Algorithm for detecting brightness differences in noisy images based on the Wilcoxon criterion

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Abstract. To detect brightness differences in grayscale images, in previous works it was proposed to use the nonparametric Wilcoxon criterion. At the same time, the obtained research results showed a sufficient effectiveness of the application of the criterion for highlighting the brightness differences in images with noise without preliminary filtering. The application of the criterion involves the transformation of the image into such an attribute as the value of the rank function for each pixel of the analyzed image area. Calculation of the extremum of the rank function makes it possible to establish the location and type of brightness differences in the image. However, as studies have shown, the value of the extremum value depends on the signal-to-noise ratio in the image and, along with the true extrema, local extrema are also distinguished, which complicates the solution of the problem of further image segmentation.

This paper presents the results of statistical studies on the use of the Wilcoxon test for detecting brightness differences in images with different signal-to-noise ratios and sample sizes. The minimum signal-to-noise ratio at which the extremum of the rank function tends to its maximum or minimum value has been established. Also, based on the statistical analysis of the behavior of the rank function to exclude the allocation of local extrema, it is proposed to use the difference of the rank functions for the analyzed rows and columns of the processed image.

1. Introduction

An important feature of the application the Wilcoxon criterion is that no pre-filtering of the noisy image is performed before its application. In different works present the studies on the application of the Wilcoxon criterion to highlight the brightness differences in grayscale images, distorted by noise of various nature and intensity [1-3].

The results of the research have shown that the criterion is not critical to the laws of noise distribution and makes it possible to distinguish brightness differences in images with signal-to-noise ratios close to unity [4,5]. The work of the algorithm based on the Wilcoxon criterion is as follows. In the analyzed row (column) of the image, samples X and Y are taken, respectively, of size n₁ and n₂ so that the size of the total sample is N = n₁ + n₂. After constructing the variation series, the sum of the ranks of the sample Y in the total sample relative to the analyzed pixel is calculated. Thus, the brightness of a pixel in the image is replaced with an attribute such as rank (the sum of the ranks).
When the total sample moves along the line (column) of the image, its rank (the sum of the ranks) is calculated for each pixel. The result is a random rank function. For \( n_1 = n_2 = 5 \) (\( N = 10 \)), the maximum value of the sum of ranks in the rank function at the point of brightness difference is 40 units (increase in brightness), the minimum value of the sum of ranks at the point of difference in brightness is 15 units (decrease in brightness). At signal-to-noise ratios close to unity, the maximum and minimum values of the rank function decrease and increase, respectively; at ratios greater than unity, they tend to their extreme values.

The analysis of the research results showed that when visually assessing the location of the brightness drop, the extremum of the function takes place at the drop point with an error up to a certain number of pixels. However, the determination of the extrema by an algorithmic method by three points (pixels), in addition to highlighting the extrema of the brightness difference, leads to the allocation of false extrema not related to the brightness difference. They tend to occur in areas of monotonous brightness. The use of five points in the algorithm reduces the likelihood of false extrema, but does not exclude their occurrence. This, in turn, significantly reduces the ability to create an image segmentation algorithm based on the proposed criterion from the point of view of identifying the contours of objects on them.

At the same time, the possibility of using the Wilcoxon criterion for the selection (detection) of small differences in brightness in noisy images predetermined further research to identify patterns in the behavior of rank functions, allowing to unambiguously determine the location of brightness differences in images with a minimum error [6,7].

2. Main part

**Average value** \( S_{cp} \) of the brightness of two adjacent image regions relative to the point of brightness difference within the sum of two samples \( n_1 \) and \( n_2 \):

\[
S_{cp} = \left( \frac{\sum_{i=1}^{n_1} X_i + \sum_{j=1}^{n_2} Y_j}{n_1 + n_2} \right)
\]

(1)

**Average deviation** \( S \) of brightness:

\[
S = \left( \frac{\sum_{i=1}^{n_1} |X_i - S_{cp}| + \sum_{j=1}^{n_2} |Y_j - S_{cp}|}{n_1 + n_2} \right)
\]

(2)

**Signal to noise ratio**:

\[
K = S/\sigma
\]

(3)

where \( S \) is an average deviation of brightness, \( \sigma \) is a standard deviation of noise for a given distribution law. The probability \( P \) of the occurrence of an extreme value of the rank function at the corresponding value of the quantity \( K \). Error \( T \) of highlighting the brightness difference relative to its true location (in pixels).

During the research, a fragment of a hypothetical halftone image was used in the form of a line with a length of 30 pixels, in which areas of the same brightness and length of 10 pixels each alternated so that the brightness first increased and then decreased. Gaussian noise \( N(0.1) \) and bipolar impulse noise with pulse amplitude equal to one and the probability of occurrence of each equal to 0.5 were used as noise. The error of the noise correspondence to their theoretical values was less than 3%. The simulation program is implemented in the C-programming language.

The results of the studies are presented in tables 1-7. In these tables, \( P_{40} \) and \( P_{15} \) denote the probability of occurrence of extreme values of the rank function; in parentheses at \( T \), the probability of occurrence of this error value is indicated. The sign (*) marks the errors of highlighting the brightness difference for two neighboring pixels with the same values of the rank function.

Table 1 shows the study results for Gaussian noise and \( N = 10 \). As can be seen from the research results, with a value of \( K \) greater than 1.5, the probability \( P \) of the appearance of extreme values of the
The rank function tends to unity, the error $T$ is no more than 2 pixels with a probability of no more than 0.45. In accordance with the task of eliminating the allocation of false extrema in the rank function, a comprehensive analysis of the behavior of the rank functions was carried out. At the same time, an important regularity was discovered: the difference between the values of the direct rank function and the counter rank function for the same image fragment at a given threshold of the value of this difference makes it possible to unambiguously highlight the true location of the brightness difference.

Table 2 shows the realizations of the direct and opposite rank functions and their difference for $K = 1.5$ and the threshold equal to 22. Table 3 shows the realizations of the direct and opposite rank functions and their difference for $K = 2.0$ and the threshold equal to ±25. A difference of ±25 units is the maximum for $N = 10$. It should be emphasized that the signal-to-noise ratio $K = 1.5$ or less characterizes low-contrast areas of images with a brightness difference of 6 or less units. Thus, for high-contrast areas of images, the difference between the extreme values of the rank functions will be ±25 units.

### Table 1. Research results for Gaussian noise. $N = 10$.

| $S$ | $K$ | $P_{40}$ | $T$ | $P_{15}$ | $T$ |
|-----|-----|---------|-----|---------|-----|
| 1.0 | 1.0 | 0.4     | 1(0.35) 2(0.15) 5(0.05) | 0.45 | 1(0.3) 2(0.1) |
| 1.5 | 1.5 | 0.8     | 1(0.3)* 0.75 | 1(0.15)* 2(0.15) 3(0.05) |
| 2.0 | 2.0 | 1.0     | 1(0.15)* 2(0.05)* | 1.0 | 1(0.3)* 2(0.05)* |
| 3.0 | 3.0 | 1.0     | 1(0.2)* 2(0.05)* | 1.0 | 1(0.4)* 2(0.05)* |

### Table 2. Values of rank functions for $K = 1.5$ and $N = 10$. Gaussian noise.

| Rank function values |
|----------------------|
| 28 32 35 35 40 35 35 29 26 22 25 18 18 19 18 20 23 23 27 27 |
| 27 27 23 23 20 18 19 18 18 25 22 26 29 35 35 40 35 35 32 28 |
| 1 5 12 12 20 22 16 17 11 1 0 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 |

### Table 3. Values of rank functions for $K = 2.0$ and $N = 10$. Gaussian noise.

| Rank function values |
|----------------------|
| 33 35 38 39 40 38 33 33 28 27 27 26 20 17 15 19 21 25 23 23 |
| 23 23 25 21 19 15 17 20 26 27 27 28 33 33 38 40 40 39 38 35 33 |
| 10 12 13 18 21 25 21 13 7 1 0 -1 -7 -13 -21 -25 -21-18 -13 -12 -10 |

Table 4 shows the research results for impulse noise and for $N = 10$. As in the case of Gaussian noise, at values of $K = 1.5$ and more, the probability $P$ of the appearance of extreme values of the rank function tends to unity. Table 5 shows the realizations of the direct and opposite rank functions and their difference for $K = 1.5$ and a threshold equal to ±25 units.
Table 4. Research results for impulse noise. $N = 10$.

| $S$ | $K$  | $P_{40}$ | $T$   | $P_{15}$ | $T$   |
|-----|------|----------|-------|----------|-------|
| 1,0 | 1,0  | 0,3      | 1(0,3) | 0,25     | 1(0,35)|
|     |      |          | 2(0,35) |          | 2(0,25)|
|     |      |          | 3(0,1)  |          | 3(0,05)|
| 1,5 | 1,5  | 1,0      | 1(0,3) | 0,9      | 1(0,25)|
|     |      |          |        |          | 2(0,05)|
| 2,0 | 2,0  | 1,0      | 1(0,25)| 1,0      | 1(0,4)|
|     |      |          |        |          | 2(0,05)|
|     |      |          |        |          | 3(0,05)|
| 3,0 | 3,0  | 1,0      | 1(0,2) | 1,0      | 1(0,4)|
|     |      |          |        |          | 2(0,05)|
|     |      |          |        |          | 3(0,05)|

Thus, the use of the difference in rank functions at a given level of its threshold uniquely makes it possible to single out the true differences in brightness in the images, regardless of the noise distribution law.

Table 5. Values of rank functions for impulse noise. $K = 1.5$ and $N = 10$.

| Rank function values |
|----------------------|
| 35 35 37 39 40 40 35 32 26 27 22 25 23 23 17 15 18 20 24 29 31 |
| 31 29 24 20 18 15 17 23 23 25 22 27 26 32 35 40 40 39 37 35 35 |
| 4 6 13 19 22 25 18 9 3 2 0 -2 -3 -9 -18 -25 -22 -19 -13 -6 -4 |

Table 6. Research results for Gaussian noise. $N = 8$.

| $S$ | $K$  | $P_{40}$ | $T$   | $P_{15}$ | $T$   |
|-----|------|----------|-------|----------|-------|
| 1,0 | 1,0  | 0,9      | 1(0,5)| 0,85     | 1(0,3)|
|     |      |          | 2(0,05)|          | 2(0,1)|
| 2,0 | 2,0  | 1,0      | 1(0,2)| 1,0      | 1(0,4)|
|     |      |          | 2(0,05)|          | 2(0,05)|
| 3,0 | 3,0  | 1,0      | 1(0,15)| 1,0     | 1(0,35)|
|     |      |          | 2(0,1)|          | 3(0,05)|

Table 7. Research results for impulse noise. $N = 8$.

| $S$ | $K$  | $P_{40}$ | $T$   | $P_{15}$ | $T$   |
|-----|------|----------|-------|----------|-------|
| 1,0 | 1,0  | 0,4      | 1(0,35)| 0,45     | 1(0,3)|
|     |      |          | 2(0,15)|          | 2(0,1)|
| 2,0 | 2,0  | 1,0      | 1(0,30)| 1,0      | 1(0,3)|
|     |      |          | 2(0,05)|          | 2(0,05)|
| 3,0 | 3,0  | 1,0      | 1(0,25)| 1,0     | 1(0,45)|
|     |      |          | 2(0,1)|          | 2(0,15)|

The research results previously carried out and presented in this work were obtained for the total sample $N = 10$. At the same time, it was important to find out the possibilities of applying the Wilcoxon test for the total sample $N = n_1 + n_2 = 4 + 4 = 8$. The research results are presented in tables
6 and 7. These results show the identity of the possibilities of applying the Wilcoxon test for $N = 8$ and $N = 10$. Thus, reducing the sampling dimension to 8 can make it possible to highlight the brightness differences of small objects in images with an acceptable accuracy.

### 3. Conclusion

The efficiency of the application of the nonparametric Wilcoxon test in the algorithm for detecting brightness differences in images can be increased by using the values of the differences of the rank functions that exceed a specified threshold for detecting them.

Reducing the dimension of the total sample to $N = 8$ compared to $N = 10$ does not reduce the probability and accuracy of identifying the location of the brightness difference for small image objects.

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