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A Simple Agro-Economic Model for Optimal Farm Nitrogen Application under Yield Uncertainty

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Abstract: Farmers in the developed world tend to over-apply fertilizer, and we explore a model for decision-making under uncertainty in yields. This article proposes an agro-economic model for farmer decision-making based on subjective expected yield and crop response to fertilization. The model explores subjective yield probability distributions that are both better suited to subjective crop yields than the previously proposed probability distribution and is easier to extract from farmers. The model allows the analysis of the impact of changes in fertilizer price and variance of expected yields. The model result is consistent with observed farmer behavior based on the rule of “fertilizing for the good years” that appears, according to our model, as rational and consistent with expected profit maximization under yield uncertainty since the cost of over-application is lower than that of the opportunity cost of under-application. The goal of increasing the efficiency of nitrogen use requires both technical innovation and an expansion of the knowledge on the socioeconomic factors underlying excessive crop fertilization that must be improved both to meet future food demands and to prevent environmental degradation and climate change.

Keywords: nitrogen management; pollution; subjective beliefs; yield distribution; nitrogen-use efficiency; over-fertilization

1. Introduction

The latest projections regarding population and food demand state that overall food production should increase by approximately 60% between 2005 and 2050 [1]. This can be achieved with higher yields, increased cropping intensity, cultivated land expansion (rainfed and irrigated), and increased irrigation water withdrawals. This will require improved technologies and public interventions in order to mitigate environmental damage caused by greenhouse gas (GHG) emissions [2] related to food production.

Certain inputs, such as water and fertilizer, are critical for the attainment of food and calorie objectives. Some of these applied inputs (i.e., nitrogen) are not removed from the system but remain locked up in it. Modern agriculture is responsible for the discharge of large quantities of agrochemicals, organic matter, sediments and salts into water bodies [2]. Agriculture is the largest consumer of nitrogen in the world, and agriculture is the largest producer by area of diffuse pollution [3]; however, in the EU, livestock production is the largest source of nitrogen water pollution [4]. Several studies estimate that agriculture is the main source of nitrogen emissions into water in the majority of the European Union (EU) regions (European Environment Agency, Indicators Report 2018), and in China [5]. Similar results were reported elsewhere. The EU has set a goal to reduce fertilizer use by 20% and fertilizer diffuse pollution by 50% by the year 2030 [6], which will require higher efficiency in nitrogen fertilization through the introduction of improvements in both technical and human factors.
This paper focuses on the human factor behind the use of fertilizers and selects nitrogen as the element nutrient to represent the challenges that society faces today. However, the models can be adapted to consider other nutrients, such as phosphorus, and with certain modifications, they can also be employed in the analysis of over-irrigation behavior.

Nitrogen is a necessary input for crop growth and production. Under natural conditions, plant-available nitrogen is usually in short supply, which limits plant growth and biomass production. The production of nitrogen fertilizers is based on a technology invented approximately 100 years ago, called the Haber-Bosch process. This process consumes approximately 5% of the world’s natural gas and approximately 1-2% of the world’s energy [7]. Between 2001 and 2014, aggregate nitrogen fertilizer consumption rose by 35% (to 110.4 Tg N) and is projected to grow to 132 Tg N by 2030 [8].

Zhang et al. [9] estimated that nitrogen-use efficiency (NUE) had increased significantly in many countries. However, the world average global NUE in crop production still needs to be improved from ~40% (current) to ~70% (2050 goal) to meet the dual goals of food security and environmental protection. The causes of low NUE include losses due to: ammonia volatilization, leaching, denitrification in flood conditions, run-off, fixation as non-exchangeable NH\textsubscript{4}, and immobilization by soil.

Nitrogen pollution is considered a critical environmental problem due to the water and air pollution incurred and its impacts on the climate. The efficient use of nitrogen could prevent greenhouse gas emissions equivalent to 5-10% of the drop in emissions necessary to reach the 2 °C target [10].

Worldwide, nearly half of N fertilizer input is not utilized by crops and is lost to the environment via volatilization and the emission of gases or by polluting water bodies, which causes environmental problems, such as greenhouse gas emissions, eutrophication, soil acidification, and a reduction in biodiversity. Nitrogen pollution in 2050 can be expected to rise to 102-156% of the 2010 value, with 60% originating from crop production [11].

Given the importance of diffuse nitrogen pollution worldwide, this paper aims to contribute by proposing a new simplified agro-economic model for optimal farm nitrogen application under yield uncertainty. The model is based on the seminal work by Babcock [12] developed for the uniform probability distribution function (PDF) and explores the use of functional forms of a more realistic nature (beta and triangular) compared to Babcock’s use of the uniform probability distribution. This distribution does not reflect the dynamics of crop yield variability, which hardly takes equiprobable values over a finite interval. Uncertainty models in risk analysis tend to be unimodal distributions over a finite range that usually have identifiable minimum and maximum values. The beta distribution is considered the most versatile distribution used in risk analysis [13] as it offers a wide variety of distribution shapes over a finite interval without the need for the normality assumption [14]. On the other hand, the triangular distribution is characterized by its ease of handling for the project planner [15], with a higher estimation precision for the mean and standard deviation than the discrete uniform approximation [16].

Additionally, our paper applies this model to Spain, which is both an EU member and a Mediterranean country and may serve as a testing ground for a location different to that of the United States where Babcock and his collaborators directed their research.

The following section reviews the state of the art regarding nitrogen use and the human factors behind the low NUE observed and explains the innovation of our approach in greater detail. The third section details material and methods, the fourth presents the results, and finally, a discussion and concluding remarks close our contribution.

2. Nitrogen-Use Efficiency and Farm Behavior

The dominant approach towards improving NUE is technological and can be defined by the ‘4Rs’ label: applying the right supply, at the right rate, at the right time, in the right place. This strategic approach is complemented with research and innovation in the fields of plant breeding, irrigation, and agronomic practice. New technological developments are also needed, such as slow-release and controlled-release fertilizers, whose aim is to
deliver N as required by the crop, thereby reducing the loss of applied N fertilizer and increasing the nitrogen-use efficiency (NUE). Additionally, certain authors promote the use of nitrification inhibitors that are now widely applied in agriculture [17]. Precision irrigation and drip irrigation also constitute a growing trend promoted by many governments [18]. The implementation of water conservation technologies also facilitates the use of fertigation (applying fertilizer via irrigation water).

The analysis of historical trends in N fertilization seems to follow the environmental Kuznets curve (EKC) with a phase where N pollution increases and then decreases with economic growth Zhang et al. [9]. In this bell-shaped curve, developing countries remain in the phase of increasing pollution (e.g., India), certain countries are in the transition phase (e.g., China), while others are in a phase of reducing pollution (e.g., the USA and the EU). According to Zhang et al. [9], the EU appears to have reached the turning point in the late 1980s due to changes in the EU Common Agricultural Policy (reduced crop subsidies and adopting the EU Nitrates Directive). On the other hand, the USA supports volunteer approaches, and improvement has been modest since the 1990s based on increased productivity (maintaining fertilizer use, increasing yields) and public and private efforts to voluntarily improve NUE.

The main proposals to reduce excess nitrates involve a limitation of the total amount of N added per hectare. Additionally, certain norms define farm quotas, issue tradable quotas, subsidies the use of lower application rates, establish a best management practice, increase research and education, and rely on voluntary responses. Regarding taxes, economists have argued that the price of fertilizer would have to be doubled in order to induce fertilizer savings [19], and the analysis of the EU experience with fertilizer taxation suggests that, overall, the effectiveness of pricing remains limited [20]. Finally, among economic tools, the introduction of tradable nutrient-reduction permits has been suggested for the Baltic Sea Action Plan [21].

The EU approach is focused on best practices and nutrient balance and promotes the adoption of nutrient management planning (NMP); however, the introduction of NMP by farmers remains slow [22]. Overall, the losses of nitrogen from agricultural land into the environment, expressed as the nitrogen balance, decreased from 2000 to 2015 and has stabilized since 2015 [23], although still 40% of surface waterbodies and more than 50% of groundwater in the EU were affected in 2015 by nutrient pollution, largely due to nitrogen fertilization and manure mismanagement. The nitrogen fertilization balance is checked by the Member States but good practices and NMP are mainly verified through document examination without any soil analysis [24]. Several socioeconomic variables (farm size, farmer’s age, etc.) are suggested as being the reason for the low adoption of NMPs. Daxini et al. [25] analyze the factors behind the low adoption and conclude that farmer psychology needs to be better understood in relation to the use of management practices that provide both environmental and financial benefits.

Recent research in Illinois found that the majority of farmers (67%) apply nitrogen above the experts’ annual recommended rate [26] by following a rule of thumb close to a 1200 kg/ha average yield even though this dose exceeds the Maximum Return to Nitrogen (MRTN) recommendations.

Enhanced efficiency in nitrogen use is hampered by uncertainty and natural and socioeconomic conditions with farmers in developed countries, who frequently use nitrogen to excess, which explains the aforementioned 60% low efficiency; part of this inefficiency may be explained by the uncertainty regarding yields [12]. On the positive side, those producers who purchase revenue insurances are likely to decrease their application of nitrogen fertilizer [27].

Further to the uncertainty of yield (due to agronomic and climatic factors), farmers also face uncertainty regarding the level of nitrogen in the soil before the crop is planted. This second source of uncertainty has also been addressed by Babcock and Blackmer [28], who found that a reduction in uncertainty with the use of a late-spring nitrate test reduces the application of N by a risk-neutral farmer (Iowa) by 38%.
Nevertheless, most of the aforementioned research assumes either an ex-post computed distribution function of yields and N availability or a model, proposed in the pioneering work by Babcock [12], which is based on the uniform (PDF) of yields. The latter work by Babcock and collaborators shows that ‘farmers (.) do not see nitrogen applications as risk-increasing, which indicates that yield variability may be a poor risk’ [29].

Our work strives to advance the understanding of farmer behavior by introducing subjective farmer expected yields to recommend a stochastic optimal nitrogen application. Our review of literature on farmer decision-making for nitrogen application did not find subjective yield distributions based on triangular or beta PDFs, which is what we propose in this section. Our model is ‘normative’; that is, it strives to determine the optimal dose based on farmer expectations and knowledge. The descriptive power of the model is not the objective of our research.

3. Materials and Methods

This paper proposes an ex-ante model of farmer decision-making for fertilizer use under uncertainty. The proposal is based on an agro-economic model, and farmers’ decisions are defined by their subjective expectation of yield and the information available on nitrogen response functions.

The response of crops to the addition of fertilizer has frequently been modelled as a Linear Response and Plateau (LRP) function that increases linearly until it reaches the maximum yield \(y_m\), at which point its increase comes to a halt. This is assumed as the standard agronomic model for the response to water [30] and fertilizer, especially in the case of nitrogen, where Liebig’s law of the minimum performed very well [31,32]. Although other response functions have been proposed, such as the quadratic and Mitscherlich functions [33], the LRP function yields optimal results.

Our analytical model is based upon the pioneering work by Babcock [12], although alternative subjective expected yields are also incorporated in order to integrate uncertainty and to evaluate fertilizer demand functions as responses to price. Babcock initially assumes uniform yield distribution (which is unrealistic) and later extended this and implicitly assumes a normal distribution [29]. This study builds upon the precedents and improves upon them by proposing distribution functions of a more realistic nature (triangular and beta), which are both easier to handle for analysts and closer to farmers’ real expectations. To the best of our knowledge, the important work carried out by Babcock [12] on the economics of fertilizer use has not explored PDFs close to farmers’ perception of yield variability [34]. Furthermore, this model will be applied to Spanish conditions to illustrate its applicability, where this type of study remains scarce.

The LRP function under the context of certainty is illustrated in Figure 1. The optimal solution is reached exactly when fertilizer use reaches \(W_m\) (maximum yield \(Y_m\)).

![Figure 1. Deterministic response function.](image-url)
Optimal fertilizer use is defined by the marginal production value and cost, as shown in Equation (1), which relates profit yield and input cost:

$$\Pi = p \cdot Y(w) - c(w)$$  \hspace{1cm} (1)

One consequence of the linear nature of the crop response is the solution to this equation that gives the optimal input use as zero or \(w_m\), as stated in Equation (2):

$$W_{mj}(year \ 'j') = \begin{cases} \frac{Y_{mj} - a_0}{a_1}, & \text{if } p a_1 > c \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (2)

Therefore, when fertilizer is not the constraining factor (as may happen for subsistence farmers) in certain developing countries [35]), the marginal value of fertilizer \((p \cdot a_1)\) is generally greater than cost \(c_{iw}\), and therefore farmers use fertilizer to achieve the maximum yield at exactly the point \(W_{mj}\).

The maximum yield \(\hat{Y}_m\) varies from year \(j\) to year \(k\) due to uncontrollable factors such as the weather and pest infestations. This variability is represented in Figure 2 by the dotted lines above and below the continuous line, where the deterministic maximum yield \(Y_m\) in Figure 1 has become the average maximum yield, as represented in Equation (3):

$$Y_{mj} = \mu_m + \epsilon$$  \hspace{1cm} (3)

![Figure 2. Stochastic response function.](image)

The maximum yield can be either higher or lower than the historical average. When growing conditions are favorable, the upper level \(Y_h\) is reached, and when growing conditions are adverse, then \(Y_l\) is attained, as shown in Figure 2.

The actual yield may be equal to \(Y_m\) when the applied fertilizer is equal to or greater than \(W_m\), or the actual yield may be below maximum \(Y_m\) in accordance with the linear production function when the applied input \((W)\) is lower than \(W_m\):

$$Y = \min(a_0 + a_1 \cdot W; Y_m)$$  \hspace{1cm} (4)

The optimum input ‘\(W\)’ under stochastic yields now becomes:

$$W_{mj} = \frac{Y_{mj} - a_0}{a_1}$$  \hspace{1cm} (5)
Note that the model parameters are defined by the crop response to the application of fertilizer, where the fertilizer crop response \( a_1 \) and residual soil nitrogen \( a_0 \) are the assumed parameters of the model. The farmer aims to maximize the net value of the factor given by the expected profit from the following variables:

\[
D = \text{"ex-post" optimal input year } j;\quad D = W_j^* = \frac{Y_{mj} - a_0}{a_1}
\]

\( W = \text{nitrogen used by a farmer in year } j \) (decision variable).

When nitrogen supply falls below the optimum input \( W < W_j^* \), then, according to Liebig’s law of the minimum, nitrogen becomes the limiting factor, and the potential yield is higher than that achieved by fertilizer application. In contrast, if the amount of fertilizer applied is greater than that required \( W > W_j^* \), then the excess nitrogen is 'lost' and incurs a cost that cannot be recovered. This decision context is similar to that of the classic inventory management problem known as the ‘news-vendor’ [36], where, if \( W < W_j^* \), then there is a ‘shortage penalty’ since potential yield \( Y_{mj} \) is more significant than that expected by the farmer and consequently profit is also lower than the potential.

The expression of the expected profit is given by [37], where the value of production is either: equal to the minimum of the stochastic yield \( X \); or determined by the input \( W \), decision variable; or given by the stochastic value of optimal nitrogen use that year \( D \) (stochastic variable, optimum fertilizer use), which is determined by the stochastic yield, as Equation (6) shows:

\[
\pi(W) = p a_1 \cdot E(\min\{W, D\}) - c_w W
\]

The decision made by the farmer is made on the horizontal axis where the optimal fertilizer demand \( W^* \) is determined by the behavior of \( D^* \) as a function of \( Y_{mj} \). The stochastic model is based on the maximized expected profit, which requires the elicitation of the fertilizer demand \( D \) distribution function:

Let \( f(x) \) be the PDF of \( D \),

and let

\[
F(x) = \text{Prob}(D \leq W) = \int_0^x f(x) dx
\]

be the cumulative distribution function (CDF) of \( D \). It can therefore be assumed that \( f(x) \) is continuous in \([0, \infty)\) in the following proof. The annual yield is determined either by the minimum value \( \min(D, W) \) (optimal 'ex-post fertilizer rate') or by the production \( Y(W) \) (yield limited by the fertilizer available). By implementing the expectations, we obtain:

\[
E[\min(D, W)] = \int_0^\infty \min(w, x) f(x) d(x) = \int_0^w x f(x) dx + W \int_w^\infty f(x) dx
\]

The profit function \( \pi(w) \) becomes:

\[
\pi(w) = (p a_1 - c_w) \left[ \int_0^w x f(x) dx W \int_w^\infty f(x) dx - c_w \int_0^w (W - x) f(x) dx \right]
\]

In order to find the maximum profit \( \pi(w) \), an input use \( w \) must be found that satisfies

\[
\frac{d\pi(w)}{dw} = 0.
\]

From the fundamental theorem of calculus:

\[
\frac{d\pi(w)}{dw} = (p a_1 - c_w) \int_0^w f(x) dx - c_w \int_w^\infty f(x) dx = (p a_1 - c_w)(1 - F(W)) - c_w F(W)
\]

By setting \( \frac{d\pi(w)}{dw} = 0 \), \( W^* \) must satisfy:

\[
F(W^*) = \frac{p a_1 - c_w}{p a_1}
\]
The solution in (11) is known as the ‘critical fractile’ from the well-known mathematical problem formulated by [36]. The ratio in (11) balances the cost of over-application (‘lost’ fertilizer) and the opportunity cost of under-application (lost yield).

Farmer decision-making is based on perceptions, values, and beliefs. The literature provides numerous alternatives for historical target yield distributions (such as conditional beta, Weibull, inverse Gaussian, and normal distributions). For the decision-making model, the subjective yield expectation is more relevant since it explains farmer behavior better than do ‘objective’ distributions. Smith and Mandac [38] compared subjective and objective yield and found certain coincidences regarding mean nitrogen response, although farmers tend to seriously underestimate the year-to-year variability in yield response and the likelihood of very low yield events.

Subjective probabilities are defined as beliefs held by individuals measured as the probability that an event will occur [39], and the most frequent subjective distributions proposed are the triangular and beta distributions, which share the existence of a minimum, maximum, and most frequent value [39,40]. Turvey et al. [41] propose the Beta-PERT function as suitable for the simulation of farmer expectations. Other subjective distributions found in the literature include the normal, beta, and Weibull distributions. However, subjective functions need to be bounded (maximum, minimum) and are frequently asymmetric, which means that unbounded functions, such as those of Weibull and normal, are unsuitable for practical implementation.

The model is applied to the case of a region with abundant water under Mediterranean conditions, as is the case of irrigated maize in the Guadalquivir basin. The parameters used for the linear response to irrigated maize under Mediterranean conditions found by [42] are \( a_0 = 5.10 \text{ Mg/ha} \) and \( a_1 = 0.033 \text{ kg/Mg} \) \((P < 0.0001)\) with yield expressed in Mg ha\(^{-1}\) and fertilizer in kg ha\(^{-1}\). We assume that parameters obtained by [42] are representative of Mediterranean (as authors claim) and Spanish conditions. In future applications of our analytical model, it will be convenient to estimate a local nitrogen yield response function to estimate parameters \( a_0 \) and \( a_1 \) with higher local significance, or alternatively to elicit farmer subjective yield response to nitrogen.

The yield distribution is based on the subjective expectation of farmers in the Guadalquivir basin, obtained through a small survey (conducted in January 2021, on a sample of 10 farmers, selected as ‘representative medium-sized farms’). The pilot survey questions are presented in Appendix A.

4. Results

The parameters obtained from our survey are: median = 14,000 kg ha\(^{-1}\); min = 12,000 kg ha\(^{-1}\); max = 16,000 kg ha\(^{-1}\); mean = 14,500 kg ha\(^{-1}\); Std.Dev = 1260 kg ha\(^{-1}\) (Table 1). The results of the model are tested with selected distributions for the evaluation of the effect of yield variability on optimal fertilizer use and for the generation of the fertilizer demand as a function of cost, where a price of 0.20 EUR kg\(^{-1}\) of maize is assumed [43]. We use the PDF that have been previously mentioned in the literature: Uniform (initially proposed by Babcock [12]) and Normal distributions are included. There are several limitations with the functions proposed by Babcock; the normal distribution is unbounded and does not correspond to any real historical PDF, and the uniform distribution is unrealistic and captures neither crop yield distribution nor farmer expectations. We, therefore, propose asymmetric bounded functions (triangular and Beta-PERT), which have all been simulated for illustrative purposes.
Figure 3 shows the demand for fertilizer under the aforementioned stochastic distributions compared to the ‘demand under certainty in the average expected yield’. As Figure 3 shows, the price needs to reach 3.0 EUR/kg of N (vs. the current Spanish level of 0.94 EUR/N) to induce farmers to apply nutrients close to the recommended levels, thereby preventing over-fertilization.

The nitrogen/crop price ratio lies within the range of 0.1 to 0.25 for maize for the majority of countries [9] during the period of 1960–2012. Consequently, according to this range of cost/price ratio and assuming a price of maize at $P = 0.20$ EUR/kg, prices of units of nitrogen move in the range of 0.80 to 2.0 EUR/kg. In this range, all the functions under study are very close except for the uniform distribution. Bounded functions stop at approximately the maximum yield ($\sim 337$ kg/ha), although the normal distribution is unbounded and simulates a higher level of fertilizer use.

The model explains why farming optimization drives the farmer to fertilize ‘for the good years’. As an example, we try the nitrogen cost at 0.92 EUR/NU as the average price given by Spanish Ministry of Agriculture for the year 2020 by computing price of different N fertilizer by the average N contain (see details in MAPA [44]). The model shows that when nitrogen fertilizer costs around 0.92 EUR/N the farmer stochastic optimal fertilizer (for all functions except the Normal) is 314 KgN, i.e., 10% over the ‘certainty demand (285 Kg/N). Details of these findings are laid out in the following section.
5. Discussion and Conclusions

According to Figure 4, which represents a typical farmer in a developed country, the criteria of maximizing expected profit induces over-fertilization compared to the ‘certainty average yield’ by approximately 14% when price is 1.4 EUR/UN (see Figure 3 responses for triangular, beta, and normal distributions) for a fertilizer to crop price of approximately 1.4 EUR/kg (median of the range 0.1 to 0.25 of maize price, typical of developed economies). This prediction is consistent with empirical findings, such as those by [45], who found an excess of nitrogen of approximately 17% on farms of Minnesota growers. Rajsic et al. [46] found a similar result for maize farmers in Ontario, where the rate of over-fertilization is approximately 14% over the recommended rate to maximize expected profit; these authors also concluded that a higher level of risk aversion reduces the optimal over-fertilization rate.

It is reasonable to think that farmers should follow different fertilization patterns based on the nature of the production systems in each country/region and how they perceive risk. For instance, most of the corn produced in the USA is genetically modified (GMO), meanwhile, in Spain, it is hybrid with GMO accounting for only 1/3 of the total corn area [47]. We assume that farm behavior and decision making under uncertainty is similar in developed market economies (such as the USA and EU) and we believe that our result can be valuable for all developed economies were input supply and the cost is not a limiting factor, probably this model should require some adaptation for developing economies were farmers food self-sufficiency and constraint to nitrogen availability, and cost may require some adaptation of our analytical model. Our results show that risk aversion in Spanish case study leads farmers to behave according to Babcock’s rule “fertilize for the good years” regardless of the region and the production system used, similarly to USA and Canada findings. Numerous studies advocate lack of information as the cause of farmers’ non-rational behavior [48].

The model is based on the LRP function of the crop response, which, according to certain authors, is insensitive to input price changes [12]. This is correct in a certainty context as expressed in Equation (2). Fortunately, our model overcomes this problem since the complete stochastic model gives a demand function response to price demand that mimics the farmer’s behavior and is downward and continuous.

Based upon a uniform function, Babcock [12] found that risk-neutral farmers react to uncertainty by increasing nutrient applications above the average rate required for crop...
uptake, and indicated that yield uncertainty may explain 25% of nutrient over-application compared with objective expected average yields. This is correct, although our model shows that a uniform distribution behaves differently to functions of a more realistic nature (triangular, beta). We do not recommend the uniform distribution, firstly because it fails to represent the subjective yield expectation of farmers, and secondly, since the use of this PDF involves unrealistically high over-fertilization.

Rajsic et al. [46], when comparing ex-post vs. recommended N rate for a set of farmers in Ontario in the period 1993 to 2001, found that the application of 20% more nitrogen than recommended leads to an improvement in expected profit (they use a quadratic–plateau model). This behavior observed in Canada is close to the 1.2 average yield rule of thumb reported by the nitrogen use recommended rate by extension services [26].

In this paper, the uniform distribution has been analyzed for illustrative purposes. Along the same lines, the normal function has been included although there is strong evidence against the existence of normality in yield distribution [49]. As Figure 3 shows, the impact is significant for very low fertilizer prices but remains close to other distributions in the typical range of nitrogen values (0.75 to 1.25 EUR/kgN).

In developed countries, such as the EU, NUE is at approximately 60% [23], and our model may explain why farmers may implement approximately a 14% over-application of fertilizers based on expected mean yield. If this is the case, this behavior may explain a significant part (approximately 1/3) of the NUE in developed countries, with the remaining part due to technical factors.

Zhang et al. [9] found that the results for maize, which presents the most data available, indicate that the fertilizer/maize price ratio is positively correlated with NUE, and our model accordingly predicts a similar conclusion, since a decreasing demand for fertilizer appears as the price increases, thereby reducing the excess application of fertilizer.

Studies into fertilizer demand have found that elasticity with respect to fertilizer price depends on the crop sectors. Examples of estimations of fertilizer demand elasticity include −0.36 for Greece [50], and 0.21 to −0.25 for short-term demand and −0.31 to −0.41 for long-term demand in the US [51]. Price elasticity of demand is a measure of how sensitive the quantity demanded of it is to its price. When the price rises, quantity demanded falls for almost any good, but it falls more for some than for others. The price elasticity gives the percentage change in quantity demanded when there is a one percent increase in price, and generally is higher (more responsiveness to price changes) at long vs. short term (as observed in the mentioned references) and elasticity is higher when the good (N fertilizer in our case) has more alternatives, as maybe in case of drinks for example. In our model, using Cw = 0.92 EUR/kgN as a reference [44], the elasticity yields a value of approximately −0.08; this is elasticity lower compared to the found in literature by econometric analysis may be explained by the fact that our model includes no trade-off nor substitution effects with other inputs such as water, phosphorus, chemicals, etc. If we introduce the possibility of input substitution in the model, the elasticity will be higher and closer to empirical estimates. The lower elasticity of price for analytical model vgs. Empirical econometric estimation has also been observed in the domain of water price [52] with similar behavior as ours and for similar reasons (higher empirical econometric elasticity vs. analytical models computations). In spite of this, the empirical elasticities and our model prediction (and the still lower estimates found by empirical models quoted above) support the argument that the effectiveness of tax on pesticides remains limited [19,20].

The impact of reducing variability is greater in terms of reducing over-fertilization. Figure 4 illustrates the impact of increased uncertainty by modifying parameters of the triangular distribution.

The impact of altering certain parameters in the model is straightforward. The farmer reacts to the increased variance of the distribution by increasing the demand for fertilizer. One of the effects of irrigation includes the reduction in the variability of yields, which constitutes one of the benefits of irrigation, and this is also perceived in the subjective PDF
in empirical work that finds that elicited probabilities lead to a significantly higher variance for rainfed vs. irrigated crops [53].

A general result from analyzing empirical elicited subjective distributions reveals the existence of farmer perception bias. There is a common finding that farmers underestimate the downside risks since they tend to fail to consider historical minimum yields. Buzby et al. [54] mention this bias, and although the historical means were close to farmer beliefs, on average the subjective means exceeded the historical measures, which indicates that farmers tend to slightly overestimate their expected yields. This finding is consistent with other empirical analyses of farmer beliefs [55] and indicates that producers thus appear to underestimate their true yield variability (upward bias) and tend to slightly overestimate their expected yields. Finally, Turvey et al. [41] also found overconfidence from farmers regarding distributions of maize yields (higher expected yield and lower expected risk than historical data). Better information and education are critical for the improvement of the expectation of farmers: recent research in China found that those farmers who receive education increase NUE in wheat production by approximately 4% of the control population [56].

The model described in this article applies both to rainfed (80% of cultivated land, 60% of food production) and irrigated areas. However, the close relationship between the use of irrigation water and fertilizer deserves mention since over-irrigation usually implies over-fertilization, both for behavioral reasons similar to those studied herein and because excess water produces diffuse pollution by exporting salts and nutrients from the fields. On the other hand, precision irrigation (including fertigation) increases NUE according to Berbel et al. [57], who report that the implementation of water conservation in irrigation schemes had successfully reduced chemical exports (nitrogen and others) from the plots by 80% compared to the levels before ‘modernization’ had been implemented. This improvement is explained by the fact that fertigation and precision irrigation reduce nitrogen application to crops by 25% compared to previous contexts, and by the implementation of the 4Rs strategy to fertilization.

Our model focuses on the use of chemical fertilizers on crops. The analysis of manure spreading and livestock requires other instruments. However, an indication regarding farmer behavior may involve the assumption of very low prices with a trend towards a zero-marginal cost since the management of manure spreading requires the transportation of large volumes. Farmers frequently use manure spreading on their own land without any serious consideration of the opportunity cost implied [58]. The inclusion of manure as a source of nitrogen would predict even greater over-fertilization. Nevertheless, the extension of our model to include manure management requires further development. Cameira et al. [59] study the 10-year implementation of the EU Nitrogen Directive in the Tagus river basin (Portugal), and find that surplus N has decreased following the implementation of this directive and the Nitrogen Directive in regions dominated by irrigated crop production, while nitrogen balance has not decreased in municipalities with intensive livestock production.

Our results are significant for agricultural and environmental policy since it has been found that the over-application of fertilizers by risk-neutral farmers is considered rational behavior as their response to variability and uncertainty of yields. Paulson and Babcock [29] investigate the impact of risk aversion on fertilizer use and reveal a small reduction (6–11%) versus neutral-risk behavior when the focus is on yield uncertainty. However, this difference is smaller when the focus is on the uncertainty in pre-planting nitrogen soil availability, which lies outside the scope of our paper and will therefore be addressed in future research.

The model proposes a stochastic optimum that supports over-application when compared to average yield. We should mention that the dose that optimizes ex-ante stochastic profit recommends an over-application that is close to empirical findings in developed economies: in the range of 14% to 17% (see [26] estimated ex-post against average yield. Additionally, our model confirms the empirical findings that suggest an inelastic demand,
which in turn implies that the effectiveness of pricing is limited since a major price increase (by means of an ecotax) would be required to attain any significant reduction in fertilizer demand.

The proposed model of farmer behavior under yield uncertainty has been employed to support optimal decision-making by farmers under a variety of subjective crop yield PDFs. We use this model to explore the impact of increasing input prices, and to ascertain the effects of different levels of uncertainty. The conclusion is that the parameter that exerts the greatest impact on the over-application of fertilizers is that of perceived yield PDF (expected average yield and variability).

In our opinion, the model developed herein captures many of the complexities of farmers’ decision-making regarding the optimal input use under uncertainty. Future research should explore behavioral economics (nudges) to align private decision-making (farmer optimization) with the minimization of social and environmental externalities produced by the over-application of nutrients and related inputs, such as irrigation water and other nutrients. Supporting and understanding the decision-making process by farmers is fundamental for the improvement of fertilizer management [25,60]. We hope that this work has contributed towards the better understanding of the reasons why farmers may over-apply fertilizers. This critical issue needs further exploration for it to influence farmers’ nutrient management decisions for their own welfare and for collective well-being. To this end, we aim to research our simple decision-support model’s descriptive power and consequently use it as a model to define policies that induce farmers to improve nitrogen application and consequently reduce externalities.

As noted in the introduction of this paper, there are many anthropogenic sources of water pollution. In the EU, livestock production is the largest source of nitrogen pollution of water, although as it is point pollution, it may be addressed with economic and normative instruments. Therefore, further research that helps to decrease any type of water pollution is required.

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Appendix A

Survey

Evaluación Eficiencia Uso Nitrógeno vs Decisión Agricultor  Fecha: 

A: CULTIVO

1. Variedad de semilla utilizada: ________________________________

2. Preparación del terreno:
   - [ ] Laboreo Convencional ¿Retira el residuo? [ ] Sí
     [ ] No ¿conoce cantidad y [N] del residuo? ____________________
   - [ ] No Laboreo ¿conoce cantidad y [N] del residuo? ______________

3. Fecha de preparación: ____________________ Fecha de Siembra: __________

4. Distancia de siembra entre surcos (cm) y entre plantas (cm): ________________

5. Sistema de Riego: _________________________________

   - Utiliza Fertirrigación: [ ] Sí [ ] No

6. Fuente de agua de riego [ ] Canal [ ] Pozo [ ] Otros

7. Análisis Agua [Nitratos]:
   - [ ] Sí Contenido de nitratos a comienzo de campaña ______________
   - [ ] No

8. Cultivo anterior ________________________________

9. Análisis de suelo [Nitratos]:
   - [ ] Sí Contenido de nitratos a comienzo de campaña ______________
   - [ ] No

10. Urea: Unidad F. Nitrógeno/Ha ________ Momento Aplicación __________
    Unidad F. Nitrógeno/Ha ________ Momento Aplicación __________

11. Fertilizante complejo: ______________________________
    Kg Aplicados ________ Momento Aplicación __________

12. ¿Utilizó Inhibidores de nitrógeno? ______________
    Kg Aplicados ________ Momento Aplicación __________

13. ¿Por qué aplicó esta dosis de Nitrógeno (urea + complejo + inhibidor)?
   - [ ] Recomendación técnica
   - [ ] Referencia de otro agricultor
   - [ ] Práctica habitual propia
   - [ ] Otros ______________________________

14. UF N/ha empleadas más habitual en las últimas 10 campañas: ______________

15. UF N/ha empleadas máximo en las últimas 10 campañas: ______________

16. UF N/ha empleadas mínimo en las últimas 10 campañas: ______________
**B: RENDIMIENTO (HISTORICO)**

17. Rendimiento (Kg/ha) esperados en la última campaña: 
18. Rendimiento (Kg/ha) obtenido en la última campaña: 

**C: RENDIMIENTO (PREVISTO en base a su histórico)**

19. Rendimiento ESPERADO más probable (Kg/ha) 
20. Rendimiento MÍNIMO esperado (Kg/ha) 
21. Rendimiento MÁXIMO esperado (Kg/ha) 

**D: RESPUESTA ESPERADA AL ABONADO**

22. ¿Qué rendimiento (kg/ha) esperaría usted obtener para los siguientes niveles de fertilización nitrogenada?:
   - 230 UF Nitrogénio/ha__________ (kg grano/ha)
   - 390 UF Nitrogénio/ha__________ (kg grano/ha)
   - 550 UF Nitrogénio/ha__________ (kg grano/ha)

**E: OTROS**

23. Perteneces a alguna asociación de productores:
   - [ ] Sí: ____________________________ [ ] No

24. Ha recibido asistencia técnica para el cultivo de maíz:
   - [ ] Sí: ____________________________ [ ] No

**F: PRODUCTOR/ENCUESTADO**

Encuestado: [ ] Productor [ ] Otro (especificar) 

25. Nombre 

26. Edad ______ Estudios/Formación:

27. Teléfono ____________________________

28. Municipio ____________________________

29. Finca ____________________________

30. Coordenadas Geográficas UTM (X e Y) ____________________________

31. Años de experiencia cultivando maíz ______ Área (ha) sembrada de maíz ______

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