ON THE EFFECTIVE EVOLUTION FOR THE INFLATON*

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The dynamics of the inflaton field is studied in the context of its interaction with bosonic and fermionic fields modeled by a minimal SUSY like model.

Despite the many studies on the dynamics of inflation, much still remains to be done in the context of understanding several aspects concerning the microscopic dynamics underlying many of the models for inflation, particularly in those cases where the inflaton is coupled to several other fields, like in hybrid inflation models and supersymmetric (SUSY) model extensions. In most of these models it is a fact that large regions of parameter space still remain unexplored and that can be feasible to inflation and new phenomena. This is also true when we consider the common approximations used to study the different aspects of inflation, which most of the time have been restricted to particular perturbative and linear regimes. This is understandable since when nonperturbative and nonlinear effects may become important standard techniques may not always apply, which have slowed down progress in that direction. At the same time methods and techniques developed in quantum field theory devoted to the description of nonequilibrium dynamics have become essential to study these new phenomena, like in those cases where there can be non-negligible particle and radiation production, e.g., in the description of the preheating phase after isentropic inflation or the study of the emergence of non-isentropic inflation (or warm inflation) scenarios, where the scalar inflaton field dissipates non-negligible amounts of radiation during inflation 1, 2, 3, 4.

Of special interest as concerned to non-isentropic inflationary scenarios is the process of how the inflaton can dissipate its energy during inflation.

*Work partially supported by FAPERJ and CNPq.
We have recently identified an efficient mechanism for that in the context of a nonlinear and nonperturbative regime for the inflaton dynamics and elaborated on its details in [5, 6, 7]. We consider the inflaton field \( \phi \) in interaction with other scalar and fermion fields, with standard interaction terms of the form

\[
L_{\text{int}} = -\frac{1}{2} g^2 \phi^2 \chi^2 - g' \phi \bar{\psi} \chi \psi \chi - h \bar{\psi} d \psi \chi,
\]

(1)

for field masses satisfying \( m_{\chi} > \min(2m_{\psi_d}, m_{\phi}) \) and \( m_{\phi} < \min(m_{\psi_d}, m_{\chi}) \) so there are kinematically allowed decay channels of the scalar \( \chi \) into the \( \psi_d \), \( \bar{\psi}_d \) fermions. The decaying of the inflaton field in this model is an indirect effect, interpreted in terms of the effective theory for \( \phi \) (after integrating over the \( \chi, \psi_d, \bar{\psi}_d, \psi, \bar{\psi}_\chi \) fields), which shows that the inflaton does not interact for instance with vacuum like \( \chi \) excitations but rather with the collective \( \chi \) excitations which can decay into light fermions. These are scattering like processes through which the inflaton transfers its energy (or radiates) and that can be efficient in nonlinear and nonperturbative regimes and can happen even deep inside the inflationary phase, as shown in [7].

The role of the spinors coupled to \( \phi \) in (1) is to mimic SUSY, keeping the quantum corrections \( \Delta V_{\text{eff}}(\phi) \) to the effective potential for the inflaton under control, so preserving the flatness of its potential. In fact, as shown in [6], a minimal SUSY model that reproduces the above interactions and decay mechanism has the superpotential \( W = \sqrt{\lambda} \Phi^3 + g \Phi X^2 + f X^3 + m X^2 + h X Y^2 \), where \( \Phi, X, Y \) are chiral superfields. Here, even for SUSY breaking, for \( \phi \neq 0 \) and \( H \neq 0 \), we still can have \( \Delta V_{\text{eff}}(\phi) \ll V_0(\phi) \), where \( V_0(\phi) \) is the tree level potential for the inflaton, that we will take to be a quartic potential with self-coupling \( \lambda \).

The effective equation of motion (EOM) that emerges for a homogeneous classical inflaton field, \( \phi \equiv \varphi(t) \), from (1) and with parameters satisfying the adiabatic (or slow) dynamics for the inflaton, \( \lambda \sim O(10^{-15}), \ g \sim g' \sim h \gtrsim O(10^{-1}) \) and \( \varphi \sim O(m_{\text{Pl}}) \), was shown in Refs. [5, 7] to be given by

\[
\ddot{\varphi}(t) + 3H(t) \dot{\varphi}(t) + \frac{dV_{\text{eff}}(\varphi)}{d\varphi} + \xi \varphi(t) R(t) + 4g^4 \varphi(t) \int_{t_0}^{t} dt' \varphi(t') K_{\chi}(t, t') \chi(t, t'),
\]

(2)

where we are working in a FRW background metric, \( R(t) \) is the scalar of curvature, with coupling \( \xi \) of the inflaton to the gravitacional field and \( K_{\chi}(t, t') \) is a nonlocal (dissipative) kernel that results from the interaction of the inflaton with the scalar \( \chi \) within the relevant scattering like term at one-loop order,
\[ K_{\chi}(t, t') = \int_{t_0}^{t'} \frac{d\tau}{a(t)\tau} \int \frac{d^3q}{(2\pi)^3} \sin \left( 2 \int_{\tau}^{t} dt'' \omega_{\chi}(q, t'') \right) e^{-2 \int_{\tau}^{t} dt'' \Gamma_{\chi}(q, t'')} |_{t > t'} \]

where \( \omega_{\chi}(q, t) = \left[ q^2/a(t)^2 + M^2_\chi(t) \right]^{1/2} \), \( M^2_\chi(t) = m^2_\chi + g_\phi^2 \varphi(t)^2 + (\xi - 1/6) R(t) \) and \( \Gamma_{\chi}(q, t) \approx h^2 M^2_\chi(t)/[8\pi \omega_{\chi}(q, t)] \) for \( m_\chi \gg m_\psi \).

In Refs. [6, 7] we have studied the dynamics of the inflaton through the full numerical solution of (2) (which is numerically implementable, since \( \Gamma_{\chi} > H \) and so the highly oscillatory nonlocal kernel is effectively damped). We also have shown that approximating the kernel as a nonexpanding one (in the Minkowski approximation of Refs. [4, 5]) is a very good approximation for the exact numerical dynamics as well (which is expected since for our parameters \( M_\chi \gg H \) and so curvature effects are subleading). Finally we also have shown that a Markovian (local) approximation for (2) is as well an excellent approximation to describe the evolution for the inflaton (which again is expected, since during inflation and parameters we consider, \( \dot{\varphi}/\varphi, H < \Gamma_{\chi} \) and so the dynamics is effectively adiabatic).

A representative example of the effects of dissipation in the inflaton’s EOM as a result of its interactions to other fields, in the relevant region of parameters for our mechanism of dissipation to work, is shown in Fig. 1.

Figure 1. Evolution for \( \varphi(t) \) for \( g = h = 0.5, \xi = 0, \lambda = 10^{-13}, m_\chi = 10^{13}\text{GeV}, \varphi(0) = m_{\text{Pl}}, \dot{\varphi}(0) = 0 \) and \( a(0) = 1 \).

Figure 1 is obtained by numerically solving Eq. (2) in the Markovian approximation simultaneously with the acceleration equation for the scale
factor. For comparison we also show the result for $\varphi(t)$ when the dissipation due to the nonlocal term in Eq. (2) is absent, for the case of a quartic effective potential, $V_{\text{eff}} = \lambda \varphi^4/4$ (dotted line). In the absence of the nonlocal term in Eq. (2), inflation for the quartic potential with the parameters of Fig. 1 ends when $\varphi_{\text{end}} \sim 0.47 m_{\text{Pl}}$, or by the time $t_{\text{end}} \sim 10^{-12} \text{ GeV}^{-1}$. At this time and well after the inflaton starts oscillating around its minimum value of the potential, in the presence of the effective dissipation term in Eq. (2) the inflaton is still in the inflationary regime (the solid curve in Fig. 1), which ends by the time $\sim 1.1 \times 10^{-11} \text{ GeV}^{-1}$, when $\varphi_{\text{end}} \sim 0.17 m_{\text{Pl}}$. Till the end of inflation the dynamical regime for the inflaton is overdamped, dominated by the nonlocal dissipative kernel. Despite the noticeable change in behavior due to field dissipation, the overall amount of radiation energy density produced is only a fraction of the inflaton’s energy density. For the parameters of Fig. 1, the radiation energy density reaches a peak value $\rho_r/\rho_\varphi \sim 10^{-2}$ at an early time, decaying next till reaching an approximate constant fraction value of $\sim 10^{-5}$. We have also checked that the adiabatic approximation used to derive Eq. (2), $\dot{\omega}/\omega^2 \ll 1$ and the Markovian approximation for the nonlocal kernel, $\dot{\varphi}/(\varphi \Gamma_\chi) \ll 1$ (see Ref. [7]), are both very robust, breaking down, for the parameters used in Fig. 1, at a time $\sim 3 \times 10^{-11} \text{ GeV}^{-1}$ and therefore well after the end of inflation.

Figure 2 compares how the number of e-folds of inflation, $N_e$, changes when we vary either the initial inflaton’s amplitude or couplings $g, h$ for the interactions terms in Eq. (1) and again contrast the results with those obtained for the quartic potential for the inflaton, in the absence of the interactions $L_{\text{int}}$. For $\varphi(0) = 4.4 m_{\text{Pl}}$, which for the inflaton’s self-interaction $\lambda = 10^{-13}$ results in $N_e \simeq 60$ in the absence of dissipation effects in the inflaton’s EOM (dotted line in both plots shown in Fig. 2), the inner plot in Fig. 2 shows the number of e-folds when we vary $g$ (taking also $h = g$). We observe that the interaction terms (1) start to influence the inflaton’s evolution in an appreciable way for $g = h \gtrsim 0.2$, with number of e-folds quickly raising up as the couplings are increased. Increasing the number of fields $\chi$ or fermions $\psi_d$ has also similar effect of increasing rather quickly the number of e-folds or the duration of the inflationary phase.

The results discussed above show that in typical multi-field inflation models there are parameter regions feasible to inflation for which nonlinear and nonperturbative effects can become important and that can lead to important changes in the dynamics for the inflaton, with the emergence of effective strong dissipative effects that alone can sustain inflation long enough and with observational effects on density perturbations $^6, ^7$. We
Figure 2. The number of e-folds $N_e$ in terms of $\varphi(0)$, considering the effective evolution (solid line) and without dissipation (dotted line). Parameters are the same as in Fig. 1. Inner plot shows $N_e$ as a function of the coupling constant, with fixed $\xi = 0$, $\lambda = 10^{-13}$, $m_\chi = 10^{13}$GeV, $\varphi(0) = 4.4m_{Pl}$, $\dot{\varphi}(0) = 0$.

should note that the appearance of strong dissipative effects in our mechanism is not related to a direct decay for the inflaton field, but it is a consequence of decaying modes for fields coupled to the inflaton that results in an effective dissipation in the inflaton’s EOM, whose magnitude can be expressive for nonlinear and nonperturbative regimes. These dissipative mechanisms discussed here have found several uses in the recent literature, like in alleviating many of the problems associated with typical inflation models (the $\eta$ problem, graceful exit, quantum-to-classical transition, large inflaton amplitude, initial conditions), in the study of baryogenesis during nonisentropic inflation, among other studies.

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