Velocity bias in the distribution of dark matter halos

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The standard formalism for the co-evolution of halos and dark matter predicts that any initial halo velocity bias rapidly decays to zero. We argue that, when the purpose is to compute statistics like power spectra etc., one should interpret the two-fluids approximation as describing effective/mean-field quantities and, therefore, momentum conservation must be modified accordingly. We will explain how the Euler equation should be modified, and demonstrate that the new two-fluids approximation predicts the (time) constancy of any statistical halo velocity bias present in the initial conditions. We test this prediction by studying the evolution of a conserved halo population in N-body simulations. We establish that the initial simulated halo density and velocity statistics show distinct features of the peak model and, thus, deviate from the simple local Lagrangian bias. We demonstrate, for the first time, that the time evolution of their velocity is in tension with the rapid decay expected in the standard approach.

The three dimensional late time matter distribution in the Universe has the potential to place stringent constraints on cosmological parameters and fundamental physics. The interpretation of the observed galaxy distribution is, however, hampered by the fact, that galaxies and their host halos are imperfect tracers of the matter distribution. The local bias model has been successful in explaining the varying clustering amplitude observed for different tracers of the large scale structure [1]. With data from current and upcoming surveys, one would like to push the maximum wavenumber in the analysis into the weakly non-linear regime, requiring consistent bias descriptions that go beyond the simple local bias model. In this regard, it was recently demonstrated that the local density bias is not even consistent with gravitational evolution. Namely, tracers which are initially locally biased will always develop nonlocal bias contributions. [2, 3].

In this letter, we focus on a different extension of the local bias model known as the peak approach [4], which has its roots in the initial distribution of the regions that will eventually form a dark matter halo. The peak model predicts the existence of a linear, statistical halo velocity bias which remains constant with time [5, 6]. If true, this would have important consequences for our description of redshift space statistics such as the power spectrum [7]. However, there is thus far no evidence for such an effect from N-body simulations. Furthermore, the coupled-fluids approximation for the coevolution of dark matter and halos, which is widely used to compute the time evolution of bias [8, 9], predicts that any initial velocity bias rapidly decays to zero [10]. This stands in conflict with predictions from the peak approach.

Here, we will show that these two seemingly contradictory results can be reconciled if one recognizes that the gravitational force acting on biased tracers of the large scale structure is itself biased. Consequently, when the purpose is to compute correlators like power spectra etc., one should interpret the two-fluids approximation as describing effective/mean-field quantities and, therefore, momentum conservation must be modified accordingly. We will explain how the Euler equation should be modified, and demonstrate that the new two-fluids approximation predicts the (time) constancy of any statistical halo velocity bias present in the initial conditions. We test this prediction by studying the evolution of a conserved halo population in N-body simulations. We establish that the initial simulated halo density and velocity statistics show distinct features of the peak model and, thus, deviate from the simple local Lagrangian bias. We demonstrate, for the first time, that the time evolution of their velocity is in tension with the rapid decay expected in the standard approach.
where the superscript $E$ labels evolved quantities and $D_+(a)$ is the linear growth rate normalized to unity at the collapse redshift. The linear Lagrangian bias $c_1(k)$ decays to the linear velocity bias, which remains constant throughout time.

This prediction stands in sharp contrast with that of the coupled fluid approximation to halos and dark matter. In this approach, halos and matter are modelled as pressureless fluids co-evolving in the potential determined by the matter distribution solely. Upon linearizing the continuity and momentum conservation equations for the Fourier modes of the halo and dark matter density fields, $\delta_h(k)$ and $\delta_m(k)$, and velocity divergences, $\theta_h(k)$ and $\theta_m(k)$, the time evolution of any initial $c_1(k)$ and $b_v(k)$ is

$$c_1^E(k, a) = 1 + D_+^{-1}(a)(c_1(k) + 2b_v(k) - 3) + 2D_+^{-3/2}(a)(1 - b_v(k)) \ldots$$

$$b_v^E(k, a) = 1 + D_+^{-3/2}(a)(b_v(k) - 1).$$

In the absence of an initial velocity bias, $b_v^E = b_v = 1$ and we recover the usual decay of the linear Lagrangian bias, $c_1^E = 1 + D_+^{-1}(c_1 - 1)$ [8]. In general, the above equations predict the rapid decay of any non-vanishing initial velocity bias.

Can we reconcile these two apparently contradictory results? Firstly, we should bear in mind that this fluid approximation aims at predicting correlators of halos and dark matter. This is the reason why we are allowed to relate $\delta_h$ with $\delta_m$ through some (local or nonlocal) bias expansion. In other words, $\delta_h$ and $\delta_h$ should in fact be thought of as being effective or mean-field quantities given a realization of $\delta_m$, in analogy with the interpretation of the peak bias expansion described in [22]. Consequently, the Euler equation for halos describes the conservation of momentum of the halo mean-field density field $\delta_h(\delta_m)$. The halo momentum conservation equation will be the same as that of the dark matter only if the gravitational force acting on halos is statistically unbiased relative to the force acting on dark matter particles.

In general, we argue that the Euler equation for halos should be changed into

$$\frac{\partial \theta_h(k)}{\partial \eta} + H\theta_h(k) + \frac{3}{2} b_a(k) H^2 \Omega_m \delta_m(k) = \text{m.c.}$$

Here, $\eta$ is the conformal time, “m.c.” designates second-order mode-coupling terms and we have omitted the dependence of $\theta$ and $\delta$ on $\eta$. $b_a(k)$ is the linear gravitational force or acceleration bias. It is important to note that our modification to the Euler equation does not contradict the equivalence principle since it only makes sense statistically: on an object-by-object basis, the biased tracers still satisfy the usual momentum conservation.

Since, in the linear regime, the acceleration is parallel to the initial velocity and since gravity mode-coupling cannot induce any linear contribution (by definition), $b_a(k)$ must be equal to the linear, initial velocity bias $b_v(k)$. Solving the fluid equations for halos and dark matter with this new halo momentum conservation equation, we find that $c_1^E(k, a)$ and $b_v^E(k, a)$ evolve in accordance with the peak predictions Eqs. (3) – (4). To convince ourselves that this must be true, consider the set of points (top to bottom) in the upper (lower) panel. The horizontal dashed lines show the scale-independent local bias, while the solid lines show the peak model fits. Halo mass is in the range $8 \times 10^{12} - 6 \times 10^{14} h^{-1} M_\odot$ and increases from bottom to top (top to bottom) in the upper (lower) panel.

![FIG. 1. Initial peak density bias $c_1(k)$ (upper panel) and velocity bias $b_v(k)$ (lower panel) at $z_i = 99$. Having fit $b_{10}$, $b_{01}$ and $R$ from the density bias, we find irrefutable evidence for a non-zero $R_v$ in the velocity bias. To highlight this detection, we overplot the damping introduced by the peak smoothing $R$ alone in the lower panel as dash-dotted lines. The horizontal dashed lines show the scale-independent local bias, while the solid lines show the peak model fits. Halo mass is in the range $8 \times 10^{12} - 6 \times 10^{14} h^{-1} M_\odot$ and increases from bottom to top (top to bottom) in the upper (lower) panel.](image-url)
So far, there has not been any conclusive evidence for a statistical velocity bias in the distribution of virialized dark matter halos, except for a couple tentative measurements from [13] [14]. These suffered from the fact that simulations are sampling the cosmic density field with a finite number of discrete tracer particles. Hence, it is difficult to define a velocity field throughout the whole simulation volume, especially for the rare, massive dark matter halos. Here, we work instead with the number-weighted (for halos) and density-weighted (for dark matter) velocity fields [15]. These “momentum” fields \( j_h = (1 + \delta_h)v_h \) and \( j_m = (1 + \delta_m)v_m \) are well defined everywhere. To extract statistical information about them, we will measure the density-momentum correlators \( \langle \delta_{m,i}^{\ast}j_{m,h}^{\ast} \rangle \), where \( j_{m,h}^{\ast} \) is the matter or halo momentum projected along the z-axis. Since it is a cross-correlation with \( \delta_m \), it does not suffer from shot-noise, like the halo-matter cross power spectrum \( \langle \delta_m\delta_h \rangle \). We begin by assessing whether both Lagrangian \( c_1(k) \) and \( b_c(k) \) have \( k^2 \)-dependencies in agreement with that predicted by peak theory. In this regard, we consider a suite of 16 collisionless dark matter simulations. The initial conditions for the 1024\(^3 \) particles in the \( V = 1500^3 h^{-3}\text{Mpc}^3 \) box were set up at \( z = 99 \) using 2LPT [16] and subsequently evolved using GADGET2 [17]. Halos are identified at \( z = 0 \) with a FoF halo finder of linking length 0.2 and their constituent particles were traced back to the initial conditions to define the Lagrangian halo distribution (or proto-halos). We repeat this procedure for a set of intermediate time steps. This provides us with the non-linear time evolution of a strictly conserved set of tracers of large scale structure. The mean mass of the five bins is \( 7.8 \times 10^{12}, 2.3 \times 10^{13}, 6.9 \times 10^{13}, 2.0 \times 10^{14} \) and \( 5.7 \times 10^{14} \) \( h^{-1}\text{M}_\odot \).

We then measure the ratio of the initial halo-matter and matter-matter density-density and density-momentum power spectra. In the peak model, at the lowest order, these quantities are given by

\[
\frac{\langle \delta_{m,i}(k)\delta_{h,i}(-k) \rangle}{\langle \delta_{m,i}(k)\delta_{m,i}(-k) \rangle} = (b_{10} + b_{01}k^2)W_R(k) \\
\frac{\langle \delta_{m,i}(k)j_{m,i}^\ast(-k) \rangle}{\langle \delta_{m,i}(k)j_{m,i}^\ast(-k) \rangle} = (1 - R^2_0k^2)W_R(k) .
\] (8) (9)

In a first step we fit the linear density bias from the density correlator on large scales, then we jointly fit for the scale of the Gaussian peak selection function \( R \) and the \( k^2 \) bias term \( b_{01} \) in the same statistic. Subsequently we use the filter with the same scale to fit the scale-dependence of the velocity bias and find strong evidence for a non-zero initial \( R_\infty \), Fig. 1 shows that this parametrization is able to reproduce the scale-dependence of the proto-halo density and velocity bias reasonably well. The choice of a Gaussian for the peak selection function is motivated by the sole requirement that the spectral moments of the Gaussian field should be convergent. This would not be the case for a top-hat window, but generalized window functions might provide a better fit and still yield convergent moments.

Next, we turn to the time-dependence of the halo velocity bias. To this purpose, we consider the time evolution of the linear density-density and density-momentum correlators. These linear correlators are obtained by cross-correlating the evolved halo positions and momenta with the linear Gaussian matter density field \( \delta_m^{(1)} = D_\Delta(z)D_m(z)\delta_m(z) \). Considering cross-correlations with the non-linear matter field would contaminate the statistics with the poorly understood late time matter distribution and, thus, undermine a clear isolation of the scale-dependencies induced by the peak constraint. We also refrain from using halo-halo correlators, since these are likely plagued by non-trivial shot noise effects [18].

We assume that peaks move according to their initial velocity as in Zeldovich approximation. We calculate the resulting correlators by writing the evolved peak positions as \( 1 + \delta_h(x) = \bar{n}_h^{-1} \sum_{i \neq h} \delta^{(1)}(x - x_i) = \bar{n}_h^{-1} \int d^3q \delta^{(1)}(x - q - \Psi(q)) \sum_{i \neq h} \delta^{(1)}(q - q_{pk}) \) following the steps laid out in [6], where \( \Psi(q) \) is the displacement field at Lagrangian position \( q \). Since the initial matter fluctuations are Gaussian, we only select the linear terms in the bias relation. We finally obtain

\[
\langle \delta_{m,1}^{(1)}(k)\delta_{h,1}(-k) \rangle = D_x^2c_{E}^2(k,a)G_{pk}(k)P(k)W_R(k) ,
\]

\[
\langle \delta_{m,1}^{(1)}(k)j_{m,1}^\ast(-k) \rangle = \left( b_c(k) - D_x^2\sigma_{pk}^2c_{E}^2(k,a)k^2 \right)
\]

\[
\times H_{\perp}D_x^2 \left( \frac{k \cdot \hat{z}}{R^2} \right) G_{pk}(k)P(k)W_R(k) ,
\]

where \( G_{pk}(k) = e^{-1/\sigma_{pk}^2k^2D_x^2} \) is the peak propagator and \( \sigma_{pk}^2 \) is the peak displacement dispersion (extrapolated to the collapse epoch), given by \( \sigma_{pk}^2 = \sigma_\perp^2 - \sigma_\parallel^2/\sigma_\perp^2 \) with \( \sigma_\perp^2 = 1/3 \int d^3k/(2\pi)^3 k^2P(k)W^2(k) \). It is reduced relative to the linear matter displacement dispersion because i) \( \sigma_\perp \) is smaller for halos than for matter due to the finite smoothing scale \( R \) (which is zero for the matter), and ii) the dark matter preferentially flows onto the peaks, so statistically the peaks are more at rest than the dark matter, and the term \(-\sigma_\parallel^2/\sigma_\perp^2 \) accounts for that. The analogy with the Eulerian coevolution model in Eqs. 3 - 4 can only be seen in the low-\( k \) limit due to the resummation of the displacement dispersions. In principle third order bias parameters contribute to the density correlator at late times. However, we refrain from considering these loop contributions since they are of percent level and hardly distinguishable from the above model [19] [20].

The evolution of the density and velocity bias in our simulations for the mass bin II is shown in Fig. 2. We divide the cross-correlators of halo density/momentum with the linear density/momentum by those of the linear matter density/momentum with the linear matter density. On large scales, the density bias evolves from the La-
the level of linear bias, the Zel’dovich evolved peaks provide a very good description of the time evolution of the halo-matter correlators. While the interior structure of a dark matter halo is highly non-linear, the effective dynamical evolution of its center of mass is more amenable to perturbative treatments than the dark matter itself. Finally, we insist on the interpretation of \( \delta_h, \theta_h \) in the halo-matter fluid approximation as mean-field/effective quantities (and not counts-in-cells), as long as the purpose is to compute ensemble averages.

Our measurement of a halo velocity bias constant throughout time may have important consequences for the modeling of halo clustering in redshift space, see e.g. [5]. However, further developments are required until peak theory can provide an accurate template for the halo-halo correlation function. Since the full, non-perturbative initial peak-peak correlation function correctly accounts for exclusion effects [21], it should lead to a comprehensive description of all the scale-dependent bias effects. On a final note, the effective field theory descriptions of halo statistics [22, 23] may lead to a functional form similar to that predicted by the peak model, but the model presented in this letter complements it by a dynamical perspective and provides physical arguments for the values of the operators.

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**FIG. 2.** Evolution of the linear density and velocity bias for bin II \((M = 2.3 \times 10^{13} h^{-1} M_{\odot})\). \(D_v\) is the velocity growth factor \(D_v = D_s f_s H\). The lines show our evolution model to linear order, Eqs. (10)–(11). The dashed lines assume that the initial velocity bias decays according to the Eulerian co-evolution model, Eq. (5), and are undoubtedly in tension with the simulation data for both statistics even at redshift \(z = 20\). At low redshifts the damping is dominated by the propagator for intermediate wavenumbers, such that this tension becomes less significant for the momentum statistic. The velocity bias is unity on large scales as required by mass and momentum conservation. On smaller scales, we see a sharpening of the dip until redshift \(z = 5\) when it starts being damped by the exponential. We also overplot the prediction of the simple linear co-evolution model Eq. (5). This model is clearly incompatible with the non-linear evolution measured in the data. The other mass bins show a similar behaviour, on which we will report in more detail in a forthcoming paper. We have focused on modeling correlators with the linear, Gaussian density field in order to isolate the effects of the scale-dependent, linear peak bias. These statistics are admittedly not those that will be measured in observations. However, they allow us to understand key properties of the joint evolution of halos and matter under gravity and, therefore, they will help predicting the true observables more accurately and reliably. Our result also shows that, at the level of linear bias, the Zel’dovich evolved peaks provide a very good description of the time evolution of the halo-matter correlators. While the interior structure of a dark matter halo is highly non-linear, the effective dynamical evolution of its center of mass is more amenable to perturbative treatments than the dark matter itself. Finally, we insist on the interpretation of \( \delta_h, \theta_h \) in the halo-matter fluid approximation as mean-field/effective quantities (and not counts-in-cells), as long as the purpose is to compute ensemble averages.

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