On the nature of long-range letter correlations in texts

D. Yu. Manin, maninpobox.com

November 29, 2016

Abstract

The origin of long-range letter correlations in natural texts is studied using random walk analysis and Jensen–Shannon divergence. It is concluded that they result from slow variations in letter frequency distribution, which are a consequence of slow variations in lexical composition within the text. These correlations are preserved by random letter shuffling within a moving window. As such, they do reflect structural properties of the text, but in a very indirect manner.

1 Introduction

Statistical properties of numerical and symbolic sequences derived from naturally occurring phenomena are of interest in many different areas. To name just a few examples, human language texts [1, 2, 3, 4, 5, 6], music [4, 7], DNA sequences [8, 9], and heartbeat recordings [10] have been subject to such examinations. There appears to be a common theme in these studies: the sequences in question are certainly not “random” in some sense, they are produced by “complex systems” (we use quotes here to convey an intuitive, non-terminological status of these statements), and so their statistical properties should depart from those of random sequences, thus revealing the regularities. One of the hopes is that we can find some general characteristics of information-bearing sequences. If this is possible, not only would we achieve new understanding of the systems in question, but also it would be possible to apply the analysis to systems of uncertain status, such as the non-coding regions of DNA, to determine whether they carry information or not.

In this work we take a closer look at one particular area where natural-language texts were found to depart from randomness: long-range letter correlation. Various authors suggested that such correlations, observed on distances of $10^3–10^4$ characters, are indicative of stylistic and conceptual (semantic) coherence of the text [5], or, more cautiously, “are of structural origin” [11]. We will demonstrate that (1) the long-range letter correlation arises from slow changes in letter distribution along the text, (2) which in turn result from slow changes in lexical composition, (3) the primary role being played by the more frequent words.
2 Random walk transformation

A popular method proposed in [9] for assessing the degree of randomness in a numerical sequence \( \{x_i\}, 0 < i < N \), as it is usually presented, is to consider its members as sequential steps of a one-dimensional random walk and calculate the mean-square displacement as a function of time interval:

\[
y_{i,k} = \sum_{j=i}^{i+k} x_j
\]

\[
F(k) = \langle y_{i,k}^2 \rangle_i - (\langle y_{i,k} \rangle_i)^2
\]

where the angle brackets \( \langle \ldots \rangle_i \) denote the average over all initial positions \( i \) in the sequence, and \( k \) is the interval length assumed to be much shorter than the total sequence length \( k \ll N \).

Or, if we subtract the mean from the data, \( \xi_i = x_i - \langle x \rangle \), then \( F(k) \) becomes the mean-square of the partial sums of the resulting sequence,

\[
F(k) = \langle S_{ik}^2 \rangle_i, \quad S_{ik} = \sum_{j=i}^{i+k} \xi_j
\]

If each \( x_i \) results from an independent trial of a random variable with variance \( \sigma^2 \), \( F(k) = k\sigma^2 \) and thus grows linearly with \( k \). If, on the other hand, there are correlations in the sequence, i.e. some averages \( \langle x_i x_{i+k} \rangle_i \) do not vanish, the growth of \( F(k) \) may depart from linearity. Generally speaking, power-law growth

\[
F(k) \sim k^\alpha
\]

may indicate fractal structure of some sort in the data sequence. The quantity \( \alpha \) is the H"{o}lder (or Hurst) exponent of order 2.

There are many possible ways to convert a natural text to a numerical sequence in order to calculate \( F(k) \). One can use a binary representation of characters and consider consecutive bit values in it [2], or assign a numerical value to each letter [1]. One can also work on the level of words and replace each word by its frequency rank [6], or build a binary sequence where 1 (resp., 0) corresponds to the transition to a longer (resp., shorter) word or to a more frequent (resp., less frequent) word [3]. Regardless of the method used, the cited authors found departures from the linear growth of the displacement function \( F(k) \). We will loosely follow the method of [1] here to demonstrate the result. We use one of the texts analyzed in that work, Herman Melville’s magnum opus Moby Dick having a respectable volume of about \( 1.2 \cdot 10^6 \) letters. For comparison, we also utilize Dickens’ David Copperfield with over \( 2 \cdot 10^6 \) letters. Before processing, the texts were converted to lowercase and non-alphabetic character sequences were collapsed to single spaces (i.e., in regular expression terms, \( s/[^a-z]+/ /g \)). The resulting character sequence is the subject of all further analysis.

To obtain a numerical sequence from the text for the random walk analysis, following [1], we select a letter and convert all instances of that letter to ones, and all other characters to zeros. Fig 1 shows \( F(k) \) for three letters ’a’, ’v’, and ’x’ representative of high, middle, and low frequency characters.
In the range of $10 < k < 600000$, where end effects can be neglected, there are three more or less distinct regions in the chart Fig. 1. Roughly between 10 and 200 characters, $F(k)$ exhibits linear growth indicating lack of correlations. Between 200 and 1200 characters (for 'a'), it is consistent with a power-law growth with exponent $\alpha \approx 1.2$ (consistent with the value reported in [1]). Above that, $F(k)$ seems to return to linear growth. Note the strikingly similar behavior of all three letters. It should be noted that the authors of [1] studied the displacement function averaged over all letters. It is interesting though that long-range correlations are revealed even in the sequences obtained from individual letters.

This result of [1] was corroborated there with other measures (see also [3], where detrended fluctuation analysis technique was applied to Moby Dick). The question we are concerned with here is what this result actually means. Ebeling et al. [1] demonstrated that if the text is randomly shuffled — whether on the level of letters, whole words, or complete sentences — the correlations are destroyed, and $F(x)$ returns to the linear growth in the entire valid range of $k$. This demonstrates that neither intra-word letter correlations, nor intra-sentence syntactic and semantic relations are responsible for the observed large-scale behavior. What is, then?

### 3 Slow distribution changes

The sentence-level shuffling of the text preserves the syntax and semantics of the language, but destroys the overall narrative with its plot and composition. Since it also destroys long-range correlation, it could be tempting to conclude that the correlation is a direct consequence of the narrative structure. It is easy, however, to disprove this notion by shuffling the text within a moving window. Namely, consider the original text as a sequence of characters $T = \{c_i\}$ and derive from it a character sequence $T'$, where the $i$-th position is occupied by a character randomly selected from $\{c_j | i - n/2 < j < i + n/2\}$, where $n$ is the window size. This sequence preserves the overall letter distribution of $T$ and, in addition, any slow changes in this distribution, but completely destroys everything else; $T'$ is not a natural language text, and $T$ can not be reconstructed from it. Fig. 2 compares the behavior of $F(k)$ for the original text and for the sequence shuffled with window $n = 3000$. Clearly, all the features of the random walk are preserved by the shuffling. With increasing window size, as expected, the linear region extends to the right, and eventually long-range correlations disappear altogether as window size exceeds the maximum correlation length.

This leaves little room for speculation: obviously, the specific behavior of the displacement function is solely a result of bulk letter distribution in the text, unrelated to any structural features that distinguish an arbitrary character sequence from a text in a natural language.

To further demonstrate this, we generate another character sequence which has no relationship to the Moby Dick text, and results from a random process that generates
the letter 'a' with probability $p$ and a blank space character with probability $1-p$, where $p = 0.062$ except in a short range of length 6250, where $p = 0.1054$ (these parameters were selected to obtain the desired qualitative behavior of the displacement function; they have no significance beyond that). Again, the displacement function exhibits the same qualitative features as for Moby Dick, as shown in Fig. 3. Of course, the distribution of the letter 'a' in Moby Dick is very different, but the point is that $F(k)$ does not reveal the difference.

We can conclude that the behavior of the displacement function $F(k)$ results from slow changes in bulk letter distribution in the text on the scales of the order $10^3$–$10^4$ characters. But why would the letter distribution be changing at all? It would seem a priori that it should be a rather stable feature of a given language, or at the very least, a given language subset. For example, children’s books may have a lower frequency of such letters as 'q' or 'x', which in English appear mostly in the “long” words of Latin origin. But there is no apparent reason why the frequency of such a neutral and common letter like 'a' should be subject to slow fluctuations.

To investigate this issue, we turn to a different tool.

4 Jensen–Shannon information divergence

We want to compare letter distributions in different segments of the text. A convenient measure for this is provided by Jensen–Shannon information divergence (JSD) [8]. Let $p = \{p_i\}$ and $q = \{q_i\}$, $1 < i < n$ be two frequency distributions of the same dimensionality $n$, normalized so that $\sum p_i = \sum q_i = 1$. Define

$$D(p, q) = H((p + q)/2) - (H(p) + H(q))/2$$

where $H$ is the entropy of the distribution

$$H(p) = \sum_i p_i \log p_i$$

and $(p + q)/2$ is a shorthand for the distribution $r$, such that $r_i = (p_i + q_i)/2$. (If $p$ and $q$ are determined from different numbers of trials, they should be weighted with corresponding coefficients in (5), but we don’t need this generalization here.) This measure is related to the mutual information between the two distributions, and vanishes if they are identical.

JSD was applied in [8] to DNA sequences and in [7] to texts and music for the purpose of segmentation, i.e. splitting a sequence into parts maximizing the difference in composition (whether in terms of “letters”, “words”, “keywords”, etc). Here we have a different application in mind. We want to find out whether the letter distribution undergoes statistically significant changes along the text. To this end, we will compare two adjacent, equal length, regions of the text, and we need to determine whether the two observed frequency distributions in them are likely to result from the same underlying probability distribution. Consequently, we need to calculate the fluctuation
level, i.e. the expected JSD between two realizations of the same probability distribution. General statistical properties of JSD were obtained by Grosse et al. [8], and we’ll briefly reproduce the derivation for the particular case at hand.

Let \( p \) be the probability distribution and \( q \) the observed frequency distribution obtained from \( N \gg n \) trials. The variance of each \( q_i \) is then \( \sigma_i^2 = 1/(p_i N) \). Assuming that it is small, \( \sigma_i \ll p_i \), we can represent \( q_i = p_i (1 + \epsilon_i) \), \( \epsilon_i = O((p_i N)^{-1/2}) \) and estimate for each term of the sum in (5)

\[
D_i(p, q) = \frac{1}{2} \left( p_i \log p_i + q_i \log q_i - (p_i + q_i) \log \frac{p_i + q_i}{2} \right) \tag{7}
\]

\[
= \frac{p_i}{2} ((1 + \epsilon_i) \log(1 + \epsilon_i) - (2 + \epsilon_i) \log(1 + \epsilon_i/2)) \tag{8}
\]

\[
\sim \frac{1}{2} p_i \epsilon_i^2 \tag{9}
\]

\[
= O(1/8N) \tag{10}
\]

where the first two terms in the Taylor expansion of \( \log(1 + x) \) were used (assuming natural logarithms to simplify the expressions). Since the deviation here is unidirectional, i.e. JSD can not be negative, the estimate for the sum in (5) is to be multiplied by \( n - 1 \), the number of degrees of freedom. Finally, if both \( p \) and \( q \) are realizations of an unknown probability distribution, this adds another factor of 2, and we arrive at

Fluctuation level of \( D(p, q) = \frac{n - 1}{4N} \) \tag{11}

where, again, \( N \) is the total number of trials in each of \( p, q \), and \( n \) is the number of possible outcomes\(^1\).

[Figure 4 about here.]

Fig. 4 shows how JSD between adjacent segments of length \( n \) varies along the text for two window sizes, \( n = 1000 \) and 100000 characters (intermediate sizes not shown to avoid clutter). For the shortest window size, JSD is at the fluctuation level (which confirms the estimate (11) as a side effect). With larger window size, however, systematic variations in letter composition stand out from the decreasing statistical noise and become significant. It may be interesting to see whether the peaks in the figure match some compositionally meaningful locations in the text, but for the purposes of this work what’s important is that JSD is comfortably above the fluctuation level practically everywhere, albeit in some places more so than in others.

Obviously, in the natural text, letters come in packages — words — and any changes in letter composition along the text must result from the changes in lexical composition. It is well known that words in the language are distributed in a highly skewed fashion, with many instances of a small number of frequent word types and increasingly larger number of rare word types. The distribution is approximately described by Zipf’s law \([11]\)

\[ f_k \sim 1/k \] \tag{12}

\(^1\)Interestingly, the probabilities themselves do not enter this estimate at all. This is in an apparent contradiction with the fact that a pdf with some number of possible outcomes \( n \) and \( p_n = 0 \) is completely equivalent to a pdf with \( n - 1 \) possible outcomes, while the estimate (11) will be different for them. However the estimate is not valid when some \( p_i \) tends to zero, because this violates the assumption of small \( \sigma_i^2 \).
where $f_k$ is the frequency of the word with rank $k$, and the rank is the word’s sequence number in a dictionary where words are ordered by decreasing frequency. The top positions in such a dictionary are occupied by grammatical words (articles, prepositions, personal pronouns, conjunctions, etc.) and high-frequency significant words (nouns like man, adjectives like old), which are common for all texts, and by select “content words” peculiar to a particular text (ship, Ahab, whale in Moby Dick). The top of the dictionary is relatively stable, while the rest of it is much more subject to changes from text to text and within texts, depending on style, topic, etc. It is not clear a priori which part(s) of the lexicon are responsible for the changes in letter composition of the text: the less frequent words are, generally speaking, more variable, but because there are many more of them, the law of large numbers should ensure a more random mixing of the letters; the more frequent words are less variable, but any change in their distribution would have a larger impact, because there is a small number of frequent word types. In the next section we focus on this question.

5 Lexical composition and its impact on letter distribution

To investigate the effect of different parts of the vocabulary, we applied the analysis of the previous section separately to the words in different frequency ranges. The frequency dictionary of the text was subdivided into 5 ranges so that words in each range are responsible for 20% of the total number of letters each. For each range, the rest of the words were blanked out, and JSD between adjacent 100000-letter segments were calculated (blanks were not counted). Fig. 5 shows the average JSD normalized by the fluctuation level for Moby Dick and David Copperfield. Interestingly, it is somewhat above fluctuation level even for the most infrequent words, but the biggest contribution in both cases is due to words that are close to the top of the dictionary, but not the most frequent ones. It is still a relatively small number of word types (135 word types for MB and 80 word types for DC). Most names of the major characters fall into this frequency range. It is a more idiosyncratic set of words than the top of the lexicon, but it is still limited enough that the letters are not well mixed according to probabilities.

It is easy to see qualitatively why the slowly changing composition of the top $\sim 10^2$ words will lead to corresponding changes in letter distribution and long-range letter correlation. As a simple model, suppose that 100 “content” words are responsible for 20% of all letters in a $10^5$-letter text segment. These 100 words are selected from the lexicon of the language, which, as a whole, is characterized by some letter frequency distribution $p_i$. The content words are selected by the writer according to the topic and style, but the resulting selection of letters is essentially random (except for very rare cases of highly alliterated prose). However due to the small number of “trials”, it will have a considerable variance. For example, the average frequency of the letter a in English is about 10%, hence out of the $100 \cdot 4.5 = 450$ letters in the 100 content words there will be about 45 ‘a’s. Depending on which 100 words are chosen, the expected
variance is on the order of \( \sqrt{45} \approx 7 \), i.e. as much as 15%. Even if the variance in the remaining 80% of the text is negligible, the frequency of 'a's will fluctuate much stronger than for a Poissonian process on the characteristic lengths where the “content words” are stable.

6 Discussion

From the analysis we presented in this paper, it follows that long-range letter correlation in natural texts results from the interplay of the following factors:

1. a significant portion of the letters in texts is contributed by a relatively small class of “content” words with high frequency and high variability in the text;
2. slow variation in the composition of the “content” words causes corresponding slow variation in the letter distribution;
3. this translates to long-range correlation between letters, which is invariant with respect to letter shuffling within sliding window of length 3000.

The variation in lexicon may reflect various properties of the text. For example, here are some of the differences observed between the first and the second halves of *Moby Dick*:

1. increased frequency of the word *whale* in the second half reflects topical differences;
2. the word *is* is more frequent than *was* in the first half, but less frequent in the second half, reflecting the difference in narrative structure;
3. the ratio of articles *the* to *a* increases from 2.7 in the first half to 3.5 in the second half, which may, for example, indicate the trend from general statements to concrete narrative.

The long-range letter correlations can serve as an indirect and indiscriminate indicator of slow variations of character frequency distribution. In natural texts, these variations result from the corresponding slow variations of lexical composition, which in turn reflect various structural properties of the text. However in the case of symbolic and numerical sequences of a different origin, such variations in and of themselves do not necessarily indicate “complexity” or information-bearing nature.
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