QUANTUM STATE OF WORMHOLES AND TOPOLOGICAL ARROW OF TIME

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Abstract

This paper studies the time-symmetry problem in quantum gravity. The issue depends critically on the choice of the quantum state and has been considered in this paper by restricting to the case of quantum wormholes. It is seen that pure states represented by a wave functional are time symmetric. However, a maximal analytic extension of the wormhole manifold is found that corresponds to a mixed state describable by a nondegenerate density-matrix functional that involves an extra quantum uncertainty for the three-metric, and is free from the divergences encountered so far in statistical states formulated in quantum gravity. It is then argued that, relative to one asymptotic region, the statistical quantum state of single Euclidean wormholes in semiclassical approximation is time-asymmetric and gives rise to a topological arrow of time which will reflect in the set of all quantum fields at low energies of the asymptotically flat region.

1 Introduction

There has been some controversy concerning whether quantum gravity is time symmetric [1-4]. The debate has been most emphasized around the ideas of Hawking and Penrose. The case for a time-symmetric quantum gravity was first presented by Hawking who suggested [1], by considering the state of thermal equilibrium of a black hole in a large container with perfectly reflecting walls, that since the essential physical theories involved are time-symmetric, the equilibrium state ought to be time-symmetric too. Penrose has argued [2] against this conclusion, and suggested in turn his own which favours a time-asymmetric quantum gravity, as expressed in terms of his general Weyl curvature hypothesis [4]. However, before worrying about time-symmetry in quantum gravity, one should make it clear that the very concept of time is far from being well defined.
in quantum gravity. Actually, such a concept is under active investigation and some proposals have been advanced in recent years [5].

Although originally aimed at showing that white holes ought to be physically indistinguishable from black holes [1], Hawking gave more recently [3] the idea of a time-symmetric quantum gravity a new more general form. By contrast to how the Penrose’s universal boundary condition associated with the Weyl curvature hypothesis, implied a time-asymmetric quantum gravity, the no boundary proposal [6] suggested by Hawking [7] enabled him to show [3] that the quantum state of the universe must be \( CTP \) and \( T \) invariant simultaneously. This is not the case, however, for the other nowadays major competitor proposal for the initial conditions advanced by Vilenkin [8].

On the other hand, in any theory of quantum gravity one should allow baby universes to branch off from our nearly flat region of spacetime through wormholes [9]. It is the aim of this work to investigate whether the quantum state of Euclidean wormholes is \( T \), \( CT \) and \( CTP \) invariant and, in particular, what could be the effects of these wormholes in the \( T \)-invariance properties of the whole set of the resulting effective quantum field theories at low energy in the asymptotic region.

2 THE QUANTUM STATE OF WORMHOLES

The question that arises is, what is the quantum state of a wormhole or little baby universe? It was first considered [10] that the quantum state of a wormhole should be given by a wave functional of the three-metric, \( h_{ij} \), and matter field configurations, \( \phi \), on the inner three-manifold, which is given by a path integral

\[
\Psi[h_{ij}, \phi_0] = \int_{C_\Psi} Dg_{\mu\nu}e^{-I[g_{\mu\nu}, \phi]},
\]

where \( I \) is the Euclidean wormhole action and the integration is over the class \( C_\Psi \) of asymptotically flat Euclidean four-geometries and asymptotically vanishing matter field configurations which match the prescribed data on an inner three-surface \( S \) with topology \( S^3 \) which divides the whole manifold into two disconnected parts. Thus, the wormhole is in a pure state. The condition that matter fields should vanish asymptotically arises from the boundary conditions [11]. For asymptotically Euclidean four-geometries, the Euclidean action expressed in Hamiltonian form, \( \tilde{I}_H \), amounted with an additional surface term at time \( \tau \to \infty \) to make the action finite, must be invariant under reparametrizations on the three-surface [12]. One should then remove the resulting gauge freedom by imposing that the induced variation of the action, \( \delta \tilde{I}_H \), be exactly zero. This, in turns implies that, if no cosmological term is present and the gravitational part of the Hamiltonian constraint goes to zero as \( \tau \to \infty \), the matter field potential should strictly vanish asymptotically, leaving no matter excitation on the large region.
More recently, however, it has been suggested [13] that the most general quantum state of wormholes need not be in a pure state. Actually, because of lack of any precise knowledge about the baby universe system, a statistical mixed state should represent better these wormholes, and therefore, a non-factorizable density matrix functional $\rho$ was proposed [13], rather than $\Psi$, to describe the quantum state of wormholes. Thus, this state is made most general by considering a path integral

$$\rho[h_{ij}, \phi_0; h'_{ij}, \phi'_0] = \int_{C_\rho} Dg_{\mu\nu} \delta\phi e^{-I[g_{\mu\nu}, \phi]},$$

(2.2)

where the integration is over a class $C_\rho$ of asymptotically flat four-geometries and asymptotically vanishing matter field configurations which match the prescribed data $[h_{ij}, \phi_0]$ on its inner three-surface $S$, and the orientation reverse of the configuration $[h'_{ij}, \phi'_0]$ on its copy three-manifold $S'$, $S$ and $S'$ together forming the inner boundary of each four-geometry in the path integral. The proposal includes those contributions coming from connected four-geometries joining $S$ and $S'$, where the inner boundary does not divide the manifold.

Both the wave function and the density matrix for wormholes have been worked out explicitly from the Wheeler DeWitt equation, subjected to suitable boundary conditions. For the most general scalar-field wormhole, they are given in terms of harmonic oscillator wave functions [10]

$$\Psi[a, f] = H_n(a)e^{-\frac{1}{2}a^2}H_m(f)e^{-\frac{1}{2}f^2}$$

(2.3)

(in which for the sake of simplicity in the formulation only the lowest homogeneous harmonic mode in the expansion of the scalar field has been considered) and the propagator [13]

$$K_0[a, f, 0; a', f', \tau] = \sum_m \sum_n \Psi_{mn}[a, f]\Psi_{mn}[a', f']e^{-\epsilon_{mn}\tau},$$

(2.4)

where $a$ and $a'$ are the radius of the inner three-spheres $S$ and $S'$, $f$ and $f'$ are the mode coefficients of the scalar harmonics in terms of which we expand the fields $\phi$ and $\phi'$, $H(x)$ denotes Hermite polynomials, $\tau$ measures the Euclidean time separation between any two three-surfaces $S$ and $S'$ for which the arguments of the functional are given, and $\epsilon_{mn}$ is expressed in terms of the energy levels of the matter fields, $E_m^f$, and gravitational field, $E_n^a$, as

$$\epsilon_{mn} = E_m^f - E_n^a.$$  

(2.5)

For observers living inside the Lorentzian light cone of one asymptotic region, $S$ and $S'$ may have any Euclidean time separation and, therefore, the proper density matrix operator would be obtained by integrating $K_0$ over all positive values of $\tau$ between 0 and $\infty$. In this case, however, one would obtain a divergent result if $\epsilon_{mn}$ is allowed to vanish (Note that $\epsilon_{mn} = 0$ would actually correspond to the ground-state wave function where one should not need to integrate over $\tau$ as the two resulting submanifolds are now disconnected) or to take on negative values as one should actually do because $\epsilon_{mn}$ is a discontinuous quantity. Clearly,
divergences come about because both $\epsilon_{mn}$ and $\tau$ enter the exponent in $K_0$ linearly and there is no nonzero vacuum zero-point energy preventing $\epsilon_{mn}$ to vanish.

If the wave functional would be evaluated by a path integration using semiclassical approximation and higher, inhomogeneous field modes were excited, then generally there will be a saddle point for each configuration. The effect of allowing excitation of higher, homogeneous modes on the density matrix would manifest by the presence of a product $\prod H_{m_j}(f_j)e^{-\frac{1}{2}f_j^2}$, instead of the single $H_m(f)e^{-\frac{1}{2}f^2}$, in the functionals entering (2.4), so as by a more complicate exponent in the propagator, $\epsilon_{m_jn}\tau = (\sum E_{m_j}^f - E_n^a)\tau$, instead of (2.5), with $\tau$ also varying between 0 and $\infty$. Furthermore, one should sum over each $m_j$ in the propagator. These modifications do not change however the conclusions of the discussion made for the case when only wormholes with the lowest, homogeneous scalar field mode $j = 1$ excited, are considered.

It can be argued [14] that the analogy between the standard theory and the gravitational theory used to formulate the propagator (2.4) is incomplete. This can be seen by considering that in the current derivation of the ground-state wave functional from the standard propagator

$$\sum_{n} \sum_{m} \Psi_{mn}(x)\Psi_{mn}(x') \exp[i\epsilon_{mn}(t-t')]$$

one currently makes a rotation $t \rightarrow -i\tau$ and takes the limit as $\tau \rightarrow -\infty$ [6]. Then, only the term $\epsilon_{mn} = 0$ would survive if all other $\epsilon_{mn} > 0$, but the functional does diverge for all those eigenenergies which are negative, $\epsilon_{mn} < 0$. If we Wick rotate $t \rightarrow +i\tau$ [14], the situation would become just the opposite, with the divergences arising then from all $\epsilon_{mn} > 0$ if we take the limit $\tau \rightarrow -\infty$, or from $\epsilon_{mn} < 0$ in the limit $\tau \rightarrow +\infty$. In the next section we will devise a method for obtaining a consistent propagator in superspace which is free from such divergences.

3 QUANTUM STATE AND SPHALERONS

Many field theories have a degenerate vacuum structure showing more than one potential minimum. Such a complicate vacuum structure makes it possible the occurrence at zero energy of quantum transitions describable as instantons between states lying in the vicinity of different vacua. Instantons are localized objects which correspond to solutions of the Euclidean equations of motion with finite action [15]. On the other hand, for nonzero energies, transitions between distinct vacua may also occur classically by means of sphalerons. Whereas instantons tunnel from one potential minimum to another by going below the barrier, sphalerons do the transit over the barrier. Sphalerons correspond to the top of this barrier and are unstable classical solutions to the field equations which are static and localized in space as well [16,17].

In this section, we shall first review the arguments that make it possible the existence of new sphaleron-like transitions which are classically forbidden though they may still take place in the quantum-mechanical realm [18], relating then
these quantum-sphaleron transitions with the admissible quantum states of wormholes. Such transitions would typically pertain to nonlinear systems showing bifurcations phenomena. In order to see how these nonclassical transitions may appear, let us consider a theory with the Euclidean action

$$S_E(\varphi, x) = -\frac{1}{2} \int_{\eta_i}^{\eta_f} d\eta \varphi^2 ((\frac{dx}{d\eta})^2 + x^2 - \frac{1}{2}m^2\varphi^2x^4), \quad (3.1)$$

where \( m \) is the generally nonzero mass of a dimensionless constant scalar field \( \varphi \) and \( \eta \) denotes a dimensionless Euclidean time \( \eta = \int \frac{dx}{x} \). The sign for the Euclidean action would be positive when we choose the usual Wick rotation \( t \to -i\tau \) (clockwise) or negative for a Wick rotation \( t \to +i\tau \) (anti-clockwise). If the 'energy' of the particles is positive, then one should use clockwise rotation, but if it is negative the rotation would be anti-clockwise [14]. It will be seen later on that (3.1) corresponds to the case of a massive field conformally coupled to gravity, with \( x \) playing the role of the Robertson-Walker scale factor. Thus, since the gravitational energy associated with the scale factor is negative, one should rotate \( t \) not to \(-i\tau\), but to \(+i\tau\), such as it is done in (3.1), and therefore the semiclassical path integral involving action \( S_E, e^{-SE} \), may be generally interpreted as a probability amplitude for quantum tunnelling [14]. In the classical case, if the scalar field \( \varphi \) is axionic, i.e. if \( \varphi \) is assumed to be a hypothetical light particle in the dynamical mechanism that solves the strong \( CP \) problem of \( QCD \), then it becomes pure imaginary, i.e. \( \varphi = i\varphi_0 \), with \( \varphi_0 \) real. In this case, the potential becomes

$$V(x) = \varphi_0^2\left(\frac{1}{2}x^2 + \frac{1}{4}m^2\varphi_0^2x^4\right), \quad (3.2)$$

the theory has just one zero-energy vacuum, and the solution to the classical equations of motion is \( \bar{x} = 0 \), with Euclidean action \( S_E = 0 \).

Expanding about the classical solution we obtain the usual path integral at the semiclassical limit [17]

$$< 0 | e^{-HT/\hbar} | 0 > \sim N[\text{det}(-\partial^2_{\tau} + \omega^2)]^{-\frac{1}{2}}(1 + O(\hbar)), \quad (3.3)$$

where \( N \) is a normalization constant and \( \omega^2 = V''(0) \), with \( ' \) denoting differentiation with respect to \( x \). The ground-state solution to the wave equation for the operator \(-\partial^2_{\tau} + \omega^2\) which corresponds here to a harmonic oscillator with substracted zero-point energy can be written as

$$\psi_0 = \omega^{-1}e^{-\omega T}\sinh[\omega(\tau + \frac{T}{2})].$$

Then, the path integral becomes constant and given by

$$< 0 | e^{-HT/\hbar} | 0 > \sim (\frac{\omega}{\pi\hbar})^{\frac{3}{2}}(1 + O(\hbar)). \quad (3.4)$$

If the zero-point energy had not been substracted, then Eqn. (3.4) would also contain the well-known time-dependent factor \( e^{-\frac{1}{2}\omega T} \) [17].
Let us now consider $\varphi^2$ as being the control parameter for the nonlinear dynamic problem posed by action (3.1). All the values of $\varphi^2$ corresponding to an axionic classical field will be negative and, in the classical case, can be continuously varied first to zero (a zero potential critical point) and then to positive values (the field $\varphi$ has become real, no longer axionic). The associated variation of the dynamics will represent a typical classical bifurcation process that can finally lead to spontaneous breakdown of a given symmetry [18]. In the semiclassical theory this generally is no longer possible however. Not all values of the squared field $\varphi^2$ are then equally probable. For most cases, large ranges of $\varphi^2$-values along the bifurcation itinerary are strongly suppressed, and hence the bifurcation mechanism would not take place. Nevertheless, there could still be sudden reversible quantum jumps from the negative values to the corresponding positive values of $\varphi^2$. Such jumps would be equivalently expressible as analytic continuations in $x$ to and from its imaginary values, lasting a very short time. The system will first go from the bottom of potential (3.2) for $\varphi^2 < 0$ to the sphaleron point of potential (see Fig.1)

$$V(x) = \varphi_0^2(-\frac{1}{2}x^2 + \frac{1}{4}m^2\varphi_0^2x^4), \quad (3.5)$$

without changing position or energy, and then will be perturbed about the sphaleron saddle point to fall into the broken vacua where the broken phase condenses for a short while, to finally redo all the way back to end up at the bottom of potential (3.2) for $\varphi^2 < 0$ again. The whole process may be denoted as a quantum sphaleron transition and it is assumed to last a very short time and to occur at a very low frequency along the large time $T$. Therefore, one can use a dilute sphaleron approximation which is compatible with our semiclassical approach. Thus, for large $T$, besides individual quantum sphalerons, there would be also approximate solutions consisting of strings of widely separated quantum sphalerons. In analogy with the instanton case [17], we shall evaluate the functional integral by summing over all such configurations, with $n$ quantum sphalerons centered at Euclidean times $\tau_1, \tau_2, \ldots, \tau_n$. If it were not for the small intervals containing the quantum sphalerons, $V^n$ would equal $\omega^2$ over the entire time axis, and hence we would obtain the same result as in (3.4). However, the small intervals with the sphalerons correct this expression. In the dilute sphaleron approximation, instead of (3.4), one has

$$\left(\frac{\omega}{\pi\hbar}\right)^{\frac{1}{2}}(-K_{sph})^n(1 + O(\hbar)), \quad (3.6)$$

where $K_{sph}$ is an elementary frequency associated to each quantum sphaleron, and the sign minus accounts for the feature that particles acquire a negative energy below the sphaleron barrier (Fig.1). Note that the action changes sign as one goes from potential (3.2) to potential (3.5) below such a barrier. After integrating over the locations of the sphaleron centers, the sum over $n$ sphalerons
produces a path integral
\[
<0 | e^{-HT/h} | 0 >_{sph} \sim (\frac{\omega}{\pi \hbar})^{\frac{1}{2}} e^{-K_{sph}T(1 + O(h))},
\]
which is proportional to the semiclassical probability of quantum tunnelling from \( \bar{x} = 0 \) first to \( \bar{x}_\pm = \pm (m\varphi_0)^{-1} \) and then to \( \bar{x} = 0 \) again.

In Eqn. (3.7) we have summed over any number of sphalerons, since all the small time intervals start and finish on the axis \( x = 0 \), at the bottom of potential (3.2). Approximation (3.7) corresponds [17] to a ground-state energy \( E_0 = \hbar K_{sph} \). Thus, the effect of the quantum sphalerons should be the creation of an extra nonvanishing zero-point energy which must correspond to a further level of quantization which is over and above that is associated with the usual second quantization of the harmonic oscillator.

As pointed out before, an Euclidean action with the same form as (3.1) arises in a theory where a scalar field \( \Phi \) with mass \( m \) couples conformally to Hilbert-Einstein gravity. Restricting to a Robertson-Walker metric with scale factor \( a \) and Wick rotating anti-clockwise, the Euclidean action for this case becomes, after integrating by parts and adding a suitable boundary term,
\[
I = -\frac{1}{2} \int d\eta N (\frac{\chi^2}{N^2} + \chi^2 - \frac{\bar{a}^2}{N^2} - a^2 + m^2 a^2 \chi^2),
\]
where the overhead dot means differentiation with respect to the conformal time \( \eta = \int d\tau/a \), \( \chi = (2\pi^2 \sigma^3)^{\frac{1}{2}} a \Phi \), \( N \) is the lapse function and \( \sigma^2 = 2G/3\pi \). The equations of motion derived from (3.8) are (in the gauge \( N = 1 \)) \( \ddot{\chi} = \chi + m^2a^2\chi \) and \( \ddot{a} = a - m^2 \chi^2 a \). We note that these two equations transform into each other by using the ansatz \( \chi = ia \). Invariance under such a symmetry manifests not only in the equations of motion, but also in the Hamiltonian action and four-momentum constraints. The need for Wick rotating anti-clockwise becomes now unambiguous [14, 19], since all the variable terms in the action become associated with energy contributions which are negative if symmetry \( \chi = ia \) holds. Without loss of generality, the equations of motion can then be written as the two formally independent expressions \( \ddot{\chi} = \chi - m^2 \chi^3 \) and \( \ddot{a} = a + m^2 a^3 \). If \( \chi = ia \), then \( \Phi \) becomes a constant axionic field \( \Phi = i(2\pi^2 \sigma^3)^{-\frac{1}{2}} \). Re-expressing action \( I \) in terms of the field \( \chi \) alone, one can write for the Lagrangian in the gauge \( N = 1 \)
\[
L(\varphi, a) = -\left( \frac{1}{2} \varphi^2 \bar{a}^2 + \frac{1}{2} \varphi^2 a^2 - \frac{1}{4} m^2 \varphi^4 a^4 + \frac{1}{2} R_0^2 \right),
\]
where \( \varphi = \frac{\varphi}{m_p} \), \( m_p \) is the Planck mass, and \( R_0^2 \) is an integration constant which has been introduced to account for the axionic character of the constant field implied by \( \chi = ia \); such an imaginary field would require a constant additional surface term to contribute the action integral. Besides this constant term \( \frac{1}{2} R_0^2 \), (3.9) exactly coincides with the Lagrangian in (3.1). For the axionic field case where the symmetry \( \chi = ia \) holds, \( \varphi^2 = -\varphi_0^2 \), with \( \varphi_0 \) real. Then the solution to the equations of motion and Hamiltonian constraint is
\[
a(\tau) = (m\varphi_0)^{-1} [(1 + 2m^2 R_0^2)^{\frac{1}{2}} \cosh(2^{\frac{1}{2}} m\varphi_0 \tau) - 1]^{\frac{1}{2}}, \chi = ia(\tau),
\]
which represents an axionic nonsingular wormhole spacetime.

If we rewrite (3.9) as a Lagrangian density $L(\Phi, a) = m_p^2 L(\varphi, a)/a^4$, it turns out that, although the Lagrangian density as written in the form $L(\Phi, a)$ looks formally similar (except the last term in the potential) to that for an isotropic and homogeneous Higgs model in Euclidean time, it however preserves all the symmetries of the theory intact. Indeed, the field $\Phi$ is not but a simple imaginary constant $\Phi = i(2\pi^2 \sigma^2)^{-\frac{1}{2}}$. None the less, if $\Phi$ is shifted by some variable $\rho$, such that $\Phi \rightarrow \phi = i\xi + \rho$, where $\xi = (2\pi^2 \sigma^2)^{-\frac{1}{2}}$, while keeping the same real scale factor, then symmetry $\chi = ia$ would become broken. In such a case, the Lagrangian density $\tilde{L} \equiv L(\phi, a)$ resulting from $L(\Phi, a)$ is seen (Note that if we let $\Phi$ to be time-dependent, then the kinetic part of the Lagrangian $L(\Phi, a)$ becomes $-\left[\frac{1}{4}(\dot{\phi})^2 + \frac{1}{2}(\Phi \dot{\phi} a + \frac{1}{2}(\dot{\phi} a)^2)\right]$) to describe a typical Higgs model in the isotropic and homogeneous euclidean framework for a charge-$Q$ field $\phi$, a massless gauge field $A \equiv A(\tau) = \frac{T r K}{e Q}$ (with $e$ an arbitrary gauge coupling and $K$ the second fundamental form), and variable tachyonic mass $\mu = \frac{1}{a}$, as now the last term in the potential does not depend on $\phi$ and becomes thereby harmless for the Higgs model. In principle, the Lagrangian could have been written as well in any of the infinite number of forms, other than (3.9), which also have both $\Phi = \text{Const}$ as a solution and the symmetry $\chi = ia$. However, the system will spontaneously choose breaking the symmetry from the particular Lagrangian form given by (3.9) for two reasons: (i) because, in so doing, it attains the most stable vacuum configuration, and (ii) because, out of the infinite number of possible Lagrangians, it is only $L(\Phi, a)$ which is invariant under the isotropic and homogeneous Euclidean $(t \rightarrow +i\tau)$ Abelian group $U(1)$ of transformations

$$A \rightarrow A - i\zeta, \Phi \rightarrow \Phi e^{i\zeta},$$

since only for $L(\Phi, a)$ integration of $A$ yields $a \rightarrow a e^{-i\zeta}$, so that $\chi$ is also invariant under these transformations. Note that the equivalent rotation in scale factor to generally complex values in the representation obtained by re-expressing $I$ in terms of $a$ alone will lead to a breakdown of diffeomorphism invariance [20]. Clearly, all theories satisfying $\chi = ia$ must contain an axionic surface term $-\int d\eta N R_0^2$ which is of course invariant under such a symmetry. Then, it is easy to show that both, the equations of motion and the Hamiltonian constraint derived from (3.8) satisfy solution (3.10). It is in this sense that the scalar field theory and the theory which shows sphaleron transitions (3.9) are equivalent, though only the latter is invariant under the transformations of the gauge group $U(1)$.

The integration constant $R_0^2$ in (3.9) corresponds to the inclusion of axionic surface terms in the action integral. For a constant real scalar field, such surface terms should produce a generally different integration constant $(R_0')^2$ whose sign is the opposite to that for $R_0^2$. Therefore, the solutions to the equations of motion and Hamiltonian constraint corresponding to a constant real scalar field $\varphi^2 = \varphi_0^2$ are

$$a_{\pm}(\tau) = (m\varphi_0)^{-1}[1 \pm (1 - 2m^2(R_0')^2)^{\frac{1}{2}} \cosh(2\frac{1}{2}m\varphi_0\tau)]^{\frac{1}{2}}. \quad (3.11)$$
Solutions $a_+$ and $a_-$ lie in the two disconnected, classically allowed regions for which the potential is negative. It has been shown [21] that (3.11) represents a wormhole with doubly connected inner region whose quantum state should be given by a statistical density matrix.

The real classical solution corresponding to the equilibrium minimum of the potential for axionic fields is $\bar{a} = 0$, and that for real fields are $\pm (m\varphi_0)^{-1}$. Therefore, relative to the simply connected inner topology implied by (3.10), the doubly-connected topology implied by (3.11) represents an actual pitchfork bifurcation for $R_0 = R_0'$.

We can see now why not all values of $\varphi_0$ are allowed semiclassically. Although one actually could also gauge the imaginary part of $\varphi$ to any value other than $\xi$, since the action $I$ depends on $\varphi_0^2$ through solution (3.10), very small values of the constant fields will make this action very large and hence the semiclassical probability $e^{-I}$ becomes vanishingly small. Thus, interpreting $\varphi^2$ as a control parameter it turns out that the bifurcation process cannot be reached in a continuous, deterministic way.

Disregarding the constant term of the potential in (3.9) to make this potential vanish at its minima, we obtain the same situation as in Fig.1, with $\omega \sim 1$ in Planck units. If it were not for the short intervals where sphalerons are quantically induced, the contribution of the path integral $<0|e^{-HT/\hbar}|0>$ to the quantum state of the system would simply be a constant factor. This would correspond to a pure quantum state representable by a probability functional factorizable in a product of wave functions. However, if such sphaleron transitions (essentially consisting of rotations of the metric to generally complex values) are taken into account, then the contribution of the path integral $<0|e^{-HT/\hbar}|0>_{sph}$ will introduce a time-dependent factor like (3.7), with $\omega \sim 1$, in the full quantum state. Now, since time separation between the initial and final states cannot be known, one should integrate over $T$ and the full quantum state becomes (here and hereafter we omit for the sake of simplicity the small term depending on $O(\bar{\hbar})$)

$$\frac{1}{\tau_p} \int_0^\infty dT \Psi(a)\Psi'(a')e^{-K_{sph}T}, \quad (3.12)$$

where $\tau_p$ is the Planck time and the $\Psi$'s are the wave functions for the initial and final states. From the Euclidean action (3.8), after applying $\chi = ia$, we obtain a Wheeler DeWitt equation

$$H_{WDW}\Psi = (\frac{\partial^2}{\partial a^2} + V(a, m, R_0))\Psi = 0, \quad (3.13)$$

where the potential $V(a, m, R_0) = R_0^2 - a^2 - \frac{1}{2}m^2a^4$. In harmonic approximation $(m \to 0)$, we obtain for the quantum state

$$\Psi_j(a) = H_j(a)e^{-\frac{1}{2}a^2}; \quad (3.14)$$

i.e., after symmetry $\chi = ia$, the wormhole wave function is given by either $\Psi(a)$, or by the analogous function $\Psi(\chi)$, but not by their product.
The contribution from the strings of single quantum sphalerons bouncing at the vacua of potential (3.5) are not the only objects that should contribute the path integral. Once the system has returned to the top of this potential after one transition is done, it may yet undergo further sphaleron-assisted bifurcations, before completing transition to the bottom of potential (3.2) within the given small time interval. This would lead to a differentiation of the transition strings: there will be strings formed by \( n_1 \) single transitions consisting of just one quantum sphaleron, strings formed by \( n_2 \) single transitions consisting of two quantum sphalerons and, in general, strings formed by \( n_j \) single transitions consisting of \( j \) quantum sphalerons. So, considering the contributions of all such transition strings leads to the maximal analytic extension of the wormhole manifold. In what follows we shall show how such contributions may enter the density matrix for wormholes. We shall use first-order time-dependent perturbation theory in our Euclidean framework in order to independently derive an expression for the density matrix in terms of states (3.14). Let us first define the factorizable pure-state density matrix elements as \( \rho_j^{(0)} = \Psi_j(a)\Psi_j(a') \), so that \( H_{WDW}\rho_j^{(0)} = 0 \).

Considering the wormhole to be off shell one can introduce an energy perturbation \( H(\tau) \). We have then

\[
(H_{WDW} + H(\tau))\rho(a, \tau) = \hbar \frac{\partial \rho(\tau)}{\partial \tau}.
\] (3.15)

In the limit of very small \( m \), using the current solution of the perturbed equation \( \rho(\tau) = \sum_j a_j(\tau)\rho_j^{(0)} \), we obtain after multiplying for \( \rho_k^{(0)} \) and integrating over the scale factor

\[
\hbar 2^{2k}(k!)^2 \pi^2 \frac{da_k(\tau)}{d\tau} = \sum_j H_{jk}(\tau)a_j(\tau),
\] (3.16)

where

\[
H_{jk}(\tau) = \int \rho_j^{(0)}H(\tau)\rho_k^{(0)} dada' = H_{jk}(0)e^{-\frac{1}{\hbar}(E_j + E_k)\tau},
\] (3.17)

in which \( E_j \) and \( E_k \) are the energies of, respectively, the \( j \)th and \( k \)th harmonic-oscillator levels. At least for the nonsupersymmetric case under study, the wormhole oscillators have no zero-point energy. Thus, choosing \( E_k = 0 \) and \( H(\tau) = \epsilon_p f(\tau) \), so that \( H_{jk}(0) = 2^{2j}(j!)^2 \pi^2 \epsilon_p \), where \( \epsilon_p \) is the Planck-scale energy, we obtain in first order

\[
\hbar \frac{da_j^{(1)}}{d\tau} = \epsilon_p e^{-\frac{1}{\hbar}E_j\tau},
\] (3.18)

For the most probable wormholes with the Planck size, \( E_j = j\epsilon_p \) in the harmonic approximation. Hence,

\[
\rho(\tau) = \frac{1}{\tau_p} \sum_{j=1}^{\infty} \int d\tau \Psi_j[a]\Psi_j[a'] e^{-\frac{i\epsilon_p\tau}{\hbar}}.
\] (3.19)

The integrand in (3.19) will correspond, for large \( \tau \), to semiclassical approximation of the path integral for our Euclidean framework. The lower limit of index
$j$ is taken to be 1 because index $k$ stands for the zero-energy ground state and $j \neq k$. Thus, the argument [17] that the path integral in semiclassical approximation must equal the first term of its expansion in energy eigenvalues should reflect here in such a way that (3.12) will be the same as (3.19) for $j = 1$, and hence $\hbar K_{sph} = \epsilon_p$. It follows then ($\tau = T$)

$$\rho(T) = \frac{1}{\tau_p} \sum_{j=1}^{\infty} \int_{0}^{\infty} dT \Psi_j[a] \Psi_j[a'] e^{-jK_{sph}T}, \quad (3.20)$$

which is the expression that one should obtain by summing up all contributions from the different strings formed by $n_j$ single elementary quantum sphalerons with $K_{sph} = \epsilon_p$ in the harmonic approximation.

As discussed in Section 2, density matrices defined as the propagator (2.4) [22] are divergent [23] and generally not positive definite [13]. This is due to the fact that integration over $\tau$ leaves a factor $\epsilon_{mn}^{-1}$ in the summation over $m$ and $n$, producing a divergent term when $\epsilon_{mn} = 0$. Now, since $\Psi[a]$ does not depend on $T$, $K_{sph} > 0$, and the lower bound for index $j$ is unity, to the extend that $\Psi$ be convergent, the state (3.20) will give an also convergent, positive definite density matrix for nonsimply connected wormholes. On the other hand, in the limit where either $\hbar \rightarrow 0$ or $G \rightarrow 0$, or both, taken after integrating over $T$, the path integral (3.20) vanishes, so leaving no wormhole state. This implies (3.20) to be a quantum state with no classical counterpart, i.e. the density matrix of wormholes is a nonclassical state similar to those occurring in quantum optics. Moreover, $K_{sph}$ corresponds to a kind of zero-point energy which does not take place in usual quantum field theories and, therefore, even though the sphaleron contribution has been worked out here in a semiclassical approximation, it must still represent an extra level of quantization above that is contained in the quantum theories. The vanishing of the path integral in the nongravitational limit suggests that the extra level of quantization introduced by quantum sphalerons can only appear when quantum-gravity effects are considered, or in other words, quantum gravity involves an extra level of quantization which is over and above that is contained in nongravitational quantum theory. We see that the introduction of this extra quantization produces three main effects: (i) it converts pure states in mixed states describable by a density matrix, (ii) it breaks energy degeneracy (all those quantum numbers, $n$ and $m$ in (2.5), leading to the same eigenenergy $\epsilon_{mn}$) by reducing the set of eigenenergies $\epsilon_{mn}$ to the subset $\epsilon_j = jK_{sph}$, and (iii) it makes the density matrix finite since $K_{sph} > 0$.

The extra degree of quantization involved in (3.20) can be thought of as being originated from a discretization of time separation [24] so that $| \tau | \geq \tau_p$, where $\tau_p$ is the Planck time, at the times a quantum sphaleron occurs. In fact, work done in quantum gravity [25-27] predicts that it is altogether impossible to get a space-time resolution better than the Planck scale. If we take Planck-sized wormholes, then it follows [24] that, as one approaches the time resolution $\Delta \tau = \tau_p$ at any $\tau$-time point along an initially single wormhole tube, a topologically nontrivial bifurcation will develop at the given point. Thus, expliciting the discrete charac-
ter of \( \tau \) at (to keep things in semiclassical and dilute bifurcation approximations) widely separated points converts any simply connected wormhole in a multiply connected wormhole with any number of (in the harmonic approximation) identical bifurcations. The quantum state of the resulting system is given by (3.20) and would be equivalent \([24]\) to that for a single wormhole-tube with \textit{continuous} time \( \tau \) having contributions from all possible discrete energies \( j\epsilon_p \), with \( j > 0 \) for \( \tau \equiv T > 0 \), \( j < 0 \) for \( \tau \equiv T < 0 \), and \( 1 \leq |j| \leq \infty \), where \( \epsilon_p = K_{sph} \) denotes Planck energy for a Planck-sized wormhole. Discreteness in time \( \tau \) can thus be transformed into discreteness in wormhole energy.

A caveat is worth mentioning here. It has been pointed out \([19]\) that a rotation \( t \to +i\tau \) would imply a repulsive gravitational regime. Nevertheless, a probability functional which is factorizable as a product of equal wave functions \([10]\) can no longer represent the ground state if diffeomorphism invariance is preserved, for all eigenenergies are strictly zero \([14]\). However, if diffeomorphism invariance is broken, so that a ground state as (3.20) becomes well defined, then such a ground state would be below the barrier for \( t \to +i\tau \), in a situation where the Euclidean action is negative and corresponds therefore to a positive gravitational constant and hence to attractive gravity.

A key feature generally distinguishing density matrices from wave functions in the context of quantum cosmology is that, whereas the Wheeler DeWitt operator, \( H_{WDW} \), annihilates the state \( \Psi \), that is no longer the case when it acts on \( \rho \), i.e. \( H_{WDW}\rho = \epsilon_{mn} \neq 0 \), meaning that the total energy of the baby universe sector is no longer zero (i.e. a virtual baby universe is no longer a closed universe \([28]\)). In that case, gravitational flow lines from the baby universes would in fact connect them to the asymptotic region \([29]\). This effective topological connection between the baby universe spacetime manifold and the main large manifold should affect the whole wormhole four-manifold \( M \), so that, even if this was originally conventionally divided into two parts, \( M_+ \) and \( M_- \), by the three-surfaces, the resulting \textit{effective} wormhole four-manifold would become connected through the large regions, making thereby the resulting density matrix non-factorizable as a product of wave functions \( \Psi_+ \) and \( \Psi_- \) \([13,29]\). Or in other words, an observer who wants to measure the metric and matter fields on any set of connected three-surfaces in the inner boundary must approach the Planck-scale resolution and hence produce, by act of measurement, a corresponding set of bifurcations that will ultimately make it impossible to divide the four-manifold in two separated parts, and so forth.

4 WORMHOLES AND TIME SYMMETRY

The trace of the second fundamental form, \( K \), or some monotonic function of it, may be taken to make an extrinsic time notion in cosmology \([3,30]\). In order to investigate the time-symmetry properties of the quantum state of wormholes as given by a density matrix, one can replace the dependence of the functional (2.2)
on $h^{\frac{1}{2}}$, the square root of the determinant of the three-metric, on $S$ and $S'$, by their respective conjugate momenta, $K$ and $K'$, using the Laplace transform

$$
\rho[\bar{h}_{ij}, \phi_0; \bar{h}'_{ij}, \phi'_0, K']
= N \int_0^\infty d[h^{\frac{1}{2}}] d[h'^{\frac{1}{2}}] e^{-\frac{i}{16\pi\hbar} (\int K h^{\frac{1}{2}} dx + \int K' h'^{\frac{1}{2}} dx')} \rho[h_{ij}, \phi_0; h'_{ij}, \phi'_0], \quad (4.1)
$$

where $N$ is some normalization constant, $\bar{h}_{ij}$ and $\bar{h}'_{ij}$ are the three-metrics on $S$ and $S'$ up to the conformal factors $h^{-\frac{1}{2}}$ and $h'^{-\frac{1}{2}}$, respectively, and we have restricted to the simplest case where $S$ and its copy $S'$ are both connected three-surfaces. The density matrix $\rho[h_{ij}, \phi_0; h'_{ij}, \phi'_0]$ is given by (2.2) and can, in turn, be obtained from (4.1) by the inverse Laplace transform

$$
\rho[h_{ij}, \phi_0; h'_{ij}, \phi'_0]
= -\frac{i}{24\pi} \int d[K'] \int_{N'} d[K] e^{\frac{i}{16\pi\hbar} (\int K h^{\frac{1}{2}} dx + \int K' h'^{\frac{1}{2}} dx')} \rho[h_{ij}, \phi_0, K; h'_{ij}, \phi'_0, K'], \quad (4.2)
$$

where, to ensure positivity of $h^{\frac{1}{2}}$ and $h'^{\frac{1}{2}}$, the contours of integrations $\Lambda$ and $\Lambda'$ must run from $-i\infty$ to $+i\infty$ to the right of any singularity of (4.1) in the complex planes $K$ and $K'$.

Yet, the Euclidean action $I[g_{\mu\nu}, \phi]$ is not bounded from below and above. Therefore, in order to make the path integral convergent, one should deform the contour of integration in (2.2) from Euclidean to complex metrics [31], with the arguments of the functional defined on the inner boundary $\partial_2 M$ and the asymptotically flat boundary $\partial_1 M$. The need for an inner boundary for wormhole manifolds would incorporate the quantum-gravity idea [26] that there exists a finite maximum spacetime resolution limit which restricted the density of spacetime foliation to be always finite for finite three-geometries. Then for a conformal transformation $\bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, $\bar{\phi} = \Omega^{-1}\phi$, where $\Omega = 1 + iy$, $y$ should be subjected to the boundary conditions, $y \to 0$, as $M \to \partial_1 M$, and $y \to y_0 \neq 0$, as $M \to \partial_2 M$. This choice, which leads to a complexified fixed $h_{ij}$ and hence to a complexified second fundamental form $K_{ij}$, so as a complexified matter field, on the inner boundary, reflects the feature that, for a wormhole manifold, one should replace a point at which the three-geometry degenerates by a minimal nonzero three-surface $\partial_2 M$ at the Planck scale [25]. One would allow the value of $y$ to be strictly zero only at the classical boundary $\partial_1 M$ where the three-geometry becomes infinite, at least in the dilute wormhole approximation [9,10]. Actually, the insertion of the Planck three-sphere [25] is equivalent to quantizing the conformal factor so that conformal fluctuations would result in a "zero-point" length $l_p$, according to $a^2 = a^2_{\text{class}} + l_p^2$ and $\Delta a \geq l_p$, where $\Delta a$ denotes quantum uncertainty in the value of $a$ [27]. But if this uncertainty holds then the baby universes can no longer be topologically closed [28], and this will amount to a nonzero imaginary part of the conformal factor also on the inner boundary. A more technical reason for the need of complex metrics on the inner boundary of multiply connected wormholes stems from the discussion of section 3 where it was seen that...
a nonfactorizable probability functional can only be obtained if we analytically continue the dynamical equations in the metric to generally complex values. No multiple connectedness of the inner manifold can be preserved otherwise. The shape of the potentials for the pure and mixed states tend to coincide as $a \to \infty$ and can therefore be both defined in terms of a real scale factor only asymptotically. It follows then that the metric for multiply connected wormholes should be complexified everywhere in the manifold, except asymptotically. Making $y$ different from zero also on the inner boundary is associated with the spontaneous symmetry breaking process whose broken vacuum phase would fix the value of $y_0$; the symmetry is restored asymptotically while the Higgs mass is stored in the baby universe sector.

Because the Laplace transform (4.1) is holomorphic for $\text{Re}(K) > 0$ [3], one can analytically continue $\rho[\tilde{h}_{ij}, \phi_0, K]$ in $\bar{K}$ and $K'$ to the Lorentzian values $K_L = iK$ and $K'_L = iK'$. Now, since under conformal transformation of the metric $\bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, we have $\bar{K} = \Omega^{-1} K$ and $\bar{K}' = \Omega^{-1} K'$, it follows

$$\bar{K}_L = \frac{(1 \mp iy_0)K_L}{1 + y_0^2},$$  \hspace{2cm} (4.3)

(and likewise for $\bar{K}'_L$) where the upper sign corresponds to complex metrics, and the lower sign, to complex conjugate metrics. The choice of time orientation on the inner three-manifold must be based on a causal analysis requiring the definition of suitable covering manifold [32], i.e. it is required that not more than one of the possible several disjoint components of the three-boundary is joined at $S$ to its respective copy $S'$.

Using a conformal metric, after analytically continuing (4.1) in $K$ and $K'$ to their Lorentzian values, we obtain from (4.3)

$$\rho^{(c)}[\tilde{h}_{ij}, K_L, \phi_0; \bar{h}'_{ij}, K'_L, \phi'_0] = N \int_{C^{(c)}} d[(\bar{h}^{(c)})^{\frac{1}{2}}] d[(\bar{h}'^{(c)})^{\frac{1}{2}}]$$

$$\times e^{-\frac{1}{12\pi\Omega} \int d^3x (i+y_0) K_L (\bar{h}^{(c)})^{\frac{1}{2}} + \frac{1}{1+y_0^2} \int d^3x' (i+y'_0) K'_L (\bar{h}'^{(c)})^{\frac{1}{2}}} \rho^{(c)}[\tilde{h}_{ij}, \phi_0; \bar{h}'_{ij}, \phi'_0],$$  \hspace{2cm} (4.4)

$$\rho^{(cc)}[\tilde{h}_{ij}, K_L, \phi_0; \bar{h}'_{ij}, K'_L, \phi'_0] = N \int_{C^{(cc)}} d[(\bar{h}^{(cc)})^{\frac{1}{2}}] d[(\bar{h}'^{(cc)})^{\frac{1}{2}}]$$

$$\times e^{-\frac{1}{12\pi\Omega} \int d^3x (i-y_0) K_L (\bar{h}^{(cc)})^{\frac{1}{2}} + \frac{1}{1+y_0^2} \int d^3x' (i-y'_0) K'_L (\bar{h}'^{(cc)})^{\frac{1}{2}}} \rho^{(cc)}[\tilde{h}_{ij}, \phi_0; \bar{h}'_{ij}, \phi'_0],$$  \hspace{2cm} (4.5)

where the superscripts $(c)$ and $(cc)$ mean quantities obtained when, respectively, only complex metrics and complex conjugate metrics are used. Integrations over contours $C^{(c)}$ and $C^{(cc)}$ respectively extend $(\bar{h}^{(c)})^{\frac{1}{2}}$ and the orientation reverse of $(\bar{h}'^{(c)})^{\frac{1}{2}}$ from 0 to $\infty$ in the complex plane for contour $C^{(c)}$, and $(\bar{h}^{(cc)})^{\frac{1}{2}}$ and the orientation reverse of $(\bar{h}'^{(cc)})^{\frac{1}{2}}$ also from 0 to $\infty$, but in the complex conjugate plane for contour $C^{(cc)}$. The lower integration limits have been chosen to be zero.
to allow for a total quantum freedom both for the three-geometry and slicing. Note that the values of the three-metrics, scalar fields and trace of the second fundamental form induced on $\partial_2 M$ are all still complex.

Applying complex conjugation ($\ast$) and the operation of $K_L$-reversal to states (4.4) and (4.5), we obtain

$$\rho^{(c)}[\tilde{h}_{ij}, -K_L, \phi_0; \tilde{h}'_{ij}, -K'_L, \phi'_0] = \rho^{(cc)}[\tilde{h}_{ij}, K_L, \phi_0; \tilde{h}'_{ij}, K'_L, \phi'_0]$$  \hspace{1cm} (4.6)

$$\rho^{(cc)}[\tilde{h}_{ij}, -K_L, \phi_0; \tilde{h}'_{ij}, -K'_L, \phi'_0] = \rho^{(c)}[\tilde{h}_{ij}, K_L, \phi_0; \tilde{h}'_{ij}, K'_L, \phi'_0].$$  \hspace{1cm} (4.7)

It follows that if the full density matrix contained equal contributions from metrics with a complex action and from metrics with a complex conjugate action, then that density matrix would be $T$ symmetric. However, fixing prescribed complex values for the boundary metric and scalar field makes the density matrix and its Laplace transform holomorphic functions, i.e. any contribution from metrics with complex conjugate action to the path integral in (4.4), or from metrics with a complex action to the path integral in (4.5) must be ruled out. It follows then that, fixing a complex value for the boundary metric and the boundary scalar field, contributions from complex conjugate four-metrics can never enter the path integral or its Laplace transform, and likewise, fixing the corresponding complex conjugate values for the boundary arguments will prevent any four complex-metric to contribute the path integral or its Laplace transform.

Since the Wheeler-DeWitt operator annihilates any pure-state wave function of wormholes, for such states we must sum over both complex and complex conjugate four-metrics which induce the prescribed boundary real metric. Thus, by analytically continuing in the metric the single wave functions for wormholes, $\Psi_+$ and $\Psi_-$, should respectively map into the two distinct density matrices $\rho^{(c)}$ and $\rho^{(cc)}$, and their Laplace transform respectively into (4.4) and (4.5). We have,

$$\rho^{(c)}[\tilde{h}_{ij}, K_L, \phi_0; \tilde{h}'_{ij}, K'_L, \phi'_0] \neq \rho^{(c)*}[\tilde{h}_{ij}, -K'_L, \phi_0; \tilde{h}'_{ij}, -K'_L, \phi'_0],$$  \hspace{1cm} (4.8)

$$\rho^{(cc)}[\tilde{h}_{ij}, K_L, \phi_0; \tilde{h}'_{ij}, K'_L, \phi'_0] \neq \rho^{(cc)*}[\tilde{h}_{ij}, -K_L, \phi_0; \tilde{h}'_{ij}, -K'_L, \phi'_0],$$  \hspace{1cm} (4.9)

and

$$\Psi[\tilde{h}_{ij}, K_L, \phi_0] = \Psi^*[\tilde{h}_{ij}, -K_L, \phi_0].$$  \hspace{1cm} (4.10)

If we interpret $\rho^{(c)}$ as a functional relative to observers in one of the two asymptotic regions, and $\rho^{(cc)}$ as a functional relative to observers in the other asymptotic region, then Eqns. (4.8) and (4.9) are a statement of $T$ noninvariance for the quantum state of wormholes for independent observers in the regions which are connected by the wormhole. This would imply that, for an observer in one large region, the probability for creating baby universes is not the same as that for destroying them. The wormhole state representation $\Psi$ should be time-symmetric however; for (4.10) contains equal contributions from metrics with a
complex action and from metrics with a complex conjugate action so that the wave function is real.

Moreover, if following Hawking [3], we replace the real scalar field by a complex one, and introduce a triad of covectors on the three-surface, so as a fermion field, one can show that, whereas a pure quantum state of wormholes also is CT and CTP invariant, wormholes describable by a density matrix must be noninvariant under these operations.

There is yet an important property [33] which could modify the extent at which the rhs and lhs of (4.8) and (4.9) are different. It is that the complex conjugate, $\bar{\phi}$, of a scalar field $\phi$ may be replaced by an independent scalar field $\bar{\phi}$ which is not related to $\phi$ by any conjugation. The same applies to spinor fields. However, since such a property would not obviously affect the complex values of the three-metrics and extrinsic curvature tensors, even if it applied to the time-symmetry treatment [3], the above analysis would only be modified in such a way that inequalities (4.8) and (4.9) will still hold.

5 THE TOPOLOGICAL ARROW OF TIME

The demand of locality on quantum fields in the asymptotic regions is made possible by the $T$ invariance of state $\Psi$, and implies that the effective interaction Hamiltonian density in Minkowski space $H_i A_i$ must commute for nonzero spacelike separations. Here, the $H_i$s are scalar-field interaction operators terms, the $A_i$s are baby universe operators which are given in terms of Fock operators [21,34], and $i$ is a discrete index characterizing baby universes. We note now that the quantum states $\rho^{(c)}$ ($\rho^{(oc)}$): (i) are Lorentz invariant because wormhole spacetime is asymptotically flat (Notice that baby universes being branched off and in through wormholes whose quantum state is not factorizable as a product of wave functions are always gravitationally connected to the asymptotic regions), and (ii) have positive definite energy which is given by $\hbar K_{sph}$. Hence, for states $\rho^{(c)}$ and $\rho^{(oc)}$, the lack of $T$, $CT$ and $CTP$ invariance must be addressed [35] to a breakdown of local causality in the baby universe sector,

$$[A_i(x), A_j(y)] \neq 0, (x - y)^2 < 0.$$  \hspace{1cm} (5.1)

From the overall demand of locality, it follows then,

$$[H_i(x), H_j(y)] \neq 0, (x - y)^2 < 0,$$ \hspace{1cm} (5.2)

which, in turn, implies that the set of all possible quantum fields in the large region able to interact with wormholes is not $CTP$ invariant, as far as these fields are Lorentz invariant and have positive energy. Clearly, although $CTP$ invariance appears to be violated in both, the baby universe and low-energy matter subsystems separately, overall demand of locality will ensure $CTP$ invariance for the total, joint system because of its Lorentz invariance and energy positiveness.

Now, as in most situations the effects of any $C$ or $P$ noninvariance can be neglected, (5.2) actually implies noninvariance of the quantum fields in the large
regions under $T$. Since it appears that all quantum fields in the large regions should interact with wormholes, associated with the time-asymmetry of $\rho^{(c)}$ (or $\rho^{(cc)}$) there will be a universal arrow of time which could be named topological because of its origin. That topological arrow, which would indicate the direction in which the two wormhole large regions are created and evolve relative to each other, becomes the basic relevant information contained in the density matrix of wormholes which is accessible to observers in the large regions, and makes $\rho^{(c)}$ and $\rho^{(cc)}$ and their Laplace transforms actual quantum states for wormholes by themselves.

If (5.1) and (5.2) hold, then there will be a topological arrow and also some direction in which entropy increases in the large regions, for these equations imply as well a loss of quantum coherence of the quantum fields at low energy [21,34], and this means a thermodynamical arrow. This universal decoherence process will furthermore induce the creation of bunched or antibunched particle states [36]. Bunched particle states would be expected as a property of incoherent matter whose particles are detected not at randomly distributed times but in the form of clusters or bunches, with a second-order coherence function larger than unity. On the other hand, there could also exist quantum particle states which do not admit any classical description, showing an opposite behaviour [37]. For such antibunched states the probability of detecting a coincident pair of particles is less than that from a fully coherent field with a random Poisson particle distribution: it would be as if the particle detection events had some sort of mutual repulsion.

For zero or very small particle number densities, the second-order coherence function for matter fields can be approximated as [30]

$$g^{(2)}_0 \simeq \exp(-7 \sinh[2(x - y)^2 l_p^2] t).$$

At any time $t > 0$, there will appear antibunched states characterized by non-classical $g^{(2)}_0 < 1$ for given spacelike separations $(x - y)^2 < 0$, and this means increasing time separations between detection events, just as if the sources were radially receding isotropically from any observer.

For large particle densities, the second-order coherence function becomes [30]

$$g^{(2)}_n \simeq \frac{1}{2}(1 + \exp(2 \sinh[2(x - y)^2 l_p^2] t)).$$

It appears that in this case one obtains bunched particles states. Such a behaviour is expected for classical states which evolve from being pure initially into thermal distribution. Here, time separations between detection events tend to decrease because of particle bunching. Therefore, the topological arrow could also imply directions in which the whole large regions expanded or contracted, that is a cosmological arrow, and direction in which some restricted fluctuating regions shrank or expanded, that is directions in which galaxies, galaxy clusters, etc, and voids could develop. Thus, it may be thought that the strong connection [3,38]
between the cosmological and thermodynamical arrows and the electromagnetic and psychological arrows could make the suggested topological arrow the actual cause of all known time directions.

Work on wormholes in the realm of Euclidean quantum gravity is certainly not free from serious problems. Therefore, although we have alleviated our framework of some of these problems, the part of this paper that refers to this subject still has a rather speculative nature.

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LEGEND FOR FIGURE

Fig. 1.- Classical bifurcation itinerary for the potential in Eqn. (3.1) for $m = 0.5$. 
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