CONSISTENT CONSTRUCTION OF REALISTIC
ONE-BODY DENSITY MATRIX IN NUCLEI

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Abstract

A phenomenological method based on the natural orbital representation is applied to construct the ground state one-body density matrix which describes correctly both density and momentum distributions in $^4\text{He}$, $^{16}\text{O}$ and $^{40}\text{Ca}$ nuclei. The parameters of the matrix are fixed by a best fit to the experimental density distribution and to the correlated nucleon momentum distribution. The method allows the natural orbitals, the occupation probabilities and the depletion of the Fermi sea to be obtained. Ground-state characteristics of $^4\text{He}$, $^{16}\text{O}$ and $^{40}\text{Ca}$ nuclei, such as rms radii and mean kinetic energies are calculated, as well.

1 Introduction

The one-body density matrix (OBDM) plays an important role in the nuclear structure theory. It is known that the nuclear wave function contains much more information than it is accessible to observation \cite{1,2}. All measurable characteristics of the nuclear ground state (apart from the energy) correspond to single-particle operators whose expectation values can be expressed in terms of the OBDM elements. The ground state energy can be calculated by a sum rule analysis \cite{3,4,5} also by means of the OBDM. Thus the determination of the OBDM from first principles \cite{6} is one of the most essential aims of the nuclear theory.
As shown in [7, 8] (and discussed in [3, 4, 5, 6]), an appropriate criterion for the proximity of the OBD $\rho_0 (r; r^0)$ corresponding to a single-Slater determinant wave function (i.e. $\beta = 0$, for instance in the Hartree-Fock approximation) to the OBD $\rho (r; r^0)$ of the true (correlated) ground state is that the "mean-square deviation per particle"

$$= A^{-1} \text{Tr}[(\rho - \rho_0)^2]$$

(1)

(where $A$ is the mass number) should be minimal. It has been shown in [3] that this is minimal when $\rho_0 (r; r^0)$ corresponds to single Slater determinant wave function constructed with natural orbitals, i.e. with the single-particle wave functions which diagonalize the OBD $\rho (r; r^0)$ [13]. As was shown in [3], if in the Hartree-Fock approximation the OBD diagonal elements $\rho_{0F} (r; r^0)$ are fitted to reproduce correctly the exact density distribution $\rho (r; r)$ (i.e. $\rho_{0F} (r; r^0)$), the extremum property is in leads inevitably to an increasing deviation between the non-diagonal elements of these two matrices $(\rho (r; r^0)$ and $\rho_{0F} (r; r^0)$ at $r \neq r^0$. The non-diagonal elements are related, however, to the nucleon momentum distribution (NMD):

$$n (k) = \frac{Z}{(2\pi)^3} \exp [\text{ik}(r - r^0)] dr dr^0.$$  

(2)

This leads to the conclusion [3] that generally the mean-field approximation is unable to give simultaneously a correct description of the two basic nuclear ground state characteristics, namely the density and momentum distributions. This has been supported by the calculations in the case of $^4$He nucleus [14].

The role of the nucleon-nucleon correlations in the nuclear system can be seen by the analyses of the quantity [13]. While in the Hartree-Fock approximation vanishes and the contribution of the random-phase approximation to $\Sigma$ is about 0.002, for realistic N-N forces the short- and medium-range correlation effects lead to a value of $\Sigma^{\text{min}} = 0.02 \pm 0.03$ that implies a Fermi sea depletion of about 10–15% . The latter is confirmed by the experimental data [15, 16]. The short-range and tensor correlations are responsible for the existence of high-momentum components in the realistic momentum distributions in nuclei which is not the case in the mean-field approximation.

A phenomenological method to construct a more realistic OBD $\rho_0 (r; r^0)$ is suggested in [3, 4] for the cases of $^{16}$O and $^{40}$Ca nuclei, both with fractional occupation numbers and full occupation of the first A natural orbitals. The latter are expanded in terms of harmonic-oscillator wave functions. The expansion coefficients, as well as the occupation numbers are fixed by a best fit to the experimental density distributions of $^{16}$O and $^{40}$Ca minimizing the mean percentage deviation between the experimental and trial radial density. A satisfactory description of g.s. properties, such as the g.s. energy and rms radii, is obtained.
using the OBDM constructed within this method, in contrast with the predictions of the Hartree-Fock approximation.

Here we would like to emphasize that according to the discussion given above, the OBDM obtained in the method from [1,2] cannot be a realistic one because it is obtained by a best only of its diagonal elements to the experimental density distribution (which is not very sensitive to the N-N correlations) and is not aimed to describe the nucleon momentum distribution which is related also to the non-diagonal elements of the OBDM and is mostly affected by the presence of short-range and tensor N-N correlations in the nuclear system.

The aim of this work is to construct the g.s. OBDM in $^4$He, $^{16}$O and $^{40}$Ca nuclei in a consistent way following the method from [1,2] but providing in an optimal way a correct description of both density and momentum distributions in nuclei considered. Solving this problem, we show that, in principle, the expansion of the natural orbitals in terms of harmonic oscillator functions (in the truncated s.p. space) cannot give a correct description of the realistic high-momentum components of the nucleon momentum distribution. This imposes to use in the expansion of the natural orbitals another set of s.p. functions, for instance that one corresponding to in the square-well potential.

The method suggested in this work enables us to construct a more realistic OBDM which includes that part of the nucleon correlations which are responsible for the high-momentum components of the nucleon momentum distribution and is consistent with the nuclear density distribution.

Some basic relations are given in Section 2. The phenomenological expression for the OBDM is presented in Section 3. The results of the numerical calculations are given and discussed in Section 4.

## 2 Nuclear ground-state properties

In order to evaluate the single-particle characteristics of the nuclear ground state one needs the OBDM which is defined by the expression:

$$
(r; r') = A \int (r; r_2; \ldots) (r'; r_2; \ldots) dr_2 dr_3 \ldots dr_A; 
$$

where $(r_1; r_2; \ldots; r_A)$ is the normalized A-nucleon ground-state wave function. The integration in Eq. (3) is carried out over the radius vectors and summation over spin and isospin variables is implied.

The one-body density matrix has the simplest form in the so called natural orbital representation [3]:

$$
(r; r') = X_i \partial_i (r) \partial_i (r'); 
$$

where
where \( i \) are the occupation numbers (eigenvalues of \( \hat{\mathbf{H}} \)) corresponding to the natural orbitals \( \hat{\mathbf{H}}(r) \) (eigenfunctions of \( \hat{\mathbf{H}} \)) which form a complete orthonormal set. The values of \( i \) satisfy the general conditions:

\[
0 < i < 1 \quad \text{and} \quad i = A.
\]

Usually there are \( A \) orbitals \( \hat{\mathbf{H}}(r) \) for which the occupation probabilities \( i \) are significantly larger than those for the others. As in the mean-field approach, these are called the hole-state orbitals while the others are called particle-state orbitals [7].

In the momentum space the OBDM (Eq.(3)) reads

\[
\sum_{k} A \sim \langle k; k_2; \ldots; k_A \rangle \sim \langle k_0; k_2; \ldots; k_A \rangle dk_2 \ldots dk_A;
\]

where \( \sim \langle k_1; k_2; \ldots; k_A \rangle \) is the Fourier transform of the A-nucleon wave function \( \langle r_1; r_2; \ldots; r_A \rangle \). Its diagonal elements determine the nucleon momentum distribution (Eq.(2)). The associated natural orbital representation

\[
n(k;k^0) = \prod_{i} \sim \langle k \rangle \sim \langle k_i \rangle
\]

depends on the same occupation numbers \( i \) entering Eq.(4) and the Fourier transform of the natural orbitals \( \sim \langle k \rangle \) in the momentum space

\[
\sim \langle k \rangle = Z \prod_{i=1}^{A} \int r \exp \{ikr\} dr:
\]

For the purposes of our work a particular attention will be paid to the local density distribution (the diagonal elements of the OBDM in the coordinate space):

\[
\langle r; r \rangle = \prod_{i} \sim \langle i; i \rangle \sim \langle i \rangle
\]

and the nucleon momentum distribution (the diagonal elements of the OBDM in the momentum space):

\[
n(k) A = \prod_{i} \sim \langle i; i \rangle \sim \langle i \rangle
\]

The second moment of \( \langle r \rangle \) and \( n(k) \) define respectively the rms radius of the nucleus

\[
\langle r^2 \rangle = Z \prod_{i=1}^{A} \int (r)^2 dr
\]

and its mean kinetic energy

\[
\langle \hat{\mathbf{H}} \rangle = \frac{\hbar^2}{2m} \int n(k)k^2 dk:
\]

The knowledge of a realistic one-body density matrix would allow to describe correctly the nuclear characteristics, such as the nucleon momentum and density distributions, rms radii and mean kinetic energies taking into account the effects of the nucleon-nucleon correlations in the nuclear system.
3 Phenomenological method for the constructing of the one-body density matrix

In this Section a phenomenological expression for the OBDM is given. The parameters in this expression can be obtained by a numerical procedure by fitting the local density \( n(r) \) and the momentum distribution \( n(k) \) to the corresponding experimental data (or to realistic theoretical estimates) \[13,18,20\].

For closed-shell nuclei with equal numbers of protons and neutrons it is reasonable to assume that the protons and neutrons have the same one-body density matrix. For such nuclei (with total spin \( J = 0 \)) the OBDM has to be diagonalized in the \( ljm \) subspace of the complete space of the natural orbitals \[21\] (\( ljm \) being the quantum numbers corresponding to the s.p. orbital and total momentum and its projection). Due to the spherical symmetry of the considered nuclei the natural orbitals can be looked for in the form:

\[
i_r(nlm)(r) = R_{nl}(r)Y_{lm}(\gamma');
\]

It is known that the radial part of the natural orbitals \( R_{nl}(r) \) and especially of the particle-state orbitals differ from the single-particle wave functions obtained within the mean field approximation to the ground state of a fermion system \[17,18,23,24\]. Following exactly the method suggested in \[23,24\], we expand the radial part of the natural orbitals of the OBDM in terms of three single-particle functions preserving all the usual symmetries for spherical nuclei:

\[
R_{\alpha}(r) = \sum_{i=1}^{X^3} C_{i}^{\alpha'}(r); \quad (\alpha n l)
\]

In Eq.(13) \( f_{nl}(r)g \) is a set of orthonormal s.p. wave functions. The expansion coefficients \( C_{i}^{\alpha} \) are fitting parameters satisfying the orthonormalization conditions. For the numerical calculations it is convenient to re-express them using polar coordinates:

\[
C_{1}^{\alpha} = \cos_{\alpha} \\
C_{2}^{\alpha} = \sin_{\alpha} \cos'_{\alpha} \\
C_{3}^{\alpha} = \sin_{\alpha} \sin'_{\alpha}:
\]

In this way the normalization is automatically guaranteed. The orthogonality among states of the same symmetry reduces the number of parameters involved in Eq.(14).

By means of Eqs.(12-14) one can obtain the expression for the one-body density matrix (Eq.(4)) within this method. The diagonal elements of the OBDM in coordinate space determine the local density.
distribution:

\[ r = \frac{1}{4} X_{n_l}^{(2l+1)} n_l R_{nl}(r) \]  \hspace{1cm} (15)

The occupation numbers \( n_l \) satisfy the condition:

\[ X_{n_l}^{(2l+1)} n_l = A \]  \hspace{1cm} (16)

and the normalization of \( r \) is:

\[ Z \int r dr = A \]  \hspace{1cm} (17)

An expression similar to that for \( r \) determines the nucleon momentum distribution:

\[ n(k) = \frac{1}{4} X_{n_l}^{(2l+1)} n_l R_{nl}(k) f_n; \] \[ \int n(k) dk = A \]  \hspace{1cm} (18)

In Eq.(18)

\[ R_{nl}(k) = \sum_{i=1}^{a} C_i^nl \ell_{-1}(k); \] \( \ell_{-1} \) \hspace{1cm} (19)

where \( \ell_{-1}(k) \) are the orthonormalized s.p. wave functions in the momentum space. Obviously, due to the truncation of the expansion \((13)\) (and the corresponding one \((19)\)) up to \( \ell = 3 \), the radial part of the natural orbitals depends strongly on the choice of the single-particle wave functions \( f'_{nl}g \). We use three sets of s.p. wave functions \( f'_{nl}g \) corresponding to: 1) the harmonic-oscillator potential \((HO)\):

\[ V(r) = V_0 + \frac{1}{2} m w^2 r^2; \] \( V_0 > 0 \) \hspace{1cm} (20)

2) the square-well potential with finite walls \((SW)\):

\[ V(r) = \begin{cases} \frac{8}{3} V_0; & r < x; \ V_0 > 0 \\ 1; & r > x \end{cases} \] \hspace{1cm} (21)

and 3) the modified harmonic-oscillator potential \((MHO)\) \cite{23,24,25}:

\[ V(r) = \begin{cases} V_0 + \frac{1}{2} m w^2 r^2 + \frac{B}{r^2}; & V_0 > 0; B > 0 \end{cases} \] \hspace{1cm} (22)

The modified harmonic-oscillator potential \((Eq.(22))\) behaves like a harmonic-oscillator one for large values of \( r \) but it has in addition a repulsive term \( B \) which is the dominant one at short distances from the origin. One can expect that this term simulates to some extent the effects of strong repulsion in the nucleon-nucleon interaction at all distances. The single-particle potential \((22)\) had been suggested and used \cite{23,24,25} for the study of charge form factors and nucleon momentum distributions of light nuclei. One of the main advantages of this potential is that analytical expressions are derived for the single-particle wave functions and for other useful quantities.
Though the M HO potential (22) leads to an infinite repulsion at the origin \((r = 0)\), the corresponding single-particle wave functions which are used in the present work are correct for all values of \(r\).

In the natural orbital representation the local density and the momentum distributions (Eqs.(15) and (18)) are expressed as sums over the whole single-particle space. In the numerical calculations we include in these sums all hole-state orbitals and several particle-state orbitals. In this way the expression for the OBDM is determined and it can be used within a minimization procedure which leads to the constructing of a realistic one-body density matrix.

4 Numerical calculations. Results and discussion

The phenomenological method for constructing the realistic one-body density matrix given in the previous Section is applied to the case of the double-closed shell nuclei \(^4\text{He}\), \(^{16}\text{O}\), and \(^{40}\text{Ca}\). Obviously, the expansion (4) should be truncated. In the case of \(^4\text{He}\) and \(^{16}\text{O}\) we include the 1s, 1p, 1d and 2s states and for \(^{40}\text{Ca}\) nucleus-the 1s, 1p, 1d, 2s, 2p, 1f and 1g states. The parameters of the OBDM, namely the occupation numbers \(f_{nlg}\), the coefficients \(C_{nlg}\) and the parameters of the sets of the orthonormal single-particle functions corresponding to the potentials (20)-(22) (the oscillator parameter \(\beta = (\hbar^2 m w)_{1/2}\), the radius of the well \(x\) and the parameter \(b = 8mB\)) are fitting parameters. The values of the parameters are fixed by a best fit of the diagonal elements of the OBDM in the coordinate space and in the momentum space (Eqs.(15) and (18)) to the experimental local density \(\rho\) obtained in \([26,27,28,29]\) and to the nucleon momentum distribution \(n(k)\). In the case of \(^4\text{He}\) we use the experimental data for \(n(k)\) obtained by means of the y-scaling analysis \([20]\) relying on the assumption that the 1=q expansion is valid. Theoretical estimations for \(n(k)\) in \(^{16}\text{O}\) and \(^{40}\text{Ca}\) obtained within the Jastrow correlation method (JCM) \([18,19,30]\) are used instead of experimental ones due to the lack of empirical data for these nuclei. Though the low-order approximation to the JCM has some deficiencies, we use it due to the obtained simple analytical expressions for the one-body nuclear characteristics, in particular, for the \(n(k)\). The latter reproduce the results from the complicated exp (S)-calculations \([31]\) including the high-momentum tail of \(n(k)\). It is possible also more realistic theoretical predictions for \(n(k)\) in the nuclei considered to be used.

Our fitting criterion is to minimize the relative deviation between the experimental and theoretical
local density distributions

\[ \frac{2^D}{q} = \sum_{q} \frac{\exp(q)}{\exp(q)} \left( \frac{\exp(q)}{\exp(q)} \right)^{\text{#}_2} \quad ; \tag{23} \]

where \( q \) is the number of points under which minimization is performed, and between the theoretical nucleon momentum distribution for \( ^{16}\text{O} \) and \( ^{40}\text{Ca} \) (or experimental \( n(k) \) for \( ^4\text{He} \)) and that corresponding to the OBDM proposed in this work. As far as the latter is concerned it is preferable to consider \( \ln(k) \) rather than \( n(k) \) because the values of \( n(k) \) rapidly decrease by several orders of magnitude with the increase of the momentum \( k \):

\[ \frac{2^M}{q} = \sum_{q} \frac{\ln^{2CM}(k)_{q}}{(\ln^{\text{th}}(k)_{q})_{q}} \quad ; \tag{24} \]

Otherwise, our minimization criterion would not be sensitive to both parts (low-momentum and high-momentum region) of the nucleon momentum distribution. Taking into account Eqs. (23) and (24) we minimize the expression:

\[ F = \frac{2^D}{q} + \frac{2^M}{q} \quad ; \tag{25} \]

The minimization is accomplished using the program M INUIT. There have been involved in \( \leq 12 \) (or 13 when M HO sp. wave functions are used) fitting parameters in the case of \( ^4\text{He} \) and \( ^{16}\text{O} \) and 20 parameters for \( ^{40}\text{Ca} \) nucleus.

The quality of the minimization procedure is illustrated in Table 1. There are listed the values of \( \frac{2^D}{q} \) (Eq. (23)), \( \frac{2^M}{q} \) (Eq. (24)) per fitting point \( q \) and \( F \) (Eq. (25)). For all nuclei examined it is seen that the minimal value of \( F \) is achieved using square-well sp. wave functions. For example, in the case of \( ^{16}\text{O} \) the local density distribution (15) and the nucleon momentum distribution (18) are approximated with almost the same accuracy. A lower value of \( F \) is also obtained when modified harmonic-oscillator sp. wave functions are used but \( \frac{2^D}{q} = q \) and \( \frac{2^M}{q} = q \) differ by two orders of magnitude. It means that in this case the local density distribution is described better than the nucleon momentum distribution. As is well known, the harmonic-oscillator sp. wave functions are appropriate to describe the local density distribution of \( ^{16}\text{O} \) within the simple shell model. Considering the values of \( \frac{2^D}{q}, \frac{2^M}{q} \) and \( F \) one can see that the proposed method in this case is unable to reproduce simultaneously the local density and the nucleon momentum distribution. Similar calculations can be drawn considering the minimization procedure for \( ^{40}\text{Ca} \). The best value of \( F \) is obtained using square-well sp. wave functions. The local density distribution and the nucleon momentum distribution are described satisfactorily to almost the same accuracy.

The values of the occupation numbers \( n_l \), of the coefficients \( C_{l}^{n_1} \) and of the parameters of the sp. wave functions in the expansion of the OBDM for \( ^4\text{He} \), \( ^{16}\text{O} \) and \( ^{40}\text{Ca} \) are given in Tables 2, 3 and 4.
respectively. Thus the one-body density matrix is fully determined within the method proposed. The values of the occupation numbers \( n_1 \) are compared with the occupation numbers \( n_1 \) obtained within the natural orbital representation of the OBDM in the low-order approximation of the Jastrow correlation method which can be considered as an "exact limit" of the method proposed in the present paper. It is seen that the values of \( n_1 \) in the case of square-well s.p. wave functions are in good agreement with the natural occupation numbers \( n_1 \).

The depletion of the Fermi sea in \(^4\text{He}\) is 18.6%, 13.2% and 11.3% in the cases when harmonic-oscillator, modified harmonic-oscillator-and square-well s.p. wave functions are used, respectively, in the expansions (13) and (19). Its corresponding values using the same types of s.p. wave functions in the case of \(^{16}\text{O}\) are 18%, 16.6% and 12.4%, respectively. In the case of \(^{40}\text{Ca}\) the depletion is 20% and 15% when harmonic-oscillator and square-well s.p. wave functions are used, respectively. It can be seen again that the best description of the OBDM within the proposed procedure is achieved when square-well s.p. wave functions are used because the depletion in this case is minima and it is the closest to the corresponding value in the natural orbital representation [13] 6% for \(^4\text{He}\), 4% for \(^{16}\text{O}\) and 6.3% for \(^{40}\text{Ca}\) and to the experimental one [32] 9.4% for \(^{40}\text{Ca}\). We would like to mention here that the differences of our results from those obtained in [13] and from the experimental ones for \(^{40}\text{Ca}\) [32] have to be considered bearing in mind the defferences of the low-order approximation to the Jastrow correlation method used in [19], the model dependence of the analyses of the empirical data and, of course, the limitations of the suggested method. Nevertheless, we emphasize that our results are within the theoretical limits for the depletion of the Fermi sea which is expected to be approximate at least 5 and 15%.

In Figs.1 and 2 the results for the local density and momentum distribution in \(^4\text{He}\), \(^{16}\text{O}\) and \(^{40}\text{Ca}\) obtained by means of the one-body density matrix determined in the model are presented. The best solution is reached by using of square-well s.p. wave functions. As is seen from Fig.1 an acceptable quantitative agreement with the experimental data for the density distribution \((r)\) (at least in the surface region) can be achieved almost independently on the choice of the s.p. wave function sets. However, only the use of the SW s.p. wave functions corresponding to the potential (21) gives a correct description of the realistic high-momentum components of the nucleon momentum distribution (see Fig.2). As is expected, the use of HO s.p. wave functions does not reproduce realistically \(n(k)\). It can be also seen from Figs.2b and 2c that although the OBDM obtained in the method from [3] gives a correct description of the density distribution, it is unable to describe the high-momentum behaviour of the momentum distribution which is sensitive to the short-range N-N correlations. In the cases of \(^4\text{He}\) and \(^{16}\text{O}\) (Figs.2a and 2b)
the use of $M$ HO s.p. wave functions corresponding to the potential (22) leads to an improvement of the behaviour of $n(k)$ at higher momenta which is due to the existence of the repulsive term in the potential.

In Table 5 the calculated rms radii $\hbar^2r_i^2$ (Eq.(10)) and mean kinetic energies per nucleon $\hbar^2 = A$ (Eq.(11)) of $^4$He, $^{16}$O and $^{40}$Ca and their values deduced from the JCM are presented. The values of the rms radii for all nuclei examined are in accord with the corresponding experimental values.

Due to the fact mentioned above that the expansion of the natural orbitals in terms of HO functions cannot give a correct description of the realistic high-momentum components of the nucleon momentum distribution (see Fig.2c), it can be seen in Table 5 a quite increased value of $\hbar^2 = A$ for $^{40}$Ca in comparison with the result obtained by using SW s.p. wave function set. Therefore, we have demonstrated that the SW s.p. wave functions are the best ones among the other used in the sense of correct description of both density and momentum distributions in $^4$He, $^{16}$O and $^{40}$Ca. Here we would like to mention that our choice of the truncation in the expansions (13) and (19) up to $i = 3$ was imposed from one side by the desire to follow exactly the method previously suggested in [1,2] in order to compare our results with those from the mentioned works. On the other hand, it is well-known that the expansions (13) and (19) in terms of the complete set of s.p. wave functions would be exact expressions for the natural orbitals. Of course, this is impossible to be done and in the practical applications we have to find a proper value of $i$ which would lead to a reasonable number of the minimization parameters. The inclusion of higher terms in the expansions ($i > 3$) would make the results of the method less sensitive to the particular choice of the s.p. wave function set, but at the same time this would increase enormously the difficulties of the practical minimization procedure. Hence, the applications of the method would be hardly possible. In our opinion, the choice $i = 3$ is an acceptable compromise within the method.

### 5 Conclusions

The results of this work can be summarized as follows:

1) A phenomenological method for a consistent construction of the one-body density matrix is suggested. In contrast to the method from [1,2], our procedure allows to obtain an one-body density matrix which gives realistically both the local density and the nucleon momentum distribution of $^4$He, $^{16}$O and $^{40}$Ca nuclei.

2) The method gives the natural orbitals, the occupation probabilities and the depletion of the Fermi sea in the nuclei considered.

3) The rms radii and mean kinetic energies of $^4$He, $^{16}$O and $^{40}$Ca are calculated with the method.
iv) Consideration of both non-diagonal and diagonal elements of \((r; r')\) makes it possible to treat in a consistent way the effects of the N-N short-range correlations in nuclei on the one-body density matrix and the related ground state characteristics.

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Figure Captions

Figure 1. Local density distribution in $^4$He (a), $^{16}$O (b) and $^{40}$Ca (c) obtained by using different sets of s.p. wave functions in Eq.(15). The normalization is \( Z \int r \, dr = A = 2 \).

Figure 2. Nucleon momentum distribution in $^4$He (a), $^{16}$O (b) and $^{40}$Ca (c) obtained by using different sets of s.p. wave functions in Eq.(18). The normalization is \( Z \int n(k) \, dk = 1 \).
Table 1. $^2$-values pertaining to the local density (Eq. (23)) and the nucleon momentum distribution (Eq. (24)) and the F-value (Eq. (25)) for $^4$He, $^{16}$O and $^{40}$Ca.

| Nuclei | S.p. wave | $^2_0$ =q | $^2_\text{M}$ =q | F     |
|--------|-----------|-----------|----------------|-------|
| $^4$He | HO        | 0.0040    | 0.0379         | 1.204 |
|        | M HO      | 0.0031    | 0.0260         | 1.027 |
|        | SW        | 0.0068    | 0.0036         | 0.932 |
| $^{16}$O | HO      | 0.0098    | 1.2458         | 6.239 |
|        | M HO      | 0.0019    | 0.1271         | 2.104 |
|        | SW        | 0.0199    | 0.0170         | 1.888 |
| $^{40}$Ca | HO      | 0.1805    | 0.6678         | 8.147 |
|        | SW        | 0.0168    | 0.0352         | 2.190 |
Table 2. Occupation numbers $n_l$, expansion coefficients $C_{i}^{n_l}$ ($i=1,2,3$), parameters of the s.p. wave functions and natural occupation numbers $n_l$ for $^4$He.

| $n_l$ | C_{i}^{n_l} | $n_l$ | C_{i}^{n_l} | $n_l$ | C_{i}^{n_l} |
|-------|-------------|-------|-------------|-------|-------------|
| 1s    | 0.8138      | 0.1937| 0.8679      | 0.2435| 0.8873      | -0.2191| 0.9428 |
|       | -0.0308     |       | 0.0001      |       | 0.0182      |       |       |
|       | -0.5893     |       | -0.9643     |       | 0.1411      |       |       |
| 1p    | 0.0392      | -0.5011| 0.0145      | -0.1136| 0.0063      | -0.9576| 0.0097 |
|       | -0.6337     |       | 0.2392      |       | 0.2513      |       |       |
|       | 0.1848      |       | -0.3959     |       | -0.3405     |       |       |
| 1d    | 0.0033      | 0.1256| 0.0064      | 0.1115| 0.0187      | 0.5185| 0.0027 |
|       | -0.9747     |       | 0.9115      |       | -0.7843     |       |       |
|       | 0.1319      |       | -0.1974     |       | -0.1848     |       |       |
| 2s    | 0.0521      | -0.7674| 0.0566      | 0.7864| 0.0002      | -0.7222| 0.0027 |
|       | -0.6275     |       | 0.5854      |       | 0.6079      |       |       |
Table 3. Occupation numbers \( n_l \), expansion coefficients \( C_{i}^{n_l} \) \((i=1,2,3)\), parameters of the sp. wave functions and natural occupation numbers \( n_l \) \([3]\) for \(^{16}O\).

| \( n_l \) | \( C_{1}^{n_l} \) | \( C_{2}^{n_l} \) | \( C_{3}^{n_l} \) | \( n_l \) |
|-------------|-------------|-------------|-------------|-------------|
| \( 1s \)    | 0.9174      | -0.2686     | 0.9359      | 0.9959      |
|             | 0.8002      | -0.2936     | 0.2783      | 0.9928      |
|             | 0.9959      | -0.9928     | -0.0808     | 0.9919      |
| \( 1p \)    | 0.8278      | 0.0008      | 0.8000      | 0.3746      |
|             | -0.0899     | 0.1194      | 0.0214      |             |
|             | 0.2225      | 0.5289      | 0.4743      |             |
| \( 1d \)    | 0.1249      | 0.9353      | 0.973       | 0.9959      |
|             | 0.5028      | 0.3651      | 0.8731      |             |
|             | 0.1464      | 0.0780      | -0.0315     |             |
| \( 2s \)    | 0.0920      | -0.4582     | 0.1774      | 0.9959      |
|             | -0.8767     | -0.0032     | -0.4831     |             |
| \( b = 0.731 \) | 0.582 fm | 0.582 fm | 0.582 fm | 0.582 fm |
Table 4. Occupation numbers $n_l$, expansion coefficients $C_i^{nl}$ ($i=1,2,3$), parameters of the s.p. wave functions and natural occupation numbers $n_l$ for $^{40}$Ca.

|                | HO s.p. wave functions | SW s.p. wave functions |
|----------------|------------------------|------------------------|
| $n_l$          | $x = 6.627 \text{ fm}^{-1}$ | $x = 6.640 \text{ fm}$ |
| $n_l$          | $C_i^{nl}$             | $n_l$                  | $C_i^{nl}$             |
| 1s             | 0.8027                 | 0.7972                 | 0.9227                 | 0.8385                 | 0.8898                 |
|                | -0.2849                |                        | -0.1939                |
|                | 0.9733                 |                        | 0.9959                 |
| 1p             | 0.8000                 | -0.0609                | 0.8718                 | -0.0717                | 0.9376                 |
|                | -0.2214                |                        | -0.0560                |
|                | 0.8023                 |                        | 0.8548                 |
| 1d             | 0.8005                 | -0.5923                | 0.8013                 | -0.5181                | 0.9463                 |
|                | 0.0746                 |                        | 0.0335                 |
|                | -0.8440                |                        | 0.8517                 |
| 2s             | 0.8020                 | 0.5261                 | 0.9558                 | 0.4586                 | 0.9579                 |
|                | -0.1045                |                        | -0.2536                |
|                | 0.2197                 |                        | -0.0203                |
| 2p             | 0.1005                 | 0.5283                 | 0.1686                 | -0.7750                | 0.0168                 |
|                | 0.8201                 |                        | 0.6317                 |
|                | 0.9913                 |                        | 0.2266                 |
| 1f             | 0.000002               | -0.1106                | 0.0465                 | 0.2341                 | 0.0127                 |
|                | -0.0717                |                        | 0.9454                 |
|                | 0.0453                 |                        | 0.0589                 |
| 1g             | 0.1879                 | 0.5765                 | 0.0742                 | -0.2858                | 0.0087                 |
|                | 0.8158                 |                        | 0.9565                 |
Table 5. Rms radii and mean kinetic energies per nucleon of $^4$He, $^{16}$O and $^{40}$Ca calculated by using different sp. wave function sets for the construction of the one-body density matrix.

| Nuclei | $\hbar^2 r^2 / A$ | $\hbar T i / A$ |
|--------|-----------------|---------------|
|        | (fm) | (MeV) |
| $^4$He |     |       |
| HO     | 1.66 | 12.33 |
| MHO    | 1.65 | 12.93 |
| SW     | 1.67 | 21.47 |
| EXP [26]| 1.67 |       |
| JCM [30]| 1.60 | 25.86 |
| $^{16}$O |     |       |
| HO     | 2.70 | 20.08 |
| MHO    | 2.71 | 16.29 |
| SW     | 2.60 | 26.52 |
| EXP [33]| 2.71 |       |
| JCM [30]| 2.63 | 21.34 |
| $^{40}$Ca |     |       |
| HO     | 3.46 | 38.29 |
| SW     | 3.42 | 26.94 |
| EXP [33]| 3.48 |       |
| JCM [30]| 3.50 | 23.36 |