$pp \rightarrow pp\pi^0$ near threshold in pionless effective theory

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The total cross section of the $pp \rightarrow pp\pi^0$ reaction near threshold is calculated in pionless effective field theory with a di-baryon and external pions. The amplitudes for a leading one-body and subleading contact neutral pion production vertex are obtained including the initial and final state interactions. After estimating a low-energy constant in the contact vertex, we compare our results for the total cross section with the experimental data.

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1. Introduction

The study of neutral pion production in proton-proton collision near threshold, $pp \rightarrow pp\pi^0$, has been inspired by precise measurements of the near-threshold cross section [1, 2]. Surprisingly, the measured cross section turned out to be \( \sim 5 \) times larger than the early theoretical predictions [3, 4]. Main theoretical difficulties in describing the threshold cross section may stem from the suppression mechanisms in both one- and two-body $S$-wave neutral pion production operators and the typical large momentum transfer $k$ between two protons at the threshold, $k \simeq \sqrt{m_\pi m_N} (k^{-1} \sim 0.55 \text{ fm})$, where $m_\pi$ and $m_N$ are the pion and nucleon mass, respectively. Subsequently, some mechanisms to account for the threshold experimental data have been suggested in model calculations: one is the short-range effect of heavy-meson exchanges [5] and another is the off-shell effect of $\pi N$ $S$-wave isoscalar amplitude in the one-pion exchange production operator [6].

Heavy-baryon chiral perturbation theory (HB\(\chi\)PT) is a low-energy effective field theory (EFT) of QCD and provides us a systematic perturbation scheme in terms of $Q/\Lambda_\chi$ where $Q$ denotes small external momentum and/or symmetry breaking term $m_\pi$ and $\Lambda_\chi$ denotes the chiral scale $\Lambda_\chi = 4\pi f_\pi \simeq 1 \text{ GeV}$; $f_\pi$ is the pion decay constant. (For reviews, see, e.g., Refs. [7, 8, 9, 10, 11].) In Refs. [12, 13, 14] the total cross section of $pp \rightarrow pp\pi^0$ was calculated at the tree-level, where the production operators were derived from the HB\(\chi\)PT Lagrangian up to next-to-leading order (NLO) with the Weinberg’s counting rules [15], and the matrix element was obtained by DWBA: thus the operators were sandwiched between the initial and final two-nucleon wavefunctions obtained by solving the Schrödinger equation with the accurate phenomenological $NN$ potentials. Up to one-loop order, i.e., next-to-next-to leading order (NNLO), the production operators were derived in Ref. [16] and the total cross section was obtained by including the initial and final state interactions in Refs. [17, 18]. The other approaches, e.g., a tree-level calculation including heavy-mesons [19] and a relativistic calculation including a part of the one-loop diagrams [20] were also reported. For a recent review, see Ref. [21] and references therein.

Though many works on the $pp \rightarrow pp\pi^0$ reaction near threshold in HB\(\chi\)PT have been done, some issues in theoretically describing the process have not been fully clarified. In the NLO HB\(\chi\)PT calculations [12, 13, 14], a significant enhancement of the off-shell $\pi\pi NN$ vertex function obtained from the NLO HB\(\chi\)PT Lagrangian is found. However, the two-body (one-pion-exchange) matrix element with the off-shell $\pi\pi NN$ vertex is almost exactly canceled with the one-body matrix element. Thus the experimental data cannot be reproduced in the NLO calculations. In the NNLO HB\(\chi\)PT calculations [16, 17, 18], a significant contribution comes out of the NNLO corrections and a moderate agreement with the experimental data is obtained [17]. However, the chiral series based on the standard Weinberg’s counting rules shows poor convergence. A modification of the original Weinberg’s counting rules to account for the large momentum transfer, $k \simeq \sqrt{m_\pi m_N}$, is discussed in Ref. [13]. The production operators at NLO using the modified counting rules are estimated, and it was reported that the NLO contributions exactly cancel among themselves [22]. Recently, some detailed issues for the loop calculations, such as a concept of reducibility [23], a representation invariance of the chiral fields among the loop
diagrams, and a proper choice of the heavy-nucleon propagator [24], were also studied.\(^2\)

The problem in those NNLO HB\(\chi\)PT calculations for the process is that no hierarchy in constructing the production operators is found. A plausible explanation for this situation is that along with the suppression of the leading order Weinberg-Tomozawa term in the \(\pi \pi NN\) vertex for the one-pion exchange contribution, the one pion exchange propagator, which is usually counted as \(Q^{-2}\), gains another suppression factor \(Q/m_N\) because of the typical large momentum transfer \(k^2 \simeq m_\pi m_N\) and thus is counted as \(Q^{-1} \sim (m_\pi m_N)^{-1}\). Therefore the one-pion-exchange diagram for the process could gain the two suppression factors \((Q/\Lambda_\chi)^2\) and thus become the same order as the one-loop diagrams in the original Weinberg’s counting rules. Apparently the contributions of the intermediate two-pion exchange and the short-range contact diagrams play almost the same role as that of the long-range one-pion-exchange one. A possible way to improve the situation would be to calculate higher order corrections employing the modified counting rules or to employ a relativistic formalism which we will discuss later. Because performing higher order loop corrections is a formidable task, an easy way to circumvent the problem might be to employ a pionless theory in which virtual pions exchanged between the two nucleons are integrated out; in this pionless theory, the one-pion exchange, two-pion exchange and contact terms in HB\(\chi\)PT are subsumed in a contact term.

In this work we employ a pionless effective field theory (EFT) with a di-baryon [27, 28]\(^3\) and external pions [31] to calculate the total cross section of the \(pp \to pp\pi^0\) process. A main motivation of this work stems from the observation that some of the HB\(\chi\)PT calculations failed in reproducing the experimental data. Meanwhile, it is known that the energy dependence of the experimental data can be well reproduced in terms of the final state interaction and the phase space [2]. The pionless theory would be a “minimal” formalism to take account of these two features. Furthermore, after taking these two features into account, the difference between the theory and experiment appears in the overall factor and the experimental data can be easily reproduced by fitting an unknown constant that appears in a contact vertex.

This paper is organized as follows. In Sec. 2 the pionless effective Lagrangian with a di-baryon and external pions is introduced. In Sec. 3 we fix low-energy constants (LECs) for the final \(^1S_0\) and initial \(^3P_0\) \(NN\) states. In Sec. 4 the amplitudes of \(pp \to pp\pi^0\) for a leading one-body and subleading contact vertex are obtained including the strong initial state interaction and the strong and Coulomb final state interactions. We estimate in Sec. 5 the value of an LEC in the external neutral pion production contact vertex from HB\(\chi\)PT, and in Sec. 6 we show our numerical results of the total cross section and compare with the experimental data. Finally, in Sec. 7, the discussion and conclusions of this work are given.

2. Pionless effective Lagrangian with a di-baryon and external pions

\(^2\)Some other issues, a role of three-particle singularities for the \(\pi NN\) system (using a toy model) [25] and the use of the effective low-momentum \(NN\) potential \(V_{\text{low-k}}\) for the process [26], were also studied.

\(^3\)We have studied \(np \to d\gamma\) cross section at BBN energies [29] and neutron-neutron fusion process [30] employing this formalism.
An effective Lagrangian without virtual pions and with a di-baryon and external pions for describing the $pp \rightarrow pp\pi^0$ reaction may read

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_s + \mathcal{L}_{NS\pi} + \mathcal{L}_{NN}^P,$$

where $\mathcal{L}_N$ is the one-nucleon Lagrangian interacting with the external pions, $\mathcal{L}_s$ is that for the $1S_0$ channel di-baryon field interacting with two-nucleon, $\mathcal{L}_{NS\pi}$ is the contact interaction Lagrangian for external pion-dibaryon-two-nucleon, and $\mathcal{L}_{NN}^P$ is the two-nucleon Lagrangian for the $^3P_0$ channel.

The one-nucleon Lagrangian $\mathcal{L}_N$ in heavy-baryon formalism reads

$$\mathcal{L}_N = N^\dagger \left\{ iv \cdot D + 2ig_A S \cdot \Delta + \frac{1}{2m_N} \left[ (v \cdot D)^2 - D^2 \right] + 2g_A \{ v \cdot \Delta, S \cdot D \} + \cdots \right\} N,$$

where $v^\mu$ is a velocity vector with a condition $v^2 = 1$ where $v^\mu = (1, \vec{0})$, and the spin operator $S^\mu$ is $S^\mu = (0, \vec{\sigma}/2)$. $D_\mu = \partial_\mu + \Gamma_\mu$ with $\Gamma_\mu = \frac{i}{2} \{ \xi^\dagger, \partial_\mu \xi \}$ and $\Delta_\mu = \frac{1}{2} \{ \xi^\dagger, \partial_\mu \xi \}$. The nonlinearly realized external pions in the $\xi$ field are given by $\xi = \exp[i\vec{f} \cdot \vec{\pi}/(2f_\pi)]$. $g_A$ is the axial vector coupling.

The effective Lagrangian for the two-nucleon part may read [27, 28, 32, 33]

$$\mathcal{L}_s = \sigma_s s^\dagger \left\{ iv \cdot D + \frac{1}{4m_N} [(v \cdot D)^2 - D^2] + \delta_s \right\} s_a - y_s \left\{ s^\dagger (N^T P_a^{(1S_0)} N) + \text{h.c.} \right\},$$

$$\mathcal{L}_{NS\pi} = \frac{d^{(2)}_\pi}{\sqrt{8m_N r_0}} \left\{ i \epsilon_{abc} s^\dagger \left[ N^T \sigma_2 \vec{\sigma} \cdot i(D - \vec{D}) \tau_2 \tau_a N \right] (iv \cdot \Delta_c) + \text{h.c.} \right\},$$

$$\mathcal{L}_{NN}^P = C_2^0 \delta_{ij} \delta_{kl} \frac{1}{4} \left( N^T \mathcal{O}_{ij,a}^{1P} N \right)^\dagger \left( N^T \mathcal{O}_{kl,a}^{1P} N \right) + \cdots,$$

with

$$\mathcal{O}_{ij,a}^{1P} = i(D_i P_{j,a}^{(P)} - P_{j,a}^{(P)} \bar{D}_i), \quad P_{i,a}^{(P)} = \frac{1}{\sqrt{8}} \sigma_2 \sigma_i \tau_2 \tau_a,$$

where $s_a$ is the (spin singlet) $1S_0$ channel di-baryon field with the isospin index $a$, $\sigma_s$ is the sign factor $\sigma_s = \pm 1$, $\delta_s$ is the mass difference between the di-baryon mass $m_a$ and two-nucleon mass, i.e., $m_a = 2m_N + \delta_s$, $y_s$ is the coupling constant of the di-baryon and two-nucleon interaction, $P_{i,a}^{(S)}$ are the projection operators for $S = 1S_0$ and $^3P_0$ channels;

$$P_a^{(1S_0)} = \frac{1}{\sqrt{8}} \sigma_2 \tau_2 \tau_a, \quad P_a^{(3P_0)} = \frac{1}{\sqrt{8}} \sigma_2 \vec{\sigma} \cdot \vec{\tau}_2 \tau_a, \quad \int d\Omega_\pi \frac{1}{4\pi} \sum_{\text{pol.avg}} \text{Tr} \left( P^{(S)\dagger} P^{(S')} \right) = \frac{1}{2} \delta^{SS'},$$

where $\sigma_i$ (or $\tau_a$) is the spin (isospin) operator. $d^{(2)}_\pi$ is an unknown LEC of the (external) pion-(spin singlet) dibaryon-nucleon-nucleon ($\pi S NN$) interaction. $r_0$ is the effective range in the $1S_0$ ($pp$) channel, and $\Delta^\mu = \frac{1}{2} \Delta a^\mu$. $C_2^0$ is the LEC for the $P$-wave $NN$ scattering in the $^3P_0$ channel.

3. Fixing LECs of the initial and final $NN$ interactions
Figure 1: Diagrams for the dressed di-baryon propagator including the Coulomb interaction. A double-line with a filled circle denotes the renormalized dressed di-baryon propagator. Double-lines without a filled circle and single-line-curves denote the bare di-baryon propagators and nucleon propagators, respectively. The two-nucleon propagator with a shaded blob denotes the Green’s function including the Coulomb potential. A (spin-singlet) dibaryon-nucleon-nucleon \((sNN)\) vertex is proportional to the LEC \(y_s\).

In this section, we calculate the \(S\)- and \(P\)-wave \(NN\) scattering amplitudes to fix the LECs in the two-nucleon part. In Fig. 1, diagrams for the dressed \(^1S_0\) channel di-baryon propagator are shown where the two-nucleon bubble diagrams including the Coulomb interaction are summed up to the infinite order. The inverse of the propagator in the center of mass (CM) frame is given by

\[
iD_s^{-1}(p) = i\sigma_s(E + \delta_s) - iy_s^2 J_0(p),
\]

with

\[
J_0(p) = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{d^3\vec{q}}{(2\pi)^3} \langle \vec{q} | \hat{G}_C^{(+)}(E) | \vec{k} \rangle,
\]

where \(\hat{G}_C^{(+)}\) is the outgoing two-nucleon Green’s function including the Coulomb potential,

\[
\hat{G}_C^{(+)}(E) = \frac{1}{E - \hat{H}_0 - \hat{V}_C + i\epsilon},
\]

where \(E\) is the total CM energy, \(E = \frac{p^2}{2m_N} + \cdots\), \(\hat{H}_0\) is the free Hamiltonian for two-proton, \(\hat{H}_0 = \frac{\vec{p}^2}{2m_N}\), and \(\hat{V}_C\) is the repulsive Coulomb force \(\hat{V}_C = \alpha/r\): \(\alpha\) is the fine structure constant. Employing the dimensional regularization in \(d = 4 - 2\epsilon\) space-time dimensions, we obtain [34, 35]

\[
J_0(p) = \frac{\alpha m_N^2}{8\pi} \left[ \frac{1}{\epsilon} - 3\gamma + 2 + \ln \left( \frac{\pi\mu^2}{\alpha^2 m_N^2} \right) \right] - \frac{\alpha m_N^2}{4\pi} \eta(\eta) - C_\eta^2 \frac{m_N}{4\pi} (ip),
\]

where \(\mu\) is the scale of the dimensional regularization, \(\gamma = 0.5772 \cdots\), and

\[
\eta(\eta) = \text{Re} \psi(i\eta) - \ln \eta, \quad \text{Re} \psi(\eta) = \eta^2 \sum_{\nu=1}^{\infty} \frac{1}{\nu(\nu^2 + \eta^2)} - \gamma,
\]

\[
C_\eta^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1}, \quad \eta = \frac{\alpha m_N}{2p}. \tag{12}
\]
Thus the inverse of the renormalized dressed di-baryon propagator is obtained as
\[
iD_s^{-1}(p) = iy_s^2\frac{m_N}{4\pi} \left[ \frac{4\pi\sigma_s\delta^R_s}{m_N y_s^2} + \frac{4\pi\sigma_s}{m_N y_s^2} p^2 + \alpha m_N h(\eta) + ip C^2_\eta \right],
\]
where $\delta^R_s$ is the renormalized mass difference between the di-baryon and two nucleons,
\[
\sigma_s\delta^R_s = \sigma_s \delta_s - \frac{y_s^2}{\alpha m^2_N} \left[ \frac{1}{\epsilon - 3\gamma + 2} + \ln \left( \frac{\pi \mu^2}{\alpha^2 m^2_N} \right) \right].
\]
We fix it by using the scattering length $a_C$ below.

In Fig. 2, a diagram of the $S$-wave $pp$ scattering amplitude with the Coulomb interaction is shown and thus we have the $S$-wave scattering amplitude as
\[
iA_s = (-iy_s\psi_0)iD_s(p)(-iy_s\psi_0)
= \frac{i4\pi}{m_N} \frac{C^2_\eta e^{2i\sigma_0}}{\frac{4\pi\sigma_s\delta^R_s}{m_N y_s^2} - \frac{4\pi\sigma_s p^2}{m_N y_s^2} - \alpha m_N h(\eta) - ip C^2_\eta},
\]
with
\[
\psi_0 = \int \frac{d^3k}{(2\pi)^3} \langle \vec{k} | \psi_{\vec{p}}^{(+)} \rangle = \int \frac{d^3k}{(2\pi)^3} \langle \psi_{\vec{p}}^{(-)} | \vec{k} \rangle = C_\eta e^{i\sigma_0},
\]
where $\langle \vec{k} | \psi_{\vec{p}}^{(\pm)} \rangle$ are the Coulomb wave functions obtained by solving the Schrödinger equation $(\hat{H} - E) | \psi_{\vec{p}}^{(\pm)} \rangle = 0$ with $\hat{H} = \hat{H}_0 + \hat{V}_C$ and represented in the $| \vec{k} \rangle$ space for the two protons. $\sigma_0$ is the $S$-wave Coulomb phase shift $\sigma_0 = \arg \Gamma(1 + i\eta)$. The $S$-wave amplitude $A_s$ is given in terms of effective range parameters as
\[
iA_s = \frac{i4\pi}{m_N} \frac{C^2_\eta e^{2i\sigma_0}}{\frac{1}{a_C} + \frac{1}{2} r_0 p^2 + \cdots - \alpha m_N h(\eta) - ip C^2_\eta},
\]
where $a_C$ is the scattering length, $a_C = -7.8063 \pm 0.0026$ fm, $r_0$ is the effective range, $r_0 = 2.794 \pm 0.014$ fm, and the ellipsis represents the higher order effective range corrections.
Now it is easy to match the LECs with the effective range parameters. Thus we have
\[ \sigma_s = -1 \text{ and } y_s = \pm 2 \sqrt{\frac{2\pi}{m_N r_0}}. \]

We note that the sign of the LEC \( y_s \) cannot be determined by the effective range parameters.

In Fig. 3, diagrams for the \( P \)-wave \( NN \) scattering are shown. Because the momenta of the two protons are quite large for the pion production reaction, we cannot treat the \( P \)-wave vertex correction in a perturbative way. Thus the two-proton bubble diagrams are summed up to the infinite order without including the Coulomb interaction and the LEC \( C_0^2 \) is renormalized by a phase shift at the threshold energy. The scattering amplitude for the \( ^3P_0 \) channel is obtained as
\[ iA_p = \frac{4\pi}{m_N} \frac{ip^2}{m_N C_0^2} - i\delta^3 \],
where we have used the nucleon propagator, \( iS_N(k) = i/[k_0 - \vec{k}^2/(2m_N) + i\epsilon] \), where \( k^\mu \) is the residual nucleon momentum \( k^\mu = P^\mu - m_N v^\mu \). \( P^\mu \) is the nucleon momentum \( P^2 = m_N^2 \), and we have employed the dimensional regularization for the loop calculation. The LEC \( C_0^2 \) is fixed by the phase shift of \( ^3P_0 \) channel at pion production threshold, \( \delta_p(p_{th}) \approx -7.5^\circ \) at \( p_{th} \approx \sqrt{m_\pi m_N} \). Thus we have
\[ \frac{4\pi}{m_N C_0^2} \approx p_{th}^3 \cot \delta_p(p_{th}). \]

4. Amplitudes for \( pp \rightarrow pp\pi^0 \) near threshold

In Figs. 4 and 5, we show diagrams for \( pp \rightarrow pp\pi^0 \) near threshold. In diagram (a) in Fig. 4 and (c) in Fig. 5, the pion is emitted from the one-body \( \pi NN \) vertex. Because the \( S \)-wave pion production is considered, the \( \pi NN \) vertex (the vertex with “X” in the figures) is obtained from the \( 1/m_N \) Lagrangian. In the diagram (b) in Fig. 4 and (d) in

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\(^4\)An effective Coulomb interaction is denoted by \( \eta = \alpha M/(2p) \). For the high momenta \( p \geq \alpha M/2 \approx 3.42 \text{ MeV}, \eta \leq 1 \) and the Coulomb interaction can be treated in a perturbative way.
Figure 4: Diagrams for $pp \rightarrow pp\pi^0$ near threshold with the strong and Coulomb final state interactions and without the initial state interaction. $S$-wave neutral pion is emitted from $\pi NN$ vertex with “X” at the $1/m_N$ order in (a), whereas the pion is emitted from a pion-dibaryon-nucleon-nucleon contact vertex in (b) which is proportional to LEC $\tilde{d}^{(2)}_{\pi}$.

![Diagram](image-url)

Figure 5: Diagrams for $pp \rightarrow pp\pi^0$ with the strong initial and the strong and Coulomb final state interactions. See the caption of Fig. 4 for more details.

![Diagram](image-url)

Fig. 5, the pion is emitted from the pion-(spin singlet) dibaryon-nucleon-nucleon ($\pi sNN$) contact vertex which is proportional to the unknown LEC $\tilde{d}^{(2)}_{\pi}$. The strong and Coulomb final state interactions are considered in all of the diagrams (a), (b), (c) and (d) in Figs. 4 and 5, whereas the strong initial state interaction is considered $^5$ in the diagrams (c) and (d) in Fig. 5.

The one-body amplitude from the (a) and (c) diagrams and the two-body (contact) amplitude from the (b) and (d) diagrams are obtained as

\begin{align}
 i A_{(a+c)} &= -\frac{4\pi g_A}{m_N f_\pi} \left( \frac{1}{a_c} - \frac{1}{2} r_0 p'^2 + \alpha m_N h(\eta') + ip' C^2_{\eta'} \right) \frac{1}{1 - \frac{m_N C^2_{\eta'}}{4}\pi i p'^3}, \\
 i A_{(b+d)} &= 4 \sqrt{\frac{2\pi}{m_N f_\pi}} \left( \frac{1}{a_c} - \frac{1}{2} r_0 p'^2 + \alpha m_N h(\eta') + ip' C^2_{\eta'} \right) \frac{1}{1 - \frac{m_N C^2_{\eta'}}{4}\pi i p'^3},
\end{align}

where we have used the nucleon propagator, $iS_N(k) = i/[k_0 - \vec{k}^2/(2m_N) + i\epsilon]$ in the calculation of the (a), (c), (d) diagrams. $2\vec{p}$ and $2\vec{p}'$ are the relative three momenta between incoming and outgoing two protons, respectively; $p = |\vec{p}|$ and $p' = |\vec{p}'|$. $\eta' = \alpha m_N/(2p')$.

$^5$See the footnote 4.
Figure 6: Diagram for a one-pion exchange contribution to the \( pp \rightarrow pp\pi^0 \) process for estimation the LEC \( \tilde{d}_\pi^{(2)} \) in the contact vertex. A double-line, single-line, and dashed-line denote a di-baryon, nucleon, and pion, respectively. A \( \pi NN \) vertex with a dot is obtained from \( L_{\pi N}^{(1)} \) and a \( \pi\pi NN \) vertex with “X” is from \( L_{\pi N}^{(2)} \) and \( L_{\pi N}^{(3)} \).

and \( \omega_q \) is the energy of outgoing pion, \( \omega_q = \sqrt{q^2 + m_\pi^2} \); \( q \) is the outgoing pion momentum. In the loop calculations for the diagrams (c) and (d) in Fig. 5, we have employed the dimensional regularization, and we have neglected terms involving \( q \) and \( p' \), which are considered to be small as compared to \( p \approx \sqrt{m_\pi m_N} \). We note that there remain no unknown parameters in the amplitudes except for the LEC \( \tilde{d}_\pi^{(2)} \) in the two-body (contact) amplitude in Eq. (22).

5. Estimate of the LEC \( \tilde{d}_\pi^{(2)} \) from HB\( \chi \)PT

In this section we estimate an order of magnitude of the LEC \( \tilde{d}_\pi^{(2)} \) from HB\( \chi \)PT. We here consider a one-pion-exchange (OPE) diagram shown in Fig. 6. This diagram is the lowest order OPE contribution in the standard Weinberg counting rules. We include a higher order (relativistic) correction to the \( \pi\pi NN \) vertex which is found to be important [20] and is, in the modified counting rules, of the same order as the lowest order diagram. Because other diagrams, e.g., two-pion exchange diagrams would give comparable contributions to that of the LEC \( \tilde{d}_\pi^{(2)} \) (see Refs. [17, 18]), we expect that the estimate of the diagram in Fig. 6 would be reliable only for the order of magnitude estimation for the LEC \( \tilde{d}_\pi^{(2)} \).

The effective chiral Lagrangian relevant to this purpose reads

\[
\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\pi N},
\]

(23)

where \( \mathcal{L}_\pi \) is the \( \chi \)PT Lagrangian for the pions, and \( \mathcal{L}_{\pi N} \) is the HB\( \chi \)PT Lagrangian for the pions and nucleon. These Lagrangians are expanded as

\[
\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \cdots, \quad \mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \cdots,
\]

(24)

where \( \mathcal{L}_\pi^{(2)} \) is the standard chiral Lagrangian for pions at LO. The expression of \( \mathcal{L}_{\pi N}^{(1)} \) and some part of \( \mathcal{L}_{\pi N}^{(2)} \) are given in Eq. (2). To calculate the isoscalar \( \pi\pi NN \) interaction in the diagram in Fig. 6, we consider the interaction Lagrangian \[36\]

\[
\mathcal{L}_{\pi N}^{(2)} = N^\dagger \left[ c_1 \text{Tr}(\chi^+) + \left( \frac{g_A^2}{2m_N} - 4c_2 \right) (v \cdot \Delta)^2 - 4c_3 \Delta \cdot \Delta \right] N + \cdots.
\]

(25)

\[
\mathcal{L}_{\pi N}^{(3)} = N^\dagger \left[ \frac{g_A^2}{4m_N^2} (iv \cdot \Delta \Delta \cdot D + h.c.) - \frac{2c_2}{m_N} [i\text{Tr}(v \cdot \Delta \Delta \mu) D^\mu + h.c.] \right] N + \cdots.
\]

(26)
where we have included relativistic corrections in the higher order Lagrangian \( \mathcal{L}^{(3)}_{\pi NN} \) as mentioned above. The values of the LECs \( c_1, c_2 \) and \( c_3 \) are fixed in the tree-level calculations \([37]\) as

\[
c_1 = -0.64, \quad c_2 = 1.79, \quad c_3 = -3.90 \text{ [GeV}^{-1}] .
\]  

(27)

Thus a contribution to the \( \pi sNN \) vertex function from the diagram in Fig. 6 is obtained as

\[
i\Gamma_{\pi sNN} = -\frac{m_Ny_sg_A|\vec{p}|\epsilon_{abc}}{16\pi f_\pi^3} \left[ \left(-4c_1 + 2c_2 - \frac{3g_A^2}{16m_N} + c_3\right) \frac{m_N^2}{m_N} \int_0^1 dx \frac{1 - x}{\sqrt{F}} \right. \\
\left. - \left( c_2 - \frac{g_A^2}{16m_N} \right) m_N^2 \int_0^1 dx \frac{x^2(1 - x)}{\sqrt{F}} \right] (28)
\]

with

\[
F = x(1 - x)p^2 + x m_N^2 ,
\] 

(29)

where \( a \) and \( b \) (in \( \epsilon_{abc} \)) are the isospin indices for the initial and final two-nucleon state, respectively, and \( c \) is that for the outgoing pion. In the calculation of the diagram in Fig. 6, we have used the nucleon propagator, \( iS_N(k) = i/[k_0 - \vec{k}^2/(2m_N) + i\epsilon] \), and the “potential” pion propagator, \( i\Delta_\pi(k) = -i/[(\vec{k}^2 + m_\pi^2 - i\epsilon)] \) because the typical momentum and energy transfer between two protons are \( \vec{k}^2 \simeq m_\pi m_N \) and \( k_0 \simeq m_\pi/2 \), respectively. Furthermore because \( p^2 \simeq m_\pi m_N >> m_N^2 \), we take an approximation \( F \simeq x(1 - x)p^2 \) and thus have

\[
i\Gamma_{\pi sNN} \simeq \mp i\frac{\sqrt{2\pi}g_A m_N^2}{16f_\pi\sqrt{70}f_\pi^2} \left(-4c_1 + 2c_2 - \frac{3g_A^2}{16m_N} + c_3\right) \epsilon_{abc} .
\]  

(30)

Therefore the value of the LEC \( \bar{d}_\pi^{(2)} \) from the loop diagram in Fig. 6 is obtained as

\[
\bar{d}_\pi^{(2)} \simeq \pm \frac{\sqrt{2\pi}g_A m_N^2}{32m_N^2f_\pi^2} \left(-4c_1 + 2c_2 - \frac{3g_A^2}{16m_N} + c_3\right) \simeq \pm 0.140 \text{ fm}^{5/2} ,
\]  

(31)

where the different signs for \( \bar{d}_\pi^{(2)} \) have been obtained because of the LEC \( y_s \) in Eq. (18).

6. Numerical results

Total cross section of \( pp \to pp\pi^0 \) near threshold is calculated using the formula

\[
\sigma = \frac{1}{2} \int_0^{q_{\text{max}}} dq \frac{d\sigma}{dq} , \quad \frac{d\sigma}{dq} = \frac{1}{v_{\text{lab}}} \frac{m_N q^2 p'}{16(2\pi)^3 \omega_{\text{spin}}^2} \sum_{i} |A_i|^2 ,
\]  

(32)

with

\[
p' = |\vec{p}'| \simeq \sqrt{m_N(T - \sqrt{m_\pi^2 + q^2}) - q^2/4} , \quad q_{\text{max}} \simeq \sqrt{T^2 - m_\pi^2} ,
\]  

(33)

\(^6\)Values of the LECs \( c_1, c_2, \) and \( c_3 \) fixed in the one-loop calculations are quite different from those in the tree-level ones. See, e.g., Refs. [11, 38] for details.
where \( q \) is the outgoing pion momentum, \( q = |\vec{q}| \), \( 2p \) and \( 2p' \) are relative momenta of the initial and final two protons, respectively. \( T \) is the initial total energy \( T \simeq \vec{p}^2/m_N \) and \( v_{\text{lab}} \simeq 2p/m_N \). We have expanded the proton energies in the phase factor in terms of \( 1/m_N \) and kept up to the \( 1/m_N \) order.

\( \mathcal{A} \) is the amplitude \( \mathcal{A} = \mathcal{A}_{(a+c)} + \mathcal{A}_{(b+d)} \) where \( \mathcal{A}_{(a+c)} \) and \( \mathcal{A}_{(b+d)} \) are obtained in Eqs. (21) and (22), respectively. Note that the factor \( 1/2 \) in Eq. (32) is the symmetry factor for the final two protons.

In Fig. 7 we plot our results of the total cross section as a function of \( \eta_\pi = |\vec{q}|_{\text{max}}/m_\pi \). The solid curve and long-dashed curve have been obtained by using \( \bar{d}_\pi^{(2)}(2) = \pm 0.140 \text{ fm}^{5/2} \) fixed from the one-pion exchange diagram in Fig. 6 in the previous section. The LEC \( \bar{d}_\pi^{(2)}(2) \) is also fixed by using the experimental data as

\[
\bar{d}_\pi^{(2)\text{fitted}} = -0.12, \quad +0.55 \text{ fm}^{5/2},
\]  

(34)

where we have two values of \( \bar{d}_\pi^{(2)} \) with different signs. The short-dashed curve is obtained by using \( \bar{d}_\pi^{(2)\text{fitted}} = 0.12 \text{ fm}^{5/2} \), The dotted line corresponds to the case where only the contribution from the one-body amplitude \( \mathcal{A}_{(a+c)} \) is considered. The experimental data are also included in the figure.

We find that the experimental data are reproduced reasonably well with the value of \( \bar{d}_\pi^{(2)} = -0.14 \text{ fm}^{5/2} \). By contrast, we obtain almost vanishing total cross sections with the value \( \bar{d}_\pi^{(2)} = +0.140 \text{ fm}^{5/2} \) because the two-body amplitude with \( \bar{d}_\pi^{(2)} = +0.140 \text{ fm}^{5/2} \) is

Figure 7: Estimated total cross section of \( pp \rightarrow pp\pi^0 \) as a function of \( \eta_\pi = |\vec{q}|_{\text{max}}/m_\pi \). See the text for details.

where we have two values of \( \bar{d}_\pi^{(2)} \) with different signs. The short-dashed curve is obtained by using \( \bar{d}_\pi^{(2)\text{fitted}} = -0.12 \text{ fm}^{5/2} \), The dotted line corresponds to the case where only the contribution from the one-body amplitude \( \mathcal{A}_{(a+c)} \) is considered. The experimental data are also included in the figure.

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Figure 7: Estimated total cross section of \( pp \rightarrow pp\pi^0 \) as a function of \( \eta_\pi = |\vec{q}|_{\text{max}}/m_\pi \). See the text for details.
almost canceled with the amplitude from the one-body contribution. On the other hand, for the whole energy range the experimental near-threshold cross section data are well reproduced with the use of the fitted parameter $d^{(2)}_{\pi}^{fitted} = -0.12 \text{ fm}^{5/2}$.

We also find that approximately a half of the experimental data comes from the one-body (IA) amplitude in the pionless theory. Whereas, in the previous DWBA calculations, about 1/5 of the experimental data comes from the one-body matrix element. The different one-body contributions in the pionful and pionless theory stem from the different short range contributions in the pionless and pionful theory. The importance of the short range contributions in the one-body part of the DWBA calculations has been pointed out (see, e.g., in Fig. 10 in Ref. [13]). In the pionless theory, we conjecture that some short range part of the one-body matrix element in the DWBA calculations is integrated out and held in the contact LEC $d^{(2)}_{\pi}$.

7. Discussion and conclusions

In this work we calculated the total cross section for $pp \rightarrow pp\pi^0$ near threshold in pionless EFT with the di-baryon and external pion fields. The leading one-body amplitude and subleading contact amplitude were obtained including the strong initial state interaction and the strong and Coulomb final-state interactions. After we fix the LECs for the $NN$ scatterings, there remains only one unknown constant, $d^{(2)}_{\pi}$, in the amplitude. We estimated it from the one-pion exchange diagram in the pionful theory. Although this method does not allow us to fix the sign of $d^{(2)}_{\pi}$, we have found that one of the two choices for $d^{(2)}_{\pi}$ leads to the cross sections that agree with the experimental data reasonably well. On the other hand, the whole range of the experimental data near threshold can be reproduced by adjusting the only unknown LEC in the theory, $d^{(2)}_{\pi}$. As discussed in Introduction, this is an expected result because the energy dependence of the experimental total cross section is known to be well described by the phase factor and the final-state interaction [2], which have been taken into account in this work, and the overall strength of the cross section can be adjusted by the value of $d^{(2)}_{\pi}$. This feature would be the same in the NNLO HBχPT calculations because an unknown constant appears in the contact $\pi NNNN$ vertex and can be adjusted so as to reproduce the experimental data (though there are many other corrections coming out of the pion loop diagrams).

The main problem in the NNLO HBχPT calculations that the contributions of formally different chiral orders become of comparable magnitude has not been fully clarified in this work. Though the main difficulties in the HBχPT calculations may stem from the suppression mechanisms in the production operator, another difficulty would stem from the heavy-baryon formalism involving the typical large momentum transfer. The equations of motion of the heavy-field, $v \cdot p \simeq \vec{p}^2/(2m_N)$, makes a connection between the terms in the different orders, one derivative term $v \cdot p$ and two-derivative term $\vec{p}^2/(2m_N)$, and thus the order counting rules become not transparent.\footnote{When there is one heavy-particle, we can avoid the problem by choosing the velocity vector $v^\mu$ so as to $v \cdot p = 0$. But if there are two heavy-particles, we cannot avoid the problem.} This is the reason why we needed to take account of the relativistic corrections: e.g., the term proportional to $v \cdot \Delta \Delta \cdot D$ in $\mathcal{L}^{(3)}_{\pi N}$ in Eq. (26) is a relativistic correction to the term proportional to $(v \cdot \Delta)^2$ in $\mathcal{L}^{(2)}_{\pi N}$ in
Eq. (25). As mentioned in Introduction, one way to solve the problem will be to employ the modified counting rules (see Refs. [23, 24] as well) and calculate the cross section collecting all of the pieces up to NNLO. However, because a lot of higher order terms will be involved in the NNLO HB\chipt calculation with the modified counting rules, another possible way to calculate the production operator may be to employ a manifestly Lorentz invariant baryon chiral perturbation theory [39] with an additional subtraction scheme, such as the infrared renormalization scheme [40] or the extended on-mass shell scheme [41]. That would be worth studying in order to clarify the issues pertaining to the HB\chipt calculation of the near-threshold \( pp \rightarrow pp\pi^0 \) reaction up to one-loop order.

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