SPATIAL DISCRETIZATION OF RESTRICTED GROUP C*-ALGEBRAS

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Abstract. We consider spatial discretizations by the finite sections method of the restricted group C*-algebra of a finitely generated discrete group, which is represented as a concrete operator algebra via its left-regular representation. Special emphasis is paid to the quasicommutator ideal of the algebra generated by the finite sections sequences and to the stability of sequences in that algebra. For both problems, the sequence of the discrete boundaries plays an essential role. Finally, for commutative groups and for free non-commutative groups, the algebras of the finite sections sequences are shown to be fractal.

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