Towards balanced clustering - part 1 (preliminaries)

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The article contains a preliminary glance at balanced clustering problems. Basic balanced structures and combinatorial balanced problems are briefly described. A special attention is targeted to various balance/unbalance indices (including some new versions of the indices): by cluster cardinality, by cluster weights, by inter-cluster edge/arc weights, by cluster element structure (for element multi-type clustering). Further, versions of optimization clustering problems are suggested (including multicriteria problem formulations). Illustrative numerical examples describe calculation of balance indices and element multi-type balance clustering problems (including example for design of student teams).

Keywords: balanced clustering, combinatorial optimization, heuristics, applications

Contents

1 Introduction 1
2 Basic balanced structures 4
3 Combinatorial balancing problems 6
4 Balance/unbalance indices for clustering solution 7
5 Optimization problem formulations for balanced clustering 9
6 Illustrative examples 10
   6.1 Illustration example for description of cluster structure 10
   6.2 Network-like illustration example 10
   6.3 Cluster structure based balanced clustering for student teams 13
7 Conclusion 15
8 Acknowledgments 15

1. Introduction

Balancing processes play central roles in many theoretical and practical fields (Fig. 1) [9,10,12,31,33,36,40,54,66,68,91,93,101,120,129,134,139,150]. The corresponding balancing problems are basic ones in various engineering domains, for example: manufacturing systems, computing systems, power/electricity systems, radio engineering systems, communication systems, and civil engineering systems. Similar balancing problems are examined in engineering management (e.g., coordination science), communications (e.g., synchronization as time-based balancing), organization science, psychology. In recent decades, the balancing approaches are very significant from the viewpoint of system modularity [16,65,86,103,112].

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The basic balancing problem consists in partitioning the element set (while taking into account element parameters, structure over the elements) into interconnected element groups (clusters) which are balanced (by cardinality of cluster elements, weight of cluster as a total weight of cluster elements, weight of cluster interconnections, structure of cluster, etc.). A general framework of balancing problems domain is depicted in Fig. 2.

It is reasonable to point out the following main application domains with using the balanced structures:
1. hierarchical organization of storage and search processes in information systems and in computer systems (e.g., various balanced search trees BSTs: AVL trees, red-black trees, splay trees) [2][3][8][9][10][11][12][13];
2. balanced parallel scheduling of computing processes (e.g., computing in multi-processor systems, distributed computing) [12][13][14];
3. service partitioning in a grid environment [19][20];
4. balanced partitioning in networks (e.g., modularization), balanced allocation of centers in networks [18][21];
5. balanced partitioning of algorithms [22];
6. balanced manufacturing scheduling in manufacturing systems (e.g., assembly line balancing problems) [23][24][25][26][27][28];
7. design of balanced hierarchical structures in organizations [29][30];
8. design and management in communication systems (network design, routing, etc.) based on balanced hierarchical structures [31][32], and
9. design of distributed defence systems [33][34].

Evidently, balanced clustering problems (including combinatorial clustering, constrained clustering, etc.) are used as the basic balancing combinatorial models for all domains [19][20][31][32]. Some basic balanced clustering problems are pointed out in Table 1.

In general, the clustering problem is the following. Let \( A = \{a_1, ..., a_j, ..., a_n\} \) be the initial set of elements (items, objects). Usually, the following characteristics are examined:

1. parameters of each element \( a_j \in A \) as vector \( \mathbf{p}(a_j) = (p_{1}(a_j), ..., p_{i}(a_j), ..., p_{m}(a_j)) \);
2. structure(s) (binary relation(s)) over the elements set \( A \): \( G = (A, E) \) where \( E \) is a set of edges/arc.

A clustering solution consists of a set of clusters (e.g., without intersections) [30][31]:
\( \bar{X} = \{X_1, ..., X_\lambda, ..., X_\lambda\} \).
i.e., dividing the set $A$ into clusters: $X_i \subseteq A \forall i = 1, \lambda$, $\eta_i = |X_i|$ is the cluster size (cardinality of cluster $X_i$, $i \in [1, \lambda]$).

| No. | Basic types of initial data | Results | Some source(s) |
|-----|-----------------------------|---------|----------------|
| 1.  | Set of elements, element parameters | Set of balanced clusters | 103, 107, 110, 104, 106 |
| 2.  | Set of elements with relations: |         |                |
| 2.1 | Set of elements with precedence relation | (1) chain over balanced clusters (balanced one-processor scheduling) | 111, 114, 141, 154 |
|     |                              | (2) parallel balanced element groups (precedence over elements in each group, multiprocessor scheduling) | 115, 144, 17, 195 |
| 2.2 | Set of elements with various binary relation(s) (e.g., precedence, inclusion) | (1) Tree/forest/hierarchy over balanced clusters (search structure, hierarchical storage paging, balanced broadcasting) | 102, 103, 128 |
|     |                              | (2) $k$-layer network architecture (hierarchical communications, hierarchical distributed computing) | 142, 151, 152, 154 |
| 3.  | Set of element chains (trajectories) | Special time interval balanced scheduling of element chains (scheduling in homebuilding) | 35 |
| 4.  | Set of element structures (e.g., programs, data structures, groups of technological operations, teams) | Set of balanced clusters of element structures (e.g., distributed computing, technological plans) | 104, 106 |

The basic balance (or unbalance/mismatch) index (parameter) for clustering solution is the difference between the maximal cardinality of a cluster and minimal cardinality of a cluster (in the considered clustering solution):

$$B^c(\tilde{X}) = \max_{i=1,\lambda} \eta_i - \min_{i=1,\lambda} \eta_i.$$ 

Thus, the balanced clustering problem is targeted to search for the clustering solution with minimum balance index (e.g., index above). Clearly, additions to the balanced problem statement can involve the following: (a) some constraints: the fixed number of clusters, restriction(s) to cluster sizes (including specified integer interval), etc.; (b) objective function(s) (e.g., minimization of total interconnections weight between clusters, maximization of total element connections weight in clusters).

This material contains an author preliminary outline of balanced clustering problems. The basic balanced structures are pointed out. New balance/unbalance indices for clustering solutions are suggested. Formulations of balanced optimization problems are described (including multicriteria problem formulations). Several numerical examples illustrate calculation of balance/unbalance indices for clustering solutions. A special example for design of student teams (as element multi-type cluster structure balanced clustering problem) is described. The material can be considered as a continuation of the author preprint on combinatorial clustering [104] and the corresponding article [105].
2. Basic balanced structures

The following basic balanced structures can be pointed out:

1. Balanced set partition (by cardinality, by element weights, element structure; in the general case, the obtained subsets can have intersections): (a) a basic illustration (Fig. 3), (b) balancing by element structure in each subset (Fig. 4).

2. Chain of balanced clusters (balanced manufacturing line, balanced chain of computing tasks groups, etc.) (Fig. 5).

3. Balanced packing of bins (e.g., multi-processor scheduling) (Fig. 6).

4. Balanced β-layer clustering (i.e., partitioning the initial element set into clusters and layers; e.g., to obtain a two-layer hierarchy with balances at the layer of clusters and at the layer of cluster groups, this is a two-balancing case) (Fig. 7).

5. Balanced trees \([3,49,93]\):
   - 5.1. balanced trees: by height (Fig. 8), by degree (Fig. 9),
   - 5.2. balanced search trees (as search index structures): (a) AVL trees, self-balancing (or height balancing) binary search tree \([2]\), (b) \(B\)-trees (path length from the root to each leaf equals \(b\) or \(b+1\) (where \(b\) is a constant) (Fig. 8) \([20,49,93]\), (c) Red-Black trees (symmetric binary B-tree) \([21,49]\), etc.

6. Balanced graph partition (e.g., partition of a graph into balanced subgraphs/communities while taking into account vertex weights or/and edge weights) (Fig. 10).

7. \(k\)-optimal partition of directed graph (path partition) \([22,26,27]\) (Fig. 11).

8. Balanced signed graph \([4,75,79,133,134]\) (Fig. 12).

9. Multi-layer structure of balanced clusters in networking: 9.1 hierarchy over balanced clusters (networking) (Fig. 13); 9.2 balanced clustering based multi-layer network structure (Fig. 14).

10. Balanced matrices \([25,47,131]\).
### 3. Combinatorial balancing problems

A list of basic balancing combinatorial optimization problems is presented in Table 2.

#### Table 2. Basic balancing combinatorial (optimization) problems

| No. | Combinatorial problem                                                                                           | Some sources                                                                 |
|-----|-----------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| I.  | Balanced partitioning problems:                                                                                  |                                                                              |
| 1.1 | Balanced partition problems (partition of set into balanced subsets)                                              | 75[4,115]                                                                   |
| 1.2 | $k$-balanced partitioning problem (partition of set into $k$ balanced subsets)                                     | 69[73]                                                                      |
| 1.3 | Partitioning (hierarchically clustered) complex networks via size-constrained graph clustering                      | 104[116]                                                                   |
| 1.4 | Uniform $k$-partition problem                                                                                    | 58                                                                          |
| 1.5 | Quadratic cost partition problem                                                                                 | 76                                                                          |
| 1.6 | Multi-constraint partitioning problem                                                                             | 81                                                                          |
| 1.7 | Tree-balancing problems                                                                                        | 213[88,143]                                                                |
| 1.8 | Minimum graph bisection problem                                                                                  | 75[83]                                                                     |
| 1.9 | Simple graph partitioning problem                                                                                | 75[144]                                                                    |
| 1.10| Balanced graph partitioning (partition of graph into balanced components)                                         | 81[2,12,25,35]                                                             |
| 1.11| Balanced partitioning of trees                                                                                    | 66[72,113]                                                                  |
| 1.12| Balanced partitioning of grids and related graphs                                                                 | 71[100,103]                                                                |
| 1.13| Directed acyclic graph (DAG) partition                                                                           | 68                                                                          |
| 1.14| Graph-clique partitioning problem                                                                                | 29[99]                                                                     |
| 1.15| Multiply balanced $k$-partitioning of graphs                                                                     | 30[32,43,63,75]                                                             |
| 1.16| Partitioning a sequence into clusters with restrictions on their cardinalities                                   | 29[24,53,90]                                                                |
| 1.17| General partition data model                                                                                     | 117                                                                         |
| II. | Some basic balanced combinatorial optimization problems:                                                          |                                                                              |
| 2.1 | Balanced knapsack problems, knapsack load balancing                                                                | 15[75,114]                                                                  |
| 2.2 | Balanced bin packing, container loading                                                                           | 53[57,75]                                                                   |
| 2.3 | Balanced parallel processor scheduling, balanced job scheduling in grids                                          | 42[75,145]                                                                  |
| 2.4 | Balanced allocation                                                                                              | 11[24,33,90]                                                                |
| 2.5 | Balanced $k$-center problem, balanced $k$-weighted center problem                                                | 18[70,108]                                                                  |
| 2.6 | Matrix balancing                                                                                                | 17[11,115,137,153]                                                          |
| 2.7 | Load balancing problems in distributed systems:                                                                   |                                                                              |
| 2.7.1| Load balancing in computer systems, in multiple processor systems                                                | 45[56,106,109]                                                              |
| 2.7.2| Load balancing in telecommunications networks                                                                     | 12[1]                                                                      |
| 2.7.3| Load balancing in structuring P2P systems                                                                        | 13[8]                                                                      |
| 2.7.4| Load balancing in web-server systems                                                                             | 13[0]                                                                      |
| 2.7.5| Load balancing in sensor networks                                                                               | 8[4]                                                                       |
| 2.7.6| Special partition of multi-hop wireless network (via graph coloring) for scheduling                             | 78                                                                          |
| 2.8 | Assembly line balancing problems                                                                                 | 59[36,37]                                                                   |
| 2.9 | Balanced graph matching                                                                                        | 30[136]                                                                    |
| 2.10| Route balancing problems                                                                                        | 87[96,124,124,125]                                                          |
| 2.11| Partition and balancing in meshes                                                                                | 59[60,146,147,148]                                                           |
| 2.12| Balanced combinatorial cooperative games                                                                         | 6[7]                                                                       |
4. Balance/unbalance indices for clustering solution

Here, balance/unbalance indices are described for clustering solution $\tilde{X} = \{X_1,...,X_{\lambda}\}$. Let $p_{bal}(X_i)$ be a total parameter for cluster $X_i$ ($i = 1,...,\lambda$) (e.g., the number of elements, total weight). In general, a balance/unbalance index for clustering solution $\tilde{X}$ can be defined via two methods:

**Method 1:** The balance (unbalance) index is defined as the following:

$$B(\tilde{X}) = \max_{i=1,\lambda} p_{bal}(X_i) - \min_{i=1,\lambda} p_{bal}(X_i).$$

Note, assessment of $p_{bal}(X_i)$ can be based on various scales: quantitative, ordinal, poset-like (e.g., multiset-based) [101][103]. Now, the following additional element parameters are considered:

1. The weight of item $w(a_j) \forall a_j \in A$ (e.g., $w(a_j) \geq 0$).
2. The weight of edge/arc between items $v(a_{j_1},a_{j_2}) \forall a_{j_1},a_{j_2} \in A$, $(a_{j_1},a_{j_2}) \in E$ (e.g., $v(a_{j_1},a_{j_2}) \geq 0$).
3. The structure of cluster by elements types is defined as a special multiset estimate [101][103]. As a result, the number of components in each multiset estimate of cluster structure will be the same.

In addition, a special “empty” element type is considered with the number $\theta_0 = 1$.

As a result, the following balance indices for clustering solution can be examined (for method 1):

(1.1) balance index by cluster cardinality is:

$$B^c(\tilde{X}) = \max_{i=1,\lambda} |X_i| - \min_{i=1,\lambda} |X_i|.$$

(1.2) balance index by total cluster weight:

$$B^w(\tilde{X}) = \max_{i=1,\lambda} \sum_{a_j \in X_i} w(a_j) - \min_{i=1,\lambda} \sum_{a_j \in X_i} w(a_j).$$

(1.3) balance index by total inter-cluster edge/arc weight:

$$B^v(\tilde{X}) = \max_{i=1,\lambda} \sum_{a_{j_1},a_{j_2} \in X_i} v(a_{j_1},a_{j_2}) - \min_{i=1,\lambda} \sum_{a_{j_1},a_{j_2} \in X_i} v(a_{j_1},a_{j_2}).$$

(1.4) balance index by total cluster structure:

$$B^\omega(\tilde{X}) = \max_{i=1,\lambda} e(X_i) - \min_{i=1,\lambda} e(X_i).$$

**Method 2.** Here, the balance/unbalance index is defined as the following:

$$\hat{B}(\tilde{X}) = \max_{i=1,\lambda} |p_{bal}(X_i) - p_0|,$$

where $p_0$ is a special specified (e.g., average) parameter of a special reference (e.g., average) cluster $X^0$ (size, weight, interconnection weight, structure estimate). Thus, the following balance indices for clustering solution can be examined:
The balance index by cluster cardinality is:

\[ \hat{B}_c(\tilde{X}) = \max_{i=1, \ldots, \lambda} | |X_i| - p_{iX_0}| |, \]

where \( p_{iX_0} \) is a special specified (e.g., average) cluster size.

The balance index by total cluster weight:

\[ \hat{B}_w(\tilde{X}) = \max_{i=1, \ldots, \lambda} | \sum_{a_j \in X_i} w(a_j) - p_{w0} | |, \]

where \( p_{w0} \) is a special specified (e.g., average) cluster weight.

The balance index by total inter-cluster edge/arc weight:

\[ \hat{B}_v(\tilde{X}) = \max_{i=1, \ldots, \lambda} | \sum_{a_{j1}, a_{j2} \in X_i} v(a_{j1}, a_{j2}) - p_{e0} | |, \]

where \( p_{e0} \) is a special specified (e.g., average) cluster weight of inter-cluster interconnections (i.e., edges/arcs).

The balance index by total cluster structure:

\[ \hat{B}_s(\tilde{X}) = \max_{i=1, \ldots, \lambda} | e(X_i) - e_{p0} | |, \]

where \( e_{p0} \) is a special specified (e.g., average) multiset estimate of cluster structure.

Table 3 contains the list of the considered balance indices. Note, a hybrid balancing (i.e., by several balance parameters) can be examined as well.

| No. | Description | Notation |
|-----|-------------|----------|
| I.  | Method 1 (difference between maximal and minimal values of cluster total parameters): |          |
| 1.1 | Balance index by cluster cardinality | \( B^c(\tilde{X}) \) |
| 1.2 | Balance index by total cluster weight | \( B^w(\tilde{X}) \) |
| 1.3 | Balance index by total inter-cluster edge/arc weight | \( B^v(\tilde{X}) \) |
| 1.4 | Balance index by cluster element structure | \( B^s(\tilde{X}) \) |
| II. | Method 2 (maximal difference between cluster total parameters and special reference specified (fixed) cluster total parameter): |          |
| 2.1 | Balance index by cluster cardinality | \( \hat{B}^c(\tilde{X}) \) |
| 2.2 | Balance index by total cluster weight | \( \hat{B}^w(\tilde{X}) \) |
| 2.3 | Balance index by total inter-cluster edge/arc weight | \( \hat{B}^v(\tilde{X}) \) |
| 2.4 | Balance index by cluster element structure | \( \hat{B}^s(\tilde{X}) \) |
5. Optimization problem formulations for balanced clustering

Thus, the balanced clustering problem is targeted to search for the clustering solution with minimum balance index (e.g., index above). Clearly, additions to the balanced problem statement can involve the following: (a) some constraints: the fixed number of clusters, restriction(s) to cluster sizes (including specified integer interval), etc.; (b) objective function(s) (e.g., minimization of total interconnections weight between clusters, maximization of total element connections weight in clusters).

The described balanced indices for clustering solutions can be used as objective functions and a basis for constraints in formulations of optimization balanced clustering models, for example:

**Problem 1:**

\[
\min B^c(\tilde{X}) \quad \text{s.t.} \quad B^w(\tilde{X}) \leq w^0, \quad B^s(\tilde{X}) \preceq e^0,
\]

where \( w^0 \) is a constraint for cluster weight difference, \( e^0 \) is a constraint for cluster structure difference.

**Problem 2:**

\[
\min B^w(\tilde{X}) \quad \text{s.t.} \quad B^c(\tilde{X}) \leq c^0, \quad B^s(\tilde{X}) \preceq e^0,
\]

where \( c^0 \) is a constraint for cluster cardinality difference, \( e^0 \) is a constraint for cluster structure difference.

**Problem 3:**

\[
\min B^w(\tilde{X}) \quad \text{s.t.} \quad B^c(\tilde{X}) \leq c^0, \quad |\tilde{X}| \leq \lambda^0,
\]

where \( c^0 \) is a constraint for cluster cardinality difference, \( \lambda^0 \) is a constraint for numbers of clusters in the clustering solution.

Evidently, multicriteria optimization models can be examined as well, for example:

**Problem 4:**

\[
\min \tilde{B}^c(\tilde{X}), \quad \min \tilde{B}^w(\tilde{X}) \quad \text{s.t.} \quad B^w(\tilde{X}) \leq w^0, \quad \tilde{B}^s(\tilde{X}) \preceq \tilde{e}^0,
\]

where \( w^0 \) is a constraint for cluster weight difference, \( \tilde{e}^0 \) is a constraint for cluster structure difference based on a reference value.

**Problem 5:**

\[
\min \tilde{B}^c(\tilde{X}), \quad \min \tilde{B}^w(\tilde{X}), \quad \min \tilde{B}^v(\tilde{X}) \quad \text{s.t.} \quad \tilde{B}^s(\tilde{X}) \preceq \tilde{e}^0, \quad |\tilde{X}| \leq \lambda^0,
\]

where \( \tilde{e}^0 \) is a constraint for cluster structure difference based on a reference value, \( \lambda^0 \) is a constraint for numbers of clusters in the clustering solution.

**Problem 6:**

\[
\min B^w(\tilde{X}), \quad \min \tilde{B}^w(\tilde{X}) \quad \text{s.t.} \quad B^c(\tilde{X}) \leq c^0, \quad \tilde{B}^s(\tilde{X}) \preceq \tilde{e}^0,
\]

where \( c^0 \) is a constraint for cluster cardinality difference, \( \tilde{e}^0 \) is a constraint for cluster structure difference based on a reference value.
6. Illustrative examples

6.1. Illustration example for description of cluster structure
Table 4 contains a description of cluster structures and corresponding multiset estimates for example from Fig. 4 (seven clusters, tree types of elements and additional “empty” element type). Table 5 contains the simplified proximity (or distance) between the multiset estimates of the clusters: \( \delta(e(X_i_1), e(X_i_2)) \), \( i_1, i_2 \in \{1, ..., 7\} \).

### Table 4. Cluster structures for example from Fig. 4

| Cluster \( X_i \) | Element type: \( \theta_1, \theta_2, \theta_3, \theta_4 \) (“empty” element) | Multiset estimate \( e(X_i) \) |
|-------------------|---------------------------------|-----------------------------|
| \( X_1 \)         | 1 1 3 2                         | (1,1,3,2)                  |
| \( X_2 \)         | 1 1 3 2                         | (1,1,3,2)                  |
| \( X_3 \)         | 1 1 4 1                         | (1,1,4,1)                  |
| \( X_4 \)         | 1 1 3 2                         | (1,1,3,2)                  |
| \( X_5 \)         | 1 1 2 3                         | (1,1,2,3)                  |
| \( X_6 \)         | 2 1 4 0                         | (2,1,4,0)                  |
| \( X_7 \)         | 1 2 4 0                         | (1,2,4,0)                  |

### Table 5. Proximity for cluster structures (example from Fig. 4): \( \delta(e(X_i_1), e(X_i_2)) \)

| Cluster \( X_i \) | Cluster \( X_j \) | \( X_1 \) | \( X_2 \) | \( X_3 \) | \( X_4 \) | \( X_5 \) | \( X_6 \) | \( X_7 \) |
|-------------------|-------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \( X_1 \)         | \( X_1 \)         | 0         | 0         | 1         | 0         | 1         | 4         | 3         |
| \( X_2 \)         | \( X_2 \)         | 0         | 0         | 1         | 0         | 1         | 4         | 3         |
| \( X_3 \)         | \( X_3 \)         | 1         | 1         | 0         | 1         | 2         | 3         | 2         |
| \( X_4 \)         | \( X_4 \)         | 0         | 0         | 1         | 0         | 1         | 4         | 3         |
| \( X_5 \)         | \( X_5 \)         | 1         | 1         | 1         | 1         | 0         | 5         | 4         |
| \( X_6 \)         | \( X_6 \)         | 4         | 4         | 3         | 4         | 5         | 0         | 1         |
| \( X_7 \)         | \( X_7 \)         | 3         | 3         | 2         | 3         | 4         | 1         | 0         |

For clustering solution from Fig. 4, the following balance indices are obtained:
\( B^c(\tilde{X}) = 7 - 4 = 3 \), \( B^s(\tilde{X}) = e(\tilde{X}_6) - e(\tilde{X}_5) = (2, 1, 4, 0) - (1, 1, 2, 3) = \delta(e(\tilde{X}_6), e(\tilde{X}_5)) = 5 \).

Further, cluster \( X_1 \) is considered as a reference cluster with corresponding its parameters. As a result, the following balance indices are obtained:
\( \hat{B}^c(\tilde{X}) = |X_6| - |X_1| = 2 \), \( \hat{B}^s(\tilde{X}) = \delta(e(\tilde{X}_6), e(\tilde{X}_1)) = \delta((2, 1, 4, 0), (1, 1, 3, 2)) = 4 \).

6.2. Network-like illustration example
In general, a wireless sensor network (WSN) can be considered as a multi-layer system (Fig. 15): (1) sensors, (2) clusters, (3) cluster heads, (4) sink, (5) database server, and (6) decision/control center. The design process of the architecture is based on element multi-type clustering (i.e., sensor/end nodes, cluster heads, etc).

![Multi-layer architecture of WSN](image-url)
Here, a numerical example from Fig. 10 is considered (three types of elements: ordinary nodes, nodes for relay, cluster heads): $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$. Parameters of elements and their interconnection are presented in Table 6 (element weights, element types) and Table 7 (weights of edges, symmetric weighted binary relation). The considered four-cluster solution is: $\bar{X}^i = \{X_1^i, X_2^i, X_3^i, X_4^i\}$ where $X_1^i = \{8, 9, 13, 14\}$, $X_2^i = \{1, 3, 4\}$, $X_3^i = \{2, 5, 6, 10\}$, and $X_4^i = \{7, 11, 12, 15\}$. Proximities between cluster structures are presented in Table 8. Fig. 16 depicts the poset-like scale $P_{4.4}$ for cluster structure, where multiset estimate of the cluster structure is: $e(X_p) = (\alpha_{\theta_1}, \alpha_{\theta_2}, \alpha_{\theta_3}, \alpha_{\theta_4})$.

Fig. 16. Poset-like scale $P_{4.4}$ for $e(X_p) = (\alpha_{\theta_1}, \alpha_{\theta_2}, \alpha_{\theta_3}, \alpha_{\theta_4})$.

The resultant integrated parameters of the clusters are contained in Table 9.

Finally, the following balance indices on the basis of method 1 are obtained (clustering solution $\bar{X}^i = \{X_1^i, X_2^i, X_3^i, X_4^i\}$): $B^c(\bar{X}^i) = 1$, $B^p(\bar{X}^i) = 6.7$, $B^\omega(\bar{X}^i) = 13.9$, $B^\theta(\bar{X}^i) = 4$.

Further, the following reference specified parameters for clusters are considered: $p_{|X_0|} = 4$, $p_{w_0} = 12.0$, $p_{v_0} = 15.0$.

The reference cluster structure is: $e_p^r = (1, 1, 2, 0)$. Thus, the balance indices based on method 2 are: $\tilde{B}^c(\bar{X}^i) = 1$, $\tilde{B}^w(\bar{X}^i) = 4.7$, $\tilde{B}^\omega(\bar{X}^i) = 7.3$, $\tilde{B}^\theta(\bar{X}^i) = 2$. 

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The reference cluster structure is: $e_p^r = (1, 1, 2, 0)$. Thus, the balance indices based on method 2 are: $\tilde{B}^c(\bar{X}^i) = 1$, $\tilde{B}^w(\bar{X}^i) = 4.7$, $\tilde{B}^\omega(\bar{X}^i) = 7.3$, $\tilde{B}^\theta(\bar{X}^i) = 2$. 

The resultant integrated parameters of the clusters are contained in Table 9.
Table 6. Parameters of elements (example from Fig. 10)

| Element $a_j$ | Cluster number $\iota$ ($X'_{\iota}$) | Element weight $w(a_j)$ | Element type $\theta(a_j)$ |
|---------------|--------------------------------------|-------------------------|--------------------------|
| $a_1$         | 2                                    | 4.2                     | 1                        |
| $a_2$         | 3                                    | 5.1                     | 1                        |
| $a_3$         | 2                                    | 1.1                     | 3                        |
| $a_4$         | 2                                    | 2.0                     | 3                        |
| $a_5$         | 3                                    | 3.1                     | 2                        |
| $a_6$         | 3                                    | 3.2                     | 2                        |
| $a_7$         | 4                                    | 1.0                     | 3                        |
| $a_8$         | 1                                    | 3.4                     | 2                        |
| $a_9$         | 1                                    | 5.0                     | 1                        |
| $a_{10}$      | 3                                    | 0.9                     | 3                        |
| $a_{11}$      | 4                                    | 4.5                     | 1                        |
| $a_{12}$      | 4                                    | 4.8                     | 1                        |
| $a_{13}$      | 1                                    | 0.8                     | 3                        |
| $a_{14}$      | 1                                    | 3.4                     | 2                        |
| $a_{15}$      | 4                                    | 3.7                     | 2                        |

Table 7. Interconnection edge weights for example from Fig. 10

| $a_{j_1}$ | $a_{j_2}$: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|-----------|------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
|           |            | 1 | 4.1 | 2.1 |   |   |   |   |   |   |    |    |    |    |    |    |
|           |            | 2 |   | 4.4 | 4.5 |   |   |   |   |   |    |    |    |    |    |    |
|           |            | 3 |   |   | 4.1 | 1.5 |   |   |   |   |    |    |    |    |    |    |
|           |            | 4 | 2.1 | 1.5 | 3.0 | 2.9 | 0.7 |   |   |   |    |    |    |    |    |    |
|           |            | 5 |   | 4.4 |   | 3.6 |   | 3.0 | 1.0 |   |    |    |    |    |    |    |
|           |            | 6 |   | 4.5 |   |   | 3.6 |   |   | 0.8 |   |    |    |    |    |    |
|           |            | 7 |   |   |   |   |   | 2.5 | 3.2 |   |    |    |    |    |    |    |
|           |            | 8 |   |   | 3.0 |   | 4.0 |   |   | 3.2 |   |    |    |    |    |    |
|           |            | 9 |   | 2.9 | 3.0 | 4.0 | 1.6 |   | 3.1 | 6.0 |   |    |    |    |    |    |
|           |            | 10|   | 0.7 | 1.0 | 0.8 | 1.6 | 5.0 |   |    | 3.3 |   |    |    |    |    |
|           |            | 11|   | 2.5 |   | 5.0 | 6.2 |   | 4.3 |   |    |    |    |    |    |    |
|           |            | 12|   | 3.2 |   | 6.2 |   | 4.2 |   |    |    |    |    |    |    |    |
|           |            | 13|   |   |   | 3.1 |   |   | 5.0 |   |    |    |    |    |    |    |
|           |            | 14|   |   | 3.2 | 6.0 |   |   | 5.0 | 2.5 |   |    |    |    |    |    |
|           |            | 15|   |   |   | 3.3 | 4.3 | 4.2 | 2.5 |   |    |    |    |    |    |    |

Table 8. Proximity for cluster structures (example from Fig. 10): $\delta(e(X'_{\iota_1}), e(X'_{\iota_2}))$

| Cluster $X'_{\iota_1}$ | Cluster $X'_{\iota_2}$: | $X_1'$ | $X_2'$ | $X_3'$ | $X_4'$ |
|-------------------------|-------------------------|--------|--------|--------|--------|
| $X_1'$                  | 0                       | 2      | 0      | 1      |
| $X_2'$                  | 2                       | 0      | 3      | 4      |
| $X_3'$                  | 0                       | 3      | 0      | 1      |
| $X_4'$                  | 1                       | 4      | 1      | 0      |

Table 9. Parameters of clusters (example from Fig. 10)

| Cluster | $\eta_i(X'_{\iota}) = |X'_{\iota}|$ | $\sum_{a_{j_1} \in X'_{\iota}} w(a_{j_1})$ | $\sum_{a_{j_1}, a_{j_2} \in X'_{\iota}} v(a_{j_1}, a_{j_2})$ | $e(X'_{\iota})$ |
|---------|----------------------------------|---------------------------------|------------------------------------------------|----------------|
| $X_1'$  | 4                                | 12.6                            | 21.3                                         | (1, 2, 1, 0)   |
| $X_2'$  | 3                                | 7.3                             | 7.7                                          | (1, 0, 2, 1)   |
| $X_3'$  | 4                                | 12.3                            | 15.4                                         | (1, 2, 1, 0)   |
| $X_4'$  | 4                                | 14.0                            | 21.6                                         | (2, 1, 1, 0)   |

The second considered four-cluster solution is: $\bar{X}'' = \{X''_1, X''_2, X''_3, X''_4\}$ where $X''_1 = \{4, 8, 9, 13, 14\}$, $X''_2 = \{1, 3\}$, $X''_3 = \{2, 5, 6\}$, and $X''_4 = \{7, 10, 11, 12, 15\}$.

Proximities between cluster structures are presented in Table 10. The resultant integrated parameters of the clusters are contained in Table 11.
Finally, the following balance indices on the basis of method 1 are obtained (clustering solution $X'' = \{X'_1, X''_2, X''_3, X''_4\}$): $B^c(X''') = 3$, $B^w(X''') = 9.6$, $B^v(X''') = 25.8$, $B^s(X''') = 6$.

The basic reference specified parameters for clusters are considered as for previous clustering solution (i.e., $X'$): $p_{X'_{10}} = 4$, $p_{X'_{20,1}} = 12.0$, $p_{X'_{20,2}} = 15.0$.

The reference cluster structure is: $e_{ρ_ρ} = (1, 1, 3, 0)$. Thus, the balance indices based on method 2 are: $\tilde{B}^c(X''') = 2$, $\tilde{B}^w(X''') = 6.7$, $\tilde{B}^v(X''') = 11.9$, $\tilde{B}^s(X''') = 4$.

Table 12 contains the obtained balance indices for the considered clustering solutions $\tilde{X}'$ and $\tilde{X}''$.

Table 10. Proximity for cluster structures (example from Fig. 10): $δ(e(X''_i), e(X''_j))$

| Cluster $X''_i$ | Clustering $X''_j$ | $X''_1$ | $X''_2$ | $X''_3$ | $X''_4$ |
|-----------------|-------------------|---------|---------|---------|---------|
| $X''_1$         | 5                 | 0       | 5       | 2       | 1       |
| $X''_2$         | 5                 | 0       | 3       | 6       |
| $X''_3$         | 2                 | 3       | 0       | 3       |
| $X''_4$         | 1                 | 6       | 3       | 0       |

Table 11. Parameters of clusters (example from Fig. 10)

| Cluster $X''_i$ | $\eta_i(X''_i) = \sum_{a_{i,j} \in X''_i} w(a_{i,j})$ | $\sum_{a_{i,j, a_{j,2}} \in X''_i} v(a_{i,j, a_{j,2}})$ | $e(X''_i)$ |
|-----------------|------------------------------------------------|-------------------------------------------------|-----------|
| $X''_1$         | 5                                               | 14.6                                           | (1, 2, 0, 0) |
| $X''_2$         | 2                                               | 5.3                                            | (1, 0, 1, 3) |
| $X''_3$         | 3                                               | 11.4                                           | (1, 2, 0, 2) |
| $X''_4$         | 5                                               | 14.9                                           | (2, 1, 2, 0) |

Table 12. Balance indices of clustering solutions $\tilde{X}'$ and $\tilde{X}''$

| No. | Description                       | Solution $X'$ | Solution $X''$ |
|-----|-----------------------------------|---------------|---------------|
| I.  | Method 1:                         |               |               |
| 1.1 | Balance index by cluster cardinality | $B^c(\tilde{X}') = 1$ | $B^c(\tilde{X}'') = 3$ |
| 1.2 | Balance index by total cluster weight | $B^w(\tilde{X}') = 6.7$ | $B^w(\tilde{X}'') = 9.6$ |
| 1.3 | Balance index by total inter-cluster edge/arc weight | $B^v(\tilde{X}') = 13.9$ | $B^v(\tilde{X}'') = 25.8$ |
| 1.4 | Balance index by cluster element structure | $B^s(\tilde{X}') = 4$ | $B^s(\tilde{X}'') = 6$ |
| II. | Method 2:                         |               |               |
| 2.1 | Balance index by cluster cardinality | $\tilde{B}^c(\tilde{X}') = 1$ | $\tilde{B}^c(\tilde{X}'') = 2$ |
| 2.2 | Balance index by total cluster weight | $\tilde{B}^w(\tilde{X}') = 4.7$ | $\tilde{B}^w(\tilde{X}'') = 6.7$ |
| 2.3 | Balance index by total inter-cluster edge/arc weight | $\tilde{B}^v(\tilde{X}') = 7.3$ | $\tilde{B}^v(\tilde{X}'') = 11.9$ |
| 2.4 | Balance index by cluster element structure | $\tilde{B}^s(\tilde{X}') = 2$ | $\tilde{B}^s(\tilde{X}'') = 4$ |

6.3. Cluster structure based balanced clustering for student teams

The considered numerical example is a modification of an example from [104]. There is a set of 13 students $A = \{a_1, ..., a_j, ..., a_{13}\}$ (Table 13) in the field of radio engineering. Four inclination/skill properties of the students are considered as parameters/criteria (student estimates are shown in Table 13): (i) inclination (skill) for mathematics $C_1$, (2) inclination (skill) for theoretical radio engineering $C_2$, (3) skill for technical works in radio engineering (usage of radio devices, design of scheme, analysis of signals, etc.) $C_3$, (4) writing skill (e.g., to prepare a laboratory work report/paper) $C_4$. The following scale is used: $[0, 1, 2, 3]$, where 0 corresponds to absent inclination/skill, 1 corresponds to low level of inclination/skill, 2 corresponds to medium level of inclination/skill, 3 corresponds to high level of inclination/skill). As a result, each student $j (j = 1, 13)$ has a vector (4 component) estimate $\xi(a_j) = (\xi^1(a_j), \xi^2(a_j), \xi^3(a_j), \xi^4(a_j))$. It is assumed, each student has as minimum one “positive” inclination/skill estimate (or more).

A symmetric weighted binary relation of student friendship $R^f = \{e(a_{j_1}, a_{j_2})\} (j_1, j_2 = 1, 13)$ (ordinal scale $[0, 1, 2, 3]$, 0 corresponds to incompatibility) is contained in Table 14.

Some notations are as follows. The structure of cluster is $X_i = \{b_1, ..., b_r, ..., b_{\mu_i}\}$.

Then the following cluster characteristics are examined:
1. The total vector estimate of the cluster structure is: $\xi(X_i) = (\xi^1(X_i), \xi^2(X_i), \xi^3(X_i), \xi^4(X_i))$. 
where $\xi^k(X_i) = \max_{r=1}^{\mu_i} \xi^k(b_r)$, $\forall k = 1,4$ (index of criteria/parameter).

2. The total estimate of the cluster by quality of intercluster compatibility is:

$\varepsilon(X_i) = \sum_{b_{\tau_1},b_{\tau_2}\in X_i} \xi^k(b_{\tau_1},b_{\tau_2})$.

### Table 13. Items/students, estimates upon criteria

| Item (student) | $C_1$ ($\xi^1(a_j)$) | $C_2$ ($\xi^2(a_j)$) | $C_3$ ($\xi^3(a_j)$) | $C_4$ ($\xi^4(a_j)$) | Vector estimate $\xi(a_j)$ |
|---------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Student 1 ($a_1$) | 1 2 3 3 | (1,2,3,3) |
| Student 2 ($a_2$) | 0 1 2 1 | (0,1,2,1) |
| Student 3 ($a_3$) | 2 3 3 2 | (1,3,3,2) |
| Student 4 ($a_4$) | 2 2 1 3 | (3,2,1,3) |
| Student 5 ($a_5$) | 0 1 2 1 | (0,1,2,1) |
| Student 6 ($a_6$) | 3 3 3 3 | (3,3,3,3) |
| Student 7 ($a_7$) | 0 1 1 1 | (0,1,1,1) |
| Student 8 ($a_8$) | 0 2 2 2 | (0,2,2,2) |
| Student 9 ($a_9$) | 0 2 3 1 | (0,2,3,1) |
| Student 10 ($a_{10}$) | 3 3 2 3 | (3,3,2,3) |
| Student 11 ($a_{11}$) | 0 1 3 2 | (0,1,3,2) |
| Student 12 ($a_{12}$) | 0 2 3 1 | (0,2,3,1) |
| Student 13 ($a_{13}$) | 0 1 1 1 | (0,1,1,1) |

### Table 14. Ordinal estimates of student friendship (compatibility) $R^f = \{\varepsilon(a_{j1},a_{j2})\}$

| $a_{j1}$ / $a_{j2}$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ | $a_{13}$ |
|---------------------|------|------|------|------|------|------|------|------|------|--------|--------|--------|--------|
| $a_1$               | 2    | 2    | 3    | 1    | 2    | 2    | 2    | 2    | 2    | 3      | 3      | 2      |
| $a_2$               | 2    | 3    | 1    | 2    | 2    | 2    | 2    | 0    | 0    | 2      | 2      | 1      |
| $a_3$               | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 2      |
| $a_4$               | 1    | 1    | 2    | 2    | 2    | 2    | 2    | 3    | 3    | 3      | 3      | 3      |
| $a_5$               | 3    | 3    | 3    | 2    | 3    | 3    | 2    | 3    | 3    | 3      | 2      | 3      |
| $a_6$               | 1    | 1    | 2    | 0    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |
| $a_7$               | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |
| $a_8$               | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |
| $a_9$               | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |
| $a_{10}$            | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |
| $a_{11}$            | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |
| $a_{12}$            | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |
| $a_{13}$            | 3    | 3    | 3    | 2    | 3    | 3    | 3    | 3    | 3    | 3      | 3      | 3      |

The clustering problem can be considered as the following:

Find the clustering solution $\tilde{X} = \{X_1,...,X_\lambda\}$ (i.e., a set of student teams as clusters/groups without intersection) for implementation of special laboratory work(s) (e.g., in radio engineering) while taking into account some requirements:

(a) constraints for the number of elements (students) in each cluster (i.e., cluster cardinality):

$\eta_{\text{min}} \leq |X_i|$

(b) constraint for total inclination/skill in the cluster: $\xi(X_i) \geq \xi_0 = (2,3,3,2)$

(c) the objective function 1 is to minimize the balance index by cluster cardinality $B^c(\tilde{X})$:

(d) the objective function 2 is to maximize (multicriteria case) the worst cluster structure estimate;

(e) the objective function 3 is to maximize the worst inter-cluster cluster estimate of element compatibility.

The version of the optimization problem for the example above can be considered as follows:

$$\min B^c(\tilde{X}) = \max_{i=1,\lambda} |X_i| - \min_{i=1,\lambda} |X_i|, \quad \max_{i=1,\lambda} \xi(X_i), \quad \max_{i=1,\lambda} \varepsilon(X_i)$$

s.t. $3 \leq |X_i| \leq 4 \quad \forall i = 1,\lambda, \quad \xi(X_i) \geq \xi_0 = (2,2,3,2) \quad \forall i = 1,\lambda$. 
The combinatorial optimization models of this kind are very complicated (i.e., NP-hard). Thus, enumerative algorithms or heuristics (metaheuristics) are used.

Now, it is reasonable to describe a simplified heuristic (for the example above):

**Stage 1.** Counting an approximate number of clusters (e.g., 4).

**Stage 2.** Selection of the most important criteria: 1st choice: criterion 3, 2nd choice: criterion 2.

**Stage 3.** Selection of the best elements (about 4) from the viewpoints of the selected criteria (e.g., by Pareto-rule) as kernels (“domain(s) leaders”) of the future clusters/teams. In general, small cliques or quasi-cliques can be considered as the kernels. In the example, the elements are: $a_1, a_3, a_6, a_9$.

**Stage 4.** Extension of each kernel above by other elements while taking into account element compatibility. As a result, the following clustering solution can be considered: $	ilde{X} = \{X_1, X_2, X_3, X_4\}$ ($B^v(\tilde{X}) = 1$), where the clusters are:

1. $X_1 = \{a_1, a_2, a_4\}$, $\xi(X_1) = (2, 2, 3, 3)$, $\varepsilon(X_1) = 8$;
2. $X_2 = \{a_3, a_7, a_8\}$, $\xi(X_2) = (2, 3, 3, 2)$, $\varepsilon(X_2) = 8$;
3. $X_3 = \{a_6, a_5, a_{11}\}$, $\xi(X_3) = (3, 3, 3, 3)$, $\varepsilon(X_3) = 8$;
4. $X_4 = \{a_9, a_{10}, a_{12}, a_{13}\}$, $\xi(X_4) = (3, 3, 3, 3)$, $\varepsilon(X_4) = 15$.

Note, balance index by total inter-cluster edge weight is $B^v(\tilde{X}) = 7$.

Fig. 17 depicts the obtained four clusters solution $\tilde{X} = \{X_1, X_2, X_3, X_4\}$ (edge weight/compatibility estimates are pointed out).

7. **Conclusion**

The paper describes the author preliminary outline of various combinatorial balancing problems including new balance indices for clustering solutions, multicriteria combinatorial models and examples. It may be reasonable to point out some prospective future research directions:

1. special investigation of balance indices for balanced structures (degree of balance, etc.);
2. study of various balancing problems in combinatorial optimization (e.g., balanced knapsack-like problems, balanced allocation problems, balanced bin packing problems);
3. taking into account uncertainty in the balancing models;
4. examination of balancing clustering problems in networking (e.g., design of multi-layer communication networks, routing);
5. study of augmentation approaches to balance structures while taking into account the improvement of the structures balance; and
6. usage of the described approaches in CS/engineering education.

8. **Acknowledgments**

The research was done in Institute for Information Transmission Problems of Russian Academy of Sciences and supported by the Russian Science Foundation (grant 14-50-00150, “Digital technologies and their applications”).

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