Modelling of Alpha and Beta for Rain Rate Prediction for Radio Propagation Systems

Y. K. Sanusi, O. Oyeleke, A. O. J. Abiodun and G. A. Alagbe

Abstract — The effect of rain in the design of satellite and terrestrial microwave radio links is of interest to Engineers and Scientists. It is good to have a reliable design that guarantees high level of accuracy of the rain rate distribution from the lowest rain rate value to the highest. The present work proposes a model that expresses rain rate as a function of alpha and beta obtained at 0.01% of time. When tested, the results obtained with the measurement perform well for the stations considered at a rain rate between 5mm/h to 200 mm/h. Thus, α and β, the empirical models that were obtained through them could be a useful tool for the radio design engineers for high rain rate areas.

Index Terms — Radio propagation, Rain Rate, Model, Attenuation, cumulative Distribution.

1 INTRODUCTION

Attenuation due to rainfall plays significant roles in the design of earth satellite radio link at frequencies above 10GHz [1]. The increasing demand for these frequencies for telecommunication and broadcasting satellite has arouse interest in the study of radio wave attention due to rainfall on earth-satellite radio paths. Most of the attenuation studies on earth-satellite path have been carried out in the temperature regions of the world [2].

The precipitation characteristics in the tropics differ appreciably from those of the temperate regions in that empirical relationship obtained in the temperate region may not be very suitable for the system design in the tropical region. Rain attenuation can be obtained directly from rain drop size distribution [2]. It is well known that rain rates and the consequent high rain attenuation in the tropics is arguably the greatest constraint to the usability of Ku and Ka bands in the tropics [3]. However, in this work, rainfall intensity R at (mm/h) exceeded for 0.01% of the time in different locations are studied and a model that expresses rain rate as a function of alpha and beta was proposed. α and β parameters at that percentage of time are very important tools for radio wave attenuation predictions both on line-of-sight and Earth-space links.

A. Rain Distribution Models

[4] has proposed a model of rain distribution of the from:

\[
P(R \geq r) = \frac{ae^{-bT}}{r^\alpha} = r \geq 2mm / h
\]

Where \( P( R>r) \) is the probability that a rain rate, \( R \) exceeds a certain threshold \( r \), “\( a \)” and “\( b \)” are parameter depending on rain intensity for which one variant is given by:

\[
a = 10^{-4}R^2 0.001 \exp (U^{0.01})
\]

\[
b = 8.22R^{0.584} \_0.01
\]

[5] proposed a prediction procedure of rainfall rate cumulative distribution which is probability law leading to a more accurate prediction of rainfall rate cumulative distribution for all climates. The rate of rain data obtained using a rain gauge with an integration time of 10 seconds at Ile-Ife in Nigeria from September 1979 to December 1981 were used to study the dependence of rain statistics on integration time. The power law between the rain of different integration given by [2] was:

\[
R_i = aR^{\beta}_T
\]

Where \( R \) is the rain rate \( T \) is the integration time, at which the rain rate is available, “\( a \)” and “\( b \)” are parameters for location. [2] obtained the lognormal model using method of moment regression over a range of 0.25mm/h to 150mm/h. The model is given by:

\[
N(D) = \frac{N_T}{\alpha D^{2\beta}} \exp \left[ -\frac{1}{2} \left( \frac{D - \mu}{\sigma} \right)^2 \right]
\]

With Ile-Ife data, [2] obtained the following relation

\[
N_T = 108R^{0.363}
\]

\[
\mu = 0.195 + 0.199 \ln R
\]

\[
\sigma = 0.137 + 0.013 \ln R
\]

[3] – [5] carried out an inter-comparison of raindrop size distribution models presented by [8,9,10,11].
[5] – [7] used the raindrop spectra data obtained at Ile-Ife three years to investigate the dependence of drop size distribution on rain type and location. The conclusion was that the negative exponential model is adequate for drizzle whereas, the lognormal provides a better fit for the widespread shower and thunderstorm rains.

II METHODS

In view of the above, rain data were collected at the university of Ilorin over a period of two years 1st January 2004 to 31st December 2005 in the physics department metrological station and cumulative distribution of rain rate computed.

A. Rain Rate Model

From the graph displayed by rain rate computed against the percentage of time, the equation 9, below was obtained.

\[
R_t = \frac{\alpha}{(\%t)^{\beta}}
\]  

(9)

Where \((\%t)\) can be replaced using \(\Gamma\) which gives \(R_t = \frac{\alpha}{\Gamma^{\beta}}\) [6]. For each station, \(\alpha\) and \(\beta\) were obtained as shown in Table 1.

| Location  | \(\alpha\)  | \(\beta\)  | No of Years |
|-----------|-------------|------------|-------------|
| Brazilia  | 5.255       | -0.365     | 1           |
| Nairobi   | 6.3396      | -0.435     | 1           |
| Daresslam | 8.1138      | -0.406     | 1           |
| Ile-Ife   | 13.4246     | -0.3738    | 1           |
| Ilorin    | 16.626      | -0.3691    | 2           |

B. Cumulative Distribution of Rainfall

The cumulative distribution of the rainfall intensity was evaluated using the following procedure:

Yearly percentage of time expressed as:

\[
(\%t) = \frac{t}{T} \times 100
\]  

(10)

Where \(t\) is the total time during which rain rate \(R\) is executed and \(t\) is the overall period of interest taken to be one year in evaluating the yearly cumulative distribution for a non-leap year the percentage of time for the first minute will be:

\[
(\%t) = \frac{t}{T} \times 100 = \left(\frac{1}{365 \times 24 \times 60}\right) \times 100 = 1.9 \times 10^4 / hr
\]

For monthly cumulative distribution;

\[
(\%t) = \frac{t}{T} \times 100 = \left(\frac{t}{N_{days} \times 24 \times 60}\right) \times 100
\]

III RESULTS AND DISCUSSION

A. Behaviour of alpha and beta at \(R_{0.01}\)

Table 2 below shows the values of parameters ‘\(\alpha\)’ and ‘\(\beta\)’ computed by the use of regression programme. The rain rate obtained for the different locations at \(R_{0.01}\) are provided.

| Location | Values of Parameters | \(R_{0.01}\) (mm/h) |
|----------|----------------------|---------------------|
| Brazilia | 5.255 -0.365         | 50                  |
| Nairobi  | 6.3396 -0.435        | 58                  |
| Daresslam| 8.1138 -0.406        | 72                  |
| Ile-Ife  | 13.4246 -0.3738      | 80                  |
| Ilorin   | 16.626 -0.3691       | 90                  |

Fig 1: The graph of parameter \(\beta\) against rainfall intensity at 0.01%

In Fig 1, the parameters ‘\(\beta\)’ plotted against the values of rainfall intensity during the 0.01% of the time for each location in Table 2. The corresponding point can be fitted by the equation 11:

\[
\beta_p = 0.0025R_{0.01} - 0.5826
\]  

(11)

For all the stations considered, there is a good agreement between the values obtained from the predicted equations and the values obtained from the measurement. \(\beta_m\) is measurement value and \(\beta_p\) is predicted value of \(\beta\).

TABLE 2: THE VARIABILITY OF PARAMETERS ‘\(\alpha\)’ AND ‘\(\beta\)’ WITH RAIN RATE AT DIFFERENT LOCATIONS

| Location | Values of Parameters | \(R_{0.01}\) (mm/h) |
|----------|----------------------|---------------------|
| Brazilia | 5.255 -0.365         | 50                  |
| Nairobi  | 6.3396 -0.435        | 58                  |
| Daresslam| 8.1138 -0.406        | 72                  |
| Ile-Ife  | 13.4246 -0.3738      | 80                  |
| Ilorin   | 16.626 -0.3691       | 90                  |

TABLE 3: THE VALUES \(\beta\) OF BOTH PREDICTED AND MEASUREMENT WITH RAIN RATE

| Parameters | \(R_{0.01}\) (mm/h) | \(\beta_m\) | \(\beta_p\) | % error |
|------------|---------------------|-------------|-------------|---------|
| 90         | -0.3691             | -0.3576     | 3.1         |
| 80         | -0.3728             | -0.3826     | -2.4        |
| 72         | -0.4060             | -0.4026     | 0.8         |
| 58         | -0.4350             | -0.4376     | -0.6        |
| 50         | -0.40650            | -0.4876     | 0.2         |

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Parameters ‘α’ plotted against corresponding $R_{0.01}$ (mm/h) has resulted into a linear equation and the empirical equation is now written as equation (12):

$$\alpha_p = 0.288 \times R_{0.01} - 10.199$$

Equation 12 was then used to generate predicted values ‘α’ results into table 4

**TABLE 4**: The Values $\alpha$ Of Both Predicted and Measurement With Rain Rate

| Parameters | $R_{0.01}$ (mm/h) | $\alpha_m$ | $\alpha_p$ | % error |
|------------|-------------------|------------|------------|---------|
| 90         | 16.625            | 15.721     | -5.5       |
| 80         | 13.446            | 12.841     | -4.5       |
| 72         | 8.110             | 10.537     | 30.0       |
| 58         | 6.359             | 6.0505     | 2.3        |
| 52         | 5.255             | 4.201      | 20.1       |

$\alpha_m$ is measurement value and $\alpha_p$ is predicted value of $\alpha$.

Tables 3 and 4 revealed the consistency of the empirical equation obtained from the graphs (Fig. 1 and 2) of Parameter ‘β’ versus $R_{0.01}$ (mm/h) and Parameter ‘α’ versus $R_{0.01}$ (mm/h).

Substituting these values of ‘α’ and ‘β’ and varying the percentage of time, the following tables were obtained using $\alpha_p = 0.288 \times R_{0.01} - 10.199$ and $\beta_p = 0.0025 \times R_{0.01} - 0.5826$ for the stations considered.

Comparing Rain–rate obtained by the use of predicted equation for ‘α’ and ‘β’.

**TABLE 6 (A)**: ILORIN: R0.01 = 90 mm/h, $\alpha = 15.72$, $\beta = -0.3691$

| % of time | $R_m$ (mm/h) | $R_p$ (mm/h) | % error |
|-----------|-------------|-------------|---------|
| 0.001     | 198         | 201.3       | -1.7    |
| 0.003     | 152         | 134.2       | -11.7   |
| 0.01      | 90          | 86.0        | -4.4    |
| 0.03      | 60          | 57.4        | -4.3    |
| 0.1       | 45          | 36.8        | 18.2    |
| 0.3       | 23          | 24.5        | 6.5     |

**TABLE 6 (B)**: ILE-IFE: R0.01 = 80 mm/h, $\alpha = 12.841$, $\beta = -0.3826$

| % of time | $R_m$ (mm/h) | $R_p$ (mm/h) | % error |
|-----------|-------------|-------------|---------|
| 0.001     | 140         | 180.5       | 29      |
| 0.003     | 125         | 118.5       | -5.2    |
| 0.01      | 80          | 74.8        | -6.5    |
| 0.03      | 65          | 49.1        | -24.5   |
| 0.1       | 38          | 31.9        | -16.8   |
| 0.3       | 15          | 20.4        | 36      |

**TABLE 6 (C)**: BRAZILIA R0.01 = 50 mm/h, $\alpha = 4.201$, $\beta = 0.4876$

| % of time | $R_m$ (mm/h) | $R_p$ (mm/h) | % error |
|-----------|-------------|-------------|---------|
| 0.001     | 95          | 121.9       | 28.3    |
| 0.003     | 64          | 71.4        | -11.7   |
| 0.01      | 50          | 39.7        | 20.6    |
| 0.03      | 32          | 23.2        | -27.5   |
| 0.1       | 16          | 12          | 25.0    |
| 0.3       | 6           | 7.6         | 26.7    |

**TABLE 6 (D)**: NAIROBI R0.01 = 58 mm/h, $\alpha = 6.3391$, $\beta = 0.435$

| % of time | $R_m$ (mm/h) | $R_p$ (mm/h) | % error |
|-----------|-------------|-------------|---------|
| 0.001     | 95          | 127.9       | 34.6    |
| 0.003     | 76          | 79.4        | 4.3     |
| 0.01      | 58          | 47          | 19      |
| 0.03      | 40          | 29          | -27.5   |
| 0.1       | 20          | 12          | 15      |
| 0.3       | 08          | 10.8        | 26.7    |

Fig 2. The graph of parameter ‘α’ against rainfall intensity at 0.01%.

Fig 3. Comparison between parameter ‘β’ obtained by measurement and prediction.

Fig 4. Comparison between parameter ‘α’ obtained by measurement and prediction.

$\alpha_p$ and $\beta_p$ obtained by measurement and varying the percentage of time.
The initially proposed power law model of the form $R = \alpha R^\beta$ gave a very good prediction for the rain-rate up to 200mm/h. The new parameters later proposed at rain-rate 0.01% of time, gives a very good representation of the parameters for the stations considered. The linear relationship that exists between ‘$\alpha$’ and ‘$\beta$’ at the rain rate 0.01% of time makes the prediction for cumulative rain rate distribution easy and gives a very good fit for the medium and high rain rate locations, especially in the tropical regions.

The linear models for ‘$\alpha$’ and ‘$\beta$’ are:

$$\beta = M_0 R_{0.01} - C_0$$

Where $M_\alpha$ and $M_\beta$ are the slopes of the graph plotted for ‘$\alpha$’ versus $R_{0.01}$ and ‘$\beta$’ versus $R_{0.01}$ respectively; $C_\alpha$ and $C_\beta$ are the intercepts obtained on their vertical axis respectively.

For all the stations considered in this work, the consistency test conducted holds well for the measurement of about 5mm/h to about 200mm/h. In addition to the behaviour of the parameters, $\alpha$ and $\beta$, the empirical models that were obtained through them could be a useful tool for the radio design engineers for high rain rate areas.

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