Numerical Simulation of Long Duration Sloshing Using Smoothed Particle Hydrodynamics

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Abstract. The sloshing phenomenon is one of the events in a liquid carrier vehicle such as airplanes and ships. Sloshing is a dangerous phenomenon because sloshing can effect ship motions that create excessive motion in liquid carrier vehicles. Long duration sloshing is challenging problems to solve using smoothed particle hydrodynamics (SPH). Stable, accurate, and reliable computation time is tried to achieve by many researchers. In this study, long-duration sloshing in the prismatic tank is tried to reproduce using single-phase and two-phase SPH. Firstly, the experiment is carried out using prismatic tank, with three pressure sensors, and a forced oscillation machine. In this study, only roll motion is used to reproduce hydrodynamics pressure with a low filling ratio. The results show SPH could reproduce fairly hydrodynamics pressure with spurious pressure oscillation. Static pressure is well reproduced by SPH.

1. Introduction

The sloshing phenomenon is one of the natural events in a liquid carrier vehicle such as airplanes, trucks, and ships. Sloshing can define as the resonance of fluid inside a tank caused by external oscillations. In big quantity of liquid carrier vehicle such as LNG ships, sloshing is a dangerous phenomenon. Because sloshing can effect ship motions that are creating excessive motions that will exist caused by fluid movement inside the tank. When ship carried out volatile fluids for an instance liquefied natural gas (LNG) impact pressure during severe sloshing can lead to explosions. Recently the demand for LNG ship is increasing. It makes sloshing analysis increased significantly for ship design.

There are many studies of sloshing had been done using experiment, empirical, and numerical approaches. Recently, computer technology is increasing dramatically in the decade, it made the numerical approach more desirable. Computational fluid dynamics (CFD) is one of the numerical approaches that mostly used for sloshing. In this study, meshless CFD and Lagrangian scheme so-called smoothed particle hydrodynamics (SPH) is used to overcome sloshing in prismatic tank. SPH was invented for astrophysical problems [1]. It was developed for free surface flows by Monaghan [2] to overcome of the dam break and water waves. It was showed SPH has a promising method to apply in free surface flow. Domínguez et al [3] were showed SPH has good accuracy predicting solitary waves in different method generations. Trimulyono et al [4] were used SPH to generate water waves in large numerical wave tanks (NWT).
Recently, Green and Peiró [5] performed a long duration of sloshing in a rectangular tank using SPH with low filling ratio and high stretching. Green and Peiró did the sloshing simulation using single-phase SPH in 2D. They study were showed SPH has a good agreement to predict wave elevation, pressure, and hydrodynamics force. Trimulyono et al [6] used SPH to make experiment validation of hydrodynamics pressure using a prismatic tank. The sloshing simulation was performed both in 2D and 3D including two-phase simulation. In addition Trimulyono et al [7] used low pass filter to reduce pressure oscillation in pressure field. The present study is carried out long duration sloshing using single-phase and two-phase SPH in the prismatic tank. The dynamic pressure of SPH was validated using the experiment result. The same duration sloshing was made with an experiment in SPH simulation. In this study, open-source SPH solver DualSPHysics version 4.2 is used to carry out sloshing simulation. The multi-phase SPH was developed by Mokos et al [8] that included in DualSPHysics version 4.2. DualSPHysics is a set of C++ code that has implemented parallel computation using general-purpose computing on graphics processing units (GPGPU) technology [9]. All simulations in this study were executed using GPU GTX 1080 ti to accelerate computation time. In this study, SPH was showed fairly accuracy regarding dynamic pressure both in single and two-phase SPH.

2. Methodology

2.1. SPH

SPH is a fully Lagrangian meshless method that adopts an interpolation scheme to approximate the physical values and derivatives of a continuous field using discrete evaluation points. These evaluation points are identified as particles that contain mass, velocity, and position values. These quantities are obtained as weighted averages of adjacent particles within a smoothing length $h$ to reduce the range of contributions from remote particles. The main features of the SPH method, which is based on integral interpolants, are described in detail in references Liu and Liu [10]. In SPH, the field function $A(r)$ in a domain $\Omega$ can be approximated using the integral approximation, therefore the particle approximation shows in equation (1) where $W$ is kernel function and $r$ is a position vector.

$$A(r_a) \approx \sum_b A(r_b) W(r_a - r_b, h) \frac{m_b}{\rho_b}$$  \hspace{1cm} (1)

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v},$$  \hspace{1cm} (2)

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla P + \mathbf{g} + \tau,$$  \hspace{1cm} (3)

$$\frac{dr}{dt} = \mathbf{v},$$  \hspace{1cm} (4)

In this study, Quintic kernel function [11] is used that showed in equation (5). The momentum equation shows in equation (6) for the water-phase and equation (7) for air-phase. The delta-SPH ($\delta_{ab}$) shows in equation (8) based on works of Molteni and Colagrossi [12], and the continuity equation becomes equation (8). The equation of state is shown by equation (9) and equation (10).

$$W(q) = \alpha_p \left(1 - \frac{q}{2}\right)^4 \left(2q + 1\right) \hspace{1cm} 0 \leq q \leq 2$$ \hspace{1cm} (5)

$$\frac{dv_a}{dt} = -\sum_b m_b \left(\frac{P_a + P_b}{\rho_a \rho_b} + \Pi_{ab}\right) \nabla_a W_{ab} + g,$$  \hspace{1cm} (6)

$$\frac{dv_a}{dt} = -\sum_b m_b \left(\frac{P_a + P_b}{\rho_a \rho_b} + \Pi_{ab}\right) \nabla_a W_{ab} - 2\alpha_p \sum_b \frac{m_b}{\rho_b} \nabla_a W_{ab},$$  \hspace{1cm} (7)
where $\Pi_{ab} = \begin{cases} \frac{-\alpha c_{ab} h_{ab}}{r_{ab}} v_{ab} \cdot r_{ab} < 0, \\ 0 \quad v_{ab} \cdot r_{ab} > 0 \end{cases}$

$$\frac{d\rho_a}{dt} = \sum_b m_b v_{ab} \cdot V_a W_{ab} + 2 \delta \phi h c_0 \sum (\rho_b - \rho_a) b \frac{r_{ab} v_a W_{ab}}{v_{ab}} m_b \rho_b$$  \hspace{1cm} (8)

$$P = b \left[ (\frac{\rho}{\rho_a})^\gamma - 1 \right]$$  \hspace{1cm} (9)

$$P = b \left[ (\frac{\rho}{\rho_a})^\gamma - 1 \right] + X - a \rho^2$$  \hspace{1cm} (10)

Figure 1 depicts the sketch of the prismatic tank in 3D and 2D. In this study, only 2D prismatic tank was used because the pressure sensor located in the middle of the tank with regular motion was used in the experiment as a consequence two-dimensional simulation is sufficient to reproduce dynamics pressure. The breadth of the tank is 0.3 m, the height of the tank is 0.21 m, and the water depth is 0.0525 for the filling ratio 25%. Only the pressure sensor located in the bottom (P1) was used to compare with the experiment. In the SPH simulation addition time simulation, two seconds utilize to settle down particle. This additional time also was used to get static pressure and comparison was made with an analytic solution.

2.2. Experiment setup

The experiment was conducted in National Research Institute Fisheries Engineering (NRIFE) using a forced oscillation machine (for the detail information see ref. 6). Figure 2 depicts of experiment condition with a filling ratio 25%, figure 2(a) shows experiment setup in the laboratory. Figure 2(b) shows a prismatic tank with a filling ratio 25%. This can be seen that the pressure sensor in the filling ratio 25% was nearly in the free surface. The pressure here defines as dynamic pressure that has been subtracted with hydrostatic pressure. Figure 3 is the time histories of tank displacement in the experiment. The time histories of tank movement will directly impose in the SPH simulation. Therefore the displacement of a tank in SPH was the same as the experiment. The amplitude of roll was 8.66° with a frequency of oscillation is 1.04 Hz.
Figure 2. Experiment condition with filling ratio 25% (a) overview of experiment (b) tank with filling ratio 25%.

Figure 3. The time history of tank movement in roll motion.

3. Results and discussion
SPH method was invented about two decades ago with the application of the free surface was began in 1994 by Monaghan [2]. Because SPH originally is used for astrophysics which is compressible flow. To accommodate free surface flow, the fluid becomes weakly compressible. It makes density easy to fluctuate caused by the speed of sound that suppressed in SPH. The pressure in SPH will have a spurious pressure oscillation caused by density fluctuation especially for weakly compressible SPH (WCSPH). In this study, we were used DualSPHysics that is based on the WCSPH scheme. Table 1 shows parameter setup that was used in SPH, initial particle distance is 0.8 mm with total particle in single-phase 22,997, and 90,057 for two-phase SPH. Time simulation is 70 seconds that included a transient movement to steady movement. In sloshing simulation speed of sound has a significant impact to pressure as shown in equation (9) and equation (10). Because in this study we utilize two-phase flow SPH, the ratio of the speed of sound of liquid and gas is important. Mokos et al [13] are suggested to keep ratio above 7.5. Coefh is a coefficient smoothing length, and delta-SPH is an
additional term to reduce pressure oscillation. As mentioned in Trimulyono et al [6] that SPH has a good accuracy to predict static pressure showed by figure 4.

**Table 1. Parameters setup**

| Parameters                | Value            |
|---------------------------|------------------|
| Kernel function           | Wendland         |
| Time step algorithm       | Sympletic        |
| Artificial viscosity coeff. α | 0.07            |
| Coef. sound for water & air | 65 & 478        |
| Particle spacing (mm)     | 0.8              |
| Coef. h                   | 0.95             |
| CFL number                | 0.2              |
| Delta-SPH (δ_b)           | 0.1              |
| Simulation time (s)       | 70.0             |

Figure 5 depicts a comparison of dynamic pressure experiments and single-phase SPH. Blueline represents an experiment result and the red line represents SPH results. Figure 5(a) shows the dynamic pressure in 70 seconds. Because by default DualSPHysics is based on WCSPH, as results the pressure has a significant noise. The implementation incompressible SPH (ISPH) developed by Chow et al [14] could remove spurious pressure oscillation. The filtering technique is needed to reduce or perhaps remove the noise using a low pass filter. Figure 5(b), (c) and (d) show dynamic pressure in steady condition with different time simulation. The time simulation was separated at the beginning, middle, and edge. In this figure, it can be seen that dynamic pressure in edge simulation has significant noise compare with other parts. In this study, we use single-precision for numerical setup because double-precision will make time computation longer, especially for two-phase SPH. On the other hand, Green and Peiró use the double-precision and ghost particles in their computation and better results they had. This could be a reason why we get different accuracy compare to them.

![Figure 4. Comparison of static pressure in single-phase (a) and two-phase (b).](image-url)
Figure 5. Comparison of dynamic pressure between experiment and single-phase SPH.

Figure 6 shows a comparison result between experiments with two-phase SPH. Blackline represents two-phase SPH and the blue line represents experiment results. It shows two-phase SPH could reproduce better than single-phase SPH. It could be caused by air-phase in two-phase SPH that effected the fluid compressibility. Two-phase SPH is also could capture phenomena of the tail pressure after the peak pressure in contrary single-phase SPH could not capture this event. Figure 7 depicts the comparison of free surface evolution between SPH and experiment. It can be seen two-phase SPH matches with experiment. Single-phase SPH depicts that fluid particle easy detaches and less run-up during impact with the sidewall of the tank.
Figure 6. Comparison of dynamic pressure between experiment and two-phase SPH.
4. Conclusions
Long duration sloshing is carried out using SPH model both single-phase and two-phase. The present study shows some advanced results for SPH study especially for long duration sloshing using SPH. Stable and accurate results of SPH are tried reproduced for sloshing with a low filling ratio. It is found that SPH can reproduce hydrostatic pressure and dynamic pressure. Dynamic pressure reproduces with long duration sloshing in a low filling ratio using experimental data. Spurious pressure oscillation exists though the tendency of pressure is similar to the experiment. Air-phase in sloshing simulation shows essential to match free surface evolution with experiment. It shows that SPH fairly reproduces the sloshing phenomenon in a prismatic tank.

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