Testing the distance scale of the Gaia TGAS catalogue by kinematic method

V.V. Bobylev and A.T. Bajkova

Pulkovo Astronomical Observatory, Russian Academy of Sciences,
Pulkovskoe sh. 65, St. Petersburg, 196140 Russia

Abstract—We have studied the simultaneous and separate solutions of the basic kinematic equations obtained using the stellar velocities calculated on the basis of data from the Gaia TGAS and RAVE5 catalogues. By comparing the values of $\Omega'_0$ found by separately analyzing only the line-of-sight velocities of stars and only their proper motions, we have determined the distance scale correction factor $p$ to be close to unity, $0.97 \pm 0.04$. Based on the proper motions of stars from the Gaia TGAS catalogue with relative trigonometric parallax errors less than 10% (they are at a mean distance of 226 pc), we have found the components of the group velocity vector for the sample stars relative to the Sun $(U, V, W)_\odot = (9.28, 20.35, 7.36) \pm (0.05, 0.07, 0.05)$ km $s^{-1}$, the angular velocity of Galactic rotation $\Omega_0 = 27.24 \pm 0.30$ km $s^{-1}$ kpc$^{-1}$, and its first derivative $\Omega'_0 = -3.77 \pm 0.06$ km $s^{-1}$ kpc$^{-2}$; here, the circular rotation velocity of the Sun around the Galactic center is $V_0 = 218 \pm 6$ km $s^{-1}$ kpc (for the adopted distance $R_0 = 8.0 \pm 0.2$ kpc), while the Oort constants are $A = 15.07 \pm 0.25$ km $s^{-1}$ kpc$^{-1}$ and $B = -12.17 \pm 0.39$ km $s^{-1}$ kpc$^{-1}$, $p = 0.98 \pm 0.08$. The kinematics of Gaia TGAS stars with parallax errors more than 10% has been studied by invoking the distances from a paper by Astraatmadja and Bailer-Jones that were corrected for the Lutz–Kelker bias. We show that the second derivative of the angular velocity of Galactic rotation $\Omega''_0 = 0.864 \pm 0.021$ km $s^{-1}$ kpc$^{-3}$ is well determined from stars at a mean distance of 537 pc. On the whole, we have found that the distances of stars from the Gaia TGAS catalogue calculated using their trigonometric parallaxes do not require any additional correction factor.

DOI: 10.1134/S1063773718020020

INTRODUCTION

The high expectations of specialists in studying the structure and kinematics of the Galaxy are associated with the determination of highly accurate trigonometric parallaxes and proper motions for hundreds of millions of stars whose observations are planned in the Gaia project (Prusti et al. 2016). The first results were already published in 2016. In particular, the proper motions of stars were found by comparing their positions measured from the Gaia satellite with those from the Hipparcos/Tycho (1997) catalogue with an epoch difference of about 24 years. This version is designated as TGAS (Tycho–Gaia Astrometric Solution, Brown et al. 2016; Lindegren et al. 2016) and contains the trigonometric parallaxes and
proper motions of \( \sim 2 \) million stars. The proper motions of \( \sim 90,000 \) stars common to the Hipparcos catalogue were measured with a mean error of \( \sim 0.06 \) mas yr\(^{-1}\), while for the remaining stars this error is \( \sim 1 \) mas yr\(^{-1}\) (Brown et al. 2016).

The mean random measurement error of the trigonometric parallaxes in the TGAS catalogue is \( \sim 0.3 \) mas, to which a systematic component of \( \sim 0.3 \) mas should be added (Brown et al. 2016). Having analyzed the kinematic characteristics of 19 open star clusters, van Leeuwen et al. (2017) concluded that the parallaxes in this catalogue have a good quality. It should be particularly noted that the new distance to the Pleiades, \( 134 \pm 4 \) pc, agrees better with its known reliable determinations, \( 133.7 \pm 1.2 \) pc (An et al. 2007; Kim et al. 2016), in particular, with its VLBI measurements, \( 136.2 \pm 1.2 \) pc (Melis et al. 2014), than with its value in the Hipparcos catalogue, \( 120 \pm 2 \) pc (van Leeuwen 2009). Fortunately, the Pleiades distance problem turned out to be local in the Hipparcos catalogue.

A comparison of the distances to Cepheids and RR Lyr variables from the TGAS catalogue with their distances estimated by other methods (for example, based on the period–luminosity relation or from Hubble Space telescope astrometry) has shown excellent agreement between the results up to distances of \( \sim 2 \) kpc (Casertano et al. 2017; Benedict et al. 2017; Clementini et al. 2017). Based on the kinematic method, Bobylev and Bajkova (2017) showed that the distances to the stars from a sample of OB stars (within \( \sim 3–4 \) kpc of the Sun) do not require the introduction of any correction factors.

However, there are reports of several authors about the detection of a systematic offset in the stellar parallaxes from the TGAS catalogue. In particular, Stassun and Torres (2016) detected such an offset, \(-0.25 \pm 0.05 \) mas, with respect to 158 calibration eclipsing binary stars. Having analyzed nearby stars (within 25 pc), Jao et al. (2017) found that the TGAS parallaxes are, on average, \( 0.24 \pm 0.02 \) mas smaller than the trigonometric parallaxes for these stars measured from the ground. Applying the statistical method revealed local anomalies in the distances when analyzing the space velocities of stars calculated from a combination of RAVE–TGAS and LAMOST–TGAS data (Schönherr and Aumer 2017). The RAVE (RADial Velocity Experiment, Steinmetz et al. 2006) and LAMOST (Large sky Area Multi-Object Fiber Spectroscopic Telescope, Luo et al. 2015) catalogues contain the results of large-scale line-of-sight velocity measurements for stars predominantly in the southern and northern hemispheres of the celestial sphere, respectively. Thus, we see that the properties of the TGAS distance scale have not yet been completely studied and, therefore, an analysis of this distance scale with the application of independent approaches is topical.

The goal of this paper is to study the TGAS stellar velocity field based on the separate solutions of the basic kinematic equations from the line-of-sight velocities and proper motions of stars. Since the line-of-sight velocities and proper motions of stars are determined by fundamentally different methods, our separate solutions of the basic kinematic equations allow the consistency between these data to be investigated from a kinematic viewpoint.

**METHODS**

We know three stellar velocity components from observations: the line-of-sight velocity \( V_r \) and the two tangential velocity components \( V_l = 4.74r\mu_l \cos b \) and \( V_b = 4.74r\mu_b \) along the Galactic longitude \( l \) and latitude \( b \), respectively, expressed in km s\(^{-1}\). Here, the coefficient 4.74 is the ratio of the number of kilometers in an astronomical unit to the number of seconds in a tropical year, and \( r \) is the stellar heliocentric distance in kpc. The proper motion components \( \mu_l \cos b \) and \( \mu_b \) are expressed in mas yr\(^{-1}\).
To determine the parameters of the Galactic rotation curve, we use the equations derived from Bottlinger’s formulas, in which the angular velocity \( \Omega_0 \) is expanded into a series to terms of the second order of smallness in \( r/R_0 \):

\[
V_r = -U_\odot \cos b \cos l - V_\odot \cos b \sin l - W_\odot \sin b + R_0 (R - R_0) \sin l \cos b \Omega_0' + 0.5 R_0 (R - R_0)^2 \sin l \cos b \Omega_0'' ,
\]

\[
V_l = U_\odot \sin l - V_\odot \cos l - r \Omega_0 \cos b + (R - R_0) (R_0 \cos l - r \cos b) \Omega_0' + 0.5 (R - R_0)^2 (R_0 \cos l - r \cos b) \Omega_0'' ,
\]

\[
V_b = U_\odot \cos l \sin b + V_\odot \sin l \sin b - W_\odot \cos b - R_0 (R - R_0) \sin l \sin b \Omega_0' - 0.5 R_0 (R - R_0)^2 \sin l \sin b \Omega_0'' ,
\]

Here, \( R \) is the distance from the star to the Galactic rotation axis:

\[
R^2 = r^2 \cos^2 b - 2 R_0 r \cos b \cos l + R_0^2 .
\]

The quantity \( \Omega_0 \) is the angular velocity of Galactic rotation at the solar distance \( R_0 \), \( \Omega_0' \) is its first derivative, \( V_0 = |R_0 \Omega_0| \); the Oort constants \( A \) and \( B \) can be found from the expressions

\[
A = -0.5 \Omega_0' R_0, \quad B = -\Omega_0 + A ,
\]

written in such a way that the following relations hold: \( A - B = \Omega_0 \) and \( A + B = - (\Omega_0 + \Omega_0' R_0) \). In this paper we adopt \( R_0 = 8.0 \pm 0.2 \) kpc that Vallée (2017) found in his recent review as the most probable value.

When simultaneously solving the system of conditional equations (1)–(3) by the least-squares method, we find and six unknowns \( U_\odot, V_\odot, W_\odot, \Omega_0, \Omega_0', \Omega_0'' \). We can find five unknowns \( U_\odot, V_\odot, W_\odot, \Omega_0', \Omega_0'' \) from Eq. (1) by analyzing only the line-of-sight velocities. When analyzing only the proper motions, it is convenient to solve the system of equations (2)–(3), which allows all six unknowns to be determined.

**DATA**

Our sample includes stars from the RAVE5 catalogue (Kunder et al. 2017) with measured line-of-sight velocities and with estimated trigonometric parallaxes and proper motions from the Gaia DR1 catalogue. Note that the RAVE5 catalogue contains more than 60 000 stars for which the line-of-sight velocities were measured several times. Therefore, we ultimately produced a sample in which each star is presented once. In the case where a star has several line-of-sight measurements, we did not perform any averaging but took the measurement with the smallest line-of-sight velocity error. This sample includes a total of \( \sim 200 \) 000 stars.

There are stars with large line-of-sight velocities \( |V_r| > 600 \) km \( s^{-1} \) in the RAVE5 catalogue. Such values were typically obtained from low-quality spectra, with a small signal-to-noise ratio. Therefore, we do not use the stars with such velocities, nor do we use the stars with large random errors in the line-of-sight velocity \( \sigma_{V_r} \). As a result, for the selection of candidates without significant random observational errors we took the stars that satisfied the following criteria:

\[
|V_r| < 600 \text{ km s}^{-1} , \quad \sigma_{V_r} < 5 \text{ km s}^{-1} , \quad |\mu_\alpha \cos \delta| < 400 \text{ mas yr}^{-1} , \quad |\mu_\delta| < 400 \text{ mas yr}^{-1} , \quad \sqrt{U^2 + V^2 + W^2} < 200 \text{ km s}^{-1} ,
\]

where the velocities \( U, V, \) and \( W \) were freed from the Galactic differential rotation, i.e., they are the residual ones. Any known Galactic rotation curve, for example, from Bobylev and Bajkova (2017) or Rastorguev et al. (2017), is suitable to perform this procedure.
Table 1: The Galactic rotation parameters found from stars with measured line-of-sight velocities and proper motions provided that the relative trigonometric parallax errors $\sigma_\pi/\pi$ are less than 10%.

| Parameters | $V_\rho, V_\phi, V_b$ | $V_\rho$ | $V_\phi, V_b$ |
|------------|---------------------|---------|---------------|
| $U_\odot$, km s$^{-1}$ | $9.15 \pm 0.18$ | $9.59 \pm 0.29$ | $8.91 \pm 0.22$ |
| $V_\odot$, km s$^{-1}$ | $20.41 \pm 0.15$ | $19.94 \pm 0.27$ | $20.55 \pm 0.19$ |
| $W_\odot$, km s$^{-1}$ | $7.74 \pm 0.12$ | $8.26 \pm 0.18$ | $7.51 \pm 0.17$ |
| $\Omega_0$, km s$^{-1}$ kpc$^{-1}$ | $27.6 \pm 1.4$ | — | $27.9 \pm 1.7$ |
| $\Omega_0'$, km s$^{-1}$ kpc$^{-2}$ | $-3.89 \pm 0.23$ | $-3.70 \pm 0.38$ | $-3.83 \pm 0.29$ |
| $\sigma_0$, km s$^{-1}$ | $27.51$ | $26.54$ | $27.98$ |
| $N_\star$ | $53173$ | $53173$ | $53173$ |
| $A$, km s$^{-1}$ kpc$^{-1}$ | $15.56 \pm 0.91$ | $14.81 \pm 1.52$ | $15.33 \pm 1.15$ |
| $B$, km s$^{-1}$ kpc$^{-1}$ | $-12.06 \pm 1.68$ | — | $-12.62 \pm 2.04$ |

RESULTS

Table 1 gives the kinematic parameters found from stars with measured proper motions and trigonometric parallaxes from the Gaia DR1 catalogue. The parameters in this table were calculated at relative trigonometric parallax errors $\sigma_\pi/\pi < 10\%$. We used a total of 53 173 stars. At such small radii of the sample the second derivative of the angular velocity of Galactic rotation is determined very poorly. Therefore, the table provides the results of the solution of Eqs. (1)–(3) without $\Omega'$. At small $\sigma_\pi/\pi$ the influence of the Lutz–Kelker (1973) bias is negligible, while at large $\sigma_\pi/\pi$ this bias should be taken into account (Astraatmadja and Bailer-Jones 2016a, 2016b). The first, second, and third columns in the table present the results of the simultaneous solution, those obtained only from the line-of-sight velocities, and those obtained only from the proper motions, respectively. Table 1 also gives the error per unit weight $\sigma_0$ determined when solving the conditional equations (1)–(3) by the least-squares method, which is close in its meaning to the residual velocity dispersion for the sample of stars being analyzed averaged over all directions. The number of stars used $N_\star$ is specified, the Oort constants $A$ and $B$ calculated from Eqs. (5) are given.

The values of $\Omega_0'$ obtained in the separate solutions are of interest for checking the distance scale used. This method is based on the fact that the line-of-sight velocity errors do not depend on the distance errors, while the errors in the tangential components depend on the latter. Therefore, a comparison of the values of $\Omega_0'$ found by various methods allows the distance scale correction factor $p$ to be determined (Zabolotskikh et al. 2002; Rastorguev et al. 2017). According to the data in Table 1, this correction factor is $p = (-3.70)/(-3.83) = 0.97 \pm 0.04$; its error was calculated from the relation

$$\sigma_p = (\sigma_{\Omega'_0}/\Omega'_0) - (\Omega'_0/\sigma_{\Omega'_0}/\Omega'_0)^2,$$

where the quantity $\Omega'_0(\rho, \phi)$ is denoted by $\Omega'_0$. Note that Bobylev and Bajkova (2017) found $p = 0.96$ from the kinematics of distant OB stars. Having analyzed the line-of-sight velocities and proper motions of more nearby red giants from the RAVE5 catalogue, Vityazev et al. (2017) found this correction factor to be 0.96, but its value from main-sequence dwarfs was 0.72.

Table 2 gives the Galactic rotation parameters found from the proper motions of all stars (from both southern and northern hemispheres of the celestial sphere) from the Gaia
Here, the mean distance of the sample stars is $\tau = 226$ pc, the number of stars used is $N_s = 424251$, the linear circular rotation velocity at the Galactocentric distance of the Sun is $V_0 = 218 \pm 6$ km s$^{-1}$ (for the adopted distance $R_0 = 8.0 \pm 0.2$ kpc), and the Oort constants are $A = 15.07 \pm 0.25$ km s$^{-1}$ kpc$^{-1}$ and $B = -12.17 \pm 0.39$ km s$^{-1}$ kpc$^{-1}$, $p = 0.98 \pm 0.08$.

To determine the Galactic rotation parameters, of course, it is more interesting to use stars at great distances. To use the data on stars with relative parallax errors more than 10%, we took their distances from Astraatmadja and Bailer-Jones (2016b). These distances were calculated using the trigonometric parallaxes of stars from the Gaia TGAS catalogue and contain the corrections for the Lutz–Kelker bias. We use the corrections calculated for two stellar density distributions: (a) an exponential drop (away from the Sun) with a radial scale

http://heasarc.nasa.gov/W3Browse/all/tycho2.html
length of 110 pc and (b) an anisotropic distribution (dependent on the heliocentric distance $r$ and coordinates $l, b$) close to the Milky Way model. For case (b) the model parameters were selected (Astraatmadja and Bailer-Jones 2016a) not from the spatial distribution of stars but from the observed photometric characteristics (typical for Gaia stars) by taking into account the peculiarities of the distribution of absorbing matter in the Galaxy. We will call case (b) the Milky Way model for short.

Tables 3 and 4 give the Galactic rotation parameters found only from the proper motions of stars from the Gaia TGAS catalogue using the calculated distances corrected for the Lutz–Kelker bias for cases (a) and (b), respectively. When producing our samples, we calculated the ratio $\sigma_\pi/\pi$ from the trigonometric measurements; therefore, the corresponding columns in Tables 3 and 4 give the parameters derived from the same stars. When solving Eqs. (2)–(3), we used the constraints on the proper motions specified in (6).

**DISCUSSION**

It is interesting to compare the Galactic rotation parameters found in this paper with the results obtained in other papers. For example, Bobylev and Bajkova (2016) analyzed stars with measured line-of-sight velocities from the RAVE4 catalogue and proper motions from the UCAC4 catalogue. The following kinematic parameters were found from a sample of more than 145 000 stars: $(U, V, W) = (9.1, 20.8, 7.7) \pm (0.1, 0.1, 0.1) \text{ km s}^{-1}$, $\Omega_0 = 28.7 \pm 0.6 \text{ km s}^{-1} \text{ kpc}^{-1}$, and $\Omega'_0 = -4.3 \pm 0.1 \text{ km s}^{-1} \text{ kpc}^{-2}$, where $V_0 = 230 \pm 12 \text{ km s}^{-1}$ (for the adopted distance $R_0 = 8.0 \pm 0.4 \text{ kpc}$), as well as the Oort constants $A = 17.1 \pm 0.5 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $B = -11.6 \pm 0.8 \text{ km s}^{-1} \text{ kpc}^{-1}$.

Rastorguev et al. (2017) determined the Galactic rotation parameters from 136 masers with measured trigonometric parallaxes. They used the VLBI measurements of water and methanol masers at frequencies from 6 to 22 GHz that were performed by several
Table 3: The Galactic rotation parameters found only from the proper motions of stars \((V_l, V_0)\) from the Gaia TGAS catalogue using the calculated distances from Astraatmadja and Bailer-Jones (2016) corrected for the Lutz–Kelker bias for an exponential drop in stellar density

| Parameters | \(\sigma_\pi / \pi < 0.15\) | \(\sigma_\pi / \pi < 0.20\) | \(\sigma_\pi / \pi < 0.30\) | \(\sigma_\pi / \pi < 0.50\) |
|------------|----------------|----------------|----------------|----------------|
| \(U_\odot\), km s\(^{-1}\) | 9.10 ± 0.04 | 8.90 ± 0.03 | 8.57 ± 0.03 | 8.25 ± 0.02 |
| \(V_\odot\), km s\(^{-1}\) | 20.01 ± 0.05 | 19.58 ± 0.04 | 19.02 ± 0.03 | 18.62 ± 0.03 |
| \(W_\odot\), km s\(^{-1}\) | 7.15 ± 0.03 | 6.90 ± 0.03 | 6.53 ± 0.02 | 6.21 ± 0.02 |
| \(\Omega_0\), km s\(^{-1}\) kpc\(^{-1}\) | 27.83 ± 0.16 | 27.77 ± 0.11 | 27.72 ± 0.08 | 27.15 ± 0.06 |
| \(\Omega'_0\), km s\(^{-1}\) kpc\(^{-2}\) | −3.835 ± 0.035 | −3.799 ± 0.024 | −3.788 ± 0.017 | −3.696 ± 0.013 |
| \(\Omega''_0\), km s\(^{-1}\) kpc\(^{-3}\) | 5.789 ± 0.168 | 4.397 ± 0.095 | 3.298 ± 0.054 | 2.803 ± 0.037 |
| \(\sigma_0\), km s\(^{-1}\) | 25.38 | 24.78 | 23.96 | 23.23 |
| \(A\), km s\(^{-1}\) kpc\(^{-1}\) | 15.34 ± 0.14 | 15.20 ± 0.10 | 15.15 ± 0.07 | 14.78 ± 0.05 |
| \(B\), km s\(^{-1}\) kpc\(^{-1}\) | −12.48 ± 0.21 | −12.58 ± 0.15 | −12.57 ± 0.10 | −12.36 ± 0.08 |
| \(\tau\), pc | 297 | 348 | 410 | 461 |
| \(N_\star\) | 773520 | 1036878 | 1386239 | 1708304 |
| \(p\) | 0.97 ± 0.09 | 0.97 ± 0.09 | 0.98 ± 0.10 | 1.00 ± 0.10 |

Scientific teams using radio interferometers in the USA, Japan, Europe, and Australia. These sources cover a wide range of Galactocentric distances \(R : 0 - 16\) kpc. For example, for the C1 model (the model of a constant radial velocity dispersion) they found \((U, V, W)_\odot = (11.0, 19.6, 8.9) \pm (1.4, 1.2, 1.1) \) km s\(^{-1}\), \(\Omega_0 = 28.4 \pm 0.5 \) km s\(^{-1}\) kpc\(^{-1}\), \(\Omega'_0 = −3.83 \pm 0.08 \) km s\(^{-1}\) kpc\(^{-2}\), \(\Omega''_0 = 1.17 \pm 0.05 \) km s\(^{-1}\) kpc\(^{-3}\), and \(V_0 = 235 \pm 7 \) km s\(^{-1}\) (for the adopted \(R_0 = 8.27 \pm 0.13 \) kpc).

From the velocities of 260 Cepheids with measured proper motions from the Gaia DR1 catalogue Bobylev (2017) found \((U, V, W)_\odot = (7.9, 11.7, 7.4) \pm (0.7, 0.8, 0.6) \) km s\(^{-1}\), \(\Omega_0 = 28.8 \pm 0.3 \) km s\(^{-1}\) kpc\(^{-1}\), \(\Omega'_0 = −4.1 \pm 0.1 \) km s\(^{-1}\) kpc\(^{-2}\), and \(\Omega''_0 = 0.81 \pm 0.07 \) km s\(^{-1}\) kpc\(^{-3}\) (for the adopted \(R_0 = 8.0 \pm 0.2 \) kpc), the circular velocity \(V_0 = 231 \pm 6 \) km s\(^{-1}\), as well as \(A = 16.2 \pm 0.4 \) km s\(^{-1}\) kpc\(^{-1}\) and \(B = −12.6 \pm 0.5 \) km s\(^{-1}\) kpc\(^{-1}\).

From 238 OB stars using the proper motions from the Gaia DR1 catalogue, Bobylev and Bajkova (2017) found \((U, V, W)_\odot = (8.2, 9.3, 8.8) \pm (0.7, 0.9, 0.7) \) km s\(^{-1}\), \(\Omega_0 = 31.5 \pm 0.5 \) km s\(^{-1}\) kpc\(^{-1}\), \(\Omega'_0 = −4.4 \pm 0.1 \) km s\(^{-1}\) kpc\(^{-2}\), \(\Omega''_0 = 0.71 \pm 0.10 \) km s\(^{-1}\) kpc\(^{-3}\), the Oort constants are \(A = 17.8 \pm 0.5 \) km s\(^{-1}\) kpc\(^{-1}\) and \(B = −13.8 \pm 0.7 \) km s\(^{-1}\) kpc\(^{-1}\), and \(V_0 = 252 \pm 8 \) km s\(^{-1}\) (for the adopted \(R_0 = 8.0 \pm 0.2 \) kpc).

The circular rotation velocity at the Galactocentric distance of the Sun \(V_0 = 218 \pm 6 \) km s\(^{-1}\) found in the solution (7) is typically 15–25 km s\(^{-1}\) smaller than the above values obtained from samples of young stars. Such a discrepancy is primarily attributable to the manifestation of Strömgberg’s asymmetry effect (an increase in the lag of the mean rotation velocity of a population of stars with increasing velocity dispersion of this population). Since we use stars of all ages, we obtain a reduced velocity \(\dot{V}_0\).

Using only the proper motions of \(~300,000\) nearby \((<250 \) pc\) main-sequence stars from the Gaia DR1 catalogue, Bovy (2017) estimated the Oort constants \(A, B, C,\) and \(K\) that describe the peculiarities of the local kinematics based on the Oort–Lindblad model. In particular, he obtained the following estimates: \(\Omega_0 = 27.1 \pm 0.5 \) km s\(^{-1}\) kpc\(^{-1}\), \(A = 15.3 \pm 0.5 \) km s\(^{-1}\) kpc\(^{-1}\) and \(A = 15.3 \pm 0.5 \) km s\(^{-1}\) kpc\(^{-1}\), and \(V_0 = 219 \pm 4 \) km s\(^{-1}\). It can be estimated that the distance \(R_0 = 8.1 \) kpc was used here, while \(\Omega'_0\) (see Eq. (5)) is −3.8 km
s⁻¹ kpc⁻². Note that our estimates of the kinematic parameters in the solution (7), first, are as accurate as the estimates of Bovy (2017) and, second, are more reliable methodologically. Indeed, the Taylor expansion is used twice in the Oort–Lindblad model that was used by Bovy. In the first case, just as we did, the angular velocity Ω₀ is expanded into a series to terms of the first order of smallness in r/R₀ and then the distance R is expanded into a series in powers of r/R₀ based on Eq. (4). In our approach the distance R is calculated from the exact formula (4) using the measured trigonometric parallaxes of stars. Note that in our case (in contrast to the Oort–Lindblad approach), the distance errors σ_R₀ should be taken into account when calculating the errors in the Oort constants A and B, as follows from Eqs. (5). In spite of this, in the solution (7) we obtained smaller errors in the Oort constants A and B than their errors derived by Bovy (2017).

In this paper we considered the samples of stars that are not very far from the Sun and, therefore, the second derivative of the angular velocity of Galactic rotation Ω₀'' is determined very poorly. As can be seen from the results presented in Tables 3 and 4, the behavior of Ω₀'' in the second case (Table 4) is most logical: they tend to ≈ 0.8 km s⁻¹ kpc⁻³ known from the analysis of completely different data (for example, those listed at the beginning of this section) with increasing radius of the sample. There are no other significant differences between the parameters corresponding to one another presented in Tables 3 and 4. Note that having compared the distances derived by them for Cepheids, Astraatmadja and Bailer-Jones (2016b) concluded that case (b) (the Milky Way model) has an advantage for stars from the range of heliocentric distances less than 2 kpc.

In this paper we applied a method that allows a “multiplicative” systematic error in the parallaxes to be detected. The “additive” error due to the error in the parallax zero point is of great interest. To estimate this error, we found Ω_Ω for subsamples of stars with parallaxes in five different ranges. The results are reflected in Table 5. However, the “multiplicator” p turned out to have no significant dependence on the mean parallax π′. In other words, based on the data from the last two rows in Table 5, we attempted to determine the correction Δπ from a linear equation like π′ = A · p + Δπ, but it turned out to be zero.
Table 5: The values of $\Omega_0$ found from the proper motions of stars from the TGAS catalogue using the calculated distances from Astraatmadja and Bailer-Jones (2017) corrected for the Lutz–Kelker bias for the Milky Way model in five distance ranges

| $\tau$  | 140 pc | 251 pc | 349 pc | 448 pc | 609 pc |
|---------|--------|--------|--------|--------|--------|
| $\Omega_0$ | $-3.841 \pm 0.160$ | $-3.865 \pm 0.085$ | $-3.871 \pm 0.060$ | $-3.928 \pm 0.050$ | $-3.928 \pm 0.050$ |
| $N_*$   | 193692 | 222917 | 215426 | 176114 | 227294 |
| $p$     | 0.96 ± 0.09 | 0.96 ± 0.07 | 0.96 ± 0.08 | 0.94 ± 0.08 | 1.00 ± 0.09 |
| $\pi$   | 7.14 mas | 3.98 mas | 2.87 mas | 2.23 mas | 1.64 mas |

We used stars with $\sigma_\pi/\pi < 0.20$, the values of $\Omega_0$ are given in km s$^{-1}$ kpc$^{-2}$.

We conclude that using the stellar proper motions from the Gaia TGAS catalogue allows us to estimate the kinematic parameters of our model in good agreement with the results of our analysis of independent data. In this case, the distance scale of the Gaia TGAS catalogue does not require using any additional correction factor in the range of heliocentric distances less than $\approx 1.5$ kpc under consideration.

CONCLUSIONS

We considered the space velocities of stars calculated using their highly accurate proper motions and trigonometric parallaxes from the Gaia TGAS catalogue in combination with their line-of-sight velocities from the RAVE5 catalogue. We obtained both simultaneous and separate solutions of the basic kinematic equations, i.e., we considered the equations that were set up using either only the line-of-sight velocities of stars, or only their proper motions, or a combination of all velocities. This allowed us to trace the consistency between the data from a kinematic viewpoint.

Based on a sample of 53 173 Gaia–RAVE stars with relative trigonometric parallax errors $\sigma_\pi/\pi$ less than 10%, we found the following kinematic parameters by simultaneously solving the equations: the components of the group velocity vector for the sample stars relative to the Sun $(U, V, W)_\odot = (9.15, 20.41, 7.74) \pm (0.18, 0.15, 0.12)$ km s$^{-1}$, the angular velocity of Galactic rotation $\Omega_0 = 27.62 \pm 1.40$ km s$^{-1}$ kpc$^{-1}$, and its first derivative $\Omega'_0 = -3.89 \pm 0.23$ km s$^{-1}$ kpc$^{-2}$. By comparing the values of $\Omega'_0$ found by separately analyzing the line-of-sight velocities $(\Omega'_0 = -3.70 \pm 0.38$ km s$^{-1}$ kpc$^{-2}$) and proper motions $(\Omega'_0 = -3.83 \pm 0.29$ km s$^{-1}$ kpc$^{-2}$) under the same constraints on the parallax error, we determined the distance scale correction factor $p$ to be close to unity, $p = 0.97 \pm 0.04$.

In the second part of the paper, we analyzed stars only from the Gaia TGAS catalogue. Thus, we considered stars covering the entire celestial sphere. From the proper motions of 424 250 stars with relative trigonometric parallax errors less than 10% we found $(U, V, W)_\odot = (9.28, 20.35, 7.36) \pm (0.05, 0.07, 0.05)$ km s$^{-1}$, $\Omega_0 = 27.24 \pm 0.30$ km s$^{-1}$ kpc$^{-1}$, and $\Omega'_0 = -3.77 \pm 0.06$ km s$^{-1}$ kpc$^{-2}$; here, the circular rotation velocity of the Sun around the Galactic center is $V_0 = 218 \pm 6$ km s$^{-1}$ (for the adopted distance $R_0 = 8.0 \pm 0.2$ kpc) and the Oort constants are $A = 15.07 \pm 0.25$ km s$^{-1}$ kpc$^{-1}$ and $B = -12.17 \pm 0.39$ km s$^{-1}$ kpc$^{-1}$, $p = 0.98 \pm 0.08$. All of the parameters obtained in this solution are new, among the most reliable ones to date.

The sample of 424 250 Gaia TGAS stars was divided into four subgroups: giants (I),
main-sequence dwarfs (II), giants (III), and subdwarfs (IV). A comparison of $\Omega''_0$ found by analyzing the proper motions of the stars from these subgroups with its value found previously only from the line-of-sight velocities of stars from the RAVE5 catalogue showed the distance scale correction factor to be $p > 0.93$.

The kinematics of more distant stars (with relative parallaxes errors more than 10%) was studied by invoking the distances from Astraatmadja and Bailer-Jones (2016), which were calculated using the trigonometric parallaxes of stars from the Gaia TGAS catalogue and contain the corrections for the Lutz-Kelker bias. We considered the distances to which the corrections were applied for two stellar density distributions: (a) an exponential drop with a radial scale length of 110 pc and (b) an anisotropic distribution close to the real Milky Way model. For both stellar density distributions we determined the kinematic parameters for samples with various relative distance errors (15%, 20%, 30%, 50%) and found the distance scale correction factor $p$ to be always close to unity. Finally, we showed that at mean parallax errors for the sample $\sigma_\pi/\pi < 50\%$ more reliable kinematic parameters are obtained for case (b). For example, in contrast to case (a), at a small mean stellar distance $\pi = 537$ pc the calculated second derivative of the angular velocity of Galactic rotation $\Omega''_0 = 0.864 \pm 0.021$ km s$^{-1}$ kpc$^{-3}$ is in good agreement with the results of other authors.

On the whole, we found that the distances to the stars (both giants and dwarfs) from the Gaia TGAS catalogue that were calculated using their trigonometric parallaxes do not require using any additional correction factor.

ACKNOWLEDGMENTS

We are grateful to the referee for the useful remarks that contributed to an improvement of the paper. This work was supported by the Basic Research Program P–7 of the Presidium of the Russian Academy of Sciences, the “Transitional and Explosive Processes in Astrophysics” Subprogram.

REFERENCES

1. D. An, D. M. Terndrup, M. H. Pinsonneault, D. B. Paulson, R. B. Hanson, and J. R. Stauffer, Astrophys. J. 655, 233 (2007).
2. T. L. Astraatmadja and C. A. L. Bailer-Jones, Astrophys. J. 832, 137 (2016a).
3. T. L. Astraatmadja and C. A. L. Bailer-Jones, Astrophys. J. 833, 119 (2016b).
4. G. F. Benedict, B. E. McArthur, E. P. Nelan, and T. E. Harrison, Publ. Astron. Soc. Pacif. 129, 2001 (2017).
5. V. V. Bobylev and A. T. Bajkova, Astron. Lett. 42, 90 (2016).
6. V. V. Bobylev, Astron. Lett. 43, 152 (2017).
7. V. V. Bobylev and A. T. Bajkova, Astron. Lett. 43, 159 (2017).
8. J. Bovy, Mon. Not. R. Astron. Soc. 468, L63 (2017).
9. A. G. A. Brown, A. Vallenari, T. Prusti, J. de Bruijne, F. Mignard, R. Drimmel, et al. (GAIA Collab.), Astron. Astrophys. 595, A2 (2016).
10. S. Casertano, A. G. Riess, B. Bucciarelli, and M. G. Lattanzi, Astron. Astrophys. 599, 67 (2017).
11. G. Clementini, L. Eyer, V. Ripepi, M. Marconi, T. Muraveva, A. Garofalo, L. M. Sarro, M. Palmer, et al., Astron. Astrophys. 605, 79 (2017).
12. E. Hog, C. Fabricius, V. V. Makarov, U. Bastian, P. Schwendiek, A. Wicenec, S. Urban, T. Corbin, and G. Wycoff, Astron. Astrophys. 355, L27 (2000).
13. W.-C. Jao, T. J. Henry, A. R. Riedel, J. G. Winters, K. J. Slatten, and D. R. Gies, Astrophys. J. 832, L18 (2017).
14. B. Kim, D. An, J. R. Stauffer, Y. S. Lee, D. M. Terndrup, and J. A. Johnson, Astrophys. J. Suppl. Ser. 222, 19 (2016).
15. A. Kunder, G. Kordopatis, M. Steinmetz, T. Zwitter, P. McMillan, L. Casagrande, H. Enke, J. Wojno, et al., Astron. J. 153, 75 (2017).
16. F. van Leeuwen, Astron. Astrophys. 497, 209 (2009).
17. F. van Leeuwen, A. Vallenari, C. Jordi, L. Lindegren, U. Bastian, T. Prusti, J. H. J. de Bruijne, A. G. A. Brown, C. Babusiaux, et al. (GAIA Collab.), Astron. Astrophys. 601, 19 (2017).
18. L. Lindegren, U. Lammers, U. Bastian, J. Hernandez, S. Klioner, D. Hobbs, A. Bombrun, D. Michalik, et al., Astron. Astrophys. 595, A4 (2016).
19. A.-Li Luo, Y.-H. Zhao, G. Zhao, Li-C. Deng, X.-W. Liu, Yi-P. Jing, G. Wang, H.-T. Zhang, et al., Res. Astron. Astrophys. 15, 1095 (2015).
20. T. E. Lutz and D. H. Kelker, Publ. Astron. Soc. Pacif. 85, 573 (1973).
21. C. Melis, M. J. Reid, A. J. Mioduszewski, J. R. Stauffer, and G. C. Bower, Science 345, 1029 (2014).
22. T. Prusti, J.H. J. de Bruijne, A. G. A. Brown, A. Vallenari, C. Babusiaux, C. A. L. Bailer-Jones, U. Bastian, M. Biermann, et al. (GAIA Collab.), Astron. Astrophys. 595, A1 (2016).
23. A. S. Rastorguev, M. V. Zabolotskikh, A. K. Dambis, N. D. Utkin, A. T. Bajkova, and V. V. Bobylev, Astrophys. Bull. 72, 122 (2017).
24. R. Schönrich and M. Aumer, MNRAS 472, 3979 (2017).
25. K. G. Stassun and G. Torres, Astrophys. J. 831, 74 (2016).
26. M. Steinmetz, T. Zwitter, A. Siebert, F. G. Watson, K. C. Freeman, U. Munari, R. Campbell, M. Williams, et al., Astron. J. 132, 1645 (2006).
27. J. P. Vallée, Astrophys. Space Sci. 362, 79 (2017).
28. V. V. Vityazev, A. S. Tsvetkov, V. V. Bobylev, and A. T. Bajkova, Astrophysics 60, 462 (2017).
29. M. V. Zabolotskikh, A. S. Rastorguev, and A. K. Dambis, Astron. Lett. 28, 454 (2002).
30. The Hipparcos and Tycho Catalogues, ESA SP–1200 (1997).