Computation of mass outflow rate from relativistic quasi-spherical accretion on to black holes

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ABSTRACT

We compute the mass outflow rate $R_m$ from relativistic matter that is accreting quasi-spherically on to the Schwarzschild black holes. Taking the pair-plasma pressure-mediated shock surface as the effective boundary layer (of the black hole) from where the bulk of the outflow is assumed to be generated, computation of this rate is done using combinations of exact transonic inflow and outflow solutions. We find that $R_m$ depends on the initial parameters of the flow, the polytropic index of matter, the degree of compression of matter near the shock surface and the location of the shock surface itself. We thus not only study the variation of the mass outflow rate as a function of various physical parameters governing the problem, but also provide a sufficiently plausible estimation of this rate.

Key words: accretion, accretion discs – black hole physics – hydrodynamics – shock waves – galaxies: nuclei – quasars: general.

1 INTRODUCTION

Computation of mass outflow rates from the advective accretion discs around black holes and neutron stars has been done very recently (Chakrabarti 1998; Das 1998a,b; Das & Chakrabarti 1998, hereafter DC98) by self-consistently combining the exact transonic accretion and wind solutions. Rigorously justifying the fact that most of the outflowing matter comes out of the centrifugal pressure-supported boundary layer (CENBOL), it has been shown that the Rankine–Hugoniot shock location or the location of the maximum polytropic pressure acts as the CENBOL. However, for some black hole models of active galactic nuclei (AGNs), the inflow may not have an accretion disc (Rees 1977 and references therein). Accretion is then quasi-spherical, having almost zero or negligible angular momentum [Bondi (1952)-type accretion] and the shock is not of the Rankine–Hugoniot type. On the other hand, absence of angular momentum rules out the possibility of formation of the polytropic pressure maxima. So, for quasi-spherical accretion, absence of intrinsic angular momentum of the accreting material does not allow CENBOL formation. It has been shown that (Meszaros & Ostriker 1983; Kazanas & Ellison 1986, hereafter KE86) for quasi-spherical accretion on to black holes, a steady-state situation may develop where a standing collisionless shock may form as a result of the plasma instabilities and for non-linearity in the flow introduced by small density perturbation. This is because, after crossing the sonic point, the infalling matter (in plasma form) becomes highly supersonic. Any small perturbation and slowing down of the infall velocity will create a piston and produce a shock. A spherically symmetric shock produced in such a way will accelerate a fraction of the inflowing plasma to relativistic energies. The shock-accelerated relativistic particles suffer essentially no Compton loss and are assumed to lose energy only through $p$–$p$ collision. These relativistic hadrons are not readily captured by the black hole (Protheroe & Kazanas 1983), instead a considerable high-energy density of these relativistic protons would be maintained to support a standing, collisionless, spherical shock around the black hole (see KE86 and references therein). Thus, a self-supported standing shock may be produced even for accretion with zero angular momentum. In this work, we take this pair-plasma pressure-mediated shock surface as the alternative of the CENBOL, which can be treated as the effective physical hard surface that, in principle, mimics the ordinary stellar surface regarding the mass outflow. The condition necessary for the development and maintenance of such a self-supported spherical shock is satisfied for the high Mach number solutions (Ellison & Eichler 1984). Keeping this in the back of our mind, for our present work, we concentrate only on low energy accretion to obtain high shock Mach numbers. Considering low-energy ($E \leq 0.001$) accretion, we assume that particles accreting towards a black hole are shock-accelerated via first-order Fermi acceleration (Axford, Leer & Skadron 1977) producing relativistic protons. Those relativistic protons usually being scattered several times before being captured by the black hole. These energized particles, in turn, provide sufficient outward pressure to support a standing, collisionless shock. A fraction of the energy flux of infalling matter is assumed to be converted into radiation at the shock stand-off distance through hadronic ($p$–$p$) collision and mesonic ($\pi^-,\pi^0$) decay. Luminosity produced by this fraction is used to obtain the shock location for the present work. Our

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approach for formulating the expression for the shock location is somewhat similar to that of KE86.

At the shock surface, the density of the post-shock material shoots up and the velocity decreases, so the infalling matter starts piling up on the shock surface. The post-shock relativistic hadronic pressure then gives a kick to the piled-up matter, which results in the ejection of outflow from the shock surface. For this type of inflow, accretion is known to proceed smoothly after a shock transition, because successful subsonic solutions have been constructed (Flammang 1982) for accretion on to black holes embedded within normal stars with the boundary condition $u = c$, where $u$ is the infall velocity of matter and $c$ is the velocity of light in vacuum. The fraction of energy converted, the shock compression ratio $R_{\text{comp}}$ (in the notation of DC98), along with the ratio of post-shock relativistic hadronic pressure to infalling ram pressure at a given shock location are obtained from the steady-state shock solution of Ellison & Eichler (1984, 1985). The shock location as a function of the specific energy $E$ of the infalling matter and accretion rate is then self-consistently obtained using the above-mentioned quantities. We then calculate the amount of mass outflow rate $R_{\text{sh}}$ from the shock surface using a combination of exact transonic inflow and outflow solutions, and study the dependence of $M_{\text{in}}$ on various physical entities governing the inflow–outflow system. We thus quantitatively compute the mass outflow rate only from the inflow parameters. In this way we analytically connect the accretion and wind-type topologies self-consistently. As there is no such attempt available in the literature that computes the mass-loss rate from zero angular momentum accretion, our work, for the first time we believe, could shed light on the nature of the outflow from the quasi-spherical Bondi-type accretion, our work, for the relevant physical quantities can be expressed likewise. We ignore the self-gravity of the flow and the calculation is done using the Paczyński–Wiita (Paczyński & Wiita 1980) potential, which mimics the surrounding of the Schwarzschild black hole. The equations (in dimensionless geometric units) governing the inflow are as follows.

(i) Conservation of specific energy is given by:

$$E = \frac{u(r)^2}{2} + n a(r)^2 - \frac{1}{2(r-1)}.$$  \hspace{1cm} (1)

(ii) Mass conservation equation is given by,

$$M_{\text{in}} = \Theta_{\text{in}}(r)u(r)r^2.$$  \hspace{1cm} (2)

As already mentioned, we assume that a steady, collisionless shock forms at a distance $r_{sh}$ (measured in units of the Schwarzschild radius) owing to the instabilities in the plasma flow. We also assume that for our model, the effective thickness of the shock $\Delta_{\text{sh}}$ is small enough compared with the shock stand-off distance, i.e.

$$\Delta_{\text{sh}} \ll r_{sh}.$$  

Accreting particles with infall velocity $u(r)$ are then assumed to be shock-accelerated via first-order Fermi acceleration. Because of this process, relativistic protons will be produced. These relativistic protons suffer essentially no Compton loss and hence are not readily swallowed by the black hole. Instead, they usually scatter several times before being captured by the black hole, thus providing sufficient outward pressure necessary to support the shock. These protons, in turn, produce pions through post-shock inelastic nuclear collisions

$$p + p \rightarrow p + p + \pi^+ + \pi^-.$$  

Pions generated by this process decay into relativistic electrons, neutrinos and antineutrinos, and produce high-energy $\gamma$-rays.

$$\pi^0 \rightarrow 2\gamma$$  

$$\pi^- \rightarrow e^- + \nu_e$$  

$$\pi^+ \rightarrow e^+ + \nu_e.$$  

These electrons produce the observed non-thermal radiation by synchrotron and inverse Compton scattering. The overall efficiency of this mechanism depends largely on the shock location. It has been shown (Eichler 1979) that almost half of the energy flux that goes into the relativistic particles is lost owing to neutrinos.

From equation (2), the density of infalling matter $\rho(r)$ is

$$\rho(r) = \frac{M_{\text{in}}}{\Theta_{\text{in}}(r)r^2}.$$  \hspace{1cm} (3)

When the shock occurs, the density of matter rapidly increases and the inflow velocity decreases abruptly. If $(p_-, u_-)$ and $(p_+, u_+)$ are the pre- and post-shock densities and velocities, respectively, then

$$\frac{p_+}{p_-} = R_{\text{comp}} = \frac{u_+}{u_-},$$  \hspace{1cm} (4)

where $R_{\text{comp}}$ is the shock compression ratio (in the notation of DC98). For the high shock Mach number solution (which is
compatible with our low-energy accretion model), the expression for $R_{\text{comp}}$ can be well approximated as

$$R_{\text{comp}} = 1.44 M_{\text{sh}}^{3/4},$$

(5)

where $M_{\text{sh}}$ is the shock Mach number and equation (5) holds true for $M_{\text{sh}} \approx 4.0$ (Ellison & Eichler 1985). However, the scattering mean free paths of the relativistic hadrons produced by this process are assumed to be small enough that they could encounter the full shock compression ratio while crossing the shock.

The hadronic interaction characteristic time-scale $\tau_{pp}$ may be expressed as

$$\tau_{pp} = \frac{1}{\rho_p \sigma_{ppc}},$$

(6)

where $\sigma_{pp}$ is the collision cross-section for relativistic protons. Using equation (4), $\tau_{pp}$ can also be expressed as

$$\tau_{pp} = \frac{1}{R_{\text{comp}} \rho_p \sigma_{ppc}}.$$  

(7)

If we now assume that a fraction $\epsilon_F$ of the infalling energy is converted into radiation through the hadronic collision ($p-p$) and mesonic ($\pi^\pm, \pi^0$) decay, this $\epsilon_F$ will allow convergent steady-state solutions and in the case of quasi-spherical infall, allows the development and maintenance of a standing, collisionless shock at a fixed distance $r_{\text{sh}}$ measured in units of Schwarzschild radius.

The luminosity obtained from this energy $\epsilon_F$ at the shock location $r_{\text{sh}}$ is assumed to be $L_1$. For a spherical shock surface and taking care of neutrino losses, the expression for $L_1$ can be written as

$$L_1 = 2 \pi r_{\text{sh}}^2 \rho_0 n_p m_H^3 \epsilon_F,$$

(8)

where $m_p$ is the mass of the proton and subscript ‘sh’ indicates that the respective quantities are measured at the shock location.

If we assume that the pressure of the relativistic particles $P_{\text{rel}}$ (uniform inside the shock), then the average energy density is $3P_{\text{rel}}$, so alternatively the luminosity can be expressed in terms of the volume integral of the emissivity $\epsilon$ owing to the hadronic ($p-p$) collision where

$$\epsilon = \frac{3P_{\text{rel}}}{\tau_{pp}}.$$  

Thus the alternative expression for the luminosity obtained as a function of $P_{\text{rel}}$ would be

$$L_2 = \frac{4 \pi r_{\text{sh}}^2 P_{\text{rel}}}{\tau_{pp}}.$$  

(9)

Defining $\delta$ as the ratio of downstream relativistic particle pressure to incoming ram pressure at the shock, we obtain

$$P_{\text{rel}} = \delta \rho_{\text{sh}} m_p^2.$$  

(10)

Equating equations (8) and (9) and substituting the values of $\rho$, $P_{\text{rel}}$, and $\tau_{pp}$ from equations (3), (7) and (10), respectively, we obtain an expression for the shock location as a function of various

1 As only a fraction of the accreting matter is shock energized, the value of $\rho_p$ used to calculate $\tau_{pp}$ is, in reality, less than that of the actual $\rho_p$; giving a higher value of $\tau_{pp}$. The accurate value of $\tau_{pp}$ can be calculated using the cosmic-ray energy spectrum and coupling the relativistic and non-relativistic parts of the accreting plasma, which will be presented elsewhere. Nevertheless, our rough estimation assures that even with the accurate value of $\tau_{pp}$ (which is higher than that used here), the conditions of shock formation are well satisfied.

inflow parameters:

$$r_{\text{sh}} = \frac{3 \sigma_{ppc} M_{\text{in}}}{\rho_0 \Theta_{\text{in}} (\delta \epsilon_F)^{1/2}}.$$  

(11)

The ratio $\delta / \epsilon_F$ as a function of the shock Mach number $M_{\text{sh}}$ for a high shock Mach number solution (low-energy inflow) is obtained from the empirical solution deduced by Ellison & Eichler (1984), after suitable modification required for our model.

2.2 Outflow model

In ordinary stellar mass loss computations (Tarafdar 1988 and references therein) the outflow is assumed to be isothermal until the sonic point. This assumption is probably justified, because copious photons from the stellar atmosphere deposit momenta on the slowly outgoing and expanding outflow and possibly make the flow close to isothermal. This need not be the case for outflow from black hole candidates. Our effective boundary layer, being pretty close to the black hole, are very hot and most of the photons emitted may be swallowed by the black hole itself instead of coming out of the region and depositing momentum on to the outflow. Thus, the outflow could be cooler than the isothermal flow in our case. We choose polytropic outflow with a different polytropic index $\gamma_0 \leq \gamma$ owing to momentum deposition. Nevertheless, it may be advisable to study the isothermal outflow to find out the behaviour of the extreme case. Modelling the isothermal outflow is in progress and will be communicated as the next work.

As a fraction of the infalling energy density $\epsilon_F$ is converted into radiation, specific energy of the outflow is somewhat less than that of the inflow. Nevertheless, the outflow specific energy is also kept constant throughout the flow.

The following two conservation laws are valid for the outflow:

$$\epsilon' = \frac{\epsilon(r)^2}{2} + n' a(r)^2 = \frac{1}{2(r - 1)}$$

(12)

$$\dot{M}_{\text{out}} = \Theta_{\text{out}} \rho u(r) \epsilon(r)^2.$$  

(13)

where $\epsilon'$ is the specific energy of the outflow, $\epsilon' < \epsilon$ and $n' = (\gamma_0 - 1)^{-1}$ is the polytropic constant of the outflow. $\Theta_{\text{out}}$ is the solid angle subtended by the outflow and $u(r)$ is the velocity of the outflow.

For simplicity of the calculation, we assume that the outflow is also quasi-spherical and $\Theta_{\text{out}} \approx \Theta_{\text{in}}$. Defining $R_{\text{in}}$ as the mass outflow rate, we obtain

$$R_{\text{in}} = \frac{\dot{M}_{\text{out}}}{M_{\text{in}}}.$$  

(14)

It is obvious from the above discussion that $R_{\text{in}}$ should have some complicated functional dependence on the following parameters

$$R_{\text{in}} = \Psi(\dot{\epsilon}, M_{\text{in}}, r_{\text{sh}}, R_{\text{comp}}, \gamma, \gamma_0).$$  

(15)

2.3 Procedure to solve the inflow and outflow equations simultaneously

Before we proceed in further detail, a general understanding of the transonic inflow–outflow system in the present case is essential to understand the basic scheme of the solution procedure. Let us consider the transonic accretion first. Infalling matter becomes supersonic after crossing a saddle-type sonic point, the location of which is determined by the inflow parameters such as the specific energy $\epsilon, M_{\text{in}}$ (in units of Eddington rate) and $\gamma$ of the inflow. This
supersonic flow then encounters a shock (if present) location of which \((r_{sh})\) is determined from equation (11). At the shock, part of the incoming matter, having higher entropy density, is likely to return back as wind through a sonic point rather than the one through which it just entered. Thus, a combination of transonic topologies, one for the inflow and one for the outflow (passing through a different sonic point and following a completely different topology to that of the ‘self-wind’ of the accretion), is required to obtain a full solution. So it turns out that finding a different topology to that of the ‘self-wind’ of the accretion), is through a different sonic point and following a topology, one for the inflow and one for the outflow (passing back as wind through a sonic point rather than the one return back as wind through a sonic point rather than the one

\[
\left. \frac{d u}{d r} \right|_{c} = \left[ \frac{2 a(r)^{2}}{r - 1} \right] \frac{a(r) - a(r)}{a(r)} \left( \frac{d a}{d r} \right) \hat{r}_{c} 
\]

At the sonic point, the numerator and denominator separately vanish, giving rise to the so-called sonic-point condition:

\[
u_{c} = a_{c} = \frac{r_{c}}{2 \left( 2 c - 1 \right)}
\]

where the subscript ‘c’ represents the quantities at the sonic point. The derivative at the sonic point \((d u/d r)_{c}\) is computed using the L'Hospital rule. The expression for \((d u/d r)_{c}\) is obtained by solving the following polynomial,

\[
\left( \frac{2 n + 1}{n} \right) \left( \frac{d a}{d r} \right)_{c}^{2} + 3 n c_{r} \left( \frac{d a}{d r} \right)_{c} \left( \frac{1}{r_{c} - 1} \right) - \frac{2 a_{c}^{2} r_{c}}{n + 1} = 0.
\]

Using the fourth-order Runge–Kutta method, \(a(r)\) and \(a(r)\) are computed along the inflow from the inflow sonic point till the position where the shock forms. The shock location is calculated by simultaneously solving equations (1), (2) and (11). With the known values of \(E'\) and \(\gamma_{m}\), it is easy to compute the location of the sonic point of the outflow from equations (12) and (13). At the outflow sonic point, the outflow velocity \(v_{o}\) and polytropic sound velocity \(a_{c}\) are computed in the same way as for the inflow. Using equations (12) and (13), \((d e/d r)\) and \((d e/d r)\) are computed as was done for the inflow. The Runge–Kutta method is then employed to integrate from the outflow sonic point towards the black hole to find out the outflow velocity \(v_{o}\) and density \(\rho_{o}\) at the shock location. The outflow rate is then computed using equation (14).

3 RESULTS

Fig. 1 shows a typical solution which combines the accretion and the outflow. The input parameters are \(E = 0.001, M_{in} = 1.0\) Eddington rate \(E_{d}\) stands for the Eddington rate in the figure) and \(\gamma = 4/3\) corresponding to relativistic inflow. The solid curve with an arrow represents the pre-shock region of the inflow and the solid vertical line with the double arrow at \(X_{pp}\) (the subscript ‘pp’ stands for pair-plasma-mediated shock) represents the shock transition. The location of the shock is obtained using equation (4) for a particular set of inflow parameters mentioned above. Three dotted curves show the three different outflow branches corresponding to different polytropic indices of the outflow as \(\gamma_{o} = 1.3\) (left-hand curve), 1.275 (middle curve) and 1.25 (right-hand curve). It is evident from the figure that the outflow moves along the solution curves in a completely different way to that of the ‘wind solution’ (solid line marked with an outward directed arrow) of the inflow which passes through the sonic point \(P_{s}\). The mass-loss ratio \(R_{m}\) for these cases is 0.0023, 0.00065 and 0.00014, respectively. In Fig. 2, we have plotted the variation of \(R_{m}\) with incoming specific energy \(E\) for a set of values of \(M_{in}\) (measured in units of Eddington rate shown as \(E_{d}\) in the figure) shown in the figure. It is observed that \(R_{m}\) monotonically increases with energy. This is because as \(E\) increases, keeping the Eddington rate of the inflow fixed, the shock Mach number \(M_{sh}\) decreases, resulting in the decrement of the shock location \(r_{sh}\) and post-shock density (via equation 5) but the increment of the post-shock fluid velocity \(v_{sh}\) with which the matter leaves the shock surface. The outflow rate \(R_{m}\), which is the product of these three quantities, in general increases monotonically with \(E\) owing to the combined tug of war of these three quantities. Moreover, the closer the shock forms to the black hole, the greater the amount of gravitational potential will be. This is available to be transferred to the relativistic hadrons to provide more outward pressure at the shock boundary, which gives a stronger ‘kick’ to the accreting matter, the result of which is the increment in \(R_{m}\). All these points are manifested in Fig. 3 where we have shown the variation of \(R_{m}\) as a function of the compression ratio \(R_{comp}\) (solid curve), the shock location \(r_{sh}\) (dotted curve) and the injection velocity of the outflow \(v_{sh}\) (dashed curve). The figure is drawn for a fixed \(\gamma = 4/3\) and \(\gamma_{o} = 1.3\). \(R_{comp}\) and \(v_{sh}\) are scaled as \(R_{comp} \to (R_{comp} - 5.890) \times 10^{3}\) and \(v_{sh} \to 4 \times 10^{-6}\). The unequal gaps between the curves with different \(E_{d}\) in Fig. 2 imply that when the inflow energy \(E\) is kept constant, \(R_{m}\) non-linearly increases with the Eddington rate. This is because, as \(E\) is kept constant while \(M_{in}\) is varied, the amount of infalling energy converted to produce high-energy protons is also fixed. So the higher the value of \(M_{in}\) (in units of Eddington rate),
increases, resulting in the increment in $R_{in}$. However, $R_{in}$ anticorrelates with $\gamma$, which is shown in Fig. 4(a).

4 CONCLUDING REMARKS

In the present paper, we have computed the mass outflow rate from the relativistic low-energy matter quasi-spherically accreting on to the Schwarzschild-type black holes. The free parameters chosen for the inflow are the specific energy $E$, mass inflow rate $\dot{M}_{in}$ (in units of Eddington rate) and the polytropic index $\gamma$ of the inflow. Only one extra parameter was supplied for the outflow, which is its polytropic index $\gamma_o$. We have computed the mass outflow rate $R_{out}$ using the inflow parameters only (except $\gamma_o$), thus we could analytically connect the accretion and wind-type topologies self-consistently. In our computation, we could investigate the dependence of $R_{in}$ as a function of different physical quantities governing the inflow.

The main conclusions of this paper are as follows:

(i) It is possible that outflows for quasi-spherical Bondi-type accretion on to a Schwarzschild black hole come from the pair-plasma pressure-mediated shock surface.

(ii) The outflow rate monotonically increases with the specific energy of the inflow and non-linearly increases with the Eddington rate of the infalling matter.

(iii) $R_{in}$, in general, correlates with $\gamma_o$, but anticorrelates with $\gamma$.

(iv) Generally speaking, as our model deals with high shock Mach number (low-energy accretion) solutions, outflows in our work are always generated from the supersonic branch of the infow, i.e. the shock is always located inside the sonic point.

(v) Unlike the mass outflow from the accretion disc around black holes (Das 1998a,b; DC98) here we found that the value of $R_{in}$ is distinguishably small. This is because matter is ejected out, owing to the pressure of the relativistic plasma pairs, which is sufficiently less in comparison with the pressure generated owing to the presence of significant angular momentum. However, in the present work we have dealt only with high Mach number solutions, which means matter is accreting with very low energy (cold inflow, as it is described in the literature). This is another possible reason for obtaining a low mass-loss rate. If, instead of high Mach number solutions, we use a low Mach number solution, e.g. high-energy accretion, the mass outflow would be considerably higher (this is obvious because it has already been established in the present work that $R_{in}$ increases with $\delta$). In our next work, we will present this type of model by calculating $R_{comp}$ and $(\delta E_p^o)$ for low Mach number solutions.

In the literature, we did not find any numerical simulation work that deals with the type of outflow we discuss in this paper (outflow from zero angular momentum inflow). On the other hand, observationally it is very difficult to calculate the outflow rate exactly from a real system, as it depends on too many uncertainties, such as filling factors and projection effects. So, at present, we do not have any offhand results in this field with which our result could be compared.

There are a number of possible improvements which could be made to this preliminary work. For instance, the effect of radiation pressure on both the inflow and the outflow could be taken into account. A preliminary investigation shows that the effect of radiation force, when included in the basic conservation equations [equations (1), (2) and (12), (13)], decreases the value of $R_{in}$. This is probably because the introduction of any radiation term that is proportional to $r^{-2}$, weakens the gravitational force and pushes
the shock location outwards, the combined effect of which is the decrement of $R_m$. Another possible improvement is to include the magnetic field, to give the outflow an appropriate geometry. In our model, we assumed the outflow to be quasi-spherical, like the inflow. The introduction of the magnetic field would probably collimate the outflow, thus helping to further the investigation of the structure of the jet related to the model of AGN without accretion discs. Finally, we did not self-consistently compute $\gamma_o$ as a function of inflow parameters and $\gamma_o$ was supplied as a free parameter. In future work, we will be presenting the self-consistent calculation of $\gamma_o$ to reduce the number of free parameters in the problem.

So far, we have made the computation around a Schwarzschild black hole. Our work could be extended to study the mass outflow in Kerr space time as well. This is under preparation and will be presented elsewhere.

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REFERENCES

Axford W. I., Leer E., Skadron G., 1977, Proc. 15th Int. Cosmic Ray Conf. (Plovdiv), Vol. 11, p. 132

Bondi H., 1952, MNRAS, 112, 195

Chakrabarti S. K., 1998 in Chakrabarti S. K., ed., Observational Evidence for Black Holes in the Universe. Kluwer, Dordrecht, p. 19

Das T. K., 1998a, in Chakrabarti S. K., ed., Observational Evidence for Black Holes in the Universe. Kluwer, Dordrecht, p. 113

Das T. K., 1998b, in Aminova A. V., ed., Proc. Xth Int. Summer School Seminar on Recent Problems in Theoretical & Mathematical Physics. Kazan, in press

Das T. K., Chakrabarti S. K., 1998, CQG, submitted (astro-ph/9809109) (DC98)

Eichler D., 1979, ApJ, 232, 106

Ellison D. C., Eichler D., 1984, ApJ, 286, 691

Ellison D. C., Eichler D., 1985, Phys. Rev. Lett., 55, 2735

Flammang R. A., 1982, MNRAS, 199, 833

Kazanas D., Ellison D. C., 1986, ApJ, 304, 178 (KE86)

Meszaros P., Ostriker J. P., 1983, ApJ, 273, L59

Paczynski B., Wiita P. J., 1980, A&A, 88, 23

Protheroe R. J., Kazanas D., 1983, ApJ, 265, 620

Rees M., 1977, QJRAS, 18, 429

Taraftar S. P., 1988, ApJ, 331, 932

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