Analytic parametrizations of the $\gamma^*N \rightarrow N(1440)$ form factors inspired by light-front holography

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We present analytic parametrizations for the $\gamma^*N \rightarrow N(1440)$ form factors derived from light-front holography in leading twist approximation. The new parametrizations describe the electromagnetic form factors using analytic functions dependent on the masses of the $\rho$ meson and respective radial excitations, as well as the masses of the nucleon and the resonance $N(1440)$. The free parameters of the model, associated with three independent couplings, are interpreted as bare couplings, and are fixed by the nucleon data for large $Q^2$. The proposed parametrizations compare remarkably well with the empirical data for $Q^2 > 2$ GeV$^2$, corroborating the dominant role of the valence quark degrees of freedom in the $\gamma^*N \rightarrow N(1440)$ transition.

I. INTRODUCTION

It was found recently that the combination of the 5D gravitational anti-de-Sitter (AdS) space and conformal field theories (CFT) can be used to study strong coupling theories like QCD in the confining regime. This application is possible due to the correspondence between the results from AdS/CFT and the results from light-front dynamics based on a Hamiltonian that includes the confining mechanism of QCD. The correspondence between AdS/QCD and light-front dynamics is the consequence of the mapping description of the hadronic modes in AdS space and the Hamiltonian formulation of QCD in physical space-time quantized on the light-front. This connection (duality) is referred as Light-Front holography or Holographic QCD. As a consequence of this duality, for hadrons with massless quarks, one can relate the AdS holographic variable $z$ with the impact separation variable $\zeta$, which measures the distance of the constituent partons inside the hadrons, and calculate wave functions of the hadrons in terms of these variables.

Over the past few years light-front holography has been used to study the properties of the hadrons, such as the mass spectrum, parton distribution functions, and structure form factors of mesons and baryons. The reduction from the 5D action to 4D introduces the holographic mass scale, $\kappa$, which establishes the scale of the meson and baryon spectrum. Light-front holography provides a good description of the physical spectrum when we consider the soft-wall model associated with a confinement potential $U(z) = \kappa^2 z^2$. The scale $z \sim 1/\kappa$ separates the ultraviolet conformal limit $z \rightarrow 0$ (perturbative region) from the infrared region (large $z$) regulated by the confinement (non-perturbative region). In the limit $\kappa \rightarrow 0$ the chiral symmetry is exact; when $\kappa$ is very large the conformal symmetry is broken.

The study of the electromagnetic structure of the baryons can be performed in light-front holography noting that the three-quark systems can be regarded as two-body systems with an active quark and a spectator quark-pair, with masses and wave functions determined by the light-front wave equations. In previous works it was found that light-front holography provides good estimates for the electromagnetic form factors of the pion, the nucleon, and the Roper, when one uses wave functions based on the first Fock states. The inclusion of higher order Fock states means that corrections to the three-valence quark approximation (leading twist approximation), such as the meson cloud effects, are taken into account. It was however found, that good estimates for the form factors can also be obtained in leading twist approximation, such as the meson cloud effects, are taken into account. Although the leading twist approximation may look as a rough simulation of the real world, it may still provide an excellent first estimate, since the confinement is effectively taken into account in the light-front wave functions.

The transition current associated with the $\gamma^*N \rightarrow N^*$ transitions, where $N^*$ is a $J^P = \frac{1}{2}^+$ resonance (spin 1/2 and positive parity), is characterized by two independent functions of $Q^2$: the Dirac (spin-nonflip) and the Pauli (spin-flip) form factors.

Brodsky and Teramond have shown that, in leading twist approximation, an accurate description of the form factors for the nucleon and the $\gamma^*N \rightarrow N(1440)$ Dirac form factor can be obtained using analytic expressions based on the $\rho$ meson masses.

In the present work, we propose simple analytic parametrizations for both $\gamma^*N \rightarrow N(1440)$ transition form factors. In those parametrizations, the $\gamma^*N \rightarrow N(1440)$ transition form factors are expressed in terms...
of the $\rho$ meson masses, as well as the nucleon and Roper masses. The parametrizations compare well with empirical data for $Q^2 > 2 \text{ GeV}^2$. Here $Q^2$ is defined by $Q^2 = -q^2$, where $q$ is the momentum transfer.

II. LIGHT-FRONT HOLOGRAPHY

In light-front holography, the electromagnetic interaction with the hadrons is defined in terms of a 5D action which includes the coupling with the electromagnetic field $V_M$ ($M = 0, 1, 2, 3, 4$), and it is at the end reduced to the 4D current $J_\mu$. The simplest possible coupling is the minimal Dirac coupling, which has the form $\bar{\Psi} \gamma^\mu V_M \Psi$. The charge transition operator and $\Gamma^M$ is a 5D gamma matrix. With the minimal Dirac coupling one obtains only contributions for the Dirac form factor [16]. One can generate contributions for the Pauli form factor when we include a non-minimal coupling of the form $\bar{\Psi} \gamma_\nu \gamma_5 \gamma^\mu V_M \gamma_5 \Psi$, where $\gamma_\nu$ is a parameter associated with the proton ($N = p$) or neutron ($N = n$) anomalous magnetic moment, and $\gamma_5$ is the Pauli isospin operator. This new coupling is absent in the 4D action, but can appear in the 5D action, inducing additional contributions to the Dirac form factor. Alternative non-minimal couplings are discussed in Ref. [22].

In holography, the calculation of the transition currents between two baryon states is performed considering the overlap of the holographic electromagnetic field, which includes the electromagnetic couplings mentioned above, with the baryon fields $\Psi_B(x, z)$ associated with the initial and final states. In this notation $x$ is the 4D space-time coordinate and $z$ the holographic variable. The fields $\Psi_B(x, z)$ are determined by equations of motion that can be reduced to Schrödinger-type wave equations in the variable $z$. Check Refs. [4, 12, 13, 16] for a review. From the transition currents we can extract the holographic expressions for the electromagnetic form factors which depend on the minimal-type coupling ($g_V$) and the two non-minimal couplings ($\eta_p$ and $\eta_n$), which characterize the electromagnetic interaction with the quarks inside the hadrons. The parameters $g_V$, $\eta_p$ and $\eta_n$ are related with the intrinsic properties of the valence quarks and represent therefore bare couplings.

Calculations of the nucleon electromagnetic form factors based on the Dirac minimal coupling and the non-minimal couplings can be found in Refs. [4, 12, 14–18]. Calculations that include the minimal-type coupling $g_V$ for the nucleon and the Roper are presented in Refs. [12, 13, 14].

In light-front holography the expressions for the baryon electromagnetic form factors include functions with poles on $Q^2$ associated with the masses $M^2 = 4(n+1)\kappa^2$, where $n = 0, 1, \ldots$. Some authors prefer to represent those functions using a pole structure based on the vector meson dominance [4, 8, 14–16], which is expected to appear in high energy processes. This representation is possible because the high energy physical photons have hadronic components that can be expressed as intermediate particles with the quantum numbers of the physical vector mesons [11, 21, 27].

The simultaneous description of the vector meson and nucleon radial excitation spectrum based on only a mass scale $\kappa$ is a difficult task [4]. The vector meson spectrum is well explained with $\kappa \approx 0.385 \text{ GeV}$, however, the nucleon and Roper masses are only roughly approximated by the holographic estimates in leading twist approximation [12, 13]. For that reason different representations of the baryon transition form factors have been used, as mentioned above. One representation keeps the pole structure $M^2 = 4(n+1)\kappa^2$ and fixes $\kappa$ by the nucleon mass or the rho mass $\kappa = \sqrt{M^2 - m^2}$ [3, 12, 13, 14]. In a second representation there is a re-interpretation of the poles, which are shifted to the vector meson physical poles, $M \rightarrow 2\sqrt{2}n + \Gamma_N$ ($n = 0, 1, \ldots$), as proposed by Brodsky and Teramond [4, 11].

In the present work we propose an alternative representation of the $\gamma^*N \rightarrow N(1440)$ transition form factors, where the mass poles are re-interpreted as vector meson poles or as mass poles of the nucleon and Roper. This re-interpretation is based on the holographic estimates of the masses in leading twist approximation, expressed in terms of the mass scale $\kappa$, and in the empirical masses of the nucleon and the nucleon radial excitations. The proposed expressions are not strictly derived from light-front holography, but they are still inspired by light-front holography.

We consider then a bottom-up approximation to QCD, starting with a well defined interaction Lagrangian, and use the phenomenology to correct the expressions for the mass poles derived from first principles. Examples of ad hoc corrections to the light-front holography estimates are common in the literature. The minimal-type coupling ($g_V$) and the Pauli coupling ($\eta_V$) were introduced for phenomenological purposes [16, 17, 19]. The calculation of the nucleon and Roper form factors in the model from Brodsky-Teramond [4, 13] is improved when we use holographic vector masses instead of the empirical masses. Those corrections to the light-front holography results motivate the following representation of the $\gamma^*N \rightarrow N(1440)$ transition form factors.

III. $\gamma^*N \rightarrow N(1440)$ TRANSITION FORM FACTORS

The $\gamma^*N \rightarrow N(1440)$ transition form factors have been calculated using light-front holography based on the nucleon and the Roper wave functions with Fock states up
to the twist order \( \tau = 5 \). The leading twist component (\( \tau = 3 \)) corresponds to the three-quark (3q) state; the following twist component (\( \tau = 4 \)) represents the gluon excitation of the three-quark state, (3q)g; and the \( \tau = 5 \) correspond to the quark-antiquark excitation (qq) of the three-quark state. Thus, \( \tau = 3 \) represents the valence quark approximation and \( \tau = 5 \) takes into account the meson cloud excitation of the three-quark core.

In the leading twist approximation (\( \tau = 3 \)) one can express the \( \gamma^*N \to N(1440) \) transition form factors in the form:

\[
F_{1N}^* = \frac{1}{12\sqrt{2}} (1 + g_\nu) \delta_N \frac{Q^4}{m_p^4} G_2 + \\
\frac{1}{24} (c_1 + c_2 g_\nu) \delta_N \frac{Q^2}{m_p^2} G_2 + \\
\frac{1}{60} \eta_N \left( 2\sqrt{2} \frac{Q^2}{m_p^4} - c_3 \frac{Q^2}{m_p^2} + c_4 \right) \frac{Q^2}{m_p^2} G_3,
\]

(1)

\[
F_{2N}^* = \frac{\sqrt{2}}{4} \eta_N \left( \frac{M_{N1} + M_N}{M_{N1}} \right)^2 \left( \frac{c_5 Q^2}{m_p^2} - 4 \right) G_2,
\]

(2)

where \( \delta_N = \pm \) (\( \delta_p = +1 \), \( \delta_n = -1 \)) and the \( c_i \) coefficients are \( c_1 = 4\sqrt{2} + 3\sqrt{3}, c_2 = 4\sqrt{2} - 3\sqrt{3}, c_3 = 9(\sqrt{3} - \sqrt{2}), c_4 = 3\sqrt{3} - 5\sqrt{2} \) and \( c_5 = 2\sqrt{6} \). As discussed next, the functions \( G_2, G_3 \) can be written as

\[
G_2 = \frac{1}{\left( 1 + \frac{Q^2}{m_p^2} \right) \left( 1 + \frac{Q^2}{m_p^2} \right) \left( 1 + \frac{Q^2}{m_N^2} \right) \left( 1 + \frac{Q^2}{m_N^2} \right)},
\]

(3)

\[
G_3 = \frac{1}{\left( 1 + \frac{Q^2}{m_p^2} \right) \left( 1 + \frac{Q^2}{m_p^2} \right) \left( 1 + \frac{Q^2}{m_p^2} \right) \left( 1 + \frac{Q^2}{m_N^2} \right)}
\times \frac{1}{\left( 1 + \frac{Q^2}{m_N^2} \right) \left( 1 + \frac{Q^2}{m_N^2} \right)},
\]

(4)

where \( M_N, M_{N1} \) are the mass of the nucleon and the nucleon first radial excitation (Roper), and \( m_p, m_{p,n} (n = 1, 2) \) are the \( p \) and the mass of the first \( p \) excitation. All the masses are interpreted as empirical masses.

Equations (1) and (2) are derived in Ref. 13 based on the expressions from Ref. 13. The difference between the present model and Ref. 19 is that we represent here the functions \( G_2 \) and \( G_3 \) given by Eqs. (3) and (4) in terms of the empirical masses of the hadrons.

The motivation for the present representation comes from the expressions for the masses of the \( \rho \)-mesons and the nucleon radial excitations derived from light-front holography. For the nucleon and \( \rho \), the holographic expressions for the masses are \( M_N = 2\sqrt{2} \kappa \) and \( m_p = 2\kappa \), respectively [4, 16]. As for the \( \rho \) excitations, we follow the parametrization \( m_{p,n} = 2\kappa \sqrt{n + \tau} \) for \( n = 1, 2, \ldots \), derived in Refs. 4, 14, 17, where the twist-2 mass poles are shifted to their physical values. A good description of the \( \rho \) masses is obtained when \( \kappa = 0.385 \) GeV (10% error).

In the equations for \( G_2 \) and \( G_3 \) there is a term \( \frac{Q^2}{M_{N1}^2} \), where \( M_{N1} \) is interpreted as the Roper mass \( (M_R) \). In this case we replace \( 4\kappa \approx 1.52 \text{ GeV} \) by \( M_{N1} \), where \( M_{N1} \) is a rough estimation of the empirical mass of the Roper \( M_R \approx 1.44 \text{ GeV} \). This replacement constitutes an exception to the identification of the hadron masses with the masses determined by light-front holography, and it is justified by the result \( M_R \approx 4\kappa \) (5% deviation). The corollary of the present parametrizations for \( G_2 \) and \( G_3 \) is the explicit dependence of the \( \gamma^*N \to N(1440) \) transition form factors on the Roper empirical mass.

In Eqs. (1) and (2) the only unknown parameters are \( g_\nu, \eta_p \) and \( \eta_n \). In previous works [4, 12, 16, 17] these parameters were determined by the nucleon data near the photon point. However, since the coefficients \( g_\nu, \eta_p \) and \( \eta_n \) are related with the intrinsic properties of the valence quarks, we believe that these coefficients are more accurately determined by the large-\( Q^2 \) data, since in that case the effect of the meson cloud is significantly reduced. In Ref. 19 the nucleon elastic form factor data for \( Q^2 > 1.5 \text{ GeV}^2 \) were studied using a light-front holographic model. It was concluded that the values \( g_\nu = 1.42 \pm 0.15, \eta_p = 0.39 \pm 0.03 \) and \( \eta_n = -0.34 \pm 0.02 \) provide a good description of the \( Q^2 > 1.5 \text{ GeV}^2 \) data [28, 30]. In the present work, we consider the central values. Since in the large-\( Q^2 \) region, the meson cloud effects are expected to be small, the values obtained for \( g_\nu, \eta_p \) and \( \eta_n \) can be interpreted as bare couplings [19].

The holographic estimates can also be compared with the transverse \((A_{1/2})\) and longitudinal \((S_{1/2})\) amplitudes in the Roper rest frame. Those helicity amplitudes are related with the transition form factors through [31, 33]

\[
A_{1/2} = \mathcal{R}(F_{1N}^* + F_{2N}^*),
\]

(5)

\[
S_{1/2} = \frac{\mathcal{R}}{\sqrt{2}} |q| \frac{M_R + M}{Q^2} (F_{1N}^* - \tau F_{2N}^*),
\]

(6)

where \( \tau = \frac{M^2 + M_R^2}{(M_R + M)^2} \) and \( |q| \) is the photon three-momentum in the Roper rest frame. The factors \( \mathcal{R} \) and \( |q| \) are respectively

\[
\mathcal{R} = \sqrt{\frac{\pi \alpha Q^2}{M_R M K}} , \quad |q| = \sqrt{Q_1^2 Q_2^2 / 2M_R},
\]

(7)

where \( Q_{1,2}^2 = (M_R \pm M)^2 + Q^2, K = M_R^2 + M^2 \) and \( \alpha \approx 1/137 \) is the fine structure constant.

The results for the nucleon to Roper transition for the proton target \((N = p)\) are presented in Figs. 1 and 2 for the form factors and helicity amplitudes, respectively. The model estimates are compared with the data from CLAS/Jefferson Lab [31, 32]. For the amplitudes we include also the result for \( A_{1/2} \) at the photon point from Particle Data Group [34] and the recent result for \( S_{1/2} \) for \( Q^2 \approx 0.1 \text{ GeV}^2 \) from the A1 collaboration [37]. We do not discuss the results for the neutron target \((N = n)\), since one has data only for \( Q^2 = 0 \).
The results presented in Fig. 1 for the transition form factors are very interesting because they show that the experimental data can be described by simple parametrizations based on the empirical masses of the hadrons. The agreement between the parametrization and the data is very good particularly for $Q^2 > 2$ GeV$^2$, where the valence quark degrees of freedom are dominant.

For low $Q^2$, we should not expect the parametrizations to be so accurate, because as mentioned, Eqs. (1) and (2) do not include the contributions from the meson cloud, which may be in general significant for small $Q^2$, and also because the bare parameters $g_V, \eta_p$, and $\eta_p$ are estimated by the large-$Q^2$ nucleon data [10]. In the upper panel of Fig. 1 we present our estimates to the Roper Pauli form factor $F_{2p}$. As far as we know, this is the first time that a simple analytic expression is presented for $F_{2p}$. Although the calculations are expected to be valid only for large $Q^2$, it is nevertheless interesting to note the remarkable description of the function $F_{2p}$ in the region $0.2–3.5$ GeV$^2$.

From the results for the helicity amplitudes, presented in Fig. 2 we can confirm that those parametrizations are also accurate for $Q^2 > 1.5$ GeV$^2$. This conclusion could have been anticipated from the analysis of the form factors. For low $Q^2$ we can observe some deviations from the data, particularly for $S_{1/2}$. This result is mainly the consequence of the model overestimation of the function $F_{1p}$ for $Q^2 < 1$ GeV$^2$, since the effect of $F_{2p}$ is reduced by the factor $\tau$, according with Eq. (6). As for the amplitude $A_{1/2}$, the deviation from the data happens only near $Q^2 = 0$, as a consequence of the results for the function $F_{2p}$, since $F_{1p}^p(0) = 0$. In both cases the deviation from the data at low $Q^2$ can be the result of no inclusion of meson cloud contributions, or a limitation of the holographic formalism. Recall that the holographic estimates are calculated in the limit of zero quark masses.

In the graph for $S_{1/2}$ it is possible to note the small value obtained at $Q^2 \simeq 0.1$ GeV$^2$ by the A1 collaboration [37]. This result may be an indication that the amplitude $S_{1/2}$ vanishes at the pseudothreshold, when $Q^2 = -(M_R - M)^2 \simeq -0.25$ GeV$^2$, according to the long-wavelength constraint [37, 42].

Once confirmed the accuracy of the parametrizations from Eqs. (1) and (2) for the $\gamma^*N \to N'(1440)$ Dirac and Pauli transition form factors, it is worth checking if
for the case of the nucleon one can also obtain simple analytic expressions based on the masses of hadrons, and if those parametrizations describe well the nucleon form factor data.

### Nucleon elastic form factors

The nucleon electromagnetic form factors can be parametrized as

\[
F_{1N} = e_N G_1 + \frac{1}{6} (e_N + \delta_N g_V) Q^2 m_p \frac{Q^2}{m_p^2} G_1
\]

\[
-\frac{1}{6} \eta_N \frac{Q^2}{m_p^2} \left( \frac{1}{2} - \frac{Q^2}{m_p^2} \right) G_2,
\]

\[
F_{2N} = 8 \eta_N G_1,
\]

where \(e_N\) is the nucleon charge and

\[
G_1 = \frac{1}{\left(1 + \frac{Q^2}{m_p^2}\right) \left(1 + \frac{Q^2}{m_{p1}^2}\right)} \left(1 + \frac{Q^2}{M_N^2}\right).
\]

Equations (8)–(10) are obtained from Ref. [19], replacing the holographic nucleon mass \((2\sqrt{2}\kappa)\) by the nucleon physical mass, combined with the re-interpretation of the poles of the function \(G_1\) as the empirical hadron masses \((m_p, m_{p1} \text{ and } M_N)\), as discussed for the case of the Roper.

The results associated with the previous analytic expression for the electric \(G_{EN} = F_{1N} - \frac{Q^2}{4M_N^2} F_{2N}\) and magnetic \(G_{MN} = F_{1N} + F_{2N}\) form factors are presented in Fig. 3 up to 8 GeV\(^2\). We use a logarithm scale for \(G_{Ep}\), \(G_{Mp}\) and \(G_{Mn}\) in order to better observe the falloff of those form factors for larger values of \(Q^2\).

In Fig. 3 one can see that Eqs. (8) and (9) provide accurate representations of the nucleon form factor data for \(Q^2 > 1.5\) GeV\(^2\), except for the proton electric form factor \((G_{Ep})\), where the agreement with the data happens only for \(Q^2 > 4\) GeV\(^2\). A better description of \(G_{Ep}\) can be obtained if we take into account in \(F_{2p}\) the difference between the nucleon physical and the holographic estimate (15% correction) [19]. Considering all the form factors, the failure for \(Q^2 < 1.5\) GeV\(^2\) may have been anticipated, since the bare parameters \((g_V, \eta_p \text{ and } \eta_n)\) are determined from fits to the large-\(Q^2\) data.

The results for \(G_{Ep}\) suggest that the meson cloud effect may play a more important role in this specific form factor. \(G_{Ep}\) is particularly sensible to the meson cloud effect, because in the case of the proton, the effect modifies the charge of the three valence-quark component, which is reduced by a factor \(Z_N < 1\), due to the normalization of the nucleon dressed wave function [43–45]. In that case, the meson cloud component contributes with the charge \((1 - Z_N)e\), where \(e\) is the elementary charge.

In principle, the description of the low \(Q^2\) region can be improved with the inclusion of high order Fock states, in particular with the term \(\tau = 5\) as in Ref. [17]. In the present work, however, our focus is in the derivation of simple analytic expressions for the \(\gamma^* N \rightarrow N(1440)\) transition form factors, valid for large \(Q^2\), which are expected...
to be dominated by the valence quark contributions.

Concerning the proton form factors, one can compare our results directly with the ratio $\gamma G_{Ep}/G_{M\gamma}$, which can be measured by polarization transfer experiments at Jefferson Lab [28, 31]. The comparison with the data analysis from Refs. 23, 32 presents the lower panel of Fig. 1. From that data on one can also estimate the ratio $F_{Ep}/F_{Mp}$ scaled by the factor $Q^2$. The results are presented in the upper panel of Fig. 1.

In both cases one can observe a deviation from the data below 3 GeV$^2$ and a good agreement for larger values of $Q^2$. The discrepancy for $\gamma G_{Ep}/G_{M\gamma}$ is a consequence of the discrepancies noted already for $G_{Ep}$ and $G_{M\gamma}$ (underestimation for $G_{Ep}$ and overestimation for $G_{M\gamma}$). The highest datapoint ($Q^2 \approx 8.5$ GeV$^2$) differs from the model estimate only by 1.3 standard deviations. For $Q^2 > 4$ GeV$^2$, one can observe an almost perfect scaling between $Q^2F_{Ep}$ and $F_{Mp}$.

From the results, one concludes that the nucleon elastic form factors at large $Q^2$ can also be described by simple analytic expressions dependent on the bare couplings, the nucleon mass and the $\rho$ meson masses. The parametrization for the neutron electric form factor, in particular, may be tested in the near future, once the results from the JLab-12 GeV upgrade become available for $Q^2 > 4$ GeV$^2$ [31].

IV. SUMMARY AND CONCLUSIONS

In the present work we use previous results from light-front holography to derive analytic parametrizations of the nucleon and $\gamma^*N \rightarrow N(1440)$ form factors in leading twist approximation. In the leading twist approximation the nucleon and the nucleon excitations are described as three valence-quark systems. The falloff of the Dirac and Pauli form factors are consistent with the behavior expected from perturbative QCD [19, 40, 41].

The parametrizations presented here are based on two main ingredients: (i) the couplings associated with the electromagnetic interaction to the quarks are interpreted as bare couplings (no meson cloud contamination); (ii) the analytic structure of the Dirac and Pauli form factors may be represented in terms of the masses of the vector mesons ($\rho$ mesons) and the masses of the nucleon and the nucleon first radial excitation (Roper).

In particular the parametrizations associated with the Roper form factors, based on Eqs. (1)-(2), give a very good description of the data for $Q^2 > 2$ GeV$^2$, corroborating the idea that, in fact, the resonance $N(1440)$ is the first radial excitation of the nucleon, as suggested by several authors [32, 33, 47]. At low $Q^2$, however, the effect of the meson cloud effects cannot be ignored. Meson cloud effects are also fundamental to explain the mass of the Roper and its decay widths [17, 42].

A simple analytic expression was previously proposed for the Roper Dirac form factor [1, 13], in close agreement with the large-$Q^2$ data. However, to the best of our knowledge, this is the first time that a simple analytic expression is presented for the Roper Pauli form factor. The deviations from the data at low $Q^2$ can be interpreted as the result of the meson cloud effects, which are omitted in leading twist approximation. Surprisingly, our estimate of the Roper Pauli form factor is also very accurate for small $Q^2$, more specifically in the region $Q^2 = 0.2-1$ GeV$^2$.

Light-front holography may in the future be used for the study of other $\gamma^*N \rightarrow N^*$ transitions, where $N^*$ is a generic $J^P$ resonance (spin $J$ and parity $P$) in leading twist approximation. The parameters associated with the electromagnetic interaction are already fixed by the study of the nucleon form factors and were successfully tested for the case of the Roper. One can then expect that analytic expressions for the transition form factors may also be derived for other $\gamma^*N \rightarrow N^*$ transitions. It is also likely that new analytic expressions for the transition form factors based on the empirical hadron masses may be derived.

To summarize, the formalism presented in this work for the Roper can in the near future be extended to higher mass nucleon excitations. The use of the light-front holography in leading twist approximation provides a natural method to estimate the valence quark contributions for the transition form factors. The effect of the meson cloud can then be estimated from the comparison with the experimental data. In addition the light-front holographic parametrization may be tested in the near future by the results from the JLab 12-GeV upgrade [31] for $Q^2 > 6$ GeV$^2$, a region where we expect the estimates to be very accurate.

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