Phase and periodicity of Aharonov-Bohm oscillations: effect of channel mixing

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Abstract

We take a negative delta function impurity in one arm of a quasi one dimensional Aharonov-Bohm ring and demonstrate abrupt phase changes across a quasi bound state of the negative delta function potential. We give a new mechanism for conductance oscillations with the strength of the negative delta potential. We also show that coupling to evanescent modes can result in a $\hbar c/2e$ flux periodicity at certain energies. These observations were made in a recent ingenious experiment by D. Mailly et al [1].

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In a recent experiment [1] the conductance oscillations of an Aharonov-Bohm ring with a gate voltage in one of the arms is studied. The main results of the experiment can be stated as follows. As the gate voltage is varied the phase of Aharonov-Bohm oscillations undergo abrupt phase changes of $\pi$. Also at some gate voltages the Aharonov-Bohm oscillations show $\phi_0/2$ periodicity, where $\phi_0 = h/e$. The conductance is found to oscillate with the gate voltage and this is believed to be not due to the fact that the gate voltage changes the wave vector under the gate. A theoretical calculation is also presented [1] that shows such abrupt phase changes. A phase difference of $\pi$ between successive channels added to the wavefunction of electrons as it passes the gate in one of the arms produces such phase changes as well as the $\phi_0/2$ periodicity in the theoretical calculation. Such a phase difference of $\pi$ is an assumption. If the phase difference is made random instead of $\pi$, the agreement is not good. But what is completely neglected in the theoretical calculations is the fact that the gate voltage can lead to mixing of the transverse modes.

In this work we show that in absence of such phase randomization, mixing of transverse modes alone can give rise to the results observed in the experiment. In our calculations, in order to demonstrate this we put a negative delta function potential in the upper arm of the ring. We chose the Fermi energy such that only one mode is propagating and the others are evanescent. We consider only one such evanescent mode but the effect of other evanescent modes will be included in the end. The negative delta function potential in 1D has only one bound state. But in a quasi 1D wire with 2 transverse channels, it has 2 bound states, one belonging to each channel. The separation between these bound states is therefore same as that of the channel quantization. The bound states that lie below the propagating threshold are true bound states. Whereas if the bound state of the second transverse channel is degenerate with the first subband scattering channel, then it is a quasi bound state [2]. When incident energy matches with this quasi bound state we get a Fano resonance. Such resonances are characterized by a zero pole pair[3]. Across the zeros of this resonance the phase of Aharonov-Bohm oscillations will change by $\pi$. We also give a completely new mechanism of conductance oscillations with the strength of the delta
potential. A delta function potential cannot modify the wave vector. Besides we show that
coupling to evanescent modes can result in $\phi_0/2$ periodicity of conductance oscillations.

A negative gate voltage in a small region narrows the thickness of the channel over this
small region. But due to box quantization quasi bound states are formed in this narrow
region [4]. Now such a quasi bound state belonging to one subband can be degenerate with
a scattering state of another subband because although their energy is the same, their wave
vectors will depend on the zero point energy of the respective subbands, and can be very
different. And then we can observe the same phenomenon as that observed with our negative
delta potential. As the gate voltage is changed it changes the dimension of the narrow region,
thus shifting the resonances in energy. If the length of the gate is small then the separation
between the resonances in the beginning will be same as that of channel quantization. Besides
impurities under the gate voltage can behave as a negative delta function potential of slowly
varying strength. In fact any bound state in the system due to some reason, under the gate
can cause the effects reported here.

A schematic diagram of the Aharonov Bohm ring with a delta function potential at site
X is shown in fig. 1. Various regions and various length parameters are also defined in
the figure. Except for the delta function potential the potential inside the system is zero
everywhere. Distance of the delta potential from the central line between the inner and outer
radius of the ring is $y_i$. At the edges of the system the potential is infinite which enforces
hard wall boundary conditions at the edges of the system. The widths of the quantum
wires making up the system is $w$. Scattering properties and bound states of a delta function
potential in a quasi 1D wire has been solved by Bagwell [5]. Since along the width of the
wires the system has perfect symmetry, different subbands will never mix, except at the
delta function potential. Some amount of mixing can be there at the junctions of the leads
but that will not cause any additional effect because the scattering effects of junctions can
be included in the scattering effects of the delta function potential (see eqn 2 in Ref. [1]). So
the evanescent mode in the lower arm of the ring and those in the leads drop out from our
calculations. Since only two subbands of opposite parity are considered here, it also drops
out in the upper arm if the delta function potential is situated at the center along the width of the upper arm, i.e., \( y_i = 0 \). So by shifting the position of the delta function potential in the upper arm we can include or exclude the role of channel mixing. The solutions of Schrodinger equation, in the absence of magnetic field, in the various regions are written down below. The choice of coordinates and origin of coordinates is easy to understand but the conventions are the same as that in ref [6] and [7]. We have set \( \hbar = 2m = 1 \).

\[
\psi_{pI}(x,y) = \xi_1(y)(e^{ikx} + R e^{-ikx}) \tag{1}
\]

\[
\psi_{pII}(x,y) = \xi_1(y)(A e^{ikx} + B e^{-ikx}) \tag{2}
\]

\[
\psi_{eII}(x,y) = \xi_2(y)(C e^{-qx} + D e^{qx}) \tag{3}
\]

\[
\psi_{pIII}(x,y) = \xi_1(y)(E e^{ikx} + F e^{-ikx}) \tag{4}
\]

\[
\psi_{eIII}(x,y) = \xi_2(y)(G e^{-qx} + H e^{qx}) \tag{5}
\]

\[
\psi_{pIV}(x,y) = \xi_1(y)(J e^{ikx} + K e^{-ikx}) \tag{6}
\]

\[
\psi_{pV}(x,y) = \xi_1(y)T e^{ikx} \tag{7}
\]

Here the superscripts ‘\( p \)’ and ‘\( e \)’ stands for ‘propagating’ and ‘evanescent’ modes, respectively. Also here \( \xi_n(y) = \sin \frac{\pi n}{w} (y + w/2) \), \( k = \sqrt{En - E1} \) and \( q = \sqrt{E2 - En} \), where \( w \) is the width of the quantum wires, \( En \) is the incident energy, \( E1 = (\pi/w)^2 \) is the propagating threshold of the first subband and \( E2 = (2\pi/w)^2 \) is the propagating threshold of the second subband. We use the three way splitter of ref [8] to match boundary conditions at the junctions \( J_1 \) and \( J_2 \), and the formalism of Bagwell to match the wave functions across the delta function potential. Magnetic field is treated in the way described in ref [6]. The expression for transmission coefficient across the whole system is too long to be produced here. However
below we give an analytical expression for the phase $\theta$ of transmission amplitude across a
delta function potential in a quantum wire with one propagating mode and one evanescent
mode in the wire.

$$\theta = \arctan \left[ \frac{-q\gamma \sin^2 \frac{\pi}{w}(y_i + w/2)}{k(2q + \gamma \sin^2 \frac{2\pi}{w}(y_i + w/2))} \right]$$

(8)

Here $\gamma$ is the strength of the delta function potential. It must be noted that this expression is valid only if $y_i \neq 0$ or else the evanescent mode gets decoupled from the propagating mode and the problem reduces to that of a delta potential in 1D. The condition for the bound state of the second subband for the delta function potential is given by

$$(2q + \gamma \sin^2 \frac{2\pi}{w}(y_i + w/2) = 0$$

(9)

and when the incident energy satisfy this condition $\theta$ abruptly changes by $\pi$. Such abrupt phase changes may have two fold consequences. It can lead to violation of Leggett’s conjecture [9] and can also lead to abrupt phase changes of conductance oscillations. To check if it leads to violation of Leggett’s conjecture one has to check if there are any discontinuities in $\text{Real}[1/T]$ [9]. One can check that $\text{Real}[1/T]=1$ for all energies and so there is no discontinuity. However the above mentioned phase $\pi$ does cause an abrupt change in the phase of conductance oscillations. We choose $w$ as the unit of length and all other length units are scaled by this. All energies are also scaled by $1/w^2$. So from now on we will only give their numerical values without mentioning the units. If we choose $l_1 = l_2 = .25$, $l_3 = .5$, $\epsilon = 4/9$, $\gamma=-13.968446982793$ and $y_i = w/3$ then the bound state occur at $E_n=12$ which is in between $E_1$ and $E_2$. In fig. 2 we show that conductance oscillations with $\alpha = 2\pi\phi/\phi_0$, where $\phi$ is the flux through the ring, at $E_n=12-.1$ and $E_n=12+.1$ are of opposite phases. This happens for infinitesimal $y_i$ or infinitesimal mixing of modes. The conductance oscillations are so small in amplitude because we are very close to a zero in the transmission coefficient of the upper arm. For all $E_n$ on the higher (lower) side of this value the conductance oscillations are in phase. Hence when every subband has a quasi bound state separated by the typical channel quantization value, oscillatory behavior in conductance with $\gamma$ is a straightforward conclusion.
We also find another striking feature of this coupling to evanescent modes. In absence of any mode coupling, i.e., when the system reduces to a 1D system, the conductance oscillations have a $\phi_0$ periodicity. Most of the time they exhibit one maxima(or minima) at $\alpha=0$ and one minima (or maxima) at $\alpha = \pi$. But then at some energies additional minima can occur due to the resonances of the ring [11]. But generally these additional minima are very different in strength (or amplitude) compared to the others. These diversities are typical features of ballistic systems that are extremely sensitive to boundary conditions. Coupling to evanescent modes smooths out these diversities by reducing sensitivity to boundary conditions. We find a very general feature of the coupling to evanescent modes is that it makes the oscillations of comparable strengths. And this in turn gives the conductance fluctuations the appearance of $\phi_0/2$ periodicity. A typical example of this is shown in fig 3. The dotted curve is plotted for $l_1 = l_2 = .25, l_3=.5, y_i = 0, En = 26.5, \gamma = -13.968446982793$ and $\epsilon = 4/9$. Whereas the solid curve is plotted for the same parameters except that $y_i = w/2.5$. So for the dotted curve there is no coupling to evanescent modes whereas for the solid curve there is. The solid curve has almost exact $\phi_0/2$ periodicity. If we move away from this incident energy then again the periodicity slowly changes. Given a position of $y_i$ one can play with the incident energy to find a value where the same happens. Or else one can keep incident energy fixed and vary $\gamma$ to get it. They are equivalent. Another set of parameter values for which this happens are mentioned below. If $l_1 = l_2 = .25, l_3=.5, \gamma=-13.968446982793, \epsilon=4/9$ and $y_i = w/3.5$ then this happens around an energy of $En = 24.5$.

The effect of other evanescent modes in the system will be to renormalize the strength of coupling between the two subbands considered here, making channel mixing stronger, apart from giving rise to more quasi boundstates that are separated by the typical channel quantization value. The scheme of renormalization is given in ref 5.

Hence most of the experimental observations are obtained in a simple model by considering existence of quasi bound states and coupling to evanescent modes. It is not necessary to assume a phase difference of $\pi$ between successive channels. It also shows a new mechanism for oscillatory behaviour with gate voltage. The study also justifies that the phase changes
in conductance oscillations obtained by Yacoby et al [12] are similar and arise due to elastic scattering and Fano resonances associated with the special geometry in that case [13].

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FIGURE CAPTIONS

Fig. 1 A quasi one dimensional Aharonov-Bohm ring with a delta function potential at site marked X in the upper arm of the ring. A magnetic flux penetrates the ring.

Fig. 2 Plot of transmission coefficient versus $\alpha = 2\pi\phi/\phi_0$ for parameter values described in the text, for two values of incident energies on opposite sides of a quasi bound state.

Fig. 3 Plot of transmission coefficient versus $\alpha = 2\pi\phi/\phi_0$ with no channel coupling (dashed line) and with channel coupling (solid line).
transmission coefficient

\[ \text{transmission coefficient} \]