Embedding of Virtual Network Requests over Static Wireless Multihop Networks

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Abstract

Network virtualization is a technology of running multiple heterogeneous network architecture on a shared substrate network. One of the crucial components in network virtualization is virtual network embedding, which provides a way to allocate physical network resources (e.g., CPU and link bandwidth) to virtual network requests. Despite significant research efforts on virtual network embedding in wired and cellular networks, little attention has been paid to that in wireless multi-hop networks, which is becoming more important due to its rapid growth and the need to share these networks among different business sectors and users. In this paper, we first study the root causes of new challenges of virtual network embedding in wireless multi-hop networks, and propose a new embedding algorithm that efficiently uses the resources of the physical substrate network. We examine our algorithm’s performance through extensive simulations under various scenarios. Due to lack of competitive algorithms, we compare the proposed algorithm to five other algorithms, mainly borrowed from wired embedding or artificially made by us, partially with or without the key algorithmic ideas to assess their impacts.

Keywords: wireless network virtualization; virtual network embedding; static wireless multi-hop network

1. Introduction

Network virtualization is a powerful technology that allows multiple heterogeneous network architectures over a shared physical network, shortly called a substrate network (SN). Major applications include testing of new network protocols, such as PlanetLab \cite{1} and Emulab \cite{2}. Network virtualization also enables multiple service providers to offer customized services with different requirements and features, e.g., a streaming video network with low delay and...
high bandwidth or a financial service network with high reliability and security
guarantees, over a common physical substrate network.

In a virtualized network environment, the SN providers accept and run multi-
tiple virtual networks (VNs), where the substrate nodes serve the nodes in a
virtual network and the physical paths configure the virtual links. Since multi-
ple VNs are bound to share the common underlying physical resources, the VN
embedding problem that finds an efficient embedding of each VN to substrate
resources, is very important in order to support as many requests as possible
and/or minimize the substrate network resources for embedding.

In this paper, we consider a VN embedding problem over wireless multi-
hop networks. We focus on the key difference–inter-link interference–in wireless
multi-hop networks from wired networks, whose challenges are summarized next.
A typical embedding algorithm consists of the following procedures: (a) search
plausible candidate embeddings for a given VN request, (b) assess their qualities
in terms of a target objective, e.g., minimization of substrate resource usage or
revenue maximization, and (c) select the best candidate among the feasible
ones. From the above procedures, we see that two primitives are necessary
in embedding: (i) checking the feasibility of a candidate embedding and (ii)
quantifying the embedding’s quality. Note that these two primitives are easy
to support in wired networks. Feasibility can be checked by comparing the
physical layer’s remaining resource with the resource required by the existing
and new embeddings. The quality of the embedding is typically quantified by
computing the total amount of resource occupied by the embedding. There,
embedding challenge is due to computational intractability just coming from
the large search space of candidate embeddings.

However, in wireless embedding, both primitives are harder to check, and
even need to be defined appropriately. This is because the resource allocated
over a link makes an indirect impact on the actual remaining resource over other
neighboring links due to inter-link interference, and both primitives are coupled
with the underlying MAC. A MAC protocol (e.g., 802.11), has its unique capa-
bility of supporting the assigned rates over each link, meaning that the feasibility
check will lead to different results, depending on the underlying MAC. Regard-
ing the quality comparison metric, as an example, consider two embeddings \(E_1\)
and \(E_2\), where the aggregate amount of resource required by \(E_1\) exceeds that
by \(E_2\). However, \(E_1\) can be preferable if the embedded nodes and links in \(E_1\)
are in less interfering regions, because it is likely that more future requests will
be accepted.

Our approach and main contributions towards efficient embedding in wireless
multi-hop networks are summarized as follows:

1) **Feasibility check.** We propose two solutions: (i) sufficiency-based approach
and (ii) simulation-based approach with smart embedding. First, in the
sufficiency-based approach, we use a graph-theoretical sufficient condition

\(^1\)We say that an embedding is feasible when the physical substrate network has enough
resource to support the embedding.
based on weighted graph coloring, which, if met, feasibility is guaranteed. Second, in the simulation based-approach, we limit the space of candidate embeddings, so that the conflict graph of the embedded substrate network always satisfies a specific pattern, called PBG (Polynomially Bounded Growing) graph. To check the feasibility of a candidate embedding, we simulate the substrate network, and the PBG property enables us to check the feasibility polynomially and performs arbitrarily close to the optimal one.

2) Quality comparison metric. We choose a simple quality metric (for a candidate embedding) that is designed to decrease when less overall loads are imposed on the SN by the embedding. While VN nodes are one-to-one mapped to the SN nodes, a VN link can be mapped to a path in the SN and the metric is designed so that the amount of link-interference of in the path is minimized.

3) Efficient candidate searching. The key to an efficient embedding algorithm lies in how to smartly search a limited set of “good” candidate embeddings. We repeatedly test a candidate embedding that is chosen by merging the node and link mapping process for a limited number of times. The joint link and node mapping process simultaneously considers the amount of available node resources in the node mapping and the degree of newly generated interference to the network in the link mapping. This selection of nodes and links are coupled with the comparison metric.

Potential applications of VN embedding over wireless multi-hop networks are as follows: With increasing number of mobile users, accelerated by proliferation of smart phones, wireless access technologies are becoming diverse, widespread, and broadband. Of many types of access networks, wireless multi-hop networks, are expected to be used as an inexpensive way to provide last-mile Internet access. In fact, several cities are currently deploying municipal wireless mesh networks. Virtualization can serve plenty of uses over multi-hop networks too. In the mobile network market, a growing number of Mobile Virtual Network Operators (MVNO), that reaches over 430 worldwide in 2010. MVNOs do not own the wireless network infrastructure, and lease part of the physical infrastructure from Mobile Network Operators (MNO) to provide customized mobile services, in which case MNOs having single/multi-hop wireless physical networks may need to virtualize their networks for the business with MVNOs. In addition, similar to PlanetLab, Orbit which is a wireless network testbed consisting of 20×20 nodes can be another example of virtualization over the wireless multi-hop network. Besides above applications, there will be other futuristic scenarios in which multi-hop virtualization is utilized, when wireless multi-hop networks come into wide use.

We consider only inter-link interference modeled based on a graph-theoretic

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2It is the graph that represents interference relationships among wireless links. Refer to for a more detailed explanation.

3It is a subgraph of the substrate network consisting only of nodes and links which serve some virtual network requests.
relationship, and do not consider wireless links’ time-varying characteristics due to e.g., SN nodes’ mobility, i.e., the capacities of links are assumed to be fixed. Although this does not reflect the practice perfectly, our work can be an important step towards efficient embedding over wireless multi-hop networks, since handling inter-link interference is one of the major obstacles there, such as the approach in wireless link scheduling research (see e.g., [2] for a survey). We expect that our work is connected to research on more practical algorithms reflecting the full wireless features in the future.

2. Related Work

Recently, there has been research interest regarding virtual network embedding over wired networks, e.g., [7, 8, 9, 10, 11, 12] and/or embedding in single-hop cellular networks [13], where the embedding problem turns out to require computationally intractable complexity for optimality, and thus various heuristics have been proposed. More challenging issues in multi-hop networks are involved mainly due to the complex interference among links and its severe coupling with network topology. These challenges require a new design of embedding algorithms.

Recently, NVS in [13] is proposed for virtualization on wireless single-hop networks. NVS provides an effective wireless resource allocation over cellular networks, by separating slice scheduling and flow scheduling. However, the VN embedding problem essentially asks the mapping correspondence between a VN node/link and SN node/path, beyond the scope of resource allocation.

Related work on embedding in wired networks mainly focuses on addressing computational challenges by restricting the problem space in different dimensions or proposing heuristic algorithms [7, 8, 9]. For example, only bandwidth requirements are considered in [7, 8] or all VN requests are assumed to be given in advance [8, 9]. The authors in [7, 8, 9] also considered the substrate network with infinite capacity, accepting all incoming VN requests. On embedding problems over SN with limited resources, multicommodity flow detection based algorithms are proposed in [10, 11], where node embedding methods also consider their relation to the link embedding stage like our work. However, all algorithms in [8, 9, 10, 11] separate node and link embedding process, i.e., all VN nodes are embedded before embedding the VN links. A single-stage wired embedding algorithm is also proposed in [12] which tries to find a subgraph isomorphism of the VN via a backtracking method. There, the algorithm imposes a limit on the length of substrate paths that will embed virtual links and checks feasibility whenever a new VN link is embedded: if it is infeasible, it is backtracked to the last feasible embedding.

We extend the domain of the embedding problem to wireless multi-hop networks, keeping considerations for more general cases such as online requests and SN with limited capacity.
3. Wireless VN Embedding Problem

3.1. Model

3.1.1. Substrate network

**Network model.** Denote the wireless substrate network (SN) by an undirected graph $G^S = (N^S, L^S, A^S_N, A^S_L, I)$

where $N^S$ and $L^S$ are the sets of nodes and links, respectively. The $A^S_N$ refers to the set of node resources, e.g., CPU resource or hard-disk. We assume that only CPU resource is considered in this paper, which can be readily extended to other resources. The $A^S_L$ is the set of link resources. We restrict our attention to the case of providing the long-term average capacity to the link resource requirement in a VN request. Thus, we model the link capacity to be fixed, which is a time-averaged value over time-varying link channels. This seems to be reasonable since the time-scale of embedding arrival and departure is much slower than that of channel variations.

Let $CPU^S(n)$ be the amount of CPU resource of node $n \in N^S$, and $CAP^S(l)$ be the capacity of wireless link $l \in L^S$. We also denote by $P^S$ the set of all paths in $G^S$.

**Interference model.** The matrix $I$ is the $|L^S| \times |L^S|$ matrix which represents the interference relationships for wireless links, where $I_{ij} = 1$ if links $i$ and $j$ interfere with each other, and 0 otherwise. Denote by $d_l$ the number of interfering links with $l$ in the SN. The interference matrix depends on the physical layer techniques as well as the employed MAC. In literature on modeling wireless networks, a hop-based interference model is popularly used, e.g., one-hop for FH-CDMA and two-hop for 802.11-like systems. However, our description can be readily extended to the general cases.

**MAC model.** We assume a MAC with $\epsilon$-throughput-optimality for some $\epsilon > 0$. Roughly, a MAC is said to be throughput-optimal if it can stabilize any arrival rate vector over the SN links whenever possible (see the seminal paper [14] for a formal definition). $\epsilon$-throughput-optimality means that a MAC can support only a $\epsilon$-reduced version of the arrival rates supported by a throughput-optimal MAC. We can regard $\epsilon$ as a reduction factor that is due to any kind of implementation overhead, e.g., message passing. We note that recently there also exists a research on so-called optimal CSMA, e.g., [15, 16, 17], that is close to throughput-optimal by simply and locally controlling CSMA parameters.

3.1.2. Virtual network

A virtual network (VN) request is denoted by an undirected graph $G^V = (N^V, L^V, C^V_N, C^V_L, D^V)$, where $N^V$ and $L^V$ are the set of nodes and links of the virtual network, respectively. $C^V_N$ and $C^V_L$ denote the set of node and link requirements. As an example, let $CPU^V(n)$ and $BW^V(l)$ be the CPU and bandwidth requirements for the virtual node $n \in N^V$ and the virtual link $l \in L^V$. These requirements can be also interpreted as constraints for embedding.

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4 Throughout this paper, we use the superscripts $S$ and $V$ to refer to the notations related to substrate and virtual networks, respectively.
i.e., to accept a VN request, the amount of allocated substrate resources for each virtual node and link should be more than required. $D^V$ implies that the VN request should be served for the duration of $D^V$. With a little abuse of notation, we also denote by $L(n)$ the set of the links connected to the node $n$ in both $G^V$ and $G^S$.

3.2. Problem Formulation: Virtual Network Embedding

We define an embedding $E$ from $G^V$ to a subset of $G^S$ as a mapping

$$E : G^V \rightarrow (N^*, P^*, R^N, R^L)$$

where $N^* \subseteq N^S$, $P^* \subseteq P^S$ and $R^N, R^L$ are the node/link resources allocated to the $G^V$ by embedding $E$.

We consider an online virtual network requests scenario, where a sequence of VN requests arbitrarily arrive and stay in the network over time. We consider a time-slotted system, indexed by $t = 0, 1, \ldots$.

For an embedded VN request $G^V$, the SN provider earns revenue $R(G^V)$ which is proportional to the total amount of the requested resources in the VN request, i.e.,

$$R(G^V) = \sum_{n \in N^V} \text{CPU}^V(n) + \alpha \sum_{l \in L^V} \text{BW}^V(l),$$

(1)

where the constant $\alpha$ reflects the relative importance in node and link resources, chosen by a virtual network operator. This weighted method is one of the typical ways to handle multi-objective optimization problem, like in the previous works [10, 11].

Then, the SN provider’s revenue $R(t)$ at time slot $t$ becomes

$$R(t) = \sum_{g \in G^V(t)} R(g),$$

(2)

where $G^V(t)$ be the set of VN requests served at time slot $t$. Since VN requests arrive and depart arbitrarily over time, we set a goal of our embedding algorithm to maximize the time-averaged revenue, given by:

$$\max \lim_{T \rightarrow \infty} \frac{\int_0^T R(t) \, dt}{T}. \quad (3)$$

4. Challenges in Wireless Embedding

4.1. Handling Online VN Requests

The VN request arrivals and departures are not known in advance, and may be quite random. In this unpredictable situation, the on-line embedding algorithm to achieve the objective (3) requires the statistical properties of VN requests and the large search space, where a mathematical tool such as dynamic programming will be used. Clearly, the issue of handling online, unpredictable VN requests also exists in the embedding over wired substrate networks.
4.2. Hardness of Embedding Quality Comparison

After searching a multiple of candidate embeddings for a given VN, the next step is to quantify how good the embeddings are. Comparing quality among candidate embeddings has the following requirements:

R1. **Simplicity.** Short computation time for comparison is necessary due to possibility of existence of a large number of candidate embeddings.

R2. **Efficiency.** The devised metric should appropriately reflect the changes in the available substrate resources for future requests.

In wired substrate networks, R1 and R2 are easy to meet, since the amount of available substrate resource is just the original amount of the resource subtracted by the resource assigned in embedding previous requests. However, in wireless multi-hop substrate networks, as discussed earlier, measuring the amount of the available resource is ambiguous. We can interpret the available (or remaining) bandwidth resource of a wireless link as its capacity multiplied by the time portion that it can be activated additionally. Therefore, the amount of available resource over each link is determined by the employed link scheduling. However, wireless networks operate a dynamic scheduling algorithm which decides a set of activated links over time. This makes exact calculation of the amount of available resources practically impossible.

4.3. Feasibility Checking Problem

**Wired vs. Wireless.** We consider an example in Fig. 1 to explain the unique challenges in the feasibility check of wireless embedding. Note that feasibility check is an important primitive in embedding to check if there is enough resource for a candidate embedding to be supported in the SN. The SN in Fig. 1(b) can be interpreted either as a wired SN or a wireless SN. We assume that no prior VN requests are served in the SN, and the interference in the wireless SN is one-hop based, i.e., any two links with one distance interfere. Consider a candidate embedding in both of wired and wireless SNs in which

\[
\text{Node mapping: } \quad x \rightarrow A, \quad y \rightarrow B, \quad z \rightarrow D \\
\text{Link mapping: } \quad (x, y) \rightarrow (A, B), \quad (x, z) \rightarrow (A, C, D),
\]

Recall that \((A, C, D)\) is a path in the SN. In the wired SN, feasibility check can be easily done by individually checking the feasibility of node and link resources. Although the mapping is feasible in wired SN, the same embedding is **infeasible** in the wireless SN, because there is no way of serving either \((A, C)\) and \((C, D)\) at one instant and providing the (long-term) bandwidth of 30 at each link. This is because link \((A, C)\) and link \((C, D)\) should be acted as the virtual link \((x, z)\) during \(0.6(=\frac{30}{50})\) portion of the time, but they cannot be activated simultaneously due to interference. This shows the difficulty of checking feasibility, where individual resource check at nodes and links is not enough, and a more complex checking procedure should be considered.

**Formalism.** Fundamental root causes of feasibility check in wireless embedding can be understood more clearly by introducing a notion of **conflict graph**. A
conflict graph graphically captures the interference relation between any pair of links by transforming the original graph for a given interference matrix such as: a link becomes a node and two links are connected if they interfere. For a candidate embedding, consider the potential normalized load of a substrate link \( l \), \( \lambda_l = \frac{\text{Req}(l)}{\text{CAP}(l)} \), where \( \text{Req}(l) \) is the required aggregate bandwidth of VN requests already being served and a new candidate embedding. Then, the candidate embedding becomes feasible if there exists a scheduling scheme which provides the long-term average rates of \( \lambda_l \). This has been traditionally studied in the context of weighted graph coloring for the conflict graph, known to be an NP-hard problem. Another way of understanding this problem from the control and networking theoretic perspective is whether the system can be stabilized or not by some scheduling algorithm, assuming stochastic packet arrivals with mean \( \lambda_l \) over each link [14].

4.4. Searching candidate embeddings

Section 4.1 discussed the challenge of unpredictable on-line requests. Sections 4.2 and 4.3 deal with the issue of comparing embeddings’ quality and checking feasibility. The final embedding algorithm remaining is to determine which of the candidate embeddings to try. This is because the entire searching of all candidates embeddings is computationally impossible. Obviously, this problem also appears in the wired embedding, yet it is coupled with feasibility check as well as quality comparison metric. Thus, based on the appropriate comparison metric definition and the feasibility checking mechanism, the question which embedding to try first and which embedding is finally selected will be decided accordingly.

5. Embedding Algorithm

5.1. Algorithm Overview

The framework of our algorithm is described in Fig. 2 and WEM (Wireless EMbedding). First, we explain how dynamic requests are processed, and then elaborate on the embedding algorithm for a VN request in the subsequent subsections, which is our major focus of this paper.

On-line requests. Serving incoming VN requests as fast as possible may be one criterion of handling on-line requests. However, a more crucial capability required by SN is to prevent the VN request from being blocked due to inefficient SN resource management. To that end, we divide the time into a sequence of windows whose duration determines how frequently the embedding process is run. This is dependent on the SN provider’s operation decision, e.g., an hour or a day. Over one time window, we collect a group of arriving requests and process one-by-one. Since it may be impossible to accommodate all the incoming requests within a certain time window, we need to smartly decide the requests that will be served. We take an approach that the SN provider may prioritize the VN requests to maximize the earned revenues by serving the requests in the order of revenues. This approach is similarly taken in wired embedding [10].
Algorithm description for a VN request. Once the sequence of VN requests to be processed in a time window is determined, we try to maximize the efficiency of using SN resources by employing a smart embedding algorithm for a VN request. We now describe our algorithm, called **WEM**, the embedding algorithm for a VN request as follows:

**WEM: Wireless EMbedding algorithm for a VN request**

**Step 1.** Decide the embedding order of VN nodes.

**Step 2.** Select the \(K\) SN “root” nodes, each of which maps the first VN node (chosen by **Step 1**) in the \(K\) candidate embedding trials.

**Step 3.** Choose \(K\) candidate embeddings, where each candidate embedding process starts from each root node (chosen in **Step 2**) and sequentially embeds other remaining VN nodes and links according to the order in **Step 1**.

**Step 4.** Evaluate \(K\) candidate embeddings based on a comparison metric, and take the highest-quality embedding candidate that passes feasibility check.

**Key ideas.** Our algorithm is designed to ensure to tackle the challenges in Section 4. Scarce resources in wireless networks should be efficiently utilized, whereas the impact of wireless interference in many algorithmic components are appropriately handled while striking a balance between efficiency and running time. In our algorithm, we first choose \(K\) embeddings (**Step 3**), where \(K\) is the search number, and in each candidate embedding, we choose the “root” SN node (**Step 2**) that maps to the first VN node (which is chosen by a selection rule, **Step 1**). Afterwards, VN links and nodes are mapped simultaneously. This is because interference is coupled with both nodes and links, e.g., a node with many incident links is likely to be in the region with severe interference. Finally, we choose the one that has the best quality and passes the feasibility check (**Step 4**). We now provide the details in each step in conjunction with the design rationales. To help the readers to understand, we will use the example in Fig. 3 in all explanations.

5.2. VN Node Sequence and SN Node Selection

**VN node sequence.** In **Step 1**, we first determine the mapping sequence of VN nodes in any embedding candidate. To that end, we define the notion of extended required node resource: for a VN node \(n\), \(\text{CPU}^V(n) + \alpha \sum_{l \in L(n)} \text{BW}^V(l)\). This notion captures the CPU resource plus the aggregate bandwidth resource required by the node’s incident links. The node with highest extended required resource is firstly embedded due to higher embedding difficulty. We embed the rest of the nodes in ascending order of hop distance from the first node.

**SN node selection.** In **Step 2**, we choose a starting root node in SN for each candidate embedding. Since we try \(K\) embeddings, \(K\) starting root nodes
should be selected. We use a similar metric to that in Step 1. We sort all the SN nodes in the decreasing order of the following extended remaining node resource: for an SN node \( n \), \( \text{CPU}_{\text{Rem}}^S(n) + \alpha \sum_{l \in L(n)} \text{BW}_{\text{Rem}}^S(l) \), where \( \text{CPU}_{\text{Rem}}^S(n) \) is the remaining CPU resource available for new VN requests (similarly, \( \text{BW}_{\text{Rem}}^S(l) \)). The extended remaining node resource quantifies the amount of remaining node resource considering the bandwidth resource of connected links to the corresponding node. It means that we prefer to try the root node which has enough resource.

**Example.** In Fig. 3(a), since node \( a \) has the highest extended required resources, the embedding sequence is \( a, c, b \). Both \( b \) and \( c \) are one hop apart from \( a \), here, we give a higher priority to \( c \) because its extended required resources is higher than \( b \)'s. It can be easily shown that node \( C \) has the highest extended remaining resources in Fig. 3(b).

5.3. Searching Candidate Embeddings

**Overview.** In this subsection, we elaborate on Step 3, where, starting with a given root SN node in an embedding candidate, we finish the embedding candidate by mapping other remaining VN nodes and links. The overview of the process is as follows: We define a notion of influence weight assigned to all SN links (to quantify interference in their neighborhoods). We then map the VN nodes to the SN nodes, so that the required link bandwidth between two VN nodes are satisfied as well as minimize the aggregate influence weight of the mapped SN path. This process is a joint link and node embedding that takes into account our intention to prefer less interfering regions with higher capacity (and thus more efficiently utilizing SN resources).

**Influence weight and distance.** The influence weight \( d_I(l) \) for an SN link \( l \) is the number of its interfering links (including itself) divided by \( \text{CAP}^S(l) \), i.e., \( d_I(l) = (d_l + 1)/\text{CAP}^S(l) \). Then, the influence weight for a path is the sum of influence weights of all links in the path. The influence distance of two SN nodes is the minimum influence weight among the paths connecting those two SN nodes. Fig. 3(c) illustrates an example of the influence weights.

**Joint VN node and link mapping.** We now explain how the VN nodes are embedded in conjunction with the VN links. Assume that we handle \( i \)-th VN node \( n_i^V \) in the sequence by Step 1. Let \( A(n) \) be the set of already embedded VN nodes adjacent to \( n \) in \( G^V \). Assuming that the SN node for \( n_i^V \) is decided (whose rule will be explained shortly), the embedding of added VN links between \( n_i^V \) to all the VN nodes in \( A(n) \) is done by the shortest paths in terms of influence weight subject to each path has enough path bandwidth to satisfy the bandwidth requirement of the corresponding VN link added due to node embedding.

We now describe the rule for selecting the SN node \( n_i^S \) which will embed \( n_i^V \):

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**Embedding of \( i \)-th VN node \( n_i^V \) with added VN links.**
where $a, b$ is the virtual link between $a$ and $b$, $E(u)$ is the SN node that serves a VN node $u$ and $d_I(E(u), n)$ is the influence distance between $E(u)$ and $n$.

The node embedding rule is designed to minimize the aggregate “load stress” added by embedding a new VN node. The load stress is measured by the aggregate bandwidth requirement weighted by the influence distance over the new SN paths. Weighting with the influence weight is due to our design rationale of avoiding the region with less capacity and high interference.

Example. We now illustrate the process using Fig. 4 that magnifies the algorithm of this subsection from the example in Fig 3. We assume that we consider a candidate embedding, where the first VN node $a$ and the root SN node is $C$. Since the next VN node is $c$, we find the SN node which has the shortest influence distance from $C$, which is node $B$. $c$ is embedded to $B$ and VN link $(a, c)$ is mapped to link $(C, B)$ (Fig. 4(b)). We next search the SN node which will serve node $b$. We calculate the (bandwidth requirement) weighted sum of influence distances from node $C$ and $B$ to each of nodes $A$, $D$, and $E$. That is: for $A$, $5 \cdot 0.1 + 3 \cdot 0.067 = 0.701$, for $D$, $5 \cdot 0.08 + 3 \cdot (0.04 + 0.08) = 0.76$ and finally for $E$, $5 \cdot (0.08 + 0.038) + 3 \cdot (0.067 + 0.08) = 1.031$. Thus, node $b$ is embedded to node $A$, and link $(a, b)$ and $(c, b)$ are embedded to link $(C, A)$ and $(B, A)$, respectively (Fig. 4(c)).

5.4. Comparison Metric

$K$ candidate embeddings are now ready, and we are in the stage of quantifying their qualities based on a metric which we explain in this subsection (Step 4). For a candidate embedding $E$, we define the embedding comparison metric $\sigma(E)$:

$$\sigma(E) \triangleq \sum_{l \in L^E} (d_l + 1) \times \lambda_l(E),$$

where $\lambda_l(E)$ is the potential normalized load of link $l$ for $E$, and recall that $d_l$ is the number of interfering links with $l$. Note that link $l$ forbids $d_l$ links to be activated at least $\lambda_l$ portion of the time. The potential normalized loads weighted by $d_l + 1$ captures the amount of offered loads considering interference (including itself). Following such intuition, we prefer a candidate embedding with smaller value of $\sigma$.

The metric $\sigma$ is devised to be fully compatible with our joint node and link embedding algorithm in Section 5.3. Suppose that a VN link $l^V$ is embedded to an SN path $P$. Then, the increment of $\sigma$ is given by:

$$\sum_{l \in P} \frac{\text{BW}^V(l^V)}{\text{CAP}^S(l)} \cdot (d_l + 1) = \text{BW}^V(l^V) \sum_{l \in P} \frac{d_l + 1}{\text{CAP}^S(l)} = \text{BW}^V(l^V) \sum_{l \in P} d_l(l).$$
Comparing the equation above, we observe the equivalence in this equation. In other words, the searching process in Section 5.3 tries different regions to search $K$ embeddings by starting from the root nodes in different regions, and their quality comparison is based on the metric that is instilled in node and link embedding.

As an example, consider two candidate embeddings $E_1$ and $E_2$ in Fig. 5. Note that virtual link $(a, b)$ is embedded to path $(E, D, C)$ in $E_2$. Since $\sigma(\cdot)$ can also be expressed by $\sum_{l \in L} Req(l) \times d_I(l)$, $\sigma(E_1) = 10 \cdot 0.04 + 5 \cdot 0.1 + 3 \cdot 0.067 = 1.101$ and $\sigma(E_2) = (10 + 5) \cdot 0.038 + (3 + 5) \cdot 0.08 = 1.21$. Thus, $E_1$ is preferred by the metric.

5.5. Checking Feasibility

We finally select the embedding with the highest quality metric, which passes feasibility check (Step 4). We now provide two candidate ways of checking feasibility.

Exploiting Sufficient Conditions. As discussed in Section 4, checking feasibility is computationally intractable. One can apply a sufficient condition for a given potential normalized load (recall its definition in Section 4.3). A well-known sufficient condition is that for any link $l$ (a vertex in the conflict graph), the sum of normalized loads of $l$ and $l$’s connected vertices in the conflict graph is less than or equal to 1 (see e.g., [18] for the formal proof). Clearly, the fact that the sufficient condition is not met does not imply that the tested embedding is infeasible. However, using this sufficient condition is not a bad idea, because adding new requests should be conservative such that existing virtualization service should not be interrupted and also the approach of sufficient condition is computationally attractive.

Simulation via Smart Embedding. In spite of computational merit of the sufficient condition, its quality can be bad for some network topologies, i.e., missing feasible embeddings. Also, for a VN request, an embedding algorithm may not need to produce the result very fast. If we spend a reasonable amount of time, yet achieving more accurate feasibility check, the SN provider is expected to earn larger revenue.

We can examine embedding feasibility by actually simulating a MAC (or its variant) for the potential normalized load $\lambda = (\lambda_l)$. In other words, we generate stochastic arrivals over each SN link $l$, where the arrival mean is same as $\lambda_l$. However, just performing simulation does not solve computational intractability. When $\epsilon = 1$, simulating the so-called Max-Weight can provides us with the result of feasibility check. However, it is widely-known that Max-Weight requires to solve an NP-hard problem (MWIS: Maximum Weight Independent Set problem) at each time instance. Note that our underlying MAC is $\epsilon$-throughput-optimal. For a general $\epsilon > 0$, which requires an $\epsilon$-approximate algorithm of MWIS, the technical challenge is that MWIS does not allow PTAS (Polynomial Time Approximation Scheme) [19].

To achieve efficiency in conjunction with a reasonable complexity, (e.g., polynomial), we perform smart embedding. The idea of smart embedding is to
restrict the use of SN nodes and links so that the conflict graph of the embedded substrate network satisfies a special geometrical property — *polynomially bounded growth*\(^5\). Recall that the embedded substrate network is a subgraph of \(G^S\) consisting only of nodes and links serving some VN requests. With polynomial growth, we can find a polynomial time algorithm which arbitrarily approximates the original problem. For example, we allow a suboptimality gap \(\epsilon > 0\), then the complexity, which is a function of \(\epsilon\), is polynomial with network size. We refer the readers to, *e.g.* [19, 20] for \(\epsilon\)-approximation algorithms of MWIS in PBG graphs.

**Comparison.** Two methods have different design rationales: (i) sufficient condition—using the entire space of SN and coarse feasibility checking or (ii) simulation via smart embedding—using a limited space of SN but finer feasibility checking at cost of increasing complexity. It is interesting how two methods perform, which will be presented in Section 6.

## 6. Performance Evaluation

We develop a wireless embedding simulator (available in public [21]) to evaluate the proposed algorithm and analyze the impact of its key features.

### 6.1. Simulation Environment

**Substrate network.** We set up a grid square (100 \(\times\) 100), where an SN will be configured. We randomly place 50 substrate nodes on the grid square, where each node has some random transmission range and any two nodes are connected if they are within the transmission range of each other. In addition to this random topology, we will show the results for other topologies, shown in Fig. 6. We use random topologies with *middle* density, unless explicitly specified. To see the impact of SN densities, we also vary the transmission range so that the number of SN links ranges about 80 \(\sim\) 240. The CPU resources of nodes and link capacities are set to follow a uniform distribution between 100 and 300 units. In the revenue earned by a VN request, we give higher priority to the links, where we choose \(\alpha = 10\). This choice is due to the fact that in wireless multi-hop networks, link capacity is the more scarce resource, which has a larger impact on the SN’s revenue. A two-hop interference model, known to suitably capture the MAC with RTS/CTS-like control messages, is adopted in our evaluations. However, we verified through simulation (not presented due to space limitation) that the overall trends do not severely depend on interference models. We assume that \(\epsilon = 0.3\), *i.e.* 30% of MAC’s suboptimality and overheads, which is used in simulation-based feasibility check.

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\(^5\) Let \(G = (V, E)\) be a graph and \(d(u, v)\) be the hop-distance between node \(u\) and \(v\). Then \(r\)-neighborhood of a node \(v\) is denoted by \(\Phi(v, r) = \{v \in V \mid d(u, v) \leq r\}\). We say that \(G(V, E)\) is a polynomially bounded growing (PBG) graph with a polynomial function \(p(r)\), if \(|\Phi(v, r)| \leq p(r)\) for any \(v\) and \(r\).
**VN requests.** In a VN request, we randomly select the number of VN nodes between 4 and 10. The probability of connecting a pair of virtual nodes is uniformly distributed over the interval $[0.2, 0.6]$. Each CPU resource requirement and bandwidth resource requirement are also uniformly distributed between 1 and 10 units. The arrival process of VN requests is modeled to follow a Poisson process with an average of 5 per time window, unless explicitly specified. Each VN request stays at the SN during the holding time following an exponential distribution with a mean of 4 time windows.

**Tested algorithms.** To the best our knowledge, there does not exist competitive embedding algorithms in wireless multi-hop networks. However, we devise some possible algorithms to provide the readers fair comparisons. In all algorithms, we select 8 candidate embeddings for each VN request (i.e., $K = 8$). The reason for this value will shortly be provided in the simulation results. We tested six algorithms to see the effect of the key features in our algorithm, as summarized in Table 2.

Recall that there are two main features of our algorithms: (F1) joint node/link embedding, and (F2) a notion of influence weight that quantifies a link weight which is used to select an embedded SN path. Regarding (F1), we artificially make three classes (none, intermediate, and full) based on existing algorithms in wired networks, and two classes for (F2) (none and interference). In (F1), none corresponds to the greedy algorithm in [10]. Intermediate corresponds to a slightly modified algorithm in [9]. Both intermediate and full, share the feature that embedded SN nodes should be closely placed. The difference is that in intermediate, the inter-distance between VN nodes is not considered, whereas in full, two VN nodes with shorter distance are embedded to two SN nodes with shorter distance. In (F2), no algorithms in wired networks consider interference, e.g., [12]. Thus, to purely focus on how the influence weight affects the performance, we choose algorithms without the influence weight as none, so the SN path is computed just by considering the number of hops.

### 6.2. Simulation Results

**Impact of feasibility check methods.** We first consider the results of comparing two feasibility checking methods, shown in Fig. 7. We observe that the simulation-based method significantly outperforms the sufficient condition based method (about two times). From this, we can see that rather than fully utilizing SN resources with an inaccurate feasibility check, it is more desirable to apply a strict checking method even with a slightly limited usage of SN resource. We apply the simulation based feasibility check to the rest of the simulations. Note that all tested algorithms are equipped with the same feasibility checking method for fair comparison.

**Impact of SN density.** We now start to compare the performance of tested algorithms. We look at the impact of different SN densities, shown in Fig. 8(a). In all graphs, Alg6 outperforms other algorithms. An interesting observation here is that as SN density increases, e.g., see the high density case, the importance of considering interference in the link weight becomes stronger. Thus, the
performance gap between Alg6 and Alg5 is small, whereas Alg3’s performance gap from Alg6 increases, compared to other lower SN density cases.

**Impact of VN topology.** VNs may often have special topological structures such as tree, hub-and-spoke, and star. This topology depends on the type of virtualization service. For example, a game service with a single server is likely to form a star topology. We study this topological impact. Fig. 8(b) shows the avg. revenue comparison for tree, star, and random topologies. We observe that in star topology, four algorithms, Alg2, 3, 5, 6, do not lead to a large performance gap. We analyze this observation as follows: In star topology, all links are connected to a “center” VN node. Then, severe local concentration is experienced around a node: all VN links should be embedded to the paths concentrated around the SN node which embeds the center VN node. This load concentration prevents a big star topology from being embedded in all algorithms, and only small star topologies are accepted. This trend is supported by the results for the tree topologies, whose degree of concentration is between random and star topology, where the performance gap is in between those two topologies. However, we still observe that joint consideration of node and link is crucial in improving the performance.

**Impact of VN arrival intensity.** We also vary the VN arrival intensities by testing various arrival rates, ranging from 1 to 8, as shown in Fig 8(c). First, in all algorithms, the revenue curves are concave with “diminishing returns.” This is because for low arrival rates, bigger VN topologies (and thus, bigger revenues) can dominate the total earned revenue. However, for high arrival rates, in addition to those big VNs, only small VNs take effect in increasing revenues. Second, Alg1-6 are grouped into three classes in terms of average revenue performance, where (i) joint link/node embedding is crucial, (ii) interference-aware link weights leads to additional revenue increase. This implies the importance of the two key features in Alg6.

**Impact of search number K.** The search number should strike a balance between the running time and efficiency. Fig. 9 shows the average revenue as well as running time as K increases. We vary the number of SN nodes using middle SN density. Fig. 9 shows that after K = 8, the revenue saturates, whereas the computation time increases linearly. The computation time also increases more sharply with the increasing number of SN nodes, because the number of SN links also increases due to the fixed SN density. The choice of this search number K may depend on the SN and VN sizes. The simulation results imply that a small search number out of the huge search space may be sufficient enough. Fig. 9 also implies that our algorithm is computationally tractable, because the algorithm generates the embedding results in the order of seconds, which is reasonable in practical applications.

**Summary.** First, the effect of joint node/link embedding is large. The node embedding in a “non-coupling” algorithm does not consider the neighborhood interference of SN nodes, and thus, with a limited search number, it is highly likely to choose inefficient embedding candidates. Second, a more accurate feasibility check is necessary for efficient embedding. This accuracy comes with
the cost of additional time, but it is still reasonable in practical cases. Third, considering wireless interference as the link weight is also important, where in some cases, the performance gap amounts to about 35% for the algorithms that are unaware of interference. Fourth, VN topology may significantly impact the effect of embedding algorithms, especially in wireless multi-hop networks, due to resource concentration at nodes that leads to the generation of bottlenecks.

7. Concluding Remarks

In this paper, we propose an embedding algorithm over wireless multi-hop networks. The key challenges in wireless embedding originate from inter-link interference, which makes the issues of feasibility check and candidate embedding search. The main features of the proposed algorithm are joint node/link embedding and interference-aware link weight. Our proposed algorithm may leave some rooms for further improvement, but our findings on the key features are expected to provide useful implications to the embedding algorithm research in wireless multi-hop networks. One can extend our algorithm to more practical MACs, e.g., 802.11 DCF, where our key ideas can be utilized except for feasibility check. Although it is not the scope of this paper, feasibility check in 802.11 may be borrowed from many research papers on admission control in 802.11-based multi-hop networks, see e.g., [22], which is left as an interesting future work.

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Table 1: Summary of Major Notations

| Notation | Description |
|----------|-------------|
| $G^S$    | substrate network |
| $N^S, L^S$ | set of substrate nodes and links |
| $\text{CPU}^S(n)$ | CPU resources of SN node $n \in N^S$ |
| $\text{CAP}^S(l)$ | capacity of SN link $l \in L^S$ |
| $d_l$ | number of interfering links with SN link $l \in L^S$ |
| $G^V$ | virtual network |
| $N^V, L^V$ | set of virtual nodes and links |
| $\text{CPU}^V(n)$ | CPU requirement of VN node $n \in N^V$ |
| $\text{BW}^V(l)$ | bandwidth requirement of VN link $l \in L^V$ |

Table 2: Tested algorithms: Alg6 corresponds to our proposed algorithm

| Link weight | Link/node coupling |
|-------------|--------------------|
| none        | none               |
| interference| Alg4               |

| Algorithm | Link/node coupling |
|-----------|--------------------|
| Alg1      | Alg2               |
| Alg3      | Alg4               |
| Alg5      | Alg6 (ours)        |
Figure 1: (a) A VN request (b) An embedding in SN. The SN is either wired or wireless. If it is wireless, the interference is one-hop based, *e.g.*, links (A, C) and (C, D) interfere. The numbers in the box and on the links represent the amount of CPU resources and link bandwidths.

Figure 2: Embedding framework for dynamic VN requests
Figure 3: An example of VN and SN to explain WEM. No existing VNs in service and one-hop interference model are assumed in the SN.

Figure 4: Example of making a candidate embedding

Figure 5: Two candidate embeddings for a given VN request
Figure 6: Tested substrate network topologies: (a), (b), and (c) are random instances with different densities. The grid topology includes 49 (7×7) SN nodes. In (a), (b), and (c), the transmission ranges are uniformly random over [20, 40], [15, 30], and [10, 20], respectively.

Figure 7: Avg. revenue for feasibility check methods

Figure 8: Comparison of tested algorithms for different scenarios
Figure 9: Effect of the number of considering candidates

(a) Avg. revenue

(b) Running time per VN request