EXTENSIONS OF THE N=2 SUPERSYMMETRIC a=-2 BOUSSINESQ HIERARCHY

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Abstract. We present two different Lax operators for a manifestly N=2 supersymmetric extension of ”a=-2” Boussinesq hierarchy. The first is the supersymmetric generalization of the Lax operator of the Modified KdV equation. The second is the generalization of the supersymmetric Lax operator of the \( N = 2 \) supersymmetric a=-2 KdV system. The gauge transformation of the first Lax operator provide the Miura link between the ”small” N=4 supersymmetric conformal algebra and the supersymmetric \( W_3 \) algebra.

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1 Introduction

The integrable hierarchies of differential equations in 1+1 dimensions occupy an important place in diverse branches of theoretical physics as exactly solvable models of fundamental physical phenomena ranging from nonlinear hydrodynamics to string theory [1-3]. Recently N=2 supersymmetric integrable hierarchies have attracted much attention (see [5-17] for example). This interest is motivated by both pure mathematical reasons and possible physical applications of these systems in non-perturbative 2D supergravity, matrix models, etc.

The integrable supersymmetric extensions of the KdV hierarchy bear an intimate relation to superconformal (super Virasoro) algebras via their second Hamiltonian structure [5]. This connection has been extended to more complicated algebra as N=2 super W3 algebra. Due to it three different super N=2 Boussinesq equations have been found [11-12]. The Lax formulation of these systems has been given however only for two cases [12,13,14,9]. Recently Liu [15] constructed the N=1 super W3 algebra via the second Gel'fand-Dickey bracket from the N=1 supersymmetric Lax operator. Moreover he conjectured that this Lax operator producesome N=2 supersymmetric extension of the Boussinesq equation and the corresponding algebra coincides with the N=2 super W3 algebra.

On the other side up to now, most efforts were focused on studying N=1 or N=2 supersymmetric systems. The higher supersymmetry N=4 have been used to supersymmetrization of the Liouville and Korteweg - de Vries system. Recently Delduc, Ivanov and Krivonos [7] has shown that the N=4 super KdV equation could be written down in terms of the N=2 superfields. Such system are called ”quasi” N=4 Susy KdV hierarchies for which the Lax operator has been found also [8]. It appeared that the so called ”small” supersymmetric conformal algebra (SCA) is responsible for the second hamiltonian structure of this hierarchy. Moreover this algebra is connected via the Miura transformation with the N = 2 supersymmetric W3 algebra [8].

In this letter we would like to study the relationship between the generalized Boussinesq and ”quasi” N=4 Susy KdV hierarchy . We show that similarly to the KdV hierarchy it is possible to construct new hierarchy for the supersymmetric a = −2 Boussinesq equation. The investigations of such generalizations is interesting from the several reasons. One of them is the possibility of the construction of a new generalization of the constrained Kadomtsev-Petviashvili hierarchy [18]. Indeed, in the so called bosonic sector of our generalized supersymmetric Boussinesq hierarchy we recover new integrable hierarchy, different than this considered by Melnikov [19].

In order to end this we consider two different Lax operators. The first is a new and in the particular case coincides with the Lax operator considered by Liu [15]. The second is the generalization of the Lax operator considered by Delduc and Gallot [8].

Moreover as the byproduct of our analysis we show that, in the particular case, our first operator give us new Lax representation of the ”quasi” N = 4 supersymmetric KdV system also. We show that the Lax operator of the supersymmetric generalization of the Boussinesq equation is gauge equivalent with the Lax operator of the ”quasi” N = 4 supersymmetric KdV system. This gauge transformation define us the Miura transformation between ”small” SCA algebra and supersymmetric W3 algebra.

Finally we show that the modification of the chirality conditions in the Lax operator
of the "quasi" $N = 4$ Susy KdV hierarchy create new Lax operator which generate the super

symmetric Boussinesq hierarchy.

The hierarchies constructed by these operators constitute the hamiltonian structures
and are connected by simple transformation among themselves. We explicitely constructed
first Hamiltonian structure for these hierarchies.

Let us mention that the similar construction us our for the supersymmetric $N = 2$
Boussinesq equation but for the $a = -1/2$ case has been carried out in [10]. However
there is a basic difference between our aproach and this presented in [10] where authors
included to the generalization, different from our, conformal dimensional chiral and antichiral superfields.

All computation presented in this paper has been carried out with the help of the symbolic computer language Reduce [20] and utilizing the package Susy2 [21].

2 Notation

The basic objects in the supersymmetric analysis are superfields and the supersymmetric
derivatives. The Taylor expansion of the superfield with respect to the $\theta$ is

$$\phi(x, \theta_1, \theta_2) = w(x) + \theta_1 \xi_1 + \theta_2 \xi_2 \theta_2 u,$$ (1)

where the fields $w, u$ are to be interpreted as the boson (fermion) fields for the superboson (superfermion) field while $\xi_1, \xi_2$, as the fermion (boson) for the superboson (superfermions) respectively. The superderivatives are defined as

$$D_1 = \partial_{\theta_1} - \frac{1}{2} \theta_2 \partial, \quad (2)$$
$$D_2 = \partial_{\theta_2} - \frac{1}{2} \theta_1 \partial, \quad (3)$$

and satisfy $D_1^2 = D_2^2 = 0$ and $D_1 D_2 + D_2 D_1 = -\partial$.

Below we shall use the following notation: $(D_i F)$ denotes the outcome of the action
of the superderivative on the superfield $F$, while $D_i F$ denotes the action itself of the superderivative on the superfield $F$.

We use, in the next an obvious relation

$$\int dxd\theta_1d\theta_2 A = \int dxd\theta_1(D_2 A), \quad (4)$$

valid for an arbitrary superfunction $A$ which rapidly vanishes when $x \to \pm \infty$. Notice that
this identity can be interpreted as the supersymmetric analog of the Stokes formula [4].

3 Supersymmetric N=2 Boussinesq equation

This equation is written down as [12]
\[
\frac{d}{dt} V = 2T_x + a(2([D_1, D_2]V_x) + 4VV_x), \\
\frac{d}{dt} T = -2V_{xxx} + ([D_1, D_2]T_x) + 10(D_1V)(D_2V)_x - 2(V[D_1, D_2]V)_x + 4V^2V_x \\
-2V_x([D_1, D_2)V] + 4V^2V_x + (5 - 2a)((D_1V)(D_2T) + (D_2V)(D_1T)) + \\
(8 + 4a)V_xT + (3 + 2a)VT_x,
\]

Here \( a \) is an arbitrary parameter. As was shown in [11,12] for three values of parameter \( a = -2, 1/2, -5/2 \) this equation possesses at least five nontrival conserved currents. The Lax operator has been found only for two values of \( a \) \( (a = -1/2, -2) \) [13,14,12,9] and hence, for these values, this system is integrable.

In the next we consider the \( a = -2 \) case only. Then the equations (5-6) can be reduced to much simpler form. Indeed if we shift the \( T \) superboson to the new one \( W \)
\[
T \rightarrow W + 2([D_1, D_2]V) + 16V^2,
\]
and rescale in an appropriate way the time, we obtain the following system of equations
\[
\frac{\partial}{\partial t} V = 2W_x, \\
\frac{\partial}{\partial t} W = ([D_1, D_2]W_x) + VW_x + (D_2W)(D_1V) + (D_1W)(D_2V).
\]

We present here two different Lax operator for this equation.

The first is
\[
L = \partial^2 + V\partial - (D_2V)D_1 + W + D_1\partial^{-1}(D_2W),
\]
while the second is
\[
L = [D_1, D_2]\partial - D_1VD_2 - D_2VD_1 + \frac{1}{2}[D_1, D_2]\partial^{-1}W + \frac{1}{2}W[D_1, D_2]\partial^{-1}.
\]

The corresponding Lax pair which, produces equation (8-9) for both cases is
\[
\frac{d}{dt} L = [L_{\geq 1}, L],
\]
where \( \geq 1 \) denotes purely (super) differential part of the operator. We show, in the last chapter, the connection of the operator (11) with the Lax operator considered in [9].

As we see our first Lax operator (10), strictly speaking, is the \( N = 1 \) supersymmetric. It possess the first Susy-derivative only. Form the point of view of \( N = 2 \) supersymmetric theory it is degenerated susy-differential operator. Assuming that \( (D_2V) = H \) and \( (D_2W) = T \) we recover the Lax operator considered by Liu in [15].

The conserved currents, for the first Lax operator are defined by the standard (N=1) residue form as
\[
I_n = \int dxd\theta_1 tr L^\theta
\]
where \( tr \) denotes the coefficient standing before \( \partial^{-1}D_1 \). Now using the formula (4) we can cast these currents to the evident N=2 supersymmetric form.
Indeed. It is easy to noticed that an arbitrary power of our Lax operator (10) commutes with the $D_2$ operator. Let us present symbolically the expansion of

$$L^k \rightarrow ...B + D_1 \partial^{-1} F + ...$$

(14)

and

$$[L^k, D_2] \rightarrow ... (D_2 B) + F + D_1 \partial^{-1} (D_2 F) + ... = 0,$$

(15)

for the zero and first pseud-Susy-derivative terms only. The superfunction $B$ and $F$ could be explicitly computed from the Lax operator (10). We conclude, from these expansions, that $F = -(D_2 B)$. Therefore we can use the analog of Stokes formula (4) in order to bring the currents to the evidently $N = 2$ supersymmetric form.

This supersymmetric generalization is bihamiltonian. The second hamiltonian structure is connected with the supersymmetric $W_3$ algebra [12] while first has simple form and reads as [13,12]

$$P = \begin{pmatrix} 0, & 2\partial \\ 2\partial, & [D_1, D_2] \partial + V \partial - (D_1 V) D_2 - (D_2 V) D_1 \end{pmatrix}.$$  

(16)

4 Susy N=2 Boussinesq hierarchy

The first generalization which we would like to study is connected with the Lax operator (10) and has the following form

$$L = \partial^2 + V \partial - (D_2 V) D_1 + W + \partial^{-1} D_1 (D_2 W) - \sum_{i=1}^{m} (F_i D_1 \partial^{-1} (D_2 G_i) - \partial^{-1} D_1 (F_i (D_2 G_i))).$$

(17)

Here $m$ pairs of superbosons chiral and antichiral fields $F_i$ and $G_i$ satisfying

$$(D_1 G_i) = (D_2 F_i) = 0.$$  

(18)

with dimension 1 were introduced.

We have checked that this Lax pair gives rise through the Lax pair (12) to the new hierarchy of the evolution equation. When all $F_k = 0$ and $G_k = 0$, the Lax operator reduces to the (10). Explicitly the first three flows are

$$\frac{\partial}{\partial t_1} V = V_x, \quad \frac{\partial}{\partial t_1} W = W_x, \quad \frac{\partial}{\partial t_1} F_i = F_{ix}, \quad \frac{\partial}{\partial t_1} G_i = G_{ix},$$

(19)

$$\frac{\partial}{\partial t_2} V = 2W_x.$$  

(20)
\[
\frac{\partial}{\partial t_2} W = ([D_1, D_2] W_x) + W_x V + (D_1 W)(D_2 V) + (D_2 W)(D_1 V) - 2 \sum_{i=1}^{m} ((D_2 G_i)(D_1 F_i))_x, \tag{21}
\]

\[
\frac{\partial}{\partial t_2} F_i = F_{ixx} + V F_{ix} - (D_2 V)(D_1 F_i), \tag{22}
\]

\[
\frac{\partial}{\partial t_2} G_i = -G_{ixx} + V G_{ix} - (D_1 V)(D_2 G_i). \tag{23}
\]

For the third flow we scaled the time \( t \mapsto -\frac{1}{4} t \) and obtained

\[
\frac{\partial}{\partial t_3} V = \partial(-V_{xx} + 3(D_2 V)(D_1 V) + \frac{1}{2} V^3 - 6([D_1, D_2] W) - 6W V + 12 \sum_{i=1}^{m} (D_2 G_i)(D_1 F_i)), \tag{24}
\]

\[
\frac{\partial}{\partial t_3} W = -4W_{xxx} - \frac{3}{2} W_x V^2 - 6W_x W - 3 \left( (D_1 W)(D_2 V) \right)_x - 3(D_1 W_x)(D_2 V) + 3 \left( (D_2 W)(D_1 V) \right)_x + 3(D_2 W_x)(D_1 V) - 3 \left( (D_1, D_2] W V \right)_x - 3([D_1, D_2] W_x)V - 3V \left( (D_1 W)(D_2 V) + (D_2 W)(D_1 V) \right) + 6 \sum_{i=1}^{m} \left( 2V \left( (D_2 G_i)(D_1 F_i) \right)_x - 2 \left( G_{ix} F_{ix} \right)_x \right) (D_2 V)(D_1 F_i) G_{ix} - (D_1 V)(D_2 G_i) F_{ix}, \tag{25}
\]

\[
\frac{\partial}{\partial t_3} F_i = -4F_{ixxx} - 6WF_{ix} - 3 \left( VF_{ix} \right)_x - 3V F_{ixx} + 3 \left( (D_2 V)(D_1 F_i) \right)_x + 3(D_2 V)(D_1 F_{ix}) + 6(D_2 W)(D_1 F_{ix}) + 3(D_2 V)(D_1 F_{ix}) V - \frac{3}{2} V^2 F_{ix}, \tag{26}
\]

\[
\frac{\partial}{\partial t_3} G_i = -4G_{ixxx} - 6WG_{ix} + 3 \left( VG_{ix} \right)_x + 3V G_{ixx} - 3 \left( (D_1 V)(D_2 G_i) \right)_x - 3(D_1 V)(D_2 G_{ix}) + 6((D_1 W)(D_2 G_i) + 3(D_1 V)(D_2 G_{ix}) V - \frac{3}{2} V^2 G_{ix}. \tag{27}
\]

If we transform our \( W \) superfield to the following form

\[
W \mapsto \sum_{i=1}^{m} F_i G_i, \tag{28}
\]

then we obtain that the transformed Lax operator
\( L = \partial^2 + V \partial - (D_2 V) D_1 + \sum_{i=1}^{m} \left( F_i G_i + F_i \partial^{-1} D_1 (D_2 G_i) \right), \)  
\( (29) \)
generates for \( m = 1 \) the so called "quasi" N=4 Susy KdV system considered in [8]. In that manner we obtained different Lax representation, than this considered in [8] for these equations.

Now let us assume that \( m = 1 \) and eliminate the field \( F_1 \) in the operator (29). In order to do it we assume that \( F_1 \neq 0 \) and gauge this operator to the new one
\[ \mathcal{L} \Rightarrow \frac{1}{F_1} LF_1. \]
\( (30) \)
We see that this transformed Lax operator has the same structure as the operator (10) if we make the following identification
\[ V = V + 2 \frac{F_{1x}}{F_1}, \]
\( (31) \)
\[ W = F_1 G_1 + \frac{F_{1xx}}{F_1} + \frac{V F_{ix}}{F_1} - \frac{(D_2 V)(D_1 F_1)}{F_1}. \]
\( (32) \)
These formulas define us the Miura transformation between "small" \( N = 4 \) supersymmetric conformal algebra and supersymmetric \( W_3 \) algebra. This transformation coincides with this considered in [8].

We succeed find the first Hamiltonian structure for our equations (20-23). It has the following form
\[ PP = \begin{pmatrix} P & 0 \\ 0 & II \end{pmatrix}, \]
\( (33) \)
where \( P \) is defined by (16) while \( II \) is \( 2m \) dimensional matrix
\[ II = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \]
\( (34) \)
and \( I \) is an identity \( m \)-dimensional matrix.

This Poisson tensor produces a new hierarchy of integrable equations of the form
\[ \frac{\partial}{\partial t_n} (V, W, F_1, \ldots G_1, \ldots)^{tp} = PP \text{grad}(I_n), \]
\( (35) \)
where \( tp \) denotes transposition, \( I_n \) is defined by (13) and \( \text{grad} \) denotes the functional gradient. Explicitly for our equations (20-23) it has the following form
\[ I_2 = \frac{1}{2}(W^2 + 2 \sum_{k=1}^{m} (G_{kx} F_{kx} - V G_k F_{kx} + G_k (D_2 V)(D_1 F_k))). \]
\( (36) \)
It is interesting to check what kind of the hierarchy we obtain if we add chiral and antichiral fields to the Lax operator (11), in a similar manner as for Lax operator (10). We have checked several possibilities and concluded that this generalization can be cast into the form
\[ L = [D_1, D_2] \partial + D_1 V D_2 + D_2 V D_1 + [D_1, D_2] \partial^{-1} W + W[D_1, D_2] \partial^{-1} + \]

If we assume the chirality conditions on the superfields $F_i$ and $G_i$ (eq.18) then we obtain the following second flow:

\[
\begin{align*}
\frac{\partial}{\partial t} V &= 2W_x + 2\sum_{i=2}^{m} (F_i G_i)_x, \\
\frac{\partial}{\partial t} W &= -([D_1, D_2]W_x) + W_x V + (D_1 W)(D_2 V) + (D_2 W)(D_1 V), \\
\frac{\partial}{\partial t} F_i &= F_{i xx} + V F_{i x} - (D_1 V)(D_2 F_i), \\
\frac{\partial}{\partial t} G_i &= -G_{i xx} + V G_{i x} - (D_2 V)(D_1 G_i).
\end{align*}
\]

Interestingly these equations are equivalent with the equations (20-23) if we transform the superboson $W$

\[ W \rightarrow W + \sum_{k=1}^{m} G_k F_k. \]

Hence we can state that this system is also the hamiltonian system.

## 5 From SUSY KdV Hierarchy to the Boussinesq Hierarchy

We have seen in the previous section that it was possible to describe two different hierarchies, ”quasi” $N = 4$ Susy Kdv and Boussinesq, using one Lax operator only. We now show that our generalized Boussinesq hierarchy can be described also by the Lax operator of the ”quasi” $N=4$ Susy KdV with much weaker condition than the chirality assumption (18).

In order to do it, let us first consider the Lax operator (29) where we do not assume any chirality conditions. Then in order to obtain the self-consistent equation of motion, it appeares that we can assume much weaker conditions than (18). Indeed, it is enough to assume that

\[ (D_2 F_i) = 0, \quad F_{i x} (G_{i x} + (D_1 D_2 G_i)) = 0, \]

for all $i$.

We can find two different solutions for the last equations. The first one is

\[ F_1 = \text{const}, \quad G_1 = W, \]

while for $i = 2, 3, ..m$ we assume the conditions (18). The second solution is given by the conditions (18) for all $i$. In the first case we obtain the generalization of the Boussinesq hierarchy, while for the second case we got new Lax operator for the ”quasi” $N=4$ Susy KdV system.

Let us now consider our second Lax operator which we rewrite it as follow

\[
L = [D_1, D_2] \partial + D_1 V D_2 + D_2 V D_1 +
\]
\[ + \sum_{i=1}^{m} \left( F_i \partial^{-1} [D_1, D_2] G_i + G_i \partial^{-1} [D_1, D_2] F_i \right). \]  

We do not assume the (anti)chirality conditions (18) on the fields \( F_i \) and \( G_i \). This Lax operator generates the self-consistent equations provided

\[ (D_2 F_i)(D_1 D_2 G_i) + (D_2 G_i)(D_1 D_2 F_i) = 0, \]  

\[ \left( (D_1 F_i)((D_1 D_2 G_i) + G_{ix}) + (D_1 G_i)((D_1 D_2 F_i) + F_{ix}) \right) = 0, \]

for all \( i \).

We obtained weaker conditions than (18) also. Moreover, the constraints (46-47) have the same solution as in the previous case. Therefore, similarly to the previous case we conclude that our first solution give us the generalization of the Boussinesq equation while the second solution the "quasi" N=4 Susy KdV Lax operator.

Finally let us present the connection of our Lax operator (11) with the Lax operator of the Boussinesq equation considered in [9]. First let us notice that the Susy N=2 a = −2 KdV possesses four different Lax operators

\[ L_1 = \partial^2 + D_1 V D_2 - D_2 V D_1, \]  

\[ L_2 = D_1 (\partial + 2V) D_2, \]  

\[ L_3 = \partial^2 + D_1 V D_2, \]  

\[ L_4 = \partial^2 + V \partial - (D_2 V) D_1. \]

These Lax operators generate the same Susy \( N = 2 \ a = -2 \) KdV equation

\[ \frac{\partial}{\partial t} V = \partial \left( -V_{xx} + 6(D_2 V)(D_1 V) + 2V^3 \right). \]

The first two Lax operators have been considered in [13,16] and they are connected each other as

\[ L_1 \rightarrow L_2 + L_2^*, \]

where * denotes the hermitean conjugation.

The last one is simple the supersymmetric version of the Lax operator of the Modified KdV equation and its generalization has been considered in the previous sections.

We succeed to generalize the second Lax operator and obtained Lax operator

\[ L = D_1 (\partial + 2V) D_2 + \sum_{i=1}^{m} G_i \partial^{-1} D_1 D_2 F_i, \]

which have been considered in [8] but now we do not assume any chirality. However it appeare that we have to assume such in order to obtain the self-consistent equations. In order to weak this assumption let us now consider a new Lax operator \( LL \) constructed as

\[ LL = L - L^*. \]

This form has predicted the unexpected behaviour of the supersymmetrical solitonic equations. We encounter several new phenomena, compare to the classical case, during the
supersymmetrization. One of them is the observation that sometimes the linear combination of Lax operator and its hermitean conjugation gives us the equation which is different than this produced by Lax operator only. For example, the Lax operator [17]

\[ L = \partial + \partial^{-1}[D_1, D_2]V, \]  

produces the Susy \( N = 2 \) \( a = 1 \) KdV system while its linear combination as (54) gives the Susy \( N = 2 \) \( a = 4 \) KdV system.

The operator (55) where \( L \) is defined by (54) coincides with the Lax operator (11). Indeed we can repeat the previous arguments to this Lax operator to establish the same connection with the Boussinesq hierarchy also.

On the other hand it is also possible to decompose this Lax operator in a different manner. Namely, we introduce the operator

\[ N = D_1(\partial + V + D_2\partial^{-1}W\partial^{-1}D_1 + \sum_{i=1}^{m} G_i\partial^{-1}F_i)D_2, \]  

where we now assume the chirality conditions (16) on the fields \( F_i \) and \( G_i \). We quickly realize that it is possible to write \( LL \) as \( LL = N - N^* \). When all \( F_i = 0 \) and \( G_i = 0 \) than the operator \( N \) is exactly the Lax operator introduced in [9].

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