Way to observe the implausible “trion-polariton”

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Abstract – Using the composite boson (coboson) many-body formalism, we determine under which conditions “trion-polariton” can exist. Dipolar attraction can bind an exciton and an electron into a trion having an energy well separated from the exciton energy. Yet, the existence of long-lived “trion-polariton” is \textit{a priori} implausible not only because the photon-trion coupling, which scales as the inverse of the sample volume, is vanishingly small, but mostly because this coupling is intrinsically “weak”. Here, we show that a moderately dense Fermi sea renders its observation possible: on the pro side, the Fermi sea overcomes the weak coupling by pinning the photon to its momentum through Pauli blocking; it also overcomes the dramatically poor photon-trion coupling by providing a volume-linear trion subspace to which the photon is coherently coupled. On the con side, the Fermi sea broadens the photon-trion resonance due to the fermionic nature of trions and electrons; it also weakens the trion binding by blocking electronic states relevant for trion formation. As a result, the proper way to observe this novel polariton is to use a doped semiconductor having long-lived electronic states, a highly bound trion and a Fermi energy as large as a fraction of the trion binding energy.

Exciton-polaritons have recently attracted very much attention because of claimed observations of Bose-Einstein condensation [1–4]. When the coupling between a photon and an exciton is strong, exciton-polaritons are formed [5,6]. By contrast, when this coupling is weak, photons are absorbed. In the former case, the exciton lifetime is long compared to the Rabi oscillation period and the Q exciton recombines into the same Q photon (see fig. 1(a)). When it is short, the exciton changes momentum before recombination occurs and the initial photon Q cannot be re-emitted; it gets absorbed along the Fermi golden rule, the small exciton lifetime producing a broadening of the exciton discrete level, which plays the role of a continuum [7].

We here consider another semiconductor bound state, the trion, and determine under which conditions this composite fermion can strongly couple to a photon to form a polariton. To focus on the trion physics, we consider the material in a “strong” coupling regime with a large exciton momentum lifetime compared to the Rabi oscillation period. Many objections lead us to first reject the idea: i) the trion coupling to photon is intrinsically weak because the emitted photon can have a momentum different from its initial value, even when the trion lifetime is long; ii) the photon-trion coupling is vanishingly small because it scales as the inverse of the sample volume; iii) an increase of the number of electrons available for pairing broadens the photon-trion resonance due to the fermionic nature of trions and electrons; iv) it also reduces the trion binding by Coulomb screening and by Pauli blocking the electronic states relevant for trion formation. Despite all these objections, we will show that there exists a narrow window in which “trion-polariton” can be formed.

Semiconductor trions [8–14] are made of two conduction electrons and one valence hole, or one conduction electron and two valence holes. We will here focus on the former case, denoted as $X^-$. Since orbital wave functions of ground states are even with respect to particle exchange, the two conduction electrons of a $X^-$ ground-state trion must be in a spin-singlet state; they thus have opposite spins. While excitons result from the attraction of a conduction electron by a valence hole, trions result from the attraction of a conduction electron by an excitonic dipole;
First objection. – In addition to the exciton coupled to the Q photon, the formation of a trion requires a free electron (see fig. 1(b)). Just from momentum conservation, we see that the re-emitted photon can have a momentum different from Q, even if the trion has a lifetime long enough to keep its Q momentum. The resulting different photon states (Q + q) lead to an intrinsic broadening of the emitted photon energy very similar to the extrinsic broadening that prevents the formation of exciton-polaritons in the case of the aforementioned weak coupling. To form a trion-polariton, additional physics is required to prevent photon absorption by forcing the photon back to its initial momentum. This physics is Pauli blocking with the electrons of a conduction band Fermi sea. When a k_e electron is taken from the Fermi sea to form a trion, it leaves a conduction hole. The trion–conduction-hole pair can either transform back into the same Q photon with the k_e electron refilling the (−k_e) hole, as in (c), or into a photon having a far larger energy than the Q photon if the released electron goes above the Fermi sea, as in (d).

Second objection. – Overcoming the above objection is not enough to produce a trion-polariton because the coupling between a trion and a photon plus a k_e electron is vanishingly small: two plane waves, one for the photon and one for the electron, then transform into the plane wave of the bound-trion center of mass. The formation of a bound trion requires the localization of an electron, originally delocalized over the sample volume V, into the trion relative-motion volume a_T^2. This brings a dramatic reduction factor (a_T^2/L^2) to the photon-trion coupling compared to photon-exciton value [20–22].

As shown more in detail below, the Fermi sea overcomes this vanishingly small coupling by providing a volume-linear trion-hole subspace to which the Q photon is coherently coupled, and which can render the photon-trion effective coupling of the order of the photon-exciton coupling.

Third objection. – Because electrons are fermions, the Fermi sea produces an energy broadening of the photon resonance to trion, which occurs at

\[ \omega_Q + \frac{k_e^2}{2m_e} \approx E_{gap} + \mathcal{E}_Q^{(T)}(k_e,\eta_0) \]

if we forget the interaction between the trion and the Fermi-sea electrons. \( E_{gap} \) is the band gap, and \( \mathcal{E}_Q^{(T)}(k_e,\eta_0) \) is the energy of a trion with center-of-mass momentum (Q + k_e) and relative-motion ground state index \( \eta_0 \). So, for
a typical photon momentum $|\mathbf{Q}| \ll |\mathbf{k}| \leq k_F$, the trion-polariton energy $\omega_{Q}$ has an energy broadening that varies from 1 to 2\,/\,3 Fermi energy, depending on the electron-hole mass ratio $m_e/m_h$. Many-body interactions with Fermi-sea electrons further produce a sharp low-energy side to the trion resonance due to low-energy excitations close to the Fermi level, and a long high-energy tail [23].

Furthermore, when a large number of trions are simultaneously present due to the absorption of many photons, these trions, which are fermions, have different energies; this constitutes another source of trion-resonance broadening.

**Fourth objection.** – In the absence of Fermi sea, the momentum $\mathbf{k}$ of the electron attracted by the exciton to form a bound trion is predominantly smaller than $1/\alpha_T$, the trion Bohr radius $a_T$ being a few times larger than the exciton Bohr radius $a_X$. In the presence of a Fermi sea, the $\mathbf{k}$ states with $0 \leq |\mathbf{k}| \leq k_F$ are Pauli-blocked. This blocking decreases the trion binding energy. To keep a sizable binding, the Fermi momentum $k_F$ has to be smaller than $1/\alpha_T$. Coulomb screening by Fermi-sea electrons also tends to reduce the trion binding.

Figure 3 shows the calculated two-dimensional (2D) energy of a ground-state exciton $|\mathbf{E}_{\nu}(k_F)|$ and a ground-state trion $|\mathbf{E}_{\nu}(k_F)|$, in the presence of a spin-polarized Fermi sea having electronic spin opposite to the spin of the electron making the exciton (see appendix for details). Screening by the 2D Fermi sea with Fermi momentum $k_F$ is included in a standard way, by replacing the 2D Coulomb potential $V_q = 2\pi e^2/\epsilon_{sc} L^2 q$ by $V_q/\kappa(q)$ with $\kappa(q) = 1 + (2/q a_X)[1 - \Theta(q - 2k_F)\sqrt{1 - (2k_F/q)^2}]$, where $\Theta(x)$ is the Heaviside function [24].

Because we have chosen to take Fermi sea electrons with a spin different from the spin of the electron making the exciton, Pauli blocking does not affect the exciton energy; so, in the absence of screening, $|\mathbf{E}_{\nu}(k_F)|$ stays equal to $4\,R_X$ where $R_X$ is the 3D exciton Rydberg unit. By contrast, Pauli blocking reduces the binding energy of the ground-state trion $|\mathbf{E}_{\nu}(k_F)|$, starting from $4.47 R_X$ [25–27] in the absence of Fermi sea when the hole mass is infinite. The inclusion of Fermi-sea screening reduces even more the trion energy. It also affects the exciton energy, although the reduction is smaller. The two energies get very close for $k_F a_{X} \sim 0.6$, making the Pauli-blocked electron of the trion essentially unbound: the trion is no longer a valid description and the exciton has to be taken into consideration. So, in order to have a real trion well separated from the exciton, it is necessary to have a moderately dense Fermi sea with a Fermi energy smaller than the trion binding, in agreement with qualitative arguments.

Figure 3 also shows the ratio of the calculated photon-trion coupling $\Omega_{\nu,0,\mathbf{Q}=0,\mathbf{k}}$ (see eq. (9)) for $|\mathbf{k}| \rightarrow k_F$ over the photon-exciton coupling $\Omega_{\nu,0,\mathbf{Q}}$, multiplied by $\sqrt{N}$ where $N = L^2 k_F^2/4\pi$ is the number of electron states in the

**2D Fermi sea.** Its value increases with the Fermi energy as expected, with a very small screening effect. The photon-trion coupling $\Omega_{\nu,0,\mathbf{Q}=0,\mathbf{k}}$ is insensitive to $\mathbf{k}$, if the Fermi momentum $k_F$ is not too large.

*All this leads to an optimal Fermi sea* for trion-polariton formation not too large in order for the trion subspace coupled to a $\mathbf{Q}$ photon to be quasi-degenerate while enhancing the reduction factor up to $N(a_T/L)^D$ of the order of 1 for the photon-trion coupling to be of the order of the exciton-photon coupling. Since the electron density $N/L^D$ scales as $k_F^D$, this corresponds to an electron Fermi energy $E_F$ of the order of a fraction of the trion binding. Note that a far smaller Fermi sea would be sufficient to pin the photon momentum to its original $\mathbf{Q}$ value. Also note that the positively charged conduction-hole can further form a bound state with the negatively charged trion (see fig. 2(b)); the trion-hole polariton would then transform into an excitonic dipole interacting with the dipole of a conduction electron-hole pair, or more generally, a cloud of such pairs. However, as the binding between trion and hole stays very small for $k_F a_{X} \ll 1$, the trion-hole object is a more accurate description when the Fermi sea is dilute, than the exciton dressed by a cloud of conduction electron-hole pairs, as proposed in refs. [15,28].

**Additional comments.** – The above trion-polariton picture corresponds to a coupling between two composite fermions, namely a photon-electron pair and a trion. The same physics can be described in a more standard way in terms of bosons: one just has to consider the Fermi sea as filled, and allows it to “boil” with excitation of conduction electron-hole pairs. The $\mathbf{Q}$ photon then transforms...
(see fig. 1(c)) into a bosonic pair of \((Q + k, \eta)\) trion
and \((-k)\) conduction-hole, the resulting trion-polariton
being a quite complex coboson, with a promise of very
interesting many-body physics.

The Fermi sea with a \(k\) state empty is surely shaken
up by Coulomb interaction with the trion: this interaction
can scatter the empty state from \(k\) to \(k \neq k\), or
excite additional conduction-electron pairs. While
the former process does not change the dimensionality
of the photon-coupled subspace made of \(Q + k\) trion
and \((-k)\) conduction hole, the recombination of a trion in the
presence of conduction-electron pairs produces pho-
tons having an energy much higher than the \(Q\) photon
energy. So, the inclusion of such Coulomb processes does
not change our conclusions.

**Formalism.** – We now outline how the above under-
standing follows from the coboson many-body formalism.
The key hinges on the description of a trion as an exciton
interacting with an electron [22,29–32].

The first step is to write the trion on the exciton-
electron basis. For two electrons located at \((r_x, r_y)\)
and one hole located at \(r_h\), the relevant trion coordinates
[22,29] are the trion center-of-mass position \(R_T\), the
electron-hole distance \(r = r_x - r_y\) in the \((e, h)\) exciton
and the distance \(u = r_x - (m_x r_x + m_h r_h)/(m_x + m_h)\)
between the \(e^\prime\) electron and the exciton center of mass.
For opposite-spin electrons in singlet \((S = 0)\) or triplet
\((S = 1)\) state and hole “spin” \(m = \pm 3/2\), the trion
creation operator can be written in terms of electron
and exciton creation operators, \(a^\dagger\) and \(B^\dagger\), as
\[
T^\dagger_{k,\eta,S,m} = \sum_{p,\nu} a^\dagger_{p,\nu} B^\dagger_{p,\beta X} B^\dagger_{p,\beta X} \langle \nu|\eta,S\rangle,
\]
where \(K\) is the trion center-of-mass momentum and \(\beta = 1 - \beta X = m_x/(2m_x + m_h)\) in order for \(p\) to be the
relative-motion momentum. The trion relative-motion wave
function \((u, r)|\eta, S\rangle\) appears in the above equation through its
Fourier transform
\[
\langle \nu|\eta, S\rangle = \int \text{dudr} \langle \nu|u\rangle \langle u|r\rangle |\eta, S\rangle.
\]

 Conversely, the creation operator for an electron-exciton
pair can be written in terms of trion operators as
\[
a^\dagger_{p,\nu} B^\dagger_{p,\beta X} B^\dagger_{p,\beta X} \langle \eta|S,=0(1)\rangle = \sum_{\eta,=0(1)} T^\dagger_{\eta,=0(1)} \langle \eta,=0(1)|\nu,p\rangle.
\]

The photon semiconductor Hamiltonian reads
\[
H = H_{ph} + H_{se} + W_{ph,sc} = \sum_{Q,=1} \alpha_Q B^\dagger_{Q,=1} B^\dagger_{Q,=1} + \text{h.c.}
\]
of electrons, Pauli blocking forces the electron momentum \( \mathbf{k} - \mathbf{q} \) in the above sum to be either equal to \( \mathbf{k} \), or above the Fermi level — the latter being excluded because too far away from the initial energy. This reduces the \( \mathbf{q} \) sum to its \( \mathbf{q} = 0 \) term: the photon then keeps its initial momentum \( Q \), in the same way as for the exciton-polariton with strong coupling; however, this condition is here fulfilled with the help of Pauli blocking with the Fermi sea.

Still, the coupling \( |\Omega_{n_{0},0,Q,k}|^{2} \) between the photon \( Q \) and the trion-conduction-hole pair \( (Q + k_{1}, -k_{1}) \) is \( (aT/L)^{D} \) smaller than the one for the exciton-polariton. A volume-linear factor is required to overcome this reduction factor. To get it, we note that the \( Q \) photon has similar vanishingly small couplings to the \( N \) Fermi-sea electrons \( k_{j} \). Diagonalization within this coupled subspace transforms

\[
(\omega_{Q} + \varepsilon_{N} - E)(E_{Q}^{(T)}(k_{1} + k_{0}, 0) - \varepsilon_{k}^{(e)} + \varepsilon_{N} - E) = |\Omega_{n_{0},0,Q,k}|^{2}
\]

that gives the system energy when the Fermi sea contains a single electron \( k \), into

\[
(\omega_{Q} + \varepsilon_{N} - E)\prod_{i=1}^{N}(E_{Q}^{(T)}(k_{i}, n_{0}, 0) - \varepsilon_{k_{i}}^{(e)} + \varepsilon_{N} - E) = \sum_{i=1}^{N}|\Omega_{n_{0},0,Q,k}|^{2} \prod_{j \neq i}(E_{Q}^{(T)}(k_{j}, n_{0}, 0) - \varepsilon_{k_{j}}^{(e)} + \varepsilon_{N} - E).
\]

By taking the trion-conduction-hole states as quasi-degenerate, eq. (12) then reduces to eq. (11) with \( |\Omega_{n_{0},0,Q,k}|^{2} \) multiplied by \( N \). This leads to a trion-polariton splitting as large as the exciton-polariton splitting for \( N(aT/L)^{D} \sim 1 \).

In the above calculations, we neglected Coulomb processes between trions and Fermi sea electrons; these processes either change \( k_{1} \) into \( k_{0} \), or create additional conduction electron-hole pairs, but they do not qualitatively change the above results, as previously explained. We also took the Fermi sea as fully polarized to elucidate the physics of this novel polariton in the simplest way. Adding spin-\((-1/2)\) electrons reduces the exciton binding due to Pauli blocking with same-spin electrons, but mostly broadens the trion resonance. Consequences raised by these issues, including the Fermi-edge singularity \([33–37]\) associated with the sudden appearance of a trion in the presence of a dense Fermi sea, will be studied elsewhere.

The strong-coupling regime in the presence of an unpolarized conduction Fermi sea has been recently investigated using transition metal dichalcogenide (MoSe\(_{2}\)) monolayers embedded in microwaviness \([15]\). The novel state found below the exciton-polariton has been partially explained in terms of “Fermi polaron-polariton” using a model based on a rigid exciton interacting with photon and electrons. Moreover, Coulomb screening and mostly Pauli blocking induced by Fermi electrons on the trion and the exciton have been neglected or included in a crude way.

A more sophisticated model \([28]\) but along the same line has been proposed to study the optical absorption. These models do miss the rich fermion-exchange physics associated with the trion structure, which is crucial, and often dominant, in problems dealing with composite bosons, like excitons, and composite fermions, like trions. These recent experiments have been unfortunately performed in a quite complex configuration; so, it is rather difficult to make a direct comparison with the present study which mostly focuses on the fundamental aspects of photons and trions in the presence of a Fermi sea.

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Appendix on numerical calculations. – To obtain the trion ground state in the presence of a frozen Fermi sea \( |F_{N,1/2}\rangle \), we consider for simplicity the valence-hole with an infinite mass, hence as fixed. We then expand the \( (k, -1/2) \) and \( (k', 1/2) \) electronic states of the trion a \( \sum_{k} \sum_{|k'\rangle > k_{F}} G_{k,k'} a_{k,-\epsilon}^{(e)} a_{k',+\epsilon}^{(c)} \) with \( k' \) outside the Fermi sea due to the Pauli blocking it induces.

To minimize the Hamiltonian mean value, we have expanded the trial function \( G_{k,k'} \) on products of 2D cylindrical functions which, for \( r = (r, \phi) \) and \( \ell = (0, \pm 1, \pm 2, \cdots) \), read as \( \Phi_{\ell,\lambda}(r) = e^{i\ell \phi} e^{-\delta r^{2} \ell^{2}} \delta_{\ell,\lambda} \) or better its Fourier transform \( \Phi_{\ell,\lambda,k} \). The 2D exciton ground state corresponds to \( (\ell = 0, \lambda = 2) \) for \( r \) in 3D Bohr radius \( a_{X} \). So, \( G_{k,k'} = \sum_{\ell,\lambda} \sum_{\ell',\lambda'} \Phi_{\ell,\lambda,k} \Phi_{\ell',\lambda,k'} \). In practice, we can simplify the \( \ell \) to \( (0, \pm 1, \pm 2) \) and take \( \ell' = -\ell \) since the ground state has zero angular momentum. We have also used an even-tempered set of seven (\( \lambda, \lambda' \)) parameters. We then used the calculated trion ground-state wave function to obtain the ratio of the photon-trion coupling \( \Omega_{n_{0},0,Q,k} \) for \( |k| \rightarrow k_{F} \) to the photon-exciton coupling \( \Omega_{Q=0} \), shown in fig. 3.

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