New Window on Matter-Antimatter Differences from BESIII

Xiaorong Zhou

State Key Laboratory for Particle Detection and Electronics, Hefei, 230026, China
Department of Modern Physics, University of Science and Technology of China, Hefei, 230026, China

Stephen Lars Olsen

University of Chinese Academy of Science, Beijing 100049, China
Institute for Basic Science, Daejeon 34126, South Korea

Recent measurements of $CP$-related comparisons of the weak-interaction parameters that describe the decays of the $\Xi^-$ and the $\bar{\Xi}^+$ doubly strange hyperon and antihyperon are reviewed and their significance is discussed.

Keywords: $J/\psi$ meson, Strange hyperons, $CP$ violation

PACS numbers:

1. Introduction

The strange particles played an illustrious role in the history of high energy physics and the development of the Standard Model (SM), the hugely successful theory that describes the properties and mutual interactions of the most elementary constituents of matter. In fact, the emergence of high energy physics as an area of research that is distinct from nuclear physics was the construction and operation of the Cosmotron accelerator at Brookhaven National Laboratory in 1952 and the Bevatron accelerator at Berkeley in 1955 for the expressed purpose of producing and studying the “New Particles,” i.e. the mysterious “V” particles that had been discovered in cosmic-ray interactions.

Studies of these “strange” new particles that occurred during the next fifteen years under the controlled laboratory conditions provided by these and follow-up accelerators uncovered many of the fundamental principles that make up the SM, including:

- **Flavor Quantum Numbers:** “Strangeness” was the first of the SM’s four “flavors,” strange, charm, bottom, & top;
- **Particle-antiparticle Mixing:** $K^0 \leftrightarrow \bar{K}^0$-like transitions also occur with charmed and bottom particles, and related processes occur with neutrinos;
- **Parity Violation:** left-right symmetry violating processes were first seen in $K$-meson decays and are now a defining characteristic of the Weak Interactions;

$^{*}$zxrong@mail.ustc.edu.cn
$^{†}$solsensnu@gmail.com
**CP Violation**: the first sign of matter and antimatter differences was the observation of \(K_L^0 \rightarrow \pi^+\pi^-\) decays;

**Flavor mixing**: the suppression of the \(\Lambda \rightarrow p e^-\bar{\nu}\) decay rate below that inferred from the half-life for \(n \rightarrow p e^-\bar{\nu}\) decay was the first sign of flavor-mixing, and the precursor of the CKM quark-mixing and PMNS neutrino-mixing matrices;

**Quark Model**: the Isospin-Strangeness patterns in the baryon octet & decuplet and the meson octets led to the discovery of \(SU(3)\) and fractionally charged quarks;

**GIM Mechanism**: the strong suppression of strangeness-changing neutral-current kaon decays led to prediction of the existence of the \(c\)-quark.

Another particle that played a key role in the development of our current understanding of nature is the \(J/\psi\) meson that was discovered nearly simultaneously by experiments at Brookhaven\(^3\) and SLAC\(^4\) in November 1974. Its appearance at SLAC was especially dramatic, where it showed up as a spectacular narrow peak in the inclusive cross-section for \(e^+e^- \rightarrow \text{hadrons}\) from its well established continuum level of \(\sigma_{\text{had}} \approx 25\) nb to \(\sigma_{\text{had}} \approx 2500\) nb; a factor of \(\sim 100\) jump contained within a \(\sim 0.1\%\) interval of center-of-mass energy. Since the \(J/\psi\) is an \(SU(3)\)-singlet, its decay strengths to strange particle and non-strange particle final states are similar, and this, together with its large production cross-section, make the reaction \(e^+e^- \rightarrow J/\psi \rightarrow Y\bar{Y} (Y = \Lambda, \Sigma, \Xi)\) a prolific source of strange hyperons.

After the discovery of the \(J/\psi\), studies of strange particles faded from the limelight as interest in the new charmed and, shortly thereafter, bottom-flavored particles attracted more and more of the particle physics community’s attention. Although the development of dedicated hyperon beams at Fermilab and CERN resulted in precise measurements of hyperon magnetic moments & semileptonic decays, and heroic efforts at the two labs resulted in the discovery of a direct \(CP\)-violating component of \(K^0 \rightarrow \pi\pi\) decays (i.e., a non-zero value of the \(CP\) parameter \(\varepsilon'\)), the activities with the highest priorities were those aimed at measuring the parameters and testing the validity of the SM with neutrinos, the new heavy-quarks, the intermediate-vector bosons and the Higgs particle.

So far, the SM has prevailed and the main focus of the field has switched to searches for cracks in the theory that might provide clues as to what physics are not properly addressed by it might look like. In this arena, precision measurements of strange particle decay properties are as well positioned for finding deviations from SM predictions as those for any of the newer particles. Moreover, the hyperons and antihyperons that are produced in the \(J/\psi \rightarrow YY\) decay process are quantum-correlated, which make them particularly well suited for some of these searches. Maybe history will repeat itself and once again it will be strange particles that teach us the physics that lies beyond the SM.

2. **The role of \(CP\) violations in BSM-physics searches**

In spite of the SM’s success at providing precise quantitative explanations for virtually all experimentally observed phenomena, including \(CP\)-symmetry violations in the decays of strange and bottom mesons, it has some obvious deficiencies that indicate that it is not a complete theory. Notable among these deficiencies is the SM’s failure to provide a descrip-
tion of how the current all-matter universe evolved from what had to have been a matter-
antimatter symmetric condition that existed shortly after the Big Bang. The SM mechanism
for \(CP\) violation fails to account for the observed baryon asymmetry of the current uni-
verse by some ten orders of magnitude. Therefore it is generally considered that the new,
“Beyond the Standard Model” (BSM) physics that will supersede the SM must include at
least one new source of \(CP\) violation.

Another important issue that confronts virtually all searches for BSM physics is the
fact that the Quantum Chromodynamics sector of the SM, which in principle explains all
of strong interaction physics, is, in practice, woefully inadequate at producing reliable high-
precision descriptions of low-energy processes that involve hadrons. As a result, the sen-
sitivity of most searches for BSM new physics are ultimately limited by QCD-associated
uncertainties. However, thanks to stringent limits on the neutron’s electric dipole moment\(^5\)
\((d_n < 2 \times 10^{-26} \text{ ecm})\), the validity of \(CP\) symmetry in QCD has been established at the part
in \(10^{10}\) level\(^6\). As a result, searches for new physics sources of \(CP\) are relatively immune
to QCD-related uncertainties. Searches for anomalous \(CP\) violations in the \(c\)- and \(b\)-quark
sectors are major motivations for the LHCb experiment\(^2\) at the CERN Large Hadron Col-
lider and the Belle II experiment\(^8\) at the KEK Super B Factory \(e^+e^-\) collider. But, since,
by definition, nothing is known about what the new sources of \(CP\) violation might be,
there is no \textit{a priori} reason that they won’t first show up at measurable levels in the \(s\)-quark
sector. So far, however, that hasn’t happened; previous searches for anomalous \(CP\) viola-
tions in kaon\(^9\) and hyperon\(^10\) decays have reached the sensitivity limits of their respective
experimental methods and found no unexpected results.

3. \(CP\) studies with quantum-correlated \(\Xi^-\bar{\Xi}^+\) pairs produced via \(J/\psi\) decay

In this context, we direct the reader to a recent publication from the BESIII experiment at
the Beijing Institute of High Energy Physics BEPCII \(e^+e^-\) collider that reports a new ap-
proach to the search for a BSM source of \(CP\) violation in the \(s\)-quark sector. This involves
a comparison of the weak interaction decay parameters of the doubly strange \(\Xi^-\) hyperons
and \(\bar{\Xi}^+\) anti-hyperons that are produced in quantum mechanically entangled pairs via the
\(e^+e^- \to J/\psi \to \Xi^-\bar{\Xi}^+\) reaction.\(^{11}\)

![Quantum correlated \(\Xi^-\bar{\Xi}^+\) decay chains produced in \(J/\psi\) decays.](image-url)
In the BESIII measurement, the Ξ hyperons occur in the reaction chains shown in Fig.1. Here the $J/\psi$ mesons are produced (nearly) at rest in the laboratory by counter-circulating $e^+$ and $e^-$ beams in the BEPCII collider and the $\Xi^-$ & $\bar{\Xi}^+$ are produced back-to-back, in a spin-correlated quantum-entangled state that persists until one of them decays, which usually occurs within a few centimeters of the production point. In the selected events, both of the Ξ hyperons decay to a charged pion and a $\Lambda$ hyperon that, in turn, decays into a charged pion and a proton also within about a few centimeters of the parent Ξ decay points. The events show up as in the detector as $p\pi^-$ and $\bar{p}\pi^+$ decay products of nearly back-to-back $\Lambda$ and $\bar{\Lambda}$ hyperons that are accompanied by low momentum $\pi^-$ and $\pi^+$ tracks. The detected events have a very clear topology, with highly constrained kinematics and no particle-assignment ambiguities. Moreover, the background levels are very small; the signal to background ratio in the BESIII measurement is 400:1.

The overall process involves five different reactions that are most simply described by considering five different reference frames. The first is the $J/\psi$ rest frame where, since the spin=1 $J/\psi$ mesons are produced in $e^+e^-$ annihilation via a single virtual photon, they are spin-aligned along the direction of the $e^+e^-$ beamline (taken as the $z$ axis). This means that equal numbers of them are produced in the $|J; J_z\rangle = |1; +1\rangle$ and $|1; -1\rangle$ angular momentum states, but none with $|J; J_z\rangle = |1; 0\rangle$. The $J/\psi \to \Xi^-\Xi^+$ decay matrix element is the sum of two helicity amplitudes. The interference between these two amplitudes and the absence of any initial-state $J_z = 0$ component produces a $\theta_\psi$-dependent production cross section for $\Xi^-\Xi^+$ hyperons that are spin-polarized (with polarization vector $P_{\Xi}$) along the direction perpendicular to the production plane:

$$\frac{1}{N} \frac{dN}{d\cos \theta_\psi} = \frac{3}{4\pi} \frac{1 + \alpha_\psi \cos^2 \theta_\psi}{3 + \alpha_\psi}$$

$$P_{\Xi} = \sqrt{1 - \alpha_\psi^2 \sin \theta_\psi \cos \theta_\psi \sin \Delta \Phi} \frac{1 + \alpha_\psi \cos^2 \theta_\psi}{1 + \alpha_\psi \cos \theta_\psi \sin \Delta \Phi},$$

where $\alpha_\psi$ is a parity- and CP-conserving parameter that is proportional to the product of the two helicity amplitudes, $\Delta \Phi$ is their relative phase, and $P_{\Xi}$ is the vector modulus of $P_{\Xi}$. The polarizations of the hyperons are equal, i.e., $P_{\Xi} = P_{\bar{\Xi}}$.

The next two reference frames to consider are the $\Xi^-$ and $\bar{\Xi}^+$ rest frames. In these frames the decay angle $\theta_\Lambda(\theta_{\bar{\Lambda}})$ relative to $P_{\Xi}$ direction, and the polarization vectors of the $\Lambda$ daughters, $P_{\Lambda}(P_{\bar{\Lambda}})$, as illustrated in Fig.2, are distributed according to:

$$\frac{dN}{d\cos \theta_{\Lambda}} \propto 1 + \alpha_{\Xi} P_{\Xi} \cos \theta_{\Lambda}$$

$$P_{\Lambda} = \frac{(\alpha_{\Xi} + P_{\Xi} \cos \theta_{\Lambda}) \hat{x} + P_{\Xi} \beta_{\Xi} \hat{y} + P_{\Xi} \gamma_{\Xi} \hat{z}}{1 + \alpha_{\Xi} P_{\Xi} \cos \theta_{\Lambda}},$$

where the unit vectors $\hat{x}, \hat{y}, \hat{z}$ are oriented as indicated in the figure and $\alpha, \beta, \gamma$ are the Lee-Yang decay parameters that are described in the following subsection.

---

*The laboratory momentum ranges of the $p$ & $\bar{p}$ and those for the pions have very little overlap.*
a) A strong interaction phase shifts, \( S \) distinct types of phases. One type corresponds the operator (\( \alpha \)) where the minus sign in the expression for \( \alpha \) for particles and antiparticles. If \( \alpha \) where \( \theta \) in \( \theta \) angle for polarized grammar.

Thus, \( \alpha \) weak\(-\) or \( CP \)-wave, and are characterized by the (real) Lee-Yang parameters \( \alpha, \beta, \gamma \), where \( \alpha \) not measured, only the \( \alpha \) and \( \alpha \) parameters are determined.

3.1. The Lee-Yang \( \alpha, \beta, \gamma \) parameters

The \( \Xi \rightarrow \Lambda \pi \) and \( \Lambda \rightarrow p \pi \) decays are weak interaction processes that involve both \( S \)- and \( P \)-waves, and are characterized by the (real) Lee-Yang parameters \( \alpha_Y, \beta_Y \) and \( \gamma_Y \), where

\[
\alpha_Y = \frac{2 \text{Re}(S_Y P_Y)}{|S_Y|^2 + |P_Y|^2}, \quad \beta_Y = \frac{2 \text{Im}(S_Y P_Y)}{|S_Y|^2 + |P_Y|^2}, \quad \gamma_Y = \frac{|S_Y|^2 - |P_Y|^2}{|S_Y|^2 + |P_Y|^2}.
\]

The decay angle \( \theta_p \) for polarized \( \Lambda \rightarrow p \pi \) decays.

The decay angle \( \theta_p \) for polarized \( \Lambda \rightarrow p \pi \) decays. The last two reference frames are the \( \Lambda \) and \( \bar{\Lambda} \) rest frames. Here the distribution of events in \( \theta_p (\theta_p) \), the direction of the proton relative to the \( \Lambda \) polarization vector (see Fig. 2), is

\[
\frac{dN}{d \cos \theta_p} \propto 1 + \alpha \Pi \cos \theta_p.
\]

where \( \Pi \) is the vector modulus of \( \Pi \). Since the proton and antiproton polarizations are not measured, only the \( \alpha \) and \( \alpha \) parameters are determined.

3.1. The Lee-Yang \( \alpha, \beta, \gamma \) parameters

The \( \Xi \rightarrow \Lambda \pi \) and \( \Lambda \rightarrow p \pi \) decays are weak interaction processes that involve both \( S \)- and \( P \)-waves, and are characterized by the (real) Lee-Yang parameters \( \alpha_Y, \beta_Y \) and \( \gamma_Y \), where

\[
\alpha_Y = \frac{2 \text{Re}(S_Y P_Y)}{|S_Y|^2 + |P_Y|^2}, \quad \beta_Y = \frac{2 \text{Im}(S_Y P_Y)}{|S_Y|^2 + |P_Y|^2}, \quad \gamma_Y = \frac{|S_Y|^2 - |P_Y|^2}{|S_Y|^2 + |P_Y|^2}.
\]

The decay angle \( \theta_p \) for polarized \( \Lambda \rightarrow p \pi \) decays. The last two reference frames are the \( \Lambda \) and \( \bar{\Lambda} \) rest frames. Here the distribution of events in \( \theta_p (\theta_p) \), the direction of the proton relative to the \( \Lambda \) polarization vector (see Fig. 2), is

\[
\frac{dN}{d \cos \theta_p} \propto 1 + \alpha \Pi \cos \theta_p.
\]

where \( \Pi \) is the vector modulus of \( \Pi \). Since the proton and antiproton polarizations are not measured, only the \( \alpha \) and \( \alpha \) parameters are determined.

The decay angle \( \theta_p \) for polarized \( \Lambda \rightarrow p \pi \) decays. The last two reference frames are the \( \Lambda \) and \( \bar{\Lambda} \) rest frames. Here the distribution of events in \( \theta_p (\theta_p) \), the direction of the proton relative to the \( \Lambda \) polarization vector (see Fig. 2), is

\[
\frac{dN}{d \cos \theta_p} \propto 1 + \alpha \Pi \cos \theta_p.
\]

where \( \Pi \) is the vector modulus of \( \Pi \). Since the proton and antiproton polarizations are not measured, only the \( \alpha \) and \( \alpha \) parameters are determined.

The decay angle \( \theta_p \) for polarized \( \Lambda \rightarrow p \pi \) decays. The last two reference frames are the \( \Lambda \) and \( \bar{\Lambda} \) rest frames. Here the distribution of events in \( \theta_p (\theta_p) \), the direction of the proton relative to the \( \Lambda \) polarization vector (see Fig. 2), is

\[
\frac{dN}{d \cos \theta_p} \propto 1 + \alpha \Pi \cos \theta_p.
\]

where \( \Pi \) is the vector modulus of \( \Pi \). Since the proton and antiproton polarizations are not measured, only the \( \alpha \) and \( \alpha \) parameters are determined.
and, if \( CP \) is not conserved, \( \xi_Y^V \neq \xi_Y^S \), and any significant difference between \( A_{CP}^V \) and zero would unambiguous evidence for a \( CP \) violation.

However, it is evident from eqn. (8) that even if \( \xi_Y^V - \xi_Y^S \) is non-zero, \( A_{CP}^V \) would be zero if \( \delta_{\pi\pi}^0 - \delta_{\pi\pi}^S = 0 \). Although it is unlikely that these strong-interaction phase-shifts are exactly zero, they are known to be small – the measured values for the \( \pi\pi \) shifts are (and \( \Lambda\pi \) shifts are) \( \delta_{\pi\pi}^P - \delta_{\pi\pi}^S = -7.2^\circ \pm 0.2^\circ \) while the theoretical values for the \( \Lambda\pi \) shifts are \( \delta_{\Lambda\pi}^P - \delta_{\Lambda\pi}^S = 8.0^\circ \pm 0.2^\circ \) and these will suppress the values of the measured \( A_{CP}^V \) asymmetry by an order of magnitude. This is not the case for measurements of the parameter \( \beta_{\Xi} \) that can be determined from the measurement of \( P_\Lambda \) in \( \Xi \to \Lambda\pi \) decays. Here, we apply the above-described calculation to a different asymmetry parameter \( B_{CP}^\Xi \), where

\[
B_{CP}^\Xi = \frac{\beta_{\Xi} + \beta_{\Xi}'}{\alpha_{\Xi} - \alpha_{\Xi}'} = \tan(\xi_{\Xi}^P - \xi_{\Xi}^S) \approx \xi_{\Xi}^P - \xi_{\Xi}^S. \quad (9)
\]

(Here \( \beta_{\Xi} + \beta_{\Xi} \) is divided by \( \alpha_{\Xi} - \alpha_{\Xi} \) and not \( \beta_{\Xi} - \beta_{\Xi} \) in order to avoid the perils of a near-zero denominator.) In this case the measured asymmetry doesn’t suffer any dilution by small strong-interaction phase-shifts. Since the \( \beta_{\Xi} \) and \( \gamma_{\Xi} \) parameters are not independent, BESIII reports measurements of \( \phi_{\Xi} \) (and \( \phi_{\Xi} \)) that are defined by the relation \((10)\)

\[
\beta_{\Xi} = \sqrt{1 - \alpha_{\Xi}^2} \sin \phi_{\Xi} \quad \text{and} \quad \gamma_{\Xi} = \sqrt{1 - \alpha_{\Xi}^2} \cos \phi_{\Xi}. \quad (10)
\]

In terms of \( \Delta \phi_{CP}^\Xi \) and \( \phi_{CP}^\Xi \): \( \xi_{\Xi}^P - \xi_{\Xi}^S = \frac{\sqrt{1 - \alpha_{\Xi}^2}}{\alpha_{\Xi}} \Delta \phi_{CP}^\Xi \)

\[
\delta_{\Lambda\pi}^P - \delta_{\Lambda\pi}^S = \frac{\sqrt{1 - \alpha_{\Xi}^2}}{\alpha_{\Xi}} (\phi_{CP}^\Xi). \quad (11)
\]

The only direct \( CP \)-violation ever seen in the strange particle sector is the kaon parameter \( \epsilon' \) that was measured as a small, non-zero effect in \( K^0 \to \pi\pi \) decay by the NA48 experiment at CERN\(^{15} \) and the KTeV experiment at Fermilab\(^{20} \). This parameter quantifies the \( CP \)-violating phase difference between the Isospin=2 and Isospin=0, parity-violating \( S \)-wave amplitudes in \( K^0(\bar{K}^0) \to \pi\pi \) decays and is suppressed in magnitude by a nominal factor of \( 1/22.5 \) by the weak-interaction’s \( \Delta I = 1/2 \) rule. The \( CP \)-phase of the \( (S\)-wave) amplitude \( K \to \pi\pi \) that is deduced from the measured value of \( \epsilon' \) and adjusted to account for the \( \Delta I = 1/2 \) rule suppression is \( \xi_{K \to \pi\pi}^S \approx 0.005^\circ \), and is within errors of the SM prediction\(^{21} \), \( \xi_{K \to \pi\pi}^S (SM) \approx 0.004^\circ \). This measured magnitude of the \( CP \)-phase in (parity-violating) \( K \to \pi\pi \) decay and its agreement with theory has been translated\(^{20\,^{22}} \) into stringent limits on contributions from BSM new physics sources to \( \xi_{\Xi}^S \) (and \( \xi_{\Xi}^S \)) and

\( ^{b} \)This is determined from \( \xi_{K \to \pi\pi}^S \approx 22.5 \sqrt{2} \text{Re} (\epsilon'/\epsilon) \times \epsilon, \) where by definition \( |\text{Im } A_{K \to \pi\pi}^V| = \sqrt{2} \text{Re}(\epsilon'/\epsilon), \) of which \( \text{Re}(\epsilon'/\epsilon) = (1.66 \pm 0.23) \times 10^{-3} \) is what is measured, \( \epsilon = (2.228 \pm 0.011) \times 10^{-3} \) is the neutral kaon mass-matrix \( CP \)-violating parameter, and the factor of 22.5 accounts for the \( \Delta I = 1/2 \) rule suppression.
this implies that if any \( CP \) violation is seen in hyperon decay at the current levels of experimental sensitivity, it would most likely correspond to enhancements in \( \xi_P^P \), \textit{i.e.} to the parity-conserving \( P \)-wave amplitude.

### 3.2. Advantages of the \( e^+e^- \rightarrow J/\psi \rightarrow \Xi^-\bar{\Xi}^+ \) environment

In the BESIII measurement, which uses fully reconstructed events of the above-described decay chains, the \( \Xi^- \) and \( \Xi^+ \) are produced in equal numbers, with equal polarizations and momenta, and under the same experimental conditions. Moreover, the decay products are detected in a low mass Helium-filled tracking volume that minimizes the main irreducible systematic differences between the \( \Xi \) and \( \bar{\Xi} \) decay chains, which are the different nuclear interaction probabilities of \( p/\bar{p} \) and \( \Lambda/\bar{\Lambda} \) daughter particles in the material of the detector. In BESIII the influence of these differences are reduced to small second-order effects.

A second important feature of this reaction is the BESIII observation of substantial \( \Xi \)-hyperon polarization in the \( J/\psi \rightarrow \Xi^-\bar{\Xi}^+ \) decay process\(^{11}\) as shown in Fig. 3, where the rms average is \( \langle P_{\Xi} \rangle_{\text{rms}} = 22.3\% \). This is important because the \( \alpha_{\Xi} \) and \( \phi_{\Xi} \) measurement sensitivities are directly proportional to the magnitude of \( P_{\Xi} \). Note that according to eqn. \( [4] \) the polarization of the daughter \( \Lambda \) in \( \hat{z} \) direction is \( P_{\Lambda} \propto \alpha_{\Xi} + P_{\Xi} \cos \theta_{\Lambda} \), and, since \( \alpha_{\Xi} \approx -0.37 \), the \( \Lambda \) hyperons that are produced in \( \Xi^- \rightarrow \Lambda\pi^- \) decays are substantially polarized even at production angles where \( P_{\Xi} = 0 \).

---

**Fig. 3.** \( \Xi \) polarization \( P_y \) and \( \Xi-\bar{\Xi} \) spin correlations \( C_{xx}, C_{yy} \) and \( C_{zz} \) in quantum entangled \( J/\psi \rightarrow \Xi\bar{\Xi} \) from BESIII\(^{13}\)

---

A third feature of this reaction is that the \( \Xi \) & \( \bar{\Xi} \) are quantum entangled, with strongly correlated \( x, y \) & \( z \) spin components. These correlations are characterized by the \( \theta_{\psi} \)-dependent quantities \( C_{xx}, C_{yy} \) & \( C_{zz} \) shown in Fig. 3 where it can be seen that they have similar magnitudes as the linear polarization \( P_y \). However, unlike the \( P_y \) terms, which are multiplied by functions of either the \( \Xi \) or the \( \bar{\Xi} \) decay angles but not both, the \( C_{jj} \) terms are multiplied by functions of both the \( \Xi \) and \( \bar{\Xi} \) decay angles. In inclusive (so-called “single-tag”) measurements, where the decay angles of \textit{only} the \( \Xi \) or \textit{only} the \( \bar{\Xi} \) are considered, the effects of the \( C_{ij} \) terms average out to zero and only the \( P_y \) terms survive\(^{23}\). In contrast, in exclusive (“double-tag”) measurements, where the decay angles of \textit{both} the \( \Xi \) and the \( \bar{\Xi} \) and their correlations, are considered event-by-event, the non-zero \( C_{ij} \) terms have the effect of increasing the \textit{effective} polarization and thereby enhance the experimental sensitivity. This is a big effect. In the BESIII measurement, these correlations enhance the statistical sensitivity of the \( \Delta\Xi_{CP} \) measurement over that for polarized, but unentangled, \( \Xi-\bar{\Xi} \) pairs, by a
factor of \( \sim 2.5 \); this is equivalent to a six-fold increase in the number of events.\(^{23}\) This is the huge benefit of having quantum entangled \( \Xi \) pairs.

The fourth big advantage of this reaction is that the \( \Lambda \) and \( \bar{\Lambda} \) polarizations are determined event-by-event and, thus, \( \phi_\Xi \) and \( \phi_{\bar{\Xi}} \) can be measured. This is important because, according to eqn. \(11\), \( \xi_{P_\Xi} - \xi_{S_\Xi} \) can be directly determined from \( \phi_\Xi \) and \( \phi_{\bar{\Xi}} \) without any dilution from the small \( \Lambda \pi \) strong interaction phase shifts. This increases the experimental sensitivity to \( CP \) phases over that for measurements based solely on \( A_{CP}^{\Xi_\ell} \) measurements by an order of magnitude.

### 3.3. Analysis and results

The simplicity of the event topology belies a rather intricate analysis. Nine angular measurements are needed to completely specify the kinematics of the event: the production angle, \( \theta_\psi \), and a pair of angles, \( \theta_Y \) & \( \phi_Y \) for each of the four hyperon decay vertices. The function that describes these highly correlated angular distributions is characterized by eight parameters: \( \alpha_\psi \) & \( \Delta \Phi \) for the \( J/\psi \rightarrow \Xi^- \Xi^+ \) decay vertex, two parameters for each \( \Xi \) decay and one parameter for each \( \Lambda \) decay (in which the proton polarization is not measured). The measured quantities are expressed as a nine-component vector \( \xi \) and the angular distribution by an eight-component vector \( \omega \):

\[
\xi = (\theta_\psi, \theta_\Lambda, \varphi_\Lambda, \theta_\bar{\Lambda}, \varphi_\bar{\Lambda}, \theta_p, \varphi_p)
\]

\[
\omega = (\alpha_\psi, \Delta \Phi, \alpha_\Xi, \alpha_{\Xi}, \alpha_{\bar{\Xi}}, \varphi_{\bar{\Xi}}, \alpha_{\bar{\Lambda}}).
\]

Perotti et al.\(^{12}\) provide a modularized expression for the nine dimensional probability distribution that is in a convenient form for symbolic computing:

\[
W(\xi : \omega) = C_{\mu \nu} \left( \sum_{\mu' = 0}^{3} a^{\Xi}_{\mu \nu} a^{\Lambda}_{\mu' 0} \right) \left( \sum_{\nu' = 0}^{3} a^{\bar{\Xi}}_{\nu \nu'} a^{\bar{\Lambda}}_{\nu' 0} \right),
\]

where \( C_{\mu \nu} \) is a 4 \times 4 matrix that contains the \( \theta_\psi \)-dependent \( \Xi \) & \( \bar{\Xi} \) polarizations and spin correlations, and each \( a^{\gamma}_{\alpha \alpha'} \) represents a 4 \times 4 matrix with elements that contain expressions that involve \( \alpha_\gamma \) & \( \phi_\gamma \) parameters and Wigner D-functions of the \( \theta_\gamma \), \( \varphi_\gamma \) angular variables. The sums in eqn. \( \ref{eqn:15} \) involve a total of 256 expressions, of which 100 are non-zero.

The BESIII measurement\(^{11}\) are based on an analysis of 73k fully reconstructed events found in a data sample that contains a total of 1.3B \( J/\psi \) decays to all modes. The results include the world’s most precise measurement of \( \alpha_\Xi \) and the first ever measurements of \( \alpha_{\bar{\Xi}} \) and \( A_{CP}^{\Xi_\ell} \). The precision level of the \( \alpha_{\Xi} \) and \( A_{CP}^{\Xi_\ell} \) measurements based on 73k \( J/\psi \rightarrow \Xi^- \Xi^+ \) events is very nearly the same as that for previous BESIII measurements of \( \alpha_{\bar{\Lambda}} \) and \( A_{CP}^{\bar{\Lambda}} \) that were based on a sample of 420k \( J/\psi \rightarrow \Lambda \bar{\Lambda} \) events.\(^{23}\)

\(^{4}\)For inclusive single-tag \( \Xi \) measurements the probability distribution is

\[
W(\xi : \omega) = C_{\mu 0} \left( \sum_{\mu' = 0}^{3} a^{\Xi}_{\mu \nu} a^{\Lambda}_{\mu' 0} \right),
\]

and only the first column of the \( C_{\mu \nu} \) matrix, which contains \( P_{\gamma} \) but no \( C_{jj} \) terms, contributes.
This reflects the benefits of the event-by-event baryon-antibaryon spin correlations that are mentioned above. Moreover, the 73k \( \Xi^- \Xi^+ \) events provided independent measurements of \( \alpha_\Lambda, \alpha_\bar{\Lambda} \) and \( A^\Lambda_{CP} \) with similar precision as those from the earlier higher-statistics BESIII measurements. This is because the rms average polarization of the \( \Lambda \) and \( \bar{\Lambda} \) hyperons produced via \( \Xi \) decays is about twice that of those produced directly in \( J/\psi \to \Lambda \bar{\Lambda} \) decays. These new \( \alpha_\Lambda \) and \( \alpha_\bar{\Lambda} \) measurements provide independent confirmation of the earlier BESIII results that are \( 5\sigma \) higher than their (45 year old) pre-2018 world average values.

Of particular note are the BESIII measurements of \( \phi_\Xi \) and \( \phi_\bar{\Xi} \):

\[
\phi_\Xi = 0.6^\circ \pm 1.1^\circ \pm 0.5^\circ \quad \text{and} \quad \phi_\bar{\Xi} = -1.2^\circ \pm 1.1^\circ \pm 0.4^\circ, \tag{16}
\]

that, according to eqns. (11) translate into

\[
\xi^\Xi_P - \xi^\Xi_S = 0.7^\circ \pm 2.0^\circ \quad \in \{-2.6^\circ, +4.0^\circ\} \quad \text{(90\% C.L.)} \tag{17}
\]

\[
\delta^\Lambda_{\pi \pi} - \delta^\bar{\Lambda}_{\pi \pi} = -2.3^\circ \pm 2.1^\circ \quad \in \{-5.8^\circ, +1.2^\circ\} \quad \text{(90\% C.L.)} \tag{18}
\]

where the errors are the quadratic sums of statistical and systematic errors, with correlations between the \( \phi_\Xi \) and \( \phi_\bar{\Xi} \) taken into account, and our estimates of the 90\% confidence level allowed intervals are indicated. These limits teach us a few important things:

- The eqn. (17) \( CP \)-phase limit interval, which is based on an analysis of the 73k \( \Xi^- \Xi^+ \) events found in a sample of 1.3B \( J/\psi \) decays, is quite similar to the one based on a recently submitted BESIII measurement of the \( \Lambda \) asymmetry parameter with 3.2M fully reconstructed \( \Lambda \bar{\Lambda} \) events that were found in a sample of 10B \( J/\psi \) decays.\(^{25}\)

\[
A^\Lambda_{CP} = -0.0025 \pm 0.0047 \tag{19}
\]

\[
\xi^\Xi_P - \xi^\Xi_S = (-1.1 \pm 2.1)^\circ \quad \in \{-4.5^\circ, +2.1^\circ\} \quad \text{(90\% C.L.).}
\]

The \( \Delta \phi^\Xi_{CP} \)-measurements achieve the same sensitivity as those based on \( A^\Lambda_{CP} \) with a factor of 40 fewer events.

- There is still considerable room left for the possible influence of new physics in non-leptonic hyperon decay processes before the level of SM effects start to show up. The SM values for the \( \xi^\Xi_P \) and \( \xi^\Xi_S \) \( CP \)-violating phases have nearly equal but opposite-sign values with the net result\(^{21}\)

\[
(\xi^\Xi_P - \xi^\Xi_S)_{\text{SM}} = -(1.4 \pm 1.2) A^2 \lambda^\Xi \eta = -0.01^\circ \pm 0.01^\circ, \tag{20}
\]

where \( A^2 \lambda^\Xi \eta \) is the imaginary part of the \( V_{ts}^* V_{td} \) CKM matrix element product in the Wolfenstein parametrization.\(^{25}\)

- Since BESIII’s measured value for the \( \delta^\Lambda_{\pi \pi} - \delta^\bar{\Lambda}_{\pi \pi} \) strong-phase difference is < 0.1 rad (eqn. (18)), the limit on \( \xi^\Xi_P - \xi^\Xi_S \) that can be inferred from \( A^\Xi_{CP} \) is almost an order of magnitude less stringent than that derived from the \( \Delta \phi^\Xi_{CP} \) measurement.

The only previous \( CP \)-related experimental result that involved \( \Xi \) hyperons is a HyperCP measurement\(^{22}\) of \( A^\Xi_{CP} \equiv (\alpha_\Xi - (\alpha_\Xi + \alpha_\bar{\Xi})) / (\alpha_\Xi + \alpha_\bar{\Xi}) = (0.0 \pm 0.7) \times 10^{-3} \), using \( \Xi^- \to \pi^- \Lambda (\pi^- p) \) decays in a \( \Xi^- \) beam with an average polarization of \( \langle P_\Xi \rangle \approx 0.035 \), and corresponding measurements with a \( \Xi^+ \) beam. This is a mixture of \( \Xi \) and \( \Lambda \)
asymmetry parameters that cannot be directly compared to the BESIII individual $A_{CP}^\Xi$ and $A_{CP}^\Lambda$ measurements, and are also subject to the same provisos about dilution by small strong-interaction phase shifts. HyperCP also used their $\Xi^-$ beam data to make the first determination of $\phi_{\Xi} = -2.4^\circ \pm 0.8^\circ$, which differs from the BESIII measurement by about 2.5 standard deviations.

4. Future prospects

The BESIII results described here are based on a 1.3B $J/\psi$ event sample. At the current time, BESIII has accumulated a total of 10B $J/\psi$ events that are still being analyzed. Thus, we can expect an improvement in the experimental sensitivity for $\xi_P^\Xi - \xi_S^\Xi$ to the $\sim 1^\circ$ level in the not so distant future. Moreover, the above-described analysis applies equally well to the similar number of $J/\psi \rightarrow \Xi^0\Xi^0$ events that exist in the BESIII 10B $J/\psi$-event sample that, when analyzed, should have a similar level of $CP$ sensitivity.

However, this level of sensitivity is still two orders of magnitude above the SM expectations that are described above, and new physics may be lurking anywhere in between. Substantial improvements on the existing BESIII limit and those expected in the near future using the above-described techniques would require data samples that contain trillions of $J/\psi$ events.

In their most recent data-taking run at the peak energy of the $J/\psi$ ($E_{cm} = 3.096$ GeV), the BESIII group collected about 1B $J/\psi$-events/month. Although this was a remarkable achievement, that rate is well below what would be needed to accumulate trillions of $J/\psi$ events. The BEPCII collider design was optimized for studies of charmed mesons that are produced in the process $e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$ at $E_{cm} = 3.773$ GeV with a peak luminosity of $10^{33}$ cm$^{-2}$ s$^{-1}$, which, at that time (circa 2005), was the state-of-the-art for $e^+e^-$ colliders at that energy. In BEPCII, the $e^+$ and $e^-$ beam bunches are about 1 cm in length and cross at small angles ($\pm 0.6^\circ$) in the horizontal plane. In this case, the luminosity suffers from the so-called “hour-glass effect” that restricts how tightly the beams can be compressed at the collision point; this limits the maximum luminosity to be at the $10^{33}$ cm$^{-2}$ s$^{-1}$ level. During the two decades that followed the time that BEPCII was designed, there have been a number of innovations in $e^+e^-$ collider technology that would, in principle, allow for further reductions of the beam sizes at the interaction point that could evade hour-glass effect limitations and potentially enhance the luminosity by as many as two orders of magnitude. These include:

**Nanobeam collisions:** a scheme in which very flat beams cross at large angles ($\sim \pm 1.7^\circ$) in the horizontal plane.

**Crab-waist collision scheme:** Where the focus point (i.e., the “waist”) for particles on one of the horizontal sides of a beam is advanced, while that on the other side of the same beam is retarded so that, in spite of the large crossing angle the waists of all of the particles in both of the two beams are synchronized, thereby avoiding the hourglass effect.

Although the incorporation of these schemes in the DAFNE and the SuperKEKB have
provided proof-of-principle for these luminosity-enhancement schemes, they have not yet been translated into the hoped for order-of-magnitude luminosity improvement. The full exploitation of these schemes, or variations of them, in a working $e^+e^-$ collider is a high priority issue in the world’s accelerator physics community\(^3\) the critical factors that are limiting the luminosity have been identified and steady progress is being made to solve them.\(^4\) This progress and the potential payoff in higher luminosity have inspired proposals in China\(^3\) and in Russia\(^5\) for futuristic “Super $\tau$-charm factories” with luminosity goals that are $\sim 50\times$ that of BEPCII. With either, or both, of these facilities, the accumulation of trillion (or more) $J/\psi$-event data samples would be feasible, and an order magnitude improvement on the BESIII sensitivity for non-SM CP-violation detection using the $e^+e^- \rightarrow \Xi\Xi$ process would be possible.

5. Summary

The BESIII experiment has opened up a new portal in the quest to identify new, beyond-the-Standard Model sources of CP violation. However, the limits they that have now, and will set soon, still leave two orders of magnitude of unexplored parameter space before SM-level effects are expected. The measurement technique that is described here is specific to the $J/\psi \rightarrow \Xi^-\Xi^+$ and $\Xi^0\Xi^0$ reactions, and the main issue impeding searches with improved sensitivities is statistics; trillion $J/\psi$ event data samples are needed, and these could be provided by new facilities that have been proposed in China and in Russia.

Acknowledgments

This work is supported in part by the National Natural Science Foundation of China (NSFC) under Contracts No. 12122509, the Korean Institute For Basic Science under project code IBS-R016-D1 and the Chinese Academy of Science President’s International Fellowship Initiative (PIFI) program.

References

1. L. Leprince-Ringuet and M. Lheritier, *Compt. Rend. Acad. Sci.* **219**, 618 (1944).
2. G. Rochester and C. Butler, *Nature* **160**, 856 (1947).
3. SLAC-SP-017 Collaboration (J. E. Augustin et al.), *Phys. Rev. Lett.* **33**, 1406 (1974), doi:10.1103/PhysRevLett.33.1406.
4. E598 Collaboration (J. J. Aubert et al.), *Phys. Rev. Lett.* **33**, 1404 (1974), doi:10.1103/PhysRevLett.33.1404.
5. nEDM Collaboration (C. Abel et al.), *Phys. Rev. Lett.* **124**, 081803 (2020), doi:10.1103/PhysRevLett.124.081803.
6. V. Baluni, *Phys. Rev. D* **19**, 2227 (1979), doi:10.1103/PhysRevD.19.2227.
7. LHCB Collaboration (A. A. Alves, Jr. et al.), *JINST* **3**, S08005 (2008), doi:10.1088/1748-0221/3/08/S08005.
8. Belle II Collaboration (T. Aushev et al.) (2010), arXiv:1002.5012.

\(^{a}\)Many of the issues that limit the luminosity in low-energy “flavor-factory” colliders also apply to proposed high energy colliders, such as the FCC-ed\(^{13}\) at CERN and the CEPC\(^{13}\) in China.
9. CPLEAR Collaboration (A. Angelopoulos et al.), *Phys. Rept.* **374**, 165 (2003), doi:10.1016/S0370-1573(02)00367-8.
10. E756 Collaboration (K. B. Luk et al.), *Phys. Rev. Lett.* **85**, 4860 (2000), doi:10.1103/PhysRevLett.85.4860.
11. BESIII Collaboration (M. Ablikim et al.), *Nature* **606**, 64 (2022), doi:10.1038/s41586-022-04624-1.
12. E. Perotti, G. Fåldt, A. Kupsc, S. Leupold and J. J. Song, *Phys. Rev. D* **99**, 056008 (2019), doi:10.1103/PhysRevD.99.056008.
13. T. D. Lee and C.-N. Yang, *Phys. Rev.* **108**, 1645 (1957), doi:10.1103/PhysRev.108.1645.
14. J. F. Donoghue and S. Pakvasa, *Phys. Rev. Lett.* **55**, 162 (1985), doi:10.1103/PhysRevLett.55.162.
15. E. Matsinos, [arXiv:2208.09207 [nucl-th]].
16. B.-L. Huang, J.-S. Zhang, Y.-D. Li and N. Kaiser, *Phys. Rev. D* **96**, 016021 (2017), doi:10.1103/PhysRevD.96.016021.
17. J. F. Donoghue, X.-G. He and S. Pakvasa, *Phys. Rev. D* **34**, 833 (1986), doi:10.1103/PhysRevD.34.833.
18. NA48 Collaboration (A. Lai et al.), *Eur. Phys. J. C* **22**, 231 (2001), doi:10.1007/s100520100822.
19. KTeV Collaboration (A. Alavi-Harati et al.), *Phys. Rev. Lett.* **83**, 22 (1999), doi:10.1103/PhysRevLett.83.22.
20. A. J. Buras, *Acta Phys. Polon. B* **52**, 7 (2021), doi:10.5506/APhysPolB.52.7.
21. J. Tandean and G. Valencia, *Phys. Rev. D* **67**, 056001 (2003), doi:10.1103/PhysRevD.67.056001.
22. X.-G. He, H. Murayama, S. Pakvasa and G. Valencia, *Phys. Rev. D* **61**, 071701 (2000), doi:10.1103/PhysRevD.61.071701, arXiv:hep-ph/9909562.
23. N. Salone, P. Adlarson, V. Batozskaya, A. Kupsc, S. Leupold and J. Tandean, Study of CP violation in hyperon decays at Super Charm-Tau Factories with a polarized electron beam, in 2022 Snowmass Summer Study, (3 2022). arXiv:2203.03035.
24. BESIII Collaboration (M. Ablikim et al.), *Nature Phys.* **15**, 631 (2019), doi:10.1038/s41567-019-0494-8.
25. BESIII Collaboration (M. Ablikim et al.) (4 2022), arXiv:2204.11058.
26. L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983), doi:10.1103/PhysRevLett.51.1945, the PDG 2020 central values are $\eta = 0.355$, $\lambda = 0.355$, $\Lambda = 0.355$, $\gamma = 0.355$, $\beta = 0.355$, & $\lambda = 0.2245$.
27. HyperCP Collaboration (T. Holmstrom et al.), *Phys. Rev. Lett.* **93**, 262001 (2004), doi:10.1103/PhysRevLett.93.262001, arXiv:hep-ex/0412038.
28. HyperCP Collaboration (M. Huang et al.), *Phys. Rev. Lett.* **93**, 011802 (2004), doi:10.1103/PhysRevLett.93.011802.
29. P. Raimondi, *Conf. Proc. C* **0606141**, 104 (2006).
30. DAFNE Collaboration (M. Zobov), *J. Phys. Conf. Ser.* **747**, 012090 (2016), doi:10.1088/1742-6596/747/1/012090, arXiv:1608.06150.
31. SuperKEKB accelerator group Collaboration (Y. Onishi), *PoS ICHEP2020*, 695 (2021), doi:10.22323/1.390.0695.
32. I. Agapov et al., Future Circular Lepton Collider FCC-ee: Overview and Status, in 2022 Snowmass Summer Study, (3 2022). 2203.08310.
33. H. Geng et al., Status of CEPC Lattice Design, in 55th ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e+e- Colliders – Higgs Factory, (2015), p. THT4B4.
34. D. Zhou (2021), https://www-kekb.kek.jp/MAC/2021/Report/Zhou.pdf.
35. J. Q. Lan, Q. Luo, C. Zhang, W. W. Gao, Y. Xu and Y. Bai, *JINST* **16**, T07001 (2021), doi:10.1088/1748-0221/16/07/T07001.
36. E. Levichev, *Phys. Part. Nucl. Lett.* **5**, 554 (2008), doi:10.1134/S1547477108070030.