Abstract

Iceberg meltwater is a critical freshwater flux from the cryosphere to the oceans. Global climate simulations therefore require simple and accurate parameterisations of iceberg melting. Iceberg shape is an important but often neglected aspect of iceberg melting. Icebergs have an enormous range of shapes and sizes, and distinct processes dominate basal and side melting. We show how different iceberg aspect ratios and relative ambient water velocities affect melting using a combined experimental and numerical study. The experimental results show significant variations in melting between different iceberg faces, as well as within each iceberg face. These findings are reproduced and explained with novel multiphysics numerical simulations. At high relative ambient velocities melting is largest on the side facing the flow, and mixing during vortex generation causes local increases in basal melt rates of over 50%. Double-diffusive buoyancy effects become significant when the relative ambient velocity is low. Existing melting parameterisations do not reproduce our findings. We propose improvements to capture the influence of aspect ratio on iceberg melting.

1 Introduction

Iceberg meltwater provides an important flux of freshwater from ice sheets to oceans [31, 25], making up 45% of Antarctic freshwater loss [28], and dominating freshwater production in Greenland fjords [13]. Melting also releases nutrients that boost biological productivity and carbon sequestration [32]. Where and when meltwater and nutrients are released depends on how quickly icebergs melt. Understanding how icebergs influence the climate therefore requires accurate predictions of iceberg melt rates. We present an experimental and numerical investigation of an often neglected aspect of melting — iceberg shape.

Icebergs display enormous variation in shape and size [7, 33, 34, 12, 31, 2]. Horizontal extents range from several meters to the record iceberg B-15 at 300 km × 40 km [3]. Depths vary considerably but almost never exceed 600 m [10]. Rolling instability further constrains realistic shapes [35, 6]. Icebergs can tumble when the aspect ratio, the ratio of length \( L \) to submerged depth \( D \), is smaller than \( \sqrt{0.92 + 58.32/D} \), where \( D \) is expressed in metres. Aspect ratios may therefore range anywhere from 1 to 1000. The overall melting will depend strongly on aspect ratio whenever bottom and side melt rates differ.
Using empirical relations for turbulent heat transfer over a flat plate [11], Weeks and Campbell developed a commonly used parameterisation for iceberg melt rates [35] (hereafter the WC model). The WC model predicts an iceberg melt rate \( v \) (in dimensional units of speed) of

\[
v = 0.037 \left( \frac{\rho_w \nu^{-7/15} \kappa^{2/3} c_p}{\Lambda} \right) \frac{U^{0.8} \Delta T}{L^{0.2}}.
\]

Here, \( \rho_w \) and \( \rho_i \) are the respective densities of water and ice, \( \nu \) and \( \kappa \) are the respective diffusivities of momentum and temperature, \( c_p \) is the heat capacity at constant pressure of seawater, \( \Lambda \) is the latent heat of ice melting, \( U \) is the relative speed between the ambient water and the iceberg, \( L \) is the iceberg length, and \( \Delta T \) is a characteristic temperature difference. Unfortunately the model makes several incorrect predictions, such as predicting that melting stops for zero relative velocity. This shortcoming was addressed recently by including the effect of meltwater-plume entrainment for low relative ambient velocities [14]. More importantly, the model fails to treat side and base melting separately, which are expected to be dominated by different dynamical processes [6].

Our goal is to understand the effect of aspect ratio on iceberg melting in a series of laboratory experiments and numerical simulations. We compare our findings with predictions of the WC model and suggest improvements to account for the influence of aspect ratio on melting. In section 2 we describe the experimental method, and summarise the findings in section 3. Using a recent numerical method [19], summarised in section 4, we reproduce the laboratory experiments in a series of fluid-solid simulations, which allow us to identify and discuss key physical processes controlling melting in section 5. We summarise geophysical implications and discuss possible improvements to parameterisations in section 6. We conclude and discuss future directions in section 7.

2 Experimental Methods

![Experimental schematic](image)

*Figure 1: (a) Experimental schematic. A dyed ice block of dimensions with length \( L \), width \( W \), and immersed depth \( D \) is fixed in a flume with a relative ambient velocity \( U \), temperature \( T_w \) and salinity \( C_w \). (b) Post-experiment photographs (top) were filtered and reoriented (waterline in red) to estimate the average melting of each face.*

The experiments immersed ice blocks of different lengths in a recirculating salt water flume with different ambient water velocities (fig. 1 (a)). The central section of the flume measured 76.5 cm long by 42 cm wide by 33.5 cm deep, and recirculated salt water with salinity \( C_w = 30 \text{ g/kg} \) and temperatures \( T_w = 18 \) to 21 °C. The 26 distinct experiments held the depth fixed at \( D = 3 \text{ cm} \). We considered 5 lengths \( L = 10, 15, 20, 25, 33 \text{ cm} \) and three relative ambient flow
velocities $U = 0, 1.5, \text{ or } 3.5 \text{ cm s}^{-1}$. The width varied from $W = 10$ to 22.5 cm. The water of each ice block was dyed blue and left to de-gas overnight before freezing at $T_i = -30^\circ\text{C}$. The ice block was weighed, positioned in the flume, and each experiment ran for 10 minutes. The ice block was then removed, reweighed to calculate mass loss, and photographed from each side.

Post-experiment photographs determined the melting of each face (fig. 1 (b)). A blue filter highlighted the block, then thresholding returned a binary image, and an opening transform removed the noise. The edges of the waterline (red line in fig. 1 (b)) are defined as the lowest points with less than 15 pixels of melting, and the images were rotated to level the waterline. Pixels were converted to cm using rulers in each photo. The left profile was calculated as a function of depth, and the left face defined as the portion with slope greater than 1. The depth average of the left face determined the average melt rate of that face. Average bottom and right melt rates were calculated similarly.

3 Experimental results

3.1 Qualitative observations

High relative ambient velocity — Figure 2 shows a time series of two experiments at $U = 3.5 \text{ cm s}^{-1}$. The melting on both front faces (left side of each frame) is much larger than on the base and sides. This result agrees with previous studies showing that flow perpendicular to an ice face leads to greater heat transport and melt rates than flow parallel to an ice face [24]. We therefore expect aspect ratio to influence melting at large relative ambient flow speeds.

The second apparent feature is the non-uniformity of melting within each face of each experiment (fig. 2). The front melting increases with depth, decreasing the slope of the leading face over time. The basal melting also has a pronounced non-uniform profile. Starting from upstream, a darker region of increased dye concentration is pooled just behind the front of the base, and the melt rate is low. This concentration suggests a stagnant zone that does not mix with the incoming flow, which is typical when flow separation occurs. Immediately
behind this region we observe increased turbulence and basal melt rates in both experiments. At the rear the longer ice block (fig. 2, right) returns to a regime of lower, uniform melting. The dye patterns toward the rear of the longer block (right) become less mixed, suggesting less turbulent flow. The melting pattern is similar for all blocks, and echoes understanding that other liquid-solid phase change problems can evolve to self-similar shapes [17, 26, 22].

The flow field helps explain the basal melting pattern. As fluid moves past a forward-facing step, vorticity separates from the leading edge. This configuration produces a region of unsteady recirculation and subsequent reattachment of the flow. The few studies examining heat transfer in flow past a forward-facing step find a maximum in convective heat transfer at the point of reattachment [1, 23]. Different estimates for the reattachment length exist (summarised in [30]) but all find that it is roughly 3 to 5 times the step height for Reynolds numbers Re = 10^3 to 10^5 (our experiment is at Re ≡ UD/ν = 800, based on the step height). Actual icebergs in Greenland fjords may experience Reynolds numbers up to 2 × 10^7, assuming a draft of 200 m, and local relative velocities up to 0.1 m s^−1 [15]. We believe some kind of turbulent-flow separation-reattachment scenario to continue to much higher Reynolds numbers. We, therefore, expect to see similar behaviour in real icebergs.

The free surface behind the large block is much darker than for the short block, suggesting a larger amount of meltwater is reaching the surface. Turbulent flows underneath the blocks help explain this observation. The flow underneath the shorter block is entirely turbulent, thoroughly mixing the meltwater. The flow underneath the longer block is instead more laminar, meaning most meltwater does not mix with the ambient water and rises to the free surface. The increased turbulence at the rear of the short block also explains the greater melt rate of the rear face.

**Low relative ambient velocity** — The left column of fig. 3 shows a time series from an experiment performed at 1.5 cm s^−1. Many of the same trends are apparent from the U = 3.5 cm s^−1 experiments. The basal melt is at a maximum behind the leading (left) edge, followed by a return to laminar flow and more uniform melt rate further downstream. The stagnant region size is comparable to that in the higher relative velocity experiments, which supports a Re independent scaling of reattachment length [30]. The primary difference between the U = 1.5 and 3.5 cm s^−1 experiments is lower overall melting and reduced turbulence (dye streaks appear mostly laminar). Meltwater preferentially pools near the free surface, as there is less mixing.
with ambient water than at higher relative velocities $U$ (fig. 2).

**No relative ambient velocity** — The zero relative velocity experiments lack any local increases in the melt rate. Most meltwater flows slowly along the base of the ice block before rising to the surface. However some dyed fluid sinks from the base of the ice as a dense plume. This is best understood as a double-diffusive effect [16]. During the melting, the adjacent salt water absorbs latent heat. The cooling occurs faster than salinity diffusion. With weak ambient flow, the saltwater can cool enough to sink and entrain dyed meltwater along with it.

### 3.2 Quantitative results

![Final side profiles cm](image)

Figure 4: The top, middle, and bottom figures plot final side profiles of each experiment relative to the bottom of the front face at the start of each experiment (origin), for respective ambient velocities $U = 3.5, 1.5, 0 \text{ cm s}^{-1}$. Profiles are coloured according to initial aspect ratio. The dashed lines show an averaged profile over all experiments at a given relative ambient velocity $U$. The approximate edges of each face are shown with circles. The grey bar illustrates a standard deviation in average total basal melt after 10 minutes calculated from table 1.

Figure 4 plots the post-experiment profile of each ice block, and supports previous observations. Melt rates differ between faces, with lowest melting on the base, and highest melting on the sides. Larger ambient velocities $U$ cause larger melt rates, particularly for the front face. Significant non-uniformities in melt rate exist on each face. These non-uniformities are the same for different experiments. Front and basal profiles at each relative ambient velocity $U$ collapse on to a single average profile (dashed) for all initial aspect ratios.

Figure 5 averages over the non-uniform melting of each face. The first three panels plot the average melt rate of each face for each experiment at relative ambient velocity $U = 3.5, 1.5$ and $0.0 \text{ cm s}^{-1}$, as a function of initial aspect ratio. These calculations, derived from post-experiment photographs, quantify previous qualitative observations. Estimated melt rates for each face averaged over all experiments at a given relative ambient velocity $U$ are shown in solid lines in fig. 5. The uncertainties (coloured bars) are defined as twice the standard deviation of the melt rates. These statistical summaries are reproduced in table 1. The final plot in fig. 5 compares the estimated volume loss for each experiment assuming average melt rates (table 1) with the measured volume loss from weighing before and after experiments. The
Figure 5: The first three panels show estimated average melt rates of each face for each experiment, for each ambient velocity $U$. The solid lines plot the average melt rate for each face averaged over all experiments. The uncertainties (coloured regions) are twice the standard deviation in average melt rates for each side. The final panel compares the volume loss estimated from average melt rates (table 1) with the volume loss measured by weighing. Exact agreement is indicated with the dashed line, and the uncertainties in the estimated volume loss are the range of volume changes using high and low estimates for each melt rate.

Photo estimates are close to an exact correspondence (dashed), supporting the averages of table 1.

| Velocity $U$ cm$^{-1}$ | Front melt rate $v_f$ cm min$^{-1}$ | Side melt rate $v_s$ cm min$^{-1}$ | Rear melt rate $v_r$ cm min$^{-1}$ | Basal melt rate $v_b$ cm min$^{-1}$ | WC melt rate $v$ cm min$^{-1}$ |
|-------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 3.5                     | $0.39 \pm 0.06$                 | $0.21 \pm 0.05$                 | $0.16 \pm 0.07$                 | $0.13 \pm 0.04$                 | $0.092 - 0.115$                 |
| 1.5                     | $0.19 \pm 0.04$                 | $0.15 \pm 0.04$                 | $0.14 \pm 0.07$                 | $0.08 \pm 0.018$                 | $0.047 - 0.058$                 |
| 0.0                     | $0.13 \pm 0.04$                 | $0.15 \pm 0.04$                 | $0.13 \pm 0.04$                 | $0.08 \pm 0.016$                 | 0                               |

Table 1: Average melt rates of each face for each flow speed $U$ from fig. 5. The uncertainties are twice the standard deviation in average melt rates in fig. 5. The WC model melt rates are given for upper and lower lengths $L_{max} = 32.5$ cm and $L_{min} = 10$ cm, with $\rho_i = 0.9167$ g cm$^{-3}$, $\rho_w = 1.021$ g cm$^{-3}$, $\nu = 1.00 \times 10^{-2}$ cm$^2$s$^{-1}$, $\kappa = 1.42 \times 10^{-3}$ cm$^2$s$^{-1}$, $c_p = 4.182$ J g$^{-1}$°C$^{-1}$, $\Lambda = 334$ J g$^{-1}$, and $\Delta T = 20$°C, appropriate for ambient water at 20°C [4].

The second through fifth columns of table 1 give experimental melt rates for the front, side, rear and basal melt rates respectively, at each relative velocity $U$. The melt rate of each face differs at all relative velocities $U$. Aspect ratio can therefore affect overall melting by changing the relative areas of these faces. The melt rate of each face increases slightly from $U = 0$ cm s$^{-1}$ to $U = 1.5$ cm s$^{-1}$, and significantly from $U = 1.5$ cm s$^{-1}$ to the more turbulent 3.5 cm s$^{-1}$ experiments. This result agrees with [14], which observed roughly constant melt rates below a threshold relative fluid velocity of 2.5 cm s$^{-1}$. Though FitzMaurice et al.’s findings were based on dominant side melting, and so are not directly applicable to basal melting, the existence of a similar threshold velocity for basal melting is likely.

We give estimated melt rates using the WC model$^1$ in the rightmost column of table 1. The upper and lower values are for the longest ($L = 32.5$ cm) and shortest ($L = 10$ cm) block lengths. The WC model underestimates melt rates of all faces at all velocities, and

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$^1$ Note that we do not use an average internal ice temperature of $-15^\circ$C (the midpoint of the melting temperature 0°C and freezer temperature $-30^\circ$C), as was done in [14]. Using an internal ice temperature implies that colder blocks will melt faster. This contradicts the fact that colder blocks reduce the heat flux difference at the interface, and therefore melt more slowly. We instead use the physically justified melting temperature of the interface.
the disagreement worsens for lower velocities, as found in [14]. The only estimate within experimental uncertainties is for basal melt rates at $U = 3.5 \text{ cm s}^{-1}$. The updated model of [14] accounts for larger melt rates at lower velocities, however it is based on side plumes that are not applicable to basal melting. The WC model is also unable to capture the elevated melt rates of the vertical faces. An accurate parameterisation must therefore account for both magnitude and orientation of relative ambient flow.

4 Computational methods

Existing melting parameterisations do not agree with our experimental findings. We use Direct Numerical Simulation (DNS) of melting ice in warm salty water to investigate the full flow dynamics and further our understanding of our laboratory observations. We simulate this challenging problem with a recent phase-field approach developed for coupled fluid flow, melting, and dissolution [19]. This method builds on previous methods for fluid-solid interactions [20] and simulations of melting in fresh water [9, 27].

4.1 Phase-field model of ice in warm salt water

Ice melting in salt water is often modelled as a moving boundary problem [37]. In the fluid, the temperature $T$ and salinity $C$ satisfy advection-diffusion equations, and the fluid velocity $u$ and pressure $p$ satisfy incompressible Navier-Stokes equations with vertical buoyancy forcing $-g\rho(T, C)\hat{z}$. The solid temperature follows a diffusive equation. Boundary conditions at the melting interface complete the system. The temperature is continuous and equal to the melting temperature, and Stefan, Robin, and Dirichlet boundary conditions conserve energy, salt, and mass respectively.

Phase-field models are a smoothed approximation of the moving boundary formulation that is physically motivated and simple to simulate [5]. Distinct phases are represented with a smooth phase field $\phi$ that is forced to $\phi \approx 1$ in the solid and $\phi \approx 0$ in the fluid. A thin region of size $\varepsilon$ separates the fluid and solid. The phase-field equations augment the bulk equations with smooth source terms that reproduce the boundary conditions in the limit $\varepsilon \to 0$,

$$
\frac{\varepsilon^2}{6} \frac{\Lambda}{c_p\kappa} \partial_t \phi - \gamma \nabla^2 \phi + \frac{1}{\varepsilon^2} \phi(1 - \phi) \left( \gamma(1 - 2\phi) + \varepsilon(T + \lambda C) \right) = 0,
$$

$$
\partial_t T + \nabla \cdot \left( (1 - \phi) u T - \kappa \nabla T \right) = \frac{\Lambda}{c_p} \partial_t \phi,
$$

$$
\partial_t \left( (1 - \phi + \delta) C \right) + \nabla \cdot \left( (1 - \phi + \delta) (u C - \mu \nabla C) \right) = 0,
$$

$$
\partial_t u + u \cdot \nabla u - \nu \nabla^2 u + \nabla p + \frac{g\rho(T, C)}{\rho_0} \hat{z} = -\frac{\nu}{\eta} \phi u,
$$

$$
\nabla \cdot u = 0.
$$

Here $\nu, \kappa,$ and $\mu$ are momentum, thermal, and salt diffusivity, $\Lambda$ is the latent heat, $c_p$ is the heat capacity of water, and $\lambda$ is a slope coefficient of the liquidus, which is assumed linear for simplicity [19]. The damping time $\eta \ll 1$ suppresses advection in the solid, $\gamma \ll 1$ expresses curvature dependence of the melting temperature, and $\delta \ll 1$ regularises the salinity equation within the ice. Replacing moving boundary conditions with smooth source terms allows simple numerical implementations that converge to the moving boundary formulation with error of order $\varepsilon^2$ [19].

Note that this model omits second order thermodynamic effects. We ignore density changes during melting, consider constant viscous, thermal, and solutal diffusivities, and describe buoyancy with the Boussinesq approximation (using the EOS-80 equation of state of seawater). We
also use two-dimensional simulations to reduce computational costs. While the experiments
did not vary much in the spanwise direction orthogonal to the flow, the reduced dimension is
known to generate larger vortices than in three dimensions.

4.2 Numerical method and simulation parameters

We perform two series of simulations. The first series considers large relative ambient velocity
$U = 3.5 \text{ cm s}^{-1}$, and compares a simulation with temperature, salinity, and buoyancy effects
turned on ($T+C$) and a simulation with salinity and buoyancy forcing neglected ($T$ only). The
second series investigates double-diffusive effects with no relative ambient velocity $U = 0 \text{ cm s}^{-1}$
by comparing simulations with equal temperature and salt diffusivities (single diffusion: SD)
and different temperature and salt diffusivities (double diffusion: DD). All simulations specify
the initial ice temperature as $T_i = 0 \degree C$, the initial water temperature as $T_w = 20 \degree C$, the
initial salinity as $C_w = 30 \text{ g kg}^{-1}$, the liquidus slope as $\lambda = 0.056 \degree C \text{ kg g}^{-1}$, the latent heat
as $\Lambda = 3.34 \times 10^2 \text{ J g}^{-1}$, and the heat capacity as $c_p = 4.2 \text{ J g}^{-1} \degree C^{-1}$. The relative ambient
velocity $U$, domain dimensions $\ell \times d$, block dimensions $L \times D$, and diffusivities $\nu, \kappa, \mu$ are given
in table 2. We use a realistic Prandtl number $Pr = \nu / \kappa$ of $7$. Due to computational constraints,
we are limited to larger salt diffusivities than reality, so the Schmidt number $Sc = \nu / \mu$ is at
most 50 rather than 500.

All simulations are performed using the spectral code Dedalus [8]. The equations are dis-
cretised with Fourier series in the horizontal direction, and trigonometric series in the vertical
direction. This corresponds to periodic horizontal boundary conditions; homogeneous Neumann
vertical boundary conditions for the horizontal velocity $u$, temperature $T$, salinity $C$, pressure
$p$, and phase field $\phi$; and homogeneous Dirichlet vertical boundaries for the vertical velocity $w$.
The simulations at $U = 3.5 \text{ cm s}^{-1}$ use a volume penalised ‘sponge layer’ at the beginning of the
domain to force the fluid temperature, salinity, and velocity to ambient values [20]. The parity
constraint of the buoyancy term in the vertical momentum equation is enforced by tapering
buoyancy near the vertical boundaries over a length scale of $0.05 \text{ cm}$. The system is integrated
using a second order implicit-explicit Runge-Kutta timestepper. The horizontal mode number
$n_x$, vertical mode number $n_z$, time step size $\Delta t$ and numerical phase field parameters are given
in table 3. All code is available online [18].

Table 2: Model parameters for flow and no-flow simulations.

| Simulation      | $U$ (cm s$^{-1}$) | $\ell$ (cm) | $d$ (cm) | $L$ (cm) | $D$ (cm) | $\nu$ (cm$^2$ s$^{-1}$) | $\kappa$ (cm$^2$ s$^{-1}$) | $\mu$ (cm$^2$ s$^{-1}$) |
|-----------------|------------------|-------------|-----------|----------|----------|------------------------|--------------------------|--------------------|
| Flow $T+C$      | 3.5              | 50          | 15        | 30       | 3        | $1.3 \times 10^{-2}$   | $1.86 \times 10^{-3}$   | $9.3 \times 10^{-4}$   |
| Flow $T$ only   | 3.5              | 50          | 15        | 30       | 3        | $1.3 \times 10^{-2}$   | $1.86 \times 10^{-3}$   | N/A                |
| No-flow DD      | 0                | 20          | 10        | 10       | 3        | $1.3 \times 10^{-2}$   | $1.86 \times 10^{-3}$   | $2.6 \times 10^{-4}$   |
| No-flow SD      | 0                | 20          | 10        | 10       | 3        | $1.3 \times 10^{-2}$   | $1.86 \times 10^{-3}$   | $1.86 \times 10^{-3}$   |

Table 3: Numerical parameters for flow and no-flow simulation series.

| Simulation series | $n_x$ | $n_z$ | $\Delta t$ (s) | $\varepsilon$ (cm) | $\gamma$ (cm$^3$ C$^{-1}$) | $\eta$ (s) | $\delta$ (s) |
|-------------------|-------|-------|-----------------|---------------------|--------------------------|------------|--------------|
| Flow              | 6144  | 1536  | $1.6 \times 10^{-4}$ | 0.01                | 0.2                      | $1.75 \times 10^{-3}$ | $5 \times 10^{-3}$ |
| No-flow           | 2048  | 1024  | $5 \times 10^{-4}$  | 0.01                | 0.2                      | $1.75 \times 10^{-3}$ | $1 \times 10^{-4}$ |
5 Computational results

5.1 High relative ambient velocity

The first series simulates the experiments at high relative ambient velocity $U = 3.5 \text{cm s}^{-1}$. The $T + C$ simulation uses the equation of state of seawater (EOS-80) to capture salinity and buoyancy effects from experiments, while the $T$ only simulation ignores these effects.

Figure 6 shows a time series of temperature in the $T + C$ and $T$ only simulations. Both simulations display similar patterns and reproduce several qualitative features of the experiment. The largest melting occurs on the front (left) face, where warm ambient water collides with the ice. The front melting increases with depth, causing a decrease in slope over time. The base melts most rapidly at the centre, but is overall slower than the front. These features are present in both simulations and therefore cannot be controlled by the buoyancy of the meltwater. At high ambient velocities heat transported by the flow determines melting.

Figure 7 quantifies the front, basal, and rear melting for each simulation. The front melting is steady in both simulations (fig. 7 top left), with little deviation between instantaneous and cumulative melt rates (fig. 7 bottom left), and an increase in front melting with depth (fig. 7 bottom left). The $T + C$ simulation reproduces the sloped front of experiments, while the $T$ only simulation has a vertical gradient at the top of the front. This difference in slope occurs because cool, buoyant meltwater pools at the top of the $T+C$ simulation, thickening the thermal boundary layer and slowing melting. Despite this difference, both simulations reproduce the experimental average front melt rate of $0.39 \pm 0.06 \text{ cm min}^{-1}$, with $0.46 \pm 0.14 \text{ cm min}^{-1}$ for the $T + C$ simulation, and $0.45 \pm 0.09 \text{ cm min}^{-1}$ for $T$ only (fig. 7 centre left).

The basal melting shows significant variation in space and time (fig. 7 top middle). Both simulations show the same distinct regions of basal melting. Just behind the leading edge, melt rates are low and steady. This stagnant region is evident in the time series plots of fig. 6. Limited mixing with the warm ambient water causes reduced melting near the leading edge. Behind this region, pooled meltwater becomes unstable due to the strong velocity shear, leading to vortex generation and shedding around the centre of the block. The unsteady flow circulates warm ambient water to the base, and is associated with the transition to larger melt rates past the centre of the base. Beyond this point the melting is on average lower, with intermittent periods of high melting occurring as vortices are shed downstream. The transition to higher melting slowly moves backward over time. This is partly because the front itself is receding, due to melting. More importantly the slope of the front face reduces over time, delaying the instability of the shear layer generated at the separation point. However the region of maximum cumulative melting does appear to saturate around $x = 28 \text{ cm}$ at late times.

The localised spatial and temporal features of the basal melting lead to nontrivial statistical properties, summarised in histograms of instantaneous melt rates in the second row of fig. 7. There is large variance and skewness in the basal melt rates, as expected from the different melting regions. The average basal melt rates of the simulations ($T + C : 0.17 \pm 0.08 \text{ cm min}^{-1}, T$ only : $0.16 \pm 0.08 \text{ cm min}^{-1}$) agree with the upper range of experimental results ($0.13 \pm 0.04 \text{ cm min}^{-1}$), supporting the validity of the simulations.

The time averaged cumulative melt rates are given in row three, and clearly reproduce the localised increase in basal melting from the experiments. The cumulative basal melt varies with distance by a factor of two, highlighting the importance of localised flow features in melting predictions. The location of maximum melting is further downstream than observed in experiments, however this may be due to small domain size, or two rather than three dimensional turbulence. Nevertheless it is clear that the buoyancy plays little role for large ambient velocities. It is advection of heat that drives melting.

Figure 8 illustrates the mechanism by which the flow induces melting hot spots. The two left figures show snapshots of the temperature, vorticity, and velocity at $t = 75 \text{ s}$. The temperature
Figure 6: Time series, at one minute intervals, of temperature for melting simulations in warm salt water at relative ambient velocity $U = 3.5$ cm s$^{-1}$. The $T + C$ simulation (left) includes temperature, salinity, and buoyancy, while the $T$ only simulation (right) omits salinity and buoyancy effects.

Figure 7: The left, middle, and right columns summarise front, base, and rear melting respectively for $U = 3.5$ cm s$^{-1}$ simulations ($T + C$ and $T$ only). The front is defined where the interface slope is less than $-1$, the base where the slope is between $-1$ and $1$, and the rear where the slope is greater than $1$. The top row plots instantaneous melt rates in space and time. The centre row gives histograms, mean, and standard deviation of instantaneous melt rates. The bottom row plots cumulative melt rates over time.
Figure 8: The left panels give snapshots of the temperature (top) and vorticity (bottom) for the $T + C$ simulation at 75 s. The vorticity plot also shows the fluid velocity (black arrows). The melt rate (solid red line) is enhanced where vortices generate upwelling. The right panel plots the basal melt rate over space and time. Vortex locations (grey) are shown where the depth integrated vorticity is less than $-2$. The time $t = 75$ s is shown in black.

Field shows a cool stagnant region that persists past the separation point at the front edge. Mixing between this region and the ambient fluid is slow and intermittent, and the melt rate (red curve) remains low. The vorticity plot shows prominent vortices being generated behind the stagnant region due to Kelvin-Helmholtz instability of the shear layer. The fluid velocity (in arrows) reveals upwelling downstream from the vortices, which coincides with increased basal melting (red curve). This explains both the mechanism by which vortices enhance melting – upwelling of warm ambient water – and accounts for the offset between vortex position and enhanced melting.

The right panel of fig. 8 shows the instantaneous basal melt rate over space and time. On top of this plot, we highlight the location of vortices in grey by thresholding the depth integrated vorticity. This plot clarifies the mechanism by which vortices enhance melting. Basal melt rates are low (roughly 0.1 cm min$^{-1}$) in the stagnant region where there are no vortices to generate mixing. Behind this region vortices are being created. These vortices transport warm ambient water to the base of the ice by upwelling to their right. This is clear from the large melt rates (almost 0.5 cm min$^{-1}$) to the right of the vortex paths (grey stripes) in fig. 8. The vortices then reach a maximum size and are rapidly advected away by the flow. In this stage they are no longer close enough to the base to enhance melting and the melt rate returns to a lower average value of around 0.2 cm min$^{-1}$. We therefore do not expect a recurrence of localised melting without a new source of instability to generate vortices and enhanced mixing.

The rear melting also shows noticeable variation in space and time (fig. 7 top right), with an important difference from experiments. While the rear face in experiments sloped to the right (fig. 4), the rear face of the $T + C$ simulation is on average vertical, and the $T$ only rear face slopes toward the left (fig. 7 bottom right). The incorrect sloping of the rear face in the simulations is likely due to large coherent vortices that remain behind the block, circulating warm ambient water toward the rear face from above (fig. 6). It is possible these coherent vortices are two-
dimensional features which would break apart in three dimensions. However, average rear melt rates are similar to the experimental value of $0.16 \pm 0.07 \text{cm min}^{-1}$, with $0.18 \pm 0.13 \text{cm min}^{-1}$ for the $T + C$ simulation, and $0.20 \pm 0.11 \text{cm min}^{-1}$ for the $T$ only simulation.

5.2 No relative ambient velocity

The next simulation series investigates melting in an initially quiescent fluid ($U = 0 \text{cm s}^{-1}$). Experiments showed some meltwater sinking beneath the block, despite the high salinity of the ambient water. We show how double-diffusivity causes this sinking by comparing a double-diffusive (DD) simulation with different thermal and salinity diffusivity (Lewis number $Le = \kappa/\mu = 50/7$) to a single-diffusive (SD) simulation with equal diffusivities ($Le = 1$). True Lewis numbers $Le$ are much larger for salt water, however these simulations are enough to demonstrate the existence of double-diffusive effects. Both simulations use identical buoyancy functions (EOS-80 of seawater) with equivalent dependence on temperature and salt.

Figure 9 plots time series of temperature and salinity of the two simulations of melting ice in initially stationary salt water. Both simulations show a clear tendency of meltwater (dark red/purple) to rise and pool near the free surface, reducing melting in that region at late times (fig. 10 top left). But there are important differences between the simulations. The layer of meltwater is more diffuse for equal diffusivities (SD). And it is only with different diffusivities (DD) that sinking plumes emerge beneath the ice block. This confirms the double-diffusive origin of sinking plumes in experiments. Initially the sinking plumes are concentrated at the sides of the ice block, where the geometry aids the instability. However the sinking tends over time to a persistent downward plume beneath the centre of the ice block (as in experiments). In contrast, no sinking plumes occur beneath the ice block for the equal diffusivity simulation. Some sinking is observed beneath the pooled meltwater, but this is because the rising water beneath the block sets up a recirculating flow that sinks and entrains meltwater near (periodic) horizontal boundaries. All meltwater around the ice block rises without double-diffusion. This leads to large differences in flow patterns, affecting melt rates.

Figure 10 quantifies the melting of the two simulations. The first row plots a colour time series of the instantaneous melt rates on the side and base of each block. Fine-scale localised melting behaviour occurs on all faces in the double-diffusive simulation, whereas the equal diffusivity simulation shows little spatial or time variation. This difference follows from the intermittent plumes generated via double diffusion. The second row of fig. 10 shows that these distributions result from processes with nontrivial spatial and temporal structure, and are not simple Gaussian processes. The mean and standard deviation of melt rates are larger for both faces for the double-diffusive simulation (DD). The double-diffusive simulation has a side melt rate ($0.11 \pm 0.10 \text{cm min}^{-1}$ (DD)) that is closer to experimental melt rates ($0.13 \pm 0.04 \text{cm min}^{-1}$) than the equal diffusive simulation ($0.09 \pm 0.08 \text{cm min}^{-1}$ (SD)). The slight underestimate of side melting is an understandable consequence of the restricted domain size, which causes pooling meltwater to reduce side melt rates at late times. The basal melting of the double-diffusive simulation ($0.09 \pm 0.10 \text{cm min}^{-1}$ (DD)) and equal diffusivity simulation ($0.07 \pm 0.02 \text{cm min}^{-1}$ (SD)) are both close to experimental basal melt rates ($0.08 \pm 0.016 \text{cm min}^{-1}$). In the final row (fig. 10) we give the cumulative melt rates, which show faster side melting for the DD simulation at early times, which become slower at late times due to pooled meltwater. Both simulations predict larger side melt rates than basal melt rates, as in experiments. The difference in melt rates and flow patterns demonstrate the potential importance of double-diffusive effects for icebergs at low relative flow velocities.
Figure 9: Time series, at one minute intervals, of temperature (left) and salinity (right) for simulations in quiescent salt water. The left simulation in each column uses a lower salt diffusivity (DD), while the right simulation in each column has equal salt and thermal diffusivity (SD).

Figure 10: The left and right columns summarise side and basal melting respectively for the no relative ambient velocity simulations. The first row shows instantaneous melt rates over space and time. The second row gives a histogram, average, and standard deviation of instantaneous melt rates. The final row plots cumulative melt rate during the simulation. The right face is analogous to the left face and omitted.
6 Geophysical application

The laboratory and numerical results highlight that melt rates vary on each ice block face, and that different faces have different average melt rates. The average melt rates of each face are also significantly higher than predicted by commonly used parameterisations. These differences in melt rates matter for melting of real icebergs.

The basal melt rate $v_b$ for each velocity is plotted in dashed lines, and annotated on the plot.

We expect melt rates to differ between faces for real icebergs, as different faces will be exposed to different fluid velocities, and experience different buoyancy effects. Even relatively low velocities towards an ice face cause higher melt rates than velocities parallel to the ice face [24]. Different melt rates for different faces should occur even without relative ambient flows, as buoyancy driven plumes induce larger velocities and melt rates on the sides [14].

Aspect ratios of geophysical icebergs also vary significantly, affecting the relative importance of melt rates on each face. The B-15 iceberg had an estimated length of 300 km and width of 40 km [3]. Using a depth of 600 m as an upper limit [10] suggests a minimum aspect ratio of approximately 180 (using the geometric mean of the two horizontal length scales). At the other extreme, stability suggests a minimum aspect ratio of 1.4 for a 50 m deep iceberg. This means the basal area of B-15 comprises 97% of the total submerged area while the basal area of a smaller marginally stable iceberg is only 27%. Different melt rates on each face will therefore matter for aspect ratios close to unity, whereas large aspect ratio icebergs will be dominated by basal melting.

We illustrate the influence of aspect ratio on melting using a simple geometric model of a melting block of ice that assumes each face melts uniformly. The length $L$, width $W$, and submerged depth $D$ are related to melt rates of the front $v_f$, rear $v_r$, side $v_s$, and base $v_b$ via $L = -v_f - v_r$, $W = -2v_s$, and $D = -v_b$ (denoting time derivatives with dots). The immersed volume $V$ and area $A$ are $V = LWD$, and $A = 2LD + 2WD + LW$. The geometrically averaged melt rate $v_{av}$ normalises the volume loss rate by the surface area, $v_{av} \equiv \frac{1}{A} \frac{d}{dt} |V| = \left( v_f + v_r \right) \frac{WD}{A} + 2v_s \frac{LD}{A} + v_b \frac{LW}{A}$, which weights the melt rate of each face by its relative proportion of the total area.

Figure 11 plots the geometrically averaged melt rate $v_{av}$ as a function of aspect ratio $L/D$, using average melt rates from experiments (table 1) and simplifying the horizontal dimensions.
as equal \((W = L)\). Figure 11 shows significant variation in the geometrically averaged melt rate \(v_{av}\) from aspect ratio 1 to 50. Ice blocks with unit aspect ratio \(L = D\) melt more than 50\% faster than large aspect ratio ice blocks, with elevated overall melting (relative to the dashed basal melt rates) apparent even for aspect ratio 10.

The toy model also offers a simple criterion for side-dominated versus base-dominated melting. An iceberg will experience side-dominated melting if the aspect ratio decreases over time, and base-dominated melting if the aspect ratio increases over time. If we simplify to two dimensions, length \(L\) and depth \(D\), with respective melt rates \(v_s\) and \(v_b\), then the time derivative of the aspect ratio is

\[
\frac{d}{dt} \left( \frac{L}{D} \right) = \frac{LD - L \dot{D}}{D^2} = \frac{L}{D} \left( \frac{v_s}{L} - \frac{v_b}{D} \right)
\]

The time derivative of aspect ratio changes sign when the ratio of side to base melting \(v_s/v_b\) is equal to the aspect ratio \(L/D\). Therefore even icebergs with larger side melting can become dominated by basal melting if the aspect ratio is sufficiently large. Returning to three dimensions, if the melt rate differs between different side faces (as for the experiments with relative ambient velocity), different regimes can be defined for each orientation of the iceberg.

We also expect non-uniform melt rates on each face to affect geophysical icebergs. The most common iceberg length of tabular icebergs in the Southern Ocean is 400 m, and the most common freeboard \(f\) is 35 m \[29\]. Using the empirical relationship \(D = 49.4 f^{0.2} \) \[29\] gives a most common depth \(D\) of approximately 100 m. Our experiments find a maximum in melting at a distance of two and a half times the depth, similar to studies of heat transfer past a forward facing step \[1\]. This predicts a maximum in basal melting at approximately 250 m, a significant proportion of the basal length of the modal tabular iceberg. This acts to further enhance overall melt rates for smaller aspect ratio icebergs. Of course, confirming that this proportionality holds at larger scales, with ocean stratification, would require further work. Nevertheless studies of flow past a forward facing step find this length scale is a small multiple of the step height over a large range of Reynolds numbers \[30\].

## 7 Conclusions

Typical existing parameterisations of iceberg melting ignore the aspect ratio of icebergs \[36, 21\]. We conducted a series of laboratory experiments and numerical simulations to examine the dependence of the melt rate on iceberg size and shape for three different ambient velocities. We find that geometry has a strong effect on the melt rate of icebergs.

Melt rates are highest on the forward facing side (with respect to the ambient flow), followed by the remaining lateral sides, with slowest melting occurring at the base of the iceberg. Changing the relative area of each face will thus change the overall melt rate, with significant variation between small and large aspect ratio icebergs.

Furthermore, the melt rate of each face is itself spatially non uniform, with localised increases in basal melt rates of up to 50\% observed. These localised regions correspond to the reattachment zones of flow past a forward-facing block, and occur at a distance approximately two to three times the depth of the block \[1\]. Our numerical investigation showed that these non-uniformities in basal melt are caused by the generation of vortices, which lead to upwelling of warm water during their formation. This non-uniformity exists at large relative speeds, and is not influenced by buoyancy. We therefore expect these non-uniformities to increase melting for small-aspect ratio icebergs.

To improve melting estimates, we emphasise that models of melt rates must depend on both the speed and orientation of the background flow, and that differing melt rates must be weighted according to the shape and aspect ratio of an iceberg.
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