On the Velocity of Light Signals in the Deep Underwater Neutrino Experiments

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Abstract

During the last few years deep underwater neutrino telescopes of a new generation with dimensions close to 100 m or more were taken into operation. For the correct track reconstruction of events and for the interpretation of light pulses from calibration lasers one has to use the group velocity for light signals. The difference between group velocity and the traditionally used phase velocity leads to an additional delay of about 10 ns for a distance of 100 m between light source and photomultiplier. From the time of the appearance of the first projects of deep underwater neutrino telescopes in the middle of 70th this fact was never mentioned in the literature.

During the last few years the new large-scale neutrino telescopes NT-200/1/ and AMANDA/2/ have been taken into operation. Two deep underwater arrays, NESTOR/3/ and ANTARES/4/, are under construction.

The Cherenkov light from high energy muons, electromagnetic and hadronic showers can be recorded at distances of 20 – 100 m depending upon the light absorption of water or ice. Impulse laser light sources are widely used in these arrays for calibration. Light pulses from such sources can be recorded over even larger distances.

At such distances, the difference between group and phase velocity of light in water or ice is essential. As far as I know, this fact hasn’t been mentioned in the literature from the time of the first neutrino telescope projects in the middle of the seventies/5,6/ and hasn’t been taken into account for the data analysis and MC calculations.

The velocity of light pulses in matter is given by the group velocity/7,8/:

\[ V_{gr} = d\omega/dk \]

(1)

where \( \omega \) is the frequency, \( k \) is the wave vector.

The connection between \( \omega \) and \( k \) can be written in the following form (dispersion equation):

\[ \omega = c \cdot k/n(\omega) \]

(2)

where \( c \) is the velocity of light in vacuum, \( n \) is the refraction index.

As it is known, refraction indices given in handbooks like/9/ correspond to the phase velocity due to the method they are measured/10/:

\[ n(\omega) = c/V_{ph}(\omega) \]

(3)

where \( V_{ph} \) is the phase velocity.
It is reasonable to accept a group refraction index for the group velocity similar to the phase index of refraction:

$$n_{gr}(\omega) = c/V_{gr}(\omega)$$

(4)

where $V_{gr}$ is the group velocity.

Using (2) and replacing the $\omega$ dependence of $n$ by a wavelength dependence, the relation between $n_{gr}$ and $n$ can be derived:

$$n_{gr}(\lambda) = n(\lambda) - \lambda dn/d\lambda$$

(5)

where $\lambda$ is the wavelength.

$dn/d\lambda$ is negative for water over the visible light range, and so $n_{gr}$ is larger than $n$. Fig.1 shows the wavelength dependence of $n$ (curve 1) for distilled water at 20°C /9/, and the same for $n_{gr}$ (curve 2).

The density of water influences the refraction index at large depths of neutrino telescopes. But this correction is rather small due to the comparatively small compression coefficient of water. For the conditions of NT-200 (depth 1100 m, water temperature 4°C) the variation of the refraction index is about 0.002 only. This is more than 10 times smaller than the difference between $n_{gr}$ and $n$. This correction of $n$ would reach 0.01 for the arrays to be placed in the ocean depths of 4 - 5 km, but even in this case the depth correction is significantly smaller than that due to the difference between group and phase velocities. The curve 3 on fig.1 shows the dependence of $n_{gr}$ on $\lambda$ for the conditions of the telescope NT-200.

All curves at fig.1 were calculated for distilled water. The light absorption of natural water differs from that of distilled water, and so there is a difference in the image part of their dielectric penetration. As Kramers-Kroning equation connects the real part of the dielectric
penetration with its imaginary part /7/, the real refraction index differs, in principle, from that of distilled water. Such a correction would be non-negligible only if there are the intensive lines of absorption in the visible light absorption spectrum.

Fig. 2 shows the dependencies of delays on the distance between light source and receiver which emerge from the difference between the phase and group velocities for different wavelengths.

For arrays with angular resolution 1-2° replacement of the phase velocity by the group velocity probably would not lead to essential changes in track reconstruction. For the projects which claim an angular accuracy 0.1-0.2° and absorption length of ≥ 50 m/4/ the use of group velocity in track reconstruction procedures seems to be absolutely necessary.

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