Fatigue Resistance Models of Structural for Risk Based Inspection

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Abstract

The current stage of civil engineering is characterized by special attention to the safety of structures with a long service life. Such objects were designed several decades ago and their safe operation was ensured by significant safety margins. Now this approach to safety has been replaced by the concept of acceptable risk. It forms the basis of a risk based inspection (RBI) maintenance strategy. The transition from preventive maintenance strategies to a technical condition maintenance is substantiated. Complex indicators of technical condition, suitable for RBI maintenance, are considered. The methodology of the resource safety index (RSI) is proposed. The latter is used as an indicator of risk. Special models of fatigue resistance is required for its control. The purpose of this paper is to build fatigue models for critical structural elements that are serviced according to the RBI concept. Instead of the traditional S-N curve, the lifetime general equation (first model) be used, where by the arguments are the main influence factors. Along with this, a modified ε - N equation is proposed for deformation criteria. The novelty of this equation is that it uses the rate of S-N curve (slope) obtained in the first model with high cycle fatigue. The second model, combining the results of fatigue tests, is the equation for the dispersion of durability. The third model is the accumulated damage function under overloads. The efficiency of the RSI method is demonstrated by the example of the reliability assessment of the high strength bolts. Thanks to RSI method forecasting, during RBI-maintenance, parts can be used 3-5 times longer than with traditional methods.

Keywords: Risk; Overload; Damage; Lifetime; Safety Index.

1. Introduction

At the contemporary stage of engineering development, fatigue resistance models have begun to be applied to objects whose safety was earlier considered in an absolutely static aspect. These objects began to include bridges, buildings, pipelines, supporting structures of industrial equipment. For example, the crash of a viaduct in Genoa (2018), scientists explain on the underestimated phenomenon of very-high cycle corrosion fatigue in existing civil infrastructures. The brittle destruction of the bridge’s cable triggered the collapse of the whole structure. The aggressive environment, as well as the structural size effect, both may change Wöhler’s curve (a model of resistance fatigue), translating it downwards and eliminating the horizontal asymptote at the basis of the concept of fatigue limit [1].

Structures whose failures are related with significant consequences can be designed with a large margin of safety. As a result, they have a long, but unfortunately an indefinite (uncertainty), period of operation. The final decision on the issue of service life, as a rule, falls to the stage of exploitation. By the beginning of the XXI century in the circles of specialists engaged in the service of industrial equipment, the look at maintenance was shaped as an integral part of enterprise [2]. The issues of service and repair of mechanical equipment always were important for the industrial sector.

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For example, in the total value of metallurgical products, more than half of the costs are related to the maintenance of equipment.

The share of repair personnel can reach up to 30% of the metallurgical enterprise staff. If the average share of maintenance costs for all costs in general industrial production is 5%, then this share for chemical production is 6.8%, and for steelmaking it is already 12.8%. In metallurgy, maintenance costs are 8.6% of investments in production (against 3.8% in chemistry) [2, 3].

In order to optimize expenses in this area, the management of enterprises gradually began to move away from the traditional preventive repair strategy (PM) with its rigid schedule. At some enterprises, the cramping in the funds for maintenance and repair led to a corrective maintenance strategy (CM), when repairs are made after failures. In both strategies, spending is far from optimal. The use of PM and CM-strategies can be justified only at certain stages of exploitation. Minimizing the costs of maintaining the equipment and maximizing its availability is facilitated by the technical condition maintenance (TCM) strategy. This strategy involves the use of a flexible schedule of repairs and is characterized by the active application of methods of technical diagnostics. Sometimes such strategies are referred to as proactive (predictive) maintenance [2, 3]; imperfect maintenance [4]. As a rule, TCM servicing is performed by means of parameter monitoring. A type of strategy with the control of complex reliability indicators (Reliability centered maintenance - RCM) is also widely used [2]. There is already a task of assessing the consequences of failures, which leads to the creation of a strategy with risk and safety control (Risk based inspection concept - RBI). Considering the above, manufacturers of metallurgical equipment (for example, SMS group, Danieli, Eirich) began to equip it with built-in systems of monitoring technical condition. Typically, the automated system includes a subsystem for monitoring the torque of the drive and the subsystem of vibrodiagnostics of rotor assemblies. With their help, maintenance personnel receive recommendations on the dates of replacement of nodes. This trend is relevant, as a result of the new-fangled outsourcing from the equipment is deleted experts-mechanics who are constantly observing it. But the amount of information characterizing the technical condition increases and becomes more complicated.

The purpose of this paper is to build fatigue models for critical structural elements that are serviced according to the RBI concept. Such models should be suitable for assessing the current values of complex indicators of technical condition according to the indications of monitoring systems.

The aim of the paper includes:

- Development of a complex indicator of the technical condition for RBI maintenance;
- Directly the development of fatigue resistance models;
- Demonstration of the practical use of the developed algorithms.

2. Complex Indicators of Technical Condition

The probability of survival operation $P$ (PS) is used as a complex diagnostic indicator for mass production systems. In the classical formulation PS characterizes the relative number of failures. On the one hand, the algorithms for predicting this indicator should not be too sensitive to the growth of the number of elements of a technical system. The trend of growth in the number of calculated and diagnosed elements can lead to an unjustifiably low projected level of PS for the whole system. As a result, the cost of the object increases. On the other hand, the PS should react to the operating time if it acts as a diagnostic parameter. Both requirements contradict each other. This circumstance forces us to seek for better indicators.

Among them the risk $\rho$ is present, in which the probability of failure is ranked by the level of its criticality $u$. In this way, failures of the technical system are reduced to the same scale. But it is not possible to completely avoid the shortcomings inherent in PS. To avoid these contradictions, a Resource Safety Index is used, representing the logarithm of the guaranteed margin of lifetime $n_{NR}$ for safety $R$ [5]:

$$\beta_R = \ln n_{NR} = \ln \frac{T_Q}{T_R} = \ln T_Q - \ln t_R$$

(1)

Where $T_Q$ is the minimum longevity value obtained from the lifetime distribution function (LDF) for the failure probability $Q = 1 - R$; $t_R$ is the maximum value of the operating time, determined from the distribution function of the operating time for the probability $R \geq 0.5$.

As a result of the use of the Resource Safety Index, the kinetic model of the degradation process such as depletion of the resource (decreasing function) is divided into two sections. At the initial stage of operation, the technical condition is monitored according to the safety index diagram, which is in double logarithmic coordinates by a straight line (Figure 1).
It leaves the point $\lg T_0 = \beta_{b0}$ and falls into a point with the same lifetime for $\beta_R = 0$. Controlling the operating time $t$ of the most dangerous place exposed to the most intense damaging process (the weak link principle, simple system), when approaching $\beta_R \to 0$, and $t \to T_0$, it is advisable to assign an inspection of the object and control of the technical state (check, Figure 1). For $\beta_R > 0$, the object is in an operable state (G, Figure 1). At $\beta_R < 0$, the technical state is controlled not by the safety index but by the reliability function $P(t)$, since in this region it is sensitive to the running time. At $P > 0.5$, the object falls into the zone of preventive replacements (repairs) (PM, Figure 1). During further operation, when the probability of survival operation drops to $P < 0.5$, and the risk rises to $\rho \to 1$, the object is in the zone of emergency failures (CM, Figure 1).

The transition to the method of the Resource Safety Index (RSI method) becomes clear in the paradigm of the amalgamated (classical) and individual (structural) approaches to the reliability assessment [6-8]. The transition from classical reliability methods to individual (structural) methods is accompanied by the replacement of mathematical-statistical models by models based on failures physics. Reliability of the system is estimated by individual indicators of its elements reliability. Structural reliability methods are used, mainly, for RBI-maintenance. With RCM-maintenance, the methods of classical reliability remain effective.

For a system of $i$ elements and $k$ degradation processes, the general safety index $\beta_{ER}$ is determined through the individual indices $\beta_{iR}$ and the criticality of the element and process $u_{ik}$ as:

$$\beta_{ER} = \lg \left( \sum_{i=1}^{i=k} \frac{u_{iR}}{\sum_{i=1}^{i=k} u_{iR}} \right)$$

(2)

The advanced service policy on technical conditions to which belongs RBI (risk based inspection)-maintenance, does not involve the decommissioning of a subject through a fixed. After a specified period, technical condition check is provided, after which a decision is taken on further operation of the facility. This period, according to the principle of ALARP, depending on the probability and severity of the failure is no less than 1 year [9, 10].

This does not mean that in the period between inspections, one should forget about monitoring the state of the structure. Otherwise, RBI service will turn to preventive maintenance. At RBI, the problem of increasing the degree of exhaustion of a physical resource is solved. However, the required level of security is respected. The evaluation of the technical condition is carried out by checking the diagnostic parameters that react to the action of damaging processes. However, fatigue damage is still difficult to control by direct (physical) methods. In practice, this is done through load monitoring. This indirect method needs regular, not periodic, registration. Organization a similar procedure is difficult. Therefore, the principle of Standardised Load Histories is used, due to which it is possible to refuse continuous monitoring, but to control the accumulation of damages [11, 12].

With the correct application of diagnostic algorithms, the forecast reliability indicators are adequate to the actual ones. The required level of safety is observed. Its decrease, as a rule, is due to the appearance of rare overloads. Then the loading process can be modeled as block, consisting of the main background process with damage per cycle $d_b$ and overloads with damage $d_a$. This approach requires the availability of extended data on fatigue resistance. This article is devoted to their consideration.

3. Models of Fatigue Resistance

From the above description of the RSI method, the requirements for fatigue models follow:

- The ability to quickly find of the lifetime distribution functions (LDF) at different stages of time operation;
- The identification of fatigue models should increase the accuracy of the LDF forecast, which is tantamount to increased safety.

In order to quickly determine the damage $d$ through the durability of $N$ as $d = 1 / N$, instead of the traditional S-N curve, the lifetime general equation (LGE) should be used, where the arguments are the main influence factors - amplitude stress $\sigma_a$, cycle asymmetry coefficient (stress ratio) $R$, theoretical coefficient stress concentration $a_c$.
\[ l g N = b_0 - m l g \sigma_a - b_r R_\sigma - b_a \alpha_\sigma + b_{rr} R_\sigma^2 - b_{aa} \alpha_\sigma^2 + b_{ma} \alpha_\sigma \cdot l g \sigma_a \] (3)

Where \( b_i \) - sensitivity coefficients to the effect of factors; \( m \) - rate of \( S-N \)-curve (slope) or sensitivity to fatigue stress amplitude.

For fixed values of \( R_\sigma \) and \( \alpha_\sigma \), the LGE are transformed into \( S-N \)-curves; for fixed values of \( N \) and \( \sigma_a \), analogues of limiting amplitudes are obtained; from the fixed values of \( \sigma_a \) and \( \sigma_m \), \( R_\sigma \), the effective stress concentration coefficients are calculated from the function \( N (\alpha_\sigma) \) (Figure 2). The LGE (3) is presented for the strength criteria of fatigue, taking into account the main factors of influence. With the use of more universal deformation and energy fatigue criteria, the number of members of the LGE will be reduced. The modified Basquin-Manson-Coffin model was successfully applied to the supporting structures [13]:

\[ \varepsilon_a = \frac{1}{4N\alpha^2} \cdot ln \left( \frac{1}{1-\psi} + \frac{\sigma_{\alpha R}}{E} \cdot \left( \frac{N_a}{N} \right)^{\frac{1}{m}} \right) \] (4)

Where \( \psi \) is the coefficient of relative constriction; \( \sigma_{\alpha R} \) is the endurance limit based on \( N_\sigma \) cycles; \( E \) is the modulus of elasticity of steel; \( \varepsilon_a \) is the amplitude deformation of the cycle.

The novelty of this equation is that it uses the rate of \( S-N \)-curve (slope) \( m \) obtained in the LGE model with high cycle fatigue.

The second basic relationship, combining the results of fatigue tests, is the equation for the dispersion of durability (EDD). It is represented by the linearized function of the lifetime logarithm standard deviation \( S_{lgN} \) from its median value \( l g N \). Its sloping section is described by the equation (Figure 3):

\[ S_{lgN} = B + k_l (l g N - l g N_A) \] (5)

The parameters of the equation \( B, k_l \), \( l g N_A \) are determined experimentally when obtaining a fatigue reference curve, which precedes the experiments to obtain the LGE. This dependence is necessary for the search the lifetime distribution function, which is involved in the Resource Safety Index assessment.

Figure 2. Transformation of LGE plots

Figure 3. The diagram for assessing the dispersion of durability under strength (\( B_c \)) and deformation (\( B_d \)) criteria
The third model is the accumulated damage function \(a_0\):

\[
a_0 = a_{ol} + \Delta a_b + \Delta a_R
\]  

(6)

Where \(a_{ol} = a_0(X_{ol})\) – basic function of accumulated damage; \(\Delta a_b\) – correction of the base function from the factor \(X_b = d_{bl} / d_b\); \(\Delta a_R\) – correction of the base function from the stress ratio \(R_\sigma\).

The main factor that affects the accumulation of damage during overloads is its relative value \(X_{ol} = d_{ol} / d_b\) (Figure 4). The basic function has no monotonic behavior and can be represented as:

\[
a_{ol} = \log \left[ 10^{p}\phi_{ol}^{m_2} \exp \left( - \frac{X_{ol}}{X_0} \right) + 10^4 \phi_{ol}^{m_2} \left( 1 - \exp \left( - \frac{X_{ol}}{X_0} \right) \right) \right]
\]  

(7)

The parameters of the equation \(p, A, X_0, m_1, m_2\) are partially determined experimentally, partially selected by recommendations. Details of the algorithm for the correction of the accumulated damage, its relationship with models of cracks retarding during overloads, is given in [14].

The combination of three models of fatigue resistance allows not only to locate accurately the LDF, but also to solve the problem of very high cycle fatigue [15, 16]. For this region it is difficult to establish experimentally the limits of endurance. The idea of using block-program tests for this is not new. Due to it there are accelerated fatigue tests. In this aspect, the function \(\Delta a_b\) is actual. Knowing its behavior at \(X_b < 1\), we can estimate the lifetime at very small amplitudes of the baseline process (Figure 4).

![Figure 4. Change of accumulated damage under the influence of the main overload parameters on the areas of low cycle (LCF), high cycle (HCF) and very high cycle (VHCF) fatigue](image)

**4. Search of the General Safety Index**

The effectiveness of the RSI method demonstrated by the example of high-strength M18 bolts for connecting structural components (Figure 5). Inattention to threaded connections can lead to sad consequences. Fatigue failure of the threaded rods of the turbine cover led to catastrophe on Sayano-Shushenskaya hydroelectric power plant (2009, Russia). Losses are estimated at hundreds of millions of US dollars [17].

Another case of inattention to bolts is associated with a blast furnace (BF) accident at a metallurgical plant in Port Talbot (2001, United Kingdom). The BF superstructure lifted 0.75 m from its supporting framework. Some 200 ton of hot materials were released out, killing three employees and seriously injuring many others. There had been no regular maintenance of the column head bolts. Prior to incident, all bolts, save for those on one column head, had fractured sometime prior. There was therefore less restriction on the shell BF movement during the incident than might have been the case with intact bolts [18].

The examined bolts are a critical part. In the bolts manufacture it undergo a complex process of mechanical and thermal treating. The thread is strengthened by rolling, and the fillet is also improved by surface-plastic deformation. With quality technologies, the destruction of bolts occurs in the thread (1, Figure 5). In case of non-observance of the technological regimes, the bolts do not pass the fatigue test and the destruction is shifted to the threaded edge 2 or to the fillet 3 (Figure 5). Another degradation process is caused by the appearance of fretting fatigue cracks in the surface of the head (4, Figure 5). Their development period is relatively long, but they also cause the bolt to collapse, though it may retain some bearing capacity.

For dangerous places bolt where fractures occurs (1, 2, 3, 4, Figure 5) obtained the fatigue models (Table 1). Fractures
in places 2 and 3 are typical for imperfect bolts. The first model is the LGE (Eq.3) in the form:

\[ \lg N = b_0 + m \lg \Delta F + b_1 R_\sigma \]  

(8)

Where \( \Delta F \) is the double amplitude (swing) of the force acting on the bolt in kN.

### Table 1. Parameters of fatigue models for dangerous bolt places and their criticality (LGE is obtained for twice the amplitude of the forces \( \Delta F \) (kN))

| Dangerous place | LGE | EDD | \( u_\Delta \) |
|-----------------|-----|-----|--------------|
| Thread, 1       | 30.7| -13.2| 0.16 | 0.16 | 5.0 | 0.66 |
| Head surface, 4 | 14.5| -4.5 | 0.16 | 0.16 | 5.0 | 0.11 |
| Fillets, 2,3    | 21.5| -8.6 | 0.30 | 0.24 | 5.6 | 0.20 |

Figure 5. The initial safety indices \( \beta_{\Sigma P0} \) of the bolt M18 with the tightening forces \( F = 75.6 \text{ kN (F1), F = 151.2 \text{ kN (F2)}} \) and the coefficients of the variation of the double amplitude of the load \( \Delta F \) \( V \) = 0.05 (V1), 0.1 (V2), 0.2 (V3)

The task was to find the cyclic lifetime of \( N_R \) which guarantees the safety of \( R = PS = 0.98 \). For this purpose, according to (2), the value \( \beta_{\Sigma P0} = \lg N_R \) is determined (Figure 1). The calculation is performed for a fixed tightening forces \( F \) generated in the body of the bolt tension 0.3 and 0.6 of yield strength. Changing the value of external loading \( \Delta F \) alters stress ratio \( R_\sigma \). Therefore, the use of LGE is convenient.

On the basis of the developed algorithm, the diagrams of the initial RSI \( \beta_{\Sigma P0} \) were obtained (Figure 5). Since during construction, the external load is considered as a variative, this diagram is, in essence, a fatigue curve for the random load at the PS = 0.98.

Given the guaranteed lifetime of the bolt under the several damaging process (Figure 6), \( u_\Delta \) criticality index can be considered as a powerful tool for regulating systems reliability. This conclusion follows from the fact that the guaranteed lifetime increases 4-5 times during the transition from the situation \( u_\Delta = 1 \) to the algorithm with actually calculated \( u_\Delta <1 \). That is, in the first situation, the amalgamated safety index is significantly lower than the average \( \beta_{\Sigma P} \) between the individual indices: \( \beta_{\Sigma P} < \beta_{\Sigma P0} \). In the second situation, the principle \( \beta_{\Sigma P} \to \beta_{\Sigma P0} \) is formalized. In this case, the method of amalgamation LDF (LDF\( \Sigma \), Figure 6) gives a very conservative result. Therefore, this method should not be considered universal.
Studies have shown that the factor of variation in external loading of $V_F$ has a more significant effect on the guaranteed lifetime than the tightening force $F$. It should be noted that the increase in the tightening force reduces the fatigue strength to a certain limit. After that, the negative effect of the tightening disappears, because at the root of the first tread there is a local plastic deformation \cite{19}. For large tightening forces $F$, the intensity of falling diagram $\beta_{\Sigma R_0}(\Delta F)$ increases, and for small $F$, this intensity decreases. This can be explained by the fact that at high values of $F$ and small $\Delta F$ there is a high asymmetry of the cycle $R_\sigma$. In such conditions, high-strength steels lose sensitivity to it. With increasing $\Delta F$ and constancy of the force of tightening $F$ the value of $R_\sigma$ decreases. Therefore, at $R_\sigma < 0.5$, the difference in durability becomes noticeable. Thus, increasing the tightening force is effective.

The diagrams obtained allow 3-5 times longer use of the details than with the forecast by traditional means (Figure 6). Considering the defined guaranteed lifetime of the object under of the system of damaging processes, we can consider the index of criticality $u_{\text{ik}}$ as a powerful instrument for the regulation of amalgamated reliability. Such a conclusion follows from the fact that the guaranteed lifetime increases 4-5 times in the transition from the situation $u_{\text{ik}} = 1$ to the algorithm with the actual calculated $u_{\text{ik}} < 1$ (Figure 6).

5. Conclusion

The Resource Safety Index is a complex indicator of the technical state that unites damaging processes of various nature. Its application is decisive to the basic requirement of RBI- maintenance: operation of the facility to a pre-failure state at an acceptable level of risk. Three proposed models of fatigue resistance allow to find individual safety indexes of a technical system elements at different stages of the life cycle. Thanks to RSI method forecasting, during RBI-maintenance, parts can be used 3-5 times longer than with traditional methods.

It is proposed to present the results of stationary fatigue tests with two basic models: the lifetime general equation and the lifetime dispersion equation. The first model allows you to abandon laborious tests to determine the endurance limit. The second model makes it possible to establish the parameters of the LDF without resorting to the procedure term-by-term variation of the fatigue curve parameters. As a result, the forecast is more accuracy. The relationship between the parameters of the second model for deformation and strength criteria is shown.

The risk and safety levels are significantly affected, not so much by the designed operating conditions as by deviations from them. That is, extreme conditions, which include overloads. Their influence is taken into account in the third model of fatigue resistance. Based on a generalization of the results of a materials resistance study under cyclic loading with overloads, an algorithm for correcting accumulated damage has been developed. It reflects the nonmonotonic behavior of the accumulated damage $a_0$ under the load of the main background process and the relative overloads level, as well as depending on their stress ratio cycle. The intensity of damage accumulation during loading with overloads depends both on the parameters of the mode, which affect the path of the function $a_0$ and the value of $a_{\text{max}}$, and on the properties of the material, which determine the value of $a_{\text{max}}$. The parameters of the loading mode are already set directly in the damage. This ensures the universality of the proposed correcting algorithm.
The algorithm for designing the diagram "general safety index $β_{SP}$ - load parameter (in this case - $ΔF$)" is a definite alternative to the damage summarization procedure. The latter is relevant in the earlier stages of design, when the uncertainty of the load forces the choice of spectra with wide variability. After the stages of working out of strength, reliability, and even more so at the stage of operation, when the loading process is monitored, its parameter variation is significantly reduced. An opportunity is created without summarizing the damage right away directly to control the exhaustion of the resource.

6. Conflicts of Interest

The authors declare no conflict of interest.

7. References

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