CKM global fit and constraints on New Physics in the $B$ meson mixing

O. Deschamps [CKMFit group]

Laboratoire de Physique Corpusculaire, IN2P3/CNRS, Clermont-Ferrand, France

An update profile of the CKM matrix is given, providing numerical and graphical constraints on the CKM parameters in the Standard Model. Constraints on additional parameters accounting for possible new physics contribution in a model-independent analysis are also reported with emphasis on the leptonic decay of the $B_d$ and on the $B_s$ mixing phase.

1. INTRODUCTION

Within the Standard Model (SM), the quark flavor mixing is described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The 3x3 unitary CKM matrix can be fully parametrized by four independent parameters among which a single non-vanishing complex phase accounts for the violation of the CP symmetry. Inspired from the one proposed by Wolfenstein [2] the following parametrization, phase-convention independent and unitary-exact to all orders in $\lambda$, is used throughout this document:

$$ \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad A^2 = \frac{|V_{cb}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. $$

While the parameters $\lambda$ and $A$ are accurately determined through the measurement of $|V_{ud}|$ from the super-allowed nuclear transitions, $|V_{us}|$ from the semileptonic kaon decay and $|V_{cb}|$ from the semileptonic $B$ decay with charm, the $(\bar{\rho} + i\bar{\eta})$ complex parameter, being the apex coordinates of the unitarity triangle (UT) related to the first and third quark families, is less constrained. The metrology of the UT apex via the determination of its sides and angles allows to measure the size of the CP violation and to validate the overall consistency of the KM scenario within the Standard Model. Any inconsistency would suggest contributions from physics beyond the Standard Model.

2. THE CKM FIT INPUTS

The CKM global fit is performed within a frequentist statistical approach including a specific treatment to handle the theoretical uncertainties on some of the inputs (RFit 3). The inputs used for the fit are the most recent available

| input | source | value | (ref) | update | UT constraint | associated theory | parameters |
|-------|--------|-------|-------|--------|--------------|-------------------|------------|
| $|V_{ud}|$ | Nuclear decays | 0.97418 ± 0.00026 | (4) | | $|V_{ud}|$ | $|V_{ud}|$ form factor |
| $|V_{us}|$ | SL kaon decays | 0.2246 ± 0.0012 | (5) | | | $|V_{us}|$ | $|V_{us}|$ form factor |
| $|V_{cb}|$ | SL charmed $B$ decays | $(40.60 \pm 0.35 \pm 0.58) \times 10^{-3}$ | (6) | | $|V_{cb}|$ | $|V_{cb}|$ form factor and/or OPE |
| $|V_{ub}|$ | SL charmless $B$ decays | $(3.76 \pm 0.10 \pm 0.47) \times 10^{-3}$ | (6) | | | $|V_{ub}|$ | $|V_{ub}|$ form factor and/or OPE |
| BR($B^+ \to \tau^+\nu$) | Leptonic $B$ decays | $(1.80 \pm 0.33) \times 10^{-4}$ | (7) | * | | leptonische amplitude [$f_{B_s}$, $f_B$, $f_{B_s}$] |
| $\Delta m_s$ | $B_sB_s$ mixing | $(17.77 \pm 0.12) \times \text{ps}^{-1}$ | (8) | | | $\Delta m_{s}^\ast$ | $\Delta B=2$ amplitude [$B_s$, $f_{B_s}$, $m_{\tau}$, $\eta_{B_s}$] |
| $\Delta m_d$ | $B_dB_d$ mixing | $(0.507 \pm 0.005) \times \text{ps}^{-1}$ | (9) | | | $\Delta m_{d}^\ast$ | $\Delta B=2$ amplitude [$B_d$, $f_{B_d}$, $f_{B_d}$, $f_{B_d}$, $\eta_{B_d}$] |
| $|e_K|$ | KK mixing | $(2.229 \pm 0.010) \times \text{ps}^{-1}$ | (9) | * | $f(\bar{\rho}, \bar{\eta})$ | $\Delta S=2$ amplitude [$B_K$, $\eta_B$, $\eta_A$, $\eta_{\text{til}}$] |
| $\sin(2\beta)$ | Charmonium $B$ decays | $(0.672 \pm 0.024) \times \text{ps}^{-1}$ | (9) | * | $\beta$ | |
| BR & $A_{CP}$ | $B \to \pi\pi, \rho\rho, \omega\omega$ decays | B-factors average | (10) | * | $\alpha$ | SU(2) |
| BR & $A_{CP}$ | $B \to DK$ decays | B-factors average | (10) | * | $\gamma$ | GLW/ADS/GGSZ |

Table I: Most relevant inputs for the global CKM fit. The left part addresses the observable inputs with their value and reference. The right part addresses the constraint on UT parameters derived from the observable. The upper part of the table contains CP-conserving observables while the CP-violating are listed in the lower part.
value of the observables from the $K$ and $B$ meson physics for which the theoretical conversion into CKM parameters is under control. The most relevant input observables of the global CKM fit, with their value and reference, are listed in the table. The observable values that have been updated for this analysis with respect to the previous similar analysis are indicated.

The right side of the table addresses the observables values used for this analysis for comparison purpose, some alternative and more recent values proposed in the reference are quoted in the last column (note that the statistical and theoretical uncertainties have been assumed quadratically summable by the authors).

A more detailed review of the CKM global fit inputs can be found in

### 3. THE SM CKM FIT RESULTS

The left part of the figure displays the result of the global fit together with the 95% CL contours of the individual constraints.

![figure](image_url)

Figure 1: Left figure: individual and global constraints in the $(\rho, \eta)$ plane from the global CKM fit. The hashed region of the global combination corresponds to 68% CL. The constraints from the experimentally dominated observables and observables inducing larger theoretical uncertainties are shown on the right-top and the right-bottom figure, respectively.

A satisfactory agreement is observed from the various individual contributions at the $2\sigma$ level, establishing the KM mechanism as the dominant source of the CP violation in the $B$ meson system.

As shown on the right side of the figure, a slight tension is however revealed when comparing the global constraint
coming from the observables dominated by the experimental measurements (UT angles) with the constraint derived from the observables inducing larger theoretical uncertainties ($|V_{ub}|$, $BR(B^+ \rightarrow \tau^+ \nu)$, $|\epsilon_K|$, $\Delta m_d/\Delta m_s$). This classification of the observables is for illustration purpose and should not be used to draw any conclusion about the possible origin of the tension: a similar discrepancy is also visible when comparing the constraint from the tree-level processes with the mixing loop-induced or the constraint from the CP-conserving observables versus the CP-violating ones.

Figure 2: Left: Confidence Level contour in the 2D ($\sin(2\beta), BR(B^+ \rightarrow \tau^+ \nu)$) plane for the fit prediction. The cross indicates the 68% CL interval for the direct experimental measurement. Middle: the corresponding 1D projection for $BR(B^+ \rightarrow \tau^+ \nu)$ compared to the direct measurement. For comparison purpose, the prediction for the branching ratio of the decay is displayed on the right plot (the decay constant value $f_{D_s} = 241 \pm 3$ MeV has been used [13]).

As illustrated on the left plot of the figure 2, the bulk of the tension is located in the correlations between the CP-conserving, UT-side related, theory-dependent observable, $BR(B^+ \rightarrow \tau^+ \nu)$ (central value of which slightly increases with the summer 2008 update [7]) and the CP-violating, UT-angle related, theory-free observable, $\sin(2\beta)$ (central value of which slightly decreases [8]).

Quantitatively, the minimal $\chi^2$ of the global fit decreases by 2.9 $\sigma$ (2.6 $\sigma$) when removing $BR(B^+ \rightarrow \tau^+ \nu)$ ($\sin(2\beta)$) from the list of the fit inputs. Obviously, the disagreement is not large enough to exclude the possibility of a statistical origin of the tension. However, it is worth mentioning that a similar discrepant pattern is also observed for the $D_s \rightarrow \tau^+ \nu$ leptonic decays (see for instance [20, 21]), suggesting a common origin. As an illustration, the expected and measured branching fraction of the leptonic $B^+$ and $D_s$ decays to $\tau^+ \nu$ final state are displayed on the right side of the figure 2.

The possibility of a generic problem with the lattice prediction for the theoretical parameters related to the $B$ meson has been investigated. Although the tree-level expression for the amplitude of the the $B^+ \rightarrow \tau^+ \nu$ annihilation decay is directly proportional to the decay constant $f_{B_d}$, it can be shown that an under-estimation of the lattice prediction for the parameters product $f_{B_d} \times (f_{B_d}/f_{B_s})$ as used in this analysis, can not by itself explain the observed tension. Indeed, the $BR(B^+ \rightarrow \tau^+ \nu)$ together with the $\Delta m_d$ observables provide a $f_{B_d}$-independent constraint in the $(\rho, \eta)$ plane in which a clear tension remains as illustrated on the figure 3 for the two set of the LQCD parameters values quoted in the table 11.

More precisely, the $f_{B_d}$-independent ratio:

$$\frac{BR(B^+ \rightarrow \tau^+ \nu)}{\Delta m_d} = \frac{3\pi}{4} \frac{m_\tau^2}{m_W^2 S(x_t)} \left(1 - \frac{m_\tau^2}{m_{B^+}^2} \right)^2 \tau_{B^+} \left(\frac{\sin \beta}{\sin \gamma} \right)^2 \frac{1}{|V_{ud}|^2} \frac{1}{B_{B_d}}$$

allows for a theory parameter-free prediction of the bag parameter, $B_{B_d}$, from the experimental measurement of $\beta, \gamma$ (or $\alpha = \pi - \beta - \gamma$), $|V_{ud}|$, $BR(B^+ \rightarrow \tau^+ \nu)$ and $\Delta m_d$. The resulting prediction is 2.4 $\sigma$ away from the
Figure 3: Constraint in the $(\bar{\rho}, \bar{\eta})$ plane from the BR$(B^+ \to \tau^+ \nu)$ and $\Delta m_d$ observables only using the LQCD parameters value quoted in the 2nd column of the table II (left) and using the alternative values quoted in the third column (middle). In the latter case, both the theoretical and statistical uncertainties are assumed to be Gaussianly distributed, resulting in a more aggressive impact of the LQCD parameters. On both plots the global constraint from the whole set of observables is indicated by the shaded area around the apex of the unitary triangle. Right: theory-free prediction for the bag parameter $\hat{B}_{B_d}$ (see text) compared to the LQCD calculation quoted in the second column of the table II.

lattice calculation [11, 12] as shown in the right plot of the figure 3. This deviation is essentially dominated by the experimental uncertainties on the $B^+ \to \tau^+ \nu$ branching ratio and on the UT angles $\alpha$ and $\gamma$.

A lattice origin of the tension would thus involve the bag parameter $B_{B_d}$ that controls the correlations between BR$(B^+ \to \tau^+ \nu)$ and the angle $\beta$, as explicitly shown in the above formula. Further investigations are needed to check whether the possible correlations between the theoretical errors affecting the lattice parameters, generally not provided by the lattician community and therefore neglected in this analysis, could account for the observed tension. Eventually, the hypothesis of new contributions beyond the SM is discussed in the next section.

4. CONSTRAINTS ON NEW PHYSICS IN THE $B_{d,s}$ MESONS MIXING

4.1. Fit procedure

New Physics (NP) is expected to affect the amplitude of the neutral $B$ mesons mixing in many scenarii (see for instance [16]). Assuming NP to contribute mostly to the short-distance part of the $\Delta F = 2$ processes, a model-independent parametrization has been proposed [17, 18]:

$$\langle B_q | M_{12}^{\text{SM+NP}} | \bar{B}_q \rangle = \Delta_{q}^{NP} \langle B_q | M_{12}^{\text{SM}} | \bar{B}_q \rangle$$

where the label $q$ stands for the $d$ or $s$ flavor of the neutral $B$ meson and the complex parameter $\Delta_{q}^{NP} = \text{Re} \Phi_{q}^{NP}$ accounts for the NP contribution. Assuming in addition that the tree-level mediated decays proceeding through a Four Flavor Change get only SM contributions (SM4FC hypothesis [15, 22]), the observables $|V_{ij}|$, $\gamma$ and $\gamma(\alpha) = \pi - \beta_{\text{UT}} - \alpha$\footnote{where $\beta_{\text{UT}}$ means the $\beta$ UT angle extracted from the analysis of the charmonium $B_d$ meson decays.} are not affected by the NP contribution and can be used in a (SM+NP) global fit to fix the SM parameters. The oscillations parameters, the weak phases, the semileptonic asymmetries and the $B$ meson lifetime-differences are...
affected by the phase and/or the amplitude of the NP contribution as quoted in the table III and allow to constrain the NP deviation to SM parametrized with \( \Delta_{q}^{NP} \).

| parameter                  | prediction in the presence of NP                                      |
|---------------------------|------------------------------------------------------------------------|
| \( \Delta m_{q} \)        | \(|\Delta_{q}^{NP}| \times \Delta m_{q}^{SM} \)                       |
| 2\( \beta \)              | \(2\beta_{SM}^{NP} \)                                                  |
| 2\( \beta_{s} \)          | \(2\beta_{s}^{SM} - \Phi_{q}^{NP} \)                                  |
| 2\( \alpha \)             | \(2(\pi - \beta_{SM}^{NP} - \gamma) - \Phi_{q}^{NP} \)               |
| \( \Phi_{12,q} \)         | \(\frac{\Gamma_{12,q}}{M_{12,q}^{SM}} \times \sin(\Phi_{12,q}^{SM} + \Phi_{q}^{NP})\) |
| \( A_{SL}^{q} \)          | \(\frac{\Gamma_{12,q}}{M_{12,q}^{SM}} \times \sin(\Phi_{12,q}^{SM} + \Phi_{q}^{NP})\) |
| \( \Delta \Gamma_{q} \)   | \(2|\Phi_{12,q}^{NP}| \times \cos(\Phi_{12,q}^{NP} + \Phi_{q}^{NP})\)    |

Table III: Theoretical prediction for the B physics observables in the presence of NP in mixing. Note the opposite sign convention for the \( B_{d} \) and \( B_{s} \) mixing phase.

The resulting constraints in the \((\text{Re}(\Delta_{q}^{NP}), \text{Im}(\Delta_{q}^{NP}))\) plane are summarized in the left and right side of the figure 4 for the \( B_{d} \) and \( B_{s} \) case, respectively. For a more complete review of the constraints from the global fit of NP in the B meson mixing see [23].

![Figure 4](image)

Figure 4: Individual and global constraints in the \((\text{Re}(\Delta_{q}^{NP}), \text{Im}(\Delta_{q}^{NP}))\) plane for the (SM+NP) fit. Due to the large uncertainties, only the 68% CL are shown for the individual constraints. Both 68% and 95% contours are shown for the global constraint.

### 4.2. The \( B_{d} \) case

For the \( B_{d} \) meson case, the dominant constraints come from the mixing phase, \( \beta \), and the oscillation frequency \( \Delta m_{d} \). Both reasonably agree with their SM prediction. A 2.1 \( \sigma \) deviation is obtained for the 2-dimensional SM hypothesis \( \Delta_{q}^{NP} = 1 \). The 1-dimension hypothesis based on the phase only, \( \Phi_{d}^{NP} = 0 \), corresponding the phase expectation in the Standard Model as well as in the Minimal Flavour Violation scenario, results in a 1.5 \( \sigma \) deviation. This slight disagreement with the SM hypothesis is essentially the translation of the tension in the SM global fit discussed in the previous section (obviously the discrepancy in the \( D_{s} \) leptonic decays mentioned in the previous section is unaffected
by the NP scenario in the $B_d$ mixing assumed here).
When removing $\text{BR}(B^+ \rightarrow \tau^+ \nu)$ observable from the fit, the deviation reduces down to 0.9 $\sigma$ for both the 1D null NP-phase hypothesis and the 2D SM hypothesis.

### 4.3. The $B_s$ case

The fit for New Physics in the $B_s$ mixing also exhibits a 2.1 $\sigma$ deviation for the SM 2D hypothesis, $\Delta s_{NP}^2 = 1$ and a 2.5 $\sigma$ for the 1D null NP-phase hypothesis, $\Phi_s^{NP} = 0$.

Contrarily to the $B_d$ case, the source of the discrepancy is obvious: the deviation is fully dominated by the direct measurement by the CDF and the D0 experiments of the correlated weak mixing phase and lifetime difference, $(\Delta \Gamma_s, 2\beta_s)$ through the time-dependent angular analysis of the $B_s \rightarrow J/\Psi \Phi$ decay. Both collaborations report a large phase with respect to the SM expectation \cite{26}. The Heavy Flavor Averaging Group \cite{6} obtains a 2.2 $\sigma$ deviation for the SM hypothesis in the D0/CDF $(\Delta \Gamma_s, 2\beta_s)$ combination. The other contributions to the NP fit only provide weak constraint:

- the oscillation frequency, $\Delta m_s$, only sensitive to the module of $\Delta s_{NP}^2$ is consistent with its SM prediction.
- the semileptonic asymmetries, $A_{3L}^s$, and the Flavor-Specific proper-time, $\tau_{FS}^s$ do not display any significant sensitivity to the NP fit parameters in the current state of their determination.
- the theoretical lifetime difference proportional to the cosine of the weak phase: $\Delta \Gamma_s \simeq 2|\Gamma_{12}^s| \times \cos(\Phi_s^{NP})$ slightly tends to push the NP phase toward the zero value as the measured $\Delta \Gamma_s$ is larger than the SM expectation for $2|\Gamma_{12}^s|$ \cite{17}.

### 5. SUMMARY AND CONCLUSIONS

The current data successfully fit the KM mechanism establishing it as the dominant source of CP violation in the $B_d$ sector. The current limitation of the CKM consistency tests are of three origins: LQCD quantities, the UT angle $\gamma$ and the $B_s$ measurements. With the recent update of the inputs, a slight tension appears in the global CKM fit within the SM hypothesis. This tension is mainly driven by the larger $\text{BR}(B^+ \rightarrow \tau^+ \nu)$ updated value and the smaller $\sin(2\beta)$ value that B-factories recently reported and results from a non-trivial correlation that we have explicited in the ratio $\text{BR}(B^+ \rightarrow \tau \nu)/\Delta m_d$. Both updates are consistent with the previous measurements but slightly go away from their SM prediction. Besides the possible statistical fluctuation of the experimental measurements, further investigations are necessary to understand the role LQCD quantities may play in the origin of the tension.

Assuming the presence of New Physics in the $B$ mixing, a model-independent fit procedure has been applied. As a translation of the observed tension in the SM CKM fit, the SM hypothesis is 2.1 $\sigma$ away from the preferred NP fit solution in the $B_d$ case. A 2.1 $\sigma$ deviation to SM is also observed in the $B_s$ sector, fully dominated by the direct experimental measurement of the $B_s$ mixing phase by the Tevatron experiments. Within a joint fit for NP in the $B_d$ and $B_s$ mixing, the 4-dimension SM hypothesis $(\Delta d_{NP}^2 = \Delta s_{NP}^2 = 1)$ results in a 2.9$\sigma$ deviation with respect to the preferred fit solution.

A more complete update with all the data eventually available at the end of the summer 2008 is to appear in \cite{15}. In the near future, significant improvements in the lattice QCD predictions as well as more precise experimental measurements can be expected in both the $B_d$ and the $B_s$ sectors. These data will be scrutinized accurately to confirm or not the appearing tension in the $B_d$ sector and the large mixing phase observed in the $B_s$ mixing.

\footnote{A deviation in excess of 3 $\sigma$ in the fit for NP in the $B_s$ mixing had previously been reported by the UTFit collaboration \cite{24} and has recently been updated to a lower value \cite{25}.}
Eventually a major step in the consistency tests of the KM mechanism will arise with the imminent LHC era and in particular with the dedicated LHCb experiment.

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References

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 40, 652 (1973)
[2] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983);
[3] The CKMfitter Group (J. Charles, et al.), Eur. Phys. C41, 1 (2005);
[4] I.S. Towner and J.C. Hardy, Phys. Rev C77 , 025501 (2008);
[5] FlaviaNet Working Group on kaon decays, arXiv:0801.1817 [hep-ph] (2008);
[6] The Heavy Flavour Averaging Group (HFAG), http://www.slac.stanford.edu/xorg/hfag/ and references therein.
[7] The BABAR Collaboration (B. Aubert, et al.), Phys.Rev. D77, 011107,2008.
   C.J. Schilling, "Experimental results on b → sl+l− and B+ → τ+ν", invited talk at the 5th Workshop on the CKM Unitary Triangle (CKM 2008), Roma, Italy, September 2008.
   The Belle Collaboration (K. Ikado, et al.), Phys. Rev. Lett. 97, 251802 (2006);
   The Belle Collaboration (I. Adachi, et al.), arXiv:0809.3834 [hep-ex] (2008);
[8] The CDF Collaboration, Phys. Rev. Lett. 97, 242003(2006);
[9] The Particle Data Group (C. Amsler et al.), Physics Letters B667, 1 (2008) and references therein;
[10] Y. Kolomensky, "CKM phases: experimental status", invited talk at the 34th International Conference on High-Energy Physics (ICHEP 2008), Philadelphia, PA, August 2008.
[11] N. Tantalo, "Lattice calculations for B and K mixing", invited talk at the 4th Workshop on the CKM Unitary Triangle (CKM 2006), Nagoya, Japan, December 2006; arXiv:0703.2411[hep-ph] (2007);
[12] U. Nierste, private communication (2003)
[13] S. Descotes-Genon, "CKMfitter - The winter 2008 collection", invited talk at the Rencontres de Moriond (2008) and [15]
[14] V. Lubicz et al., arXiv:0807.4605 [hep-latt] (2008);
[15] The CKMfitter group (http://www.slac.stanford.edu/xorg/ckmfitter/)
[16] R. Fleischer et al., JHEP 0305, 053 (2003)
[17] A. Lenz and U. Nierste, arXiv:0612.167 [hep-ph] (2006);
[18] Y. Grossman et al., Phys. Lett. B380, 99 (1996);
[19] E. Follana et al., arXiv:0706.1726 [hep-ph] (2008);
[20] E. Kronfeld, "Non-Standard Physics in (Semi)Leptonic Decays", the XXVI International Symposium on Lattice Field Theory (Lattice 2008), Williamsburg, Virginia (2008).
[21] E. G´amiz, "Heavy flavour phenomenology from Lattice QCD", the XXVI International Symposium on Lattice Field Theory (Lattice 2008), Williamsburg, Virginia (2008).
[22] T. Goto, et al., Phys. Rev. D53, 6662 (1996)
[23] J. Charles, "CKMFitter update, short status of New Physics in B\bar{B} mixing", Capri 2008 conference, Anapri, Capri island (2008). To appear in the Proceedings Supplements of Nuclear Physics B.
[24] The UTFit Collaboration (M. Bona, et al.), arXiv:0803.0659 [hep-ph] (2008)).
[25] Maurizio Pieriemi, "Update on the unitarity triangle (UTFit)”, this conference (2008).
[26] The CDF Collaboration (T. Aaltonen, et al.), Phys. Rev. Lett. 100, 161802 (2008); The D0 Collaboration (V.M. Abazov, et al.), arXiv:0802.2255 [hep-ex] (2008).