Delta\nu_{RD0}/2$). Then $K_0D(0) = K_{L0}\beta_D D_DU = 2.5 \cdot 10^5$.

At operating frequency we have $K_L\beta_D D_DU \ll 1$, $K_0D = 1$ and $U_{SDD} = D_DU\delta\nu_L$.

For cut-off frequency of 100 Hz we believe $|K_L|\beta_D D_DU = 1$. From comparison with $K_0D(0)$ we find $1 + j\omega\tau^2 = 2.5 \cdot 10^5$ and $\omega\tau = 5 \cdot 10^5$. Hence, the numerical value $\tau$ and entire frequency dependence becomes perfectly defined.

Vibration random process is observed in test at output of the synchronous detector. In formulation of the task it is represented by the expression: $U_{SDD} = K_D D_D(\delta\nu_L - \delta\nu_D)$. We introduce the difference: $\delta\nu_{LD} = \delta\nu_L - \delta\nu_D$. The solution takes the form:

$$\delta\nu_{LD} = \frac{\delta\nu_L}{K_0D(\omega)}.$$

It can be seen that significantly more powerful vibration interference income the discriminator as a rejector filter for lowest frequencies. Spectral components of interference close to cut-off frequency are not suppressed effectively.

Power LF noise is carried by vibration interference. To estimate this effect random process has impeded by harmonic voltage at output of the synchronous detector [25]: $D_DU\Delta\nu_m \sin \Omega_It$, where $\Delta\nu_m$ is amplitude of process $\delta\nu_{LD}(t)$. Taking into account power modulating stochastic process we write out:

$$U_{SDD} = D_DU\Delta\nu_m \sin \Omega_It[1 + \xi(t)] = D_DU\Delta\nu_m \sin \Omega_It + \xi(t)D_DU\Delta\nu_m \sin \Omega_It.$$

Here the last member presents a new noise source. It contains infra LF modulating function; averaging has given the coefficient $2/\pi \approx 0.6$. The derived formulas take the form

$$S_{\nu}^{1/2} \simeq (2/\pi)\Delta\nu_m m_{\nu_L}^{1/2}, \quad G_{XD}^{1/2} \simeq 0.6(L_G/\nu_L)\Delta\nu_m m_{\nu_L}^{1/2}.$$

For the value $\Delta\nu_m = 7$ kHz there has been obtained $(S_{\nu})^{1/2} = 4.5 \cdot 10^{-3}$ Hz/Hz$^{1/2}$; it determines estimation of displacement readout resolution of $3 \cdot 10^{-15}$ cm/Hz$^{1/2}$ [25].

According to tests in Moscow, the sum of vibration noise and laser frequency suppressed noise had reached a level of $2 \cdot 10^{-15}$ cm/Hz$^{1/2}$. Since the task is to reduce this noise contribution to value of $2 \cdot 10^{-16}$ cm/Hz$^{1/2}$, the interference average amplitude should be reduced by $\sim 26$dB. This random process can be suppressed significantly by means of circuitry technique when improving tracking performance: by increasing speed of action and accuracy of tracking. There is proposed to increase feedback gain in the pointed range. Frequency dependence of the gain has low slope. Also, the analysis has shown that spectrum components of $(30 \div 100)$ Hz are not sufficiently reduced. The simplest action is to increase the cut-off frequency; this should make tracking system faster.

"The set of improved mirrors for the antennae in BNO INR has been manufactured by Laboratoire Materiaux Avances, Lyon, France" [1]. They are intended to fastening on bars after preparedness of the laminar room around the OGRAN installation. Significant narrowing
of discriminator cavity band and increase of decrement $D_D$ are expected. This may require further reduction of interference level (dispersion), and a reserve is further increase of cut-off frequency from 100 Hz by 30-100 times. This means that the condition $K_{OD} = 1$ at the operating frequency of 1.3 kHz is not executed. From the equation decision we can conclude that implementation of values $|K_{OD}| = 3 \div 5$ at frequency of 1.3 kHz does not reduce potential sensitivity of the installation. It is determined by ratio of signal $D_D \delta \nu$ and noise $U_N$ and we have $\delta \nu_{min} = U_N/D_D$. In this mode normal steep slope of feedback gain frequency dependence is achieved, that is, while decreasing factor $K_L(f)$ is increased relatively harshly.

References

[1] Bagaev S N, Bezrukov L B, Kvashnin N L, Motylev A M, Oreshkin S I, Popov S M, Rudenko V N, Samoilenko A A, Skvortsov M N and Yudin I S 2015 Instrum. & Exper. Tech. 58 pp 257-67
[2] Kulagin V V, Oreshkin S I, Popov S M, Rudenko V N and Yudin I S 2016 Physics of Atomic Nuclei 79 pp 1552-9
[3] Braginsky V B 1970 Physical Experiments with Test Bodies (Moscow)
[4] Braginskii V B 1968 Soviet physics JETP 26 pp 831-4
[5] Braginskii V B, Mitrofanov V P and Rudenko V N 1971 Prib. Tekh. Eksp. 4 pp 241-3
[6] Braginskii V B, Manukin A B, Popov E I and Rudenko V N 1974 Sov. Phys. JETP 39 pp 387-92
[7] Rudenko V N and Panov V I 1979 Radiotechnika i Elektronika 24 pp 1036-43
[8] Krysanov V A, Kuklachov M I and Rudenko V N 1979 Prib. Tekh. Eksp. 4 pp 240-3
[9] Gusev A V, Kulagin V V and Rudenko V N 1996 Gravitation & Cosmology 5 pp 68-70
[10] Gavrilyuk Yu M, Gusev A V, Krysanov V A, Kulagin V V, Motylev A M, Oreshkin S I, Rudenko V N, Silin V A and Tsepkov A N 2012 Astronomy Reports 56 pp 638-52
[11] Bezrukov L B, Popov S M, Rudenko V N, Serebolskii A V and Skvortsov M N 2004 Proc. Int. Conf. "Astrophysics & Cosmology after Gamow" (Preprint gr-qc/0411083v1)
[12] Conti L, Cerdonio M, Taffarello L, Zendi J P, Rizzo C, Ruoso G, Prodi G A, Vitale S, Cantatore G and Zavattini E 1998 Rev. Sci. Instrum. 69 pp 554-8
[13] Panov V I and Petnikov V G 1975 Vestnik Moscow Univ. 3 2 pp 212-5
[14] Braginskii V B, Panov V I and Popel’nik V D 1981 JETP Lett. 33 pp 405-7
[15] Paik H J 1976 J. Appl. Phys. 47 pp 1168-78
[16] Krysanov V A and Rudenko V N 1980 Proc. 9th Int. Conf. on Gen. Relat. & Grav. ed E Schmutzer 2 394
[17] Krysanov V A and Rudenko V N 1984 Prib. Tekh. Eksp. 3 pp 199-203
[18] Rudenko V N, Popov S M, Samoilenko A A, Oreshkin S I and Cheprasov S A 2008 Proc. Int. meeting “PIRT-2007” ed M Duffy (Moscow: BMSTU) pp 49-54
[19] Krysanov V A, Motylev A M, Oreshkin S I and Rudenko V N 2011 Measurement Techniques 56 pp 86-90; 2015 Measurement Techniques 57 pp 1416-22
[20] Black E D 2001 Am. J. Phys. 69 pp 79-87
[21] Kulagin V V, Polnarev A G and Rudenko V N 1986 Sov. Phys. JETP 64 pp 915-21
[22] Vishnyakov V I, Ignatovich S M, Kvashnin N L, Popov S M, Rudenko V N, Samoilenko A A, Skvortsov M N and Yudin I S 2013 Tech. Digest Int. Symp. “MPLP-2013” (Novosibirsk Russia)
[23] Popov S M, Samoilenko A A, Cheprasov S A and Yudin I S 2011 Measurement Techniques 57 pp 1416-22
[24] Okhapkin M V, Skvortsov M N, Belkin A M, Kvashnin N L and Bagayev S N 2002 Optics Communications 203 pp 359-62
[25] Krysanov V A 2018 Journal of Physics Conf. Ser. 1051 012020
[26] Bezrukov L B, Kvashnin N L, Motylev A M, Oreshkin S I, Popov S M, Rudenko V N, Samoilenko A A, Skvortsov M N and Yudin I S 2014 Proc. Int. meeting “PIRT-2013” ed M Duffy (Moscow BMSTU) pp 23-9