Steering Control of Dual Electric Drive Tracked Vehicle Based on Model Predictive Control

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Abstract. A model predictive control (MPC) algorithm is designed to solve problem in the steering control of dual electric drive tracked vehicle. Firstly, the driver manipulated model is constructed after analyzing tracked vehicle dynamics. Based on which, the predictive control of vehicle speed and yaw rate is achieved by establishing predictive model, rolling optimization, and correcting feedback. Finally, co-simulation model of vehicle multi-body dynamics and steering control is constructed to simulate the steering conditions of variable speed and variable radius. The simulation results show that the MPC algorithm can quickly and accurately track the target speed and yaw rate, and the output motor torque is smooth. Which can realize the stable steering of vehicles under various working conditions.

1. Introduction

With the advantages of good maneuverability, engine-off operation ability, flexible spatial layout and low fuel consumption, electric drive tracked vehicles have gradually become an important research and development direction for future tracked vehicles [1, 2]. In the dual electric drive scheme, the track movements are independent of each other. The vehicle motion presents the characteristics of nonlinearity and strong coupling due to the dramatic changes of the road structure, parameters and the involvement of the vehicle body [3]. Therefore, effective coordinated control must be performed on the dual motor drive system to achieve the desired motion performance.

Recently, some research results on the steering control have been proposed for dual electric drive tracked vehicles. Zou proposed electronic differential control and direct torque control according to the characteristics of the driving scheme. The simulation results under different working conditions show the effectiveness of the algorithm [4-6]. For electronic differential control, the driving motor cannot track the target speed quickly when vehicle load changes in a wide range, which will result in unstable power output and poor driver manipulated experience. Direct torque control adopted the open-loop control structure, which will increase working strength of driver manipulation and cannot guarantee steering trajectory accurately.

To this end, a model predictive control (MPC) algorithm is proposed in this paper to achieve accurate and stable tracking for vehicle target speed and yaw rate, based on dynamic model of dual electric drive tracked vehicle. The effectiveness of the MPC algorithm is verified by constructing the co-simulation model of controller and multi-body dynamics.
2. Vehicle scheme and dynamics analysis
The scheme of dual electric drive system is consist of engine, generator, super capacitor, power battery, DC-DC converter, motors, steering controller etc. The ground reference coordinate system \( XOY \) and the vehicle body reference coordinate system \( xCy \) are established with point \( O \) and vehicle center \( C \) as the origin respectively. The stable steering force condition of tracked vehicle is shown in Figure 1.

![Figure 1. Steering Force Condition of Tracked Vehicle](image)

\( L \) represents the connection length of the track, \( B \) represents the distance between dual tracks, \( F_r \) and \( F_r' \) represent the driving force of sprocket on both sides respectively, \( F_\beta \) and \( F_\beta' \) represent rolling resistance of track on both sides respectively, \( M_\mu \) represents the steering resistance torque, the point \( O' \) represents the instantaneous steering center, \( R \) represents the turning radius, \( v \) and \( \omega \) represent the longitudinal speed and the yaw rate respectively.

According to the theory of vehicle dynamics, the dynamic model of tracked vehicle is obtained as follows [7]:

\[
\begin{aligned}
F_l + F_r - F_\beta - F_r' - F_s - F_w &= m\dot{v} \\
\frac{(F_r - F_r') B}{2} - M_\mu - M_\lambda &= I_\omega \dot{\omega}
\end{aligned}
\]  

(1)

Where \( F_s \) represents the longitudinal resistance opposite to the driving direction of the vehicle when climbing, \( F_w \) represents air resistance, \( M_\lambda \) represents the change of the steering resistance torque, which is mainly caused by the lateral deviation of the instantaneous steering center in working condition of lateral slope, longitudinal slope steering and high speed. \( M \) represents the weight of the vehicle, \( I_\omega \) represents the moment of inertia of vehicle. The equations of relevant forces and torques can be expressed as follows:

\[
\begin{aligned}
F_l &= \frac{T_{e,\omega} i}{r} \\
F_\beta &= 0.5 f m g \\
F_s &= m g \sin \alpha \quad M_\mu = \frac{\mu m g L}{4} \\
F_w &= C_D A v^2 / 21.15 \\
M_\lambda &= M_\mu \left(\frac{2 \lambda}{L}\right)^2
\end{aligned}
\]  

(2)
Where \( i \) represents reduction ratio, \( r \) represents the radius of sprockets, \( T_i \) represents output torque of the left and right motors, and \( a \) represents the front area of tracked vehicle. \( C_o, f \) and \( \mu \) represent air resistance coefficient, rolling resistance coefficient and steering resistance coefficient respectively. \( \rho \) represents the relative steering radius, \( \alpha \) represents the gradient of road.

### 2.1. Manipulated Signal Analysis

To achieve vehicle control goals such as vehicle speed and yaw rate, the manipulated signal should be interpreted according to the driver actual manipulated habits. It is defined that any angle after the accelerator pedal is pressed corresponds to the target vehicle speed, and the vehicle speed has a linear relationship with the increase of the pedal angle. Its signal is interpreted as follows:

\[
v^* = \frac{\phi - \phi_0}{\phi_{\text{max}} - \phi_0} v_{\text{max}} \quad (\phi_0 \leq \phi \leq \phi_{\text{max}})
\]

Where \( \phi, \phi_0, \phi_{\text{max}} \) represent the real-time angle displacement, free travel angle displacement and maximum displacement of the accelerator pedal respectively, which \( v_{\text{max}} \) represents the maximum speed.

The signal of the steering wheel is interpreted as the target turning radius of the vehicle. Taking the right turning as an example, the target turning radius corresponding to the steering angle is \( R^* \). According to vehicle manipulated habits, the relationship between steering angle and the target turning radius is nonlinearity. The corresponding target turning radius is large when the steering angle is small, and the corresponding turning radius is small when the steering angle is large. In practice, a large number of experiments are needed to calibrate the relationship between the steering angle and the target turning radius. Considering the boundary conditions and the actual requirements, the signal of the steering wheel steering signal can be interpreted as follows[8]:

\[
\begin{align*}
R^* &= k \log_{0.5} (-\psi - 5) + 250 (-\psi_{\text{max}} \leq \psi < -\psi_0) \\
R^* &= \infty \quad (-\psi_0 \leq \psi \leq \psi_0) \\
R^* &= k \log_{0.5} (\psi - 5) + 250 (\psi_0 < \psi \leq \psi_{\text{max}})
\end{align*}
\]

Where \( k = -249.0075 / \log_{0.5} 80 \). According to the calculation equation of turning radius, yaw rate of the target can be obtained as follows:

\[
\omega^* = v^*/R^*
\]

### 3. Design of model predictive controller

According to the formula (1) and (2), the dynamic equation of the vehicle is rewritten as the system state space expression as follows:

\[
\left( \begin{array}{c}
\dot{v} \\
\dot{\omega}
\end{array} \right) = \left( \begin{array}{c}
i \\
-i
\end{array} \right) \frac{m r}{i B} \left( \begin{array}{c}
f_i \\
-f_i
\end{array} \right) + \frac{m}{2I_z} \left( \begin{array}{c}
f - F_w - F_z \\
-M_{\mu} - M_{\Delta}
\end{array} \right)
\]

The steering control of electric drive tracked vehicle has high requirements for the accurate tracking of vehicle speed and yaw rate, as well as the ability to overcome external disturbances such as road parameters. MPC can predict the output of a certain time in the future based on historical information and
future input of the system [9, 10], which is suitable for solving the problem of constrained multivariable control. The double input and double output (DIDO) nonlinear system shown in the formula (6) can obtain the optimal motor torque on both sides through modeling prediction, rolling optimization and correcting feedback.

3.1. Establishment of Prediction Model
Firstly, the nonlinear system should be linearized. Define \( x = [v \quad \omega]^T \), \( u = [r \quad \tau]^T \), the error between the control target and the actual state is \( \tilde{e} = x - x^* \), and the error between the control input and the actual input is \( \tilde{u} = u - u^* \). Then formula (6) can be rewritten as follows:

\[
\dot{x} = f(x, u) = f_1(x) + Gu
\]  

(7)

After expanding formula (7) by the Taylor formula at the reference target point and ignoring higher-order terms, the equation (7) can be set as the following form:

\[
\dot{x} = f(x^*, u^*) + \frac{\partial f}{\partial x}(x - x^*) + \frac{\partial f}{\partial u}(u - u^*) = \dot{x}^* + \frac{\partial f}{\partial x} \tilde{e} + \frac{\partial f}{\partial u} \tilde{u}
\]  

(8)

The derivation of \( \tilde{e} \) is:

\[
\dot{\tilde{e}} = \dot{x} - \dot{x}^*
\]  

(9)

According to formulas (8) and (9), the new state space equation of system error is set as follows:

\[
\dot{\tilde{e}} = A\tilde{e} + B\tilde{u}
\]  

(10)

Where \( A = \frac{\partial f}{\partial x} \), \( B = \frac{\partial f}{\partial u} \). In order to apply the system error model to the design of model predictive controller, the forward Euler method is used to discretize the formula (10):

\[
\dot{\tilde{e}} = (\tilde{e}(k + 1) - \tilde{e}(k)) / T
\]  

(11)

Where \( T \) represent sampling time. According to formulas (10) and (11), the linear time-varying discrete state space equation of the system is obtained as follows:

\[
\tilde{e}(k + 1) = A(k)\tilde{e}(k) + B(k)\tilde{u}(k)
\]  

(12)

Where \( A(k) = I + TA \), \( B(k) = TB \).

To comprehensively consider the system status and control input, define \( \tilde{\xi}(k) = [\dot{\tilde{e}}(k) \quad \tilde{u}(k - 1)]^T \), the formula (12) is rewritten as follows:

\[
\tilde{\xi}(k + 1) = \tilde{A}\tilde{\xi}(k) + \tilde{B}\Delta\tilde{u}(k)
\]  

(13)

Where \( \tilde{A} = \begin{bmatrix} A(k) & B(k) \\ 0_{2 \times 2} & I_2 \end{bmatrix}, \tilde{B} = \begin{bmatrix} B(k) \\ I_2 \end{bmatrix} \), \( \Delta\tilde{u}(k) = \tilde{u}(k) - \tilde{u}(k - 1) \).
3.2. Rolling Optimization of Objective Function

The input of the system changes \( N_c \) steps from the beginning of the time \( k \), according to the prediction model of formula (12), the vehicle state in the future can be predicted under the input motor torque on both sides. It can be described in the form of vector matrix as follows:

\[
X(k) = \Psi \hat{\xi}(k) + \theta U(k)
\]  

(14)

Where

\[
X(k) = \begin{bmatrix}
\hat{e}(k+1) \\
\vdots \\
\hat{e}(k+N_p)
\end{bmatrix}, \Psi = \begin{bmatrix}
\tilde{A} \\
\tilde{A}^2 \\
\vdots \\
\tilde{A}^{N_p}
\end{bmatrix}, \theta = \begin{bmatrix}
\tilde{B} \\
\tilde{A}\tilde{B} \\
\tilde{A}^2\tilde{B} \\
\vdots \\
\tilde{A}^{N_p-1}\tilde{B}
\end{bmatrix},
\]

and

\[
U(k) = \begin{bmatrix}
\Delta\tilde{u}(k) \\
\Delta\tilde{u}(k+1) \\
\vdots \\
\Delta\tilde{u}(k+N_p)
\end{bmatrix}
\]

The objective function needs to ensure that vehicle speed and yaw rate can track the target value stably and quickly, and the input motor torque is as small as possible. Which can be define as \( X \rightarrow X_{ref} \) and \( \min \Delta U \). The performance index function using the weighted sum of squares can be described as:

\[
\min \left[ (X - X_{ref})^T Q (X - X_{ref}) + U^T R U \right]
\]  

(15)

Where \( Q \) and \( R \) represent weight matrix. The first term of formula (15) reflects the tracking performance of the vehicle speed and yaw rate, and the second term reflects the constraint optimization of the input motor torque. To solve formula (15) readily, this define error as \( E = \Psi e - \Psi e_{ref} = \Psi e - X_{ref} \).

\( X - X_{ref} \) In first term can be obtained according to formula (14) as follows:

\[
X - X_{ref} = E + \theta U
\]  

(16)

By substituting the formula (16) into the formula (15), the performance index function can be transformed into a typical quadratic programming problem for solving:

\[
\min \left[ (X - X_{ref})^T Q (X - X_{ref}) + U^T R U \right]
\]  

(17)

The output capacity of motors is limited during actual steering of the vehicle. In order to prevent motors from entering the deep saturation state, the system performance is deteriorated when motors enter the deep saturation state. To avoid the problem, input of the control system needs to be constrained as follows:

\[
|u(t + k)| \leq T_{max} \quad (k = 0, 1, \cdots, N_c - 1)
\]  

(18)
3.3. Feedback Correction

The increment of control system input in quadratic programming formula (17) is set as $U$, in which $\Delta u(k) = \bar{u}(k) - \bar{u}(k-1) = u(k) - u^*(k) - (u(k-1) - u^*(k-1))$. Therefore, the input of control system at time $k$ can be described as follows:

$$u(k) = u^*(k) + (u(k-1) - u^*(k-1)) + \Delta u(k)$$  \hspace{1cm} (19)

4. Modelling and Simulation

4.1. Construction of Co-simulation Model

In order to verify the control effect of the model predictive control algorithm, this paper uses Simulink and the multi-body dynamics software RecurDyn to build the co-simulation model for dual electric drive tracked vehicle. The model structure is shown as Figure 2, which mainly follows the modular modeling idea and can be divided into steering control module, driving motor module and vehicle dynamics module.

![Figure 2. Structure of Co-Simulation Model](image)

The steering control and drive motor modules are built by Simulink, and the parameters of each module are independent of each other. The input and output interfaces are reserved for information exchange during packaging. The vehicle dynamics module is built by the Track-HM toolkit in RecurDyn. After defining the PIN and POUT of the vehicle model, the dynamics module is transformed into m file and embedded in the Simulink model to form co-simulation model.

The specific parameters of drive motor module and vehicle dynamics module are shown in Table 1:

| Parameter                  | Value          | Parameter                  | Value          |
|----------------------------|----------------|----------------------------|----------------|
| Vehicle mass $m$ (kg)      | 8000           | Rated power (Kw)           | 100            |
| $L$ (m)                    | 2.975          | Maximum torque (Nm)        | 600            |
| $B$ (m)                    | 1.985          | Rated torque (Nm)          | 300            |
| $f$                        | 0.05           | Maximum speed (rpm)        | 7500           |
| $\mu_{max}$                | 0.65           | Rated speed (rpm)          | 3200           |
| $r$ (m)                    | 0.25           | $i$                        | 10             |

Table 1. Parameters of Vehicle Dynamics Module and Drive Motor Module
4.2. Simulation analysis

Prediction horizon and control horizon in the steering controller can be set as $N_p = 4$ and $N_c = 3$. In order to verify the tracking performance of MPC for the target speed, the initial speed target value is set as 10km/h, the target value increases to 30km/h at 10s, 40km/h at 20s and 50km/h at 35s. To verify the anti-interference performance of MPC, the road parameter changes from asphalt road to sandy road at 14s, to simulate the disturbance of ground deformation resistance coefficient. The road gradient changes to 15° at 25s, to simulate the large disturbance of road structure. The simulation results are shown as Figure 3.

![Figure 3. Variable Speed Driving Condition](image)

It can be seen from Figure 3 that the algorithm can track the target speed value with high steady-state tracking accuracy. The tracking accuracy is almost unaffected when road disturbance is small. When road disturbance is large, the algorithm can quickly suppress the disturbance and return to the target value, which has good robustness.

To verify the tracking performance of MPC for yaw rate, experiments of small radius and large radius steering conditions at different speeds are simulated. The vehicle speed is set as 10km/h during 0-12. The radius is given 2.5B during 4-6s, 5B during 10-12s. The target speed is set as 30km/h during 12-25s, the radius is given 15B during 18-20s, 30B steering during 22s-24s. The simulation results are shown as Figure 4.

![Figure 4. Steering Conditions at Different Speeds](image)

(A) Vehicle Yaw Rate Tracking Curve  (B) Output Torque Curve of Motors on Both Sides

It can be seen from Figure 5 that the algorithm can realize the stable tracking of the target yaw rate and vehicle speed. The torque of the motor only fluctuates greatly when entering the turning. When entering steady-state turning, the torque fluctuation is small, the output power is stable, and the response time of turning and returning are all within 0.25s.

5. Conclusion

This paper mainly proposes a model predictive control algorithm based on the construction of vehicle dynamics and driver manipulated model for steering control problem of dual electric drive tracked vehicle. The vehicle speed and yaw rate are tracked through establishing predictive model, rolling
optimization, and correcting feedback. To verify the effectiveness of MPC, co-simulation model of vehicle dynamics and steering control is constructed. The simulation results show that MPC algorithm can quickly and accurately track driver's intentions, the motor output torque is smooth, and the vehicle can run stably.

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