Enhancement of Fluctuation-Induced Electromagnetic Phenomena in dynamically nonequilibrium systems at Resonant Photon Emission

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We study the resonances in Casimir friction, radiative heat transfer and heat generation for two plates sliding relative to each other. Resonances have a different origin in the frequency range of the normal (NDE) and anomalous (ADE) Doppler effect. In the frequency range of NDE, resonances are associated with resonant photon tunnelling between surface phonon/plasmon polaritons of plates. For two identical plates, such resonances exist only at a relative sliding velocity $v=0$. However, for different plates such resonance can exist at $v \neq 0$. In the frequency range of ADE, resonances are associated with the creation of excitations in both plates. While in the frequency range of NDE the photon emission rate has an upper limit, in the frequency range of ADE, the photon emission rate can diverge even in the presence of dissipation in the system. We consider resonances for the identical and different sliding plates. We discuss the possibility to detect Casimir friction with its limiting case of quantum friction using an atomic force microscope in graphene structures.

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I. INTRODUCTION

Quantum fluctuations of the electromagnetic field manifest themselves in a wide variety of fields of physics. For example, the Lamb shift of atomic spectrum and anomalous magnetic moment of the electron were explained with the help of this idea. Most directly, these fluctuations are manifested through the Casimir effect. In the late 1940s Hendrik Casimir predicted\(^1\) that two macroscopic non-magnetic bodies with no net electric charge (or charge moments) can experience an attractive force much stronger than gravity. The existence of this force is one of the direct macroscopic manifestations of quantum mechanics. Hendrik Casimir based his prediction on a simplified model involving two parallel perfectly conducting plates separated by vacuum. A unified theory of both the van der Waals and Casimir forces between plane parallel material plates, in thermal equilibrium and separated by a vacuum gap, was developed by Lifshitz (1955)\(^2\). To calculate the interaction force Lifshitz used Rytov's theory of the fluctuating electromagnetic field\(^3\). At present the interest of Casimir forces is increasing because they dominate the interaction between nanostructures, and are often responsible for the friction between moving parts in small devices such as micro- and nanoelectromechanical systems, and can be considered as practical mechanisms for the actuation of such devices. Due to this practical interest, and the fast progress in force detection techniques, experimental and theoretical investigations of Casimir forces have experienced an extraordinary renaissance in the past few years (see\(^4\) for a review and the references therein).

Another manifestation of quantum fluctuations of the electromagnetic field is the noncontact quantum friction between bodies in relative motion\(^5-7\). The noncontact friction determines the ultimate limit to which the friction force can be reduced and, consequently, also the force fluctuations because they are linked to friction via the fluctuation-dissipation theorem. The force fluctuations (and hence friction) are important for the ultrasensitive force detection. Friction is usually a very complicated process. The simplest case consists of two flat surfaces, separated by a vacuum gap (see Fig. 1), sliding relative to each other at $T = 0$ K, where the friction is generated by the relative movement of quantum fluctuations\(^6-15\). The thermal and quantum fluctuations of the current density in one body induces the current density in other body; the interaction between these current densities is the origin of the Casimir interaction. When two bodies are in relative motion, the induced current will lag slightly behind the fluctuating current inducing it, and this is the origin of the Casimir friction. At present the Casimir friction is attracting a lot of attention due to the fact that it is one of the mechanisms of noncontact friction between bodies in the absence of direct contact\(^6-7\). The Casimir friction was studied in the configurations plate-plate\(^6-16\), neutral particle-plate\(^6,7,14-17,29\), and neutral particle-blackbody radiation\(^6,7,14-27,30-34\). While the predictions of the theory for the Casimir forces were verified in many experiments\(^6\), the detection of the Casimir friction is still challenging problem for experimentalists. However, the frictional drag between quantum wells\(^35,36\) and graphene sheets\(^37,38\), and the current-voltage dependence of nonsuspended graphene on the surface of the polar dielectric SiO\(^29\), were accurately described using the theory

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of the Casimir friction. At present, the frictional drag experiments were performed for weak electric field when the induced drift motion of the free carriers is smaller than the threshold velocity for quantum friction. Thus in these experiments the frictional drag is dominated by the contributions from thermal fluctuations. However, the measurements of the current-voltage dependence were performed for high electric field, where the drift velocity is above the threshold velocity, and where the frictional drag is dominated by quantum fluctuations.

Quantum friction is associated with creation of excitations of different kind. For transparent dielectrics such excitations are photons and quantum friction is associated with quantum Vavilov-Cherenkov radiation. Thus there is a close connection between quantum friction and the quantum Vavilov-Cherenkov radiation – both of these phenomena are related with the instability of the quantum state against spontaneous production of elementary excitations in the conditions of the anomalous Doppler effect. According to Landau the same instability breaks superfluid order beyond a certain velocity. Quantum Vavilov-Cherenkov radiation was first described by Frank and Ginzburg and Frank (see also for review of these work). Quantum friction is associated with creation of surface phonon polaritons for dielectrics or surface plasmon polaritons for metals. If an object has no internal degrees of freedom (e.g., a point charge), then the energy of the radiation is determined by the change of the kinetic energy of the object. However, if an object has internal degrees of freedom (say, an atom), then two types of excitations are possible. If the frequency of the excitation in the lab reference frame $\omega > 0$, then in the rest frame of an object, due to the Doppler effect, the frequency of the radiation $\omega' = \omega - k_r v$. In the normal Doppler effect frequency region, when $\omega' < 0$, the excitation energy is determined by the decrease of the internal energy. For example, for an atom the state may changes from the excited state $|1\rangle$ to the ground state $|0\rangle$. The region of the anomalous Doppler effect corresponds to $\omega' < 0$ in which case an object becomes excited when it creates an excitation in the lab reference frame. For example, an atom could experience the transition from the ground state $|0\rangle$ to the excited state $|1\rangle$ when it radiates. In such a case energy conservation requires that the energy of the excitations result from a decrease of the kinetic energy of the object. That is, the self-excitation of a system is accompanied by a slowing down of the motion of the object as a whole. For a neutral object the interaction of the object with the matter is determined by the fluctuating electromagnetic field due to the quantum fluctuations inside the object.

While a constant translational motion requires for the emission of the radiation at least two bodies in relative motion (otherwise it is not possible due to Lorenz invariance), a single accelerated object can radiate and experience friction. Quantum fluctuations of the electromagnetic field are determined by virtual photons that are continuously created and annihilated in the vacuum. Using a metal mirror in accelerated motion, with velocities near the light velocity, virtual photons can be converted into real photons, leading to radiation emitted by the mirror. This is the dynamic Casimir effect; which recently was observed in a superconducting waveguide. Radiation can be also emitted by a rotating object. In fact this phenomenon is closely related to the prediction by Zeldovich of an amplification of certain waves during scattering from a rotating body. Rotation quantum friction acting on a nanoparticle rotating near the surface was studied in . The authors showed that quantum friction increases strongly at resonant generation of surface phonon polaritons at the rotation frequency $\Omega = \omega_1 + \omega_2$, where $\omega_1$ and $\omega_2$ are the frequencies of surface polaritons for the particle and surface, respectively. In it was shown that quantum resonance satisfies the condition for which the electromagnetic field becomes classical. Stationary rotation is impossible at resonance because the friction torque increases unlimitedly with time. However, stationary rotation is possible near resonance. In this case, fluctuation-induced electromagnetic effects increase strongly. Singular resonance at the relative sliding of two identical plates was first predicted in and was then studied in . Singular resonance for two rotating nanoparticles was first predicted in . Singular resonance for a body rotating in vacuum is impossible because it requires at least two bodies. However, singular resonance is possible for a body rotating in a cavity. This effect was first predicted by Zeldovich and was recently studied for a cavity with rotating walls. Resonant photon tunnelling enhancement of the radiative heat transfer between two plates at rest was considered in Ref. In this article more general study of the resonances which exist in Casimir friction, radiative heat transfer and heat generation for two plates sliding relative to each other is presented. We consider the resonances which exist in the frequency range of the normal and anomalous Doppler effect.

II. CASIMIR FRICTION BETWEEN TWO PLATES

A. General theory

We consider two plates sliding relative to each other with velocity $v$ and separated by a distance $d$ (see Fig.1). According to the theory of the Casimir friction, the contributions of the evanescent waves (which dominate for short separation between plates, large velocities and low temperatures) to the friction force $F_x$, and the heat power $P_1$ absorbed by plate 1 in the nonrelativistic and nonretarded limit in the lab reference frame, are determined by formulas
Other with the velocity $v$ obtained from the preceding terms by permutation of indexes $R$ - polarized electromagnetic wave.

FIG. 1: Two semi-infinite bodies with plane parallel surfaces separated by a distance $d$. The upper solids moves parallel to other with the velocity $v$.

\[
\left( \frac{F_{1z}}{P_1} \right) = \int_0^\infty \frac{d^2q}{(2\pi)^2} \int_0^\infty \frac{d\omega}{2\pi} \left( \frac{\hbar q_x}{\hbar \omega} \right) \Gamma_{12}(\omega, q) \text{sgn}(\omega^-) [n_2(\omega^-) - n_1(\omega)]
\]

where the positive quantity

\[
\Gamma_{12}(\omega, q) = 4 \text{sgn}(\omega^-) \left\{ \frac{\text{Im} R_{1p} \text{Im} R_{2p}^-}{|\Delta_{pp}|^2} + (p \leftrightarrow s) \right\} e^{-2qy}
\]

can be identified as a spectrally resolved photon emission rate,

\[
\Delta_{pp} = 1 - e^{-2qy} R_{1p} R_{2p}^-.
\]

$n_1(\omega) = [\exp(\hbar \omega/k_B T_1) - 1]^{-1}$, $R_{1p(s)}$ is the reflection amplitude for surface 1 in the rest reference frame for a $p(s)$ - polarized electromagnetic wave, $R_{2p(s)} = R_{2p(s)}(\omega^-, q)$ is the reflection amplitude for surface 2 in the rest frame of plate 2 for a $p(s)$ - polarized electromagnetic wave, $\omega^+ = \omega - q_x v$. The symbol $(p \leftrightarrow s)$ denotes the terms that are obtained from the preceding terms by permutation of indexes $p$ and $s$. In the domains of the normal Doppler effect ($\omega - q_x v > 0$) the last factor in the integrand in Eq. (1) can be written in the form

\[
\text{sgn}(\omega^-)[n_2(\omega^-) - n_1(\omega)] = n_2(\omega^-)[1 + n_1(\omega)] - n_1(\omega)[1 + n_2(\omega^-)].
\]

Thus, in this domain the energy and momentum transfer are related as when the excitations are annihilated in one body and created in other body. Such processes are only possible at $T \neq 0$ K, i.e. they are associated with thermal radiation. On the other hand, in the case of the anomalous Doppler effect ($\omega - q_x v < 0$)

\[
\text{sgn}(\omega^-)[n_2(\omega^-) - n_1(\omega)] = 1 + n_2(|\omega^-|) + n_1(\omega) = [1 + n_2(|\omega^-|)][1 + n_1(\omega)] - n_2(|\omega^-|) n_1(\omega).
\]

In this case the excitations are created and annihilated simultaneously in both bodies. Such processes are possible even at $T = 0$ K and are associated with quantum friction.

At $T_1 = T_2 = 0$ K the propagating waves do not contribute to the friction and the radiative heat transfer. However, the contribution from the evanescent waves does not vanish. Taking into account that $n_1(\omega) = 0$ at $T_1 = T_2 = 0$ K and $\omega > 0$,

\[
n_2(\omega^-) = \begin{cases} -1 & \text{for } 0 < \omega < q_x v \\ 0 & \text{for } \omega > q_x v \end{cases}
\]

Eq. (1) is reduced to the formula obtained by Pendry in Ref. 2

\[
\left( \frac{F_x}{P_1} \right) = -\frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_z \int_0^{q_x v} d\omega \left( \frac{\hbar q_x}{\hbar \omega} \right) \left( \frac{\text{Im} R_{1p} \text{Im} R_{2p}^-}{|\Delta_{pp}|^2} + \frac{\text{Im} R_{1s} \text{Im} R_{2s}^-}{|\Delta_{ss}|^2} \right) e^{-2qy},
\]

Equation (3) is reduced to the formula obtained by Pendry in Ref. 2.
B. Resonances in the Casimir friction

1. The frequency range of the normal Doppler effect

In the frequency range of the normal Doppler effect, when \( \omega > 0 \) and \( \omega - \Omega > 0 \), we can write \( R_{1p}(\omega) = |R_{1p}(\omega)\exp(i\phi_1)| \) and \( R_{2p}(\omega - q_x v) = |R_{2p}(\omega - q_x v)\exp(i\phi_2)| \), and the photon emission rate can be written in the form

\[
\Gamma_{12} = \frac{4|R_1(\omega)|^2 |R_2(\omega - q_x v)| e^{-2qd} |1 - e^{-2qd} R_p(\omega)|^2}{1 + |R_1(\omega)|^2 |R_2(\omega - q_x v)|^2 |1 - 2|R_1(\omega)||R_2(\omega - q_x v)e^{-2qd}||\cos(\phi_1 + \phi_2)|},
\]

which is maximal (\( \Gamma_{\text{max}} = 1 \)) for \( |R_1(\omega)||R_2(\omega - q_x v)e^{-2qd}| = 1 \) and \( \phi_1 = \phi_2 \). Thus at \( v = 0 \) \( P_1 \leq P_{\text{max}} \), where

\[
P_{\text{max}} = \frac{k_B^2}{4\varepsilon_0 h} (T_2^2 - T_1^2) q_c^2
\]

where \( q_c \) is a cut of in \( q \), determined by the properties of the materials. The largest possible \( q_c \sim 1/b \) where \( b \) is an inter-atomic distance. Thus the ratio of the maximal heat flux connected with evanescent waves to heat flux due to black body radiation \( P_{\text{max}}/P_{\text{BB}} \sim (\lambda T/b)^2 \) where \( \lambda_T = c h/k_B T \). At room temperature, the maximal contribution to the heat flux from the evanescent waves will be approximately in \( 10^8 \) times larger than the contribution from black body radiation.

The radiative heat transfer between two plates is strongly enhanced in the case of the resonant photon tunneling. The reflection amplitude for the dielectric plate for \( d < \varepsilon / (\varepsilon_1 |\epsilon_1|) \)

\[
R_{ip} = \frac{\varepsilon_i - 1}{\varepsilon_i + 1},
\]

where \( \varepsilon_i \) and \( \omega_i \) are the dielectric function and the phonon polariton frequency for dielectric \( i \). The dielectric surfaces have resonances at \( \omega_i'(\omega) = -1 \) where \( \omega_i' \) is the real part of \( \omega_i \). For a polar dielectric \( \omega_i \) determines the frequency of the surface phonon polariton. Near the resonance at \( \omega \approx \omega_i \) and \( \omega - q_x v \approx \omega_i \) for \( 2/\varepsilon_i'' >> 1 \) in the frequency region of the normal Doppler effect the reflection amplitudes for dielectrics can be written in the form

\[
R_{1p}(\omega) = -\frac{a_1}{\omega - \omega_1 + i\Gamma_1}, \quad R_{2p}(\omega - q_x v) = -\frac{a_2}{\omega - q_x v - \omega_2 + i\Gamma_1},
\]

where

\[
a_i = \frac{2}{(d/d\omega)\varepsilon_i'(\omega)_{|\omega=\omega_i}}, \quad \Gamma_i = \frac{\varepsilon_i''(\omega_i)}{(d/d\omega)\varepsilon_i'(\omega)_{|\omega=\omega_i}}.
\]

For two identical surfaces, when \( \omega_1 = \omega_2 = \omega_0 \), \( a_1 = a_2 = a \) and \( \Gamma_1 = \Gamma_2 = \Gamma \), the resonant condition \( \phi_1 = \phi_2 \) can be only satisfied at \( v = 0 \). The second resonance condition \( |R_p(\omega)|^2 e^{-2qd} = 1 \) is satisfied at \( \omega = \omega_\pm \) where

\[
\omega_\pm = \omega_0 \pm \sqrt{a^2 e^{-2qd} - \Gamma^2}.
\]

Close to the resonance the photon emission rate for two identical dielectric plates can be written in the form

\[
\Gamma_{12} = \frac{4(|\text{Im}R_{de}e^{-qd}|^2)^2}{|1 - e^{-2qd} R_p|^2} \approx \frac{4(a^2 e^{-qd})^2}{(|\omega - \omega_+|^2 + \Gamma^2)[(\omega - \omega_-)^2 + \Gamma^2]}.
\]

where

\[
\omega_{\pm} = \omega_0 \pm ae^{-qd}.
\]

Thus \( \Gamma_{12} = 1 \) at \( \omega = \omega_\pm \). Using Eq. (10) in Eq. (11) gives the resonant contribution to the heat transfer

\[
P_1 \approx \frac{\hbar \omega_0 \Gamma}{2\pi} \int_0^\infty dq q \frac{[2e^{-qd}/\varepsilon''(\omega_0)]^2}{[2e^{-qd}/\varepsilon''(\omega_0)]^2 + 1} \approx \frac{\hbar \omega_0 \Gamma}{2\pi} \int_0^{q_c} dq q
\]
condition for resonance requires that at

\[ q \omega = \omega \]

by \( \omega \). For example, the dielectric function of amorphous SiO\(_2\) at \( T = 300K \) and \( R \) for the reflection amplitudes given by Eq. (7)

\[ (10) \]

Using (10) in (13) gives the resonant contribution to the friction coefficient

\[ \gamma \approx \frac{\hbar^2 T}{4\pi k_B T} \int_0^\infty dq q^3 \frac{[2e^{-qd}/\epsilon''(\omega)]^2}{\sinh^2 \left( \frac{\hbar \omega}{2k_B T} \right)} \approx \frac{\hbar^2 T}{4\pi k_B T} \int_0^\infty dq q^3 \]

where \( \epsilon'' \) is the imaginary part of \( \epsilon \), \( q_c = \ln[2/\epsilon''(\omega)]/d \). It was assumed that \( 2/\epsilon''(\omega) \gg 1 \), \( \omega \gg a\exp(-qd) \). To linear order in the velocity \( v \) the friction force \( F = \gamma v \) where at \( T_1 = T_2 = T \), the friction coefficient

\[ \frac{h\omega_0}{4\pi} \Gamma_0^2 \left[ n_2(\omega_0) - n_2(\omega) \right], \quad (12) \]

At small frequencies far from the resonance (\( \omega \ll \omega_0 \)) \( \Gamma_{12} \approx (\omega/\omega^*)^2 \) and the off-resonant contribution to the friction coefficient

\[ \gamma_{\text{off res}} \approx \frac{\hbar}{16\pi d^3} \left( \frac{k_B T}{\hbar \omega^*} \right)^2. \quad (15) \]

For example, the dielectric function of amorphous SiO\(_2\) can be described using an oscillator model (13)].

\[ \epsilon(\omega) = \epsilon_\infty + \sum_{j=1}^{2} \frac{\sigma_j}{\omega^0_{\omega,j} - \omega^2 - i\omega\gamma_j}, \quad (16) \]

where parameters \( \omega_{0,j}, \gamma_j \) and \( \sigma_j \) were obtained by fitting the actual \( \epsilon \) for SiO\(_2\) to the above equation, and are given by \( \epsilon_\infty = 2.0014, \sigma_1 = 4.4767 \times 10^{27} \text{s}^{-1}, \omega_{0,1} = 8.6732 \times 10^{13} \text{s}^{-1}, \gamma_1 = 3.3026 \times 10^{12} \text{s}^{-1}, \sigma_2 = 2.3584 \times 10^{28} \text{s}^{-2}, \omega_{0,2} = 2.0219 \times 10^{14} \text{s}^{-1}, \) and \( \omega_{0,2} = 1.1 \times 10^{13} \text{s}^{-1}, \epsilon''(\omega) = 1, \omega^* = 2.3 \times 10^{16} \text{s}^{-1} \). With these parameters Eqs. (27) and (15) at \( T = 300K \) and \( d = 1\text{nm} \) give \( \gamma_{\text{res}} = 3.5 \times 10^{-2} \text{kgs}^{-1} \text{m}^2 \) and \( \gamma_{\text{off res}} = 1.8 \times 10^{-5} \text{kgs}^{-1} \text{m}^2 \). For two different plates resonance is possible when the real part of the reflection amplitudes \( R''(\omega_i) = 0 \) at the frequency of the surface phonon/plasmon polariton \( \omega_i \). In this case \( \phi_1 = \phi_2 = \pi/2 \) for \( q_x v = \omega_1 - \omega_2 \). The second condition for resonance requires that at \( q_x = (\omega_1 - \omega_2)/v \) and \( q_y = 0 \)

\[ R''(\omega_1)R''(\omega_2)\exp\left( -2|\omega_1 - \omega_2|/v \right) = 1 \quad (17) \]

where \( R''(\omega) \) is the imaginary part of the reflection amplitude. From (17) follows that resonance is possible for \( v > v_c \) where the critical velocity

\[ v_c = \frac{|\omega_1 - \omega_2|d}{\ln|R''(\omega_1)R''(\omega_2)|^{1/2}}. \quad (18) \]

For the reflection amplitudes given by Eq. (18) \( R''(\omega) = a_i/\Gamma_i = 2/\epsilon''(\omega) \) and the critical velocity

\[ v_c = \frac{|\omega_1 - \omega_2|d}{\ln[2/\epsilon''(\omega_1)\epsilon''(\omega_2)]^{1/2}}. \quad (19) \]

For \( \omega \approx \omega_1 \) and \( \omega - q_x v \approx \omega_2 \) the photon emission rate can be written in the form

\[ \Gamma_{12} \approx \frac{2\Gamma_1\Gamma_2a_1a_2e^{-2qd}}{(\Gamma_1 + \Gamma_2)^2(\omega - \omega_c)^2 + \left[ \Gamma_1\Gamma_2 \left( \frac{q_x v - \omega_1 + \omega_2}{1 + i\Gamma_2} \right) \right]^2 - (\omega - \omega_c)^2 + \left( \frac{(\omega - \omega_1 + \omega_2)(\Gamma_1 - \Gamma_2)(\omega - \omega_0) + \Gamma_1\Gamma_2 + a_1a_2e^{-2qd}}{1 + i\Gamma_2} \right)^2} \]
where
\[
\omega_c = \frac{\Gamma_1(q_x v + \omega_2) + \Gamma_2\omega_1}{\Gamma_1 + \Gamma_2}.
\] (21)

The photon emission rate given by Eq. (20) has a maximum \(\Gamma_{12}^{\text{max}} = 1\) at \(\omega = \omega_c\), \(q_x v = \omega_1 - \omega_2\) when the condition (17) is fulfilled. Using (29) in (1) for \(\omega - q_x v > 0\) gives the resonant contributions at \(v = v_c\) to the heat transfer rate and friction force for \(|\omega_1 - \omega_2|\gg \sqrt{\Gamma_1 \Gamma_2}\) from the frequency range of the normal Doppler effect

\[
\left(\frac{F_x}{P_1}\right) \approx \frac{1 + e^{i\phi_1} e^{-2qvd}}{R_2(\omega - q_x v)e^{2qvd} - 2|\phi_1| |\phi_2| e^{-2qvd} |\cos(\phi_1 - \phi_2)|}.
\] (22)

where \(c = \log_2(1/\varepsilon_1''(\omega_1)\varepsilon_2''(\omega_2))^{1/2}\) and \(q_c = c/d\).

2. The frequency range of the anomalous Doppler effect

In the frequency range of the anomalous Doppler effect, when \(\omega > 0\) and \(\omega - \Omega < 0\), we can write \(R_{1p}(\omega) = |R_1(\omega)| \exp(i\phi_1)\) and \(R_2(\omega - q_x v) = R_2^*(\omega - q_x v)\exp(-i\phi_2)\), and the photon emission rate can be written in the form

\[
\Gamma_{12} = \frac{4\text{Im}R_1(\omega)\text{Re}R_2(\omega - q_x v)e^{-2qvd}}{|1 - e^{-2qvd}R_1(\omega)R_2(\omega - q_x v)|^2} = \frac{4|R_1(\omega)||R_2(\omega - q_x v)|e^{-2qvd} \sin\phi_1 \sin\phi_2}{1 + |R_1(\omega)|^2|R_2(\omega - q_x v)e^{-2qvd}|^2 - 2|R_1(\omega)||R_2(\omega - q_x v)e^{-2qvd}| \cos(\phi_1 - \phi_2)},
\] (23)

which diverges (\(\Gamma_{12} = \infty\)) for \(|R_1(\omega)||R_2(\omega - q_x v)e^{-2qvd}| = 1\) and \(\phi_1 = \phi_2\). For two identical plates \(\phi_1 = \phi_2\) for \(q_x v = 2\omega\) or \(q_x v = 2\omega/v\) because in this case \(R_2(\omega - q_x v) = R_1(-\omega) = R_1^*(\omega)\). The second condition for the occurrence of singular resonance at \(q_x v = 2\omega/v\) takes the form\(49,65\)

\[
|R_p(\omega)|^2 e^{-2qvd} < |R_p(\omega_0)|^2 e^{-\frac{4\omega d}{v}} = 1
\] (24)

where \(\omega_0\) is the frequency of the surface phonon/plasmon polariton at which \(R_p(\omega_0) > 1\). From (24) follows that the photon emission rate and, consequently, the heat generation and quantum friction diverge for \(v > v_c\) where

\[
v_c = \frac{2\omega_0 d}{\ln|R_p(\omega_0)|}\]
(25)

The origin of this divergence is associated with the electromagnetic instability when above the threshold velocity \(v_c\) the electromagnetic field increases indefinitely with time even in the presence of a dissipation in the system\(24\).

For two different plates singular resonance is possible when the real part of the reflection amplitudes \(R_{1p}(\omega_1) = 0\) at the frequency of the surface phonon/plasmon polariton \(\omega_1\). In this case \(\phi_1 = \phi_2 = \pi/2\) for \(q_x v = \omega_1 + \omega_2\) and the critical velocity is given by

\[
v_c = \frac{(\omega_1 + \omega_2)d}{\ln[R_{1p}'(\omega_1)R_{2p}'(\omega_2)]^{1/2}}
\] (26)

where \(R_{1p}'(\omega)\) is the imaginary part of the reflection amplitude.

Near the resonance at \(\omega \approx \omega_1\) and \(\omega - q_x v \approx -\omega_2\) for \(2/\varepsilon_1''(\omega_1) >> 1\) the reflection amplitudes for dielectrics can be written in the form

\[
R_{1p}(\omega) \approx \frac{a_1}{\omega - \omega_1 + i\Gamma_1}, \quad R_{2p}(\omega - q_x v) = R_{2p}^*(q_x v - \omega) \approx \frac{a_2}{q_x v - \omega - \omega_2 - i\Gamma_1}.
\] (27)

At the resonance \(R_{1p}'(\omega_1) = a_i/\Gamma_i = 2/\varepsilon_1''(\omega_1)\) and the critical velocity

\[
v_c = \frac{(\omega_1 + \omega_2)d}{\ln[\varepsilon_1''(\omega_1)\varepsilon_2''(\omega_2)]^{1/2}}
\] (28)
For $\omega \approx \omega_2$ and $\omega - \Omega \approx -\omega_1$ the photon generation rate for can be written in the form

$$\Gamma_{12} \approx \frac{4\Gamma_1\Gamma_2a_1a_2e^{-2qd}}{(\Gamma_1 + \Gamma_2)^2(\omega - \omega_c)^2 + \left[ \Gamma_1\Gamma_2 \left( \frac{q_xv - \omega_l - \omega_2}{(1 + \Gamma_2)} \right)^2 - (\omega - \omega_c)^2 + \frac{(\Omega - \Omega_0)(\Gamma_2 - \Gamma_1)(\omega - \omega_b)}{1 + \Gamma_2} + \Gamma_1\Gamma_2 - a_1a_2e^{-2qd} \right]^2}$$

(29)

where

$$\omega_c = \frac{\Gamma_1(q_xv - \omega_2) + \Gamma_2\omega_1}{\Gamma_1 + \Gamma_2}$$

(30)

For $v > v_c$ the photon generation rate diverges at $\omega = \omega_c$ and $q_xv = \Omega^\pm$, where

$$\Omega^\pm = \omega_1 + \omega_2 \pm (\Gamma_1 + \Gamma_2)\sqrt{\frac{4}{\varepsilon(\omega_1)\varepsilon(\omega_2)} e^{-\frac{2(\omega_1 + \omega_2)d}{v}} - 1}$$

(31)

Close to the resonance when

$$\frac{\Gamma_1\Gamma_2}{(\Gamma_1 + \Gamma_2)^2} \left( \frac{q_xv - \omega_1 - \omega_2}{(1 + \Gamma_2)} \right)^2 + 1 - \frac{4}{\varepsilon(\omega_1)\varepsilon(\omega_2)} e^{-\frac{2(\omega_1 + \omega_2)d}{v}} \ll 1,$$

(32)

using (29) in (31) gives the resonant contributions to the quantum heat generation rate and quantum friction from the frequency range of the anomalous Doppler effect

$$\left( \frac{F_x}{P_1} \right) \approx \frac{\Gamma_1\Gamma_2e^{3/2}}{\pi d^2(\omega_1 + \omega_2)} \left( \frac{h\omega_c}{\hbar \omega_1} \right) \ln \left( \frac{v_c - v}{v_c} \right)$$

(33)

where $c = \ln2/[(\varepsilon(\omega_1)\varepsilon(\omega_2)]^{1/2}$ and $q_c = c/d$.

Fig. 2 shows the dependence of the friction force, acting between two SiO$_2$ plates sliding relative to each other with the relative velocity $v$ at the separation $d = 1$nm and for the different temperatures. The friction force $F = F(0) + F_T$ where $F(0)$ is the contribution from quantum fluctuations which exist even at $T = 0K$ (this friction is denoted as quantum friction). $F_{friction}(T = 0K) = F(0)$ and $F_T$ is the contribution from the thermal fluctuations which exists only at finite temperatures. The thermal contribution dominates for $v < k_BTd/h$ and quantum contribution dominates for $v > k_BTd/h$. For SiO$_2$ plates the critical velocity determined by (28) $v_c = 2.7 \cdot 10^8 \text{m/s}$. For $v > v_c$ uniform sliding of the plates is impossible because the friction force indefinitely increases with time. Close to the critical velocity quantum friction significantly exceeds the thermal contribution to the friction force even for $T = 600K$.

III. CASIMIR FRICTION IN THE TIP-PLATE CONFIGURATION

An atomic force microscope tip with the radius of curvature $R \gg d$, at a distance $d$ above a flat sample surface, can be approximated by a sphere with radius $R$. In this case the friction force between the tip and the plane surface can
be estimated using the approximate method of Derjaguin\textsuperscript{25}, later called the proximity force approximation (PFA).\textsuperscript{25} According to this method, the friction force in the gap between two smooth curved surfaces at short separation can be calculated approximately as a sum of forces between pairs of small parallel plates corresponding to the curved geometry of the gap. Specifically, the sphere-plane friction force is given by

$$ F = 2\pi \int_0^R d\rho f(z(\rho)), $$

(34)

where \( z(\rho) = d + R - \sqrt{R^2 - \rho^2} \) denotes the tip-surface distance as a function of the distance \( \rho \) from the tip symmetry axis, and the friction force per unit area \( f(z(\rho)) \) is determined in the plate-plate configuration. This scheme was proposed in\textsuperscript{25,27} for the calculation of the conservative van der Waals interaction; in this case the error is not larger than 5-10\% in an atomic force application, and 25\% in the worst case situation\textsuperscript{25}. We assume that the same scheme is also valid for the calculation of the Casimir friction. However, as it was discussed in Sec. II for two plates the Casimir friction diverges at the velocities above the critical velocity \( v_c \). However, for different plates the friction force can be finite even above the threshold velocity if at resonance the real part of the reflection amplitude is not exactly zero. For the SiO\textsubscript{2} the threshold velocity \( v_c \approx 2.7 \cdot 10^4 m/s \). Thus in the present study the numerical calculations are performed at the velocities below the threshold velocity when one can assume that the PFA gives sufficiently accurate estimation of the Casimir friction including the quantum friction. During the last few years, the most general method available for calculating both Casimir force and radiative heat transfer between many bodies of arbitrary shapes, materials, temperatures and separations was obtained which expresses the Casimir force and radiative heat transfer in terms of the scattering matrices of individual bodies.\textsuperscript{4} Specifically, the numerically exact solution for the near-field radiative heat transfer between a sphere and an infinite plane was first performed using the scattering matrix approach. In principle the same approach can be used for the calculation of the Casimir friction. We assume that the tip has a paraboloid shape given [in cylindrical coordinates \((z, \rho)\)] by the formula: \( z = d + \rho^2/2R \), where \( d \) is the distance between the tip and the flat surface. If

$$ f = \frac{C}{(d + \rho^2/2R)^n}, $$

(35)

we get

$$ F = \frac{2\pi R}{n-1} \frac{C}{d^{n-1}} = \frac{2\pi R d}{n-1} f(d) \equiv A_{\text{eff}} S(d), $$

(36)

where \( A_{\text{eff}} = 2\pi R d/(n-1) \) is the effective surface area. In a more general case one must use numerical integration to obtain the friction force.

From Eq. \( \text{(34)} \) in the proximity force approximation the friction coefficient in the tip-plate configuration

$$ \Gamma_{\text{res}} \approx \frac{\hbar^2 \Gamma q_e^4}{8\pi k_B T \sinh^2 \left( \frac{\hbar \omega_{\text{res}}}{2k_B T} \right)} Rd, $$

(37)

In an experiment \( \Gamma \) is determined by measuring the quality factor of the cantilever vibration parallel to the substrate surface.\textsuperscript{25} At present can only be detected the friction coefficient in the range \( 10^{-12} - 10^{-13} \text{kg/s} \). For a SiO\textsubscript{2} tip and a SiO\textsubscript{2} plate at \( R = 1\mu m \) and \( d = 1\text{nm} \) the friction coefficient calculated using Eq. \( \text{(37)} \) is below \( 10^{-16} \text{kg/s} \) thus it can not be tested by the modern experimental setup. However, it has been predicted in Ref.\textsuperscript{26} that for the same configurations involving adsorbates the Casimir friction coefficient can be large enough to be measured by state-of-art non-contact force microscope.

During the cantilever vibration the velocity of the AFM tip does not exceed \( 1\text{m/s} \). However, the Casimir friction force can be strongly enhanced at the large relative sliding velocity. This friction force can be detected if it produces sufficiently large bending of the cantilever. Fig. \( \text{(A)} \) shows the dependence of the friction force, acting on the SiO\textsubscript{2} tip with the radius of the curvature \( R = 1\mu m \), on the relative velocity \( v \) between the tip and the SiO\textsubscript{2} plate at the separation \( d = 1\text{nm} \) and for the different temperatures. The friction force \( F = F_0 + F_T \) where \( F_0 \) is the contribution from quantum fluctuations which exist even at \( T = 0\text{K} \) (this friction is denoted as quantum friction\textsuperscript{8}, \( F_{\text{friction}}(T = 0\text{K}) = F_0 \)) and \( F_T \) is the contribution from the thermal fluctuations which exist only at finite temperature. The thermal contribution dominates for \( v < k_B T d/\hbar \) and quantum contribution dominates for \( v > k_B T d/\hbar \). On Fig. \( \text{(A)} \) \( F > 10^{-12} \text{N} \) at \( v > 10^6 \text{m/c} \). In the modern experiment\textsuperscript{21} the spring constant of the cantilever are between \( k_0 = 30 \) and \( k_0 = 50\mu N/m \). The friction force \( \approx 10^{-12} \text{N} \) will produce the displacement of the tip of the order \( 10^2 \text{nm} \) which can be easily detected. However, at present there is no experimental setup with the relative sliding velocity between the tip and substrate \( \sim 10^3 \text{m/s} \). An alternative method for the detection of the Casimir friction is possible for the SiO\textsubscript{2} + graphene - SiO\textsubscript{2}...
FIG. 3: (a) A SiO\(_2\) tip and a SiO\(_2\) plate. The plate is moving relative to the tip with the velocity \(v\). (b) The dependence of the different contributions to the friction force on the relative sliding velocity \(v\) between a SiO\(_2\) tip and a SiO\(_2\) plate. The blue and red lines show the thermal contributions at \(T = 600\)K and \(T = 300\)K, respectively. The green line shows the quantum contribution (\(T = 0\)K). The radius of the curvature of the tip \(R = 1\)\(\mu\)m. The separation between the tip and plate \(d = 1\)nm.

FIG. 4: (a) A SiO\(_2\) tip and a SiO\(_2\) plate covered by a graphene sheet. DC current with a drift velocity \(v_{\text{drift}}\) of free charge carriers is induced in the graphene sheet. (b) The dependence of the different contributions to the friction force on the drift velocity \(v_{\text{drift}}\) of the free charge carriers in the graphene sheet. The blue and red lines show the thermal contributions at \(T = 600\)K and \(T = 300\)K, respectively. The green line shows the quantum contribution (\(T = 0\)K). The radius of the curvature of the tip \(R = 1\)\(\mu\)m. The separation between the tip and substrate \(d = 1\)nm.

configuration (see Fig.\textsuperscript{4}). For this configuration inducing current in a graphene sheet with the drift velocity of the free charge carriers \(v_{\text{drift}}\) will produce the fluctuating electromagnetic field which is similar to the electromagnetic field due to the mechanical motion of the sheet with the velocity \(v = v_{\text{drift}}\). Due to the high mobility of the charge carriers in graphene, in a high electric field electrons (or holes) can move with very high velocities (up to \(10^6\) m/s). The drift motion of charge carries in graphene will result in a modification of dielectric properties of graphene due to the Doppler effect. Fig.\textsuperscript{4} shows the friction force for the SiO\(_2\)+graphene-SiO\(_2\) configuration which is of the same order of magnitude as the friction force for the SiO\(_2\)-SiO\(_2\) configuration (see Fig.\textsuperscript{3}).
IV. SUMMARY

We have studied the Casimir friction, the radiative heat transfer and heat generation between two plates sliding relative to each other. We have found that for dynamically nonequilibrium systems in fluctuation-induced electromagnetic phenomena - Casimir friction, radiative heat transfer and heat generation, resonances that do not exist for equilibrium systems are possible. The origin of these resonances is different in the frequency regions of the normal (NDE) and anomalous (ADE) Doppler effect. While in the region of NDE resonances are associated with the resonant photon tunnelling, in the region of ADE they are associated with the resonant photon emission when excitations are created in both moving media. In sharp contrast with resonances in the NDE frequency range, where resonances are finite, in the ADE frequency range singular resonances are possible. We have discussed the possibility to detect Casimir friction in graphene structures using an atomic force microscope.
42 M.F. Maghrebi, R. Golestanian and M. Kardar, Phys.Rev. A 88, 042509 (2013).
43 M.G. Silveirinha, Phys. Rev. X 4, 031013 (2014).
44 I.M. Frank, J. Phys. USSR 7, 49 (1943).
45 I.M. Frank and V.L. Ginzburg, J.Phys. (USSR) 9, 353 (1945).
46 V.L. Ginzburg, Phys.-Usp. 39, 973 (1996).
47 L.D. Landau, Zh. Eksp. Theor. Fiz. 11, 592 (1941).
48 I.E. Tamm, Usp. Phys. Nauk 68, 387 (1959); Science textbf131, 206 (1960).
49 G.T. Moore, J. Math. Phys. 11, 2679 (1970).
50 S.A. Fulling and P.C.W. Danies, Proc. R. Soc. London Ser. A 348, 393 (1976).
51 C.M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J.R. Johansson, T. Duty, F. Nori and P. Delsing, Nature 479, 376 (2011).
52 A. Manjavacas and F.J. García de Abajo, Phys. Rev. Lett. 105, 113601 (2010).
53 A. Manjavacas and F.J. García de Abajo, Phys. Rev. A 82, 063827 (2010).
54 M. F. Maghrebi, R. L. Jaffe, and M. Kardar, Phys. Rev. Lett. 108, 230403 (2012).
55 M. F. Maghrebi, R. L. Jaffe, and M. Kardar, Phys. Rev. A 90, 012515 (2014).
56 H. Bercegol and Jaffe, and L. Lehoucq, Phys. Rev. Lett. 115, 090402 (2015).
57 Y.B. Zel’dovich, JETP lett. 14, 180 (1971).
58 R. Zhao, A. Manjavacas and F.J. García de Abajo, and J.B. Pendry, Phys. Rev. Lett. 109, 123604 (2012).
59 G. V. Dedkov and A. A. Kyasov, EPL, 99, 64002 (2012).
60 A. Manjavacas, F. J. Rodriguez-Fortuo, F. J. Garca de Abajo, and A. V. Zayats, Phys. Rev. Lett. 118, 133605 (2017).
61 G. V. Dedkov and A. A. Kyasov, Tech. Phys. 62, 1266 (2017).
62 A.I. Volokitin, EPL. 122, 14003 (2018).
63 A.I. Volokitin, JETP Lett. 108, 147 (2018).
64 Y. Guo and Z. Jacob, J. Opt., 16, 114023 (2014).
65 Y. Guo and Z. Jacob, Opt. Express, 22, 21 (2014).
66 A. I. Volokitin, Phys. Rev. B, 94, 235450 (2016).
67 A. I. Volokitin and E. V. Dubas, JETP Lett., 105, 733 (2017).
68 A. I. Volokitin, Phys. Rev. A, 96, 012520 (2017).
69 Y. B. Zeldovich, JETP Lett., 14, 180 (1970).
70 Y. B. Zeldovich, Sov. Phys. JETP 35, 1085 (1972).
71 S. Lannebere and M. G. Silveirinha, Phys. Rev. A 94, 033810 (2016).
72 A.I. Volokitin and B.N.J. Persson, Phys. Rev. B 69, 045417 (2004).
73 D.Z.A. Chen, R. Hamam, M. Soljacic, J.D. Joannopoulos and G. Chen, Appl. Phys. Lett. 90, 181921 (2007).
74 M. G. Silveirinha, New J. Phys., 16, 063011 (2014).
75 B. Derjaguin, Kolloid-Z. 69, 155 (1934).
76 Blocki, J., J. Randrup, W. J. Swiatecki, and C. F. Tsang, Ann. Phys. (N.Y) 105, 427 (1977).
77 U. Hartmann, Phys. Rev. B 42, 1541 (1990); 43, 2404 (1991).
78 P.Johansson and P.Apell, Phys. Rev. B 56, 4159 (1997).
79 E. Gnecco and E. Meyer, Elements of friction theory and nanotribology, ed. by E. Gnecco and E. Meyer (Cambridge University Press 2015).
80 A. I. Volokitin and B. N. J. Persson, and H. Ueba, Phys. Rev. B 73, 165423 (2006).
81 A. Mehlin, F. Xue, D. Liang, H. F. Du, M. J. Stolt, S. Jin, M. L. Tian, and M. Poggio, Nano Lett. 15, 4839 (2015).