Galaxy Dynamics Predictions in the Nonsymmetric Gravitational Theory

J. W. Moffat and I. Yu. Sokolov

Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7

(August 13, 2018)

Abstract

In the weak field approximation, the nonsymmetric gravitational theory (NGT) has, in addition to the Newtonian gravitational potential, a Yukawa-like potential produced by the exchange of a spin $1^+$ boson between fermions. If the range $r_0 = \mu^{-1}$ is 25 kpc, then this additional potential due to the interaction with matter in the halos of galaxies can explain the flat rotation curves of galaxies and the Tully-Fisher law ($L \sim v^4$) without the dark matter hypothesis. Possible fits to clusters of galaxies and gravitational lensing observations are discussed. The results are based on a linear approximation to a new perturbatively consistent version of the NGT field equations, which does not violate the weak equivalence principle.
I. INTRODUCTION

After two decades there has not been any observation of exotic dark matter candidates. Recent observational results using the HST have excluded faint stars as a significant source of dark matter in the solar neighborhood [1]. However, the galaxy dynamics observations continue to pose a serious challenge to gravitational theories. The data are in sharp contradiction with Newtonian dynamics, for virtually all spiral galaxies have rotational velocity curves which tend towards a constant value [2–4].

As in the case of anomaly problems in solar dynamics of the past century, concerning Uranus and Mercury, there are two ways to circumvent the problem. The most popular is to postulate the existence of dark matter [5]. It is assumed that dark matter exists in massive almost spherical halos surrounding galaxies. About 90% of the mass is in the form of dark matter and this can explain the flat rotational velocity curves of galaxies. However, the scheme is not economical, because it requires three or more parameters to describe different kinds of galactic systems and no satisfactory model of galactic halos is known.

The other possible explanation for the galactic observations is to say that Newtonian gravity is not valid at galactic scales. This has been the subject of discussion in recent years [6–13]. We know that Einstein's gravitational theory (EGT) correctly describes solar system observations and the observations of the binary pulsar PSR 1913+16 [14]. Therefore, any explanation of galactic dynamics based on gravity must be contained in a modified gravitational theory that is consistent with EGT. The following constraints on a classical gravitational theory are:

(1) The theory must be generally covariant, i.e., the field equations should be independent of general coordinate transformations and should reduce to special relativity dynamics in flat Minkowskian spacetime.

(2) The theory should be derivable from a least action principle in order to guarantee the consistency of the theory.

(3) The linear approximation should be consistent, i.e., there should not be
any ghost poles, tachyons or higher-order poles and the asymptotic flat space boundary conditions should be satisfied.

(4) The equations of motion of test particles should be consistent with local equivalence principle tests.

(5) All solar system tests of gravity and the observed rate of decay of the binary pulsar should be predicted by the theory.

We shall now consider the predictions for galaxy dynamics in a new version of the non-symmetric gravitational theory which can satisfy all the above criteria [15–17]. The theory has a linear approximation free of ghost poles, tachyons and higher-order poles with field equations for a massive spin $1^+$ boson with a range parameter, $r_0 = \mu^{-1}$, corresponding to Proca-type equations for an antisymmetric potential. The expansion of the field equations about an arbitrary EGT background metric is also consistent and satisfies the physical boundary conditions at asymptotically flat infinity.

II. TEST PARTICLE ACCELERATION IN NGT

A derivation of the equations of motion of test particles yields the following additional NGT test particle radial acceleration in the weak field approximation [18]:

$$a_{\text{ngt}}(r) = \frac{\lambda \gamma(r)c^2}{\alpha(r)} \left(1 - \frac{\lambda \gamma(r)f'(r)}{\sqrt{r^4 + f^2(r)}} \right)^{-1} \frac{d}{dr} \left(\frac{\gamma(r)f'(r)}{\sqrt{r^4 + f^2(r)}}\right),$$  \hspace{1cm} (1)

where $r$ is the radial distance from a point source, $\lambda$ is a coupling constant, $c$ is the velocity of light, and

$$\gamma(r) \approx \alpha(r)^{-1} = 1 - \frac{2GM}{c^2r}. \hspace{1cm} (2)$$

Moreover, $f(r)$ is defined by

$$g_{[23]}(r) = f(r) \sin \theta, \hspace{1cm} (3)$$

where $g_{[23]}$ is the only non-zero component of the antisymmetric part $g_{[\mu\nu]}$ of the nonsymmetric tensor $g_{\mu\nu}$, defined by $g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]}$. 

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For $r \geq 0.2$ kpc (see Section VI), we have [18]:

$$f(r) \approx C \exp(-r/r_0)(1 + r/r_0), \quad (4)$$

where $C$ is a constant of dimension (length)$^2$. Substituting (4) into (1), we get the total radial acceleration experienced by a test particle in a static spherically symmetric gravitational field:

$$a(r) = -\frac{GM}{r^2} + \frac{\lambda C c^2}{r_0^2} \frac{\exp(-r/r_0)}{r^2}(1 + r/r_0). \quad (5)$$

For $r \ll r_0$, we find that [18]:

$$f(r) \approx C \left[ 1 - \frac{1}{2} \left( \frac{r}{r_0} \right)^2 + \frac{2GM}{c^2r} \right]. \quad (6)$$

Substituting this expression into (1), we obtain the additional NGT acceleration:

$$a_{\text{ngt}}(r) = \frac{4\lambda GM(C^2 + 2r^4)}{r^3(C^2 + r^4)^{3/2}}. \quad (7)$$

In the new version of NGT, the weak equivalence principle is satisfied for the motion of test particles, whereas the strong equivalence principle is not satisfied. Thus, test particles fall in a local gravitational field independently of their composition, while the non-gravitational laws of physics in local Minkowskian frames are not equivalent to one another. We therefore expect to satisfy all the Eötvös-type experiments for which bodies with baryon or lepton number charges fall at the same rates in a gravitational field [14][19][20].

We shall apply these formulas to try to explain the observed long-standing paradox of the flatness of the rotation curves of galaxies, as well as the Tully-Fisher law [21], without postulating large amounts of dark matter in galaxies.

The above formulas will only be used for the acceleration due to points sources. A more extensive solution requires, for example, an integration over a disk density profile. We shall not attempt to do such a fit in the following, but leave this problem for a future publication.
III. FITTING THE ROTATION CURVES

As one can see from the previous section, there are two free parameters, $\lambda$ and $r_0$, and a constant $C$ that remains to be fixed. We shall choose the constant $C$ to be

$$C \propto \sqrt{M}.$$  \hfill (8)

We then get the total radial acceleration on a test particle:

$$a(r) = -\frac{G_\infty M}{r^2} + \sigma \sqrt{M} \frac{\exp(-r/r_0)}{r^2} (1 + r/r_0),$$  \hfill (9)

where $\sigma$ is a constant, $G_\infty$ is defined to be the gravitational constant at infinity. We are assuming that the gravitational constant can be different at small and large distance scales. We shall also assume that the range parameter $r_0$ is compatible with galaxy scales.

Let us assume that Newton’s law of gravity is valid with the standard gravitational constant, $G_0 = 6.673 \times 10^{-11} N/kg/m^2$, for distances that are smaller than the galaxy scale $\sim 10$ kpc. It is known that the deviations of the rotation curves of spiral galaxies from the simple Newtonian law (if we do not assume the dark matter hypothesis), take place for distances starting at $\sim 1$-4 kpc, depending on the galaxy mass. For the sake of simplicity, let us put $\sigma = G_0 \sqrt{M_0}$, where $M_0$ is a constant mass parameter. In order to guarantee that we obtain the usual Newtonian law for sufficiently small $r$, we set

$$G_\infty = G_0 \left( 1 + \sqrt{\frac{M_0}{M}} \right).$$  \hfill (10)

We then get the expression for the total radial test particle acceleration:

$$a(r) = -\frac{G_0 M}{r^2} \left\{ 1 + \sqrt{\frac{M_0}{M}} [1 - \exp(-r/r_0)(1 + r/r_0)] \right\}.$$  \hfill (11)

We can now find the dependence of the rotation velocity on the mass $M$ and the distance $r$ by using the formula: $a = v^2/r$. We have the two free constants $M_0$ and $r_0$ to fit the rotation curves of galaxies. Since our formulas are valid only for point mass sources, it means that we can fit only velocities of stars and hydrogen gas that rotate quite far from the disk of galaxy matter. Fortunately, the problem of the flatness of the rotation curves is associated
with this region of the galaxies. For distances less than $1 - 4$ kpc, the standard Newtonian law of gravity will apply.

Taking the information about rotation curves from Refs. [2,3], we have found at least 9 spiral galaxies in which the rotation curves were measured far from the disk of galaxy matter. They are: N2403, N2903, N3109, N3198, N5033, N5055, U2259, N3031 and N7331. The flat regions of the dependence of rotation velocities on the distance from the galaxy center (plateaus) is $\sim 10$ to $40$ kpc, so Eq.(11) must generate a flat function of $r$ for such distances. A similar problem was solved by Sanders [8] for one of the variants of modified Newtonian dynamics (MOND). In analogy, we find for the case of Eq.(11) that it yields flat curves if $r_0 \sim 20$ kpc and $5 < \sqrt{M_0/M} < 100$.

In the actual comparison with observations, we shall parameterize the velocity not by the mass $M$ but by the luminosity $L$, which is proportional to the mass. We shall use $L/M \sim 3$ and this gives for the rotational velocity:

$$v^2 = \frac{3G_0L}{r} \left\{ 1 + \sqrt{\frac{L_0}{L}} \left[ 1 - \exp(-r/r_0)(1 + r/r_0) \right] \right\}, \quad (12)$$

where $L_0 = 3M_0$.

Fig.1 shows an example of fitted curves for three galaxies from the above list for which there were enough data (at least six points to plot). For the best fit, we found the parameters to be: $L_0 = 250 \times 10^{10}L_\odot$ and $r_0 = 25$ kpc. In these fits we treated $L$ as a free parameter for all the galaxies. From the fits we obtained a best set of $L$'s for the galaxies, which we called $L_{\text{theor}}$. In Fig.2, we display the dependence of $L_{\text{theor}}$ on the observed $L_{\text{observ}}$, plotted for all the nine galaxies we have considered. The solid line corresponds to $L_{\text{theor}} = L_{\text{observ}}$. As we can see, the agreement is very good. The scattering of the results around
the line \( L_{\text{theor}} = L_{\text{observ}} \) is normal for this kind of data and can be explained by the measurement errors and by the existence of some fraction of dark baryonic matter.

![Graph showing \( \log(L_{\text{theor}}) \) vs. \( \log(L_{\text{observ}}) \)]

**Fig. 2**

**IV. THE TULLY-FISHER LAW**

Eq. (12) gives a good fit to the Tully-Fisher law, for which \( L^\beta \sim \nu^4 \) where \( \beta = 1 \). Fig. 3 presents a graph of the value of the parameter \( \beta \) as a function of \( r \) and the absolute magnitude \( H \), defined by \( H = -2.5 \log(L/L_\odot) + b \), where \( b \) is a constant. We choose \( b = 0 \) and consider the interval of \( H \) corresponding to \( 10^8 < L/L_\odot < 5 \times 10^{10} \), which covers all the galaxies considered here. The case \( \beta = 1 \) corresponds to the exact Tully-Fisher law, if we put the maximum difference between radial velocities in a galaxy equal to \( 2\nu \) (the difference between the forward and backward velocities). In the same figure, we show the plane \( \beta = 1.1 \). If we take into account that bigger \( r \) corresponds to bigger \( L \) (bigger absolute value of \( H \)), which
means that the larger galaxies are more luminous, we see from the figure that the deviation from $\beta = 1$, i.e. the Tully-Fisher law is about $10 - 15\%$.

We recall that we have used the point mass approximation for the galaxies. Presumably in an exact calculation, the repulsive NGT force would give a larger contribution \cite{22}, which means that an exact calculation would give a diminished radial velocity for a given $L$. This would result in a decrease of $\beta$.

Let us now consider the approximation that the maximum difference between radial velocities in a galaxy is equal to $2v$. From Fig.4, we see that for large galaxies with a big luminosity (mass) ($L > 5 \times 10^{10}L_\odot$) the maximum velocity is bigger than the plateau velocity, whereas for small galaxies the maximum velocity is equal to the plateau velocity (this follows from the use of the approximation of a localized central mass). If we consider the normal mass distribution (not localized at the center), we see that the rotation velocity has the tendency to decrease with diminishing radius, because of the diminishing effective mass near the center.
From these two considerations, it is reasonable to expect to have the maximum velocity difference equal to double the plateau velocity for not very large galaxies. This indicates that the Tully-Fisher law is true for smaller galaxies ($L < 5 \times 10^{10} L_\odot$).

V. THE DARK MATTER PROBLEM OUTSIDE THE GALAXY AND GRAVITATIONAL LENSING

It is known that the nearest giant spiral galaxy M31 and our Galaxy form the so-called Local Group. It is also known [23] that the center of M31 is approaching the center of the Galaxy at a velocity $\sim 119$ km/sec. This was an unexpected result, for most galaxies are moving apart with the general Hubble law expansion. A natural explanation of this fact was given by Kahn and Woltjer [24]. They suggested that the relative Hubble expansion of M31 and the Galaxy has been halted and reversed by their mutual gravitational attraction. As was shown by them, the total mass of the Local Group should be abnormally large, namely, the mass-to-light ratio should be about $100 (M/L)_\odot$. Such a big ratio is usually explained...
by the dark matter hypothesis.

Let us consider whether it is possible to explain this phenomenon without the dark matter hypothesis, using instead the gravity predictions of NGT. Because the distance between M31 and the Galaxy is $\sim 700$ kpc, we can safely use the point mass approximation. Nevertheless, we have to consider our formulas as an approximation, because the masses of the galaxies are not much different. Namely, the mass of M31 is about twice the mass of the Galaxy. However, it is reasonable to apply our formalism, since the disturbance due to the second mass is caused by the background metric which is rather small. Moreover, if we apply our model for this situation, the force of mutual attraction will be practically independent of what we treat as the source and the test particle.

If we now calculate the gravitational attraction, we find that the additional exponential force is vanishingly small. What is left contains the renormalized gravitational constant, Eq.(10), which contains the factor:

$$1 + \sqrt{\frac{M_0}{M}} = 1 + \sqrt{\frac{L_0}{L}} \sim 17.$$  

(13)

Thus, for the gravitational attraction we obtain:

$$a(r) = -\frac{G_0 M^*}{r^2},$$

(14)

where $M^* \sim 17M$ and from the “observed” mass-to-light ratio:

$$\frac{M^*}{L} \sim 100\left(\frac{M}{L}\right)_\odot,$$

we predict

$$\frac{M}{L} \sim 6\left(\frac{M}{L}\right)_\odot,$$

which agrees with the estimated ratio for luminous matter without the hypothesized dark matter.

Gravitational lensing experiments could give us new information about the mass of galaxies [25]. We could estimate the value of the mass expected in such an experiment by using
our model. Let us consider the lensing effect produced by a galaxy. Because we consider a point mass approximation, the distance $R$ between the galaxy center and the deflected light ray must be greater than the size of the galaxy. For this case, we find the angle of deflection $\Delta \phi$, obtained from a post-Newtonian expansion of the field equations is [14]:

$$\Delta \phi = \frac{4G_0 \left( 1 + \sqrt{M_0/M} \right) M}{c^2 R},$$

(15)

where $M$ is the mass of the galaxy.

We see that we anticipate a larger light deflection than is expected to be produced by the luminous mass of the galaxy. This prediction is close to the one that follows from the dark matter hypothesis.

VI. TERRESTRIAL AND SOLAR SYSTEM EFFECTS

We have to consider the two questions:

1. What is the range of $r$ for which the approximate result, Eq.(7), is valid?

2. Can we estimate the corrections to the gravitational force in the solar system and for terrestrial experiments?

The answer to the first question comes from Eq.(7). The first correction comes from the exponential force, while the second correction comes from the exact, static Wyman solution of the vacuum NGT field equations [15,16] valid in the long-range approximation, $\mu \approx 0$. The region in which the exponential force dominates is in the interval:

$$\frac{1}{2} \left( \frac{r}{r_0} \right) > \frac{2G_\infty M}{c^2 r}.$$ 

For $r_0 = 25$ kpc, one has

$$r > 0.2 \left( \sqrt{\frac{M}{10^{10}M_\odot}} \right) \text{kpc.}$$
Thus, we can use the exponential force formula as a correction up to $r \geq 1$ kpc for a galaxy with $M = 10^{10} M_\odot$, which means that our approximation scheme is self-consistent.

Let us now address the second question. Choosing the constant $\lambda C$ obtained from our fits to the galaxies, we find that

$$\lambda C \sim 10^{51} \sqrt{\frac{M}{M_\odot}} m^3. \quad (16)$$

We see that the constant $C$ cannot be very large, since otherwise it will lead to a big modification of the Schwarzschild solution of GR. If we take the value of $\lambda C$ from Eq. (13), then we see that the correction is much bigger than the Newtonian acceleration which is unacceptable in our linear approximation. This means that we need to solve the NGT field equations more exactly in order to guarantee that the corrections are properly accounted for. In particular, we must be able to renormalize the gravitational constant $G$ in the manner of Eq. (14), within the small $r$ expansion of the exact NGT solution, to guarantee that the terrestrial and solar system corrections are small.

On the other hand, let us consider Eq. (13) as an exact solution. If we calculate the limit of this acceleration formula for small $r$, we get the result:

$$\delta a(r) = \frac{a(r) - a_{\text{ngt}}(r)}{a_{\text{Newton}}(r)} \approx \frac{1}{2} \sqrt{\frac{M_0}{M}} \left( \frac{r}{r_0} \right)^2. \quad (17)$$

For $r_0 \sim 25$ kpc, we find that for solar and terrestrial experiments, $\delta a < 10^{-13}$. This deviation from the Newtonian force law is too small to be detected with current experiments intended to search for differences from Newtonian gravity, as well as a new “fifth force” in nature (see, e.g., [25,27]).

**VII. CONCLUSIONS**

From the linear weak field approximation to a generally covariant theory of gravitation, which is free from ghost poles and tachyons, we have fitted the flat rotation curves of galaxies and the Tully-Fisher law, $L \sim v^4$. The fits were obtained for large as well as small galaxies.
We also found that it is reasonable to expect that we can explain some of the cluster dynamics effects normally attributed to large amounts of dark matter, although further work has to be carried out to improve on the point source force law, before definite conclusions can be drawn about the fits to clusters of galaxies.

We also showed that gravitational lensing predictions can be made on the basis of the light deflection predicted at galaxy scales, since the renormalized gravitational constant produces effects that simulate those thought to be due to significant amounts of dark matter in galaxy halos.

The Yukawa-like force produced in the weak field limit of the NGT field equations is a non-additive force that does not violate the weak equivalence principle, and which appears to be consistent with solar system and terrestrial experiments when considered as a self-consistent solution of the NGT field equations.

ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada. We thank L. Demopoulos and S. Tremaine for helpful and stimulating discussions.
REFERENCES

[1] J. N. Bahcall, C. Flynn, A. Gould and S. Kirhakos, Ap. J. 435 (Letters) L51, (1994).

[2] S. M. Kent, Ap. J. 93, 816, 1993.

[3] M. Aaronson et al., Ap. J. Suppl. Series, 50, 241 (1982).

[4] R. H. Sanders, Astr. Ap. Rev. 2, 1 (1990).

[5] K. M. Ashman, Publications of the Astron. Soc. of the Pacific 104, 1109 (1992).

[6] For a review and references, see: J. D. Bekenstein, in Proc. 2nd Canadian Conf. on General Relativity and Relativistic Astrophysics, eds. C. Dyer and T. Tupper (Singapore, World Scientific, 1988), p.68.

[7] R. H. Sanders, Astr. Ap. 136, L21-L23 (1984).

[8] R. H. Sanders, Mon. Not. R. Aston. Soc. 223, 539 (1986).

[9] R. H. Sanders, Astr. Ap. 154, 135-144 (1986).

[10] M. Milgrom, Ap. J. 302, 617 (1986).

[11] D. H. Eckhardt, Phys. Rev. D 48, 3762 (1993).

[12] P. D. Mannheim, Found. Phys. 24, 487 (1994); P. D. Mannheim and D. Kazanas, Gen. Relativ. Gravit. 26, 337 (1994).

[13] V. V. Zhytnikov and J. M. Nester, National Central University, Chung-Li, Taiwan, preprint, gr-qc/9410002, 1994.

[14] C. W. Will, Theory and Experiment in Gravitational Physics, Cambridge University Press, Cambridge, revised edition 1993.

[15] J. W. Moffat, Phys. Letts. B355, 447 (1995).

[16] J. W. Moffat, J. Math. Phys. 36, 3722 (1995); errata, J. Math. Phys., to be published.
[17] J. Légaré and J. W. Moffat, Gen. Rel. Grav. 27, 761 (1995).

[18] J. Légaré and J. W. Moffat, Toronto preprint, UTPT-95-19, 1995. gr-qc/9509035.

[19] C. W. Stubbs et al., Phys. Rev. Lett. 58, 1070 (1987); E. G. Adelberger et al., ibid 59, 849 (1987).

[20] T. M. Niebauer, M. P. McHugh, and J. E. Faller, Phys. Rev. Lett. 59, 609 (1987); in Neutrinos and Exotic Phenomena, Moriond Meetings 1988 and 1990. Editions Frontières, Gif-sur-Yvette, France.

[21] R. B. Tully and J. R. Fisher, Astr. Ap. 54, 661 (1977).

[22] see, e.g., V. M. Mostepanenko and I. Yu. Sokolov, Phys. Lett. A132, 313 (1988).

[23] J. Binney and S. Tremaine, Galaxy Dynamics, Princeton University Press, 1987.

[24] F. D. Kahn and L. Woltjer, Ap. J. 130, 705 (1959).

[25] J. Tyson, F. Valdes and R. Wenk, Ap. J. (Letters) 349, L19 (1990).

[26] E. Fischbach et al., Metrologia 29, 213 (1992).

[27] V. M. Mostepanenko and I. Yu. Sokolov, Phys. Rev. D 47, 2882 (1993).