Semi-classical magnetoresistance in weakly modulated magnetic fields

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In recent years there has been an increased interest in hybrid systems which promise to increase the functionality of present day semiconductor devices. One example of such type of systems are those in which semiconductors and magnetic materials are combined where the magnetic material provides a local magnetic field which influences locally the motion of the electrons in the semiconductor. The latter is usually a heterostructure which contains a two-dimensional electron gas (2DEG). The 2DEG acts as a detector measuring the magnetic state of the magnetic material. Previously, the coupling between such a non homogeneous magnetic field and the 2DEG was demonstrated in the case the magnetic field (B) was directed perpendicular to the 2D plane. In this case one has a modulation of the B-field on top of a homogeneous background field and the influence of the B-field modulation on the 2DEG is relatively weak.

When the magnetic field is directed parallel to the 2DEG the magnetic material is magnetized parallel to the 2D plane. Because now there is no background perpendicular magnetic field for the 2DEG the influence on the resistance of such magnetic barriers is much more pronounced and large increases in the magnetoresistance have been found.

In the present work we investigate the magnetotransport in weak modulated magnetic fields for which the average B-field is zero. For the case the typical magnetic energy is much smaller than the Fermi energy a semi-classical analysis is applicable. We find that in the diffusive regime the correction to the magnetoresistance exhibits a non analytical behavior in the limit of small magnetic field amplitudes which differs from the behavior in the ballistic regime.

We consider electrons moving in a two-dimensional (2D) xy-plane. The magnetic field, directed along the z-direction, is periodic along the x-direction \( B(x) = B(0,0,b(x/l_0)) \) with period \( l_0 \), where \( b(x) \) is a periodic \( (b(x+1) = b(x)) \) dimensionless function describing the magnetic field modulation with zero average value.

In a semi-classical analysis the electron motion in a magnetic field is described by the following Hamiltonian (or its energy):

\[
\varepsilon = \frac{1}{2m} \left\{ p_x^2 + \left[ p_y - \frac{eB_0}{c} a(x/l_0) \right]^2 \right\},
\]

where \( m \) is the electron effective mass, \( \vec{p} = (p_x, p_y) \) is the electron canonical momentum, and the dimensionless periodic function

\[
a(x) = \int_0^x dx' b(x'),
\]

characterizes the vector potential \( \vec{A} = B(l_0(0,a(x/l_0),0) \) which is taken in the Landau gauge. The quantum energy spectrum of electrons in modulated magnetic fields was studied in Refs.

We restrict our analysis to the case where the electron transition through a single period is ballistic, i.e. the mean free path \( l = v_F \tau \gg l_0 \) ( \( v_F \) is the electron Fermi velocity, and \( \tau \) the relaxation time) and the motion in the sample is diffusive, i.e. the mean free path is smaller than the size of the sample. (\( l \ll L_x, L_y \)). For diffusive transport and in the limit of small magnetic fields \( (\omega_c \tau \ll 1) \) the expression for the average conductivity tensor is given by the following integral over the electron phase space \( (x, p_x, p_y) \)

\[
\sigma_{ij} = -\frac{e^2}{(2\pi \hbar)^2 L_z} \int_0^{L_z} dx \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \tau v_i v_j \frac{\partial f_F}{\partial \varepsilon},
\]

where the symbol \( f_F(\varepsilon) = \{\exp((\mu - \varepsilon)/kT) + 1\}^{-1} \) stands for the equilibrium electron Fermi-Dirac distribution function with \( T \) the temperature and \( \mu \) the chemical potential which equals the Fermi energy \( E_F = \pi \hbar^2 n/m \) in the zero temperature limit, where \( n \) is the 2D electron density. Note that the coordinate \( y \) is excluded from the phase space as the system is homogeneous in that direction. The expressions for the electron velocities follow from the Hamilton equations of motion.
\[ v_x = \frac{\partial \varepsilon}{\partial p_x} = \frac{1}{m} p_x, \]
\[ v_y = \frac{\partial \varepsilon}{\partial p_y} = \frac{1}{m} \left\{ p_y - \frac{eB_0}{c} a(x/l_0) \right\}. \]  

(4)

First, let us consider the conductivity in the zero temperature limit when the derivative of the Fermi function reduces to a \( \delta \)-function. The component \( \sigma_{yy} \) can be calculated straightforwardly, as the sample is homogeneous along the \( y \)-direction all trajectories have to be taken into account. Inserting expression (3) into the conductivity expression (6) we obtain

\[ \sigma_{yy} = \sigma_0 = \frac{e^2}{(2\pi\hbar)^2 L_x} \int_0^{L_x} dx \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \frac{1}{m^2} \]
\[ \times \left\{ p_y - \frac{eB_0}{c} a(x/l_0) \right\}^2 \]
\[ \times \delta \left\{ E_F - \frac{1}{2m} \left( p_x^2 + \left( p_y - \frac{eB_0}{c} a(x/l_0) \right)^2 \right) \right\} \].  

(5)

which leads to the well-known magnetic field independent result

\[ \sigma_{yy} = \sigma_0 = \frac{e^2 E_F \tau}{2\pi\hbar^2}. \]  

(6)

We assumed that the relaxation time depends only on the electron energy and therefore for the case of zero temperature it can be replaced by the constant value \( \tau = \tau(E_F) \). Thus the weak magnetic field modulation in the \( x \)-direction does not change the conductivity in the \( y \)-direction, as expected.

The calculation of \( \sigma_{xx} \) is more complicated because for sufficiently strong magnetic fields (or small electron velocity, but such that \( \hbar \omega_c \ll E_F \)) some electrons can be forced into snake orbits. The electron motion on such orbits oscillates in the \( x \)-direction around the average value \( x_0 \). Therefore, such electrons do not contribute to \( j_x \), and consequently, in the expression for the conductivity \( \sigma_{xx} \) those snake orbits have to be excluded. The classification of all possible electron orbits are given in Fig. 4 where the Fermi surface \( \varepsilon(p_x, p_y, x) = E_F \) is plotted. In the case of zero temperature only electrons with trajectories on the Fermi surface contribute to the conductivity integral. As the energy (3) does not depend on \( y \), the momentum \( p_y \) is conserved, and consequently the trajectories are defined by the intersection of the above Fermi surface with the \( p_y = C^{\text{sn}} \) planes. It is apparent from Fig. 4 that there are two types of trajectories. The trajectories as indicated by symbol \( D \) are able to run along the whole \( x \)-axis. Such electrons are moving along open trajectories and they will contribute to the current along the \( x \)-direction. The other trajectories, indicated by symbol \( E \), are closed and they correspond to the snake orbits. Thus we can separate the conductivity into two parts

\[ \sigma_{xx} = \sigma_0 - \sigma_{xx}^{\text{sn}}. \]  

(7)

where the symbol \( \sigma_{xx}^{\text{sn}} \) stands for the snake orbit contribution. The latter term defines the decrement of the conductivity due to the modulated magnetic field, and in the limiting case of small magnetic fields it is proportional to the increase of the magnetoresistance due to the magnetic field modulation.

The snake orbits are located above the plane \( A \) and below the plane \( B \). Those planes are defined by \( p_y(A) = \sqrt{2mE_F} + (eB_0/c)a_{\text{min}} \) and \( p_y(B) = -\sqrt{2mE_F} + (eB_0/c)a_{\text{max}} \), with \( a_{\text{min}} = \min\{a(x)\} \) and \( a_{\text{max}} = \max\{a(x)\} \) the extremal points of the vector potential \( a(x) \). Thus we can write

\[ \sigma_{xx}^{\text{sn}} = \sigma_A + \sigma_B, \]  

(8)

where the contribution from the snake orbits above the \( A \) plane is given by

\[ \sigma_A = \frac{e^2}{(2\pi\hbar)^2 L_x} \int_0^{L_x} dx \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \tau \]
\[ \times \delta \left\{ E_F - \frac{1}{2m} \left( p_x^2 + \left( p_y - \frac{eB_0}{c} a(x/l_0) \right)^2 \right) \right\} \]
\[ \times \left\{ p_y - \frac{eB_0}{c} a_{\text{min}} - \frac{2\pi m E_F}{\hbar^2} \right\}, \]
\[ = \frac{e^2 E_F \tau}{2\pi\hbar^2 L_x} \int_0^{L_x} dx \int_{-\varphi_0}^{\varphi_0} \sin^2(\varphi) d\varphi. \]  

(9)

Here the angular integration has to be performed over the white sections of the circle \( C \) in Fig. 4 or equivalently, along the bold arch shown in Fig. 4. The limiting angle is defined as \( \varphi_0 = \arccos(1 - \Delta p_y / \sqrt{2mE_F}) \), with \( \Delta p_y = (eB_0/c) \{a(x/l_0) - a_{\text{min}}\}. \) In the asymptotic case of small magnetic field modulations it becomes \( \varphi_0 = \sqrt{2\Delta p_y / \sqrt{2mE_F}}. \) In the latter case the integration in expression (8) leads to

\[ \sigma_A = \frac{e^2 E_F \tau}{2\pi\hbar^2 L_x} \int_0^{L_x} dx \int_{-\varphi_0}^{\varphi_0} \varphi^2 d\varphi = \sigma_{yy} c_A \left( \frac{B}{B_0} \right)^{3/2}, \]  

(10)

where \( B_0 = (2\pi c \sqrt{mE_F}) / (e\ell_0) \), and

\[ c_A = \frac{8 \cdot 2^{1/4} \sqrt{\pi}}{3} \int_0^1 dx \{a(x) - a_{\text{min}}\}^{3/2}. \]  

(11)

The integration over the snake orbits below the plane \( B \) leads to the same expression (10), except that now the coefficient \( c_A \) has to be replaced by

\[ c_B = \frac{8 \cdot 2^{1/4} \sqrt{\pi}}{3} \int_0^1 dx \{a_{\text{max}} - a(x)\}^{3/2}. \]  

(12)

Thus the resistance change due to the modulated magnetic field becomes \( \Delta R_{xx}/R_0 = (c_A + c_B)(B/B_0)^{3/2} \) where \( R_0 \) is the resistance in the absence of any magnetic field modulation.
As a special case let us consider a simple cosine periodic magnetic field modulation (with period $l_0$) $b(x) = \cos(2\pi x)$, which leads to $a(x) = \sin(2\pi x)/2\pi$. In this case the coefficients can be easily evaluated

$$c_0 = c_A + c_B = \frac{8}{3\pi} \cdot \frac{2^{3/4}}{9\pi^2} 
\approx 0.86.$$  \hspace{1cm} (13)

In the present classical ballistic situation an electron which passed through the first magnetic barrier will also pass through the other barriers. As a consequence the above result can also be applied to the one barrier situation. The periodic oscillating Fermi surface shown in Fig. 1 reduces now to a single step. The trajectories of $D$-type (i.e. the open orbits) contribute to the $\sigma_{xx}$ conductivity, but the snake orbits (trajectories of $A$-type) are replaced by trajectories which reflect from the barrier. The integral in expression (13) must now be evaluated over those reflected trajectories. When the barrier thickness $l_0 \ll L_x$, the integral $\int_{-L_x/2}^{L_x/2} dx (a(x)/l_0 - a_{\text{min}})^{3/2}$ becomes the sample length multiplied by the total vector potential increment over the barrier: $L_x (a_{\text{max}} - a_{\text{min}})^{3/2}$, where $a_{\text{max}} = a(L_x/2)$ and $a_{\text{min}} = a(-L_x/2)$.

A single magnetic field barrier in a 2DEG, created experimentally by parallel magnetization of magnetic strips placed on top of the 2DEG, can be represented by

$$B_z(x) = \frac{\mu_0 M}{4\pi} \ln \frac{x^2 + d^2}{x^2 + (d + D)^2}.$$ \hspace{1cm} (14)

The magnetic strip has a thickness $D$ with magnetization $M$ and is placed a distance $d$ from the 2D electron system. The vector potential is obtained by integrating (14) over $(-L_x/2, L_x/2)$ which gives $A_{\text{max}} - A_{\text{min}} = \mu_0 M D/2$ where use has been made of $L_x \gg d$, which is valid in typical experimental situations and $A = B l_0$. Finally, we obtain for the contribution of the reflected trajectories to the conductivity

$$- \frac{\sigma_{xx}^{\perp}}{\sigma_{yy}} = \frac{\Delta R_{xx}}{R_0} = \frac{2^{1/4}}{3\pi} \left( \frac{\epsilon \mu_0 M D}{\sqrt{\mu_0 M D} E_F} \right)^{3/2}.$$ \hspace{1cm} (15)

The main feature of the obtained magnitoresistance is its non analytical behavior $B^{3/2}$ for small magnetic field amplitudes. It is remarkable that it does not depend on the actual form of the modulating field, but it is determined by the density of snake orbits (or reflected trajectories) at the Fermi surface.

For non zero temperature, the previous effect will be suppressed by thermal fluctuations. To generalize our results to non zero temperature we have to replace any function $G(E_F)$ which depends on the Fermi energy $E_F$ by the corresponding average over the derivative of the Fermi function

$$G(\mu) = \frac{1}{T} \int_0^\infty d\varepsilon \exp \left( \frac{(\varepsilon - \mu)/k_B T}{\exp (\varepsilon - \mu)/k_B T + 1} \right)^2.$$ \hspace{1cm} (16)

Consequently, taking into account $\sigma_{yy}$ and the dependence of $B_0$ on $E_F$ we obtain for the snake orbit contribution to the conductivity

$$\sigma_{xx}^{\perp}(T) = \left( \frac{e^2 \tau c_0}{2\pi \hbar^2} \right) \left( \frac{B l_0}{c \sqrt{\mu_0 M D}} \right)^{3/2} \frac{1}{T} \int_0^\infty d\varepsilon \exp \left( \frac{(\varepsilon - \mu)/k_B T}{\exp (\varepsilon - \mu)/k_B T + 1} \right)^{1/4},$$ \hspace{1cm} (17)

where we assumed that the relaxation time does not depend on the energy. In the limit of small temperatures the integral can be evaluated analytically and we arrive at the final expression for the conductivity along the direction of the magnetic field modulation

$$\frac{\sigma_{xx}}{\sigma_0} = \left\{ 1 - c_0(T) \left( \frac{B}{B_0} \right)^{3/2} \right\},$$ \hspace{1cm} (18)

where

$$c_0(T) = c_0 \left\{ 1 - \frac{\pi^2}{32} \left( \frac{k_B T}{\mu} \right)^2 \right\}.$$ \hspace{1cm} (19)

Notice that temperature decreases the nonanalyticity coefficient $c_0$ (with about 30% when $k_B T = \mu$) but does not influence the power law dependence.

Tunneling through magnetic barriers in the ballistic regime was studied numerically in Refs. [4,5]. For weak barrier only electrons impinging on the magnetic barrier under an angle $\theta \approx \pi/2$ (i.e. $\cos(\theta) \approx (\pi/2 - \theta) \approx \phi$) are reflected and thus we obtain from Eq. (6) of Ref. [8] the conductance change due to tunneling through a weak magnetic barrier

$$- \Delta G \approx G_0 \int_0^{\phi_0} \phi d\phi,$$ \hspace{1cm} (20)

where $G_0 = e^2 n v_F L_0 / h^2$. Notice that the difference with the diffusive case is the power of the angle $\phi$ in the expression for the conductance/conductivity. In the case of ballistic transport the conductance is proportional to $v_x$, while for diffusive transport, see Eq. (5), we have $v_x^2$. This difference leads to a linear $B$-dependence in the ballistic regime

$$- \frac{\Delta G}{G_0} = \frac{\Delta R_{xx}}{R_0} = \frac{\Delta \rho_y}{\sqrt{2 m E_F}} = \frac{e \mu_0 M D}{2 c \sqrt{2 m E_F}}.$$ \hspace{1cm} (21)

The above expression is obtained for the single barrier [11] and agrees with Eq. (3) of Ref. [8].

Let us compare these results with the experiments of Refs. [4-6]. In Ref. [8] a Co strip of thickness $D = 90nm$ was placed a distance $d = 35nm$ from a 2DEG formed in a GaAs-heterostructure with $E_F = 15.7meV$. The
magnetization of the Co strip \( \mu_0 M \approx 9B \) with a saturation magnetization of \( 1.6T \). In this experiment one is in the diffusive regime and inserting these values in Eq. (15) we obtain \( \Delta R_{xx}/R_0 = 4.3B(T)^{3/2} \). The estimated zero field resistance \( \mu_0 M \) was \( R_0 = 2.35\Omega \) which results into \( \Delta R_{xx}(\Omega) = 10.1B(T)^{3/2} \) and agrees with the experimental \( \mu_0 M \) low magnetic field behavior \( \Delta R_{xx}(\Omega) = 9B(T)^{3/2} \).

In contrast, the experiments of Vančura et al. [10] are closer to the ballistic regime (they have been performed on shorter samples, i.e. \( L_y = 34\mu m \)). They placed rectangular Co dots of thickness \( D = 100\text{nm} \) above a 2DEG of width \( L_y = W = 20\text{nm} \) in a GaAs-heterostructure with \( E_F = 19.7\text{meV} \). Inserting these values in Eq. (19) we obtain \( \Delta R_{xx}/R_0 = 3.6B(T) \) where \( R_0 = 1/G_0 = 1.01\Omega \). This gives \( \Delta R_{xx}(\Omega) = 3.96B(T) \) which is a factor 2.2 smaller than the experimental result \( \Delta R_{xx}(\Omega) = 8.75B(T) \). At the saturation field of \( 1.5T \) we find theoretically \( \Delta R_{xx} = 5.94\Omega \) which compares with the experimental result \( \Delta R_{xx} = 3.5\Omega \) and which is a factor 1.7 smaller. Thus on the average we find a reasonable agreement between theory and experiment but it is clear that all details are not yet fully understood. The discrepancy may be due to the fact that the Co dots are not homogeneously magnetized. Nevertheless, the linear magnetic field dependence is nicely reproduced.

In conclusion we obtained the change in the magneto-resistance due to the presence of magnetic barriers with zero average magnetic field. We found that for diffusive transport the magneto-resistance increases as \( c(T)B^{3/2} \) with the amplitude of the magnetic barrier \( (B) \) where the coefficient \( c(T) \) decreases with increasing temperature. This result is different from the ballistic regime where the increase is linear in \( B \).

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FIG. 1. Fermi surface (thick solid curve) in phase space. Planes \( A \) and \( B \) are surfaces for the constant of motion \( p_y \) delimiting the open and closed orbits. Curve \( E(D) \) is an example of a closed (open) orbit.

FIG. 2. Contour for angular integration in expression (3). Line \( A \) corresponds to the Fermi surface intersection indicated by plane \( A \) in Fig. 1.