Modeling and controlling a quarter-vehicle active suspension model

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Abstract. This paper presents an application of the LQR active suspension control algorithm for a vertical planar oscillation model developed for ¼ of a vehicle. The wheel smoothness and dynamics with the road surface are two parameters to provide control signals. A simulation model is developed here based on MATLAB software to compare and evaluate the LQR active suspension model with the passive suspension. The results obtained here shows an improvement for a number of parameters when utilizing the active suspension model including fluctuating amplitude; oscillation damping time; the displacement acceleration of the active suspension body.

1. Introduction
A vehicle is an oscillating system that is in close contact with the complex contour of the road. Oscillation of the vehicle affects not only people, transported goods, but also the durability of urban complexes. The oscillation has a negative effect on the vehicle and especially on the driver's experience. The suspension is needed to solve the problems of driving comfort and safety of the vehicle [1, 3]. The passive suspension only reacts to certain road bumps. The damping factor of the passive suspension still conflicts with safety and ride comfort. In order to meet the criteria for driving comfort and safety on all types of roads, the characteristics need to be changed while driving in accordance with the characteristics of the road. One of the main areas of development that well-known car manufacturers are striving for today is the active suspension. The term "positive" can be understood as a suspension system in which the operating parameters can change during operation. The electronic active suspension control system automatically changes the operating parameters.

In this article, we mainly study controlling model and controlling method for active suspension model. The controlling model given here is a quarter-vehicle model. In the controlling method part, optimization of the controlling model will be addressed.

2. Establishment of simulation model and method of primary control
The operating principle of an active suspension model: vehicle height sensors continuously monitor the distance between the body and the struts to determine the chassis height, the speed sensor records and sends signals to the suspension ECU. The suspension ECU is responsible for receiving signals from all sensors to
control the damping force and spring rate, vehicle height, depending on the operating conditions of the vehicle, through the drive changing the damping force and suspension stiffness. The electronically controlled drive responds precisely to constant changes in vehicle operating conditions.

2.1 Establishing an active suspension model for ¼ vehicle

Initially, we created a model of an active suspension system, including springs, shock absorbers, interaction force \( U_a \), tires and suspension weight (Figure 1). We configure the model in accordance with the parameters in Table 1.

| Table 1 - Calculation parameters for ¼ vehicle suspension model |
|-----------------|----------|----------|
| Vehicle body weight          \( M_2 \) | 290      | Kg       |
| Tire and suspension weight   \( M_1 \)     | 59       | Kg       |
| Tire hardness                \( K_t \)    | 190000   | N / m    |
| Spring hardness              \( K_s \)     | 16812    | N / m    |
| Vibration damping factor     \( C_s \)     | 1000     | Ns / m   |

![Figure 1. ¼ Model of the vehicle suspension system [2].](image)

The system includes: the part \( M_2 \) of the vehicle mass acting on the spring; the tire and suspension weight, \( M_1 \), is below the spring; spring rate \( K_s \); coefficient of reduction of displacement of hydraulic damping \( X_s \), \( X_w \), distance \( r \) of the corresponding position of the mass. Using Newton’s second law, we obtain the equation of dynamics as follows:

\[
-K_t (X_w - r) - K_s (X_w - X_s) - C_s (\dot{X}_w - \dot{X}_s) + U_a = M_1 \ddot{X}_w \quad (1)
\]

\[
-K_s (X_s - X_w) - C_s (\dot{X}_s - \dot{X}_w) - U_a = M_2 \ddot{X}_s \quad (2)
\]

In order to design a control system based on linear optimal integration standards, control parameters were chosen to determine the minimum for the quality function. Parameters shown in equations (1) and (2) are estimated using the following correlations:

\[
\dot{X}_1 = \dot{X}_s - \dot{X}_w \approx X_2 - X_4; \dot{X}_2 = \dot{X}_s; \dot{X}_3 = \dot{X}_w - \dot{r} \approx X_4 - \dot{r}; \dot{X}_4 = \dot{X}_w.
\]

2.2 Establishing the LQR control algorithm

LQR is one of the most common control methods commonly used to study active suspension model. Many studies have been conducted and it has been concluded that the LQR control method will give the best results in terms of the vehicle’s ride on the road [1, 2].
A suitable linear full-state feedback control law used as,

$$u(t) = -Kx(t)$$  \hspace{1cm} (3)

K - the state of the feedback matrix

The optimization process consists of identifying the control input $U$, which helps to minimize performance indicators. Performance index $J$ represents performance requirements as well as controller input limits. An optimal system controller is defined as a controller design that minimizes the following performance index as shown in the following equation:

$$J = \frac{1}{2} \int_0^t (x^T Q x + u^T R u) dt$$  \hspace{1cm} (4)

Amplifier factor $K$ is denoted as:

$$K = R^{-1}B'P$$

Matrix $B$ shall satisfy the abbreviated Riccati equation:

$$A'P + PA - PBR^{-1}B'P + Q = 0$$  \hspace{1cm} (5)

Then the information controller $U$

$$u(t) = -(R^{-1}B'P)x(t) = -Kx(t)$$  \hspace{1cm} (6)

To design a control system based on an optimal control method (LQR), control parameters are selected from the conditions of minimum effort for the quality function. The values of the parameters $X_s, X_w, r$ are determined using the following correlations:

$$X_1 = X_s - X_w; X_2 = X_s; X_3 = X_s - r; X_4 = X_w;$$

From there, the above equation can be rearranged as:

$$\dot{X}(t) = Ax(t) + B_u(u(t)) + f(t)$$

We get:

$$\dot{X}_1 = \dot{X}_2 = \dot{X}_3 = \dot{X}_4 = \dot{X}_w$$

From a continuous feedback state model: $U = -Kx$

Where $K$ is a feedback matrix designed in such a way that the objective quality function is minimal. We will use Matlab to find the response matrix $K$ in accordance with linear optimal integration standards:

$$K = lqr (A, B, Q, R);$$

Using the above command in MATLAB, we determine the result of the feedback matrix $K$. From the calculation parameters, Q,R, and K can be computed as following:

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

Value $R = 0.00001$

Determining the value of the feedback matrix

$$K = \begin{bmatrix} 2750 & 9720 & -206400 & -8240 \end{bmatrix}$$
3. Application of MATLAB / SIMULINK to simulate the dynamics of a suspension system

3.1 Setting the road profile signal simulation
When analyzing the suspension model, it is necessary to determine how the vehicle performs in different road conditions. To match the actual traffic conditions, a simulation model for the road profiles is established (Figure 2) [4, 5].

![Figure 2. An example of a road profile with a single bump.](image)

3.2 Designing an optimal control system for an active suspension system
SIMULINK is a MATLAB extension that aims to model and research dynamic systems. The graphical interface on the SIMULINK screen allows you to represent the system in signal diagrams with commonly used function blocks. From the data warehouse of the SIMULINK model, it is possible to generate a control diagram, based on the optimal model of the problem of controlling the active suspension of a vehicle to establish the optimal control model in accordance with the block diagram (Figure 3) [6, 8, 10].

![Figure 3. Block diagram of active suspension control.](image)
3.3 Simulation results

![Graphs showing force and mixing suspended mass](image)

**Figure 4.** The generate force and displacement of the suspended mass along the profile of the road of category 1.

![Graphs showing body displacement acceleration and wheel deflection](image)

**Figure 5.** Change in the acceleration of body displacement and wheel deflection in time with the profile of a road of category 1.

![Graphs showing wheel deflection and suspension travel](image)

**Figure 6.** Wheel deflection and suspension travel in time with road profiles of category 1.

4. Conclusion

By comparing the characteristics of the passive and active suspension using the LQR control technique, it is possible to display the lower amplitude of the active suspension and oscillation damping time, with the control algorithm developed, it is possible to reduce the amplitude and shorten the time compared to the...
passive suspension. Body acceleration is also improved, with less vibration amplitude than the acceleration created when utilizing the passive suspension. Wheel displacement of an active suspension system has a smaller amplitude than that of the passive suspension, but still provides a quick damping time.

The use of an active suspension in a vehicle offers much better advantages than a passive suspension, since the active suspension needs to control the force acting mainly in the 5000N range. Application of SIMULINK in the field of optimal control is a modern control method that is widely used in the field of automatic control.

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