Research on the Mathematical Model of Standard Betting Procedures in Portfolio Selection

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Abstract. It is a potential subject research on venture capital. Everyone always hopes to get as more as possible and as far as possible in a short period of time after the gains. However, this article studies it from the standard betting procedures in portfolio selection according the mathematical model studied them. Finally got the more important case, the risk is minimized independently of expected return.

1. Introduction

Of course, everyone would like to know how to pick winning stocks[1-4] but there is no such mathematical theory[5-6], nor is a guaranteed qualitative method of success available to us. Given one risky asset, how much should one then bet on it? According to the famous Gambler’s Ruin[2-3] we should bet the whole amount if winning is essential for survival. If, however, one has a time horizon beyond the immediate present then maybe the amount gambled should be less than the amount required for survival in the long run. Given two or more risky assets, we can ask hard question, which is more precise: can we choose the fractions invested in each in such a way as to minimize the risk, which is defined by the standard deviation of the expected return?

2. Risk and Return

A so-called risk-free asset is one with a fixed interest rate, money market account or a treasury bill. Barring financial disaster, you are certain to get your money back, plus interest. A risky asset is one that fluctuates in price, one where retrieving the capital cannot be guaranteed, especially over the long run. In all that follows we work with returns $x = \ln \left( \frac{p(t)}{p(0)} \right)$ instead of prices $p$.

Averages

$$R = <x> = \ln \left( \frac{p(t)}{p(0)} \right)$$

are understood always to be taken with respect to the empirical distribution unless we specify that we are calculating for a particular model distribution in order to make a point. The empirical distribution is not an equilibrium one because its moments change with time without approaching any constant limit. Finance texts written from the standpoint of neo-classical economics assume “equilibrium,” but statistical equilibrium would require time independence of the empirical distribution, and this is not found in financial markets. In particular, the Gaussian model of returns so beloved of economists is an example of a non-equilibrium distribution.

Consider first a single risky asset with expected return $R_1$ combined with a risk-free asset with known return $R_0$. Let $f$ denote the fraction invested in the risky asset. The fluctuating return of the
portfolio is given by  

\[ x = f x' + (1 - f) R_0 \]

and so the expected return of the portfolio is  

\[ R = f R' + (1 - f) R_0 = R_0 + f \Delta R \]  \hspace{1cm} (2) \]

where \( \Delta R = R' - R_0 \). The portfolio standard deviation, or root mean square fluctuation, is given as  

\[ \sigma = f \sigma_1 \]  \hspace{1cm} (3) \]

where  

\[ \sigma_1 = \left\langle (x - R')^2 \right\rangle^{1/2} \]  \hspace{1cm} (4) \]

is the standard deviation of the risky asset. We can therefore write  

\[ R = R_0 + \frac{\sigma}{\sigma_1} \Delta R \]  \hspace{1cm} (5) \]

which we will generalize later to include many uncorrelated and also correlated assets.

![Figure 1. Return R vs ‘Risk’/Standard Deviation, \( \sigma \) for a Portfolio Made Up of One Risky Asset and One Risk-Free Asset.](image)

In this simplest case the relation between return and risk is linear (Figure 1): the return is linear in the portfolio standard deviation. The greater the expected return the greater the risk. If there is no chance of return then a trader or investor will not place the bet corresponding to buying the risky asset.

Based on the Gambler’s Ruin, we argued in the following that “buy and hold” is a better strategy than trading often. However, one can lose all one’s money in a single throw of the dice (for example, had one held only Enron). We now show that the law of large numbers can be used to reduce risk in a portfolio of \( n \) risky assets. The Strategy of Bold Play and the Strategy of Diversification provide different answers to different questions.

### 3. Diversification and Correlations

Consider next \( n \) uncorrelated assets; the \( x_k \) are all assumed to be distributed statistically independently. The expected return is given by  

\[ R = \sum_{k=1}^{n} f_k R_k \]  \hspace{1cm} (6) \]

and the mean square fluctuation by
\[ \sigma^2 = \left( \sum_{k=1}^{n} f_k x_k - R \right)^2 = \sum f_k^2 \sigma_k^2 \]  

(7)

where \( f_k \) is the fraction of the total budget that is bet on asset \( k \).

As a special case consider a portfolio constructed by dart throwing (a favorite theme in \([5]\)):

\[ f_k = \frac{1}{n} \]  

(8)

Let \( \sigma_1 \) denote the largest of the \( \sigma_k \). Then

\[ \sigma \leq \frac{\sigma_1}{\sqrt{n}} \]  

(9)

This shows how risk could be reduced by diversification with a statistically independent choice of assets. But statistically independent assets are hard to find. For example, automobile and auto supply stocks are correlated within the sector, computer chips and networking stocks are correlated with each other, and there are also correlations across different sectors due to general business and political conditions.

Consider a portfolio of two assets with historically expected return given by

\[ R = f R_1 + (1-f) R_2 = R_2 + f (R_1 - R_2) \]  

(10)

and risk-squared by

\[ \sigma^2 = f^2 \sigma_1^2 + (1-f)^2 \sigma_2^2 + 2f(1-f) \sigma_{12} \]  

(11)

where

\[ \sigma_{12} = \langle (x_1 - R_1)(x_2 - R_2) \rangle \]  

(12)

describes the correlation between the two assets. Eliminating \( f \) via

\[ f = \frac{R - R_2}{R_1 - R_2} \]  

(13)

and solving

Figure 2. The Efficient Portfolio, Showing the Minimum Risk Portfolio as the Left-Most Point on the Curve.
\[ \sigma^2 = \left( \frac{R - R_2}{R_1 - R_2} \right)^2 \sigma_1^2 + \left( 1 - \frac{R - R_2}{R_1 - R_2} \right)^2 \sigma_2^2 + 2 \frac{R - R_2}{R_1 - R_2} \left( 1 - \frac{R - R_2}{R_1 - R_2} \right) \sigma_{12} \]  

(14)

for reward \( R \) as a function of risk \( \sigma \) yields a parabola opening along the \( \sigma \)-axis, which is shown in Figure 2.

4. One Important Case

Now, given any choice for \( f \) we can combine the risky portfolio (as fraction \( w \)) with a risk-free asset to obtain

\[ R_r = (1 - w)R_0 + wR = R_0 + w\Delta R \]  

(15)

With \( \sigma_r = w\sigma \) we therefore have

\[ R_r = R_0 + \frac{\sigma_r \Delta R}{\sigma} \]  

(16)

The fraction \( w = \frac{\sigma_r}{\sigma} \) describes the level of risk that the agent is willing to tolerate. The choice \( w = 0 \) corresponds to no risk at all \( R_r = R_0 \), and \( R_r = R_1 \), and \( w = 1 \) corresponds to maximum risk, \( R_r = R_1 \).

Next, let us return to equations (14)-(16). There is a minimum risk portfolio that we can locate by using (14) and solving

\[ \frac{d\sigma^2}{dR} = 0 \]  

(17)

Instead, because \( R \) is proportional to \( f \), we can solve

\[ \frac{d\sigma^2}{df} = 0 \]  

(18)

to obtain

\[ f = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \]  

(19)

Here, as a simple example to prepare the reader for the more important case, risk is minimized independently of expected return.

5. Conclusions

This is the beginning of the analysis of the question of risk vs reward via diversification. We are forewarned that this article is written on the assumption that the future will be statistically like the past, that the historic statistical price distributions of financial markets are adequate to predict future expectations like option prices. This assumption will break down during a liquidity crunch, and also after the occurrence of surprises that change market psychology permanently.
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