Current-Induced Resonant Motion of a Magnetic Vortex Core: Effect of Nonadiabatic Spin Torque

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The current-induced resonant excitation of a magnetic vortex core is investigated by means of analytical and micromagnetic calculations. We find that the radius and the phase shift of the resonant motion are not correctly described by the analytical equations because of the dynamic distortion of a vortex core. In contrast, the initial tilting angle of a vortex core is free from the distortion and determined by the nonadiabaticity of the spin torque. It is insensitive to experimentally uncontrollable current-induced in-plane Oersted field. We propose that a time-resolved imaging of the very initial trajectory of a core is essential to experimentally estimate the nonadiabaticity.

A spin-polarized current can exert torque to a ferromagnet by transferring spin-angular momentum, i.e. spin-transfer torque. The spin-transfer torque provides full magnetization reversal, steady-state precession motion, and domain wall movement \cite{1,2}. It is composed of adiabatic and nonadiabatic spin torque terms in continuously varying magnetization. The adiabatic spin torque arises from the conduction electron spin whose projection on the film plane follows the direction of a local magnetization, whereas the nonadiabatic torque arises from a mismatch of the direction as a result of the momentum transfer or the spin relaxation \cite{2,3,4}.

Until now, the experimental threshold current density $J_C$ to steadily move a domain wall has been reported to be about $10^8 A/cm^2$, too large for an application. In addition to the resonant depinning \cite{6} and the use of perpendicular anisotropy \cite{7}, an increase of $\beta$, the ratio of the nonadiabatic spin torque to the adiabatic one can reduce $J_C$ \cite{8}. Despite its importance, however, the exact value of $\beta$ is still under debate even in theories \cite{6,10,11,12,13}; $\beta = 0$, $\beta = \alpha$, and $\beta \neq \alpha$ where $\alpha$ is the Gilbert damping constant. This debate is also related to which mechanism between the Landau-Lifshitz damping and the Gilbert one is correct to describe the energy dissipation \cite{14}. Experimental determination of $\beta$ is essential to resolve this debate, but experimentally estimated values are also distributed; $\beta = 8\alpha$ \cite{6}, $\beta = \alpha$ \cite{13}, $\beta > \alpha$ \cite{16}, $\beta = 2\alpha$ \cite{17}, and $\beta \neq \alpha$ \cite{18}. Since most experiments have used the same material (Permalloy), this wide distribution is caused by origins irrelevant to the material itself.

The wide distribution can originate from the Joule heating, the edge roughness of nanowire, and the in-plane component of the current-induced Oersted field $H_{\text{Oe}}^{\text{In}}$. The Joule heating significantly affects $J_C$ and wall velocity \cite{13,19}. Therefore, it is difficult to precisely estimate $\beta$ when the Joule heating is not negligible, i.e. $J_C > 10^8 A/cm^2$. In a magnetic nanowire, the edge roughness distorts the domain wall \cite{20} and thus prevents a proper interpretation of experimental data using theories derived for an ideal nanowire. A way to avoid the above issues is to experimentally study resonant motions of a magnetic vortex core (VC) in a patterned disk by injecting an alternating current of the order of $10^7 A/cm^2$. The magnetic vortex is an ideal system for the resonant motion study since VC can be considered as a topological point charge which efficiently responses to external forces \cite{21,22,23}. It was experimentally confirmed that the VC can be resonantly excited by an ac current \cite{6,16,24,25}. Even in this case, however, a very small in-plane component of the ac current-induced alternating Oersted field $H_{\text{Oe}}^{\text{In}}$ inhibits a precise estimation of the spin torque parameters \cite{25,26}. Note that the $H_{\text{Oe}}^{\text{In}}$ is not a current-induced field along the thickness direction of the disk, but an in-plane field caused by any geometrical symmetry-breaking of the system. The driving force due to the $H_{\text{Oe}}^{\text{In}}$ of only 0.3 Oe is as large as 30% of the total resonant excitation \cite{25}. Such a small $H_{\text{Oe}}^{\text{In}}$ is difficult to remove since it is caused by an uncontrollable nonuniform current distribution due to a geometrical symmetry-breaking such as electric contacts or notches. Therefore, an experimental way to estimate the $\beta$ which is safe from the Joule heating and the edge roughness, and also insensitive to the $H_{\text{Oe}}^{\text{In}}$ is highly desired.

In this letter, we propose that a direct imaging of the very initial trajectory of VC induced by an ac current is a plausible way to experimentally estimate $\beta$. We find that $\beta$ does not change the resonant frequency, but affects the phase of resonant motion. The phase-shift is $\beta$-dependent since $\beta$ determines the tilting angle of very initial trajectory measured from the direction of the electron-flow. On the other hand, the $H_{\text{Oe}}^{\text{In}}$ with a typical magnitude does not change the tilting angle although it affects the steady resonant motion. More importantly, the initial tilting angle is only one physical observable which can be directly compared to the analytical result, whereas the others such as the radius and the phase shift are not correctly described by the analytical equations because of the dynamic distortion of VC.

The current-induced motion of VC is calculated using...
the Thiele’s equation with the spin-transfer torque terms (Eq. 1) \[ \mathbf{G}(p) \times (\mathbf{u} - \dot{\mathbf{r}}) = -\frac{\delta U(\mathbf{r})}{\delta \mathbf{r}} - \alpha \mathbf{D} \mathbf{r} + \beta \mathbf{D} \mathbf{u} \] (1)

where \( G(p) = -G_0 pe_z \) is the polarity \((p \pm 1)\) dependent gyrovector, \( G_0 \) is obtained from the spin texture as

\[ G_0 = \frac{M_S}{\gamma} \int dV \sin(\theta) (\nabla \theta \times \nabla \psi) \cdot e_z, \] (2)

\( \theta (\psi) \) is the out-of-plane (in-plane) angle of the magnetization, \( M_S \) is the saturation magnetization, \( \gamma \) is the gyromagnetic ratio, \( u = u_0 \exp(\imath \omega t) e_y \) is the amplitude of adiabatic spin torque, \( \omega \) is the angular frequency of the ac current, \( \mathbf{r}(t) = X(t) + Y(t) \) is the time-dependent position vector of VC, and \( U(\mathbf{r}) \) is the potential well. The damping tensor \( D \) is also obtained from the integration of spin texture as

\[ D = -\frac{M_S}{\gamma} \int dV\left[ (\nabla \theta \nabla \theta) + \sin^2(\theta) (\nabla \psi \nabla \psi) \right]. \] (3)

When VC is at the static equilibrium position, \( G_0 \) is \( 2\pi L M_S/\gamma \) and \( D \) is \( G_0 \ln(R/\delta)/2 \) where \( L \) is the thickness of disk, \( R \) is the vortex radius, and \( \delta \) is the core diameter.

With \( X(t) = X_0 \exp(\imath \omega t), Y(t) = Y_0 \exp(\imath \omega t) \) and \( 1 + (\alpha D/G_0)^2 \sim 1 \), the solutions are in the form of \( X(t) = X_1 \cos(\omega t) - X_2 \sin(\omega t) \) and \( Y(t) = Y_1 \cos(\omega t) - Y_2 \sin(\omega t) \) where

\[ X_1 = A \omega_r\left[ (\omega_r^2 - \omega^2) + 2C^2\alpha(\beta - \alpha)\omega^2 \right], \]
\[ X_2 = A \omega C[(\beta - \alpha)(\omega_r^2 - \omega^2) + 2\alpha \omega_r^2], \]
\[ Y_1 = A \omega C[(\omega_r^2 - \omega^2) + 2\alpha(1 + C^2\alpha\beta)\omega^2], \]
\[ Y_2 = A \omega(1 + C^2\alpha\beta)(\omega_r^2 - \omega^2) - 2C^2\alpha\beta\omega_r^2. \] (4)

Here, \( A = u_0/[(\omega_r^2 - \omega^2) + (2C\alpha \omega_r \omega)^2] \), \( \omega_r = \kappa/G_0 \) is the resonance frequency, \( \kappa = (dU/dr)/R \) is the effective stiffness coefficient of the potential well, and \( C \) is \( D/G_0 = \ln(R/\delta)/2 \). From the eqs. (4), one finds the radius \( \alpha(t) = \sqrt{X(t)^2 + Y(t)^2} \), and the phase shift \( \phi \) between the phase of the core gyration and that of the ac current;

\[ \phi = \tan^{-1}\left( \frac{X_1}{Y_1} \right) \]
\[ = \tan^{-1}\left( \frac{1 - (\omega_r/\omega)^2 + 2C^2\alpha(\beta - \alpha)}{C\beta(\omega_r/\omega)^2 - 1 + 2C\alpha(1 + C^2\alpha\beta)} \right). \] (5)

To verify the validity of the analytical solutions, the micromagnetic simulation is also performed by means of the Landau-Lifshitz-Gilbert equation including the spin torque terms;

\[ \frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_S} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \]
\[ + u_0 \mathbf{V} \mathbf{M} \times \nabla \mathbf{M} \] (6)

where \( \mathbf{H}_{eff} \) is the effective field including the external, the magnetostatic, the exchange, and the current-induced Oersted field. The model system is a circular Permalloy disk with the thickness of 10nm and the diameter of 270nm (Fig. 1(a)). The unit cell size is \( 2 \times 2 \times 10nm^3 \). The ac current with the maximum current density of \( 1.25 \times 10^7 A/cm^2 \) is applied along the y-axis. Standard material parameters for Permalloy are used; \( M_S = 800emu/cm^3, \gamma = 1.76 \times 10^7 Oe^{-1}s^{-1}, \alpha = 0.01, P = 0.7, \) and the exchange constant \( A_{ex} = 1.3 \times 10^{-6}erg/cm \).

First, we assume \( H_{0x}^{ln} = 0 \) in order to investigate the effect originating exclusively from \( \beta \) on the resonant motion. We will recall the effect of \( H_{0x}^{ss} \) in the latter part. In Fig. 2 (a) and (b), we show analytical and modeling results of the time averaged value \( \langle Y(t) \cdot I_0 \sin(\omega t) \rangle/I_0 \) at various \( \beta \)-terms. To obtain analytical results, we use \( C = 1.3 \) because \( R \) is 135nm and \( \delta \) is 10nm, determined from the micromagnetic configuration at the initial equilibrium state. \( \langle Y(t) \cdot I_0 \sin(\omega t) \rangle/I_0 \) shows a peak at the resonance frequency \( \omega_r \) of 360MHz. \( \omega_r \) does not change with \( \beta \) whereas the peak structure becomes more asymmetric as \( \beta \) increases. In spite of qualitative agreement, however, analytical results are quantitatively different from modeling ones. This is because the radius of gyroscopic motion is different between the modeling result and the analytic solution (inset of Fig. 2(b)). We attribute this difference to a dynamic distortion of VC. As shown in Fig. 2(c), the VC shape in the initial equilibrium state is symmetric whereas it in a dynamic motion is asymmetric. The distortion changes the gyroscopic parameter \( G_0 \), the damping tensor \( D \) (thus, the parameter \( C \)) and the effective stiffness coefficient \( \kappa \), but all parameters are determined by details of the spin texture (See eqs. (2), (3), and the definition of \( \kappa \)). From micromagnetic spin textures of the vortex in the dynamic motion, we find that both \( C \) and \( \kappa \) increase from the
initial equilibrium values because of the dynamic distortion (Fig. 2(d)). The increase of $\kappa$ is much larger than that of $C$, and responsible for the reduced radius in the modeling results. This increase of $\kappa$ occurs at the very initial time stage, indicating that the assumption of the rigid VC and potential well is invalid except for the very initial trajectory. Therefore, even when $H_{Oe}^I$ is zero, it may be difficult to deduce important parameters such as $P$ by directly comparing analytical solutions with experimental measurements of the steady gyroscopic motion.

Nevertheless, it is worthwhile investigating how $\beta$ induces the asymmetry in the peak structure. Fig. 3(a) shows analytical phase shifts at various $\beta$-terms. As $\beta$ increases, the absolute value of the phase shift $\phi$ decreases (increases) for the frequency smaller (higher) than $\omega_r$. In other words, a vertical offset $\phi(\beta) - \phi(\beta = \alpha)$ of the phase shift increases with increasing $\beta$. From the eq. (11) with $\alpha \ll 1$ and $\beta \ll 1$, one can find that the vertical offset is approximately $\tan^{-1}[C(\beta - \alpha)]$ and thus dependent on $\beta$. This is why the peak structure becomes more asymmetric as increases. However, quantitative disagreement between analytic solution and modeling result was again observed (inset of Fig. 3(a)). The difference becomes larger as the radius of core gyration increases, i.e. the frequency approaches $\omega_r$. This is also caused by the dynamic distortion as explained above.

The vertical offset is $\beta$-dependent since the initial tilting angle $\theta_{int}$ is determined by $\beta$ (Fig. 3(b)). VC initially moves along the direction of the electron-flow. Because of the imbalance of magnetostatic field, VC experiences the centripetal force and starts to undergo a gyration motion. In the absence of $H_{Oe}^I$, $\theta_{int}$ of the initial trajectory can be obtained from the eq. (1) by dropping the potential gradient term since VC is initially at the bottom of the potential well where the gradient is zero. When $H_{Oe}^I$ is nonzero, the potential gradient is no longer zero and could affect $\theta_{int}$. In order to investigate the effect of $H_{Oe}^I$ on $\theta_{int}$, we perform micromagnetic simulations for the initial trajectory with and without taking into account $H_{Oe}^I$. We assume that an alternating $H_{Oe}^I$ is applied along the x-axis and its magnitude is 0.3Oe which is similar with the estimated value in the experiment of the Ref. [25]. As shown in Fig. 4(a), the effect of $H_{Oe}^I$ on the very initial trajectory is negligible whereas the difference in trajectories between the two cases becomes larger and larger as the time evolves. This insensitivity of the initial trajectory and thus $\theta_{int}$ to $H_{Oe}^I$ is valid for a different $\beta$ (not shown). Thus, we drop the potential gradient term in the eq. (11) to derive $\theta_{int}$. With $p = +1$ and the direction of the initial current along the +y-axis, one finds

$$\alpha DX - G_0 dY = G_0 u_0,$$
$$G_0 X + \alpha D Y = -\beta D u_0$$  

(7)

where $X$ and $Y$ are the velocity along the x- and y-axis, respectively. By solving the eqs. (7) for $X$ and $Y$, we get

$$X = G_0 u_0 D - \frac{\alpha - \beta}{\alpha^2 D^2 + G_0^2},$$
$$Y = -u_0 \frac{G_0^2 + \alpha \beta D^2}{\alpha^2 D^2 + G_0^2}.$$  

(8)

FIG. 2: (Color online) The $<Y(t) : I_0 \sin(\omega t) \rangle / I_0$ as a function of the frequency obtained from (a) analytic solution of Thiele’s equation and (b) micromagnetic simulation. (c) Comparison of the shape of vortex core, and (d) variation of the frequency obtained from (a) analytic solution of Thiele’s equation and (b) micromagnetic simulation. (c) and (d) are the velocity along the x- and y-axis, respectively. By solving the eqs. (7) for $X$ and $Y$, we get
The initial tilting angle $\theta_{\text{int}}$ is given by

$$\theta_{\text{int}} = \tan^{-1}\left(\frac{X}{Y}\right) = \tan^{-1}\left(\frac{C(\alpha - \beta)}{1 + \alpha\beta C^2}\right).$$  \quad (9)

Note that the eq. (9) is equivalent to the equation for the vertical phase-shift with considering the sign of the initial ac current and $\omega = \omega_\text{r}$. It confirms that the vertical shift originates from the $\beta$-dependent $\theta_{\text{int}}$. Fig. 4(b) shows $\theta_{\text{int}}$ as a function of $\beta/\alpha$ for various values of $C$. It should be noted that $\theta_{\text{int}}$’s obtained from modeling are in excellent agreement with analytical ones in contrast to the radius and the phase shift. It is because VC retains its equilibrium shape at the very initial time stage where $C$ and $\kappa$ hardly changes. For the tested Permalloy disk ($C = 1.3$), the difference in $\theta_{\text{int}}$ between $\beta = 0$ and $\beta = 8\alpha$ is about 6 degree which may be small to experimentally measure. However, the $\beta$-dependent $\theta_{\text{int}}$ becomes larger as the parameter $C$ increases. For instance, the difference in $\theta_{\text{int}}$ between $\beta = 0$ and $\beta = 8\alpha$ is about 18 degree at $C = 4$. The $C$ increases as the disk diameter (core diameter) increases (decreases). The core diameter $\delta$ decreases with decreasing the disk thickness [31]. The equilibrium value of $C$ can be determined by micromagnetic calculation or MFM imaging for a given disk geometry. Consequently, a time-resolved magnetic imaging with high spatial resolution such as X-ray microscopy [31] for observation of the very initial trajectory of VC in a wider and thinner disk is a possible way to experimentally estimate $\beta$.

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