Nuclear matter and surface properties are carefully studied for a model based on an effective hadronic lagrangian with vector-vector and scalar-vector self-interactions. The results of the model are compared with those of the successful standard non-linear $\sigma - \omega$ model.

1 The model.

Quantum Hadrodynamics (QHD) and the relativistic treatment of nuclear systems has been a subject of growing interest during recent years. The $\sigma - \omega$ model of Walecka\cite{1} plus extensions have been widely used to this end. Traditionally, the hadronic lagrangian was required to be renormalizable. To parametrize the density dependence of the interaction, cubic and quartic self-interactions of the scalar field\cite{2} were introduced. At the mean field (Hartree) level, the model has been very successful in describing many properties of the atomic nucleus.

But recently it has been proposed to look at QHD from a different perspective, as an effective field theory\cite{3}. The lagrangian has to be generalized including all the terms, non-renormalizable in general, that are consistent with the symmetries of the underlying theory, QCD. In principle, this lagrangian has infinite terms. Therefore, it is necessary to find suitable expansion parameters and a scheme of truncation of the generated series. In the nuclear matter problem the meson fields divided by the nucleon mass and their gradients are small and can be used to this end\cite{1,3}. The guide in this expansion should be the concept of naturalness: it means that all the unknown couplings of the theory when written in appropriate dimensionless form should be of order unity. In Ref.\cite{3} the free coefficients were fitted to several nuclear properties and it was found that the best fits to nuclei were indeed natural and that it is enough to go to fourth order in the expansion.

In this work we will study the influence of these new couplings on the surface properties of nuclei. We will work in the semi-infinite nuclear matter geometry which is more convenient to study surface effects.
2 The Energy Functional

The energy functional in mean field approximation for semi-infinite nuclear matter reads

\[ E(z) = \sum_i \varphi_i^\dagger(z) \{-i \mathbf{\alpha} \cdot \nabla + \beta [M - \Phi(z)] + W(z)\} \varphi_i(z) \]

\[ + \frac{1}{2 g_s^2} \left( 1 + \alpha_1 \frac{\Phi(z)}{M} \right) (\nabla \Phi(z))^2 + \left( \frac{1}{2} + \frac{\kappa_3 \Phi(z)}{3! M} + \frac{\kappa_4 \Phi^2(z)}{4! M^2} \right) \frac{m^2}{g_s^2} \Phi^2(z) \]

\[ - \frac{1}{2 g_v^2} \left( 1 + \alpha_2 \frac{\Phi(z)}{M} \right) (\nabla W(z))^2 - \frac{\zeta_0}{4! g_v^4} W^4(z) \]

\[ - \frac{1}{2} \left( 1 + \eta_1 \frac{\Phi(z)}{M} + \frac{\eta_2 \Phi^2(z)}{2 M^2} \right) \frac{m^2}{g_v^2} W^2(z), \quad (1) \]

where the index \( i \) runs over all occupied states of the positive energy spectrum, \( \Phi \equiv g_s \phi_0 \) and \( W \equiv g_v V_0 \) (notation as in Ref. [3]). Except for the terms with \( \alpha_1 \) and \( \alpha_2 \), the functional (1) is of fourth-order in the expansion. We retain the fifth-order terms \( \alpha_1 \) and \( \alpha_2 \) because in Refs. [1] and [3] they have been estimated to be numerically important at the surface.

Though we have studied several surface properties such as the density profile, the central potential, the spin-orbit potential, etc, here we will focus on the surface energy coefficient \( E_s \) Ref. [4] and on the surface thickness \( t \) (90%-10% fall-off distance of the density profile).

3 Results.

3.1 Volume terms: \( \zeta_0, \eta_1, \eta_2 \).

Figure 1a shows the change of \( E_s \) and \( t \) against the parameter \( \eta_0 \) that is related with the quartic vector self-interaction coefficient \( \zeta_0 \) through \( \eta_0 = \sqrt{6 m_s^2 / (g_v^2 \rho_0 \zeta_0)} \) (Ref. [5]), for the saturation properties \( \rho_0 = 0.152 \text{ fm}^{-3}, a_v = -16.42 \text{ MeV}, K = 200 \text{ MeV and } M_{\infty}^*/M = 0.6 \) or 0.7, with \( m_s = 490 \text{ MeV} \). We can see that the overall effect is small for the natural zone (that roughly is \( 2 \leq \eta_0 \leq \infty \)). Nevertheless, there is a change of tendency with \( M_{\infty}^*/M \). For \( M_{\infty}^*/M = 0.6 \), both \( E_s \) and \( t \) decrease while for \( M_{\infty}^*/M = 0.7 \), \( E_s \) increases and \( t \) decreases. This effect, that can also be achieved changing \( K \), can help to find parametrizations with both \( E_s \) and \( t \) lying in the empirical region [6].

The effect of the mixed scalar-vector self-interactions can be seen in Figure 1b. The figure depicts the contours of constant \( E_s \) (solid lines) and \( t \) (dashed lines) in the plane \( \eta_1 - \eta_2 \) for the same saturation properties as before (with \( M_{\infty}^*/M = 0.6 \)). On the one hand, the verticality of the lines indicates that
the influence of the $\eta_2$ coupling is negligible compared to the coupling $\eta_1$. On the other hand, the slope of the lines of constant $E_s$ and $t$ is very similar. Therefore, it is not possible to change $t$ keeping a constant $E_s$ (or viceversa) from the interplay of the $\eta_1$ and $\eta_2$ coefficients. We also have checked that these global trends are the same with different values of $K$ and $M^*/M$.

Figure 1: (a). $E_s$(solid line) and $t$ (dashed line) against $\eta_0$ for the saturation properties indicated in the text. (b) Contours of constant $E_s$ (solid) and $t$ (dashed) in the $\eta_1 - \eta_2$ plane.

3.2 Gradient terms : $\alpha_1$, $\alpha_2$.

Figure 2a shows the lines of constant $E_s$ and $t$ in the plane $\alpha_1 - \alpha_2$, for the same nuclear matter properties as before. Now, the slopes of the contours of constant $E_s$ and $t$ are slightly different. Then, keeping a constant $E_s$ it is possible to change $t$ in a small but appreciable margin. Also, it can be seen that the range of variation of both $E_s$ and $t$ is wider than for the volume terms. This justifies the inclusion in the energy functional of the gradient terms. Figure 2b displays the influence on the spin-orbit strength $V_{so}(z)$ (Ref. 6) of these terms. The effect is not negligible, at least for $M^*/M = 0.6$ and greater than for the volume terms, but it is small. For $M^*/M = 0.7$ it is not possible to achieve the same spin-orbit potential as for the $M^*/M = 0.6$ case. To obtain this effect a tensor coupling of the vector field has to be added.

4 Conclusions.

We have studied the influence on the surface properties of the new terms arising from an energy functional based on relativistic effective field theory.
Figure 2: (a) Contours of constant $E_s$ (solid line) and $t$ (dashed line) in the $\alpha_1 - \alpha_2$ plane. (b) Spin-orbit potential for the cases indicated in the legend.

The volume terms (quartic vector and scalar-vector self-interactions) show a small influence on the surface properties. Nevertheless, the introduction of terms proportional to the gradients of the meson fields can be used to improve the surface properties of a given parametrization taking into account that these terms do not play any role on the saturation properties. Concerning the spin-orbit potential, the influence of all these terms is small. The results of this investigation as well as the study of the properties of asymmetric nuclear matter will be presented elsewhere.

References

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