Time-dependent reliability-based design optimization of structures with fuzzy uncertainty

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Abstract. For addressing the optimal design parameter for the time-dependent structure involving fuzzy variables, a novel reliability-based design optimization (RBDO) model with the constraints of time-dependent failure possibility (TDFP) is proposed. The solution of the RBDO is a double loop method, the outer loop is deterministic optimization, and the inner loop performs TDFP analysis. For evaluating TDFP in the inner loop, firstly the double-loop nested optimization method (DLOM) is investigated, but the high computational cost of DLOM is impractical, thus the single-loop optimization method is adopted. After the detailed implementation of the proposed RBDO is given, several examples containing one numerical example and two engineering examples are introduced to show the rationality of the proposed approach.

1. Introduction

The uncertainty is evitable in practical structure systems design, such as variation of material properties, manufacturing tolerance and fluctuation of load applied on the structure systems[1]. These uncertainties must lead to the variability of the structure systems performance which influences the quality of the structure systems. Consequently, considering the reliability of the structure systems performance is very important in product design to ensure the safety of the structure. Traditional design based on safety factor stemmed from past experience and engineering judgment is inappropriate or obsolete and thus results in potential functional fault[2]. Thus, the reliability-based design optimization (RBDO) is developed to deal with uncertainties systematically during structure design and remarkable academic achievement have been made in this field[3][4][5][6]. However, most of the RBDO researches assume that the uncertainty parameters are represents by probability density functions which is difficult to acquire in initial design stage.

In view of the defects of random uncertainty assumption under the insufficient data, many attempts have been made to adopt fuzzy set to handle uncertainties in human behavior and expert judgment[7][8]. Several conceptions of fuzzy-based structural reliability indexes have been introduced, and then applied it into the optimal design model. The fuzzy optimum design of aseismic structures was developed by Wang and Wang[9], and a two-step approach that the first step is to solve the minimum cost design
point and the second step is to trade off the construction cost and earthquake-caused loss expectation is presented. Du and Choi[10] proposed a possibility-based design optimization using the performance measure approach, and the maximal possibility search method is proposed to improve efficiency and accuracy of the inverse possibility analysis. Considering the statistical and fuzzy uncertainties, a mixed-variable design optimization method using the performance measure approach was presented, and a most probable/possible point search method which is an integration of the enhanced hybrid mean value method and maximal possibility search method was proposed to conduct the inverse analysis[11]. Tang et al. proposed a possibilistic safety index-based design optimization model under the fuzzy uncertainty, and target performance-based design approach was proposed to improve the efficiency of the solution of the optimal parameters[12]. Wang[13] et al. presented a reliability-based optimization model under the random, interval and fuzzy uncertainties, furthermore, the level-cut performance function was quantified to represent the safety state by interval ranking strategy, a subinterval vertex method is proposed to improve the computational efficiency.

The RBDO models mentioned above are completely time-independent reliability constraints for the structure systems. However the variability of the geometry size caused by corroded, the degradation of material properties and the dynamic loading are time-dependent reliability problem[14][15]. Therefore, establishing a reasonable time-dependent RBDO model of the structure under the fuzzy uncertainty is very essential. The research works of the time-dependent RBDO involving stochastic uncertainty have already been very well, and several methods of the solution have been presented[16][17][18][19]. But the time-dependent RBDO under the fuzzy uncertainty is promising but mostly unexplored. Consequently, the model of the time-dependent RBDO involving fuzzy uncertainty is established by using the time-dependent failure possibility constraints[20], and the TDFP index-based approach is investigated to solve the optimal parameters. The double-loop optimization method is used to analysis the TDFP in the inner loop of the optimization model, but the computational burden is prohibitive for complex structure. Therefore, the single-loop optimization method based on the extreme value transformation is established.

The paper is constructed as follows: the TDFP index is firstly reviewed in section 2. A novel time-dependent RBDO model is established by using the TDFP index constraints, and the TDFP index-based approach is proposed in section 3. In section 4, three examples are provided to verify the rationality of the presented method, and the paper is concluded with a brief summary at last.

2. Time-dependent failure possibility index for structures with fuzzy variables

The uncertainties of an structural system are identified by the variations of the fuzzy variables $X$. The fuzzy distribution of $X$ is characterized by the membership function $u_x(x)$. Suppose the time-dependent performance function of the structural system is $G(X,t)$, where the system fails if $G(X,t) \leq 0$ in a specified time interval $t \in [t_s, t_e]$, the mathematical expression of the TDFP is showed as

$$\pi_{\alpha \epsilon [t_s, t_e]} = \text{Poss}\{G(X,t) \leq 0\} = \sup \{\alpha | G(X,t) \leq 0, \exists t \in [t_s, t_e]\}$$

(1)

where $\text{Poss}\{}$ is the possibility of an event, $\alpha$ is the membership level of the output response, $\sup \{}$ means the supremum of set.

TDFP is the possibility of the time-dependent performance function less than zero at any time instant over the service time interval $t \in [t_s, t_e]$, and $\pi_{\alpha \epsilon [t_s, t_e]}$ is the supremum of the membership level $\alpha$ of the time-dependent performance function as the failure exists, i.e.,
\{G(X, t) \leq 0, \forall r \in (t_s, t_e)\} \). The definition of the TDFP indicates that the time-dependent structure is not failure if the membership level \( \alpha \) of the performance function is larger than \( \pi_{\beta_d(t_s, t_e)} \). It is known from the definition of TDFP that \( \pi_{\beta_d(t_s, t_e)} \) is the maximum failure membership level \( \alpha \) of the time-dependent structure under the fuzzy uncertainty, hence it clearly demonstrates the safety degree of the time-dependent structure involving the fuzzy variables.

Based on the mathematical model of the TDFP, two basic properties can be concluded as follows. Firstly, the value of the failure possibility of the time-dependent structure is located in the interval \([0, 1]\), secondly, if the lower boundary \( t_s \) of the service time interval is fixed, \( \pi_{\beta_d(t_s, \cdot)} \) is non-decreasing function of the upper boundary \( t_e \), the detailed property proof can be refers to reference[19].

3. Time-dependent reliability-based design optimization (RBDO) of the structure involving fuzzy uncertainty

In this section, the formulation of the time-dependent RBDO is firstly discussed. Then time-dependent failure possibility-index-based approach is given. Finally, the detailed implementations of solving the optimization design parameters are given.

3.1 RBDO problem of the time-dependent structure formulation

The initial value for the RBDO is the solution of the deterministic optimization (DO) in this contribution, thus the DO model are discussed before the RBDO formulation for the time-dependent structure involving the fuzzy uncertainty is given.

\[
\begin{align*}
\text{find } & \mathbf{d} \\
\min_{\mathbf{d}} & \quad f(\mathbf{d}) \\
\text{s.t.} & \quad \min_{t} g_i(\mathbf{d}, t) \geq 0 \\
& \quad \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U, t \in [t_s, t_e] \\
& \quad i = 1, 2, \ldots, N_c
\end{align*}
\]

(2)

where \( \mathbf{d} \) is the \( n_d \)-dimensional deterministic design variables, \( f \) stands for the objective function to be minimized, and \( g_i(\mathbf{d}) \) denotes the \( j \)th deterministic constraint, \( \mathbf{d}^L \) and \( \mathbf{d}^U \) are the lower and upper boundary of the design variable respectively, \( i \) represents the number of the constraints.

Assume the time-dependent structure is the function of the fuzzy input variables \( \mathbf{X} = (X_1, X_2, \ldots, X_n) \) and the time variable \( t \), and the \( j \)th constraint is expressed as TDFP index. Consequently, the time-dependent RBDO of the structure considering TDFP index constraints is mathematically defined as

\[
\begin{align*}
\text{find } & \mathbf{d}, \mathbf{X}' \\
\min_{\mathbf{d}} & \quad f(\mathbf{d}, \mathbf{X}') \\
\text{s.t.} & \quad \pi_{\beta_d(t_s, t_e)} = \text{pos}(g_i(\mathbf{d}, \mathbf{X}, \mathbf{p}, t) \leq 0, \forall r \in [t_s, t_e]) \leq \bar{\pi}_{\beta_d} \\
& \quad \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \\
& \quad \mathbf{X}^L \leq \mathbf{X}' \leq \mathbf{X}^U \\
& \quad 0 \leq \bar{\pi}_{\beta_d} \leq 1 \\
& \quad i = 1, 2, \ldots, N_c
\end{align*}
\]

(3)
where $d$ is the $n_d$-dimensional deterministic design variables, $X^c$ represents the nominal value vector of $n_c$-dimensional fuzzy input variables $X$, $p$ is $n_p$-dimensional fuzzy vector, $\pi_{\text{TDFP}(t,\alpha)}$ and $\pi_{\text{TDFP}}$ denote the TDFP index and target TDFP index of the $i$th constraint, respectively. The value of the target TDFP is bounded in the interval $[0,1]$.

3.2 TDFP-index-based approach

From the mathematical definition of the RBDO of the time-dependent structure with fuzzy variables expressed by Eq. (3), the solution of RBDO is a double-loop nested optimization, the outer loop is a deterministic optimization process, and the inner loop is TDFP-index analysis process. Thus double-loop nested optimization method (DLOM) in the inner loop is firstly discussed.

The TDFP expressed by Eq. (1) is the maximum membership level of the time-dependent structure failure under the specified time interval, i.e. $\{G(X,t) \leq 0, \exists t \in (t_s, t_e)\}$. Consequently, the DLOM firstly estimates time-independent failure possibility by the discretization of the service time interval $t \in (t_s, t_e)$, then the TDFP is the maximum of the values of the time-independent failure possibility for all discretized time instant. The membership function of the time-independent performance is calculated by the $\alpha$-level optimization method[8], then the corresponding time-independent failure possibility is estimated with the simple nonlinear interpolation.

Overall, the DLOM of the time-dependent structure is a double-loop nested optimization problem shown by Eq. (3), the RBDO is also a double-loop nested problem of minimizing the objective function and the TDFP-index analysis. Thus, the RBDO of the time-dependent structure obviously becomes a triple-loop nested problem. The computational cost of the RBDO of the time-dependent structure is very huge for the complex engineering problem, therefore the single-loop optimization method (SLOM) is adopted to efficiently address the TDFP-index analysis.

The TDFP can be transform to time-independent failure possibility based on the extreme value theory, that is to say, TDFP can be rewritten to Eq.(4) as follows

$$
\pi_{\text{TDFP}(t,\alpha)} = \text{Poss} \{ \min_{t \in [t_s, t_e]} G(X,t) \leq 0 \}
= \text{Poss} \{ G_{\min}(X) \leq 0 \} = \sup \{ \alpha \mid G_{\min}(X) \leq 0 \}
\tag{4}
$$

where $G_{\min}(X) = \min_{t \in [t_s, t_e]} G(X,t)$ is the minimum of the time-dependent performance function $G(X,t)$ in the specified time interval $t \in (t_s, t_e)$.

Eq. (4) indicates that the TDFP can be computed by the extreme value of the time-dependent performance, which is significantly improve the computational efficiency. Thus the RBDO of the time-dependent structure becomes a double-loop nested optimization, the outer loop is the deterministic optimization, and the inner loop is a single-loop optimization process. The corresponding implementations of the two algorithms to compute the RBDO of the time-dependent structure are established in next section.

3.3 implementations of solving the optimization design parameters

In this contribution, an available executable optimizer $\text{fmincon}$ which is used to solve the Eq. (3) has been provided by the MATLAB 2018b. Flowchart of the proposed RBDO of the time-dependent structure based on DLOM is illustrated in Fig.1, and its procedures are presented as follows:

1. Initialize the design variable $d^{(0)}$ and $X^{(0)}$, set the iteration counter $k = 0$, and keep the default optimization convergence parameters in $\text{fmincon}$. 

4
(2) Conduct DO without considering the uncertainty in the Eq.(2), and the initial design solution is recorded as \( d_{d}^{(0)} \) and \( X_{d}^{(0)} \).

(3) Perform deterministic optimization in the outer loop expressed in the Eq.(3). Compute the TDFP index constraint with DLOM in the inner loop.

(4.1) Discretize the given time interval \( t \in [t_{1}, t_{n}] \) to a set \( [t_{1}, t_{2}, \ldots, t_{m}] \).

(4.2) Calculate the membership function of the output response \( G(d, X, p, t) \) by the optimizer and the corresponding \( \pi_{\beta_{j}} \) with the nonlinear interpolation.

(4.2.1) For the computation of \( \pi_{\beta_{j}} \), discretize \( \alpha \in [0,1] \) to \( m_{j} \) levels as \( \alpha_{j_{j}} \) \( (j_{j} \in \{1,2,\ldots,m_{j}\}) \), and estimate the membership interval \( [p_{k}^{(L)}(\alpha_{j_{j}}), p_{k}^{(U)}(\alpha_{j_{j}})] \) \( (k_{k} = 1,2,\ldots,n_{p}) \) of \( p_{k} \) by the membership function \( u_{p_{k}}(p_{k}) \) of \( p_{k} \), the same as fuzzy variable \( X \).

(4.2.2) Calculate the interval \( [G_{a_{h}}^{(L)}(d, X, p, t), G_{a_{h}}^{(U)}(d, X, p, t)] \) of the performance function \( G(d, X, p, t) \) corresponding to the membership level \( \alpha_{j} \) \( (j_{j} = 1,2,\ldots,m_{j}) \) by the optimization algorithm.

(4.2.3) Estimate the membership function \( u_{G(d,X,p,t)} \) of the performance function \( G(d, X, p, t) \) \( j_{j} \in \{1,2,\ldots,m_{j}\} \) and obtain the corresponding failure possibility \( \pi_{\beta_{j}} \) by the interpolation.

(4.3) Calculate the \( \pi_{\beta_{j}} \) \( (i = 1,\ldots,m) \) by the loop \( i \in (1,\ldots,m) \), and estimate the TDFP by the equation
\[
\pi_{\beta(a_{i},x)} = \max_{i=1}^{m} \pi_{\beta_{j}}.
\]

(5) If all TDFP-index constraints fulfill and the optimization is convergent, then stop the iteration, if not, let \( k = k + 1 \) and return step (3).
Discretize \([0,1]\) into \(m\) values \((1,2,...,m)\), and let \(\mathbb{I}(s)\), \(s = 1, i\)

For \(t_c\), discretize \(\alpha \in [0,1]\) into \(m\) values 

Let \(i = 1\)

(4.2) Estimate the lower boundary \(G_{\min}^{(L)}(d,X,\alpha_i,p(\alpha_i))\) of the minimum \(G_{\min}^{(L)}(d,X,\alpha_i,p(\alpha_i))\) of the performance \(G(d,X,p,t)\) with respect to time \(t \in [t_s,t_e]\) at the membership level \(\alpha_j\) \((i = 1,2,...,m)\).

(4.3) Calculate the TDFP by \(G_{\alpha_i}^{(L)}(d,X,\alpha_i,p(\alpha_i)))\) \((i = 1,2,...,m)\).

(5) If all TDFP-index constraints fulfil and the optimization is convergent, then stop the iteration, otherwise, let \(k = k + 1\) and return step (3).

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**Figure 1.** Triple-loop configuration of the RBDO of the time-dependent structure

Flowchart of the proposed RBDO of the time-dependent structure based on SLOM is illustrated in Fig. 2, and the detailed operation flow is shown as follows:

1. Initialize the design variable \(d^{(0)}\) and \(X^{(0)}\), set \(k = 0\), and keep the default optimization convergence parameters in \(fmincon\).
2. Conduct DO without considering the uncertainty in the Eq.(2), and the initial design solution is recorded as \(d^{(0)}\) and \(X^{(0)}\).
3. Perform the deterministic optimization in the outer loop expressed in the Eq.(3).
4. Compute the TDFP index constraint with SLOM in the inner loop.
   - (4.1) Discretize the membership level \(\alpha \in [0,1]\) to a set \([\alpha_1,\alpha_2,\ldots,\alpha_m]\), calculate the membership interval \([X^{(L)}(\alpha), X^{(U)}(\alpha)](k = 1,2,...,n_i)\) of \(X_k\) by the membership function \(u_{X_k}(x_k)\) of \(X_k\), the same as fuzzy variable \(p\).
   - (4.2) Estimate the lower boundary \(G_{\min}^{(L)}(d,X,\alpha_i,p(\alpha_i))\) of the minimum \(G_{\min}^{(L)}(d,X,\alpha_i,p(\alpha_i))\) of the performance \(G(d,X,p,t)\) with respect to time \(t \in [t_s,t_e]\) at the membership level \(\alpha_j\) \((i = 1,2,...,m)\).
   - (4.3) Calculate the TDFP by \(G_{\alpha_i}^{(L)}(d,X,\alpha_i,p(\alpha_i)))\) \((i = 1,2,...,m)\).
5. If all TDFP-index constraints fulfill and the optimization is convergent, then stop the iteration, otherwise, let \(k = k + 1\) and return step (3).
Discretize \([0,1]\) into a vector \((i=1,\ldots,\frac{1}{m})\), let 
\[
\mathbf{d} = (\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_m)
\]
\[
\mathbf{d}_i \leq \mathbf{d} \leq \mathbf{d}_i + \mathbf{d}_i, \quad 0 \leq \mathbf{m}_i \leq 1, \quad \mathbf{X}^l \leq \mathbf{X} \leq \mathbf{X}^u
\]

Estimate \((\mathbf{X}(\mathbf{d}), \mathbf{X}(\mathbf{d}, \mathbf{p}))\) at \(X_1, X_2, X_3\) and \([X_1^l, X_1^u] \times [X_2^l, X_2^u] \times [X_3^l, X_3^u]\) and interpolation algorithm.

Let \(d^{(0)}, X^{(0)}, k = 0\)

\[
\text{find } \mathbf{d}, \mathbf{X}^*
\]

\[
\min f(\mathbf{d}, \mathbf{X}^*)
\]

\[
\text{s.t. } \text{pos}\{g_i(\mathbf{d}, \mathbf{X}, \mathbf{p}, t) \leq 0, \exists t \in [t_1, t_2]\} \leq \mathbf{z}_i(t = 1, 2, 3)
\]

where \(g_1(\mathbf{X}, t) = X_1^2 - 5X_1 + (X_2 + 1)^2 - 20\)

\[
\frac{90}{30} \leq \frac{120}{30} \leq 0 \leq \mathbf{X} \leq 10, \quad \mathbf{z}_i = 0.5(i = 1, 2, 3)
\]

\[
\text{where two fuzzy design parameters } X_1 \text{ and } X_2 \text{ are normal distribution with the standard deviation } \sigma_{X_1} = 0.6 \quad (i = 1, 2), \quad \text{i.e., } u_{X_1}(x_i) = \exp[-(\frac{X_1 - X_1^c}{\sqrt{2}\sigma_{X_1}})^2]\]

\[
\text{and } u_{X_2}(x_i) = \exp[-(\frac{X_2 - X_2^c}{\sqrt{2}\sigma_{X_2}})^2].\]

The specified time interval is \([0, 5]\) and the target TDFP value is 0.5 for these three constraints.

The optimization solutions and the computational cost statistics computed by the DO, the DLOM and the SLOM are given in Table 1. The DO, the DLOM and the SLOM stand for deterministic optimization method, TDFP-index-based approach involving the DLOM, TDFP-index-based approach.
involving the SLOM. The evaluated TDFP of the \(i\)th constraints at the optimum are presented in Table 2. The initial optimal parameters in DO model are \([X_{1}^{(0)}, X_{2}^{(0)}] = [5, 5]\), and the initial design parameters in RBDO is the solution in DO model.

Table 1 show that the optimal solutions calculated by the DLOM and the SLOM mutually match very well, which demonstrates the rationality of the RBDO model and the accuracy of the TDFP-index-based approach and two methods result in the same optimal objective value. The number of the function calls for TDFP analysis in the DLOM is 963472, while the computational cost in the SLOM is 18969, which demonstrates that the SLOM is more efficient than the DLOM to analyze the TDFP constraints. It can be seen that the computation time cost in SLOM is less than the DLOM.

Table 1. Summary of the optimization results for the Example 4.1

| methods | objective | Design variable | Number of function calls | Computation time/second |
|---------|-----------|-----------------|--------------------------|-------------------------|
|         |           |                 | objective | constraint | total     |                        |
| DO      | 6.5769    | [3.1218,3.4551] | 9         | 108        | 117       | 0.3                   |
| DLOM    | 7.9918    | [3.8293,4.1625] | 24        | 963472     | 963496    | 307                   |
| SLOM    | 7.9897    | [3.8281,4.1616] | 21        | 18969      | 18990     | 3.7                   |

From Table 2, we can conclude that all the TDFP index constraints in RBDO model satisfy the target TDFP value at the optimum at the same level, is inactive at the optimum, and the corresponding TDFP index is very closed to zero. Overall, the proposed RBDO model with the TDFP constraints is reasonable and the SLOM method is more efficient than the DLOM.

Table 2. TDFP index of constraints at optimal design parameters of example 4.1

| Methods | \(\pi_{b1}\) | \(\pi_{b2}\) | \(\pi_{b3}\) | \(\tilde{\pi}_{b(i)}(i = 1,2,3)\) |
|---------|-------------|-------------|-------------|------------------|
| DLOM    | 0.4990      | 0.4214      | 0.0071      | 0.5              |
| SLOM    | 0.4999      | 0.4223      | 0.0069      | 0.5              |

4.2 A cantilever beam

![Figure 3. A cantilever beam](image)
As shown in Fig.3, the optimization design objective is to minimize the weight of a cantilever beam\([21][16]\) which is free at the right end and anchored at the left end. The length of the beam \(L\) is 100 in and the considering service time period \([t_i,t_f]\) is \([0,10]\) years. The load on the free end of the beam is a vertical time-dependent force \(F_1=0^0\sin t\) and a horizontal time-dependent force \(F_2=0^0\sin t\). The width \(w\) and the thickness \(h\) of the cross-section are deterministic design parameters, the time-independent force \(F_1^0\) and \(F_2^0\), the Young’s Modulus \(E\) and yield stress \(\gamma\) are fuzzy parameters which is shown in table 3. Two time-dependent failure possibility constraints are considered in the engineering problem, and the first failure mode is that the maximum stress is greater than yield stress \(\gamma\), the second failure mode is that the replacement of the free end of the beam is larger than the allowable displacement \(D_0=2.5\) in. The time-dependent RBDO of the cantilever beam is shown as follows:

\[
\begin{align*}
\text{min} & \quad f(w,h) = w \times h \\
\text{s.t.} & \quad \text{prob}[g_i(d,X,t) \leq 0, \forall t \in [0,10]] \leq \frac{\alpha}{\nu}(i=1,2) \\
\text{where} & \quad g_i(d,X,t) = \frac{600F_1^0 \sin t}{wh^2} + \frac{600F_2^0 \sin t}{w^2h} \\
& \quad d = [w,h], X = [\gamma, E, F_1^0, F_2^0] \\
& \quad w > 0 \text{ in}, 0 < h < 5 \text{ in}, \frac{\alpha}{\nu} = 0.3 (i=1,2)
\end{align*}
\]

\(6\)

**Table 3.** The distribution parameters of the input variables of example

| Fuzzy variable | Distribution | Fuzzy mean | Fuzzy standard variance |
|----------------|--------------|------------|-------------------------|
| \(\gamma\) psi | Gaussian     | 4000       | 4000                    |
| \(E\) psi     | Gaussian     | 2.9e7      | 2.9e6                   |
| \(F_1^0\) lb  | Gaussian     | 1000       | 100                     |
| \(F_2^0\) lb  | Gaussian     | 500        | 50                      |

The initial optimal parameters are set to be \([w^0,h^0]=2,4\] for the DO model. The optimal solution and computational cost statistics calculated by the DLLOM and the SLOM are given in Table 4. It can be seen from Table 4 that all the method result in the same optimal objective value and two optimal solutions mutually agree very well, and the SLOM which only needs 75048 calls to estimate the TDFP index is more efficient than the DLLOM; furthermore, the computational time cost in the SLOM is less than the DLLOM.

**Table 4.** Summary of the optimization results for the Example 4.2

| methods | objective | Design variable | Number of function calls | Computation time/second |
|---------|-----------|----------------|--------------------------|------------------------|
|         |           |                | objective constraint total |                        |
The TDFP index constraints at the optimum are evaluated and presented in Table 5. From Table 5, we can conclude that all the TDFP index constraints satisfy the target TDFP index at the optimum at the same level.

Table 5. TDFP index of constraints at optimal design parameters of example 4.2

| Methods | $\pi_{\beta_1}$ | $\pi_{\beta_2}$ | $\pi_{\mu_i}(i = 1, 2)$ |
|---------|-----------------|-----------------|-------------------------|
| DLOM    | 0.3004          | 0.0389          | 0.3                     |
| SLOM    | 0.3000          | 0.0849          | 0.3                     |

Through the two examples, it is seen that the optimal design model with the TDFP index constraints is rational. Comparing with the DLOM and the SLOM methods, we can conclude that the SLOM is much more efficient than the DLOM under the acceptable precision.

5. Conclusions
In this study, a time-dependent reliability-based design optimization model with TDFP index constraints is established. Furthermore, the proposed model can be extended to address the RBDO of the time-dependent structure by considering both fuzzy and probabilistic uncertainties. For estimating the time-dependent RBDO involving fuzzy variables, the TDFP-index based approach is proposed. The DLOM is a brute force strategy to analyze the TDFP index constraints in the inner loop of the RBDO, thus, the computational cost is very high for the complex engineering problem. In order to improve the efficiency of the computation of the TDFP index constraints, the SLOM which is based on the extreme value transformation theory is introduced in the computation of the TDFP index constraints. Consequently, the computational cost of the RBDO of the time-dependent structure is extremely improved.

Several examples were calculated in order to verify the accuracy and efficiency of the RBDO model and the TDFP-index based approach. Through the results we can conclude that the RBDO model can guide the engineering design when the insufficient data cannot construct the probability distribution function, and the TDFP-index based approach with the SLOM is very efficient to solve the complex structure.

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References
[1] W. Yao, X. Chen, W. Luo, M. Van Tooren and J. Guo 2011 Review of uncertainty-based multidisciplinary design optimization methods for aerospace vehicles Prog. Aerosp. Sci. 47 450–79 https://doi.org/10.1016/j.paerosci.2011.05.001
[2] G.I. Schuëller and H.A. Jensen 2008 Computational methods in optimization considering uncertainties—An overview Comput. Methods Appl. Mech. Eng. 198 2–13 https://doi.org/10.1016/j.cma.2008.05.004

[3] J. Tu, K.K. Choi and Y.H. Park 1999 A new study on reliability-based design optimization J. Mech. Des. Trans. ASME 121 557–64 https://doi.org/10.1115/1.2829499

[4] X. Du and W. Chen 2004 Sequential optimization and reliability assessment method for efficient probabilistic design J. Mech. Des. Trans. ASME 126 225–33 https://doi.org/10.1016/1.1649968

[5] G.B. Dantzig 2016 Linear Programming under Uncertainty Stable URL: http://www.jstor.org/stable/2627159 April–July 1955 Science 1 197–206

[6] i.R.H. Sues and M.A. Cesare 2000 An innovative framework for reliability-based MDO 41st Struct. Dyn. Mater. Conf. Exhib.

[7] D. Dubois and H. Prade 1987 The mean value of a fuzzy number, Fuzzy Sets Syst. 24 279–300 https://doi.org/10.1016/0165-0114(87)90028-5

[8] B. Möller, W. Graf and M. Beer 2000 Fuzzy structural analysis using α-level optimization Comput. Mech. 26 547–65 https://doi.org/10.1007/s004660000204

[9] W. Guang-Yuan and W. Wen-Quan 1985 Fuzzy optimum design of aseismic structures Earthq. Eng. Struct. Dyn. 13 827–37 https://doi.org/10.1002/eqe.4290130607

[10] L. Du, K.K. Choi and B.D. Youn 2006 Inverse possibility analysis method for possibility-based designoptimization AIAA J. 44 2682–90 https://doi.org/10.2514/1.16546

[11] L. Du and K.K. Choi 2008 An inverse analysis method for design optimization with both statistical and fuzzy uncertainties 107–19 https://doi.org/10.1007/s00158-007-0225-0

[12] Z.C. Tang, Z.Z. Lu and J.X. Hu 2014 An efficient approach for design optimization of structures involving fuzzy variables Fuzzy Sets Syst. 255 52–73 https://doi.org/10.1016/j.fss.2014.05.017

[13] C. Wang, Z. Qiu, M. Xu and Y. Li 2017 Novel reliability-based optimization method for thermal structure with hybrid random, interval and fuzzy parameters Appl. Math. Model. 47 573–86 https://doi.org/10.1016/j.apm.2017.03.053

[14] Z. Hu and S. Mahadevan 2016 A Single-Loop Kriging Surrogate Modeling for Time-Dependent Reliability Analysis J. Mech. Des. 138 061406. https://doi.org/10.1115/1.4033428

[15] E. Zio 2009 Reliability engineering: Old problems and new challenges Reliab. Eng. Syst. Saf. 94 125–41 https://doi.org/10.1016/j.ress.2008.06.002

[16] C. Jiang, T. Fang, Z.X. Wang, X.P. Wei and Z.L. Huang 2017 A general solution framework for time-variant reliability based design optimization Comput. Methods Appl. Mech. Eng. 323 330–52 https://doi.org/10.1016/j.cma.2017.04.029

[17] Z. Hu and X. Du 2016 Reliability-based design optimization under stationary stochastic process loads Eng. Optim. 48 1296–312 https://doi.org/10.1080/0305215X.2015.1100956

[18] Y. Shi, Z. Lu, L. Xu and Y. Zhou 2019 Novel decoupling method for time-dependent reliability-based design optimization Struct. Multidiscip. Optim. https://doi.org/10.1007/s00158-019-02371-7

[19] Z. Wang and P. Wang 2012 A nested extreme response surface approach for time-dependent reliability-based design optimization J. Mech. Des. Trans. ASME. 134 1–14 https://doi.org/10.1115/1.4007931

[20] C. Fan, Z. Lu and Y. Shi 2019 Time-dependent failure possibility analysis under consideration of fuzzy uncertainty Fuzzy Sets Syst. 367 19–35 https://doi.org/10.1016/j.fss.2018.06.016

[21] J. Liang, Z.P. Mourelatos and J. Tu 2008 A single-loop method for reliability-based design optimisation Int. J. Prod. Dev. 5 76–92 https://doi.org/10.1504/IJPD.2008.016371