Biases on the cosmological parameters and thermal Sunyaev–Zel’dovich residuals

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ABSTRACT
We examine the biases induced on cosmological parameters when the presence of secondary anisotropies is not taken into account in Cosmic Microwave Background analyses. We first develop an exact analytical expression for computing the biases on parameters when any additive signal is neglected in the analysis. We then apply it in the context of the forthcoming Planck experiment. For illustration, we investigate the effect of the sole residual thermal Sunyaev–Zel’dovich signal that remains after cluster extraction. We find in particular that analyses neglecting the presence of this contribution introduce on the cosmological parameters $n_s$ and $\tau$ biases, at least ~ 6.5 and 2.9 times their one $\sigma$ confidence intervals. The $\Omega_b$ parameter is also biased to a lesser extent.

Key words: methods: statistical – galaxies: clusters: general – cosmic microwave background – cosmological parameters – cosmology: theory.

1 INTRODUCTION
Future Cosmic Microwave Background (CMB) experiments, which are designed to be cosmic variance limited, will allow us to determine the cosmological parameters with a relative precision of the order of, or better than, one percent. It will be made possible in particular through the measurement of CMB temperature and polarisation anisotropies with unequaled sensitivities, exquisite angular resolution and optimal frequency coverage. In this context, additional contributions to the signal (galaxies, point sources, secondaries arising from the interaction of CMB photons with matter after decoupling, etc.) can no more be neglected. More specifically, a precise quantification of the biases on the parameters and of their sources is now needed.

The study of biases in cosmology is receiving growing attention. In the context of weak lensing, Amara & Réfrégier (2008a) have derived a method based on a Fisher matrix type analysis for quantifying systematic biases. In CMB analyses, Miller, Shimon & Keating (2008) examined the biases introduced by beam systematics for five upcoming experiments that will measure the B-mode polarisation (Planck, PolarBear, Spider, Q/U Imaging Experiment (QUIET)+Clover and CMBPol). Similarly, using Planck characteristics, previous studies have estimated the biases induced by the contribution from patchy reionisation (Zahn et al., 2005). They concluded that the biases, depending on the model of reionisation, can be as high as a few in units of the one sigma error. More recently, Serra et al. (2008) have focused on the contribution from clustered IR point sources and its effect on the cosmological parameters. Those two studies used quite different approaches. While the Serra et al. (2008) analysis was based on Monte Carlo Markov Chains (MCMC), Zahn et al. used an approximate analytic computation of the biases (see also Hutner et al. 2006).

In this study, we present an analytical derivation of the biases on the cosmological parameters that goes beyond the aforementioned approximations. It is a method valid when the primary signal and secondary contribution (astrophysical or systematic) are additive and it is exact as it can be applied even when the secondary signal is dominant over the primary. The method presented here is applied to the estimate of the cosmological parameters with the CMB power spectrum. We furthermore focus on one single source of additional anisotropies: those associated with undetected clusters. The Sunyaev–Zel’dovich (SZ) effect of galaxy clusters (Sunyaev & Zel’dovich, 1972) is indeed the major source of secondary temperature anisotropies (Aghanim, Majumdar & Silk, 2008, and references therein). The SZ effect is two-fold: the thermal SZ due to the inverse Compton scattering of photons off the hot electrons in the intra-cluster gas (e.g. Rephaeli, 1995; Itoh, Kohyama & Nozawa, 1998), and the kinetic effect due to the Doppler shift caused by the proper motion of the clusters in the CMB reference frame. The upcoming large multi-frequency surveys will be able to detect and extract galaxy clusters using their specific thermal SZ spectral signature. We nevertheless expect some level of residual SZ contribution from undetected clusters in the temperature anisotropy maps. Such a residual signal might be the cause of the excess of power measured by small scale CMB experiments like BIMA (Berkeley Illinois Maryland Association Array), CBI, ACBAR (Dawson et al. 2002; Pearson 2002; Kuo 2004; Readhead 2004). The excess could be also due to unremoved
point sources (Toffolatti et al. 2005; Douspis, Aghanim & Langer 2006).

Our article is organised as follows, we present in Section 2 the method to calculate the biases on cosmological parameters. We then apply our method to estimate the biases induced by the SZ residuals. We present our results in Section 3 and discuss them in the following section. Finally, we conclude in Section 5. Throughout the article, we assume that the SZ signal is made of primary only (Case 2), one obtains a set of biases on cosmological parameters is the sum of a primary signal \[ C_{\ell}^{\text{add}} \] that may, or may not, contain cosmological information. If such an additional signal is taken into account in the parameter estimation analysis (Case 1) then one recovers the “true” cosmological parameters, \( \hat{\theta} \). If, on the contrary, one uses the total signal but assumes that it is made of primary only (Case 2), one obtains a set of biased cosmological parameters, \( \hat{\theta} \). In the following, we derive an analytical expression of this bias, \( \hat{b} = \hat{\theta} - \theta \).

We consider that the errors associated with the “data” \( C^P \) are distributed according to a Gaussian law. The likelihood function is then written as \( \mathcal{L} = e^{-\chi_1^2/2} \). For Case 1, the \( \chi_1^2 \) is:

\[
\chi_1^2(\hat{\theta}) = \sum_{\ell \neq X} \text{cov}_{\ell X}^{-1}(C^{CMB}_{\ell \ell}) \left[ C_{\ell}^{\text{DX}} - C_{\ell}^{\text{mod}}(\hat{\theta}) \right] \times \left[ C_{\ell}^{\text{DX}} - C_{\ell}^{\text{mod}}(\hat{\theta}) \right]
\]

where \( C_{\ell}^{\text{mod}} = C_{\ell}^{\text{CMB}} + C_{\ell}^{\text{add}} \). For Case 2, the \( \chi_2^2 \) is given by:

\[
\chi_2^2(\hat{\theta}) = \sum_{\ell \neq X} \text{cov}_{\ell X}^{-1}(C^{CMB}_{\ell \ell}) \left[ C_{\ell}^{\text{DX}} - C_{\ell}^{\text{mod}}(\hat{\theta}) \right] \times \left[ C_{\ell}^{\text{DX}} - C_{\ell}^{\text{mod}}(\hat{\theta}) \right]
\]

where \( C_{\ell}^{\text{mod}} = C_{\ell}^{\text{CMB}} \) only, and \( X, Y = \text{TT}, \text{EE}, \text{TE} \). We consider the temperature and E-mode auto-correlations, TT and EE, and the cross-correlation, TE, for which the coefficients of the covariance matrix are \( \text{cov}(C^{CMB}_{\ell \ell}) \) (see, e.g. Zaldarriaga & Seljak 1997).

Assuming the "data" represent the sum of primordial and additional signals, \( C^P = C^{\text{CMB}}(\hat{\theta}) + C^{\text{add}} \), the set of parameters \( \hat{\theta} \) minimises \( \chi_2^2 \) and the parameter set \( \theta \) minimises \( \chi_1^2 \):

\[
\forall \ell \frac{\partial (\chi_1^2)}{\partial \theta_i} \bigg|_{\hat{\theta}} = \frac{\partial (\chi_2^2)}{\partial \theta_i} \bigg|_{\hat{\theta}} = 0.
\]

An ensemble average of a second order approximation of equation (3) gives:

\[
\langle \chi_2^2(\hat{\theta}) \rangle = \langle \chi_2^2(\hat{\theta}) \rangle + \sum_{\ell} b_j \left[ \frac{\partial^2 (\chi_2^2)}{\partial \theta_j \partial \theta_i} \bigg|_{\hat{\theta}} \right] + \frac{1}{2} \sum_{ij} b_i b_j \left[ \frac{\partial^2 (\chi_2^2)}{\partial \theta_i \partial \theta_j} \bigg|_{\hat{\theta}} \right].
\]

Using equations (3) and (4), we obtain:

\[
\forall \ell \left[ \frac{\partial (\chi_1^2)}{\partial \theta_i} \bigg|_{\hat{\theta}} \right] = - \sum_{j} b_j \left[ \frac{\partial^2 (\chi_2^2)}{\partial \theta_i \partial \theta_j} \bigg|_{\hat{\theta}} \right]
\]

that can be written in the compact form

\[
V = G \hat{b}
\]

wherefrom the bias vector follows simply,

\[
\hat{b} = G^{-1} V.
\]

From equation (2), in the context of CMB angular power spectra measurements,

\[
G_{ij} = \sum_{\ell, X} \text{cov}_{\ell X}^{-1}(C^{CMB}_{\ell \ell}) \left[ \frac{\partial C^{\text{mod}}_{\ell \ell}}{\partial \theta_i} \bigg|_{\hat{\theta}} \right] \left[ \frac{\partial C^{\text{mod}}_{\ell \ell}}{\partial \theta_j} \bigg|_{\hat{\theta}} \right] + \frac{\partial C^{\text{mod}}_{\ell \ell}}{\partial \theta_i} \bigg|_{\hat{\theta}} \frac{\partial C^{\text{mod}}_{\ell \ell}}{\partial \theta_j} \bigg|_{\hat{\theta}} - C_{\ell}^{\text{add}} \frac{\partial^2 C^{\text{mod}}_{\ell \ell}}{\partial \theta_i \partial \theta_j} \bigg|_{\hat{\theta}} - C_{\ell}^{\text{add}} \frac{\partial^2 C^{\text{mod}}_{\ell \ell}}{\partial \theta_i \partial \theta_j} \bigg|_{\hat{\theta}} \bigg|_{\hat{\theta}} \bigg|_{\hat{\theta}} \bigg|_{\hat{\theta}}
\]

and

\[
V_i = \sum_{\ell, X} \text{cov}_{\ell X}^{-1}(C^{CMB}_{\ell \ell}) \left[ C_{\ell}^{\text{mod}} + \frac{\partial C^{\text{mod}}_{\ell \ell}}{\partial \theta_i} \bigg|_{\hat{\theta}} \right] + C_{\ell}^{\text{add}} \frac{\partial C^{\text{mod}}_{\ell \ell}}{\partial \theta_i} \bigg|_{\hat{\theta}}.
\]

Equation (5) thus allows us to calculate the bias on a parameter \( \theta_i \) as a function of the additional signal and of the first and second derivatives of the primary signal. The computational advantage is that any additive contribution can be readily inserted without the need of a re-computation of the derivatives of the primary signal.

In the case of CMB (temperature and polarisation) observations, when an additional signal is ignored, the biases induced on cosmological parameters are obtained from equations (7–9). The derived formula (equation 5) can, however, be used in many other cases (matter power spectrum, weak lensing power spectrum, etc.), as it accounts quite generally for additive contributions or systematics. Furthermore, it is the most general expression for the biases (for additive signals) as it applies to cases where the secondary signal is dominant over the primary. In contrast, previous studies (e.g. Zahn et al. 2005; Huterer et al. 2006) of the bias on cosmological parameters used approximations applicable only when the additional signal is negligible with respect to the primary signal.

The biases on the investigated parameters become relevant only if they are larger than the expected confidence intervals. The latter can be computed through a Fisher matrix analysis. The 68.3 per cent confidence interval on one parameter (the others being known) is given by \( \delta \theta_j = \sqrt{F_{jj}^{-1}} \).

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where the matrix coefficients are:

\[ F_{ij} = \sum_i \sum_x \sum_x \text{cov}_{i,x}^{-1}(C_{i,x}^X C_{i,x}^Y \frac{\partial C_{i,x}^X}{\partial \theta_i} \frac{\partial C_{i,x}^Y}{\partial \theta_j}) \]  

(11)

The numerical values for the instrumental noise and the beam used to compute the covariance matrix can be found in the *The Scientific Programme of Planck* (also known as the *Planck Blue Book*, *The Planck Collaboration* 2006).

### 3 COMPUTATION OF THE THERMAL SZ RESIDUAL

The *Planck* satellite will measure CMB anisotropies with an unprecedented precision, in temperature and polarisation, over the full sky and from the largest scales down to five arcminutes. Foregrounds and secondary anisotropies are expected to contribute to the signal. Taking advantage of the multi-frequency observation, cleaning algorithms are developed to disentangle the primary signal from contaminants. On small scales, the SZ effect from galaxy clusters will be one of the major secondary contributions to the signal. Fortunately, the characteristic spectral signature of the thermal SZ effect makes it easily detectable. On the one hand, one can remove some of the thermal SZ signal from the CMB maps in order to recover the best primary CMB angular power spectrum, and on the other hand a cluster catalogue can be built (e.g. Schaefer et al. 2006). Nevertheless, some level of residual, unresolved SZ signal will remain in the temperature maps. We thus apply the method described in the previous section to estimate the bias induced by that residual signal in the CMB maps on the six ‘standard’ cosmological parameters.

We now compute the additional contribution to the CMB signal that enters equation (7). Here, it is simply the power spectrum of the residual SZ signal that remains after cluster extraction. We focus on the Poisson contribution to the SZ angular power spectrum following Komatsu & Seljak (2002) and assume that the contribution from correlated halos is negligible (valid for \( \ell > 300 \), Komatsu & Kitayama 1999):

\[ C_\ell = f^2(x) \int_0^{r_{\text{max}}} dz dV_a \sum_{M_{\text{min}}}^{r_{\text{max}}} dM \frac{dn(M,z)}{dM} |\tilde{y}_\ell(M,z)|^2, \]  

(12)

where \( \frac{dV_a}{dz} \) is the comoving volume per unit redshift and solid angle and \( n(M,z) dM \frac{dn}{dM} \) is the probability of having a galaxy cluster of mass \( M \) at a redshift \( z \) in the direction \( d\Omega \). In the present study, we use the Sheth & Tormen (1999) mass function \( n(M,z) \).

The SZ frequency dependence is encoded in the function \( f(x) = \frac{d\nu}{dz} \), where \( x = \frac{\nu}{T_e} \) is the dimensionless frequency, and \( \tilde{y}_\ell = \tilde{y}(M,z) \) is the two-dimensional Fourier transform on the sphere of the 3D radial profile of the Compton \( \gamma \)-parameter of individual clusters,

\[ \tilde{y}_\ell = \frac{4\pi}{D_A} \int_0^{r_{\text{max}}} d\gamma(y_\ell(r/D_A) \sin(r/D_A) r^2 dr, \]  

(13)

with \( y_\ell(r) = \frac{\sigma_T}{2\pi} \frac{\beta}{\sigma_b} n_b(r) \) and \( D_A = D_A(z) \) the proper angular-distance diameter. To model the SZ signal from individual clusters, we assume the electron density \( n_e(r) \) within the cluster virial radius \( r_{\text{vir}} \) follows a spherical isothermal \( \beta \)-profile (Cavaliere & Fusco-Femiano 1978) with core radius \( r_c = 0.1 r_{\text{vir}} \) and \( \beta = 2/3 \) for all clusters for simplicity. The isothermal temperature of the intra-cluster gas \( T_e \) is set equal to the virial temperature.

In order to evaluate equation (12), we choose \( z_{\text{max}} = 7 \) and \( M_{\text{min}} = 10^{13} M_\odot \). The angular power spectrum of the residual SZ is the contribution from undetected clusters. It is computed by setting \( M_{\text{max}} = M_{\text{lim}}(z) \), the cluster detection limit. The selection function determines the limiting mass \( M_{\text{lim}}(z) \) as a function of redshift for a cluster to be detected by a given instrument characterised by its beam, sensitivity and frequency coverage. We compute the selection function similarly to Bartelmann (2001). A galaxy cluster is detected if at the same time its beam-convolved Compton parameter \( \tilde{y}(\theta) \) emerges from the confusion noise, and if its integrated signal is above the instrument sensitivity. This translates into the following condition on the flux variation:

\[ \Delta F(\theta) = g(x) I_0 \int \tilde{y}(\theta) d\Omega \geq \lambda g(x) I_0 \tilde{y}_{\text{lim}}, \]  

(14)

where \( g(x) = \frac{\alpha^2 n_b(r)}{\sigma_b n_b(r)} \) and \( I_0 = \sqrt{\frac{\nu_0}{T_{\text{map}}} \nu^3} \). The integral is calculated over the lines of sight for which \( \tilde{y}(\theta) \geq 3 \Delta y_{\text{bg}} \), and the sensitivity limit \( F_{\text{lim}} \) is derived from the antenna temperature sensitivity of the instrument.

The background SZ signal responsible for the confusion is computed assuming a Poisson distribution of clusters with masses between \( 10^{13} M_\odot \) and \( 5 \times 10^{16} M_\odot \):

\[ \Delta y_{\text{bg}} = \sqrt{\int \frac{dV_a}{d\Omega} dM n(M,z) \left( \int d\Omega \tilde{y}(\theta, M,z) \right)^2}. \]  

Finally, the flux in equation (14) is integrated over a top hat function to account for the frequency response of the instrument, and the integer \( \lambda \) is the detection threshold in terms of the instrumental noise \( \sigma_f \).

In order to compute the SZ medium temperature power spectrum that enters equations (7) and (6), we derive an idealised cluster selection function for *Planck* (not accounting for the effects of foregrounds and point sources) considering only the channels relevant for the SZ measurement, namely 100, 143 and 353 GHz (217 GHz is the frequency for which the thermal SZ signal is null). We consider that an SZ cluster is detected only if the selection criterion (4) is satisfied in all three frequency channels simultaneously.

### 4 RESULTS

We now illustrate the application of our formalism by taking the future *Planck* satellite as an example of high sensitivity high resolution CMB experiment. The instrumental characteristics that we used can be found in the *Planck Blue Book* (The *Planck Collaboration* 2006). We quantify simultaneously the biases induced on the cosmological parameters \( \Omega_{\Lambda}, \Omega_0, H_0, n_s \) and \( \tau \) by the residual SZ contribution at 100 GHz. As a reference model, we take the cosmological parameters obtained by the WMAP team (Dunkley 2003) and the corresponding \( C^{\text{CMB}}_\ell + C^{\text{res}}_\ell \), where \( C^{\text{res}}_\ell \) is the residual SZ signal.

Undetected clusters contribute as a residual signal at high multipoles where they dominate over the primordial CMB for \( \ell \) higher than 2000 approximately (Figure 4, left panel). At \( \ell > 1000 \), we show on the right panel of figure 4 that the residual SZ contribution after extracting clusters above \( 3\sigma_r \) already represents more than 10 per cent of the total signal and exceeds the instrumental noise.

In equations (7) and (6), we do not consider polarised residuals at high \( \ell \) since the polarisation induced by galaxy clusters is negligible compared to the primary CMB polarisation at those scales (e.g. Liu da Silva & Aghanim 2005). Therefore

\[ C^{\text{res}}_\ell(\hat{\theta}) = \begin{cases} C^{\text{CMB}}_\ell(\hat{\theta}) & \text{if } X = \text{TT} \\ 0 & \text{otherwise} \end{cases} \]  

(16)

We show the induced biases in figure 2. The black solid lines...
Figure 1. Left-hand panel: At 100 GHz, primary CMB (solid line), SZ angular power spectrum of the whole cluster population (long-dashed green line), residual SZ spectrum after the extraction of clusters above $3\sigma_Y$ simultaneously at 100, 143 and 353 GHz (dot-dashed red line), primordial CMB + residual SZ spectrum (dashed blue line). The black dotted lines represent the $1\sigma$ error bars for the 100 GHz Planck channel. Right-hand panel: Contribution of the residual SZ power to the total signal after $3\sigma_Y$ cluster detections (solid black line), $1\sigma_Y$ (green dotted line) and $5\sigma_Y$ (dashed red line). The envelop represents the instrumental sensitivity and the cosmic variance.

Figure 2. 68.3 per cent joint confidence regions for $\Omega_\Lambda$, $\Omega_b$, $H_0$, $n_s$, $\sigma_8$ and $\tau$ (solid line) obtained with the expected Planck TT, TE and EE spectra computed for the reference cosmological model. The dotted (red) and long dashed (green) shifted ellipses represent the 68.3 per cent joint confidence regions around the biased parameters if respectively the $1\sigma_Y$ and $5\sigma_Y$ residual SZ signal is not taken into account.

represent the 68.3 per cent joint confidence level ellipses on the parameters $\Omega_\Lambda$, $\Omega_b$, $H_0$, $n_s$, $\sigma_8$ and $\tau$ using the TT, TE and EE power spectra centered on the reference model. As shown in the figure, the expected constraints on cosmological parameters are at a few percent precision. We show, in addition, the biased values of the parameters due to the residual SZ signal. We consider different cluster detection limits (1, 3, and $5\sigma_Y$) and thus obtain various levels of residuals in the signal. The results for all parameters are summarised in Table 1. In figure 2 the dotted red and long dashed green ellipses, shifted with respect to those in solid black, represent
the 68.3 per cent joint confidence regions if the SZ residual signal that is obtained when clusters are extracted above 1 or 5σ is not modelled properly.

First, and unsurprisingly, the biases induced on Ωb and H0 are negligible. At most, i.e. for a large residual contribution, they reach roughly 0.1 and 0.6 in units of the 1σ errors on the parameters. Then, as expected, σb, and to a larger extent nσ, Ωb and τ, are on the contrary significantly affected by the residual SZ signal. Those parameters are indeed, by nature, the most sensitive to an excess of power at small scales; additionally they are degenerate. An excess of power at high ℓ, due to SZ residuals, can be accounted for by over-estimating the value of σb. Since the amplitude of the CMB power spectrum strongly depends on σb, a relatively small variation of σb is enough to fit the power excess. As a result, the bias on this parameter is rather small. The excess of power at small scales decreases the ratio between the amplitudes of the forth and fifth CMB peaks and slightly shifts them towards higher ℓ values (Fig. 3) mimicking the effects induced by an increase of Ωb. We find that the bias is roughly 1.4 and 2.4 in units of the error on Ωb, for the 1 and 5σ cases respectively. Higher Ωb and σb values imply a higher amplitude of the primary CMB spectrum at all multipoles. However, the SZ residual signal contributes significantly to the CMB spectrum only at multipoles higher than 500, leaving lower multipoles almost unaffected. The effects at low multipoles of simultaneously higher Ωb and σb, namely an overall rise of the power, are compensated by an increase of the spectral index ns, which raises (lowers) the power at multipoles larger (smaller) than ~ 1200. They are also compensated by a decrease of the optical depth τ, that reduces the CMB power. As a consequence, fitting data containing primary CMB and SZ secondary residual with a pure primary CMB spectrum induces quite an important bias on ns and τ. The bias on τ is between 2.8 and 6 times the expected precision for cluster detection limits 1 and 5σ, respectively, and the bias on ns is between 6.5 and 10.4 times the expected precision for 1 and 5σ cluster thresholds. All these biases are summarized in Fig. 4 as functions of the SZ cluster detection threshold.

5 DISCUSSION

We show that an unremoved thermal SZ contribution in future high sensitivity high resolution CMB experiments, measuring both temperature and polarisation, introduces excess of power at small angular scales. This in turn induces biases on the cosmological parameters associated with inflation, the spectral index ns and, to a lower extent, the normalisation σb, as well as on the density of baryons Ωb and the optical depth τ. The other two parameters of the standard model remain essentially unaffected.

The amplitude of the biases strongly depends on the amplitude and the detailed shape of the residual power spectrum. In order to estimate the residual thermal SZ power spectrum, we used a theoretical SZ cluster selection function built from the expected capabilities of Planck and not accounting for foregrounds and point sources. The detected clusters are expected to have masses of a few 10^13M⊙ at redshifts 0.4 < z < 0.7. Although, the theoretical selection function does not coincide totally with that obtained from simulations (false detections, confusion with point sources, etc., J.-B. Melin, private communication), it gives us a reasonable estimate of the residual thermal SZ power spectrum used in our analysis.

It is important to bear in mind that other astrophysical contributions need to be taken into account in future CMB experiments observing at small scales. In that respect, we have also estimated the biases induced on the cosmological parameters by the kinetic SZ anisotropies of the whole cluster population. Such a signal cannot be separated out from the primary signal as they both share the same spectral signature. The kinetic SZ signal, although one order of magnitude smaller, will dominate over the thermal SZ at 217 GHz where the latter vanishes. We found that the bias on the spectral index ns is less than 2σ, whereas the other parameters are essentially unaltered by the additional contribution. The channel at 217 GHz could seem more appropriate for the determination of cosmological parameters. However, one needs to consider additionally the contribution from extra-galactic point sources, radio or IR galaxies and their clustering. A recent study by Serra et al. (2008) has extended the work of Douspis, Aghanim & Langer (2006) by focusing on IR galaxy clustering with WMAP and ACBAR data, as well as with Planck. They showed (at 143 GHz) that the expected biases on Ωb, ns and σb are of the order of 2.

We have also applied our method to estimate possible biases...
on the cosmological parameters with WMAP. Due to its frequency coverage ($\nu < 94$ GHz) we can safely consider that the thermal SZ residual signal in the data is constituted of the contribution from all the clusters (none of them being actually detected and extracted). The angular resolution of WMAP is limited to $\ell < 1000$. In this range, the contribution of the SZ signal to the CMB power does not exceed 10 per cent. As a result, although the contamination by thermal SZ is maximum in the WMAP case, we found that the derived cosmological parameters are unbiased.

Given the possibility that large biases could arise due to various systematics, it is important to find a way to reduce their impact. First, one could consider only the multipoles for which the primary contribution to the TT signal is dominant, i.e. typically $\ell < 1000$. This was done by Zahn et al. (2005) who suggested to take into account the full polarisation power spectra but to use the TT power spectrum only out to $\ell \sim 1000$. We have tested such an approach on our issue: the SZ residuals. We find that truncating the measurements above $\ell = 1000$ artificially increases the error-bars on $\Omega_\Lambda$, $\Omega_b$, and $H_0$ by approximately 10 per cent. This effect is even more important for $n_s$ as it increases the error-bars by more than 30 per cent. As expected, the cosmological parameters are then less biased. Nevertheless, $n_s$ remains significantly affected as the bias is still more than 3 times its expected accuracy. The bias on $\sigma_8$ is also significant. Consequently, a more appropriate analysis would rather consist in fitting the CMB data with a coherent model accounting for both the primary and the residual signal with full dependency on the cosmological parameters. For that purpose, one would need to use the cluster selection function, the understanding of which is rather complex (instrumental effects, foregrounds, limits due to the component separation techniques and cluster extraction methods, etc.). In that case, a better solution would be rather to use the total signal including the primary CMB and the secondary anisotropies to determine the cosmological parameters in a coherent way, that is taking into account the full cosmological dependence of the secondary signal. The latter is crucial, as it was shown (Douspis, Aghanim & Langer 2006) that modelling the SZ contribution by a fixed-shape power spectrum with a varying amplitude $A_{SZ}$ is not sufficient and biases the cosmological parameters.

### 6 CONCLUSIONS

In this study, we develop an analytical method to calculate the biases on the cosmological parameters. Our method applies to any contamination provided the primary signal and the contamination are additive. Additionally, it is an exact derivation to the second order with the advantage of being applicable even when the contaminant dominates over the primary signal. The next generation of CMB experiments will measure, with a high sensitivity, the signal at small angular scales where secondary contributions intervene. We apply our method to the case of a contribution from undetected thermal SZ clusters to the primary CMB signal (assuming no contribution from foregrounds or point sources).

For illustration, we take the characteristics (noise, beam size) of Planck and compute the residual SZ signal from the undetected clusters assuming all clusters above 3 or 5$\sigma$ are detected simultaneously in the channels 100, 143 and 353 GHz. The residual SZ signal contributes more than 10 per cent of the total signal at multipoles higher than 1000. The higher the SZ cluster detection threshold, the higher the contamination.

We perform a bias estimation simultaneously on six cosmological parameters ($\Omega_\Lambda$, $\Omega_b$, $H_0$, $n_s$, $\sigma_8$ and $\tau$) using temperature and polarisation anisotropy TT, TE and EE power spectra. This quantifies the effect of fitting the data, that include a residual contribution, with a model that ignores it. We then compare the biases to the expected $1\sigma$ errors on each parameter. We find that the biases induced by the thermal SZ residual signal on $\Omega_\Lambda$ and $H_0$ are negligible. At most, they are of the order of 0.08 and 0.6 in units of the $1\sigma$ error on the parameters. On the contrary, the determination of $\Omega_b$, $n_s$, and $\tau$ is significantly altered by the residual SZ signal. The biases are $2.4, 6.$ and $10.4\sigma$ respectively. This is easily understood as they are the most sensitive parameters to an excess of power at small scales and, moreover, they are degenerate.

We point out the importance of taking into account the SZ residuals in the analysis of the small scale high sensitivity CMB data. The SZ residual power spectrum depends on the cosmological parameters and on the cluster selection function. A joint analysis of primary and secondary CMB signal will provide additional constraints on the cosmological parameters and thus reduce the biases arising from the SZ residual power excess at high multipoles. A coherent analysis, including full cosmological parameters dependency of the primary and secondary signal, allow one to use the whole range of multipoles, including the highest ones.

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