Greening File Distribution: Centralized or Distributed?

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Abstract—Despite file-distribution applications are responsible for a major portion of the current Internet traffic, so far little effort has been dedicated to study file distribution from the point of view of energy efficiency. In this paper, we present a first approach at the problem of energy efficiency for file distribution. Specifically, we first demonstrate that the general problem of minimizing energy consumption in file distribution in heterogeneous settings is NP-hard. For homogeneous settings, we derive tight lower bounds on energy consumption, and we design a family of algorithms that achieve these bounds. Our results prove that collaborative p2p schemes achieve up to 50% energy savings with respect to the best available centralized file distribution scheme. Through simulation, we demonstrate that in more realistic cases (e.g., considering network congestion, and link variability across hosts) we validate this observation, since our collaborative algorithms always achieve significant energy savings with respect to the power consumption of centralized file distribution systems.

I. INTRODUCTION

The need for a reduction in the carbon footprint of all human activities while satisfying an ever growing energy demand has triggered the interest on the design of novel energy-efficient solutions in several domains. Specifically, recent studies reveal that the ICT (Information and Communications Technologies) sector is becoming a major contributor to the worldwide energy consumption, comparable to the aviation sector [1]. Furthermore, the energy consumption of the ICT sector is expected to double in the next decade [2], unless new mechanisms and solutions are implemented. This situation has motivated the research community to investigate novel mechanisms and solutions for saving energy in ICT, to be deployed by telecommunication network operators, Internet Service Providers (ISPs), content providers, and datacenter owners [3]–[6]. The proposed approaches in the field of energy efficient networking at either the device level (e.g., new hardware design [7]) or the system level (energy efficient routing [8], [9] or sleep modes in wired and wireless networks [10], [11]) aim to achieve an “energy proportional” network. This is, making the energy consumed by the network proportional to its traffic load. Specifically, hosts (servers and user terminals) are responsible of the major portion of the whole Internet power consumption [2].

Current energy efficient strategies in this domain aim at making the energy consumed proportional to the level of CPU or network activity of hosts, and often imply switching off or to a low power mode the devices when not active. However, energy proportionality of hardware does not suffice to define a complete energy efficient framework for hosts. Indeed, new solutions must be found that implement energy efficient services (e.g. file sharing, web browsing, etc.) to optimize the utilization of hosts and network resources.

In this paper, we focus on the file distribution service, which is one of the most widespread services on the Internet. Indeed, some of the existing file distribution services, such as peer-to-peer (p2p), one-click-hosting (OCH), software release, etc., represent a major fraction of current Internet traffic [12]–[14]. Despite of the importance of these services, to the best of the authors’ knowledge, little effort has been dedicated to understanding and achieving energy-efficiency in the context of file distribution applications. In addition, within the context of corporate/LAN networks, other operations such as software updates are also file distribution processes. All this makes essential to deeply investigate energy-efficiency in file distribution, in order achieve a truly Green Internet.

This paper is a first step into this direction. Our aim is to define the analytical and algorithmic basis for the design of energy efficient file distribution protocols. For this purpose, we first prove that the general problem of minimizing energy consumption in a file distribution process is NP-hard. Hence, we analytically study restricted versions of the problem, yet maintaining a balance between simplicity and applicability in real scenarios. Our analysis defines lower bounds and proposes collaborative p2p optimal (and near-optimal) algorithms for reducing energy consumption in the studied file distribution scenarios. Afterwards, we present an empirical evaluation through simulation, that allows us to (i) validate our analytical results and (ii) relax several assumptions imposed in the analytical study. Simulations show that, even in more realistic cases (considering energy costs associated to on-off state transitions or network congestion), our collaborative p2p schemes achieve significant energy savings with respect to centralized file distribution systems. These savings range between 50% and two order of magnitude depending on the centralized scheme under consideration.

In summary, the main contributions of this paper are the following:

• We prove that the general problem of minimizing energy consumption in a file distribution process is NP-hard.
• We derive lower bounds for the energy consumed in a file distribution process for simple yet realistic scenarios.
• We design algorithms that achieve optimal (or near-
optimal) energy consumption for these simple scenarios.

- We demonstrate that the proposed collaborative p2p scheme is an appropriate approach to reduce the energy consumption in a file distribution process showing an improvement factor of at least 50% with respect to any centralized file distribution schemes in the studied scenarios.
- We perform an empirical simulation study that validates all the previous statements and quantify the energy savings achievable with our algorithms on a representative set of scenarios.

The rest of the paper is structured as follows. Section II provides the network and energy model along with definitions and terminology used throughout the paper. Section III presents theoretical results obtained, in the form of bounds and file distributions schemes. In Section IV we present our simulation study. Section V revises the related work and Section VI concludes the paper.

II. SYSTEM MODEL, PROBLEM DEFINITION AND ASSUMPTIONS

A. System Model and Assumptions

We consider a system of \( n + 1 \) hosts \((n \geq 1)\) that are fully connected via a wired network. One of these hosts, called the server and denoted by \( S \), has initially a file of size \( B \) that it has to distribute to all the other hosts, which we call the clients. We assume that the file is divided into \( \beta \geq 1 \) blocks of equal size \( s = B/\beta \). The set of hosts is denoted as \( \mathcal{H} = \{S, H_0, H_1, \ldots, H_{n-1}\} \), and the set of blocks as \( \mathcal{B} = \{b_0, b_1, \ldots, b_{\beta-1}\} \). We will also use in this paper a set of indexes, defined as \( \mathcal{I} = \{S, 0, \ldots, n-1\} \). For simplicity of notation and presentation, we will often use an index \( i \in \mathcal{I} \) to denote a host, and even talk about host \( i \) instead of host \( H_i \) (or \( S \) when \( i = S \)).

All the hosts in \( \mathcal{H} \) can potentially upload blocks of the file to other hosts (initially only \( S \) can do so). A client can start uploading block \( b_i \) only if it has received \( b_i \) completely. Hosts have upload capacity \( u_i \) and download capacity \( d_i \), for \( i \in \mathcal{I} \). (Observe that the server has upload capacity \( u_S \).) We assume that all capacities are integral. All the hosts are assumed to be identical with respect to processing speed, and to have enough memory to sustain the distribution process. No host can upload more than a block at any given time instant, but can simultaneously upload and download from other hosts. Moreover, it can simultaneously download from multiple hosts as long as the download capacity allows it. We also assume that hosts always upload at their full capacity.

We assume that time in the file distribution process is slotted. Each block transmission between hosts starts and finishes within the same slot. We assume that no host uploads to more than one host in one slot. In general, the slot duration may vary from one slot to the next. However, unless otherwise stated, we will assume during the rest of the paper that all slots have the same duration \( \gamma \). Then, if the process of file distribution starts at time \( t = 0 \), the time interval \([0, \gamma]\) corresponds to slot \( \tau = 1 \) and, in general, slot \( \tau \) spans the time interval \([（τ-1）\gamma, τ\gamma]\). In each slot of a scheme, a host is assigned another host to serve (if any), and the set of blocks it will serve during that slot. Note that hosts can only serve blocks that have been received completely.

In this work we consider only the energy consumed by hosts during the file distribution process. We do not consider the energy consumed by other network devices. In our model, the energy consumption has the following three components:

1. Each host \( i \in \mathcal{I} \), just for being on, consumes power \( P_i \) (when a host is off, we assume that it consumes no power).
2. In addition, each host consumes \( \delta_i \geq 0 \), \( i \in \mathcal{I} \) for each block served and/or received.
3. A host consumes energy while being switched on or off. If host \( i \in \mathcal{I} \) takes time \( \alpha_i \) to switch on or off, the energy consumed by switching is given by \( P_i \alpha_i \).

B. Problem and its Complexity

We define a file distribution scheme, or scheme for short, as a schedule of block transfers between hosts such that, after all the transfers, all the hosts have the whole file. Observe that a scheme must respect the model previously defined. Then, the problem we study in this paper is defined as follows.

Definition 1: The file distribution energy minimization problem is the problem of finding or designing a file distribution scheme that minimizes the total energy consumed.

The bad news is that this problem is NP-hard even if switching on and off is free and there is no additional energy consumption per block (i.e., \( \alpha_i = \delta_i = 0, \forall i \in \mathcal{I} \)). Please refer to Appendix A for the NP-hardness proof. The good news is that, as will be shown later, even though the general problem is NP-hard, by making a few simplifying but still realistic assumptions, we can solve the file distribution energy minimization problem optimally.

C. Additional Assumptions

Henceforth, we assume that all the hosts have the same upload capacity \( u \), and the same download capacity \( d \). We also assume that \( \frac{d}{u} = k \) for some positive integer \( k \). Unless otherwise stated, we assume that hosts are switched on and off instantaneously, i.e., \( \alpha_i = 0, \forall i \), and hence switching consumes no energy.

The uniformity of capacities results in a uniform slot duration, equal to \( \gamma = \frac{\Delta_i}{u} \), for all the block transfers. A host is said to be active in a time slot if it is receiving or serving blocks in the slot. Otherwise, it is said to be idle. The energy \( \Delta_i \) consumed by an active host \( i \in \mathcal{I} \) in one slot can be computed as follows.

\[
\Delta_i = P_i \gamma + \delta_i = \frac{P_i S}{u} + \delta_i = \frac{P_i B}{u \beta} + \delta_i. \tag{1}
\]

Without loss of generality, we assume that \( \Delta_0 \leq \cdots \leq \Delta_{n-1} \).

In some cases below we will assume that the system is energy-homogenous. This means that all hosts have the same energy consumption parameters, i.e., \( P_i = P \) and \( \delta_i = \delta \), for
all \( i \in I \). In such a homogeneous system, also all hosts have the same value of \( \Delta_i = \Delta \). Note that, unless otherwise stated, we assume a heterogeneous system.

Let us consider parameters \( n, k, \beta \) of the file distribution energy minimization problem. Let us define the set of all possible schemes with these parameters by \( Z_{k}^{n, \beta} \). Let \( E(z) \) be the energy consumed by scheme \( z \in Z_{k}^{n, \beta} \).

**Definition 2:** A scheme \( z_0 \in Z_{k}^{n, \beta} \) is energy optimal (or optimal for short) if \( E(z_0) \leq E(z), \forall z \in Z_{k}^{n, \beta} \).

Hence, our objective in the rest of the paper is to find optimal (or quasi-optimal) schemes.

### D. Normal Schemes

To rule out redundant and uninteresting schemes, we will consider only what we call normal schemes. Observe that the block transfers of a scheme \( z \) in a slot \( \tau \) can be modeled as a directed transfer graph with the hosts as vertices and block transfers as edges (see Fig. 1). Then, a normal scheme is a distribution scheme in which there are no idle hosts, there are no slots without active hosts, and each slot has a connected transfer graph. We denote the set of normal schemes with parameters \( n, \beta, \) and \( k \) by \( Z_{k}^{n, \beta} \). From now onwards, we will consider only normal schemes. It is easy to observe that any optimal scheme can be transformed into a normal scheme that is also optimal. Hence, we are not losing anything by concentrating only on normal ones.

Observe that in a transfer graph the out-degree of each vertex is at most 1 (by the upload constraint). Thus, the transfer graph of a slot in a normal scheme can either be a tree (Fig. 1(a)) or a graph with exactly one cycle (Fig. 1(b)). Note also that in a slot with cycle all hosts upload blocks, while in a tree slot there are hosts that do not upload.

#### E. Costs

Let us consider scheme \( z \in Z_{k}^{n, \beta} \). Denote with \( I_{z}^{\tau} \subseteq I \) the indexes of the set of active hosts in time slot \( \tau \) under scheme \( z \).

**Definition 3:** The cost of slot \( \tau \) under scheme \( z \), denoted \( c_{z, \tau} \), is the energy consumed by all active hosts \( I_{z}^{\tau} \) in \( \tau \), i.e.,

\[
c_{z, \tau} = \sum_{i \in I_{z}^{\tau}} \Delta_i
\]

Let \( \tau_{z}^{\tau} \) be the makespan of scheme \( z \), i.e., the time slot in which the distribution of the file is completed. Then, the energy consumed by the scheme \( z \) can be obtained as

\[
E(z) = \sum_{\tau=1}^{\tau_{z}^{\tau}} \sum_{i \in I_{z}^{\tau}} \Delta_i
\]

The cost of a slot, as defined above, does not take into account which host is serving which block to which host. However, the total energy consumption of a scheme also depends on this. Thus, for a better insight on the schemes, we also associate a cost to a block transfer.

We denote the set of blocks downloaded by host \( i \in I \) in slot \( \tau \) under scheme \( z \) by \( S_{i, \tau}^{z} \) and the index of the host serving \( b_j \in S_{i, \tau}^{z} \) as \( serv(j, i) \).

**Definition 4:** We define the cost \( c_{z, i, \tau} \) of a block \( b_j \) received by \( H_i \) under scheme \( z \) as,

\[
c_{z, i, \tau} = D_{j, i}^{\tau} \cdot \Delta_i + U_{i, \tau}^{z} \cdot \Delta_{serv(j, i)}
\]

where, if \( b_j \) is received by \( H_i \) in slot \( \tau \),

\[
D_{j, i}^{\tau} = \begin{cases} 1 & \text{if } j = \min \{ j' | b_{j'} \in S_{i, \tau}^{z} \} \\ 0 & \text{Otherwise} \end{cases}
\]

\[
U_{i, \tau}^{z} = \begin{cases} 1 & \text{if } S_{serv(j, i), \tau}^{z} = \emptyset \\ 0 & \text{Otherwise} \end{cases}
\]

\( D_{j, i}^{\tau} \) accounts for the energy consumption of host \( H_i \) (in units of \( \Delta_i \)) that is receiving the block. A block contributes to the energy consumed by \( H_i \) if it is downloading. If a host is downloading more than one block in parallel, then we assume that only one block adds to the cost, as the rest of the blocks can be received without incurring any further cost. \( U_{i, \tau}^{z} \) accounts for the energy consumption of the host that is serving the block when \( S_{serv(j, i), \tau}^{z} = \emptyset \) (the host that is serving \( b_j \) to \( H_i \) is not downloading any block).

With the above definition, the sum of the costs of all blocks transferred in slot \( \tau \) should be equal to the cost of the slot \( \tau \), \( c_{z, \tau} \). The next result establishes that this is indeed true for all the schemes. The proof can be found in Appendix B.

**Theorem 1:** The sum of the costs of all the blocks transferred during slot \( \tau \) is equal to the cost of that slot, i.e.,

\[
\sum_{i \in I_{z}^{\tau}} \sum_{b_j \in S_{i, \tau}^{z}} c_{z, i, \tau} = c_{z, \tau}
\]

Thus, we can express the energy of a scheme \( z \) in terms of the cost of blocks \( c_{z, i, \tau} \) as

\[
E(z) = \sum_{i=0}^{n-1} \sum_{j=0}^{\beta-1} (\Delta_i \cdot D_{j, i}^{\tau} + \Delta_{serv(j, i)} \cdot U_{i, \tau}^{z})
\]
III. Theoretical Analysis

In this section we provide analytical results for the file distribution energy minimization problem, under the additional assumptions described previously. The results in this section are classified depending on the ratio \( k \) between the download and upload capacities. First, we derive lower bounds on the energy consumption, and provide optimal schemes for the case \( k = 1 \). For \( k > 1 \), we provide optimal and near-optimal bounds and algorithms.

A. Download Capacity = Upload Capacity

In this setting, a host can download at most one block during a slot. We first provide lower bounds on the energy consumed by any scheme. Then, we present several optimal schemes, and we derive the value of \( \beta \) that minimizes the energy of optimal schemes in energy-homogenous systems.

1) Lower Bound: The following theorem provides a lower bound on the energy consumed by any distribution scheme when \( k = 1 \).

**Theorem 2**: The energy required by any scheme \( z \) to distribute a file divided into \( \beta \) blocks among \( n \) clients when \( k = d/u = 1 \), satisfies

\[
E(z) \geq \beta \left( \Delta_S + \sum_{i=0}^{n-1} \Delta_i \right) + \max\{0, n - \beta\} \min\{\Delta_S, \Delta_0\}
\]

The key observation behind this result is that each host has to be active for at least \( \beta \) slots to receive the file, whereas the server has to be active for at least \( \beta \) slots to upload one copy of each block among the clients. The proof of the theorem can be found in Appendix C.

2) Optimal Distribution Schemes: We now present optimal schemes achieving the lower bound of Theorem 2. We distinguish among three cases, depending on the relation between \( n \) and \( \beta \), and we indicate the resulting schemes as Algorithms 1, 2, and 3. Note that in pseudocode, the transfer of block \( b_j \) from host \( H \) to host \( H' \) is expressed as \( H \xrightarrow{b_j} H' \). Also, all the transfers that occur in the same slot are enclosed by the lines \texttt{begin slot} and \texttt{end slot}. While the three algorithms could be merged into a single one, we have chosen to present them separately for clarity.

We now provide some intuition on the algorithms. We start from Algorithm 1 which assumes that the number of clients is equal to the number of blocks. As each host has to be active at least \( \beta \) slots to receive the complete file, Algorithm 1 makes sure that the hosts are active for exactly \( \beta \) slots. In the first \( n \) slots of the algorithm, the server uploads a different block of the file to each of the \( n \) clients. Since \( n = \beta \), the server can upload the whole file to the clients in \( n \) slots. Then the server goes off. At this point, all the hosts have one block and they all need to get the remaining \( n - 1 \) blocks. Each client chooses a client to serve, in a way that the resulting transfer graph is a cycle of \( n \) nodes. All the hosts start uploading the latest block they have received, and this process continues for \( \beta - 1 \) slots, until all the hosts have all the blocks.

Algorithm 2 which assumes \( n < \beta \), is more involved, but uses similar ideas as Algorithm 1. In Fig. 2, we present a toy example of an scheme obtained from Algorithm 2. In Algorithm 3, the number of clients is larger than the number of blocks. Thus some hosts will have to upload the same block more than once. In this algorithm, after that the server has served the first \( \beta \) blocks, the host with the smallest energy consumption per slot uploads block \( b_0 \) to those hosts without any block.

**Theorem 3**: When \( d = u \), Algorithms 1, 2, 3 describe optimal distribution schemes, with energy

\[
E(z) = \beta \left( \Delta_S + \sum_{i=0}^{n-1} \Delta_i \right) + \max\{0, n - \beta\} \min\{\Delta_S, \Delta_0\}
\]

For the proof, please refer to Appendix D. In what follows, with \( \text{Opt}(n, \beta) \) we indicate the algorithm corresponding to the values of \( n \) and \( \beta \).

![Fig. 2. Example of Algorithm 2 for \( n = 3 \) and \( \beta = 4 \). The label on each arrow is the index of the block being served.](image-url)

**Algorithm 1** Optimal scheme for \( \beta = n 

```
1: for \( j = 0 : n-1 \) do 
2: \( \text{begin slot} \)
3: \( S \xrightarrow{b_j} H_j \)
4: \( \text{end slot} \)
5: end for
6: for \( j = n : 2n-2 \) do
7: \( \text{begin slot} \)
8: for \( i = 0 : n-1 \) do
9: \( H_i \xrightarrow{b_{(i+j) \mod n}} (H_{(i-1) \mod n} \text{ end for}) \)
10: \( \text{end for} \)
11: \( \text{end slot} \)
12: end for
```

3) Optimal Number of Blocks in Energy Homogenous Systems: In this section we consider an energy-homogenous system, in which all hosts have the same energy consumption parameters, i.e., \( P_i = P \) and \( \delta_i = \delta \), for all \( i \in I \). In this system we want to find the optimal value of \( \beta \) into which the file should be divided for minimum energy consumption. Intuitively, the number of blocks into which the file must be divided depends on the value of \( \delta \). If \( \delta \) is very large, then it is better to divide the file in a small number of blocks, since each block transmission consumes additional energy \( \delta \). On the other hand, if \( \delta \) is small, we can divide the file into a number of blocks such that the energy consumed is reduced due to concurrent transfers.

The following theorem presents the optimal value of \( \beta \).

**Theorem 4**: In an energy-homogenous system with \( k = d/u = 1 \), the value of \( \beta \) that minimizes the energy consump-

Algorithm 2 Optimal scheme for $\beta > n$

1: for $j = 0 : n - 1$ do
2: begin slot
3: $S \xrightarrow{\beta} H_j$
4: end slot
5: end for
6: for $j = n : \beta - 1$ do
7: begin slot
8: $S \xrightarrow{\beta} H_{n-1}$
9: for $i = 1 : n - 1$ do
10: $H_i \xrightarrow{\beta} H_{i-1}$
11: end for
12: end slot
13: end for
14: for $j = \beta : \beta + n - 2$ do
15: begin slot
16: $H_i \xrightarrow{\beta} H_{i+1}$
17: for $i = 1 : n - 1$ do
18: $H_i \xrightarrow{\beta} H_{i+1}$
19: end for
20: end slot
21: end for

Algorithm 3 Optimal scheme for $\beta < n$. $H_{\min}$ is the host with smallest $\Delta_j$. ($H_{\min} \in \{S, H_0\}$.)

1: for $j = 0 : \beta - 1$ do
2: begin slot
3: $S \xrightarrow{\beta} H_j$
4: end slot
5: end for
6: for $j = \beta : n - 1$ do
7: begin slot
8: $H_{\min} \xrightarrow{\beta} H_{k-1}$
9: for $i = 1 : \beta - 1$ do
10: $H_{i+j-\beta} \xrightarrow{\beta} H_{i+j-1}$
11: end for
12: end slot
13: end for
14: for $j = n : n + \beta - 2$ do
15: begin slot
16: $H_{2n-(j+1)} \xrightarrow{\beta} H_{n+j+1}$
17: for $i = 0 : \beta - 2$ do
18: $H_{(n+i-j) \mod n} \xrightarrow{\beta} H_{n+i-j}$
19: end for
20: end slot
21: end for

The derivation of this bound is based on proving that the required number of tree slots is at least $n$, because there are $n$ clients. For the complete proof, please refer to Appendix F.

2) (Quasi-)Optimal Distribution Schemes: Observe that the energy consumption of Algorithms 1 and 2 in an energy homogenous system with $\beta \leq n$ is exactly $n(\beta + 1)\Delta$ (Theorem 5). Hence, these algorithms describe optimal schemes for this system. However, if $\beta > n$, the algorithm for $k = 1$ (Algorithm 2) is not optimal anymore if $k > 1$. In this section we present an algorithm, namely Algorithm 4 that describes a distribution scheme for this case. In fact, the scheme works with $k = 2$, as no host has more than two downloads in parallel.

Algorithm 4 distributes the file among the clients using ideas from Algorithms 1 and 2. We represent the state of process with a two dimensional array $A$ of size $n \times \beta$ (Fig. 3) with the rows and the columns representing the clients and the blocks, respectively. We set an entry $A_{ij} = 1, i \in \{0, 1, \ldots, n-1\}, j \in \{0, 1, \ldots, \beta - 1\}$ if and only if $H_i$ has received $b_j$, and 0 otherwise. At the beginning, all the entries are 0 and after the completion of the algorithm they all should be 1. Furthermore, imagine the array $A$ divided in $\lfloor \frac{\beta}{n} \rfloor - 1$ square subarrays of size $n \times n$ and one rectangular subarray of size $n \times (n + b)$. (Note that this is just a conceptual division to understand Algorithm 4 in terms of Algorithms 1 and 2.)

After the first loop, the diagonal of the first square subarray is set to 1, i.e., $A_{ii} = 1, \forall i \in \{0, 1, \ldots, n-1\}$. Additionally, after the second loop, the top left corner position (see Fig. 3) of each subarray has also been set to 1, i.e., $A_{ij} = 1, \forall j \in \{0, n, 2n, \ldots, (\lfloor \frac{\beta}{n} \rfloor - 1)n\}$. In each iteration of the for loop at Line 12, the elements of one of the subarrays of $n \times n$ are set to 1 by serving in the same fashion as in Algorithm 1 while the server completes serving the diagonal of the next square/rectangular subarray. When Line 22 is reached, all the elements of all the square subarrays are marked as 1. The remaining blocks are served using Lines 20-22 of Algorithm 2 with an appropriate relabeling of the blocks.

We present the bounds achieved in this section in the following theorem. The proof of the second claim can be found in Appendix G.

Theorem 6: In a homogeneous system with $k > 1$,

- If $\beta \leq n$, then Algorithms 1 and 2 describe optimal distribution schemes with energy $E(z) = n(\beta + 1) \cdot \Delta$.

Note that if the value of $\sqrt{\frac{PB}{u\delta}}$ is not an integer, it has to be rounded to one of the two closest integer values, such that $E(\beta)$ is minimum.

B. Download Capacity > Upload Capacity

In this subsection, we consider an energy homogenous system in which $k > 1$.

1) Upper Bound: In this section, we present a lower bound on the energy of a schedule in an energy homogenous system with $k > 1$. In this setting, the possibility to download more than one block in a slot implies that the minimum number of slots in which a host has to be on can be less than $\beta$. 

$$\beta = \min \left\{ \sqrt{\frac{PB}{u\delta}} \right\} \quad (6)$$
In this section we briefly present a description of the scheme used to distribute a file across hosts.

2) File Distribution Schemes: The file distribution schemes that we have considered in the performance evaluation are:

- **Opt**: This is the file distribution scheme detailed in Section III-A. It is a distributed scheme, where the upload capacity for distributing the file is made available by the same hosts that are downloading the file.

- **Parallel**: This is a centralized scheme, in which all users download the same file at the same time from the same server in parallel. This is one of the most common architectures for file distribution, and it models a large number of file distribution services present in the current Internet (e.g., One Click Hosting systems such as Megaupload or RapidShare).

- **Serial**: In this centralized scheme, the server uploads in sequence the complete file to the hosts involved in the file distribution process. That is, the server uploads the complete file to the first host. Once it finishes, it uploads the file to the second host, and so on. We consider this scheme because when \( u_i = d_i \) it minimizes the amount of time each host is

\( u = d = 10 \text{ Mbps} \). Finally, unless otherwise stated, we consider a scenario with one server and 200 hosts.

This homogeneous scenario models a corporate network in which both the network infrastructure and the whole set of devices belong to the same company/organization, and are centrally managed. Typical file distribution processes in this context are software updates (e.g., OS, antivirus), which are usually centrally coordinated by system administrators. These environments are typically characterized by a relatively high uniformity in the network infrastructure and in the user terminals, especially if compared with the Internet. It is expected that communications among hosts in this type of intranet scenario happen at high bit rates, and that the bottleneck for file transfers happens at the terminals rather than in the network. Finally it is worth to mention that, in these settings, energy expenditure is a concern for the organization, as it directly impacts the OPEX of the IT infrastructure.

- **Heterogeneous scenario**: In this setting, we analyze the impact of heterogeneity in host configurations on the performance of our schemes. This scenario captures the case in which hosts are typical Internet nodes (including home users), and it is therefore characterized by a significant variability across hosts in both the energy consumption profile and the observed network performance (i.e., different access speed and congestion conditions). In this case, the file distribution process is represented by, for instance, a software being released\(^1\) (e.g., a new Linux distribution). In this scenario, the incentive for saving energy comes from corporate and individual sensibility towards reducing the carbon footprint, since the potential economical benefits for a single host are usually negligible.

In this setting we assume \( u_i = d_i, \forall i \in I \). In order to simplify our study, in our experiments we consider separately the effect of heterogeneity in power consumption and the effect of varying network conditions.

2) File Distribution Schemes: The file distribution schemes that we have considered in the performance evaluation are:

- **Opt**: This is the file distribution scheme detailed in Section III-A. It is a distributed scheme, since the upload capacity for distributing the file is made available by the same hosts that are downloading the file.

- **Serial**: This is a centralized scheme, in which all users download the same file at the same time from the same server in parallel. This is one of the most common architectures for file distribution, and it models a large number of file distribution services present in the current Internet (e.g., One Click Hosting systems such as Megaupload or RapidShare).

- **Parallel**: This is a centralized scheme, in which all users download the same file at the same time from the same server in parallel. This is one of the most common architectures for file distribution, and it models a large number of file distribution services present in the current Internet (e.g., One Click Hosting systems such as Megaupload or RapidShare).

\(^1\)Other applications such as entertainment content (video, music) file distribution also fit into this scenario.
active in order to receive a file, and therefore the amount of energy spent by each host in the distribution process. This is realized at the expense of the server, who has to remain on for the whole duration of the scheme.

3) Energy Model: For our experiments we considered two different energy models. In a first one, the hosts only have two power states: an OFF state, in which they do not consume anything, and an ON state, in which they consume the full nominal power, equal to 80W (typical nominal power consumption for notebooks and desktop PCs lies in the range 60W-80W [15]). Unless otherwise stated, this is the default energy model for our experiments.

In order to understand the impact of load proportional energy consumption in our schemes, we consider a model that fits most of the current network devices [15], in which the energy consumed has some dependency on the CPU utilization and network activity. This energy model is characterized by four states. Besides the OFF state, the other states are: the IDLE state, in which the device is active but not performing any task, and consuming 80% of the nominal power; the TX-or-RX state, in which the device is active and either transmitting or receiving, and consuming 90% of the nominal power; the TX-and-RX state, in which the device is active and both transmitting and receiving, and consuming its full nominal power. We considered this model to analyze the impact of load proportionality on the overall energy consumption of the schemes considered in our experiments.

In Section IV-C1 we analyze the effect of having devices with heterogeneous power consumption profiles. For this purpose we use the previously described two-state model, but we assume that for each host its nominal power consumption is drawn from two different distribution: (i) a Gaussian distribution with an average of 80 W and a standard deviation of 20 W, and (ii) an exponential distribution, with an average of 80 W.

Note that, despite large servers typically present a larger nominal power, in our experiments we assign to the server the same nominal power as a regular host. This assumption is consistent with our intention to be conservative in our study, since our schemes require the server to be active far less time than the serial and parallel schemes.

4) Goodness Metric: The goodness metric we have used in order to compare the energy consumption of different file distribution schemes is energy per bit, computed as the ratio of the total amount of energy consumed by the distribution process, divided by the sum of the sizes of all the files delivered in the scheme.

B. Homogeneous Scenario

1) Validation of the Analysis: In Fig. 3 we have plotted the energy per bit consumed by the file distribution process as function of the size of the file, for the three different file distribution schemes considered. As we can see, our schemes perform consistently better than both serial and parallel schemes. In particular, by maximizing the amount of time in which hosts serve while being served, our schemes tend towards reducing by half the total energy cost of serving a block with respect to the serial scheme. This performance improvement with respect to the serial scheme is due to the use of (p2p-like) distribution, and indeed it decreases as the file size (and the number of blocks into which it is split) decrease. With respect to the serial scheme, our optimal schemes make the most out of the energy consumed by all hosts which are active and being served at a given time, by having them contributing as much as possible to the file distribution. As a consequence, despite each host spends more time in an active state than in the serial scheme, the net effect is a decrease of the total energy.

Moreover, we can also observe how the parallel scheme performs consistently worse than any other scheme, consuming up to two orders of magnitude more than the serial scheme. Since the utilization of this parallel scheme is widespread in the current Internet, our observations confirm the great potential of distributed schemes for saving energy.

Fig. 4 also depicts the performance of our Opt algorithm for different number of hosts (50, 200, and 400). We observe that the energy per bit consumed by our algorithm as well as by the serial scheme are not affected by the number of hosts in the scheme. Hence for the rest of the section we will present results exclusively for a setting with 200 hosts.

Finally, it is worth noting that, for the optimal scheme, the nonsmooth variation of the energy per bit with file size, observable at low values of file size, is due to quantization in the number of blocks. The serial and parallel schemes (for which there is no partition of the file into blocks) have a smoother behavior with respect to file size.

2) Block Size: The impact of the total number of blocks on the energy consumed by our Opt scheme can be seen in Fig. 3 where we plotted the energy per bit consumed with Opt for variable file sizes, and for a total of 200 hosts. The green curve corresponds to the case in which a fixed block size, equal to 256 kB, is used, while the lower red one is obtained by using an optimal block size, according to the formula in Section III-A3. We see how the use of an optimal block size leads to an increment in energy savings mainly for small file sizes. The reason is that for small file sizes a fixed block size leads to a small number of blocks, and consequently to exploit less the distributed (p2p-like) mechanisms which, in our scheme, improve the efficiency of the distribution process.

3) ON/OFF Energy Costs: As seen in previous sections, our optimal algorithms develop in rounds. Typically, not every host is on in every round (i.e., some go on and off more than once during the file distribution process). In a realistic scenario, a host takes some time to both go off (or into a very low power mode), and to get back to active mode. Usually, this on/off time is in the order of a few seconds [16]. The additional amount of energy consumed while switching between these power states (that we call here “on/off costs”) has potentially an important impact on the energy performance of a scheme, penalizing specifically those schemes in which host activity is more “discontinuous” over time.

In order to mitigate the negative impact of on/off costs,
in our simulations we implement the following mechanism.
When a host $A$ has finished its activity (i.e. uploading or downloading a block) in an slot $t_1$, and has no activity until slot $t_2$, it computes the energy cost of staying on ($\text{cost}_{\text{on}}$) until the slot $t_2$ and the cost of going off during the rest of slot $t_1$ and switching on at the beginning of slot $t_2$ ($\text{cost}_{\text{off(on)}}$. Hence, if $\text{cost}_{\text{on}} \leq \text{cost}_{\text{off(on)}}$, $A$ decides to stay on. Otherwise, it goes off for its non-active period between slots $t_1$ and $t_2$.

Fig. [9] presents the energy consumed by our scheme in comparison to the serial scheme considering a switch on/off time equal to 2 and 4s. As expected, the on/off costs increase the energy per bit consumed by all schemes. This increment is more pronounced for small file sizes, where we see that on/off costs make the performance of our scheme closer (but still better) to the serial scheme. Conversely, for medium/large file sizes, the contribution of on/off costs to the total energy consumed by a scheme becomes marginal, and the performance of both the optimal scheme and the serial approaches the one in the case without on/off costs. Note the widening of the gap between the serial scheme and our scheme for file sizes around 50MB is due to the different behavior that our scheme has for the case $n < \beta$ and for the other case.

4) Load Dependency: In this set of experiments, we have analyzed the impact of the four-states energy model described in Section [IV-A3] which implies some degree of energy proportionality of the host devices. The research community is putting a lot of effort in energy proportionality. Hence, in the future it is expected that network devices will consume energy proportionally to the supported load. Fig. [7] shows that with the four-states energy model the percentual decrease in the energy per bit consumed by our $Opt$ scheme and by the serial one is the same. This suggests that even with load proportional hardware our scheme enables significant energy savings with respect to the serial one.

C. Heterogeneous Scenario

In this subsection we consider two separated heterogeneous scenarios. On the one hand, we study the case in which different hosts present different power consumption profiles. On the other hand, we address the scenario in which each host observes different network conditions (i.e., different access speed and congestion level).

1) Heterogeneous Power Consumption: In Section [III-A] we have proved analytically that our $Opt$ algorithm minimizes the overall power consumption of the file distribution process, even in a heterogeneous scenario in which each host presents a different energy consumption (as long as all the nodes have the same upload and download rate). To validate this statement, in this subsection we have run experiments in which the nominal power consumed by the hosts varies according to either a Gaussian or an exponential distribution as defined in Section [IV-A3]. Then, the energy consumption has been compared with a homogeneous scenario. The results, presented in Fig. [8] validate our analysis, since the three curves for the $Opt$ scheme overlap perfectly. We also observe that heterogeneous power consumption has some minor impact in the case of the serial scheme. Finally, it is worth to note that confidence intervals have been calculated for each curve (but not shown for clarity), being in any case lower than 5%.
2) **Heterogeneous Network Conditions:** In the results presented we have considered (i) similar upload/download access speed for all host and (ii) no network congestion. In this subsection we relax these assumptions, and consider a heterogeneous scenario where hosts have different access speeds and observe different network state (e.g., congestion). This scenario accurately models a content distribution process in the Internet.

In particular, in the simulations we model the different nominal access speed of hosts using an exponential distribution, based on realistic speed values provided in [17]. Additionally, in order to model the variation in link speed over time due to network conditions (i.e., congestion) we multiply the nominal access speed by a positive factor taken from a Gaussian distribution with average 1 and standard deviation 0.07. Fig. 9 presents the results for these heterogeneous network conditions, for both our Opt scheme and the serial scheme, and compares them with the homogeneous case. The results show that both schemes suffer from an increment in the power consumption, with respect to the homogeneous case. However, the relative difference between the Opt and serial schemes increases. This suggests that even in heterogeneous network conditions the proposed algorithm outperforms any centralized scheme.

Moreover, we observe that the energy per bit consumed is constant for both Opt and serial schemes when considering heterogeneous network conditions. This occurs because none of the considered schemes takes into account host upload/download capacity in determining the schedule for file distribution.

Finally, note that confidence intervals have been obtained for the different curves and all of them present less than 5% difference to the average value in the figure.

**V. RELATED WORK**

**Energy-Efficiency in Networks:** In order to reduce the overall energy consumption of the Internet, many dimensions for energy savings have been explored. The main efforts include turning off the devices that are unnecessarily on [10], [11], aggregating traffic streams to send data in bulk [10], [18], [19], network planning [20], energy efficient routing [8], [9] and virtualization and migration of routers [21]. Furthermore, some works have addressed specific aspects of energy-efficiency in datacenters [5], [22], [23].

**Optimization problems in file-distribution processes:** An important amount of effort has been dedicated to study the completion download time in a file distribution process [24–26]. The minimization of the average finish time in P2P networks is considered in [27–29]. Of interest to this paper, [30] presents a theoretical study to derive the minimum time associated to a P2P file distribution process. However, an scheme guaranteeing a file distribution with minimum time does not generally leads to minimize the energy consumption. Moreover, schemes with similar distribution time may have different energy costs.

**Energy-Efficiency in file distribution:** To the best of the authors knowledge energy consumption in file distribution processes has received little attention so far. On the one hand, practical studies [6], [31]–[34] have discussed and compared the energy consumed by different content distribution architectures or protocols. However none of them relies on an analytical basis nor aims to design optimal algorithms, as is the case of our paper. On the other hand, Mehyar et al. [35] and Sucevic et al. [36] (similarly as we do) address the energy-efficiency in file-distribution from an analytical point of view. However, their studies are restricted to P2P schemes whereas the current paper cover both centralized and distributed approaches in order to identify the most efficient scheme. In addition, their analysis is limited to networks of at most 3 nodes. For bigger network sizes, they provide heuristics and use simulations to evaluate energy efficiency. Instead, our analysis is valid for an arbitrary number of nodes. Finally, it is worth to mention that, to the best of our knowledge, we are the first on providing a proof of the NP-hardness of the energy-efficiency optimization problem for file-distribution processes.

**VI. CONCLUSIONS**

This paper presents one of the first dives into a novel and relevant field that has received little attention so far: energy-efficiency in file distribution processes. We present a theoretical framework that constitutes the analytical basis for the design of energy-efficient file distribution protocols. Specifically, this framework reveals two important observations: (i) the general problem of minimizing the energy consumption in a file distribution process is NP-hard and (ii) in all the studied scenarios there exists always a collaborative (i.e. p2p-like) distributed algorithm that reduces the energy consumption of any centralized counterpart. This suggests that in those file distribution processes in which reducing the energy consumption is of significant importance (e.g. software update over night in a corporative network) a distributed algorithm should be implemented.

**REFERENCES**

[1] ICT and CO2 emissions. [Online]. Available: http://www.parliament.uk/documents/post/post1319.pdf

[2] M. Pickavet, W. Vereecken, S. Demeyer, P. Audenaert, B. Vermeulen, C. Develder, D. Colle, B. Dhoedt, and P. Demeester, “Worldwide energy needs for ICT: The rise of power-aware networking,” in ANTS, 2008.

[3] L. Chiariavaggio, M. Mellia, and F. Neri, “Minimizing isp network energy cost: Formulation and solutions,” IEEE/ACM Transactions on Networking, 2011.

[4] E. Goma, M. Canini, A. Lopez Toledo, N. Laoutaris, D. Kostić, P. Rodriguez, R. Stanoević, and P. Yagüe Valentin, “Insomnia in the access: or how to curb access network related energy consumption,” in ACM SIGCOMM, 2011.

[5] B. Heller, S. Seetharaman, P. Mahadevan, Y. Yikoumis, P. Sharma, S. Banerjee, and N. McKeown, “ElasticTree: saving energy in data center networks,” in NSDI, 2010.

[6] V. Valancius, N. Laoutaris, L. Massoulie, C. Diot, and P. Rodriguez, “Greening the internet with nano data centers,” in ACM CoNEXT, 2009.

[7] R. Bolla, R. Bruschi, F. Davoli, and F. Cucchietti, “Energy efficiency in the future internet: A survey of existing approaches and trends in energy-aware fixed network infrastructures,” Communications Surveys & Tutorials, IEEE, vol. 13, no. 2, pp. 223–244, 2011.

[8] J. Restrepo, C. Gruber, and C. Machuca, “Energy profile aware routing,” in Communications Workshops, IEEE ICC 2009., 2009, pp. 1–5.
[9] M. Andrews, A. Fernández Anta, L. Zhang, and W. Zhao, “Routing for energy minimization in the speed scaling model,” in Transactions of Networking. Accepted for publication in 2011, DOI: 10.1109/TNET.2011.2159864, 2010.

[10] M. Gupta and S. Singh, “Greening of the internet,” in SIGCOMM, 2003.

[11] Y. Agarwal, S. Hodges, R. Chandra, J. Scott, P. Bahl, and R. Gupta, “Somniloquy: augmenting network interfaces to reduce pc energy usage,” in NSDI, 2009, pp. 365-380.

[12] C. Gkantisidis, T. Karagiannis, and M. Vojnovic, “Planet scale software updates,” in ACM SIGCOMM, 2006.

[13] C. Labovitz, S. Lekel-Johnson, D. McPherson, J. Oberheide, and F. Jahanian, “Internet inter-domain traffic,” in ACM SIGCOMM, 2010.

[14] Sandvine fall 2011 global internet phenomena report,” http://www.sandvine.com/news/global_broadband_trends.asp.

[15] B. Nordman and K. J. Christensen, “Greener pcs for the enterprise,” IT Professional, vol. 11, no. 4, pp. 28-37, 2009.

[16] “in windows 7 use sleep to resume the os in 2 seconds,” http://news.softpedia.com/news/In-Windows-7-Use-Sleep-To-Resume-the-OS-in-2-Seconds-101290.shtml.

[17] “The real connection speeds for internet users across the world (charts),” http://royal.pingdom.com/2010/11/12/real-connection-speeds-for-internet-users-across-the-dyi-world/.

[18] S. Nedevschi, L. Popa, G. Iannaccone, S. Ratnasamy, and D. Wetherall, “Reducing network energy consumption via sleeping and rate-adaptation,” in ACM SIGCOMM, 2008.

[19] M. Andrews, A. Fernández Anta, L. Zhang, and W. Zhao, “Routing and scheduling for energy and delay minimization in the powerdown model,” in IEEE INFOCOM, 2010.

[20] J. Chabarek, J. Sommers, P. Barford, C. Estan, D. Tsang, and S. Wright, “Power awareness in network design and routing,” in INFOCOM, 2008, pp. 457-465.

[21] Y. Wang, E. Keller, B. Biskeborn, J. van der Merwe, and J. Rexford, “Virtual routers on the move: live router migration as a network-management primitive,” in ACM SIGCOMM, 2008.

[22] D. Abts, M. R. Marty, P. M. Wells, P. Klausler, and H. Liu, “Energy proportional datacenter networks,” SIGARCH Comput. Archit. News, vol. 38, pp. 338-347, June 2010.

[23] B.-G. Chun, G. Iannaccone, G. Iannaccone, R. Katz, G. Lee, and L. Niccolini, “An energy case for hybrid datacenters,” SIGOPS Oper. Syst. Rev., vol. 44, pp. 76-80, March 2010.

[24] R. Kumar and K. Ross, “Peer-assisted file distribution: The minimum distribution time,” in IEEE Workshop on Hot Topics in Web Systems and Technologies (HOTWEB ’06), 2006, pp. 1-11.

[25] M. Lingjum, P. Tsang, and K. Lui, “Improving file distribution performance by grouping in peer-to-peer networks,” IEEE Transactions on Network and Service Management, vol. 6, no. 3, pp. 149-162, 2009.

[26] T. Langner, C. Schindelhauer, and A. Soura, “Optimal file-distribution in heterogeneous and asymmetric storage networks,” SOFSEM 2011: Theory and Practice of Computer Science, pp. 368-381, 2011.

[27] S. Sanghavi, B. Hajek, and L. Massoulie, “Gossiping with multiple messages,” IEEE Transactions on Information Theory, vol. 53, no. 12, pp. 4640-4654, 2007.

[28] L. L. A. G. Matthew Ezovski, Ao Tang, “Minimizing average finish time in p2p networks,” in IEEE Infocom, 2009.

[29] K.-S. L. Pui-Sze Tsang, Xiang Meng, “A novel grouping strategy for reducing average distribution time in p2p file sharing,” in IEEE ICC, 2010.

[30] J. Munding, R. Weber, and G. Weiss, “Optimal scheduling of peer-to-peer file dissemination,” Journal of Scheduling, vol. 11, no. 2, pp. 105-120, 2008.

[31] U. Lee, I. Rimac, D. Kilper, and V. Hilt, “Toward energy-efficient content dissemination,” Network, IEEE, vol. 25, no. 2, pp. 14-19, 2011.

[32] A. Feldmann, A. Gladisch, M. Kind, C. Lange, G. Smarkagakis, and F. Westphal, “Energy trade-offs among content delivery architectures,” in IEEE Telecommunications Internet and Media Techno Economics (CTIE), 2010, pp. 1-6.

[33] J. Blackburn and K. Christensen, “A simulation study of a new green bittorrent,” in Communications Workshops, ICC, 2009, pp. 1-6.

[34] A. P. Giuseppe Anastasi, Ilaria Gianiotti, “A bittorrent proxy for green internet file sharing: Design and experimental evaluation,” Computer Communications, vol. 33, no. 7, pp. 794-802, 2010.

[35] M. Mehryar, W. Gu, S. Low, M. Effros, and T. Ho, “Optimal strategies for efficient peer-to-peer file sharing,” in Acoustics, Speech and Signal Processing, ICASSP, vol. 4, 2007.

[36] A. Sucevic, L. Andrew, and T. Nguyen, “Powering down for energy efficient peer-to-peer file distribution,” 2011.

APPENDIX

A. NP-hardness

We show in the section that a general version of the problem considered in this paper is NP-hard. The following theorem summarizes the result.

Theorem 7: Assume that time is slotted, that hosts must upload at their full capacity, and that no host can upload to more than one host in the same slot. The problem of minimizing the energy of file distribution is NP-hard if hosts can have different upload capacities and power consumptions, even if \( \alpha_i = \delta_i = 0, \forall i \).

Proof: We use reduction from the partition problem. The input of this problem is a set of integers (we assume all of them to be positive) \( A = \{x_0, x_2, \ldots, x_{k-1}\}, k > 1 \). Let \( M = \sum_{x_i \in A} x_i \) to be even. The problem is to decide whether there is a subset \( A' \subset A \) such that \( \sum_{x_i \in A'} x_i = M/2 \).

We reduce an instance of the partition problem to an instance of our problem as follows. The file to distribute has \( M \) blocks of size 1. There are \( n = k+3 \) hosts: server \( S \), hosts \( T \) and \( R \), and hosts \( H_i \), for \( i \in [0, k-1] \). All hosts have fixed setup energy \( \delta_i = 0 \) and no cost for switching on and off, i.e., \( \alpha_i = 0 \). Server \( S \) has upload capacity \( M \) and power \( P \). Host \( T \) has download and upload capacity \( M \), and power \( P \). Hosts \( H_i \), \( i \in [0, k-1] \), have download capacity \( M \), upload capacity \( u_i = x_i \), and power consumption \( P \). Host \( R \) has download capacity \( M/2 \) and power consumption \( P' > 2P(2k+1) \). The slot length is one unit of time.

Observe that there is always a feasible solution that respects the assumptions of the model. It works as follows. First, \( S \) serves the whole file to \( T \) in one slot. Then, \( T \) serves the whole file to hosts \( H_i \), \( i \in [0, k-1] \), in consecutive slots. Finally, each host \( H_i \), \( i \in [0, k-1] \), serves \( x_i \) different blocks to \( R \) in consecutive slots.

We claim that the subset \( A' \) that satisfies \( \sum_{x_i \in A'} x_i = M/2 \) exists if and only if the file distribution problem can be solved with energy smaller than \( 3P' \). Hence, the energy minimization problem is NP-hard.

If subset \( A' \) exists, the following schedule is feasible. First, \( S \) serves \( T \) the whole file in one slot. Then, \( T \) serves each host \( H_i \), \( i \in [0, k-1] \), the whole file in consecutive slots. Let \( U = \cup_{x_i \in A'} \{H_i\} \), then the hosts in \( U \) upload the file to \( R \) in two slots, half the file in each slot. The total energy consumed is

\[
E = 2P + 2Pk + 2(|A'|P + P') \leq 2P(2k+1) + 2P' < 3P'.
\]

Assume now that there is a schedule with energy less than \( 3P' \). Then, \( R \) has been up two slots. Since they cannot upload at full capacity to \( R \), and they cannot serve more than one host, neither \( S \) nor \( T \) can serve \( R \). Then, looking at the first slot in which \( R \) is up, \( R \) must have been served by a subset of hosts \( H_i \) whose aggregate upload capacity is exactly \( M/2 \). This proves the existence of \( A' \).
B. Proof of Theorem 2

We transform the cost of a block as defined in Equation 3 to the following one. For each host \( i \in \mathcal{I}_z \), define \( \phi_i \) and \( \psi_i \) as

\[
\phi_i = \begin{cases} 
\Delta_i & \text{if } S_{z,\tau}^i \neq \emptyset \\
0 & \text{otherwise} 
\end{cases}
\]

\[
\psi_i = \begin{cases} 
\Delta_i & \text{if } S_{z,\tau}^i = \emptyset \\
0 & \text{otherwise} 
\end{cases}
\]

Note that \( \sum_{j \in \mathcal{S}_{z,\tau}^i} D_{z,j,i}^\tau = 1 \) iff \( |S_{z,\tau}^i| \geq 1 \) (i.e., when \( \phi_i = \Delta_i \)). It is easy to see that \( U_{z,j,i}^\tau = 1 \) iff \( \psi_{\text{serv}(j,i)} = \Delta_{\text{serv}(j,i)} \), i.e., \( S_{\text{serv}(j,i),\tau}^i = \emptyset \). Therefore, for a host \( i \in \mathcal{I}_z \), either \( \phi_i = \Delta_i \) or \( \psi_i = \Delta_i \), never both 0 or both \( \Delta_i \). Hence,

\[
\sum_{i \in \mathcal{I}_z} (\phi_i + \psi_i) = \sum_{i \in \mathcal{I}_z} \Delta_i
\]

C. Proof of Theorem 2 (Lower Bound for most block in a time slot \( \tau \))

We now prove the claim. In order to compute the minimum energy consumption, we need to lower bound Equation 5 from Lemma 2 and Corollaries 1 and 2

\[
\sum_{i=0}^{n-1} \sum_{j=0}^{\beta-1} \Delta_{\text{serv}(j,i)} U_{z,j,i}^\tau \geq \beta \Delta_S + \max\{0, n-\beta\} \min\{\Delta_S, \Delta_0\}
\]

Adding Equations 10 and 11 the claim follows.

D. Proofs of Correctness and Optimality for \( k = 1 \)

For the correctness and optimality proofs of a scheme \( z \) (described by an algorithm), we define the state \( \sigma_{z,\tau}^i \), of a host \( i \in \mathcal{I} \) at the end of slot \( \tau \) as the set of blocks held by that host at the time. Thus, to start with, initially for \( S \) we have, \( \sigma_{z,0}^i = \emptyset \), and, for each client \( i \in \{0, ..., n-1\} \), \( \sigma_{z,0}^i = \emptyset \).

If \( z \) is correct, after the makespan of \( z \) (\( \tau^f \) slots) the state of every client \( i \in \{0, ..., n-1\} \) must be \( \sigma_{z,\tau^f}^i = \emptyset \). We omit \( z \) and \( \tau \) when clear from the context.

1) Algorithm 2: Let us denote the scheme described by Algorithm 1 as \( z_1 \). This scheme has the following properties.

Observation 1: After the for loop at Lines \([15]\) the state of client \( i \) is \( \sigma_i = \{b_i\}, \forall i \in \{0, ..., n-1\} \).

Observation 2: After the \( q^{\text{th}} \) iteration of the loop at Lines \([6,12]\) for \( q \in \{0, ..., n-1\} \), each host \( H_i, i \in \{0, ..., n-1\} \) has

\[
\sigma_i = q \cup \{b_{i+p} \mod n\} \quad (12)
\]

Proof: We prove the claim by induction on \( q \). The base case \( (q = 0) \) holds from the observation: After the for loop at lines \([15]\) \( \sigma_i = \{b_i\} \).

Assuming the hypothesis to be true for \( q-1 \), in the \( q^{\text{th}} \) iteration \( H_i \) receives block \( b_{i+q+1} \mod n \). In this iteration, the value of \( j \) is \( j = q + n - 1 \). Hence, \( H_i \) receives \( b_{i+q} \mod n \), and the state after the \( q^{\text{th}} \) iteration is

\[
\sigma_i = q \cup \{b_{i+p} \mod n\} = q \cup \{b_{i+p} \mod n\} \quad (13)
\]

Lemma 5: In every iteration of the for loop at Lines \([6,12]\) host \( H_i, i \in \{0, ..., n-1\} \) serves one of the blocks it has already downloaded.

Proof: In the \( q^{\text{th}} \) iteration, \( q \geq 1 \), \( H_i \) serves block \( b_{i+q} \mod n \). From the previous lemma, after the \( (q-1)^{\text{th}} \) iteration, the state of \( i \) is

\[
\sigma_i = q \cup \{b_{i+p} \mod n\} \quad (14)
\]

which includes \( b_{i+q-1} \mod n \). Hence the claim follows.

Theorem 8: After the termination of Algorithm 1 each client \( H_i, i \in \{0, ..., n-1\} \), has received all the blocks \( b_j \mod n \) with optimal energy \( E(z_1) = n(\Delta_S + \sum_{i=0}^{n-1} \Delta_i) \).

Proof: It follows from Lemma 4 that after the \( (n-1)^{\text{th}} \) iteration of the loop at Lines \([6,12]\) each host has received all the blocks. The scheme is then correct, since each host serves a block it has already downloaded (Lemma 5). Each host (including the server) is active exactly \( n \) slots. Then, the
total energy consumed is \( E(z_i) = n(\Delta_S + \sum_{i=0}^{n-1} \Delta_i) \), which is optimal since it matches the lower bound.

2) Algorithm 2. Let us denote the scheme described by Algorithm 2 as \( z_2 \). This scheme has the following properties.

**Observation 2:** After the for loop at Lines 15, the state of client \( i \) is \( \sigma_i = \{b_i\} \), \( \forall i \in \{0, ..., n-1\} \).

**Lemma 6:** After the \( q \)-th iteration of the loop at Lines 6, for \( q \in \{0, 1, ..., \beta - n\} \), each host \( H_i, i \in \{0, ..., n-1\} \), has state

\[
\sigma_i = \bigcup_{p=0}^{q-1} \{b_{i(p)+q}\} \cup \{b_{i(q)+1}\} = \bigcup_{p=0}^{q} \{b_{i(p)+q}\}
\]

**Proof:** We use induction on \( q \) to prove the lemma. The base case \( (q = 0) \) follows from the observation.

Induction step: Assume the hypothesis to be true for the \((q-1)\)-th iteration. Client \( H_i, i \in \{0, ..., n-2\} \) receives block \( b_{q(i)+q} \) in the \( q \)-th iteration, while client \( H_{n-1} \) receives block \( b_{q(n-1)+1} \) from the server. Thus, \( \forall i \in \{0, ..., n-1\} \), the state of client \( H_i \) after the \( q \)-th iteration is

\[
\sigma_i = \bigcup_{p=0}^{q-1} \{b_{i(p)+q}\} \cup \{b_{i(q)+1}\} = \bigcup_{p=0}^{q} \{b_{i(p)+q}\}
\]

**Lemma 7:** After the \( q \)-th iteration of the loop at Lines 14, for \( q' \in \{0, 1, ..., n-1\} \), each host \( H_i, i \in \{0, 1, ..., n-1\} \), has state

\[
\sigma_i = \bigcup_{p=0}^{q' + \beta - n} \{b_{i(p)+q'} \mod \beta\}
\]

**Proof:** We use induction on \( q' \) to prove the claim. The base case \( (q' = 0) \) follows from Lemma 6 with \( q = \beta - n \). Let the claim (induction hypothesis) be true for the \((q' - 1)\)-th iteration. In the \( q \)-th iteration, the value of \( j = q' + 1 - \beta \). Hence, \( H_i \) receives block \( b_{i(q'} + \beta - n\). Thus, the state of client \( H_i \) after the \( q \)-th iteration is

\[
\sigma_i = \bigcup_{p=0}^{q' - 1 + \beta - n} \{b_{i(p)+q'} \mod \beta\} \cup \{b_{i(q') + \beta - n} \mod \beta\} = \bigcup_{p=0}^{q' + \beta - n} \{b_{i(p)+q'} \mod \beta\}
\]

**Lemma 8:** During the execution of Algorithm 2, each host \( H_i, i \in \{0, ..., n-1\} \) serves a block that it has already downloaded.

**Proof:** Let us consider the loops at Lines 6 and Lines 14-20 in sequence. In the \( q \)-th iteration of these loops, host \( H_i \) serves block \( b_{i(q-1) \mod \beta} \). From the previous lemmas, after the \((q-1)\)-th iteration of these loops, host \( H_i \) has state

\[
\sigma_i = \bigcup_{p=0}^{q-1} \{b_{i(p) \mod \beta}\}
\]

which includes \( b_{i(q-1) \mod \beta} \). Hence, the claim follows.
Lemma 11: During the execution of Algorithm 3 each host \( H_i, i \in \{0, 1, \ldots, n - 1\} \), serves a block that it has already downloaded.

Proof: In the for loop at Lines 6-13 during iteration \( q = j + 1 - \beta, q \in \{1, \ldots, n - \beta\} \), block \( b_{r_{q}} \) is served by \( H_{r_{q-1}} \). It has it because after iteration \( q - 1 \),
\[
\zeta_{r} = \bigcup_{p=0}^{q-1} \{H_{r+p}\},
\]
which includes \( H_{r_{q-1}} \). \( H_0 \) always serves \( b_0 \), if any, which it has from the above observation.

In the for loop at Lines 14-21 during iteration \( q' = j + 1 - n, q' \in \{1, \ldots, \beta - 1\} \), block \( b_{\beta - 1} \) is served by \( H_{n-q'} \). It has it because after iteration \( q' - 1 \),
\[
\zeta_{\beta - 1} = \bigcup_{p=0}^{q'-1} \{H_{\beta+p-1}\} \bigcup \{H_{\beta-1-p}\} \bigcup \{H_{\beta-1-q}\}
\]
which includes \( H_{n-q'}, \forall q' \in \{1, 2, \ldots, \beta - 1\} \).

Block \( b_{r}, r \in \{0, 1, \ldots, \beta - 2\} \) is served by \( H_{(r-(q'-1)) \mod n} \). It has it because after iteration \( q' - 1 \),
\[
\zeta_{r} = \bigcup_{p=0}^{n-\beta} \bigcup_{p=0}^{q'-1} \{H_{r+p}\} \bigcup \{H_{(r-p) \mod n}\}
\]
which includes \( H_{(r-(q'-1)) \mod n} \). Hence, the claim follows.

Theorem 10: After the termination of Algorithm 3 each host \( H_i, i \in \{0, 1, \ldots, n - 1\} \) has received all the blocks \( b_r \in B \) with optimal energy \( E(z_3) = \beta \left( \Delta_S + \sum_{i=0}^{n-1} \Delta_i \right) + (n - \beta) \min \{\Delta_S, \Delta_0\} \).

Proof: It follows from Lemma 10 that each host has received all the blocks as it has already downloaded (Lemma 11).

We need to bound now the energy consumed. Let us denote \( \Delta_{\min} = \min \{\Delta_S, \Delta_0\} \). The energy consumed in the loop at Lines 13 is easily observed to be
\[
E_1 = \beta \Delta_S + \sum_{i=0}^{\beta - 1} \Delta_i
\]

The energy consumed in the loop at Lines 6-13 is
\[
E_2 = \sum_{j=\beta}^{n-1} \left( \Delta_{\min} + \Delta_{j+1-\beta} + \sum_{i=1}^{\beta - 1} \Delta_{i+j+1-\beta} \right)
= (n - \beta) \Delta_{\min} + \sum_{j=\beta}^{n-1-\beta} \Delta_{j+1-\beta} + \sum_{j=\beta - 1}^{n-\beta - 1} \Delta_{j+1-\beta}
= (n - \beta) \Delta_{\min} + \sum_{j=0}^{\beta - 1} \sum_{i=0}^{\beta - 1} \Delta_{i+j+1}
\]

Finally, the energy consumed in the loop at Lines 14-21 is
\[
E_3 = \sum_{j=n}^{n+\beta - 2} \Delta_{n+\beta-j-2} + \sum_{i=0}^{\beta - 1} \Delta_{n+i-j-1} \mod n
= \sum_{j=n}^{n+\beta - 2} \beta-1 \sum_{i=0}^{\beta - 1} \Delta_{n+i-j-1} \mod n
= \sum_{j=0}^{\beta - 1} \sum_{i=0}^{\beta - 1} \Delta_{i+j-1} \mod n
\]

Adding Equation 20, 21 and 22 we get,
\[
E(z_3) = E_1 + E_2 + E_3
= \beta \Delta_S + (n - \beta) \Delta_{\min} + \sum_{i=0}^{\beta - 1} \Delta_i
+ \sum_{j=0}^{\beta - 1} \sum_{i=0}^{\beta - 1} \Delta_{i+j+1} \mod n
= \beta \Delta_S + (n - \beta) \Delta_{\min} + \sum_{j=0}^{\beta - 1} \sum_{i=0}^{\beta - 1} \Delta_{i+j+1}
\]

which is optimal.

E. Proof of Theorem 7

From Theorems 2 and 3 the energy consumption of an optimal scheme \( z \) in an energy homogeneous system is
\[
E(z) = (n \beta + \max [n, \beta]) \cdot \frac{PB}{u \beta + \delta}
\]

To find the optimal value of \( \beta \), we need to minimize the right hand side of Equation 23. This can be written as a function of \( \beta \) as
\[
E(\beta) = \begin{cases} 
\frac{PB}{u} (n + 1) + \frac{\delta (n+1)}{\beta}, & \beta \geq n \\
\frac{nPB}{u} \left( 1 + \frac{1}{\beta} \right) + \delta n (\beta + 1), & \beta \leq n
\end{cases}
\]

Note that in Equation 24 the first term is a constant and the second is linear in \( \beta \). This is a straight line with positive slope \( \delta (n+1) \). Hence, the function attains the minimum at the lower extreme \( \beta = n \), where it intersects Equation 25. Hence it is enough to consider Equation 25 for \( \beta \leq n \). Minimizing Equation 25 with respect to \( \beta \) we get,
\[
\beta = \sqrt{\frac{PB}{u \delta}}
\]

When this value is larger than \( n \) the value \( \beta = n \) has to be used.
**F. Proofs of Theorem 5**

**Proof:** It can be easily observed that every slot in which a host receives its first block is a tree slot (since it does not serve anyone). Additionally, no two clients can receive their first block in the same slot in a normal scheme. Then, there are at least \( n \) tree slots.

According to Definition 1, the cost \( c_{z,j,i} \) of a block can only take values 0, \( \Delta \), or \( 2\Delta \). Let us consider a slot \( \tau \). We denote with \( \#0 \), \( \#1 \), and \( \#2 \) the number of blocks whose cost is 0, \( \Delta \), and \( 2\Delta \) in \( \tau \), respectively. Then, we can prove that if \( \tau \) is a tree slot, then \( \#2 = \#0 + 1 \), while if \( \tau \) is a slot with a cycle, then \( \#2 = \#0 \). The proof of this claim goes as follows. From Theorem 1, the cost of all blocks in \( \tau \) add up to the cost of \( \tau \). Since all hosts have the same \( \Delta \), then 
\[
0 \cdot \#0 + 1 \cdot \#1 + 2 \cdot \#2 = |I_{z\tau}|
\]
Hence the claim follows.

This implies that, if \( x \) blocks are served in slot \( \tau \), the cost of \( \tau \) is 
\[
c_{z\tau} = x\Delta \text{ if } \tau \text{ is a slot with a cycle, and } c_{z\tau} = (x+1)\Delta \text{ if } \tau \text{ is a tree slot.}
\]
Since the total number of blocks served is \( n\beta \) and there are at least \( n \) tree slots, the bound follows. 

**G. Proofs of Algorithm 4**

The proof of correctness of Algorithm 4 can be divided in essentially four parts. (We use the array abstraction for clarity.) The first claim is that, after the first loop (Lines 2-6), the diagonal of the first subarray has been filled. (I.e., \( A_{ii} = 1 \), \( \forall i \in \{0,...,n-1\} \).) This claim follows trivially by inspection. The second claim is that after the second loop (Lines 7-11), the top left corner position of each subarray has also been set to 1. (I.e., \( A_{ij} = 1 \), \( \forall j \in \{0,n,2n,...,([\frac{\beta}{n}] - 1)n\} \).) This claim also follows by inspection.

The third claim is that, after the \( q^{th} \) iteration of the third loop (Lines 12-21), the whole \( q^{th} \) subarray and the diagonal of the \((q+1)^{th}\) subarray have been set to 1 (and the blocks served by a host were available at the host for being served). This can be shown by induction on \( q \), where the base case is the first claim above. In the induction step, the proof that the whole \( q^{th} \) subarray is set to 1 is similar to the proof of Algorithm 1. The proof that the diagonal of the \((q+1)^{th}\) subarray is set follows from the second claim above and Line 15 of the algorithm.

Finally, the fourth claim is that the process described in Line 22 completes the array. The proof of this claim is very similar to the proof of Algorithm 4.

Let us now compute the energy consumed by the scheme described by the algorithm. The first loop consumes energy 
\[
E_1 = 2n\Delta.
\]
The second loop consumes 
\[
E_2 = 2(\lfloor \beta/n \rfloor - 1)\Delta.
\]
The third loop uses energy 
\[
E_3 = \Delta \sum_{i=0}^{\lfloor \beta/n \rfloor - 2} \sum_{j=0}^{n-2} (n+1) = \Delta((\lfloor \beta/n \rfloor - 1)(n^2 - 1))
\]
Finally, the energy consumed by the process described in