Tracking of Maneuvering Star-Convex Extended Target Using Modified Adaptive Extended Kalman Filter

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ABSTRACT

Tracking extended targets aims to estimate the kinematic state and shape of the target of interest with a varying number of noisy detections received by a sensor. The key challenge in this problem stems from its nonlinearity and high dimensionality due to the target maneuver and model complexities. This paper presents a modified adaptive extended Kalman filter based on the random hypersurface model (RHM) to address this problem. First, the target maneuver is judged by using the input estimate (IE) chi-square detector. Then, the magnitude of the target maneuver is used to modify the prior of the shape parameters. Based on the prior information, we derive an extended Kalman filter for a closed-form recursive measurement update. The simulation and experimental results demonstrate the usefulness of the proposed method for tracking the maneuvering star-convex extended target.

INDEX TERMS

Extended target, random hypersurface model, maneuver target, nonlinear filters, input estimate chi-square detector.

I. INTRODUCTION

The purpose of extended target tracking is to simultaneously estimate the kinematic state and shape of the extended target from a sequence of noisy sensor measurements. With the development of high-resolution sensors, extended target tracking technology is becoming increasingly important for critical military and civilian applications, such as autonomous driving [1], motion and scene analysis [2], and maritime surveillance [3], [4]. In contrast to point target tracking, the high-resolution sensors for extended target tracking such as X-band radar [5] provide a strongly fluctuating number of spatially distributed measurements per scan from the surface or the boundary of the extended target. The assumption that the target can be treated as a point source for which the physical characteristics (e.g., size, shape, and orientation) can be ignored is no longer valid in this case. Thus, new methods have recently emerged to address the problem of extended target tracking.

Various shape models for representing the contour of an extended target have been investigated. For a rectangular or elliptical extended target, the two shape parameters encode the lengths of the major and minor axis or the length and width of the target, respectively [6]. In [7] and [8], the random matrix approach models the ellipsoid with a Gaussian distribution for which the covariance matrix is represented by the inverse Wishart density. Although these models have been proven to be effective in real scenes, the shapes of other extended targets such as relatively large ships or aircraft are different from convex shapes. In [9] and [10], the random hypersurface model (RHM) was used to construct a parametric representation of the shape contour, which can be used to model a basic elliptical shape as well as a star-convex shape. Similarly, the shape contour describing the extent was modeled with a Gaussian process (GP) in [11]. Other advanced contour description methods for arbitrary shapes have also been proposed. These approaches include mixture RHM [12], level-set RHM [13], and multiple ellipsoidal subobjects [14], [15].
For the abovementioned models, the shape parameters are estimated using Gaussian estimators such as the extended Kalman filter for ellipses based on RHM (RHM-EKF) [16], extended Kalman filter for star-convex shapes based on GP (GP-EKF) [11], and unscented Kalman filter for star-convex shapes based on RHM (RHM-UKF) [17]. In many practical cases, the maneuvers of the target and model complexities give rise to a high-dimensional problem with strongly nonlinear behavior. This issue dramatically degrades the performance of the traditional Gaussian estimators, and the estimations no longer track the target trajectory. This problem has motivated the use of Monte Carlo methods. In [18], a Rao-Blackwellized particle filter is designed to exploit the conditional linear Gaussian structure of the GP parameters. In addition, Aftab et al. [19] proposed a Gaussian process convolution particle filter that does not depend on any prior knowledge of the measurement statistics and an analytical expression of the likelihood function. Freitas et al. [20] designed the box particle filter, which replaces traditional multiple measurements with a rectangular region of the nonzero volume in the state space to deal with extended targets; however, the computational complexity of these filters increases rapidly with increasing state dimension. In a recent study [21], Yang used multiplicative noise to model the spatial extent of an extended target and designed a second-order extended Kalman filter for the estimation of an elliptical shape. Generally, a target maneuver leads to a change in the target orientation, which will inevitably degrade the tracking performance of these estimators. Further, there is no a priori knowledge on the extent of target. To deal with this problem, an adaptive tracking algorithm for the arbitrary shape that extracts the extended target contour by the current measurements as the shape prior is proposed in [22]. In [23], Sun constructed the coupled dynamics model of both the kinematic state and shape based on the Minkowski sum and adopted the UKF to jointly estimate the centroid kinematic state and target extension.

Inspired by [21], we propose a modified adaptive EKF based on a random hypersurface model (RHM-AEKF) for tracking a maneuvering star-convex extended target. The main contributions of this work are threefold: First, we introduce the input estimate chi-square detector to judge whether the target is maneuvering and adopt the modified prior to update the shape parameters of a moving target, rather than use the original shape parameters. Second, we design an adaptive EKF based on the random hypersurface model to solve the high-dimensional nonlinear estimation problem. Third, we build an experimental platform for tracking a star-convex-shaped ground moving mobile target to evaluate the proposed algorithm.

The rest of the paper is structured as follows. Section II introduces the basic measurement and dynamic models that are used to track the star-convex extended target. In Section III, the input estimate chi-square detector for judging the target maneuver is first presented. Then, we modify the shape prior and derive the compact closed-form expressions for a recursive measurement update. Section IV describes the simulation setup in detail and presents the obtained results. Finally, the conclusions and future work are described in Section V.

Notation: The notation used throughout the paper is standard. \( \mathbb{R}^n \) denotes the n-dimensional Euclidean space. For a matrix \( A, A^T \) and \( A^{-1} \) represent its transpose and inverse, respectively. \( \mathbb{E}[x] \) denotes the expectation of random variable \( x \). \( I \) and \( O \) represent the identity matrix and the zero matrix with appropriate dimensions, respectively. \( \text{diag}(A_1, A_2, \ldots, A_n) \) represents a block-diagonal matrix with matrices \( A_1, A_2, \ldots, A_n \) on the diagonal.

II. PROBLEM FORMULATION

This section introduces the state value and shape parameterization, measurement model, and dynamic model for the star-convex extended target.

A. STATE VALUE AND SHAPE PARAMETERIZATION

The kinematic state of the extended target at time \( k \) is given by

\[
x_k = [m_k^T, \dot{m}_k^T, \ldots]^T
\]

and is specified by both kinematic parameters, i.e., center \( m_k \in \mathbb{R}^2 \), velocity \( \dot{m}_k \) and possibly additional variables. The star-shaped extended target \( S(p_k) \) can be described in parametric form \( r(p_k, \phi) \) with the RHM.

\[
S(p_k) = \{r(p_k, \phi) \cdot e(\phi) \cdot \rho | \phi \in [0, 2\pi]\}
\]

where \( p_k \) denotes the shape parameter vector, \( e(\phi) = [\cos(\phi), \sin(\phi)]^T \) is a unit vector with angle \( \phi \), and \( \rho \in (0, 1) \) is a scaling factor that specifies the relative distance of the current center from the measurement source.

Assuming \( r(p_k, \phi) \) to be periodic in \( \phi \) with the period \( [0, 2\pi] \), the Fourier series of degree \( n_f \) becomes

\[
r(p_k, \phi) \approx \frac{1}{2} a_k^{(0)} + \sum_{j=1, \ldots, n_f} \left( a_k^{(j)} \cos(j\phi) + b_k^{(j)} \sin(j\phi) \right)
\]

\[
= \mathbf{R}(\phi) \cdot \mathbf{p}_k
\]

\[
\mathbf{p}_k = [a_k^{(0)}, a_k^{(1)}, b_k^{(1)}, \ldots, a_k^{(n_f)}, b_k^{(n_f)}]^T
\]

where \( \mathbf{r}(p_k, \phi) \) represents a radius function, which gives the distance from the target center to a contour point depending on the angle \( \phi \) and a parameter vector \( \mathbf{p}_k \). Fourier coefficients with small indices encode the information about the coarse features of the shape, and Fourier coefficients with larger indices encode the finer details [9]. Here, the Fourier series of degree is 11.

B. MEASUREMENT MODEL

At each time step \( k \), the extended target generates a fluctuating number of two-dimensional Cartesian measurements \( \mathbf{Z}_k = [\mathbf{z}_{k,i}]^T_{i=1} \). Each individual measurement \( \mathbf{z}_{k,i} \) originates from a measurement source \( \mathbf{y}_{k,i} \) lying on the target extent, which is corrupted by an additive Gaussian white noise \( \mathbf{v}_{k,i} \).
with covariance matrix $C^p_k$. Here, each measurement source $y_{k,i}$ follows a uniform spatial distribution. Any measurement $z_{k,i}$ lying on the target extent can be represented as

$$z_{k,i} = y_{k,i} + v_k = R(\hat{\phi}_{k,i}) \cdot p_k \cdot e(\hat{\phi}_{k,i}) \cdot \rho_{k,i} + v_{k,i} \quad (5)$$

**Remark 1:** The measurement sources are uniformly distributed over the surface, and the scaling factor $\rho_{k,i}$ is also uniformly distributed. As we will estimate the kinematic state and extent by using the Kalman filter, we assume that the scaling factor $\rho_{k,i}$ is a Gaussian distribution with mean $\bar{\rho}$ and covariance matrix $C^\rho_k$. In [12], it can be approximated by a Gaussian distribution with mean 0.8 and covariance 1/12.

**Remark 2:** The state estimate is calculated by the most likely angle $\phi_{k,i}$. To avoid the treatment of an uncertain angle, we assume that the point estimate $\hat{\phi}_{k,i}$ is obtained by calculating the angle between the vector from current kinematic state estimate $\hat{x}_k$ to the measurement $z_{k,i}$ and the $x$-axis, $\hat{\phi}_{k,i} = \angle(z_{k,i} - H_i \hat{x}_k)$.

For a star-convex extended target with shape parameters $p_k$ and center $m_k$ at time $k$, the Cartesian position measurements are obtained using the following measurement equation:

$$z_{k,i} = m_k + R(\hat{\phi}_{k,i}) \cdot p_k \cdot e(\hat{\phi}_{k,i}) \cdot \rho_{k,i} + v_{k,i} = U_{k,i} \quad (6)$$

where $U_{k,i}$ denotes the Cartesian position of the $i$-th measurement source that lies on the boundary of the shape. Note that $R(\hat{\phi}_{k,i}) \cdot p_k$ is a polar representation of the contour of the extended target. These polar points are transformed to Cartesian coordinates by the standard coordinate conversion.

In a compact form, Eq. (6) can be rewritten as

$$z_{k,i} = H_k x_k + U_{k,i} \cdot \rho_{k,i} + v_{k,i} \quad (7)$$

where $H_k = [I \ O]$. An illustration of the measurement model is shown in Fig. 1.

**C. DYNAMIC MODEL**

The kinematic state and shape parameters of the extended target are assumed to follow a Markov model:

$$x_{k+1} = F_k x_k + G_k u_k + w_k$$

where $F_k$ denotes the state transition matrix, $g(\cdot)$ is a nonlinear function, $u_k$ is an unknown input modeling the target maneuvers, $G_k$ is the input gain, and $w_k$ and $w_k^p$ are the zero-mean Gaussian process noise with covariance matrices $C^w_k$ and $C^{w_p}_k$, respectively.

**Remark 3:** As described in the introduction, the kinematic state of the target changes with time. Usually, the orientation of the target is unknown and should be considered. In this letter, we assume that the extended target is aligned along its velocity vector.

**III. MODIFIED ADAPTIVE EXTENDED KALMAN FILTER**

The target extent cannot be obtained with a linear estimator that uses the original measurement; that is, the shape parameters do not change when updated with a single measurement $z_{k,i}$ in the Kalman filter. To address this problem, we construct a pseudomeasurement from the original measurement and combine the linearization and analytic moment calculation techniques to obtain the pseudomeasurement covariance. Because the orientation of the target changes with the kinematic state, we modify the shape parameters based on the maneuver onset time that is estimated by using the IE detector. A block diagram of the proposed RHM-AEKF algorithm is depicted in Fig. 2.

**A. MANEUVER DETECTION**

Because of the variation in the maneuvering characteristics of the extended target, it is necessary to determine the maneuver onset time. Here, the purpose of maneuver onset detection is to determine the maneuver and estimate the onset time $k$; this can be formulated as a binary hypothesis testing problem. $H_0$: the target is not maneuvering; $H_1$: the target is maneuvering.
at the onset time \( k \). We employ the IE detector to make a decision based on the chi-square test of the unknown input \( \hat{u} \). The normalized input residual squared \( \varepsilon_k^u \) can be written as

\[
\varepsilon_k^u = \hat{u}_k^T (\Sigma_k^u)^{-1} \hat{u}_k
\]  

(9)

where \( \varepsilon_k^u \) is \( \chi^2 \) distribution with \( n = \dim(\hat{u}_k) \) for any \( n \)-dimensional Gaussian random vector \( \hat{u}_k \sim \mathcal{N}(0, \Sigma_k) \). The chi-square test provides a justification of the goodness of fit for the determination of the maneuver onset detection. Assume the input \( \hat{u}_k \) is constant in the interval \([k-s, \ldots, k] \), i.e., \( u_i = \hat{u}_k, i = k-s, \ldots, k-1 \). It can be shown that \( \hat{u}_k \) follows from the linear model

\[
\xi = \Psi u + \nu
\]  

(10)

where \( \xi = [\xi_k^{T-1}, \ldots, \xi_{k-s+1}^T, \Psi = [\Psi_k^{T-1}, \ldots, \Psi_{k-s+1}^T]^T \), \( \Psi_i = H \sum_{j=k-s}^{i} \prod_{m=0}^{i-j-1} \Phi_{i-m} \) \( G \), \( \Phi_i = F [I - K, H] \). Here, \( F, G \), and \( H \) represent the state transition matrix, input gain, and measurement matrix, respectively. For simplicity, we omit the time index. Then, the maneuver input is calculated by using least-squares batch estimation.

\[
\begin{align*}
\dot{u} &= (\Psi^T S^{-1} \Psi)^{-1} \Psi^T S^{-1} \xi \\
\Sigma &= \text{cov}(\dot{u}) = (\Psi^T S^{-1} \Psi)^{-1} \\
S &= \text{diag}(S_{k-s+1}, \ldots, S_i), S_i = \text{cov}(\xi_i)
\end{align*}
\]  

(11)

The chi-square test-based maneuver detector decides \( H1 \) if

\[
\varepsilon_k^u > \lambda = \chi^2_n(\alpha)
\]  

(12)

where \( \alpha \) is the false alarm rate.

Remark 4: Our method takes into account the maneuver onset time and then modifies the shape prior depending on the maneuver onset time. Additionally, a time delay exists in the IE detector. Once the maneuver is detected at time step \( k-s \), we modify the shape prior information and estimate the duration of the target at time \( k-s \).

B. PREDICTION STAGE

According to the standard Kalman filter equations, the estimation of the kinematic state and covariance can be calculated as follows:

\[
\begin{align*}
x_k &= F_{k-1} x_{k-1} \\
C_k^x &= F_{k-1} C_{k-1}^x F_{k-1}^T + Q_{k-1}
\end{align*}
\]  

(13)

(14)

Because the temporal evolution of the shape parameters follows a nonlinear model, the Fourier coefficients change with the target rotation. If the updated shape parameters are used as the prior information, then the performance of the estimator will be degraded. Hence, we modify the shape prior based on the magnitude of the target maneuver. First, the target contour is divided into \( N \) subareas according to the angle \( \theta = 2\pi / N \). The finite contour sequence \( f_k \) is reconstructed by applying the centroid contour distance method [24] for the shape parameters \( p_k, f_k = [z_{k,i}]_{i=1}^N \). The original contour is rotated to reformulate the new contour sequence \( f_{\hat{z}} = [\hat{z}_{k,j}]_{j=1}^N \), where

\[
\hat{z}_{k,j} = z_{k,j} \begin{bmatrix} \cos \theta_k & -\sin \theta_k \\ \sin \theta_k & \cos \theta_k \end{bmatrix}
\]  

(15)

where \( \theta_k \) denotes as the angle between two velocity vectors \( \dot{m}_k \) and \( \dot{m}_{k-1} \). The modified shape parameters can be obtained by processing the rotated contour sequence with the discrete Fourier transform.

\[
\hat{p}_k = \frac{2}{N} \sum_{j=0}^{N-1} d_z \cdot e^{-j(\frac{2\pi}{N})jN}
\]  

(16)

where \( d_z = \| \hat{z}_{k-1,j} - m_{k-1} \| \) represents the Euclidean distance from the rotated contour point to the kinematic center of the extended target.

C. UPDATE STAGE

Once the measurement set \( Z_k \) is obtained, the posterior means and covariances of the kinematic state and shape parameters are calculated. Let

\[
\hat{x}_k^{(i)}, \hat{p}_k^{(i)} \quad \text{and} \quad C_k^{x(i)}, C_k^{p(i)}
\]  

denote the updated estimates for the kinematic state \( \hat{x}_k \) and shape parameters \( \hat{f}_k \) that incorporate all measurements up to the \( i \)-th measurement \( Z_k^{(i)} \) at time \( k \) based on the previous estimates

\[
\hat{x}_k^{(i-1)}, \hat{p}_k^{(i-1)} \quad \text{and} \quad C_k^{x(i-1)}, C_k^{p(i-1)}
\]  

where the notation \( (\hat{\circ})_k^{(0)} \) represents the prediction for time \( k \), having not yet incorporated a measurement for time \( k \).

Note that the kinematic state and shape of the extended target are independent.

1) KINEMATIC STATE UPDATE

By using the Kalman filtering algorithm [27], we obtain the expected value of Eq. (7), measurement covariance \( C_k^{zz(i)} \), and the cross-covariance \( C_k^{xz(i)} \) [16].

\[
\mathbb{E} (z_{k,i}) = H_k \hat{x}_k^{(i-1)}
\]  

(17)

\[
C_k^{zz(i)} = \mathbb{E} [(z_{k,i} - \mathbb{E} (z_{k,i}))(z_{k,i} - \mathbb{E} (z_{k,i}))^T]
\]

\[
= H_k C_k^{x(i-1)} H_k^T + U_k C_k^{q(i)} U_k^T + C_k^v
\]  

(18)

\[
C_k^{xz(i)} = \mathbb{E} ([x_k - \hat{x}_k^{(i-1)}](z_{k,i} - \mathbb{E} (z_{k,i}))^T)
\]

\[
= C_k^{x(i-1)} H_k^T
\]  

(19)

The standard Kalman filter measurement update results in the following equations:

\[
\hat{x}_k^{(i)} = \hat{x}_k^{(i-1)} + C_k^{xz(i)} (C_k^{zz(i)})^{-1} (z_{k,i} - \mathbb{E} (z_{k,i}))
\]  

(20)

\[
C_k^{x(i)} = C_k^{x(i-1)} - C_k^{xz(i)} (C_k^{zz(i)})^{-1} C_k^{xz(i)}^T
\]  

(21)
2) SHAPE PARAMETERS UPDATE

Since as shown in [25], the shape parameters cannot be obtained with a linear measurement update stage with the original measurement, we build a pseudomeasurement by using the 2-fold Kronecker product of the original measurement, and the expectation and covariance of the pseudomeasurement can be approximated by the second and fourth central moments of the original measurement. The pseudomeasurement is defined as follows:

\[
\tilde{z}_{k,i} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \{ (z_{k,i} - H_i \hat{x}_{k,i}^{(0)}) \otimes (z_{k,i} - H_i \hat{x}_{k,i}^{(0)}) \}
\]

\[
= \begin{bmatrix}
(z_{k,i,1} - H_{k,1}\hat{x}_{k,1}^{(0)})^2 \\
(z_{k,i,2} - H_{k,2}\hat{x}_{k,2}^{(0)})^2 \\
(z_{k,i,1} - H_{k,1}\hat{x}_{k,1}^{(0)})(z_{k,i,2} - H_{k,2}\hat{x}_{k,2}^{(0)})
\end{bmatrix}
\]

(22)

Note that Eq. (22) is an uncorrelated transformation [26], which means that the pseudomeasurement is uncorrelated with the original measurement.

Similar to the kinematic state update, the shape parameters are updated with the pseudomeasurement \( \tilde{z}_{k,i} \) by using the Kalman filter update formulas

\[
\hat{p}_k^{(i)} = \hat{p}_k^{(i-1)} + C_k^{\tilde{z}(i)}(C_k^{\tilde{z}(i)-1}(\tilde{z}_{k,i} - E[\tilde{z}_{k,i}])
\]

\[
C_k^{\tilde{z}(i)} = C_k^{\tilde{z}(i)-1} - C_k^{\tilde{z}(i)}(C_k^{\tilde{z}(i)-1})^T
\]

(23)

(24)

where \( C_k^{\tilde{z}(i)} \) denotes the covariance of the pseudomeasurement, and \( C_k^{\tilde{z}(i)-1} \) is the cross-covariance between the shape parameters \( p_k^{(i)} \) and pseudomeasurement \( \tilde{z}_{k,i} \). Given the covariance matrix of the original measurement \( C_k^{\tilde{z}(i)} \),

\[
C_k^{\tilde{z}(i)} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} \\
\sigma_{12} & \sigma_{22}
\end{bmatrix}
\]

(25)

where \( \sigma_{11} \) represents the covariance of \( z_{k,1,m} \) and \( z_{k,1,m} \). Based on the moment match method [28] and the Isserlis method [29], the expectation and covariance of the pseudomeasurement can be calculated as follows:

\[
E[\tilde{z}_{k,i}] = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{22}
\end{bmatrix}^T
\]

(26)

\[
C_k^{\tilde{z}(i)} = \begin{bmatrix}
2\sigma_{11}^2 & \sigma_{11}\sigma_{22} & 2\sigma_{12}\sigma_{22} & 3\sigma_{11}\sigma_{12} & 3\sigma_{11}\sigma_{12} & 3\sigma_{12}\sigma_{22} & 2\sigma_{12}\sigma_{22}
\end{bmatrix}
\]

(27)

The derivation of Eq. (27) is shown in Appendix A. The cross-covariance is approximated by using a Taylor series expansion to linearize the pseudomeasurement:

\[
\tilde{z}_{k,i} \approx g(\hat{x}_{k,i}^{(i-1)}, \hat{p}_k^{(i-1)}, \rho_k, v_k, k_i)
\]

\[
+ J_k(\hat{p}_k, v_k, k_i)(x_k - \hat{x}_{k,i}^{(i-1)}) + J_p(\hat{p}_k, v_k, k_i)\hat{p}_k - P_k^{(i-1)}
\]

(28)

where \( J_k(\rho_k, v_k, k_i) \) and \( J_p(\hat{p}_k, v_k, k_i) \) are the Jacobians of Eq. (22) evaluated at \( \hat{x}_{k,i}^{(i-1)} \) and \( P_k^{(i-1)} \) with respect to \( x_k \) and \( \hat{p}_k \). According to the assumption that the extension and kinematic state are independent, the expectation of \( J_k(\rho_k, v_k, k_i) \) is zero. Hence, the cross-covariance can be calculated as follows:

\[
C_k^{\tilde{z}(i)} = C_k^{p(i-1)}(M_k^{(i-1)})^T
\]

(29)

with

\[
M_k = \begin{bmatrix}
2S_1\hat{b}^2J_{S1} + 2S_1C_k^{p}\hat{J}_{S1} \\
2S_2\hat{b}^2J_{S2} + 2S_2C_k^{p}\hat{J}_{S2} \\
S_1\hat{b}^2J_{S1} + S_1C_k^{p}\hat{J}_{S2} + S_2\hat{b}^2J_{S1} + S_2C_k^{p}\hat{J}_{S1}
\end{bmatrix}
\]

The derivation of Eq. (29) is shown in Appendix B.

IV. EXPERIMENTAL RESULTS

In this section, we evaluate our method for tracking a star-convex extended target in numerical simulation and test environment based on RGB data using the modified Hausdorff distance. In the experiment, we compare the proposed algorithm with the random matrix method [7], RHM-EKF (elliptical model) [16], GP-EKF [11], and RHM-UKF (star-convex model) [17].

Shape estimation evaluation of an extended target is considered a problem of matching the estimated shape and the true shape. Therefore, the modified Hausdorff distance can be adopted to measure the degree of their resemblance [30]. The modified Hausdorff distance is written as

\[
d_H(S_A(p_k), S_A(\hat{p}_k)) = \max\{d(S_A(p_k), S_A(\hat{p}_k)), d(S_A(p_k), S_A(\hat{p}_k))\}
\]

(30)

And

\[
d(S_A(p_k), S_A(\hat{p}_k)) = \max_{a \in S_A(p_k)} \{ d_E(a, S_A(\hat{p}_k)) \}
\]

\[
d(S_A(\hat{p}_k), S_A(p_k)) = \max_{a \in S_A(p_k)} \{ d_E(\hat{a}, S_A(p_k)) \}
\]

where \( d_E(\hat{a}, S_A(p_k)) \) denotes the Euclidean distance between \( a \) and \( \hat{a} \) in \( S_A(p_k) \). \( S_A(\hat{p}_k) \) and \( S_A(p_k) \) represent the discrete estimated shape set and true set via uniform angle sampling with \( \{\frac{2\pi}{N_s}, i = 1, 2, \ldots, N_s\} \); \( N_s \) is the number of samples. Clearly, the shorter the modified Hausdorff distance is, the closer the estimated shape is to the true shape.

A. EXPERIMENT 1: NUMERICAL SIMULATION

In this section, we consider a simple 2D extended target tracking scenario over the surveillance area \([0, 100 m] \times [0 m, 200 m]\) for a period of \( T = 30 s \) time steps. The sampling interval is \( k = 1 \ s \). The target starts from \( x_0 = \{0 m, 10 m/s, 0 m, 0 m/s\} \) and no process noise. It performs a constant velocity motion except maneuvering with the turn rate \( \omega_1 = 0.314 \ rad/s \) and \( \omega_2 = -0.7854 \ rad/s \) for the time intervals [6, 10] and [21, 25], respectively. At each time step, the measurements are generated from a uniform distribution on the target extent, and the number of measurements follows a Poisson distribution with mean \( \lambda_p = 30 \). The measurement noise covariance is assumed as \( \text{diag}(0.1 \ m, 0.1 \ m) \), and the false alarm rate is 0.02. For both the RHM-EKF and random matrix method, the standard deviation of the
orientation noise is 0.02 rad, and the semiaxis variances are diag((0.05 m^2, 0.05 m^2)).

The trajectory and one sample run of these five estimators are depicted in Fig. 3. It is observed that the proposed method gives a more precise shape estimate than the RHM-UKF, GP-EKF, RHM-EKF, and random matrix method. This result indicates that the proposed approach is effective.

The average position RMSEs over 100 Monte Carlo trials are plotted in Fig. 4 and indicate that the performance of the RHM-AEKF is better than that of the GP-EKF. This is because GP-EKF is sensitive to the errors due to the target dynamic model mismatch during the maneuver. As observed from the average velocity RMSEs in Fig. 5, the RHM-UKF algorithm performs slightly better than RHM-AEKF. This outcome is unsurprising because UKF directly uses the unscented transformation to approximate the probability density of the state distribution, overcoming the linearization error involved in the EKF.

The modified Hausdorff distances for these five algorithms are shown in Fig. 6. It is observed that RHM-AEKF displays better performance than the other four algorithms, especially during and after the maneuver (for k = 21 through 25). The obvious reason for this result is the consideration of both the onset time and magnitude of the target maneuver that are estimated in the proposed RHM-AEKF. Moreover, the onset time and magnitude have been used to calculate the modified shape parameters in order to eliminate the impact of the shape prior. Not surprisingly, the random matrix method performs worse when the target maneuvers, which is because the random matrix method assumes an elliptical shape for the target’s extent and cannot consider the effect of the extension change caused by the changes in the orientation or size.

The results presented in Fig. 7 show the modified Hausdorff distances with different numbers of measurements. It is clear that the modified Hausdorff distance of the RHM-AEKF is smaller than those obtained by the other four algorithms. That is, as the number of the measurements increases, the modified Hausdorff distances decreases. This is because a lower number of measurements cannot provide enough information for shape estimation when using a smaller level n_k = 5.

**B. EXPERIMENT 2: RGB DATA**

In this section, we build a real-time target tracking system to evaluate the performance of the proposed algorithm. The work considers a single moving target on the ground with a monocular camera observing the scene from a bird’s-eye view. The surveillance area is on the ground with a toy train...
that travels on tracks. The spatial extent of the extended target is simulated by an LED array light source. The following signal processing algorithm is used to detect moving points. First, the original image is grayed to remove the background; in the second step, moving points are detected using the feature point extraction methods called the scale-invariant feature transform (SIFT) algorithm.

The original image supplied by the monocular camera and the extracted measurements are shown in Fig. 8. The shape estimates provided by the five algorithms are depicted in Fig. 9. As seen in Fig. 9(a) and (b), RHM-AEKF provides a smooth and precise estimate of the target extent, while RHM-UKF suffers from an imprecise estimate (due to the target maneuver). The main reason is that the uncertainty of the shape prior caused by the target maneuver leads to an inaccurate estimation of the target extent. In addition, the kinematic state is independent of the extension, so the estimation error of the kinematic state does not affect the error of the extension. The modified Hausdorff distances for the five algorithms are given in Fig. 10. It is evident that RHM-AEKF outperforms the other algorithms.

V. CONCLUSION AND FUTURE WORKS

In this paper, we have proposed a robust filter to address the problem of the kinematic state and shape estimation of a maneuvering star-convex extended target. The IE detector is used to calculate the maneuver onset time, and the shape prior parameters are modified by the magnitude of the target maneuver. In the update stage, we constructed the pseudomeasurement and adopted linearization and analytic moment calculation techniques to obtain the pseudomeasurement covariance. The simulation and experimental results verified that the proposed method outperforms the RHM-EKF, RHM-UKF, GP-EKF, and random matrix methods. In the future, we will investigate two-dimensional and three-dimensional target tracking for an arbitrary shape. Furthermore, multiple extended target tracking will be considered.

APPENDIX A
DERIVATION OF PSEUDOMEASUREMENT COVARIATION

Here, the covariance of the pseudomeasurement can be calculated by using the Isserlis theorem [29]. For Eq. (22), we omit the time index. The pseudomeasurement can be rewritten as
Based on the Isserlis theorem, \[ \mathbb{E}[\tilde{z}_1^2] = \mathbb{E}[(z_1 - \tilde{H}_1 x_1)^2] = 3 \mathbb{E}[(z_1 - \tilde{H}_1 x_1)^2] = 3 \bar{\rho}_1^2 \]
\[ \mathbb{E}[\tilde{z}_2^2] = \mathbb{E}[(z_2 - \tilde{H}_2 x_2)^2] = 3 \mathbb{E}[(z_2 - \tilde{H}_2 x_2)^2] = 3 \bar{\rho}_2^2 \]
\[ \mathbb{E}[\tilde{z}_3^2] = \mathbb{E}[\tilde{z}_1 \tilde{z}_2] = \mathbb{E}[\tilde{z}_1 \tilde{z}_2] = 2 \mathbb{E}[(z_1 - \tilde{H}_1 x_1)(z_2 - \tilde{H}_2 x_2)]^2 = \sigma_{12}^2 + 2 \sigma_{12} \]
\[ \mathbb{E}[\tilde{z}_1 \tilde{z}_3] = 3 \mathbb{E}[(z_1 - \tilde{H}_1 x_1)^2] \mathbb{E}[(z_1 - \tilde{H}_1 x_1)(z_2 - \tilde{H}_2 x_2)] = 3 \sigma_{12} \bar{\rho}_1 \]
\[ \mathbb{E}[\tilde{z}_2 \tilde{z}_3] = 3 \mathbb{E}[(z_1 - \tilde{H}_2 x_2)^2] \mathbb{E}[(z_1 - \tilde{H}_1 x_1)(z_2 - \tilde{H}_2 x_2)] = 3 \sigma_{22} \bar{\rho}_1 \]

**APPENDIX B**

**DERIVATION OF PSEUDEMEASUREMENT EQUATION**

Here, we omit the time index, letting \( S_1 \) and \( S_1 \) denote the first and second row of the shape matrix \( \Sigma_{X1} \) and \( \Sigma_{X2} \) denote the first and second row of the measurement matrix \( \Theta \); then, the pseudomeasurement equation in Eq. (22) can be rewritten as

\[
\psi(x, \rho) = \begin{bmatrix}
(H_1 x_1 + S_1 \rho - \tilde{v}_1 - \tilde{z}_1)^2 \\
(H_2 x_2 + S_2 \rho - \tilde{v}_2 - \tilde{z}_2)^2 \\
(H_1 x_1 + S_1 \rho - \tilde{v}_1 - \tilde{z}_1)(H_2 x_2 + S_2 \rho - \tilde{v}_2 - \tilde{z}_2)
\end{bmatrix}
\]

(32)

The cross-covariance of the pseudomeasurement is approximated by

\[
C_{\psi \rho} = \mathbb{E} \left[ \frac{\partial \psi}{\partial \rho} \frac{\partial \psi}{\partial \rho} \right]
\]

Here, the Jacobi matrix \( \frac{\partial \psi}{\partial \rho} \) is given by (34), as shown at the top of the page.

For the first row of Eq. (34), we have

\[
\frac{\partial \psi}{\partial \rho_1} = 2H_1 x_1 \rho^T J_{S_1} + 2S_1 \rho J_{S_1} - 2v_1 \rho^T J_{S_1} - 2\tilde{z}_1 \rho^T J_{S_1}
\]

(35)

The scaling factor \( \rho \) obeys the Gaussian distribution with mean \( \bar{\rho} \) and covariance \( C_{\rho}^{\rho} \). We can take the expectation of Eq. (35).

\[
\mathbb{E} \left[ \frac{\partial \psi}{\partial \rho_1} \right]
\]

follows:

\[
\tilde{z} = \begin{bmatrix}
\tilde{z}_1 \\
\tilde{z}_2 \\
\tilde{z}_3
\end{bmatrix} = \begin{bmatrix}
(z_1 - \tilde{H}_1 x_1)^2 \\
(z_2 - \tilde{H}_2 x_2)^2 \\
(z_1 - \tilde{H}_1 x_1)(z_2 - \tilde{H}_2 x_2)
\end{bmatrix}
\]

(31)

A similar process is used to process the second and third rows. Finally, we have

\[
M_k = \begin{bmatrix}
2S_1 \rho^2 J_{S_1} + 2S_1 C_{\rho}^{\rho} J_{S_1} \\
2S_2 \rho^2 J_{S_2} + 2S_2 C_{\rho}^{\rho} J_{S_2} \\
S_1 \rho^2 J_{S_1} + S_1 C_{\rho}^{\rho} J_{S_1} + S_2 \rho^2 J_{S_2} + S_2 C_{\rho}^{\rho} J_{S_2}
\end{bmatrix}
\]

(37)

According to the above results, the \( mn \)-th term of the covariance of pseudomeasurement can be obtained as

\[
\text{cov} \{ \tilde{z}_m, \tilde{z}_n \} = \mathbb{E} \{ \tilde{z}_m \tilde{z}_n \} - \mathbb{E} \{ \tilde{z}_m \} \mathbb{E} \{ \tilde{z}_n \}
\]

Hence, we can obtain the covariance of the pseudomeasurement as given by Eq. (27).

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