Adaptive data fusion method based on Kalman filter and its application in piston shape and position detection

Leiguang Nan1, XinqiangMa1, HailongGe1, QingzengMa1, YanleiLi1, Wei Cheng†*†

1Laser Research InstituteQilu University of Technology (Shandong Academy of Sciences)Jinan, Shandong, China
*chengw@sdas.org
†Corresponding author’s e-mail:chengweijob@163.com

Abstract. In order to meet the needs of production, this paper studies the detection data processing algorithm for the angle of the piston shape and position, the groove width and the runout of the groove bottom. An adaptive data fusion method based on Kalman filtering is proposed. This algorithm can use the adaptive method to calculate the different weight coefficients of each data point according to the standardized sampling data and the smoothed data points obtained by the cubic B-spline approximate fitting algorithm. This paper obtain the fusion data; then calculate the groove angle, groove width and groove bottom runout of the piston according to the conversion formula. Finally, experimental data verification is carried out, which shows the effectiveness and good robustness of the data processing of this algorithm. At the same time, a piston shape and position detection system was developed. The system rotates a circle at 90 degrees per second to obtain the measurement data. After the system software and hardware processing, the groove angle detection accuracy can reach ±2 minutes, the groove width and the groove bottom. The detection accuracy of the runout amount is within ±2 microns, and the measurement items in the measurement time and accuracy side meet the customer's technical document requirements and achieve satisfactory results.

1. Preface
For the safe operation of the engine, the piston shape and position manufacturers need to perform precise inspections on the groove width, groove angle and runout of the piston shape and position produced to screen out unqualified pistons, so as to avoid engine production due to piston machining accuracy problems serious consequences[2]. Related industries at home and abroad have earlier proposed and applied the detection methods of groove width, groove angle and runout to measure the process quality of the piston shape and position[3]. The main direction of this paper is to calculate the runout of 4 measurement items (1# slot angle, 2# slot width, 3# slot width and 3# slot bottom), judge whether the piston shape and position are qualified by the result size, and mark the position of the relevant measurement parameters, as shown in Figure 1.
In order to make the algorithm parameters easy to quantify and fix in order to enhance the stability of the algorithm [4], firstly, the sampled data is subjected to limiting filtering method to eliminate bad points and standardization processing; secondly, the cubic B-spline approximate fitting algorithm [5] is used for curve fitting. In order to smooth the sampling curve, obtain the overall change amount and change trend of the data; then an adaptive data fusion method based on Kalman filtering [6] is proposed. This algorithm can also retain the sampling points as much as possible during the smoothing filtering process. In order to retain the true value as much as possible, the robustness of the algorithm is enhanced [7].

2. Data preprocessing

2.1. Dead pixel removal
Define the piston to perform 360-degree (1 revolution) rotation sampling, each pulse rotates 0.005 degrees, read the current value of the encoder to read and record the current sampling value every 10 pulses, so that the piston shape and position rotate 1 revolution to get 7200 Sampling value, in order to eliminate random interference, the commonly used limiting filter method is used to eliminate bad pixels. The relevant formula is as follows:

\[
y_j(i) = \begin{cases} y_j(i), & y_j(i) - y_j(i-1) \leq A \\ y_j(i-1), & y_j(i) - y_j(i-1) > A \end{cases}
\]

Basic method: definition \( j(0 < j \leq 4; \text{j is an integer}) \)

respectively represent all the sampling values of 1# slot angle, 2# slot width, 3# slot width or 3# slot bottom, the i-th sample value of the measurement item \( y_j(i) \) ( \( 0 < i \leq 7200 \) ), i is an integer. Compare the two sampled values \( y_j(i) \) and \( y_j(i-1) \) of adjacent i and i-1, determine the maximum deviation allowed for two samples \( A \) (According to the test article \( A=0.025 \)), more than the \( A \), \( y_j(i-1) \) insted \( y_j(i) \); If not more than \( A \), then keep the sampled value \( y_j(i) \).

2.2. Data standardization
In order to make the parameters of the follow-up Kalman algorithm and cubic B-spline approximation fitting algorithm easy to solidify and enhance the good adaptability of the algorithm, it is very necessary to standardize the collected data. For convenience, the following linear standardization method is used [8]:

\[
w_j(i) = \frac{y_j(i) - \min\{f_j\}}{\max\{f_j\} - \min\{f_j\}}
\]

Among them, the symbol \( \{f_j\} \) and \( \{f_j\} \) respectively represent the maximum and minimum values of one cycle sampling value of the j-th measurement item among the selected 4 measurement items, \( y_j(i) \) its i-th sampled value. After the above standardization operation, the value
ranges of all features are mapped to the [0 1] interval. At the same time, add value \( w_j(7201) = w_j(1) \), by \( W_j = [w_j(1), w_j(2), \ldots w_j(n)] \), heren = 7201, get 4 measurement item matrix representation: \( W = [W_1, W_2, W_3, W_4] \). Definition \( F_j = \max \{f_j\} - \min \{f_j\} \). available: \( F = [F_1, F_2, F_3, F_4] \).

3. Cubic B-spline approximate fitting algorithm
In this paper, the cubic B-spline approximate fitting algorithm [9] is used to perform curve fitting on each point of \( n=7201 \), using this algorithm from \( w_j(1) \) start, fit a curve with every 4 consecutive points, and repeat this process for the next curve from the 4-th point. So 7201 points can fit 2400 curves in total, here each curve ism (0 < m ≤ 2400) said. This article the equation of the m-th cubic B-spline curve of each measurement item is as follows:

\[
P_{m,j}(t) = \sum_{i=3m-2}^{3m+1} w_j(i) E_{1,3}(t)
\]

At the same time, according to the basis function formula in the cubic B-spline curve equation, the following equation can be obtained:

\[
E_{3m-2,3}(t) = \frac{1}{6}(1 - t)^3
\]

\[
E_{3m-1,3}(t) = \frac{1}{6}(3t^3 - 6t^2 + 4)
\]

\[
E_{3m,3}(t) = \frac{1}{6}(-3t^3 + 3t^2 + 3t + 1)
\]

\[
E_{3m+1,3}(t) = \frac{1}{6}t^3
\]

According to the above formula, the m-th cubic B-spline curve of the final j-th measurement item can be obtained\( P_{m,j}(t) \). According to the above formula, the first \( j \) all curves fitted by each measurement item. Then \( t \) equalsw_j(3m - 2),w_j(3m - 1),w_j(3m) and \( w_j(3m + 1) \) bring in the curve generated by the fit\( P_{m,j}(t) \). available points\( p_j(3m - 2),p_j(3m - 1),p_j(3m) \) and \( p_j(3m + 1) \). then the m-th curve of the j-th measurement item\( P_{m,j}(t) \) calculated 4-th point\( p_j(3m + 1) \) and the m+1-th curve\( P_{m+1,j}(t) \) calculated first point\( p_j(3(m + 1) - 2) \) perform average calculation to get \( v_j(3m + 1) \), however\( p_j(3m - 1) \) and\( p_j(3m) \) no operation (for ease of subsequent expression, add this 2 points to\( v_j(3m - 1) \) and \( v_j(3m) \) said). What needs to be explained is: the first curve\( P_{1,j}(t) \) and 2400th curve \( P_{2400,j}(t) \) calculated points \( p_j(1) \) and \( p_j(7201) \) no calculations, to\( v_j(1) \) and \( v_j(7201) \) said. Therefore, the value of each point calculated after fitting and smoothing can be obtained\( v_j(i) \), the vector is expressed as \( V_j = [v_j(1), v_j(2), \ldots v_j(n)] \), here \( n = 7201 \).

4. Adaptive data fusion method based on Kalman filter [10]
the adaptive data fusion method proposed in this paper uses formula (4.1) for data fusion to obtain\( r_j(i) \), this value represents the optimal value after data fusion. Through standardized sampling dataw_j(i) and the value obtained after B-spline fitting\( v_j(i) \) the value obtained by the difference is\( \theta_j(i) \). \( \theta_j(i) \) it can reflect the deviation between the sampled value and the fitted and smoothed value. The adaptive data fusion method is determined by the Kalman filtering methodj-th scale factor of the i-th point of the measurement item\( \alpha_j \). \( \tau_j(i) \) represents the slope value of the i-th point of the j-th measurement item.

\[
\tau_j(i) = \alpha_j(i)w_j(i) + (1 - \alpha_j(i))v_j(i) \quad (4.1)
\]

At the same time make the following definitions:

\[
\theta_j(i) = w_j(i) - \tau_j(i) \quad (4.1.1)
\]

\[
\theta_j(i) = \dot{v}_j(i) \quad (4.1.2)
\]
\[ a_j(i) = \frac{\dot{\theta}_j(i)}{\dot{\theta}_j(i)} \tag{4.1.3} \]

The symbols in this article are as follows: with superscript ‘~’ the amount represents the predicted value. With superscript ‘\(~\)’ the amount represents Kalman calculated optimal value. For the convenience of presentation, all the following Step1~Step6 represent the runout of 1# slot angle, 2# slot width, 3# slot width or 3# slot bottom measurement item \( j(0 < j \leq 4): j \) is an integer omits, the default is to take the \( j \)-th measurement item to perform the next measurement item calculation, and the 4 measurement items are respectively performed the following calculations, independent of each other, and have no correlation. The entire Kalman filtering process is divided into a prediction stage and a correction stage. The detailed steps (Step1~Step6) are as follows:

**Prediction phase:**

**Step1:** Define the predicted value: \( \hat{X}(i) = \begin{bmatrix} \hat{\theta}_i \\ \hat{\dot{\theta}}_i \end{bmatrix}^T \), calculated optimal value \( \bar{X}(i) = \begin{bmatrix} \hat{\theta}_i \\ \hat{\dot{\theta}}_i \end{bmatrix}^T \), \( A = \begin{bmatrix} 1 & -dt \\ 0 & 1 \end{bmatrix} \), \( B = \begin{bmatrix} dt \\ 0 \end{bmatrix} \), \( G = \dot{\theta}_i(i) \) represents acceleration value, data sampling interval: \( dt = 0.005 \), from the Kalman prior estimation formula: \( \hat{X}(i) = A\bar{X}(i-1) + BU(i) \), bring the definition into it to get \( \hat{X}(i) \):

\[
\begin{bmatrix} \hat{\theta}_i \\ \hat{\dot{\theta}}_i \end{bmatrix} = \begin{bmatrix} 1 & -dt \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\theta}_{i-1} \\ \hat{\dot{\theta}}_{i-1} \end{bmatrix} + \begin{bmatrix} dt \\ 0 \end{bmatrix}
\]

**Step2:** According to the Kalman error covariance estimation formula, according to the formula: \( \bar{P}(i) = \bar{A}(i-1)A^T + Q \), get \( \bar{P}(i) \). In the formula, \( Q \) is the covariance matrix of vector \( A \), and because of the \( \theta_i \) and \( \dot{\theta}_i \) are independent of each other, so you can get:

\[
Q = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}
\]

\( S_1 \) and \( S_2 \) represent cooperative parties. According to experiments and calculations, in this article: \( S_1 = 0.001, S_2 = 0.003 \).

**Calibration phase:**

**Step3:** According to the Kalman gain formula, according to the formula: \( Kg(i) = \bar{P}(i)H^T/(H\bar{P}(i)H^T + R) \), find the Kalman gain coefficient \( Kg(i) \), \( Kg(i) \) it is a two-dimensional vector, defined as follows:

\[
Kg(i) = [Kg_\theta(i) Kg_\dot{\theta}(i)]^T
\]

Because \( \theta_i \) and \( \dot{\theta}_i \) there is no correlation, so the measurement equation of the formula is deduced by the Kalman filter, which can make \( H = [1 \ 0] \). At the same time in this article, \( R = [\dot{\theta}_i(i) \ \dot{\theta}_i(i)]^T \).

**Step4:** Definition: \( X(i) = [\theta_i \ \dot{\theta}_i]^T \), modified formula by Kalman: \( \bar{X}(i) = \bar{X}(i) + Kg(i)(X(i) - H\bar{X}(i)) \), the best value can be obtained \( \bar{X}(i) \).

**Step5:** Update formula from Kalman error covariance: \( \bar{P}(i) = (1 - Kg(i)H)\bar{P}(i) \), the best estimated covariance can be obtained \( \bar{P}(i) \). According to the definition of Kalman filtering, \( I \) is the following identity matrix.

\[
I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

**Step6:** If \( i < n \) (This article \( n < 7201 \) ), return to Step1 to calculate the next optimal value \( \bar{X}(i + 1) \).

After the above steps, you can get \( \dot{\theta}_i(i) \) and \( \ddot{\theta}_i(i) \), calculated according to formula (3.1.3), we can get \( \ddot{\theta}_i(i) \). Finally, the data fusion is obtained by formula (3.1) \( \tau_j(i) \) the vector is expressed as: \( \tau_j = [\tau_j(1), \tau_j(2), \ldots \tau_j(n)], heren = 7201 \).

Find the maximum value of the \( j \)-th measurement item \( \max\{r_j\} \) and minimum \( \min\{r_j\} \), corresponding maximum \( \max \{\max \} \) the point number is \( i_{\max} \). The jitter of the \( j \)-th measurement item can be obtained by formula (4.2) \( d_j \). Among them \( F_j = \max\{\{\} - \min\{\{\} \} \), is the difference between the maximum and minimum values obtained in the data standardization step.
\[ d_j = (\max\{r_j\} - \min\{r_j\}) \cdot F_j \quad (4.2) \]

5. Experimental data and analysis

In order to verify the effectiveness of the adaptive Kalman data fusion method proposed in this paper, the laboratory standard piston shape and position are selected for verification test. The actual groove angle, groove width and groove bottom runout value of this standard piston shape are shown in Table 1.

Table 1. Actual values of standard piston shape and position parameters

| NO. | 1# slot angle | 2# slot width (mm) | 3# slot width (mm) | 3# slot bottom runout (mm) |
|-----|---------------|---------------------|--------------------|---------------------------|
| 1   | 15°02’04’’    | 3.0689              | 4.0305             | 0.0253                    |
| 2   | 15°01’36’’    | 3.07                | 4.0298             | 0.0252                    |
| 3   | 15°02’30’’    | 3.0699              | 4.0306             | 0.0253                    |
| 4   | 15°01’46’’    | 3.0699              | 4.0305             | 0.0255                    |
| 5   | 15°01’47’’    | 3.07                | 4.0305             | 0.0253                    |
| 6   | 15°02’30’’    | 3.0699              | 4.0294             | 0.0242                    |
| 7   | 15°01’48’’    | 3.0699              | 4.0308             | 0.0268                    |
| 8   | 15°02’13’’    | 3.07                | 4.0297             | 0.0252                    |
| 9   | 15°01’52’’    | 3.07                | 4.0303             | 0.0256                    |
| 10  | 15°02’40’’    | 3.0699              | 4.0302             | 0.0256                    |
| 11  | 15°03’4’’     | 3.0699              | 4.0297             | 0.0256                    |
| 12  | 15°02’28’’    | 3.0699              | 4.0306             | 0.0253                    |
| Mean| 15°02’14’’    | 3.069925            | 4.030475           | 0.02538                   |

Table 2 shows the measured values of the groove angle, groove width and runout of the groove bottom 12 times of the standard piston shape and position at a rotation speed of 90 degrees/sec using the algorithm proposed in this paper. The average value is the average angular position of the 12 measurements. Observing and analyzing the data, it can be seen that a better effect has been achieved.

Table 2. Detected values of standard piston shape and position parameters (90 degrees/sec)

| NO. | 1# slot angle | 2# slot width (mm) | 3# slot width (mm) | 3# slot bottom runout (mm) |
|-----|---------------|---------------------|--------------------|---------------------------|
| 1   | 15°02’04’’    | 3.0689              | 4.0305             | 0.0253                    |
| 2   | 15°01’36’’    | 3.07                | 4.0298             | 0.0252                    |
| 3   | 15°02’30’’    | 3.0699              | 4.0306             | 0.0253                    |
| 4   | 15°01’46’’    | 3.0699              | 4.0305             | 0.0255                    |
| 5   | 15°01’47’’    | 3.07                | 4.0305             | 0.0253                    |
| 6   | 15°02’30’’    | 3.0699              | 4.0294             | 0.0242                    |
| 7   | 15°01’48’’    | 3.0699              | 4.0308             | 0.0268                    |
| 8   | 15°02’13’’    | 3.07                | 4.0297             | 0.0252                    |
| 9   | 15°01’52’’    | 3.07                | 4.0303             | 0.0256                    |
| 10  | 15°02’40’’    | 3.0699              | 4.0302             | 0.0256                    |
| 11  | 15°03’4’’     | 3.0699              | 4.0297             | 0.0256                    |
| 12  | 15°02’28’’    | 3.0699              | 4.0306             | 0.0253                    |
| Mean| 15°02’14’’    | 3.069925            | 4.030475           | 0.02538                   |

Table 3 shows the difference between the actual value of the standard piston 1# groove angle, 2# groove width, 3# groove width and 3# groove bottom runout shape parameter actual value and the standard piston shape parameter detection value. After analysis, it can be concluded that the detection accuracy of the groove angle reaches ±2 minutes, and the detection accuracy of the groove width and the runout of the groove bottom reaches within ±2 microns.

Table 3. Standard piston shape and position deviation

| NO. | 1# slot angle | 2# slot width | 3# slot width | 3# slot bottom runout |
|-----|---------------|---------------|---------------|-----------------------|
| 1   | +0°01’00”    | +0.0011       | +0.0015       | +0.0015               |
| 2   | -0°01’28”    | null          | -0.0015       | -0.0011               |

At the same time, in order to measure the effectiveness of the adaptive data fusion method based on Kalman filtering, this paper uses the mean square error as the evaluation index [11], groove angle, groove width and groove bottom runout, the accuracy evaluation formula of mean square error is as follows:
\[ d(MES) = \frac{1}{n} \sum_{i=1}^{n} (d_j(i) - \hat{d}_j(i))^2 \]

Table 4. Mean square error comparison of two filtering algorithms

| NO. | Standard Kalman | Adaptive Kalman data fusion |
|-----|-----------------|-----------------------------|
| 1   | 2.3             | 1.5                         |
| 2   | 2.5             | 1.6                         |
| 3   | 1.7             | 1.4                         |
| 4   | 1.9             | 1.4                         |
| 5   | 2.7             | 1.8                         |
| **average value** | **2.22** | **1.54** |

This article takes 5 consecutive (n=5) measurement data of 3# slot bottom runout (j=4), where the amount with the superscript "=" represents the measured slot angle, slot width and runout of the slot bottom the true value of the quantity. At the same time, the standard Kalman filtering algorithm and the adaptive Kalman data fusion method in this paper use the mean square error as the evaluation index [12] to obtain data as shown in Table 4. Through comparison, it can be found that the Kalman filter-based adaptive data fusion method proposed in this paper has better advantages than the standard Kalman filter algorithm, and can significantly improve the measurement accuracy. This is because this algorithm can also retain the detailed information of each sampling point as much as possible in the smoothing and filtering process [13], thus obtaining a better detection effect.

6. Concluding remarks

Based on the current piston shape and position detection groove angle, groove width detection and groove bottom runout measurement project, this paper also studies the reasons that affect the detection accuracy combined with practical applications, and uses the cubic B-spline approximate fitting algorithm to smooth Curve [14] obtains the overall change trend of the data, and at the same time proposes an adaptive data fusion method based on Kalman filtering, and conducts experimental verification. In the later stage, it is necessary to carry out experiments according to different piston shape and position specifications [15] and at the same time improve and optimize the execution algorithm and algorithm parameters in order to continue to enhance the fault tolerance of the detection system.

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