Homography Loss for Monocular 3D Object Detection

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Abstract

Monocular 3D object detection is an essential task in autonomous driving. However, most current methods consider each 3D object in the scene as an independent training sample, while ignoring their inherent geometric relations, thus inevitably resulting in a lack of leveraging spatial constraints. In this paper, we propose a novel method that takes all the objects into consideration and explores their mutual relationships to help better estimate the 3D boxes. Moreover, since 2D detection is more reliable currently, we also investigate how to use the detected 2D boxes as guidance to globally constrain the optimization of the corresponding predicted 3D boxes. To this end, a differentiable loss function, termed as Homography Loss, is proposed to achieve the goal, which exploits both 2D and 3D information, aiming at balancing the positional relationships between different objects by global constraints, so as to obtain more accurately predicted 3D boxes. Thanks to the concise design, our loss function is universal and can be plugged into any mature monocular 3D detector, while significantly boosting the performance over their baseline. Experiments demonstrate that our method yields the best performance (Nov. 2021) compared with the other state-of-the-arts by a large margin on KITTI 3D datasets.

1. Introduction

Monocular 3D object detection is a fundamental task in computer vision, where the goal is to localize and estimate 3D bounding boxes, parameterized by location, dimension, and orientation, of objects from a single image. It can be applied to various scenes, such as autonomous driving, robotic navigation, etc. However, it is an ill-posed and challenging problem since a single image cannot provide explicit depth information. To acquire such resources, most existing methods resort to LiDAR sensors to obtain accurate depth measurements \cite{17}, or stereo cameras for stereo depth estimation \cite{15}, but they will increase the cost of practical usages. In comparison, the monocular camera is cost-effective.

Most of the existing monocular 3D object detection methods have already achieved remarkable high accuracy with fixed camera settings. However, in their training strategies, each 3D object in the scene is treated as an individual sample without considering the mutual relationships with other neighboring objects, for example, as shown in Fig. 1(a). Assuming that, if the predicted 3D box of a single object obviously deviates from its ground truth, without additional constraints, it is usually hard for the network to refine and correct the estimated position of this specific sample. To handle this, apart from the regression loss defined by minimizing the discrepancies between the predicted 3D boxes and the ground truths, many algorithms propose projection loss \cite{15, 17, 25, 26} to constrain the optimization of 3D boxes with the supervision of corresponding projected 2D ground truth boxes. However, the 3D localization of a single object is still independent of the others. Differently, MonoPair \cite{7} exploits the object relationships and builds scene graph to enhance the mutual connections of objects during training and inference. They fully leverage the spatial relationships between close-by objects instead of individually focusing on the information-constrained single object. An obvious drawback is that an object can only locally connect with its nearest neighbor.

On the other hand, a large percent of approaches are effective for normal objects. In reality, only the foreground...
objects can be detected easily, because they are fully visible and have rich recognizable features. Therefore, these approaches still struggle to handle the occluded objects or small ones that are far away from the camera, and those objects usually occupy a higher proportion in the scene. Limited improvement is achieved since little information is helpful to solve the problem. A straightforward way to improve the 3D detection is to correct the results by the foreground objects or even the 2D detection results. The most relevant work, MonoFlex [42], which leverages the distribution of different objects and proposes a flexible framework to decouple the truncated objects and adaptively combine multiple approaches for 3D detection. However, it is also limited to training the network for each individual sample.

Moreover, due to the perspective projection, objects with different depths may block each other in image space. Thus, OFTNet [33] and ImVoxelNet [34] propose to regress 3D positions on Bird’s Eye View (BEV), since objects on the projected BEV plane do not intersect with each other and can be distinguished.

In general, to be concrete as shown in Fig. 1, our core idea is to build the connections between all the objects and globally optimize their 3D positions. Besides, we also associate BEV with image view through inverse projective mapping and apply 2D detection results as guidance to improve the 3D localization in BEV. To achieve the goal, we propose Homography Loss to combine 2D and 3D information and globally balance the mutual relationships to obtain more accurate 3D boxes. By doing so, our loss function is able to effectively encode necessary geometric information in both 2D and 3D space, and the network will be enforced to explicitly capture the global geometric relationships between objects which are demonstrated to be helpful for 3D detection. Because of the differentiability and interpretability, our loss function can be plugged into any monocular 3D detector. Practically, we take ImVoxelNet [34] and MonoFlex [42] as examples, and integrate the novel homography loss during training phase, experiments demonstrate that our method outperforms the state-of-the-arts by a large margin on KITTI 3D detection benchmark (Nov. 2021). The main contributions can be summarized as follows:

1. We propose a novel loss function, termed as homography loss, to exploit geometric relationships of all the objects in the scene and globally constrain their mutual locations, by using the homography between the image view and the Bird’s Eye View. At the same time, the geometric consistency in both 2D and 3D space will be well preserved. To the best of our knowledge, this is the first work that fully leverages the global geometric constraints in monocular 3D object detection.

2. The proposed monocular 3D detector based on homography loss achieves the state-of-the-art performance on KITTI 3D detection benchmark, and surpasses the results of all the other monocular 3D detectors, which implies the superiority of our loss.

- We apply this loss function to several popular monocular 3D detectors. Without any additional inference cost, the training is more stable and easier to converge, achieving higher accuracy and performance. It can be a plug-and-play module and be adapted to any monocular 3D detector.

2. Related Work

We first review methods on monocular 3D object detection, followed by a brief introduction of geometric constraints that are commonly used during training phase.

Monocular 3D object detection is an ill-posed problem because of lacking depth clues of the monocular 2D image. When compared with stereo images [15] or LiDAR-based methods [23, 27, 30, 32, 39, 40], in some earlier works, auxiliary information are necessary for monocular 3D detection to achieve competitive results. These prior knowledge usually includes ground plane assumption [5], morphable wire-frame model hypothesis [13] or 3D CAD model [3, 14], etc.

Moreover, some other works only take a single RGB image as input. For example, Deep3DBox [25] estimates the 3D pose and dimension from the image patch enclosed by a 2D box. Afterwards, the network with a 3D regression head [9, 18, 26] is used to predict the 3D box while searching and filtering the proposal whose 2D projection has the threshold overlap with the ground-truth 2D box. MonoGRNet [31] detects and localizes 3D boxes via geometric reasoning in both the observed 2D projection and the unobserved depth dimension. MonoDIS [36] leverages a novel disentangling transformation for 2D and 3D detection losses. M3D-RPN [1] reformulates the monocular 3D detection problem as a standalone 3D region proposal network. Unlike previous methods, which depend on 2D proposals, SMOKE [19] argues that the 2D detection network is redundant and introduces non-negligible noise in 3D detection. Thus, it predicts a 3D box for each object by combining a single keypoint estimated with regressed 3D variables via a single-stage detector, and similarly, RTM-3D [17] predicts nine perspective keypoints of a 3D box in the image space. Specifically, MonoFlex [42] proposes a flexible framework for monocular 3D object detection that explicitly decouples the truncated objects and adaptively combines multiple approaches for depth estimation.

However, image-based training and inference will introduce non-linear perspective distortion where the scale of objects varies drastically with depth, which makes it hard to accurately predict the relative distance and location of the object of interest. To handle this, OFTNet [33] proposes orthographic feature transform by mapping image-based fea-
3.1. Motivation

We have two key observations: 1) the 2D detection can serve as a guidance to constrain and supervise the training of 3D localization, 2) the position of a single object should be globally influenced by the surrounding objects, as detailed in Fig. 2 and 3. To handle those problems, we propose homography loss to implement the conversion from 2D image space to 3D BEV space, and simultaneously constrain the globally geometric relationships of all the objects.

3.2. Revisiting of Homography

A homography is a mapping between two planar surfaces which preserves collinearity. The homography matrix \( H \in \mathbb{R}^{3 \times 3} \) between two 2D planes maps \( p_1 \) in the plane 1 to \( p_2 \) in the plane 2 up to a scale factor \( s \). It satisfies:

\[
s p_2 = H p_1,
\]

where \( p = [x, y, 1]^T \) is the homogeneous coordinate of a 2D point in a plane. Since the homography matrix has 8 degrees of freedom, at least 4 corresponding point pairs are necessary for recovering the matrix. Inspired by ImVoxelNet [34], the projections of objects on BEV plane do not intersect with each other and accordingly contain more information about 3D localization, we define the homography matrix between the image plane and BEV plane, in order to implicitly transform coordinates from 2D to 3D space. More details will be illustrated in Sec. 3.3. Then, let us explain why homography is a global geometric constraint. Firstly, all pairs of corresponding points will involve in solving the homography matrix from Eq. 1, and the solution is guaranteed to be globally optimal. In other words, the constraint enforced by arbitrary pair of corresponding points will finally affect the whole optimization process. Thus, homography is a global constraint. Secondly, in projective geometry, a homography is an isomorphism of projective spaces, which correlates a group of points on one plane to the other and preserves geometric properties, e.g., collinearity. So, homography is also a geometric constraint.
3.3. Homography Loss

Inspired by those observations, we propose a global loss function, termed as homography loss, aiming to establish the geometric connections among all the objects by leveraging the homography matrix. Assuming that we already have a monocular 3D object detector that could predict 3D boxes under the supervision of the ground truths, in addition to the regular classification and regression loss in the common pipelines, our homography loss penalizes the wrong relationship among all the predicted boxes and refines the final locations. The major steps are listed as follows.

Candidate Points Modeling. Suppose we have the predicted boxes box\textsubscript{pred} obtained from the arbitrary 3D detector and the corresponding ground truth boxes box\textsubscript{gt}. As mentioned in Sec. 3.2, we opt to use the homography matrix to describe the projection relationship between the image plane and the BEV plane. For a single object, as demonstrated in Fig. 4, we pick up five bottom points q\textsubscript{pred} = [x\textsubscript{pred}, y\textsubscript{pred}, z\textsubscript{pred}]\textsuperscript{T} of box\textsubscript{pred} as representatives, including one bottom center point and four bottom corner points. We also assume that all the objects are always on the flat ground, the bottom points on the BEV plane can thus be simplified as Q\textsubscript{pred} = [x\textsubscript{gt}, y\textsubscript{gt}, z\textsubscript{gt}]\textsuperscript{T} obtained from box\textsubscript{gt}. After the camera projection, the ground truth 3D box will be transformed into the image space, which is defined by:

\[ q = K [R|t] Q, \]

where K is the intrinsic matrix and [R|t] are the extrinsic matrices, and q = [u, v]\textsuperscript{T} represents the projected pixel on the image plane, which is suitable for both box\textsubscript{pred} and box\textsubscript{gt}. Therefore, if there exist N objects, we can get 5N pairs of candidate points q\textsubscript{pred}, Q\textsubscript{pred} for box\textsubscript{pred} and q\textsubscript{gt}, Q\textsubscript{gt} for box\textsubscript{gt}, respectively, which are prepared for calculating the homography matrix.

Calculating Homography. To implicitly constrain relative positions of each object, without loss of generality, we select q\textsubscript{gt} and Q\textsubscript{pred}. Specifically, we use the ground truth coordinates q\textsubscript{gt} in 2D image view as guidance, to correct the final positions Q\textsubscript{pred} in 3D space. The formulation is defined, up to a scale factor (omitted here) with homoge-neous coordinates, as follows,

\[ \tilde{Q}_{pred} = Hq_{gt}, \left( \begin{array}{c} x_{pred} \\ y_{pred} \\ 1 \end{array} \right) = H \left( \begin{array}{c} u_{gt} \\ v_{gt} \\ 1 \end{array} \right). \]

Here, H stores the mutual relationships of all the objects by mapping between two views. We use singular value decomposition (SVD) to calculate the homography matrix H as it can be easily implemented in PyTorch \cite{28}.

In practice, the homography matrix in Eq. 3 is estimated since Q\textsubscript{pred} may deviate a lot from the ground truth at the very beginning of training. We denote it as \( \hat{H} \), and represent \( Q_{homo} = \hat{H}q_{gt} \). As the training progresses, the estimated value \( Q_{homo} \) will approach \( Q_{pred} \) and \( Q_{gt} \).

Loss Function. The homography matrix \( \hat{H} \) implicitly contains the correspondences between two different views and the relative positions of all the objects. Previously, 3D detection is treated as an independent task for each object, which is constrained by regression loss, such as \( L_{reg} = \text{L1}(\hat{Q}_{gt} - Q_{pred}) \). Here, we propose a novel loss function, named as homography loss, to optimize the locations with strong spatial constraints. The homography loss is defined as follows,

\[ L_{homo} = \text{SmoothL1}(\hat{Q}_{gt} - \hat{Q}_{homo}) \]
\[ = \text{SmoothL1}(Q_{gt} - \hat{H}q_{gt}). \]

Different from the regression loss, calculating the homography matrix \( \hat{H} \) will take all pairs of corresponding points into consideration. It is therefore a global loss for geometric constraint, which is used to guide the prediction of 3D positions from the ground truth 2D localization. On the other hand, by optimizing Eq. 4, \( \hat{H} \) is also enforced to be closer to the ground truth homography matrix. Another advantage of homography loss is that it is differentiable. It can be a plug-and-play module for any monocular 3D object detector, and serves as a strong spatial constraint for 3D localization of objects.

3.4. Case Study

As our novel homography loss can be plugged into any 3D object detector, we take the state-of-the-art detectors, ImVoxelNet \cite{34} and MonoFlex \cite{42}, as examples, and illustrate how to seamlessly integrate our loss function into the network. As the main algorithm has been explained in Sec. 3.3, more details of the selection of predicted boxes and training strategies are presented here.

Anchor based method. ImVoxelNet \cite{34} is a one-stage anchor-based monocular 3D detector, which transforms 2D image features into 3D space and regresses the positions of objects in BEV like LiDAR-based 3D detectors. Anchors
with IoU > 0.6 will be considered as positives for training and each ground truth object will be assigned by several positive anchors that are served as potential proposals.

To calculate homography, we need to specify one-to-one matching point pairs for the predicted boxes and the ground truth boxes. Therefore, we choose the one with the highest classification score from positive proposals as a representative, which also keeps the consistency between classification and regression. As anchor-based detectors always produce stable proposals during training, we add the homography loss at the beginning of training and train the network from scratch. The loss function defined below consists of four parts, i.e., location loss \(L_{\text{loc}}\), focal loss for classification \(L_{\text{cls}}\), cross-entropy loss for direction \(L_{\text{dir}}\), and additional homography loss \(L_{\text{homo}}\):

\[
L = \frac{1}{N_{\text{pos}}} (\lambda_{\text{cls}} L_{\text{cls}} + \lambda_{\text{loc}} L_{\text{loc}} + \lambda_{\text{dir}} L_{\text{dir}} + \lambda_{\text{homo}} L_{\text{homo}}),
\]

where \(N_{\text{pos}}\) is the number of positive anchors, \(\lambda_{\text{cls}} = 1.0, \lambda_{\text{loc}} = 2.0, \lambda_{\text{dir}} = 0.2, \lambda_{\text{homo}} = 0.2\). Note that, apart from \(L_{\text{homo}}\), other loss terms and balancing weights are all adopted from [34].

**Anchor-free based method.** MonoFlex [42] is a one-stage monocular 3D detector based on CenterNet [43], which predicts projected 3D center, box (including depth, dimension, and orientation), and keypoints in different heads. As it is an anchor-free detector, the location of the representative box is automatically assigned as the 3D projected center in the heatmap head without selection. And the depth is regressed in the final head. The main difference is the training policy.

As 3D projected center and depth can define the coordinates in the image view and Bird’s Eye View, these two components are the main contributors for homography loss. But the depth head is very unstable at the beginning of training, and the locations in the Bird’s Eye View is also of low confidence, making the homography matrix distorted. Therefore, two strategies are proposed to solve the problem. Firstly, we make a delay by adding our homography loss after 40 epochs when the depth head is consistent and reliable. Secondly, we replicate the predicted boxes by using one of the components (3D projected center and depth), while replacing the other one with its ground truth values. Therefore, homography loss can be replicated three times and ensembled together. The main loss function can be described as a combination of classification loss for heatmap \(L_{\text{hm}}\), regression loss for box size and rotation \(L_{\text{box}}\), regression loss for keypoints of 3D boxes \(L_{\text{kp}}\), and additional homography loss \(L_{\text{homo}}\):

\[
L = \frac{1}{N_{\text{pos}}} (\lambda_{\text{hm}} L_{\text{hm}} + \lambda_{\text{box}} L_{\text{box}} + \lambda_{\text{kp}} L_{\text{kp}} + \lambda_{\text{homo}} L_{\text{homo}}),
\]

where \(N_{\text{pos}}\) is the number of positive predictions, \(\lambda_{\text{hm}} = 1.0, \lambda_{\text{box}} = 1.0, \lambda_{\text{kp}} = 1.0, \lambda_{\text{homo}} = 0.2\).

| Method            | Extra Data | \(\text{AP}_{\text{3D}}/\text{Box}\) | \(\text{AP}_{\text{BEV}}/\text{Box}\) | Time(s) |
|-------------------|------------|------------------------------------|--------------------------------------|---------|
| Mono-PLiDAR [40]  | Depth      | 10.76 7.50 6.10 21.27 13.92 11.25| 0.10                                 |         |
| PatchNet [22]     | Depth      | 15.68 11.12 10.17 22.97 16.86 14.97| 0.40                                 |         |
| D4LICN [8]        | Depth      | 16.65 11.72 9.51 22.51 16.03 12.55| 0.20                                 |         |
| MonoRUn [4]       | Depth      | 19.65 12.30 10.58 27.94 17.34 15.24| 0.07                                 |         |
| Kinematic3D [2]   | Temporal   | 19.07 12.72 9.17 26.69 17.52 13.10| 0.12                                 |         |
| DDMP-3D [17]      | Depth      | 19.71 12.78 9.80 28.08 17.89 13.44| 0.18                                 |         |
| Aug3D-RPN [11]    | Depth      | 17.82 12.99 9.78 26.00 17.89 14.18| 0.08                                 |         |
| DFR-Net [45]      | Depth      | 19.40 13.63 10.35 28.17 19.17 14.84| 0.18                                 |         |
| CaDDN [32]        | LiDAR      | 19.17 13.41 11.46 27.94 18.91 17.19| 0.63                                 |         |
| MonoEF [44]       | Depth      | 21.29 13.87 11.71 29.03 19.70 17.26| 0.03                                 |         |
| Autoshape [20]    | Shape      | 22.47 14.17 11.36 30.06 20.08 15.59| 0.04                                 |         |
| MID-RPN [1]       |            | 14.76 9.71 7.42 21.02 13.67 10.23| 0.16                                 |         |
| SMOKE [19]        |            | 14.03 9.76 7.84 20.83 14.49 12.75| 0.03                                 |         |
| MonoPair [7]      |            | 13.04 9.99 8.65 19.28 14.83 12.89| 0.06                                 |         |
| RTM3D [17]        |            | 14.41 10.34 8.77 19.17 14.20 11.99| 0.05                                 |         |
| PGD-FCOS3D [38]   |            | 19.05 11.76 9.39 26.89 16.51 13.49| 0.03                                 |         |
| M3DSSD [21]       |            | 17.51 11.46 8.98 24.15 15.93 12.11| 0.16                                 |         |
| MonodLE [24]      |            | 17.23 12.26 10.29 24.79 18.89 16.00| 0.04                                 |         |
| MonoRCNN [35]     |            | 18.36 12.65 10.03 25.48 18.11 14.10| 0.07                                 |         |
| ImVoxelNet [34]   |            | 17.15 10.97 9.15 25.19 16.37 13.58| 0.20                                 |         |
| ImVoxelNet+(homo) |            | 20.10 12.99 10.50 29.18 19.25 16.21| 0.20                                 |         |
| MonoFlex [42]     |            | 19.94 13.89 12.07 28.23 19.75 16.89| 0.03                                 |         |
| MonoFlex+(homo)   |            | 21.75 14.94 13.07 29.60 20.68 17.81| 0.03                                 |         |
Table 2. 3D object detection performance of Car category on KITTI validation set.

| Method       | AP_{3D,R_{40}} | AP_{BEV,R_{40}} |
|--------------|----------------|-----------------|
| M3D-RPN [1]  | 14.53          | 8.65            |
| MonoPair [7] | 16.28          | 10.42           |
| MonoRCNN [35]| 16.61          | 10.65           |
| MonoDLE [24] | 17.45          | 11.68           |
| ImVoxelNet(+homo) | 21.44       | 12.08           |
| MonoFlex(+homo) | 23.04       | 14.90           |

Table 3. 3D object detection performance of Pedestrian and Cyclist on KITTI test set.

| Method            | Pedestrian AP_{3D,R_{40}} | Cyclist AP_{3D,R_{40}} |
|-------------------|---------------------------|------------------------|
| PGD-FCOS3D [38]   | 2.28                      | 1.38                   |
| MonoEF [64]       | 4.27                      | 2.79                   |
| D4LCN [8]         | 4.55                      | 3.42                   |
| M3D-RPN [1]       | 4.92                      | 3.84                   |
| DDMP-3D [37]      | 4.93                      | 3.55                   |
| DFR-Net [45]      | 6.09                      | 3.62                   |
| M3DSSD [21]       | 5.16                      | 3.87                   |
| Aug3D-RPN [11]    | 6.01                      | 4.71                   |
| MonoFlex [42]     | 9.43                      | 6.31                   |
| MonoPair [7]      | 10.02                     | 6.68                   |
| MonoRUn [4]       | 10.88                     | 6.78                   |
| ImVoxelNet(+homo) | 12.47                     | 7.62                   |
| MonoFlex(+homo)   | 11.87                     | 7.66                   |

4. Experiments

4.1. Setup

Dataset and Evaluation Metrics. Our proposed method is evaluated on KITTI 3D Object Detection benchmark [10], which includes 7481 images for training and 7518 images for testing. The training set is split into 3712 samples for training and 3769 samples for validation as suggested in [6]. The classes are Car, Pedestrian, and Cyclist with three difficulty levels for each class, i.e., Easy, Moderate, and Hard. The official KITTI leaderboard is ranked on Moderate difficulty. Our method is evaluated on KITTI test set by submitting the detection results to the official server. For a fair comparison with other methods, we use official metrics, average precision (AP) with an IoU threshold of 0.7 for Car and 0.5 for both Pedestrian and Cyclist. In all experiments, the AP_{3D,R_{40}} results are reported for a comprehensive comparison with previous studies.

Implementation Details. We use the official implementations of ImVoxelNet [34] with ResNet50 [12] and MonoFlex [42] with DLA34 [41] as their backbones. We follow all the experimental settings of the original code and add our homography loss as an auxiliary loss. For ImVoxelNet [34], we add the loss at the beginning and train 24 epochs. As for MonoFlex [42], the homography loss is added after 40 epochs and we train the network 80 epochs in total. We name these two new implementations as ImVoxelNet(+homo) and MonoFlex(+homo), respectively.

4.2. Quantitative Results

Results of Car category on KITTI test set. As demonstrated in Tab. 1, the proposed method MonoFlex(+homo) achieves superior results on Car category compared with the previous methods, even including those with extra data, such as depth or LiDAR point clouds. To be specific, MonoFlex(+homo) achieves 1.81%, 1.05%, and 1.00% gains on the easy, moderate and hard settings, respectively. Besides, our ImVoxelNet(+homo) achieves 2.95%, 2.02%, and 1.45% gains over the original baseline, which shows its robustness and effectiveness.

Results of Car category on KITTI validation set. We also present our model’s performance on the KITTI validation set in Tab. 2. Specifically, our method achieves the SOTA performance compared with the previous methods. Compared to MonoPair [7], our ImVoxelNet(+homo) and MonoFlex(+homo) get performance gain by 2.58%/4.59% for moderate setting at the 0.7 IoU threshold. This shows that our method is more capable of detecting hard examples in autonomous driving scenes by adding homography loss as an additional constraint.

Pedestrian/Cyclist detection on KITTI test set. For Pedestrian and Cyclist, we present the detection performance in Tab. 3. Our method MonoFlex(+homo) leads to the competitive performance in both categories. This shows our homography loss can also improve the performance for detecting small objects, e.g., human. MonoFlex(+homo) outperforms all other approaches in the Pedestrian category, with an 0.88% improvement from the previous best method (7.66% vs 6.78%). A possible reason is that human’s standing point is a more reliable reference for computing the homography matrix.

4.3. Ablation Study

We conduct ablation studies to analyze the effects of our loss on Car category of the KITTI validation set. The default evaluation metric is AP_{3D|R_{40}}.

4.3.1 Calculating Homography

To calculate the homography matrix, we use q_{gt} and Q_{pred} (Type 1) to construct the geometric constraints. Similarly, q_{pred} and Q_{gt} (Type 2) can also be selected. Therefore, we compare the performance of these two types in ImVoxelNet(+homo) and MonoFlex(+homo). The results are listed in Tab. 4 and 5. We can see that for those methods that predict in BEV domain like ImVoxelNet, Type 2 is more suit-
able. As for those who predict in 2D images like MonoFlex, Type 1 gets higher performance. Therefore, the prediction domain can affect the final performance. So how to choose a proper type will finally depend on the specific application.

### 4.3.2 Representative Proposals

In anchor-based methods like ImVoxelNet, several anchors will be assigned to the same ground truth box based on IoU threshold. Therefore, we need to select the representative proposal from these positive proposals. Here, we have three strategies of selection: 1) the proposal with the highest classification score, 2) the proposal with the highest IoU score, 3) the average proposal of all positive anchors. We conduct the ablation experiment in Table 4. The result shows that the one with the highest classification score achieves the best performance at 14.88% of the moderate setting. It also shows our homography loss can strengthen the consistency between regression and classification heads.

### 4.3.3 Replicated Losses

For anchor-free methods, such as MonoFlex, the depth regression head can be very unstable at the beginning of training. To solve this problem, we refer to the replicated strategy in [27] and propose a replicated proposal strategy here to strengthen the robustness. The homography loss is replicated 3 times in total to get a reliable homography matrix. We conduct the ablation by four different settings: 1) $q_{pred} + \text{Depth}_{pred}$ (the predicted depth), 2) $q_{pred} + \text{Depth}_{gt}$ (the ground truth depth), 3) $q_{gt} + \text{Depth}_{pred}$, 4) ensemble by adding the aforementioned three losses together. The results are shown in Table 5. We observe that the ensemble strategy has a better result due to sufficient constraints.

### 5. Discussions

#### 5.1. Difference with Projection Loss

As shown in Eq. 2, the calibration parameters of the camera can be used to project a single predicted 3D keypoint onto a 2D image plane which will be further constrained by its corresponding 2D ground-truth value. It means that each training sample is considered individually, and the predicted 3D positions are also refined and optimized independently during network training. This is the key idea of the commonly used projection loss. However, for calculating the homography matrix, all pairs of correspondences will be involved in the computation, each pair of corresponding 2D/3D points will contribute two linear equations for solving Eq. 3. During backpropagation of the gradient of Eq. 4, $H$ is gradually optimized, that is to say, all the predicted $Q_{pred}$ that are used for calculating the homography matrix will also be refined according to the chain rule. Therefore, homography loss can be leveraged to globally constrain the optimization of 3D localization. We compare projection loss with the proposed homography loss as shown in Table 4.
Figure 5. We visualize the results of 3D object detection using ImVoxelNet(+homo) on KITTI val set, where the orange represents the ground truth and our predicted results are colored in blue. The left column shows results of the network trained on the Car category only, and the right column is trained on three categories including Car, Pedestrian, and Cyclist. It is worth noting that, with the homography loss, it is possible to detect small targets and even truncated objects.

Table 6. Depth range statistics at the 0.7 and 0.5 IoU threshold.

| Metric       | ImVoxelNet | Depth Range (m) |
|--------------|------------|-----------------|
|              |            | 0-10  10-20  20-30 >30 |
| KITTI        | baseline@0.7 | 35.45 17.48 1.23 0.17 |
| Moderate     | +homo@0.7   | 35.99 20.48 2.17 0.20 |
| AP_{3D|R|40}  | baseline@0.5 | 78.57 59.34 15.24 3.31 |
|              | +homo@0.5   | 81.57 61.68 18.44 4.07 |

5.2. Depth Range Statistics

In order to investigate why homography loss is useful for improving the accuracy of 3D detection. We design an experiment that divides the depth range into several segments as shown in Tab. 6 and gets the statistics for each interval. For fairness, the evaluation metric is also AP_{3D|R|40} on the Car category of the KITTI validation set with the difficulty of moderate level. Obviously, we can see that in the area of 10 meters away, the effect of the detection algorithm with homography loss is much better than that of the baseline. Especially in the range of 10-20m, we obtain 3.0% and 2.34% gains over the baseline method with different IoU thresholds, respectively. This shows that our loss function is more effective for small target detection. The reason is that, as elaborated in Eq. 3, the ground truth 2D position q_{gt} on the image plane is used as guidance to correct the predicted 3D position \( \hat{Q}_{pred} \). The relative geometric relationship of objects on the image plane will be transferred to the corresponding 3D objects on BEV plane by homography loss. In other words, the improvement of the detection effect of distant objects in Tab. 6 is due to the homography relationship, which refines the inaccurate estimated 3D positions to satisfy the overall geometric constraint.

5.3. Limitations

As stated in Sec. 3.3, we assume that the ground plane is flat and use the simplified 2D coordinates \( Q = [x, y]^T \) on BEV plane to replace the original 3D points \( Q = [x, y, z]^T \). However, in practice, as pointed out in [44], usually the road is not smooth and has slight fluctuation, it will influence the accuracy of 3D detection.

6. Conclusion

In this paper, we propose a differentiable loss function, named as homography loss, which is a plug-and-play module that can be integrated into any monocular 3D detector, to help globally optimize the 3D positions of all the objects, instead of taking each object as an independent sample during training. Homography loss also fully exploits the inherent connection between 2D image space and 3D Bird’s Eye View and constrains the optimization of 3D positions under the guidance of 2D localization, which is demonstrated to be useful for detecting small targets or highly truncated objects. In the future work, we will consider how to avoid the assumption of flatness of the ground.

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