Measuring anomalous $Wtb$ couplings at $e^-p$ collider

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ABSTRACT: We study the physics potential of the proposed Large Hadron electron Collider (LHeC) by estimating the accuracy with which it can measure the anomalous $Wtb$ couplings in the single anti-top quark production through $e^-p$ collisions. We consider the generic lowest order $CP$ conserving Lagrangian for the $Wtb$ interaction, which allows a right-handed vector, as well as left- or right-handed tensor couplings. We examine the one dimensional distributions of the various kinematic observables and their asymmetries corresponding to all anomalous couplings in both hadronic and leptonic decay modes of $W^-$. We find that at $95\,\%\,$C.L. the anomalous coupling associated with the left handed vector current can be measured at an accuracy of the order of $\sim 10^{-2} - 10^{-3}$, while those associated with the right handed vector/tensor and left handed tensor currents are sensitive at the order of $\sim 10^{-1} - 10^{-2}$ corresponding to the systematic uncertainty varying between 10%-1% at an integrated luminosity of 100 fb$^{-1}$. We further analyzed the combined covariance matrix derived from all one dimensional distributions of kinematical observables in the hadronic and leptonic decay modes of $W^-$ to compute the errors of anomalous couplings and their correlations. The impact of the luminosity uncertainty on the errors and their correlations are also studied.
1 Introduction

The top quark being the heaviest particle in the Standard Model (SM) provides an excellent opportunity for the study of electroweak symmetry breaking mechanism as well as to provide glimpse of new physics (NP) beyond the SM. The top (anti-top) quark decays almost exclusively in the $t \rightarrow bW^+ (\bar{t} \rightarrow \bar{b}W^-)$ channel. As a consequence of its lifetime ($\sim 10^{-25}$ s), being one order of magnitude smaller than the typical hadronization time scale ($\sim 10^{-24}$ s), its spin information is transferred to the decay products. The kinematic distributions of decayed particles from top (anti-top) quark provide the information about the $W^+tb$ ($W^-\bar{b}$) vertex and associated new physics potentiality with the top (anti-top) quark production mechanism.

Within the SM, the $Wtb$ vertex is purely left-handed, and its amplitude is given by the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{tb}$, related to weak interaction between a top and a $b$-quark and assuming $|V_{td}|^2 +...
$|V_{ts}|^2 \ll |V_{tb}|^2$. The most general, lowest dimension, $CP$ conserving (in effect of which couplings are real), Lagrangian for the $Wtb$ vertex is given by \[1–3\]

$$\mathcal{L}_{Wtb} = \frac{g}{\sqrt{2}} \left[ W_\mu \bar{t} \gamma^\mu (V_{tb}) f_1^L P_L + f_1^R P_R \right] b - \frac{1}{2 m_W} W_{\mu \nu} \bar{t} \sigma^{\mu \nu} \left( f_2^L P_L + f_2^R P_R \right) b + h.c. \quad (1.1)$$

where $f_1^L \equiv 1 + \Delta f_1^L$, $W_{\mu \nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $P_L,R = \frac{1}{2} (1 \pm \gamma_5)$ are left- and right-handed projection operators, $\sigma^{\mu \nu} = i/2 (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$ and $g = e/\sin \theta_W$. Since in the SM $|V_{tb}| f_1^L \simeq 1$, the $\Delta f_1^L$ along other couplings $f_2^L, f_1^R, f_2^R$ vanish at tree level, while there non-vanishing values are generated at the one loop level \[4\].

**Figure 2.** Single anti-top quark production cross section at the LHec with the variation of electron energy $E_e$ and fixed proton energy $E_p = 7 \text{ TeV}$. The top two curves depict the cross-section for $e^- p \to \nu_e l$ from the 80% polarized and unpolarized $e^-$ beam, respectively. The third and the fifth curve corresponds to the branching of the unpolarized cross-section into hadronic and leptonic decay modes of $W^-$. The first and the third curve from the below corresponds to the cross-section for $e^- p \to l \nu_e b$ branching to the leptonic and hadronic decay modes of $W^-$, respectively.

$Wtb$ anomalous couplings $f_i$ are constrained from flavor physics. The magnitudes of the right-handed vector and tensor couplings can be indirectly constrained by the measured branching ratio of the $b \to s \gamma$ process because they receive large contributions from loops involving the top quark and the $W$ boson. Current 95% C.L. bounds based on the CLEO data give $|f_1^R| \leq 4.0 \times 10^{-3}$ at the 2-\(\sigma\) level \[5–7\]. The branching ratio (BR) $BR(b \to s \gamma)$ is computed by neglecting the terms proportional to the square of $f_i$ in the matrix element squared. Assuming only one anomalous coupling to be non-zero at a time, the upper and lower limits for $|V_{tb}| f_1^L, f_1^R, f_2^L$ and $f_2^R$ obtained from the $B$ decays are $-0.13 \leq |V_{tb}| \Delta f_1^L \leq 0.03$, $-0.0007 \leq f_1^R \leq 0.0005$, $-0.0015 \leq f_2^L \leq 0.0004$ and $-0.15 \leq f_2^R \leq 0.57$, respectively \[8\]. If more than one coupling are taken non-zero simultaneously, their magnitudes in principle are not bound by $b \to s \gamma$ alone and the limits can be very different. Combining the analysis on $B_{d,s} = \bar{B}_{d,s}$ mixing and $B \to X_s l^+ l^-$, authors of reference \[9\] constrained $Wtb$ couplings within an effective field theory framework.
The sensitivity of anomalous $Wtb$ couplings can also be measured from $W^\pm$ helicity distributions arising from top or anti-top decays to their dominant $Wb$ mode in the top-antitop pair production processes [10]. It can also be measured from the observed single top or anti-top quark production cross section through $W$-boson exchange and has both the linear and quadratic terms in the effective couplings. Although the single top/anti-top production in the SM is comparable (only a little less than half) to the $t\bar{t}$ pair production, it is quite challenging to make the extraction due to considerable backgrounds at the Tevatron [11, 12] and the LHC [13, 14]. Recently DØ with 5.4 fb$^{-1}$ data reported a combined analysis of $W$ boson helicity studies and the single top quark production cross section exclusively through $Wtb$ vertex i.e. $|V_{td}|^2 + |V_{ts}|^2 \ll |V_{tb}|^2$. This sets upper limits on anomalous $Wtb$ couplings at 95% C.L. assuming $|V_{tb}| f^L_1 = 1$, $|f^L_2| \leq 0.224$, $|f^R_1| \leq 0.548$, $|f^R_2| \leq 0.347$ [15]. Sensitivity of the anomalous $Wtb$ couplings on the cross-section of the associated $tW$ production are also studied at LHC through $γp$ collision [16] and provide $|f^L_2| \leq 0.22$, $|f^R_1| \leq 0.55$, $|f^R_2| \leq 0.35$. The study of coefficients of dimension six operators affecting $Wtb$ couplings from electroweak precision measurements [17, 18], suggest that the upper limits on these couplings are one order of magnitude weaker, to those obtained directly from the helicity fraction study of the top decay at NLO QCD [19].

The sensitivity of the effective couplings in (1.1) corresponding to the different chiral vector and tensor current can be studied through one-dimensional distributions of kinematic observables. These distributions manifest a certain amount of associated asymmetry depending on the specific Lorentz structure, which can then be used as a discriminator to constraint these anomalous couplings. Based on associated asymmetries generated from the measured angular distributions of $\cos \theta^*$ defined in [20], the ATLAS collaboration [21] set limits on single anomalous couplings at 95% C.L. to be $\text{Re}(f^R_1) \in [-0.44, 0.48]$, $\text{Re}(f^L_2) \in [-0.24, 0.21]$ and $\text{Re}(f^R_2) \in [-0.49, 0.15]$. A combined constraint on anomalous

\[^1\]The cosine of the angle $\theta^*$ between the momentum direction of the charged lepton from the $W$-boson decay and the reversed momentum direction of the $b$ quark from top-quark decay, both boosted into the $W$-boson rest frame.
Figure 4. The variation of helicity fractions $F_-, F_0$ and $F_+$ as defined in the text with the anomalous coupling $f_i$.

couplings from CMS and ATLAS [22] shows the sensitivity of these couplings with respect to the helicity fraction in the top quark decays. Constraints on $Wtb$ vertex based on the angular asymmetries constructed from ATLAS data and the t-channel single top cross section in CMS have been analyzed in [23]. A projected sensitivity of all anomalous top couplings have also been studied in reference [24].

Effects of anomalous coupling on angular distributions of the $b$-quark and $\mu^+$ have been studied in $e^+e^-$ linear collider with one specific semileptonic channel in the double resonance approximation for the $t$ and $t$ production [25–28]. A preliminary study of the sensitivity of $Wtb$ anomalous couplings on the single top quark production cross-section in $e^-p$ collision for TESLA+HERA and LHC+CLIC energies has been performed in [29].

Recently a deep inelastic electron-nucleon scattering facility is proposed at the LHC, known as LHeC. It is proposed that an electron beam of 60 GeV will collide with 7 TeV proton beam simultaneous to the existing proton-proton collision experiments at the LHC [30–32]. The LHeC is expected to test the rich electroweak physics with precision. There has been some work on the physics goals of the collider [30–35]. The working group involved in the synergy between the LHC and the LHeC brought out an excellent report showing the inter-dependencies of the physics reach and goals of both these colliders [36]. The LHeC is going to provide an unprecedented platform for studying the single top quark production
as this has an advantage over the LHC and the Tevatron in terms of providing (a) a clean environment with suppressed background from strong interaction initiated processes, and (b) a kinematic reach for lepton-nucleon scattering at c.m. energy around 1.3 TeV. Thus it is worthwhile to study the single top quark production and probe the $Wtb$ anomalous couplings at the LHeC.

In Sec. 2 we analyze and study the single anti-top quark production and potential backgrounds, their yield, choice of selection cuts and kinematic distributions at the LHeC. We introduce kinematic asymmetries as estimators in Sec. 3, provide the exclusion contours based on bin analysis of distributions involving kinematic observables and finally using the method of optimal variables we give error correlation matrices and exclusion contours with 1% luminosity uncertainty. We discuss the impact of the luminosity uncertainty on the measurement of the couplings and their correlations. The summary and analysis of our observations are given in Sec. 4.

2 Single anti-top quark production

In hadron colliders, the SM single top quark production at leading order is studied through three disparate non-interfering modes via $s$, $t$- and $Wt$- channels, respectively and details can be found in [37]. The $t$- channel through charge current (CC) interactions dominates over all the other production mechanism. In the LHeC we can study the single top quark production only through $t$ channel process $e^- b \to \nu_e t + X$ as shown in Figure 1. In sharp contrast to the LHC the absence of pile-up and underlying event effects at the LHeC, high rates of single anti-top production is expected to provide a better insight on $Wtb$ anomalous couplings. The sensitivity of the $Wtb$ couplings are also investigated through the sub-dominant associated $tW$ production in references [38, 39].

We have implemented $Wtb$ effective couplings corresponding to both chiral vector and tensor structures given by the Lagrangian (1.1) in MadGraph/MadEvent [40] using FeynRules [41]. The partonic cross sections are convoluted with CTEQ6L1 parton distribution functions (PDF) keeping factorization and renormalization scale $\mu_F = \mu_R = m_t = 172.5$ GeV. The mass of $b$-quark $m_b = 4.7$ GeV and $W^\pm$ boson $m_W = 80.399$ GeV, assuming the SM value for $|V_{tb}|/f_L^t = 1$.

The total top decay width which is is one of the fundamental property of top physics is measured with precision from the partial decay width $\Gamma(t \to W b)$ in the $t$ channel of the single top quark production. The effect of anomalous $Wtb$ couplings in evaluating the decay width of the anti-top quark is consistently taken into account throughout our analysis for the signal cross-section.

Considering the five flavor constituents of proton we study the $2 \to 2$ process $e^- p \to \nu_e t + X$ and probe the accuracy with which the anomalous couplings can be measured. The variation of the cross-section of the single top production in SM is studied with respect to the center of mass energy and electron energy in Figure 2 and we are in agreement with the earlier results given in [29]. We also show the effect of taking 80% beam polarization for electron, which results in the enhancement of the SM single top production cross section as the cross-section scales as $(1 + P_{e-})$, $P_{e-}$ being the degree of polarization of the electron.
We also depict the varying contribution of $2 \to 3$ process $e^- p \to \bar{t} \nu_e b$ from the four flavor proton where the gluon splits into $b$, $\bar{b}$ and $\bar{b}$ participates in the interaction while $b$ quark is produced in final state as a spectator quark. This process is however suppressed in comparison to the $2 \to 2$ process $e^- p \to \bar{t} \nu_e$. This signal can be vetoed out by demanding the exclusion of two $b$ jets. We do not consider this process for our analysis.

For the rest of the analysis we compute all cross-sections for the proposed LHeC with $E_{e^-} = 60$ GeV and $E_p = 7$ TeV as per recommendations given in the LHeC conceptual design report [30]. The total events are estimated with an integrated luminosity $L = 100$ fb$^{-1}$.

The new physics effect can arise either at the production vertex of the anti-top in the process $e^- p \to \bar{t} \nu_e \to \bar{b}W^- \nu_e$ or at the decay vertex. Figure 3 depicts the interplay of the interference terms for the left handed current and shows the variation of the cross section with respect to the variation in the anomalous couplings. We observe that the cross-section corresponding to the left handed vector current mediated process varies as $\left[(1 + \Delta f^L_1)|V_{tb}|\right]^2$. The cross-section of the right handed current mediated process varies as $|f_i^R|^2$ for $i = 1, 2$ with the respective anomalous coupling where we have assumed the exclusive decay channel of top to $Wb$ channel.

We estimate and study the $W^-$ helicity distributions arising from NP effects. As mentioned earlier, the $W$ polarization distribution can be a sensitive observable to distinguish the contribution of anomalous couplings. We study the behaviour of the helicity fractions of the $W^-$ in terms of ratios of the number of events $F_- = N_-/N$, $F_+ = N_+/N$ and $F_0 = N_0/N$ where $N_-$, $N_+$ and $N_0$ are the left, right and longitudinally polarized $W^-$ events and $N = N_+ + N_- + N_0$. We vary the coupling and study its effect through the variation on these ratios in Figure 4. We observe that

(a) The $F_-$ and $F_+$ corresponding to the positive and negative polarized $W$'s show opposite trend with the variation of all effective couplings except $|v_{tb}| \Delta f^L_1$.

(b) The helicity fractions $F_i$ associated with the left handed tensor current is most sensitive as it interferes with the SM and has a larger momentum dependence. Right handed vector chiral current shows an appreciable sensitivity w.r.t. $F_i$ helicity distribution.

The helicity fractions $F_-$ and $F_0$ are also sensitive to the change in the coefficient of the right handed tensor current.

Finally we analyze the anti-top through the hadronic and leptonic decay modes of $W$'s. Henceforth, we have multiplied the cross-section (for processes having $b$ or $\bar{b}$ as its final state) with $b$, $\bar{b}$ tagging efficiency $\epsilon_b = 0.6$.

2.1 Sensitivity in the Hadronic Mode

In order to study the sensitivity of the anomalous couplings introduced in equation (1.1), we examine the process $e^- p \to \bar{t} \nu_e, (\bar{t} \to W^- \bar{b}, W^- \to jj)$, $j \equiv u, d, c, s$ at the LHeC and its potential backgrounds. We impose standard selection cuts as follows

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Table 1. Cross-section of all background processes in pb for the hadronic channel with selection cuts. The effective background cross-section $\sigma_{\text{eff}}$ is computed in the fifth column by multiplying b/$\bar{b}$ tagging efficiency and/or faking probability 1/10 and 1/100 corresponding to final state charm/anti-charm and light jets $j \equiv u, \bar{u}, d, \bar{d}, s, \bar{s}, g$, respectively.

| Event Selection | $p_{T,j,b} \geq 20$ GeV | $|\eta_j| \leq 5, |\eta_b| \leq 2.5$ | $\Delta R_{j,b} \geq 0.4$ | $|m_{j1,j2} - m_W| \leq 22$ GeV | $\sigma_{\text{eff}}$ |
|-----------------|------------------------|-------------------|------------------------|-------------------------|-----------------|
| SM              | $3.2 \times 10^4$      | $2.3 \times 10^4$ | $2.2 \times 10^4$      | $66.7 \%$               |                 |
| SM+$\sum_i$ Bkg | $6.5 \times 10^4$      | $5.0 \times 10^4$ | $4.0 \times 10^4$      | $61.5 \%$               |                 |
| $|V_{tb}| \Delta f^j_t = .5$ | $7.3 \times 10^4$      | $5.0 \times 10^4$ | $5.0 \times 10^4$      | $68.0 \%$               | 1.92            |
| $f^j_t = .5$    | $4.6 \times 10^4$      | $3.2 \times 10^4$ | $3.2 \times 10^4$      | $69.7 \%$               | 1.43            |
| $f^j_t = -.5$   | $4.9 \times 10^4$      | $3.6 \times 10^4$ | $3.6 \times 10^4$      | $73.2 \%$               | 1.55            |
| $f^j_b = .5$    | $3.4 \times 10^4$      | $2.3 \times 10^4$ | $2.3 \times 10^4$      | $69.6 \%$               | 1.40            |
| $f^j_b = .5$    | $5.7 \times 10^4$      | $4.1 \times 10^4$ | $4.1 \times 10^4$      | $72.3 \%$               | 1.69            |

Table 2. Yield with selection cuts in the hadronic channel corresponding to the chosen anomalous coupling value of 0.5 at integrated luminosity $L = 100$ fb$^{-1}$. The yield corresponding to SM+$\sum_i$ Bkg signifies the total cumulative events of SM and all backgrounds after taking into account the b, $\bar{b}$ faking/tagging efficiency.

(i) Minimum transverse momentum for jets, $\bar{b}$-antiquark $p_{T,j} \geq 20$ GeV, $p_{T,b} \geq 25$ GeV and minimum missing transverse energy $E_T \geq 25$ GeV.

(ii) The pseudo-rapidity region for leptons and $\bar{b}$-antiquark $|\eta_{b,l}|$ is taken to be $\leq 2.5$, however for jets $|\eta_j| \leq 5$.

(iii) Isolation cuts for lighter, heavy quarks and lepton require $\Delta R_{ij} \geq 0.4$ where $i, j \equiv$ leptons, jets and $\bar{b}$ anti-quark.

In addition, we impose the following cuts to reduce the background

(iv) The difference of azimuthal angle between missing energy $E_T$ and jets, leptons, $\bar{b}$-antiquark should be $\Delta \phi \geq 0.4$. 

| No. | Background Process | $p_{T,j,b} \geq 20$ GeV | $|\eta_j| \leq 5, |\eta_b| \leq 2.5$ | $\Delta R_{j,b} \geq 0.4$ | $|m_{j1,j2} - m_W| \leq 22$ GeV | $\sigma_{\text{eff}}$ |
|-----|-------------------|------------------------|-------------------|------------------------|-------------------------|-----------------|
| 1   | $e^- p \to \nu_e W^- b$ without anti-top line | $7.5 \times 10^{-3}$ | $6.8 \times 10^{-3}$ | $4.5 \times 10^{-3}$ | $2.7 \times 10^{-3}$ | 
| 2   | $e^- p \to \nu_e jjj$ | $4.2 \times 10^6$ | $3.6 \times 10^5$ | $2.4 \times 10^4$ | $7.2 \times 10^{-2}$ | 
| 3   | $e^- p \to \nu_e cjj$ & $e^- p \to \nu_e cjj$ | $1.5 \times 10^6$ | $1.2 \times 10^5$ | $8.6 \times 10^4$ | $8.6 \times 10^{-2}$ | 
| 4   | $e^- p \to \nu_e c\bar{c}j$ | $5.8 \times 10^{-2}$ | $5.0 \times 10^{-2}$ | $3.2 \times 10^{-2}$ | $6.7 \times 10^{-3}$ | 
| 5   | $e^- p \to \nu_e bbj$ | $2.5 \times 10^{-2}$ | $2.2 \times 10^{-2}$ | $1.5 \times 10^{-2}$ | $1.5 \times 10^{-3}$ | 
| 6   | $e^- p \to c\bar{c}e$ | $2.5 \times 10^{-2}$ | $2.2 \times 10^{-2}$ | $1.5 \times 10^{-2}$ | $1.5 \times 10^{-3}$ | 

$(\bar{c} \to W^- \bar{s})$
(v) To further reduce the background in the hadronic channel we reconstruct $W^-$ from di-jets assuming the jet energy resolution $\approx \frac{5}{F} = \frac{0.6}{\sqrt{F}}$. In this setup the di-jet invariant mass resolution around the $W^-$ mass is approximately 7%. Thus a mass window around 28% (4 times of this resolution at 2$\sigma$ level) of the $W$ mass $\approx 22$ GeV is taken into consideration and hence di-jet invariant mass is allowed to satisfy $|m_{jj_1j_2} - m_W| \leq 22$ GeV.

The cross-section of the background processes and the effect of these selection cuts are given in the Table 1. The effective cross-section given in the fifth column is calculated after multiplying the $b\bar{b}$ tagging efficiency of 0.6. The $b\bar{b}$ faking probability is taken to be 1/100 for $u,d,s$ quarks, antiquarks and 1/10 for $c,\bar{c}$ quarks. We observe that

(a) The dominant background process is $e^- p \rightarrow \nu_c \bar{c} (jj)$ where $j \equiv u, \bar{u}, d, \bar{d}, s, \bar{s}, g$. The effective irreducible cross-section of the this background after imposing all cuts is $\approx 0.1$ pb. The other dominant background is $e^- p \rightarrow \nu_c jj$ which along with the first one constitute almost 94% of the total irreducible background 169 fb.

(b) The cross-section of $e^- p \rightarrow \nu_c W^- \bar{b}$ is dominated by diagrams wherein the $W^- \bar{b}$ is generated from anti-top quarks. However, after multiplying with the appropriate branching ratio for the hadronic mode of $W^-$ the cross-section is reduced to the order of $10^{-3}$ pb. We have also found that the potential background due to mis-tagging of one of the double $b,\bar{b}$ events arising from the process to $e^- p \rightarrow \nu_c j \bar{b}b$ is negligibly small.

| No. | Background Process | $p_T j_{k,l} \geq 20$ GeV, $\Delta R_{j_kj_l} \geq 0.4$, $\not{E}_T \geq 25$ | $|\eta_j| \geq 5$, $|\eta_{b,l}| \geq 2.5$ | $\Delta \Phi_{b,j} \geq 0.4$ | $\sigma_{\text{eff.}}$ |
|-----|-------------------|---------------------------------|---------------------------------|------------------|-------------|
| 1   | $e^- p \rightarrow \nu_e j$ | $1.5 \times 10^{-4}$ | $1.4 \times 10^{-4}$ | $1.4 \times 10^{-4}$ |            |
| 2   | $e^- p \rightarrow \nu_e j$ | $6.6 \times 10^{-4}$ | $6.1 \times 10^{-4}$ | $6.1 \times 10^{-4}$ |            |
| 3   | $e^- p \rightarrow \nu_e j$ & $e^- p \rightarrow \nu_e b$ | $3.6 \times 10^{-3}$ | $3.2 \times 10^{-3}$ | $1.9 \times 10^{-3}$ |            |
|     | $\nu_{\bar{c}} j$ | Without top line | | | |
| 4   | $e^- p \rightarrow e^- \nu_e j$ | $1.5 \times 10^{-4}$ | $6.9 \times 10^{-4}$ | $6.9 \times 10^{-4}$ |            |
| 5   | $e^- p \rightarrow e^- \nu_e j$ | $1.2 \times 10^{-4}$ | $5.5 \times 10^{-4}$ | $5.5 \times 10^{-4}$ |            |

Table 3. Cross-section of all background processes in pb for the leptonic channel with selection cuts. The effective background cross-section $\sigma_{\text{eff.}}$ is computed in the fourth column by multiplying $b/\bar{b}$ tagging efficiency and/or faking probability 1/10 and 1/100 corresponding to final state charm/anti-charm and light jets $j \equiv u, \bar{u}, d, \bar{d}, s, \bar{s}, g$, respectively. The background processes with two charged leptons are taken into consideration where one get lost in the beam pipe.

To probe the effect of these cuts on the yield, we study all kinematic distributions in SM, other non-top backgrounds and compare them with contributions from new physics cases with the representative value of the effective coupling at 0.5. The analysis is summarized in Table 2 and the overall fiducial efficiencies of additional cuts are presented. The significance $S/\sqrt{S+B}$ give the sensitivity of the cross-section corresponding to these representative values.
The characteristics of the highest $p_T$ jet $j_1$, the final state $\bar{b}$ and the missing transverse energy $E_T$ are likely to bear the signature of the $Wtb$ couplings at the production/decay vertex. We reconstruct the $W^-$ from jets at the final states to study the azimuthal angle separation between $W^-$ and $\bar{b}$ and missing energy $E_T$. We study one dimensional distributions of azimuthal angle (angle between the planes) $\Delta \phi_{{E_T,j_1}}$, $\Delta \phi_{{E_T,b_j}}$, $\Delta \phi_{{E_T,W}}$ and $\Delta \phi_{b,W}$ along with the $\cos \theta_{bj_1}$ and $\Delta \eta_{bj_1}$, where all angles are defined in the lab frame.
Event Selection & \( p_T^{j,b} \geq 20 \text{ GeV} \) & \( \Delta \Phi_{\nu,j} \geq 0.4 \) & \( \Delta \Phi_{\nu,b} \geq 0.4 \) & \text{Fiducial} & \( S/\sqrt{S + B} \) \\
SM & \( 1.2 \times 10^4 \) & \( 1.1 \times 10^4 \) & 92.0 \% & – \\
SM + \sum \text{Bkg}_i & \( 1.3 \times 10^4 \) & \( 1.2 \times 10^4 \) & 92.0 \% & – \\
\( |V_{tb}| \Delta f^T_l = .5 \) & \( 4.5 \times 10^4 \) & \( 2.5 \times 10^4 \) & 92.6 \% & 1.55 \\
f^T_0 = .5 & \( 2.8 \times 10^4 \) & \( 1.6 \times 10^4 \) & 94.1 \% & 1.23 \\
f^T_1 = .5 & \( 3.1 \times 10^4 \) & \( 1.7 \times 10^4 \) & 89.5 \% & 1.27 \\
f^T_2 = -.5 & \( 1.8 \times 10^4 \) & \( 1.0 \times 10^4 \) & 90.9 \% & 0.95 \\
f^T_3 = .5 & \( 3.6 \times 10^4 \) & \( 2.0 \times 10^4 \) & 90.9 \% & 1.38 \\

Table 4. Yield with selection cuts in the leptonic channel corresponding to the chosen anomalous coupling value of 0.5 at integrated luminosity \( L = 100 \text{ fb}^{-1} \). The yield corresponding to \( \text{SM} + \sum \text{Bkg}_i \) signify the total cumulative events of SM and all backgrounds after taking into account the \( b, \bar{b} \) faking/tagging efficiency.

Figure 5 exhibit these distributions. To study the distribution profile and shape variation, all histograms are normalized to unity and are drawn for a anomalous coupling representative value 0.5. The normalized distributions corresponding to \( |V_{tb}| \Delta f^T_l = \pm 0.5 \) is identical to that of SM. However, on consideration of backgrounds the distribution profile of kinematical variable generated from \( |V_{tb}| \Delta f^T_l = \pm 0.5 \) shows distortion when compared to that of pure SM. In most of the distributions the new physics couplings play a significant role and clear distinction has been seen in profile with respect to combined effect SM and backgrounds. We observe from Figure 5 that the contribution of left and right handed tensorial lorentz structures are distinguishable in most distributions. The distributions corresponding to (a) azimuthal angle between missing energy and highest \( p_T \) jet \( j_1 \) and (b) cosine of the angle between massive \( b \) quark and \( j_1 \) show a noticeable difference in the profile with respect to the right handed vector chiral current.

### 2.2 Sensitivity in the Leptonic Mode

Similarly we study the yield of the leptonic decay mode of \( W^- \) through the process \( e^- p \rightarrow l \nu_e, \ (l \rightarrow W^- \bar{b}, \ W^- \rightarrow l^- \nu_l) \), \( l^\pm \equiv e^-, \mu^- \) at the LHeC. We impose the standard selection cuts are same as those given in 2.1. The effects of these selection cuts are given in Table 3. The effective cross-section is given in the fourth column of this table. In general all backgrounds processes are sub-dominant. Reading this Table 3, we observe that

(a) processes with a charged lepton, \( E_T \) and light jets, where the light jets can fake a \( b \) jet of the signal becomes negligibly small once they are screened through the selection cuts and multiplied by the appropriate faking probability factor.

(b) background processes with two charged leptons where one of them vanishes in the beam pipe is negligible after the imposition of the selection cuts.

The fiducial efficiencies due to the additional cuts are computed for the representative value of couplings at \( \pm 0.5 \) corresponding to the coefficient of the different chiral and Lorentz structures as given in (1.1). They are shown along with the significance in Table 4.
Figure 6. Normalized distributions of $\Delta \phi_{E_T l_1}$, $\Delta \phi_{E_T b}$, $\cos \theta_{bl_1}$ and $\Delta \eta_{bl_1}$ for leptonic decay mode of $W^-$ corresponding to SM and an anomalous coupling of 0.5. Here $l_1$ is the highest $p_T$ charged lepton. The normalized distributions corresponding to $|V_{tb}| \Delta f_1^L = \pm 0.5$ is identical to that of SM. All kinematic observables are measured in lab frame.

In the leptonic mode the final state charged lepton along with $\bar{b}$ shows the characteristic features of the anomalous couplings. Further we study the sensitivity of the couplings through one dimensional distributions corresponding to azimuthal angle $\Delta \phi_{E_T l_1}$, $\Delta \phi_{E_T b}$, $\Delta \phi_{E_T W^-}$ and $\Delta \phi_b W^-$ along with the polar angle $\cos \theta_{bl_1}$ and difference of pseudo-rapidity $\Delta \eta_{bl_1}$ between $\bar{b}$ and the charged lepton with highest $p_T$ designated as $l_1$. Figure 6 depict these distributions. As mentioned before all normalized distributions corresponding to $|V_{tb}| \Delta f_1^L = \pm 0.5$ are identical to that of SM single top production. We observe that $f_2^L$ shows a distinguishable profile over others. However, the distribution $\Delta \phi_{E_T l}$ is sensitive to all anomalous couplings.

3 Estimators and $\chi^2$ analysis

3.1 Angular Asymmetries from Histograms

We construct the asymmetry from the distribution of kinematic observables in both the hadronic and leptonic modes. These asymmetries can be sensitive discriminators to distinguish the contribution from the different Lorentz structure due to their characteristic
momentum dependence. We study the angular asymmetries with respect to the polar angle \( \cos \theta_{ij} \), rapidity difference \( \Delta \eta_{ij} \) and azimuthal angle difference \( \Delta \phi_{ij} \), where \( i, j \) may be any partons (including \( \bar{b} \)-antiquark), charged lepton or missing energy. The associated asymmetries \( A_{\theta_{ij}}, A_{\Delta \eta_{ij}} \) and \( A_{\Delta \phi_{ij}} \) are defined as

\[
A_{\theta_{ij}} = \frac{N_A^+ (\cos \theta_{ij} > 0) - N_A^- (\cos \theta_{ij} < 0)}{N_A^+ (\cos \theta_{ij} > 0) + N_A^- (\cos \theta_{ij} < 0)}
\]

\[
A_{\Delta \eta_{ij}} = \frac{N_A^+ (\Delta \eta_{ij} > 0) - N_A^- (\Delta \eta_{ij} < 0)}{N_A^+ (\Delta \eta_{ij} > 0) + N_A^- (\Delta \eta_{ij} < 0)}
\]

\[
A_{\Delta \phi_{ij}} = \frac{N_A^+ (\Delta \phi_{ij} > \frac{\pi}{2}) - N_A^- (\Delta \phi_{ij} < \frac{\pi}{2})}{N_A^+ (\Delta \phi_{ij} > \frac{\pi}{2}) + N_A^- (\Delta \phi_{ij} < \frac{\pi}{2})}
\]

with \( 0 \leq \Delta \phi_{ij} \leq \pi \). The asymmetry \( A_\alpha \) and its statistical error for \( N_A^+ \) and \( N_A^- \) events where \( N = (N_A^+ + N_A^-) = L \cdot \sigma \) is calculated by using the following definition based on binomial distribution:

\[
A_\alpha = a \pm \sigma_a, \quad \text{where} \quad a = \frac{N_A^+ - N_A^-}{N_A^+ + N_A^-} \quad \text{and} \quad \sigma_a = \sqrt{\frac{1-a^2}{L \cdot \sigma}}; \quad (\alpha = \cos \theta_{ij}, \Delta \eta_{ij}, \Delta \Phi_{ij})
\]

Here \( \sigma \equiv \sigma (e^- p \to \bar{t} \nu, \bar{t} \to W^- \bar{b}) \times BR(W^- \to jj/l^- \bar{\nu}) \times \epsilon_b \) is the total cross-section in the respective channel after imposing selection cuts and \( \epsilon_b = 0.6 \) is the \( b \bar{b} \) tagging efficiency.

Based on the one dimensional histograms given in Figures 5 and 6, we look for the asymmetry within a distribution generated due to the interplay of the SM, Background channels and a given anomalous coupling for two distinct hadronic and leptonic modes of \( W^- \) decay. Any large deviation from the combined asymmetry due to SM and background processes would then imply that the associated kinematic observable is an optimal variable in determining the sensitivity of the given anomalous coupling. We provide these asymmetries constructed from the distributions in Table 5 and 6 for the hadronic and leptonic channels, respectively a representative value of the anomalous coupling 0.5. As in absence of background processes, any asymmetry with respect to distributions corresponding to \( |V_{ib}| \Delta f_1^L \) is identical to the one in SM, we do not exhibit them in the table.

| \( A_{\Delta \Phi_{\ell, jj}} \) | \( A_{\Delta \Phi_{\ell, ji}} \) | \( A_{\Delta \Phi_{\ell, w^-}} \) | \( A_{\Delta \Phi_{W^- inj}} \) | \( A_{\eta_{ij}} \) | \( A_{\Delta \eta_{ij}} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| SM+Bkg. \( f_1^b = +.5 \) | \( .532 \pm .003 \) | \( .282 \pm .005 \) | \( .503 \pm .004 \) | \( .799 \pm .003 \) | \( .023 \pm .001 \) | \( -.712 \pm .003 \) |
| \( f_2^b =+.5 \) | \( .327 \pm .004 \) | \( .231 \pm .004 \) | \( .564 \pm .004 \) | \( .778 \pm .003 \) | \( .005 \pm .004 \) | \( -.806 \pm .003 \) |
| \( f_3^b = +.5 \) | \( .528 \pm .004 \) | \( .082 \pm .004 \) | \( .716 \pm .003 \) | \( .748 \pm .003 \) | \( -.196 \pm .004 \) | \( -.868 \pm .002 \) |
| \( f_4^b = +.5 \) | \( .390 \pm .005 \) | \( .269 \pm .004 \) | \( .585 \pm .004 \) | \( .683 \pm .004 \) | \( .106 \pm .005 \) | \( -.795 \pm .003 \) |
| \( f_5^b = +.5 \) | \( .330 \pm .004 \) | \( .363 \pm .004 \) | \( .566 \pm .003 \) | \( .656 \pm .003 \) | \( -.197 \pm .004 \) | \( -.823 \pm .002 \) |

Table 5. Asymmetries and its error associated with the kinematic distributions in Figure 5 at an integrated luminosity \( L = 100 \text{ fb}^{-1} \). These asymmetries are computed for a representative value of the anomalous coupling 0.5 along with SM.

Asymmetries shown in Tables 5 and 6 are good estimators for preliminary studies. They give a handle for judging the ability of the measured observable to distinguish the
Table 6. Asymmetries and its error associated with the kinematic distributions in Figure 6 at an integrated luminosity $L = 100$ fb$^{-1}$. These asymmetries are computed for a representative value of the anomalous coupling 0.5 along with SM and all background processes.

| Coupling | $A_{\Delta \Phi_{l1}}$ | $A_{\Delta \Phi_{l2}}$ | $A_{\Delta \eta_{l1}}$ | $A_{\Delta \eta_{l2}}$ |
|----------|---------------------|---------------------|---------------------|---------------------|
| SM + $\sum_i$ Bkg. | .384 ± .004 | .710 ± .003 | .551 ± .006 | -.765 ± .007 |
| $f^R_1 = +.5$ | .484 ± .004 | .702 ± .003 | .332 ± .006 | -.821 ± .003 |
| $f^L_2 = -.5$ | .526 ± .004 | .620 ± .003 | .410 ± .006 | -.831 ± .002 |
| $f^L_2 = +.5$ | .353 ± .005 | .812 ± .003 | .392 ± .007 | -.850 ± .003 |
| $f^R_2 = +.5$ | .424 ± .004 | .684 ± .003 | .507 ± .005 | -.809 ± .003 |

contribution from an anomalous term in the Lagrangian. We observe in Table 5 that the couplings are sensitive in magnitude as well as sign of the asymmetry generated by $\cos \theta_{b,j_1}$ distribution. But they may not be sensitive enough for the couplings which are one order of magnitude smaller than the representative value. In fact the whole distribution is essentially divided into two halves which correspond to only two bins with large bin-width. The asymmetries induced by the tiny new physics couplings among these two large bins is likely to be seen.

3.2 Exclusion contours from bin analysis

In this subsection the sensitivity of couplings are obtained through $\chi^2$ analysis, where we compute the sum of the variance of events over all bins. Thus more bin information is likely to yield better sensitivity than the asymmetries which are generated essentially by dividing the whole distribution into two equal bins.

To make the analysis more effective we switch on two effective anomalous couplings at a time with SM and all possible background processes with same final states. The $\chi^2$ becomes a function of two effective anomalous couplings $f_i, f_j$ and defined as

$$\chi^2(f_i, f_j) = \sum_{k=1}^{N} \left( \frac{N_{k}^{\text{exp}} - N_{k}^{\text{th}}(f_i, f_j)}{\delta N_{k}^{\text{exp}}} \right)^2$$

(3.6)

where $N_{k}^{\text{th}}(f_i, f_j)$ and $N_{k}^{\text{exp}}$ are the total number of events predicted by the theory involving $f_i, f_j$ and measured in the experiment for the $k$th bin. $\delta N_{k}^{\text{exp}}$ is the combined statistical and systematic error $\delta_{sys}$ in measuring the events for the $k$th bin. If all the coefficients $f_i$’s are small, then the experimental result in the $k$th bin should be approximated by the SM and background prediction as

$$N_{k}^{\text{exp}} \approx N_{k}^{\text{SM}} + \sum_i N_{k}^{\text{Bkg}_i} = N_{k}^{\text{SM}+\sum_i \text{Bkg}_i}.$$

(3.7)

The error $\delta N_{k}^{\text{SM}}$ can be defined as

$$\delta N_{k}^{\text{SM}+\sum \text{Bkg}_i} = \sqrt{N_{k}^{\text{SM}+\sum \text{Bkg}_i} \left( 1 + \delta_{sys}^2 N_{k}^{\text{SM}+\sum \text{Bkg}_i} \right)}.$$

(3.8)

The $\chi^2$ analysis due to luminosity uncertainty etc. is studied for three representative values of $\delta_{sys}$ at 1%, 5% and 10 %, respectively.
68.3 % C.L. exclusion contours on the plane of $|V_{tb}| \Delta f_1^R - f_1^R$, $|V_{tb}| \Delta f_2^R - f_2^R$, $f_1^R - f_2^R$, and $f_1^L - f_2^L$ and based on combined bin analysis of all kinematic observables in the hadronic decay mode of $W^-$. A $\chi^2$ analysis is performed by taking into account the deviation from SM and background process with the systematic error of 1%, 5% and 10%, respectively at an integrated luminosity of $L = 100 \text{ fb}^{-1}$.

The analysis is performed for both hadronic and leptonic observables which depend on the distributions shown in Figures 5 and 6. Using this definition of $\chi^2$ in (3.6), we draw the exclusion contours on the six different two dimensional planes defined by the anomalous couplings $|V_{tb}| \Delta f_1^L$, $f_1^R$, $f_2^L$ and $f_2^R$. 68.3% and 95% C.L. Exclusion contours for the hadronic and leptonic channels are shown in Figures 7, 8 and 9, 10, respectively.
For each pair of the couplings, the effect of the overall systematic uncertainty (includes luminosity measurement error etc.) is sketched for three representative values of δSys = 1%, 5% and 10%, respectively at an integrated luminosity of L = 100 fb$^{-1}$.

On examination of the exclusion contours in both decay modes we find that

(a) The sensitivity of measuring all anomalous couplings are affected by the systematic

Figure 8. 95 % C.L. exclusion contours on the plane of $|V_{tb}|\Delta f_1 - f_1^R$, $|V_{tb}|\Delta f_1 - f_2^R$, $f_1^R - f_2^R$, $f_1^L - f_2^L$ and based on combined bin analysis of all kinematic observables in the hadronic decay mode of $W^-$. A $\chi^2$ analysis is performed by taking into account the deviation from SM and background process with the systematic error of 1%, 5% and 10%, respectively at an integrated luminosity of $L = 100$ fb$^{-1}$. 

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Figure 9. 68.3 % C.L. exclusion contours on the plane of $|V_{tb}| \Delta f_1^L - f_2^R$, $|V_{tb}| \Delta f_1^L - f_2^R$, $f_1^R - f_2^R$, $f_1^R - f_2^R$ and $f_1^L - f_2^R$ and based on combined bin analysis of all kinematic observables in the leptonic decay mode of $W^-$. A $\chi^2$ analysis is performed by taking into account the deviation from SM and background process with the systematic error of 1%, 5% and 10%, respectively at an integrated luminosity of $L = 100 \, fb^{-1}$.

uncertainty $\delta_{Sys}$.

(b) The sensitivity of $|V_{tb}| \Delta f_1^L$ at 95% C.L. is of the $\sim 5 \times 10^{-3}$ and $\sim 3 \times 10^{-2}$ with systematic error of 1% and 10 %, respectively. The order of the sensitivity for other anomalous couplings varies as $\sim 10^{-2} - 10^{-1}$ at 95 % C.L. with the $\delta_{Sys}$ varying between .01 to 0.1.
Figure 10. 95% C.L. exclusion contours on the plane of $|V_{tb}| \Delta f_1^L - f_1^R$, $|V_{tb}| \Delta f_1^L - f_1^L$, $|V_{tb}| \Delta f_2^L - f_2^R$, $|V_{tb}| \Delta f_2^L - f_2^L$, $|V_{tb}| \Delta f_1^R - f_1^R$, $|V_{tb}| \Delta f_1^R - f_1^L$, $|V_{tb}| \Delta f_2^R - f_2^R$, $|V_{tb}| \Delta f_2^R - f_2^L$ and based on combined bin analysis of all kinematic observables in the leptonic decay mode of $W^-$. A $\chi^2$ analysis is performed by taking into account the deviation from SM and background process with the systematic error of 1%, 5% and 10%, respectively at an integrated luminosity of $L = 100 fb^{-1}$.

3.3 Errors and correlations

In order to constrain anomalous $Wtb$ couplings further the method of optimal observables by using the full information from the distribution of kinematic observables can be adopted. In this method, all the anomalous couplings $f_i$, having different shape profiles, from each other can be constrained simultaneously. For a given integrated Luminosity $L$, the statistical
errors in the $f_i$ and the correlations of the errors among anomalous coupling measurement can be obtained from the $\chi^2$ which is a function of all anomalous couplings. Redefining the $\chi^2$ of equation (3.6) in terms of the two anomalous couplings and the covariance matrix $V$ we have

$$\chi^2(f_i, f_j) = \chi^2_{\text{min}} + \sum_{i,j} (f_i - \bar{f}_i)(V^{-1})_{ij} (f_j - \bar{f}_j)$$  \hspace{1cm} (3.9)$$

A total of ten inverse covariant matrices $V^{-1}$ can be generated from six and four distinct distributions of kinematic observables in hadronic and leptonic modes, respectively, using the approximation (3.7). If the SM prediction along with all dominant backgrounds gives a reasonably good description of the data in most of the phase space region, then the statistical errors $\Delta f_i$ of $f_i$ and their correlations are determined solely in terms of the these six covariance matrices $V$ as

$$f_i - \bar{f}_i = \pm \Delta f_i = \pm \sqrt{V_{ii}}, \quad \rho_{ij} = V_{ij}/\sqrt{V_{ii}V_{jj}}.$$  \hspace{1cm} (3.10)$$

$\rho_{ij}$ gives correlation coefficient between two distinct anomalous couplings $f_i$ and $f_j$ and gives the absolute error for a given anomalous coupling $f_i = f_j$. $\Delta f_i$ gives the uncertainty with which these couplings will be measured at the LHeC. $\bar{f}_i$ is the expected mean value in SM, which is zero for all anomalous couplings $f_i$.

Subsequently, an optimal analysis with an integrated luminosity $L = 100$ fb$^{-1}$ is made after combining all kinematic observables in both hadronic and the leptonic modes, respectively.

The inverse of the covariance matrix $V^{-1}_{ij}$ is generated from one dimensional histogram of each sensitive kinematic observable and the corresponding respective correlation matrix is computed. We combine all inverse covariant matrices to compute the combined $\chi^2$ in the hadronic and leptonic modes separately.

The combined $\chi^2$ reads as

$$\chi^2_{\text{comb.}}(f_i, f_j) = \sum_{k=1}^{n} \chi^2_{\text{min}} + \sum_{k} \sum_{i,j} (f_i - \bar{f}_i)(V^{-1})_{ij}^{k} (f_j - \bar{f}_j)$$  \hspace{1cm} (3.11)$$

Here $k \equiv$ number of distributions corresponding to the kinematic observables. $n = 6$ and 4 for hadronic and leptonic channels, respectively. We thus provide the accuracy with which anomalous couplings can be measured from each of these distributions. The correlation matrices and the absolute errors in each and every couplings in the hadronic and leptonic modes are given below:

\[
\begin{pmatrix}
|Vtb| \Delta f_1^L = \pm 4.5 \times 10^{-4} & 1 \\
\frac{1}{f_1^R} = \pm 7.2 \times 10^{-4} & -.07 & 1 \\
\frac{1}{f_2^L} = \pm 4.7 \times 10^{-4} & -.04 & -.07 & 1 \\
\frac{1}{f_2^R} = \pm 3.2 \times 10^{-4} & -.03 & .06 & -.02 & 1
\end{pmatrix} ;
\begin{pmatrix}
|Vtb| \Delta f_1^L = \pm 4.6 \times 10^{-4} & 1 \\
\frac{1}{f_1^R} = \pm 7.2 \times 10^{-4} & -.02 & 1 \\
\frac{1}{f_2^L} = \pm 8.3 \times 10^{-4} & -.05 & -.06 & 1 \\
\frac{1}{f_2^R} = \pm 4.3 \times 10^{-4} & -.01 & .09 & -.07 & 1
\end{pmatrix}
\]

(a) hadronic mode  
(b) leptonic mode

(3.12)
Up till now we have considered the hadronic and leptonic modes of single anti-top production at the LHeC to be two different probes for measuring these anomalous couplings. We now combine observations from both channels in terms of combined inverse covariance matrix. The global errors and correlations from the corresponding global combined covariance matrix is then given as

\[
\begin{align*}
|V_{tb}| \Delta f^L_1 &= \pm 3.2 \times 10^{-4} \\
 f^R_1 &= \pm 4.6 \times 10^{-4} \\
 f^L_2 &= \pm 4.2 \times 10^{-4} \\
 f^R_2 &= \pm 2.6 \times 10^{-4}
\end{align*}
\]

\[\begin{pmatrix}
1 & -0.05 & 1 \\
-0.05 & 1 & 0.04 & -0.06 & 1 & -0.02 & 0.03 & -0.06 & 1
\end{pmatrix}; \quad (3.13)
\]

It is worthwhile to mention that we have not yet considered any systematic error in the covariance analysis. On comparing the errors given in equations (3.12a) and (3.12b) corresponding to hadronic and leptonic modes, respectively and errors for the global combined analysis in equation (3.13) with those in section 3.2, we find that the sensitivity of \(|V_{tb}| \Delta f^L_1\) and others are found to have increased by one and two orders of magnitude, respectively.

### 3.3.1 Luminosity Error

So far we have computed all the errors and their correlations based on the optimal observables method have been computed by assuming the true luminosity \(L\). An error in the measurement of luminosity is however, is likely to affect the measurements of some effective couplings. It is thus instructive to study the impact of uncertainty in luminosity measurement on the sensitivity of anomalous \(Wtb\) couplings. The true luminosity \(L\) can be estimated as

\[
L \equiv \beta \bar{L}, \quad \beta = 1 \pm \Delta \beta
\]

where \(\bar{L}\) is the measured mean value, and \(\Delta \beta\) is its one \(\sigma\) uncertainty. With the inclusion of the luminosity uncertainty the \(\chi^2_{\text{comb}}\) definition given in (3.9) is modified to

\[
\chi^2_{\text{comb}}(f_i, f_j) \rightarrow \chi^2_{\text{comb}}(f_i, f_j, \beta) \equiv \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} (f_i - \bar{f}_i) [V^{-1}]_{ij}^k (f_j - \bar{f}_j) + \left(\frac{\beta_k - 1}{\Delta \beta_k}\right)^2
\]

Here \([V^{-1}]_{ij}^k\) is now \((n + 1) \times (n + 1)\) matrix with \(f_0 = \beta - 1\). The luminosity uncertainty \(\Delta \beta_k \equiv \Delta \beta\) is same for all kinematic observables at a given collision energy. Here \(n \equiv 0, 1, 2, 3, 4\) corresponding to luminosity factor \(\beta\) and four anomalous couplings. \(m = 6\) (4) corresponds to the number of kinematic observables for hardonic (leptonic) mode.

It is straightforward to integrate out the \(f_0 = 1 - \beta\) dependence and obtain the probability distribution of the parameters \(f_1\) to \(f_4\) in the presence of the luminosity uncertainty. \(|V_{tb}| \Delta f^L_1\) is the only coupling whose weight function is identical to the SM distribution at tree level. The other effective couplings get the SM contribution at the one-loop level and it is thus likely that the statistical errors dominate over systematics. Therefore errors coming from the luminosity uncertainty can then be safely neglected for the other three couplings namely \(f^R_1, f^L_2\) and \(f^R_2\).
The impact of the luminosity uncertainty can thus be accounted for algebraically by using the $\chi^2$ functions written in term of $\Delta f_i^L$. Redefining our $\chi^2_{\text{comb.}}$ function as

$$\chi^2_{\text{comb.}}(f_i, f_j, \beta) = \chi^2_{\text{comb.}} \left( \Delta f_i^L \rightarrow \Delta f_i^L' = \Delta f_i^L + \frac{\beta - 1}{2} \right) \left( \frac{\beta - 1}{\Delta \beta} \right)^2 \tag{3.16}$$ \hspace{1cm}

$$= \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} f'_i [V^{-1}]^k_{ij} f'_j + \left( \frac{\beta_k - 1}{\Delta \beta_k} \right)^2 \tag{3.17}$$

where $\Delta f_i^L' = \Delta f_i^L + (\beta - 1)/2$ and $f'_i \equiv f_i$ (for $i \neq 1$). The luminosity uncertainty in the $\chi^2_{\text{comb.}}$ function in equation (3.17) can be factored out as

$$\chi^2_{\text{comb.}} = \left[ \frac{\beta - 1}{\Delta \beta_{\text{eff}}} + \Delta \beta_{\text{eff}} R \right]^2 + \tilde{\chi}^2_{\text{comb.}}, \quad \text{where}$$

$$\left\{ \Delta \beta_{\text{eff}} \right\}^{-2} = \frac{1}{\Delta \beta^2} + \frac{1}{4} |V^{-1}|_{11}; \quad \text{and} \quad R = \frac{1}{2} \sum_{a=1}^{4} f_a |V^{-1}|_{1a}; \tag{3.18}$$

The new $\tilde{\chi}^2_{\text{comb.}}$ is the reduced combined $\chi^2$ function, which can be re-written as

$$\tilde{\chi}^2_{\text{comb.}} = \xi_{\text{comb.}}^2 - \left( \Delta \beta_{\text{eff}} \right)^2 R^2 \tag{3.19}$$

The reduced $\chi^2$ function can now be used to study the constraints on the effective couplings in the presence of the luminosity uncertainty. It is worth mentioning that correlations between the $|V_{tb}| \Delta f_i^L$ with other couplings are affected due to the presence of the second term in equation (3.19).

Following the optimal analysis by incorporating the luminosity uncertainty and the reduced $\chi^2_{\text{comb.}}$, we get $4 \times 4$ covariance matrix. The modified correlation matrices based on the combined study of the six and four kinematical distributions from hadronic and leptonic modes, respectively at an integrated luminosity of $L = 100$ fb$^{-1}$ can now be computed for different luminosity uncertainty factor $\beta$. We give a spectrum of three correlation matrices corresponding to the three choices for $\Delta \beta = 1\%$, $5\%$ and $10\%$, respectively:

\begin{equation}
\begin{pmatrix}
|V_{tb}| \Delta f_1^L = \pm 5.0 \times 10^{-3} \\
 f_1^R = \pm 4.7 \times 10^{-4} \\
 f_2^L = \pm 4.2 \times 10^{-4} \\
 f_2^R = \pm 2.6 \times 10^{-4}
\end{pmatrix}
\begin{pmatrix}
1 \\
-0.003 & 1 \\
-0.003 & -0.068 & 1 \\
-0.002 & 0.032 & -0.041 & 1
\end{pmatrix};
\begin{pmatrix}
|V_{tb}| \Delta f_1^L = \pm 2.5 \times 10^{-2} \\
 f_1^R = \pm 4.6 \times 10^{-4} \\
 f_2^L = \pm 4.2 \times 10^{-4} \\
 f_2^R = \pm 2.6 \times 10^{-4}
\end{pmatrix}
\begin{pmatrix}
1 \\
0 & 1 \\
0.000 & -0.068 & 1 \\
0.002 & -0.032 & -0.041 & 1
\end{pmatrix};
\end{equation}

(a) $\Delta \beta = 1\%$ 
(b) $\Delta \beta = 5\%$

\begin{equation}
\begin{pmatrix}
|V_{tb}| \Delta f_1^L = \pm 5.0 \times 10^{-3} \\
 f_1^R = \pm 4.7 \times 10^{-4} \\
 f_2^L = \pm 4.2 \times 10^{-4} \\
 f_2^R = \pm 2.6 \times 10^{-4}
\end{pmatrix}
\begin{pmatrix}
1 \\
0 & 1 \\
0.000 & -0.068 & 1 \\
0.002 & -0.032 & -0.041 & 1
\end{pmatrix};
\end{equation}

(c) $\Delta \beta = 10\%$ 
(3.20)
It is observed from equations (3.20a), (3.20b) and (3.20c) that the sensitivity of all couplings except $|V_{tb}| \Delta f_1^L$ remain same as before given in (3.13). The sensitivity of $|V_{tb}| \Delta f_1^L$ which has same weight function as SM is however, reduced by one order of magnitude $\sim 10^{-3}$ corresponding to luminosity uncertainty 1%. The error in $|V_{tb}| \Delta f_1^L$ is now comparable to that obtained in the bin analysis with 1% systematic error. Following the same suite of bin analysis the sensitivity further worsens by an order of magnitude $\sim 10^{-2}$ with increased luminosity uncertainty at 5% -10% uncertainty.

On assumption that the statistical error might dominate over the systematics in the determination of all other couplings, we observe that they are not affected due to the varying $\Delta \beta$ as mentioned in the definition of $\chi^2_{\text{Comb}}$. This is in sharp contrast to that observed in bin analysis where all couplings are affected by the systematic uncertainty.

The correlations of $f_1^R$, $f_2^L$ and $f_2^R$ with $|V_{tb}| \Delta f_1^L$ are drastically reduced for $\Delta \beta = .01$ and finally becomes vanishingly small for $\Delta \beta = .05$ and $\Delta \beta = 0.10$. However, the correlations among $f_1^R$, $f_2^L$ and $f_2^R$ remain same as given in equation (3.13).

As an illustration, we study the variation in total error measurement of $|V_{tb}| \Delta f_1^L$ based on this optimal analysis with a fixed luminosity uncertainty $\Delta \beta$. In Figure 11, the variation of the total error in the estimation of $|V_{tb}| \Delta f_1^L$, with the luminosity for a given $\Delta \beta$ is shown. Thus the error in the anomalous coupling not only depends on the high magnitude of the luminosity, but also on its measured value.

4 Observations and Discussion

An attempt has been made to study and investigate the sensitivity of the measurement of anomalous $Wtb$ couplings associated with the left or right vector and tensor chiral currents. The LHeC being comparatively clean with respect to $pp$ and $p \bar{p}$ colliders, provides an excellent environment to study the electroweak production of single anti-top. We analyze the effect of anomalous couplings in the $Wtb$ vertex by examining its one dimensional distributions.

4.1 Observations

We summarize our observations as follows:
(i) We observed high yields of single anti-top quark production with fiducial efficiency of \( \sim 70\% \) and \( \sim 90\% \) in the hadronic and leptonic decay modes of \( W^- \), respectively after imposing additional selection cuts. The yield and significance for all effective couplings at the representative values of \( \pm 0.5 \) are given in Tables 2 and 4, respectively.

(ii) The helicity distribution of \( W^- \) with the variation of anomalous couplings (excluding \( |V_{tb}| \Delta f_1^L \)) from anti-top quark decay (Figure 4), shows generically that the negative helicity \( F_- \) and positive helicity \( F_+ \) fraction of events have opposite trends with the variation of all anomalous couplings. \( F_- \) is highly sensitive to both the tensor currents while the rate of variation of \( F_+ \) with \( f_2^L \) is much steeper than others. The helicity fraction of longitudinally polarized \( W^- \) bosons grows with the increase in the couplings corresponding to the left handed tensor and right handed vector currents.

(iii) Asymmetries of kinematic variables are constructed from the one dimensional distribution as preliminary estimators for the sensitivity. It is found that asymmetries constructed from distributions can discriminate the effect of non-SM contribution through new vector and tensor chiral currents except for \( |V_{tb}| \Delta f_1^L \), as shown in Tables 5 and 6, provided anomalous couplings are of the order of \( \sim 10^{-1} \). The asymmetry study suggests that the distribution of the cosine of angle between the tagged \( \bar{b} \) quark and the highest \( p_T \) jet \( j_1 \) in the hadronic decay mode of \( W^- \) to be the most sensitive observable.

(iv) We have conducted a \( \chi^2 \) analysis based on the differential events of the kinematic observables and with systematic error \( \delta_{\text{Sys}} = 1\%, 5\% \) and \( 10\% \) with an integrated luminosity of 100 fb\(^{-1} \) data. We combine the \( \chi^2 \) corresponding to all kinematic variables for hadronic and leptonic modes SM and background channels. This gives the exclusion contours on the six different two dimension planes defined by the four anomalous couplings. Contours at 68 % are provided for both hadronic and leptonic decay modes of \( W^- \) in Figures 7 and 9, respectively. Contours at 95 % C.L. are given in Figures 8 and 10 for hadronic and leptonic modes, respectively.

The sensitivity of \( |V_{tb}| \Delta f_1^L \) at 95% C.L. is found to be of the order of \( \sim 10^{-3} - 10^{-2} \) with the corresponding variation of 1% - 10% in the systematic error (which includes the luminosity error). The order of the sensitivity for other anomalous couplings varies between \( \sim 10^{-2} - 10^{-1} \) at 95 % C.L.

(v) Adopting the technique of the optimal observable we extract the inverse of the covariance matrices corresponding to each of the six kinematic observables by using the full information of all the four anomalous couplings in the \( \chi^2 \) as defined in equation (3.9). The effect of all observables are encoded in the combined inverse covariance matrices corresponding to the hadronic and leptonic modes, respectively. The hadronic and leptonic combined errors and correlations are then computed in equations (3.12a) and (3.12b), respectively. We combine the \( \chi^2 \)'s of the hadronic and leptonic mode by adding their inverse covariance matrices. Inverting the global 4 \( \times \) 4 covariant matrix we get the global error sensitivity with their correlations in equation (3.13) with an
integrated luminosity of 100 fb\(^{-1}\) data corresponding to the LHeC center of mass energy \(\sqrt{s} \approx 1.3\) TeV. The global combined error sensitivity of all couplings given in equation (3.13) is of the order of \(\sim 10^{-4}\) in absence of any systematic uncertainty.

(vi) Lastly, we have extended our optimal analysis to include the luminosity uncertainty factor in addition to four anomalous couplings and computed the resulting covariance matrix to give the correlation matrix and absolute error with which these couplings are going to be measured. The increasing luminosity error reduces the sensitivity of \(|V_{tb}| \Delta f_{L}^{f}\) and its correlation with other couplings. On combining the results from both hadronic and leptonic modes and computing global correlation matrices and errors with luminosity uncertainty at 1% we find that the error sensitivity of \(|V_{tb}| \Delta f_{L}^{f}\) becomes comparable to that observed in the bin analysis. The sensitivity is further reduced to the order \(10^{-2}\) for luminosity uncertainty greater than 5%. However, sensitivity of all other couplings are unchanged at \(\sim 10^{-4}\). The variation of the sensitivity of \(|V_{tb}| \Delta f_{L}^{f}\) w.r.t. luminosity for a fixed luminosity error at 1% is also studied and given in Figure 11, which shows that for a given luminosity uncertainty the error stabilizes for large values of the luminosity.

4.2 Comparison and Analysis

We compare our results with those quoted in the joint report TOPLHCNOTE [22], based on the recent experimental data at \(\sqrt{s} = 7\) TeV and integrated luminosity of 35 pb\(^{-1}\) to 2.2 fb\(^{-1}\). They found the sensitivity of \(\text{Re}(f_{R}^{2}) = 0.10 \pm 0.10\) \text{(stat.)} + 0.07 \text{(syst.)}\). Performing the analysis for the LHeC with \(E_{p} = 7\) TeV, \(E_{e} = 60\) GeV and integrated luminosity of 100 fb\(^{-1}\), we find the upper limit on anomalous coupling \(|f_{R}^{2}| \approx 0.011\) and 0.01 for the hadronic and leptonic modes, respectively.

Alternatively, one constrains the \(C_{tW}/\Lambda^{2}\), a coefficient of dimension six operator \(O_{tW} = (\bar{q} \sigma^{\mu \nu} \tau^{I} t) \ \hat{\phi} W_{\mu \nu}^{I}\) that contribute to the \(Wtb\) anomalous coupling. By translating the upper bound on the \(f_{L}^{R}\) on the upper limit of the coefficient corresponding to the dimension six operator, we find \(|C_{tW}/\Lambda^{2}| \leq 0.13\) TeV\(^{-2}\).

However, the limit from low energy electroweak precision data on the above operator is much stronger \(\sim |C_{tW}/\Lambda^{2}| \leq 0.4 \pm 1.2\) TeV\(^{-2}\) [17] than the present LHC bound and till date it provides the benchmark upper limit on the coefficient for this operator. Electroweak precision data also constrains the other coefficient \(C_{bW}/\Lambda^{2}\) \(\leq 11 \pm 13\) associated with dimension six operator \(O_{bW} = (\bar{q} \sigma^{\mu \nu} \tau^{I} b) \ \phi W_{\mu \nu}^{I}\). Translating the upper bound from the coefficient \(f_{L}^{R}\), we find that the proposed LHeC will improve the bound to a level of \(10^{-2}\) as evident from equation (3.20).

Recently, a detailed study on the top anomalous couplings for LHC at 14 TeV with 10 fb\(^{-1}\) data [24] is done. They have computed the effect of the anomalous couplings at both production and decay vertices into the full \(t\) channel matrix element of the single top quark production and illustrated that one sigma contours on the plane of the anomalous couplings lie within order of magnitude \(\sim 10^{-1}\). Therefore, the accuracy with which these couplings are measured at LHC can then be improved upon in the proposed LHeC as shown in Figures 7, 8, 9, 10 from the bin analysis.
At present the stringent upper bound on the magnitude of the anomalous couplings exist from the low energy $B$ physics experiments as mentioned in the introduction and given in references [5–9]. On comparing with these limits we find that the LHeC might be able to measure these anomalous couplings at the same level of accuracy or even can do better with a high luminosity facility having luminosity uncertainty $\leq 1\%-2\%$.

Our analysis shows that we can probe the $Wtb$ vertex at the LHeC to a very high accuracy and can obtain much more stringent upper limits on anomalous couplings, in comparison to existing limits from the LHC, electroweak physics and $B$ meson decays. We hope that our report will be useful in studying the physics potential of the LHeC project.

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