A Cooper-Pair Box Architecture for Cyclic Quantum Heat Engines

Andrew Guthrie,†, * Christoforus Dimas Satrya,†, † Yu-Cheng Chang,†
Paul Menczel,‡ Franco Nori,§,∥ and Jukka P. Pekola‡,*

1Pico group, QTF Centre of Excellence, Department of Applied Physics, Aalto University School of Science, P.O. Box 13500,00076 Aalto, Finland
2Theoretical Physics Laboratory, RIKEN Cluster for Pioneering Research, Wako-shi, Saitama 351-0198, Japan
3Department of Physics, The University of Michigan, Ann Arbor, Michigan 48109, USA
4Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia

(Dated: September 8, 2021)

Here we present an architecture for the implementation of cyclic quantum thermal engines using a superconducting circuit. The quantum engine consists of a gated Cooper-pair box, capacitively coupled to two superconducting coplanar waveguide resonators with different frequencies, acting as thermal baths. We experimentally demonstrate the strong coupling of a charge qubit to two superconducting resonators, with the ability to perform voltage driving of the qubit at GHz frequencies. By terminating the resonators of the measured structure with normal-metal resistors whose temperature can be controlled and monitored, a quantum heat engine or refrigerator could be realized. Furthermore, we numerically evaluate the performance of our setup acting as a quantum Otto-refrigerator in the presence of realistic environmental decoherence.

I. INTRODUCTION

The study of quantum heat engines and refrigerators plays a key role in the investigation of the fundamental relationship between quantum mechanics and thermodynamics [1–4]. However, experimental realizations of cyclic quantum thermal engines have remained elusive. Such systems, in their most basic form, consist of a working substance with quantized energy levels which can be selectively coupled to a series of thermal reservoirs, and are capable of transporting heat [5]. By modulation of the working substance energy-level separation, for example, the system can be tuned to interact with each thermal reservoir sequentially. Moreover, by periodic modulation of the system energy levels, a quantum heat engine, or quantum refrigerator can be actualized [6, 7].

A multitude of platforms have been proposed and explored to realize quantum thermal machines. In a seminal paper [8], an ion held in a linear Pauli trap was used to extract work by alternate exposure between a white noise electric field (hot reservoir), and a laser cooling beam (cold reservoir). More recently, a solid state quantum dot was operated as a ‘particle exchange’ heat engine, where the dot can control a thermally driven flow of charge carriers [9]. In a further development, a $^{13}$C nuclear spin has been utilized to implement an Otto cycle using a nuclear magnetic resonance setup [10]. Additionally, an electron spin impurity has been shown to act as an analogue heat-engine, with the ‘thermal’ reservoirs inferred by the relative chemical potential in the leads [11]. However, in all such systems the heat current cannot be probed directly, and must be inferred from an additional parameter.

Circuit quantum electrodynamics (c-QED) using superconducting qubits remains a highly promising platform for realizing such a thermal machine, owing largely to the exceptional control which experimentalists have over the collective quantum degrees of freedom [12–14]. c-QED has enjoyed a striking period of advancement, with numerous studies demonstrating strong coupling of photons to various types of qubits [15–17], with a broad range of applications [18–20]. Furthermore, modern nanofabrication techniques allow the integration and characterization of superconducting qubits coupled to normal-metal dissipative elements [21], creating hybrid c-QED systems capable of probing thermal transport in quantum systems. Such systems differentiate themselves from previous attempts on quantum heat engines via their unambiguous implementation of thermal reservoirs, which naturally define the bath temperature, and possess a multitude of techniques for both primary and secondary thermometry [22, 23].

Superconducting quantum circuits involving dissipative elements have already platformed several pioneering experiments in quantum heat transport. A transmon qubit coupled to two superconducting resonators, terminated by normal metal resistors was used to measure DC heat transport modulated by magnetic flux threading a superconducting quantum interference (SQUID) loop. By using both identical and non-identical resonator frequencies, this led to the realization of a quantum heat valve [24] and a quantum heat rectifier [25]. Despite the remarkable control exhibited by such systems, high frequency cyclic driving of transmon qubits has proven experimentally difficult due to the large power dissipated by on-chip flux bias lines. Additionally, the performance of transmon qubits in thermal systems is limited by the relatively weak anharmonicity due to the large ratio of Josephson energy to charging energy, $E_J/E_C$. Weak anharmonicity removes the ability to properly isolate a qubit transition, meaning contributions from higher en-
FIG. 1. (a) Design overview of the measured device. Two quarter-wavelength resonators of differing frequency are inductively coupled to a common feedline for readout. The voltage antinode of each resonator is capacitively coupled to a common superconducting island, connected to ground through a single Josephson junction (shown in the inset), and controllable by a nearby voltage gate. (b) Equivalent circuit for the measured sample with the feedline excluded. The quantum heat engine could then be operated by adding resistive normal-metal elements at the inductive ends of the resonators. (c) Calibrated $|S_{21}|(20 \text{ mK})/S_{21}(4 \text{ K})$ transmission through the feedline showing signals from two resonators and no spurious modes. (d) Calculated energy spectrum of the Cooper-pair box showing the first three energy levels for $E_c/h = 6.8 \text{ GHz}$ and $E_J/h = 3.5 \text{ GHz}$, clearly showing high anharmonicity around the degeneracy point. The blue and red arrows indicate the level spacing corresponding to the cold and hot resonator frequencies respectively.

ergy levels create undesired parasitic coupling [25]. Moreover, the relatively long coherence time of a transmon qubit, a key asset in quantum information applications, can limit the performance of cyclic quantum engines due to the build up of coherences in a process known as ‘quantum friction’ [26, 27].

The theoretical operation of superconducting qubits as thermal machines has been explored extensively, with promising proposals for implementing both refrigerators [6, 7, 28–30] and heat engines [3, 31–33]. The Otto refrigerator cycle remains the most explored, and is a practically achievable implementation of a quantum refrigerator. The Otto cycle consists of sequential interactions between a two-level system and a cold ($f_c$) and hot ($f_h$) reservoir. It has four branches: an adiabatic stroke of the qubit frequency from $f_c$ to $f_h$, thermalization with the hot bath at frequency $f_h$, an adiabatic stroke back from $f_h$ to $f_c$, and finally thermalization with the cold bath at frequency $f_c$. Cooling is achieved under the condition $f_h/f_c > T_h/T_c$, where $T_h$, $T_c$ are the temperatures of the normal-metal elements shunting the hot and cold resonators to ground, respectively. In addition, there already exist several proposals for advanced control techniques, including counter-diabatic driving techniques to compensate quantum friction [34–37]. Despite the theoretical progress, the realization of such a quantum refrigerator remains an experimental challenge.

In this work, we address the aforementioned problems to realize cyclic quantum heat engines by utilizing a Cooper-pair box (charge qubit) with a small $E_J/E_C$ as the working substance. A charge qubit, in its simplest form, consists of a nanoscale superconducting island, grounded through a Josephson junction [38]. Due to the small island dimension, the qubit frequency can now be tuned via the offset charge, $N_g$, on the island - controllable via the voltage of a nearby gate. Charge qubits have seen extensive study, both individually [39–41] and embedded in microwave cavities [42–45]. The strong cou-
plling of a single photon to a charge qubit was demonstrated in a pioneering work of c-QED [15]. Furthermore, modulated DC heat transport through a charge sensitive superconducting single electron transistor has been realized and explained with a simple theoretical model [46]. Charge qubits have experienced diminishing popularity in recent years since their charge sensitivity [47] leads to high dephasing rates in quantum information applications. We exploit their charge sensitive properties to create an efficient working substance which can be operated with remarkably small input signals. We further note that a charge sensitive qubit connecting two cavity resonators could be a fundamental component in the field of quantum information processing. The setup could allow interactions between distant transmon qubits to be controlled using voltage gates rather than magnetic flux lines - significantly reducing the power required to realize two-qubit gates, compared to tunable flux qubits or SQUIDs [48].

We experimentally demonstrate a charge sensitive qubit capacitively coupled to two $\lambda/4$ resonators of differing frequency. We utilize a charge qubit consisting of an 8 nm thick and 12 $\mu$m long superconducting island connected to ground through a single-Josephson junction. Using a single junction, rather than the more common two junction SQUID approach, allows us to achieve higher $E_c$ whilst reducing the sensitivity of the qubit frequency to stray magnetic fields. Furthermore, we implement the ability for GHz driving of the qubit via an on-chip voltage gate in close proximity. Through a novel gate-line filtering scheme, we can prevent microwave leakage from the resonators, and decay of the qubit, whilst maintaining the ability to drive at GHz frequencies. Our results show that, despite the sub-µm island dimensions, we can realize strong coupling of the qubit with two high-Q resonators without creating any spurious hybridized modes of the system. We go on to perform numerical simulations using a Markovian master equation [49, 50], in the presence of realistic experimental decoherence. Our work paves the way towards the operation of a two-level qubit as a quantum heat engine or refrigerator.

II. COOPER-PAIR BOX COUPLED TO TWO RESONATORS

The working substance of the quantum thermal device consists of a charge qubit operating deep in the charge sensitive regime ($E_c \sim 2E_j$). Here, the energy states correspond closely to the charge states, $|N\rangle$, on the island, and the Hamiltonian of the bare qubit is given by [38]

$$H_Q = \sum_N \left[ 4E_c(N-N_g)^2|N\rangle\langle N| - \frac{E_j}{2} (|N\rangle\langle N+1| + |N+1\rangle\langle N|) \right],$$

where $N_g = C_{gate}V_{gate}/2e$ is the dimensionless offset charge controlled by the nearby gate voltage $V_{gate}$, with capacitance $C_{gate}$. At sufficiently low temperature and with restricted $N_g$ around the degeneracy point ($N_g = 0.5$), we can consider only two charge states: $|0\rangle$ and $|1\rangle$, and approximate the charge qubit Hamiltonian as a two-level system, described by

$$H_Q = -2E_c(1-2N_g)\sigma^- - \frac{E_j}{2} \sigma^z,$$

where $\sigma^z, \sigma^x$ are the corresponding $2 \times 2$ Pauli matrices in the charge basis. The energy transition of the qubit will be

$$\hbar \omega_Q = \sqrt{16E_c^2(1-2N_g)^2 + E_j^2}.$$ 

In order to couple the working substance to the heat reservoirs, the qubit interacts capacitively with both voltage antinodes of two quarter wavelength resonators. The Hamiltonian of each resonator is $H_{R,i} = \hbar f_i a_i^\dagger a_i$ for $i = \{c, h\}$, with the qubit-resonator interaction terms given by [16]

$$H_{I,i} = g_{i,0}(a_i^\dagger + a_i)[1-2N_g - \cos(\theta)\sigma^z + \sin(\theta)\sigma^x],$$

where $\theta = \arctan(E_j/4E_c(1-2N_g))$ is the qubit mixing angle. The atom cavity coupling at the degeneracy point is determined by the zero-point energy fluctuations of the cavity electric potential, and given by [47]

$$g_{i,0} = \frac{eC_i}{C_{\Sigma}} \sqrt{\frac{\hbar f_i}{l_i e C_{\Sigma}}},$$

where $C_i, C_{\Sigma}$ are the coupler-island and island total capacitances respectively, $l_i$ is the resonator length, and $\tilde{c}$ is the capacitance per unit length. By adding a term accounting for the resonator-resonator coupling, $\propto \tilde{g}$, the full system Hamiltonian is therefore

$$H = H_Q + \sum_{i=c,h} (H_{R,i} + H_{I,i}) + \tilde{g}(a_{c,h}^\dagger a_{c,h} + a_{c,c}^\dagger a_{c,c}).$$

Operating close to the degeneracy point the full Hamiltonian can be reduced to a simplified two-cavity Jaynes-Cummings-type Hamiltonian. The effective coupling strength $g$, i.e., the coupling strength when operating away from the degeneracy point in resonance with the respective reservoir, is reduced by a factor $\sin(\theta)$ in this framework. The system could then be operated as a cyclic Otto refrigerator by implementing a time dependent $N_g(t)$ field, to modulate the qubit transition frequency, $\omega_Q(t)$, back and forth between the two resonators $f_c$ and $f_h$, further discussed in Sec. VII.

III. EXPERIMENTAL SETUP AND DEVICE

The measured device consists of a charge qubit, formed by a superconducting island connected to ground through a tunnel junction, capacitively coupled to two quarter
The model is calculated for $E_{\text{c}}/h = 6.8$ GHz, $E_{\text{j}}/h = 3.5$ GHz, $g_{\text{c,0}}/2\pi = 140$ MHz, $g_{\text{h,0}}/2\pi = 250$ MHz, and $\bar{g} = 0$.

The inset of Fig. 1(a) presents a scanning electron micrograph of the qubit, showing the 12 $\mu$m superconducting island, two Nb couplers and gate line. The Josephson energy, $E_{\text{j}} = 3.5$ GHz is controlled via the parameters of the oxidation, and can be estimated by measuring the normal-state resistance ($R_N = 42$ k$\Omega$) of replica junctions fabricated alongside the main structures. By using fork-shaped coupling structures on either side of the small superconducting island we maximize the coupling strength by increasing the ratio $C_{\text{i}}/C_{\Sigma}$, as shown in Eq. (5). Using COMSOL simulations we estimate the capacitances to be $C_{\text{c}} = C_{\text{h}} = 460$ aF, $C_{\text{gate}} = 5$ aF, and the measured $C_{\Sigma} = 2.7$ fF, allowing a remarkable $C_{\text{i}}/C_{\Sigma} = 17\%$ to be achieved each, competitively large even when compared with single-resonator systems [43, 47].

The two resonators are terminated to ground close to a common feedline, creating inductive coupling to allow excitation and readout. By coupling both resonators to a single feedline, we perform single-tone spectroscopy of the qubit in a wide frequency range, confirming the interaction of the qubit with each resonator. The resonators are read out through a notch-type measurement, by measuring the scattering parameter, $S_{21}$ [52]. Both resonators are overcoupled to the feedline (i.e. the coupling quality factor $Q_c$ is less than the internal quality factor $Q_i$), to allow straightforward measurements in the single-photon regime. All measurements are performed in a cryogen-free dilution refrigerator, with a base temperature $20$ mK. A detailed diagram and further description of the measurement setup can be found in App. B. Figure 1(c) shows the calibrated $S_{21}$ data as measured through the common feedline. Calibration is performed by measuring the same sample close to the critical temperature of the Nb film, and then correcting via plotting $|S_{21}(20 \text{ mK})/S_{21}(4 \text{ K})|$. In this way, impedance mismatches due to the circuit are removed and only the temperature dependent resonator structures remain. We can identify two peaks at $f_c = 4.718$ GHz and $f_h = 8.001$ GHz, corresponding to the two $\lambda/4$ resonators. We note the absence of any parasitic or hybridized modes, suggesting the resonator-resonator cross-talk is small.

**IV. SPECTROSCOPY OF THE RESONATOR-QUBIT-RESONATOR SYSTEM**

To confirm the interaction of the qubit with each resonator, we perform simultaneous one-tone spectroscopy of the resonator-qubit-resonator system. We use a vector network analyzer (VNA) to probe a frequency, $f_{\text{probe}}$, in the vicinity of the two resonator frequencies, whilst varying the DC gate voltage. The number of photons in the cavity is estimated to be less than 1 in both cases. Figure 2 shows the one-tone spectra in the proximity of the two frequencies, as a function of the dimensionless offset charge, $N_g$. The Rabi splitting associated with the interaction of a two-level system with a cavity is shown clearly for the hot resonator (Fig. 2(a)) and the cold resonator (Fig. 2(b)). Two periods are presented, symmetrically around $N_g = 1$. The solid white lines are fits using the numerical solutions to Eq. (6), with $E_{\text{c}}/h = 6.8$ GHz, $E_{\text{j}}/h = 3.5$ GHz, with excellent agreement with the theoretical model.

In our Cooper-pair box, the spatial profile of the superconducting gap energy is controlled by a thickness difference between the Al-island and Al-lead to suppress the quasiparticle-tunneling rate across the junction [53]. As a result, we observe one-Cooper-pair periodicity of the Rabi splitting in the one-tone spectroscopy. Never-
the effective coupling strengths are
$g$ where $\Delta$ at the degeneracy point and calculated by
$g$ by measuring the dispersive shift, $\chi$ the two lowest energy levels of the Cooper-pair box. The
decoherence, two-tone spectroscopy is performed to map
the performance of heat engines. In order to quantify the qubit
dissipation associated with charge sensitive devices when
compared with transmon type qubits. Although the raw
couplings, $g_{c,0}$, $g_{h,0}$, are large, due to the charge sensi-
tivity the effective couplings are reduced by a factor of
$\sin \theta$, and decrease as we move away from the degener-
acy point. We extract the effective coupling strengths by measuring the dispersive shift, $\chi$, of the resonance
at the degeneracy point and calculate by $g_i = \sqrt{\chi_i \Delta_i}$, where $\Delta_i = (f_i - E_J/h)$ is the detuning at degeneracy.
The effective coupling strengths are $g_h/2\pi = 125$ MHz and $g_c/2\pi = 76$ MHz, corresponding to 1.6% and 2% of
the resonance frequency respectively. The high coupling
strengths are of paramount importance to the operation
of the device as a quantum refrigerator, since the cool-
ing power in such systems has been shown to scale with
the square of the coupling strength [6]. Furthermore, the
extracted $E_J$ and $E_C$ can be confirmed experimentally
by using a two-tone spectroscopy technique, to probe the
exact qubit transition in the vicinity of the degeneracy
point, discussed in detail in Sec. V.

Importantly, due to the large $E_C$ and $C_{gate}$ in the mea-
sured device, we can achieve qubit control in a large
frequency range using remarkably small signals. Based
upon the measured system parameters, the qubit could be
driven sinusoidally between the two resonators using
a signal amplitude $V_{\text{rms}} = 1.2$ mV, corresponding to just
10 nW power at 50 $\Omega$. This presents a four orders-of-magnitude improvement over a comparable driving
scheme using an on-chip flux bias line. The presented
data is, to the authors knowledge, the first use of a charge sensitive qubit as a coupling element between two super-
conducting resonators.

\section{Decoherence of the Charge Qubit}

Energy loss from the working substance to the sur-
rounding media could play a fundamental role in the per-
formance of heat engines. In order to quantify the qubit
decoherence, two-tone spectroscopy is performed to map
the two lowest energy levels of the Cooper-pair box. The

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{(a) Two-tone spectroscopy of the charge qubit in the vicinity of the degeneracy point ($N_g = 0.5$), with fixed
probe tone $f_{\text{probe}} = 8.001$ GHz, and sweeping pump tone
$f_{\text{pump}}$ from 3.4 to 4.4 GHz. The solid white line indicates the
transition energy of the two-level system qubit solved from
the Hamiltonian Eq. (6) using SCQubits, for $E_c/h = 6.8$ GHz
and $E_J/h = 3.5$ GHz. The inset shows a frequency slice of
2D data, as indicated by the red arrow, and is fitted us-
ing a Lorentzian function. (b) Squared spectral linewidth
of the Lorentzian as a function of pump power, $P_{\text{pump}}$, av-
eraged by repeated sampling of the curves. The dashed
orange line presents a fit using Eq. (7). By extrapolation
($P_{\text{pump}} \rightarrow 0$, $n_p \rightarrow 0$), we estimate the dephasing time to be
$\Gamma_{2,1}/2\pi = 24$ MHz.}
\end{figure}
obtaining $E_c/h=6.8$ GHz and $E_j/h = 3.5$ GHz, in agreement with the results obtained in Sec. IV. The inset in Fig. 3(a) shows a slice at $N_p = 0.5$ (blue line) and is fitted by a Lorentzian function (orange line) with a linewidth $\delta f$. The spectral linewidth measured in this way relates to the longitudinal-relaxation rate, $\Gamma_{1,\perp}$, and phase-decoherence rate, $\Gamma_{2,\perp}$, of the qubit by

$$2\pi\delta f = \Gamma_{2,\perp}\sqrt{1 + \frac{4n_p\phi^2}{\Gamma_{1,\perp}\Gamma_{2,\perp}}}, \quad (7)$$

where $n_p$ is the pump photon number.

Figure 3(b) presents the dependence of the spectral linewidth squared for varying pump power, $P_{\text{pump}}$, showing the expected power dependence, as $n_p \propto P_{\text{pump}}$. The dashed-orange line shows a fit to the data using Eq. (7), allowing us to extract the spectral linewidth as $P_{\text{pump}} \to 0$ by extrapolation. Here, due to the low power of the pump signal, $n_p \to 0$, the linewidth $\delta f$ is dominated by qubit dissipation [58], thus $2\pi\delta f \sim \Gamma_{2,\perp}$, which is the qubit decoherence rate. The dissipation measured by this approach can be decomposed to the two relaxation processes, longitudinal relaxation, $\Gamma_{1,\perp}$, and pure dephasing, $\Gamma_\varphi$, related through the expression

$$\Gamma_{2,\perp} = \frac{\Gamma_{1,\perp}}{2} + \Gamma_\varphi. \quad (8)$$

In Sec. VII, we will consider the effect of both processes on the Otto refrigerator performance. Although our measurement technique does not allow us to extract the relative size of each contribution, previous experiments suggest that longitudinal relaxation is dominant close to the degeneracy point, where quasiparticle tunneling is the major contributor [59].

VI. VOLTAGE DRIVING AND SUPPRESSION OF MICROWAVE LEAKAGE

To operate our cyclic quantum refrigerator, sinusoidal or arbitrary voltage driving must be applied through the gate line to modulate the qubit energy level between the cold ($f_c$) and hot ($f_h$) resonator frequency, without introducing noise or microwave leakage through the driving line. Quarter-wave ($\lambda/4$) resonators, which have a voltage maximum at the open side, interact capacitively with the gate line, introducing some microwave leakage to the line. Filtering the operating range of the qubit (4-8 GHz) whilst allowing an AC-signal up to few GHz is pivotal for the operation of a quantum heat engine. We utilize a superconducting LCL-circuit acting as a band-pass filter [60], enabling us to prevent microwave leakage with around 20 dB attenuation and to drive the qubit up to a cutoff frequency of few GHz. Figure 4(a) shows the schematic circuit of an LCL-filter consisting of two series inductors slanted at the center by a capacitor. In the fabricated device, as shown in Fig. 4(b), the filter is realized by a meandering-line inductor ($L_{\text{LCL}}$) and an interdigitated capacitor ($C_{\text{LCL}}$) both with a central width and line-spacing of 4 $\mu$m.

To understand the transmission properties, the filter is separately characterized at 20 mK, as shown in Fig. 4(c), by measuring $S_{21}$ of a sinusoidal signal with frequency $f_{\text{drive}}$ between port 1 and port 2. The blue dots in Fig. 4(c) show the calibrated $S_{21}$ signal, and exhibit close to 100% transmission up to $\sim 2.3$ GHz and $> 20$ dB attenuation within the range of 4-10 GHz. Above 10 GHz, the attenuation starts to decrease and reaches 0 dB at $\sim 13$ GHz. A lumped circuit model of transmission derived from the ABCD matrix (dashed black line), discussed in detail in App. C, predicts the filter’s behavior up to $\sim 4$ GHz and deviates above it due to the parasitic capacitance of the meandering inductors, which is not taken into account in the model. From the fitting, the cut-off frequency $f_{\text{cutoff}} = \sqrt{2/L_{\text{LCL}}C_{\text{LCL}}}$ is obtained to be 2.3 GHz with $L_{\text{LCL}} = 5.9$ nH and $C_{\text{LCL}} = 1.7$ pF. A finite element simulation using SONNET.

![FIG. 4.](image-url) (a) Schematic lumped circuit of LCL-filter consisting of two series inductors grounded in the middle by a capacitor. (b) Scanning electron microscope (SEM) image of the fabricated LCL-filter comprising of two Nb-film meandering-line inductors and an interdigitated capacitor. (c) Transmission spectra, $S_{21}$, between port 1 and port 2 for driving frequencies $f_{\text{drive}}= 0.1-14$ GHz. Blue dots are measured data at 20 mK, the solid orange line and black dashed line are the SONNET simulation and analytical model respectively. The LCL cutoff frequency, $f_{\text{cutoff}}= 2.3$ GHz, is shown by the blue dashed line.
Following time dependence in the offset charge $Q(t)$, we consider Eq. (2) however allow-
ting cyclic quantum heat engines.

We consider driving the system with a truncated trapezoidal pattern, 
which $\Gamma_{\uparrow,\downarrow}$, $\Gamma_{\uparrow,\downarrow}$, $\Gamma_{\uparrow,\downarrow}$ and $\Gamma_{\uparrow,\downarrow}$ are the instantaneous jump opera-
tors of the system and $\{a, b\}$ defines the anti-commutator operation. Following [6, 37] we consider the dissipators to take the form of Johnson-Nyquist noise generated by a normal metal resistor, and spectrally filtered by the Lorentzian function of the resonators. The transition rates are therefore described by

$$\Gamma_{\uparrow,\downarrow} = \frac{g_i}{4\pi} \frac{1}{1 + Q_i^2(\omega_i/\omega_Q - \omega_Q/\omega_i)^2} \frac{\omega_Q}{1 - e^{-\hbar\omega_Q/k_B T_i}},$$

where $\omega_i = 2\pi f_i$ is the resonator frequency, $Q_i$ is the associated quality factor, $\omega_Q(t)$ is the instantaneous qubit frequency, and $T_i$ is the temperature of each normal-metal resistor terminating the resonator. The rates coming from the two heat baths, and the environmental bath, obey the detailed balance condition as

$$\Gamma_{\uparrow,\downarrow} = \Gamma_{\downarrow,\uparrow} e^{-\hbar\omega_Q/k_B T_i}.$$

The equivalent heat flow diagram, including two heat baths, an environmental bath, and the corresponding rates, is shown in Fig. 5. We simplify Eq. (11) by transforming to the rotating frame using $\tilde{\rho} = V(t)^\dagger \rho V(t)$ [61], where $V$ is the unitary matrix diagonalizing $H_Q(t)$. Then, parameterizing in terms of the Bloch equation elements, $x(t)$, $y(t)$, $z(t)$, the evolution results in the compact expressions

$$\rho(t) = \frac{-i}{\hbar} \left[ H_Q(t), \rho(t) \right] + \Gamma_\uparrow(t) \left[ \sigma_+ \rho(t) \sigma_- - \frac{1}{2} \{ \sigma_- \sigma_+, \rho(t) \} \right] + \Gamma_\downarrow(t) \left[ \sigma_- \rho(t) \sigma_+ - \frac{1}{2} \{ \sigma_+ \sigma_-, \rho(t) \} \right],$$

with $N_{g,\uparrow}$ and $N_{g,\downarrow}$ the start and initial state, respectively, obtained from rearranging Eq. (3).
\[ \dot{x}(t) = -\frac{\Gamma_\downarrow(t) + \Gamma_\uparrow(t)}{2} x(t) - \omega_Q(t) y(t) - \frac{8E_cE_J\dot{N}_g(t)}{\hbar^2\omega_Q^2} z(t), \tag{14} \]
\[ \dot{y}(t) = \omega_Q(t) x(t) - \frac{\Gamma_\downarrow(t) + \Gamma_\uparrow(t)}{2} y(t), \tag{15} \]
\[ \dot{z}(t) = \frac{8E_cE_J\dot{N}_g(t)}{\hbar^2\omega_Q(t)^2} x(t) - [\Gamma_\downarrow(t) + \Gamma_\uparrow(t)] z(t) - [\Gamma_\downarrow(t) - \Gamma_\uparrow(t)]. \tag{16} \]

![Plot](image)  
**FIG. 6.** Numerical simulations of cooling power in a Cooper-pair box acting as a quantum Otto refrigerator, for various values of the environment temperature. We use realistic values of the resonators quality factor $Q_e = Q_h = 3$, $T_e = T_h = 300$ mK, and with $\alpha = 2$. For the most pessimistic case of a 1 K environmental temperature we can achieve $\sim 40$ aW of cooling power, detectable using standard normal metal-insulator-superconductor thermometry techniques.

Furthermore, we can write the exact heat currents from each bath in terms of the elements of the Bloch vector as

\[ \dot{Q}_i(t) = \frac{\hbar \omega_Q(t)}{2} \left( [\Gamma_{i,\downarrow}(t) + \Gamma_{i,\uparrow}(t)] z(t) + [\Gamma_{i,\downarrow}(t) - \Gamma_{i,\uparrow}(t)] \right), \tag{17} \]

allowing us to characterize the refrigerator properties fully in the rotating-frame. Figure 6 shows the average power extracted from the hot and cold baths $\bar{\dot{Q}} = \frac{1}{T} \int_0^T \dot{Q}_i(t) dt$ as the red and blue solid lines for the measured system parameters, with $\Gamma_{2,\downarrow}/2\pi = 24$ MHz, as discussed in Sec. V, for a range of environmental bath temperatures, $T_e$. Variation of $T_e$ has the effect of changing the relative size of the excitation rate from the environment, $\Gamma_{2,\uparrow}$. The heat bath temperatures are set to be $T_e = T_h = 300$ mK, with $Q_e = Q_h = 3$.

For the unlikely case $T_e < T_{e,h}$, cooling is observed even at zero drive, as energy leaks from the baths to the environment, mediated by the qubit. When $T_e > T_{e,h}$, the environment has a detrimental effect, meaning some finite drive is required to compensate for the additional heat load of the surroundings. Importantly, even in the hottest case of $T_e = 1$ K, by $f_{\text{drive}} = 500$ MHz the heat extracted from the cold bath is already 40 aW, detectable using standard normal metal-insulator-superconductor thermometry techniques [22].

Conversely, we can briefly consider the unlikely situation in which pure-dephasing is the dominant contribution to the observed spectral width in Fig. 3 ($\Gamma_{1,\downarrow} \ll \Gamma_\phi$). In this case, dephasing due to the baths far exceeds any additional qubit dephasing rate. The pure dephasing is therefore negligible and the simulated heat current collapses to trivial case in which $\Gamma_{2,\downarrow} = \Gamma_{2,\uparrow} \sim 0$.

Fascinatingly, the driving rate in our system is high enough to observe quantum behavior in the refrigerator cooling power, whereby off-diagonal terms in the density matrix, $\rho$, could begin to affect the refrigerator performance. In the simulations, the quantum effects are clearly visible by sharp oscillations in the cooling power at high values of $f_{\text{drive}}$, as has been seen in previous theoretical studies involving qubit Otto refrigerators [6]. Dips are created when the frequency of the free qubit rotation about the Bloch sphere matches the driving frequency. In the future, to suppress this behavior, a counter-diabatic driving protocol could be implemented, which generally consists of an additional field to ‘guide’ the qubit along an adiabatic trajectory [37].

**VIII. CONCLUSION**

In summary, the presented device marks a significant milestone towards the realization of a cyclic quantum refrigerator using a c-QED platform. A charge sensitive qubit has been coupled to two superconducting coplanar waveguides for the first time, with the ability to drive the qubit over a large frequency range using remarkably small excitations. Additionally, the measured effective coupling strength of the qubit to each resonator remains exceptionally high, competitive with the highest previously measured in charge sensitive devices. Furthermore, we demonstrate that despite the close proximity of the various coupling elements, our system can be simply described within the framework of a two-level qubit interacting with two resonators. By adding normal-
metal resistors shunting each resonator to ground, the measured system could be operated as a quantum refrigerator. Utilizing the extracted device parameters, we have simulated the performance of our device acting as a quantum Otto refrigerator, and show cooling powers of the order \( \sim 40 \text{ aW} \). The calculated cooling power is easily detectable using normal metal-insulator-normal metal (NIS) thermometry. Additionally, the measured system could be used to realize a highly effective heat rectifier, owing to the large anharmonicity of the charge qubit, allowing the isolation of a single qubit transition. Our work truly lays the technical foundation towards the first studies of cyclic quantum heat engines within the c-QED framework, and opens the door towards a multitude of future studies in the field of quantum thermodynamics.

Appendix A: Fabrication Details

The fabrication of the device is done in a multi-stage process on a 675 \( \mu \text{m} \)-thick and highly resistive silicon substrate. The fabrication consists of two main steps: patterning microwave structures on a Nb film, and Josephson-junction elements on an Al film. A 40 nm-thick \( \text{Al}_2\text{O}_3 \) layer is deposited onto a silicon substrate using atomic layer deposition, followed by a deposition of a 200 nm-thick Nb film using DC magnetron sputtering. Positive electron beam resist, AR-P6200.13, is spin-coated with a speed of 5500 rpm for 60 s, and is post-baked for 9 minutes at 150°C, which is then patterned by electron beam lithography (EBL) and etched by reactive ion etching. A shadow mask defined by EBL on a 1 \( \mu \text{m} \)-thick poly(methyl-metacrylate)/copolymer resist bilayer is used to fabricate the Al island and Josephson junction using a two-angle deposition technique at 0° and 32° sequentially. Before the deposition, the Nb surface is cleaned in-situ by Ar ion plasma milling for 45 s, followed by first 8 nm-thick Al island deposition. The island then is oxidized at pressure 2.5 mbar for 2.5 minutes to form a tunnel barrier before depositing the second 100 nm Al film. Finally, after liftoff in acetone and isopropyl alcohol, the substrate is cut by an automatic dicing-saw machine to the size 7 \( \times \) 7 mm and wire-bonded to an RF-holder for the low-temperature characterization.

Appendix B: Experimental Details

Measurements are performed in a cryogen-free dilution refrigerator with a base temperature of 20 mK. Using a VNA, a probe microwave tone is supplied to the feedline through a 80 dB of attenuation distributed at the various temperature stages of the cryostat. The probe signal is then passed through two cryogenic circulators, before being amplified first by a 40 dB cryogenic amplifier, and secondly by a 40 dB room-temperature amplifier. The offset charge, \( N_o \), is supplied by a nearby voltage gate, with DC component passed though an low temperature

![Figure 7: Schematic of the experimental setup used for one and two-tone spectroscopy measurements.](image)

RC filter, LC filter, and eccosorb filter, and connected to an isolated voltage source at room temperature. The device is mounted in a tight cooper holder and covered by an Al-shield to protect from stray magnetic field and incident radiation.

Appendix C: LCL-Filter Transmission

The 2 \( \times \) 2 transmission matrix, or ABCD matrix, of a network is constructed by multiplying the ABCD matrices of each individual two-port element sequentially [62]. In the case of LCL network, the ABCD matrix is given by the multiplication of ABCD matrices of the inductor (\( Z_L \)) in series, capacitor (\( Z_C \)) in parallel and inductor (\( Z_L \)) in series, as schematically shown in Fig. 4(a) and formulated as follows

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
= \begin{pmatrix}
1 & Z_L(f) \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
1/Z_C(f) & 1
\end{pmatrix}
\begin{pmatrix}
1 & Z_L(f) \\
0 & 1
\end{pmatrix}
\]

(C1)

The voltage ratio between port 2 and port 1 (\( S_{21} \)), can then be calculated as

\[
S_{21}(f) = \frac{2}{A + B/Z_0 + CZ_0 + D}
\]

(C2)
where \( Z_L = j2\pi f L_{LCL}, Z_C = j/(2\pi f C_{LCL}), \) and \( Z_0 = \frac{1}{50} \Omega; L_{LCL}, \) and \( C_{LCL} \) are inductance and capacitance values of the \( LCL \)-filter.

### ACKNOWLEDGMENTS

We acknowledge Dr. Joonas Peltonen, Dr. Dmitry Golubev, Dr. George Thomas, Dr. Neill Lambert and Ilari Mäkinen for technical support and insightful discussions. This work is financially supported through the Foundational Questions Institute Fund (FQXi) via Grant No. FQXi-IAF19-06. Academy of Finland grants 312057, the Russian Science Foundation (Grant No. 20-62-46026) and from the European Union’s Horizon 2020 research and innovation programme under the European Research Council (ERC) (Grant No. 742559). We acknowledge the provision of facilities by Micronova Nanofabrication Centre and OtaNano - Low Temperature Laboratory of Aalto University to perform this research. We thank VTT Technical Research Center for sputtered Nb films.

### DATA AVAILABILITY STATEMENT

All data used in this paper are available upon request to the authors, including descriptions of the data sets, and scripts to generate the figures.

---

[1] R. Kosloff and A. Levy, “Quantum heat engines and refrigerators: Continuous devices,” Annu. Rev. Phys. Chem. 65, 365–393 (2014).

[2] R. Alicki, “The quantum open system as a model of the heat engine,” J. Phys. A: Math. Gen. 12, L103–L107 (1979).

[3] H. T. Quan, Yu-xi. Liu, C. P. Sun, and F. Nori, “Quantum thermodynamic cycles and quantum heat engines,” Phys. Rev. E 76, 031105 (2007).

[4] F. Binder, L. A. Correa, C. Gogolin, J. Anders, and G. Adesso, Thermodynamics in the Quantum Regime (Springer-Verlag GmbH, 2019).

[5] R. Kosloff, “Quantum thermodynamics: A dynamical viewpoint,” Entropy 15, 2100–2128 (2013).

[6] B. Karimi and J. P. Pekola, “Otto refrigerator based on a superconducting qubit: Classical and quantum performance,” Phys. Rev. B 94, 184503 (2016).

[7] A. O. Niskanen, Y. Nakamura, and J. P. Pekola, “Information entropic superconducting microcooler,” Phys. Rev. B 76, 174523 (2007).

[8] J. Roßnagel, S. T. Dawkins, K. N. Tolazzi, O. Abah, E. Lutz, F. Schmidt-Kaler, and K. Singer, “A single-atom heat engine,” Science 352, 325–329 (2016).

[9] M. Josefsson, A. Svilans, A. M. Burke, E. A. Hoffmann, S. Fahlvik, C. Thelander, M. Leijnse, and H. Linke, “A quantum-dot heat engine operating close to the thermodynamic efficiency limits,” Nat. Nanotechnol. 13, 920–924 (2018).

[10] J. P. S. Peterson, T. B. Batalhão, M. Herrera, A. M. Souza, R. S. Sarthour, I. S. Oliveira, and R. M. Serra, “Experimental characterization of a spin quantum heat engine,” Phys. Rev. Lett. 123, 240601 (2019).

[11] K. Ono, S. N. Shevechenko, T. Mori, S. Moriyama, and F. Nori, “Analog of a quantum heat engine using a single-spin qubit,” Phys. Rev. Lett. 125, 166802 (2020).

[12] J. Q. You and F. Nori, “Superconducting circuits and quantum information,” Phys. Today 58, 42–47 (2005).

[13] X. Gu, A. F. Kockum, A. Miranowicz, Y.-x. Liu, and F. Nori, “Microwave photonics with superconducting quantum circuits,” Phys. Rep. 718-719, 1–102 (2017).

[14] M. Kjaergaard, M. E. Schwartz, J. Braumüller, P. Krantz, J. I.-J. Wang, S. Gustavsson, and W. D. Oliver, “Superconducting qubits: Current state of play,” Annu. Rev. Condens. Matter Phys. 11, 369–395 (2020).

[15] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, “Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics,” Nature 431, 162–167 (2004).

[16] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, “Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation,” Phys. Rev. A 69, 062320 (2004).

[17] D. I. Schuster, A. A. Houck, J. A. Schreier, A. Wallraff, J. M. Gambetta, A. Blais, L. Frunzio, J. Majer, B. Johnson, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, “Resolving photon number states in a superconducting circuit,” Nature 445, 515–518 (2007).

[18] J. Clarke and F. K. Wilhelm, “Superconducting quantum bits,” Nature 453, 1031–1042 (2008).

[19] A. Montanaro, “Quantum algorithms: an overview,” npj Quantum Inf. 2, 15023 (2016).

[20] G. Wendin, “Quantum information processing with superconducting circuits: a review,” Rep. Prog. Phys. 80, 106001 (2017).

[21] Y.-C. Chang, B. Karimi, J. Senior, A. Ronzani, J. T. Peltonen, H.-S. Goan, C.-D. Chen, and J. P. Pekola, “Utilization of the superconducting transition for characterizing low-quality-factor superconducting resonators,” Appl. Phys. Lett. 115, 022601 (2019).

[22] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, “Opportunities for mesoscopics in thermometry and refrigeration: Physics and applications,” Rev. Mod. Phys. 78, 217–274 (2006).

[23] J. P. Pekola and B. Karimi, “Colloquium: Quantum circuit,” Nature 431, 2406-2409 (2007).

[24] A. Ronzani, B. Karimi, J. Senior, Y.-C. Chang, J. T. Peltonen, C. Chen, and J. P. Pekola, “Tunable photonic heat transport in a quantum heat valve,” Nat. Phys. 14, 991–995 (2018).
erators,” Phys. Rev. B 99, 224306 (2019).

[62] D. M. Pozar, Microwave engineering; 3rd ed. (Wiley, Hoboken, NJ, 2005).