From Bipedal Walking to Quadrupedal Locomotion:
Full-Body Dynamics Decomposition for Rapid Gait Generation

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Abstract—This paper systematically decomposes a quadrupedal robot into bipeds to rapidly generate walking gaits and then recomposes these gaits to obtain quadrupedal locomotion. We begin by decomposing the full-order, nonlinear and hybrid dynamics of a three-dimensional quadrupedal robot, including its continuous and discrete dynamics, into two bipedal systems that subject to external forces. Using the hybrid zero dynamics (HZD) framework, gaits for these bipedal robots can be rapidly generated (on the order of seconds) along with corresponding controllers. The decomposition is achieved in such a way that the bipedal walking gaits and controllers can be composed to yield dynamic walking gaits for the original quadrupedal robot — the result is the rapid generation of dynamic quadrupedal gaits utilizing the full-order dynamics. This methodology is demonstrated through the rapid generation (3.96 seconds on average) of four stepping-in-place gaits and one diagonally symmetric ambling gait at 0.35 m/s on a quadrupedal robot — the Vision 60, with 36 state variables and 12 control inputs — both in simulation and through outdoor experiments. This suggested a new approach for fast quadrupedal trajectory planning using full-body dynamics, without the need for empirical model simplification, wherein methods from dynamic bipedal walking can be directly applied to quadrupeds.

I. INTRODUCTION

The control of quadrupedal robots has seen great experimental success in achieving locomotion that is robust and agile, dating back to the seminal work of Raibert [26]. These results have been achieved despite the fact that quadrupedal robots have more legs, degrees of freedom, and more complicated contact scenarios when compared to their bipedal counterparts. Bipedal robots (while seeing recent successes) still have yet to experimentally demonstrate the dynamic walking behaviors in real-world settings that quadrupeds are now displaying on multiple platforms. Yet, due to the lower degrees of freedom and, importantly, simpler contact interactions with the world, gait generation for bipedal robots based upon the full-order dynamics has a level of rigor not yet present in the quadrupedal locomotion literature (which primarily leverages heuristic and reduced-order models). It is this gap between bipedal and quadrupedal robots that this paper attempts to address: can the formal full-order gait generation methods for bipeds be translated to quadrupeds while preserving the positive aspects quadrupedal locomotion?

To achieve quadrupedal walking, controller design has widely adopted model-reduction techniques. For example, the massless leg assumption [6], [9], linear inverted pendulum model [20], [10] and assuming the 3D quadrupedal motion can be reduced to a planar motion [7], [11] are often utilized to mitigate the computational complexity of the quadrupedal dynamics so that online control techniques such as QP, MPC, LQR can be applied [8]. While these methods are effective in practice, it often requires some add-on layers of parameter tuning due to the gap between model and reality. This tuning is particularly prevalent for bigger and heavier robots, whose “ignored” physical properties may play a more significant role.

In the context of bipedal robots, due to their inherently unstable nature, detailed model and rigorous controller design have been long been developed. A specific methodology that leverages the full-order dynamics of the robot to make formal guarantees is Hybrid Zero Dynamics (HZD) [31], [5], [2] which has seen success experimentally for both walking and running [29], [23], [27]. A key to this success has been the recent developments in rapid HZD gait generation using collocation methods [17], with the ability to generate gaits for high-dimensional robots in some cases in seconds [19]. Recently, the HZD framework was translated to quadrupedal robots both for gait generation and controller design [22], [3]. Although the end result was the ability generate walking, ambling and trotting for the full-order model, the high dimensional and complex contacts of the system made the gait generation complex with the fast gait being generated in 43 seconds and hours of post-processing needed to guarantee stability. The goal of this paper is, therefore, to translate the positive aspects of HZD gait generation to quadrupeds while mitigating the aforementioned drawbacks.
Pioneers in robotics have discerned the correlation between bipedal and quadrupedal locomotion. For example, [26], [25] applied several bipedal gaits on quadrupedal robots; [11], [12] provided stability analysis for a planar abstract hopping robot. The ZMP condition of two bipeds was used to synthesis stability criteria for a quadruped in [21]. However, these results rely on model reduction methods —the hip, upper and lower links. Utilizing the floating base convention [15], the configuration space is chosen as \( q = (q_0^T, \theta_1^T, \theta_2^T, \theta_3^T)^T \in \mathbb{Q} \subseteq \mathbb{R}^{18} \), where \( q_0 \in \mathbb{R}^{3 \times SO(3)} \) represents the Cartesian position and orientation of the body linkage, and \( \theta_i \in \mathbb{R}^3 \) represents the three joints: hip roll, hip pitch and knee on the leg \( i \in \{0, 1, 2, 3\} \). All of these leg joints are actuated, with torque inputs \( u_i \in \mathbb{R}^3 \). This yields the system’s total DOF \( n = 18 \) and control inputs \( u = (u_0^T, u_1^T, u_2^T, u_3^T)^T \in \mathbb{R}^m \), \( m = 12 \). Further, we can define the state space \( \mathcal{X} = T\mathbb{Q} \subseteq \mathbb{R}^{2n} \) with the state vector \( x = (q^T, \dot{q}^T)^T \), where \( T\mathbb{Q} \) is the tangent bundle of the configuration space \( \mathbb{Q} \).

1) State space and inputs: The robot begin considered —the Vision 60 V3.2 in Fig. 2—is composed of 13 links: a body link and 4 limb links, each of which has three sublinks—the hip, upper and lower links. The continuous-time dynamics \( \dot{q} \) is the tangent bundle of the configuration space \( \mathbb{Q} \).

\[
\begin{align*}
D(q) \ddot{q} + H(q, \dot{q}) &= Bu + J_1^T(q) \lambda_1 + J_2^T(q) \lambda_2 \\
J_1(q) \ddot{q} + \dot{J}_1(q, \dot{q}) \dot{\dot{q}} &= 0 \\
J_2(q) \ddot{q} + \dot{J}_2(q, \dot{q}) \dot{\dot{q}} &= 0
\end{align*}
\]

with the domain \( D \) \( \equiv \{ x \in \mathcal{X} : \dot{h}_1(q, \dot{q}) = \dot{\dot{h}}_2(q, \dot{q}) = 0, h_2 (q) = h_2 (q) = 0 \} \). In this formulation, we utilize the following notation: \( D(q) \in \mathbb{R}^{n \times n} \) is the inertia-mass matrix; \( H(q, \dot{q}) \in \mathbb{R}^n \) contains Coriolis forces and gravity terms; \( h_1(q), h_2(q) \in \mathbb{R}^3 \) are the Cartesian positions of toe1 and toe2, their Jacobians are \( J \) \( = \partial h/\partial \theta \); \( h_2(q), h_2(q) \) are these toes’ height; \( \lambda_1, \lambda_2 \) are the ground reaction force on toe1 and toe2; \( B \in \mathbb{R}^{n \times m} \) is the actuation matrix.

In this section, we decompose the full body dynamics and control of quadrupedal robots into two identical bipedal systems. The nonlinear model of quadrupedal locomotion is a hybrid dynamical system, which is an alternating sequence of continuous- and discrete-time dynamics. The order of the sequence is dictated by contact events.

A. Quadrupedal Dynamics

The full-body dynamics of quadrupedal robots have been detailed in [22] and will be briefly revisited here to setup the problem properly. Note that in this section, we only focus on the most popular quadrupedal robotic behavior — the diagonally supporting amble (see Fig. 3).

Fig. 2: On the left is the robot in MuJoCo, and on the right is the illustration of the configuration coordinates for the robot. The leg indices \( l \) are shown on the vertices of the body link. Each leg has three actuated joints and equipped with a point contact toe.

Fig. 3: The cyclic directed graph for the single-domain hybrid dynamics of the diagonally supporting amble behavior.
Essentially, we use a set of differential algebraic equations (DAEs) to describe the dynamics of the quadrupedal robot that is subject to two holonomic constraints on toe1 and toe2.

3) The discrete dynamics: On the boundary of domain $D$ we impose discrete-time dynamics to encode the perfectly inelastic impact dynamics as toe0 and toe3 impact the ground (and suppressing the dependence of $D$ and $J_r$ on $q$ and $\dot{q}$):

$$\begin{align*}
\dot{D}(q^+ - q^-) &= J_0^T \Lambda_0 + J_3^T \Lambda_3 \\
J_0\dot{q}^+ &= 0 \\
J_3\dot{q}^+ &= 0
\end{align*}$$

(2)

by using conservation of momentum while satisfying the next domain’s holonomic constraints, which is that toe0 and toe3 stay on the ground after the impact event. We denoted $\dot{q}^+$ and $\dot{q}^-$ as the pre- and post-impact velocity terms, $\Lambda_0, \Lambda_3 \in \mathbb{R}^3$ are the impulses exerted on toe0 and toe3.

B. Continuous dynamics decomposition

We now decompose the quadrupedal full body dynamics into two bipedal robots. First, as shown in Fig. 1 the open-loop dynamics can be equivalently written as:

$$\text{OL-Dyn} \triangleq \begin{cases} 
D_1 \ddot{q}_1 + H_1 = J_1^T \lambda_2 + B_1 u_t - J_1^T \lambda_c \\
D_2 \ddot{q}_2 + H_2 = 0 \\
J_2 \dot{q}_2 + J_1 \dot{q}_1 = 0 \\
\dot{q}_b = \dot{q}_b = 0
\end{cases}$$

(3)

wherein we utilized the following notation: $q_b, \dot{q}_b \in \mathbb{R}^3 \times SO(3)$ are the coordinates for the body linkages of the front and rear bipeds (see Fig. 1): $q_t = (q_{b1}, \dot{q}_{b1}, \dot{q}_c)^T$ and $q_r = (q_{b2}, \dot{q}_{b2}, \dot{q}_c)^T$ are the configuration coordinates for the front and rear bipeds; $D_1(q_t), D_1(q_r) \in \mathbb{R}^{12 \times 12}$ are the inertia-mass matrices of the front and rear biped robots; The Jacobians $J_{1z} = \partial h_{1z} / \partial \dot{q}_t, J_{2z} = \partial h_{2z} / \partial \dot{q}_t$ with the Cartesian positions of toe2 $-h_{1z}(q_t)$ and toe1 $-h_{2z}(q_t)$; The Jacobian matrix for the connection constraint (7) is $J_c = \partial (q_t - q_b) / \partial \dot{q}_t; u_t = (u_{t1}, u_{t2})^T$ and $u_c = (u_{c1}, u_{c2})^T$. Note that the Cartesian position of toe2 only depends on $q_t$, which is due to the floating base coordinate convention.

Proposition 1. The dynamical system (OL-Dyn) is equivalent to the system (1).

Proof. We can write (4) and (5) as:

$$\begin{align*}
[D_1 \ddot{q}_1 + H_1 = J_1^T \lambda_2 + B_1 u_t - J_1^T \lambda_c] &+ [D_2 \ddot{q}_2 + H_2 = 0] = 0 \\
[D_1 \ddot{q}_1 + H_1 = J_1^T \lambda_2 + B_1 u_t - J_1^T \lambda_c] &+ [D_2 \ddot{q}_2 + H_2 = 0] = 0 \\
[D_1 \ddot{q}_1 + H_1 = J_1^T \lambda_2 + B_1 u_t - J_1^T \lambda_c] &+ [D_2 \ddot{q}_2 + H_2 = 0] = 0 \\
[D_1 \ddot{q}_1 + H_1 = J_1^T \lambda_2 + B_1 u_t - J_1^T \lambda_c] &+ [D_2 \ddot{q}_2 + H_2 = 0] = 0
\end{align*}$$

where each entry has a proper dimension to make the equations consistent. Expanding them yields:

$$\begin{align*}
\begin{bmatrix} D_{b1} & D_{b2} & \ddot{q}_b \\
D_{b0} & D_0 & 0 \\
D_{b3} & 0 & D_3 \\
\end{bmatrix} \begin{bmatrix} \ddot{q}_b \\
q_0 \\
\dot{q}_3 \\
\end{bmatrix} + \begin{bmatrix} H_{b1} \\
H_0 \\
H_3 \\
\end{bmatrix} = B_1 u_t + J_1^T \lambda_2 - J_1^T \lambda_c \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{align*}$$

(8)

Combining these two equations, and using the fact that $q_{b1} - q_{b2} \equiv 0$ (holonomic constraint) yields the dynamics given in (1).

Note that (7) can be equivalently replaced by summing the first 6 equations of (3) and (5):

$$(D_1 + D_2)\ddot{q}_b + \sum_{j=0}^{3} D_{b,j}\ddot{q}_j + H_b + H_c = J_{1b}^T \lambda_1 + J_{2b}^T \lambda_2$$

Denoted by: $h_{c}(q_t, \dot{q}_t, \dot{q}_c, \lambda_2, q_t, \dot{q}_t, \dot{q}_c, \lambda_1) = 0$

(9)

which is a dynamical system with feedforward terms $(q_t, \dot{q}_t, \dot{q}_c, \lambda_1)$. The dynamics of the rear biped (r), can be similarly obtained using (5), (6), and (8). We have thus decomposed the dynamics of a quadrupedal robot (1) to two bipedal dynamical systems (f) and (r), as shown in Fig. 4.

**Example 1.** The idea of dynamics decomposition can be illustrated using a simple example in Fig. 5. Note that each subsystem is not subject to any constraints. The half-body dynamics of a single cart with an inverted pendulum are:

$$\begin{align*}
\begin{bmatrix} M + m & -ml \cos \theta_1 \\
-ml \cos \theta_1 & ml^2 \\
\end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\
\ddot{\theta}_1 \\
\end{bmatrix} + \begin{bmatrix} ml \dot{\theta}_1 \sin \theta_1 \\
-ml \dot{\theta}_1 \sin \theta_1 \\
\end{bmatrix} = \begin{bmatrix} \pm \lambda_1 \\
\pm \lambda_1 \\
\end{bmatrix}
\end{align*}$$

where $i \in \{f, r\}$. The sign for $\lambda_c$ is negative for the front system and positive for the rear system. We can use a joint-space PD controller $u(t_1, \dot{\theta}_1, u(t_1, \dot{\theta}_1)$ to achieve a desired behavior such that the two invert pendulums vibrate symmetrically, i.e., $\dot{\theta}_1 = -\dot{\theta}_1$. Then using (8) we have

$X(t) = Y(t)$ for all $t$ they are defined on.
We define outputs (virtual constraints) for the biped stories representing quadrupedal behaviors. The algorithm to word, when the two invert pendulums move symmetrically, \( \lambda \) decomposed bipeds subject to control as follows:

\[
\begin{align*}
D_t \ddot{q}_t + H_t &= J_t^T \lambda_2 + B_t u_t^r - J_t^c \lambda_c \\
J_t \ddot{q}_t + J_t \dot{q}_t &= 0 \\
\dot{y}_t &= k_1 \dot{y}_t + k_2 y_t \\
h_c(q_t, \dot{q}_t, \ddot{q}_t, \lambda_2, q_1, \dot{q}_1, \dot{q}_1, \lambda_1) &= 0 \\
\end{align*}
\]

(10)

In particular, the output dynamics implemented here is an implicit version of input-output feedback linearization, details of this implementation can be found in [16, 4].

However, to design trajectories and determine the control inputs for a biped such as (f), we need to know all of the feedforward terms \((\dot{q}_t, \ddot{q}_t, \dot{q}_1, \lambda_1)\) for the time \(t \in [0, T]\), with \(T\) the time duration of a step. Therefore, the following equation is used to encode the desired correlation between the front and rear bipeds:

\[
B_t(t) = MB_t(t) + b.
\]

(12)

Further, we consider a widely used motion of quadrupedal robots — the diagonally symmetric gait, where the joints of leg3 is a mirror of leg0 and those of leg1 is a mirror of leg2. In this case we have \(M\) a diagonal matrix whose diagonal entries are \(-1, 1, 1, -1, 1, 1\) and \(b = 0\). Note that one can specify other motions as well, for example, a torso-leaned motion can be achieved by offsetting \(b\). Since the connection constraint \(q_0 = q_b\) is always satisfied by mechanical wrenches \(\lambda_c\), then on the zero dynamics (2D) surface [28], i.e., \(y(t) = y_0(q_t) - B_t(t) \equiv 0\), we have the following correlation between the two bipeds:

\[
q_t \equiv Aq_{\bar{t}} + b, \text{ where } A = \begin{bmatrix} I & \end{bmatrix}.
\]

(13)

Additionally, to determine \(\lambda_1\) of the biped (r), we also need to impose the constraint \(\bar{o}\) to the system in (10). Then subtract the dynamics of biped (f) from \(\ddot{q}_t\) to have the closed-loop dynamics of the front biped subject to the connection wrench \(\lambda_c\) as:

\[
\text{CL-Dyn-f} \triangleq \begin{cases}
D_t \ddot{q}_t + H_t = J_t^c \lambda_c + B_t u_t^r - J_t^c \lambda_c \\
\dot{y}_t = k_1 \dot{y}_t + k_2 y_t \\
\end{cases}
\]

(14)

\[
J_t \ddot{q}_t + J_t \dot{q}_t = 0 \\
h_c(q_t, \dot{q}_t, \ddot{q}_t, \lambda_2, q_1, \dot{q}_1, \dot{q}_1, \lambda_1) = 0 \\
\end{cases}
\]

(15)

The proof is similar to that of continuous dynamics decomposition, thus omitted. On the ZD surface, where both of the bipedal systems (f) and (r) have zero tracking errors \((q_2 - B_t(t) \equiv 0)\), we can similarly expand the impact dynamics (2) as:

\[
\begin{align*}
D_t \ddot{q}_t + H_t &= J_t^c \lambda_c \\
J_t \ddot{q}_t &= 0 \\
\dot{y}_t &= k_1 \dot{y}_t + k_2 y_t \\
h_c(q_t, \dot{q}_t, \ddot{q}_t, \lambda_2, q_1, \dot{q}_1, \dot{q}_1, \lambda_1) = 0 \\
\end{align*}
\]

(16)

\[
\begin{align*}
D_t \ddot{q}_t + H_t &= J_t^c \lambda_c \\
J_t \ddot{q}_t &= 0 \\
\dot{y}_t &= k_1 \dot{y}_t + k_2 y_t \\
h_c(q_t, \dot{q}_t, \ddot{q}_t, \lambda_2, q_1, \dot{q}_1, \dot{q}_1, \lambda_1) = 0 \\
\end{align*}
\]

(17)

(18)

D. Impact dynamics of the decomposed system

With the continuous dynamics written as (OL-Dyn), we can similarly expand the impact dynamics [2] as:

\[
\Delta \triangleq \begin{cases}
D_t \ddot{q}_t^\prime - \ddot{q}_t^\prime &= J_t^c \lambda_c \\
\dot{y}_t^\prime &= k_1 \dot{y}_t^\prime + k_2 y_t^\prime \\
h_c(q_t^\prime, \dot{q}_t^\prime, \ddot{q}_t^\prime, \lambda_2, q_1, \dot{q}_1, \dot{q}_1, \lambda_1) = 0 \\
\end{cases}
\]

(19)

Note that although system (20) is an overdetermined system, removing the redundant equations is not desirable in practice, as it may result in an ill-posed problem. This issue can be more severe for robots with light legs. Moreover, the implicit optimization method in the latter section can solve this system accurately and efficiently.

III. Decomposition-based Optimization

Past work has investigated the formal analysis and controller design for the full-body dynamics of quadrupeds [3], [22]. Although we were able to produce trajectories that are stable solutions to the closed-loop multi-domain dynamics for walking, ambling, and trotting, the computational complexity makes realizing these methods difficult in practice: it typically takes minutes to generate a trajectory and hours to post-process the parameters to guarantee dynamic stability. However, by using the dynamics decomposition method, we can produce bipedal walking gaits that can be composed to obtain quadrupedal locomotion while maintaining the
efficiency of computing the lower-dimensional dynamics of bipedal robots. In this section, we detail this process using nonlinear programming (NLP).

Given the constrained bipedal dynamics (CL-Dyn-f) and the impact dynamics (20), the target is to find a solution to the closed-loop dynamical system efficiently. The nonlinear program is formulated as:

\[
\min \sum_{j=1}^{2N+1} \|q_{t_j}\|^2 \quad (21)
\]

s.t. C1. dynamics (CL-Dyn-f) \[ j = 1, 3, \ldots, 2N + 1 \]
C2. collocation constraints \[ j = 2, 4, \ldots, 2N \]
C3. impact dynamics (20) \[ j = 2N + 1 \]
C4. periodic continuity \[ j = 1, 2N + 1 \]
C5. physical feasibility \[ j = 1, 2, \ldots, 2N + 1 \]

with the following notation: \(2N+1=11\) is the total number of collocation grids; the decision variable is defined as

\[
Z = (\alpha, t^i, q^i, q^i_t, u^i, \lambda_1^i, \lambda_2^i, \lambda_3^i, \lambda_4^i, \lambda_5^i, \lambda_6^i, \lambda_7^i, \lambda_8^i)
\]

and \(\alpha \in \mathbb{R}^{36}\) are the coefficients for the Bezier polynomial that defines the desired trajectory \(B(t)\); \(\square^i\) is the corresponding quantities at time \(t^i\) with \(t^{2N+1} = T\). In short, the cost function is to minimize the body’s vibration rate to achieve a more static torso movement. The constraints C1-C3 solve the hybrid dynamics of bipedal robots subject to external forces. Details regarding the numerical optimization can be found in [16]. In particular, the Hermite-Simpson collocation formulation can be found in equations (C1,C2) in [16]. Here, the periodic continuity constraint C4 enforces state continuity through an edge, i.e., the post-impact states \(q^-, q^+\), are equivalent to the initial states \(q^1, q^2\). Therefore, the resultant trajectory is a periodic solution to the bipedal dynamics. C5 imposed some feasibility conditions on the dynamics, including torque limits \(\|u_i\|_\infty \leq 50\), joint feasible space \((q_i, q^i_t) \in \mathcal{X}\), foot clearance and the friction pyramid conditions. Note that we posed these constraints conservatively to reduce the difficulties implementing the optimized trajectories in experiments.

To solve the optimization problem (21) efficiently, we used a toolbox FROST [17], [18], which parses a hybrid control problem as a NLP based on direct collocation methods, in particular, Hermite-Simpson collocation. It is worthwhile to mention that a critical reason for the high efficiency of FROST comes from the implicit formulation of the dynamics. Matrix inversion is avoided in every step due to its computational complexity: \(O(n^3)\), with \(n\) the dimension of a matrix. Inspired by this, we remark the dynamics decomposition method proposed in this paper also only used differential algebra equations (DAEs) instead of ordinary differential equations (ODEs), which requires matrix inversion both for the inertia matrix and the closed-loop controller formulation.

Once the optimization (21) converged to a set of parameters \(\alpha\) for the four bipedal robots’ walking gait \(B_i(t)\), we can use (12) to obtain the trajectory for the rear biped \(B_i(t)\) and then recompose them to get the parameters for the quadrupedal locomotion. For validation, we simulated an ambling step of the quadrupedal dynamics using the composed bipedal gait. As shown in Fig. 5 we have the joint angles and constraint wrench (ground reaction force) on toe1 \(\lambda_{z,2}\), and toe2 \(\lambda_{z,2}\) of the quadruped matched with those corresponding external force to the bipedal dynamics.

We now take advantage of the efficient, decomposition-based optimization to generate several walking patterns for the front biped, then recompose them to obtain quadrupedal stepping-in-place behaviors. By adjusting the constraint bounds in the NLP (21), such as the upper and lower bound \(T_{\text{max}}, T_{\text{min}}\) of time duration \(T\), or the bounds of the nonstance foot height \(\delta_{\min} \leq h_{\text{f,z}}(q_f) \leq \delta_{\max}\), we can obtain gaits with different stepping frequency and foot clearance. Further, we remove the constraint that the nonstance foot lands at the origin to generate a diagonally ambling gait with a speed of 0.35 m/s. See Fig. 6 for the tiles of these gaits. The result of the methods presented is the ability to generate quadrupedal gaits rapidly. We benchmark the performance by considering computing speed for each of the quadrupedal locomotion patterns generated, as is shown in Table I. In summary, with the objective tolerance and equality constraint tolerance configured as \(10^{-8}\) and \(10^{-5}\) respectively, we have the average computation time as 3.96 second, and time per iteration averages 0.039 second. In comparison with the regular full-model based optimization methods from [22], the decomposition-based optimization is an order of magnitude faster.

IV. SIMULATION AND EXPERIMENTS

One of the motivations for realizing rapid gait generation using the full-body dynamics of the quadruped, i.e., without computing performance of gait generation. This is performed on a Linux machine with an i7-6820HQ CPU @2.70 GHz and 16 GB RAM.

| Behaviors | gait1 | gait2 | gait3 | gait4 | amble |
|-----------|-------|-------|-------|-------|-------|
| frequency (Hz) | 2.5   | 2.3   | 2.2   | 2.6   | 2.83  |
| clearance (cm)  | 11    | 12    | 15    | 13    | 13    |
| # of iterations | 96    | 122   | 98    | 46    | 147   |
| time of IPOPT (s) | 1.60  | 2.10  | 1.02  | 0.81  | 2.59  |
| time of evaluation (s) | 1.94  | 3.24  | 2.10  | 0.94  | 2.36  |
| NLP time(s)   | 3.54  | 5.34  | 3.72  | 1.75  | 5.35  |
model simplifications, is to allow for the seamless translation of gaits from theoretical simulation to hardware. In this context, we first validated the dynamic stability of the gaits produced by the decomposition-based optimization problem using a third party physics engine — MuJoCo. These gaits include a diagonally symmetric ambling and four stepping in place behaviors. Then we conducted experiments, walking on a outdoor tennis court, using the same control law as that in simulation in outdoor environments. In particular, we used a PD approximation of the input-output linearizing controllers to track the time-based trajectories given by the optimization,

\[ u(q_a, \dot{q}_a, t) = -k_1(y_a - \bar{B}(t)) - k_2(y_a - B(t)). \]  

Note that the event functions (switching detection) are also given by the optimized trajectories, meaning the walking controller switches to the next step when \( t = T \). We report that for all given optimal gaits, the PD gains are picked as \( k_p = 230, 230, 300, k_d = 5 \) for the hip roll, hip pitch, knee joints, respectively. The averaged absolute joint torque inputs are logged in Table II, all of which are well within the hardware limitations. The tracking of the ambling gait in simulation and experiment are shown in Fig. 7.

The result is that the Vision 60 quadruped can step and amble in an outdoor tennis court in a sustained fashion. Importantly, this is without any add-on heuristics and achieved by only uploading different gait parameters \( \alpha \) for each experiment (obtained from the different NLP optimization problems with different constraints). See [1] for the video of Vision 60 in both simulation and experiments. As demonstrated in the video, we remark that the proposed method has rendered a good level of robustness against rough terrain with slopes, wet dirt and surface roots. Hence periodic stability has been obtained in both simulation and experiment. Fig. 6 shows a side to side comparison of the simulated amble and experimental data (in cyan) in the form of phase portrait using 18 seconds’ data. HR: hip roll joint; HP: hip pitch joint; K: knee joint.

TABLE II: Average torque inputs in experiments and simulations.

| Experiments | gait1 | gait2 | gait3 | gait4 | amble |
|-------------|-------|-------|-------|-------|-------|
| MuJoCo(N-m)| 5.04  | 4.83  | 4.16  | 5.14  | 7.11  |
| MuJoCo(N-m)| 3.65  | 5.24  | 5.26  | 3.77  | 6.28  |
| MuJoCo(N-m)| 16.45 | 16.30 | 16.86 | 16.95 | 18.36 |

V. CONCLUSION

In this paper, we decomposed the full-body dynamics of a quadrupedal robot — the Vision 60 with 18 DOF and 12 inputs — into two lower-dimensional bipedal systems that are subject to external forces. We are then able to solve the constrained dynamics of these bipeds quickly through the HZD optimization method, FROST, wherein the gaits can be recomposed to achieve locomotion on the original quadruped. The result is the ability to generate walking gaits rapidly. Specifically, by changing a constraint, we can produce different bipedal and, thus, quadrupedal walking behaviors from stepping to ambling in 3.9 seconds on average. Furthermore, the implementation in simulation and experiments used a single simple controller, without the need for additional heuristics.

Without sacrificing the model fidelity of the full-body dynamics of the quadruped, the ability to exactly decompose these dynamics into equivalent bipedal robots makes it possible to rapidly generate gaits that leverage the full-order dynamics of the quadruped. Importantly, this allows for the rapid iteration of different gaits necessary for bringing quadrupeds into real-world environments. Moreover, the fact that these gaits can be generated on the order of seconds suggests that with code optimization on-board and real-time gait generation may be possible soon. The goal is to ultimately use this method to realize a variety of different dynamic locomotion behaviors on quadrupedal robots.
