Probabilistic Framework for Constrained Manipulations and Task and Motion Planning under Uncertainty

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Abstract—Logic-Geometric Programming (LGP) is a powerful motion and manipulation planning framework, which represents hierarchical structure using logic rules that describe discrete aspects of problems, e.g., touch, grasp, hit, or push, and solves the resulting smooth trajectory optimization. The expressive power of logic allows LGP for handling complex, large-scale sequential manipulation and tool-use planning problems. In this paper, we extend the LGP formulation to stochastic domains. Based on the control-inference duality, we interpret LGP in a stochastic domain as fitting a mixture of Gaussians to the posterior path distribution, where each logic profile defines a single Gaussian path distribution. The proposed framework enables a robot to prioritize various interaction modes and to acquire interesting behaviors such as contact exploitation for uncertainty reduction, eventually providing a composite control scheme that is reactive to disturbance. The supplementary video can be found at https://youtu.be/CEaJdVlSZyo.

I. INTRODUCTION

Manipulation planning problems often involve two major difficulties, namely high-dimensionality and discontinuous contact dynamics, which prohibit widely-used motion planning algorithms such as sampling-based planning [1], [2] or trajectory optimization [3], [4], [5] from being directly applicable. To handle such difficulties, hybrid approaches have been proposed, where additional variables that explicitly represent discrete aspects of problems are incorporated into optimization and jointly optimized [6], [7], [8]. For example, contact invariant optimization [6] relaxes the discontinuity of contact dynamics and utilizes the additional continuous-valued variable to express contact activity that enforces the trajectory to be consistent with physics. In [7], the additional integer variables describe hybrid contact activities and the resulting Mixed-Integer Programming is solved with optimization algorithms involving branch-and-bound. Logic-Geometric Programming (LGP) [8] adopts logic rules to describe discrete aspects of problems on a higher level than typical contact activities, e.g., touch, hit, push, or more general tool-use. A sequence of these logic states (called a skeleton) directly implies contact activities over time, which imposes equality/inequality constraints for smooth trajectory optimization. The expressive power of logic enables LGP to enumerate valuable local optima of the planning problem by searching over the logic space.

In this work, we present a probabilistic framework of such hybrid trajectory optimization by extending LGP to stochastic domains, where the dynamics is described stochastically and the cost function is the expectation over all possible trajectories. The corresponding problems can be formulated as stochastic optimal control (SOC), which gives rise to some important and interesting features. First, when comparing various local optima (plans) with different skeletons, the robustness of the plans should be taken into account. For example, consider a planning problem in Fig. 1 whose objective is to push some object on a table towards target area using a single finger or two fingers. Both plans might incur similar costs in a deterministic sense, but the single-finger push strategy is much less favorable in reality because it is more vulnerable to disturbances and uncertainties. Second, we can observe contact exploitation behaviors. To cope with actuator disturbance, a robot might want to fix some parts of its body, e.g., elbow, on the desk to reduce the uncertainty of end-effector’s position. Lastly, depending on the deviation from the plan, a robot might decide whether or not to switch to another plan or even to stay in-between them. The original LGP formulation is only deterministic so, even though it can find various feasible plans, a robot cannot help but choose one plan to execute based on the deterministic path cost. In contrast, the probabilistic framework in this work allows a robot for prioritizing different plans by taking robustness as well as deterministic path cost into account and for acquiring interesting contact exploitation behaviors. Furthermore, a composite reactive controller constructed from various plans enables a robot to adaptively choose which plan(s) to follow.

The technical aspect of this work is based on the duality between control and inference [9], [10], [11]. Under this duality, motion planning is equivalent to inference of posterior path distribution. As in trajectory optimization, the gradient and Hessian can accelerate the inference procedure, which relates to the Laplace approximation [12]. Given the fact that the prescribed logics provide smoothness of sub-problems,
we interpret LGP in stochastic domains as fitting a mixture of Gaussians to the posterior path distribution, which expresses a single Gaussian path distribution.

II. BACKGROUND

A. Stochastic optimal control as KL-minimization

Consider the configuration space $\mathcal{X} = \mathbb{R}^{d_x}$ of an $d_x$-dimensional robot and an initial configuration $x_0 \in \mathcal{X}$ and a velocity $\dot{x}_0$ are given. Let $z = (x, \dot{x}) \in \mathbb{R}^{2d_x}$ be a state vector, $u \in \mathbb{R}^{d_u}$ be a control vector which represents torques or desired accelerations of actuated joints ($d_u \leq d_x$), and $w$ be a $d_x$-dimensional Wiener process that is injected into the robot's actuators. Then the robot dynamics can be written as the following stochastic differential equation (SDE), which is affine in the control input and the disturbance:

$$dz(t) = f(z(t))dt + G(z(t)) u(t)dt + \sigma dw(t),$$

where $f : \mathbb{R}^{2d_x} \to \mathbb{R}^{2d_x}$ is the passive dynamics and $G : \mathbb{R}^{2d_x} \to \mathbb{R}^{d_x \times d_u}$ is the control transition matrix function. With an instantaneous state cost function $V : \mathbb{R}^{2d_x} \to \mathbb{R}^+$, an SOC problem is formulated as follow:

$$J = \mathbb{E}_{p_u}[V_T(z(T)) + \int_0^T V(z(t)) + \frac{1}{2\sigma^2} \lambda^T u(t)^T u(t) dt],$$

where $p_u$ is the probability measure induced by the controlled trajectories in $[1]$, with $z(0) = x_0 = (x_0, \dot{x}_0)$. The objective of an SOC problem is then to find a control policy $u(t) = \pi^*(z(t), t)$ that minimizes the cost functional $[2]$.

The above types of SOC problems, which are defined with control/disturbance-affine dynamics and quadratic control cost, are called linearly solvable optimal control problems and have interesting properties [13, 11]. In particular, they can be transformed into Kullback-Leibler (KL) divergence minimization [14, 15] using the following theorem:

**Theorem 1 (Girsanov’s Theorem [16]):** Suppose $p_u$ is the probability measure induced by the uncontrolled trajectories from $[1]$ with $z(0) = (x_0, \dot{x}_0)$ and $u(t) = 0 \ \forall t \in [0, T]$. Then, the Radon-Nikodym derivative of $p_u$ with respect to $p_0$ is given by

$$\frac{dp_u}{dp_0} = \exp \left( \frac{1}{2\sigma^2} \int_0^T \|u(t)\|^2 dt + \frac{1}{\sigma} \int_0^T u(t)^T dw(t) \right),$$

where $w(t)$ is a Wiener process for $p_0$.

With Girsanov’s theorem, the objective function $[2]$ can be rewritten in terms of KL divergence:

$$J = \mathbb{E}_{p_u}[V_T(z(T)) + \int_0^T V(z(t)) + \frac{1}{2\sigma^2} u(t)^T u(t) dt + \log \frac{dp_u(z(t))}{dp_0(z(t))}]$$

$$= \mathbb{E}_{p_u}[V_T(z(T)) + \int_0^T V(z(t)) dt + \log \frac{dp_u(z_0, T)}{dp_0(z_0, T)}]$$

$$= \mathbb{E}_{p_u}[\log \frac{dp_u(z_0, T)}{dp_0(z_0, T)}] - \mathbb{E}_{p_u} [\exp (-V(z_0, T))] / \xi - \mathbb{E}_{p_u} [\exp (-V(z_0, T))]$$

$$= D_{KL} (p_u(z_0, T) \| p^*(z_0, T)) - \log \xi,$$

where $z_{0:T} \equiv \{z(t); \forall t \in [0, T]\}$ is a state trajectory, $V(z_{0:T}) = V_T(z(T)) + \int_0^T V(z(t)) dt$ is a trajectory state cost and $\xi = \int \exp (-V(z_0, T)) dp_0(z_0, T)$ is a normalization constant. Because $\xi$ is a constant, $p^*(z_{0:T})$ can be interpreted as the optimally-controlled trajectory distribution that minimizes the cost functional [2].

$$dp^*(z_{0:T}) = \frac{\exp (-V(z_{0:T})) dp_0(z_{0:T})}{\int \exp (-V(z_{0:T})) dp_0(z_{0:T})},$$

and the corresponding optimal cost is given by

$$J^* = -\log \int \exp (-V(z_{0:T})) dp_0(z_{0:T}).$$

Once the optimal trajectory distribution is obtained, the optimal control can be recovered by enforcing the controlled dynamics to mimic the optimal trajectory distribution, e.g., via moment matching. By applying Girsanov’s theorem to [5], the optimal trajectory distribution can be expressed as:

$$dp^*(z_{0:T}) \propto dp_u(z_{0:T}) \exp (-V_u(z_{0:T})),$$

where $V_u(z_{0:T}) = V(z_{0:T}) + \frac{1}{2\sigma^2} \int_0^T \|u(t)\|^2 dt + \int_0^T u(t)^T dw(t)$. The SOC framework that utilizes the importance sampling scheme to approximate this distribution is called path integral control [17, 14]. It samples a set of trajectories $z^i_{0:T} \sim p_u(\cdot)$, assigns their importance weights as $w^i \propto \exp (-V_u(z^i_{0:T}))$, and computes the optimal control by matching the first (and second) moments of $p_u$ to $p^*$.

B. Laplace approximation of path distributions

Instead of relying on sampling schemes for approximating $p^*$, this work builds on the efficient local optimization methods by investigating a close connection between second order trajectory optimization algorithms [3, 4, 5] and the Laplace approximation. The Laplace approximation fits a normal distribution to the first two derivatives of the log target density function at the mode. Let $x = x_{1:N} = (x_1, x_2, ..., x_N) \in \mathbb{R}^{N \times d_x}$ be a path representation of $z_{0:T}$, which is a path of $N$ time steps in the configuration space $\mathcal{X}$. In this path representation, the velocity and acceleration (and control inputs) of the joints can be computed from two and three consecutive configurations, respectively, using the finite difference approximation [4]. Slightly abusing the notation, the uncontrolled path distribution and the state trajectory cost are then expressed as functions of $x$:

$$p_0(x) \propto \exp \left(- \sum_{n=1}^{N-1} f_0(x_{n-2:n}) \right),$$

$$\exp (-V(x)) = \exp \left(- \sum_{n=1}^{N} f_V(x_{n-1:n}) \right).$$

1. Note that the second term in the exponent of [3] disappears when taking expectation w.r.t. $p_0$, i.e. $\mathbb{E}_{p_0}[\|u(t)\|^2 dw(t)] = 0$.

2. Our method especially builds upon the framework of k-order Motion Optimization (KOMO) [5] which has the same efficiency as the others while being able to address more general problems.

3. The path representation significantly reduces the size of optimization problems, which leads to better numerical stability [18, 19, 5].
for an appropriately given prefix \( x_{-1:0} \). Then, the problem of finding the mode \( x^* \) of \( p^*(x) \propto p_0(x) \exp(-V(x)) \) is an unconstrained nonlinear program (NLP):

\[
\min_{x_{1:N}} \sum_{n=1}^{N} f_0(x_{n-2:n}) + f_V(x_{n-1:n}),
\]

which can be solved using the Newton-Raphson algorithm, i.e.,

\[
x^{k+1} = x^k - \nabla^2 f(x^k)^{-1} \nabla f(x^k)
\]

where \( f(x) = \sum_{n=1}^{N} f_0(x_{n-2:n}) + f_V(x_{n-1:n}) \). With a solution, \( x^* \), and a Hessian at the solution, \( \nabla^2 f(x^*) \), the resulting Laplace approximation is given by:

\[
p^*(x) \approx N(x|x^*, \nabla^2 f(x^*)^{-1}).
\]

The optimal cost is also approximated similarly from [6]:

\[
J^* \approx f(x^*) - \frac{1}{2} \log \frac{|\nabla^2 f_0(x^*)|}{|\nabla^2 f(x^*)|}.
\]

Note that the covariance of the approximate distribution has full rank for fully-actuated robots \( (d_u = d_x) \), but not for underactuated \( d_u < d_x \). In such cases, the NLP should be formulated with equality constraints that restrict the uncontrolled subspace to be consistent with the dynamics [1]. Details will be addressed in Section III-B.

III. ESTIMATING THE PATH DISTRIBUTION FOR CONSTRAINED TRAJECTORY OPTIMIZATION

A. Logic geometric programming in stochastic domains

We now consider more general manipulation planning problems where the configuration space \( \mathcal{X} = \mathbb{R}^{d_x \times SE(3)^m} \) involves \( m \) rigid objects as well as a \( d_x \)-dimensional robot. The dynamics in [1] becomes complicated in this case, because it should express interactions between the robot and the objects. The objects are controllable only when they are in contact with the robot, thus the dynamics [1] is discontinuous around the contact activities. Local optimization methods are no longer effective since they cannot utilize the well-defined gradient and Hessian along the directions of contact switching. Logic Geometric Programming (LGP) addresses this difficulty by augmenting the formulation with additional logic decision variables, \( (s_{1:K}, a_{1:K}) \), which describe discrete aspects of dynamics in a higher level, e.g., touch, hit, push, or more general tool-use. A mode \( s_k \) imposes a set of equality/inequality constraints for the prescribed contact activities to the optimization while a switch \( a_k \) represents transitions between the modes. We can formulate an LGP problem in stochastic domains as follows:

\[
\min_{u_{0:T}, a_{1:K}, s_{1:K}} \mathbb{E}_{p_u} \left[ V_T(z(T)) + \int_0^T V(z(t)) + \frac{1}{2g^2} ||u(t)||^2 dt \right]
\]

s.t. \( \forall t \in [0, T] : h_{path}(z(t), s_k(t)) = 0, g_{path}(z(t), s_k(t)) \leq 0 \)
\[
dz(t) = f_{s_{k}(t)}(z(t)) dt + g_{s_{k}(t)}(z(t)) (u(t) dt + \sigma \, dw(t))
\]

\[
\forall K_{k=1} : h_{switch}(z(t), a_k(t)) = 0, g_{switch}(z(t), a_k(t)) \leq 0, s_k \in \text{succ}(s_{k-1}, a_k).
\]

Here, the SDE is conditioned on \( s \) so that it can be defined only in the remaining subspace which is not constrained by the path constraints, \( (h, g)_{path} \). For example, when a mode specifies manipulation of a particular object, the SDE represents the robot’s dynamics constrained for that specified interaction and the dynamics of the manipulated object is defined by path constraints. Because the contact activities are prescribed by the skeleton \( a_{1:K} \) and the smooth switch constraints \( (h, g)_{switch} \), the dynamics in (13) is now smooth w.r.t. the state and the control inputs, thereby making the corresponding SOC, \( \mathcal{P}(a_{1:K}) \), to be smooth.

LGP problems are often addressed with a two-level hierarchical approach [20], [8], where a higher-level module proposes a skeleton \( a_{1:K} \) using, e.g., tree search and a lower-level NLP solver returns a solution of \( \mathcal{P}(a_{1:K}) \). LGP in stochastic domains (13) has two distinctive features: While \( \mathcal{P}(a_{1:K}) \) is evaluated for a single optimal trajectory in the deterministic case, the path distribution should instead be considered, which results in the additional stochastic cost term (Sec. III-B). In addition, various modes allow for constructing the composite reactive control law (Sec. IV-B).

B. Probabilistic LGP as fitting a mixture of Gaussians

Let \( \{a_i = a_{1:K}^{(i)}; i = 1, ..., N_a \} \) be a set of candidate skeletons for an LGP problem (13). We now attempt to approximate the optimal path distribution as a mixture distribution, where each skeleton defines one mixture component:

\[
p^*(x) \propto \sum_{i=1}^{N_a} p^*(a_i) p^*(x|a_i).
\]

The mixture component \( p^*(x|a_i) \) corresponds to the SOC problem \( \mathcal{P}(a_i) \), of which support is defined by the constraints of the original problem, i.e.:

\[
p^*(x|a_i) \propto p_0(x|s_i) \exp(-V(x))
\]

s.t. \( h_{path}(x, s) = 0 \), \( g_{path}(x, s) \leq 0 \), \( h_{switch}(x, a) = 0 \), \( g_{switch}(x, a) \leq 0 \), \( \forall K_{k=1} : s_k \in \text{succ}(s_{k-1}, a_k) \).

Given that \( \mathcal{P}(a_{1:K}) \) is a smooth SOC problem, we can use the Laplace approximation to represent each mixture component \( p^*(x|a_i) \) as a Gaussian distribution. Apart from the unconstrained NLP in (10), however, \( \mathcal{P}(a_{1:K}) \) yields a more complicated trajectory optimization problem; e.g., the dynamics of moving or manipulated objects are defined by equality constraints and the resting objects are just imposing inequality constraints for collision avoidance. Such problems should be formulated as a constrained NLP:

\[
\min_{x_{1:N}} \sum_{n=1}^{N} f_0(x_{n-2:n}) + f_V(x_{n-1:n})
\]

s.t. \( \forall n_{1:N} : h(x_{n-1:n}) = 0 \), \( g(x_{n-1:n}) \leq 0 \),

which can be addressed by any constrained optimization methods, such as Augmented Lagrangian Gauss-Newton.

Suppose we have found \( x^*_i \) an NLP solution for the \( i \)th skeleton. We then approximate the \( i \)th mixture density as:

\[
p^*(x|a_i) \approx N(x|x^*_i, \Sigma^*_i).
\]
Because of the equality/inequality constraints imposed in \[16\], this distribution is degenerate; i.e., deviations from \( x^*_i \) can only lie in the kernel of the equality and active inequality constraint Jacobians, thereby making the above distribution only span a lower-dimensional subspace. Let a column of matrix \( W \) denote an orthonormal basis of the nullspace of 
\[
 J = \begin{bmatrix} \nabla h(x^*) \\ \nabla \text{diag}(\lambda) g(x^*) \end{bmatrix}, \lambda \text{ is the dual variables on the inequality constraints. Then, } \Sigma_i^* \text{ is given by the inverse of the projected second derivatives of } \log p^*(x) \text{ at } x^*:\n\]
\[
\Sigma_i^* = W (W^T \nabla^2 f(x^*) W)^{-1} W^T. \quad (18)
\]
To complete the mixture approximation, we also need to compute the mixture weights, \( p^*(a_i) \). Because a skeleton imposes different constraints to the corresponding NLP \[16\], making each mixture component span different subspaces, we can assume that the modes are widely separated, which enables the mixture weights to be computed independently \[21, Chapter 12\]:
\[
p^*(a_i) \propto p_0(x_i^*|s_i) \exp(-V(x_i^*)) \{(2\pi)^d |\Sigma_i^*| + 1\}^{1/2} \propto \exp(-f(x_i^*)) \{|\Sigma_i^*| + 1|/|\Sigma_i^*| + 1\}^{1/2}, \quad (19)
\]
where \( \Sigma_i = W (W^T \nabla^2 f_0(x_i^*) W)^{-1} W^T \) is the covariance of the (degenerate) uncontrolled path distribution \( p_0(x|a_i) \); \( d = \text{rank}(\Sigma_i^*) \), and \(|. . .|_+\) denotes a pseudo-determinant. Note that all the gradients (and Hessian) of \( f, h, g \) are already computed while solving the NLP \[16\]. The minimization in KODP \[21\] is then written as:
\[
\delta x_n^* = \text{argmin}_{x_n} f(\delta x_{n-2:n}) + J_{n+1}(\delta x_{n-1:n}) \text{ s.t. } h_n(\delta x_{n-2:n}) = 0, g_n(\delta x_{n-2:n}) \leq 0, \quad (22)
\]
which has the form of a quadratic program (QP) with \[5\].
\[
f_n + J_{n+1} \equiv \frac{1}{2} \begin{bmatrix} \delta x_{n-2:n-1} \\ \delta x_n \end{bmatrix}^T \begin{bmatrix} D_n & C_n \\ C_n^T & E_n \end{bmatrix} \begin{bmatrix} \delta x_{n-2:n-1} \\ \delta x_n \end{bmatrix} + d_n \begin{bmatrix} \delta x_{n-2:n-1} \\ \delta x_n \end{bmatrix} + c_n, \text{ s.t. } \begin{bmatrix} l_n^T \\ m_n^T \end{bmatrix} \begin{bmatrix} \delta x_{n-2:n-1} \\ \delta x_n \end{bmatrix} = 0. \quad (23)
\]
Given the cost-to-go function at the next time step \( J_{n+1} \), the solution of the above QP can be represented linearly around \( \delta x = 0 \) (which corresponds to the solution of the original problem) using the sensitivity analysis of NLPs \[22, 23\]:
\[
\begin{bmatrix} E_n & m_n^T \\ m_n & 0 \end{bmatrix} \begin{bmatrix} \delta x_n \\ \delta x_{n-2:n-1} \end{bmatrix} = - \begin{bmatrix} -C_n & 0 \\ C_n^T & E_n \end{bmatrix}^{T} \begin{bmatrix} \delta x_{n-2:n-1} - e_n \\ -d_n \delta x_{n-2:n-1} \end{bmatrix}. \quad (24)
\]
The above directly implies the linear feedback control law:
\[
\begin{bmatrix} \delta x_n \\ \delta x_{n-2:n-1} \end{bmatrix} = \begin{bmatrix} E_n & m_n^T \\ m_n & 0 \end{bmatrix}^{-1} \begin{bmatrix} e_n \\ C_n^T \end{bmatrix} \delta x_{n-2:n-1} = u^* + K_n \delta x_{n-2:n-1}. \quad (25)
\]
The cost-to-go functions along the whole time horizon can be derived from the Bellman equation \( J_n = \min_{x_n} \left[ f_n + J_{n+1} \right] \) which results in the following backward matrix recursion:

\[
V_n = D_n + 2 \left[ C_n \ l_n \ H_n \left[ C_n^{T} \right] - \left[ C_n \ l_n \right] H_n \left[ C_n^{T} \right] 0 \right]
\]

\[
v_n = d_n - \left[ C_n \ 0 \right] H_n \left[ c_n \right] + \left[ C_n \ l_n \right] (H_n - H_n) \left[ c_n \right]
\]

\[
\bar{v}_n = c_n + \frac{1}{2} \left[ c_n \ 0 \right] (\bar{H}_n - 2H_n) \left[ c_n \right],
\]

where \( H_n = \left[ E_n \ m_{n,T} \right]^{-1} \) and \( \bar{H}_n = H_n \left[ E_n \ 0 \ 0 \right] H_n \).

Note that, in the 1st-order unconstrained case, the above recursion (26) is equivalent to the Riccati equation of LQR.

B. Composite optimal control policy

During the execution, the mixture weights can be updated on the fly. Let \( J_n^{(i)} \) be the cost-to-go function w.r.t. an \( i \)th skeleton at a time step \( l = n \) and \( \Sigma \) be a covariance matrix that is only for the future trajectory \( l = n, ..., N \) which only takes submatrix of \( \nabla^2 f(x_n^*) \) or \( \nabla^2 f_0(x_n^*) \). Then, the mixture weight of an \( i \)th skeleton is given as:

\[
p_n^*(\mathbf{a}_i) \propto \exp \left( -J_n^{(i)}(x_{n-2:n-1}) \right) \left( ||\Sigma^*_i|| + ||\Sigma_i|| \right)^{1/2}
\]

(27)

where \( x_{n-2:n-1} \) is the two past configurations that the robot observed. With these mixture weights, we introduce two different methods to construct the composite control policies from all skeletons, \( u_n^{(i)} \) in (25).

- **Blending**: As suggested in [24], [25], the control input can be computed as a linear combination, i.e.,

\[
u_n^* = \sum_i p_n^*(\mathbf{a}_i) u_n^{(i)},
\]

(28)

which minimizes forward KL divergence \( D_{KL}(p^*||p_n) \).

- **Switching**: The blending method can cause undesirable smoothing effects in practice because it mixes behaviors for different contact activities. An alternative is to simply take the best expected policy as:

\[
u_n^* = u_n^{(i^*)}, \quad i^* = \arg\max_i p_n^*(\mathbf{a}_i).
\]

(29)

We briefly show the different resulting behaviors of two methods in the following section.

The overall framework for planning and control can be summarized as follows: (i) In the offline planning phase, trajectories w.r.t. the different candidate skeletons proposed by a logic-level planner are optimized by solving (10), and for each skeleton, the cost-to-go function as well as the linear feedback policy are computed from KODP (20) and (25).
B. Robust planning: Single- vs two-finger push

The second example involves an object to be manipulated, where the goal is to push the object into the target position/orientation using one or two fingers. The robot’s dynamics is modeled as a double integrator with 7 degrees of freedom and the motion of the pushed object is defined by the quasi-static dynamics [26], [27], [28]. Under the optimal policy, both plans result in similar deterministic costs, and similarly small RMS errors of the final box configuration, 1.6568 × 10⁻⁵ and 1.9720 × 10⁻⁵, respectively. Fig. 3 shows that the two-finger push is inherently more stable, thereby having a lower stochastic cost; the robust strategy can be chosen only when the stochastic cost is also considered.

C. Reactive controller: Elbow-on-table & Banana

For the Elbow-on-table example, Fig. 4 shows the executed trajectories with two composite control laws, blending (28) and switching (29). We reduced the control cost weight for KODP to encourage the switching behavior. In both cases, the robot chooses the most constrained strategy, (Fixing Joint1&2), in earlier phases to reduce the uncertainty and shifts to less constrained modes. The blending controller, however, does not make the elbow completely put on the table while the switching does; the robot cannot fully exploit the uncertainty reduction benefit of (Fixing Joint1&2) because of this undesirable smoothing effect.

The last example is a so-called banana problem; to catch a banana that is high up, a robot has to move a box first and then climb on it. As depicted in Fig. 5, the robot in the considered scenario can use either the blue or the red box, and (Using Blue Box) has a lower cost since it is closer. We injected disturbances in the direction of the red box before the robot takes the first step, and considered the switching composition scheme. Fig. 6 shows that, if the disturbance is small, the robot stays in (Using Blue Box), but switches to (Using Red Box) if it is large.

VI. Conclusion

This work has proposed a probabilistic framework for manipulation planning in stochastic domains. By connecting hybrid trajectory optimization and approximate posterior inference, we have built the optimal path distribution as a mixture of Gaussians. The proposed framework can evaluate plans not only in the deterministic sense but also in the sense of robustness, allowing for a reactive composite controller.

There is a close connection between this work and LQR-trees [29]. By expanding a backward tree from the goal like LQR-trees, the reactive controller would become able to
consider various plans more efficiently. Also, the exponential combinatorial complexity of skeletons (and thus the number of mixture components) can be addressed using deep architectures like [30] while the proposed method also can provide more sensible measures for learning such architectures.

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