Locally Lorentz-Covariant Theory of Gravity Founded on Inertial Frame of Center of Mass

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Abstract: A Locally Lorentz-covariant theory of gravity which is equivalent to general relativity in weak gravitational field is present. The space-time standard in local gravitational field is modified in terms of equivalence principle to keep consistent with that of inertial frame. The static metric in our theory agrees with Schwarzschild metric at first order approximation. The gravitational vector potential produced by a moving body is obtained by applying local Lorentz transformations in gravitational field. On the other hand, we still regard inertial frame, i.e. center of mass of the system, as the preferred frame of reference. This is because Newton’s laws of motion hold only for inertial frames. The apsidal motion of binary system and the expansion of the universe can be explained reasonably when observed from inertial frame. Black holes and singularities do not exist in our theory.

Keywords: Equivalence principle; Lorentz-covariant; General relativity; Inertial frame of reference
PACS: 04.20.Cv, 04.25.–g

1 Introduction

Einstein’s general relativity is a generally covariant theory of gravity. While successfully explaining some phenomena that cannot be explained in Newtonian theory, it still remains some puzzles, such as black holes and singularities [1-3]. We know that the combination of special relativity and Coulomb’s law can obtain the electromagnetic force produced by a moving charge [4]. Based on the similarity between the law of universal gravitation and Coulomb’s law, we wish to deal with the law of universal gravitation in the same way. But we find this method fails for the prescription of gravitational force. This is due to the different influences of electromagnetic and gravitational fields on the light. Photons have no electric charge; the electromagnetic force does not change the frequency and wavelength of light. Thus we can establish synchronous clocks and unified rulers anywhere in space. But we cannot do so in the presence of gravitational field, because the effective masses of photons will change. In the following we propose a simple theory which is equivalent to general relativity in weak field by modifying the local space-time standard.

2 The metric in static gravitational field

2.1 The mass variation of a body moving in static gravitational field

We know that Coulomb’s law applies to the interaction between rest charges, but can be extended to the interaction of a rest charge on a moving charge. Similarly, we suppose Newton’s law of universal gravitation also holds for the interaction of a rest gravitational source on a moving body.

We suppose, for simplicity, that a point mass $M$ is fixed at point o, as shown in Fig. 1, and a body with the gravitational mass of $m$ is falling freely in the static gravitational field produced by $M$. As it falls, the work done on it by the gravitational field will be converted into its kinetic energy, and its inertial mass increases. Then according to the principle of equivalence, its gravitational mass increases and it will experience a larger gravitational force.
Fig. 1 A body falls freely in the static gravitational field.

Suppose that the body \( m \) moves a differential distance of \( dr \), and the variation of mass is \( dm \). Based on the relation between work and energy, we have

\[
dmc^2 = -\frac{GMm}{r^2} dr, \quad (1)
\]

\[
dm = -\frac{GM}{r^2c^2} dr. \quad (2)
\]

Suppose the rest mass of the body at infinity is \( m_0 \). Integrating on both sides of the equation, we find

\[
m = m_0 e^{-\frac{GM}{rc^2}}. \quad (3)
\]

Eq. (3) is the formula of mass variation for a body moving in static gravitational field. The reason we emphasize static gravitational field is that we regard \( M \) as a rest inertial frame and \( m \) a test body. In fact, \( M \) will also be attracted by \( m \). So \( M \) is actually not an inertial frame. Eq. (3) holds only for the situation of \( M \gg m \). In this case, the acceleration of \( M \) is negligible compared with that of \( m \) and we may think \( M \) is at rest. If \( m \) and \( M \) are comparable, we must observe the motions of the two bodies from the center of mass of the system, which is an inertial reference frame. The detailed discussion may see section 4.

The effective mass of a photon is \( \frac{\hbar\omega}{c^2} \). Suppose the frequency of light at infinity is \( \omega_0 \). As it travels in the gravitational field, according to Eq. (3), its frequency will be \( \omega = \omega_0 e^{\frac{GM}{rc^2}} \). Likewise, if the frequency of light at the surface of a star with the radius \( R \) is \( \omega_0 \), it will become \( \omega = \omega_0 e^{-\frac{GM}{Rc^2}} \approx \omega_0 (1 - \frac{GM}{Rc^2}) \) as it propagates to infinity, which is the formula of gravitational redshift.

The expression of gravitational potential is \( -\frac{GM}{r} \) in Newtonian mechanics. Now we recalculate it. By definition, the potential of a body equals to the negative value of the work done by the gravitational field as it falls from infinity. Then we have

\[
E_p = -\int_0^\infty \frac{GMm}{r^2} dr = \int_0^\infty \frac{GMm_0}{r^2} e^{-\frac{GM}{rc^2}} dr = m_0c^2 \left(1 - e^{-\frac{GM}{rc^2}}\right). \quad (4)
\]

Thus the static gravitational potential is \( \phi = c^2 \left(1 - e^{-\frac{GM}{rc^2}}\right) \).

### 2.2 The effects of time dilation and length contraction in static gravitational field

As light travels in the gravitational field, it will be influenced by the gravitational field, as we have shown above. Thus we cannot define synchronous clocks as in inertial frame. As pointed out in [1], the time dilation effect cannot be observed by merely measuring the time interval of the ticks of a clock and comparing it with the time standard defined by the maker, because the gravitational influence on the time standard is equal to that on the clock. That is to say, if a clock reads one second for a physical process without gravitation, it will still read one second in the gravitational field, for the gravitational field has the same influence on the clock and the process.
In order to synchronize the clocks at different positions in the gravitational field, we compare the rest clock in the gravitational field with the clock in inertial frame. The gravitational force at infinity is zero and here is an inertial frame of reference. Suppose a sequence of oscillating wave propagates from infinity to \( r \), whose frequency is \( \omega_0 \) and the time interval between two adjacent wave crests is \( \Delta t = \frac{2\pi}{\omega_0} \) at infinity. Then the interval is still \( \Delta t \) as measured by the rest clock in the gravitational field, as stated above. But compared with the frequency at infinity, the local frequency of the light now becomes \( \omega = \omega_0 e^{-GM/ rc^2} \). Thus the local time interval between two adjacent wave crests is \( \Delta t e^{-GM/ rc^2} \) when observed form inertial frame of reference. Now that the time interval measured by the local clock is \( \Delta t \), the local clock slows down. In order to be consistent with the time standard in the inertial frame, we must multiply the local time interval by a factor of \( e^{-GM/ rc^2} \).

We then turn to the length contraction effect. Suppose there are a standard ruler and a pole. When there is no gravitational field, the length of the pole measured by the ruler is one meter. Then if we place the pole and the ruler along the radial direction of the gravitational field, the length that the ruler measures is still one meter, i.e. the ruler and the pole experience the same contraction. In order to see the length contraction effect, we suppose the wavelength of light at infinity is \( \lambda_0 \) and the distance it travels within unit time is \( n\lambda_0 \). As it propagates to \( r \), there are still \( n \) waves within unit time (due to the fact that gravitational field has the same effect on process and space-time). Now that the local frequency increases, the local wavelength will become \( \lambda_0 e^{-GM/ rc^2} \) when observed from inertial frame, and the distance that light travels within unit time is \( n\lambda_0 e^{-GM/ rc^2} \). Length is defined to be the distance that light travels within a given time interval. Thus compared with the length in inertial frame, the local length contracts. In order to be consistent with the length standard in inertial frame, we should multiply the local length in the radial direction of the gravitational field by a factor of \( e^{-GM/ rc^2} \).

### 2.3 The metric in static gravitational field

As stated above, there are no synchronous clocks and unified rulers in gravitational field due to the time dilation and length contraction effects. In order to establish unified space-time standard in the gravitational field, we transform the local time interval and length to the corresponding quantities in inertial frame. Then we must multiply the local time interval by a factor of \( e^{-GM/ rc^2} \) and the local radial length a factor of \( e^{GM/ rc^2} \). By doing so, the infinitesimal interval will be equal anywhere in the gravitational field, just as the instance in inertial frame, and the metric in static spherically symmetric gravitational field can be written as

\[
ds^2 = c^2 e^{-\frac{2GM}{rc^2}} dt^2 - e^{-\frac{2GM}{rc^2}} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2,
\]

which becomes Schwarzschild metric at first order approximation

\[
ds^2 = c^2(1 - \frac{2GM}{rc^2}) dt^2 - (1 - \frac{2GM}{rc^2})^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2.
\]

The motion equations in static gravitational field can be obtained by solving the following equation

\[
\delta \int ds = 0.
\]

For the motion of light, we have another equation

\[
ds = 0.
\]
In weak gravitational field, our metric is the same as Schwarzschild metric. Thus for the classical tests of gravity theory, such as the perihelion advance of the planets, the light deflection, the delay of radar echo, the results in our theory will agree with those of general relativity.

3 The gravitational vector potential produced by a moving body

3.1 The gravitational vector potential produced by a moving body

We know that a rest charge produces only electrostatic field, while a moving charge produces simultaneously electric and magnetic fields. Let’s first see how to derive magnetic force from Coulomb’s law and special relativity [4]. Suppose a charge $Q$ moves with a uniform velocity $u$ relative to a rest inertial frame $S$, and another charge $q$ moves with a velocity $v$ with respect to $S$. We first write down the interaction of $Q$ on $q$ in the inertial frame $S’$ moving with charge $Q$, it is electrostatic force according to Coulomb’s law. Then according to the transformation formulas of force between inertial frames, we obtain the interaction of $Q$ on $q$ in the rest frame $S$. The corresponding magnetic force term will appear.

We wish to deal with the law of universal gravitation in the same way. However, Lorentz transformations cannot be directly applied to large distance scale in the presence of gravity. Fortunately, we can use $g_{0i}$ in the expression of metric to obtain the gravitational vector potential in local gravitational field.

As Lorentz transformations are convenient to apply in rectangular coordinate system, we first derive the expression of metric in rectangular coordinate system. Suppose $r = r_1f(r_1)$, where $f(r_1)$ is a function to be determined. Then we have

$$dr = f(r_1)dr_1 + r_1 f'(r_1)dr_1 = dr_1[f(r_1) + r_1 f'(r_1)].$$

Let

$$[f(r_1) + r_1 f'(r_1)] e^{\frac{GM}{r_1 f(r_1)}} = f(r_1),$$

then Eq. (5) becomes

$$ds^2 = c^2 e^{\frac{2GM}{r_1 f(r_1)}} dt^2 - f^2(r_1)(dr_1^2 + r_1^2 d\theta^2 + r_1^2 \sin^2 \theta d\phi^2).$$

Let $x = r_1 \sin \theta \cos \phi$, $y = r_1 \sin \theta \sin \phi$, $z = r_1 \cos \theta$, we find

$$dx = \sin \theta \cos \phi dr_1 + r_1 \cos \theta \cos \phi d\theta - r_1 \sin \theta \sin \phi d\phi,$$

$$dy = \sin \theta \sin \phi dr_1 + r_1 \cos \theta \sin \phi d\theta + r_1 \sin \theta \cos \phi d\phi,$$

$$dz = \cos \theta dr_1 - r_1 \sin \theta d\theta.$$

Then we have

$$dx^2 + dy^2 + dz^2 = dr_1^2 + r_1^2 d\theta^2 + r_1^2 \sin^2 \theta d\phi^2.$$ Substituting it into Eq. (11), we have

$$ds^2 = c^2 e^{\frac{2GM}{r_1 f(r_1)}} dt^2 - f^2(r_1)(dx^2 + dy^2 + dz^2).$$

The remaining problem is to solve for $f(r_1)$ with Eq. (10). Its analytical solution is difficult to find. But it is easy for Schwarzschild metric. We make the following transformations [2]
then the Schwarzschild metric becomes

\[
\begin{align*}
    ds^2 &= \left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 - \left(1 + \frac{2GM}{rc^2}\right)(dx^2 + dy^2 + dz^2) \\
    &\approx \left(1 - \frac{2GM}{r_1c^2}\right)c^2dt^2 - \left(1 + \frac{2GM}{r_1c^2}\right)(dx^2 + dy^2 + dz^2).
\end{align*}
\]  

(15)

In the weak gravitational field, it follows from Eq. (14) that \( r_1 \approx r \), then we have

\[
    ds^2 = \left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 - \left(1 + \frac{2GM}{rc^2}\right)(dx^2 + dy^2 + dz^2)  
\]  

(16)

Suppose a coordinate system S is at rest relative to a gravitational source \( M \), another coordinate system \( S' \) moves with a speed of \( v \) along \( x \) axis relative to \( M \). Then both S and \( S' \) are not inertial frames. In order to apply Lorentz transformations between the two frames, we must transform the time intervals and distances measured in the two frames into the corresponding quantities in inertial frame, that is

\[
\begin{align*}
    dt_0 &= dt e^{-GM/rc^2} & dt'_0 &= dt' e^{-GM/rc^2} \\
    dx_0 &= dx e^{GM/rc^2} & dx'_0 &= dx' e^{GM/rc^2} \\
    dy_0 &= dy e^{GM/rc^2} & dy'_0 &= dy' e^{GM/rc^2} \\
    dz_0 &= dz e^{GM/rc^2} & dz'_0 &= dz' e^{GM/rc^2}
\end{align*}
\]  

(17)

where \( dt_0, dx_0, dy_0, dz_0, dt'_0, dx'_0, dy'_0, dz'_0 \) are respectively the quantities in inertial frames, and they satisfy Lorentz transformations

\[
\begin{align*}
    dx_0 &= \frac{dx'_0 + vt_0'}{\sqrt{1 - \beta^2}} \\
    dt_0 &= \frac{dt'_0 + v}{c^2}dx'_0, \\
    dy_0 &= dy'_0 \\
    dz_0 &= dz'_0
\end{align*}
\]  

(18)

Substituting Eq. (17) into Eq. (18), we get
Considering the instance of weak-field and low-speed, where \( \frac{GM}{r c^2} \ll 1 \), \( \beta^2 \ll 1 \), we have

\[
\begin{align*}
\begin{cases}
\frac{dx}{dt} = \frac{dx'}{dt'} + v dt', \\
\quad \frac{dy}{dt} = \frac{dy'}{dt'} + \frac{v}{c^2} dx', \\
\quad \frac{dz}{dt} = \frac{dz'}{dt'}.
\end{cases}
\end{align*}
\]

Then in the coordinate system \( S' \), the metric is

\[
d s^2 = d s^2 = \left(1 - \frac{2GM}{rc^2}\right)c^2 dt^2 - \left(1 + \frac{2GM}{rc^2}\right)\left(dx^2 + dy^2 + dz^2\right)
= c^2\left(1 - \frac{2GM}{rc^2}\right)\left(dt' + \frac{v}{c^2} dx\right)^2 - \left(1 + \frac{2GM}{rc^2}\right)\left(dx' + v dt'\right)^2 + dy'^2 + dz'^2
\approx c^2\left(1 - \frac{2GM}{rc^2}\right) dt'^2 - \left(1 + \frac{2GM}{rc^2}\right)\left(dx'^2 + dy'^2 + dz'^2\right) - \frac{8GMv}{rc^2} dt' dx' .
\]

According to the definition of gravitational vector potential \( A_i = -g_{0i}/\sqrt{g_{00}} \) [2], we have \( A'_i = 4GMv/\sqrt{1 - 2GM/rc^2} \), \( A'_i = A'_i = 0 \), which implies that when a body moves in the gravitational field, it will experience a gravitational vector potential. As the motion is relative, we may take this vector potential as produced by a moving gravitational source with the velocity \( -v \). Then we have \( A_x = -4GMv/rc^2 \), \( A_y = A_z = 0 \), which agree with the results of post-Newtonian approximation in general relativity [1]. When compared with electromagnetic vector potential, we find that gravitational vector potential increases by a factor of 4. This is due to the effects of time dilation and length contraction in gravitational field.

3.2 The motion of a body in gravitational field

Now we consider the motion of a body in gravitational field. Similar to the electromagnetic force \( F = qE + qv \times B \), we have

\[
F = mE_g = mv \times B_g,
\]

for a body in gravitational field, where \( E_g = d\phi / dr \), \( B_g = \nabla \times A_g \).

General covariance principle takes inertial and non-inertial frames as equally valid. But the results in non-inertial frames are usually not simple. For example, when we observe the planetary orbits from the earth, their trajectories are very complicated. While viewed from the sun, they move in simple elliptical orbits. Furthermore, if we use Newton’s second law of motion to describe a body’s motion in
gravitational field, inertial frame of reference must be adopted because it only holds for inertial frame; otherwise the motion of the non-inertial frame itself should be taken into account. Newton defines inertial frame as frame without force. In the presence of matter, we can take infinity as inertial frame where the gravitational force is zero. In addition, it can be proven that the center of mass of the system is also an inertial frame [5]. For two-body motion, if \( M \gg m \), then body \( M \) may be regarded as an inertial frame, just like the case of the sun and the earth.

As an example, we analyze the precession of the spin of the gyroscope orbiting the earth [1, 6]. As the gyroscope is moving around the earth, it is a non-inertial frame of reference. In order to calculate its precession relative to an inertial frame of reference (distant star), the Thomas precession must be taken into account. We first see the instance of electromagnetic field. Suppose a charged particle rotates with respect to a laboratory inertial frame. The charged particle’s rest frame of coordinate is defined as a co-moving sequence of inertial frames whose successive origins move at each instant with the velocity of the charged particle. The total time rate of the spin with respect to the laboratory inertial frame, or more generally, any vector \( \mathbf{G} \) is given by the well-known result [7]

\[
\frac{d\mathbf{G}}{dt} = \left( \frac{d\mathbf{G}}{dt} \right)_{\text{rest frame}} + \mathbf{\omega}_T \times \mathbf{G},
\]

where \( \mathbf{\omega}_T \) is the angular velocity of rotation found by Thomas.

\[
\mathbf{\omega}_T = \frac{\gamma^2}{\gamma + 1} \frac{\mathbf{a} \times \mathbf{v}}{c^2} \approx \frac{1}{2} \frac{\mathbf{a} \times \mathbf{v}}{c^2}.
\]

In the charged particle’s rest frame, the equation of motion of the spin is

\[
\left( \frac{d\mathbf{J}}{dt} \right)_{\text{rest frame}} = \mathbf{\mu} \times \mathbf{B}',
\]

where \( \mathbf{\mu} \) is the magnetic moment of the charged particle, \( \mathbf{B}' \) the magnetic induction intensity in the charged particle’s rest frame. The classical relation between \( \mathbf{\mu} \) and angular momentum \( \mathbf{J} \) is

\[
\mathbf{\mu} = \frac{q}{2m} \mathbf{J}.
\]

Thus the motion equation of the spin of the charged particle with respect to the laboratory inertial frame is

\[
\frac{d\mathbf{J}}{dt} = \frac{q}{2m} \mathbf{J} \times \mathbf{B}' + \mathbf{\omega}_T \times \mathbf{J}.
\]

Based on the similarity between gravitational and electromagnetic forces, we replace charge \( q \) with mass \( m \), and \( \mathbf{B}' \) with \( \mathbf{B}'_g \), where \( \mathbf{B}'_g \) is the gravitomagnetic intensity observed from the rest frame of the gyroscope. When viewed from the rest frame of the gyroscope, the earth not only moves around the gyroscope but also rotates about its spin axis. Accordingly \( \mathbf{B}'_g = \nabla \times \mathbf{A}'_g \) consists of two terms, i.e. geodetic term and frame-dragging term.

We first see the geodetic effect, as in Fig. 2. According to the above analysis, the components of vector potential experienced by the gyroscope are \( A'_x = A'_z = 0 \), \( A'_y = 4GMv/xc^2 \), respectively. Then we have
Fig. 2. The relative motion between the gyroscope and the earth.

\[
\mathbf{B}'_g = \nabla \times \mathbf{A}'_g = \left\{ 0, 0, \left( \frac{4GMv}{xc^2} \right)' \right\} = \left\{ 0, 0, \frac{4GMv}{x^2c^2} \right\}.
\]  

(28)

Substituting it into Eq. (27), we get

\[
d\mathbf{J} = -\frac{1}{2} \frac{4GMJv}{x^2c^2} + \frac{1}{2} \mathbf{a} \times \mathbf{v} \times \mathbf{J} = \frac{1}{2} \mathbf{J} \times \frac{4a \times v}{c^2} + \frac{1}{2} \mathbf{a} \times \mathbf{v} \times \mathbf{J} = -\frac{3a \times v}{2c^2} \times \mathbf{J},
\]

(29)

where \( \mathbf{a} \) and \( \mathbf{v} \) are the acceleration and velocity vectors of the gyroscope, respectively. Thus the geodetic precession of the gyroscope is \(-\frac{3a \times v}{2c^2}\), which agrees with the result in general relativity [1]. Similarly, we can calculate the vector potential produced by the spin of the earth. The detailed calculation may refer to [1]. We only give the result

\[
\mathbf{A}_g = \frac{2G}{x^2c^2}(x \times \mathbf{J}_e),
\]

(30)

where \( \mathbf{J}_e \) is the spin angular momentum of the earth. Then the frame-dragging precession of the gyroscope is \(-\frac{1}{2} \nabla \times \mathbf{A}_g\), which also agrees with the result of general relativity.

It should be noted that since the sun is an accurate inertial frame in the solar system, the geodetic precession arising from the rotation of the gyroscope around the sun should also be taken into account. A simple calculation indicates that the rate is 19 milliarcsec/yr. As the included angular between the equatorial and orbital planes of the earth is 23.5°, only a fraction of \(19 \times \cos 23.5° = 17.4\) milliarcsec/yr can lead to the precession of the gyroscope. The other fraction cannot lead to the precession for its direction is parallel to the spin axis of the gyroscope. For detailed discussion one may refer to [6].

3.3 Gravitational radiation

It is well known that as an electrical charge makes accelerated motion, it produces electromagnetic radiation. Likewise, we expect that a body will produce gravitational radiation as it makes accelerated motion. For an isolated charge system, the strongest radiation is electric dipole moment radiation. The dipole moment is

\[
\mathbf{d}_e = \sum_i e_i \mathbf{r}_i,
\]

(31)

where \( e_i \) and \( \mathbf{r}_i \) are the charge and the position vector of particle \( i \), respectively. The radiant intensity of dipole moment is proportional to \( \mathbf{d}_e \). If we replace \( e_i \) with \( m_i \), we obtain the mass dipole moment of an isolated system

\[
\mathbf{d}_m = \sum_i m_i \mathbf{r}_i,
\]

(32)
whose first order derivative is the total momentum of the system

$$\dot{\mathbf{d}}_m = \sum_i m_i \dot{\mathbf{r}}_i.$$

(33)

Because the total momentum of an isolated system is conserved, we have $\dot{\mathbf{d}}_m = \dot{\mathbf{p}} = 0$. Thus mass dipole moment radiation cannot exist in gravity physics.

The second strongest radiations are magnetic dipole moment radiation and electric quadrupole moment radiation, respectively. The radiant intensity of magnetic dipole moment is determined by its second order derivative. The magnetic dipole moment can be written as

$$\mathbf{\mu} = \frac{1}{2} \sum_i \mathbf{r} \times (e_i \mathbf{v}_i).$$

(34)

Replacing $e_i$ with $m_i$, we obtain the gravitomagnetic dipole moment

$$\mathbf{\mu}_g = \frac{1}{2} \sum_i \mathbf{r} \times (m_i \mathbf{v}_i),$$

(35)

which is just half of the angular momentum of the system. Due to the conservation of the angular momentum of an isolated system, there does not exist gravitomagnetic dipole moment radiation.

The gravitational radiation similar to electric quadrupole moment radiation does exist. For an isolated system, the main gravitational radiation is mass quadrupole moment radiation. The mass quadrupole moment is

$$D_{\alpha \beta} = \int \rho (3x^\alpha x^\beta - \delta^\beta_\alpha x^\gamma x^\gamma) d^3x.$$

(36)

The total power radiated is

$$\frac{dE}{dt} = -\frac{G}{45c^5} \dot{\mathbf{d}}_m^2.$$

(37)

The above simple discussion can refer to [3]. More detailed discussions may see [1, 2]. Our theory is equivalent to general relativity in weak field and low speed instance. Thus the two give a same result for the gravitational radiation of isolated system.

4 Comparison with general relativity

Although the theoretical foundations and formalisms of our theory and general relativity are different, the two are actually equivalent for the instance of weak gravitational field. Deviation occurs only in strong gravitational field. This is because $G_{\alpha \nu}$ only comprises metric and its first and second derivatives in Einstein’s gravitational field equations. As noted in [1], $G_{\alpha \nu}$ must have the dimensions of a second derivative. Other terms of type $N \neq 2$ appear multiplied with a constant having the dimension of length to the power $N - 2$. If infinite order derivative of metric are included, the two theories will be exactly equivalent. On the other hand, disagreements arise when different frames of reference are adopted. An apparent example is the motion of binary system, which will be discussed below.

4.1 On the apsidal motion of binary system

The motion of the apsidal line of binary arises from the classical tidal interaction and axial rotation of the components as well as relativistic contribution. We first see the classical terms. The equations of the relative motion of the binary are [8]
\[ r^2 \dot{\theta} = h \]

where \( \delta = \frac{k_1 R_1^5}{G m_1} \dot{\theta}_1 + \frac{k_2 R_2^5}{G m_2} \), \( \delta' = \frac{6m_2}{m_1} k_1 R_1^5 + \frac{6m_1}{m_2} k_2 R_2^5 \). Let \( u = 1/ r \), we get

\[
\frac{d^2 u}{d\theta^2} + u = \frac{G(m_1 + m_2)}{h^2} (1 + \delta u^2 + \delta' u^5),
\]

After one revolution, the periastron longitude has increased by

\[ 2\pi e \approx \frac{G(m_1 + m_2)}{Ah^2} \int_0^{2\pi} (\delta u^2 + \delta' u^5) \cos \theta d\theta = 2\pi [\delta u^2 + \frac{5}{2} \delta' u^5 (1 + \frac{3}{2} e^2 + \frac{1}{8} e^4)], \]

where \( A = l/e \), and \( l = a(1 - e^2) \), \( a \) is semi-major axis of relative motion orbit, \( e \) the eccentricity.

Now we derive the motion equations of component star \( m_1 \) with respect to the center of mass of the system. As \( m_1 r_1 = m_2 r_2 \), \( r_1 + r_2 = r \), we have \( r = (1 + m_1 / m_2) r_1 \). Substituting it into the first equation of (38), we obtain the radial equation of \( m_1 \). The second equation is the conservation of angular momentum, which should also be relative to the center of mass of the system. Then we have

\[
\begin{align*}
\dot{r}_1 - r_1 \dot{\theta}_1 &= -\frac{G m_2}{(m_1 + m_2)^2} (1 + \delta r_1^{-2} + \delta' r_1^{-5}) , \\
r_1^2 \dot{\theta}_1 &= \dot{h}_1 ,
\end{align*}
\]

where \( \delta_1 = \left( \frac{m_1 + m_2}{m_1} \right)^{-2} \), \( \delta'_1 = \left( \frac{m_1 + m_2}{m_1} \right)^{-5} \). Let \( u_1 = 1/ r_1 \), we get

\[
\frac{d^2 u_1}{d\theta^2} + u_1 = \frac{G m_2}{(m_1 + m_2)^2} h_1^{-2} (1 + \delta_1 u_1^2 + \delta'_1 u_1^5) ,
\]

Comparing Eq. (39) with (42) and using the relations of \( \delta u^{-2} = \delta_1 l_1^{-2} \) and \( \delta u^{-5} = \delta_1 l_1^{-5} \), we can easily find that the rate of apsidal motion of component \( m_1 \) is just \( m_2 / (m_1 + m_2) \) times the result of relative motion. Similarly, the rate of apsidal motion of component \( m_2 \) is \( m_1 / (m_1 + m_2) \) times the result of relative motion. Then the rate of the apsidal motion of binary agrees with the result of relative motion.

We then see the apsidal motion of relativistic effect. The metric for relative motion is

\[
ds^2 = c^2 \left[ 1 - \frac{2G(m_1 + m_2)}{rc^2} \right] dt^2 - \left[ 1 - \frac{2G(m_1 + m_2)}{rc^2} \right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 .
\]

The rate of apsidal motion of the system is \( 6\pi G(m_1 + m_2)/ac^2(1 - e^2) \) \[9\]. We now observe the motion of binary from the center of mass of the system. For component \( m_1 \), according to Eq. (42) we may imagine that there exists a mass of \( m_2 / (m_1 + m_2) \) at the center of mass while the other component \( m_2 \) does not exist, and \( m_1 \) can be regarded as a test body. Then the metric of \( m_1 \) with respect to the center of mass of the system is

\[
ds^2 = c^2 (1 - \frac{2GM'}{r c^2}) dt^2 - (1 - \frac{2GM'}{r c^2})^{-1} dr^2 - r_1^2 d\theta^2 - r_1^2 \sin^2 \theta d\phi^2 ,
\]
where $M' = m_2^2/(m_1 + m_2)^2$. The apsidal motion of component $m_i$ with respect to the center of mass of the system is $6\pi GM'/(a_i c^2 (1-e^2))$, where $a_i = m_2 a_i/(m_1 + m_2)$. Then the rate of apsidal motion of $m_i$ decreases by a factor of $m_i^2/(m_1 + m_2)^2$ compared to that of relative motion. Similarly, the rate of apsidal motion of $m_2$ decreases by a factor of $m_i^2/(m_1 + m_2)^2$. Then the rate of apsidal motion of binary system is $(m_1^2 + m_2^2)/(m_1 + m_2)^2$ times that of relative motion.

As the center of mass lies in the connecting line between the two components, Kepler’s second law holds also for the relative motion of binary, so the classical contribution to apsidal motion is the same for the two cases. But for relativistic effect, the results are different when different frames of reference are adopted.

When the inertial frame of center of mass is adopted, the discrepancies between the theoretical and observed values of the apsidal motions of some binary systems will decrease. DI Herculis ($m_1 = 5.15$ M$_\odot$, $m_2 = 4.52$ M$_\odot$, where M$_\odot$ is the solar mass) is an example. When viewed from relative motion, the theoretical values of classical and relativistic apsidal motion are $1.93\pm0.100$ yr and $2.34\pm100$ yr, respectively. The observed apsidal motion is $0.65\pm0.18/100$ yr [9]. The new observation reveals a larger value of $1.30\pm0.14/100$ yr [10]. Our theoretical value for relativistic apsidal motion is $(5.15^2+4.52^2)/(5.15+4.52)^2 \times 2.34 = 1.17/100$ yr. On the other hand, if we suppose the rotational axes of components are not perpendicular to the orbital plane, the theoretical value will further decrease, the detailed discussions may see [11, 12], and there are evidences indicating that this situation is true for some binary systems [13].

The researches on 62 binary systems in [14] indicate that the cases in which the theoretical estimate exceeds the observed value are several times more frequent than the cases in which the theoretical value is lower than the observed one. For 20 of the 62 systems there is agreement to within the errors, for 28 systems the theoretical values exceed the observed ones, and for 14 systems the observed rates are higher.

When the inertial frame of center of mass is adopted, we recalculate the theoretical values of the 62 systems. The errors are taken from the data in [14], and the masses of the components of the binary systems are adopted from [15]. As there are two opposite sets of data for HR8584 in [14], we use the data of the remaining 61 systems. The results are as follows: for 16 systems theoretical values are equal to observed data within the errors, for 26 systems theoretical values exceed observed ones, for 19 systems the observed rates are higher. It can be seen that the asymmetry between the case in which theoretical values are higher and the case in which observed data are higher will decrease when the inertial frame of center of mass is adopted.

### 4.2 On the gravitational radiation of binary system

We first consider the instance of circular motion. The power radiated by a mass $m$ making a circular motion with a radius $R$ is [1, 2]

$$-\frac{dE}{dt} = \frac{32}{5} \frac{G}{c^5} \rho^6 m^2 R^4.$$  \hspace{1cm} (45)

For binary system, we have $\rho^2 = G(m_1 + m_2)/R^3$, where $R$ is the distance between the two component stars. The term $m$ in Eq. (45) should be replaced by reduced mass $m_1 m_2/(m_1 + m_2)$, then the power radiated by the binary system is [2, 3]
\[-dE = \frac{32G^4}{5c^5}
\begin{array}{c}
m_1^2m_2^2(m_1+m_2)/R^5.
\end{array}
\] (46)

When the binary system makes an elliptical motion, the radiant power should be multiplied by a factor 
\[f(e) \quad [16]\]
\[
f(e) = \frac{1 + \frac{7}{24}e^2 + \frac{37}{96}e^4}{(1-e^2)^{3/2}}. \] (47)

In this case, \(R\) should be replaced by the semi-major axis of the relative motion orbit \(a\), and \(e\) is the 
eccentricity. As the center of mass of the binary system is an inertial frame of reference, we should 
calculate respectively the radiant power of the two components with respect to the center of mass, and 
then add them up. The radiant power of component \(m_1\) with respect to center of mass is
\[
-dE_1 = \frac{32G^4}{5c^5} \omega^6 m_1^2 a_1^4 f(e). \] (48)

The radiant power of component \(m_2\) with respect to center of mass is
\[
-dE_2 = \frac{32G^4}{5c^5} \omega^6 m_2^2 a_2^4 f(e). \] (49)

In above equations, \(a_1, a_2\) are the orbital semi-major axes of components \(m_1\) and \(m_2\) with respect 
to the center of mass of binary system, respectively, and we have \(m_1a_1 = m_2a_2, \ a_1 + a_2 = a, \)
\(\omega^2 = G(m_1 + m_2)/a^3\). The total radiant power of the binary system is
\[
-dE = \frac{32G^4}{5c^5} \omega^6 \left( m_1^2 a_1^4 + m_2^2 a_2^4 \right) \left( m_1 + m_2 \right) f(e)
\] (50)

\[
= \frac{32G^4}{5c^5} \frac{m_1^2 m_2^2 \left( m_1^2 + m_2^2 \right)}{(m_1 + m_2)a^5} f(e).
\] (50)

Compared with the result of relative motion, our theoretical value decreases by a factor of \((m_1^2 + m_2^2)/(m_1 + m_2)^2\), just like the instance of the relativistic effect for apsidal motion of binary system.

In the case of \(m_1 = m_2\), our theoretical value is half that of the relative motion instance.

**4.3 On the TOV equation and mass of neutron star**

Now we consider the metric inside the neutron star. The metric is actually determined by the local 
gravitational field intensity or gravitational potential. Inside a static spherically symmetric body, there is 
no vector potential, and the metric has the following general form [1]
\[
dr^2 = \frac{1}{a_1^2 \theta^2} \left[ \cos^2 \theta \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) + b(r)dt^2. \right.
\] (51)

For a static spherically symmetric body, the local gravitational field intensity inside the body is 
related to the radius \(r\) and the mass \(M(r)\) inside the radius. In terms of Gauss’s law in gravitational 
field, we find the metric inside the neutron star is just the same as that of in vacuum, so we have
\[
ds^2 = -e^{2GM(r)/rc^2} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + e^{-2GM(r)/rc^2} c^2 dt^2. \] (52)

The hydrostatic equilibrium equation for perfect fluid in gravitational field is [1]
\[ \frac{b'}{b} = -\frac{2p'}{\rho + p/c^2}. \]  

(53)

With \( b(r) = \exp(-2GM(r)/rc^2) \), we have

\[ \frac{dp}{dr} = \frac{GM(r)}{r^2}(\rho + p/c^2), \]  

(54)

where \( \rho \) and \( p \) are proper density and pressure, respectively. It can be seen that the above equation is just the same as that when the gravitational influence on fluid is not considered. In order to explain this issue clearly, let’s start with Eq. (1), i.e., \( dmc^2 = -GMmdr/r^2 \), where \( dm \) and \( M \) are proper quantities (the quantities without the presence of gravity). Now we take into account the gravitational influence. In this case, \( m \), \( dr \) and \( r \) will change as follows: \( m \to me^{GM/rc^2}, \) \( dr \to dre^{GM/rc^2}, \) \( r \to re^{GM/rc^2}. \) Substituting these into Eq. (1), we find Eq. (1) remains unchanged. Now we prescribe Eq. (54) in the same way, that is, \( dp \to dpe^{GM/rc^2}, \) \( \rho + p/c^2 \to (\rho + p/c^2)e^{GM/rc^2}, \) \( dr \to dre^{GM/rc^2}, \) \( r \to re^{GM/rc^2}. \) then we find Eq. (54) still remains unchanged.

For comparison, the Tolman-Oppenheimer-Volkoff (TOV) equation is [1]

\[ -\frac{dp}{dr} = \frac{GM(r)}{r^2}(\rho + p/c^2)(1 + \frac{4\pi^2 p}{M(r)c^2})(1 - \frac{2GM(r)}{rc^2})^{-1}. \]  

(55)

We see that discrepancy arise between Eqs. (54) and (55), this is because \( G_{uv} \) only comprises metric and its first and second derivatives in Einstein’s gravitational field equations. In higher order instance, deviation will appear.

For the equations of state of ideal gas of neutrons, a parameter \( t \) is introduced according to the method of Oppenheimer and Volkoff [2]

\[ t = 4\ln[x + (1 + x^2)^{1/2}]. \]  

(56)

The equations of state are

\[ \rho = \kappa(\text{sh}t - t)/c^2, \]  

(57)

\[ p = \frac{1}{3}\kappa(\text{sh}t - 8\text{sh}\frac{1}{2}t + 3t), \]  

(58)

where \( \kappa = \frac{\pi n^4 c^5}{4h^3}. \) For the mass equation of \( M(r), \) we have

\[ \frac{dM(r)}{dr} = 4\pi r^2 \]  

(59)

According to above equations, our computed neutron masses as well as the results calculated by TOV equation are listed in Table 1. Our numerical evaluation uses fourth-order Runge-Kutta method.

In Tab. 1, the results in column a are calculated by TOV equation, while the results in column b are calculated according to our theory. Note that we must be careful of the case with a larger \( t. \) In the case of \( t=3, \) the core density of pure neutron star is \( 4\times10^{15}\text{g/cm}^3. \) If \( t=5, \) the core density will be \( 4\times10^{16}\text{g/cm}^3. \) Then the equations of state for neutron star may be inapplicable, since there will be other fundamental particles and matter phases in the core. For the maximum mass of neutron star, one may see [17, 18].
Table 1 Computed results for the neutron star masses

| t  | $M/M_\odot^a$ | $M/M_\odot^b$ |
|----|---------------|---------------|
| 1  | 0.30          | 0.32          |
| 2  | 0.61          | 0.80          |
| 3  | 0.71          | 1.20          |
| 4  | 0.64          | 1.40          |
| 5  | 0.55          | 1.41          |
| 8  | 0.37          | 0.97          |
| 10 | 0.42          | 0.76          |
| 15 | 0.42          | 0.93          |

4.4 On black holes and singularities

From Schwarzschild solution of Einstein’s gravitational field equations, one finds gravitational radius $r = \frac{2GM}{c^2}$, which raises the puzzles of black holes and singularities. In our theory, since gravitational radius disappears, black holes and singularities do not exist. It is known that the electron degeneracy pressure resists the gravity of white dwarf; while the neutron degeneracy pressure resists the gravity of neutron star. It can be expected that in the primordial universe, the degeneracy pressure of elementary particles (quarks and leptons) resists the strong gravity inside the universe.

Now it is believed that there is a massive black hole at the center of each galaxy. In fact, these black holes are small primordial universes. They have extremely high temperatures, but they do not radiate light and heat to the outside, so there is no difference between black holes and small primordial universes in appearance. There is a certain probability that these small primordial universes explode, that is, a Little Bang will take place. Do we have any astronomical observation evidences for the Little Bang? A huge radio source in Ophiuchus galaxy cluster may be a candidate [19]. It is the strongest galaxy explosion ever observed since the Big Bang, and is 5 times more powerful than the strongest explosion observed so far. On the other hand, its galaxy nucleus is a cold galaxy nucleus rather than a normal active galaxy nucleus. These agree with the characteristics of Little Bang. Intense X-rays and radio emission indicate that it is in a radiation-dominated era. There are numerous massive stars in the universe, most of them will eventually form a small primordial universe (the mass of a neutron star is no more than several solar masses), so we will have more opportunities to see the Little Bang or Mini Bang events in the universe.

4.5 On the expansion of the universe

The first question to be discussed is: the expansion of the universe is relative to what? General relativity takes the universe’s expansion as expansion of “space” itself. But since all the celestial objects are expanding relative to others, we must observe the motion of objects from a static, non-expanding reference frame. As the center of mass of the universe is a rest inertial frame, the expansion of the universe must be relative to it. Speaking strictly, only when we observe the universe from its center of mass is the space homogeneous and isotropic. Considering the giant cosmological scale, the space is almost homogeneous and isotropic given the point of observation is not far from the center of the universe. Although the Milky Way may not be the center of the universe, the observations of distribution of galaxies and CMBR show that the space around us is highly homogeneous and isotropic [20]. So the Milky Way must be located near the center of the universe but not at the edge of the universe. For convenience, we might as well suppose we are at the center of the universe.
At the beginning of the Big Bang, all particles went outwards at almost the speed of light relative to the center of mass of the universe. Under the influence of gravitational interactions, their velocities gradually slow down. It is doubtless that the universe undergoes decelerated expansion after Big Bang. In fact, Hubble’s law is just the demonstration of the decelerated expansion of the universe. As the observational velocities actually represent the velocities of the celestial bodies at past times, it is reasonable to deduce that the farther the distances of the celestial bodies, the faster their receding velocities with respect to the center of the universe, which agrees qualitatively with Hubble’s law. It should be noted that Hubble’s law only hold within certain distance scale. Its validity is unverified on the whole cosmological scale. That is, Hubble constant may vary with different evolution stage of the universe with respect to the center of the universe.

We now associate the decelerated expansion of the universe with the observational data of supernovas. The observational data of supernovae indicate that the luminosity distances of distant supernovae are larger than their Hubble distances (see, e.g. [21, 22]). This can be explained with a varying Hubble constant, or the expansion rate in the past times is larger than the current rate. So the supernovas evidence has nothing to do with cosmological constant and dark energy.

5 Conclusion

In general, our theory is equivalent to general relativity in weak gravitational field. Compared with general relativity, our method is more natural and simple. We only need to modify the standards of space-time in local gravitational field to let it be consistent with that of inertial frame, and the correct metric will be obtained. In addition, we can use Lorentz transformations in local gravitational field to derive gravitational vector potential.

Our theory is founded on inertial frame. For the description of motion of bodies in gravitational field, inertial frame of reference should be adopted. The apsidal motion and gravitational radiation of binary star, the expansion of the universe, all should be observed from their respective frames of center of mass. In this case, our calculated results for apsidal motion and gravitational radiation of binary system are different from those of general relativity.

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