Towards the assignment for the $4^1S_0$ meson nonet

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Abstract

The strong decays of the $\pi(2070)$, $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, and $\eta(2225)$ as the $4^1S_0$ quark-antiquark states are investigated in the framework of the $^3P_0$ meson decay model. It is found that the $\pi(2070)$, $\eta(2100)$, and $\eta(2225)$ appear to be the convincing $4^1S_0$ $q\bar{q}$ states while the assignment of the $\eta(2010)$ and $\eta(2190)$ as the $4^1S_0$ isoscalar states is not favored by their widths. In the presence of the $\pi(2070)$, $\eta(2100)$, and $\eta(2225)$ being the members of the $4^1S_0$ meson nonet, the $4^1S_0$ kaon is phenomenologically determined to have a mass of about 2153 MeV. The width of this unobserved kaon is expected to be about 197 MeV in the $^3P_0$ decay model.

Key words: mesons, $^3P_0$ model

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1 Introduction

From PDG2006[1], the $1^1S_0$ meson nonet ($\pi$, $\eta$, $\eta'$, and $K$) as well as the $2^1S_0$ members [$\pi(1300)$, $\eta(1295)$, and $\eta(1475)$] has been well established. In Ref.[2], we suggested that the $\pi(1800)$, $K(1830)$, together with the $X(1835)$ and $\eta(1760)$ observed by BES Collaboration[3, 4] constitute the $3^1S_0$ meson nonet. More recently, we argued that the $\eta(2225)$ with a mass of $(2240^{+30}_{-20}+30^{-20}_{-20})$ MeV and a width of $(190\pm30^{+40}_{-60})$ MeV observed by the BES Collaboration[5] could be the $4^1S_0$ $s\bar{s}$ in Ref. [6] where the other members of the $4^1S_0$ meson nonet were not discussed. In the present work, we shall address the possible assignment for the $4^1S_0$ meson nonet.

With the assignment of the $\eta(2225)$ as the $s\bar{s}$ member of the $4^1S_0$ meson nonet, one can expect that both the isovector and another isoscalar members of the $4^1S_0$ meson nonet should be lighter than the $\eta(2225)$. Experimentally, in the mass region 2000-2225 MeV, the pseudoscalar states $\pi(2070)$ [Mass: $2070\pm35$ MeV, Width: $310^{+100}_{-50}$ MeV], $\eta(2010)$ [Mass: $2010^{+35}_{-60}$ MeV, Width: $270\pm60$ MeV], $\eta(2100)$ [Mass: $2103\pm50$ MeV, Width: $187\pm75$ MeV], and $\eta(2190)$ [Mass: $2190\pm50$ MeV, Width: $850\pm100$ MeV] are reported[1]. Theoretically, some predicted values for the $\pi(4^1S_0)$ mass are 2.15 GeV by QCD sum rules[7, 8], 2.009 GeV by the spectrum integral equation[9], 2.193 GeV by a covariant quark model[10], 2.039 GeV by a relativistic independent quark model[11], and 2.07 GeV by Regge phenomenology[12]. In addition, the mass of the third radial excitation of the $\eta$ is predicted to be about 2.267 GeV by a covariant quark model[10] or 2.1 GeV by Regge phenomenology[12]. The $\pi(2070)$ mass is similar to the predicted $\pi(4^1S_0)$ mass, and all the masses of the $\eta(2010)$, $\eta(2100)$, and $\eta(2190)$ are close to the predicted mass range of the third radial excitation of the $\eta$. Only the mass information of these states is insufficient to classify them. The main purpose of this work is to discuss whether these reported pseudoscalar states can be assigned as the members of the $4^1S_0$ meson nonet or not by investigating their decay properties in the $3P_0$ meson decay model.

The organization of this paper is as follows. In section 2, the brief review of the $3P_0$ decay model is given (for the detailed review see e.g. Refs.[13, 14, 15, 16].) In sections 3 and 4, the decay widths of the $\pi(2070)$, $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, and $\eta(2225)$ as the $4^1S_0$ $q\bar{q}$ state are presented. The decay widths of the $4^1S_0$ kaon are predicted in section 5, and the summary and
conclusion are given in section 6.

2 The $^{3}P_0$ meson decay model

The $^{3}P_0$ decay model, also known as the quark-pair creation model, was originally introduced by Micu[17] and further developed by Le Yaouanc et al.[13]. The $^{3}P_0$ decay model which (in several variants) is the standard model for strong decays at least for mesons in the initial state, has been widely used to evaluate the strong decays of hadrons[18, 19, 20, 21, 22, 23, 24, 25, 26, 27], since it gives a good description of many of the observed decay amplitudes and partial widths of the hadrons. The main assumption of the $^{3}P_0$ decay model is that strong decays take place via the creation of a $^{3}P_0$ quark-antiquark pair from the vacuum. The new produced quark-antiquark pair, together with the $q\bar{q}$ within the initial meson regroups into two outgoing mesons in all possible quark rearrangement ways, which corresponds to the two decay diagrams as shown in Fig.1 for the meson decay process $A \rightarrow B + C$.

![Diagram](image)

Figure 1: The two possible diagrams contributing to $A \rightarrow B + C$ in the $^{3}P_0$ model.

The transition operator $T$ of the decay $A \rightarrow BC$ in the $^{3}P_0$ model is given by

$$T = -3\gamma \sum_m (1m1 - m|00) \int d^3\vec{p}_3 d^3\vec{p}_4 \delta^3(\vec{p}_3 + \vec{p}_4) \chi^m_{1} \frac{(\vec{p}_3 - \vec{p}_4)}{2} \chi^{34}_{1-m} \phi^{34}_{0} \omega^{34}_{0} b^{\dagger}_{3}(\vec{p}_3) d^{\dagger}_{4}(\vec{p}_4),$$

(1)

where $\gamma$ is a dimensionless parameter representing the probability of the quark-antiquark pair $q_3\bar{q}_4$ with $J^{PC} = 0^{++}$ creation from the vacuum, $\vec{p}_3$ and $\vec{p}_4$ are the momenta of the created quark $q_3$ and antiquark $\bar{q}_4$, respectively. $\phi^{34}_{0}$, $\omega^{34}_{0}$, and $\chi^{34}_{1-m}$ are the flavor, color, and spin wave functions of the $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\chi_{1}^m(\vec{p}) \equiv |p|Y_{1}^{m}(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\bar{q}_4$.

For the meson wave function, we adopt the mock meson $|A(n_{A}^{2S_{A}+1}L_{A}J_{A}M_{A})⟩(\vec{P}_A)$ defined
by\cite{28}

\[ |A(n^2_A L_A J_{A M_J A})(\vec{P}_A)\rangle = \sqrt{2E_A} \sum_{M_{L_A},M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A}|J_{A M_J A}\rangle \]

\[ \times \int d^3\vec{p}_A \psi_n A L_A M_{L_A}(\vec{P}_A) \chi^{12}_{S_A M_{S_A}} \phi^{12}_A \omega^{12}_A \]

\[ \times |q_1(\frac{m_1}{m_1+m_2}\vec{p}_A + \vec{p}_A)q_2(\frac{m_2}{m_1+m_2}\vec{p}_A - \vec{p}_A)\rangle, \quad (2) \]

where \( m_1 \) and \( m_2 \) are the masses of the quark \( q_1 \) with a momentum of \( \vec{p}_1 \) and the antiquark \( \vec{q}_2 \) with a momentum of \( \vec{p}_2 \), respectively. \( n_A \) is the radial quantum number of the meson \( A \) composed of \( q_1 \vec{q}_2 \). \( \vec{S}_A = \vec{s}_{q_1} + \vec{s}_{q_2}, \vec{J}_A = \vec{L}_A + \vec{S}_A, \vec{s}_{q_1} (\vec{s}_{q_2}) \) is the spin of \( q_1 (q_2), \vec{L}_A \) is the relative orbital angular momentum between \( q_1 \) and \( q_2 \). \( \vec{P}_A = \vec{p}_1 + \vec{p}_2, \vec{p}_A = \frac{m_1\vec{p}_1 - m_2\vec{p}_2}{m_1 + m_2}. \)

\[ \langle L_A M_{L_A} S_A M_{S_A}|J_{A M_J A}\rangle \] is a Clebsch-Gordan coefficient, and \( E_A \) is the total energy of the meson \( A \). \( \chi^{12}_{S_A M_{S_A}}, \phi^{12}_A, \omega^{12}_A, \) and \( \psi_n A L_A M_{L_A}(\vec{P}_A) \) are the spin, flavor, color, and space wave functions of the meson \( A \), respectively. The mock meson satisfies the normalization condition

\[ \langle A(n^2_A L_A J_{A M_J A})(\vec{P}_A)|A(n^2_A L_A J_{A M_J A})(\vec{P}_A')\rangle = 2E_A\delta^3(\vec{P}_A - \vec{P}_A'). \quad (3) \]

The S-matrix of the process \( A \rightarrow BC \) is defined by

\[ \langle BC|S|A\rangle = I - 2\pi i\delta(E_A - E_B - E_C)\langle BC|T|A\rangle, \quad (4) \]

with

\[ \langle BC|T|A\rangle = \delta^3(\vec{P}_A - \vec{P}_B - \vec{P}_C)\mathcal{M}^{M_J A M_J B M_J C}, \quad (5) \]

where \( \mathcal{M}^{M_J A M_J B M_J C} \) is the helicity amplitude of \( A \rightarrow BC \). In the center of mass frame of meson \( A \), \( \mathcal{M}^{M_J A M_J B M_J C} \) can be written as

\[ \mathcal{M}^{M_J A M_J B M_J C}(\vec{P}) = \gamma\sqrt{S_E A E_B E_C} \sum_{M_{L_A},M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A}|J_{A M_J A}\rangle \]

\[ \times \langle L_B M_{L_B} S_B M_{S_B}|J_B M_{J_B}\rangle \langle L_C M_{L_C} S_C M_{S_C}|J_C M_{J_C}\rangle \]

\[ \times \langle 1m_1 - m|00\rangle \chi^{14}_{S_B M_{S_B}} \chi^{32}_{S_C M_{S_C}} \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1 - m} \]

\[ \times [f_1(\vec{P}, m_1, m_2, m_3) + (-1)^{1+S_A+S_B+S_C} f_2(\vec{P}, m_2, m_1, m_3)], \quad (6) \]
with \( f_1 = \langle \phi_B^{14} \phi_C^{32} | \phi_A^{12} \phi_0^{34} \rangle \) and \( f_2 = \langle \phi_B^{32} \phi_C^{14} | \phi_A^{12} \phi_0^{34} \rangle \), corresponding to the contributions from Figs. 1 (a) and 1 (b), respectively, and

\[
I(\vec{P}, m_1, m_2, m_3) = \int d^3\vec{p} \psi_n^* \varphi_{nLAMLA}(\vec{P}_B + \vec{p}) \times \psi_n \varphi_{nLAMLA}^*(\vec{P}_B + \vec{p}) Y_{LM}^m(\vec{p}),
\]

(7)

where \( \vec{P} = \vec{P}_B = -\vec{P}_C, \vec{p} = \vec{p}_3, m_3 \) is the mass of the created quark \( q_3 \).

The spin overlap in terms of Winger’s 9j symbol can be given by

\[
\langle \chi_{SB}^{14} \chi_{SC}^{32} | \chi_{SA}^{12} \chi_{SA}^{34} \rangle = \sum_{S,M_S} \langle S_B | S_C \rangle \langle S_C | S_A \rangle \langle S_A | 1-m | S_M \rangle \times (-1)^{S+1} \sqrt{3(2S_A + 1)(2S_B + 1)(2S_C + 1)} \left\{ \begin{array}{ccc}
\frac{1}{2} & \frac{1}{2} & S_A \\
\frac{1}{2} & \frac{1}{2} & 1 \\
S_B & S_C & S
\end{array} \right\}.
\]

(8)

In order to compare with the experiment conventionally, \( \mathcal{M}^{MJA,MJB,MJC}(\vec{P}) \) can be converted into the partial amplitude by a recoupling calculation[29]

\[
\mathcal{M}^{LS}(\vec{P}) = \sum_{M_JA,MJB,MJC} \langle LM_L | S_M | S_A \rangle \langle J_B | J_C \rangle \langle J_A | S_M \rangle \times \int d\vec{p} Y_{LM_L}^* \mathcal{M}^{MJA,MJB,MJC}(\vec{P}).
\]

(9)

If we consider the relativistic phase space, the decay width \( \Gamma(A \to BC) \) in terms of the partial wave amplitudes is

\[
\Gamma(A \to BC) = \frac{\pi P}{4M_A^2} \sum_{LS} |\mathcal{M}^{LS}|^2.
\]

(10)

Here \( P = |\vec{P}| = \sqrt{M_A^2 - (M_B + M_C)^2} \left[ (M_A^2 - (M_B - M_C)^2) \right]/2M_A \), \( M_A, M_B, \) and \( M_C \) are the masses of the meson \( A, B, \) and \( C \), respectively.

The decay width can be derived analytically if the simple harmonic oscillator (SHO) approximation for the meson space wave functions is used. In momentum-space, the SHO wave function is

\[
\psi_{nLM_L}(\vec{p}) = R_{nL}(p)Y_{LM_L}(\Omega_p),
\]

(11)
where the radial wave function is given by

$$P_{nL}^{\text{SHO}} = \frac{(-1)^n(-i)^L}{\beta^{\frac{L}{2}}} \sqrt{\frac{2n!}{\Gamma(n + L + \frac{3}{2})}} \left(\frac{p}{\beta}\right)^L e^{-\frac{p^2}{2\beta^2}} L_n^{L+\frac{1}{2}}(\frac{p^2}{\beta^2}).$$  \hspace{1cm} (12)

Here $\beta$ is the SHO wave function scale parameter, and $L_n^{L+\frac{1}{2}}(\frac{p^2}{\beta^2})$ is an associated Laguerre polynomial.

The SHO wave functions cannot be regarded as realistic, however, they are a de facto standard for many nonrelativistic quark model calculations. Moreover, the more realistic space wave functions such as those obtained from Coulomb, plus the linear potential model do not always result in systematic improvements due to the inherent uncertainties of the $^3P_0$ decay model itself[19, 20, 22]. The SHO wave function approximation is commonly employed in the $^3P_0$ decay model in literature. In the present work, the SHO wave function approximation for the meson space wave functions is taken.

Under the SHO wave function approximation, the parameters used in the $^3P_0$ decay model involve the $q\bar{q}$ pair production strength parameter $\gamma$, the SHO wave function scale parameter $\beta$, and the masses of the constituent quarks. In the present work, we take $\gamma = 8.77$ and $\beta_A = \beta_B = \beta_C = \beta = 0.4$ GeV, the values recently obtained by fitting 32 experimentally well-determined decay rates with the $^3P_0$ decay model$^1$, and $m_u = m_d = 0.33$ GeV, $m_s = 0.55$ GeV[26]. The meson masses used to determine the phase space and final state momenta are$^2$

$M_\pi = 138$ MeV, $M_K = 496$ MeV, $M_\eta = 548$ MeV, $M_\eta' = 958$ MeV, $M_\rho = 776$ MeV, $M_{K^*} = 894$ MeV, $M_{\omega} = 783$ MeV, $M_\phi = 1019$ MeV, $M_{a_2}(1320) = 1318$ MeV, $M_{K^*_2}(1430) = 1429$ MeV, $M_{f_2}(1270) = 1275$ MeV, $M_{f_2'}(1525) = 1525$ MeV, $M_{\pi}(1300) = 1240$ MeV, $M_{a_0}(1450) = 1474$ MeV, $M_{K^*}(1430) = 1414$ MeV, $M_{f_0}(1370) = 1370$ MeV, $M_{\rho}(1450) = 1459$ MeV, $M_{\omega}(1420) = 1420$ MeV, $M_{K^*}(1580) = 1580$ MeV, $M_{\rho}(1700) = 1720$ MeV, $M_{K^*}(1680) = 1717$ MeV, and $M_{K^*_4}(1780) = 1776$ MeV.

$^1$Our value of $\gamma$ is higher than that used by Ref.[26] (0.505) by a factor of $\sqrt{96\pi}$ due to different field conventions, constant factor in $T$, etc. The calculated results of the widths are, of course, unaffected.

$^2$The assignment the $K^*(1410)$ as the $2^3S_1$ kaon is problematic[25, 30]. Quark model[31] and other phenomenological approaches[32] consistently suggest the $2^3S_1$ kaon has a mass about 1580 MeV, here we take 1580 MeV as the mass of the $2^3S_1$ kaon [$K^*(1580)$]. Also, we assume that the $a_0(1450)$, $K_0^*(1430)$, and $f_0(1370)$ are the ground scalar mesons as Refs.[23, 24, 25].
3 Decays of the $\pi(2070)$

From (10), the numerical values of the partial decay widths of the $\pi(2070)$ as the $4^1S_0$ isovector state are listed in Table 1. The initial state mass is set to 2070 MeV.

Table 1: Decays of the $\pi(2070)$ as the $4^1S_0$ isovector state in the $3^P_0$ model.

| Mode       | $\Gamma_i$ (MeV) | Mode       | $\Gamma_i$ (MeV) |
|------------|------------------|------------|------------------|
| $\rho\omega$ | 3.1              | $\rho\pi$  | 5.9              |
| $\pi(1300)\rho$ | 52.0            | $\rho(1700)\pi$ | 3.6           |
| $\rho(1450)\pi$ | 112.5           | $f_2(1270)\pi$ | 48.0          |
| $f_0(1370)\pi$ | 0.3             | $a_2(1320)\eta$ | 8.8           |
| $a_0(1450)\eta$ | 5.1             | $KK^*$     | 8.5             |
| $K^*K^*$      | 14.9            | $K_0^*(1430)K$ | 10.3        |
| $K_2^*(1430)K$ | 4.6             |            |                 |

$\Gamma_{thy} = 277.6$ MeV, $\Gamma_{expt} = 310^{+100}_{-50}$ MeV

Figure 2: The predicted total width of the $\pi(2070)$ as the $4^1S_0$ isovector versus the initial state mass.

Table 1 indicates that the total width of the $\pi(2070)$ as the $4^1S_0$ isovector state predicted by the $3^P_0$ decay model is about 278 MeV, consistent with the observation ($310^{+100}_{-50}$) MeV within errors, and the dominant decay modes are expected to be $\pi(1300)\rho$, $\rho(1450)\pi$ and $f_2(1270)\pi$.

Also, in order to check the dependence of the theoretical result on the initial state mass, the predicted total width of the $\pi(2070)$ is shown in Fig. 2 as the function of the initial state mass. Fig. 2 shows that when the initial state mass varies from 2035 to 2105 MeV, the total width of
the $4^1S_0$ isovector state varies from about 230 to 320 MeV, generally in accord with the width range of the $\pi(2070)$. Both the mass and width of the $\pi(2070)$ are consistent with the predicted $4^1S_0$ isovector state, which therefore suggests that the assignment of the $\pi(2070)$ as the $4^1S_0$ isovector state seems plausible.

## 4 Decays of the $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, and $\eta(2225)$

In the presence of the $\eta(2225)$ being one isoscalar member of the $4^1S_0$ meson nonet, the $\eta(2010)$, $\eta(2100)$, and $\eta(2190)$ would complete another isoscalar member. It is well known that in a meson nonet, the two physical isoscalar states can mix. The mixing of the two isoscalar states can be parameterized as

$$\eta(x) = \cos \phi \ n\bar{n} - \sin \phi \ s\bar{s},$$

$$\eta(2225) = \sin \phi \ n\bar{n} + \cos \phi \ s\bar{s},$$

where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ are the pure $4^1S_0$ nonstrange and strange states, respectively, and $\eta(x)$ denotes the $\eta(2010)$, $\eta(2100)$, or $\eta(2190)$.

According to (10), the partial widths of $\eta(x)$ and $\eta(2225)$ become with mixing

$$\Gamma(\eta(x) \to BC) = \frac{\pi P}{4M_{\eta(x)}^2} \sum_{LS} |\cos \phi M_{n\bar{n}\to BC}^{LS} - \sin \phi M_{s\bar{s}\to BC}^{LS}|^2,$$  \hspace{1cm} (15)

$$\Gamma(\eta(2225) \to BC) = \frac{\pi P}{4M_{\eta(2225)}^2} \sum_{LS} |\sin \phi M_{n\bar{n}\to BC}^{LS} + \cos \phi M_{s\bar{s}\to BC}^{LS}|^2.$$  \hspace{1cm} (16)

Based on (15) and (16), the predicted total widths of the $\eta(2010)$, $\eta(2100)$, $\eta(2190)$, and $\eta(2225)$ are shown in Fig. 3 as functions of the initial state mass and the mixing angle $\phi$. From Fig. 3, one can see that with the variations of the initial state mass and $\phi$, only the measured widths of the $\eta(2100)$ and $\eta(2225)$ are possible to be reasonably reproduced in the $3^3P_0$ model. We therefore suggest that the assignment of the $\eta(2010)$ and $\eta(2190)$ as the $4^1S_0$ isoscalar states seems unfavorable. We shall focus on the possibility of the $\eta(2100)$ being the partner of the $\eta(2225)$. Taking $M_{\eta(2100)} = 2103$ MeV and $M_{\eta(2225)} = 2240$ MeV, we list the numerical values of the partial decay widths of the $\eta(2100)$ and $\eta(2225)$ in Table 2. The variation of the theoretical total widths of the $\eta(2100)$ and $\eta(2225)$ with the mixing angle $\phi$ is shown in Fig. 4.
From Fig. 4, we find that if the $\eta(2100) - \eta(2225)$ mixing angle $\phi$ lying in the range from about $-0.6$ to $+0.7$ radians, both the measured widths of the $\eta(2100)$ and $\eta(2225)$ can be reasonably reproduced. In order to check whether the possibility of $-0.6 \leq \phi \leq +0.7$ radians exists or not, below we shall estimate the $\eta(2100)$-$\eta(2225)$ mixing angle phenomenologically.

In the $n\bar{n}$ and $s\bar{s}$ basis, the mass-squared matrix describing the $\eta(2100)$ and $\eta(2225)$ mixing can be written as

$$M^2 = \begin{pmatrix} M_{n\bar{n}}^2 + 2A_m & \sqrt{2}A_mX \\ \sqrt{2}A_mX & M_{s\bar{s}}^2 + A_mX^2 \end{pmatrix},$$

where $M_{n\bar{n}}$ and $M_{s\bar{s}}$ are the masses of the states $n\bar{n}$ and $s\bar{s}$, respectively, $A_m$ denotes the total
Table 2: Decays of the $\eta(2100)$ and $\eta(2225)$ as the $4^1S_0$ isoscalar states in the $^3P_0$ model. $c \equiv \cos \phi$, $s \equiv \sin \phi$.

| Mode               | $\eta(2100)$                | $\eta(2225)$                |
|--------------------|-----------------------------|-----------------------------|
| $\rho \rho$        | $2.1c^2$                    | $1.4s^2$                    |
| $\omega \omega$    | $0.9c^2$                    | $0.3s^2$                    |
| $\phi \phi$        | $9.7s^2$                    | $20.1c^2$                   |
| $a_2(1320)\pi$     | $143.7c^2$                  | $158.8s^2$                  |
| $a_0(1450)\pi$     | $0.1c^2$                    | $4.5s^2$                    |
| $KK^*$             | $7.4c^2 - 15.0cs + 7.6s^2$  | $14.4c^2 + 12.7cs + 2.8s^2$ |
| $K^*K^*$           | $15.4c^2 - 13.4cs + 2.9s^2$ | $0.8c^2 - 6.6cs + 13.6s^2$  |
| $KK_0^*(1430)$     | $8.8c^2 - 7.9cs + 1.8s^2$   | $2.5c^2 - 4.7cs + 2.2s^2$   |
| $KK_2^*(1430)$     | $7.7c^2 - 31.1cs + 31.3s^2$ | $69.4c^2 + 91.9cs + 30.4s^2$|
| $KK^*(1580)$       | $5.4c^2 + 17.2cs + 13.6s^2$ | $90.7c^2 - 158.1cs + 68.9s^2$|
| $KK^*(1680)$       |                             | $1.6c^2 - 1.4cs + 0.3s^2$   |

\( \Gamma_{\text{th}} = 191.5c^2 - 50.2cs + 66.9s^2 \) \quad \Gamma_{\text{th}} = 199.4c^2 - 66.2cs + 283.2s^2

\( \Gamma_{\text{expt}} = 187 \pm 75 \) \quad \Gamma_{\text{expt}} = 190 \pm 30^{+40}_{-60} \)

Annihilation strength of the $q\bar{q}$ pair for the light flavors $u$ and $d$, $X$ describes the $SU(3)$-breaking ratio of the nonstrange and strange quark masses via the constituent quark mass ratio $m_u/m_s$.

The masses of the two physical states $\eta(2100)$ and $\eta(2225)$ can be related to the matrix $M^2$ by the unitary matrix $U = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}$

\[ U M^2 U^\dagger = \begin{pmatrix} M^2_{\eta(2100)} & 0 \\ 0 & M^2_{\eta(2225)} \end{pmatrix}. \tag{18} \]

$n\bar{n}$ is the orthogonal partner of the $\pi(4^1S_0)$, the isovector state of $4^1S_0$ meson nonet, and one can expect that $n\bar{n}$ degenerates with $\pi(4^1S_0)$ in effective quark masses, here we take $M_{n\bar{n}} = M_{\pi(4^1S_0)} = M_{\pi(2070)}$. With the help of the Gell-Mann-Okubo mass formula $M_{ss}^2 = 2M_{K(4^1S_0)}^2$ --
the following relations can be derived from (18)

\[ 8X^2(M_{K(4^1S_0)}^2 - M_{\pi(2070)}^2)^2 = [4M_{K(4^1S_0)}^2 - (2 - X^2)M_{\pi(2070)}^2 - (2 + X^2)M_{\eta(2100)}^2] \]
\[ \times [(2 - X^2)M_{\pi(2070)}^2 + (2 + X^2)M_{\eta(2225)}^2 - 4M_{K(4^1S_0)}^2], \] (19)

\[ A_m = \frac{(M_{\eta(2225)}^2 - 2M_{K(4^1S_0)}^2 + M_{\pi(2070)}^2)(M_{\eta(2100)}^2 - 2M_{K(4^1S_0)}^2 + M_{\pi(2070)}^2)}{2(M_{\pi(2070)}^2 - M_{K(4^1S_0)}^2)X^2}. \] (20)

If the SU(3)-breaking effect is not considered, i.e., \( X = 1 \), relation (19) can be reduced to Schwinger’s original nonet mass formula[36]. Taking \( X = m_u/m_s = 0.33/0.55 = 0.6 \), from (19) and (20) we have

\[ M_{K(4^1S_0)} = 2.153 \text{ GeV}, \ A_m = 0.07 \text{ GeV}^2. \] (21)

Based on the values of the above parameters involved in (17), the unitary matrix \( U \) can be given by

\[ U = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} = \begin{pmatrix} +0.995 & -0.104 \\ +0.104 & +0.995 \end{pmatrix}, \] (22)

which gives \( \phi = +0.1 \) radians, just lying in the range from about \(-0.6\) to \(+0.7\) radians. From Table 2, this estimated mixing angle leads to \( \Gamma_{\text{thy}}(\eta(2100)) = 185.2 \text{ MeV} \) and \( \Gamma_{\text{thy}}(\eta(2225)) = 193.7 \text{ MeV} \), both in good agreement with the experimental results. The \( \eta(2100) \) and \( \eta(2225) \), together with the \( \pi(2070) \) therefore appear to be the convincing \( 4^1S_0 \) states.

### 5 Decays of the \( 4^1S_0 \) kaon

The above two sections show that in the presence of the \( \pi(2070) \), \( \eta(2100) \), and \( \eta(2225) \) belonging to the \( 4^1S_0 \) meson nonet, the total widths of these three states can be naturally accounted for in the \( ^3P_0 \) decay model, and the \( 4^1S_0 \) kaon is expected to have a mass of about 2153 MeV by the mass formula (19). Below the \( K(2150) \) denotes the \( 4^1S_0 \) kaon. We note that the \( K, K(1460)^3, K(1830) \), and \( K(2150) \) approximately populate a common trajectory as shown in Fig. 5. The quasi-linear trajectories at the \( (n, \text{Mass-squared}) \)-plots turned out to be able to describe the light mesons with a good accuracy[12]. Fig. 5 therefore indicates that the

\( ^3 \)The \( K(1460) \) mass is taken 1400 MeV reported by[37].
$K(1460)$, $K(1830)$, and $K(2150)$ could be the good candidates for the $2^1S_0$, $3^1S_0$, and $4^1S_0$ kaons, respectively.

![Diagram](image)

**Figure 5:** The (n, Mass-squared)-trajectory for the $K$.

The $K(2150)$ is not related any current experimental candidate. The predicted decay widths of the $K(2150)$ are listed in Table 3. The initial state mass is set to 2153 MeV. The total width of the $K(2150)$ is predicted to be about 197 MeV, and the dominant decay modes of the $K(2150)$ are expected to be $K^*_2(1430)\pi$, $K^*(1580)\pi$, $\rho(1459)K$ and $a_2(1320)K$. These results could be of use in searching the candidate for the $4^1S_0$ kaon experimentally.

## 6 Summary and conclusion

With the assignment of the $\eta(2225)$ recently observed by the BES Collaboration as the $s\bar{s}$ member of the $4^1S_0$ meson nonet, the possibility of the $\pi(2070)$, $\eta(2010)$, $\eta(2100)$, and $\eta(2190)$ being the $4^1S_0$ $q\bar{q}$ states is discussed. With respect to the $\pi(2070)$, its assignment to the $4^1S_0$ isovector state is not only favored by its mass, but also by its width. The assignment of the $\eta(2010)$ and $\eta(2190)$ as the $4^1S_0$ isoscalar states is not favored by their widths. Both the widths of the $\eta(2100)$ and $\eta(2225)$ can be reasonably reproduced with the mixing angle lying in the range from about $-0.6$ to $+0.7$ radians. The assignment of the $\pi(2070)$, $\eta(2100)$, and $\eta(2225)$ as the members of the $4^1S_0$ meson nonet not only leads to that the $\eta(2100)$-$\eta(2225)$ mixing angle is about $+0.1$ radians which naturally accounts for the widths of the $\eta(2100)$ and $\eta(2225)$, but also gives that the $4^1S_0$ kaon has a mass of about 2153 MeV. The $K$, $K(1460)$, $K(1830)$, and $K(2150)$ approximately populate a common (n, Mass-squared)-trajectory. We
Table 3: Decays of the $K(2150)$ as the $4^1S_0$ isodoublet in the $^3P_0$ model.

| Mode   | $\Gamma_i$ (MeV) | Mode   | $\Gamma_i$ (MeV) |
|--------|------------------|--------|------------------|
| $\rho K$ | 3.9             | $\omega K$ | 1.3          |
| $\phi K$ | 1.3             | $\rho K^*$ | 0.08         |
| $\omega K^*$ | 0.04           | $\phi K^*$ | 12.0        |
| $\pi K^*$ | 4.5             | $\eta K^*$ | 1.5          |
| $\eta' K^*$ | 0.09           | $K^*_0(1430)\pi$ | 1.6     |
| $K^*_2(1430)\pi$ | 29.7          | $K^*(1580)\pi$ | 34.5      |
| $K^*(1680)\pi$ | 2.5           | $K^*_3(1780)\pi$ | 1.0      |
| $\pi(1300)K^*$ | 6.3           | $\rho(1450)K$ | 42.8    |
| $\omega(1420)K$ | 14.6          | $a_2(1320)K$ | 26.7     |
| $f_2(1270)K$ | 9.9            | $f'_2(1525)K$ | 2.9      |

$\Gamma_{thy} = 197.2$ MeV

tend to conclude that the observed pseudoscalar states $\pi(2070), \eta(2100), \eta(2225)$, together with the unobserved $K(2150)$ appear to be the good candidates for the members of the $4^1S_0$ meson nonet. The $K(2150)$ width is predicted to be about 197 MeV, and the dominant decay modes of the $K(2150)$ are expected to be $K^*_2(1430)\pi$, $K^*(1580)\pi$, $\rho(1459)K$ and $a_2(1320)K$. These results could be of use in searching the candidate for the $4^1S_0$ kaon experimentally.

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