Stability Analysis and Flutter Suppression of Ultrasonic Elliptical Vibration Milling of Ti-6Al-4V Alloy

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Stability analysis and flutter suppression of ultrasonic elliptical vibration milling of Ti-6Al-4V alloy

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Abstract

Ti-6Al-4V alloy is typical aircraft materials to be difficult machined. Ultrasonic elliptical vibration-assisted milling has been proved to greatly enhance machining performance. In ultrasonic elliptical vibration milling (UEVM), the cutting edge moves in 3D space, reducing extrusion friction between the flank and machined surface, and playing an important role in high precision-efficient manufacturing of difficult-to-cut materials. This paper focused on the stability analysis and flutter suppression of ultrasonic elliptical vibration milling of Ti-6Al-4V alloy. There are separated and unseparated types of ultrasonic elliptical vibration milling systems, which stability is analyzed in this study by simulating their tool tip motion paths. The essence of the tip and workpiece at high-frequency separation is revealed through analysis of cutter tooth path in UEVM. Next, the coordinate method is used to set up the window function to facilitate the derivation of true instantaneous chip thickness. The dynamic model of UEVM is established to study the system stability and plot its stability prediction lobes diagrams. The effect of machining parameters on the system stability is assessed and discussed in detail. The prediction results show that the stability region and axial depth of cut grow with the increase of feed rate per tooth and decrease of ultrasonic frequency. Among the six groups of ultrasonic amplitude parameters studied, the optimal system's stability corresponds to the ultrasonic amplitudes of 5 and 3μm in the x-and y-directions, respectively. Finally, stability tests of UEVM are performed, which results well match the numerical simulation predictions. This implies that the proposed window function is adequate, the theoretical model is valid, and the predicted stability curve is feasible and reliable.

Keywords: Stability analysis; Ultrasonic elliptical vibration milling; Machining parameters; Flutter
1. Introduction

Titanium alloys are widely used in aerospace, biomedical, and chemical industries due to their excellent high-temperature strength, heat resistance, and corrosion resistance[1]. However, machining of titanium alloys is quite problematic due to large cutting forces and tool yield, as well as low machining accuracy. To make full use of titanium alloy materials, scholars worldwide tried many ways to improve their machining quality and efficiency.

As a typical non-conventional cutting, ultrasonic vibration-assisted machining is used to treat titanium alloys and improve their cutting performance[2-5]. Qin et al.[6] developed a predictive cutting force model in ultrasonic vibration-assisted grinding of titanium alloys. It served as a useful template and foundation for developing cutting force models in ultrasonic vibration-assisted grinding to predict torque, cutting temperature, tool wear, surface roughness, and so on. Sui et al.[7] studied to machine extra-large aspect ratio of Ti6Al4V aviation deep-hole with the axial ultrasonic vibration-assisted boring method. The results showed that boring assisted with ultrasonic vibration-assisted cutting has been proved to greatly enhance machining performance. Kandi and Sahoo[8] carried out experimental investigations for conventional turning and ultrasonic vibration-assisted turning to demonstrate the effects of various inputs on the output responses like cutting forces, surface roughness, tool wear, and temperature rise. The results showed that ultrasonic vibration-assisted turning reduced cutting forces and surface roughness. Lu and Zhang [9] introduced high-pressure coolant to the high-speed ultrasonic vibration cutting process. They studied its effects on cutting performance of high-speed ultrasonic vibration cutting of titanium alloys, including tool life and wear mechanism, surface quality, cutting temperature, and cutting force. The results showed that the tool life in high-speed ultrasonic vibration could exceed that in conventional cutting by 7.3 times at a 400 m/min cutting speed. Compared to conventional cutting, the temperature can be reduced by 55% at 300 m/min. Chen et al. [10][11] studied the material removal mechanism, geometrical texture, and surface integrity in ultrasonic vibration helical milling (UVHM) of Ti-6Al-4V alloy. The results showed that surface roughness was reduced, and the compressive stress in the hole surface increased in UVHM compared to helical milling. Compared with the helical milling process, UVHM provided a larger hardness at the subsurface of machined holes (20–200 μm) due to the material deformation caused by ultrasonic vibration. Niu et al. [12][13] designed a series of longitudinal-torsion ultrasonic vibration milling experiments to evaluate the effect of machining parameters on the milling performance of Ti-6Al-4V. It was found that longitudinal-torsion ultrasonic vibration effectively generated surface compressive stresses. Xing [14] also studied the surface friction performance of titanium alloys in ultrasonic vibration-assisted milling. Ni et al. [15] applied vibrations at 20 kHz along the feed direction from the workpiece side and reported the improvement of surface finish and reduced cutting forces in milling of Ti-6Al-4V. Zhao and Li [16] studied fractal characterization of surface micro-texture of Ti-6Al-4V subjected to ultrasonic vibration-assisted milling. The experimental results indicated that smaller characteristic fractal dimensions corresponded to more regular surface microtextures and contour curves. Zhu et al.[17] investigated the micro-texture generation mechanism and the effect of textured surface characteristics on tribological properties of machined surfaces in the ultrasonic vibration-assisted milling process. Experiment results showed that uniform micro-texture could significantly improve surface morphology and surface quality. Gao et al. [18] investigated the interaction mechanism between the tool flank wear and the cutting force and compared the
longitudinal-torsional ultrasonic-assisted milling in dry cutting with conventional milling of Ti-6Al-4V alloy.

Elliptical vibration-assisted cutting proposed by Moriwaki and Shamoto synchronized the vibrations in tangential and thrust directions in diamond turning and milling [19]. They also developed an elliptical vibration milling machine using an eccentric sleeve to deliver vibrations at 167 Hz and reported achieving a better surface finish in the milling of hardened steel[2]. Further studies used ultrasonic elliptical vibration milling of titanium alloys. Jiang et al.[20]utilized an original ultrasonic vibration tool holder and experimented with titanium alloys. They reported that the milling force in ultrasonic elliptical vibration milling was reduced by 50% compared with conventional milling and significantly improved shape precision. Liu et al. [21][22]introduced rotary ultrasonic elliptical vibration-assisted milling(UEVM) to side milling of Ti-6Al-4V. The experimental results showed improved surface integrity in terms of larger compressive residual stress, more pronounced plastic deformation, and promoted work hardening of the machined surface. Zhang et al. [23]investigated the effects of tool vibration on surface integrity in UEVM of Ti-6Al-4V from low cutting speed to high cutting speed, including surface topography and roughness, microstructural alterations, micro-hardness, and residual stresses. Results showed that the values of surface roughness (Ra) grew from 0.160 to 0.758μm with increased cutting speed and vibration amplitude. Then, they also studied the surface characteristics and sub-surface microstructure in UEVM of Ti-6Al-4V. The results showed that the uniform textures in the form of ridges mapped on the machined surface varied with the cutting speed[24].Jung and Shamoto [25] proved that the ultrasonic elliptical vibration cutting with cemented carbide tools could be applied to precision machining of Ti-6Al-4V alloy under suitable cutting conditions. This method significantly reduced cutting forces and improved the surface quality. Wan[26] investigated the tool-workpiece separation caused by the modified tooltip path with vibration assistance and developed a dynamic model with varying time delay. Due to the complexity of the ultrasonic elliptical vibration that is classified as a two-dimensional motion, the relevant literature is scarce.

The above findings were mainly obtained experimentally, while dynamic models of the vibration-assisted machining process require further development. Good surface integrity was attained under system stability, which affected the quality and productivity of machining and the life of cutting tools and machine tools. The stability of the cutting system is especially crucial for the low-stiffness components in precision and ultra-precision machining[27]. The flutter stability of conventional machining operations has been studied extensively in the literature [28-31], in contrast to ultrasonic vibration-assisted milling. Ma and Shamoto [32] investigated the regenerative flutter suppression in the turning process by applying the ultrasonic elliptical vibration to the cutting tool. They proved experimentally and theoretically the effectiveness of this approach. Tang and Zhao [33] studied separated longitudinal-torsional composite ultrasonic milling and determined the optimal characteristics required to reduce the cutting force, promote chip removal, and improve the tool life. Zhang and Zhao [34] studied the stability of separated feed ultrasonic milling and reported that it outperformed conventional milling. However, studies on the stability of ultrasonic elliptical vibration milling are scarce.

This paper uses the coordinate method to develop the window function. This allows one to use the unified dynamic model of instantaneous chip thickness for separated and unseparated cutting states and simplify its solution. The time delay differential equations of UEVM are deduced using the linear theory of nonlinear periodic function and solved by the full-discretization method. Then, the stability prediction lobes diagrams of UEVM systems are plotted via
MATLAB software. They are differentiated by separated and unseparated types and discussed in detail. Finally, flutter experiments on Ti-6Al-4V alloy specimens are conducted under UEVM, and their results are further verified by the surface quality tests.

2. UEVM stability analysis

Establishing an accurate cutting force model is crucial for assessing system stability. The milling force is mainly determined by the instantaneous cutting thickness, which is closely related to the motion paths of the current and previous cutter teeth. In ordinary milling, the cutting of a single cutter tooth is continuous, and there is an intermittent cutting between cutter teeth. Researchers have established a relatively robust milling force model according to this characteristic. However, the intermittent cutting may occur in machining a single cutter tooth in ultrasonic elliptical vibration milling. Starting from the motion track of cutter teeth, this study furnishes a solution to the true instantaneous cutting thickness and establishes the dynamic model of ultrasonic elliptical vibration milling.

2.1. UEVM tool tip path

In the UEVM, the cutter is analyzed to derive the relative motion between the workpiece and the tool. As shown in Fig.1, the fixed coordinate system XOY is set up with the center of the milling cutter as the coordinate origin, and the moving coordinate system HOV is set up at the center of the tool tip.

![Fig.1. The coordination systems of the UEVM tool tip](image)

At the initial moment, the two coordinate axes of the moving and fixed coordinate systems coincide. For down-milling, at arbitrary time \( t \), the cutter turning angle is derived as \( \alpha = \omega t \), and the tip path equation in the fixed coordinate system can
be expressed as follows:

\[
\begin{align*}
\dot{x}_j &= r\sin(w_i t) + v_i t + a\sin(2p ft)\cos(w_i t) \\
&\quad + b\cos(2p ft)\sin(w_i t) \\
\dot{y}_j &= r\cos(w_i t) - a\sin(2p ft)\sin(w_i t) \\
&\quad + b\cos(2p ft)\cos(w_i t)
\end{align*}
\]

(1)

where \( w_i \) is spindle rotational speed of the cutter, \( v_i \) is feed rate, \( r \) is the cutter radius.

Separated and unseparated UEVM tool tip paths are depicted in Fig.2. According to \( j \)-th and \((j-1)\)-th cutter tooth paths in Fig.2 (b), it can be observed that the cutter teeth rotate while moving with the ultrasonic ellipse. The solid line in the figure shows the path left on the workpiece during the cutting process, while the dashed line represents the cutter teeth separated from the workpiece. The points A, B, C, D, and E in Fig.2(c) denote the motion of the cutter teeth per cycle of separated ultrasonic elliptical vibration milling(SUEVM). During the cycle, the A-B section corresponds to the preparation of the cutter teeth before the cutting stage, when they start to move in the workpiece’s direction. The B-C section is the cutting-in stage, during which the cutter teeth enter into the workpiece, with a gradual cutting thickness increase. The C-D section is the cutting-out stage, where the cutter teeth separate from the workpiece, and the cutting thickness gradually decreases. As the tool tip reaches point D, the cutter teeth become completely separated from the workpiece. The motion cycle is completed when the tool tip reaches point E. Then, the cutter teeth start to prepare for the next cycle. During the whole period of the elliptical ultrasonic vibration, the A-B and D-E sections correspond to the idling stages, with zero cutting forces on the cutter teeth and zero instantaneous cutting thickness.

Fig.2. Path simulations of two subsequent cutter teeth in UEVM

It can be seen from Fig.2 that the cutter tooth path is a continuous curve at any time, with no intersections between the cutter tooth paths at two adjacent moments. This implies that the cutter tooth and the workpiece are in constant contact. In addition, judging from the path position of \( j \)-th and \((j-1)\)-th cutter teeth, the current cutter tooth position is always above...
the previous cutter tooth, and thus their instantaneous cutting thickness is meaningful at any moment.

2.2 Derivation of the instantaneous cutting thickness

Based on the cutter tooth path analysis, its characteristics were used to judge the existence of the instantaneous thickness \( h_{st} \). The full-discretization method was applied to assess the stability of SUEVM. The cutter tooth pass period \( T_d \) included the ultrasonic elliptical vibration period \( T \) repeated \( m \) times, i.e., \( T_d=mT \). During each cycle, the initial angle of the ultrasonic elliptical vibration was adjusted to guarantee that any elliptic ultrasonic cycle would start from the extreme point \( Q \) and ends at another extreme point \( Q' \) in Fig.2(c). Thus, each ultrasonic elliptical cycle contains only one key point, \( K_p(i=1,2,...,m) \). They are cutting-in and cutting-out points, i.e., the equivalent points of the path function at different times. They are also the maximum or minimum extreme values, where the cutter tooth path is separated from the workpiece at any cycle within the coordinate system depicted in Fig.1. According to these characteristics, the function value of key points can be derived. Then, the instantaneous thickness \( h_{st} \) at \( j \)-th and \((j-1)\)-th cutter teeth is assessed using key points' function value. If the function value of discrete points is smaller than that of the key point, the discrete points are left on the workpiece. The instantaneous chip thickness is significant; the judgment function \( L(t) \) value equals 1; otherwise, it is zero. As shown in Fig.2, points \( X_{mT+iT} \) and \( X_{mT-(i+1)T} \) on the \( j \)-th and \((j-1)\)-th cutter teeth paths, respectively, their function values are smaller than that of the key point of the corresponding period, therefore connecting \( X_{mT+iT} \) with \( X_{mT-(i+1)T} \), the radial distance between them along the spindle rotation angle is the instantaneous thickness \( h_{st} \) at time \( t \) during the cycle. The motion paths before and after \( X_{mT+iT} \) and \( X_{mT-(i+1)T} \) are expressed by dotted lines because these two points are not on the path, and the instantaneous chip thickness does exist (i.e., is meaningless). In this case, based on full-discretization, a judgment function is set up to judge the separation of the cutter tooth and workpiece at any time and further assess the instantaneous chip thickness for the stability analysis. The window function, which indicates whether the separation of the cutter tooth and workpiece exists or not, has the following form:

\[
L(t) = \begin{cases} 
1, & y_{j,T} \leq y_{K_p,T}, y_{j-1,T} \leq y_{K_p,T} \quad (j = I; N, I = 1L, m) \\
0, & \text{else} 
\end{cases}
\]  

where \( x_{j,T}, x_{j-1,T}, y_{j,T}, y_{j-1,T} \) are the coordinate values of the \( j \)-th and \((j-1)\)-th cutter teeth at any period \( T_i \), respectively, while \( x_{K_p,T}, y_{K_p,T}, x_{K_p,T}, y_{K_p,T} \) are the key point coordinate values of the \( j \)-th and \((j-1)\)-th teeth at any period \( T_i \), respectively.

The flowchart of the proposed method is shown in Fig.3.
2.3 Dynamic model of UEVM

Figure 4 presents the ending milling, in which the tool vibration model is reduced to the 2DoF (two-degrees-of-freedom) mass-damping-spring mechanical system along the x- and y- directions.
The comprehensive analysis of Figs. 4 and 1 revealed that the cutting, tangential, and radial forces acting on the cutter tooth in x- and y-directions could be expressed as follows:

\[
m_i \ddot{x} + c_i \dot{x} + k_i x = F_i(t) \quad (3)
\]

\[
F_i = \frac{c_i}{\xi} \cos(f_j(t)) - \frac{c_i}{\xi} \sin(f_j(t)) \dot{u} + \frac{c_i}{\xi} \dot{u} + \frac{c_i}{\xi} \dot{u} + \frac{c_i}{\xi} \dot{u} \quad (4)
\]

Where \( m_i, c_i, k_i (i=x, y) \) are the system mass, damp, and kinetic coefficients in x- and y-direction respectively, \( F_i(t) \) are the cutting force components acting on the tool in both directions at time \( t \), while \( u(t) = [x(t) y(t)]^T \) is the displacement of the cutter in x- and y-directions at time \( t \). Besides, \( f_j(t) \) is the rotation angle of the cutter tooth \( j \) at time \( t \):

\[
f_j(t) = \frac{2 \pi W}{60} - (j - 1) \frac{2 \pi}{N}, \quad j=1,2,\ldots,\Omega
\]

is the spindle speed, \( F_j^t \) and \( F_j^r \) are the tangential and radial forces acting on the \( j \)-th cutter tooth, respectively.

The tangential and radial forces acting on the cutter tooth \( j \) are derived as follows:

\[
\frac{\dot{F}_j^t}{\xi} \dot{u} + \frac{\dot{K}_t}{\xi} \dot{u} = a_p (\frac{\dot{F}_j^t}{\xi} \dot{u} + \frac{\dot{K}_t}{\xi} \dot{u}) + \frac{\dot{K}_r}{\xi} \dot{u} + \frac{\dot{F}_j^r}{\xi} \dot{u} + g(f_j(t)) \quad (5)
\]

where \( K_t \) and \( K_r \) are tangential and radial cutting parameters, respectively; \( a_p \) is axial cutting depth; \( K_u \) and \( K_v \) are tangential and radial parameters of the cutting edge, respectively; \( q \) is the cutting constant; \( h_j(t) \) is the dynamic chip thickness considering the regeneration effect; \( g(f_j(t)) \) is the window function used to judge whether a tooth is in the cut or not. The tooth is in cut if \( f_s, f_f \notin [f_j(t), f_f] \).

This function is expressed by Eq.(6):

\[
g(f_j(t)) = \begin{cases} 1, & f_s \notin [f_j(t), f_f] \\ 0, & \text{else} \end{cases} \quad (6)
\]

where \( f_s \) and \( f_f \) are the start and exit angles, respectively. According to the coordinate frame shown in Fig.2, for up-milling, \( f_s = 0 \) and \( f_f = \arccos(1 - 2a_p) \); for down-milling, \( f_s = \arccos(1 - 2a_p) \) and \( f_f = p \), where \( a_p = a_e / D \) is the radial immersion ratio.

The instantaneous chip thickness \( h_j \) is composed of static \( (h_{js}) \) and dynamic \( (h_{jd}) \) thickness components. The instantaneous, static, and dynamic chip thickness can be expressed as follows:

\[
h_j(t) = L(t) * (h_{js}(t) + h_{jd}(t)) \quad (7)
\]
h_{ss}(t) = \sin(f(t)) \ast f_1 + \sin(f(t)) \ast A_x \sin 2pf + \cos(f(t)) \ast B_y \cos 2pf \quad (8)

where $A_x$ and $B_y$ are ultrasonic amplitudes in $x$- and $y$-directions, $f$ is ultrasonic frequency, $t$ is the tooth passing interval, $L(t)$ is the window function that determines the existence or absence of cutter tip-workpiece interaction. Its value is unity when the cutter tip contacts the workpiece; otherwise, it is zero.

The model of UEVM forces adopts an exponential pattern to predict the system stability, using the linear theory of nonlinear periodic function [29]. After the instantaneous chip thickness is transformed by the Taylor expansion and trigonometry, the 2DoF dynamic model of elliptical ultrasonic milling takes the following form (with no account of the coupling effect):

\[
\begin{align*}
\dot{\text{m}}_{xx} &= 0 \quad \ddot{\text{m}}_{xx} \omega_{m}^2 \quad 0 \quad \ddot{\text{m}}_{xx} \omega_{m}^2 \quad 0 \quad \ddot{\text{m}}_{xx} \\
\ddot{\text{m}}_{xx} &= 0 \quad \ddot{\text{m}}_{xx} \omega_{m}^2 \quad 0 \quad \ddot{\text{m}}_{xx} \omega_{m}^2 \quad 0 \quad \ddot{\text{m}}_{xx} \\
0 &= \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \\
0 &= \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \\
0 &= \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \quad \frac{\ddot{h}_{xx}}{\omega_{XX}^2} \\
\end{align*}
\]

where $h_{xx} = L(t) \sum_{j=1}^{N} g(f_x(t)) q(f_x^* s + 2pf) fT(A_x s + B_y) e^{fT - f} \sin(f_j(t)) \cos(f_j(t)) + K_x \sin(f_j(t))$.

$\begin{align*}
\dot{h}_{xy} &= 0 \quad \ddot{h}_{xy} \omega_{m}^2 \quad 0 \quad \ddot{h}_{xy} \omega_{m}^2 \quad 0 \quad \ddot{h}_{xy} \\
\ddot{h}_{xy} &= 0 \quad \ddot{h}_{xy} \omega_{m}^2 \quad 0 \quad \ddot{h}_{xy} \omega_{m}^2 \quad 0 \quad \ddot{h}_{xy} \\
0 &= \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \\
0 &= \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \\
0 &= \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \quad \frac{\ddot{h}_{xy}}{\omega_{XY}^2} \\
\end{align*}$

where $h_{xy} = L(t) \sum_{j=1}^{N} g(f_x(t)) q(f_x^* s + 2pf) fT(A_x s + B_y) e^{fT - f} \cos(f_j(t)) \sin(f_j(t)) + K_y \sin(f_j(t))$.

Here $s = \sin(f_j(t))$ and $c = \cos(f_j(t))$, while $m_i, z_j$, and $w_{i,j} (i=x,y)$ are the model mass, relative damping, and natural angular frequency, respectively.

Combining the above path method and full-discretization method, the state transition matrix of the elliptical ultrasonic milling system was acquired. Then, the Floquet theory was used to assess the eigenvalue mode of the state transition matrix: if it was less than 1, the system was stable; otherwise, the system was unstable. The number of iterations of 40 and a radial immersion ratio of 0.05 were selected, while the stability charts were calculated over a 400 x 200-sized grid of parameters. Computer programs of the proposed approach were all written in MATLAB7.1 and implemented on a PC (Intel Core (TM) i5-3470CPU, 3.2 GHz, 4GB). The computation time was about 390 s.

3. Effect of machining parameters on the UEVM stability

The feed per tooth, ultrasonic amplitude, and ultrasonic frequency directly impact the cutting separation characteristics
and milling force in the UEVM. Given the high hardness values of the titanium alloy, the step of feed per tooth of 0.004mm was selected. As to the ultrasonic frequency, Zhao [35] study revealed that the material damage decreased with frequency under 35kHz, and the damage degree was the lowest at 35kHz. Later, the damage increased with frequency during the grinding experiment. In addition, authors [34] reported that the best stability of feed ultrasonic milling was attained at ultrasonic frequencies below 35 kHz. Therefore, ultrasonic frequencies exceeding 35 kHz were selected to study the effect of machining parameters on stability.

3.1 Feed per tooth effect on the UEVM stability

To clarify the effect of feed per tooth on the stability of UEVM, the machining parameters were selected as follows: ultrasonic amplitudes of 5 and 3μm in the x- and y-directions, respectively, an ultrasonic frequency of 35 kHz, a radial immersion rate of 0.05, a tool radius of 10 mm, and a number of tool teeth of 4. According to Section 2.2, the stability prediction lobes of UEVM under different feed per tooth were plotted in Fig.5. It can be seen from Fig.5 that the stability area and axial cutting depth of the system gradually rose with the feed rate per tooth. The maximum axial cutting depth was enhanced by 37% at a spindle speed of 3000r/min. However, the increased feed per tooth strongly influenced the system's stability at first, but this effect was weakened after a certain level was reached, as shown in Fig 5 by a slight stability area variation from 0.004mm (green area) to 0.016mm of feed per tooth). Therefore, in the UEVM system, the feed per tooth increase by small steps would be conducive to system stability improvement. In Fig.5, the red and black lines correspond to the separated and unseparated UEVM systems. When the feed of each tooth was increased, higher spindle speeds made it easier to separate the cutter teeth from the workpiece. In comparison, the effect of feed per tooth on the unseparated system was stronger than that on the separated one.

![Fig.5. Stability prediction lobes of UEVM for different feed per tooth values](image-url)
3.2 Ultrasonic amplitude effect on the UEVM stability

An ultrasonic frequency of 35kHz, a 0.012mm feed per tooth, and a 0.05 radial immersion rate were chosen, while all other parameters were the same as above. The stability prediction diagram of the UEVM system for different ultrasonic amplitudes was plotted, as shown in Fig.6.

Firstly the ultrasonic amplitude in the \( x \)-direction (\( A_x \)) was fixed at 5μm, while that in they-direction (\( B_y \)) took values of 3, 4, and 6μm, respectively. It can be seen from Fig.6 that the stability curve of the system did not rise systematically but fluctuated. At \( B_y=3 \)μm, the stability curve of the system was above the amplitude of 4μm, and the limit values of the axial cutting depth of both reached their maxima at about 3000 r/min. However, the axial cutting depth limit value with \( B_y=3 \)μm was ten times higher than that \( B_y=4 \)μm. For the major half-axis of the ellipse oriented in the \( x \)-direction, the smaller the short half-axis was, the better was the system's stability. The stability curve for \( B_y=6 \)μm was higher than that at \( B_y=4 \)μm. Under the stability curve for \( B_y=3 \)μm, the system’s limit value of the axial cutting depth was the highest near 3100r/min, exceeding that at \( B_y=4 \)μm by about 300%.

Next, \( B_y \) was fixed at 5μm, while \( A_x \) took values of 3, 4, and 6μm, respectively. It can be observed from Fig. 6 that the stability curve of the system with \( A_x=4 \)μm was the lowest. This stability region was very small, while the stability curves of the system with \( A_x=3 \)μm and \( A_x=6 \)μm were tightly intertwined and continuously intersected. Their limit values of the axial cutting depth were equal, and the stability zone areas (shown in green and pink in Fig.6) were the same. In terms of the above six groups of parameters, it can be seen that the parameters \( A_x=5 \)μm and \( B_y=3 \)μm were optimal. The stability region of the system (highlighted in blue in Fig.6) and the limit value of axial cutting depth were the largest. However, for this group of parameters, the stability curves of the system were very smooth at low and medium speeds, while a slight flip bifurcation phenomenon appeared in the curve at relatively high speeds, reducing
the system’s stability area. Thus, at constant values of $A_x$, variations of $B_y$ strongly influenced the system’s stability. However, at constant values of $B_y$, the effect of variations of $A_x$ on the system stability was less pronounced. At $A_x=5\mu m$, the critical point of cutting separation occurred at about 2100r/min with variations of $B_y$. However, at $B_y=5\mu m$, the critical point of the cutting separation phenomenon was permanently shifted to the right: higher spindle speeds corresponded to larger values of $A_x$. This indicated that the ultrasonic amplitude in the $x$-direction ($A_x$) had a stronger effect on the system’s cutting separation than that in the $y$-direction ($B_y$) because the former was in line with the feed direction, so their interaction affected the cutting speed direction of the cutter teeth.

### 3.3 Ultrasonic frequency effect on the UEVM stability

The machining parameters were preset as follows: $A_x=5\mu m$, $B_y=3\mu m$, a feed rate per tooth of 0.004 mm, a radial immersion rate of 0.05, a tool radius of 10 mm, and a number of tool tooth of 4. The stability prediction diagrams of UEVM for different ultrasonic frequencies, namely 35, 45, and 55 kHz, are depicted in Fig.7. It can be seen in Fig.7 that with an increase in ultrasonic frequency, the overall stability prediction curve of the system declined, the stability regions (blue and pink in Fig.7) reduced, and the limit value of the system’s axial cut depth also decreased. The axial cut depth reached the maximum at about 3100 r/min, corresponding to the above three frequencies. However, with the frequency increase from 35 to 45 kHz, the maximum value of the axial cut depth dropped by about three times, while with the frequency increase from 45 to 55 kHz, it decreased by 1.75 times. It can be concluded that for ultrasonic elliptical vibration, the higher the frequency, the worse the stability. As far as the current parameters are concerned, the frequency of 35 kHz was the best for the system’s stability. In addition, it can also be seen from Fig.7 that with the increased ultrasonic frequency, the critical point of cutting separation shifted to the right. As a whole, whether separated or not, the system’s stability decreased with the ultrasonic frequency. The decreasing range of the separated type was smaller than that of the unseparated one. The stability region of the unseparated system changed more dramatically. With increased spindle speed, the stability area of the unseparated type system decreased significantly. The stability curve analysis revealed that the system was the most suitable for operating at 3000r/min, where the optimal stability was observed.

![Fig.7. Stability prediction lobes of UEVM under different ultrasonic frequencies](image-url)
4. Experimental setups and procedures

The UEVM tests were divided into three operations: modal parameter identification tests, coefficients' identification tests of cutting forces, and flutter identification tests. All tests were conducted on a VMC850 CNC machining center, but the experimental setups were slightly different since the processing software packages used in these operations varied.

In the modal parameter identification tests, the method of obtaining the frequency response function of the tool tip was adopted. That is, the piezoelectric acceleration sensors (YD39, 0.5Hz~10kHz) were fixed at the tool tip. Then, it was hammered by an impact hammer (8207). The acceleration signal of the tool tip was picked up using the DASP modal analysis software, which determined the frequency response function (FRF) of the cutter tip. The schematic diagram of the hammering test is presented on the right of part of Fig.8.

![Schematic diagram of hammering test](image)

**Fig.8**. The experimental setup

The modal parameters were determined with no account of coupled modes and summarized in Table 1.

| $M$ (kg) | $C$ (N$\cdot$s/m) | $K$ (N/m) |
|---------|-----------------|----------|
| 0.06954 | 0.948 $\frac{u}{u}$ | 1.042 $\cdot 10^5$ $\frac{u}{u}$ |
| 0.06568 | 2.035 $\frac{u}{u}$ | 1.288 $\cdot 10^5$ $\frac{u}{u}$ |

The cutting force coefficient identification test was used to measure the three-dimensional cutting force in the milling process under specific processing conditions (including specific workpiece tool material matching, tool geometry, etc.). The workpiece material was Ti-6Al-4V alloy and the tool material was polycrystalline diamond (PCD) with TiAlN coating. Mechanical and thermal parameters of test materials and tool geometry are listed in Tables 2 and 3, respectively.

| Parameters | Value |
|-----------|-------|
| Density/kg/m$^3$ | 4.51×10^3 |
| Elastic modulus/GPa | 110 |
| Poisson's ratio | 0.33 |
| Thermal conductivity/W/m$\cdot$K | 7.955 |
| Linear expansion coefficient/ /℃ | 7.89×10^-6 |
| Specific heat / cal/g·℃ | 0.612 |
Table 3. Tool geometry parameters

| Parameters            | value |
|-----------------------|-------|
| Edge diameter /mm     | 10    |
| Stem diameter /mm     | 10    |
| Edge length /mm       | 30    |
| Total length /mm      | 75    |
| Helix angle /degree   | 35    |
| Tool tooth            | 4     |

The cutting force coefficients were assessed according to the adopted model of workpiece and tool. More detail on the applied method can be found elsewhere[36]. The milling force was measured by a dynamometer (9257B, Kistler). The electrical signal from the dynamometer was amplified by an amplifier (5070A) and then fed to a data recorder(2825A). The recorded data were then saved to PC and displayed using Kistler’s Dyno Ware commercial software package. Under the above conditions, cutting force coefficients were as follows: $K_t=6.16\times10^8$ N/m$^2$, $K_r=1.8\times10^8$ N/m$^2$, $K_{te}=9.31$ N/m and $K_{re}=4.931$ N/m.

There are two kinds of methods used in flutter identification. The first method transforms the time domain signal of the experimentally obtained dynamic cutting force into the frequency domain through the fast Fourier transform. According to the vibration frequency, judge whether there is flutter or not. The second method is to assess the surface quality of the workpiece. In this study, the surface roughness of the workpiece was measured via a three-dimensional white interferometer (Talysurf CCI6000, Taylor Hobson). Three points were gauged for each set of parameters to gain reliable data, and the average value was taken as the final result. The obtained milling test parameters are listed in Table 4, and the processing site is shown on the left side of Fig.8.

Table 4. Processing parameters during milling

| Parameters            | Conditions                                                                 |
|-----------------------|-----------------------------------------------------------------------------|
| VMCS800E              | Maximum torque: 35.8 Nm; Maximum spindle speed: 8000 r/min; Motor power:11kW|
|                       | Maximum travel of worktable: X 850mm  Y 510mm  Z 540mm                      |
|                       | Knife handle:MAS403 BT40                                                     |
| Workpiece             | Length x width x height: 20mmx15mmx10mm                                     |
| Spindle speed $n$     | $1000,2000,3000,4000$ r/min                                                |
| Feed per tooth $f_z$  | 0.004,0.008,0.012,0.016 mm/z                                               |
| Milling depth $a_p$   | 0.05, 0.1, 0.15, 0.2 mm                                                     |
| Frequency             | 35 kHz                                                                      |
| Amplitude             | $A_x=5\mu$m, $B_y=3\mu$m/ $A_x=3\mu$m, $B_y=5\mu$m                        |

The milling tests were carried out with two combinations of ultrasonic amplitudes: (1) $A_x=5\mu$m,$B_y=3\mu$m, and (2) $A_x=3\mu$m,$B_y=5\mu$m. Four values of spindle speed, feed per tooth, and axial cutting depth were preset in each case, forming 64 groups of experiments, which were performed via down-milling with no flood coolant. The flutter presence or absence was judged by the workpiece surface roughness, time-domain, and spectrum signals of the cutting forces.
5. Results and discussion

The stability prediction lobes of UEVM were plotted according to the dynamic model’s solutions, as shown in Fig. 9.

In Fig. 9, the red and black lobes denote separated and unseparated UEVM modes, respectively. The stability areas of SUEVM, highlighted in green in Fig. 9, were less than those of unseparated UEVM (blue areas). This implied that the tool and workpiece did not separate with increased spindle speed. Flutter test results are also shown in Fig. 9, where symbols ○ and □ correspond to flutter and stability in UEVM, respectively. The four points A, B, C, and D were tested. The following parameters were preset: spindle speeds of 1250, 1500, 2000, and 3000r/min; axial cutting depths of 0.05, 0.1, and 0.15mm; feed per tooth value of 0.004mm; ultrasonic frequency of 35kHz; ultrasonic amplitudes $A_x$=5mm and $B_y$=3mm. Under the four groups of parameters, the time-domain signals of cutting forces in $x$- and $y$-directions and the spectrum of $y$ were plotted, as shown in Fig. 10.
According to the cutting force spectrum, points B and D with spindle speeds of 1500 and 3000 r/min and axial cutting depths of 0.05 and 0.15mm exhibited a power jump only in the cutter tooth frequency or double frequency without flutter. On the contrary, point A with a spindle speed of 1250 r/min and an axial cutting depth of 0.05mm at frequencies of 778.997 and 888.65Hz, and point C with a spindle speed of 2000 r/min and a cutting depth of 0.1mm at frequencies of 597.63 and 864.03 Hz displayed a power jump. Flutter occurred at point A since these frequencies differed from cutter tooth and spindle rotation frequencies. It also can be seen that the stability of the points on the boundary of the stability lobes of UEVM was not as good as that of the unseparated UEVM. Probably, vibration between the workpiece and the
cutter teeth occurred between their separation and contact in the separated system, influencing the system stability.

The UEVM flutter test results were consistent with the stability prediction curves, which implied that the limit value of the axial cutting depth grew with the rotation speed. At about 3000r/min, this value reached its maximum. With the further rotation speed increase, the stability region first increased and then declined. The time-domain simulation and experimental results matched very well. The stability prediction curve provided a stable band for UEVM rather than a stable or unstable region. This stable band can provide a useful reference for selecting processing parameters for practical applications.

Additionally, the surface roughness of tested workpieces was observed and measured. The machined surfaces in four test points A, B, C, and D, presented a strip-shaped microstructure, as shown in Fig. 11(a)-(d).

The surface roughness values of stable points B and D were smaller than those of unstable points A and C. The experimental results were consistent with the flutter test ones. Besides, the experimentally obtained machining parameters were used in the orthogonal experiment design to study their effect on flutter. The respective results are shown in Fig.12.

![](image)

**Fig.11.** The macrostructure and surface roughness of test points

It can be seen in Fig.12 that under the same processing parameters, the surface quality of the workpiece with ultrasonic amplitudes $A_x=5\mu m$ and $B_y=3\mu m$ was significantly better than that with $A_x=3\mu m$ and $B_y=5\mu m$. In both cases, the workpiece’s surface roughness decreased first and then grew with the spindle speed. The surface quality was the best at a spindle speed of 3000r/min. Compared with 1000r/min, the maximum reduction rate of roughness value was 38% at $A_x=3\mu m$, $B_y=5\mu m$. The axial cutting depth effects on the surface quality were different for the two sets of ultrasonic amplitudes. At $A_x=5\mu m$ and $B_y=3\mu m$, the surface roughness first decreased and then grew with the axial cutting depth, while at $A_x=3\mu m$ and $B_y=5\mu m$, the surface roughness systematically grew. Compared with the axial cutting depth of 0.05mm, the maximum surface roughness value decreased by 38.47% at $A_x=5\mu m$ and $B_y=3\mu m$. 
increasing by 63.28% at ultrasonic amplitudes $A_x=3\mu m$ and $B_y=5\mu m$. For these two pairs of amplitudes, the feed per tooth effects on the surface quality were the same. With an increased feed rate per tooth, the surface roughness first dropped and then grew, but the fluctuation range was small. These experimental findings further confirmed the previous simulation results.

![Graphs showing the effect of machining parameters on surface roughness and axial cutting depth.](image)

**Fig.12.** The revealed effect of machining parameters on flutter

### 6. Conclusions

This study elaborated the dynamic model for the ultrasonic elliptical vibration milling (UEVM) system with separated and unseparated cutter tip motion paths. This model was used to simulate the effects of various machining parameters, such as ultrasonic amplitudes, feed per tooth, and ultrasonic frequency, on the system’s stability. The milling and flutter tests on the Ti-6Al-4V alloy workpiece and polycrystalline diamond (PCD) tool with TiAlN coating were performed, which proved the proposed model’s feasibility. The results obtained made it possible to draw the following conclusions:

1. The coordinate method was comprehensively combined with the discrete method to analyze the ultrasonic milling system stability. This extension of the discrete method opened up a new way of judging the instantaneous chip thickness. On this basis, deep analyses of the UEVM stability were provided.

2. In the UEVM system, the system’s stability curve gradually increased with the feed per tooth, but the amplitude increment decreased. At about 3000r/min, the maximum increase of the axial cutting depth reached 37%.
(3) When ultrasonic amplitude in the x-direction $A_x$ was fixed, a change in the ultrasonic amplitude in the y-direction $B_y$, strongly influenced the system’s stability region and the limit value of the axial cutting depth. At a fixed value of $B_y$, changes in $A_x$ only slightly affected the system’s stability. The system’s stability was the best at $A_x=5\mu m$ and $B_y=3\mu m$.

(4) The stability of separated and unseparated UEVM systems decreased with ultrasonic frequency. For the preset range of machining parameters, the ultrasonic frequency of 35kHz ensured the best stability of the system.

(5) The proposed approach and application of system stability prediction diagrams for selecting machining parameters in the actual milling process are instrumental in improving milling efficiency and quality.

**Author contribution** Yuemin Zhang: Methodology, Investigation, Formal analysis, Writing-review & editing. Xiaobo Wang: Project administration, Formal analysis, Writing-review & editing. Ying Niu: Data curation, Investigation. Bo Zhao: Data curation, Investigation.

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**Data availability** The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Declarations**

**Ethical approval** I would like to declare on behalf of my co-authors that the work described was an application that has not been published previously, and not under consideration for publication elsewhere, in whole or in part.

**Consent to participate** All authors know and agree to be co-authors.

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