Investigating an online social network using spatial interaction models

Conrad Lee\textsuperscript{a,1,*}, Thomas Scherngell\textsuperscript{b}, Michael J. Barber\textsuperscript{b}

\textsuperscript{a}Clique Research Cluster, University College Dublin, Ireland
\textsuperscript{b}AIT Austrian Institute of Technology GmbH, Foresight and Policy Development Department, Vienna, Austria

Abstract

Using a spatial interaction modeling approach, we investigate the German collegiate social network site StudiVZ. We focus on identifying factors that foster strong inter-institutional linkages, testing whether the acquaintanceship rate between institutions of higher education is related to various geographic and institutional attributes. We find that acquaintanceship is most significantly related to geographic separation: measuring distance with automobile travel time, acquaintanceship drops by 91\% for each additional 100 minutes. Institution type and the former East-West German divide are also related to this rate with statistical significance.

Keywords: online social networks, spatial interaction modeling, generalized linear models, separation measures, StudiVZ

1. Introduction

Recent research of social network data indicates that the strength of flow between cities can be approximated by a gravity law. That is, the flow between a pair of cities—which may represent people, communication, goods, or money—is proportional to the product of their sizes divided by the square of their distance \cite{Lambiotte2008, Krings2009}. However, flows between cities depend upon more than their sizes and the distance that separates them; they also depend upon cultural factors such as language, region, or economic similarity. For example, the work of Blondel et al. \cite{Blondel2008} suggests that the Belgian mobile communication network splits rather cleanly into two large-scale communities: French and Dutch speakers. Using the network of flows of currency between cities, Brockmann \cite{Brockmann2010} found that flows do not continuously decrease as a function of distance, but rather are affected by sharp cultural boundaries embedded in geographic space.

\*Corresponding author
\textsuperscript{1}E-mail: conradlee@gmail.com

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Given a social network that contains data on both geographic location and various other node attributes such as language, it is natural to hypothesize that some node attribute is related to geographic flow. In this paper, we argue that spatial interaction models are useful for statistically testing such hypotheses. Spatial interaction modeling focuses on estimating the variation of flows between locations in geographic space, such as regions or cities (see, for example, Sen and Smith, 1995). While spatial interaction modeling is widely used in the fields of economics and economic geography to understand and predict interaction behavior in a system of spatial units (for an overview, see Fotheringham and O’Kelly (1983)), it has only rarely been applied to social network data (Scherngell and Barber, 2009).

We use spatial interaction models to investigate data from the social network site (SNS) StudiVZ, a collegiate online social network popular in German-speaking countries. In particular, we consider acquaintanceships between students at the University of Bielefeld (UniBi) and students at other institutions. We determine how various geographic and institutional attributes—including institution type and region—affect the number of acquaintanceships between these institutions.

We use these attributes to define several measures of separation between institutions, with their importance assessed by comparing to the data. We consider two similar models, the first with fewer and rougher separation measures, and the second with more and finer separation measures. For both models, we find that the most pronounced separation measure is geographic distance, with institution type (e.g., university, technical college, art school) also playing an important and significant role. Regional factors play a less pronounced, yet statistically significant, role.

In Section 2, we give an overview of StudiVZ and describe how we collected the data on its linkage structure. We begin Section 3 by briefly introducing spatial interaction modeling in general; we then explain the details of the two particular models that we propose. Estimation results for the models are presented in Section 4. We summarize and discuss our results in Section 5.

2. StudiVZ

2.1. Overview

StudiVZ is a social network site for German post-secondary students that was created in October 2005. As reported by Bakst (2006), StudiVZ was inspired by and intentionally imitated the leading U.S. social networking service for students, Facebook, and the site therefore shares many features with Facebook. Within months of its creation, StudiVZ became the dominant social networking service for students in Germany, Austria, and regions of Switzerland.

Social network sites have been defined by Boyd and Ellison (2007) as web services that allow individuals to (1) construct a public or semi-public profile, (2) articulate a list of other users with whom they are connected, and (3) view and traverse connections made by others. In addition to these basic capabilities, important functions of StudiVZ and most other SNSs include semi-public
photo-sharing, “walls” (i.e., message boards), and status updates—all linked to individuals’ profile pages. Users of SNSs are often highly engaged; for example, at the time we collected data, StudiVZ users averaged about one visit per user per day, and about 30 page impressions per visit (Hensen, 2008).

While one might expect much of the interaction between users of these sites to take place between people who have met only on the internet (as is often the case for message boards and chat rooms), Boyd and Ellison note that the relationships articulated in SNSs tend to have an offline origin. This claim has been supported by the findings of Mayer and Puller (2008), who report that only 0.4% of the Facebook friendships they studied appear have originated as “merely online friendships,” as well as other empirical studies (Haythornthwaite, 2005; Lampe et al., 2006; Ellison et al., 2007). This offline basis suggests that conclusions based on SNS data are more likely to be generalizable to everyday, personal social interaction than data which comes from other online services such as chat rooms or message boards, where social interaction is often anonymous.

Despite the correspondence between online and offline social contact, it is difficult to interpret the meaning of SNS relationships, labeled in StudiVZ as “friendships.” In the StudiVZ data that we analyze in this paper, over half of users listed more than 48 “friends,” and over a quarter of users had more than 86 friends. In a recent dataset from the more mature Facebook, where users have had more time to accumulate friends, over half of users listed more than 100 friends (Lewis et al., 2008). Such large degrees indicate that many of these so-called friendships are not close, active friendships, but rather latent friendships or acquaintanceships, such as old high-school friends or colleagues from previous jobs. Nevertheless, for the rest of the paper, all of these ties will be referred to as friendships.

Within the StudiVZ system, users associate themselves with the institutions they attend. As we collected data in January 2008, StudiVZ reported 4.5 million members (Hensen, 2008). If one considers that a total of 2.47 million students were enrolled at higher-learning institutions in Germany, Austria, and Switzerland in the academic year 2007/2008, it appears that not only had StudiVZ largely saturated its target audience of students, but also that the site had attracted users from other audiences, perhaps former students, exchange students, and young people who were not students. These users presumably associated themselves with institutions that they did not attend.

2.2. Data Collection

We retrieved data in late January, 2008. The data collected that is relevant to this paper is the friendship data, which at the time of collection was publicly visible for all profiles. As mentioned above, a “friendship” in StudiVZ is the primary indicator of relation, and is formed when one user requests another

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2 In the academic year 2007/2008, Germany reported 1.97 million post-secondary students (Hochschulrektorenkonferenz, 2008), Austria reported 272,003 (Gumpoldsberger and Nitsch, 2009), and Switzerland reported 225,862 thousand (Bundesamt für Statistik, 2009).
user to confirm him as a friend. The information concerning the direction of the request (i.e., who requested whom) is not public, and therefore we treat all friendships as undirected.

We collected the friendship data using snowball sampling, such that we were left with all friendship data of UniBi users in the giant connected component of StudiVZ's UniBi subgraph. This data set included 29,192 users. The users we identified at UniBi listed over 1.4 million friendships. Around 350,000 of these friendships are among the 29,192 UniBi users; we refer to these as internal friendships. Approximately 1.05 million friendships were between UniBi students and some 367,000 students associated with other higher-learning institutions; we call these external friendships. Because at the time of collection StudiVZ enjoyed such widespread popularity among German college students, we contend that a considerable proportion of all acquaintanceships between the students of the University of Bielefeld and students at other institutions is included in the external friendships.

Here we examine only the external friendships aggregated at the level of institutions of higher education (henceforth labeled as simply “institutions”). More specifically, a weight was determined for the connection between each of 304 German institutions and UniBi. The weight between UniBi and some other institution corresponds to the number of friendships between all StudiVZ users in UniBi and all StudiVZ users at that institution.

We exclude from consideration the four nearest institutions, each with a travel time of under thirty minutes to UniBi, because these institutions have special relationships with UniBi (e.g., shared dormitories, libraries, and presumably other, unknown arrangements). These relationships are not accounted for in our model, leading it to drastically under-predict the rate of friendship with these four institutions. Other institutions that are nearby, but outside this thirty-minute limit, do not show a marked mismatch with the model predictions. Additionally, we consider only institutions in Germany, for which information on addresses, institution type, and enrollment size is consistently available from the website of the German Rectors’ Conference.

3. Spatial Interaction Model Definition

To investigate the StudiVZ network, we adopt a spatial interaction modeling perspective. Spatial interaction modeling focuses on estimating the variation of flows between locations in geographic space, such as regions or cities (see, for example, Sen and Smith [1995]). Said flows, known as spatial interactions, can be of many forms, including people, goods, money, or knowledge. Spatial interaction modeling is widely used in the fields of economics and economic geography to understand and predict interaction behavior in a system of spatial units; for an overview, see Fotheringham and O’Kelly [1989]. Here, we will investigate flows of social interactions, in the form of online friendships.

http://www.hochschulkompass.de/hochschulen/download.html
The specific technique we will use is that of generalized linear models. The usual approach is to assume that the spatial interaction $F_{ij}$ between an origin location $i$ and a destination location $j$ has a product form given by

$$F_{ij} = A_i B_j S_{ij}, \quad (1)$$

where $A_i$ is the origin function, $B_j$ is the destination function, and $S_{ij}$ is the separation function. As StudiVZ friendships are inherently symmetric, we can without loss of generality assume that UniBi is always the destination, reducing the model to a one-dimensional version given by

$$F_i = c A_i S_i. \quad (2)$$

In Eq. (2), $F_i$ and $S_i$ are the source-only analogs of $F_{ij}$ and $S_{ij}$, respectively. The destination function $B_j$ has been replaced by a constant term $c$, reflecting the contribution from UniBi. As $A_i$ and $S_i$ both have the same dependence on $i$, the two terms could be merged into one, but we will treat them separately to maintain the distinction between properties of just the origin institution and properties dependent upon the separation between the origin and destination (i.e., UniBi) institutions.

We take the origin function $A_i$ to be

$$A_i = a_i^\alpha, \quad (3)$$

where $a_i$ is the number of students enrolled at institution $i$ during the 2007/2008 academic year, accounting for differences in the sizes of the institutions; $\alpha$ is a parameter to be determined. We take the separation function $S_i$ to include $N_d$ measures of separation, with a general form of

$$S_i = \exp \left( \sum_{k=1}^{N_d} \beta^{(k)} d_i^{(k)} \right). \quad (4)$$

The parameters $\beta^{(k)}$ weigh various separation measures $d_i^{(k)}$ against one another. For notational convenience, we similarly rewrite the destination constant as an exponential, $c = \exp \gamma$.

With the origin and separation functions chosen as in Eqs. (3) and (4), the spatial interaction model in Eq. (2) is a generalized linear model. A common approach for determining the model parameters is standard OLS estimation (Bergkvist and Westin, 1997). However, as the $f_i$ are non-negative integers, OLS estimation is inappropriate, being equivalent to assuming the residuals $F_i - c A_i S_i$ for Eq. (2) are normally distributed. In the present case, a discrete data generating process would then be approximated by a misrepresentative continuous process. Instead, we assume a negative binomial model specification

$$P(f_i) = \frac{\Gamma(f_i + \delta^{-1})}{\Gamma(f_i + 1) \Gamma(\delta^{-1})} \left( \frac{\delta^{-1}}{c A_i S_i + \delta^{-1}} \right)^{f_i} \left( \frac{c A_i S_i}{c A_i S_i + \delta^{-1}} \right)^{\delta^{-1}}, \quad (5)$$
where $\Gamma(\cdot)$ denotes the Gamma function and $\delta$ is a dispersion parameter; in the limit as $\delta \to 0$, Eq. (5) reduces to a Poisson model specification. We estimate model parameters with maximum likelihood procedures using Newton-Raphson.

For more detail on the derivation and properties of negative binomial models like those in Eq. (5), see Cameron and Trivedi (1998).

The separation function constitutes the core of a spatial interaction model, and is of central importance in the context of the research questions of the current study. We use a multivariate exponential separation function (Sen and Smith, 1995), as defined in Eq. (4), that provides a flexible framework for representing different kinds of separation. In the current paper, we focus on $N_d = 10$ separation measures.

We distinguish between geographic separation effects, separation effects related to the federal states of Germany (which will henceforth be referred to simply as states), and institutional separation effects. Geographic separation effects are captured by $d_{i}^{(1)}$, the logarithm of the automobile travel time in minutes between UniBi and the other institutions. We augment this with $d_{i}^{(2)}$, a dummy variable that measures separation effects related to the states of Germany. It is defined as a binary variable set to one if institution $i$ is located in North Rhine-Westphalia, the same state as UniBi, and zero otherwise.

Further, we take into account separation effects related to the type of the institutions. The dummy variable $d_{i}^{(3)}$ accounts for the type of institution, with its value set to one if one institution $i$ is the same type of institution as UniBi, and zero otherwise. Here we use information on the organization of the German post-secondary education system, which contains several types of institutions. Our dataset includes general universities, Fachhochschulen (officially translated as “universities of applied sciences”, and roughly described as technical colleges or trade schools—these will be referred to as “applied universities”); religious institutions; art schools, including music schools; teacher colleges; and private institutions.

With this information we further refine $d_{i}^{(3)}$ into separate dummy variables for each institution type, more precisely reflecting how each type of institution affects interaction probability. Thus, we set a value of one for $d_{i}^{(4)}$ if institution $i$...
is an applied university, for $d_i^{(5)}$ if institution $i$ is a religious, for $d_i^{(6)}$ if institution $i$ is an art school, for $d_i^{(7)}$ if institution $i$ is a teacher college, for $d_i^{(8)}$ if institution $i$ is a private institution, and for $d_i^{(9)}$ if institution $i$ is a general university.

Due to the enormous political and institutional changes during the reunification of the Federal Republic of Germany and the German Democratic Republic in 1990, we considered a variable that reflects predominant barriers between these major blocks. Thus, we introduce $d_i^{(10)}$, a dummy variable that is set to one if institution $i$ is located in the former German Democratic Republic, and zero otherwise.

4. Estimation Results

In this section, we discuss the estimation results of the negative binomial spatial interaction model, as defined by Eq. (5). The dependent variable is the observed number of friendships between institution $i$ and the University of Bielefeld. The independent variables are origin and separation measures as defined in Section 3. We consider a basic model version including separation measures for the logarithmic geographic distance $d_i^{(1)}$, the same state $d_i^{(2)}$, and the same category $d_i^{(3)}$. In an extended model version, we replace $d_i^{(3)}$ with specific dummy variables for each category ($d_i^{(k)}$ for $k = 4, 5, \ldots, 9$). The extended model serves to check robustness of the other model parameters $d_i^{(1)}$ and $d_i^{(2)}$, while also providing additional information on specific effects related to different institution types.

In Table 1, we present the sample estimates of the spatial interaction model parameters, along with tests of statistical significance (Greene, 2003). The second and third columns contain modeling results for the basic and extended models, respectively. There are 304 observations. The estimate of the dispersion parameter $\delta$ is 0.51 in the basic model version and 0.45 in the extended model version; the estimates are statistically significant. Thus, we conclude that unobserved heterogeneity, not captured by the covariates, leads to overdispersion (Cameron and Trivedi, 1998) and that the negative binomial specification of Eq. (4) is a more appropriate choice than would be, e.g., a simpler Poisson specification.

The model diagnostics underpin the statistical significance of the models. The likelihood ratio test is significant for both model versions. To compare the two models, we draw on the Bayesian Information Criterion (BIC), which is widely used for comparing model fit of non-nested models (see, for instance, Raftery, 1995). It can be seen that the extended model version fits better to the data. This is also reflected by McFadden’s $R^2$ and Nagelkerke’s $R^2$ in terms of the explained variance by the covariates (for a formal description of the model diagnostics used, see Long and Freese, 2001).

Turning our attention first to the results of the basic model in the second column, we note that increasing travel time between the University of Bielefeld and other institutions $i$ has a significant negative effect on the number of friendships.
| Model Version | Basic          | Extended        |
|---------------|---------------|-----------------|
| Destination constant $\gamma$ | $9.37(0.49)^{***}$ | $9.59(0.55)^{***}$ |
| Origin exponent $\alpha$ | $0.76(0.03)^{***}$ | $0.71(0.05)^{***}$ |
| Log geographic distance $\beta^{(1)}$ | $-1.45(0.09)^{***}$ | $-1.43(0.08)^{***}$ |
| Same state $\beta^{(2)}$ | $-0.32(0.16)^{*}$ | $-0.32(0.15)^{*}$ |
| Same category $\beta^{(3)}$ | $-0.58(0.12)^{***}$ | $-$ |
| Applied University $\beta^{(4)}$ | $-$ | $-0.56(0.16)^{***}$ |
| Religious Institution $\beta^{(5)}$ | $-$ | $-0.98(0.24)^{***}$ |
| Art School $\beta^{(6)}$ | $-$ | $-0.73(0.21)^{***}$ |
| Teacher College $\beta^{(7)}$ | $-$ | $-0.26(0.31)$ |
| Private $\beta^{(8)}$ | $-$ | $-0.28(0.21)$ |
| General University $\beta^{(9)}$ | $-$ | $0.20(0.17)$ |
| Former GDR $\beta^{(10)}$ | $-$ | $-0.27(0.11)^{*}$ |
| Dispersion parameter $\delta$ | $0.51(0.04)^{***}$ | $0.45(0.04)^{***}$ |
| McFadden’s Adjusted $R^2$ | 0.115 | 0.139 |
| Cragg-Uhler (Nagelkerke) $R^2$ | 0.846 | 0.900 |
| Log-likelihood | $-2106.52$ | $-2085.83$ |
| Likelihood ratio $\chi^2$ | 659.99 | 701.37 |
| Akaike Information Criterion | 14.998 | 13.802 |
| Bayesian Information Criterion | 2609.460 | 2502.287 |

*p < 0.05  **p < 0.01  ***p < 0.001

Table 1: Parameter estimates for the spatial interaction models. Fit parameters for the spatial interaction models are based on 304 institutions. Parenthetical values show the standard errors. Asterisks indicate the statistical significance of the parameter fits. Performance statistics indicate that the extended model better explains the data.
This result indicates that StudiVZ is not a virtual world in which geographic location is unimportant. The parameter estimate of $\beta^{(1)} = -1.45$ in the basic model indicates that for each additional 100 minutes of travel time to the University of Bielefeld, the mean friendship probability decreases by 91%. The parameter $\beta^{(2)}$ is estimated to be -0.3 and is statistically significant, indicating that the rate of acquaintanceship decreases when institution $i$ is located in a different state than the University of Bielefeld. However, more important than whether $i$ is located in the same state as the University of Bielefeld is whether $i$ is in the same institutional category. The parameter $\beta^{(3)}$ is estimated to be -0.7, and reveals an interesting tendency: students at the University of Bielefeld, which is a general university, tend to have a higher rate of acquaintanceship with students from other general universities than with students attending other types of institutions.

The parameter estimates of the extended model, displayed in the third column, include more detailed information about how region and institution type affect the rate of acquaintanceship. While the estimate for the same state remains unchanged at -0.3, our estimate for the $\beta^{(10)}$ parameter, also -0.3, indicates that those $i$ located in former East Germany tend to have a lower rate of acquaintanceship than those $i$ located in former West Germany. Furthermore, the more specific treatment of institution type in the extended model reveals that UniBi has especially low acquaintanceship rates with religious and art-focused institutions, while the negative effects associated with applied technical colleges, private institutions, and teacher colleges are less pronounced.

In the extended model version, the parameter estimate of log travel time $\beta^{(1)}$ slightly decreases to a value of $-1.43$, indicating that the estimated parameter is quite robust. The slight decrease might suggest that travel time is to some extent a proxy for other unobserved characteristics in the basic model that are better reflected by additional independent variables in the extended model.

5. Conclusion

In this paper, we have used spatial interaction models to investigate StudiVZ friendships aggregated at the level of institutions of higher education. We have focused identifying institutional attributes that are related to the strength of ties between institutions. Our results indicate that geographic distance is the most important separation variable: for each additional 100 minutes of drive time separating UniBi from some other institution, the mean acquaintanceship rate with that institution drops by approximately 90%. StudiVZ is thus not a virtual world in which physical distance is insignificant. This finding is in agreement with a previous study of LiveJournal, a blogging service with SNS features [Liben-Nowell et al., 2005].

Additionally, we found that students at UniBi are most likely to form relationships with students at “peer institutions,” i.e., students at other universities and technical universities, and that they are less likely to form acquaintanceships with students at teacher colleges, applied technical colleges, art schools,
and religious institutions. Acquaintanceship is also (albeit less prominently) affected by regional characteristics: students at UniBi are more often acquainted with students in their own federal state (North Rhine-Westphalia), and to the same extent they more frequently are friends with students from former West Germany than students from former East Germany.

One important line of future work is to confirm these results with network data that is more complete. Our results are based on only those edges that have one end connected to a student at UniBi, and they are therefore skewed to reflect the preferences of students at that institution. With complete network data one could test whether these preferences closely resemble those of German students in general.

More broadly, the spatial interaction modeling approach taken in this paper could be applied to other networks and with other separation measures. Depending on the data available, models of greater sophistication—either conceptually or methodologically—may become relevant. For example, spatial interaction models may include production constraints or attraction constraints [Sen and Smith, 1995], and spatial autocorrelation may be taken into account using spatial filtering techniques [Fischer and Griffith, 2008].

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