Analysis on harmonic characteristic of transformer DC bias caused by metro stray current

Xinxiang Cheng1,a*, Yanru Ni1,b, Kun Yu1,c, Xiangjun Zeng1,d, Chao Zhuo1,e and Wei Han2,f

1Changsha University of Science and Technology, Changsha, Hunan Province, China
2State Grid Taiyuan Power Supply Company, Taiyuan, Shanxi Province, China
achxinee028@stu.csust.edu.cn, byanruni@qq.com, kunyu0707@163.com,
dexjzeng@163.com, ezhuochoa106@qq.com
f674065110@qq.com
*Corresponding author’s e-mail: chxinee028@stu.csust.edu.cn

Abstract—With the continuous improvement of the economic level, urban rail transit has been vigorously developed. However, the metro stray current will be generated during the actual operation of urban rail, which will cause transformer DC bias along the metro line. In order to effectively evaluate the influence of the transformer on the DC bias under urban rail operation, the stray current distribution model with single and double-ended power supply mode was established, and the numerical calculation of the stray current was realized. Based on the real-time dynamic change characteristics of the stray current, a synchronous simulation model of the transformer DC bias was established, and the harmonic characteristics of the transformer excitation current were further analyzed. The simulation test shows that when the direct current is injected into the neutral point of the transformer, the excitation current waveform is asymmetrical along the positive and negative semi-axes, and the odd and even harmonics coexist. And the influence of transformer DC bias is weakened when transformer air core reactance is increased. The characteristic quantities such as excitation current amplitude, waveform characteristics, and harmonic content have been quantitatively analyzed in this paper, which has certain guiding significance for evaluating the influence of transformer DC bias.

1. Introduction

The construction of urban rail transit is related to the people's livelihood. It is not only an important support for building a smart city and a transportation power, but also a leading field to open the comprehensive construction of a socialist modern power. The 14th Five-Year Plan puts forward the goal of vigorously developing urban rail transit and adding 3000 km of urban rail running mileage in the construction project of transportation power [1]. By June 2021, the running mileage of urban rail transit in China has exceeded 7900 km. In recent years, a new round of urban rail transit construction planning of 35 cities has been approved by the National Development and Reform Commission, the construction of urban rail transit has maintained a trend of rapid development [2].

During the actual running of urban rail, it will have an adverse impact on the surrounding environment and facilities. Traction substation converts three-phase high-voltage AC into low-voltage DC suitable for urban rail running, forming a current loop between substation, catenary, rail and substation. Due to the existence of transition resistance, the rail can not be completely insulated from
the ground, and stray current will be generated during the running of urban rail. Direct current has electrochemical corrosion characteristics, and buried metals are corroded under long-term electrochemical action [3]. In addition, the stray current entering the ground flows through the grounding device of the AC power grid. It will cause the neutral grounding transformer DC bias along the metro line, leading to transformer vibration noise and heating. This can also damage to the transformer internal structure, which seriously threaten the safe and stable operation of the power grid. At present, the research work on DC bias of transformer caused by stray current mainly focuses on vibration, noise, overheating and suppression of transformer [4-5]. Reference [6] studies the frequency spectrum characteristics of transformer noise under DC bias, and comes to the conclusion that the surge of excitation current leads to the overheating of magnetic circuit and the high-frequency vibration of transformer caused by odd wave noise; On the basis of summarizing the previous DC bias suppression technology, reference [7] proposed a suppression measure with dual protection effect. Reference [8-9] analyzes the influence of train running state and soil resistivity on stray current, and establishes a stray current simulation model. However, the research on the characteristics of transformer excitation current under DC bias has not been in-depth. Reasonable and efficient evaluation of transformer DC bias caused by urban rail is helpful to the prevention of urban rail stray current and the suppression of DC bias at the power grid side.

In this paper, the numerical model of urban rail stray current is built, and the numerical variation rule of stray current under the change of random vehicle operation is obtained. Based on this, the DC magnetic bias simulation model of transformer is established. The time-frequency domain distribution characteristics of transformer electrical quantity are studied according to the change of DC magnetic bias with the running state of urban rail, time and locomotive status. By changing transformer parameters, the DC bias of transformer caused by the running of urban rail is evaluated reasonably.

2. Numerical modeling of metro stray current
At present, the power source of urban rail is DC power supply system, which starts from the positive pole of traction substation and returns to the negative pole of substation through contact network and rail. However, the rail can not completely insulate the ground, which leads to some traction current leaks into the ground to form stray current. The schematic diagram of metro stray current as shown in Fig.1.

![Fig.1 Schematic diagram of stray current](image)

Under different power supply modes, the stray current of urban rail is modeled numerically. Single ended power supply means that the train runs only when one traction substation provides traction current and the power source of the locomotive is single-ended power supply. On the other hand, the traction current obtained by the locomotive from two traction substations is double-ended power supply. In the actual running of the train, the numerical modeling of the stray current on the urban rail is limited by some factors such as soil resistivity, rail location, locomotive speed and so on in different sections. The ideology of mathematical calculus and Kirchhoff's law are used for theoretical derivation. It is assumed that the longitudinal resistance and transition resistance are evenly distributed in the loop, and the parameters of two traction substations are the same in the two-end power supply mode.
2.1. Stray current distribution model of urban rail under single terminal power supply

Under the mode of single-end power supply. The mathematical model of rail ground double-layer resistance distribution network is established, which are shown in Fig.2. Assuming that x=0 at traction substation, parameter settings are shown in Table 1.

| Parameter | Definition       | Unit       | Parameter | Definition       | Unit       |
|-----------|------------------|------------|-----------|------------------|------------|
| x         | Distance         | km         | I         | Traction current | A          |
| u(x)      | Rail voltage     | V          | R_p      | Rail longitudinal resistance | Ω / km |
| i(x)      | Rail current     | A          | R_g      | Rail to ground transition resistance | Ω·km |
| i_s(x)    | Stray current    | A          | L        | Distance between train and substation | km |

![Fig.2 Single-end power supply resistance distribution diagram](image)

![Fig.3 Distribution diagram of track voltage and current under single terminal power supply mode](image)

According to Kirchhoff's law and mathematical calculus in Fig.3. The KVL equations are written from Fig.3(a) node voltage diagram:

\[ i(x) \cdot R_p \cdot dx + u(x) - [u(x) + du(x)] = 0 \]  \( (1) \)

\[ u(x) = R_g \cdot di(x) / dx \]  \( (2) \)

\[ u(x) \cdot \frac{R_p}{R_g} = \frac{d^2u(x)}{dx^2} \]  \( (3) \)

Figure \( \alpha^2 = \frac{R_p}{R_g} \), the following can be obtained:

\[ u(x) = a \cdot c \alpha x + b \cdot s \alpha x \]  \( (4) \)
Taking the derivation of (4), bring the boundary conditions
\[
\begin{aligned}
  i(x) &= I \\
  i'(x) &= \alpha
\end{aligned}
\]
into (4) to obtain:
\[
\begin{aligned}
a &= -\sqrt{R_p R_g} \cdot I \cdot \text{th} \frac{\alpha}{2} \frac{L}{2} \\
b &= \sqrt{R_p R_g} \cdot I \\
u(x) &= I \sqrt{R_p R_g} \frac{\text{sh} \alpha \left( x - \frac{L}{2} \right)}{\text{ch} \alpha L / 2} \\
i(x) &= I \frac{\text{ch} \alpha \left( x - \frac{L}{2} \right)}{\text{ch} \alpha L / 2}
\end{aligned}
\]

In summary, metro stray current can be obtained:
\[
i_w(x) = I - I \frac{\text{ch} \alpha \left( x - \frac{L}{2} \right)}{\text{ch} \alpha L / 2}
\]

Considering the actual situation of train operation, \( I = 1000 \text{A}, L = 2 \text{km}, R_p = 0.025 \Omega/\text{km}, R_g = 14 \Omega \cdot \text{km} \).
Through parameter simulation calculation:

![Fig.4 Stray current distribution in single-end power supply mode](image)

As can be seen from Fig.4. The simulation results show that the metro stray current has a hyperbolic distribution, and the metro stray current at the middle position between the traction substation and the locomotive reaches its peak value, which close to 1A. In addition, the traction current is the sum of track current and metro stray current at any time and position during locomotive running.

2.2. Stray current distribution model of urban rail under two terminal power supply mode

Under double terminal power supply mode, parameter settings are shown in Table 2.

| Parameter | Definition | Unit |
|-----------|------------|------|
| \( I_1 \) | Traction current of substation 1 | \( A \) |
| \( I_2 \) | Traction current of substation 2 | \( A \) |
| \( L_1 \) | Distance between train and substation 1 | \( km \) |
| \( L_2 \) | Distance between train and substation 2 | \( km \) |
The rail ground resistance distribution model under two terminal power supply mode can be equivalent to the numerical model under two single terminal power supply modes, which are shown in Fig.5.

Fig.5 Double terminal power supply resistance distribution diagram

Taking the locomotive as the center and referring to the calculation process under single terminal power supply mode. Considering the locomotive running in different sections, the metro stray current can be obtained as:

When $x \in [0, L_1]$:

$$i_{x1}(x) = I_1 - I_2 \frac{\text{cha} \left( x - \frac{L_1}{2} \right)}{\text{cha} L_1 / 2}$$  

(10)

When $x \in [L_1, L]$:

$$i_{x2}(x) = I_2 - I_1 \frac{\text{cha} \left( L - x - \frac{L_2}{2} \right)}{\text{cha} L_2 / 2}$$  

(11)

Considering the actual situation of train operation, $I=1000A$, $I_1=750A$, $I_2=250A$, $L_1=0.5km$, $L_2=1.5km$, $L=2km$, $R_p=0.025 \Omega/km$, $R_g=14 \Omega \cdot km$. Through parameter simulation calculation, the numerical law of stray current varying with train operation are shown in Fig.6.

The distribution curve shows that the metro stray current reaches its maximum value at the midpoint of each interval, and its peak value is smaller in double-ended mode than in single-ended mode, which proves that under double-ended mode, the leakage current of rail is smaller and the stray current entering the ground is reduced.
3. Simulation of DC magnetic bias in transformers

3.1. Principle of transformer DC bias
The stray current generated by urban rail running is injected into the ground, resulting in the change of substation ground potential in the nearby AC power grid, which leads to potential difference between substations at different locations. When there are multiple neutral grounded transformers in different positions, the potential difference makes the DC current pass through the neutral point of the transformer, resulting in DC bias of the transformer. When the transformer has no DC injection, the excitation current is sine wave, which are shown in Fig.7. The magnetization curve of transformer core has nonlinear characteristics. When DC current passes through transformer winding, DC bias occurs in transformer. Under the superposition of DC flux and AC flux, the flux in the same direction as the bias increases and the flux in the opposite direction decreases, resulting in the distortion of the excitation current waveform, the positive and negative half axis waveforms are asymmetric [10].

![Fig.7 Mechanism diagram of transformer DC bias](image_url)

3.2. Simulation of transformer DC bias
In order to study the time-frequency distribution characteristics of transformer electrical quantities, especially the characteristic quantities of excitation current, PSCAD / EMTDC platform is selected for transformer DC bias simulation test. PSCAD / EMTDC adopts time-domain analysis method, which is suitable for solving power system differential equations. While changing parameters to simulate different test conditions, it can read test results and curves through output charts, so as to improve the efficiency of scientific research. The topology diagram of transformer DC magnetic bias is shown in Fig.8.

![Fig.8 Transformer DC bias topology circuit diagram](image_url)
Two three-phase star double winding transformers T1 and T2 are selected as the simulation objects. The controlled current source is connected in series between the T1 star winding #2 and T2 star winding #1 neutral point to simulate the injection of DC. The value of the current source is controlled by the slider. By adjusting the slider, different values of DC are injected into the neutral point of the transformer to simulate the DC bias of the transformer under various conditions. The excitation current of three-phase transformer is approximately no-load current, parameter settings are shown in Table 3.

| Parameter | Definition | Unit |
|-----------|------------|------|
| \( f \)   | System frequency | 50 Hz |
| \( U_0 \) | Three phase voltage source value | 15 kV |
| \( S_1 \) | Rated capacity of transformer T1 | 40 kVA |
| \( S_2 \) | Rated capacity of transformer T2 | 10 kVA |
| \( U_{1,1} \) | T1 winding #1 line voltage | 15 kV |
| \( U_{1,2} \) | T1 winding #2 line voltage | 0.2 kV |
| \( U_{2,1} \) | T2 winding #1 line voltage | 0.2 kV |
| \( U_{2,2} \) | T2 winding #2 line voltage | 0.2 kV |

When the injected DC value is 0, the transformer has no DC bias and no excitation current distortion, and its waveform is positive and negative semi-axisymmetrical sine wave. The fast Fourier transform module (FFT) is used to convert the excitation current waveform. When \( I_{DC} = 0 \) A, the excitation current does not contain even harmonics, which are shown in Fig.9.

[Fig.9 Analysis diagram of excitation current harmonic characteristics when \( I_{DC} = 0 \) A]

When direct current is injected into the neutral point of the transformer, DC bias occurs in the transformer, which leads to the magnetic flux density of half cycle wave with the same direction of total flux and bias flux increases greatly, and the amplitude of excitation current waveform shifts towards the positive half axis, the excitation current waveform is asymmetric along the positive and negative half axes, along with odd and even harmonics coexist. Gradually increasing the injected DC value, the distortion degree of excitation current is higher and higher, and the harmonic content increases gradually, which are shown in Fig.10-12.

[Fig.9 Analysis diagram of excitation current harmonic characteristics when \( I_{DC} = 0 \) A]
The simulation results show that the transformer excitation current has no distortion when there is no DC injection in the transformer winding, and its waveform is about positive and negative semi-axisymmetrical sine wave, and only contains odd harmonics. With the increase of injected DC, the density of magnetic flux in the same direction as the biased magnetic flux increases, and the dc and AC magnetic flux are superimposed. The higher the distortion degree of excitation current waveform is, the content of even harmonics increases gradually, and even and odd harmonics coexist.

4. Influencing factors and evaluation of transformer DC bias
When direct current is injected into the winding, the excitation current waveform is distorted. Changing transformer parameters to limit the impact of direct current can weaken the impact of DC bias of transformer to a certain extent. When $I_{DC}=10A$, the influence of air core reactance on DC bias of transformer is evaluated by changing the parameters of transformer air core reactance, which are shown in Fig.13.
Curve II shows the excitation current waveform when the transformer air core reactance is 0.09pu. The excitation current is seriously distorted, the wave crest is sharp, and the waveform is completely asymmetric along the positive and negative half axes. Curve I shows the excitation current waveform when the air core reactance of the transformer is 5pu. When the injected DC current is 10A, it can still maintain a relatively complete sinusoidal waveform. It is proved that increasing the value of transformer air core reactance can weaken the influence of transformer DC bias to a certain extent.

5. Conclusion
Firstly, this paper models the distribution characteristics of urban rail stray current under two different power supply modes, studies the distribution law of stray current random vehicle operation state change, simulates the DC bias of transformer based on PSCAD / EMTDC platform, analyzes the harmonic characteristics of excitation current, and comes to the conclusion: when there is no DC injection in excitation winding, the DC bias of transformer does not occur. The excitation current waveform is a sine wave symmetrical along the positive and negative semi axes, which contains only odd harmonics at this time. When DC power is injected into the transformer winding, the excitation current waveform is distorted, the flux density in the same direction as the bias increases, the AC flux and DC flux are superimposed, the waveform is asymmetric along the positive and negative half axes, and odd and even harmonics exist at the same time. By changing the transformer parameters, the influence degree of transformer DC bias is simulated. When designing the transformer parameters, increasing the transformer air core reactance can reduce the influence of DC bias to a certain extent.

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