Remeasuring the anomalously enhanced $B(E2; 2^+ \rightarrow 1^+)$ in $^{8}$Li and comparison to ab initio predictions

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The large reported $E2$ strength between the $2^+$ ground state and $1^+$ first excited state of $^{8}$Li, $B(E2; 2^+ \rightarrow 1^+) = 55(15)\,e^2\text{fm}^4$, presents a puzzle. Unlike in neighboring $A = 7$–9 isotopes, where enhanced $E2$ strengths may be understood to arise from deformation as rotational in-band transitions, the $2^+ \rightarrow 1^+$ transition in $^{8}$Li cannot be understood in any simple way as a rotational in-band transition. Moreover, the reported strength exceeds ab initio predictions by an order of magnitude. In light of this discrepancy, we revisited the Coulomb excitation measurement of this strength, now using particle-γ coincidences, yielding a revised $B(E2; 2^+ \rightarrow 1^+) = 25(8)(3)\,e^2\text{fm}^4$. We explore how this value compares to what might be expected in rotational, Elliott SU(3), and ab initio descriptions, including no-core shell model (NCSM) calculations with various internucleon interactions. While the present value is a factor of 2 smaller than previously reported, it remains anomalously enhanced.

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I. INTRODUCTION

Large $E2$ transition strengths found in the $A = 7–9$ mass region [1, 2] suggest these nuclei have a significant deformation, which gives rise to rotational structure as a dominant feature in the low-lying spectrum [3, 4]. The strong transitions between the ground state and first excited state in $^7$Li, $^7$Be, and $^9$Be, or between excited states in $^8$Be [5], are interpreted as in-band rotational transitions. This deformation, in turn, is understood to arise from cluster molecular structure [6–10], e.g., with $^7$Be as a $^3$He $+ ^4$He dimer, and the heavier Be isotopes as $^4$He $+ ^4$He plus neutrons.

In general, $E2$ strengths provide a probe of nuclear structure and its evolution [11]. For light $p$-shell nuclei, which are accessible to ab initio nuclear theory by a variety of approaches, the ab initio calculations can provide qualitative insight into the structural origin of the $E2$ strengths [12–15]. Meanwhile, experimental measurements can provide quantitative validation of the ability of the calculations to faithfully describe the nuclear system [16].

Taken in this light, the large reported $E2$ strength between the $2^+$ ground state and $1^+$ first excited state of $^8$Li, $B(E2; 2^+ \rightarrow 1^+) = 55(15) e^2$fm$^4$ [17], presents a puzzle. This strength corresponds to $\sim 58$ Weisskopf units, which would be considered collective even in much heavier mass regions.

It is not unexpected that $^8$Li would be deformed. For instance, the ground state of the mirror nuclide $^8$B is suggested to have proton halo structure [18–21] as a deformed $^7$Be core, with a loosely-bound proton in a spatially-extended molecular orbital [22]. The ground-state spectroscopic quadrupole moment of $^8$Li is similar in magnitude to those of its deformed neighbors [23].

Nonetheless, an enhanced $2^+ \rightarrow 1^+$ transition cannot be easily understood, as an in-band rotational transition, like the transitions in neighboring nuclei. As least in a conventional axially-symmetric rotational picture [24, 25], if the $2^+$ ground state is the band head of a $K = 2$ band, there is no $1^+$ band member, and the transition to the $1^+$ excited state is at most an interband transition (between $K = 2$ and $K = 1$ band heads). Moreover the reported strength exceeds ab initio Green’s function Monte Carlo (GFMC) predictions [14] by nearly two orders of magnitude, despite a quadrupole moment prediction for the $2^+$ ground state, from the same calculations, which is in line with experiment.

The large reported $E2$ strength in $^8$Li [17] was measured in Coulomb excitation with a radioactive beam of $^8$Li, where the inelastically-scattered $^8$Li nuclei were detected by measuring their energy using Si detectors. Such an experiment is ostensibly susceptible to events coming from
That is produced in its excited state in the primary reaction rather than those coming from the Coulomb excitation of \(^8\text{Li}\) in the secondary target, which would result in an inflated measured Coulomb excitation cross section and thus the extracted \(E2\) strength. To eliminate such a possible source of error, we revisit this radioactive beam Coulomb excitation measurement, but now with gamma-ray detection capability, to impose a coincidence requirement between the detection of the inelastically-scattered \(^8\text{Li}\) nucleus and the \(1^+ \rightarrow 2^+\) deexcitation gamma ray. We use an array of high-efficiency LaBr\(_3\) gamma-ray detectors, in coincidence with a Si particle detector centered on the secondary target. While we measure a smaller value than previously reported, the change is not sufficient to bring experiment in line with current theoretical understanding of the structure.

To shed light on possible structural interpretations, we delve more deeply into possible interpretations of the \(2^+ \rightarrow 1^+\) transition, both within the rotational picture and in \textit{ab initio} calculations. We begin by considering the enhanced rotational transitions in the neighboring \(A = 7\text{–}9\) nuclei, to provide context for the more enigmatic case of \(^8\text{Li}\). Here it is helpful that ground state quadrupole moments are well-measured in \(^7\text{Li}, \text{~}^8\text{Li}, \text{~}^9\text{Be}\) [23]. These moments provide a rotational normalization for the \(E2\) strength: ratios of the form \(B(E2)/(eQ^2)\) within a rotational band are given by the rotational Alaga rules [24–26]. Taking such ratios of \(E2\) matrix elements cancels out systematic normalization uncertainties arising (from incomplete convergence) in \textit{ab initio} calculations [16, 27, 28].

Beyond the simple rotational model, we examine the predictions of Elliott’s SU(3) symmetry [29, 30] and discuss the results of no-core shell model (NCSM) [31] calculations with several choices of internucleon interaction. A consistent description is obtained for the rotational transitions in the neighboring nuclei, and an enhanced rotational \(2^+ \rightarrow 3^+\) strength is expected to the more highly-excited \(3^+\) state in \(^8\text{Li}\). However, the large measured \(2^+ \rightarrow 1^+\) in \(^8\text{Li}\) is consistently inexplicable.

We first outline the present radioactive beam experiment with the \textit{TwinSol} low-energy radioactive nuclear beam apparatus at the University of Notre Dame Nuclear Science Laboratory [32] (Sec. II) and detail the subsequent analysis used to extract the \(2^+ \rightarrow 1^+\) strength from Coulomb excitation (Sec. III), including an assessment of two-step and other possible contributions. We then discuss this strength in the context of rotational or Elliott SU(3) (Sec. IV A) and \textit{ab initio} (Sec. IV B) descriptions. These results were reported in part in Ref. [33].
II. EXPERIMENT

In order to produce a beam of $^8\text{Li}$, the 10 MV Tandem Van De Graaff at the University of Notre Dame Nuclear Science Laboratory (NSL) was used to accelerate a 4.5 $\mu$A beam of $^7\text{Li}$ $3^+$ ions to 26 MeV. This beam was steered to a production gas cell, which had a 12 $\mu$m thick $^9\text{Be}$ foil placed on the downstream side of the gas cell to serve as the production target. The gas cell was filled with 300 Torr of He gas to help cool the $^9\text{Be}$ target and a 4 $\mu$m Ti foil on the upstream side of the gas cell was used to contain the gas. The $^8\text{Li}$ beam was produced by the $^7\text{Li}(^9\text{Be},^8\text{Be})^8\text{Li}$ reaction at an energy of 22.7(5) MeV, along with other isotopes produced by competing reactions. This cocktail beam was sent into the TwinSol apparatus [32]. TwinSol consists of two superconducting solenoid magnets that are used as magnetic lenses to focus the radioactive beam of interest and eliminate contaminants. More details on how the TwinSol apparatus was used in this experiment are given in Ref. [16]. The first of these two solenoids was set to 2.43 T to best focus the $^8\text{Li}$ beam through a 9 mm diameter collimator placed between the solenoids, which eliminated the majority of the contaminants. The second solenoid was set to 1.40 T, to refocus the $^8\text{Li}$ beam after it had passed through the collimator.

After exiting TwinSol, the $^8\text{Li}$ beam was sent into our scattering chamber. The first element the beam encountered was a 9 mm radius collimator, to eliminate divergent aspects of the beam and define the beam spot. The collimator and the experimental setup is shown in Fig. [1]. Directly downstream of the collimator was a target ladder with a 1 $\mu$m thick $^{197}\text{Au}$ target, an empty frame, and a Si surface barrier detector for determining beam purity. During beam development, the beam was sent onto the surface barrier detector for particle identification and the final beam components were identified as being 98% $^8\text{Li}$ with $^7\text{Be}$ and scattered $^7\text{Li}$ making up the majority of the contaminants. The optimized radioactive beam was sent onto the the gold foil for the duration of the experiment. Placed 34.6 mm downstream from the gold foil, a 1000 $\mu$m-thick S7 annular Si detector, made by Micron Semiconductor Limited [34], was used to measured the beam that scattered from the $^{197}\text{Au}$ target in the angular range of 20.6° - 45.3° degrees. The upstream side of the Si detector is segmented into 45 0.5 mm concentric rings from its 13 mm inner radius to its 35 mm outer radius while the downstream side of the detector is segmented into 16 radial sectors of 22.5° each, giving precise radial positions and the ability to measure beam offsets. Due to the limited number of electronic channels available, the ring electronic channels were combined in pairs to make 22 rings, each effectively 1 mm wide.
FIG. 1. (Color online) The experimental setup is shown. Ten LaBr$_3$ detectors from the HAGRiD array are placed at 30$^\circ$, 60$^\circ$, 90$^\circ$, 120$^\circ$, and 150$^\circ$ with respect to the beam axis surrounding the Au target, a brass collimator, and a S7 Si detector inside the scattering chamber (not shown).

Outside of the scattering chamber, 10 LaBr$_3$ detectors with 2 in. x 2 in. cylindrical crystals from the HAGRiD array [35] were placed 17.2 cm away from the target, in order to measure $\gamma$ rays emitted by Coulomb-excited $^8$Li nuclei. The arrangement of the LaBr$_3$ detectors can be seen in Fig. 1. The detectors were placed symmetrically around the chamber at 30$^\circ$, 60$^\circ$, 90$^\circ$, 120$^\circ$, and 150$^\circ$ on both sides of the beam axis. The $\gamma$-ray peaks from the intrinsic radiation of the LaBr$_3$ detectors, $\beta$ decay of $^{138}$La, were used to gain match the different detectors before the experiment and were monitored throughout the experiment.

The signals from the Si and LaBr$_3$ detectors were run through preamplifiers and into a digital data acquisition (DAQ) system using Pixie-16 modules from XIA, LLC [36]. In this experiment, a hit in any detector channel defined an event in the DAQ, with a hit in any another detector channel being considered coincident and packaged together into the same event if it occurred within a 500 ns time window of the original event. In the second half of the experiment, this timing window was reduced to 100 ns, to reduce the number of random coincidences in the $\gamma$-ray spectrum. The experiment was run for a total of 5 days with a beam rate of approximately $4 \times 10^5$ pps.

At the end of the experiment, a 1.44 $\mu$Ci $^{152}$Eu source was placed at the target location and the multiple $\gamma$ rays from its decay were used to calibrate each LaBr$_3$ detector in energy and determine its $\gamma$-ray efficiency. The entire array was found to have a total $\gamma$-ray efficiency of 0.75% at 1 MeV. The energy resolutions of the LaBr$_3$ detectors in the array were 1.6 – 2% at 1408 keV. These resolutions were sufficient to cleanly resolve our $\gamma$-ray of interest (981 keV) since this energy is
FIG. 2. (Color online) The Doppler-corrected $\gamma$-ray spectrum summed over the entire experiment and gated on coincidence with the scattered-particle peak in the silicon detector. The spectrum is binned to 8 keV/bin. The $\gamma$-ray peak corresponding to the $2^+ \rightarrow 1^+$ transition of $^8\text{Li}$ is seen at 983(3) keV.

far enough away from the background $\gamma$-rays seen in the Doppler uncorrected spectrum.

III. ANALYSIS

The first step in this analysis was to determine the yield of $\gamma$ rays from the first excited state of $^8\text{Li}$. Afterwards, the integrated beam flux over the course of the experiment was determined by comparing the measured rates of $^8\text{Li}$ in our Si detector to Monte Carlo simulations. With these two values as inputs into the Winther-De Boer Coulomb excitation code [37], the $E2$ transition strength was then calculated.

A coincidence gate was placed on the scattered $^8\text{Li}$ peak seen in our Si detector in order to reduce the number of $\gamma$ rays from room background and the intrinsic radioactivity of the LaBr$_3$ detectors. As the Si segment and LaBr$_3$ detector responsible for the coincidence event were known, the geometric angle between the scattered $^8\text{Li}$ particle and the emitted $\gamma$ ray was determined and used to correct for the Doppler shift of the $\gamma$-ray energy. During the analysis, a large amount of beam background was observed in the two innermost rings of the Si detector, which were therefore not included in this analysis.
Additionally, some of the random coincidences seen in the spectrum were eliminated by requiring a tight time coincidence between the LaBr and Si detector signals. The original coincidence window used in the experiment was 500 ns wide, but an additional timing gate was added in the offline analysis. The timing offsets between different rings of the Si detector and different LaBr$_3$ detectors were aligned and a 30 ns timing gate was placed over the position of the particle-$\gamma$ coincidences in the time difference spectrum. As the number of particle-$\gamma$ coincidences were low, this position was determined by observing the Doppler-corrected $\gamma$-ray spectrum while using a moving gate in the time-difference spectrum. The position and width were chosen to provide a robust $\gamma$-ray peak of interest with minimum background level. A $\gamma$-ray peak with a centroid value of 983(3) keV and a full-width-at-half-maximum (FWHM) of 20 keV was visible above the background, as shown in Fig. 2. This energy is in agreement with the literature value of 980.80(10) keV \cite{38} for the energy of the $1^+ \rightarrow 2^+$ transition of $^8$Li.

To extract the total counts from our peak, we fit it with a Gaussian function combined with a linear fit to the local background. This fit yielded a total peak area of 43(14) counts. Using the efficiency of the LaBr$_3$ array, determined during the energy calibration, we were able to determine the absolute $\gamma$-ray yield for the experiment.

After measuring the $\gamma$-ray yield, we had to determine the total, integrated $^8$Li beam current on the Au target to calculate the $^8$Li $B(E2)$ value. The process of producing an in-flight beam with TwinSol typically leads to an extended spot size on target. A fixed collimator upstream of the gold foils restricted our beam spot to a 9 mm radius, which was chosen to match the size of our gold target. Due to the proximity of the target to the Si detector, a diffuse beam will cause the $^8$Li ions seen in the rings of the Si to come from a range of scattering angles. The Si detector also shows some up-down asymmetry in the measured rates in the sectors, indicating that the beam was offset to some degree. Because of the diffuse and asymmetric nature of the beam, comparing the distribution of the particles in the rings of the detector to a Rutherford distribution does not yield an accurate fit. We used a Geant4 \cite{39,40} simulation to model the width and offset of the beam on target, to then be able to determine the beam rate. The radius of the beam and its offset from the beam axis were varied in the simulation over a range of values (1-7 mm for the radius and 1-4 mm for the offset) to reproduce the distribution seen in both the rings and sectors. The simulation also tracks the scattering angle of each particle that hits the simulated Si detector (see Fig. 4), which proved useful later in the analysis when determining the detector efficiencies. We found a 7 mm beam radius and a 1 mm offset best reproduced the shape of the Si detector ring
FIG. 3. (Color online) A plot of $^8$Li ions measured in the rings of the Si detector (circles) and the Geant4 simulated data (squares). The angles given are the angle for the center of each ring of the Si detector with the horizontal uncertainties showing the full angular extent of each ring.

and sector data. The data seen in the Si rings and the corresponding reproduction of the data are shown in Fig. 3. After the simulation reproduced the shape of the experimental data, it was scaled to have the same magnitude as the total counts measured during the experiment and a beam rate of $4.1(3) \times 10^5$ pps was extracted from the scaling. The beam rate uncertainty was estimated by changing the beam parameters and the beam scaling until the simulation results exceeded the experimental uncertainties.

The final step was to determine the $^8$Li $B(E2; 2^+ \rightarrow 1^+)$ value from the total $\gamma$-ray yield in the experiment. We used a version of the Winther-De Boer Coulomb excitation code, which is based on the semi-classical theory of Coulomb excitation. This version of the code was capable of calculating electric dipole to hexadecapole transitions and also multiple excitations. After inputting an $E2$ matrix element, the code outputs a differential cross section which can be used to match the output of the $\gamma$-ray yield measured in the experiment. Due to the shape of the $^8$Li beam, the scattering angles and detector geometric efficiencies were non-uniform and would be difficult to account for in a normal calculation. Instead, we used the Geant4 simulation to use the event-by-event simulated angle to determine the proper Coulomb-excitation probability given by the Winther-De Boer calculations. The $^8$Li $B(E2; 2^+ \rightarrow 1^+)$ value we obtained is $25(8) e^2fm^4,$
FIG. 4. (Color online) A simulation of the angular distribution of \(^{8}\)Li particles observed in the first ring of the Si detector used in our analysis when using the beam spot determined by our simulation. The number of particles seen here is not representative of the total particles seen during the experiment, though the shape of the distribution should be accurate.

which includes the \(\gamma\)-ray yield statistical and beam rate uncertainties.

The two sources of systematic uncertainties are the M1 component of the excitation probability and contributions from electric dipole polarizability, i.e., virtual \(E1\) excitations to collective structures at high energy. Based on a study of \(^{7}\)Li [43], which should be a fairly close analogue to \(^{8}\)Li, the M1 contribution at the forward angles in our experiment is between 2-3%. Much less easily understood is the effect of the \(E1\) dipole polarizability. Unlike \(^{7}\)Li, which has a strong virtual excitation to the break up of \(^3\)H+\(\alpha\), \(^{8}\)Li mainly virtually excites to levels that decay by neutron emission. Studies on the \(E1\) dipole polarization effect in \(^6\)Li and \(^7\)Li suggest that this effect is small, on the order of less than 10% at forward angles [43, 44]. With a conservative estimate of 3\% uncertainty due to the M1 excitation and a 10\% uncertainty due to the effect of \(E1\) dipole polarizability, we adopt a systematic uncertainty of 3 e\(^2\)fm\(^4\). More precise measurements in the future will need to more carefully consider and estimate the strength of the virtual \(E1\) breakup. In addition, a calculation was performed using a coupled-channel code to estimate the effect of \(E2\) excitations to the unbound \(^3\)\(^+\) level, the level closest to the neutron threshold. It was found that the cross section for this excitation was a factor of 1000 less likely than to the first excited \(^1\)\(^+\) level. It seems unlikely then that two step processes to known levels would significantly contribute to our deduced B(E2) value.

Last, a possible mechanism for the population of the first \(^1\)\(^+\) state in \(^8\)Li is the excitation from
the non-resonant continuum in the Coulomb excitation process. The possible magnitude of this contribution is currently unknown. A future reaction theory calculation using, for example, the Extended Continuum Discretized Coupled Channels method \cite{45,47}, would be able to estimate such a contribution.

IV. DISCUSSION

A. Rotation, clustering, and Elliott SU(3) in $^8$Li and its neighbors

Let us examine first the rotational interpretation for the enhanced $E2$ strengths in the neighboring nuclides $^7$Be and $^9$Be, to provide a more quantitative baseline for what might be expected in $^8$Li, then see what a rotational picture may have to say regarding $E2$ strengths in $^8$Li itself. While the basic rotational relations are independent of the intrinsic structure which gives rise to the deformation, we shall find it helpful to keep in mind the likely cluster structures underlying the rotational spectra in these nuclei. We then compare with a complementary microscopic interpretation of rotation in these nuclei through Elliot SU(3) symmetry \cite{29,30}. This understanding in terms of simple collective descriptions provides physical context when we consider the \textit{ab initio} predictions for the $E2$ strengths in $^8$Li and its neighbors below (Sec. IV B).

In a cluster molecular picture, the Be isotopes are dimers \cite{6,8}: $^7$Be may be viewed as $^4$He + $^3$He, while $^9$Be is $^4$He + $^4$He + $n$, where the residual neutron occupies a molecular orbital surrounding the two $^4$He clusters \cite{49} (see, \textit{e.g.}, Figs. 14, 15, and 33 of Ref. \cite{8} for visualizations). They may thus be expected to behave as axially-symmetric rotors.

For an axially-symmetric rotor, the usual rotational relations \cite{24,27} dictate that the in-band $E2$ strengths should be proportional to the square of the intrinsic $E2$ matrix element for that band or, equivalently, to the square of the intrinsic quadrupole moment $[eQ_0 \equiv (16\pi/5)^{1/2}\langle \phi_K | Q_{2,0} | \phi_K \rangle]$. This proportionality constant is given simply in terms of Clebsch-Gordan coefficients (see Fig. 1 of Ref. \cite{27} for a quantitative overview).

The ground state quadrupole moment has been precisely measured for many nuclei \cite{23}, providing a valuable reference point for $E2$ strengths. As the quadrupole moments for band members and, in particular, the ground state, are also proportional to the intrinsic quadrupole moment, the $B(E2)$ should in turn be proportional to the squared ground state quadrupole moment. We thus consider dimensionless ratios of the form $B(E2)/(eQ)^2$. 
Since the ground state quadrupole moment is well-known experimentally in \(^{7}\text{Li}\), the mirror nuclide to \(^{7}\text{Be}\), it is natural to start with this nuclide, which is understood to have the analogous cluster structure \(^{4}\text{He} + ^{3}\text{H}\). In both these mirror nuclides, the 3/2\(^{-}\) ground state and 1/2\(^{-}\) excited state are interpreted as members of a \(K = 1/2\) rotational band, where the energy order is inverted due to Coriolis staggering [25] (see, e.g., Fig. 3 of Ref. [50]).

For \(^{7}\text{Li}\), combining \(Q(3/2^{-}) = -4.00(3)\text{fm}^2\) [23] with \(B(E2; 3/2^{-} \rightarrow 1/2^{-}) = 8.3(5)\text{e}^2\text{fm}^4\) [1], which corresponds to a comparatively modest 10.4(6) W.u., we have \(B(E2; 3/2^{-} \rightarrow 1/2^{-})/[eQ(3/2^{-})]^2 = 0.52(3)\). This experimental ratio is consistent with the rotational predicted ratio of \(\approx 0.497\).

For \(^{7}\text{Be}\), although the ground state quadrupole moment is not experimentally known [23], we may make a primitive estimate. Simply substituting \(^{3}\text{He}\) for \(^{3}\text{H}\) in the cluster description gives a quadrupole moment for \(^{7}\text{Be}\) larger than that of \(^{7}\text{Li}\) by a factor of \(\sim 1.47\) [28]. To preserve the rotational value for the ratio \(B(E2)/[eQ]^2\), we would thus need the \(E2\) strength in \(^{7}\text{Be}\) to be larger than that in \(^{7}\text{Li}\) by a factor of \(\sim 2.3\). This is indeed the case, albeit to within rather generous uncertainties, for the measured \(B(E2; 3/2^{-} \rightarrow 1/2^{-})\) in \(^{7}\text{Be}\) noted above [16], which is 3.1(8) times that in \(^{7}\text{Li}\). We thus see that the disparity in scale between the \(E2\) strengths in \(^{7}\text{Li}\) and \(^{7}\text{Be}\), though perhaps striking, is not inconsistent with what would be expected in a simple cluster picture.

Finally, in \(^{8}\text{Be}\), the 3/2\(^{-}\) ground state and 5/2\(^{-}\) excited state are the first two members of a \(K = 3/2\) rotational band (see, e.g., Fig. 1 of Ref. [50]). The \(E2\) strengths are known from the 3/2\(^{-}\) ground state to both the 5/2\(^{-}\) and 7/2\(^{-}\) members of the \(K = 3/2\) ground-state rotational band. Combining \(Q(3/2^{-}) = +5.29(4)\text{fm}^2\) [23] with the \(B(E2; 3/2^{-} \rightarrow 5/2^{-})\) quoted above gives \(B(E2; 3/2^{-} \rightarrow 5/2^{-})/[eQ(3/2^{-})]^2 = 1.50(11)\), roughly consistent with the rotational prediction of \(\approx 1.279\), though not to within experimental uncertainties. Similarly, the experimental \(B(E2; 3/2^{-} \rightarrow 7/2^{-}) = 19(8)\text{e}^2\text{fm}^4\) [2] gives \(B(E2; 3/2^{-} \rightarrow 7/2^{-})/[eQ(3/2^{-})]^2 = 0.71(3)\), consistent to within uncertainties with the rotational prediction of \(\approx 0.711\) (see also Sec. III D of Ref. [27]).

Returning to \(^{8}\text{Li}\), there is at least a prospective cluster rotational description for the low-lying spectrum. The mirror nuclide \(^{8}\text{B}\) has been proposed to have a \(^{7}\text{Be} + p\) molecular structure, that is, consisting of a \(^{4}\text{He} + ^{3}\text{He}\) dimer plus a proton in a spatially-extended molecular orbital. The

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1 Ab initio predictions [14, 28] similarly suggest that the quadrupole moment for \(^{7}\text{Be}\) should be larger than that of \(^{7}\text{Li}\) by a factor of \(\sim 1.6–1.7\).
resulting proton halo structure [22] could contribute to the enhanced quadrupole moment in $^8$B [18, 51], measured to be $Q(2^+) = +6.34(14)\text{ fm}^2$ [23]. By mirror symmetry, $^8$Li would then have a $^7$Li + $n$ molecular structure, that is, consisting of a $^4$He + $^3$H dimer plus a neutron. We may therefore again attempt an axial rotor description.

In this description, the $2^+$ ground state and $3^+$ excited state (a narrow resonance just above the neutron separation threshold, at 2.2 MeV) are members of the same $K = 2$ band. The $2^+ \to 3^+$ transition in $^8$Li is thus an in-band transition, and expected to be enhanced. The rotational prediction of $\approx 0.609$ for $B(E2; 2^+ \to 3^+)/[eQ(2^+)]^2$, combined with the measured $Q(2^+) = +3.14(2)\text{ fm}^2$ [23], gives an estimated $2^+ \to 3^+$ transition strength of $\sim 6\text{ e}^2\text{fm}^4$.

However, the transition to the $1^+$ excited state in $^8$Li would ostensibly be an interband transition, requiring a change in the rotational intrinsic wave function. More specifically, in the cluster molecular orbital description, we may expect the valence neutron to be in a $\pi$ orbital with $K = 3/2$, as in the isotone $^9$Be [52]. The $K$ quantum number adds algebraically, so the $K = 2$ ground state band is obtained from aligned coupling of this neutron with the $K = 1/2$ $^7$Li core, while the $1^+$ excited state is ostensibly the band head of a $K = 1$ band arising from the antialigned coupling. The $2^+ \to 1^+$ transition strength thus depends, in a rotational picture, upon the interband intrinsic matrix element $\langle \phi_{K=1}|Q_{2,-1}|\phi_{K=2}\rangle$. Even if this matrix element were identical to the diagonal intrinsic matrix element $\langle \phi_{K=2}|Q_{2,0}|\phi_{K=2}\rangle$ which determines the intrinsic quadrupole moment, as a generous upper limit [25] we would estimate a $B(E2; 2^+ \to 1^+)/[eQ(2^+)]^2$ of $\lesssim 0.24$, yielding a $2^+ \to 1^+$ transition strength of $\lesssim 2\text{ e}^2\text{fm}^4$. Conversely, the present measured $E2$ strength (Sec. III) yields the much larger ratio $B(E2; 2^+ \to 1^+)/[eQ(2^+)]^2 = 2.5(9)$.

Thus, the $E2$ strengths in the $A = 7$ and 9 nuclides neighboring $^8$Li are generally understandable in relation to the ground state quadrupole moments of these nuclei, in a rotational picture. Even

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2 In the molecular orbital picture, this intrinsic matrix element $\langle \phi_{K=1}|Q_{2,-1}|\phi_{K=2}\rangle$ involves changing the alignment of the core and neutron $K$ quantum numbers. Since the neutron is uncharged, the quadrupole operator can only change the alignment of the $^7$Li core and neutron by flipping the angular momentum projection of the core along the symmetry axis. Thus the relevant intrinsic matrix element is the off-diagonal $\langle \phi_{K=1/2}|Q_{2,-1}|\phi_{K=7/2}\rangle$, involving the conjugate intrinsic state $|\phi_{K=7/2}\rangle$ for the $^7$Li core. Such off-diagonal matrix elements are not traditionally expected to be enhanced [25], but could in principle be estimated within a microscopic cluster model description. See also Sec. II C of Ref. [27] for a discussion of how this matrix element may be constrained from ab initio NCSM calculations of rotational $E2$ strengths.
the extraordinarily strong $3/2^- \rightarrow 5/2^-$ transition in $^{9}\text{Be}$ has an experimental central value which is within 20\% of the rotational prediction. However, even accounting for possible cluster rotation in $^{8}\text{Li}$, an enhanced $1^+ \rightarrow 2^+$ strength in $^{8}\text{Li}$ is not readily explained in the same fashion as the enhanced transitions in the neighboring nuclides.

Rotation may also be understood, purely within the context of the shell model, through Elliott’s SU(3) symmetry [29, 30]. Although this description is restricted to correlations in the shell model valence (or $0\hbar\omega$) space, it is not necessarily incompatible with the cluster description. Some, but not all, cluster correlations may be reproduced in the $0\hbar\omega$ shell model space (e.g., $\pi$ molecular orbitals, but not $\sigma$ orbitals, may be approximately described in the $p$ shell) [8].

The low-lying states obtained in \textit{ab initio} calculations for a variety of $p$-shell nuclei have been found to have a $0\hbar\omega$ component which is remarkably well-described by Elliot’s SU(3) symmetry [53–56]. Thus, by considering the SU(3) description for $^{8}\text{Li}$ and its neighbors, we may hope to obtain some physically relevant understanding of the rotational structure. In particular, the SU(3) description provides concrete predictions for the strengths of interband transitions in relation to the in-band transitions. The SU(3) structure of the ground state rotational bands in the neighboring nuclei $^{7}\text{Li}$ (or $^{7}\text{Be}$) and $^{9}\text{Be}$ has already been discussed in some detail [4, 50, 55, 57], but we review the essential elements underlying the $B(E2)/(eQ)^2$ predictions obtained below.

Elliott’s SU(3) scheme reorganizes the shell model space into irreducible representations (or irreps), each of which consists of states sharing the same quadrupole deformation. (These states are correlated states, that is, linear combinations of the original shell model configurations.) Under Elliott’s schematic quadrupole-quadrupole ($-Q \cdot Q$) Hamiltonian, the “leading” irrep, consisting of the most highly-deformed states, comes lowest in energy. These states are, in the basic Elliott description, $LS$-coupling states, with definite total orbital angular momentum $L$ and total spin $S$, which then combine to yield various angular momenta $J$.

The quadrupole-quadrupole Hamiltonian yields a rotational energy splitting only by $L$, as $L(L+1)$. The resulting low-lying level-schemes for $^{8}\text{Li}$ and its neighbors are shown in Fig. 5. Note the twofold degeneracies which result in $^{7}\text{Li}$ (or $^{7}\text{Be}$) [Fig. 5(a)] and $^{9}\text{Be}$ [Fig. 5(c)], and higher-fold degeneracies in $^{8}\text{Li}$ [Fig. 5(b)].

However, the microscopic spin-orbit interaction perturbs this simple scheme, so as to mix states of different $L$ (but the same $S$ and $J$) within an irrep, and break the degeneracies. As elaborated by Elliott and Wilsdon [29, 30], the primary result is to reproduce traditional rotational bands, comprised of states with good angular momentum projection $K$ along the intrinsic axis, and energies
FIG. 5. (Color online) Low-lying spectrum provided by an Elliott SU(3) quadrupole-quadrupole Hamiltonian, consisting of the states from the highest-deformation (leading) SU(3) irrep in the $0\hbar\omega$ shell-model space of (a) $^7\text{Li}$ (or $^7\text{Be}$), (b) $^8\text{Li}$, and (c) $^9\text{Be}$. In each panel, the orbital angular momenta $L$ arising within the given SU(3) irrep $(\lambda,\mu)$ are shown at left (with the corresponding rotational $K_L$ quantum number indicated at bottom), and then the resulting degenerate levels of total angular momenta $J$ arising from the angular momentum coupling with spin $S$ are shown at right. Degenerate levels are split in the diagram for readability.

obeying the standard rotational energy formula (including Coriolis energy staggering for $K = 1/2$ bands) as a function of $J$.

For $^7\text{Li}$ [Fig. 5(a)], the leading SU(3) irrep has Elliott quantum numbers $(\lambda,\mu) = (3,0)$. Such quantum numbers of the form $(\lambda,0)$ are identified with a purely axially-symmetric prolate deformation. The orbital angular momenta $L = 1,3$ combine with spin $S = 1/2$, to give ground state $K = 1/2$ rotational band members with $J = 1/2,3/2,5/2,7/2$. The wave function for each $J$ is unique, within the leading irrep, and is simply an unmixed $LS$-coupled state. The microscopic spin-orbit interaction in the Elliot-Wilsdon rotational picture serves only to break the degeneracies within the $1/2-3/2$ and $5/2-7/2$ doublets, to give something akin to the experimentally-observed splitting (e.g., Fig. 3 of Ref. [50]), without affecting the wave functions. The resulting ratio, $B(E2; 3/2_{K=1/2} \rightarrow 1/2_{K=1/2})/[eQ(3/2_{K=1/2})]^2$, is identical to that obtained for the standard axially-
symmetric rotor considered above.

For $^9$Be [Fig. 5(c)], in the leading SU(3) irrep $(3, 1)$, orbital angular momenta $L = 1, 2, 3, 4$ combine with spin $S = 1/2$, to give angular momenta $J = 3/2, 5/2, 7/2, 9/2$ from the aligned couplings, or $J = 1/2, 3/2, 5/2, 7/2$ from the antialigned couplings. Notice, therefore, that, except for the extremal angular momenta $J = 1/2$ and $9/2$, there are two possible $LS$-coupled wave functions for each $J$. Under the quadrupole-quadrupole Hamiltonian, $e.g.$, the lower $J = 3/2$ state would be a pure $L = 1$ state and the higher would have $L = 2$, as in Fig. 5(c). However, in general, we may expect this situation to be perturbed, and the two states of different $L$ to mix. In the Elliott-Wilsdon picture, the mixing is generally modest, $e.g.$, an $\approx 19\%$ $L = 2$ component to the $J = 3/2$ ground state $[4, 57]$, a conclusion generally supported by $ab\ initio$ calculations as well $[50, 58]$. The resulting states in the Elliott-Wilsdon picture for $^9$Be form an yrast $K = 3/2$ band and an yrare $K = 1/2$ band (see, $e.g.$, Fig. 1 of Ref. [50] for experimental counterparts). Taking this Elliott-Wilsdon prediction for the mixing $[59]$, we obtain $B(E2; 3/2_{K=1/2} \rightarrow 5/2_{K=3/2})/[eQ(3/2_{K=1/2})]^2 \approx 1.227$, which deviates by only a few percent from the rotational value of $\approx 1.279$ noted above, and $B(E2; 3/2_{K=1/2} \rightarrow 7/2_{K=3/2})/[eQ(3/2_{K=1/2})]^2 \approx 0.547$, which falls more noticeably below the rotational value of $\approx 0.711$ noted above. By comparison, in the SU(3) picture, the interband transition to the $K = 1/2$ band head is expected to be two orders of magnitude weaker, with $B(E2; 3/2_{K=1/2} \rightarrow 1/2_{K=1/2})/[eQ(3/2_{K=1/2})]^2 \approx 0.0058$ (experimentally, no $\gamma$ transition has been observed connecting the ground state and first $1/2^-$ state $[21]$).

Finally, for $^8$Li [Fig. 5(b)], the leading SU(3) irrep $(2, 1)$ comes with two possible total spins, both $S = 1$ and $S = 0$. All told, there are four different ways to make a $J = 2$ state, or three ways to make a $J = 1$ state. For the $J = 2$ ground state, we might at least expect, from the quadrupole-quadrupole Hamiltonian, that the component with $L = 1$ and $S = 1$ will be the predominant contribution. But, for the $J = 1$ excited state, the $L = 1$ contributions with $S = 1$ and $S = 0$ are initially degenerate. We may expect this degeneracy to be broken with a realistic interaction (or, schematically, by the spin-orbit interaction). However, from the viewpoint of two-state mixing $[11, 60]$, we are starting with degenerate unmixed states. We may thus expect the resulting mixing to be highly sensitive to the details of the interaction.

Nonetheless, to provide a simple (and likely simplistic) estimate, we may simply ignore the possible $S = 0$ contributions, not only to the $J = 2$ ground state, but to the excited states as well. Then, in the Elliott-Wilsdon rotational picture, the states within the $(2, 1)$ irrep with $S = 1$ form rotational bands with $K = 0, 1, 2$. For the in-band $2^+ \rightarrow 3^+$ transition, then we obtain
FIG. 6. (Color online) Convergence of \textit{ab initio} NCSM calculations for $^8$Li, with the Daejeon16 interaction: (a) the $2^+$ ground state energy, (b) the $B(M1)$ for decay of the $1^+$ excited state, (c) the $B(E2)$ for excitation of the $1^+$ excited state, and (d) the ratio $B(E2)/(eQ)^2$ of this strength to the squared $2^+$ quadrupole moment. Calculated values are shown as functions of the basis parameter $\hbar \omega$, for successive even value of $N_{\text{max}}$ (increasing symbol size and longer dashing), from $N_{\text{max}} = 4$ (short dashed curves) to 14 (solid curves). The experimental $B(M1)$ value [2] is also shown (square, at left) in panel (b).

$B(E2; 2_{K=2} \rightarrow 3_{K=2})/(eQ(2_{K=2}))^2 \approx 0.675$, or 11% above the rotational estimate of $\approx 0.609$ discussed above. For the interband $2^+ \rightarrow 1^+$ transition, the Elliott-Wilsdon picture gives an exceedingly weak $B(E2; 2_{K=2} \rightarrow 1_{K=1})/(eQ(2_{K=2}))^2 \approx 0.0098$, well below the crude rotational upper bound suggested above.

B. \textit{Ab initio} predictions

Let us now see what might be expected in an \textit{ab initio} description, for these $E2$ strengths and, in particular, $B(E2)/(eQ)^2$ ratios. We present calculations obtained via the no-core configuration interaction (NCCI), or no-core shell model (NCSM), approach [31], as well as comparing with prior
Green’s function Monte Carlo (GFMC) calculations [14]. After highlighting essential features of the convergence of the calculations, we again start by examining predictions for the enhanced $E2$ strengths in the neighboring $A = 7$ and 9 nuclides, before considering the $^8\text{Li}$ $E2$ predictions in detail. From prior work, the \textit{ab initio} predictions for the neighboring nuclides are already known to conform at least approximately to a rotational picture [27, 50, 61].

In the NCSM approach, the nuclear many-body problem is solved in a basis of antisymmetrized products (Slater determinants) of harmonic oscillator single-particle states. In principle, if unlimited oscillator excitations could be included in the basis, the true results of solving the many-body problem, for the given internucleon interaction, would be obtained to unlimited accuracy. However, in practice, the basis must be truncated. Therefore, only approximate results are obtained for energies, $E2$ strengths, and other observables, and these results depend upon the particular choice of truncated basis: the maximum number $N_{\text{max}}$ of oscillator excitations allowed for the basis states and the oscillator length (or, equivalently, the oscillator parameter $\hbar \omega$ [62]) of the underlying single-particle states. Convergence towards the true value is recognized when the calculated values become insensitive to further increases in $N_{\text{max}}$ and to variation in $\hbar \omega$ (as illustrated in, \textit{e.g.}, Refs. [16, 28, 50, 63–65]).

For $^8\text{Li}$ specifically, let us examine the results for various observables involving the states of interest, as shown in Fig. 6: the $2^+$ ground state energy eigenvalue (\textit{i.e.}, negative binding energy) [Fig. 6(a)], $2^+ \rightarrow 1^+ M1$ strength [Fig. 6(b)], and $2^+ \rightarrow 1^+ E2$ strength [Fig. 6(c)]. These calculations, obtained using the NCSM code MFDn [66, 67], are based on the Daejeon16 internucleon interaction [68]. Each curve represents the results of calculations carried out with the same basis truncation $N_{\text{max}}$ (from short dashes for $N_{\text{max}} = 4$ to solid lines for $N_{\text{max}} = 14$), while varying the oscillator parameter $\hbar \omega$.

For the calculated energies [Fig. 6(a)], note the convergence towards a “floor”, as dictated by the variational principle for energies. The variational minimum here is in the vicinity of $\hbar \omega = 15$ MeV, but the location in general depends upon the nuclide, state, and choice of interaction, and may drift with increasing $N_{\text{max}}$.

Then, for the $M1$ strength [Fig. 6(b)], note the rapid convergence with $N_{\text{max}}$ over a wide range of $\hbar \omega$ values. Reassuringly, this result is consistent with the experimental value of $B(M1; 2^+ \rightarrow 1^+) = 3.0(10) \mu_N$ [2]. (Here we simply use the impulse-approximation $M1$ operator, while further corrections to $M1$ strengths in $^8\text{Li}$ are expected from meson exchange currents, as explored in Ref. [14].)
FIG. 7. (Color online) Ratios of the form $B(E2)/(eQ)^2$, calculated with the Daejeon16 interaction, for ground state band rotational transitions in the neighboring nuclides: (a) $^7$Li/$^7$Be and (b) $^9$Be (b). Calculated values are shown as functions of the basis parameter $\hbar \omega$, for successive even value of $N_{\text{max}}$ (increasing symbol size and longer dashing), from $N_{\text{max}} = 4$ (short dashed curves) to the maximum value for that nuclide (solid curves), indicated at bottom.

Calculations of $E2$ matrix elements are sensitive to the long-distance tails of the wave functions. Thus, $E2$ moments and transition strengths are poorly convergent in NCSM calculations. For a comparatively “soft” interaction, such as Daejeon16, the calculations may be suggestive of an approximate result. For instance, the convergence pattern for the $2^+ \rightarrow 1^+$ $E2$ strength considered here [Fig. 6(c)] is suggestive of a value of $\sim 1-2 e^2 \text{fm}^4$. However, commonly even an order-of-magnitude estimate is elusive (see, e.g., Ref. [28], for further examples).

Nonetheless, we have seen (Sec. IV A) that the structural information of interest may be contained in ratios of calculated $E2$ matrix elements. In such ratios, the errors in the individual matrix elements arising from truncation of the many-body space can cancel, leading to much more rapidly and robustly convergent results for the ratio. Such has been found for calculated ratios of $E2$ matrix elements within a rotational band [27, 50], or between rotational bands with related structures [69], as well as for isobaric analog $E2$ transition strengths [16] and quadrupole moments [28].

Thus we may also hope to obtain meaningful NCSM predictions for the $B(E2)/(eQ)^2$ ratios of present interest in $^8$Li and its neighbors. This ratio is shown for the $^8$Li calculations in Fig. 6(d): that is, each point represents the ratio of a calculated $E2$ strength [from Fig. 6(c)] and a squared quadrupole moment [from Fig. 4(a) of Ref. [28]], giving $B(E2; 2^+ \rightarrow 1^+)/(eQ(2^+))^2$. Indeed,
a generally more tractable convergence is obtained for this ratio, which rapidly converges to a value independent of \(N_{\text{max}}\) and \(\hbar \omega\) at the scale shown. Similar results for the \(B(E2)/(eQ)^2\) ratios involving in-band transitions in the neighboring nuclides \(^7\text{Li}, \, ^7\text{Be},\) and \(^9\text{Be}\) are shown in Fig. 7.

Let us start by considering these results for the neighboring nuclides. For the \(A = 7\) nuclides [Fig. 7(a)], we see that the calculated values of \(B(E2; 3/2^- \to 1/2^-)/(eQ(3/2^-))^2\) for \(^7\text{Be}\) (upper curves) converge rapidly towards a value in the vicinity of 0.55. The calculated values for \(^7\text{Li}\) (lower curves), which are not as robustly converged, appear to be approaching a lower value, between 0.50 and 0.55.

Then, for \(^9\text{Be}\) [Fig. 7(b)], the calculated values for \(B(E2; 3/2^- \to 5/2^-)/(eQ(3/2^-))^2\) (upper curves) are largely independent of \(\hbar \omega\), over most of the \(\hbar \omega\) range shown, suggesting a prediction of \(\approx 1.4\). Although the variation with \(N_{\text{max}}\) is small at the scale shown, the calculated values are still steadily increasing with \(N_{\text{max}}\). Then, while the calculated values for \(B(E2; 3/2^- \to 7/2^-)/(eQ(3/2^-))^2\) (lower curves) show some indication of flattening with respect to \(\hbar \omega\), they continue to increase in nearly equal steps with \(N_{\text{max}}\). Therefore, these calculations only provide a likely lower bound on the true result.

To facilitate a quantitative comparison, we take a “slice” through these NCSM results, at fixed \(\hbar \omega\) chosen based on the approximate location of the variational minimum, in Fig. 9. Here we...
FIG. 9. (Color online) Ratios of the form $B(E2)/(eQ)^2$, for excitation to low-lying states in $^8\text{Li}$ and its neighbors, obtained with the Daejeon16 (circles), JISP16 (squares), and LENPIC (diamonds) interactions, at fixed $\hbar\omega$ (15 MeV, 20 MeV, and 25 MeV, respectively, for the three interactions). Calculated values are shown for successive even values of $N_{\text{max}}$ (increasing symbol size), from $N_{\text{max}} = 4$ to the maximum value for that nuclide, indicated at top. Also provided for comparison, where applicable, are: GFMC AV18+IL7 predictions [14] (crosses), experimental values (horizontal lines and shaded error bands), and axially-symmetric rotational (asterisks) and Elliott-Wilsdon SU(3) (hexagons) predictions (see text).

... may readily compare with experiment (horizontal lines and shaded error bands) and the predictions from the simple rotational (asterisks) and SU(3) (hexagons) descriptions discussed above (Sec. IV A).

In addition to the results obtained with the Daejeon16 interaction (circles), as discussed so far, we also evaluate these ratios for the JISP16 [70] (squares) and LENPIC [71, 72] (diamonds) interactions in Fig. [9] Further details, including illustrations of the convergence properties of $E2$ observables obtained with these different interactions, may be found in Ref. [28].

Still focusing on the neighboring nuclides to $^8\text{Li}$, observe that the $B(E2)/(eQ)^2$ ratios are largely

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3 Specifically, for the LENPIC interaction, which is a modern chiral effective theory interaction, we take the two-body part, at N$^2$LO, with a semi-local coordinate-space regulator of length scale $R = 1$ fm.
Insensitive to the choice of interaction. The calculated ratios in $^7\text{Li}$ [Fig. 9(a)] and $^7\text{Be}$ [Fig. 9(b)] may differ slightly, with the latter tending above the rotational value of $\approx 0.497$, as already seen in Fig. 7(a), but these differences are at the $\lesssim 10\%$ scale. For the ratio involving the $5/2^-$ transition in $^9\text{Be}$ [Fig. 9(e)], the NCSM predictions are larger than the rotational value, lying closer to experiment (and at the edge of the experimental uncertainties). For the ratio involving the $7/2^-$ transition in $^9\text{Be}$ [Fig. 9(f)], the values at the highest $N_{\text{max}}$ calculated are closely comparable to the rotational value and in the center of the experimental range, but we must allow for their continued increase with $N_{\text{max}}$.

Furthermore, quadrupole moments and transition matrix elements have previously been reported [14] from ab initio Green’s function Monte Carlo (GFMC) [73] calculations, based on the Argonne $v_{18}$ (AV18) two-nucleon [74] and Illinois-7 (IL7) three-nucleon [75] potentials. The predictions are subject to Monte Carlo statistical errors, so the calculational uncertainties are of a qualitatively different nature from those entering into the NCSM calculations (where, rather, convergence errors are introduced by truncation of an oscillator basis). In particular, the GFMC calculated values for the ground state quadrupole moments and $E2$ transition strengths may meaningfully be compared directly with experiment, without taking a ratio to cancel truncation errors, and, in these neighboring nuclides to $^8\text{Li}$, they are consistent to within the combined experimental and GFMC statistical uncertainties.

Nonetheless, for comparison with the NCSM results, we may recast these GFMC results as $B(E2)/(eQ)^2$ ratios (crosses) in Fig. 9. In the $A = 7$ nuclides [Fig. 9(a,b)], note the general consistency with the rotational estimate and the NCSM predictions. For the ratio involving the $5/2^-$ transition in $^9\text{Be}$ [Fig. 9(e)], the NCSM predictions lie between the rotational value and the GFMC value. However, recall that the NCSM values are still increasing slowly with $N_{\text{max}}$, that is, in the direction of the GFMC result.

Thus, for the neighbors to $^8\text{Li}$, there is general agreement, in the $B(E2)/(eQ)^2$ ratios, between the predictions obtained across several choices for the internucleon interaction. Furthermore, these predictions come from calculations carried out with two different ab initio calculational methods.

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4 After converting gamma widths to $B(E2)$ values, the GFMC calculations of Ref. [14] yield: for $^7\text{Li}$, $Q(3/2^-) = -4.0(1)$ fm$^2$ and $B(E2; 3/2^- \rightarrow 1/2^-) = 7.8(4)$ e$^2$fm$^4$; for $^7\text{Be}$, obtained from the $^7\text{Li}$ wave function assuming mirror symmetry, $Q(3/2^-) = -6.7(1)$ fm$^2$ and $B(E2; 3/2^- \rightarrow 1/2^-) = 22.2(11)$ e$^2$fm$^4$; for $^8\text{Li}$, $Q(2^+) = +3.3(1)$ fm$^2$ and $B(E2; 2^+ \rightarrow 1^-) = 0.83(7)$ e$^2$fm$^4$; and, for $^9\text{Be}$, $Q(3/2^-) = +5.1(1)$ fm$^2$ and $B(E2; 3/2^- \rightarrow 5/2^-) = 38.4(9)$ e$^2$fm$^4$. 
NCSM and GFMC, having fundamentally different convergence properties. These calculated values, like the experimental ratios where available, are approximately consistent with either the simple axial rotor model or the more microscopic Elliott-Wilsdon SU(3) description.

Now let us return to $^8$Li. We have already seen that the calculated values for the ratio $B(E2; 2^+ \rightarrow 1^+)/[eQ(2^+)]^2$ obtained with the Daejeon16 interaction [Fig. 6(d)] converge rapidly with $N_{\text{max}}$, giving results largely independent of $\hbar \omega$ (at least for $\hbar \omega \gtrsim 10$ MeV). These suggest a value in the vicinity of 0.18.

We compare now to the results obtained with the JISP16 and LENPIC interactions, shown in Fig. 8. For the JISP16 interaction [Fig. 8(a)], while the results exhibit greater $\hbar \omega$ dependence, especially for lower $N_{\text{max}}$, they likewise appear to robustly converge towards a result. In this case, however, the value is lower by nearly a factor of two, near 0.10. For the LENPIC interaction [Fig. 8(b)], while there is markedly less $\hbar \omega$ dependence, there is however no clear sign of $N_{\text{max}}$ independence. The values are increasing slowly but steadily with $N_{\text{max}}$ over much of the $\hbar \omega$ range. Nonetheless, with this caveat, the values are again in the vicinity of 0.10.

These results are displayed side-by-side in Fig. 9(c). While the NCSM predictions for the $B(E2)/[eQ]^2$ ratio vary with interaction, in the range $\sim 0.10–0.18$, these results are all consistent with a modest strength for the $2^+ \rightarrow 1^+$ transition, an order of magnitude smaller than either the prior or present experimental values. Taken in conjunction with the experimental ground state quadrupole moment, they suggest a $B(E2; 2^+ \rightarrow 1^+)$ of $1–2e^2\text{fm}^4$. Conversely, recall (Sec. IV A) the present measurement yields the much larger ratio $B(E2; 2^+ \rightarrow 1^+)/[eQ(2^+)]^2 = 2.5(9)$ [Fig. 9(c)].

Comparing with the GFMC predictions [14] (see footnote 4), obtained with the AV18+IL7 potentials, the predicted $Q(2^+) = +3.3(1)\text{fm}^2$ is only marginally in tension with the experimental value of $+3.14(2)\text{fm}^2$ (above). Yet the predicted $B(E2; 2^+ \rightarrow 1^+) = 0.83(7)e^2\text{fm}^4$ gives $B(E2; 2^+ \rightarrow 1^+)/[eQ(2^+)]^2 = 0.076(8)$ [cross in Fig. 9(c)], below even the lowest of the NCSM estimates.

5 The earlier NCSM calculations of Maris et al. [76], based on the chiral N$^3$LO two-nucleon interaction of Entem and Machleidt [77], together with the N$^2$LO three-nucleon interaction of Navrátíl [78], carried out using a basis with $N_{\text{max}} = 8$ and $\hbar \omega = 13$ MeV, and calculated with an Okubo-Lee-Suzuki renormalized effective interaction, give $Q(2^+) = 2.648\text{fm}^2$ and $B(E2; 2^+ \rightarrow 1^+) = 0.714e^2\text{fm}^4$, similarly yielding a ratio of $B(E2; 2^+ \rightarrow 1^+)/[eQ(2^+)]^2 \approx 0.10$. 
It is interesting to contrast these calculated ratios with the results for the ostensibly in-band $2^+ \rightarrow 3^+$ transition [Fig. 9(d)]. As the $3^+$ state is a narrow resonance above the neutron threshold, a Coulomb excitation measurement for the $E2$ strength would require neutron detection, and only the $M1$ decay width is known from $^7\text{Li}(n,\gamma)$ [2, 81]. The NCSM calculations, as obtained with all three interactions, suggest ratios $B(E2; 2^+ \rightarrow 3^+)/[eQ(2^+)]^2$ in the range $\sim 0.8$–1.0. The lower end of this range is consistent with the GFMC results, while lying modestly above the rotational and SU(3) values. In conjunction with the known quadrupole moment, they suggest a comparatively collective $B(E2; 2^+ \rightarrow 3^+)$ of $8$–$10 \, e^2\text{fm}^4$.

That the \textit{ab initio} predictions for the $2^+ \rightarrow 1^+$ transition show some variation is perhaps not surprising. The transition involved is (predicted to be) a “weak” (noncollective) transition. Physically, one may also take the perspective that the $E2$ ratio is not “constrained”, as in the other examples in the neighboring nuclides, by the symmetry considerations which apply to in-band transitions in an axially symmetric rotor, but rather is sensitive, even in a cluster rotational picture, to the detailed microscopic structure of rotational intrinsic states and molecular orbitals. We alternatively interpret this sensitivity in terms of shell model SU(3) bases and mixing above (Sec. IV A).

Yet the \textit{ab initio} predictions, which in the neighboring nuclides are consistent with each other \textit{and} with experiment, are in $^8\text{Li}$ still consistent with each other in order of magnitude, but not with experiment. The \textit{ab initio} results confirm the expectation from simple arguments (Sec. IV A), that the $2^+ \rightarrow 1^+$ transition would have no reason to be collectively enhanced. We are thus left with the question of whether there could be some genuinely unexpected structural phenomenon which mixes collective strength into the $2^+ \rightarrow 1^+$ transition, or whether the explanation for the discrepancy lies elsewhere, either in the Coulomb excitation mechanism or the experiment proper.

\section*{V. SUMMARY AND OUTLOOK}

We have performed a radioactive-beam Coulomb excitation experiment to remeasure the $2^+ \rightarrow 1^+$ $E2$ strength in $^8\text{Li}$, now making use of particle-$\gamma$ coincidences. Compared to the previously-reported $B(E2; 2^+ \rightarrow 1^+) = 55(14) \, e^2\text{fm}^4$ [17], our measured value of $25(8)(3) \, e^2\text{fm}^4$ is smaller by approximately a factor of two, but remains anomalously large compared to what is expected from theory.

We have reexamined the structural interpretation of the ground state $E2$ transition strengths in $^8\text{Li}$ and, for context, in its $A = 7$ and 9 neighbors (namely, $^7\text{Li}$, $^7\text{Be}$, and $^9\text{Be}$), both in terms of
collective rotational descriptions and through *ab initio* NCSM calculations. In order to extract meaningful predictions, given the slow convergence of $E2$ observables with respect to basis truncation in NCSM calculations, we focus on ratios of $E2$ matrix elements, much in the spirit of traditional rotational spectroscopy. Errors introduced by basis truncation systematically cancel in the ratio, leading to comparatively rapid convergence. In particular, the ground-state quadrupole moments are experimentally well-known [23], providing a reference scale for the $E2$ strengths and suggesting that we consider ratios of the form $B(E2)/(eQ)^2$.

The enhanced $E2$ strengths in the neighboring $A = 7$ and 9 nuclei are naturally understood in terms of in-band rotational transitions, and we find that they are well-described by *ab initio* predictions. However, there is no simple way to explain an enhanced $2^+ \rightarrow 1^+$ transition in $^8$Li. Such a transition is not naturally viewed as a rotational in-band transition, and it is unnaturally large for an interband transition, a view supported by the Elliott SU(3) picture. The present *ab initio* predictions, obtained for a variety of internucleon interactions (Daejeon16, JISP16, and LENPIC) are consistent, in order of magnitude, both with each other and with the prior GFMC result [14], giving $B(E2; 2^+ \rightarrow 1^+)/[eQ(2^+)]^2 \approx 0.1–0.2$. However, they are an order of magnitude smaller than the the measured $E2$ strength, which yields a ratio of $\sim 2.5$.

Although the large measured $B(E2)$ in $^8$Li is thus seen to be difficult to reconcile with theoretical understanding, the magnitude of possible contributions from the non-resonant continuum to the $E2$ strength extracted from experiment is unknown. Calculations from reaction theory to estimate the contributions from the continuum would clarify if these contributions could explain the magnitude of the experimental result. Possible contributions to the cross section from two-step Coulomb excitation (or dipole polarization), involving virtual excitation of higher-lying states above the neutron separation threshold, as well as from indirect feeding involving direct excitation of these states, are estimated to be insufficient to explain the discrepancy between experiment and theory. The use of particle-$\gamma$ coincidences in the present experiment eliminates any significant contribution from $^8$Li $1^+ \rightarrow 2^\gamma$ rays that are produced in the primary reaction removing a possible experimental inflation of the $B(E2)$ value. In conclusion, our studies confirm that there is no easy interpretation for a large $B(E2)$ in $^8$Li, and our experimental result reinforces the tension between experiment and theory suggested by the previous work.
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