ACCRETION DISC THEORY: FROM THE STANDARD MODEL UNTIL ADVECTION

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Abstract

Accretion disc theory was first developed as a theory with the local heat balance, where the whole energy produced by a viscous heating was emitted to the sides of the disc. One of the most important new invention of this theory was a phenomenological treatment of the turbulent viscosity, known as "alpha" prescription, when the \( (r\phi) \) component of the stress tensor was approximated by \( (\alpha P) \) with a unknown constant \( \alpha \). This prescription played the role in the accretion disc theory as well important as the mixing-length theory of convection for stellar evolution. Sources of turbulence in the accretion disc are discussed, including non-linear hydrodynamical turbulence, convection and magnetic field role. In parallel to the optically thick geometrically thin accretion disc models, a new branch of the optically thin accretion disc models was discovered, with a larger thickness for the same total luminosity. The choice between these solutions should be done of the base of a stability analysis. The ideas underlying the necessity to include advection into the accretion disc theory are presented and first models with advection are reviewed. The present status of the solution for a low-luminous optically thin accretion disc model with advection is discussed and the limits for an advection dominated accretion flows (ADAF) imposed by the presence of magnetic field are analysed.

1 Introduction

Accretion is served as a source of energy in many astrophysical objects, including different types of binary stars, binary X-ray sources, most probably quasars and active galactic nuclei (AGN). While first development of accretion theory started long time ago (Bondi and Hoyle, 1944; Bondi, 1952), the intensive development of this theory began after discovery of first X ray sources (Giacconi et al, 1962) and quasars (Schmidt, 1963). Accretion into stars, including neutron stars, is ended by a collision with an inner boundary, which may be a stellar

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surface, or outer boundary of a magnetosphere for strongly magnetized stars. We may be sure in this case, that all gravitational energy of the falling matter will be transformed into heat and radiated outward.

Situation is quite different for sources containing black holes, which are discovered in some binary X-ray sources in the galaxy, as well as in many AGN. Here matter is falling to the horizon, from where no radiation arrives, so all luminosity is formed on the way to it. The efficiency of accretion is not known from the beginning, contrary to the accretion into a star, and depends strongly on such factors, like angular momentum of the falling matter, and magnetic field embedded into it. It was first shown by Schwarzman (1971), that during spherical accretion of nonmagnetized gas the efficiency may be as small as $10^{-8}$ for sufficiently low mass fluxes. He had shown that presence of magnetic field in the accretion flux matter increase the efficiency up to about 10%, and account of heating of matter due to magnetic field annihilation in the flux rises the efficiency up to about 30% (Bisnovatyi-Kogan, Ruzmaikin, 1974). In the case of a thin disc accretion when matter has large angular momentum, the efficiency is about 1/2 of the efficiency of accretion into a star with a radius equal to the radius of the last stable orbit. Matter cannot emit all the gravitational energy, part of which is absorbed by the black hole. In the case of geometrically thick and optically thin accretion discs the situation is approaching the case of spherical symmetry, and a presence of a magnetic field playes also a critical role.

Here we consider a developement of the theory of a disk accretion, starting from creation of a so called "standard model", and discuss recent trends, connected with a presence of advection.

2 Developement of the standard model of the disc accretion into a black hole

Matter falling into a black hole is gathered into a disc when its angular momentum is sufficiently high. It happens when the matter falling into a black hole comes from the neighbouring ordinary star companion in the binary, or when the matter appears as a result of a tidal disruption of the star which trajectory of motion approaches sufficiently close to the black hole, so that forces of selfgravity could be overcomened. The first situation is observed in many galactic X-ray sources containing a stellar mass black hole (Cherepashchuk, 1996). A tidal disruption happens in quasars and active galactic nuclei (AGN), if the model of supermassive black hole surrounded by a dense stellar cluster of Lynden-Bell (1969) is true for these objects.

The models of the accretion disc structure around a black hole had been investigated by Lynden-Bell (1969), Pringle and Rees (1972). The modern "standard" theory of the disc accretion was formulated in the papers of Shakura (1972), Novikov and Thorne (1973) and Shakura and Sunyaev (1973). It is im-
important to note, that all authors of the accretion disc theory from USSR were students (N.I.Shakura) or collaborators (I.D.Novikov and R.A.Sunyaev) of academician Ya.B.Zeldovich, who was not among the authors, but whose influence on them hardly could be overestimated.

The equations of the standard disc accretion theory were first formulated by Shakura (1972); some corrections and generalization to general relativity (GR) were done by Novikov and Thorne (1973), see also correction to their equations in GR made by Riffert & Herold (1995). The main idea of this theory is to describe a geometrically thin non-self-gravitating disc of the mass $M_d$, which is much smaller then the mass of the black hole $M$, by hydrodynamic equations averaged over the disc thickness $2h$.

2.1 Equilibrium equations

The small thickness of the disc in comparison with its radius $h \ll r$ indicate to small importance of the pressure gradient $\nabla P$ in comparison with gravity and inertia forces. That leads to a simple radial equilibrium equation denoting the balance between the last two forces occuring when the angular velocity of the disc $\Omega$ is equal to the Keplerian one $\Omega_K$,

$$\Omega = \Omega_K = \left( \frac{GM}{r^3} \right)^{1/2}. \quad (1)$$

Note, just before a last stable orbit around a black hole, and of course inside it, this suggestion fails, but in the "standard" accretion disc model the relation (1) is suggested to be fulfilled all over the disc, with an inner boundary at the last stable orbit.

The equilibrium equation in the vertical $z$-direction is determined by a balance between the gravitational force and pressure gradient

$$\frac{dP}{dz} = -\rho \frac{GMz}{r^3} \quad (2)$$

For a thin disc this differential equation is substituted by an algebraic one, determining the half-thickness of the disc in the form

$$h \approx \frac{1}{\Omega_K} \left( \frac{2P}{\rho} \right)^{1/2}. \quad (3)$$

The balance of angular momentum, related to the $\phi$ component of the Euler equation has an integral in a stationary case written as

$$\dot{M}(j - j_{in}) = -2\pi r^2 2ht_{\phi}, \quad t_{\phi} = \eta r \frac{d\Omega}{dr}. \quad (4)$$

Here $j = v_{\phi}r = \Omega r^2$ is a specific angular momentum, $t_{\phi}$ is a component of the viscous stress tensor, $\dot{M} > 0$ is a mass flux per unit time into a black hole, $j_{in}$ is
an integration constant having, after multiplication by $\dot{M}$, a physical sense of difference between viscous and advective flux of the angular momentum, when $j_{in}$ itself is equal to the specific angular momentum of matter falling into a black hole. In the standard theory the value of $j_{in}$ is determined separately, from physical considerations. For the accretion into a black hole it is suggested, that on the last stable orbit the gradient of the angular velocity is zero, corresponding to zero viscous momentum flux. In that case

$$j_{in} = \Omega_K r_{in}^2,$$

(5)
corresponding to the Keplerian angular momentum of the matter on the last stable orbit. During accretion into a slowly rotating star which angular velocity is smaller than a Keplerian velocity on the inner edge of the disc, there is a maximum of the angular velocity close to its surface, where viscous flux is zero, and there is a boundary layer between this point and stellar surface. In that case (5) remains to be valid. The situation is different for accretion discs around rapidly rotating stars with a critical Keplerian speed on the equator. Here there is no extremum of the angular velocity of the disc which smoothly joins the star. In stationary self-consistent situation when the accreting star remains to rotate critically during the process of a disc accretion, the specific angular momentum of matter joining the star is determined by a relation (Bisnovatyi-Kogan, 1993):

$$j_{in} = \frac{dJ}{dM}|_{crit},$$

where the derivative is taken along the states of the star having a Keplerian equatorial speed. For stars with a polytropic structure, corresponding to equation of state $P = K\rho^{\gamma}$, this derivative is calculated numerically giving the value $j_{in} = 0.176\Omega_K r_{in}^2$ for $n = 1.5$; 0 for $n = 2.5$; and negative values of $j_{in}$ for larger $n$.

Note, that in the pioneering paper of Shakura (1972) the integration constant $j_{in}$ was found as in (3), but was taken zero in his subsequent formulae. Importance of using $j_{in}$ in the form (5) was noticed by Novikov and Thorne (1973), and became a feature of the standard model.

### 2.2 Viscosity

The choice of the viscosity coefficient is the most difficult and speculative problem of the accretion disc theory. In the laminar case of microscopic (atomic or plasma) viscosity, which is very low, the stationary accretion disc must be very massive and very thick, and before its formation the matter is collected by disc leading to a small flux inside. It contradicts to observations of X-ray binaries, where a considerable matter flux along the accretion disc may be explained only when viscosity coefficient is much larger then the microscopic one. In the paper of Shakura (1972) it was suggested, that matter in the disc is turbulent, what determines a turbulent viscous stress tensor, parametrized by a pressure

$$t_{r\phi} = -\alpha \rho v_r^2 = -\alpha P,$$

(6)
where $v_s$ is a sound speed in the matter. This simple presentation comes out from a relation for a turbulent viscosity coefficient $\eta_t \approx \rho v_t l$ with an average turbulent velocity $v_t$ and mean free path of the turbulent element $l$. It follows from the definition of $t_{r\phi}$ in (6), when we take $l \approx h$ from (3)

$$t_{r\phi} = \rho v_t h r \frac{d\Omega}{dr} \approx \rho v_t v_s = -\alpha \rho v_s^2,$$

(7)

where a coefficient $\alpha < 1$ is connecting the turbulent and sound speeds $v_t = \alpha v_s$. Presentations of $t_{r\phi}$ in (6) and (7) are equivalent, and only when the angular velocity differs considerably from the Keplerian one the first relation to the right in (7) is more preferable. That does not appear (by definition) in the standard theory, but may happen when advective terms are included.

Development of a turbulence in the accretion disc cannot be justified simply, because a Keplerian disc is stable in linear approximation to the development of perturbations. It was suggested by Ya.B.Zeldovich, that in presence of very large Reynolds number $\text{Re} = \frac{\rho v_t l}{\eta}$ the amplitude of perturbations at which nonlinear effects become important is very low, so in this situation a turbulence may develop due to nonlinear instability even when the disc is stable in linear approximation. Another source of viscous stresses may arise from a magnetic field, but it was suggested by Shakura (1972), that magnetic stresses cannot exceed the turbulent ones.

Magnetic plasma instability as a source of the turbulence in the accretion discs has been studied extensively in last years (see review of Balbus and Hawley, 1998). They used an instability of the uniform magnetic field parallel to the axis in differentially rotating disc, discovered by Velikhov (1959). It could be really important in absence of any other source of the turbulence, but it is hard to believe that there is no radial or azimuthal component of the magnetic field in matter flowing into the accretion disc from the companion star. In that case the field amplification due to twisting by a differential rotation take place without necessity of any kind of instability.

It was shown by Bisnovatyi-Kogan and Blinnikov (1976, 1977), that inner regions of a highly luminous accretion discs where pressure is dominated by radiation, are unstable to vertical convection. Development of this convection produce a turbulence, needed for a high viscosity. Other regions of a standard accretion disc should be stable to development of a vertical convection, so other ways of a turbulence exitation are needed there. With alpha- prescription of viscosity the equation of angular momentum conservation is written in the plane of the disc as

$$\dot{M}(j - j_{in}) = 4\pi r^2 \alpha P_0 h.$$

(8)

When angular velocity is far from Keplerian the relation (8) is valid with a coefficient of a turbulent viscosity

$$\eta = \alpha \rho v_s \delta h,$$

(9)
where values with the index "0" denote the plane of the disc.

### 2.3 Heat balance

In the standard theory a heat balance is local, what means that all heat produced by viscosity in the ring between $r$ and $r + dr$ is radiated through the sides of disc at the same $r$. The heat production rate $Q_+$ related to the surface unit of the disc is written as

$$Q_+ = h_t r \phi_\Omega dr = \frac{3}{8\pi} \frac{GM}{r^3} \left( 1 - \frac{j_m}{J} \right). \quad (10)$$

Heat losses by a disc depend on its optical depth. The first standard disc model of Shakura (1972) considered a geometrically thin disc as an optically thick in a vertical direction. That implies energy losses $Q_-$ from the disc due to a radiative conductivity, after a substitution of the differential equation of a heat transfer by an algebraic relation

$$Q_- \approx \frac{4}{3} acT^4 \kappa \Sigma. \quad (11)$$

Here $a$ is a constant of a radiation energy density, $c$ is a speed of light, $T$ is a temperature in the disc plane, $\kappa$ is a matter opacity, and a surface density $\Sigma = 2\rho h$. Here and below $\rho, T, P$ without the index "0" are related to the disc plane. The heat balance equation is represented by a relation

$$Q_+ = Q_-, \quad (12)$$

A continuity equation in the standard model of the stationary accretion flow is used for finding of a radial velocity $v_r$

$$v_r = \frac{\dot{M}}{4\pi rh\rho} = \frac{\dot{M}}{2\pi r \Sigma}. \quad (13)$$

Equations (3), (8), (12), completed by an equation of state $P(\rho, T)$ and relation for the opacity $\kappa = \kappa(\rho, T)$ represent a full set of equations for a standard disc model. For power low equations of state of an ideal gas $P = P_g = \rho RT$ ($R$ is a gas constant), or radiation pressure $P = P_r = \frac{2P_g}{3}$, and opacity in the form of electron scattering $\kappa_e$, or Karammers formulae $\kappa_k$, the solution of a standard disc accretion theory is obtained analytically (Shakura, 1972; Novikov, Thorne, 1973; Shakura, Sunyaev, 1973). Checking the suggestion of a large optical thickness confirms a self-consistency of the model. One of the shortcoming of the analytical solutions of the standard model lay in the fact, that solutions for different regions of the disc with different equation of states and opacities are not matched to each other.
2.4 Optically thin solution

Few years after appearance of the standard model it was found that in addition to the optically thick disc solution there is another branch of the solution for the disc structure with the same input parameters \( M, \dot{M}, \alpha \) which is also self-consistent and has a small optical thickness (Shapiro, Lightman, Eardley, 1976). Suggestion of the small optical thickness implies another equation of energy losses, determined by a volume emission \( Q_0 \approx q \rho \bar{h} \), where due to the Kirghoff law the emissivity of the unit of a volume \( q \) is connected with a Plankian averaged opacity \( \kappa_p \) by an approximate relation \( q \approx a c T_0^4 \kappa_p \). Note, that Krammers formulae for opacity are obtained after Rosseland averaging of the frequency dependent absorption coefficient. In the optically thin limit the pressure is determined by a gas \( P = P_g \). Analytical solutions are obtained here as well, from the same equations with volume losses and gas pressure. In the optically thin solution the thickness of the disc is larger then in the optically thick one, and density is lower.

While heating by viscosity is determined mainly by heavy ions, and cooling is determined by electrons, the rate of the energy exchange between them is important for a thermal structure of the disc. The energy balance equations are written separately for ions and electrons. For small accretion rates and lower matter density the rate of energy exchange due to binary collisions is so slow, that in the thermal balance the ions are much hotter then the electrons. That also implies a high disc thickness and brings the standard accretion theory to the border of its applicability. Nevertheless, in the highly turbulent plasma the energy exchange between ions and electrons may be strongly enhanced due to presence of fluctuating electrical fields, where electrons and ions gain the same energy. In such conditions difference of temperatures between ions and electrons may be negligible. Regretfully, the theory of relaxation in the turbulent plasma is not completed, but there are indications to a large enhancement of the relaxation in presence of plasma turbulence, in comparison with the binary collisions (Quataert, 1997).

2.5 Accretion disc structure from equations describing continuously optically thin and optically thick disc regions

In order to find equations of the disc structure valid in both limiting cases of optically thick and optically thin disc, and smoothly describing transition between them, Eddington approximation had been used for obtaining formulae for a heat flux and for a radiation pressure (Artemoma et al., 1996). The following expressions had been obtained for the vertical energy flux from the disc \( F_0 \), and the radiation pressure in the symmetry plane.
\[ F_0 = \frac{2acT_0^4}{3\tau_0} \left( 1 + \frac{4}{3\tau_0} + \frac{2}{3\tau_*^2} \right)^{-1}, \quad P_{rad,0} = \frac{aT_0^4}{3} \left( 1 + \frac{4}{3\tau_0} + \frac{2}{3\tau_*^2} \right), \] (14)

where \( \tau_0 = \kappa_e \rho h \), \( \tau_* = (\tau_0 \tau_{a0})^{1/2} \), \( \tau_{a0} \approx \kappa_p \rho h \). At \( \tau_0 \gg \tau_* \gg 1 \) we have (11) from (14). In the optically thin limit \( \tau_* \ll \tau_0 \ll 1 \) we get

\[ F_0 = acT_0^4 \tau_{a0}, \quad P_{rad,0} = \frac{2}{3} acT_0^4 \tau_{a0}. \] (15)

Using \( F_0 \) instead of \( Q_+ \) and equation of state \( P = \rho RT + P_{rad,0} \), the equations of accretion disc structure together with equation \( Q_+ = F_0 \), with \( Q_+ \) from (10), have been solved numerically by Artemova et al. (1996). It occurs that two solutions, optically thick and optically thin, exist separately when luminosity is not very large. Two solutions intersect at \( \dot{m} = \dot{m}_b \) and there is no global solution for accretion disc at \( \dot{m} > \dot{m}_b \) (see Fig.1). It was concluded by Artemova et al (1996), that in order to obtain a global physically meaningful solution at \( \dot{m} > \dot{m}_b \), account of advection is needed.

3 Accretion discs with advection

Standard model gives somewhat nonphysical behaviour near the inner edge of the accretion disc around a black hole. For high mass fluxes when central regions are radiation-dominated \( (P \approx P_r, \kappa \approx \kappa_e) \), the radial dependence follows relations (Shakura, Sunyaev, 1973)

\[ \rho \sim r^{3/2} J^{-2} \to \infty, \quad T \sim r^{-3/8}, \]
\[ h \sim J \to 0, \quad \Sigma \sim r^{3/2} J^{-1} \to \infty, \quad v_r \sim r^{-5/2} J \to 0, \]

where limits relate to the inner edge of the disc with \( r = r_{in}, J = 1 - \frac{\dot{M}}{\dot{M}_r} = 1 - \sqrt{\frac{\dot{M}}{\dot{M}_r}} \). At smaller \( \dot{M} \), when near the inner edge \( P \approx P_g, \kappa \approx \kappa_e \), there are different type of singularities

\[ \rho \sim r^{-33/20} J^{2/5} \to 0, \quad T \sim r^{-9/10} J^{2/5} \to 0, \]
\[ h \sim r^{21/20} J^{1/5} \to 0, \quad \Sigma \sim r^{-3/5} J^{3/5} \to 0, \quad v_r \sim r^{-2/5} J^{-3/5} \to \infty. \] (17)

This results from the local form of the equation of the thermal balance (12). It is clear from physical ground, that when a local heat production due to viscosity goes to zero, the heat brought by radial motion of matter along the accretion disc becomes more important. In presence of this advective heating (or cooling term, depending on the radial entropy \( S \) gradient) written as

\[ Q_{adv} = \frac{\dot{M}}{2\pi r} T \frac{dS}{dr}, \] (18)
Figure 1: The dependences of the optical depth $\tau_0$ on radius, $r_\ast = r/r_g$, for the case $M_{BH} = 10^8 M_\odot$, $\alpha = 1.0$ and different values of $\dot{m}$. The thin solid, dot-triple dash, long dashed, heavy solid, short dashed, dotted and dot-dashed curves correspond to $\dot{m} = 1.0, 3.0, 8.0, 9.35, 10.0, 11.0, 15.0$, respectively. The upper curves correspond to the optically thick family, lower curves correspond to the optically thin family.
the equation of a heat balance is modified to $Q_+ + Q_{adv} = Q_-$. In order to describe self-consistently the structure of the accretion disc we should also modify the radial disc equilibrium, including pressure and inertia terms

$$r(\Omega^2 - \Omega_k^2) = \frac{1}{\rho} \frac{dP}{dr} - v_r \frac{dv_r}{dr}. \quad (19)$$

Appearance of inertia term leads to transonic radial flow with a singular point. Conditions of a continuous passing of the solution through a critical point choose a unique value of the integration constant $j_{in}$. First approximate solution for the advective disc structure has been obtained by Paczynski and Bisnovatyi-Kogan (1981), but a corresponding set of equations had been discussed earlier (Hoshi and Shibazaki, 1977; Liang and Thompson, 1980). Attempts to find a solution for advective disc structure (see e.g. Matsumoto et al., 1984; Abramovich et al., 1988) gave the following results. For moderate values of $\dot{M}$ a unique continuous transonic solution was found, passing through singular points, and corresponding to a unique value of $j_{in}$. The number of critical points in the radial flow happens always to be more than unity. This is connected with two reasons. First, the gravitational potential $\phi_g$ in papers dealing with advective disc solutions was different from Newtonian one (Pacziński and Wiita, 1980):

$$\phi_g = \frac{GM}{r - r_g}, \quad r_g = \frac{2GM}{c^2}. \quad \text{The advantage of this potential is a realistic approximation of the general relativistic (GR) effects, namely, infinitive gravitational attraction at a gravitational radius } r_g, \text{ and existence of the stable circular orbits only up to } r = 3r_g, \text{ like in exact GR. Appearance of two critical points for a radial flow in this potential was analysed by Chakrabarti and Molteni (1993). The second reason of multiplicity of singular points is connected with using of equations averaged over a thickness of the disc. That changes a structure of hydrodynamic equations, leading to a position of singular points not coinciding with a unit Mach number point, and increasing a number of critical points.}

When $\dot{M}$ is becoming so high, that radiation pressure starts to be important, still unresolved problems appear in a construction of the advective disc model. These problems are connected with increasing of a number of a critical points from one side, and loss of uniqueness of the transonic solution from another. So, with increasing of $\dot{M}$ the solution becomes nonunique at some parameters, or was not found at all (see Matsumoto et al., 1984; Abramovich et al., 1988; Artemova et al., 1996a). At high $\dot{M}$ the integral curves are very sensitive to input conditions: form of viscosity stresses (4) or (6), choice of boundary conditions etc. The system of equations has a very small resource of stability, so it cannot be excluded, that the failures are connected with an improper choice of a numerical method and development of numerical instabilities prevents of finding a unique physical solution. In addition to continuous solutions, solutions with standing shock waves have been investigated (Chakrabarti, 1996).
3.1 Two-temperature advective discs

In the optically thin accretion discs at low mass fluxes the density of the matter is low and energy exchange between electrons and ions due to binary collisions is slow. In this situation, due to different mechanisms of heating and cooling for electrons and ions, they may have different temperatures. First it was realized by Shapiro, Lightman, Eardley (1976) where advection was not included. It was noticed by Narayan and Yu (1995), that advection in this case is becoming extremely important. It may carry the main energy flux into a black hole, leaving rather low efficiency of the accretion up to $10^{-3} - 10^{-4}$ (advective dominated accretion flows - ADAF). This conclusion is valid only when the effects, connected with magnetic field annihilation and heating of matter due to it are neglected.

In the ADAF solution the ion temperature is about a virial one $kT_i \sim GMm_i/r$, what means that even at high initial angular momentum the disc becomes very thick, forming practically a quasi-spherical accretion flow. It is connected also with an "alpha" prescription of viscosity. At high ion temperatures, connected with a strong viscous heating, the ionic pressure becomes high, making the viscosity very effective. So, due to suggestion of "alpha" viscosity in the situation, when energy losses by ions are very low, some kind of a "thermo-viscous" instability is developed, because heating increases a viscosity, and viscosity increases a heating. Development of this instability leads to formation of ADAF.

A full account of the processes, connected with a presence of magnetic field in the flow, is changing considerably the picture of ADAF. It was shown by Schwarzman (1971), that radial component of the magnetic field increases so rapidly in the spherical flow, that equipartition between magnetic and kinetic energy is reached in the flow far from the black hole horizon. In the region where the main energy production takes place, the condition of equipartition takes place. In presence of a high magnetic field the efficiency of a radiation during accretion of an interstellar matter into a black hole increase enormously from $\sim 10^{-8}$ up to $\sim 0.1$ (Schwarzman, 1971), due to efficiency of a magnetobremstrahlung radiation. So possibility of ADAF regime for a spherical accretion was noticed long time ago. To support the condition of equipartition a continuous magnetic field reconnection is necessary, leading to annihilation of the magnetic flux and heating of matter due to Ohmic heating. It was obtained by Bisnovatyi-Kogan and Ruzmaikin (1974), that due to Ohmic heating the efficiency of a radial accretion into a black hole may become as high as $\sim 30\%$. The rate of the Ohmic heating in the condition of equipartition was obtained in the form

$$T \frac{dS}{dr} = -\frac{3 B^2}{2 8 \pi \rho r}.$$  \hspace{1cm} (20)

In the supersonic flow of the radial accretion equipartition between magnetic
and kinetic energy was suggested by Schwarzman (1971):

\[
\frac{B^2}{8\pi} \approx \frac{\rho v_r^2}{2} = \frac{\rho GM}{r}.
\]

(21)

For the disc accretion, where there is more time for a field dissipation, almost equipartition was suggested (Shakura, 1972) between magnetic and turbulent energy, what reduces with account of "alpha" prescription of viscosity to a relation

\[
\frac{B^2}{8\pi} \sim \frac{\rho v_r^2}{2} = \frac{3}{2} \alpha_m^2 P,
\]

(22)

where \(\alpha_m\) characterises a magnetic viscosity in a way similar to the turbulent \(\alpha\) viscosity. It was suggested by Bisnovatyi-Kogan and Ruzmaikin (1976) the similarity between viscous and magnetic Reynolds numbers, or between turbulent and magnetic viscosity coefficients

\[
Re = \frac{\rho v l}{\eta}, \quad Re_m = \frac{\rho v l}{\eta_m},
\]

(23)

where the turbulent magnetic viscosity \(\eta_m\) is connected with a turbulent conductivity \(\sigma = \frac{\rho c^2}{4\pi \eta_m}\). Taking \(\eta_m = \frac{\alpha_m}{\alpha} \eta\), we get a turbulent conductivity

\[
\sigma = \frac{c^2}{4\pi \alpha_m l v_s}, \quad v_s^2 = \frac{P_g}{\rho}
\]

(24)

in the optically thin discs. For the radial accretion the turbulent conductivity may contain mean free path of a turbulent element \(l_t\), and turbulent viscosity \(v_t\) in (24) instead of \(h\) and \(v_s\). In ADAF solutions, where ionic temperature is of the order of the virial one the two suggestions (21) and (22) almost coincide at \(\alpha_m \sim 1\).

The heating of the matter due to an Ohmic dissipation may be obtained from the Ohm’s law for a radial accretion in the form

\[
\dot{T} \frac{dS}{dr} = \frac{\sigma E^2}{v_r} \approx -\sigma v_t^2 B^2 = -\frac{B^2 v_t}{4\pi \alpha_m v_r l_t},
\]

(25)

what coincides with (24) when \(\alpha_m = \frac{4 rv_t}{3v_r l_t}\), or \(l_t = \frac{4rv_t}{3v_r \alpha_m}\). Here a local electrical field strength in a highly conducting plasma is of the order of \(E \sim \frac{mB}{e}\) for the radial accretion.

Equations for a radial temperature dependence in the accretion disc, separate for the ions and electrons are written as (Bisnovatyi-Kogan and Lovelace, 1997)

\[
\frac{dE_i}{dt} - \frac{P_i d\rho}{\rho^2 dt} = \mathcal{H}_{ni} + \mathcal{H}_{Bi} - Q_{ie},
\]

(26)

\[
\frac{dE_e}{dt} - \frac{P_e d\rho}{\rho^2 dt} = \mathcal{H}_{ne} + \mathcal{H}_{Be} + Q_{ie} - C_{brem} - C_{cyc},
\]

(27)
Here \( \frac{d}{dt} = \frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r} \). A rate of a viscous heating of ions \( \mathcal{H}_{\eta i} \) is obtained from (10) as

\[
\mathcal{H}_{\eta i} = \frac{2\pi r}{M} Q_+ = \frac{3}{2} \alpha \frac{v_K v_e^2}{r}, \quad \mathcal{H}_{\eta e} = \sqrt{\frac{m_e e}{m_i}} \mathcal{H}_{\eta i},
\]

where \( v_K = r \Omega_K \). The rate of the energy exchange between ions and electrons due to the binary collisions was obtained by Landau (1937), Spitzer (1940) as

\[
Q_{ie} \approx \frac{4(2\pi)^{\frac{3}{2}} n e^4}{m_i m_e} \left( \frac{T_e}{m_e} + \frac{T_i}{m_i} \right)^{-\frac{1}{2}} \ell n \Lambda (T_i - T_e),
\]

with \( \ell n \Lambda = O(20) \) the Coulomb logarithm. The electron bremsstrahlung \( C_{brem} \) and magneto-bremsstrahlung \( C_{cyc} \) cooling are taken into account. The expression for an Ohmic heating in the turbulent accretion disc may be written in different ways, using different velocities \( v_E \) in the expression for an effective electrical field \( \mathcal{E} = \frac{v_E B}{r} \). A self-consistency of the model requires, that expressions for a magnetic heating of the matter \( \mathcal{H}_B \), obtained from the condition of stationarity of the flow (24), and from the Ohm’s law (25), should be identical. That gives some restrictions for the choice of a characteristic velocity \( v_E \). Comparison between (20) and (25) shows the identity of these two expressions at \( v_E = v_r, \quad \alpha \frac{v_E}{v_K} = \frac{3\sqrt{2}}{4} \). So, the model is becoming self-consistent at the reasonable choice of the parameters. Note, that in the advective models \( J \) is substituted by another function which is not zero at the inner edge of the disc. The heating due to magnetic field reconnection \( \mathcal{H}_B \) in the equations (26), (27), may be written as

\[
\mathcal{H}_B = \frac{3}{16\pi} \frac{B^2}{r \rho} v_r = \frac{1}{2J} \mathcal{H}_{\eta i} \left( \frac{v_B}{v_K} \right).
\]

So, at \( v_B = v_K \) the expressions for viscous and magnetic heating are almost identical. The distribution of the magnetic heating between electrons and ions has a critical influence on the model, if we neglect the influence of a plasma turbulence on the energy relaxation, and take into account only the energy exchange by binary collisions from (24). Observations of the magnetic field reconnection in the solar flares show (Tsuneta, 1996), that electronic heating prevails.

It follows from the physical picture of the field reconnection, that transformation of the magnetic energy into a heat is connected with the change of the magnetic flux, generation of the vortex electrical field, accelerating the particles. This vortex field has a scale of the turbulent element and suffers rapid and chaotic changes. The accelerating forces on electrons and protons in this fields are identical, but accelerations themselves differ \( \sim 2000 \) times, so during a sufficiently short time of the turbulent pulsation the electron may gain much larger energy, then the protons. Additional particle acceleration and heating
happens on the shock fronts, appearing around turbulent cells, where reconnection happens. In this process acceleration of the electrons is also more effective than of the protons. In the paper of Bisnovatyi-Kogan and Lovelace (1997) the equations (26), (27) have been solved in the approximation of nonrelativistic electrons, $v_B=v_B$, what permitted to unite a viscous and magnetic heating into a unique formula. The combined heating of the electrons and ions were taken as $H_e = (2 - g)H_\Omega$, $H_e = gH_\Omega$. In the expression for a cyclotron emission self-absorption was taken into account according to Trubnikov (1973). The results of calculations for $g = 0.5 \div 1$ show that almost all energy of the electrons is radiated, so the relative efficiency of the two-temperature, optically thin disc accretion cannot become lower then 0.25. Note again that accurate account of a plasma turbulence for a thermal relaxation and corresponding increase of the term $Q_{ie}$ may restore the relative efficiency to its unity value, corresponding to the optically thick discs.

4 Discussion

Observational evidences for existence of black holes inside our Galaxy and in the active galactic nuclei (Cherepashchuk, 1996, Ho, 1998) make necessary to revise theoretical models of the disc accretion. Large part of high energy radiation indicates to its origin close to the black hole, where standard accretion disc model is not a appropriate. The improvements of a model are connected with account of advective terms and more accurate treatment of the magnetic field effects. Conclusions about existence of ADAF solution for an optically thin accretion disc at low mass flux are connected with an incomplete account of the effects connected with magnetic field annihilation. Their account does not permit to make a relative efficiency of the accretion lower then $\sim 0.25$ from the standard value. It is expected that more accurate treatment of the relaxation connected with the plasma turbulence will even more increase the efficiency, making it close to unity (see also Fabian and Rees, 1995).

Some observational data which were interpreted as an evidence for the existence of the ADAF regime have disappered after additional accumulation of data. The most interesting example of this sort is connected with the claim of the proof of the existence of event horizon of the black holes due to manifestation of the ADAF regime of accretion (Narayan et al., 1997). Analysis of the more complete set of the observational data (Chen et al., 1997) had shown disapperence of the statistical effect clamed as an evidence for ADAF. This example shows how dangerous is to base a proof of the theoretical model on the preliminary observational data. It is even more dangerous, when the model is physically not fully consistent. Then even a reliable set of the observational data cannot serve as a proof of the model. The classical example from astrophysics of this kind gives the theory of the origin of the elements presented in the famous book of G.Gamov (1952), where the model of the hot universe was developed. In
addition to rich advantages of this model, the author also wanted to explain the origin of heavy elements in the primodial explosion, neglecting the problems connected with an absence of the stable elements with the number of barions equal to 5 and 8. G. Gamov considered a good coincidence of his calculations, where the mentioned problem was neglected, and the observational curve, as a best proof of his theory of the origin of the elements. The farther developements have shown that his outstanding theory explains lot of things, except the origin of the heavy elements, which are produced due to stellar evolution.

It looks like it is difficult to use ADAF for solution of the problem of existence of underluminous AGN, where the observed flux of the energy is smaller, then the expected from the standard accretion disc models. Two possible ways may be suggested. One is based on a more accurate estimations of the accretion mass flow into the black hole, which could be overestimated. Another, more attractive possibility, is based on existence of another mechanisms of the energy losses in the form of accelerated particles, like in the radio-pulsars, where their losses exceed strongly a radiation losses. This is very probable to happen in a presence of a large scale magnetic field which may be also responsible for a formation of the observed jets. To extend this line, we may suggest, that underlumilnous AGN loose main part of their energy to the formation of jets. The search of the correlation between existence of jets and lack of the luminosity could be very informative.

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