Rock Triaxial Tests: Global Deformation vs Local Strain Measurements—Implications

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Received: 6 July 2020 / Accepted: 25 January 2021 / Published online: 5 April 2021
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Abstract
Accurate stress–strain measurements in triaxial tests are critical to compute reliable mechanical parameters. We focus on compliance at the interfaces between the specimen and endcaps, and test specimens under various triaxial conditions using different instrumentation protocols. The tested materials include aluminum, Eagle Ford shale, Berea sandstone, and Jubaila carbonate. Results obtained following common practice reveal that surface roughness at the specimen-endcap interfaces leads to marked seating effects, affects all cap-to-cap based measurements and hinders ultrasonic energy transmission. In particular, cap-to-cap deformation measurements accentuate hysteretic behavior, magnify biases caused by bending and tilting (triggered by uneven surfaces and misalignment), and affect the estimation of all rock parameters, from stiffness to Biot’s α-parameter. Higher confining pressure diminishes seating effects. Local measurements using specimen-bonded strain gauges are preferred (Note: mounting strain gauges on sleeves is ill-advised). We confirm that elastic moduli derived from wave propagation measurements are higher than quasi-static moduli determined from local strain measurements using specimen-bonded strain gauges, probably due to the lower strain level in wave propagation and preferential high-velocity travel path for first arrivals.

Keywords Strain gauge · LVDT · Triaxial testing · Ultrasonic · Static modulus · Dynamic modulus

1 Introduction
Mechanical parameters such as stiffness, Poisson’s ratio and Biot’s α-parameter are used for analyses and designs in the geotechnical, petroleum, and mining sectors. Inherent biases within triaxial tests can obscure and distort measured values and produce unrepresentative rock properties.

Current best practice involves strain measurements conducted mid-height using strain gauges mounted directly on top of the rock specimen and that extend across at least ten-grain diameters (ASTM International D7012-14 (2014)). However, common practice uses global deformation sensors mounted either on both endcaps across the entire length of the specimen or coupled on top of the jacket to measure deformations within the central zone along with the specimen height. Table 1 lists strain and deformation sensors used in geomechanics laboratories.

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Measurements using cap-to-cap deformation sensors include seating effects due to topographic differences at the specimen-endcap interfaces and non-uniform strains along with the specimen due to restraints at the interface (Baldi et al. 1988). Local measurements avoid the impact of unwanted deformations at the specimen-endcap interfaces and result in computed stiffness values between 15% to 45% higher than cap-to-cap measurements (Isah et al. 2018; Yimsiri et al. 2005; Kung 2007; Xu et al. 2014; Xu 2017; Kumar et al. 2016).

In this study, we conduct axial compression tests with cylindrical aluminum and intact-rock specimens. We compare stress–strain plots obtained with local measurements using strain gauges against computed strains from cap-to-cap and local LVDTs deformation measurements. The study includes high-resolution surface roughness measurements and ultrasonic velocity data to gain an enhanced understanding of specimen-endcap contact effects. We use these results to quantitatively address seating effects and understand discrepancies in estimated moduli obtained from local and global measurements.
2 Experimental Studies

We tested standard aluminum rods and three lithologically different intact-rock specimens prepared from outcrop samples. Experimental details follow.

2.1 Test Protocol

All tests start with a detailed characterization of the specimen and endcaps surfaces using an optical profilometer. The chromatic confocal interferometer has a spot size of $d = 7 \, \mu m$ and provides a lateral resolution of $\Delta x \approx 15 \, \mu m$ and vertical resolution of $0.2 \, \mu m$ (NANOVEA Jr25). For comparison, we also gather topographic data using the standard procedure that involves a $d = 1 \, mm$ tip size and elevation data every $\Delta x = 3 \, mm$ separation (ASTM International D4543-08 (2008)).

Then, we mount two strain gauges diametrically opposite to each other at the mid-height of the specimen (installation details in ASTM International D7012-14 (2014)). We use linear or rosette type strain gauges in this study. The rosette gauges have a $3.18 \, mm$ length, $4.57 \, mm$ grid width, gauge factor of $2.15$, resistance of $350 \, \Omega$ and $\pm 3 \times 10^{-2}$ strain range; these rosettes allow the simultaneous collection of axial and radial strain data for the accurate determination of Poisson’s ratio. The linear strain gauges are $3.18$ and $12.7 \, mm$ long with $\pm 3 \times 10^{-2}$ strain range; both linear gauges have a similar gauge width of $4.57 \, mm$, resistance

| Strain measurement devices/techniques | Measurement Ranges and Resolutions | Benefits/Limitations | References |
|---------------------------------------|-------------------------------------|----------------------|------------|
| Local measurements/techniques         |                                     |                      |            |
| Strain Gauges                         | 0.3 – 150 mm range Less than 1 \mu m resolution | Flexible, submersible, Requires complete sealing, not reusable, strain drifting, time intensive installation | Howarth (1984), van Heerden (1987) |
| Local Linear Variable Differential Transformer (LVDT) | Up to 10 mm range Less than 1 \mu m resolution | Good resolution, stability, linear calibration, reusable, submersible | Atkinson (2000), Wild et al. (2017) |
| Electro Level Inclinometer            | Resolution up to 1 \mu m | Submersible, compatible with unconsolidated specimens | Jardine et al. (1984), Symes and Burland (1984) |
| Hall Effect Local Strain Transducers  | 4–6 mm range Less than 0.1 \mu m resolution | Fixed or floating type | Clayton and Khatrush (1986) |
| Digital Image Correlation using Digital Camera | Ranges and resolutions dependent on camera specifications | Straight forward operation | Bhandari et al. (2012), Li et al. (2016) |
| Local Strain Transducers-LST          | Up to 1.5 mm range 1 \mu m resolution | Reusable | Tatsuoka (1988) |
| Proximity Sensors                     | Up to 5 mm range Up to 10 \mu m resolution | Fixed/floating, high resolution | Hird and Yung (1989) |
| Fiber Bragg Grating—FBG               | Up to 80 mm range. 4.45 \mu m resolution | Less expensive than LDT | Xu (2017) |
| Global measurements/techniques        |                                     |                      |            |
| Cap-to-cap LVDT                       | Up to 1 m range Less than 1 \mu m resolution | Submersible, easy to install, linear calibration, reusable | Filho (1985), Tisato and Madonna (2012) |
of 350 Ω, and gauge factor of 2.175. We test two different mounting types: gauges attached directly onto the specimen i.e., beneath the Viton sleeve (Fig. 1a), and gauges bonded onto the Viton sleeve (Fig. 1b). The Viton sleeve has a 0.5 mm thickness, a maximum shrink ratio of 2:1, and a working temperature range from – 55 °C to 200 °C. We also examine two installations for the LVDTs: cap-to-cap (Configuration A and B) and local (Configuration C in Fig. 1).

In addition to the local strain and global deformation measurements, we acquire ultrasonic transit times for all specimens during loading/unloading cycles. The instrumentation involves P-wave crystals and two orthogonally polarized S-wave transducers. At each stress level, we gather and stack 512 signals to enhance the signal-to-noise ratio with minimal preconditioning.

The loading sequence involves (1) three loading/unloading cycles to 40 MPa at different strain rates (Fig. 2); and (2) three loading/unloading cycles to 60 MPa. We repeat the last test at four confining stress levels: 0 MPa (unconfined), 10 MPa, 30 MPa and finally 60 MPa. The selected confining stress levels reflect field conditions encountered in infrastructure and geo-energy applications. Load cycles allow us to investigate the effect of consecutive axial loading/unloading on both seating effects and stiffness, and the relationship between seating effects and axial deviatoric stress.

2.2 Standard Aluminum Specimen

The experimental study starts with a standard aluminum 6061-T6 specimen given its well-known properties (Table 2).
We explore the impact on seating effects by testing (1) the original unpolished specimen, and (2) the specimen after shaving both end surfaces using a precision CNC milling machine (herein called “polished specimen”–ASTM International D4543-08 (2008)). Before testing, we measured surface roughness in both cases using the optical profilometer (detailed above—NANOVEA Jr25).

2.3 Rock Specimens

We extend the experimental study to rock specimens using the same test protocols described above. The selected specimens include lower Eagle Ford shale (Western Gulf outcrop, USA), late Upper Jurassic Jubaila carbonates (Riyadh outcrop, Saudi Arabia), and Berea sandstone (Waverly Group outcrop, USA). The Eagle Ford shale specimen has a bedding angle perpendicular to the specimen’s axis. The Jubaila carbonate specimens have minor stylolite streaks perpendicular to the specimen’s axis. The Berea sandstone is a homogeneous specimen with 19.9% porosity. Table 2 summarizes specimen properties, peak axial stresses, and applied confining pressures.

3 Experimental Results

Experimental results obtained with the standard aluminum specimen isolate boundary effects for a well-known homogenous material. We present these results first, followed by data gathered with rock specimens.

3.1 Local Strain vs. Global Deformation

The stress–strain curves computed using cap-to-cap LVDTs deformation measurements show hysteresis. The computed stiffness $E \approx 59\, \text{GPa}$ is markedly below the reference Young’s modulus for aluminum $E \approx 69\, \text{GPa}$ (ASM Handbook 1990, Fig. 2), and deviations from linearity are more

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Table 2: Specimen specifications, confining pressures and peak stresses used during the loading/unloading cycles

| Specimen                  | Length [mm]$^a$ | Diameter [mm]$^b$ | L:D Ratio | Mass [g]$^c$ | Estimated UCS [MPa] | Confining Pressure [MPa] | Peak Stress [MPa] |
|---------------------------|-----------------|------------------|-----------|--------------|---------------------|------------------------|------------------|
| Aluminum                  | 50.230          | 25.105           | 2.00:1    | 67.132       | 310$^a$             | 0, 10, 30, 60          | 60 60 60         |
| Eagle Ford Shale          | 50.896          | 25.435           | 2.00:1    | 65.639       | 130–150             | 0                      | 58 58 58         |
| Berea Sandstone           | 51.349          | 25.385           | 2.02:1    | 55.194       | 62–78               | 0                      | 35 35 –          |
| Jubaila Carbonate 1 (Packstone) | 57.841        | 26.813           | 2.16:1    | 78.752       | 24–48               | 0                      | 21 21 –          |
| Jubaila Carbonate 2 (Wackestone) | 53.953         | 26.714           | 2.02:1    | 72.989       | 24–48               | 0                      | 10 15 20         |

Note: all rock specimen come from outcrop samples

$^a$UCS value for Aluminum 6061-T6 obtained from ASM Handbook (1990). Typical aluminum 6061-T6 composition: 95.8–98.6% Al, 0.04–0.35% Cr, 0.15–0.4% Cu, Max 0.7% Fe, 0.8–12% Mg, Max 0.15% Mn, 0.4–0.8% Si, Max 0.15% Ti, Max 0.25% Zn

$^b$Uncertainty in length and diameter are 5 μm

$^c$Uncertainty in mass is 0.5 mg. The average top and bottom surface roughness for the rock specimens is ~10 μm
pronounced at lower axial loads. Both polished and unpolished specimens exhibit seating effects on cap-to-cap LVDTs data, albeit with different degrees (Fig. 3).

Seating effects and hysteresis do not appear on strain data gathered with specimen-bonded strain gauges (Fig. 3a). The measured stiffness agrees with the reference Young’s modulus for aluminum, and strain rates do not affect the measured Young’s modulus ($\varepsilon = 0.5 \times 10^{-6} \text{ s}^{-1}$, $10^{-6} \text{ s}^{-1}$ and $2 \times 10^{-6} \text{ s}^{-1}$—Fig. 2).

Strains measured from the two diametrically opposite strain gauges plot on top of each other indicating equal strain levels on both sides and, therefore, no specimen bending (Fig. 3). However, deformations measured with the two cap-to-cap LVDTs are different and imply cap tilting. Cap tilting effects are most pronounced in unconfined compression test data, i.e., low initial contact stress.

Local strain and cap-to-cap deformation measurements with rock specimens show similar trends (relevant specimen characteristics listed in Table 2). Seating effects are apparent in all specimens when using cap-to-cap LVDTs deformation measurements (Fig. 4). While there are not seating effects on local measurements gathered with specimen-bonded strain gauges, the Berea sandstone shows an initial non-linear convex response suggesting the presence of microcracks and some grain debonding. The Berea sandstone is a high porosity rock with weak grain-to-grain bonding. In contrast, the Jubaila and Eagle Ford specimens have lower porosity values with well-cemented matrices; the measured response suggests that these rocks did not experience debonding during sampling and exhumation.

### 3.2 Instrumentation on the Sleeve—Compliance

Leakage and related experimental difficulties associated with strain gauges affixed to the rock specimen are overcome by mounting instruments on the sleeve (Note: this strategy applies to standard triaxial systems and is avoided in Hoek-cells).

We mounted “local LVDTs” on the specimen mid-height with a separation of 26 mm between clamps (Configuration C, Fig. 1). The local LVDTs data provide stiffness values near the standard Young’s modulus ($E \approx 69 \text{ GPa}$) but exhibit some hysteresis probably due to slippage and compliance effects at the clamp-sleeve-specimen interface (Fig. 5).

Strain gauges mounted on the sleeve show more pronounced compliance issues: the measured strains are markedly smaller than the specimen strains and result in unreasonably high apparent stiffness (Fig. 6b: the estimated Young’s modulus $E \approx 419 \text{ GPa}$ of the light green line is six times higher than the standard value of $E \approx 69 \text{ GPa}$). Higher confining pressures reduce compliance effects (Fig. 6). Nevertheless, mounting strain gauges—even long ones—on top of the sleeve is ill-adviced.

Specimen-bonded strain gauges are the best option for pre-peak strain measurements (Figs. 5 and 6). However, strain...
gauges fixed directly on the specimen require properly sealed cables to avoid leakage. In our setup, we pierce a small hole through the Viton sleeve directly at the location where the cables attach to the strain gauges and fill the hole with both silicon sealant and polyurethane. This method is still prone to leaks; one-in-four of our specimens required resealing.

### 3.3 Surface Roughness

Surface topography gathered with the large $d = 1$ mm tip (ASTM International D4543-08 (2008)) renders a smoother surface than the more accurate and higher resolution interferometer data (spot size $d = 7$ μm). In fact, the large tip acts as a non-linear low-pass filter (Table 3 see also Fig. 7a–c).

Circumferential profiles gathered at 5 and 10 mm radii on the aluminum specimen end face reveal the effect of polishing on surface topography: the unpolarized surfaces have a ~150 μm elevation range compared to ~38 μm for the

**Table 3** Surface roughness for aluminum and rock specimens according to ASTM standard and profilometer measurements

| Specimen                 | ASTM International D4543-08 (2008) (1 mm tip) | Profilometer (7 μm spot size) |
|--------------------------|-----------------------------------------------|-------------------------------|
|                          | Max amplitude | Standard deviation | Max amplitude | Standard deviation |
| Aluminum                 | 23            | 12               | 38            | 13               |
| Eagle Ford shale         | 11            | 5                | 20            | 6                |
| Jubaila carbonate        | 10            | 4                | 13            | 4                |
| Berea Sandstone          | 23            | 11               | 41            | 11               |
| Endcaps                  | 21            | 11               | 34            | 11               |
polished specimen surface (Figs. 7). Note that both values exceed the maximum 25 μm range recommended in ASTM International D4543-08 (2008).

Endcaps have their own topography. Orthogonal radial profiles on the endcaps exhibit a clear concave geometry with the inner section ~ 14 μm higher than the perimeter (Fig. 8). The following section explores implications on inferred stress–strain data.

Fig. 7 Surface topography: Polished and rough aluminum specimens. Circumferential profiles of the (a) top and (b and c) bottom surfaces. Optical profilometer: rough surface (blue lines) and polished surface (orange lines). Measurements following ASTM International D4543-08 (2008) standards shown as dashed blue lines. Sketches show measurement locations in light red color. d Stress–strain curves measured with cap-to-cap LVDTs for the polished (orange) and rough aluminum specimens (blue). The light green line shows specimen-bonded strain gauge measurements (Color figure online)

Fig. 8 Triaxial endcaps surface profile – Optical profilometer. Top (a) and bottom (b) endcaps. Roughness measurements along two orthogonal paths (see sketches)
4 Analyses and Discussion

4.1 Seating Effects and Surface Roughness

Results in Sect. 3 show that the specimen and endcap topographies are uneven along the perimeter and cause tilting, as hinted by the differential deformation measured with the diametrically opposite cap-to-cap LVDTs. Furthermore, asperities experience stress concentration

\[ \sigma_{\text{asp}} = \sigma \left( \frac{A_s}{A_{\text{asp}}} \right) \]

where the applied external stress \( \sigma \) is amplified by the area ratio \( \frac{A_s}{A_{\text{asp}}} \) resulting in the localized deformation

\[ \delta_{\text{asp}} = \varepsilon \cdot h = \frac{\sigma_{\text{asp}}}{E} h_{\text{asp}} = \frac{\sigma}{E} \left( \frac{A_s}{A_{\text{asp}}} \right) h_{\text{asp}} \]

(1)

where subscripts refer to asperities “asp” and specimen “s”. Then, the total strain \( \varepsilon_{\text{comp}} \) computed from cap-to-cap LVDTs deformation measurements is

\[ \varepsilon_{\text{comp}} = \delta_s + 2 \delta_{\text{asp}} = \frac{\sigma}{E} \left( \frac{A_s}{A_{\text{asp}}} \right) h_{\text{asp}} h_s \approx \frac{\sigma}{E} \left( 1 + \frac{A_s}{A_{\text{asp}}} \frac{h_{\text{asp}}}{h_s} \right) = \varphi \frac{\sigma}{E} = \varphi_s \]

(2)

Finite element modeling allows us to simulate the complete deformation of the endcap-specimen-endcap stack (COMSOL Multiphysics). The model parameters for elastoplastic aluminum behavior are elastic modulus \( E = 69 \) GPa, Poisson’s ratio \( \nu = 0.33 \) and yield stress \( \sigma_{ys0} = 276 \) MPa. Similarly, \( E = 113.8 \) GPa, \( \nu = 0.34 \) and \( \sigma_{ys0} = 880 \) MPa for the titanium endcaps (ASM Handbook 1990; Boyer et al. 1994). We use the von Mises yield criterion

\[ F = \sigma_{\text{mises}} - \sigma_{ys} \leq 0 \]

(3)

and isotropic linear hardening where the yield stress follows

\[ \sigma_{ys} = \sigma_{ys0} + E_{iso} \varepsilon_{pe} \]

(4)

\[ E_{iso} = \frac{1}{\left( \frac{1}{E_{\text{Tiso}}} - \frac{1}{E} \right)} \]

(5)

This first-order approximation captures the main parameters that determine seating effects, herein combined into the single \( \varphi \)-factor. Experimental results show that the \( \varphi \)-factor ranges between \( \varphi = 1.7 \) and \( \varphi = 2.5 \) in this study.

\[ \sigma_{ys} = \sigma_{ys0} + E_{iso} \varepsilon_{pe} \]

(6)

where \( \varepsilon_{pe} \) is the effective plastic strain, \( E_{iso} \) the isotropic elastic modulus and \( E_{\text{Tiso}} \) the post-yield tangential modulus; for aluminum: \( E_{\text{Tiso}} = 562 \) MPa (Thota et al. 2015); titanium: \( E_{\text{Tiso}} = 1.25 \) GPa (Ziaja 2009). We
simulate two interface macro-topographies inspired by the measured surfaces (Fig. 9): (1) convex-aluminum surface on flat endcaps, and (2) concave endcaps against a flat aluminum surface (the flattened circular contact diameter $d_{\text{contact}} = 10 \text{ mm}$ avoids numerical instability). Both of them are spherical sectors of radius $R$ and height $h$. Simulations consider three different [$h, R$] pairs: [80 $\mu$m, 1000 mm], [40 $\mu$m, 2000 mm], and [20 $\mu$m, 4000 mm]. Results in Fig. 9 show the seating effects on the stress–strain response computed from cap-to-cap deformation measurements. Clearly, convexity/concavity at interfaces has a pronounced effect on the inferred specimen response and its stiffness. Numerical results reveal a 29% decrease in estimated stiffness for 20 $\mu$m concavity height, reaching as much as 50% stiffness reduction for 80 $\mu$m concavity height. For comparison, experimental results show ~8% diminished elastic modulus for a measured concavity height of ~14 $\mu$m.

4.2 Ultrasonic Velocity

Ultrasonic data gathered during the triaxial tests also show the effect of specimen-endcap interfaces by hindering energy transmission. We used the integral of the energy within a 10 $\mu$s time window and normalized the energy in a given signal by the energy in the signal gathered at 60 MPa. Figure 10 shows the normalized energy in P-wave signals as a function of deviatoric stress for both rough and polished aluminum specimens. The data reveal enhanced coupling and higher P-wave received energy at high axial stresses. In contrast, energy transmission drops by one (polished) to two (rough) orders of magnitude at lower deviatoric stress levels. Clearly, P-wave transmission emerges as a sensitive indicator of the evolving specimen-endcap interces.

We use coda wave analysis to accurately determine changes in first arrival times (Dai et al. 2013). The stretching factor $\theta$ is proportional to the time shift of the first arrival time $t$ between two signals $a$ and $b$ in a cascade:

$$\theta = \frac{\Delta t}{t_a} = \frac{(t_a - t_b)}{t_a}$$  \hspace{1cm} (6)

Figure 11a and b display cascades of P- and S-wave signatures acquired for the polished aluminum specimen (a subset of the signals is shown for clarity). The red markers indicate first arrival times obtained from coda wave analysis where the signal at the highest deviatoric stress is used as the base trace. Figures 11c and d show the computed P- and S-wave velocities using the instantaneous specimen length, as a function of deviatoric stress during loading (red) and unloading (blue). While the transmitted acoustic energy changes with stress level (Fig. 10), coda wave analysis recovers changes in travel time even for low deviatoric stresses. The data show that propagation velocities increase with axial deviatoric stress following a Hertzian-type power function:

$$V = a \left( \frac{\sigma_d}{1 \text{ MPa}} \right)^\beta$$  \hspace{1cm} (7)

where $a$ is the wave velocity at 1 MPa, and the $\beta$-exponent reflects the stress sensitivity. The low-stress sensitivity of wave velocity in aluminum $\beta = 0.003$ to 0.007 is caused by changes in interatomic distance. On the other hand, rock specimens tend to exhibit higher stress sensitivity than aluminum specimens due to inter-particle contact deformation (clastics), closure of micro-cracks and re-contact between debonded grains. Our measurement results show: Eagle Ford shale $\beta \approx 0.004$, Jubaila carbonate $\beta = 0.006$ to 0.013, and Berea sandstone $\beta = 0.029$ to 0.032 (Fig. 12). In comparison, the stress sensitivity of the propagation velocity is much higher in fractured rocks and granular media where contact mechanics controls stiffness and the $\beta$-exponent can range from $\beta \approx 0.2$ to $\beta > 0.5$ (Cha et al. 2014).
The quasi-static modulus is a low-frequency measurement, where the wavelength is much longer than the specimen height $\lambda > L$, and the duration of the measurement is much longer than any internal time scale in the system $T_{\text{meas}} > T_{\text{int}}$. Typically, the static modulus is obtained from uniaxial compression stress–strain data (ASTM International D7012-14 (2014)). On the other hand, the dynamic modulus results from high-frequency measurements, and short wavelengths so that the perturbation may interact with internal time and spatial scales in the specimen (impulse response, ASTM International E (1876)-15 (2015); ultrasonic ASTM International E494-15 (2015); sonic resonance, ASTM International E (1875)-13 (2013); resonance column, ASTM International D4015-15e1 (2015); cyclic loading, ASTM International D5311/D5311M-13 (2013)). Table 4 summarizes test conditions for commonly used dynamic techniques.

The dynamic modulus obtained with high-frequency wave propagation is typically higher than the static modulus because of: (1) ballistic wave propagation along high velocity pathways that avoid low-velocity zones in heterogeneous media including grain debonding, and cavities (Zisman 1933; Ide 1936; Simmons and Brace 1965; Cheng 1981); (2) global-vs-local drainage conditions (Biot 1956; Dvorkin et al. 1995); and (3) strain level (Fjær 2019).
Table 4 Overview of dynamic measurement techniques to derive elastic moduli

| Methods                     | Frequency range                        | Measurements                      | Excitation                                                                 |
|-----------------------------|----------------------------------------|-----------------------------------|----------------------------------------------------------------------------|
| Impulse response            | 100 Hz–50 kHz (depends on specimen stiffness and size) | Fundamental resonant frequency    | Mechanically by a singular elastic strike with an impulse tool |
| ASTM International E (1876)-15 (2015) |                                        |                                   |                                                                            |
| Ultrasonic                  | 1–20 MHz                               | Pulse transit time                | Low strain pulse using piezo transducers                                   |
| ASTM International E494-15 (2015) |                                        |                                   |                                                                            |
| Sonic resonance             | 100–30,000 Hz                          | Resonant frequency                | Variable-frequency audio oscillator. A speaker of the tweeter type or a magnetic cutting head |
| ASTM International E (1875)-13 (2013) |                                        |                                   |                                                                            |
| Resonance column            | 0.1–300 Hz (for soils)                 | Fundamental resonant frequency    | Flexural and longitudinal vibration using electromagnetic devices         |
| ASTM International D4015-15e1 (2015) |                                        |                                   |                                                                            |
| Cyclic loading              | 0.1–2 Hz                               | Strain and phase shift            | Loading–unloading using tri/uniaxial compression device                   |
| ASTM International D5311/D5311M-13 (2013) |                                        |                                   |                                                                            |

Figure 13 displays the static tangential modulus derived from strain gauge data (4 MPa deviatoric stress increment, strain level between 0.0005 and 0.0006), and the dynamic modulus computed from ultrasonic P- and S-wave velocities as a function of the deviatoric stress. Static and dynamic moduli follow the same loading trend with deviatoric stress \( \sigma_d \), however, the dynamic modulus is higher than the quasi-static modulus, \( E_{dy} / E_{st} \geq 1 \) in both the aluminum and shale specimen. For completeness, we include the static modulus estimated from cap-to-cap LVDTs deformation measurements: this static modulus is significantly lower and exhibits a markedly higher stress sensitivity due to the non-linear seating effects discussed above.

Figure 14 plots static \( E_{st} \) and dynamic \( E_{dy} \) moduli compiled from the literature; the different symbols distinguish local and cap-to-cap instrumentations (empirical relations for \( E_{dy} / E_{st} \) can be found in King 1983; Eissa and Kazi 1988; Christaras et al. 1994; Brautigam et al., 1998; Mockovčiaová and Pandula 2003; Soroush and Fahimifar 2003 and Brotons et al. 2014 and 2016). The figure includes values obtained as part of this research (Data shown at 15 MPa axial deviatoric stress only). Differences between dynamic and static moduli reduce when local strain measurements are used; in fact, previously reported \( E_{dy} / E_{st} \) ratios are due in part to inherent measurement biases. However, the ratio remains \( E_{dy} / E_{st} \geq 1.0 \) even when specimen-bonded strain gauges are used; therefore, internal spatial and time scales, strain level and propagation phenomena do affect the stiffness measured using wave propagation techniques.

Quasi-static strain rosette data and dynamic \( V_p/V_S \) measurements allow us to compute Poisson’s ratios. The dynamic values are slightly higher than the static values in the tested shale and carbonate: \( \nu_{dy} = 0.27 \) versus \( \nu_{st} = 0.25 \).
for the Eagle Ford shale, and $\nu_{\text{dyn}} = 0.24$ versus $\nu_{\text{st}} = 0.21$ for the Jubaila carbonates.

### 4.4 Impact on Biot’s $\alpha$-Parameter

Biot’s $\alpha$-parameter relates the applied total stress $\sigma$ and pore fluid pressure $P_p$ to the effective stress $\sigma'$ (Biot 1941),

$$\sigma' = \sigma - \alpha P_p$$  \hspace{1cm} (8)

Biot’s $\alpha$-parameter requires careful measurement of the skeleton $K_s$ and the grain $K_m$ bulk moduli:

$$\alpha = 1 - \frac{K_s}{K_m}$$  \hspace{1cm} (9)

Results presented above show that cap-to-cap LVDTs deformation measurements lead to a lower skeleton bulk modulus $K_s$ due to seating effects which result in higher $\alpha$-parameter values.

$$0 < \alpha_{\text{local}} < \alpha_{\text{global}} < 1$$  \hspace{1cm} (10)

While local strain measurements avoid the seating effects on stiffness, compliance effects at specimen-endcap interfaces still affect the pore pressure generation. Therefore, accurate polishing of endcaps and specimen surfaces will play a critical role in the determination of the $\alpha$-parameter.

### 5 Conclusions

1. Surface roughness at the specimen-endcap interfaces causes high local deformation, tilting and even specimen bending. These seating effects affect the measurement of all mechanical properties, from stiffness and Poisson’s ratio to Biot’s poroelastic $\alpha$-parameter. Seating effects are most pronounced at low normal stresses (relative to the specimen stiffness), that is, during the early stages of confinement and deviatoric loading.

2. Cap-to-cap LVDTs deformation measurements are particularly affected by seating effects. The measured stress–strain trends reflect significantly reduced stiffness and exhibit hysteresis. Measurement errors do not cancel when considering the average of diametrically opposite LVDT deformation measurements. The installation of deformation transducers over sleeve faces complies issues and biases measurements.

3. Local strain measurements using specimen bonded strain gauges avoid seating effects and most of its consequences. Sealing is critical and prone to leakage in standard triaxial configurations. Mounting strain gauges on sleeves is ill-advised due to compliance effects.

4. Proper surface polishing limits the impact of seating effects. Polishing the specimen and endcaps may be important even when using specimen-bonded strain gauges. For example, compliance at the specimen-endcap interfaces affects pore pressure generation and the determination of Biot’s $\alpha$-parameter.

5. High-resolution profilometry measurements of specimen surfaces and triaxial endcaps provide a direct assessment of the topography and its potential impact on triaxial test data. Small spot-size profilometer data identifies differences in surface roughness often missed with the large mechanical tips currently used in standard testing protocols.

6. Surface roughness and the mismatch at specimen-endcap interfaces reduce the transmitted ultrasonic energy. Whilst this does not affect ultrasonic first arrivals per se, low energy transmission may impact the identification of first arrivals and affect the computed modulus.

7. Accurate measurements of deformation and strain in triaxial tests allow the study of important rock phenomena, such as differences between dynamic and static moduli, and the stress sensitivity of the dynamic modulus observed even in intact rock specimens.

**Acknowledgements** Support provided by the KAUST Endowment at King Abdullah University of Science and Technology for this research. Gabrielle E. Abelskamp edited the manuscript.
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References

ASM Handbook (1990) Properties and selection: nonferrous alloys and special-purpose materials, 10th edn. ASM International, Materials Park, West Conshohocken, p 1300

ASTM International D4015–15e1 (2015) Standard test methods for modulus and damping of soils by fixed-base resonant column devices. ASTM, West Conshohocken. https://doi.org/10.1520/D4015-15E1

ASTM International D4543-08 (2008) Standard practices for preparing rock core as cylindrical test specimens and verifying conformance to dimensional and shape tolerances. ASTM, West Conshohocken. https://doi.org/10.1520/D4543-08

ASTM International D5311/D5311M-13 (2013) Standard test method for load controlled cycled triaxial strength of soil. ASTM, West Conshohocken. https://doi.org/10.1520/D5311_D5311M-13

ASTM International D7012-14 (2014) Standard test methods for compressive strength and elastic moduli of intact rock core specimens under varying states of stress and temperatures. ASTM, West Conshohocken. https://doi.org/10.1520/D7012-14

ASTM International E1875-13 (2013) Standard test method for dynamic young’s modulus, shear modulus, and poisson’s ratio by sonic resonance. ASTM, West Conshohocken

ASTM International E1876-15 (2015) Standard test method for dynamic young’s modulus, shear modulus, and poisson’s ratio by impulse excitation of vibration. ASTM, West Conshohocken. https://doi.org/10.1520/E1876-15

ASTM International E494–15 (2015) Standard practice for measuring ultrasonic velocity in materials. ASTM, West Conshohocken. https://doi.org/10.1520/E0494-15

Atkinson JH (2000) Non-linear soil stiffness in routine design. Géotechnique 50(5):487–508

Baldi G, Hight DW, Thomas GE (1988) State-of-the-art Paper: a Reevaluation of Conventional Triaxial Test Methods. In: Donaghe RT, Chaney RC, Silver ML (eds) Advanced Triaxial Testing of Soil and Rock. ASTM STP 97, Philadelphia, pp 219–263

Bhandari A, Powrie W, Harkness R (2012) A digital image-based deformation measurement system for triaxial tests. ASTM GeoTech Test J 35(2):209–226

Biot MA (1941) General theory of three-dimensional consolidation. J Appl Phys 12(2):155–164. https://doi.org/10.1063/1.1712886

Biot MA (1956) Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range. J Acoust Soc Am 28(2):168–178

Boyer R, Welsch G, Collings EW (1994) Materials properties handbook: titanium alloys. ASM International, Materials Park

Brautigam T, Knochel A, Lehne M (1998) Prognosis of uniaxial compressive strength and stiffness of rocks based on point load and ultrasonic tests. Otto-Graf-9:61–79

Brotons V, Tomas R, Ivorra S (2014) Grediaga A (2014) Relationship between static and dynamic elastic modulus of calcarenite heated at different temperatures: the San Julia’s stone. Bull Eng Geol Environ 73:791–799. https://doi.org/10.1007/s10064-014-0583-y

Brotons V, Tomas R, Ivorra S, Grediaga A, Martinez MJ, Benavente D, Gomez-Heras M (2016) Improved correlation between the static and dynamic elastic modulus of different types of rocks. Mater Struct 49:3021–3037

Cha M, Santamarina JC, Kim H, Cho G (2014) Small-strain stiffness, shear-wave velocity, and soil compressibility. J Geotech Geoenviron Eng 140(10):060140114

Cheng CH (1981) Dynamic and static moduli. Geophys Res Lett 8(1):39–42

Christaras B, Auger F, Mosse E (1994) Determination of the moduli of elasticity of rocks. Comparison of the ultrasonic velocity and mechanical resonance frequency methods with direct static methods. Mater Struct 27:222–228

Clayton CRI, Khatrush SA (1986) A new device for measuring local axial strains on triaxial specimens. Géotechnique 36(4):593–597

Dai S, Wuttte F, Santamarina JC (2013) Coda wave analysis to monitor processes in soils. J Geotech Geoenviron Eng 139(9):1504–1511

Dvorin J, Marko G, Nur A (1995) Squirt flow in fully saturated rocks. Geophysics 60(1):97–107

Elkataty A, Mahmoud M, Mohamed I, Abdulaheem A (2018) Development of a new correlation to determine the static Young’s modulus. J Pet Explor Prod Technol 8:17–30. https://doi.org/10.1007/s13202-017-0316-4

Fei W, Huiyun G, Jun Y, Yonghao Z (2016) Correlation of dynamic and static elastic parameters of rock. Electron J Geotech Eng 21(4):1551–1560

Filho LMC (1985) Measurement of axial strains in triaxial tests on London Clay. Geotech Test J 8(1):3–13. https://doi.org/10.1520/GTT10851J

Fjær E (2019) Relations between static and dynamic moduli of sedimentary rocks. Geophys Prospect 67:128–139

Gong F, Di B, Wei J, Ding P, Tian H, Han J (2019) A study of the anisotropic static and dynamic elastic properties of transversely isotropic rocks. Geophysics 84(6):C281–C293

Hird CC, Yung PCY (1989) Use of proximity transducers for local strain measurements in triaxial tests. Geotech Test J 12(4):292–296

Howarth DF (1984) Apparatus to determine static and dynamic elastic moduli. Tech Note Rock Mech Rock Eng 17:255–264

Ide JM (1936) Comparison of statically and dynamically determined Young’s modulus of rocks. Proc Natl Acad Sci 22:81–92

Isah BW, Mohamad H, Harahap ISH (2018) Measurement of small-strain stiffness of soil in a triaxial setup, review of local instrumentation. Int J Adv Appl Sci 5(7):15–26. https://doi.org/10.21833/ijaas.2018.07.003

Jardine RJ, Symes NJ, Burland JB (1984) The Measurements of soil stiffness in the triaxial apparatus. Géotechnique 34(3):323–340

King MS (1983) Static and dynamic elastic properties of rocks form the Canadian shield. Tech Note Int J Rock Mech Min Sci Geomech 20(5):237–241. https://doi.org/10.1016/0148-9622(84)90037-8

Kumar SS, Khrisna AM, Dey A (2016) Local strain measurements in triaxial tests using on Sample transducers. Indian Geotechnical Conference IGC2016, pp 1–4

Kung GTC (2007) Equipment and testing procedures for small strain triaxial tests. J Chin Inst Eng Series A Chung-Kuo Kung Ch’eng
Hsuch K’an 30(4):579–591. https://doi.org/10.1080/02533839.2007.9671287

Li L, Zhang X, Chen G, Lytton R (2016) Measuring unsaturated soil deformations during triaxial testing using a photogrammetry-based method. Can Geotech J 53(3):472–489. https://doi.org/10.1139/cjg-2015-0348

Mockovčiaková A, Pandula B (2003) Study of the relation between the static and dynamic moduli of rocks. Metalurgija 42(1):37–39

Simmons G, Brace WF (1965) Comparison of static and dynamic measurements of compressibility of rocks. J Geophys Res 70(22):5649–5656. https://doi.org/10.1029/JZ070i022p05649

Sone H, Zoback MD (2013) Mechanical properties of shale-gas reservoir rocks-part 1: Static and dynamic elastic properties and anisotropy. Geophysics 78(5):D381–D392

Soroush H, Fahimifar A (2003) Evaluation of some physical and mechanical properties of rocks using ultrasonic pulse technique and presenting equations between dynamic and static elastic constants. ISRM 2003–Technology roadmap for rock mechanics, pp 1113–1118

Symes M, Burland J (1984) Determination of local displacements on soil samples. Geotech Test J 7(2):49. https://doi.org/10.1520/GTJ10593J

Tatsuoka F (1988) Some recent developments in triaxial testing systems for cohesionless soils. In: Donaghe RT, Chaney RC, Silver ML (eds) Advanced triaxial testing of soil and rock. ASTM, West Conshohocken, pp 7–67

Thota J, Trabia M, O’Toole B (2015) Computational prediction of low impact shock propagation in a lab-scale space bolted frame structure. Int J Comput Methods Eng Sci Mech 3(2):139–149

Tisato N, Madonna C (2012) Attenuation at low seismic frequencies in partially saturated rocks: measurements and description of a new apparatus. J Appl Geophys 86:44–53. https://doi.org/10.1016/j.jappgeo.2012.07.008

van Heerden WL (1987) General relations between static and dynamic moduli of rocks. Int J Rock Mech Min Sci Geomech 24(6):381–385. https://doi.org/10.1016/0148-9062(87)92262-5

Wild KM, Barla M, Turinett G, Amann F (2017) A multi-stage triaxial testing procedure for low permeable geomaterials applied to Ophiolite Clay. J Rock Mech Geotech Eng 9(3):519–530

Xu DS (2017) A new measurement approach for small deformations of soil specimens using fiber Bragg grating sensors. Sensors 17(1016):1–13. https://doi.org/10.3390/s17051016

Xu DS, Borana L, Yin JH (2014) Measurement of small strain behavior of a local soil by fiber Bragg grating-based local displacement transducers. Acta Geotech 9(6):935–943. https://doi.org/10.1007/s11440-013-0267-y

Yimsiri S, Soga K, Chandler SG (2005) Cantilever-type local deformation transducer for local axial strain measurement in triaxial test. Geotech Test J 28(5):445–451. https://doi.org/10.1520/GTJ11432

Ziaja W (2009) Finite element modelling of the fracture behaviour of surface treated Ti-6Al-4V alloy. Assoc Comput Mater Sci Surf Eng 1(1):53–60

Zisman WA (1933) Comparison of the statically and seismologically determined elastic constants of rocks. Proc National Acad Sci 19:680–686

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