Modeling selected mechanical properties of magnetorheological elastomers

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Abstract. Magnetorheological elastomers are an important group of non-classical engineering materials. The aim of the works carried out is to study the influence of magnetic field on a series of mechanical properties of composite materials in this group. Based on prior analyses, a limited usability of the viscoelastic material model was identified for the purpose of describing the stress-strain relationship in magnetorheological elastomers. Therefore, it has become necessary to propose changes to this rheological model. The present article includes theoretical considerations in relation to the mathematical derivation of functions allowing to account for the influence of strain amplitude on the progressive character of the mechanical hysteresis loops in the materials under discussion. It also provides a detailed analysis of the derived formulas together with a discussion, supplemented with the description of employed simplifying assumptions.

1. Introduction
Magnetorheological elastomers are composite materials combining the characteristics of polymers with ferromagnetic materials. Their features include the possibility to alter certain mechanical characteristics which are controllable via magnetic field. In recent years, one can observe a growing interest in this group of materials. Consequently, the number of suggested applications for these non-classical engineering materials is constantly increasing. An analysis of solutions provided in subject literature allows to determine that the major use area of magnetorheological elastomers is vibration absorption equipment [1–4]. This area of use includes not only active, linear vibration absorbers which utilize the discussed materials in compression [5] or shear [6, 7] work mode, but also active torsion vibration absorbers [8–10]. Effective use of magnetorheological elastomers is not possible without mathematical models allowing to predict and evaluate their characteristics under various conditions. Consequently, it is justified to search for new material models or to modify existing ones in order to obtain the greatest degree of accuracy. The works carried out are a continuation of study on the properties and possible use of magnetorheological elastomers in the broadly understood machine engineering [11].

2. Building the mathematical model
One of the most often used models for the description of characteristics of magnetorheological elastomers is the Kelvin-Voigt model (the viscoelastic material). However, subject literature includes works employing other rheological models [12]. As provided earlier [13, 14], the classic Kelvin-Voigt model describes changes occurring in the mechanical characteristics of magnetorheological elastomers to a satisfying degree only in a certain range of frequency and strain amplitude. It was therefore decided...
that the model requires modification. As the baseline point, the effect of strain amplitude on the set of obtained result was to be included. Naturally, changes in mechanical characteristics caused by input frequency, in general, must not be omitted. Further study in this area is planned in the future.

Analyzing the theoretical considerations included in [15], it was determined that the use of its guidelines pertaining the calculations for rubber components is possible. The influence of strain value on the hardening of the tested samples can be accounted for using high flexibility potential $W_e$, assuming incompressible composite shroud material. This means that the volume is maintained regardless of the applied stress. Such an assumption necessitates that in the considered example, Poisson’s ratio should be $\nu = 0.5$. The differences in directional physical properties of the elastomer are lower than the accuracy of the calculations, therefore for the purpose of further analysis, it is assumed that the shroud material is isotropic [15].

The actual stress value in the extended sample $\sigma_{rr}$ may be defined as a ratio of extending force $F_r$ to actual cross-section area $A_r$:

$$\sigma_{rr} = \frac{F_r}{A_r}. \quad (1)$$

As a result of problems with measuring the actual sample cross-section area during the performance of the experiment, common engineering practice utilizes an agreed upon stress value $\sigma_r$ defined using the initial surface area $A$:

$$\sigma_r = \frac{F_r}{A}. \quad (2)$$

Using actual and initial sample length, the following formula can be established:

$$A_r l_r = A l_0. \quad (3)$$

Introducing degree of deformation $\lambda$ as a ratio of actual length $l_r$ to initial length $l_0$ we arrive at:

$$A_r = A \frac{l_0}{l_r} = \frac{A}{\lambda}. \quad (4)$$

Substituting the function (2) and (4) to the formula (1), we are able to determine the function binding the actual stress $\sigma_{rr}$ and agreed stress $\sigma_r$ values:

$$\sigma_{rr} = \sigma_r \lambda. \quad (5)$$

Assuming values $\lambda_1, \lambda_2, \lambda_3$ as degrees of strain in three orthogonal directions, it is possible to establish the value of high flexibility potential as the following formula:

$$W_e = \frac{1}{2} N k T(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3). \quad (6)$$

The product in formula (6) can be simplified accounting for that:

$$G = N k T, \quad (7)$$

where: $G$ stands for Kirchhoff’s modulus, $N$ is the number of chains in polymer sample, $k$ stands for Boltzmann’s constant and $T$ for temperature. Considering the mono-axial material load (e.g. extension), as provided on figure 1 below, one can formulate the following:

$$\lambda_1 = \lambda, \quad \lambda_2 = \lambda_3 = \lambda^{-\frac{1}{2}}. \quad (8)$$
After introducing formulas (7) and (8) into the expression (6), we arrive at the following:

\[ W_ε = \frac{1}{2} G \left( \frac{\lambda^3 - 3\lambda + 2}{\lambda} \right). \]

The function binding stress and strain can be established from the equation:

\[ σ_r = \frac{\partial W_ε}{\partial \lambda} = G \left( \lambda - \frac{1}{\lambda^2} \right). \]

Introducing the equation (5) to the expression (10), we arrive at the following:

\[ σ_{rr} = G \left( \lambda^2 - \frac{1}{\lambda} \right). \]

The deformation degree \( \lambda \) can also be expressed as a formula:

\[ \lambda = ε + 1. \]

If the deformation degree value \( \lambda \) is close to 1, therefore in expression (12) the \( ε \) value is very low. According to [15], the area of major deformation includes deformation degree in the range \( \lambda \in (1.75 \div 3.75) \). Equating formulas (11) and (5), after prior introducing to them the formula (12), we arrive at the following:

\[ σ_r = \frac{3Gε}{(1 + 2ε + ε^2)}. \]

Using the theory of elasticity, it is possible to formulate the following function for an isotropic material:

\[ ν = \frac{E}{2G} - 1, \]

where \( E \) stands for Young’s modulus. Utilizing the earlier assumed condition of incompressibility, for the value \( ν = 0.5 \) the equation (14) is afterwards simplified to the following form:

\[ E = 3G. \]

For the equation (13), the higher order elements are dismissed as negligible (due to low strain value). Afterwards, the expression (15) is introduced into the equation (13), to achieve the expression to describe tensile stress:

\[ σ_r \approx \frac{Eε}{1 + 2ε}. \]

An analogous course may be demonstrated for compressive stress, and therefore the final form of the equation (16) shall be as follows:

\[ σ_{r/s} \approx \frac{Eε}{1 \pm 2ε}. \]

where: the “+” sign is used for tensile stress \( σ_r \), and “−” shall apply for compressive stress \( σ_s \).

A breakdown of results obtained from equation (17) and Hooke’s Law are provided in figure 2.
What follows from the included considerations, for specific assumptions and a limited range of strain, it is possible to include the progressive characteristics of the compressed elastomer component which is a result of uneven distribution of stress.

On the basis of provided equations, the rheological model of the viscous-flexible material model was modified with the equation (18):

\[ \sigma = E\varepsilon + \eta \dot{\varepsilon}, \]  

where: \( \sigma \) stands for stress, \( \eta \) stands for viscosity and \( \dot{\varepsilon} \) shall be deformation velocity (linear deformation velocity). The introduced changes allow for modeling stress characteristics. Using the equation (17), the rigidity value in the model is dependent on magnetic field and deformation. Given the function as below:

\[ E_{mod} = f(B, \varepsilon) = \frac{pE}{1 - q\varepsilon}, \]  

(19)

where: \( p \) and \( q \) are shape coefficients of the hysteresis loop, and \( B \) stands for magnetic induction.

Given the values \( p, q \) and \( E \) as a function of magnetic induction \( B \). Such modifications introduced to the model allow to account for the influence of magnetic field and deformation on the determined stress value. Therefore, the equation (19) shall have the following form:

\[ \sigma = E_{mod}\varepsilon + \eta \dot{\varepsilon}. \]  

(20)

The diagram of the modified Kelvin-Voigt substance model is provided in figure 3.

3. Analysis of the formulated mathematical model

A detailed analysis of the modified Kelvin-Voigt model can be carried out by establishing a forced model. Assuming a given a sinusoidal strain value over time, the expression to characterize the resultant stress in the material shall be as follows:

\[ \sigma(t) = \frac{p}{1 - q\varepsilon_0 \sin(\omega t)} \cdot E\varepsilon_0 \sin(\omega t) + \eta \omega \varepsilon_0 \cos(\omega t). \]  

(21)

What follows from the above equation, the value of \( p \) parameter must fall in the range as below:

\[ q \in \left( -\frac{1}{\varepsilon_0}, \frac{1}{\varepsilon_0} \right). \]  

(22)
In order to carry out an analysis of the influence of the respective parameters on the course of the stress, a series of curves was established for different values of parameters $p$ and $q$.

Figure 4 presents a comparison of stress characteristics as a function of time for different values of $q$ coefficient and constant values of the remaining parameters of the modified Kelvin-Voigt model. For reference, a curve established using a classic viscous-flexible model was also provided. On the other hand, figure 5 presents a similar comparison for different values of the $p$ parameter.

**Figure 4.** Juxtaposition of stress as a function of time for different values of the $q$ parameters in the modified and classical Kelvin-Voigt model; the characteristics were determined based on the following constant values: $f = 0.05$ Hz, $E = 3.5$ MPa, $\eta = 2.6$ MPa s$^{-1}$, $\varepsilon_0 = 0.3$ and $p = 1$.

**Figure 5.** Juxtaposition of hysteresis loops as a function of time for different values of the $p$ parameter in the modified and classical Kelvin-Voigt model; the characteristics were determined based on the following constant values: $f = 0.05$ Hz, $E = 3.5$ MPa, $\eta = 2.6$ MPa s$^{-1}$, $\varepsilon_0 = 0.3$ and $q = 0$.

Based on the obtained results, a series of hysteresis loops were established, which are used as characteristics for the established mathematical model binding stress and strain. The comparison of hysteresis loops for different values of $q$ and $p$ parameters for the classical and modified Kelvin-Voigt model is provided in figures 6 and 7.
Figure 6. The juxtaposition of hysteresis loops for different values of the $p$ parameter in the modified and classical Kelvin-Voigt model; the characteristics were determined based on the following constant values: $f = 0.05$ Hz, $E = 3.5$ MPa, $\eta = 2.6$ MPa s$^{-1}$, $\varepsilon_0 = 0.3$ and $q = 0$.

Figure 7. The juxtaposition of hysteresis loops for different values of the $q$ parameter in modified and classical Kelvin-Voigt model; the characteristics were determined based on the following constant values: $f = 0.05$ Hz, $E = 3.5$ MPa, $\eta = 2.6$ MPa s$^{-1}$, $\varepsilon_0 = 0.3$ and $p = 1$.

The analysis of the provided graphs indicates that the increase in the value of the $q$ parameter causes the increase in the stress value necessary to cause the same strain. Changing the sign of the $q$ parameter results in a different characteristic of the modeled material. Positive values result in a progressive characteristic; whereas a negative value results in a regressive characteristic. One needs to point out that for alternating strain, the hysteresis loops are symmetrical in relation to the ordinate axes. The analysis of the provided graphs indicates that the change in the $p$ parameter value linearly affects the change of value of maximum stress. The change of sign of the $p$ parameter causes a phase displacement, by the periodic value, of the stress graph. Such changes result in the change of inclination angle values of respective hysteresis loops. Therefore, the role of the $p$ parameter in the equation (21) is similar to rigidity $E$. However, it is directly dependent on magnetic induction, whereas the $E$ value depends on it indirectly through changing the angle of mechanical loss $\varphi$. The introduction of this parameter aims to
as accurately match the model as possible to actual values. Consider that for $p = 0$, the hysteresis loop assumes a characteristic position in which the ordinates of the stress-strain graph are its semi-axes.

4. Conclusion

As part of the study, modifications were introduced to the Kelvin-Voigt rheological model. The aim of such modifications was to account for, in the presented mathematical apparatus, the influence of strain on the progressive characteristic of the registered stress value when compressing magnetorheological elastomers in a magnetic field. The functions were determined at several simplifications which assumed incompressibility and isotropy of the composite shroud material as well as omitting the influence of input frequency. It is emphasized that future research into determining the effects of this parameter on incompressibility and isotropy of the composite shroud material as well as omitting the influence of elastomers in a magnetic field. The functions were determined at several simplifications which assumed incompressibility and isotropy of the composite shroud material as well as omitting the influence of input frequency. It is emphasized that future research into determining the effects of this parameter on the measured results are planned. The established mathematical model benefits from the same advantages as the classical Kelvin-Voigt model. This includes a small number of easily determinable material constants as well as a simplicity of interpretation. One needs to point out that the set of presented equations should be considered as a proposition which can be modified or improved so as to achieve the best possible match of the calculated values and measured data.

5. References

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