Evolution of superconducting correlations within magnetic-field-decoupled CuO$_2$ layers of La$_{1.905}$Ba$_{0.095}$CuO$_4$

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We explore the evolution of superconductivity in La$_{2-x}$Ba$_x$CuO$_4$ with $x = 0.095$ in magnetic fields of up to 35 T applied perpendicular to the CuO$_2$ planes. Previous work on this material has shown that perpendicular fields enhance both charge and spin stripe order within the planes. We present measurements of the resistivity parallel and perpendicular to the planes, as well as the Hall effect. Measurements of magnetic susceptibility for fields of up to 15 T applied both parallel and perpendicular to the planes provide complementary measures of the superconductivity. We show that fields sufficient to destroy pair tunneling between the planes do not disrupt the superconducting correlations within the planes. In fact, we observe an onset of large amplitude but phase disordered superconductivity within the planes at approximately 30 K that is remarkably insensitive to field. With further cooling, we observe a phase-transition-like drop in the in-plane resistivity to an apparent state of superconductivity, despite the lack of phase coherence between the layers. These observations raise interesting questions concerning the identification of the upper critical field, where pairing is destroyed, in underdoped cuprates.

I. INTRODUCTION

Since Emery and Kivelson first suggested that phase fluctuations might limit the transition temperature $T_c$ of superconductors with low superfluid density, there has been considerable research aimed at determining where and how strong superconducting correlations turn on in cuprates. This issue applies both to the onset of superconductivity on cooling and to the loss of superconductivity in an increasing magnetic field. The topic has both theoretical and practical relevance. On the theoretical side, the question of the onset of pairing correlations might be connected with the pseudogap phenomena in underdoped cuprates. On the practical side, if there are regions of temperature and field where only phase fluctuations limit superconducting order, one might hope to find ways to enhance phase order so as to extend the useful range of superconducting order.

Experimental evidence for superconducting fluctuations at temperatures far above $T_c$ have been provided by Nernst effect and torque magnetometry measurements on a variety of cuprates by Ong and coworkers. This work motivated theoretical suggestions of a possible phase-disordered 2D superconducting state (2D vortex liquid) that might exist above $T_c$. In contrast, recent studies of superconducting contributions to magnetoresistance and low-frequency optical conductivity, as well as further torque magnetometry studies, indicate that strong superconducting correlations are found only within a relatively narrow range ($\sim 10$ K) above $T_c$; the response at higher temperatures is quite weak relative to expectations for a 2D vortex-liquid state.

There is general agreement that pairing interactions within the CuO$_2$ layers are responsible for the development of superconducting correlations in the cuprates. Josephson coupling between the layers leads to the onset of three-dimensional (3D) superconductivity as soon as the correlation length for superconducting order within the layers becomes sizable. The recent observation that strong superconducting correlations appear only in a regime that is reasonably close to $T_c$ is consistent with the expectation that 3D order should appear as soon as 2D superconducting correlations become substantial. In fact, in a previous study of La$_{2-x}$Ba$_x$CuO$_4$ (LBCO) with $x = 0.095$, evidence was found indicating that superconducting correlations between layers start to develop locally before superconducting correlations diverge within the layers. Nevertheless, there remains a question as to whether one might be able to observe the 2D vortex-liquid state by suppressing the interlayer Josephson coupling with a magnetic field applied perpendicular to the layers.

Another question concerns the evolution of superconducting correlations as order is suppressed by a strong magnetic field. For a type-2 superconductor, the initial onset of finite resistivity corresponds to the flow of vortices; destruction of Cooper pairs should occur at a higher field, conventionally labeled $H_{c2}$. Given the large magnitude of the superconducting gap in underdoped cuprates, one might expect $H_{c2}$ to be much larger than the field at which resistivity appears; however, a variety of recent transport measurements on YBa$_2$Cu$_3$O$_{6+x}$ have been interpreted in terms of a rather low $H_{c2}$, especially in the...
behavior reported for LBCO with charge-stripe order, indicates a frustration of the interlayer Josephson coupling state, whether true superconducting order is achieved, as well as in the normal state. Indeed, a specific-heat study indicates that the quantum oscillations do occur within the superconducting mixed state. To explain the measured cyclotron frequency, reconstruction of the Fermi surface by competing order has been invoked. Several recent experiments have provided direct evidence for charge order that is enhanced when superconductivity is depressed by a strong magnetic field. The relationship between the superconductivity, charge order, and normal quasiparticles remains a hot topic of debate.

In this article, we present a study of the superconductor La$_{1.905}$Ba$_{0.095}$CuO$_4$ ($T_c = 32$ K) in strong magnetic fields $H_{\perp}$ applied perpendicular to the CuO$_2$ planes. In previous work, it has been shown that the weak charge stripe order present in zero field is enhanced by $H_{\perp}$. Here we demonstrate that a strong enough $H_{\perp}$ can completely destroy the phase coherence between neighboring layers without destroying the superconducting correlations within the layers. Evidence for the layer decoupling is obtained from measurements of the resistivity perpendicular to the layers, $\rho_{\perp}$, while evidence for the survival of the superconductivity is provided by measurements of magnetic susceptibility, Hall effect, and resistivity parallel to the layers, $\rho_{\parallel}$. We find that the onset of strong superconducting correlations within the decoupled layers occurs at approximately 30 K, with little variation due to $H_{\perp}$ up to our maximum of 35 T. We label this a layered vortex liquid (LVL) state; it is essentially a 2D superfluid liquid state, but there could be electromagnetic interactions between the layers associated with the vortices, resulting in 3D correlations.

Within the LVL state, $\rho_{\parallel}$ has a finite magnitude consistent with that expected for a 2D superconductor without phase order. On cooling in fixed field, $\rho_{\parallel}$ decreases in a fashion suggesting critical behavior similar to that predicted for a 2D superconductor on the approach to the phase-ordering transition of Berezinskii and Kosterlitz and Thouless (BKT). (We note that the theory applies only to the case of zero field.) Following the variation of $\rho_{\parallel}$ with $H_{\perp}$ at fixed temperature, we observe behavior suggesting a transition to a state with negligible $\rho_{\parallel}$ despite an absence of phase coherence between the layers. We label this state a layered, phase-decoupled superconductor (LPD-SC). Our results are summarized in Fig. 1.

The occurrence of the LPD-SC state (regardless of whether true superconducting order is achieved), as well as the complete decoupling of the layers in the LVL state, indicates a frustration of the interlayer Josephson coupling by some mechanism other than thermal vortex fluctuations. Together with the field-enhanced charge-stripe order, there is a clear parallel with behavior reported for LBCO with $x = 1/8$, where LVL and LPD-SC states associated with stripe order were observed in zero field. In that case, the frustration of the Josephson coupling has been explained in terms of a proposed pair-density-wave (PDW) superconductor. The similar phenomenology suggests that field-induced PDW order could be relevant to the $x = 0.095$ sample.

The rest of this paper is organized as follows. The experimental methods are described in the following section. In Sec. III A, we present the resistivity data and analyze the normal-state magnetoresistance. Evidence for the onset of strong superconducting correlations within the CuO$_2$ layers from Hall effect and magnetic susceptibility measurements is presented in Secs. III B and III C, respectively. Analysis of $\rho_{\perp}(H_{\perp})$ and the decoupling of the layers is described in Sec. III D, while the evidence from $\rho_{\parallel}$ measurements for the transition to the LPD-SC
state is given in Sec. III. The paper concludes with a summary and discussion in Sec. IV.

II. EXPERIMENTAL METHOD

The crystals, grown by the traveling-solvent floating-zone method, have been characterized in several previous studies. Most of the present experiments were performed in the 35-T dc magnet at the National High Magnetic Field Laboratory (NHMFL). The crystals for the measurements of $\rho_\parallel$ and $\rho_\perp$ are the same as those used in a previous transport study and the contact configurations are described there. The resistance was measured using an ac resistance bridge with an excitation current of 1 mA. All measurements were done after field-cooling from above $T_c$, sweeping the field from 35 T to 0 while holding the temperature fixed. (Note that sweeping the field at fixed temperature minimizes the energy consumption of the magnet compared to sweeping the temperature at fixed field.)

A third crystal was prepared for measurements of the Hall effect. The geometry was similar to that of the crystal for the $\rho_\parallel$ measurement, except that the voltage contacts were on opposite edges of the crystal in order to measure the Hall voltage $V_H$ in the direction transverse to the current flow. For each measurement, the sample was cooled in zero field from above $T_c$. Once the temperature was stabilized at the desired value the field was swept from 0 to 35 T, back to 0, down to $-35$ T, and back to 0 again. In order to eliminate the magnetoresistance contribution due to imperfect alignment of the voltage contacts, the net Hall voltage was calculated as:

$$V_H = \frac{[V(+B \uparrow) - V(-B \downarrow)]}{2},$$  \hspace{1cm} (1)

where $V(+B \uparrow)$ corresponds to the up-sweep from 0 to 35 T and $V(-B \downarrow)$ corresponds to the down-sweep from 0 to $-35$ T. (We checked that the results were the same using the opposite set of field sweeps.)

The results are expressed in terms of the Hall coefficient $R_H$:

$$R_H = \frac{V_H d}{IB} \hspace{1cm} (2)$$

where $d$ is the sample thickness and $I$ is the longitudinal current. To reduce the noise in the data, the measurements were averaged over windows of 1.75 T in width. Also, in some cases there were anomalous features at low field, so we present the results just for fields above 3.5 T.

Magnetic susceptibility measurements on a fourth crystal were performed with fields of 7 and 15 T using a vibrating sample magnetometer (VSM) located at the IFW Dresden. These data have been compared with previous measurements at fields of 1 and 7 T obtained with a SQUID (superconducting quantum interference device) magnetometer at Brookhaven. The temperature dependence of the different measurements at 7 T are in good agreement, but there are small rigid shifts between the data sets. For presentation, the VSM data have been shifted ($< 0.05 \times 10^{-4}$ emu/mol) to match the SQUID data at 100 K, where the magnetization is linear in the applied field.

III. DATA AND ANALYSIS

A. Normal state and superconducting fluctuations

The results for resistivity vs. $H_\perp$ obtained for a range of temperatures are shown in Fig. 2. Let us first consider the data for $T > T_c$. As discussed by Rullier-Albenque et al., one expects that the in-plane magnetoresistance increases as $H_\perp^2$ in the normal state. Plotting $\rho_\parallel$ versus $H_\perp^2$ in Fig. 3, we see that the expected behavior is approached at high fields. The dashed lines for 50 K and above are fits to the high-field data corresponding to

$$\rho_{\parallel,n} = \rho_{\parallel,n}(H_\perp = 0) + a_\rho(H_\perp^2).$$  \hspace{1cm} (3)

The deviations at low field are attributed to superconducting fluctuations.
\[ \sigma_{SF}(H, T) = 1/\rho_\parallel - 1/\rho_{\parallel,n}. \quad (4) \]

The results are shown in Fig. 5 for a range of temperatures. At high temperatures, \( \sigma_{SF} \) decreases substantially with increasing field. In contrast, there is a distinct change as one develops 3D superconducting correlations at 40 K and below. Not only does \( \sigma_{SF}(H = 0) \) rapidly grow large, but one also observes that the maximum magnetic field is not sufficient to fully suppress \( \sigma_{SF} \).

For comparison, we have plotted \( \rho_\perp \) versus \( H^2 \) in Fig. 6. We see that the magnetoresistance is relatively small at 50 K, and a negative magnetoresistance (at high fields) develops on cooling. For low fields, there is positive magnetoresistance at 40 K and 35 K, indicative of 3D superconducting fluctuations. The change in sign of the magnetoresistance indicates that the 3D superconducting fluctuations are suppressed at high field; at 35 K, the suppression occurs for \( \mu_0 H_c > 20 \) T.

Returning to Fig. 5, we see that \( \sigma_{SF} \) at 35 K drops significantly on applying a relatively small field, corresponding to suppression of the 3D correlations, but it remains substantial at high field. A similar pattern is apparent at lower temperatures, as well. The conductivity due to superconducting fluctuations that survives at high field must occur only within the CuO layers. To approximate the magnitude of \( \sigma_{SF} \), we can compare with the formula obtained by Aslamazov and Larkin for a 2D superconductor according to Eq. (5) from Ref. 17.

Assuming a two-fluid model, we can extract the conductivity due to superconducting fluctuations as

\[ \sigma_{SF}(H, T) = 1/\rho_\parallel - 1/\rho_{\parallel,n}. \quad (4) \]

The fitted parameters \( a_\rho \) and \( \rho_{\parallel,n}(H = 0) \) are plotted in Figs. 4 (a) and (b), respectively. The coefficient \( a_\rho \) is observed to vary as \( T^{-0.5} \). The quantity \( \rho_{\parallel,n}(H = 0) \) varies linearly with temperature for the data from 50 to 100 K. The downward deviation of \( \rho_{\parallel,n}(H = 0) \) at 40 K is correlated with the onset of 3D superconducting fluctuations as demonstrated by Wen et al. To approximate the normal-state behavior, we will use the linear extrapolation of \( \rho_{\parallel,n}(H = 0) \) from the trend at \( T \geq 50 \) K. The extrapolated result at 40 K leads to the dashed line shown in Fig. 3.

Using the extrapolated normal state behavior and assuming a two-fluid model, we can extract the conductivity due to superconducting fluctuations as

\[ \sigma_{SF}(H, T) = 1/\rho_\parallel - 1/\rho_{\parallel,n}. \quad (4) \]

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Using the extrapolated normal state behavior and assuming a two-fluid model, we can extract the conductivity due to superconducting fluctuations as

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superconductor:
\[ \sigma_{SF} = e^2/16d\hbar\tau, \]  
where \( d \) is the thickness of the superconductor and \( \tau = (T - T_c)/T_c \). We take the thickness to be equal to the layer spacing, \( s = 6.6 \, \text{Å} \). The formula was nominally derived for zero field; we will assume that the only impact of the applied field is to reduce \( T_c \), as illustrated in Fig. 1(b). Evaluating the formula for \( T = 30 \, \text{K} \) and \( \mu_0H \geq 20 \, \text{T} \) yields the dashed line shown in Fig. 5, which falls about a factor of two below the data curve. Thus, at 30 K and below, the magnitude of the experimentally-determined \( \sigma_{SF} \) at high fields is larger than the prediction for fluctuation pairing in 2D layers. At higher temperatures, the magnitude of \( \sigma_{SF} \) falls off much faster than predicted by Eq. (5); this is consistent with the conclusion of Rullier-Albenque et al.\[12\] for superconducting fluctuations in the normal state of \( \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \).

**B. Hall coefficient, \( R_H \)**

Before continuing the analysis of the resistivity data to lower temperatures, let us consider the Hall data. The full set of measurements is plotted in Fig. 7. The field dependence of \( R_H \) is relatively small compared to the temperature dependence. To further illustrate this, we compare the temperature dependence of \( R_H \) for fields of 6 and 34 T in Fig. 8. In the normal state, \( R_H \) increases on cooling. It reaches a maximum near 40 K, below which it rapidly drops in magnitude. This drop is independent of field. Very similar behavior has been observed previously for the in-plane thermopower divided by temperature, for fields up to 9 T.\[23\] It appears that this drop is due to the rapid growth of in-plane superconducting correlations. The drop in \( R_H \) is insensitive to the presence of 3D superconducting correlations, as the same the initial drop in \( R_H \) occurs for \( \mu_0H_\perp = 34 \, \text{T} \), where, as we will see, it must be due to superconducting correlations that are restricted to the \( \text{CuO}_2 \) layers.

We also observe that \( R_H \) goes negative at low temperature. Previous studies\[22,26\] of \( \text{LBCO} \) have reported a negative \( R_H \) below \( T_{co} \) for \( x = 0.10 \) and 0.11, but \( R_H \) tends toward zero (without going negative) for \( x = 0.083 \) and 0.12. For our \( x = 0.095 \) sample, the regime of negative \( R_H \) corresponds to the LPD-SC state, where superconducting order appears within but not between the layers.
planes, as indicated in Fig. 1(d). A change in sign from the normal state due to superconducting fluctuations has been predicted theoretically.

C. Magnetic susceptibility

To confirm our analysis of superconducting contributions to $R_H$, we present in Fig. 9 measurements of the magnetic susceptibility in fields up to 15 T. One can see from $\chi(H \parallel ab)$, measured with fields parallel to the planes, that the normal state susceptibility decreases roughly linearly with temperature due to the paramagnetic response of Cu spins. In contrast, there is a growing diamagnetic drop in $\chi(H \perp ab)$ as one cools, especially below $\sim 40$ K. The kink between 30 and 34 K is associated with a structural transition, discussed in Ref. 20, that enables the appearance of weak stripe order, even in zero field.

To extract the diamagnetic response, a linear fit to $\chi(H \perp ab)$ between 80 and 100 K, representing the paramagnetic contribution $\chi_{\text{pm}}$, has been extrapolated and subtracted from the data. Multiplying by the field, we obtain the diamagnetic magnetization, $M_{\text{dia}}$, that is plotted versus temperature for several $H_\perp$ in Fig. 10. We first note that the structural transition has a modest impact on the thermal evolution of the diamagnetism, which continues to grow on cooling below 30 K. More significantly, one can see that $-M_{\text{dia}}$ grows with field for $T \gtrsim 35$ K but decreases with field for $T \lesssim 25$ K. Such behavior is qualitatively consistent with the predicted response of a stack of decoupled superconducting layers with $T_{\text{BKT}} \sim 30$ K. The observed response is also similar to that measured in magnetization studies of La$_{2-x}$Sr$_x$CuO$_4$ with $x = 0.09$ (Ref. 59) and $x = 0.10$ (Ref. 57).

D. Interlayer resistivity, $\rho_\perp(H_\perp, T)$

Let us now consider $\rho_\perp(H_\perp)$ in the superconducting regime, as illustrated back in Fig. 2(a). For $T \lessapprox 30$ K, increasing $H_\perp$ initially causes $\rho_\perp$ to become finite, followed by rapid growth and eventual saturation, followed by a gradual decrease beyond the maximum. Such behavior has been studied previously, especially in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ and Bi$_2$Sr$_{2-x}$La$_x$CuO$_{6+\delta}$. The rise of $\rho_\perp$ with increasing $H_\perp$ is due to suppression of the conduction channel associated with interlayer pair tunneling; on crossing the maximum, single-particle transport dominates. The region of negative magnetoresistance at high field has been attributed to the impact of $H_\perp$ on the pseudogap, reducing the antinodal gap increases the density of normal carriers that can move between planes. Parallels have also been drawn with the field-tuned superconductor-insulator transition observed in disordered thin-films of various metals. By this latter analogy, the resistive transition in $\rho_\perp$ can be viewed as a transition to a Cooper pair insulator phase at high $\mu_0 H_\perp$. In our case, the Cooper pairs are localized along the $c$ axis, becoming restricted to the CuO$_2$ layers.

To emphasize the striking difference between $\rho_\perp$ and $\rho_\parallel$ in an applied field, we compare their temperature dependences in Fig. 11 for $\mu_0 H_\perp = 0, 20$, and 35 T. For $\rho_\perp$, the field appears to shift the superconducting transition to low temperature. In contrast, $\rho_\parallel$ shows a substantial drop near 30 K even in the highest field, and it continues towards zero on further cooling. There is clearly a broad regime in which the superconducting layers are decoupled in terms of coherent Cooper-pair transport.

Now we want to be a bit more quantitative in defining transitions and crossovers. The regime of 3D superconductivity ends when $\rho_\perp$ becomes finite. We label the field...
at which this occurs as $H_{c}^\perp$. Our determination of $H_{c}^\perp$ is indicated by the triangles in Fig. 1(a).

To analyze the growth of $\rho_\perp$ with field, we start with the model of a stack of Josephson junctions between superconducting CuO$_2$ layers. It has been argued by several groups that the field-induced rise in $\rho_\perp$ can be understood in terms of phase fluctuations in the interlayer Josephson junctions due to thermal noise. In this interpretation, the relevant quantity is the extensive resistance per Josephson junction. Hettinger et al. demonstrated empirically that the effective junction area corresponds to $A = \Phi_0 / (B_\perp + B_0)$, where $\Phi_0$ is the flux quantum and $B_0$ is a parameter. Some of us have shown previously that this approach gives a good description of the evolution of $\rho_\perp(T, H_\perp)$ in our sample for $T < T_c$ with $B_\perp = \mu_0 H_\perp$ and $B_0 = 2.2$ T. The effective junction resistance is then $R_\perp = \rho_\perp s / A$, where $s$ is the interlayer spacing (6.6 Å).

According to Halperin et al., the criterion for a Josephson junction to become effectively insulating is that it exceed $R_Q = \hbar / (4e^2) = 6.45$ kΩ, the quantum of resistance for Cooper pairs. We define $H_Q$ as the field

![FIG. 11. (color online) Measurements of (a) $\rho_\perp$ and (b) $\rho_\parallel$ as a function of temperature for $\mu_0 H_\perp = 0$, 20, and 35 T. The dashed lines in (a) are guides to the eye. The solid lines in (b) are calculations of 2D flux-flow resistivity, as discussed in the text. Inset of (b) shows $\Delta \rho_\parallel / \Delta T$ as a function of $H_\perp$ and $T$.](image)

at which $R_\perp = R_Q$; the $T$ dependence of $H_Q$ is shown by the squares in Fig. 1(a). As one can see, the separation between $H_Q$ and $H_c^\perp$ is rather modest.

Looking at Fig. 2(a), it appears that there is a common shape to $\rho_\perp(H_\perp)$ measured at different temperatures. In Fig. 12, we show that $R_\perp / R_Q$ scales as $[(H_\perp - H_c^\perp)/(H_Q - H_c^\perp)]^{\alpha(T)}$, with the $T$ dependence of the exponent $\alpha$ displayed in the inset. The scaling is motivated by a calculation from Konit and for $\rho_\perp$ in a model of weak Josephson coupling between 2D layers; he predicts $\alpha_K = \frac{4}{3}(1 + t)$, with $t = (T_c - T)/T_c$, which is represented by the dashed line in the inset.

### E. In-plane resistivity, $\rho_\parallel(H_\perp, T)$

We have already noted that the Hall effect and susceptibility measurements indicate the onset of strong superconducting fluctuations below 40 K, in a fashion that is surprisingly independent of field. We see related behavior of $\rho_\parallel$ in Fig. 11(b), where the high-field data show a rapid drop at $\sim 30$ K. To emphasize this behavior, the inset of Fig. 11(b) shows the ratio of finite differences $\Delta \rho_\parallel / \Delta T$, as a function of field and temperature. For $\mu_0 H_\perp > 1$ T, one can see that the maximum of this approximate derivative occurs at $30 \pm 2$ K, which we identify as $T_c^{\parallel \text{2D}}$, the onset of the LVL state. The finite resistivity at lower temperature indicates a lack of phase order, as we discuss next.

![FIG. 12. (color online) Plot of $R_\perp / R_Q$ vs. $[(H_\perp - H_c^\perp)/(H_Q - H_c^\perp)]^{\alpha(T)}$, as described in the text, for data at temperatures from 1.6 to 27 K. The dashed straight line is for reference. The $T$ dependence of the exponent $\alpha$ is plotted in the inset. Dashed line represents $\alpha_K$, as described in the text; solid line corresponds to $\alpha = \alpha_K + 0.35 + 4.5t^2$, with $t = (T_c - T)/T_c$, and $T_c = 31.5$ K.](image)
BKT predicted that, in a 2D system with vortex-like excitations, it is possible to have a topological transition from an ordered to a phase-disordered state. The transition can be described as an unbinding of thermally-excited vortex-antivortex pairs. The nature of the transition depends crucially on having the interaction energy of a pair of vortices vary logarithmically as their separation distance. For a superconductor, the logarithmic interaction applies only at distances shorter than the magnetic penetration depth; at larger distances it is screened. In a thin film, it is possible to enhance the effective screening length but this can still be smaller than the sample size. Attempts to observe BKT transitions in thin films have been controversial and there have been analyses showing that effects near the edges of a thin film could give the appearance of BKT behavior even when it is absent in the bulk of the film.

This history would make it appear that consideration of BKT-like effects in our bulk sample would be inappropriate. It turns out, however, that the presence of many adjacent, phase-decoupled layers restores the conditions necessary for a BKT-like transition. Raman, Oganesyan, and Sondhi have shown that, due to the electromagnetic interactions of pancake vortices in different layers, the interaction energy between vortices remains logarithmic to long distances. They find that the system does exhibit a phase disordering transition, though there are small quantitative corrections relative to the predictions of the 2D theory. This analysis provides an explanation for the BKT-like transition observed in LBCO $x = 1/8$ in zero field. 

With that context, let us consider Fig. 13 where we plot $\rho_{||}(H_{\perp})$, normalized to the normal-state $\rho_{n}(H_{\perp})$ evaluated in Sec. III A for a number of temperatures. At each temperature, there is a phase-transition-like rise in the normalized resistivity as $H_{\perp}$ increases. To make an initial estimate of the transition field, which we label $H_{c}\perp$, we choose the point at which the $\rho_{||}/\rho_{n}$ reaches $10^{-3}$, as indicated by the squares superimposed on the data in Fig. 13. The obtained transition fields correspond approximately to the circles plotted in the phase diagram Fig. 1(b). (The corrected values of $H_{c}\perp$ are discussed below.) Comparing with Fig. 1(a), we see that $H_{c}\perp > H_{Q}$ for any $T$, so the apparent transition occurs in the regime where there is no coherent Josephson tunneling between layers. Thus, on cooling in a field of $\mu_{0}H_{\perp} \gtrsim 1$ T, we see behavior consistent with a BKT-like transition from the LVL state to the LPD-SC state.

To go further, we test the functional form of critical behavior of the resistivity. Above a BKT transition at $T_{\text{BKT}}$, the resistivity, which is proportional to the product of the density of free vortices and the vortex mobility, is predicted to have the form

$$\frac{\rho_{||}}{\rho_{n}} = a e^{-b/T^\gamma},$$

where $\gamma = (T - T_{\text{BKT}})/T_{\text{BKT}}$, $a$ and $b$ are constants of order one, and $\gamma = 0.5$. In the mixed state, the applied $H_{\perp}$ will increase the density of free vortices, and hence will increase the resistivity. In a study of 2D-like superconductivity in LBCO $x = 1/8$, it was found empirically that $\rho_{||}(T)$ in a field could still be described by Eq. (6), provided that one takes account of the reduction of $T_{\text{BKT}}$ by the field. Theoretically, one does not expect $T_{\text{BKT}}$ to remain finite in the mixed state of a 2D superconductor, however, our system is never truly 2D. Perhaps the electromagnetic interactions between pancake vortices in neighboring layers, considered by Raman et al. in the zero-field limit, are sufficient to maintain a finite $T_{\text{BKT}}$ in large $H_{\perp}$.

Without theoretical justification, we take Eq. (6) as a useful functional form. We use it to model the field-dependent data of Fig. 13 by inserting the field dependence through $T_{\text{BKT}}(H_{\perp})$; the latter corresponds to $H_{c}\perp(T)$, which we have already estimated. Of course, our estimates $H_{c}\perp$ were determined at $\rho_{||}/\rho_{n} = 10^{-3}$; we need to adjust these values for the finite cutoff. Empirically, we find that an effective one-parameter formula for the correction is $\mu_{0}H_{\perp} = \mu_{0}H_{c}\perp - C_{i}/T$ with $C_{i} = 44.8$ T K. Each $H_{c}\perp$ value corresponds to a particular temperature, which can be viewed as $T_{\text{BKT}}$ for that field value. We fit $T_{\text{BKT}}(H_{\perp})$ with a cubic polynomial in $H_{\perp}$. Plugging these values into Eq. (6), we obtain the curves indicated by the solid (black) lines in Fig. 13 using the fixed set of parameters ($a = 2.5, b = 1.5, \gamma = 0.6$). We see that the data are reasonably well described simply by accounting for the variation in transition temperature with field.

We can use the same formula and parameters to describe the temperature dependence of the resistivity in the LVL phase at fixed field. The solid lines through the data points in Fig. 11(b) correspond to such calculations, with $T_{\text{BKT}} = 16.5$ and 12.5 K for $\mu_{0}H_{\perp} = 20$ and 35 T.
Along with other recent work \textsuperscript{14,15} T > T explain features of the pseudogap phase at \( \rho \) like drop in conducting state. This is detected through a transition-apparent transition to a layered, phase-decoupled superconducting susceptibility. With further cooling, there is an apparent transition to a layered, phase-decoupled superconducting state. This is detected through a transition-like drop in \( \rho_{\parallel} \); the phase decoupling is clear from the behavior of \( \rho_{\perp} \).

A vortex-liquid state has previously been proposed to explain features of the pseudogap phase at \( T > T_{dx} \). \textsuperscript{11,17} Along with other recent work \textsuperscript{12-15}, our results suggest that such a scenario is overly optimistic. The rise of \( \rho_{\parallel} \) on warming through \( T_{c2D}^\parallel \) indicates a loss of uniform superconducting correlations in the normal state. \( T_{c0} \) is slightly larger than \( T_{c2D}^\parallel \), suggesting that, upon cooling in zero field, 3D order develops just before the individual layers would become superconducting in the absence of Josephson coupling, as others have observed in Bi\textsubscript{2}Sr\textsubscript{2}CaCu\textsubscript{2}O\textsubscript{8+\delta}. \textsuperscript{79,81}

The LVL state that we observe here only at finite \( H \parallel \) is equivalent to the state previously detected in LBCO \( x = 1/8 \) below 40 K in zero field. \textsuperscript{82} We compare the phase diagrams of these two compositions in Fig. 14. In both cases, the LPD-SC state is reached at lower temperature. The observation of such a state is only possible when the interlayer Josephson coupling is frustrated. We note that one proposed origin of the frustration is the development of pair-density-wave (PDW) order in association with charge and spin stripe order. \textsuperscript{42,43} Neutron and x-ray diffraction measurements on LBCO \( x = 0.995 \) have demonstrated that both charge and spin stripe order are enhanced by \( H \parallel \) for \( T \lesssim T_{c2D}^\parallel \) (Refs. \textsuperscript{84} and \textsuperscript{30}); however, it should be noted that, while the occurrence of PDW order would explain the loss of 3D superconducting order, it would not, by itself, explain the apparent stability of the LPD-SC state in large \( H \parallel \).

The modulated pair wave function of the PDW state provides a way for the superconductivity to coexist with local antiferromagnetic order: the superconducting wave function has zeros at the extrema of the spin density wave, and vice versa. This is consistent with experimental evidence that long-range commensurate antiferromagnetic order and superconductivity do not coexist in LBCO or La\textsubscript{2−x}Sr\textsubscript{x}CuO\textsubscript{4}. \textsuperscript{82,83} At the same time, the modulation makes the PDW state quite sensitive to disorder, consistent with the strongly depressed bulk \( T_{c} \) in LBCO \( x = 1/8 \). For \( x = 0.995 \), the superconducting order develops at much higher \( T_{c0} \), and \( T_{c2D}^\parallel \) is virtually independent of \( H \parallel \) even for \( \mu_{0}H_{\perp} \) as high as 35 T. The stability indicates that there are at least quantitative differences from \( x = 1/8 \).

One possible interpretation of the field-independence of \( T_{c2D}^\parallel \) is that this crossover is determined by competition between different correlations. For example, Emery, Kivelson, and Zachar\textsuperscript{85} originally proposed that superconductivity in a stripe-ordered system would involve in-phase Josephson coupling between neighboring charge stripes. In contrast, the PDW state is proposed to have antiphase coupling. \textsuperscript{82,83} Before relative phase order is established, there may be a competition between the interactions that favor in-phase vs. anti-phase coupling. In this scenario, strong pairing correlations would exist within the fluctuating charge stripes at \( T > T_{c2D}^\parallel \), but they would have only a weak impact on measurable quantities. The onset of interstripe phase coherence might be relatively insensitive to \( H \parallel \). Theoretical analysis is necessary to determine whether this speculation is realistic. We note that there is an empirical correlation between the onset of spin-stripe order and \( T_{c0} \) in several cuprates, \textsuperscript{80,85} with the onset of spin-stripe order exhibiting minimal dependence on \( H \parallel \). This suggests that the correlations within spin stripes can impact the development of superconducting phase order, in samples with varying types of superconducting correlations.

We noted in Sec. \textsuperscript{11} that the the temperature dependence of \( \rho_{\perp} \) in large \( H \parallel \) displayed in Fig. 11 with a rapid rise and gradual fall off with increasing \( T \) has been seen previously in other cuprates, \textsuperscript{83,87} There have also been studies of \( \rho_{\perp} \) in underdoped YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{6+x} for transverse fields up to 60 T by Vignolle \textit{et al.}\textsuperscript{89} They have been able to measure into the regime where \( \rho_{\perp} \) remains finite and large down to 2 K. Assuming that they have reached the normal state, they make a correction for normal-state magnetoresistance and obtain re-
sults suggesting a crossover to coherent c-axis conduction at low temperature. While we have not measured to such high fields, we nevertheless have observed a regime in which $\rho_\perp$ exhibits an insulator-like temperature dependence while superconducting correlations are present within the planes. When there is a metallic-like temperature dependence of $\rho_\perp$, it is due to superconductivity within the layers. While Vignolle et al. have provided self-consistent arguments to support their identification of normal state behavior at high-fields in YBa$_2$Cu$_3$O$_{6+x}$, we suggest that the possibility of hidden superconducting correlations within the CuO$_2$ bilayers that begin to impact $\rho_\perp$ at sufficiently low temperature should be considered as a possible alternative explanation.

Another intriguing observation in YBa$_2$Cu$_3$O$_{6+x}$ at high field and low temperature is the negative value of $R_H$, with a magnitude even larger than at $T > T_c$. This behavior has been interpreted as evidence for electron pockets associated with the high-field normal state. While we have also observed a regime of negative $R_H$ in our LBCO $x = 0.095$ sample, the magnitude of $R_H$ is much smaller. The correspondence of this regime with the LPD-SC state, as indicated in Fig. 1(d), suggests in our case that it may be associated with superconducting fluctuations.

Ramshaw et al. have recently made the interesting observation for YBa$_2$Cu$_3$O$_{6+x}$ that, at low temperature, the magnetic field at which $\rho_\perp$ becomes finite, which they label $H_{c2}$, is a minimum for a hole concentration of $\sim 0.12$. It appears that LBCO also follows this pattern, and it will be interesting to see whether other cuprate families follow it. In terms of notation, the field at which the resistivity becomes finite is more commonly labelled as the irreversibility field, $H_{ir}$; $H_{c2}$ should correspond to the field at which pairing is completely eliminated. For underdoped LBCO, at least, our results suggest that $H_{c2}$ cannot be readily determined from measurements of $\rho_\perp$. Even when measurements sensitive to in-plane correlations are made, it appears that $H_{c2}$ for LBCO $x = 0.095$ is quite large for temperatures at all the way to $T_{c0}$.

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