Bayesian Statistics as a New Tool for Spectral Analysis: I. Application for the Determination of Basic Parameters of Massive Stars

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ABSTRACT

Spectral analysis is a powerful tool to investigate stellar properties and it has been widely used for decades now. However, the methods considered to perform this kind of analysis are mostly based on iteration among a few diagnostic lines to determine the stellar parameters. While these methods are often simple and fast, they can lead to errors and large uncertainties due to the required assumptions.

Here we present a method based on Bayesian statistics to find simultaneously the best combination of effective temperature, surface gravity, projected rotational velocity, and microturbulence velocity, using all the available spectral lines. Different tests are discussed to demonstrate the strength of our method, which we apply to 54 mid-resolution spectra of field and cluster B stars obtained at the Observatoire du Mont-Mégantic. We compare our results with those found in the literature. Differences are seen which are well explained by the different methods used. We conclude that the B-star microturbulence velocities are often underestimated. We also confirm the trend that B stars in clusters are on average faster rotators than field B stars.

Key words: massive stars, B stars, spectral analysis, Bayesian statistics

1 INTRODUCTION

Massive stars are bright and short-lived objects. Therefore they are relatively easy to observe and remain close to their birth place. Their influence on their environment is important, from photoionizing HII regions, polluting their environment with enriched chemical elements, to expelling material out of galaxies. But they also represent an important candidate, due to their high level of internal radiation pressure, for the general study of stellar evolution. Therefore an accurate characterization of the fundamental and spectral parameters of massive stars is greatly needed in astrophysics.

One of the most useful tools to investigate massive stars is quantitative spectroscopy, which involves the comparison of a stellar spectrum with synthetic data created by state-of-the-art atmospheric models. There is a wide variety of comparison techniques presented in the literature. But whether it is by fitting spectral lines “by eye” (Searle et al. 2008), by using $\chi^2$ minimization (Daflon et al. 2007), or by measuring equivalent widths (Lefever et al. 2010), most of these methods rely on an iterative process, due to the large number of parameters involved in the analysis. For B stars, these parameters are the effective temperature $T_{\text{eff}}$, the surface gravity $\log(g)$, the projected rotational velocity $v\sin(i)$, the micro- and macro-turbulence velocity $\xi$ and $\zeta$, and of course the abundances of the chemical elements. The iterative analysis usually starts by adopting best-estimate values for all parameters. It then alternatively refines each parameter individually while using specific spectral indicators that are mostly sensitive to a sub-sample of parameters. For instance, in B stars the hydrogen lines H\textbeta and H\textgamma are mostly sensitive to $T_{\text{eff}}$ and $\log(g)$, while ionized silicon lines are mostly sensitive to $T_{\text{eff}}$, $\xi$, and its abundance. Therefore, one can start by assuming initial values for $T_{\text{eff}}$, $\log(g)$, and $\xi$ for instance, and then use the Balmer lines to determine a new $\log(g)$ value. This new result combined with the Si lines is then used to infer a new effective temperature and, going back to the Balmer lines, to update $\log(g)$, and so on until all the parameter values are stable. Of course, this does not mean that each indicator is completely insensitive to the other parameters. On the contrary, there is a highly non-trivial coupling between all the involved parameters (Nieva & Przybilla 2010). And due to this coupling, the use of an iterative process based on a few spectral indicators can lead to a local solution rather than a global solution that takes into account all the parameters (Mugnes & Robert 2012).

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Moreover, depending on the diagnostic lines used and on the initial parameter values, the same method may not give the same results. Also, due to the large number of parameters involved and in order to reduce computation time, many works in the literature have focused on a reduced number of parameters while assigning fixed values for the others (for instance by adopting solar abundance values, or assuming $\xi = 2 \text{ km s}^{-1}$ for main sequence stars and $10 \text{ km s}^{-1}$ for evolved stars). Fixed values are often based either on theoretical estimates or on results from other works. For example, Searle et al. (2008) used the projected rotational velocities of Howarth et al. (1997). But as discussed in the present work, the final results depend on the choice of the parameter that is fixed and on the choice of the value adopted. Even for a given atmospheric model and a given set of observed spectra, all the simplifications done (i.e. the adoption of initial estimated values, the use of fixed parameters, or an iterative process with only a few specific indicators for a reduced number of parameters) have an impact on the results and their accuracy and make it difficult to compare or to compile results from different works in a consistent way. It is therefore important to find a method which does not rely on initial estimations of the parameters, that will simultaneously constrain all the parameters using most of (if not all) the available information contained in an observed spectrum, and will work in a reasonable amount of time.

Thanks to the development of computer power, along with its recent success in many scientific fields, Bayesian statistics has been more and more used in astrophysics during the last decades. Studies found in the literature cover a large range of astrophysical topics, from redshift estimations with photometry [Benítez 2000] to the study of magnetic fields in interstellar clouds [Crutcher et al. 2010], including cosmological parameter selection [Trotta 2007]. Among these studies, a few authors applied Bayesian statistics to the determination of stellar parameters, hereby illustrating some of its advantages. For instance Pont &Everl (2004) used Bayesian statistics to redefine the age-metallicity relation in field dwarfs. They showed that most of the scatter found in the previously derived age-metallicity relation was due to a simplified treatment which, among other issues, did not properly take into account systematic biases. Slesidy et al. (2007) performed a complex and complete study of the impact of both systematic and statistical measurement errors on stellar parameters and their uncertainties. More recently, Schönrich & Bergemann (2014) developed a method which allow the combined use of photometric, spectroscopic, and astrometry informations for the determination of stellar parameters and metallicities. Nevertheless, none of these studies were applied on massive stars, nor did they performed a spectral analysis involving more than three parameters at the same time (the spectroscopic part of the work of Schönrich & Bergemann 2014 focused only on the effective temperature, surface gravity, and microturbulence velocity).

In this work, we use both a large pre-calculated grid of synthetic spectra and small interpolated and tailored grids in order to reduce the computation time. Bayesian statistics are applied to allow for a logical and numerical connection between the two grids as well as to constrain all the parameters at the same time while using nearly all the available information given by a spectrum. In the present paper, as a way to introduce our method and to illustrate the parameter interdependency, we focus on four stellar parameters (effective temperature, surface gravity, projected rotational velocity, and microturbulence velocity). In a following paper, we will add individual chemical abundances and the global metallicity to the analysis.

The next section summarises the ingredients and vocabulary used in Bayesian statistics. This is followed by a description of our technic and different tests done to demonstrate its reliability. Stellar parameters are obtained for a sample of B stars and are compared to other works.

## 2 DESCRIPTION OF THE METHOD

We here briefly describe the key elements of Bayesian statistics involved in our method. For a complete description of Bayesian statistics see Gregory (2010).

### 2.1 The Bayes Theorem

#### 2.1.1 Definition

Let us make an experiment or an observation that allow us to retrieve some data $D$ (a stellar spectrum for instance) in order to test a given hypothesis $H$ (for example, the hypothesis that a given atmospheric model reproduce correctly the stellar spectrum). And let us also assume that we have some prior knowledge or information $I$ (the stellar parameters given by an approximate spectral type for instance). Then by using the Bayes theorem we can assign a plausibility degree (or probability) to the hypothesis $H$. Namely, the Bayes theorem states that the probability $P$ of $H$ being true given $D$ and $I$ is called the posterior probability and is given by:

$$P(H|D,I) = \frac{P(H|I) \cdot P(D|H,I)}{P(D|I)}, \quad (1)$$

where $P(H|I)$ is the prior probability which is obtained over a previous set of data or from previous knowledge, $P(D|H,I)$ is the likelihood, i.e. the result of the actual data $D$, and where the global likelihood $P(D|I)$ is a normalization factor. Simply put, according to the Bayes theorem, our final state of knowledge (the posterior probability) is the product of what we already knew (the prior probability) with what the experiment (the likelihood) tells us.

#### 2.1.2 Application to a Stellar Spectrum

In this work we adopt the Bayes theorem to stellar spectroscopy considering three main concepts:

1) Let us consider an observed spectrum of a given star that contains several spectral lines. We then artificially divide the whole spectrum into smaller spectra, each centered on an individual spectral line (Fig. 1a). Each smaller spectrum can be seen as an individual observation. As the original spectrum can be described as a data set $D$ composed of all the spectral resolution elements (or flux points) contained in the whole spectrum, then each smaller spectrum $k$ can also be described as a data set $D_k$ composed of all the spectral resolution elements contained in the spectrum $k$ (Fig. 1b).

Thus for a given star, we have $K$ independent spectra, where each spectrum $k$ is centered on the spectral line $k$ and is
described by a data set $D_k$ (where $1 \leq k \leq K$, and $K$ is the total number of lines contained in the original observed spectrum).

2) As we want to fit an observed spectrum over a grid of synthetic spectra, we can replace the hypothesis $H$ by a set of hypothesis $M_i$ (with $i = 1, 2, ..., N_{\text{models}}$) representing the propositions stating that each synthetic spectrum calculated with an atmosphere model $M$ with a set of parameters $i$ is true (i.e. correctly reproduces the observation). Then for a given data set $D_k$, we can assign a probability for each synthetic spectrum. Since the propositions $M_i$ are mutually exclusive, we can write: $\sum_i P(M_i|D_k) = 1$.

3) Finally each line $k$ (or experiment $k$) has an associated prior information $I_k$ and a posterior probability reflecting our state of knowledge respectively before and after doing the experiment $k$.

We can then rewrite the Bayes theorem for each line $k$ as:

$$P(M_i|D_k, I_k) = \frac{P(M_i|I_k) \cdot P(D_k|M_i, I_k)}{P(D_k|I_k)}.$$  \hspace{1cm} (2)

If we use the posterior probability given by the line $k-1$ as the prior probability of the line $k$, then we can define the prior information of the line $k$ with $I_k = D_{k-1}, I_{k-1}$ (i.e. the combined information given by the previous data set $D_{k-1}$, or line $k-1$, and its associated prior information $I_{k-1}$). The Bayes theorem then becomes:

$$P(M_i|D_k, I_k) = \frac{P(M_i|D_{k-1}, I_{k-1}) \cdot P(D_k|M_i, I_{k-1})}{P(D_k|I_k)},$$  \hspace{1cm} (3)

which can be understood as: the probability that the synthetic spectrum $M_i$ reproduce well the observed line $D_k$ is proportional to the quality of the fit (likelihood) between the observed line $D_k$ and the corresponding synthetic line from the spectrum $M_i$, times the probability that the same synthetic spectrum $M_i$ reproduce the previous line $D_{k-1}$ with the previous corresponding synthetic line.

Here $P(M_i|D_k, I_k)$ is the posterior probability for one synthetic spectrum $M_i$. When we consider all the synthetic spectra at the same time, $\{P(M_i|D_k, I_k)\}_{i=1..N_{\text{models}}}$ is thus called the posterior probability distribution (the same obviously applies for the likelihood and the prior).

In other words, if we apply this theorem to all the lines in the observed spectrum (each time using the last posterior probability distribution as a prior for the next line) and considering all the synthetic spectra, we obtain, in the end, a final posterior probability distribution that simultaneously takes into account all the constraints given by all the lines in the observed spectrum for all the considered parameters. Figure 2 shows an example of the process when we apply our method on 4 spectral lines using 25 synthetic spectra. Note here that the order of the lines to which we apply the theorem does not matter. Indeed, if we neglect the normalisation factors, the final posterior probability is simply the product of the first prior probability with the likelihoods of all the lines involved.

2.1.3 Constructing the Likelihood

In order to construct a proper likelihood, we need to make a few basic assumptions. First, each data set $D_k$ (i.e. line $k$) is composed of $N_k$ independent spectral resolution elements $D_k = \{d_{n,k}\} = d_{1,k}, d_{2,k}, ..., d_{N_k,k}$ (Fig. 1(a)). Then we can do the same for the model spectrum $M_{i,k} = \{m_{n,k,i}\} = m_{1,k,i}, m_{2,k,i}, ..., m_{N_k,k,i}$ (Fig. 1(b)). Finally let us suppose that each observed spectral resolution element can be associated with a spectral resolution element from a model, given a certain flux error $e_{n,k,i} = d_{n,k,i} - m_{n,k,i}$ (Fig. 1(c)). If we suppose the errors $\{e_{n,k,i}\}$ to be independent and represented by a Gaussian distribution centered on zero with a standard deviation of $\sigma_{n,k,i}$, then we can write the likelihood as:

$$P(D_k|M_{i,k}) = \prod_{n,k} \frac{1}{\sqrt{2\pi\sigma_{n,k,i}^2}} e^{-\frac{(d_{n,k,i} - m_{n,k,i})^2}{2\sigma_{n,k,i}^2}}.$$
\[ P(D_k|M, I_k) = \prod_{n=1}^{N} \left( \frac{1}{\sqrt{2\pi \sigma_{n,k}}} \right) \exp \left[ -\sum_n \frac{(d_{n,k} - m_{n,k,i})^2}{2\sigma_{n,k}^2} \right] = C_k \exp \left[ -\frac{\chi^2_{i,k}}{2} \right]. \]  

The errors can represent the noise in the data but also the incompleteness of the model (due to simplifications, approximations, or unknown variables) or even some errors in the reduction process of the data (bad normalisation for instance). To take these sources into account, we define the standard deviation of the Gaussian distribution as follow:

\[ \sigma_{n,k} = \sqrt{\left( \sigma_{\text{noise}} \right)_{n,k}^2 + \left( \sigma_{\text{model}} \right)_{n,k}^2}, \]  

where \( \left( \sigma_{\text{noise}} \right)_{n,k} \) refers to the errors coming from the noise in the data, and \( \left( \sigma_{\text{model}} \right)_{n,k} \) refers to the other sources of errors (model incompleteness and reduction errors). In this work, for simplicity, we assume \( \left( \sigma_{\text{noise}} \right)_{n,k} = \sigma_{\text{noise}} \), a constant for all the spectrum, defined as the standard deviation in regions of the normalised continuum of the observed spectrum. And for each line, \( \left( \sigma_{\text{model}} \right)_{n,k} \) is given by:

\[ \left( \sigma_{\text{model}} \right)_{n,k} = \min_{\nu_i} \left( \sqrt{\frac{1}{N_k} \sum_n (d_{n,k} - m_{n,k,i})^2} \right) = \left( \sigma_{\chi} \right)_{k}. \]  

In other words, for each line \( k \), \( \left( \sigma_{\text{model}} \right)_{k} \) is the minimal RMS between the observed line and all the corresponding model spectrum \( M_i,k \). With these definitions, \( \sigma_{n,k} \) is the same for all the spectral resolution elements \( n \) of a line, but is different for each line. Thus:

\[ \sigma_{n,k} = \sqrt{\sigma_{\text{noise}}^2 + \left( \sigma_{\text{model}} \right)_{n,k}^2} = \left( \sigma_{\chi} \right)_{k}. \]  

Note that \( \left( \sigma_{\chi} \right)_{k} \) is a very important variable as it ultimately links physical imperfections and theoretical incompleteness to the uncertainty of the final parameters. And with this decomposition, if we were to work with perfect data \( \left( \sigma_{\text{noise}} \rightarrow 0 \right) \), then the uncertainty of the final parameters would be related to the limitations of the atmospheric or atomic models used. Inversely, if we have a perfect model \( \left( \left( \sigma_{\text{model}} \right)_{k} \rightarrow 0 \right) \), the final uncertainties would then be directly related to the quality of the data.

2.1.4 Global Likelihood and First Prior

Now, since \( \sum_i P(M_i|D_k, I_k) = 1 \), the global likelihood \( P(D_k|I_k) \) is then given by:

\[ P(D_k|I_k) = \sum_i \left( P(M_i|D_{k-1}, I_{k-1}) \ast P(D_k|M_i, I_k) \right). \]  

And the Bayes theorem (Eq. 3) becomes:

\[ P(M_i|D_k, I_k) = \frac{P(M_i|D_{k-1}, I_{k-1}) \ast C_k \exp \left[ -\frac{\chi^2_{i,k}}{2} \right]}{\sum_i \left( P(M_i|D_{k-1}, I_{k-1}) \ast C_k \exp \left[ -\frac{\chi^2_{i,k}}{2} \right] \right)}. \]  

Finally, when we apply the theorem (Eq. 9) for the first time, we assume that all models \( M_i \) are equiprobable: \( P(M_i|D_0, I_0) = 1/Na_{\text{model}} \) (i.e. we consider a flat prior distribution as in Fig. 3), and \( D_0 \) represents no data. This means that, at first, we do not give preference to any model (or set of parameters). Note here that we are not performing a model comparison but a parameter estimation and as we use large data samples, the results are not affected by the choice of the prior (Gregory 2010).

2.2 Parameter Estimation

In this work we focus on the stellar parameters \( T_{\text{eff}}, \log(g), v \sin(i) \), the projected rotational velocity, and \( \xi \) the microturbulence velocity. We thus need to retrieve, from the final posterior probability mentioned above, an estimation for each parameter by considering all the other parameters. For this, we use another major feature of the Bayesian statistics, the marginalization. This is done by defining the model \( M_i \) as: the synthetic spectrum computed with the model \( M \) using the set of parameters \( T_{\text{eff}}, \log(g), v \sin(i), \xi \), which we write \( M_i = T_i, l_i, v_i, \xi_i, M \). Since we suppose our model \( M \) to be true, our posterior probability thus becomes:

\[ P(M_i|D_k, I_k) = P(T_i, l_i, v_i, \xi_i|M, D_k, I_k), \]  

which can be read as the joint probability for \( T_i, l_i, v_i, \) and \( \xi_i \) to be true if the model \( M \), the data \( D_k \), and our prior knowledge \( I_k \) are true.

Since our parameters have discrete values in our analysis, we can retrieve the marginal posterior probability distribution for each parameter by using the marginalization equation. Here we give an example for \( T_i \):

\[ P(T_i|M, D_k, I_k) = \sum_a \sum_b \sum_c P(T_i|I_0, a_b, c|M, D_k, I_k), \]  

which is the marginal probability distribution of \( T_i \) considering all the values of the other parameters. As shown in Figure 3 from this distribution we obtain the most probable value and the mean value, as well as the associated uncertainties which are defined as the values between which the sum of the marginal posterior probability distribution is equal to or greater than a certain confidence level \( C = \sum_{T_i} P(T_i|M, D_K, I_K) \). In this work we use \( C = 0.99 \) which can be seen as a 3\( \sigma \) uncertainty (recall that \( \sum_i P(T_i|M, D_K, I_K) = 1 \)). Note that these uncertainties are related to the variations of the other parameters but also, as stated in 2.1.3, to possible data or model imperfections.

2.3 Preventing Numerical Biases

Due to the fact that we want to treat all available lines equally, we need to calculate the \( \chi^2 \) for the same number of spectral resolution elements in each line. Otherwise, broad features (such as hydrogen and helium lines in B stars) will dominate the solution over the narrow features (metal lines for instance) in the posterior probability.

Indeed, given the same differences \( d_{n,k} - m_{n,k,i} \) between the observed and synthetic line and the same \( \sigma_k \) for both a broad and a narrow line, only the number of spectral resolution elements will have an impact on the \( \chi^2 \) value. The more
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Figure 2. Illustration of the Bayesian analysis step by step. For this example, we consider 4 data sets (or lines) $D_k$ and 25 synthetic spectra calculated with only two free parameters: $T_{\text{eff}}$ and $\xi$. The black plus signs in columns 2, 3, and 4 indicate the value used for these two parameters. The colour of the surface contours is scaled to the relative probability of each distribution: the dark red regions being the most probable and the white ones the less probable. The yellow squares indicate the global maximum of each distribution. The first column shows every data set considered, and the second column the likelihood obtained for each data set as in Equation 4. The third column represents the prior probability distribution known at each step $k$, and the last column shows the posterior probability distribution which is the normalised product of the prior and the likelihood of a given step $k$. The method goes as follow: considering the first line $k = 1$ (in (a)), we calculate its likelihood using Equation 4 (in (b)), and use, as a first prior, a flat distribution (in (c)) assuming we have no knowledge of the best pair $[T_{\text{eff}}, \xi]$. We then apply the Bayes theorem (Eq. 3) and obtain the first posterior probability distribution (in (d)). For the next data set ($k = 2$), we perform the same operation but we use the posterior distribution of the previous step (in (d)) as a new prior distribution (in (g)) since our “state of knowledge” has been changed after the first step. In the end, the final posterior probability distribution (in (p)) represents the solution combining all the information given by the 4 lines. Note that since the method can be summarized as a series of products, the order in which we consider the data sets have no influence on the shape of the final posterior distribution (see Fig. A1).

2.4 Synthetic Spectra

We use atmospheric models from the metal line-blanketed, NLTE, plane-parallel, and hydrostatic code TLUSTY [Lanz & Hubeny 2007]. With these models, we create synthetic spectra by running the program SYNOPS. More precisely, we use the SYNPLOT package generously offered to us by Hubeny and Lanz, which contains a TLUSTY pre-calculated B-star grid (BSTAR2006) and the latest version of SYNSPEC and SYNPLOT which allows the user to interpolate more specific models from the closest BSTAR2006 models. The BSTAR2006 grid considers 16 values of the effective temperature, between 15000 K and 30000 K with a 1000 K step; 13 surface gravities, from 1.75 to 4.75 in log($g$) with a step of 0.25 dex; and 6 chemical compositions, $Z/Z_{\odot} = 2, 1, 1/2, 1/5, 1/10$ (where $Z_{\odot}$ is the solar composition); a solar helium abundance, He/H = 0.1 by number, and a microturbulence velocity of 2 and 10 km s$^{-1}$.

For this paper we consider only the solar chemical composition, as established by Asplund et al. [2009], and we...
use SYN Spec to create a grid of synthetic spectra (hereafter called BaseGrid) with the same values of the effective temperature and surface gravity as those of the BSTAR2006 grid with an extended range of 7 microturbulence velocities, from 0 to 30 km s$^{-1}$ with a step of 5 km s$^{-1}$. We then convolve these spectra to take into account rotational broadening with 21 different values of v sin(i), from 0 to 400 km s$^{-1}$ with a 20 km s$^{-1}$ step, to create our starting grid for the analysis: BSTAR. Note that the spectra of BaseGrid are not convolved with instrumental and rotational broadening and are used, in this work, as the basis for the creation of a new spectrum or new refined grids by directly interpolating the non-convolved spectra (because this is faster than interpolating the BSTAR2006 models) to the desired values of $T_{\text{eff}}$, log(g), and v sin(i).

For each star, we begin, as previously mentioned, by assuming a uniform prior probability over all the BSTAR models. We then apply our Bayesian analysis over all the selected lines in the spectrum of a star using all the models of the BSTAR grid. This first analysis gives us, after marginalization, a reduced parameter space by excluding all values outside the 99% confidence level. We then construct another refined grid and redo the analysis, and so on. The program stops either when the parameter space remains unchanged or when the parameter steps used to create a new grid are equal to or smaller than a certain limit. Here, as a compromise between accuracy and computing time, we set this limit at 100 K in $T_{\text{eff}}$, 0.03 dex in log(g), 5 km s$^{-1}$ in v sin(i), and 1 km s$^{-1}$ in $\xi$.

3 TESTING THE METHOD ON ARTIFICIAL STELLAR SPECTRA

As a way to test the efficiency of the method as well as its robustness to noise and uncertainties, we perform three tests.

For the first test, we create an artificial observational spectrum using synthetic spectra with the following specifications: $T_{\text{eff}} = 21350$ K, log(g) = 3.80, v sin(i) = 45 km s$^{-1}$, $\xi = 6$ km s$^{-1}$, and solar chemical composition. The spectrum is also convolved with a Gaussian instrumental profile with a full width at half maximum (FWHM) of 2.3 Å. Random noise is also added in order to emulate a standard deviation, $\sigma_{\text{noise}}$, that is typical of what we see in our actual observations (with $\sigma_{\text{noise}} \approx 0.004$ in the normalised flux). Then, we compare the artificial spectrum with our models using the method described in §2.4. But, as we are in the situation of a perfect model and imperfect data, we set $(\sigma_{\text{model}})_k$ to 0 and we substitute $\sigma_{\text{noise}}$ by $x\sigma_{\text{noise}}$ where $x = 1, 2, 3, \ldots$ in Equation 7. This way, while not changing the actual noise level in the spectrum, we can overestimate its value in the calculation by increasing x, and thus, we can test the robustness of the method to noise over-estimation. This analysis is then repeated 10 times, each time increasing x by 1. The results are summarized in Figure 4.

![Figure 4](image-url)
is well under our minimal uncertainty (100 K). The runs for different values of $\sigma_\chi/\sigma_{\text{noise}}$ then give parameter values which are all very close to the original values used to create the artificial spectrum.

Increasing the ratio $\sigma_\chi/\sigma_{\text{noise}}$ means overestimating the noise level in the artificial spectrum, which should logically lead to greater parameter uncertainties since $\sigma_\chi$ acts as a tolerance parameter. Here, the parameter uncertainties increase with $\sigma_\chi$ roughly as $\sigma_{\text{noise}}/2$, as shown in Figure 5. This means that even when increasing the flux uncertainty by a factor of 2 or 3, there will be nearly no incidence on the method accuracy. Note also that, up to $4\sigma_{\text{noise}}$, the accuracy of $v \sin(i)$ is largely higher than the adopted resolution (around 77 km s$^{-1}$ for a FWHM of 2.3 Å). Usually, with classical methods, the uncertainty for $v \sin(i)$ with a value smaller than half the FWHM should be of the order of FWHM/2 since instrumental broadening becomes more important than rotational broadening under this limit. These effects are a direct consequence of cumulating the information from one line to the others and thus having multiple simultaneous constraints on each parameter. Note that when we use large $\sigma_\chi$ values, we find the classical uncertainties (close to the $v \sin(i)$ value when $v \sin(i)$ is below the classical limit) since, with such a huge tolerance parameter, most of the constraints coming from the lines disappears.

Since spectral analysis usually tends to be more difficult and less accurate for fast rotating stars due to the blending of multiple lines, we repeat the first test for a higher projected rotational velocity, $v \sin(i) = 145$ km s$^{-1}$. Uncertainty and line definitions are the same as in Figure 5.

Figure 5. Plot of the relative stellar parameter uncertainties $\Delta_{\chi i}/\Delta_{\min}$ with increasing $\sigma_\chi/\sigma_{\text{noise}}$. For each parameter, $\Delta_{\chi i}$ is the length of the error bars from Figure 4 and $\Delta_{\min}$ the length of the smallest error bar found for the 10 runs. The thin black dashed line is a linear fit of the mean of all the parameter uncertainties.

For the second test, we again use an artificial spectrum (with the following parameters: $T_{\text{eff}} = 21350$ K, log($g$) = 3.80, $v \sin(i) = 125$ km s$^{-1}$, $\xi = 3$ km s$^{-1}$, and solar chemical composition) but this time we change the noise level added to it in order to create different signal-to-noise ratios (SNR), ranging from 25 to 350 per resolution element with a step of 25. We estimate the SNR using the der_snr program of [Stoehr et al. 2008]. On this scale, a SNR of 250 corresponds to a $\sigma_{\text{noise}}$ of approximately 0.004 in the normalized continuum. For each value of SNR, we create 10 noisy artificial spectra. For the 140 new spectra thus created, we apply our method with the original definition of $\sigma_\chi$ (Eq. 7). This way we simulate a real analysis even though we still are in the case of a perfect model and imperfect data. Of course, in this case $\sigma_\chi$ is overestimated by roughly a factor of 2, since for a given line $k$, $(\sigma_{\text{model}})_k \approx \sigma_{\text{noise}}$, but as explained above, it has nearly no incidence on the method accuracy. The final parameters returned by the 140 runs are shown in Figure 7 and the overall success rate of the method is shown in Figure 8. This test allows us to check for the robustness of the method against noisy data, and we can see that for SNR around 100-125 and higher, the method returns correct and accurate results (with uncertainties equal or close to the minimal uncertainties and with a mean success rate of 97.5%). For lower SNR ($\leq 100$), the parameter uncertainties become naturally larger but the real parameter values are still found nearly 84% of the time (within the uncertainties). Considering all the SNR values, the overall success rate is around 94%.

The third test is the same as the second one, except that the artificial spectrum is created with the atmospheric model ATLAS9 [Castelli & Kurucz 2004], rather than with TLUSTY, using the same stellar parameters. This test is close to an analysis of real data since we are in the case of imperfect data and inexact models. The results are shown in Figure 9 and the overall success rate of the method in Figure 10. Compared to Figure 7, we can see from Figure 9 that the best parameters returned by the method for the spectrum without noise (orange lines) are different than the real parameters (green lines) used to create the spectrum. It is especially true for the effective temperature (the other parameters are within the minimal uncertainties), where $T_{\text{eff,real}} = 21930$ K while $T_{\text{eff,est}} = 21350$ K. This discrepancy is to be expected when comparing full non-LTE (i.e. non-LTE atmospheric models with non-LTE radiative
transfert) spectra with hybrid LTE/non-LTE (i.e. LTE atmospheric models with non-LTE radiative transfert) spectra since, non-LTE line profiles tend to be slightly stronger and broader than those derived from LTE atmospheres, thus giving a lower temperature when considering a LTE treatment (Huang, Gies & McSwain 2010, Lanz & Hubeny 2007). Here the artificial spectrum is created by a hybrid model while the grid of synthetic spectra are calculated using a full non-LTE treatment. Therefore our method finds a higher effective temperature than the one used to create the hybrid spectrum. Note that this discrepancy is not taken into account by $\sigma_{\text{model}}$ since this parameter reflects the model incapacity to correctly reproduce the lines (whatever parameter values are considered). Here, our model is able to reproduce nearly perfectly most of the lines but for a different effective temperature than the one used to create the hybrid lines. With this in mind, if we consider the best parameter values found for the hybrid spectrum without noise as the reference values (rather than the real values used to create the hybrid spectrum), we find a mean success rate of nearly 93% (Fig. 10) which is equivalent to the success rate found in the previous test (Fig. 5). Note that if we consider the original parameter values, the mean success rate for all the parameters is around 75%, and around 96% when we exclude $T_{\text{eff}}$. In tests 2 and 3, if we multiply the derived parameter uncertainties by a factor 1.5 or 2, the overall success rate (which is around 94% in both case) jumps to nearly 98% and 99%, respectively. Thus, in terms of classical statistics, our results are given with an accuracy close to a $2\sigma$ uncertainty.

![Figure 8](image8.png)

**Figure 8.** Success rate of the method, to derive the correct parameter values (within the uncertainties) used to create the original artificial spectrum, as a function of the signal-to-noise ratio. Percentages are mean values gathered from all the parameters for each SNR value.

![Figure 9](image9.png)

**Figure 9.** Same test as in Figure 7 when using an artificial spectrum created by the atmospheric model ATLAS9. The green lines show the real parameter values used to create the original artificial spectrum ($T_{\text{eff}} = 21350$ K, log$(g) = 3.80$, $v \sin(i) = 125$ km s$^{-1}$, and $\xi = 3$ km s$^{-1}$). The orange lines show the best parameters returned for the original spectrum without noise added ($T_{\text{eff}} = 21930$ K, log$(g) = 3.81$, $v \sin(i) = 125$ km s$^{-1}$, and $\xi = 4$ km s$^{-1}$). The orange line overlaps the green line in the case of $v \sin(i)$.

4 **TESTING OUR METHOD ON REAL STELLAR SPECTRA**

4.1 Observational Data

The selection criteria for our sample were as follow: 1) only mid-B to early-B type stars, since the pre-calculated BSTAR2006 atmospheric models only cover the range of effective temperature for these subtypes; 2) relatively bright stars to allow a high SNR; 3) no binaries or peculiar stars to avoid complications while testing our method; 4) relatively close objects in order to work with a solar chemical abundance approximation (chemical element abundances will be the topic of a following paper); and 5) well known objects to allow the comparison of our method with other works. The main studies used here for the comparison are from Huang, Gies & McSwain (2010, hereafter HG10), Lefever.
We test our method on a sample of 54 mid-B to early-B stars, where 38 are from the field and 16 from clusters. The field stars are all nearby dwarf and giant B stars, with a visible magnitude $V \leq 8$. The cluster stars are members of two open clusters: 8 B stars from NGC1960 (M36) with $8.8 \leq V \leq 10.8$, and 8 B stars from NGC884 ($\chi$ Persei) with $8.9 \leq V \leq 11.6$. These two clusters have a colour excess $E(B-V) = 0.22$ and 0.56, respectively, and due to their proximity, are assumed to have a metallicity close to solar (Kharchenko et al. 2005). The spectra have been collected at the $1.6$ m telescope of the Observatoire du Mont-Mégantic at 3 epochs: February 2011, August 2013, and December 2013. They cover the wavelength range from 3500 to 5500 Å at a spectral resolution of 2.3 Å and a mean SNR of 250 per pixel in the continuum (with very little variation from the blue to the red end). We select all the observed lines available for the spectral analysis, except when they are not re-produced by the models, when they are barely distinct from the noise, or when they are superimposed on bad CCD pixels. Table 1 lists all the major useful lines. Note that not all these lines are available for every stars since they may appear or disappear depending on the spectral type. Note also that due to our modest resolution, and sometimes due to strong rotational velocities, more minor lines are included in the analysis.

### Table 1. Principal spectral lines used for the analysis

| Species | Wavelength (Å) |
|---------|----------------|
| H I     | 3712, 3722, 3734, 3750, 3771, 3798, 3835, 3889, 3970, 4102, 4340, 4861 |
| He I    | 3785, 3820, 3867, 3872, 3927, 3936, 4009, 4026, 4121, 4144, 4169, 4388, 4438, 4471, 4713, 4922, 5016, 5048 |
| C II    | 4267, 4374, 4619, 4619, 5133, 5145, 5648 |
| N II    | 3995, 4044, 4228, 4237, 4447, 4601, 4607, 4631, 4803, 4994, 5667, 5680 |
| O II    | 3912, 3954, 3982, 4070, 4079, 4085, 4153, 4185, 4277, 4304, 4317, 4320, 4367, 4415, 4515, 4591, 4642, 4649, 4662, 4676, 4705, 4907, 4943 |
| Si II   | 3856, 3863, 4131, 5056 |
| Si III  | 4553, 4568, 4575, 4683, 4829, 5740 |
| Si IV   | 4089 |
| S II    | 4294, 4525, 4816, 5032, 5103, 5321, 5433, 5454 |
| S III   | 4254, 4285, 4362 |
| Fe II   | 5169, 5260 |
| Fe III  | 5074, 5087 |

Concerning NGC1960 SAB30 (Fig. 12 upper panel), the poor quality of the fit (by far the worse case in our sample) is mainly due to the fact that this star has an effective temperature beyond the reach of our model. Its effective temperature is most likely around 13000 K (HG06), while our grid can only go down to 15000 K. This is also the case for NGC1960 SAB23 (Fig. 12 second panel) but to a lesser extent, since its effective temperature seems to be closer to 15000 K (HG06) than for SAB30.

For HD40160 (Fig. 12 lower panel), we encountered some difficulty during the reduction process, due to the presence of humidity spots on the CCD at the time of the observation, making the Hβ line unusable, altering the shape of Hγ (i.e. a shallower centroid), and reducing the number of usable lines in general. We still performed the analysis of this star as an extreme test for the sensitivity of our method to the number and quality of the lines used. While our results for HD40160 are not extremely different from those found in the literature, they present one of the largest discrepancy of our results with other works, and therefore are not considered in the remaining of this paper. In particular, we find $T_{eff} = 20010^{+290}_{-240}$ K, $\log(g) = 4.05^{+0.07}_{-0.05}$, $v \sin(i) = 128^{+13}_{-8}$ km s$^{-1}$, and $\xi = 5^{+1.5}_{-2}$ km s$^{-1}$, while HG06 found $T_{eff} = 16453 \pm 250$ K, $\log(g) = 3.735 \pm 0.027$, $v \sin(i) = 115 \pm 35$ km s$^{-1}$, and $\xi = 2$ km s$^{-1}$.

Figure 13 compares our results with those from other groups. All the numerical values can be found in Tables B1 (for field stars) and Table B2 (for cluster stars), where the
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Figure 11. Examples of the best fits obtained for four B stars in our sample. These fits have the smallest averaged rms of our sample (from top to bottom, the averaged rms are 0.0043, 0.0042, 0.0041, and 0.0038). Observed spectra are in black, best synthetic spectra are in red, and the offset rms are in green. Regions of spectra not used for the analysis are in lighter colour.

uncertainties given by our method (as explained in §2.2) are generally smaller than those found in the literature, because of the cumulative effect of our method (as explained in Fig. 2). While most of the time there is a good agreement between our results and those from other works, there are some important discrepancies. These are discussed in the following subsections.

4.2.1 Chemical Abundances

In this paper we want to do a study of the impact of the spectral synthesis method used on the stellar parameters, therefore it is important that we first demonstrate that the discrepancies in Figure 13 are not associated to the chemical abundances considered by the different analysis. Some of the other works used here for comparison derived the abundances of a few chemical elements at the same time that they determined the basic stellar parameters, while we are considering a solar abundance, based on Asplund et al. (2009), as the standard chemical composition for the models for all the stars in our sample. Nevertheless, the variations of the abundances found by the other authors are small, the overall metallicity is solar, and this distinction is not the main reason for the discrepancies found in Figure 13. To demonstrate this, we repeat the analysis of the stars in our sample using the abundances derived by the other works. Specifically, we adopt the abundances of the following chemical elements: He and Si from LEF10 (Table B3), O and Ne from TAK10 (Table B4), C, N and O from SEA08 (Table B5), He and Si from MAR08 (Table B6), C, N, O, Mg, Al, and Si for DAF07 (Table B7), He and Mg from LYU05 (Table B8), and C and N from AND99 (Table B9) for the stars that we have in common. HG10 and HG06 did not derive the abundances for any specific elements but they used the atmospheric model ATLAS9 with the solar abundances from Grevesse & Sauval (1998), as we also redo for the corresponding stars (Tables B10 and B11). Note that HG10 and HG06 derived $T_{\text{eff}}$ and log($g$) using TLUSTY models and as such, one should consider only Figure 13 for a proper comparison of $v \sin(i)$. Figure 14 shows the comparison of our new results, with the other papers, obtained when we are using their abundance data. When comparing Figures 13 and 14, we see a small and marginal difference between the two set of results (except for the stars studied by HG10 which we discuss in more details in §4.2.2). We believe that the reason
for the small change between the two figures is mainly related to the fact that the element abundances derived by the other authors were tailored to a few lines only and have little impact on the other parameters obtained simultaneously by fitting the whole spectrum as is done with our method. As discussed in the following subsections, we find that the impact of the method used and the choice of the diagnostic lines is more important to explain the discrepancies seen in Figures 13 or 14. A complete analysis with our method, including abundances determination and their effect on the basic stellar parameters, will be done in a following paper.

4.2.2 Effective Temperature and Surface Gravity

Figure 13 or 14 often shows a large scattering for the effective temperature and the surface gravity. Among others, we see a quasi-systematic difference between our estimates of $T_{\text{eff}}$ and $\log(g)$ and those of HG10 and HG06. We believe that the main reason for this is related to the fact that these authors used only H$\gamma$ as a diagnostic line. For the 30 stars that we have in common with HG10 and HG06, we perform our analysis using the atmospheric model ATLAS9 and the solar chemical composition of Grevesse & Sauval (1998) with all the available spectral lines (these results are presented in Fig. 14), but also, with only H$\gamma$ as a diagnostic line. Figure 15 shows for the 30 stars the effect on $T_{\text{eff}}$ and $\log(g)$ of the number of diagnostic lines used, as well as the effect of the different models. From the upper panels of Figure 15, we see that a change of atmospheric model or chemical composition (TLUSTY with the solar composition of Asplund et al. 2009 in red versus ATLAS9 and the solar composition of Grevesse & Sauval 1998 in green) has little impact on the discrepancies between our results and those of HG10 and HG06: the discrepancies remain important. Whereas the lower panels shows that considering only H$\gamma$ as a diagnostic line drastically reduces the variations in $T_{\text{eff}}$, and greatly affect those in $\log(g)$. For instance, the 3 stars in Figure 13 with a large difference in $T_{\text{eff}}$, when compared to HG10, are the most extreme examples of this effect (they are the stars number 23, 24, and 25 in Figure 15). It is clear in this case that the number and choice of diagnostic lines used has a greater impact on the determination of $T_{\text{eff}}$ and $\log(g)$ than the choice of the atmospheric model or the solar chemical composition.

Furthermore, the choice of the diagnostic line, when only one is used, also affects the results. Figure 16 shows the best values of $T_{\text{eff}}$ and $\log(g)$ that we obtain for differ-
Figure 13. Comparison of our results for $T_{\text{eff}}$, log($g$), $v\sin(i)$, and $\xi$ with other works from the literature. Atmospheric models from TLUSTY with the solar composition from Asplund et al (1999) have been used. Open diamonds are cluster stars and the other symbols are field stars. Mean error bars for these data are shown in black. The black full line indicates a one-to-one match. Note that some of our stars have been studied in more than one paper and thus appear multiple times in the graphics. The abbreviations in the legend correspond to the following papers: HG10 for Huang, Gies & McSwain (2010); LEF10 Lefever et al. (2010); TAK10 Takeda et al. (2010); SEA08 Searle et al. (2008); MAR08 Markova & Puls (2008); DAF07 Daflon et al. (2007); HG06 Huang & Gies (2006a,b); LYU05 Lyubimkova et al. (2005); and AND99 for Andrievsky et al. (1999).

ent values of $v\sin(i)$ when only H$\beta$, H$\gamma$, or H$\delta$ is considered for the analysis. This analysis is done using an artificial spectra with $T_{\text{eff}} = 21350$ K, log($g$) = 3.8, $\xi = 3$ km s$^{-1}$, and $v\sin(i) = 15$ km s$^{-1}$. In Figure 16, we can see that each line has a different sensitivity over $v\sin(i)$ and that two lines rarely give the same results for a given $v\sin(i)$. Moreover, $T_{\text{eff}}$ and log($g$) given by H$\gamma$ are lower than the real values by 2350 K and 0.3 dex, respectively, when $v\sin(i)$ is close to its real value. These differences are of the order of the quasi-systematic difference seen between HG10, HG06, and our work.

When comparing our results for the evolved stars with those obtained by Searle et al. (2008) in Figure 13, we find similar temperatures for the stars below 20000 K, and less scatter than Searle et al. (2008) for the stars above (the effective temperature goes from 23500 K to roughly 25000 K in our work, while the authors found temperatures going from 21000 K to 30000 K). The log($g$) values are, within the uncertainties, quite identical, with log($g$) below 3 in our case. We also find higher microturbulence velocities and a larger scatter for the rotational velocity. Note that performing our analysis with the C, N, and O abundances derived by Searle et al. (2008) has little to no impact on the observed discrepancies (as shown when comparing Figures 13 and 14). The reason for the differences between our results and those of Searle et al. (2008) may be related to two important effects:

1) First of all, Searle et al. (2008) used a combination of TLUSTY and CMFGEN in order to include wind effects which we are not doing. For instance, in B supergiant, the Balmer lines could suffer from wind contamination, where photons emitted by hydrogen atoms in the wind “fill in” the absorption profiles making them shallower than any profiles calculated with a photospheric model (see Searle et al. 2008). It is especially true for H$\alpha$ and H$\beta$ and to a lesser extent for H$\gamma$ and the other Balmer lines. Other photospheric lines can also be influenced by winds, such as the Si IV $\lambda\lambda4089$ line and the Si III $\lambda\lambda4552,4568,4575$ multiplet (Hillier et al. 2003; Dufton et al. 2005; Searle et al. 2008). But since this
Figure 14. Comparison as in Figure 13 but with the individual chemical abundances fixed, in our analysis, to the values given by other papers. The modified chemical elements are as follow: He and Si for a comparison with LEF10; O and Ne for TAK10; C, N, and O for SEA08; He and Si for MAR08; C, N, O, Mg, Al, and Si for DAF07; He and Mg for LYU05; and C and N for a comparison with AND99. No individual chemical abundance modifications were considered for the stars in common with HG10 and HG06, but we used the atmospheric model ATLAS9 with the solar abundances of Grevesse & Sauval (1998) in this case. Note that we find some stars with $T_{\text{eff}} < 15000$ K since the grid calculated with ATLAS9 has a broader range in $T_{\text{eff}}$ than our grid created with TLUSTY. Also, while HG10 derived $T_{\text{eff}}$ and $\log(g)$ using ATLAS9, these authors used TLUSTY to derive $v \sin(i)$, the comparison for this parameter is thus irrelevant here.

2) Furthermore, Searle et al. (2008) used, in an iterative way, different diagnostic lines for different parameters each time, while fixing the other parameters and adopting a rotational velocity found in the literature. While vastly used, due to its simplicity and swiftness, this iterative method may provide a local maximum in the probability space rather than a global maximum because it considers each parameter independently. For instance, Searle et al. (2008) used Si IV λ4089 and Si III λλ4552,4568,4575 as primary diagnostic lines for the effective temperature with fixed values of $v \sin(i)$ and $\xi$. Then they used the effective temperature obtained and the Hγ line to determine $\log(g)$. And finally they adopted their pair $[T_{\text{eff}}, \log(g)]$, and a value of $v \sin(i)$ found in the literature, to determine a new microturbulence velocity using the Si III λλ4552,4568,4575 lines. The whole process was repeated until small changes in the parameters were seen. With this iterative method, the effective temperature returned by the silicon lines greatly depends on the values used for the microturbulence and rotational velocities. And $\log(g)$ obtained from Hγ also depends on the effective temperature. Finally, the microturbulence is significantly affected by $T_{\text{eff}}$ and $\log(g)$. To illustrate the impact of an iterative method, we create another artificial stellar spectrum with the following specifications: $T_{\text{eff}} = 25000$ K, $\log(g) = 2.8$, $v \sin(i) = 100$ km s$^{-1}$, and $\xi = 14$ km s$^{-1}$.
Figure 15. $T_{\text{eff}}$ and log($g$) variations for different models and different number of lines for the 30 stars in common with the sample of Huang, Gies & McSwain (2010) and Huang & Gies (2006a). The variations are defined as $\Delta T_{\text{eff}} = T_{\text{eff}}(\text{this work}) - T_{\text{eff}}(\text{Huang et al.})$ and $\Delta \log(g) = \log(g)(\text{this work}) - \log(g)(\text{Huang et al.})$. Upper panels: Variations when considering all the lines but different atmospheric models and solar chemical composition. Lower panels: Variations when considering all the available lines or only H$\gamma$.

Figure 16. The most probable pair $[T_{\text{eff}}, \log(g)]$ as a function of $v\sin(i)$ for different lines (as indicated in the plot) for the artificial spectrum. The parameters used to create the artificial spectrum are: $T_{\text{eff}} = 21350$ K, log($g$) = 3.8, $\xi = 3$ km s$^{-1}$, and $v\sin(i) = 15$ km s$^{-1}$. The behaviour as a function of $v\sin(i)$ is shown projected in the plane $[T_{\text{eff}}, \log(g)]$. The likelihoods are marginalized over all $\xi$ values. Note in particular the most probable pair $[T_{\text{eff}} 19000$ K, log($g$) = 3.5] given by H$\gamma$ when $v\sin(i) \leq 150$ km s$^{-1}$.

We then calculate the posterior probability for the 4 silicon lines from this artificial spectrum with the BSTART grid. After the marginalization over the log($g$) domain, we find the most probable effective temperature for each given value of $\xi$ and $v\sin(i)$ (Fig. 17, left panel). We then perform the same operation for H$\gamma$ and marginalize over $v\sin(i)$ and $\xi$ (since both parameters have little influence on the returned log($g$) value, showing the marginalized probability is enough in this case) to obtain the most probable log($g$) estimate given each value of $T_{\text{eff}}$ (Fig. 17, middle panel). Finally we redo the same analysis, but this time for only the 3 lines Si$\text{III}$ $\lambda\lambda 4552,4568,4575$ and with a marginalization over $v\sin(i)$ in order to find the most probable value of $\xi$ for each value of $T_{\text{eff}}$ and log($g$) (Fig. 17, right panel). To conclude, we can see that the chosen silicon lines almost always overestimate the effective temperature (except when $\xi = 10$ km s$^{-1}$ and $v\sin(i) \geq 200$ km s$^{-1}$). Figure 17 also shows a clear anti-correlation: the effective temperature decreases as the rotational velocity increases. Thus if we were to underestimate (respectively overestimate) $v\sin(i)$, we would overestimate (respectively underestimate) $T_{\text{eff}}$. As log($g$) increases with the effective temperature, we would therefore overestimate (respectively underestimate) log($g$). Finally, we would overestimate or underestimate the microturbulence velocity depending on how we estimate log($g$). This behaviour is what we observe between our results and those of Searle et al. (2008) for the stars hotter than 18000 K. For the cooler stars, Searle et al. (2008) used Si$\text{III}$ $\lambda\lambda 4128,4130$ rather than Si$\text{IV}$ $\lambda 4089$, which seems to
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Figure 17. Illustration of the interdependency between \( T_{\text{eff}} \), log(\( g \)), and \( \xi \) when individual diagnostic lines are considered in an iterative method. The parameters of the artificial spectrum used in this example are: \( T_{\text{eff}} = 25000 \) K, log(\( g \)) = 2.8, \( v \sin(i) = 100 \) km s\(^{-1} \), and \( \xi = 14 \) km s\(^{-1} \). Left Panel: Most probable effective temperature as a function of \( v \sin(i) \) for 3 different microturbulence velocities using Si IV \( \lambda 4089 \) and Si III \( \lambda\lambda 4552,4568,4575 \). Orange squares and thin orange dashed lines indicate the extreme solutions found when underestimating and overestimating \( v \sin(i) \) compared to the parameters used to create the artificial spectrum (thin green dotted lines in all panels). Middle panel: Most probable log(\( g \)) values as a function of \( T_{\text{eff}} \), after marginalizing over \( v \sin(i) \) and \( \xi \) while using H\( \gamma \). Orange squares and orange dashed lines indicate the solution found when overestimating and underestimating \( T_{\text{eff}} \) from the left panel. Right Panel: Most probable microturbulence velocities as a function of \( T_{\text{eff}} \), after marginalizing over \( v \sin(i) \), for 3 different log(\( g \)) estimates. The full green, dashed blue, and dot-dashed red lines represent the possible values of log(\( g \)) found in the middle panel when overestimating and underestimating \( T_{\text{eff}} \). The thin orange dashed lines indicate overestimated and underestimated \( T_{\text{eff}} \) from the left panel.

induce a smaller effective temperature discrepancy with our method.

Results from the other authors in our comparison list are in a reasonably good agreement with our results even though a wide variety of techniques and methods were used. Takeda et al. (2010), Lyubimkov et al. (2005), and Andrievsky et al. (1999) used the \( uvby \) photometric system to derive \( T_{\text{eff}} \) and log(\( g \)). Our results are quite consistent with theirs. Daflon et al. (2007) derived \( T_{\text{eff}} \) using the UVB photometric system and obtained log(\( g \)) by adjusting the observed H\( \gamma \) line wings with models from ATLAS9. Note that we do not observe the same discrepancies in log(\( g \)) with Daflon et al. (2007), as seen between our results and those of HG10, since on the one hand, Daflon et al. (2007) used \( T_{\text{eff}} \) values close to ours, and on the other hand, they did not fit the core of the H\( \gamma \) line where non-LTE effects are stronger. Markova & Puls (2008) used a similar approach than Searle et al. (2008): models including stellar wind, log(\( g \)) from Balmer lines, and \( T_{\text{eff}} \) from Si lines. Our results agree well with theirs since the stars we have in common have weak stellar winds and \( T_{\text{eff}} \) below or around 18000 K. Finally, Lefever et al. (2010) used a semi-automated iterative method which is close to our simultaneous approach because groups of parameters are derived using a large number of lines simultaneously. Our respective results are in a reasonable agreement considering that Lefever et al. (2010) used models including stellar wind (FASTWIND) and an iterative method.

4.2.3 Projected Rotational Velocity

A substantial difference in Figure 13 between our results and those published in the literature, concerns \( v \sin(i) \). The most important discrepancy is found for 4 cluster stars that are also in the sample of HG10 (as mentioned earlier, they also used TLUSTY to derive \( v \sin(i) \)). We indeed derive higher \( v \sin(i) \) values (around 300 km s\(^{-1} \)) by a factor of

Figure 18. Comparison of the best fits for the lines He I \( \lambda 4471 \) and Mg II \( \lambda 4481 \) for 2 stars with large rotational velocities. Individual data points with error bars represent our observations. In full red: models based on our best set of parameters. In dashed green: models using the parameters of HG10. The \( v \sin(i) \) values are: 288 km s\(^{-1} \) (red) and 77 km s\(^{-1} \) (green) for BD+34 1103; and 315 km s\(^{-1} \) (red) and 199 km s\(^{-1} \) (green) for SAO58329. The respective complete sets of parameters are listed in Table B2.
Figure 19. Comparison of the fits for 4 helium lines obtained with our best set of parameters and those from Lyubimkov et al. (2005), for HD184171. Individual data points with error bars represent our observations. In full red: our best model with $T_{\text{eff}} = 16890$ K, $\log(g) = 3.89$, $\xi = 3$ km s$^{-1}$, and $v \sin(i) = 3$ km s$^{-1}$. In dotted green: model using the parameters of Lyubimkov et al. (2005): $T_{\text{eff}} = 16100$ K, $\log(g) = 3.62$, $\xi = 0$ km s$^{-1}$, and $v \sin(i) = 29$ km s$^{-1}$.

Interestingly, while the differences between our results for $v \sin(i)$ and those of HG10 and HG06 are generally greater for cluster stars than for field stars, and even though our sample is not large, we still find the same conclusion as HG06, i.e. cluster stars are globally faster rotators than field stars. From our results, cluster stars are even faster rotators.

Another issue concerning $v \sin(i)$ in Figure 13 or 14 appears at relatively low rotational velocities ($v \sin(i) \leq 100$ km s$^{-1}$). In this case, there is a rather large scatter between our results and those from other authors. Some of the largest differences can either be explained by our limited range in temperature (for example, Takeda et al. 2010 found $v \sin(i) = 15$ km s$^{-1}$ for HD185330, while we find $v \sin(i) = 115$ km s$^{-1}$ because the effective temperature is most certainly below 15000 K), or by the parameters and spectral lines considered. For instance, the $v \sin(i)$ values of Lyubimkov et al. (2005) are mostly between 15 and 50 km s$^{-1}$, while ours are between 0 and 10 km s$^{-1}$ with an uncertainty around 10 km s$^{-1}$ for the same stars. Lyubimkov et al. (2005) derived their velocities using fixed values of $T_{\text{eff}}$ and $\log(g)$, free values of $v \sin(i)$, $\xi$, and helium abundances, and considered 6 helium lines, while we use solar helium abundance, free values of $T_{\text{eff}}$, $\log(g)$, $v \sin(i)$, and $\xi$, and more spectral lines. While helium abundance has an obvious impact on the shape of helium lines, relatively small changes in $T_{\text{eff}}$, $\log(g)$, and $\xi$ also have an influence on the resulting $v \sin(i)$ value. For instance, when consider-

at least 1.5 for BD+34 1103s, NGC1960 SAB20, NGC1960 SAB30, and SAO58239. Of these stars, SAB20 and SAB30 have effective temperatures below our limit of 15000 K and the lowest SNR (around 200 and 75 respectively) objects of our sample. Consequently, the best fits for these stars are among the worst in our sample (Fig. 12) even when using ATLAS9 models, we thus consider that the associated results are not reliable. On the contrary, the other two stars, BD+34 1103s and SAO58239, have excellent SNR and rms (see Fig. 11 for SAO58239). Here, the differences in $v \sin(i)$ cannot be explained simply either in terms of the number or the choice of lines, or in terms of the atmospheric models used, since HG10 also used TLUSTY and SYNSPEC to determine $v \sin(i)$ and we could not reproduce their results using the same lines. One plausible explanation would be the difference in the treatment of $v \sin(i)$ (HG10 used a grid of models based on a three-dimensional parameterization of $T_{\text{eff}}$, $\log(g)$, and the cosine of the angle between the surface normal and the line of sight) but, in this case, there should be a systematic effect between our derived values and theirs for all the cluster stars that we have in common, which we do not observe. Moreover, when we reproduce the lines He I λ4481 and Mg II λ4481, used by HG10 to derive $v \sin(i)$, we clearly obtain an excellent fit of our own data with our parameters (Fig. 18). We are thus confident of our derived parameters for these two stars even though we have no clear explanation for these discrepancies.
Lyubimkov et al. (2005) found a solar helium abundance, \( T_{\text{eff}} = 16100 \) K and \( \log g = 3.62 \), \( v \sin(i) = 29 \) km s\(^{-1}\), and \( \xi = 0 \) km s\(^{-1}\), while we obtain \( T_{\text{eff}} = 16890 \) K, \( \log g = 3.89 \), \( v \sin(i) = 3 \) km s\(^{-1}\), and \( \xi = 3 \) km s\(^{-1}\) for a solar helium abundance. Figure 19 shows the models for these 2 sets of parameters for 4 helium lines (that are among the 6 helium lines used by Lyubimkov et al. [2005]). From this figure, it is clear that, with our set of parameters, a higher value of \( v \sin(i) \) would not improve the fits. Note also that our parameters are derived from the analysis of more than these 4 helium lines (27 in this case), and as such are more constrained, thus explaining the small uncertainties. Indeed, if we perform an analysis using only these 4 helium lines, with \( T_{\text{eff}}, \log(g), \) and \( \xi \) fixed to their best values, the uncertainty for \( v \sin(i) \) goes roughly from \( \pm 10 \) to \( \pm 20 \) thus reducing the discrepancy between our results and those of Lyubimkov et al. (2005).

There is also an interesting point concerning low rotational velocities and their respective uncertainties. Indeed, HG10 noted that depending on the resolution of a given spectrum, and for \( v \sin(i) \) less than half the FWHM of the instrumental broadening function, the instrumental broadening dominates the rotational broadening, implying uncertainties of the order of \( \pm \)FWHM/2 for low \( v \sin(i) \) values. In the work of HG10, FWHM \( \approx 0.8 \) and \( 1.5 \) Å resulting in uncertainties of 29 and 48 km s\(^{-1}\), respectively, for \( v \sin(i) \) below these values. Our data have a FWHM of 2.3 Å, and thus the theoretical limit is around 80 km s\(^{-1}\). But, as our method cumulates the constraints of a large number of lines, it allows us to significantly reduce this limit. Figure 20 shows, for each star, the maximum uncertainty of \( v \sin(i) \) as a function of \( v \sin(i) \). Here, we see that the method generally and naturally returns larger uncertainties for \( v \sin(i) \) \( \approx 80 \) km s\(^{-1}\) than for higher values, but with a maximum uncertainty (for low \( v \sin(i) \)) of roughly 30 km s\(^{-1}\) well below the limit of 80 km s\(^{-1}\).

As a further demonstration, we apply our method on higher spectral resolution data of two stars in our sample, obtained with a different grating at the Observatoire du Mont-Mégantic (in February 2011). These two stars are HD77770 and HD44700, whose spectra have an effective resolution of 0.9 Å and a wavelength range spanning from 3600 to 4400 Å. The results for \( T_{\text{eff}}, \log(g), \) and \( \xi \) obtained for these two stars are the same (within the uncertainties) at moderate and high resolution. But for \( v \sin(i) \), HD77770 gives \( 0^{+13}_{-0} \) km s\(^{-1}\) at high resolution, rather than \( 14^{+4}_{-1} \) km s\(^{-1}\) at moderate resolution, and HD44700 gives \( v \sin(i) = 4^{+10}_{-4} \) km s\(^{-1}\), at high resolution, rather than \( 28^{+15}_{-22} \) km s\(^{-1}\) at moderate resolution.

### 4.2.4 Microturbulence Velocity

Finally, one of the most obvious difference in Figure 13 or 14 lies with the microturbulence velocity but it is simply explained by the fact that, in most works, the microturbulence velocity is not a free parameter. Usually \( \xi \) is fixed at about 3 km s\(^{-1}\) and 10 km s\(^{-1}\) for main-sequence stars and for evolved stars, respectively (HG10; Takeda et al. [2010]).

And, when the microturbulence velocity is not fixed, it is estimated with only one or two lines in the spectrum (Lyubimkov et al. [2005]) or with only one species (Lefever et al. [2010]). When considering all the available lines to fit the microturbulence velocity simultaneously with the other parameters, we find a higher estimate than what is generally used. We obtain a mean microturbulence velocity of \( 6.1 \) km s\(^{-1}\) with a standard deviation of \( 2.6 \) km s\(^{-1}\) for the main-sequence stars, and a mean value of \( 21.4 \) km s\(^{-1}\) with a standard deviation of \( 7.6 \) km s\(^{-1}\) for the evolved stars. Note that our results are given for solar abundances, but even though we find slightly lower values of microturbulence velocity (by nearly 2 km s\(^{-1}\)) when we use the abundances derived by the various authors, our values are still higher than those from the literature.

Fixing or constraining the microturbulence velocity with only a few lines can lead to large errors for the other parameter estimates as well. Figure 21 shows an example of the most probable pairs \( [T_{\text{eff}}, \log(g)] \) for \( \xi = 2, 6, 10, 14, \) and \( 18 \) km s\(^{-1}\) when calculating the likelihoods with the lines He I \( \lambda 4388, \) He I + Mg II around 4476 Å, Si II \( \lambda \lambda 4128,4130, \) and Si III \( \lambda \lambda 4552,4567,4574 \) of the artificial spectrum with the following specifications: \( T_{\text{eff}} = 21350 \) K, \( \log(g) = 3.8 \), \( \xi = 3 \) km s\(^{-1}\), and \( v \sin(i) = 15 \) km s\(^{-1}\). This shows that each line has a different sensitivity to the microturbulence velocity and, depending on the line or the group of lines chosen as a diagnostic, the result differs in \( T_{\text{eff}} \) and \( \log(g) \) but also in \( \xi \). For instance here, the most probable value of \( \xi \) given by the helium lines is 10 km s\(^{-1}\) (when marginalizing over all the other parameters) while the silicon lines give 2 km s\(^{-1}\). Thus, it is very important to consider \( \xi \) as a free parameter as is usually done with \( T_{\text{eff}}, \log(g), \) and \( v \sin(i), \) and to constrain these four parameters simultaneously with as many lines as possible. Also it is even more important to consider the microturbulence velocity as a free parameter when studying the abundance, since it has a significant impact on this parameter as well (Nieva & Przybilla [2010]).

### 5 CONCLUSIONS

We described here a spectral analysis method based on Bayesian statistics that simultaneously constrains four stellar parameters (effective temperature, surface gravity, pro-
the modest resolution (2.3 Å) of our data. Even though nearly all the lines are heavily blended due to the line profile, it is still able to give accurate results with small uncertainties. The method cumulates the constraints of all available lines, which is different from the global fits using a classical iterative method. One important point is that our method considers all the parameters and all the elements (an iterative method versus our simultaneous method) and can lead to substantial errors on the other parameters. Also, our method is self-consistent and efficient even when it is applied to spectra with high noise level or when the number and choice of the diagnostic lines used (few lines for specific parameters versus, as we did, all available lines with the same weight for all the parameters). One important behaviour is pointed out in this study: the microturbulence velocity is often underestimated for the B dwarfs as well as for the giants and supergiants. Furthermore, we also confirmed that cluster B stars are on average faster rotators than field B stars.

Now that we have demonstrated that we can successfully apply our method to gather stellar parameters using solar abundances, we will investigate in a future paper the impact and efficiency of our method for the abundance determination of various chemical elements.

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Figure 21. The most probable pair \(T_{\text{eff}}, \log(g)\) as a function of \(\xi\) for different lines (as indicated in the plot) for the same artificial spectrum as in Figure 10. The behaviour as a function of \(\xi\) is shown in the plane \(T_{\text{eff}}, \log(g)\). The likelihoods are marginalized over all \(v \sin(i)\) values.
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Figure A1. Illustration of the commutativity of the analysis. The lines from Fig 2 are used here with a similar grid of synthetic spectra where all the parameters other than $T_{\text{eff}}$ and $\xi$ have been marginalized. The green dash-dotted frame on the left shows each line considered with its respective likelihood, while the blue dashed frames holds the posterior probability distribution obtained when considering several possible combinations of the likelihoods. From the top to the bottom, each column of the blue frame shows the evolution of the posterior probability distribution as new lines are added to the analysis. While the order in which the lines are added differs from column to column, the final posterior probability distribution (bottom row) remains the same in each case. Note that the products written in each graphic (such as $L1*L2*L3$) are simplified for clarity. In fact, each posterior distribution is the normalised product of the likelihoods as stated in equation 9.
## APPENDIX B: TABLES OF STELLAR PARAMETERS

| Star Id. | $T_{\text{eff}}$ | log($g$) | $v\sin(i)$ | $\xi$ | ref |
|----------|-----------------|---------|------------|------|-----|
|          | (K)             |         | (km s$^{-1}$) |      |     |
| HD886    | 21250 ± 130     | 3.98 ± 0.04 | 31$^{+9}_{-11}$ | 8 ± 1 | TW  |
| 21600 ± 1000 | 3.7 ± 0.3 | 8 ± 2 | 3.3       |    |     |
| 23000 ± 1000 | 3.8 ± 0.1 | 8 ± 1 | 1$^{+1}_{-1}$ |    |     |
| 21676 ± 650 | 3.8 ± 0.2 | 0 ± 9 | 3 ± 2     |    |     |
| HD3827   | 29440$^{+170}_{-240}$ | 4.27 ± 0.04 | 155 ± 5 | 6 ± 1 | TW  |
| 24678 ± 450 | 3.836 ± 0.038 | 164 ± 10 | 2     |    |     |
| HD5882   | 18250 ± 130     | 4.27 ± 0.04 | 330$^{+5}_{-10}$ | 7$^{+1}_{-2}$ | TW  |
| 15626 ± 150 | 3.731 ± 0.021 | 358 ± 5 | 2     |    |     |
| HD32249  | 20000 ± 130     | 4.26 ± 0.13 | 90 ± 10 | 5 ± 1 | TW  |
| 17081 ± 150 | 3.859 ± 0.025 | 60 ± 12 | 2     |    |     |
| 18500 ± 300 | 3.9 ± 0.1 | 46 | 0.5$^{+0.5}_{-0.5}$ |    |     |
| 18500 ± 1000 | 4. ± 0.3 | 48 ± 8 | 4     |    |     |
| 18890 ± 567 | 4.1 ± 0.2 | 30 ± 9 | 3 ± 2     |    |     |
| HD32672  | 20950$^{+130}_{-150}$ | 3.93 ± 0.04 | 37 ± 13 | 4$^{+1}_{-2}$ | TW  |
| 18089 ± 300 | 3.434 ± 0.034 | 32 ± 14 | 2     |    |     |
| HD33816  | 22430$^{+320}_{-240}$ | 3.94 ± 0.06 | 8$^{+12}_{-8}$ | 7 ± 1 | TW  |
| 21613 ± 500 | 3.818 ± 0.049 | 24 ± 24 | 2     |    |     |
| HD35912  | 19400$^{+230}_{-100}$ | 4.23$^{+0.05}_{-0.04}$ | 15$^{+17}_{-15}$ | 1$^{+2}_{-1}$ | TW  |
| 19000 ± 1000 | 4.1 ± 0.3 | 22 ± 4 | 4     |    |     |
| HD36280  | 28000 ± 130     | 4.02 ± 0.04 | 203$^{+7}_{-8}$ | 9$^{+2}_{-1}$ | TW  |
| 23611 ± 600 | 3.613 ± 0.053 | 206 ± 9 | 2     |    |     |
| HD37032  | 30000 ± 4$^{+250}_{-250}$ | 4.07 ± 0.04 | 63 ± 15 | 10$^{+1}_{-2}$ | TW  |
| 22499 ± 300 | 3.442 ± 0.028 | 70 ± 14 | 2     |    |     |
| HD40160  | 20010$^{+290}_{-240}$ | 4.05$^{+0.07}_{-0.05}$ | 128$^{+7}_{-8}$ | 5$^{+1}_{-2}$ | TW  |
| 16433 ± 250 | 3.735 ± 0.027 | 115 ± 13 | 2     |    |     |
| HD42782  | 15000$^{+250}_{-20}$ | 4.02 ± 0.04 | 60$^{+27}_{-21}$ | 0$^{+2}_{-1}$ | TW  |
| 15133 ± 150 | 3.790 ± 0.023 | 44 ± 21 | 2     |    |     |
| HD44700  | 17380 ± 130     | 4.24 ± 0.04 | 28$^{+18}_{-12}$ | 5 ± 1 | TW  |
| 17520$^{+130}_{-160}$ | 4.24 ± 0.13 | 4$^{+10}_{-4}$ | 6 ± 1 |    |     |
| 15190 ± 150 | 3.907 ± 0.026 | 26 ± 26 | 2     |    |     |
| 17500 ± 500 | 3.8 ± 0.1 | 8 ± 1 | 6 ± 2     |    |     |
| 16551 ± 497 | 4.2 ± 0.2 | 0 ± 9 | 3 ± 2     |    |     |
| HD45418  | 16750 ± 130     | 4.27 ± 0.04 | 247$^{+5}_{-7}$ | 6 ± 1 | TW  |
| 16000 ± 500 | 4. ± 0.2 | 237 ± 28 | 3 ± 2     |    |     |
| HD77770  | 19250 ± 130     | 3.77 ± 0.04 | 14$^{+6}_{-13}$ | 7 ± 1 | TW  |
| 19000 ± 130 | 3.53 ± 0.04 | 0$^{+1}_{-0}$ | 4$^{+1}_{-2}$ |    |     |
| 16935 ± 200 | 3.386 ± 0.023 | 49 ± 49 | 2     |    |     |
| HD160762 | 17050 ± 200     | 3.83 ± 0.03 | 3$^{+1}_{-3}$ | 8 ± 1 | TW  |
| 17000 ± 250 | 3.77 ± 0.12 | 15 | 1.3$^{+1}_{-1.3}$ |    |     |
| 19500 ± 500 | 4.1 ± 0.1 | 8 ± 2 | 3 ± 2     |    |     |
| 17440 ± 523 | 3.9 ± 0.2 | 0 ± 9 | 3 ± 2     |    |     |
| HD166182 | 19130$^{+120}_{-130}$ | 3.54 ± 0.04 | 0$^{+10}_{-9}$ | 10 ± 1 | TW  |
| 20000 ± 600 | 3.54 ± 0.16 | 44 | 4.7$^{+4}_{-4.7}$ |    |     |
| HD184171 | 16980$^{+150}_{-230}$ | 3.89$^{+0.05}_{-0.07}$ | 3$^{+7}_{-3}$ | 3 ± 1 | TW  |
| 16100 ± 700 | 3.62 ± 0.15 | 29 | 0 ± 0     |    |     |
| 15858 ± 476 | 3.5 ± 0.2 | 15 ± 9 | 3 ± 2     |    |     |

Continued on next page
## Table B1 – continued from previous page

| Star Id. | T_{eff} | log(g) | v sin(i) | ξ | ref |
|----------|---------|--------|---------|-----|-----|
| HD185330 | 15000 $^{+250}_{-200}$ | 3.70 ± 0.04 | $115^{+11}_{-10}$ | 1 ± 1 | TW |
|          | 16167 ± 485 | 3.7 ± 0.2 | 15 ± 9 | 3 ± 2 | TAK10 |
| HD188892 | 15120 $^{+130}_{-120}$ | 3.57 ± 0.04 | $3^{+7}_{-3}$ | 4 ± 1 | TW |
|          | 14080 ± 420 | 3.3 ± 0.2 | 30 ± 9 | 3 ± 2 | TAK10 |
| HD189944 | 15500 ± 130 | 4.02 ± 0.04 | $0^{+10}_{-9}$ | 6 ± 1 | TW |
|          | 14793 ± 444 | 4.0 ± 0.2 | 10 ± 9 | 3 ± 2 | TAK10 |
| HD190066 | 23560 $^{+190}_{-180}$ | 2.90 ± 0.04 | $99^{+6}_{-9}$ | 25 ± 2 | TW |
|          | 21000 ± 1000 | 2.88 ± 0.25 | 82 ± 20 | 15 | SEA08 |
| HD191243 | 15130 $^{+120}_{-130}$ | 2.73 ± 0.04 | $3^{+7}_{-3}$ | 11 ± 1 | TW |
|          | 13923 ± 418 | 2.5 ± 0.2 | 30 ± 9 | 3 ± 2 | TAK10 |
| HD190066 | 23560 $^{+190}_{-180}$ | 2.90 ± 0.04 | $99^{+6}_{-9}$ | 25 ± 2 | TW |
|          | 21000 ± 1000 | 2.88 ± 0.25 | 82 ± 20 | 15 | SEA08 |
| HD191243 | 15130 $^{+120}_{-130}$ | 2.73 ± 0.04 | $3^{+7}_{-3}$ | 11 ± 1 | TW |
|          | 13923 ± 418 | 2.5 ± 0.2 | 30 ± 9 | 3 ± 2 | TAK10 |
| HD190066 | 23560 $^{+190}_{-180}$ | 2.90 ± 0.04 | $99^{+6}_{-9}$ | 25 ± 2 | TW |
|          | 21000 ± 1000 | 2.88 ± 0.25 | 82 ± 20 | 15 | SEA08 |
Table B1 – continued from previous page

| Star Id. | $T_{\text{eff}}$ (K) | log($g$) | $\nu \sin(i)$ (km s$^{-1}$) | $\xi$ (km s$^{-1}$) | ref |
|----------|----------------------|----------|-------------------------------|-------------------|-----|
| HD215371 | 20380$^{+120}_{-130}$ | 4.27 ± 0.04 | 165 ± 5 | 8 ± 1 | TW |
| 17688 ± 250 | 3.806 ± 0.024 | 138 ± 10 | 2 | HG10 |

1: Spectrum with an effective resolution of 0.9 Å.

References: TW = This Work using models from TLUSTY with the solar composition from Asplund et al. (2009); HG10 = Huang, Gies & McSwain (2010); LEF10 = Lefever et al. (2010); TAK10 = Takeda et al. (2010); SEA08 = Searle et al. (2008); MAR08 = Markova & Puls (2008); DAF07 = Daflon et al. (2007); HG06 = Huang & Gies (2006a,b); LYU05 = Lyubimkov et al. (2005); and AND99 = Andrievsky et al. (1999).
| Cluster Id. | Star Id. | \(T_{\text{eff}}\) (K) | \(\log(g)\) | \(v\sin(i)\) (\(\text{km s}^{-1}\)) | \(\xi\) ref (\(\text{km s}^{-1}\)) | ref |
|------------|----------|----------------|-----------|-----------------|----------------|-------|
| NGC1960 BD+34 1103s | 19660 \(+220\)^{+160}_{-100} | 4.30 ± 0.04 | 288 \(+9\)^{+6}_{-16} | 4 \(+1\)^{+2}_{-2} | TW |
| SAB20 | 15000 \(+250\)^{+100}_{-0} | 4.20 \(+0.04\)^{+0.08}_{-0.08} | 301 \(+11\)^{+9}_{-9} | 6 \(+3\)^{+2}_{-2} | TW |
| SAB22 | 15000 \(+130\)^{+100}_{-0} | 4.35 ± 0.04 | 118 \(+7\)^{+8}_{-8} | 3 \(+2\)^{+2}_{-2} | TW |
| SAB23 | 15000 \(+250\)^{+100}_{-0} | 4.5 \(+0.06\) | 300 ± 20 | 0 \(+4\)^{+4}_{-4} | TW |
| SAB30 | 15240 \(+760\)^{+240}_{-240} | 4.34 \(+0.16\)^{+0.28}_{-0.28} | 351 \(+43\)^{+45}_{-45} | 13 \(+8\)^{+13}_{-13} | TW |
| SAO58227 | SAB20 15000 \(+250\)^{+100}_{-0} | 4.07 ± 0.03 | 79 ± 24 | 2 | HG10 |
| SAO58230 | 20830 \(+140\)^{+240}_{-240} | 4.09 ± 0.04 | 95 \(+10\)^{+9}_{-10} | 6 \(+1\)^{+1}_{-1} | TW |
| SAO58239 | 20600 \(+160\)^{+130}_{-130} | 4.06 ± 0.04 | 315 ± 5 | 5 \(+1\)^{+1}_{-1} | TW |
| NGC884 BD+56 556 | 25130 \(+140\)^{+250}_{-250} | 3.97 \(+0.04\)^{+0.08}_{-0.08} | 140 ± 5 | 10 \(+1\)^{+1}_{-1} | TW |
| BD+56 604 | 23250 ± 130 | 3.83 ± 0.04 | 136 \(+9\)^{+9}_{-9} | 4 \(+1\)^{+1}_{-1} | TW |
| HD14321 | 23972 ± 500 | 3.999 ± 0.048 | 97 ± 10 | 2 | HG10 |
| HD14476 | 24630 \(+120\)^{+130}_{-130} | 3.45 ± 0.04 | 105 ± 5 | 13 \(+1\)^{+1}_{-1} | TW |
| HG2048 | 19720 \(+110\)^{+120}_{-120} | 4.31 ± 0.04 | 203 \(+7\)^{+7}_{-7} | 6 \(+1\)^{+2}_{-2} | TW |
| HG2255 | 21670 \(+240\)^{+140}_{-140} | 3.83 ± 0.04 | 329 \(+6\)^{+6}_{-6} | 6 \(+1\)^{+1}_{-1} | TW |
| HG2794 | 22510 ± 260 | 3.78 \(+0.07\)^{+0.07}_{-0.05} | 140 ± 10 | 8 \(+1\)^{+1}_{-1} | TW |
| W168 | 21370 \(+130\)^{+120}_{-120} | 4.27 ± 0.04 | 29 \(+10\)^{+10}_{-10} | 5 \(+1\)^{+1}_{-1} | TW |
| 19259 ± 350 | 3.98 ± 0.04 | 44 ± 14 | 2 | HG10 |

References: TW = This Work using models from TLUSTY with the solar composition from Asplund et al. (2009); HG10 = Huang, Gies & McSwain (2010); and HG06 = Huang & Gies (2006a,b).
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Table B3: Basic Parameters of Field B Stars with Modified He and Si Abundances from [Lefever et al. (2010)]

| Star Id. | $T_{\text{eff}}$ (K) | log($g$) | $v \sin(i)$ (km s$^{-1}$) | $\xi$ (km s$^{-1}$) | log($^{12}\text{He}/^{1}\text{H}$) + 12 | log($^{12}\text{Si}/^{1}\text{H}$) + 12 | ref |
|----------|-----------------------|----------|-----------------------------|----------------------|-------------------------------------|-------------------------------------|-----|
| HD160762 | 18090 $^{+130}_{-120}$ | 3.97 ± 0.04 | $3^{+7}_{-3}$ | 9 ± 1 | 11 | 7.14 | TW |
|          | 19500 ± 500           | 4.1 ± 0.1 | $8^{+2}_{-3}$ | 3 ± 2 | | | LEF10 |
| HD44700  | 17250 ± 130           | 4.22 ± 0.04 | $8^{+16}_{-8}$ | 6 ± 1 | 11 | 7.07 | TW |
|          | 17500 ± 500           | 3.8 ± 0.1 | $8^{+2}_{-3}$ | 6 ± 2 | | | LEF10 |
| HD45418  | 16750 ± 130           | 4.27 ± 0.04 | 247 $^{+8}_{-7}$ | 6 ± 1 | 11 | 7.51 | TW |
|          | 16000 ± 500           | 4. ± 0.2 | 237 ± 28 | 3 ± 2 | | | LEF10 |
| HD886    | 21120 $^{+140}_{-240}$ | 3.95 ± 0.04 | 34 ± 8 | 9 ± 1 | 11 | 7.16 | TW |
|          | 23000 ± 1000          | 3.8 ± 0.1 | 10 ± 1 | $1^{+2}_{-1}$ | | | LEF10 |

References: TW = This Work; LEF10 = [Lefever et al. (2010)].
## Table B4: Basic Parameters of Field B Stars with Modified O and Ne Abundances from \[\text{Takeda et al. (2010)}\]

| Star Id. | \(T_{\text{eff}}\) (K) | \(\log(g)\) | \(\nu \sin(i)\) (km s\(^{-1}\)) | \(\xi\) (km s\(^{-1}\)) | \(\log(O/H) + 12\) | \(\log(N_e/H) + 12\) | ref |
|----------|--------------------------|--------------|-------------------------------|-----------------|----------------|----------------|------|
| HD160762 | 16880 ± 130              | 3.81 ± 0.04  | \(3.12 \pm 0.1\)             | 8.74 ± 0.1      | 8.08           | TW            |      |
|          | 17440 ± 523              | 3.9 ± 0.2    | 0 ± 9                         | 3 ± 2           |                | TAK10        |      |
| HD184171 | 17000 ± 130              | 3.86 ± 0.04  | \(3.7 \pm 0.1\)              | 8.69 ± 0.1      | 8.01           | TW            |      |
|          | 15858 ± 476              | 3.5 ± 0.2    | 15 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD185330 | 15000 \(+250\) -0  \(+60\) | 3.7 ± 0.04   | 120 ± 12                     | 2.1 ± 2         | 8.35 ± 0.1     | TW            |      |
|          | 16167 ± 485              | 3.7 ± 0.2    | 15 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD188892 | 15120 \(+130\) -120 \(+30\) | 3.57 ± 0.04  | \(3.7 \pm 0.1\)             | 4 ± 1           | 8.67 ± 0.1     | TW            |      |
|          | 14008 ± 420              | 3.3 ± 0.2    | 30 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD189944 | 15500 ± 130              | 4.02 ± 0.04  | \(0.10 \pm 0.0\)            | 6 ± 1           | 8.79 ± 0.1     | TW            |      |
|          | 14793 ± 444              | 4.0 ± 0.2    | 10 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD191243 | 15230 \(+150\) -230 \(+30\) | 2.76 ± 0.05  | \(3.7 \pm 0.1\)             | 10.1 ± 2        | 8.66 ± 0.1     | TW            |      |
|          | 13923 ± 418              | 2.5 ± 0.2    | 30 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD196035 | 18350 \(+100\) -110 \(+30\) | 4.20 ± 0.03  | \(3.12 \pm 0.1\)            | 8 ± 2           | 8.7 ± 0.1      | TW            |      |
|          | 17499 ± 525              | 4.3 ± 0.2    | 20 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD198820 | 16750 \(+130\) -120 \(+30\) | 3.77 ± 0.04  | \(3.7 \pm 0.1\)             | 6 ± 1           | 8.8 ± 0.1      | TW            |      |
|          | 15852 ± 476              | 3.8 ± 0.2    | 15 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD199578 | 15000 \(+250\) -0  \(+60\) | 4.05 ± 0.04  | \(3.7 \pm 0.1\)             | 6 ± 2           | 8.7 ± 0.1      | TW            |      |
|          | 14480 ± 434              | 4.0 ± 0.2    | 30 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD206540 | 15000 \(+250\) -0  \(+60\) | 4.21 ± 0.07  | \(4.16 \pm 0.4\)            | 4 ± 1           | 8.72 ± 0.1     | 8 ± 0.1       | TW    |
|          | 13981 ± 419              | 4.0 ± 0.2    | 20 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD209419 | 15000 \(+130\) -0  \(+60\) | 3.95 ± 0.04  | \(4.10 \pm 0.1\)            | 5 ± 1           | 8.73 ± 0.1     | 7.99          | TW    |
|          | 14404 ± 432              | 3.8 ± 0.2    | 20 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD32249  | 19980 \(+130\) -120 \(+30\) | 4.26 ± 0.13  | 86 ± 8                        | 5 ± 1           | 8.69 ± 0.1     | 7.95          | TW    |
|          | 18890 ± 567              | 4.1 ± 0.2    | 30 ± 9                        | 3 ± 2           |                | TAK10        |      |
| HD44700  | 17370 \(+130\) -120 \(+30\) | 4.24 ± 0.04  | 29 ± 10                       | 5 ± 1           | 8.73 ± 0.1     | 8.04          | TW    |
|          | 16551 ± 497              | 4.2 ± 0.2    | 0 ± 9                         | 3 ± 2           |                | TAK10        |      |
| HD886    | 20930 \(+140\) -150 \(+30\) | 3.95 ± 0.04  | \(31.9 \pm 1.1\)            | 8 ± 1           | 8.77 ± 0.1     | 8.09          | TW    |
|          | 21667 ± 650              | 3.8 ± 0.2    | 0 ± 9                         | 3 ± 2           |                | TAK10        |      |

References: TW = This Work; TAK10 = \[\text{Takeda et al. (2010)}\].
**Table B5: Basic Parameters of Field B Stars with Modified C, N, O Abundances from Searle et al. (2008)**

| Star Id.  | $T_{	ext{eff}}$ (K) | log($g$) | $v$ sin($i$) (km s$^{-1}$) | ξ (km s$^{-1}$) | log($^{12}$C/$^{12}$O) + 12 | log($^{14}$N/$^{12}$O) + 12 | log($^{15}$N/$^{12}$O) + 12 | ref   |
|-----------|---------------------|----------|-----------------------------|----------------|-------------------------------|--------------------------------|--------------------------------|-------|
| HD190066  | 23750 ± 190 21000 ± 1000 | 2.90 ± 0.04 2.88 ± 0.25 | 95 ± 9 82 ± 20 | 25 ± 1 11 ± 1 | 7.88 7.70 | 8.15 8.65 | 8.53 8.73 | TW   |
| HD191243  | 15200 ± 130 14500 ± 1000 | 2.70 ± 0.04 2.75 ± 0.25 | 3 $^{+7}_{-3}$ 38 ± 20 | 11 ± 1 20 ± 20 | 7.70 7.65 | 7.51 8.73 | 8.37 TW | SEA08 |
| HD192660  | 25140 ± 160 30000 ± 1000 | 2.88 ± 0.03 3.25 ± 0.25 | 140 ± 5 94 ± 20 | 29 $^{+1}_{-2}$ 20 | 8.02 7.66 | 8.15 8.73 | 8.45 TW | SEA08 |
| HD193183  | 19610 ± 240 18500 ± 1000 | 2.65 ± 0.04 2.63 ± 0.25 | 32 ± 12 68 ± 20 | 23 ± 1 20 ± 20 | 7.66 7.65 | 8.15 8.73 | 8.45 TW | SEA08 |
| HD198478  | 16130 ± 120 17500 ± 1000 | 2.04 ± 0.04 2.25 ± 0.25 | 0 $^{+10}_{-6}$ 61 ± 20 | 33 ± 1 20 ± 20 | 7.86 7.65 | 8.15 8.73 | 8.45 TW | SEA08 |
| HD206165  | 18630 ± 150 18000 ± 1000 | 2.55 ± 0.04 2.50 ± 0.25 | 75 ± 5 73 ± 20 | 23 ± 1 20 ± 20 | 7.96 7.65 | 8.15 8.73 | 8.45 TW | SEA08 |
| HD213087  | 24620 ± 130 27000 ± 1000 | 2.86 ± 0.04 3.13 ± 0.25 | 103 ± 8 88 ± 20 | 24 ± 1 20 ± 20 | 8.00 7.65 | 8.15 8.73 | 8.45 TW | SEA08 |

References: TW = This Work; SEA08 = Searle et al. (2008).

**Table B6: Basic Parameters of Field B Stars with Modified He and Si Abundances from Markova & Puls (2008)**

| Star Id.  | $T_{	ext{eff}}$ (K) | log($g$) | $v$ sin($i$) (km s$^{-1}$) | ξ (km s$^{-1}$) | log($^{14}$He/$^{16}$O) + 12 | log($^{28}$Si/$^{16}$O) + 12 | ref   |
|-----------|---------------------|----------|-----------------------------|----------------|-------------------------------|--------------------------------|-------|
| HD191243  | 15240 ± 140 14800 ± 1000 | 2.77 $^{+0.04}_{-0.08}$ 2.6 ± 0.2 | 3 $^{+7}_{-3}$ 38 ± 10 | 11 ± 1 8 ± 5 | 10.954 11.176 | 7.48 7.58 | TW   |
| HD198478  | 16270 ± 230 17500 ± 1000 | 2.08 $^{+0.08}_{-0.04}$ 2 $^{+28}_{-2}$ | 26 $^{+1}_{-1}$ 39 ± 10 | 8 ± 5 8 ± 5 | 11.176 11.176 | 7.58 7.58 | TW   |
| HD206165  | 19380 ± 120 19300 ± 1000 | 2.63 ± 0.04 2.5 ± 0.2 | 76 $^{+6}_{-6}$ 45 ± 10 | 16 $^{+2}_{-2}$ 8 ± 5 | 11.176 11.176 | 7.58 7.58 | TW   |

References: TW = This Work; MAR08 = Markova & Puls (2008).
Table B7: Basic Parameters of Field B Stars with Modified C, N, O, Mg, Al, and Si Abundances from Daflon et al. (2007)

| Star Id. | $T_{\text{eff}}$ (K) | log($g$) | $v$ sin($i$) (km s$^{-1}$) | $\xi$ (km s$^{-1}$) | $\epsilon$(C) | $\epsilon$(N) | $\epsilon$(O) | $\epsilon$(Mg) | $\epsilon$(Al) | $\epsilon$(Si) | ref |
|----------|-----------------|----------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----|
| hd202347 | 23730 $^{+240}_{-260}$ | 4.27 $^{+0.04}_{-0.08}$ | 96 $^{+9}_{-6}$ | 9 $^{+1}_{-1}$ | 7.54 | 8.54 | 7.35 | 6.28 | 7.13 | TW |
|          | 23280 $^{+260}_{-900}$ | 4.1 $^{+0.1}_{-0.1}$ | 119 $^{+6}_{-6}$ | 6 $^{+1}_{-1.5}$ |              |                |                |                |                |                | DAF07 |
| hd205794 | 25010 $^{+120}_{-140}$ | 3.95 $^{+0.04}_{-0.05}$ | 3 $^{+17}_{-3}$ | 11 $^{+1}_{-1}$ | 8.09 | 7.57 | 8.63 |              | 7.09 | TW |
|          | 26890 $^{+1000}_{-100}$ | 4.2 $^{+0.1}_{-0.1}$ | 14 $^{+3}_{-3}$ | 8 $^{+1.5}_{-1.5}$ |              |                |                |                |                | DAF07 |

References: TW = This Work; AND99 = Daflon et al. (2007). Here, $\epsilon$(X) = log($X$/H) + 12.

Table B8: Basic Parameters of Field B Stars with Modified He and Mg Abundances from Lyubimkov et al. (2005)

| Star Id. | $T_{\text{eff}}$ (K) | log($g$) | $v$ sin($i$) (km s$^{-1}$) | $\xi$ (km s$^{-1}$) | log($\frac{He}{H}$) + 12 | log($\frac{Mg}{H}$) + 12 | ref |
|----------|-----------------|----------|-----------------|-----------------|-----------------|-----------------|-----|
| HD160762 | 17280 $^{+120}_{-130}$ | 3.88 $^{+0.04}_{-0.04}$ | 3 $^{+7}_{-3}$ | 7 $^{+1}_{-1}$ | 11.057 | 7.62 | TW |
|          | 17000 $^{+250}_{-250}$ | 3.77 $^{+0.12}_{-0.12}$ | 15 $^{+1}_{-1}$ | 1.3 $^{+1}_{-1.3}$ |              |                | LUY05 |
| HD166182 | 19150 $^{+230}_{-150}$ | 3.55 $^{+0.07}_{-0.07}$ | 0 $^{+10}_{-0}$ | 9 $^{+1}_{-1}$ | 11.045 | 7.39 | TW |
|          | 20000 $^{+600}_{-600}$ | 3.54 $^{+0.16}_{-0.16}$ | 44 $^{+4.7}_{-4.7}$ |              |                |                | LUY05 |
| HD184171 | 16880 $^{+120}_{-130}$ | 3.85 $^{+0.04}_{-0.04}$ | 3 $^{+7}_{-3}$ | 2 $^{+1}_{-1}$ | 11.041 | 7.89 | TW |
|          | 16100 $^{+700}_{-700}$ | 3.62 $^{+0.15}_{-0.15}$ | 29 $^{+0}_{-0}$ |              |                |                | LUY05 |
| HD196035 | 18210 $^{+170}_{-210}$ | 4.18 $^{+0.04}_{-0.04}$ | 3 $^{+7}_{-3}$ | 6 $^{+2}_{-1}$ | 11.065 | 7.66 | TW |
|          | 17100 $^{+250}_{-250}$ | 4.0 $^{+0.08}_{-0.08}$ | 34 $^{+2}_{-2}$ |              |                |                | LUY05 |
| HD198820 | 16500 $^{+130}_{-130}$ | 3.73 $^{+0.04}_{-0.04}$ | 3 $^{+7}_{-3}$ | 3 $^{+1}_{-1}$ | 11.11 | 8.01 | TW |
|          | 15600 $^{+300}_{-300}$ | 3.65 $^{+0.15}_{-0.15}$ | 35 $^{+0.1}_{-0.1}$ |              |                |                | LUY05 |
| HD209419 | 15000 $^{+250}_{-0}$ | 3.95 $^{+0.04}_{-0.04}$ | 60 $^{+5}_{-5}$ | 4 $^{+1}_{-1}$ | 11.093 | 8.03 | TW |
|          | 14100 $^{+600}_{-600}$ | 3.70 $^{+0.12}_{-0.12}$ | 17 $^{+0}_{-0}$ |              |                |                | LUY05 |
| HD32249  | 18880 $^{+120}_{-120}$ | 4.11 $^{+0.04}_{-0.04}$ | 90 $^{+5}_{-5}$ | 2 $^{+1}_{-1}$ | 11.124 | 8.02 | TW |
|          | 18300 $^{+300}_{-300}$ | 3.9 $^{+0.1}_{-0.1}$ | 46 $^{+0.5}_{-0.5}$ |              |                |                | LUY05 |

References: TW = This Work; LUY05 = Lyubimkov et al. (2005).
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Table B9: Basic Parameters of Field B Stars with Modified C and N Abundances from Andrievsky et al. (1999)

| Star Id. | \( T_{\text{eff}} \) (K) | \( \log(g) \) | \( v \sin(i) \) (km s\(^{-1}\)) | \( \xi \) (km s\(^{-1}\)) | \( \log(C^N_{\text{eff}}) + 12 \) | \( \log(N^N_{\text{eff}}) + 12 \) | ref |
|----------|------------------|-------------|------------------|-------------|------------------|------------------|-----|
| HD32249  | 20000 ± 130      | 4.26 ± 0.13 | 86 ±11/7         | 5 ± 1       | 8.19             | 7.8              | TW  |
|          | 18500 ± 1000     | 4.1 ± 0.3   | 48 ± 8           | 4           |                  |                  | AND99 |
| HD35912  | 19610 ±140/240   | 4.27 ± 0.04 | 17 ±15/17        | 1 ±2        | 8.28             | 7.76             | TW  |
|          | 19000 ± 1000     | 4.1 ± 0.3   | 22 ± 4           | 4           |                  |                  | AND99 |
| HD886    | 21250 ± 130      | 3.97 ± 0.04 | 4 ±16/4          | 8 ± 1       | 8.11             | 7.55             | TW  |
|          | 21600 ± 1000     | 3.7 ± 0.3   | 8 ± 2            | 3.3         |                  |                  | AND99 |

References: TW = This Work; AND99 = Andrievsky et al. (1999).

Table B10: Basic Parameters of Field B Stars Returned by ATLAS9 with Solar Composition from Grevesse & Sauval (1998)

| Star Id. | \( T_{\text{eff}} \) (K) | \( \log(g) \) | \( v \sin(i) \) (km s\(^{-1}\)) | \( \xi \) (km s\(^{-1}\)) | ref |
|----------|------------------|-------------|------------------|-------------|-----|
| HD214263 | 19760 ±120/140   | 4.08 ± 0.04 | 88 ±7/8          | 1 ±2/1     | TW  |
|          | 18570 ± 350      | 3.87 ± 0.06 | 63 ± 2           |            | TWHG |
|          | 19027 ± 250      | 3.807 ± 0.025 | 63 ± 16     |            | HG10 |
| HD214432 | 18870 ±130/120   | 4.47 ± 0.04 | 132 ±8/7        | 5 ± 1       | TW  |
|          | 15560 ±350/560   | 3.95 ± 0.08 | 137 ± 2         |            | TWHG |
|          | 16124 ± 200      | 3.89 ± 0.028 | 137 ± 9        |            | HG10 |
| HD215191 | 20500 ±130/120   | 3.91 ± 0.04 | 200 ± 5         | 6 ± 1       | TW  |
|          | 20490 ± 360      | 3.88 ± 0.06 | 195 ± 2         |            | TWHG |
|          | 21487 ± 350      | 3.83 ± 0.03 | 195 ± 11        |            | HG10 |
| HD215371 | 19380 ±240/140   | 4.23 ± 0.04 | 160 ± 5         | 5 ± 1       | TW  |
|          | 18131 ± 450      | 3.99 ± 0.1  | 138 ± 2         |            | TWHG |
|          | 17688 ± 250      | 3.806 ± 0.024 | 138 ± 10     |            | HG10 |
| HD32249  | 18890 ± 130      | 4.25 ± 0.04 | 95 ± 5          | 4 ± 1       | TW  |
|          | 16490 ± 450      | 3.84 ± 0.11 | 60 ± 2          |            | TWHG |
|          | 17081 ± 150      | 3.859 ± 0.025 | 60 ± 12     |            | HG10 |
| HD32672  | 19770 ±200/250   | 3.87 ±0.07/0.05 | 18 ±2/18 | 1 ± 1       | TW  |
|          | 18290 ± 570      | 3.69 ± 0.08 | 32 ± 2          |            | TWHG |
|          | 18089 ± 300      | 3.43 ± 0.034 | 32 ± 14     |            | HG10 |
| HD33816  | 20780 ±130       | 3.85 ± 0.04 | 8 ±12/8         | 4 ± 1       | TW  |
|          | 21110 ± 1540     | 3.91 ± 0.06 | 24 ± 2          |            | TWHG |
|          | 21613 ± 500      | 3.818 ± 0.049 | 24 ± 24     |            | HG10 |
| HD36280  | 24330 ±160/170   | 3.91 ± 0.04 | 214 ± 6         | 8 ± 1       | TW  |
|          | 24020 ± 530      | 3.84 ± 0.1  | 206 ± 2         |            | TWHG |
|          | 23611 ± 600      | 3.613 ± 0.053 | 206 ± 9     |            | HG10 |
| HD37032  | 29540 ±240/290   | 4.28 ±0.07/0.05 | 43 ±19/11 | 5 ± 1       | TW  |
|          | 22110 ± 1300     | 3.56 ± 0.17 | 70 ± 2          |            | TWHG |
|          | 22499 ± 300      | 3.442 ± 0.028 | 70 ± 14     |            | HG10 |
| HD3827   | 29500 ± 500      | 4.43 ± 0.04 | 165 ± 5         | 6 ± 1       | TW  |
|          | 24100 ± 810      | 3.93 ± 0.14 | 164 ± 2         |            | TWHG |
|          | 24678 ± 450      | 3.836 ± 0.038 | 164 ± 10     |            | HG10 |

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Table B10 – continued from previous page

| Star Id. | $T_{\text{eff}}$ (K) | log(g) | $v$ sin(i) (km s$^{-1}$) | $\xi$ (km s$^{-1}$) | ref |
|----------|-----------------|--------|----------------|-----------------|-----|
| HD42782  | $13040^{+210}_{-170}$ | 3.72 $^{+0.07}_{-0.05}$ | $155^{+17}_{-15}$ | 0 $^{+2}_{-0}$ | TW |
|          | $14570 \pm 590$    | 3.9 $\pm 0.16$  | 44              | 2               | TWHG |
|          | $15133 \pm 150$    | 3.790 $\pm 0.023$ | 44 $\pm 21$    | 2               | HG10 |
| HD44700  | $16900^{+260}_{-150}$ | 4.23 $^{+0.08}_{-0.04}$ | 82 $^{+8}_{-7}$  | 3 $^{+2}_{-1}$  | TW |
|          | $15010 \pm 510$    | 3.91 $\pm 0.12$  | 26              | 2               | TWHG |
|          | $15190 \pm 150$    | 3.907 $\pm 0.026$ | 26 $\pm 26$    | 2               | HG10 |
| HD5882   | $17500 \pm 130$    | 4.27 $\pm 0.04$  | 328 $\pm 6$    | 6 $\pm 3$      | TW |
|          | $14760 \pm 300$    | 3.71 $\pm 0.09$  | 358             | 2               | TWHG |
|          | $15826 \pm 150$    | 3.731 $\pm 0.021$ | 358 $\pm 5$    | 2               | HG10 |
| HD77770  | $18500 \pm 130$    | 3.78 $\pm 0.04$  | 15 $^{+5}_{-10}$ | 0 $^{+4}_{-9}$  | TW |
|          | $15590 \pm 420$    | 3.31 $\pm 0.07$  | 49              | 2               | TWHG |
|          | $16935 \pm 200$    | 3.386 $\pm 0.023$ | 49 $\pm 49$    | 2               | HG10 |

References: TW = This Work; TWHG = This Work using H$\gamma$ only; HG10 = Huang, Gies & McSwain (2010). Numbers between parenthesis refer to the 'Star id n' in Figure 15.

Table B11: Stellar Parameters of Cluster B Stars Returned by ATLAS9 with Solar Composition from Grevesse & Sauval (1998)

| Cluster Id. | Star Id. | $T_{\text{eff}}$ (K) | log(g) | $v$ sin(i) (km s$^{-1}$) | $\xi$ (km s$^{-1}$) | ref |
|-------------|----------|-----------------|--------|----------------|-----------------|-----|
| NGC1960 BD341103s | 18870 $^{+130}_{-120}$ | 4.31 $\pm 0.04$ | 290 $^{+9}_{-6}$ | 2 $\pm 1$ | TW |
|             | 20030 $\pm 920$    | 4.40 $\pm 0.17$  | 77    | 2               | TWHG |
|             | 17297 $\pm 150$    | 3.983 $\pm 0.015$ | 77 $\pm 7$ | 2               | HG10 |
| SAB20      | 12970 $^{+150}_{-150}$ | 3.87 $^{+0.05}_{-0.07}$ | 274 $^{+14}_{-10}$ | 1 $^{+2}_{-1}$ | TW |
|             | 14560 $\pm 620$    | 4.25 $\pm 0.15$  | 141   | 2               | TWHG |
|             | 14845 $\pm 100$    | 4.242 $\pm 0.018$ | 141 $\pm 7$ | 2               | HG10 |
| SAB22      | 14500 $\pm 130$    | 4.33 $\pm 0.04$  | 126 $^{+10}_{-1}$ | 3 $\pm 1$ | TW |
|             | 13500 $\pm 210$    | 4.09 $\pm 0.06$  | 135   | 2               | TWHG |
|             | 13579 $\pm 100$    | 4.071 $\pm 0.019$ | 135 $\pm 6$ | 2               | HG10 |
| SAB23      | 12780 $^{+270}_{-130}$ | 4.07 $^{+0.13}_{-0.08}$ | 244 $^{+16}_{-14}$ | 4 $\pm 2$ | TW |
|             | 12910 $\pm 1220$   | 4.18 $\pm 0.3$   | 302   | 2               | TWHG |
|             | 13715 $\pm 150$    | 4.267 $\pm 0.028$ | 302 $\pm 11$ | 2               | HG10 |
| SAB30      | 11920 $^{+710}_{-820}$ | 4.89 $^{+0.11}_{-0.19}$ | 261 $^{+37}_{-50}$ | 12 $\pm 3$ | TW |
|             | 12160 $\pm 840$    | 4.88 $\pm 0.3$   | 79    | 2               | TWHG |
|             | 13651 $\pm 150$    | 4.07 $\pm 0.03$  | 79 $\pm 24$ | 2               | HG10 |
| SAO58227   | 16600 $^{+150}_{-230}$ | 4.52 $^{+0.05}_{-0.07}$ | 123 $^{+7}_{-11}$ | 5 $\pm 1$ | TW |
|             | 14790 $\pm 250$    | 4.19 $\pm 0.08$  | 72    | 2               | TWHG |
|             | 14625 $\pm 100$    | 4.031 $\pm 0.017$ | 72 $\pm 7$ | 2               | HG10 |
| SAO58230   | 20190 $^{+290}_{-180}$ | 4.11 $\pm 0.04$ | 98 $\pm 12$ | 2 $^{+2}_{-1}$ | TW |
|             | 19420 $\pm 630$    | 4.05 $\pm 0.1$   | 57    | 2               | TWHG |
|             | 19671 $\pm 300$    | 3.879 $\pm 0.026$ | 57 $\pm 11$ | 2               | HG10 |
| SAO58239   | 19120 $^{+130}_{-120}$ | 3.95 $\pm 0.04$ | 315 $\pm 5$ | 2 $\pm 1$ | TW |
|             | 21000 $\pm 560$    | 4.15 $\pm 0.08$  | 199   | 2               | TWHG |
|             | 19582 $\pm 200$    | 3.911 $\pm 0.018$ | 199 $\pm 7$ | 2               | HG10 |
Table B11 – continued from previous page

| Cluster Id. | Star Id. | \( T_{\text{eff}} \) (K) | \( \log(g) \) | \( v \sin(i) \) (km s\(^{-1}\)) | \( \xi \) (km s\(^{-1}\)) | ref |
|------------|----------|-----------------|-----------|----------------|----------------|----|
| NGC884     | BD56556  | \( 24390^{+230}_{-140} \) | \( 4.25 \pm 0.25 \) | \( 4.25 \pm 0.25 \) | \( 4.25 \pm 0.25 \) | TW |
|            | (1)      | \( 25000 \pm 2300 \)   | \( 145 \pm 5 \)   | \( 145 \pm 5 \)   | \( 145 \pm 5 \)   | TWHG |
|            |          | \( 25937 \pm 401 \)    | \( 69 \pm 9 \)    | \( 69 \pm 9 \)    | \( 69 \pm 9 \)    | HG06 |
| BD56604    |          | \( 22470^{+120}_{-140} \) | \( 3.86 \pm 0.04 \) | \( 3.86 \pm 0.04 \) | \( 3.86 \pm 0.04 \) | TW |
|            | (2)      | \( 24960 \pm 880 \)    | \( 150 \pm 5 \)   | \( 150 \pm 5 \)   | \( 150 \pm 5 \)   | TWHG |
|            |          | \( 23972 \pm 500 \)    | \( 97 \pm 2 \)    | \( 97 \pm 2 \)    | \( 97 \pm 2 \)    | HG10 |
| HD14321    |          | \( 21750 \pm 130 \)    | \( 150 \pm 5 \)   | \( 150 \pm 5 \)   | \( 150 \pm 5 \)   | TW |
|            | (3)      | \( 22470 \pm 1030 \)   | \( 106 \pm 2 \)   | \( 106 \pm 2 \)   | \( 106 \pm 2 \)   | TWHG |
|            |          | \( 23601 \pm 550 \)    | \( 106 \pm 10 \)  | \( 106 \pm 10 \)  | \( 106 \pm 10 \)  | HG10 |
| HD14476    |          | \( 25400^{+230}_{-150} \) | \( 3.73 \pm 0.04 \) | \( 3.73 \pm 0.04 \) | \( 3.73 \pm 0.04 \) | TW |
|            | (4)      | \( 26870 \pm 1620 \)   | \( 40 \pm 2 \)    | \( 40 \pm 2 \)    | \( 40 \pm 2 \)    | TWHG |
|            |          | \( 26006 \pm 800 \)    | \( 14 \pm 1 \)    | \( 14 \pm 1 \)    | \( 14 \pm 1 \)    | HG10 |
| HG2048     |          | \( 18870 \pm 130 \)    | \( 199 \pm 5 \)   | \( 199 \pm 5 \)   | \( 199 \pm 5 \)   | TW |
|            | (5)      | \( 19530 \pm 510 \)    | \( 125 \pm 2 \)   | \( 125 \pm 2 \)   | \( 125 \pm 2 \)   | TWHG |
|            |          | \( 18007 \pm 198 \)    | \( 125 \pm 7 \)   | \( 125 \pm 7 \)   | \( 125 \pm 7 \)   | HG06 |
| HG2255     |          | \( 19670 \pm 170 \)    | \( 329 \pm 6 \)   | \( 329 \pm 6 \)   | \( 329 \pm 6 \)   | TW |
|            | (6)      | \( 21220 \pm 760 \)    | \( 318 \pm 2 \)   | \( 318 \pm 2 \)   | \( 318 \pm 2 \)   | TWHG |
|            |          | \( 20360 \pm 178 \)    | \( 318 \pm 10 \)  | \( 318 \pm 10 \)  | \( 318 \pm 10 \)  | HG06 |
| HG2794     |          | \( 21500 \pm 130 \)    | \( 148 \pm 7 \)   | \( 148 \pm 7 \)   | \( 148 \pm 7 \)   | TW |
|            | (7)      | \( 22410 \pm 1060 \)   | \( 79 \pm 2 \)    | \( 79 \pm 2 \)    | \( 79 \pm 2 \)    | TWHG |
|            |          | \( 21624 \pm 291 \)    | \( 79 \pm 8 \)    | \( 79 \pm 8 \)    | \( 79 \pm 8 \)    | HG06 |
| W168       |          | \( 19790^{+120}_{-130} \) | \( 41 \pm 11 \)  | \( 41 \pm 11 \)  | \( 41 \pm 11 \)  | TW |
|            | (15)     | \( 18870 \pm 680 \)    | \( 44 \pm 2 \)    | \( 44 \pm 2 \)    | \( 44 \pm 2 \)    | TWHG |
|            |          | \( 19259 \pm 350 \)    | \( 44 \pm 14 \)   | \( 44 \pm 14 \)   | \( 44 \pm 14 \)   | HG10 |

References: TW = This Work; TWHG = This Work using H\(_\gamma\) only; HG10 = Huang, Gies & McSwain (2010); and HG06 = Huang & Gies (2006a,b).

Numbers between parenthesis refer to the ‘Star id n°’ in Figure 15.