ANALYTICAL STUDY OF THERMONUCLEAR REACTION
PROBABILITY INTEGRALS

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Abstract. An analytic study of the reaction probability integrals corresponding to the
various forms of the slowly varying cross-section factor $S(E)$ is attempted. Exact expressions
for reaction probability integrals are expressed in terms of the extended gamma functions.
1. Introduction

Nuclear reactions govern major aspects of the chemical evolution of galaxies and stars (Fowler, 1984, Morel et al., 1999). The analytic study of the reaction rates and reaction probability integrals was undertaken by Critchfield (1972), Anderson et al. (1994), Haubold and Mathai (1998), and Chaudhry (1999). A proper understanding of the nuclear reactions that are going on in hot cosmic plasma, and those in the laboratories as well, requires a sound theory of nuclear-reaction dynamics (Clayton, 1983, Bergstroem et al., 1999). The rate \( r_{ij} \) of unlike reacting nuclei \( i \) and \( j \) in the case of nonrelativistic nuclear reactions taking place in nondegenerate environment is expressed as (Clayton, 1983, Lang, 1999)

\[
r_{ij} = n_i n_j \left( \frac{8}{\pi \mu} \right)^{1/2} \left( \frac{1}{kT} \right)^{3/2} \int_0^{\infty} E \sigma(E) e^{-E/kT} dE,
\]

where \( n_i \) and \( n_j \) denote the particle number densities of the reacting nuclei \( i \) and \( j \), \( \mu = \frac{m_im_j}{m_i + m_j} \) is the reduced mass of the reacting nuclei, \( T \) is the temperature, \( k \) is the Boltzmann constant, \( \sigma(E) \) is the cross-section for the reaction under consideration, and \( v = \left( \frac{2E}{\mu} \right)^{1/2} \) is the relative velocity (for reactions between like nuclei \( i = j \), \( n_i n_j \) has to be replaced by \( n^2 \)). Thus (Clayton, 1983, Lang, 1999)

\[
r_{ij} = n_i n_j \lambda := n_i n_j \langle \sigma v \rangle := \int_0^{\infty} \sigma(E) v(E) \psi(E) dE
\]

is the definition of the reaction probability integral \( \lambda \), that is, the probability per unit time that two nuclei, confined to a unit volume, will react with each other. The reaction probability is written in the significant form \( \langle \sigma v \rangle \) to indicate that it is an appropriate average of the product of the reaction cross section and relative velocity of the interacting nuclei. If the reacting mixture is in thermal equilibrium this quantity depends only on the temperature.

For nonresonant nuclear reactions between nuclei of charges \( z_i \) and \( z_j \) at low energies (below the Coulomb barrier), the reaction cross-section has the form (Clayton, 1983, Rowley
\[ \sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta(E)} \]  
\[ \eta(E) = \left(\frac{\mu}{2}\right)^{1/2} \frac{z_i z_j e^2}{\hbar E^{1/2}} \]

is the Sommerfeld parameter, \( \hbar \) is Planck’s quantum of action, and \( e \) is the quantum of electric charge. Eq. (1.3) allows the extrapolation of measured reaction cross sections down to astrophysical energies by introducing the S-factor.

It is to be noted that \( S(E) \), a residual function of energy, represents intrinsically nuclear parts of the probability for the occurrence of a nuclear reaction (Clayton, 1983). It is often found to be constant or a slowly varying function over a limited energy range when the interaction energy of the pair of nuclei is not nearly equal to an energy at which the two nuclei resonate in a quasi-stationary state.

The normalized energy distribution, representing the isotropic velocity distribution of the reacting nuclei, is given by (Clayton, 1983, Lang, 1999)

\[ \psi(E) dE = \frac{2}{\sqrt{\pi}} \frac{E}{kT} \exp \left(-\frac{E}{kT}\right) \left(\frac{dE}{(kT E)^{1/2}}\right) \]  

(for reaction rates with anisotropic distributions see Imshennik, 1990). The substitution of \( \psi(E)dE \) in (1.2) yields

\[ \lambda = \left(\frac{8}{\mu \pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp \left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) dE. \]

where \( b = \left(\frac{\mu}{2}\right)^{1/2} \frac{z_i z_j e^2}{\hbar} \). If the residual function \( S(E) \) is considered to be constant, \( S_0 \), the corresponding reaction probability integral is given by

\[ \lambda = \left(\frac{8}{\mu \pi}\right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^\infty \exp \left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) dE. \]
In effect, the reaction probability is governed by the average of the Gamow penetration factor over the Maxwell-Boltzmann distribution. Anderson et al. (1994) considered for $S(E)$ the Maclaurin series expansion

$$S(E) = S(0) + \frac{dS(0)}{dE}E + \frac{1}{2} \frac{d^2S(0)}{dE^2}E^2$$

(1.8)

and solved (1.6) in terms of $G$- and $H$-functions (Mathai and Saxena, 1973, 1978).

It is to be noted that the residual function $S(E)$ may not admit the representation (1.8) if the thermonuclear fusion plasma is not in the thermodynamic equilibrium, that is, the case in which there is a depletion or cut-off of the high energy tail of the Maxwell-Boltzman distribution in (1.5) (Lapenta and Quarati, 1993). Some of these cases have been considered in Anderson et al. (1994) and Haubold and Mathai (1998). In this paper we consider representations for the residual function given by

$$S_1(E) = S_0 \delta(E - E_0),$$

(1.9)

$$S_2(E) = S_0 \left( \frac{E}{E_0} \right)^{\alpha-1},$$

(1.10)

$$S_3(E) = S_0 \left( \frac{E}{E_0} \right)^{\alpha-1} H(E - E_1),$$

(1.11)

$$S_4(E) = S_0 \left( \frac{E}{E_0} \right)^{\alpha-1} H(E_1 - E),$$

(1.12)

$$S_5(E) = S_0 \left( \frac{E}{E_0} \right)^{\alpha-1} \exp(C(E - E_1)),$$

(1.13)

where

$$H(E - E_1) := \begin{cases} 1, & \text{if } E > E_1, \\ 0, & \text{if } E < E_1 \end{cases}$$

(1.14)

is the unit step function and

$$\delta(E - E_1) := \frac{d}{dE} (H(E - E_1))$$

(1.15)

is the Dirac delta function (Jennings and Karataglidid, 1998).
We discuss the analytic representations of the corresponding reaction probability integrals. The integral corresponding to (1.9) may be called instantaneous reaction probability integral.

2. The Astrophysical Thermonuclear Functions

The theoretical and experimental verification of nuclear cross-sections leads to the derivation of the closed-form representation of the thermonuclear reaction rates. These rates are expressed in terms of the four astrophysical thermonuclear functions (given in the notation chosen by Anderson et al., 1994)

\[ I_1(z, \nu) := \int_0^\infty y^{\nu} \exp(-y - \frac{z}{\sqrt{y}})dy, \tag{2.1} \]
\[ I_2(z, d, \nu) := \int_0^{d} y^{\nu} \exp(-y - \frac{z}{\sqrt{y}})dy, \tag{2.2} \]
\[ I_3(z, t, \nu) := \int_0^\infty y^{\nu} \exp(-y - \frac{z}{\sqrt{y} + t})dy, \tag{2.3} \]
\[ I_4(z, \delta, b, \nu) := \int_0^\infty y^{\nu} \exp(-y - by^\delta - \frac{z}{\sqrt{y}})dy, \tag{2.4} \]

where \( y = \frac{E}{kT} = \frac{\mu^2}{2kT} \) relates \( E \) or \( v \), respectively, to the mean thermal velocity and \( z = 2\pi \left( \frac{\mu}{2kT} \right)^{1/2} \frac{z_i z_j v^2}{h} = 2\pi \left( \frac{\mu^2}{2kT} \right)^{1/2} \alpha z_i z_j \), were the velocity of light was introduced to make the dimension more apparent and to show the dependence on Sommerfeld’s fine structure constant \( \alpha \). The closed-form representations of these integral functions, in terms of G- and H-functions, asymptotic values and, numerical results are discussed in Anderson et al. (1994) and Chaudhry (1999). Accounts about the developments of Meijer’s G-function and of Fox’s H-function are given in Mathai and Saxena (1973) and Mathai and Saxena (1978), respectively. Originally the investigations on these generalized hypergeometric functions were confined to theoretical results such as their analytical properties, integral representations, and asymptotic expansions, and then to the study of symmetric Fourier kernels, the solution of certain functional equations and other mathematical topics. Later, both the G-function
and the H-function have been also used in the fields of statistical and astrophysical sciences, including extensive studies of the thermonuclear functions in eqs. (2.1)-(2.4) (Mathai, 1993).

3. The Extended Gamma Functions

The study of the astrophysical thermonuclear functions led to the development of a new class of special functions (Chaudhry and Zubair, 1998). In particular, we define the extended gamma functions by

\[
\Gamma(\alpha, x; b; \beta) := \int_x^\infty t^{\alpha-1} e^{-t-b/t^\beta} dt, 
\]

and

\[
\gamma(\alpha, x; b; \beta) := \int_0^x t^{\alpha-1} e^{-t-b/t^\beta} dt. 
\]

It is to be noted that the functions (3.1) and (3.2) are special cases of the general class of extended gamma functions introduced in Chaudhry and Zubair (1998).

In fact we have

\[
\Gamma(\alpha, x; b; \beta) = \Gamma_{0.2}^{2.0} \left[ (b, x) \mid ((0, 1), (\alpha, \beta)) \right] := \frac{1}{2\pi} \int_{c-i\infty}^{c+i\infty} \Gamma(s) \Gamma(\alpha + \beta s, x) b^{-s} ds, 
\]

and

\[
\gamma(\alpha, x; b; \beta) = \gamma_{0.2}^{2.0} \left[ (b, x) \mid ((0, 1), (\alpha, \beta)) \right] := \frac{1}{2\pi} \int_{c-i\infty}^{c+i\infty} \gamma(s) \Gamma(\alpha + \beta s, x) b^{-s} ds. 
\]

These functions satisfy the decomposition formula (Chaudhry and Zubair, 1998).

\[
\gamma(\alpha, x; b, \beta) + \Gamma(\alpha, x; b; \beta) = H_{0.2}^{2.0} \left[ (b, x) \mid (0, 1), (\alpha, \beta)) \right]. 
\]
The astrophysical thermonuclear functions (2.1) – (2.4) are special cases of the extended gamma functions (3.1) and (3.2) (Chaudhry, 1999)

\[ I_1(z, \nu) = \Gamma \left( \nu + 1, 0; z; \frac{1}{2} \right) \]  
\[ I_2(z, d, \nu) = \gamma \left( \nu + 1, d; z; \frac{1}{2} \right) \]  
\[ I_3(z, t, \nu) = e^t \sum_{r=0}^{\nu} \left( \frac{-t}{r} \right)^r \Gamma \left( \nu + t, 1, z; \frac{1}{2} \right) \]  
\[ I_4(z, \delta, b, \nu) = \sum_{r=0}^{\nu} \frac{(-b)^r}{r!} \Gamma \left( \nu + r\delta + 1, 0; z; \frac{1}{2} \right) \].

It is straightforward to note that the transformation theorem

\[ \Gamma (\alpha, x; b; \beta) = \frac{1}{\beta} [\Gamma(0, b; \beta) - \Gamma(0, 0, b; \beta)] \]

for the extended gamma function reveals as a special case that

\[ \Gamma (\alpha, 0; b; \beta) = H_{0.2}^{2.0} \left[ (b^{1/\beta}, x) \right] \left[ (0, 1), \left( \alpha, 1 \right) \right] \]

\[ \times G_{0, n+1}^{n+1, 0} \left[ \frac{(b^{1/\beta})^n}{n!} \right] \left[ \left( 0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, \frac{\alpha}{n} \right) \right]. \]

The relation (3.11) yields directly the closed form representation (Anderson et al., 1994)

\[ I_1(z, \nu) = \Gamma \left( \nu + 1, 0; z; \frac{1}{2} \right) \]

\[ = \pi^{-1/2} G_{3.0}^{2.0} \left[ \frac{z^2}{4} \right] \left[ 0, \frac{1}{2}, 1 + \nu \right]. \]

The asymptotic representation of the extended gamma function \( \Gamma (\alpha, 0; b; \beta) \) for small and large values of \( b \) are given as follows (Chaudhry and Zubair, 1998)

\[ \Gamma (\alpha, 0; b; \beta) \sim \left\{ \begin{array}{ll} \Gamma (\alpha) + \frac{1}{\beta} \Gamma \left( -\frac{\alpha}{\beta} \right) b^{\alpha/\beta} + o(b), & \text{for small } b, \alpha \neq 0, \\ -\frac{1}{\beta} \ln b, & \text{for small } b, \alpha = 0, \end{array} \right. \]

and

\[ \Gamma (\alpha, 0; b; \beta) \sim \frac{1}{\beta} \left( \frac{2\pi \beta}{1 + \beta} \right)^{1/2} \beta^{(2a+\beta)/2(1+\beta)} \delta^{(2a-1)/2(\beta+1)} \]

\[ \times \exp \left[ -(1 + \beta)^{\beta/(1+\beta)} b^{1/(1+\beta)} \right], \]  
\[ \text{for large } b. \]
The representations (3.13) and (3.14) are useful in finding the asymptotic values of the thermonuclear reaction probability integrals.

4. Thermonuclear Reaction Probability Integrals

In this section we study the closed form representations of the thermonuclear reaction probability integrals corresponding to the various forms (1.9) – (1.13) of the residual function $S(E)$.

(4.1) Instantaneous Residual Function, $S(E) = S_0 \delta(E - E_0)$. The substitution of this value of $S(E)$ in (1.6) yields an elementary representation

$$
\lambda_1 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{(kT)^{3/2}} \exp \left(- \frac{E_0}{kT} - \frac{b}{\sqrt{E_0}} \right)
$$

of the corresponding thermonuclear probability integral.

(4.2) Power Type Residual Function, $S(E) = S_0 (E/E_0)^{\alpha-1}$. The substitution of the power type residual function in (1.6) yields

$$
\lambda_2 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^\infty (E/E_0)^{\alpha-1} \exp \left(- \frac{E}{kT} - \frac{b}{\sqrt{E}} \right) dE.
$$

The transformation $t = E/kT$ in (4.2) yields

$$
\lambda_2 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{1}{2}} \int_0^\infty t^{\alpha-1} \exp \left(- t - \frac{b}{\sqrt{kT} \sqrt{t}} \right) dt,
$$

which is available from Haubold and Mathai (1998) and can be written in terms of the extended gamma function, i.e.,

$$
\lambda_2 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{1}{2}} \Gamma \left( \alpha, 0; \frac{b}{\sqrt{kT} \sqrt{1/2}} \right).
$$
In view of the decomposition relation (3.5) we can simplify (4.4) to obtain

$$\lambda_2 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{3}{2}} H_{2,0}^{2,0} \left[ \frac{b}{\sqrt{kT}} \left| - \right. \right] \left( \left. 0, 1 \right), (\alpha, 2) \right]$$  \hspace{1cm} (4.5)

that can further be simplified to give

$$\lambda_2 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{3}{2}} G_{3,0}^{3,0} \left[ \frac{b^2}{kT} \left| 0, \frac{1}{2} \alpha \right. \right].$$  \hspace{1cm} (4.6)

(4.c) Power Type Delayed Residual Function, \( S(E) = S_0(E/E_0)^{\alpha-1}H(E - E_1) \). The substitution of the power type delayed residual function in (1.6) gives

$$\lambda_3 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{1}{2}} \int_{E_1}^{\infty} \frac{1}{t^{\alpha-1}} \exp \left( -\frac{t}{kT} - \frac{b}{\sqrt{E}} \right) dt.$$ \hspace{1cm} (4.7)

The transformation \( t = E/kT \) in (4.7) yields

$$\lambda_3 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{1}{2}} \int_{E_1/kT}^{\infty} \frac{1}{t^{\alpha-1}} \exp \left( -t - \frac{b}{\sqrt{kT} \sqrt{t}} \right) dt,$$  \hspace{1cm} (4.8)

or

$$\lambda_3 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{1}{2}} \Gamma \left( \alpha, \frac{E_1}{kT}, \frac{b}{\sqrt{kT}}, \frac{1}{2} \right).$$  \hspace{1cm} (4.9)

The asymptotic representations of \( \lambda_3 \) for small and large values of \( b/\sqrt{kT} \) can be found from (3.13) and (3.14). In fact for small value of \( b/\sqrt{kT} \) and \( \alpha \neq 0 \)

$$\lambda_3 \sim \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{3}{2}} \left\{ \Gamma(\alpha) + \frac{2\Gamma(-\alpha)b^2}{kT} \right\}.$$ \hspace{1cm} (4.10)

(4.d) Power Type Cut-off Residual Function, \( S(E) = S_0(E/E_0)^{\alpha-1}H(E_1 - E) \). The substitution of the power type cut-off residual function in (1.6) gives

$$\lambda_4 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^{E_1} \frac{1}{t^{\alpha-1}} \exp \left( -\frac{E}{kT} - \frac{b}{\sqrt{E}} \right) dE.$$

that can similarly be simplified in terms of the extended gamma function to give

$$\lambda_4 = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{S_0}{\sqrt{E_0}} \left( \frac{kT}{E_0} \right)^{\alpha - \frac{3}{2}} \gamma \left( \alpha, \frac{E_1}{kT}, \frac{b}{\sqrt{kT}}, \frac{1}{2} \right).$$ \hspace{1cm} (4.12)
(4.e) Exponential Type Residual Function, $S(E) = S_0(E/E_0)^{\alpha-1} \exp(-C(E - E_1))$, \((C > -\frac{1}{kT})\). The substitution of the exponential type residual function in (1.6) yields
\[
\lambda_5 = \left(\frac{8}{\mu \pi}\right)^{1/2} \frac{S_0}{(kT)^3/2} \exp(C E_1) \int_0^\infty (E/E_0)^{\alpha-1} \left(-\left(C + \frac{1}{kT}\right) E - \frac{b}{\sqrt{E}}\right) dE. \tag{4.13}
\]
The transformation
\[
t = \left(C + \frac{1}{kT}\right) E, \quad \left(C > -\frac{1}{kT}\right)
\]
in (4.13) yields
\[
\lambda_5 = \left(\frac{8}{\mu \pi}\right)^{1/2} \frac{S_0}{(kT)^3/2} \left(\frac{CkT + 1}{kTE_0}\right)^{\alpha-1} \exp(C E_1) \int_0^\infty t^{\alpha-1} \exp \left(-t - \frac{b}{\sqrt{\frac{kT}{CkT + 1} t}}\right) dt. \tag{4.14}
\]
The integral in (4.14) is solvable in terms of extended gamma function to give
\[
\lambda_5 = \left(\frac{8}{\mu \pi}\right)^{1/2} \frac{S_0}{(kT)^3/2} \left(\frac{CkT + 1}{kTE_0}\right)^{\alpha-1} \exp(C E_1) \Gamma \left(\alpha, 0; \frac{b}{\sqrt{\frac{kT}{CkT + 1} t}}\right). \tag{4.15}
\]
The equation (4.15) can be simplified further in terms of the H-function to give
\[
\lambda_5 = \left(\frac{8}{\mu \pi}\right)^{1/2} \frac{S_0}{(kT)^3/2} \left(\frac{CkT + 1}{kTE_0}\right)^{\alpha-1} \exp(C E_1) H_{0,2}^{2,0} \left[\frac{b}{\sqrt{\frac{kT}{CkT + 1} t}}\right]^{\alpha-1} \left(0, 1; (\alpha, 2)\right). \tag{4.16}
\]

5. Conclusion

Nuclear reactions govern major aspects of the chemical evolution of galaxies and stars. The analytic study of the reaction rates and reaction probability integrals is important in astrophysics. The development of the new class of extended gamma functions by Chaudhry and Zubair has facilitated in convenient notations for these analytic representations. We have considered various forms of the residual function occurring in the reaction probability integrals and have written them analytically in terms of the extended gamma functions.
Acknowledgment. The first author is indebted to King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia for excellent research facilities.

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