A glimpse of objectivity in bipartite systems for non-entangling pure dephasing evolutions

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We study separable system-environment evolutions of pure dephasing type in the context of objectivity and find that it can lead to the natural emergence of Spectrum Broadcast Structure (SBS) states at discrete instances of time. Contrary to the standard way of obtaining SBS states which requires entanglement with the observed environment, reaching such states here does not require decoherence (no unobserved environments are necessary). Yet the biggest difference is the basis with respect to which the SBS states are formed. Here it is not the pointer basis of the system given by the interaction with the environment, but an equal superposition basis of said pointer states. The price to pay is the momentary character of the formed SBS structures, hence the term “glimpse”.

I. INTRODUCTION

Objectivity in quantum mechanics [1–3] is the property of a composite quantum system which allows for the determination of the state of some part of said system (central system) via measurements on other parts of it (environments) by different, independent observers in such a way that the measurements yield the same results regardless of the observer and do not destroy (on average) the state of the whole system. The property is virtually guaranteed in classical physics, but in quantum mechanics it can only be seen in very specific situations. Quite recently, building on the earlier ideas of quantum Darwinism [4, 5], the mathematical structure a state of the whole system would have to have for objective behavior in quantum mechanics has been proposed [4, 7] (for the relationship to quantum Darwinism see Ref. [6]). The structure is called Spectrum Broadcast Structure (SBS), and states which retain it are a special class of zero-discord states with respect to all environments and the system of interest (the quantum discord is by definition asymmetric and a system can be discordant only with respect to one subsystem [7]).

The quantum discord [7–9] quantifies quantum correlations in a given state from the perspective of measurement. If a system state is discordant with respect to one subsystem it means that the state of said subsystem cannot be fully determined by local measurements on this subsystem without disturbing the state [4]. Obviously then if a state has no discord with respect to any of its parts, the measurement of any part does not disturb the whole state. For the state to allow objective determination of the state of the central system via measurements on any of its environments, the state must additionally contain strong classical correlations between the parts. Hence, the SBS state is of the form [4, 5]

\[
\hat{\sigma}_{SBS} = \sum_i p_i \ket{i} \bra{i} \otimes \hat{\rho}_k^i,
\]

where \(\hat{\rho}_k^i\) denotes density matrices in the subspace of environment \(k\) with \(\hat{\rho}_k^{i'} \hat{\rho}_k^i = 0\) for \(i \neq i'\). It is easy to check that such a state must be zero-discordant and that not all states with zero discord can be put in SBS form, because the density matrices of each subsystem must be orthogonal with respect to one another. For such a state the outcome \(i\) on environment \(k\) means that the state of the system is the state \(\ket{i}\). A series of measurements on environment \(k\) which allows the determination of the state of this environment also yields the knowledge of the state of the system of interest.

Typically, the search for SBS states is performed in system-multiple-environments scenarios where the interaction is limited to the kind which can only lead to pure dephasing decoherence on the system alone [10–14]. The reason for this is that in such evolutions there is an naturally chosen system basis (namely the pointer basis [13, 14]) for the decomposition into an SBS structure, which does not change over time. Obtaining SBS states in this manner requires the addition of unobserved environments which lead to the decoherence of parts of the system-environments density matrix necessary [10–12]. Furthermore, it has recently been shown that orthogonalization of the environmental states requires the generation of entanglement between the system and its observed environments [13, 14].

We study the same type of interaction between a system and its environment(s), so the evolution of the system alone undergoes pure dephasing in some pointer basis chosen by the Hamiltonian. The difference is that we limit ourselves to the situation when the joint system-environment evolution does not involve the generation of entanglement between them (is separable). We find that
SBS states can emerge in such situations as long as there is only a single observed environment present in the system, only not in the pointer basis of the system, $|i\rangle$, but in a complementary basis, composed of equal superposition states. Such bases are known as Mutually Unbiased Bases (MUBs); see e.g. Refs [17, 18).

This is a peculiar property of non-entangling system-environment evolutions of pure dephasing type, since it is enough to ensure two-way zero-discordance to glimpse an SBS state, as we will demonstrate. It turns out that the orthogonality condition of environmental states is always fulfilled in the class of evolutions studied. This is contrary to the findings of the standard way of obtaining SBS states, where for the studied family of evolutions [13] orthogonality requires entanglement, and therefore orthogonality is always harder to obtain than proper decoherence.

The SBS states obtained this way are qualitatively different from the ones usually found, not only because of the extreme change of the distinguishable central system states, but also since no decoherence (stemming from unobserved environments) is necessary for them to occur. These are naturally zero-discord states which emerge throughout the evolution. The lack of discord with respect to the system at most times is the most important issue, since pure dephasing separable evolutions always have zero discord with respect to the environment [10].

This type of zero-discordance can only occur at specific instances of time, and it is natural to expect that an increase of the complexity of the system or environment will lead to the glimpses of SBS states to become more sparse, unless special symmetries are present in the system. Nevertheless, we have shown that SBS states can and will occur in systems of one or many qubits without the aid of decoherence, and in this situation, separability of the system-observed-environment evolution is a requirement, rather than an obstacle for SBS states to emerge.

The paper is organized as follows. In Sec. III we comment on the standard procedure of searching for SBS states, and outline the necessities for the SBS structure to occur, namely the unobserved environments for decoherence and entanglement for orthogonality. In Sec. III we describe the system under study, which is composed of a system and its (possibly multiple) environments, which evolve together causing decoherence in the system alone, but without generating system-environment entanglement. In Sec. IV we show that such an evolution is capable of producing SBS states at discrete instances of time for a bipartite system (as long as there is only one observed environment). In Sec. V we show that for multiple environments the procedure will not generate glimpses of objectivity. Sec. VI concludes the paper.

II. obtaining SBS States

The standard way of obtaining an SBS state during a joint evolution of a system and its environments involves the addition of unobserved environments [10, 14] for the purpose of obtaining enough decoherence to reach the zero-discord form of the system-observed-environments density matrix [11]. The most efficient decoherence for this purpose is of pure-dephasing type, since it nullifies (after the unobserved environments are traced out) the terms proportional to the off-diagonal elements in the system of interest, directly leading to the structure of eq. (1), possibly without satisfying the orthogonality conditions. The characteristic of this type of interaction is that there exists a chosen basis for the system of interest of so called pointer states [12, 16], which do not undergo decoherence. The fully decohered state is diagonal in the pointer basis.

For the choice of the pointer basis to be definite, the interaction of the system with its observed environments must be of pure dephasing type [16, 20, 21]. Such an evolution does not disturb system-of-interest occupations and has a particularly simple form, even when the observed environments are taken into account in the density matrix. Note that this is by no means the only way for an SBS state to be reached, it is merely the most straightforward idea how to reach those type of states. The limitation of the study to such interactions is reasonable from the perspective of physics, since pure dephasing commonly the dominant source of environmentally-induced noise for solid state qubits [22, 23].

It has recently been shown that the necessary conditions for the emergence of SBS states in such a scenario, when the central system is an initially pure qubit, is the generation of system-environment entanglement with the observed environments [13]. This is because the lack of said entanglement precludes orthogonalization of the environmental density matrices $\rho_{ik}$ conditional on the pointer state $|i\rangle$, since the separability condition for entanglement between the system and environment $k$, necessary and sufficient for qubits [51], while only sufficient for larger systems [22], is

$$\forall_{i,j} \rho_{ii}^{k} = \rho_{jj}^{k},$$

Hence, in the studied scenario of [13], emergence of objectivity is irrefutably linked with entanglement generation. As we show in the following, there exist situations when the emergence of objectivity can occur without system-environment entanglement generation but, quite surprisingly, with respect to a different observable than what one would guess from the Hamiltonian.

III. Non-Entangling Pure-Dephasing Evolution

In the following we assume that the central system of interest is of dimension $d_Q$ while each environment
is of dimension $d_k$. A local Hamiltonian describing the interaction between the central system and all of its environments (in the context of objectivity, local means that the interaction between the central system and its environments can be described by a separate term in the Hamiltonian for each environment) which can only lead to pure dephasing in the qubit subspace must be of the form

$$\hat{H} = \sum_i \varepsilon_i |i\rangle \langle i| + \sum_i |i\rangle \langle i| \otimes \sum_k \hat{V}_k^i,$$  

(3)

where $\varepsilon_i$ describe the free evolution of the central system, while each term $\hat{V}_k^i$ describes both the free evolution of environment $k$ and its interaction with the system. Here we assumed that the environments do not interact directly with each other. We do not differentiate between observed and unobserved environments, as the distinction will not be necessary for the description of the emergence of SBS states during separable evolution.

Following Refs. [31, 32], it is possible to find the general form of the density matrix of the qubit and its environments at time $t$. If the initial state of the whole system is in product form with respect to the qubit and each environment, and we additionally assume that the initial state of the central system is pure

$$|\psi(0)\rangle = \sum_i a_i |i\rangle,$$  

(4)

then this density matrix may be written at any time $t$ as

$$\hat{\sigma}(t) = \sum_{ij} |i\rangle \langle j| a_i(t) a_j^\dagger(t) \otimes \hat{\rho}_{ij}^k(t),$$  

(5)

where $a_i(t) = a_i e^{-i\varepsilon_i t}$ contain the free evolution of the central system. The matrices describing the evolution of the environmental parts in the density matrix are defined as

$$\hat{\rho}_{ij}^k(t) = \hat{\omega}_{ij}^k(t) \hat{\rho}(0) \hat{\omega}_{ij}^k(t),$$  

(6)

where

$$\hat{\omega}_{ij}^k(t) = e^{-iV_{ij}^k t}.$$  

(7)

Note that to obtain the SBS state in the system pointer basis some of the environments $k$ would have to be unobserved, so that tracing out their degrees of freedom would cancel out the off-diagonal matrices (in terms of the central system) in the density matrix. Furthermore all diagonal density matrices corresponding to observed environments would have to be mutually orthogonal $\hat{\rho}_{00}^k(t) \hat{\rho}_{11}^k(t) = 0$, possibly after an application of the macrofraction technique [4]. This orthogonality condition is hard to fulfill, since here it requires strong entanglement [13].

For a given environment $k$, for the qubit environment evolution to be separable (non-entangling) at time $t$, the condition of separability of eq. [2] must be fulfilled [31], which for the density matrix [3] translates into

$$\hat{\rho}_{ij}^k(t) = \hat{\rho}_{ij}^k(t) \text{ for all } i \neq j \text{ and all environments } k. \text{ If the central system is larger than a qubit then an additional set of separability criteria must be fulfilled [32], namely we must have}$$

$$\left[\hat{\omega}_{ij}^k(t) \hat{\omega}_{ji}^k(t), \hat{\omega}_{in}^k(t) \hat{\omega}_{mj}^k(t)\right] = 0$$

(8)

for all $i, j, n,$ and $m$.

The density matrix can then be transformed into an obviously separable form with the use of the fact that the separability condition [32] also guarantees the commutation of $\hat{\rho}_{00}^k(t)$ and the operator products $\hat{\omega}_{ij}^k \hat{\omega}_{ji}^k$ and the separability condition [32] guarantees that they commute among themselves [32]. This means that all operator products and the conditional density matrix of environment $k$ can be diagonalized in the same basis,

$$\hat{\rho}(0) = \sum_n p_n |\tilde{n}_k(t)\rangle \langle \tilde{n}_k(t)|,$$  

(11)

where $\{|\tilde{n}_k(t)\rangle\}$ may be time-dependent, while the probabilities $p_n$ are not. They are in fact the same coefficients which enter the decomposition of the initial density matrix of a given environment

$$\hat{\rho}(0) = \sum_n p_n |\tilde{n}_k(t)\rangle \langle \tilde{n}_k(t)|.$$  

(11)

Let us for the moment restrict the central system to a qubit and consider only one environment $k$, so we are considering a bipartite system: The central qubit plus a single environment. The obviously separable form of eq. [1] under [2] (the condition [3] is superfluous for a qubit) is then given by

$$\hat{\sigma}(t) = \sum_{n=0}^{d-1} p_n |\psi_n(t)\rangle \langle \psi_n(t)| \otimes |n(t)\rangle \langle n(t)|,$$  

(12)

with

$$|\psi_n(t)\rangle = a_0(t) |0\rangle + a_1(t) e^{-i\phi_n(t)} |1\rangle,$$  

(13)

where we have omitted the environmental subindex $k$. Obviously, only the phase factor $\phi_n(t) = \phi_n^{\text{env}}(t)$ in state [13] depends on the index labeling the corresponding environmental states $n$.

The full state [12] is not only separable, but has zero-discord with respect to the environment, since the states $|n(t)\rangle$ form an orthonormal basis in the subspace of this environment at any moment $t$ [13].

IV. A GLIMPSE OF OBJECTIVITY FOR A SINGLE ENVIRONMENT

An interesting thing can now happen. Let us first study a central system composed of only a qubit, as the
qubit, which must be in an equal superposition state, a strong condition on the initial state of the central environment. This requirement imposes the condition

\[ |w_n(t)\rangle = \hat{V}^1 |w_n(t)\rangle \]

must be fulfilled, so \( \phi_0(t) - \phi_1(t) = (2m + 1)\pi \), with \( m = 0, 1, 2, \ldots \). Evidently the condition cannot be fulfilled for all instances of time, since \( \phi_n(0) = 0 \). When the above conditions are met, \( |\psi_1(t)\rangle = \hat{V}^1 |\psi_1(t)\rangle \) and \( |0,1\rangle \) are mutually unbiased bases for the central qubit.

The question now is, do there exist separable evolutions of pure dephasing type that exhibit instances of time when the condition is met? To show that they do, let us study the situation of an asymmetric qubit-environment coupling, meaning that \( \omega_0(t) = 1 \) and only \( \omega_1(t) \) drives the dephasing. This is a particularly simple case, since now \( \hat{\rho}_0(t) = \hat{\rho}_0(0) = \hat{\rho}_0 \), so the environmental density matrix conditional on the qubit being in state \( |0\rangle \) is equal to the initial density matrix of the environment and its eigenstates (not only eigenvalues as before) are constant in time. For a separable evolution, this also means that the operator \( \hat{w}_1(t) \) commutes with the initial state of the environment \( \hat{\rho}_0 \) at all times. Hence, it can always be written in the basis of initial environmental eigenstates, and since it is a direct function of the Hermitian operator \( \hat{V}^1 \) [see the Hamiltonian \( H \)], we will have a linear time dependence of the phase-factors

\[ \hat{w}_1(t) = \sum_{n} e^{-i\frac{\omega_n}{2}t} |\bar{n}\rangle \langle \bar{n}|, \]

where \( \omega_n \) are the eigenvalues of \( \hat{V}^1 \) corresponding to each eigenstate of the initial density matrix of the environment, \( |\bar{n}\rangle \). Hence, \( \phi_0(t) - \phi_1(t) = (\omega_0 - \omega_1)t \) and instances of time for which the phase factors \( e^{-i\omega_0(t)} \) and \( e^{-i\omega_1(t)} \) have opposite signs must occur periodically in all cases when \( \omega_0 \neq \omega_1 \). In such an instant of time we are dealing with an SBS state between two qubits, which can be written explicitly in the form

\[ \hat{\sigma}(t) = \sum_{n=0}^{\infty} p_n |\psi_n\rangle \langle \psi_n| \otimes |\bar{n}\rangle \langle \bar{n}|, \]

where the environmental qubit states are \( |\bar{n}\rangle = |0\rangle, |1\rangle \), are the basis states in which the environment was initially diagonal, with corresponding initial occupations \( \rho_0 \) and \( \rho_1 \), while the two central qubit states are orthogonal and are some trivial variation on the states

\[ |\psi_0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle), \]

\[ |\psi_1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle); \]

\(|0\rangle \) and \(|1\rangle \) denote the pointer states of the central qubit. Note, that the situation when \( \psi_0^0 = 1 \) is trivial in the sense that it leads to no dephasing, and the existence of the environment has no effect on the qubit.

### A. Larger environments

Let us now consider the situation when the central system is still a qubit, but the environment is larger dimensional, of dimension \( d \). Imagine there is a time moment \( t \) such that the \( d \) environmental states \( |\psi_n(t)\rangle \) divide into two groups \( I \) and \( II \) such that: i) all the states within each group are identical up to a global phase factor:

\[ |\psi_{n_{I/II}}(t)\rangle = e^{i\theta_{n_{I/II}}}|\psi_{I/II}(t)\rangle \]

for all \( n_{I} \in \text{Group } I \) and \( n_{II} \in \text{Group } II \); ii) the states in the two groups are orthogonal with respect to each other

\[ \langle \psi_{n_{I}} | \psi_{n_{II}} \rangle = |a_{0}(0)|^2 + |a_{1}(0)|^2 e^{(\phi_{n_{I}} - \phi_{n_{II}})} = 0, \]

Then for this moment \( t \) the state

\[ \hat{\sigma}(t) = p_{I} |\psi_I(0)\rangle \langle \psi_I(0)| \otimes \hat{\rho}_I(t) + p_{II} |\psi_{II}(0)\rangle \langle \psi_{II}(0)| \otimes \hat{\rho}_{II}(t), \]

where \( p_{I/II} = \sum_{n_{I/II}} p_{n_{I/II}} \) are the probabilities that the environment is found in either of the two groups, and the corresponding environmental density matrices conditional on either of the groups are given by

\[ \hat{\rho}_{I/II}(t) = \sum_{n_{I/II}} \frac{p_{n_{I/II}}}{p_{I/II}} |n(t)\rangle \langle n(t)|. \]

We note that by the construction \( \hat{\rho}_I(t) \) and \( \hat{\rho}_{II}(t) \) are supported on orthogonal subspaces and \( p_I + p_{II} = 1 \). Thus the state \( (21) \) surprisingly takes a (bipartite) SBS form. This bipartite SBS form is however with respect to a different basis than the pointer one in the interaction Hamiltonian \( H \) in direct accordance with the results corresponding a qubit environment. It is not immediately obvious that this can happen as the summation over \( n \) in \( (12) \) is a summation over the basis of the environment that can be arbitrarily large. The equality...
and orthogonality conditions impose the same limitations on the initial state of the central qubit as before, namely, \(|a_0(0)| = |a_1(0)| = 1/\sqrt{2}\). In terms of phase factors, the fulfillment of the equality condition requires all phase factors in each group to align \(\phi_{m_{i'j}i'}(t) - \phi_{m_{ij}i}(t) = 2j\pi\), for some integer \(j\), while the fulfillment of the orthogonality condition requires strong misalignment of the phases from different groups with respect to each other \(\phi_{m_{i'j}i'}(t) - \phi_{m_{ij}i}(t) = (2m + 1)\pi\). These conditions are equivalent to the phase condition for a qubit environment (the equality condition was automatically fulfilled in that case, since there was only one state in each group), but obviously the number of phase conditions that have to be simultaneously met grows strongly with the size of the environment. Hence, it is harder to obtain the situation when all conditions are met for a larger environment, even though there are no additional constraints on the division of environmental states between the two groups (as long as two groups exist).

To see that such glimpses of objectivity at discrete times are possible even for larger environments, it is again convenient to consider the asymmetric coupling case, similarly as in case of the environment size restricted to one qubit. Then the different phases are linear functions of time and only the differences between the eigenvalues of \(V^1\), \(v^1_{i'}\), are relevant. Obviously, if \(V^1\) is degenerate and has only two different eigenvalues, the situation operationally reduces to the qubit environment case (and the states are permanently divided into the two groups by these different values). Otherwise, at different instances of time different subgroups of environmental states can align, leading to a more complicated pattern of SBS state emergence (typically, for randomly chosen \(v^1_{i'}\), SBS states will be glimpsed more sparsely throughout the evolution with growing size of the environment). In time instances that the phase factors corresponding to the effect of the environment align into two groups, leading to opposite phase factors, the SBS state will emerge, and will be of the form \(|\psi_{i'j'i}\rangle\), where the central qubit states \(|\psi_{i'j'i}\rangle\) can still only be a trivial variation of the states given in eqs (18).

B. Beyond the qubit

The fulfillment of all separability criteria, \(|\psi\rangle\) and \(\sigma\), allows to transform the full system-environment density matrix into an obviously separable form as in the case of the qubit. For a single environment it looks exactly the same as eq. (12), but the qubit states are now replaced by states of the system dimension,

\[
|\psi\rangle = \sum_{i=0}^{d_Q-1} a_i(t)e^{-i\phi_{m_{ij}i}(t)}|i\rangle,
\]

(23)

where \(|i\rangle\) labels the pointer states of the system as before, \(a_i(0)\) are the initial occupations of the system state and the coefficients \(a_i(t)\) contain the free evolution of the system and \(d_Q\) denotes the dimension of the system.

The state still has zero discord with respect to the environment and the environmental states \(|n(t)\rangle\) still form an orthonormal basis. Therefore what is necessary for the emergence of SBS states is that the qubit \(|\psi_n\rangle\) states would form an orthonormal basis. For this to occur the initial central system occupations must all be equal, \(a_i(0) = 1/\sqrt{d_Q}\) (since all states \(|\psi_n\rangle\) have the same occupations) while the different phase factors in eq. (22) must align so that the \(|\psi_n\rangle\) states form a basis. As a result, they form a MUB with respect to the \(|i\rangle\) base \(|0\rangle, |1\rangle\). For systems which can be decomposed into qubits, so their dimension is \(d_Q = 2^m\), with integer \(m\), such bases can be easily constructed by tensor multiplication of the qubit equal-superposition basis. Note that for glimpses of objectivity to be possible, the dimension of the environment must be of the same size or larger than that of the central system.

For a central system state larger than a qubit (plus environment) to be a true SBS state, all \(d_Q\) central system MUB states must be present in the decomposition (12), one should therefore expect the natural occurrence of SBS states to be less frequent also with the growing size of the central system, unless the environment is specially structured, so the phase relations between the different pointer states in the states \(|\psi_n\rangle\) oscillate with frequencies which are not random.

V. MULTIPLE ENVIRONMENTS

Let us now consider more environments than just one, while again restricting the central system to a qubit. Assuming the initial state of the environment is a product, \(\hat{\rho}^E(0) = \bigotimes_k \hat{\rho}^k(0)\), we obtain a very similar form of the time evolved qubit-environments density matrix to the single-environment density matrix (12),

\[
\hat{\sigma}(t) = \sum_{n_1,\ldots,n_N} \left( \prod_{k=1}^N p_{nk} \right) |\psi_{n_1n_2\ldotsn_N}(t)\rangle\langle \psi_{n_1n_2\ldotsn_N}(t)| \bigotimes_{q=1}^N |n_q(t)\rangle\langle n_q(t)|,
\]

(24)

with the states of the different environments now resolved and their evolved states are obtained from the initial eigenstates of each density matrix (11) as previously, \(|n_q(t)\rangle = \tilde{w}_q^k(t)|\tilde{n}_q\rangle\), where \(\tilde{w}_q^k(t)\) is given by eq. (7). The qubit states also retain a form similar to the single-environment case, with the exception that the phases now accumulate from different environments,

\[
|\psi_{n_1n_2\ldotsn_N}(t)\rangle \equiv a|0\rangle + b(t)e^{-i\sum_k \phi_{m_{ij}i}|1\rangle}.
\]

(25)

For the qubit-multiple-environments density matrix (24) to be of SBS form (1), firstly the conditions for
the orthogonality qubit states $|\psi_{n_1...n_N}(t)\rangle$ should be fulfilled. These conditions have been specified in the previous section, and the ones which apply here are those pertaining to a single larger environment. Namely each qubit state $|\psi_{n_1...n_N}(t)\rangle$ must at the given time $t$, when objectivity is to be glimpsed, be one of two orthogonal states as in Sec. [IV.A]. The only difference is that the phases accumulated by each qubit state are now a sum of the phases stemming from the evolution of each environment.

If the SBS conditions for the qubit part of the density matrix are met, there is still the question if the SBS form is retained on the side of each environment. For a single environment this was always the case, since at any time $t$ the environmental states $|n(t)\rangle$ form an orthonormal basis, so states corresponding to either group of qubit states can be combined into a diagonal density matrix in this basis, and the two density matrices must be orthogonal to one another.

In case of multiple environments, this simple method does not work, because the states $\otimes_{q=1}^N |n_q(t)\rangle$, although they are composed of parts which are orthogonal for every environment, will in general loose that property when grouped as in eq. (22). To see this, let us consider two environments composed of a single qubit each and denote the environmental states $|n_1(t)\rangle \otimes |n_2(t)\rangle$ corresponding to a time $t$ when the central qubit fulfils the SBS conditions as $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. Let us further denote the two states of the central qubit as $|+\rangle$ and $|-\rangle$. In the symmetric case, when both of the environmental states correspond to central qubit state $|+\rangle$, say $|00\rangle$ and $|01\rangle$, and the other two to the $|-\rangle$ state, the states of the central qubit will be fully distinguishable by measurements on one qubit, but completely indistinguishable by measurements on the other. For the allotment as chosen above, the density matrices of each environment that enter eq. (1) are given by $\hat{\rho}_0^1 = |0\rangle\langle 0|$, $\hat{\rho}_1^1 = |1\rangle\langle 1|$, and $\hat{\rho}_0^2, \hat{\rho}_1^2 \sim p |0\rangle\langle 0| + (1 - p) |1\rangle\langle 1|$ so the states of the second environment are clearly not orthogonal to each other. In the asymmetric case, when only one state is allotted to state $|+\rangle$, say $|00\rangle$, and the other three to state $|-\rangle$, the qubit-environments density matrix can no longer be written in the SBS form (1) corresponding to two en-

environments, since we have

$$
\hat{\rho}_{SBS} = \frac{1}{2} \left[ p_+ |+\rangle\langle +| \otimes |0\rangle\langle 0| \otimes |0\rangle\langle 0|.
\right.
$$

$$
+ p_+ |-\rangle\langle -| \otimes |0\rangle\langle 0| \otimes |1\rangle\langle 1|.
$$

$$
+ \frac{p_B}{2} |0\rangle\langle 0| \otimes |1\rangle\langle 1| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|).
$$

Here, the measurement $|0\rangle$ or $|1\rangle$ on either environmental qubit can correspond to either central qubit state.

The situation only becomes more complicated for larger central systems, but the outcome is the same and SBS states obtained in the described way can occur in nonentangling evolutions only if there is one observed environment.

VI. CONCLUSION

We have studied a class of nonentangling evolutions which lead to pure dephasing of the central system due to the interaction with one or multiple environments. We have shown that system-environment entanglement is not necessary for the emergence of SBS states, but the situation when it does emerge is rather special. Firstly, it does not require there to be unobserved environments (no decoherence source is necessary). Secondly, it is possible only at discrete instants of time, and thirdly, it can only occur if the initial state of this system is an equal-superposition state (so only in mutually unbiased bases of the central system). Furthermore, it is only possible for one observed environment.

If the central system is a qubit initially in an equal superposition state, SBS states will occur naturally throughout the evolution at instances of time when the phases which are the outcome of the difference of the evolution of the environment conditional on the state of the qubit all reach one of two values which differ by $\pi$ (states corresponding to both types of phases must exist) yielding orthogonal qubit states for the qubit. Contrarily to the standard way of obtaining SBS states, orthogonality of environmental states is always fulfilled in the studied scenario, while the orthogonality of the qubit states is problematic (it is guaranteed when the qubit basis is its pointer basis as in the standard scenario), and can occur only at discrete points of time. We have shown that such instances must occur for asymmetric qubit-environment couplings, for which the aforementioned phases are linear functions of time. The glimpses of objectivity will occur for environments larger than a qubit, but since this requires the alignment of the number of phase factors equal to the dimension of the environment, they will occur the more rarely the bigger the environment (assuming that the different states of the environment oscillate differently).

[1] Harold Ollivier, David Poulin, and Wojciech H. Zurek, “Objective properties from subjective quantum states: Environment as a witness,” Phys. Rev. Lett. 93, 220401 (2004)
[2] Harold Ollivier, David Poulin, and Wojciech H. Zurek, “Environment as a witness: Selective proliferation of information and emergence of objectivity in a quantum universe,” Phys. Rev. A 72, 042113 (2005).

[3] Wojciech H. Zurek, “Quantum Darwinism,” Nature Physics 5, 181 (2009).

[4] J. K. Korbicz, P. Horodecki, and R. Horodecki, “Objectivity in a noisy photonic environment through quantum state information broadcasting,” Phys. Rev. Lett. 112, 120402 (2014).

[5] R. Horodecki, J. K. Korbicz, and P. Horodecki, “Quantum origins of objectivity,” Phys. Rev. A 91, 032312 (2015).

[6] Thao P. Le and Alexandra Olaya-Castro, “Strong quantum Darwinism and strong independence are equivalent to spectrum broadcast structure,” Phys. Rev. Lett. 122, 010403 (2019).

[7] Kavan Modi, “A pedagogical overview of quantum discord,” Open Syst. Inf. Dyn. 21, 1440006 (2014).

[8] H. Ollivier and W. H. Zurek, “Quantum Discord: A Measure of the Quantumness of Correlations,” Phys. Rev. Lett. 88, 017901 (2002). [quant-ph/0105072]

[9] L. Henderson and V. Vedral, “Classical, quantum and total correlations,” Journal of Physics A Mathematical General 34, 6899-6905 (2001). [arXiv:quant-ph/0105028]

[10] J. Tuiziemski and J. K. Korbicz, “Dynamical objectivity in quantum brownian motion,” EPL (Europhysics Letters) 112, 40008 (2015).

[11] P. Mironowicz, J. K. Korbicz, and P. Horodecki, “Monitoring of the process of system information broadcasting in time,” Phys. Rev. Lett. 118, 150501 (2017).

[12] Jan Tuiziemski, Aniello Lampo, Maciej Lewenstein, and Jaroslav K. Korbicz, “Reexamination of the decoherence of spin registers,” Phys. Rev. A 99, 022122 (2019).

[13] Katarzyna Roszak and Jaroslav K. Korbicz, “Entanglement and objectivity in pure dephasing models,” (2019). [arXiv:1904.08261 [quant-ph]]

[14] Guillermo Garcia-Perez, Dario A. Chisholm, Matteo A. C. Rossi, G. Massimo Palma, and Sabrina Maniscalco, “Decoherence without entanglement and quantum Darwinism,” (2019). [arXiv:1907.12447 [quant-ph]]

[15] W. H. Zurek, “Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?” 24, 1516 (1981).

[16] Wojciech Hubert Zurek, “Decoherence, einselection, and the quantum origins of the classical,” Rev. Mod. Phys. 75, 715–775 (2003).

[17] Thomas Durt, “A new expression for mutually unbiased bases in prime power dimensions,” (2004). [arXiv:quant-ph/0409090 [quant-ph]]

[18] Stephen Brierley, Stefan Weigert, and Ingemar Bengtsson, “All mutually unbiased bases in dimensions two to five,” Quantum Info. Comput. 10, 803-820 (2010).

[19] Katarzyna Roszak and Łukasz Cywiński, “Equivalence of qubit-environment entanglement and discord generation via pure dephasing interactions and the resulting consequences,” Phys. Rev. A 97, 012306 (2018).

[20] Jens Eisert and Martin B. Plenio, “Quantum and classical correlations in quantum brownian motion,” Phys. Rev. Lett. 89, 137902 (2002).

[21] Klaus Horneber, “Introduction to decoherence theory,” Lect. Notes Phys. 768, 221 (2009).

[22] Y. Nakamura, Yu. A. Pashkin, T. Yamamoto, and J. S. Tsai, “Charge echo in a cooper-pair box,” Phys. Rev. Lett. 88, 047901 (2002).

[23] Katarzyna Roszak and Pawel Machnikowski, “Complete disentanglement by partial pure dephasing,” Phys. Rev. A 73, 022313 (2006).

[24] Michael J. Biercuk, Hermann Uys, Aaron P. VanDevender, Nobuyasu Shiga, Wayne M. Itano, and John J. Bollinger, “Optimized dynamical decoupling in a model quantum memory,” Nature 458, 996 (2009).

[25] Jonas Bylander, Simon Gustavsson, Fei Yan, Fumiki Yoshihara, Khalil Harrabi, George Fitch, David G. Cory, Yasunobu Nakamuta, Jaw-Shen Tsai, and William D. Oliver, “Dynamical decoupling and noise spectroscopy with a superconducting flux qubit,” Nature Phys. 7, 565 (2011).

[26] J. Medford, L. Cywiński, C. Barthel, C. M. Marcus, M. P. Hanson, and A. C. Gossard, “Scaling of dynamical decoupling for spin qubits,” Phys. Rev. Lett. 108, 086802 (2012).

[27] T. Staudacher, P. Shi, S. Pezzagna, J. Meijer, J. Du, C. A. Meiler, F. Reinhard, and J. Wrachtrup, “Nuclear magnetic resonance spectroscopy on a (5-nanometer)³ sample volume,” Science 339, 561 (2013).

[28] Juha T. Muhonen, Juan P. Dehollain, Arne Laucht, Fay E. Hudson, Rachpon Kalra, Takeharu Sekiguchi, Kohei M. Itoh, David N. Jamieson, Jeffrey C. McCallum, Andrew S. Dzurak, and Andrea Morello, “Storing quantum information for 30 seconds in a nanoelectronic device,” Nature Nanotechnology 9, 986 (2014).

[29] Filip K. Malinowski, Frederico Martins, Łukasz Cywiński, Mark S. Rudner, Peter D. Nissen, Saeed Fallahi, Geoffrey C. Gardner, Michael J. Manfra, Charles M. Marcus, and Ferdinand Kuemmeth, “Spectrums of the nuclear environment for gaas spin qubits,” Phys. Rev. Lett. 118, 177702 (2017).

[30] P. Szankowski, G. Ramon, J. Krzywda, D. Kwiatkowski, and L. Cywiński, “Environmental noise spectroscopy with qubits subjected to dynamical decoupling,” J. Phys.:Condens. Matter 29, 333001 (2017).

[31] Katarzyna Roszak and Łukasz Cywiński, “Characterization and measurement of qubit-environment-entanglement generation during pure dephasing,” Phys. Rev. A 92, 032310 (2015).

[32] Katarzyna Roszak, “Criteria for system-environment entanglement generation for systems of any size in pure-dephasing evolutions,” Phys. Rev. A 98, 052344 (2018).