THE DENSITY DISTRIBUTION IN TURBULENT BISTABLE FLOWS

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Received 2012 November 5; accepted 2013 January 7; published 2013 February 14

ABSTRACT

We numerically study the volume density probability distribution function ($n$–PDF) and the column density probability distribution function ($\Sigma$–PDF) resulting from thermally bistable turbulent flows. We analyze three-dimensional hydrodynamic models in periodic boxes of 100 pc by side, where turbulence is driven in the Fourier space at a wavenumber corresponding to 50 pc. At low densities ($n \lesssim 0.6$ cm$^{-3}$), the $n$–PDF is well described by a lognormal distribution for an average local Mach number ranging from $\sim 0.2$ to $\sim 5.5$. As a consequence of the nonlinear development of thermal instability (TI), the logarithmic variance of the distribution of the diffuse gas increases with $M$ faster than in the well-known isothermal case. The average local Mach number for the dense gas ($n \gtrsim 7.1$ cm$^{-3}$) goes from $\sim 1.1$ to $\sim 16.9$ and the shape of the high-density zone of the $n$–PDF changes from a power law at low Mach numbers to a lognormal at high $M$ values. In the latter case, the width of the distribution is smaller than in the isothermal case and grows slower with $M$. At high column densities, the $\Sigma$–PDF is well described by a lognormal for all of the Mach numbers we consider and, due to the presence of TI, the width of the distribution is systematically larger than in the isothermal case but follows a qualitatively similar behavior as $M$ increases. Although a relationship between the width of the distribution and $M$ can be found for each one of the cases mentioned above, these relations are different from those of the isothermal case.

Key words: hydrodynamics – instabilities – ISM: structure – methods: numerical – turbulence

Online-only material: color figures

1. INTRODUCTION

The density distribution, and particularly the density probability distribution function (PDF), has become a crucial ingredient in theories about molecular cloud and core formation and evolution and for star formation theories (e.g., Padoan & Nordlund 2002; Krumholz & McKee 2005; Elmegreen 2002, 2008; Hennebelle & Chabrier 2008; Elmegreen 2011; Padoan & Nordlund 2011; Zamora-Avilés et al. 2012).

The volume density PDF in turbulent compressible flows has been widely studied for the isothermal case. In particular, numerical experiments of driven and decaying turbulence (Vázquez-Semadeni 1994, 1998; Padoan et al. 1997; Stone et al. 1998; Klessen 2000; Ostriker et al. 2001; Boldyrev et al. 2002; Beetz et al. 2008; Federrath et al. 2008) have shown that this distribution has a lognormal shape in a large number of situations. From the theoretical point of view, the development of a lognormal distribution for isothermal flows has been explained as a consequence of the multiplicative central limit theorem, assuming that individual density perturbations are independent and random (Vázquez-Semadeni 1994; Passot & Vázquez-Semadeni 1998; Nordlund & Padoan 1999). Furthermore, Passot & Vázquez-Semadeni (1998) showed that the density PDF develops a power-law tail at high (low) densities for values of the polytropic index $\gamma$ smaller (larger) than 1. Also for the isothermal case, numerical simulations at a fixed rms Mach number $M$ show an empirical relationship between the width of the density PDF, as measured by its variance or its standard deviation $\sigma$ and $M$ (e.g., Padoan et al. 1997; Passot & Vázquez-Semadeni 1998). For the distribution of $\ln n/\bar{n}$, this relationship reads

$$\sigma^2 = \ln(1 + b^2 M^2),$$

where $b$ is a constant of the order of unity whose value is not clearly established. In the literature, it goes from 0.26 (Kritsuk et al. 2007) to 1 (Passot & Vázquez-Semadeni 1998) and seems to vary depending on the relative degree of compressible and solenoidal modes of the forcing (Federrath et al. 2008, 2010). Recently, Price et al. (2011) found that for $M$, up to $20 b = 1/3$ is a good fit to numerical simulations with solenoidal forcing and is in agreement with other recent numerical results (Federrath et al. 2008, 2010; Lemaster & Stone 2008).

On the other hand, the condensation of diffuse gas produced by the isobaric mode of thermal instability (TI; Field 1965) triggered in colliding flows has been recognized as playing an important role for molecular cloud formation (e.g., Hennebelle & Pérault 1999; Vázquez-Semadeni et al. 2006; Heitsch & Hartmann 2008), especially in early phases. However, the density PDF resulting in turbulent bistable flows, which preserves the bimodal nature that is the signature of the development of TI, and the behavior of the distribution for each phase have not been systematically studied.

The density PDF resulting from non-isothermal simulations has nevertheless been reported in some papers. Using two-dimensional simulations of turbulent thermally bistable flows, Gazol et al. (2005) have shown that the effective polytropic index of the gas increases with $M$, and that for Mach numbers between 0.5 and 1.25 (with respect to the gas at $10^4$ K), it remains $<1$, with specific values also depending on the forcing scale going from $\approx 0.2$ to $\approx 0.4$ for large-scale forcing. They also found that the bimodal nature of the distribution becomes less pronounced as the $M$ increases. The same behavior has been reported from three-dimensional (3D) simulations (Seifried et al. 2011). Audit & Hennebelle (2010) compare the density distribution resulting from a bistable simulation and the one resulting when a single polytropic equation of state with $\gamma = 0.7$ is used. In the latter case, they find that the low-density part of the PDF is well described by a lognormal distribution, while for the higher densities the PDF is a power law with an exponent of about $-1.5$, which is consistent with the distribution expected for a very
compressible gas with $\gamma < 1$ predicted by Passot & Vázquez-Semadeni (1998). For the cooling run, they find that the density distribution of the cold gas, whose effective polytropic index is close to 0.7, is well fitted by a lognormal for values larger than $\sim 300$ cm$^{-3}$. They attribute the difference in the behavior to possible resolution effects and raise the question of whether or not the choice of a lognormal distribution in models of molecular clouds is adequate.

The density distribution resulting from more complex systems including a variety of physical ingredients has been studied by a large number of authors. Here, we just mention some examples that illustrate the activity in this direction. In the context of galactic disks, the effect of varying the polytropic index has been numerically studied by Li et al. (2003), who find that for self-gravitating gas, the density distribution shows an imperfect lognormal shape whose width decreases as $\gamma$ increases. Wada & Norman (2007) use 3D hydrodynamic simulations of a globally stable, inhomogeneous cooling interstellar medium (ISM) in galactic disks and report density PDFs that are well fitted by a single lognormal function (with larger dispersions for more gas-rich systems) over a wide density range. However, Robertson & Kravtsov (2008) include a detailed model of cooling and heating for temperatures $< 10^4$ K and account for the equilibrium abundance of H$_2$. They conclude that each thermal phase in their model galaxies has its own lognormal density distribution, implying that using a single lognormal PDF to build a model of global star formation in galaxies is likely an oversimplification. Density distributions with well-defined peaks have also been obtained from models of vertically stratified ISM dominated by supernovae (e.g., de Avillez & Breitschwerdt 2005; Joung & MacLow 2006).

Observationally, PDFs of the average volume density in diffuse interstellar gas ($n_{HI} < 1$ cm$^{-3}$ for the atomic gas) have been reported by Berkhuijsen & Fletcher (2008). For the atomic gas, they used 375 lines of sight (LOSs) from the sample given by Diplas & Savage (1994) and they estimate the volume density using the column density and the distance to each star. They find that the PDF tends to be a lognormal but cannot be well fitted by a single curve. In particular, different widths and means are obtained depending on the position (in the disk or away from it) and on the type of sampled gas. Specifically, they find that a mixture of cool and warm gas along the LOSs causes an increase in the dispersion.

An associated tool is the column density PDF ($\Sigma$–PDF), which has been numerically studied mainly in the context of isothermal turbulent flows. Vázquez-Semadeni & García (2001) studied the relationship between a lognormal volume density PDF ($n$–PDF) and the resulting column density PDF, finding that when the number of decorrelated density structures along the LOS is small enough, then the $\Sigma$–PDF is representative of the $n$–PDF and also have a lognormal shape. As the number of decorrelated density structures increases, the column density PDF slowly transits to a Gaussian distribution, passing through an intermediate stage where the distribution shows an exponential decay. As the relationship between the $n$–PDF and the $\Sigma$–PDF depends on the applicability of the central limit theorem, the authors argue that for non-isothermal flows with a volume density PDF that has a well-defined variance, this theorem should apply and the $\Sigma$–PDF should converge to a Gaussian for LOSs with a large enough number of decorrelated density structures. More recently, Burkhart & Lazarian (2012) used solenoidaly driven isothermal MHD simulations to investigate the presence of an empirical relationship between the variance of the column density distribution and the sonic Mach number. They found a relationship with the same form as Equation (3), namely,

$$\sigma_{\ln(\Sigma/z_{ini})} = A_{\Sigma} \ln \left(1 + \frac{b_{\Sigma}}{1} M^2\right), \tag{2}$$

where the scaling parameter $A_{\Sigma} = 0.11$ and $b_{\Sigma} = 1/3$, which is also close to the value reported for the 3D density distribution in the isothermal case.

On the other hand, Ballesteros-Paredes et al. (2011) numerically studied the evolution of the $\Sigma$–PDF resulting from colliding self-gravitating flows in the presence of TI, which are intended to model the formation process of molecular clouds. They found that a very narrow lognormal regime appears when the cloud is being assembled. However, as the global gravitational contraction occurs, the initial density fluctuations are enhanced, resulting first in a wider lognormal $\Sigma$–PDF, and later in a power-law $\Sigma$–PDF. These results suggest an explanation for the observational fact that clouds without star formation seem to possess a lognormal distribution, while clouds with active star formation develop a power-law tail at high column densities (e.g., Kainulainen et al. 2009).

In this work, we quantitatively study the behavior of the density and the column density distributions resulting in thermally bistable turbulent flows. For this purpose, we analyze simple numerical 3D experiments that include only a cooling function appropriate for the diffuse neutral interstellar gas and turbulent forcing. The paper is organized as follows. The model we use is described in Section 2. Then, in Section 3, we present the analysis of the behavior of both the volume density PDF and the column density PDF. These results are discussed in Section 4. Finally, in Section 5, we present our conclusions.

2. THE MODEL

We use the same model as in Gazol & Kim (2010), where a MUSCL-type scheme (Monotone Upstream-centered Scheme for Conservation Laws) with HLL Riemann solvers (Harten et al. 1983; Toro 1999) is employed to solve hydrodynamic equations in three dimensions within a cubic computational domain with a physical scale of 100 pc by side. The turbulence is randomly driven in Fourier space at large scales corresponding to $1 \leq k \leq 2$, where $k$ is the magnitude of the wave vector, $\mathbf{k}$. We use a purely solenoidal forcing for two reasons. First, it is the most common kind of forcing used in the isothermal case and second, also for the isothermal case, it is the kind of forcing for which the density PDF is better described by a lognormal distribution (Federrath et al. 2008). For further details concerning the turbulent forcing, see Gazol & Kim (2010) and for a discussion on the effects of forcing see Section 4.4.3. In the real ISM, turbulent motions are driven by spatially localized sources, and since the forcing in Fourier space is applied at any time in all of the grid points, this kind of driving is not very realistic. However, we chose this kind of driving method because it allows us to compare our results with previous work performed for the isothermal case which includes Fourier space forcing. As an additional advantage, with this method we can study the density distribution for the dense gas as well as the density distribution for the diffuse gas (see Section 3). We utilize the radiative cooling function presented by Sánchez-Salcedo et al. (2002), which is based on the standard $P$ versus $\rho$ curve of Wolfire et al. (1995).

For all simulations, we show in the next section that the resolution is 512$^3$, the boundary conditions are periodic, the gas is initially at rest, and the initial density and temperature are
uniform with $n_0 = 1 \text{ cm}^{-3}$ and $T_0 = 2399 \text{ K}$, which correspond to thermally unstable gas.

3. RESULTS

In this section, we analyze the density distribution resulting from one set of seven simulations with different Mach numbers. The values of $M$ for those simulations are 0.28, 0.73, 1.86, 2.60, 3.29, 5.50, and 6.23, where $M$ is computed as the mean value of the local Mach number. In what follows, we call this value $\langle M \rangle$ and we use $M$ as a generic abbreviation for Mach number in situations that are independent of the specific way used to compute it or in situations where Mach numbers calculated in different ways are included.

3.1. The Volume Density Distribution

Volume density histograms resulting from these simulations are displayed in Figure 1 along with lognormal fits to the high- and low-density parts of the distribution. As expected from previous works (e.g., Gazol et al. 2005; Seifried et al. 2011), when the Mach number increases, the PDF becomes wider and its bimodal nature, which is a consequence of TI development, becomes less pronounced. These two facts have been explained as a consequence of the decrease in the local ratio between the turbulent crossing time and the cooling time $\eta$ produced by the increase of $M$ (Gazol et al. 2005). This behavior was proved by Seifried et al. (2011), who, using test particles, measured the time spent by a particle in the unstable regime as well as the frequency with which a test particle is perturbed, finding that both quantities increase with $M$ (i.e., both quantities increase as $\eta$ decreases). Concerning the shape of each part of the distribution, there are four things to note from a simple inspection. First, the low-density zone of the PDF can be relatively well described by a lognormal (see dotted lines in Figure 1). Second, the width of this lognormal increases with $\langle M \rangle$. Third, for the smallest values of $\langle M \rangle$ that we consider, the zone of the distribution corresponding to high densities is not well described by a lognormal (see dashed lines in Figure 1). Fourth, when a lognormal is an appropriate fit to the high-density part, its width does not seem to systematically increase with the value of $M$ as rapidly as in the diffuse case.

In Figure 2, we show the width of the lognormal fits $\sigma_s$ (note that $\sigma_s$ is the width of the logarithmic density distribution $\ln n/\bar{n}$) that we obtain for the gas at low densities as a function of the local Mach $M_w$, which is computed as the average value of the local Mach number in points with densities within the range where the fit has been computed. The dotted lines in this figure are plots of the relationship between $\sigma_s$ and $M$ for isothermal gas (see Equation (1)) for two different values of the parameter $b$. Although for low $M_w$ values $\sigma_s$ is close to the
Figure 3. Lognormal widths as a function of $M_c$ for fits corresponding to low-density gas in PDFs displayed in Figure 1. Dashed black lines plot $\sigma_t = \ln(1 + b^2 M^2)^{1/2}$ for $b = 1/2$ and $1/3$, whereas dashed red line corresponds to the fit described in the text. Mach number values are average values of the local Mach number for the diffuse gas.

(A color version of this figure is available in the online journal.)

Figure 4. Effective polytropic index $\gamma$ as a function of the corresponding averaged local Mach number for the whole simulation (solid line, $\langle M \rangle$), the diffuse gas (dashed line, $M_{w_{diff}}$), and the dense gas (dotted line, $M_{w_{dense}}$).

$b = 1/2$ isothermal case, for our simulations it grows faster with $M_{w_{diff}}$. In fact, the red dashed line represents a fit of the form

$$\sigma_t^2 = A \ln(1 + b^2 M^2),$$

where $A = 2.25$ and $b = 0.33$. This value of $b$ is close to the typical values found for isothermal turbulence with solenoidal forcing.

For the dense gas, the behavior of $\sigma_t$ with $M_c$ (computed as the average value of the local Mach number in points with $n > 7.1$ cm$^{-3}$, this density corresponds to the thermal equilibrium value below which the cooling function implies that the gas is thermally unstable) is qualitatively different from Equation (3) (see Figure 3). In this case, the distribution is narrower than in the isothermal case and its width grows more slowly with $M_c$ than in the isothermal case. In fact, due to the large difference between the widths we obtain and the ones implied by Equation (3), we do not include both cases in the figure. For instance, for $M = 10$ in the isothermal case, $\sigma_t = 1.58$ is expected if $b = 1/3$.

As mentioned in Section 1, previous works have found that the shape of the density PDF in non-isothermal turbulent flows depends on the value of the effective polytropic index $\gamma$ (Passot & Vázquez-Semadeni 1998). For our simulations, we first measured this index as the slope of a linear least-squares fit of the whole log $P$ versus log $n$ distribution and we plotted it as a function of $\langle M \rangle$ (Figure 4, solid line). As expected from Gazol et al. (2005), $\gamma$ increases with $\langle M \rangle$ and remains $< 1$. As can be seen from the log $P/k$ versus log $n$ distributions displayed in Figure 5, for low values of $\langle M \rangle$, TI can develop almost without disturbance and the dense as well as the diffuse gas are predominantly in thermal equilibrium, with some of the gas transiting isobarically between stable branches. On the other hand, at high $\langle M \rangle$, the mean pressure in low-density gas decreases below the thermal equilibrium value while the mean pressure in high-density gas increases above its thermal equilibrium value, producing a neat positive slope of the whole log $P/k$ versus log $n$ distribution. If the value of $\gamma$ obtained in this way were a suitable parameter to infer the shape of the density PDF, then we would expect a power law at high density for all of our simulations. Also, from Figure 5, it is clear that for low values of $\langle M \rangle$ almost all of the dense gas ($n > 7.1$ cm$^{-3}$) is in thermal equilibrium, implying that its thermodynamic state is approximately well described by a polytropic relation with $\gamma = 0.53$, which is the power corresponding to the thermal equilibrium curve and which, according to the Passot & Vázquez-Semadeni (1998) theory, should exhibit a power-law PDF. The dotted line in Figure 4 corresponds to the slope of the log $P/k$ versus log $n$ distribution computed for densities larger than 7.1 cm$^{-3}$ and plotted as a function of the average local Mach number for dense gas $M_c$. This slope shows a decrease with $M_c$ from $\sim 0.53$ for $\langle M \rangle = 0.28$ ($M_c = 1.07$) to $\sim 0$ for $\langle M \rangle = 6.23$ ($M_c = 16.91$). This decrease is due to the increased dispersion at the lower density values of high-density gas, and implies that the polytropic description of the dense gas becomes less appropriate for large $M_c$ values.

3.2. The Column Density Distribution

As far as we know, the column density distribution resulting from turbulent thermally unstable simulations has not been...
systematically studied. It is thus interesting to study the $\Sigma$–PDF associated with the $n$–PDFs discussed in previous section.

In Figure 6, we show the column density distribution resulting from our set of simulations. The high-density part of the distribution can be well described by a lognormal (dotted lines) for all of the values of $\langle M \rangle$ that we consider, even for the two smaller values (0.28 and 0.73) for which the volume density PDF has a behavior approaching a power law. For these two values, the bimodal nature of the $n$–PDF is preserved in the $\Sigma$–PDF. For larger values of $\langle M \rangle$, the distribution becomes single peaked and the lognormal also fits an important fraction of the low-density part. The widths of the lognormals, plotted as a function of the Mach number in Figure 7 (solid line), are systematically larger than the corresponding values found by Burkhart & Lazarian (2012) for the isothermal case (dotted line), but they follow a qualitatively similar behavior with $M$. In fact, this behavior can be fit by a function of form (2) with $A_{\Sigma} = 0.084$ and $b_{\Sigma} = 12.5$ (dashed red line), obtaining an error in the fit of 2.17%. This set of parameters is, however, not unique. As an example, we also display the curve for $A_{\Sigma} = 0.081$ and $b_{\Sigma} = 14.29$ (dashed blue line), which has an error of 2.22%. Considering the fact that errors in fitting lognormals are greater than the difference between the errors resulting from the two previous fits, we can consider them as being equivalent. Note that in Figure 6, the Mach number is the rms value at the mean temperature $M_{\text{rms}}$. We choose this value because the average local Mach is dominated by the warm gas ($T > 6100$ K), whose volume fraction is between $\sim 30\%$, for very turbulent simulations, and $\sim 80\%$, for the smaller value of $M$, which is very large when compared with that of the cold gas ($T < 310$ K), which goes from $\sim 2\%$ to 8%.

4. DISCUSSION

4.1. Diffuse Gas

The approximate lognormal shape of the diffuse gas density PDF is consistent with the observations reported by Berkhuijsen & Fletcher (2008) for galactic low-density neutral gas. However, the fact that the lognormal fit fails at very low densities is also consistent with predictions by Passot & Vázquez-Semadeni (1998) for polytropic flows with $\gamma < 1$, according to which the PDF is expected to decrease faster than a lognormal at low densities.

The behavior of $\sigma_n$ with $M$, which grows faster than in the isothermal case, shows the effects of the presence of TI. In fact, as discussed in Vázquez-Semadeni et al. (2003), the increase of $M$ has two effects on the development of TI. First, as the turbulent crossing time decreases, it becomes shorter than the growth time for linear TI, which is the time for the gas to form condensations, increasing the fraction of unstable gas (e.g., Gazol et al. 2001) and producing a larger drift from the thermal equilibrium. On the other hand, the presence of velocity fluctuations generates adiabatic density perturbations that are linearly unstable only when the cooling time is shorter than the dynamical time (which for the supersonic case is the turbulent crossing time), i.e., only at large scales. However, in the nonlinear regime, reached as $M$ increases, due to the local increase of density in forcing generated compressions, the cooling time can locally decrease, implying that even initially small-scale fluctuations can become thermally unstable. The consequence of this nonlinear development of TI is an enhancement of the density contrast that could lead the width of the density PDF to grow faster with $M$ than in the isothermal case.

4.2. The Dense Gas

The results we obtained for the high-density gas reconcile the theoretical prediction from Passot & Vázquez-Semadeni (1998) with previous numerical results for thermally bistable flows (Audit & Hennebelle 2010). Following the former, for a cooling function adapted to describe the dense atomic gas, with an effective polytropic index $\gamma < 1$ in thermal equilibrium, the density PDF is expected to develop a power law at high values. On the other hand, Audit & Hennebelle (2010) report that the dense gas density PDF resulting from thermally bistable colliding flows seems to be better described by a lognormal than by a power law. From results presented in Section 3.1, it is clear that the dense gas PDF can behave as a power law or as a lognormal depending on the $M$ value, and more specifically on the amount of dense gas out of thermal equilibrium. The transition between these behaviors could be due to the fact that the higher is the amount of gas out of thermal equilibrium, the lesser is an adequate polytropic equation of state as a thermodynamic description of the gas. In fact, Passot & Vázquez-Semadeni (1998) suggest that the development of a power law for $\gamma \neq 1$ is the consequence of density jumps depending on the local density. In our simulations, the dense gas drifting away from thermal equilibrium is in fact a consequence of the density fluctuations being determined by velocity fluctuations. A remarkable
The difference between the power laws we obtain at high densities for low \( M \) simulations and the predictions from Passot & Vázquez-Semadeni (1998) are the logarithmic slope behavior. We get slopes of \(-4.43\) and \(-2.41\) for \( \langle M \rangle = 0.28 \) and \( \langle M \rangle = 0.73 \), respectively. This implies that the power laws resulting from our simulations are steeper than the power law they predict for \( \gamma = 0.5 \), which is expected to have a logarithmic slope of \(-1.2\). Note, however, that this value corresponds to the large \( M \) limit.

The development of a lognormal at high densities in high \( M \) simulations suggests that in some cases, the use of this kind of distribution as an initial condition in the molecular cloud formation process represents an adequate choice. Nevertheless, for our simulations, the relationship \( \sigma_{\Sigma} - M \) is very different from Equation (3), implying that in early times during the cloud formation process, the density distribution cannot be related with the dynamical state of the gas through the isothermal version of this equation. In particular, for a given Mach number, we find a narrower distribution. It is important to note that the presence of self-gravity, which is not included in our models, could have noticeable effects on the distribution of dense gas even in this early phase. This problem is going to be addressed in a future work.

4.3. The Column Density

The dense branch of the column density PDF is well described by a lognormal regardless of the \( M \) value. This is partially consistent with recent observational and numerical works suggesting that in clouds where the effects of gravity are not dominant in determining the cloud structure, the column density PDF has a lognormal shape (Kainulainen et al. 2011; Ballesteros-Paredes et al. 2011). In those works, however, turbulence is invoked as the main agent in shaping the column density PDF. From our results, it is possible to suggest that even when low levels of turbulence are present, the cooling properties of the gas could produce a lognormal column density PDF. In fact, Heitsch et al. (2008) investigate the expected timescales of the TI and...
dynamical instabilities leading to the rapid fragmentation of gas swept up by large-scale flows and compare them with global gravitational collapse timescales. They identify parameter regimes in gas density, temperature, and spatial scale within which a given instability dominates, finding that the thermally dominated parameter regime has a remarkably large extent due to the fact that outside the strictly thermally unstable regions, cooling could still be the dominant agent leading to fragmentation in the presence of an external (in this case ram) pressure.

On the other hand, we find that the width of the column density distribution is much larger than in the isothermal case studied by other authors for MHD flows (Kowal et al. 2007; Burkhart & Lazarian 2012). This is a natural consequence of the large density dynamical range produced by the presence of TI. The resulting $\sigma_{\ln(\Sigma/\Sigma_0)} - M$ relationship has to include the fact that the density contrast is large even for low Mach numbers, requiring a very rapid growth that implies parameter values completely different from the ones reported in the isothermal case, namely, $A_\Sigma = 0.11$ and $b_\Sigma = 1/3$.

4.4. Numerical Issues

Several numerical factors can affect in some way the results obtained in the present work.

4.4.1. Resolution

We have performed some higher resolution simulations in order to see the effects on the density PDF. For similar Mach numbers, we find that the main consequence of increasing the resolution to 1024$^3$ seems to be a deviation of the high-density tail in moderate Mach simulations. In particular, for $(M) \sim 1.9$, the distribution falls faster than a lognormal for high $n$. Unfortunately, we do not have enough snapshots to quantify this effect and to measure $\sigma$ in high-resolution simulations. Although our PDFs could not be fully converged and higher resolution simulations could potentially lead to results quantitatively different from those presented in previous sections, the fact that the diffuse gas PDF can be well described by a lognormal and that the dense gas PDF is well described by this kind of distribution only for large enough values of $M$ does not seem to change with resolution.

4.4.2. Model

Even if our model allows both the study of diffuse gas distribution and a direct comparison with isothermal numerical experiments reported by other authors, there are some choices that can potentially affect our results. The periodic boundary conditions maintain a fixed amount of gas in the box and this fact can artificially regulate the gas segregation. Also, the particular choice of the cooling function can affect the gas segregation. This function could change because of physical reasons, such as abundance variations, heating rate variations, or an additional cooling process. Finally, additional physics such as the presence of self-gravity and magnetic fields could also modify the quantitative behavior of the density distribution.

4.4.3. Forcing

Federrath et al. (2008) and Federrath et al. (2010) found that for isothermal gas, the width of the PDF depends not only on the rms Mach number but also on the relative degree of compressible and solenoidal modes in the turbulence forcing, with $b = 1/3$ appropriate for purely solenoidal and $b = 1$ for purely compressive forcing. For the thermally bistable case, the effects of forcing could be more complex due to the interplay between TI and turbulence. Although simulations with different kinds of forcing have been presented by Seifried et al. (2011), the energy transfer between the solenoidal and the compressive modes for turbulent bistable flows have not been addressed. As shown in Gazol & Kim (2010) for purely solenoidal forcing, the presence of TI does significantly affect the density as well as the velocity power spectrum. In fact, it is well known that the development of TI produces turbulent motions with typical velocities of the order of tenths of km s$^{-1}$ (Kritsuk & Norman 2002; Piontek & Ostriker 2004) but, as stated earlier, the presence of turbulence does in turn modify the development of TI. A detailed analysis of the effects of forcing, also comparing the resulting velocity power spectrum and the density-weighted velocity power spectrum for the compressible as well as for the solenoidal modes, should be done in order to address this problem, but it is out of the scope of the present work.

5. SUMMARY AND CONCLUSIONS

In the present work, we have presented numerical experiments showing that:

1. At low densities, the volume density PDF resulting from turbulent thermally bistable flows can be well described by a lognormal distribution whose width increases with the Mach number.
2. A relationship between the width of the distribution and the average local Mach number can be found, however, this relationship is not the same as in the isothermal case. The value of the parameter $b$, included in the isothermal case, is, for our simulations, surprisingly close to the one obtained for purely solenoidal forcing in isothermal gas, but the nonlinear development of TI, producing a more efficient rarefaction of gas, causes a faster growth of the distribution width as $M$ is increased. The consequence of this enhanced growth in the mathematical form of the $\sigma_{\ln(\Sigma/\Sigma_0)} - M$ relation is the presence of a scale factor distinct from 1.
3. At high densities, the form of the volume density PDF depends on the value of the Mach number. For simulations

Figure 7. Lognormal widths as a function of $M_{\text{rms}}$ for fits corresponding to column density distributions displayed in Figure 6. In this plot Mach number values are $\sigma_{\ln(\Sigma/\Sigma_0)}$ at the mean temperature. The dotted black line plots $A_{\Sigma} = 0.11, b_{\Sigma} = 1/3$, whereas the dashed red and dashed-dotted blue lines correspond to the fits described in the text. Mach number values are the rms values at the mean temperature $M_{\text{rms}}$.

(A color version of this figure is available in the online journal.)
with transonic or weakly supersonic average velocities in dense gas, the distribution is a power law, while in the presence of highly supersonic velocities the distribution becomes lognormal.

4. For simulations that develop a high-density distribution with a lognormal shape, the width of the distribution is smaller than in the isothermal case and grows slower with the Mach number.

5. At high densities, the column density PDF resulting from our simulations can be described by a lognormal for all of the Mach numbers we consider. As $M$ increases, the density range where the lognormal fit is adapted expands and the lognormal becomes wider.

6. The width of the column density distribution resulting from our simulations is systematically larger than the width obtained in the isothermal case. A relationship between the width of the column density distribution and the rms Mach number at the mean temperature can be found. This relationship has the same form as the one reported in the literature for the isothermal case, but the parameters resulting from our fit are very different.

From these results, it is clear that when studying the diffuse and/or the dense atomic interstellar gas in order to relate the density structure, and in particular the width of its PDFs, with the dynamical state of the gas, characterized by the Mach number, the use of results obtained from isothermal turbulent flows is not an adequate choice. Specific relations between $\sigma_\tau (\Sigma_{mc}, \Sigma_{\nu\sigma})$ and $M$ for thermally bistable flows should be taken into account in any observational or theoretical work using the density (column density) PDF as a measure of the Mach number.

The relationships obtained in the present work could be affected by the inclusion of additional physics such as self-gravity, magnetic fields, or variations in the cooling function due to variations of the heating rate and the gas abundances.

The work of A.G. was partially supported by UNAM-DGAPA grant IN106511. The work of J.K. and A.G. was partially supported by the International Research & Development Program (RADIONET3 grant No. 2012049606) of the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (MEST) of Korea. The numerical simulations were performed at the cluster Platform 4000 (KanBalam) at DGSCA, UNAM and at the high performance computing cluster in the Korean Astronomy and Space Science Institute.

REFERENCES

Audit, E., & Hennebelle, P. 2010, A&A, 115, 76
Ballesteros-Paredes, J., Vázquez-Semadeni, E., Gazol, A., et al. 2011, MNRAS, 416, 1436
Beetz, C., Schwarz, C., Dreher, J., & Grauer, R. 2008, PhLA, 372, 3037
Berkhuijsen, E. M., & Fletcher, A. 2008, MNRAS, 390L, 19
Boldyrev, S., Nordlund, A., & Padoan, P. 2002, PhRvL, 89, 031102
Burkhart, B., & Lazarian, A. 2012, ApJL, 755, L19
de Avillez, M. A., & Breitschwerdt, D. 2005, A&A, 436, 585
Diplas, A., & Savage, B. D. 1994, ApJS, 93, 211
Elmegreen, B. G. 2002, ApJ, 577, 206
Elmegreen, B. G. 2008, ApJ, 672, 1006
Elmegreen, B. G. 2011, ApJ, 731, 61
Federrath, C., Klessen, R. S., & Schmidt, W. 2008, ApJL, 688, L79
Federrath, C., Roman-Duval, J., Klessen, R. S., Schmidt, W., & Mac Low, M.-M. 2010, A&A, 512, A81
Field, G. B. 1965, ApJ, 142, 531
Gazol, A., & Kim, J. 2010, ApJ, 723, 482
Gazol, A., Vázquez-Semadeni, E., & Kim, J. 2005, ApJ, 630, 911
Gazol, A., Vázquez-Semadeni, E., Sánchez-Salcedo, F. J., & Scalo, J. 2001, ApJL, 557, L121
Harten, A., Lax, P. D., & van Leer, B. 1983, SIAMR, 25, 35
Heitsch, F., & Hartmann, L. 2008, ApJ, 689, 290
Heitsch, F., Hartmann, L., & Burkert, A. 2008, ApJ, 683, 786
Hennebelle, P., & Chabrier, G. 2008, ApJ, 684, 395
Hennebelle, P., & Pénaud, M. 1999, A&A, 351, 309
Joung, M. K. R., & McKee, C. F. 2005, ApJ, 653, 1266
Kainulainen, J., Beuther, H., Banerjee, R., Federrath, C., & Henning, T. 2011, A&A, 530, A64
Kainulainen, J., Beuther, H., Henning, T., & Plume, R. 2009, A&A, 508, L35
Klessen, R. S. 2000, ApJ, 555, 869
Kowal, G., Lazarian, A., & Beresnyak, A. 2007, ApJ, 658, 423
Kritsuk, A., & Norman, M. L. 2002, ApJL, 569, L127
Kritsuk, A. G., Norman, M. L., Padoan, P., & Wagner, R. 2007, ApJ, 665, 416
Krumholz, M. R., & McKee, C. F. 2005, ApJ, 630, 250
Lemaster, M. N., & Stone, J. M. 2008, ApJ, 682, 97
Li, Y., Klessen, R. S., & Mac Low, M. M. 2003, ApJ, 592, 975
Nordlund, A. A., & Padoan, P. 1999, in Interstellar Turbulence, Proc. of the 2nd Guillermo Haro Conference, ed. J. Franco & A. Carramiñana (Cambridge: Cambridge Univ. Press), 218
Ostriker, E. C., Stone, J. M., & Gammie, C. F. 2001, ApJ, 546, 980
Padoan, P., & Nordlund, A. 2002, ApJ, 576, 870
Padoan, P., & Nordlund, A. 2011, ApJ, 730, 40
Padoan, P., Nordlund, A., & Jones, B. J. T. 1997, MNRAS, 288, 145
Passot, T., & Vázquez-Semadeni, E. 1998, PhRvE, 58, 4501
Piontek, R. A., & Ostriker, E. C. 2004, ApJ, 601, 905
Price, D. J., Federrath, C., & Brunt, C. M. 2011, ApJL, 727, L21
Robertson, B. E., & Kravtsov, A. V. 2008, ApJ, 680, 1083
Sánchez-Salcedo, F. J., Vázquez-Semadeni, E., & Gazol, A. 2002, ApJL, 577, 768
Seifried, D., Schmidt, W., & Niemeyer, J. C. 2011, A&A, 506, A14
Stone, J. M., Ostriker, E. C., & Gammie, C. F. 1998, ApJL, 508, L99
Toro, E. F. 1999, Riemann Solvers and Numerical Methods for Fluid Dynamics: A Practical Introduction (Berlin: Springer)
Vázquez-Semadeni, E., García, N. 2001, ApJL, 555, 727
Vázquez-Semadeni, E., Gazol, A., Sánchez-Salcedo, F. J., & Passot, T. 2003, in Lecture Notes in Physics, Vol. 614, Turbulence and Magnetic Fields in Astrophysics, ed. T. Passot & E. Falgarone (Berlin: Springer-Verlag), 213
Vázquez-Semadeni, E., Ryu, D., Passot, T., González, R. F., & Gazol, A. 2006, ApJ, 643, 245
Wada, K., & Norman, C. A. 2007, ApJ, 660, 276
Wolff, M. C., Hollenbach, D., McKee, C. F., Tielens, A. G. G. M., & Bakes, E. L. O. 1995, ApJ, 443, 152
Zamora-Avilés, M., Vázquez-Semadeni, E., & Colin, P. 2012, ApJ, 751, 77