Once more on extra quark-lepton
generations and precision
measurements
(dedicated to L.B. Okun’s 80th birthday)

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Abstract

Precision measurements of $Z$-boson parameters and $W$-boson and $t$-quark masses put strong constraints on non $SU(2) \times U(1)$ singlet New Physics. We demonstrate that one extra generation passes electroweak constraints even when all new particle masses are well above their direct mass bounds.

1 Introduction

Nine years ago in paper [1] it was noted that contrary to the common belief expressed in review paper [2] the precision electroweak data do not exclude the existence of extra quark-lepton generations. A year after in important

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paper [3] it was found that heavy Higgs boson does not contradict electroweak data as soon as extra generation exists.

The purpose of the present paper is twofold. Firstly, to update old results taking into account the present values of electroweak observables. Secondly, the existence of the fourth generation does not contradict precision data even if the mass of new neutral lepton is much larger than $m_Z/2$ contrary to the statement of the update of [2] published in [4].

2 Standard Model fit

In Table 1 we present the results of the data fit performed by LEPTOP code [5].

| Observable       | Exper. data  | LEPTOP fit  | Pull |
|------------------|--------------|-------------|------|
| $\Gamma_Z$, GeV  | 2.4952(23)   | 2.4963(15)  | -0.5 |
| $\sigma_h$, nb   | 41.540(37)   | 41.476(14)  | 1.8  |
| $R_l$            | 20.771(25)   | 20.743(18)  | 1.1  |
| $A^l_{FB}$       | 0.0171(10)   | 0.0164(2)   | 0.8  |
| $A_h$            | 0.1439(43)   | 0.1480(11)  | -0.9 |
| $R_b$            | 0.2163(7)    | 0.2158(1)   | 0.7  |
| $R_c$            | 0.172(3)     | 0.1722(1)   | -0.0 |
| $A^L_{FB}$       | 0.0992(16)   | 0.1037(7)   | -2.8 |
| $A^c_{FB}$       | 0.0707(35)   | 0.0741(6)   | -1.0 |
| $s^2_l(Q_{FB})$  | 0.2324(12)   | 0.2314(1)   | 0.8  |
| $A_{LR}$         | 0.1513(21)   | 0.1479(11)  | 1.6  |
| $A_b$            | 0.923(20)    | 0.9349(1)   | -0.6 |
| $A_c$            | 0.670(27)    | 0.6682(5)   | 0.1  |
| $m_W$, GeV       | 80.398(25)   | 80.377(17)  | 0.9  |
| $m_t$, GeV       | 172.6(1.4)   | 172.7(1.4)  | 84$^{+32}_{-24}$ |
| $m_H$, GeV       |               |             |      |
| $\hat{\alpha}_s$| 1.184(27)    | 0.1184(27)  |      |
| $1/\bar{\alpha}$| 128.954(48)  | 128.940(46) |      |
| $\chi^2/n_{d.o.f.}$ | 18.1/12     |             |      |

Standard Model fit by LEPTOP, summer 2008.
The quality of the fit is characterized by $\chi^2/n.d.f. = 18/12$ and is reasonably good. The central value of the Higgs boson mass is well below LEP II direct search bound: $m_H \geq 114$ GeV at 95% C.L.

3 Fits with the fourth generation

Introducing the fourth generation we get many new parameters: quark and lepton masses and mixing with three existing generations. To simplify the analysis let us suppose that mixing is small.

In order to investigate 5-dimensional parameter space ($m_H, m_U, m_D, m_E, m_N$) we use the results of [6] where steep and flat directions in the dependence of $\chi^2$ on new particle masses were found. We fix the values of the sum of new quark masses $m_U + m_D = 600$ GeV to avoid Tevatron direct search bounds and the value of the charged lepton mass $m_E = 200$ GeV and look for a minimum of $\chi^2$ for the fixed values of $m_H$, varying neutral lepton mass and the difference of Up- and Down-quark masses. The results of the fit are presented in Fig. 1 for $m_H = 120$ GeV, in Fig. 2 for $m_H = 600$ GeV and in Fig. 3 for $m_H = 1000$ GeV. We see that the quality of the fits is close to that for the Standard Model without additional generation.

4 How many new generations?

The next question which we address is the number of additional generations allowed by precision data\footnote{To simplify the analysis we assume the degeneracy of new particles with the identical quantum numbers: $m_{E_1} = m_{E_2} = ..., m_{N_1} = m_{N_2} = ..., m_{U_1} = m_{U_2} = ..., m_{D_1} = m_{D_2} = ....$}. To study this problem we fix $m_E = 200$ GeV, $m_U = m_D = 300$ GeV, since in Figures 1, 2 and 3 minimum of $\chi^2$ corresponds to almost degenerate quarks, and allow the number of extra generation $N_g$ and mass of neutral lepton $m_N$ to vary, bounding Higgs boson mass to be above direct search bound, $m_H > 114$ GeV. The levels of $\chi^2$ are shown in Fig. 4.

As we already stressed the value of $\chi^2$ for Standard Model and for $N_g = 1$ are almost the same, while three and more additional generations are excluded.
Figure 1: *Exclusion plot on the plane* $m_N, m_U - m_D$ *for fixed values* $m_H = 120$ GeV, $m_E = 200$ GeV, $m_U + m_D = 600$ GeV. *χ² minimum shown by the star* corresponds to $χ^2/d.o.f. = 17.7/11$. *The borders of the regions show the domains allowed at the level* $Δχ^2 = 1, 4, 9, 16$ etc.

In order to understand which masses of Higgs bosons correspond to different regions in Fig. 4 we draw Fig. 5. We see that for the case $N_g = 1$ higgs mass rapidly grows when the neutral lepton mass diminishes below 100 GeV.

5 **S, T, U versus Vₘ, Vₐ, Vᵣ – formulas**

$S, T$ and $U$ variables were suggested in [7] for the analysis of New Physics contribution to electroweak observables. In paper [7] it was supposed that New Physics contributes only to the vector bosons polarization operators and
masses of new particles are much heavier than $m_Z$, $m_W$. Extra generations not mixed with three “light” ones satisfy the first requirement; concerning the second one it depends on the masses of new particles. The expressions for $S$, $T$ and $U$ introduced in [7] take into account the values of the polarization operators and their first derivatives at $q^2 = 0$. In this way the beautiful formulas were obtained with a clear physical meaning: $S$ characterizes the violation of chiral symmetry, while $T$ and $U$ characterize the isotopic symmetry violation.

Radiative corrections to electroweak observables were expressed in LEP-TOP through three functions $V_i$:

$$\frac{m_W}{m_Z} = c + \frac{3\bar{c}c}{32\pi s^2(c^2 - s^2)} V_m , \quad (1)$$
Figure 3: The same as Fig. 1 for $m_H = 1000$ GeV. $\chi^2$ minimum shown by the star corresponds to $\chi^2/d.o.f. = 18.4/11$.

\[ g_A = -\frac{1}{2} - \frac{3\bar{\alpha}}{64\pi c^2 s^2} V_A, \]
\[ \frac{g_V}{g_A} = 1 - 4s^2 + \frac{3\bar{\alpha}}{4\pi(c^2 - s^2)} V_R, \]
\[ s^2 c^2 \equiv \sin^2 \theta_W \cos^2 \theta_W = \frac{\pi \bar{\alpha}}{\sqrt{2} G_F m_Z^2}, \quad \bar{\alpha} \equiv \alpha(m_Z) = (128.87)^{-1}, \]

where $g_A$ and $g_V$ are axial and vector couplings of Z-boson with charged leptons. Functions $V_i$ depend both on Standard Model and New Physics contributions. To make comparison with $S,T,U$ approach let us separate these two contributions in $V_i$:

\[ V_i \equiv V_i^{SM} + \delta_{NP} V_i. \]
Figure 4: Exclusion plot on the plane \(N_g, m_N\) for fixed values \(m_U = m_D = 300\ GeV, m_E = 200\ GeV\). \(\chi^2\) minimum is shown by the star. The condition \(m_H > 114\ GeV\) is imposed.

The definitions of \(S, T\) and \(U\) used in [4] differ from the original one used in papers [7]: instead of the derivatives of polarization operators at \(q^2 = 0\) their values at \(q^2 = m_W^2\) and \(q^2 = m_Z^2\) were used. These new definitions coincide with our functions \(V_i\) with one exception: instead of derivative of \(Z\)-boson polarization operator at \(q^2 = m_Z^2\) which enters function \(V_A\), the difference \(\Pi_Z(m_Z^2) - \Pi_Z(0)\) is used in [4].

Comparing the definitions of \(S, T\) and \(U\) presented in Eqs. (10.61a - 10.61c) from [4] with definitions of our functions \(V_i\) presented in [1] Eqs. (1)
Figure 5: Values of Higgs boson masses on the plot of Fig. 4 are shown. Each strip corresponds to the definite value of $m_H$.

- (3) we get\(^2\):

$$T = \frac{3}{16\pi s^2 c^2} \delta_{NP}V_A + \Delta \equiv T' + \Delta, \quad (6)$$

$$S = \frac{3}{4\pi} [\delta_{NP}V_A - \delta_{NP}V_R] + 4s^2 c^2 \Delta \equiv S' + 4s^2 c^2 \Delta, \quad (7)$$

$$S + U = \frac{3}{4\pi (c^2 - s^2)} (\delta_{NP}V_m - \delta_{NP}V_R) \equiv S' + U', \quad (8)$$

$$\Delta \equiv \frac{1}{\alpha} \left[ \Pi'_Z(m^2_Z) - \frac{\Pi_Z(m^2_Z)}{m^2_Z} + \frac{\Pi_Z(0)}{m^2_Z} \right], \quad (9)$$

\(^2\)Let us note that our definition of polarization operators differs by sign from that used in [4]; also functions $V_R$ and $V_m$ contain derivative of photon polarization operator at $q^2 = 0$, while $S$ and $U$ contain $\Pi')(m^2_Z)/m^2_Z$. 

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where we introduce new set of functions \( S', T' \) and \( U' \) which are directly related to \( \delta_{NP}V_i \). If the authors of [4] use these primed functions in the expressions for physical observables instead of \( S, T \) and \( U \), then their formulas would be exact for arbitrary values of the new particles masses. The accuracy of the formulas for physical observables through \( S, T \) and \( U \) is good as long as new particles are considerably heavier than \( Z \)-boson, since in this case \( \Delta \) is suppressed as \( (m_Z/2m)^2 \) (we substituted \( 2m \) instead of \( m \) since cut of \( Z \)-boson polarization operator starts at \( q^2 = (2m)^2 \)). The authors of [4] recognize that for the light fourth generation particles their approach based on \( S, T \) and \( U \) can be wrong.

Our next goal is to present explicit formulas for the fourth generation contributions to \( T', S \) and \( \Delta \). Let us start from the expression for \( \delta_{NP}V_A \):

\[
\delta_{NP}V_A = \frac{16\pi^2 \alpha^2}{3a} \left[ \frac{\Pi_Z(m^2_Z)}{m^2_Z} - \frac{\Pi_W(0)}{m^2_W} - \frac{\Pi'_Z(m^2_Z)}{m^2_Z} \right] \quad (10)
\]

and compare it with that for \( T \) from [4], Eqs. (10.61a, 10.63, 10.59):

\[
\bar{\alpha}T = \frac{\Pi_Z(0)}{m^2_Z} - \frac{\Pi_W(0)}{m^2_W} = \frac{3\bar{\alpha}}{16\pi c^2 s^2} \left[ u + d - \frac{2ud}{u - d} \ln \left( \frac{u}{d} \right) \right] + \frac{\bar{\alpha}}{16\pi c^2 s^2} \left[ N + E - \frac{2NE}{N - E} \ln \left( \frac{N}{E} \right) \right], \quad (11)
\]

where \( u \equiv m^2_U/m^2_Z, d \equiv m^2_D/m^2_Z, N \equiv m^2_N/m^2_Z \) and \( E \equiv m^2_E/m^2_Z \). With the help of Eq.(6) we obtain:

\[
\Delta = -\frac{3}{16\pi s^2 c^2} \delta_{NP}V_A + \frac{1}{\bar{\alpha}} \left[ \frac{\Pi_Z(0)}{m^2_Z} - \frac{\Pi_W(0)}{m^2_W} \right] = \frac{3}{16\pi s^2 c^2} \left\{ \frac{Nq_{u}^{4}}{3} [F'(u) + F'(d)] + N_{q_{d}}^{4} \left( \frac{4}{9}s^2 + \frac{1}{9} \right) \times \right. \\
\times [2uF'(u) - (1 + 2u)F'(u) + 2dF(d) - (1 + 2d)F'(d)] + \\
+ \frac{16}{9}N_{q_{d}}^{4}s^4 Q_{u}^{2} \left[ (1 + 2u)F'(u) - 2uF(u) \right] + \\
+ \frac{1}{9}N_{q_{d}}^{4}s^2 Y_{q_{d}}^{4} \times \\
\left. \times \left[ (1 + 2d)F'(d) - 2dF(d) + 2uF(d) - (1 + 2u)F'(u) \right] \right\}, \quad (12)
\]
where the contributions of quarks and leptons should be summed up, \( N_q^c = 3, N_l^c = 1, Y_q = Q_u + Q_d = 1/3, Y_l = Q_N + Q_E = -1, \)

\[
F(u) = 2 \left[ 1 - \sqrt{4u - 1} \arcsin \frac{1}{\sqrt{4u}} \right], \tag{13}
\]

\[
F'(u) \equiv -u \frac{d}{du} F(u) = \frac{1 - 2uF(u)}{4u - 1},
\]

see Eqs. (5), (9), (10) from [1].

From Eqs. (7) and (12) we get:

\[
S = -\frac{3}{4\pi} \delta_{NP} V_R + \frac{4s^2c^2}{\tilde{\alpha}} \left[ \frac{\Pi_Z(0)}{m_Z^2} - \frac{\Pi_W(0)}{m_W^2} \right], \tag{14}
\]

and with the help of Eq. (11) and Eq. (6) from [1] we finally get:

\[
S = \frac{3}{4\pi} \left\{ \frac{2N_q^c}{3} [uF(u) + dF(d)] - \frac{16}{9} N_q^c s^2c^2 \left[ Q_u^2 \left( (1 + 2u)F(u) - \frac{1}{3} \right) + Q_d^2 \left( (1 + 2d)F(d) - \frac{1}{3} \right) \right] - \frac{2N_q^c Y_q^c}{9} [(1 + 2d)F(d) - (1 + 2u)F(u) + \ln \left( \frac{u}{d} \right)] \right\}, \tag{15}
\]

where contributions of quarks and leptons should be summed up.

6 S, T, U versus \( V_m, V_A, V_R \) – numbers

In Section 3 we found three points of \( \chi^2 \) minimum in the fourth generation parameter space, which corresponds to \( m_H = 120 \text{ GeV}, m_H = 600 \text{ GeV} \) and \( m_H = 1000 \text{ GeV} \). The values of quark and lepton contributions to \( S, S', T, T', U \) and \( U' \) at these points are presented in Table 2.
Table 2

|        | $m_H = 120$ |         | $m_H = 600$ |         | $m_H = 1000$ |
|--------|-------------|---------|-------------|---------|--------------|
| $m_U$  | 310         | $m_N$  | 120         | $m_U$  | 300          | $m_N$  | 50          | $m_U$  | 315          | $m_N$  | 53          |
| $m_D$  | 290         | $m_E$  | 200         | $m_D$  | 300          | $m_E$  | 200         | $m_D$  | 285          | $m_E$  | 200         |
| $T'$   | 0.02        | 0.11    | 0           | 0.24    | 0.05         | 0.27   |
| $T$    | 0.02        | 0.11    | 0           | 0.37    | 0.05         | 0.36   |
| $S'$   | 0.15        | -0.01   | 0.16        | -0.23   | 0.15         | -0.19  |
| $S$    | 0.15        | -0.01   | 0.16        | -0.14   | 0.15         | -0.13  |
| $U'$   | 0           | 0.02    | 0           | 0.20    | 0            | 0.16   |
| $U$    | 0           | 0.01    | 0           | 0.11    | 0            | 0.10   |

Quark and lepton contributions to $S, T, U$ and $S', T', U'$ at the points of $\chi^2$ minimum in Figures 1,2 and 3. All masses are in GeV.

For heavy higgs the mass of a neutral lepton is close to $m_Z/2$, that is why the values of $S', T'$ and $U'$ considerably differ from $S$, $T$ and $U$. For light higgs ($m_H = 120$ GeV) the masses of all new fermions are far from $m_Z/2$, that is why $S$, $T$ and $U$ almost coincide with $S'$, $T'$ and $U'$.

In order to get whether these sets of the fourth generation particle masses are allowed by precision data in the framework of $S$, $T$, $U$ approach, we should look at Fig. 10.4 from [4]. Standard Model corresponds to $S = T = 0$, and this point is just at the border of 90 % C.L. allowed domain for $m_H = 117$ GeV.

Summing quarks and leptons contributions for the fourth generation point with $m_H = 120$ GeV we get $S = 0.14$, $T = 0.13$. It is at the border of the same domain, so its level of $\chi^2$ is the same as in the case of SM – the result coinciding with that of $V_i$ analysis presented in Section 3.

Next we should consider the point with heavy higgs, $m_H = 1000$ GeV. At this point $S = 0.02$, $T = 0.41$, which is a bit outside the allowed domain for 1000 GeV higgs from Fig. 10.4. However since at this point $S' = -0.04$, $T' = 0.32$ we see that primed quantities are again at the border of 90 % C.L. allowed domain.
7 Conclusions

The possibility to include an extra quark-lepton generation(s) into Standard Model has been studied. We have found that the electroweak data do not contradict the existence of one extra family with specially adjusted masses. Three examples corresponding to light and heavy higgs bosons are presented. The properly made analysis based on $S, T, U$ (for $m_H = 120$ GeV) and $S', T', U'$ (for $m_H = 1000$ GeV) confirms the results of the analysis based on $V_i$. Let us note that in paper [9] a set of masses of the fourth generation particles was found which pass electroweak tests and corresponds to $m_H = 115 - 300$ GeV.

This paper is an expanded version of the presentation at the “Beyond the 3 SM generation at the LHC era” workshop at CERN, September 4-5 2008. We dedicate it to Lev Okun, who initiated our common electroweak project many years ago.

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