Working group summary: $\pi N$ sigma term

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Key-Words: Chiral perturbation theory, Pion nucleon interactions, Sigma term

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INTRODUCTION

The denomination “sigma term” stands, in a generic way, for the contribution of the quark masses $m_q$ to the mass $M_h$ of a hadronic state $|h(p)\rangle$. According to the Feynman-Hellmann theorem [1], one has the exact result (the notation does not explicitly take into account the spin degrees of freedom)

$$\frac{\partial M^2}{\partial m_q} = < h(p)|(\bar{q}q)(0)|h(p) > .$$

(1)

In practice, and in the case of the light quark flavours $q = u, d, s$, one tries to perform a chiral expansion of the matrix element of the scalar density appearing on the right-hand side of this formula. In the case of the pion, for instance, one may use soft-pion techniques to obtain the well-known result [4] (here and in what follows, $O(M^n)$ stands for corrections of order $M^n$ modulo powers of $\ln M$)

$$\frac{\partial M^2}{\partial m_q} = \frac{< \bar{q}q >_0}{F_0^2} + O(m_u, m_d, m_s), q = u, d, s, \text{ and } \frac{\partial M^2}{\partial m_q} = 0 + O(m_u, m_d, m_s),$$

(2)

where $< \bar{q}q >_0$ denotes the single flavour light-quark condensate in the $SU(3)_L \times SU(3)_R$ chiral limit, while $F_0$ stands for the corresponding value of the pion decay constant $F_0 = 92.4$ MeV.

In the case of the nucleon, the sigma term is defined in an analogous way, as the value at zero momentum transfer $\sigma \equiv \sigma(t = 0)$ of the scalar form factor of the nucleon $(t = (p' - p)^2)$,

$$\pi_N(p')\mu_N(p)|\sigma(t) = \frac{1}{2M_N^2} < N(p')|\hat{m}u + \bar{d}d)(0)|N(p) > ,$$

(3)

and contains, in principle, information on the quark mass dependence of the nucleon mass $M_N$. Most theoretical evaluations of the nucleon sigma term consider the isospin symmetric limit $m_u = m_d$, but this is not required by the definition (3).

Another quantity of particular interest in this context is the relative amount of the nucleon mass contributed by the strange quarks of the sea,

$$y \equiv 2\frac{< N(p)|(|\bar{s}s)(0)|N(p) >}{< N(p)|(|\bar{u}u + \bar{d}d)(0)|N(p) >} .$$

(4)

Large-$N_c$ considerations (Zweig rule) would lead one to expect that $y$ is small, not exceeding $\sim 30\%$. The ratio $y$ can be related, via the sigma term and the strange to non-strange quark mass ratio, to the nucleon matrix element of the $SU(3)_V$ breaking part of the strong hamiltonian,

$$\sigma(1 - y) \left( \frac{m_s}{\hat{m}} - 1 \right) = \frac{1}{2M_N^2} < N(p')|(m_s - \hat{m})(\bar{u}u + \bar{d}d - 2\bar{s}s)(0)|N(p) > .$$

(5)

For the standard scenario of a strong $< \bar{q}q >_0$ condensate, $m_s/\hat{m} \sim 25$, the evaluation of the product $\sigma(1 - y)$ in the chiral expansion gives $\sim 26$ MeV at order $O(m_q)$ [3], $\sim 35 \pm 5$ MeV at order $O(m_q^{3/2})$ [3], and $\sim 36 \pm 7$ MeV at order $O(m_q^2)$ [3].

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THE NUCLEON SIGMA TERM AND $\pi N$ SCATTERING

Although the nucleon sigma term is a well-defined QCD observable, there is, unfortunately, no direct experimental access to it. A link with the $\pi N$ cross section (for the notation, we refer the reader to Refs. [4,4]) at the unphysical Cheng-Dashen point, $\Sigma = F^{\pi^+}_\pi \mathcal{D}^+(\nu = 0, t = 2M^2_\pi)$, is furnished by a very old low-energy theorem [8].

$$\Sigma = \sigma(1 + O(m_q^{1/2})). \quad (6)$$

A more refined version of this statement [2] relates $\Sigma$ and the form factor $\sigma(t)$ at $t = 2M^2_\pi$,

$$\Sigma = \sigma(2M^2_\pi) + \Delta_R, \quad (7)$$

where $\Delta_R = O(m_q^2)$. The size of the correction $\Delta_R$, as estimated within the framework of Heavy Baryon Chiral Perturbation Theory (HBChPT), is small [10], $\Delta_R \leq 2 \text{ MeV}$ (an earlier calculation to one-loop in the relativistic approach [11] gave $\Delta_R = 0.35 \text{ MeV}$).

In order to obtain information on $\sigma$ itself, one thus needs to pin down the difference $\Delta_\sigma \equiv \sigma(2M^2_\pi) - \sigma(0)$, and to perform an extrapolation of the $\pi N$ scattering data from the physical region $t \leq 0$ to the Cheng-Dashen point, using the existing experimental information and dispersion relations. The analysis of Refs. [1,12,13], using a dispersive representation of the scalar form factor of the pion, gives the result $\Delta_\sigma = 15.2 \pm 0.4 \text{ MeV}$. On the other hand, from the subthreshold expansion

$$\mathcal{D}^+(\nu = 0, t) = d^+_{00} + td^+_{01} + \cdots \quad (8)$$

one obtains $\Sigma = \Sigma_d + \Delta_D$, with $\Sigma_d = F^{\pi^+}_\pi(d^+_{00} + 2M^2_\pi d^+_{01})$, and $\Delta_D$ is the remainder, which contains the contributions from the higher order terms in the expansion (8). In Ref. [13], the value $\Delta_D = 11.9 \pm 0.6 \text{ MeV}$ was obtained, so that the determination of $\sigma$ boils down to the evaluation of the subthreshold parameters $d^+_{00}$ and $d^+_{01}$. Their values can in principle be obtained from experimental data on $\pi N$ scattering, using forward dispersion relations [3,4]

$$d^+_{00} = \mathcal{D}^+(0,0) = \mathcal{D}^+(M_\pi,0) + \mathcal{J}_D(0), \quad d^+_{11} = \mathcal{E}^+(0,0) = \mathcal{E}^+(M_\pi,0) + \mathcal{J}_E(0), \quad (9)$$

where $\mathcal{J}_D(0)$ and $\mathcal{J}_E(0)$ stand for the corresponding forward dispersive integrals, while the subtraction constants are expressed in terms of the $\pi N$ coupling constant $g_{\pi N}$ and of the S- and P-wave scattering lengths as follows:

$$\mathcal{D}^+(M_\pi,0) = 4\pi(1+x)a^+_{0+} + \frac{g^2_{\pi NN} x^3}{M_\pi(4-x^2)}, \quad \mathcal{E}^+(M_\pi,0) = 6\pi(1+x)a^+_{1+} - \frac{g^2_{\pi NN} x^2}{M_\pi(2-x)}, \quad (10)$$

The dispersive integrals $\mathcal{J}_D(0)$ and $\mathcal{J}_E(0)$ are evaluated using $\pi N$ scattering data, which exist only above a certain energy, and their extrapolation to the low-energy region using dispersive methods. In the analysis of Ref. [4], the two scattering lengths $a^+_{0+}$ and $a^+_{1+}$ are kept as free parameters of the extrapolation procedure. In the Karlsruhe analysis, their values were obtained from the iterative extrapolation procedure itself [3]. Using the partial waves of [4,5], the authors of Ref. [6] obtain the following simple representation of $d^+_{00}$ and $d^+_{01}$ (with $a^+_{0+}, t = 0,1$, in units of $M_\pi^{-1-2t}$),

$$d^+_{00} = -1.492 + 14.6(a^+_{0+} + 0.010) - 0.4(a^+_{1+} - 0.133),$$
$$d^+_{01} = 1.138 + 0.003(a^+_{0+} + 0.010) + 20.8(a^+_{1+} - 0.133). \quad (11)$$

This leads then to a value $\sigma \sim 45 \text{ MeV}$, corresponding to $y \sim 0.2$ [12]. Further details of this analysis can be found in Refs. [1,4,5].
THEORETICAL ASPECTS

In the framework of chiral perturbation theory, the sigma term has an expansion of the form

\[ \sigma \sim \sum_{n \geq 1} \sigma_n M_n^{n+1}. \]  

(12)

The first two terms of this expansion were computed in the framework of the non-relativistic HBChPT in Ref. [17],

\[ \sigma_1 = -4c_1, \quad \sigma_2 = -\frac{9g_A^2}{64\pi F_\pi^2}. \]  

(13)

The determination of the low-energy constant \(c_1\), which appears also in the chiral expansion of the \(\pi N\) scattering amplitude, is crucial for the evaluation of \(\sigma\). Earlier attempts, which extracted the value of \(c_1\) from fits to the \(\pi N\) amplitude extrapolated to the threshold region using the phase-shifts of Refs. [14,15], obtained rather large values, \(\sigma \sim 59\) MeV [18] \((c_1 = -0.94 \pm 0.06\) GeV\(^{-1}\)), or even \(\sigma \sim 70\) MeV [19] \((c_1 = -1.23 \pm 0.16\) GeV\(^{-1}\)), as compared to the result of Ref. [12].

The threshold region in the case of elastic \(\pi N\) might however correspond to energies which are already too high in order to make these determinations of \(c_1\) stable as far as higher order chiral corrections are concerned. A new determination of \(c_1\), obtained by matching the \(\mathcal{O}(q^3)\) HBChPT expansion of the \(\pi N\) amplitude inside the Mandelstam triangle with the dispersive extrapolation of the data leads to a smaller value [20,21], \(c_1 = -0.81 \pm 0.15\) GeV\(^{-1}\), corresponding to \(\sigma \sim 40\) MeV. It remains however to be checked that higher order corrections do not substantially modify this result. Let us mention in this respect that the higher order contribution \(\sigma_3\) (which contains a non-analytic \(\mathcal{O}(M_n^2 \ln M_n/M_N)\) piece) in the expansion [12] has been computed in the context of the manifestly Lorentz-invariant baryon chiral perturbation theory in Ref. [22], (see also [23]). Once the expression of the \(\pi N\) amplitude is also known with the same accuracy [23], a much better control over the chiral perturbation evaluation of \(\sigma\) should be reached.

Finally, let us also mention that the results quoted above were based on the \(\pi N\) phase-shifts obtained by the Karlsruhe group [3]. Using instead the SP99 phase-shifts of the VPI/GW group, the authors of Ref. [20] obtain a very different result, \(c_1 \sim -3\) GeV\(^{-1}\), which leads to \(\sigma \sim 200\) MeV. Needless to say that the consequences of this last result \((y \sim 0.8)\) would be rather difficult to accept.

EXPERIMENTAL DEVELOPMENTS

We next turn to the discussion of several new experimental results which have some bearing on the value of the nucleon sigma term. All numerical values quoted below use \(M_\pi = 139.57\) MeV and \(F_\pi = 92.4\) MeV.

Let us start with the influence of the scattering length \(a_{1+}\) on the value of the sub-threshold parameter \(d_{00}^+\), using Eq. (11) and \(a_{1+} = 0.133 M_\pi^{-3}\). The first line of Table 1 gives the result obtained from the value of the phase-shift analysis of Ref. [3]. In the second line of Table 1, we show the value reported at this conference [24] and obtained from the data on pionic hydrogen, \(10^5 M_\pi \times a_{0+}^+ = 1.6 \pm 1.3\). The analysis of Loiseau et al. [25] consists in extracting the combinations of scattering lengths \(a_{\pi^-p} + a_{\pi^-n}\) from the value of pion deuteron scattering length \(a_{\pi-d}\) obtained from the measurement of the strong interaction width and lifetime of the 1S level of the pionic deuteron atom [26,27]. Assuming charge exchange symmetry \((a_{\pi+p} = a_{\pi-n})\), they find \(10^5 M_\pi \times a_{0+}^+ = -2 \pm 1\) (third line of Table 1). Another determination of \(a_{0+}^+\) is also possible using the GMO sum rule (we use here the form presented in [27], with the value of the total cross section dispersive integral \(J^-= -1.083(25)\), expressed in mb and \(a_{\pi^-p}, a_{0+}^+\) expressed in units of \(M_\pi^{-1}\))

\[ g_{\pi N}^2/4\pi = -4.50 J^- + 103.3 a_{\pi^-p} - 103.3 a_{0+}^+. \]  

(14)

Using the value \(a_{\pi^-p} = 0.0883 \pm 0.0008\) obtained by [25] and the determination \(g_{\pi N} = 13.51 \pm 12\) from the Uppsala charge exchange \(np\) scattering data [23], one obtains \(a_{0+}^+ = -0.005 \pm 0.003\). The resulting effect on \(\Sigma_d\) is shown on the fourth line of Table 1.
Table 1. $d_{00}^+$ for different values of the scattering length $a_{0+}^+$.

|                | $a_{0+}^+ \times 10^3 m$ | $F_2^2 d_{00}^+$ (MeV) | $\Delta \Sigma_d$ (MeV) |
|----------------|--------------------------|-------------------------|--------------------------|
| KH            | $-9.7$                   | $-91.0$                 | 0                        |
| $A_{\pi-d}$   | $+2 \pm 1$               | $-80 \pm 1$             | +11                      |
| $A_{\pi-d}$+GMO | $-2 \pm 1$               | $-84 \pm 1$             | +7                       |
| $g_{\pi N}$+GMO | $-5 \pm 3$               | $-87 \pm 3$             | +4                       |

Several new determinations of the $\pi N$ coupling constant $g_{\pi N}$ have also been reported at this meeting, with values which differ from the “canonical” value obtained long ago. Since most of these recent determinations do not result from a complete partial-wave analysis of $\pi - N$ scattering data, we can only compare the effect of variations in the value of $g_{\pi N}$ on the subtraction terms. The results are shown in Tables 2 and 3, respectively. Again, we take the value of $\pi N$ coupling constant, and for fixed value of the scattering length $a_{0+}^+ \times 10^3 m = -9.7$.

Table 2. The subtraction constant $D^+ (M_{\pi}, 0)$ of Eq. (10) for different values of the $\pi N$ coupling constant, and for fixed value of the scattering length $a_{0+}^+ \times 10^3 m = -9.7$.

|                | $g_{\pi N}^2/4\pi$ | $F_2^2 D^+ (M_{\pi}, 0)$ (MeV) | $\Delta \Sigma_d$ (MeV) |
|----------------|---------------------|-------------------------------|--------------------------|
| KH            | $14.3 \pm 0.2$      | 0.53                          | 0                        |
| VPI/GW        | $13.73 \pm 0.07$    | 0.16                          | -0.37                    |
| $A_{\pi-d}$+GMO | $14.2 \pm 0.2$      | 0.46                          | -0.07                    |
| Uppsala       | $14.52 \pm 0.26$    | 0.67                          | +0.14                    |

Table 3. The subtraction constant $E^+ (M_{\pi}, 0)$ of Eq. (14) for different values of the $\pi N$ coupling constant, and for fixed value of the scattering length $a_{i+}^+ \times 10^3 m = 133$.

|                | $g_{\pi N}^2/4\pi$ | $F_2^2 M_\pi^2 E^+ (M_{\pi}, 0)$ (MeV) | $\Delta \Sigma_d$ (MeV) |
|----------------|---------------------|----------------------------------------|--------------------------|
| KH            | $14.3 \pm 0.2$      | 105                                    | 0                        |
| VPI/GW        | $13.73 \pm 0.07$    | 108                                    | +6                       |
| $A_{\pi-d}$+GMO | $14.2 \pm 0.2$      | 105                                    | +1                       |
| Uppsala       | $14.52 \pm 0.26$    | 104                                    | -2                       |

Finally, we have summarized the various results in Table 4, where now the complete results for the determination of the dispersive integrals $J_D$ and $J_E$ have been included where possible, i.e., in the case of the KH and of the VPI/GW analyses (see also Table 1 in [31]). The corresponding values of $\Sigma_d$ are given in the last column of Table 4. The analysis of the VPI/GW group increases the value of the sigma term by more than 25%, as compared to the value extracted from the KH phase-shift analysis. This would lead to a value of $y \sim 0.5$, which is rather difficult to understand theoretically. It should also be
noticed that this large difference is due for a large part to the value \(d_{01}^+ = (1.27 \pm 0.03)M_{\pi}^{-3}\) (including a shift in the value of the scattering length \(a_{1+}\), which by itself accounts for half of the difference between KH and VPI/GW in the \(d_{01}^+\) contribution in Table 4) as quoted by the VPI/GW group and obtained from fixed-\(t\) dispersion relation. A similar analysis, but based on so-called interior dispersion relation (see for instance [31] and references therein), yields a much smaller value, \(d_{01}^+ = 1.18M_{\pi}^{-3}\) [32], which lowers the VPI/GW value of \(\Sigma_d\) in Table 4 by 10 MeV. It remains therefore difficult to assess the size of the error bars that should be assigned to the numbers given above. Also, the VPI/GW phase-shifts have sometimes been criticized as far as the implementation of theoretical constraints (analyticity properties) is concerned (see for instance [33]). Furthermore, the issue of having a coherent \(\pi N\) data base remains a crucial aspect of the problem. The VPI/GW partial wave analyses include data posterior to the analyses of the Karlsruhe group, but which are not always mutually consistent (see e.g. [16] and references therein). Hopefully, new experiments (see [34]), will help in solving the existing discrepancies.

Table 4. Comparison of the values of the subthreshold parameters \(d_{00}^+\) and \(d_{01}^+\) according to differences in the input discussed in the text.

|            | \(F_{\pi}^2d_{00}^+\) (MeV) | \(2M_{\pi}^2F_{\pi}^2d_{01}^+\) (MeV) | \(\Sigma_d\) (MeV) |
|------------|-----------------------------|----------------------------------|------------------|
| KH         | -89.4                       | 139.2                            | 50               |
| VIP/GW     | -77.3                       | 155.2                            | 50 +12+16        |
| \(A_{\pi-d} + GMO\) | -83                          | –                                | 50+6             |
| Uppsala   | -86                         | –                                | 50+3.5           |

Finally, it should be stressed that the above discussion is by no means a substitute for a more elaborate analysis, along the lines of Ref. [7], for instance (see also [31] and [30]) Such a task would have been far beyond the competences of the present author, at least within a reasonable amount of time and of work. Nevertheless, very useful discussions with G. Höhler, M. Pavan, M. Sainio and J. Stahov greatly improved the author’s understanding of this delicate subject. The author also thanks R. Badertscher and the organizing committee for this very pleasant and lively meeting in Zuoz.

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