Impacts of Number of Cloud Condensation Nuclei on Two-Dimensional Moist Rayleigh Convection

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Abstract

The impacts of the number density of cloud condensation nuclei (CCN) and other thermodynamic quantities on moist Rayleigh convection were examined. A numerical model, consisting of a simple two-dimensional equation for Boussinesq air and a sophisticated double moment microphysics scheme, was developed. The impact of the number of CCN is most prominent in the initially formed convection, whereas the convection in the quasi-steady state does not significantly depend on the number of CCN. It is suggested that the former convection is driven by a mechanism without a background circulation, such as parcel theory. In contrast, the latter convection appears to be driven by the statically unstable background layer.

Incorporating the cloud microphysics reduces the integrated kinetic energy and number of convective cells (increases the distance between the cells), with some exceptions, which are consistent with previous studies. These features are not largely sensitive to the number of CCN. It is shown in this study that the reduction in kinetic energy is mainly due to condensation (evaporation) in the upper (lower) layer, which tends to stabilize the fluid.

The ensemble simulation shows that the sensitivity of the moist processes to changes in the temperature at the bottom boundary, temperature lapse rate, water vapor mixing ratio, and CCN is qualitatively similar to that in the control simulation. The impact becomes strong with increasing temperature lapse rate. The number of convective cells in a domain decreases with the degree of supersaturation or an increase in the domain-integrated condensate.

Keywords convection; cloud microphysics

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1. Introduction

Low–level clouds at the top of the atmospheric boundary layer (~ 1 km) play an important role in the radiation budget of Earth’s atmosphere and, therefore, in climate projection (Klein and Hartmann 1993; Klein et al. 1995; Wood 2012; Sato et al. 2015; Zelinka et al. 2016). It has been found that the cloud structure is highly sensitive to the number of cloud condensation nuclei (CCN) (Xue et al. 2008; Stevens and Feingold 2009). A larger number of CCN results in more water droplets with a smaller radius once condensation occurs, which tend not to fall down as precipitation. Because the presence of precipitation or drizzle greatly affects the cloud structure, CCN play an important role in determining the cloud structure. Satellite observations indicate that low–level clouds are distributed as a cellular structure, implying that the flow field is driven by Rayleigh convection.

Rayleigh convection is driven by buoyancy and is characterized by upward and downward motions that are distributed horizontally as a cellular field. A steady state is achieved when the buoyancy and the viscous force are balanced. Rayleigh convection should be affected by the phase changes of the substances contained in the fluid, because this changes the buoyancy, which changes the kinetic energy of the fluid. One of the most characteristic features of the atmosphere is the phase change of water. Once water changes from gas to liquid phase, it releases heat into the surrounding air, which increases the buoyancy and vertical velocity.

It has been experimentally found that internal heating during the upward motion of air expands the wavelength of convection (Tritton and Zarraga 1967; Roberts 1967). The structure can be explained by a vertically nonuniform heating profile (Krishnamurti 1975). Helfand and Kalnay (1983) derived analytical solutions for two–dimensional Rayleigh–Bénard (RB) convection, indicating that a heating profile with heating at the bottom and cooling at the top results in a vertically tilted cell structure. The effects of background rotation have also been examined for a deeper understanding of clouds in the atmosphere (Nakajima and Matsuno 1988; Sakai 1997; Satoh 1998). More recently, realistic flow fields of low–level clouds have been successfully simulated by developing a model that solves for buoyancy by simply incorporating the effects of the phase change of water (Cieszelski 1998; Feingold et al. 2010; Weidauer et al. 2010; Schmidt et al. 2011; Pauluis and Schumacher 2010, 2011, 2013; Schumacher and Pauluis 2010; Weidauer and Schumacher 2012; Weiss and Ahlers 2013). Pauluis and Schumacher (2011) has found that the circulation intensity of RB convection is significantly reduced by including the effects of phase change. It has also been found that the phase change of water greatly changes the flow structures of RB convection (Bretherton 1987; Stevens 2005; Pauluis 2008; Vallis et al. 2019). By developing a Boussinesq fluid model with a simplified Clausius–Clapeyron relation, Vallis et al. (2019) showed that convection occurs under low Rayleigh numbers as long as condensation occurs.

However, few studies have been conducted to investigate the effects of CCN on Rayleigh convection. As the proposed impacts of CCN include many complicated processes, it is mathematically difficult to include the CCN effects in simple models, and all the previous studies without explicitly solving cloud microphysics ignore the effects of CCN (and hence precipitation), to the best of the authors’ knowledge. Vallis et al. (2019) assumed that all the condensate is immediately removed once it is produced (i.e., there is no liquid phase). Other simplified models such as those constructed by Bretherton (1987, 1988) or Pauluis and Schumacher (2010) consider the liquid phase of water while it is not precipitating. The number of CCN controls the number and size of cloud droplets and hence the fall speed of droplets, which is hardly represented in a simple theoretical model. Thus, the impacts of CCN need to be examined by solving fully nonlinear equations including the cloud microphysics.

The purpose of this paper is to examine the impact of CCN and parameters associated with the cloud microphysics on moist convection. For this purpose, a simple two–dimensional fluid model, combined with a sophisticated cloud microphysics model, was developed, and a series of numerical experiments were conducted by systematically changing the parameters. The rest of the paper is organized as follows. Model equations and the experimental setup are introduced in Section 2. The results of the parameter–sweep experiment are shown in Section 3 and discussed in Section 4. The concluding remarks are provided in Section 5.

2. Model and experimental setup

2.1 Two–dimensional fluid model with sophisticated cloud microphysics

The fluid system considered in this study is a two–dimensional (x, z), Boussinesq air for moist Rayleigh convection. The mass conservation, momentum, and thermodynamic equations are the dimensional forms of those used in Miyamoto et al. (2015), except for the source term due to the phase change in the thermody-
namic equation \(- (L_v/c_p) Q_q\), precipitation flux terms, and the adiabatic term. The equations are

\[
\frac{\partial u}{\partial t} + \frac{1}{\rho_0} \frac{\partial \rho'}{\partial x} + \frac{\partial u q V_w}{\partial z} = \frac{K_m}{\rho_0} \nabla^2 u, \tag{1}
\]

\[
\frac{\partial v}{\partial t} = - \frac{1}{\rho_0} \frac{\partial \rho'}{\partial y} + \frac{\partial u q V_w}{\partial z} + K_m \nabla^2 w, \tag{2}
\]

\[
\frac{\partial T}{\partial t} = w (\Gamma - \Gamma_d) - \frac{\partial T q V_w}{\partial z} + K_t \nabla^2 T - \frac{L_v}{c_p} Q_q, \tag{3}
\]

where \((u, w)\) are the velocities in the \((x, z)\) directions, respectively; \(t\) is time; \(d/dt \equiv \partial_t + u \partial_x + w \partial_z\) is the total derivative; \(p\) is the pressure; \(\rho_0\) is the air density; \(K_m\) and \(K_t\) are the momentum viscosity and temperature diffusivity, respectively; \(\alpha\) is the thermal expansion coefficient; \(g\) is the gravitational acceleration; \(V_w\) is the fall velocity of water; \(\Gamma\) is the fall velocity of water vapor; \(\Gamma_d \equiv g/c_p\) is the dry lapse rate; \(c_p\) is the specific heat at a constant pressure; \(L_v\) is the latent heat; \(\nabla^2 \equiv \partial^2_x + \partial^2_z\) is a differential operator; and the bar and prime indicate the background state and its deviation, respectively. The derivation of the precipitation flux terms is shown in the appendix.

The equation for the conservation of water, including the mixing ratio and number density but not the rain water mixing ratio, is

\[
\frac{d \phi}{dt} = K_m \nabla^2 \phi + Q_q, \tag{5}
\]

whereas the conservation equation of the mixing ratio and number density of rain water is

\[
\frac{d \phi}{dt} = K_t \nabla^2 \phi + Q_q - \frac{\partial \phi q V_w}{\partial z}, \tag{6}
\]

where \(\phi_i \in \{q, q_c, q_r, N_c, \phi, q, q_r\}\), \(q_c\), \(q_r\), \(N_c\), \(N_r\), and \(q\) are the mixing ratios of water vapor, cloud water, and rain water, respectively; \(q_r = q_c + q\) is the mixing ratio of the condensate (sum of cloud water and rain water); and \(N_c\) and \(N_r\) are the number densities of cloud water and rain water, respectively. The mass of water removed from the domain by precipitation is not recovered, and hence the total mass will decrease over time. However, the mass removed by precipitation is less than 0.1% of the total mass at the end of the simulation with heavy precipitation. The effects of the variation of water vapor, that is by using the virtual temperature, and water loading have been tested. Nevertheless, the impacts were found to be insignificant and hence they have been neglected in the governing equations.

The equations were discretized in time and space; the spatial derivatives were approximated by a fourth–order central difference, except for the grids next to the boundaries at the top and bottom of the domain. Miyamoto et al. (2015) analytically showed that the fourth–order central difference was accurate enough to simulate a realistic wavenumber of Rayleigh convection given a horizontal grid spacing less than half that of the vertical domain. The time derivative was discretized using the Adams–Bashforth scheme. The code was parallelized using the message passing interface, and the numerical domain was divided in the \(x\) direction.

The water quantity source terms were solved using the double–moment microphysics scheme of Seiki and Nakajima (2014). The scheme solves the phase change process by taking into consideration the number of CCN. See Seiki and Nakajima (2014) for details regarding the cloud microphysics scheme. Hereafter, the source term of water vapor \(Q_q\) is simply represented as \(Q\) because this study focuses only on the source and sink of water vapor.

### 2.2 Experimental setting

The parameters in the governing equations, among which the sensitivities were investigated in this study, are listed in Table 2. The equations were integrated for 10800 s \((= 3.0\ h)\) with a time step of 0.2 s. Figure 1 shows the computational domain and vertical profiles of the background temperature, mixing ratio of water vapor, and CCN. The domain covers 60 km × 0.8 km in the \(x\) and \(z\) directions with uniformly distributed grid spacings of 50 and 20 m, respectively. The values of prognostic quantities were fixed to the initial values.
at the top and bottom boundaries ($z = 0$ and 0.8 km). The periodic boundary condition was applied to the lateral boundaries ($x = 0$ and 60 km). In the control experiment, the Rayleigh number is 1338. The Rayleigh number was below the range of bifurcation, even when the temperature gradient, which is the only parameter controlling the Rayleigh number in the present sensitivity experiment, reached its maximum value (11.5 K km$^{-1}$). Thus, this study focuses not on turbulent or transient convection, but rather on laminar flows.

A random white noise with an amplitude of less than 0.1 K was added to the temperature at the initial time. The noise did not change the boundary condition for $T'$ at the top and bottom boundaries. Three ensemble simulations were conducted; each had different initial random noise components, while all other numerical conditions were fixed. In all, 120 experiments were executed (number of parameters × number of experiments for each parameter × number of members with a different random noise = 4 × 10 × 3 = 120) for both dry and moist conditions.

### 2.3 Calculation of convection scale

Since this study focuses on the impact of microphysics, which appears to result in the highly asymmetric structure of convection, that is, narrow upward and wide downward regions, the wavenumber may not be able to capture the spatial scale of convection. Instead, we explicitly calculated the number of convective cells based on the spatial distribution of vertical velocity at the middle of the domain ($z = 0.4$ km). To remove small-scale fluctuations, the vertical velocity field at $z = 0.4$ km was smoothened by taking a 1–2–1 average 20 times. The local peaks were then detected. The number of convective cells in the numerical domain can be a measure of distance between convective cells or the spatial scale of convection and hence it is used for the following analyses.

### 3. Results

#### 3.1 Control experiment

First, the results of the dry and moist experiments at the control settings ($T_s = 284$ K, $\Gamma = 11$ K km$^{-1}$, $q_{t,0} = 9$ g kg$^{-1}$, and CCN = 100 cm$^{-3}$) are examined. Dry experiments are defined as experiments in which cloud microphysics is not solved, that is, $Q \rightarrow 0$ in (4) and (6). Figure 2 displays the $x$–$z$ cross section of the temperature $T'$ and velocity vector in the dry and moist experiments. A number of pairs of upward and downward motions exist at a nearly constant distance in the entire numerical domain. The temperature deviation is positive in the upward–moving regions, whereas it is negative in the downwards–moving regions. The magnitude of the temperature deviation from the horizontal average is larger in the dry experiment than in the moist experiment.

The $x$–$z$ section of condensate $q_t$ and the diabatic heating rate $Q$ in the moist experiment are depicted in Fig. 2c. The heating rate $Q$ is positive (negative) especially in the upward (downward) region, whereas the fluid layer is filled with the condensate $q_t$, which increases with height. The heating and cooling distributions appear to be reasonable as they are co–located with the upward and downward motions, respectively. The vertical gradient of $q_t$ is consistent with the temperature gradient, because a lower temperature results in a smaller saturation mixing ratio of $q_t$ and produces a larger amount of water.

Figure 3 displays the $x$–$z$ cross section of the quantities in the both the dry and moist experiments with $\Gamma = 9$ K km$^{-1}$. Since $\Gamma = 9$ K km$^{-1}$ is less than the dry adiabatic lapse rate $\Gamma_d$, convection does not form in the dry experiment. In contrast, the convection does

| Parameter | Unit | Control value | Range      | Interval | Description                     |
|-----------|------|---------------|------------|----------|---------------------------------|
| CCN       | cm$^{-3}$ | 100 | 100–1000 | 100      | Cloud Condensation Nuclei       |
| $T_s$     | K    | 284 | 278–287  | 1.0      | Temperature at the basal boundary|
| $\Gamma$ | K km$^{-1}$ | 10.5 | 7.0–11.5 | 0.5      | Temperature lapse rate          |
| $q_{t,0}$ | g kg$^{-1}$ | 8.0 | 7.6–9.4  | 0.2      | Mixing ratio of water vapor     |
form in the moist experiment, which is likely because of the conditional instability driven by condensation heating. Note that the distance of convection is wide and the intensity is slightly weak compared with those in the control experiment. The negative heating region that appeared nearby the convective updraft in the control experiment is not observed in the present case. The temperature deviation is negative in the downwind region, which appears to be due to the fact that the deviation is the difference from the horizontal mean at each time. Since the diabatic heating is strong at the upward region, the deviation in the downward region tends to be negative. Furthermore, the diabatic cooling is present in the downward region, whereas that the magnitude is small.

Figure 4 shows the time series of the number of convective cells and domain–integrated kinetic energy \( \rho_0 (u^2 + w^2)/2 \) for the control experiment. The number of convective cells sharply decreases immediately after the initiation, and becomes nearly steady after \( t = 1 \) h. The numbers of convective cells in the dry and moist experiments are 27 and 10 at the end of the simulation, respectively. This indicates that the horizontal scale of the convective cell is larger in the moist experiment than that in the dry experiment. The decrease in the number of convective cells may be due to the increased contribution of nonlinear terms. It was shown that the dominant wavenumber, i.e., convective cell, is large in linear theory, and it decreases in a
finite-amplitude state (Busse and Whitehead 1971). The decrease in wavenumber has also been observed in barotropic instability (Niino 1982) and baloclinic instability (Yoshizaki 1982a, b).

The kinetic energy increases rapidly with time until \( t = 0.4 \) h in the moist experiment, it then decreases sharply in the moist experiment, and it becomes nearly constant after \( t = 1.3 \) h. Meanwhile, the kinetic energy gradually increases with time and approaches a constant value in the dry experiment. The kinetic energy behaves differently in the two experiments: It was 8.7 kJ at the end of the dry experiment, and only 5.9 kJ in the moist experiment. In sum, adding the microphysical process decreases the number of convective cells (increases the spatial scale of convection) and the kinetic energy.

Figure 5 shows the vertical profiles of quantities horizontally averaged in the \( x \) direction in the dry experiment. The temperature deviates positively from the initial state in the upper layer and negatively in the lower layer. The second moment of the vertical velocity has a peak in the middle of the domain, whereas the third moment has both positive and negative peaks at \( z = 0.40 \) and 0.64 km, respectively. The kinetic energy has two peaks close to the vertical boundaries, and it is minimum in the middle of the domain (\( z = 0.4 \) km). The vertical heat flux has a peak in the middle of the vertical domain but is equal to 0 at the top and bottom boundaries. The water vapor flux is 0 in the domain, as the water vapor is conserved in Lagrangian motion and is constant in the domain at the initial time. The mixing ratios and number densities of cloud water and rain water are 0 across the entire domain.

The vertical profiles of the quantities in the moist experiment are shown in Fig. 6. \( Q \) has positive and negative peaks at \( z = 0.60 \) and 0.20 km, respectively. Thus, the diabatic processes associated with the cloud microphysics tend to heat and cool the upper and lower layers, respectively. The peak of the second moment of the vertical velocity is located at \( z = 0.48 \) km, which is slightly higher than that in the dry experiment. The third moment of the vertical velocity also has a positive peak at \( z = 0.48 \) km, indicating that the vertical velocity has large positive values in the narrow region. The kinetic energy still has a sharp peak at the top boundary, whereas the peak at the bottom boundary is much smaller than that in the dry experiment. The sensible heat flux reaches a maximum at \( z = 0.54 \) km and a minimum at \( z = 0.20 \) km, that is, the same altitude as the minimum of the diabatic heating rate (cf. Fig. 6b). The vertical flux of the latent heat is positive with a peak at \( z = 0.42 \) km. Although the magnitude of the sensible heat flux is very small in the moist experiment, the order of magnitude of the sum of the sensible and latent heat fluxes is comparable with that of the dry experiment. \( q_v \) decreases almost linearly with height. \( q_r \) is close to 0 up to \( z = 0.20 \) km and has large values, with a peak at \( z = 0.50 \) km. \( q_r \) increases linearly with height, while it is two orders of magnitude larger than \( q_v \). The profiles of the number densities are similar to those of the mixing ratios; the peak of \( N_c \) is located at \( z = 0.64 \) km, whereas \( N_r \) vertically increases, and its magnitude is much smaller than that of \( N_c \).

Figures 7a, b depicts the \( x-z \) cross sections of the stream function and temperature deviation composited for all the detected cells at the final simulation time in both the dry and moist experiments. The stream function \( \psi \) is defined as

\[
\psi = \frac{\partial w}{\partial z}, \quad u = -\frac{\partial \psi}{\partial x}.
\]

The composite flow field has a circulation: upward at the center, outward in the upper layer, downward
Fig. 5. Vertical profile of (a) the temperature $T'$, (b) the diabatic heating rate $Q$, (c) the second moment of the vertical velocity $w^2$, (d) the third moment of the vertical velocity $w^3$, (e) the kinetic energy, (f) the vertical sensible heat flux, (g) the vertical flux of water vapor, (h) the water–vapor mixing ratio, (i) the cloud water and rain water mixing ratios, and (j) the number densities of cloud water and rain water. The quantities are horizontally averaged in the dry experiment. The gray dashed line in (a) indicates the initial profile. The black and gray lines in (i) and (j) represent the cloud water and rain water, respectively.

Fig. 6. Same as Fig. 5, but for the moist experiment. The gray dashed line in (h) indicates the initial profile. In (i) and (j), the black and gray lines stand for the cloud–water and rain–water, respectively.
in the outer regions, and inward in the lower layer. The distance between the upward and downward regions in the dry experiment is approximately 0.96 km (the aspect ratio of the cell is approximately 1.2). In contrast, the circulation is asymmetric in the moist experiment, with a narrow upward region and a wide downward region. The high-temperature deviation is co-located with the narrow upward region, whereas the temperature is low in the downward region outside. In the upper layer, the temperature deviation is negative and positive at the upward and downward branches, respectively. The vertical velocity and hence
circulation are stronger in the dry experiment. The asymmetric flow structure is consistent with the previous studies, which obtained analytical solutions for a single convective cell (Kuo 1961; Bretherton 1987).

Figure 7c depicts composites of the diabatic heating rate with temperature. High $\dot{Q}$ is located in the narrow upward region, whereas $\dot{Q}$ is negative in the downward region, and its magnitude is smaller than the positive values. This suggests that the negative temperature deviation in the upper layer of the upward branch is due to the adiabatic cooling associated with the upward velocity. Figure 7d shows the mixing ratios of the cloud water $q_c$ and rain water $q_r$. $q_c$ is concentrated in the positive diabatic heating region, which corresponds to the upward region. Specifically, it is largest at the peak of the vertical velocity ($x = 0$ km) at all altitudes.

The number densities of the cloud water and rain water, $N_c$ and $N_r$, respectively, are depicted in Fig. 7e. Large number densities are co-located with large mixing ratios. The exceptions are the large $N_r$ in the lower layer of the upward branch ($z \sim 0.3$ km), where $q_r$ is not large, and the small $N_c$ outside the peak of the vertical velocity (e.g., $x = 0.1$ km at $z = 0.5$ km). Thus, the droplet size of cloud water is small in the upper layer and that of rain water is largest outside the peak of the vertical velocity. It is suggested that the rain water droplets tend to fall down immediately outside the peak of the vertical velocity, but not in the downward region, indicating that the cloud water and rain water do not move along with the flow motion.

### 3.2 Sensitivity experiments

Figure 8 represents the time series of the domain-averaged kinetic energy in the experiments on sensitivity to the number of CCN. In all the members, the kinetic energy rapidly increases with time from $t = 0.2$ to 0.4 h and then sharply decreases to a constant value. There are two notable features in this figure: (1) the peak kinetic energy increases with the number of CCN, and (2) the kinetic energy is less sensitive to the number of CCN at the end of the simulation. These features will be discussed later.

The left panels of Fig. 9 show the number of convective cells as functions of the experimental parameters in the dry and moist experiments, which are averaged temporally during the last 4000 s ($\sim 1.1$ h) of each simulation for three ensemble members. Overall, the number of convective cells is lower in the moist experiments except when the bottom temperature $T_s$ is greater than 284 K and the temperature lapse rate $\Gamma$ is less than 10.0 K km$^{-1}$.

In the dry experiments, the number of convective cells is sensitive only to $\Gamma$. Convection does not form when $\Gamma$ is less than 10.0 K km$^{-1}$. The number of convective cells does not depend on $T_s$, $q_v$, or the number of CCN as implicated in classical theory. In the moist experiments, the number of convective cells increases with $T_s$ when $T_s$ is greater than 284 K and decreases with $q_v$, whereas it is not so strongly sensitive to $\Gamma$ and CCN.

The right panels of Fig. 9 show the domain-integrated kinetic energy. In all the experiments, the kinetic energy decreases when the cloud microphysics is included, except when $\Gamma$ is greater than 8.5 K km$^{-1}$ and less than 11.0 K km$^{-1}$. In the dry experiments, as was the case for the number of convective cells, the kinetic energy does not depend on $T_s$, $q_v$, or CCN, whereas it monotonically increases with $\Gamma$ when it is larger than 10.0 K km$^{-1}$. In the moist experiments, the kinetic energy decreases slightly with $T_s$ when $T_s \leq 283$ K, has a maximum at $T_s = 284$ K, and decreases with $T_s$. The kinetic energy increases and decreases with $\Gamma$ and $q_v$, respectively. The dependence of the kinetic energy on CCN is not significant; it increases slightly as the number of CCN is large, up to 100 cm$^{-3}$, whereas it does not depend on CCN when CCN $\geq 200$ cm$^{-3}$.

Figure 10 shows the vertical temperature and water vapor fluxes as functions of the experimental parameters, which are averaged spatiotemporally. In all the experiments, the vertical temperature flux is significantly small in the moist experiments. Note that as seen in Fig. 6f, the temperature flux is not zero in
the moist experiments. The vertical temperature flux is sensitive only to $\Gamma$ when $\Gamma$ is greater than 10.0 K km$^{-1}$. In contrast, the latent heat flux is large and sensitive to the experimental parameters in the moist experiments. The water vapor flux is largest at $T_s = 286$ K, whereas it is very small when $T_s \leq 283$ K. The water vapor flux monotonically increases with $\Gamma$ and decreases with $q_{v0}$. The flux does not depend on the number of CCN. The latent heat flux appears to be correlated with the kinetic energy (cf. Fig. 9).

Figure 11 shows the mixing ratios of water vapor $q_v$ and total condensate $q_t$ (sum of cloud water and rain water). By solving the cloud microphysics, $q_v$ is reduced from the initial value, and the magnitude of decrease in $q_v$ is large as $T_s$ is small, $\Gamma$ is large, or $q_{v0}$ is large. Thus, it is indicated that the reduction of $q_v$ from the dry setting is significant when the degree of supersaturation is large. Moreover, $q_v$ is large as the reduced amount of water vapor in the dry setting increases. This is suggested to be simply because a higher
degree of supersaturation produces a larger amount of condensate. It is noted that \( q_t \) mainly consists of \( q_r \), whereas \( q_c \) is one to two orders of magnitude smaller than \( q_r \).

Figure 12 shows the domain–averaged \( q_c \) and diabatic heating rate \( Q \). \( q_c \) is large when \( T_s > 283 \text{ K} \) compared with that when \( T_s \leq 283 \text{ K} \), monotonically increases with \( \Gamma \), and decreases with \( q_{v_0} \), whereas it is not sensitive to CCN. The vertical averages of \( Q \) in the upper and lower layers are positive and negative, respectively, whereas the domain average is close to zero. In other words, the diabatic processes tend to heat and cool the air in the upper and lower layers in most of the moist experiments. The difference between the upper and lower layers increases with \( \Gamma \) and decreases with \( q_{v_0} \), whereas the differences are relatively insensitive to the number of CCN. The domain average of \( Q \) is slightly negative when \( T_s \geq 284 \text{ K} \). The difference between the upper and lower layers is largest at \( T_s = 285 \text{ K} \).

4. Discussion

The results show the sensitivity of convection to the number of CCN. Figure 8 indicates that more CCN
produces stronger convection, whereas convection at the end of simulation is not so sensitive to CCN. When the number of CCN is not large enough, the size of the water droplet is large at their formation and droplets tend to fall down, which likely preclude the classical mechanism of convection such as that explained by parcel theory. By contrast, if the number of CCN is large enough (CCN $\geq 100$ cm$^{-3}$), convection can increase, and the peak energy is not sensitive to the number of CCN. In other words, the strong convection can hardly form in the pristine condition as proposed in previous studies such as Rosenfeld et al. (2008). In this particular system, the critical value of CCN, which is the value below which the convection is affected by the number of CCN, is approximately 100 cm$^{-3}$.

The insensitivity of the kinetic energy to the number of CCN at the end of the simulation appears to be due to the fact that the present experiments have a statically unstable lapse rate. After the peak of the kinetic energy forms, the circulation is suggested to be driven by the unstable background temperature gradient. Because the temperature gradient is the same in the set of experiments, the kinetic energy does not largely depend on the number of CCN. In sum, the effects of CCN are significant in the initially formed convection, and the convection that subsequently forms is similar to Rayleigh convection, which is driven by the vertical temperature gradient.

Incorporating the cloud microphysics reduces the integrated kinetic energy and number of convective cells (increases the distance of the cells), with some
exceptions. The former and latter are consistent with previous studies (Pauluis 2008; Pauluis and Schumacher 2011; Kuo 1961; Bretherton 1987). To discuss the possible causes of the impacts, we will focus on the two notable features: (1) the upward latent heat flux, in addition to the sensible heat flux, and (2) the vertical profile of diabatic heating. Both of them tend to stabilize the fluid. The sum of the sensible and latent heat fluxes in the moist experiments (Fig. 6) is comparable with, or smaller than, that in the dry experiment (Fig. 5). Thus, the latent heat flux helps to transport heat at a given degree of instability. This would result in a small sensible flux and a low kinetic energy.

A comparison of the dependences of the kinetic energy and the vertical fluxes on the parameters (Figs. 9, 10) suggests that larger sensible and latent heat fluxes result in a larger kinetic energy. Figure 13a shows the relationship between the kinetic energy in the moist experiments and the vertical water vapor flux. The kinetic energy is positively correlated with the latent heat flux, which appears to be similar to the dry cases with sensible heat flux. In the moist experiments, latent heat fluxes are much larger than sensible heat fluxes.

The parameter dependences of $q_c$ in Fig. 12 appear to be similar to those of the kinetic energy and the vertical latent flux. In fact, there is a positive relationship between the kinetic energy and $q_c$ (Fig. 13b). By contrast, $q_r$ is not correlated with the kinetic energy (figure not shown). This is due to the effects of precipitation that is a direct product of $q_r$ and will
be discussed below. $q_c$ is produced by condensation, which heats the fluid and causes buoyancy, and removed by evaporation, which cools fluid and causes negative buoyancy. In other words, $q_c$ appears to accelerate the circulation, whereas the effect of $q_r$ is not straightforward. A large $q_r$ in the downward branch of convection tends to accelerate the circulation. However, it can tend to decelerate the circulation when $q_r$ is large in the upward motion, as it has a large mass and nonzero falling velocity.

On the other hand, the vertical distribution of diabatic heating would also play a role in determining the structure of convection. Diabatic heating and cooling are, respectively, present in the upper and lower layers (Figs. 7, 6b), which tends to stabilize the fluid. Figure 13c shows scatter plots of the domain-averaged kinetic energy in the moist experiments as a function of the vertical difference in diabatic heating. Diabatic heating is averaged in the upper layer ($z > 0.4$ km) and lower layer ($z < 0.4$ km). The kinetic energy and difference in diabatic heating have a clear positive correlation. Thus, incorporating the cloud microphysics promotes diabatic heating and cooling by condensation and evaporation in the upper and lower layers, respectively, which tends to stabilize the fluid and hence to reduce the kinetic energy.

The number of convective cells is generally small in the moist experiments. This is because the diabatic processes tend to stabilize the fluid, and the total amount of heat to be transported vertically is reduced. The classical linear theory of convection predicts that the number of convective cells increases with the temperature lapse rate. Thus, the number of convective cells would decrease in the stabilized fluid.

The microphysical process in this system is basically initiated from supersaturation at the initial state. The scatter plots in Fig. 14a show the difference in the number of convective cells between the moist and dry experiments as a function of the degree of supersaturation. The supersaturation is calculated from the temperature and $q_v$ in the middle layer ($z = 0.4$ km). The difference in the number of convective cells decreases with supersaturation; that is, the number of convective cells in the moist experiments is low as the supersaturation increases. The relationship of the difference in the number of convective cells with $q_t$ also has a clear negative correlation (Fig. 14b). This is because larger supersaturation produces more $q_t$. As a result, the cell distance, compared with that in the dry experiments, is related to the amount of condensate; the lower the amount of condensate, the larger the difference between the dry and moist experiments.

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**Fig. 13.** Scatter plots for the kinetic energy in the moist experiment (ordinate) with (a) the vertical difference in the diabatic heating rate, (b) the mixing ratio of cloud water, and (c) the vertical flux of the water vapor mixing ratio. The parameters in the abscissa as well as the kinetic energy are averaged in the domain. The vertical difference in diabatic heating is calculated after estimating the averages of heating in the upper half and lower half of the domain. Each point indicates a single sensitivity experiment. The solid line indicates the linear regression, and the correlation coefficient is shown in the panel.
By contrast, the relationship with \( q_c \) is opposite, and large \( q_c \) results in a large number of convective cells in the moist experiments (Fig. 14c). Specifically, \( q_c \) is large when \( T_s \geq 285 \text{ K} \) and \( q_v \leq 7.7 \text{ g kg}^{-1} \) (cf. Fig. 9). The amount of \( q_c \) is much smaller than that of \( q_r \). As \( q_r \) has no falling velocity, it is advected by fluid motion. The change in \( q_c \) owing to the phase change of water would change the flow speed; if \( q_c \) is produced (removed) in the upward (downward) motion, it tends to accelerate the circulation. The faster circulation would result in a larger kinetic energy, which tends to produce a larger number of convective cells as seen in the dry Rayleigh convection. Unlike \( q_c \), \( q_r \) has a nonzero falling velocity, and \( q_r \) in the upward region tends to decelerate the vertical velocity. Hence a large \( q_r \) weakens the convection.

5. Conclusions

The effects of the number of CCN and other thermodynamic quantities on two-dimensional Rayleigh convection were investigated by conducting a series of numerical simulations utilizing a simple fluid model and a sophisticated cloud microphysics model. The fluid was assumed to be a Boussinesq air with constant eddy diffusivities for momentum, heat, and scalars.

It was found that the convection cannot have strong intensity when the number of CCN is less than 100 cm\(^{-3}\), which appears to be because of the effect of large precipitable droplets on the formation. However, the convection subsequently formed because of the statically unstable background state. Overall, the number of convective cells and domain-integrated kinetic energy in the quasi-steady state are surprisingly insensitive to CCN.

Explicitly solving the cloud microphysics decreased the number of convective cells and domain-integrated kinetic energy in the Rayleigh convection. The decrease in the kinetic energy was caused by the positive vertical latent heat flux and diabatic heating and cooling in the upper and lower layers, respectively. The latter diabatic effects tend to stabilize the layer and hence weaken the buoyant motions. The decrease in the number of convective cells, or the increase in the horizontal scale of the cells, was suggested to be due to the stabilization effects. Meanwhile, the decrease in the number of convective cells in the moist experiments was reduced when \( q_r \) was large, which resulted from a high degree of supersaturation. Once \( q_r \) became large in the upward branch of the circulation path, the effect of diabatic heating in the upward motion was weakened, and the number of convective cells

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Fig. 14. Same as Fig. 13, but for the relationships of the difference in the number of convective cells between the dry and moist experiments with (a) supersaturation, (b) \( q_r \), and (c) \( q_c \). Both variables are averaged in the domain. The crossings indicate experiments in which the domain-averaged \( q_r \) is greater than 1.0 \( \times \) 10\(^{-3} \text{ g kg}^{-1} \) (cf. Fig. 11). The solid line indicates the linear regression of the points except for the small kinetic energy whose correlation coefficient is shown in the panel. The dashed line indicates the regression for the crossings.
decreased.

The ensemble simulations also showed that the sensitivity of the vertical sensible heat flux to the experimental parameters was almost the same as that of the kinetic energy, whereas the sensitivity of the vertical latent heat flux was large as the sensible flux decreased in the dry experiments. $q_v$ increased with $\Gamma$ and $q_{bh}$, whereas it decreased with $T_a$. Thus, $q_v$ increased with the degree of supersaturation. This was thought to be due to the large number of CCN resulting in a large number of small water droplets through condensation and making it unlikely that they would grow into rain droplets.

One possible way to further investigate the impact of CCN is to examine the vertical gradient of the water vapor mixing ratio, which was assumed to be constant at the initial time of the simulation.

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Appendix: Vertical flux associated with precipitation

The appendix provides the derivation of the precipitation flux terms that appeared in the governing equations. Water droplets with sufficiently large size fall down to the surface because of gravity, and thus they have nonzero vertical motion relative to the air speed. The vertical flux of a quantity $\phi_1$ that has the same value in both air and water droplets in the present incompressible system can be written as

$$F_{\phi_1} = w\Phi_1 + (w + V_w)\Phi_1 q_{bh}.$$  (8)

where $q_{bh}$ is the mixing ratio of rain water having nonzero fall velocity $V_w$ ($< 0$). The first term represents the vertical flux of $\Phi_1$ in an air volume whose velocity and volume relative to the total are $w$ and $1 - q_{bh}$, respectively. The second term represents the vertical flux of $\Phi_1$ along with water droplets whose velocity and relative volume are $w + V_w$ and $q_{bh}$. Equation (8) can be rewritten as

$$F_{\phi_1} = w\Phi_1 + V_w\Phi_1 q_{bh}.$$  (9)

The convergence of the vertical flux changes $\Phi_1$ in a volume. The first term is the vertical advection, and the second is the convergence of the precipitation flux, which appears on the right–hand side of the governing equations.

For quantities associated only with precipitable droplets, such as the mixing ratio or number concentration, the first term disappears, and the flux is simply represented as

$$F_{\phi_1} = (w + V_w)\phi_1.$$  (10)

For the temperature, the flux includes adiabatic heating and cooling as

$$F_T = wT(1 - q_{bh}) + (w + V_w)Tq_{bh} + wT_a(1 - q_v).$$  (11)

where the last term of the right–hand side is the flux associated with the adiabatic process. We have ignored the temperature change because of the adiabatic process of water droplets. The vertical derivative of $F_T$ is

$$\frac{\partial F_T}{\partial z} = \frac{\partial wT}{\partial z} + \frac{\partial wT_a(1 - q_v)}{\partial z} + \frac{\partial V_wTq_{bh}}{\partial z}.$$  (12)

Using the continuity equation (1) and combining the horizontal flux convergence, the first and second terms on the right–hand side may be $w\partial T_a$ and $w\partial [T_a(1 - q_v)]$, respectively, in the temperature equation. Decomposing the temperature into the background $T$ and its deviation $T'$, the right–hand side of (12) becomes

$$w\frac{\partial T'}{\partial z} - w(\Gamma - \Gamma_a) + \frac{\partial V_wTq_{bh}}{\partial z},$$  (13)

where we have assumed that the mixing ratio of precipitable water is much smaller than 1; that is, $q_{bh} \ll 1$ and $wT_a(1 - q_v) \approx wT_a$.

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