An exact solution is obtained in the tetrad theory of gravitation. This solution is characterized by two-parameters $k_1, k_2$ of spherically symmetric static Lorentzian wormhole which is obtained as a solution of the equation $\rho = \rho_t = 0$ with $\rho = T_{ij}u^i u^j, \rho_t = (T_{ij} - \frac{1}{2} T g_{ij})u^i u^j$ where $u^i u_i = -1$. From this solution which contains an arbitrary function we can generates the other two solutions obtained before. The associated metric of this spacetime is a static Lorentzian wormhole and it includes the Schwarzschild black hole, a family of naked singularity and a disjoint family of Lorentzian wormholes. Calculate the energy content of this tetrad field using the gravitational energy-momentum given by Møller in teleparallel spacetime we find that the resulting form depends on the arbitrary function and does not depend on the two parameters $k_1$ and $k_2$ characterize the wormhole. Using the regularized expression of the gravitational energy-momentum we get the value of energy does not depend on the arbitrary function.
1. Introduction

At present, teleparallel theory seems to be popular again, and there is a trend of analyzing the basic solutions of general relativity with teleparallel theory and comparing the results. It is considered as an essential part of generalized non-Riemannian theories such as the Poincaré gauge theory [1] ∼ [7] or metric-affine gravity [8] as well as a possible physical relevant geometry by itself-teleparallel description of gravity [9, 10]. Teleparallel approach is used for positive-gravitational-energy proof [11]. A relation between spinor Lagrangian and teleparallel theory is established [12]. In [13] it is shown that the teleparallel equivalent of general relativity (TEGR) is not consistent in presence of minimally coupled spinning matter. The consistency of the coupling of the Dirac fields to the TEGR is demonstrated [14]. However, it is shown that this demonstration is not correct [15, 16].

For a satisfactory description of the total energy of an isolated system it is necessary that the energy-density of the gravitational field is given in terms of first- and/or second-order derivatives of the gravitational field variables. It is well-known that there exists no covariant, nontrivial expression constructed out of the metric tensor. However, covariant expressions that contain a quadratic form of first-order derivatives of the tetrad field are feasible. Thus it is legitimate to conjecture that the difficulties regarding the problem of defining the gravitational energy-momentum are related to the geometrical description of the gravitational field rather than are an intrinsic drawback of the theory [17, 18]. Møller has shown that the problem of energy-momentum complex has no solution in the framework of gravitational field theories based on Riemannian spacetime [19]. In a series of papers, [19]∼[22] he was able to obtain a general expression for a satisfactory energy-momentum complex in the teleparallel spacetime.

It was recognized by Flamm [23] in (1916) that our universe may not be simply connected, there may exist handles or tunnels now called wormholes, in the spacetime topology linking widely separated regions of our universe or even connected us with different universes altogether. That such wormholes may be traversable by humanoid travellers was first conjectured by Morris and Thorne [24], thereby suggesting that interstellar travel and even time travel may some day be possible [25, 26].

Morris and Thorne (MT) wormholes are static and spherically symmetric and connect asymptotically flat spacetimes. The metric of this wormhole is given by

\[ ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

where \( \Phi(r) \) being the redshift function and \( b(r) \) is the shape function. The shape function describes the spatial shape of the wormhole when viewed. The metric (1) is spherically symmetric and static. The geometric significance of the radial coordinate \( r \) is that the circumference of a circle centered on the throat of the wormhole is given by \( 2\pi r \). The coordinate \( r \) is nonmonotonic in that it decreases from \( +\infty \) to a minimum value \( b_0 \), representing the location of the throat of the wormhole, and then it increases from \( b_0 \) to \( +\infty \). This behavior of the radial coordinate reflects the fact that the wormhole connects two separate external universes. At the throat \( r = b = b_0 \), there is a coordinate singularity where the metric coefficient \( g_{rr} \) becomes divergent but the radial proper distance

\[ l(r) = \pm \int_{b_0}^{r} \frac{dr}{\sqrt{1 - b(r)/r}}, \]

must be required to be finite everywhere [27]. At the throat, \( l(r) = 0 \), while \( l(r) < 0 \) on the left side of the throat and \( l(r) > 0 \) on the right side.
have no horizon which implies that $g_{tt}$ must never allowed to be vanish, i.e., $\Phi(r)$ must be finite everywhere.

Traversable Lorentzian have been in vogue ever since Morris, Thorn and Yurtsever [28] came up with the exciting possibility of constructing time machine models with these exotic objects. (MT) paper demonstrated that the matter required to support such spacetimes necessarily violates the null energy condition. Semiclassical calculations based on techniques of quantum fields in curved spacetime, as well as an old theorem of Epstein et. al. [29], raised hopes about generation of such spacetimes through quantum stresses.

There have been innumerable attempts at solving the ”exotic matter problem” in wormhole physics in the last few years [25, 30]. Alternative theories of gravity [31] evolving wormhole spacetimes [32] with varying definitions of the throat have been tried out as possible avenues of resolution.

It is the aim of the present work to derive a wormhole in Møller’s tetrad theory of gravitation. To do so we first begin with a tetrad having spherical symmetry with three unknown functions of the radial coordinate [37]. Applying this tetrad to the field equations of Møller’s theory we obtain a set of non linear partial differential equations. It is our aim to find a general solution to these differential equations and discuss its physical properties. In §2 a brief survey of Møller’s tetrad theory of gravitation is presented. The exact solution of the set of non linear partial differential equations is given in §3. In §4 the energy content of the tetrad field is calculated and the form of energy depends on the arbitrary function and does not depend on the two parameters $k_1$ and $k_2$ that characterize the wormhole is obtained. In §5 the energy recalculated using the regularized expression of the gravitational energy-momentum. Discussion and conclusion of the obtained results are given in §6.

2. Møller’s tetrad theory of gravitation

In a spacetime with absolute parallelism the parallel vector fields $e_i^\mu$ define the nonsymmetric affine connection

$$\Gamma^\lambda_{\mu\nu} \overset{\text{def.}}{=} e_i^\lambda e_{\mu, \nu}^i,$$

where $e_i^\mu, \nu = \partial_\nu e_i^\mu$. The curvature tensor defined by $\Gamma^\lambda_{\mu\nu}$ is identically vanishing, however.

Møller’s constructed a gravitational theory based on this spacetime. In this theory the field variables are the 16 tetrad components $e_i^\mu$, from which the metric tensor is derived by

$$g^{\mu\nu} \overset{\text{def.}}{=} \eta^{ij} e_i^\mu e_j^\nu,$$

where $\eta^{ij}$ is the Minkowski metric $\eta_{ij} = \text{diag}(+1, -1, -1, -1)$.

We note that, associated with any tetrad field $e_i^\mu$ there is a metric field defined uniquely by (4), while a given metric $g^{\mu\nu}$ does not determine the tetrad field completely; for any local Lorentz transformation of the tetrads $b_i^\mu$ leads to a new set of tetrads which also satisfy (4). The Lagrangian $L$ is an invariant constructed from $\gamma_{\mu\nu\rho}$ and $g^{\mu\nu}$, where $\gamma_{\mu\nu\rho}$ is the contorsion tensor given by

$$\gamma_{\mu\nu\rho} \overset{\text{def.}}{=} e_i^\mu e_{\nu; \rho}^i,$$

where the semicolon denotes covariant differentiation with respect to Christoffel symbols. The most general Lagrangian density invariant under the parity operation is given by the form [22]
where
\[ g \overset{\text{def}}{=} \det(g_{\mu\nu}), \] (7)
and \( \Phi_\mu \) is the basic vector field defined by
\[ \Phi_\mu \overset{\text{def}}{=} \gamma^\rho_{\mu\rho}. \] (8)
Here \( \alpha_1, \alpha_2, \) and \( \alpha_3 \) are constants determined by Møller such that the theory coincides with general relativity in the weak fields:
\[ \alpha_1 = -\frac{1}{\kappa}, \quad \alpha_2 = \frac{\lambda}{\kappa}, \quad \alpha_3 = \frac{1}{\kappa}(1 - 2\lambda), \] (9)
where \( \kappa \) is the Einstein constant and \( \lambda \) is a free dimensionless parameter\(^*\). The same choice of the parameters was also obtained by Hayashi and Nakano [36].

Møller applied the action principle to the Lagrangian density (6) and obtained the field equation in the form
\[ G_{\mu\nu} + H_{\mu\nu} = -\kappa T_{\mu\nu}, \quad F_{\mu\nu} = 0, \] (10)
where the Einstein tensor \( G_{\mu\nu} \) is the Einstein tensor, \( H_{\mu\nu} \) and \( F_{\mu\nu} \) are given by
\[ H_{\mu\nu} \overset{\text{def}}{=} \lambda \left[ \gamma_{\rho\sigma\mu} \gamma_{\rho\sigma\nu} + \gamma_{\rho\sigma\mu} \gamma_{\nu\rho\sigma} + \gamma_{\rho\sigma\nu} \gamma_{\mu\rho\sigma} + g_{\mu\nu} \left( \gamma_{\rho\sigma\tau} \gamma_{\tau\sigma\rho} - \frac{1}{2} \gamma_{\rho\sigma\tau} \gamma_{\rho\sigma\tau} \right) \right], \] (11)
and
\[ F_{\mu\nu} \overset{\text{def}}{=} \lambda \left[ \Phi_{\mu,\nu} - \Phi_{\nu,\mu} - \Phi_\rho \left( \gamma^\rho_{\mu\nu} - \gamma^\rho_{\nu\mu} \right) + \gamma_{\mu\nu \rho} \right], \] (12)
and they are symmetric and skew symmetric tensors, respectively.

Møller assumed that the energy-momentum tensor of matter fields is symmetric. In the Hayashi-Nakano theory, however, the energy-momentum tensor of spin-1/2 fundamental particles has non-vanishing antisymmetric part arising from the effects due to intrinsic spin, and the right-hand side of antisymmetric field equation (10) does not vanish when we take into account the possible effects of intrinsic spin.

It can be shown [9] that the tensors, \( H_{\mu\nu} \) and \( F_{\mu\nu} \), consist of only those terms which are linear or quadratic in the axial-vector part of the torsion tensor, \( a_\mu \), defined by
\[ a_\mu \overset{\text{def}}{=} \frac{1}{3} \epsilon_{\mu\nu\rho\sigma} \gamma^{\nu\rho\sigma}, \quad \text{where} \quad \epsilon_{\mu\nu\rho\sigma} \overset{\text{def}}{=} \sqrt{-g} \delta_{\mu\nu\rho\sigma}, \] (13)
where \( \delta_{\mu\nu\rho\sigma} \) being completely antisymmetric and normalized as \( \delta_{0123} = -1 \). Therefore, both \( H_{\mu\nu} \) and \( F_{\mu\nu} \) vanish if the \( a_\mu \) is vanishing. In other words, when the \( a_\mu \) is found to vanish from the antisymmetric part of the field equations (10), the symmetric part will coincides with the Einstein field equation in teleparallel equivalent of general relativity.
3. Spherically Symmetric Solutions

Let us begin with the tetrad \([37]\)

\[
(e_{\mu}^\nu) = \begin{pmatrix}
A & Dr & 0 & 0 \\
0 & B \sin \theta \cos \phi & \frac{B}{r} \cos \theta \cos \phi & -\frac{B \sin \phi}{r \sin \theta} \\
0 & B \sin \theta \sin \phi & \frac{B}{r} \cos \theta \sin \phi & \frac{B \cos \phi}{r \sin \theta} \\
0 & B \cos \theta & -\frac{B}{r} \sin \theta & 0
\end{pmatrix},
\]  

(14)

where \(A, D, B\) are functions of the radial coordinate \(r\). The associated metric of the tetrad (14) has the form

\[
ds^2 = -\frac{B^2 - D^2 r^2}{A^2 B^2} dt^2 - 2 \frac{Dr}{AB^2} dr dt + \frac{1}{B^2} dr^2 + \frac{r^2}{B^2} (d\theta^2 + \sin^2 \theta d\phi^2).
\]  

(15)

As is clear from (15) that there is a cross term which can be eliminated by performing the coordinate transformation

\[
dT = dt + \frac{ADr}{B^2 - D^2 r^2} dr,
\]  

(16)

using the transformation (16) in the tetrad (14) we obtain

\[
(e_{\mu}^\nu) = \begin{pmatrix}
\frac{A}{1 - D^2 R^2} & \frac{RD - R^2 DB'}{1 - D^2 R^2} & 0 & 0 \\
\frac{ARD \sin \theta \cos \phi}{1 - D^2 R^2} & \frac{(R - RB') \sin \theta \cos \phi}{1 - D^2 R^2} & \frac{\cos \theta \cos \phi}{R} & -\frac{\sin \phi}{R \sin \theta} \\
\frac{ARD \sin \theta \sin \phi}{1 - D^2 R^2} & \frac{(R - RB') \sin \sin \phi}{1 - D^2 R^2} & \frac{\cos \theta \sin \phi}{R} & \frac{\cos \phi}{R \sin \theta} \\
\frac{ARD \cos \theta}{1 - D^2 R^2} & \frac{(R - RB') \cos \theta}{1 - D^2 R^2} & -\frac{\sin \theta}{R} & 0
\end{pmatrix},
\]  

(17)

where \(A, D\) and \(B\) are now unknown functions of the new radial coordinates \(R\) which is defined by

\[
R = \frac{r}{B}, \quad B' = \frac{dB(R)}{dR}.
\]  

(18)

Applying the tetrad fields (17) to the field equations (10) we obtain the following non linear partial differential equations

\[
\rho(R) = \frac{1}{\kappa} \left[ 2 \left( R^2 D^2 B' - R^2 D^2 - R B' + 1 \right) B'' + \left( 2 R^2 D D' + 5 R^2 D^2 - 3 \right) B'^2 \\
- 2 \left( 2 R^2 D D' + 4 R D^2 - \frac{2}{R} \right) B' + \left( 2 R D' + 3 D \right) D, \right.
\]

\[
\tau(R) = \frac{1}{\kappa} \left[ 2 R^2 A D D' - 2 R^3 D^2 A' + 3 R^2 A D^2 + 2 R A' - A \right] B R^2 - 2 \left( 2 R^3 A D D' - 2 R^3 D^2 A' \right) B R^2.
\]
Using (20) and (21) in (10) we can get the components of the energy-momentum tensor turn out to be non-vanishing. From the first equation of (19) we have

\[ A = \sqrt{\frac{2m}{R^3} + \frac{B'}{R} (RB' - 2)}, \]

where
\[ \rho(R) = T^0_0, \quad \tau(R) = T^1_1, \quad p(R) = T^2_2 = T^3_3, \]

with \( \rho(R) \) being the energy density, \( \tau(R) \) is the radial pressure and \( p(R) \) is the tangential pressure. (Note that \( \tau(R) \) as defined above is simply the radial pressure \( p_r \), and differs by a minus sign from the conventions in [24, 25].)

Now let us try to solve the above differential equations (19).

**The General Solution**

It is our purpose to find a general solution to the differential equations (19) when the stress-energy momentum tensor is not vanishing. From the first equation of (19) when \( \rho(R) = 0 \), we can get the unknown function \( D(R) \) in terms of the unknown functions \( B(R) \) to have the form

\[ D(R) = \frac{1}{1 - RB'} \sqrt{\frac{2m}{R^3} + \frac{B'}{R} (RB' - 2)}, \]

substitute (20) into (19) we can obtain the unknown function \( A(R) \) in terms of the unknown function \( B(R) \) to have form

\[ A(R) = \frac{1}{(1 - RB')} \left( k_2 + \frac{k_1}{\sqrt{1 - \frac{2m}{R}}} \right), \]

As is clear from (20) and (21) that the solution depends on the arbitrary function \( B \), i.e., we can generate the previous solutions obtained before by Nashed [38] by choosing the arbitrary function \( B \) to have the form

\[ B(R) = \ln \left\{ R \left( R - m + R \sqrt{1 - \frac{2m}{R}} \right) \right\} - 2 \sqrt{1 - \frac{2m}{R}}, \quad \text{and} \quad B(R) = 1. \]
to have the form
\[ \rho(R) = 0, \quad \tau(R) = -\frac{1}{\kappa} \left( \frac{2m k_1}{R^3 (k_1 + k_2 \sqrt{1 - \frac{2m}{R}})} \right), \quad P(R) = \frac{1}{\kappa} \left( \frac{m k_1}{R^3 (k_1 + k_2 \sqrt{1 - \frac{2m}{R}})} \right). \]  

The weak
\[ \rho \geq 0, \quad \rho + \tau \geq 0, \quad \rho + P \geq 0, \]  
and null energy conditions
\[ \rho + \tau \geq 0, \quad \rho + P \geq 0 \]
are both violated as is clear from (23). The violation of the energy condition stems from the violation of the inequality \( \rho + \tau \geq 0 \).

The complete line element of the above solution (20) and (21) is
\[ ds^2 = -\eta_1(R) dT^2 + \frac{dR^2}{\eta_2(R)} + R^2 d\Omega^2, \quad \text{where} \quad \eta_1(R) = \left( k_1 + k_2 \sqrt{1 - \frac{2m}{R}} \right)^2, \quad \eta_2(R) = \left( 1 - \frac{2m}{R} \right), \]
with \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \). If one replacing \( k_2 \) by \(-k_2\) at the above solution (20) and (21), the resulting form will also be a solution to the non linear partial differential equations (19). All the above solutions have a common property that their scalar Ricci tensor vanishing identically, i.e.,
\[ R(\{\}) = 0. \]

The metric (26) makes sense only for \( R \geq 2m \) so to really make the wormhole explicit one needs two conditions patches
\[ R_1 \in (2m, \infty), \quad R_2 \in (2m, \infty), \]
which we then have to sew together at \( R = 2m \). More discussion for such wormholes can be found in [34]. We are interested in the evaluation of energy since it is the most important test for any gravitational energy expression, local or quasi-local, since the geometrical setting corresponds to an intricate configuration of the gravitational field [18].

4. Energy content

The superpotential is given by
\[ U_{\mu}^{\nu \lambda} = \frac{(\mathcal{g})^{1/2}}{2\kappa} P_{\chi \rho \sigma}^{\tau \nu \lambda} \left[ \phi^{\rho} \mathcal{g}^{\sigma \chi} g_{\mu \tau} - \lambda g_{\tau \mu} \xi^{\chi \rho \sigma} - (1 - 2\lambda) g_{\tau \mu} \nabla^{\rho \chi} \right], \]  
where \( P_{\chi \rho \sigma}^{\tau \nu \lambda} \) is
\[ P_{\chi \rho \sigma}^{\tau \nu \lambda} \overset{\text{def.}}{=} \delta_{\chi}^{\tau} g_{\rho \sigma}^{\nu \lambda} + \delta_{\rho}^{\tau} g_{\sigma \chi}^{\nu \lambda} - \delta_{\sigma}^{\tau} g_{\chi \rho}^{\nu \lambda} \]  
with \( g_{\rho \sigma}^{\nu \lambda} \) being a tensor defined by
The energy is expressed by the surface integral [39, 40, 41]

\[
E = \lim_{r \to \infty} \int_{r=\text{constant}} U_0^{0\alpha} n_\alpha dS,
\]

where \( n_\alpha \) is the unit 3-vector normal to the surface element \( dS \).

Now we are in a position to calculate the energy associated with solution (20) and (21) using the superpotential (27). As is clear from (30), the only components which contribute to the energy is \( U_0^{0\alpha} \). Thus substituting from solution (20) and (21) into (27) we obtain the following non-vanishing value

\[
U_0^{0\alpha} = \frac{2x^\alpha}{k r^3} \left(2m - R^2B' + R^3B'^2\right).
\]

Substituting from (31) into (30) we get

\[
E(R) = 2m - R^2B' + R^3B'^2.
\]

We accept the formula of the energy to depend on the physical quantities but we do not accept the formula to depend on an arbitrary function [18]. Now we are going to follow a procedure similar to that follow by Brown-York formalism [42].

5. Regularized expression for the gravitational energy-momentum

An important property of the tetrad fields that satisfy the condition

\[
e_{i\mu} \cong \eta_{i\mu} + (1/2)h_{i\mu}(1/r),
\]

is that in the flat space-time limit \( e_i^\mu(t, x, y, z) = \delta_i^\mu \), and therefore the torsion tensor defined by

\[
T^\lambda_{\mu\nu} \stackrel{\text{def}}{=} e_a^\lambda T_{\mu\nu}^a = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu},
\]

is vanishing, i.e., \( T^\lambda_{\mu\nu} = 0 \). Hence for the flat space-time it is normally to consider a set of tetrad fields such that \( T^\lambda_{\mu\nu} = 0 \) in any coordinate system. However, in general an arbitrary set of tetrad fields that yields the metric tensor for the asymptotically flat space-time does not satisfy the asymptotic condition given by (33). Moreover for such tetrad fields the torsion \( T^\lambda_{\mu\nu} \neq 0 \) for the flat space-time [18, 43, 44]. It might be argued, therefore, that the expression for the gravitational energy-momentum (30) is restricted to particular class of tetrad fields, namely, to the class of frames such that \( T^\lambda_{\mu\nu} = 0 \) if \( e_i^\mu \) represents the flat space-time tetrad field [43]. To explain this, let us calculate the flat space-time tetrad field of (14) with (20) and (21) which is given by

\[
(E_i^\mu) = \begin{pmatrix}
(1 - RB') & \sqrt{R^2B'^2 - 2RB'} & 0 & 0 \\
\sqrt{R^2B'^2 - 2RB'} \sin \theta \cos \phi & (1 - RB') \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi R \\
\sqrt{R^2B'^2 - 2RB'} \sin \theta \sin \phi & (1 - RB') \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi R \\
\sqrt{R^2B'^2 + RB^2} & \sqrt{R^2B'^2 + RB^2} \cos \phi & 1 & 0 \\
\end{pmatrix}.
\]
Expression (35) yields the following non-vanishing torsion components:

\[ T_{001} = B', \quad T_{112} = -r \cos(\theta) \cos \phi B', \quad T_{113} = \sin(\theta) \sin \phi B', \quad T_{114} = -\frac{\sin(\theta) \cos \phi \sqrt{B'(1 - B')}}{\sqrt{R^2 B' - 2R}}, \]
\[ T_{124} = \cos \theta \cos \phi \sqrt{R^2 B'^2 - 2RB'}, \quad T_{134} = -\sin(\theta) \sin \phi \sqrt{R^2 B'^2 - 2RB'}, \quad T_{212} = -R \cos(\theta) \sin \phi B', \]
\[ T_{213} = -R \sin(\theta) \cos \phi B', \quad T_{214} = -\frac{\sin \theta \sin \phi \sqrt{B'(1 - B')}}{\sqrt{R^2 B' - 2R}}, \quad T_{224} = \cos(\theta) \sin \phi \sqrt{R^2 B'^2 - 2RB'}, \]
\[ T_{234} = \sin(\theta) \cos \phi \sqrt{R^2 B'^2 - 2RB'}, \quad T_{312} = R \sin(\theta) B', \quad T_{314} = -\frac{\cos(\theta) \sqrt{B'(1 - B')}}{\sqrt{R^2 B' - 2R}}, \]
\[ T_{324} = -\sin(\theta) \sqrt{R^2 B'^2 - 2RB'}. \quad (36) \]

Maluf et al. [18, 43, 44] discussed the above problem in the framework of TEBGR and constructed a regularized expression for the gravitational energy-momentum in this frame. They checked this expression for a tetrad field that suffers from the above problems and obtain a very satisfactory results [43]. In this section we will follow the same procedure to derive a regularized expression for the gravitational energy-momentum defined by Eq. (30). It can be shown that one can define the gravitational energy-momentum contained within an arbitrary volume \( V \) of the three-dimensional spacelike hypersurface in the form [39, 40]

\[ P_\mu = \int_V d^3x \partial_\alpha U_\mu^{\alpha}, \quad (37) \]

where \( U_\mu^{\alpha\lambda} \) is given by Eq. (27). Expression (37) bears no relationship to the ADM (Arnowitt-Deser-Misner) energy-momentum [44]. \( P_\mu \) transforms as a vector under the global SO(3,1) group.

Our assumption is that the space-time be asymptotically flat. In this case the total gravitational energy-momentum is given by

\[ P_\mu = \oint_{S \to \infty} dS_\alpha U_\mu^{\alpha}. \quad (38) \]

The field quantities are evaluated on a surface \( S \) in the limit \( r \to \infty \).

In Eqs. (37) and (38) it is implicitly assumed that the reference space is determined by a set of tetrad fields \( e^i_\mu \) for flat space-time such that the condition \( T^\lambda_{\mu\nu} = 0 \) is satisfied. However, in general there exist flat space-time tetrad fields for which \( T^a_{\mu\nu} \neq 0 \). In this case Eq. (37) may be generalized [43, 44] by adding a suitable reference space subtraction term, exactly like in the Brown-York formalism [42].

We will denote \( T^a_{\mu\nu}(E) = \partial_\mu E^a_\nu - \partial_\nu E^a_\mu \) and \( U_\mu^{\alpha}(E) \) as the expression of \( U_\mu^{\alpha} \) constructed out of the flat tetrad \( E^i_\mu \). The regularized form of the gravitational energy-momentum \( P_\mu \) is defined by

\[ P_\mu = \int_V d^3x \partial_\alpha \left[ U_\mu^{\alpha}(e) - U_\mu^{\alpha}(E) \right]. \quad (39) \]

This condition guarantees that the energy-momentum of the flat space-time always vanishes. The reference space-time is determined by tetrad fields \( E^i_\mu \), obtained from \( e^i_\mu \) by requiring the vanishing of the physical parameters like mass, angular momentum, etc. Assuming that the space-time is asymptotically flat then Eq. (39) can have the form

\[ P_\mu = \oint_{S \to \infty} dS_\alpha \left[ U_\mu^{\alpha}(e) - U_\mu^{\alpha}(E) \right]. \quad (40) \]
where the surface $S$ is established at spacelike infinity. Eq. (40) transforms as a vector under the global SO(3,1) group [22]. Now we are in a position to proof that the tetrad field (14) with (20) and (21) yields a satisfactory value for the total gravitational energy-momentum.

We will integrate Eq. (40) over a surface of constant radius $x^1 = r$ and require $r \to \infty$. Therefore, the index $\alpha$ in (40) takes the value $\alpha = 1$. We need to calculate the quantity $U^0_{01}$ and we find

$$U^0_{01}(e) \approx -\frac{1}{4\pi} R \sin(\theta) \left( \frac{2m}{R} - RB' + R^2 B'^2 \right),$$

and the expression of $U^0_{01}(E)$ is obtained by just making $m = 0$ in Eq.(41). it is given by

$$U^0_{01}(E) \approx -\frac{1}{4\pi} R \sin(\theta) (R^2 B'^2 - RB').$$

Thus the gravitational energy contained within a surface $S$ of constant radius $r$ is given by

$$P_0 \approx \int_{R \to \infty} d\theta d\phi \frac{1}{4\pi} \sin(\theta) \left\{ -R \left( \frac{2m}{R} - RB' + R^2 B'^2 \right) + (R^2 B'^2 - R^2 B') \right\} = 2m,$$

this value of $2m$ is the value obtained by several different approach [40, 41]

6. Discussion and conclusion

In this paper we have applied the tetrad having spherical symmetry with three unknown functions of the radial coordinate [37] to the field equations of Møller’s tetrad theory of gravitation [22]. From the resulting partial differential equation we have obtained an exact non vacuum solutions. This solutions is characterized by an arbitrary function $B(R)$ and from it one can generates the other two solutions . The solutions in general are characterize by three parameters $m$, $k_1$ and $k_2$. If the two parameters $k_1 = 0$ and $k_2 = 1$ then one can obtains the previous solutions [45]. The energy-momentum tensor has the property that $\rho = 0$. The line element associated with these solutions has the form (26).

To make the picture more clear we discuss the geometry of each solution. The line element of this solution in the isotropic form is given by

$$ds^2 = -\eta_1(R)dT^2 + \frac{dR^2}{\eta_2(R)} + R^2 d\Omega^2, \text{ where } \eta_1(R) = \left( k_1 + k_2 \sqrt{1 - \frac{2m}{R}} \right)^2, \eta_2(R) = \left( 1 - \frac{2m}{R} \right).$$

If $g_{tt} = 0$ one obtains a real naked singularity region. Outside these regions naked singularity does not form and one obtains a traverse wormhole. The throat of this wormhole $g_{tt}(R = 2m)$ gives the conditions that $g_{tt} = -k_1^2$ ⇒ ($k_1 \neq 0$ is required to ensure the traversability). The properties of this wormhole are discussed by Dadhich et. al. [34].

We calculate the energy content of the solution (20) and (21) using the energy-momentum complex given by [39, 40]. We find that the energy does not depend on the two parameters $k_1$ and $k_2$ characterize the wormhole (32). On contrary it depends on the arbitrary function $B(R)$. This is in fact not acceptable we accept the energy to depend on the physical quantities like mass $m$ and the charge $q$ etc.

Maluf et al. [18, 43, 44] have derived a simple expression for the energy-momentum flux of the gravitational field. This expression is obtained on the assumption that Eq.(37) represent the
energy-momentum of the gravitational field on a volume $V$ of the three-dimensional spacelike hypersurface. They [43, 44] gave this definition for the gravitational energy-momentum in the framework of TEGR, which require $T^\lambda_{\mu\nu}(E) = 0$ for the flat space-time. They extended this definition to the case where the flat space-time tetrad fields $E^a_\mu$ yield $T^\lambda_{\mu\nu}(E) \neq 0$. They show that [44] in the context of the regularized gravitational energy-momentum definition it is not strictly necessary to stipulate asymptotic boundary conditions for tetrad fields that describe asymptotically flat space-times.

Using the definition of the torsion tensor given by Eq. (34) and apply it to the tetrad field (35) we show that the flat space-time associated with this tetrad field has a non-vanishing torsion components Eq. (36). However, using the regularized expression of the gravitational energy-momentum Eq. (40) and calculate all the necessary components we finally get Eq. (43) which shows that the total energy of the tetrad field (14) with (20) and (21) does not depend on the arbitrary function.

As a punchline we obtain a traversable wormhole in tetrad theory of gravitation given by Møller [22] using a spherical symmetric tetrad given by Robertson [37] without using the line element given by Eq. (1) [24]. Lemos et. al. [46] has studied Morris-Thorne wormholes with a cosmological constant using the tetrad form of the line element (1) in the diagonal form. Now one can do the same procedure with the non diagonal tetrads given by the solutions (21) and (22).

**Acknowledgment**

The author would like to thank Professor J.G. Pereira Universidade Estadual Paulista, Brazil.
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