Thermal network model and experimental validation for a motorized spindle including thermal–mechanical coupling effect

Changjiang Zhou¹ · Zefeng Qu¹ · Bo Hu¹,² · Shengbo Li³

Received: 20 January 2021 / Accepted: 3 May 2021 / Published online: 10 May 2021
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Abstract
Thermal deformation caused by temperature rise affects the contact status of bearings in motorized spindles. In turn, the change in the contact status will affect the temperature rise and thermal deformation of the system. However, the latter has been rarely focused on in the previous literature. Therefore, a thermal network model of motorized spindle is enhanced by considering the thermal–mechanical coupling effect. Then, an iterative method is presented to solve the coupled equations, and a temperature test rig of the motorized spindle is set up to verify the proposed model. The predicted results by the proposed model are compared with the experimental results and the predicted results by the previous model. The relative error between the predicted and experimental results at two test points decreases by 9.56% and 3.44%, respectively, after considering the thermal–mechanical coupling effect. The temperature changes the contact angle and contact load of the bearings, thereby causing changes in frictional heat and temperature field. Such changes cause the difference between the proposed and previous models. The comparison with the experimental results shows that the proposed model with thermal–mechanical coupling effect can obtain a more accurate temperature field than the previous model.

Keywords Motorized spindle · Thermal network model · Thermal-mechanical coupling · Temperature field

1 Introduction
Motorized spindle is a core functional component of computer numerical control (CNC). High transmission accuracy is implemented by the spindle system because the spindle is driven by the motor directly. However, the thermal deformation of the spindle system, caused by uneven temperature rise, reduces the machining precision and reliability of a motorized spindle [1]. Temperature affects not only the execution accuracy of the mechanical system [2] but also the machining accuracy of the machine tool [3, 4]. To compensate for the error induced by thermal deformation, an accurate temperature field prediction model of the motorized spindle is essentially studied.

The temperature rise of the spindle system usually focuses on thermal characteristics. To date, considerable efforts have been dedicated on the thermal characteristics of the spindle system. As the bases of calculation for the temperature in the spindle system, a thermal–dynamic model of bearings was established by Palmgren [5] and the frictional heat of bearings was calculated by Jones [6]. Thereafter, the thermal characteristics of the spindle system have attracted increasing attention. The temperature in the spindle system was determined through a finite difference model (FDM) by Harris [7], but the thermal deformation of the spindle system was not included in their model. Later, a thermal characteristic model with thermal deformation was developed [8]. Finite element method (FEM) was also used to predict the temperature field of the spindle system [9–11].

Recently, motorized spindles, as a core part of CNC, have been a concern. An FDM with heat transfer mechanism of motorized spindle was proposed by Bossmanns [12], and the temperature at several locations matched the measured values.
Chen et al. [13] developed a thermal error model of motorized spindle and found that spindle temperature has the characteristics of time-varying, nonlinearity, and strong dependence on rotation speed. Affected by these factors, the thermal characteristics of motorized spindles are considerably complicated. Afterward, an integrated thermal–mechanical coupled dynamic model of motorized spindle was developed. Li et al. [14, 15] predicted the temperature of the spindle and bearings at different preloads, moments, and speeds. The analysis results showed that the predicted temperature matched the measured one, except for the temperature of the shell near the motor. Chen [16] analyzed the effects of speed, bearing configuration, and coolant flow on the temperature through a temperature rise experiment. The result indicated that reasonable bearing configuration, low-viscosity lubricating oil, and high cooling water flow are important for the thermal performance of the motorized spindle. FEM has been gradually adopted and developed in temperature prediction of spindle systems. Lee et al. [17] analyzed the temperature and thermal deformation of the spindle system, and the results showed that the convection should be considered. Zivkovic et al. [18] investigated the interaction between temperature and bearing stiffness. The results showed that the bearing stiffness of the spindle system is enhanced as the rotation speeds increase. Than et al. [19] combined the FEM and the conjugate gradient method and estimated the time-varying heat sources in the high-speed spindle. Their results show that this method can be used for reference in thermal compensation design. To decrease the thermal error caused by temperature rise, Yang et al. [20] developed an FEM model using experimental design. Liu et al. [21] established a thermal–structure model by considering the bearing thermal preload. Zhang et al. [22] proposed a thermal deformation prediction model for the 100 MD60Y4 motorized spindle by optimizing the heat transfer coefficient. Yan et al. [23] investigated the variation rules of the spindle temperature field and found that the variation process can be divided into nonuniform and uniform variation stages. Their findings can provide a new perspective for temperature variable selection and coefficient adjustment. In addition, more factors have been considered in the predicted model of temperature field, such as cooling and lubrication [21, 24], thermal contact resistance (TCR) [21, 25–28], heat transfer coefficient [29], and dynamic effect [30, 31].

In addition to the complicated FDM and FEM, a thermal network model based on thermal resistance has recently been applied to predict the temperature field of the spindle system. Liu et al. [32] developed a thermal resistance network model of spindle–bearing–pedestal for motorized spindle and analyzed the effects of thermal resistance on the steady temperature field. Nevertheless, the importance of time-varying thermal parameters for motorized spindle analysis was underrated in the model. A thermal deformation coupling model in the thermal network for the spindle–bearing system was studied by Yan et al. [33]. In this model, other time-varying parameters, such as thermal deformation and lubrication viscosity, were considered, and the temperature of the bearing outer ring was approximate to the measured result at mid and high speeds. In view of the effect of forced vibration on bearing heat generation, the bearing heat generation model was improved by Zheng et al. [34]. The bearing temperature rise can be better forecasted by using this model. Furthermore, the influence of the cooling system on the temperature variation of the motorized system has captured the attention of researchers. Liu et al. [35] developed an improved thermal resistance network model by considering the temperature variation of the cooling system and analyzed the effects of coolant flow rate and speed on the temperature variation of the spindle system. The results indicated that the coolant flow rate has a considerable effect on the temperature of bearings at high rotation speeds.

The abovementioned works have enriched the research on the thermal characteristics of motorized spindles and are of great significance to the improvement of their thermal performance. In fact, the temperature and contact performance of bearings in motorized spindles are coupled and inseparable and include two points: (1) the thermal deformation caused by the temperature rise will affect the contact performance of the bearings and (2) the change in the contact status will in turn affect the temperature rise and thermal deformation of the system. However, the mentioned works have mainly focused on content (1), whereas content (2) has rarely been covered. Therefore, an enhanced thermal network model for a motorized spindle is proposed in this work by considering the thermal–mechanical coupling effect. A new method for solving the bearing heat is given, in which the effect of temperature rise is considered. A temperature experiment is performed to verify the proposed model.

The rest of this paper is organized as follows. In Section 2, the thermal network model for motorized spindle is established, and a new method for solving bearing heat is provided. In Section 3, the computation flowchart is given to acquire the steady temperature field of the motorized spindle, and a comparison analysis with the experimental results is performed. Thermal displacement, contact angle, contact load, friction heat, and temperature for bearings are investigated in Section 4 to interpret the change law of temperature with speeds. Lastly, the conclusions are indicated in Section 5.

2 Model

2.1 Geometric model

A three-dimensional geometry model of a motorized spindle is shown in Fig. 1. Bolts, bolt holes, cooling pipes, and lubrication pipes are omitted, and some small chamfers and rounded
The rotor is connected with the rotating shaft by pressure matching, and the helical cooling jacket is fitted on the stator surface. Angular contact ball bearings are utilized to support the rotating shaft. Herein, three B7018C bearings are installed at the front of the shaft, and one B7011C bearing is installed at the rear of the shaft, as shown in Fig. 1. The heat generated by the stator is carried away by the coolant in the helical cooling jacket, and part of the friction heat in the bearing raceway is taken away by grease.

The motorized spindle is axisymmetric; hence, its thermal characteristics can be analyzed by establishing a two-dimensional heat transfer structure. In Fig. 2, the red lines with arrows represent heat transfer paths, and the heat diffuses outward to the environment in the arrow direction. Points A and B represent test points 1 and 2 near the outer ring of the bearing presented in Section 3.2, respectively.

The thermal network model based on thermal resistance is established as follows. (1) In accordance with the number of balls in the bearing, the motorized spindle is equally divided along the circumference. (2) The thermal network of the motorized spindle is established based on the heat transfer mechanism. (3) The system thermal equilibrium equations are derived using Kirchhoff’s current law (KCL). (4) The steady temperature field of the motorized spindle is obtained by coupling the equations of temperature field and bearing heat.

### 2.2 Thermal network model

In accordance with the heat transfer mechanism of the motorized spindle, the global (Fig. 3) and local (Fig. 4) thermal network models are established based on thermal resistance. In these models, the thermal resistance of the motorized spindle mainly includes the thermal convection resistance of the solid and environmental fluid, the thermal conduction resistance inside the solid, and the TCR amongst the contact surfaces.

As shown in Fig. 4a, the thermal resistance $R_b$ of the ball and $R_{L_b}$ or $R_{L_o}$ of grease are connected in parallel for grease ball bearing. $H_i$ and $H_o$ represent the frictional heat generated by the inner and outer raceways of the bearing, respectively, and the heat is transferred outward through thermal resistance. $R_{ih}$ and $R_{oh}$ denote the TCR between the bearing inner ring and the spindle and between the bearing outer ring and the bearing housing, respectively. In Fig. 4b, $R_{rs}$ and $R_{sc}$ represent the TCR between the rotor and spindle and between the stator and cooling pipe, respectively. The heat $H_e$ of the stator can be greatly removed using the oil in the cooling pipe and the turbulent gas in the air gap of the motor, and the heat $H_r$ of the rotor is mainly taken away by the rotor rotation effect.

As shown in Fig. 5, KCL with thermal resistance can be applied to solve the thermal network model of the motorized spindle. On the basis of KCL, the heat flow into a node is
positive; otherwise, it is negative. The algebraic sum of all the heat flow involved in the node is equal to zero, which can be expressed as
\[ \sum H = 0, \quad (1) \]

\( N \) temperature nodes are assumed to exist in the system, and each node is connected with \( m \) heat transfer routes. The thermal equilibrium equation of the nodes is written as
\[ H_i + \sum_{j=1}^{m} \frac{T_j - T_i}{R_{ij}} = 0, \quad (2) \]

where \( i, j = 1, 2, \ldots, N; T_i \) and \( T_j \) denote the temperatures of nodes \( i \) and \( j \), respectively; \( R_{ij} \) represents the thermal resistance between nodes \( i \) and \( j \); and \( H_i \) describes the thermal flow generated by node \( i \), which is regarded as the inflow thermal flow.

The thermal equilibrium equations of all nodes can be expressed in the following matrix form:
\[ [H] + [R]^{-1}[T] = 0 \quad (3) \]

where \([H]\) denotes the heat flow matrix, which is determined using bearing friction and motor loss; \([R]\) represents the thermal resistance matrix; and \([T]\) describes the temperature distribution matrix.

The thermal network parameters include the heat flow matrix \([H]\), thermal resistance matrix \([R]\), and temperature distribution matrix \([T]\). \([T]\) can be determined using \([H]\) and \([R]\). \([H]\) includes the heat generated by the bearing and motor, and \([R]\) includes the thermal conduction resistance, thermal convection resistance, and TCR. Under the influence of the cooling system, the effect of thermal radiation on the temperature field can be ignored for the motorized spindle.

In this work, the bearing housing, inner ring, outer ring, stator, and rotor of the motor are simplified into a hollow cylinder, whereas the solid spindle is simplified into a cylinder. The thermal conduction resistance can be divided into axial and radial resistance. From the Fourier theorem, the axial thermal conduction resistance of the cylinder can be expressed

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Fig. 3 Global thermal network model of motorized spindle

Fig. 4 Local thermal network model for a bearings and b motor

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where \( L \), \( k \), and \( S_a \) denote the axial length, thermal conductivity, and axial cross-sectional area of the cylinder, respectively.

The radial thermal conduction resistance of the hollow cylinder is expressed as

\[
R_2 = \frac{\ln(d_2/d_1)}{2\pi kL}
\]

where \( d_1 \) and \( d_2 \) are the inner and outer diameters of the cylinder, respectively.

The radial thermal conduction resistance of a solid cylinder can be written as

\[
R_3 = \frac{1}{\pi kL}
\]

The thermal conduction resistance of the ball is expressed as

\[
R_4 = \frac{2}{\pi k_b D_b}
\]

where \( k_b \) is the thermal conductivity of the ball.

The thermal conduction resistance of grease is described as

\[
R_5 = \frac{D_b}{2k_L \left( \frac{\pi d_i B_i}{4} \frac{1}{\pi Z D_b} \right)}
\]

\[
R_6 = \frac{D_b}{2k_L \left( \frac{\pi D_o B_o}{4} \frac{1}{\pi Z D_b^2} \right)}
\]

where \( k_L \) is the thermal conductivity of grease; \( d_i \) and \( D_o \) denote the diameters of the bearing inner and outer raceways, respectively; and \( B_i \) and \( B_o \) represent the width of the bearing inner and outer rings, respectively.

On the basis of Fourier theorem, the axial and radial thermal convection resistance for a cylinder is written as

\[
R_7 = \frac{1}{hS}
\]

where \( h \) denotes the convective heat transfer coefficient and \( S \) is the area of convective heat transfer between the cylinder and fluid.

The convective heat transfer coefficient on surfaces can be calculated by

\[
h = \frac{n u \lambda}{D_b}
\]

where \( N_u \) and \( \lambda \) denote the Nusselt number and fluid thermal conductivity, respectively, and \( D_b \) is the geometric characteristic dimension of the heat exchange. For highly turbulent flow, \( N_u \) is described as \([12]\)

\[
N_u = 0.0225 h_c 0.8 \Pr 0.3
\]

When the following conditions are met, Eq. (12) is valid:

\[
\left\{ \begin{array}{l}
Re > 10000 \\
0.7 < \Pr < 100 \\
L/h_{gap} > 60
\end{array} \right.
\]

where \( L \) is the length of the cross section for the heat exchange surface, \( h_{gap} \) represents the clearance, \( \Pr \) is the Prandtl number, and \( R_e \) denotes the Reynolds number, which can be written as

\[
R_e = \frac{u D_b}{\eta_f}
\]

where \( u \) is the average velocity of fluid and \( \eta_f \) is the kinematic viscosity of fluid.

If the part is rotating at high speed, the heat transfer between its end face and the fluid will become forced convection, such as the end face of the rotating shaft or the motor rotor. The convective heat transfer coefficient between the end face and the air can be further rewritten as \([36]\)

\[
h_d = 28 \left( 1 + \sqrt{\frac{0.225 \omega_s d_e}{p}} \right)
\]

where \( \omega_s \) denotes the angular speed of the rotating shaft, \( d_e \) is the equivalent diameter of the end face of the rotating shaft, and \( p \) refers to the pole pairs of the motor.

During the operation of the motorized spindle, the convective heat transfer coefficient between the stationary
surface on bearing housing and the environmental fluid is described as [36]

\[ h_s = 9.7 \text{ (W/}(m \cdot ^\circ \text{C})) \]  

(16)

The actual contact between the mating mechanical surfaces only occurs in the finite asperities on the contact surface. Hence, as the heat passes through the interfacial clearance, additional resistance is produced, which is defined as TCR. The TCR is written as

\[ R_s = \frac{1}{h_s A^*}, \]  

(17)

where \( h_s \) denotes the contact thermal conductivity of fluid in interfacial clearance and \( A \) is the nominal contact area between the mating mechanical surfaces.

In consideration of the thermal conduction effect of asperities and interfacial fluid medium, the contact thermal conductivity is expressed as [25]

\[ h_c = \frac{1}{L_g} \left[ A^* \left( \frac{2k_1k_2}{k_1 + k_2} \right) + (1-A^*)k_f \right], \]  

(18)

where \( L_g \) is the thickness of the interfacial clearance, \( k_1 \) and \( k_2 \) denote the thermal conductivity of the mating components, \( k_f \) represents the thermal conductivity of the interfacial clearance medium, \( A^* \) is the dimensionless actual contact area and \( A^* = A/A_c \), and \( A_c \) denotes the actual contact area [25]. The calculation methods for \( L_g \) and \( A^* \) are presented in Appendix 2.

2.3 Computation method of heat

As an important part of the motorized spindle, the built-in permanent magnet synchronous motor has the characteristics of high efficiency, small volume, light weight, and low temperature rise. In accordance with classical core loss separation theory, the losses of the built-in permanent magnet synchronous motor mainly include the stator winding copper, iron, and rotor eddy current losses. The stator winding copper loss is determined using the resistance of the stator winding and the current in the winding, which can be written as

\[ H_{Cu} = ml^2R_s \]  

(19)

where \( H_{Cu}, m, l, \) and \( R_s \) denote the copper loss, phase number, current, and resistance of the stator winding, respectively.

From the classical model of core loss separation [37], the core loss caused by any magnetic density distribution includes hysteresis, eddy current, and additional losses, which can be expressed as

\[ H_{Fe} = H_h + H_e + H_{exc} \]  

(20)

where \( H_{Fe} \) denotes the core loss and \( H_h, H_e, \) and \( H_{exc} \) represent the hysteresis, eddy current, and additional losses, respectively. These parameters can be expressed as

\[ H_h = M_s (k_h B_m^3) \]  

(21)
\[ H_e = M_s (k_f f^2 B_m^2) \]  

(22)
\[ H_{exc} = M_s (k_{exc} f^{1.5} B_m^{1.5}) \]  

(23)

where \( M_s, f \) are the stator mass and working frequency of the motor, respectively; \( k_h, k_f, \) and \( k_{exc} \) are the coefficients of hysteresis, eddy current, and additional losses, respectively; \( B_m \) is the maximum magnetic flux density; and \( \alpha \) is the dimensionless hysteresis loss parameter.

The rotor eddy current loss of the permanent magnet synchronous motor is expressed as

\[ H_r = \rho_r l \int J^2 ds \]  

(24)

where \( H_r \) denotes the eddy current loss of rotor; \( \rho_r \) and \( l \) are the resistivity of the rotor material and the axial length of the rotor, respectively; \( J \) denotes the induced eddy current density; and \( s \) represents the cross-sectional area of the rotor.

As a crucial parameter in the thermal network model, the frictional heat generated by the bearing should be considered and calculated accurately. The definition of the position angle for the bearing balls is shown in Fig. 6, and the position of ball and the inner and outer rings before and after loading is illustrated in Fig. 7. Without loss of generality, the curvature center of the bearing outer raceway is assumed to be fixed in this work. Under the action of force \( \mathbf{F} = [F_x, F_y, F_z, M_x, M_y] \), the relative displacement \( \delta = \left[ \delta_x, \delta_y, \delta_z, \theta_x, \theta_y \right] \) in the bearing occurs. In consideration of frictional heat, the thermal
displacement $u$ ($u = [u_a, u_r]$) in the bearing is generated. At high speed, the centrifugal displacement $u_c$ is produced by the bearing.

As shown in Fig. 7, at the position angle $\psi_j$, the relative axial distance $A_{1j}$ and radial distance $A_{2j}$ between the bearing inner and outer raceways can be expressed as follows:

$$A_{1j} = BD_b \sin \alpha_0 + \Delta_a$$  \hspace{1cm} (25)

$$A_{2j} = BD_b \cos \alpha_0 + \Delta_r + u_c$$  \hspace{1cm} (26)

where parameter $B$ is written as $B = f_i + f_o - 1$; $f_i$ and $f_o$ represent the curvature radius coefficients of the inner and outer raceways, respectively; $D_b$ is the diameter of the ball; $\alpha_0$ is the initial contact angle of the bearing; $\Delta_a$ and $\Delta_r$ denote the total relative axial and radial displacements in the bearing, respectively; and $u_c$ represents the centrifugal displacement of the inner raceway, which has been calculated in Jorgensen and Shin [8].

For diverse configurations in Fig. 8, $\Delta_a$ is determined using different methods, and it can be described as

$$\Delta_a = \begin{cases} 
\delta_z + R_j \theta_j \cos \psi_j - R_i \theta_i \sin \psi_j + u_c \\
-\delta_z - R_i \theta_i \cos \psi_j + R_j \theta_j \sin \psi_j + u_c 
\end{cases}$$  \hspace{1cm} (27)

As shown in Fig. 8, $\Delta_a$ in bearings 1 and 4 is determined using Eq. (27a), and that in bearings 2 and 3 is calculated using Eq. (27b). In Eq. (27), $u_a$ denotes the axial thermal displacement, which can be written as

$$u_a = \begin{cases} 
\delta_z - u_c - \beta_b \Delta T_b D_b (\sin \alpha_i + \sin \alpha_o) \\
\delta_z - u_c - \beta_b \Delta T_b D_b (\sin \alpha_i + \sin \alpha_o) 
\end{cases}$$  \hspace{1cm} (28a) \hspace{1cm} (28b)

where $\delta_z$ denotes the relative axial displacement caused by the bearing preload; $R_i$ represents the radius of the curvature in the bearing inner raceway; $\theta_x$ and $\theta_y$ are the relative angular displacements of the bearing in $x$ and $y$ directions, respectively; $\psi_j$ denotes the position angle of the ball; and $u_c$ in bearings 1 and 2 is determined using Eq. (28a), and that in bearings 3 and 4 is calculated using Eq. (28b). In Eq. (28), $\beta_b$ represents the thermal conductivity of the balls; $\Delta T_b$ denotes the temperature rise of the balls; $\alpha_i$ is the contact angle between the ball and the inner raceway; $\alpha_o$ is the contact angle between the ball and the outer raceway under operation; and $u_h$ and $u_s$ denote the thermal displacement of the bearing housing and spindle, respectively. Notably, bearings 1, 2, and 3 form an O-type configuration, and bearings 3 and 4 constitute an X-shaped configuration in this work, as shown in Fig. 1.

For the bearing housing and spindle without internal heat source, $u_h$ and $u_s$ can be expressed as follows:
where $c_1$, $c_2$, $c_3$, and $c_4$ are the integral constants. $c_3$ and $c_4$ are determined using temperature boundary conditions, whereas $c_1$ and $c_2$ can be obtained using displacement boundary conditions.

In Eq. (25), the total relative radial displacement $\Delta r$ is described as

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} + u_r,$$

where $\delta_x$ and $\delta_y$ are the relative displacement caused by the preload in $x$ and $y$ directions, respectively; $\theta_r$ denotes the angle between the ball with the maximum deformation and that with the position angle of $0^\circ$, which is expressed as $\theta_r = \arccos(\Delta x / \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2})$; and $u_r$ represents the radial thermal displacement in the bearing and is expressed as

$$u_r = \gamma_x \Delta T_x d_x + \left[\gamma_y \Delta T_y (1 + \mu_y) - \gamma_z \Delta T_z \right] d_y^2 - \gamma_h \Delta T_h (1 + \mu_h) D_y - \gamma_h \Delta T_h D_y (\cos \alpha_j + \cos \alpha_o)$$

where $\gamma_x$, $\gamma_y$, and $\gamma_z$ denote the thermal expansion coefficient of the inner ring, spindle, and bearing housing, respectively; $\Delta T_x$, $\Delta T_y$, and $\Delta T_h$ represent the temperature variation of the inner ring, spindle, and bearing housing, respectively; $\mu_x$ and $\mu_y$ are the Poisson’s ratios of the spindle and bearing housing, respectively; $d_x$ and $d_y$ are the diameters of the bearing inner and outer raceways, respectively.

From Fig. 7, the geometrical compatibility equation in the bearing can be written as

$$(A_{ij} - X_{ij})^2 + (A_{jx} - X_{jx})^2 - (f_j - 0.5)D_b + \delta_{ij}^2 = 0,$$

where $X_{ij}$ and $X_{jx}$ denote the axial and radial distances between the ball and curvature centers of the outer raceway, respectively, and $\delta_{ij}$ and $\delta_{ij}^2$ are the contact deformation values of the inner and outer raceways contacting with the ball, respectively.

As shown in Fig. 9, force is loaded on the ball. On the basis of outer raceway control theory, the gyroscopic moment on the ball is prevented by the contact friction between the ball and the outer raceway at high speed; otherwise, it is equally shared by the friction in the contact area.

On the basis of Hertz point contact, the force balance for the ball can be described as

$$u(x) = \frac{c_1}{2} x^2 + c_2 x + c_3 x + c_4,$$

where $\gamma$ denotes the thermal expansion coefficient of the components; and $c_1$, $c_2$, $c_3$, and $c_4$ are the integral constants. $c_3$ and $c_4$ are determined using temperature boundary conditions, whereas $c_1$ and $c_2$ can be obtained using displacement boundary conditions.
the position angle $\Psi_j$, the friction moment on the inner and outer raceways can be expressed as

$$M_{ij} = 0.675\xi_0\left(\eta_0\omega_{cj}\right)^{2/3}D_b^3 + \xi_1\left(\frac{Q_{ij}}{Q_{ij_{\text{max}}}}\right)^{1/3} Q_{ij}D_b,$$

and

$$M_{oj} = 0.675\xi_0\left(\eta_0\omega_{cj}\right)^{2/3}D_b^3 + \xi_1\left(\frac{Q_{oj}}{Q_{oj_{\text{max}}}}\right)^{1/3} Q_{oj}D_b,$$

where $\xi_0$ and $\xi_1$ denote the coefficients related to the bearing type, lubrication, and load; $\eta_0$ is the kinematic viscosity of the lubricant; $\omega_{cj}$ represents the common angular velocity of ball $j$; and $Q_{ij_{\text{max}}}$ and $Q_{oj_{\text{max}}}$ denote the maximum contact stress between the ball and inner or outer raceway, where $Q_{ij_{\text{max}}} = (K_{ij}\delta_{ij}^{1.5})_{\text{max}}$ and $Q_{oj_{\text{max}}} = (K_{oj}\delta_{ij}^{1.5})_{\text{max}}$. $Q_{ij}$ and $Q_{oj}$ can be determined by simultaneously solving Eqs. (32)-(40).

The spin friction moment on the ball at the inner raceway is expressed as [8]

$$M_{sij} = \frac{3\mu_{ij}Q_{ij}a_{ij}\Sigma_{ij}}{8}$$

where $\mu_{ij}$ denotes the friction coefficient between the ball and the inner raceway, $a_{ij}$ is the half width of the Hertzian contact between ball $j$ and the inner raceway, and $\Sigma_{ij}$ represents the second type of complete integral for the Hertzian contact ellipse between ball $j$ and the inner raceway.

The bearing friction heat is determined using friction moment and angular velocity, which can be described as

$$H_{ij} = \omega_{ij}M_{ij} + \omega_{bj}M_{sij}$$

and

$$H_{oj} = \omega_{oj}M_{oj}$$

where $H_{ij}$ and $H_{oj}$ denote the friction heat of the contact area, $H_b$ represents the total friction heat of the bearing, and $\omega_{bj}$ represents the angular velocity of the ball.

In summary, to acquire the friction heat of the bearings, the contact load is required by solving Eqs. (31)–(39). The contact load of the bearings is closely related to the temperature field of the system and the thermal deformation of the bearings. Then, the friction heat of the bearings must be initially gained to predict the temperature field of the system (Eq. (3)). Therefore, the friction heat of the bearings and the temperature field of the system are coupled with each other and cannot be solved independently. An iterative algorithm is provided to solve the coupled equations in the next section.

### 3 Numerical solution and experimental verification

#### 3.1 Numerical solution

The solution flowchart of the thermal network model for the motorized spindle is illustrated in Fig. 10. Firstly, the motor power loss $H_m$ and bearing friction heat $H_{b0}$ are determined using the methods introduced in Subsection 2.3, where $H_{b0}$ is calculated when $\Delta_a$ and $\Delta_r$ are 0. Secondly, the temperature field $[T]$ of the motorized spindle can be obtained by substituting the heat into Eq. (3). Thirdly, the thermal displacement of
the bearings can be acquired from the temperature field, and Eqs. (32)–(40) are solved to gain the bearing friction heat $H_{b1}$ by substituting the thermal displacement. The steady temperature field $[T]$ of the motorized spindle is outputted until the convergence criterion is satisfied, where the convergence criterion can be given as

$$\frac{|H_{b1} - H_{b0}|}{H_{b0}} \leq \varepsilon$$

(47)

Otherwise, $H_{b0} = H_{b1}$ is assigned, and Eq. (3) is resolved. $\varepsilon$ is the user-defined error threshold, and $\varepsilon = (10)^{-4}$ is found to be exact for the example analysis presented in the next section by repeatedly comparing the simulated results.

### 3.2 Experimental verification

The temperatures near the outer ring of bearings 3 and 4 are tested using a thermocouple sensor to verify the thermal network model (Fig. 11). The test rig is shown in Fig. 11a, and the position dimensions of the measuring points are $L_1 = 212.5 \text{ mm}$, $L_2 = 80 \text{ mm}$, $L_3 = 61.25 \text{ mm}$, and $L_4 = 63 \text{ mm}$ (Fig. 11b). In this work, the spindle runs for more than 3 h to ensure that the spindle is in thermal balance. Then, the temperature value on the temperature indicator can be recorded.

In this test, the rated speed of the motorized spindle is 3000 rpm, and the ambient temperature is 26 °C. The cooling oil volume flow rate of the stator is 4 L/min, and the inlet oil temperature is 21 °C. All bearings are preloaded by positioning, and the initial preload is 150 N. The bearing parameters and materials of the components are shown in Tables 1 and 2, respectively.

Notably, the temperature is predicted and measured under unloaded condition. During the test, it is very difficult to maintain the same grinding force for a long time under loaded condition. To avoid the measured error caused by the changes of the grinding force, the temperature is measured under unloaded condition.

#### Table 1 Basic parameters of bearings

| Parameters                              | Bearing 3 (B7018C) | Bearing 4 (B7011C) |
|-----------------------------------------|---------------------|---------------------|
| Medium diameter of bearing/mm          | 115                 | 72.5                |
| Diameter of balls/mm                   | 13.75               | 9.27                |
| Curvature radius coefficient of the inner raceway | 0.57               | 0.57                |
| Curvature radius coefficient of the outer raceway | 0.54               | 0.54                |
| Initial contact angle/°                 | 15                  | 15                  |
| Number of balls                         | 25                  | 20                  |
The measured and predicted temperatures near the outer ring of bearings 3 and 4 are illustrated in Fig. 12 and Table 3. The current results are calculated using the proposed model, whereas the previous results refer to the calculated result without the thermal–mechanical coupling effect. The measured and predicted results illustrate that the temperature near the outer ring becomes large with the increase in rotational speed. Given that the thermal–mechanical coupling effect and iterative computation are considered in this work, the prediction temperature by the proposed model is closer to the measured temperature than that by the previous model. In detail, the maximum relative errors (MREs) of temperature near bearings 3 and 4 for the previous result are 12.33% and 7.09, respectively. By contrast, the MREs for the current result decrease to 2.77% and 3.65%. After the thermal–mechanical coupling effect is considered, the temperature rise is affected by the change in contact status of the bearing, such as contact angle and contact load. The reasons for the difference between the previous and current results will be elucidated in the next section.

### 4 Results and discussions

Other parameters of the motorized spindle need to be qualitatively analyzed to understand the reasons for the difference between the previous and current results. In consideration of the position of measured points, the thermal parameters of bearings 3 and 4 are analyzed in this section.

The contact angle and load of bearings 3 and 4 are shown in Fig. 13. The change law of the contact angle and load of bearing 3 predicted using the current model is the same as that of bearing 4. However, a difference exists between the current results of bearings 3 and 4. For bearing 3, the centrifugal force of the balls becomes large as the rotation speeds increase, thereby increasing the inner contact angle and decreasing the outer contact angle. The outer contact angle of bearing 4 also decreases with the increase in speeds, whereas its inner contact angle initially decreases and then increases. The diameter of the balls of bearing 4 is smaller than that of bearing 3; thus, the centrifugal force of the balls of bearing 4 is not dominant at low speeds. On the contrary, the axial thermal

### Table 2 Materials of parts in the motorized spindle system

| Parts      | Spindle | Bearing ball | Bearing ring | Stator       | Rotor       | Bearing housing |
|------------|---------|--------------|--------------|--------------|-------------|-----------------|
| Material   | 38CrMoAlA | Si3N4        | GCr15        | Silicon steel| Nd2Fe14B    | 40Cr            |

### Fig. 12 The predicted and measured temperature at a point 1 and b point 2

The measured and predicted temperatures near the outer ring of bearings 3 and 4 are illustrated in Fig. 12. The current results are calculated using the proposed model, whereas the previous results refer to the calculated result without the thermal–mechanical coupling effect. The measured and predicted results illustrate that the temperature near the outer ring becomes large with the increase in rotational speed. Given that the thermal–mechanical coupling effect and iterative computation are considered in this work, the prediction temperature by the proposed model is closer to the measured temperature than that by the previous model. In detail, the maximum relative errors (MREs) of temperature near bearings 3 and 4 for the previous result are 12.33% and 7.09, respectively. By contrast, the MREs for the current result decrease to 2.77% and 3.65%. After the thermal–mechanical coupling effect is considered, the temperature rise is affected by the change in contact status of the bearing, such as contact angle and contact load. The reasons for the difference between the previous and current results will be elucidated in the next section.

### Table 3 The measured and predicted temperature

| Speed (rpm) | Temperature at Point 1 (°C) | Temperature at Point 2 (°C) |
|-------------|-----------------------------|-----------------------------|
|             | Measured | Previous | Current | Measured | Previous | Current |
| 900         | 26.9     | 26.5     | 26.6     | 28.2     | 26.2     | 27.2    |
| 1500        | 27.1     | 26.8     | 26.8     | 28.4     | 26.4     | 27.5    |
| 2100        | 27.3     | 27.3     | 27.2     | 28.5     | 26.8     | 28.0    |
| 2700        | 27.7     | 28.0     | 27.7     | 28.8     | 27.3     | 28.6    |
| 3300        | 27.9     | 28.8     | 28.4     | 29.1     | 27.8     | 29.3    |
| 3900        | 28.7     | 29.8     | 29.1     | 29.9     | 28.4     | 30.0    |
| 4500        | 29.2     | 30.9     | 30.0     | 30.6     | 29.1     | 30.8    |
| 5100        | 30.3     | 32.2     | 31.0     | 31.0     | 29.9     | 31.6    |
| MRE (%)     | /        | 12.33    | 2.77     | /         | 7.09     | 3.65    |
displacement of bearing 4 changes sharply at low speeds (Fig. 15 in Appendix 1), thereby causing a reduction in the inner contact angle in the range of 900–2100 rpm. Likewise, the contact load of bearing 3 decreases at low speeds, which is mainly caused by the axial thermal displacement. With the increase in rotation speeds, the centrifugal force and radial thermal displacement play dominant roles, thereby increasing the inner and outer contact loads of bearing 3. The influence of centrifugal force is insufficient to reverse the influence of thermal displacement on contact loads in the range of 900–5100 rpm. Therefore, the inner and outer contact loads of bearing 4 decrease with the increase in rotation speeds.

As shown in Fig. 14, the predicted results by the previous and current models illustrate that the friction heat of the bearings increases with the rotational speed. For bearing 3, the value of...
frictional heat predicted by the previous model is larger than that predicted by the current model. However, the value of frictional heat predicted by the previous model is smaller than that predicted by the current model for bearing 4. Comparison of Figs. 12 and 14 indicates that the variation law of the bearing temperature is almost the same as that of the bearing friction heat.

As shown in Fig. 13, the contact angle and load obtained using the two methods are relatively different. The contact angle and load of the bearings are affected by thermal deformation, thereby changing the frictional heat. The outer contact angle of bearing 3 with thermal-mechanical coupling effect is smaller than that without thermal-mechanical coupling effect, thereby causing a smaller contact load and frictional heat. Therefore, the temperature acquired by the previous model is higher than that of the current model. The comparison analysis between the current and previous results shows that the temperature rise and thermal deformation have considerable effects on the contact performance of bearings, and the change in the contact status will in turn affect the temperature rise and thermal deformation of the system. These analysis results explain why the current result is more accurate than the previous one.

5 Conclusions

An enhanced thermal network model of motorized spindle was proposed by considering the thermal-mechanical coupling effect. In accordance with this model, the temperature field of the motorized spindle was obtained, and the temperature near the bearings at different rotation speeds was analyzed. Then, the proposed model was verified via temperature experiment. The conclusions are summarized as follows.

(1) The temperature predicted by the proposed model is more accurate than that predicted by the previous model (without thermal-mechanical coupling effect). The measured temperature is regarded as a baseline. The MREs of temperature near bearings 3 and 4 decrease by 9.56% and 3.44%, respectively, by considering the thermal-mechanical coupling effect. The comparison result shows that the thermal-mechanical coupling effect needs to be included to predict the temperature field of motorized spindle systems.

(2) The temperature changes the contact performance of the bearings (e.g., contact angle and contact load), thereby causing changes in the frictional heat and temperature field. Such changes cause the difference between the current and previous results.

In comparison with the previous model, the enhanced thermal network model can obtain a more accurate temperature. Therefore, the proposed model can be efficiently applied to investigate the thermal performance of motorized spindles, and the analysis results are valuable to the design and improvement of such systems.

Author contribution Zhou C.: Methodology, Formal analysis, Investigation, Resources, Data curation, Writing—review and editing, Visualization, Project administration, Funding acquisition. Qu Z.: Methodology, Software, Investigation, Writing—original draft. Hu B.: Conceptualization, Methodology, Validation, Investigation, Resources, Writing—review and editing, Supervision, Project administration, Funding acquisition. Li S.: Conceptualization, Methodology, Writing—review and editing.

Funding This work was supported by the National Natural Science Foundation of China (Grants Nos. 52005051 and 52075153) and the Key Research and Development Program of Hunan Province (Grants Nos. 2020WK2032 and 2016JC2001).

Availability of data and material Not applicable.

Code availability Not applicable.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

Conflict of interest The authors declare no competing interests.
Appendix 1

Fig. 15 Thermal displacement and centrifugal displacement of (a) bearing 3 and (b) bearing 4

Appendix 2

When the fractal dimension of the surface $D < 1.5$, $A^*$ in Eq. (18) can be expressed as

$$A^* = \left( \frac{3\sqrt{2\pi}}{4} \right)^{2/(3-D)} \left( \frac{L_u}{G} \right)^{(2D-2)/(3-D)} g_4(D) \left( \frac{P}{E} \right)^{2/(3-D)}$$

(48)

otherwise,

$$A^* = \left[ g_3(D) \right]^{-2/D} \left( \frac{P}{E} \right)^{-2/D} \left\{ \frac{4}{3\sqrt{2\pi}} \left( \frac{G}{L_u} \right)^{D-1} \left[ -g_1(D) \right] \left( a_c^* \right)^{(3-2D)/2} + \frac{H}{E} g_2(D) \left( a_c^* \right)^{(2-D)/2} \right\}^{-2/D}$$

(49)

In Eq. (18), $L_g$ can be described as

$$L_g = 2 \left[ e^{-\psi(D-2)/2} G^{D-1} \left( \frac{4-2D}{D} \right)^{(2-D)/2} \left( A^* \right)^{2-D} L_u \right]^{2-D}$$

(50)

where $G$ is the fractal roughness parameter, $L_u$ is the upper limit of the sample length, $D$ is the fractal dimension of the surface ($1 < D < 2$), $p$ is the apparent pressure, $E$ is the equivalent elastic modulus of the mating surface, $a_c^*$ is the normalized critical micro contact area, and $\psi$ is the domain extension factor for micro contact size distribution. $H$ is the hardness of the softer material between two parts, $z$ is the maximum height difference of the asperities of two surfaces, and $g(D)$ is the function of intermediate variable. Detailed calculation of the above parameters can be found in reference [25, 38].

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