Multiwavelength Signatures of Episodic Nullpoint Reconnection in a Quadrupolar Magnetic Configuration and the Cause of Failed Flux Rope Eruption

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Abstract

In this paper, we present multiwavelength observations of the triggering of a failed-eruptive M-class flare from active region NOAA 11302 and investigate the possible reasons for the associated failed eruption. Photospheric observations and nonlinear force-free field extrapolated coronal magnetic field revealed that the flaring region had a complex quadrupolar configuration with a preexisting coronal nullpoint situated above the core field. Prior to the onset of the M-class flare, we observed multiple periods of small-scale flux enhancements in GOES and RHESSI soft X-ray observations from the location of the nullpoint. The preflare configuration and evolution reported here are similar to the configurations presented in the breakout model, but at much lower coronal heights. The core of the flaring region was characterized by the presence of two flux ropes in a double-decker configuration. During the impulsive phase of the flare, one of the two flux ropes initially started erupting, but resulted in a failed eruption. Calculation of the magnetic decay index revealed a saddle-like profile where the decay index initially increased to the torus-unstable limits within the heights of the flux ropes, but then decreased rapidly and reached negative values, which was most likely responsible for the failed eruption of the initially torus-unstable flux rope.

Unified Astronomy Thesaurus concepts: Active sun (18); Solar activity (1475); Solar active region filaments (1977); Solar flares (1496); Active solar corona (1988)

Supporting material: animation

1. Introduction

Solar eruptive phenomena are violent activities occurring in the solar atmosphere that include catastrophic energy releases within a short time in a localized region, i.e., flares (Priest & Forbes 2002; Fletcher et al. 2011; Benz 2017), along with the expulsion of plasma and magnetic field into the interplanetary space, i.e., coronal mass ejections (CMEs; Chen 2011; Green et al. 2018). Earth-directed CMEs inflict hazardous effects in the near-Earth environment that include damage to satellites, disruption of the telecommunication system, and damage of the electrical power grids on Earth (space weather; see, Moldwin 2008; Koskinen et al. 2017; Lanzerotti 2017). While most of the major flares are associated with CMEs (i.e., eruptive flares), a significant number of flares does not lead to CMEs (i.e., confined flares; see, e.g., Yashiro et al. 2005; Baumgartner et al. 2018; Li et al. 2020). Observationally, a particular variant of confined flares also involves so-called failed flux rope eruptions, where a flux rope is initially activated from the source region, but subsequently fails to escape from the overlying layers of the solar corona, and the material eventually falls back (Gilbert et al. 2007, see also Ji et al. 2003; Alexander et al. 2006; Liu et al. 2009; Kushwaha et al. 2015). With an ever-increasing urge to understand the factors leading to CME eruptions and develop methods to predict space weather, the observational and theoretical studies of failed eruptions have recently gained much attention and have become an important research topic in contemporary solar physics (see, e.g., Cheng et al. 2011; Amari et al. 2018; Sarkar & Srivastava 2018).

Eruptive flares are usually characterized by the formation of two parallel, ribbon-like brightenings followed by the development of a coronal flare arcade connecting the two ribbons (Svestka & Cliver 1992). To explain these two-ribbon flares, the standard flare model, also known as the CSHKP model, was developed. It includes the pioneering works of Carmichael (1964), Sturrock (1966), Hirayama (1974), and Kopp & Pneuman (1976). In recent years, this model has been extended to three dimensions (Aulanier et al. 2012, 2013; Janvier et al. 2013). According to this model, when a preexisting magnetic flux rope (MFR) is triggered for upward eruption, inflow of magnetic field occurs beneath the MFR, where a current sheet is eventually generated. Magnetic reconnection at these current sheets rapidly converts previously stored magnetic energy into plasma heating and kinetic energy of accelerated particles, and also adds magnetic flux to the erupting flux rope, prolonging the driving Lorentz force (Vršnak 2016; Veronig et al. 2018). The CSHKP model is quite successful in explaining commonly observed large-scale features of eruptive flares, e.g., footpoint and looptop hard X-ray (HXR) sources; the expansion of flare ribbons in opposite polarities observed in optical, extreme-UV (EUV), and soft X-ray (SXR) wavelengths; the formation of highly structured loop arcades observed in optical, EUV, and SXR wavelengths that connect the flare ribbons; and the formation of hot cusp-like structures (see, e.g., Tsuneta et al. 1992; Masuda et al. 1994; Sui et al. 2006; Veronig et al. 2006; Miklenic et al. 2007; Joshi et al. 2017; Gou et al. 2019; Mitra & Joshi 2019). However, some important aspects related to solar eruptions remain beyond the scope of the standard flare model,
e.g., the processes of flux rope formation; onset or triggering of eruptions; and the initial dynamical evolution of the CME or flux rope (Green et al. 2018, see also Joshi et al. 2019).

Theoretically, an MFR is defined as a set of magnetic field lines that are wrapped around a central axis (Gibson & Fan 2006). Observationally, MFRs can be identified in the form of filaments or prominences, coronal cavities, coronal sigmoids, hot coronal channels, etc. (see the review by Patsourakos et al. 2020). Filaments are dark, thread-like structures observed in chromospheric images of the Sun (Martin 1998). When filaments are observed above the limb of the Sun, they appear as bright structures, and therefore, they are called prominences (see the review by Gibson 2018). Although the exact structure of filaments is still debated (see, e.g., Antiochos et al. 1994), it is now believed that MFRs form the basic structures of active region filaments. Coronal cavities represent the transverse cross-sectional view of MFRs, where the accumulation of plasma can be observed at the bottom of the dark cavity, providing important insights into the relation between MFRs and filaments (Gibson 2015). Coronal sigmoids are S (or reverse-S) shaped structures observed in SXR (Manoharan et al. 1996; Rust & Kumar 1996) and EUV wavelengths (Joshi et al. 2017; Mitra et al. 2018), and they are interpreted as manifestations of highly twisted MFRs (Green et al. 2018). Hot channels are coherent structures observed in the high-temperature passband EUV images of the solar corona, which indicate activated, quasi-stable MFRs (Zhang et al. 2012; Cheng et al. 2013; Nindos et al. 2015; Joshi et al. 2018; Hernandez-Perez et al. 2019; Mitra & Joshi 2019; Sahu et al. 2020; Kharat et al. 2021). Two possible scenarios of flux rope formation have been proposed: emergence of MFRs from the convection zone of the Sun into the solar atmosphere by magnetic buoyancy (Archontis & Hood 2008; Chatterjee & Fan 2013), and the formation of MFRs from sheared arcades in response to small-scale magnetic reconnection in the corona (Aulanier et al. 2010; Inoue et al. 2018; Mitra et al. 2020a).

Successful eruptions of MFRs are essential for the generation of CMEs. In order to explain the triggering of MFRs toward an eruption, different models have been proposed that can be largely classified into two groups: models relying on ideal magnetohydrodynamics (MHD) instability (kink and torus instability), and models relying on magnetic reconnection (tether-cutting model, breakout model, etc.). According to the torus instability (Kliem & Török 2006), a toroidal current ring may be triggered to erupt if the ambient poloidal magnetic field decreases with height faster than a critical rate. The magnetic decay index (n), defined as \( n = -\frac{d \log (B_p)}{d \log (z)} \), where \( B_p \) and \( z \) are used to represent the poloidal magnetic field and height, respectively, is used to quantify the decay of the magnetic field with height above the relevant polarity inversion line. Theoretically, it was found that an MFR is subject to torus instability when it reaches a region characterized by \( n > 1.5 \) (Bateman 1978). However, a number of observational studies revealed that the threshold value of \( n \) for the torus instability lies within the range [1.1–1.75] (e.g., Liu 2008; Kliem et al. 2013; Zuccarello et al. 2015). Kink instability suggests that an MFR may be destabilized if its twist increases beyond \( \approx 3.7 \pi \) (Török et al. 2004). The tether-cutting model explains the triggering of solar eruptions from a highly sheared bipolar magnetic configuration in which the initial magnetic reconnection takes place deep within the sheared core field (Moore & Roumeliotis 1992). Here, the flux rope is developed from the sheared arcades in response to these preflare magnetic reconnections, and the successful eruption of the flux rope depends on the flux content of the sheared core field relative to the overlying envelope field (Moore et al. 2001). In contrast to the tether-cutting model, the breakout model involves complex multipolar topology with one or more nullpoints situated well above the core field (Antiochos et al. 1999). As the photospheric magnetic field evolves (in the form of flux emergence, shearing motion, etc.), the core field extends outward, stretching the nullpoint, which leads to the formation of breakout current sheets (Karpen et al. 2012). The initial reconnection (breakout reconnection) takes place on these breakout current sheets, which reduces the downward strapping force of the enveloping field and allows the core field to erupt successfully. Notably, once the triggered flux rope attains upward eruptive motion, standard flare reconnection (as explained in the CSHKP model) sets in beneath the flux rope, a process that is common to all the triggering mechanisms.

The presence of the coronal nullpoint in the preflare configuration is an essential requirement in the breakout model as the breakout current sheet is formed by stretching the nullpoint. Theoretically, magnetic nullpoints are defined as locations where all the three components of magnetic field become zero and increase linearly with distance from it (Chapter 6 in Priest 2014). Thus, nullpoints create discontinuities in the coronal magnetic field and separate different domains of flux regions. In general, regions of strong gradients in continuous magnetic fields are identified as quasi-separatrix layers (QSLs; Priest & Démoulin 1995). The gradient of magnetic connectivity can be quantified by computing the degree of squashing factor \( Q \) by calculating the norm \( N \) of the field line mapping of magnetic domains (Demoulin et al. 1996; Pariat & Démoulin 2012). Theoretically, Titov et al. (2002) showed that in all physical scenarios, \( Q = 2 \) is the lowest possible value of \( Q \), and QSLs are therefore characterized by \( Q \geq 2 \) (Aulanier et al. 2005), while \( Q \to \infty \) is representative of nullpoints. Due to local diffusion, magnetic fields in QSLs can constantly change their connectivities (Aulanier et al. 2006), which can observationally be identified as apparent slipping motion of flare kernels in imagery of the lower solar atmosphere, which is called “slipping” or “slip-running” reconnection (see, e.g., Janvier et al. 2013).

In this paper, we report an M4.0 flare from active region NOAA 11302 on 2011 September 26, which was associated with the failed eruption of a filament. Different multiwavelength aspects of this event with the main focus on the nonthermal energy evolution were studied by Kushwaha et al. (2014). Their observations suggest that the impulsive phase of the flare was characterized by two short-lived microwave (MW) peaks in 17 and 34 GHz observed by the Nobeyama Radioheliograph (NoRH; Nakajima et al. 1994; Takano et al. 1997). Interestingly, while the first MW burst occurred contemporaneously with a sudden peak in hard X-ray (HXR) wavelengths observed by the Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI; Lin et al. 2002) with energies up to \( \approx 200 \) keV following a power law with a hard photon spectral index \( \gamma \) \( \approx 3 \), the second nonthermal burst observed in MW was much less pronounced in HXRs. Importantly, the onset of the X-ray emission during the flare occurred immediately after the emergence of a pair of small-scale magnetic transients of opposite polarities in the inner core.
region, which led them to conclude that small-scale changes in the magnetic structure may play a crucial role in triggering the flare process by disturbing the preflare magnetic configurations. In view of the rapid temporal evolution of the HXR and MW flux during the early impulsive phase of the flare, they concluded that an abrupt energy release via spontaneous magnetic reconnection was responsible for the occurrence of the flare. In order to investigate whether this apparent spontaneous magnetic reconnection was influenced by topological features, e.g., flux ropes or nullpoints, we revisited the event considering a longer period of observation and analysis. Our study readily revealed that the flare was preceded by a number of subtle SXR flux enhancements, suggesting that a possible influence of external factors is responsible for triggering the flare. By employing a nonlinear force-free field (NLFFF) extrapolation model, complemented by EUV observations of the Atmospheric Imaging Assembly (AIA; Lemen et al. 2012) on board the Solar Dynamics Observatory (SDO; Pesnell et al. 2012), we investigate the causal connection between the activities during the extended preflare phase and the main flare, and we investigate the factors responsible for the failed filament eruption. The structure of the paper is as follows: Section 2 provides a brief description of the observational data sets and different analysis techniques used in this work. We discuss the temporal evolution of X-ray and EUV emission from the flaring active region in Section 3. Section 4 gives a detailed overview of the photospheric magnetic configuration of the active region and discusses the coronal magnetic configuration of the flaring region. Section 5 presents the observational results on the basis of a coronal and chromospheric imaging analysis. The results obtained by modeling coronal magnetic field are explained in Section 6. We discuss and interpret the results in Section 7.

2. Observational Data and Analysis Techniques

2.1. Data

In this work, we have combined multiwavelength observational data from different sources. For EUV imaging of the Sun, we used observations provided by the AIA on board SDO. In particular, we extensively used the full disk 4096 × 4096 pixel solar images in 304 Å (log(T) = 4.7), 171 Å (log(T) = 5.8), 94 Å (log(T) = 6.8), and 131 Å (log(T) = 5.6, 7.0) channels at a pixel scale of 0.56 and a cadence of 12 s. In order to enhance the fine structures, all the AIA images have been filtered with the unsharp_mask image processing algorithm. Chromospheric observations of the Sun in the Hα passband were obtained from the Global Oscillation Network Group (GONG; Harvey et al. 1996). GONG provides 2096 × 2096 pixel full-disk Hα images of the Sun with a pixel scale of approximately 1″0 (Harvey et al. 2011). To study the evolution of photospheric magnetic structures, we used data from the Helioseismic and Magnetic Imager (HMI; Schou et al. 2012) on board SDO. Among the different products of HMI, we have used the 4096 × 4096 pixel full-disk intensity images and line-of-sight (LOS) magnetograms at a pixel scale of 0.55 and cadence of 45 s. We used X-ray observations of the RHESSI (Lin et al. 2002), which has a spatial resolution (as fine as ~2″3) and energy resolution (1–5 keV) over the energy range 3 keV−17 MeV. To construct RHESSI X-ray images, we used the PIXON algorithm (Metcalf et al. 1996) with the natural weighting scheme for front detector segments 2–9 (excluding 7).

2.2. Numerical Analysis Techniques

In order to investigate the coronal magnetic configuration of the active region with the aim to understand the cause of the failed eruption associated with the event, we carried out coronal magnetic field modeling using the optimization-based NLFFF extrapolation technique developed by Wiegelmann & Inhester (2010) and Wiegelmann et al. (2012). As the boundary condition for the extrapolation, we used the vector magnetogram of 2011 September 26 04:58 UT from the hmi_sharp_cea_720s series of HMI/SDO with a reduced resolution of 1″0 pixel−1. The extrapolation was made in a Cartesian volume of 616 × 288 × 256 pixels, which corresponds to physical dimensions of 447 × 209 × 186 Mm3. According to the theory of the NLFFF model, the average value of the Lorentz force, i.e., [J × B]1, is 0. However, because the extrapolation approach is based on numerical techniques, the NLFFF-reconstructed magnetic field is expected to return nonzero values of [J × B]. Therefore, to assess the quality of the coronal magnetic field reconstruction, the average value of the fractional flux ratio (fj) = (|∇ · B|)/(|B|/Δx), weighted between J and B can be considered (see DeRosa et al. 2015). In this study, we obtained the values of the residual errors by averaging these parameters over the entire computational domain, i.e., 616 × 288 × 256 pixels, with pixel dimensions physically translating into ≈0.725 Mm. The residual errors were found to be

\[ f_j \approx 4.06 \times 10^{-4} \]

\[ \frac{|J \times B|}{|J \cdot B|} \approx 0.14 \]

\[ \theta_j \approx 6.76 \]  

In general, NLFFF solutions are considered good solutions if they return the values \( f_j \approx 2 \times 10^{-3} \) and \( \theta_j \lesssim 10^\circ \) (see, e.g., DeRosa et al. 2015).

Using the NLFFF extrapolation results, we calculated the degree of squashing factor (Q̂) and the twist number (T̂) within the extrapolation volume by employing the code developed by Liu et al. (2016). In order to locate 3D nullpoints within the extrapolation volume, we used the trilinear method (Haynes & Parnell 2007) by dividing the whole active region volume into grid cells of dimension 2 × 2 × 2 pixels. If any of the three components of the magnetic field vector have same sign at all the eight corners of the grids, it is considered that the corresponding grid cell cannot contain any nullpoint within it, and therefore, the corresponding cell is excluded from further analysis. Each of the remaining other cells is then further divided into 100 × 100 × 100 subgrid cells and the nullpoint is located by using the Newton–Raphson method for finding roots of equations. This iterative method was continued until the uncertainty in the solution reached to \( \lesssim 2 \) subgrid cell width. To visualize the results obtained from the NLFFF extrapolation, we used the software called Visualization and Analysis Platform for Ocean, Atmosphere, and Solar Researchers (VAPOR; Clyne et al. 2007).

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6. See http://halpha.nso.edu.

7. http://fourier.eng.hmc.edu/e176/lectures/NM/node21.html
3. Temporal Evolution of X-Ray and EUV Emission

In Figure 1 we compare the temporal evolution of the X-ray fluxes in the 1–8 Å and 0.5–4 Å channels of GOES and multiple energy channels of RHESSI covering the energy range 3–25 keV (Figure 1(a)) and EUV fluxes derived from different AIA channels (Figure 1(b)) during 2011 September 26 04:00 UT—05:15 UT, which included an extended phase prior to the onset of the M4.0 flare as well as the impulsive and a part of the gradual phase of it. GOES did not observe the Sun during 05:15 UT—06:25 UT, which covered most of the declining phase of the flare.\(^8\) RHESSI missed the initial phase of our observing period (until \(\sim\)04:13 UT) due to RHESSI night. From Figure 1(a), we find a number of episodes of small emission enhancements prior to the M-class flare during \(\sim\)04:04 UT—04:53 UT, which were most prominent in GOES 0.5–4 Å and RHESSI channels up to 12 keV. Multiwavelength EUV and X-ray imaging (discussed in detail in Sections 5.1) suggested that a localized region undergoes compact brightening during these flux-enhancing periods. Based on the intensity and compactness of the EUV and X-ray sources, we divided the whole period into two phases: Phase 1 from 04:04 UT—04:39 UT (highlighted by light yellow background in Figure 1), and Phase 2 from 04:39 UT—04:53 UT (highlighted by light purple background in Figure 1), and we identify the flux peaks by P1\(_1\)–P1\(_8\) and P2\(_1\)–P2\(_4\), respectively. The onset of the flare occurred at \(\sim\)05:06 UT, which was followed by an impulsive rise in flare emission. The flare reached its peak at \(\sim\)05:08 UT after undergoing a brief impulsive phase lasting for only \(\sim\)2 minutes. AIA EUV intensities displayed a general agreement with the X-ray flux variation, although the small-scale enhancements observed in the GOES 0.5–4 Å and RHESSI light curves up to energies \(\approx\)12 keV during Phase 1 were much less pronounced in EUV light curves. Notably, the AIA 171 and 304 Å intensities showed a significant enhancement during the P2\(_2\) peak (indicated by the green arrows in Figure 1(b)), while they remained completely unchanged during P1\(_1\). Intensities in the AIA 94 and 131 Å channels showed mild enhancements during both the aforementioned X-ray peaks (indicated by the black arrows in Figure 1(b)).

It is worth mentioning that the soft and hard X-ray light curves obtained from GOES and RHESSI (Figure 1(a)) are disk integrated, i.e., they include emission originating from the full solar disk. On the other hand, the EUV light curves from

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\(^8\) See https://www.swpc.noaa.gov/products/goes-x-ray-flux.
SDO/AIA displayed in Figure 1(b) were derived by integrating the counts within the flaring region of with a field of view (FOV) of [(-405°0:−645°0), (30°0:230°0)], shown in Figure 2(d). Despite the difference in the regions used to construct the light curves in Figures 1(a) and (b), their general agreement suggests that the overall activity of the Sun was primarily dominated by the flaring activities occurring within active region NOAA 11302. This is expected because the intensities in the EUV, SXR, and HXR domains from flaring active regions increase by orders of magnitude during flares.
and thus become the dominant contribution to the changes in the full-Sun light curves.

4. NOAA 11302: δ-sunspot Region and Quadrupolar Configuration

During the M-class flare under study, active region NOAA 11302 was centered at the heliographic location ≈N15E35. The white-light image of the active region (Figure 2(a)) shows that the active region was comprised of three separate sunspot groups that were distributed along the northeast–southwest (NE–SW) direction. In addition to the three prominent sunspots, the active region also contained a few small pores. Comparison of a cotemporal magnetogram of the active region (Figure 2(b)) with the white-light image suggests that the easternmost sunspot group was of predominantly positive polarity, while the westernmost sunspot group was mostly composed of negative flux, although a few dispersed positive flux regions were distributed around it. The middle sunspot group is particularly interesting as it had bipolar magnetic structures. The intensity contours over the magnetogram shown in Figure 2(b) show that this sunspot group had fragmented umbrae of opposite polarities within a single penumbra, which suggests that the active region is a δ-type AR. Notably, the M-class flare and associated failed eruption reported in this paper originated from this sunspot group. The chromospheric Hα image shows a very small, faint filament with one leg attached to the middle sunspot group (indicated by the red arrow in Figure 2(c)).

Coronal images of the active region, particularly in the hot AIA EUV channels, e.g., 94 Å and 131 Å, revealed an interesting structure at the northern end of the filament (indicated by the red arrow in Figure 2(d)), which was composed of multiple small coronal loops connecting different polarity regions of the middle sunspot group. The NLFFF extrapolation results readily validated the observed coronal structures at large and small scales. In Figure 2(e) we show the modeled coronal configuration associated with the central sunspot group. Notably, the coronal loops shown in blue, green, pink, and teal constituted a well-defined quadrupolar coronal configuration. Importantly, our analysis further confirms the presence of a coronal nullpoint in the quadrupolar configuration above the central sunspot group (shown by the red patch in Figure 2(e) and indicated by the black arrow). The NLFFF extrapolation also demonstrated the presence of a flux rope associated with the central sunspot region (shown by the yellow lines in Figure 2(e)).

5. Multiwavelength Imaging of Coronal Energy Release: Onset and Consequences

In Figure 3 we present an overview of the prominent episodes of energy release in multiple EUV channels of AIA (171, 131, and 304 Å) with respect to the X-ray flux evolution obtained from the GOES 0.5–4 Å channel and RHESSI 6–12 keV energy band. AIA images during the P1 peak (see Figure 1) suggest a localized brightening (indicated by the red arrow in Figure 3(c)) situated at the northern end of the filament (indicated by the white arrows in Figures 3(b)–(d)). Notably, this brightening was most prominent in the high-temperature AIA 131 Å channel compared to the AIA filters that sample plasma at lower temperatures, e.g., 171 and 304 Å. Emission from this localized region significantly increased in all the AIA EUV channels during the P21 and P22 peaks (indicated by the red arrows in Figures 3(e)–(f) and the teal arrow in Figure 3(g)). This localized region was identified as a coronal nullpoint configuration in the NLFFF extrapolation results (Figure 2), suggesting nullpoint reconnection to be responsible for the repetitive flux enhancements during the extended periods of Phase 1 and Phase 2. During this time, we observed signatures of a second filament (indicated by the green arrows in Figures 3(e) and (g)) close to the filament previously observed (indicated by white arrows in Figures 3(e) and (g) and also in Figures 3(b)–(d)).

The onset of the impulsive phase of the M4.0 flare was characterized by the activation of the first filament, which resulted in intensified emission (the filament appearance changed from absorption to emission, indicated by the white arrows in Figures 3(h) and (j)). During the impulsive phase, the filament initially displayed eruptive motion and extended spatially along its length. However, the eruption of the filament ceased to continue within ≈2 minutes, after which we observed a complex structure in which the two activated filaments became intertwined with each other during the peak phase of the flare. In Figures 3(k) and (m) we indicate the two activated intertwined filaments by the white and green arrows. Notably, during this time, we observed quite intense emission from a second location as well, which was situated remotely to the north of the filaments (indicated by the teal arrows in Figures 3(k) and (m)). A summary of the different evolutionary phases under study is provided in Table 1.

5.1. Episodic Energy Release at Nullpoint Topology

Comparison of Figures 1(a) and (b) readily suggests that the small-scale flux enhancements observed in X-ray channels were more in agreement with hot AIA channel intensity variations (i.e., 94 and 131 Å) compared to the low-temperature channels (i.e., 304 and 171 Å). Therefore, in order to have a thorough understanding of the activities during Phase 1 and Phase 2, we examine the AIA 131 Å images of the flaring region in Figures 4 and 5, respectively. Hot AIA EUV imaging clearly revealed that repetitive X-ray/EUV peaks during Phases 1 and 2 were essentially linked with an episodic, impulsive brightening of the compact loop structure. Notably, as discussed in Section 4 (see also Figure 2), the region of these compact bright loops has been identified with the location of the coronal nullpoint topology.

In Figure 4(a) we show AIA 131 Å image of the region, in which we outline the bright loops by dashed red curves. From the overplotted line-of-sight (LOS) magnetogram contours (Figure 4(a)), it becomes evident that the bright loops were connecting different opposite-polarity regions of the central sunspot group (see also Figures 2(b), d). During the subsequent emission peaks of Phase 1, i.e., P11–P16, we observed clear X-ray sources up to 25 keV that originated from the location of the compact loop system associated with the nullpoint. Despite continued X-ray emission and enhanced loop brightening in the EUV, there was no significant change in the morphology of the compact loop system. This brightness of the compact EUV loop system increased during Phase 2 (Figure 5), which is also evident from the EUV light curves (Figure 1(b)). Notably, RHESSI sources were identified to be more compact during Phase 2 than in Phase 1. Furthermore, during Phase 2, we found continued X-ray emission at higher energy (12–25 keV). Thus, the morphology and intensity of X-ray and EUV sources during Phase 2 suggest that the reconnection activities at the nullpoint are more energetic. Moreover, we observed eruptions of small loop-like structures from the location of the nullpoint during the peaks P21 and P22 (indicated by the yellow arrows in Figures 5(b) and (c)), which point toward the
restructuring of the magnetic configuration in the vicinity of the nullpoint as a result of nullpoint reconnection.

5.2. Failed Eruption of the Filament and Associated M4.0 Flare

After a prolonged period of repetitive events of small-scale coronal energy release, the main M4.0 flare was initiated at \( \approx 05:06 \) UT. In Figure 6 we show a series of AIA 94 Å images of the core region displaying different phases of the main flare. The preflare filament was not prominently visible (indicated by the red arrow in Figure 6(a)) in the AIA 94 Å channel images. It samples high-temperature coronal plasma \( \approx 6 \) MK, but during the onset of the highly impulsive rise phase of the flare, we observe narrow, stretched bright emission beneath the filament along its entire length (indicated by the black arrow in Figure 6(b)). During the impulsive phase, the activated filament went through an eruptive motion toward the projected eastern direction (see arrows in Figures 6(b)–(e)). We observed strong X-ray sources up to energy \( \approx 50 \) keV from the location of the

![Figure 3](image-url)
activated filament. Notably, the X-ray sources seem to coincide well with the initial location of the spatially elongated filament, which was most likely generated from the newly formed post-reconnection arcade (Figure 6(d)). During the gradual phase, the erupting filament interacted with a set of closed low-coronal loops lying above the core region, which most likely caused the eruption of the filament to cease, leading to a classification of the flare in the “failed eruptive” category. The gradual phase of the flare was characterized by intense emission from these closed low-coronal loops (indicated by the blue arrow in Figure 6(f)) as well as from the postflare arcade following the erupting filament (indicated by the black arrow in Figure 6(f)). A comparison of the AIA 94 Å image with the overplotted contours of the cotemporal HMI LOS magnetogram (Figure 6(f)) reveals that the overlying coronal loops connected the negative polarity regions of the central sunspot region to the dispersed positive-polarity region situated north of the central sunspot group.

In order to investigate the evolution in the low atmospheric layers of the Sun, we look at AIA 304 Å images (Figure 7), where we readily observe signatures of two quite prominent filaments that appeared to be separated spatially prior to the onset of the flare (indicated by the yellow arrows in Figure 7(a)). During the impulsive phase, as the filament displayed a brief period of eruption, we observed intense emission from the core of the active region (Figures 7(b)–(c)). During the peak phase of the flare, the two filaments appeared to become intertwined with each other, resembling a double-decker flux rope system. The two hot filaments of the double-decker system are indicated by the green and teal arrows in Figure 7(d). The double-decker structure shrank during the early gradual phase of the flare (indicated by the yellow arrows in Figure 7(e)), which was further followed by the formation of flare ribbons and postflare arcades (within the dashed green box in Figure 7(f)).

Interestingly, during the peak phase of the M4.0 flare (≈05:08 UT), we observed multiple ribbon-like brightenings from regions situated north of the location of the filaments (indicated by the white arrows in Figure 7(d)). During the gradual phase, emission from the ribbon-like structures significantly intensified (see Figures 7(d), (e), (f)). Topological analysis was carried out to explore the physical connections of these remote brightenings with the flaring processes (Section 6.1).

6. Coronal Magnetic Field Modeling

6.1. Preflare Coronal Configuration and Postflare Arcade

In Figure 8 we plot NLFFF-extrapolated coronal field lines in the active region prior to the onset of the M4.0 flare and compare them with the coronal loops observed in EUV images. For convenience, we plot cotemporal LOS magnetogram and AIA 171 Å image of the active region in Figures 8(a) and (b), respectively. From the contours of the HMI LOS magnetogram plotted on top of the AIA 171 Å image (Figure 8(b)), it becomes clear that in addition to the flaring region, the active region was characterized by a set of large coronal loops that extended up to high coronal heights, which is indicated by the teal arrows. We further note the presence of a different filament within the active region that was not involved in the reported flaring activity. In Figure 8(b) we indicate the filament by a black arrow.

In Figure 8(c) we display a few sets of modeled coronal field lines in the overall AR. We find that the teal field lines correlate well with the coronal loops indicated by the teal arrows in Figure 8(b). In Figures 8(d)–(f) we focus on the NLFFF-extrapolated field lines within the flaring region alone, where we readily identify three flux ropes that are shown in bright yellow, pink, and light yellow. Interestingly, the flux rope shown in light yellow is precisely co-spatial with the filament indicated by the black arrow in Figure 8(b), which is further demonstrated by the AIA 304 Å image in the background of Figure 8(d). Furthermore, the two flux ropes shown in bright yellow and pink constitute a double-decker flux rope configuration that is exactly co-spatial with the two filaments that were involved in the M4.0 flare (Figures 3, 7).

Here we recall that, during the gradual phase of the flare, multiple ribbon-like brightenings situated north of the core region became very bright (indicated by the white arrow in Figure 7(f); Section 5.2). NLFFF-extrapolation results suggest that a set of field lines connect this region to the northern leg of the double-decker flux rope system; they are shown in green.
Notably, a few of the green lines, exactly above the northern leg of the flux rope shown in bright yellow, were characterized by high values of the squashing factor \((Q = \log_3)\), which is represented by the teal patch in Figures 8(d)–(f). Furthermore, the footpoints of the green model coronal loops were characterized by high \(Q\)-values, which is evident from the background red patches representing \(\pm 400, 1400 \text{ G}\) in Figures 8(e)–(f). Notably, the coronal loops involved in the quadrupolar configuration are shown by the blue lines in Figures 8(c)–(e).

In Figure 9 we show the association of the double-decker flux ropes and the quadrupolar coronal configuration from different viewing angles. From the top and side views of the whole configuration, we readily understand that the quadrupolar configuration including the nullpoint (indicated by the green, orange, and black arrows in Figures 9(a)–(c), respectively) was only lying over the yellow flux rope. NLFFF-modeled field lines, complemented by AIA observations, clearly demonstrate that the nullpoint is situated above the central part of the yellow flux rope (see the green and red arrows indicating the nullpoint and the northern leg of the yellow flux rope, respectively, in Figure 9(a)). Furthermore, we could identify a different set of model field lines (shown in teal in Figure 9(d)) that manifest a structure similar to the dense
coronal arcade (indicated by the blue arrow in Figure 6(f)) that developed during the gradual phase of the flare. During the impulsive phase of the flare, the flux rope (shown in bright yellow) might have undergone eruptive motion toward the east, interacted with the teal lines, giving rise to the enhanced emission from the highly structured teal lines.

6.2. Decay Index and Twist Number

EUV observations revealed that the eruption of the filament was constrained within a short while after initiation, leading to a failed eruption. In order to investigate the coronal conditions responsible for the failed eruption, we calculated the magnetic decay index ($n$) within the extrapolation volume.

Theoretically, if a current ring of major radius $R$ is embedded in an external magnetic field, then the ring experiences a radially outward hoop force because of its curvature. In stable conditions, this hoop force is balanced by the inwardly directed Lorentz force. If the Lorentz force due to the external field decreases faster with $R$ than the hoop force, the system becomes unstable due to the torus instability (Bateman 1978). The decay rate of the external magnetic field is quantified by the magnetic decay index ($n$). Considering the fact that the
toroidal component (directed along the axis of the flux rope) of the external magnetic field does not contribute to the strapping force (a nice visual depiction is provided in “Extended Data Figure 2” in Myers et al. 2015), in the ideal current-wire approach, the magnetic decay index is calculated as

\[ n = -\frac{d \log(B_p)}{d \log(h)} \]  

(2)

where \( B_p \) is the external poloidal field (along the transverse direction of the flux rope axis), and \( h \) is the height.

However, in reality, flux ropes are observed to significantly differ from the simplified shape of a semicircular current wire (see, e.g., Démoulin & Aulanier 2010; Fan 2010; Olmedo & Zhang 2010). Therefore, in order to calculate the magnetic decay index, we manually determined the approximate 2D projection of the flux rope axis on the photosphere (shown by the red curve in Figure 8(a)). The 3D magnetic field vectors at each pixel on the vertical surface above the axis were then decomposed into two components: along the direction of the axis, i.e., the toroidal component, and perpendicular to the axis, i.e., the poloidal component. This perpendicular
Figure 8. Panel (a): HMI LOS magnetogram of active region NOAA 11302. Panel (b): Cotemporal AIA 171 Å image. The flaring region is outlined by the white box. The teal arrows indicate a set of large, high-lying coronal loops above the flaring region. Contours of the LOS magnetograms are plotted over the AIA 171 Å image at levels of ±[500, 1000, 1500] G. Red and cyan contours refer to positive and negative polarities, respectively. Panel (c): NLFFF-extrapolated magnetic field lines showing the important coronal connectivities in the overall active region. In panels (d)–(f), we show only the model field lines in the flaring region. The bright yellow and pink lines represent two flux ropes. The northern part of the flux rope shown in bright yellow was enveloped by the blue lines. A set of field lines (shown in green) situated north of the flux rope shown in bright yellow was associated with a high squashing factor. The teal patch in panels (d)–(f) within the region of green lines is characterized by $\log(Q) = 3$. The light yellow lines in panels (d) and (e) represent another flux rope that was present in the active region, but did not take part in the flaring activity. In panel (f), we show the same model coronal loops except for the blue and light yellow lines to better visualize the green lines. The color template used in the background magnetograms is same as in Figure 2(b). The background of panel (d) is the AIA 304 Å image. The reddish patches over the background in panels (e)–(f) are characterized by $\log(Q) = 2$. The red curve in panel (a) approximately denotes the axis of the bright yellow flux rope, which is used for the magnetic decay index analysis shown in Figure 10.
component was used in Equation (2) to compute the magnetic decay index.

From the distribution of the magnetic decay index above the flux rope axis (Figure 10), we find that within a very low height above the flux rope, we found an extended region of high magnetic decay index (depicted by the red patch in Figure 10(a)). This region was immediately enveloped by another region in which the decay index was as low as $\approx -3$ (the region shown in blue in Figure 10(a)). Above this region, the value of the decay index slowly increased and again reached 1.5 at a quite high coronal layer (indicated by the yellow curve in the top portion of Figure 10(a)). In Figure 10(b) we plot the variation in decay index averaged over the axis of the double-decker filament with height, where we find that the average value of decay index in the high corona reached values of 1.0 and 1.5 at heights of $\approx 43$ Mm and $\approx 64$ Mm, respectively.

In order to investigate the applicability of the kink instability as the triggering mechanism of the flux ropes, we calculated the twist number within the extrapolation volume using the IDL-based code developed by Liu et al. (2016). The twist number ($T_w$; Berger & Prior 2006) associated with a flux rope is defined as

$$ T_w = \int L \left( \nabla \times B \right) \cdot \frac{B}{4\pi B^2} \, dl, $$

where $L$ denotes the length of the flux rope. Our calculations suggest that the two flux ropes in the double-decker flux rope system were associated with a positive twist. The average value of the twist number associated with the flux rope shown in bright yellow was found to be $\approx 1.5$, while the twist number associated with the flux rope shown in pink was found to be $\approx 1.7$. A statistical survey conducted by Duan et al. (2019) revealed the critical value of $|T_w|$ for kink instability to be 2, which suggests that the kink instability was not responsible for the activation of the filament during the onset of the M4.0 flare.

7. Discussion

In this paper, we have investigated the triggering and subsequent failed eruption of a small filament from active region NOAA 11302. The morphology of the active region was highly interesting as three prominent and distinct sunspot groups in this active region were distributed in an almost linear manner. While the central sunspot group was the smallest of the three, it was the only bipolar sunspot group containing $\delta$ spots. Notably, the reported flare occurred from the central sunspot group.

While one filament underwent a failed eruption, EUV images of the flaring region prior to the onset of the M4.0 flare...
clearly revealed signatures of two distinct filaments (Figures 3, 7(a)). During the impulsive phase of the flare, both filaments were activated, extended spatially, and developed into a complex structure in which both filaments were intertwined with each other (Figures 3(k)–(m), 7(d)). NLFFF extrapolation results suggest the presence of two flux ropes in the flaring region in a double-decker flux rope configuration. The contemporary concept of the double-decker flux rope system was first reported by Liu et al. (2012), and only a handful of articles have reported such complex structures since then (e.g., Cheng et al. 2014; Kliem et al. 2014; Dhakal et al. 2018; Tian et al. 2018; Wang et al. 2018; Awasthi et al. 2019; Zheng et al. 2019; Mitra et al. 2020b; Mitra & Joshi 2021). Despite the small number of reportings, double-decker flux rope systems can be mostly divided into two categories. While most of the reported double-decker flux rope systems were identified as two vertically well separated filaments, one of them undergoing eruption (e.g., Liu et al. 2012; Dhakal et al. 2018; Tian et al. 2018; Awasthi et al. 2019), the second category comprises complex preflare sigmoidal structures.

Figure 10. Panel (a): Distribution of magnetic decay index with height along the vertical surface above the axis of the yellow flux rope (indicated by the red curve in Figure 8(a)). The yellow curves refer to $n = 1.5$. The variation in decay index with height, averaged over the flux rope axis, is shown in panel (b). The dotted green and red lines indicate the critical heights corresponding to $n = 1.0$ and $n = 1.5$, respectively. The dash-dotted blue and teal lines indicate the maximum heights of the yellow and pink flux ropes, respectively (see Figure 9).
involving intertwined flux ropes (Cheng et al. 2014; Mitra et al. 2020b; Mitra & Joshi 2021). In the first category, triggering and eruption of flux ropes are generally caused by photospheric activity, i.e., shearing and/or rotating motion. However, in this case, the activation of one flux rope in the double-decker system may or may not influence the stability of the other flux rope (see, e.g., Liu et al. 2012). On the other hand, in the case of intertwined flux ropes, interaction between the two flux ropes mostly causes an activation and eruption of the system. The two filaments reported in this article were spatially well separated during the preflare phase. During the impulsive phase, they extended longitudinally and became intertwined with each other. Furthermore, the onset of the flare in this case followed clear preflare activities that occurred at a separate location from the filaments, which was identified both in the SXR flux evolution and in the AIA EUV images. In view of this, the double-decker system reported in this study seems to have characteristics of both the categories of double-decker systems discussed above. This complex and intriguing behavior of magnetic flux ropes challenges our general understanding of solar magnetic fields and their evolution during transient activities.

The SXR flux evolution prior to the onset of the M-class flare revealed multiple episodes of flux enhancements (Figure 1). Comparison of SXR light curves with EUV images of the active region suggested that these preflare flux enhancements are associated with a localized brightening that originated above the northern end of the filaments. The photospheric magnetic field of the region manifested a complex distribution of magnetic polarities involving δ spots (i.e., the central sunspot group; see Figure 2). The NLFFF extrapolation revealed a multiflux topology in the flaring region that forms a quadrupolar configuration with a coronal nullpoint (Figures 2(e), 9). This preflare coronal configuration is in agreement with the scenario prescribed in the breakout model of solar eruptions (Antiochos et al. 1999; Karpen et al. 2012). According to this model, the small-scale preflare reconnections at the nullpoint will remove the overlying (constraining) magnetic flux by transferring it to the side-lobe field lines (see Figure 1 in Karpen et al. 2012). In view of this, the observed preflare episodic brightening of the coronal loops associated with the nullpoint is well consistent with the initiation process of the eruptive flux ropes invoked in the breakout model. We note intense and structured emission during the preflare phase (see Figure 5) that continued until the earliest development of the M-class flare (Figure 6(a)). This structured emission can be readily perceived as the brightening of multiple low-lying loops of the quadrupolar configuration in and around the coronal nullpoint, providing credence to our proposed scenario of preflare energy release by nullpoint reconnection.

Notably, the location of the nullpoint above the filament by identifying the region of highest value of the degree of squashing factor \(Q\) within the extrapolation volume. Nullpoints are essentially singular points in the corona in which all the components of the magnetic field vanish (Aschwanden 2005). Theoretically, a nullpoint can be characterized by \(Q \rightarrow \infty\) as it works as a separator between different topological domains of the magnetic field (see, e.g., Longcope & Klapper 2002). However, numerical techniques used to calculate \(Q\)-values will always return finite values, thus we can expect a coronal nullpoint to be associated with high-degree \(Q\)-values (Pariat & Démoulin 2012). In a number of studies concerning coronal nullpoints and their evolution, the coronal nullpoints were found to be associated with a wide range of \(Q\)-values (\(\sim 10^4\)–\(10^{12}\); see Yang et al. 2015; Liu et al. 2020; Qiu et al. 2020; Prasad et al. 2020). In the present study, we observed the nullpoint to be associated with a value of \(Q \approx 10^5\). Regions with smaller \(Q\) are identified as quasi-separatrix layers (QL; Aulanier et al. 2006). Here we recall that during the gradual phase of the M4.0 flare, multiple ribbon-like brightenings were observed from locations north of the flaring region (Figures 7(d)–(f)). The NLFFF extrapolation results revealed the presence of a set of coronal loops that connect the remote region with the region situated at the northern footpoint of the flux rope (shown by the green lines in Figures 8(d)–(f)). Calculation of \(Q\) has further revealed that the green lines are associated with strong \(Q\)-values (Figures 8(d)–(f)). These findings demonstrate that the remote brightening during the peak phase of the main flare is an observational manifestation of slip-running reconnection influenced by the QSLs.

The most important aspect of this study is the investigation of the failed eruption of the flux ropes. Analysis of the magnetic decay index \(n\) suggests the presence of an extended region just above the flux ropes characterized by \(n > 1.5\) (Figure 10), which suggests that the flux ropes were subject to torus instability. The topological configuration of the active region corona in our event presented a special scenario in which a coronal nullpoint existed in the core region and the region of torus instability resided immediately below the nullpoint (Figure 9(c)). High decay of magnetic field is expected below a nullpoint, which in turn increases the value of the decay index. However, the coronal region within the height range \(\approx 7–20\) Mm above the flux ropes (Figure 10(c)) is of particular interest in view of the negative magnetic decay index within this height range. Notably, the variation of average decay index with height presents a so-called saddle-like profile where a local maximum of \(n\) in lower height is followed by a local minimum of \(n\) at relatively larger height (Wang et al. 2017). Such a saddle-like profile of the decay index produces a favorable condition for a failed eruption of flux ropes because the toroidal strapping force (responsible for constraining the erupting magnetic structure) increases in height after an initial decrease (see, e.g., Wang et al. 2017; Filippov 2020b). However, a number of studies have reported successful eruption of flux ropes even though the coronal decay index profile was saddle-like (e.g., Cheng et al. 2011; Wang et al. 2017; Filippov 2020a), suggesting the involvement of other factors toward the successful eruption. Among the number of parameters such as the Lorentz force and the nonpotentiality of the source region, the value of decay index at the saddle bottom is believed to be important in determining the eruption profile of flux ropes (Liu et al. 2018). Inoue et al. (2018) investigated a successful flux rope eruption through a saddle-like decay index profile by numerical simulation and found that the feedback relation between the eruption of the flux rope and reconnection rate beneath it enabled the flux rope to pass through the torus-stable zone. The event reported here is a good example of a failed eruption due to a saddle-like decay index profile of the corona above the flux rope. This event is special through the negative decay index region above the torus-unstable zone within a low height (\(\leq 20\) Mm; Figures 10(c), (e)), which is rarely observed (see Filippov 2020a). The negative value of the magnetic decay index not only decelerated the eruption of the flux rope, but also repelled it downward.

In the present study, we analyze and interpret an intriguing case of the failed eruption of a torus-unstable, complex double-decker flux rope system. We show that the continuation of a flux rope eruption against the overlying coronal fields is
essentially controlled by the extended decay index profile characterizing the strength of the strapping field through the larger coronal heights above the flux rope axis. Investigations of the failed eruptions of torus-unstable flux ropes have emerged as one of the most crucial topics in the contemporary solar physics because of their importance in the space-weather prediction. In this context, in addition to the classical torus instability, some additional factors have been assessed to understand the cause of the failed eruption, which are worth discussing (Myers et al. 2015; Zhong et al. 2021). Based on the results of laboratory experiments, Myers et al. (2015) found that the failed eruption can also occur when the guide magnetic field, i.e., the toroidal field (the ambient field that runs along the flux rope) is strong enough to prevent the flux rope from kinking. Under these conditions, the guide field interacts with electric currents in the flux rope to produce a dynamic toroidal field tension force that halts the eruption. The study by Zhong et al. (2021) presents a data-driven MHD simulation for a confined eruption. They showed a Lorentz force component, resulting from the radial magnetic field or the nonaxisymmetry of the flux rope, which can essentially constrain the eruption. In the light of the above studies and our work, we understand that the ultimate fate of solar eruptions is determined by a complex interplay of coronal magnetic fields involving the magnetic flux rope and the core and envelope fields. To this end, we plan to carry out a series of subsequent studies of failed eruptions of torus-unstable flux ropes in order to reach further understanding of the mechanism(s) of successful or failed solar eruptions.

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Appendix
Calculation of the Twist Number

For a smooth, non-self-intersecting curve $x(s)$ parameterized by arclength $s$ and a second such curve $y(s)$ surrounding $x$, jointly defining a ribbon, the twist number $\tau_g$ is given by (Equation (12) in Berger & Prior 2006)

$$\tau_g = \frac{1}{2\pi} \int_s \hat{T}(s) \cdot \hat{V}(s) \times \frac{d\hat{V}(s)}{ds} ds,$$

(A1)

where $\hat{T}(s)$ represent the unit tangent vector to $x(s)$ and $\hat{V}(s)$ denotes the unit vector normal to $x(s)$. Here, any point on $y(s)$ is related to $x(s)$ by

$$y(t) = x(t) + \epsilon V(t).$$

(A2)

In the context of the present analysis, $\hat{T} = \frac{\hat{B}}{|\hat{B}|}$ and $J = \frac{\epsilon}{B_0} \hat{V} \times \hat{B}$. A detailed calculation conducted by Liu et al. (2016) reveals

$$\frac{d\tau_g}{ds} \approx \frac{\mu_0 J_0}{4\pi |\hat{B}|} + \frac{c}{2\pi |\hat{B}|}$$

(A3)

$$\Rightarrow \tau_g = \int_s \frac{\mu_0 J_0}{4\pi |\hat{B}|} ds + \int_s \frac{c}{2\pi |\hat{B}|} ds.$$  

(A4)

where $J_0 = \frac{\int \hat{B} ds}{|\hat{B}|^2}$ and $c$ is a constant dependent on $\hat{V}$ and on the spatial variation of $\hat{B}$. The first term in the right-hand side approaches $\tau_g$ close to the axis of the flux rope, i.e.,

$$\lim_{\epsilon \to 0} \tau_w(\epsilon) = \tau_g - \int_s \frac{c}{2\pi |\hat{B}|} ds.$$  

(A5)

From Equation (A5), it becomes clear that $\tau_w$ provides an underestimation of the true twist number of flux ropes. However, calculation of $\tau_g$ requires including the exact geometry of the flux rope. Finding the exact geometry of a flux rope from the extrapolated magnetic field is practically not possible. Considering the flux rope to have approximated uniform twist, i.e., a uniform $\alpha$-flux rope (Lundquist 1950), $c \approx 0$, i.e., $\tau_w \approx \tau_g$ (Liu et al. 2016).

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