Subjecting dark matter candidates to the cluster test

Theodorus Maria Nieuwenhuizen\textsuperscript{1,2}

\textsuperscript{1}Institute for Theoretical Physics, University of Amsterdam, Science Park 904, 1090 GL Amsterdam, The Netherlands
\textsuperscript{2}International Institute of Physics, Federal University of Rio Grande do Norte, Natal, Brazil

Galaxy clusters, employed by Zwicky to demonstrate the existence of dark matter, pose new stringent tests. If merging clusters demonstrate that dark matter is self-interacting with cross section $\sigma/m \sim 2\text{ cm}^2/\text{gr}$, MACHOs, primordial black holes and light axions that build MACHOs are ruled out as cluster dark matter. Recent strong lensing and X-ray gas data of the quite relaxed and quite spherical cluster A1835 allow to test the cases of dark matter with Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac distribution, next to Navarro-Frenk-White profiles. Fits to all these profiles are formally rejected at over 5$\sigma$, except in the fermionic situation. The interpretation in terms of (nearly) Dirac neutrinos with mass of $1.61^{+0.39}_{-0.30}$ eV/c$^2$ is consistent with results on the cluster A1689, with the WMAP, Planck and DES dark matter fractions and with the nondetection of neutrinoless double $\beta$-decay. The case will be tested in the 2018 KATRIN experiment.

I. INTRODUCTION

The existence of dark matter (DM), or some equivalent effect, is beyond doubt and proves the existence of new degrees of freedom. The usual suspects are WIMPs, axions and sterile neutrinos. The standard model of cosmology ΛCDM explains Big Bang Nucleosynthesis (BBN), the Cosmic Microwave Background (CMB) and the Baryon Acoustic Oscillations (BAO), as recently supported by the Dark Energy Survey \cite{1}. But there are several issues, such as: The DM particle has been sought intensively but not found \cite{2}, neither is there a hint for supersymmetry at the LHC. BBN faces the $^7$Li problem \cite{3}, the CMB has a small Hubble constant \cite{4} and faces foreground issues \cite{5,6}. Red-and-dead galaxies require early structure formation \cite{7}, as does a dusty galaxy at $z \sim 7$ with some $3 \times 10^{11} M_\odot$ in gas \cite{8}. Lyman-\alpha clouds are supposed to be stabilized by a high temperature plasma, which should be easy to detect but never was.

These and other sobering results motivate to reconsider other DM options, like primordial black holes (PBHs) or MACHO dark matter. PBHs were thought to be ruled out, but became fashionable again after the discovery of gravitational waves from BH mergers, to meet fresh criticism \cite{9}. A MACHO can be e.g. a planet or a solar mass object, that may consist of normal matter, but also stand for a self-gravitating Bose-Einstein condensate (BEC) of axions or axion-like particles (ALPs). From another angle, our studies of lensing by the cluster A1689 consistently yield good fits for neutrino DM \cite{10-12}.

Supposing that DM does not exist but that Newton’s law gets modified below a critical value of the acceleration has been fruitful for the description for galactic rotation curves \cite{13}. However, it has been demonstrated that these theories, in particular MOND, Emergent Gravity, $f(R)$ and MOG, run into serious troubles for galaxy clusters. The fairly relaxed cluster Abell 1689 posed problems for these theories \cite{14}, as did a second relaxed cluster, A1835 \cite{15}. To function in clusters, MOND and EG would need additional DM, e.g., in the form of $\sim 2$ eV thermal neutrinos. This hot DM is known to induce free streaming in the early Universe, thus suppressing structure formation. They are considered as ruled; in fact the sum of neutrino masses is estimated to lie in the $0.1 - 0.3$ eV range. Nevertheless, a rarely considered question is: has structure formation indeed been linear?

With the road for non-Newtonian gravity essentially closed in our contribution to FQMT’15 \cite{14}, the way forward is to study implications of particle dark matter theories in galaxy clusters. In contrast to CMB and BAO theories, relaxed clusters have simple physics: one may assume that some kind of equilibrium has been reached, so that the history needs not be considered. As such, they put important bench marks.

The paper is composed as follows. In section 2 we consider the effect of DM self-interaction. In section 3 we discuss data for the cluster A1835 and their binning. This is applied to NFW fits in section 4 and to thermal fits in section 5. The paper ends with a summary and an outlook. Throughout the paper we use the reduced Hubble constant $h = 0.7$.

II. ON DARK MATTER SELF-INTERACTION

A. MACHOs and PBHs

In clusters there are too few baryons to account for all the DM but MACHOs may consist of axions or ALPs, or be PBHs. Let us look at a specific cluster, the “train wreck” cluster Abell 520. It reflects the past collision of at least three sub-clusters, which are on their exit. Surprisingly, it it

\[ \frac{\sigma}{m} \sim 1.7 \text{ cm}^2/\text{gr} \]

A similar estimate comes from the Bullet Cluster \cite{19}. MACHOs and PBHs can not have this; for 1 Earth mass, e. g., they would need the gigantic value $\sigma \sim 50 \text{ AU}^2$. If SiDM exists, MACHOs are ruled out as the cluster DM. For both clusters the existence of SiDM has been questioned, however \cite{20,21}. But also the cluster A3827 yields a mild indication for self-interaction, $(\sigma/m) \times \cos i = 0.68^{+0.28}_{-0.29} \text{ cm}^2/\text{gr}$, where $i$ is an inclination angle \cite{22},
### B. WIMPs

The same argument applies to WIMPs, though in a much weaker form. Intuitively, scattering occurs by contact interaction if particles come within their Compton radius. The condition $\sigma_{\text{sc}} < (\hbar/mc)^2$ then leads to

$$m \lesssim \left( \frac{\hbar^2 \text{cm}^2}{2e^2 \text{gr}} \right)^{1/3} = \frac{40 \text{ MeV}}{c^2},$$

which would explain why no WIMP has been observed in the GeV regime. To go beyond this puts a constraint on theories.

### C. Sterile neutrinos

In recent years attention has been payed to sterile neutrinos, so-called warm DM. In particular the report of a 3.5 keV $\gamma$-ray line, possibly related to a 7 keV sterile neutrino, has been inspiring [23, 24]. For elastic scattering the value $\sigma/m \sim 2 \text{cm}^2/\text{gr}$ may not look problematic, but actually they should hardly interact at all, since sterile-sterile neutrino scattering happens indirectly via their mixing with standard ‘active’ neutrinos. For an active-sterile mixing angle $\theta_{14}$, the cross section can be estimated as $\sigma \sim \theta_{14}^2 G_F^2 (m_e e^2)^2 = 1.4 \times 10^{-44} \text{ cm}^2$ [25]. With $m = 7.02$ keV and $\sin^2 2\theta_{14} = 0.69 - 2.29 \times 10^{-10}$ [26] it follows that $\sigma/m = 10^{-37} - 10^{-36} \text{ cm}^2/\text{gr}$. If sterile neutrinos are to make up SiDM, they need another, strong scattering mechanism.

### D. Axions and axion-like particles

ALPs may be as light as $10^{-22} \text{ eV}$; with eV masses they will be thermal; if heavier, they act as WIMPs. Light ones may form Bose-Einstein condensates (BECs). It has been proposed that very light ones, $m \sim 10^{-22} \text{ eV}$, build BECs which act as MACHOs [27]. However, MACHO scenarios can not act as SiDM.

Let us see whether perhaps the whole cluster DM can be one Mpc-sized BEC constituted by ALPs. Its ground state wavefunction satisfies the Schrödinger equation

$$\frac{\hbar^2}{2m} \nabla^2 \psi_0 + m \varphi \psi_0(r) = E \psi_0(r),$$

and the Poisson equation, which relates the gravitational potential $\varphi$ to the mass density $\rho_G$ of the Galaxies, the $\rho_\psi$ of the X-ray gas and the $\rho_{DM}$ of the DM,

$$\nabla^2 \varphi = 4\pi G \rho, \quad \rho = \rho_\psi + \rho_{DM}, \quad \rho_\psi = \rho_G + \rho_\phi.$$  

Here $\rho_{DM} = \psi_0^2$ with normalisation $\int d^3r \psi_0^2 = M_{\text{bdc}}$. In the cluster centre the mass density is known to stem mainly from the brightest cluster galaxy, so $\psi_0^2(0) \ll \rho_G(0)$. Hence the potential is harmonic, $\varphi \approx \frac{\hbar^2}{2m} \omega^2 r^2$. With $\rho_G(0) \sim 10^{12}$ $M_\odot/(10 \text{ kpc})^3$ it has a frequency $\omega \approx [4\pi G \rho_G(0)/3]^{1/2} \approx 1/10^7 \text{ yr}$. This problem is solved in every quantum mechanics textbook. Its characteristic length

$$\ell_0 = \sqrt{\frac{\hbar}{m \omega}} = \frac{410^{-12}}{\sqrt{mc^2/\text{eV}}} \text{ kpc}$$

is tiny on the cluster scale, so the condensate must basically act as a point mass, maximally equal to

$$M_{\text{bdc}} < \rho_G(0) \ell_0^3 \sim \frac{5}{(mc^2/\text{eV})^{1/2}} 10^{-26} M_\odot,$$

which even for $m \sim 10^{-22} \text{ eV}$ is less than $10^8 M_\odot$ and thus negligible. Extended DM distributions must thus have many BECs acting as MACHOs, a scenario discussed already. Hence light axions and ALPs are problematic as SiDM.

### III. A1835 DATA AND THEIR BINNING

For the cluster A1835 theories of DM can be tested on recent data for $M_{2d}(r)$, the mass in a cylinder around the cluster centre [15]. From the observed strong lensing arcs mass maps are generated; this being an underdetermined problem, an ensemble $N = 1001$ of compatible 2d mass maps is produced and from them their $M_{2d}$ values at radii $r_n \sim a^n$ with $n = 1, \cdots, 149$, such that $(r_1, r_{149}) = (4.03, 11200) \text{ kpc}$. In the centre only a few arcs occur, hence only $N = 117$ of the $r_n$ contain data for $\hat{\Sigma}_n = \langle M_{2d}(r_n)/r_n^2 \rangle$ and their covariances $\Gamma_{mn}$ [15]; the index $n = 1, \cdots, N$ is relabelled accordingly. The matrix $\Gamma$ has a big spread of eigenvalues, roughly between 0.5 and $5 \times 10^{-15} \text{ gr}^2/\text{cm}^4$. The standard definition of $\chi^2$ involves $\Gamma^{-1}$ but small eigenvalues should not matter and have to be regularised. Here we shall merely employ the data themselves.

As first step to eliminate the small eigenvalues, the $N$ data points are grouped in $N_{\text{bin}} = 17$ bins with in principle $n_i = 7$ points, but not all bins can be full. Choosing $n_9 = 5$ or $n_{10} = 5$ we minimize bias around the bin 9, which has the smallest errors. We can now relabel the index $n \rightarrow \{ik\}$, according to the bin number $i = 1, \cdots, N_{\text{bin}}$ and the location $k = 1, \cdots, n_i$ inside the bin; this defines $r_{ik}, \hat{\Sigma}_{ik}$ and $\Gamma_{ik,jl}$. As bin centre $r_i$ we take the geometrical average $r_i = (\prod_{k=1}^{n_i} r_{ik})^{1/n_i}$.

As a new step, we divide out the theoretical value in the binning. Given a theoretical or empirical $\Sigma(r)$, the data is binned as

$$\hat{\Sigma}_i = \Sigma(r_i) \frac{1}{n_i} \sum_{k=1}^{n_i} \frac{\hat{\Sigma}_{ik}}{r_{ik}}, \quad i = 1, \cdots, N_{\text{bin}}.$$  

The standard binning with $\Sigma(r) \rightarrow 1$ would do less justice to the data than the best $\Sigma(r)$ fit, and hence lead to a loss of information. Moreover, the binning (6) makes the choice of $r_i$ as good as any other. The binned covariances read

$$\Gamma_{ij} = \frac{\Sigma(r_i) \Sigma(r_j)}{n_i n_j} \frac{1}{n_k \prod_{l=1}^{n_i} (r_{ik}/\Sigma(r_i)) \Sigma (r_{jl}) \Sigma (r_{jl})}.$$  

$\Gamma_{ij}$ has eigenvalues typically from 0.07 to $5 \times 10^{-14} \text{gr}^2/\text{cm}^4$, hardly better than $\Gamma$. The way to proceed is by noting that eq. (6) puts forward a measure for the intra-bin fluctuations,

$$\gamma_i = \frac{\sum_{k=1}^{n_i} \left| \frac{\hat{\Sigma}_{ik}}{\Sigma (r_{ik})} - \frac{\hat{\Sigma}_{ik}}{\Sigma (r_i)} \right|}{\left| \Sigma (r_i) \right|}.$$  

This is actually a square; without absolute values, it would vanish. As final step, we add the $\gamma_i$ as diagonal regulator and define the total binned covariance matrix $C$,

$$C_{ij} = \Gamma_{ij}^{\text{bin}} + \delta_{ij} \gamma_i.$$
The eigenvalues of $C$ go down to $\sim 10^{-7}$ g$^2$/cm$^4$, so further regularization with an ad hoc constant $\delta \gamma_i = \gamma$ [11, 12, 28] is not needed. As measure for the goodness of the fit we take

$$
\chi^2(\Sigma) = \sum_{i,j=1}^{N_{\text{bin}}} \left[ \Sigma^{\text{bin}}_{ij} - \Sigma(r_{ij}) \right]^2 \frac{1}{\sigma_{i,j}^2} \left[ \Sigma^{\text{bin}}_{ij} - \Sigma(r_{ij}) \right].
$$

(10)

It differs from the standard $\chi^2$ in that the data and the covariances are binned employing the fit function $\Sigma(r)$.

To estimate the errors in fit parameters $p_1, p_2, \cdots$ we assume that the data involve Gaussian errors. Denoting $\Delta_i = \Sigma^{\text{bin}}_{ij} - \Sigma(r_{ij})$ and the errors by $\delta$, the leading Gaussian errors of $\chi^2(\Sigma) = \Delta C^{-1} \Delta$ are collected symbolically as

$$
\delta \chi^2(\Sigma) = \langle \delta \Delta \rangle = (\delta C - \delta C^{-1} \delta C)^{-1} (\delta \Delta - \delta C^{-1} \Delta),
$$

(11)

where $\delta C_i = \sum_k (\partial C_i/\partial p_k) \delta p_k$, and likewise for $\delta C_{ij}$. The covariances are defined from $\delta \chi^2(\Sigma) \equiv \sum_{k,l} (X^{-1})_{kl} \delta p_k \delta p_l$ as $(\delta p_k \delta p_l) = X_{kl}$ and the errors in the $p_k$ as $\Delta p_k = (X_{kk})^{1/2}$.

IV. NFW FITS

We first apply this to the Navarro-Frenk-White (NFW) profile [29],

$$
\rho_{\text{NFW}} = \frac{A R^3}{r(r + R)^2} = \frac{200 \rho_c (1 + z_{\text{Ar835}})^3}{3 \log(1 + c) - c/(1 + c)} \frac{R^3}{r(r + R)^2},
$$

(12)

From any mass density $\rho$, the tested quantity is

$$
\Sigma(r) = \frac{4}{r^2} \int_0^r ds \rho(s) + \int_r^\infty ds \frac{4s \rho(s)}{s + \sqrt{s^2 - r^2}}.
$$

(13)

As best fit to $\chi^2(\Sigma)$ we find for NFW with $n_8 = 5$

$$
A = 0.4330 \pm 0.0088 m_N/cm^3, \quad R = 159.0 \pm 1.9 \text{kpc}.
$$

(14)

Using $z_{\text{Ar835}} = 0.253$ this corresponds to concentration $c = 9.55 \pm 0.08$. With $\nu = 17 - 2$, $\chi^2/\nu = 5.5$ and $q = 2.0 \times 10^{-11}$, the case is formally ruled out at 6.7 $\sigma$.

The generalization “gNFW” involves a power $n \neq 1$ [30],

$$
\rho_{\text{gNFW}} = \frac{A R^3}{r^n(r + R)^{3-\sigma}}.
$$

(15)

The best gNFW fit again occurs for $n_8 = 5$,

$$
A = 0.2976 \pm 0.067 m_N/cm^3, \quad R = 180 \pm 19 \text{kpc},
$$

(16)

and $n = 1.135 \pm 0.036$, so that $c = 8.18 \pm 0.72$. This fit has $\chi^2/\nu = 5.8$, $q = 1.6 \times 10^{-11}$ and is formally ruled out at 6.8 $\sigma$.

V. THERMAL PARTICLES

A. Generalities

We turn to thermal bosons for $g$ species of mass $m$ and chemical potential $\mu_g$, at temperature $m\sigma^2$. Setting $p = mv$, the Bose-Einstein mass density reads

$$
\rho_{\text{DM}}(r) = \int \frac{d^3v}{(2\pi)^3} \exp\left(\frac{m_g}{T} + \varphi(r) - \mu_g/\sigma^2\right) - s,
$$

(17)

with $s = 1$. For $s = 0$ this describes isothermal classical particles, while for $s = -1$ thermal fermions.

The data for the X-ray gas in A1835 fit well to [15]

$$
\rho_\varphi(r) = \frac{\sigma_\varphi^2(r^2 + R_{g1}^2)}{2\pi G (r^2 / R_{g1}^2)(r^2 + R_{g2}^2)},
$$

(18)

with $\sigma_g = 496.6 \pm 6.4$ km/s; $\{R_{g0}, R_{g1}, R_{g2}\} = \{91 \pm 13, 31.8 \pm 2.9, 169 \pm 15\} \text{kpc}$. We model the galaxy mass density as [28]

$$
\rho_c(r) = \frac{\rho_c^0}{(1 + r^2 / R_c^2)^2} + (1 + r^2 / R_t^2)^2.
$$

(19)

Solving the Poisson equation (3) we may now determine $\Sigma$ from (13), which can also be expressed as [10]

$$
\Sigma(r) = \frac{1}{\pi G} \int_0^\infty ds \varphi'(r \cosh s).
$$

(20)

With $\varphi' > 0$ and varying less than $\rho$, this relation is numerically better behaved.

B. Isothermal classical particles or objects

Minimizing $\chi^2(\Sigma)$ with respect to the free parameters in (17) and (19) we have $\nu = 12$. Treating $m^4$ and $\sigma^4$ as independent Gaussian variables, we obtain for the case $n_{t0} = 5$

$$
m = 4.07^{+93}_{-4.071} \text{g}^{-1/4} \text{e}^{-\nu/\sigma^2} \text{eV} / c^2, \quad \sigma = 1464 \pm 1160 \text{km/s}, \quad (21)
$$

$$
\rho_c^0 = 24 + 2512 \frac{\text{M}_\odot}{\text{cm}^3}, \quad \{R_c, R_t\} = \{1.5 \pm 45.4, 122 \pm 151\} \text{kpc}.
$$

The large error estimates and its $\chi^2/\nu = 6.05$ express that the fit is bad. It corresponds to $q = 1.0 \times 10^{-10}$ and being formally ruled out at 6.5 $\sigma$.

C. Thermal bosons

Let us return to the BE case (17) for axions, ALPs and dark photons. It is instructive to minimize $\chi^2(\Sigma)$ for $n_{t0} = 5$ at fixed $\mu$, so that $\nu = 12$. The worst case occurs at $\mu = 0$,

$$
m = 7.6^{+1.0}_{-0.7} \text{g}^{-1/4} \text{eV} / c^2, \quad \sigma = 1210^{+2000}_{-1210} \text{km/s}, \quad (22)
$$

$$
\rho_0 = 286 \pm 1520 \frac{\text{M}_\odot}{\text{cm}^3}, \quad \{R_c, R_t\} = \{1.2 \pm 3.4, 120 \pm 380\} \text{kpc}.
$$

(Its $\chi^2/\nu = 12.5$ and $q = 5.8 \times 10^{-26}$ mean formal ruling out at 10.6 $\sigma$. For $\mu$ taking increasingly negative values, $\chi^2$ diminishes until for $\mu \ll -\sigma^2$ the BE distribution approaches a MB one, with its large $m$ from (21). Hence minimizing $\chi^2$ as function of $\mu$ will drive the best boson fit towards the classical isothermal limit, where it is still formally ruled out at 6.5 $\sigma$.}
After all these negative findings, we test eq. (17) for fermions. Successful fermion fits to data sets of the cluster A1689 have been reported [10–12]. For A1835 this case again yields a good fit. For $n_{\text{fit}} = 5$ the value $\chi^2(\Sigma)/\nu = 1.82$ with $\nu = 11$ and $g = 0.046$ is perfectly acceptable and proves the adequacy of our approach. The parameters are

$$
\begin{align*}
\sigma &= 1164 \pm 39 \text{ km/s}, \quad \mu = 5.8 \pm 1.3 \times 10^6 \text{ km}^2/\text{s}^2, \\
\rho_0 &= 18 \pm 4 \text{ mN/cm}^3, \\
R_c &= 7.2 \pm 9.2 \text{ kpc}, \\
R_t &= 123 \pm 160 \text{ kpc}.
\end{align*} 
(23)
$$

For the DM density the parameters are reasonably constrained, but for the galaxies not. The mass takes the value

$$
m = 1.61^{+0.19}_{-0.30} \times 10^{-11} \text{ eV}/c^2. 
(24)
$$

The fit is presented in fig. 1. The residues have a systematic trend, again induced by the small errors around 100 kpc and minimized by choosing bin 10 as the one with 5 points.

In our approach the dark matter is fitted together with the galaxies. One may wonder whether this induces a bias towards fermions. However, dropping the galaxies mass density and only fitting the last 6 bins again brings fermions as best fit, be it with mass of $2.14(12/g)^{1/4} \text{ eV}$.

**VI. SUMMARY**

After recalling that modifications of Newton’s law do not solve the dark matter problem in galaxy clusters [14, 15], we consider the performance of the most studied DM candidates in clusters. An important question is whether DM is self-interacting (SiDM). If this is indeed the case, its elastic cross section $\sigma/m \sim 2 \text{ cm}^2/\text{gr}$ puts strong constraints: MACHOs and primordial black holes are ruled out, together with light axion-like particles that have to build MACHOs. It would also put constraints on other particle models, for instance, axions and sterile neutrinos should, at best, scatter very weakly. Hence the establishment or ruling out of SiDM in cluster collision is of major interest.

In contrast to CMB and BAO analyses, relaxed clusters provide a simple cosmological test, because their history has just led to a certain relaxed shape for the DM and can be disregarded. To compare to our previous works on A1689, we consider here the cluster A1835, for which strong lensing and X-ray data were presented [15]. We introduce a new, parameter-free method to regularize the small eigenvalues of the covariance matrix: binning and accounting for the intra-bin variations. We present results for one particular way of binning and fitting; other ones produced the same trend. Within this approach we analyze several options for dark matter. NFW models and classical isothermal models do not fare well for the small errors in the data and seem eliminated at more than 6$\sigma$; hence even if DM turns out not to be self-interacting, MACHOs and PBHs seem to be ruled out. Thermal bosonic models perform even less well unless they are in their classical isothermal limit; this severely questions whether thermal axions or ALPs can constitute the DM.

Thermal fermionic DM, however, does offer a good match. They have to represent (nearly) Dirac neutrinos with a mass $\lesssim 2.8 \times 10^{-3} \text{ eV}$. The exclusion of more than one sterile neutrino in (nearly) this mass and a (nearly) vanishing Majorana mass has to be proven. Indeed, GERDA gives as most recent result $m^{\text{res}} < 0.15 - 0.33 \text{ eV}$ [31], where, in the usual notation [25],

$$
m^{\text{res}}_{\beta\beta} \equiv |c_{12}c_{13}|^2 m_1 + |c_{12}s_{13}|^2 m_2 + |s_{12}s_{13}|^2 m_3.
(25)
$$

For equal $m_{1,2,3} = m_\nu$ and $\eta_{1,2} = \pi$, the known mixing angles [25] yield the value $0.37 m_\nu$, so that in general $m_\nu \lesssim 2.8 m^{\text{res}}_{\beta\beta}$. Violating this bound for any $g \lesssim 110$, our neutrinos must be of (nearly) Dirac type [12]. Up to the small effects of neutrino oscillations, the active neutrinos have (nearly) equal mass, also 3 sterile partners with (nearly) this mass and a (nearly) zero sterile Majorana mass matrix [25]. With the antineutrinos there are $g = 12$ fermion species or $3 + 3$ fermion families.

The number density is $56 \text{ cm}^{-3}$ for each species [25], so if the cold dark matter fraction $\Omega_c$ actually stems from neutrinos, the WMAP value [22] corresponds to $m = 1.80 \pm 0.08 \text{ eV}$ and the Planck value [33] to $m = 1.88 \pm 0.03 \text{ eV}$. DES Y1 [1] implies $m = 1.68^{+0.25}_{-0.15} h^{-2} \text{ eV}$. Within $1.5 \sigma$ these cases are covered by (24) and support our findings for A1689 [10–12].

**D. Thermal fermions**

The fermionic case likely refers to neutrinos and antineutrinos. Indeed, they act as $g/2$ relativistic degrees of freedom during the BBN, which poses new issues, so it is economic that some of them are known particles.

Active neutrinos are in principle Majorana particles, but with eV mass, neutrinoless double $\beta$-decay should have been discovered. Indeed, GERDA gives as most recent result $m^{\text{res}}_{\beta\beta} < 0.15 - 0.33 \text{ eV}$ [31], where, in the usual notation [25],

$$
m^{\text{res}}_{\beta\beta} \equiv |c_{12}c_{13}|^2 m_1 + |c_{12}s_{13}|^2 m_2 + |s_{12}s_{13}|^2 m_3.
(25)
$$

For equal $m_{1,2,3} = m_\nu$ and $\eta_{1,2} = \pi$, the known mixing angles [25] yield the value $0.37 m_\nu$, so that in general $m_\nu \lesssim 2.8 m^{\text{res}}_{\beta\beta}$. Violating this bound for any $g \lesssim 110$, our neutrinos must be of (nearly) Dirac type [12]. Up to the small effects of neutrino oscillations, the active neutrinos have (nearly) equal mass, also 3 sterile partners with (nearly) this mass and a (nearly) zero sterile Majorana mass matrix [25]. With the antineutrinos there are $g = 12$ fermion species or $3 + 3$ fermion families.

The number density is $56 \text{ cm}^{-3}$ for each species [25], so if the cold dark matter fraction $\Omega_c$ actually stems from neutrinos, the WMAP value [22] corresponds to $m = 1.80 \pm 0.08 \text{ eV}$ and the Planck value [33] to $m = 1.88 \pm 0.03 \text{ eV}$. DES Y1 [1] implies $m = 1.68^{+0.25}_{-0.15} h^{-2} \text{ eV}$. Within $1.5 \sigma$ these cases are covered by (24) and support our findings for A1689 [10–12].

E. Interpretation in terms of neutrinos

The fermionic case likely refers to neutrinos and antineutrinos. Indeed, they act as $g/2$ relativistic degrees of freedom during the BBN, which poses new issues, so it is economic that some of them are known particles.

Active neutrinos are in principle Majorana particles, but with eV mass, neutrinoless double $\beta$-decay should have been discovered. Indeed, GERDA gives as most recent result $m^{\text{res}}_{\beta\beta} < 0.15 - 0.33 \text{ eV}$ [31], where, in the usual notation [25],

$$
m^{\text{res}}_{\beta\beta} \equiv |c_{12}c_{13}|^2 m_1 + |c_{12}s_{13}|^2 m_2 + |s_{12}s_{13}|^2 m_3.
(25)
$$

For equal $m_{1,2,3} = m_\nu$ and $\eta_{1,2} = \pi$, the known mixing angles [25] yield the value $0.37 m_\nu$, so that in general $m_\nu \lesssim 2.8 m^{\text{res}}_{\beta\beta}$. Violating this bound for any $g \lesssim 110$, our neutrinos must be of (nearly) Dirac type [12]. Up to the small effects of neutrino oscillations, the active neutrinos have (nearly) equal mass, also 3 sterile partners with (nearly) this mass and a (nearly) zero sterile Majorana mass matrix [25]. With the antineutrinos there are $g = 12$ fermion species or $3 + 3$ fermion families.

The number density is $56 \text{ cm}^{-3}$ for each species [25], so if the cold dark matter fraction $\Omega_c$ actually stems from neutrinos, the WMAP value [22] corresponds to $m = 1.80 \pm 0.08 \text{ eV}$ and the Planck value [33] to $m = 1.88 \pm 0.03 \text{ eV}$. DES Y1 [1] implies $m = 1.68^{+0.25}_{-0.15} h^{-2} \text{ eV}$. Within $1.5 \sigma$ these cases are covered by (24) and support our findings for A1689 [10–12].

**Figure 1.** Data for $\Sigma$ in A1835 (black points, gray error bars) and binned data (red). Upper line: best fit for thermal fermion model (blue). Lower lines: contributions from neutrinos, galaxies and X-ray gas, respectively. Lower pane: fit residuals.
streaming road block of linear structure formation. But the notorious the $^7$Li problem in the BBN may as well require nonlinearities. The latter could be restricted to the cluster scale and down to the galaxy scale or lower, and have not much impact on the CMB. Neutrinos with eV mass have no impact inside galaxies, but the solution could lie in MOND or gravitational hydrodynamics [35].

An effect similar to dark matter self-interaction in cluster-cluster collision may be caused by the Pauli principle acting in the collision of such quantum degenerate “neutrino stars”.

VII. OUTLOOK

The question raised by previous studies of A1689 and now confirmed for A1835 becomes pressing: What is the reason for singling out degenerate fermions as best fit for cluster lensing? While it is desirable to study more clusters, preferably relaxed spherical ones, one may already wonder: Is there a conspiracy, or is simply the neutrino, after all, just the dark matter particle, and $\Lambda$CDM only an effective theory? And is the neutrino a Dirac fermion just having its right handed partner? The answer will come from the test of the electron antineutrino mass in the KATRIN experiment [36]; for the prediction of 1.5 – 1.9 eV two months of data taking in 2018 [37] should suffice. If such a detection is indeed made, the neutrino sector of the standard model is basically determined and the cluster dark matter riddle solved.

Acknowledgments We thank A. Morandi, M. Limousin and E.F.G. van Heusden for discussion.

[1] M. Troxel, N. MacCrann, J. Zuntz et al. arXiv preprint arXiv:1708.01538 (2017).
[2] G. Arcadi, M. Dutra, P. Ghosh et al. arXiv preprint arXiv:1703.07364 (2017).
[3] R. H. Cyburt, B. D. Fields, K. A. Olive, and T. H. Yeh Reviews of Modern Physics 88(1), 015004 (2016).
[4] W. L. Freedman arXiv preprint arXiv:1706.02739 (2017).
[5] G. Verschuur and J. Schmelz The Astrophysical Journal 832(2), 98 (2016).
[6] V. Vavryčuk Monthly Notices of the Royal Astronomical Society: Letters 470(1), L44–L48 (2017).
[7] D. J. Croton and G. R. Farrar Monthly Notices of the Royal Astronomical Society 386(4), 2285–2289 (2008).
[8] M. Strandet, A. Weiß, C. De Breuck et al. arXiv preprint arXiv:1705.07912 (2017).
[9] S. M. Koushiappas and A. Loeb arXiv preprint arXiv:1704.01668 (2017).
[10] T. M. Nieuwenhuizen EPL (Europhysics Letters) 86(5), 59001 (2009).
[11] T. M. Nieuwenhuizen and A. Morandi Monthly Notices of the Royal Astronomical Society p.stt1216 (2013).
[12] T. M. Nieuwenhuizen J. Phys.: Conf. Ser. 701, 012022 (2016).
[13] M. Milgrom The Astrophysical Journal 270, 371–389 (1983).
[14] T. M. Nieuwenhuizen Fortschritte der Physik 65(6-8) (2017).
[15] T. M. Nieuwenhuizen, A. Morandi, and M. Limousin to be published (2017).
[16] M. Jee, A. Mahdavi, H. Hoekstra et al. The Astrophysical Journal 747(2), 96 (2012).
[17] D. Clowe, M. Markevitch, M. Bradač et al. The Astrophysical Journal 758(2), 126 (2012).
[18] M. J. Jee, H. Hoekstra, A. Mahdavi, and A. Babul The Astrophysical Journal 783(2), 78 (2014).
[19] M. Markevitch, A. Gonzalez, D. Clowe et al. The Astrophysical Journal 606(2), 819 (2004).
[20] A. Robertson, R. Massey, and V. Eke Monthly Notices of the Royal Astronomical Society p.stw2670 (2016).
[21] A. Peel, F. Lanusse, and J.L. Starck arXiv preprint arXiv:1708.00269 (2017).
[22] R. Massey arXiv preprint arXiv:1708.04245 (2017).
[23] E. Bulbul, M. Markevitch, A. Foster et al. The Astrophysical Journal 780(1), 13 (2014).
[24] A. Boyarsky, O. Ruchayskiy, D. Iakubovskyi, and J. Franse Physical review letters 113(25), 251301 (2014).
[25] J. Lesgourgues, G. Mangano, G. Miele, and S. Pastor, Neutrino cosmology (Cambridge University Press, 2013).
[26] N. Cappelluti, E. Bulbul, A. Foster et al. arXiv preprint arXiv:1701.07932 (2017).
[27] L. Hui, J. P. Ostriker, S. Tremaine, and E. Witten arXiv preprint arXiv:1610.08297 (2016).
[28] M. Limousin, J. Richard, E. Jullo et al. The Astrophysical Journal 668(2), 643 (2007).
[29] J. F. Navarro, C. S. Frenk, and S. D. White The Astrophysical Journal 783(2), 78 (2014).
[30] Y. Jing and Y. Suto The Astrophysical Journal 789(6-8), A13 (2016).
[31] GERDA-Collaboration Nature 544(7648), 47–52 (2017).
[32] G. Hinshaw, D. Larson, E. Komatsu et al. The Astrophysical Journal Supplement Series 208(2), 19 (2013).
[33] P. A. Ade, N. Aghanim, M. Arnaud et al. Astronomy & Astrophysics 594, A13 (2016).
[34] C. Giunti Nuclear Physics B 908, 336–353 (2016).
[35] T.M. Nieuwenhuizen, C.H. Gibson, and R.E. Schild EPL (Europhysics Letters) 88(4), 49001 (2009).
[36] E. W. Otten and C. Weinheimer Reports on Progress in Physics 71(8), 086201 (2008).
[37] A. Cho Science(June) (June, 29, 2017).