Understanding the evolution of massive stars

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ABSTRACT

To try to develop an intuitive understanding of why stars evolve in the way that they do, we compute sequences of snapshot stellar structure models. These models allow us to properly isolate the key internal properties that drive the evolution of stars in different phases. Our models can mimic the effects of a very wide range of physical processes including metallicity, internal mixing, binary interaction and mass loss. We focus on massive stars during the main sequence, the expansion across the HR diagram and core helium burning. With our novel approach, we isolate and quantify several important internal properties that set the surface properties at different evolutionary stages. For the main sequence, we find that models with the same mass and very similar surface properties can have different internal distributions of hydrogen and convective core masses. We discuss why stars expand after the main sequence and the fundamental reasons for why stars become red, blue or yellow supergiants. For the post-main sequence, we demonstrate that small changes in the abundance profile can have very large effects on the surface properties, discuss why stars with large envelope masses tend to be blue supergiants and study the cause of blue loops. We also show that massive stars with lower metallicity tend to be more compact for two reasons, lower CNO abundances in the burning regions and lower opacity in the envelope.

Key words: stars: evolution – stars: massive

1 INTRODUCTION

Stars evolve due to changes in their internal chemical composition, ultimately driven by nuclear fusion. Their interiors are also affected by many other physical processes and change in multiple ways simultaneously. This can make it challenging to determine which properties of the stellar interior are primarily driving the overall evolution at any given point in its life. It is also often mysteriously difficult to provide satisfactory, straightforward explanations for why a star evolves in a given way, beyond describing what happens in stellar evolution models. In this paper we aim to (i) develop an intuitive understanding for why stars evolve in the way that they do and (ii) to isolate the key internal properties that set the luminosity and effective temperature and drive the evolution of stars during different evolutionary stages.

These aims are worth pursuing for many important reasons and we list some below. First, an improved understanding of the cause and effect in stellar evolution models and in the evolution of actual stars can lead to a more satisfactory understanding of how stars evolve. Second, understanding the key properties that set the luminosity and effective temperature can provide a new way to interpret observations of individual stars and stellar populations in terms of the structural properties that favour a given set of observed properties. Third, the impact of stars on their immediate surroundings and the rest of the universe through the emission of photons, the release of mechanical energy by stellar winds and the release of chemical elements (van der Kruit & Freeman 2011) is regulated by the evolution of the luminosity, \( L \), and effective temperature, \( T_{\text{eff}} \). Fourth, the timing of binary interaction and its outcome is affected by the evolution of \( L \) and \( T_{\text{eff}} \). Additionally, the radius and \( T_{\text{eff}} \) of a star is also relevant for the structure and binding energy of the envelope which has important implications for pulsational pair instability eruptions and other transients. Finally, the outputs from stellar evolution models are often counter-intuitive and sensitive to initial conditions and we might want to understand exactly why. These reasons also connect to studies of stellar populations (e.g. Hurley et al. 2000; Eldridge et al. 2008; de Mink et al. 2013), the pre-supernova structure and appearance of massive stars (e.g. Heger et al. 2003; Groh et al. 2013), supernova explosion properties, massive star nucleosynthesis (e.g. Woosley & Weaver 1995; Chieffi & Limongi 2004; Nomoto et al. 2013) and interpreting gravitational wave observations (e.g. Abbott et al. 2019).

To provide context for our study, we review some of the key classical results from studies of how physical processes affect the evolution of \( L \) and \( T_{\text{eff}} \) in massive stars (see Chiosi & Maeder 1986; Heger et al. 2003; Langer 2012; Smith 2014, for more detailed

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reviews). One of these key processes is mass loss (Chiosi & Nasi 1974a; Chiosi & Maeder 1986; Meynet et al. 1994). Stellar evolution models show that mass loss can modify the location of the terminal-age main sequence in the Hertzsprung-Russell (HR) diagram (De Loore et al. 1977; Chiosi et al. 1978) and favour a lower $T_{\text{eff}}$ during core helium burning, near the Hayashi line (Hayashi et al. 1962; Stothers & Chin 1979; Maeder & Meynet 1987). Significant mass loss in higher mass stars can cause evolution to higher $T_{\text{eff}}$ during core helium burning if the envelope becomes stripped from the star (Maeder 1981a; Sreenivasan & Wilson 1983; Maeder & Meynet 1987; Salasnich et al. 1999; Vanbeveren et al. 2007; Yoon & Cantiello 2010; Georgy 2012; Groh et al. 2013; Meynet et al. 2015). Stellar rotation is also important for massive star evolution (Heger & Langer 2000; Maeder & Meynet 2000; Maeder et al. 2009). In general, moderate rotation favours evolution to higher luminosities and lower $T_{\text{eff}}$ during both the hydrogen and helium burning phases (Meynet & Maeder 2000; Chielli & Limongi 2013). Fast rotation can produce evolution towards higher $T_{\text{eff}}$ during hydrogen burning if the star becomes chemically homogeneous (e.g. Maeder & Meynet 1987; Langer 1992; Yoon & Langer 2005; Brott et al. 2011).

Convective mixing is one of the most important (and uncertain) physical processes in stellar interiors (Shaviv & Salpeter 1973; Stothers & Chin 1973; Maeder 1985). Both the convective stability criterion and the nature of convective boundary mixing are known to have important impacts on the evolution of $L$ and $T_{\text{eff}}$. For example, a moderate amount of convective core overshooting during core hydrogen burning results in higher $L$ and lower $T_{\text{eff}}$ at the end of the main sequence and therefore an extended main sequence width in the HR diagram (Maeder 1975, 1976, 1981b; Alongi et al. 1993). The choice of the Ledoux or Schwarzschild criterion for convective stability affects $L$ and $T_{\text{eff}}$ during core helium burning (Oke & Schwarzschild 1952; Saslaw & Schwarzschild 1965; Stothers & Chin 1975, 1976; Georgy et al. 2014, 2021). Convective overshooting in envelopes and the value of the mixing length parameter can also impact the evolution of $T_{\text{eff}}$, particularly for red supergiants (Alongi et al. 1991; Chun et al. 2018). In regions that are stable with respect to the Schwarzschild criterion, but unstable to the Ledoux criterion, semi-convection can occur (Langer et al. 1983). Semi-convection can affect the evolution of $L$ and $T_{\text{eff}}$ in multiple ways and has been shown to favour core helium ignition as a red supergiant rather than a blue supergiant (Langer et al. 1985; Schootemeijer et al. 2019). Additionally, interaction with a binary companion can affect the evolution of $T_{\text{eff}}$ (see Eldridge et al. 2017, and citations therein). In addition to these evolutionary processes, the initial metallicity is known to play a very important role in the evolution of the surface properties (e.g. Schlesinger 1969; Trimble et al. 1973; Chiosi & Nasi 1974b; Schaller et al. 1992; Langer & Maeder 1995; Groh et al. 2019). Low metallicity stars tend to spend more of their post-main sequence evolution in the blue region of the HR diagram, have more extended blue loops (Schaller et al. 1992) and produce more blue supergiant supernova progenitors (Langer 1991).

These classical results, combined with more recent studies, have lead to great advances in the understanding of the evolution of massive stars. However, it is still not always obvious what drives a star to evolve to the blue or the red in the HR diagram. In addition to computing standard stellar evolution models, previous studies have studied this question using a variety of techniques. The question of why stars evolve to become red giants after core hydrogen depletion has been studied by artificially modifying stellar evolution models (Hoppner & Weigert 1973; Sugimoto & Fujimoto 2000; Stancil et al. 2009). Other studies have analysed stellar evolution models in terms of the effect of specific internal properties, such as the effect of the hydrogen gradient above the helium core on the surface properties (Schootemeijer & Langer 2018). Another technique, and the one which we will use in this paper, is to combine static stellar structure models with stellar evolution models (Giannone 1967; Giannone & Weigert 1967; Lauterborn et al. 1971a,b; Farrell et al. 2020a,b). In Farrell et al. (2020b), we introduced a method to compute “snapshot” stellar structure models in hydrostatic and thermal equilibrium and used them to investigate the connections between the core mass, envelope mass and core composition on $L$ and $T_{\text{eff}}$. We applied these models in several ways, including to study red supergiants and stripped stars.

In this paper, we extend our method to compute “snapshot” stellar models to study the connections between the stellar interior and the surface properties during different evolutionary phases. In Sec. 2, we describe our method and our detailed numerical stellar models. In Sec. 3, we use current theory of stellar structure and evolution to produce a new framework to understand why stars evolve the way they do. Following this, we describe the important effects for the two main long-lived nuclear burning stages – hydrogen burning (Sec. 4) and helium burning (Sec. 5). We then describe what happens during the short-lived phase between hydrogen and helium burning that often lead to an expansion and the formation of a red (super)giant (Sec. 6). In Sec. 7, we present a numerical test that demonstrates clearly why stellar evolution models are so sensitive to small changes in initial conditions. Finally, we apply our results to explain cause and effect in some representative stellar evolution models (Sec. 8) and discuss other implications of our results (Sec. 9).
nuclear burning region. $L_{\text{actual}}$ is the actual internal luminosity profile in a star. $L_{\text{actual}}$ can be affected by (i) the hydrostatic structure of the star and (ii) the energy transport within the star. These are described by the equations for hydrostatic equilibrium and energy transport respectively,
\[
\frac{dP}{dm} = -\frac{Gm}{4\pi r^2}
\]  
\[
\frac{dT}{dm} = -\frac{GmT}{4\pi r^4} \nabla
\]
and where $\nabla$ is the opacity and all other variables have their usual meaning. For transport by radiation, $\nabla = \nabla_{\text{rad}}$ or the gradient that is obtained from mixing-length theory. $L_{\text{actual}}$ can be affected by any property that affects hydrostatic equilibrium and energy transport including the mass, the opacity or the presence of convection. The value of $L_{\text{actual}}$ is larger if hydrostatic equilibrium imposes a larger luminosity or if more energy can be transported, and vice versa. It is also worth noting that $L_{\text{actual}}$ is equivalent to the maximum luminosity that can be transported. Rearranging Equation 1, and assuming $\epsilon_r \ll \epsilon_{\text{nuc}}$ (which is valid for the vast majority of a star’s lifetime), we get
\[
\epsilon_{\text{grav}} = \frac{dL_{\text{actual}}}{dm} - \epsilon_{\text{nuc}} = \frac{dL_{\text{actual}}}{dm} - \frac{dL_{\text{nuc}}}{dm}
\]  
Starting in thermal equilibrium, $\epsilon_{\text{grav}} = 0$, an increase in $dL_{\text{nuc}}/dm$ or decrease in $dL_{\text{actual}}/dm$ will lead to $\epsilon_{\text{grav}} < 0$ and expansion, and vice versa. The picture can be simplified by considering that an increase/decrease in $L_{\text{nuc}}$ or $L_{\text{actual}}$ at a given point in the star will also result in a local increase/decrease in $dL_{\text{nuc}}/dm$ or $dL_{\text{actual}}/dm$ respectively. $L_{\text{nuc}}$ and $L_{\text{actual}}$ provide a framework to qualitatively understand the evolution of the radius and luminosity of the star. However, one should keep in mind that the gradients are ultimately the factors affecting expansion and contraction and need to be used to recover the quantitative effects.

When a star is in perfect hydrostatic and thermal equilibrium, $dL_{\text{nuc}}/dm = dL_{\text{actual}}/dm$ and $L_{\text{actual}} = L_{\text{nuc}}$ at all points in the star. Any change that causes either an increase of $L_{\text{nuc}}$ or a decrease of $L_{\text{actual}}$ will favour evolution to a larger radius (usually lower $T_{\text{eff}}$). Conversely, any property that causes a decrease of $L_{\text{nuc}}$ or an increase of $L_{\text{actual}}$ will favour evolution to a smaller radius (usually higher $T_{\text{eff}}$). The evolution of the luminosity is determined by how the surface value of $L_{\text{actual}}$ changes when the star relaxes to thermal equilibrium. It typically increases with changes that increase $L_{\text{actual}}$ and vice versa. However, the change in luminosity can be difficult to predict a priori due to the possibility of the formation of convective zones. In summary, stars $^1$ can contract or expand on nuclear or thermal timescales due to changes in:

(i) The temperature, density or fuel supply of a nuclear burning region (affects $L_{\text{nuc}}$)
(ii) The hydrostatic structure of the star e.g. a decrease in the envelope mass due to mass loss (affects $L_{\text{actual}}$)

$^1$ Note that we are not considering stars that are out of hydrostatic equilibrium, e.g. pulsating stars or luminous blue variables in eruption, which behave very differently to stars evolving on nuclear or thermal timescales.

### Table 1. Sequences of snapshot stellar structure models (S-) and numerical test models (T-) in the paper.

| Sequence | Isolated property |
|----------|-------------------|
| S-1      | Average mean molecular weight ($\mu_{\text{avg}}$) |
| S-2      | CNO abundance in the core (CNO_{core}) |
| S-3      | Fuel supply in the core (Fuel_{core}) |
| S-4      | Metal abundance in the envelope ($Z_{\text{env}}$) |
| S-5      | Total stellar mass |
| S-6      | Homogeneity of hydrogen profile ($H_{\text{profile}}$) |

### Core Helium Burning

| Sequence | Isolated property |
|----------|-------------------|
| S-7      | Hydrogen gradient in H-shell |
| S-8      | Excess He mass in H-shell with same gradient |
| S-9      | Distribution of He in H-shell with same excess He mass |
| S-10     | H abundance in envelope with same H-shell profile |
| S-11     | H abundance in envelope with same gradient |
| S-12     | H abundance in envelope with same inner envelope |
| S-13     | Helium abundance in the core ($Y_{\text{core}}$) |
| S-14     | Core mass, with very shallow hydrogen gradient |
| S-15     | Core mass, with shallow hydrogen gradient |
| S-16     | Core mass, with medium hydrogen gradient |
| S-17     | Core mass, with steep hydrogen gradient |
| S-18     | Core mass, with very steep hydrogen gradient |
| S-19     | CNO abundance in H-shell (CNO_{shell}) |
| S-20     | Metal abundance in the envelope ($Z_{\text{envelope}}$) |

### Crossing of the HRD after the MS

| Sequence | Description |
|----------|-------------|
| T-21     | 12 M$_\odot$ stellar evolution model |
| T-22     | Artificially suppressing contraction of the core |
| T-23     | Effect of different $Y_{\text{shell}}$ |
| T-24     | Effect of different CNO_{shell} |

### 3 A FRAMEWORK TO EXPLAIN THE EVOLUTION OF THE SURFACE PROPERTIES OF STARS

Stars evolve because they are never in perfect thermal equilibrium. Expansion and contraction in stars can be encapsulated by $\epsilon_{\text{grav}}$ in the equation of energy conservation
\[
\frac{dL_{\text{actual}}}{dm} = \epsilon_{\text{nuc}} - \epsilon_r + \epsilon_{\text{grav}}
\]  
where $\epsilon_{\text{grav}}$ can be expressed as
\[
\epsilon_{\text{grav}} = \frac{\epsilon_r}{T} \left( \frac{1}{T} \frac{dT}{dt} - \nabla_{\text{rad}} \frac{dP}{dm} \right)
\]  
and where $\epsilon_{\text{nuc}}$ is the rate of nuclear energy generation per unit mass and $\epsilon_r$ represents neutrino losses. A local value of $\epsilon_{\text{grav}} > 0$ indicates a local contraction, $\epsilon_{\text{grav}} < 0$ indicates a local expansion, while $\epsilon_{\text{grav}} = 0$ indicates local thermal equilibrium. The expansion/contraction determined by $\epsilon_{\text{grav}}$ operates on the thermal timescale indicated by the partial derivatives with respect to time in Equation 2. Understanding the behaviour of $\epsilon_{\text{grav}}$ is critical to understanding why a star evolves to a particular $L$ and $T_{\text{eff}}$.

To get an intuitive understanding for what sets $\epsilon_{\text{grav}}$ throughout a star as it evolves, we divide the factors that affect the value of $\epsilon_{\text{grav}}$ into two components, $L_{\text{nuc}}$ and $L_{\text{actual}}$. We define $L_{\text{nuc}}$ as the cumulative internal luminosity profile produced by nuclear reactions,
\[
L_{\text{nuc}}(m) = \int_0^m \epsilon_{\text{nuc}} dm'.
\]  
$L_{\text{nuc}}$ can be affected by anything that changes the nuclear energy generation rates, i.e. the temperature, density or fuel supply in a
The efficiency of energy transport e.g. a change in opacity or the presence of convection (affects $L_{\text{actual}}$)

Any change to the internal abundance profile, the hydrostatic structure or the energy transport that causes a mismatch between $L_{\text{nuc}}$ and $L_{\text{actual}}$ will change the surface properties of a star. The star will relax towards thermal equilibrium and a new $L$ and $T_{\text{eff}}$. In the following sections, we will use $L_{\text{nuc}}$ and $L_{\text{actual}}$ to explain what sets the values of $L$ and $T_{\text{eff}}$ during core hydrogen burning (Sec. 4), core helium burning (Sec. 5) and the expansion across the HR diagram between the hydrogen and helium burning phases (Sec. 6).

4 THE CORE HYDROGEN BURNING PHASE

4.1 What sets the luminosity and effective temperature of a star on the main-sequence?

Stars generally increase in luminosity and expand during the core hydrogen burning phase, in the absence of strong internal mixing or large amounts of mass loss. While this is well-known, it is arguably difficult to provide a simple explanation for why stars evolve like this. Such an explanation would ideally also encompass other known evolutionary results. For instance, we know that lower metallicity main-sequence stars favour higher $T_{\text{eff}}$ and that moderate convective overshooting or rotation can extend the terminal-age main-sequence to lower $T_{\text{eff}}$. Keeping these evolutionary effects in mind, we try to develop a comprehensive understanding for why main-sequence stars evolve in the way that they do.

To do this, we need to disentangle the different effects of the stellar interior that ultimately drive the evolution. This is usually difficult to do with stellar evolution models because the evolution of the stellar interior is quite complex and many different properties change simultaneously. We overcome this problem by using snapshot stellar structure models. As discussed above, the key advantage of snapshot models is that they allow us to vary one internal property at a time while holding the others constant. In a somewhat analogous process to principal component analysis, we selected important components of the stellar interior and isolated and quantified their effect on the stellar structure and surface properties. We identified six key properties for main sequence stars; these are the average mean molecular weight ($\mu_{\text{avg}}$), the CNO abundance in the convective core ($\text{CNO}_{\text{core}}$), the fuel supply in the convective core ($\text{Fuel}_{\text{core}}$), the metal abundance in the envelope ($Z_{\text{env}}$), the total stellar mass and the homogeneity of the hydrogen abundance profile ($H_{\text{profile}}$).

To demonstrate the effect of each of these properties, we begin with a reference model of a 15 M$_\odot$ solar metallicity star at the middle of the main sequence phase. We then compute a sequence of models in which only one internal property is changed at a time, with the exception of the $H_{\text{profile}}$, in which two properties change. The mean molecular weight and the fuel supply are unusual because they cannot be isolated by simply modifying the internal abundance profiles. To isolate these properties, we introduce an artificial element called ‘non-burning hydrogen’ which has the mean molecular weight and opacity properties of hydrogen but that (rather unsurprisingly) doesn’t participate in nuclear burning. These sequences of models are presented in Fig. 1. The points on each line indicate the location of each individual snapshot model and the arrows indicate the direction of increase/decrease. The legend on the right lists the numerical values of each property for the reference model plotted in black and for the modified models indicated by the large coloured points at the other end of each model sequence. See Appendix B for the internal abundance profiles of the model sequences in Fig. 1. In the following paragraphs, we discuss the effects of each of these properties in detail and consider how they affect $L_{\text{nuc}}$ and $L_{\text{actual}}$.

To change the average mean molecular weight ($\mu_{\text{avg}}$), we convert some fraction of the helium in the convective core to ‘non-burning hydrogen’. This has the desired effect of reducing the mean molecular weight without modifying the fuel supply. It does have the small side effect of changing the opacity in the interior but this has a quantitatively small effect on the overall structure of the star. Note also that while the set of models plotted in Fig. 1 does only modify $\mu$ in the convective core, we also computed models in which $\mu$ is changed both in and above the core and found similar results. Through the effect of the equation of state, a higher $\mu_{\text{avg}}$ favours a larger radius $R$, lower $T_{\text{eff}}$ and higher luminosity $L$. This result can be understood by considering how $\mu_{\text{avg}}$ affects $L_{\text{nuc}}$. For a higher $\mu$ the equation of state requires a higher temperature and/or density to maintain hydrostatic equilibrium. Due to the higher temperature and density in the nuclear burning region, the rate of nuclear energy generation increases, thus increasing $L_{\text{nuc}}$. This increase in $L_{\text{nuc}}$ favours a larger radius, as discussed in Sec. 3. The increase in the surface luminosity $L$ with $\mu_{\text{avg}}$ is due to the higher temperature and density in the central burning region once the star has relaxed to thermal equilibrium. One can derive a consistent conclusion using homologous relations, where $L \propto \mu^2$ (e.g. Kippenhahn & Weigert 1990).

We modify the CNO abundances in the core by scaling down the abundances of all the C, N and O isotopes from the reference model (with a solar metallicity) and replacing them with hydrogen. The primary impact of this on the stellar structure is to modify the abundance of CNO isotopes available to act as catalysts in the CNO cycle. We find that a lower CNO abundance in the core ($\text{CNO}_{\text{core}}$) favours a smaller $R$, higher $T_{\text{eff}}$ and higher $L$. The effect on the radius can be understood by considering that a decrease in $\text{CNO}_{\text{core}}$ decreases $L_{\text{nuc}}$ (because the nuclear energy generation rates scale with the abundance of CNO elements) which favours a smaller radius. So, why does the luminosity increase? One might have guessed that the higher temperatures in the core required to produce enough luminosity to support the star would exactly balance the effect of the lower CNO abundances. Our models show that the hydrostatic structure of a star in thermal equilibrium with a lower abundance of CNO elements in the core has an internal temperature and density profile that drops off slower as you move away from the center of the star. This results in a higher energy generation rate in the outer parts of the nuclear burning region. An interesting consequence of this is that if you remove fuel, either hydrogen or CNO abundances, from the core, and keep other quantities constant, a star will actually become more luminous.

The fuel supply in the core ($\text{Fuel}_{\text{core}}$) can be modified by converting some fraction of the hydrogen in the core to ‘non-burning hydrogen’. We find that a decrease in $\text{Fuel}_{\text{core}}$, holding everything else constant, favours a smaller $R$, higher $T_{\text{eff}}$ and higher $L$. Due to the effect on nuclear energy generation a decrease of $\text{Fuel}_{\text{core}}$ will decrease $L_{\text{nuc}}$, favouring a smaller radius. As in the case of the $\text{CNO}_{\text{core}}$, because a lower value of $\text{Fuel}_{\text{core}}$ favours a more compact star in thermal equilibrium, the temperatures and densities throughout the nuclear burning region (in particular the outer parts) are slightly higher. This results in a higher surface luminosity.

A lower abundance of metals in the envelope ($Z_{\text{env}}$) favours a smaller $R$, higher $T_{\text{eff}}$ and higher $L$. This is due to the effect of $Z_{\text{env}}$ on the opacity of the envelope. A decrease in opacity affects the energy transport and increases $L_{\text{actual}}$ in the envelope, favouring a smaller radius, higher $T_{\text{eff}}$ and higher luminosity.

We isolate the effect of the mass by keeping the same normalised
abundance profile but modifying the total stellar mass. A larger mass causes a larger $R$, higher $T_{\text{eff}}$ and higher $L$. While this is very well-known (especially at the zero-age main sequence) and relatively intuitive to understand, it is also worth understanding the result in terms of $L_{\text{nuc}}$ and $L_{\text{actual}}$, especially to contrast to helium burning stars later on. A larger mass requires a higher luminosity to maintain hydrostatic equilibrium, resulting in an increase in $L_{\text{actual}}$. This actually favours a contraction to a smaller radius. However due to the magnitude of the increase of $L_{\text{actual}}$, the contraction in the center of the star is large enough to significantly increase the core temperature and the nuclear energy generation rate, causing a feedback effect on $L_{\text{nuc}}$. The balance between the competing effects on $L_{\text{nuc}}$ and $L_{\text{actual}}$ results in an overall increase in the radius with mass. We will come back to this point when discussing the effects of increasing the envelope mass of a core helium burning star.

Finally, we compare the reference model to a model with the same total mass of hydrogen and helium, but distributed homogeneously throughout the star. While this does increase the fuel supply in the core, it is still a good representation of the effect of the distribution of hydrogen throughout the star ($H_{\text{profile}}$). The abundance profile of hydrogen in the envelope affects the opacity. Similar to $Z_{\text{env}}$, a decrease in opacity allows a higher luminosity to be transported and increases $L_{\text{actual}}$ in the envelope, favouring a smaller radius and higher $T_{\text{eff}}$. Our models indicate that the hydrogen profile has very little impact on the luminosity. We elaborate further on the impact of the internal distribution of hydrogen in Sec. 4.2.

Given the above discussion, we can understand why models with moderately high overshooting or rotation evolve to lower $T_{\text{eff}}$ at the terminal-age main sequence. When these models reach the lowest value of $T_{\text{eff}}$ on the MS, internal mixing has allowed a larger proportion of the hydrogen in the star to be converted to helium. As a result, the value of $\mu_{\text{avg}}$ is larger. As we have discussed above, a larger value of $\mu_{\text{avg}}$ favours a larger radius and lower $T_{\text{eff}}$. Another way of looking at this is to imagine picking up a model with higher overshooting at the TAMS and compressing it so that it is located at the TAMS location of a model with lower overshooting (at higher $T_{\text{eff}}$). The star would then be out of thermal equilibrium as $L_{\text{nuc}}$ would be larger than $L_{\text{actual}}$. This would favour an expansion back to larger radii and lower $T_{\text{eff}}$. For more fully mixed models, corresponding to strong rotation or very high overshooting, the competing effect of opacity due to the amount of hydrogen near the surface of the star dominate over the effect of $\mu_{\text{avg}}$, favouring a smaller radius and higher $T_{\text{eff}}$. It’s also interesting to note that the isolated effect of metallicity on stellar structure of stars at this mass is composed of two significant components, the CNO abundance in the core and the metal abundance in the envelope. Our models show that the effect of opacity is not the only reason that lower metallicity stars have a higher $T_{\text{eff}}$.

### 4.2 Degeneracy between internal hydrogen profile and surface properties

We now examine in more detail the effect of distribution of hydrogen/helium within the star, holding the total mass of hydrogen/helium constant. Fig. 2 compares four core hydrogen burning stellar models with the same mass (15 $M_\odot$) and the same total helium mass, i.e. same $\mu_{\text{avg}}$, but with different internal distributions of hydrogen. All of the models are in hydrostatic and thermal equilibrium. Models T, U and V each have different internal distributions of hydrogen (and helium) and different convective core masses. However they have very similar surface properties, differing by at most 0.01 dex in $\log T_{\text{eff}}$ and 0.002 in $\log L/L_\odot$. These models demonstrates a degeneracy between the internal distribution of hydrogen and the surface properties for stars with the same total mass and $\mu_{\text{avg}}$. The degeneracy applies only in the limit that the effects of opacity in the outer layers of the envelope do not dominate. When the star is more fully mixed and homogeneous (Model W), the effects of opacity in the outer layers of the star dominate and it has a significantly higher $T_{\text{eff}}$. Our results here are consistent with conclusions from simplified models of main sequence stars (e.g. Schwarzschild & Härm 1958).

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**Figure 1.** The coloured lines represent sequences of models (S-1 to S-6) in which the impact of different properties on the surface properties of a 15 $M_\odot$ main sequence star are isolated. These are the average mean molecular weight ($\mu_{\text{avg}}$), the CNO abundance in the convective core (CNO$_{\text{core}}$), the fuel supply in the core (Fuel$_{\text{core}}$), the metal abundance in the envelope ($Z_{\text{env}}$), the total stellar mass and the homogeneity of the hydrogen abundance profile ($H_{\text{profile}}$). An evolutionary track of a 15 $M_\odot$ star with solar metallicity ($Z = 0.020$) is plotted in grey for reference. See Appendix B for plots of the internal abundance profiles.

| Reference | Modified |
|-----------|----------|
| $\mu_{\text{avg}}$ | 0.71 | 0.63 |
| CNO$_{\text{core}}$ | 0.0150 | 0.0001 |
| Fuel$_{\text{core}}$ | 0.35 | 0.02 |
| $Z_{\text{env}}$ | 0.02 | 0.00 |
| Mass | 15 $M_\odot$ | 16 $M_\odot$ |
| $H_{\text{profile}}$ | "standard" | homogenous |
The evolution of massive stars after the main sequence is complicated. Some of the fundamentals are well understood after decades of research, however many important questions remain unanswered. Stellar evolution models are one of the key tools used to tackle these questions. Understanding exactly what drives a post-main sequence massive stellar model to a blue or red solution is often very difficult. Stars can evolve to the right and left in the HR diagram in ways that are difficult to understand, sometimes executing loops. The stellar interior changes in multiple ways simultaneously, which makes it difficult to distinguish cause and effect. In addition, very small changes to the stellar interior from earlier evolutionary phases can have a significant effect on the subsequent evolution (Iben 1974; Weiss 1989; Chin & Stothers 1990; Maeder & Meynet 1994; Ritossa 1996). For this reason, Rudolf Kippenhahn referred to the post main sequence stage as a “sort of magnifying glass, also revealing relentlessly the faults of calculations of earlier phases” (Kippenhahn & Weigert 1990). In this section, we isolate and analyse some of the important internal properties that set $L$ and $T_{\text{eff}}$ of massive stars during core helium burning. Because stellar interiors can get quite complicated and are ultimately described by a set of vectors describing the internal abundance profiles of each isotope, we cannot fully describe every internal property that affects the surface properties. Instead, we try to select the important properties that are modified by evolutionary processes. These are the effect of the helium abundance in the hydrogen shell, the hydrogen abundance in the envelope, the helium abundance in the core, the core mass ratio, the CNO abundance in the hydrogen shell and the metallicity in the envelope which we discuss in Sections 5.1 - 5.5 below.

5 THE CORE HELIUM BURNING PHASE

The evolution of massive stars after the main sequence is complicated. Some of the fundamentals are well understood after decades of research, however many important questions remain unanswered. Stellar evolution models are one of the key tools used to tackle these questions. Understanding exactly what drives a post-main sequence massive stellar model to a blue or red solution is often very difficult. Stars can evolve to the right and left in the HR diagram in ways that are difficult to understand, sometimes executing loops. The stellar interior changes in multiple ways simultaneously, which makes it difficult to distinguish cause and effect. In addition, very small changes to the stellar interior from earlier evolutionary phases can have a significant effect on the subsequent evolution (Iben 1974; Weiss 1989; Chin & Stothers 1990; Maeder & Meynet 1994; Ritossa 1996). For this reason, Rudolf Kippenhahn referred to the post main sequence stage as a “sort of magnifying glass, also revealing relentlessly the faults of calculations of earlier phases” (Kippenhahn & Weigert 1990). In this section, we isolate and analyse some of the important internal properties that set $L$ and $T_{\text{eff}}$ of massive stars during core helium burning. Because stellar interiors can get quite complicated and are ultimately described by a set of vectors describing the internal abundance profiles of each isotope, we cannot fully describe every internal property that affects the surface properties. Instead, we try to select the important properties that are modified by evolutionary processes. These are the effect of the helium abundance in the hydrogen shell, the hydrogen abundance in the envelope, the helium abundance in the core, the core mass ratio, the CNO abundance in the hydrogen shell and the metallicity in the envelope which we discuss in Sections 5.1 - 5.5 below.

5.1 Helium Abundance Profile in the Hydrogen Burning Shell

The fact that the hydrogen gradient at the interface between the core and the envelope impacts the value of $T_{\text{eff}}$ has been known since some of the earliest stellar models were computed (e.g. Lauerborn et al. 1971b). A steeper hydrogen gradient is found to favour a bluer star while a shallower gradient favours a redder star (Stothers & Chin 1968; Robertson 1971; Fricke et al. 1971; Lauerborn et al. 1971a; Stothers & Chin 1976; Schlesinger 1977; Langer et al. 1985; Walmswell et al. 2013; Schwoetemeier & Langer 2018; Schwoetemeier et al. 2019). The reason why the hydrogen gradient has this effect is not immediately obvious. An investigation by Walmswell et al. (2015) isolated the effects of the hydrogen profile on the opacity, mean molecular weight and fuel supply and how each affected $T_{\text{eff}}$. They found that the increased mean molecular weight and decreased effect of opacity associated with a shallower hydrogen gradient favour a redder star, the reduced fuel supply favours a bluer star, and concluded that the effect of the increased mean molecular weight dominates. Following this, one may still wonder why a gradient of mean molecular weight at the core/envelope interface affects $T_{\text{eff}}$ like this. In this section, we investigate this question.

To encapsulate the abundance profile of hydrogen at the core/envelope interface, previous stellar evolution studies have used the term “hydrogen gradient”. This is a simple, useful quantity, sufficient for some purposes. However, as is well known, the abundance profiles can be quite complicated e.g. due to convective shells during the main sequence phase, semi-convection (Langer et al. 1985), mass gainers (Braun & Langer 1995) and mergers (Glebbeek et al. 2013). Therefore, we will instead refer to this in a more general way as the helium abundance profile in the region of the hydrogen burning shell, denoted by $Y_{\text{shell}}$. We will demonstrate why this is necessary below.

In stellar evolution models, $Y_{\text{shell}}$ is primarily set by the receding convective core during the MS phase (e.g. Robertson 1971) and
by internal mixing between the end of core hydrogen burning and the beginning of core helium burning (e.g. Langer et al. 1985). During core helium burning, Yshell evolves as the hydrogen shell burns through the layers at the base of the envelope. In addition, several different physical processes can modify the shape of the profile. Firstly, the implementation of convective core overshooting during the main sequence phase and choice of the free overshooting parameter (αeff or fα) can cause a steeper or shallower gradient of hydrogen and helium in the H-shell (Stothers & Chin 1991; Langer 1991). Semi-convective mixing between core hydrogen depletion and the beginning of helium burning can also have an important effect (Langer et al. 1985; Langer 1991; Schootemeijer et al. 2019).

A combination of reasonable choices for the free parameters for convective overshooting and semi-convective efficiency can produce a very wide variety of hydrogen/hehelium profiles in the H-shell (Schootemeijer et al. 2019). Rotation has also been shown to modify the heli profile (Maeder & Meynet 2001) and this depends on the initial rotational velocity and choice of the diffusive mixing parameter. In binary systems, mass gainers and stellar mergers during the MS and the post-MS (e.g. Braun & Langer 1995; de Mink et al. 2013; Glebbeek et al. 2013) and the reorganisation of the star after a merger event could produce abundance profiles that differ significantly to single stars. In addition, current 1D stellar evolution models likely do not fully or accurately capture the mixing processes that affect Yshell. All of this is to say that the helium abundance profiles in the hydrogen shell that are produced in stellar evolution models are subject to many different uncertain processes.

We compute several sets of snapshot stellar structure models that isolate the effect of Yshell on L and Teff, presented in Fig. 3. Each set of models isolates Yshell in different ways, keeping some other quantity constant. The lower panels in Fig. 3 show the corresponding values of Teff for each of the three models in the upper panels (black circles) as well as for several intermediate models that are not plotted in the upper panels (white circles). Lines of constant radius are also plotted in grey. The dash-dot lines connect the two models that are closest to the bi-stability transition between a blue and a red supergiant, similar to the one found in Farrell et al. (2020b).

The first set of models in Fig. 3, Models A, B and C have the same core and envelope mass and isolate the effect of a linear abundance profile of hydrogen/helium. We choose to parameterise each model in terms of the hydrogen gradient as defined by Schootemeijer & Langer (2018). Models A, B and C indicate clearly that a shallower hydrogen gradient favours a redder star with a larger radius, while a steeper gradient favours a bluer star with a smaller radius. This reproduces the well-known results regarding the hydrogen gradient that we discussed above. For moderate changes in the gradient, our models indicate that the radius can change by a factor of ~20 and Teff can vary from 22 000 K to 4 000 K.

The second set of models, Models D, E and F, have the same hydrogen gradient and a different ‘excess mass’ of helium in the shell. By ‘excess mass’, we mean the extra mass of helium as compared to a perfect step profile at the core/envelope interface with the same surface abundance. Despite the fact that the hydrogen gradient is the same, the value of Teff and the stellar radius can vary quite significantly for models with a different mass of helium in the shell. Although this phenomenon is well known from stellar evolution models, Models E and F clearly demonstrate that very small changes in the stellar interior make a huge difference in Teff. Although they differ by just 0.05 M⊙ in their total mass of helium, a tiny fraction of the total stellar mass of 15 M⊙, but E is a blue supergiant and F is a red supergiant. The sudden transition between blue and red supergiants due to a small change in stellar structure is similar to the one discussed in Farrell et al. (2020b). The difference between Models D, E and F is smaller than typical uncertainties in the internal mixing in a stellar evolution model and yet they differ in radius by a factor of 12. As they are located in different parts of the HR diagram, they may be interpreted as having different evolutionary histories, despite their very similar internal structure. Although it has been emphasised by many others, we would like to reiterate the importance of being cautious when using the results of an individual stellar evolution model. A related consequence of these results is that the helium abundance profile in the shell cannot be represented by a single parameter. Even for the same hydrogen gradient (e.g. as defined by Schootemeijer et al. 2019), Teff can vary significantly due to the effect of the helium profile (compare models E and F). Although this complicates things, is worth keeping it in mind when interpreting stellar evolution models in future.

Finally, Models G, H and I have the same total mass of helium but distributed differently throughout the envelope. We parameterise the distribution of helium in the envelope in terms of a new parameter DHe, where a value of DHe = 0 means helium is distributed perfectly evenly throughout the envelope and DHe = 1 means all of the helium is concentrated at the base of the envelope and all the hydrogen is at the surface (quite an artificial situation). Models with a higher proportion of helium distributed towards the base of the envelope favour redder stars with larger radii, while those with more evenly distributed helium favour bluer stars with smaller radii. For different distributions of helium, the effective temperature can range from 23 000 K to 4 000 K.

To understand why the helium abundance profile has these effects on Teff, consider how Yshell affects Lshell and Lactual. An increase of the helium abundance in the region of the H-burning shell causes an increase in the mean molecular weight and, through the equation of state, in the temperature and density (the same effect demonstrated in detail later in the numerical test in Sec. 7). This causes an increase in the nuclear energy generation rate in the hydrogen burning shell, increasing Lnuc. As discussed in Sec. 3, an increase in Lnuc favours a larger radius and, in this case, a redder star. All of the trends presented in Fig. 3 can be understood in this way. Any process that causes an increase in the helium abundance in the nuclear burning region of the hydrogen burning shell favours a redder star with a larger radius. In addition, we can also use the concept of Lnuc and Lactual to understand the separate effects of the fuel supply and the opacity on Teff found by Walmswell et al. (2015). A decreased fuel supply causes a decrease in Lnuc which favours a bluer star. A decreased opacity in the H-burning shell initially favours a local increase in Lactual causing a local contraction in the hydrogen burning shell which actually increases Lnuc. The increase in Lnuc dominates, favouring a redder star. Unsurprisingly, we found the same qualitative results for the effect of the mean molecular weight and fuel supply on the Teff of core hydrogen burning stars in Sec. 4.

5.2 Hydrogen Abundance in the Envelope

The abundance profile of hydrogen in the envelope of a core helium burning star (Xenvelope) can be affected by several different factors. The initial abundance of hydrogen and helium can vary with metallicity. Internal convective zones and semi-convective mixing in the envelope during the transition between core hydrogen and core helium burning can also alter the hydrogen profile in the envelope. This can also happen, for instance, if a star becomes a red supergiant and its convective envelope extends deep enough to a region of decreased hydrogen abundance in a dredge-up episode. Accretion of
A decrease in the hydrogen abundance in the envelope (e.g. compare Models A, B & C from the upper panel (black circles), as well as several similar models that are excluded from the upper panel for clarity (white circles). The dash-dot line indicates the bi-stability transition between a blue and red supergiant. Middle Column: Models with the same total mass of helium mass, distributed differently throughout the envelope (S-9). Right Column: Models with the same total mass of helium mass, distributed differently throughout the envelope (S-9).

5.3 Helium Abundance in the Core

During the core helium burning phase, the helium abundance in the core, \( Y_c \), is primarily modified by nuclear burning. Its evolution as a function of time can also be affected by processes such as convective core overshooting, rotational mixing and, in evolutionary models, the \( ^{12}C(\alpha, \gamma)^{16}O \) reaction rate. Using snapshot models, we can isolate the effect of \( Y_c \) on \( T_{\text{eff}} \) and the stellar radius for a set of snapshot models with identical abundance profiles in the rest of the star outside the convective core Fig. 5. To support our discussion, we also plot the impact of \( Y_c \) on the radius of the helium core in green, defined as where \( X < 10^{-3} \), and the radius of the peak of the hydrogen burning shell in blue.

We find that \( Y_c \) affects \( T_{\text{eff}} \) in two competing ways. First, a decrease in \( Y_c \) causes an increase in the mean molecular weight of the core \( \mu_{\text{core}} \) as helium is converted to C and O. The increase in \( \mu_{\text{core}} \) favours an increase in the radius of the helium core (Fig. 5). This is for the same reason that the radius of a core hydrogen burning star increases with increasing \( \mu_{\text{core}} \). A higher \( \mu_{\text{core}} \) favours a higher temperature and density through the equation of state, increasing \( L_{\text{nuc}} \) and therefore favouring a larger helium core radius. However, a larger helium core radius favours a smaller total stellar radius. This is because as the core radius increases, so does the radius of the hydrogen burning shell, as it is located just above the helium core. As the radius of the hydrogen shell increases, the temperature and density throughout the burning region decrease, which leads to a decrease in \( L_{\text{nuc}} \) and favours a smaller radius. The second impact of \( Y_c \) on \( T_{\text{eff}} \) is through the decrease in the available fuel supply in the core for the triple alpha reaction. Again, similar to core hydrogen burning stars, a decrease in the fuel supply causes a decrease in \( L_{\text{nuc}} \) and favours a smaller helium core radius. A smaller helium
core radius moves the radius of the hydrogen burning shell inwards, causing an increase in the temperature, density and $L_{\text{nucl}}$, favouring a larger total stellar radius. In summary, as $Y_c$ decreases, the increase in $\mu_{\text{core}}$ initially favours a decrease in the stellar radius while the decrease in the available fuel supply in the core subsequently favours an increase in the stellar radius.

For our 15 $M_\odot$ representative model, the impact of $Y_c$ on $\mu_{\text{core}}$ dominates for $1 < Y_c < 0.50$ while the impact on the fuel supply dominates for $0.50 \leq Y_c < 0$. Therefore, the effect of $Y_c$ on the surface properties in non-monotonic. For $1 < Y_c < 0.50$ a decrease in $Y_c$ favours an increase in $T_{\text{eff}}$, while for $0.50 \leq Y_c < 0$ a decrease in $Y_c$ favours a decrease in $T_{\text{eff}}$. We find that the quantitative impact of $Y_c$ on $T_{\text{eff}}$ is smaller for large values of $Y_c$, i.e. the beginning of helium burning, and larger towards the end of helium burning for $Y_c < 0.50$. The impact of the fuel supply on $T_{\text{eff}}$ in core hydrogen burning stars only dominates over the effect of the mean molecular weight for very low values of $X_c \leq 0.05$. However, for core helium burning stars it can dominate for $X_c \leq 0.50$. This is likely related to a combination of the larger relative change in mean molecular weight when converting from $^1$H to $^4$He and from $^4$He to $^{12}$C as well as the smaller dependence of the CNO cycle on the density of hydrogen compared to the dependence of the triple alpha reaction on the density of He.

We tested our interpretation of the effect of $Y_c$ on $T_{\text{eff}}$ by computing a similar set of snapshot models to those in Fig. 5 but in which we keep the mean molecular weight constant, and just decrease the fuel supply with decreasing $Y_c$. This is achieved using a similar method as for the core hydrogen burning models described in Sec. 4. In these models, we found that the radius of the core and H-shell decreased monotonically with decreasing $Y_c$ and the total stellar radius increased monotonically. This implies that the effect of a decreased fuel supply is indeed to favour an overall increase of the stellar radius in a core helium burning star, supporting our analysis above.

In the context of blue loops in the HR diagram Lauterborn et al. (1971a) discussed the impact of the radius of the helium core on $T_{\text{eff}}$ in terms of the parameter $\Phi_c = M_c / R_c$, the ratio of the mass of the core to the radius of the core. Consistent with our results, they found that a decrease of the radius of the core at constant mass favours an increase in the total radius, and vice versa. Our analysis builds on this by explaining the non-monotonic impact of $Y_c$ on the core radius and total radius, as well as a framework for understanding why the radius of the core impacts the total radius in the way that it does.

### 5.4 Core Mass Ratio

The combination of the core mass $M_{\text{core}}$ and envelope mass $M_{\text{env}}$ of a star during helium burning can be affected by many different physical processes. Core masses are intrinsically larger for higher mass stars due to the formation of larger convective core masses during the core hydrogen burning phase. The core mass can also be increased by mixing during core hydrogen burning due to convective boundary mixing and rotation. The envelope mass can decrease due to mass loss and can be dramatically modified by binary interaction via stripping, mass accretion or a merger. In Farrell et al. (2020b), we computed snapshot models to isolate the effect of the envelope mass on the surface properties for constant helium core mass and abundance. Here, we extend this analysis by studying how the helium abundance in the hydrogen shell affects the relationship between the core mass, envelope mass and the surface properties.

We present five sequences of snapshot models, each with the same helium core mass and varying envelope mass. Each sequence has a different hydrogen/helium abundance profile in the hydrogen burning shell, as indicated in Fig. 6. For a given helium core mass,
the envelope mass affects $T_{\text{eff}}$ in two competing ways. On one hand, an increase in the mass of hydrogen in the envelope increases the effect of opacity. This decreases the amount of energy that can be transported, causing a decrease in $L_{\text{actual}}$ and favouring a larger radius. On the other hand, an increase in the mass of the envelope modifies the hydrostatic structure of the star and requires a larger value of $L_{\text{actual}}$ in the envelope to support a higher mass. The increase in $L_{\text{actual}}$ favours a bluer star with a smaller radius. For all of the model sequences in Fig. 6, the core mass ratio has a non-monotonic effect on $T_{\text{eff}}$. For core mass ratios $M_{\text{core}}/M_{\text{total}} \geq 0.6$, the effect of opacity dominates and therefore $T_{\text{eff}}$ decreases with increasing envelope mass (decreasing $M_{\text{core}}/M_{\text{total}}$). For core mass ratios $M_{\text{core}}/M_{\text{total}} \leq 0.6$, the effect of increasing mass on the hydrostatic structure dominates and $T_{\text{eff}}$ increases with increasing $M_{\text{env}}$ (increasing $M_{\text{core}}/M_{\text{total}}$). The fact that massive stars during the post-main sequence with higher envelope masses tend to favour a blue supergiant solution rather than a red supergiant solution has been known in the literature for a long time. However, a simple explanation for this has not always been clear.

Fig. 6 also clearly illustrates how the helium profile in the hydrogen burning shell affects the relationship between the core mass ratio and the surface properties. In the stripped star regime (i.e. low envelope masses), at the same core mass ratio a larger abundance of helium in the hydrogen shell favours a bluer star. This is simply due to the fact that the envelope mass is so small that the helium lowers the opacity of the envelope, favouring a smaller radius. For intermediate and higher envelope masses, a high $Y_{\text{shell}}$ at the same core mass ratio favours a redder star. In these models, the effect of the helium abundance on the H-shell energy generation dominates, increasing $L_{\text{nuc}}$ and favouring a redder star as discussed in Sec. 5.1.

At this point, the reader may wonder why it is that if you continue to add mass onto a typical core helium burning star, the radius decreases, but if you add mass to a typical main sequence star, the radius increases? It appears that this is due to a sort of a boundary condition effect. The boundary at the inner edge of the hydrogen burning shell in stars during helium burning is set by the hydrostatic and thermodynamic properties of the helium core. However, the boundary at the center of main sequence stars is not constrained in the same way. This allows the temperature and density to increase with increasing mass, increasing $L_{\text{nuc}}$ and favouring a larger radius.

### 5.5 Metallicity: CNO in the hydrogen shell and Z in the envelope

Stellar evolution models show that core helium burning stars with a lower initial metallicity tend to have a higher $T_{\text{eff}}$ (e.g. Stothers & Chin 1968). The initial metallicity of a star can impact its evolution in several ways including by modifying its mass loss rate, the efficiency of rotational mixing, the internal temperature structure and the convective core mass. All of these evolutionary effects have complex feedback effects. Because of this, it is difficult to use evolutionary models to isolate the effect of metallicity on post-MS stellar structures. Our snapshot models indicate that both the CNO abundance in the hydrogen burning shell ($\text{CNO}_{\text{shell}}$) and the metal opacity in the envelope have important impacts on $T_{\text{eff}}$. The value of $\text{CNO}_{\text{shell}}$ is predominantly determined by the initial metallicity but can be modified by rotational mixing if C and O are mixed from the core into the H-shell. This occurs most dramatically so in the case of low or zero metallicity stars (e.g. Ekström et al. 2008).

Using snapshot models, we isolate the effect of $\text{CNO}_{\text{shell}}$ on $T_{\text{eff}}$ for a representative 15 $M_\odot$ model, in which we scale the abundance of all the CNO elements in the H-shell, while keeping all other internal properties constant (Fig. 7). The CNO elements are converted to hydrogen to conserve mass. An increase in $\text{CNO}_{\text{shell}}$ favours an increase in the stellar radius and a lower $T_{\text{eff}}$. This can be understood by considering that a higher $\text{CNO}_{\text{shell}}$ causes a higher $L_{\text{nuc}}$ (due to the effect on the CNO cycle), favouring a redder star. We also test the effect of the abundance of the metals in the non-burning region of the envelope, above the hydrogen shell (Fig. 7). As expected, a higher abundance of metals in the envelope increases the effect of opacity, decreases $L_{\text{actual}}$ and favours a larger radius. The internal abundance profiles for these models are included in Appendix C1 and C2 in the online supplementary material.

Many previous works have studied and discussed the evolutionary effects of metallicity on $T_{\text{eff}}$ of core helium burning stars (e.g Schaller et al. 1992; Langer & Maeder 1995). It has been pointed out by Stothers & Chin (1968) that the $T_{\text{eff}}$ of massive post-main sequence stellar models may be lowered by increasing only the CNO abundances in the star. This can be connected to the effect...
Understanding how stars evolve

Figure 6. The effect of the combination of core mass ratio $M_{\text{core}}/M_{\text{total}}$ on the surface properties. **Left panel:** The helium abundance profile in the hydrogen shell region for each sequence of models (S-14 to S-18). **Right panel:** The value of $T_{\text{eff}}$ as a function of core mass ratio for each sequence of models with the same core mass and varying envelope mass.

Figure 7. **Left panel:** Effect of the CNO abundance in the hydrogen burning shell on $T_{\text{eff}}$ for a representative set of stellar models with the same core mass, envelope mass, core composition and helium abundance profile (S-19). **Right panel:** Effect of the metallicity in the envelope above the hydrogen burning shell (S-20).

of CNO$_{\text{shell}}$. Additionally, Schaller et al. (1992) found that a lower initial metallicity favours a more extended blue loops in the HR diagram during helium burning. This can also be understood in terms of CNO$_{\text{shell}}$ which, holding all other properties constant, favours a bluer star.

5.6 Comparison of effects in the HR diagram

Fig. 8 summarises and compares the effects of the five internal properties discussed above on the values of $L$ and $T_{\text{eff}}$. For each property, we select representative models which demonstrate the important effects. The internal hydrogen and helium abundance profiles of two or three representative snapshot models are plotted in the left panels and their location in the HR diagram are plotted in the right panels (blue, red and green circles), as well as several intermediate models that are not included in the abundance profiles.
Figure 8. Isolating the effect of internal properties on the location of a 15 M_⊙ star in the HR diagram. In each case, we plot the internal abundance profiles of hydrogen and helium for two or three stellar models in which only one property changes and the rest of the star remains the same. We also plot the location in the HR diagram of each of these models (circles) with a black line joining intermediate models (not plotted in the upper panels). The dash-dot line indicates the bi-stability transition between a BSG and RSG.
When hydrogen is exhausted in the core at the end of the main sequence, the core (empty circles). The dash-dot lines in the HR diagram indicate the approximate location of the bi-stability transition between a blue and a red supergiant, in which intermediate stellar models in hydrostatic and thermal equilibrium do not exist. As these results relate only to the internal structure of stars in hydrostatic and thermal equilibrium, they are not affected by the prior evolution. Therefore, they apply to all stars regardless of the mass loss history, any internal mixing or binary interaction. Fig. 8 is also relevant for the crossing of the HR diagram between the core hydrogen and helium burning phases, as we will discuss in Sec. 6.

In summary, Fig. 8 demonstrates the following: First, a larger envelope abundance profile in the region of the H-burning shell, $Y_{\mathrm{shell}}$, favours a larger radius and lower $T_{\mathrm{eff}}$. In some cases, the stellar radius can change by a factor of 4 for a change in the helium mass of only 0.05 $M_\odot$. $Y_{\mathrm{shell}}$ does not modify the luminosity significantly except when the star is on the Hayashi track. As $Y_{\mathrm{shell}}$ increases, the stellar radius increases, the envelope cools and more of the envelope becomes convective. A higher convective mass in the envelope increases the rate at which energy can be transported, increasing $L_{\mathrm{actual}}$, resulting in a higher surface luminosity. Second, the CNO abundance in the shell, $CNO_{\mathrm{shell}}$, has a similar effect on $L$ and $T_{\mathrm{eff}}$ to $Y_{\mathrm{shell}}$, for similar reasons. We see again that a small change in the internal abundance profile can have a large effect on the surface properties. An increase of $CNO_{\mathrm{shell}}$ by a factor of 6 decreases $T_{\mathrm{eff}}$ from 11 000 K to 4 000 K. Third, an increase in $X_{\mathrm{envelope}}$ also favours a lower luminosity due to the fact that an increase in the opacity causes a decrease in $L_{\mathrm{actual}}$, resulting in a lower surface luminosity. A decrease of the hydrogen abundance in the envelope ($X_{\mathrm{envelope}}$) from 0.70 to 0.60 can cause a very large increase in $T_{\mathrm{eff}}$ i.e. from 4 000 K to 20 000 K. Fourth, the effect of $Y_c$ on $T_{\mathrm{eff}}$ is non-monotonic but most important during the second half of the core helium burning phase when it favours a decrease in $T_{\mathrm{eff}}$ with decreasing $Y_c$. The effect on the luminosity is similar to the previous properties, for the same reasons. Finally, the core mass ratio $M_{\mathrm{core}}/M_{\mathrm{total}}$ also has a non-monotonic effect on $T_{\mathrm{eff}}$. For $M_{\mathrm{core}}/M_{\mathrm{total}}$ decreasing from 1 to $\sim$ 0.6 (i.e. corresponding to increasing envelope mass), the radius increases and decreases for further decreasing $M_{\mathrm{core}}/M_{\mathrm{total}}$ (i.e. further increasing envelope mass), the radius decreases and $T_{\mathrm{eff}}$ increases. The luminosity gradually increases with increasing envelope mass due to the larger luminosity produced/required by the hydrogen burning shell.

6 THE CROSSING OF THE HR DIAGRAM AFTER THE MAIN SEQUENCE

When hydrogen is exhausted in the core at the end of the main sequence, most stars expand significantly to become giants or supergiants. A simple explanation for why this happens currently appears somewhat elusive, despite extensive discussion in the literature (Hoppner & Weigert 1973; Eggleton et al. 1981; Yahil & van den Horn 1985; Applegate 1988; Eggleton & Cannon 1991; Renzini et al. 1992; Iben 1993; Sugimoto & Fujimoto 2000; Stancliffe et al. 2009; Ball et al. 2012). To investigate this expansion in massive stars, we perform numerical tests on representative 12 $M_\odot$ stellar models and apply our understanding of what sets $L$ and $T_{\mathrm{eff}}$ from Sec. 3, 4, 5. One point we would like to emphasise is that any compelling explanation for why stars expand after the main sequence should also describe why some stars (or stellar models) expand more than others. For example, in Fig. 15 the 16 $M_\odot$ stellar model expands to about 80 $R_\odot$ at SMC metallicity ($Z = 0.002$), compared to 600 $R_\odot$ at solar metallicity ($Z = 0.020$).

When the central hydrogen abundance decreases below about $X_c = 0.05$ in our standard (unmodified) 12 $M_\odot$ stellar evolution model, the entire star contracts due to the decrease in the central fuel supply, decreasing the total (gravitational + internal) energy of the star. The region above the hydrogen depleted core heats up and begins to burn hydrogen, forming the hydrogen burning shell. The hydrogen burning shell prevents that region from contracting due to the stabilising feedback of nuclear burning. However, the hydrogen depleted core continues to contract as it has no other way to support
This excess energy cannot be transported by the envelope, so it cools and expands.

At any given point during the expansion, the envelope can transport a certain amount of energy. As the shell heats up and increases $L_{\text{nuc}}$ and the core releases energy into the envelope, the envelope cannot transport the excess energy and absorbs it. As a result, it cools and expands. In our 12 $M_\odot$ model, 60% of the increase of the energy of the envelope is accounted for by the decrease in the energy of the core and 40% is accounted for by increased nuclear energy generation in the hydrogen shell as a result of the contraction. Of course, these fractions will be different for models of different masses. A consequence of this is that the effect of a given amount of core contraction on the total stellar radius will depend on the internal properties of the star including $Y_{\text{shell}}$, CNO$_{\text{shell}}$, $N_{\text{envelope}}$, and $M_{\text{core}}/M_{\text{total}}$. Any property of the envelope that favours a larger radius when the whole star is in thermal equilibrium will also cause the star to expand by more for a given amount of contraction by the core (i.e. a given change in the central temperature or radius of the core).

To test this explanation, we perform three tests using modified stellar evolution models. Fig. 10 shows the effect on the surface properties if we artificially suppress the contraction of the core during the crossing of the HR diagram after core hydrogen exhaustion (green line) compared to the usual evolution (black line) in a representative 12 $M_\odot$ star at solar metallicity. The evolution during the main sequence phase is plotted in grey. Just after core hydrogen exhaustion (green circle), we effectively “remove the core” by inserting fixed inner boundary conditions equal to the values at the boundary of the core. For this purpose, we define the core as the central region that is contracting. Maintaining fixed inner boundary conditions, we follow the response of the star in the HR diagram, indicated by the green line in Fig. 10. The star does not expand, remaining in the blue region of the HR diagram. The luminosity increases slightly due to the position of the hydrogen shell artificially moving slightly inwards. However, the star does not expand. This supports the understanding that if the hydrogen shell is not continually forced to contract and heat up by the core, the star will simply not expand. We also perform a test (not plotted) in which we turn off the hydrogen shell burning. In this test, the star does not expand, but rather stays in the blue region of the HR diagram and contracts. The lack of the stabilising feedback provided by nuclear burning in the hydrogen shell means that the whole star continues to contract.

We now investigate how the abundance profile in the hydrogen shell ($Y_{\text{shell}}$ from Sec. 5.1) affects the expansion across the HR diagram. Along with the standard 12 $M_\odot$ evolutionary model from Fig. 10, we compute two models with larger and smaller amounts of helium in the hydrogen burning shell. In Fig. 11, we plot the value of $T_{\text{eff}}$, representing the expansion of the envelope across the HR diagram, as a function of the central temperature, representing the contraction of the core. In all three models, the timescale of the expansion is the same. The expansion of the star proceeds on the Kelvin-Helmholtz timescale of the core. As the core mass is the same for each model, the timescale of the expansion is also the same for each model. The expansion of the standard, unmodified model is plotted in black. The high $Y_{\text{shell}}$ model with a higher abundance of helium in the hydrogen shell expands at a faster rate as a function of the contraction of the core than the standard, unmodified model (in black). Conversely, the low $Y_{\text{shell}}$ model expands at a slower rate than the standard model and actually begins core helium burning as a blue rather than a red supergiant. This can be understood by considering that a given contraction of the core will cause a given increase in the temperature and density in the hydrogen shell.

**Figure 11.** The effect of the helium abundance in the H-burning shell $Y_{\text{shell}}$ on the expansion of a 12 $M_\odot$ star from the end of the MS to the start of core helium burning (T-23). The usual stellar evolution model is shown in black as well as models with a higher (red) and lower (blue) value of $Y_{\text{shell}}$.

**Figure 12.** Effect of the CNO abundance in the H-burning shell CNO$_{\text{shell}}$ on the expansion across the HR diagram after core hydrogen exhaustion in a 16 $M_\odot$ model at a metallicity of $Z = 0.002$ (T-24). The usual evolution to the end of the MS to the start of core hydrogen exhaustion is plotted in black. The high $Y_{\text{shell}}$ model expands at a slower rate than the standard model and actually begins core helium burning as a blue rather than a red supergiant. This can be understood by considering that a given contraction of the core will cause a given increase in the temperature and density in the hydrogen shell.
However, the quantitative effect on $L_{\text{nuc}}$, and therefore on the stellar radius, depends on the properties of the hydrogen shell. As discussed in Sec. 5.1, a larger value of $Y_{\text{shell}}$, i.e. a shallower hydrogen gradient, favours a redder star.

We perform a similar investigation for the abundance of the CNO elements in the hydrogen shell CNO$_{\text{shell}}$ (Fig. 12). We begin with a $16 \, M_\odot$ stellar model with a metallicity of $Z = 0.002$. Just after core hydrogen exhaustion (black point), we increase the abundance of the CNO elements just in the region of the hydrogen shell (similar to Fig. 8) by a factor of 6. We then observe the response of the model as it expands to begin core helium burning (red line). It expands to much larger radii and lower $T_{\text{eff}}$ than the original model, beginning helium burning as a red supergiant rather than a blue supergiant. This can be understood in a similar way to the effect on $Y_{\text{shell}}$. Stellar evolution models of low metallicity stars tend to favour beginning core helium burning as blue supergiants, compared to higher metallicity models which favour core-He ignition as a RSG. This is not because their cores are hotter so they take less time to contract to become hot enough to ignite helium and therefore have less time to expand. Rather it is the effect of the lower CNO abundance in the hydrogen burning shell, combined with the lower metal opacity in the envelope. In fact, there is kind of a coincidence here: the core is hotter because there is a lower CNO abundance in the core and the star begins helium burning at a higher $T_{\text{eff}}$ because there is a lower CNO abundance in the hydrogen shell. Of course, the lower CNO abundances are both due to the lower metallicity.

7 WHY ARE STELLAR EVOLUTION MODELS SOMETIMES SO SENSITIVE TO SMALL CHANGES IN THE INITIAL CONDITIONS?

To demonstrate how a star responds when $L_{\text{nuc}} \neq L_{\text{actual}}$, and why stellar evolution models are sometimes so sensitive to small changes, we perform a numerical test. The initial state for the test consists of a stellar structure model of a $15 \, M_\odot$ blue supergiant at the middle of the core helium burning phase (Fig. 13, Model I). The initial model is in both hydrostatic and thermal equilibrium and $L_{\text{nuc}} = L_{\text{actual}}$ at all points in the star (Fig. 13b). We perturb the initial model by modifying the internal abundance profile by hand in the region of the hydrogen burning shell, replacing a small amount of hydrogen with helium (Fig. 13d). This new model could, for example, correspond to what one would obtain assuming a slightly different implementation of the (uncertain) internal mixing processes. We then put the model back into $\text{MESA}$ to find a solution to the stellar structure equations, enforcing hydrostatic equilibrium, and study how it relaxes to thermal equilibrium.

The new stellar model that we obtain (Model II) is initially out of thermal equilibrium and $L_{\text{nuc}} \neq L_{\text{actual}}$ (Fig. 13e). The surface properties of Model II have not changed compared to Model I (Fig. 14) as the changes in the interior have not yet propagated to the surface. The value of $L_{\text{nuc}}$ in the envelope is larger in Model II than Model I. This is because a larger helium abundance increases the mean molecular weight ($\mu$) in the H-shell. For a higher $\mu$, the equation of state requires a higher temperature and/or density to maintain hydrostatic equilibrium. In this case, both the temperature and density in the H-shell are higher in Model II compared to Model I (Fig. 13f). Due to the higher temperature and density, the rate of nuclear energy generation in the H-shell increases, therefore increasing $L_{\text{nuc}}$. In Model II, the value of $L_{\text{nuc}}$ is greater than $L_{\text{actual}}$ above the H-shell. This means the luminosity entering the envelope above the H-shell is greater than the luminosity that can be transported by the envelope. The excess energy that cannot be transported will be absorbed into the envelope and cause it to cool and expand, as expected by the virial theorem. As long as $L_{\text{nuc}} \neq L_{\text{actual}}$, the star will continue to expand and move to the right in the HR diagram (Fig. 14). The expansion will stop when the energy produced in the H-shell is exactly equal to the energy that the envelope can transport, i.e. $L_{\text{nuc}} = L_{\text{actual}}$.

As the star expands, the values of $L_{\text{nuc}}$ and $L_{\text{actual}}$ change. $L_{\text{nuc}}$ gradually decreases because the expansion of the envelope causes the temperature and density in the outer regions of the H-shell to decrease (Fig. 13h, i). In the envelope, $L_{\text{actual}}$ initially decreases and then subsequently increases. This is reflected in the change of the surface luminosity between Model III and IV (Fig. 14). The initial decrease is a result of the decrease in the temperature gradient $dT/dr$ which decreases the efficiency of energy transport by radiation and, hence, decreases $L_{\text{actual}}$. The decrease of $L_{\text{actual}}$ actually makes it more difficult for the star to reach thermal equilibrium because it causes the difference between $L_{\text{nuc}}$ and $L_{\text{actual}}$ to increase. As the radius increases, an increasing proportion of the envelope begins to transport energy by convection. This is a result of cooling of the envelope and an increase in $\nabla_{\text{rad}}$. As convection is more efficient at transporting energy, it favours an increase in $L_{\text{actual}}$, which occurs once enough of the envelope becomes convective. This is one way to think about why the luminosity of a star increases as it expands along the Hayashi track. It is important to note that the formation of the convective envelope is an effect of the expansion, not the cause. The star will expand regardless of the onset of convection.

Another way to think about the expansion is in terms of the change in the internal and gravitational energy (Fig. 14). As discussed above, in Model II the H-shell is producing more luminosity than the envelope can transport. The excess energy is therefore absorbed into the envelope, and it cools and expands. The cooling and expanding of the envelope corresponds to a decrease in the total internal energy of the star of $4.5 \times 10^{49}$ erg and an increase in its total gravitational potential energy of $7.0 \times 10^{49}$ erg, reflecting the virial theorem for stars. As a result, the total (gravitational + internal) energy of the star increases by $2.5 \times 10^{49}$ erg. This net increase comes from nuclear energy generation in the H-shell, so that energy is conserved in total. The increase in the total energy is almost entirely due to the increase in the energy of the envelope. In general, processes which increase the total energy of the envelope also tend to favour an increase in the total stellar radius.

8 CAUSE AND EFFECT IN THE HR DIAGRAM

In this section, we apply our results from Sections 4, 5 and 6 to explain which internal properties dominate the change in $L$ and $T_{\text{eff}}$ at different points in the evolution of a star. We select three representative cases at intermediate and high masses: a $16 \, M_\odot$ star at solar metallicity ($Z = 0.020$), a $16 \, M_\odot$ star at SMC metallicity ($Z = 0.002$) and a $6 \, M_\odot$ star at solar metallicity.

8.1 The Evolution of a 16 Solar Mass Star at $Z = 0.020$

A $16 \, M_\odot$ star at solar metallicity ($Z = 0.020$) begins its life on the zero-age main sequence (Fig. 15a, point I) with a convective core and a radiative envelope. As it evolves, hydrogen is burned to helium via the CNO cycle, increasing the average mean molecular weight of the star $\mu$ and decreasing the fuel supply of hydrogen in the core. Both the increase in $\mu$ and the decrease in $X_\text{C}$ favour an increase in the overall stellar luminosity. However, there is a competition between the increasing $\mu$ favouring an overall expansion and the decreasing $X_\text{C}$ favouring an overall contraction. For most of the evolution, the
Figure 13. A numerical test to examine the response of a star to changes in its abundance profile. Model I: the original blue supergiant in thermal equilibrium. Model II: stellar model immediately after the helium is inserted to the H-shell by hand, indicated by the black arrow in (d). Model III: a few timesteps after Model II after the star has expanded to a radius of 49 R\(_{\odot}\). Model IV: when the star has reached thermal equilibrium as a red supergiant. For each model, we plot the internal profiles of the \(^4\)He, temperature (\(\log T\) scaled by a factor 0.2), the cumulative nuclear energy generation profile \(L_{\text{nuc}}\), the luminosity imposed by hydrostatic equilibrium \(L_{\text{actual}}\) and the ratio with Model I of the internal temperature, density and \(\epsilon_{\text{nuc}}\) from CNO burning (scaled by a factor of 0.15) profiles.

Figure 14. **Left panel:** The change in \(L\) and \(T_{\text{eff}}\) during the numerical test shown in Fig. 13. The locations of Models I, II, III and IV from Fig. 13 are indicated. **Right panel:** The change in the total internal, gravitational and total energy of the star from the test as it expands from a blue to a red supergiant and eventually reaches thermal equilibrium on the right-hand side of the figure. The bump at \(\sim 440 R_{\odot}\) is due to convection in the envelope.
effect of increasing $\mu$ dominates over the effect of decreasing $X_c$. As a result, the star expands. The combination of the increase in luminosity and the increase in radius causes the star to evolve to lower $T_{\text{eff}}$ in the HR diagram.

When the central hydrogen mass fraction $X_c$ drops below about $X_c = 0.05$ (point II), the entire star contracts, causing the star to evolve to the left in the HR diagram (the Henyey hook). At this point, the decrease in $X_c$ dominates over the change in $\mu$. The luminosity continues to increase due to the additional energy released from the gravitational contraction in the outer layers of the star.

The contraction of the star increases the temperature in all layers of the star. When the region above the hydrogen depleted core heats up enough to burn hydrogen via the CNO cycle, the hydrogen burning shell is formed (point III). The shell will contribute to the luminosity of the star until at least the end of core helium burning. As the core is depleted in hydrogen and not hot enough to burn helium, the only way it can continue to support itself against gravity is to contract toward the blue in the HR diagram, due to the effect of increasing mean molecular weight. At point V, the evolution in the HR diagram reverses and the star evolves back to higher $T_{\text{eff}}$, due to the effect of increasing mean molecular weight in the core, $\mu_{\text{core}}$. At point VI, the star depletes its core helium.

8.2 The Evolution of a 16 Solar Mass Star at $Z = 0.002$

A 16 M$_\odot$ star at SMC metallicity ($Z = 0.002$) begins its evolution with a smaller radius at a higher $T_{\text{eff}}$ and a higher luminosity than at solar metallicity (Fig. 15b, point 1). A lower abundance of CNO elements in the center of the star and of metals in the envelope means result in a smaller radius that at solar metallicity. The higher luminosity is a result of higher $T$ and $\rho$ in the outer parts of the nuclear burning region. The evolution during the MS is the similar to the solar metallicity model until core hydrogen depletion (point III). After the main sequence, the rate of expansion of the envelope with the contraction of the core is smaller than for the solar metallicity model. Again, this is due to lower CNO abundances in the H-burning shell and metals in the envelope.

Due to the lower CNO abundance in the H-burning shell, the low metallicity model begins burning helium as a BSG (point IV). Similar to the solar metallicity model, the H-shell burns through the profile left behind from the MS and the expansion after the MS, changing the amount of helium in the H-shell, $Y_{\text{shell}}$. The envelope contracts and the star evolves to higher $T_{\text{eff}}$ as $Y_{\text{shell}}$ decreases. Once $Y_{\text{shell}}$ remains relatively constant, the contraction stops and the star begins to evolve back towards the red region of the HR diagram. The increase in $M_{\text{core}}/M_{\text{total}}$ as a result of the growth of the mass of the core drives the evolution to lower $T_{\text{eff}}$. Subsequently, the decrease in $Y_c$ dominates, which also drives evolution to lower $T_{\text{eff}}$.

8.3 The Evolution of a 6 Solar Mass Star at $Z = 0.020$

A intermediate mass 6 M$_\odot$ star evolves similarly to the 16 M$_\odot$ model beginning of helium burning (Fig. 15c). Once core helium burning begins, the hydrogen shell burns through the profile above the core driving the star to lower radii and higher $T_{\text{eff}}$, as for 16 M$_\odot$ model. However in the 6 M$_\odot$ case, due to the lower core mass ratio $M_{\text{core}}/M_{\text{total}}$, the star evolves to much lower radii than the 16 M$_\odot$ model. It evolves away from the Hayashi line and toward the blue region of the HR diagram and the beginning of a blue loop is formed. The star spends about 70% of its helium burning lifetime as a BSG at higher $T_{\text{eff}}$. The combination of the subsequent increase in $M_{\text{core}}/M_{\text{total}}$ and decrease in $Y_c$ makes the star evolve back to the Hayashi line.

9 OTHER DISCUSSION POINTS

9.1 Blue Loops in the Hertzsprung-Russell Diagram

Stellar evolution models in the mass range 5 - 12 M$_\odot$ sometimes exhibit a “blue loop”, in which a star evolves from a RSG to the blue region of the HR diagram and back to the red, completing a loop in the HR diagram (e.g. Fig. 15c). Their existence has been known for a long time (Hayashi & Cameron 1962; Hofmeister et al. 1964) and they have been extensively discussed in the literature (Schlesinger 1977; Stothers & Chin 1979; Maeder & Mermilliod 1981; Walmswell et al. 2015). The properties of blue loops are known to be highly sensitive to processes such as convective overshooting, mass loss and semi-convection. They are especially important in the context of the production of Cepheids. In the following paragraph, we use the model from Fig. 15c and our results from Sec. 5 to explain why blue loops occur.

As a RSG evolves, its helium core increases in mass as hydrogen burns in a thin shell around the core. As it burns outwards, the helium abundance profile in the hydrogen shell may change, which can affect the stellar radius (Sec. 5.1). In typical stellar evolution models at these masses, the change in the helium profile during helium burning favours a bluer star. At the beginning of core helium burning, the helium profile in the shell favours a large radius due to its high helium content. As the core moves outwards through the helium profile, the profile in the burning region changes to favour a blue star. If this effect outweighs the other effects of the core mass ratio, affected by e.g. convective overshooting, mass loss and rotation, and the CNO abundance in the hydrogen shell, then the star evolves back towards the blue and may become a BSG. After the initial decrease in radius, the combined effect of decreasing $Y_c$ and increasing core mass ratio due to hydrogen shell burning favours evolution to a larger radius. At some point, this effect wins out and the star evolves back.
Figure 15. Top Panel: The evolution of a 16 $M_\odot$ star at solar metallicity ($Z = 0.020$) in the HR diagram, with arrows and text indicating the primary cause for the evolution in each direction. The dashed line indicates the transition from the MS to core helium burning. The following six stages of the evolution are highlighted: I is at the zero-age main-sequence, II is at the terminal-age main-sequence when the star begins to contract, III is when the star begins to expand and cross the HR diagram, IV is the beginning of core helium burning, V is when $Y_c = 0.30$ and VI is at the end of core helium burning. Middle Panel: Same as top panel, but for a 16 $M_\odot$ star at SMC metallicity ($Z = 0.002$). Bottom Panel: Same as top panel, but for a 6 $M_\odot$ star at solar metallicity ($Z = 0.020$).
to the red. Our results suggest that blue loops in intermediate and massive stars should be favoured at relatively lower masses because of their intrinsic lower core mass ratios. This is consistent with what we find in stellar evolution models and in observations of Cepheids. Our models also suggest that processes that increase the core mass ratio such as increased convective overshooting or post-MS mass loss will disfavour the production or extent of blue loops, which is also consistent with previous studies.

The “mirror effect” has been invoked as a phenomenological description of the blue loops (Hayashi & Cameron 1962; Hofmeister et al. 1964; Sandage & Schwarzschild 1952). In this description the expansion of the core causes a contraction of the envelope as the cause of the star evolving back to the blue, in the opposite way to a star expanding across the HR gap. Our results indicate that the slight expansion of the edge of the core is an effect of the changing helium profile above the core star, rather than a cause of the blue loop.

9.2 Why do stars tend to become more luminous and expand as they evolve?

The temperature and density in the nuclear energy generation regions increase as a star evolves. This is primarily due to either an increased higher mean molecular weight in the burning region or a change in the hydrostatic structure due to the contraction of the core when it runs out of fuel. The increased temperature and density cause an increase in $L_{\text{nuc}}$. As a consequence, stars usually produce slightly more energy than they can transport, causing a cooling and expansion of the envelope. The luminosity goes up if the increase in $L_{\text{nuc}}$ outweighs the effect of the expansion on the burning region. This is always the case during MS and sometimes the case during post-MS. Additionally, the evolution of most of the internal properties e.g. M_{\text{nuc}}/M_{\text{total}}, Y_c, Y_{\text{shell}} favour a larger radius as the star becomes more evolved. This means that the more a star evolves, the harder it is to maintain a small radius.

10 CONCLUSIONS

In this paper, we aimed to develop an intuitive understanding of why stars evolve in the way that they do and to isolate the key internal properties that drive stellar evolution during different evolutionary stages. We summarise the key findings below.

(i) We proposed a new way to think about cause and effect in stellar evolution that is ultimately based on an argument from conservation of energy. We discussed how changes in $L_{\text{nuc}}$, the cumulative internal luminosity distribution generated by nuclear reactions, and $L_{\text{actual}}$, the actual internal luminosity distribution, can help to provide an intuitive understanding for why a star evolves to a given $L$ and $T_{\text{eff}}$. Beginning in thermal equilibrium, any change to the internal abundance profile, the hydrostatic structure or the energy transport that causes an increase in $L_{\text{nuc}}$ or a decrease in $L_{\text{actual}}$ will favour evolution to a larger radius, and vice versa.

(ii) We isolated and quantified the key internal properties that set the surface properties for stars during the main sequence, the core helium burning phase and the short-lived expansion in between. Our results provide a new way to interpret observations of individual stars and stellar populations in terms of the structural properties that favour a given set of observed properties.

(iii) Massive stars with lower metallicity tend to have higher $T_{\text{eff}}$ for two reasons: (i) lower CNO abundances in the core (hydrogen burning) and the H-shell (helium burning) which affects nuclear energy generation and (ii) lower opacity in the envelope. During the post-main sequence, the effect of the CNO abundances dominates for BSGs while the effect of opacity dominates for RSGs.

(iv) Models of massive main sequence stars with the same mass and very similar surface properties can have different internal distributions of hydrogen and convective core masses. This degeneracy might be broken with current and future asteroseismology observations.

(v) Massive stars expand after the main sequence because the contraction of the core heats up the hydrogen-burning shell, generating more energy than the envelope can transport. Whether a star begins helium burning as a blue or red supergiant depends on the helium and CNO abundances in the hydrogen shell, the core mass ratio and opacity due to hydrogen and metals in the outer envelope. Each of these properties affect the rate of expansion of the envelope for a given contraction of the core.

(vi) We explain the cause of blue loops in the HR diagram during the post-main sequence and why some stellar evolution models exhibit blue loops and others do not. This has important implications for interpreting observations of Cepheids.

(vii) We present a numerical test that clearly demonstrates that small changes in the stellar interior can cause very large changes in the surface properties. We conclude that much of the sensitivity seen in post-main sequence massive star models is ultimately due to the strong dependence of the CNO cycle energy generation rate on temperature in the hydrogen burning shell, which can be modified by very small changes in the helium abundance profile.

Consistent with previous work, our results show that a careful analysis of the internal abundance profiles is important for understanding how stars evolve. Given current uncertainties in the internal mixing of massive stars, it is possible that the abundance profiles in the current state-of-the-art stellar evolution models differ substantially from actual stars. This may have significant impacts for our overall picture of stellar evolution, including studies of binary interaction and gravitational wave progenitors.

DATA AVAILABILITY

The derived data generated in this research will be shared on reasonable request to the corresponding author.

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APPENDIX B: MAIN SEQUENCE MODEL PROFILES

APPENDIX C: CORE HELIUM BURNING INTERNAL PROFILES
**Figure B5.** Internal abundance profiles for main sequence models from Fig. 1 in the total stellar mass sequence.

**Figure C1.** Internal abundance profiles for helium burning models with varying CNO abundance in the hydrogen shell.

**Figure B6.** Internal abundance profiles for main sequence models from Fig. 1 in the hydrogen profile sequence.

**Figure C2.** Internal abundance profiles for helium burning models with varying metal abundance in the envelope.