ABSTRACT

In this paper, we propose a generalized opinion dynamics model (GODM), which can dynamically compute each person’s expressed opinion, to solve the internal opinion maximization problem for social trust networks. In the model, we propose a new, reasonable and interpretable confidence index, which is determined by both person’s social status and the evaluation around him. By using the theory of diagonally dominant, we obtain the optimal analytic solution of the Nash equilibrium with maximum overall opinion. We design a novel algorithm to maximize the overall with given budget by modifying the internal opinions of people in the social trust network, and prove its optimality both from the algorithm itself and the traditional optimization algorithm-ADMM algorithms with $l_1$-regulations. A series of experiments are conducted, and the experimental results show that our method is superior to the state-of-the-art in four datasets. The average benefit has promoted $67.5\%$, $83.2\%$, $31.5\%$, and $33.7\%$ on four datasets, respectively.

Index Terms— Opinion Maximization, Confidence index, Social Trust Networks, Optimization, Limited budget

1. INTRODUCTION AND MOTIVATION

The study of how people form their opinions through social interactions with others has long been the subject of research in the field of social science. On social networks, users can express their opinions on a product or a real-time new event. By quantifying these opinions, and forming the expressed opinions through social interactions with others, the overall opinion of a network on the product or event can be determined. This allows one to quantitatively study interactions of opinions on a large scale, and to improve them through targeted interventions. Generally speaking, for the merchant, the overall opinion of the network on the product as high as possible is expected. In order to realize it, if possible, the overall opinion can be adjusted through a certain budget. For the government, when a positive real-time new event occurs, it hopes that the overall opinion of this event as high as possible. Conversely, when a negative one happens, the government hopes that the overall opinion as low as possible. It is obviously that the minimization of the overall opinion under the premise of a certain budget can be achieved by converting the original network to a negative network.

1.1. Background

Social trust network is a relatively stable network of relationships among certain groups (people, enterprises and organizations). A social trust network and its graph representation can be seen in Fig. 1. Many opinion formation models have been proposed and studied based on the influence through social interaction, and the issue of opinion or influence maximization has been widely studied in [1] and [2]. Chen et al. [1] proposed an algorithm to find $K$ nodes which perform the greatest common influence on the network. Cong et al. [3] considered an issue called active opinion maximization, whose goal is to find a subset of seed users to maximize the overall opinion dissemination of the target product in a multi-round campaign. In addition to considering interventions that directly modify people’s internal opinions, Abebe et al. [4] also considered interventions that to change people’s sensitivity to persuasion. The goal was to modify individual sensitivities to maximize the overall opinion at Nash equilibrium. And [2] proposed a new continuous-valued opinion dynamics model for social trust networks, which was more consistent...
1.2. Contributions in this paper

- We develop a generalized opinion dynamics model for social trust networks to precisely calculate the opinion everyone express. And we introduce a novel confidence index $\alpha_i$, which is determined by a person’s social status and the evaluation of people around him. In the traditional dynamics model, a person’s opinion is usually expressed as a weighted average of his internal opinion and the opinions of those around him. For different human groups, the range of $\alpha_i$ will be different. For example, when there are mostly young people in the group, the value of $\alpha_i$ tends to be relatively low, and when most people in the group are elderly, the value of $\alpha_i$ will be higher. From the theory and experiments, we can see that when $\alpha_i = 1/2$, our model GODM is the same one as the model in [2].

- By using the theory of diagonally dominant and Friedkin-Johnsen model, we give the analytic solution in GODM when the Nash equilibrium is reached.

- We introduce a novel algorithm and prove that the result is optimal. In addition, we introduce a mature optimization algorithm named ADMM, and solve the problem with special form derived from this algorithm-$\ell_1$ regulation and achieve the same result as our algorithm, which further verifies the optimality of our algorithm.

2. PRELIMINARIES

2.1. Definition and proposition used in our model

**Definition 1** (Strictly diagonally dominant). Square matrix $A = (a_{ij})_{n \times n}$ is strictly diagonally dominant if the absolute value of its main diagonal element is greater than the sum of the absolute values of the other elements in the row, i.e., $|a_{ii}| > \sum_{j \neq i} |a_{ij}|$, $i = 1, \ldots, n$.

**Proposition 1.** Let $A = (a_{ij})_{n \times n}$ be a square matrix that is strictly diagonally dominant, then $\det(A) \neq 0$.

**Proof.** Suppose $\det(A) = 0$, then the system of linear equations $Ax = 0$ has non-zero solution $x = (x_1, x_2, \ldots, x_n)^T$. Suppose the largest one of $|x_1|, \ldots, |x_n|$ is $|x_k|$, then $|x_k| > 0$. Known by $\sum_{j=1}^n a_{kj}x_j = 0$, we have

$$\sum_{j \neq k} a_{kj}x_j = -a_{kk}x_k = |a_{kk}| |x_k| > |x_k| \sum_{j \neq k} |a_{kj}| \geq \sum_{j \neq k} |a_{kj}||x_j| = \sum_{j \neq k} |a_{kj}x_j|,$$

which is a contradiction. \hfill $\square$

2.2. Friedkin-Johnsen Model

We can represent a social trust network with a directed signed weighted graph $G = (V, E)$, where $V = \{1, 2, \ldots, n\}$ is the set of nodes and $E$ is the set of arcs (directed edges). Each directed edge from $i$ to $j$, $(i, j) \in E$, is associated with the weight $\omega_{ij}$, which shows the extent of the relationship. Let $A = (a_{ij})$ be the adjacency matrix of $G$, where $a_{ij} = \omega_{ij}$ if $(i, j) \in E$, $a_{ij} = 0$ otherwise. Let $D$ be a diagonal matrix with diagonal entry $d_{ii} = \sum_{j=1}^n |a_{ij}|$, and $L = D - A$ be the Laplacian matrix.

The Friedkin-Johnsen model [3] is a famous dynamics model of continuous-valued opinion. In this model, each node $i \in V$ has a fixed internal opinion $s_i$ that stays the same. During each iteration, the expressed opinion $z_i$ of node $i$ is updated as follows:

$$z_i = s_i + \sum_{j \in N^+(i)} \omega_{ij} z_j \left(1 + \sum_{j \in N^+(i)} \omega_{ij}\right),$$

where the values of $s_i$ and $z_j$ are in the interval $[-1, 1]$. $N^+(i)$ denotes the set of successors of node $i$, and $\omega_{ij}$ denotes the weight of the directed edge $(i, j)$. In a social game, the result of repeated average converges to the Nash equilibrium [6].

2.3. Generalized Opinion Dynamics Model (GODM) For Social Trust Networks

It can be seen from the Friedkin-Johnsen model that for each node $i$, its expressed opinions are the weighted average of its internal opinions and the opinions of the people around it. However, in real life, people’s personalities are different, which lead to the different degrees they are affected by other people. So we develop a generalized opinion dynamics model for social trust networks.

The consensus cost function of the social game is computed as follows:

$$c(z_i) = \alpha_i (z_i - s_i)^2 + (1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}| (z_i - \text{sgn}(i, j) z_j)^2,$$

where $\text{sgn}(i, j)$ denotes the sign of nonzero $\omega_{ij}$, and $\alpha_i$ denotes the confidence index of node $i$. The confidence index $\alpha_i$ reflects how confident everyone in a social network is. In social networks, a higher $\alpha_i$ indicates that a person is more likely to listen to his or her own thoughts when expressing opinions, rather than being easily influenced by others. In a real social network, the $\alpha_i$ value is related to its importance in the network and how others evaluate it.
Theorem 2.1. Let \( \Lambda = \text{diag}(\alpha_1, \ldots, \alpha_n) \) be a diagonal matrix, where \( \alpha_i \) is the confidence index of node \( i \) in a social trust network, and let \( L \) be the Laplacian matrix of the social trust network. Then \( \Lambda + (I - \Lambda)L \) is invertible.

**Proof:** It is clear that

\[
(\Lambda + (I - \Lambda)L)_{ii} = \alpha_i + (1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}|.
\]

The sum of the absolute values of the off-diagonal elements of \( \Lambda + (I - \Lambda)L \) is equal to the sum of the absolute values of the off-diagonal elements of \( (I - \Lambda)L \), which is \((1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}|.\) For \( 0 < \alpha_i < 1, \) \( \Lambda + (I - \Lambda)L \) is the diagonally dominant matrix, as desired. \( \square \)

**Proposition 2.** Let \( s \) be the internal opinion vector, whose \( i \)-th component is \( s_i \), and \( z^* \) be the expressed opinion vector at the Nash equilibrium. We have \( z^* = (\Lambda + (I - \Lambda)L)^{-1}s \).

**Proof.** At the Nash equilibrium, we compute expressed opinion \( z^*_i \) that minimizes its cost for each node \( i \), i.e., \( c'(z^*_i) = 0 \). Hence,

\[
c'(z^*_i) = 2\alpha_i(z^*_i - s_i) + 2(1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}|(z^*_i - s_n(i, j)z^*_j) = 0.
\]

It follows that

\[
z^*_i = \frac{\alpha_i s_i + (1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}| z^*_i}{\alpha_i + (1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}|}.
\]

We rewrite Equation \( \text{(2.3)} \) as:

\[
(\alpha_i + (1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}|)z^*_i = \alpha_i s_i + (1 - \alpha_i) \sum_{j \in N^+(i)} |\omega_{ij}| z^*_j.
\]

By introducing the opinion vectors \( s \) and \( z^* \), we can transform Equation \( \text{(2.3)} \) as its matrix form: \((\Lambda + (I - \Lambda)L)z^* = \Lambda s\). According to the Theorem 2.1, \( \Lambda + (I - \Lambda)L \) is invertible. Therefore, we obtain that \( z^* = (\Lambda + (I - \Lambda)L)^{-1}s \). \( \square \)

**PageRank:** PageRank is a technology which is calculated by search engines based on the hyperlinks between pages \( [7] \). It is used to show the relevance and importance of web pages.

For a page \( v_i \), \( r(v_i) = \frac{d_i}{\sum d_j} + d \left( \frac{r(u_1)}{c(u_1)} + \cdots + \frac{r(u_N)}{c(u_N)} \right) \), where \( r(v_i) \) denotes the PageRank value of page \( v_i \), \( u_i \) refers to one of the pages that points to \( v_i \), \( c(u_i) \) denotes the number of edges that point to other pages for page \( u_i \), \( N \) is the number of nodes in the social network. \( d \) is the damping coefficient, usually \( d = 0.85 \). In a large network, the PageRank value for each point may be very small. So we normalized the PageRank values of points by dividing by the maximum PageRank value among all points.

**Mean evaluation:** In social networks, each node plays two roles, the output node and the input node. As an input node, its edge weight and edge sign represent the evaluation that other nodes on it. Here, for a node \( i \), we use mean evaluation to express the average evaluation that other nodes on it, so for node \( i \), the calculation formula of mean evaluation that others on it can be expressed as: \( m(i) = \frac{\sum_{j \in N^-(i)} \omega_{ij}}{d^+_i} \), where \( m(i) \) denotes the mean evaluation of node \( i \), \( N^-(i) \) is the predecessors of node \( i \), and \( d^+_i \) denotes the amount of the predecessors of node \( i \).

**Compute the confidence index \( \alpha_i \):** Here, let’s consider two scenarios. First, the confidence index \( \alpha_i \) is fixed, which doesn’t change with the structure of the graph. Second, the confidence index \( \alpha_i \) is adjustable. Here we let the confidence index \( \alpha_i \) weighted by PageRank value and mean evaluation, which is calculated by the following formula:

\[
\alpha_i = q m(i) + (1-q)r(i),
\]

where \( m(i) \) and \( r(i) \) are the mean evaluation and PageRank value of node \( i \), respectively, and \( 0 \leq q \leq 1 \) is a constant. When \( q = 1 \), the value of \( \alpha_i \) is completely determined by the PageRank value, and when \( q = 0 \), the value of \( \alpha_i \) is determined by what other people say about it. In this case, let’s take \( q = 0.5 \).

In real social trust networks, such as a network of users’ reviews of a website. Due to the trust and distrust between users, this may result in a negative mean evaluation of a user. And the PageRank value may be very small, which may result in the calculated \( \alpha_i \) value, the confidence index, being negative. This is unreasonable, so we introduce the Rectified Linear Unit, which restricts the value of \( \alpha_i \) to \([0, 1]\). So Equation \( \text{(2.4)} \) becomes the following:

\[
\alpha_i = \text{ReLU}(q m(i) + (1-q)r(i)),
\]

where \( \text{ReLU}(x) = \max(0, x) \). From Equation \( \text{(2.5)} \), we can see that the more important a person is in the network and the higher people’s evaluation of him, the higher his confidence index will be.

**Definition 2 (Overall opinion).** The overall opinion \( p(z^*) \) of a social trust network is the sum of the expressed opinions at Nash equilibrium: \( p(z^*) = z^* \), where \( 1 \) is the all-ones column vector.

**3. INTERNAL OPINION MAXIMIZATION IN SOCIAL TRUST NETWORKS**

**Definition 3 (Internal opinion problem).** Given a social trust network \( G \), the internal opinion vector \( s \), and the budget amount \( \mu \), our goal is to find out the modification of internal opinions \( \Delta s \) to maximize the overall opinion \( p(z^*) \):

\[
\max 1^T(\Lambda + (I - \Lambda)L)^{-1}s + \Delta s,
\]

s.t.

\[
\begin{align*}
-1 & \leq s_i + \Delta s_i \leq 1, \quad (i = 1, 2, \ldots, n), \\
\|\Delta s\|_1 & \leq \mu.
\end{align*}
\]
According to the Proposition 2, the expressed opinion vector is in the column space of $\Lambda + (I - \Lambda)L$ and the coordinates are stored in the vector of internal opinions. Then, we name $g = 1^T(\Lambda + (I - \Lambda)L)^{-1}\Lambda$ the contribution index vector. As $p(z^*) = 1^T(\Lambda + (I - \Lambda)L)^{-1}\Lambda s = gs$, the $i$-th element of $g$ represents the contribution of the internal opinion of node $i$ to the overall opinion. Specifically, the contribution index is defined as follows.

**Definition 4 (Contribution index).** The contribution index $g_i$ of node $i$ represents how $i$’s internal opinion contributes to the overall opinion and is quantified by:

$$g_i = (1^T(\Lambda + (I - \Lambda)L)^{-1}\Lambda)_i.$$  

To maximize the overall opinion, we change the internal opinion with the largest absolute value of the contribution index of the node, then the maximum benefit can be obtained under the same budget. Our approach is summarized in Algorithm 1. It is further proved that Algorithm 1 outputs the optimal result of the internal opinion problem.

### Algorithm 1: Solving Internal Opinion Problem

**Input:** Social trust network $G = (V,E)$; Budget $\mu$; Randomly generate internal opinions $s$.

**Output:** The modification $\Delta s$.

begin
  Compute $g_i = (1^T(\Lambda + (I - \Lambda)L)^{-1}\Lambda)_i$;
  Set $\text{sign} = \frac{g_i}{|g_i|}$;
  while $\mu > 0$ do
    Find the largest $|g_i|$, cost $= 1 - \text{sign} \cdot s_i$;
    if cost $> \mu$ then
      $\Delta s = \text{sign} \cdot \mu$, $\mu = 0$;
    else
      $\Delta s = \text{sign} \cdot \text{cost}$, $\mu = \mu - \text{cost}$;
  end
end

**Theorem 3.1.** Algorithm 1 outputs the optimal result of the internal opinion problem.

**Proof.** According to the Algorithm 1, we know the benefit is $g\Delta s$. For any intervened node $i$, we can adjust the corresponding modification $\Delta s_i$ to $(\Delta s_i - v)$ if $\Delta s_i > 0$, or $(\Delta s_i + v)$ otherwise, where $v > 0$. Then we have an unused budget $v$ and randomly find a node $j$ that is not intervened before. As the internal opinion of node $j$ is changed, the benefit becomes $g\Delta s + (|g_j| - |g_i|)v$. We know from Algorithm 1 that $|g_j| \leq |g_i|$, thus $g\Delta s \geq g\Delta s + (|g_j| - |g_i|)v$. □

### 4. SOLVE INTERNAL OPINION MAXIMIZATION PROBLEM WITH THE ADMM ALGORITHM

#### 4.1. ADMM algorithm

Alternating Direction Method of Multipliers (ADMM) is a convergent algorithm combining the factorability of dual ascending and the advantages of multiplier method, which was proposed by Boyd [8]. When there are multiple variables in the target of a convex optimization problem, it is difficult to solve, so we use variable separation to divide the problem into several simple subproblems. Then we can solve these subproblems in parallel and coordinate the subproblems to get the global solution of original problem.

#### 4.2. Steps of the ADMM algorithm

Suppose the optimization problems are as follows:

$$\min_{x,z} f(x) + g(z)$$

s.t. $Ax + Bz = c$,

where $x \in \mathbb{R}^n$, $z \in \mathbb{R}^m$, $A \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{p \times m}$, $c \in \mathbb{R}^p$. Its augmented Lagrangian function is as follows:

$$L_\rho(x, z, y) = f(x) + g(z) + y^T(Ax + Bz - c) + \frac{\rho}{2} \|Ax + Bz - c\|_2^2.$$  

The update iteration form of ADMM is as follows:

$$x^{k+1} := \arg\min_x L_\rho(x, z^k, y^k),$$

$$z^{k+1} := \arg\min_z L_\rho(x^{k+1}, z, y^k),$$

$$y^{k+1} := y^k + \rho(Ax + Bz - c),$$

where $\rho > 0$, $c \in \mathbb{R}^p$ is a dual variable, and this is the original iteration form of ADMM algorithm.

#### 4.3. Solve the internal opinion problem with the ADMM algorithm

The internal opinion problem we want to solve is as follows:

$$\max 1^T(\Lambda + (I - \Lambda)L)^{-1}\Lambda(s + \Delta s)$$

s.t. $\begin{cases} -1 \leq s_i + \Delta s_i &\leq 1, \quad (i = 1, 2, \ldots, n), \\ \|\Delta s\|_1 &\leq \mu. \end{cases}$

Because $1^T(\Lambda + (I - \Lambda)L)^{-1}\Lambda s$ is a constant, we can rewrite the problem as:

$$\max 1^T(\Lambda + (I - \Lambda)L)^{-1}\Lambda \Delta s,$$

s.t. $\begin{cases} -1 - s_i \leq \Delta s_i &\leq 1 - s_i, \quad (i = 1, 2, \ldots, n), \\ \|\Delta s\|_1 &\leq \mu. \end{cases}$

We can translate the problem into the following form:

$$\min I(x) + \lambda \|x\|_1,$$
where \( l(x) = -1^T (\Lambda + (I - \Lambda) L)^{-1} \Lambda \), \( x = \Delta s \), \( a_i = -1 - s_i, b_i = -1 + s_i \). We then further translate this into the following form that can be solved by ADMM:

\[
\min l(x) + \lambda \|z\|_1, \quad \text{s.t.} \begin{cases} a \leq x \leq b, \\ x = z. \end{cases}
\]

In this case, the variables \( x, z \) and \( u \) are updated as follows:

\[
x^{k+1} := \arg\min_x \left( l(x) + \frac{\rho}{2} \|x - z^k + u^k\|_2^2 \right), \\
z^{k+1} := S_{\lambda/\rho}(x^{k+1} + u^k), \\
u^{k+1} := u^k + x^{k+1} - z^{k+1}.
\]

From the above formula, the analytic solution of each step iteration can be written:

\[
x_i^{k+1} := \begin{cases} a_i, & \text{if } z_i^k - u_i^k - \frac{A_i}{\rho} < a_i, \\
z_i^k - u_i^k - \frac{A_i}{\rho}, & \text{if } z_i^k - u_i^k - \frac{A_i}{\rho} < b_i, \\
b_i, & \text{if } b_i < z_i^k - u_i^k - \frac{A_i}{\rho}, \\
z_i^{k+1} := \begin{cases} z_i^{k+1} - u_i^k - \frac{\lambda}{\rho}, & \text{if } z_i^{k+1} - u_i^k > \frac{\lambda}{\rho}, \\
0, & \text{if } -\frac{\lambda}{\rho} < z_i^{k+1} - u_i^k < \frac{\lambda}{\rho}, \\
z_i^{k+1} - u_i^k + \frac{\lambda}{\rho}, & \text{if } z_i^{k+1} - u_i^k < -\frac{\lambda}{\rho}, \\
u_i^{k+1} := u_i^k + x_i^{k+1} - z_i^{k+1}.
\end{cases}
\]

The algorithm converges to the global optimal solution, so after a certain number of iterations, we can get the optimal result.

5. EXPERIMENTS

In this section, we conduct a series of experiments to evaluate the proposed method. We carry out experiments from two aspects. First, we compare the results obtained by our model with those in [2]. To further illustrate the superiority of our model, our method is compared with other heuristic methods, and the experimental results show that our method has overwhelming advantages.

5.1. Datasets

The datasets we used in the experiments are as follows: (i) Alpha; (ii) OTC [9, 10]; and we normalize the trust values (i.e., edge weights) to the interval \([-1, 1]\) on Alpha and OTC. (iii) Elec; (iv) Rfa [11]; and the relationships in the last two networks are closely related to trust.

5.2. Internal opinion initialization

We randomly select values that match a particular distribution to initialize the internal opinion vector \( s \) to simulate different situations. For each network, we use three ways to initialize the internal opinions. (i) We initiate the internal opinions to follow a uniform distribution (i.e., \( s \sim U(-1, 1) \)). (ii) We initiate the internal opinions to follow a standard normal distribution (i.e., \( s \sim N(0, 1) \)). (iii) We initialize the internal opinions of a node to positively correlated with the column connectivity of that node (i.e., \( s \sim \sum_j |a_{ij}| \)).

5.3. The value of \( \alpha_i \)

We experimented with two different ways of selecting value of \( \alpha_i \): (i) Fixed \( \alpha_i \). In this case, we fixed the value of \( \alpha_i \), which means that everyone’s confidence index is the same in the network. (ii) Adjusted \( \alpha_i \). In this case, \( \alpha_i \) changes with the internal structure of each group. When \( \alpha_i \in \{2/3, 1/2, 1/3, 1/4\} \), we carried out a set of tests, and the experimental results are shown in Table 1.

| Network | \( \alpha_i \) | 2/3 | 1/2 | 1/3 | 1/4 |
|---------|----------------|-----|-----|-----|-----|
| Alpha   | Fixed          | 254 | 277 | 293 | 343 |
|         | Adjusted       | 481 | 461 | 429 | 479 |
| OTC     | Fixed          | 270 | 318 | 367 | 386 |
|         | Adjusted       | 546 | 600 | 578 | 555 |
| WikiElec| Fixed          | 1165| 1115| 1144| 1149|
|         | Adjusted       | 1538| 1468| 1499| 1510|
| WikiRfa | Fixed          | 906 | 1000| 1121| 1098|
|         | Adjusted       | 1340| 1341| 1375| 1293|

As can be seen from Table 1, when \( \alpha_i = 1/2 \), that is, the proportion of the opinions of everyone inside and those of the people around is the same, and the experimental results are consistent with those in [2]. When \( \alpha_i > 1/2 \), the experimental result is lower than the result in [2], and when \( \alpha_i < 1/2 \), the experimental result is higher than the result in [2]. This situation is also reasonable in real life. A larger \( \alpha_i \) indicates that the opinions expressed by each person are less likely to be influenced by those around them. The smaller \( \alpha_i \) is, the more susceptible the expressed influence is to those around it. Therefore, changing the internal opinions of a small number of people will affect the overall opinions of the whole network to change greatly.

5.4. Comparative Methods

In order to better reflect the advantages of our results, as with [2], we compared it to several other three heuristics. (i) Rand.
Table 2. Average benefit compared with other heuristics

|       | Alpha | OTC | WikiElec | WikiRfa |
|-------|-------|-----|----------|---------|
| Uniform | Ours  | 463 | 532      | 1550    | 1340    |
|        | Rand  | 41  | 21       | 13      | 29      |
|        | Trust | 225 | 83       | 275     | 298     |
|        | IO    | 23  | 25       | 144     | 34      |
| Normal | Ours  | 452 | 578      | 1581    | 1341    |
|        | Rand  | 26  | 20       | 87      | 14      |
|        | Trust | 250 | 112      | 205     | 378     |
|        | IO    | 36  | 23       | 233     | 15      |
| Degree | Ours  | 454 | 569      | 1545    | 1375    |
|        | Rand  | 18  | 14       | 29      | 40      |
|        | Trust | 250 | 92       | 297     | 329     |
|        | IO    | 260 | 120      | 8       | 174     |

Fig. 2. Average benefit of different methods on four datasets.

We sort the nodes randomly. (ii) Trust. We represent the trust sum of the nodes by calculating the sum of the corresponding columns of the adjacency matrix. Nodes with a large amount of trust may have a strong ability to influence the opinions of other nodes. Therefore, we sort the nodes in descending order of the node trust sum. (iii) IO. If we can persuade those negative internal opinions to have positive internal opinions, then the overall opinion may increase. Therefore, we take the internal opinions of the node and sort the corresponding nodes in ascending order. In order to ensure the fairness of the experimental results, we reproduced the other methods mentioned in [2] and carried out the experiment on the same computer. In the experiment, we set the parameter $\mu = 200$, and the result is shown in Table 2 and Fig. 2.

As can be seen from the results, our approach has an overwhelming advantage over all datasets. A part of the performance curves are shown in Fig. 3. The $x$-axis shows the budget amount, the left $y$-axis shows the overall opinion, and the right $y$-axis shows the unit benefit.

6. CONCLUSION

In this paper, we have proposed a generalized opinions dynamics model for social trust networks to solve the internal opinion maximization problem. Different from other methods, our model GODM introduces a new confidence index and proposes a novel algorithm, which proved to be optimal. Extensive experimental results on four datasets demonstrate the effectiveness of our model.

7. REFERENCES

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