THE SCATTERING TRANSFORM NETWORK WITH GENERALIZED MORSE WAVELETS AND ITS APPLICATION TO MUSIC GENRE CLASSIFICATION

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Abstract:

We propose to use the Generalized Morse Wavelets (GMWs) instead of commonly-used Morlet (or Gabor) wavelets in the Scattering Transform Network (STN), which we call the GMW-STN, for signal classification problems. The GMWs form a parameterized family of truly analytic wavelets while the Morlet wavelets are only approximately analytic. The analyticity of underlying wavelet filters in the STN is particularly important for nonstationary oscillatory signals such as music signals because it improves interpretability of the STN representations by providing multiscale amplitude and phase (and consequently frequency) information of input signals. We demonstrate the superiority of the GMW-STN over the conventional STN in music genre classification using the so-called GTZAN database. Moreover, we show the performance improvement of the GMW-STN by increasing its number of layers to three over the typical two-layer STN.

Keywords:
Generalized Morse Wavelets; Analytic Wavelet Transform; Scattering Transform; Music Genre Classification

1. Introduction

A Convolutional Neural Network (CNN), in particular, its “deeper” version, a Deep Neural Network (DNN), has been shown to be effective in the extraction of hierarchical features for many applications with large training datasets [1]. However, there is a lack of interpretability of the DNN outputs. Also, the DNN does not demonstrate a good performance without having a large dataset due to model overfitting.

On the other hand, the Scattering Transform Network (STN), which has a similar architecture with the CNN, can generate more interpretable representations of input data. In addition, the STN works reasonably well on smaller datasets because the convolutional filters are pre-constructed using the established techniques from signal processing. Originally, Mallat proposed the STN to connect the wavelet theory with the CNN, and showed that the STN generates a quasi-translation-invariant signal representation with a cascade of wavelet filtering and modulus nonlinearities [2]. Bölcskei and Wiatowski showed that with increasing depth, the STN achieves better translation invariance [3]. Numerically, Bruna and Mallat illustrated the capability of the STN for texture image classification [4, 5].

Most of these previous works on the STN used the Morlet (or Gabor) wavelet filters [6, Sec. 4.3], which are only approximately analytic. The analyticity of the wavelet filters is quite important for input signals that are nonstationary and oscillatory because it allows us to represent them in terms of amplitude, phase, and frequency in a multiscale manner [7].

In this paper, we propose to use the so-called Generalized Morse Wavelet (GMW) filters [8, 9] as the wavelet filters in the STN instead of the commonly-used Morlet wavelet filters. The GMW provides a parameterized family of truly analytic wavelets, and adopting this in the STN framework should give us better performance in classifying input signals and interpreting their STN representations.

A major application of our proposed method is music genre classification. The input data here are recorded digital music signals, which are nonstationary and quite oscillatory. Hence, we should be able to see clear advantages of using the GMWs over the Morlet wavelets.

2. Wavelet and Scattering Transforms

In this section, we will introduce the mathematical methods that are essential to help us process the music/audio signals. These also serve as the foundation to understand the motivation of our new method for music signal classification.
2.1. Generalized Morse Wavelets (GMWs)

Let us review the concept of analytic wavelets for signal analysis. We will also describe the properties of the Generalized Morse Wavelets (GMWs) and the Continuous Wavelet Transform (CWT), both of which are crucial for the STN in Section 2.2.

A (mother) wavelet \( \psi(t) \in L^2(\mathbb{R}) \) is a function whose dilated and translated versions provide a method to perform localized time-frequency analysis of nonstationary oscillatory signals, such as audio and music signals [6, Sec. 4.3]. A wavelet is said to be analytic if it is complex-valued with vanishing support on negative frequencies, see, e.g., [6, Sec. 4.3]. The Continuous Wavelet Transform (CWT) of a signal \( g(t) \in L^2(\mathbb{R}) \) with respect to the mother wavelet \( \psi \) is given by

\[
W_\psi g(a, b) := \frac{1}{\sqrt{a}} \int_{\mathbb{R}} g(t) \psi \left( \frac{t-b}{a} \right) dt,
\]

for any \( a \in \mathbb{R}_+ := \{ t \in \mathbb{R} \mid t > 0 \}, b \in \mathbb{R} \). The CWT is called the Analytic Wavelet Transform (AWT) when \( \psi \) is analytic.

A follow-up question that we have to address is which analytic wavelet is suitable in practice. The Morlet wavelet was extensively used in the STN literature and software implementation. However, Lilly and Olhede [9, 10] demonstrated numerically that even small leakage to negative frequencies in the Morlet wavelet can lead to abnormal transform phase variation.

On the contrary, the Generalized Morse Wavelets (GMWs) is a promising superfamily of truly analytic wavelets [8, 9]. In the frequency domain, the GMW is defined as

\[
\Psi_{\beta, \gamma}(\omega) := \int_{\mathbb{R}} \psi_{\beta, \gamma}(t) e^{-i\omega t} dt = H(\omega) \alpha_{\beta, \gamma} \omega^\beta e^{-\omega^\gamma},
\]

where \( \beta > 0, \gamma > 1 \) are two main parameters, \( \alpha_{\beta, \gamma} \) is a normalization constant, and \( H(\omega) \) is the Heaviside step function. The parameters \( \beta \) and \( \gamma \) control the time-domain and frequency-domain decay, respectively. The peak frequency \( \omega_{\beta, \gamma} := (\beta/\gamma)^{1/\gamma} \) is the frequency at which the derivative of \( \Psi_{\beta, \gamma} \) vanishes [9, 10]. The numerical implementation and experiment by Lilly and Olhede [9, 10] illustrate that the GMWs are supported only on positive frequencies unlike the Morlet wavelets. Thus the statistical properties will not be destroyed due to departures from analyticity if one adopts the GMWs.

2.2. Scattering Transform Network (STN)

The architecture of the Scattering Transform Network (STN) is a tree-like analog of a convolutional neural network. Figure 1 illustrates its typical architecture.

At the \( m \)th layer of the STN, we denote \( \lambda_m = (j_m - J_m)/Q_m, j_m \in \{0, 1, \ldots, J_m\} \), the index for a multiscale wavelet filter where \( Q_m > 0 \) is the so-called quality factor and \( 2^j_m/Q_m \) is the largest scale of interest at the \( m \)th layer. Hence, \( j_m = 0 \) corresponds to the coarsest scale/lowest frequency band whereas \( j_m = J_m \) corresponds to the finest scale/highest frequency band. By dilating a mother wavelet \( \psi \), we can generate multiscale wavelet filters, i.e.,

\[
\psi_{\lambda_m}(t) := 2^{\lambda_m} \psi(2^{\lambda_m} t) \Leftrightarrow \Psi_{\lambda_m}(\omega) = \Psi(2^{-\lambda_m} \omega),
\]

where we assume \( \Psi_{\lambda_m} \in L^1(\mathbb{R}) \cap L^2(\mathbb{R}) \). These correspond to receptive fields of a CNN [3].

Let \( f \in L^2(\mathbb{R}) \) be an input signal of interest. We define a contraction operator \( M_m \) which is Lipschitz continuous, and satisfies the condition \( M_m f(t) = 0 \Rightarrow f(t) = 0 \). One popular choice of \( M_m \) is the modulus operator, i.e., \( M_m f(t) := |f(t)| \), which we will use in our experiments in Section 3. Let \( \Lambda_m \) be the set of indices \( \{\lambda_m\} \) for the \( m \)th layer. Each internal layer operator \( U_m : \Lambda_m \times L^2(\mathbb{R}) \rightarrow L^2(\mathbb{R}) \) is defined by three operations: 1) the (linear) wavelet transform; 2) the contraction operator; 3) subsampling:

\[
U_m[\lambda_m] f(t) := M_m (f * \psi_{\lambda_m})(r_m t),
\]

where \( r_m \geq 1 \) is the subsampling rate. Therefore, there is a path of indices \( \Lambda \in \Lambda_0 \times \cdots \times \Lambda_1 \) such that

\[
U[\lambda] f(t) := U_m[\lambda_m] U_{m-1}[\lambda_{m-1}] \cdots U_1[\lambda_1] f(t).
\]

We define the operator \( S_m \) for each layer \( m \) to generate the robust multiscale features of the input signal \( f(t) \), which we also call “coefficients” or “representations.”

\[
S_m[\lambda] f(t) := (\varphi_m * U[\lambda] f)(r_m t),
\]

where \( \varphi_m \) represents the averaging function, or father wavelet at a certain scale corresponding to the mother wavelet \( \psi_m \). After the averaging stage, we can subsample again at rate \( r_m' \geq 1 \) since the averaging wavelet is a lowpass filter. Note that in particular, for layer \( m = 0 \), we have \( S_0[0] f(t) := (\varphi_0 * f)(r_0 t) \).

Note that the GMWs were originally implemented in MAT-LAB® and disseminated as the JLAB package [10]. We implemented the STN with the option of using either the Morlet wavelets or the GMWs [11] in the Julia programming language [12], and we refer to the STN with the latter option as the GMW-STM. The doubtlet of parameters \( (\beta, \gamma) \) of the GMWs in our implementation is set as \( (4, 2) \) considering the balance of the time-frequency decays.
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of the intermediate signal representations within CNN s. Hence, 

of the STN, in particular, the GMW-STN. 

3.2. Interpreting music features in different layers

In our three-layer STN, we set the subsampling rates \( r_m = 8 \) 

and \( r_m' = 32 \), which generated the following size of the 

STN coefficients for each input signal of length 110,250: the 

0th layer: 3,445; the 1st layer: (431, 33); the 2nd layer: 

(54, 14, 33); and the 3rd layer: (7, 10, 14, 33). We also set the 

quality factors as \((Q_1, Q_2, Q_3) = (8, 4, 4)\), and \((J_1, J_2, J_3) = 

(32, 13, 9)\). These numbers mean that we used 33 scales of the 

form \(2^{(32−j_3)/8}\) with \(j_3 = 0 : 32\) in the 1st layer, 14 scales of 

the form \(2^{(13−j_3)/4}\) with \(j_3 = 0 : 13\) in the 2nd layer, and 10 

scales of the form \(2^{(9−j_3)/4}\) with \(j_3 = 0 : 9\) in the 3rd layer. 

The first numbers, 3, 445, 431, 54, and 7, are the size of the 

output coefficients in each path in the respective layers.

FIGURE 1. A typical STN architecture

3. Music Genre Classification using the GMW-STN

Categorizing recorded music signals into different genres 

such as classical, country, hiphop, jazz, pop and so on is a 

difficult task, in part because classification of music genres by 

human judgment can be subjective and ineffective. In such a 

classification problem, several CNN-based methods have been 

proposed, e.g., [13] among others. Although these methods of­

served some escape routes, they cannot completely escape from 

the following two fundamental problems (as discussed in Intro­

duction) when they are applied to music signal databases of rel­

atively small size: 1) model overfitting; and 2) interpretability 

of the intermediate signal representations within CNNs. Hence, 

such music genre classification with a music signal database of 

small size is an ideal application to demonstrate the advantage 

of the STN, in particular, the GMW-STN.

3.1. The GTZAN database and data preparation

In our experiments, we used the so-called GTZAN 

database [14] contains 1,000 audio/music tracks each of which 

is 30 second long and was sampled at 22,050Hz. The tracks 

are evenly distributed into ten music genres: blues; classical; 

country; disco; hiphop; jazz; metal; pop; reggae; and rock. For 

each music genre, the 100 tracks were recorded under different 

conditions.

In our experiments, we split each 30-second track into a 

set of 15 overlapping segments each of which is 5 second 

long. Let \( k \) denote the index of such a music segment. The 

time interval (indexed by samples) of the \( k \)th music segment is 

\([kL/3 + 1, kL/3 + L]\) for \( k = 0 : 14\), where \( L = 22050 \cdot 5 \) 

samples; that is, the hop size is \( L/3 \), i.e., the two adjacent segments 

have \(2/3 \cdot 5 \approx 3.33\) second overlap.

In our three-layer STN, we set the subsampling rates \( r_m = 8 \) 

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The first numbers, 3, 445, 431, 54, and 7, are the size of the 

output coefficients in each path in the respective layers.

3.3 3 second overlap.
3.3. Classification experiment settings

We employed the three-fold cross validation scheme and repeated this ten times. In each experiment, the 1,000 music tracks were shuffled and grouped into three folds of 340, 330, and 330 tracks. We used two folds for training and one fold for testing. Then we iterated the process three times by permuting the folds in each of those ten experiments. In other words, we ran the classification experiments 30 times in total. Each fold contains all music genres that are evenly distributed. For instance, there are 34 music files for each music genre in the first fold containing 340 files. In each fold, we further split each music track into 15 segments as described in Section 3.1.

In the training stage, we extracted the STN outputs from all segments in the training set. Then we compressed these outputs using the PCA implemented in the MultivariateStats.jl. We chose the top 1,000 principal components based on the experimental performance. Then we fed these into a classifier. In the testing stage, we assigned each input file a label based on the majority vote among the labels of its 15 segments predicted by the trained classifier.

In our numerical experiments, we mainly used the Support Vector Machine (SVM) [15, Sec. 3.6] as a classifier of choice. It transforms the input features to new representations in which the different classes are separated with margins that are as wide as possible. The PCA-compressed STN coordinates were fed to the SVM classifier of a polynomial kernel of degree 1 implemented in the LBSVM.jl package [16]. Along with the SVM, we also used the GLMNet [15, Chap. 3] in our experiments in order to interpret the classification results in a more intuitive manner. The GLMNet fits a generalized linear model with Lasso regularization through penalized maximum likelihood, and the GLMNet coefficients are denoted by $\theta$. The regularization path corresponding to the Lasso penalty was computed using cyclic coordinate descent. The significance of the STN coefficients in distinguishing the music genre can be captured by $\theta$ when the mean loss is minimized in the GLMNet.jl package [17].

3.4. Classification results and evaluation

We evaluated performance of various methods by comparing the predicted labels and the ground truth of the music tracks. We computed the classification accuracy by first computing the average accuracy under one experiment of the three-fold cross validation, and then computing the mean of these average accuracies of these ten repeated experiments.

Table 1 shows the superior performance of the GMW-STN with SVM compared to the Morlet-STN with SVM. The novel incorporation of the GMWs into the STN increased the accuracy by more than 4% using the 3rd layer outputs. Moreover, this table indicates that as the number of layers of the STN increases, the classification accuracy also increases regardless of the wavelet filters. The increase in accuracy is most significant from the 1st layer to the 2nd layer.

In comparison to the SVM classifier, the GLMNet classifier performed slightly worse (~3%). However, we will show in Section 4 that the GLMNet classifier can explain the results and shed light on the music information collected from the STN coefficients, which is impossible with the SVM. Figure 3 displays the performance of the GMW-STN with SVM for each individual music genre. The GMW-STN with SVM performed best in classifying classical music (94.9%) followed by metal (88.2%), jazz (84.9%), and blues (81.4%). However, it did not perform well in classifying pop (66.8%) and rock (59.6%).
TABLE 1. Average classification accuracy on music genres using GMW-STN and Morlet-STN

| Layer | GMW GLMNet SVM | Morlet SVM |
|-------|----------------|------------|
| 1     | 52.3711% 53.0529% 48.8776% | 70.2504% 73.7329% 70.0517% |
| 2     | 74.5500% 77.9088% | 73.7178% |

The average accuracy of each music genre:

See Section 4 for the explanation on the difference of the accuracies among the music genres. Our result is comparable to the reported accuracy (76.02%) without data augmentation and (80.93%) with data augmentation using 1D CNN [13]. However, we can uniquely provide the explanation of the results based on the additional music information retrieved in the layers, by visualizing and interpreting the corresponding STN coefficients. On the other hand, it is quite difficult to explain the results and interpret the intermediate representations in deep learning. Thus, interpretability is a main advantage of our approach over CNNs.

4. Significance of STN Coefficients

As we mentioned in the previous subsection, the SVM cannot indicate which subset of the STN coefficients mainly contributed to the correct classification results. Unlike the SVM, the GLMNet can provide such information. Since the GLMNet coefficient vector $\theta$ for each genre was computed on the top 1,000 PCA components of the 3rd layer STN coefficients, we first inverted the PCA to get the corresponding 3rd layer STN coefficients. Then, we normalized these coefficients so that the maximum value became 1, which we call the significance scores. Figure 4 displays these significance scores of ten music genres with the lower bound clamped to 0.4. It shows that information quantified in the 3rd layer STN is critical in the low frequency portion, especially in the $(j_3, j_2) = (0, 1)$ blocks. In general, the concentration in the lower frequency region is positively associated to the high classification rate. For instance, from Figure 4, the classical music has the highest scores in the lower frequency portion of the 3rd layer coefficients while pop and rock have more dispersed score distributions, which can be attributed to the variations of the music patterns in these genres, which in turn may have contributed to the lower classification accuracies for these genres.

5. Conclusions

We demonstrated that the GMW-STN outperformed the conventional STN using the Morlet wavelets. It can be explained by the importance of analyticity of the underlying wavelet transform. In addition, the classification accuracy became higher with more layers in the STN since we could retrieve the more relevant music information that are stable with respect to local deformations in the deeper layer STN coefficients. We could illustrate the connection between the music information retrieved from the GMW-STN and the classification results, which would be impossible using CNNs/DNNs. In addition, it turned out that the lower frequency portion of the music information retrieved from the 3rd layer STN coefficients mainly contributed to the music genre classification performance. In the near future, we plan to explore a 2D STN applied to the spectrograms of the input music tracks.

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FIGURE 4. Significance scores from the 3rd layer STN coefficients for each music genre

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