Stueckelberg Bosons as an Ultralight Dark Matter Candidate

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Abstract

In this letter, we propose a new model of fuzzy dark matter based on Stueckelberg theory. Dark matter is treated as a Bose-Einstein condensate of Stueckelberg particles and the resulting cosmological effects are analyzed. Fits are understood for the density and halo sizes of such particles and comparison with existing models is made. Certain attractive properties of the model are demonstrated and lines for future work are laid out.

Dark matter is a hypothesized form of invisible matter that is used to explain the anomalous rotation curves of galaxies. Several models (see [1, 2, 3, 4, 5, 6, 7] and references therein) have been suggested, including WIMPS, neutrino candidates, supersymmetric particles and axions. Fuzzy dark matter [4, 5, 6] however is a model which suggests that dark matter is composed of ultralight particles having Compton wavelength on cosmological scales, with mass comparable in order of magnitude to $10^{-22}eV$.

Phenomenologically, this model presents attractive features in the intra-galactic domain, and much of this is described in detail in [6, 8, 9]. Such an ultralight dark matter proposal requires a reasonable candidate of mass $\sim 10^{-22}eV$. Conventional discussions of fuzzy dark matter involve introducing new particles not included in the Standard Model. For example, the QCD axion is one such candidate, which presents a problem in terms of its high mass. Dark photons are another candidate, again outside the scope of
the Standard Model. However, as given Reviews in Particle Physics PDG [1], there is an essentially unique candidate viz. the photon. Unfortunately, the photon interacts electromagnetically with matter, ruling it out as a suitable candidate. The massive photon however has spin 1, thus having three degrees of freedom. A mass term would normally break gauge invariance, which is also not attractive. Stueckelberg [10] added another scalar particle to maintain gauge invariance while supplying the photon a mass. The question of photon mass was considered by Schrödinger with the question ‘Must the photon be massless?’ [11], who gave a negative answer as well as an upper bound of $10^{-16} \text{eV}$, which is determined based on the geomagnetic field. This has been improved to $10^{-18} \text{eV}$ as pointed out by Goldhaber et al. [1] using solar magnetic field upto Pluto’s orbit.

In this letter, we examine the scalar component of the massive photon as a possible dark matter candidate, as it exhibits the necessary properties. The low mass of the particle results in a high critical temperature, enabling Bose-Einstein condensation at low energies. Furthermore, in [12], a previous work by the authors, it was shown that at low energies or in length scales of galaxies, these particles only interact gravitationally. These properties make this particle an ideal dark matter candidate. In the following sections, some of the cosmological implications of this model are investigated.

1 Stueckelberg Theory and Fuzzy Dark Matter

Stueckelberg theory is the model obtained when $U(1)$ gauge QED supplemented with an Abelian Higgs mechanism undergoes spontaneous symmetry breaking. The Stueckelberg Lagrangian makes reference to two types of fields, the photon $A_\mu$ and the Stueckelberg field $\phi$ and is given by,

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_\gamma^2 \left( A_\mu - \frac{1}{m_\gamma} \partial_\mu \phi \right) \left( A^\mu - \frac{1}{m_\gamma} \partial^\mu \phi \right).$$

Here, the photon mass has been indicated by $m_\gamma$. Manifest gauge invariance is exhibited by this Lagrangian under the transformations $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$ and $\phi \rightarrow \phi + m_\gamma \lambda$. One point which merits notice here is the fact that the formalism so described is true for pure QED, while the real world is described by the Weinberg Salam model. Two resolutions are possible. One approach is to first break the electroweak symmetry and then
add the Stueckelberg mechanism, which is an unattractive option. The second possibility involves generalizing the Stueckelberg mechanism to operate alongside the Higgs mechanism, which is described in [13]. This approach leads to the hypercharge field $B_{\mu}$ developing a small mass. However, it can be shown that after spontaneous symmetry breaking of electroweak symmetry, the same small mass is acquired by the photon.

Experimentally as pointed out earlier, using the Sun’s magnetic field, the mass of the photon is strongly constrained with by upper bound of $10^{-18}$eV (see [1, 14]). In addition to this fact, as we noted in [12], the Stueckelberg field $\phi$ decouples at low energies from physical processes and only exhibits gravitational interactions.

These observations suggest that the following proposal may be made regarding the structure of dark matter. In the fuzzy dark matter picture, we may treat the constituent particles as these Stueckelberg particles which do not interact. Furthermore, this particle will be such a candidate only if they form a Bose-Einstein condensate. This latter suggestion stems from the fact that the formula for the critical temperature of a collection of such bosons having density $\rho$ will be given by,

$$T_c = \frac{\hbar c}{k_B} \left( \frac{\rho \pi^2}{m_{\gamma} \zeta(3)} \right)^{1/3}.$$  \hspace{1cm} (2)

The small mass of the Stueckelberg particles implies a critical temperature large enough to accommodate Bose-Einstein condensation during any epoch of interest.

To obtain bounds on the critical temperature, input regarding the density needs to be supplied. One has to consider the change in density effected as a consequence of the Friedman expansion. Let the dark matter condensate be described by an initial density of $\rho_0$ at the time of decoupling, before the radiation dominated era begins. The epochs that follow are,
Figure 1: Epochs and Scale Factors

Electroweak symmetry breaking is between $10^{-12}$ to $1s$ with temperatures around $10^{20}K$.

Using these, the time evolution into the current density $\rho_{\text{final}}$ is carried out,

$$\rho_{\text{final}}^{\frac{1}{3}} = \rho_0^{\frac{1}{3}} \frac{1}{(1.2 \times 10^{12})^{\frac{2}{3}}} \left( \frac{47000}{9.8 \times 10^9} \right)^{\frac{2}{3}} \frac{1}{1.377}. \quad (3)$$

Employing this in (2), we obtain the following rough relation between the observed dark matter density $\rho$ and the critical temperature required to achieve Bose-Einstein condensation,

$$\rho \sim 10^{-22} m_\gamma T_c^3 \quad (4)$$

which we have expressed as an order of magnitude relation. In SI units, the observed dark matter density is $10^{-22} kg/m^3$ or 1 proton/cc is recovered if we take $m_\gamma \sim 10^{-19} eV$ and $T_c \sim 10^{17} K$. The corresponding estimate for $m_\gamma = 10^{-22} eV$ would be $T_c \sim 10^{19} K$.

This is a naive estimate based on assuming that the average distribution of dark matter is substantially composed of a fuzzy component made up of Stueckelberg particles.

Further motivating this point of view is the annihilation properties of dark matter. Due to the nature of its coupling, dark matter will have interaction terms with conserved currents of the form $j^\mu \partial_\mu \phi$ and with the graviton of the form $h^{\mu \nu} \partial_\mu \phi \partial_\nu \phi$. The former will not contribute non trivial annihilation channels due to current conservation. Consequently, only the graviton will supply an annihilation channel that will lead to a differential cross section that is non vanishing.
The annihilation of two Stueckelberg particles into photons may be studied using an order of magnitude estimate. Schematically, the Stueckelberg graviton vertex goes as \( \sim p^2 \), the photon graviton vertex as \( \sim p^2 \) along with a propagator. In the infrared regime in which we are interested, the magnitudes of these are comparable to the photon mass squared. This kind of estimate shows that the differential cross section for the process goes as

\[
\sim \frac{m_\gamma^2}{M_{Planck}^4}.
\] (5)

Evidently, the minuteness of this cross section indicates that primordial Stueckelberg particles would still abound, and if they formed a substantial portion of the observed density, they will continue to do so today.

This calculation also reveals in hindsight why the density of dark matter would evolve governed by the free Boltzmann equation, indicating the validity of the calculation that gave rise to (3).

In the analysis presented above, we have assumed that the current dark matter density can be entirely explained in terms of a condensate of Stueckelberg particles. If we are to generalize this model, we may consider the half radii of condensates as well as their masses. Following Hui et al., the formulae (Eqs.29,30 of the paper[6]) for these are given by,

\[
r_\frac{1}{2} = 3.925 \frac{h^2}{Gm_\gamma^2},
\] (6)

and

\[
\rho_c = 4.4 \times 10^{-3} \left( \frac{Gm_\gamma^2}{h^2} \right)^3 M^4,
\] (7)

where \( \rho_c \) is the central density of the halo and \( M \) is the soliton mass (for further details the reader may consult [6]).

It is desirable now to refine our hypothesis and study the theory space of the model. There are three parameters to be taken into account, viz., mass of the photon, the critical temperature and the self interactions. For the moment we ignore the last one. We need to compare with the central density and half radius parameters of the dwarf galaxies. We exhibit this constraint and that due to the upper bound of the photon mass by a parameter \( x \), and consider Stueckelberg particles of mass \( 10^{-17-x} eV \).
The critical temperature required for condensation in our model also enjoys some freedom. The high temperature predicted by the previous estimate can be generalized to a temperature $10^{50^{-y}K}$.

Within this model, we are confined to the region spanned by the variables $0 \leq x \leq 4$ and $y \leq 5$. Using these parameters, we present the sample spaces for the density, soliton mass and the half radius:

Figure 2: $\log \left( \frac{\rho}{1\text{GeV}/c^2} \right) = 5 + x - 3y$

Figure 3: $\log \left( \frac{r_1}{\text{1 parsec}} \right) = 1.59 + 0.75x + 0.75y$
From the plot, the physically realizable regions may be read off. The median half radius $0.25$ kpc is exhibited near the lower end of FIG. 3, where $x+y=1.06$. This is however just the expectation, as more detailed estimates may be made in the Stueckelberg Standard Model where self couplings will be included. Since Stueckelberg field is related to mass generation through Higgs mechanism, the bounds require modifications as mentioned in PDG itself. Such an ultralight massive particle has several predictions as pointed out in literature \cite{6, 8, 9} like size and mass of dwarf galaxies dominated by dark matter, new boson stars (which may be termed Stueckelberg stars). Our future program will bring out these by applying to realistic models.

Comparison may be made with existing models of dark matter in the ultralight domain, which often involve the hypothesis of axion-like forms of matter. In the case of QCD axions, the problem of the particles exhibiting self interactions is persistent. Furthermore, the mass of such particles ($10^{-3}$ eV) is substantially above the mass domains considered in this letter. If one considers a model of dark matter with the mass similar to what was treated here, similar constraints are obtained \cite{8}, with which our results maintain consistency.

Finally, in light of such models, it may be noted that a similar class of ultralight dark matter theories was considered in \cite{9} as well. In our framework as well, the gas of dark matter particles behave ultra-relativistically at most epochs, since $mc^2 \ll k_B T$. However, the soft modes corresponding to a
nonrelativistic component will dominate at low temperatures, or equivalently, large times in the universe history.

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