On the theory of plateau-plateau transitions in Quantum Hall Effect

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Abstract

The Lagrangian (action) formulation of the Chalker-Coddington network model for plateau-plateau transitions in quantum Hall effect is presented based on a model of fermions hopping on Manhattan Lattice (ML). The dimensionless Landauer resistance is considered and its average is calculated over the random $U(1)$ phases with constant distribution on the circle. The Lagrangian of the resultant model on $ML$ is found and the corresponding $R$-matrix is written down. It appeared, that this model is integrable, rising hope to investigate physics of plateau-plateau transitions by the exact method of powerful Algebraic Bethe Ansatz (ABA).

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1. The nature and the mechanism of plateau-plateau transitions in the Integer Hall Effect (IHE) is still one of open problems in the way of its complete understanding. In the article [1] J.Chalker and P.Coddington (CC) have introduced a network model to deal with this problem and have studied it numerically. The model is phenomenological and uses the Transfer Matrix formalism. It based on the quasi-classical picture of noninteracting electrons in two space dimensions moving along the boundaries of the droplets(edge currents), which are formed by domains of the constant perpendicular magnetic fields. This domains appear due to disorder character of the magnetic field, presence of which is crucial for the Quantum Hall Effect (QHE). Close to critical point the droplets approaching each other and the quantum tunnelling of edge electrons from one to other droplet becomes essential. This phenomena causes the appearance of de-localized state after disorder over random $U(1)$ phases are taken into account.

The careful numerical investigations of the critical conductivity and the correlation length index $\nu$ within the CC network model in a numerous articles [1, 2, 3, 4] have shown excellent coincidence with the well established experimental value $\nu \approx 2.3 \pm 0.1$ (may be precisely $7/3$) [5, 6], which essentially has motivated the considerable interest to the model and stimulated it’s further investigation up to our days [7, 8, 9, 10, 11, 13, 14, 15, 16, 19, 20].

The main problem in this direction off course is the problem of formulation of the field theory, which corresponds to critical limit of CC model after quenched disorder is taken into account. But already before the disorder one needs to find the equivalence of CC Transfer Matrix formulation on a some kind of lattice.

In the early stage of investigations of the CC model D.H.Lee [7] mapped original model to an antiferromagnetic spin chain. Later Lee, together with Wang [8], have extended the model to the replica limit of the associated Hubbard model. In the articles [10] authors have developed further this ideas by mapping the problem of localization to the problem of diagonalization of some one-dimensional non-Hermitian Hamiltonian of interacting bosons and fermions. But it is absolutely clear, that one should consider this mappings as an alternative description of he CC model, rather than exact correspondence.

At the same time the technique of supersymmetry was used by M.Zirnbauer to average over the disorder in CC model in the article [11], where he has reduced the problem to the supersymmetric $\sigma$-model and an antiferromagnetic supersymmetric spin-chain.

In the articles [12] the $U(1)$ CC model was extended to spin case to describe the spin QH transitions in a system of noninteracting quasi-particles in 2D. Authors of the articles [13] have used a supersymmetry representation of such models to obtain a mapping onto the 2D classical bond percolation transition and get some critical exponents and universal ratios analytically. It is also necessary to mention recent investigation of the chiral universality class of the localization problem and corresponding modification of the original network model [20].

In the interesting article [14] the authors, instead of mapping CC model into some kind of Hamiltonian, are analyzing the hierarchy of network models by real space renormalization technique in order to understand the nature of localization-delocalization transition. The multifractal properties of the generalized CC model was investigated in the articles [15].

Some variant of CC network model was used in the articles [16, 17, 18, 19] in order to study numerically (and analytically) the random bond 2D Ising model by mapping one onto the other.

Analyzing carefully the mentioned upper articles, where authors have reduced the original Transfer Matrix formulation of the CC network model (just for which the correlation length
index was proved is coinciding with the experimental value of QHE plateau transitions) to some kind of supersymmetric spin chain problem one can find out that their exact equivalence remains obscure.

In this article, developing the results obtained in our previous works \cite{21, 22, 23}, we give the exact action (Lagrangian) formulation of the original CC Transfer Matrix model as a field theory on 2D lattice. First we show, that before the disorder over $U(1)$ phases is taken into account the CC model is equivalent to some inhomogeneous modification of the $XX$ model in the background of $U(1)$ field. Basic element of this construction is the fermionized version of the standard $R$-operator of the $XX$ model, but the Transfer Matrix is the staggered product of $\pi/2$-rotated $R$-operators. We prove, that in one particle sector the matrix elements of the Transfer Matrix of the defined model precisely reproduces the Transfer Matrix of the CC model. Then, by introducing the fermionic coherent states, we write down the action of the CC model on the 2D Manhattan Lattice ($ML$). Further, in order to investigate the presence of delocalized states, we consider the Landauer resistance in the model and take into account the disorder over $U(1)$ phases. As it was argued in the articles \cite{24}, the averaged Landauer resistance in quasi-one dimensional systems defines the double of localization length of the theory. We have considered homogenous distribution of random $U(1)$ phases over the circle, calculated the average $\langle T \otimes T^\dagger \rangle$ ($T$ is the Transfer Matrix of the CC model) and found the $R$-matrix and the action of corresponding model. The result is written in terms of two type (spin up and spin down) Fermi fields. It appeared, that the $R$-matrix we have found is fulfilling the Yang-Baxter Equations ($YBE$), which were defined in the article \cite{22} for the models with staggered disposition of $R$-matrices. This means that the resultant model is integrable. It rises hopes, that by use of powerful technique of Algebraic Bethe Ansatz (ABA) \cite{25, 26} one can investigate and calculate the critical properties of the QHE exactly.

2. The basic element of our construction is the $R_{i,j}$-matrix(operators) of the Algebraic Bethe Ansatz technique, which acts on a direct product of the linear spaces $V_i$ and $V_j$ of the quantum states at the chain sites $i$ and $j$ respectively.

$$\tilde{R}_{i,j} = V_i \otimes V_j \rightarrow V'_i \otimes V'_j. \quad (1)$$

Let us attach the Fock spaces $V_j$ of scalar fermions $c^+_j, c_j$ to each site of the chain and consider the operator forms of two types of $R$-matrices of the $XX$-model in braid formalism

$$\tilde{R}_{2j,2j+1}^\pm =$$
$$= a^\pm n_{2j} n_{2j+1} + a_\pm (1 - n_{2j})(1 - n_{2j+1}) + n_{2j}(1 - n_{2j+1})$$
$$+ (a^\pm a_\pm + b^\pm b_\pm) n_{2j+1}(1 - n_{2j}) + b_{\pm} c_{2j+1}^+ c_{2j} + b_{\pm} c_{2j}^+ c_{2j+1} +$$
$$+ c_{\pm} c_{2j,2j+1}^+ (a_{\pm} - 1)c_{2j,2j+1}^+ (1 - a_{\pm}) c_{2j,2j+1} \quad (2)$$

corresponding to two type of acts of scattering in the CC model, as it is drown in Figure1.

In the expression \cite{22} the symbol $: :$ means normal ordering of fermionic operators in the even sites and anti-normal (hole) ordering in the odd sites. The convenience of this choose will be clear later. The dot lines in the picture represents standard view of the $R$-matrices, while solid lines are convenient in a language of fermions with hopping parameters $a_{\pm}$ and $b_{\pm}$. Each
of the $R^\pm$ operators are nothing, but the fermionized versions of $R$-matrices of the ordinary $XX$ models

$$\tilde{R}_\pm = \begin{pmatrix} a_\pm & 0 & 0 & 0 \\ 0 & 1 & b_\pm & 0 \\ 0 & b_\pm & (a_\pm a_\mp + b_\pm b_\mp) & 0 \\ 0 & 0 & 0 & a_\pm \end{pmatrix},$$
(3)

which can be found by Jordan-Wigner transformation [27], or by the alternative technique, developed in [28].

Let us consider now Monodromy matrices $M_1$ and $M_2$ as a following products of $R$-matrices

$$M_1 = \prod_{j=1}^{N} \tilde{R}_{2j-1,2j}^+, \quad M_2 = \prod_{j=1}^{N} \tilde{R}_{2j,2j+1}^-,$$
(4)

which are corresponding to neighbor columns of the scatterings in the Figure 2. $M_{1,2}$ are discrete time evolution operators of the quantum states of the chain $|\Psi(t)\rangle \in \bigotimes_j V_j$ on one step $|\Psi(t+1)\rangle = M |\Psi(t)\rangle$. They are acting consequently and, therefore, the translational invariance in a time and chain directions exists only for even lattice spacing translations.

The dot lines in the Figure 2. represents standard $CC$ model, while solid lines define so called Manhattan Lattice ($ML$) due to the disposition of arrows on links. We have assigned the links of $ML$ with corresponding hopping parameter.

Let us consider now the one-particle sector of the Fock space of the chain

$$|i\rangle = |0,0..1_i,..0\rangle = c_i^+ |0,0..0,..0\rangle, \quad i = 1,..2N$$
(5)

and calculate the matrix elements of the operators $M_1$ and $M_2$ in this basis. After the parametrization of the hopping parameters as

$$a_- = 1/\cosh \theta', \quad b_- = 1/\tanh \theta', \quad a_+ = a = 1/\cosh \theta, \quad b_+ = b = 1/\tanh \theta$$
(6)

and unessential rescaling of $M_1$ and $M_2$ by the factors $a_+^N$ and $a_-^N$ respectively, one easily can recover the $2N \times 2N$ Transfer matrices $B$ and $C$, introduced in the article [1] by J.Chalker and
Figure 2: The dot lines represent the CC model. The solid lines represents the Manhattan Lattice, where equivalent formulation of CC can be given.

P. Coddington. In order to have 2D rotational invariance they have fixed $a_+ = b_- = a$, $b_+ = a_- = b$, which gives $\sinh \theta = \sinh \theta'$.

The random $U(1)$-phase factors ($A$ and $C$ matrices in [1]) can be introduced via operators $U_{\{a\}} = \exp[i \sum_{j=1}^{2N} \alpha_j n_j] = U_{\alpha_{\text{even}}} U_{\alpha_{\text{odd}}}$. The Transfer Matrix for the slice will look now as $M = M_1 U_1 M_2 U_2$ and it defines the $N$-step evolution operator by $T = (M_1 U_1 M_2 U_2)^N = (U_{\alpha_{\text{even}}} M_1 U_{\alpha_{\text{odd}}} U_{\alpha_{\text{even}}} M_2 U_{\alpha_{\text{odd}}})^N$. The phase factors in the operator $U_{\alpha_{\text{even}}} M_1 U_{\alpha_{\text{odd}}}$ appear near the $a_\pm, b_\pm$ parameters of the expression (2) for the $R$-operators, and we obtain

$$\tilde{R}^\pm_{2j,2j+1} = e^{i \alpha n_j} \tilde{R}^\pm_{2j,2j+1} e^{i \alpha' n_j}$$

As we see from this formula, the phase factors are the same on the exiting from the point links of the graphical representation of the $R$-matrix (see Fig.1).

\[\text{It is necessary now to make some remark. The rotational invariance indeed demands this relations between the hopping parameters, as it can be seen in the Figure 2, by rotating the } R\text{-matrices on } \pi/2, \text{but the disposition of signs will be different. However, since it does not changes the result after the calculation of the disorder we left it as in [1].}\]
Now we would like to present a field theory formulation of the CC model on ML and define an action for it. In Lagrangian formulation the calculation of phase disorder for the Landauer resistance will be straightforward. Let us introduce fermionic coherent states according to articles \cite{29} and pass to fermionic Transfer Matrix as it is done in \cite{21, 23}.

\begin{equation}
|\psi_{2j}\rangle = e^{\psi_{2j}c_{2j}^0}|0\rangle, \quad \langle \bar{\psi}_{2j}| = \langle 0|e^{c_{2j}\bar{\psi}_{2j}}
\end{equation}

for the even sites of the chain and

\begin{equation}
|\bar{\psi}_{2j+1}\rangle = (c_{2j+1}^+ - \bar{\psi}_{2j+1})|0\rangle, \quad \langle \psi_{2j+1}| = \langle 0|\bar{\psi}_{2j+1}
\end{equation}

for the odd sites. These states are designed as an eigenstates of creation-annihilation operators of fermions $c^+_j, c_j$ with eigenvalues $\psi_j$ and $\bar{\psi}_j$.

\begin{equation}
\begin{aligned}
c_{2j} | \psi_{2j}\rangle = -\psi_{2j} | \psi_{2j}\rangle, \quad \langle \bar{\psi}_{2j}| c_{2j}^+ = -\langle \bar{\psi}_{2j}| \bar{\psi}_j, \\
c_{2j+1}^+ \bar{\psi}_{2j+1}\rangle = \bar{\psi}_{2j+1} | \psi_{2j+1}\rangle, \quad \langle \psi_{2j+1}| c_{2j+1} = -\langle \bar{\psi}_{2j+1}| \psi_{2j+1}.
\end{aligned}
\end{equation}

This states are designed as an eigenstates of creation-annihilation operators of fermions $c^+_j, c_j$ with eigenvalues $\psi_j$ and $\bar{\psi}_j$.

It is easy to calculate the scalar product of this states

\begin{equation}
\langle \bar{\psi}_{2j}| \psi_{2j}\rangle = e^{\bar{\psi}_{2j}\psi_{2j}}, \quad \langle \psi_{2j+1}| \bar{\psi}_{2j+1}\rangle = e^{\bar{\psi}_{2j+1}\psi_{2j+1}}
\end{equation}

and find the completeness relations

\begin{align}
\int d\bar{\psi}_{2j}d\psi_{2j} | \psi_{2j}\rangle \langle \bar{\psi}_{2j}| e^{\bar{\psi}_{2j}\psi_{2j}} &= 1, \\
\int d\bar{\psi}_{2j+1}d\psi_{2j+1} | \bar{\psi}_{2j+1}\rangle \langle \psi_{2j+1}| e^{\bar{\psi}_{2j+1}\psi_{2j+1}} &= 1.
\end{align}

Let us now pass to the coherent basis \cite{5, 12} in the space of states $\prod_j V_j$ of the chain and calculate the matrix elements of the $R_{2j, 2j+1}^\pm$-operators between the initial $| \psi_{2j}\rangle \in V_{2j}$, $| \bar{\psi}_{2j+1}\rangle \in V_{2j+1}$ and final $| \psi'_{2j}\rangle \in V'_{2j}$, $| \bar{\psi'}_{2j+1}\rangle \in V'_{2j+1}$ states. By use of properties of coherent states it is easy to find from the formula \cite{7}, that

\begin{align}
R_{2j, 2j+1}^\pm | \psi_{2j}, \bar{\psi}_{2j+1}\rangle = & \langle \psi'_{2j+1}| R_{2j, 2j+1}^\pm \bar{\psi}_{2j+1}\rangle \\
= & e^{e^{i\alpha_2}\bar{\psi}_2\bar{\psi}_{2j+1}| \psi_{2j}, \bar{\psi}_{2j+1}\rangle} \\
= & e^{-S^\pm(\psi_{2j}, \bar{\psi}_{2j+1}, \bar{\psi}_{2j+1}', \psi_{2j+1}')}
\end{align}

This formula clarifies the convenience of introduction of the solid lines in the picture for $R$-matrix (Fig.1) since the parameters $a$ and $b$ getting a meaning of hopping parameters for fermions on it.

The completeness relations \cite{12} define the multiplication rule of $R$-operators \cite{13} and one can now express the Partition Function $Z$ of the model before the disorder calculations as functional integral over the classical Grassmann fields $\{\psi\}$

\begin{equation}
Z = Tr T^N = \int D\{\bar{\psi}\}D\{\psi\}e^{-\sum_{R\text{-matrices}} S^\pm(\psi_{2j}, \bar{\psi}_{2j+1}, \bar{\psi}_{2j+1}', \psi_{2j+1}') + \sum_j \bar{\psi}_j \psi_j}
\end{equation}
with the action defined on the \( ML \) (see \cite{21,23} for details). We would like to emphasize now that though we have started with the \( R \)-matrix of the \( XX \) model (or, which is the same, \( 2D \) Ising model), but the theory we have formulated is essentially different from \( XX \) model, since the \( R \)-matrices are disposed in the Transfer Matrix (or in the action) inhomogeneously.

In order to analyze the localization properties of the \( CC \) model we will consider now the dimensionless Landauer resistance, which is the ratio of reflection over the transmission coefficients and is nothing but the particular matrix elements of the direct product of the Transfer Matrix \( T \) with its Hermitian conjugate \( T^\dagger \). It was argued in the articles \cite{24} that the average of Landauer resistance defines the double of inverse of the scaling localization length. Since each phase factor appears locally only in one \( R \)-matrix and the disorder is full (there are no correlations between different points in the distribution of the phase factors), it is clear from the formulas \cite{24} that

\[
\langle T \otimes T^\dagger \rangle = \left( \prod_j \langle \bar{R}_{2j,2j+1}^+ \otimes (\bar{R}_{2j,2j+1}^+)^\dagger \rangle \prod_i \langle \bar{R}_{2i-1,2i}^- \otimes (\bar{R}_{2i-1,2i}^-)^\dagger \rangle \right)^N
\]

This average is easy to calculate in the \( \psi \)-basis of coherent states and we need to introduce two copies of Grassmann fields, say \( \psi_1 \) and \( \psi_2 \), for \( T \) and \( T^\dagger \) respectively. The effective theory is convenient to represented graphically on double \( ML \), as in Figure 3, expressing the hoppings of fermions separately.

We think that the Hermitian conjugation in the formula \cite{15} can be defined in a generalized way. Namely, in \( \bar{R} \) the phases \( e^{i\alpha} \) of the links in the expression \cite{13} of \( R \)-matrix can be changed by \( e^{i\bar{\phi}}e^{-i\alpha} \) (rather than by \( e^{-i\alpha} \)), expressing the possibility that our system is in vacuum \( \phi \)-flux background. Then, the average \( \langle \bar{R}_{2j,2j+1} \otimes \bar{R}_{2j,2j+1}^\dagger \rangle \) in case of Gaussian distribution \( \mathcal{P}(\{\alpha_j\}) = \prod_j \frac{1}{\kappa\sqrt{\pi}} \exp\left(-\frac{\alpha_j^2}{\kappa^2}\right) \) of phases can be calculated easily and one will find the \( R \)-matrix of the Hubbard model (see \cite{23}). But we do not have ordinary Hubbard model in a result, because, as it shown in Figure 3, in the Partition Function two type of \( R \)-matrices have to bee disposed in a staggered way. We will write down now only the expression of \( R \)-matrix for the background flux \( \phi = \pi \) and in the strong coupling limit \( (\kappa \to \infty) \), corresponding to homogenous distribution of phases over the circle, because, as it appeared, in this case the final model is integrable.

\[
(R^\pm)|_{\psi_1,\sigma}^{\psi_2,\sigma,\dagger} = \exp \left\{ e^{i\bar{\phi}}|_{\psi_1,\sigma}^{\psi_2,\sigma,\dagger}(a_+\psi'_{2j+1,\dagger} - b_+\psi_{2j,\dagger} - b_-\psi_{2j+1,\dagger} + a_-\psi'_{2j,\dagger}) + e^{-i\bar{\phi}}|_{\psi_1,\sigma}^{\psi_2,\sigma,\dagger}(a_+\psi_{2j,\dagger} + b_+\psi'_{2j+1,\dagger} - b_-\psi_{2j+1,\dagger} + a_-\psi'_{2j,\dagger}) \right\}
\]

Due to rotational invariance one should take \( a_+ = b_- = a \), \( b_+ = a_- = b \) and this two matrices have a chess like disposition on the \( ML \), as it is shown in the Figure 3a.

The \( R \)-matrices \cite{16} define an integrable model of the type developed recently in a chain of articles \cite{22,23}. First, it is easy to make an \( U(1) \) gauge transformation and pass from the distribution of hopping parameters of the model as it defined by the formula \cite{16} and shown on the Fig.3a, to the one, which is drown on Fig.3b. This gauge transformation is possible, because the distribution of fluxes through the plaquettes of \( ML \) is the same in both cases. Then, following
Figure 3: The CC model after disorder was taken into account. The hopping parameters of two type fermions are marked separately. Pictures a) and b) differs by gauge transformation.

[22], let us define a new Transfer Matrix of the model as a product of $R$-matrices along a chain, which is rotated on angle $\pi/4$ with respect to vertical access. From the Fig.3b one can see, that we have two different lines of Transfer Matrices

$$T_1 = \prod_j R_{2j,2j+1}^- R_{2j-1,2j}^+,$$
$$T_2 = \prod_j R_{2j,2j+1}^- R_{2j-1,2j}^+ \ast R_{1\ast,2\ast}^- \ast R_{1\ast,2\ast}^+.$$

The $R$-matrices of the first line (marked $T_1$ in the Fig.3b) are defined by the formulas (16) but without factors $e^{i\pi/4}$ in all hopping parameters. In the second line (marked $T_2$ in the Fig.3b) the $R$-matrices have minus sign in front of vertical hopping parameters of one of fermions (say spin-up), which we marked by the subscript $\iota$ in the expression (17). But this is precisely the $\iota$ operation defined in the articles [22]. We have an inhomogeneous model. Besides of the $\iota$ operation differing neighbor chains, the $R$ matrices are staggered along of each chain. It was shown in [22], that this $Z_2$ graded structure demands two sets of Yang-Baxter’s Equation ($YBE$) in order to ensure the commutativity of the Transfer Matrices with different spectral parameters $u$ and $v$.

$$R_{23}(u,v)R_{12}^{\ast,\iota}(u)R_{23}^{-}(v) = R_{1,2}^{-}(v)R_{2,3}(u)R_{1,2}^{\ast}(u,v),$$
\begin{align}
R_{2,3}(u,v)R_{1,2}^{+,i}(u)R_{2,3}^{+}(v) &= R_{1,2}^{+,i}(v)R_{2,3}^{+}(u)R_{1,2}(u,v). \tag{18}
\end{align}

One can check by direct calculations, that the YBE \((18)\) for intertwinners \(R_{i,i+1}^{+,i}(u,v)\) and \(R_{i,i+1}^{-,i}(u,v)\) have a solution of the same form as \((16)\) with
\begin{align*}
b_2(u,v) &= (b_2(u))^2 - (b_2(v))^2, \\
a_2(u,v) &= 1 - b_2(u,v). \tag{19}
\end{align*}

Therefore our model is integrable. This nontrivial fact gives us the possibility to use the powerful method of ABA \([25, 26]\) and investigate QH plateau-plateau transitions by the exact technique. This is the subject of further investigations.

In conclusion I would like to point out the main results of this work. The exact Lagrangian formulation of the CC model before the disorder is taken into account is presented based on staggered XX model. After calculation of the average of the Landauer resistance with constant distribution of random phases over the circle we have obtained a new type integrable model with two fermions (spin up/down). The integrability will allow to use the powerful method of ABA in order to investigate the plateau-plateau transitions in QHE exactly. If the bosonic partner of this construction will be found one can also formulate the corresponding supersymmetric model, which will allow to average the free energy (rather than Landauer resistivity) of the CC model.

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