On-chip black hole: Hawking radiation and curved spacetime in a superconducting quantum circuit with tunable couplers

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Hawking radiation is one of quantum features of a black hole that can be understood as a quantum tunneling across the event horizon of the black hole, but it is quite difficult to directly observe the Hawking radiation of an astrophysical black hole. Remarkable experiments of analogue black holes on various platforms have been performed. However, due to the experimental difficulty of accurately constructing curved spacetime and precisely measuring the thermal spectrum, Hawking radiation and its quantum nature, such as entanglement, have not been adequately investigated. Based on the recent architecture breakthrough of tunable couplers for superconducting processors, we experimentally realize an analogue black hole using our new developed chip with a chain of 10 superconducting transmon qubits interactions mediated by 9 transmon-type tunable couplers.

By developing efficient techniques to engineer the couplings between qubits via tuning couplers, we achieve both the flat and curved spacetime backgrounds. The quantum walks of quasi-particle in the curved spacetime reflect the gravitational effect near the black hole, resulting in the behavior of Hawking radiation. By virtue of the state tomography measurement of all 7 qubits outside the analogue event horizon, we show that Hawking radiation can be verified. In addition, an entangled pair is prepared inside the horizon and the dynamics of entanglement in the curved spacetime is directly measured. Our results would stimulate more interests to explore the related features of black holes using programmable superconducting processor with tunable couplers.

Introduction.—In the classical picture, a particle falls into a black hole horizon and the horizon prevents the particle from turning back, then escape becomes impossible. However, taking into account quantum effect, the particle inside the black hole is doomed to gradually escape to the outside, leading to the Hawking radiation [1]. The problem is that direct observation of such a quantum effect of a real black hole is difficult in astrophysics. For a black hole with solar mass, the associated Hawking temperature is only \( \sim 10^{-5} \) K and the corresponding radiation probability is astronomically small. Given by this, various analogue systems were proposed to simulate a black hole and its physical effects in laboratories [2]. Over the past years, the theory of Hawking radiation has been tested in experiments based on various platforms engineered with analogue black holes, such as using supersonic fluid [2–7], Bose-Einstein condensates [8–10], optical metamaterials and light [11–13], etc.

On the other hand, the developments of superconducting processor enable us to simulate various intriguing problems of many-body systems, molecules, and to achieve quantum computational supremacy [14–17]. However, constructing an analogue black hole on a superconducting chip is still a challenge, which requires wide-range tunable and site-dependent couplings between qubits to realize the curved spacetime [18]. Coincidentally, a recent architectural breakthrough of tunable couplers for superconducting circuit [19], which has been exploited to implement fast and high-fidelity two-qubit gates [20–22], offers an opportunity to achieve specific coupling distribution analogous to the curved spacetime. We develop such a superconducting processor integrated with a one-dimensional (1D) array of 10 qubits with interaction couplings controlled by 9 tunable couplers, see Fig. 1, which can realize both flat and curved spacetime backgrounds. The quantum walks of quasi-particle excitations of superconducting qubits are performed to simulate dynamics of particles in a black hole background, including dynamics of entangled pair inside the horizon. By using multiqubit state tomography, Hawking radiation is measured which is in agreement with theory prediction. This new constructed analogue black hole then facilitates further investigations of other related problems of black hole.
**FIG. 1. On-chip analogue black hole.** a. False-color image of superconducting processor and schematic analogue black hole. Ten transmon qubits, $Q_1 \sim Q_{10}$, shown as cross, are integrated along a chain with nearest-neighbor couplings. Each nearest-neighbor two qubits are coupled via a coupler, $C_1 \sim C_5$, realized by a transmon with only a flux bias line. All the transmons are frequency-tunable, but only qubit has XY control line and readout resonator. The schematic image represents the background of curved spacetime simulated by this superconducting chip. The red cartoon spin located at upper-left denotes the evolution of one quasi-particle that are initially in the black hole and the outward-going radiation. b. Schematic representation of the site-dependent effective coupling strengths $\kappa_j$. In the experiment, the coupling $\kappa_j$ is designed according to Eq. (3). There is a boundary analogous to the event horizon of a black hole, where the coupling changes its sign at site $Q_1$. Thus qubits $Q_1$ and $Q_2$ can be considered as the interior of black hole, $Q_3$ is at the horizon, and $Q_4 \sim Q_{10}$ are in the outside black hole. c. Experimental pulse sequence for observing dynamics of entanglement. The pulse sequences of frequency are presented for preparing an entangled pair on $Q_1Q_2$, which are mediated by coupler $C_1$, in the analogue black hole.

**Model and setup.**—Consider a general two-dimensional spacetime background with a fixed static metric $g_{\mu \nu}$, the metric in the Schwarzschild coordinates $(t_s, x)$ reads $ds^2 = f(x)dt_s^2 - f^{-1}(x)dx^2$. To describe a black hole with nonzero temperature in 2-dimensional spacetime, we require that the function $f$ has a root at $x = x_h$ with $f'(x_h) > 0$ and $f(x) > 0$ for $x > x_h$ standing for the exterior of the black hole, while $f(x) < 0$ for $x < x_h$ for the interior. The horizon of black hole then locates at $x = x_h$. For our purpose and experimental setups, we transform above metric into “advanced Eddington-Finkelstein coordinates” by the coordinates transformation $t = t_a + \int f^{-1}(x)dx$. In the coordinates $\{t, x\}$, the metric now becomes $ds^2 = f(x)dt^2 - 2dxdt$. A brief introduction on the differences between the “time-orthogonal coordinates” and “advanced Eddington-Finkelstein coordinates” can be found in our Supplementary Material.

To consider the effects of quantum matters in curved spacetime, we consider a Dirac field, of which the Dirac equation is written as [23, 24]

$$i\gamma^a e_\mu^a \partial_\mu \psi + \frac{i}{2} \gamma^a \frac{1}{\sqrt{-g}} \Theta^a \left( \sqrt{-g} e_\mu^a \right) \psi - m \psi = 0,$$

where $g$ is the determinate of $g_{\mu \nu}$, the vielbein $e_\mu^a$ satisfies the orthonormal condition $e_\mu^a e_{\nu}^{\mu} = \delta^\mu_\nu$ and the $\gamma$-matrices in the two-dimensional case are chosen to be $\gamma = (\sigma_z, i\sigma_y)$. In the Eddington-Finkelstein coordinates $\{t, x\}$ and in the massless limit $m \to 0$, such a Dirac field can be quantized into a discrete XY lattice model with site-dependent hopping couplings. The effective Hamiltonian reads (see Supplementary and Ref. [18])

$$\frac{\hat{H}}{\hbar} = -\sum_j \kappa_j (\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \hat{\sigma}_j^- \hat{\sigma}_{j+1}^+) - \sum_j \mu_j \hat{\sigma}_j^+ \hat{\sigma}_j^-,$$

where $\hbar$ is the reduced Planck constant (for convenience $\hbar$ will be assumed to be 1 in the following), $\sigma_j^+$ ($\sigma_j^-$) is the raising (lowering) operator of the $j$-th qubit, $\mu_j$ denotes the on-site potential, the site-dependent coupling $\kappa_j$ takes the form $\kappa_j \approx f((j - j_h + 1/2)d)/4d$ with $d$ being the lattice constant. Here, the function $f(x)$ is related to spacetime metric, which is given in the Eddington-Finkelstein coordinates $\{t, x\}$ as $ds^2 = f(x)dt^2 - 2dxdt$. The spatial position $x$ is discretized as $x_j = (j - j_h)d$. Since the horizon locates at $f(x_h) = 0$ with $f'(x_h) > 0$, the horizon in our analogues model is then defined at site $j = j_h$ where $f(x_h) = 0$, but the sign of $\kappa_j$ is different on its two sides of the horizon resulting in a black hole spacetime structure. One side of horizon is considered
FIG. 2. Quantum walks in a 1D array of 10 superconducting qubits. a, Results of the quantum walks in a flat spacetime for four different initial states, i.e., $|\psi(0)\rangle = |1000000000\rangle, |1100000000\rangle, |0100000000\rangle$ and $|0010000000\rangle$ with $|0\rangle$ representing the ground state of a qubit and $|1\rangle$ the excited state. The case of black hole spacetime is presented in b. The heatmap denotes the probabilities of excited-state for $Q_i$ in time. The horizontal axis is indexed as qubit number $i$, the vertical axis is the evolution time. Here we show both the numerical simulation and experiment data to compare the difference between the flat spacetime ($\kappa_j \approx 2.94$ MHz) and the curved spacetime ($\beta \approx 4.39$ MHz). c, The fidelity of quantum walks in the curved spacetime. d, The probability of finding a particle outside the horizon on qubits $\{Q_4, Q_5, Q_6, Q_7, Q_8, Q_9, Q_{10}\}$. Error bars are 1 SD calculated from all probability data of the $k = 1$-50 pulse sequences.

as the interior of the black hole, while the opposite side represents the exterior of the black hole.

We perform the experiment to simulate the black hole using a superconducting processor with a chain of 10 qubits $Q_1$-$Q_{10}$, which represents the Hamiltonian (2), additionally with 9 tunable couplers interspersed between every two nearest-neighbor qubits, see Fig. 1. The effective hopping coupling $\kappa_j$ between qubits $Q_j$ and $Q_{j+1}$, which derives from their direct capacitive coupling and the indirect virtual exchange coupling via the coupler in between, can be tuned arbitrarily via programming the frequency of the corresponding coupler $C_j$ (see Supplementary and Ref. [19]). Additionally, we develop an efficient and automatic calibration for multi-qubit devices with tunable couplers to achieve accurate control of couplings (see Supplementary for details). To describe the curved spacetime experimentally, we change the frequencies of all the couplers by using the arbitrary waveform generator (AWG) to generate various flux-bias Z pulses applied to them, and design the effective coupling distribution as

$$\kappa_j = \frac{\beta \tanh ((j - j_h + 1/2)d)}{4d}$$

with $j_h = 3$, $d = 0.35$ a.u. and $\beta/(2\pi) \approx 4.39$ MHz. Here we choose $f(x) = \beta \tanh(x)$. As shown in Fig. 1b, the coupling $\kappa_j$ goes monotonically from negative to positive from $Q_3$’s left to right side. In this way, the information of the static curved spacetime background is encoded into the site-dependent coupling distribution. Thus, the site $Q_3$ where the sign of the coupling reverses can be analogous to the event horizon of the black hole, the side of negative coupling ($Q_1$-$Q_2$) can be considered as the interior of the black hole, and $Q_4$-$Q_{10}$ are outside the horizon. For comparison, we also realize a uniform coupling distribution with $\kappa_j/(2\pi) \approx 2.94$ MHz to realize a flat spacetime.

In the experiment, we first prepare an initial state $|\psi(0)\rangle$ with quasi-particle excitations, i.e., exciting qubits or creating an entangled pair. The evolution of the initial state known as quantum walk will be governed by
Schrödinger equation $|\psi(t)| = e^{-iHt} |\psi(0)|$ based on 1D programmable controlled Hamiltonian (2). The dynamics of the prepared states then behaves the behavior of quasi-particle in the studied (1+1)-dimensional spacetime with designed flat or curved structure.

Quantum walks in curved spacetime.—Figs. 2a and 2b show the propagation of quasi-particle in flat and curved spacetimes, respectively. Here we initialize the system by preparing four different single-particle or two-particle states, including $|\psi(0)| = |0000000000\rangle, |1100000000\rangle, |0010000000\rangle$ and $|0001000000\rangle$ with $|0\rangle$ representing the ground state of a qubit and $|1\rangle$ the excited state. Once the initial state is prepared, we apply the rectangle Z pulses on each qubit to make the on-site potential $\mu_j$ of all qubits at a reference frequency $\omega_{\text{ref}}/(2\pi) \approx 5.1$ GHz. During the quench dynamics, the hopping couplings $\kappa_j$ between qubits are fixed as Eq.(3) (curved spacetime) or a constant (flat spacetime) via biasing couplers. After time $t$, all qubits are biased back to idle points for readout. The occupation of quasi-particle density distribution $p_j(t) := \langle \psi(t)| \hat{\sigma}_j^+ \hat{\sigma}_j^- |\psi(t)\rangle$ is measured by averaging 5000 repeated single-shot measurements, as shown in Figs. 2a and 2b for four initial states.

Fig. 2a shows that the propagation of quasi-particle in the flat spacetime is unimpeded, corresponding to the result of conventional quantum walk with diffusive expansion [25–28]. In contrast, the particle is mainly trapped in our on-chip black hole due to the analogue gravity around the horizon $Q_3$, as shown in Fig. 2b with the initial state $|\psi(0)| = |0000000000\rangle$ and $|\psi(0)| = |1100000000\rangle$. Due to the infalling Eddington-Finkelstein coordinates we took, our model only simulates the outgoing modes of the particle (see Supplementary). Hence, the interior and exterior of black hole is equivalent so that the same phenomenon can be observed in this case where the particle is initially prepared in the exterior of black hole ($|\psi(0)| = |0001000000\rangle$).

Here, we also present the result of the particle initialized at the horizon in Fig. 2b, i.e., $|\psi(0)| = |0010000000\rangle$. In the continuous curved spacetime, the particle initialized at the horizon is bound to the horizon forever due to the zero couplings on both sides of the horizon. However, in the finite-size lattice, the coupling strengths on both sides of the horizon are not strictly zero even though they are very small ($\approx 0.54$ MHz). The particle seems to be localized at the horizon for a very short time, but it is doomed to escape from the constraints due to the finite-size effects.

To show the accuracy of the experimental results of quantum walk in the curved spacetime, we present the fidelity $F(t) = \sum_{j=1}^{10} \sqrt{p_j(t)q_j(t)}$ between the measured and theoretical probability distributions $p_j(t)$ and $q_j(t)$ in Fig. 2c. The high fidelity, greater than 97% within 400 ns experiment time, implies that our experimental results are consistent with the theoretical predictions as also demonstrated by the similarity between experimen-

![Image](image-url)

**Fig. 3. Observation of analogue Hawking radiation.**

a. The 7-qubit density matrix at $t = 0$ ns. Initially, only $Q_1$ is prepared in $|1\rangle$ and all the qubits outside the horizon are almost in $|0\rangle$. b. The 7-qubit density matrix at $t = 1000$ ns after the quench dynamics. Due to the Hawking radiation, radiation states can be detected with small probabilities. The fidelity between ideal and experimental density matrix at $t = 0$ and 1000 ns are 99.2% and 88.1%, respectively. c. The logarithmic probability of finding a particle outside the horizon $P_0$ vs. its energy $E_n$. d. The logarithm of average radiation probability vs. the energy of particle when $E_n > 0$. Error bars are 1 SD calculated from the tomography data at the same energy. The slope of the red line represents the reciprocal of Hawking temperature without noise, where the Hawking temperature here is given by $T_H/(2\pi) = 3/(8\pi^2) \approx 0.35$ MHz or $\approx 1.7 \times 10^{-5}$ K in Kelvin temperature. The experimental results are in agreement with the simulated data for low energy, but diverge at high energy due to experiment noises.

The theory of Hawking radiation points out that the probability of radiation satisfies a canonical blackbody
where $E$ denotes the energy of particle outside the horizon, $T_H/(2\pi) = g_h/(4\pi^2)$ is defined as the effective temperature of the Hawking radiation, and $g_h = \frac{1}{2}\alpha'(x_h) = \beta/2$ represents the surface gravity of the black hole [18]. The derivation of Eq. (4) can be constructed by using the picture of quantum tunneling to obtain the tunneling rate of particle [30–32]. We use this picture in this work to understand the Hawking radiation. Such a picture is equivalent to a field theoretical viewpoint of “particle-antiparticle pairs” created around the horizon: the antiparticle (negative energy) falls into the black hole and annihilates with this particle inside the black hole, the particle outside the horizon is materialized and escapes into infinity (see Supplementary). Also, Eq. (4) can be viewed as the detailed balance relation between creation and annihilation of particle around the horizon in a thermal environment [33].

In Fig. 3a and 3b, we present the density matrix of 7 qubits $Q_k-Q_{10}$ by quantum state tomography (QST) at $t = 0$ and $t = 1000$ ns, such a final time is long enough so that the particle inside the black hole has finished its tunelling to the outside but the boundary effect is negligible to the results. Here, the initial state is $|\psi(0)\rangle = |1000000000\rangle$, i.e., a particle in the black hole has a certain position. When $t = 0$ ns, no radiation can be detected and all the qubits outside the horizon are almost in $|0\rangle$, see Fig. 3a. After a long time $t = 1000$ ns, one may have a small chance to probe the particle outside the horizon, see Fig. 3b. The corresponding probabilities of radiation can be extracted from the measured 7-qubit density matrix. Assuming that $|E_n\rangle$ is the $n$-th eigenenergy of total Hamiltonian and $\rho_{out}$ is the density matrix outside obtained by QST, then the probability of finding a particle of energy $E_n$ outside the horizon can be obtained as,

$$P_n = \langle E_n|\rho_{out}|E_n\rangle. \quad (5)$$

Although there are $2^{10} = 1024$ eigenstates for 10-qubit Hamiltonian Eq. (2) and the same number of $P_n$, the radiation states involve only 10 single-particle excited eigenstates due to the particle number conservation. As a consequence, only those $P_n$ that are corresponding to single-particle excited eigenstates have non-zero values, as shown in Fig. 3c. Therefore, we take the average of $P_n$ with the same positive energy $E_n$ as $\bar{P}_n$ to describe the average probability of finding a particle outside with $E_n > 0$. It can be expected that the relation between $\bar{P}_n$ and $E_n$ will agree with the theoretical prediction in Eq. (4). In Fig. 3d, the simulated results show that the logarithm of the average radiation probability is approximately linear in energy with Hawking temperature $1.7 \times 10^{-5}$ K. The fitted Hawking temperature of experimental data is around $7.7 \times 10^{-5}$ K, showing validity with the same order of magnitude. The deviation between experimental data and ideal simulation data is mainly caused by the evolution of the imperfect initial state. The fidelity between the imperfect initial state in experiment and ideal initial state is 99.2%, which may derive from the experimental noises including XY crosstalk, thermal excitation, leakage, etc. We substitute such a experimental state for the ideal initial state in the numerical simulation of Hawking radiation, then the results of numerical simulation agree with experimental results better.

Note that the simulated black hole here is one-dimensional in space. It may be difficult to define mass. If we consider a black hole in four-dimensional spacetime with the same Hawking temperature $T_H$, its mass can be calculated by $M/M_s = 6.4 \times 10^{-8} K/T_H$ [1], where $M_s \approx 2 \times 10^{30}$ kg is the solar mass. For the simulated black hole here, its mass is about one thousandth of the solar mass.

**Dynamics of entanglement in the analogue black hole.**—Hawking predicted that the entanglement entropy increases when a black hole forms and evaporates due to the Hawking radiation. Each Hawking particle is entangled with a partner particle in the black hole. Such kind of quantum feature plays a crucial role in studying black holes and quantum information [34].

To investigate the dynamical entanglement and non-local correlation both in flat spacetime and curved spacetime, we perform the time-dependent QST measurement on reduced quantum states of two qubits ($Q_1$&$Q_2$) in the black hole. We focus on two quantities, entanglement en-
tropy and concurrence. Both of them can be computed by using the two-qubit density matrix $\rho_{ab}(t)$ with time $t$ obtained by the results of QST. Here, the entanglement entropy is given by $S(t) = -\text{Tr}[\rho(t) \ln \rho(t)]$ quantifying the entanglement between the interior of black hole and the exterior [35]. The concurrence represents the entanglement just between the two qubits in the black hole, which is calculated as $E(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ with $\lambda_i$ being the square roots of the eigenvalues of matrix $\tilde{\rho}$ in decreasing order, where $\tilde{\rho} = (\hat{\sigma}_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \hat{\sigma}_y)$ is the spin-flipped state of $\rho$ with $\sigma_y$ being Pauli matrix [36].

As shown in Fig. 1c, we first prepare an entangled pair in our on-chip black hole by implementing a $\pi/2$ pulse (along the $Y$-axis of the Bloch sphere) on control qubit $Q_1$ and a subsequent controlled-NOT (CNOT) gate on the pair. The CNOT gate includes a $-\pi/2$ pulse on target qubit $Q_2$, an adiabatic controlled-phase (CZ) gate and a $Y/2$ gate on $Q_2$ in sequence. We realize the CZ gate with the tunable coupler by using the recent developed approach of adiabatic CZ gate [20–22]. The mean fidelity of prepared Bell state $\langle (00) + (11) \rangle / \sqrt{2}$ is 99.15%.

Figure 4 shows the dynamics of such an initial entangled state in both the flat and curved spacetime. Here we choose a relatively small $\beta$ to reduce the propagation speed of particle that escapes from the black hole and the finite size effect thus will be reduced during the quench dynamics. In Fig. 4a, the entanglement entropy in the case of curved spacetime progressively increases due to the Hawking radiation, while in the flat spacetime it has two wavefronts resulting from the quantum interference and reflection respectively [27]. On the other hand, the concurrence decreases with time in both cases, reflecting the process of entanglement being lost into the environment. However, the entanglement between the pair in the black hole is protected by the analogue gravity so that it can slow the decrease of the concurrence, see Fig. 4b.

Summary and outlook.—In summary, we have experimentally simulated a curved spacetime of black hole and observed Hawking radiation in a superconducting processor with tunable couplers. Our results would stimulate further interests to explore related problems such as different designed curved spacetime and time-evolving black holes [34, 37–39]. A more advanced processor with more qubits in 1D array may reduce the finite-size effects, and provides more accurate data in simulating black holes. A 2D quantum processor may facilitate the simulation of other properties of black holes such as gravitational lensing and slingshot, the relationship between entropy and black hole horizon area.

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Supplementary Material: On-chip black hole: Hawking radiation and curved spacetime in a superconducting circuit with tunable couplers

INTRODUCTION ON EDDINGTON-FINKELSTEIN COORDINATES

In this paper, the spacetime geometry is present by advanced Eddington-Finkelstein coordinates (AEFC) \{t, x\}. Though the coordinate “t” plays the role of “time” in this system, there are a few of differences compared with the usually time coordinate. In this appendix, we will given a basic introduction.

One simple way to obtain an intuition of advanced Eddington-Finkelstein coordinates {v, x} is to consider the wave propagating in flat spacetime. Let us consider a Minkowski spacetime. The usual Minkowski coordinates (MC) is \{t_m, x\}, of which the metric reads
\[
\text{ds}^2 = \text{dt}^2_m - \text{dx}^2
\]  
(S1)
The massless scalar field then will satisfies
\[
\partial^2_{t_m}\phi - \partial^2_x\phi = 0
\]  
(S2)
The solution of this equation in general is given by following “traveling wave solution”,
\[
\phi(t_m, x) = \phi_1(t_m + x) + \phi_2(t_m - x)
\]  
(S3)
where \(\phi_1(t_m + x)\) stands for the advanced solution and \(\phi_2(t_m - x)\) stands for the outgoing solution.

To covert the advanced Eddington-Finkelstein coordinates \{t, x\}, we consider a coordinates transformation
\[
t = t_m + x,
\]  
(S4)
and so the metric then reads
\[
\text{ds}^2 = \text{dt}^2 - 2\text{dtdx}
\]  
(S5)
It needs to note that, though \(dt_m \neq dt\), we still have
\[
\partial_{t_m}h|_{MC} = \partial_t h|_{AEFC}
\]  
(S6)
for arbitrary function \(h\), i.e. the time derivatives are the same as that in the usual Minkowski coordinates and advanced Eddington-Finkelstein coordinates. Thus, the growth rate of a quantity in usual Minkowski coordinates can also be computed according to the time derivative of advanced Eddington-Finkelstein coordinates. On the contrary, if one wants to compute the spatial derivative, then the two coordinate systems will have different results
\[
\partial_x h|_{MC} \neq \partial_x h|_{AEFC}
\]  
(S7)

FIG. S1. The anti-particle flow of negative energy infalling toward the interior of black hole can always be interpreted as a particle flow of positive energy outgoing from the interior.

in general since the derivative in left-side fixes \(t_m\) but the derivative of right-side fixes \(t = t_m + x\).

The propagators of wave are also very different in these two coordinate systems. From Eq. (S3), one sees that the “traveling wave solution” in advanced Eddington-Finkelstein coordinates reads
\[
\phi(t, x) = \phi_1(t) + \phi_2(t - 2x),
\]  
(S8)
This can also be obtained from the wave equation \(2\partial_t\partial_x\phi + 2\partial^2_x\phi = 0\). It is a little surprising that the advanced wave \(\phi_1\) has no propagator! In fact the infalling mode now becomes a boundary condition rather than a propagator. If we impose a boundary condition
\[
\phi(t, \pm \infty) = 0,
\]  
(S9)
then we have \(\phi_1(t) = 0\) and so there is only outgoing mode. Thus, the advanced Eddington-Finkelstein coordinates with boundary condition (S9) can only represent the propagator of outgoing modes. In other words, the advanced Eddington-Finkelstein coordinates play the role of selector to choose only outgoing modes.

Though we assume that the spacetime is flat and the matter is scalar field in above discussion, the basic physical picture will still be true if we consider a curved 2-dimensional spacetime and Dirac field. From the Fig. 2 in our main-context, one can see that our model only simulates the outgoing modes, just as we expected in above discussion. In general, the wave in gravitational fields contains both advanced modes and outgoing modes. The Hawking radiation is an energy flux towards infinity, i.e. carried by outgoing modes. This is one reason why this paper uses advanced Eddington-Finkelstein coordinates to study the Hawking radiation.

MORE EXPLANATION ON THE “TUNNELING PICTURE” OF HAWKING RADIATION

In this paper, we use the picture of “quantum tunneling” to understand the Hawking radiation [30–32].
Though this is also a wide spread picture to understand Hawking radiation in the community of black hole physics, it may not be familiar to the other readers. Here we make a brief introduction on this picture.

A first glance, the tunneling picture is very different from picture of “pair creation” outside horizon. However, they are equivalent in physics. Based on the picture of “pair creation” in Hawking radiation, “particle-antiparticle pairs” can be created around the horizon. The antiparticle (negative energy) falls into the black hole and annihilates with positive energy particle inside the black hole, the particle outside the horizon is materialized and escapes into infinity. Note that the pair creation/annihilation is a virtual process, and the really materialized result is that the original particle inside the black hole disappears but an identical particle appears outside the horizon. The anti-particle of negative energy infalling the interior of black hole can always be interpreted as a particle of positive energy outgoing from the interior, see schematic diagram Fig. S1. This leads to an equivalent picture to understand Hawking radiation via quantum tunneling: the particle inside the horizon escapes to outside by quantum tunneling. Thus, the “tunneling picture” and “pair creation picture” are just two different pictures to understand the same physical phenomenon. Note that this “tunneling picture” does not violate causality since the spectrum is thermal and no information is carried.

Let us explain in detail about how to use this tunneling picture to understand the spectrum of radiation and corresponding temperature. For the spacetime with a black hole, the metric in the Schwarzschild coordinates \( \{ t_s, x \} \) is given by \( ds^2 = f(x)dt_s^2 - f^{-1}(x)dx^2 \). We consider an outgoing mode with positive energy (corresponding to the observers of infinity) of a massless scalar or Dirac field, which can be written as

\[
\Phi(t_s, x) \propto \exp \left[ -i\omega \left( t_s - \int \frac{dx}{f(x)} \right) \right]. \quad (S10)
\]

By using the Eddington-Finkelstein coordinates we have

\[
\Phi(t, x) \propto \exp \left[ -i\omega \left( t - 2 \int \frac{dx}{f(x)} \right) \right]. \quad (S11)
\]

Since \( f(x) \) has a root at \( f(x_h) \), we separate integration into two parts,

\[
\int \frac{dx}{f(x)} = F(x) + \int \frac{dx}{2gh(x - x_h)} = F(x) + \frac{1}{2gh} \ln |x - x_h|, \quad (S12)
\]

where

\[
F(x) = \int \left[ \frac{1}{f(x)} - \frac{1}{2gh(x - x_h)} \right] dx \quad (S13)
\]

is regular at \( x = x_h \). Here \( gh = f'(x_h)/2 \) is the surface gravity. Thus, the outgoing positive mode then reads

\[
\Phi(t, x) \propto e^{-i\omega |x - x_h|} e^{i\omega / gh}. \quad (S14)
\]

This outgoing mode has an infinite number of oscillations as \( x \to x_h \) and therefore cannot be straightforwardly extended to the inner region from the region outside horizon. As argued in Ref. [30], we can use analytic continuation to connected two branches in complex plane: the wave function \( \Phi \) describing a particle state (positive frequencies) can be analytically continued to complex plane (see Fig. S2). Then we obtain

\[
\Phi(t, x) = \begin{cases} 
    e^{-i\omega |x - x_h|} e^{i\omega / gh}, & x > x_h \\
    e^{-i\omega |x - 2F(x)|} e^{i\omega / gh} e^{i\omega / gh}, & x < x_h 
\end{cases}. \quad (S15)
\]

This gives us the tunneling rate

\[
P = e^{-2i\omega / gh}, \quad (S16)
\]

which is identical to the detailed balance relation for transition rates in a thermal environment [33]. Note that the tunneling rate \( (S16) \) stands for the rate of a single particle. It is possible that the tunneling of multiple particles happens simultaneously. For bosons we have

\[
\text{particle number}: \quad 0 \quad 1 \quad 2 \quad 3 \quad \ldots \\
\text{probability}: \quad 1 \quad P \quad P^2 \quad P^3 \quad \ldots \quad (S17)
\]

Thus the occupation number of energy \( \omega \) reads

\[
n(\omega) = \sum_{k=0}^{\infty} \frac{k! P^k}{\sum_{k=0}^{\infty} P^k} = \frac{1}{e^{2\pi\omega/gh} - 1}. \quad (S18)
\]

This gives us the expected distribution of bosons and the temperature reads \( T = gh/(2\pi) \) as predicted by Hawking. For fermions, if there is no other internal degree of freedom, the Pauli exclusion principle implies that there is at most one particle of same energy. Thus, Eq. \( (S17) \) is replaced by

\[
\text{particle number } n : \quad 0 \quad 1 \\
\text{probability}: \quad 1 \quad P \quad (S19)
\]

Thus the occupation number of energy \( \omega \) reads

\[
n(\omega) = \sum_{k=0}^{1} \frac{k! P^k}{\sum_{k=0}^{\infty} P^k} = \frac{1}{e^{2\pi\omega/gh} + 1}. \quad (S20)
\]
FIG. S3. The theoretical framework of simulating quantum field theory in (1+1)-D curved spacetime with quantum many-body systems. The “time” $t$ in the infalling Eddington-Finkelstein metric is given by $t = t_x + \int f^{-1} dx$ where $\{t_x, x\}$ are the Schwarzschild coordinates. In (1+1)-D configurations, the massless scalar field $\phi$ and the massless Dirac field $\psi$ can be discreted into the Bose-Hubbard model and the spinless Fermi-Hubbard model respectively, where the static curved spacetime background is encoded into the site-dependent hopping coupling distribution $\kappa_j$ satisfying Eq. (S41). Here the on-site potential $\mu$ is an arbitrary constant. Note that both of the Bose-Hubbard model and the spinless Fermi-Hubbard model can be transformed into the XY model, which implies their equivalence in (1+1)-D spacetime. This schematic diagram is a brief summary of the theory in Ref. [18].

This gives us the expected distribution of fermions and the temperature still reads $T = g_b/(2\pi)$.

In addition, the composite system of the interior of black hole and the exterior is isolated. The interior and the exterior exchange energy and particles via the horizon. Hence, the occupations Eq. (S18) and (S20) can be viewed as the statistical averages of grand canonical distributions concerning bosons and fermions, respectively.

\[
\hat{H} = \hat{H}_Q + \hat{H}_C + \hat{H}_{Q-C} + \hat{H}_{Q-C} + \hat{H}_{C-C}, \quad \text{(S21)}
\]
\[
\hat{H}_Q/\hbar = \sum_{j=1}^{10} \omega_j \hat{b}_j \hat{\bar{b}}_j + \frac{\alpha_{q_{j+1}}}{2} \hat{b}_j \hat{\bar{b}}_{j+1} \hat{b}_{j+1} \hat{\bar{b}}_j, \quad \text{(S22)}
\]
\[
\hat{H}_C/\hbar = \sum_{j=1}^{9} \omega_{c_j} \hat{c}_j \hat{\bar{c}}_j + \frac{\alpha_{c_{j+1}}}{2} \hat{c}_j \hat{\bar{c}}_{j+1} \hat{c}_{j+1} \hat{\bar{c}}_j, \quad \text{(S23)}
\]
\[
\hat{H}_{Q-C}/\hbar = \sum_{j=1}^{9} g_{q_j, c_{j+1}} (\hat{b}_j \hat{\bar{b}}_{j+1} + \hat{b}_{j+1} \hat{\bar{b}}_j), \quad \text{(S24)}
\]
\[
\hat{H}_{C-C}/\hbar = \sum_{j=1}^{9} g_{c_j, c_{j+1}} (\hat{c}_j \hat{\bar{c}}_{j+1} + \hat{c}_{j+1} \hat{\bar{c}}_j), \quad \text{(S25)}
\]
\[
\hat{H}_{Q-C}/\hbar = \sum_{j=1}^{9} g_{q_j, c_j} (\hat{b}_j \hat{\bar{c}}_j + \hat{b}_j \hat{\bar{c}}_j) + g_{q_{j+1}, c_j} (\hat{b}_{j+1} \hat{\bar{b}}_j + \hat{b}_{j+1} \hat{\bar{b}}_j), \quad \text{(S26)}
\]

where $\hbar$ is the reduced Planck constant (for convenience $\hbar$ will be assumed to be 1 in the following), $\hat{b}_j$ ($\hat{c}_j$) and $\hat{b}_j$ ($\hat{c}_j$) denote the annihilation and creation operators of the $j$-th qubit (coupler), respectively. The corresponding frequencies and anharmonicities are $\omega_j$ ($\omega_j$) and $\alpha_q$ ($\alpha_q$). Every pair of two neighbouring qubits and their middle coupler are coupled through exchange-type interactions with coupling strengths $g_{q_j, c_j}$, $g_{q_{j+1}, c_j}$, $g_{q_{j+1}, c_{j+1}}$.

MODEL AND HAMILTONIAN

Our experiment is performed on a superconducting quantum processor which consists of 10 transmon qubits ($Q_1 \sim Q_{10}$) and 9 transmon-type couplers ($C_1 \sim C_9$). Each qubit and coupler are frequency-tunable, but only qubits have the XY control line and the readout resonator. The total Hamiltonian of this all-transmon system can be expressed as
and $g_{q_j,q_{j+1}}$. Here, the total Hamiltonian has three parts, including qubit-qubit interaction $\hat{H}_{Q-Q}$, coupler-coupler interaction $\hat{H}_{C-C}$ and qubit-coupler interaction $\hat{H}_{Q-C}$. The total system is equivalent to a 19-qubit Bose-Hubbard model.

In our experiment, the strong dispersive condition $g_{q_j,c_j} \ll |\Delta_{q_j,c_j}|$ is satisfied, where $\Delta_{q_j,c_j} = \omega_{q_j} - \omega_{c_j}$ is the frequency detuning. By virtue of the so-called Schrieffer-Wolff transformation

\[
\hat{U} = \exp \left[ \sum_{j=1}^{9} \frac{g_{q_j,c_j}}{\Delta_{q_j,c_j}} \left( \hat{b}_{q_j} \hat{b}_{c_j} + \hat{b}_{q_j} \hat{b}_{c_j} \right) + \frac{g_{q_j+1,c_j}}{\Delta_{q_j+1,c_j}} \left( \hat{b}_{q_j+1} \hat{b}_{c_j} + \hat{b}_{q_j+1} \hat{b}_{c_j} \right) \right],
\]

one can obtain the effective qubits Hamiltonian

\[
\hat{H} = \hat{U} \hat{H} \hat{U}^\dagger = \hat{H}_Q + \hat{H}_{Q-Q} = \sum_{j=1}^{10} \omega_{q_j} \hat{b}_{q_j}^\dagger \hat{b}_{q_j} + \frac{\alpha_{q_j}}{2} \hat{b}_{q_j}^\dagger \hat{b}_{q_j}^\dagger \hat{b}_{q_j} \hat{b}_{q_j} + \sum_{j=1}^{9} \tilde{g}_{q_j,q_{j+1}} \left( \hat{b}_{q_j}^\dagger \hat{b}_{q_{j+1}} + \hat{b}_{q_j} \hat{b}_{q_{j+1}}^\dagger \right),
\]

with the corresponding dressed frequency

\[
\tilde{\omega}_{q_j} = \begin{cases} 
\omega_{q_j} + \frac{g_{q_j,c_j}}{\Delta_{q_j,c_j}}, & j = 1 \\
\omega_{q_j} + \frac{g_{q_j,c_j}}{\Delta_{q_j-1,c_j}} + \frac{g_{q_{j+1},c_j}}{\Delta_{q_{j+1},c_j}}, & 1 < j < 10 \\
\omega_{q_j} + \frac{g_{q_j,c_{j-1}}}{\Delta_{q_j,c_{j-1}}}, & j = 10
\end{cases}
\]

and effective coupling strength

\[
\tilde{g}_{q_j,q_{j+1}} = g_{q_j,q_{j+1}} + \frac{g_{q_j,c_j} g_{q_{j+1},c_j}}{\Delta_{q_j,q_{j+1}}},
\]

where $\Delta_{q_j,q_{j+1}} = 2/(1/\Delta_{q_j,c_j} + 1/\Delta_{q_{j+1},c_j})$ is the harmonic mean of the frequencies detuning between the $j$-th coupler and its nearest neighbor qubits. Eq. (S30) implies that the effective qubit-qubit coupling is derived from their direct capacitive coupling and the indirect virtual exchange coupling via the coupler in between. If the frequency of coupler is above the frequencies of qubits, $\Delta_{q_j,q_{j+1}} < 0$ holds so that the effective coupling $\tilde{g}_{q_j,q_{j+1}}$ can be tuned from positive to negative monotonically with gradually decreasing the frequency of coupler. Experimentally, we use the arbitrary waveform generator (AWG) to generate various fast-bias voltages applied to the corresponding couplers. These pulses on the Z control lines change the frequencies of couplers and then make it possible for the superconducting circuit with tunable couplers to engineer arbitrary coupling distribution.

With $\tilde{g}_{q_j,q_{j+1}} \ll \alpha_{q_j}$, the effective Hamiltonian Eq. (S28) can be rewritten as a site-dependent XY model:

\[
\hat{H} = -\sum_{j=1}^{9} \kappa_j (\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \hat{\sigma}_j^- \hat{\sigma}_{j+1}^+) - \sum_{j=1}^{10} \mu_j \hat{\sigma}_j^+ \hat{\sigma}_j^-,
\]

where $\hat{\sigma}_j^+$ ($\hat{\sigma}_j^-$) is the raising (lowering) operator of the $j$-th qubit. Here we choose

\[
\kappa_j = -\tilde{g}_{q_j,q_{j+1}}, \quad \mu_j = -\tilde{\omega}_{q_j}.
\]

For the Hamiltonian Eq. (S31), one can map the spin variables to spinless fermion operators by introducing the Jordan-Wigner transformation [40]:

\[
\hat{\sigma}_j^+ = \hat{c}_{j+1}^\dagger \exp \left\{ \pi \sum_{k=1}^{j-1} \hat{c}_{k}^\dagger \hat{c}_{k} \right\} \hat{c}_{j}, \quad \hat{\sigma}_j^- = \exp \left\{ -\pi \sum_{k=1}^{j-1} \hat{c}_{k}^\dagger \hat{c}_{k} \right\} \hat{c}_{j},
\]

where the operators $\hat{c}^\dagger$ and $\hat{c}$ satisfy the commutation relations of fermions, i.e., $\{\hat{c}_j, \hat{c}_k\} = \{\hat{c}_j^\dagger, \hat{c}_k^\dagger\} = 0$ and $\{\hat{c}_j, \hat{c}_k^\dagger\} = \delta_{jk}$. Hence, the effective Hamiltonian is mapped into a spinless fermion lattice model as

\[
\hat{\tilde{H}} = -\sum_{j=1}^{9} \kappa_j (\hat{c}_{j+1}^\dagger \hat{c}_{j+1} + \hat{c}_j^\dagger \hat{c}_j) - \sum_{j=1}^{10} \mu_j \hat{c}_j^\dagger \hat{c}_j.
\]

CORRESPONDENCE WITH TWO-DIMENSIONAL CURVED SPACETIME

Considering the Heisenberg equation $i \hbar \frac{d}{dt} \hat{c}_j = [\hat{c}_j, \hat{H}]$, the evolution equation for the operator $\hat{c}_j$ can be given by

\[
\frac{d}{dt} \hat{c}_j = -\kappa_j \hat{c}_{j+1} - \kappa_{j-1} \hat{c}_{j-1} - \mu \hat{c}_j.
\]

By introducing a variable transformation $\hat{\tilde{c}}_j(t) = (-i)^j e^{-\mu t} \hat{c}_j$, we obtain

\[
\frac{d}{dt} \hat{\tilde{c}}_j(t) = -\kappa_j \hat{\tilde{c}}_{j+1}(t) + \kappa_{j-1} \hat{\tilde{c}}_{j-1}(t).
\]

Here $\hat{\tilde{c}}_j(t)$ can be viewed as a quantized operator of a discrete field $\varphi_j(t)$, where $d$ denotes the lattice constant and the spatial position can be discretized as $x = x_j = x_0 + j d$ with $x_0 = j_0 d$ and $j \in \mathbb{Z}^+$. Now let us recover the continuous field $\varphi(t, x)$. If we define a function $f$ that is dependent of the spatial position $x_j$ and substitute $\kappa_j$ as,

\[
\kappa_j = f(x_{j+1}) + f(x_j) - \frac{f(x_{j+1}) + f(x_j - 1)}{8d},
\]

according to Eq. (S35), $\varphi(t, x_j) \rightarrow \hat{\tilde{c}}_j(t)/\sqrt{d}$ will obey the following relation in the continuum limit,
\[ \frac{\partial}{\partial t} \varphi(t, x) = -\frac{f(x_{j+1}) + f(x_j)}{8d} \varphi(t, x_{j+1}) + \frac{f(x_j) + f(x_{j-1})}{8d} \varphi(t, x_{j-1}) \]

\[ = -\frac{f(x_j)}{4} \varphi(t, x_{j+1}) - \varphi(t, x_{j-1}) - \frac{1}{4} \cdot f(x_{j+1})\varphi(t, x_{j+1}) - f(x_{j-1})\varphi(t, x_{j-1}) \]

\[ = -\frac{f(x)}{4} \varphi(t, x) - \frac{f(x)}{4} \varphi(t, x). \]  

(S37)

In fact, Eq. (S37) can be considered as a special case of Dirac equation in the massless limit \( m \to 0 \) if we decompose the Dirac field operator into \( \psi = \frac{1}{\sqrt{2}} (\xi + \varphi, \xi - \varphi)^\top \). In the light of Refs [24, 41], the Dirac equation in (1+1)-dimensional curved spacetime with the metric \( g_{\mu\nu} \) is written as

\[ i\gamma^a \epsilon^{(a)}_\mu \partial_\mu \psi + \frac{i}{2} \gamma^a \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \epsilon^{(a)}_\mu) \psi - m \psi = 0, \]  

(S38)

where the dyad \( \epsilon^{(a)}_\mu \) satisfies the orthonormal condition \( \epsilon^{(a)}_\mu \epsilon^{(a)}_\nu = \delta_\mu^\nu \) and the \( \gamma \)-matrices in the two-dimensional case are \( \gamma^0 = \sigma_z \) and \( \gamma^1 = i \sigma_y \). When the dyad is chosen as

\[ \epsilon^{(a)}_\mu = \begin{pmatrix} -1 & 1 \\ 1-f & 1+f \end{pmatrix}. \]  

(S39)

Thus, Eq. (S38) can be decomposed into two independent equations,

\[ \partial_\nu \varphi = -\frac{f}{2} \partial_\nu \varphi - \frac{f'}{4} \varphi + \frac{i}{2} m \xi, \quad \partial_\nu \xi = -im \varphi. \]  

(S40)

In the massless limit \( m \to 0 \), one can find that Eq. (S40) is in accord with Eq. (S37). Hence, what the effective Hamiltonian Eq. (S33) describes is equivalent to a two-dimensional static curved spacetime governed by the massless Dirac equation if we set \( \kappa_j \) as

\[ \kappa_j = \frac{f(x_{j+1}) + f(x_j)}{8d} \approx \frac{f(x_j + d/2)}{4d}. \]  

(S41)

There is only one single nondegenerate horizon \( x_h \) so that \( f(x_h) = 0 \) and \( f(x) > 0 \) when \( x > x_h \) and

\[ g_h = \frac{1}{2} f'(x_h) > 0, \]  

(S42)

where \( g_h \) is the surface gravity of the horizon, which gives the Hawking temperature \( T_H = g_h/(2\pi) \). In the main text, we set \( f(x) = \beta \tanh x \) with corresponding Hawking temperature \( T_H = \beta/(4\pi) \) and

\[ \kappa_j = \frac{\beta \tanh ((j - j_h + 1/2)d)}{4d}. \]  

(S43)

What we have shown above is the correspondence between XY model and the (1+1)-D Dirac field. The case of scalar field governed by Klein-Gordon equation is similar. For a complete presentation, one can refer to the earlier theoretical work [18]. Here we briefly summarize the theoretical framework, as shown in Fig. S3.

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**EXPERIMENTAL SETUP AND DEVICE PARAMETERS**

Our superconducting quantum processor is placed in a BlueFors dilution refrigerator. The base temperature of mixing chamber (MC) is about 9 mK. All qubits share one readout line equipped with a Josephson parametric amplifier (JPA) and a high-electron-mobility transistors (HEMT). Pulse on the readout transmission line is first generated as a mixture of local oscillation (LO) and the envelopes from an arbitrary waveform generator (AWG) and then demodulated by an analog digital converter (ADC). In this experiment, we replace the DC bias with a long Z square pulse generated by AWGs. Both X and Z control signals are programmed in advance before being uploaded to AWGs. A schematic diagram of experiment setup is given in Fig. S4.

The device parameters are briefly shown in Table S1. All the parameters are characterized by various relatively efficient and automatic methods, especially the parameters concerning couplers. Details of those methods in our experiment will be introduced in the following.

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**EFFICIENT AND AUTOMATIC CALIBRATION FOR MULTI-QUBIT DEVICES WITH TUNABLE COUPLERS**

Before carrying out our experiment for simulating an analogue black hole, we need calibrate all 10 qubits and find the useful parameters of 9 couplers. This is far more difficult and time-consuming than calibrating a typical 10-qubit sample without tunable couplers. In order to measure and characterize device parameters more efficiently, we adopt an automatic calibration technology based on a combination of physical models and optimization methods.

**Spectrum of qubit and frequency calibration**

First and foremost, all the qubits are individually brought up through the standard single-qubit calibration (from identifying the readout resonator frequency to calibrating \( \pi \) pulse). If a qubit is brought up at a certain fre-
corresponding readout frequency \( \omega_{0j} \), its frequency can be given by \( \omega_{0j} \propto \delta \). The magnetic field produced by Z pulse is approximately proportional to Zp \( \text{a} \). Thus, the mapping between qubit Zpa and \( I_{0j} \rangle \) is the unit flux, \( E_{13} \) is the charging energy of the Josephson junctions connected by a loop which is in series with a capacitor. The critical currents of two junctions are \( I_{c1} \) and \( I_{c2} \) and \( E_{C} \) denotes the charging energy of capacitor. By using the perturbation theory, the transition frequency can be approximately written as [42–44]

\[
\omega(\Phi) \approx \sqrt{8E_{13}E_{C}\sqrt{\delta^2 + \cos^2\left(\frac{\pi\Phi}{\Phi_0}\right)} - E_{C}}, \quad (S44)
\]

where \( \Phi_0 = h/(2e) \) is the unit flux, \( E_{13} = g_{13}/(2\sqrt{I_{c1}I_{c2}}) \) represents the junction asymmetry. Here, the total magnetic flux \( \Phi \) is in direct proportion with the strength of the magnetic field threading the loop and this weak magnetic field induced by Z pulse is approximately proportional to Zpa (\( \Phi \propto \text{Zpa} \)). Thus, the mapping between qubit Zpa and its frequency can be given by

\[
\omega(\text{Zpa}) \approx \sqrt{8E_{13}E_C\sqrt{\delta^2 + \cos^2(A \cdot \text{Zpa} + \phi)} - E_{C}}, \quad (S45)
\]

and

\[
\text{Zpa}(\omega) \approx \arccos\left[ \pm \sqrt{\frac{(\omega + E_C)^2}{(8E_{13}E_C)^2} - \delta^2} - \phi \right] \frac{A}{\Phi_0}, \quad (S46)
\]

where \( E_C \) can be measured by two-photon excitation experiment (double difference between two-photon excitation frequency and qubit frequency), the remaining parameters \( E_{13}, \delta, A \) and \( \phi \) will be obtained by fitting the two-dimensional spectrum of qubits. Here parameter \( A \) describes the efficiency of qubit bias which depends on the attenuation on the Z control line, while \( \phi \) is the initial flux shift. Even if the refrigerator temperature rises and cools again, only \( \phi \) may have some displacement. As long as the circuit wiring does not change, parameter \( A \) will keep its value. Notice, however, that Eq. (S45) and Eq. (S46) need to be modified by the crosstalk of Z control lines if multi-qubit case is involved.

When we design the multi-qubit levels, Eq. (S46) will be beneficial to obtain the corresponding Zpa according to the target frequency. A more accurate frequency calibration can be implemented by Ramsey fringe experiment.

Calibration of pulse distortion and Z crosstalk

Although the Z pulse generated by AWG is designed carefully, the shape of the pulse is distorted when it interacts with the qubit. To calibrate the distortion of step

| qubit | \( Q_1 \) | \( Q_2 \) | \( Q_3 \) | \( Q_4 \) | \( Q_5 \) | \( Q_6 \) | \( Q_7 \) | \( Q_8 \) | \( Q_9 \) | \( Q_{10} \) |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| \( \tilde{\omega}_{0j} \)/2\( \pi \) (GHz) | 5.300 | 4.760 | 5.330 | 4.805 | 5.278 | 4.830 | 5.231 | 4.705 | 5.180 | 4.655 |
| \( \tilde{\omega}_{0j} \)/2\( \pi \) (GHz) | 6.667 | 6.687 | 6.708 | 6.733 | 6.749 | 6.772 | 6.792 | 6.815 | 6.840 | 6.857 |
| \( E_{Cj} \)/2\( \pi \) (MHz) | 195.8 | 194.5 | 195.4 | 198.4 | 197.0 | 196.2 | 195.7 | 201.8 | 199.7 | 203.2 |
| \( E_{13j} \)/2\( \pi \) (GHz) | 20.69 | 19.88 | 19.78 | 18.55 | 19.30 | 19.52 | 19.39 | 17.66 | 18.64 | 18.00 |
| \( F_{0j} \) | 0.965 | 0.958 | 0.974 | 0.938 | 0.956 | 0.940 | 0.982 | 0.952 | 0.949 | 0.943 |
| \( F_{1j} \) | 0.885 | 0.901 | 0.907 | 0.887 | 0.885 | 0.910 | 0.871 | 0.875 | 0.904 | 0.906 |
| \( T_{1j} \) (\( \mu s \)) | 24.9 | 20.1 | 18.6 | 27.8 | 27.9 | 25.4 | 24.1 | 23.5 | 37.4 | 27.7 |
| \( T_{2j} \) (\( \mu s \)) | 4.2 | 1.5 | 5.4 | 2.0 | 5.9 | 1.9 | 5.2 | 2.7 | 5.9 | 2.0 |
| \( T_{\text{echo}} \) (\( \mu s \)) | 7.2 | 3.4 | 17.9 | 4.4 | 10.7 | 4.6 | 11.3 | 5.3 | 9.0 | 4.1 |

TABLE S1. List of device parameters. Here, \( \tilde{\omega}_{0j} \) is \( |0\rangle \rightarrow |1\rangle \) transition frequency of the \( j \)-th qubit (coupler) with the corresponding readout frequency \( \tilde{\omega}_{0j} \). \( E_{Cj} \) and \( E_{13j} \) denote the charging energy and the Josephson energy. \( F_{0j} \) and \( F_{1j} \) are measure fidelities of \( |0\rangle \) and \( |1\rangle \), respectively. \( T_{1j} \) represents the energy relaxation time of \( q_j \) at the idle point. The dephasing time \( T_{2j} \) is characterized by the Ramsey fringe experiment, while \( T_{\text{echo}} \) is measured by spin echo sequence with an inserted \( \pi \) pulse. The coupling strengths of exchange-type interactions between qubits and the corresponding coupler are \( g_{q_j, c_j} \) and \( g_{q_{j+1}, c_j} \), and the direct coupling of qubits is \( g_{q_j, q_{j+1}} \).
FIG. S4. A schematic diagram of the experimental system and partial wiring information.

For the crosstalk of Z control lines between qubits and qubits or qubits and their non-nearest neighbor couplers, a routine Z crosstalk measurement with a small scanning range is adopted, which can be used to estimate the crosstalk coefficients by measuring the frequency response to the Z control lines. However, it may be better to extend to a wider range of scanning when it comes to the Z crosstalk of couplers to qubits. If the frequency of coupler approaches the frequency of qubit, the effect of anti-crossing will be amplified due to the strong coupling between coupler and its nearest neighbor qubit, leading to a distinctly non-linear relationship between coupler Zpa and qubit Zpa (as shown in Fig. S7b). To correct the crosstalk from classical flux crosstalk of Z control lines that basically meet the linear relationship, we first select a range of data away from the resonance points to use linear fitting, and constantly fine-tune the corresponding crosstalk coefficient until a symmetrical anti-crossing pattern is obtained. For a more accurate Z crosstalk calibration, we still take advantages of Ramsey fringe experiment, but proximity to the resonance points should be avoided. Here we emphasize that in our procedure of calibration, Z crosstalk of couplers to qubits must be corrected in order to more accurately measure spectrum of coupler and coupling strengths, as explained in the following.

Spectrum of coupler and anti-crossing of energy levels

As what mentioned above, it is difficult to directly excite and measure a coupler because it has no XY control line and readout resonator. Therefore, we make use of two qubits ($Q_j$ and $Q_{j+1}$) that are adjacent to the cou-

FIG. S5. Experimental data of qubit automatical spectroscopy measurement. Here we take the spectrum of $Q_1$ as an example. The black area is unscanned in order to save time. We first scan a small square area (about four columns data) around Zpa = 0 and then use polynomial curves to fit the peaks of these data. The corresponding polynomial fitting coefficients will help predict the next peak of qubit spectrum. By constantly measuring, fitting and predicting, we obtain the experimental data of qubit spectrum with a wide range. The mapping between qubit Zpa and its frequency can be obtained by fitting the experimental data based on Eq. (S45).
The spectrum of the local $Q_j$ in Fig. S8 which are actually the first three eigen-spectra (red lines in the inset of Fig. S8) due to its variation of frequency and its population of excited state will be decreased [20, 21] (see pulse sequence calibrated pulse (up) is distorted, while the corrected result (down) shows a stationary step response.

To observe a tilted anti-crossing pattern of qubit and its nearest neighbor coupler. By constantly fine-tuning the crosstalk coupling strengths only by scanning the three-body spectrum due to the broadening of spectral lines and some impure peaks [46, 47]. For more accurate measurement, we scan two extra anti-crossing spectrums of two-body systems $Q_jC_j$ and $Q_{j+1}C_j$, as shown in Fig. S8b. Truncated to two energy levels, the Hamiltonian of a qubit $Q_j$ coupled to a coupler $C_j$ can be expressed in the subspace basis $\{|01\rangle, |01\rangle\}$ as

$$\hat{H}_{Q_jC_j} = \begin{pmatrix} \omega_{q_j} & g_{q_j,c_j} \\ g_{q_j,c_j}^* & \omega_{c_j} \end{pmatrix},$$

(S48)

and its eigen-energy spectra are

$$\omega_{q_j,c_j}^\pm = \frac{\omega_{q_j} + \omega_{c_j}}{2} \pm \sqrt{\frac{g_{q_j,c_j}^2}{4} + \frac{(\omega_{q_j} - \omega_{c_j})^2}{4}}.$$ (S49)

FIG. S7. Experimental data of automatical Z crosstalk calibration. a. Pulse sequence for measurement of Z crosstalk of coupler to qubit. b. Before Z crosstalk is corrected, one can observe a tilted anti-crossing pattern of qubit and its nearest neighbor coupler. By constantly fine-tuning the crosstalk coefficient and applying the Zpa to compensate crosstalk from coupler $Z$ line, a symmetrical anti-crossing pattern will be obtained after corrected. The black area is unscanned, while the red lines are the results of linear fitting. Here we show the experimental data of Z crosstalk calibration c. All the coefficients of Z crosstalk. Compared with the high crosstalk from couplers $Z$ line to qubits, the absolute coefficients of Z crosstalk between qubits are all at a low level ($< 2\%$).
Similarly, the eigen-energy spectra of $Q_{j+1}C_j$ are

$$\omega_{Q_{j+1}C_j}^\pm = \frac{\omega_{Q_{j+1}} + \omega_{C_j}}{2} \pm \sqrt{\frac{g_{Q_{j+1},C_j}^2}{4} + \left(\frac{\omega_{Q_{j+1}} - \omega_{C_j}}{2}\right)^2}.$$  

Combining the above two equations with the diagonalization result of Eq. (S47), one can finally determine the coupling strengths between coupler and qubits (i.e., $g_{Q_j,C_j}$ and $g_{Q_{j+1},C_j}$) and the mapping between coupler frequency $\omega_{C_j}$ and its Zpa. Actually, this is a multi-objective optimization problem of simultaneously fitting 3 spectroscopy results via 5 parameters. We utilize the optimization function scipy.optimize.minimize in the Python module SciPy to solve this problem.

Measurement of the effective coupling

To measure the effective coupling strength $\tilde{g}_{Q_j,Q_{j+1}}$, we measure the joint probability as a function of qubit-qubit swapping time $t$ and the Zpa of coupler [46, 47], as shown in Fig. S8b. Similar to Eq. (S48), the swapping Hamiltonian of $Q_jQ_{j+1}$ in the subspace effective basis $\{\ket{01}, \ket{01}\}$ is

$$\hat{H}_{Q_jQ_{j+1}} = \left(\tilde{\omega}_{Q_j} \tilde{g}_{Q_j,Q_{j+1}} \tilde{\omega}_{Q_{j+1}}\right).$$  \hspace{1cm} (S51)

If $Q_jQ_{j+1}$ is initially in state $\ket{01}$, the time-dependent joint probability $P_{01}(t) = \bra{01} e^{-i\hat{H}_{Q_jQ_{j+1}}t} \ket{01}$ can be expressed as

$$P_{01}(t) = \frac{1}{2} \cos \left[\sqrt{4g_{Q_j,Q_{j+1}}^2 + (\tilde{\omega}_{Q_j} - \tilde{\omega}_{Q_{j+1}})^2} t\right] + \frac{1}{2},$$  \hspace{1cm} (S52)

FIG. S8. Experimental data of coupler automatical spectroscopy measurement. The red curves are numerical simulation results for fitting the peaks of spectroscopy data, which is based on a multi-objective optimization. a, The spectrum of the local $Q_jC_j$ of three-body system. The black area is unscanned, while the experimental data consists of the blue area. When the frequency of coupler is far from the qubits frequency, we only need to scan a very narrow width like single-qubit spectroscopy measurement. As it approaches the anti-crossing points, we increase the scan width to reduce the impact caused by predictive error, which ensures a clear three-body spectrum and saves time simultaneously. b, Experimental data of anti-crossing spectrums of $Q_jC_j$ (left) and $Q_{j+1}C_j$ (right). Here the results of $C_2$ are taken as an example.

FIG. S9. Experimental data of the effective coupling strength measurement. a, Pulse sequence for measurement of swapping between qubits while changing the Zpa of coupler. b, Measured joint probability $P_{01}$ of qubits vs Zpa of coupler (or corresponding frequency) and the swapping time. c, The Fourier transform of b, where the heatmap represents the normalized Fourier amplitude. The relation between absolute effective coupling strength $|\tilde{g}_{Q_j,Q_{j+1}}|$ and coupler Zpa (or corresponding frequency) is given by each peak of normalized Fourier amplitude. The red dash line is the fitting curve of $|\tilde{g}_{Q_j,Q_{j+1}}|$ by using Eq. (S54), while white dot lines denote two decoupling points $|\tilde{g}_{Q_j,Q_{j+1}}| = 0$. As coupler frequency decreases, $|\tilde{g}_{Q_j,Q_{j+1}}|$ decreases from positive to zero. Once it passes the decoupling point, $\tilde{g}_{Q_j,Q_{j+1}}$ becomes negative and its absolute value will increase rapidly, in especial approach to resonance point of qubits.
which is reduced to
\[ P_{01}(t) = \frac{1}{2} \cos(2\tilde{\omega}_{q_j}t) + \frac{1}{2} \] (S53)
when the two qubits are resonant, namely \( \tilde{\omega}_{q_j} = \tilde{\omega}_{q_{j+1}} \).
Thus, the effective coupling strength can be calculated as half the Fourier frequency of probability \( P_{01}(t) \). It needs to be emphasized that decoherence may cause the damping amplitude of swapping probability but does not affect the Fourier frequency.

For each \( Z_{pa} \) of coupler (related to its frequency), we calculate \( \tilde{\omega}_{q_j},q_{j+1} \) via measuring \( P_{01}(t) \) and performing Fourier transform (as shown in Fig. S9c). Subsequently, one can utilize Eq. (S30) to draw the mapping between the effective coupling strength and coupler \( Z_{pa} \):
\[ g_{q_j,q_{j+1}}(Z_{pa}) = g_{q_j,q_{j+1}} + \frac{g_{q_j,c_j}g_{q_{j+1},c_j}}{\omega_{c_j}(Z_{pa}) - \omega}, \] (S54)
where \( \omega = \omega_{q_j} = \omega_{q_{j+1}} \) is the resonant frequency of qubits, the direct coupling \( g_{q_j,q_{j+1}} \) is the fitted value and the coupler frequency \( \omega_{c_j}(Z_{pa}) \) obeys Eq. (S45). Hence, if the \( Z_{pa} \) of coupler is given, the effective coupling strength can be computed via Eq. (S54); or given a target coupling, one can estimate the \( Z_{pa} \) of coupler by
\[
Z_{pa}(\tilde{g}_{q_j,q_{j+1}}) \approx \frac{1}{A} \arccos \left[ \frac{\pm \frac{1}{8E_{JJ}E_C} \sqrt{\left( \omega + E_C + \frac{g_{q_j,c_j}g_{q_{j+1},c_j}}{\tilde{g}_{q_j,q_{j+1}} - g_{q_j,q_{j+1}}} \right)^2 - \left( \frac{8E_{JJ}E_C\delta}{(8E_{JJ}E_C)^2} \right)^2} - \phi}{A} \right]. \] (S55)

where \( E_{JJ} \) and \( E_C \) are the Josephson energy and the charging energy of coupler, respectively. Eq. (S55) is a crucial foundation for engineering arbitrary coupling distribution in a superconducting circuit with tunable couplers.

**FIG. S10.** The effects of two typical disorders on Hawking radiation. a, The logarithm of average radiation probability vs. the positive energy of particle with different disorder strengths of \( g_{NNN} \). b, The logarithm of average radiation probability vs. the positive energy of particle with different disorder strengths of \( \mu \). Here, the red solid line represents the theoretical result.

**FIG. S11.** Quantum walks of different coupling distribution. a, The couplings keep the same amplitude inside and outside of the black hole, where the black solid line represents all couplings keeping positive and the red dash line denotes all couplings remaining negative. These two cases are equivalent and the same results are shown in c. b, Non-zero coupling inside the black holes (black solid line and red dash line imply positive coupling and negative coupling respectively) and zero coupling between all sites outside the black hole, where the corresponding results of quantum walks are shown in d. Here we take the initial state \( |1000000000\rangle \) as an example.

**ADDITIONAL DISCUSSION**

For further discussion, we perform additional numerical simulations to compare and supplement with our results in this paper. In the following, the effects of disorders, different coupling distribution and finite size are investigated.

**The effects of disorders**

In reality, qubits are doomed to be disturbed by various disorders, leading to the nuance between experimental conditions and theoretical assumptions. For a 1D-array of qubits, one can consider two disorders about next-nearest-neighbor (NNN) coupling \( g_{NNN} \) and on-site...
FIG. S12. Simulation of a 300-qubit chain with horizons at different locations. Here the coupling \( \kappa_j \) takes the form of Eq. (S43), where \( d = 0.35 \) and \( \beta/(2\pi) = 4.39 \) MHz. From a to c, the corresponding horizons are located at \( Q_{25} \), \( Q_{50} \) and \( Q_{150} \), respectively. \( P_{\text{out}} \) is defined as the sum of probabilities of all the qubits outside the horizon.

FIG. S13. The finite-size effects on the horizon. The particle is initialized at the horizon. a, Simulation of a 20-qubit chain with the horizon located at \( Q_{10} \). b, Simulation of a 300-qubit chain with the horizon located at \( Q_{150} \). Here the coupling \( \kappa_j \) takes the form of Eq. (S43), where \( d = 0.35 \) and \( \beta/(2\pi) = 4.39 \) MHz. c, Simulation of a 300-qubit chain in flat spacetime with \( \kappa_j = 3.14 \) MHz. \( P_{\text{out}} \) is defined as the sum of probabilities of all the qubits outside the horizon.

potential \( \mu \) with the corresponding disorder strengths \( W_{g_{\text{NNN}}} \) and \( W_{\mu} \). Specifically, the Hamiltonian of disorders is

\[
\hat{H}_{\text{dis}} = \sum_{j=1}^{8} g_{\text{NNN}}^j (\hat{\sigma}_j^{+} \hat{\sigma}_{j+2}^{+} + \hat{\sigma}_j^- \hat{\sigma}_{j+2}^-) - \sum_{j=1}^{10} \mu_j \hat{\sigma}_j^+ \hat{\sigma}_j^-,
\]

where the site-dependent \( g_{\text{NNN}}^j \) and \( \mu_j \) are in \([-W_{g_{\text{NNN}}}, W_{g_{\text{NNN}}}]\) and \([-W_{\mu}, W_{\mu}]\), and their distribution are assumed to be uniform. To probe the effects of disorders, we numerically model the dynamics of Hawking radiation by considering \( \hat{H}_{\text{dis}} \). In Fig. S10a and b, one can note that both disorders diverge the probability spectrum and theoretical results of Hawking radiation, especially in the condition of strong disorder. However, we measure the NNN coupling of \( g_{\text{NNN}}^j \approx 0.1 \) MHz and the frequencies difference of \( |\mu_j - \omega_{\text{ref}}| < 0.2 \) MHz with reference frequency \( \omega_{\text{ref}}/(2\pi) \approx 5.1 \) GHz. According to Fig. S10, such small degree of disorders have little impact on the results of Hawking radiation in experiment. In fact, we measure the initial density matrix of 7 qubits outside the horizon. The fidelity between the imperfect initial state in experiment and ideal initial state is 99.2\% (see Fig. 3a in the main text), which may be caused by the XY crosstalk, thermal excitation, leakage, etc. We substitute such an experimental state for the ideal initial state in the numerical simulation of Hawking radiation, then the results of numerical simulation agree with experimental results better (see Fig. 3d in the main text).

Different coupling distribution

Admittedly, our model does not mandate flipping the sign of coupling \( \kappa_j \) near the horizon. In the main text, we request that the coupling goes monotonically from negative to positive (or vice versa) from the left of horizon to
its right side. This is based on the realistic consideration of the smoothness of \( f(x) \). In fact, if instead of flipping the sign of couplings they were all kept positive (or negative) inside and outside of the black hole (Fig. S11a), all of the results would be similar, as shown in Fig. S11c. For the case with non-zero coupling inside the black hole but zero coupling between all sites outside the black hole (Fig. S11b), one can find the results inside black hole are also similar with the results in case of flipping the sign of couplings, but it is quite different for the results outside black hole. Due to the zero coupling between all sites outside, no particles can travel in the exterior (Fig. S11d) and thus no radiation can be detected by the observer outside.

The finite-size effects

Here we perform the numerical simulation of a 300-qubit chain to show the finite-size effects more clearly when we initialize the system by preparing a particle in the black hole. When the particle arrives at the horizon, it is going to be reflected back into the black hole in all probability but has a little chance to appear in the outside. The horizon is similar to a ‘membrane’ with certain transmittance, see Fig. S12. The probability of finding the particle outside \( P_{\text{out}} \) shows a general upward trend due to the Hawking radiation. However, the particle will be reflected when it arrives at the boundary (\( Q_1 \) or \( Q_{400} \)) due to the finite-size effects. When the particle reflected by the boundary of the black hole reaches the horizon again, it has a certain probability to escape into the outside and \( P_{\text{out}} \) thus increases again (see Fig. S12a and Fig. S12b). Conceivably, if there are no boundaries, \( P_{\text{out}} \) will be increase to a certain value and eventually the particle reaches a steady state of radiation.

In addition, the finite size can also affect the horizon. In the continuous curved spacetime, the particle initialized at the horizon is bound to the horizon forever due to the zero couplings on both sides of the horizon. However, in the finite-size lattice, the coupling strengths on both sides of the horizon are not strictly zero even though they are very small. As shown in Fig. S13a and Fig. S13b (also Fig. 2b in the main text), although the particle seems to be localized at the horizon for a very short time, it is doomed to escape from the constraints. When the particle is far from the horizon, its behavior is similar to that in flat spacetime (see Fig. S13b and Fig. S13c).

**EXTENDED DATA**

In addition, we design an analogue black hole at the center of our 1D array superconducting qubits, see Fig. S14a. Here the coupling \( \kappa_j \) is designed as \( \beta (\tanh((j-7/2)d)-\tanh((j-13/2)d)+1)/(4d) \) with \( d=1 \) a.u. and \( \beta/(2\pi) \approx 13.2 \) MHz. The qubits \( Q_5 \) and \( Q_6 \) are in the interior of the black hole, \( Q_4 \) and \( Q_7 \) are at the horizon, and other qubits are in the outer space. In Fig. S14b, we prepare four initial states to show quantum walks in such a curved spacetime, i.e., |0000100000>, |0000110000>, |0001010000> and |1000000000>, respectively. The results of quantum walks of black hole at the center in the chain are similar to the results of black hole at the leftmost boundary in the main text. However when the black hole is at the center, finite size effect in the experimental 10-qubit 1D array becomes obvious, for which the tunneling out particle soon reaches the boundary and is reflected back.

FIG. S14. Quantum walks in a 1D array of 10 superconducting qubits with black hole at the center. a, Schematic representation of the black hole at the center and the corresponding coupling distribution. b, Quantum walks in such a curved spacetime. The heatmap denotes the probabilities of excited-state for \( Q_i \) in time. The horizontal axis is indexed as qubit number \( i \), the vertical axis is time. Here we show both the numerical simulation and experiment data. c, Fidelity of the experimental data compared to ideal numerical simulation of quantum walks.

![Diagram](image-url)