Controlling Flexible Rotor Vibrations Using Parametric Excitation

Lawrence Atepor

Department of Mechanical Engineering, University of Glasgow, G12 8QQ, UK

katepor@yahoo.com

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Abstract: This paper presents both theoretical and experimental studies of an active vibration controller for vibration in a flexible rotor system. The paper shows that the vibration amplitude can be modified by introducing an axial parametric excitation. The perturbation method of multiple scales is used to solve the equations of motion. The steady-state responses, with and without the parametric excitation terms, is investigated. An experimental test machine uses a piezoelectric exciter mounted on the end of the shaft. The results show a reduction in the rotor response amplitude under principal parametric resonance, and some good correlation between theory and experiment.

1. Introduction

Unbalance forces are the main source of vibration in rotating machines, but perfect balance is almost impossible to achieve. Moreover the distribution of unbalance can change in time because of wear or depositions all machines are subjected to. Reduction of rotor vibration is very important for safe and efficient functioning of all rotating machines. This paper proposes an active vibration control scheme for controlling transverse vibration of rotor shaft due to mass unbalance and presents both theoretical and experimental studies. The use of piezoelectric actuators in active vibration control has been considered in the past by Palazzolo et al.[1] and Barret et al.[2]. Yabuno et al.[3] used a piezoelectric actuator to stabilize the parametric resonance induced in a cantilever beam and to control bifurcation resulting in the shift of the bifurcation set and the expansion of the stable region. Carmignani et al.[4] developed an adaptive hydrodynamic bearing made up of a mobile housing mounted on piezoelectric actuators. In their work they showed that imposing a harmonic displacement on the mobile bearing, in two orthogonal directions, a rotating force, and a correcting moment can be produced on the shaft of a rotor system to reduce the bending caused by the unbalance. Das et al.[5] proposed an active vibration control scheme for controlling transverse vibration of a rotor shaft due to unbalance. These authors worked on the vibration control of rotors due to unbalance by placing electromagnetic exciters, at convenient location on the span of the rotor away from the bearings. They showed that, locations, distant from discs are in general convenient for exciters as they do not interfering in any way with the rotor operation. Similar electromagnetic exciters were conceptualised by Janik et al.[6] and were used to excite a rotor-shaft system for extracting the modal information by Janik et al.[7] and Irretier et al.[8].

No attempt of vibration control of rotor systems by the use of an axially placed piezoelectric exciter has, however been reported to the authors’ knowledge. There are many natural phenomena in which excited parametric and self-excited vibrations interact with one another. Examples are flow-induced vibrations and vibrations in forced rotor systems. The responses of nonlinear excited systems to parametric excitations have been investigated by Nayfeh et al.[9]. Several authors, including Skalak and Yarymovych[10],
Struble[11], Dugundji et al.[12], Chester[13] and Cartmell[14] have studied the effects of combined parametric and forced vibrations in dynamic systems. Frolov[15] examined a mechanical system excited simultaneously by parametric and periodic forced excitations. Frolov[15] showed that the resonant amplitude can be reduced by random variation of the system parameters. Mustafa and Ertas[16] theoretically and experimentally examined the effect of a pendulum (attached to the tip of a parametrically excited cantilever beam) whose natural frequency is tuned to be commensurable with a frequency of the beam in order to generate autoparametric resonance. For chosen external and internal resonance combinations, where the excitation frequency is twice the natural frequency of the first beam mode, and the linearised pendulum frequency is one-half that of the first beam mode, the results showed that, in some parametric excitation frequency ranges, the pendulum acts as a vibration-absorbing device in the same manner as the pendulum attached to the main system under external excitation. Nguyen and Ginsberg[17] studied vibration control of a simple pendulum using parametric excitation. They showed that with judicious selection of the parametric excitation, a parametric frequency that is very high relative to the highest contemplated excitation frequency can substantially reduce the forced vibration response at any lower excitation frequency. The above ideas have led to the design of the piezoelectric exciter and the deliberate introduction of parametric excitations into a flexible rotor-bearing system axially to moderate the response of the pre-existing mass-unbalance vibration inherent to the rotor. The idea here is to use piezoelectric stack actuator to put axial excitations into the shaft to investigate the interactions between forced vibrations, which emanate from rotor unbalance and parametric excitations which results from the periodic stiffness variation caused by a periodic axial excitations from the actuator. A practically implementable strategy is proposed in which the inherent and predominant instabilities in the flexible rotor-bearing system are manipulated in such a way that their effects on the overall performance of the rotor system can be effectively controlled. In justifying this work, a program of research has been carried out and the results show reductions in the resonant amplitudes for forward whirl in the flexible rotor-bearing system.

2. Theoretical work
2.1 Equations of motion
The rotor is assumed to be simply supported at both ends and the expression for the displacement are expressed in the generalised coordinates using the Rayleigh-Ritz method as $u(x,t) = f(y)q(t) = f(y)q_1$ and $w(x,t) = f(y)q_2(t) = f(y)q_2$, where $q_1$ and $q_2$ are generalised independent coordinates, $u$ and $w$ are displacements with respect to $x$ and $u, f(y)$ is the displacement function, and it is chosen as the normalised first mode shape of a beam with a constant cross section in bending and simply supported at both ends, and is given as $f(y) = \sin\left(\frac{\pi y}{L}\right)$. The system energies are

$$T = \frac{1}{2}m\left[q_1^2 + q_2^2\right] + \Omega^2\left[\frac{1}{2}I_d + \rho IL\right] - \Omega a_2\left[q_1\sin(\Gamma q_2)\right] + m_u \int_0^l q_1^2 \cos(\Omega t - q_2 \sin(\Omega t) \right]$$

and

$$U = \frac{1}{2}k_2\left[q_1^2 + q_2^2\right], \text{where} \ \Gamma = g(l_1), \ m = M_d f^2(l_1) + \rho s \int_0^l f^2(y) dy, \ P = I_d g^2(l_1) + \rho I \int_0^l g^2(y) dy,$$

$$a_1 = I_d g^2(l_1) + \rho I \int_0^l g^2(y) dy, \ a_2 = I_d g^2(l_1) + 2\rho I \int_0^l g(y) dy, \ f(l_1) = \pi L, \ \text{and} \ \ g(l_1) = \frac{\pi L}{L} \cos\pi L,$$

$$k_2 = EI \int_0^h f^2(y) dy + F_0 \int_0^h g^2(y) dy \ \text{and} \ \ \text{is the stiffness of the shaft}, \ h(y) = \frac{d^2f(y)}{dy^2}, \ l_1 \ \text{is the position of the disk on the shaft and} \ L \ \text{is the shaft length and} \ \text{are obtained from the experimental rig}. \ M_d \ \text{is the mass of disk,} \ s \ \text{is the cross-sectional area,} \ P, a_1, a_2, a_3 \ \text{are geometric constants,} \ I \ \text{is the moment of inertia and} \ \rho \ \text{is mass per unit volume. The equations of motion are derived by means of Lagrangian analysis as follows,}$$
\[
\ddot{q}_1 + \dot{c}_q \dot{q}_1 - \Omega \dot{a}_q q_2 + \omega^2 q_1 + \dot{b} q_1^3 - \dot{F}_{ax} q_1 = \mu d \Omega^2 \sin \Omega t \\
(1)
\]
\[
\ddot{q}_2 + \dot{c}_q \dot{q}_2 + \Omega \dot{a}_q q_1 + \omega^2 q_2 + \dot{b} q_2^3 - \dot{F}_{ax} q_2 = \mu d \Omega^2 \cos \Omega t \\
(2)
\]
where \( \dot{a}_q = \frac{a_q}{m} \omega^2 = \frac{k}{m} \), \( \dot{b} = \frac{b}{m} \dot{c}_q = \frac{c}{m} \), \( \mu = \frac{m_a}{m} \), \( \dot{F}_0 = \frac{F_{ax}}{m} \), \( F_{ax} q_i \) is the axial excitation force term and the dots denote differentiation with respect to \( t \).

2.2. Solutions to the equations of motion

The approximate solution for the model of the flexible rotor system with and without parametric excitation terms are obtained using the method of multiple scales as

\[
\ddot{q}_1 = 2 p \cos \left( \frac{\Omega t}{2\omega} \right) - 2 q \sin \left( \frac{\Omega t}{2\omega} \right) + \frac{\dot{b}}{4\omega^2} p^3 \cos \left( \frac{3\Omega t}{2\omega} \right) - \frac{3\dot{b}}{4\omega^2} \omega q q^2 \sin \left( \frac{3\Omega t}{2\omega} \right) + \frac{\dot{b}}{4\omega^2} q^3 \sin \left( \frac{3\Omega t}{2\omega} \right) \\
(3)
\]
\[
\ddot{q}_2 = 2 r \cos \left( \frac{\Omega t}{2\omega} \right) - 2 s \sin \left( \frac{\Omega t}{2\omega} \right) + \frac{\dot{b}}{4\omega^2} r^3 \cos \left( \frac{3\Omega t}{2\omega} \right) - \frac{3\dot{b}}{4\omega^2} \omega s s^2 \sin \left( \frac{3\Omega t}{2\omega} \right) + \frac{\dot{b}}{4\omega^2} s^3 \sin \left( \frac{3\Omega t}{2\omega} \right) \\
(4)
\]
\[
\ddot{q}_1 = 2 p \cos \left( \frac{\Omega t}{2\omega} \right) - 2 q \sin \left( \frac{\Omega t}{2\omega} \right) + \frac{\dot{b}}{4\omega^2} \omega q^3 \sin \left( \frac{3\Omega t}{2\omega} \right) + \frac{3\dot{b}}{4\omega^2} \omega q^2 \sin \left( \frac{3\Omega t}{2\omega} \right) + \frac{2\dot{F}_{ax} q}{2\Omega + 4\Omega \omega^2} \sin \left( \frac{5\Omega t}{4\omega} \right) \\
+ \frac{2\dot{F}_{ax} q}{2\Omega + 4\Omega \omega^2} \sin \left( \frac{5\Omega t}{4\omega} \right) - \frac{2\dot{F}_{ax} Q q}{2\Omega + 4\Omega \omega^2} \sin \left( \frac{5\Omega t}{4\omega} \right) \\
(5)
\]
\[
\begin{align*}
\ddot{x}_2 &= 2r \cos \left( \frac{\Omega t}{2\omega} \right) - 2x \sin \left( \frac{\Omega t}{2\omega} \right) + \dot{b} \frac{t}{4\omega} r^3 \cos \left( \frac{3\Omega t}{2\omega} \right) - 3\dot{b} \frac{t}{4\omega} r^2 x \sin \left( \frac{3\Omega t}{2\omega} \right) \\
&- \frac{3\dot{b} t}{4\omega^2} r^2 \cos \left( \frac{3\Omega t}{2\omega} \right) + \dot{b} \frac{t}{4\omega} r^3 \sin \left( \frac{3\Omega t}{2\omega} \right) - 2\dot{F}_{\text{act}} \cos \left( \frac{\Omega t}{2\omega} \right) \\
&+ \frac{2\dot{F}_{\text{act}}}{2\Omega} \sin \left( \frac{5\Omega t}{2\omega} \right) - \frac{2\dot{F}_{\text{act}} \Omega \dot{2} \dot{c} k r}{2\Omega \omega^2 + 4\Omega \omega^3} \\
&- \frac{2\dot{F}_{\text{act}} \Omega \dot{2} \dot{c} k s}{2\Omega \omega^2 + 4\Omega \omega^3} \cos \left( \frac{5\Omega t}{2\omega} \right) - \frac{2\dot{F}_{\text{act}} \dot{c} k r}{2\Omega \omega^2 + 4\Omega \omega^3} \sin \left( \frac{5\Omega t}{2\omega} \right) \tag{6}
\end{align*}
\]

where, \( k = \left( \begin{array}{c}
\frac{1}{-\Omega^2} \\
\frac{2\Omega}{\omega^2}
\end{array} \right) \) and \( Q = \left( \begin{array}{c}
\frac{1}{-\Omega^2} \\
\frac{4\Omega}{\omega^2} - \frac{4\Omega^2}{\omega^2}
\end{array} \right) \). Equations (3) and (4) are full time-domain solutions of the equations of motion (1) and (2) without parametric excitation terms and equations (5) and (6) are the solutions with parametric excitation terms. \( P, Q, r \) and \( s \) are amplitudes, \( \Omega \) is the excitation frequency and \( \Omega_2 \) is the principal parametric resonance frequency and \( \Omega_2 = 2\Omega \).

3. Experimental Work

A commercial rotor-kit (Bently Nevada rotor kit RK4) and a piezoelectric exciter specifically developed for this research are used for the experiment. The rotor kit provided a rotor supported by journal bearings, an electrical drive to run the rotor with a separate control box from which the desired rotational speed is selected. The torque is transmitted from the electrical motor to the rotor by means of a solid coupling. Provided are displacement transducers to measure the movements of the rotor, and a rigid V-shaped base, to which any components could be easily attached. The rotor kit is equipped with the piezoelectric exciter designed for active vibration control. The critical parts of the exciter unit are, a piezoelectric actuator supported by a helical compression spring, all placed inside a linear sliding bearing, and an aluminium casing. The piezoelectric actuator is driven by a function generator through a piezoelectric actuator amplifier. To avoid direct contact between the shaft and piezoexciter, and to allow free rotation and movement of the shaft end, a small self-aligned ball bearing is fixed in between the shaft and the piezoexciter. The vibration response of the rotor is then measured by means of a Polytec Laser Vibrometer allowing the displacement responses to be identified and monitored by a multi-channel data acquisition analyser. Figures 1 and 2 show the experimental configuration for activating the flexible rotor system. The leading principle here is to control, axially, the vibrations of the rotor, supported on conventional bearings, by using the piezoelectric actuator.
3.1. Test Setup

The objective of this work has been the design and construction of a test rig to verify the feasibility of active control of vibration in rotor dynamics using a piezoelectric actuator. In particular the possibility of reducing the amplitude of vibrations of a flexible dynamically unbalanced rotor within acceptable levels is investigated. This is carried out by designing a Piezoexciter excited by a high frequency drive. The active Piezoexciter comprises a sliding bearing which houses the piezoelectric stack actuator which is serially attached to a compression spring. Since the actuator operates only in expansion, with small displacement, the reaction spring is set up against it. The spring is adjusted to the required length by the spring compressor and voltage is applied through a piezoelectric voltage amplifier to the actuator which in turn develops the parametric excitation at a frequency of twice the first whirl frequency of the rotor system. The exciter is driven by a function generator through a high voltage amplifier. Activating the piezoelectric actuator at twice the excitation frequency of the rotor system generates the parametric excitation force to be introduced to the shaft, axially. The vibration response of the rotor-bearing system is then measured by means of the laser vibrometer. A multi-channel data acquisition analyser is then used to analyse the response. The compression spring of the exciter unit and the rotor-bearing system are set to their required length and first whirl resonance frequency respectively, and the response of the rotor is measured. The piezoelectric actuator is then activated, first at a frequency twice the first whirl frequency of the rotor system. A series of timed tests are performed and average readings are taken. Sweep tests around the first whirl frequency are then performed, first without activating the piezoeexciter, and then with the exciter activated at the parametric excitation frequency.
3.2. Theoretical Results

![Figure 3: Schematic of the piezoelectric exciter.](image)

**Figure 3: Schematic of the piezoelectric exciter.**

**Figure 4: Amplitudes of the response as functions of the frequency**

In figure 4, considering plots indicated by a, both the responses in the first mode of $\tilde{q}_1$ and $\tilde{q}_2$ show hardening characteristics, jump phenomena and both stable and unstable solutions when the equations of motion contain no parametric force term. Including the parametric force terms, the results in figure 4, plots indicated by b show 23% reduction in amplitude, elimination of the jump phenomena and stable solutions.
3.3. Experimental results

![Graph showing amplitude vs. frequency for different states of piezoexciter activation](image)

**Figure 5:** The amplitudes of disc vibration versus frequency with a spring compression length of 25.2 mm.

In figure 5 the combined effects due to the existing forced vibration due to mass unbalance and also an additional parametric excitation in principal parametric resonance provided by the piezoexciter resulted in the moderation of the responses of the pre-existing mass unbalance vibration inherent to the rotor, with an approximately 13% reduction in the critical whirl amplitude.

4. Conclusion

Comparison between the results from the theoretical analysis and the experimental benchmark summarised here shows evidence of a consistent phenomenon. The methods of investigating and identifying the response behaviour of rotor systems have all shown similar trends with regards to the effects of introducing parametric force terms. Prototypical experimental results from tests on rotor systems conclude that the novel piezoelectric exciter concepts could be successfully applied to industrial applications, particularly installations where axial loading on the rotor shaft is also an inherent part of the control actuation.

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