RADIATIVE CORRECTIONS TO THE TOP QUARK WIDTH

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ABSTRACT

Calculations of radiative corrections to the top quark width are reviewed. QCD effects are discussed for $t - \bar{t}$ systems produced in $e^+ e^-$ annihilation near the energy threshold.

1. Single Quark Decay

1.1. Total Rate

The top is the first heavy quark whose mass can be measured to better than 1% precision at a future $e^+ e^-$ collider [1, 2, 3]. Therefore, measurements of its width will not only test the standard model at the Born level, but also the QCD radiative corrections which are of order 10% [4]. This is in contrast to $b$ and $c$ quarks, where uncertainties in the masses and non-perturbative effects preclude this possibility.

The one loop electroweak corrections to the total rate have been also calculated. These corrections evaluated in the narrow width approximation [5, 6] turned out to be positive and rather small (1-2%). A reason for that is that the contribution from the Higgs field Yukawa coupling remains very small for realistic top quark masses [7]. The effect of finite $W$ width [8] is comparable in size to the narrow width electroweak correction but of the opposite sign for $m_t$ above 110 GeV.

Formulae for the QCD and electroweak corrections to the top quark width in the standard model are given in Ref.[8] as well as a more comprehensive list of papers on this subject. In Table 1 we summarize the results [8] for the corrections obtained from different approximations as well as for the total decay rate $\Gamma_t$ and
its narrow width Born approximation $\Gamma_{nw}^{(0)}$. We give the ratios of the corrections to the zeroth-order result $\Gamma_{nw}^{(0)}$, i.e. we define

$$\delta^{(i)} = \frac{\Gamma^{(i)}}{\Gamma_{nw}^{(0)}} - 1$$

(1)

where $i = 0, 1$ corresponds to the Born and the QCD corrected rate respectively, and the widths in the numerators include the effects of the W propagator. Analogously we define $\delta_{nw}^{(1)}$ which is given by the ratio of the QCD corrected and the Born widths, both evaluated in the narrow width approximation, $\delta_{nw}^{(1)}(0)$ for massless $b$ quark and $\delta_{ew}$ for the electroweak narrow width result [5].

| $m_t$ (GeV) | $\alpha_s(m_t)$ | $\Gamma_{nw}^{(0)}$ (GeV) | $\delta^{(0)}$ (%) | $\delta_{nw}^{(1)}(0)$ (%) | $\delta_{nw}^{(1)}$ (%) | $\delta^{(1)}$ (%) | $\Gamma^{(1)}$ (GeV) | $\delta_{ew}$ (%) | $\Gamma_t$ (GeV) |
|-----------|----------------|-----------------|----------------|-----------------|----------------|----------------|-----------------|----------------|--------------|
| 90.0      | .118           | .0234           | 11.69          | 7.88            | -3.81          | 6.56           | .0249           | 0.81           | .0251        |
| 100.0     | .116           | .0931           | 0.16           | -4.56           | -6.91          | -6.89          | .0867           | 1.04           | .0876        |
| 110.0     | .115           | .1955           | -1.44          | -6.81           | -7.83          | -9.22          | .1775           | 1.20           | .1796        |
| 120.0     | .113           | .3265           | -1.78          | -7.61           | -8.20          | -9.89          | .2942           | 1.33           | .2982        |
| 130.0     | .112           | .4849           | -1.82          | -7.97           | -8.37          | -10.08         | .4360           | 1.43           | .4423        |
| 140.0     | .111           | .6708           | -1.77          | -8.15           | -8.44          | -10.10         | .6031           | 1.51           | .6122        |
| 150.0     | .110           | .8852           | -1.69          | -8.25           | -8.47          | -10.05         | .7962           | 1.57           | .8087        |
| 160.0     | .109           | 1.130           | -1.60          | -8.31           | -8.49          | -9.99          | 1.017           | 1.62           | 1.033        |
| 170.0     | .108           | 1.405           | -1.52          | -8.34           | -8.49          | -9.91          | 1.266           | 1.67           | 1.287        |
| 180.0     | .107           | 1.714           | -1.45          | -8.35           | -8.48          | -9.84          | 1.546           | 1.70           | 1.572        |
| 190.0     | .106           | 2.059           | -1.39          | -8.36           | -8.47          | -9.77          | 1.857           | 1.73           | 1.890        |
| 200.0     | .106           | 2.440           | -1.33          | -8.36           | -8.46          | -9.70          | 2.203           | 1.76           | 2.242        |

Table 1: Top width as a function of top mass and the comparison of the different approximations.

A number of intrinsic uncertainties remains. It should be noted that the size of the electroweak corrections is comparable to the uncertainties from as yet uncalculated $O(\alpha_s^2)$ correction. The present uncertainty in $\alpha_s$ and the ignorance concerning the the second order QCD correction limit the accuracy of the prediction to about 1-2%. One has to take into account the experimental and theoretical errors in the determination of the top mass which may lead to uncertainties of similar magnitude, in particular for lower allowed values of $m_t$. At present the best place for a precise determination of $\Gamma_t$ is believed to be the threshold region for $t\bar{t}$ production in $e^+e^-$ annihilation. The most optimistic current estimate of the relative precision is 5% [9], so at present the theory seems to be in good shape. However, in the future when $e^+e^-$ 500 becomes a reality it will be mandatory to give the theory prediction including the $O(\alpha_s^2)$ contribution. Bound state effects in the threshold region, c.f. next section, may in principle enhance this correction. Needless to say such a calculation is necessary when one aims, as many people do, to use a precise measurement of the top width as a consistency check of the standard model.

‡M.J. thanks Andrzej Buras for a helpful discussion on this subject
In fact a number of calculations have been performed studying electroweak effects on the top width in theories extending the standard model. In particular it has been found \[10\] that the additional corrections from the extended Higgs sector of the minimal supersymmetric standard model are significantly smaller than 1%.

The situation changes drastically when the channel $t \rightarrow H^+ b$ is kinematically allowed. QCD corrections to the corresponding partial width have been recently calculated \[11\] as well as the electroweak ones \[12\].

1.2. Differential Distributions

The calculations of QCD corrections to the differential decay distributions have been reviewed in \[13\]. Recently a calculation \[14\] of the $W$ mass distribution in $t \rightarrow b \bar{f} f'$ including $O(\alpha_s)$ corrections has been repeated and a fast Monte Carlo generator for these decays has been written \[15\].

2. Width of $t - \bar{t}$ system near threshold

2.1. Motivation

From Table 1 and the present Fermilab lower limits on the top quark mass we conclude that the $t$ quark is a short–lived particle, and its width $\Gamma_t$ is of the order of several hundred MeV. As a consequence the cross section for $t\bar{t}$ pair production near energy threshold has a rather simple and smooth shape. In particular, it is likely that in $e^+ e^-$ annihilation only the $1S$ peak survives as a remnant of toponium resonances. Nevertheless, the excitation curve $\sigma(e^+ e^- \rightarrow t\bar{t})$ allows a precise determination of $m_t$ and the strong coupling constant $\alpha_s$ \[16, 17\]. The idea \[16, 17\] to use the Green function instead of summing over overlapping resonances has been also applied in calculations of differential cross sections, in particular for intrinsic momentum distributions of top quarks in $t\bar{t}$ systems \[18, 19\]. It has been argued \[20\] and demonstrated \[3\] that the combined measurements of the total and the differential cross sections in $e^+ e^- \rightarrow t\bar{t}$ offer a very promising method for a simultaneous determination of $m_t$ and $\alpha_s$. A possible problem is related to the fact that when produced near energy threshold $t$ and $\bar{t}$ cannot be considered as free particles. The binding energy and the ‘intrinsic’ kinetic energy of the $t - \bar{t}$ system tend to reduce the available phase space for the decay. Although the effect is only $O(\alpha_s^2)$ the suppression is large \[21\], especially for $m_t$ slightly above the threshold for real $W$ decay. Apparently in a high precision calculation one has to consider the width $\Gamma_{t-\bar{t}}(p)$ as a non-trivial function of the intrinsic momentum $p$. It is not surprising at all that when the phase space suppression is taken into account one finds \[18\] that the effects of the momentum dependent width are quite large and may show up in the annihilation cross section $\sigma(e^+ e^- \rightarrow t\bar{t})$. An immediate question is: to what extent do theoretical model assumptions spoil the precision of determination of $m_t$ and $\alpha_s$?

2.1. Models and Results

The width of the $t - \bar{t}$ system depends on the intrinsic momentum, of say $t$ quark, because both the matrix element and the phase space available for the decay products depend on it. The phase space effect tends to reduce the decay rate of
bound top quarks relative to free ones \cite{21}, and the effect is enhanced, because for short–lived particles the distribution of intrinsic momentum is broad. However, for the same reason the decays take place at short relative distances, where the wave functions of $b$ and $\bar{b}$ quarks originating from the decays are distorted (enhanced) by Coulomb attraction. Therefore, when calculating the amplitude of \( t \to bW \) transition, one should use Coulomb wave functions rather than plane waves for $b$ quarks. This effect clearly increases the rate. A third factor is the time dilatation: a top quark moving with velocity $v$ lives longer in the center–of–mass laboratory frame. While phase space reduction and time dilatation can be implemented in a straightforward way Coulomb enhancement cannot be easily taken into account. In principle one has to replace the plane wave functions for $b$ quarks by relativistic Coulomb functions when evaluating the amplitude for the $t \to bW$ transition. One may hope, however, that the following observation, valid for muons bound in nuclei \cite{23}, holds also for chromostatic attraction in $t – \bar{t}$ systems: the phase space suppression and the Coulomb enhancement nearly cancel each other. For light nuclei the result is well described by the time dilatation suppression alone.\footnote{For $\mu^-$ bound in a nucleus of charge $Z$ one obtains}

$$\Gamma = \Gamma_{\text{free}} \left[ 1 - 5(Z\alpha)^2 \right] \left[ 1 + 5(Z\alpha)^2 \right] \left[ 1 - (Z\alpha)^2 / 2 \right]$$

where the first correction factor comes from the phase space suppression, the second from the Coulomb enhancement, and the third one from time dilatation. Thus there is no first order correction to the total rate from the rescattering in the nucleus potential \cite{23}. A similar result has been recently obtained for the final state rescattering in $t – \bar{t}$ threshold region \cite{24}.

We show our results for $m_t = 120$ GeV. This is likely to be the most difficult case. For higher masses the effects of the momentum dependent width are smaller. For lower $m_t$ more information is available from peaks in the total cross section. We compare the total $\sigma(e^+e^- \to t\bar{t})$ (Figure 1) and differential (Figure 2) $d\sigma/dp$ cross sections. The dashed lines are obtained assuming constant momentum independent width. The dotted lines correspond to Model 1 where the momentum dependent width $\Gamma_{t-\bar{t}}(p)$ is significantly reduced for intermediate and large momenta $p$, mainly as a consequence of phase space suppression. The solid lines (Model 2) have been obtained assuming cancelation of phase space suppression and Coulomb enhancement. It can be seen that the results of this model are quite close to those obtained assuming constant width. It is noteworthy that even for Model 1 the $1S$ peak (threshold position) in $\sigma(e^+e^- \to t\bar{t})$ and the position of the maximum for $d\sigma/dp$ are not much affected by the momentum dependent width. Since the idea of \cite{3} is to combine these observables the resulting theoretical errors in determination of $m_t$ and $\alpha_s$ are quite small.
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