Low-Latency Digital Downconversion for Control Applications

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Abstract—The slowly varying complex envelope of sinusoidal signals can be estimated in real-time using digital downconversion. In this paper, we discuss the requirements on digital downconversion for control applications. Two low-latency downconversion filters are compared with respect to performance and implementation aspects.

I. INTRODUCTION

Estimating the complex envelope of digitized sinusoidal signals in real-time is necessary in many control applications. Some examples are: control of electromagnetic fields in particle accelerators [1]–[6], MEMS gyroscopes [7], and laser stabilization [8]. For real-time computation of complex envelopes it is typical to use digital downconversion (DDC), see Fig. 1. There is a vast literature on DDC for telecommunications [9]–[12], but as we will see, it has limited applicability to control applications.

The focus of this paper is the filter \( H(z) \) in Fig. 1. The filter \( H(z) \) should both reject the double-frequency component of the mixer output and reduce aliasing. In telecommunications, its design is optimized with respect to passband flatness, stopband attenuation, and resource usage. For control applications, the latency, or more precisely the phase-drop at the feedback loop’s cross-over frequency, is more important.

We present two types of low-latency DDC filters that are suitable for control applications: moving averaging and two-sample reconstruction [1], [2]. These filters have previously been used for field control in particle accelerators [1]–[5]. In this paper, we study their frequency-domain characteristics, compare their performance, and consider implementation aspects. A key to our analysis is the realization that two-sampling reconstruction corresponds to Fig. 1 with a two-tap complex-coefficient filter \( H(z) \). The complex-valued perspective is particularly insightful in conjunction with complex-signal control analysis [13], [14].

Notation: The complex conjugate of \( a \in \mathbb{C} \) is denoted \( a^* \).

Remark: Another interesting example of DDC, with its own set of challenges, is single-phase phasor estimation in three-phase power systems [15].

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II. BACKGROUND

A. Introduction to Digital Downconversion

A sinusoid with carrier frequency \( \omega_c \), whose amplitude \( A(t) \) and phase \( \phi(t) \) vary slowly, has the form

\[
y_c(t) = A(t) \cos(\omega_c t + \phi(t)) = \text{Re}\{A(t)e^{i\phi(t)}e^{i\omega_c t}\} = \text{Re}\{y(t)e^{i\omega_c t}\}.
\]

The complex signal \( y(t) = A(t)e^{i\phi(t)} \) is known as the (slowly varying) complex envelope or the equivalent baseband signal, of \( y_c \). If \( y_c(t) \) is sampled with a period \( h \), at time instances \( t_k = kh \), then a sampled version of \( y \) can be recovered by DDC as shown in Fig. 1.

To better understand Fig. 1, note that the signals at the indicated points are given by

1. \( y_c[k] = \text{Re}\{y[k]e^{i\omega_c t_k}\} = (y[k]e^{i\omega_c t_k} + y[k]^*e^{-i\omega_c t_k})/2 \)

2. \( y[k] = y[k]e^{-2i\omega_c t_k} \)

The low-pass filter removes the double-frequency component at \(-2\omega_c\), giving the estimate \( \hat{y}[k] \approx y[k] \) at 3.

DDC can be interpreted in the frequency domain as a translation by \(-\omega_c\) followed by truncation of high frequencies, see [12, Fig. 18.14].

B. Baseband Model of DDC

For control analysis it is convenient to transform the model in Fig. 1 to the baseband. This yields the model in Fig. 2 that relates the complex envelope \( y \) of the input signal to the downconverted signal \( \hat{y} \). In the baseband, it is seen that DDC amounts to adding complex-valued, cyclostationary noise \( n \) and filtering by a linear, time-invariant (LTI) filter \( H(z) \).
For control applications, the filter $H(z)$ should have low latency and hence little impact on closed-loop performance. It is nevertheless prudent to include the dynamics $H(z)$ and the measurement noise $n$ in analysis.

### C. DDC for Telecommunications

In telecommunications, the signals of interest have a relatively flat spectrum over the channel bandwidth. To estimate such signals without distortion, while rejecting spectral content outside the channel, calls for a filter $H(z)$ that is almost unity across the channel and has a fast transition to a well-attenuated stop band. The desired features $H(z)$ can be summarized as

- **T1)** flat amplitude response in the passband, and
- **T2)** excellent stopband suppression.

In particular the second item requires significant engineering efforts since it is often difficult to achieve sufficient attenuation of adjacent channels and spurious interference.

Typically, $H(z)$ is taken as an FIR filter $H(z) = h_0 + h_1 z^{-1} + \cdots + h_{N-1} z^{-(N-1)}$ with (conjugate-)symmetric coefficients $(h_{N-1-k} = h_k^*)$. The symmetry ensures that $H(z)$ is a linear-phase filter with the same phase response as a delay of $(N-1)/2$ samples.

The impulse and frequency responses of a typical filter $H(z)$ for telecommunications are shown in Figs. 3 and 4.

In addition to the requirements T1 and T2, the design of $H(z)$ should allow an implementation with

- **T3)** low power consumption, and
- **T4)** few hardware resources.

For these reasons, the filter $H(z)$ is typically implemented as a cascade of filters at different rates, where the first one is typically a cascaded-integrator-comb filter [16]. For details, see [11], [12].

To reduce the computations in the downstream baseband processing, the output of $H(z)$ is typically decimated as much as possible. This can be done without loss of information due to that the spectrum of the baseband signal is centered around zero frequency.

In telecommunications, delay requirements are typically a minor concern\(^1\).

\(^1\)Personal communication with B. Bernhardsson, former filter expert at Ericsson.

### III. DDC for Control Applications

#### A. Requirements on DDC for Control Applications

The requirements on DDC for control applications are quite different from those on DDC for telecommunications. Typically, the signal of interest is very narrowband due to feedback, and hence it gives rise to ADC and mixer harmonics, as well as a strong double-frequency component. It is important to avoid that decimation folds these products onto the signal of interest. However, unlike in telecommunications, there is no need to worry about folding of spectral content from adjacent channels.

A main concern is instead to attenuate measurement noise (mostly ADC quantization noise) since it drives control errors and control signal activity. However, perhaps most important is to maintain sufficient phase margins. The desired features of $H(z)$ for low-latency control applications could be summarized as

- **C1)** small phase drop around the closed-loop bandwidth,
- **C2)** suppression of (ADC) measurement noise,
- **C3)** rejection of the double-frequency component,
- **C4)** rejection of the DC-offset spur, and
- **C5)** rejection of mixer and ADC harmonics.

The trade-off between C1 and C2 is studied in the control literature [17, Sec. 7.4]. The following subsections introduce filters that address points C3–C5, although additional low-pass filtering might be necessary to achieve C2. The filters $H(z)$ that we consider are rather simple, so resource usage should be a minor concern.

**Remark:** Requirement C2 is usually understood in a 2-norm sense, while requirement T2 in Sec. II-C is understood in a sup-norm sense.

#### B. Decimation

Since power and hardware resources tend to be less scarce in control applications, it could be feasible to use

\(^2\)If a heterodyne architecture is used.
little, or no, decimation after the filter \( H(z) \). Decimation
by a factor \( N \) gives a controller period \( h_{\text{reg}} = Nh \),
which effectively corresponds to a delay \( h_{\text{reg}}/2 \) relative
to continuous-time control [17]. For this reason \( h_{\text{reg}} \)
should be kept small. However, a small \( h_{\text{reg}} \), increases
the required resolution of coefficients and computations
in the controller. Trade-offs in the selection of \( h_{\text{reg}} \) are
discussed in [17].

C. IQ Sampling

A simple, low-latency approach to DDC is to take \( f_e = f_s/4 \) and \( H(z) = 1 + z^{-1} \), where \( f_e := \omega_c/(2\pi) \) and \( f_s := 1/h \) is the sampling frequency. This is approach
is called IQ (in-phase and quadrature) sampling [1] or \( f_s/4 \)
sampling [12]. The zero of \( H(z) \) in \( -1 \) eliminates the
double-frequency component at \(-f_s/2\). IQ sampling can
be implemented without multiplications and additions,
which makes it attractive in terms of resource usage and
power consumption [12].

A major drawback of IQ sampling is that odd mixer
and ADC harmonics of the spectrally narrow input signal
alias to the zero baseband frequency [1]. This problem
makes IQ sampling unsuitable for high-precision control.

D. Non-IQ Sampling

The aliasing problems of IQ sampling are avoided by using non-IQ (near-IQ) sampling [1], [4] where \( N \)
samples are taken over \( M \) periods of the carrier, i.e.,
\( M/f_e = N/f_s \).

A filter \( H(z) \) for non-IQ sampling should at least reject
the double-frequency component at \(-2\omega_c\). Two such filter
are presented in the next two subsections.

It will be convenient to introduce the phase advance
between consecutive samples as
\[
\Delta := \omega_c h = 2\pi M/N.
\]
Note that \( \Delta \) corresponds to \( \omega_c \) in normalized angular
frequency.

E. Non-IQ DDC Filter: Moving Averaging

For non-IQ sampling, an \( N \)-sample moving average filter
\[
H_{\text{MA}}(z) = \frac{1}{N} \left( 1 + z^{-1} + \ldots + z^{-(N-1)} \right) = \frac{1 - z^{-N}}{1 - z^{-1}}
\]
is a common choice [4], [5].

The impulse and frequency responses of an \( 11 \)-sample
moving average filter are shown in Figs. 5 and 6. An
attractive feature of the moving average filter (3) is that
it has zeros at the frequencies of the double-frequency
component, the DC-offset spur, and all ADC and mixer
harmonics (except multiples of \( N \)).

Decimation by a factor \( N \) after the moving aver-
age filter (3) corresponds to a single-stage cascaded-
integrator–comb (CIC) filter [16]. The high side lobes in
Fig. 6 indicate that aliasing could be a problem, but for
decimation by \( N \), these side lobes are neatly folded away
from the zero frequency, enabling them to be rejected by
a lower-rate low-pass filter [16].

DDC with a moving average filter has two some-
what illuminating interpretations: as a short-time dis-
crete Fourier transform with a single bin at frequency
\( \omega_c \) [5]; and as the least-squares estimate of \( y \) given \( N \)
observations \( y_c[k], \ldots, y_c[k+N-1] \) [4].

F. Non-IQ DDC Filter: Two-Sample Reconstruction

Two-sample reconstruction estimates the complex en-
velope \( y \) of a signal \( y_c \) based on only two samples. We
start by showing that this method is of the form in
Fig. 1 which allows for easier analysis than in [2], [3],
[6]. Assume that \( y_c \) is sampled at times \( t_k-1 = (k-1)h \)
and \( t_k = kh \), and that \( y \) does not change between these
two samples, i.e.,
\[
y_c[k-1] = \text{Re}\{ye^{i\omega_c t_k-1}\} = \frac{1}{2}\left(ye^{i\omega_c t_k-1} + ye^{-i\omega_c t_k-1}\right)
\]
\[
y_c[k] = \text{Re}\{ye^{i\omega_c t_k}\} = \frac{1}{2}\left(ye^{i\omega_c t_k} + ye^{-i\omega_c t_k}\right).
\]

Taking (4b) times \( e^{-i\omega_c t_k} \) minus (4a) times
\( e^{-2i\Delta} e^{-i\omega_c t_k-1} \) gives
\[
e^{-i\omega_c t_k} y_c[k] - e^{-2i\Delta} e^{-i\omega_c t_k-1} y_c[k-1] = \frac{1}{2} (y - e^{-2i\Delta} y).
\]

Thus, \( y \) can be recovered as
\[
y = \frac{2}{1 - e^{-2i\Delta}} \left(e^{-i\omega_c t_k} y_c[k] - e^{-2i\Delta} e^{-i\omega_c t_k-1} y_c[k-1]\right)
\]
which corresponds to filtering the signal \( 2e^{-i\omega_c t_k} y_c[k] \)
through the filter
\[
H_{\text{2SR}}(z) = \frac{1}{1 - e^{-2i\Delta}} \left[1 - e^{-2i\Delta} z^{-1}\right]
\]
\[= \frac{e^{i\Delta}}{2i \sin \Delta} \left[1 - e^{-2i\Delta} z^{-1}\right]
\]
This filter should only be used if $\Delta \approx \pi/2$ to avoid amplifying measurement noise. Magnitude responses of the filter $H_{\text{DCR}}(z)$ together with $H_{\text{SR}}(z)$ are shown in Fig. 7.

An alternative is to reject the DC component at the mixer input by a high-pass filter $(z-1)/(z-p)$ where $p$ is a real number slightly smaller than one. In the baseband, this filter corresponds to an IIR notch filter. A filter of this type with $p = 15/16$ is used for the field control systems at the Linac Coherent Light Source II.

G. Non-IQ DDC Filter: IIR Notch Filter

An alternative to moving averaging and two-sample reconstruction is to use a first-order IIR notch filter with a notch at $-2\omega_c$. This approach avoids the latency of moving averaging and the constraint $|\sin \Delta| \approx 1$ for two-sample reconstruction. However, implementing an IIR filter at the sampling rate of the ADC could be technically challenging, and we are not aware that this approach has been used in practice. For this reason, we will not consider it further.

IV. Additional Low-Pass Filtering

The noise rejection of the filters $H_n(z)$ in the previous section is often insufficient. In this section we analyze two aspects related to additional low-pass filtering by a filter $F_{\text{LP}}(z)$, see Fig. 8.

A. Moving Averaging vs. Two-Sample Reconstruction

Which of the two filters in the previous section is the better choice? Two-sample reconstruction has a shorter latency, but moving averaging provides better high-frequency roll off. For a comparison, we considered the setup in Fig. 8 where $n$ is white noise and $F_{\text{LP}}(z)$ is a first-order low-pass filter with bandwidth $\omega_{L_P}$,

$$F_{\text{LP}}(z) = \frac{1-a}{1-a z^{-1}}, \quad a = e^{-\omega_{L_P} h}. \quad (8)$$

For each filter $H_n(z)$, the lowpass bandwidth $\omega_{L_P}$ was tuned for three levels of noise rejection $||H_n(z) F_{\text{LP}}(z)||_2^2$. The frequency responses of the resulting filter combinations are shown in Fig. 9. It is seen that two-sample reconstruction enables slightly faster roll off with less phase retardation, which enables better feedback performance. The difference is quite small at low frequencies, but becomes noticeable if a high cross-over frequency is desired. In particular, for long moving average filters (note the dashed line in Fig. 9).

That moving averaging is usually suboptimal for control applications is well-known and can be intuitively understood from that equal weight is placed on the most recent sample and the $N$th most recent sample.

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3 The offset error can be on the order of 1% of the maximum sine-wave amplitude. For example, LTC2175 from Linear Technologies has an offset error of up to 1.2% of the maximum sine-wave amplitude.

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![Graph](image_url)
B. Order of Low-Pass Filtering and Decimation

If the output signal $\hat{y}$ is to be decimated, one may ask whether this should be done before or after the low-pass filter $F_{\text{LP}}(z)$ (Fig. 10). Decimating by a factor $N$ before low-pass filtering enables a lower-rate filter implementation $\tilde{F}_{\text{LP}}(z)$ (as in (8)) but with $a = e^{-\omega_{\text{LP}}N\Delta}$, which allows slightly better performance, in particular for extremely fast feedback. However, with two-sample reconstruction (6) (if $|\sin \Delta| \approx 1$), this is not overly problematic.

Fig. 11 shows how the order of filtering and decimation affects the resulting noise level as a function of the bandwidth $\omega_{\text{LP}}$ of the low-pass filter. The output variance of the periodic system in Fig. 11b was computed as $||F_{\text{LP}}(z^N)H(z)||^2$ (see noble identities in [10]).

In the case of moving average filters, we see from Fig. 11a that the order of low-pass filtering and decimation has little impact on the noise rejection if the bandwidth $\omega_{\text{LP}}$ is lower than 100 kHz. For long moving average filters (and consequently high decimation ratios) it is seen that there is much to gain from low-pass filtering with $F_{\text{LP}}(z)$ before decimation.

For two-sample reconstruction, we see from Fig. 11b that the order of decimation and low-pass filtering makes little difference if $|\sin \Delta| \approx 1$ (which was necessary for two-sample reconstruction to be viable in the first place, see Sec. III-F).

V. EXAMPLES

LCLS-II: The field control systems for the Linac Coherent Light Source II use non-IQ sampling with $M/N = 7/33$ ($f_c = 20$ MHz and $f_s = 94.29$ MHz) together with two-sample reconstruction [18]. Due to the high feedback gain ($K \approx 1000$), an additional first-order low-pass filter with a bandwidth between 50 kHz–200 kHz is used to limit the control signal activity.

ESS: The field control systems for the European Spallation Source use non-IQ sampling with $M/N = 3/14$ ($f_c = 25.16$ MHz and $f_s = 117.40$ MHz) together with moving averaging and decimation by $N$.

VI. CONCLUSIONS

We have discussed DDC for control applications and how it differs from DDC for telecommunications. We considered two types of DDC filters that are suitable for control applications: moving averaging, which is a decent and trouble-free choice; and two-sample reconstruction which allows slightly better performance, in particular for extremely fast feedback. However, with two-sample...
reconstruction it is necessary to be mindful of harmonics, the DC-offset spur, and that $|\sin \Delta| \approx 1$.

Typically, the output of these two filters needs to be low-pass filtered and decimated. It is preferable to low-pass filter before decimating, but the noise increase from doing it the other way around is small, except for long moving averaging filters or if little filtering is applied.

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