Effective mass of composite fermion: a phenomenological fit in with anomalous propagation of surface acoustic wave

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We calculate the conductivity associated with the anomalous propagation of a surface acoustic wave above a two-dimensional electron gas at $\nu = 1/2$. Murthy-Shankar’s middle representation is adopted and a contribution to the response functions beyond the random phase approximation is taken into account. We give a phenomenological fit for the effective mass of composite fermion in with the experimental data of the anomalous propagation of surface acoustic wave at $\nu = 1/2$ and find the phenomenological value of the effective mass is several times larger than the theoretical value $m^*_{\text{th}} = 6\varepsilon/e^2l_{\text{A}}$ derived from the Hartree-Fock approximation. We compare the phenomenological value of the composite fermion effective mass with that measured in the experiments of the activation energy and the Shubnikov-de Haas oscillations. It is found that the comparison is fairly well.

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1. INTRODUCTION

The composite fermion (CF) theory for fractional quantum Hall effect has been evolved for a decade since Jain first proposed this complex of the electron and flux quanta in connection with the trial wave functions of the prominent quantum Hall states $1 \nu$. An important step in the development of the CF theory was stridden by Haplerin, Lee and Read (HLR) as well as Kalmeyer and Zhang, in which the fermion-Chern-Simons (FCS) theory is applied to describe the physics at the even-denominator filling fractions, e.g., $\nu = \frac{1}{2}$. HLR successfully predicted the existence of the CF Fermi surface at such filling fractions, which indicated a CF Fermi liquid or a modified Fermi liquid. However, HLR recognized a crucial difference between the FCS theory and the conventional Fermi liquid, namely the effective mass of CF is divergent at the Fermi surface. This draws forth many subsequent studies of HLR. Most recently, it was recognized that the charge carried by the CF is obviously different from the real physical excitation at $\nu = 1/2$. This stirs up a series of recent studies of them, the dipolar neutral CF is attracting particular attentions from many authors, which stems from Read’s conceptual paper. Based on the neutral CF, the effective mass problem was carefully reconsidered. Shankar and Murthy found a way to define the CF effective mass and showed that it is of the order of the electron-electron interaction, namely, $1/m^* \sim (1/\nu)^{2/3}e^2l_{\text{A}}/\varepsilon$ in the Hartree-Fock approximation.

Beside the prediction of the CF Fermi surface, another celebrated success of HLR’s theory is the explanation for the anomaly of the surface acoustic wave (SAW) propagation above a two-dimensional electron gas in GaAs/AlGaAs heterostructures. In their experiment, Willett et al observed the anomalous maximum and minimum at $\nu = \frac{1}{2}$ of the attenuation rate and the velocity shift of the SAW, respectively. Correspondingly, a maximum of the conductivity appears at $\nu = \frac{1}{2}$. In the clear range where the wavelength of the SAW is shorter than the CF’s mean free path, the conductivity has a linear-dependence on the wave vector. The theoretical results from the FCS theory agree qualitatively with these experimental observations.

However, from the beginning, there is a systematic discrepancy between the theoretical and experimental values for the magnitude of the conductivity $\sigma_{xx}(q)$. In the FCS theory, there is no adjustable parameter to enhance $\sigma_{xx}(q)$ such that its theoretical value is approximately a factor of 2 smaller than the conductivity observed in experiments. We notice that this non-adjustability in the FCS theory is due to the perturbative random phase approximation (RPA). For a generic field theory, the RPA may or may not provide a full description for the low energy behavior of the system. Particularly, for the FCS theory, the important low energy information might be lost by only taking the RPA. It is because, firstly, the Chern-Simons coupling constant, which is one of the parameters in the perturbative expansion, is not small; Secondly, the fluctuations of the Chern-Simons gauge field is subject to the well-known constraint for the physical states which will be shown explicitly below in the FCS theory, the important low energy behavior of the system.
tion [10,17,19,21], one can fit the effective mass and finds that it is in several times of Shankar-Murthy’s theoretical estimation, \( m^* = (e^2 l_1/3 / 6\varepsilon)^{-1} \). We argue that it is reasonable that the effective mass increases for \( \nu \to 1/2 \) because, theoretically, the gauge fluctuations always raise the effective mass with respect to its Hartree-Fock value and experimentally, the increase of the effective mass has been observed much faster than the current theoretical predictions [22,23]. We also compare our phenomenological effective mass to those in measurements of the activation energy and the Shubnikov-de Haas oscillation [24,25,26].

This paper is organized as follows. In Sec. II, we review Murthy-Shankar’s theory and set up Feynman’s rules for the perturbative theory. In Sec. III, various response functions are calculated. In Sec. IV, the conductivity including a non-RPA correction is derived. In Sec. V, the CF effective mass is fit in with the SAW propagation experiments and is compared to the known theoretical and experimental results. The section VI consists of our conclusions.

II. HAMILTONIAN AND PERTURBATIVE THEORY

There are essentially two kinds of formulations in the study of the CF theory. One of them is proposed, by construction, within the subspace of the lowest Landau level (LLL) [17,19,21]. These formulations automatically incorporate the feature that the energy scale and effective mass are set by the electron-electron interaction and in the subspace of the lowest Landau level. The other kind of formulations begins with an enlarged quantum state space. The FCS theory is the basis of the later one. In the present paper, we shall relate the effective mass to the conductivity in the SAW propagation. The FCS theory already gave the anomaly of the CF theory. One of them is proposed by Murthy and Shankar [10]. Enlightened by Bohm and Pines [27], Murthy and Shankar chose a gauge for the FCS Hamiltonian and named this Hamiltonian the ‘middle representation’ of their theory. For the CF excitation with a correct charge, they applied a canonical transformation to the middle representation and arrived at their ‘final representation’. In their final representation, the CF excitation is neutral at \( \nu = 1/2 \) and the effective mass is of the order of the electron-electron interaction and independent of the band mass. On the other hand, it has been shown that if we do not set foot in the quasiparticle charge, we can also get this same value of the effective mass by using the middle representation [13]. In this paper, we would like to fit the effective mass in with the experiment of the SAW propagation. Thus, we will employ Murthy-Shankar’s middle representation.

A. Hamiltonian

We start from a two dimensional interacting electron system that is placed in a uniform magnetic field \( B \) perpendicular to the two dimensional plane imbedded in a uniform positive background. We assume that all electrons are spin-polarized. For the two-body interaction potential \( V \), the \( N \)-electron Hamiltonian reads,

\[
H_e = \frac{1}{2m_b} \sum_i \left[ -i\hbar \nabla_i + \frac{e}{c} \vec{A}_i(\vec{x}_i) \right]^2 + \sum_{i<j} V(\vec{x}_i - \vec{x}_j),
\]

where the vector potential \( \vec{A} \) corresponds to the magnetic field \( B \) and \( m_b \) is the band mass of the electrons. Hereafter, we will use the unit \( \hbar = \hbar = 1 \). At this stage, we do not confine the electrons in the LLL. The attraction between the electrons and the uniform background is not explicitly shown up.

Following a common treatment, we make an anyon transformation [28] for the electron wavefunction \( \Phi(\vec{r}_1, \ldots, \vec{r}_N) \) with \( \vec{r}_j \) being the position of the \( j \)-th electron. The transformed wavefunction is given by

\[
\Psi_{cs}(z_1, \ldots, z_N) = \prod_{i<j} \left[ \frac{z_i - z_j}{|z_i - z_j|} \right]^{\frac{\phi}{2\pi}} \Phi(z_1, \ldots, z_N),
\]

where \( z_j = x_j + iy_j \), and \( \phi \) is an even integer. \( \Psi_{cs} \) is the wavefunction for the transformed fermion (so called the Chern-Simons fermion). The Hamiltonian corresponding to the transformation becomes

\[
H_{cs} = \frac{1}{2m_b} \sum_i \left[ -i\hbar \nabla_i + \vec{A}_i(\vec{x}_i) - \vec{a}_i(\vec{x}_i) \right]^2 + \sum_{i<j} V(\vec{x}_i - \vec{x}_j),
\]

where \( \vec{a} \) is a statistical gauge potential. i.e.,

\[
\vec{a}(\vec{x}_i) = \frac{\phi}{2\pi} \sum_{j \neq i} \frac{\hat{z} \times (\vec{x}_i - \vec{x}_j)}{|\vec{x}_i - \vec{x}_j|^2},
\]

which satisfies the following constraint

\[
\nabla \times \vec{a}(\vec{x}) = 2\pi \rho(\vec{x}) \equiv \delta(\vec{x}).
\]

The mean field approximation for the FCS theory can be achieved as

\[
\vec{a}_{MF} = \vec{A},
\]

where \( \vec{A} = (B/2) \hat{z} \times \hat{r} \). Around the mean field state, there is an important gauge fluctuation \( \delta \vec{a} = \vec{A} - \vec{a} \). For convenience, we denote \( \delta \vec{a} \) as \( \vec{a} \) which is the fluctuation of the statistic gauge field around the mean field. If we introduce the second quantization notation with the Chern-Simons fermion field \( \psi_{cs} \), then the Hamiltonian around the mean field reads,
\[ H = \int d^2x \frac{1}{2m_b} \left| -i \nabla + \hat{a}(\vec{x}) \right| \psi_{cs}^2 \]
\[ + \frac{1}{2} \int d^2x d^2x' \delta \rho(\vec{x}) V(\vec{x} - \vec{x}') \delta \rho(\vec{x}'). \]

with \( \delta \rho = \rho - \rho_0 \). The gauge fluctuation obeys the constraint \( \Box \rho = 0 \) associated with the density fluctuation \( \delta \rho \). Notice that here the Hamiltonian is written in the Coulomb gauge \( \nabla \cdot \vec{a} = 0 \). It is well-known that the FCS theory has a gauge symmetry corresponding to the gauge transformation of \( \vec{a} \) if we consider the bulk states only so that the Hamiltonian can also be written as a gauge invariant form. Then, one can choose other gauges to deal with the system. For our purpose, we choose a gauge that was used by Shankar and Murthy \[9\], enlightened by Bohm-Pines’ gauge choice for the three dimensional electron gas in the real electromagnetic field \[27\]. In this gauge, the Hamiltonian, what is called the middle representation, takes its form as

\[ H_{\text{eff}} = H_0f + H_{0a} + H_i + H_{ia} + H_{sr}, \]

(2.8)

where \( H_{sr} \) is the non-dynamic short-range gauge fluctuation and

\[ H_0f = \frac{1}{2m_b} \int d^2x |\nabla \psi|^2, \]
\[ H_{0a} = \frac{\rho_0}{2m_b} \int d^2x (a_x^2 + a_y^2) \]
\[ + \frac{1}{8\pi^2\rho_0} \int d^2x d^2x' [\nabla \times \hat{a}(\vec{x})] V(\vec{x} - \vec{x}') [\nabla' \times \hat{a}(\vec{x}')] , \]
\[ H_i = \int d^2x \vec{a} \cdot \frac{i}{2m_b} (\psi^\dagger \nabla \psi - \nabla \psi^\dagger \psi), \]
\[ H_{ia} = \frac{1}{2m_b} \int d^2x \delta \rho \vec{a}^2. \]

(2.9)

Here \( H_0f \) and \( H_{0a} \) stand for the free Hamiltonian of the CF and the gauge fluctuation, respectively. \( H_i \) and \( H_{ia} \) are the interactions. The Fourier component \( \vec{a} \) of the gauge field obeys the commutation relation

\[ [a_x(\vec{q}), a_y(\vec{q}')] = i \delta^{(2)}(\vec{q} - \vec{q}'), \]

(2.10)

and is restricted by the constraint

\[ \left. \frac{\vec{q}a_x}{2\pi^2} - \delta \rho \right|_{\text{Phys}} \geq 0, \quad 0 < q < Q. \]

(2.11)

with the cut-off \( Q = k_F \). All short range fluctuations are included in \( H_{sr} \), which is ignored in discussing the low energy physics we are interested in. Murthy-Shankar’s formulation begins with the above Hamiltonian after dropping \( H_{sr} \). Finally, they ignored the non-RPA interaction \( H_{ia} \) and employed a canonical transformation to cancel \( H_i \) and reach their final representation. Here, instead of using the final representation, we exert their middle representation which is favorite for the perturbative calculation. However, to directly deal with \( H_{ia} \) in the perturbative theory is difficult. Solving \( \delta \rho \) by the constraint \( \Box \rho = 0 \), \( H_{ia} \) is transformed into

\[ H'_{ia} = \frac{1}{4\pi \rho m_b} \int d^2x (\nabla \times \vec{a}) \vec{a}^2. \]

(2.12)

Now, we have the Hamiltonian

\[ H = H_{0f} + H_{0a} + H_i + H'_{ia} \]

(2.13)

to work on.

**B. Feynman’s Rules**

The perturbative theory starts from to set up Feynman’s rules. Since we have neglected the short wave length gauge fluctuations, the gauge field wave vector is restricted to \( q < k_F \) in all following perturbative calculation. The free CF propagator (Fig. 2(a)) is

\[ G_0(k, \omega) = \frac{\theta(k - k_F)}{\omega - \epsilon_x + i\delta} + \frac{\theta(k_F - k)}{\omega - \epsilon_x - i\delta}, \]

(2.14)

and the gauge fluctuation propagates (Fig. 2(b)) can be read out

\[ D^0(q, \omega) = U, \]

(2.15)

with the matrix \( U \) defined by

\[ U^{-1} = \begin{pmatrix}
-\frac{\rho_0}{m^*} & i\omega \\
\frac{i\omega}{2\pi \phi} & -\frac{\rho_0}{m^*} (1 + \frac{e^2q}{2\omega e^2})
\end{pmatrix}. \]

(2.16)

Here we have taken the 2 \times 2 matrix description of the gauge propagator with \( D_{11}^0 = U_{11} \) and \( D_{22}^0 = U_{22} \) and so on. The interaction has been specified as the Coulomb interaction \( V(q) = \frac{2\pi e^2}{eq} \) which is taken throughout in the whole paper. The indices ‘1’ and ‘2’ correspond to the components parallel and perpendicular to the wave vector \( \vec{q} \), respectively. The interaction vertex is shown as (Fig. 2(c))

\[ g_a = \frac{1}{m^*} ((\vec{k} + \frac{\vec{q}}{2}) \cdot \vec{q}, (\vec{k} + \frac{\vec{q}}{2}) \times \vec{q}). \]

(2.17)

In eqs. (2.10) and (2.21), we have simply replaced the band mass by a phenomenological effective mass as did by HLR \[3\]. The gauge field self-interaction vertex figured in Fig. 2(d) can be read out from the Hamiltonian

\[ f_{122}(-\vec{q} - \vec{q}', \vec{q}, \vec{q}') = \frac{i}{8\pi m_b} (q_2 + q_2'), \]
\[ f_{211}(-\vec{q} - \vec{q}', \vec{q}, \vec{q}') = -\frac{i}{8\pi m_b} (q_1 + q_1'). \]

(2.18)

We call the interaction described by the vertex (2.18) the non-RPA interaction. It has been pointed out that this interaction vertex is not renormalizable. So, neither the band mass in (2.18) because it is not relevant to the CF kinetic energy \[1\].
III. RESPONSE FUNCTIONS

A. Non-interacting Response Functions

The calculation of the response functions is the central task of this section. The simplest response functions are the free CF’s (Fig. 3), which are defined as [2]

\[
K_{11}^0 = \int \frac{d^2k}{(2\pi)^2} \frac{k_\perp^2}{m^*} \left( \frac{f(\omega_{k+q/2}) - f(\omega_{k-q/2})}{\omega - \omega_{k+q/2} + \omega_{k-q/2} + i\delta} \right),
\]

(3.1)

where \( f(\omega_k) \) is the Fermi factor. In the current-current response function, we do not subtract \( \frac{q}{q^2} \) because it has appeared in (2.16). The static response functions for \( q < < k_F \) are calculated as

\[
K_{00}^0(q,0) = \frac{m^*}{2\pi} + O(q^2),
\]

(3.2)

\[
K_{11}^0(q,0) = \frac{\rho_0}{m^*} + O(q^4),
\]

\[
K_{22}^0(q,0) = \frac{\rho_0}{m^*} - \frac{q^2}{24\pi m^*} + O(q^4).
\]

For \( \omega < < v_F q \), the imaginary parts of \( K^0 \) are calculated as follows:

\[
\text{Im} K_{00}^0(q,\omega) \approx \frac{m^* \omega}{2\pi v_F q},
\]

(3.3)

\[
\text{Im} K_{11}^0(q,\omega) \approx \frac{\rho_0 \omega^2}{2\pi v_F q^2},
\]

\[
\text{Im} K_{22}^0(q,\omega) \approx \frac{2\rho_0 \omega}{k_F q}.
\]

Meanwhile there is a non-zero real part of \( K_{11}^0 \)

\[
\text{Re} K_{11}^0(q,\omega) \approx \frac{m^* \omega^2}{2\pi q^2}.
\]

(3.4)

It is seen that

\[
K_{11}^0 - \frac{\rho_0}{m^*} = \frac{\omega^2}{q^2} K_{00}^0,
\]

(3.5)

which simply recovers the physics of the continuous equation \( j_1 = (\omega/\gamma) j_0 \). Comparing the imaginary ones with the real ones in eqs. (3.2)-(3.4), \( \text{Im} K_{00}^0 \) and \( \text{Im} K_{11}^0 \) are small and can be neglected while \( \text{Im} K_{22}^0 \) should be kept.

On the other hand, for \( \omega > > v_F q \), it is easy to have

\[
K_{11}^0 \sim K_{22}^0 \sim O\left(\frac{(v_F q)^2}{\omega^2}\right).
\]

B. Bare Non-RPA Response Functions

Another kind of the response functions which may be relevant is so-called bare non-RPA response functions (see Fig. 4). These response functions are determined by the non-RPA interaction vertex \( \Gamma \) and the bare gauge fluctuation propagator \( D_0 \) (see Fig. 2(b)). The calculating results for these response functions in \( q < < k_F \) are as follows. Because there is no pole for \( \omega < v_F q \) in the integration, one can easily see that

\[
K_{11}^0 = K_{22}^0 = K_{12}^0 = 0,
\]

(3.7)

i.e., these static response functions vanish. For \( \omega > > v_F q \), one has

\[
K_{11}^0(q,\omega) = K_{22}^0(q,\omega) = -\frac{q^2}{16\pi m_b} \frac{1}{4-\frac{m^*}{m}}.
\]

(3.8)

with \( z = \omega/\omega_c \). Because the poles of the bare non-RPA response functions are in the high energy region, they will not contribute to the low energy behavior.

C. RPA Response Functions and RPA Gauge Propagator

The RPA equation for \( K \) (Fig. 5) may be written as

\[
K_R = K^0 - K^0 [K^0 + U^{-1}]^{-1} K^0.
\]

(3.9)

According to the calculations in the previous two subsections, we see that the low frequency limit of \( K_R \) is not modified if one replaces \( K^0 \) by \( K^0 + K_b^0 \). Meanwhile, although there is the anomalous pole \( \omega = 2\omega_c \) in the bare non-RPA response functions for \( \omega > > v_F q \), they will not violate Kohn’s theorem since they are as small as the order of \( q^2 \).

Therefore, the bare non-RPA response functions will not affect the physical properties we are interested in. According to such a result , the RPA gauge propagator \( D^r \) (the thick wave line in Figures) can also be defined by neglecting \( K_b^0 \). That is, \( D^r = (K^0 + U^{-1})^{-1} \). For the \( \omega < < v_F q \) case it can be written as

\[
D_{11}^r(q,\omega) = \frac{2\pi}{m^* (\omega + i\delta)^2},
\]

\[
D_{22}^r(q,\omega) = \frac{q}{i\omega \gamma - q^2 \chi},
\]

(3.10)

\[
D_{12}^r(q,\omega) = -D_{12}^r = -\frac{iq^2}{2m^* \omega + i\delta} \frac{1}{i\omega \gamma - q^2 \chi},
\]

(3.11)

where \( \gamma = \frac{k_F}{2\pi} \) and \( \chi = \frac{\epsilon^2}{8\pi}\).
D. Response Functions beyond RPA

So far, we find no interesting result beyond the RPA although the bare non-RPA response functions are calculated in subsection B. We did not find out a nontrivial contribution to the response functions for \( \omega << v_F q \). On the other hand, we calculated the RPA gauge propagator in the last subsection. One may ask what could be happened if the bare gauge propagator in the bubble of the bare non-RPA response function is replaced by the RPA gauge propagator? Instead of Fig. 4, a response function beyond the RPA is represented as Fig. 1 and defined by

\[
K_{aa}^{nr}(q, \omega) = \int \frac{d^2q'}{(2\pi)^2} \frac{d\omega'}{2\pi i} f_{abc} f_{a'b'c'} \times D_{1b'}^{r}(q' + q', \omega + \omega') D_{c'}^{r}(q', \omega').
\]  

(3.12)

Before going to the details, we recall some useful properties: The gauge propagator \( D_{12}^{r} = -D_{21}^{r} \) and the coupling constants \( f_{112}(q', \theta)f_{122}(q', \theta) = -f_{112}(q, -\theta)f_{122}(q, -\theta) \). Those properties immediately lead to \( K_{22}^{nr} = K_{22}^{nr} = 0 \). So, we are only interested in \( K_{22}^{nr} \) for \( a = 1, 2 \), e.g.,

\[
K_{22}^{nr} = \int \frac{d^2q'}{(2\pi)^2} \frac{d\omega'}{2\pi i} \left[ -(f_{211}(q' + q)D_{11}^{r}(q' + q, \omega + \omega')D_{11}^{r}(q', \omega') - f_{212}(q' + q)D_{11}^{r}(q' + q, \omega + \omega')D_{12}^{r}(q', \omega') - f_{221}(q' + q)D_{22}^{r}(q' + q, \omega + \omega')D_{11}^{r}(q', \omega') \right],
\]

(3.13)

where we have dropped many vanishing terms (e.g., the terms with respect to \( D_{12}^{r} \) due to the property \( D_{12}^{r} = -D_{21}^{r} \), etc). Substituting the non-RPA interaction vertex and the RPA gauge propagator into the above equation, and after a variable shift, the leading term in the case \( q << k_F \) can be derived as

\[
K_{22}^{nr} \approx \frac{2\pi}{m^* 8\pi m_b} \int \frac{d^2q'}{(2\pi)^2} \frac{d\omega'}{2\pi i} q_2^2 q^3 \left[ \frac{1}{(\omega' + \omega + i\delta)^2} + \frac{1}{(\omega' - \omega - i\delta)^2} \right] \times \frac{1}{i\gamma' \omega' - \chi q^2} - O(\frac{\omega^2}{k_F}),
\]

(3.14)

where the factor \( q_2^2 \) comes from \( f_{221}^2 = f_{212}^2 \) and \( q^3 \) from the propagators; the \( q \)-dependent contributions are of the higher order. After the integration, one can attract a pure imaginary constant contribution to \( K_{22}^{nr} \),

\[
K_{22}^{nr} \approx -i k_F Q^{3/2} e^2 \left( 1 - O(\frac{\omega^2}{k_F^2}) \right),
\]

(3.15)

where \( Q \) is the cut-off in integrating over \( q' \) and is taken to be \( Q = k_F \) according to the gauge choice. \( O(\frac{\omega^2}{k_F^2}) \) is positive so that it reduces a small value of \( K_{22}^{nr} \). Because

\[ \boxed{[15]} \]

is invariant if one replaces \( q_2^2 \) by \( q_1^2 \), a similar calculation shows that

\[
K_{22}^{nr} = K_{22}^{nr},
\]

(3.16)

in their leading terms. Our calculation provides a correction to the imaginary part of the response function. In some sense, we may use

\[
K^n = K^0 + K^{nr},
\]

(3.17)

to replace \( K^0 \).

IV. THE CONDUCTIVITY TENSOR

The conductivity tensor is one of the most important observables in the experiment of the transport property of the system. For the FCS system, the conductivity tensor has been defined by Halperin, Lee and Read in their seminal paper [3]. We here take their definition with a minor modification. Basically, HLR’s definition of the conductivity tensor is valid for \( \omega \sim v_F q \) with \( v_F \) being the SAW propagation velocity and the tensor reads

\[
\sigma_{xx}(\vec{q}, \omega) = \frac{i q^2}{\omega} \left[ \frac{1}{\Pi_{00}(q, \omega)} - \frac{1}{\Pi_{00}(q, 0)} \right],
\]

(4.1)

\[
\sigma_{yy}(\vec{q}, \omega) = -\frac{i}{\omega} \left[ \Pi_{22}(\vec{q}, \omega) - \text{Re}(\Pi_{22}(\vec{q}, 0)) \right],
\]

\[
\sigma_{xy}(\vec{q}, \omega) = -\sigma_{yx}(\vec{q}, \omega) = \frac{i}{q} \Pi_{02}(\vec{q}, \omega),
\]

where a modification has been made in defining \( \sigma_{yy} \) with the real part of \( \Pi_{22}(\vec{q}, 0) \) being subtracted. Similarly, the “quasiparticle conductivity tensor” is defined by

\[
\sigma_{xx}(\vec{q}, \omega) = \frac{i q^2}{\omega} \left[ \frac{1}{K_{00}(q, \omega)} - \frac{1}{K_{00}(q, 0)} \right],
\]

\[
\tilde{\sigma}_{yy}(\vec{q}, \omega) = -\frac{i}{\omega} \left[ \tilde{K}_{22}(\vec{q}, \omega) - \text{Re}(\tilde{K}_{22}(\vec{q}, 0)) \right],
\]

\[
\sigma_{xy}(\vec{q}, \omega) = -\sigma_{yx}(\vec{q}, \omega) = \frac{i}{q} \tilde{K}_{02}(\vec{q}, \omega),
\]

The matrix \( \Pi \) in (4.1) consists of the sum of all Feynman diagrams for the full response function matrix \( \Pi \) which is irreducible with respect to the Coulomb interaction, while \( \tilde{K} \) in (4.2) contains only those diagrams which are irreducible with respect to both the Chern-Simons interaction and the Coulomb interaction. In the regime where the definition of the conductivity tensors are available, \( \sigma \) and \( \tilde{\sigma} \) are related to the corresponding resistivities \( \rho \) and \( \tilde{\rho} \) through the matrix equations

\[
\sigma \equiv \rho^{-1},
\]

\[
\tilde{\sigma} \equiv \tilde{\rho}^{-1},
\]

\[
\rho = \rho + \rho_{cs},
\]

\[
\tilde{\rho} = \tilde{\rho} + \rho_{cs},
\]

\[
\rho_{cs} = \frac{4\pi \hbar}{e^2} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},
\]

(4.3)
respectively. Considering the case where \( \sigma_{xy} = -\sigma_{yx} = 0 \), we have

\[
\tilde{\rho}_{xx}(q, \omega) = \frac{1}{\sigma_{xx}(q, \omega)},
\]

\[
\tilde{\rho}_{yy}(q, \omega) = \frac{1}{\sigma_{yy}(q, \omega)}.
\]  

(4.4)

Using the above definition of the conductivity tensor, the formula for \( \sigma_{xx}(q) \) may then be rewritten as

\[
\sigma_{xx}(q) = \rho_{yy}(q)/\rho_{xy}^2,
\]  

where \( \rho_{xy} = 4\pi\hbar/e^2 \), and

\[
\frac{1}{\rho_{yy}(q)} = e^2 \lim_{\omega \to 0} \frac{1}{\omega} \Im \tilde{K}_{22}(q, \omega).
\]  

(4.6)

It is impossible to get the exact form of \( \tilde{K} \). The RPA definition of the conductivity tensor consists of replacing \( \tilde{K} \) by the free CF response function \( K^0 \). We may improve the RPA definition by replacing \( \tilde{K} \) by \( K^m \). Thus, we have,

\[
\frac{1}{\rho_{yy}(q)} = e^2 \lim_{\omega \to 0} \frac{1}{\omega} \Im K_{22}^m(q, \omega).
\]  

(4.7)

V. SURFACE ACOUSTIC WAVE PROPAGATION

A. Anomaly of the Surface Acoustic Wave Propagation

The experimental observation for the SAW propagation exhibits an anomaly in the relevant conductivity which deviates from the normal macroscopic DC value \[10\]. This phenomenon may be observed through the measurement of the SAW velocity shift \( \Delta v_s \) and the attenuation rate \( \kappa \) for the SAW amplitude,

\[
\frac{\Delta v_s}{v_s} = \frac{\alpha^2}{2} \frac{1}{1 + [\sigma_{xx}(q)/\sigma_m]^2},
\]

\[
\kappa = \frac{q_0^2}{2} \frac{[\sigma_{xx}(q)/\sigma_m]^2}{1 + [\sigma_{xx}(q)/\sigma_m]^2},
\]  

(5.1)

where \( \sigma_m = \frac{\rho_{xx}}{\rho_{yy}} \) and \( \alpha \) is a constant proportional to the piezoelectric coupling of GaAs. The experiments of the SAW’s propagation were performed in high quality GaAs/AlGaAs heterostructures with \( q \ll k_F \) and \( \omega = v_s q \ll v_F q \). The experiment result shows that the longitudinal conductivity is linearly dependent on the SAW’s wavevector \[10\], which is qualitatively in agreement with the prediction of the FCS theory approach \[2\]. However, to compare in details with the experimental data, it is necessary to use a value of \( \sigma_m \) approximately four times larger than the theoretical one. This discrepancy remains not explained so far. Moreover, there is an additional quantitative discrepancy between the theoretical prediction and experimental data in the longitudinal conductivity. In the RPA, HLR arrived at \[2\]

\[
\rho_{yy}(q) = \frac{2\pi}{k_F^2} e^{-C q^2}, \quad \text{for } q \gg \frac{2}{l},
\]

\[
\rho_{yy}(q) = \frac{4\pi}{k_F} e^{-C q^2}, \quad \text{for } q \ll \frac{2}{l},
\]  

(5.2)

where \( l \) is the CF transport mean free path at \( \nu = \frac{1}{2} \). As emphasized by the authors of ref. \[2\] there is no adjustable parameters in the RPA of \[16\] while the theoretical values of \( \sigma_{xx} \) are approximately a factor of 2 smaller than the experimental values obtained by Willett et al. That is, the theoretical result \[16\] is ‘too small’ in a factor of 2. We notice that the unadjustability comes from the replacement of \( \tilde{K} \) by \( K^0 \) in the RPA. To make it be adjustable, we suggest that \( \tilde{K} \) in \[16\] should be approximated by \( K^m \) instead of \( K^0 \). Hence, the inverse of the transverse resistivity of the CF in the clear range is given by

\[
\frac{1}{\rho_{yy}(q)} = e^2 \lim_{\omega \to 0} \frac{1}{\omega} \Im K_{22}^m(q, \omega)
\]

\[
\approx \frac{k_F}{4\pi q} (2 - C) e^2 \hbar \frac{2}{l}, \quad \text{for } q \gg \frac{2}{l},
\]  

(5.3)

where the physical dimension has been restored and the constant \( C \) is defined by

\[
C = \frac{v_F^2/v_s}{3(m_b/m_{\text{coul}})^2} = \frac{k_F^2 h/m^* v_s}{3(m_b/m_{\text{coul}})^2}
\]  

(5.4)

with \( m_{\text{coul}} = \frac{z e^2}{\hbar^2} \) being a mass scale induced by the Coulomb interaction. Different from the result given by \[16\], therefore, \[5.3\] includes an adjustable parameter \( C \) in the \( q \)-dependent conductivity \( \sigma_{xx}(q) \). When the experimental parameters are fixed, \( C \) is entirely determined by the CF effective mass. To be fit in with these series of experimental results, we should have \( C \approx 1 \). Then, we have to check with the CF effective mass to see whether it is consistent with such a fit or not.

B. Phenomenological Fit of the Effective Mass

In this subsection, we use the experiment data in a set of the SAW propagation experiments to fit the CF effective mass and compare the fit result to the established values of \( m^* \) both theoretically and experimentally.

For GaAs/AlGaAs heterostructures, we take the dielectric constant \( \varepsilon = 12.6 \) and the electron band mass \( m_b \approx 0.07 m_e \). We refer to several sets of experimental data by Willett et al as follows.

(A) The two-dimensional electron density \( n_e = 6.6 \times 10^{10}\text{cm}^{-2} \), the frequency of SAW \( f = 2.4\text{GHz} \) and the corresponding wavelength \( \lambda = 1.2\text{\mu m} \) \[10\].

(B) \( n_e = 6 \times 10^{10}\text{cm}^{-2}, f = 1.5\text{GHz} \) and \( \lambda = 2.0\text{\mu m} \) \[7\]:

6
\[ n_e = 7 \times 10^{10} \text{cm}^{-2}, \quad f = 0.36 \text{GHz} \text{ and } \lambda = 7.8 \mu \text{m} \]
\[ n_e = 1.0 \times 10^{11} \text{cm}^{-2}, \quad f = 1.2 \text{GHz} \text{ and } \lambda = 8 \mu \text{m} \]
\[ n_e = 1.6 \times 10^{11} \text{cm}^{-2}, \quad f = 10.7 \text{GHz} \text{ and } \lambda = 0.27 \mu \text{m} \]

First, let us compare \( m^* \) with the theoretical effective mass \( m_{th}^* \). We take the theoretical value of the CF effective mass to be the Hartree-Fock one \([9,11]\), i.e.,
\[ m_{th}^* = 6m_{coul} \quad (5.5) \]
which is in good agreement with the result of the numerical simulation \([24]\). If one takes \( C \approx 1 \) in \([5,6]\), one gets the effective mass corresponding to the experimental data,
\[ m_{(A)}^* \approx 2.50m_{th}^*, \quad m_{(B)}^* \approx 2.17m_{th}^*, \quad m_{(C)}^* \approx 2.67m_{th}^*, \]
\[ m_{(D)}^* \approx 3.00m_{th}^*, \quad m_{(E)}^* \approx 6.00m_{th}^* \quad (5.6) \]
where \( m_{(A)}^* \) corresponds to the experimental data in (A) etc.

The experimental data of the effective mass from Willett et al are not available. To have an instructive understanding, we compare the phenomenological effective mass with the effective mass measured in other experiments. In the activation energy gap measurement, Du et al \([22]\) gave \( m_{exp}^* = 0.57m_e \) and Manoharan et al gave \( m_{exp}^* = 1.4m_e \) \([20]\). (Here \( m_e \) is the bare mass of the electron.) The Shubnikov-de Haas effective provides \( m_{exp}^* \approx 0.7m_e \) \([23]\) and even a much larger value (divergence) \([25]\) by Du et al while Leadley et al gave \( m_{exp}^* = 0.51m_e \) \([24]\). Our phenomenological results are
\[ m_{(A)}^* = 0.91m_e, \quad m_{(B)}^* = 0.74m_e, \quad m_{(C)}^* = 0.99m_e, \]
\[ m_{(D)}^* = 1.34m_e, \quad m_{(E)}^* = 3.38m_e \quad (5.7) \]
Examining \((5.6)\), the phenomenological fit of the effective mass seems to be ‘heavier’ in several times than theoretical one. There may be several possible explanations to this result:

(1). It is possible that there are still some important diagrams that are not taken into account to approximate \( K \) and are expected to further improve our calculation.

(2). The higher order terms are neglected in the calculation of \( K^0 \) and \( K^{nr} \), which may give a further adjustment for the effective mass. For example, \( O(\omega^2/k_F^2) \) in \([1,15]\) decreases \( K^{nr} \) such that \( m^* \) decreases in order to keep \( C = 1 \).

(3). The larger effective mass at \( \nu = 1/2 \) coincides with the increase (even divergence) of the effective mass at this filling fraction.

(4). The theoretical value of the effective mass should be, in fact, larger than Hartree-Fock one because the gauge fluctuations always enlarge the effective mass. The experimental measurement of the effective mass supports this point because all \( m_{exp}^* \) we quoted are larger than \( m_{th}^* \) in a ratio \( m_{exp}^*/m_{th}^* \geq 1.58 \).

On the other hand, the comparison between the phenomenological effective mass showed in \((5.7)\) and the experimental values is fairly well although \( m^* \) is little bit larger than \( m_{exp}^* \). However, \( m_{(E)}^* \) seems to be extraordinarily large. The reason for this remains unknown.

**VI. CONCLUSIONS**

In conclusions, we have shown a possibility to resolve the discrepancy between theory and experiment in the longitudinal conductivity dependent on the wavevector of the SAW propagating above the 2DEG. The key point is to take a non-RPA correction to the response function from the self-interaction among the gauge fluctuations into account. This correction increases the theoretical value of the longitudinal conductivity and gives an adjustable parameter which relates the CF effective mass to the conductivity. Thus, the phenomenological effective mass of CF is estimated by using the data in the SAW propagation experiments and one found that it is fairly consistent with the effective mass measured in the experiments relating to the activation energy and the Shubnikov-de Haas oscillations. Although the phenomenological value of the CF effective mass is several times larger than the theoretical one obtained by a Hartree-Fock approximation, we explained the possible sources to cause this.

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**FIGURE CAPTIONS**

Fig. 1: One-loop diagram of fluctuations from the self interaction of the gauge field, where the gauge propagator is taken to be the RPA one.

Fig. 2: (a) The free CF propagator; (b) The bare gauge propagator; (c) The CF-gauge fluctuation interaction vertex; (d) The self-interaction vertex of the gauge fluctuations.

Fig. 3: The non-interaction CF response function.

Fig. 4: The bare non-RPA response function.

Fig. 5: The RPA response function.