Prediction of ground vibrations induced by rail traffic in Lisbon urban area

João Manso*, Jorge Gomes, João Marcelino

National Laboratory for Civil Engineering, Av. do Brasil 101, Lisbon, Portugal

HIGHLIGHTS

- Performance of a vibration measurement program, which included several trains.
- Characterisation of profiles with different slopes and retaining concrete walls.
- Development of a numerical model capable of modelling the propagation of vibrations.
- Calibration of the numerical model and forecast the introduction of high speed trains.
- The implementation of high-speed trains will lead to a slightly vibration increase.

ARTICLE INFO

Keywords:
High-speed trains
Ground vibrations
Measurement campaigns
2D finite element
Vehicle-track-soil dynamic interaction
Track irregularities

ABSTRACT

The possibility of introducing high-speed trains in Portugal will challenge the regular train circulation of existing railways tracks. To carry this traffic, improvements and maintenance programmes will be intensified together with the evaluation of the railway performance and its consequences on neighbourhood structures, including the execution of several vibration measurement tests. One of them was performed in Lisbon’s urban area for measure of ground-borne vibration due to the train traffic. It was used to develop an efficient numerical model, which allowed to study the influence of introducing high-speed vehicles on the generation and propagation of vibrations through the free field and the impact on the wayside buildings. To access the effect of introducing such trains in the existing tracks, three bi-dimensional numerical models for each measurement profile were developed. Numerical models were calibrated using the monitoring records. Then, new simulations were performed with Thalys HST train allowing to conclude that the introduction of high-speed trains will not lead to an increase of level of vibrations, for the adopted traffic conditions.

1. Introduction

High-speed train lines have been developed throughout Europe, North America, and East Asia, answering the need of populations who are demanding broader networks capable of reaching newer territories [1]. At the same time, train speeds have been increasing, reaching now more than 500 km/h [2]. This calls for new railway lines or rebuild existing ones that may render crossing urban areas unavoidable. Highly populated areas are particularly sensitive to excessive vibration from any kind of trains (traditional, high-speed or freight). Such motions are not only worrisome for their human disturbance and environmental impacts, but also raise concerns about people’s safety near the train-tracks, degradation of the surrounding buildings, and structure-borne noises [3, 4].

* Corresponding author.
E-mail address: jmanso@lnec.pt (J. Manso).

https://doi.org/10.1016/j.heliyon.2022.e10001
Received 25 October 2021; Received in revised form 6 January 2022; Accepted 14 July 2022

2405-8440/© 2022 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Moving loads such as trains have long been recognized as potential sources of ground vibration [2, 5, 6]. The ground response to a moving load is dictated largely by its characteristics and the relation between the load speed and the characteristic wave velocities of the ground medium [5]. At low speeds, the ground response from a moving load is essentially quasi-static. That is, the displacements and stress fields are essentially the static fields under the load simply moving with it. However, as the speed of the load increases, dynamic phenomena gradually take over and dominate the response [7]. The train velocity in urban areas are usually limited to small values, which could generally be considered as being in the subseismic regime. Analytical studies on the response of a half-space to moving loads have shown that this regime represents a quasi-static condition [8].

This subject has been studied for several years and during that time various formulations have been proposed to predict the propagation of train-induced ground vibration [1, 8, 9, 10, 11, 12, 13, 14, 15]. Analytical and numerical models using the finite and boundary element method have been developed to study wave propagation and dynamic soil-structure interaction. They usually differ in two aspects: the considered excitation mechanisms (quasi-static axle loads and track or vehicle irregularities) and the track–soil coupling. Degrange and Lombaert [16] measured free field vibrations and track response of the Thalys train at different speeds (223 to 314 km/h). Then, they compared these results with numerical results obtained using Krylov’s prediction model [1]. The adopted model only incorporated quasi-static loading and disregarded other excitation mechanisms, such as, rail and wheel irregularities. This aspect could have been one of the causes which led to an underestimation in the mid-frequency range, when the lower and higher bands showed good predictive capabilities. Sheng et al. [17] defined a theoretical ground vibration model, which considered the quasi-static and dynamic mechanisms of excitation. They computed the model using multi-layer models on a homogeneous half-space and compared the results with the measured data from three different sites. They found that the dynamic mechanism of vibration generation has a major contribution for environmental vibration than the quasi-static axle loads. Auersch [18] focused on the excitation process of the railway vibration and proposed theoretical models for each subsystem (vehicle, track and soil) and studied their interaction. He used a mixed boundary-element finite-element model to analyse the dynamic compliance of the track and multi-body models for the vehicle. Then, he performed some tests and recorded runs of the Intercity Experimental and compared them with the model results. His major findings included that the static axle load component is important for the track and close surroundings, but its amplitude decrease considerably with distance, being less important for ground vibrations. He also observed that the vehicle irregularities were dominant at high frequencies and the track irregularities main contribution was to the lower frequencies. Yang et al. [19] proposed a 2.5D finite/infinite procedure where they considered an additional degree of freedom (DOF), comparing with conventional plane strain elements, to account for the out-of-plane wave transmission. The profile of the half-space was divided in two parts, one corresponding to the near field and simulated by finite elements, and another that covered the far field using infinite elements. Recently, Correia dos Santos et al. [20] developed a three-dimensional numerical model to study problems involving vibrations induced by railway traffic, considering the dynamic interaction of the vehicle-structure-ground system. Despite the versatility of the model, which allowed to study problems where a variety of discontinuities take place (irregularities of the track, transition zones between a slab track and a ballasted track, buildings located in the vicinity of the track), the main drawback is related to the computational costs and time involved in the analysis. Due to this very important aspect, bi-dimensional models still presents as a valid alternative to study such problems, specially when time is a crucial factor.

This paper briefly outlines the numerical models adopted for the prediction of quasi-static response from trains in the high-speed rail network, particularly, a part localised in Lisbon’s urban area. Each model considered three subsystems: train, track and ground, and have been validated by in situ measurements performed by LNEC. Then, they have been used to characterise the actual situation (considering regular train traffic) and explore the future situation due to the passage of trains (high-speed typology), as well as to evaluate the possibility of applying mitigation strategies to attenuate the effect of vibrations.

2. Ground vibration testing program

The vibration measurement program was included in the preliminary study for the Third Crossing of the river Tagus, which made part of the high-speed rail network between Lisbon and Madrid. LNEC has performed the test runs in three different locations at a densely populated area of Lisbon, namely Marvila, Braço de Prata and Parque das Nações. The sites were located along two railway lines, Internal Circular and Northern Lines, between central Lisbon and Oriente Station (Fig. 1) and they have been chosen based on their singularities. The first one, located in Marvila, was a regular measurement profile, almost horizontal. In Braço de Prata existed a concrete retaining wall, which was located near the train track, and the last one was confined between two concrete walls. LNEC carried out an extensive programme of measurements, recording the regular traffic, in order to determine the current level of vibration at different distances from the track [21].

2.1. Profiles

2.1.1. Profile I - Marvila

The measurement profile was placed in an area called Marvila, near the Internal Circular Line (Fig. 2b). The profile extends 150 m in the SE-NW direction and 70 m in the NW-SE direction, perpendicularly measured from track 1 of the Internal Circular Line. The area was surrounded by buildings up to four stories, however the ones located 10 m from the railway only had two.

Axial accelerometers were used in twelve points and, in two of them considered relevant, horizontal components of acceleration were also measured (2N and 3N), making up a total of 16 measurements as outlined in Fig. 2c.

The measurement profile was considered horizontal, without any major singularity (Fig. 2c). In this particular area the train track was curved and the trains maximum velocity was limited to 80 km/h.

Tables 1 and 2 present the gathered information, considering the geotechnical works done during the Internal Line quadruplication. Among the available information two trial pits (P115 and P116) and two boreholes (S119 and S120) were chosen due to their proximity with the measured profile. Results show that the soil was relatively heterogeneous and constituted mainly of sands and clays, with low to medium plasticity.

Table 1. Marvila – Information collected in the trial pits P115 and P116.

| Depth (m) | LL (%) | LP (%) | IP (%) | \( \omega_0 \) (rad/s) | \( \gamma_0 \) (kN/m²) |
|----------|--------|--------|--------|----------------------|----------------------|
| P115     | 0.25–1.50 | 24     | 18     | 6                    | 11.4                 | 19.0                |
| P116     | 0.20–1.20 | 34     | 21     | 13                   | 14.8                 | 17.6                |

Table 2. Marvila – Information collected in the boreholes S119 and S120.

| Depth (m) | Material         | \( N_{xy} \) |
|----------|------------------|-------------|
| S119     | 0.0–1.5 Sandy and silty soil | -- |
|          | 1.5–12.0 Sandy and silty soil | 60 |
|          | 0–5.5 Clayey     | 15          |
|          | and              | 12          |
|          | 5.5–9.5 Sandy soil with clay | 2 |
|          | 9.5–14.0 Sandy and silty soil | 60 |
|          | 14.0–15.0 Clayey and silty soil | 60 |
2.1.2. Profile II - Braço de Prata

The second site was located in Braço de Prata near the Northern Line, between St. Apolónia and Oriente station (Fig. 3b). This profile extended 45 m in the E-W direction and 40 m in the W-E direction, perpendicularly measured from track 1 of the Northern Line. The vertical accelerations in 8 points were registered as well as the transversal component of the point located in the rail. In points 2N, 3N and 4N the horizontal components of acceleration were considered relevant and were also measured, making up a total of 13 measurements as outlined in Fig. 3c.

This profile was constituted by an horizontal part (near the train tracks) and a slope with a retaining concrete wall, which had a height of 5.5 m (Fig. 3c). This wall was placed less than 10 m from the track and its influence was studied with the points 3N and 4N. 7 stories buildings surrounded the area at a minimum distance of 25 m from the track and the vibration level was registered with point 5N.

Table 3. Braço de Prata – Information collected in the trial pit P108.

| Depth (m) | LL (%) | IP (%) | Fines (%) | $\omega$ (%) | $\omega_{opt}$ (%) | $\gamma_{max}$ (kN/m$^2$) |
|-----------|--------|--------|-----------|--------------|-------------------|--------------------------|
| 0.50–0.55 | 35     | 16     | 73        | 7.3          | —                 | 11.5                     |
| 0.80–0.90 | 32     | 10     | 81        | 7.3          | 11.5              | 18.7                     |

Table 4. Braço de Prata – Information collected in the borehole S126.

| Depth (m) | Material                  | $N_{syz}$ |
|-----------|---------------------------|-----------|
| 0.5–1.0   | Silty sand                | 56        |
| 1.5–6.0   | Clayey soil               | 51        |
| 6.0–9.0   | Clayey soil               | 17        |
| 9.0–14.0  | Sandstones and clay       | 37        |
| S126      |                           | 60        |
In this area, the modernisation of the Northern Line took place in the 90’s. Considering the geotechnical prospecting that resulted from this work, one trial pit (P108), one borehole (S126) and one excavated tranche (V103) were chosen due to their proximity to the measured profile. Tables 3, 4 and 5 show that the foundation is relatively competent, constituted mainly by silts and clays with medium plasticity.

2.2.3. Profile III - Parque das Nações

The last measurement site, located at Parque das Nações, was also near the Northern Line, close to Oriente station (Fig. 4b), a major transport hub in Lisbon. This profile extended 2 m in the E-W direction and 40 m in the W-E direction, perpendicularly measured from track 1. Vertical accelerations were registered in 7 points and the transversal component was measured in the point located in the rail. In point 2N the horizontal components of acceleration were also registered, due to their relevance, making up a total of 10 measurements as outlined in Fig. 4c.

In this profile the train tracks were confined between two 5.5 m high retaining walls made of concrete (Fig. 4c). The area was surrounded by one story buildings, which were mainly industrial facilities, at a distance of 10 m from the track. However, major concern was related to residential buildings, located near the southern retaining wall. This wall was placed at about 25 m from the measured track, therefore only the vertical component was registered (4S and 5S).

Tables 6 and 7 present the collected information considering the geotechnical works done during the modernisation of the Northern Line. Among the available information one trial pit (P230A) and one borehole (S228) were chosen to characterise the soil present in the track foundation. Results have shown that the soil was heterogeneous, consisting mainly of silts and clays with high plasticity.

2.2. Trains

The normal rail traffic registered during the measurement campaigns included several trains (passenger and freight), single locomotives and environmental noise. After comparing all the records, we have chosen the Portuguese’s UQE2300 and UTE2240 passenger trains, which we considered the most significant.

UQE2300 rolling stock consisted of 2 coaches (axle spacing of 2.6 m and pivot spacing of 1.42 m) and 2 engines (axle spacing of 2.6 m and pivot spacing of 1.42 m), with four axles each. The mass of the powered axles was around 16 tons and 12 tons for the unpowered axles (Fig. 5a). UTE2240 consisted of 2 engines (axle spacing of 2.5 m and pivot spacing of 1.4 m) and one coach (axle spacing of 2.8 m and pivot spacing of 1.4 m), with four axles each. The mass of the powered axles was around 8 tons and 16 tons for the unpowered axles (Fig. 5c).

In this area the maximum speed of the trains was limited to 80 km/h and the registered train velocities varied from 25 to 60 km/h. Thus, for modelling purposes was adopted a velocity of 60 km/h.

3. Numerical modelling

The excitation sources of a passing train are the moving axle loads (quasi-static) together with stationary dynamic forces and moving dynamic forces [22]. Still, slow heavy-axle load traffic on conventional
lines continues to represent the major component of environmental vibration and are the source of the majority of complaints about vibration in line-side buildings [17].

In some situations, the dynamics of vehicles and the track quality have an effect on the level of vibration. Models that consider only quasi-static loads tend to underestimate the response level for higher frequencies component [23]. Several authors [17, 24, 25, 26, 27] have identified the relevant role on dynamic loads that occur at wheel–rail contact points caused by track irregularities, discontinuities at welds and joints, such as other vehicle defects. Those variations introduce a relative displacement input in the vehicle and track systems [28].

In order to perform a comprehensive analysis of ground vibration generation and propagation in the measurement sites, the developed models should be capable of replicate the interactions between vehicles, track and ground, and consider the effect of train speeds and the particularities in ground geometry, such as concrete retaining walls [29, 30]. This was accomplished by simulating an infinite track coupled to
a semi-analytical model for a two-dimensional layered ground. Each model consisted of three subsystems: train, track and foundation.

In the models presented in this work, viscous boundaries built in FLAC2D were used. The modelling of geom mechanics problems, that rely on the discretization of a finite region of space, require the definition of appropriate conditions at the artificial numerical boundaries. Although in static analyses, fixed or elastic boundaries can be realistically placed at some distance from the region of interest, in dynamic problems, such boundary conditions cause the reflection of outward propagating waves back into the model. The use of a larger model can minimize the problem; however, this solution considerably increases computational costs. The alternative is to use quiet (or absorbing) boundaries.

The viscous boundary used in FLAC2D was developed by [31]. It involves damps pots attached independently to the boundary in the normal and shear directions. The method practically absorbs all the body waves, approaching the boundary, at angles of incidence greater than 10°. For lower angles of incidence, or for surface waves, the method is not so effective, although there is still energy absorption. Nonetheless, one of the main advantages is that it operates in the time domain. Its effectiveness has been demonstrated in both finite-element and finite-difference models [32].

### 3.1. Modelling of the track

In a Winkler foundation model [33] a continuous elastic foundation is replaced by a series of spring elements. Analytical solutions have been developed [34] to analyse the beam response on the Winkler foundation to a single concentrated load acting on an infinite Euler-Bernoulli beam and moving at a constant velocity. If it is assumed that the response remains elastic, the response of the beam subjected to multiple concentrated loads, moving at a constant velocity, can be calculated by using superposition [35, 36, 37, 38, 39].

The passage of the train’s successive axles over the track constitutes the quasi-static excitation, which corresponds to constant forces moving along the track with train speed, $v$.

The axle loads, $F_i$, of each train were modelled considering the Winkler beam foundation model. The displacements caused by a moving load, $F_s$, in a quasi-static state, for an undamped situation, can be determined using:

$$F(s) = \frac{F}{2L} e^{-\frac{sL}{2}} \left( \cos \frac{sL}{L} + \sin \frac{sL}{L} \right)$$

(1)

where $s$ denotes the absissa of a moving referential that is equal to zero in the axle position and $L$ represents the characteristic length.

The relation:

$$s = \frac{1}{L} \left( x - v_0 t \right)$$

characterises the correspondence between the spatial referential, $x$, and the referential that follows the moving load, $s$, considering the train speed, $v_0$, and the time, $t$. The characteristic length is a relation between the bending stiffness, $EI$, and the foundation coefficient:

$$L = \sqrt{\frac{4EI}{k}}$$

(3)

Considering that 60% of the axle load are transmitted to the track, in the position $s = 0$ the characteristic length is equal to 0.83(3).

The load distribution, function of the time and the position of the spatial referential, can be rewritten relating Eqs. (1) and (2):

$$F(t, x) = \frac{F_s}{2L} e^{-\frac{v_0 t + x}{2L}} \left( \cos \frac{x - v_0 t}{L} + \sin \frac{x - v_0 t}{L} \right)$$

(4)

The effect of a moving train is given by the sum of all the axles, according to its load distribution and assuming a linear elastic response.

### 3.2. Modelling of the track

The model adopted for the railway track, which referred to the effect of combined wheel–rail irregularities, was the one used by [25]. It applies a frame of reference that moves with the vehicle [40, 41]. More details of the compliance formulation can be found in the work of [40].

A wave number, $\lambda$, characterises rail roughness for a given speed and then it was introduced by adding it to the moving load (considering that the model was in the elastic regime). A vertical force due to the wheel–rail interface, $f$, function of the angular frequency, $\omega$, is given by:

$$\left[ C^w(\omega) + C^r(\omega) \right] \hat{g}(\omega) = \hat{u}_{tj}(\omega)$$

(5)

where $\hat{u}_{tj}(\omega)$ represents the combined roughness of the wheel and the rail, $C^w(\omega)$ and $C^r(\omega)$ are the vehicle and track rigidity compliances.

The track roughness, $\hat{u}_{tj}(\omega)$, can be calculated using a single-sided power spectral density (PSD), $\hat{G}_{ij}(n_j)$, which can be rewritten as a function of the inverse of the wave length, $n_j = f/v = \lambda^2$, as:

$$\hat{G}_{ij}(n_j) = \frac{A^2 n_j^2 (n_j^2 + n_y^2)}{n_j^2 (n_j^2 + n_y^2)}$$

(6)

where $y$ represents an observer’s position in an artificial profile, $f$ is the linear frequency, $\lambda^2$ represents the wavelength, $A^2$ represents a constant dependent of the track class, $n_j, n_y$ and $n_z$ represent the cyclic wave numbers.

To obtain the PSD as a function of the wave number, $k^2$, it is necessary to divide eq. (6) by $2\pi$ as:

$$\hat{G}_{ij}(k^2) = \frac{G_{ij}(n_j)}{2\pi}$$

(7)

The Federal Railroad Administration (FRA) defined 6 track classes with a classification ranging from 1 (poor quality) to 6 (good quality). Accordingly to Gupta et al. [25] $n_y$ and $n_z$ are almost constant for each class and can be admitted to be equal to 0.0233 cycles/m and 0.1312 cycles/m, respectively. On the other hand the constant $A^2$ varies with the track class according with Table 8.

| Table 8. Roughness parameter of the track class, following the FRA. |
|-----------------------------|-----------------------------|
| Track class | 6 | 5 | 4 | 3 | 2 | 1 |
| $A^2 [10^{-4} m/ cycle]$ | 1.06 | 1.69 | 2.96 | 5.29 | 9.52 | 16.72 |

Fig. 6 shows the single-sided PSD of the rail roughness for the FRA track classes 1 and 6.

An artificial profile, $u_{tj}(y)$, can be generated by defining the PSD $G_{ij}(k)$ for an interval $[k_{ij}, k_{ij}]$, considering the relevant frequencies for this study and the train velocity, and dividing the latter interval in $N$ subintervals, with a width of $\Delta k_{ij}$ and a centre in the wave number $k_{ij}$. This profile is obtained summing co-sin functions with phase angles, $\theta_i$, created aleatory in the interval $[0, 2\pi]$, as:

$$u_{tj}(y) = \sum_{i=1}^{N} a_i \cos(k_{ij}y - \theta_i)$$

(8)

where the parameters $a_i$ are obtained imposing the area under the PSD, $G_{ij}(k)$, for each interval $\Delta k_{ij}$, with center $k_{ij}$, to be equal to the square root of the artificial profile, $a$:

$$a_i = \sqrt{2G_{ij}(k_{ij})\Delta k_{ij}}$$

(9)

In the numerical models was adopted the FRA track class 6, corresponding to the superior rail quality, to generate the artificial roughness profile $u_{tj}(y)$. However, instead of using it as function of position $y$, it was convenient to use it as function of time, taking into consideration the train speed, $u_{tj}(t)$.
3.3. Modelling of the ground

To model the ground, it had been assumed that the material and geometric properties of the system remained identical along the train-moving direction. The adopted profiles consisted of a near field of finite irregular region and a semi-infinite region in the far field, which were traditional plane elements with 2 degrees of freedom per node, to account for the horizontal and the vertical responses. Here, both the rails and the sleepers were not geometrically represented, instead they were considered as equivalent forces distributed along the sleepers’ length.

The adopted soil geometry consisted of a number of layers, \(n\), superposed on a half-space. Properties for each layer were determined from soil characterisation performed in previous works, namely, Internal Circular Line quadruplication and Northern Line modernisation. The soil samples were taking using a variety of samplers, such as, trial pits, machine excavated tranches and boreholes. They were distributed along both tracks and they were chosen based on their proximity with the testing sites. The information was sparse and essentially consisted of Standard Penetration tests SPT. This test main purpose was to provide an indication of the relative density of granular soils and, besides its many flaws, it had widespread due to its simplicity and cost. It is usual to correlate SPT, which involves some uncertainties. The reason being that SPT results are often the only test results available. Different correlations are proposed for granular and cohesive soils between \(N_{SPT}\) and the soil’s friction angle (\(\phi\)), the Young’s modulus (\(E\)) and the shear modulus (\(G\)) [42]. Fonseca [43] introduced a relation between \(N_{SPT}\) and the soil’s friction angle for sedimentary sandy soils normally consolidated, which are gathered in Table 9.

![Fig. 6. PSD curves of the rail roughness for track classes 1 and 6 ([25]).](image)

**Table 9. Relation between \(N_{SPT}\) and soil’s friction angle ([43]).**

| \(N_{SPT}\) | \(\phi^\circ\) (*) |
|-----------|----------------|
| 5         | 28             |
| 10        | 32             |
| 15        | 35             |
| 20        | 37             |
| 25        | 38.5           |
| 30        | 40             |
| 40        | 41.5           |
| 50        | 43             |

According to Bowles [44], for clay soils Young’s modulus determined from the bi-layer should be between the values presented in Table 10.

The range for the values presented in Table 10 is quite large considering the dependence of the stress history, water content, density and age. Therefore, in cases of sparse information, the adopted values should reflect the observed conditions if possible.

**Table 10. Values of Young’s modulus for clay soils (Bowles, 1996).**

| Clay | \(E\) (MPa) |
|------|-------------|
| Very soft | 2–15       |
| Soft | 5–25        |
| Medium | 15–50      |
| Stiff | 50–100      |
| Sandy | 25–250      |

4. Calibration of the numerical model

The option of using a bi-dimensional model to simulate the problem requires the consideration of certain assumptions. The track and the foundation layers are continuous with infinite extension in the direction perpendicular to the plane. This option is valid if the model geometry remains constant for a distance compatible with the studied phenomenon. Therefore, a bi-dimensional model is only acceptable for tracks of great length, which is adequate for this problem. In singularities, such as, curves, transitions between bridges and plain tracks and geotechnical singularities it should not be adopted. The problem was investigated using a numerical simulation in two-dimensional (2D) explicit finite-difference program FLAC2D [45]. Since only the vertical displacements are studied in this paper, this representation is sufficient to predict the reflection in the railway structure (ballast to sub-grade), to calculate the forces acting on the soil surface, and to evaluate the ground vibrations induced on the neighbourhood by the passing of railway vehicle [46, 47].

The vibration propagation is a three-dimensional effect and in order to consider it in the model, some corrections were adopted. During propagation, geometrical damping occurs in a cylindrical front. The wave front of this damping is characterised by an energy decrease as the wave front moves further away from the vibration source, since the wave successively propagates in larger volumes. This type of damping is of foremost importance, as it causes a decay of both volumetric and sur-
face waves, even in perfectly elastic materials [48]. Although the results show a good fit with reality, in terms of arrival times and wave shape, the amplitude of the waves suffers a greater attenuation with distance. Therefore, as the two-dimensional model does not consider the geometrical damping of the three-dimensional phenomenon, it is necessary to correct the numerical results. The correction factor was applied directly to the results and it was equal to the inverse of the mean square of the distance from the point to the vibration source [9].

Soils have some energy dissipation capacity for load-unload cycles, even for small deformations. In time-domain software, Rayleigh damping is commonly used to provide damping that is frequency-independent over a restricted range of frequencies. Although Rayleigh damping embodies two viscous elements (in which the absorbed energy is dependent on frequency), the frequency-dependent effects are arranged to cancel out at the frequencies of interest.

Hysteretic damping is an alternative damping option. This form of damping allows strain-dependent modulus and damping functions to be incorporated directly into the FLAC simulation. However, hysteretic damping provides almost no energy dissipation at very low cyclic strain levels, which may be unrealistic. To avoid low-level oscillation, Rayleigh damping should be added when hysteretic damping is used in a dynamic simulation.
Rayleigh damping is specified in FLAC with the parameters centre frequency $f_{\text{min}}$ in Hertz (cycles per second) and damping ratio $\zeta_{\text{min}}$.

In the absence of specific parameters, typical values of a ballasted track with monobloc sleepers were assigned to the track structure. The adopted solution was the same presented by Correia et al. [49] (Table 11).

### 4.1. Marvila’s profile

Based on the gathered information, we defined the parameters for the numerical model, which are presented in Table 12. The layer’s thickness follow the results of the drills S119 and S120 and the unit weight results from the wells P115 and P116. Values of damping ratio of 1 and 3% were considered for the track structure and the soil layers, respectively, whereas the centre frequency for this profile was 4.26 Hz.

Although several railway track models have been developed before, various difficulties arise when it is necessary to adequately represent the properties and loading conditions of the track structure. Usually, in numerical models the track substructure is generally assumed to be linear and modelled as an elastic foundation or a series of discrete linear springs and dampers in the vertical direction [50]. Nevertheless, previous experimental field tests have shown that the track support system has non-linear mechanical behaviour, specially after a large number of load cycles.

In every model, it was decided to define an additional 50 meters from the last measurement point. Considering the geometric simplification of the measured profile in Marvila, Fig. 7 shows a detail of the adopted numerical model in the zone of the rail embankment. It has a total of about 200 m in the SE-NW and in the NW-SE directions, for symmetry, excluding the track width, totalling about 410 meters wide. The horizontal displacements of the vertical boundaries were restrained, as well as the displacements in the lower horizontal boundary. Both boundaries (vertical and horizontal) were also viscous boundaries to avoid the so called “box effect”.

Fig. 8 indicates the distribution of the maximum ground vibration levels, induced by the passing UQE2300 train at Marvila site, versus the distance to the train track. The highest vibration velocity is 1.14 mm/s (point 2S at 7.8 m) followed by 0.31 mm/s (point 2N at 3.5 m) which are clearly perceptible to humans according to LNEC [51]. Then, the measured ground vibration level at 10, 20, 40, 80, 160 m stop being perceptible to humans, with a main trend that attenuates with the increase of the distance to the point vibration source. Close to the track, up to 10 m, every bogie can be easily identified as a pulse, whereas further away the whole train is present only as a single up and downward wave. For distances greater than 55 m we could only register environmental noise. Clearly, the points located south of the track, with notation S, present higher values than the ones located north. The difference is not noted in the model due to geometric factors. The ground has a slope pending to south and the outer part of the track curve is in that direction. Furthermore, the accelerometers were fixed to wood blocks, which were previously glued to the elements that were intended to be moni-

### Table 11. Characteristics adopted for the track structure.

| Element | Thickness (m) | $\gamma_{\text{mat}}$ (kN/m$^3$) | Young’s modulus (MPa) |
|---------|---------------|---------------------------------|-----------------------|
| Ballast | 0.3           | 1800                            | 200                   |
| Sub-ballast | 0.2       | 2200                            | 300                   |
| Subgrade | 0.5          | 2200                            | 400                   |

### Table 12. Characteristics adopted for Marvila’s ground.

| Element | Thickness (m) | $\gamma_{\text{mat}}$ (kN/m$^3$) | Young’s modulus (MPa) |
|---------|---------------|---------------------------------|-----------------------|
| Layer 1 | 1.5           | 1830                            | 33                    |
| Layer 2 | 8.0           | 1830                            | 25                    |
| Layer 3 | 5.5           | 1830                            | 177                   |

**Fig. 8.** Marvila – measurement values and corresponding numerical results (UQE 2300).
In the railway track, the wood blocks were directly fixed to the sleeper, or to the rail, using an appropriate glue tested in similar situations. For the other monitoring points, the wood blocks were connected to piles driven into the ground, so that the measured values were representative of the ground accelerations. Two types of piles were used. Some of them were metallic, for hard soils, while others were made of wood, for softer soils. In point 2N a metallic pile was used, whereas for point 2S a wooden pile was chosen. It was expected that the maximum value registered in this profile was in the point near the source of excitation (point 2N), which was not the case. Therefore, the choice of fixation to the ground might affect results, regarding the in situ measurements. Those factors, not reflected in the numerical model, might explain the differences.

4.2. Braço de Prata’s profile

Following the same method, we defined the parameters for the numerical model of Braço de Prata (Table 13). The layer’s thickness follow the results of the drills S126 and the unit weight values from the well P108. Comparing with the last profile, clay materials were more relevant and the ground presented higher SPT values. Values of ramping ratio of 1 and 3% were considered for the track structure and the soil layers, respectively, whereas the centre frequency for this profile was 6.16 Hz.

As referred before there was a concrete wall sustaining an embankment near the train track. This was taking into account by restraining the horizontal displacements of the wall. The remaining restraints followed the same method as in Marvila. Fig. 9 shows a detail of the...
adopted numerical model. Considering the additional 50 m wide in both directions, this model has a total of about 90 m in the E-W and in the E-W directions, excluding the track width, totalling about 190 meters wide.

Fig. 10 indicates the distribution of the maximum ground vibration levels, induced by the passing UQE2300 train at Braço de Prata site, versus the distance to the train track. The accelerometer located in the beam registered vibration levels capable of producing severe cracking on buildings (26.6 mm/s). This might be an issue related to the conditions of fixation of accelerometers to the sleeper. A reason for this behaviour might be related to the integrity of the sleeper, which was not characterised, or the fixation of the accelerometers. Regarding the fixation of the equipment, the same methodology was applied in other measurements performed before with superior results. What is more, this measurement was made directly in the source of excitation and its disturbing character, which can cause this behaviour. Apart from that, the nearest points of the track present values of 0.53 mm/s that were clearly perceptible by humans (point 2N at 2.1 m) becoming less perceptible further from the track: 0.25 mm/s at point 3N at 7.6 m and 0.14 mm/s at point 2S at 9.6 m. It is worth noting that at point 4N, which is located less than 10 m from the track on top of the concrete wall (8.6 m), the velocity experiences a severe drop. Comparing it with point 2S, which is located almost at the same distance, but south from the track, the registered value was only 25% and equal to 0.03 mm/s. The values clearly show the drop that the embankment produces in the vibration velocity. The attenuation promoted by the concrete wall continue to reflect further from the track, where points located north from the track present smaller velocity values (0.005 and 0.011 mm/s) than point 3S 0.015 mm/s (located at 39.5 m from the vibration source).

As in Marvila’s site, close to the track, up to 10 m, every bogie can be easily identified as a pulse. Further away the whole train is present only as a single up and downward wave, but for distances greater than 25 m (for points located north of the track) the environmental noise component becomes relevant. It is worth noting that in points 5N (25 m) and 3S (40 m) the registered velocities are very similar.

In the case of Braço de Prata the numerical results do not reflect correctly the effect of the retaining wall. For points located south, the model gives acceptable values, for points located north differences start to appear. Firstly, for those points close to the wall, the vibration drop registered experimentally is not reflected numerically. Then, for greater distances the differences start to decrease. Therefore, some aspects should be implemented in order to improve the results from the numerical model.

4.3. Parque das Nações’ profile

The last profile, located in Parque das Nações area, had some differences, compared to the previous ones. In this profile, the train track was situated above an embankment and our main concern was with the area located at south of the track, where the majority of buildings stood. Taking into account the geometric simplifications followed in the other examples, Fig. 11 shows a detail of the adopted numerical model. As in the later profile, due to the additional 50 m wide in both directions, this model has a total of about 90 m in the E-W and in the E-W directions, excluding the track width, totalling about 190 meters wide. The parameters defined for the numerical model of Parque das Nações are presented in Table 14. Values of ramping ratio of 1 and 3% were considered for the track structure and the soil layers, respectively, whereas the centre frequency for this profile was 8.27 Hz.

Fig. 12 indicates the distribution of the maximum ground vibration levels, induced by the passing UTE2240 train, versus the distance to the train track. Like in the Braço de Prata profile, the accelerometers located in the train track registered the highest values. The one in the rail registered values capable of produce severe cracking in a building (21.7 mm/s in point 1C), whereas the measurements in the sleeper

| Table 13. Characteristics adopted for Braço de Prata’s ground. |
|-------------------------|-------------------------|-------------------------|
| Element | Thickness (m) | $f_{ref}$ (4 N/m) | Young’s modulus (MPa) |
| Layer 0 | 6.0 | 1870 | 25 |
| Layer 1 | 3.0 | 1870 | 157 |
| Layer 2 | 5.0 | 1870 | 115 |
| Layer 3 | 5.0 | 1870 | 155 |
could lead to falling of plaster from building walls (4.5 mm/s in point 1T). Apart from those, the nearest points from the track had velocities clearly perceptible by humans (0.95 mm/s in point 2N and 0.45 mm/s in point 2S) and the rest of them stop being perceptible, with values small than 0.11 mm/s.

Once again, every bogie can be easily identified as a pulse close to the track. However, in this profile further than 20 m from the track the whole train is present only as a single up and downward wave, but with a relevant environmental noise component. The presence of the concrete retaining wall, separating points 4S and 5S, produces a drop of almost 30% in the registered values.

Apart from the points located in the track, the numerical model gives higher values than the ones measured. However, the trend was successfully simulated, specially in the points 3S, 4S and 5S. In the point 6S the registered value was almost the same as in 5S, but this was due to the environmental noise component, not considered in the numerical model.

5. Forecast for the introduction of Thalys HST trains

One of the objectives of this test program has been to characterise the actual vibrations and establish numerical models that can reliably simulate the observed track response. Once validated, such models can be used to investigate the introduction of high speed trains in the same tracks and access its effect on the surrounding buildings.

Table 14. Characteristics adopted for the ground of Parque das Nações’s.

| Element | Thickness (m) | $\gamma_{\text{max}}$ (kN/m$^2$) | Young’s modulus (MPa) |
|---------|---------------|---------------------------------|-----------------------|
| Layer 1 | 2.3           | 1770                            | 12                    |
| Layer 2 | 8.2           | 1770                            | 34                    |
| Layer 3 | 4.0           | 1770                            | 122                   |
For this study we should have considered the train which would be applied in Portugal, however it is yet to be decided. Therefore, the choice fell on one of the trains riding in Europe. Thalys HST is a high-speed train that circulates between Belgium and France. Its rolling stock consists of 2 engines with four axles each (axle spacing 3.0 m and pivot spacing 11.0 m), 2 adjacent coaches with three axles (axle spacing 3.0 m and pivot spacing 15.7 m) and another 6 coaches with four axles (axle spacing 3.0 m and pivot spacing 15.7 m). The mass of the powered axles and the four axles coaches is around 17.0 tons and three axles coaches 14.5 tons (Fig. 13a). For this simulation, instead of adopting the maximum speed registered during the measurements, we decided to use 80 km/h, which was the allowed maximum speed. This speed is smaller than the ground’s lowest shear wave velocity and represents a subseismic condition. Then, the displacement field is quasi-static. Apart from the differences regarding the modelling of Thalys HST, the track and the ground were modelled as in the previous analysis (for every site and trains, UQE2300 and UTE2240), in order to allow the characterisation of introducing high speed trains in the studied sites.

Figs. 14 to 16 compare the passage of a high speed train with the actual train (UQE 2300 or UTE 2240), regarding the distribution of the maximum ground vibration levels of the simulated vertical track displacement. Generally, the results from the high-speed train are slightly larger than the considered regular train (UQE 2300 or UTE 2240). This situation was expected considering the differences between the axle loads of each train and the adopted speed for the Thalys train (80 km/h instead of 60 km/h).

It can be seen that the introduction of high speed trains in the profile of Braço de Prata will be practically unnoticed, since the differences between simulated trains were small. They were almost coincident, apart for points near the vibration source (rail). In Marvila’s profile it is expected that the passage of the Thalys train will slightly increase the ground vibration levels, specially in the near and the far field. It can be observed that for distances to the vibration source (rail) of around 40 m the ground vibration levels will be practically the same. Finally, the last profile (Parque das Nações) has proved to be more sensitive to the passage of Thalys train, where its introduction will increase the vibration levels since the near field.

The adoption of a homogeneous medium with an elastic behaviour in the numerical models can be considered appropriate for a region far from the vibration source, due to the loads small intensity. However, one should take into consideration that the adoption of a homogeneous continuum medium can create some discrepancies in the results analysis. The vibration propagation cross several heterogeneities causing an increase on the wave damping. This leads to numerical values greater than the experimental values, so the adoption of these hypotheses put us in the side of security.

In conclusion, the implementation of high-speed trains would lead to a slightly vibration increase (which magnitude varies with the distance to the vibration source), directly related to the circulation speeds allowed and with the type of train that will be chosen.

6. Conclusions

Train traffic has long been recognized as a potential source of ground vibration and has been studied for several years. Various formulations, including 2.5D finite/infinite procedures and three-dimensional numerical models, have been proposed to predict the propagation of train-induced ground vibration with very good results. Despite the versatility of such models and formulations, frequently its computational cost and time involved in the analysis are not suitable to improvements and maintenance programmes held in train tracks. Due to this very important aspect, bi-dimensional models still presents as a valid
alternative to study such problems, specially when time is a crucial factor. Therefore, this study is of interest to both structural dynamics and railway dynamics, since it presented an efficient and easy to implement numerical model.

The experimental data presented in this paper are complementary to other data sets published in literature. Especially the fact that measurements have been made at 3 different sites that included slopes and concrete walls near the train track, makes this data set unique. Nonetheless, a major shortcoming of the present data set is that, due to time and budget limitations, no in situ experiments have been made to determine the characteristics of the foundations of each wall. Only limited data are also available on the stratification of the soil and the variation of dynamic soil characteristics with depth, especially material damping. This compromises the quantitative validation of numerical prediction models.

The accelerometers were fixed to wood blocks, which were previously glued to the elements that were intended to be monitored. In the railway track, the wood blocks were directly fixed to the sleeper using an appropriate glue, tested in similar situations. At the other monitoring points located on the ground, the wood blocks were connected to piles driven into the ground, so that the measured values were representative of the ground accelerations. Two types of piles were used. Some of them were metallic, for hard soils, while others were made of wood, for softer soils. The choice of fixation to the ground might affect results, regarding the in situ measurements. In Marvila profile, the point closer to the track where the train passed did not return the maximum value. This behaviour was observed in the other two profiles.

The forecast for the introduction of the high-speed train was made considering the Thalys HST. If another train is chosen, the analysis should be redone. The circulation speed that was considered for Thalys train was slightly greater than the velocity that is currently implemented. Results showed that the effective velocities obtained using the numerical models for the actual situation (reference) were slightly lower than the values previewed for the high-speed train. However, this increase was meaningless and it can be concluded that, generally, the new train moving loads will not significantly alter the actual situation.

Although there was scarcely any information of the train track foundation and no in situ tests were expected to be performed and instead of trying to match the experimental results by modifying the input parameters in a trial-and-error procedure, a qualitative assessment of the predictions has been made. The values obtained in the performed measurements presented some dispersion, for points close to the excitation source (rail), that tended to decrease with increasing distance. This phenomenon is believed to be due to the proximity to the source of excitation and its disturbing character. In the profiles of Marvila and Parque das Nações, the numerical values are, in general, higher than the measured values, while in the profile of Braço de Prata there was no such trend. Since the model was bi-dimensional, loading extends to infinity and has a cylindrical damping, whereas in reality, damping is spherical, and loading has a finite length. The adoption of a homogeneous medium with an elastic behaviour in the numerical models can be considered more appropriate for a region far from the vibration source, due to the load’s small intensity. Furthermore, the developed bi-dimensional models returned very good results, when compared to experimental data. Consequently, the methodology applied in this work can be adopted by other geotechnical engineers to perform preliminary studies in the design of new tracks or in the rehabilitation of existing ones, when ground information is sparse.

**Declarations**

**Author contribution statement**

João Manso: Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Jorge Gomes: Conceived and designed the experiments.
Fig. 14. Marvila – campaign measurements and numerical results (Thalys).

Fig. 15. Braço de Prata – campaign measurements and numerical results (Thalys).
João Marcelino: Contributed reagents, materials, analysis tools or data.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability statement

Data will be made available on request.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

References

[1] V. Krylov, C. Ferguson, Recent progress in the theory of railway-generated ground vibrations, Proc. Inst. Acoust. 17 (4) (1995) 55–68.
[2] P.A. Costa, A. Colaço, R. Calçada, A.S. Cardoso, Critical speed of railway tracks. Detailed and simplified approaches, Transp. Geotech. 2 (2015) 30–46.
[3] A. Garinei, G. Ristano, L. Scappaticci, Experimental evaluation of the efficiency of trenches for the mitigation of train-induced vibrations, Transp. Res., Part D, Transp. Environ. 32 (2014) 303–315.
[4] P. Coulie, G. Lombaert, G. Degrande, The influence of source-receiver interaction on the numerical prediction of railway induced vibrations, J. Sound Vib. 353 (12) (2014) 2520–2538.
[5] X. Bian, H. Jiang, C. Chang, J. Hu, Y. Chen, Track and ground vibrations generated by high-speed train running on ballastless railway with excitation of vertical track irregularities, Soil Dyn. Earthq. Eng. 76 (2015) 29–43.
[6] L. Auersch, Simultaneous measurements of the vehicle, track, and soil vibrations at a surface, bridge, and tunnel railway line, Shock Vib. 2017 (2017) 31–33.
[7] V.V. Krylov, Focusing of ground vibrations generated by high-speed trains travelling at trans-Rayleigh speeds, Soil Dyn. Earthq. Eng. 100 (2017) 389–395.
[8] A. Kaynia, C. Madhus, P. Zackrison, Ground vibration from high-speed trains: prediction and countermeasure, J. Geotech. Geoenviro. Eng. 126 (6) (2000) 531–537.
[9] J. Manso, Aplicações de modelos bidimensionais ao estudo da geração e propagação de vibrações, Ph.D. thesis, Instituto Superior Tecnico, Universidade de Lisboa, 2011.
[10] G. Kouroussis, D.P. Connolly, K. Vogiatzis, O. Verlinden, Modelling the environmental effects of railway vibrations from different types of rolling stock: a numerical study, Shock Vib. 2015 (2015) 16.
[11] G. Kouroussis, D.P. Connolly, G. Alexiou, Field study of train-induced vibration by wheel and rail singular defects, Veh. Syst. Dyn. 53 (10) (2015) 1500–1519.
[12] G. Kouroussis, D.P. Connolly, B. Olivier, O. Laghrrouch, P. Alves, Science of the total environment railway cuttings and embankments: experimental and numerical studies of ground vibration, Sci. Total Environ. 557–558 (2016) 110–122.
[13] B. Olivier, D.P. Connolly, P.A. Costa, G. Kouroussis, The effect of embankment on high speed rail ground vibrations, Int. J. Rail Transit 4 (4) (2016) 229–246.
[14] K. Vogiatzis, H. Mouzakis, Ground-borne noise and vibration transmitted from subway networks to multi-storey reinforced concrete buildings, Transport 33 (2) (2018) 446–453.
[15] M. Germonpré, G. Degrande, G. Lombaert, Periodic track model for the prediction of railway induced vibration due to parametric excitation, Transp. Geotech. 17 (2018) 98–108.
[16] G. Degrande, G. Lombaert, High-speed train induced free field vibrations: in situ measurements and numerical modelling, in: Proceedings of the International Workshop Wave 200, December 2000, pp. 29–41.
[17] X. Sheng, C. Jones, D. Thompson, A comparison of a theoretical model for quasi-statically and dynamically induced environmental vibration from trains with measurements, J. Sound Vib. 267 (3) (2003) 621–635, http://linkinghub.elsevier.com/retrieve/pii/S0022554703000784.
[18] L. Auersch, The excitation of ground vibration by rail traffic: theory of vehicle-track-soil interaction and measurements on high-speed lines, J. Sound

Fig. 16. Parque das Nações – campaign measurements and numerical results (Thalys).
Mechanical vibrations induced by railway traffic: experimental validation of a 3D numerical model, Soil Dyn. Earthq. Eng. 97 (March 2017) 324–344.

J. Gomes, J. Marcelino, J. Manso, Campanha de medição de vibrações devido ao tráfego ferroviário realizada na estação do Oriente. Measurement campaign of vibrations induced by railway traffic performed in Oriente train station, in: XIII Congresso Nacional de Geotecnia, Sociedade Portuguesa de Geotecnia, Lisboa, 2012, p. 8.

X. Sheng, C. Jones, M. Petyt, Ground vibration generated by a load moving along a railway track, J. Sound Vib. 228 (1) (1999) 129–156, http://linkinghub.elsevier.com/retrieve/pii/S0022460X99924069.

C. Lai, A. Callerio, E. Faccioli, A. Martinó, Mathematical modelling of railway-induced ground vibrations, in: Proceedings of the International Workshop Wave 2000, 2000, pp. 99–110.

C. Esveld, Modern Railway Track, 2001.

S. Gupta, W. Liu, G. Degrande, G. Lombaert, W. Liu, Prediction of vibrations induced by underground railway traffic in Beijing, J. Sound Vib. 310 (3) (2008) 608–630, http://linkinghub.elsevier.com/retrieve/pii/S0022460X0700573X.

J. Manso, J. Marcelino, Geração e propagação de vibrações produzidas pela passagem de comboios de alta-velocidade, in: 3as Jornadas Hispano Portuguesas Sobre Geotecnia Nas Infraestruturas Ferroviárias, CEDEX, Madrid, 2009, p. 8.

A.M. Kaynia, J. Park, K. Norén-Congriff, Effect of track defects on vibrations from high speed train, Proc. Eng. 199 (2017) 2681–2686.

L.S. Tang, H.K. Chen, H.T. Sang, S.Y. Zhang, J.Y. Zhang, Determination of traffic-load influenced depths in clayey subsoil based on the shake down concept, Soil Dyn. Earthq. Eng. 77 (2015) 182–191.

G. Kouroussis, D.P. Connolly, O. Verlinden, Railway-induced ground vibrations – a review of vehicle effects, Int. J. Rail Transp. 2 (2) (2014) 69–110.

X. Bian, H. Jiang, C. Cheng, Y. Chen, R. Chen, J. Jiang, Full-scale model testing on a ballastless high-speed railway under simulated train moving loads, Soil Dyn. Earthq. Eng. 66 (2014) 368–384.

J. Lynner, R.L. Kuhlemeyer, Finite dynamic model for infinite media, J. Eng. Mech. Div. 95 (4) (1969) 859–877.

R. Kunar, P. Beresford, Effects of site conditions on floor response spectra, in: Proceedings of the Sixth World Conference on Earthquake Engineering [held at], Vol. 3, New Delhi, January 10–14, 1977, 1977.

E. Winkler, The Doctrine of Elasticity and Strength: with a Special Focus on Its Application in Technology, for Polytechnic Schools, Building Academies, Engineers, Mechanical Engineers, Architects, etc., H. Dominicus, 1867.

J. Kenney Jr., Steady-state vibrations of beam on elastic foundation for moving load, J. Appl. Mech. 21 (4) (1954) 359–364.

Z. Cai, G. Raymond, R. Bathurst, Natural vibration analysis of rail track as a system of elastically coupled beam structures on winker foundation, Comput. Struct. 53 (6) (1994) 1427–1436.

Z. Cai, G. Raymond, Modelling the dynamic response of railway track to wheel/rail impact loading, Struct. Eng. Mech. 2 (1) (1994) 95–112.

J. Wang, Z. Shi, H. Xiang, G. Song, Modeling on energy harvesting from a railway system using piezoelectric transducers, Smart Mater. Struct. 24 (10) (2015) 105017.

C.A. Murray, W.A. Take, N.A. Hoult, Measurement of vertical and longitudinal rail displacements using digital image correlation, Can. Geotech. J. 52 (2) (2015) 141–155.

Y. Zhang, X. Liu, Response of an infinite beam resting on the tensionless winker foundation subjected to an axial and a transverse concentrated loads, Eur. J. Mech. A, Solids 77 (2019) 103819.

G. Lombaert, G. Degrande, J. Kogut, S. François, The experimental validation of a numerical model for the prediction of railway induced vibrations, J. Sound Vib. 297 (3–5) (2006) 512–535.

D. Cloutere, G. Degrande, G. Lombaert, Numerical modelling of traffic induced vibrations, Meccanica 36 (4) (2001) 401–420.

J. Martínez, T. Miranda, Ensaios de Penetração nos Solos Graníticos da Região Norte de Portugal - algumas correlações, 2003.

A. Fonseca, Geomecânica dos solos residuais do granito do Porto. Critérios para dimensionamento das fundações directas, 1996.

J. Bowles, Foundation Analysis and Design, 1996.

ITASCA ITASCA, FLAC2D Version 3.4, 1998, in: Fast Lagrangian Analysis of Continua in 2 Dimensions, 1998.

K.L. Knorre, S.L. Grassie, Modelling of railway track and vehicle / track interaction at high frequencies, Veh. Syst. Dyn. 22 (1993) 209–262.

G. Kouroussis, O. Verlinden, C. Conti, On the interest of integrating vehicle dynamics for the ground propagation of vibrations: the case of urban railway traffic, Veh. Syst. Dyn. 3114 (2010) 1744–1759.

L. Hall, Simulations and analyses of train-induced ground vibrations, vol. 4, Doktorarbeit, Division of Soil and Rock Mechanics, Royal Institute of Technology, Stockholm, 2000, p. 69.

A. Correia, N. Araújo, J. Martins, J. Cunha, Interaction Soil-Railway Track for High Speed Trains, 2006.

J. Sadeghi, H. Askarinejad, Development of nonlinear railway track model applying modified plane strain technique, J. Transp. Eng. 136 (12) (2010) 1068–1074.

LNEC, Aspectos Regulamentares e Normativos no domínio do Ruído e Vibrações. Curso Leccionado no LNEC, 2002.