Attractive Interaction between Vortex and Anti-vortex in Holographic Superfluid

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Annihilation process of a pair of vortices in holographic superfluid is numerically simulated. The process is found to consist of two stages which are amazingly separated by vortex size $2r$. The separation distance $\delta(t)$ between vortex and anti-vortex as a function of time is well fitted by $\alpha(t_0 - t)^n$, where the scaling exponent $n = 1/2$ for $\delta(t) > 2r$, and $n = 2/5$ for $\delta(t) < 2r$. Then the approaching velocity and acceleration as functions of time and as functions of separation distance are obtained. Thus the attractive force between vortex and anti-vortex is derived as $f(\delta) \propto 1/\delta^3$ for the first stage, and $f(\delta) \propto 1/\delta^4$ for the second stage. In the end, we explained why the annihilation rate of vortices in turbulent superfluid system obeys the two-body decay law when the vortex density is low.
I. INTRODUCTION

Turbulence can be roughly defined as a spatially and temporally complex state of fluid motion\textsuperscript{[1]}. It can be found everywhere, such as rapid stream, smog, airflow, superfluid helium of heat convection, cold atoms being stirred, etc. Meanwhile, troubled with nonlinearity, far beyond linear response and interaction of many length and time scales, turbulence become one of the biggest problems in modern science. Both the classical turbulence and quantum turbulence are popular and active research directions. Specially, in recent decades, due to the great development of cooling and controlling techniques, quantum turbulence has established itself in the turbulence community, new light is shed on both the quantum and classical aspects of turbulence.

Vortices are essential objects in turbulence. They play a crucial role in various phenomena. In quantum turbulence, such as superfluid helium, atomic Bose-Einstein condensates, vortices are topological defects which are quantized. Reconnections of vortex line in three dimensional superfluid, or annihilation of vortex and anti-vortex in two dimensional superfluid are important phenomena which reduce the topological defects, dissipate energy and randomise the velocity field\textsuperscript{[2]}. During the reconnection process, the separation distance $\delta(t)$ between two vortices varying with respect to time is deeply investigated. It is firstly reported by de Waele and Aarts\textsuperscript{[3]} that

$$\delta(t) = \left(\frac{\kappa}{2\pi}\right)^{1/2} \sqrt{t_0 - t},$$  \hspace{1cm} (1)

where $\kappa$ is the circulation quantum, $t_0$ is the reconnection time. This numerical work is based on Schwarz’s vortex filament model\textsuperscript{[4]}, which assumes a very small vortex core. Thus the above result breaks down when $\delta(t)$ is smaller than the vortex size. Anyway, this $(t_0 - t)^{1/2}$ scaling is confirmed in He II experiment\textsuperscript{[5, 6]}, and by an approximate analytic solution of the Gross-Pitaevskii Equation (GPE)\textsuperscript{[7]}. However, there are also many numerical studies report similar fitting functions with modified scaling exponents. For example, the fitting functions

$$\delta(t) = A_1(t_0 - t)^{A_2},$$  \hspace{1cm} (2)

and

$$\delta(t) = B_1(t_0 - t)^{1/2}[1 + B_2(t_0 - t)]$$  \hspace{1cm} (3)
are reported in Refs.[2,8–11]. Here scaling exponent $A_2$ is found to be varying around $1/2$, and $B_2$ is small.

All of the above studies are about three-dimensional situations. The two-dimensional cases, such as the oblate Bose-Einstein condensates where vortices are annihilated rather than reconnected, are also important and interesting. Do they have similar behaviors and scaling exponents? In this paper, we try to answer this question. We numerically simulate and study the annihilation process of vortex and anti-vortex based on the holographic duality instead of GPE. Holographic duality[12–14] is an alternative theoretical framework to deal with strongly coupled quantum many-body systems which is encoded in a classical gravitational system with one extra dimension. Holographic superfluid model is established in Refs.[15,16], soliton and vortex solutions are studied in Refs.[17–19], holographic superfluid turbulence are studied in Refs.[20–23]. Holographic superfluid model is non-perturbation, thus it allows a first-principles investigation of the annihilation process. However, the one extra dimension makes it almost impossible for us to deal with the three-dimensional superfluid turbulence, as numerical simulation of a dynamical gravity system with 4+1 dimensional spacetime consumes too much computing resources.

This paper is organised as follows. Holographic superfluid model and relevant numerics are introduced in Sec. II. Numerical results and attractive interaction between vortex and anti-vortex are analysed and discussed in Sec. III. Sec. IV devotes to summarize our conclusions and suggest future investigations.

## II. INTRODUCTION TO THE HOLOGRAPHIC SUPERFLUID AND RELEVANT NUMERICS

For the two dimensional superfluid, a simple holographic model is a gravitational system in asymptotically AdS$_4$ spacetime. The corresponding bulk action is written as[15,16]

$$S = \frac{1}{16\pi G} \int_M d^4x \sqrt{-g} (R + \frac{6}{L^2} + \frac{1}{q^2} \mathcal{L}_{\text{matter}}),$$

where the matter Lagrangian reads

$$\mathcal{L}_{\text{matter}} = -\frac{1}{4} F_{ab} F^{ab} - |D\Psi|^2 - m^2 |\Psi|^2.$$  

Here $D = \nabla - iA$ with $\nabla$ the covariant derivative compatible to the metric. $A_a$ is a dynamical U(1) gauge field and $\Psi$ is a complex scalar field with mass $m$ and charge $q$. In
order to simplify the problem, we usually work in the probe limit, namely the large $q$ limit which decouples the matter fields from gravity. Thus the Schwarzschild black brane can be written in the infalling Eddington coordinates as

$$ds^2 = \frac{L^2}{z^2}(-f(z)dt^2 - 2dtdz + dx^2 + dy^2),$$

where the factor $f(z) = 1 - (\frac{z}{z_h})^3$ with $z = z_h$ the horizon and $z = 0$ the AdS boundary. Then the equations of motion for the matter fields can be written as

$$D_a D^a \Psi - m^2 \Psi = 0, \nabla_a F^{ab} = i(\overline{\Psi} D_b \Psi - \Psi D^b \overline{\Psi}).$$

In what follows, we will take the units in which $L = 1, 16\pi G q^2 = 1$, and $z_h = 1$, and work with $m^2 = -2$ in the standard quantization case. In the axial gauge $A_z = 0$, the asymptotic solutions of $A$ and $\Psi$ near the AdS boundary can be expanded as

$$A_\mu = a_\mu + b_\mu z + o(z), \quad \Psi = z[\beta + \psi z + o(z)].$$

According to the holographic dictionary, the temperature, the expectation values of the conserved current $j^\mu$ and the condensate operator $O$ in the superfluid are given by

$$T = \frac{3}{4\pi},$$

$$\langle j^\mu \rangle = \left. \frac{\delta S_{\text{ren}}}{\delta a_\mu} \right|_{z \to 0} = \lim_{z \to 0} \sqrt{-g} F^{\mu z},$$

$$\langle O \rangle = \left. \frac{\delta S_{\text{ren}}}{\delta \beta} \right|_{z \to 0} = -\lim_{z \to 0} z \sqrt{-\gamma} n a D^a \overline{\Psi} + \overline{\Psi}$$

$$= \overline{\psi} - \dot{\overline{\beta}} - i a t \overline{\beta},$$

where the dot denotes the time derivative, and the renormalized action is given by

$$S_{\text{ren}} = S - \int_B \sqrt{-\gamma} |\Psi|^2$$

with the counter term added to make the original action finite.

Here we switch off the sources of the operators by setting

$$a_x = 0, a_y = 0, \beta = 0.$$
Increasing the chemical potential $\mu \equiv a_t$, the system will undergo a phase transition process spontaneously, and the superfluid condensation $\langle O \rangle = \overline{\psi}$ will change from zero to non-zero. The critical chemical potential $\mu_c \approx 4.07$. Then the superfluid velocity is defined as
\[
\mathbf{u} = \frac{\mathcal{J}}{|\psi|^2}, \quad \mathcal{J} = i \left( \overline{\psi} \partial \psi - \psi \partial \overline{\psi} \right),
\]
and the winding number $\sigma$ of a vortex is
\[
\sigma = \frac{1}{2\pi} \oint_c d\mathbf{x} \cdot \mathbf{u},
\]
where $c$ denotes a counterclockwise oriented path surrounding a single vortex. In the following we determine the position of the vortex by calculating the winding number of each point in the system.

To investigate the annihilation process of a vortex pair with winding number $\sigma = \pm 1$, we consider a $30 \times 30$ square box with a periodic boundary on which a pair of vortices are placed to allow them to evolve freely. The initial position of the vortices are randomly placed. What’s more, the system are added a random perturbation velocity field. As a result, the vortices have random initial velocities. By defining a new function $\Phi = \frac{\psi}{z}$, the later time behavior of the system is determined by the following rewritten equations of motion
\[
\partial_t \partial_z \Phi = i A_t \partial_z \Phi + \frac{1}{2} \left[ i \partial_z A_t \Phi + f \partial^2 \Phi + f' \partial_z \Phi \right. \\
\left. + (\partial - i A)^2 \Phi - z \Phi \right],
\]
\[
\partial_z (\partial_z A_t - \partial \cdot \mathbf{A}) = i (\overline{\Phi} \partial_z \Phi - \Phi \partial_z \overline{\Phi}),
\]
\[
\partial_t \partial_z \mathbf{A} = \frac{1}{2} \left[ \partial_z (\partial A_t + f \partial_z \mathbf{A}) + (\partial^2 \mathbf{A} - \partial \partial \cdot \mathbf{A}) \\
- i (\overline{\Phi} \partial \Phi - \Phi \partial \overline{\Phi}) \right] - \mathbf{A} \overline{\Phi} \Phi,
\]
\[
\partial_t \partial_z A_t = \partial^2 A_t + f \partial_z \partial \cdot \mathbf{A} - \partial_t \partial \cdot \mathbf{A} - 2 A \overline{\Phi} \Phi \\
+ i f (\overline{\Phi} \partial_z \Phi - \Phi \partial_z \overline{\Phi}) - i (\overline{\Phi} \partial_t \Phi - \Phi \partial_t \overline{\Phi}).
\]

The related numerical methods used in this paper include pseudo-spectral method and Runge-Kutta method. The pseudo-spectral method is used to represent the above functions with 25 Chebyshev modes in the $z$ direction and 241 Fourier modes in the $x, y$ direction. The Runge-Kutta method is used to evolve the equations in time direction with the time step $\delta t = 0.05$. For a detail review of these methods, one can refer to Ref. \cite{20, 22, 23, 25}.
FIG. 1: The configuration of vortex for chemical potential $\mu = 6$. We define the vortex radius as $r$, such that $|\langle O(r) \rangle| = 0.99|\langle O \rangle|_{\text{max}}$. Here $r = 2.05$.

III. NUMERICAL RESULTS

Vortex and anti-vortex have the same configuration. The only difference is that one has clockwise rotation and the other has counterclockwise rotation. The static vortex solution can be obtained from the above equations of motion in the cylindrical coordinate[18]. When the chemical potential of the superfluid is $\mu = 6 > \mu_c$, the vortex configuration is shown in Fig.1. We define the vortex radius as $r$, such that $|\langle O(r) \rangle| = 0.99|\langle O \rangle|_{\text{max}}$. $|\langle O \rangle|_{\text{max}}$ is the condensation of homogeneous superfluid solution. The vortex radius is found to be $r = 2.05$.

A. separation distance between vortex and anti-vortex as a function of time

We achieved long-time simulations of the vortex annihilation process and obtained 10 sets of data. As an example, the superfluid configurations for different time $t = 400, 594, 640, 660$ are shown in Fig.2. The graph is especially shown at time $t = 594$, when the vortex and anti-vortex just touched, that is $\delta(t = 594) = 2r = 4.1$. The initial relative motion speed is small. The separation distance $\delta(t)$ between vortex and anti-vortex decreases with time until the vortices are annihilated.

$\delta(t)$ is shown in Fig.3. The blue dots represent the separation distance between the vortices at different times. Our aim is to study the interaction between vortex and anti-vortex without being affected by other effects. So the separation distance is recorded every $\Delta t$ from
FIG. 2: The superfluid configurations for different time $t = 400, 594, 640, 660$. Red arrow indicates the direction of velocity. The separation distance $\delta(t)$ between vortex and anti-vortex decreases with time until the vortices are annihilated. The graph is especially shown at time $t = 594$, when the vortex and anti-vortex just touched, that is $\delta(t = 594) = 2r = 4.1$.

Time $t = 330$, when the initially added perturbation modes are basically all dissipated. From $t = 330$ to $t = 600$, $\Delta t = 2$. While for the rest of time, $\Delta t = 1$. It can be seen from the figure that the separation distance of the vortices becomes smaller with time, and the relative motion between them becomes faster and faster, as if there is a mutually attractive force. In particular, from their contact $\delta(t) < 2r = 4.1$, the acceleration becomes larger, that is, the attractive force becomes larger. As a result, this annihilation process can be divided into two stages, the first part is the stage before the vortex pair contact, and the second part is the stage after the contact. In Fig. 3, the solid red line is the fitting curve for the first stage ($\delta(t) > 4.1$), and the solid brown line is the fitting for the second stage ($\delta(t) < 4.1$). The
FIG. 3: The blue dots represent the separation distance between the vortices at different times. The solid red line is the fitting curve for the first stage ($\delta(t) > 4.1$), and the solid brown line is the fitting for the second stage ($\delta(t) < 4.1$).

First stage is well fitted by function

$$\delta(t) = A(t_0 - t)^{1/2}, \quad (20)$$

Here $A = 0.529$ is the fitting constant, $t_0 = 649.6$ is the time when the vortex pair annihilated.

The second stage is well fitted by function

$$\delta(t) = B(t_0 - t)^{2/5}, \quad (21)$$

Here $B = 0.799$ is the fitting constant, $t_0 = 649.6$ is the time when the vortex pair annihilated.

In the 10 sets of simulation experiments, the average value of $A$ is 0.529, and $B$ is 0.798.

A comparison of our results with the results of previous work is interesting. First of all, the scaling law $1/2$ of the separation distance between vortices for $\delta(t) > 2r$ seems to be universal. It is predicted by simple dimensional analysis [3, 11]. It is also confirmed by experiments [5, 6], and by an approximate analytic solution [7]. Although some numerical studies report small modified scaling exponents [2, 8, 11]. Secondly, when $\delta(t) < 2r$, another scaling exponent $2/5$ is obtained in this paper. Due to the small vortex radius, minor differences between the two scaling exponents $1/2$ and $2/5$ may have not yet been observed in the other work. Thirdly, our work is based on holographic duality, and the vortex is two dimensional. The other work are based on vortex filament model or Gross-Pitaevskii model or experiments, and the vortex line or ring are three dimensional.
B. attractive interaction as a function of separation distance

Eq. (20) and Eq. (21) can be rewritten into the following form,

\[ \delta(t) = \alpha(t_0 - t)^n. \]  

(22)

Then the velocity that the vortices are approaching to each other is derived as

\[ v(t) = \frac{d\delta(t)}{dt} = -n\alpha(t_0 - t)^{n-1}. \]  

(23)

The expression of velocity about distance can be obtained as

\[ v(\delta) = -n\alpha^{\frac{1}{n}}\delta^{1-\frac{1}{n}}. \]  

(24)

The expression of acceleration about time is derived as

\[ a(t) = \frac{d^2\delta(t)}{dt^2} = n(n-1)\alpha(t_0 - t)^{n-2}. \]  

(25)

The expression of acceleration about distance can be obtained as

\[ a(\delta) = n(n-1)\alpha^{\frac{2}{n}}\delta^{1-\frac{2}{n}}. \]  

(26)

If the vortices are loosely treated as particles, then according to the expression of the acceleration with respect to the separation distance, we can get the attractive force between the vortex and anti-vortex as a function of separation distance,

\[ f(\delta) \propto |a(\delta)| = |n(n - 1)\alpha^{\frac{2}{n}}\delta^{1-\frac{2}{n}}|. \]  

(27)

For the first stage, \( n = 1/2 \), the attractive force is

\[ f(\delta > 2r) \propto \frac{1}{\delta^3}. \]  

(28)

For the second stage, \( n = 2/5 \), the attractive force is

\[ f(\delta < 2r) \propto \frac{1}{\delta^4}. \]  

(29)

For the second stage, the approaching velocity as a function of time and as a function of separation distance is shown in Fig. 4. The acceleration as a function of time and as a function of separation distance is shown in Fig. 5. When the separation distance is far away, the velocity increases very slowly and the acceleration is extremely small.
FIG. 4: For the second stage, left graph is $v(t)$, right graph is $v(\delta)$. When the separation distance is far away, the velocity increases very slowly.

FIG. 5: For the second stage, left graph is $a(t)$, right graph is $a(\delta)$. When the separation distance is far away, the acceleration is extremely small.

C. vortex annihilation rate in superfluid turbulence with low vortex density

It can be seen that only when the separation distance is very small, even less than the vortex size $2r = 4.1$, the acceleration is larger. This explains why the annihilation process of vortices in a superfluid turbulent system obeys the two-body decay law when the vortex density is low [22, 23, 26]. As the separation distance of vortices are large, the attractive force between vortex and anti-vortex is small and the acceleration is small. Also because of the chaotic motion of superfluid turbulence, the effect of the long distance attractive force between vortex and anti-vortex is weakened. Therefore the average velocity of the vortex can be regarded as a constant $v_0$ for $\delta > 2r$. So the annihilation probability for a pair of vortices will be

$$P_1 \propto \frac{v_0 2r}{l^2/N(t)},$$  \hspace{1cm} (30)

where $v_0 2r$ is the area swept by the vortex per unit time, $l^2$ is the area of the system, $N(t)$ is total vortex number and $l^2/N(t)$ is the area occupied by a single vortex. Then the total
Annihilation rate should be

\[
\frac{dN(t)}{dt} = -N(t)P_1 \propto -N(t)\frac{v_02r}{l^2/N(t)} = -\frac{2rv_0}{l^2}N(t)^2, \tag{31}
\]
or rewritten in terms of the vortex number density

\[
\frac{dn(t)}{dt} \propto -2rv_0n(t)^2 = -Cn(t)^2, \tag{32}
\]
where \(C\) is a constant. The above equation denotes the two-body decay law. \(n(t)\) is obtained as

\[
n(t) = \frac{1}{Ct + 1/n_0} \propto (t + \alpha)^\beta, \tag{33}
\]
here \(\beta = -1\) which is confirmed in Ref.\[22, 23, 26\] when the vortex number density is low.

IV. CONCLUSION AND DISCUSSION

In this paper, annihilation process of vortices in two-dimensional superfluid is numerically simulated based on holographic duality. We find that the approaching velocity of vortex and anti-vortex increases with time, and finally the vortices annihilate into a crescent shaped wave (gray soliton). Firstly, the separation distance between vortex and anti-vortex as a function of time is recorded. The function is well fitted by \(\alpha(t_0 - t)^n\), where the scaling exponent \(n = 1/2\) for \(\delta(t) > 2r\), and \(n = 2/5\) for \(\delta(t) < 2r\). Thus the annihilation process can be divided into two stages which are separated by the vortex size \(2r\).

Secondly, the approaching velocity as a function of time and as a function of separation distance are shown in Fig.4. The acceleration as a function of time and as a function of separation distance are shown in Fig.5. When the separation distance is far away, The velocity increases very slowly and the acceleration is extremely small. If the vortices are loosely treated as particles, then according to the expression of the acceleration \(a(\delta)\), we can obtain the attractive force between the vortex and anti-vortex as a function of separation distance. \(f(\delta) \propto 1/\delta^3\) for the first stage, and \(f(\delta) \propto 1/\delta^4\) for the second stage.

Thirdly, according to the characteristics of acceleration, we can reasonably assume that the average velocity of vortex in turbulent state is a constant when \(\delta > 2r\). Then the annihilation probability for a pair of vortices is derived as \(P_1 \propto \frac{v_02r}{l^2/N(t)}\), and the total annihilation
rate is $\frac{dn(t)}{dt} \propto -Cn(t)^2$ which is called the two-body decay law. Thus we successfully explained why the annihilation process of vortices in a superfluid turbulent system obeys the two-body decay law when the vortex density is low\[22, 23, 26\].

In the end, we would like to emphasize that the scaling exponent $1/2$ for $\delta(t) > 2r$ in two-dimensional superfluid is same with many results\[3, 5–7, 11\] obtained in three-dimensional cases. But another scaling exponent $2/5$ is obtained for $\delta(t) < 2r$ in this paper. All of the above results are based on holographic duality. The two-dimensional cases in other models deserve to be disclosed in the future research. We will give the two-dimensional result within the GPE model in the near future. Investigation of the interaction between two same kind of vortices is also interesting. According to the symmetry, the interaction should be repulsive. When the separation distance is large, the repulsive behavior should be the same as the attractive force. When the separation distance is small, these two forces should behave differently. Anyway, the final result deserved to be revealed.

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