Preparation and detection of a mechanical resonator near the ground state of motion

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Cold, macroscopic mechanical systems are expected to behave contrary to our usual classical understanding of reality; the most striking and counterintuitive predictions involve the existence of states in which the mechanical system is located in two places simultaneously. Various schemes have been proposed to generate and detect such states1–2, and all require starting from mechanical states that are close to the lowest energy eigenstate, the mechanical ground state. Here we report the cooling of the motion of a radio-frequency nanomechanical resonator by parametric coupling to a driven, microwave-frequency superconducting resonator. Starting from a thermal occupation of 480 quanta, we have observed occupation factors as low as 3.8 ± 1.3 and expect the mechanical resonator to be found with probability 0.21 in the quantum ground state of motion. Further cooling is limited by random excitation of the microwave resonator and heating of the dissipative mechanical bath. This level of cooling is expected to make possible a series of fundamental quantum mechanical observations including direct measurement of the Heisenberg uncertainty principle and quantum entanglement with qubits.

Naively treating the motion of a mechanical resonator quantum mechanically produces the elementary result that the energy should be quantized: \( E_n = \hbar \omega_m (n + 1/2) \), where \( n \) is an integer, \( \omega_m \) is the resonant frequency (\( m \) denoting the mechanical resonator) and \( \hbar \) is Planck’s constant divided by 2\( \pi \). In thermal equilibrium, an average occupation factor is expected to follow the Bose–Einstein distribution: 
\[
\bar{n}_m^T = \left( e^{\hbar \omega_m / kT} - 1 \right)^{-1},
\]
where \( T \) and \( k_b \) are the temperature and Boltzmann’s constant, respectively. Cooling a resonator into the quantum regime where \( \bar{n}_m^T \ll 1 \), and measuring the very small motions, has been challenging for a number of technical reasons; not only are very low temperatures necessary to freeze out the mode, but detection with sensitivity at the quantum zero-point level, that is, on length scales \( \sqrt{\hbar / 2m \omega_m} \), is required. Furthermore, this strong position measurement must not heat the mode with measurement back-action3.

Many strategies have been proposed4–8 and applied to realize the quantum regime, with increasing success. Experiments with nanoelectromechanical structures have been able to reach a mechanical occupation of \( \bar{n}_m = 25 \) by passively cooling a nanomechanical resonator4, detected with a superconducting single-electron transistor. (The mechanical occupation is not necessarily in equilibrium with the thermal occupation, \( \bar{n}_m^T \).) Researchers experimenting with optomechanical systems have been able to use extremely sensitive optical detection and radiation pressure to both cool and detect \( \bar{n}_m = 65 \) in a toroidal resonator5, \( \bar{n}_m = 37 \) in microsphere resonator10 and \( \bar{n}_m = 35 \) in an optical cavity11.

The technique we use to both cool and detect the motion of a nanomechanical resonator close to the ground state involves parametrically coupling the motion to a superconducting microwave resonator (SMR)11,12 (Fig. 1). The nanomechanical resonator has a fundamental in-plane flexural resonance of \( \omega_m = 2\pi \times 6.3 \) MHz and is capacitively coupled to a symmetric, two-port, half-wave SMR that resonates at \( \omega_{SMR} = 2\pi \times 7.5 \) GHz. The device is located in a dilution refrigerator and pumped through carefully filtered and cooled leads. The thermal occupation of the SMR, \( \bar{n}_m^{SMR} \), is expected to be 0.09 at 146 mK.

The nanomechanical-resonator damping rate, \( \Gamma_m \), has an unusual linear temperature dependence below 600 mK, reaching a resonator quality factor of \( Q = 10^6 \) at 100 mK. The SMR damping rate, \( \kappa = 2\pi \times 600 \) kHz, is essentially temperature independent below 700 mK and is a factor of 2.4 higher than expected from design owing to internal losses.

The Hamiltonian that describes the coupled resonators is given by7,8
\[
\hat{H} = \hbar \left( \omega_m + \alpha \hat{a}_m + \frac{1}{2} \right) \hat{b}_m^2 + \frac{1}{2} \hbar \omega_m \left( \hat{a}_m^2 + \hat{b}_m^2 \right) + \hbar \omega_{SMR} \left( \hat{a}_{SMR}^2 + \hat{b}_{SMR}^2 \right)
\]
where \( \hat{a}_m \) and \( \hat{a}_m^\dagger \) are respectively the nanomechanical-resonator annihilation and creation operators, and \( \hat{b}_m \) and \( \hat{b}_m^\dagger \) are those of the SMR. The first term shows the ponderomotive-like coupling of the SMR’s frequency to the mechanical motion: \( \hat{x} = x_{SMR} \left( \hat{a}_m + \hat{a}_m^\dagger \right) \) and \( \hat{\dot{x}} = \omega_{SMR} \left( \hat{a}_m - \hat{a}_m^\dagger \right) \) (the effective coupling capacitance and \( C_t = 260 \) fF is the SMR’s total effective capacitance). The term proportional to \( \hat{x}_m \) results from the electrostatic frequency-pulling of the mechanical resonator by the SMR13, where \( \lambda = \left( \omega_{SMR}/2C_t \right) \left( \hat{\phi}_m \hat{\phi}_m^\dagger \right) \), and is responsible for parametric instabilities under certain pump configurations.

When pumping the SMR at \( \omega_p = \omega_{SMR} - \omega_m \) harmonic motion of the nanomechanical resonator preferentially up-converts microwave photons to frequency \( \omega_{SMR} \), extracting one radio-frequency nanomechanical-resonator quantum for each up-converted microwave SMR photon, a process that both damps and cools the nanomechanical resonator’s motion7,8,15–17. This cooling process is analogous to Raman scattering and the process used to cool an atomic ion to the quantum ground state of motion8,18. In the sideband-resolved limit, \( \kappa \ll \omega_m \) the rate of this up-conversion process is given by
\[
\Gamma_{opt} = 4\pi C_t \left( \hat{\phi}_m \hat{\phi}_m^\dagger \right)^2 \frac{\kappa \omega_m}{\sqrt{\kappa^2 + \omega_m^2}}.
\]
From detailed balance, the nanomechanical-resonator occupation factor is expected to follow
\[
\bar{n}_m = \frac{\Gamma_m \bar{n}_m^T + \Gamma_{opt} \bar{n}_{SMR}}{\Gamma_m + \Gamma_{opt}}
\]
where \( \bar{n}_{SMR} = (\kappa/4\omega_m)^2 + \bar{n}_{SMR}^T \left( 1 + 2(k/4\omega_m)^2 \right) \) is the effective occupancy associated with the SMR’s back-action when \( \Gamma_{opt} < \kappa \) (ref. 19). The first term in the expression for \( \bar{n}_{SMR} \) is due to the

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quantum fluctuations of the pump field, and the second term is due to the thermal occupation of the SMR, $n_{\text{SMR}}$. The expressions above show that the minimum mechanical occupation possible is the effective occupation of the SMR.

The first realization of cooling in a parametrically coupled, electromechanical microwave system was with a kilogram-scale gravitational wave transducer\(^{20}\), cooling from $n_m = 10^8$ to $n_m = 10^9$. Cooling of a nanomechanical resonator with an SMR was recently demonstrated\(^{21}\) and achieved cooling from $n_n = 700$ to $n_n = 120$ using a scheme similar to that presented here. Our results are made possible by improvements in device engineering (the coupling strength between the nanomechanical resonator and the SMR, $g$, and the maximum SMR occupation, $n_{\text{SMR}}$), which leads to an improvement in $T_{\text{opt}}$ and resulting cooling rates of two orders of magnitude.

The up-converted noise power is calibrated by applying a weak pump signal ($I_{\text{opt}} < I_m^*)$ and measuring the resulting integrated sideband power, $P_{\text{sum}}$, normalized by the applied microwave pump power, $n_p$, as a function of refrigerator temperature, $T$ (Fig. 1b). For temperatures above $\sim 150 \text{ mK}$, we observe the expected behaviour consistent with equipartition and use this curve to establish the relationship between measured output noise power and $n_m$. For temperatures below $150 \text{ mK}$, we observe fluctuations in $n_m$ apparently due to a non-thermal, intermittent force noise at the level of $10^{-18} \text{ N Hz}^{-1/2}$, which is observed in other similar samples\(^{22}\) and is similar to anomalous heating effects in other systems\(^{23}\). Furthermore, the linear temperature dependence of $I_{\text{opt}}^T$ causes the nanomechanical resonator to decouple from the thermal environment at the lowest measured temperatures. Although the behaviour of the nanomechanics seems consistent with a non-thermal force, we do not currently understand its source.

The measured signal powers are consistent with our knowledge of the attenuation and gain of our measurement circuit and estimates of the device parameters. We find that $g/2\pi = 84 \pm 5 \text{ kHz nm}^{-1}$, which is the largest coupling strength so far demonstrated in a system of this type. From measurements of $\omega_m$ versus $n_p$ and pump frequency, we determine that $\lambda/2\pi = 2.1 \pm 0.7 \text{ kHz nm}^{-2}$.

With the refrigerator stabilized at $T = 146 \text{ mK}$ ($n_m^* = 480$), we measure output noise spectra, $S_n(\omega)$, versus the SMR pump occupation, $n_p$. $S_n(\omega)$ is referred to the oscillator position using the nanomechanical-resonator thermal noise calibration, and is composed of up-converted microwave photons due to $n_m$, SMR noise due to $n_{\text{SMR}}$, and HEMT amplifier noise. We measure $n_{\text{SMR}}$ directly by observing the noise spanning the SMR resonance. Back-action correlations between the nanomechanical-resonator motion and the SMR field are important in our measured noise spectra at the lowest mechanical occupation factors. Fluctuations in the SMR voltage, due to $n_{\text{SMR}}$, together with the pump, produce forces at frequency $\omega_m$. The resulting motion, together with the pump, produces noise at frequency $\omega_{\text{SMR}}$, but $180^\circ$ out of phase with the original SMR fluctuations. This correlation results in an inverted noise peak, similar to noise squashing\(^{24}\), which adds incoherently to the noise power driven by the thermal bath. The resulting observed noise peak or dip, $n_{\text{eff}}$, is calibrated using thermal noise. Our analysis shows that the nanomechanical resonator occupation factor is given by $n_m = n_{\text{eff}} + 2n_{\text{SMR}}$ (Supplementary Information). Figure 2 shows measurements of $S_n(\omega)$ in three cases at low occupation factors: $n_{\text{eff}} > 0$, $n_{\text{eff}} = 0$ and, showing the squashed output noise, $n_{\text{eff}} < 0$.

**Figure 2** | Measured noise spectra. The noise squashing effect on $S_n(\omega)$ due to the finite occupation of the SMR can be seen in three situations: $n_{\text{eff}} > 0$ (top), $n_{\text{eff}} = 0$ (middle) and $n_{\text{eff}} < 0$ (bottom). The red curves show Lorentzian fits through the mechanically up-converted sideband.
Taking the effects of $n_{\text{SMR}}$ into account in this way, the lowest mechanical occupation we have observed is $n_a = 3.8 \pm 1.3$, shown in Fig. 2, with the uncertainty dominated by the uncertainty in $n_{\text{SMR}}$. At this low occupation factor, the resonator is expected to be found in the ground state with probability $P_0 = 1/(n_a + 1) = 0.21$. The cooling power of this refrigeration technique is $Q = \hbar \omega_{\text{opt}} \Gamma_{\text{opt}} = 10^{-2.2} \text{W}$.

We lowered the refrigerator temperature to 20 mK and did not observe a decrease in the minimum $n_a$ value. Using the detailed balance relationship and the measured values of $n_{\text{opt}}, n_{\text{SMR}}$ and $\Gamma_{\text{opt}}$ values, we can compute the bath heating rate, $\dot{n}_T = T_m^{-1} n_{\text{SMR}}$ as a function of $n_a$ (Fig. 3). It is clear that as $n_a$ increases above $3 \times 10^7$, $\dot{n}_T$ begins to increase, nullifying the benefit of starting at low temperatures. This level of heating is consistent with ohmic losses in the metal film on top of the nanomechanical resonator, and the thermal conductance of a normal-state electron gas.

To check the behaviour of our system (nanomechanical resonator and SMR) in a range where $n_{\text{SMR}}$ is not a complicating factor, we applied radio-frequency electrostatic force noise at the nanomechanical-resonator frequency. Starting from $n_a = 2.5 \times 10^8$, we observe cooling, by a factor of 3,000, to $n_a = 80$, which closely follows the expected cooling curve over the full range of $n_a$ (Fig. 4). This suggests that the increase in bath rate at high pump power is due primarily to an increase in bath temperature and not a significant increase in $I_m$. To check the stability of our system, we cycled the device from millikelvin temperatures to 77 K and back. To within a few per cent, we observed no change in the microwave signal levels generated by thermal noise, and no change in $I_{\text{opt}}$ as a function of $n_a$. We also found the out-of-plane mechanical resonance to be 150 kHz lower than the in-plane resonance used in this work. This additional resonance is sufficiently different in frequency that we do not expect any significant interaction.

These measurements identify three effects that work against the cooling process: excess fluctuations of the SMR ($n_{\text{SMR}}$), heating of the nanomechanical resonator thermal bath at high pump powers, and the non-thermal force noise at low temperatures.

We believe that the excess SMR occupation, $n_{\text{SMR}}$, is not a result of phase or amplitude noise of our microwave source: the pump signal is filtered using tunable, copper microwave cavities (one at 300 K ($Q = 9.5 \times 10^3$) and a second at 77 K ($Q = 2.6 \times 10^5$)) achieving a microwave power measured 6.3 MHz from, and relative to $\delta_0$, $c_0$, of less than $\sim 195 \text{ dB, Hz}^{-1}$ (where dB denotes units of noise power measured relative to the pump power) and contributing less than 0.04 photons into the SMR at our highest value of $n_a$. Without these cavities, the SMR would be excited to $n_{\text{SMR}} = 35$. We also believe that this excess SMR occupation is not due to ohmic heating of, and resulting thermal radiation from, the cryogenic attenuator network because $n_{\text{SMR}}$ increases only weakly over a wide range of $n_a$ values. Tests of Nb SMR devices at 1.2 K before the surface micromachining of the nanomechanical resonator do not show excess dissipation and suggest that the excess losses are related to our fabrication process.

Increasing $I_{\text{opt}}$ by engineering a larger coupling strength, $g$, and/or a smaller $\kappa$ value should be very beneficial as it will lead to higher cooling rates at lower pump powers, minimizing the effect of excess bath heating, $\dot{n}_T$. By increasing $I_{\text{opt}}$ by a factor of ten, maintaining the same value of $\kappa$ and $\dot{n}_T$, we expect to observe $n_a \approx 0.5$ with $P_0 = 0.67$. This approach will be limited when $I_{\text{opt}}$ becomes comparable to $\kappa$, which constrains the rate of cooling $g^3 / (2 \pi)^2$.

The deep quantum limit, $n_a \ll 1$, will be accessible when it is possible to reach lower refrigerator temperatures and lower mechanical
damping rates at these temperatures. Understanding and eliminating the excess bath heating and the non-thermal force noise will be required, although it should be pointed out that, even without these improvements, the device described here should achieve $n_{\text{th}} < 0.5$ if excess SMR occupation is reduced. Furthermore, superconducting metals on the nanomechanical resonator also appear to be required owing to the expected mechanical force noise from transport and electron momentum scattering in diffusive conductors. For a normal-state conductor, each electron scattering event imparts a momentum change on the order of the Fermi momentum. The dif-fusing scattering of current through the beam directly produces force noise on the nanomechanical resonator. We estimate that this heating mechanism will result in a limit of $n_{\text{th}} > 3$ at $n_p = 3 \times 10^9$, assuming our current device parameters and a resistance of $100\Omega$ through the nanomechanical resonator.

These measurements show that detection with sensitivity to resolve motions approaching the ground state is possible with existing HEMT-based amplifiers. Eliminating internal SMR losses, unbalancing the SMR couplings and using improved microwave amplifiers would significantly reduce the measurement time.

Nonetheless, the production and detection of a nanomechanical resonator with $n_{\text{th}} = 3.8$ is sufficient to allow future experiments. Owing to uncertainty-principle fluctuations of the mechanical motion, and resulting spontaneous emission, the rate of micro-wave-photon up-conversion is expected to differ from the rate of down-conversion. This difference can be used as a fundamental thermometry technique, and would allow the quantitative measurement of the zero-point motion of a mechanical structure.

This level of cooling is essential for the formation of entangled states between superconducting quantum bits and the motion of a nanomechanical device. Proposed schemes allow the generation and detection of nanomechanical resonator/qubit entanglement using a Jaynes–Cummings-type interaction with the mechanical resonator at thermal occupations of the level shown here. Similar to procedures in atomic physics, such an experiment would involve preparing the cold state of the mechanical device and, after the refrigeration is complete, turning the cooling off. The state of the cold beam could then be manipulated before thermalization of the motion. In our implementation of this process, we expect cooling from $n_{\text{th}} = 500$ to $n_{\text{th}} = 4$ in $\sim 200\mu$s, and we expect one thermal quantum to enter the resonator in $t = 1/\hbar \gamma = 2\mu$s, which exceeds superconducting qubit manipulation times and is comparable to qubit measurement and relaxation times.

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**Supplementary Information** is linked to the online version of the paper at www.nature.com/nature.

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