Non-trivial Tests of EW Corrections via $\alpha$, $G_F$ and $M_{W,Z}$

Zenrō HIOKI

Institute of Theoretical Physics, University of Tokushima
Tokushima 770, JAPAN

ABSTRACT

The standard electroweak theory is tested at non-trivial quantum correction level through $\alpha$, $G_F$ and the latest data of the weak-boson masses. The improved-Born approximation and the non-decoupling top-quark effects are studied without depending on the CDF data of $m_t$, while the bosonic effects are examined by fully taking account of it.

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$^\ast$) Talk presented at INS Workshop “Physics of $e^+e^-$, $e^-$\gamma and $\gamma\gamma$ collisions at linear accelerators”, Inst. for Nucl. Study (INS), Univ. of Tokyo, Japan, December 20-22, 1994 (to appear in the Proceedings).

$^{**}$) E-mail: hioki@ias.tokushima-u.ac.jp
Many particle physicists now believe that the standard electroweak theory (plus QCD) describes correctly phenomena below \( O(10^2) \) GeV. In fact, there has been observed no discrepancy between experimental data and the corresponding predictions by this theory with radiative corrections. Novikov et al. claimed in [1], however, that the Born approximation based on \( \alpha(M_Z) \) instead of \( \alpha \) ("improved-Born" approximation) explains all electroweak precision data up to 1993 within the 1\( \sigma \) accuracy, where \( \alpha \) and \( \alpha(M_Z) \) are the QED coupling constants at \( m_e \) and \( M_Z \) scales respectively. This means that the electroweak theory had not been tested by that time at "non-trivial" level.

After their work, a new experimental value of \( M_W \) was reported (\( M_W^{\exp} = 80.23 \pm 0.18 \) GeV) [4], and furthermore CDF collaboration at FNAL tevatron collider obtained some evidence on the top quark (\( m_t^{\exp} = 174 \pm 17 \) GeV) [3]. Being stimulated by them, I started to study the present issue, and worked up the results into three papers [4, 5, 6]. At this workshop, I showed the main point of these works.

What I studied is "structure of EW(electroweak) corrections". The EW corrections consist of several parts with different properties, and I examined via \( \alpha \), \( G_F \) and \( M_{W,Z} \) what would happen if each of them would not exist. More concretely, I examined whether the improved-Born approximation still works or not, and then focused on the top-quark contribution which does not decouple, i.e., becomes larger and larger as \( m_t \) increases. It is very significant to test it because the existence of such effects is a characteristic feature of theories in which particle masses are produced through spontaneous symmetry breakdown plus large Yukawa couplings. Furthermore, I also studied the bosonic contribution to the whole corrections. From a theoretical point of view, this is another important test since the bosonic part includes the gauge-boson- and Higgs-boson-loop effects.

Through the \( O(\alpha) \) corrections to the muon-decay amplitude, \( \alpha \), \( G_F \) and \( M_{W,Z} \)
are connected as

\[ M_W^2 = \frac{1}{2} M_Z^2 \left( 1 + \sqrt{1 - \frac{2\sqrt{2\pi\alpha}}{M_Z^2 G_F (1 - \Delta r)}} \right). \]  

(1)

Here \( \Delta r \) expresses the corrections, and it is a function of \( \alpha, G_F, M_Z, m_f \) and \( m_\phi \). This formula, the \( M_W-M_Z \) relation, is the main tool of my analyses. Before proceeding to the actual analyses, let me show by using this formula how the theory with the full corrections is successful, although it is already a well-known fact. The \( W \)-mass is computed thereby as

\[ M_W^{(0)} = 80.941 \pm 0.005 \text{ GeV} \quad \text{and} \quad M_W = 80.33 \pm 0.11 \text{ GeV} \]  

(2)

for \( M_W^{(0)} = 91.1888 \pm 0.0044 \) GeV [3], where \( M_W^{(0)} \) and \( M_W \) are those without and with the corrections respectively, and \( M_W \) is for \( m_t^{\exp} = 174 \pm 17 \) GeV [3], \( m_\phi = 300 \) GeV and \( \alpha_{QCD}(M_Z) = 0.118 \). We can find that the theory with the corrections is in good agreement with the experimental value \( M_W^{\exp} = 80.23 \pm 0.18 \) GeV, while the tree prediction fails to describe it at more than 3.9\( \sigma \) (99.99 \% C.L.).

We are now ready. First, it is easy to see if taking only \( \alpha(M_Z) \) into account is still a good approximation. The \( W \)-mass is calculated within this approximation by putting \( \Delta r = 0 \) and replacing \( \alpha \) with \( \alpha(M_Z) \) in Eq.(1), where \( \alpha(M_Z) = 1/(128.87 \pm 0.12) \) [10]. The result is

\[ M_W[\text{Born}] = 79.957 \pm 0.017 \text{ GeV}, \]  

(3)

which leads to

\[ M_W^{\exp} - M_W[\text{Born}] = 0.27 \pm 0.18 \text{ GeV}. \]  

(4)

\(^{51}\) Strictly speaking, Eq.(1) is not complete: It is a formula based on the one-loop calculations (with resummation of the leading-log terms by the replacement \((1+\Delta r) \rightarrow 1/(1-\Delta r)\)). Over the past several years, some corrections beyond the one-loop approximation have been computed. They are two-loop top-quark corrections [3] and QCD corrections up to \( O(\alpha^2_{QCD}) \) [8] for the top-quark loops. As a result, we have now a formula including \( O(\alpha \alpha^2_{QCD} m_t^2) \) and \( O(\alpha^2 m_t^4) \) effects. In the following, \( M_W \) is always computed by incorporating all of these higher-order terms as well, although I will express the whole corrections with these terms also as \( \Delta r \) for simplicity.
This means that $M_W[\text{Born}]$ is in disagreement with the data now at $1.5\sigma$, which corresponds to about 86.6 % C.L.. Although the precision is not yet sufficiently high, it indicates some non-Born terms are needed which give a positive contribution to the $W$-mass. It is noteworthy since the electroweak theory predicts such positive non-Born type corrections unless the Higgs is extremely heavy (beyond TeV scale). A similar result was obtained also in [1].

The next test is on the non-decoupling top-quark effects. Except for the coefficients, their contribution to $\Delta r$ is

$$\Delta r[m_t] \sim \alpha (m_t/M_Z)^2 + \alpha \ln(m_t/M_Z).$$

(5)

According to my strategy, I computed the $W$-mass by using the following $\Delta r'$ instead of $\Delta r$ in Eq.(1):

$$\Delta r' \equiv \Delta r - \Delta r[m_t].$$

(6)

The resultant $W$-mass is denoted as $M'_W$. The important point is to subtract not only $m_t^2$ term but also $\ln(m_t/M_Z)$ term, though the latter produces only very small effects unless $m_t$ is extremely large. $\Delta r'$ still includes $m_t$ dependent terms, but no longer diverges for $m_t \to +\infty$ thanks to this subtraction. I found that $M'_W$ takes the maximum for the largest $m_t$ and the smallest $m_\phi$. That is, we get an inequality

$$M'_W \leq M'_W[m_t^{\text{max}}, m_\phi^{\text{min}}].$$

(7)

which holds for any experimentally-allowed values of $m_t$ and $m_\phi$.

Although the CDF report on the top-quark is quite exciting, but its final establishment must come after D0 collaboration confirms it. Therefore, I took a conservative position and calculated the right-hand side of the above inequality for $m_t^{\text{max}} \to +\infty$. Concerning $m_\phi^{\text{min}}$, on the other hand, we can use the present experimental bound $m_\phi^{\text{exp}} > 61.5$ GeV [12]. The accompanying uncertainty for
$M'_W$ is estimated at most to be about 0.03 GeV. We have then

$$M'_W < 79.865(\pm 0.030) \text{ GeV and } M^{\exp}_W - M'_W > 0.36 \pm 0.18 \text{ GeV,} \tag{8}$$

which show that $M'_W$ is in disagreement with $M^{\exp}_W$ at more than $2.0\sigma$ (=95.5 \% C.L.). This means that 1) the electroweak theory is not able to be consistent with $M^{\exp}_W$ whatever values $m_t$ and $m_\phi$ take if the non-decoupling top-quark corrections $\Delta r[m_t]$ would not exist, and 2) the theory with $\Delta r[m_t]$ works well, as shown before, for experimentally-allowed $m_t$ and $m_\phi$. Combining them, we are led to an interesting phenomenological indication that the latest experimental data of $M_{W,Z}$ demand, independent of $m_\phi$, the existence of the non-decoupling top-quark corrections. It is a very important test of the electroweak theory as a renormalizable quantum field theory with spontaneous symmetry breakdown.

Finally, let us look into the bosonic contribution. It was pointed out in \cite{13} by using various high-energy data that such bosonic electroweak corrections are now inevitable. I studied whether we could observe a similar evidence in the $M_W-M_Z$ relation. In this case, we have to compute $M_W$ taking account of only the pure-fermionic corrections $\Delta r[f](=\Delta r-\Delta r[\text{boson}])$. Since $\Delta r[f]$ depends on $m_t$ strongly, it is not easy to develop a quantitative analysis of it without knowing $m_t$. Therefore, I used the CDF data on $m_t$. I express thus-computed $W$-mass as $M_W[f]$. The result became

$$M_W[f] = 80.44 \pm 0.11 \text{ GeV.} \tag{9}$$

This value is of course independent of the Higgs mass, and leads to

$$M_W[f] - M^{\exp}_W = 0.21 \pm 0.21 \text{ GeV,} \tag{10}$$

which tells us that some non-fermionic contribution is necessary at 1\sigma level.

It is of course too early to say from Eq.(10) that the bosonic effects were confirmed. Nevertheless, this is an interesting result since we could observe nothing before: Actually, the best information on $m_t$ before the CDF report was the

\footnote{Of course, it is conservative in this case to use the CDF data.}
bound $m_t^{exp} > 131$ GeV by D0 [14], but we can thereby get only $M_W[f] > 80.19$
$(\pm 0.03)$ GeV (i.e., $M_W[f] - M_W^{exp} > -0.04 \pm 0.18$ GeV). We will be allowed therefore to conclude that “the bosonic effects are starting to appear in the $M_W$-$M_Z$
relation”.

We have seen that the standard electroweak theory seems now very happy. Isn’t there any problem in this theory, then? Najima and I pointed out one thing in [3]. I showed that the $W$-mass with the whole corrections for $m_t^{exp} = 174 \pm 17$
GeV and $m_\phi = 300$ GeV is consistent with the data. However, in order for $M_W|_{m_t=174\ \text{GeV}}$ to reproduce the central value of $M_W^{exp}$ (80.23 GeV), the Higgs mass needs to be 1.1-1.2 TeV [3]. Even if we limit discussions to perturbation calculations, such an extremely-heavy Higgs will cause several problems [15, 16]. Moreover, the present LEP and SLC data require a light Higgs boson: $m_\phi \lesssim 300$
GeV [17]. This means that we might be caught in a kind of dilemma.

At present, it is never serious since $m_\phi$ as low as 60 GeV is also allowed if we take into account $\Delta m_t^{exp} = \pm 17$ GeV and $\Delta M_W^{exp} = \pm 0.18$ GeV ($M_W - M_W^{exp} = 0.20 \pm 0.21$ GeV for $m_\phi = 60$ GeV). Still, this definitely shows that more precise measurements of $M_W$ and $m_t$ are considerably significant not only for precision tests of the electroweak theory but also for new-physics searches beyond this theory.

ACKNOWLEDGEMENTS

I am grateful to R. Najima for collaboration in [3], on which a part of this talk is based. I also would like to thank S. Matsumoto for stimulating discussions on the data of $M_W$. 

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