THERMAL FEATURES FAR FROM EQUILIBRIUM: PRETHERMALIZATION

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The phenomenon of prethermalization and the subsequent steps of thermalization
are analyzed in the framework of the chiral quark model. We solve the quantum
equations of motion of the field theory derived from the 2PI effective action and
study the time scales of equilibration. We find that already after a 0.6 fm/c long
period of time some equilibrium features appear, even though the system is still
far from equilibrium. This might be an ingredient for understanding the success
of ideal hydrodynamic description.

1. Introduction

One of the crucial issues in heavy ion physics is the early thermalization of
the excited quark and gluon fields: The extreme success of hydrodynamic
calculations started earlier than 1 fm/c require the theoretical establishment
of the early existence of an equation of state.1

The proper description of equilibration is a longstanding challenge in
particle physics. Kinetic theories are popular tools for studying the series of
incoherent collisions. By neglecting the off-shell processes the easiest way is
to follow the elastic collisions only, but these deliver thermalization times by
orders of magnitudes larger than desired. Equilibration may be enhanced
by inelastic scatterings so that it completes before chemical freeze-out2.
This way, thermalization times reduce to 2-3 fm/c, which is still longer
than the early use of flow equations would require3.

Recent field theoretical developments enable the nonequilibrium treat-
ment of quantum fields by performing a systematic ladder resummation4.
The available numerical techniques and resources enable us to solve the
time evolution of 3+1 dimensional quantum fields in a self-consistent app-
proximation scheme. These methods are not limited to scalar fields any
more, nowadays the 3+1 dimensional chiral quark model (with two quark
flavors and 4 scalars) may also be routinely solved5.
Thermalization time is not a single number, of course. Different degrees of thermalization (LTE, kinetic and chemical equilibration) can have very different time scales ranging from the rapid convergence of bulk quantities to the slow equilibration of energy spectra. The model we consider is built up from two massless fermionic degrees of freedom and four scalars interacting according to the following Lagrangian:

\[
L = \bar{\psi} \partial \psi + \frac{1}{2} \left[ (\partial \sigma)^2 + (\partial \pi)^2 \right] + \frac{g}{\sqrt{2}} \bar{\psi} \left[ \sigma + i \gamma_5 \tau^a \pi^a \right] \psi - V(\sigma^2 + \pi^2),
\]

where \(\tau^a\) denote the standard Pauli matrices. We consider a quartic scalar self-interaction \(V(\sigma^2 + \pi^2) = m^2 (\sigma^2 + \pi^2) / 2 + \lambda (\sigma^2 + \pi^2)^2 / (4! N_f^2)\). The employed couplings are taken to be \(g = 1\) and \(\lambda = 1\). Though these couplings are smaller than heavy ion applications would require, we will be able to draw conclusions for the physical values since our main result will turn out to be coupling independent. The smaller-than-natural coupling leads to a spectacular separation of the various time scales.

We use the two-particle irreducible (2PI) effective action to two-loop order, which includes direct scattering as well as off-shell and memory effects. We numerically solve the nonequilibrium gap equation and obtain the evolution of the propagator \(G(t_1, t_2, |\vec{x}_1 - \vec{x}_2|)\) for each degree of freedom. To simplify our equations we assume homogeneity and isotropy in space.
2. Time scales of thermalization

Thermalization is loss of initial information. Our dynamics is conservative, still, virtually, we loose information if we do not read out all the internal variables but only the time-local propagator $G(t_1 \approx t_2, |\vec{x}_1 - \vec{x}_2|)$. This time-local propagator stores the information on the particle number distribution\textsuperscript{5}. The evolution of these occupation numbers for three different momenta is shown in Fig. 1left. We also show the convergence for two different initial spectra with equal energy density. Although the details of the initial distribution are mainly washed out in the rather early evolution (15$|m|^{-1}$), the spectrum is still far from equilibrium. It takes by an order of magnitude longer to approach the thermal spectrum (for small coupling).

We may split the observed time scales to three epochs. The separation of these scales we find for perturbative couplings only, in a strong coupling application the first two might coincide. (The time values correspond to the parameters used in the presented plots.)

1) Prethermalization — loss of coherence ($t < 5|m|^{-1}$)

Prethermalization is a universal far-from-equilibrium phenomenon, its time scale is given by the relevant scales of the dynamics (temperature, mass) but it is independent of the coupling, as shown in Fig. 1right.
Contrary to the mode-by-mode occupation numbers (Fig. 1 left) the bulk, averaged quantities are often insensitive to the details of the spectrum as it is also observed in classical field theory. We find that this short time is enough for very rapid establishment of an almost constant ratio of pressure over energy density (Fig. 1 right), as well as a kinetic temperature based on average kinetic energy (Fig. 2 left, for definitions see Ref. 6). These are important ingredients of the early equation of state. This level of equilibration is one of the important preconditions of the use of hydrodynamic equations.

(2) **Damping** — decay of individual modes ($t < 15|m|^{-1}$)

The individual excitations are washed out so that a smooth but still far-from-continuum spectrum is formed. We find that this relaxation time agrees with the inverse width ($\gamma(\vec{p}; t)$) of the nonequilibrium spectral function. We know that the damping time (obtained from the imaginary part of the self energy or from the width of the spectral function) is the decay rate of an individual signal excited over the ensemble. This relaxation time may be much shorter than thermalization time. In the leading order perturbation theory relaxation time is proportional to the coupling-squared. We also see that this relaxation time scale is not a subject of spectacular evolution.

Within this time scale the generically independent symmetric propagator ($F$) and spectral function ($\rho$) evolve to fulfill a relation (in Wigner coordinates): 

$$F(\omega, \vec{p}; t) = \pm (n(\omega; t) + \frac{1}{2})\rho(\omega, \vec{p}; t),$$

which is known as fluctuation–dissipation relation in equilibrium. Here the nontrivial issue is the momentum ($\vec{p}$) independence of $n(\omega; t)$. This enables us to translate it to $n(\vec{p}; t)$ using the dispersion relation defined by the peak of the spectral function. The existence of the relation above provides a particle number definition $n(\vec{p}; t)$ not tainted by quasi-particle picture. Fig 2 right shows by shaded bands how consistently the particle number $n(\vec{p}; t)$ is determined from the evolution of different $\vec{k}$ modes with $\omega_k \approx \omega_p \pm \gamma_p$.

(3) **Thermalization** — loss of spectral information ($t \lesssim 500|m|^{-1}$)

Kinetic equilibration is a result of the collective evolution of the particle spectra. At the same time the particle abundances in the various species are equilibrated as well. Whether this longer epoch fits into the short time range before chemical freeze-out is still a matter of discussion.
3. Discussion

Direct heavy ion applications require higher couplings than in the example above. Tuning the coupling to $g = 2.5 - 3$ one observes that both relaxation and thermalization time shrinks considerably, approaching the prethermalization time. The elementary oscillators are then over-damped, and the estimated prethermalization time will also give a hint for the damping time scale. With these couplings one can easily obtain thermal spectra in the fermionic sector within $1 - 3 \text{ fm}/c$.

We find that prethermalization time can be estimated by the inverse average kinetic temperature$^6$. In particular, we obtained the following quantitative relation: $T t_{pt} \approx 2 - 2.5$. If one attempts to use the concept of prethermalization for the case of heavy ion collisions one has to replace this temperature by the relevant scale of the dynamics. This we assume to be the saturation scale: $Q_s \approx 1 \text{ GeV}$ at RHIC energies. Estimating the prethermalization time by $3$ times $1/Q_s$ one has: $t_{\text{prethermalization}} \approx 0.6 \text{ fm}/c$.

This is the very close to the instant that is usually taken as initial time for the hydrodynamic evolution$^{11,12}$. If the complete local equilibration cannot be established, it is worth thinking if this partial equilibration can already establish the use of flow equations. We do believe that the study of partial equilibration can help us overcome the discrepancy between the long thermalization time predicted by kinetic theories and the short times used by hydrodynamic models.

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