Utilizing geogebra in financial mathematics problems: didactic experiment in vocational college

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Abstract. GeoGebra application offers users to solve real problems in geometry, statistics, and algebra fields. This study determines the effect of utilizing GeoGebra on students understanding skill in the field of financial mathematics. This didactic experiment study used pre-test-post-test control group design. Population of this study were vocational college students in Banking and Finance Program of Balikpapan State Polytechnic. Two classes in the first semester were chosen using cluster random sampling technique, one class as experiment group and one class as control group. Data were analysed used independent sample t-test. The result of data analysis showed that students understanding skill with learning by utilizing GeoGeobra is better than students understanding skill with conventional learning. This result supported that utilizing GeoGebra in learning can assist the students to enhance their ability and depth understanding on mathematics subject.

1. Introduction

The development of information and communication technology (ICT) affects many sectors all over the world, including higher education \cite{1, 2}. The ICT in higher education are being used for developing course material; delivering and sharing content; communication between learners, teachers and the outside world; creation and delivery of presentation and lectures; academic research; administrative support; student enrolment etc. \cite{2}. ICT can be an effective tool in supporting teaching and learning when it is used appropriately \cite{3, 4}.

Utilizing ICT especially computer application in college students will enhance understanding skill and assist a depth discussion in the higher education classroom. When working with multimedia applications, students are engaged in activities which develop higher order thinking skills such as problem solving, reflecting, recognizing relationship, analyzing and creatively synthesizing information, as they deal with complex and realistic information \cite{5}. These activities will facilitate students to understand the problems in particular subject, including mathematics.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge \cite{6}. Understanding has, in effect, become a means and a goal of mathematics education \cite{7}. Furthermore, it seems reasonable to characterize mathematical understanding as a matter of having a world of dynamic mental models that are consistent with the conceptual systems of mathematics. The dynamic nature of mathematical understanding and the corresponding needs for multiple representations serve as a theoretical foundation for the integration of technological tools such as GeoGebra \cite{8}.
GeoGebra is a community-supported open-source mathematics learning environment that integrates multiple dynamic representations, various domains of mathematics, and a rich variety of computational utilities for modeling [9] and simulation [10]. It is dynamic mathematics software for all levels education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package. GeoGebra includes a number of features-most notably, sliders and CAS capabilities—that aren’t available in typical spreadsheet programs such as Excel or Calc [11]. This study focuses on effect of utilizing GeoGebra on students understanding skill in the field of financial mathematics.

2. Difficulties in financial mathematics

Financial mathematics is one of compulsory subjects in Banking and Finance Program, Polytechnic State of Balikpapan, East Borneo, Indonesia. The students attend this course in the first semester. Several students who attend this course were still had difficulties to understand the materials. One of the reasons is some students do not have enough background on mathematics in their high school. In addition, most problems in this field are real problems so students must make a mathematical model from the problems before they solve it.

The short definition of mathematical model and modeling is “translation between mathematics and reality”. The reality according to “rest of the world” outside mathematics including nature, society, everyday life and other scientific discipline [12]. In this paper, GeoGebra will be used to visualize mathematical model based on the problems about Compound Value, Present Value, and Future Value. The descriptions of the materials as follows:

2.1. Compound Value, Present Value and Future Value

Let $P$ (Principal) refers to principal of loan which is charged by compound interest, $S$ is the Compound Value, $n$ is the number of compounding periods, and $i$ is the monthly interest in its decimal form. Then the formula of Compound Value is:

$$S = P(1 + i)^n$$

Let $PV$ (Present Value) mean total amount of loan, $A$ is the instalment amount, $n$ is the number of the instalment, and $i$ is the monthly interest in its decimal form. The $PV$ equation is:

$$PV = \frac{(1-(1+i)^{-n})}{i}A$$

And let $FV$ (Future Value) means total savings, and $A$ is amount of each annuity payments (saving), then the formula is:

$$FV = \frac{(1+i)^{n}-1}{i}A$$

The examples of problems in the particular materials above as follows:

2.2. Problem 1

The following is a table of Credit for Businesses installment ($Kredit Usaha Rakyat$, or KUR) issued officially by Bank Negara Indonesia (BNI). The information listed states that the interest rate is 0.412% flat per month. How much is the real interest rate of the annuity considering that the system used by bank is the annuity interest rate?
Figure 1. Table of credit for businesses installment (Kredit Usaha Rakyat, or KUR) issued officially by BNI.

To determine the interest rate annuity from the installment table manually needs a quite long iteracy. From equation (2) above, it is obtained that the polynomial equation of degree $n + 1$:

$$PV(1 + i)^n - A(1 + i)^n + A = 0 \quad (4)$$

By replacing equation (2) with the number of the installment $n = 12$, $PV = 10,000,000$ and $A = 874,515$ from the table (see Figure 1), it will form the polynomial equation of degree 13:

$$10,000,000 (1 + i)^{12} - 874,515 (1 + i)^{12} + 874,515 = 0$$

The features of CAS on GeoGebra can be utilized to solve the equation above. Computer Algebra System (CAS) is one of features in GeoGebra which is possible to perform mathematical calculation in a manner similar to manual one. The presentation of CAS feature to solve the problem above is as follows.

Figure 2. Determining value of $i$ through CAS feature in GeoGebra
It is seen clearly from Figure 2 that a positive solution of polynomial equation \(10,000,000(1 + i)^2 - 874,515(1 + i)^2 + 874,515 = 0\) is \(i = 0.0075\). So it is concluded that the effective interest rate in a year = 0.0075 x 12 x 100% = 9%.

2.3. Problem 2

Determine the loan balance on tenor 15 years between loan Rp500,000.00 which is charged by compound interest 8% p.a, and fixed savings Rp4,000.00 permonth with interest rate annuity 8% p.a, and when the total of loan will be paid off?

The Compound Value \((S)\) on the Problem 2 can be obtained by replacing equation (4) with parameters \(i = \frac{8}{12}\) p.a, \(P = 500,000\) and \(n = 15 \times 12 = 180\), such that:

\[
S = 500,000(1 + \frac{0.08}{12})^{180} = 1,653,460.7
\]

And the Future Value \((FV)\) can be obtained by replacing equation (4) with \(i = \frac{9}{12}\) p.a, \(A = 4,000\) and \(n = 15 \times 12 = 180\), such that:

\[
FV = \frac{\left(1+\frac{0.09}{12}\right)^{180}-1}{\frac{0.09}{12}}4,000 = 1,384,152.9
\]

A loan balance is the amount left to pay on the loan. Every loan that is taken out will have a loan balance up until the loan is completely paid off [10]. The formula of loan balance can be generated as follows. Let \(a\) means to the value of loan balance, then

\[
a = S - FV
\]

Such that, \(a = 1,653,460.7 - 1,384,152.9 = 269,307.9\). So the first question in Problem 2 is solved.

The next question is to determine when the total of loan will be paid off. The debt settlement is achieved when the value of loan balance equal to zero \((a = 0)\). It can be written as follows.

\[
S - FV = 0
\]

Then:

\[
S = FV
\]

Such that:

\[
P(1 + i)^n = \frac{((1+i)^n-1)}{i}A
\]

After replacing equation (7) with parameter \(P = 500,000\), \(A = 4,000\), and \(i = 0.08\), we get

\[
500,000 \left(1 + \frac{0.08}{12}\right)^{269.66} - 4,000 = 3000,000
\]

We can solve equation above numerically, so that the value of \(n = 269.66\) months = 22.4717 years \(\approx 22.5\) years. Furthermore, we can compute the Compound Value \((S)\) which is equal to Future Value \((FV)\), such that

\[
S = 500,000 \left(1 + \frac{0.08}{12}\right)^{269.66} \approx 3000,000
\]

Several steps to solve Problem 2 above can be visualized in GeoGebra and can be analyzed geometrically as follows.
Based on the Figure 3 above, the value of interest rate, instalment, and principal of loan are movable using slider in GeoGebra. The value of loan balance will be obtained automatically by moving parameter value $i$, $A$, $P$ and $n$. According to Problem 2 above, it is seen that the value of loan balance obtained is 269,307.9 and the intersection between the Compound Value and the Future Value is achieved when $n = 22.5$. It means that the compound loan will be paid off on the 23rd year. In other words, it produces the same value as previous calculation without using GeoGebra. However, GeoGebra can show the dynamic movements of the interest rate, annuity instalment, principal of loan so it offers wider explorations to see the influence of characteristics of these three parameters on the value of loan balance.

3. Method
This experiment study used pre-test-post-test control group design. Population of this study were vocational college students in Banking and Finance Program of Balikpapan State Polytechnic. Two classes in the first semester were chosen as sample of the study using cluster random sampling. Class 1KP1 was chosen as control group with conventional learning and Class 1KP2 was chosen as experiment group with learning by utilizing GeoGebra. A pre-test was conducted to measure the initial ability of two groups before the treatment. Then, a post-test was conducted to measure students understanding skill after the treatment. Independent sample t-test was used to analyse the data from the post-test. The fulfilment of the assumptions is required before running the analysis. The assumptions were normality and homogeneity test. The normality of the post-test score was assessed by Kolmogorov-Smirnov test and Levene test were used to assess the homogeneity of variance from the post-test score.

4. Result and discussion
4.1. Result
Descriptive statistics were conducted to describe the mean, standard deviation and standard error mean from the post-test scores. The result is provided on Table 1.
Table 1. Result of descriptive statistics

| Performance          | Group    | N   | Mean   | Std. Deviation | Std. Error Mean |
|----------------------|----------|-----|--------|----------------|-----------------|
| Financial Mathematics| Control  | 33  | 7.2061 | 2.05502        | .35773          |
| Post-test            | Experiment | 32  | 8.1750 | 1.67120        | .29543          |

According to Table 1, mean value of control and experiment groups are 7.2061 and 8.1750 respectively. It means that the mean value of experiment group is higher than the mean of control group.

The t-test is used to discover whether there are statistically significant differences between the means of two groups. In this study, the independent t-test was used to determine the differences between control and experiment groups. The experiment group used GeoGebra while the control group did not. Data of post-test score was normally distributed and had equal variances based on the result of Kolmogorov Smirnov test and Levene test. The result of independent sample t-test showed that the value of $t_0$ is -2.082 and the value of $p$ is 0.041. It means that there is significant difference between the means of two groups, control and experiment groups.

4.2. Discussion

The study was conducted in Banking and Finance Program of Balikpapan State Polytechnic at the first semester. Before the treatment, both groups of students were given a pre-test to measure their initial abilities. The result of pre-test showed that both groups had same initial abilities. During the treatment, the experiment group discussed the real problems about financial mathematics using GeoGebra. Students were fully engaged in the learning process as they were interested with the tools. Apagu and Wakili’s study [15] revealed that using ICT in technical college makes the teaching and learning become interesting.

GeoGebra software also can be used as a powerful tool to visualize relationship between compound value, present value, and future value. During the discussion, students made a mathematical model from the real problems given using GeoGebra. The dynamic movements of the interest rate, annuity instalment, and compound value of the main loan can enhance wider exploration on the influence of characteristic parameters. It means that GeoGebra helped students in modelling the real problem and in broadening their explorations and visualization skills through the process of constructing visual images to analyse the problem, taking into account their informal and visual conceptions [16].

In the last meeting, they were given a post-test to measure their understanding skills in the course. The result of independent sample t-test showed that there is difference on students understanding skill between group with learning by utilizing GeoGeobra and group with conventional learning. In addition, the mean value of experiment group is higher than the mean of control group. It showed that students understanding skill with learning by utilizing GeoGeobra is better than students understanding skill with conventional learning. This result supports the assistance of GeoGebra to enhance the ability and depth understanding of student on mathematics concept especially in financial mathematics course [17, 18].

5. Conclusion

Based on the result and discussion, it can be concluded that students understanding skill with learning by utilizing GeoGeobra is better than students understanding skill with conventional learning. Modelling activities and utilizing GeoGebra should be taken into consideration to enhance vocational college students understanding skill. This study recommends that utilizing GeoGebra in learning should be continued in the classroom and further research about utilizing GeoGebra in other subjects and populations should be conducted by other researchers.
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