Analysis of the in situ stress contrast impact on a planar hydraulic fracture propagation using the fully coupled poroelastic model

A N Baykin$^{1,2}$ and S V Golovin$^{1,2}$

$^1$ Novosibirsk State University, Novosibirsk, Russia
$^2$ Lavrentyev Institute of Hydrodynamics SB RAS, Novosibirsk, Russia

E-mail: alexey.baykin@gmail.com, sv giov olin@gmail.com

Abstract. In this paper, we use a fully coupled model for 3D planar fracture propagation within a poroelastic reservoir in order to analysis the non-symmetry of the fracture as it develops in a 3-layer reservoir with different compressive stresses in each layer. We demonstrate that the fracture predominantly develops in the layer with the lower compressive stress until it gets arrested due to the equality of the total leakoff to the volume of pumped fluid.

1. Introduction

Mathematical modeling of hydraulic fracturing attracts great attention in the scientific literature due to its importance in applied fields such as intensification of hydrocarbon production. Recent advances in the modeling are presented in the review papers [1, 2]. Roughly, the single-fracture models are classified as 2D and pseudo 3D ones that use various simplifying assumptions related to the geometry of the fracture, and also planar 3D and fully 3D models that use a solution of the complete 3D problem in order to model the spatial propagation of the fracture. On the other hand, most of the models are based on either numerical or analytical solutions of pure elastic equations coupled with lubrication equation for the fluid flow within the fracture, and the approximate Carter formula for estimation of the fluid leakoff from the fracture to the reservoir. More advanced approach consists of observing the poroelasticity model that has an essential advantage due to correct description of the fluid exchange between the fracture and the reservoir, and also due to account for the coupled influence of the pore fluid pressure on the stress state near the fracture, and the modification of filtration due to inhomogeneous stress.

In papers [3, 4], we demonstrated that inhomogeneity of the permeability may significantly influence the dynamics of the fracture’s development. This analysis was based on the numerical model for the hydraulic fracture propagation in poroelastic medium such that in each section of the fracture by the vertical plane the width of the fracture remains constant (the same assumption as used in the classical KGD model). In present paper we generalize our numerical model to the 3D case by assuming the a planar fracture propagation without any limitations to the width of the fracture in all directions along the plane. At present, we do not model the fracture toughness by assuming that the fracture propagates in the viscosity-dominated regime. The fracture is growing due to the pressure action of the fluid pumped into the fracture through the horizontal well that intersects the fracture at a given point. As the reservoir itself is regarded
as a porous medium, there is a natural fluid exchange between the reservoir and the fracture. During the fracture growth its area increases and, consequently, the volume of fluid that leaks off from the fracture to the reservoir increases as well. After some time, the fracture comes to the state when the volume of leaked fluid coincides with the volume of fluid pumped into the fracture. After this moment, the fracture is arrested \[5\] by this fluid balance and does not propagate further, provided the pumping rate of fracturing fluid does not grow.

As a simple analysis of the model’s capabilities, we analyze the shape of the fracture at the time of fracture’s arrest as it propagates in a 3-layered reservoir with different confining stresses in each layer. The confining stress is higher in the surrounding layers in comparison with the central layer, where the source of fracturing fluid is located. It is shown that even for the small contrast of the confining stresses, the fracture develops mostly in the central layer until it gets arrested. Note, that this effect might be distorted by the introduction of a finite fracture toughness, especially, if the toughness would be different in each layer.

2. Mathematical formulation of the problem

The mathematical model presented in this paper is a direct generalization to the 3D case of the hydraulic fracture model presented and analyzed in our earlier works \[3, 6, 4\].

We consider a planar hydraulic fracture propagating due to the pressure of fluid that is pumped into the fracture through the horizontal wellbore intersecting the fracture at a given point. We use a Cartesian system of coordinates such that Oxz plane coincides with the fracture’s plane, Oy axis is directed upwards, and Oz is orthogonal to the fracture plane. The coordinates’ origin coincides with the position of the well.

The reservoir is considered to be poroelastic meaning that it is represented as a two-phase continuum characterized by the displacement vector \(\mathbf{u}\) of the elastic solid matrix (skeleton) and the pressure \(p\) of the fluid within the pore space. The elastic response to the applied stress is assumed to be linear with the moduli \(\lambda\) and \(\mu\) as the skeleton’s parameters. The filtration properties are characterized by the porosity \(\phi\) and the permeability \(k_r\). The speed \(q\) of the fluid flow in the porous medium is governed by the Darcy law \(q = -(k_r/\eta_f)\nabla p\) for a single-phase Newtonian fluid with the effective viscosity \(\eta_f\).

It is supposed that the fluid filtrating from the fracture to the reservoir has the same viscosity as the pore fluid. However, the fluid injected into the fracture has different viscosity \(\eta_f\). It corresponds to the normal situation in hydraulic fracturing when the fracturing fluid is a high-viscous gel and only its low-viscous base fluid is filtrated into the reservoir. The geological prestress state of the reservoir is taking into account by introducing the initial stress tensor \(\mathbf{\sigma}_0\).

Following \[3, 6, 4\] the mechanics of the poroelastic medium in quasi-static state is governed by the following system of equations:

\[
\text{div} \mathbf{\tau} = 0, \quad \mathbf{\tau} = \mathbf{\sigma}_0 + \lambda \text{div} \mathbf{u} \mathbf{I} + 2\mu \mathbf{E}(\mathbf{u}) - \alpha \rho \mathbf{I}, \quad (1)
\]

\[
S_\varepsilon \frac{\partial p}{\partial t} = \text{div} \left( \frac{k_r}{\eta_f} \nabla p - \alpha \frac{\partial \mathbf{u}}{\partial t} \right), \quad (2)
\]

where \(\mathbf{E}(\mathbf{u})\) is the Cauchy’s strain tensor \(2\mathbf{E}(\mathbf{u})_{ij} = \partial u_i/\partial x_j + \partial u_j/\partial x_i\) \((i, j = 1, 2)\), \(\mathbf{I}\) is the identity tensor, \(\alpha\) is the Biot coefficient responsible for the solid-fluid coupling. The storativity \(S_\varepsilon\) reflects the dependence of the Lagrangian porosity \(\phi\) on \(\epsilon = \text{tr} \mathbf{E}\) and \(p\) as in \[7\]:

\[
\frac{\partial \phi}{\partial t} = \alpha \frac{\partial \epsilon}{\partial t} + S_\varepsilon \frac{\partial p}{\partial t}, \quad S_\varepsilon = \frac{(\phi_0 - \alpha)(1 - \alpha)}{K}, \quad (3)
\]

where \(K = \lambda + 2\mu/3\) is the bulk modulus, \(\phi_0\) is the initial porosity.

Symmetry of the problem with respect to Ox-axis makes it possible to solve equations (1)–(2) within domain \(\Omega = [-L_x/2, L_x/2] \times [-L_y/2, L_y/2] \times [0, L_z/2]\) which cross-section is shown in
Figure 1. The cross-section of the fracture by plane $x = 0$.

Figure 1. Over the outer boundary $\Gamma_R = \partial \Omega \setminus \{z = 0\}$, the confining far-field stress $\sigma_\infty$ and the constant pore pressure $p = p_\infty$ are applied:

$$\Gamma_R: \quad p = p_\infty, \quad \tau \langle n \rangle = \sigma_\infty, \quad (\tau \langle n \rangle)_i = \tau_{ij} n_j. \quad (4)$$

Henceforth, $n$ and $s$ denote the outer normal and tangential unit vectors to the boundary of the domain $\Omega$; the summation over the repeating index is implied. Passing to the limit $t \to 0$ implies that the presstress $\tau_0$ satisfies the same boundary condition: $\tau_0 \langle n \rangle|_{\Gamma_R} = \sigma_\infty$. On the part of the boundary occupied by the fracture $\Gamma_f$ it is set the condition that the fracture wall is subjected to the normal load generated by the fluid pressure $p_f$. The 2D fluid flow inside the fracture is governed by the mass conservation law in the lubrication theory approximation:

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial x} \left( \frac{w^3}{3\eta_f} \frac{\partial p_f}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{w^3}{3\eta_f} \frac{\partial p_f}{\partial y} \right) + q_\ell + Q \delta(x, y, z), \quad (5)$$

where the leak-off rate $q_\ell$ is proportional to the pressure $p$ gradient over the fracture’s wall. The pumping rate $Q$ is a multiplier of the Kronecker delta function concentrated at the origin. Over the remaining boundary part $\Gamma_s = \{z = 0\} \setminus \Gamma_f$ the symmetry conditions hold. The lag between the fracture front and fluid front inside the fracture is neglected. Also for the sake of simplicity we limit ourselves to the zero fracture toughness case so that the flow is assumed to be in the viscosity-dominated regime.

3. Numerical algorithm
For the numerical solution we apply the Finite Element Method. To this purpose, we use the weak formulation of the problem extended by the penalty integral term for prevention of the interpenetration of fracture walls. All the details of the numerical implementation of the algorithm are similar to the ones described in papers [3, 6, 4]. As in the cited algorithm, we resolve the nonlinear terms via the fixed-point iterations. The numerical code is implemented in the open-source software package FreeFem++ [8].

4. Case studies
The numerical model proposed in previous sections is employed to study the influence of the non-uniformness of closure stress on the fracture propagation. In our numerical experiment we consider the reservoir consists of three layers with different closure stress (see Figure 2). It corresponds to the case when the reservoir’s pay zone, where the fracture is initiated, is confined by two symmetrical stress barriers with higher in situ stresses. This condition is realized by the
special choice of the pre-stress $\tau_0$ that satisfies the following condition over the fracture path [4]:

$$n \cdot \tau_0(n)|_{\Gamma f} = \begin{cases} 
\sigma_{pay}, & |y| < H_{pay}/2, \\
\sigma_{barrier}, & |y| \geq H_{pay}/2.
\end{cases}$$  \hspace{1cm} (6)

In simulations the following set of parameters is used: the reservoir sizes $L_x = 80$ m, $L_y = 80$ m, $L_z = 60$ m, the pay zone height $H_{pay} = 10$ m, the in situ stress in the pay zone $\sigma_{pay} = 10$ MPa and in the confining layers $\sigma_{barrier} = 10-12$ MPa, Young’s modulus $E = 17$ GPa, Poisson’s ratio $\nu = 0.2$, initial porosity $\phi_0 = 0.2$, permeability $k_r = 10^{-14}$ m$^2$, Biot coefficient $\alpha = 0.75$, reservoir fluid viscosity $\eta_r = 0.001$ Pa·s, fracturing fluid viscosity $\eta_f = 0.1$ Pa·s, pumping rate $Q = 0.02$ m$^3$/s.

The calculations continue until the fracture is arrested by the equality of the total leakoff and the fluid inflow. The confining stress within the stress barrier is changed from 10 to 12 MPa. The purpose of the numerical experiment is to estimate the deviation of the fracture from the circular one for different values of the confining stress contrast $\Delta \sigma = \sigma_{barrier} - \sigma_{pay}$ between the pay zone and the barriers.

![Figure 2](image.png)

**Figure 2.** Schematics of the reservoir: the pay zone is surrounded by two barrier layers.

Results of calculations are demonstrated in Figures 3–6. One can see that the profile of the area occupied by the fracture barely depends on the confining stress above the limit of $\Delta \sigma = \sigma_{barrier} - \sigma_{pay} = 0.4$ MPa. For the higher values of the stress contrast, the fracture becomes shorter in the upper direction and longer along the horizontal axis.

5. Conclusion

A numerical model of a planar hydraulic fracture propagating in a three dimensional poroelastic medium based on the approach developed in in [3, 6] is presented. The model accounts for the influence of the pore fluid and the elastic rock deformation on the growth of the fracture. The fluid exchange between the reservoir and the fracture is naturally incorporated into the model. Also the model is able to take into account the inhomogeneity of reservoir properties and geological stresses [4].

The model is employed to analyze the influence of the in situ stress contrast in the confining layers on the final fracture geometry. Modeling of the three-layer reservoir with the stress contrast between the central and two surrounding layers demonstrates that even a small contrast of confining stresses leads to propagation of fracture only within the layer with a smaller confining stress. Increase of the value of stress contrast above 20% does not significantly change the fracture dynamics. It is also shown that in every case fracture comes to a stationary state as the total volume of the leakoff coincides with the volume of fluid pumped into the fracture.
Figure 3. Hydraulic fracture half-width distribution at $t = 2400$ s. The in situ stress contrast is $\Delta \sigma = 0.8$ MPa.

Figure 4. Hydraulic fracture half-width distribution at $t = 2500$ s. The in situ stress contrast is $\Delta \sigma = 0.8$ MPa.

Figure 5. Fracture half-width profiles along line $\{z = 0\}$ at $t = 2500$ s for different in situ stress in confining layers.

Figure 6. Fracture half-width profiles along line $\{y = 0\}$ at $t = 2500$ s for different in situ stress in confining layers.

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