Affleck–Dine leptogenesis via multiscalar evolution in a supersymmetric seesaw model

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Abstract. A leptogenesis scenario in a supersymmetric standard model extended with introducing right-handed neutrinos is reconsidered. Lepton asymmetry is produced in the condensate of a right-handed sneutrino via the Affleck–Dine mechanism. The $LH_u$ direction develops a large value due to a negative effective mass induced by the right-handed sneutrino condensate through the Yukawa coupling of the right-handed neutrino, even if the minimum during the inflation is fixed at the origin. The lepton asymmetry is nonperturbatively transferred to the $LH_u$ direction by this Yukawa coupling.

Keywords: neutrino properties, baryon asymmetry

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1. Introduction

The origin of the baryon asymmetry of the universe is one of the unsolved problems. The existence of the baryon asymmetry is confirmed in several ways. Among them, the observation of the cosmic microwave background constrains the amount of the baryon asymmetry with considerable accuracy. It is given in terms of the baryon-to-entropy ratio as [1]

\[ \frac{n_B}{s} = (8.7 \pm 0.3) \times 10^{-11} \]  

where \( s \) is the entropy density of the universe.

The origin of neutrino masses is another problem of the standard model (SM). By observation of neutrino oscillations [2,3], it is confirmed that at least two flavors of neutrinos have non-zero masses. On the other hand, the sum of the masses of the three flavors of neutrinos is constrained as \( \sum m_\nu < 2 \text{ eV} \) by cosmological observations [1,4]. Hence, the SM should be extended to explain nonzero neutrino masses and the smallness of them. Introducing heavy right-handed Majorana neutrinos provides a good explanation for the problem of neutrino masses via the seesaw mechanism [5].

Heavy right-handed Majorana neutrinos also provide an attractive solution of the origin of the baryon asymmetry by leptogenesis [6]. In the leptogenesis scenario, lepton asymmetry is generated at first, and then the sphaleron process partially transfers it into the baryon asymmetry. In the thermal leptogenesis scenario, the lepton asymmetry is generated via the out-of-equilibrium decay of heavy right-handed Majorana neutrinos produced in the primordial thermal bath [6,7]. Therefore, this scenario requires the
reheating temperature $T_R$ after the inflation to be higher than the mass of the lightest right-handed neutrino. However, $T_R \lesssim 10^8$ GeV is required to avoid the overproduction of the gravitino in supersymmetric (SUSY) theories [8]. The production of the sufficient lepton asymmetry is very difficult in SUSY models due to this constraint [7].

Hence, several alternative leptogenesis scenarios have been considered in SUSY theories. The Affleck–Dine mechanism [9]–[11], which produces asymmetry between particle and antiparticle in the condensate of a scalar field, is one of the interesting scenarios, since many species of scalar particles exist in SUSY models. In the Affleck–Dine leptogenesis along the $LH_u$ flat direction [12], the source of lepton number violation originates in the operator of the neutrino mass. However, in the SUSY seesaw model, if the mass of a right-handed neutrino is smaller than the Hubble parameter at the end of the inflation $H_{\text{inf}}$, the $LH_u$ direction is not flat due to the $F$-term potential from the Yukawa coupling of the right-handed neutrino$^2$. The right-handed neutrino mass should be less than about $10^{13}$ GeV for $m_\nu \sim 1$ eV if the Yukawa coupling is less than unity. For the chaotic inflation, which predicts $H_{\text{inf}} \sim 10^{13}$ GeV, it is severe that the right-handed neutrino masses dominate over $H_{\text{inf}}$.

In addition to right-handed neutrinos, there are right-handed sneutrinos $\tilde{N}$ in the SUSY seesaw model. The potential for $\tilde{N}$ is flat except for SUSY mass terms, if $R$-parity is conserved. Therefore, quantum fluctuation of a right-handed sneutrino can be large at the end of the inflation [12], as long as the right-handed neutrino mass is smaller than $H_{\text{inf}}$. Hence, the right-handed sneutrino can have the large number density at the end of the inflation. Thus, if $CP$-violation in the $\tilde{N}$ decay into leptons and anti-leptons is large enough, sufficient lepton asymmetry can be produced.

Later, this scenario was drastically changed after SUSY breaking in the early universe was reported [14]. Due to the negative Hubble mass squared of a right-handed sneutrino, minima of the potential of the right-handed sneutrino are largely deviated from the origin during inflation. Therefore, the right-handed sneutrino has a large value. After the inflation, this scalar field condensate begins coherent oscillations. If there exists $CP$-violation in the potential, e.g. $B$ term of $\tilde{N}$, this condensate acquires the particle number asymmetry for the right-handed sneutrino via the Affleck–Dine mechanism. In [15], one scenario along with the multidimensional Affleck–Dine mechanism [16] is reported. The asymmetry of $\tilde{N}$ is nonperturbatively transferred to the $LH_u$ direction, if the $LH_u$ direction is approximately flat, that is, both the $\tilde{N}$ and $LH_u$ directions have a large value during the inflation. In this scenario, the lepton asymmetry can be generated without $CP$-violation in the right-handed sneutrino decay.

On the other hand, in [17], Allahverdi and Drees reported that the particle number asymmetry of $\tilde{N}$ can be transferred by perturbative decay of $\tilde{N}$ even if the $LH_u$ direction is not approximately flat or has a positive Hubble mass squared, i.e. without the evolution of the scalar field along the $LH_u$ direction. Because of the Majorana nature of the right-handed neutrino, right-handed sneutrinos decay with generating $\pm 1$ lepton number in the same decay rate if SUSY is conserved. These two decay rates are deviated from each other because of SUSY breaking by thermal effects in the early universe. Therefore, non-zero lepton number can be generated in compensation for some parameter tuning.

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$^2$ If a neutrino Yukawa coupling is very tiny, e.g. in the Dirac neutrino model, the $LH_u$ direction is approximately flat and can have a large value during inflation. The Affleck–Dine leptogenesis in the Dirac neutrino model can explain the baryon asymmetry without the lepton number violation except for the sphaleron process [13].
However, we found in this work that the evolution of the \( LH_u \) direction is induced by a negative effective mass given by the right-handed sneutrino condensate, even if the \( LH_u \) direction is not approximately flat or has a positive Hubble mass squared. Therefore, the evolution of the \( LH_u \) direction cannot be neglected in the broad parameter region. Thus, the evolution of the scalar fields and the lepton asymmetry is complicated, like the scenario in [15]. In this paper, we reconsider the scenario discussed in [17], including the evolution of the \( LH_u \) direction. We will see that the lepton asymmetry in the right-handed sneutrino condensate can be nonperturbatively transferred to the \( LH_u \) direction condensate via the interaction between these scalar fields. We also summarize the condition that the evolution of the \( LH_u \) direction is relevant.

The rest of this paper is organized as follows. In the next section, we summarize the set-up of this scenario. In section 3, we discuss the evolution of scalar fields. In section 4, we consider the evolution of lepton asymmetry. The resultant baryon asymmetry is discussed in section 5. We also comment on the difference between this scenario and the previous one. Finally, we summarize the result in section 6.

2. Model

We consider the SUSY seesaw model. The superpotential is given by

\[
W = W_{MSSM} + y_{\nu} N LH_u + \frac{M_N}{2} NN + \frac{\lambda}{4 M_{Pl}} N N N N,
\]

where \( W_{MSSM} \) is the superpotential of the minimal SUSY SM (MSSM), \( y_{\nu} \) is the coupling between right-handed neutrinos and left-handed leptons, and \( M_N \) is the mass matrix of right-handed neutrinos. We included the non-renormalizable superpotential of \( N \) with coupling constant given by \( \lambda \), which is important for successful baryogenesis in this scenario. For simplicity, only one flavor of right-handed sneutrino is considered. This can be naturally realized by assuming that three neutrino masses satisfy the condition, \( M_N = M_{N_1} < H_{\text{inf}} < M_{N_2} < M_{N_3} \). Hence, we consider only one flavor of left-handed leptons coupling to the right-handed sneutrino. We choose \( y_{\nu} \) and \( M_N \) to be real and positive by redefining superfields. We assign lepton number \(-1\) to \( \tilde{N} \) although right-handed neutrinos cannot carry \( U(1) \) charge because of their Majorana nature. The lepton number of \( \tilde{N} \) is violated by \( B \) terms and non-renormalizable terms, but this violation is suppressed by SUSY breaking effects or cutoff scale \( M_{Pl} \). Therefore, we can assign the lepton number to \( \tilde{N} \) in this work.

A light neutrino mass is given by the seesaw mechanism as

\[
m_{\nu} = \frac{y_{\nu}^2 v^2}{M_N} \sim 10^{-2} \text{eV} \left( \frac{y_{\nu}}{10^{-2}} \right)^2 \left( \frac{10^{11} \text{GeV}}{M_N} \right).
\]

Here \( v \sim \mathcal{O}(100) \text{GeV} \) is the vacuum expectation value (vev) of \( H_u \).

We consider the evolution of the right-handed sneutrino \( \tilde{N} \) and the \( LH_u \) direction parametrized by a complex scalar field \( \phi \), namely

\[
\tilde{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}.
\]
The $\text{LH}_a$ direction is $D$-flat, but not $F$-flat due to the Yukawa coupling of the neutrinos. Nevertheless, the evolution of this direction gives an important effect on the leptogenesis. Hence, in this work we take this direction into consideration.

Including Hubble-induced SUSY-breaking effects and thermal corrections, the scalar potential is given by

$$V(\phi, \tilde{N}) = \frac{y_v^2}{4}|\phi|^4 + M_N^2|\tilde{N}|^2 + \sum_{a=1}^{4} y_a^2|\phi|^2|\tilde{N}|^2 + \frac{\lambda^2}{M_{\text{Pl}}^2}|\tilde{N}|^6$$

$$+ \left[ \left( \frac{y_v}{2} M_N \phi^2 \tilde{N}^* + \frac{y_v \lambda}{2 M_{\text{Pl}}} \phi^2 \tilde{N}^* \tilde{N} \right) + h.c. \right]$$

$$+ c_\phi H^2 |\phi|^2 - c_N H^2 |\tilde{N}|^2$$

$$+ \left[ \left( \frac{b H}{2} M_N \tilde{N}^2 + \frac{a_\phi y_v}{2} H \phi^2 \tilde{N} + \frac{a_\lambda}{4 M_{\text{Pl}}} H \tilde{N}^4 \right) + h.c. \right] + V_{\text{th}}(\phi),$$  \hspace{1cm} (5)

where $H$ is the Hubble parameter and we ignored low-energy SUSY-breaking terms, which are not relevant to this leptogenesis. The first and second lines are the $F$-term potential from the superpotential (2), while the third and fourth lines except for the last term are Hubble-induced SUSY breaking terms. Here we assumed that the Hubble-induced mass of $\phi$ is positive in order to set it at the origin during the inflation even for small $y_v$, while that of $\tilde{N}$ is negative in order to give large values. Therefore, the real parameters $c_\phi \sim 1$ and $c_N \sim 1$ are both assumed to be positive. Complex parameters $a_\phi$, $a_\lambda$ and $b$ determine the magnitude of the Hubble-induced $A$ terms and $B$ term, respectively. Absolute values of these constants are typically $O(1)$ during the inflation. However, since these terms are generally induced by coupling with the inflaton, they oscillate rapidly by the oscillation of the inflaton after inflation. Thus, effective values of these terms vanish if they are averaged over a timescale much longer than the period of oscillation of the inflaton. Therefore, we simply assume $a_\phi = a_\lambda = b = 0$ after the inflation. The last term in the fourth line is thermal-mass corrections [18]. These are given by

$$V_{\text{th}}(\phi) \equiv \sum_{f_k |\phi| < T} c_k f_k^2 T^2 |\phi|^2.$$  \hspace{1cm} (6)

Here, $f_k$ denotes coupling constants of left-handed leptons or up-type Higgs, and $c_k$ are determined by degrees of freedom of these particles. The temperature of the thermal plasma $T$ before the reheating ends is estimated as

$$T \sim \left( g_*^{-1/2} H T_{\text{Pl}}^2 M_{\text{Pl}} \right)^{1/4},$$  \hspace{1cm} (7)

where $g_* \simeq 200$ is the effective total degree of freedom of the thermal bath and $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. There are also thermal-log corrections [19], but they do not have dominant effects in this leptogenesis.

During the inflation, the potential of $\tilde{N}$ has minima at $|\tilde{N}| \simeq \sqrt{H} M_{\text{Pl}} / \lambda$. However, starting from these minima, $\tilde{N}$ is eventually trapped at minima of the $F$-term potential, $|\tilde{N}| \simeq \sqrt{M_N M_{\text{Pl}} / \lambda}$. In order to avoid this disastrous consequence, we assume that $\tilde{N}$ is in a multiplet of the $SO(10)$ grand unified theory (GUT). When $|\tilde{N}| > M_{\text{GUT}}$, the steep $D$-term potential of the GUT gauge group appears. Therefore, the initial value of $\tilde{N}$ is given by $|\tilde{N}_{\text{ini}}| = M_{\text{GUT}} = 10^{16}$ GeV. In addition, it is required that the local maximum of
the $F$-term potential of the radial direction of $\tilde{N}$, which is given by $|\tilde{N}| \simeq \sqrt{M_N M_{\text{Pl}}/(3\lambda)}$, is placed at $|\tilde{N}| > M_{\text{GUT}}$. This requirement can be rewritten by

$$M_N/\lambda > 1.2 \times 10^{14} \text{ GeV}.$$  \hfill (8)

Provided with these conditions, $\tilde{N}$ rolls down towards the origin and oscillates around there after $H < M_N$.

3. Evolution of scalar fields

The evolution equations of the scalar fields are given by

$$\ddot{N} + (3H + \Gamma_N)\dot{N} + \frac{\partial V}{\partial N^*} = 0,$$  \hfill (9)

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0,$$  \hfill (10)

where $\Gamma_N = y_\nu^2 M_N/(4\pi)$ is the decay width of $\tilde{N}$. The initial condition of $\tilde{N}$ is $|\tilde{N}_{\text{ini}}| = M_{\text{GUT}}$. On the other hand, the initial phase of $\tilde{N}$ is dependent on whether or not the Hubble parameter $H_{\text{inf}}$ is much larger than the effective mass of the phase direction during inflation. The potential is dominated by the $A$ or $B$ term under the condition (8). If the effective mass of the phase direction is much smaller than $H_{\text{inf}}$, $\theta_{\tilde{N}}$ is randomly displaced from minima of the potential. Otherwise, $\theta_{\tilde{N}}$ may be fixed at one of the minima of the potential. In both cases, the successful baryogenesis can be realized as we will see later. In the former case, $H_{\text{inf}}$ is constrained to be $H_{\text{inf}} < 3 \times 10^{12}$ GeV for small enough baryonic isocurvature perturbation as discussed in section 5.2. In the latter case, large $M_N$ or $\lambda$ is required to derive the large effective mass of the phase direction. Meanwhile, $\phi$ has positive Hubble-induced mass and large effective mass $y_\nu M_{\text{GUT}}$ via the neutrino Yukawa coupling. Therefore, $\phi$ is trapped at the origin $\phi = 0$ and only has quantum fluctuation suppressed by its large effective mass. Although the value $\langle \phi \rangle$ averaged over the universe is expected to vanish, a finite value $\langle \phi^2 \rangle$ remains.

3.1. Before destabilization of $\phi$

For $H > M_N$, $\tilde{N}$ remains as $|\tilde{N}| = |\tilde{N}_{\text{ini}}|$. When $H$ becomes $H < M_N$, the mass term dominates the evolution of $\tilde{N}$ and $\tilde{N}$ begins oscillation around the origin. The amplitude of the oscillation decreases with

$$|\tilde{N}| \sim M_{\text{GUT}} \frac{H}{M_N}.$$  \hfill (11)

The cross-term in the $F$-term potential $(y_\nu M_N \phi^2 \tilde{N}^* + \text{h.c.})$ gives a negative contribution to the effective mass of $\phi$. Therefore, when $H$ decreases to $H_1$, $\phi$ becomes tachyonic and acquires large amplitude because the mass squares from $H^2|\phi|^2$, $y_\nu^2|\phi|^2|N|^2$ and the cross-term decreases as $H^2$, $H^2$ and $H$, respectively. The negative mass dominates over the Hubble-induced mass when $H < y_\nu M_{\text{GUT}}$, while it dominates over the effective mass from the quartic coupling when $H < M_N^2/(y_\nu M_{\text{GUT}})$. The $LH_u$ direction $\phi$ becomes tachyonic when both conditions are satisfied. Therefore, we define the Hubble parameter
at the destabilization \( H_1 \) as

\[
H_1 = \begin{cases} 
\frac{y_\nu M_{\text{GUT}}}{M_N^2} & \text{if } m_\nu < 10^{-8} \text{ eV} \left( \frac{M_N}{10^{11} \text{ GeV}} \right) \\
\frac{M_N^2}{y_\nu M_{\text{GUT}}} & \text{if } m_\nu > 10^{-8} \text{ eV} \left( \frac{M_N}{10^{11} \text{ GeV}} \right).
\end{cases}
\] (12)

Note that always \( H_1 < M_N \). The negative contribution from \( \phi^2 \tilde{N}^* \tilde{N} \) term cannot be dominant under the condition (8).

Thermal-mass terms give positive contributions to the effective mass of \( \phi \), while thermal-log corrections are ineffective because \( \phi \) does not have a large value. The destabilization of \( \phi \) is prevented if thermal-mass corrections dominate over the negative mass contribution. Otherwise, the same scenario as discussed in [17] is realized. This condition is given by

\[
H_1 > H_1' \simeq 10^7 \text{ GeV} \left( \frac{y_\nu}{200} \right)^{-1/2} \left( \frac{T_R}{10^9 \text{ GeV}} \right)^2 \left( \frac{y_\nu}{10^{-2}} \right)^{-2} \left( \frac{M_{\text{GUT}}}{10^{16} \text{ GeV}} \right)^{-2},
\] (13)

where we consider the thermal-mass correction from top (s)quarks in the thermal bath.

In addition, the right-handed sneutrino must not decay before the destabilization. This constraint is given as \( \Gamma_N < H_1 \). If \( H_1 = y_\nu M_{\text{GUT}} \), this is rewritten as \( M_N < 10^{16} \text{ GeV}(m_\nu/10^{-2} \text{ eV})^{-1/3} \). On the other hand, for \( H_1 = M_N^2/(y_\nu M_{\text{GUT}}) \), this constraint is given as \( M_N < 10^{16} \text{ GeV}(m_\nu/10^{-2} \text{ eV})^{-3} \), Thus, this constraint is ignorable in both cases.

In [17], it is also discussed that the parametric resonance [20,21] may cause fast decay of \( |\tilde{N}| \). Since \( \tilde{N} \) oscillates with large amplitude, it gives large oscillating effective mass of \( \phi \), unless \( \tilde{N} \) has a circle-like trajectory in the complex plane due to large angular momentum. This may result in exponential amplification of the fluctuation of \( \phi \) and rapid decrease of the amplitude of the \( \tilde{N} \) oscillation. If \( \tilde{N} \) decays via the parametric resonance before the destabilization of \( \phi \), the destabilization cannot take place. However, this is not serious for \( m_\nu > 10^{-8} \text{ eV}(M_N/10^{11} \text{ GeV}) \), because \( \phi \) can decay into other particles. The typical amplification rate under broad parametric resonance is estimated to be proportional to \( \exp(0.175M_N t) \) [21]. On the other hand, the decay rate \( \Gamma_\phi \) has the smallest value just before the destabilization of \( \phi \), \( \Gamma_\phi \sim \sum g_i^2 M_N/(8\pi) \), where \( g_i \) are Yukawa and gauge couplings, and we sum over all final states. Hence, the parametric resonance is safely negligible since \( \Gamma_\phi > 0.175M_N \) in the MSSM. On the other hand, parametric resonance may take place for \( m_\nu < 10^{-8} \text{ eV}(M_N/10^{11} \text{ GeV}) \), because \( \Gamma_\phi > 0.175M_N \) is not guaranteed. Since the detail of the parametric resonance is involved, we do not discuss whether the parametric resonance actually gives fast decay of \( |\tilde{N}| \). For simplicity, we neglect the case \( m_\nu < 10^{-8} \text{ eV}(M_N/10^{11} \text{ GeV}) \).

### 3.2. After destabilization of \( \phi \)

After \( H = H_1 \), the potential of \( \phi \) has two distinct minima in opposite phase directions:

\[
|\phi| \sim \sqrt{\frac{2M_N|\dot{\tilde{N}}(H_1)|}{M_{\text{GUT}}}} \sim \sqrt{\frac{2M_N H_1}{y_\nu}}.
\] (14)

The initial condition at the destabilization is determined by quantum fluctuation during inflation and the subsequent evolution. After the destabilization of \( \phi \), long wavelength
modes of fluctuation of $\phi$ get large values via tachyonic instability \cite{22,23}. Once the value of $\phi$ begins to track the minimum of the potential, it can be interpreted as a classical field in a local patch of the universe. Hereafter we assume that the universe consists of patches in which the condensate of $\phi$ has various initial values. Hence, the resultant lepton asymmetry is estimated by averaging over results from various values of initial quantum fluctuation. Since a typical comoving wavelength of this quantum fluctuation is negligibly small compared with the present horizon scale, this fluctuation has no cosmologically observable consequence. In other words, the initial fluctuation of $\phi$ averaged over cosmological scale is suppressed to be negligibly small, because such a scale is far larger than the horizon scale at the epoch $H = H_1$. Indeed, the comoving length $k^{-1}_1$ of the latter scale is estimated to be

$$k^{-1}_1 \sim \mathcal{O}(10) \text{ km} \times \left( \frac{H_1}{10^8 \text{ GeV}} \right)^{-1/3} \left( \frac{T_R}{2 \times 10^6 \text{ GeV}} \right)^{-1/3} \left( \frac{y_\nu}{100} \right)^{-1/12}. \quad (15)$$

Note that the problem of domain walls separating two minima is not serious since they disappear when $\phi$ evaporates.

With the minimization of the cross-term, the dominant contributions in this epoch can be rewritten by

$$V_F(\phi, \tilde{N}) = \left( \frac{y_\nu}{2} |\phi|^2 - M_N |\tilde{N}| \right)^2 + y_\nu^2 |\phi|^2 |\tilde{N}|^2. \quad (16)$$

Though this potential has the global minimum at $|\phi| = |\tilde{N}| = 0$, there is a valley on the trajectory $y_\nu |\phi|^2 = 2M_N |\tilde{N}|$, lifted by $y_\nu^2 |\phi|^2 |\tilde{N}|^2$. Therefore, $\phi$ and $\tilde{N}$ oscillate around the origin approximately satisfying the relation, $y_\nu |\phi|^2 = 2M_N |\tilde{N}|$. The amplitudes of $|\phi|$ and $|\tilde{N}|$ decrease proportionally to $H^{1/2}$ and $H$, respectively.

The condensate of $\tilde{N}$ decays by the neutrino Yukawa coupling at $H \sim \Gamma_N$. Since the decay rate acts as a friction term in the evolution equation, $\tilde{N}$ is strongly fixed at the minimum of the potential soon after $H = \Gamma_N$. Hence, the evolution of the scalar fields is determined by that of $\phi$. After $|\tilde{N}|$ is integrated out by $|\tilde{N}| = y_\nu |\phi|^2/(2M_N)$, the effective potential of $\phi$ is given by

$$V_{\text{eff}}(\phi) \simeq \frac{y_\nu^4}{4} \frac{|\phi|^6}{M_N^2} + V_{\text{th}}(\phi), \quad (17)$$

where we included thermal potentials. The $LH_u$ direction does not decay before $\tilde{N}$ does, since all decay modes with coupling constants larger than $y_\nu$ are kinematically forbidden as long as $H > \Gamma_N$. Since the first term decreases as $H^0$, thermal corrections usually dominate the evolution of $\phi$, soon after the $\tilde{N}$ decay starts. Afterward, $\phi$ oscillates around the origin and eventually evaporates.

In figure 1, we show an example of the evolution of the scalar fields. The black (grey) line show the evolution of $|\phi|$($|\tilde{N}|$). Here, parameters are taken as $M_N = 10^{11}$ GeV, $y_\nu = 10^{-2}$, $T_R = 2 \times 10^6$ GeV, $c_\phi = c_N = 1$ and $\lambda = 10^{-4}$. We assumed $a_y = a_N = b = 0$ as mentioned above. As the initial condition, we assumed $|\tilde{N}_{\text{ini}}| = M_{\text{GUT}}$ (arg $\tilde{N}_{\text{ini}} = \pi/4$) and $|\phi_{\text{ini}}| = 10^{11}$ GeV. Here we evaluated typical values of quantum fluctuation by averaging over the horizon scale after the inflation with $H_{\text{inf}} = 10^{13}$ GeV, because the destabilization of $\phi$ is not instantaneous. In order to take the randomness of the initial condition into account, we iterated the same calculation for various initial amplitudes and phases of $\phi$.
This figure is the result for arg($\phi_{\text{ini}}$) = 0. It can be seen that $|\tilde{N}|$ is fixed at $|\tilde{N}| = M_{\text{GUT}}$ until $H \approx M_N/3$ and begins oscillation at $H \approx M_N/3$ from this figure. We can also confirm that the destabilization of $\phi$ completes at $H \sim H_1 = 10^8$ GeV. Afterward, $|\phi|$ decreases proportionally to $H^{1/2}$, while $|\tilde{N}|$ does proportionally to $H$. Finally, $\tilde{N}$ decays at $H \sim \Gamma_N$.

4. Evolution of lepton asymmetry

The evolution of the lepton asymmetry is determined by the evolution of the scalar fields. We define lepton asymmetries in the condensate of the right-handed sneutrino $L_{\tilde{N}}$ and in that of the $LH_u$ direction $L_\phi$ as follows:

$$L_{\tilde{N}} \equiv -i(\dot{\tilde{N}}^* \tilde{N} - \tilde{N} \dot{\tilde{N}}^*)$$

$$L_\phi \equiv \frac{1}{2}(\phi^* \phi - \dot{\phi} \phi^*)$$

Note that we assigned lepton number $-1$ to $\tilde{N}$. Evolution equations of these lepton asymmetries are given from the evolution equations of these scalar fields, equations (9) and (10),

$$\frac{d}{dt} \left( \frac{L_{\tilde{N}}}{H^2} \right) + \Gamma_N \frac{L_{\tilde{N}}}{H^2} = -\frac{1}{H^2} \left[ y_\nu M_N \text{Im}(\phi^* \tilde{N}) + \frac{4\lambda_{\nu}}{M_{\text{Pl}}} \text{Im}(\tilde{N}^3 \tilde{N}^*) + \frac{3y_\nu \lambda}{M_{\text{Pl}}} \text{Im}(\phi^* \tilde{N}^3) \right]$$

$$\frac{d}{dt} \left( \frac{L_\phi}{H^2} \right) = -\frac{1}{H^2} \left[ y_\nu M_N \text{Im}(\phi^* \tilde{N}) + \frac{y_\nu \lambda_{\nu}}{M_{\text{Pl}}} \text{Im}(\phi^* \tilde{N}^3) \right]$$

Figure 1. The evolution of the values of $|\phi|$ (black) and $|\tilde{N}|$ (grey) as a function of $H$. We take $M_N = 10^{11}$ GeV, $y_\nu = 10^{-2}$, $T_R = 2 \times 10^6$ GeV and $\lambda = 10^{-4}$.
where Hubble-induced phase-dependent terms are dropped as mentioned above. The right-hand side of these equations are source terms of the lepton asymmetry. If a dominant source term is simply scaling with \( t^\gamma \propto H^{-\gamma} \) on average, where the index \( \gamma \) is a constant, the evolution of the lepton asymmetry is simple: if \( \gamma > -3 \), the lepton asymmetry increases proportionally to \( t^{\gamma+3} \), while if \( \gamma < -3 \), the lepton asymmetry is fixed. In the case of \( \gamma = -3 \), the lepton asymmetry increases proportionally to \( \log t \). Thus, the lepton asymmetry is also considered to be almost fixed in this case.

The evolution of these lepton asymmetries are very complicated. However, the evolution of the left–right asymmetry \( L_\phi - L_\tilde{N} \) is simpler. The evolution of \( L_\phi - L_\tilde{N} \) is given by the equation

\[
\frac{d}{dt} \left( \frac{L_\phi - L_\tilde{N}}{H^2} \right) = \frac{1}{H^2} \left[ \frac{4 \lambda M_N}{M_{Pl}} \text{Im} \left( \tilde{N}^3 \tilde{N}^* \right) + \frac{2 y_\nu \lambda}{M_{Pl}} \text{Im} \left( \phi^{*2} \tilde{N}^3 \right) \right] + \frac{\Gamma_N}{H^2} L_\tilde{N}.
\]

(22)

Thus, the dominant contribution from the \( \phi^{*2} \tilde{N} \) term to the left–right asymmetry cancels. Since the other terms attenuate sufficiently fast, the evolution of the left–right asymmetry is fixed after \( \tilde{N} \) begins oscillation.

4.1. Before destabilization of \( \phi \)

In this era, the decay width is negligible because \( H \gg \Gamma_N \). The second and third terms in the right-hand side of equation (22) are also negligible, since the fluctuation of \( \phi \) is suppressed by its large effective mass. Thus, the first term is dominant because the phase direction of \( \tilde{N} \) is generally displaced from minima determined by the \( \tilde{N}^3 \tilde{N}^* \) term. Therefore, \( L_\phi - L_\tilde{N} \) is determined only by the evolution of \( \tilde{N} \). The magnitude of left–right asymmetry at \( H = M_N/3 \) is estimated by

\[
\frac{|L_\phi - L_\tilde{N}|}{s'} \sim \frac{|L_\tilde{N}|}{s'} \sim \frac{6 \lambda M^4_{GUT} T_R}{M_{Pl}^2 s'^2} \delta_{\text{eff}},
\]

(23)

where the entropy parameter \( s' \) is defined as \( s' \equiv 4M_{Pl}^2 H^2/T_R \), and \( \delta_{\text{eff}} \lesssim 1 \) is the phase factor. Note that \( s' \) is normalized by the entropy after the reheating completes.

4.2. After destabilization of \( \phi \)

After \( \phi \) acquires a large value, it takes part in the evolution. In equations (20) and (21), the first term of the right-hand side becomes dominant. Since this term induces rapid exchange between \( L_\tilde{N} \) and \( L_\phi \), these asymmetries oscillate rapidly.

However, \( L_\phi - L_\tilde{N} \) is fixed because all the source terms for left–right asymmetry attenuate with \( H^4 \) after \( \phi \) gets a large value. At the epoch \( \phi \) is rolling down to the displaced minimum, the contribution to the \( L_\phi - L_\tilde{N} \) from the second term of equation (22) is difficult to estimate because of the fast and nonlinear evolution. Furthermore, this contribution depends on the initial phase of the random fluctuation of \( \phi \). However, we confirmed that this contribution is subdominant if the contribution from the first term is sufficiently large, since the contribution from the second term almost vanishes on average over the cosmological scale, which contains a large number of patches of horizon scale at the epoch.
Therefore this direction is the same over the cosmological scale. Since \( \sim \) direction of the rotational evolution of \( \hat{\varphi} \) depends on the initial value of \( \sim \) of fixed asymmetry of \( \varepsilon \). This implies \( \delta \) on the phase of the oscillation of \( \sim \) the asymmetry of \( \hat{\varphi} \) for \( H > M \) does not change because of approximate conservation of the angular momentum. Thus, the final direction of the rotation of \( \hat{\varphi} \) is the minimum that \( \varphi \) is induced by this rotation of two minima, \( \hat{\varphi} \) rotates in one direction. Because the rotational minima after destabilization due to coupling with \( \hat{\varphi} \) evaporates. The fact that the final lepton asymmetry has a non-vanishing value can also be understood qualitatively by the following discussion. The potential of \( \hat{\varphi} \) has two distinct minima after destabilization due to coupling with \( \hat{N} \). The crucial observation is that the direction of the rotational evolution of \( \hat{N} \) is determined only by initial evolution of \( \hat{N} \). Therefore this direction is the same over the cosmological scale. Since \( \hat{N} \) rotates in a definite direction, these two minima also rotate in one direction. Because the rotational motion of \( \hat{\varphi} \) is induced by this rotation of two minima, \( \hat{\varphi} \) rotates in one direction whichever is the minimum that \( \hat{\varphi} \) is trapped in. After \( \hat{N} \) decays, the direction of the rotation of \( \hat{\varphi} \) does not change because of approximate conservation of the angular momentum. Thus, the final direction of the rotation of \( \hat{\varphi} \), which is equivalent to the sign of the final lepton asymmetry \( \hat{L}_\phi \), is the same all over the universe.

We show the evolution of asymmetries in figure 2 for the same parameter set as we chose for figure 1. The solid black line, the solid grey line and the dashed grey line indicate evolutions of \( |\hat{L}_\phi|/s' \), \( |\hat{L}_N|/s' \) and \( |\hat{L}_\phi - \hat{L}_N|/s' \), respectively. It can be seen that for \( H > M_N \), \( |\hat{L}_N| \) increases, while \( |\hat{L}_\phi| \) is negligible. At \( H \sim H_1 \), \( \hat{\varphi} \) receives a fraction of the asymmetry of \( \hat{N} \). We can also confirm from this figure that for \( H < \Gamma_N \simeq 10^6 \) GeV, \( \hat{L}_N \) decreases exponentially and \( \hat{L}_\phi \) is fixed. For the parameter used in our calculation, the estimation using equation (23) gives \( |\hat{L}_\phi|/s = 8.3 \times 10^{-11} \delta eff \varepsilon \). We iterated the same calculation for 25 various initial phases, and the result is \( |\hat{L}_\phi|/s = 5.2 \times 10^{-11} \) on average. This implies \( \delta eff \varepsilon \sim 0.6 \) in this case. We also confirmed that other choices of initial amplitude \( |\phi ini| \) do not change the result.

5. Resultant baryon asymmetry

5.1. Estimation of baryon asymmetry

The lepton asymmetry released into the SM sector is transferred to the baryon asymmetry through the sphaleron process by the ratio \( n_B = (8/23)n_L \) [24], where \( n_B \) is the resultant baryon asymmetry in the thermal bath and \( n_L \) is the lepton asymmetry produced by this scenario. Hence, the baryon asymmetry is estimated as

\[
\frac{n_B}{s'} \sim \frac{48 \lambda M_{\rm GUT}^4 T_R}{23 M_N^2 M_{Pl}^4} \delta eff \varepsilon. \tag{24}
\]
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Figure 2. The evolution of lepton asymmetries $|L_\phi|/s'$ (solid black), $|L_\tilde{N}|/s'$ (solid grey) and the left–right asymmetry $|L_\phi - L_\tilde{N}|/s'$ (dashed grey). Since these asymmetries oscillate, we show magnitudes of them. The parameters are the same as for figure 1.

In figure 3, we show the parameter region in which the right amount of baryon asymmetry is produced and some constraints on the $M_N-T_R$ plane. Solid lines with dotted parts show the region in which resultant baryon asymmetry (24) is $n_B/s = 8.7 \times 10^{-11}$ for various $\lambda$. We assumed that $\delta_{\text{eff}} \epsilon = 1$. The baryon asymmetry is simply proportional to this factor. The dotted parts of these lines are excluded because of the constraint (8). The dashed lines indicate the constraint that thermal corrections must not dominate over the negative mass contribution of $\phi$ for several neutrino masses (see equations (12) and (13)). Note that neutrino masses are determined by equation (3). We can see that this constraint is not stringent. Above these lines, the scenario discussed in [17] can be realized. In addition, if initial $\theta_\tilde{N}$ is not fixed at any minima determined by Hubble-induced $A$ or $B$ term, $M_N < H_{\text{inf}} < 3 \times 10^{12}$ GeV is required in order that baryonic isocurvature perturbation should be sufficiently small (see section 5.2). This result indicates that successful baryogenesis via this scenario favors larger $M_N$ and higher $T_R$ for large $\lambda$ and smaller $M_N$ and lower $T_R$ for small $\lambda$.

As we can see in equation (24), the resultant baryon asymmetry is proportional to $|\tilde{N}_{\text{ini}}|^4 = M_{\text{GUT}}^4$. If $\tilde{N}$ is the multiplet of the subgroup of $SO(10)$ GUT, e.g. $SU(2)_R$, $|\tilde{N}_{\text{ini}}| = M_{SU(2)_R} < M_{\text{GUT}}$ should be used. Unless $\lambda$ is extremely small, it is difficult to explain the origin of baryon asymmetry by this scenario for $|\tilde{N}_{\text{ini}}| = M_{SU(2)_R} < M_{\text{GUT}}$.

Finally, we summarize the difference between our scenario and that in [17]. The latter scenario may be realized above the dashed lines in figure 3. Our scenario can explain the baryon asymmetry for lower $T_R$ than that in [17]. On the other hand, the scenario in [17] has an advantage that sufficient baryon asymmetry can be generated if
Figure 3. Solid lines show the parameter region in which the resultant baryon asymmetry is $n_B/s = 8.7 \times 10^{-11}$ for various values of $\lambda$. We assumed that $\delta_{\text{eff}} \epsilon = 1$. The dotted parts of these lines are excluded by the condition $M_N/\lambda > 1.2 \times 10^{14}$ GeV. The dashed lines show the constraint $H > H_1'$ for several $m_\nu$, which is not stringent. Above these lines, the scenario discussed in [17] can be realized.

Another advantage of their scenario is that non-renormalizable terms in the superpotential are not required. On the other hand, the most important advantage of our scenario is that the successful baryogenesis can be realized without fine-tuning. The scenario in [17] requires some tuning between the decay width of $\Gamma_N$ and the magnitude of $B$ terms. However, any parameter tuning is not required in our scenario.

5.2. Baryonic isocurvature perturbation

In the case that the potential of the phase direction $\theta_\tilde{N}$ of $\tilde{N}$ is sufficiently flat during the inflation, $\theta_\tilde{N}$ is randomly displaced from the minima of the potential. Then, $\theta_\tilde{N}$ has isocurvature perturbation:

$$\delta \theta_\tilde{N} = \frac{H_{\text{inf}}}{\sqrt{2k^3 |N|}}. \quad (25)$$

for Fourier mode $k$. Since $L_\tilde{N}$ is produced via the displacement of $\theta_\tilde{N}$ from minima determined by $\tilde{N}^3\tilde{N}^*$ term, this isocurvature perturbation results in the isocurvature perturbation of $L_\tilde{N}$, which is finally transferred to the isocurvature perturbation of baryon asymmetry. Since $n_B \propto \delta_{\text{eff}}$ and $\delta_{\text{eff}}$ can be estimated by $\delta_{\text{eff}} \sim \sin(2\delta \theta_\tilde{N})$, the amplitude of the baryonic isocurvature perturbation can be estimated to be

$$\frac{\delta n_B}{n_B} \sim 2\delta \theta_\tilde{N}. \quad (26)$$
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According to the constraint on the baryonic isocurvature perturbation in terms of the ratio between the power spectrum of matter isocurvature perturbation and that of curvature perturbation [25]

\[ B_a \equiv \sqrt{\frac{P_S}{P_R}} = \sqrt{\frac{1}{2.4 \times 10^{-9}}} \left( \frac{k^3 \Omega_b^2}{2 \pi^2 \Omega_m^2} \left\langle \frac{\delta n_B^2}{n_B^2} \right\rangle \right) < 0.31, \]  \hspace{1cm} (27)  

the Hubble parameter during the inflation is constrained to be \( H_{\text{inf}} < 3 \times 10^{12} \) GeV. Note that this constraint can be avoided if initial \( \theta_S \) is fixed at one of the minima determined by Hubble-induced \( A \) or \( B \) term. This can be realized if the value of \( b \) or \( a \) during the inflation satisfies the following condition, \( bM_N \sim H_{\text{inf}} \) or \( \lambda a > (H_{\text{inf}}/10^{14}) \) GeV.

6. Summary

We reconsidered the leptogenesis scenario in the SUSY seesaw model by including the evolution of the \( LH_u \) direction. We found that the \( LH_u \) direction acquires a large value due to a negative effective mass induced by the right-handed sneutrino condensate through the Yukawa coupling, even if the minimum of \( \phi \) is fixed at the origin during the inflation, unless the reheating temperature \( T_R \) is sufficiently high. The lepton asymmetry is first produced in the condensate of \( \tilde{N} \) via the Affleck–Dine mechanism, then transferred nonperturbatively to the condensate of \( \phi \) by the Yukawa coupling. This lepton asymmetry is released into the baryon asymmetry in the SM sector by the sphaleron process. In this scenario, the appropriate amount of baryon asymmetry can be generated for low \( T_R \) avoiding the gravitino problem without parameter tuning.

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