Optimal design of inerter-integrated vibration absorbers for seismic retrofitting of a high-rise building in Colombia

Yuan Li\textsuperscript{1}, Luca Lombardi\textsuperscript{2}, Flavia De Luca\textsuperscript{2}, Yosef Farbiarz\textsuperscript{3}, John Jairo Blandon\textsuperscript{4}, Luis Augusto Lara\textsuperscript{4}, Juan Felipe Rendon\textsuperscript{4}, Jason Zheng Jiang\textsuperscript{1}, Simon Neild\textsuperscript{1}

\textsuperscript{1}Department of Mechanical Engineering, University of Bristol, Bristol BS8 1TR, UK
\textsuperscript{2}Department of Civil Engineering, University of Bristol, Bristol BS8 1TR, UK
\textsuperscript{3}Departamento de Materiales y Minerales. Facultad de Minas, Universidad Nacional de Colombia sede Medellin Cr80 No. 65-223, Colombia
\textsuperscript{4}Departamento de Ingenieria Civil. Facultad de Minas, Universidad Nacional de Colombia sede Medellin Cr80 No. 65-223, Colombia

E-mail: z.jiang@bristol.ac.uk

Abstract. Retrofitting of existing buildings with seismic protection devices is an important approach which is commonly adopted by the civil engineering community. This paper investigates the potential of inerter-integrated seismic vibration absorber for building structures. A real case study of a high-rise building in Colombia is considered herein to demonstrate the effectiveness of the proposed device. An inerter generates a force proportional to the relative acceleration between its two terminals and it fundamentally enlarges the range of realisable passive mechanical impedances. In this work, an interter-integrated device is modelled into the open-source structural engineering software \textit{OpenSees} and implemented into a simplified two-dimensional (2D) version of the Colombian test-bed structure. The performances of the structure retrofitted with the proposed absorber are compared to those of the same structure retrofitted with a traditional damper device, a more classical retrofitting. This preliminary work demonstrates the effectiveness of the inerter-integrated retrofitting strategies considering the non-structural damage caused by a moderate earthquake. It has been found that with an optimal inerter-integrated device, the maximum inter-storey drift of the building will be reduced by 45.64\% comparing with the structure without any retrofitting, which is 20.8\% further reduction comparing with the structure retrofitted with an optimal damper.

1. Introduction

Numerous buildings in many seismic regions worldwide do not conform with modern seismic design standards \cite{1}. For example, a vast number of existing buildings in Colombia were constructed before the entry into force of the first Colombian code for Earthquake Resistant Design and Construction issued in 1984 (CCCSR-84, \cite{2}). It has been reported that such substandard buildings are vulnerable to earthquake motions \cite{3}. To strengthen the seismic performance of existing buildings, retrofitting techniques are often required. Regardless of the retrofitting strategy adopted, the main target in any retrofitting intervention is to improve the
seismic performances of structures over their life-time. It has been shown that over the life-cycle of structures both rare high magnitude earthquakes (i.e., representative of ultimate limit states scenarios) and frequent low magnitude earthquakes (i.e., representative of serviceability limit states scenarios) are important; as the former influence the performances related to the no-collapse or life-safety limit states while the latter influence damage limitation and operativity limit states having a significant impact on loss estimation [4].

Reducing the seismic effects on the buildings with seismic protection devices is becoming increasingly attractive to the structural engineering community [5]. Supplemental seismic protection devices can be broadly divided into four categories: passive, active, semi-active and hybrid ones, based on the operational strategies [6]. Comparing with the other three methods, passive devices are more widely used in engineering practise since they are typically simpler, more reliable and requiring no power source [7]. Development of passive energy dissipative devices, such as dissipative bracing [8–10], fluid viscous damper [7, 11], metallic dampers [12, 13] and friction dampers [14, 15], have been investigated. Another commonly accepted means of seismic protection is to use passive isolators, which normally shift the structure’s fundamental frequency away from the dominant frequency of ground motion making use of rubber bearings [16]. Retrofitting of passive tuned mass damper (TMD) has also been extensively studied, see [17–19] as examples. Although in general a larger attached mass results in a more effective TMD [20], it is often impractical to install a huge mass to the primary structure [21].

An inerter, firstly introduced by Smith [22] in 2002, has recently attracted great research attention in multiple engineering fields. The inerter is a two-terminal device which generates a force proportional to the relative acceleration between its terminals and the proportionality is called inertance. This device helps to complete the mechanical-electrical analogy and hence the range of passive vibration absorbers is fundamentally enlarged [22]. Comparing with the mass, the inerter has another notable advantage that is the achievable inertance can be significantly larger than the device mass with the gearing mechanism. For example, a full-scale inerter-integrated device called tuned viscous mass damper (TVMD) with 5400 t inertance was tested for seismic control [23]. This advantage makes the inerter more attractive to civil structures [24]. The earliest work on inerter-integrated building suspension was conducted by Wang et al. [25, 26] and the benefits of several inerter-integrated layouts on suppressing the traffic- and earthquake-induced vibrations were identified. Significant contributions to the application of inerter-integrated devices as seismic protection systems were made in Japan and most of them focus on the TVMD device [23, 27, 28]. Lazar et al. [24] proposed a tuned inerter damper (TID) by replacing the mass to an inerter in a TMD and suggested that comparing with a conventional TMD, the TID can provide enhanced seismic control responses with a much smaller device mass. Another notable inerter-integrated building suspension is tuned mass-damper-inerter (TMDI) introduced in [29]. Employing the network synthesis method, Zhang et al. [30] proposed a series of layouts with fixed-sized inerter for seismic control of a three-storey building.

Even though many researchers have investigated the benefits of inerter-integrated seismic protection systems, the models they considered are extremely simplified with reduced lumped masses and only focused on the building dynamics in one specific direction aligned with base excitations. In this regard, the potential of a passive inerter-integrated seismic absorber is investigated herein considering a 2D model using finite element method (FEM) extracted from a real three-dimensional (3D) high-rise reinforced concrete structure in Colombia. The performance advantages of the inerter-integrated device are compared with those provided by a conventional damper.

In the following, Section 2 introduces the modelling of inerter property in OpenSees ([31], http://opensees.berkeley.edu), and comparative validation of this new element using OpenSees and Matlab is demonstrated with a single-degree-of-freedom (SDOF) system. A candidate inerter-integrated absorber, TVMD (labelled as $L_i$ in this work), is considered. Section 3
provides the description of the test-bed structure in Colombia and its linear model in OpenSees including the $L_i$ device into the structure. The earthquake loading is chosen in Section 4 and the optimisation problem is then formulated. Based on the optimisation, the performance advantages of the proposed inerter-integrated absorber are identified comparing with an optimal damper. Note that as a preliminary study, the optimisation at this stage is performed considering a single moderate earthquake waveform that produces inter-storey drifts resulting in non-structural damage. Conclusions are finally drawn in Section 5.

2. Modelling of inerter-integrated device in OpenSees

In this section, the modelling of the inerter property in OpenSees is presented, which builds on the source codes firstly introduced in [32]. Considering a SDOF system, the inerter property modelled in OpenSees is validated by comparing with that in Matlab. One inerter-integrated vibration absorber layout $L_i$ is proposed as candidate in this analysis and its modelling in OpenSees is also introduced with the SDOF system.

2.1. Introduction of Inerter material into OpenSees

To numerically simulate the seismic responses of the building retrofitted with an inerter-integrated absorber in OpenSees, the first step is to introduce the inerter property into OpenSees. Similar to the ‘Elastic’ (spring) or ‘Viscous’ (damper) material, the inerter can be constructed as a new 'UniaxialMaterial' object using C++ programming language and integrated into OpenSees.

The source code to define an ideal linear ‘Inerter’ property in OpenSees has been partly published in [32] where the inerter is termed as a Gyromass damper. The force-deformation relationship described in this source code can be expressed as

$$ F = b \cdot \ddot{u} $$

where $F$ represents the force and $u$ is the relative deformation, and the material parameter is the inertance $b$. In this code, the local accelerations of elements $\ddot{u}$ are approximated with the Newmark-Beta integration approach [33], with the setting $\gamma = 0.5$ and $\beta = 0.25$ (see a detailed derivation in [32]). Note that in [32], the author mentioned the damping tangent for this material was chosen based on a trial and error process. However, based on the Newmark-Beta approach [33], the approximated damping tangent can be derived by

$$ \frac{\partial F}{\partial \ddot{u}} \approx \frac{\partial b\ddot{u}}{\partial \ddot{u}} = b \frac{\partial \ddot{u}}{\partial \ddot{u}} $$

$$ \frac{\Delta \dot{u}}{\Delta t} \approx \frac{\Delta u_i}{\Delta t} = \frac{\gamma \Delta t \Delta u_i - \gamma \ddot{u}_i + \Delta t (1 - \gamma) \dot{u}_i}{\Delta t} = \Omega, $$

where $\Delta t$ is the integration time step, $\Delta u_i$ is the increment of the displacement, $\dot{u}_i$, $\ddot{u}_i$ are the velocity and acceleration for the current time step. So taking $\gamma = 0.5$ and $\beta = 0.25$ into Eqs. 2 and 3, we obtain

$$ \frac{\partial F}{\partial \ddot{u}} \approx b \frac{\partial \Omega}{\partial \ddot{u}} = -b \frac{\gamma}{\beta \Delta t} = -\frac{2b}{\Delta t}. $$
To validate the inerter material introduced by the source code in OpenSees, a SDOF system as shown in Fig. 1(a) is considered, with a single inerter installed. In the model of this SDOF system in OpenSees, only the horizontal dynamics are considered and the ground is modelled as a fixed node N₁ while the moving mass as N₂. The system is excited by a harmonic excitation $\ddot{x}_g$. The inerter material is assigned to a ‘ZeroLength’ element which connects two nodes N₁ and N₂. A Simulink/Matlab model for this SDOF system is also built for comparison, in which an inerter is represented by the ideal force-acceleration $(F - \Delta a)$ relationship $F = b \cdot \Delta a$ is modelled. Fig. 2 shows the time-series responses obtained with OpenSees compared with those with Simulink/Matlab. It can be found that the obtained responses are very close to each other, for example with the displacement difference in the order of $10^{-4}$ m (Fig. 2(a)). A linear relationship between the applied force and the relative acceleration of the inerter is observed for both models (Fig. 2(d)).

Figure 1. (a) SDOF system used for validation of inerter modelling in OpenSees, (b) candidate inerter-integrated device layout Lᵢ (TVMD), with the big green dot representing the extra internal node labelled as IN.

Figure 2. Comparisons of the time histories of the inerter’s relative (a) displacement, (b) velocity, (c) acceleration, (d) the force-acceleration relationship of inerter obtained with OpenSees and Matlab.
2.2. Modelling of candidate inerter-integrated layout
After being integrated into OpenSees, the inerter material can be simulated together with other mechanical components, such as spring and damper, to function as an inerter-integrated vibration absorber. In this work, a particular candidate inerter-integrated vibration absorber, \( L_i \), is proposed, as shown in Fig. 1(b). This layout has shown to be beneficial for seismic protection of buildings [23, 27, 28], as introduced in Section 1 and will be analysed in this work.

It should be noted that the ‘inerter’ material in OpenSees introduced here can be combined in parallel, but not in series with other materials. Hence, an extra node, \( IN \), needs to be introduced when the inerter is in series with spring or damper, as the big green dot shown in Fig. 1(b).

To model \( L_i \) in the SDOF system, two ‘ZeroLength’ elements are considered: (i) the first is characterised with an elastic material (i.e., the spring), (ii) the second is characterised by a parallel material including the interter-damper properties. The two ‘ZeroLength’ elements are connected through the extra internal node \( IN \), to the ground and to the moving-mass node, \( N_1 \) and \( N_2 \), respectively. In this way, a close match between the responses obtained with OpenSees and Simulink/Matlab is also observed.

3. Modelling of a benchmark building and the integration of inerter-integrated absorber
In order to show a practical example of the usage of inerter-integrated vibration absorber, an existing building located in downtown Medellin (Colombia) is considered as a benchmark in this paper (see Fig. 3). In this section, a brief description of the test-bed building is presented and a 2D linear FEM model is extrapolated from the 3D structure and implemented in OpenSees. The \( L_i \) layout introduced in Section 2.2 is included in the OpenSees model.

![Figure 3](image)

**Figure 3.** The high-rise reinforced-concrete test-bed building located in downtown Medellin (Colombia): (a) photo of the building, (b) plan view of a typical storey, and (c) geometric configuration of the north-facade frame analysed in this paper.
3.1. Building description
The considered reinforced concrete benchmark building was built between 1964 and 1969 (see Fig. 3(a)) according to the ACI 318-63 standards (ACI 318 1963 [34]). The building is characterised by moment resisting frames along the two main directions and two shear walls next to the staircase, centrally located (see Fig. 3(b)). In 1982, another storey, which is Storey 11 according to the definitions provided in Fig. 3(c), was added at the top level. This storey is structured with a reinforced-concrete frame made of 30×30 cm$^2$ columns having similar reinforced concrete of the pre-existing building. Research attention have also been made to investigate the structural responses of an existing building with added storeys, which is called hybrid structure, for example, in [35]. Fig. 3(c) shows the north-facade frame of the building analysed in this paper. The construction of this building predated the Colombian first seismic design code CCCSR-84 [2].

3.2. FEM building model in OpenSees
The building described in Section 3.1 is analysed in OpenSees. The 2D FEM model of the frame in Fig. 3(c) is built up through elastic beam elements (‘elasticBeamColumn’ elements in OpenSees) characterised by a Youngs Modulus equal to 19.38 GPa. Cross sections of elements are assigned according to the technical drawings and account for reduction of flexural stiffness, due to concrete cracking, of 65% and 30% for beams and columns respectively. Storey diaphragms are assigned to each floor. Floor loads and elements’ self-weight are assigned through distributed loads. Masses are assigned at the ends of each element and they are evaluated according to the tributary area of the seismic weights distribution criterion. For simplicity the columns of the additional storey (Storey 11 in Fig. 3(c)) are considered fixed to the upper storey in the original configuration dating back to 1960s (Storey 10 in Fig. 3(c)) and the basement is not accounted in this FEM model. Some key system parameters used in the FEM model are summarised in Table 1.

| Parameters       | Values | Units |
|------------------|--------|-------|
| Storey Mass      |        |       |
| Storey 1         | 4.41   | t     |
| Storey 2         | 26.17  | t     |
| Storey 3         | 36.48  | t     |
| Storey 4         | 39.69  | t     |
| Storey 5-9       | 25.75  | t     |
| Storey 10        | 29.73  | t     |
| Storey 11        | 9.52   | t     |
| Storey Height    |        | m     |
| Storey 1         | 2.72   | m     |
| Storey 2         | 3.40   | m     |
| Storey 3-11      | 2.8    | m     |

Using modal analysis, modal properties of the building are reported in Table 2. Gravity load analysis is preliminarily performed to simulate the initial condition of the structure before an earthquake is applied. When subject to an earthquake, the FEM model is analysed through linear time-history analysis in OpenSees by using Newmark Integrator and the default analysis
parameters $\gamma = 0.50$ and $\beta = 0.25$. Rayleigh damping model with 5% damping coefficient applied to the first mode is assumed to evaluate damping forces in the linear time-history analysis.

### 3.3. Integration of the inerter-integrated absorber

In this work, the inerter-integrated vibration absorber $L_i$ will be installed between the two floors experiencing the maximum inter-storey drift under a seismic loading. As shown in Fig. 4(a), the absorber will connect two frame nodes across two bays horizontally (FN1 and FN2 in Fig. 4). Different from the inerter in the SDOF system which only works horizontally, the $L_i$ layout needs to work along the diagonal direction between two frame nodes. Hence, $L_i$ is modelled using two ‘twoNodeLink’ elements with finite lengths ($l$ in Fig. 4(b)) to define its diagonal orientation in the building frame. In detail, one of the ‘twoNodeLink’ elements has the material modelled as a spring and the other one as the material with inerter and damper working in parallel. A rigid element is connected between the two ‘twoNodeLink’ elements which are connected between the frames nodes (FN1 and FN2) and extra internal nodes (IN1 and IN2), as shown in Fig. 4(b). These ‘twoNodeLink’ elements have the two material properties along their longitudinal direction. The transversal direction of the ‘twoNodeLink’ elements is considered fixed while the rotation is considered free.

![Figure 4](image)

**Figure 4.** Illustrations of (a) the equivalent element of the absorber within the building frame, and (b) modelling detail of the $L_i$ layout adopted in *OpenSees* in this work.

### 4. Optimisation problem and results

In this section, the dynamic behaviour of the building is investigated through linear time-history analyses. For the sake of simplicity, the analyses are performed considering only one earthquake in order to show the benefits of the proposed inerter-integrated device. The time-domain optimisation problem and procedure are then introduced. Both a traditional damper (labelled as $L_c$) and the inerter-integrated device $L_i$ are considered and will be located where the maximum inter-storey drift occurs subject to the chosen earthquake. Based on the optimisations, the parameter values of the optimal damper $L_c$ and the inerter-integrated $L_i$ configurations are identified, with the performance advantages on reducing the maximum inter-storey drift quantified. Further optimisations are carried out to investigate the performance benefits of the optimal $L_i$ with a reduced inertance.

#### 4.1. Earthquake waveform selection

The benchmark building is analysed subject to the M5.3 Dursunbey earthquake 1979 because there is not a Colombian earthquake that excites the building properly. Fig. 5(a) shows the pseudo-acceleration spectrum of one of the two horizontal ground motion components, provided
by the European Strong Motion Database (http://isesd.hi.is/), compared to the Colombian code spectrum at damage limit state (NSR-10 [36]) together with the building’s periods of vibration. This earthquake represents a waveform with dominant frequency content localised in correspondence of the building’s higher modes. Transient time-history analysis is carried out using the case-study building subject to the chosen earthquake. The peak inter-storey drift (in displacement) plot is illustrated in Fig. 5(b).

Since the dominant frequency of the chosen earthquake does not affect the fundamental period of the building ($T_1$) which presents a pseudo-acceleration at $T_1$ equal to 0.06g against 0.2g of the damage code spectrum, this earthquake does not represent a significant case for the benchmark building. However, it can cause large inter-storey drifts at the top levels (Fig. 5(b)), which can lead to non-structural damage and hence the economic loss during earthquake events [37]. It can be seen from Fig. 5(b) that the maximum drift occurs between the 9th and 8th storeys (according to the storey numbering proposed in Fig. 3(c)), which equals 8.83 mm. For damage limit state verifications, the Colombian code imposes an inter-storey drift ratio limit (which is given by the inter-storey drift over the inter-storey height) equal to 0.4% for new concrete frame buildings having non-structural partitions separated from the main structure (i.e., frame).

For the case-study building, the inter-storey height between the 9th and 8th storeys is 2.8 m, resulting in an inter-storey drift limit equal to 11.2 mm. However, concrete frame buildings built before the existence of the Colombian code usually include unreinforced masonry partitions built without any separation from the main structure, making them particularly vulnerable to earthquakes. Previous research works have shown that unreinforced masonry partitions present levels of damage from values of the inter-storey drift ratio equal to 0.2% [38]. In this case, it leads to an inter-storey drift limit equal to 5.6 mm while the maximum inter-storey drift of the case-study building is about 1.6 times above the limit. In seismic retrofitting, one of the objectives can be reducing the inter-storey drifts through specific interventions [39]. In the following section, the absorber presented above is included between the 9th and 8th storey in order to investigate its benefits in reducing inter-storey drifts.

Figure 5. (a) Comparison of the design spectrum for damage limit state in NSR-10 [36] and the pseudo-acceleration spectrum of Dursunbey earthquake 1979, and (b) the peak inter-storey drift plot of the benchmark building subject to the chosen earthquake, where the maximum drift is represented by a thicker red line.
4.2. Optimisation problem and procedure

In this work, the objective function for the optimisation is the maximum inter-storey drift among all the storeys, which can be expressed as

\[ J = \max(|\Delta \delta_i|), \quad i = 0, \ldots, 11 \]  

(5)

where \( |\Delta \delta_i| \) represents the peak inter-storey drift between two consecutive storeys with respect to time. Both a linear damper \( L_c \) and the inerter-integrated device \( L_i \), located between the 9th and 8th storeys with the orientation shown in Fig. 4(a), will be optimised to minimise the objective function.

For the optimisation carried out in the present work, Matlab optimisation commands, patternsearch first and then fminsearch for fine-tuning, are used. The initial design parameters for the devices, the parameter boundaries and the objective function will be defined in Matlab. The objective function is evaluated with different parameters generated by the optimisation command for loads of iterations during the optimisation run. For each iteration, Opensees will be called to perform the numerical simulation with the parameters passed from Matlab and return the value of the objective function.

4.3. Optimisation results

Table 3 summarises all the optimisation results considering the \( L_c \) and \( L_i \) layouts. Note that two improvement measures are used in Table 3: one is a comparison between the benchmark building without any retrofitting and the building with retrofitting, and the other one is a comparison between the building with an optimal damper \( L_c \) and the inerter-integrated device \( L_i \) (in the brackets of Table 3).

It can be seen from Table 3 that with an optimal damper \( L_c \), the maximum inter-storey drift can be reduced by 31.4\%, comparing with the benchmark building. Further improvement is found if using an optimal \( L_i \), with up to 45.64\% comparing with the benchmark building and 20.8\% comparing with the case with the optimal \( L_c \). The damping value needed in this optimal \( L_i \), 2.48 KNS/mm, is significantly smaller than that of the optimal \( L_c \) (13.69 KNS/mm). The inertance required in the optimal \( L_i \) is 103 t, which is 4 times the mass of 9th storey, 25.75t.

Although the inertance can be significantly amplified by the gearing mechanism and 113 t is realisable in applications for civil structures, the device size and weight are still dependent on the inertance value. In practical use, it is desirable to have a smaller inertance resulting in the absorber having a feasible size and weight. Hence, it is worthwhile to investigate how much performance improvement will be compromised if reducing the inertance of \( L_i \). Herein, to measure the inertance value, we define an inertance-to-mass ratio \( \mu_b \) as expressed by

\[ \mu_b = \frac{b}{\text{Mass}_9}, \]

(6)

where \( \text{Mass}_9 \) is the mass of the 9th storey and \( \mu_b = 4 \) for the optimal \( L_i \) case without a limit on the inertance. Further optimisations of \( L_i \) are carried out with \( \mu_b = 2, 1, 0.5 \) and the results are illustrated in Table 3. It can be seen that with reduced inertance, the performance improvement will be reduced, for example, with half of the storey mass, only 4.79\% further reduction on the objective function can be obtained comparing with the optimal damper. This suggests a trade-off between the performance and the device’s inertance. The comparison of inter-storey drifts in the benchmark building and the optimal solutions provided in Table 3 is shown in Fig. 6. It can be observed that with an optimal \( L_c \) or \( L_i \), the maximum inter-storey drifts occur between the 8th and 7th storeys while for the benchmark building it is between the 9th and 8th storeys. The time-series inter-storey drifts between such storeys are compared in Fig. 7. It can be seen that with the help of the optimal \( L_i \) with \( \mu_b = 4 \), the time-series drifts are significantly smaller than those of the benchmark building and the building with the optimal \( L_c \).
Table 3. Optimisation results using $L_c$ and $L_i$

| Cases         | $J$, mm | Impro. over the benchmark, % | Parameter values | KN/mm, KNs/mm, t |
|---------------|---------|--------------------------------|------------------|------------------|
| Benchmark     | 8.83    | -                              |                  |                  |
| $L_c$         | 6.06    | 31.4(-)                        | $c = 13.69$      |                  |
| $L_i$         | 4.80    | 45.64 (20.8)                   | $c = 2.48, \ k = 7.0 \times 10^7, \ b = 103$ |                  |
| $L_i, \mu_b = 2$ | 5.26    | 40.43(13.2)                    | $c = 2.20, \ k = 1.0 \times 10^8, \ b = 51.5$ |                  |
| $L_i, \mu_b = 1$ | 5.63    | 36.24(7.1)                     | $c = 3.43, \ k = 2.3 \times 10^8, \ b = 25.75$ |                  |
| $L_i, \mu_b = 0.5$ | 5.77    | 34.65(4.79)                    | $c = 3.5, \ k = 7.8 \times 10^8, \ b = 12.88$ |                  |

Figure 6. Comparisons of the peak inter-storey drifts of the benchmark building and the building retrofitted with all optimised solutions. The maximum drifts are represented by thicker lines.

Figure 7. Comparisons of the time-series inter-storey drifts of the 9th and 8th storeys for the benchmark building and the 8th and 7th storeys for the building with the optimal $L_c$ and $L_i$, $\mu_b = 4$. 
5. Conclusions
This paper analyses the performance benefits of an inerter-integrated device for seismic retrofitting of a real-life high-rise building in Colombia. First, the inerter is introduced into OpenSees as a new material and its property has been validated with that in Matlab using a SDOF system. A candidate inerter-integrated device is proposed and the validation of its modelling in OpenSees is also demonstrated. A simplified 2D FEM model of the test-bed structure retrofitted with the proposed inerter-integrated absorber is implemented in OpenSees. A specific earthquake waveform is selected to excite the building. Based on the time-domain optimisations, it has been found that the optimal inerter-integrated absorber can help to reduce the maximum inter-storey drift among all the storeys, by 45.64% comparing with the structure without any retrofitting and 20.8% further reduction comparing with the employment of an optimal damper. The trade-off between the inertance and the performance gain has then been investigated.

6. Acknowledgement
The authors would like to acknowledge the support of the Engineering and Physical Sciences Research Council (EPSRC): Jason Zheng Jiang is supported by an EPSRC grant EP/P013456/1.

References
[1] Fardis M N 2009 Seismic design, assessment and retrofitting of concrete buildings: based on EN-Eurocode 8 vol 8 (Springer Science & Business Media)
[2] 1984 Colombian code of constructions earthquake resistant, decree-Law 1400 from 1984 (Colombian Association of Seismic Engineering, Bogota, Colombia)
[3] Tirca L, Serban O, Lin L, Wang M and Lin N 2015 Journal of Structural Engineering 142 C4015003
[4] Miranda E, Aslani H and Taghavi S 2004 International Workshop on Performance-Based Seismic Design: Concepts and Implementation (Pacific Earthquake Engineering Research (PEER) Center, University of California Berkeley) pp 149–160
[5] Cardone D and Flora A 2016 Earthquake Engineering & Structural Dynamics 45 91–111
[6] Saaed T E, Nikolakopoulos G, Jonasson J E and Hedlund H 2015 Journal of Vibration and Control 21 919–937
[7] Wang S and Mahin S A 2018 Journal of Structural Engineering 144 04018091
[8] Mazzolani F M, Della Corte G and D’Aniello M 2009 Journal of Civil Engineering and Management 15 7–19
[9] Bosco M and Marino E M 2013 Earthquake Engineering & Structural Dynamics 42 1243–1263
[10] Gerami M and Sivandi-Pour A 2014 The Structural Design of Tall and Special Buildings 23 881–896
[11] Symans M and Constantinou M 1998 ISET Journal of Earthquake Technology 35 185–206
[12] Bagheri S, Barghian M, Saieri F and Farzinfar A 2015 Structures vol 3 (Elsevier) pp 163–171
[13] Moreschi L and Singh M 2003 Earthquake Engineering & Structural Dynamics 32 1291–1311
[14] Mualla I H and Belev B 2002 Engineering Structures 24 365–371
[15] Kim J, Choi H and Min K W 2011 The Structural Design of Tall and Special Buildings 20 515–537
[16] Anajafi H and Medina R A 2018 Engineering Structures 158 110–123
[17] Rana R and Soong T 1998 *Engineering Structures* 20 193–204
[18] Sadek F, Mohraz B, Taylor A W and Chung R M 1997 *Earthquake Engineering & Structural Dynamics* 26 617–635
[19] Hoang N and Warnitchai P 2005 *Earthquake Engineering & Structural Dynamics* 34 125–144
[20] De Angelis M, Perno S and Reggio A 2012 *Earthquake Engineering & Structural Dynamics* 41 41–60
[21] Ikago K, Sugimura Y, Saito K and Inoue N 2012 *Proceedings of the 15th World Conference on Earthquake Engineering* vol 1575
[22] Smith M C 2002 *Proceedings of the 41st IEEE Conference on Decision and Control* vol 2 (IEEE) pp 1657–1662
[23] Watanabe Y, Ikago K, Inoue N, Kida H, Nakaminami S, Tanaka H, Sugimura Y and Saito K 2012 *Proceedings of the 15th World Conference on Earthquake Engineering* vol 1206
[24] Lazar I, Neild S and Wagg D 2014 *Earthquake Engineering & Structural Dynamics* 43 1129–1147
[25] Wang F C, Chen C W, Liao M K and Hong M F 2007 *Proceedings of the 46th IEEE Conference on Decision and Control* (IEEE) pp 3786–3791
[26] Wang F C, Hong M F and Chen C W 2010 *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 224 1605–1616
[27] Ikago K, Saito K and Inoue N 2012 *Earthquake Engineering & Structural Dynamics* 41 453–474
[28] Ikago K, Sugimura Y, Saito K and Inoue N 2012 *Journal of Asian Architecture and Building Engineering* 11 375–382
[29] Marian L and Giaralis A 2014 *Probabilistic Engineering Mechanics* 38 156–164
[30] Zhang S Y, Jiang J Z and Neild S 2017 *Structural Control and Health Monitoring* 24 e1887
[31] Mazzoni S, McKenna F, Scott M H, Fenves G L et al. 2006 *Pacific Earthquake Engineering Research (PEER) Center* 264
[32] Mirza Hessabi R 2017 Application of Real-Time Hybrid Simulation Method in Experimental Identification of Gyromass Dampers Ph.D. thesis
[33] Chopra A K 2007 *Dynamics of structures: theory and applications to earthquake engineering* vol 3 (Pearson/Prentice Hall Upper Saddle River, NJ)
[34] Standard A 1963 *Journal of the American Concrete Institute*
[35] Sivandi-Pour A, Gerami M and Khayroddin A 2015 *Iranian Journal of Science and Technology. Transactions of Civil Engineering* 39 81
[36] 2010 *Standards of design and construction earthquake resistant, NSR-10, decree 926 of 2010 and other regulatory decrees* (Colombian Seismic Engineering Association, Bogota, Colombia)
[37] Ramirez C, Liel A, Mitran-Reiser J, Haselton C, Spear A, Steiner J, Deierlein G and Miranda E 2012 *Earthquake Engineering & Structural Dynamics* 41 1455–1475
[38] Farbiarz J 1999 *Proceedings of the 8th North American Masonry Conference*
[39] Del Gobbo G M, Blakeborough A and Williams M S 2018 *Bulletin of Earthquake Engineering* 1–24