TESTS FOR TAU’S CHARGED-CURRENT STRUCTURE

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Abstract

The Lorentz structure of the tau lepton’s charged-current can almost be completely
determined by use of stage-two spin-correlation functions for the \( \{\rho^-, \rho^+\} \) and \( \{a_1^-, a_1^+\} \) decay
modes. It is possible to test for a “(V – A) + something” structure in the \( J_{\text{ChargedLepton}} \)
current, so as to bound the scales \( \Lambda \) for “new physics” such as arising from tau weak mag-
etism, weak electricity, and/or second-class currents. In practice, only limited information
can be obtained from the \( \tau \rightarrow \pi \nu \) channels.

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Based on the assumption of a mixture of V and A couplings in the $\tau$ charged-current, experiments at $e^-e^+$ colliders have been setting limits on the presence of $(V+A)$ couplings in $\tau^- \rightarrow A^-\nu_\tau$ decay for $A = a_1, \rho, \pi, (l\bar{\nu}_l)$. The mixture of V and A couplings can be characterized by the value of the “chirality parameter” $\xi_A \equiv \frac{|g_L|^2 - |g_R|^2}{|g_L|^2 + |g_R|^2} = \frac{2\text{Re}(v_Aa_A^*)}{|v_A|^2 + |a_A|^2}$. Note that $\xi_A = -\langle h_{\nu_\tau} \rangle$, twice the negative of the $\nu_\tau$ helicity, in the special case for a spin- one $A^-$ particle of only V and A couplings and $m_\nu = 0$. Using spin-correlations, the ARGUS [1], ALEPH [2], and CLEO [3] collaborations have measured $\xi_A$. The current world average is $\xi_A = 1.002 \pm 0.032$ [3]. So the leading contribution in the tau’s charged-current is $(V - A)$ to better than the 5% level.

Therefore, the focus of this paper is on tests for “something” in a “$(V - A) + something$” structure in the tau’s $J^{\text{Charged Lepton}}$ current [4]. This extra contribution can show up experimentally because of its interference with the $(V - A)$ part which, we assume, arises as predicted by the standard lepton model. More precisely, the idea is to search for “additional structure” due to additional Lorentz couplings in $J^{\text{Charged Lepton}}$ by generalizing the $\tau$ spin- correlation function $I(E_\rho, E_{\bar{\rho}})$ by including the $\rho$ polarimetry information [3, 5] that is available from the $\rho^{ch} \rightarrow \pi^{ch}\pi^0$ decay distribution [7]. The symbol $B = \rho, \pi, l$. Since this adds on spin-correlation information from the next stage of decays in the decay sequence, we call such an energy- angular distribution a stage-two spin-correlation (S2SC) function. Similarly, $a_1$ polarimetry information can be included from the $\tau^- \rightarrow a_1^-\nu \rightarrow (\pi^-\pi^-\pi^+)\nu, (\pi^0\pi^0\pi^-)\nu$ decay modes [8].

The simplest useful S2SC is for the $CP$-symmetric decay sequence $Z^o$, or $\gamma^* \rightarrow \tau^-\tau^+ \rightarrow (\rho^-\nu_\tau)(\rho^+\bar{\nu}_\tau)$ followed by both $\rho^+ \rightarrow \pi^+\pi^0$, $I(E_\rho, E_{\bar{\rho}}, \tilde{\theta}_1, \tilde{\theta}_2) = |T (\pm) |^2 \rho_{++}\bar{\rho}_{--} + |T (-+) |^2 \rho_{--}\bar{\rho}_{++}$. 

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$$+ |T(++)|^2 \rho_{++} \bar{\rho}_{++} + |T(--)|^2 \rho_{--} \bar{\rho}_{--}$$  \hspace{1cm} (1)$$

If we think in terms of probabilities, the quantum-mechanical structure of this expression is apparent, since the $T(\lambda_{\tau^-}, \lambda_{\tau^+})$ helicity amplitudes describe the production of the $(\tau^- \tau^+)$ pair via $Z^0$, or $\gamma^* \rightarrow \tau^- \tau^+$. For instance, in the 1st term, the factor $|T(+,-)|^2 =$ “Probability to produce a $\tau^-$ with $\lambda_{\tau^-} = \frac{1}{2}$ and a $\tau^+$ with $\lambda_{\tau^+} = -\frac{1}{2}$” is multiplied by the product of the decay probability, $\rho_{++}$, for the positive helicity $\tau^- \rightarrow \rho^- \nu \rightarrow (\pi^- \pi^o) \nu$ times the decay probability, $\bar{\rho}_{--}$, for the negative helicity $\tau^+ \rightarrow \rho^+ \bar{\nu} \rightarrow (\pi^+ \pi^o) \bar{\nu}$.

The kinematic variables in $I_4$ are the usual “spherical” ones which naturally appear in the helicity formalism in describing such a decay sequence. The 1st stage of the decay sequence $\tau^-, \tau^+ \rightarrow (\rho^- \nu_\tau)(\rho^+ \bar{\nu}_\tau)$ is described by the 3 variables $\theta_1^\tau, \theta_2^\tau, \cos \phi$ where $\phi$ is the opening $\angle$ between the two decay planes. These are equivalent to the $Z^0$, or $\gamma^*$ center-of-mass variables, $E_\rho, E_{\bar{\rho}}, \cos \psi$. Here $\psi =$ “opening $\angle$ between the $\rho^-$ and $\rho^+$ momenta in the $Z/\gamma^*$ cm”. When the Lorentz “boost” to one of the $\rho$ rest frames is directly from the $Z/\gamma^*$ cm frame, the 2nd stage of the decay sequence is described by the usual 2 spherical angles for the $\pi^{ch}$ momentum direction in that $\rho$ rest frame: $\bar{\theta}_1, \bar{\phi}_1$ for $\rho_1^- \rightarrow \pi_1^- \pi_1^o$, and $\bar{\theta}_2, \bar{\phi}_2$ for $\rho_2^+ \rightarrow \pi_2^+ \pi_2^o$. (See figures in [1].)

In (1), the composite decay density matrix elements are simply the decay probability for a $\tau_1^-$ with helicity $\frac{h}{2}$ to decay $\tau^- \rightarrow \rho^- \nu \rightarrow (\pi^- \pi^o) \nu$ since $dN/d(\cos \theta_1^\tau) d(\cos \bar{\theta}_1) = \rho_{hh} (\theta_1^\tau, \bar{\theta}_1)$ and for the decay of the $\tau_2^+$, $\bar{\rho}_{hh} = \rho_{-h, -h}$ (subscripts $1 \rightarrow 2, a \rightarrow b$). For a $\tau_1^-$ with helicity $\frac{h}{2}$ to decay $\tau^- \rightarrow \rho^- \nu_L \rightarrow (\pi^- \pi^o) \nu_L$

$$\rho_{hh} = (1 + h \cos \theta_1^\tau) \left[ \cos^2 \omega_1 \cos^2 \bar{\theta}_1 + \frac{1}{2} \sin^2 \omega_1 \sin^2 \bar{\theta}_1 \right]$$

$$+ \frac{r^2}{2} (1 - h \cos \theta_1^\tau) \left[ \sin^2 \omega_1 \cos^2 \bar{\theta}_1 + \frac{1}{2} \left( 1 + \cos^2 \omega_1 \right) \sin^2 \bar{\theta}_1 \right]$$
nal state particles are given by the substitution rules: 
\[ \rho \rightarrow \rho, \bar{\rho} \rightarrow \bar{\rho}, \beta \rightarrow \bar{\beta}, \bar{\beta} \rightarrow \beta, \]
with the Wigner rotation angle \( \omega_1 = \omega_1(E_\rho) \). The dynamical parameters to be experimentally measured are the polar parameters \( \beta_a = \phi_a, \beta_b = \phi_b \), and \( r_a = |A(-1, -\frac{1}{2})|/|A(0, -\frac{1}{2})| \), \( r_b = |B(1, \frac{1}{2})|/|B(0, \frac{1}{2})| \). In the standard lepton model with a pure \((V - A)\) coupling, the predicted values are \( \beta_{a,b} = 0, r_{a,b} = \frac{\sqrt{2m_\rho}}{E_\rho + \mu_\rho} \sim \frac{\sqrt{2}m_\rho}{m_\tau} \sim 0.613 \). Note that the above \( I_4 \) spin-correlation function only depends on 4 of the above 7 kinematic variables. Refs. [7, 8] give its generalization, \( I_7 \), which also depends on \( \cos \phi, \tilde{\phi}_1 \), and \( \tilde{\phi}_2 \). We use \( I_4 \) in this paper because it is less complicated, has a useful sensitivity level, and sometimes \( I_7 \) is not significantly better.

For the \( \tau^- \rightarrow a_1 \nu_L \rightarrow (\pi^- \pi^- \pi^+) \nu_L, (\pi^0 \pi^0 \pi^-) \nu_L \) modes,
\[
\rho_{hh} = \left(1 + h \cos \theta_1^2 \right) \left[ \sin^2 \omega_1 \cos^2 \bar{\theta}_1 + \left(1 - \frac{1}{2} \sin^2 \omega_1 \right) \sin^2 \bar{\theta}_1 \right] \\
+ \frac{r_a^2}{2} \left(1 - h \cos \theta_1^2 \right) \left[ (1 + \cos^2 \omega_1) \cos^2 \bar{\theta}_1 + \left(1 + \frac{1}{2} \sin^2 \omega_1 \right) \sin^2 \bar{\theta}_1 \right] \\
- h \frac{r_a}{\sqrt{2}} \cos \beta \sin \theta_1 \sin 2\omega_1 \left[ \cos^2 \bar{\theta}_1 - \frac{1}{2} \sin^2 \bar{\theta}_1 \right] \tag{3}
\]
Here \( \bar{\theta}_1 \) specifies the normal to the \((\pi^- \pi^- \pi^+)\) decay triangle, instead of the \( \pi^- \) momentum direction used for \( \tau^- \rightarrow \rho^- \nu \). The Dalitz plot for \((\pi^- \pi^- \pi^+)\) has been integrated over so that [7] it is not necessary to separate the form-factors for \( a_1^- \rightarrow (\pi^- \pi^- \pi^+) \).

It is straightforward to include \( \nu_R \) and \( \bar{\nu}_L \) couplings in S2SC functions since
\[
I \left(E_\rho, E_\rho, \bar{\theta}_1, \bar{\theta}_2 \right)_{|_{\nu_R, \bar{\nu}_L}} = I_4 + (\lambda_R)^2 I_4 \left( \rho \rightarrow \rho^R \right) + (\bar{\lambda}_L)^2 I_4 \left( \bar{\rho} \rightarrow \bar{\rho}^L \right) \\
+ (\lambda_R \bar{\lambda}_L)^2 I_4 \left( \rho \rightarrow \rho^R, \bar{\rho} \rightarrow \bar{\rho}^L \right) \tag{4}
\]
where \( \lambda_R \equiv |A(0, \frac{1}{2})|/|A(0, -\frac{1}{2})| \), \( \bar{\lambda}_L \equiv |B(0, \frac{1}{2})|/|B(0, \frac{1}{2})| \) give the moduli’s of the \( \nu_R \) and \( \bar{\nu}_L \) amplitudes versus the standard amplitudes. The \( \rho_{hh} \)’s for \( \tau \rightarrow \rho \nu \) with \( \nu_R \) and \( \bar{\nu}_L \) final state particles are given by the substitution rules:
\[
\rho_{hh}^R = \rho_{-h,-h} \left( r_a \rightarrow \rho_a^R, \beta_a \rightarrow \bar{\beta}_a^R \right), \rho_{hh}^L =
\]
\( \rho_{-h,-h} \left( r_b \rightarrow r_b^L, \beta_b \rightarrow \beta_b^L \right) \). The helicity amplitudes\(^2\) for \( \tau^- \rightarrow \rho^- \nu_{L,R} \) for both \( (V \mp A) \) couplings and \( m_\nu \) arbitrary are given in \([7]\).

Historically in the study of the weak charged-current in muonic and in hadronic processes, it has been important to determine the “complete Lorentz structure” directly from experiment. Here the \( I_4 \) and \( I_7 \) functions can be used to do this for the \( \tau \) charged-current since these functions depend directly on the 4 helicity amplitudes for \( \tau^- \rightarrow \rho^- \nu \) and on the 4 amplitudes for the \( CP \)-conjugate process. In this paper, for \( I_4 \) we report the associated “ideal” sensitivities. We first consider the “traditional” couplings for \( \tau^- \rightarrow \rho^- \nu \) which characterize the most general Lorentz coupling

\[
\rho_\mu^* \bar{u}_{\nu\tau}(p) \Gamma^\mu u_\tau(k), \quad k_\tau = q_\rho + p_\nu.
\]

It is convenient to treat separately the vector and axial vector matrix elements. We introduce a parameter \( \Lambda = \) “the scale of New Physics”. In effective field theory this is the scale at which new particle thresholds are expected to occur. In old-fashioned renormalization theory it is the scale at which the calculational methods and/or the principles of “renormalization” breakdown, see e.g. \([10]\). While some terms of the following types do occur as higher-order perturbative-corrections in the standard model, such SM contributions are “small” versus the sensitivities of present tests in \( \tau \) physics in the analogous cases of the \( \tau \)'s neutral-current and electromagnetic-current couplings, c.f. \([11]\). For charged-current couplings, the situation should be the same.

In terms of the “traditional” tensorial and spin-zero couplings

\[
V_{\nu\tau}^\mu \equiv \langle \nu | v^\mu(0) | \tau \rangle = \bar{u}_{\nu\tau}(p) \left[ g_{V} \gamma^\mu + \frac{f_M}{2\Lambda} \sigma^{\mu\nu} \right] u_\tau(k) + \frac{g_s^-}{2\Lambda} (k-p)^\mu u_\tau(k) \tag{5}
\]

\[
A_{\nu\tau}^\mu \equiv \langle \nu | a^\mu(0) | \tau \rangle = \bar{u}_{\nu\tau}(p) \left[ g_{A} \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda} \sigma^{\mu\nu} \right] u_\tau(k) + \frac{g_p^-}{2\Lambda} (k-p)^\mu \gamma_5 u_\tau(k) \tag{6}
\]

Notice that \( \frac{f_M}{2\Lambda} \) is a “tau weak magnetism” type coupling, and \( \frac{f_E}{2\Lambda} \) is a “tau weak electricity” type coupling. Both the scalar \( g_s^- \) and pseudo-scalar \( g_p^- \) couplings do not contribute for \( \tau^- \rightarrow \rho^- \nu \).
since $\rho_\mu q^\mu = 0$, nor for $\tau^- \to a_1^- \nu$. By Lorentz invariance, there are the equivalence theorems for the vector current

$$ S \equiv V + f_M, \quad T^+ \equiv -V + S^- $$

and for the axial-vector current

$$ P \equiv -A + f_E, \quad T_5^+ \equiv A + P^- $$

where

$$ \Gamma_V^\mu = g_V \gamma^\mu + \frac{f_M}{2\Lambda} \epsilon^{\mu\nu\rho\sigma}(k-p)_\nu + \frac{g_{S^-}}{2\Lambda} (k-p)^\mu + \frac{g_{S^+}}{2\Lambda} (k+p)^\mu + \frac{g_{T^+}}{2\Lambda} \epsilon^{\mu\nu\rho\sigma}(k+p)_\nu $$

$$ \Gamma_A^\mu = g_A \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda} \epsilon^{\mu\nu\rho\sigma}(k-p)_\nu \gamma_5 + \frac{g_{P^-}}{2\Lambda} (k-p)^\mu \gamma_5 + \frac{g_{P^+}}{2\Lambda} (k+p)^\mu \gamma_5 + \frac{g_{T_5^+}}{2\Lambda} \epsilon^{\mu\nu\rho\sigma}(k+p)_\nu \gamma_5 $$

The matrix elements of the divergences of these charged-currents are

$$ (k-p)_\mu V^\mu = [g_V(m_\tau - m_\nu) + \frac{g_{S^-}}{2\Lambda} q^2 + \frac{g_{S^+}}{2\Lambda} (m_\tau^2 - m_\nu^2) + \frac{g_{T^+}}{2\Lambda} (q^2 - [m_\tau - m_\nu]^2)] \bar{u}_\nu u_\tau $$

$$ (k-p)_\mu A^\mu = [-g_A(m_\nu + m_\tau) + \frac{g_{P^-}}{2\Lambda} q^2 + \frac{g_{P^+}}{2\Lambda} (m_\nu^2 - m_\tau^2) + \frac{g_{T_5^+}}{2\Lambda} (q^2 - [m_\tau + m_\nu]^2)] \bar{u}_\nu \gamma_5 u_\tau $$

Both the weak magnetism $\frac{f_M}{2\Lambda}$ and the weak electricity $\frac{f_E}{2\Lambda}$ terms are divergenceless. On the other hand, since $q^2 = m_\rho^2$, when $m_\nu = m_\tau$ there contributions from $S^-, T^+, A, P^-, T_5^+$.

Table 1 gives the limits on these additional couplings assuming a “$(V-A) +$something” structure for the $\tau$ charged-current assuming real coupling constants. At $M_Z$ the scale of $\Lambda \approx$ few 100GeV can be probed; and at 10GeV or at 4GeV the scale of $1-2TeV$ can be probed.

The tables list only the ideal statistical errors, and assume respectively $10^7 Z^0$ events and $10^7 (\tau^- \tau^+)$ pairs. For the $\rho$ mode, we use $B(\tau \to \rho \nu) = 24.6\%$. For the $a_1$ mode we sum the charged plus neutral pion $a_1$ final states so $B(\tau \to a_1^{ch-neu} \nu) = 18\%$, and use $m_{a_1} = 1.275GeV$.  


The results in these tables simply follow by using (7-8) and from the dependence of the helicity amplitudes for $\tau^- \rightarrow \rho^- \nu$ on the presence of $(S \pm P)$ couplings with $m_\nu$ arbitrary:

$$A(0, -\frac{1}{2}) = g_{S^+P'}\frac{2E_\nu}{m_\rho} \sqrt{m_\tau (E_\nu + q_\rho)} + g_{S^-P'}\frac{2E_\nu}{m_\rho} \sqrt{m_\tau (E_\nu - q_\rho)}, \quad A(-1, -\frac{1}{2}) = 0 \quad (13)$$

and

$$A(0, \frac{1}{2}) = g_{S^+P'}\frac{2E_\nu}{m_\rho} \sqrt{m_\tau (E_\nu - q_\rho)} + g_{S^-P'}\frac{2E_\nu}{m_\rho} \sqrt{m_\tau (E_\nu + q_\rho)}, \quad A(1, \frac{1}{2}) = 0 \quad (14)$$

In compiling the entries in Table 1, we have adopted the idea of 1st and 2nd class currents [12]. This is suggested by a 3rd-family perspective of a possible “$\tau \leftrightarrow \nu_\tau$ symmetry” in the structure of the tau lepton currents. At the level of the masses, this truly is a badly broken symmetry$^2$.

But heeding the precedent historical successes of the SM in regard to current-versus-mass symmetry distinctions, we believe that this symmetry might nevertheless be relevant to 3rd-family currents. Therefore, we assume that the effective charged-current $J_{Lepton}^{Charged}$ is Hermitian and has such an SU(2) symmetry, so that we can identify the $\nu_\tau$ and the $\tau^-$ spinors. Thereby, we obtain for the “traditional couplings” and real form factors that the “Class I” couplings are $V, A, f_M, P^-$, and that the “Class II” couplings are $f_E, S^-$ if we define $J_{Lepton}^\mu = J_{I}^\mu + J_{II}^\mu$ where for $U = \exp(i\pi I_2)$

$$(J_{I}^\mu)^\dagger = -UJ_{I}^\mu U^{-1} \quad \text{First Class}$$

$$(J_{II}^\mu)^\dagger = UJ_{II}^\mu U^{-1} \quad \text{Second Class}$$

This classification is particularly useful in considering the reality structure of the charged-current [14]. As show in Table 2 there is a “clash” between the “Class I and Class II” structures and the consequences of time-reversal invariance. In particular, there are the useful theorems that (a) $(\tau \leftrightarrow \nu_\tau \text{ symmetry}) + (T \text{ invariance}) \implies \text{Class II currents are absent}$, (b) $(\tau \leftrightarrow \nu_\tau \text{ symmetry}) + (\text{existence of } J_{I}^\mu \text{ and } J_{II}^\mu) \implies \text{violation of } T \text{ invariance}$, and (c) $(\text{existence of } J_{II}^\mu) + (T \text{ invariance}) \implies (\tau \leftrightarrow \nu_\tau \text{ symmetry}) \text{ in } J_{Lepton}^\mu \text{ is broken.}$

6
Table 3 shows the limits on such couplings assuming a pure-imaginary coupling constant. In the case of $(V - A)$ the limits on the $\beta$'s in Refs. [7, 8, 4] cover this situation. The limits here are in $(\Lambda)^2$ with $\Lambda \sim$ few $10 GeV$'s because this is not a S2SC interference effect.

Besides the 3rd-family perspective of a possible $\tau \leftrightarrow \nu_\tau$ symmetry, it is also instructive to consider “additional structure” in the $\tau$ charged-current from the viewpoint of “Chiral Combinations” of the various Lorentz couplings. This is especially interesting because the $S \pm P$ couplings do not contribute to the transverse $\rho$ or $a_1$ transitions. Tables 4 and 5 give the limits on $\Lambda$ in the case of purely real and imaginary coupling constants for the “Chiral Couplings”.

Finally, Table 6, the helicity amplitudes themselves provide a simple framework for characterizing a “complete measurement” of $\tau^- \to \rho^- \nu$ and of $\tau^- \to a_1^- \nu$: For either, when only $\nu_L$ coupling’s exist, there are only 2 amplitudes, so 3 measurements, of $r_a, \beta_a$, and $|A(0, -\frac{1}{2})|$ via $\{\rho^-, B^+\} |_{B \neq \rho}$, will provide a “complete measurement”. When $\nu_R$ coupling’s also exist, then there are 2 more amplitudes, $A(0, \frac{1}{2})$ and $A(1, \frac{1}{2})$. Then to achieve an “almost” complete measurement, 3 additional quantities must be determined, e.g. by the $I_4$: $r_a^R, \beta_a^R$ and $\lambda_R \equiv \frac{|A(0, \frac{1}{2})|}{|A(0, -\frac{1}{2})|}$. But to also measure the relative phase of the $\nu_L$ and $\nu_R$ amplitudes, $\beta_a \equiv \phi_a^R - \phi_a^L$ or $\beta_1 \equiv \phi_1^R - \phi_1^L$, requires, e.g., the occurrence of a common final state which arises from both $\nu_L$ and $\nu_R$.

For comparison, Table 7 shows what can be learned from the $\tau^\pm \to \pi^\pm \nu$ decay mode. The $\xi_\pi$ parameter and the $\Gamma(\tau \to \pi \nu)$ partial width are the only observables for these modes. While the $f_M(q^2)$ and $f_E(q^2)$ couplings do not contribute to this decay mode, useful bounds can be obtained for the $V + A$, $S + P$, and $T^+ + T_5^+$ chiral couplings. In principle this channel is particularly important for the $S^- \pm P^-$ couplings contribute here whereas they do not for the $\rho$ and $a_1$ modes. However, in the $\tau^\pm \to \pi^\pm \nu$ decay amplitudes, each such coupling appears multiplied.
by a suppression factor of $m_\pi^2/(m_\tau^2 - m_\nu^2)$. Hence, one conclusion of this paper is that both the present and potential experimental bounds on $S^- \pm P^-$ couplings are exceptionally poor or non-existent from measurements of the $\pi, \rho$ and $a_1$ modes in tau lepton decays!

In conclusion, $(\tau^- \tau^+)$ spin correlations with $\rho$ and $a_1$ polarimetry observables can be used to probe for “additional structure” in the tau’s charged-current. For example, tau weak magnetism, $f_M(q^2)$, and tau weak electricity, $f_E(q^2)$, can be probed to new physics scales of $\Lambda_{RealCoupling} \sim 1.2-1.5 TeV$ at 10, or $4 GeV$ and $\Lambda_{Imag.Coupling} \sim 28-34 GeV$ at 10, or $4 GeV$. By spin-correlation techniques the Lorentz structure of the $\tau$ charge-current can almost be completely determined from the $\{\rho^-, \rho^+\}$and $\{a_1^-, a_1^+\}$ modes.

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Footnotes

1. For testing for $(V + A)$ versus $(V - A)$, the use of $I_7$ for $\{\rho^-, \rho^+\}$ gives less than a 1% improvement over $I_4$ at $M_Z$, 10$ GeV$, or 4$ GeV$. If in addition the $\tau^-$-momentum direction is known via a SVX detector, there is only an $\sim 11\%$ improvement. The same numbers occur for $\{a_1^-, a_1^+\}$. In contrast, by using $I_4$, instead of the simpler 2 variable $I(E_\rho, E_{\bar{\rho}})$ spin-correlation function, there is about a factor of 8 improvement at $M_Z$.

2. Note $\frac{m_\tau}{m_\nu} \sim \frac{5}{174} \sim 3\%$, and $\frac{m_\nu}{m_\tau} < \frac{2.8}{1777} \sim 1.4\%$ so this symmetry is badly broken in the
masses for the 3rd family. However, for the other leptons this symmetry may be more strongly broken since $\frac{m_{\nu_e}}{m_e} < 10^{-5}$, and $\frac{m_{\nu_\mu}}{m_\mu} < 0.15\%$ from the current empirical bounds. From phenomenological mass formulas, e.g. [13], such as the GUT mass formula, $\nu_\tau : \nu_\mu : \nu_e \sim m_\tau^2 : m_e^2 : m_\mu^2$, the tau leptons are also the least asymmetric since then $\frac{m_{\nu_\tau}}{m_\tau} \approx 10^{-8}$, $\frac{m_{\nu_\mu}}{m_\mu} \approx 10^{-11}$, and $\frac{m_{\nu_e}}{m_e} \approx 3 \cdot 10^{-14}$ for the normalization $m_{\nu_e} = 20eV$.

3. Details on the analysis of the $\tau \to \pi \nu$ modes will be reported elsewhere [8].

4. The tests in this paper use $(\tau^- \tau^+) \text{ spin- correlations}$ as it is assumed that the $e^-$ and $e^+$ colliding beams are not longitudinally-polarized. Recently, Y.-S. Tsai [15, 16] has shown that in tau decays the sensitivities of tests for $CP$ violation, and for other types of “new physics”, are substantially improved in regard to both systematic and statistical errors by the use of longitudinally- polarized beams at the $(\tau^- \tau^+)$ threshold.

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Table Captions

Table 1: Limits on $\Lambda$ in GeV for Real $g_1$'s. For $V + A$ only, the entry is for $\xi_A$.

Table 2: “Reality structure” of $J_{\mu}^{Lepton}$ current’s form factors.

Table 3: Limits on $\Lambda$ in GeV for Pure Imaginary $g_1$'s. For $V + A$ only, the entry is for $\xi_A$.

Table 4: “Chiral Couplings”: Limits on $\Lambda$ in GeV for Real $g_i$'s. For the $\rho$ and $a_1$ modes, equivalent couplings are $T^+ + T_5^+ \equiv V - A; T^+ - T_5^+ \equiv V + A$.

Table 5: “Chiral Couplings”: Limits on $\Lambda$ in GeV for Pure Imaginary $g_i$'s. For the $\rho$ and $a_1$ modes, equivalent couplings are $T^+ + T_5^+ \equiv V - A; T^+ - T_5^+ \equiv V + A$.

Table 6: Elements of error matrix for limits on $\nu_R$ and $\nu_L$ couplings in terms of the helicity amplitudes for respectively $\tau \to \rho \nu$, and $\tau \to a_1 \nu$.

Table 7: “Chiral Couplings”: Limits on $\Lambda$ in GeV from $\tau \to \pi \nu$. $\xi_\pi$ entries are for $\{\pi^-, \pi^+\}$ spin correlations at $M_Z, 10 GeV, 4 GeV$. $\Gamma(\tau \to \pi \nu)$ entries follow from current data \cite{17}.
Table 1:

|                  | $\{\rho^-, \rho^+\}$ mode |                  | $\{a_1^-, a_1^+\}$ mode |
|------------------|-----------------------------|------------------|---------------------------|
|                  | At $M_Z$ 10, or 4 GeV       | At $M_Z$ 10, or 4 GeV |
| **1st Class Currents** |                             |                  |                           |
| $V + A$, for $\xi_A$ | 0.006                       | 0.0012           | 0.010                     | 0.0018                     |
| $f_M$, for $\Lambda$ | 214 GeV                     | 1,200            | 282                       | 1,500                       |
| $S$               | 306 GeV                     | 1,700            | 64                        | 345                         |
| $T_5^+$           | 506 GeV                     | 2,800            | 371                       | 2,000                       |
| **2nd Class Currents** |                             |                  |                           |
| $f_E$, for $\Lambda$ | 214 GeV                     | 1,200            | 282                       | 1,500                       |
| $P$               | 306 GeV                     | 1,700            | 64                        | 345                         |
| $T^+$             | 506 GeV                     | 2,800            | 371                       | 2,000                       |
Table 2:

| Form Factor: | Class I Current | Class II Current | $T$ invariance |
|-------------|-----------------|------------------|----------------|
| $V, A, f_M, P^-$ | Real parts      | Imaginary parts  | $Re \neq 0, Im = 0$ |
| $f_E, S^-$    | Imaginary parts | Real parts       | $Re \neq 0, Im = 0$ |

Table 3:

|                      | 1st Class Current | 1st Class Current | 2nd Class Current | 2nd Class Current |
|----------------------|-------------------|-------------------|-------------------|-------------------|
|                      | $\{\rho^-, \rho^+\}$ mode | $\{a_1^-, a_1^+\}$ mode | $\{a_1^-, a_1^+\}$ mode | $\{a_1^-, a_1^+\}$ mode |
|                      | At $M_Z$          | 10, or 4 GeV      | At $M_Z$          | 10, or 4 GeV      |
| $V + A$, for $\xi_A$ | 0.006             | 0.0012            | 0.010             | 0.0018            |
| $f_M$, for $(\Lambda)^2$ | $(12 GeV)^2$     | $(28)^2$          | $(15)^2$          | $(34)^2$          |
| $S$                  | $(14 GeV)^2$      | $(33)^2$          | $(6)^2$           | $(13)^2$          |
| $T_5^-$              | $(22 GeV)^2$      | $(50)^2$          | $(18)^2$          | $(42)^2$          |

1st Class Currents:

2nd Class Currents:
Table 4:

|                      | \( \{\rho^-, \rho^+\} \) mode | \( \{a_{\frac{1}{2}}, a_{\frac{3}{2}}^\pm\} \) mode |
|----------------------|---------------------------------|---------------------------------|
|                      | At \( M_Z \) 10, or 4 GeV       | At \( M_Z \) 10, or 4 GeV       |
| \( V + A, \xi_A \)   | 0.006                           | 0.0012                          |
| \( S + P, \Lambda \) | 310 GeV                         | 1,700                           |
| \( S - P, (\Lambda)^2 \) | (11 GeV)^2                     | (25)^2                          |
| \( f_M + f_E, \Lambda \) | 210 GeV                         | 1,200                           |
| \( f_M - f_E, (\Lambda)^2 \) | (9 GeV)^2                       | (20)^2                          |

Table 5:

|                      | \( \{\rho^-, \rho^+\} \) mode | \( \{a_{\frac{1}{2}}, a_{\frac{3}{2}}^\pm\} \) mode |
|----------------------|---------------------------------|---------------------------------|
|                      | At \( M_Z \) 10, or 4 GeV       | At \( M_Z \) 10, or 4 GeV       |
| \( V + A, \xi_A \)   | 0.006                           | 0.0012                          |
| \( S + P, (\Lambda)^2 \) | (11 GeV)^2                     | (25)^2                          |
| \( S - P, (\Lambda)^2 \) | (11 GeV)^2                     | (25)^2                          |
| \( f_M + f_E, (\Lambda)^2 \) | (9 GeV)^2                       | (20)^2                          |
| \( f_M - f_E, (\Lambda)^2 \) | (9 GeV)^2                       | (20)^2                          |
Table 6:

|                      | \( \{\rho^-,\rho^+\} \) mode | \( \{a^-_1,a^+_1\} \) mode |
|----------------------|-------------------------------|-----------------------------|
|                      | \( M_Z \) 10, or 4 GeV        | \( M_Z \) 10, or 4 GeV      |
| Diagonal elements:   |                               |                             |
| \( a = \lambda_R \) | (8\%)\(^2\)                  | (18\%)\(^2\)               |
| \( b = \lambda_R r^R_a \) | (8\%)\(^2\)                  | (18\%)\(^2\)               |
| \( c = \)           |                               |                             |
| \( (\lambda_R)^2 r^R_a \cos \beta^R_a \) | (13\%)\(^2\)               | (41\%)\(^2\)               |
| Correlations:        |                               |                             |
| \( \rho_{ab} \)     | -0.75                         | -0.77                       |
| \( \rho_{ac} \)     | -0.27                         | -0.17, 0.06                 |
| \( \rho_{bc} \)     | 0.085                         | 0.017, 0.003                |
| From $\xi_\pi$: | From $\Gamma(\tau \to \pi \nu)$ |
|----------------|-----------------|
| $|g_i/g_L|^2$   | $|g_i/g_L|^2$    | $2\text{Re}(g_L^*g_i)$ |
| $V + A$, for $\xi_\pi$ | 0.015, 0.004, 0.009 | 0.014 |
| $S + P, T^+ + T_5^+$, for $\Lambda$ | 127 GeV |
| $S - P, T^+ - T_5^+$, for $\Lambda$ | $(10 \text{GeV})^2$, $(21 \text{GeV})^2$, $(13 \text{GeV})^2$ | $(< 1 \text{GeV})^2$ |
| $S^- + P^-$, for $\Lambda$ |  | $< 1 \text{GeV}$ |
| $S^- - P^-$, for $\Lambda$ | $(< 1 \text{GeV})^2$, $(1.6 \text{GeV})^2$, $(1 \text{GeV})^2$ | $(< 1 \text{GeV})^2$ |