Josephson admittance spectroscopy application for frequency analysis of broadband THz antennas.

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Abstract. Application of Josephson admittance spectroscopy for the spectral analysis of a broad-band log-periodic superconducting antenna was demonstrated in the frequency range from 50 to 700 GHz. The [001]-tilt YBa₂Cu₃O₇₋ₓ bicrystal Josephson junctions, integrated with sinuous log-periodic YBa₂Cu₃O₇₋ₓ antennas, were fabricated on NdGaO₃ bicrystal substrates. A real part of the antenna admittance Re¥(f) as a function of the frequency f was reconstructed from the modification of the dc current-voltage characteristic of the junction, induced by the antenna. Resonance features were observed in the recovered Re¥(f)-spectra with a periodicity in the logarithmic frequency scale, corresponding to log-periodic geometry of the antenna. The Re¥(f)-spectra, recovered by Josephson spectroscopy, were compared with the Re¥(f)-spectra, obtained by CAD simulation, and both spectra were shown to be similar in their main features. A value of 23 was obtained for an effective permittivity of the NdGaO₃ bicrystal substrates by fitting simulated data to those obtained from Josephson spectroscopy.

1. Introduction

The dc IV-curve of an autonomous Josephson junction (JJ) has a nonlinear shape, which is determined by averaging of Josephson oscillations [1]. When JJ is placed into a substance with the dispersion or attached to lumped elements with frequency-dependent characteristics, a pattern of the Josephson oscillations is modified. This affects the dc IV-curve of the junction. It was shown in previously-published papers, that JJ can be used at the microwave and millimeter-wave wavelength ranges for indication of characteristic frequencies of a Fabry-Perot resonator [2], resonances in LC-circuits and coaxial lines, attached to the junction, absorption spectral lines of various substances, deposited on the weak link [3], [4]. The effect of external circuits on the dc IV-curve of JJ was theoretically considered in [5]. The results of this work form the basis of Josephson admittance spectroscopy. It is a spectroscopy that gives an opportunity to measure the spectral function, namely, a real part of the frequency-dependent admittance of the JJ environment, rather than just the value of characteristic resonance frequencies, as it was done in the earlier experiments [2]-[4].

Currently, a number of experimental techniques are used to measure complex frequency-dependent impedance in the submillimeter wavelength range. They are based on Fourier-spectroscopy of

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transmitted/reflected radiation and more universal time-domain spectroscopy, which allows to study the absorption spectra of substances, as well as spectral characteristics of antenna and semiconductor devices [6]-[10]. These techniques need external sources of electromagnetic radiation at the terahertz frequency range or ultrashort optical pulses. Recently, it was made an attempt to use the ac Josephson effect for antenna characterization [11], but the only result was an indication of some resonance frequencies, rather than any antenna spectral function.

In the technique described below, JJ is a source, receiver and spectrum analyzer at the same time. It is located in the close vicinity of the analyzed object or connected to this object. The frequency-dependent admittance of the object can be reconstructed from the Josephson junction IV-curve modified by the object. A small size of the weak link in JJ, being much less then a wavelength, provides correct analysis of the admittance. Considerable progress has been recently achieved in technological development of high-\textit{Tc} JJ, which makes possible to extend the frequency range of Josephson admittance spectroscopy up to a few THz [12], [13].

In this paper, an application of Josephson admittance spectroscopy for spectral characterization of the broadband log-periodic antenna is presented at the frequency range of 50-700 GHz. The differential resistance \( R_d(V) \) of JJ as a function of bias voltage shows a periodic structure of features, which correspond to the resonances, determined by antenna geometry. Due to this circumstance, a log-periodic antenna is a very suitable object for such test measurements.

2. Josephson admittance spectroscopy

Let us assume, that a deviation of the dc voltage-current characteristic \( V(I) \) of JJ from the similar characteristic \( V_0(I) \), calculated according to the RSJ model [1] is small. This deviation may be caused by a variety of physical mechanisms, e.g. electromagnetic losses in materials of JJ and substrate, the influence of absorptive environment, antenna or other electromagnetic structures. The effect of all these mechanisms on JJ can be considered by a frequency-dependent complex admittance \( Y_{ext}(f) \), connected to the junction in parallel. In this case, following [5], a deviation \( \delta V(I) \) of the dc voltage-current characteristic, measured at current bias \( I \), can be described by following expression:

\[
\delta V(I) = R_n \left( IR_n - V \right) \text{Re} \left[ Y_{ext} \left( 2eV / h \right) \right]
\] (1)

where \( V = \sqrt{R_n(I^2 - I_c^2)} \) is the unperturbed voltage at the given current \( I \), which is proportional to the Josephson frequency \( f \) according to the relation \( V = h/2e, I_c \) is the critical current and \( R_n \) is the normal-state resistance of JJ. It is assumed in (1) \( Y_{ext}(f)R_n \ll 1, Y_{ext}(0) = 0 \) and \( T = 0 \).

Thus, starting from the measurements of the IV curve deviation, it is possible to reconstruct a spectral dependence of a real part of the external admittance \( Y_{ext}(f) \). If \( V_0(I) \) is used as an unperturbed IV-curve, a reconstructed dependence \( \text{Re} Y_{ext}(f) \) can contain components related to different physical mechanisms. Slow varying component is usually concerned to properties of JJ. Along with it a lower-level fast-varying component related with resonance structure of the external circuit may exist. It is rather difficult to extract this component directly from the dc IV-curve because of considerable \( 1/f \) noise in high-\textit{Tc} junctions [14]. Therefore, to recover the fast-varying part of \( \delta V(I) \) more precisely, a differential resistance \( R_d(I) \), as a function of the current can be measured, then subtract the slow-varying component and calculate \( \delta V(I) \):

\[
\delta V(I) = \int_{I_0}^{I} \delta R_d(I') dI'
\] (2)

Expression (1) was obtained without taking into account a contribution of thermal noise to the IV-curve it is not applicable at voltages less than the voltage, which corresponds to the maximum of \( R_d \). Therefore, a low integration limit \( I_0 > I_c \) in (2) is chosen to neglect the impact of fluctuations on \( R_d \).
3. Experimental details

A set of [001]-tilt bicrystal YBa$_2$Cu$_3$O$_{7-x}$ thin-film JJs, integrated with YBa$_2$Cu$_3$O$_{7-x}$ log-periodic antennas, was fabricated on (110) NdGaO$_3$ bicrystal substrates (figure 1a) [15]. The junctions were characterized by following parameters: $I_c = 0.05 - 0.4$ mA and $I_cR_n = 0.4 - 1$ mV at $T = 5$ K. Geometry of a broad-band planar antenna (figure 1b) was developed according to principles of Rumsey [16], i.e. a shape of an antenna depends on angles only, and Babinet-Booker [17], i.e. a shape of an antenna coincides with that of the complement. In addition, sinuous-type modulation of antenna edges [18] was used instead of conventional rectangular modulation.

![Figure 1](image_url)

**Figure 1.** A YBa$_2$Cu$_3$O$_{7-x}$ bicrystal Josephson junction with a log-periodic YBa$_2$Cu$_3$O$_{7-x}$ antenna on a NdGaO$_3$ bicristal substrate, where a bicrystal boundary is shown by a thick line (a) and a wide-band log-periodic antenna (b).

The antenna has 25 vibrators at each side of the antenna’s arm. Log-periodic geometry of the antenna was defined by following rules:

$$\frac{R_{n+1}}{R_n} = \tau, \quad r_n = \sqrt{\tau}$$

(3)

where $\tau = 1.25$, maximal and minimal radii $R_{\text{max}} = 1000$ µm, $R_0 = 5$ µm. In addition, the antenna meets Rumsey’s principle if

$$\varphi^1 = \varphi_0^1 + a \sin \left[ \alpha \cdot \log \left( \frac{R}{R_0} \right) \right], \quad \varphi^2 = \varphi_0^2 + a \sin \left[ \alpha \cdot \log \left( \frac{r}{r_0} \right) \right]$$

(4)

where $r_0 = R_0 \sqrt{\tau}$, $\varphi_{0,1}^2$ are the angles, which define the vibrators from each side of the arm, $a$, $\alpha$, $R_0$ are the parameters, which define antenna geometry.

The antenna was formed from the epitaxial (001) YBa$_2$Cu$_3$O$_{7-x}$ film deposited on NdGaO$_3$ bicrystal substrate (Fig. 1a). It was excited by oscillations generated in the [001]-tilt bicrystal YBa$_2$Cu$_3$O$_{7-x}$ thin-film JJ. To carry out measurements, a substrate with the antenna and JJ was mounted on a cryogenic dipstick and was placed into liquid-helium dewar. The $IV$-curves $I(V)$ and differential resistances $R_d(I)$ of JJ were measured in current-bias mode at the temperature range from 4.5 to 70 K. To reduce the impact of excess $1/f$ noise, the $R_d(I)$ dependencies were measured at the frequency of 300 KHz. A special low-noise amplifier with resonance transformer was used. A noise level, specified for the input of the transformer was equal to $5 \cdot 10^{10}$ V/Hz$^{1/2}$. 


4. Experimental results and reconstruction of antenna admittance.
The experimental $V(I)$-curve and voltage dependence of the differential resistance $R_d(V)$ for one of the junctions integrated with a log-periodic antenna are presented in figure 2 for a temperature of 4.5 K. An unperturbed dependence $V_0(I)$ was calculated according to the RSJ model and was fitted to the experimental curve. The critical current $I_c$ and normal-state resistance $R_n$ of JJ were used as fitting parameters. The best fit was provided by following values of the parameters: $I_c = 0.36 \text{ mA}$ and $R_n = 1.97 \text{ Ohm}$. A maximum relative deviation of the measured curve $V(I)$ from the calculated one was equal to 2%.

![Figure 2](image)

These deviations from the RSJ model are more visible in the voltage dependence of differential resistance $R_d(V)$, where, instead of a smooth function for RSJ model, oscillatory behavior is clearly seen at the voltage range $0.2 \text{ mV} < |V| < 1.3 \text{ mV}$ (figure 2). The relative amplitudes of oscillations in the $R_d(V)$ were as high as 50% in the middle of this voltage range. The modification of the $R_d(V)$ is more than one order higher, when compared with the 2%-deviations of the $IV$-curve, and the voltage noise in $R_d(V)$ measurements is two-three orders lower, so a signal/noise ratio in our $R_d(V)$ measurements is four orders higher than in the dc $IV$-measurements for the same external admittance.

The $R_d(V)$ curves at several temperatures from 5 K and 70 K are presented in figure 3. At all temperatures repeating features are observed and their locations on the voltage axis are not temperature depend. This circumstance indicates, that these features are related to external geometrical resonances rather than internal processes in JJ.

![Figure 3](image)

A real part of the admittance of the real environment of the ideal RSJ-like JJ with the integrated antenna was reconstructed from the experimental $IV$-curve at $T = 4.5 \text{ K}$, according to equation (1), and
shown in figure 4 as a curve 1. We suppose that electromagnetic losses in antenna material [19] and in weak link [13] give the main slow-varying contribution \( \text{Re} Y(f) \) to the \( \text{Re} \tilde{Y}(f) \).

Along with the slowly varying component, an oscillating component with significantly smaller amplitude is observed in \( \text{Re} Y(f) \) (curve (a)). With much higher signal/noise ratio, this component was recovered from the \( R_d(V) \) data and is presented by curve (b) in figure 4. To consider the impact of measurements at finite temperatures, an error related to a low-integration limit \( I_0>I_c \) in equation (2) is included into the \( \tilde{Y} \) along with the slowly-varying component \( \tilde{Y}(f) \) of the admittance.

In addition to the reconstruction of \( \text{Re} Y(f) \) considered above, a real part of the admittance for log-periodic sinuous antenna of the same geometry was obtained by computer simulations. Calculations were performed by Agilent ADS Momentum software package [19]. Both characteristics, recovered from the experiment and simulated, are presented in figure 5. A value of permittivity \( \varepsilon = 23 \) was chosen for the (110) NdGaO\(_3\) bicrystal substrate to fit positions of resonance features in the simulated \( \text{Re} Y(f) \) to those of the reconstructed one. This value is only (5÷6)% higher than the corresponding values of 21.6 ÷ 21.7 for a (110) NdGaO\(_3\) single-crystalline substrate measured at the frequencies of 300 and 600 GHz [21].

Also, a frequency-independent value of sheet resistance \( R_\parallel = 0.1 \) Ohm/square at 4.5 K was used in simulations as a loss parameter for a material of the antenna. With this value of \( R_\parallel \), peak-to-peak values of the calculated function \( \text{Re} Y(f) \) corresponded to maximum peak-to-peak values of the similar reconstructed function at the frequency of 230 GHz. The value of \( R_\parallel \) used in simulations is close to the \( R_\parallel \)-values, which might be obtained from temperature dependences of the resistivity \( \rho(T) \) for our c-axis epitaxial YBa\(_2\)Cu\(_3\)O\(_7-x\) thin films. Indeed, the linear dependence of \( \rho(T) \) is observed at the temperature range from 225 K to 125 K for YBa\(_2\)Cu\(_3\)O\(_7-x\) thin films with a slope \( \frac{d \rho(T)}{dT} \) of around 0.6-10\(^{-6}\) Ohm-cm/K [22]. A normal-state resistivity \( \rho(4.5 \text{ K}) \) of 2.7-10\(^{-6}\) Ohm-cm might be estimated from this linear trend of the \( \rho(T) \), which gives a value of \( R_\parallel \approx 0.2 \) Ohm/square for a 140nm-thick YBa\(_2\)Cu\(_3\)O\(_7-x\) thin film used as a material for the antenna.

The antenna shape provides a periodic dependence of all antenna characteristics, including \( \text{Re} Y(f) \), in the logarithm of frequency with the period \( \log(\tau) \) [16]. Hence for the frequencies \( f_n \) and \( f_m \) of the antenna resonances with numbers \( n \) and \( m \) we can obtain following scaling rule:

\[
\log_{10} \left( \frac{f_n}{f_m} \right) = 0.5 \cdot (n - m) \log_{10}(\tau). \tag{5}
\]

The coefficient 0.5 in (5) appears because the resonances arise subsequently in the vibrators at the opposite sides of the antenna arm.
The values of experimental resonance frequencies as a function of the resonance number are presented in figure 6 by filled circles. Similar data, obtained by computer simulation, are presented by open triangles and a solid line shows the same dependence, calculated according to equation (5). At frequencies above 150 GHz, the experimental points coincide with those obtained from simulation and (5) fits them with a scaling parameter \( \tau = 1.24 \pm 0.01 \). This value coincides with a corresponding geometrical parameter in equation (3).

A resolution of Josephson spectroscopy is defined by Josephson linewidth [1] and is shown by an error bar for each experimental point in figure 6. At frequencies below 150 GHz the Josephson linewidth increases, which leads to merging of neighboring resonance peaks in the curve 1 in figure 5 and violation of scaling behavior of experimental points (filled circles in figure 6), while log-periodic behavior holds for simulated points (triangles in figure 6).

At the voltages \( V < 100 \) \( \mu \text{V} \) an experimental differential resistance \( R_d(V) \) significantly differs from that of calculated from the RSJ model at \( T = 0 \) and equation (1) cannot be applied for the reconstruction of spectral dependence of the admittance \( Y(f) \) at frequencies below 50 GHz. Intensities of resonance peaks, obtained from the experiment, decrease as long as frequencies go above 350 GHz (curve (a) in figure 5). This can be explained by a kinetic inductance and an increase of electromagnetic losses in the antenna material [19] and bicrystal boundary [13]. The same intensities in simulation data (curve (b) in figure 5) demonstrate weak frequency dependence at this frequency range because a frequency-independent resistivity of the antenna material was used in simulations.

5. Conclusions
In this work it has been demonstrated an application of Josephson admittance spectroscopy to analysis of spectral properties of broadband log-periodic antenna at the frequency range 50-700 GHz. At frequencies above 150 GHz it was obtained a periodic dependence of antenna admittance versus the logarithm of the frequency and the period coincides with corresponding geometrical parameter of the antenna. At lower frequencies, a violation of the logarithmic periodicity was observed, which can be explained by the decrease of resolution. When frequency was increased above 350 GHz, the intensities of resonance peaks decreased. This can be explained by electromagnetic losses in \( \text{YBa}_2\text{Cu}_3\text{O}_{7-x} \) material and grain boundary [5], as well as kinetic inductance.
A computer simulation of the antenna was performed. A function $\text{Re}Y(f)$ was obtained and fitted to reconstructed one from measurement data. Permittivity of NdGaO$_3$ substrate material was estimated as $\varepsilon=23$.

To measure spectral characteristics of antenna in more extended frequency range, it is necessary to use JJ with lower losses. The measurement of the real part of admittance in frequency range, covering all antenna resonances, will make possible to obtain a complete complex admittance of antenna using Kramers-Kronig relations.

Josephson admittance spectroscopy can be also applied to analysis of spectral properties of the interconnect structures, microelectronic devices and absorption spectra of substances at subterahertz and terahertz frequency ranges.

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