Some electronic and optical properties of self-assembled quantum dots: asymmetries in a lens domain

Arezky H. Rodríguez and L. Meza-Montes

Instituto de Física, Universidad Autónoma de Puebla, Apdo. Postal J-48, Puebla, Pue. 72570, México.

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Abstract

The self-assembled quantum dot with lens domain has rotational symmetry but it is intrinsically asymmetric when the electron moves perpendicularly to its circular base, i.e. along the rotational axis. To characterize this asymmetry, an external electric field is applied along the positive or negative direction of the rotational axis. We report the different Stark shifts appearing in the spectra as a function of the field intensity for different lens domains. It is shown that for a flat lens domain the asymmetry effects decrease, but even for very flat lenses they can not be approximated by a cylindrical domain. Finally, some optical properties such as the dielectric constant and electroabsorption are studied. Signatures of the energy spectrum reveal in these quantities. The importance of considering the proper lens domain as long as the magnitude and direction field to tune a specific level transition is stressed.
I. INTRODUCTION

Quantum dot (QD) structures have attracted the interest of theoretical and experimental research due to the novel properties they exhibit as a consequence of spatial confinement [1, 2]. The modern growth techniques have led to the self-assembled quantum dot [3] with nearly a lens shape, which is characterized by a spherical cap shape [4]. The electronic properties of this so-called self-assembled quantum lens (SAQL) has been studied previously [5, 6]. The study has included the effects of an external electric [7, 8] and magnetic field [9]. Some optical properties of such structures have also been analyzed [10].

The lens domain is limited by the interception of two boundaries, one given by the surface of a spherical cap with certain curvature, and the other a flat boundary. This yields an asymmetric behaviour of the charge carriers as they are pushed toward either surface. Consequences of this spatial asymmetry was first reported in Ref. [11] for microphotoluminescence measurements and was qualitatively reproduced in Ref. [12].

The present work is devoted to study in more detail the asymmetry effects on the electronic and optical properties of different lens configurations. It will be shown that in the presence of an external electric field, the electronic and optical properties have different behaviour depending on whether the field is along the positive or negative direction of the axial axis.

II. ELECTRONIC STRUCTURE.

We will consider a typical SAQL that presents a circular cross section of radius \(a\) and maximum cap height \(b\) with \(b < a\). The lens base lies on the \(xy\)-plane and the axial symmetric axis is along the \(z\) axis. We want to explore the implications of the different shapes of the lens boundaries on the physical parameters of the SAQL, such as energy levels and wavefunctions. Thus, the SAQL will be under an external electric field \(F\) which will be along either the positive or negative direction of the \(z\) axis.

The exciton wave functions are solutions of

\[
\left[-\frac{\hbar^2}{2m^*_e} \nabla^2_e - \frac{\hbar^2}{2m^*_h} \nabla^2_h - e \mathbf{F} \cdot \mathbf{r}_e - e \frac{\epsilon^2}{\epsilon |\mathbf{r}_e - \mathbf{r}_h|}\right] \Psi_{N_e,N_h}(\mathbf{r}_e, \mathbf{r}_h) = (E - E_g) \Psi_{N_e,N_h}(\mathbf{r}_e, \mathbf{r}_h),
\]

where \(E_g\) is the gap energy, \(\epsilon\) is the dielectric constant, and \(m^*_i\) is the quasiparticle effective
mass with \( i = \{e, h\} \), \( e \) (\( h \)) denoting electron (hole).

Closed solutions of one-particle wave functions \( \Psi_{N,m} \) and energy levels \( E_{N,m} \) as a function of the applied electric field and lens geometry have been published elsewhere \([6, 12]\). Here, \( N \) enumerates, for a fixed value of \( m \), the electronic levels by increasing value of energy, and \( m \) is the \( z \) component of the orbital angular momentum. The excitonic correction appearing in Eq. (1) has been considered in first order perturbation theory. Details can be seen in Ref. \([10]\).

### A. Asymmetry of the ground state transition energy.

In order to explore the effect of spatial asymmetry on the lens spectra, we have defined for a given material the parameter

\[
\delta(F) = E_T(+F) - E_T(-F),
\]

where \( E_T \) is the ground state transition energy. For an electric field of magnitude \( F \) the parameter \( \delta(F) \) measures the difference between the transition energies for the fields along the positive direction \( +F \) and the negative direction \( -F \).

In Fig. 1 we show (a) the parameter \( \delta(F) \) as a function of \( b/a \) for two values of the electric field and dots different materials, InAs/GaAs and CdSe/ZnSe. The unit of energy is \( E_o = \hbar^2/2m_eV_o^{2/3} \), where \( m_e \) is the free electron mass and the unit of electric field is \( F_o = E_o/eV_o^{1/3} \). We have used \( m_e^*/m_e = 0.023 \) and \( m_{hh}^*/m_e = 0.34 \) for InAs \([13]\) whereas \( m_e^*/m_e = 0.11 \) and \( m_{hh}^*/m_e = 0.44 \) for CdSe \([14]\).

The parameter \( V_o \) is the volume of the lens domain. It is related to the deformation \( b/a \) and the radius \( a \) of the domain through the expression \( V_o = 1/2\pi a^3(b/a)(1 + 1/3(b/a)^2) \). Thus, the lens domain can be defined by three parameters: the volume \( V_o \), the radius \( a \) and the deformation \( b/a \), but only two of them are independent. The confinement regime is determined by the volume \( V_o \) of the domain and the deformation \( b/a \). If one of the them decreases, the energy levels increase. We want to study the effects on the energy levels by changes of the lens domain but not in terms of the confinement caused by the volume variation. Therefore, in Fig. 1(a) the lens volume is kept fixed as \( b/a \) is modified.

In Fig. 1(b) the values of \( \delta \) are shown, now as a function of the absolute value of the electric field for two different lens domains given by the indicated values of \( b/a \). Similarly
FIG. 1: Parameter $\delta$ as a function of (a) the lens domain deformation $b/a$ for two values of the electric field, and (b) the absolute value of the electric field $F$ for two lens configurations. For solid line the InAs parameters were used whereas the dotted line refers to CdSe SAQDS.

to Fig. (a), the volume $V_o$ has been fixed.

It can be seen in both Figures that as $b/a$ decreases, so does the value of $\delta$. This could be expected due to the fact that, as the lens domain becomes flatter, the two boundaries turn out to be more similar and then the asymmetry along the axial axis “is reduced”. Thus, higher field intensities $F$ are necessary for increasing the value of $\delta$ for small $b/a$ ratio. Notice also that for the CdSe SAQL, the values of $\delta$ in both Figures are higher compared to those of the InAs dot. This effect is produced by the effective mass which in the CdSe material is higher for both the electron and heavy hole. Then the energy of the ground state
transition is lower than the InAs case, and the contribution of the electric field to energy is higher that in the InAs case for a given value of $F$.

Higher states are less affected by the field because its energy contribution is then relatively lower [7]. Then, lower values of $\delta$ would be expected for both materials in these cases. Therefore, for a flat enough lens domain ($b/a \ll 1$) and not too high values of the electric field, the asymmetry in the values of the ground and the first excited states transition energies could be neglected. This suggests to employ simpler geometries, as we now discuss.

B. Comparison between lens and cylindrical domains.

Due to the results obtained in the previous section, one could ask if a strongly deformed lens ($b/a \ll 1$) could be approximated by a cylindrical domain with the same volume $V_o$, radius $r_{cil}$ and height $h_{cil}$. The cylinder parameters should be selected in such a way that the cylinder shape resembles the lens domain.

We have now the problem of selecting properly the values of $r_{cil}$ and $h_{cil}$ to obtain energy levels the more similar to those of the lens. It is also necessary to take into account that the energy levels in the cylindrical domain are given by

\[
E_{n,l,m}^{(cil)} = \frac{\hbar^2}{2m^*} \left( \frac{\mu_{n,l}^2}{r_{cil}^2} + \frac{m^2 \pi^2}{h_{cil}^2} \right),
\]

where $\mu_{n,l}$ is the $n$-th zero of the Bessel function $J_l$ with $l = 0, 1, 2, \ldots$ and $m = 1, 2, \ldots$ the angular momentum quantum numbers due to the rotational symmetry of the cylinder.

We impose two conditions for any values of $b/a$: (1) the cylinder and lens domains have the same volume $V_o$ which is kept constant, and (2) the ground electronic state of both domains geometries are the same. This allow us to find suitable values of $r_{cil}$ and $h_{cil}$ and to find out if the first excited states of the cylinder are rather close to those of the corresponding lens.

There are two possible cylinder configurations which fulfill the former conditions for a given value of $b/a$. One of them consists of a very high cylinder with $h_{cil} \gg b$ and small $r_{cil}$. This configuration is not considered here, instead we consider the other one since it resembles the lens domain. In Fig. 2(a) we can see the behaviour of $r_{cil}$ and $h_{cil}$ as a function of $b/a$, for a cylinder domain which fulfills the conditions (1) and (2). The ratio $r_{cil}/a$ is almost constant and we have $r_{cil}/a < 1$. Contrary, the ratio $h_{cil}/a$ strongly decreases with $b/a$. The
FIG. 2: (a) The behaviour of the \( r_{cil} \) and the \( h_{cil} \) as a function of \( b/a \) for a cylindrical configuration which fulfills conditions (1) and (2). (b) First two energy levels of the lens domain and first six energy levels of the cylindrical domain as a function of \( b/a \). Quantum numbers of the levels are also shown in the Figure. See text.
dotted line maps directly the values of \( b/a \) onto the vertical axis. Comparing this dotted line with \( h_{cil}/a \) we conclude that \( h_{cil}/a < b/a \) for the shown range of \( b/a \). Consequently, we can see that \( h_{cil} \) is also smaller than the lens height \( b \) and for \( b/a \to 0 \) we have that \( h_{cil} \to b \).

In Fig. 2 (b) we have plotted using continuous lines the first two energy levels of the lens domain as a function of \( b/a \). We have included also in dotted lines the first six energy levels of the cylindrical configuration using Eq. (3) with parameters \( r_{cil} \) and \( h_{cil} \) shown in Fig. 2(a). In this case, the energy unit is \( E_o = \hbar^2/2m^*V_o^{2/3} \), with \( m^* \) the electron effective mass. Then, the plot shows the universal behaviour of the energy levels of the lens and cylindrical domain for any material having effective mass \( m^* \). The quantum numbers of the levels are shown in the Figure.

To analyze Fig. 2 it is important to take into account that as the lens parameter \( b/a \) decreases the radius \( a \) increases, for a given value of the volume \( V_o \). The behaviour of \( r_{cil} \) and \( h_{cil} \) in Fig. 2 (a) and Eq. (3) suggest that the cylinder levels of Fig. 2 (b) can be approximated for \( b/a \ll 1 \), by the expression

\[
E_{n,l,m}^{(cil)} \approx \frac{\hbar^2}{2m^*} \frac{m^2 \pi^2}{h_{cil}^2},
\]

showing why cylinder levels with same \( m \) tend to collapse in the \( b/a \ll 1 \) regime. This contrasts with the behaviour of the energy levels of the lens domain which approach each other but remain separated. See Fig. 2 (b).

It can be concluded then that the first excited levels of a flat lens can not be approximated by those of a cylindrical domain with same volume and ground state energy.

III. LENS DOMAIN: INFLUENCE OF ASYMMETRY ON THE OPTICAL PROPERTIES.

The asymmetry found in the energy levels for positive and negative electric field is not the only possible effect of the domain asymmetry along the axial axis. The wavefunctions are also affected by the different curvatures of the boundaries, depending on the direction of the electric field. It also give rise to the asymmetry in other physical properties. See for example Fig. 2 in Ref. [12], where differences in the carrier polarizations and the oscillator strengths were also reported. One way to study the combined influence of asymmetry on the shifts of the transition energy and the oscillator strengths is through the dielectric constant.
| Parameters     | InAs |
|---------------|------|
| $E_g$ (eV)    | 0.45a|
| $m^*_{c}/m_0$ | 0.023a|
| $m^*_{hh}/m_0$| 0.34a|
| $m^*_{lh}/m_0$| 0.027a|
| $\Delta E_c$ (%) | 40%a|
| $\Delta E_v$ (%) | 60%a|
| $P^2/m_0$ (eV) | 10.0 c|
| $\gamma_{hh}$ (meV) | 3 |
| $\gamma_{lh}$ (meV) | 5 |

$^a$ Ref. [13]
$^b$ Ref. [14]
$^c$ Ref. [15]

TABLE I: Parameters used in the calculation of the dielectric constant.

In the following, we analyze the imaginary part of the dielectric constant for the case of InAs/GaAs SAQLs by comparing two lens configurations. Due to the axial symmetry of the SAQL, the interband selection rules correspond to excitonic branches such that $m_c - m_v = 0$, where $m_c(m_v)$ is the $z$-projection of the angular momentum in the conduction and valence band respectively. We have only considered incident frequencies in the range below the band offset, according to the material parameters listed in Table III where $m^*$ is the effective mass of the particle in the conduction $c$ or valence bands $hh$ and $lh$, $P$ is the interband optical matrix element between conduction and valence bands and $\gamma_{N_c,N_{hh}}$ is the broadening parameter of the Lorentzian function. The value $E_g = 1.51$ eV for GaAs has also been used [13]. The theoretical formalism to calculate the dielectric constant in SAQLs was given in Ref. [10].

Figure 3 shows the dielectric constant for a SAQL of InAs embedded in a GaAs matrix for an incident light propagating along the $z$-axis and polarized in the plane of the SAQL base. The SAQL in Fig. 3 (a) and (b) has a radius $a = 21.6$ nm and height $b = 11.0$ nm, while in Fig. 3 (c) the configuration has the same volume but $a = 17.9$ nm and $b = 16.3$ nm.
TABLE II: Data corresponding to the transitions labelled in Fig. 3 (a) and (b).

Three values of the electric field are considered: $F = 0$ (solid line) in Fig. 3 (a) and $F = 150$ kV/cm (dotted line) and $F = -150$ kV/cm (dashed line) in (b) and (c). It is not shown the complete range of the band offset, but only an energy range relevant for discussion where the first transition takes place. In Table II the quantum numbers for the transitions in Fig. 3 are denoted by $(N_c, N_v; m)$ where $N$ enumerates the electronic energies by increasing value, for a fixed value of $m$. Moreover, the numbers $(N_c, N_v; m)$ indicate that the transition takes place from level $N_v$ of the valence band ($hh$ or $lh$) to level $N_c$ at the conduction band. Also specified are the oscillator strengths at cero, -150 and 150 kV/cm. The symbol “-” means that the transition does not occur because the corresponding level at the conduction band is not present.

Comparing Figs. 3 (a) and (b) it is clearly seen the energy red-shift of the transitions 1 to 6 (Stark effect) at $F = \pm 150$ kV/cm respect to their positions at $F = 0$. Nevertheless, in Fig. 3 (b) it can be seen that these displacements are not quantitatively the same for positive or negative field. For example, the transition peaks labelled by 1 and 2 do not
FIG. 3: Dielectric constant for InAs/ GaAs SAQLs. (a) $a = 21.6$ nm and $b = 11.0$ nm. (b) Same configuration as in (a). (c) $a = 17.9$ nm and $b = 16.3$ nm. Both configurations have the same volume. Solid line represents the case with $F = 0$, while dotted (dashed) line the case $F = 150 (F = -150)$ kV/cm.
appear at the same energy leading to $\delta(F) \neq 0$ for both cases.

On the other hand, the oscillator strengths for transitions at $F = 0$ are in general higher than their corresponding values for $F = \pm 150$ kV/cm due to the mixture of states caused as the electric field is turned on. Notice that in Fig. 3(a) the scale of the dielectric constant is twice the ones used in Figs. 3 (b) and (c). Furthermore, transition 6 is originated in a light-hole valence band and comparing it in Figs. 3(a) and (b) we can conclude that the transitions for the $lh$ band are less affected by the electric field, due to the lower effective mass than in the case of the $hh$-valence band.

It can also be seen that the transitions from valence to conduction bands with $m \neq 0$ are relatively stronger than those with $m = 0$, for a given value of the electric field. Compare for example, transition 2 which has $m = 1$ (see Table II) with transition 1 with $m = 0$. It is a consequence of the excitonic breaking of degeneracy. In the case of a transition with $(N_c, N_v, m = 0)$ the excitonic couple does not break the degeneracy. However, for a transition with $(N_c, N_v, m \neq 0)$, a three-fold level splitting is obtained. See details in Ref. [10]. The splitting levels remain very close each other causing closely spaced peaks which give rise to an enhanced dielectric signal.

Forbidden transitions at zero field are allowed at $F \neq 0$ and some of them are tuned at $F = \pm 150$ kV/cm in different ways, according to the direction of the applied field. See for example transitions 7, 9, 10, 11 and 12. In the case of transition 9, its oscillator strength at $F = 150$ kV/cm is so small that does not yield a visible peak, but this transition is noticeable for $F = -150$ kV/cm. On the other hand, transition 8 only appears at $F = 150$ kV/cm because the corresponding level does not exist at the conduction band for $F = -150$ kV/cm.

In Fig. 3(c) the domain has the same volume as in Fig. 3(a) and (b), but now the parameter $b/a = 0.91$. The transitions take place this time at lower energies, as can be seen in the Figure. This shift is due to the softer confinement, for the lens has a shape almost semi-spherical. For the same reason, the effects of the electric field are reinforced, see Fig. 11. Thus, the mixing of the wavefunctions becomes stronger and, as a consequence, the oscillator strengths are more homogeneously distributed along all the possible transitions giving rise to rich peaks distributions with quantum numbers $N_v \neq N_c$. For the same reason, the peak heights are smaller compared to those of Fig. 3(b). Differences in the energies where transitions 1 and 2 occur are bigger than in Fig. 3(b), in agreement with the result.
FIG. 4: Oscillator strength as function of the electric field for electron-hole transitions 7, 10, 11 and 12 which correspond to Fig. 3

obtained in Fig. 1.

For a better understanding of the peaks distribution in the dielectric constant, in Fig. 4 the oscillator strengths for transitions 7, 10, 11 and 12 are plotted as a function of the electric field. We can see a strong variation of the oscillator strength for transitions 7, 10 and 12 in a small range of positive values of the electric field. For transitions labelled with 7 and 10 this variation is a consequence of an anticrossing effect around \( F = 100 \) kV/cm, between the corresponding energy levels \( N_v = 5 \) (in transition 7) and \( N_v = 4 \) (in transition 10) for the \( hh \)-valence band with \( m = 0 \) (see Table II). Similarly, transition 12 has a sharp variation at \( F \approx 150 \) kV/cm because of an anticrossing of the corresponding valence band energy level
at this value of the electric field. Then, it is possible to say that for some transitions the
intensity of the dielectric signal vanishes at certain values of the electric field and a given lens
configuration, leading to a non-monotonic behavior of the oscillator strengths as a function
of the electric field.

A. Electroabsorption.

The optical properties of the semiconductor structure have led to the most powerful
techniques to study the electronic properties in solids [16]. The electroabsorption is one
of these techniques used to inquire about the fundamental properties of different quantum
structures [17]. The relation between the imaginary part of the dielectric constant $\epsilon_2$ and
the absorption coefficient $\alpha$ is given by

$$\epsilon_2(\omega) = \frac{n c}{\omega} \alpha(\omega)$$

(5)

where $n$ is the real part of the refractive index and $c$ the speed of light. We obtained the
real part of the refractive index according to

$$n = \sqrt{\frac{\epsilon_1^2 + \epsilon_2^2 + \epsilon_1}{2}} + n_\infty$$

(6)

where $\epsilon_1$ and $\epsilon_2$ are the real and the imaginary part of the dielectric constant, respectively,
and $n_\infty$ is the high-frequency refractive index. For the case of an InAs SAQL we take
$n_\infty = 3.517$ [15]. $\epsilon_1$ is calculated from $\epsilon_2$ and the Kramers-Kronig relations [10].

Previous studies [17, 18] have used the absorption change $\Delta \alpha(\omega, F) = \alpha(\omega, F) - \alpha(\omega, 0)$ of
a SAQL as a tool to study low dimensional structures. In our case, we study the absorption
change for the two configurations used in Fig. 3. Different values of $\Delta \alpha$ will be obtained
according to $F$ being positive or negative. The results are shown in Fig. 5. In Fig. 5 (a) the lens configuration of Fig. 3 (a) and (b) has been used while in Fig. 5 (b) the lens configuration corresponds to Fig. 3 (c). In both cases, the dotted (dashed) line represents $\Delta \alpha$ for $F = 150$ ($F = -150$) kV/cm. Typical electroabsorption spectra are obtained and they
depend on the direction (positive or negative) of the electric field. A close correspondence is
observed among the peak positions in this figure and the transitions shown in the constant
dielectric (Fig. 3). Some transitions have been indicated for comparison. $\Delta \alpha$ depends on the
direction of the electric field due to the spatial asymmetry of the lens, confirming that not
FIG. 5: Electroabsorption as a function of the energy. (a) Configuration corresponding to Fig. 3(a) and (b), (b) Configuration corresponding to Fig. 3(c).
only the magnitude but also the direction of the field is important. Since this effect becomes important for stronger confinement, the peak heights turn out to be larger as Fig. 5 (a) shows. Thus, electroabsorption measurements would provide signatures of the electronic structure of the SAQL.

IV. CONCLUSIONS.

The present work has studied the influence of the lens asymmetry along the axial axis on its electronic properties. The difference in the energy Stark-shift was characterized with the parameter $\delta(F)$, which measures the separation of the ground state energy as the electric field has a positive or negative direction along the axial symmetry axis with same intensity. It was shown that the parameter $\delta(F)$ increases for high values of the field and diminishes as the lens deformation becomes larger (smaller ratio $b/a$). Nevertheless, even in this latter case the eigenvalues of the lens domain can not be approximated by those of a cylindrical domain with the same volume. Therefore, the full lens geometry must be considered always to obtain the correct spectrum.

The consequences of this asymmetry on the dielectric constant of the SAQL were also studied. We analyzed its behaviour as a function of the lens deformation for a fixed volume of the domain and a given absolute value of the field. Different transitions are obtained depending on the lens deformation $b/a$, the radius $a$ and the magnitude and direction of the external field. We emphasize the importance of the direction of the electric field in the optical properties, such as the absorption change, showing different behaviour according to a positive or negative direction along the axis lens. Thus, in an experimental set up, an specific transition could be tuned. Moreover, the correlation between the electroabsorption peaks (shown in this work) and the electronic structure of the quantum lens (see details in Ref. [10] and references therein) can be useful to characterize the geometrical dimensions of these semiconductor nanostructures.
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[1] A. D. Yoffe, Advances in Physics 50, 1 (2001).
[2] L. Jacak, P. Hawrylak, and A. Wojs, Quantum dots (Springer-Verlag, Berlin, 1998).
[3] I. N. Stranski and L. Krastanow, Sitzungsber. Akad. Wiss. Wien 146, 797 (1938).
[4] P. W. Fry et al., Phys. Rev. Lett. 84, 733 (2000).
[5] A. Wojs, P. Hawrylak, S. Fafard, and L. Jacak, Phys. Rev. B 54, 5604 (1996).
[6] A. H. Rodríguez, C. R. Handy, and C. Trallero-Giner, J. Phys.: Condens. Matter 15, 8465 (2003).
[7] A. H. Rodríguez and C. Trallero-Giner, Phys. Stat. Sol. (b) 230, 463 (2002).
[8] A. H. Rodríguez, L. Meza-Montes, C. Trallero-Giner, and S. E. Ulloa, Phys. Stat. Sol. (b) 242, 1820 (2005).
[9] E. Casado and C. Trallero-Giner, Phys. Stat. Sol. (b) 196, 335 (1996).
[10] A. H. Rodríguez, C. Trallero-Giner, M. Muñoz, and M. C. Tamargo, Phys. Rev. B 72, 045304 (2005).
[11] S. Raymond et al., Phys. Rev. B 58, R13415 (0098).
[12] A. H. Rodríguez and C. Trallero-Giner, J. Appl. Phys. 95, 6192 (2004).
[13] E. Menéndez-Proupin, C. Trallero-Giner, and S. E. Ulloa, Phys. Rev. B 60, 16747 (1999).
[14] M. Muñoz et al., App. Phys. Lett. 83, 4399 (2003).
[15] L.-B. Tables, Numerical Data and Functional Relationships in Science and Technology, Vol. 37a of Landolt-Börnstein New Series, Group III, edited by O. Madelung ed. (Springer-Verlag, Berlin, 1982).
[16] M. Dressel and G. Grüner, Electrodynamics of Solids: optical properties of electron in matter (University Press, Cambridge, 2002).
[17] K. Tanaka, T. Takahashi, and T. Kondo, Phys. Rev. B 71, 045312 (2005).
[18] F. Henneberger, S. Schmitt-Rink, and E. O. Gödel, Optics of Semiconductor Nanostructures (Akademie Vlg., Berlin, 1993).