A Possible Black Hole Background in c=1 Matrix Model

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Abstract

We propose a new space-time interpretation for c=1 matrix model with potential $V(x) = -x^2/2 - \mu^2/2x^2$. It is argued that this particular potential corresponds to a black hole background. Some related issues are discussed.
c=1 matrix model turns out to have very rich structure. It possesses, for example, a propagating degree of freedom, a non-trivial $S$-matrix, discrete states, a large symmetry algebra $w_\infty$, and so on. For a review, see [1]. All these have their corresponding counterparts in the continuum Liouville theory as well. Now viewed as a critical $D = 2$ string theory, the continuum theory also has a black hole solution [2], which has not been seen in matrix models. A priori, there are problems. If a black hole radiates, it should be in the physical spectrum. On the other hand, the matrix model is already unitary. Closely related is the fact that the matrix model does not introduce the dilaton and metric degrees of freedom explicitly, so it is not clear how to study the back reaction, if there is one. It is however recently argued that the Euclidean black hole mass is a superselection parameter and does not fluctuate [3]. This gives us some hope that the black hole may be represented by a specific one-body Hamiltonian of the matrix model.

There is another reason to look for different Hamiltonian than the usual inverted harmonic oscillator. Compare the work in matrix model approach and the Liouville approach, we can make identification of special operators in the two theories [4]. It turns out the “correct” Liouville dressing of a primary matter operator corresponds to a polynomial with time dependence in the matrix model. An infinitesimal black hole, being “wrongly” dressed, would correspond to a potential of negative power in the matrix model. Such a potential is a relevant perturbation in the sense that it alters drastically the critical behavior. So the correct way to solve the problem is to go beyond the usual perturbation theory and it is interesting to study this as a part of larger program: investigate the relevant perturbations and find out their physical interpretations [1].

We first briefly review what is known about the black hole solution in the continuum theory. Then we consider general coupling of special states in the matrix model, and finally

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1 Multicritical points with the potential $x^n (n > 0)$ have been considered by Gross and Miljkovic [3]. See also [6] for different approaches.
specialize to the possible black hole background.

The Euclidean black hole can be summarized by the following metric and dilaton

\[
ds^2 = (1 - Me^{-\phi})dt^2 + (1 - Me^{-\phi})^{-1}d\phi^2
\]

\[
\Phi = \phi.
\] (1)

Here we have chosen a coordinate system where dilaton is identified with the spatial coordinate, and \( M \) is the black hole mass. The world sheet action can be written as (in a flat world sheet background)

\[
S = \frac{1}{2} \int d^2\sigma \left[ \frac{1}{1 - Me^{-\phi}} \partial_z t \partial_{\bar{z}} t + (1 - Me^{-\phi}) \partial_z \phi \partial_{\bar{z}} \phi \right].
\] (2)

From (2), if we use the minisuperspace quantization, which is known to be exact for \( c \leq 1 \) in the Liouville background, we have the following Wheeler-de-Witt (WdW) wave equation

\[
\frac{1}{1 - Me^{-\phi}} \frac{\partial^2}{\partial t^2} \Psi + \frac{\partial}{\partial \phi} (1 - Me^{-\phi}) \frac{\partial}{\partial \phi} \Psi = 0.
\] (3)

We will find (1) and (3) follow from the matrix model as well.

Now we turn to \( c=1 \) matrix model. We will use the collective field theory as our starting point, keeping in mind the underlying fermion picture. This Thomas-Fermi approach has been advocated by Polchinski. The collective field seems to describe the tachyon dynamics. Its precise relation with the tachyon in the Liouville theory is however not clear. There may be some non-local field redefinition between the two. Actually the question is more general: it is not clear how to relate the spacetime picture in the two approaches. For example, the WdW equation derived from the minisuperspace corresponds to the Laplace transformed equation of motion of the linear fluctuation of the collective field, which certainly points to a subtle relationship between the two.

In what follows we find that the correspondence of our matrix model with the black
hole is much more straightforward, a point related perhaps with the dual relation between the black hole and the Liouville theory discovered in [13].

1+1 dimensional string from \( c = 1 \) matrix model is described by \( N \) fermions with Hamiltonian,

\[
H_F = \int_{-\infty}^{\infty} dx \left[ \frac{1}{2} \partial_x \psi^\dagger \partial_x \psi + V(x) \psi^\dagger \psi \right],
\]

whose bosonized form is the collective field theory with the Hamiltonian

\[
H_B = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ (\frac{p_+^3}{6} + p_+ V) - (\frac{p_-^3}{6} + p_- V) \right].
\]

In the boson form,

\[
p_\pm = \pi_\xi \pm \partial_x \xi
\]

\[
\{ \pi_\xi(x,t), \xi(x',t) \}_{P.B.} = 2\pi \partial_x \delta(x - x').
\]

Since \( \partial_x \xi \) is the density of states, it must be positive semi-definite. So we are dealing with a fluid field theory. In the usual double scaling limit, \( V(x) \) is an inverted harmonic oscillator,

\[
V_1(x) = -\frac{1}{2} x^2.
\]

We will discuss another potential,

\[
V_2(x) = -\frac{1}{2} x^2 - \frac{\mu^2}{2x^2},
\]

which coincides with \( V_1(x) \) when \( |x| \to \infty \). It can be considered as a different fine-tuning of the critical potential.

The equations of motion of \( p_\pm \) are

\[
\partial_t p_\pm = -V'(x) - p_\pm \partial_x p_\pm.
\]

A natural starting point to solve the theory would be to expand \( p_\pm \) around static back-}

\[
V'(x) + \bar{p} \partial_x \bar{p} = 0,
\]
where

\[ \bar{p}_+ = -\bar{p}_- \equiv \bar{p} \geq 0. \]  

(12)

We are interested in the case when the fermi surface is right at the top of the potential. It corresponds to zero cosmological constant on the world sheet in continuum language, an ansatz used in [2].

To figure out the space-time metric of the background (11), we must look for excitations around it, following [3]. Let

\[ \xi = \bar{\xi} + \delta\xi, \]

\[ p_\pm = \bar{p}_\pm + \delta p_\pm, \]

we have for the action \( S \),

\[ \begin{align*}
S &= \bar{S} + \int dx \, dt \left\{ \pi_\xi \delta \dot{\xi} - \frac{1}{2} \bar{p}(\delta p_+^2 - \delta p_-^2) + \frac{1}{6}(\delta p_+^3 - \delta p_-^3) \right\} \\
&= \bar{S} + \int dx \, dt \left\{ \pi_\xi \delta \dot{\xi} - \frac{1}{2} \bar{p}(\pi^2_\xi + (\partial_x \delta \xi)^2) + \frac{1}{6}(\delta p_+^3 - \delta p_-^3) \right\},
\end{align*} \]

(14)

where we have used the definition of \( p_\pm \) in the second line. Looking only at the quadratic piece of (14), we can see that it corresponds to a massless scalar field propagating in an external metric background. In order to make connection with the continuum theory, it is more convenient to use \( \phi = \ln(-x) \) as the spatial coordinate and make a scaling \( \pi_\xi \to \pi_\xi / x \).

With this done, (14) becomes

\[ \begin{align*}
S &= \bar{S} + \int dx \, dt \left\{ \pi_\xi \delta \dot{\xi} - \frac{1}{2} \bar{p}(\pi^2_\xi + (\partial_x \delta \xi)^2) + \frac{1}{6x^2}(\delta p_+^3 - \delta p_-^3) \right\}. \\
&= \bar{S} + \int dx \, dt \left\{ \pi_\xi \delta \dot{\xi} - \frac{1}{2} \bar{p}(\pi^2_\xi + (\partial_x \delta \xi)^2) + \frac{1}{6x^2}(\delta p_+^3 - \delta p_-^3) \right\}.
\end{align*} \]

(15)

The string coupling constant is obviously \( \exp(-2\phi) \). So \( \phi \) can be thought as the dilaton.

Before we plunge into the details, let us remark on the discrete state in matrix model. Consider adding the following special operator in the Hamiltonian,

\[ O_{mnl} = \int dx(p^m_+ - p^m_-)x^n e^{it}, \]

(16)
where $m,n$ and $l$ are integers. By the way, although one can construct $w_{\infty}$ generators from the above construction for both $p_+$ and $p_-$, the underlying fermion picture allows only the diagonal $w_{\infty}$ as the dynamical algebra, a fact also well-understood in the Liouville theory. Back to (15), note that not every operator is independent, however\footnote{The following remark belongs to J. Polchinski.}. In the classical theory, an perturbation of (8) that can been written as a total time divergence is considered merely as a canonical transformation, so it does not change the physics. Since

$$ \frac{d}{dt} O_{mnl} = il O_{mnl} + \frac{mn}{m+1} O_{m+1,n-1,l} + mO_{m-1,n+1,l}. $$

(17)

We can successively use (17) to relate different operators and the independent operators are labeled by only two integers. We can choose their form as, for example,

$$ O_{m1} = \int dx (p_+ - p_-) x^m e^{ilt}. $$

(18)

(18) is an irrelevant perturbation if $m > 0$. Now an infinitesimal black hole has the vertex operator

$$ B = M(\partial_z t \partial_{\bar{z}} t - \partial_z \phi \partial_{\bar{z}} \phi) e^{-2\phi}. $$

(19)

In matrix model language, we propose the following operator be identified with (19),

$$ O_{2,0} = \int dx (p_+ - p_-) x^{-2}. $$

(20)

This is a relevant perturbation, since it changes the critical behavior quite a bit. A natural thing to resolve this seems to impose suitable boundary conditions near $x = 0$. We will define the critical point to be when the chemical potential reaches the maximal of the potential. The boundary condition is such that fermion wave function is zero at the maximal. This mimics the hard wall of the Euclidean black hole.

Consider now the potential (9). The time independent background is given by

$$ p = x - \mu/x. $$

(21)
In (21) we have tuned chemical potential to be exactly at the top of the potential. This makes sense because the black hole (1) is characterized by one parameter $M$. We will see shortly that $\mu$ labels the black hole mass. The action (15) becomes

$$S = \bar{S} + \int d\phi dt [\pi_\xi \dot{\xi} - \frac{1}{2}(1 - \mu e^{-2\phi})(\pi_\xi^2 + (\partial_\phi \xi)^2) + \frac{1}{6} e^{-2\phi}(\delta p_+^3 - \delta p_-^3)].$$

We see that the linearized equation derived from (22) is exactly (3) without any field redefinition. Besides, the string coupling is obviously of the standard form $\exp(-\phi)$. So identifying $\phi$ with the dilaton background is very natural. If we accept these, we conclude that (22) describes $D = 2$ string moving in a black hole background.

It is now straightforward to use the effective action to calculate the scattering amplitudes. We leave it to a future work. Here we would like to make some general remarks.

1. Why is that for the cosmological constant background, $\phi$ is not identified with the Liouville field zero mode, whereas for the black hole it is?

2. Consequently, defining metric, dilaton and tachyon background is tricky. This fact has been appreciated by many authors. It seems that our work adds one more subtlety: we modify the Hamiltonian and make an explicit separation of the tachyon and the metric backgrounds. It is not clear how to study a combined tachyon and black hole background, since we make a direct identification of the dilaton with $\ln(-x)$, yet the relation is more indirect for the cosmological constant background. It is very important to understand this issue.

3. A closely related problem is to find a $\sigma$-model action that reproduces the exact results for both cosmological background and black hole. In [14] it is proposed to “covariantize” the inverse harmonic oscillator collective field theory action. Due to the problem mentioned above, it is not clear to what extent it can be valid for the both backgrounds. This may be related to the field redefinition issue in string theory.

4. The matrix model action seems to make sense in Minkowski space too. How would
this affect our understanding of the Minkowski black hole? Is it possible that it is stable? Also, even in Euclidean space, there is no apparent reason to compactify $t$ in the matrix model. Clearly some deeper understanding is needed, especially of the path integral measure problem.

5. Previously, an attempt has been made to study black holes in the usual $c = 1$ model matrix [15]. The idea is to study the dynamics of the formation and subsequent disappearance of black hole due to tachyon fluctuation. It was find that the tachyon self-interaction is too strong. Maybe this is the indication that the usual matrix model Hilbert space does not include black hole. One must change the dynamics in order to see it.

6. What does a more general potential $V(x)$ correspond? Several recent studies [16, 17] find that the negative power operators act as derivatives on a distribution–very singular objects. Some appropriate boundary conditions are needed to make sense of them, as we have done in this paper.

7. In order to completely confirm our proposal, we must study the matrix model black hole is to examine correlation functions and compare with the continuum theory. This is under study.

In conclusion, we have suggested a new space-time interpretation of the $c=1$ matrix model. With a modified critical potential, a possible black hole background emerges. Lots of questions remain to be answered.

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