Generation of $N$-qubit $W$ state with rf-SQUID qubits by adiabatic passage

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Abstract

A simple scheme is presented to generate $n$-qubit $W$ state with rf-superconducting quantum interference devices (rf-SQUIDs) in cavity QED through adiabatic passage. Because of the achievable strong coupling for rf-SQUID qubits embedded in cavity QED, we can get the desired state with high success probability. Furthermore, the scheme is insensitive to position inaccuracy of the rf-SQUIDs. The numerical simulation shows that, by using present experimental techniques, we can achieve our scheme with very high success probability, and the fidelity could be eventually unity with the help of dissipation.

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Besides being an essential resource for testing quantum mechanics against local hidden theory \cite{1}, entanglement also has many applications in quantum information processing (QIP) such as quantum computation \cite{2}, quantum cryptography \cite{3}, quantum teleportation \cite{4} and so on. Recently, five-photon entanglement \cite{5}, six-ion Greenberger-Horne-Zeilinger (GHZ) state \cite{6} and eight-ion \(W\) state \cite{7} have been experimentally reported. The \(W\) state, whose general form is \(W_N = \frac{1}{\sqrt{N}}|N-1,1\rangle\) with \(|N-1,1\rangle\) being all the totally symmetric states involving \(N-1\) zeros and 1 one, is famous for its entanglement robustness to local operation, even under qubit loss. As far as we know, while there are some theoretical proposals discussing the generation of \(W\) states in cavity QED using atoms \cite{8, 9, 10, 11}, quantum dots \cite{12}, and rf-SQUIDs \cite{13, 14}, we have not yet found any experimental report of the \(W\) state in cavity QED. In Refs. \cite{8, 9}, atoms are required to pass through the cavity with a certain velocity or velocity distribution, which is experimentally challenging. Measurement of leaky photons \cite{10, 15} can also lead to multi-atom entanglement, but it is only probabilistic to get the desired state in one trial and the measurement requires highly efficient detector. Ref. \cite{13} proposed to generate multi-rf-SQUID \(W\) states via nonresonant interaction with the cavity along with auxiliary measurement, which evidently restricts the speed and success probability. In Ref. \cite{14}, by Raman transition, the \(W\) state can be efficiently generated. However, like most of the schemes mentioned above, it requires precise control of the interaction time.

On the other hand, based on Josephson junction, superconducting qubits including charge qubits \cite{16, 17}, flux qubits \cite{17, 18}, and phase qubits \cite{19}, have attracted much attention due to their potential for scalability. The rf-SQUID, which consists of an enclosed superconducting loop interrupted by a Josephson junction, is the simplest design for flux qubits. When rf-SQUIDs are embedded in a cavity, the strong coupling limit of the cavity QED, i.e., \(g^2/(\Gamma \kappa) \gg 1\) (parameters explained later), which is difficult to achieve with atoms in cavity, can be easily realized \cite{20}. In this paper, we show how to generate multi-qubit \(W\) states with rf-SQUIDs in a cavity by adiabatic passage \cite{20, 21}. The biggest merit of using adiabatic passage is that it does not need precise control of the Rabi frequency and pulse duration, and the population in excited states is negligible due to their absence in the dark state used for evolution. Our scheme has the following advantages: (i) The rf-SQUIDs are fixed in the cavity, which lowers the experimental difficulty for control comparing to previous schemes \cite{8, 9, 22} with atoms flying through cavities or to schemes \cite{23} that require the
trapped atoms to be well localized; (ii) By virtue of adiabatic passage, there is no need for
precise control of the Rabi frequency and the pulse duration; (iii) By adding the microwave
pulse collinearly with the cavity mode, the scheme is insensitive to position inaccuracy of
the rf-SQUIDs; (iv) With present experimental technology, the success probability is very
high and the fidelity is eventually unity with the help of dissipation; (v) Qubits are encoded
in the two lowest flux states. So once it is prepared, the $W$ state can remain in the cavity
almost without energy relaxation.

Let us consider $N$ rf-SQUIDs, each of which has a ∧-type configuration formed by two
lowest levels and an excited level, as shown in Fig. 1. For a rf-SQUID $j$, the classical field
drives the transition resonantly between the level $|1\rangle_j$ and the level $|e\rangle_j$ with Rabi frequency
$\Omega_j$, while the cavity field couples resonantly to the level $|0\rangle_j$ and the level $|e\rangle_j$ with coupling
constant $g_j$. The Hamiltonian of a rf-SQUID $j$ with junction capacitance $C_j$, loop inductance
$L_j$ and externally applied biased flux $\Phi_{x,j}$ can be written in the usual form [17, 20, 24, 25],

$$H_{s,j} = \frac{Q_j^2}{2C_j} + \frac{(\Phi_j - \Phi_{x,j})^2}{2L_j} - E_{J,j} \cos(2\pi \frac{\Phi_j}{\Phi_0}), \quad (1)$$

where $\Phi_j$ is the magnetic flux threading the loop, $Q_j$ is the charge on the leads, and
$E_{J,j} = I_{c,j} \Phi_0/2\pi$ is the Josephson energy with $I_{c,j}$ the critical current of the junction and
$\Phi_0 = \hbar/2e$ the flux quantum. So the expressions of $g_j$ and $\Omega_j$ are given, respectively, by
[20, 25],

$$g_j = \frac{1}{L_j} \sqrt{\frac{\omega_c}{2\mu_0 \hbar}} \int_S {\cal B}_c^j(r) \cdot dS, \quad (2)$$

$$\Omega_j = \frac{1}{L_j \hbar} \int_S {\cal B}_{\mu}^j(r, t) \cdot dS,$$

here $\omega_c$ is the cavity frequency, $B_c^j(r)$ and $B_{\mu}^j(r)$ are the magnetic component of the cav-
ity mode and the classical microwave in the superconducting loop of the $j$th rf-SQUID,
respectively. We assume that all the $N$ rf-SQUIDs are identical. The Hamiltonian of $N$
rf-SQUIDs interacting simultaneously with a single cavity mode and a microwave pulse in
the interaction picture is described by (assuming $\hbar = 1$)

$$H_I = \sum_{j=1}^N g_j (a^\dagger |0\rangle_{jj} \langle e| + a|e\rangle_{jj} \langle 0|) + \Omega_j(t) (|1\rangle_{jj} \langle e| + |e\rangle_{jj} \langle 1|), \quad (3)$$
where $a^\dagger$ and $a$ are, respectively, the creation and annihilation operators for the cavity mode. Initially all the qubits are in the ground state, i.e., $|0\rangle_{1,2,\ldots,N}$ and the cavity is in the single-photon state $|1\rangle$. The dark state associated with such an initial state is

$$|D(t)\rangle \propto |0\rangle_{1,2,\ldots,N}|1\rangle - \left(\frac{g_1}{\Omega_1(t)}|1\rangle_1|0\rangle_{2\ldots}|0\rangle_{N-1}|0\rangle_N + \frac{g_2}{\Omega_2(t)}|0\rangle_1|1\rangle_2|0\rangle_{N-1}|0\rangle_N + \ldots + \frac{g_N}{\Omega_N(t)}|0\rangle_1|0\rangle_2|0\rangle_{N-1}|1\rangle_N|0\rangle\right). \quad (4)$$

If the microwave pulse is input collinearly with the cavity axis and thereby the driving pulse will have the same spatial mode structure as the cavity mode \[26\], i.e., $B_{\mu w}(r, t) \propto \tilde{\Omega}(t)B_{e}^{j}(r)$, with $\tilde{\Omega}(t)$ being a variable that reflects the time dependence of the microwave pulse’s amplitude, then we have $g_j/\Omega_j(t) = K/\tilde{\Omega}(t)$ ($j = 1, 2, \ldots, N$ and $K$ is a constant) and Eq. (4) reduces to

$$|D(t)\rangle \propto |0\rangle_{1,2,\ldots,N}|1\rangle - \frac{K\sqrt{N}}{\Omega(t)}W_{N}|0\rangle. \quad (5)$$

By reducing $\tilde{\Omega}(t)$ adiabatically, we can get the $N$-qubit $W$ state $W_{N}$ without the requirement of accurate positions of the rf-SQUIDs. To carry out our scheme, we first need to prepare the single-photon cavity state, which can be done as follows: Initially the cavity is in the vacuum state $|0\rangle$ and the first rf-SQUID is in state $|1\rangle_1$, while the other $N-1$ rf-SQUID qubits are in state $|0\rangle_2,3,\ldots,N$. After simultaneously adjusting, by the externally applied biased flux $\Phi_{x,j}$, the level spacing of the rf-SQUID qubits, from the second to the $N$th, to be decoupled from the cavity mode and the microwave, we drive $|1\rangle_1|0\rangle$, by a microwave with increasing amplitude, to evolve along the dark state $g_1|1\rangle_1|0\rangle - \Omega_1(t)|0\rangle_1|1\rangle$ to the single-photon cavity state and the state $|0\rangle_1$ of the first rf-SQUID. Then by adjusting the biased flux again, we bring back the level spacing of the other $N-1$ rf-SQUID qubits simultaneously to the original situation.

To check the feasibility of our scheme, we consider below the decaying effects on our $W$ state preparation. To this end, we will numerically simulate the system evolution by quantum jump approach \[27\] under the condition that no photon leakage actually happens, due to either the spontaneous emission from the rf-SQUID’s excited state or the decay from the cavity mode, during the preparation period. In real experiments, the values of $g_j$ and $\Omega_j(t)$ may vary from qubit to qubit, but their ratio is a constant as analysed above, which
guarantees that it has no effect on the fidelity of the generated \( W \) state. For simplicity, we assume all the qubits have the same coupling strengths, i.e., \( g_j = g, \Omega_j(t) = \Omega(t) \) \((j = 1, 2, ..., N)\). Similar to Ref. [20], we suppose \( \Omega(t) = g \times 40 \times \exp[-t^2/2\tau^2] \) with \( \tau = 4/g \) and choose the coupling \( g = 1.8 \times 10^8 \text{ s}^{-1} \), the spontaneous emission rate of the excited level \( \Gamma = 4 \times 10^5 \text{ s}^{-1} \), and the cavity decay rate \( \kappa = 1.32 \times 10^6 \text{ s}^{-1} \), which are available with the present experimental technology [20]. In Fig. 2, we show the time evolution of the three-qubit case. In Fig. 2 (a), we find small oscillations of the unwanted states. By enlarging \( \Gamma \) to \( 4 \times 10^7 \text{ s}^{-1} \) in Fig. 2 (b), we can much suppress those small oscillations. This is because larger spontaneous emission rate can damp the population of unwanted states faster, so the fidelity in (b) at the time \( t = 25/g \) is \( F = 0.9994 \), higher than that in (a) with \( F = 0.9946 \), at the cost of a lower success probability at that time point. Obviously, with the help of dissipation, the fidelity is eventually unity. We have also plotted Fig. 3 to show how our scheme works with the multi-qubit case: For a certain qubit number, the longer the evolution time, the higher the fidelity and the lower the success probability, and eventually, the fidelity could be unity and the success probability remains a constant. We show in Fig. 3 that the fidelities are almost unity at time \( t = 50/g \) for the qubit number ranging from 3 to 80, and the corresponding success probabilities are all higher than 90.5\%. The optimal success probability and fidelity for a certain qubit number can be obtained by changing the pulse maximum height and pulse duration time. For simplicity, we have assumed in our numerical simulation the same pulse for different qubit number cases, because we just want to illustrate that our scheme could work well in the case of different qubit number.

As mentioned above, there have been some proposals for preparing \( W \) state in cavity QED. For example, in Refs. [8, 9], atoms need to be well controlled with velocity in passing through the cavity due to the requirement of the interaction time, which is experimentally very difficult. In contrast, SQUIDs are static and controllable in our scheme. Ref. [11] also generated the multi-atom \( W \) state by adiabatic passage. But due to the small value of \( g^2/(\Gamma \kappa) \) regarding the atom-cavity system, the success probability is low. Moreover, as the scheme starts from the initial state, i.e., \( |1\rangle_1|0\rangle_2,3,...,N|0\rangle \), the timescale for the population transfer is ten times larger than that in our scheme [28], which further reduces the success probability. Refs. [10, 13] require auxiliary measurement, so the success probability would also be low. In contrast, our scheme can be reached with high success probability because of the achievable strong coupling limit of cavity QED with the rf-SQUIDs. Thus we can reduce
the repetitional times in experiments. Furthermore, by Raman transition, the W state can also be generated with high success probability in Ref. [14], based on the requirements of precise control of the pulse duration time and of accurate positions of the SQUIDs. However, in real experiments, these requirements are hard to be met. In this sense, our scheme is much advantageous with the adiabatic passage and the driving pulse added collinearly with the cavity mode.

In summary, we have presented a simple scheme of generating $N$-qubit W state with rf-SQUIDs by adiabatic passage. Our numerical simulation has shown that our scheme can be achieved with high success probability and unity fidelity. Moreover, it has some favorable features for experimental implementation. So we argue that our proposal might be experimentally realized with present technology and would be useful for experimental implementation of solid-state quantum information processing.

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[28] It is found by our simulation from the initial state $|1\rangle_1|0\rangle_{2,3,...,N}|0\rangle$ with the same parameters as in Fig. 2 (b).
Figure captions

FIG. 1. The level configuration for the $j$th rf-SQUID, where level $|1\rangle_j$ and the level $|e\rangle_j$ are coupled resonantly by the classical microwave, while level $|0\rangle_j$ and the level $|e\rangle_j$ are coupled resonantly by the cavity mode. $\Omega_j$ and $g_j$ are coupling strengths regarding the microwave and the cavity mode, respectively. The qubits are encoded in levels $|0\rangle_j$ and $|1\rangle_j$.

FIG. 2. Time evolution with respect to the three-qubit case, where $g = 1.8 \times 10^8$ s$^{-1}$, $\kappa = 1.32 \times 10^6$ s$^{-1}$ and $\Gamma = 4 \times 10^5$ s$^{-1}$ in (a) and $\Gamma = 4 \times 10^7$ s$^{-1}$ in (b). The dotted, dashed and solid lines represent the populations of $|0\rangle_1|0\rangle_2|0\rangle_3|1\rangle$, of the $W$ state (i.e., $|1\rangle_1|0\rangle_2|0\rangle_3|0\rangle$, $|0\rangle_1|1\rangle_2|0\rangle_3|0\rangle$, $|0\rangle_1|0\rangle_2|1\rangle_3|0\rangle$) and of the states containing the excited state $|e\rangle_j$ (i.e., $|e\rangle_1|0\rangle_2|0\rangle_3|0\rangle$, $|0\rangle_1|e\rangle_2|0\rangle_3|0\rangle$, $|0\rangle_1|0\rangle_2|e\rangle_3|0\rangle$), respectively, while the dashed-dotted line denotes the success probability $P(t)$. The values of $F$ and $P$ labeled in the plot are the fidelity and success probability at time $t = 25/g$.

FIG. 3. Success probability $P$ and fidelity $F$ versus the qubit number $N$, where $g = 1.8 \times 10^8$ s$^{-1}$, $\kappa = 1.32 \times 10^6$ s$^{-1}$ and $\Gamma = 4 \times 10^7$ s$^{-1}$. The circle and five-pointed star denote the fidelities at time $t = 25/g$ and $50/g$, respectively, and the plus sign and the asterisk are their corresponding success probabilities.
(a) $P=0.9295$
$F=0.9946$

(b) $P=0.9134$
$F=0.9994$
