THE ROLE OF SOLVABLE GROUPS
IN QUANTIZATION OF LIE ALGEBRAS

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(To be published in Zapiski Nauchn. Semin. POMI, V. 209)

Abstract.

The elements of the wide class of quantum universal enveloping algebras are proved to be Hopf algebras $H$ with spectrum $Q(H)$ in the category of groups. Such quantum algebras are quantum groups for simply connected solvable Lie groups $P(H)$. This provides utilities for a new algorithm of constructing quantum algebras especially useful for nonsemisimple ones. The quantization procedure can be carried out over an arbitrary field. The properties of the algorithm are demonstrated on examples.

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1 Supported by Russian Foundation for Fundamental Research, Grant N 94-01-01157-a.
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Algebraic approach to quantization of Lie groups \[1, 2\] implies the following sequence of constructions

\[ G \implies \text{Fun}(G) \implies U(A) \implies U_q(A) \implies (U_q(A))^* \implies G_q \]  

(1)

Here \( U(A) \) is the universal enveloping algebra for Lie algebra \( A \) of group \( G \), \((U_q(A))^*\) – the Hopf algebra dual to \( U_q(A) \). The quantum group \( G_q \) is understood as the spectrum of the Hopf algebra \((U_q(A))^*\), i.e. it corresponds to \((U_q(A))^*\) just as the group \( G \) to the algebra of functions \( \text{Fun}(G) \). In the sequence (1) the main procedure is the construction of the quantum algebra \( U(A) \implies U_q(A) \). It was stressed in \[3\] that the main object in the quantization procedure is the quantum algebra of functions \( \text{Fun}_q(G) \approx (U_q(A))^* \), moreover this interpretation is also valid for \( U_q(A) \) (but with respect to the dual group). At the same time in general situation the spectrum of a noncommutative Hopf algebra does not exist and the pair \( \text{Fun}_q(G) \implies G_q \) must be understood as unique entity.

The problem of equivalence between the categories \( U_q(A) \) and \((U_q(A))^*\) was first mentioned in \[1\], where the enveloping algebra \( U_q(A) \) was incorporated in the family of Hopf algebras having the commutative classical limit. The quantum duality principle \[4\] allows to elucidate the nature of this equivalence. If one considers the Lie bialgebra \((A, A^*)\) as the starting object \[1\] then the quantization is the simultaneous deformation of \( \text{Fun}(G) \) and \( \text{Fun}(G^*) \), where \( G, G^* \) are the Lie groups with algebras \( A \) and \( A^* \). So \( \text{Fun}_q(G^*) \) (as an algebra dual to \( \text{Fun}_q(G) \approx (U_q(A))^* \)) is equivalent to the Hopf algebra \( U_q(A) \) thus being not only a quantum algebra but also a quantum group.

In this paper the scheme is proposed for the explicit realization of \( U_q(A) \) as a quantum group. It is shown that the wide class of quantum universal enveloping algebras are factor algebras of such noncommutative Hopf algebras \( H \) that the spectrum \( Q(H) \) does exist. The ’classical limit’ of \( Q(H) \) is a simply connected solvable group \( P(H) – \) the factor group of \( G^* \). The established correspondence between \( U_q(A) \) and solvable Lie groups \( P(H) \) provides new possibilities to construct quantum Lie algebras.

In sect.1 the properties of the selected class of Hopf algebras \( U_q(A) \) are formulated. For each \( U_q(A) \) the simply connected solvable Lie group \( P(H) \) strictly corresponds. In sect.2 the inverse problem is solved – for a given group \( P \) the Hopf algebra \( H \) is obtained that can be factorized to the quantum universal enveloping algebra \( U_q(A) \). In sect.3 the exposed scheme is demonstrated on some known constructions and an example of application of the new quantization method is given.

For the elements of tensor products of Hopf algebras the abbreviated notation will be used: \( a' \equiv a \otimes 1; \quad a'' \equiv 1 \otimes a \).
1.
Consider the quantum universal enveloping algebra \( U_q(A) \) of an algebra \( A \) over an arbitrary field \( K \) with generators \( \{x_l\}, \ l = 1, \ldots, n \) and a set of quantization parameters \( q \). Set the following conditions

**u.1)** for generators \( \{x_l\} \) the tensor multipliers \( \Delta'_j \) and \( \Delta''_j \) in the coproduct

\[
\Delta(x_l) = \sum_j \Delta'_j(x_l) \Delta''_j(x_l)
\]

are either linear functions of generators, or convergent power series in \( \{h_i\} \). The elements of \( \{h_i\}, \ i = 1, \ldots, m < n \), commute.

**u.2)** in the coproduct

\[
\Delta(h_i) = \sum_j \Delta'_j(h_i) \Delta''_j(h_i)
\]

\( \Delta' \) and \( \Delta'' \) transfer \( h_i \) to the subalgebra \( U_q(h_i) \) generated by \( \{h_i, 1\} \).

**u.3)** \( \varepsilon(x_k) = 0 \).

**u.4)** the relations \( (\cdot)(S \otimes \text{id})\Delta = (\cdot)(\text{id} \otimes S)\Delta = \eta \circ \varepsilon \), applied to generators \( \{x_l\} \), can be solved (when subalgebra \( U_q(h_i) \) is commutative) to define all the \( S(x_l) \), whatever the other multiplications in \( U_q(A) \) are.

**u.5)** \( \lim_{q \to 0} \Delta(x_l) = x'_l + x''_l \).

It is known \( \exists \) that for each pair \((X, R)\) of Hopf algebra \( X \) and commutative unital algebra \( R \) the coproduct \( \Delta_X \), counit \( \varepsilon_X \) and antipode \( S_X \) induce the group structure on the set of algebraic morphisms \( \text{Hom}(X, R) \) with the multiplication

\[
(\chi_1 \star \chi_2)(x) = (\cdot)_R(\chi_1 \otimes \chi_2)\Delta_X(x); \quad x \in X; \quad \chi_1, \chi_2 \in \text{Hom}(X, R).
\]  \hspace{1cm} (2)

When \( R \) is noncommutative the previous statement fails, the map \( \chi \circ S_X \) having no more the property of inverse element. The fact is that \( S_X \) being an antihomomorphism forms the composition \( \chi \circ S_X \) that does not belong to \( \text{Hom}(X, R) \).

We shall demonstrate that for algebras \( U_q(A) \) (with the properties u.1 - u.5) this obstacle can be overcome. Let us consider \( U_q(A) \) together with such an associative algebra \( H \) (with the same set of generators) that

**h.1)** the operators \( \Delta_H, S_H, \varepsilon_H \) and \( \eta_H \) on the generators coincide with the corresponding defining compositions in \( U_q(A) \),

**h.2)** the subalgebra \( H^{(h)} \), generated by \( \{h_i, 1\} \), is equivalent to \( U_q^{(h)} \),

**h.3)** the algebra \( H \) is free modulo the relations of commutativity of \( H^{(h)} \),

**h.4)** the operators \( \Delta \) and \( \varepsilon \) are extended to \( H \) homomorphically and the antipode \( S - \text{antihomorphically} \).
These properties guarantee that $H$ is a Hopf algebra.

Let $V$ and $V^{(h)}$ be the subspaces of the vector space of $H$ – the corresponding lineals of $\{x_i\}$ and $\{h_i\}$. Consider a free associative algebra $L$ and the space of morphisms $\text{Mor}(V, L)$. In $\text{Mor}(V, L)$ define the subset $\text{Mor}^{(h)}$ such that its elements send the space $(V^{(h)})$ to the fixed commutative subalgebra in $L$. The set $\text{Mor}^{(h)}$ is obviously a vector space. Each $\zeta \in \text{Mor}^{(h)}$ is fixed by $n$ coordinates $\zeta(x_i)$. Let $\zeta^\dagger H$ be the homomorphic extension of $\zeta$ to $H$. Such extensions always exist but do not constitute the vector space anymore.

The multiplication on $\text{Mor}^{(h)}$ will be introduced similarly to (2):

$$\zeta_1 * \zeta_2 = (\cdot)_L(\zeta_1^\dagger H \otimes \zeta_2^\dagger H)\Delta. \quad (3)$$

For each $\zeta \in \text{Mor}^{(h)}$ the inverse will be given by

$$\zeta^{-1} \equiv \zeta^\dagger H \circ S. \quad (4)$$

Note that according to the definition of $\zeta$ the antipode $S$ in $\zeta^{-1}$ acts only on the linear combinations of the generators. From u.2 and u.4 it follows that $\zeta^{-1} \in \text{Mor}^{(h)}$ for any $\zeta \in \text{Mor}^{(h)}$. The map

$$\zeta(0) \equiv \eta_L \circ \varepsilon_H \quad (5)$$

is the zero vector in the space $\text{Mor}^{(h)}$ (see the property u.3).

Let $G$ be the Lie group with the algebra $A$. Denote by $Q(H)$ the space $\text{Mor}^{(h)}$ with the multiplication (3), the inversion (4) and the marked element (5).

**Proposition 1.** $Q(H)$ is a group. The 1-dimensional representation $d$ of $L$ transforms the group $Q(H)$ into the vector solvable Lie group $P(H)$ on the $n$-dimensional vector space. Groups $P(H)$ and $G^*$ are equivalent if and only if $\dim G^* = n$.

**Proof.** Consider the product

$$\zeta * \zeta^{-1} = (\cdot)_L(\zeta_1^\dagger H \otimes \zeta_2^\dagger H \otimes S^\dagger H)\Delta = (\cdot)_L(\zeta_1^\dagger H \otimes \zeta_2^\dagger H)(\text{id} \otimes S^\dagger H)\Delta = \zeta_1^\dagger H(\cdot)_H(\text{id} \otimes S^\dagger H)\Delta.$$

Note that $S^\dagger H$, used here according to the definition (3), is not the antipode of $H$. This operator coincides with $S_H$ on generators and is homomorphically extended to $H$. Nevertheless the properties u.1 and u.2 guarantee that the operator $S^\dagger H$ in the multiplication of $Q(H)$ acts either on $V$, or on power series in $U^{(h)}$. In these situations $S^\dagger H$ coincides with $S$ and the last equality can be continued:

$$\zeta * \zeta^{-1} = \zeta_1^\dagger H(\cdot)_H(\text{id} \otimes S)\Delta = \zeta_1^\dagger H\eta_H\varepsilon_H \equiv \eta_L\varepsilon_H = \zeta(0) \quad (6)$$
The properties of $\zeta(0)$ (as the unit of $Q(H)$) and the associativity of multiplication are verified similarly.

The representation $d : L \to R$ maps $L$ in the abelian algebra $R$ and induces the transformation of the space $\text{Mor}^{(h)}$ to $\text{Mor}(V, R)$. The elements

$$d \circ \zeta \in \text{Mor}(V, R)$$

are fixed by the coordinates $\{(d \circ \zeta)(x_k)\}$. Given the properties u.1 - u.3 the coordinates of the product are the analytical functions of the coordinates of factors. Thus $P(H)$ is the $n$-dimensional vector space with an analytic group multiplication law. Due to Levy theorem the vector Lie group is a solvable group with trivial tori.

**Corollary** The group $P(H)$ can be presented as a sequence of semidirect products of vector spaces (as abelian additive groups).

The obtained sequence

$$U_q(A) \Rightarrow H \Rightarrow Q(H) \Rightarrow P(H) \tag{7}$$

is unique and can serve for classification of quantum algebras $U_q(A)$. The group $Q(H)$ in (7) is the group with noncommuting coordinates. It is the quantum analogue of the group $P(H)$. The properties of the coordinate algebra $L$ are defined by the multiplication in $H$. Abelian subalgebra in $L$ is fixed by the images $\zeta(H^{(h)})$, all other multiplications in $L$ are free. For the algebra $H$ the existence of groups $Q(H)$ and $P(H)$ means that $H$ retains the properties of Hopf algebra when its multiplication is changed for abelian one while the coalgebra structure is conserved. Such algebra $H^c$ is evidently an algebra of functions on the dual group $P(H)$. $H$ itself is an algebra of noncommutative functions on $P(H)$. The Hopf algebra $U_q(A)$ can be treated as $\text{Fun}_q(G^*)$, as $\text{Fun}_q(P(H))$ and also as $\text{Fun}_q(Q(H))$. To realize the last variant one must consider the quantum commutation relations as a deformation of almost free multiplication (strictly speaking we have here the coboundary deformation, i.e. the contraction [3]).

2. Let us turn to the problem of construction of $U_q(A)$-type algebra starting with the solvable Lie group. Fix the global coordinate system on the simply connected solvable Lie group $P$. Let $Q$ be the solvable group with coordinates in an associative algebra $L$. Suppose that after the change of scalars $L \to L/L^{(1)}$ the group $Q$ becomes equivalent to $P$. (Here $L^{(1)}$ is the first derivative of $L$.) The multiplication on $L$ can have additional restrictions when the
standard group axioms on $Q$ are imposed. Consider for example

$$Q = \mathbb{R}^p \triangleright \mathbb{R}^q,$$

where the subgroups $\mathbb{R}^p$ and $\mathbb{R}^q$ are the abelian additive and the multiplication is goverened by the homomorphism $\Phi : \mathbb{R}^p \to \text{Aut}(\mathbb{R}^q),$

$$(a', b')(a'', b'') = (a' + a'', \Phi(a'')b' + b'').$$

Suppose the coordinates $\{a_s, b_t\}$ belong to $L$. Then the associativity and the inverse element properties imply the commutativity of coordinates $\{a_s\}$. We shall not discuss here the conditions under which the noncommutative algebra $L$ exists realizing the transformation of $P$ into $Q$. Suppose $L$ is such an algebra (with the necessary multiplication properties). Suppose also that the coordinate system of $Q$ is correlated with the analitic coordinates on $P$. Construct the coalgebra $H$, generated by the coordinate functions $\Psi_i$ on $Q$:  

$$(\Delta \Psi_i)(\phi' \otimes \phi'') = \Psi_i(\phi' \cdot \phi''),$$

$$\varepsilon(\Psi_i) = \Psi_i(e_Q) = 0, \quad \eta(1) = 1_H,$$

where $\phi', \phi'' \in Q$. The number of generators in $H$ is equal to $\text{dim} P$. Define the multiplication in $H$,  

$$(\Psi_i \cdot \Psi_j)(\phi) = \Psi_i(\phi)\Psi_j(\phi)$$

and the antipode

$$(S\Psi_i)(\phi) = \Psi_i(\phi^{-1}).$$

The direct verification of Hopf algebra axioms proves the validity of the following statement.

**Proposition 2.** $H$ is a Hopf algebra if the operations $\Delta$ and $\varepsilon$ are extended to $H$ by homomorphisms and $S$ – by antihomomorphisms.

**Note.** The properties (9) and (10) are true for an arbitrary element of $H$, while the antipode of a policoordinate function is not equal to the function of inverse element.

We have constructed such a noncommutative and (in general) noncocommutative Hopf algebra $H$, that $Q$ is its quantum group. Only on the subset of generators of $H$ the multiplication defined by (11) is free.

We are interested in the ideals of $H$ which guarantee that $H/J(H) = U_q(A)$. The structure of $J(H)$ must be in agreement with operations (9-12) and the multiplication (11).

$$[\Psi_i, \Psi_j] = \Phi_{ij}, \quad \Phi_{ij} \in H$$

(13)
The Jacoby identity and the correlation conditions

\[ \Delta \Phi_{ij} = [\Delta \Psi_i, \Delta \Psi_j], \quad S \Phi_{ij} = -[S \Psi_i, S \Psi_j] \]  

(14)

give the system of equations for the deforming functions \( \Phi_{ij}(\Psi_l) \). Every solution of this system provides \( H \) the properties of quantum universal enveloping algebra \( U_q \).

To find the algebra \( U(A) \) corresponding to the classical limit of \( U_q \) let us contract the group \( P \) to the Abelian vector group. This means that \( P \) is included in the \( q \)-parametric set of deformations of Abelian group \( \text{lim}(P_q) \) treated as additive. For the groups \( Q_q \) and the algebras \( H_q \) and \( U_q \) the same parametrisation is induced. If the limit \( \text{lim} U_q \equiv U(A) \) exists the algebra \( U_q \)(the quantum analogue of the group \( P \)) can also be considered as a quantum universal enveloping algebra of \( A \). Note that the dimension of \( A \) may be infinite.

Such an approach treats the quantization of a Lie algebra \( A \) as a deformation of an Abelian vector group to a solvable one.

It must be pointed out that in this approach one can pay no attention to simplicity or semisimplicity of \( A \), as well as to the properties of its main field. Thus one gets the possibility to construct quantum analogues of nonsemisimple Lie algebras and to obtain directly the real quantum algebras.

3.

3.1 For the standard quantization of simple complex Lie algebras [1, 2] the conditions u.1 - u.5 are valid. Here the solvable group \( P(H) \) can be treated as a group of upper triangular matrices

\[
\begin{pmatrix}
 e^{1/2(q_1h_1)} & 0 & \cdots & 0 & 0 & e^{1/4(q_1h_1)x_1^+} \\
0 & e^{1/2(q_1h_1)} & \cdots & 0 & 0 & e^{1/4(q_1h_1)x_1^-} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & e^{1/2(q_rh_r)} & 0 & e^{1/4(q_rh_r)x_r^+} \\
0 & 0 & \cdots & 0 & e^{1/2(q_rh_r)} & e^{1/4(q_rh_r)x_r^-} \\
0 & 0 & \cdots & 0 & 0 & 1
\end{pmatrix}
\]

(15)

with coordinates \( \{h_i, x_i^\pm\} \), and \( r \) equal to the rank of \( A \). Let \( L_0 \) be a free associative algebra with generators \( \{h_i, x_i^\pm, 1\} \). Factorize it by the ideal, generated by the commutators of \( \{h_i\} \). The group structure is preserved if the coordinates of the matrix group (15) belong to the factor algebra of \( L \). We get the group \( Q(H) \). The auxiliary Hopf algebra \( H \) is a factor of the free algebra generated by coordinate functions \( \{H_i, X_i^\pm, 1\} \) on \( Q(H) \) with operations \((\cdot), \Delta, S, \varepsilon \) and \( \eta \) given by (9 - 12). The ideal \( J(H) \) is generated by the relations
$H_i H_j = H_j H_i$. The final result – the quantum algebra $U_q(A)$ – can be obtained when the correlation equations are solved and the deformations of $U(A)$ are in agreement with the operations (9) and (12). Algebra $U_q(A)$ is thus realized as a quantum group $\text{Fun}_q(P(H))$. In the only case when the number of generators in $U_q(A)$ is equal to the dimension of $A$, the group $P(H)$ is equivalent to the dual group $G^*$. 

3.2 In [7] the solvable groups of the type (8) were used to construct quantum algebras $U_q(A)$. In the matrix form the vector group $P(H)$ that gives the Hopf algebra $H$ can be written as

$$
\begin{pmatrix}
  e^{\gamma_i h_i} & e^{-\beta_i h_i x} \\
  \vdots & \vdots \\
  0 & \cdots & 0 \\
  1 & \cdots & 1
\end{pmatrix}
$$

(16)

Here $x$ is an $m$-dimensional vector, $\{\gamma^i, \beta^i\}$ is the set of commuting $m \times m$-matrices, $i = 1, \ldots, u; u \leq m$. Parameters $\{h_i\}$ are the coordinates of the abelian matrix group $e^{(\gamma_i h_i)}$. Let the matrices (16) have the noncommutative coordinates belonging to the associative algebra $L$ with generators $\{h_i, x^i, 1\}$ and relations $h_i h_j = h_j h_i$. The coordinate functions $\{H_i, X_j, 1\}$ generate algebra $H$. Its multiplication properties are analogous to those of $L$ while the coproduct has the form

$$(\Delta X)_j = (e^{\alpha^i H_i' X'' + \beta^i H_i'' X'''}), \quad \Delta H_i = H_i' + H_i'', \quad \alpha^i \equiv \beta^i + \gamma^i.$$ 

It was shown in [7] that in virtue of such properties $H$ is a Hopf algebra. It follows from the Proposition 1 that the group structure is conserved when the numeric coordinates in $P(H)$ are substituted by the noncommutative ones. In other words there exists the group $Q(H)$ that is the spectrum of the algebra $H$. With the help of $Q(H)$ and $H$ the quantum analogues of nonsemisimple real algebras where constructed in [7] for Heisenberg and 2-dimensional flat motions algebra.

3.3 Let $P$ be the nilpotent matrix group:

$$
\begin{pmatrix}
  1 & x_1 & y \\
  0 & 1 & x_2 \\
  0 & 0 & 1
\end{pmatrix}
$$

(17)

its coordinates $x_i, y$ can belong to an arbitrary associative algebra. Consider $L$ to be freely generated by $\{x_i, y, 1\}$. The group $Q$ is thus defined. The free associative algebra $H$ with the generators $\{X_i, Y, 1\}$ (dual to $\{x_i, y, 1\}$) will be supplied by the coproduct

$$
\Delta X_i = X_i' + X_i'', \quad \Delta Y = Y' + Y'' + X_1' X_2'',
$$

(18)

7
the antipode
\[ SX_i = -X_i, \quad SY = -Y + X_1X_2, \] (19)
and the counit
\[ \varepsilon(X_i) = \varepsilon(Y) = 0. \] (20)

It is evident that the commutative Hopf algebra \( H^c \) with the properties (18 - 20) also exists. Thus we shall search for a deformation \( U_q \) of \( (H^c) \):
\[ [X_i, Y] = \Phi_i, \quad [X_1, X_2] = \Phi_{12}. \]

Equations (14) are easily solved. The verification of the Jacoby identities completes the construction of \( U_q \). It is defined by formulas (18 - 20) and the Lie composition
\[ [X_1, X_2] = X_1 + X_2, \quad [X_1, Y] = Y + (1/2)X_1X_1, \quad [X_2, Y] = -Y - (1/2)X_2X_2, \]

One can easily obtain the continuous family of groups \( P(H; q) \) with the coordinates \( \{x_i, y\} \) and the multiplication
\[ (x_i', y')(x''_i, y'') = (x_i' + x''_i, y' + y'' + qx_1'x_2''). \]

This leads to the corresponding family of algebras \( U_q \):
\[ \Delta X_i = X'_i + X''_i, \quad [X_1, X_2] = X_1 + X_2, \]
\[ \Delta Y = Y' + Y'' + qX'_1X''_2, \quad [X_1, Y] = Y + (q/2)X_1X_1, \]
\[ S(X_i) = -X_i, \quad [X_2, Y] = -Y - (q/2)X_2X_2, \]
\[ S(Y) = -Y + qX_1X_2, \quad \varepsilon(X_i) = \varepsilon(Y) = 0. \]

In the limit \( q \to 0 \) the family \( U_q \) tends to the universal enveloping algebra \( U(A) \), where \( A \) is defined by the compositions
\[ [X_1, X_1 + X_2] = X_1 + X_2, \quad [X_1, Y] = Y, \quad [X_1 + X_2, Y] = 0. \]

This proves the Hopf algebra \( U_q \) to be the quantization of the Lie algebra \( A \). At the same time \( U_q \) is realised as the quantum group for the group \( P \): \( U_q \cong \text{Fun}_q(P) \). Let \( G \) be the simply connected Lie group with algebra \( A \). The Lie algebra \( A^* \) of the group \( P \) forms together with \( A \) the Lie bialgebra. Thus the group \( P \) is equivalent to the group \( G^* \) dual to \( G \).

In the general case quantum universal enveloping algebra \( U_q \) of the type described here can be a quantum group \( \text{Fun}_q \) not only for the dual group \( G^* \) but also for its factor group \( P \). The group \( P \) is the minimal factor group in \( G^* \) that totally defines the algebra \( U_q \).

The author is grateful to P.P.Kulish for numerous stimulating discussions.
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