Magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors

Zheyu Huang, Huaisong Zhao, and Shiping Feng*
Department of Physics, Beijing Normal University, Beijing 100875, China

The weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors is studied based on the kinetic energy driven superconducting mechanism. The electromagnetic response kernel is evaluated by considering both couplings of the electron charge and electron magnetic momentum with a weak magnetic field and employed to calculate the superfluid density, then the main features of the weak magnetic field induced reduction of the low-temperature superfluid density are qualitatively reproduced. The theory also shows that the striking behavior of the weak magnetic field induced reduction of the low-temperature superfluid density is intriguingly related to both depairing due to the Pauli spin polarization and nonlocal response in the vicinity of the d-wave gap nodes on the Fermi surface to a weak magnetic field.

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The superfluid density \( \rho_s \), being proportional to the density of the supercarriers, is one of the important characteristic of the superconducting (SC) condensate. It is sensitive to the low-lying excitation spectrum, and therefore the knowledge of the superfluid density is essential to understanding the physics of the underlying mechanism responsible for superconductivity. Since cuprate superconductors are doped Mott insulators with the strong short-range antiferromagnetic correlation dominating the entire SC phase, the magnetic field can be also used to probe the doping and momentum dependence of the SC gap and spin structure of the Cooper pair. This is why the first evidence of the d-wave Cooper pairing state in cuprate superconductors was obtained from the experimental measurement for the magnetic field penetration depth \( \lambda \) (then the superfluid density \( \rho_s \equiv \lambda^{-2} \)). Experimentally, by virtue of systematic studies using the muon-spin-rotation measurement technique, some essential features of the superfluid density in cuprate superconductors have been established now for all the temperature \( T \leq T_c \) throughout the SC dome. However, there are numerous anomalies, which complicate the physical properties of the superfluid density. Among these anomalies is the magnetic field dependence of the superfluid density first observed on the cuprate superconductor YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\), where a weak magnetic field can induce an reduction of the superfluid density at the low temperatures. Later, this weak magnetic field induced reduction of the low-temperature superfluid density was also found in other families of cuprate superconductors.

The appearance of the weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors is the mostly remarkable effect, however, its full understanding is still a challenging issue. The earlier work gave the main impetus for a phenomenological description of the magnetic field dependent superfluid density in the Meissner state, where it has been argued that the weak magnetic field induced reduction of the low-temperature superfluid density arises from non-linear response of the d-wave state to a weak magnetic field. Later, a more dominant contribution to the magnetic field dependence of the single-particle excitations in the superfluid density comes from the nonlocality of the supercurrent response in the vicinity of the d-wave gap nodes on the Fermi surface in the d-wave SC state. In our recent work based on the kinetic energy driven SC mechanism, the doping and temperature dependence of the electromagnetic response in cuprate superconductors has been discussed by considering the coupling of the electron charge with a weak magnetic field, where the Meissner effect is obtained for all the temperature \( T \leq T_c \) throughout the SC dome, and then the main features of the doping and temperature dependence of the local magnetic field profile, the magnetic field penetration depth, and the superfluid density are well reproduced. In particular, it is shown that in analogy to the dome-like shape of the doping dependent SC transition temperature, the maximal superfluid density occurs around the critical doping \( \delta \approx 0.195 \), and then decreases in both lower doped and higher doped regimes. However, the coupling of the electron magnetic momentum with a weak magnetic field in terms of the Zeeman mechanism has been dropped in these discussions. In this paper, we study the weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors by considering both couplings of the electron charge and electron magnetic momentum with a weak magnetic field. Following the linear response theory, we have evaluated the magnetic field dependence of the response kernel within the kinetic energy driven SC mechanism. This response kernel is employed to calculate the superfluid density, then the main features of the weak magnetic field induced reduction of the low-temperature superfluid density are qualitatively reproduced. Our results also show that the striking behavior of the weak magnetic field induced reduction of the low-temperature superfluid density is intriguingly related to both depairing due to the Pauli spin polarization and nonlocal response in the vicinity of the d-wave gap nodes on the
Fermi surface to a weak magnetic field. We start from the t-J model on a square lattice. However, for discussions of the weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors, the t-J model can be extended by including the exponential Peierls factor and Zeeman term as,

\[
H = -t \sum_{\langle i \sigma \rangle} e^{-i \tilde{A}(l) \cdot \hat{\eta}} c_{l \sigma}^\dagger c_{l+\hat{\eta} \sigma} + \mu \sum_{l \sigma} \bar{c}_{l \sigma}^\dagger \bar{c}_{l \sigma} + J \sum_{l \bar{\eta}} \mathbf{S}_l \cdot \mathbf{S}_{l+\bar{\eta}} - \varepsilon_B \sum_{l \sigma} \sigma \bar{c}_{l \sigma}^\dagger \bar{c}_{l \sigma},
\]

(1)

supplemented by an important on-site local constraint \( \sum_{\sigma} \bar{c}_{l \sigma}^\dagger \bar{c}_{l \sigma} \leq 1 \) to remove the double occupancy, where \( \bar{\eta} = \pm \bar{x}, \pm \bar{y}, \bar{\eta}^\prime = \pm \bar{x} \pm \bar{y}, \bar{c}_{l \sigma}^\dagger (\bar{c}_{l \sigma}) \) is the electron creation (annihilation) operator, \( \mathbf{S}_l = (S_x^l, S_y^l, S_z^l) \) are spin operators, and \( \mu \) is the chemical potential. The exponential Peierls factors account for the coupling of the electron charge to a weak magnetic field in terms of the vector potential \( \mathbf{A}(l) \), while the Zeeman magnetic energy \( \varepsilon_B = g_\mu_B B \) accounts for the coupling of the electron magnetic momentum \( g_\mu_B \) with the weak magnetic field \( \mathbf{B} = \text{rot} \mathbf{A} \), with the Lande factor \( g \) and Bohr magneton \( \mu_B \). For a proper description of the electron single occupancy local constraint, the charge-spin separation (CSS) fermion-spin theory has been proposed, where the physics of no double occupancy is taken into account by representing the electron as a composite object created by \( \bar{c}_{l \sigma} = h^\dagger_{l \sigma} S_{l \sigma}^{-} \) and \( \bar{c}_{l \sigma}^\dagger = h_{l \sigma}^\dagger S_{l \sigma}^{+} \), with the spinful fermion operator \( h_{l \sigma} = e^{-i \Phi_{l \sigma}} h_{l} \) that describes the charge degree of freedom of the electron together with some effects of spin configuration rearrangements due to the presence of the doped hole itself (charge carrier), while the spin operator \( S_{l \sigma} \) represents the spin degree of freedom of the electron, then the electron single occupancy local constraint is satisfied in analytical calculations. In this CSS fermion-spin representation, the t-J model can be expressed as,

\[
H = t \sum_{l \bar{\eta}} e^{-i \tilde{A}(l) \cdot \hat{\eta}} (h_{l+\hat{\eta} \sigma}^\dagger h_{l \sigma} S_{l+\hat{\eta}}^{-} + h_{l+\hat{\eta} \sigma}^\dagger h_{l \sigma} S_{l \hat{\eta}}^{+}) \\
- t' \sum_{l \bar{\eta}^\prime} e^{-i \tilde{A}(l) \cdot \hat{\eta}^\prime} (h_{l+\hat{\eta}^\prime \sigma}^\dagger h_{l \sigma} S_{l+\hat{\eta}^\prime}^{+} + h_{l+\hat{\eta}^\prime \sigma}^\dagger h_{l \sigma} S_{l \hat{\eta}^\prime}^{-}) \\
- \mu \sum_{l \sigma} h_{l \sigma}^\dagger h_{l \sigma} + J_{\text{eff}} \sum_{l \bar{\eta}} \mathbf{S}_l \cdot \mathbf{S}_{l+\bar{\eta}} - 2 \varepsilon_B \sum_{l \sigma} S_{l \sigma}^z,
\]

(2)

where \( J_{\text{eff}} = (1 - \delta)^2 J \), and \( \delta = \langle h_{l \sigma}^\dagger h_{l \sigma} \rangle = \langle h_{l \sigma} h_{l \sigma} \rangle \) is the doping concentration.

For a microscopic description of the SC state of cuprate superconductors, the kinetic energy driven SC mechanism has been developed based on the CSS fermion-spin theory, where the charge carrier-spin interaction from the kinetic energy term in the t-J model induces a charge carrier d-wave pairing state by exchanging spin excitations in the higher power of the doping concentration, then the electron Cooper pairs originating from the charge carrier pairing state are due to the charge-spin recombination, and their condensation reveals the d-wave SC ground-state. In particular, this kinetic energy driven SC state is the conventional Bardeen-Cooper-Schrieffer (BCS)-like with the d-wave symmetry, so that all main low energy features of the SC coherence of the quasiparticle peaks have been quantitatively reproduced, although the pairing mechanism is driven by the kinetic energy by exchanging spin excitations. For discussions of the weak magnetic field induced reduction of the low-temperature superfluid density, we generalize the analytical calculation from our previous case without considering the coupling of the electron magnetic momentum with a weak magnetic field to the case in the presence of the coupling of the electron magnetic momentum with a weak magnetic field. Following the discussions in Ref. and Ref. , the mean-field (MF) spin excitation spectrum in the t-J model in the presence of the coupling of the electron magnetic momentum with a weak magnetic field can be evaluated as

\[
\omega_p^{(B)} = \sqrt{\omega_p^2 + (2 \varepsilon_B)^2},
\]

where \( \omega_p \) is the spin excitation spectrum in the case without the coupling of the electron magnetic momentum with a weak magnetic field, and has been given in Ref. Obviously, an additional spin gap \( 2 \varepsilon_B = 2 g_\mu_B B \) in the spin excitation spectrum is induced by the externally applied magnetic field \( \mathbf{B} \). In this case, the full charge carrier Green’s function can be obtained explicitly in the Nambu representation as

\[
\mathcal{G}(k, i \omega_n, B) = Z_B^{(B)} \omega_n \tau_0 + \frac{\tilde{C}_k \tau_3 - \tilde{\Delta}_B^{(B)}(k) \tau_1}{(i \omega_n)^2 - E_{1k}^{(B)}},
\]

(3)

where \( \tau_0 \) is the unit matrix, \( \tau_1 \) and \( \tau_3 \) are Pauli matrices, the renormalized charge carrier excitation spectrum \( \tilde{C}_k = Z_B^{(B)} C_k \), with the MF charge carrier excitation spectrum \( \tilde{C}_k = Z_B^{(B)} \chi_B^{(B)} \langle k \rangle \), the spin correlation functions \( \chi_1 = (S_z^{l+\hat{\eta}} S_z^{l\hat{\eta}})^2 \), \( \chi_2 = (S_z^{l+\hat{\eta} \sigma} S_z^{l \sigma})^2 \), \( \gamma_k = (1/Z) \sum_{\eta} \epsilon^{ik \eta} \gamma_k \), \( \bar{\gamma}_k = (1/Z) \sum_{\eta} \epsilon^{ik \eta} \bar{\gamma}_k \), \( Z \) is the number of the nearest neighbor or next-nearest neighbor sites, the renormalized charge carrier d-wave pair gap \( \tilde{\Delta}_B^{(B)}(k) = 2 g_\mu_B \chi_B^{(B)}(k) \), with the effective charge carrier d-wave pair gap \( \Delta_B^{(B)}(k) = \frac{\tilde{\Delta}_B^{(B)}(k)}{\cos k_x - \cos k_y}/2 \), and the charge carrier quasiparticle spectrum \( E_{1k}^{(B)} = \sqrt{\epsilon_k^2 + |\tilde{\Delta}_B^{(B)}(k)|^2} \), while the magnetic field dependence of the effective charge carrier pair gap \( \tilde{\Delta}_B^{(B)}(k) \) and the quasiparticle coherent weight \( Z_B^{(B)} \) satisfy the following two equations \( \tilde{\Delta}_B^{(B)}(k) = \Sigma_1^{(B)}(k, \omega = 0, B) \) and \( Z_B^{(B)}(k) = 1 - \Sigma_1^{(B)}(k, \omega = 0, B) \), where \( \Sigma_1^{(B)}(k, \omega, B) \) and \( \Sigma_2^{(B)}(k, \omega, B) \) are the charge carrier self-energies obtained from the spin bubble, while \( \Sigma_1^{(B)}(k, \omega, B) \) is the antisymmetric part of \( \Sigma_1^{(B)}(k, \omega, B) \). Moreover, the forms of the obtained \( \Sigma_1^{(B)}(k, \omega, B) \) and
The response kernel \( K_{\mu\nu}(\mathbf{q}, \omega, B) \) in the present case are the same as those given in Ref. 22 for the case without considering the coupling of the electromagnetic momentum with a weak magnetic field except the spin excitation spectrum \( \omega_p \) has been replaced by \( \omega_p^{(B)} \), therefore the effect of a weak magnetic field \( \mathbf{B} \) on the SC state is considered explicitly through the weak magnetic field \( \mathbf{B} \) entering in \( \Sigma^{(B)}_1(\mathbf{k}, \omega, B) \) and \( \Sigma^{(B)}_2(\mathbf{k}, \omega, B) \). These two equations should be solved simultaneously with other self-consistent equations.\(^{19,22}\) However, in the limit of the weak magnetic field \( B \ll B_c \) for a given doping concentration, the main role of the coupling of the electromagnetic momentum with a weak magnetic field is induced an additional spin gap in the spin excitation spectrum as we have mentioned above, while the effect on other spin correlation functions is negligible. In this case, we will concentrate on the effect of a weak magnetic field on the charge carrier part since the SC state is dominated by the charge carrier d-wave pairing state, where the effective charge carrier gap parameter \( \Delta_{\text{h}}^{(B)} \), the quasiparticle coherent weight \( Z_{\text{h}}^{(B)} \), the chemical potential, and other charge carrier particle-hole parameters must be solved self-consistently.

In the presence of the coupling of the electromagnetic momentum with a weak magnetic field, the magnetic field dependence of the response current density \( J_\mu \) and the vector potential \( A_\nu \) are related by a nonlocal kernel of the response function \( K_{\mu\nu} \) as\(^{18}\),

\[
J_\mu(\mathbf{q}, \omega, B) = -\sum_{\nu=1}^3 K_{\mu\nu}(\mathbf{q}, \omega, B) A_\nu(\mathbf{q}, \omega),
\]

with the Greek indices label the axes of the Cartesian coordinate system. This magnetic field dependence of the response kernel \( K_{\mu\nu} \) can be separated into two parts as \( K_{\mu\nu}(\mathbf{q}, \omega, B) = K_{\mu\nu}^{(d)}(\mathbf{q}, \omega, B) + K_{\mu\nu}^{(p)}(\mathbf{q}, \omega, B) \), where \( K_{\mu\nu}^{(d)}(\mathbf{q}, \omega, B) \) and \( K_{\mu\nu}^{(p)}(\mathbf{q}, \omega, B) \) are the corresponding diamagnetic and paramagnetic parts, respectively. In the CSS fermion-spin representation\(^{18,22}\), the vector potential \( \mathbf{A} \) has been coupled to the electron charge, which are now represented by \( C_{l\uparrow} = h_{l\uparrow}^\dagger S_{l\uparrow}^\uparrow \) and \( C_{l\downarrow} = h_{l\downarrow}^\dagger S_{l\downarrow}^\downarrow \). In this case, the electron polarization operator is expressed as \( \mathbf{P} = -e \sum_{i\sigma} \mathbf{C}_{i\sigma}^\dagger \mathbf{C}_{i\sigma} = -e \sum_{i\sigma} h_i^\dagger h_i \), then the corresponding electron current operator is obtained by evaluating the time-derivative of this polarization operator as \( \mathbf{j} = \mathbf{j}^{(d)} + \mathbf{j}^{(p)} \), with \( \mathbf{j}^{(d)} \) and \( \mathbf{j}^{(p)} \) are the corresponding diamagnetic \( (d) \) and paramagnetic \( (p) \) components of the electron current operator. Following our previous discussions\(^{18,22}\), these diamagnetic and paramagnetic parts of the magnetic field dependence of the response kernel \( K_{\mu\nu}^{(d)}(\mathbf{q}, \omega, B) \) and \( K_{\mu\nu}^{(p)}(\mathbf{q}, \omega, B) \) can be obtained in the static limit as,

\[
K_{\mu\nu}^{(d)}(\mathbf{q}, \omega, B) = -\frac{4e^2}{\hbar^2} (\chi_1 \phi_1 t - 2\chi_2 \phi_2 t') \delta_{\mu\nu} = \frac{1}{\lambda_L^2} \delta_{\mu\nu},
\]

\[
K_{\mu\nu}^{(p)}(\mathbf{q}, \omega, B) = 1 - \sum_{\mathbf{k}} \gamma_{\mu}(\mathbf{k} + \mathbf{q}, \mathbf{k}) |L_1^{(B)}(\mathbf{k}, \mathbf{q})|^2 + |L_2^{(B)}(\mathbf{k}, \mathbf{q})|^2 = K_{\mu\nu}^{(p)}(\mathbf{q}, 0, B) \delta_{\mu\nu},
\]

where the charge carrier particle-hole parameters \( \phi_1 = \langle h_{l\uparrow}^\dagger h_{l\downarrow} \rangle \) and \( \phi_2 = \langle h_{l\downarrow}^\dagger h_{l\uparrow} \rangle \), \( \lambda_L^2 = -4e^2 (\chi_1 \phi_1 t - 2\chi_2 \phi_2 t')/\hbar^2 \) is the London penetration depth, and now is doping, temperature, and magnetic field dependent, the bare current vertex \( \gamma_{\mu}(\mathbf{k} + \mathbf{q}, \mathbf{k}) \) has been given in Ref. 18, while the functions \( L_1^{(B)}(\mathbf{k}, \mathbf{q}) \) and \( L_2^{(B)}(\mathbf{k}, \mathbf{q}) \) are obtained as,

\[
L_1^{(B)}(\mathbf{k}, \mathbf{q}) = \frac{Z_{\text{h}}^{(B)} \left( 1 + \frac{\xi_k \xi_{k+\mathbf{q}} + \Delta_{\text{h}}^{(B)}(\mathbf{k}) \Delta_{\text{h}}^{(B)}(\mathbf{k} + \mathbf{q})}{E_{\text{h}}^{(B)} - E_{\text{h}}^{(B)}(\mathbf{k})} \right) }{n_F(E_{\text{h}}^{(B)} - n_F(E_{\text{h}}^{(B)}(\mathbf{k}) + 1)},
\]

\[
L_2^{(B)}(\mathbf{k}, \mathbf{q}) = \frac{Z_{\text{h}}^{(B)} \left( 1 - \frac{\xi_k \xi_{k+\mathbf{q}} + \Delta_{\text{h}}^{(B)}(\mathbf{k}) \Delta_{\text{h}}^{(B)}(\mathbf{k} + \mathbf{q})}{E_{\text{h}}^{(B)} - E_{\text{h}}^{(B)}(\mathbf{k})} \right) }{n_F(E_{\text{h}}^{(B)} + n_F(E_{\text{h}}^{(B)}(\mathbf{k}) + 1)}. \]

It is easy to show\(^{18}\) that in the long wavelength limit, i.e., \( \mathbf{q} \rightarrow 0 \), \( K_{\mu\nu}^{(p)}(\mathbf{q} \rightarrow 0, 0, B) = 0 \) at the temperature \( T = 0 \). In this case, the long wavelength electromagnetic response is determined by the diamagnetic part of the kernel only. On the other hand, at the SC transition temperature \( T = T_c \), \( K_{\mu\nu}^{(p)}(\mathbf{q} \rightarrow 0, 0, B) = -(1/\lambda_L^2) \), which exactly cancels the diamagnetic part of the response kernel\(^{18}\), and then the Meissner effect in the presence of the coupling of the electron magnetic momentum with a weak magnetic field is obtained for all \( T \leq T_c \) throughout the SC dome.

However, the result we have obtained the response kernel in Eqs. (5a) and (5b) cannot be used for a direct comparison with the corresponding experimental data of cuprate superconductors because the response kernel derived within the linear response theory describes the response of an infinite system, whereas in the problem of the penetration of the field and the system has a surface, i.e., it occupies a half-space \( x > 0 \). In such problems, it is necessary to impose boundary conditions for charge
carriers. This can be done within the simplest specular reflection model with a two-dimensional (2D) geometry of the SC plane. Taking into account the 2D geometry of cuprate superconductors within the specular reflection model, we can obtain the magnetic field penetration depth as,

\[
\lambda(T, B) = \frac{1}{B} \int_0^\infty h_z(x, B) \, dx
\]

\[
= \frac{2}{\pi} \int_0^\infty \frac{dq_x}{\mu_0 K_{yy}(q_x, 0, 0, B) + q_x^2},
\]

which therefore reflects the measurably electromagnetic response in cuprate superconductors.

![Diagram](image)

**FIG. 1:** The magnetic field penetration depth \(\Delta \lambda\) as a function of temperature at the doping concentration \(\delta = 0.09\) for the magnetic field \(B = 0\) T (solid line), \(B = 0.5\) T (dashed line), and \(B = 1.0\) T (dash-dotted line) with parameters \(t/J = 2.5\), \(t'/t = 0.3\), and \(J = 1000\)K. Inset: the corresponding experimental data for YBa\(_2\)Cu\(_3\)O\(_8\) taken from Ref. 13.

Now we are ready to discuss the magnetic field dependence of the Meissner effect in cuprate superconductors. In cuprate superconductors, although the values of \(J, t, \) and \(t'\) are believed to vary somewhat from compound to compound, however, as a qualitative discussion as in our recent work, the commonly used parameters in this paper are chosen as \(t/J = 2.5\), \(t'/t = 0.3\), and \(J = 1000\)K. Furthermore, a characteristic length scale \(a_0 = \sqrt{T^2 a / \mu_0 e^2 J}\) is introduced. Using the lattice parameter \(a \approx 0.383\)nm for YBa\(_2\)Cu\(_3\)O\(_7\)–\(y\), this characteristic length is obtain as \(a_0 \approx 97.8\)nm. In this case, the magnetic field penetration depth \(\Delta \lambda(T, B) = \lambda(T, B) - \lambda(0, 0)\) as a function of temperature at the doping concentration \(\delta = 0.09\) for the magnetic field \(B = 0\) T (solid line), \(B = 0.5\) T (dashed line), and \(B = 1.0\) T (dash-dotted line) is plotted in Fig. 1 in comparison with the corresponding experimental result \(^{13}\) of YBa\(_2\)Cu\(_4\)O\(_8\) (inset). The similar magnetic field dependence of the magnetic field penetration depth has been also observed experimentally on YBa\(_2\)Cu\(_3\)O\(_6\)–\(9.9\)%. However, YBa\(_2\)Cu\(_3\)O\(_7\)–\(δ\) contains a single CuO chain per unit cell whose oxygen content can be varied to tune the doping level on the CuO\(_2\) plane, while its close relative YBa\(_2\)Cu\(_4\)O\(_8\) has a double chain layer that is stoichiometric and a planar state that is underdoped. Although this difference of the electronic structure of the quasi-2D plane leads to some subtly different behaviors between YBa\(_2\)Cu\(_3\)O\(_7\)–\(δ\) and YBa\(_2\)Cu\(_4\)O\(_8\), where at a weak magnetic field, the experimental curves of the temperature dependent magnetic field penetration depth for YBa\(_2\)Cu\(_3\)O\(_6\)–\(9.9\) show curvature in low temperatures as in the case of zero magnetic field, while the corresponding experimental curves of the temperature dependent magnetic field penetration depth for YBa\(_2\)Cu\(_4\)O\(_8\) are linear, the qualitative properties in both YBa\(_2\)Cu\(_3\)O\(_7\)–\(δ\) and YBa\(_2\)Cu\(_4\)O\(_8\) are consistent each other. In this paper, we mainly focus on the qualitative properties of the magnetic field dependent Meissner effect in cuprate superconductors based on the simple \(t-J\) model \(^{1}\). Within the kinetic energy driven SC mechanism, the SC transition temperature \(T_c = 52\)K at \(\delta = 0.09\) for zero magnetic field. As we have shown that at the SC transition temperature \(T = T_c\), the kernel of the response function \(K_{\mu\nu}(q \rightarrow 0, 0, 0)|_{T=T_c} = 0\). In this case, we obtain the magnetic field penetration depth from Eq. (7) as \(\lambda(T_c, 0) = \infty\), which reflects that in the normal state, the external magnetic field can penetrate through the main body of the system, therefore there is no the Meissner effect in the normal state. Furthermore, our present result in Fig. 1 shows clearly that the characteristic feature of the temperature dependent \(\lambda(T, B)\) is essentially independent on a weak magnetic field, in particular, \(\lambda(T, B)\) shows a crossover from the linear temperature dependence at higher temperatures to a nonlinear one in the low temperatures as in the case of zero magnetic field, in qualitative agreement with the corresponding experimental data \(^{8}\) of YBa\(_2\)Cu\(_3\)O\(_6\)–\(9.9\). Moreover, the magnitude of \(\lambda(T, B)\) at the low temperatures is dependent on a weak magnetic field, and then it is independent on a weak magnetic field at higher temperatures. However, there is a substantial difference between theory and experiment, namely, the value of the magnetic field dependent penetration depth \(\Delta \lambda(T, B)\) at the low temperatures calculated theoretically is smaller than the corresponding value measured in the experiment. However, upon a closer examination one can see immediately that the main difference is due to fact that the calculated value of \(\Delta \lambda(T, B)\) increases slowly with magnetic field at the low temperatures. The simple \(t-J\) model can not be regarded as a complete model for the quantitative comparison with cuprate superconductors, however, as for a qualitative discussion in this paper, the overall shape seen in the theoretical result is qualitatively consistent with that observed in the experiment. To show this magnetic field dependence of \(\lambda(T, B)\) at the low temperatures clearly, we have made a series of calculations for \(\lambda(T, B)\) at differently weak magnetic fields, and the result of \(\Delta \lambda(T, B)\) as a function of magnetic field at \(\delta = 0.09\) with \(T = 4\)K is plotted in Fig. 2 in com-
comparison with the corresponding experimental result\textsuperscript{11} of YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{6.95} at temperature $T = 4.2$K (circles) and $T = 7$K (squares) taken from Ref.\textsuperscript{11}. Inset: the corresponding experimental data for YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{6.95} at temperature $T = 4.2$K (circles) and $T = 7$K (squares) taken from Ref.\textsuperscript{11}. In correspondence with the result of $\lambda(T, B)$ shown in Fig.\textsuperscript{1} the characteristic feature of the temperature dependent superfluid density is also independent on a weak magnetic field, where as in the case of zero magnetic field, $\rho_s(T, B)$ shows a linear temperature dependence at higher temperatures, and then it crosses over to a nonlinear temperature behavior at the low temperatures. However, most importantly, the magnitude of $\rho_s(T, B)$ at the low temperatures decreases with increasing magnetic field, and then it turns to be independent on a weak magnetic field at higher temperatures, in qualitative agreement with experimental data of cuprate superconductors\textsuperscript{10,12–14}. The present result also indicates that the nature of the quasiparticle excitations at the low temperatures is strongly influenced by a weak magnetic field. This weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors contrasts with the that observed from the conventional superconductors\textsuperscript{22}, where the curves of the temperature dependent superfluid density for differently weak magnetic fields were found to collapse onto a single curve since the conventional superconductors are fully gaped.

The weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors arises from both depairing due to the Pauli spin polarization and nonlocal response in the vicinity of the d-wave gap nodes on the Fermi surface to a weak magnetic field. In the framework of the kinetic energy driven SC mechanism, the d-wave SC state is mediated by the interaction of electrons and spin excitations\textsuperscript{22}, where the depairing can occur due to the Pauli spin polarization in the presence of an external magnetic field. This follows a fact that an applied magnetic field aligns the spins of the unpaired electrons, i.e., there is a tendency to induce the magnetic order, then the kinetic energy driven d-wave Cooper pairs can not take advantage of the lower energy offered by a spin-polarized state\textsuperscript{19,28}. On the other hand, the characteristic feature of the d-wave superconductors is the existence of four nodes on the Fermi surface, where the energy gap vanishes $\Delta_{\text{dd}}(k)|_{\text{at nodes}} = \Delta_{\text{dd}}(\cos k_x - \cos k_y)/2|_{\text{at nodes}} = 0$. These two special features in the kinetic energy driven d-wave SC state indicate that even small thermal energy or externally small magnetic energy can excite excitations, then the superfluid density decreases with increasing temperature or increasing magnetic field, reflecting that the weak magnetic field induced reduction of the low-temperature superfluid density is a natural consequence of the kinetic

Next we discuss the weak magnetic field induced reduc-
energy driven d-wave SC state. Since the quasiparticles selectively populate the nodal region at the low temperatures, then the most physical properties in the SC state are controlled by the quasiparticle excitations around the nodes. In this case, the Ginzburg–Landau ratio around the nodal region is no longer large enough for the system to belong to the class of type-II superconductors, and the condition of the local limit is not fulfilled\textsuperscript{17,18,29}. On contrary, the system falls into the extreme nonlocal limit, then the nonlinear characteristic in the temperature dependence of the superfluid density (then the magnetic field penetration depth) can be observed experimentally in cuprate superconductors at the low temperatures. However, when a weak magnetic field is applied to the system even at zero temperature, the quasiparticles around the nodal region become excited out of the condensate, and at the same time the electron attractive interaction for the Cooper pairs by exchanging spin excitations is weaken\textsuperscript{12}, both these effects lead to a decrease in the superfluid density. With increasing temperatures, the externally small magnetic energy due to the presence of a weak magnetic field is comparable with the small thermal energy at the low temperatures, therefore both small thermal energy and weak magnetic field induce an reduction of the superfluid density. However, at higher temperatures, this externally small magnetic energy is much smaller than the thermal energy, then the major contribution to a decrease of the superfluid density comes from the thermal energy. This is why a weak magnetic field only reduces an reduction of the superfluid density only at the low temperatures.

As we have mentioned in Eq. (1), the coupling of the electron charge to a weak magnetic field is in terms of the vector potential $A(l)$, while the coupling of the electron magnetic momentum $\mu B$ with the weak magnetic field $B = \text{rot} A$ is in terms of the Zeeman mechanism. During the above discussions, the electromagnetic response kernel $K(\omega)$ is calculated with the bare current vertex\textsuperscript{30}, where the important point is that for a weak magnetic field which orientation is the same at all spatial points the spin polarization axis along the weak magnetic field is chosen accordingly. As a consequence, we do not take into account longitudinal excitations properly\textsuperscript{3}, and the obtained results are valid only in the gauge, where the vector potential is purely transverse, e.g. in the Coulomb gauge. If we want to keep the theory gauge invariant, it is crucial to approximate the electromagnetic response kernel in a way maintaining local charge conservation\textsuperscript{2}. For the case without considering the coupling of the electron magnetic momentum with a weak magnetic field, we have shown that although the electromagnetic response kernel is not manifestly gauge invariant within the bare current vertex\textsuperscript{18}, the gauge invariance is kept within the dressed current vertex\textsuperscript{30}. However, for the present case with considering both couplings of the electron charge and electron magnetic momentum with a weak magnetic field, the gauge invariance should be kept within the dressed current vertex together with accordingly rotated spin polarization axis along the weak magnetic field. These and the related issues are under investigation now.

In conclusion, we have discussed the magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors based on the kinetic energy driven SC mechanism by considering both couplings of the electron charge and electron magnetic momentum with a weak magnetic field. Our results show that although the characteristic feature of the temperature dependent superfluid density is found to be independent on a weak magnetic field, this weak magnetic field induces an reduction of the low-temperature superfluid density in the Meissner state. Our results also show that the striking behavior of the weak magnetic field induced reduction of the low-temperature superfluid density can be attributed to both depairing due to the Pauli spin polarization and nonlocal response in the vicinity of the d-wave gap nodes on the Fermi surface to a weak magnetic field.

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\textsuperscript{*} To whom correspondence should be addressed, E-mail: spfeng@bnu.edu.cn

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