Charge- and parity-projected Hartree-Fock method for the strong tensor correlation and its application to the alpha particle

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Abstract

We propose a new mean-field-type framework which can treat the strong correlation induced by the tensor force. To treat the tensor correlation we break the charge and parity symmetries of a single-particle state and restore these symmetries of the total system by the projection method. We perform the charge and parity projections before variation and obtain a Hartree-Fock-like equation, which is solved self-consistently. We apply the Hartree-Fock-like equation to the alpha particle and find that by breaking the parity and charge symmetries, the correlation induced by the tensor force is obtained in the projected mean-field framework. We emphasize that the projection before the variation is important to pick up the tensor correlation in the present framework.

Key words: tensor force, parity mixing, charge mixing, Hartree-Fock approximation, variation after projection, alpha particle

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1 Introduction

Mean-field models with effective interactions like the Skyrme force [1] or the Gogny force [2], are widely applied to various fields in nuclear physics with great success. The relativistic mean field theory is also applied to many nuclear...
systems successfully [3,4]. These models are based on the single-particle picture of the nuclear many-body system. In this picture, a nucleon in a nucleus is moving in a nuclear mean field made by other nucleons in the nucleus. The single-particle picture is known to be appropriate in the wide range of nuclear phenomena since the establishment of the shell model [5].

Usually, the effective interactions used in nuclear mean-field models only have the central and LS parts as the nuclear two-body interactions. They do not have the tensor part. However, the tensor force is known to be very important in the structure of nuclei [6,7,8,9,10,11,12]. The tensor force is mainly mediated by the pion, which is one of the most important meson since Yukawa proposed it as a mediator of the nuclear force [13]. Many studies have demonstrated that the tensor force gives large attractive energy in light nuclei [6,7,8,9,10]. The shell model calculation shows that the tensor force can produce about a half of single-particle $ls$ splittings in light nuclei [11]. These facts show that the tensor force is an essential ingredient of the structure of light nuclei. Furthermore, the tensor force plays an important role in the saturation mechanism of nuclear matter [12]. This fact indicates that the tensor force may have an important effect also on the structure of nuclei in the heavy mass region.

The importance of the tensor force mentioned above inspires us to treat the tensor force seriously even in a mean-field model. Many people, however, believe that the effect of the tensor force can be treated by renormalizing the central and LS parts of the effective two-body interaction. This renormalization is founded by the G-matrix theory, where the effective interaction is made by renormalizing the central and LS forces to include the effect of the tensor force together with the short-range correlation. In the G-matrix theory, the effective tensor force is weakened and treated simply as the residual interaction. The main part of the tensor force is included in the $^3E$ part of the central force. We see the importance of the tensor force by comparing the theoretical results for nuclear matter and the alpha particle. In nuclear matter, the ratio of the energy contribution from the $^3E$ part of the effective interaction and that from $^1E$ is nearly equal to 1, but in the alpha particle, this ratio increases to about 1.5 [6]. This enhancement of the energy contribution from the $^3E$ component indicates that the effect of the tensor force is larger in the alpha particle than in nuclear matter, and the tensor force should be more important in finite nuclei than in nuclear matter.

There are some Hartree-Fock calculations using the nuclear forces which include the tensor force [14,15,16] or the pion [17,18] explicitly. They show that the tensor force gives a large contribution to the $ls$ splitting of single-particle states especially in the spin-unsaturated nuclei. In contrast to the conventional mean-field calculations, we recently proposed a new method to treat the pion in the relativistic mean field (RMF) theory [19]. Because the pion is a pseudoscalar meson, a single-particle state of a nucleon in a nuclear mean-field
should change its parity when the nucleon emits or absorbs a pion. It suggests that we can incorporate the pion into the RMF theory by introducing parity-mixed single-particle states. The idea of the parity-mixed single-particle state was already discussed in the references [20,21,22,23,24] many years ago. In the previous paper [19], we applied this idea in the RMF theory and found that the pion mean field becomes finite, due to which the correlation induced by the pion is appropriately taken into account. This result is not restricted to a relativistic framework. We can treat the tensor force by the introduction of the parity-mixed single-particle states in a non-relativistic framework also.

The purpose of this paper is to apply the same idea in a non-relativistic mean-field model and see the effect of the tensor force in the alpha particle. We make two major improvements compared to the previous study in the treatment of a nuclear many-body system. The first improvement is straightforward as coming from the isovector character of the pion. Because the pion is an isovector meson, a single-particle state in a mean field changes its charge state when it emits or absorbs a charged pion. This fact indicates that the isovector nature of the pion can be treated by introducing charge-mixed single-particle states, which have both proton and neutron components. The dominant part of the tensor force is $\tau_1 \cdot \tau_2$-type and therefore this improvement is important when we treat the tensor force. The second improvement is related with the restoration of symmetry. We mix the parities and the charges into a single-particle state. A total wave function made from the single-particle states with the parity and charge mixings does not have a good parity and a definite charge number. Therefore we need to perform the parity projection and the charge (number) projection to obtain a total wave function having a good parity and a definite charge number. By performing the projections we can pick up the 2-particle–2-hole, the 4-particle–4-hole, ... correlations, which are induced by the tensor force for the ground state of even-even mass nuclei [19], and are important in the binding mechanism from light nuclei to nuclear matter.

We may think of performing the charge and parity projections after we obtain a total wave function assuming the charge and parity mixings (projection after variation). Because the effect of the tensor force is strong and affect the binding mechanism of nuclei largely, we need to treat the restoration of the parity and charge symmetries carefully. Therefore, in a more sophisticated way, we should first perform the parity and charge projections, and then take a variation of the energy expectation value which is evaluated with the projected total wave function (variation after projection (VAP)). In the present study we perform calculations for the alpha particle in the VAP scheme and show that the VAP scheme is needed to treat the tensor force more appropriately. We should note that the present framework is not a simple mean-field model because we effectively superpose many Slater determinants by performing the projections. In this sense, our framework is a kind of descriptions beyond the mean field [25,26,27,28,29].
This paper is arranged as follows. In Section 2, we will formulate the charge- and parity-projected Hartree-Fock method. In Section 3 we will apply the charge- and parity-projected Hartree-Fock method to the alpha particle to show the effectiveness of our method. In the last section we will give the summary.

## 2 Charge- and parity-projected Hartree-Fock method

We present the charge- and parity-projected Hartree-Fock (CPPHF) method. In the CPPHF method, we construct a total wave function with single-particle states with the parity and charge mixings. A single-particle wave function with the charge and parity mixings in the spherical case is written as,

$$\psi_{njm}(x) = \sum_{t_z = \pm 1/2} \left( \phi_{njlt_z}(r)\mathcal{Y}_{jlm}(\Omega)\zeta(t_z) + \phi_{nj\bar{l}t_z}(r)\mathcal{Y}_{j\bar{l}m}(\Omega)\zeta(t_z) \right). \tag{1}$$

This wave function consists of four terms, each of which has a positive or negative parity, and a charge number, 1 or 0 (proton or neutron). Here, the radial wave functions $\phi$'s depend only on the radial coordinate $r$, the isospin wave functions are denoted as $\zeta(t_z)$ ($t_z = 1/2$ for proton and $t_z = -1/2$ for neutron), and $\mathcal{Y}_{jlm}$ is the eigenfunction of total spin $j = l + s$. In the spherical case with the parity and charge mixings, the good quantum numbers are $j$ and $m$. Two angular wave functions $\mathcal{Y}_{jlm}$ and $\mathcal{Y}_{j\bar{l}m}$ are related to each other as

$$\mathcal{Y}_{j\bar{l}m}(\Omega) = \sigma \cdot \hat{r} \mathcal{Y}_{jlm}(\Omega), \quad \bar{l} = \begin{cases} l + 1 & (j = l + 1/2), \\ l - 1 & (j = l - 1/2). \end{cases} \tag{2}$$

Here, $\sigma$ is the Pauli spin operator and $\hat{r}$ is the unit radial vector. Because $\sigma \cdot \hat{r}$ is a $0^-$ operator, $\mathcal{Y}_{jlm}$ and $\mathcal{Y}_{j\bar{l}m}$ have the same total angular momentum $j$ but different orbital angular momenta, i.e., $|l - \bar{l}| = 1$. It means that they have the opposite parities to each other. The symbol $n$ is introduced to label the single-particle states with the same $j$ and $m$. In the CPPHF method, $n$ does not correspond to the node quantum number because of the parity and charge mixings. We take a Slater determinant made from the charge- and parity-mixed single-particle states as an intrinsic wave function for a nucleus with the mass number $A$:

$$\Psi_{\text{intr}} = \frac{1}{\sqrt{A!}} \hat{A} \prod_{a=1}^{A} \psi_{\alpha_a}(x_a). \tag{3}$$
Here, \( \alpha_a \) denotes \( n, j, \) and \( m \) in (1) and \( \hat{A} \) is the antisymmetrization operator. As already mentioned above, \( \Psi^{\text{intr}} \) does not have a good parity and a definite charge number. We perform the projections of parity (\( \pm \)) and charge number (\( Z \)) on \( \Psi^{\text{intr}} \):

\[
\Psi^{(\pm;Z)} = \hat{P}^p(\pm) \hat{P}^c(Z) \Psi^{\text{intr}}. \tag{4}
\]

Here, \( \hat{P}^p(\pm) \) is the parity-projection operator, where \( \hat{P}^p(+) \) projects out the positive parity state and \( \hat{P}^p(-) \) projects out the negative parity one. \( \hat{P}^c(Z) \) is the charge-number-projection operator, which projects out the wave function with a charge number \( Z \). Therefore, \( \Psi^{(\pm;Z)} \) has a good parity (\( \pm \)) and a definite charge number (\( Z \)). The parity projection operator \( \hat{P}^p(\pm) \) is defined as

\[
\hat{P}^p(\pm) = \frac{1 \pm \hat{\mathcal{P}}}{2}, \tag{5}
\]

where the total parity operator \( \hat{\mathcal{P}} \) is the product of the parity operator \( \hat{\rho}_a \) for each single-particle state. The charge projection operator \( \hat{P}^c(Z) \) is defined as

\[
\hat{P}^c(Z) = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i(\hat{Z}-Z)\theta} = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{-iz\theta} \hat{C}(\theta) \quad \left( \hat{Z} = \sum_{a=1}^A \frac{1+\tau_3^a}{2} \right), \tag{6}
\]

where \( \hat{Z} \) is the charge number operator, which is the sum of the single-particle proton projection operator \((1+\tau_3^a)/2\), and the charge-phase operator is defined as \( \hat{C}(\theta) = e^{i\hat{Z}\theta} \).

We take a Hamiltonian \( \hat{H} \) in the following form,

\[
\hat{H} = -\sum_{a=1}^A \frac{\hbar^2}{2M} \Delta_a - \frac{1}{2AM} \left( \sum_{a=1}^A \left( \frac{\hbar}{i} \nabla_a \right) \right)^2
\]

\[
+ \sum_{a>b=1} \left( \hat{v}_C(x_{ab}) + \hat{v}_T(x_{ab}) + \hat{v}_{LS}(x_{ab}) + \hat{v}_{\text{Coul}}(x_{ab}) \right) \tag{7}
\]

\[
= \sum_{a=1}^A \hat{t}(x_a) + \sum_{a>b=1} \hat{v}(x_{ab}). \tag{8}
\]

Here, the first and the second terms on the right-hand side are the single-particle kinetic energy and the energy of the center of mass motion. The two-body interactions, \( \hat{v}_C, \hat{v}_T, \hat{v}_{LS}, \) and \( \hat{v}_{\text{Coul}} \) represent the central, tensor, LS, and Coulomb interactions, respectively. We define the one-body kinetic energy operator \( \hat{t}(x_a) \) and the two-body potential energy operator \( \hat{v}(x_{ab}) \) for the later
The parity-inverted and charge-rotated wave functions \( \psi \)

The charge-rotated wave function

\[
\frac{\text{determinant of the norm matrix between the original wave function } s}{t}
\]

The energy correction due to the center of mass motion in (7) is included in \( \hat{t}(x_a) \) and \( \hat{v}(x_{ab}) \).

We take the expectation value for a Hamiltonian \( \hat{H} \) with the projected wave function and obtain the energy functional,

\[
E^{(\pm;Z)} = \frac{\langle \Psi^{(\pm;Z)} | \hat{H} | \Psi^{(\pm;Z)} \rangle}{\langle \Psi^{(\pm;Z)} | \Psi^{(\pm;Z)} \rangle} = \frac{\langle \Psi_{\text{intr}} | \hat{H} | \hat{P}^c(Z) \Psi_{\text{intr}} \rangle}{\langle \Psi_{\text{intr}} | \hat{P}^c(Z) \Psi_{\text{intr}} \rangle} \equiv \frac{1}{4\pi} \int_0^{2\pi} d\theta e^{-iZ\theta} \left( E^{(0)}(\theta) \pm E^{(P)}(\theta) \right) \left( n^{(0)}(\theta) \pm n^{(P)}(\theta) \right).
\]

The denominator in the right-hand side of the above equation is the normalization of the total wave function,

\[
n^{(\pm;Z)} \equiv \langle \Psi^{(\pm;Z)} | \Psi^{(\pm;Z)} \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\theta e^{-iZ\theta} \left( n^{(0)}(\theta) \pm n^{(P)}(\theta) \right).
\]

Here, \( n^{(0)}(\theta) \) is the determinant of the norm matrix between the original wave functions \( \psi_{\alpha_a} \) and the charge-rotated wave functions \( \psi_{\alpha_a}(\theta) \). \( n^{(P)}(\theta) \) is the determinant of the norm matrix between the original wave functions \( \psi_{\alpha_a} \) and the parity-inverted and charge-rotated wave functions \( \psi_{\alpha_a}(\theta) \).

\[
n^{(0)}(\theta) \equiv \langle \Psi_{\text{intr}} | \hat{C}(\theta) | \Psi_{\text{intr}} \rangle = \text{det } B^{(0)}(\theta) \quad (B^{(0)}(\theta)_{ab} \equiv \langle \psi_{\alpha_a} | \psi_{\alpha_b}(\theta) \rangle),
\]

\[
n^{(P)}(\theta) \equiv \langle \Psi_{\text{intr}} | \hat{P}^c(Z) \hat{C}(\theta) | \Psi_{\text{intr}} \rangle = \text{det } B^{(P)}(\theta) \quad (B^{(P)}(\theta)_{ab} \equiv \langle \psi_{\alpha_a} | \psi^{(p)}_{\alpha_b}(\theta) \rangle).
\]

The charge-rotated wave function \( \psi_{\alpha_a}(x_b; \theta) \) and the parity-inverted and charge-rotated wave function \( \psi^{(p)}_{\alpha_a}(x_b; \theta) \) are defined as

\[
\psi_{\alpha_a}(x_b; \theta) \equiv e^{i\theta(1 + \tau^3_b)/2} \psi_{\alpha_a}(x_b),
\]

\[
\psi^{(p)}_{\alpha_a}(x_b; \theta) \equiv \hat{p}_b e^{i\theta(1 + \tau^3_b)/2} \psi_{\alpha_a}(x_b),
\]

where \( \hat{p}_b \) is the single-particle parity operator in (5) and \( (1 + \tau^3_b)/2 \) is the single-particle proton projection operator in (6).

The numerator in the right-hand side of (11) is the unnormalized total energy,

\[
\langle \Psi^{(\pm;Z)} | \hat{H} | \Psi^{(\pm;Z)} \rangle \equiv \frac{1}{4\pi} \int_0^{2\pi} d\theta e^{-iZ\theta} \left( E^{(0)}(\theta) \pm E^{(P)}(\theta) \right).
\]
$E^{(0)}(\theta)$ in the right-hand side of (16) has a similar form as a simple Hartree-Fock energy but the single-particle wave functions in the ket are modified by the charge rotation,

$$E^{(0)}(\theta) \equiv \langle \Psi_{\text{intr}} | \hat{H} \hat{C}(\theta) | \Psi_{\text{intr}} \rangle$$

$$= A \sum_{a=1}^{A} \langle \psi_{\alpha a} | \hat{t} | \tilde{\psi}_{\alpha a}(\theta) \rangle + A \sum_{a>b=1}^{A} \langle \psi_{\alpha a} \psi_{\alpha b} | \hat{v} | \tilde{\psi}_{\alpha a}(\theta) \tilde{\psi}_{\alpha b}(\theta) - \tilde{\psi}_{\alpha b}(\theta) \tilde{\psi}_{\alpha a}(\theta) \rangle. \quad (17)$$

Here, $\tilde{\psi}_{\alpha a}(x; \theta)$ is the superposition of $\psi_{\alpha a}(x; \theta)$ weighted by the inverse of the charge-rotated norm matrix $(B^{(0)}(\theta)^{-1})_{ba}$,

$$\tilde{\psi}_{\alpha a}(x; \theta) = \sum_{b=1}^{A} \psi_{\alpha b}(x; \theta)(B^{(0)}(\theta)^{-1})_{ba}. \quad (18)$$

This summation for $\psi_{\alpha a}(x; \theta)$ comes from the antisymmetrization of the total wave function. $E^{(0)}(\theta = 0)$ reduces to a simple Hartree-Fock energy. $E^{(P)}(\theta)$ in the right-hand side of (16) has a similar form as $E^{(0)}(\theta)$ but $\tilde{\psi}_{\alpha a}(\theta)$'s are replaced by $\tilde{\psi}^{(p)}_{\alpha a}(\theta)$'s,

$$E^{(P)}(\theta) \equiv \langle \Psi_{\text{intr}} | \hat{H} \hat{P} \hat{C}(\theta) | \Psi_{\text{intr}} \rangle$$

$$= A \sum_{a=1}^{A} \langle \psi_{\alpha a} | \hat{t} | \tilde{\psi}^{(p)}_{\alpha a}(\theta) \rangle + A \sum_{a>b=1}^{A} \langle \psi_{\alpha a} \psi_{\alpha b} | \hat{v} | \tilde{\psi}^{(p)}_{\alpha a}(\theta) \tilde{\psi}^{(p)}_{\alpha b}(\theta) - \tilde{\psi}^{(p)}_{\alpha b}(\theta) \tilde{\psi}^{(p)}_{\alpha a}(\theta) \rangle. \quad (19)$$

Here, $\tilde{\psi}^{(p)}_{\alpha a}(x; \theta)$ is the sum of $\psi^{(p)}_{\alpha a}(x; \theta)$ weighted by the inverse of the parity-inverted and charge-rotated norm matrix $(B^{(P)}(\theta)^{-1})_{ba}$,

$$\tilde{\psi}^{(p)}_{\alpha a}(x; \theta) = \sum_{b=1}^{A} \psi^{(p)}_{\alpha b}(x; \theta)(B^{(P)}(\theta)^{-1})_{ba}. \quad (20)$$

We then take the variation of $E^{(\pm; Z)}$ with respect to a single-particle wave function $\psi_{\alpha a}$,

$$\frac{\delta}{\delta \psi_{\alpha a}^{(\pm; Z)}(x_a)} \left\{ E^{(\pm; Z)} - \sum_{b,c=1}^{A} \epsilon_{bc} \langle \psi_{\alpha b} | \psi_{\alpha c} \rangle \right\} = 0. \quad (21)$$

The Lagrange multiplier $\epsilon_{ab}$ is introduce to guarantee the ortho-normalization of a single-particle wave function, $\langle \psi_{\alpha a} | \psi_{\alpha b} \rangle = \delta_{\alpha_a, \alpha_b}$. As the result, we obtain the following Hartree-Fock-like equation with the charge and parity projec-
tions (the CPPHF equation) for each $\psi_{\alpha_a}$,

$$\frac{1}{4\pi} \int_{0}^{2\pi} d\theta e^{-iZ\theta} \left[ n^{(0)}(\theta) \left\{ \hat{\epsilon}(x_a) \tilde{\psi}_{\alpha_a}(x_a; \theta) + \sum_{b=1}^{A} \langle \psi_{\alpha_b} | \hat{v}(x_{a1}) | \tilde{\psi}_{\alpha_b}(\theta) \rangle \psi_{\alpha_a}(x_a; \theta) \right. \right.$$  

$$- \sum_{b=1}^{A} \langle \psi_{\alpha_b} | \hat{v}(x_{a1}) | \tilde{\psi}_{\alpha_b}(\theta) \rangle \psi_{\alpha_a}(x_a; \theta) \right\}$$  

$$= n^{(+)}(\theta) \left\{ \hat{\epsilon}(x_a) \tilde{\psi}_{\alpha_a}(x_a; \theta) + \sum_{b=1}^{A} \langle \psi_{\alpha_b} | \hat{v}(x_{a1}) | \tilde{\psi}_{\alpha_b}(\theta) \rangle \psi_{\alpha_a}(x_a; \theta) \right.$$$$

- \sum_{b=1}^{A} \langle \psi_{\alpha_b} | \hat{v}(x_{a1}) | \tilde{\psi}_{\alpha_b}(\theta) \rangle \psi_{\alpha_a}(x_a; \theta) \right\}$$  

$$= n^{(+)}(\theta) \sum_{b=1}^{A} \epsilon_{ab} \psi_{\alpha_b}(x_a), \quad (22)$$

where $a = 1, 2, \ldots, A$. Here, $n_{ab}^{(0)}(\theta)$ and $n_{ab}^{(P)}(\theta)$ are defined as follows,

$$n_{ab}^{(0)}(\theta) \equiv \langle \psi_{\alpha_a} | \hat{\epsilon} | \tilde{\psi}_{\alpha_b}(\theta) \rangle$$  

$$+ \sum_{c=1}^{A} \langle \psi_{\alpha_b} \psi_{\alpha_c} | \hat{v} | \tilde{\psi}_{\alpha_c}(\theta) \rangle \psi_{\alpha_b}(\theta) - \tilde{\psi}_{\alpha_b}(\theta) \psi_{\alpha_a}(\theta) \rangle, \quad (23)$$

$$n_{ab}^{(P)}(\theta) \equiv \langle \psi_{\alpha_a} | \hat{\epsilon} | \tilde{\psi}_{\alpha_b}^{(p)}(\theta) \rangle$$  

$$+ \sum_{c=1}^{A} \langle \psi_{\alpha_b} \psi_{\alpha_c} | \hat{v} | \tilde{\psi}_{\alpha_c}^{(p)}(\theta) \rangle \psi_{\alpha_b}(\theta) - \tilde{\psi}_{\alpha_c}^{(p)}(\theta) \psi_{\alpha_a}(\theta) \rangle, \quad (24)$$

and the notation for the integration of the two-body matrix elements,

$$\langle \psi_{\alpha_b} | \hat{v}(x_{a1}) | \psi_{\alpha_c} \rangle_1 = \int dx_1 \psi_{\alpha_b}^\dagger(x_1) \hat{v}(x_{a1}) \psi_{\alpha_c}(x_1). \quad (25)$$

The system of the coupled equations (22) for $a = 1, \ldots, A$ is solved self-consistently. We note here that the CPPHF equation reduces to the parity-projected Hartree-Fock equation with only the parity projection by setting $\theta = 0$ in (22), which was already obtained by S. Takami et al. [25].

We give here the expressions for the expectation value of the kinetic energy $\langle \hat{T} \rangle^{(\pm;Z)}$ with the center of mass correction and that of the two-body potential
energy $\langle \hat{v}_\sigma \rangle^{(\pm;Z)}$ for $\hat{v}_\sigma$ ($\sigma = C, T, LS,$ and Coul) for the later convenience.

$$
\langle \hat{T} \rangle^{(\pm;Z)} = \frac{1}{4\pi n^{(\pm;Z)}} \int_0^{2\pi} d\theta e^{-iZ\theta} \left[ n^0(\theta) \sum_{a=1}^{A} \langle \hat{\ell}_a | \hat{p}_a \rangle \langle \hat{p}_a | \hat{\ell}_a \rangle + \sum_{a>b=1}^{A} \langle \hat{\ell}_a | \hat{p}_a \rangle \langle \hat{p}_a | \hat{\ell}_a \rangle \right]
$$

$$
\langle \hat{v}_\sigma \rangle^{(\pm;Z)} = \frac{1}{4\pi n^{(\pm;Z)}} \int_0^{2\pi} d\theta e^{-iZ\theta} \left\{ n^0(\theta) \sum_{a=1}^{A} \langle \hat{\ell}_a | \hat{p}_a \rangle \langle \hat{p}_a | \hat{\ell}_a \rangle + \sum_{a>b=1}^{A} \langle \hat{\ell}_a | \hat{p}_a \rangle \langle \hat{p}_a | \hat{\ell}_a \rangle \right\}
$$

3 Application to the alpha particle

We apply the CPPHF method to the alpha particle (A=4, Z=2), which has been studied extensively with various theoretical methods including exact calculations. As the ground state configuration of the alpha particle, we assume four states $n = 1, 2$ and $(jm) = (1/2, \pm1/2)$, which are fully occupied. Here, $j$ and $m$ are the total angular momentum and the magnetic quantum number of a single-particle state. The index $n$ labels the two different states which have the same $j$ and $m$. The intrinsic wave function (3) for the alpha particle becomes

$$
\Psi^{\text{intr}} = \hat{A} \prod_{n=1,2} \prod_{m=\pm \frac{1}{2}} \psi_{nj=\frac{1}{2}m}(x).
$$

Because we fill $m=\pm1/2$ states with $j = 1/2$ for each $n$, $\Psi^{\text{intr}}$ has the total angular momentum 0. For each $n$, $\psi_{nj=\frac{1}{2}m}$ becomes

$$
\psi_{nj=\frac{1}{2}m}(x) = \sum_{t_z=\pm \frac{1}{2}} \left( \phi_{nt=0t_z}(r) \right) \chi_{\frac{1}{2}l=0m}(\Omega) \zeta(t_z)
$$

$$
+ \phi_{nt=1t_z}(r) \chi_{\frac{1}{2}l=1m}(\Omega) \zeta(t_z).$$

(29)
The single-particle wave function \( \psi_{nj=\frac{1}{2}m} \) consists of four components, which have different parities (±) and different charges \((t_z = \pm 1/2)\). The first term on the right-hand-side has \( j = 1/2 \) and \( l = 0 \) (a s-state with positive-parity) and the second term has \( j = 1/2 \) and \( l = 1 \) (a p-state with negative-parity). The single-particle p-state probability \( P(-) \), which is a measure for how much the p-state mixes into the simple \((0s)^4\) configuration in the intrinsic state of the alpha particle, is defined as

\[
P(-) = \frac{1}{2} \sum_{n=1,2} \sum_{t_z=\pm} \int dr r^2 \hat{\phi}^\dagger_{nl=1t_z}(r) \hat{\phi}_{nl=1t_z}(r).
\]

The factor 1/2 in front of the right hand side of (30) is added to average the single-particle p-state probability over the states for \( n = 1,2 \). The four-body intrinsic wave function (28) does not have a good parity and a definite charge number; it is a mixture of positive and negative parities, and of various charge numbers \( Z=0\sim 4 \). In the alpha particle, the ground state has a positive parity and the charge number \( Z=2 \). Therefore, we act the projection operator \( \hat{P}^p(+) \hat{P}^c(2) \) on the intrinsic wave function (28).

In the numerical calculation, we expand the radial wave function \( \phi \) in (29) by the Gaussian basis with the geometric-series widths [30]. We take the number of basis as 10 and set the minimum width to 0.5 fm and the maximum width to 6.0 fm. To solve the CPPHF equation (22) self-consistently we use the gradient method [25,31]. For the central force, we take the Volkov force No. 1 [32] as a reference and introduce the multiplying factor \( x_{TE} \) for the triplet-even part.

\[
\hat{u}^{x_{TE}}(x_{ab}) = -v_A \exp\left(-\left(r_{ab}/\alpha_A\right)^2\right) \left(x_{TE}P^{3E}_{ab} + P^{3E}_{ab} + (1 - 2m_V)(P^{3O}_{ab} + P^{3O}_{ab})\right) + v_R \exp\left(-\left(r_{ab}/\alpha_R\right)^2\right) \left(P^{3E}_{ab} + P^{3E}_{ab} + (1 - 2m_V)(P^{3O}_{ab} + P^{3O}_{ab})\right).
\]

Here, \( P^{3E}_{ab} \), \( P^{3E}_{ab} \), \( P^{3O}_{ab} \), and \( P^{3O}_{ab} \) are the projection operators for the triplet-even, singlet-even, triplet-odd, and singlet-odd states, respectively. If we set \( x_{TE} = 1 \), \( \hat{u}^{x_{TE}} \) reduces to the original Volkov force. The term proportional to \( v_A \) is a middle-range attractive part and that to \( v_R \) is a short-range repulsive one. The values of the parameters are \( v_A = 83.34 \) MeV, \( \alpha_A = 1.6 \) fm, \( v_R = 144.86 \) MeV, and \( \alpha_R = 0.82 \) fm. We fix the Majorana parameter \( m_V = 0.6 \). The Volkov force reproduces the binding energy of the alpha particle without non-central forces. We can consider that the effect of the tensor force is included in the Volkov force by renormalizing the central force. This renormalization affects the middle range of the \( ^3E \) part of the central force mainly. Therefore, we reduce the \( ^3E \) part of the central force in the Volkov force by multiplying the factor \( x_{TE} \) to the attractive \( ^3E \) part of the Volkov interaction, when we include the tensor force explicitly.
As for the non-central forces (the tensor and the LS forces), the G3RS [33] force is adopted as a reference, which is determined so as to reproduce the scattering phase shift of NN scattering and have the one-pion-exchange potential tail in the tensor part. In the mean-field-type framework, we have to treat the effect of the short-range correlation on the tensor interaction and further take into account the effect of the $\Delta$ isobar-hole excitations [34], separately from the mean-field correlation. In the present study, we shall treat these effects by multiplying a factor $x_T$ to the $\tau_1 \cdot \tau_2$ part of the tensor force, which is the dominant part of the tensor force and mediated mainly by the pion. We need further studies on this factor in the near future. The tensor part of the G3RS force with the multiplying factor $x_T$ can be written as following,

$$\hat{v}^{xT}_{T}(x_{ab}) = \frac{1}{4} \left\{ \sum_{n=1}^{3} v_{T n}^{E} \exp\left(-\left(r_{ab}/\eta_{T n}^{E}\right)^2\right) + 3 \sum_{n=1}^{3} v_{T n}^{O} \exp\left(-\left(r_{ab}/\eta_{T n}^{O}\right)^2\right) \right\} + x_T \left\{ -3 \sum_{n=1}^{3} v_{T n}^{E} \exp\left(-\left(r_{ab}/\eta_{T n}^{E}\right)^2\right) + 3 \sum_{n=1}^{3} v_{T n}^{O} \exp\left(-\left(r_{ab}/\eta_{T n}^{O}\right)^2\right) \right\} \tau_a \cdot \tau_b S_{ab}^T,$$

(32)

where the tensor operator $S_{ab}^T$ is defined as,

$$S_{ab}^T = \frac{3 (\sigma_a \cdot r_{ab})(\sigma_b \cdot r_{ab})}{r_{ab}^2} - \sigma_a \cdot \sigma_b.$$

(33)

The factors, $x_{TE}$ and $x_T$ are correlated and are determined so as to reproduce the binding energy of the alpha particle.

The LS part of the G3RS force is made from the two-range Gaussians:

$$\hat{v}_{LS}(x_{ab}) = \sum_{n=1}^{2} v_{LS n}^{O} \exp\left(-\left(r_{ab}/\eta_{LS n}^{O}\right)^2\right) P^{O} L_{ab} \cdot S_{ab}.$$

(34)

The parameters of the G3RS force used in this work is summarized in Table 1. We take further the Coulomb interaction,

$$\hat{v}_{Coul}(x_{ab}) = \frac{e^2}{r_{ab}},$$

(35)

with $e$ being the charge of the electron.

We present here the results of the CPPHF calculations for the alpha particle. To see the effect of the charge and parity mixings, we calculate three cases. The first calculation is the simple Hartree-Fock (HF) scheme, in which we do not perform neither the parity nor charge projection. The second calculation is the parity-projected Hartree-Fock (PPHF) scheme, in which we only perform the parity projection. The third calculation is the charge- and parity-projected
Table 1
The parameters for the G3RS force used in this work [33]. The second, the third, and the fourth columns are the interaction strengths \( v \) in MeV and the corresponding ranges \( \eta \) in fm for the \( ^3E \) tensor and \( ^3O \) tensor and \( ^3O \) LS channels, respectively. (See Eqs. (32) and (34).)

| Tensor LS | \( ^3E \) | \( ^3O \) | \( ^3O \) |
|-----------|-----------|-----------|-----------|
| \( v_1 \) (MeV) | -7.5 | 2.5 | -800 |
| \( \eta_1 \) (fm) | 2.5 | 2.5 | 0.6 |
| \( v_2 \) (MeV) | -67.5 | 20. | 800 |
| \( \eta_2 \) (fm) | 1.2 | 1.2 | 0.4 |
| \( v_3 \) (MeV) | 67.5 | -20. |
| \( \eta_3 \) (fm) | 0.447 | 0.6 |

Hartree-Fock (CPPHF) scheme, in which we perform both the parity and charge projections. We show the calculated results in Table 2, where we take \( x_T = 1.5 \) and \( x_{TE} = 0.81 \), with which the binding energy of the alpha particle is reproduced in the CPPHF case. For comparison, we show the results for the HF calculation with the original Volkov No. 1 force \( (x_T = 1.0 \) and \( x_{TE} = 1.0) \).

In the simple HF case the energy from the tensor force \( \langle \hat{v}_T \rangle^{(\pm;2)} \) is zero. It means that the result of the HF calculation simply becomes a \((0s)^4\) configuration and there is no \( p \)-state component. In this case the expectation value of the tensor force is zero identically, because the tensor force does not act between \( s \)-states. If we perform the parity-projection (PPHF), the energy contribution from the tensor force becomes finite. The kinetic energy becomes larger because some component of the \( s \)-state is shifted up to \( p \)-states to gain the correlation caused by the tensor force. We perform then the charge projection (CPPHF) further. We see the contribution from the tensor force becomes much larger. It is reasonable because in the PPHF case only the \( \tau_1^0 \tau_2^0 \) component of \( \bm{\tau}_1 \cdot \bm{\tau}_2 \) in the tensor force is active, while in the CPPHF case all the \( \tau_1^+ \tau_2^- \), \( \tau_1^- \tau_2^0 \), and \( \tau_1^\pm \tau_2^\mp \) parts of the tensor force are active. In fact, the energy from the tensor force in the CPPHF case is about three times larger than that in the PPHF case. The results indicate that both the parity and charge projections are very important to treat the tensor force in a mean-field-type model.

We note that the variation-after-projection scheme that we take here, is needed because even if we assume the mixing of parity and charge in the simple Hartree-Fock calculation, we cannot obtain the result with mixed symmetries. The parity mixed state contains odd parity states, which require large energy jump across a major shell. In fact, the energy gap between \( 0s\)-shell and \( 0p\)-shell in the alpha particle is more than 20 MeV. Therefore, we need to per-
The results for the ground \((0^+\rangle\) state of the alpha particle for various cases. HF denotes the simple Hartree-Fock scheme. PPHF denotes the parity-projected Hartree-Fock scheme in which only the parity-projection is performed. CPPHF denotes the charge- and parity-projected Hartree-Fock scheme in which both the charge and parity projections are performed. The potential energy \(\langle \hat{v} \rangle^{(+;2)}\), kinetic energy \(\langle \hat{T} \rangle^{(+;2)}\), total energy \(E^{(+;2)}\), root-mean-square matter radius \(R_m\), and the probability of the p-state component (P(-)) are given in the table. \(\langle \hat{v}_C \rangle^{(+;2)}\), \(\langle \hat{v}_T \rangle^{(+;2)}\), \(\langle \hat{v}_{LS} \rangle^{(+;2)}\), and \(\langle \hat{v}_{Coul} \rangle^{(+;2)}\) are the expectation values for the central, tensor, LS, and Coulomb potentials, respectively. The factors, \(x_T\) and \(x_{TE}\), denote the factors multiplied to the \(\tau_1 \cdot \tau_2\)-type tensor force and the \(^3\)E central force.

|        | HF | HF | PPHF | CPPHF |
|--------|----|----|------|-------|
| \(x_T\) | 1.0 | 1.5 | 1.5  | 1.5   |
| \(x_{TE}\) | 1.0 | 0.81 | 0.81 | 0.81  |
| \(\langle \hat{v}_C \rangle^{(+;2)}\) (MeV) | -76.67 | -56.85 | -61.31 | -64.75 |
| \(\langle \hat{v}_T \rangle^{(+;2)}\) (MeV) | 0.00 | 0.00 | -10.91 | -30.59 |
| \(\langle \hat{v}_{LS} \rangle^{(+;2)}\) (MeV) | 0.00 | 0.00 | 0.67  | 1.91  |
| \(\langle \hat{v}_{Coul} \rangle^{(+;2)}\) (MeV) | 0.83 | 0.76 | 0.78  | 0.85  |
| \(\langle \hat{v} \rangle^{(+;2)}\) (MeV) | -75.84 | -56.10 | -70.76 | -92.58 |
| \(\langle T \rangle^{(+;2)}\) (MeV) | 48.54 | 39.98 | 49.67 | 64.39 |
| \(E^{(+;2)}\) (MeV) | -27.30 | -16.12 | -21.09 | -28.19 |
| \(R_m\) (fm) | 1.48 | 1.63 | 1.51 | 1.39 |
| \(P(-)\) (%) | 0.0 | 0.0 | 7.6  | 16.1 |

form the parity projection before variation to remove the spurious component mixing into the intrinsic wave function \(\Psi^{intr}\) in (28) properly. In contrast to the parity mixing, the correlations by deformation and pairing can be treated quite nicely with the projection-after-variation scheme including a configuration mixing along the quadrupole degree of freedom [26,27,28,29], where the rotation symmetry and the particle-number symmetry are broken in an intrinsic state. The correlations by deformation and pairing are in principle those within the same major shell and causes small energy loss and therefore the mixing of the angular momentum and the particle number occur even in the projection-after-variation scheme.

We check now the dependence of the results on the strength of the tensor force by changing \(x_T\) from 1.0 to 2.0. We determine \(x_{TE}\) so as to reproduce the binding energy of the alpha particle for each \(x_T\). We show the results of the CPPHF scheme in Table 3. From the table we can see that the contribution of the energy from the tensor force becomes larger with \(x_T\). The kinetic energy becomes larger also. The probability of the p-state (P(-)) changes from 10.5%
The results for the ground \((0^+\)) state of the alpha particle with the CPPHF scheme. We change \(x_T\) from 1.0 to 2.0 and determine \(x_{TE}\) accordingly so as to reproduce the binding energy of the alpha particle. The potential energy \((\langle \hat{v} \rangle^{(+;2)})\), kinetic energy \((\langle \hat{T} \rangle^{(+;2)})\), total energy \((E^{(+;2)})\), root-mean-square matter radius \((R_m)\), root-mean-square charge radius \((R_c)\), the probability of the \(p\)-state component \((P(-))\), and the probability of the \(S = 2\) component \((P(S = 2))\) are shown. \(\langle \hat{v}_C \rangle^{(+;2)}\), \(\langle \hat{v}_T \rangle^{(+;2)}\), \(\langle \hat{v}_{LS} \rangle^{(+;2)}\), and \(\langle \hat{v}_{Coul} \rangle^{(+;2)}\) are the expectation values for the central, tensor, LS, and Coulomb potentials, respectively.

| \(x_T\) | 1   | 1.25 | 1.5  | 1.75 | 2   |
|--------|-----|------|------|------|-----|
| \(x_{TE}\) | 0.93 | 0.88 | 0.81 | 0.73 | 0.64 |
| \(\langle \hat{v}_C \rangle^{(+;2)}\) (MeV) | -73.60 | -70.13 | -64.75 | -58.34 | -50.69 |
| \(\langle \hat{v}_T \rangle^{(+;2)}\) (MeV) | -12.26 | -20.23 | -30.59 | -43.86 | -60.41 |
| \(\langle \hat{v}_{LS} \rangle^{(+;2)}\) (MeV) | 0.75  | 1.26  | 1.91  | 2.72  | 3.72  |
| \(\langle \hat{v}_{Coul} \rangle^{(+;2)}\) (MeV) | 0.85  | 0.85  | 0.85  | 0.86  | 0.87  |
| \(\langle \hat{v} \rangle^{(+;2)}\) (MeV) | -84.27 | -88.26 | -92.58 | -98.62 | -106.51 |
| \(\langle \hat{T} \rangle^{(+;2)}\) (MeV) | 55.81 | 59.71 | 64.39 | 70.52 | 78.19 |
| \(E^{(+;2)}\) (MeV) | -28.46 | -28.55 | -28.19 | -28.10 | -28.32 |
| \(R_m\) (fm) | 1.43 | 1.41 | 1.39 | 1.36 | 1.33 |
| \(R_c\) (fm) | 1.64 | 1.62 | 1.60 | 1.58 | 1.56 |
| \(P(-)\) (%) | 10.5 | 13.3 | 16.1 | 18.8 | 21.4 |
| \(P(S = 2)\) (%) | 2.9  | 4.9  | 7.3  | 10.1 | 13.3 |

to 21.4%. In the last row in Table 3, we show the probability of the total spin \(S = 2\) component \(P(S = 2)\), which is defined as,

\[
P(S = 2) = \sum_{M_S = -2}^{2} \frac{\langle \Psi^{(+;2)}|\Phi_{S=2}(M_S)\rangle \langle \Phi_{S=2}(M_S)|\Psi^{(+;2)}\rangle}{\langle \Psi^{(+;2)}|\Psi^{(+;2)}\rangle},
\]

where

\[
\Phi_{S=2}(M_S) = \left[ [\chi_1 \times \chi_2]^{(1)} \times [\chi_3 \times \chi_4]^{(1)} \right]^{(2)}_{M_S}.
\]

\(P(S = 2)\) corresponds to the \(D\)-state probability, because in the present calculation the total wave function has the \(0^+\) spin-parity. \(P(S = 2)\) changes from 2.9% to 13.3% when \(x_T\) changes from 1.0 to 2.0, and is almost proportional to the tensor correlation energy \(\langle \hat{v}_T \rangle^{(+;2)}\). We note that the \(D\)-state probability of the alpha particle in the recent exact-type calculations [35] is about 15%. The matter root-mean-square radius \(R_m\) decreases with \(x_T\). The root-mean-square charge radius \(R_c\) is calculated from the proton root-mean-square radius \(R_p\) as \(R_c = \sqrt{R_p^2 + 0.64}\). This approximation for \(R_c\) corresponds to assuming the
charge radius of proton as 0.80 fm. The charge radii obtained here are slightly smaller than the experimental value $R_c = 1.676(8)$ fm [36]. The charge form factor has a dip around the momentum transfer squared $q^2 = 15$ fm$^{-2}$ for the case without the tensor force, and with the inclusion of the tensor force the position of the dip moves towards smaller momentum transfer. The position of the dip is around $q^2 = 11$ fm$^{-2}$ for $x_T = 1.5$ in accordance with the experiment, but the amount of the second bump is somewhat underestimated.

![Wave function squared](image)

**Fig. 1.** Single particle wave functions squared for the case with $x_T = 1.5$ and $x_{TE} = 0.81$ as a function of the radial coordinate $r$ (fm). The solid curve denotes the positive-parity neutron component $|\phi_{10}^{\frac{1}{2}}|^2$ ($l = 0$), the dotted curve the negative-parity neutron component $|\phi_{11}^{\frac{1}{2}}|^2$ ($l = 1$), the short-dashed curve the positive-parity proton component $|\phi_{10}^{\frac{1}{2}}|^2$ ($l = 0$), and the long-dashed curve the negative-parity proton component $|\phi_{11}^{\frac{1}{2}}|^2$ ($l = 1$). For comparison, we show the harmonic oscillator (HO) wave functions with the oscillator length 1.37 fm for the 0s state (dashed and dotted) and the 0p state (dashed and double dotted).

In Fig. 1, we show one of the intrinsic single-particle wave function (squared wave function) for the alpha particle with $x_T = 1.5$ in the CPPHF method. The dominant component of this wave function is the $s$-state (positive-parity state) and neutron. Due to the charge and parity mixings, there appear the components of the $p$-state and neutron, $s$-state and proton, and $p$-state and proton. In the wave function the probability of the negative parity component is 16% and that of the proton component is 17%. There is one more intrinsic single-particle state ($n = 2$), which has the $s$-state-and-proton component as the dominant one. In the figure the harmonic oscillator (HO) wave functions with the oscillator length $b = 1.37$ fm for the 0s and 0p state are also plotted for comparison. The 0s-state HO wave function almost corresponds to the calculated wave function in the simple HF scheme with the original Volkov
interaction \((x_T = 1.0, x_{TE} = 1.0)\). The \(s\)-state and neutron component in the CPPHF method has almost the same width as the HO wave function but the magnitude is somehow reduced. This reduction is caused by the parity and charge mixings. The mixing of parity reduces the wave function around the origin. It is interesting to see that the negative parity components have narrower widths as compared to the \(0p\)-state HO wave function [37]. If we calculate the overlap between the \(p\)-state wave function in the CPPHF method with the \(0p\)-state HO wave function changing the oscillator length \(b_{0p}\), the maximum overlap is achieved at \(b_{0p} = 0.81\) fm. It means that the negative parity component in the CPPHF method is not a simple \(0p\) state. The mixing of a higher momentum component is necessary to make the \(p\)-state have such a compact distribution. This fact suggests that the tensor force induces higher momentum components, which are not included in a simple shell model configuration. The mixing of the higher-momentum component results in the increase of the kinetic energy as seen in Table 3. The correlation induced by the tensor force produces more attractive energy and therefore the mixing of the higher-momentum component is favorable in total.

Table 4

The results for the first \(0^-\) state in the alpha particle. The potential energy \((\langle \hat{v} \rangle^{(-2)})\), kinetic energy \((\langle \hat{T} \rangle^{(-2)})\), total energy \((E^{(-2)})\), root-mean-square matter radius \((R_m)\), and the probability of the \(p\)-state component \((P(-))\) are shown. \(\langle \hat{v}_C \rangle^{(-2)}, \langle \hat{v}_T \rangle^{(-2)}, \langle \hat{v}_{LS} \rangle^{(-2)}, \langle \hat{v}_{Coul} \rangle^{(-2)}\) are the expectation values for the central, tensor, LS, and Coulomb potentials, respectively.

| \(x_T\)   | 1   | 1.25 | 1.5  | 1.75 | 2  |
|-----------|-----|------|------|------|----|
| \(x_{TE}\) |     |      |      |      |    |
| 1         | -29.05 | -29.73 | -29.36 | -28.16 | -26.03 |
| \(\langle \hat{v}_C \rangle^{(-2)}\) (MeV) |     |      |      |      |    |
| 1.25      | -4.57  | -8.68  | -14.00 | -20.49 | -28.18 |
| \(\langle \hat{v}_T \rangle^{(-2)}\) |     |      |      |      |    |
| 1.5       | 0.38   | 0.73   | 1.11  | 1.52  | 1.96 |
| \(\langle \hat{v}_{LS} \rangle^{(-2)}\) |     |      |      |      |    |
| 1.75      | 0.39   | 0.48   | 0.54  | 0.58  | 0.61 |
| \(\langle \hat{v}_{Coul} \rangle^{(-2)}\) |     |      |      |      |    |
| 2         | -32.85 | -37.21 | -41.72 | -46.55 | -51.63 |
| \(\langle \hat{v} \rangle^{(-2)}\) |     |      |      |      |    |
| \(\langle \hat{T} \rangle^{(-2)}\) |     |      |      |      |    |
| \(E^{(-2)}\) |     |      |      |      |    |
| \(R_m\) (fm) |     |      |      |      |    |
| \(P(-)\) (%) |     |      |      |      |    |

It is an interesting subject to solve the \(0^-\) state in the CPPHF method, since the \(0^-\) state is the daughter state of the \(0^+\) ground state in the parity- and charge-mixed intrinsic state. We calculate the \(0^-\) state of the alpha particle with the same parameters as the \(0^+\) state. The results are tabulated in Table 4. The attraction due to the tensor force increases with the tensor parameter \(x_T\), but the amount is less than a half of the case of the positive parity state.
The net binding energy comes out to be about 2.8 MeV. It is interesting to note, however, that the admixture of the opposite parity components $P(-)$ in the single-particle states is larger than the case of the $0^+$ state as seen in Table 3. This is related with the fact that $P(-)$ ought to be finite for the $0^-$ state even for the case without the tensor force, while for the $0^+$ case $P(-)$ becomes zero in such a case. From Table 3 and 4 the excitation energy $E_x$ from the ground state of the $0^-$ state in the CPPHF method is about 25.6 MeV in our calculation. The experimental value for the first $0^-$ state, which corresponds to the $0^-$ state calculated here, is $E_x = 21.010$ MeV with the total width $\Gamma = 0.840$ MeV [38]. In the case of the alpha particle, the energy difference between the daughter states, the positive and negative parity states, is reasonably large, although the intrinsic single-particle states have large breaking of the parity and charge symmetries.

Fig. 2. Densities for the alpha particle with $x_T = 1.5$ and $x_{TE} = 0.81$ as a function of the radial coordinate $r$ (fm). The density of the $0^+$ state (the solid curve) and that of the $0^-$ state (the dotted curve) together with that of the $0^+$ state with the original Volkov No. 1 force in the simple Hartree-Fock scheme (the dashed curve) are shown.

Finally, we show the density distributions of the $0^+$ and $0^-$ states in Fig. 2. The density of the $0^+$ in the CPPHF scheme decreases around the center and increase in the middle region (around $r = 0.8$ fm) from that of the HF scheme. This change is induced by the admixture of the $p$-state component. In fact, the wave function squared for the $p$-state in Fig. 1 have the maximum around $r = 0.8$ fm. Due to the smaller binding, the density of the $0^-$ state is much more spread over the space than the case of the $0^+$ state.
4 Summary

We have developed a mean-field framework with the projection method in order to treat the tensor force. To do this we mix parities and charges into a single-particle state, instead of using a single-particle state with a good parity and a definite charge number. We perform the projections of parity and charge from the intrinsic wave function, consisting of the parity- and charge-mixed single-particle states. We take the variation of the energy expectation value of the Hamiltonian with the projected total wave function, and obtain a Hartree-Fock-like equation, the charge- and parity-projected Hartree-Fock (CPPHF) equation.

We have applied the CPPHF equation to the alpha particle and shown that the CPPHF method is able to treat the strong tensor correlation. The variation after projection is very important to obtain the tensor correlation in the present framework, because the mixing of parity and charge does not occur if the parity and charge projections are not performed before variation.

We have obtained a large tensor correlation energy in the CPPHF method. The tensor correlation energy is about 31 MeV for $x_T = 1.5$. To obtain such large tensor correlation energy, the charge mixing and projection on top of the parity mixing and projection is very important due to the isovector character of the one-pion-exchange interaction. We have found that if we perform the mixing of both charge and parity and their projections, the tensor correlation energy becomes about three times larger than in the case where only the parity mixing and projection is performed. In the result of the alpha particle, we see that the $p$-state component has the narrower width than the $s$-state component, and does not seem like a simple $0^p$-state. It means that to obtain the correlation of the tensor force we need to mix higher-momentum components. This fact indicates the increase of the kinetic energy by the inclusion of the tensor correlation. In spite of the increase of the kinetic energy, the correlation energy from the tensor force produces more attractive energy and a net energy gain is achieved in total. The probability of the $p$-state component in the single-particle states becomes 10.5% to 21.4% with the parameters adopted here. The probability of the total spin $S = 2$ component, which corresponds to the $D$-state probability, changes 2.9% to 13.3% with the same parameters. As for the negative parity state ($0^-$), the total binding energy becomes around 2.8 MeV. We have found that the density for the $0^+$ state decreases around the center and increases in the middle-range region due to the mixing of the $p$-state component. The density distribution for the $0^-$ state is much broader than the ground state ($0^+$).

In this paper, we have pursued a mean-field (single-particle) framework with the projection method, in which the tensor force can be treated in the same
way as the central and LS forces. We have made such a framework by combining the mixings of charge and parity in a single-particle state and the charge and parity projections on the total wave function. In the present study, we have applied the CPPHF method only to the most simple closed-shell nucleus, the alpha particle. The application of the CPPHF scheme to heavier closed-shell nuclei, for example, $^{12}$C and $^{16}$O, will show the role of the tensor force in the formation of the shell structure of nuclei. The extension of the present framework to the deformed case is also interesting and will give us the insight of the role of the tensor force on the nuclear deformation and the nuclear clustering. An application of the present method on the relativistic mean field framework is under progress [39].

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