Second Quantization of Cini Model for High Order Quantum Decoherence in Quantum Measurement

D. L. Zhou, G. R. Jin and C. P. Sun

Institute of Theoretical Physics, Academia Sinica,
P. O. Box 2735, Beijing 100080, China

By making the second quantization for the Cini Model of quantum measurement without wave function collapse [M. Cini, Nuovo Cimento, B73 27(1983)], the second order quantum decoherence (SOQD) is studied with a two mode boson system interacting with an idealized apparatus composed by two quantum oscillators. In the classical limit that the apparatus is prepared in a Fock state with a very large quantum number, or in a coherent state with average quantum numbers large enough, the SOQD phenomenon appears similar to the first order case of quantum decoherence.

PACS number(s): 03.65-w, 32.80-t, 42.50-p

I. INTRODUCTION

In usual the quantum coherence is reflected by the spatial interference of two or more “paths” in terms of single particle wave function. Correspondingly the decoherence phenomenon losing coherence can be understood in term of an “which-path” detection implied by the quantum entanglement of the considered system with the environment or the measuring apparatus [1-3]. Most recently, we have shown [4,5] that this more simple, but most profound observation can be also implemented in the many particle picture to account for the losing of the high order quantum coherence (HOQC) described by the high order correlation function [6,7]. In this letter we will give a detailed study of this novel context specified for the quantum measurement problem.
To this end we first make the second quantization for the Cini Model of quantum measurement without wave function collapse [8] to obtain a modelled system — a two mode boson system interacting with an idealized measuring apparatus composed also by two quantum oscillators. Then, the second order quantum decoherence (SOQD) is studied with this model in the classical limit that the apparatus is prepared in a Fock state with a very large quantum number, or in a coherent state with average quantum numbers large enough.

The crucial point that we understand the higher order quantum decoherence problem in the “which-path” picture is to introduce the concept of the multi-particle wave amplitude (MPWA), whose norm square is just the high order correlation function [4,5]. Before the measurement, as an effective wave function, this multi-time amplitude can be shown to be a supposition of several components. When the an apparatus entangles with them to make an effective measurement, the high order quantum coherence loses dynamically. This decoherence process can be explained as a generalized “which-path” measurement for the defined multi-particle paths in the MPWA.

II. SECOND QUANTIZATION OF CINI MODEL

The original Cini model for quantum measurement emphasizes the production of quantum entanglement between the states of measured system S (a two-level system) and the measuring apparatus D — many indistinguishable particles with two possible modes $\omega_1$ and $\omega_2$. The two states $u_g$ and $u_e$ of S have different interaction strengths $d_g$ and $d_e$ with D. Then, the large number N of “ionized” particles in the “ionized” mode $\omega_2$ transiting from the “un-ionized” model $\omega_2$ shows this quantum entanglement. In the following we wish to make a second quantization for the system components to built a novel model for SOQD with Hamiltonian

$$\hat{H}_0 = \omega_e \hat{b}_e^\dagger \hat{b}_e,$$

$$\hat{V} = \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \omega_2 \hat{a}_2^\dagger \hat{a}_2 + (d_e \hat{b}_e^\dagger \hat{b}_e + d_g \hat{b}_g^\dagger \hat{b}_g)(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1),$$
where $\hat{H}_0$ is the free Hamiltonian of the system, $\hat{V}$ the free Hamiltonian of the apparatus $D$ plus the interaction between $S$ and $D$; and $\hat{b}_e^\dagger(\hat{b}_e), \hat{b}_g^\dagger(\hat{b}_g)$ the creation (annihilation) operators of two modes labelled by index $e$ and $g$. Their frequencies are $\omega_e$ and $\omega_g = 0$ respectively. The operators $\hat{a}_j^\dagger(\hat{a}_j)$ are creation (annihilation) operators of the modes which labelled by index $j$ for the mode frequency $\omega_j$, $j = 1, 2$. The frequency-dependent constant $d_e$ ($d_g$) measures the coupling constant between the $e$ ($g$) mode of the system and the apparatus.

The most important feature of the model is that $[\hat{H}_0, \hat{V}] = 0$, i.e. the system does not dissipate energy to the apparatus. Notice this model is equivalent to the generalized Cini model with many-levels given by us [9].

Starting with this concrete model we first consider the meaning of the “path” for the high order quantum correlation in the free particle case. The typical example of the higher order quantum coherence is that the single-component state $|1_e, 1_g\rangle$ of the two independent particles shows its quantum coherence in its second order quantum correlation function $G^{(2)}(t_1, t_2)$, which can just be written as the norm square $G^{(2)} = |\psi|^2$ [10] of the equivalent “two-time wave function” $\Psi(t_1, t_2)$ (it was also called the biparticle wavepacket [11] for photons), namely,

$$G^{(2)} = \langle 1_e 1_g|\hat{\phi}^\dagger(t_1)\hat{\phi}^\dagger(t_2)\hat{\phi}(t_2)\hat{\phi}(t_1)|1_e, 1_g\rangle$$

$$= |\langle 0, 0|\hat{\phi}(t_2)\hat{\phi}(t_1)|1_e, 1_g\rangle|^2 \equiv |\Psi(t_1, t_2)|^2$$

(3)

Here, we define a “measuring” field operator of two modes $g$ and $e$

$$\hat{\phi} = c_g\hat{b}_g e^{-i\omega_g t} + c_e\hat{b}_e e^{-i\omega_e t} \equiv c_g(t)\hat{b}_g + c_e(t)\hat{b}_e.$$  

(4)

The two time wave function $\Psi(t_1, t_2)$ can be understood in terms of the two “paths” picture from the initial state $|1_g, 1_e\rangle$ to the final one $|0, 0\rangle$ [5]:
Obviously, they are just associated with the two amplitudes forming a coherent superposition

$$\Psi(t_1, t_2) = \langle 0, 0 | \hat{\phi}(t_2) \hat{\phi}(t_1) | 1_e, 1_g \rangle$$

$$= c_e c_g e^{-i\omega_e t_2 - i\omega_e t_1} + c_g c_e e^{-i\omega_g t_2 - i\omega_e t_1}$$

(5)

(6)

Correspondingly, the second order correlation function

$$G^{(2)} = 2|c_e c_g|^2 [1 + \cos(\omega_g - \omega_e)[t_2 - t_1])]$$

(7)

shows the HOQC in the time domain. The above observation for the second order quantum coherence can also be discovered in the higher order case. It is noticed that our present arguments will be based on the equivalent field operator $$\hat{\Phi} = \sum c_n \hat{b}_n$$ is specified for a quantum measurement to a superposition state $$|\phi\rangle = \sum c_n |n\rangle$$. In reference [5], we have point out its observability in an idealized cavity QED experiment.

### III. MANY-PARTICLE “WHICH-WAY” DETECTION

In the case with interaction, we consider the generalized second order correlation functions

$$G[t, t', \hat{\rho}(0)] = Tr(\hat{\rho}(0) \hat{B}^\dagger(t) \hat{B}^\dagger(t') \hat{B}(t') \hat{B}(t))$$

(8)

with respect to “measuring” field operator [4]. It is defined as a functional of the density operator $$\hat{\rho}(0)$$ of the whole system for a given time 0. Here, the bosonic field operator

$$\hat{B}(t) = \exp(i\hat{H}t)[c_1 \hat{b}_g + c_2 \hat{b}_e] \exp(-i\hat{H}t)$$

$$= c_1 \exp(i\hat{V}t)[c_1 \hat{b}_g + \hat{b}_e c_2 \exp(-i\omega_e t)] \exp(-i\hat{V}t)$$

(9)

(10)

describes a specific quantum measurement with respect to the superpositions $$|+\rangle = c_1 |e\rangle + c_2 |g\rangle$$ and $$|\rangle = c_2 |e\rangle - c_1 |g\rangle$$ where $$c_1$$ and $$c_2$$ satisfy the normalization relation $$|c_1|^2 + |c_2|^2 = 1$$. Without loss of the generality, we take $$c_1 = c_2 = 1/\sqrt{2}$$ standing for a measurement as follows.
To examine whether the classical feature of the apparatus causes the second order decoherence or not, we consider the whole system in an initial state

\[ |\psi(0)\rangle = |1_g, 1_e\rangle \otimes |\phi(0)\rangle, \]  

where \(|\phi(0)\rangle\) is the initial state of the apparatus. In the case with interaction, instead of defining the equivalent “two-time wave function” in the case of free particle, we define an effective two-time state vector

\[ |\psi_B(t, t')\rangle = \hat{B}(t')\hat{B}(t)|\psi(0)\rangle. \]  

(12)

to re-write the second order correlation function as

\[ G[t, t', \hat{\rho}(0)] = \langle \psi_B(t, t')|\psi_B(t, t')\rangle \]  

(13)

It is interesting that the effective state vector can be evaluated as the superposition

\[ |\psi_B(t, t')\rangle = \frac{1}{2} e^{i\hat{V}(0,0)t'}[\exp(-i\omega_g t')e^{-i\hat{V}(1,0)t'}e^{i\hat{V}(1,0)t} + \]  

\[ \exp(-i\omega_e t')e^{i\hat{V}(0,0)t'}e^{-i\hat{V}(0,1)t'}e^{i\hat{V}(0,1)t}e^{-i\hat{V}(1,1)t'}|\{0\}_j\rangle \otimes |0_g, 0_e\rangle \]  

(14)

(15)

of two components with respect to the two paths from the initial two particle state \(|1_g, 1_e\rangle\) to the two particle vacuum \(|0_g, 0_e\rangle\). It should be noticed that the effective actions of the apparatus

\[ \hat{V}(m, n) \equiv \sum_j \hat{V}_j(m, n) = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j (d_e(\omega_j)m + d_g(\omega_j)n)(\hat{a}_j^\dagger + \hat{a}_j) \]  

(16)

can label the different paths and thus lead to the higher order quantum decoherence. The above result clearly demonstrates that, in presence of the apparatus, the different probability amplitudes (~\(\exp(-i\omega_g t')\) and \(\exp(-i\omega_e t)\)) from \(|1_g, 1_e\rangle\) to \(|0_g, 0_e\rangle\) entangle with the different states \(\frac{1}{2}e^{i\hat{V}(0,0)t'}e^{-i\hat{V}(1,0)t'}e^{i\hat{V}(1,0)t}e^{-i\hat{V}(1,1)t'}|\{0\}_j\rangle\) and \(\frac{1}{2}e^{i\hat{V}(0,0)t'}e^{-i\hat{V}(0,1)t'}e^{i\hat{V}(0,1)t}e^{-i\hat{V}(1,1)t'}|\{0\}_j\rangle\) of the apparatus. This is just physical source of the higher order quantum decoherence.

In the following calculation, the second order correlation function
\[ G[t, t', \hat{\rho}(0)] = \frac{1}{2} + e^{i\omega(t-t')} F + \frac{e^{-i\omega(t-t')}}{4} F^*, \quad (17) \]

is expressed explicitly in terms of the decoherence factor

\[ F = \langle \phi(0) | e^{i\hat{V}(1,1)t} e^{-i\hat{V}(0,1)t'} e^{i\hat{V}(0,1)t'} e^{-i\hat{V}(1,0)t} e^{i\hat{V}(1,0)t'} e^{-i\hat{V}(1,1)t} | \phi(0) \rangle \quad (18) \]

which determines the extent of coherence or decoherence in the second order case.

**IV. DYNAMIC HIGH-ORDER QUANTUM DECOHERENCE**

In the following, to given the factor \( F \) explicitly, the normal ordering technique [12] is adopted to calculate the second order decoherence factor \( F \). The calculation is carried out in six steps.

At the \( k \)-th step the evolution is dominated by the step-Hamiltonian

\[ \hat{h}^k = \alpha^k_1 \hat{a}_1^\dagger \hat{a}_1 + \alpha^k_2 \hat{a}_2^\dagger \hat{a}_2 + \beta^k (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1), \quad k = 1, 2, \cdots, 6 \]

during the time period \( t_k \). The coefficients \( \{\alpha^k_1, \alpha^k_2, \beta^k, t_k\} \) take different values in different steps:

\[ \begin{align*}
\alpha^1_1 &= \omega_1, \quad \alpha^1_2 = \omega_2, \beta^1 = d_e + d_g, \quad t_1 = t, \\
\alpha^2_1 &= -\omega_1, \quad \alpha^2_2 = -\omega_2, \beta^2 = -d_e, \quad t_2 = t, \\
\alpha^3_1 &= \omega_1, \quad \alpha^3_2 = \omega_2, \beta^3 = d_e, \quad t_3 = t', \\
\alpha^4_1 &= -\omega_1, \quad \alpha^4_2 = -\omega_2, \beta^4 = -d_g, \quad t_4 = t', \\
\alpha^5_1 &= \omega_1, \quad \alpha^5_2 = \omega_2, \beta^5 = d_g, \quad t_5 = t, \\
\alpha^6_1 &= -\omega_1, \quad \alpha^6_2 = -\omega_2, \beta^6 = -d_e - d_g, \quad t_6 = t.
\end{align*} \quad (19) \]

Assume that at the \( k \)-th step the evolution operator \( \hat{u}^k(t) \) can be written in the normal order as

\[ \hat{u}^k(t) = N \{ e^{A^k(t)\hat{a}_1^\dagger \hat{a}_1 + B^k(t)\hat{a}_2^\dagger \hat{a}_2 + C^k(t)\hat{a}_1^\dagger \hat{a}_2 + D^k(t)\hat{a}_2^\dagger \hat{a}_1} \}. \quad (20) \]
The advantage of this form is that, when calculating the average of the operator in the coherent state \(|\alpha, \beta\rangle\), we only need to replace the annihilation operators with the corresponding complex values. This evolution operators satisfy the Schrödinger equation \(i\frac{d}{dt}\hat{u}^k = \hat{h}^k\hat{u}^k\). To take the expectation values of the above equations in the coherent state \(|\alpha, \beta\rangle\), the coefficients satisfy the following system of equations:

\[
\begin{align*}
    i\frac{dA^k}{dt} &= \alpha_1^k(A^k + 1) + \beta^kD^k, \\
    i\frac{dB^k}{dt} &= \alpha_2^k(B^k + 1) + \beta^kC^k, \\
    i\frac{dC^k}{dt} &= \alpha_1^kC^k + \beta^k(B^k + 1), \\
    i\frac{dD^k}{dt} &= \alpha_2^kD^k + \beta^k(A^k + 1).
\end{align*}
\] (21)

The solution of the system of equations is

\[
\begin{align*}
    A^k + 1 &= e^{-i(\alpha_1^k + \alpha_2^k)t_k/2}(\cos(\Gamma^k t_k) + \frac{i(\alpha_2^k - \alpha_1^k)}{2\Gamma^k}\sin(\Gamma^k t_k)), \\
    B^k + 1 &= e^{-i(\alpha_1^k + \alpha_2^k)t_k/2}(\cos(\Gamma^k t_k) - \frac{i(\alpha_2^k - \alpha_1^k)}{2\Gamma^k}\sin(\Gamma^k t_k)), \\
    C^k &= D^k = -\frac{i\beta^k}{\Gamma^k}e^{-i(\alpha_1^k + \alpha_2^k)t_k/2}\sin(\Gamma^k t_k),
\end{align*}
\] (22)

where

\[\Gamma^k = \sqrt{(\alpha_2^k - \alpha_1^k)^2 + \beta^k2}\]

Notice that the above results have been before given in ref.[12], but the original ones contain some minor misprints. Here, we have corrected them. Then we obtain

\[
e^{-i\hat{h}^k t_k}|\alpha^{k-1}, \beta^{k-1}\rangle = |(A^k + 1)\alpha^{k-1} + C^k\beta^{k-1}, (B^k + 1)\beta^{k-1} + D^k\alpha^{k-1}\rangle \equiv |\alpha^k, \beta^k\rangle.
\] (23)

From the above equation, it is obvious that, when the apparatus is initially in the product coherent state \(|\alpha^0, \beta^0\rangle\), after six steps of evolution, the final state of the apparatus remains in a product coherent state \(|\alpha^{0}(\alpha^0, \beta^0), \beta^{0}(\alpha^0, \beta^0)\rangle\).
To consider the classical feature of the apparatus, two specific initial states will be studied with different classical correspondences. And we give the numerical results respectively thereafter. The first case is that the initial state of the apparatus takes $|\phi(0)\rangle = |0, \beta\rangle$. When the norm of $\beta$ goes to infinity, it corresponds to classical field in some sense. In this case, we can obtain the decoherence factor and therefore the second order correlation function becomes

$$G[t, t', \hat{\rho}(0)] = \frac{1}{2} + \left( e^{i\omega_c(t-t')} \frac{\langle 0, \beta | \alpha^6(0, \beta), \beta^6(0, \beta) \rangle}{4} + h.c. \right).$$ (24)

A typical case of the numerical result of the above equation is given in FIG.1.
FIG. 1. The horizontal axe denotes time period $t' - t$, the vertical axe denotes the second order correlation function $G[t, t', \hat{\rho}(0)]$, parameters $\omega_1 = 0.2, \omega_2 = 1.3, d_e = 0.8, d_e = 0.2, \omega_e = 1.0$, (a)$N = 10, t = 0$, (b)$N = 10, t = 10$, (c)$N = 10^2, t = 0$, (d)$N = 10^2, t = 10$, (e)$N = 10^4, t = 0$, (f)$N = 10^4, t = 10$.

In FIG.1, we observe that the second order correlation function is an explicit function of both the time interval $t' - t$ and the time $t$. With the increasing of time $t$, it obviously oscillate faster and faster. It is all observed that, as the average particle number of the coherent state increases, the second order correlation function decoheres in a shorter time scale. The decoherence rate is independent of the time $t$. The later observation implies that, when the average particle number approaches to infinity, the second order correlation function will decohere in very short time, and the quantum revivals can not be observed in a finite time period.

The second case is that the initial state of the apparatus takes a Fock number state $|\phi(0)\rangle = |0, N\rangle$. When the number $N$ approaches into infinity, it also corresponds to classical field in some sense. In this case, we can obtain the decoherence factor and therefore the second order correlation function

$$G[t, t', \hat{\rho}(0)] = \frac{1}{2} + \left( \frac{e^{i\omega_e(t-t')}}{4} \int \frac{d^2\beta}{\pi} \langle 0, N | \alpha^6(0, \beta), \beta^6(0, \beta) \rangle \langle \beta | N \rangle + \text{h.c.} \right). \quad (25)$$

A typical numerical result of the above equation with the same parameters as in FIG. 1. is given in FIG.2.
FIG. 2. The horizontal axe denotes time period $t' - t$, the vertical axe denotes the decoherence factor $G[t, t', \hat{\rho}(0)]$, parameters $\omega_1 = 0.2, \omega_2 = 1.3, d_e = 0.8, d_e = 0.2, \omega_c = 1.0$, (a) $N = 10, t = 0$, (b) $N = 10, t = 10$, (c) $N = 10^2, t = 0$, (d) $N = 10^2, t = 10$, (e) $N = 10^4, t = 0$, (f) $N = 10^4, t = 10$.

In FIG.2, we observed the similar phenomena as in FIG.1. We would like to emphasize that difference between the number state and the coherent state only manifests in the case that the number is quite small. We can expect that they may gives the same limit for infinite
V. CONCLUDING REMARK

In fact, embodying the wave nature of particles in the quantum world, the quantum coherence is usually reflected by the spatial interference of two or more “paths” in terms of single particle wave function. However, the usual quantum coherence phenomenon with the first order interference fringes does not sound very marvellous for the same circumstances can also occur in classical case, such as an usual optical interference. But in association with the Hanburg-Brown-Twiss experiment [7], Glauber’s higher order quantum coherence manifests the intrinsically quantum features of coherence beyond the classical analogue. For example, in a quantum system composed by identical particles, the quantum coherence is indeed manifested in the observation of interference fringes reflected not only by the first order correlation functions, but also by higher order ones.

On the other hand, in the present of external quantum system (i.e. an apparatus) interacting with the studied system, the quantum decoherence of the system happens as the disappearance of the first order interference fringes. This decoherence mechanism provides the essential elements in the understanding for quantum measurements and the transition from quantum to classical mechanics. Just based on this conception, our present work extends the above understanding for quantum decoherence in terms of the first order interference to the high order case. With a two mode boson model, we have studied the second order decoherence in the classical limit. Even without the factorization structure and thus the obvious the macroscopic limit, the high-order quantum decoherence still happens in the classical limit, i.e., when the quantum number to infinity. It is concluded that this decoherence process losing the higher order coherence can be also explained as a generalized “which-path” measurement for the defined multi-particle paths.

Acknowledgement This work is supported by NSF of China and the knowledged Innovation Programme(KIP) of the Chinese Academy of Science.
[1] D. Giulini, et. al., *Decoherence and Appearance of Classical World in Quantum Theory*, Springer Berlin, 1996.

[2] W. H. Zurek, Phys. Today **44**(10), 36 (1991).

[3] S. Durt, T. Nonn, and G. Rampe, Nature **395**, 33(1998).

[4] D. L. Zhou, and C. P. Sun, quant-ph/0104038, LANL preprint(2001).

[5] D. L. Zhou, P. Zhang and C. P. Sun, quant-ph/0105088, LNAL preprint(2001).

[6] R. J. Glauber, Phys. Rev. **130**, 2529(1963); **131**, 2766(1963).

[7] H. Hanburg-Brow and R. Q. Twiss, Phil. Mag. **45**, 663(1954); Nature **178**, 1046(1956); Proc. Roy. Soc. A **242**,300(1957).

[8] M. Cini, Nuovo Cimento, B73, 27 (1983).

[9] X. J. Liu, C. P. Sun, Phys. Lett. A, **198**, 371 (1995).

[10] M. O. Scully, and M. S. Zubairy, *Quantum Optics*, Cambridge University press, 1997, pp 97-129.

[11] Y. Shih, Adv. At. Mole. and Opt. Phys. Vol 41(1999), pp 1-42.

[12] W. H. Louisell, *Quantum Statistical Properties of Radiations*, John Wiley & Sons (1973).

[13] C. P. Sun, Phys. Rev. A 48, 878(1993). C. P. Sun, Chin. J. Phys. **32**, 7(1994). C. P. Sun, X. X. Yi, and X. J. Liu, Fortschr. Phys. **43**, 585. C. P. Sun, H. Zhan, and X. F. Liu, Phys. Rev. A **58**, 1810(1998).