The masses of the mesons and baryons.
Part III.  The size of the particles

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The size of the stable elementary particles is investigated with the standing wave model. The particle size follows from the magnitude of the radiation pressure. It is shown that the outward directed radiation pressure is balanced by the inward directed elastic force per unit area in the cubic nuclear lattice, provided that the sidelength of the lattice is \(10^{-13}\) cm, which agrees with the measured radius of the proton \(r = 0.8 \cdot 10^{-13}\) cm, within the uncertainty of the parameters.

1 Introduction

In a previous article [1] we have shown that it follows from the well-known decays and masses of the so-called stable elementary particles that the spectrum of the stable particles consists of a \(\gamma\)-branch and a neutrino branch, and that the masses of the \(\gamma\)-branch are integer multiples of the mass of the \(\pi^0\) meson, with an average deviation of 1.0059. In a following paper [2] we have explained the integer multiple rule with the eigenfrequencies of plane, standing, electromagnetic waves in a cubic nuclear lattice. We have found that the masses of the \(\gamma\)-branch, the \(\pi^0, \eta, \eta', \Lambda, \Sigma^0, \Xi^0, \Omega^-, \Lambda^+_c, \Sigma^0_c, \Xi^0_c\) and \(\Omega^{0}_{c}\) particles, must be integer multiples of the mass of the \(\pi^0\) meson. The ratios of the masses of these particles are, in the standing wave model, independent of the size of the lattice. However, when we determine the mass of the \(\pi^0\) meson in absolute terms, we need the number of the lattice points, which follows from the size of the lattice. In [2] we have used for the size of the lattice the known mean square radius of the proton, which is of order \(10^{-13}\) cm. We have, thereby, implied that the size of the \(\pi^0\) meson is the same as the size of the proton. In the following we will justify this assumption, showing that the size of the particles is limited to a particular value by the radiation pressure.

2 Radiation pressure and Young’s modulus

Standing electromagnetic waves in a cubic lattice with free boundaries exert an outward directed force on the ends of the lattice, the radiation pressure, which is caused by the reflection of the waves at the lattice end. The lattice will break up, at the latest, when the radiation pressure is equal to the force per unit area by
which one layer of the lattice attracts an adjacent layer. The radiation pressure
is determined as follows: The force caused by the reflection of one wave from a
boundary is given by the momentum change. It is \( \Delta p = 2 \frac{h}{\lambda} \) and \( \Delta t = \frac{1}{\nu} \),
where \( p \) stands as usual for momentum, \( h \) for Planck's quantum, \( \lambda \) for the
wavelength, \( t \) for the time, and \( c \) for the velocity of light. The force caused by
the reflection of all permissible waves with the wavelengths \( \lambda = \frac{2L}{n} \), where
\( L \) is the side-length of the lattice, and \( n \) an integer number ranging from 1 to
about 1000, is given by

\[
\sum_n \frac{\Delta p_n}{\Delta t_n} = \sum_n \frac{2h}{\lambda_n} \nu_n = \sum_n \frac{n E_n}{L},
\]
which, when applied to the entire lattice end, is

\[
F = m \frac{c^2}{L},
\]
where \( m \) is the mass of the particle. The force per unit area, or pressure \( P \) is

\[
P = m \frac{c^2}{L^3} = \rho c^2.
\]

The energy in the mass of the \( \pi^0 \) meson is 134.9 MeV = \( 2.16 \cdot 10^{-4} \) erg.
Assuming that \( L = 10^{-13} \) cm it follows that the radiation pressure acting on an
end of the cubic lattice is

\[
P = 2.16 \cdot 10^{35} \text{ dyn/cm}^2.
\]

The outward directed radiation pressure can at most be equal to the inward
acting elastic force at the lattice end. The elastic force per unit area is char-
acterized by Young’s modulus \( Y \). We have previously determined the value of
Young’s modulus of a cubic lattice held together by the weak nuclear force in
[3], Eq.(21) therein. We found

\[
Y = 2.06 \cdot 10^{35} \text{ dyn/cm}^2.
\]

Within the uncertainties of the parameters, in particular of the breaking
point of the lattice, the radiation pressure \( P \) is balanced by the elastic force
of the lattice, provided that the side-length of the lattice is \( L = 10^{-13} \) cm. \( L \)
determined from either Eq.(4) or Eq.(5) differs by only 1.6%. We will use in
the following \( L = 10^{-13} \) cm. If \( |Y| = |P| \) we can write Eq.(3) as the well-known
formula for the velocity of elastic waves in a rigid body

\[
v = \sqrt{\frac{Y}{\rho}}.
\]

The velocity in the case of the nuclear lattice is equal to the velocity of light,
which cannot be exceeded, nor can the velocity be smaller than \( c \), because the
waves making up the particles of the \( \gamma \)-branch are electromagnetic.

The considerations above apply to the lowest mode of the lattice oscillations,
that is to the \( \pi^0 \) meson. The higher modes of the lattice oscillations correspond
to the $\eta$ meson, $\Lambda$ baryon etc, whose masses are, within 3%, integer multiples of the mass of the $\pi^0$ meson, as discussed in [1], in particular Table 1 therein. The outward directed radiation pressure of the higher modes must be equal to the value of the radiation pressure $P$ of the $\pi^0$ meson times an integer number, representing the ratio of the mass of the particular $\gamma$-branch particle divided by $m(\pi^0)$, because, as we have learned in [2], the higher modes of the standing waves in the nuclear lattice differ from the waves in the $\pi^0$ meson only in the number of the waves, not in the frequencies. As the number of waves increases, so does the radiation pressure, provided that the sidelength of the lattice is the same for all particles of the $\gamma$-branch. But since the density $\rho = m / L^3$ of the particles with higher modes increases if $L = \text{const}$, the velocity $v$ would, according to Eq.(6), decrease, if Young’s modulus $Y$ of the particles with the higher modes remains constant. However, since the velocity of the standing waves in a cubic nuclear lattice is the constant velocity of light, Eq.(6) requires that Young’s modulus increases by the same amount as the density of the particles with the higher modes. Young’s modulus of the particles does indeed increase this way, because as the number of waves increases, so does the number of strings which hold the lattice together, each wave corresponding to a string. In other words, in a higher mode the density $\rho$ increases as the number of waves increases, and the radiation pressure increases by the same amount for the same reason. Young’s modulus increases likewise by the same amount, because the internal forces in the lattice are proportional to the number of waves in the lattice. The increase of $\rho$, $P$ and $Y$ is the same, provided that the sidelength of the lattice is independent of the order of the modes.

3 Conclusion

It is shown that the outward directed radiation pressure of standing, electromagnetic waves in a cubic nuclear lattice is balanced by the elastic force per unit area, provided that the sidelength of the lattice is equal to $10^{-13}$ cm, which means about equal to the size of the proton. The size of the particles is an automatic consequence of the standing wave model of the stable elementary particles of the $\gamma$-branch. Similar considerations apply for the particles of the $\nu$-branch which are held together by a neutrino lattice.

References
[1] E.L.Koschmieder, to appear in Proc. Roy. Belg. Acad. Sci.; xxx.lanl.gov/abs/hep-ph/0002179.
[2] E.L.Koschmieder, to appear in Proc. Roy. Belg. Acad. Sci.; xxx.lanl.gov/abs/hep-lat/0002016.
[3] E.L.Koschmieder, Nuovo Cim. A 101,1017 (1989).