Evaluation of well interference by correlation analysis

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Abstract: The analysis of well interference is understood as an assessment of the presence or absence of pressure communication, the calculation of the response time of production wells to the change in pumping of injection wells, as well as the evaluation of the contribution of injection wells to production wells flow rate. The problem of determining the interference of wells at the moment is solved by several methods, which can be divided into 2 groups:
1. Traditional methods. Methods of data analysis that require any technological operations on the investigated wells (observation well testing, tracer studies).
2. Alternative methods. Various methods for analyzing field data using mathematical statistics or by numerical modeling of the behavior of pressure or flow in specialized software.
The optimal option is a comprehensive approach based on the principles of the correlation method of analysis of field data, the main advantage of which is the speed of analysis and decisions, as well as the lack of large capital investments needed.

1. Introduction
Since the middle of the last century, when developing oil fields in our country, the method of artificial flooding was widespread. Its high efficiency is well known. However, the full extent of formation coverage and oil recovery decrease with an increase in the degree of formation heterogeneity. In heavily inhomogeneous reservoirs, injected water breaks into producing wells through highly permeable interlayers, leaving oil unallocated in low-permeability layers. In other cases, the production well area may not be sufficiently covered by the injection water front. To make effective decisions on adjusting the operation of the wells of the operational fund and determining additional measures for pumping water, it is necessary to have an actual, clear understanding on the wells interference. This need is related to the development of a mathematical approach to the analysis of field data that would correspond to high quantitative accuracy and efficiency in an active, dynamic system with the possibility of a complex description of the interaction of a whole group of wells, which is impossible without determining the contribution from the participation of injection wells.

2. Research
- Determination of the presence or absence of a hydrodynamic connection between injection wells and adjacent production wells.
- Determination of the contribution of injection wells to the production of adjacent production wells.
- Determination of the reaction time of the extraction fund for the operation of the formation pressure maintenance wells.
3. Results and discussion
Correlation analysis is related to the search for dependencies between the behavior of two objects. The result of the analysis is the value of the calculated coefficient, indicating the presence or absence of some dependence. In the oil and gas business, the practice of applying this approach is connected with the use of the Spearman’s rank coefficient [1, 2]. To evaluate its applicability and confirm the effectiveness, the Kendall correlation coefficient was also studied in addition to the general form of the correlation ratio [3].

The degree of dependence of the two parameters (parameters) of the operation of arbitrary wells No. 1 and No. 2 can be described on the basis of an analysis of the obtained value of the Spearman’s ratio, which in the general case varies in the range from -1 (inverse dependence) to 1 (direct dependence). The zero value of the coefficient indicates that the parameters are independent. The Spearman’s ratio is calculated by the formula:

\[
R_s = \frac{1}{6} \cdot n(n^2 - 1) - \sum d - T(X) - T(Y) \\
\sqrt{\left(\frac{1}{6} \cdot n(n^2 - 1) - 2T(X)\right) \cdot \left(\frac{1}{6} \cdot n(n^2 - 1) - 2T(Y)\right)},
\]

where \( M_x, M_y \) - the number of groups of the same values for the parameters of the X and Y objects, \( N_x, N_y \) - the number of identical elements in the corresponding group.

To estimate the correlation dependence, it is also possible to use other coefficients: the Kendall's rank coefficient (2), as well as the correlation ratio (3).

\[
R_k = \frac{\sum P - \sum I}{\sqrt{(n(n-1)/2 - U_x) \cdot (n(n-1)/2 - U_y)}},
\]

\[
U_x = \frac{1}{2} \sum_{i=1}^{M_x} (N_x^2 - N_x),
\]

\[
U_y = \frac{1}{2} \sum_{i=1}^{M_x} (N_y^2 - N_y),
\]

where \( P \) - number of proversions; \( I \) - number of inversions.

\[
R_{XY}^2 = \frac{S_{y(x)}^2}{S_y^2},
\]

\[
S_{y(x)}^2 = \frac{1}{n} \sum_{i=1}^{k} m_i (y_i - \bar{y})^2,
\]

\[
\bar{y} = \frac{1}{n} \sum_{i=1}^{k} m_i y_i, \quad \bar{y}_i = \frac{1}{m_i} \sum_{j=1}^{m} y_{ij},
\]
\[ S_y^2 = \frac{1}{n} \sum_{i=1}^{k} \sum_{j=1}^{m_i} (y_{ij} - \bar{y})^2, \]

where \( k \) - number of grouping intervals; \( m_i \) - the number of sampling points that fall in the \( i \)-th grouping interval, \((i=1, 2, \ldots, k)\).

We note that the Spearman's and Kendall's rank coefficients well describe linear dependences. The correlation relation of the form (3) is not burdened by any additional assumptions regarding the general form of the regression dependence [3], i.e. it allows taking into account, among other things, nonlinear dependencies. It takes values from 0 (no connection) to 1 (strong interconnection).

Thus, to solve the problem of determining the hydrodynamic connection between the injection and production wells, the correlation coefficients are applicable which was noted a long time ago. However, to determine the contribution of pumping of injection wells into oil well production, calculations of such coefficients were not previously applied. To determine the applicability of the correlation analysis to this task, an experiment was performed.

The idea of the experiment is to simulate the production rate of a fictitious well with different injection ratios of real injection wells (Table 1), in calculating the correlation coefficient for each case and comparing them. It is assumed that there is a dependence between the correlation coefficients and the weight ratios of injectability.

As the investigated objects, real injection wells were taken of one of the fields of Western Siberia № 757, № 857. These wells participated in the cyclic flooding program, which only contributes to better characterization of responses (Figure 1).

Table 1. Modeling options.

| Options | Well № 757 | Well № 857 |
|---------|------------|------------|
| 1       | 0.6        | 0.4        |
| 2       | 0.38       | 0.62       |
| 3       | 0.22       | 0.78       |

Modeling was carried out in a hydrodynamic simulator. To minimize the influence of various factors that may affect the results, the model was created under the following conditions:

1. An infinite homogeneous formation was considered.
2. The fictitious production well was equidistant from the injection wells and was located between them.
3. The bottomhole pressure of the producing well was assumed to be unchanged.

The results for the simulated conditions are presented in Figures 2, 3, 4.

**Figure 2.** The simulated production curve at the injection ratio of 0.6/0.4 of wells № 757 and № 857, respectively.

**Figure 3.** The simulated production curve at the injection ratio of 0.38/0.62 of well № 757 and № 857, respectively.
simulated production rate of a fictitious well, m$^3$/day;
well № 757 injection capacity (the share of self-injection – 0.3), m$^3$/day;
well № 857 injection capacity (the share of self-injection – 0.7), m$^3$/day.

Figure 4. The simulated production curve at the injection ratio of 0.22/0.78 of wells № 757 and № 857, respectively.

On the presented diagrams it is possible to distinguish 3 characteristic sites:
1. The first half of the data (before cyclic flooding) - exponential growth of production rate is associated with simultaneous launch of all wells (Figure 4 - I).
2. Period of cyclic operation of injection wells (Figure 4 - II).
3. Period after the cyclic flooding (Figure 4 - III).

It is obvious that the values of the calculated correlation coefficients may differ for different parts of the data. To identify the most representative period, the calculations were carried out for different periods of time:
1. The entire period of the data under consideration.
2. Period from the beginning of the cyclic flooding to the end of the data.
3. Period with cyclic injection only.

The calculation results are presented in Tables 2, 3, 4.

Table 2. Correlation coefficients calculated at the ratio of injections of 0.6/0.4 of wells № 757 and № 857, respectively.

|       | Well | Spearman | Kendall | Cor. Ratio (k-means) |
|-------|------|----------|---------|----------------------|
| The entire period | 757  | 0.18     | 0.13    | 0.47                 |
|       | 857  | -0.59    | -0.39   | 0.596                |
| From the beginning of the cycle to the end of the entire period | 757  | 0.66     | 0.49     | 0.67                 |
|       | 857  | -0.16    | -0.08   | 0.26                 |
| From the beginning to the end of the cycle | 757  | 0.75     | 0.56     | 0.8                  |
|       | 857  | -0.49    | -0.34   | 0.21                 |

Table 3. Correlation coefficients calculated at the ratio of injections of 0.38/0.62 of wells № 757 and № 857, respectively.

|       | Well | Spearman | Kendall | Cor. Ratio (k-means) |
|-------|------|----------|---------|----------------------|
| The entire period | 757  | -0.069   | -0.04   | 0.47                 |
Table 4. Correlation coefficients calculated at the ratio of injections of 0.22/0.78 of wells № 757 and № 857, respectively.

|                     | Well | Spearman | Kendall | Cor. Ratio (k-means) |
|---------------------|------|----------|---------|----------------------|
| The entire period   | 757  | -0.27    | -0.19   | 0.48                 |
|                     | 857  | -0.21    | -0.15   | 0.37                 |
| From the beginning of the cycle to the end of the entire period | 757  | 0.014    | 0.006   | 0.28                 |
|                     | 857  | 0.59     | 0.387   | 0.8                  |
| From the beginning to the end of the cycle | 757  | -0.33    | -0.193  | 0.74                 |
|                     | 857  | 0.702    | 0.465   | 0.85                 |

The obtained coefficients were compared with the given injection ratios. The graph below (Figure 5) presents the average errors of different calculation methods. It can be seen from the graph that the most accurate tool is the calculation of the correlation ratio.

![Figure 5. Average errors in the coefficients calculated in different ways.](image-url)
Figure 6. The average error of the correlation ratio, depending on the period of time under consideration.

Figure 6 shows the distribution of errors in the correlation ratio, depending on the data section. It can be seen that the best results were obtained in the data period from the beginning of the amplitude change in the operation mode of the injection wells to the end of the period under consideration. The average error in determining the contribution of injection wells to the productivity of the producing well by calculating the correlation ratio was about 14%.

Despite the positive nature, it should be noted that a small error and the results, in general, were obtained under fairly "ideal" conditions. When analyzing real conditions, it should be considered that, according to the basics of hydrodynamics, increase in the distance between the wells and decrease in permeability leads directly to a decrease in the connection and the quantitative estimate of the contribution.

Calculation of the correlation ratio can be considered not only an indicator of the interaction of two wells, but also a parameter with a quantitative characteristic by their interference, but without an estimate of the time of their response, the conditions will be incomplete.

One of the solutions to this problem is the grapho-analytical correlation of the operating conditions of each well, but without a well-developed mathematical base, the results are largely subjective.

To understand the mechanism for determining the response time, let us turn to the classical basics of hydrodynamic interpretation by the observation well testing. The actual result for the observation well testing is the Response time ($\Delta t$) of pressure and the Response amplitude ($\Delta P$) of pressure (Figure 7) [4].
Figure 7. Response Time and Response Amplitude during the observation well testing.

It is known that the rate of pressure distribution in the reservoir is characterized by the coefficient of piezoelectric conductivity and depends on the filtration properties of the formation and fluid. In other words, the time at which the pressure perturbation front passes through the distance \( r \) is directly proportional to the piezoconductivity of the formation. In this case, the analytical calculation of the response time (5) also depends on the production rate and the chosen value of the pressure change \( dP \), which can be fixed by measuring instruments.

\[
\Delta t = \frac{1.781}{0.00144} \frac{\phi \mu c t r^2}{k} e^{9.205 \frac{h P}{B \mu}},
\]

(5)

where \( k \) - phase formation permeability, mD; \( t \) - response time, h; \( \phi \) - formation porosity; \( \mu \) - dynamic viscosity, cPs; \( c_t \) - total compressibility of the system, atm-1; \( h \) - thickness h of the formation, m; \( q \) - flow rate, m3/day; \( B \) - volume ratio.

Thus, under the condition that the fixed change in pressure is very small \((dP\sim0)\), the response time will be characterized only by the rate of pressure distribution in the formation, i.e. piezoconductivity. In this case, the estimate of the reaction time only by the piezoconductivity will be minimal.

It follows from the earlier discussion that low values of the correlation ratio can indicate both a weak pressure communication and, respectively, a small contribution, and a small contribution with a sufficiently good pressure communication.

In connection with this, it is theoretically possible to determine the reaction time of the producing well to change the injection well mode using the correlation ratio. Following is the algorithm for finding the most probable estimate of the reaction time:

1. A time interval is selected on which the accuracy of calculating the correlation ratio is maximal, i.e. field data have less noise.
2. An iterative calculation of the correlation ratios of a pair of wells (production-injection) is performed, in which, at each iteration step, the production data of the examined wells are shifted relative to each other by a certain time step. The number of iterations depends on the preliminary evaluation of the response time.
3. The values of the calculated coefficients at each step are compared with each other and the maximum value is calculated.
4. The value of the time lag at which the maximum correlation ratio was determined is considered to be the response time.
5. The results are compared with an estimate of the response time on the basis of the \( \Delta t = \frac{\varphi c r^2}{k} \) dependence using the results of the well test data.

As a confirmation of the efficiency of the developed approach, a typical analysis of a section of one of the fields in Western Siberia with the injection wells № 757, № 857 is performed. Figure 8 shows a map of the corresponding section of the formation drained by the wells №№ 269Б, 282, 283, 1048, 758, 61П, 270.

![Figure 8. The site under study.](image)

To carry out the study, the data on the surface flow rate of well № 282 (gusher) and the injectivity of wells №№ 757, 857 for 2013 (Figure 9) were taken as the initial data.

![Figure 9. Dynamics of production of well № 282 and injection of wells №№ 757 and 857.](image)
On the production schedule, there are jumps that are most likely related to the work of injection wells. This assumption confirms the coincidence of the periods of fluctuations in production rates and injectivity. To determine which of the two injection wells has the strongest effect, a correlation analysis was performed to determine the time delays.

The preliminary evaluation of the maximum response time for wells № 857 and № 757 through the piezoelectric conductivity of the formation was 324 and 45 days, respectively.

To determine the actual response time, the corresponding calculations were performed based on the previously developed algorithm. Below is a graph of the dependence of the calculated correlation coefficients on the time delay (Figure 10).

According to the calculations, the following results were obtained:

**Table 5. Calculation results.**

| Injector | Correlation ratio | Response time, d. |
|----------|-------------------|-------------------|
| № 757   | 0.721             | 2                 |
| № 857   | 0.461             | 16                |

**Figure 10.** Dependence of the correlation ratio on the response time.

The correlation ratio between production rate of well № 282 and injection capacity of well № 757, m³/day; correlation ratio between production rate of well № 282 and injection capacity of well № 857, m³/day;

The correlation ratio between production rate of well № 282 and injection capacity of well № 757 was 0.721, which indicates a good pressure communication between these wells and the predominant contribution of well № 757 to the production of well № 282. The low value of the correlation coefficient (0.461) between wells № 282 and № 857 may indicate a weaker effect of the injection well, while the pressure communication may be good enough. As a percentage, the contribution of well № 757 is about 61%, well № 857 - 39%, which is consistent with their location relative to each other.

The results of calculations for the remaining wells in the section under consideration are shown in Table 6.

**Table 6.** The calculation results for a group of wells.

| Injector / observation well | № 757 | № 857 |
|----------------------------|-------|-------|
| Correlation ratio          |       |       |
| Response time, d.          |       |       |
The obtained results, generally, are consistent with the operation modes and their mutual position. Figure 11 summarizes and graphically depicts the interference of wells in the area under consideration.

In the graph, the arrows indicate the direction of connection between the wells. The difference in color indicates the gradation of the calculated correlation ratios. It should be noted that it is most useful to compare the values in the case when the calculated correlation coefficients refer to one production well.

![Figure 11. Wells interference.](image)

4. Conclusion
In solving the problem of analyzing well interference, a complex mathematical analysis approach was developed to analyze and process field data, based on the principles of mathematical statistics.

A) A numerical experiment was performed to substantiate the approach, based mainly on the mathematical principles of correlation methods of data processing.

B) An algorithm was developed for estimating the reaction time, estimating the pressure communication and the contribution of injection wells to the production of adjacent oil wells (correlation ratio).

C) Response time and correlation ratio for wells of the selected section of the Western Siberia deposit were estimated.

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