Geoid Deformation and Elevation Influence Caused by Building Loading

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Abstract  The paper studies the ground vertical deformation and the geoid undulation caused by loading of neighboring buildings, based on the loading tides theory. The influence on elevation is also considered. The results show that the ground vertical deformation and the geoid undulation both reach millimeter magnitude. Therefore, it is obvious that the building loading significantly affects the precise engineering surveying, and it must be seriously considered in application.

Keywords  loading tides; building loading; ground deformation; geoid undulation

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Introduction

Part of the tide in the solid earth is due to the earth’s yielding to the body forces exerted by the sun and moon. In addition to the tidal body force, the forces from the pressure of the harmonically varying surface mass act on the earth producing loading tides. The analysis of the changes in the geophysical field caused by the movement of the surface mass is the main problem of the loading tides theory. The variations of the geophysical property of the earth relating to ocean loading, atmosphere loading and Antarctic icecap loading are well researched. In contrast, the knowledge about artificial loading is much less; only the domestic scholars Zhang Kefei, Bian Shaofeng and Tao Benzao have made some studies in this field.

With the economy developing and the urban population increasing, there are more and more large and high buildings in big cities. The earth is elastic; however, any loading on its surface may lead it to elastic deformation. The deformation may cause deviation in original surveying achievements especially in leveling data. The paper discusses the computation of the geoid undulation, vertical deformation and the elevation influence caused by building loading orientated on the basis of loading tide theory using real data.

1 Deformation due to building loading

On the basis of the loading tide theory, the data of the building distribution provided by Shanghai Surveying and Mapping Institute were used to retrieve the ground and geoid deformation over Shanghai and its surrounding area.

1.1 Ground and geoid deformation caused by single building

Every building is considered as a particle for the convenience of solving the problem and the geometric center is taken as the centroid. As shown in Fig.1, in the spherical coordinate system, O is the earth’s centroid; P is the unknown point and Q is a building
with mass $m$. The distance between $P$ and $Q$ is $s$. Thus the gravitational potential of $P$ yielding to $Q$ is\(^1\)

$$\nabla = \frac{Gm}{|r_2 - r_1|} = \frac{Gm}{s} \quad (1)$$

Where $G$ is gravitational constant, $1/s$ is expanded in terms of Legendre polynomials\(^2, 3\),

$$\nabla = \frac{Gm}{s} \sum_{n=0}^{\infty} \frac{|r_1|^n}{|r_2|^{n+1}} P_n(cos \beta) \quad (2)$$

Since $P$ and $Q$ are both on the surface of the earth, $|r_1| = |r_2| = R$. $R$ is the average radius of the earth. With $g = \frac{GM}{R^2}$, we find:

$$\nabla = \frac{mgR}{M} \sum_{n=0}^{\infty} P_n(cos \beta) \quad (3)$$

![Fig.1 The position relationship among P, Q and O](image)

According to the loading tide theory, there is an additional potential caused by the deformation of the earth because of building loading. Therefore, the total gravitational potential of $P$ affected by $Q$ is

$$\nabla = \frac{mR}{M} g \sum_{n=0}^{\infty} P_n(cos \beta) (cos \theta) \quad (4)$$

Where $h'_n$ and $k'_n$ are love numbers, the solutions of the Elastic Equilibrium equation, which are used to describe the displacement of the earth by surface loading.

Based on Brons’ formula, the geoid undulation is expressed as\(^4\)

$$\delta h = \frac{\nabla}{g} = \frac{mR}{M} \sum_{n=0}^{\infty} P_n(cos \beta)(1 + k'_n - h'_n) \quad (5)$$

Similarly, the vertical displacement at $P$ caused by loading $Q$ is given by\(^2\)

$$u_r = \frac{Rm}{M} \sum_{n=0}^{\infty} k'_n P_n(cos \beta) \quad (6)$$

Where the upward is the positive direction.

### 1.2 Ground and geoid deformation caused by building complex

Since the vertical displacements are in the same direction, the total vertical deformation caused by building complex is given by the superposition of the

$$u_r = \sum u_{r_i} \quad (7)$$

The whole variation of potential response of a building complex is equivalent to the sum of the variation response of a single building. Thus, the geoid undulation caused by a building complex is expanded in

$$\delta h = \sum \delta h_i \quad (8)$$

### 1.3 The model for elevation changes

For the building complex loading, the point $P$ on the earth’s surface moves to $P'$ and the relevant elevation changes from $H$ to $H'$. Also, $u$ is the vertical displacement while $\delta h$ is the geoid undulation. The equivalent relation among these variables can be found as shown in Fig.2\(^3\),

$$H + \delta h = H' + u \quad (9)$$

Thus, the elevation change of the point $P$ is,

$$dh = H' - H = \delta h - u \quad (10)$$

![Fig.2 The elevation change of the observation point](image)

### 2 Data results

The attribute data of high-rise buildings used are provided by Shanghai Surveying and Mapping Institute including information of 6 284 buildings. The computations concerning the effect of building loading on ground and geoid over Shanghai and its surrounding areas were carried out.

As shown in Fig.3 and Fig.4, the biggest vertical displacement due to building loading may reach 1.2 mm, and the value decreases rapidly as the distance increases from the center of the building complex. Point (0, 0) is the origin of the Shanghai local reference plan frame, which is nearly the center of the city.

From Fig.5 and Fig.6, we find that the largest geoid undulation by building loading is 0.2 mm. The undulation diffuses in sine wave with its amplitude speedily weaken far from the center of the building complex.
and the deformation decreases quickly as it moves far from the center of the building complex.

3 Conclusion

The paper studied the influence/impact on vertical deformation and geoid undulation caused by urban building loading as well as the changes in elevation based on loading tide theory. Several results are achieved:

(1) The ground and geoid deformation caused by a single building as surface loading is discussed as well as the influence/impact responding to the building complex loading. The models of vertical deformation, geoid undulation and elevation changes are set up.

(2) By making use of the building data provided by Shanghai Surveying and Mapping Institute, the calculation of the deformation by building loading over Shanghai and its surrounding area were carried out. The results show that the biggest values of the vertical displacement, geoid undulation and elevation changes are $-1.3269\, \text{mm}$, $-0.25\, \text{mm}$ and $1.1052\, \text{mm}$, respectively.

(3) According to the figures, the deformations caused by building loading decrease as the distance increases from the center of the building complex. The geoid undulation attenuates in sine wave.

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