Hydrostatic buoyancy alternative correction for weighing using high-resolution single-pan balances

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Abstract. In this work we compare some methods of buoyancy corrections applied to sample weighing that require a high accuracy results. We also established and disseminate the fundamentals for a better understanding of this phenomenon. An alternative approach was applied to find a buoyancy correction equation adapted to single-pan balances. The results show a good compatibility between two of these buoyancy corrections. The approach advantage is to keep the mathematical formalism adapted to the single-pan balance measurement procedure, unlike the others methods that developed its formalism based in the equations adapted to the double-pan balances.

Keywords: single-pan balance, hydrostatic buoyancy correction, convencional mass.

1. Introduction
High-resolution balances (special accuracy class I [1]), typically 0.1 up to 100 micrograms resolution, are widely used in research and analyzes laboratories to provide weighing traceability or quality product assurance, e.g., certified reference materials. The sample weighing procedure requires a series of attentions and corrections from the deviations and uncertainty due to several external influences on the measurand and in weighing procedures. The hydrostatic buoyancy is one of those sources of deviations and uncertainty contributions in gravimetric measurements results. This force appears in every gravimetric measurement where exists any kind of atmosphere, like the air, O₂, N₂, He or any other gases combinations.

It is possible to calculate the hydrostatic buoyancy, i.e. it is not a random influence, and in many cases is interesting to consider its influence in the measurement procedure and results to improve the accuracy in the weighing results. These corrections generally have it origin on double-pan balances, the mathematical formalism and weighing equations are formulative to explain this situation. Besides this, the correction to single or double-pan balances to follows the same physical principles and work well in both cases, is useful, to better understand this phenomenon in a single-pan balances, to develop an appropriate adapted formalism to this specific situation. For instance, to a single-pan balance, we have a virtual comparison with a mass standard, in fact we never have the situation described in a double-pan balance situation where exists, at least, one mass standards participating in this procedure.
Thus we choose to adopt the formalism most indicate to the single-pan balance. The approach to obtain a buoyancy correction use an infinite series of corrections sum in a self-consistent equation. The work motivation emerges from the interaction with researchers, students and users of gravimetric measurement results, in particular, that one’s obtained on high-resolution balances. This expertise in balances calibrations allowed attesting the robust knowledge field of measurements and research works that rely on that technique such as studies parts wear, liquid evaporation, surface adsorption and desorption, Analytical Chemistry, and others. The buoyancy correction is a basic interdisciplinary and useful technique to several knowledge areas, for example, Metrology, Chemistry, Physics, Biotechnology, among others. Many balances users adopt the indicated currently methods to the buoyancy correction. It’s clear that still is important to spread the knowledge on how to take into account the hydrostatic buoyancy and to understand its physical origins or even the fundamentals of the balances calibration. This work aims are to study the most spread buoyancy corrections, to show an alternative hydrostatic buoyancy correction approach and the consolidation of important concepts about this force to give a better understanding of the phenomenon and to spread the buoyance correction correct use in gravimetric measurements results, especially in a single-pan balances that require high accuracy measurements results. The results was compatible, for this kind of measurements, with the conventional mass metrological equation [1] [2]. Is important to notify that this works is focused just to samples mass measurements and do not could be directly applied to the mass standard calibrations because this procedure have specific recommendations, technical standards and regulations.

2. Hydrostatic buoyancy
To better understand the hydrostatic buoyancy we need to point out some relevant aspects related to this phenomenon.

2.1. Basic concepts
The hydrostatic buoyancy is a contact force so it has an electromagnetic nature. This force depends on the existence of a fluid confined in a space region, the fluid properties, and at least the existence of one force field acting on the fluid in the same region of space.

In fact, the fluid may be non-static and its density may be not constant in time or in space, in any case the buoyancy force will exist. On the other hand, if we consider the fluid density as constant in time (does not need to be in the space) and that the fluid has no turbulent flows or vortices, the hydrostatic buoyancy equations become much simpler. The other aspect is the existence of a force field acting on the fluid. This aspect is often forgotten or even replaced by the planet Earth gravitational field, like this were the only way to exist the force of hydrostatic buoyancy.

One simple example of buoyancy generated by non-gravitational force field is a bottle (full of water) being turned in any direction, in this case there is an inertial force (also known as fictitious forces) the centripetal force, that causes the liquid inside the bottle is compressed against the bottom of the bottle thus creating an inertial force field (in this case centripetal) that causes the hydrostatic buoyancy inside the fluid beyond that relative to the gravitational force. The force field may have mechanical, gravitational or any other origin since it to be able to actuate on the fluid. In any case the buoyancy direction is the same as the force field or the force field resultant. The buoyancy orientation will be opposite to the force field orientation or the resultant from these forces fields. In general, high-resolution balances are subject to the presence of a specific fluid, the air, and only one force field, the gravitational force field of planet Earth.

Once we have established the conditions for our buoyancy calculation, such as the presence of a force field, from the Earth’s gravity force, the fluid with density static in time, the atmospheric air (and approximately constant in space) and without the presence of turbulent flows or vortices, we can analyze the buoyancy origin and to understand it in this scenario.
2.2. The hydrostatic buoyancy force origin

An usual explanation for the buoyancy force is the one that affirms the buoyancy force is equal to the weight of the fluid displaced by the body undergoing the force, this explanation is convenient for ease of understanding and however it does not provide a physical origin to the hydrostatic buoyancy.

Hydrostatic buoyancy is a contact force (electromagnetic) it acts on the entire surface of the body surrounded by the fluid and it is the final result of the pressure differences of the upper and bottom body surfaces (in the direction of the force field). As the pressure of the fluid in the bottom of the body is greater than on its upper surface it is clear that this force is opposite to the force field acting on the fluid. As consequence, the buoyancy has the same module as the fluid weight that would occupy that region of space if the body were not there.

2.3. The hydrostatic buoyancy effect in weighing measurements results

Before treating the hydrostatic buoyancy, we first need to establish the difference between conventional mass and true mass even as the single and double-pan balances.

2.3.1. True mass and conventional mass

The body conventional mass is the result of weighing in the air, for a weighing taken at a reference temperature of 20 °C, using a reference weight with density of 8000 kg/m$^3$ which the balances and weights are in the atmospheric air with a reference density of 1.2 kg/m$^3$ [1] [2] [3]. The relation between true mass and conventional mass [1] [2] is given by the equation (1),

$$m_c = m \left(1-\frac{\rho_0}{\rho}\right)/(1-\frac{\rho_0}{\rho_c})$$

(1)

where, $m_c$ is the conventional mass; $m$ is the true mass; $\rho_0$ is the dry air density (1.2 kg/m$^3$); $\rho$ is the sample density; $\rho_c$ is the reference density of the stainless steel mass standard (8000 kg/m$^3$).

The conventional mass, in other words, is different of true mass, if the weighing process occurs in the air, the true mass is going to be different from the conventional mass for all the samples with density different of 8000 kg/m$^3$. The true mass $m$ is the mass value after this buoyancy correction or in the vacuum weighing case. Equation (1) is the recommended to the metrological uses [2].

2.3.2. Double-pan balances

It is easy to understand the hydrostatic buoyancy effects in a double-pan balance. Consider the case with two bodies with very distinct volumes and the same mass, in the air, the balance will show an imbalanced case, of course if the balance’s sensitivity is enough to detect it. This imbalanced situation arises from the difference in pressure exerted by the fluid (the air) on both weighing plates.

One way to understand the imbalanced situation in a double-pan balance with two equal masses weights is to think that the largest volume body contributes with its mass, while the one of smaller volume contributes with its own mass and the air mass that’s occupies the difference of volume between both bodies. Thus, the imbalanced situation will occur towards the high-density weight due to the smaller volume body, and this overweight comes from the air weight due to the volume difference between the two bodies. In other wise, in the weighing balanced case (equilibrium case), exist a true mass difference between the two bodies, this difference is due to the different buoyancies forces magnitudes acting in both.

2.3.3. Single-pan high-resolution balances

We consider again the system with two bodies with the same mass (true mass) and distinct volumes but now with the analytical balance [4]. In this case, we have two situations:

a) In the first scenario, one uses a mass standard (with known true mass) to perform the weighing and then the object to be measured replaces it. By recording the balances indications we can measure the difference caused by the hydrostatic buoyance, evidently considering that there were no changes in
the air density among these measurements and that there was no balance drift or that this drift was eliminated by one of the various weighing schemes for drift correction.

b) In the second scenario, this is the usual case, the analytical balance is calibrated by a mass standard, usually made of austenitic stainless steel [1], with a density of approximately 8000 kg/m$^3$. In this case, the mass difference is not observed directly by the balance indication, however there is a virtual comparison with the mass standard used in the equipment calibration, in this case it is possible to calculate the hydrostatic buoyancy effect. In this situation, there is one more uncertain contribution source due to the differences in the environmental conditions at the calibration of the mass standard and the balance and the moment where the measurement in the balance is performed. The comparison with a virtual mass needs a functional equation to be solved, because we have just the balance indication to solve an equation that involves the buoyancy force, but the buoyancy depends upon the indications or more precisely by the corrected mass.

It is important to highlight that the high-resolution balances (special accuracy class I), are usually calibrated with high accuracy class mass standard ($E_1$, $E_2$ or $F_1$ [1]), the differences due to calibration with conventional mass values will be small. For example, for the $E_1$ accuracy class I kg mass standard this typical difference is less than 1.2 mg/kg and it is due to the difference in density of the steel used in the standards manufacture. This difference is usually smaller than that ones caused by the uncertainty of the air density and much smaller than the hydrostatic buoyancy modulus. In general, the air density in laboratories that use high-resolution balances is quite stable, so the difference in air density at the time of balance calibration and the moment of the weighing procedure is very small, and if the balance calibration certificate data contains the value of the mass and the conventional mass these uncertainties will be minimized. The air density usually varies less than 3 % and the buoyancy correction is usually less than 0.01 %. In any case, is recommend to reduce or control the air density differences between the time of the standard calibration and the weighing procedure. Another possibility is to correct that value by obtaining the necessary data for this, as well as to evaluate the calibration difference of the mass balance or conventional mass. An approach to consider this in more detail is described in OIML D28 [2], as well as its uncertainty estimative.

3. Hydrostatic buoyancy corrections

The usual application of buoyancy correction is in analytical chemistry, where its small correction becomes important, and there are already alternatives for its calculation in the literature, two of which are described in Skoog [5] and Harris [6] academic texts books. Other approaches require measurement results that are not always available in the laboratory e.g. the determination of the molar fraction of H$_2$O in the weighing environment. In this work we present the equations used in Skoog and Harris academic text books that are sufficiently accurate to guarantee an improvement in the measurements results with correction of the hydrostatic buoyancy. To provide an alternative to these approaches we apply the buoyancy correction performed by infinite contributions sum. Thus to simplify we will consider the corrections how correction $S$ to Skoog, correction $H$ to Harris, and correction $B$ to Buoyancy due to the infinite contributions sum.

3.1. Correction $S$

The equation (2) was adapted of Skoog [5] to the single-pan balance and provides the buoyancy correction in terms of the densities involved in the weighing process,

$$M = I [1 + (\rho_{air}/\rho_{spl}) - (\rho_{air}/\rho_{std})]$$

(2)

where, $M$ is the mass (corrected) of the sample; $I$ is the balance indication or the indications after the drift correction; $\rho_{air}$ is the air density; $\rho_{spl}$ is the object density; $\rho_{std}$ is the mass standard density used in the balance calibration procedure.
3.2. Correction H
The equation (3) was adapted of Harris [6] and provides the buoyancy correction in balance weighing procedure, indeed this is the same that the conventional mass equation, equation (1). Thus the conventional mass [2] and the $H$ corrections are synonyms in this work.

$$M = I [(1 - \rho_{\text{air}}/\rho_{\text{std}}) / (1 - \rho_{\text{air}}/\rho_{\text{Spl}})]$$

(3)

Where the quantities involved are adapted to the same quantities before described in equation (2). As previously mentioned, this equation provides the same correction results provided by equation (1).

3.3. Correction B
This hydrostatic buoyancy correction corrects the buoyancy effects by infinite sum of buoyancy contributions. This correction method is similar to the Feynman diagrams technique where higher order interactions are accounted for by infinite series that can be summed up exactly.

Considering two bodies with different densities, $\rho_{\text{std}}$ and $\rho_{\text{Spl}}$, we write the difference of the inverse of the densities $D$ (volume per mass) as,

$$D = (1/\rho_{\text{Spl}}) - (1/\rho_{\text{std}})$$

(4)

then we can write the first order object mass correction as a function of the $D$ quantity,

$$M g = I g + I g \rho_{\text{air}} D$$

(5)

The Earth’s gravitational acceleration $g$ can be simplified in equation (5). In its turn, this incremental hydrostatic buoyancy, must be consider because also contributes to the buoyancy force, and must be added to the value of $M$.

$$M = I + I \rho_{\text{air}} D + I \rho_{\text{air}} D \rho_{\text{air}} D$$

(6)

in fact, this process is an infinite series of corrections, then the complete equation (6) is given by (7).

$$M = I + I \rho_{\text{air}} D + I \rho_{\text{air}} D \rho_{\text{air}} D + I \rho_{\text{air}} D \rho_{\text{air}} D \rho_{\text{air}} D + I \rho_{\text{air}} D \rho_{\text{air}} D \rho_{\text{air}} D \rho_{\text{air}} D + ...$$

(7)

Reorganizing the equation (7) terms we have,

$$M = I + \rho_{\text{air}} D (I + I \rho_{\text{air}} D + I \rho_{\text{air}} D \rho_{\text{air}} D + I \rho_{\text{air}} D \rho_{\text{air}} D \rho_{\text{air}} D + ... )$$

(8)

the term inside the brackets, equation (8), is exactly equal to the value of $M$, so it can be written by,

$$M = I + \rho_{\text{air}} D M$$

(9)

or even

$$M = I / (1 - \rho_{\text{air}} D)$$

(10)

The equation (10) contains all the infinite sum of all the hydrostatic buoyancy contributions in a very simple and compact way.

4. Results
To compare all corrections we will use the standard air density, $\rho_{\text{air}} = 1.2 \text{ kg/m}^3$, however it can be measured if needed. In this section we will present the results obtained with corrections $S$, $H$ and $B$, and will be considered as an ideal case such that the atmospheric pressure, temperature, air humidity, and air density are all well defined and constant quantities and equal ones at the sample measurement
procedure as well as in the balance calibration procedure.

To the simulated performed comparisons, of each correction approach, are made with nominal values from 1 g up to 1 kg, and for each of these values we simulate density materials of 8000 kg/m³ and objects of 7000 kg/m³, 5000 kg/m³ and 1000 kg/m³ which covers almost all density values normally used in target samples of laboratory weighing. In the test with the 8000 kg/m³ sample density, all the corrections resulted null, as expected. To the 7000 kg/m³ density, was obtained a small correction, for instance to 1 kg the typical correction is 21 mg. For this values range, density near the standard weights density, the corrections are very small comparing with others uncertainties contribution involved in the mass measurement result. However, to lower densities objects the corrections became greater. Table 1 shows an intermediate material density, and the buoyancy corrections rises its importance. In this case, a kilogram object with this density (5000 kg/m³) has about 90 mg correction.

Table 1. Buoyancy correction comparison to density object of 5000 kg/m³.

| Indication (g) | B correction (g) | H correction (g) | S correction (g) | ABS(B – H) (g) | ABS(B – S) (g) | ABS(H – S) (g) |
|---------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|
| 1             | 1.00009001      | 1.00009002      | 1.00009000      | 0.00000001     | 0.00000001     | 0.00000002     |
| 2             | 2.00018002      | 2.00018004      | 2.00018000      | 0.00000003     | 0.00000002     | 0.00000004     |
| 5             | 5.00045004      | 5.00045011      | 5.00045000      | 0.00000007     | 0.00000004     | 0.00000011     |
| 10            | 10.00090008     | 10.00090022     | 10.00090000     | 0.00000014     | 0.00000008     | 0.00000022     |
| 20            | 20.00180016     | 20.00180043     | 20.00180000     | 0.00000027     | 0.00000016     | 0.00000043     |
| 50            | 50.00450041     | 50.00450108     | 50.00450000     | 0.00000068     | 0.00000041     | 0.00000108     |
| 100           | 100.00900081    | 100.00900216    | 100.00900000    | 0.00000135     | 0.00000081     | 0.00000216     |
| 200           | 200.01800162    | 200.01800432    | 200.01800000    | 0.00000270     | 0.00000162     | 0.00000432     |
| 500           | 500.04500405    | 500.04501080    | 500.04500000    | 0.00000675     | 0.00000405     | 0.00001080     |
| 1000          | 1000.09000810   | 1000.09002161   | 1000.09000000   | 0.00001350     | 0.00000810     | 0.00002161     |

Table 2 shows the results for the 1000 kg/m³ density object, this is the water density and water is used in many solutions. In this range of material density, table 2, we realize that the buoyancy correction could be measured even in semi-analytical balances. It is an important magnitude and must be corrected to ensure a better measurement results.

Table 2. Buoyancy correction comparison to density object of 1000 kg/m³.

| Indication (g) | B correction (g) | H correction (g) | S correction (g) | ABS(B – H) (g) | ABS(B – S) (g) | ABS(H – S) (g) |
|---------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|
| 1             | 1.00105110      | 1.00105126      | 1.00105000      | 0.00000016     | 0.00000110     | 0.00000126     |
| 2             | 2.00210221      | 2.00210252      | 2.00210000      | 0.00000032     | 0.00000221     | 0.00000252     |
| 5             | 5.00525552      | 5.00525631      | 5.00525000      | 0.00000079     | 0.00000552     | 0.00000631     |
| 10            | 10.01051104     | 10.01051262     | 10.01050000     | 0.00000158     | 0.00001104     | 0.00001262     |
| 20            | 20.02102207     | 20.02102523     | 20.02100000     | 0.00000316     | 0.00002207     | 0.00002523     |
| 50            | 50.05255518     | 50.05256308     | 50.05250000     | 0.00000789     | 0.00005518     | 0.00006308     |
| 100           | 100.10511037    | 100.10512615    | 100.10500000    | 0.00001579     | 0.00011037     | 0.00012615     |
| 200           | 200.21022073    | 200.21025230    | 200.21000000    | 0.00003157     | 0.00022073     | 0.00025230     |
| 500           | 500.52555183    | 500.52563076    | 500.52500000    | 0.00007893     | 0.00055183     | 0.00063076     |
| 1000          | 1001.05110366   | 1001.05126151   | 1001.05000000   | 0.00015785     | 0.00110366     | 0.00126151     |

In figure 1, the corrections values comparison to all different corrections methods for an object density of 2500 kg/m³ with nominal values from 1 g to 5 kg. In blue color, we have the B corrections, red color to H correction and yellow color to S correction. The corrections magnitude are very similar in all cases.
Figure 1. Comparison between the corrections.

One way to better realize these corrections results differences is to plot just the absolute difference between the $B$, $H$ and $S$ (figure 2). These absolute differences is to a 2500 kg/m$^3$ material density.

Figure 2. Comparison between the corrections differences (2500 kg/m$^3$).
The blue color is $\text{ABS}(B - H)$, red color is $\text{ABS}(B - S)$ and yellow color is $\text{ABS}(H - S)$, where $H$ is the correction most spread in the academic literature ($m_c$ equation (1) in Metrology). The difference between $B$ and $H$ are the smaller and, by these results, we may affirm that are more compatible in these densities range, while the correction $S$ in the less compatible. Figure 3 shows the same previous comparison but to a 1000 kg/m$^3$ sample density, it is the most common density range in the researches and analyzes laboratories because is the water density commonly used in the chemical and biological solutions.

![Figure 3. Comparison between the corrections differences (1000 kg/m$^3$).](image)

For all purposes comparing only the corrections values, we can see that $B$, $H$ and $S$ corrections for 1 kg provide compatible results. Their differences appear only in high-resolution weighing equipment, 0.1 μ to 0.1 mg, and this value order is smaller than the air density and balance calibration measurement uncertainties (the typical 1 kg (E$^1$) mass standard used to balance calibration have 100 micrograms of measurement expanded uncertainty).

5. Conclusion
All approaches studied shows equivalent results, to the proposed use, thus can be applied to the sample weighing buoyancy correction when the sample density is known. The shown differences are smaller than the involved densities measurement uncertainties for this kind of weighing procedures.

Attention should be taken with respect to the conventional mass, especially if the balance has been calibrated with lower accuracy class mass standards because these classes admit standards of materials such as brass and even cast iron. In these cases the differences between mass and conventional mass may be important, although those who need to correct the hydrostatic buoyancy with greater accuracy and very small target uncertainties generally use $F_1$ or better accuracy class standards.

Likewise, the determination of air density by the CIPM 2007 [7] equation instead of using the standard value of 1.2 kg/m$^3$ improves the accuracy of the correction as well as reduces the uncertainty associated with this correction.

This alternative hydrostatic buoyancy correction, correction $B$, obtained by infinite sum of contributions is not intended to replace International Organization of Legal Metrology (OIML)
recommendations nor the equations already used in analytical chemistry, but rather to present a way of consolidating concepts related to hydrostatic buoyancy and even as a validation method because it is a simple, compact and easy to understand the buoyancy correction in single-pan balances.

The $B$ correction proved to be compatible with the most widespread $H$ correction in academic books (and $m_c$ in the metrological ones). Both are obtained by exact solutions, the differences are due to the terms mathematical organization, but both consider the correct solution of the buoyancy correction. The B correction provide a good validation method to both equations $S$ and $H$. In otherwise the correction $S$ was the less compatible approach and its use must be analyzed carefully.

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