1 Abstract

By applying the renormalization group equation, it has been shown that the effective potential $V$ in the massless $\phi^4_4$ model and in massless scalar quantum electrodynamics is independent of the scalar field. This analysis is extended here to the massive $\phi^4_4$ model, showing that the effective potential is independent of $\phi$ here as well.
Recently, it has been shown [1] that in the massless scalar model with classical Lagrangian

\[ L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda}{4!} \phi^4 \]  

(1)

and in massless scalar quantum electrodynamics (MSQED) with classical Lagrangian

\[ L = (\partial_\mu + ieA_\mu) \phi^* (\partial^\mu - ieA^\mu) \phi - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \lambda (\phi^* \phi)^2 \]  

(2)

the effective potential \( V(\phi) \) [2-5] is either flat (ie, independent of the background field \( \phi \)) with non-analytic dependence on the coupling, or there is no spontaneous symmetry breaking (ie, \( \langle \phi \rangle \equiv v = 0 \)). This has been done by applying the renormalization group (RG) to \( V \) and requiring that \( V'(\phi = v) = 0 \).

In this note, we supplement the Lagrangian of eq. (1) with a mass term

\[ L = -\frac{1}{2} m^2 \phi^2 \]  

(3)

and apply the same techniques as were used in ref. [1] to see the effect of inclusion of this mass term on the effective potential.

This same action has been considered in refs. [6-11]. It was demonstrated in ref. [6] that the general form of the resulting effective potential is

\[ V(\lambda, x, y, \phi) = \left\{ \left( a + \frac{b}{x} \right) + \sum_{\ell=1}^\infty \lambda^{\ell+1} \sum_{m=0}^{\ell-1} x^{m-2} \sum_{n=0}^{\ell} y^n a_{\ell mn} \right\} \phi^4 \]

\[ + \left( \frac{1-x}{x} \right)^2 \sum_{k=1}^\infty b_k \lambda^k \]  

(4)

where \( a = \frac{-5}{24} \), \( b = \frac{1}{4} \) and

\[ x = \frac{1}{1 + \frac{2m^2}{\lambda \phi^2}} \quad y = \ln \left( \frac{\lambda \phi^2}{2 \mu^2} \right) = \ln \left( \frac{m^2 + \frac{\lambda \phi^2}{2}}{\mu^2} \right). \]  

(5)

Following refs. [1,12], we now expand \( V \) in powers of \( y^n \), so that

\[ V(\lambda, x, y, \mu, \phi) = \left( \sum_{n=0}^\infty A_n(\lambda, x)y^n \right) \phi^4. \]  

(6)

The RG equation for \( V \) is

\[ \mu^2 \frac{dV}{d\mu^2} = \left[ \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \gamma_m(\lambda) m^2 \frac{\partial}{\partial m^2} + \gamma_\phi(\lambda) \phi^2 \frac{\partial}{\partial \phi^2} \right] V = 0 \]  

(7)
or by eqs. (5) and (6)

\[
\left\{ -1 + \left( \frac{\beta}{\lambda} + \gamma_\phi \right) x + \gamma_m(1-x) \right\} \frac{\partial}{\partial y} + \left( \frac{\beta}{\lambda} - \gamma_m + \gamma_\phi \right) x(1-x) \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial \lambda} + 2\gamma_\phi \right\} \sum_{n=0}^{\infty} A_n(\lambda, x) y^n = 0.
\]  

(8)

Eq. (8) is satisfied order by order in \( y \) provided

\[
A_{n+1}(\lambda, x) = \frac{1}{n+1} \left[ f(\lambda, x) \frac{\partial}{\partial x} + g(\lambda, x) \frac{\partial}{\partial \lambda} + h(\lambda, x) \right] A_n(\lambda, x)
\]

(9)

where

\[
f(\lambda, x) = \frac{\left( \frac{\beta}{\lambda} - \gamma_m + \gamma_\phi \right) x(1-x)}{1 - \left( \frac{\beta}{\lambda} + \gamma_\phi \right) x - \gamma_m(1-x)}
\]

(10)

\[
g(\lambda, x) = \frac{\beta}{1 - \left( \frac{\beta}{\lambda} + \gamma_\phi \right) x - \gamma_m(1-x)}
\]

(11)

\[
h(\lambda, x) = \frac{2\gamma_\phi}{1 - \left( \frac{\beta}{\lambda} + \gamma_\phi \right) x - \gamma_m(1-x)}.
\]

(12)

Furthermore, since

\[
\frac{dV}{d\phi^2} = \left\{ \frac{1}{\phi^2} \left[ x(1-x) \frac{\partial}{\partial x} + x \frac{\partial}{\partial y} \right] + \frac{\partial}{\partial \phi^2} \right\} \left[ \sum_{n=0}^{\infty} A_n(\lambda, x) y^n \right] \phi^4
\]

\[
= \sum_{n=0}^{\infty} \left[ \left( x(1-x) \frac{\partial A_n}{\partial x} + 2A_n \right) y^n + nxA_n y^{n-1} \right] \phi^2,
\]

(13)

an extremum of \( V \) is attained at \( \phi^2 = \nu^2 \) if at each order of \( y \), eq. (13) vanishes, leading to

\[
A_{n+1}(\lambda, x_0) = \frac{1}{n+1} \left[ -\frac{2}{x} - (1-x) \frac{\partial}{\partial x} \right] A_n(\lambda, x_0)
\]

(14)

provided \( \nu^2 \neq 0 \). (We have set \( x_0 = \frac{1}{1+2\alpha^2} \).) Having both the renormalization group equation (8) and the extremum condition (13) hold order-by-order in \( y \) thus leads to a pair of distinct equations relating \( A_{n+1} \) to \( A_n \).

If eq. (14) were to hold for all \( x \), then

\[
A_n(\lambda, x) = \frac{1}{n!} \left( -\frac{2}{x} - (1-x) \frac{\partial}{\partial x} \right)^n A_0(\lambda, x)
\]

(15)
so that eq. (6) reduces to

\[ V = \sum_{n=0}^{\infty} \frac{y^n}{n!} \left( \frac{-2}{x} - (1 - x) \frac{\partial}{\partial x} \right)^n A_0(\lambda, x) \phi^4. \] (16)

If now \( z = \ln(1 - x) \), then eq. (16) becomes

\[ V = \exp \left( 2 \int_{z_0}^{z} \frac{dt}{1 - e^t} \right) \sum_{n=0}^{\infty} \frac{y^n}{n!} \left( \frac{\partial}{\partial z} \right)^n B_0(\lambda, z) \phi^4 = \exp \left( -2 \int_{z}^{z+y} \frac{dt}{1 - e^t} \right) A_0(\lambda, z+y) \phi^4 \]

(17)

where

\[ B_0(\lambda, z) = \exp \left( -2 \int_{z_0}^{z} \frac{dt}{1 - e^t} \right) A_0(\lambda, x). \] (18)

Noting that from eq. (5),

\[ z + y = \ln \left( 1 - \frac{1}{1 + \frac{2m^2}{\mu^2}} \right) + \ln \left( \frac{m^2 + \frac{\lambda^2\phi^2}{2}}{\mu^2} \right) = \ln \left( \frac{m^2}{\mu^2} \right) \] (19)

eq (17) becomes

\[ V = \frac{4(m^2 - \mu^2)^2}{\lambda^2} A_0 \left( \lambda, \ln \frac{m^2}{\mu^2} \right) \] (20)

showing that \( V \) has no dependence on \( \phi \). The function \( A_0 \) can be fixed by eqs. (9) and (14) with \( n = 0 \); this leads to the equation

\[ \left( -\frac{2}{x} - (1 - x) \frac{\partial}{\partial x} \right) A_0(\lambda, x) = \left( f(\lambda, x) \frac{\partial}{\partial x} + g(\lambda, x) \frac{\partial}{\partial \lambda} + h(\lambda, x) \right) A_0(\lambda, x). \] (21)

An alternative approach to deriving eq. (20) which does not rely on eq. (13) holding order by order in \( y \) at \( \phi^2 = v^2 \) can also be used, following a line of reasoning based on the “method of characteristic” [13,14] much as in ref. [1]. We begin by choosing \( \mu^2 \) so that \( y \) in eq. (13) vanishes when \( \phi^2 = v^2 \); eq. (14) then holds when \( n = 0 \), provided \( \lambda \) and \( x_0 \) are evaluated with \( \lambda(\mu^2) \) and \( m^2(\mu^2) \) evaluated at this value of \( \mu^2 \). In refs. [2,15] this is taken to impose a restriction on the couplings at this value of \( \mu^2 \); we instead follow ref. [1] and take this to be an equation that fixes the function \( A_1 \) in terms of \( A_0 \), since the actual values of \( \lambda(\mu^2) \) and \( m^2(\mu^2) \) can be independently varied at this value of \( \mu^2 \) by adjusting the initial conditions on these running parameters. We thus have a functional equation relating \( A_0(\lambda, x) \) to \( A_1(\lambda, x) \) rather than equation that relates the values of parameters in the theory,

\[ A_1(\lambda, x) = \left[ -\frac{2}{x} - (1 - x) \frac{\partial}{\partial x} \right] A_0(\lambda, x). \] (22)
in addition to eq. (9).

Much as was done in the discussion of MSQED in ref. [1], we now define

\[ a_n(\lambda(t), \pi(t), t) = \exp \left[ \int_{t_0}^{t} d\tau h(\lambda(\tau), \pi(\tau)) \right] A_n(\lambda(t), \pi(t)), \]  

(23)

where

\[ \frac{d\pi(t)}{dt} = f(\lambda(t), \pi(t)) \quad (\pi(t_0) = x) \]  

(24)

and

\[ \frac{d\lambda(t)}{dt} = g(\lambda(t), \pi(t)) \quad (\lambda(t_0) = \lambda). \]  

(25)

Together, eqs. (9) and (23-25) show that

\[ \frac{d}{dt} a_n(\lambda(t), \pi(t), t) = (n + 1) a_{n+1}(\lambda(t), \pi(t), t) \]  

(26)

with

\[ a_n(\lambda(t_0), \pi(t_0), t_0) = A_n(\lambda, x). \]  

(27)

If now

\[ \overline{\varphi}(t) = \ln \left( \frac{\lambda(t)\overline{\phi}^2(t)}{2\overline{\pi}^2(t)} \frac{1}{\pi(t)} \right) \]  

(28)

with

\[ \frac{1}{\overline{\phi}^2(t)} \frac{d\overline{\phi}^2(t)}{dt} = \gamma_{\phi}(\lambda(t)) \]  

\[ 1 - \left( \frac{\beta(\lambda(t))}{\lambda(t)} + \gamma_{\phi}(\lambda(t)) \right) \pi(t) - \gamma_m(\lambda(t))(1 - \pi(t)) \]  

(29)

and

\[ \frac{1}{\overline{\pi}^2(t)} \frac{d\overline{\pi}^2(t)}{dt} = \frac{1}{\left[ 1 - \left( \frac{\beta(\lambda(t))}{\lambda(t)} + \gamma_{\phi}(\lambda(t)) \right) \pi(t) - \gamma_m(\lambda(t))(1 - \pi(t)) \right]} \]  

(30)

then eqs. (24, 25, 28-30) together imply that

\[ \frac{d\overline{\varphi}(t)}{dt} = -1 \quad (\overline{\varphi}(t_0) = \varphi^2). \]  

(31)

Now defining

\[ V(t) = \sum_{n=0}^{\infty} a_n(\lambda(t), \pi(t), t)\overline{\varphi}^n(t)\varphi^4, \]  

(32)
then by eqs. (26) and (31), we have
\[ \frac{dV(t)}{dt} = 0 \quad (33) \]
with
\[ V(t_0) = V(\lambda, x, y, \mu, \phi). \quad (34) \]

From eqs. (26) and (32), it follows that
\[ V(t) = \sum_{n=0}^{\infty} \frac{y^n(t)}{n!} \left( \frac{d}{dt} \right)^n a_0(\lambda(t), \pi(t), t) \phi^4 \quad (35) \]
\[ = a_0(\lambda(t + y(t)), \pi(t + y(t)), t + y(t)) \phi^4 \quad (36) \]

which by eq. (33) and (34) leads to
\[ V(\lambda, x, y, \mu, \phi) = a_0(\lambda(t_0 + y), \pi(t_0 + y), t_0 + y) \phi^4. \quad (37) \]

Again following the steps used to analyze MSQED in ref. [1], we define
\[ \tilde{A}_0(\lambda(t), \pi(t), t) = \left[ \exp \int_{t_0}^{t} \frac{2d\tau}{\pi(\tau)} \right] a_0(\lambda(t), \pi(t), t) \quad (38) \]
which by eq. (23) is also equal to
\[ = \left[ \exp \int_{t_0}^{t} \left( \frac{2}{\pi(\tau)} + h(\lambda(\tau), \pi(\tau), \tau) \right) d\tau \right] A_0(\lambda(t), \pi(t)). \quad (39) \]

Eqs. (21), (24), (25) and (39) now show that
\[ \frac{d}{dt} \tilde{A}_0(\lambda(t), \pi(t), t) = -(1 - \pi(t)) \frac{\partial}{\partial \pi(t)} \tilde{A}_0(\lambda(t), \pi(t), t). \quad (40) \]

It is also apparent from eqs. (27) and (38) that
\[ \tilde{A}_0(\lambda(t_0), \pi(t_0), t_0) = a_0(\lambda(t_0), \pi(t_0), t_0) = A_0(\lambda, x). \quad (41) \]

We now can use eqs. (35) and (38) to write
\[ V(t) = \sum_{n=0}^{\infty} \frac{\mathcal{H}^n(t)}{n!} \left( \frac{d}{dt} \right)^n \left\{ \left[ \exp - \int_{t_0}^{t} \frac{2d\tau}{\pi(\tau)} \right] \tilde{A}_0(\lambda(t), \pi(t), t) \right\} \phi^4 \quad (42) \]
which by eq. (40) becomes
\[ = \left[ \exp - \int_{t_0}^{t} \frac{2d\tau}{\pi(\tau)} \right] \sum_{n=0}^{\infty} \frac{\mathcal{H}^n(t)}{n!} \left[ \frac{-2}{\pi(t)} - (1 - \pi(t)) \frac{\partial}{\partial \pi(t)} \right]^n \tilde{A}_0(\lambda(t), \pi(t), t) \phi^4. \quad (43) \]
Now following the same argument that led from eqs. (16) to (20), eq. (43) reduces to

\[ V(t) = \exp\left[ -\int_0^t \frac{2d\tau}{x(\tau)} \right] \exp\left( -2\int_{\tau(t)}^{\tau(t)+\Phi(t)} \frac{d\tau}{1-e^{\tau}} \right) \tilde{A}_0(\lambda(t), \tau(t) + \Phi(t), t) \phi^4. \]  

(44)

where \( \tau(t) = \ln(1 - \lambda(t)) \). We can now set \( t = t_0 \) in eq. (44); by eqs. (19), (34) and (41) we recover eq. (20).

It is of interest to consider the solution of the \( m^2 \to 0 \) limit of eq. (7),

\[ \left[ \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \gamma_\phi(\lambda) \phi^2 \frac{\partial}{\partial \phi^2} \right] S\left( \lambda, \ln \left( \frac{\lambda \phi^2}{2 \mu^2} \right) \right) \phi^4 = 0 \]  

(45)

in the case where \( \beta(\lambda) \) and \( \gamma_\phi(\lambda) \) are taken to be given exactly by the order \( \lambda^2 \) expressions \( \beta(\lambda) = b\lambda^2, \gamma_\phi(\lambda) = g\lambda^2 \). We first note that eq. (45) becomes

\[ \left[ \left( -1 + \frac{\gamma + \beta}{\lambda} \right) \frac{\partial}{\partial L} + \frac{2\gamma}{\beta} \right] S(\lambda, L) = 0. \]  

(46)

If now

\[ S(\lambda, L) = \exp\left( -\int_{\lambda_0}^\lambda dt \left( \frac{2\gamma(t)}{\beta(t)} \right) \right) T(\lambda, L), \]  

(47)

then eq. (46) reduces to

\[ \left( f(\lambda) \frac{\partial}{\partial L} + \frac{\partial}{\partial \lambda} \right) T(\lambda, L) = 0 \]  

(48)

where

\[ f(\lambda) = \frac{-1 + \gamma(\lambda) + \beta(\lambda)/\lambda}{\beta(\lambda)}. \]  

(49)

Eq. (48) can be solved using separation of variables; if we set \( T(\lambda, L) = a(\lambda)b(L) \) then

\[ \frac{b'(L)}{b(L)} = \kappa = -\frac{a'(\lambda)}{f(\lambda)a(\lambda)}. \]  

(50)

where \( \kappa \) is a constant. Integration of eq. (50) results in

\[ b(L) = Be^{\kappa L} \]  

(51)

\[ a(\lambda) = A \exp\left( -\kappa \int_{\lambda_0}^\lambda dt f(t) \right) \]  

(52)
so that
\[ V(\lambda, \phi, \mu) = \mathcal{C} \exp \left( - \int_{\lambda_0}^{\lambda} dt g(t) \right) e^{\kappa L} \exp \left( -\kappa \int_{\lambda_0}^{\lambda} dt f(t) \right) \phi^4 \] (53)
where \( \mathcal{C} = AB \) and \( g(t) = \frac{2g(t)}{\beta(t)} \).

Since \( L = \ln \left( \frac{\lambda^2}{2\mu^2} \right) \), all of the dependence on \( \phi \) in eq. (53) reduces to just \( \phi^{4+2\kappa} \); viz
\[ V(\lambda, \phi, \mu) = \mathcal{C} \exp \left[ -\int_{\lambda_0}^{\lambda} dt (g(t) + \kappa f(t)) \right] \left( \frac{\lambda \phi^2}{2\mu^2} \right)^\kappa \phi^4. \] (54)

This supports the possibility in ref. (1) of spontaneous symmetry breaking not occurring in the massless \( \phi_4 \) model when all radiative effects are taken into account.

If in eq. (54) we take \( g(t) = \frac{2g}{b} \) and \( f(t) = \frac{-1+g(t)+bt}{bt^2+bt} \), we obtain
\[ V(\lambda, \phi, \mu) = \mathcal{C} \exp \left[ -\int_{\lambda_0}^{\lambda} dt \left( g(t) + \kappa f(t) \right) \right] \left( \frac{\lambda \phi^2}{2\mu^2} \right)^\kappa \phi^4. \] (55)

We now also have the implicit dependence of \( V \) on \( \mu^2 \) governed by the equations
\[ \mu^2 \frac{d\lambda}{d\mu^2} = \beta(\lambda) = b\lambda^2 \] (56)
\[ \mu^2 \frac{d\phi^2}{d\mu^2} = \gamma_\phi(\lambda) \phi^2 = g\lambda^2 \phi^2. \] (57)

Integrating these equations leads to
\[ \lambda = \lambda(\mu^2) = \frac{\lambda_1}{1 - \lambda_1 b \ln \left( \frac{\mu^2}{\mu_1^2} \right)} \] (58)
\[ \phi^2 = \phi^2(\mu^2) = \phi_1^2 \exp \left( \frac{g\lambda_1}{b} \left( \frac{1}{1 - \lambda_1 b \ln \left( \frac{\mu^2}{\mu_1^2} \right)} - 1 \right) \right) \] (59)
where \( \lambda_1 \) and \( \phi_1^2 \) are boundary values of \( \lambda(\mu^2) \) and \( \phi^2(\mu^2) \) at \( \mu^2 = \mu_1^2 \). Substitution of eqs. (58) and (59) into eq. (55) results in
\[ V = \mathcal{C} \exp \left[ \frac{g}{b}(2 + \kappa)(\lambda_0 - \lambda_1) + \kappa \left( \frac{1}{\lambda_0} - \frac{1}{\lambda_1} \right) \right] \left[ \frac{\lambda_0 \phi_1^2}{2\mu_1^2} \right]^\kappa \phi_1^4. \] (60)

The renormalization group equation (7) ensures that \( V \) is independent of \( \mu^2 \); this is realized explicitly in eq. (60) when we use the truncated expressions for \( \beta \) and \( \gamma_\phi \) used in eqs. (56)
and (57). If the boundary values $\lambda_0$ and $\lambda_1$ are set equal to each other, $\kappa$ is set equal to zero, and $C$ is set equal to $\frac{\lambda_1}{4!}$ we obtain simply

$$V = \frac{\lambda_1}{4!} \phi_1^4. \quad (61)$$

This disappearance of dependence on $\mu^2$ is similar to what occurs in the discussion of “the method of characteristics” in ref. [14]. The example used there is the equation

$$x \frac{\partial A(x, y)}{\partial x} + y^2 \frac{\partial A(x, y)}{\partial y} = 0; \quad (62)$$

a solution of which is

$$A_0(x, y) = xe^{1/y}. \quad (63)$$

If “characteristic functions” $\varpi(t)$ and $\vartheta(t)$ are defined by

$$\frac{d\varpi(t)}{dt} = \varpi(t) \quad (\varpi(0) = x) \quad (64)$$

$$\frac{d\vartheta(t)}{dt} = \vartheta^2(t) \quad (\vartheta(0) = y) \quad (65)$$

then it is evident that $A_0(\varpi(t), \vartheta(t))$ also satisfies eq. (62) and that $\frac{dA_0(\varpi(t), \vartheta(t))}{dt} = 0$. Indeed, by eqs. (64) and (65), $\varpi(t) = xe^t$ and $\vartheta(t) = \frac{\mu}{1-\mu t}$, and it follows that $A_0(\varpi(t), \vartheta(t)) = xe^{1/y}$. All $t$ dependence has cancelled out. If however we have only considered a perturbative approximation to $A_0(x, y)$ given in eq. (63),

$$A_0^{(1)}(x, y) = x \left(1 + \frac{1}{y}\right) \quad (66)$$

then $A_0^{(1)}(\varpi(t), \vartheta(t))$ reproduces $A_0(x, y)$ only at the particular value of $t$ given by $t = \frac{1}{y}$. Much the same happens in ref. [10], where a “boundary function” for the effective potential is chosen at $L$ loop order to be the computed value of $V$ at that order with $s$ set equal to zero. (All notation is that of ref. [10].) However, this function is not a solution to the full renormalization group equation (7), even if the renormalization group functions are truncated at $L + 1$ loop order; it can be seen that a full non-perturbative solution to the renormalization group equation, even with truncated renormalization group functions (akin to eq. (63), and not eq. (66), providing a solution to eq. (62)) involves portions of all
functions $f_\ell$ appearing in the sum of eq. (13) of ref. [10]. This accounts for having to set $\overline{s} = 0$ to reproduce the results of ref. [6]; this is much like having to choose $t = \frac{1}{y}$ in order for $A_0^{(1)}(\overline{x}(t), \overline{y}(t))$ to reproduce $A_0(x, y)$.

We are in the process of examining the effective potential for models which scalars possessing a mass at the tree level couple to other fields, such as a gauge Boson.

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