Non-abelian discrete gauge symmetries and inflation

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Abstract

Obtaining a potential flat enough to provide slow roll inflation is often difficult when gravitational effects are included. Non-abelian discrete gauge symmetries can guarantee the flatness of the inflaton potential in this case, and also provide special field values where inflation can end.

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1 Introduction

Inflation provides initial conditions in the early universe with attractive and observationally motivated features \[1\]. When inflation is driven by the almost constant potential energy \( V(\phi) \) of a field \( \phi \), the equation of motion for \( \phi(t) \) (\( \phi \) becomes spatially homogeneous quickly after inflation starts) is

\[
\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V(\phi)}{\partial \phi}
\]

where

\[
\left( \frac{\dot{a}}{a} \right)^2 = H^2 = \frac{8\pi G}{3} \rho.
\]

Here \( \rho \) is the energy density, \( a(t) \) the scale factor, and the curvature scale and cosmological constant have been taken to be zero.

Approximately scale invariant perturbations are consistent with observation and can be produced with slow roll inflation\[2\]. Slow roll inflation not only requires the inflationary acceleration of the scale factor (\( \ddot{a} > 0 \) which here becomes \( V \gg \dot{\phi}^2 \)) but also that

\[
\left( \frac{V'}{V} \right)^2 \ll \frac{1}{M_{Pl}^2}
\]

and

\[
\left| \frac{V''}{V} \right| \ll \frac{1}{M_{Pl}^2}.
\]

Here \( M_{Pl} \) is the Planck constant, \( 1/\sqrt{8\pi G} \). Many models of inflation are built ignoring gravitational strength interactions, and so are implicitly setting \( M_{Pl} = \infty \). In this case, satisfying the conditions above is not possible. Keeping \( M_{Pl} \) finite implies one should include gravitational effects. To include gravitational effects, we will work in the context of supergravity. In this case, the first condition suggests we should be near a maximum, or other extremum, of the potential. The second has been found to be non-trivial \[3, 4\]. In supergravity, the potential is composed of two parts, the \( F \)-term and the \( D \)-term. If the inflationary potential energy is dominated by the \( F \)-term then one can show that \[4, 5\]

\[
\frac{V''}{V} = \frac{1}{M_{Pl}^2} + \text{model dependent terms}
\]
Unless the model dependent terms cancel the first term, the second slow roll condition, Eq. (4) above, is violated. Thus to build a model of slow-roll inflation one must be able to control the gravitational strength corrections. There have been various attempts at achieving slow-roll inflation naturally, for a discussion see [6, 7], for a recent review of inflationary models see, e.g. [8].

Discrete non-abelian gauge symmetries provide several ingredients for successful inflation. The symmetries ensure that the potential is flat enough for inflation to occur, and the discreteness of the symmetries provide exit points in the potential where inflation can end. As will be seen below, non-abelian symmetries in particular are useful for hybrid models in low energy effective field theories with supergravity corrections.

2 The idea

In hybrid[9] and mutated hybrid models, one field slowly rolls and a second is pinned until the first field reaches some critical value. E.g., for concreteness and illustration, take the slowly rolling field to be $\Phi$ and call the second field $\Psi$. The hybrid potential for $\Psi$ in the simplest case has the form

$$\left(\alpha |\Phi|^{\beta} - m^2 \right) |\Psi|^2 + O(\Psi^3)$$  \hspace{1cm} (6)

where $m^2$, $\alpha$, $\beta > 0$. The potential for $\Phi$ is not shown, its crucial feature is sufficient flatness to allow for slow roll (the conditions (3,4)). For large $\Phi$, the field $\Psi$ has a large mass and is pinned. During slow roll inflation, $\Phi$ decreases, until near the critical value $\Phi_c^{\beta} \sim m^2/\alpha$ the field $\Psi$ is freed. The rolling of $\Psi$ eventually ends inflation, often quickly after $\Phi$ reaches the critical value. The specific properties of the exit depend on the other terms in the potential as well. (Slow roll conditions for multiple fields are discussed in [11]). Supergravity corrections, as mentioned above, will generically give both $\Phi$ and $\Psi$ masses of order the Hubble constant, making slow roll for $\Phi$ hard to achieve.

This letter describes a mechanism for providing a flat potential when supergravity corrections are included, and providing a way for inflation to end via special exit points. Although hybrid models are discussed here, a variant also works for mutated hybrid models as well. Hybrid and mutated hybrid inflationary models have the advantage that the energy scale of inflation can
sometimes be quite low, which allows for a description in terms of lower energy, and thus often better controlled, theories. (The low energy scale advantage is coupled with the caveat that large fluctuations are likely at the end of inflation in many of these models[12].)

Adding a discrete non-abelian gauged symmetry in the low energy effective field theory, the context for these models, provides for both the flatness and the exit as follows. Instead of taking one field $\Phi$ and one field $\Psi$, take an N-tuple of fields, $\Phi_i$ and $\Psi_i$, with $i = 1,...,N$, and impose a discrete non-abelian gauged symmetry. This symmetry has to be chosen such that the lowest order allowed terms in the superpotential respect a larger continuous symmetry, for example SU(N) in this case. The higher order terms then break this effective continuous symmetry to a discrete subgroup. The other condition on this effective field theory is that the supersymmetry breaking comes from some other sector, and provides a vacuum energy $V_0$. As we are using an effective field theory we do not need to say where this term comes from in particular. The vacuum energy in turn generates supersymmetry breaking masses for the fields $\Phi_i$, $\Psi_i$ as well as other higher order terms.

As the lowest order terms in the superpotential respect the SU(N) symmetry, the mass terms for the fields will be functions of the SU(N) invariants $|\Phi|^2 = \sum_i |\Phi_i|^2$ and $|\Psi|^2 = \sum_i |\Psi_i|^2$. Renormalization group flow[10] will shift the masses $m_{\Phi}^2$ and $m_{\Psi}^2$ to have non-trivial minima which to leading order only depend on the SU(N) symmetric field combination $|\Phi|$ and $|\Psi|$, that is, for example,

$$V(|\Phi_i|) \sim m_{\Phi}^2(|\Phi| - |\Phi_0|)^2 + \ldots ,$$

This consequently provides for flat directions in the potential where inflation can occur. The field $|\Phi|$ will be locked, $|\Phi| \sim |\Phi_0|$ but the various components of $\Phi$ can vary, causing slow roll inflation.

If the SU(N) symmetry were exact, then the corresponding hybrid term above would have to also respect the SU(N) symmetry and be a function of $|\Phi|$, $|\Psi|$ alone, just as above:

$$V(\Phi_i, \Psi_i) \sim (\alpha|\Phi|^\beta - m_{\Psi}^2)|\Psi|^2 + \ldots$$

Although the components of $|\Phi|$, the $\Phi_i$, are free to vary, only their combination appears in the hybrid exit, and as $|\Phi|$ remains locked, the exit cannot take place. On the other hand, if the theory only respects a discrete subgroup of SU(N) this can change. For instance if the theory respects say a
permutation symmetry, the hybrid term

\[ V(\Phi_i, \Psi_i) \sim \sum_i (\alpha_i|\Phi_i|^\beta - m_\Phi^2)|\Psi_i|^2 + h.o.t. \]  

may be allowed. If \( \beta \geq 4 \), this term will be higher order than the couplings which generate the SU(N) symmetric masses, thus allowing the SU(N) symmetry to consistently pin \( |\Phi| \sim |\Phi_0| \). In this case, the individual \( |\Phi_i| \) can control the end of inflation, by decreasing to some critical value, although the sum of the \( |\Phi_i|^2 \) is fixed by the supersymmetry breaking and SU(N) respecting renormalization group effects. In this way, the discrete symmetry allows inflation both to occur (via the continuous effective symmetry) and to end with a hybrid mechanism (via the discrete subgroup). As the norms of the fields, the \( |\Phi_i| \) appear in these hybrid potentials, a non-abelian discrete symmetry is needed. An abelian discrete gauge theory would allow for changes in the phase of \( \Phi_i \), but as the exit depends on the magnitude of \( \Phi_i \) this abelian variation will not trigger the exit. The basic idea of using discrete gauged symmetries generalizes the idea used in Natural Inflation \[13\]. It is also similar to \[14\], where an approximate symmetry is provided by lower terms in the lagrangian (in particular the kinetic terms). Although orbifold constructions, such as \[3\], \[4\], \[15\] also involve a discrete gauge symmetry, the flatness of the potential is obtained differently.

### 3 Example

This basic idea can be used to construct a low energy effective model. One chooses a suitable gauge group and representations, ensuring that the gauge symmetries are not anomalous. The symmetries constrain the allowed terms in the superpotential \( W \) and Kahler potential \( K \). The couplings and supersymmetry breaking terms correspond to regions in parameter space which have constraints both from observation and underlying physics (for example the supersymmetry breaking terms will naturally have coefficients the scale of supersymmetry breaking). For this effective field theory, as mentioned earlier, we assume a hidden sector breaks supersymmetry. This generates supersymmetry breaking terms in the effective potential, including a vacuum energy \( V_0 \) and masses for the scalars. In addition, as also mentioned earlier, the renormalization group running of the supersymmetry breaking
mass term for $\Phi$ to generates a potential for $\Phi$ with non-trivial minimum $|\Phi| = \Phi_0$. The renormalization is induced (to leading order) by low dimension couplings symmetric under the extended continuous symmetry. Thus the renormalization group masses and the potential will be symmetric under the extended continuous symmetry.

A superpotential with a term $\Phi^\beta \Psi^2$, will produce a hybrid potential of the form above, eqn.(6), using the same notation for the scalar component of the superfield and the superfield itself. For example, a model based on a discrete subgroup of SU(2) can have the fields and charges shown in table one. The second discrete symmetry must have $M > 4$, the specific value is

\[
\begin{array}{cccc}
\phi_i & \psi_i & \chi_i & \rho \\
SU(2)_4 \subset SU(2) & 2 & 2 & 2 & 1 \\
Z_4 & 1 & 1 & -2 & 1 \\
Z_M & 1 & -1 & 0 & -1 \\
\end{array}
\]

Table 1: Symmetries and fields.

not important. The generators of the discrete subgroup of SU(2) are taken to be

\[
\begin{pmatrix}
0 & i \\
i & 0
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\begin{pmatrix}
e^{\frac{2\pi i}{N}} & 0 \\
0 & e^{-\frac{2\pi i}{N}}
\end{pmatrix}
\]

(10)

with $N > 4$. The holomorphic terms which respect the symmetries give the superpotential for this model:

\[
W = \lambda_0(\phi_1\chi_2 - \phi_2\chi_1)\rho + \frac{1}{2}\lambda_1(\phi_1^2\psi_2^2 + \phi_2^2\psi_1^2) + \lambda_2\phi_1\phi_2\psi_1\psi_2 + \lambda_3\chi_1^2\chi_2^2 + h.o.t.
\]

(11)

The lowest order terms respect the full SU(2), while the higher order terms only have this symmetry for special values of $\lambda_1, \lambda_2$. The leading neglected higher terms are of the form $\rho\phi\chi^3$, $\phi^2\chi^2\rho^2$ and $\phi^2\psi^2\chi^2$. Calculating the bosonic contribution to the potential (again using the same notation for the
superfields and their bosonic components) from this superpotential gives

\[ V_{\text{susy}} = |\lambda_0|^2(\phi_1\chi_2 - \phi_2\chi_1)^2 + |\phi_1\rho + \frac{\lambda_0}{\lambda_1}\chi_2\chi_1|^2 + | - \phi_2\rho + \frac{\lambda_0}{\lambda_1}\chi_2\chi_1|^2 \]

\[ + |\lambda_0\chi_2\rho + \lambda_1\phi_1\psi_2 + \lambda_2\phi_2\psi_1\psi_2|^2 + | - \lambda_0\chi_1\rho + \lambda_1\phi_2\psi_1^2 + \lambda_2\phi_1\phi_2\psi_1|^2 \]

\[ + |\lambda_1\phi_2^2\psi_1 + \lambda_2\phi_1\phi_2\psi_2|^2 + |\lambda_1\phi_2^2\psi_2 + \lambda_2\phi_1\phi_2\psi_1|^2 + h.o.t. \]

(12)

The supersymmetry breaking terms are

\[ V_{\text{break}} = V_0 + m_2^2|\chi|^2 + \tilde{m}_2^2|\phi|^2 - m_\psi^2|\psi|^2 + m_\rho^2|\rho|^2 + \mu_0(\phi_1\chi_2 - \phi_2\chi_1)\rho \]

\[ + \frac{1}{2}\mu_1(\phi_1^2\psi_2^2 + \phi_2^2\psi_1^2) + \mu_2\phi_1\phi_2\psi_1\psi_2 + h.c. + h.o.t. \]

(13)

Note that for the hybrid exit the fields \( \psi_i \) have a negative mass squared. As mentioned earlier, we take the supersymmetry breaking mass term for \( \phi \) \((|\phi|^2 = |\phi_1|^2 + |\phi_2|^2)\) to be its renormalized form

\[ \tilde{m}^2|\phi|^2 = m_0^2(|\phi| - |\phi_0|)^2 + h.o.t. \]

(14)

The mass term for \( \psi \) can have a similar form, but as it will be considered near the value \( \psi \sim 0 \), this effect is not relevant.

An extremum of the combined potential \( V_{\text{susy}} + V_{\text{break}} \) occurs when

\[ \chi_i = \rho = \psi_i = 0, \quad |\phi| \sim |\phi_0|. \]

(15)

This is the background value around which slow roll inflation is taken to occur. The mass terms for \( \psi_i \) are

\[ -m_\psi^2|\psi|^2 + \frac{1}{2}\mu_1(\phi_1^2\psi_2^2 + \phi_2^2\psi_1^2) + \mu_2\phi_1\phi_2\psi_1\psi_2 + h.c. + |\lambda_1\phi_2^2\psi_1 + \lambda_2\phi_1\phi_2\psi_1|^2 + |\lambda_1\phi_2^2\psi_2 + \lambda_2\phi_1\phi_2\psi_2|^2, \]

(16)

Taking \( \lambda_2 \ll \lambda_1 \) so that the SU(2) is strongly broken, and using that \( \mu_i \) are small, one disentangles that the leading terms in the mass of \( \psi_i \) are proportional to \( |\lambda_1\epsilon_{ij}\phi_j|^2 \). Thus as mentioned earlier \( |\phi| \) can remain fixed while \( |\phi_j| \) rolls, and thus inflation can end.

### 4 Summary

We have shown that non-abelian discrete gauge symmetries can provide for sufficient flatness for inflation to occur and special exit points where inflation
can end. The flatness comes from an approximate continuous symmetry induced by the discrete symmetry, while the special exit points and dynamics are due to the fact that only the discrete symmetry is exact and unbroken. This gives a new way for inflation to happen even when masses from supergravity corrections are taken into account. As a low energy effective field theory is used, one does not need specific information about physics at higher energies, but only the induced field content and symmetries at the scale of interest.

Integrating this mechanism into a more complete model requires several steps, which are outside the scope of this letter. The field content must be anomaly free which may require the addition of more fields. Other considerations will constrain the model’s parameter space and background field values. For instance, the stability of the potential around eq. (15) may require specific values of $\phi_0$ and perhaps couplings. The slow roll behavior of $\phi_i$ must lead $\phi_i$ to the hybrid exit, and will be controlled by higher order corrections in the potential such as those mentioned above, corrections to the Kahler potential and supersymmetric loop corrections. The exit from inflation must not introduce large fluctuations in an observable regime, as mentioned earlier. Constraints from the measured COBE normalization[17] and the scale dependence of the fluctuations (tilt) must also be imposed. These requirements fix combinations of the parameters for the model. In a longer forthcoming paper [7], we present two specific models utilizing discrete non-abelian gauge symmetry as described here. We impose these constraints for a hybrid and mutated hybrid model, including higher order corrections, and calculate detailed properties of the parameter space, the exit from inflation and features of the spectrum.

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