Holographic thermal width of a quark antiquark pair in the presence of gluon condensation

Sara Tahery,\textsuperscript{a} Xurong Chen,\textsuperscript{b} Zi-qiang Zhang\textsuperscript{c}

\textsuperscript{a,b}Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
\textsuperscript{b}University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{b}Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China
\textsuperscript{c}School of Mathematics and Physics, China University of Geosciences, Wuhan 430074, China

Abstract

For a moving heavy quark antiquark in a QGP, we use gauge/gravity duality to study both real and imaginary parts of the potential in a gluon condensate theory. The complex potential is derived from the Wilson loop by considering the thermal fluctuations of the worldsheet of the Nambu-Goto holographic string \cite{1,2}. We calculate in both cases where the axis of the moving $Q\bar{Q}$ pair is transverse and parallel with respect to its direction of movement in the plasma. For the real part of potential we find that the inclusion of gluon condensate increases the dissociation length while rapidity has the opposite effect. For the imaginary part of the potential we observe that increasing gluon condensate leads to a reduction in the onset of the imaginary potential thus decreasing quarkonia dissociation, consistent with previous results on the entropic force. We also discuss the behavior of the thermal width of a moving quarkonia with gluon condensate.

Keywords: AdS/QCD, Gluon Condensation, Wilson Loop, Thermal worldsheet fluctuation.

\textsuperscript{1}E-mail: s.tahery@impcas.ac.cn
\textsuperscript{2}E-mail: xchen@impcas.ac.cn
\textsuperscript{3}E-mail: zhangzq@cug.edu.cn
1 Introduction

The heavy ion collisions at Relativistic Heavy Ion Collisions (RHIC) and Large Hadron Collider (LHC) have produced a new state of matter called quark-gluon plasma (QGP) \[3\]–\[5\]. One of the most experimental signatures of QGP formation is the dissociation of quarkonia, like \(c \bar{c}\) in the medium \[6\]. Most studies over the past years have found that the main mechanism responsible for this dissociation is color screening \[7\], however, recent studies suggest a more important reason than the screening, the imaginary part of the heavy quark potential, \(\text{Im}V\) \[8\]. Moreover, this quantity could be used to estimate the thermal width which is an important subject in QGP. Calculations of the \(\text{Im}V\) associated with QCD and heavy ion collisions were performed for static pairs using pQCD \[9\],\[10\], also from first principle and non perturbative method in lattice QCD the study has been done in \[11\]–\[13\] before the gauge/gravity duality. Theoretical analysis and experimental data demonstrate that QGP is strongly coupled \[14\], then non-perturbative methods are required. The equation of state of the QGP at zero and finite temperature are given in \[15\]–\[16\] by using non-perturbative LQCD. Using the AdS/CFT as another non-perturbative approach is helpful, although it does not describe the real QCD.

AdS/CFT conjecture originally relates the type IIB string theory on \(AdS_5 \times S^5\) space-time to the four-dimensional \(\mathcal{N} = 4\) SYM gauge theory \[17\]. In a holographic description of AdS/CFT, a strongly coupled field theory at the boundary of the AdS space is mapped onto the weakly coupled gravitational theory in the bulk of AdS \[18\]. The SYM plasma is a conformal system while QGP produced in heavy ion collisions is non-conformal. There is a relatively broad band of temperatures where the system changes the nature of its relevant degrees of freedom, and several different pseudocritical temperatures may be associated to characteristic points of different physical observables. Although the actual QGP is clearly different from the SYM plasma, at high temperatures the trace anomaly slowly approaches zero. Notice that classical gauge/gravity models are always strongly
coupled, so such models flow at asymptotically high temperatures to a nontrivial, strongly coupled
UV fixed point. They are not asymptotically free as in actual QCD but are asymptotically safe.

The bottom-up approach begins with a five-dimensional effective field theory somehow moti-
vated by string theory and tries to fit it to QCD as much as possible. In the gravitational dual of
QCD, the presence of probe branes in the AdS bulk breaks the conformal symmetry and sets the
energy scales so corrections in $AdS_5$ are useful to find more phenomenological results.

GC (gluon condensation) model is a holographic toy model with phenomenological applicabil-
ity as an effective model for the QGP. Originally, gluon condensation was a measure of the non-
perturbative physics in zero-temperature QCD [19], it is an order parameter for (de)confinement
hence could be a condition for the phase transition. The usual order parameter for the deconfin-
ment transition at finite temperature is the Polyakov loop. Also the Wilson loop can be used to
identify the (de)confined phases of pure YM theory by its area law behavior. However, there is no
order parameter for the real-world QGP, since LQCD already established that there is no actual
phase transition at zero and moderate baryon densities. GC model is useful to study the nonper-
turbative nature of the QGP [20–29], such as in RHIC physics [30]. In the mentioned references it
is shown that QCD sum rules is used to study the nonperturbative physics of the strong interaction
at zero temperature. In this approach, the nonperturbative nature of the vacuum is summarized
in terms of quark and gluon condensates. To study hot systems, one generalizes the technique to
finite temperature. The nonperturbative physics remaining even at high temperatures, is mani-
fested through the nonvanishing of some of the vacuum condensates. This model describes some
holographic plasma in equilibrium which is traversed by a probe heavy $Q\bar{Q}$ pair. Using holographic
 gluon condensation model, the thermodynamic properties of the system is discussed in [20], in
which the well-known Stephan-Boltzmann law with no condensation case can be recovered, also
the energy density for high temperature is given but for low temperature other back ground is
dominating. In the same reference, the dilaton (or gluon condensation) contribution of the energy
momentum tensor is identified as the difference of total and the thermal gluon, the gluon condensa-
tion contributes negative energy which is a reminiscent of the zero temperature result of Shifman,
Vainstein and Zakharov [19]. In both case, the negativeness is coming from the renormalization.
Also in [20] the pressure, the trace anomaly and the entropy density are given in presence of gluon
condensation, as it is expected, the entropy in condensed state is less than that in thermal state.

The potential of the pair describes the interaction energy between quark and anti-quark and
the thermal width of the $Q\bar{Q}$ is estimated by the imaginary part of the interaction energy at
finite temperature [31,32]. Using a holographic approach [33], one can adopt the saddle point
approximation and discuss the motion of a heavy quarkonia in a plasma and its thermal width.
The thermal width of the quark antiquark pair results from the effect of thermal fluctuations due
to the interactions between quarks and the strongly coupled medium. By integrating out thermal
long wavelength fluctuations in the path integral of the Nambu-Goto action in the background
spacetime, a formula for the imaginary part of the Wilson loop can be found in this approach that is valid for any gauge theory dual to classical gravity. Different AdS/QCD models were applied to study properties of hadron physics including the thermal width of the $Q\bar{Q}$ [34–59].

In this work we extend the study of the quark potential using the holographic gluon condensate model. Our main propose is to consider gluon condensation phenomena in QGP while it affects heavy quark potential. Note that the effect of the medium in the motion of a $Q\bar{Q}$ should be taken into account and the pair’s rapidity through the plasma has some effects on their interactions. In the LHC, the heavy quarkonia are not only produced in large numbers but also with high momenta so it is essential to consider the effect of bound state speed on dissociation [60]. The gluon condensate dependency of the heavy quark potential was studied in [61] and the results indicate that the potential becomes deeper as the gluon condensate in the deconfined phase decreases and the mass of the quarkonium drops near $T_c$ (the deconfinement temperature). The gluon condensate dependency of the jet quenching parameter and drag force was analyzed in [62] and it was found that the two quantities both decrease as the gluon condensate decreases in the deconfined phase, indicating that the energy loss decreases near $T_c$. In [63] it is shown that the dropping gluon condensate near $T_c$ increases the entropic force and thus enhances the quarkonium dissociation. In [64,65] gluon condensate shows a drastic change near $T_c$, in a pure gauge YM theory with a first order phase transition corresponds to a pure gluon plasma in the deconfined phase.

This paper is organized as follows, in the section 2 we study the real part of the potential for both cases in which the dipole moves transversely and parallel to the dipole axis. We proceed to calculate the imaginary potential for the two cases mentioned in the section 3. Section 4 contains results and conclusions.

## 2 Potential of moving $Q\bar{Q}$ in presence of gluon condensation

In this section we evaluate the real part of the potential energy of the moving quark-antiquark pair. The heavy quark potential (the vacuum interaction energy) is related to the vacuum expectation value of the Wilson loop [66–68] as,

$$\lim_{T \to 0} \langle W(\mathcal{C}) \rangle_0 \sim e^{iTV_{Q\bar{Q}}(L)},$$

where $\mathcal{C}$ is a rectangular loop of spatial length $L$ and extended over $T$ in the time direction. The expectation value of the Wilson loop can be evaluated in a thermal state of the gauge theory with the temperature $T$. From this point of view $V_{Q\bar{Q}}(L)$ is the heavy quark potential at finite temperature and its imaginary part defines a thermal decay width. To estimate the thermal width mentioned, one can use worldsheet fluctuations of the Nambu-Goto action [33]. Note that although the Nambu-Goto action on top of a background with a nontrivial dilaton field contains a coupling of this field with the Ricci scalar as it is shown in [69], but the contribution
of this term is small. Therefore the well-known modified holographic model introducing the gluon condensation in the boundary theory is given by the following background action,

\[ S = - \frac{1}{2k^2} \int d^5x \sqrt{g} \left( \mathcal{R} + \frac{12}{L^2} - \frac{1}{2} \partial_\lambda \varphi \partial^\lambda \varphi \right), \]  

(2)

where \( k \) is the gravitational coupling in 5-dimensions, \( \mathcal{R} \) is Ricci scalar, \( L \) is the radius of the asymptotic \( AdS_5 \) spacetime, and \( \varphi \) is a massless scalar which is coupled with the gluon operator on the boundary. By considering the following ansatz the equations of the above action could be solved, \([70–72]\),

\[ ds^2 = \frac{R^2}{z^2} (A(z) dx_i^2 - B(z) dt^2 + dz^2), \]  

(3)

where in this dilaton black hole background, \( A(z), B(z), f \) are defined as,

\[ A(z) = (1 + f z^4)^\frac{L a}{2z^7} (1 - f z^4)^\frac{-L a}{2z^7}, \]

\[ B(z) = (1 + f z^4)^\frac{L a}{2z^7} (1 - f z^4)^\frac{-L a}{2z^7}, \]

\[ f^2 = a^2 + c^2, \]  

(4)

\( a \) is related to the temperature by \( a = \left( \frac{\pi T}{4} \right)^4 \) and the dilaton field is given by,

\[ \phi(z) = c \frac{\sqrt{3}}{f} \ln \frac{1 + f z^4}{1 - f z^4} + \phi_0. \]  

(5)

In \([3]\) \( i = 1, 2, 3 \) are orthogonal spatial boundary coordinates, \( z \) denotes the 5th dimension, radial coordinate and \( z = 0 \) sets the boundary. \( \phi_0 \) in \([5]\) is a constant. We work in the unit where \( R = 1 \). Note that the dilaton black hole solution is well defined only in the range \( 0 < z < f^{-1/4} \), where \( f \) determines the position of the singularity and \( z_f \) behaves as an IR cutoff. For \( a = 0 \), it reduces to the dilaton-wall solution. Meanwhile, for \( c = 0 \), it becomes the Schwarzschild black hole solution. Also, for both solutions, expanding the dilaton profile near \( z = 0 \) will give,

\[ \varphi(z) = \varphi_0 + \sqrt{6} c z^4 + .... \]  

(6)

c is nothing but the holographic gluon condensation parameter. As discussed in \([20]\), there exists a Hawking–Page transition between the dilaton wall solution and dilaton black hole solution at some critical value of \( a \). So the former is for the confined phase, while the latter describes the deconfined phase. The gluon condensate \( G_2 \) is the vacuum expectation value of the operator \( \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \) where \( G_{\mu\nu}^a \) is the gluon field strength tensor. A non-zero trace of the energy-momentum tensor appears in a full quantum theory of QCD. The anomaly implies a non-zero gluon condensate which can be calculated as \([26,27,73]\),

\[ \Delta G_2(T) = G_2(T) - G_2(0) = -(\varepsilon(T) - 3 P(T)), \]  

(7)
where $G_2(T)$ denotes the thermal gluon condensate, $G_2(0)$, being equal to the condensate value at the deconfinement transition temperature, is the zero temperature condensate vale, $\varepsilon(T)$ is the energy density, $P(T)$ is the pressure of the QGP system.

To account for the effect of rapidity, one starts from a reference frame where the plasma is at rest and the dipole is moving with a constant velocity so it can be boosted to a reference frame where the dipole is at rest but the plasma is moving past it \[2\]. Consider a $Q\bar{Q}$ pair moving along $x_3$ direction with rapidity $\eta$. Correspondingly, we can consider a reference frame in which the plasma is at rest and the dipole moves with a constant rapidity $-\eta$ in the $x_3$ direction. Consider the following boost to a reference frame in which the dipole is at rest but the plasma is moving past it \[2\],

\[

dt \rightarrow dt \cosh \eta - dx_3 \sinh \eta \\
\ dx_4 \rightarrow -dt \sinh \eta + dx_3 \cosh \eta,
\]

if we transform the metric \[3\] with \[9\] we obtain,

\[
ds^2 = \frac{1}{z^2} \left( A(z) \, dx_1^2 + [\cosh^2 \eta \, A(z) - \sinh^2 \eta \, B(z)] \, dx_2^2 - [\cosh^2 \eta \, B(z) - \sinh^2 \eta \, A(z)] \, dt^2 \\
- 2[A(z) - B(z)] \, \sinh \eta \cosh \eta \, dx_3 \, dt + dz^2 \right), \tag{9}
\]

from now on, we can consider the dipole in the gauge theory, which has a gravitational dual with metric \[9\].

### 2.1 Pair alignment transverse to the axis of the quarks, ReV

Consider the dipole is moving transverse to the dipole axis. The spacetime target functions are $X^\mu = (\tau = t, \sigma = x_1, \text{cte, cte}, z(x, t))$. In static gauge we take $z(x, t) = z(x)$. The heavy quark-antiquark potential energy $V_{Q\bar{Q}}$ of this system is related to the expectation value of a rectangular Wilson loop,

\[
\langle W(C) \rangle \sim e^{-iS_{\text{str}}}, \tag{10}
\]

$S_{\text{str}}$ is the classical Nambu-Goto action of a string in the bulk,

\[
S_{\text{str}} = \frac{1}{2\pi \alpha'} \int d\sigma d\tau e^{\frac{i\phi(z)}{\alpha}} \sqrt{-\det(G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu)} \tag{11}
\]

Plugging back $S_{\text{str}}$ \[11\] in \[10\] we extract the real part of $V_{Q\bar{Q}}$. Starting from the metric \[9\], dilaton field \[5\], and the mentioned $X^\mu$ we get,

\[
S_{\text{str}} = \frac{T}{2\pi \alpha'} \int_{-L/2}^{L/2} d\sigma \sqrt{f_1(z) \cosh^2 \eta - f_2(z) \sinh^2 \eta + (f_3(z) \cosh^2 \eta - f_4(z) \sinh^2 \eta) z'\sigma}. \tag{12}
\]
The quarks are located at \( x_3 = \frac{L}{2} \) and \( x_3 = -\frac{L}{2} \), \( z' = \frac{dz}{d\sigma} \) and we defined,

\[
\begin{align*}
f_1(z) &= \frac{\omega^2(z)}{z^4} A(z) B(z), \\
f_2(z) &= \frac{\omega^2(z)}{z^4} A^2(z), \\
f_3(z) &= \frac{\omega^2(z)}{z^4} B(z), \\
f_4(z) &= \frac{\omega^2(z)}{z^4} A(z),
\end{align*}
\]

and,

\[
\omega(z) = e^{\frac{\phi(z)}{2}} = \frac{1 + f(z)^4}{1 - f(z)^4} \frac{1}{2} \sqrt{\frac{1}{\tau}}.
\]

We also write,

\[
\begin{align*}
F(z) &= f_1(z) \cosh^2 \eta - f_2(z) \sinh^2 \eta, \\
G(z) &= f_3(z) \cosh^2 \eta - f_4(z) \sinh^2 \eta.
\end{align*}
\]

So action (12) could be written as,

\[
S_{str} = \frac{T}{2\pi \alpha'} \int_{-L/2}^{L/2} d\sigma \sqrt{F(z) + G(z) z'^2(\sigma)}.
\]

The action depends only on \( \sigma = x \) and the associated Hamiltonian is a constant of the motion. With the corresponding position of the deepest position in the bulk being \( z_* \), Hamiltonian is,

\[
H = \frac{F(z)}{\sqrt{F(z) + G(z) z'^2(\sigma)}} = cte = \sqrt{F(z_*)}.
\]

From the Hamiltonian (17), we can write the equation of motion for \( z(x) \) as,

\[
\frac{dz}{dx} = \left[ \frac{F(z)}{G(z)} \left( \frac{F(z)}{F(z_*)} - 1 \right) \right]^{-\frac{1}{2}}.
\]

Therefore,

\[
\frac{dx}{dz} = \left[ \frac{F(z)}{G(z)} \left( \frac{F(z)}{F(z_*)} - 1 \right) \right]^{-\frac{1}{2}} dz,
\]

and we can relate \( L \) to \( z_* \) as follows,

\[
\frac{L}{2} = \int_0^{z_*} \left[ \frac{F(z)}{G(z)} \left( \frac{F(z)}{F(z_*)} - 1 \right) \right]^{-\frac{1}{2}} dz.
\]

From (20) we find the length of the line connecting both quarks as,

\[
L = 2 \sqrt{F(z_*)} \int_0^{z_*} \left[ \frac{G(z)}{F(z)(F(z) - F(z_*))} \right]^{\frac{1}{2}} dz.
\]
In the literature [74,75] the maximum value of the above length has been used to define a dissociation length for the moving $Q\bar{Q}$ pair, where the dominant configuration for $S_{str}$ is two straight strings (two heavy quarks) running from the boundary to the horizon.

If we put (18) in (16) the action is written as follows,

$$S_{str} = \frac{T}{\pi\alpha'} \int_0^{z_*} dz \sqrt{G(z)} \left[ \frac{F(z)}{F(z_*)} - 1 \right]^{-1/2}. \quad (22)$$

To regularize the above integral, we write,

$$S_{str}^{\text{reg}} = \frac{T}{\pi\alpha'} \int_0^{z_*} dz \sqrt{G(z)} \left[ \frac{F(z)}{F(z_*)} - 1 \right]^{-1/2} - \frac{T}{\pi\alpha'} \int_0^\infty dz \sqrt{f_3^0(z)}, \quad (23)$$

where $f_3^0(z) = f_3(z)|_{a-0}$ (quark self energy). Finally, we proceed from $\text{Re} V_{Q\bar{Q}} = S_{str}^{\text{reg}} / T$ to,

$$\text{Re} V_{Q\bar{Q}} = \frac{\sqrt{\lambda}}{\pi} \int_0^{z_*} dz \sqrt{G(z)} \left[ \frac{F(z)}{F(z_*)} - 1 \right]^{-1/2} - \frac{\sqrt{\lambda}}{\pi} \int_0^\infty dz \sqrt{f_3^0(z)}, \quad (24)$$

where $\lambda = \frac{1}{\alpha'^2}$ is the ’t Hooft coupling of the gauge theory. Figure 1 shows the real part of the potential as a function of $L$ with the $Q\bar{Q}$ pair oriented transverse to the axis of the quarks, in the presence of gluon condensation. The results show that increasing rapidity leads to a decrease in dissociation length while $c$ has the opposite effect.

2.2 Pair alignment parallel to the axis of the quarks, $\text{Re} V$

In this step we consider that the dipole moves parallel to the dipole axis. The spacetime target functions are $X^\mu = (\tau = t, \sigma = cte, cte, x_3, z(x,t))$ and in the static gauge we take $z(x,t) = z(x)$.
Using steps similar to (16) we get the action with the new worldsheet as,
\[
S_{str} = \frac{T}{2\pi \alpha'} \int_{-L/2}^{L/2} d\sigma \sqrt{f_1(z) + G(z)z''(\sigma)},
\]  
(25)
where \(G(z)\) and \(f_1(z)\) are defined as (15) and (13). Similar to the transverse case, we find the line connecting both quarks as,
\[
L = 2\sqrt{f_1(z_*)} \int_0^{z_*} \left[ \frac{G(z)}{f_1(z)(f_1(z) - f_1(z_*))} \right]^{1/2} dz,
\]  
(26)
and the real part of the potential as,
\[
\text{Re } V_{Q\bar{Q}} = \frac{\sqrt{\lambda}}{\pi} \int_0^{z_*} dz \sqrt{G(z)} \sqrt{\frac{f_1(z)}{f_1(z_*)}} \left[ \frac{f_1(z)}{f_1(z_*)} - 1 \right]^{-1/2} - \frac{\sqrt{\lambda}}{\pi} \int_0^\infty dz \sqrt{f_0^3(z)}.
\]  
(27)

Figure 2: \(\text{Re } V_{Q\bar{Q}}\) as a function of \(L\) for a \(Q\bar{Q}\) pair oriented parallel to the axis of the quarks, from top to bottom for \(\eta = 0, 0.4, 0\) respectively, in the presence of gluon condensation, for a) \(c = 0.02\) GeV\(^4\) and b) \(c = 0.9\) GeV\(^4\).

Figure 2 shows the real part of potential as a function of \(LT\) for some choices of \(\eta\) where \(Q\bar{Q}\) pair oriented parallel to the axis of the quarks, in presence of gluon condensation. Similar to previous case, increasing rapidity leads to decreasing the dissociation length while \(c\) has the opposite effect.

Figure 3 shows a comparison between the real part of the potential for the parallel and the transverse cases. Although the difference is not significant, the plots show that the effect of the gluon condensation is slightly stronger for the parallel case. In other words, increasing \(c\) increases the dissociation length in both the transverse and the parallel cases (previous figures), this effect appears stronger when the dipole moves parallel to the axis of the quarks.
Figure 3: Re$V_{Q\bar{Q}}$ as a function of $L$, for fixed value of $\eta$ and fixed value of $c$, as a comparison between the parallel and the transverse cases. The solid black line shows parallel case and the dashed red line shows transverse case.

3 Imaginary potential of moving $Q\bar{Q}$ in presence of gluon condensation

In this section, we calculate the imaginary potential using the thermal worldsheet fluctuations method for both the transverse and parallel cases.

3.1 Pair alignment transverse to the axis of the quarks, Im$V$

Consider the effect of worldsheet fluctuations around the classical configuration $r = \frac{1}{z}$,

$$r(x) = r_*(x) \rightarrow r(x) = r_*(x) + \delta r(x),$$

then the fluctuations in the partition function should be considered as follows,

$$Z_{str} \sim \int D\delta r(x) e^{iS_{NC}(r_*(x)+\delta r(x))}. \quad (29)$$

Hence there is an imaginary part of the potential in the action. Dividing the interval of $x$ into $2N$ points (where $N \rightarrow \infty$) we obtain,

$$Z_{str} \sim \lim_{N \rightarrow \infty} \int d[\delta r(x_{-N})] \ldots d[\delta r(x_N)] \exp \left[ i \frac{T \Delta x}{2\pi \alpha'} \sum_j \sqrt{G r_j^2 + F} \right], \quad (30)$$
where $\tilde{G}$ and $\tilde{F}$ are functions of $r_j$. We expand $r_*(x_j)$ around $x = 0$ and keep only terms up to second order of it because thermal fluctuations are important around $r_*$ which means $x = 0$,

$$r_*(x_j) \approx r_* + \frac{x_j^2}{2} r_*''(0),$$  \hspace{1cm} (31)

considering small fluctuations we have,

$$\tilde{F} \approx \tilde{F} + \delta r \tilde{F}' + r_*''(0) \tilde{F}' \frac{x_j^2}{2} + \frac{\delta r^2}{2} \tilde{F}'',$$  \hspace{1cm} (32)

where $\tilde{F}_* \equiv \tilde{F}(r_*)$ and $\tilde{F}'_* \equiv \tilde{F}'(r_*)$. The action is written as,

$$S_{j}^{NG} = \frac{T}{2\pi\alpha'} \sqrt{C_1 x_j^2 + C_2},$$  \hspace{1cm} (33)

where $C_1$ and $C_2$ are given as follows,

$$C_1 = \frac{r_*''(0)}{2} \left[ 2\tilde{G}_* r_*''(0) + \tilde{F}'_* \right],$$  \hspace{1cm} (34)

$$C_2 = \tilde{F}_* + \delta r \tilde{F}'_* + \frac{\delta r^2}{2} \tilde{F}''_*,$$  \hspace{1cm} (35)

to have $\text{Im}V_{QQ} \neq 0$, the function in the square root (33) should be negative. Then, we consider the j-th contribution to $Z_{str}$ as,

$$I_j \equiv \int_{\delta r_{j\text{min}}}^{\delta r_{j\text{max}}} D(\delta r_j) \exp \left[ i \frac{T}{2\pi\alpha'} \sqrt{C_1 x_j^2 + C_2} \right],$$  \hspace{1cm} (36)

$$D(\delta r_j) \equiv C_1 x_j^2 + C_2(\delta r_j),$$  \hspace{1cm} (37)

$$\delta r = -\frac{\tilde{F}_*'}{\tilde{F}_*''},$$  \hspace{1cm} (38)

so, $D(\delta r_j) < 0 \implies -x_0 < x_j < x_0$ leads to an imaginary part in the square root. We write,

$$x_* = \sqrt{\frac{1}{C_1} \left[ \frac{\tilde{F}_*''^2}{2\tilde{F}_*''} - \tilde{F}_* \right]},$$  \hspace{1cm} (39)

if the square root is not real we should take $x_* = 0$. With all these conditions we can approximate $D(\delta r)$ by $D(-\frac{\tilde{F}_*'}{\tilde{F}_*''})$ in $I_j$ as,

$$I_j \sim \exp \left[ i \frac{T}{2\pi\alpha'} \sqrt{C_1 x_j^2 + \tilde{F}_* - \frac{\tilde{F}_*''^2}{2\tilde{F}_*''} \right].$$  \hspace{1cm} (40)
The total contribution to the imaginary part will be available with a continuum limit. So,

$$\text{Im} V_{Q\bar{Q}} = -\frac{1}{2\pi \alpha'} \int_{|x|<x_{*}} dx \sqrt{-x^2 C_1 - \tilde{F}_s + \frac{\tilde{F}^2_s}{2F''_s}}, \quad (41)$$

which leads to,

$$\text{Im} V_{Q\bar{Q}} = -\frac{1}{2\sqrt{2} \alpha'} \sqrt{G_s z_2^2 \left[ \frac{F_s}{z_2^2 F'_s} - \frac{z_2^2 F'_s}{4z_2^2 F'_s + 2z_2^4 F''_s} \right]}, \quad (42)$$

Note that (42) is the imaginary potential with the $r$ coordinate. Changing the variable back to the coordinate $z = \frac{1}{r}$ according to our background, we will have,

$$\text{Im} V_{Q\bar{Q}} = -\frac{1}{2\sqrt{2} \alpha'} \sqrt{G_s z_2^2 \left[ \frac{F_s}{z_2^2 F'_s} - \frac{z_2^2 F'_s}{4z_2^2 F'_s + 2z_2^4 F''_s} \right]}, \quad (43)$$

where $F$ is again a function of $z$. In (43) the following condition should be satisfied for the square root,

$$\frac{B(z_*)}{A(z_*)} > \tanh^2 \eta. \quad (44)$$

Figure 4 shows the imaginary potential as a function of $LT$ for a $Q\bar{Q}$ pair oriented transverse to the axis of the quarks, from left to right for $\eta = 0.8, 0.4, 0$ respectively, in the presence of gluon condensation, for a) $c = 0.02$ GeV$^4$ and b) $c = 0.9$ GeV$^4$.
which is consistent with the results of the reference \[76]. Thus our results show that the $Q\bar{Q}$ pair’s thermal width decreases with increasing its rapidity relative to the plasma, while $c$ has the opposite effects.

### 3.2 Pair alignment parallel to the axis of the quarks, $\text{Im} V$

Taking action (25) and using the same approach we followed to find (43), we get the imaginary potential of a pair moving parallel to the axis of the quarks as,

$$\text{Im} V_{Q\bar{Q}} = -\frac{1}{2\sqrt{2}\alpha'} \sqrt{G} z^2 \left[ \frac{f_{1*}}{z^2 f'_{1*}} - \frac{z^2 f'_{1*}}{4z^4 f'_{1*} + 2z^4 f''_{1*}} \right].$$

(45)

Figure 5: $\text{Im} V_{Q\bar{Q}}$ as a function of $LT$ for a $Q\bar{Q}$ pair oriented parallel to the axis of the quarks, from left to right for $\eta = 0.8$, 0.4, 0 respectively, in the presence of gluon condensation, for a) $c = 0.02 \text{ GeV}^4$ and b) $c = 0.9 \text{ GeV}^4$.

Figure 5 shows the imaginary potential as a function of $LT$ for some choices of $\eta$ where $Q\bar{Q}$ pair oriented parallel to the axis of the quarks, in the presence of gluon condensation. The $c$ parameter in the gravitational dual, introduces the gluon condensation in the QCD side of duality. Therefore with increasing gluon condensation the imaginary part of the potential starts to become nonzero for larger values of $LT$ and the onset of the imaginary potential happens for larger $LT$ and $\text{Im} V$. Our results thus indicate that the thermal width of the $Q\bar{Q}$ pair increases with increasing gluon condensation. Similar to the transverse case, these effects are the opposite of the rapidity effects. Figure 6 shows a comparison between the imaginary part of the potential for the parallel and the transverse cases. Similar to $\text{Re} V$ in figure 3, the plots show that the effect of the gluon condensation is stronger for the parallel case. As shown previously, increasing $c$ increases the thermal width in both the transverse and the parallel cases. This effect appears stronger when the dipole moves parallel to the axis of the quarks. While the magnetic field \[40\] and the chemical potential effects
were more important for the transverse case, in the parallel case the gluon condensation has a stronger impact.

4 Conclusions

In this work we investigated the potential of a moving quark antiquark pair in a plasma considering the effect of gluon condensation. Taking into account the thermal fluctuations of the worldsheet of the holographic Nambu-Goto string, we calculated thermal width of the moving quark antiquark pair for the cases where the axis of the moving $Q \bar{Q}$ pair is transverse and parallel with respect to its rapidity in the plasma. Our results indicate that increasing gluon condensation results in an increase in dissociation length. We found the dependency of $ImV_{Q\bar{Q}}$ on the rapidity and gluon condensation. While the thermal width of the pair is heavily suppressed as a function of $\eta$, the $c$ could be considered as an amplifying parameter of thermal width. In other words, the thermal width of the $Q\bar{Q}$ pair increases with increasing gluon condensation, which is the opposite of the rapidity effect. As it is found in [63] in the presence of gluon condensation, increasing temperature leads to easier quarkonium melting and the dropping gluon condensate near $T_c$ enhances the quarkonium dissociation which agrees with our results. It would be interesting to check analytically whether the nonzero imaginary potential found by continuing the string configurations into the complex plane [43] agrees with current results. We hope to work on this topic in the future.
Acknowledgement
Authors would like to thank Kazem Bitaghsir Fadafan for useful comments. This work was supported by Strategic Priority Research Program of Chinese Academy of Sciences (XDB34030301) and the CAS President’s International Fellowship Initiative, PIFI (2021PM0065).

References

[1] S. I. Finazzo and J. Noronha, “Estimates for the Thermal Width of Heavy Quarkonia in Strongly Coupled Plasmas from Holography”, JHEP 11 (2013) 042 [arXiv: 1306.2613 [hep-ph]].

[2] S. I. Finazzo and J. Noronha, “Thermal suppression of moving heavy quark pairs in strongly coupled plasma” JHEP 01 (2015) 051 [arXiv: 1406.2683 [hep-th]].

[3] STAR Collaboration: J. Adams, et al, “Experimental and Theoretical Challenges in the Search for the Quark Gluon Plasma: The STAR Collaboration’s Critical Assessment of the Evidence from RHIC Collisions”, Nucl.Phys.A 757 (2005) 102 [arXiv: nucl-ex/0501009].

[4] PHENIX Collaboration, K. Adcox, “Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX collaboration”, Nucl.Phys.A 757 (2005) 184 [arXiv: nucl-ex/0410003].

[5] E.V.Shuryak, “What RHIC Experiments and Theory tell us about Properties of Quark-Gluon Plasma ?”, Nucl.Phys.A 750 (2005) 64 [arXiv: hep-ph/0405066].

[6] E. V. Shuryak, “Quantum Chromodynamics and the Theory of Superdense Matter”, Phys.Rept. 61 (1980) 71.

[7] T.Matsui, H.Satzab, “J/ψ suppression by quark-gluon plasma formation”, Phys.Lett.B 178 (1986) 416.

[8] L. Thakur, N. Haque, H. Mishra, “Heavy quarkonium moving in hot and dense deconfined nuclear matter”, Phys.Rev.D 95 (2017) 036014 [arXiv: 1611.04568 [hep-ph]].

[9] M. Laine, O. Philipsen, P. Romatschke and M. Tassler, “Real-time static potential in hot QCD”, JHEP 03 (2007) 054 [arXiv: hep-ph/0611300].

[10] M. Laine, “A Resummed perturbative estimate for the quarkonium spectral function in hot QCD”, JHEP 05 (2007) 028 [arXiv: 0704.1720 [hep-ph]].

[11] A. Rothkopf, T. Hatsuda and S. Sasaki, “Complex Heavy-Quark Potential at Finite Temperature from Lattice QCD”, Phys.Rev.Lett 108 (2012) 162001 [arXiv: 1108.1579 [hep-lat]].
[12] G. Aarts, C. Allton, S. Kim, M. P. Lombardo, M. B. Oktay, S. M. Ryan, D. K. Sinclair and J. I. Skullerud, “What happens to the Upsilon and \( \eta_b \) in the quark-gluon plasma? Bottomonium spectral functions from lattice QCD”, JHEP 11 (2011) 103 [arXiv: 1109.4496 [hep-lat]].

[13] G. Aarts, C. Allton, S. Kim, M. P. Lombardo, S. M. Ryan and J. I. Skullerud, “Melting of \( P \) wave bottomonium states in the quark-gluon plasma from lattice NRQCD”, JHEP 12 (2013) 064 [arXiv: 1310.5467 [hep-lat]].

[14] E. Shuryak, “Why does the Quark-Gluon Plasma at RHIC behave as a nearly ideal fluid?”, Prog.Part.Nucl.Phys. 53 (2004) 273 [arXiv: hep-ph/0312227].

[15] S. Borsanyi, Z. Fodor, Ch. Hoelbling, S.D. Katz, S. Krieg, K.K. Szabo, “Full result for the QCD equation of state with \( 2 + 1 \) flavors”, Phys. Lett. B 370 (2014) 99 [arXiv: 1309.5258 [hep-lat]].

[16] A. Bazavov, T. Bhattacharya, C. DeTar, H.-T. Ding, S. Gottlieb, R. Gupta, P. Hegde, U.M. Heller, F. Karsch, E. Laermann, L. Levkova, S. Mukherjee, P. Petreczky, C. Schmidt, C. Schroeder, R.A. Soltz, W. Soeldner, R. Sugar, M. Wagner, P. Vranas, “The equation of state in \( (2+1) \)-flavor QCD”, Phys. Rev. D 90 (2014) 094503 [arXiv: 1407.6387 [hep-lat]].

[17] J. M. Maldacena, “The Large \( N \) limit of superconformal field theories and supergravity”, Adv.Theor.Math.Phys 2 (1998) 231 [arXiv: hep-th/9711200].

[18] E. Witten, “Anti-de Sitter space and holography”, Adv.Theor.Math.Phys 2 (1998) 253 [arXiv: hep-th/9802150].

[19] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, “QCD and resonance physics. theoretical foundations”, Nucl.Phys.B 147 (1979) 385.

[20] Y. Kim, B.-H. Lee, Ch. Park, S.-J. Sin, “Gluon Condensation at Finite Temperature via AdS/CFT”, JHEP 09 (2007) 105 [arXiv: hep-th/0702131].

[21] S.H. Lee, “Gluon condensates above \( T_c \)”, Phys.Rev.D 40 (1989) 2484.

[22] M.D. Elia, A. D. Giacomo, E. Meggiolaro, “Gauge-invariant field-strength correlators in pure Yang-Mills theory and full QCD at finite temperature”, Phys.Rev.D 67 (2003) 114504 [arXiv: hep-lat/0205018].

[23] D.E. Miller, “Lattice QCD Calculation for the Physical Equation of State”, Phys.Rept 443 (2007) 55 [arXiv: hep-ph/0608234].

[24] R. C. Quevedo, J. L. Goity, R. Trinchero, “QCD condensates and holographic Wilson loops for asymptotically AdS spaces”, Phys.Rev.D 89 (2014) 036004 [arXiv: 1311.1175 [hep-ph]].
[25] Sh. Nojiri, S. D. Odintsov, “Two-Boundaries AdS/CFT Correspondence in Dilatonic Gravity”, Phys.Lett.B 449 (1999) 39 [arXiv: hep-th/9812017].

[26] P. Colangelo, F. Giannuzzi, S. Nicotri, F. Zuo, “Temperature and chemical potential dependence of the gluon condensate: a holographic study”, Phys.Rev.D 88 (2013) 115011 [arXiv: 1308.0489 [hep-ph]].

[27] P. Castorina, M. Mannarelli, “Effective degrees of freedom and gluon condensation in the high temperature deconfined phase”, Phys.Rev.C 75 (2007) 054901 [arXiv: hep-ph/0701206].

[28] P.N. Kopnin, A. Krikun, “Wilson loops in holographic models with a gluon condensate”, Phys.Rev.D 84 (2011) 066002 [arXiv:1106.4978 [hep-th]].

[29] X. Chen, D. Li, M. Huang, “Criticality of QCD in a holographic QCD model with critical end point”, Chin.Phys.C 43 (2019) 2, 023105 [arXiv: 1810.02136 [hep-ph]].

[30] G. E. Brown, J. W. Holt, Ch. H. Lee, M. Rho, “Vector manifestation and matter formed in relativistic heavy-ion processes”, Phys.Rept 439 (2007) 161.

[31] N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, “Heavy Quarkonium in a weakly-coupled quark-gluon plasma below the melting temperature”, JHEP 09 (2010) 038 [arXiv: 1007.4156 [hep-ph]].

[32] Y. Guo and M. Strickland, “The imaginary part of the static gluon propagator in an anisotropic (viscous) QCD plasma”, Phys.Rev.D 79 (2009) 114003 [arXiv: 0903.4703 [hep-ph]].

[33] J. Noronha, A. Dumitru, “Thermal Width of the Υ at Large t’ Hooft Coupling”, Phys.Rev.Lett 103 (2009) 152304 [arXiv: 0907.3062 [hep-ph]].

[34] M. Strickland, “Thermal Upsilon(1s) and χc1 suppression in sqrt(s_{NN}) = 2.76TeV Pb-Pb collisions at the LHC”, Phys.Rev.Lett 107 (2011) 132301 [arXiv: 1106.2571 [hep-ph]].

[35] M. Strickland and D. Bazow, “Thermal Bottomonium Suppression at RHIC and LHC”, Nucl.Phys.A 879 (2012) 25 [arXiv: 1112.2761 [nucl-th]].

[36] M. Margotta, K. McCarty, C. McGahan, M. Strickland, and D. Yager-Elorriaga, “Quarkonium states in a complex-valued potential”, Phys.Rev.D 83 (2011) 105019 [arXiv: 1101.4651 [hep-ph]].

[37] G.Aarts, C. Allton, S. Kim, M. P. Lombardo, M. B. Oktay, S. M. Ryan, D. K. Sinclair and J. I. Skullerud, “S wave bottomonium states moving in a quark-gluon plasma from lattice NRQCD”, JHEP 03 (2013) 084 [arXiv: 1210.2903 [hep-lat]].
[38] M. Ali-Akbari, D. Giataganas, Z. Rezaei, “The Imaginary Potential of Heavy Quarkonia Moving in Strongly Coupled Plasma”, Phys.Rev.D 90 (2014) 086001 [arXiv: 1406.1994 [hep-th]].

[39] J. Sadeghi, S. Tahery, “The effects of deformation parameter on thermal width of moving quarkonia in plasma”, JHEP 06 (2015) 204 [arXiv: 1412.8332 [hep-th]].

[40] Z. q. Zhang, D.f Hou, “Imaginary potential in strongly coupled $\mathcal{N} = 4$ SYM plasma in a magnetic field”, Phys.Lett.B 778 (2018) 227 [arXiv: 1802.01919 [hep-th]].

[41] S. Tahery, J. Sadeghi, “The investigation of quark - antiquark potential in plasma with hyperscaling violation background”, J.Phys.G 44 (2017) 105001 [arXiv: 1509.01309 [hep-th]].

[42] Y. Q. Zhao, Zh. R. Zhu, X. Chen, “The effect of gluon condensate on imaginary potential and thermal width from holography”, Eur.Phys.J.A 56 (2020) 57 [arXiv: 1909.04994 [hep-ph]].

[43] J. L. Albacete, Y. V. Kovchegov, A. Taliotis, “Heavy Quark Potential at Finite Temperature Using the Holographic Correspondence”, Phys.Rev.D 78 (2008) 115007 [arXiv: 0807.4747 [hep-th]].

[44] K. Bitaghsir Fadafan, D. Giataganas, H. Soltanpanahi, “The Imaginary Part of the Static Potential in Strongly Coupled Anisotropic Plasma”, JHEP 11 (2013) 107 [arXiv: 1306.2929 [hep-th]].

[45] K. Bitaghsir Fadafan, S. K. Tabatabaei, “Thermal Width of Quarkonium from Holography”, Eur.Phys.J.C 74 (2014) 2842 [arXiv: 1308.3971 [hep-th]].

[46] K. Bitaghsir Fadafan, S. K. Tabatabaei, “The Imaginary Potential and Thermal Width of Moving Quarkonium from Holography”, J.Phys.G 43 (2016) 9, 095001 [arXiv: 1501.00439 [hep-th]].

[47] N. R. F. Braga, L. F. Ferreira, “Thermal width of heavy quarkonia from an AdS/QCD model”, Phys.Rev.D 94 (2016) 094019 [arXiv: 1606.09535 [hep-th]].

[48] J. Erlich, E. Katz, D. T. Son, M. A. Stephanov, “QCD and a holographic model of hadrons”, Phys.Rev.Lett. 95 (2005) 261602 [arXiv: hep-ph/0501128].

[49] G. F. de Teramond, S. J. Brodsky, “Hadronic spectrum of a holographic dual of QCD”, Phys.Rev.Lett. 94 (2005) 201601 [arXiv: hep-th/0501022].

[50] Y. Kim, J.-P. Lee, S. H. Lee, “Heavy quarkonium in a holographic QCD model”, Phys.Rev.D 75 (2007) 114008 [arXiv: hep-ph/0703172].
[51] M. Á. Escobedo, “The relation between cross-section, decay width and imaginary potential of heavy quarkonium in a quark-gluon plasma”, J.Phys.Conf.Ser. 503 (2014) 012026 [arXiv: 1401.4892 [hep-ph]].

[52] Sh.-Q. Feng, Y.-Q. Zhao, X. Chen, “A systematically study of thermal width of heavy quarkonia in a finite temperature magnetized background from holography”, Phys.Rev.D 101 026023 [arXiv: 1910.05668 [hep-ph]].

[53] T. Hayata, K. Nawa, T. Hatsuda, “Time-dependent heavy-quark potential at finite temperature from gauge-gravity duality”, Phys.Rev.D 87 (2013) 101901 [arXiv: 1211.4942 [hep-ph]].

[54] K. Hashimoto, D. E. Kharzeev, “Entropic destruction of heavy quarkonium in non-Abelian plasma from holography”, Phys.Rev.D 90 (2014) 125012 [arXiv: 1411.0618 [hep-th]].

[55] Y. Burnier, M. Laine, M. Vepsalainen, “Quarkonium dissociation in the presence of a small momentum space anisotropy”, Phys.Lett.B 678 (2009) 86 [arXiv: 0903.3467 [hep-ph]].

[56] D. Dudal, Th. G. Mertens, “Melting of charmonium in a magnetic field from an effective AdS/QCD model”, Phys.Rev.D 91 (2015) 086002 [arXiv: 1410.3297 [hep-th]].

[57] N. R. F. Braga, L. F. Ferreira, Heavy meson dissociation in a plasma with magnetic fields, Phys.Lett.B 783 (2018) 186 [arXiv: 1802.02084 [hep-ph]].

[58] L. Bellantuono, P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri, “Quarkonium dissociation in strongly coupled far-from-equilibrium matter: holographic description”, Nucl.Phys.A 982 (2019) 931.

[59] L. Bellantuono, P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri, “Quarkonium dissociation in a far-from-equilibrium holographic setup”, Phys.Rev.D 96 (2017) 034031 [arXiv: 1706.04809 [hep-ph]].

[60] Q. J. Ejaz, Th. Faulkner, H. Liu, K. Rajagopal, U. A. Wiedemann, “Quadsia J. Ejaz, Thomas Faulkner, Hong Liu, Krishna Rajagopal, Urs Achim Wiedemann”, JHEP 04 (2008) 089 [arXiv: 0712.0590 [hep-th]].

[61] Y. Kim, B. H. Lee, C. Park, S. J. Sin, “The effect of gluon condensate on holographic heavy quark potential”, Phys.Rev.D 80 (2009) 105016 [arXiv: 0808.1143 [hep-th]].

[62] Z. q. Zhang, X. Zhu, “Effect of gluon condensate on jet quenching parameter and drag force”, Eur.Phys.J.C 79 (2019) 107.

[63] Z. q. Zhang, D. f. Hou, “Entropic destruction of heavy quarkonium in quark-gluon plasma with gluon condensate”, Phys.Lett.B 803 (2020) 135301.
[64] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Luetgemeier, B. Petersson, “Thermodynamics of SU(3) Lattice Gauge Theory”, Nucl.Phys.B 469 (1996) 419 [arXiv: hep-lat/9602007].

[65] G. Boyd, D. E. Miller, “The Temperature Dependence of the SU(N) Gluon Condensate from Lattice Gauge Theory”, [arXiv: hep-ph/9608482].

[66] K. G. Wilson, “Confinement of quarks”, Phys.Rev.D 10 (1974) 2445.

[67] J. L. Gervais and A. Neveu, “The Slope of the Leading Regge Trajectory in Quantum Chromodynamics”, Nucl.Phys.B 163 (1980) 189.

[68] A. M. Polyakov, “Gauge fields as rings of glue”, Nucl. Phys. B 164 (1980) 171.

[69] U. Gursoy, E. Kiritsis, F. Nitti, “Exploring improved holographic theories for QCD: Part II”, JHEP 02 (2008) 019 [arXiv: 0707.1349 [hep-th]].

[70] A. Kehagias, K. Sfetsos, “On running couplings in gauge theories from type-IIB supergravity”, Phys.Lett.B 454 (1999) 270 [arXiv: hep-th/9902125 ].

[71] C. C. Matthew Reece, “Toward a Systematic Holographic QCD: A Braneless Approach”, JHEP 05 (2007) 062 [arXiv: hep-ph/0608266 ].

[72] D. Bak, M. Gutperle, S. Hirano, N. Ohta, “Dilatonic Repulsions and Confinement via the AdS/CFT Correspondence”, Phys.Rev.D 70 (2004) 086004 [arXiv: hep-th/0403249 ].

[73] H. Leutwyler, “in QCD 20 Years Later: Proceedings, Workshop, Aachen, Germany”, 693 (1992).

[74] H. Liu, K. Rajagopal, U. A. Wiedemann, “An AdS/CFT Calculation of Screening in a Hot Wind”, Phys.Rev.Lett 98 (2007) 182301 [arXiv: hep-ph/0607062].

[75] H. Liu, K. Rajagopal, U. A. Wiedemann, “Wilson loops in heavy ion collisions and their calculation in AdS/CFT”, JHEP 03 (2007) 066 [arXiv:hep-ph/0612168 ].

[76] K. Bitaghsir Fadafan, S. K. Tabatabaei, “Entropic destruction of a moving heavy quarkonium”, Phys.Rev.D 94 (2016) 026007 [arXiv: 1512.08254 [hep-ph]].