Propulsion mechanisms for Leidenfrost solids on ratchets
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The propulsion mechanism of self-propelling Leidenfrost drops on ratchets has been debated since the first publication of the phenomenon by Linke et al. in 2006 [1]. A simpler system that leaves out many of the complications due to the deformable nature of the drops [2–5] are Leidenfrost solids. Similar to drops, a platelet of dry ice levitating on a cushion of its own vapor over a hot ratchet surface shows directed motion [6]. Even for such a simplified system a debate is going on about what the physical mechanism responsible for propulsion could be. It has been suggested that the Leidenfrost solid is driven by a “rocket effect” originating from the recoil of the vapor produced from sublimation [6]. On the other hand, an attempt to explain the propulsion via thermal creep flow due to rarefaction effects in the vapor phase has been published [7]. Finally, a viscous mechanism due to pressure-driven flow has been suggested [1,3,8]. As sublimated vapor is drained sideways along the grooves, perpendicular to the movement of the Leidenfrost solid, it is additionally directed down the slope of the ratchet towards the deep sections of the groove, dragging the levitated solid along [8].

In this Rapid Communication, we corroborate the picture of pressure-driven viscous flow as the main propulsion mechanism for typical scenarios studied in experiments. Our work is based on a combination of model calculations and experimental studies. We propose a model based on a continuum description for the velocity, pressure, and temperature fields in the gap between the hot surface and the dry ice. The propulsion force is obtained from the viscous drag on the surface of the Leidenfrost solid, and we develop a scaling expression reflecting how this force depends on geometric parameters of the ratchet surface and properties of the sublimating solid. As such, it supersedes the scaling analysis given in [8] by taking into account the three-dimensional (3D) character of the vapor flow, allowing us to understand the series of experiments also presented here.
above parameters we get \( \text{Re} \approx 1 \). Correspondingly, since the forces are assumed to be viscous, the relevant scale for the stresses is \( \Pi_0 \sim \eta U_0 / \Lambda \sim \eta \Delta T / (\Lambda^2 \rho \Delta H_{\text{subl}}) \), containing all of the material properties of the vapor and the temperature difference.

To start with, consider only a single groove of width \( L \) and depth \( H \) (see Fig. 1). This situation can be thought of as a platelet that hovers over many grooves, of which one is studied. An ice block of width \( 2R \) is assumed to levitate at a distance \( B \) from the surface. Further, we assume the flow, pressure, and temperature fields to be periodic in the direction normal to the groove \( (x \text{ direction}) \). At the sides of the ice block \( (y = \pm R) \) the pressure is ambient and the normal derivatives of temperature and velocities vanish. Since \( R \gg L, H, B \), the details of this boundary condition do not strongly affect the flow inside the groove.

As an example, we consider the parameters \( R = 5 \) mm, \( L = 1 \) mm, \( H/L = 0.15 \) and \( B = 75 \) \( \mu \)m. We solve Eqs. (1) and (2) via finite-element discretization as implemented in COMSOL MULTIPHYSICS 4.1. The structure of the solution is as follows. The main pressure gradient is along the groove, underpinning that the groove itself acts mainly as a drain for the vapor. The temperature profile and the gradients in pressure normal to the \( y \) direction become essentially independent of \( y \). Figure 2 shows the flow field projected onto the plane normal to the groove \( (\text{arrows}) \), at position \( y = R/2 \). Again, this distribution is almost independent of \( y \). The flow field is essentially as described above: the strong flow along the groove is fed by the regions of strong sublimation at the narrow gap. The velocity field is almost parabolic, albeit slightly squeezed in the \( xz \) projection due to the evaporation mass flux at the top wall.

Since a direct computation of the flow below a realistically large platelet is beyond our reach, we propose a simplified model, assuming—in the spirit of a lubrication approximation—a parabolic flow profile everywhere, neglecting all inertial terms. Defining the average flow rate in the \( i \) direction \((i \in \{x,y\})\) by integrating over the height

\[
Q_i(x,y) = \int_0^{h(x,y)} dz \, u_i(x,y,z),
\]

where \( h(x,y) \) is the local distance between the ratchet and ice surfaces, the Reynolds (lubrication) equation becomes

\[
\partial_i p(x,y) = -\frac{12\eta}{h(x,y)} Q_i(x,y).
\]

Mass conservation dictates \( \partial_i Q_i = u_w \), with a source term due to sublimation equal to the normal velocity at the ice surface as defined before. In the spirit of the lubrication approximation we assume the local temperature gradient to be proportional to \( 1/h(x,y) \). Taking the divergence of the Reynolds equation then yields

\[
\partial_i (h^3 \partial_i p) = -12(\Pi_0 \Lambda^2)/h.
\]

We solve Eq. (5) in the case of a single groove, assuming periodicity \([p(x,y) = p(x + L,y)]\), as well as in the case of a circular platelet. As the boundary condition at the edge of the domain \( \Omega \), i.e., the circumference of the platelet, we assume \( \partial_n p = 0 \) and neglect any outlet effects beyond that. For the solution, a finite-element discretization as implemented in COMSOL MULTIPHYSICS is used. In our approximation, the wall shear stress is given by \( \tau_w = -\frac{1}{2} h \nabla \cdot p \).

The lubrication model and the 3D model for a single groove give similar shear stress distributions at the upper wall. In both cases a net shear stress is generated driving the platelet in the positive \( x \) direction. To verify the model predictions, experiments with levitated dry ice platelets have been performed. A quantity that is directly accessible experimentally is the acceleration and hence the net force on a dry ice platelet of known mass. The stress simply is this force divided by \( \pi R^2 \). For this, cylinders of dry ice with a diameter of \( 2R = 14 \pm 0.6 \) mm and of different heights are placed on a ratchet heated to 450 °C. Subsequently, the propelling force is measured using the method described in [8] (see Supplementary Material [9]). The geometry parameters of the ratchet are \( L = 1.5 \) mm and \( H = 0.25 \) mm. In Fig. 3 the experimentally obtained average wall shear stress is plotted vs the platelet thickness for values between 1 and 10 mm. In the same diagram the corresponding values obtained with the 3D model for a single groove and with the lubrication model for both a single groove and a circular disk are displayed. The model results are unanimously compatible with a scaling \( \xi_{\text{ex}} \sim H_{\text{ice}}^{1/2} \) of the shear stress with the thickness of the platelet, agreeing well with the experimental data. However, the different models predict slightly different values for the shear stress at a given thickness. For the single groove, the lubrication model predicts a slightly larger shear stress compared to the more complete 3D model. This is partly due to the neglect of inertial effects and partly due to the simplified temperature profile used in the lubrication model. Comparing the lubrication results obtained for a single groove and the more adequate circular disk, the latter predicts a larger shear stress. This is due to the fact that for the disk a portion of the vapor can also escape to the front and back. Therefore the disk hovers closer to the surface, resulting in

![FIG. 2. Projected velocity profiles at \( y = R/2 \), both \( xz \) (top) and \( yz \) velocities (bottom); since \( u_x \gg u_y \), the arrow scales are not identical [differing by a factor \( O(10) \) at this position]. \( R = 5 \) mm, \( L = 1 \) mm, \( H/L = 0.15 \), \( B = 75 \) \( \mu \)m.]

![FIG. 1. (Color online) Sketch of the geometry considered. The dry ice at temperature \( T_{\text{subl}} \) hovers a distance \( B \) above the ratchet geometry. The ratchet grooves have periodicity \( L \) and depth \( H \). In our modeling we consider a single groove of \( y \) extension \( 2R \) with periodicity in the \( x \) direction, as well as a disk of radius \( R \), covering several grooves.]
the force increases more rapidly. In this region in Fig. 5 the mean stress observed for small increasing the angle to \( \tan \alpha \) gives an example for the obtained scaling with \( \tan \alpha \) of the geometric parameters in the 3D model, albeit with slightly different exponents, considering a single groove in lubrication approximation or \( \sim H_{\text{ice}} \). A similar scaling is obtained when \( \sim \frac{\Delta P_1}{R} \). A typical \( \frac{\Delta P_1}{R} \) scaling stems from the quadratic pressure increase with length in a channel with constant inflow across permeable walls. Second, the flow escapes laterally (along the \( y \) axis in Fig. 1) with a velocity \( U_2 \sim \frac{H^2}{\eta} (\Delta P_2/R) \).

To corroborate the findings of the scaling analysis, we show in Fig. 5 the mean stress \( \tau \) measured on a platelet as a function of its radius, for a height \( H_{\text{ice}} = 5 \text{ mm} \). Corresponding results obtained in the lubrication approximation for a disk are shown in the same figure. The stress decreases as the radius increases, and the dashed line of slope \( \sim 3 \) shows a fair agreement with experiments and model results.

FIG. 5. (Color online) Average shear stress \( \tau \) as a function of the platelet radius \( R \). A platelet of height \( H_{\text{ice}} = 5 \text{ mm} \) and radius \( R = 7 \text{ mm} \) is considered. Blue circles are experimental results, red squares are results obtained in the 2D lubrication model for a disk of corresponding dimensions. The dotted line shows \( \sim R^{-3} \).

It is interesting to understand this behavior (and more generally the scalings found in this study) based on a simple intuitive picture. We divide the flow in a single groove into two parts. First, the vapor flows down the incline (along the \( x \) axis in Fig. 1). In the lubrication approximation, the velocity of this flow is \( U_1 \sim \frac{\eta}{4} (\Delta P_1/L) \) where \( \Lambda \) is a typical vertical length scale, and \( \Delta P_1 \) is deduced from mass flux conservation [see Eq. (5)]: \( \Delta P_1 \sim \frac{\eta \rho \Delta T L^2}{(\Lambda^2 p \Delta H_{\text{subl}})} \). The \( \Delta P_1 \sim L^2 \) scaling stems from the quadratic pressure increase with length in a channel with constant inflow across permeable walls. Second, the flow escapes laterally (along the \( y \) axis in Fig. 1) with a velocity \( U_2 \sim \frac{H^2}{\eta} (\Delta P_2/R) \).
The pressure associated with this flow balances the weight of the platelet \( (\Delta P_2 \sim \rho_{\text{ice}} g H_{\text{ice}}) \) where \( \rho_{\text{ice}} \) is the dry ice density.

The shear stress arises from the “first” flow, so it scales as \( \eta U_1/\Lambda \). We deduce the length \( \Lambda \) from mass conservation between the two flows: \( U_1/\Lambda R \sim U_2/\Lambda L \). Eventually, we get an expression for \( \tau \) which is

\[
\tau \sim \left( \frac{\rho \Delta H_{\text{subl}} g^3 \rho_{\text{ice}}^3}{\eta \Delta T \lambda} \right)^{1/2} \frac{H_{\text{ice}}^{3/2} L H^3}{R^3}. \tag{6}
\]

We find that the stress should vary as \( H_{\text{ice}}^{1.5}, R^{-3}, H^3 \), and \( L \), scalings very close to the ones derived from the lubrication approximation (only \( H_{\text{ice}}^{0.7} \) deviates slightly). Note that this scaling of Eq. (6) is also obtained in the lubrication approximation for a single groove in the limit of small angles (see Supplemental Material [9]).

The experimental results and the model both reveal a decrease of the propelling force with the radius of the platelet \( (F \sim \tau R^2 \sim R^{-1}) \). This behavior stands in contrast to what can be observed for Leidenfrost droplets self-propelling on ratchets, for which the force was shown to increase rapidly with the radius \([6]\). The systems are indeed quite different: a drop on a hot ratchet follows the asperities of the texture, so that the characteristic length scales involved in the scaling laws above get modified—changing the geometry in these systems has a deep implication on the resulting force.

Based on the observations above, we conclude that viscous drag from pressure-driven flow due to sublimation seems the most important driving force for self-propelling Leidenfrost solids on ratchets. However, despite the excellent agreement with scaling predictions our model results consistently lie below the measured values, indicating that some detail may not be fully captured yet. Also note that these findings do not necessarily carry directly over to Leidenfrost drops for which additional effects such as deformability or interfacial stresses come into play.

Contrasting this viscous mechanism based on pressure-driven flow, it was recently suggested that rarefaction effects, in particular thermal creep flow, could play a dominating role for the propulsion on Leidenfrost ratchets \([7]\). We have investigated these effects using Monte Carlo simulations in \([10]\). For typical geometries used in experiments it is shown that such rarefaction effects based on the finite mean free path of gas molecules only play a minor role.

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