The Schwarzschild Black-Hole Pair

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By allowing the lightcones to tip over according to the conservation laws of an one-kink in static, Schwarzschild metric, we show that there also exists an instanton which represents production of pairs of chargeless, nonrotating black holes with mass \( M \), joined on an interior surface, beyond the horizon at \( r = M \). Evaluation of the thermal properties of each of the black holes in a pair leads one to check that each black hole is exactly the antiblack hole to the other black hole in the pair. The instantonic action has been calculated and seen to be smaller than that corresponding to pair production by factors that associate with the Bekenstein-Hawking entropy and a baby universe entropy \( 2\pi^2 M^2 \). This suggests these entropies to count numbers of internal states.

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1 Introduction

The success of the thermodynamical analogy in black hole physics allows us to hope that this analogy may be even deeper, and that it is possible to develop a statistical-mechanical foundation of black hole thermodynamics [1]. However, it is not quite clear that black hole entropy may count the number of internal degrees of freedom. These would describe different internal states which may exist for the same values of the black hole external parameters. It might be [2] that in the state of thermal equilibrium the parameters for the internal degrees of freedom will depend on the temperature of the system in the universal way, thus cancelling all contributions which depend on the particular properties and number of internal fields.

Clearly, the most compelling argument in favour of the idea that black hole entropy counts the number of internal states has recently come from proposals of black hole pair creation [3-7]. By computing the exact action of the black hole pair instanton it has been shown [4] that the action for the case of nonextreme Reissner-Nordstrom black holes is smaller than that of the corresponding pair creation rate by exactly a factor of the black hole entropy $S_{BH}$; that is precisely what one would expect if black holes had $e^{S_{BH}}$ internal states. This is not the case nevertheless for extreme Reissner-Nordstrom black holes where [5] the instanton action exactly equals the rate of pair production, though this apparent contradiction has been explained by the argument that these black holes have zero entropy [6].

Apart from the fact that all these considerations are based only on the leading order semiclassical approximation and some higher order terms might be expected [7] to also contribute importantly, the above analyses show two limitations. First of all, they are restricted to deal with Reissner-Nordstrom black holes. It would appear to be of most interest if we could make a similar analysis in Schwarzschild black holes, where there is just an event horizon with unambiguous classical localization and one external parameter, i.e. the mass $M$. On the other hand, there seems to be no clear idea about where exactly the internal states may reside. They might be either inside the black hole or on the horizon and, in the event the first possibility applies, one may still wonder which region of the black hole interior does form the geometrical domain for states, so as exactly on what internal surface are identified the two black holes in a pair while preserving the appropriate contribution of the spacetime to the quantum construct.

An outcome to some of these limitations is attempted in the present paper by considering the geometrical situation that results (Section 2) from identifying a Schwarzschild black hole with a spherically symmetric, four dimensional gravitational topological defect which can move in spacetime but cannot be removed without cutting [8]. As a consequence of imposing invariance of the resulting kink number, we can obtain a maximally extended Kruskal black hole metric which is describable only by means of two coordinate patches. This construct will represent two black holes, each in a different universe, whose interior surfaces at
$r = M$, rather than the event horizons, are identified by the continuity of the tipping over of lightcones. Thus, the two black holes are joined by a nontraversable wormhole thinner than the Einstein-Rosen bridge [9], leaving only a part of their interior regions in the respective universe. By studying the process of thermal emission of such Schwarzschild black holes we have been able to show (Section 3) that they form a black hole-antiblack hole pair, connected by a Tolman-Hawking wormhole [10], even in the Lorentzian case. We have devised a method to calculate (Section 4) the Euclidean action of the corresponding instanton. It is seen that this action is in fact smaller than that associated with the semiclassical rate of pair production by factors that correspond to the entropy of the black holes and to an entropy of the resulting baby universe, $S_U = 2\pi^2 M^2$. We conclude that both the entropy of the black holes and the entropy of the baby universe would provide an actual count of the internal states which are confined to reside, respectively, in the region between the event horizon and the surface at $r = M$, and in the region beyond the latter surface, and can be projected onto their respective enclosing surfaces. We close up with a summary of the results and some further comments. Unless otherwise stated, we shall use natural units so that $\hbar = c = G = 1$, throughout the paper.

2 The Schwarzschild Kink Instanton

In this Section we shall derive an instantonic solution which corresponds to the creation of a pair of Schwarzschild black holes, joined at the internal surface $r = M$. This will be accomplished by analytically continuing into the Euclidean time the maximally extended Schwarzschild-Kruskal metric kink. We first review the way along which this metric can be obtained in a form that is suited for deriving the instanton. We start with the usual, static Lorentzian Schwarzschild metric

$$ds^2 = -(1 - \frac{2M}{r})dT^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2d\Omega^2,$$  \hspace{1cm} (2.1)

where $d\Omega^2$ is the metric on the unit 2-sphere. Metric (2.1) is now transformed into the spherically symmetric kink metric [11]

$$ds^2 = \cos 2\alpha(-dT^2 + dr^2) - 2\sin 2\alpha dt dr + r^2d\Omega^2,$$  \hspace{1cm} (2.2)

where $\alpha$ is the angle of tilt of the lightcones. We ensure one kink to exist by requiring $\alpha$ to monotonously increase from 0 to $\pi$, starting with $\alpha(0) = 0$. Metric (2.2) can be transformed into (2.1) if we choose

$$\sin \alpha = \sqrt{\frac{M}{r}},$$  \hspace{1cm} (2.3)

and introduce the change of time coordinate $T = t + g(r)$, where the function $g(r)$ must be chosen such that

$$\frac{dg}{dr} \equiv g' = \tan 2\alpha.$$  \hspace{1cm} (2.4)
$g'$ is singular at the event horizon where there appears a geodesic incompleteness. Since $\sin \alpha$ cannot exceed unity it follows that $\infty \geq r \geq M$, such that $\alpha$ varies from 0 to $\frac{\pi}{2}$ only. In order to describe a complete one-kink one need therefore a second coordinate patch for $\frac{\pi}{2} \leq \alpha \leq \pi$. The need for such an additional coordinate patch can be better seen by introducing a new time coordinate $\bar{t} = t + h(r)$, where the new function $h(r)$ is defined so that

$$\frac{dh}{dr} \equiv h' = \tan 2\alpha - \frac{k_1}{\cos 2\alpha},$$ (2.5)

with $k_1 = \pm 1$. Then $k_1 = +1$ will correspond to the first coordinate patch and $k_1 = -1$ to the second one. This time re-definition transforms metric (2.2) into the standard kink metric [11]

$$ds^2 = \cos 2\alpha d\bar{t}^2 - 2k_1d\bar{t}dr + r^2d\Omega_2^2.$$ (2.6)

In metric (2.6) the existence of two coordinate patches corresponding to the two possible values of $k_1$ is clearly manifested.

However, geodesic incompleteness at $r = 2M$ is still present in the two coordinate patches of metric (2.6). This geodesic incompleteness can be removed by using the Kruskal method. Thus, let us introduce the general metric

$$ds^2 = -2F(U, V)dUdV + r^2d\Omega_2^2,$$ (2.7)

where

$$U = \mp e^{\gamma \bar{t}} e^{2\gamma k_1 \int_{\infty/M}^r \frac{dr}{\cos 2\alpha}}$$ (2.8)

$$V = \mp \frac{1}{2\gamma M} e^{-\gamma \bar{t}}$$ (2.9)

and

$$F = \frac{4M \cos 2\alpha}{\gamma} e^{-2\gamma k_1 \int_{\infty/M}^r \frac{dr}{\cos 2\alpha}}.$$ (2.10)

The arbitrary parameter $\gamma$ will be chosen such that the resulting metric will not show any singularity other than the curvature singularity at $r = 0$, and the lower integration limit $\infty/M$ refers to the choices $r = \infty$ and $r = M$, depending on whether the case $k_1 = +1$ or the case $k_1 = -1$ is being considered. Let us then evaluate the integral appearing in the exponent of (2.8) and (2.10)

$$\int_{\infty/M}^r \frac{dr}{\cos 2\alpha} = 2M\left(\frac{1}{2} \csc^2 \alpha - \ln\left(\frac{\sin^2 \alpha}{2\sin^2 \alpha - 1}\right)\right),$$

where we have not explicited any real constants coming from lower integration limits. Such constants can always be absorbed in a normalizing factor in the definition of $U$ and $F$.

We avoid the unphysical singularity at $r = 2M$ by setting $\gamma^{-1} = 4Mk_1$. Hence,

$$U = \mp e^{\frac{t}{4 Mk_1}} e^{\frac{2M}{M}(\frac{2M - r}{M})}.$$ (2.11)
\[ V = \mp 2k_1 e^{-\frac{r}{4Mk_1}} \]  \hspace{1cm} (2.12) 

and 
\[ ds^2 = \frac{-32M^3k_1}{r} e^{-\frac{r}{4Mk_1}} dU dV + r^2 d\Omega_2^2. \]  \hspace{1cm} (2.13) 

The use of (2.11) and (2.12) transforms (2.13) into (2.6). Metric (2.13) is the same as the kinkless extension of the Schwarzschild metric unless by the sign ambiguity parameter \( k_1 \) which distinguishes the two coordinate patches.

In order to look for the instanton that corresponds to metric (2.13), we should start with the Schwarzschild instanton derived from (2.1) by the replacement \( T \rightarrow iT \). Then, \( \tilde{T} = \tilde{t} + \tilde{g}(r) \), \( \tilde{\bar{t}} = \tilde{t} + \tilde{h}(r) \), so that \( g' \rightarrow i\tilde{g}' \), \( h' \rightarrow i\tilde{h}' \) and metrics (2.2) and (2.6) respectively become 
\[ d\tilde{s}^2 = \cos 2\alpha (d\tilde{t}^2 + dr^2) - 2i \sin 2\alpha dr d\tilde{t} + r^2 d\Omega_2^2 \]  \hspace{1cm} (2.14) 
\[ d\tilde{s}^2 = \cos 2\alpha d\tilde{t}^2 - 2ik_1 dr d\tilde{t} + r^2 d\Omega_2^2. \]  \hspace{1cm} (2.15) 

The Kruskal extension of (2.15) will lead finally to the metric 
\[ d\tilde{s}^2 = + \frac{32M^3k_1}{r} e^{-\frac{r}{4Mk_1}} d\tilde{U} d\tilde{V} + r^2 d\Omega_2^2, \]  \hspace{1cm} (2.16) 

where \( \tilde{U} = U \) and \( \tilde{V} = V \). Therefore, the instantonic metric (2.16) is the same as the Lorentzian metric (2.13), unless by the sign of the first term in the rhs. In Eqn. (2.5) we chose the sign minus in front of \( k_1 \) in the second term of the rhs. A corresponding plus sign had exchanged the roles played by the two coordinate patches. This is exactly the sole effect we obtain by Euclideanizing metric (2.13). In what follows we shall discuss the meaning of the instanton (2.16) for which 
\[ \tilde{U}\tilde{V} = UV = 2k_1 e^{\frac{r}{2M}} \left( \frac{2M - r}{M} \right). \]  \hspace{1cm} (2.17) 

The Kruskal diagrams for the two coordinate patches are given in Fig. 1, in the case of the Lorentzian metric. The corresponding diagrams for the Euclidean metric can be simply obtained from those of Fig.1 by changing the sign of \( k_1 \) both in each patch and in the original regions \( (I_k_1, II_k_1) \) and the new regions created in the Kruskal extension \( (III_k_1, IV_k_1) \). We first note that the curvature singularity at \( r = 0 \) is avoided because of the continuity of the angle of tilt \( \alpha \) on \( \alpha = \frac{\pi}{2} \). This in turn implies identification of hypersurfaces \( r = M \) of the two patches, both for original and new regions, and Lorentzian and Euclidean metrics. These identifications amount to the existence of bridges connecting the two coordinate patches.

In order to analyse physical processes taking place in the diagrams it is instructive to consider paths followed by null geodesics on them. For the geodesic labelled \( a_1a_2a_3 \), the segment \( a_1 \), which starts at \( r = \infty \), crosses the \( V \) axis on the event horizon \( r = 2M \), and then passes from original region \( I_+ \) to \( II_+ \). It continues as segment \( a_2 \) passing into original region \( I_- \) of the second patch, to
cross after $t = -\infty$ ($U$ axis) into the new region $III_-$, to end up at $r = \infty$ on this region. The whole process corresponds to the transition from an asymptotically flat region in a coordinate patch to another asymptotically flat region in the other coordinate patch, both in Lorentzian spacetime. It takes place through a bridge (wormhole) with size $r = M$, in a similar fashion to that occurs in the Einstein-Rosen bridge [9]. The main an essential difference is that in the present case the connection happens at $r = M$, rather than $r = 2M$.

Complete transitions with null geodesics in our Lorentzian spacetime cannot occur however since the crossing of the horizon in patch $k_1 = -1$ will take an infinite Lorentzian time $t$ in the process considered for geodesic $a_1a_2a_3$. This leaves any null ray trapped inside the black hole of patch $k_1 = -1$ whose asymptotic region therefore will never be reached. This difficulty is no longer present in the Euclidean metric where a true instantonic tunnelling connecting the asymptotic regions of the two patches makes the transition possible. To be sure, in the Euclidean case paths would also cross the horizon $\bar{V} = 0$, but this occurs now at an infinite imaginary time, outside the lightcones. Passage through the bridge can thus be regarded as a quantum tunnelling between two distinct universes with the branching off of a baby universe the size $M$. This can better be illustrated by using, instead of $\bar{t}$, the new coordinates $\tau = f(\chi) + t$, $r = a(\tau) \sin \chi$, with $0 \leq \chi \leq \pi$, and restricting to the surface $r = M$ (i.e. $\alpha = \pi/2$), with

$$\frac{df}{d\chi} = ak \sin \chi = kM, \quad \frac{da}{d\tau} = \frac{k}{\cos \chi},$$

where again $k = \pm 1$ for the two coordinate patches. Metric (2.2) on $r = M$ can then be rewritten

$$ds^2_{W} = \left(1 - \frac{M^2}{a^2}\right)^{-1}d\tau^2 + a^2 d\Omega_3^2 = a^2 (d\eta^2 + d\Omega_3^2) \quad (2.18)$$

$$a = (M^2 + t^2)^{1/2} = Me^{\eta}, \quad f = kM \chi + \text{Const.}, \quad (2.19)$$

in which $d\Omega_3^2 = d\chi^2 + \sin^2 \chi d\Omega_2^2$ is the metric on the unit 3-sphere. We have thus attained the metric of the Tolman-Hawking wormhole [10], with the scale factor given in terms of the Lorentzian time $t$ or $\eta = \int \frac{d\tau}{\tau}$. In the Euclidean picture, from (2.14) on $r = M$ and $i\bar{\tau} = \tilde{f}(\chi) + \bar{t}$, $r = a(\tilde{\tau}) \sin \chi$, instead of $\bar{t}$, we arrive at

$$ds^2_{W} = \left(1 - \frac{M^2}{a^2}\right)^{-1}d\tilde{\tau}^2 + a^2 d\Omega_3^2 = a^2 (d\tilde{\eta}^2 + d\Omega_3^2) \quad (2.20)$$

$$a = (M^2 - \tilde{\tau}^2)^{1/2} = Me^{i\tilde{\eta}}, \quad \tilde{f} = ikM \chi + \text{Const.}, \quad (2.21)$$

which is the metric of the Tolman (baby) universe [10], now with the scale factor expressed in terms of the Euclidean time $\tilde{\tau}$ or $\tilde{\eta} = \int \frac{d\tilde{\tau}}{\tilde{\tau}}$.

It appears then that a connection between black holes and wormholes can be achieved if we let lightcones of a single black hole spacetime to tip over from $\alpha = 0$ to $\alpha = \pi/2$ in a given coordinate patch. Then, conservation of the resulting kink
number would imply the emergence of a second black hole in a different coordinate patch whose lightcones would tip over between $\frac{\pi}{2}$ and $\pi$. On the internal surface $\alpha = \frac{\pi}{2}$ the two black holes are connected through a Tolman-Hawking wormhole (in the Lorentzian case) or a baby universe (in the instantonic case).

It is known that the complete kinkless Schwarzschild-Kruskal spacetime can be described by a single coordinate patch with a functional form of the metric given by (2.13) just for $k_1 = +1$. This patch is somewhat larger than that in Fig. 1 as the upper and lower hyperbolae are located at $UV = 4$ (i.e. $r = 0$) in the kinkless case. Since this spacetime is geodesically complete on a single patch it is possible to foliate its Kruskal diagram with spacelike hypersurfaces which start from left infinity and cut the diagram all the way through, to finally reach right infinity after crossing the horizon twice. This is no longer the case when a kink is present. The hypersurfaces would then have to be continued over from the right infinity of patch $k_1 = +1$ into a similar surface starting from left infinity of patch $k_1 = -1$. Unfortunately, this continuation is disallowed for all $r \neq M$. Curves of constant $t$ passing through the two patches offer no solution as they can only foliate either the entire original region $I_+ \cup I_+ \cup I_- \cup I_-$ or the entire new region $III_+ \cup IV_+ \cup III_- \cup IV_-$, but not simultaneously both. On the other hand, these curves are not spacelike everywhere as their slope $dU/dV$ changes sign along each path. Only if the foliating hypersurfaces are on the extreme hyperbolae $r = M$ of one patch can they be identified with the similar hypersurfaces in the other patch. In the Lorentzian Kruskal diagrams these hyperbolae describe a Tolman-Hawking wormhole (2.19) which covers the whole domain of possible values for the scale factor $a$ as one moves from $t = -\infty$ to $t = +\infty$. The whole of the wormhole spacetime can then be foliated by a continuous family of surfaces with $t = T = \text{Const}$.

### 3 The Quantum Black-Hole Pair

In this Section we shall consider the process of thermal emission in the maximally extended spacetimes of the two coordinate patches of the Schwarzschild black hole kink by using the Green function method [12], thereby checking that, in fact, the two involved black holes form a pair. We will see that, quite remarkably, this does not require recoursing to any complexification of the physical time $t$. Actually, it will be shown that the procedure is equally valid and leads to exactly the same results both when applied to Lorentzian and Euclidean black holes. The approach used by Hartle and Hawking [12], according to which one should invoke a general relation between metric periodicity and gravitational temperature, has always seemed [13] rather mysterious as it gives no explanation to the use of the Euclidean version of spacetime. We shall show that in our model periodicity in the metric follows from a perfectly justifiable requirement of mathematical completeness, rather than a suggestive though not very convincing procedure. This result suggests that a proper evaluation of the thermal processes in black
holes need dealing with what would actually be the complete black hole system, i.e. a pair of black holes, each in a different universe, joined by one wormhole at their middle internal surface. Considering just one of such black holes may lead to results which are incomplete or even paradoxical.

In order to see this, let us first compute the most general analytic expression for the time $\bar{t}$ entering the definition of the Lorentzian Kruskal coordinates $U$, $V$. This will require calculating the function $h(r)$ which is obtained by integration of (2.5). Using the change of variables $p = k_1 \cot \alpha$ and the identification $\sin \alpha = \sqrt{\frac{M}{r}}$, we finally arrive at

$$\bar{t} = t + 4Mk_1 \left\{ k_1 \cot \alpha - \frac{1}{4} \csc^2 \alpha + \ln \left( \left[ \frac{(\sin \alpha - k_1 \cos \alpha) \sin^2 \alpha}{(\sin \alpha + k_1 \cos \alpha)(2 \sin^2 \alpha - 1)} \right]^{\frac{1}{2}} \right) \right\} + 2Mk_1 k_2 i(1 - k_2)\pi, \quad (3.1)$$

where the new sign ambiguity $k_2 = \pm 1$ arises from the square root in the argument of the ln, and any real integration constant has been absorbed in time $t$ which will now depend on $k_1$, $t \equiv t(k_1)$. The constant imaginary term in (3.1) is only nonzero for $k_2 = -1$ and will prove to be essential for a consistent derivation of the black hole thermal effects using the Green function approach. Now, one would again recover (2.6) from (2.7) with the same requirements as in Section 2 if we re-define the Kruskal coordinates as follows

$$U = \bar{U} = \pm k_2 e^{\frac{i}{4}iMk_1k_2} e^{\frac{2M - r}{M}}, \quad (3.2)$$

$$V = \bar{V} = \pm k_1k_2 e^{-\frac{i}{4}iMk_1k_2} \quad (3.3)$$

where

$$\bar{t}_c = \bar{t} + 4\pi iMk_1k_2, \quad (3.4)$$

with $\bar{t}$ taken to be the real part of (3.1). This choice leaves expressions for $UV = \bar{U}\bar{V}$, $F$, $r$ and the Kruskal metrics (2.13) and (2.16) real and unchanged. For $k_2 = -1$ Eqns. (2.11) and (2.12) become, respectively, the sign-reversed to (3.2) and (3.3); i.e the points ($\bar{t} - 4\pi iMk_1$, $r, \theta, \phi$) on the coordinate patches of Fig. 1 are the points on the new regions $III_{k_1}$ or $IV_{k_1}$, on the same figure, obtained by reflecting in the origins of the respective $U, V$ planes, while keeping metric (2.13) and the physical time $t$ real and unchanged. This leads to identifications of hyperbolae in the new, unphysical regions $III_{k_1}$ and $IV_{k_1}$ with hyperbolae in, respectively, original regions $II_{k_1}$ and $I_{k_1}$.

Let us derive in some detail the thermal effects in our black hole pair model for the Lorentzian Kruskal metric. The treatment for the Euclidean case is completely parallel and essentially leads to the same conclusions.

The evolution of a field along null geodesics as those in Fig. 1 can be described using a quantum propagator. If the field is scalar with mass $m$, such a propagator will be the one used by Hartle and Hawking [12] which satisfies the Klein-Gordon equation

$$(\Box_x - m^2)G(x', x) = -\delta(x, x'). \quad (3.5)$$
We note [12] that for metric (2.13) the propagator \( G(x', x) \) will be analytic on a strip of width \( 4\pi M \) which precisely is that is predicted by the imaginary constant component of (3.4), thus without any need of extending time \( t \) into the Euclidean region. Then, following Hartle and Hawking [12], the amplitude for detection of a detector sensitive to particles of a given energy \( E \), in regions \( I_+ \) and \( II_- \), would be proportional to

\[
\Pi_E = \int_{-\infty}^{+\infty} d\bar{t}_c e^{-i\bar{t}_c E} G(0, \bar{R}', \bar{t}_c, \bar{R}),
\]

(3. 6 )

where \( \bar{R}' \) and \( \bar{R} \) denote respectively \( (r', \theta', \phi') \) and \( (r, \theta, \phi) \). Since time \( \bar{t}_c \) (but not \( t \)) already contains the imaginary constant term which is exactly required for the thermal effects to appear, we need now not make the physical time \( t \) complex. From (3.4) and (3.6) one can then write

\[
\Pi_E = e^{4\pi M k_1 k_2 E} \int_{-\infty}^{+\infty} d\bar{t}_c e^{-i\bar{t}_c E} G(0, \bar{R}', \bar{t}_c + 4\pi i M k_1 k_2, \bar{R}).
\]

(3. 7 )

Let us next consider a point \( x' \) on the hyperbola \( r = M \) of region \( II_+ \), corresponding to patch \( k_1 = +1 \). Since such a hyperbola should be identified with the hyperbola at \( r = M \) of region \( I_- \) of patch \( k_1 = -1 \), the point \( x' \) can be taken to simultaneously belong to the two patches. Then, one can draw null geodesics starting at \( x' \) which connect such a point with different points \( x \) on either the original region \( I_+ \) of patch \( k_1 = +1 \) or the original region \( II_- \) of patch \( k_1 = -1 \).

In the first case, we obtain from (3.7)

\[
P^{I_+}_a(E) = e^{-8\pi M E} P^{I_+}_e(E),
\]

(3. 8 )

where \( P^{I_+}_a(E) \) denotes the probability for detector to absorb a particle with positive energy \( E \) from region \( I_+ \), and \( P^{I_+}_e(E) \) accounts for the similar probability for detector to emit the same energy also to region \( I_+ \), in the coordinate patch \( k_1 = +1 \) corresponding to the first universe. An observer in the exterior original region of patch \( k_1 = +1 \) will then measure an isotropic background of thermal radiation with positive energy, at the Hawking temperature \( T_{BH} = (8\pi M)^{-1} \).

For the path connecting \( x' \) with a point \( x \) on the original exterior region \( II_- \) of patch \( k_1 = -1 \) in the other universe, we obtain for an observer in this region,

\[
P^{II_-}_a(-E) = e^{+8\pi M E} P^{II_-}_e(-E).
\]

(3. 9 )

According to (3.9), in the exterior region \( II_- \) of the second universe there will appear as well an isotropic background of thermal radiation, also at the Hawking temperature \( T_{BH} \), which is formed by exactly the antiparticles to the particles of the thermal bath detected in region \( I_+ \). Whether or not these two backgrounds are mutually correlated is an issue which could become of crucial importance to decide on the problem of the loss of quantum coherence in black holes [14]. Clearly, if as it seems most likely they were correlated to each other, one would then expect quantum coherence to be preserved in the full process involving simultaneous evaporation of the two black holes, though relative to observers in
each universe, there would appear to be loss of coherence. This would be a typical example of an apparent paradoxical result finding a rather natural explanation when regarded in its complete framework.

In any event, an evaporation process would then follow according to which a black hole in a universe will disappear completely, taking with it all the particles that fell in to form the black hole and the antiparticles to emit radiation, by going off through the wormhole of throat radius $M$ whose other end, which opens up in other universe, is another black hole which can be regarded to have been formed from the collapse of massive antifermions, and also evaporated, giving off exactly the antiparticles to the particles of the thermal radiation emitted by the first black hole in the first universe.

For $k_2 = +1$ we obtain similar hypersurface identifications as for $k_2 = -1$. In this case, the identifications comes about in the situation resulting from simply exchanging the mutual positions of the original regions $I_{k_1}$ and $II_{k_1}$ for, respectively, the new regions $III_{k_1}$ and $IV_{k_1}$, on the coordinate patches of Fig. 1, while keeping the sign of coordinates $U, V$ unchanged with respect to those in (2.11) and (2.12); i.e. the points $(\tilde{t} + 4\pi Mk_1, r, \theta, \phi)$ on the so-modified regions are the points on the original regions $I_{k_1}$ or $II_{k_1}$, on the same patches, again obtained by reflecting in the origins of the respective $U, V$ planes, while keeping metric (2.13) and the physical time $\tilde{t}$ real and unchanged. Thus, expressions for the relations between probabilities of absorption and emission for $k_2 = +1$ are obtained by simply replacing region $I_+^+$ for $IV_{+}^+$, region $II_-$ for $III_{-}$, and energy $E$ for $-E$ in (3.8) and (3.9), so that the same Hawking temperature $T_{BH}$ is obtained in all the cases. Hence, ”observers” will detect thermal radiation with energy $E < 0$ on region $IV_{+}^+$, and with energy $E > 0$ on region $III_{-}$, in both cases at the Hawking temperature.

By starting with the instantonic Kruskal metric (2.16), and hence analytically continuing $\tilde{t} \rightarrow i\tilde{t}$ in the above analysis, one can readily see that we attain exactly the same results as in the Lorentzian treatment. This is not surprising actually. After all, the thermal emission of black holes was first discovered using a purely Lorentzian formalism [15]. The set of results achieved so far amounts to the comments included at the beginning of this Section. Moreover, these results make quite tempting to conjecture that if eventually a black hole candidate in our universe is confirmed to exist, then this would also be a proof that there is another universe whose particles are exactly the antiparticles to the particles in ours.

4 On the Origin of Black-Hole Entropy

For the reasons given in the Introduction, it appears now interesting to calculate an exact expression for the Euclidean action of a Schwarzschild pair. The issue would relate with the problem of the origin of the black hole entropy and, moreover, with the question on whether the baby universes that nucleate with the
black hole pair might carry some finite entropy. Pair production appears to be independent of Planck scale physics and therefore it should be an unambiguous consequence from quantum gravity [7].

The amplitude for production of nonrotating, chargeless black hole pairs can be calculated in the semiclassical approximation from the corresponding instanton action which is

\[ S = -\frac{1}{16\pi} \int d^4x \sqrt{g} R - \frac{1}{8\pi} \int d^3x \sqrt{h} K, \]  

(4.1)

where \( K \) denotes the trace of the second fundamental form, and \( h \) is the metric on the boundary. We wish to find the contribution of the two black holes plus their bridging wormhole to action (4.1). This contribution will correspond to substracting from (4.1) the flat metrics that the instanton asymptotically approaches.

Since variation of the tilt angle \( \alpha \) must be continuous at \( r = M \), the above contribution can be evaluated from the corresponding enforced variation \( \delta \) in the time parameter of the instanton at \( r = M \) that leads to a net variation of radial coordinate \( (\delta \ell) \) also on \( r = M \). This variation would be computed with respect to a given coordinate patch, as one passes from that patch to the other, and leaves unchanged the flat metrics. It would correspond to a variation of action (4.1), \( \delta S \), which should be finite when evaluated at \( r = M \), but would diverge on the event horizon. In what follows we shall interpret \( \delta S|_{r=M} \) as the contribution of the two black holes and the intermediate wormhole to instanton action (4.1).

Let us then evaluate \( \delta S \). Using Einstein equations we find

\[ \delta S = -\frac{1}{8\pi} \int d^3x \delta (\sqrt{h} h^{ik} K_{ik}) \]

\[ = -\frac{1}{8\pi} \int d^3x \left( \sqrt{h} (\delta h^{ik})(K_{ik} - \frac{1}{2} h_{ik} K) + \sqrt{h} h^{ik} \delta K_{ik} \right). \]  

(4.2)

The last term in the rhs of (4.2) can be written

\[ h^{ik} \delta K_{ik} = \frac{1}{2} h^{ik} \frac{\partial}{\partial t} \delta h_{ik} = \frac{1}{2} \frac{\partial}{\partial x^l} h^{ik} \delta h_{ik} = \frac{1}{2} \frac{\partial A^l}{\partial x^l}, \]

with \( A^l = h^{ik} \delta h_{ik} \) a contravariant vector, and the overhead dot meaning time derivative. Hence

\[ h^{ik} \delta K_{ik} = \frac{1}{2} \frac{\partial}{\partial x^l} (\sqrt{h} A^l), \]

so that

\[ \int d^3x \sqrt{h} h^{ik} \delta K_{ik} = \frac{1}{2} \int d^3x \frac{\partial}{\partial x^l} (\sqrt{h} A^l). \]

This can be now converted into an integral over \( A^l \) extended to the 2-surface surrounding the boundary. Since variations of the field are all zero on the boundary, this term must vanish and we have finally

\[ \delta S = -\frac{1}{16\pi} \int_{S^2} d^3x \sqrt{h} (\delta h^{ik})(2K_{ik} - h_{ik} K), \]  

(4.3)
where $S^2$ denotes the 2-sphere on $r = M$.

For the Schwarzschild metric, we then have

$$
\delta S = \frac{M}{\pi} \left[ -\frac{1}{24} \rho^{-\frac{3}{2}} + \rho^{-\frac{1}{2}} + \arctan(\rho^{\frac{1}{2}}) \right] \int_0^\pi d\theta \int_0^{2\pi} d\phi (\delta r)
$$

$$
= \left( -\frac{M}{24} + M + \frac{\pi M}{4} \right) \int_0^{2\pi} d\phi (\delta r).
$$

(4.4)

where $\rho = \frac{2M}{r} - 1$. We evaluate variation $(\delta r)$ at $r = M$ by considering null geodesics that cross each other at exactly the surface $r = M$, going always through original regions on the Kruskal diagrams for the two patches. For such geodesics we have $t_{k_1=+1} = t_{k_1=-1}$. Thus, if in passing from the original regions of patch $k_1 = +1$ to the original regions of patch $k_1 = -1$ time $t$ remains constant, time $t$ must change on $r = M$ according to (Ref. Eqn. (3.1))

$$
\delta t|_{r=M} \equiv t_{k_1=+1} - t_{k_1=-1}|_{r=M} = 2M.
$$

(4.5)

In order to calculate the corresponding rate $\frac{dr}{dt}$ from patch $k_1 = +1$, we note that

$$
\frac{dt}{dr} \biggr|_{r=M} = \left( \frac{dt}{dr} + \tan 2\alpha \frac{1}{\cos 2\alpha} \right) \biggr|_{r=M} = 0.
$$

(4.6)

Hence,

$$
\frac{dt}{dr} \biggr|_{r=M} = -1.
$$

(4.7)

We have then

$$
(\delta r)_{k_1=+1\rightarrow k_1=-1} = \delta t|_{r=M} \frac{dr}{dt} \biggr|_{r=M} = -2M.
$$

(4.8)

This would be the contribution of just the black hole in the universe described in coordinate patch $k_1 = +1$ and the half of the associated wormhole relative to the same universe. Evaluation of $(\delta r)_{k_1=-1\rightarrow k_1=+1}$, corresponding to the black hole and wormhole half for the other universe, leads to the same value as in (4.8). Therefore, the full variation of radial coordinate implied by invariance of the kink number becomes $(\delta r) = -4M$. Hence, from (4.4) we finally obtain as the exact expression for the Euclidean action of the two black holes plus the wormhole in a pair

$$
I = \delta S = \frac{\pi}{3} M^2 - 8\pi M^2 - 2\pi^2 M^2.
$$

(4.9)

The semiclassical production rate is then

$$
e^{-I} \sim \exp(-\frac{\pi}{3} M^2 + 8\pi M^2 + 2\pi^2 M^2).
$$

(4.10)

We interpret now the distinct factors in (4.10). Note first that the factor $e^{-\frac{\pi}{3} M^2}$ should give the full rate of Schwarzschild black hole pair production in the gravitational field created by a body with mass $M$ made of fermions in a universe,
and another body with the same mass, but having exactly the antifermions to the fermions of the first body, in the other universe.

The second factor in (4.10) gives the sum of the entropies of the two black holes, each being \( S_{BH} = 4\pi M^2 = \frac{1}{4}A_{BH} \), where \( A_{BH} \) is the surface area of a single black hole. This entropy can also be obtained from the black hole temperature \( T_{BH} \) derived in Section 3 by insertion into the thermodynamic formula \( T_{BH}^{-1} = \frac{\partial S}{\partial E} \).

The last factor in (4.10) should then be interpreted as the entropy of the nucleated baby universe. This would be a closed universe with volume \( V_U = 2\pi^2 R^3 \), in which \( R \) is the radius of the baby universe, \( R = M \). Since the term \( \frac{4}{3}M^2 = S_p \) has been interpreted as the action for the production rate of a Schwarzschild black hole pair and, according to (4.9), the total Euclidean action is smaller than \( S_p \) by \( 2S_{BH} \) and \( 2\pi^2 M^2 \), then each of the factors \( e^{2S_{BH}} \) and \( e^{2\pi^2 M^2} \) should count a given number of internal states. One would therefore expect the latter factor to be associated with a maximum entropy proportional to the bounded volume of a closed space, that is

\[
S_U \propto \frac{V_U}{l_p^3} = \frac{2\pi^2 M^3}{l_p^3}, \quad (4.11)
\]

where, for a moment, we have explicited the Planck length \( l_p = \sqrt{\frac{\hbar}{G}} \).

However, there is good reason to believe that gravitational entropy is proportional to the surface area and not the volume of the bounded region. In this sense, a transition from volume to area has been proposed by 't Hooft [16] by invoking a "holographic projection" according to which it is possible to describe all internal states within a bounded volume by a set of degrees of freedom which reside on the surface bounding the given volume, without any loss of actual information. Because we now know where the internal states of black holes and wormholes may reside, a particular implementation of this volume→area transition can be given. Thus, if one would assume that the internal states of a black hole are in the entire volume \( \frac{4}{3}\pi R^3 \) of a 2-sphere, the number of such states would be

\[
\frac{4}{3}\pi \frac{R^3}{l_p^3}.
\]

The actual number of black hole internal degrees of freedom could then be estimated by the following ansatz: shrink first any of the three dimensions to \( l_p \) and then restrict to the surface whose degrees of freedom result from holographic projection of the region where the states can actually reside. Recalling that black hole states should all be within the spherical shell between \( 2GM \) and \( GM \), but not within the spherical region of radius \( r = GM \), we get for the degrees of freedom whose count would produce the black hole entropy

\[
\frac{4}{3}\pi \left( \frac{R^3}{l_p^3} \right)_{2GM}^{GM} = 4\pi GM^2 = S_{BH}. \quad (4.12)
\]
When applied to a closed space with volume $2\pi^2 R^3$, this ansatz leads then to

$$S_U = 2\pi^2 R^2 \frac{G M}{l^2} \bigg|_0^{GM} = 2\pi^2 G M^2 = \frac{1}{2} \pi G A_U,$$

(4.13)

where $A_U$ is the surface area for the baby universe. Restoring full natural units, we see that (4.13) exactly coincides with the last term in the rhs of (4.9).

If we had evaluated $\delta S$ at the event horizon $r = 2M$, instead of $r = M$, in (4.4), then the action had diverged so that $\delta S \to +\infty$, as it was expected.

For such a case, moreover, the rate of pair production had vanished identically, and the entropy for black holes went to its classical infinite value, while that for baby universe vanished. It appears therefore that black hole pairs joined at the event horizon cannot be produced as they would correspond to perfectly classical spacetime constructs.

Consistency of the above interpretations can only be achieved if we look at the factors $e^{S_{BH}}$ and $e^{S_U}$ as counts of, respectively, either the number of physically relevant black hole internal states residing in between the event horizon and the interior surface at $r = M$, and the number of physically relevant internal states beyond $r = M$ of a baby universe, or rather the number of degrees of freedom on the event horizon surface and on the surface $R = M$. Positiveness of the full exponent in (4.10) leads, on the other hand, to the remarkable feature that although the rate of pair production is maximum for Planck-sized black holes, once one of such pairs is formed, the semiclassical probability (4.10) will tend to favour processes in which the mass of the black holes, and hence the size of the baby universe, increase endlessly.

5 Summary and Conclusions

In this paper we have presented arguments in favour of the existence of an instanton that describes the production of pairs of chargeless, nonrotating black holes with any mass $M$, which are joined on the interior surface $r = M$. The latter feature distinguishes these constructs from the Einstein-Rosen bridge where junction occurs on the event horizon surface, at the bifurcation point $U = V = 0$ on the kinkless Kruskal diagram. We have interpreted these Schwarzschild black hole pairs as gravitational topological defects that avoid the curvature singularity at $r = 0$.

The geodesic incompleteness at $r = 2M$ in the spacetime metric representing these pairs is removed by using the Kruskal method, both in the Lorentzian and Euclidean metrics. The only difference in the functional form of the resulting two metrics is in the sign for the metric component for Kruskal coordinates.

The process of thermal emission by a pair has been studied by employing the Green function technique in the purely Lorentzian case, without recoursing to any complex time. We obtain thus the result that the particles of the radiation emitted by one black hole are the antiparticles to the particles in the radiation
emitted by the other black hole. This result suggests that black holes can never be individually produced, but only in black hole-antiblack hole pairs, each black hole in a pair being in a different universe.

An exact expression has been calculated for the action of the instanton pair. This action is smaller than that for the corresponding rate of pair production by exactly the Bekenstein-Hawking entropy of the two black holes, and by an entropy for baby universe given by $S_U = 2\pi^2 M^2$. We conclude therefore that these entropies count the number of, respectively, physically relevant black hole internal states and baby universe internal states (i.e. the degrees of freedom on, respectively, the event horizon and the interior surface at $r = M$), and that although the rate of pair production is maximum for Planck-sized black holes, there is a small but still nonvanishing probability for the nucleation of baby universes with macroscopic sizes. On the other hand, overall positiveness of the exponent of the semiclassical probability implies that once a Planck-sized baby universe is created, it will tend to expand, in accordance with the second law of thermodynamics. From this standpoint, annihilation of black hole pairs [6] could only occur through Hawking evaporation or, statistically, in a large ensemble of pairs while the total entropy of the system still increases. The model could thus be implemented as a cosmogonic model as far as it actually implies spontaneous creation of a closed Tolman universe which can be driven to expansion. Finally, it is worth noticing that this model offers a suitable scenario where the often assumed connection between black holes and wormholes can be implemented. Actually, we have seen that in fact the metric of these wormholes is not but a natural continuation of the Lorentzian Schwarzschild half one-kink metric.

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References

1 G ’t Hooft, Nucl. Phys. B256 (1985) 727; V Frolov and I Novikov, Phys. Rev. D48 (1993) 4545; S Carlip and J Uglum, Phys. Rev. D50 (1994) 2700.

2 V Frolov, *Black Hole Entropy*, [hep-th/9412211] (1994).

3 D Garfinkle and A Strominger, Phys. Lett. 256B (1991) 146.

4 D Garfinkle, S Giddings and A Strominger, Phys. Rev. D49 (1994) 958.

5 H F Dowker, J Gauntlett, S Giddings and G T Horowitz, Phys. Rev. D50 (1994) 2662.

6 S W Hawking, G T Horowitz and S F Ross, *Entropy, Area, and Black Hole Pairs*, [gr-qc/9409013].

7 G T Horowitz, in: *Matters of Gravity*, No. 5 (1995) 10.

8 D Finkelstein and C W Misner, Ann. Phys. (N.Y.) 6 (1959) 230; D Finkelstein, in: *Directions in General Relativity I*, eds. B L Hu, M P Ryan Jr. and C V Vishveshwara (Cambridge Univ. Press, Cambridge, 1993).

9 A Einstein and N Rosen, Phys. Rev. 48 (1935) 73.

10 S W Hawking, in: *Astrophysical Cosmology: Proceedings of the Study Week on Cosmology and Fundamental Physics*, eds. H A Brck, G V Coyne and M S Longeir (Pontificiae Academiae Scintiorum Scripta Varia, Vatican City, 1982), p. 563; J J Halliwell and R Laflamme, Class. Quant. Grav. 6 (1989) 1839; P F Gonzalez-Daz, Phys. Rev. D40 (1989) 4184.

11 D Finkelstein and G McCollum, J. Math. Phys. 16 (1975) 2250.

12 J B Hartle and S W Hawking, Phys. Rev. D31 (1976) 2188.

13 N Pauchapakesan, in: *Highlights in Gravitation and Cosmology*, eds. B R Iyer, A Kembhavi, J V Narlikar and C V Vishveshwara (Cambridge Univ. Press, Cambridge, 1988).

14 S W Hawking, Phys. Rev. D14 (1976) 2460; Phys. Rev. D37 (1988) 904.

15 S W Hawking, Commun. Math. Phys. 43 (1975) 199.

16 G ’t Hooft, *Dimensional Reduction in Quantum Gravity*, Utrecht Preprint THU-93/26 (1993).
Legend for Figure

Fig. 1: Kruskal diagrams for the two coordinate patches \( (k_1 = \pm 1) \) of the one-kink extended Schwarzschild metric. Each of these patches is regarded as a different universe. Points on the diagrams represent 2-spheres. The null geodesic discussed in the text is the straight line labelled \( a_1a_2a_3 \) on the diagrams. The hyperbolae at \( r = M \) are identified on, respectively, the original regions \((II_+ \text{ and } I_-)\) and the new regions \((III_+ \text{ and } IV_-)\) created by the Kruskal extension.
This figure "fig1-1.png" is available in "png" format from:

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