Preparation of polarization entangled mixed states of two photons

Chuanwei Zhang

We propose a scheme via controlled localization of optical fibers, and mixed states of two photons. Entanglement has played a crucial role for many applications of quantum information, such as quantum teleportation [1], superdense coding [2], and quantum error correction [3], etc. To function optimally these applications requires maximal pure entanglement. However, unmaximal pure entanglement can be realized by using a fast switch to combine different modes, in addition to a pulsed pump laser. However, this method is not very practical although it can be implemented in principle. A practical replacement can be a passive coupler without switch although it will decrease the optimal success probability due to losses in the coupling. Denote $L_{i}^{A(B)}$ as the optical path lengths of paths $i_{A(B)}$ (from the BBO crystal to coupler $G_{A(B)}$). If $L_{i}^{A(B)}$ satisfy $L_{i}^{A} = L_{i}^{B}$ and $\Delta L_{ij}^{A(B)} = L_{i}^{A(B)} - L_{j}^{A(B)} \gg l_{coh}$ for different $i$ and $j$, the location modes will be traced out and we obtain a mixed state, where $l_{coh}$ is the single-photon coherence length.

The mixed state still contains information of the location modes since photons from different paths $i$ will arrive at the detector at different time. Therefore the mixed state is composed by two discrete subspace state: two photons $A$, $B$ arriving at same time and at different time. If the time window of the coincidence counter is small enough, only photons from paths with same lengths ($i_{A}$ and $i_{B}$) contribute the counts and the polarization states of photons are reduced to the subspace state with

\begin{equation}
\rho = \sum_{i=1}^{4} p_{i} |\psi_{i}\rangle_{AB} \langle \psi_{i}|,
\end{equation}

where $0 \leq p_{i} \leq 1$, $\sum_{i=1}^{4} p_{i} = 1$, and $p_{i} \geq p_{j}$ for $i \leq j$. $|\psi_{i}\rangle_{AB}$ are two-qubit pure states with same entanglement of formation (EOF) as $\rho$. Therefore $|\psi_{i}\rangle$ are same up to some local unitary operations $U_{i}$, i.e. $|\psi_{i}\rangle = U_{i} \otimes V_{i} |\Phi\rangle$, where $U_{i}$ and $V_{i}$ are local unitary operations and $|\Phi\rangle = \cos \theta |00\rangle + \sin \theta |11\rangle$ with $0 \leq \theta \leq \pi/4$.

The experimental arrangement for our scheme is described as Fig.1. First, spontaneous parametric down-conversion in two adjacent $\beta$-barium borate (BBO) crystals produces initial two photons polarization-entangled pure state $|\Phi\rangle_{AB} = \cos \theta |HH\rangle + \sin \theta |VV\rangle$ [14, 17], where $|H\rangle$ and $|V\rangle$ are horizontal and vertical polarization respectively. Then six beam splitters with variable transmission coefficients (VBS) couple the initial polarization state $|\Phi\rangle_{AB}$ to location modes and each photon has four possible optical paths $i_{A(B)}$. At each path, single-qubit polarization rotations (SPR) perform local unitary operations $U_{i}$, $V_{i}$ on the polarization mode of each photon and transform initial entangled state $|\Phi\rangle_{AB}$ to different $|\psi_{i}\rangle_{AB}$. The four paths of each photon mix on couplers $G_{A(B)}$ and become one through single-mode optical fibers [16]. In experiment, lossless mixture process can be realized by using a fast switch to combine different modes, in addition to a pulsed pump laser. However, this method is not very practical although it can be implemented in principle. A practical replacement can be a passive coupler without switch although it will decrease the optimal success probability due to losses in the coupling.

Preparation of polarization entangled mixed states of two photons

Department of Physics and Center for Nonlinear Dynamics, The University of Texas, Austin, Texas 78712-1081

PACS numbers: 03.67.-a, 42.25.Ja, 03.65.Ud, 89.70.+c

arXiv:quant-ph/0107145v2 21 Nov 2003

Figure 1: Experimental set-up for preparing an arbitrary polarization-entangled mixed state of two photons.
same arrival time. In this subspace, the state is the two-qubit mixed state $\rho$. Therefore the final SF each arm, along with PBS, enable analysis of the polar-
ization correlations in any basis, allowing tomogr;
reconstruction of the density matrix [4] 12 13.

In our scheme, we require that the coherence $k$ of pump laser is much smaller than path length difference $\Delta_{ij}$ in order to avoid two-photons interference [15]. two-photons interference can be used to real Franson-type test of Bell inequalities, where the length difference is 2 orders of magnitude smaller the coherence length of the pump laser. In our scheme,
we require that the path length difference is larger both single photon and pump laser coherence lengths to avoid both single and two photons interference. In this case, the travel time of a photon pair from laser to detector enables to resolve the different paths in principle. It is therefore important to post-select only photons arriving in coincidence but not to resolve the travel time from emission to detection in order to trace out all location modes.

The VBS in the scheme can be implemented using a one-order Mach-Zehnder interferometer [14, 20], and it transforms location modes in the following way

$$|a\rangle_{\text{initial}} \rightarrow \sqrt{\eta_i} |a\rangle_{\text{final}} + \sqrt{1 - \eta_i} |b\rangle_{\text{final}},$$

where $|a\rangle$ and $|b\rangle$ are the location modes and $\sqrt{\eta_i}$ are the transmission coefficients. The $SP_{A|B}^i$ at paths $i_{A|B}$ can be constructed with wave plate sequences \{QWP, HWP, QWP\} [20] and they perform unitary operation $U_i^A V_i^B$. The coupler $G_{A|B}$ introduces decoherence, which yields mixed state

$$\rho_0 = \sum_{i,j=1}^4 p_{ij} U_i \otimes V_j \langle \Phi | U_i^\dagger \otimes V_j^\dagger | \Phi \rangle,$$

where $p_{ij}$ is the combined probability with photons $A$ and $B$ at paths $|i\rangle_A, |j\rangle_B$ respectively. If the time window of the coincidence counter $T$ satisfies $T < \Delta_{ij}/c$ ($c$ is the velocity of the light), only photons with location modes $|i\rangle_A |j\rangle_B$ are registered by the coincidence counter, and density matrix $\rho_0$ is reduced to

$$\rho = \frac{1}{F} \sum_{i=1}^4 p_i |\psi_i\rangle \langle \psi_i|,$$

where $F = \sum_{i=1}^4 p_i$ is the successful probability of generating the mixed state $\rho$.

Denote $p_i = p_{ii}/F$, the remaining problem is to find the optimal successful probability $F$. For a given density matrix of a mixed state, there are several choices of $|\psi_i\rangle$ (to be realized using different local operation and different initial entangled states) and thus also different beamsplitter settings. The best choice is the one where the success probability is maximized. From Fig.1, we find $p_{11} = n_1 n_2 n_3 n_5, p_{22} = n_1 n_2 (1 - n_3) (1 - n_5), p_{33} = (1 - n_1) (1 - n_2) n_4 n_6$, and $p_{44} = (1 - n_1) (1 - n_2) (1 - n_4) (1 - n_6)$. Assume $p_1 \geq p_2 \geq p_3 \geq p_4, p_1 > 0$, and $A_i = p_i/p_1$, the optimal values of $F$ can be obtained using Lagrangian Multipliers and the results are classified as

1. If $A_i > 0$ for $i = 2, 3$, then $n_1 = n_2 = (1 + \sqrt{A_2})/(\sum_{i=1}^4 \sqrt{A_i}), n_3 = n_5 = 1/(1 + \sqrt{A_2}), n_4 = n_6 = \sqrt{A_5}/(\sqrt{A_2} + \sqrt{A_4})$, and $F_{\text{optimal}} = (\sum_{i=1}^4 A_i)/(\sum_{i=1}^4 \sqrt{A_i})^2$.

2. If $A_2 > 0$, $A_3 = A_4 = 0$, then $n_1 = n_2 = n_4 = n_6 = 1, n_3 = n_5 = 1/(1 + \sqrt{A_2})$, and $F_{\text{optimal}} = (1 + A_2)/(1 + \sqrt{A_2})^2$.

3. If $A_2 = A_3 = A_4 = 0$, then $n_1 = 1$, and $F_{\text{optimal}} = 1$.

So far we have described a scheme for preparing an arbitrary polarization-entangled mixed state of two photons using variable beam splitters and single mode optical fibres. In practical quantum information process, we often use a special set of mixed states with the form $\rho = p |\psi\rangle \langle \psi| + (1 - p) |\phi\rangle \langle \phi|$, where $0 \leq p \leq 1$ and $|\psi\rangle, |\phi\rangle$ are arbitrary two-qubit pure states. Our scheme can be simplified for this special mixed state. Assume $|\psi\rangle = U_1 \otimes V_1 |\Phi (\alpha)\rangle, |\phi\rangle = U_2 \otimes V_2 |\Phi (\beta)\rangle$ with $|\Phi (\theta)\rangle = (\cos \theta |HH\rangle + \sin \theta |VV\rangle)$, and $0 \leq \beta \leq \alpha \leq \pi/4$, the experimental arrangement may be described by the schematic in Fig. 2.

In Fig.2, VBS and SMOF perform same operations as those in Fig.1. The distillation filters are used for entanglement transformation [4] that is performed on location $|1\rangle_A$ or $|2\rangle_A$, depending on the initial state $(|1\rangle_A$ corresponds to transformation $|\Phi (\beta)\rangle \rightarrow |\Phi (\alpha)\rangle$ and $|2\rangle_A$ to $|\Phi (\alpha)\rangle \rightarrow |\Phi (\beta)\rangle$). The decoherence process yields dif-

![Figure 2: Experimental set-up used to generate mixed states $\rho = p |\psi\rangle \langle \psi| + (1 - p) |\phi\rangle \langle \phi|$](image)

![Figure 3: The final successful probabilities $P (a), P’ (b)$ versus transmission coefficient $n_1$. $A = 10000$ (solid), $A = 1$ (dot), $A = 0.0001$ (dash). (a) $k_1 = 0.8$; (b) $k_2 = 0.7$.](image)
In different final states for different initial states $|\Phi(\beta)\rangle$ and $|\Phi(\alpha)\rangle$, 

$$
\rho = \frac{1}{P} (k_1 \eta_1 \eta_2 |\psi\rangle \langle \psi| + (1 - \eta_1) (1 - \eta_2) |\phi\rangle \langle \phi|), \tag{5}
$$

$$
\rho' = \frac{1}{P'} (\eta_1 \eta_2 |\psi\rangle \langle \psi| + k_2 (1 - \eta_1) (1 - \eta_2) |\phi\rangle \langle \phi|),
$$

where $k_1 = \sin^2 \beta / \sin^2 \alpha$ ($k_2 = \cos^2 \alpha / \cos^2 \beta$) is the maximally feasible transformation probability in experiment from $|\Phi(\beta)\rangle$ to $|\Phi(\alpha)\rangle$ (from $|\Phi(\alpha)\rangle$ to $|\Phi(\beta)\rangle$). $P = k_1 \eta_1 \eta_2 + (1 - \eta_1) (1 - \eta_2)$ and $P' = \eta_1 \eta_2 + k_2 (1 - \eta_1) (1 - \eta_2)$ are the successful probabilities to obtain $\rho$ and $\rho'$. The transmission coefficients $\sqrt{\eta_1}$ and $\sqrt{\eta_2}$ satisfy conditions $(1 - \eta_1) (1 - \eta_2) = Ak_1 \eta_1 \eta_2$ for $\rho$ and $k_2 (1 - \eta_1) (1 - \eta_2) = A\eta_1 \eta_2$ for $\rho'$, where $A = (1 - p)/p$. The optimization of $P$ and $P'$ yields that 

$$
P = k_1 (1 + A) / \left(1 + \sqrt{Ak_1}\right)^2, \tag{6}
$$

$$
P' = k_2 (1 + A) / \left(\sqrt{k_2 + \sqrt{A}}\right)^2,
$$

with $\eta_1 = \eta_2 = 1 / (1 + \sqrt{Ak_1})$ for $\rho$ and $\eta_1 = \eta_2 = \sqrt{k_2} / (\sqrt{k_2 + \sqrt{A}})$ for $\rho'$. Direct comparison between $P$ and $P'$ shows that if $0 \leq p \leq \frac{k_1 (1 - \sqrt{\eta_2})^2}{k_1 (1 - \sqrt{\eta_2}) + k_2 (1 - \sqrt{\eta_1})}$, then $P' \leq P$ and $|\Phi(\beta)\rangle$ is chosen as initial state. Otherwise $P \leq P'$ and $|\Phi(\alpha)\rangle$ is used.

In Fig.3, we plot the successful probabilities $P$, $P'$ with respect to the transmission coefficient $\eta_1$. The probability $P$ ($P'$) reaches the maximum at certain $\eta_1$, as predicted by Eq.(6). We notice that there exist fixed points $(\eta_1, P) = (1 / (1 + k_1), k_1 / (1 + k_1))$ and $(\eta_1, P') = (k_2 / (1 + k_2), k_2 / (1 + k_2))$ for arbitrary parameter $A$. If the transmission coefficient $\eta_1$ is selected at those points, the final successful probabilities $P$, $P'$ are constants, independent of the ratio of two components $|\psi\rangle$ and $|\phi\rangle$.

In Fig.4, we plot the optimal probabilities $P$, $P'$ with respect to the parameters $A$. As predicted by Eq.(6), $P \to k_1$, $P' \to 1$ as $A \to 0$; while $P \to 1$, $P' \to k_2$ as $A \to \infty$. These two cases correspond to pure final state $\rho$. We also notice that there exist minimum values of $P$ ($P'$) at $A = k_1 (1/k_2)$.

Fig.5 shows the change of success probability $P(P')$ as $\beta$ increases from 0 to $\alpha$. If $A \leq 1$, $P$ is always less than $P'$ for any $\beta$ and initial state $|\Phi(\alpha)\rangle$ is always used; while for $A > 1$, $|\Phi(\beta)\rangle$ may be used for large $\beta$.

In conclusion, we have described an experimental scheme for producing an arbitrary polarization-entangled mixed state of two photons via controlled location decoherence. The scheme uses only linear optical devices and single-mode optical fibers and may be feasible within current optical technology. We believe the scheme may provide a useful mixed state entanglement source in the exploration of various quantum information processing.

[1] C. H. Bennett et al., Phys. Rev. Lett. 70, 1895 (1993).
[2] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).
[3] D. Gottesman, e-print quant-ph/0004072.
[4] P. G. Kwiat et al., Nature 409, 1014 (2001).
[5] J.-W. Pan et al., Nature 410, 1067 (2001).
[6] W. J. Munro, et al., Phys. Rev. A 64, 030302 (2001).
[7] R. T. Thew and W. J. Munro Phys. Rev. A 64, 022320 (2001).
[8] J. Bouda and V. Buzek, Phys. Rev. A 65, 034304 (2002).
[9] R. Cleve et al., Phys. Rev. Lett. 83, 648 (1999).
[10] G. Bowen and S. Bose, Phys. Rev. Lett. 87, 267901 (2001).
[11] F. Verstraete and H. Verschelde, Phys. Rev. Lett. 90, 097901 (2003).
[12] P. G. Kwiat et al., Science 290, 498 (2000); A. J. Berghund, quant-ph/0010001.
[13] Y.-S. Zhang et al., Phys. Rev. A 66, 062315 (2002).
[14] W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998).
[15] M. A. Nielsen, Phys. Rev. Lett. 83, 436 (1999).
[16] P. G. Kwiat et al., Phys. Rev. Lett. 75, 4337 (1995); Phys. Rev. A 60, 773(R) (1999).
[17] A. G. White et al., Phys. Rev. Lett. 83, 3103 (1999).
[18] W. Tittel et al., Phys. Rev. Lett. 81, 3563 (1998).
[19] M. Reck et al., Phys. Rev. Lett. 73, 58 (1994).
[20] C. Zhang, quant-ph/0104054.
[21] G. Vidal, Phys. Rev. Lett. 83, 1046 (1999).