Research Article

Health Condition Evaluation of Cable-Stayed Bridge Driven by Dissimilarity Measures of Grouped Cable Forces

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An index system for the health-status evaluation of cable-stayed bridges is presented based on a set of dissimilarity measures (DMs) of the cable force of grouped cables. The DMs and their labels corresponding to health status derived by the serviceability limit states of a bridge are determined by the combination of the finite element method and the influence matrix method. The Monte-Carlo method is used to determine the rational values of thresholds of proposed DMs. By comparing the indices with the thresholds, the holistic health status assessment of a cable-stayed bridge can be easily and reasonably determined. Based on eight years of historical data on the cable forces investigated from a real cable-stayed bridge, the proposed methods are applied to evaluate the health status of this cable-stayed bridge. The results show the validity of the proposed methods.

1. Introduction

The relationship between the current internal force state (action effects) and the structure’s limit state (resistance forces) bridges the gap between measureable quantities and structural health status. Thus, the availability of these quantities can facilitate the evaluation of the health status of the long-span cable-stayed bridges [1]. Stayed cables comprise the main structural components of a cable-stayed bridge, and the changes in inclined cable force significantly affect the state of the whole structure [2]. After years of bridge service, deviation attributed to tower-top displacement, foundation settlement, shrinking and creeping, and so on are likely to occur between real and designed cable forces [3]. These deviations are apparently linked to specific and holistic working statuses when the structure is in service. To detect and explore the deviation of grouped cable forces rationally, the changes in the corresponding internal force state of the whole structure can be determined [4, 5]. By associating these changes in the internal force state with the known relationships between such changes and a certain structural limit states, the health status of a cable-stayed bridge structure could be evaluated. Therefore, monitoring the grouped cable forces of long-span cable-stayed bridges and developing a safety evaluation method based on such monitoring data have become important tasks in bridge structural safety evaluation.

A simple comparison method continues to be widely used for the evaluation of cable forces. By computing the relative changes in single-cable force over time to reflect the state of cable forces, such state can be considered normal when the relative change value is less than a certain value (e.g., 5%) [6]. This estimation method has several limitations. On one hand, a rational threshold system has never been built, which makes quantitative evaluation impossible; on the other hand, considering the stress redistribution in the cable plane, the significant changes in single-cable force may not directly affect the safety of the whole structure. However, even when the cable force deviations occurred in each cable is inconsiderable and hereby jumped to a positive conclusion, the holistic cable forces would sometimes fail to meet the safety requirements of the whole structure. Therefore, to evaluate bridge safety based on cable force monitoring data, the whole cable group should be considered simultaneously.
The present synthetic evaluation methods like variable weight method [6] and gray related degree method [7] continue to be the most commonly used in evaluation of grouped cable forces. The two methods rely on certain parameters with poor physical significance, thus yielding unsatisfactory results. The former method depends on a balance coefficient representing the degree of correlation of structural components, whereas the latter is constrained by the correlation of the cable forces, where all such methods have no significant relationship with the structures' working state and thus have the disadvantage of poor interpretability. The statistical methods and pattern recognition methods recently are used in structural status evaluation [8] and health monitoring [9, 10], with a capacity of dealing with multiple quantities measured by integrated monitoring system or comprehensive bridge detection project synchronously, and so the ability to conduct synthetic evaluation of bridge condition can therefore be expected. For lack of effective channel of communication between patterns and the definition of health status of target structures, the interpretability of these kinds of methodologies still are unsatisfactory. Thus, developing a rational and interpretable technology for the evaluation of the working state of whole structures based on grouped cable forces is necessary.

Based on a comprehensive analysis of grouped cable forces, this paper proposes a rational evaluation method that considers the dissimilarity measure (DM) of grouped cable forces. The threshold system of DMs can then be obtained by using the Monte-Carlo method. Finally, based on the historical measurement data of cable forces of a long-span, cable-stayed bridge located in Guangdong, China, this paper conducts a health evaluation for same bridge and confirms the validity of the evaluation method.

2. Measure of Grouped Cable Forces

A holistic assessment of structural health status can be achieved by analyzing the data samples of grouped cable forces. However, two issues must be addressed: first, the assumption that grouped cable forces are directly related to structural health status should be proven. Second, valid measures of grouped cable forces should be determined to achieve an accurate assessment.

2.1. Correlation of Grouped Cable Forces and Structural Inner Force. As a space-stressed system, a cable-stayed bridge is characterized by a condition wherein loads are borne by the combination of the main girder and the cable tower as well as the stayed cables. Among all loads, vertical loads are primarily balanced by the vertical component of the cable forces. Considering that all cables are anchored to the same tower and main girder, the functionality of a single cable is restricted by that of other cables. All cables are closely related and thus serve a joint function of balancing the loads. Therefore, the inner force of structural components aside from the cables correlates with the holistic cable forces but not with the force of a single cable. Changes in single-cable force, whether present or not, have a negligible effect on the inner force of the whole structure.

The relationship between the distribution of grouped cable forces and the behavior of the structure is significantly affected by factors such as traffic loads, uneven settlement, and temperature. The effects of wind load and nonlinear factors on the internal forces of grouped cables are negligible [11]. Thus, the influence matrix method based on linear theory can be used to describe the effect of grouped cable forces on the behavior of the bridge structure [12].

Figure 1 shows that if the effect of wind load is disregarded, the relationship between the grouped cable forces and the target mechanical variables of the bridge structure (e.g., external load, inner force, stress, or displacement) could be described as

\[ S = C \cdot D, \]

where \( S \) = \( S_\text{tar} \), \( S_\text{config} \), \( S_\text{tr} \), \( S_\text{us} \), and \( S_\text{t} \) is a column vector of the grouped cable forces of a finished bridge. \( S_\text{G} \), \( S_\text{tr} \), \( S_\text{us} \), and \( S_\text{t} \), respectively, denote the cable forces produced by the dead loads, traffic loads, uneven settlement, and temperature; \( S_\text{config} \) stands for the additional cable forces of the group cables that emerge after reaching the destination through cable force adjustment. \( S \) is used to denote the passive controlled vectors.

\[ D = \{D_\text{G}, D_\text{config}, D_\text{tr}, D_\text{us}, D_\text{t}\}^T \]

stands for the actions applied to the structure. Index \( G \) represents the dead loads, \( \text{config} \) represents the traffic loads, \( \text{us} \) represents the uneven settlement, and \( t \) represents the temperature effect. \( D_\text{config} \) represents the initial cable force obtained after cable force is adjusted. All elements in \( D \) are row vectors, and \( D \) is column vector, which is used to denote the comprehensive active controlled vectors.

\( C \) stands for the influence matrix and is a constant under the linear hypothesis. \( C = [C_\text{G}, C_\text{config}, C_\text{tr}, C_\text{us}, C_\text{t}] \), and the submatrices in \( C \) relate some factors to corresponding cable forces. The referred factors contain dead loads, initial cable force after cable force adjustment, traffic loads, uneven settlement, temperature effect, and corresponding cable forces. The size of first dimension of \( C \) is equal to the number of grouped cables, and the size of second dimensions is equal to that of \( D \).

In the same way, the relationship between the target mechanical variables and the comprehensive active controlled vectors under the ultimate limit state (ULS) and serviceability limit state (SLS) is given by

\[ S_\text{tar} = C_\text{tar} \cdot D, \]

where \( S_\text{tar} \) is column vector of the target mechanical variables, generally refers to typical mechanical variables in typical sections of a cable-stayed bridge, including internal forces, stress and displacement; \( C_\text{tar} \) is the influence matrix between the comprehensive active controlled vectors and the target mechanical variables. The target mechanical variables are typically used in function equations which describe a certain condition under the ULS and SLS in the design stage. Let \( Z \) represent the function equation \( Z \) or function margin, which is given by

\[ Z = [S_\text{tar}] - S_\text{tar}, \]
where \([S_{\text{tar}}]\) denotes the thresholds or resistance column vectors of the corresponding target mechanical variables.

According to the influence matrix features, (1) could be rewritten as follows:

\[
S = S_D + S_Q + S_R,
\]

\[
S_D = C_G D_G + C_{\text{config}} D_{\text{config}},
\]

\[
S_Q = C_{\text{us}} D_{\text{us}} + C_{t,a} D_{t,a},
\]

\[
S_R = C_{t,\text{tr}} D_{t,\text{tr}} + C_{j,a} D_{j,a},
\]

where \(S_D\) refers to the grouped cable forces under the combination of the permanent loads; \(S_Q\) denotes the grouped cable forces under the combination of quasipermanent loads; and \(S_R\) stands for the grouped cable forces under the combination of live loads, which has a random value. “t,a” and “t,\text{tr}”, respectively, represent the average annual temperature effects and the momentary temperature effects of the structure. The former is recognized as a quasipermanent load and has high periodicity and certainty, whereas the latter is characterized by randomness.

The target mechanical variables \(S_{\text{tar}}\), which reflect the structure state, can also be rewrite as:

\[
S_{\text{tar}} = S_{\text{tar,D}} + S_{\text{tar,Q}} + S_{\text{tar,R}},
\]

\[
S_{\text{tar,D}} = C_{t,a} D_{t,a} + C_{\text{tar}} D_{\text{config}},
\]

\[
S_{\text{tar,Q}} = C_{\text{us}} D_{\text{us}} + C_{\text{tar}} D_{t,a},
\]

\[
S_{\text{tar,R}} = C_{t,\text{tr}} D_{t,\text{tr}} + C_{\text{tar}} D_{j,a}.
\]

Thus, the relationship between the target mechanical variables and the grouped cable forces could be written as the following linear combination:

\[
S_{\text{tar}} = C_{\text{tar}} \cdot C^{-1} \cdot S. \tag{6}
\]

Equations (1) to (6) show that the state of the inner forces of the structure is closely related to the grouped cable forces. Moreover, the target mechanical variables that reflect the state of the structure can be approximately expressed as a linear combination of grouped cable forces. Thus, through the grouped cable forces, the condition of the target mechanical variables can be obtained; moreover, the function margin \(Z\) can be expressed by the following equation:

\[
Z = [S_{\text{tar}}] - C_{\text{tar}} \cdot C^{-1} \cdot S. \tag{7}
\]

The resistance \([S_{\text{tar}}]\) and the effects \(S_{\text{tar}}\) in (3) are difficult to monitor, but the grouped cable forces in (7) can be measured easily and thus can serve as an agent in function equation. Thus, the original function equation based on resistance and effects is transformed into one that is based on the agent of grouped cable forces, which facilitates to form a structural reliability evaluation frame based on aforementioned monitored measurement agent.

2.2. DMs of Grouped Cable Forces. To determine structure’s states accurately, rational measures should be given for the vectors of grouped cable forces. Grouped cable forces are high dimensional and are thus difficult to deal with. Therefore, low-dimensional measures of the vectors should be introduced to simplify the analysis.

In practical applications, the most common method used for vector measurement is DM. DM determines the dissimilarity among multidimension data samples, which are essentially a group of special dissimilar functions. The differences between two sample vectors can possibly be transformed into scalars. Common DM functions include Euclidian distance, street block distance, Chebyshev distance, Canberra distance, angle separation degree, and correlation coefficient [13, 14].

The two data sample vectors can be set as:

\[
x = [x_1 \ x_2 \ \cdots \ x_i \ \cdots x_n]^T, \tag{8}
\]

\[
y = [y_1 \ y_2 \ \cdots \ y_i \ \cdots y_n]^T.
\]

Thus, DMs could constitute the index system used to evaluate the health status of a structure. When using the given DMs to measure grouped cable forces, observations \(S_0\) under specific conditions could be selected as the reference, and together with the rest observations \(S\), DMs can be derived via operation on DM operators, that is, DM\((S_0, S)\). Designed cable forces and finished-bridge cable forces could both be used as \(S_0\).

DMs are used to classify the data points and to identify the boundaries between data points under different categories. Their capability to distinguish such categories could be measured based on the value of the criteria \(J_1\), \(J_2\), and \(J_3\). The given criteria are defined in (9) [14]. A larger value of these criteria denotes a smaller within-class spread and a larger
Table 1: Functions of dissimilarity measures [6].

| Dissimilarity Measure          | Formula                                                                 |
|-------------------------------|------------------------------------------------------------------------|
| Euclidean distance DM         | $D(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$                        |
| Street block distance DM      | $D(x, y) = \sum_{i=1}^{n} |x_i - y_i|$                               |
| Chebyshev distance DM         | $D(x, y) = \max_{i} |x_i - y_i|$                                     |
| Canberra distance DM          | $D(x, y) = \sum_{i=1}^{n} |x_i - y_i| / \sqrt{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}$ |
| Angle separation degree DM    | $D(x, y) = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}}$ |
| Correlation coefficient DM    | $D(x, y) = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 \sum_{i=1}^{n} (y_i - \overline{y})^2}}$ |

between-class spread of DMs. And a larger between-class spread of DMs results in better classification performance:

$$J_1 = \text{Tr} \left\{ Q_W^{-1} Q_B \right\}, \quad J_2 = \frac{|\Sigma|}{|Q_B|}, \quad J_3 = \frac{\text{Tr} \left\{ Q_B \right\}}{\text{Tr} |Q_W|}$$

(9)

where $Q_W$ is the within-class scatter matrix, $Q_B$ is the sample between-class covariance matrix,

$$Q_W = \frac{C}{n} \sum_{i=1}^{n} \Sigma_i,$$

$$Q_B = \frac{C}{n} \sum_{i=1}^{n} (m_i - m)(m_i - m)^T,$$

(10)

where $\Sigma$ is the population covariance matrix, and $\Sigma_i$ is the covariance matrix of class $w_i$. The maximum likelihood estimates of $\Sigma$ and $\Sigma_i$ are, respectively, $\hat{\Sigma}$ and $\hat{\Sigma}_i$:

$$\hat{\Sigma} = \frac{1}{n} \sum_{j=1}^{n} (x_j - m)(x_j - m)^T,$$

$$\hat{\Sigma}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} z_{ij} (x_j - m_i)(x_j - m_i)^T,$$

(11)

where

$$z_{ij} = \begin{cases} 1, & x_j \in w_i, \\ 0, & \text{otherwise}, \end{cases}$$

(12)

$$n_i = \sum_{j=1}^{n_i} z_{ij},$$

$m_i$ is the sample mean of the class $w_i$, which is given by

$$m_i = \frac{1}{n_i} \sum_{j=1}^{n_i} z_{ij} x_j,$$

(13)

$m$ is the sample population mean

$$m = \frac{C}{N} \sum_{i=1}^{N} m_i,$$

(14)

3. Threshold Systems of DMs

To achieve an accurate and rational evaluation of the health status of bridge structures, the threshold system of the DMs of grouped cable forces should be determined. From (7), it is known that the thresholds of DMs can be obtained by comparing the real-time target mechanical variables of the significant sections with its corresponding thresholds under two limit states: the ULS and SLS [15]. The solving method is given in the following subsections.

3.1. Definition of Threshold Systems of DMs. The threshold systems of the DMs of grouped cable forces can be obtained by using (3), (6), and (7). Cable-stayed bridges typically have two limit states, and each limit state has upper and lower limits in real conditions. Equation (3) could thus be rewritten as

$$[S_{tar}]_{u,f} \leq [S_{tar}]_{u,c} \leq [S_{tar}]_{u,s},$$

(15)

where $[S_{tar}]_{u,f}$, $[S_{tar}]_{u,c}$ denote the lower and upper limit thresholds of the target mechanical variables under ULS, respectively, whereas $[S_{tar}]_{u,f}$, $[S_{tar}]_{u,c}$ denote the same thresholds under SLS, respectively. The four limit thresholds can be obtained based on structure design specifications or through finite element analysis. $S_{tar,u}$, $S_{tar,s}$ stand for the target mechanical variables calculated with grouped cable forces under conditions of ULS and SLS, which can be obtained as follows:

$$S_{tar,u} = C_{tar,u} \cdot C^{-1} \cdot S,$$

$$S_{tar,s} = C_{tar,s} \cdot C^{-1} \cdot S.$$
based on the target mechanical variables that reflect the structure state, that is,

$$[S]_i = C \cdot (C_{tar,u})^{-1} \cdot [S_{tar}]_i,$$

(17)

where subscript “i” represents the indexes $u$, $f$, $u$, $c$, $s$, $f$, and $s$, $c$. The thresholds of the DMs of grouped cable forces can then be derived as

$$[DM(\cdot)]_i = DM([S],[S_0]),$$

(18)

where $S_0$ is a specific vector of grouped cable forces used as the reference, typically the design cable force or finished-bridge cable force.

Therefore, the evaluation of grouped cable forces $S$ could be converted to the comparison of the DMs with its corresponding thresholds,

$$DM(S, S_0) = \begin{cases} > & \text{if } [DM(\cdot)]_i > \end{cases},$$

(19)

$$< \text{if } [DM(\cdot)]_i < \end{cases},$$

where $[S_{tar}]_i$, which is the threshold of the structural target mechanical variable $S_{tar}$, is usually a scalar or low-dimensional vector with dimensions that are far less than the number of group cables. Given that (17) does not have a definite solution, it cannot be used to calculate the thresholds. In practical applications, the Monte-Carlo methods can be used to generate mass of samples of grouped cable forces, and then all samples can be labeled into two classes: the safe and unsafe label classes. Subsequently, by distinguishing these labeled DMs samples, the boundary of safe and unsafe samples can be derived, and the threshold system of DMs can be further obtained.

3.2. Flowchart for Threshold Solving Using the Monte-Carlo Method. The initial comprehensive active controlled vectors $D_0$ and the comprehensive influence matrix $[C, C_{tar,u}, C_{tar,c}]$ are obtained through cable force adjustment process firstly, which is operated under the combinations of action effects under SLS, including dead loads, traffic loads, temperature, uneven settlement, and so on. Then, the values of the comprehensive active controlled subvectors corresponding to dead loads, initial cable force, uneven settlement, and global temperature effect are fixed as constants; the values of comprehensive active controlled subvectors, $D_{tr}$ and $D_f$, corresponding to traffic loads and transient temperature effect, are varied within a rational range randomly, which are then determined using the Monte-Carlo method [7]. The random comprehensive active controlled vectors $D$ are then established and used to calculate the random grouped cable force vectors and target mechanical variables via influence matrix method. The DMs of all cable forces vectors are calculated against its references, and (15) is used to determine the sample labels of all cable force vectors by comparing the target mechanical variables with their limits or thresholds. Finally, the threshold system of the DMs of grouped cable forces is determined according to the boundaries between DMs with different labels.

The steps employed by the Monte-Carlo method to determine the threshold system of DMs are shown in Figure 2.

| Mechanical variables | Floor threshold | Ceiling threshold | Limit state |
|----------------------|-----------------|-------------------|-------------|
| Bending moment (tonf-m) | -5229.62 | 32908.08 | Ultimate |
| Mid-span deflection (m) | -0.738 | 0.116 | Serviceability |

4. Engineering Application

4.1. Engineering Background. Taking a long-span cable-stayed bridge with two towers in China’s Guangdong province as a case study, seen in Figure 3, the feasibility of the method discussed above will be verified in these sections. The bridge has four cable planes, with 40 cables in each plane for a total of 160 cables. Considering that cable force detection has been conducted eight times and that the bridge is not entirely longitudinally symmetrical, the cable forces of the 80 cables of two cable planes at the upstream side are selected as data samples. Totally 16 data samples can be gathered.

All cable forces are regularly detected by the same bridge detection organization and thus the consistency and credibility of the data are guaranteed. Through an analysis of the variation of single cable force, the results show significant similarities. Except for the variations of a few cable forces of nearly $\pm 10\%$ in the previous year, the range of changes of rest cable forces was within $-5\%$ to $5\%$. At present, there are not apparent indications of abnormality occurred during its service history, which means the bridge is in a good condition.

As previously mentioned, the key to apply the Monte-Carlo method to obtain the threshold values of grouped cable force is to bond the class label to a certain limit state. This paper uses Midas Civil Software to build the finite element model (FEM) of the bridge, as shown in Figure 4. In this model, the living loads are arranged at the most unfavorable position. The bending moment of the unilateral bottom section of the tower is chosen as the mechanical variable to calculate the ultimate limit state and the mid-span deflection of girders is taken as the mechanical variable to calculate the serviceability limit state. The bending moment arrives at its ceiling with counterclockwise direction and floor value with clockwise direction when the two limit live load distributions are applied to the girder. While, for serviceability limit state, the two extremes live loads distributions are arranged in this way that the deflection of middle span has exceeded the design specifications or codes both upward and downward. The threshold values of the two mechanical variables are shown in Table 2. From the FEM model of the bridge, the initial active controlled vectors of cable force and the influence matrix can also be obtained. The two limit states are shown in Figure 5.

4.2. Monte-Carlo Statistical Test. To obtain the mass of samples of grouped cable forces and to ensure that the samples can fully reflect the structural behavior of the target bridge under real operation conditions, a traffic load model must
be properly built. In order to simplify the calculation, such modeling should differ from lane load modeling in terms of design specification in that it should be randomly distributed on the deck, but it should not stand on any certain vehicle loading manners or any random distribution types of traffic loads.

Therefore, a model of traffic with 80 independent concentrated loads located on the anchorages of the cables and girder are built and applied to the bridge FEM models. Considering

Figure 2: Flowchart of Monte-Carlo method for threshold of dissimilarity measures of grouped cable forces.

Figure 3: Case study for cable-stayed bridge (Guangdong, China).

Figure 4: FEM model of target bridge by Midas Civil.
the short-term temperature effect, the lower limit of loads should be negative. Thus, the preliminary design is that each concentrated load is evenly distributed within an interval of −70 and 210 tons. To avoid mass repetition of the FEM calculations, the influence matrix method is used to derive the cable forces in the Monte-Carlo simulations operation, under the postulate that the influence matrix can properly represent the complex functional relations among random live loads, grouped cable forces, and target mechanical variables. A total of 180,000 rounds of simulation are conducted, and the same number of cable force samples was obtained. To simplify the illustration, 5000 samples are randomly selected from the population. For every sample, the corresponding values of the target mechanical variables are also derived from the simulations simultaneously.

Figures 6 and 7 show that the grouped cable forces provided by the simulation cover the design cable forces, the region of cable forces of determined by the SLS, as well as the historical cable forces and the scope of these samples approached to the limit determined by ULS. It can be concluded that the parameters used in the simulation have good representativeness and can reflect the possible conditions of real cable forces. Similarly, the simulation values of target mechanical variables overspread the feasible engineering region, and the sample set contains the full information on structural behavior.

4.3. Comparison of DMs and Determination of Threshold System. It is reasonable to designate the finished-bridge state as the reference, which implies that the target bridge is in a health status at the very beginning of its service history; choosing the grouped cable forces on phase of finished bridge as standard, the DM of the samples of cable forces are calculated. The corresponding class labels can then be
determined based on the comparison between the simulation values and the SLS threshold values of the target mechanical variables. Therefore, when using DMs to describe grouped cable force samples, each DM is related to a class label.

According to (9), we can assess the classification capacity of DMs based on their corresponding class labels, and as shown in Table 3, it can be seen from that the best DM is Canberra distance, followed by street block distance, and Euclidean distance. However, the $J_3$ values of Chebyshev distance, angular separation, and correlation coefficient are extremely small and are thus inappropriate for classifications.

Figure 8 shows that two clear distinctions exist between the points of DMs with two different class labels in Figure 8(a) Canberra distance, Figure 8(b) street block distance, and Figure 8(c) Euclidean distance, which divides the whole measurement space into three regions: completely safe region, completely unsafe region, and mixed region. For Figure 8(d) Chebyshev distance, the completely unsafe line is clear, but the completely safe line is difficult to identify. For Figure 8(e) angular separation and Figure 8(f) correlation coefficient, the points with safe and unsafe labels are fully mixed, and almost the entire measurement space is a mixed area. Thus, the two DMs lose the capability of classification.

In Figures 8(a)–8(d), the boundaries of different regions can be determined by the following equations:

$$[\text{DM}(\cdot)]_{\text{safety}} = \min \left( \text{DM} \left( S_{\text{unsafety}}, S_0 \right) \right),$$

$$[\text{DM}(\cdot)]_{\text{unsafety}} = \max \left( \text{DM} \left( S_{\text{safety}}, S_0 \right) \right).$$

Based on the formulas of DMs, a lesser similarity of the samples denotes larger values of Euclidean distance, street block distance, Chebyshev distance, and Canberra distance but smaller values of angular separation and correlation coefficient. Combined with the $J_3$ criterion, the maximum of DMs with safe labels and the minimum of DMs with unsafe labels are chosen as the thresholds, as shown in Table 4.

4.4. Health Status Evaluation. To validate the rationality and validity of the given threshold system, the DMs in practical application are derived by samples of historical measured cable forces and finished-bridge cable forces. With eight years’ measurement results and considering 160 cables divided into two samples (80 upstream cables as the first sample and 80 downstream cables as the second), the number of samples of historical measured cable forces comes up to 16. Thus, 16 results can be obtained for each DM, which are drawn in scatter plots in Figure 9. The green points are DMs of the measures at the finished-bridge state (given that they are compared with themselves, DMs equal zero), whereas the red points are the maximum DMs, which are the closest points to the safe boundaries. The results show that all data are located at the safe region of the corresponding measurement space. This demonstrates that both the safety requirements and serviceability requirements of the bridge are met. This deduction from these three figures is consistent with the current service status of the bridge.

Figures 9(a)–9(c) show that whether Canberra distance, street block distance, or Euclidean distance, the maximum values of DMs occur in the most recent monitoring year (2010). With the increasing lifespan of the bridge, each DM value gradually increases and approximates the upper limit of the safe region. The tendency discovered by DM history...
Figure 8: DMs scatter plots of grouped cable forces from Monte-Carlo simulation (green points are labeled as a class meeting requirement of SLS, whereas red points are labeled as a class dissatisfying it).
Figure 9: Health status evaluation driven by DMs of historical real data on grouped cable forces.

Table 3: Comparison of classification capacities of DMs.

| DMs                  | Between-class spread | Within-class spread | Total spread | $J_3$ value |
|----------------------|----------------------|---------------------|--------------|-------------|
| Euclidean distance   | 30867.8              | 21905.21            | 52773        | 0.7096      |
| Street block distance| 2217641              | 1680175             | 3897816      | 0.7576      |
| Chebyshev distance   | 1446.89              | 332.481             | 1779.37      | 0.2298      |
| Canberra distance    | 6.0993               | 5.0901              | 11.1894      | 0.8345      |
| Angular separation   | $6.90E-7$            | $1.31E-7$           | $8.21E-07$   | 0.1905      |
| Correlation coefficient | $1.15E-4$           | $2.00E-5$           | $1.35E-4$    | 0.1744      |
curves demonstrates that the overall performance of the bridge is gradually degrading and the disquieting departure from baseline status is increasing continuously, which should call the attention of the bridge owner.

5. Conclusions

This paper proposed an evaluation method for the health status of a cable-stayed bridge based on a set of DMs of grouped cables forces. By mapping the relationship between grouped cable forces and health status and building the measure space of grouped cable forces, the evaluation of structure health status is realized. Likewise, the structure's health status can be grasped in a macro sense. The main research conclusions are as follows.

(1) The status of grouped cable forces corresponds to the specific inner force state of a bridge. Thus, a holistic state evaluation of bridge health can be accomplished by evaluating the state of grouped cable forces as the agent, and the measure of grouped cable forces, which are DMs, can further service for this evaluation.

(2) Based on the Monte-Carlo method and combined with the threshold of target mechanical variables under the bridge limit state, a threshold system of the DMs of grouped cable forces can be established.

(3) Among all the DMs, Canberra distance, street block distance, and Euclidean distance, and Euclidean distance exhibit better classification performance and can thus be used for similar cases.

(4) With increasing service time, the DMs values of the grouped cable forces continuously increase, which indicates that the performance of the bridge structure degrades annually.

Based on the finding of this paper, historical data on cable forces can be used to establish a safety evaluation system for cable-stayed bridges. Likewise, the data can also be used to facilitate the functions of online early warning and real-time evaluation of bridges based on the fact that the health monitoring system of bridges can acquire sufficient real-time data on cable forces.

However, this paper does not consider the influence of nonlinear factors in calculating the influence matrix. Relevant research considering more other factors will be conducted in the future.

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Table 4: Threshold of dissimilarity measures.

| DM(⋅) | [DM(⋅)]_{safety} | [DM(⋅)]_{unsafety} |
|-------|------------------|---------------------|
| Euclidean distance | 566.7726 | 168.0435 |
| Street block distance | 4679.68 | 1227.92 |
| Canberra distance | 9.0863 | 2.8368 |
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