Law of fluctuation in plant leaves thickness during day and night *

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Abstract. In order to achieve a precise control of the growth conditions for plants, the measurable properties of plants have been studied by using the measurement instruments. First of all, it is developed that the developed using modern industrial metrology methods to measure the leaves of two kinds of plant for more than 6 months. The measurements made on the measurable properties of plants for establishment of the law of daily variation in plant leaves thickness indicate that the peak takes place at the noon, which is the subject of next research and the mathematic model established in the paper provides an important theoretical basis for further research on control of plants.

Keywords. Thickness measurement of plant leaf; Plant growth control; Leaf variable law of day and night; Mathematic model of leaf thickness.

1. Foreword
People has hoped to control the growth of plants for a long time, especially after the deterioration of environments in recent years, the possibility of which depends on the development of modern science and technology. The researches on plants have made remarkable progresses in solving some key problems from the points of systematic and cybernetics views. As macro models can not meet the demand of micro control for further researches, the systems have to be made smaller and thinner. If a plant is taken as a system, and its growth parameters are taken as control variables, we can then use modern control theory to study how a plant can grow better so that people can produce more fruit using less water.

As the most important organs of plants, the changes of leaves in geometries reflect the growth status of plants, such as photosynthesis, status of water content and alimentation etc. As shown by some studies, the changes in leaves thickness is periodic and regular, during 24 hours of a day. The law of variation in leaves thickness is of great help for the study on water contents in plants. However, to the best of our knowledge, not much work has been done on accurate measurement of leaves thickness so far. The water content in leaves is very high, usually above 60%, and the leaves are very soft, and so, the best way of measurement is non-contact measurement. But this kind of method can not be used to reach the micrometer and sub-micrometer resolution. Therefore, we have to adopt a contact measurement method, i.e. the measuring device will get in touch with the examined leaf. This is actually a traditional measurement method, which can be used to achieve high precision in measuring metal parts. By contrast, it is much more complicated to measure the biology material when
it is needed to consider the deformation error as well as the influence of the environmental factors. We would like to present in this paper some very important results achieved through experimental contact measurements.

2. Measurement principle
For contact measurement of plant leaves, we have designed a kind of sensors as shown in figure 1, in which, an original integrated one-chip amplifying and converting circuit is built in the LVDI sensor, making the whole circuit compact, reliable, and cheap. Figure 1 is the block diagram of the amplifying and converting circuit built in the LVDI sensors.

The measuring leaves thickness apparatus we have developed has a high-precision differential inductance sensor TESA with a resolution of 0.1μm which is calibrated using class III gauge blocks to make the measurements traceable.

3. Experimental measurement of plant leaves thickness
The especially developed instrument has been used for experimental measurement of cluster red-pepper and two kind of Changchun flowers on the campus with a lot of data acquired in more than 6 months, including data on daily variation in plants leaves thickness are shown in table 1 and figure 3. As shown in figure 3, the maximum variation in leaves thickness takes place at noon, and the variation trend in other time is at a modest rate in contrast. It can be seen from the experimental results that the plant leaves thickness is measurable and the change in thickness is obvious.

It is at the noon when the photosynthesis effects is the strongest and the plant leaves have their peak in thickness which shows probably that the size of a plant changes when the physiological change takes place. If this idea is proved to be true, we can estimate the photosynthesis status and moisture content of plants based on the sizes of some plants organs, although this hypothesis justify further study to give a reasonable explanation.
Table 1. Measurements made with leaves.

| Measuring time (h) | 00:00 | 00:16 | 00:32 | 00:48 | 01:04 | 01:36 | 01:52 | 02:08 | 02:24 |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Thickness of leaf (μm) | 197   | 196.4 | 196.3 | 196.3 | 196.2 | 196.2 | 196.2 | 196.2 | 196.2 |
| Measuring time (h) | 02:24 | 02:40 | 02:56 | 03:12 | 03:28 | 03:44 | 04:00 | 04:16 | 04:32 |
| Thickness of leaf (μm) | 196.1 | 196   | 196   | 195.9 | 195.9 | 195.9 | 195.8 | 195.7 | 195.6 |
| Measuring time (h) | 05:20 | 05:36 | 05:52 | 06:08 | 06:24 | 06:40 | 06:56 | 07:12 | 07:28 |
| Thickness of leaf (μm) | 195.5 | 195.5 | 195.5 | 195.5 | 195.7 | 195.8 | 196   | 196.3 | 196.6 |
| Measuring time (h) | 08:00 | 08:16 | 08:32 | 08:48 | 09:04 | 09:20 | 09:36 | 09:52 | 10:08 |
| Thickness of leaf (μm) | 197.5 | 198.3 | 199.3 | 200   | 202   | 203.5 | 204   | 205.3 | 206.5 |
| Measuring time (h) | 10:40 | 10:56 | 11:12 | 11:28 | 11:44 | 12:00 | 12:16 | 12:32 | 12:48 |
| Thickness of leaf (μm) | 196   | 197.2 | 197.5 | 197.4 | 197.7 | 197.7 | 197.6 | 204.6 | 203.8 |
| Measuring time (h) | 13:20 | 13:36 | 13:52 | 14:08 | 14:24 | 14:40 | 14:56 | 15:12 | 15:28 |
| Thickness of leaf (μm) | 203.1 | 202.1 | 201   | 201   | 199.9 | 199.4 | 199.1 | 198.1 | 198.2 |
| Measuring time (h) | 16:00 | 16:16 | 16:32 | 16:48 | 17:04 | 17:20 | 17:36 | 17:52 | 18:08 |
| Thickness of leaf (μm) | 197.7 | 197.4 | 196.9 | 196.8 | 196.7 | 196.5 | 197.5 | 196.3 | 197.7 |
| Measuring time (h) | 18:40 | 18:56 | 19:12 | 19:28 | 19:44 | 20:00 | 20:16 | 20:32 | 20:48 |
| Thickness of leaf (μm) | 197.5 | 197.5 | 197.5 | 197.6 | 197.3 | 197   | 196.5 | 195.9 | 196   |
| Measuring time (h) | 21:20 | 21:36 | 21:52 | 22:08 | 22:24 | 22:40 | 22:56 | 23:12 | 23:28 |
| Thickness of leaf (μm) | 196.1 | 196.1 | 196   | 196   | 195.8 | 195.5 | 195.4 | 195.3 | 195   |

Figure 3. Scatter diagram of primary data.

4. Establishment of a mathematical model
According to the need of study on the controllability of the system, a differential equation model is formulated, as equation (1) to show the variation in plant leaves thickness(y) in time(t) for fitting the
experimental data of plant leaves \((y_i, t_i)(i = 1, 2, \ldots, n)\), of which \(a_0, a_1, \ldots, a_m\) is the undetermined parameter:

\[
\frac{dy}{dt} = a_0 + a_1 t + \cdots + a_m t^m \quad (1)
\]

For time \((t)\) is equally spaced, \(t_i = i(i = 1, 2, \ldots, n)\), which makes it easier to disperse the differential equation, and then based on the experimental data, a difference equation of the continuous differential equation model as equation (1) is set up as shown below.

\[
y_{k+1} - y_k = a_0 + a_1 t_k + a_2 t_k^2 + \cdots + a_m t_k^m \quad k = 1, 2, \ldots, n - 1 \quad (2)
\]

From the experimental data, we can formulate a system of equations for the undetermined parameters \(a_0, a_1, \ldots, a_m\) using equation (2):

\[
\begin{pmatrix}
1 & t_1 & t_1^2 & \cdots & t_1^m \\
1 & t_2 & t_2^2 & \cdots & t_2^m \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & t_{n-1} & t_{n-1}^2 & \cdots & t_{n-1}^m \\
\end{pmatrix}
\begin{pmatrix}
a_0 \\
a_1 \\
\vdots \\
a_m \\
\end{pmatrix} =
\begin{pmatrix}
y_2 - y_1 \\
y_3 - y_2 \\
\vdots \\
y_n - y_{n-1} \\
\end{pmatrix}
\]

Then we have a normal orthogonal system of equations by least square method:

\[
A\alpha = b \quad (4)
\]

In equation (4):

\[
A = B^T B, \quad b = B^T x, \quad B = 
\begin{pmatrix}
1 & t_1 & t_1^2 & \cdots & t_1^m \\
1 & t_2 & t_2^2 & \cdots & t_2^m \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & t_{n-1} & t_{n-1}^2 & \cdots & t_{n-1}^m \\
\end{pmatrix}, \quad x =
\begin{pmatrix}
y_2 - y_1 \\
y_3 - y_2 \\
\vdots \\
y_n - y_{n-1} \\
\end{pmatrix}
\]

By solving the equation, we can obtain the determined parameter \(a_0, a_1, \ldots, a_m\), and then establish the differential equation model.

According to the experimental data, the maximum leaves thickness takes place at noon, and the trend of change in other time is at a modest rate in contrast. Based on this phenomenon, we set up a segmented model of the first order differential equation by several data fittings. During 00:00 ~ 06:24 \((t_1 \sim t_{25})\):

\[
\frac{dy}{dt} = -0.2221 + 2.4264 \times 10^{-2} t - 6.6289 \times 10^{-4} t^2 \quad (5)
\]

During the period of 06:24 ~ 17:20 \((t_{25} \sim t_{66})\):

\[
\frac{dy}{dt} = -6.8946 \times 10^{-2} + 7.2648 \times 10^{-2} t + 2.6593 \times 10^{-2} t^2 - 3.2921 \times 10^{-3} t^3 + 1.1273 \times 10^{-4} t^4 - 1.2057 \times 10^{-5} t^5 \quad (6)
\]

During the period of 17:20 ~ 23:44 \((t_{66} \sim t_{90})\):

\[
\frac{dy}{dt} = 0.46647 - 2.6722 \times 10^{-2} t - 2.8457 \times 10^{-2} t^2 + 4.0585 \times 10^{-3} t^3 - 1.9379 \times 10^{-4} t^4 + 3.0903 \times 10^{-5} t^5 \quad (7)
\]

Utilizing the fourth order Runge-Kutta equation, we can calculated the fitting value \(\hat{y}(y_i)\) of the examined plant leaves thickness corresponding to experimental data \((y_i, t_i)(i = 1, 2, \ldots, n)\) from differential equations (5) ~ (7). In order to know the error of fitting value \(\hat{y}\), relative to \(y\), we examined first the interdependency of the model above with the method of \(F\) test, and calculated the residual
sum of squares ($Q$), regression sum of squares ($U$) and the value of F test of this model with, the results shown below.

$$Q = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2, \quad U = \sum_{i=1}^{n}(\bar{y} - \hat{y}_i)^2, \quad F = \frac{U \cdot (n-m-1)}{Q \cdot m}$$

If there is $F_\alpha(m, n-m-1)\leq F$ to the value of $\alpha$ and corresponding degree of freedom at different levels, the interdependency of this model has remarkable meanings at $\alpha$ level, or else it does not have. Secondly the fitting precision of the model is test with the average residual sum of squares.

$$Q_r = \frac{1}{n} \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

The value of F test and the average residual sum of squares ($Q_r$) of model (5) ~ (7) are shown as follows:

$F = 32.1, Q_r = 0.0407; F = 527.7, Q_r = 0.2233; F = 17.8, Q_r = 0.1639$

Through checking the form we have:

$F_{0.01}(2.25 - 2 - 1) = 5.72, F_{0.01}(5.42 - 5 - 1) = 3.57, F_{0.01}(5.25 - 5 - 1) = 4.17$

According to these results, the interdependencies of the above models have remarkable meaning at the level of $\alpha=0.01$, and the average residual sums of squares are small, so these models have fine interdependency and high fitting precision, and the model can be used for study on the daily changes in plant leaves thickness. The fitting precision is as shown in figure 4.

![Figure 4. Comparison between of raw data and fitted data.](image)

5. Conclusion
(1) In order to meet the need of plant growth control for the agro forestry and the garden virescence, an accurate instrument for developed measurement of plant leaves thickness has been used to measure plant leaves for more than 6 months, and the measurements proved the measurable property of some plant leaves thickness, which interconnects botany physiology.
(2) The peak characteristics of leaves and regular changes in 24 hours, indicate that the peak of leaves thickness takes place at noon, which is similar to photosynthesis, although the reason why needs further study.
(3) The differential mathematics model established for the time-domain curve of the peak provides important theoretical basis for further research.

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