Plastic analysis of steel frame structures using computer modeling

J. M. Djoković, K. M. Veljković and R. R. Nikolić*

*Technical Faculty of Bor, University of Belgrade, Vojske Jugoslavije 12, 19210 Bor, Serbia
a Polytechnic School Kragujevac, Kosovska 8, 34000 Kragujevac, Serbia
b Faculty of Engineering, University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia and
Faculty of Civil Engineering, University of Žilina, Univerzitná 8215/1, 01026 Žilina, Slovakia

Abstract

Plastic analysis is used in design of steel structures composed of various elements such as beams, frames, girders, arches etc. Here it is applied for determining the load-carrying capacity or limiting load at which the structure collapses. In this paper is presented a C# program, which computes the value of the load factor and determines whether the static distribution of the bending moment is safe, using methods of plastic analysis. The program has the capability of generating the possible mechanisms of collapse visually. Basic and combined mechanisms, generated by the program, could be viewed on screen. This is particularly important when choosing the optimal solution, because different mechanisms with the same load factors can be compared. Several examples, presented here, show how easy it is to create the model and perform the analysis. This is not only a simple analysis program, but also a program to enable the designer to create his models and to explore as many alternatives as he wishes.

© 2012 Published by Elsevier Ltd. Selection and review under responsibility of University of Žilina, FCE, Slovakia. Open access under CC BY-NC-ND license.

Keywords: Plastic analysis, Collapse mechanisms, C# program, Plastic hinge concept, Combined mechanisms

1. Introduction

Science and technique developments request increase of economics of design, what is achieved by better use of materials. In special constructions such as aircrafts, ships, etc., weight decreasing with less materials consumption is essential. These requests lead to development of the new methods for analysis of carrying structures, such as the method of plastic analysis.

Analysis based on the linear elasticity theory determines the structure strength and the criterion is the safety degree, related to stress or deformation limit. Meanwhile, the real structure strength is far from this established...
value. Plastic analysis has attempted to determine the load-carrying capacity or limiting load at which the structure collapses. Structure design by the plastic analysis leads to better use of materials and to materials savings with corresponding safety. Plastic analysis is used primarily in the design of steel structures composed of various elements such as beams, frames, girders, arches etc.

2. The basic theorems and methods of plastic analysis

As the first, the notion of the plastic hinge must be defined. Considered is the simple beam with central concentrated load. When bending moment reaches the value of plastic moment $M_p$, an increase in plastic deformation in the middle cross section occurs, what causes a great deal of fiber reduction in the compression zone and fiber extension in the extension zone. This deformation causes relative rotation of the left and of the right hand side from the middle cross section. This relative rotation allowed introduction of the hinge like frame element.

Since elastic deformation from the remaining part of the beam is small, relative to plastic deformation in this zone, the whole carrier acts like a mechanism when bending moment reaches the plastic moment. This mechanism has hinges at the end of each side and one in the middle. Theoretically speaking, appearance of the plastic hinge means the structure collapse. Therefore, in collapse the girder passes into a mechanism.

Theorems of plastic analysis or bound theorems enable obtaining of limits between which lies the true load capacity. There are static and kinematic theorem. The static theorem is based on the static equilibrium of the system. For the statically undetermined system it is possible to draw infinitely many diagrams of the moments that are balanced by the given load.

The kinematic theorem refers to collapse mechanisms of systems where it is possible to transform the system into a mechanism by inserting the plastic hinges. If the real collapse mechanism is known, the limit load is obtained by equalizing the work of external forces at virtual displacements with the work absorbed in plastic hinges. If the real collapse mechanism is not known, the various collapse mechanisms are assumed and the limit loads are determined for each of them. So, if all the possible collapse mechanisms are presented, the real limit value will be one which is the least of all the determined values of loads.

By application of the static and kinematic theorem one obtains the lower and the upper load limits, between the two will be the real load.

The assumed collapse mechanism will be the real collapse mechanism if the bending moment at no cross-section of the system is greater than the moment carrying capacity of that cross-section. For all the other collapse mechanisms this value will be exceeded either at one or at all the cross sections, what is not permissible in the structure.

The essence of the plastic analysis methods lie in the fact that for any frame structure \( n = N - R \) independent collapse mechanisms can be formed, with the corresponding collapse load factors; \( N \) is the number of possible plastic hinges, while \( R \) is the number of static variables. By superposition of the basic mechanisms one comes up with the combined mechanisms that can have lower values of the collapse load factor. The mechanism with the lowest value of the collapse load factor is the actual mechanism of collapse.

The collapse load factor is obtained by the virtual work theorem. Rotations and displacements are considered to be virtual and the internal and external virtual works are computed. The collapse load factor for a specific mechanism is the ratio of these two:

$$\lambda_c = \frac{\text{external virtual work}}{\text{internal virtual work}}$$  \hspace{1cm} (1)
The external virtual work is calculated by multiplying the joint forces by the joint displacements, while the internal virtual work is calculated by multiplying the rotations at the plastic hinges by the plastic moments of members in which the plastic hinges form.

Since the theoretical fundamentals of the plastic analysis of the frame structures are given in [2-4], here is presented only the program for plastic analysis of frame structures and the solution obtained by this program is illustrated on few examples.

3. Program for plastic analysis of structures

In this section the solving of the collapse mechanism is determined by use of the PLASTIC ANALYSIS OF PLANAR FRAME program, written in the C# programming language. The part of the code is shown below:

```csharp
private void rtb3F2F_CheckedChanged(object sender, EventArgs e)
{
    try
    {
        double Mp = double.Parse(tbMp.Text);
        double F = double.Parse(tbF.Text);
        double teta = double.Parse(tbt.Text);
        double l = double.Parse(tbl.Text);
        double Wi = 6 * Mp * teta;
        double We = (3 * F * l / 2 * teta) + (2 * F * l * teta);
        double A3 = Variable.lambda3 = Wi / We;
        tbWi.Text = Wi.ToString("0.000");
        tbWe.Text = We.ToString("0.000");
        tblambda.Text = A3.ToString("0.000");
        Graphics f = ptbFrame.CreateGraphics();
        ptbFrame.Refresh();
        BeamSingle myBeam = new BeamSingle();
        myBeam.SetColors(Color.Red);
        myBeam.SetLength(250);
        myBeam.SetThickness(5);
        myBeam.SetColors(Color.Red);
        myBeam.SingleBeamDash(f);
        Arrows arw = new Arrows();
        arw.SetColors(Color.Black);
        arw.SetThickness(2);
        arw.SetCoordinates(230, 97, 230, 40);
        arw.FrameArrowsV(f);
        arw.SetCoordinates(0, 97, 27, 97);
        arw.FrameArrowsH(f);
        Dots dot = new Dots();
        dot.SetColor(Color.Red);
        dot.SetCoordinates(63, 100);
        dot.DrawDots(f);
        dot.SetCoordinates(457, 100);
        dot.DrawDots(f);
        dot.SetCoordinates(257, 180);
        dot.DrawDots(f);
        dot.SetCoordinates(33, 350);
        dot.DrawDots(f);
        dot.SetCoordinates(427, 350);
        dot.DrawDots(f);
        myBeam.SetColors(Color.Black);
        myBeam.SetCoordinates(33, 350, 63, 100);
        myBeam.SetColors(Color.Red);
        myBeam.SingleBeamVH(f);
        VerticalForce verforce = new VerticalForce();
        verforce.SetColors(Color.Black);
        verforce.SetThickness(5);
        verforce.SetCoordinates(63, 100, 257, 180, 457, 100);
    }
    catch (Exception e)
    {
        MessageBox.Show(e.Message, "Error");
    }
}
```
This program application is illustrated on following two examples. First, the basic collapse mechanisms are created and their load factors are determined. The next step is to combine the basic mechanisms in order to obtain a mechanism with the lowest load. By clicking on the button "The true collapse mechanism", a mechanism is obtained with the lowest load factor, which also represents the real collapse mechanism.

**Example 1**: A beam, shown in Figure 1 is considered. Its configuration and loading are as shown in the figure. The beam has two independent collapse mechanisms which are shown with the corresponding collapse load factors. The true collapse mechanism is presented also.
Example 2: A frame is considered as shown in Figure 2, with presented configuration and loading. The frame has seven independent collapse mechanisms which are also shown with the corresponding collapse load factors. The last figure presented is the true collapse mechanism for this frame.
4. Conclusions

Presented C# program computes the load factor of planar frames and determines whether the static distribution of the bending moment is safe, using methods of plastic analysis. The program has the capability of generating the frames visually and observing the results by simple clicks on appropriate button. Basic and combined collapse mechanisms could be viewed on screen. This is particularly important when choosing the optimal solution, because different mechanisms with the same load factors can be compared. Several examples, presented here, show how easy it is to create the model and perform the analysis. This is not only a simple analysis program, but also a program to enable the designer to create his models and to explore as many alternatives as he wishes.

Acknowledgement

This research was partially supported by the Ministry of Education and Science of Republic of Serbia through Grants ON174001 "Dynamics of hybrid systems with complex structures. Mechanics of materials", ON174004 "Micromechanics criteria of damage and fracture" and TR 32036 "Development of software for solving the coupled multi-physical problems" and realized while Mrs. Ružica R. Nikolić was on the SAIA grant of the Slovak Republic government at University of Žilina, Slovakia.

References

[1] Mendelson A.: Plasticity: Theory and Application, The Macmillan Company New York, 1968.
[2] Nikolić R. and J. Veljković.: Plastic Analysis of Frame Steel and Coupled Structures, 3rd International Conference "Concrete and Concrete Structures", Žilina, Slovakia, 2002, pp. 405-410.
[3] Nikolić R. and J. Veljković.: Some aspects of limiting design, 11th International Scientific Conference: "Science, Education and Society", Žilina, Slovakia, Section 1, 2003, pp. 311-315.
[4] Nikolić R. and J. Veljković.: Some aspects of plastic analysis of steel frames and coupled structures, 14th International Symposium EURNEX, Žilina, Slovakia, 2006, Part 1, pp. 65-68.