Triviality Bound on Lightest Higgs Mass in NMSSM

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Abstract

We study the implication of triviality on Higgs sector in next to minimal supersymmetric model (NMSSM) using variational field theory. It is shown that mass of the lightest Higgs boson in NMSSM has an upper bound $\sim 10 M_W$ which is of the same order as that in standard model.

PACS Nos. : 14.80.Cp, 11.15.Tk, 12.60.Jv, 11.30.Pb
1. Introduction

It is now widely believed that the $\phi^4$-theory in four space-time dimensions is trivial. If one accepts this, one is forced to conclude that the GSW-model cannot be an exact field theory but at best a reasonable effective theory valid upto an energy scale of $\Lambda_c$ (i.e. all integrations over intermediate four-momenta are evaluated by putting a cut-off $\Lambda_c$). We do not have any definite idea of the scale of $\Lambda_c$ at present except theoretical conjectures. The standard model (SM) has also another mass parameter, namely the mass $M_H$ of the Higgs boson, which can take arbitrary values in the model. This particle has not yet been experimentally observed but one can keep one’s faith in the SM alive by proposing that the Higgs particle lies beyond the present experimental limits. It would then seem that as far as the Higgs sector is concerned the experimental non-observability of the Higgs upto any energy would never cast any shadow on the SM. Things are different if the SM is regarded as an effective theory valid upto some scale $\Lambda_c$. It would then be reasonable to demand that $M_H$ be not too close to $\Lambda_c$ for the effective theory to be a reasonable one; we then have a limit say $M_H < \Lambda_c/5$. With such a restriction, one can establish that the Higgs quartic coupling cannot be arbitrarily large, i.e., it has a maximum allowed value, which translates into an upper bound for $M_H/M_W$. This is the triviality bound studied by various non-perturbative methods. Results are similar, namely that $M_H$ cannot be heavier than value in the range $800$ GeV–$1$ TeV [1]. Methods used in arriving at these results include RGE equation [2], improved perturbative approach [3] and recently two of us have used a variational approach [4] to arrive at a similar result. This last approach is extremely simple and admits of easy generalisation to situations more complicated than SM.

Supersymmetric (SUSY) generalisation of the SM have been studied in recent times [5]. The most economic SUSY-extension of the SM is minimal (MSSM) one [5]. In this version, the quartic couplings are restricted by the gauge coupling with
the result that the Higgs cannot be arbitrarily heavy. At tree level, one has the relationship

\[ M_H^2 \leq M_Z^2 \cos^2 2\beta \leq M_Z^2 \] (1)

where \( \tan \beta \) is the ratio of the vacuum expectation values (VEV’s) of the neutral components of the scalar fields \( H_1 \) and \( H_2 \) that the MSSM involves. Going beyond the tree approximation does not change (1) qualitatively. Thus, Quiros [6] gives the bound

\[ M_H \leq 125 \text{ GeV} \] (2)

for \( m_t = 174 \text{ GeV} \) and a cut-off \( \Lambda_c \approx 10^{19} \text{ GeV} \).

An alternative supersymmetric model proposed is the next to minimal super-symmetric model (NMSSM) which has two \( \text{SU}(2) \otimes \text{U}(1) \) Higgs doublets and one Higgs singlet [5]. The inclusion of a Higgs singlet is suggested in many superstring models and grand unified models. The NMSSM has more coupling parameters than the MSSM and hence it is an interesting theoretical question to enquire into the upper bounds of the Higgs spectrum of the lightest of them. We expect this to be much higher than the one given in (2) and this is the subject matter of our investigation.

The method we follow here is a variational one. Starting with Hamiltonian of the NMSSM we use a gaussian trial wave functional for the ground state and obtain estimates of mass spectra in terms of the bare parameters of the theory. The strategy then is to vary bare parameters over their entire range, impose restrictions that the masses cannot get very close to the cut-off (say less than \( \Lambda_c/5 \)) and obtain the highest mass of the lightest Higgs particle. The parameter space is however very large, and we will be making specific choices of parameters in the hope that our results will be typical of the model itself.

As this investigation was in progress, a paper by Y.Y. Wu on the triviality bound in NMSSM appeared in print [7]. The approach used by this author is
different from ours and triviality requirement used by him was to ensure that the
Higgs coupling constant remains finite below the triviality scale. This leads to a
bound on Higgs mass much lower than the one obtained by us. However the author
has used perturbation theory which as he himself states may not be desirable.
We have used non-perturbative approach and this is the reason, apart from the
difference in the concept of triviality, for the disagreement in results.

2. The Model

In the NMSSM, the potential of the Higgs sector [7] is

\[ V = |hN|^2 \left( \Phi_1^+ \Phi_1 + \Phi_2^+ \Phi_2 \right) + |h \Phi_1^+ \Phi_2 + \lambda N^2|^2 + \frac{1}{8} g_1^2 (\Phi_1^+ \Phi_1 - \Phi_2^+ \Phi_2)^2 \\
+ \frac{1}{8} g_2^2 \left[ (\Phi_1^+ \Phi_1 + \Phi_2^+ \Phi_2)^2 - 4 (\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1) \right] + m_1^2 \Phi_1^+ \Phi_1 + m_2^2 \Phi_2^+ \Phi_2 \\
- m_3^2 (\Phi_1^+ \Phi_2 + h.c.) + m_4^2 N^* N + m_5^2 (N^2 + N^{*2}) \]  

(3)

Here \( \Phi_1 = (\phi_1^+, \phi_1^0) \) and \( \Phi_2 = (\phi_2^+, \phi_2^0) \) are two SU(2) \( \otimes \) U(1) doublets, N a com-
plex singlet, \( m \)’s are mass parameters, \( g \)’s gauge couplings and \( h, \lambda \) are Higgs
couplings. The last five terms represent SUSY-breaking. Equation (3) has two
coupling constants \( h \) and \( \lambda \); we will study the strong coupling behaviour when \( h \)
is very large and hence for simplicity we set \( \lambda = 0 \). Also we take \( m_1 = m_2 = m \) for
simplicity.

It is more convenient to work with the fields defined by

\[ \chi_{1,2} = \frac{1}{\sqrt{2}} (\Phi_1 \pm \Phi_2) \]  

(4)

Now the Higgs potential reduces to

\[ V = h^2 |N|^2 (\chi_1^+ \chi_1 + \chi_2^+ \chi_2) + \frac{1}{4} h^2 |\chi_1^+ \chi_1 - \chi_2^+ \chi_2 + \chi_1^+ \chi_2 - \chi_1^+ \chi_2|^2 \\
+ (m^2 - m_3^2) \chi_1^+ \chi_1 + (m^2 + m_3^2) \chi_2^+ \chi_2 + m_4^2 N^* N + m_5^2 (N^2 + N^{*2}) \]  

(5)
We now assume that the fields $\chi_{1,2}$, where

$$\chi_k \equiv \frac{1}{\sqrt{2}} \left( \chi^c_{kR} + i \chi^c_{kI} \right), \quad (k = 1, 2)$$  \hspace{1cm} (6)

(the superscripts $c$ and 0 denoting charged and neutral components), break the $\text{SU}(2) \otimes \text{U}(1)$ symmetry by assuming a non-zero vacuum expectation value (VEV)

$$\langle \chi_1 \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \chi_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$  \hspace{1cm} (7)

All other fields in (5) are assumed to have zero VEV’s. Writing $N = \frac{1}{2} (N_1 + i N_2)$, there are ten real fields in (5): $\chi^c_{1R}, \chi^c_{1I}, \chi^0_{1R}, \chi^0_{1I}, \chi^c_{2R}, \chi^c_{2I}, \chi^0_{2R}, \chi^0_{2I}, N_1$ and $N_2$, which we denote by $\eta_i (i = 1 \text{ to } 10)$ respectively and their tree level masses by $M_i$. Even after $\chi_1$ develops a non-zero VEV, the Higgs potential has a residual symmetry. To see this explicitly we define:

$$\vec{G} \equiv (\chi^c_{1I}, -\chi^c_{1R}, \chi^0_{1I}) \equiv (\eta_2, -\eta_1, \eta_4),$$

$$\vec{H} \equiv (\chi^c_{2R}, \chi^c_{2I}, \chi^0_{2R}) \equiv (\eta_5, \eta_6, \eta_7)$$

$$S_1 = \chi^0_{1R} \equiv \eta_3 \quad \text{and} \quad S_2 = \chi^0_{2I} \equiv \eta_8$$

Now shifting the fields by their VEV’s, Higgs potential can be written as

$$V = \frac{1}{4} h^2 (N_1^2 + N_2^2) \left[ \vec{G}^2 + (S_1 + v)^2 + \vec{H}^2 + S_2^2 \right]$$

$$+ \frac{1}{16} h^2 \left[ \vec{G}^2 + (S_1 + v)^2 - \vec{H}^2 - S_2^2 \right]^2 + \frac{1}{4} h^2 \left[ \vec{G}.\vec{H} - (S_1 + v) S_2 \right]^2$$

$$+ \frac{1}{2} \left( m^2 - m_3^2 \right) \left( \vec{G}^2 + (S_1 + v)^2 \right) + \frac{1}{2} \left( m^2 + m_3^2 \right) \left( \vec{H}^2 + S_2^2 \right)$$

$$+ \frac{1}{2} \left( m_4^2 + 2m_5^2 \right) N_1^2 + \frac{1}{2} \left( m_4^2 - 2m_5^2 \right) N_2^2$$

Under a SU(2) rotation wherein $\vec{G}$ and $\vec{H}$ are triplets, and $(S_1, S_2, N_1, N_2)$ all singlets, $V$ is invariant. We expect then $M_1 = M_2 = M_4$ and $M_5 = M_6 = M_7$ on account of this symmetry. We note that this symmetry is present in a two-doublet model for the special choice of our parameters. Thus in the two-doublet
model discussed in section 2 of reference [8], setting $v_1 = v_2 = \xi = 0$ and $\lambda_1 = \lambda_4$ makes $H_1^0$, $H_2^0$ and $H_4^0$ degenerate. We will build in this residual symmetry in our variational approach.

In terms of ten real fields the Higgs potential

$$V = (m^2 - m_3^2) v^2 + \frac{1}{4} h^2 v^4 + \frac{1}{2} (m^2 - m_3^2 + \frac{1}{2} h^2 v^2) (\eta_1^2 + \eta_2^2 + \eta_4^2)$$

$$+ \frac{1}{2} (m^2 - m_3^2 + \frac{3}{2} h^2 v^2) \eta_3^2 + \frac{1}{2} (m^2 + m_3^2 - \frac{1}{2} h^2 v^2) (\eta_5^2 + \eta_6^2 + \eta_7^2)$$

$$+ \frac{1}{2} (m^2 + m_3^2 + \frac{1}{2} h^2 v^2) \eta_8^2 + \frac{1}{2} (m_4^2 + 2m_5^2 + h^2 v^2) \eta_9^2$$

$$+ \frac{1}{2} (m_4^2 - 2m_5^2 + h^2 v^2) \eta_{10}^2 + \frac{1}{16} h^2 \sum_{i=1}^{8} \eta_i^4 + \frac{1}{4} h^2 (\eta_5^2 + \eta_6^2 + \eta_7^2 + \eta_8^2)$$

$$+ \frac{1}{8} h^2 \left[ \eta_1^2 (\eta_2^2 + \eta_3^2 + \eta_4^2) - \eta_5^2 - \eta_6^2 - \eta_7^2 - \eta_8^2 \right]$$

$$+ \eta_2^2 (\eta_3^2 + \eta_4^2 + \eta_5^2) - \eta_6^2 - \eta_7^2 - \eta_8^2 \right) + \eta_3^2 (\eta_4^2 - \eta_5^2 - \eta_6^2 - \eta_7^2 + \eta_8^2)$$

$$+ \eta_4^2 (-\eta_5^2 - \eta_6^2 + \eta_7^2 - \eta_8^2) + \eta_5^2 (\eta_6^2 + \eta_7^2 + \eta_8^2)$$

$$+ \eta_6^2 (\eta_7^2 + \eta_8^2) + \eta_7^2 \eta_8^2 \right] + V_{\text{linear}} + V_{\text{cubic}} \tag{8}$$

where $V_{\text{linear}}$ and $V_{\text{cubic}}$ respectively represent the terms linear and cubic in fields.

The tree level minima condition is equivalent to equating the term linear in $\eta_3$ (i.e. $\chi_{1R}^0$) in (8) to zero,

$$m^2 - m_3^2 + \frac{1}{2} h^2 v^2 = 0 \tag{9}$$

Inspection of the quadratic terms in (8) together with (9) immediately tells us that

$$M_1 = M_2 = M_4 = 0 , \tag{10}$$

indicating that $\eta_1$, $\eta_2$ and $\eta_4$ are the goldstones. Furthermore, we have for the charged Higgs triplet

$$M_5^2 = M_6^2 = M_7^2 = m^2 + m_3^2 - \frac{1}{2} h^2 v^2 \tag{11}$$
where the degeneracy is as expected. Lastly,

\[ M_3^2 = m_2^2 - m_3^2 + \frac{3}{2} h^2 v^2, \tag{12} \]

\[ M_8^2 = m_2^2 + m_3^2 + \frac{1}{2} h^2 v^2, \tag{13} \]

\[ M_9^2 = m_4^2 + 2m_5^2 + h^2 v^2 \tag{14a} \]

and

\[ M_{10}^2 = m_4^2 - 2m_5^2 + h^2 v^2 \tag{14b} \]

are the masses of neutral Higgs. For simplicity we also take \( m_5 = 0 \) so that

\[ M_9^2 = M_{10}^2 \tag{15} \]

3. Gaussian Trial Wave-Functional

In order to obtain information on masses beyond the tree level, we follow a variational method with a Gaussian trial wave-functional. Most generally, this wave functional would be the vacuum state of a set of free fields of masses \( \Omega_1, \ldots, \Omega_{10} \) with the \( \Omega \)'s (and \( v \) in equation (7)) representing the variational parameters. However, taking a variational ground state that respects the residual symmetry stated in the last section, we set \( \Omega_1 = \Omega_2 = \Omega_4 \) and \( \Omega_5 = \Omega_6 = \Omega_7 \). Further imposing the symmetry in (15) we also put \( \Omega_9 = \Omega_{10} = \Omega_N \). We thus have five independent masses \( \Omega_1, \Omega_3, \Omega_5, \Omega_8 \) and \( \Omega_N \), and of course \( v \), as variational parameters.

Following standard techniques [9], the expectation value of Hamiltonian density \( \mathcal{H} \) in our trial vacuum state wave functional is

\[
V_G = (m^2 - m_3^2)v^2 + \frac{1}{4} h^2 v^4 + 2 \left[ \frac{1}{2} (M_N^2 - \Omega_N^2)I_0(\Omega_N^2) + I_1(\Omega_N^2) \right] \\
+ \sum_{i=1}^{8} \left[ \frac{1}{2} (M_i^2 - \Omega_i^2)I_0(\Omega_i^2) + I_1(\Omega_i^2) \right] + \frac{1}{2} h^2 I_0(\Omega_N^2) \sum_{i=1}^{8} I_0(\Omega_i^2)
\]
Using equations (18), \( V_G \) reduces to

\[
+ \frac{3}{16} h^2 \left[ 5 I_0^2(\Omega_1^2) + I_0^2(\Omega_3^2) + 5 I_0^2(\Omega_5^2) + I_0^2(\Omega_8^2) \right] \\
+ \frac{1}{8} h^2 \left[ 3 I_0(\Omega_1^2) \left( I_0(\Omega_3^2) - I_0(\Omega_5^2) - I_0(\Omega_8^2) \right) \right] \\
+ 3 I_0(\Omega_8^2) \left( I_0(\Omega_8^2) - I_0(\Omega_8^2) \right) + I_0(\Omega_8^2) I_0(\Omega_8^2) \tag{16}
\]

Here

\[
I_1(\Omega) = \frac{1}{(2\pi)^4} \int d^4k_E \, \ln(k_E^2 + \Omega^2) + \text{constant} \tag{17a}
\]

and

\[
I_0(\Omega) = \frac{1}{(2\pi)^4} \int d^4k_E \, \frac{1}{k_E^2 + \Omega^2} \tag{17b}
\]

Differentiating (16) \( w.r.t. \) \( \Omega_N^2, \Omega_1^2, \Omega_3^2, \Omega_5^2 \) and \( \Omega_8^2 \) gives us five mass equations:

\[
\Omega_N^2 = M_N^2 + \frac{1}{2} h^2 \left[ 3 I_0(\Omega_N^2) + 3 I_0(\Omega_3^2) + I_0(\Omega_5^2) + I_0(\Omega_8^2) \right] \tag{18a}
\]

\[
\Omega_1^2 = M_1^2 + \frac{1}{4} h^2 \left[ 4 I_0(\Omega_1^2) + 5 I_0(\Omega_3^2) + I_0(\Omega_5^2) - I_0(\Omega_8^2) - I_0(\Omega_8^2) \right] \tag{18b}
\]

\[
\Omega_3^2 = M_3^2 + \frac{1}{4} h^2 \left[ 4 I_0(\Omega_3^2) + 3 I_0(\Omega_1^2) + 3 I_0(\Omega_5^2) - 3 I_0(\Omega_8^2) + I_0(\Omega_8^2) \right] \tag{18c}
\]

\[
\Omega_5^2 = M_5^2 + \frac{1}{4} h^2 \left[ 4 I_0(\Omega_5^2) - I_0(\Omega_1^2) - I_0(\Omega_3^2) + 5 I_0(\Omega_8^2) + I_0(\Omega_8^2) \right] \tag{18d}
\]

\[
\Omega_8^2 = M_8^2 + \frac{1}{4} h^2 \left[ 4 I_0(\Omega_8^2) - 3 I_0(\Omega_1^2) + I_0(\Omega_3^2) + 3 I_0(\Omega_5^2) + 3 I_0(\Omega_8^2) \right] \tag{18e}
\]

Using equations (18), \( V_G \) reduces to

\[
V_G = (m^2 - m_3^2) v^2 + \frac{1}{4} h^2 v^4 \\
+ \left[ 2 I_1(\Omega_N^2) + 3 I_1(\Omega_1^2) + I_1(\Omega_3^2) + 3 I_1(\Omega_5^2) + I_1(\Omega_8^2) \right] \\
- \frac{3}{16} h^2 \left[ 5 I_0^2(\Omega_1^2) + I_0^2(\Omega_3^2) + 5 I_0^2(\Omega_5^2) + I_0^2(\Omega_8^2) \right] \\
- \frac{1}{2} h^2 I_0(\Omega_N^2) \left[ 3 I_0(\Omega_1^2) + I_0(\Omega_3^2) + 3 I_0(\Omega_5^2) + I_0(\Omega_8^2) \right] \\
- \frac{1}{8} h^2 \left[ 3 I_0(\Omega_1^2) \left( I_0(\Omega_3^2) - I_0(\Omega_5^2) - I_0(\Omega_8^2) \right) \right] \\
+ 3 I_0(\Omega_8^2) \left( I_0(\Omega_8^2) - I_0(\Omega_8^2) \right) + I_0(\Omega_8^2) I_0(\Omega_8^2) \tag{19}
\]
Here $\Omega_i$’s are to be understood as depending on $v$ through (18). Differentiating $V_G$ w.r.t. $v^2$, we get

\[
d\frac{dV_G}{dv^2} = m^2 - m_3^2 + \frac{1}{2} h^2 v^2 + \frac{1}{4} h^2 \left[ 4I_0(\Omega_N^2) + 3I_0(\Omega_1^2) \right.
\]

\[
+ 3I_0(\Omega_3^2) - 3I_0(\Omega_5^2) + I_0(\Omega_8^2) \right] \tag{20}
\]

Setting $dV_G/dv^2$ to zero and using (18), we get

\[
\Omega_3^2 = h^2 v^2 \tag{21}
\]

It is clear from (20), that increasing $h$ would increase $\Omega_3^2$ without limits. However, the limit would be set by demanding that stationary solution (20) be stable, i.e. the stability matrix $\left( \partial^2 V_G \right)$ be positive definite. Stability condition is obtained by considering $\left( \partial^2 V_G \right)$ to be a function of six variables — five mass parameters ($\Omega_1^2$, $\Omega_3^2$, $\Omega_5^2$, $\Omega_8^2$ and $\Omega_N^2$) and $v^2$, and demanding all its eigenvalues to be positive. As in the case of standard model [4], we expect that as $h$ increases, this condition would no longer be satisfied beyond a certain maximum value of $h$.

Our query regarding triviality bounds does not involve the complete numerical solution. Of five independent masses $\Omega_1, \Omega_3, \Omega_5, \Omega_8$ and $\Omega_N$, $\Omega_1$ is the mass of the Goldstone bosons. This in an exact calculation is expected to be zero but in variational methods (see reference [9] for elaboration), we can get a small but non-vanishing mass. Of the remaining four Higgs masses, we wish to find out whether they can be made arbitrarily heavy relative to $v^2$ (or $M_W^2$). The only condition we would impose is the same as laid down by Hasenfratz and Nager, namely that for a cut off theory to make any physical sense, each one of the masses $\Omega_i$ must not be close to or greater than the cut-off; we put an upper limit of $\Lambda_\epsilon/5$ for definiteness, for all $\Omega_i$’s (See reference [2]).

Our task is then to set $\Omega_i$’s at their maximum possible values and determine the value of $h$ for which the eigen values of stability matrix go from positive to
negative. Since there are only three mass input parameters $m, m_3$ and $m_4$, all the five $\Omega_i$’s cannot be assigned arbitrary values by suitably choosing $m$’s. Furthermore, only $\Omega_N^2$ involves $m_4^2$, so that we can set $\Omega_N$ as the highest acceptable mass namely $\Lambda_c/5$ right away. Of the remaining four masses, we immediately have the sum rule,

$$\Omega_8^2 - \Omega_5^2 = \Omega_3^2 - \Omega_1^2 = h^2v^2 + \frac{h^2}{2} \left[ I_0(\Omega_3^2) - I_0(\Omega_1^2) + I_0(\Omega_8^2) - I_0(\Omega_5^2) \right]$$  \hspace{1cm} (22)

Also since $I_0(\Omega^2)$ is a decreasing function of $\Omega^2$, we get from (18) and (22)

$$\Omega_8^2 > \Omega_5^2 \text{ and } \Omega_3^2 > \Omega_1^2$$

The first possibility is to set

$$\Omega_8 = \Omega_3 = \Lambda_c/5;$$

$\Omega_1$ is the Goldstone and $\Omega_5$ is then the lightest Higgs mass. However in this case $\Omega_5 = \Omega_1$, and the maximum value of $\Omega_5$ turns out to be lower than in the next sequence of masses.

Next we assume $\Omega_8 = \Lambda_c/5$ and the sequence

$$\Omega_8 > \Omega_5 > \Omega_3 > \Omega_1$$

In this $\Omega_3$ is the lightest Higgs. From (22), pushing $\Omega_5$ towards $\Omega_8$ will make $\Omega_3$ go towards $\Omega_1$, i.e. will make $\Omega_3$ lighter. We expect then some kind of optimal situation to arise if $\Omega_5 = \Omega_3$. With the values of $\Omega_3$ (obtained from (18)) corresponding for various $h$, we can calculate the eigen values of stability matrix. As expected one eigen value crosses over from positive to negative at a value of Higgs coupling,

$$h \equiv h_{max} = 4.52$$  \hspace{1cm} (23)
Using this value, then the upper bound on mass of the lightest Higgs $M_{LH} = \Omega_3 = \Omega_5$ is

$$\left.\frac{M_{LH}}{M_W}\right|_{\text{max}} = 10.1$$

(24)

The Goldstone mass $\Omega_1$ for this choice, as we stated before, is not zero but smaller than other masses in the spectrum.

4. Conclusion

We have shown that in the next to minimal version of the supersymmetric model, the lightest Higgs particle has an upper bound which is $\sim 10M_W$. This is of the same order as the bounds in the standard model. We have not attempted to determine the absolute bound taking into account the full range of variation of parameters in the NMSSM (including SUSY breaking parameters). This is because our main aim was to show that in the non minimal version of supersymmetric model, one is not constrained by the rather strict limits on the Higgs mass that one obtains in the minimal model. Also limits on the Higgs mass above 1 TeV are of little interest. There is no possibility in any near future to detect any signals for such a heavy Higgs. Moreover Higgs particle with masses above 1 TeV with widths comparable to masses will make the mass parameter rather meaningless from an experimental point of view. What is more relevant is to note that in a supersymmetric theory, the Higgs mass bounds has the same features as the regular standard model.
References

1. For a summary, see K. Huang, MIT preprint, CTP # 1631, 1988.
2. P. Hasenfratz and J. Nager, Zeit. Phys. C37, 477 (1988).
3. M. Luscher and P. Weisz, Nucl. Phys. B290, 25 (1987), B295, 65 (1988).
4. S.R. Choudhury and Mamta, Delhi preprint (submitted for publication).
5. See e.g. J.F. Gunion et. al., 'The Higgs Hunter’s Guide' (Addison-Wesley, 1990).
6. M. Quiros, CERN preprint, CERN-TH. 7507/94 (November 1994).
7. Y.Y. Wu, Phys. Rev. D51, 5276 (1995)
8. J.F. Gunion and H.E. Haber, Nucl. Phys. B272, 1 (1986)
9. P.M. Stevenson, B. Allès and R. Tarrach, Phys. Rev. D35, 2407 (1987).