Process parameter optimization for thin-walled tube push-bending using response surface methodology

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Abstract
In this paper, the response surface methodology (RSM) and finite element (FE) simulation were applied to optimize the push-bending process parameters of the thin-walled tube with polyurethane mandrel. The objective of the present work is to predict the optimal set of process parameters including the length to diameter ratio of the mandrel \((L/D)\), the friction coefficient between die and tube \((\mu_1)\), the friction coefficient between polyurethane and tube \((\mu_2)\), and Poisson’s ratio of polyurethane \((\nu)\) to obtain qualified bent tubes. Three empirical models were developed to describe the relationship between process parameters and quality parameters of the bent tubes. In addition, the significant factors affecting the forming quality were analyzed using analysis of variance (ANOVA) of each model. Response surfaces were constructed to study the effect of each process parameter on the quality of the bent tubes. Finally, the process optimization window with the maximum thinning rate \((\varphi)\) less than 20\%, the maximum thickening rate \((\psi)\) less than 17\%, and the maximum cross-section ovality \((\xi)\) less than 5\% of the bent tube was established. Qualified bent tubes with diameter of 144 mm, wall thickness of 2 mm, and bending radius of 280 mm were formed experimentally by following the established process window.

Keywords Thin-walled tube · Push-bending · Process optimization · Response surface methodology

1 Introduction
With the development of modern industry, lightweighting has become more and more an urgent requirement and development direction of parts manufacturing, and large diameter thin-walled bent tubes are widely used in the aerospace, aviation, ships, and oil industries as the typical lightweight parts [1, 2]. There are four main defects in the tube bending, including over thinning of the outer wall of the bent tube, wrinkling in the inner wall, cross-sectional distortion, and spring back after unloading [3–5]. Especially as the relative wall thickness \(t_0/D\) decrease, more serious forming defects will occur, where \(D\) is initial outer diameter and \(t_0\) is initial wall thickness. Most tube bending processes are dedicated to reducing even avoiding the generation of the above defects. Many scholars have carried out researches on solving the problems in the tube bending process by using different bending methods such as NC bending [6], press bending with hydraulic pressure [7], and push-bending [8].

During tube bending, the outer wall of the bent tube is subjected to axial tensile stress and thinned while thickening even wrinkling appears in the inner wall due to axial compressive stress. At the same time, the outer side wall of the elbow also moves inward causing cross-sectional distortion. In order to describe the above three defects, the maximum thinning rate \(\varphi\), the maximum thickening rate \(\psi\), and the maximum cross-section ovality \(\xi\) are defined as:
\[ \varphi = \frac{t_0 - t_{\text{min}}}{t_0} \]  

(1)

\[ \psi = \frac{t_{\text{max}} - t_0}{t_0} \]  

(2)

\[ \xi = \frac{D - D_{\text{min}}}{D} \]  

(3)

where \( t_0 \) is the initial tube wall thickness, \( t_{\text{min}} \) is the minimum wall thickness of bent tube after bending, \( t_{\text{max}} \) is the maximum wall thickness of bent tube after bending, \( D \) is the initial outer diameter of tube, and \( D_{\text{min}} \) is the minimum outer diameter of bent tube after bending. The values of the above three defects will be used as the following optimization evaluation indicators, and the purpose of the optimization is to reduce the values of these three defects as much as possible.

No matter which kind of tube bending technology is adopted, the appropriate process parameters play a vital role in the bending forming quality of the tube. Therefore, the optimization of process parameters must be considered in the actual bending process. By optimizing the process parameters, the potential of the tube bending technology can be fully developed. Nguyen et al. [9] selected mandrel diameter, the distance between mandrel rings, and distance from the tip of the mandrel bar to the center of the base die to study their effects on the bending quality of copper JIS25A according to Taguchi’s orthogonal array and finite element simulation. Safdarian [10] studied the effect of pressure of pressure die, boosting velocity of pressure die, friction between the tube and pressure die, mandrel position and number of mandrel balls in rotary draw bending on the fracture, wrinkling, and tube’s ovality of BS 3059 steel tube. Xu et al. [11, 12] optimized the parameters such as polyurethane length and friction coefficient in the push-bending process using sectional elastomers as mandrel, and the 1Cr18Ni9 and 5A02 tubes were well-formed. To realize the control of spring back and section deformation, the multi-parameter sensitivity analysis method was used to obtain the processing parameters in the bending process of rectangular tubes [13]. Li et al. [14] provided a comprehensive diagram of tube bending limits during rotary draw bending in a range of tube sizes, material types, and processing parameters, and experimental verification was also carried out by several practical bending schemes. Lăzărescu [15] adopted finite element simulation and response surface methodology to optimize the tube bending process (relative bending radius \((R/D)\) and relative tube diameter \((D/t))\) and the relationship between the cross-section ovality and wall thinning. And the statistical models were used to establish an optimal technological window for the tube bending process in order to obtain products with an acceptable quality.

Some advanced optimization methods have been used to obtain useful regularities from the data, which will be of great value in guiding production. The response surface method (RSM) has been widely used in multi-target process parameter optimization. Response surface method is a statistical and mathematical method that processes the data by using regression analysis [16]. The empirical functions between response variables and processing parameters can be established by regression surface fitting. Generally, these mathematical models are polynomials with an unknown structure and usually this equation in second-order quadratic formula to describe interaction between factors [17]. The second-order model is shown in Eq. (4).

\[ y = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{i<j}^{k} \beta_{ij} x_i x_j + \sum_{j=1}^{k} \beta_j x_j^2 + \epsilon \]  

(4)

where \( y \) is the response, \( x_i \) is the process parameters, \( \epsilon \) is the experimental random error, \( x_i x_j \) is the interaction term between parameter \( i \) and parameter \( j \), \( x_j^2 \) is the secondary term of parameter \( j \), and \( k \) is the number of process parameters. The constant terms and coefficients in models can be obtained by fitting the experimental data by the least square method, and then the empirical function between the response and each process parameter is established. By using RSM and FEM, Lin et al. [18] developed the empirical equation to explore the relationship between the principal strain of fracture risk elements and the working parameters, and the model adequacy was checked and confirmed using analysis of variance (ANOVA). Lepadatu et al. [19] carried out the optimization of spring back in sheet metal bending processes using FEM and RSM. Liu et al. [20] used the RSM to research the influences of three experimental factors on maximum forming load and maximum tool wear depth, and the optional parameter combination was employed to redesign the billet and the forging tools for industrial practice.

For process optimization, experimental design is very important, and reasonable experimental design can ensure the accuracy of experimental results while minimizing the number of experiments. In RSM, central composite design (CCD) is a popular experimental design method shown in Fig. 1. The CCD contains an imbedded factorial or fractional factorial design with center points which is augmented with a group of “star points” that allow estimation of curvature. If the distance from the center of the design space to a factorial point is \( \pm 1 \) unit for each factor, the distance from the center of the design space to a star point is \( |\alpha| > 1 \). The value of \( \alpha \), in case of three factors is 1.682 [16].

In order to check the fitness of regression model fitting the real data points, a statistical measure \( R^2 \) is introduced. And the closer the value of \( R^2 \) to unity, the better the fit.
between the empirical model and the actual data. $F$ value is another evaluation, which is the ratio of the mean square obtained by regression to the mean square due to residual. For the $F$ value, there is a critical value $F_0$, and $P$ value represents the probability of $F < F_0$. Generally, the model is considered to be significant and the influence of the terms on the response is significant when $P < 0.05$.

A modified thin-walled tube push-bending process with polyurethane mandrel has been proven to be suitable for forming large diameter and thin-walled tubes [21]. The schematic diagram of thin-walled tube push-bending process with polyurethane mandrel is illustrated in Fig. 2 [21]. Prior to forming, a tubular blank filled with polyurethane mandrel of length $L$ is inserted into a guide die. It should be noted that a metal thick plate stopper is welded on the front end of the tube blank to seal the polyurethane mandrel. During tube bending, the punch applies a pushing force on the back end of polyurethane mandrel and then under the common constraint of the guide, polyurethane mandrel and the bending die, the bent tube with bending radius of $R$ is formed. Besides, the distribution and influence of contact pressure between polyurethane mandrel and tube were investigated to further understand the characteristics of polyurethane mandrel. The results show that the length of polyurethane, the coefficient of friction between polyurethane and tube, and Poisson’s ratio of polyurethane had a great influence on the distribution of internal pressure. In addition, the friction coefficient between the tube and the die will affect the flow of the tube material and the forming quality greatly. Hence, it is necessary to optimize these parameters. The forming quality of the tube is affected by the interaction of multiple forming parameters. Empirical or trial-and-error methods cannot meet the production of high-quality large-diameter thin-walled elbow tubes. Therefore, the length to diameter ratio of the mandrel ($L/D$), coefficient of friction between die and tube ($\mu_1$), the coefficient of friction between polyurethane and tube ($\mu_2$), and the Poisson’s ratio of polyurethane ($\nu$) were selected in this paper to optimize the process parameters. The central composite design (CCD) is introduced and adopted to develop three empirical equations based on numerical FE simulation results to describe the relationship between quality characteristics and process parameters. Three bent tube quality characteristics, including the maximum thinning rate ($\phi$), the maximum thickening rate ($\psi$), and the maximum cross-section ovality ($\xi$) were set as responses in RSM. The length to diameter ratio of the mandrel ($L/D$), coefficient of friction between die and tube ($\mu_1$), the coefficient of friction between polyurethane and tube ($\mu_2$), and the Poisson’s ratio of polyurethane ($\nu$) were four factors in RSM. The process window was given by combining parameter optimization method, in

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**Fig. 1** Central composite design for three factors

**Fig. 2** Schematics of the tube push-bending process with polyurethane mandrel a before bending and b after bending [21]
which the interaction of multiple parameters is considered. Finally, the reliability of the empirical equations is verified by experiments.

2 Experimental methods

2.1 Finite element modeling

As shown in Fig. 3, the 3D elastic–plastic FE model of the thin-walled tube push-bending process with polyurethane mandrel was built including the punch, polyurethane mandrel, tube, stopper, and die (including guide). The diameter of the tube is 144 mm and the wall thickness is 2 mm. The bending radius is 280 mm, and the bending angle is 70°. Numerical simulations were carried out using ABAQUS/Explicit. Due to symmetry, only a half FE model was adopted to reduce the computational time. All the freedom of rotation and translation of the die were constrained. The punch contact PU only. The axial displacement of the punch along the tube was applied, at the same time translation in other directions and all the freedom of rotation were also constrained. Tie constrained was used for tube and stopper to seal the PU. The die, punch, and stopper were predefined as discrete rigid. The contacts at the tube polyurethane and tube die were modeled using surface-to-surface contact algorithm. The friction behavior in the FE model was described with the classic Coulomb friction model. The tube was meshed by four-node doubly curved thin shell S4R with 5 integral points along the thickness direction. Calculate the mesh size of the tube from large to small, and obtain the maximum thinning rate. When the maximum thinning rate tended to be stable, the current mesh size was selected. The mesh size of the tube was set to 3.5 mm. It is assumed that the material used in the simulation is a homogeneity, isotropic material following the Von-Mises yield criterion, and has anisotropic work hardening. The mechanical properties of 0Cr18Ni9 stainless steel tube blank were obtained by uniaxial tensile test of the specimens cut along the longitudinal direction of the tube, as shown in Fig. 4. The mechanical properties of 0Cr18Ni9 stainless steel tube blank are presented in Table 1 and its true stress–strain curve at room temperature is shown in Fig. 4.

The Marlow strain energy potential model [21] was adopted in ABAQUS to characterize the mechanical properties of polyurethane mandrel, where only one set of test data (uniaxial, equibiaxial, or planar test data) is required. The relevant material performance testing experiments and results are shown in Fig. 5, and uniaxial compression and triaxial compression tests were carried out on polyurethane, respectively. The elastic modulus $E$ of polyurethane was 21 MPa by linear fitting of uniaxial compression test results (Fig. 5a). Meanwhile, the uniaxial compression results were also input into ABAQUS to establish the Marlow strain energy potential model. The polyurethane was put into a rigid capsule for triaxial compression tests. Due to the gap between polyurethane and rigid capsule, the polyurethane fully filled the capsule cavity after the filling stage and transition stage, and its stress state becomes three-dimensional compressive stress (Fig. 5a). And the compression modulus $E_s$ of polyurethane was 2088 MPa by linear fitting of triaxial

![Fig. 4 True stress–strain curve of 0Cr18Ni9 tubular blank](image)

| Table 1 Mechanical properties of 0Cr18Ni9 tubular blank |
|-----------------------------------------------|
| Young’s modulus $E$ (MPa) | Density (kg/m$^3$) | Elongation (%) | Tensile yield strength (MPa) | Poisson’s ratio |
| 201,000 | 7930 | 55 | 205 | 0.3 |
compression tests. Finally, Eq. (5) was used to calculate the Poisson’s ratio of polyurethane. The result of Poisson’s ratio \( \nu \) of polyurethane is 0.4983, which will be used in the Marlow strain energy potential model. The specific derivation process of Eq. (4) can refer to our previous research [21].

It is worth noting that the experimental verification of the finite element model was also carried out to ensure the reliability of the model. The wall thickness distribution of tubes obtained by experimental and FEM was compared, and the pushing force versus displacement of punch was also compared, which verified the accuracy of the FE model.

\[
v = \sqrt{\left(\frac{E_s}{E} - 1\right)^2 + \frac{8E_s(E_s - E)}{4E_s} - \left(\frac{E_s}{E} - 1\right)}
\]

(5)

### 2.2 Response surface methodology (RSM)

The central composite design response surface methodology (CCD-RSM) was used to optimize the push-bending with polyurethane mandrel process parameters in statistical modeling software Design Expert [15]. And the purpose of the integral bending forming of the seamless 0Cr18Ni9 stainless steel bent tube with outer diameter of 144 mm, wall thickness of 2 mm, bending radius of 280 mm, and bending angle of 70° was finally achieved.

The length to diameter ratio of the mandrel \((L/D)\), coefficient of friction between die and tube \((\mu_1)\), coefficient of friction between polyurethane and tube \((\mu_2)\), and Poisson’s ratio of polyurethane \((\nu)\) are considered to be the main factors affecting the forming quality of the bent tube. And the forming quality of the bent tube is mainly described by the maximum thinning rate \((\varphi)\), the maximum thickening rate \((\psi)\), and the maximum cross-section ovality \((\xi)\). Therefore, \(\varphi, \psi, \text{ and } \xi\) were selected as the responses in this study.

The range and levels of experimental variables investigated in this study are presented in Table 2. For the choice of the length to diameter ratio of the mandrel, the upper and lower limit of the length to diameter ratio of the mandrel is determined by the length of the bent tube. The lower limit length of the polyurethane is the lowest length of the bent tube with bending radius of 280 mm, bending angle of 72°, and both ends with 20-mm straight section. The upper limit length of the polyurethane is less than the maximum equipment travel. Lubrication between the tube and die can reduce the friction and improve the flow of materials, thus reducing the defects occurred in the process of bending. Since the lubricant were used in general tube bending process to reduce the friction coefficient between the tube and die as much as possible, the upper limit of coefficient of friction between die and tube is selected as 0.12 using general phosphating plus soaping, MoS\(_2\), etc. [22, 23]. A zero value of the coefficient of friction could not be reached in practice, being the lower limit and ideal case. The purpose of this ideal zero value is to set a boundary within which the optimization direction can be obtained. For example, if the friction coefficient of the optimization result is close to 0, then

| Independent variables | Symbol | Coded values \((\alpha = 1.682)\) |
|-----------------------|--------|---------------------------------|
| The length to diameter ratio of the mandrel \((L/D)\) | \(x_1\) | 2.9 3.425 3.95 4.475 5 |
| Coefficient of friction between die and tube \((\mu_1)\) | \(x_2\) | 0.00 0.03 0.06 0.09 0.12 |
| Coefficient of friction between polyurethane and tube \((\mu_2)\) | \(x_3\) | 0.06 0.12 0.18 0.24 0.3 |
| Poisson’s ratio of polyurethane \((\nu)\) | \(x_4\) | 0.49 0.4921 0.4942 0.4962 0.4983 |

![](image)

**Fig. 5** Nominal stress–strain curve of polyurethane. **a** Uniaxial compression. **b** Triaxial compression [21]
the reasonable lubrication method is expected to be found to reduce the friction as much as possible. In the practical bending process, the value in FEM is used as a reference; many studies [12, 22–24] have given possible friction coefficients for different lubricants, such as high-viscosity oil (0.06), low-viscosity oil (0.08), and polytetrafluoroethylene (0.03). These results were used for selecting suitable lubricant. The friction coefficient between the polyurethane and the tube will affect the internal pressure distribution of the polyurethane [21], and the selection range of its value is relatively wide. The upper limit is 0.3 using dry friction and lower limit is 0.06 using high-viscosity oil. Polyurethane is a material with low modulus and high elasticity, and the Poisson’s ratio of polyurethane is contained in the narrow range with a value greater than 0.49, close to 0.5, and nearly invariant in all directions [25]. According to this property of polyurethane, polyurethane with Poisson’s ratio of 0.49 to 0.483 can be obtained in this paper, being the upper and lower limit. In order to simplify the research, the coefficient of friction between die and tube is taken as independent of the coefficient of friction between polyurethane and tube, which is also what most of the research has assumed [8, 11, 12, 15, 26]. The experimental plan for the coded values in Table 2 and corresponding response values form FE simulations is shown in Table 3. The length to diameter ratio of the mandrel (L/D), coefficient of friction between die and tube (μ1), coefficient of friction between polyurethane and tube (μ2), and Poisson’s ratio of polyurethane (ν) are expressed by x₁, x₂, x₃, and x₄, respectively.

### 3 Results and discussion

#### 3.1 Analysis of REM results and discussion

##### 3.1.1 Mathematical modeling

According to the design of experiments in Table 3, different process parameter combinations are assigned to the finite element model established above, and then the simulation results were included in “The Response” in Table 3. These results were used to establish empirical models.

The second-order model was used to fit the simulation results, and regression analysis was conducted by the least square method to determine the polynomial coefficients of
the empirical functions between the three responses and the four process parameters.

The response model of the maximum thinning rate (\( \varphi \)) was obtained as:

\[
\varphi = -441.45782 - 37.47364x_1 - 102.48468x_2 - 114.75151x_3 \\
+ 2050.61784x_4 - 0.99206x_1x_2 + 2.48016x_1x_3 + 79.96299x_1x_4 \\
- 105.90278x_2x_3 + 251.12345x_2x_4 + 174.29950x_3x_4 \\
- 0.33501x_1^2 + 8.51473x_2^2 + 54.21202x_3^2 - 2305.65745x_4^2
\]  

(6)

The response model of the maximum thickening rate (\( \psi \)) was obtained as:

\[
\psi = -12093.19003 + 24.30531x_1 - 62.16865x_2 + 292.02573x_3 \\
+ 4886.06797x_4 + 1.38889x_1x_2 - 2.28175x_1x_3 - 48.14773x_1x_4 \\
+ 157.98611x_2x_3 + 69.38937x_2x_4 - 530.33307x_3x_4 + 0.011251x_1^2 \\
- 105.39320x_2^2 - 59.90252x_3^2 - 49300.16868x_4^2
\]  

(7)

The response model of maximum cross-section ovality (\( \xi \)) is obtained as:

\[
\xi = -2556.10446 + 31.86584x_1 - 2.15336x_2 + 467.93133x_3 \\
+ 10160.17023x_4 - 1.09203x_1x_2 - 3.24452x_1x_3 - 60.50560x_1x_4 \\
+ 111.55489x_2x_3 - 36.19161x_2x_4 - 885.44772x_3x_4 - 0.16056x_1^2 \\
- 113.60391x_2^2 - 53.50382x_3^2 - 10090.52354x_4^2
\]  

(8)

To test the accuracy of the REM model, four groups of process parameters were randomly selected in Table 4.

The corresponding results obtained by REM model and FE model were also in Table 4. The results were compared in Fig. 6, and the predicted values of the REM model was in good agreement with the values of the FE model. The average error of maximum thinning rate, maximum thickening rate, and maximum cross-section ovality were 0.73\%, 2.57\%, and 3.63\%, respectively.

### 3.1.2 Analysis of variance of response model

Table 5 shows the results of ANOVA applied to the individual coefficients for the maximum thinning rate response model (Eq. 6). The model \( P \) value is 0.0002 less than 0.0500, and \( R^2 \) is 0.9432, indicating that the model fits the response values with a very high degree of confidence. The significant terms in the maximum thinning rate response model include the length to diameter ratio of the mandrel \( x_1 \), coefficient of friction between polyurethane and tube \( x_2 \), Poisson’s ratio of polyurethane \( x_3 \), the interaction term between coefficient of friction between die and tube and coefficient of friction between polyurethane and tube \( x_2x_3 \), and the square of coefficient of friction between polyurethane and tube \( x_3 \), where the length to diameter ratio of the mandrel \( x_1 \) and Poisson’s ratio of polyurethane \( x_3 \) are the greatest significant items with \( P \) value less than 0.0001; therefore, the following will introduce the impact of these two on wall thickness thinning.

Table 6 shows the results of ANOVA applied to the individual coefficients for the maximum thickening rate and maximum cross-section ovality.
response model (Eq. 7). The model $P$ value is 0.0054 less than 0.0500, and $R^2$ is 0.8835, indicating that the model fits the response values with a very high degree of confidence. The significant terms in the maximum thickening rate response model include coefficient of friction between polyurethane and tube $x_3$, Poisson’s ratio of polyurethane $x_4$, and the interaction term between coefficient of friction between die and tube and coefficient of friction between polyurethane and tube $x_3x_4$, where coefficient of friction between polyurethane and tube $x_3$ is the greatest significant items with $P$ value less than 0.0001.

Table 7 shows the results of ANOVA for the maximum cross-section ovality response model (Eq. 8). The $P$ value of the model is 0.0003, which is much less than 0.05, and

| Source          | Sum of squares | Degree of freedom | Mean square | $F$ value | $P$ value | Probability > $F$ |
|-----------------|----------------|-------------------|-------------|-----------|-----------|------------------|
| Model           | 12.61          | 14                | 0.90        | 5.42      | 0.0054    | Significant      |
| $x_1$           | 0.50           | 1                 | 0.50        | 3.00      | 0.1138    |                  |
| $x_2$           | 0.41           | 1                 | 0.41        | 2.49      | 0.1457    |                  |
| $x_3$           | 6.79           | 1                 | 6.79        | 40.84     | <0.0001   | Significant      |
| $x_4$           | 2.10           | 1                 | 2.10        | 12.65     | 0.0052    | Significant      |
| $x_1x_2$        | 7.656E-003     | 1                 | 7.656E-003  | 0.046     | 0.8343    |                  |
| $x_1x_3$        | 0.083          | 1                 | 0.083       | 0.50      | 0.4967    |                  |
| $x_1x_4$        | 0.043          | 1                 | 0.043       | 0.26      | 0.6221    |                  |
| $x_2x_3$        | 1.29           | 1                 | 1.29        | 7.79      | 0.0191    | Significant      |
| $x_2x_4$        | 2.914E-004     | 1                 | 2.914E-004  | 1.754E-003| 0.9674    |                  |
| $x_3x_4$        | 0.068          | 1                 | 0.068       | 0.41      | 0.5365    |                  |
| $x_1^2$         | 1.080E-004     | 1                 | 1.080E-004  | 6.502E-004| 0.9802    |                  |
| $x_2^2$         | 0.069          | 1                 | 0.069       | 0.41      | 0.5349    |                  |
| $x_3^2$         | 0.52           | 1                 | 0.52        | 3.14      | 0.1066    |                  |
| $x_4^2$         | 0.50           | 1                 | 0.50        | 3.04      | 0.1120    |                  |
| Residual        | 1.66           | 10                | 0.17        |           |           |                  |
| Cor Total       | 14.27          | 24                | 0.64        |           |           |                  |

$R^2=0.8835$
$R^2$ is 0.9386, indicating that the model fits the response values with a very high degree of confidence. The coefficient of friction between die and tube $x_2$, coefficient of friction between polyurethane and tube $x_3$, Poisson’s ratio of polyurethane $x_4$, and the interaction term $x_2x_3$ have significant influence on maximum cross-section ovality. And coefficient of friction between die and tube $x_2$ and Poisson’s ratio of polyurethane $x_4$ are the greatest significant items with $P$ value less than 0.0001.

It can be seen that the $x_3$, i.e., the friction coefficient between the tube and the polyurethane mandrel, has the greatest impact both on the thinning rate and the thickening rate of the bent tube wall thickness. $x_3$ mainly affects the friction force between the polyurethane mandrel and the tube, thereby affecting the tangential force of the bent tube. While the thinning or the thickening of the wall thickness of the tube in the bending process is mainly due to the external tangential force, so the $x_3$ is a significant term in the maximum thinning rate response model as well as maximum thickening rate response model.

The value of Poisson’s ratio directly affects the compressibility of polyurethane mandrel. Under the condition of applying the same force, different Poisson’s ratio polyurethanes will have different contact pressure response with the tube due to different compressibility when expanding in the sealed tube. The support internal pressure provided by polyurethane mandrel not only affects the friction force, but also greatly affects the deformation of the tube cross-section. Therefore, Poisson’s ratio of polyurethane $x_4$ is the greatest significant term in the maximum thinning rate response model and the maximum cross-section ovality response model.

Also noted is that coefficient of friction between die and tube $x_2$ is another greatest significant item with $P$-value less than 0.0001 in the maximum cross-section ovality response model. The reason is that the coefficient of friction between die and tube will affect the flow of tube material in the die during bending process, which results in different cross-section ovality responses.

### 3.1.3 Response surface analysis

The 3D response surface plots were drawn to visualize the influence of the variables on the responses. There are 4 process parameters in the study, but only two of the process parameters can be shown in a 3D response surface diagram. So, the four process parameters are divided into two categories, friction coefficient (includes coefficient of friction between die and tube $x_2$ and coefficient of friction between polyurethane and tube $x_3$) and parameters of polyurethane (includes the length to diameter ratio of the mandrel $x_1$ and Poisson’s ratio of polyurethane $x_4$). $x_2$ and $x_3$ were used as a group to plot a 3D response surface ($x_1 = 4$, $x_4 = 0.4983$). $x_1$ and $x_4$ were used as another group to construct a 3D response surface ($x_2 = 0.08$, $x_3 = 0.15$).

Figures 7, 8, and 9 show the influence of $x_2$ (the coefficient of friction between the tube and die) and $x_3$ (the coefficient of friction between the tube and polyurethane) on $\phi$ (the maximum thinning rate), $\psi$ (the maximum thickening rate), and $\xi$ (the maximum cross-section ovality). The 3D response surface plots were drawn to visualize the influence of the variables on the responses. There are 4 process parameters in the study, but only two of the process parameters can be shown in a 3D response surface diagram. So, the four process parameters are divided into two categories, friction coefficient and parameters of polyurethane. $x_2$ and $x_3$ were used as a group to plot a 3D response surface ($x_1 = 4$, $x_4 = 0.4983$). $x_1$ and $x_4$ were used as another group to construct a 3D response surface ($x_2 = 0.08$, $x_3 = 0.15$).

### Table 7: Analysis of variance (ANOVA) for maximum cross-section ovality $\xi$

| Source     | Sum of squares | Degree of freedom | Mean square | $F$ value | $P$ value | Probability $> F$ |
|------------|----------------|-------------------|-------------|-----------|-----------|------------------|
| Model      | 15.39          | 14                | 1.10        | 10.91     | 0.0003    | Significant      |
| $x_1$      | 0.016          | 1                 | 0.016       | 0.16      | 0.6994    |                  |
| $x_2$      | 6.92           | 1                 | 6.92        | 68.72     | < 0.0001  | Significant      |
| $x_3$      | 2.16           | 1                 | 2.16        | 21.47     | 0.0009    | Significant      |
| $x_4$      | 4.61           | 1                 | 4.61        | 45.73     | < 0.0001  | Significant      |
| $x_1x_2$   | 4.73E-003      | 1                 | 4.73E-003   | 0.047     | 0.8328    |                  |
| $x_1x_3$   | 0.17           | 1                 | 0.17        | 1.66      | 0.2268    |                  |
| $x_1x_4$   | 0.068          | 1                 | 0.068       | 0.67      | 0.4309    |                  |
| $x_2x_3$   | 0.65           | 1                 | 0.65        | 6.40      | 0.0298    | Significant      |
| $x_2x_4$   | 7.92E-005      | 1                 | 7.92E-005   | 7.870E-004 | 0.9782 |                  |
| $x_3x_4$   | 0.19           | 1                 | 0.19        | 1.88      | 0.1999    |                  |
| $x_1^2$    | 0.022          | 1                 | 0.022       | 0.22      | 0.6503    |                  |
| $x_2^2$    | 0.12           | 1                 | 0.12        | 1.17      | 0.3056    |                  |
| $x_3^2$    | 0.42           | 1                 | 0.42        | 4.14      | 0.0693    |                  |
| $x_4^2$    | 0.021          | 1                 | 0.021       | 0.21      | 0.6567    |                  |
| Residual   | 1.01           | 10                | 0.10        |           |           |                  |
| Cor Total  | 16.40          | 24                |             |           |           |                  |

$R^2=0.9386$
rate), and $\xi$ (the maximum cross-section ovality), respectively. And Figs. 10, 11, and 12 show the influence of $x_1$ (the length to diameter ratio of the mandrel) and $x_4$ (the Poisson’s ratio of polyurethane) on $\varphi$, $\psi$, and $\xi$, respectively.
Results from previous study have shown that the internal pressure increases with the decrease of the friction coefficient between polyurethane and tube (in the range of 0.08–0.14), the increase of polyurethane length (in the range of 415–565 mm), and the increase of Poisson’s ratio (in the range of 0.4963–0.4983) when other process parameters are fixed [21]. When the internal pressure is high, the cross-section ovality of the bent tube is small. As can be seen from Fig. 9b, the maximum cross-section ovality of the bent tube decreases with the decrease of friction coefficient between polyurethane and tube when the friction coefficient between polyurethane and tube $x_3$ is in the range of 0.08–0.14 and
$x_3$ is 0.1. At the same time, the results from Fig. 12b show that the maximum cross-section ovality of the bent tube decreases with the increase of the length to diameter ratio of the mandrel $x_4$ in the range of 2.98–4.06, i.e., the length of polyurethane in the range of 415–565 mm when $x_4$ is 0.4983. The maximum cross-section ovality of the bent tube decreases with the increase of Poisson’s ratio $x_4$ in the range of 0.4963–0.4983 when $x_1$ is 4.06. The results in previous research can be verified in the response surface.

It can be found that the forming quality of the bent tube is often influenced by the multiple instabilities and multi-factor coupling effects. The method used in previous literature, that is, to obtain the results by changing a single process parameter, is not comprehensive, which illustrates the advantages of using advanced parameter optimization methods. There may be causing a change of one forming parameter on the law of forming quality of the bent tube when a change of another forming parameter. A typical result which is the influence of coefficient of friction between die and tube $x_2$ and coefficient of friction between polyurethane and tube $x_3$ on maximum thinning rate $\varphi$ (Fig. 7) illustrates this deeply. If coefficient of friction between polyurethane and tube $x_3$ is small, the maximum thinning rate increases linearly with the increase of coefficient of friction between die and tube $x_2$, but when coefficient of friction between polyurethane and tube $x_3$ is greater than a certain critical value, the maximum thinning rate will decrease linearly with the increase of coefficient of friction between die and tube $x_2$.

In a comparison between Fig. 7 and Fig. 8, it can be found that the effect of coefficient of friction between die and tube $x_2$ on the maximum thickening rate and maximum thinning rate is opposite. For example, if coefficient of friction between polyurethane and tube $x_3$ is set to 0.06, increasing coefficient of friction between die and tube $x_2$ will reduce the maximum thickening rate, but inevitably increase the maximum thinning rate. That is, by adjusting coefficient of friction between die and tube $x_2$ to mitigate one forming defect, it will exacerbate another forming defect.

The forming quality is greatly influenced by the Poisson’s ratio of polyurethane. The higher Poisson’s ratio of the polyurethane mandrel is helpful to reduce cross-section ovality of the bent tube but increases the maximum thinning rate. Therefore, the choice of polyurethane material is also a very important issue in the push-bending process with polyurethane mandrel.

### 3.2 Analysis of multi-objective process optimization and experiment results

As mentioned above, the bending forming quality of tube is affected by the interaction of multiple forming parameters, and it is not possible to reduce all forming defects by adjusting only one of the parameters. Therefore, it is useful to use the results of the response surface analysis to build multi-objective process windows.

In this study, the product process optimization window with critical maximum thinning rate of 20%, critical maximum thickening rate of 17%, and the critical maximum ovality of 5% was used as the qualified standard according to the industrial standard of QJ 919A-1998. As shown in Fig. 13, the 0Cr18Ni9 qualified stainless-steel elbow with an outer diameter of 144 mm, a wall thickness of 2 mm, a bending radius of 280 mm, and a bending Angle of 70° can be obtained by the combination of process parameters in the yellow area in the figure.

The bending test was carried out by using the process parameters of the qualified area in the process optimization window. The length to diameter ratio of the polyurethane mandrel used in the experiments is 3.92; The Poisson’s ratio of polyurethane is 0.4983; engine oil is used to lubricate the inner and outer walls of the tube blank, the die cavity, and the outer...
surface of the polyurethane mandrel. Three experiments were conducted using the same parameters.

The elbow obtained from the above experiment is shown in Fig. 14. The wall thickness distribution and section diameter distribution of the bend tube are given to indicate the forming quality of the bent tube formed from this method (Fig. 15). Hence, the maximum thinning rate $\varphi$, the maximum thickening rate $\psi$, and the maximum cross-section ovality $\xi$ of the obtained elbow were calculated according to formula 1 to formula 3. Table 8 shows the experimental results of $\varphi$, $\psi$, and $\xi$. The results show that the product was formed with high quality using push-bending process with polyurethane mandrel, and the reliability of forming process optimization window based on response surface analysis was also shown.

![Fig. 14 Bent tubes formed with optimized process parameters](image)

![Fig. 15 Wall thickness and cross-section diameter of thin-walled bending tubes](image)

### Table 8 Experimental results of $\varphi$, $\psi$, and $\xi$ of bent tubes

| Items                | Maximum thinning rate $\varphi$ (%) | Maximum thickening rate $\psi$ (%) | Maximum cross-section ovality $\xi$ (%) |
|----------------------|------------------------------------|-----------------------------------|----------------------------------------|
| Maximum allowable value | 20                                 | 17                                | 5                                      |
| Bent tube 1          | 15.5                                | 14.5                              | 2.1                                    |
| Bent tube 2          | 16                                  | 12                                | 2                                      |
| Bent tube 3          | 14.5                                | 13                                | 1.7                                    |
4 Conclusions

CCD-RSM was used to analyze tube push-bending process with polyurethane mandrel based on the FEM. The main conclusions are as follows:

(1) The empirical functions between the three forming quality indicators and four process parameters were developed.

(2) The ANOVA shows that the friction coefficient between the tube and the polyurethane mandrel has the greatest impact both on the thinning rate and the thickening rate of the bent tube wall thickness; The value of Poisson’s ratio is the greatest significant term in the maximum thinning rate response model and the maximum cross-section ovality response model; the coefficient of friction between die and tube is another greatest significant item in the maximum cross-section ovality response model.

(3) The bending forming quality of tube is affected by the interaction of multiple forming parameters, and it is not possible to reduce all forming defects by adjusting only one of the parameters.

(4) The process optimization window with critical maximum thinning rate $\phi$ of 20%, critical maximum thickening rate $\psi$ of 17%, and the critical maximum cross-section ovality $\xi$ of 5% was used as the qualified standard. And the qualified bent tubes were formed experimentally by following the established process window. The forming process optimization window based on response surface analysis has high reliability and is suitable for industrial applications.

Nomenclature

- $D$: Initial outer diameter of tube; $t_0$: Initial wall thickness of tube; $R$: Centerline radius of bent tube; $L$: Center axis length of polyurethane mandrel; $LD$ ($x_1$): The length to diameter ratio of the mandrel; $\mu_1$ ($x_2$): The friction coefficient between die and tube; $\mu_2$ ($x_3$): The friction coefficient between polyurethane and tube; $v$ ($x_1$): Poisson’s ratio of polyurethane; $\delta$: The maximum thinning rate of the bent tube; $\phi$: The maximum thickening rate of the bent tube; $\xi$: The maximum cross-section ovality of the bent tube; $t_{min}$: The minimum wall thickness of bent tube; $t_{max}$: The maximum wall thickness of bent tube; $D_{max}$: The minimum outer diameter of bent tube; $E$: The elastic modulus of polyurethane; $\sigma_c$: The compression modulus of polyurethane; $\gamma$: The response in RSM; $x_i$: The process parameter $i$ in RSM; $e$: The experimental random error in RSM; $k$: The number of process parameters in RSM; $R^2$: A statistical measure in RSM; $F$: The ratio of the mean square obtained by regression to the mean square due to residual; $F_0$: The critical value of $F$; $P$: The probability of $F < F_0$

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Declarations

Ethical approval The authors claim that none of the contents in this manuscript has been published or considered for publication elsewhere. Besides, the research contents of the article do not violate ethics.

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