An Orientifold from F Theory

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Abstract

The massless spectrum of an orientifold of the IIB string theory is computed and shown to be identical to F theory on the Calabi-Yau threefold with $h_{11} = 51$ and $h_{21} = 3$. Target space duality is also considered in this model.
1. Introduction

Many recent and some older papers [1-4] have explored orientifolds of the IIB string theory. It is interesting to expand the terrain of these orientifolds into the domains of mother (male) and father (female) theories generally known as M and F theories as we search for the ineluctably elusive universal theory.

These orientifolds have the common feature that one divides by a discrete symmetry group that includes world-sheet parity and symmetries of space-time. Cancelling tadpole anomalies in these theories usually necessitates the addition of branes but hopefully not branks. In the case of orbifolds with larger than a $\mathbb{Z}_2$ space-time symmetry, one must be careful of the definition of world-sheet parity in the twisted sectors. There are also subtleties in the application of the IIB $SL(2, \mathbb{Z})$ (S) and target space (T) dualities.

We consider a simple orientifold here that illustrates the relation to F theory and the subtlety of using T-duality. The model we calculate here is F theory on the Calabi-Yau threefold with Hodge numbers $h_{11} = 51$ and $h_{21} = 3$ or $(51, 3)$ which we show to be equivalent at the massless level to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold of IIB $(\Omega(-1)^F R_3, \Omega(-1)^F R_4)$ where $\Omega$ is world-sheet parity, $(-1)^F$ is left-handed (right-handed) space-time fermion number ($(-1)^F = e^{2\pi i s^L}$), and $R_3(R_4)$ are space-time reflections on the torus $T^3(T^4)$ of the four-torus $T^3 \times T^4 (R_3 = e^{i\pi (s_3^L + s_3^R)}$, $R_4 = e^{-i\pi (s_4^L + s_4^R)})$. We have chosen 1, 2 as space-time directions and 3, 4 as internal $T^4$ directions while $s$ denotes spin. A future paper will look at the $(3, 51)$, which adds discrete torsion to the $(51, 3)$; S-duality; and other $\mathbb{Z}_n$ orientifolds.

2. The Calculation

First we consider the $(51, 3)$ from the point of view of F theory. As noted in [5], this Calabi-Yau threefold can be obtained from an orbifold of $T^6$ in which the two generators are $g_{35}$ and $g_{45}$ where $g_{ij}$ denotes reflections of $z_i$ and $z_j$ and 5 is the 11-12 direction of F theory. The results of Morrison and Vafa imply that that the massless spectrum is 17 tensors, 4 hypermultiplets, and a gauge group $SO(8)^8$.

Now we wish to compute the IIB orientifold corresponding to F theory on the $(51, 3)$. In a generic point of the moduli space this F theory model is equivalent (by definition) to a compactification of type IIB on the base of the elliptic fibration with a space-dependent complex coupling constant identified with the complex structure of the fiber; seven-branes are required at the singularities of the fibration. As shown by Sen [3], at the orbifold
limit the coupling constant can be chosen to be space-independent giving a standard type IIB compactification. A reflection of $z_5$ is equivalent to a monodromy by $-1$ in $SL(2, \mathbb{Z})$. By studying its action on the massless fields, we recognize its equivalence to the operator $\Omega(-1)^{F_I}$ of the type IIB theory.

All the Voisin-Borcea models listed in [8] are the product of a two-torus and $K_3$ divided by a $\mathbb{Z}_2$ symmetry; they correspond to type IIB compactifications with a space-independent coupling constant. The cases in which these Calabi-Yau are orbifolds can be exactly described by type IIB orientifolds. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds are listed in [8], and they correspond to $T^6$ divided by 1) $g_{35}$ with a shift of order two in $z_4$, $g_{45}$ with a shift of order two in $z_3$; 2) the same except that $g_{45}$ is not accompanied by a shift; and 3) the same with no shifts at all. They have Hodge numbers $(11, 11), (19, 19),$ and $(51, 3)$ respectively. The rule for obtaining the type IIB orientifold is quite simple: F theory on the Voisin-Borcea model is the type IIB orientifold on $T^4$ obtained by replacing in the above $\mathbb{Z}_2 \times \mathbb{Z}_2$ action the reflection of $z_5$ by $\Omega(-1)^{F_I}$ (or $\Omega(-1)^{F_r}$ if necessary to close the algebra).

The case 1) is an F theory model with 9 tensors, 12 hypermultiplets, and no gauge fields. By changing coordinates to $y_i = z_i + 1/2$, $i = 3, 4$, the orientifold is easily seen to be equivalent to the following model: type IIB on $K_3$ divided by $\Omega(-1)^{F_I}\rho$ where $\rho$ is the Enrique’s involution [10]. This model makes sense as a closed string model. It is easy to check that the Klein bottle tadpoles cancel. The shifts introduce an extra $(-1)^m$ in the loop channel momentum sum

$$\sum_m (-1)^m e^{-tm^2/R^2}$$

which goes to zero in the limit $t \to 0$, as can be seen by a Poisson resummation. There is no necessity to introduce open strings. By looking at the action on the massless fields of $\Omega(-1)^{F_I}$ and at the action of $\rho$ on the cohomology of $K_3$, we obtain exactly 9 tensors and 12 hypermultiplets.

The case 2) has 9 tensors, 20 hypermultiplets, and gauge group $U(1)^8$ at a generic point. From the orientifold point of view, one of the two operators involving $\Omega$ contains a space-time shift and does not produce tadpoles; the other one requires 32 seven-branes. As shown by R. Gopakumar and S. Mukhi [11] this is the T-dual of the model in [3].

The model 3) is the $\mathbb{Z}_2 \times \mathbb{Z}_2$ described in the introduction and is the more interesting since it has an unbroken gauge group even at a generic point of moduli space. From the F theory point of view one gets an $SO(8)^8$ enhanced gauge symmetry from the $D_4$ singularities of the fibration. We want to understand this result from the orientifold point
of view. This model is very similar to that of Gimon and Polchinski\cite{4} except that two kinds of seven-branes are required rather than five or nine-branes. As we will discuss extensively in the next section, this model cannot be obtained as the T-dual of the one in\cite{4}.

Let us first study the closed string spectrum. The calculation is straightforward, but we list the right-moving massless states since the explicit form of the $Z_2$ twisted sector will be useful in understanding the subtleties of the model.

| Sector | State | $R$ | $SO(4)$ rep. |
|--------|-------|-----|--------------|
| NS :   | $\psi^\mu_{-1/2} | 0 >$ | 1   | $(2, 2)$      |
|        | $\psi^1_{-1/2} | 0 >$ | $-1$ | $2(1, 1)$    |
|        | $\psi^2_{-1/2} | 0 >$ | $-1$ | $2(1, 1)$    |
| R :    | $s_1 s_2 s_3 s_4 >$ |     |              |
|        | $s_1 = + s_2, s_3 = + s_4$ | 1   | $2(2, 1)$    |
|        | $s_1 = - s_2, s_3 = - s_4$ | $-1$| $2(1, 2)$    |

and for the $Z_2$ twisted sector:

| Sector | State | $R$ | $SO(4)$ rep. |
|--------|-------|-----|--------------|
| NS :   | $s_3 s_4 >, s_3 = + s_4$ | 1   | $2(1, 1)$    |
| R :    | $s_1 s_2 >, s_1 = - s_2$ | 1   | $(1, 2)$     |

We have imposed the GSO projection and decomposed the little group of the space-time Lorentz group as $SO(4) = SU(2) \times SU(2)$. The spectrum for the orientifold group is obtained by taking products of states from the left and right sectors and dividing by $(\Omega(-1)^F l R_3, \Omega(-1)^F r R_4, R)$. The action of $R$ is listed in the table; $R_3 = e^{i \pi (s^L_i + s^R_i)}$, $R_4 = e^{-i \pi (s^L_i + s^R_i)}$, and the $\Omega$ projection acts by symmetrizing left and right states in the Neveu-Schwarz–Neveu-Schwarz(NS-NS) sector, while antisymmetrizing in the Ramond-Ramond(R-R) sector.

Let us note that the $Z_2 \times Z_2$ algebra closes only up to a factor

$$(-1)^{F_l + F_r} \exp 2 \pi i (s^L_i + s^R_i)$$

Fortunately, in the closed string Hilbert space this operator is identically 1. It acts as a global $-1$ in all the right or left sectors twisted by $1/2$, but in the closed string Hilbert space it always cancels between left and right states. However, this operator signalizes an ambiguity in the definition of the $Z_2 \times Z_2$ algebra on the open string spectrum.
The closed string spectrum is now straightforward. From the untwisted sector we have the supergravity multiplet, 1 tensor and 4 hypermultiplets. The main difference between this model and that of [4] (clearly showing that the two models are not T-dual) lies in the twisted sector where \((-1)^{F_1} R_3\) introduces an extra minus sign; as a consequence, now the hypermultiplets are projected out, and we get 16 tensors, one from each fixed point.

Let us now turn to the open string spectrum. The tadpoles are essentially the same; the presence of \((-1)^{F_1}\) compensating in the Ramond sector for extra signs introduced by \(R_{3,4}\). Therefore, we omit the calculation and present only the results. First, we list the tadpoles for the untwisted R-R potentials. We have (proportional to \((1 - \frac{1}{16v_4}) \int_0^{\infty} dl\)):

\[
\text{Tr}(\gamma_{0,7})^2 - 64 \text{Tr}(\gamma_{\Omega R_3(-1)^{F_1},7} \gamma_{T \Omega R_3(-1)^{F_1},7}) + 32^2,
\]

(proportional to \((1 - \frac{1}{16v_3}) \int_0^{\infty} dl\)):

\[
\text{Tr}(\gamma_{0,7'})^2 - 64 \text{Tr}(\gamma_{\Omega R_3(-1)^{F_1},7'} \gamma_{T \Omega R_3(-1)^{F_1},7'}) + 32^2,
\]

and for the twisted potentials

\[
\frac{1}{16} \sum_{IJ} (\text{Tr} \gamma_{R7,I} - \text{Tr} \gamma_{R7',J})^2
\]

where \(I\) and \(J\) are respectively the four fixed points of \(R_3\), the four of \(R_4\), and other notations are those of [4].

The number of seven-branes is 32 for each of the two kinds. There are a couple of subtleties in the calculation which caused us a bit of trouble. Using the same argument as Gimon and Polchinski for the sign of \(\Omega^2\) in the 5–9 sector, we obtain an extra minus sign for \(\Omega^2\) acting on the seven-brane vacuum. This extra minus is confirmed by the new consistency condition discovered in [12]. This condition reads (for the model of ref. [4])

\[
\gamma_R = -\gamma \Omega^T \gamma_R \Omega^{-1},
\]

and it is obtained by considering the transition between the closed string R-R twisted ground state and the open string vector. In our case \(\Omega\) is replaced by \(\Omega(-1)^{F_1} R_3\) and \((-1)^{F_1}\) introduces an extra minus sign; therefore \(\gamma_R\) can be chosen to be symmetric. We must be careful with the definition of the operators in the open string sector since, as we noted before, there is an ambiguity in the closure of the \(\mathbb{Z}_2 \times \mathbb{Z}_2\) algebra (\([2,4]\)). This ambiguity is only revealed in the 7–7' sector. Multiplying together the two \(\mathbb{Z}_2\) operators
involving $\Omega$ gives $\Omega^2 R$ rather than $R$ for open string states in the $7 - 7'$ sector. This $\Omega^2 R$ does not affect the tadpole equations where $R$ is used since we want to cancel divergences of the NS-NS and R-R sectors of the closed string, but gives an extra minus in the $7 - 7'$ sector. The algebraic, tadpole equations, and the consistency equation discovered in [12] are satisfied while the $\mathbb{Z}_2 \times \mathbb{Z}_2$ algebra is realized if we choose the matrices operating on Chan-Paton factors equal to the identity. The twisted tadpole condition (2.7) is satisfied if we put eight seven-branes at each of the fixed points of $R_3$ and $R_4$. All the matter is projected out. So the open string sector yields simply the gauge group $SO(8)^8$. As we discussed before, we also get one tensor and four hypermultiplets from the untwisted closed string sector as well as sixteen tensors from each of the fixed points of $R$ in the $R$-twisted sector. Thus, the orientifold agrees with the result of F theory.

### 3. T-Duality

We noted before that we cannot map this model to a simpler one by a T-duality. In fact, let us apply T-duality in the three direction. The effect is to transform an operator $O$ to $e^{i\pi s_3} O e^{-i\pi s_3}$ so that operators with $\Omega$ get an extra factor $e^{i\pi (s_3^L - s_3^R)}$. Other operators are unchanged. It is easy to show that the spectrum remains unchanged under the transformation.

Notice that $\Omega$ is transformed to $\Omega R_3 e^{2\pi i s_3^L}$. The Gimon and Polchinski [4] model has a T-dual model which contains two kinds of seven-branes but differs from the $(51, 3)$ model. The difference is that in the definition of the projection operators every occurrence of $e^{2\pi i s_3^L}$ is replaced by $(-1)^{F_i} e^{2\pi i s_3^L}$. These two operators are equivalent whenever there are no twisted sectors, but they differ by a minus sign in every $Z_2$ twisted sector as is obvious from equation (2.3). We have already utilized this observation when we computed the twisted contributions to the closed string spectrum. Gimon and Polchinski had one hypermultiplet from each fixed point, while we got a tensor. Thus, we obtain a different answer from [13] because of the T-duality subtlety. The irreducible part of the gravitational anomaly is cancelled by the spectrum we have obtained. There is no Higgs mechanism as there is no charged matter. The sixteen tensors obtained in the $R$-twisted sector should suffice to cancel any left over anomalies by the methods of [14][15]. This model has illustrated the application of Gimon and Polchinski techniques in F theory and serves as a starting point for understanding other models.
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References

[1] A. Sagnotti, in Cargese '87, “Nonperturbative Quantum Field Theory”, eds. G.Mack et al. (Pergamon Press, Oxford, 1988), p.521.

[2] P. Horava, Nucl. Phys. B327 (1989) 461; Phys. Lett. B231 (1989) 251.

[3] M. Bianchi and A. Sagnotti, Phys. Lett. B247 (1990) 517; Nucl. Phys. B361 (1991) 519.

[4] E. Gimon and J. Polchinski, hep-th/9601038.

[5] A. Dabholkar and D. Park, hep-th/9602030.

[6] E. Gimon and C. Johnson, hep-th/9604129; A. Dabholkar and D. Park, hep-th/9604178.

[7] M. Berkooz and R. Leigh, hep-th/9605049.

[8] D. Morrison and C. Vafa, hep-th/9603161.

[9] A. Sen, hep-th/9605150.

[10] S. Ferrara, J. Harvey, A. Strominger, and C. Vafa, Phys. Lett. B361 (1995) 59; S. Ferrara, R. Minasian, and A. Sagnotti, hep-th/9604097.

[11] R. Gopakumar and S. Mukhi, private communication.

[12] J. Polchinski, hep-th/9606163.

[13] E. Gimon and C. Johnson, hep-th/9606176.

[14] M. Green and J. Schwarz, Phys. Lett. B149 (1984) 117.

[15] A. Sagnotti, Phys. Lett. B294 (1992) 196.