ACCRETION DISKS AND THE LYMAN CONTINUUM POLARIZATION OF QSOS

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ABSTRACT

_Hubble Space Telescope_ observations of some QSOs show a strong, abrupt increase in polarization at rest wavelength about 750 Å. The closeness of the polarization rise to the H Ⅰ Lyman edge suggests a connection, but the displacement to shorter wavelengths and the shape of the polarization rise require explanation.

We have computed the polarized spectrum of a thermally emitting accretion disk around a supermassive black hole, including the effects of the relativistic transfer function. The local stellar atmosphere spectra show a blueshifted polarization rise in the Lyman continuum, as found by Blaes & Agol in 1996. However, the relativistic transfer function adds an additional blueshift of sufficient magnitude that the model cannot explain the observations.

We show that a good fit results if the emitted radiation is assumed to have a sharp increase in polarized flux at the Lyman edge in the rest frame of the orbiting gas. Relativistic effects then cause the observed polarization to rise sharply at a wavelength substantially less than 912 Å. The blueshift depends on the angular momentum of the black hole and the inclination of the disk. A good fit to PG 1630+377 results from a simple model with a dimensionless angular momentum \( a_\ast \equiv cJ/M^2 = 0.5 \), and an observer viewing angle \( \mu_\ast \equiv \cos \theta_\ast = 0.1 \). Smaller values of \( a_\ast \) give insufficient blueshifts, and values close to \( a_\ast = 0.9982 \) require unrealistically large polarizations in the rest frame of the gas. An intermediate value of \( a_\ast \) might result from coalescence of black holes, successive accretion events, or electromagnetic extraction of angular momentum from the hole.

Subject headings: accretion, accretion disks — black hole physics — galaxies: active — polarization — quasars: general

1. INTRODUCTION

The leading model of energy production in QSOs involves an accretion disk orbiting a supermassive black hole (Blandford 1990). Evidence for black holes in the nuclei of nearby galaxies is increasing (Rees 1997), but proof of accretion disks in QSOs has been elusive.

The expected disk effective temperatures imply disk emission at wavelengths in rough agreement with the Big Blue Bump observed in energy distributions of QSOs (Shields 1978; Malkan 1983; Czerny & Elvis 1987). Accretion disk fits to the energy distributions of a number of QSOs have been quite successful (e.g., Sun & Malkan 1989; Webb et al. 1993). These models indicate masses \( M_H \) of over \( 10^9 M_\odot \) and accretion rates \( M \) above \( 1 M_\odot \) yr\(^{-1} \) for luminous QSOs. Lyman edges in the total flux should be inconspicuous because of relativistic Doppler broadening (Laor & Netzer 1989, hereafter LN; Laor 1991) and non-LTE (NLTE) effects (Coleman 1993; Shields & Coleman 1994, hereafter SC; Hubeny & Hubeny 1997). The Lyman edge of H Ⅰ will be most conspicuous in relatively cool disks (\( T_{\text{max}} \approx 25,000 \) K), but will appear as a slope change over the wavelength range \( \sim 1100 \) to 700 Å (SC) rather than as an abrupt discontinuity.

Models of accretion disks predict substantial electron-scattering opacity at optical and ultraviolet (UV) wavelengths, leading to polarization of the emitted light (see § 2.1). In contrast, observed polarizations in the optical and near UV are typically \( \sim 1\% \) or less (Stockman, Moore, & Angel 1984; Webb et al. 1993). Moreover, in radio-loud objects, where a radio extension indicates the position angle of the disk axis, the observed polarization tends to be parallel to the disk axis (Berriman et al. 1990). Electron scattering in a geometrically thin disk would produce polarization perpendicular to the axis. Predicted polarizations can be reduced by absorption opacity, Faraday rotation (Agol & Blaes 1996), general relativistic depolarization (Laor, Netzer, & Piran 1990, hereafter LNP), and disk-surface structure (Coleman & Shields 1991), but some concern still remains (Antonucci et al. 1996).

LNP computed the expected energy distribution and polarization of QSO disks with a simple model of the atmosphere and with full consideration of the general relativistic transfer function. They found that while the Lyman edge in the energy distribution is inconspicuous in general, a substantial drop in polarization in the H Ⅰ Lyman continuum should result from the increase in absorption opacity relative to scattering. Several groups undertook spectro-polarimetry with the _Hubble Space Telescope_ (HST) Faint Object Spectrograph (FOS), targeting “Lyman-edge candidates” for which _International Ultraviolet Explorer_ (IUE) spectra suggested an intrinsic drop in the flux around the Lyman edge (e.g., Koratkar, Kinney, & Bohlin 1992). Rather than finding a drop in polarization in the Lyman continuum, these observations revealed an abrupt rise in polarization at rest wavelength \( \sim 750 \) Å in several objects (Impey et al. 1995; Koratkar et al. 1995). The polarization increases by a large factor, from \( \lesssim 1\% \) at \( \lambda \gtrsim 1000 \) Å to \( \sim 5\% \) in PG 1222+228 at \( \lambda \sim 600 \) Å and \( \sim 20\% \) in PG 1630+377. The proximity of the polarization rise to the
Lyman edge is suggestive (Impey et al. 1995).

An intriguing explanation of the displaced polarization rise was offered by Blaes & Agol (1996, hereafter BA). Using LTE and NLTE models for the disk atmosphere with a hydrostatic vertical structure, they found that, for $T_{\text{eff}} \approx 20,000$ K, a similarly blueshifted, steep polarization rise resulted from a combination of effects. The polarization drops at the Lyman edge because of the increased absorption opacity, but then it increases with decreasing wavelength because of the $\lambda^{-3}$ dependence of the H i photoionization opacity and the increasingly steep source-function gradient at higher frequencies. A strong source-function gradient leads to severe limb darkening, which is needed for strong polarization (Agol, Blaes, & Ionescu-Zanetti 1998; Cheng et al. 1988). A model for a disk around a nonrotating black hole gave a promising fit to PG 1222 + 228, but polarization as strong as that observed for PG 1630 + 377 could not be achieved. However, BA did not include the relativistic transfer function, and it was unclear whether these effects would upset the fit to PG 1222 + 228.

We report here results of a theoretical study of polarization in QSO accretion disks. We focus on the steep polarization rises observed in PG 1222 + 228 and PG 1630 + 377. In § 2 we describe our stellar atmosphere models and the treatment of the relativistic transfer function. Then we describe attempts to fit the two QSOs with accretion disk atmospheres and show that general relativistic effects preclude an acceptable fit. In § 3 we show that a good fit can be achieved in a simple model with a sharp polarization jump at the Lyman edge in the rest frame of the gas, and we consider possible sources for this emission. Our conclusions are discussed in § 4.

2. ACCRETION DISK MODEL ATMOSPHERES

2.1. Computational Methods

We have calculated LTE stellar atmospheres for conditions appropriate for QSO disks. These models were supplemented by some “mixed” LTE-NLTE calculations described below. Because we are interested only in the continuum, and because a perfect description of the vertical structure of AGN disks is not available, we used simple methods to model the atmosphere. These resemble the LTE calculations of Coleman (1993), but the codes used here were written largely independently. The physics included is similar to the treatment of BA, but the computational methods are quite different.

Atmospheres were computed for a set of radii and for corresponding values of effective temperature $T_{\text{eff}}$ and vertical acceleration of gravity $g_z$, evaluated for a given $M_{\text{BH}}$, $M$, and angular momentum, $a_*=cJ/GM^2$, from the relativistic relations given by Novikov & Thorne (1973, Novikov & Thorne 1973, hereafter NT). The atmosphere was computed for a set of points (typically 91) equally spaced in the logarithm of the column density ($g$ cm$^{-2}$), ranging from log $\Sigma(z) = -6.5$ to log $\Sigma(z) = 2.5$. The hydrostatic density profile was integrated from the top down, according to the equation

$$\frac{dP_g}{d\Sigma} = g_{\text{eff}} = g_z - g_{\text{rad}},$$

where $P_g$ is the gas pressure, and $g_{\text{rad}} = \kappa_F F_\text{iso}/c$ is the radiative acceleration. Here $\kappa_F = \int_0^\infty F_\nu \kappa_\nu dv/\int_0^\infty F_\nu dv$ is the flux-weighted mean opacity per unit mass (Mihalas 1978).

Radiative transfer in the atmosphere was calculated by performing a downward integration of the equations

$$\frac{dK_\gamma}{d\tau} = H_\gamma,$$

and

$$\frac{dH_\nu}{d\tau} = \epsilon_\nu (J_\nu - B_\nu),$$

where $J_\nu$, $H_\nu$, and $K_\gamma$ are the usual Eddington moments of the radiation field ($I_\nu(\tau_g, \mu)$; $\tau_g$ is the monochromatic optical depth from the top of the atmosphere; $\epsilon_\nu$ is the total absorption and scattering opacities per unit mass, respectively. The emergent flux $H_\nu$ was adjusted in a shooting method to satisfy a lower boundary condition $J_\nu = B_\nu$ at a fiducial optical depth, typically $\tau = 8$. The first iteration used a gray-atmosphere temperature profile, the Eddington approximation ($J_\nu = 3K_\nu$), and a surface boundary condition $J_\nu = B_\nu$ at a fiducial optical depth, typically $\tau = 8$. Subsequent iterations used variable Eddington factors and a temperature profile corrected by the Unsold-Lucy method to achieve flux conservation (Mihalas 1978). We assumed constant flux, i.e., no local heating (see below). The resulting $J_\nu (\tau_g)$ was used to compute the source function $S_\nu (\tau_g)$, and new values of $J_\nu$, $K_\gamma$, and $H_\nu$ were then computed at each depth point from the usual exponential integral expressions (Mihalas 1978). These gave new values for the Eddington factors, the quantities needed for the temperature correction, and $\kappa_F$. The entire procedure (hydrostatic integration, transfer integration, moments, temperature correction) was iterated as necessary, typically three iterations. Opacity sources included were electron scattering, free-free absorption, and bound-free absorption from the lowest seven levels of H i and the ground states of He i and He ii. Ionization and H i level populations were computed from the Boltzmann-Saha equation (LTE). Convergence was good except in a thin surface layer for certain ranges of $T_{\text{eff}}$, for which H or He was partially ionized at the surface. The flux in the related Lyman continuum of H or He was negligibly small in these cases, and the fluxes at longer wavelengths were not significantly affected.

The assumption of constant flux through the atmosphere should be adequate in our case (see also BA). We describe below models using the same parameters, ($M_{\text{BH}}$, $M$, $a$), as BA. The region of the disk emitting the Lyman continuum is characterized by the radius $9.5GM/c^2$ giving maximum $T_{\text{eff}} (25,500$ K). In the vertically averaged model of NT with $a = 0.1$, the surface density of the disk (midplane to surface) is $\Sigma_c = 2000$ g cm$^{-2}$. The surface density above the point where $\tau_c = 1$ at $\lambda = 911$ Å is $\Sigma_1 = 0.8$ g cm$^{-2}$. If energy is dissipated along the vertical extent of the disk roughly in proportion to density, then only a small fraction of the flux is dissipated in the photosphere. Also, we find that, at $\tau_c = 1$, the dissipation rate per unit mass is tiny compared to the radiative heating and cooling rates in the atmosphere. These conclusions are even stronger if dissipation is proportional to gas pressure or to total pressure. Moreover, our conclusions depend upon more relativistic effects than atmospheric details.

Polarization was calculated using a straightforward iterative procedure by Voigt (1951), which was applied to the final converged atmosphere. Although Voigt studied cases with small scattering opacity fractions, we found that convergence was rapid in all cases. The exact polarization exceeds that of the first iteration by almost a factor of 2 for wavelengths at which electron scattering strongly domi-
nated the opacity. This results from multiply scattered photons that produce substantial “prepolarization” of the radiation going into the last scattering.

CS and others have emphasized that the effective gravity $g_{\text{eff}}$ in AGN disks will be very low because of close cancellation of $g_r$ and $g_{\text{int}}$. This leads to a large gas-pressure scale height, a low density in the photosphere, high ionization, and a large opacity contribution from electron scattering. Coleman (1993), Hubeny & Hubeny (1997), and others have shown that significant differences in emitted spectrum can result from use of a fully self-consistent vertical structure rather than an approximate, constant gravity solution. However, our results show that, for the polarization phenomena of primary concern here, the details of the local atmospheric emission are less important than the effect of the relativistic transfer function. Therefore, we followed the procedure of BA by using a value of $g_r$ that was 20% larger than the Eddington limit for a given $T_{\text{eff}}$. This is about the minimum $g_r$ that allowed convergence for most $T_{\text{eff}}$. For these purposes, we took $g_{\text{int}} = 1.1$, $\pi F/c$, where $\kappa_{\text{sc}}$ is the electron-scattering opacity for fully ionized pure hydrogen. (Reduction of $g_r$ to 1.1$g_{\text{int}}$, in cases where the atmosphere still converged, gave considerably lower values of $g_{\text{eff}}$, but the emergent spectrum and polarization changed only modestly.)

Sensitivity of our results to NLTE effects was evaluated in several sample cases by using our LTE atmospheric structure, $\rho(\Sigma)$ and $T(\Sigma)$, as input to an NLTE stellar atmosphere program developed by the Munich University Observatory stellar atmosphere group. This code, by using modern methods that includes accelerated lambda iteration (ALI), includes bound-bound and bound-free transitions of many levels of H and He. When set to run in LTE mode, this code gave emergent spectra in excellent agreement with our LTE code. In NLTE mode, the results showed the expected reduction of the Lyman absorption edge for relatively cool atmospheres (see Coleman 1993; Hubeny & Hubeny 1997). However, the polarization as a function of wavelength was not greatly different between the LTE and NLTE cases, in agreement with BA.

Following BA, we cut off the disk at 50$R_*$, where $R_*$ is the gravitational radius. For Schwarzschild black holes, we computed atmospheres at $r_\text{gr} \equiv R/R_g = 7.76, 9.52, 15.3, 20.3, 31, 44,$ and 50. Test cases with additional radii gave similar results. Summation over disk radii and application of the relativistic transfer function, which accounts for Doppler shifts, transverse and gravitational redshifts, and gravitational deflection of light rays, was accomplished with a modified version of the computer codes used by LNP and kindly made available by T. Piran and A. Laor. These codes use a set of 51 radii from $r_\text{gr} = 1.2346$ to $r_\text{gr} = 400$ to cover disks around either rotating or nonrotating black holes. We typically used a set of 80 frequencies uniformly spaced in log $\nu$ between 1.96 x 10$^{14}$ and 1.85 x 10$^{16}$ Hz.

The LNP transfer function assumes limb-darkening laws in the intensity and polarization for pure electron scattering (Chandrasekhar 1960), although there is provision for a scale factor in the polarization. The actual limb darkening in the polarized intensity can be quite different (see Cheng et al. 1988; Agol, Blaes, & Ionescu-Zanetti 1998, and references therein). In order to accommodate a general limb-darkening law with minimum changes to the LNP codes, we used a simple fitting procedure. The polarization intensity from a given radius and frequency can be described in terms of the orthogonal polarized intensity components $I_{\lambda}(\mu)$ and $I_{\lambda}(\mu)$. Each of these was fitted with a six-term series in integer powers of $1 - \mu$ from $n = 0$ to $n = 5$, which gave an excellent fit in all cases. Heuristically, this may be thought of as a set of 12 accretion disks, each emitting at its surface either pure $I_0$ or pure $I_\nu$, with a limb darkening $(1 - \mu)$. The corresponding 12 transfer functions were computed with the LNP code and applied to the respective fitting components, and the resulting Stokes parameters at the observer were summed to produce the observed spectrum. Our atmosphere results were interpolated to the LNP radius grid and the adopted frequency grid by using a linear interpolation in $R$ and $\nu$ for (1) the brightness temperature $T_B$ defined by $F_B = B_B(T_B)$ and (2) $I_{\lambda}(\mu)/I_{\lambda}(0)$ and $I_{\lambda}(\mu)/I_{\lambda}(0)$ at five values of $\mu$ (prior to determining the fitting coefficients). Test cases gave good agreement with a disk emitting with the electron-scattering darkening law and with results from an independent transfer function code by Agol (1997a; 1997b, private communication).

The emergent energy distribution and polarization for an atmosphere with $T_{\text{eff}} = 25,500$ and $g_{\text{int}} = 0.0$ are shown in Figure 1. The polarization rises in the Lyman edges of $H_\alpha$ and $H_\beta$ are evident, similar in the case of $H_\gamma$ to the results of BA (who did not include He). Note also the substantial polarization at optical wavelengths (see also Fig. 3 of BA). The low effective gravity leads to a low density at the optical photosphere, and electron scattering contributes substantially to the opacity ($\sim 75\%$ at $\tau_\lambda = 1$ for $\lambda = 5000$ Å). Some earlier models (e.g., LNP) used the midplane density throughout the atmosphere. The predicted optical polarizations may be a problem for disk models, given the weak observed optical polarizations for radio-quiet QSOs (e.g., Berriman et al. 1990).

2.2. Results

We have computed models for PG 1222+228 and PG 1630+377, for which the most dramatic Lyman continuum polarization rises have been reported. For PG 1222+228, following BA and Webb et al. (1993), we have used a nonrotating black hole with $M_0 \equiv M/10^5 M_\odot = 5.3$ and $M_0 \equiv M/10^3 M_\odot \text{ yr}^{-1} = 18.9$. (Actually, only the quantity

![Fig. 1](image-url)
weaker polarization; the same model for is shown for viewing angle (Larger gives resulting energy distribution is shown in for observer-Figure 2.)

This, along with determines the run of with radius, \(M\) and is determined by in our models. The overall 

\[ \frac{\dot{M}}{M_*^2} \]

matters for the shape of the energy distribution. This, along with \(a_*\), determines the run of \(T_{\text{eff}}\) with radius, and \(g_z\) is determined by \(T_{\text{eff}}\) in our models. The overall luminosity can be scaled to fit the observations by scaling \(M\). The same is true for the “toy” models of § 3. The resulting energy distribution is shown in Figure 2 for observer-viewing angle \(\mu = \cos \theta_0 = 0.35\). (Larger \(\mu\)'s give weaker polarization; the same model for \(\mu = 0.1\) is shown in Fig. 3.) The energy distribution is in fairly good agreement with the observations, as found by BA and Webb et al. (1993). However, the polarization rise is more blueshifted and more gradual than that observed. This differs from the results of BA, who did not include the relativistic transfer function (but who warned that it could modify their conclusions). Note that for \(\mu = 0.35\), more edge-on than BA’s model, the polarization still reaches the observed \(\sim 5\%\) but at a wavelength shorter than observed. The transfer function diminishes the peak polarization and adds its own blueshift to that already present in the stellar-atmosphere spectra, causing the total blueshift to exceed that observed.

Figure 3 shows the spectrum of the same disk, viewed at a more edge-on angle (\(\mu = 0.1\)) and scaled to the observed energy distribution of PG 1630 + 377. The energy distribution fits fairly well. However, even for this nearly edge-on viewing angle, the polarization does not approach the value \(\sim 20\%\) observed, and the polarization rise is again too blue-shifted in the model.

Another difficulty with the BA model may be the small ionizing luminosity from such a cool disk. The expected H\(\beta\) equivalent width for our model is only \(~ 25\ \AA\) for \(\mu = 0.35\), if the line-emitting gas absorbs all the ionizing photons. Observed equivalent widths are somewhat larger, and the covering factor for the emitting gas may be small. Moreover, the disk’s He I and He II ionizing luminosities are negligible. However, a “nonthermal” ionizing source may also be present.

We conclude that the stellar atmosphere phenomenon suggested by BA to explain the observed polarization rises fails on account of the relativistic transfer function. The actual stellar atmosphere effect is nevertheless present, and it may contribute at some level to the observed spectrum of AGN disks.

3. A POLARIZATION JUMP “AT THE EDGE”

The preceding results show that the relativistic transfer function causes a substantial blueshift of any polarization rise in the disk’s local emission. This raises the question, what will be observed if the disk gas emits a continuum with an abrupt increase in polarization right at the Lyman edge in the rest frame of the gas? We have calculated a simple model in which the disk surface, at any radius, emits a continuum. The polarization was assumed to rise from effectively zero at an arbitrary multiple of \(\mu = 0.1\) to an arbitrary multiple of \(\mu = 0.35\), simulating an atmosphere whose brightness temperature drops to the boundary temperature in the Lyman continuum. The polarization was assumed to rise from effectively zero at \(v < v_H\) to an arbitrary multiple of \(p_{\text{es}}(\mu)\) at \(v \geq v_H\), where \(p_{\text{es}}\) refers to the polarization of electron-scattering atmosphere (Chandrasekhar 1960). To this end, we define \(a_\mu\) by \(p(\mu) = a_\mu p_{\text{es}}(\mu)\).

Figure 4 shows the resulting fit to PG 1630 + 377 for several values of the black hole angular momentum and \(\mu = 0.1\). For each value of \(a_\mu\), the values of \(M_0\) and \(M_\infty\) were adjusted to fit the energy distribution, and \(a_\mu\) was adjusted to fit the maximum polarization observed. Figure 4 shows that the observed polarization rise remains quite sharp and is displaced to the blue by an amount that depends on \(a_\mu\). For larger \(a_\mu\), the disk’s inner boundary at the marginally stable radius shifts inward, from \(r_\mu = 6\) for \(a_\mu = 0\) to \(r_\mu = 1.22\) for \(a_\mu = 0.9982\) (Page & Thorne 1974). For larger \(a_\mu\), the Lyman continuum comes from smaller radii, where the relativistic effects are stronger. The wavelength and shape of the polarization rise agree with the observations for \(a_\mu = 0.5\). Evidently, this simple model gives a good fit to the observed energy distribution and to the shape of the rise in polarization.
Why does the polarization remain low for a considerable wavelength interval to the blue of the Lyman edge, and then rise abruptly? The relativistic transfer function causes the approaching, blueshifted part of the disk to appear brighter than the receding portion. In addition, light from the sideways moving portion of the disk behind the black hole is bent as it passes near the hole on its way to the observer. Even for a fairly edge-on viewing angle, this light leaves the atmosphere fairly close to the normal, so that its intensity is enhanced by limb darkening, the projected solid angle subtended by unit surface area is increased, and its polarization leaving the disk surface is low (see Agol 1997a). This light also suffers depolarization from relativistic rotation of the polarization plane, which varies rapidly with position on the part of the disk behind the black hole (Connors, Piran, & Stark 1980; Agol 1997a). The light from the part of the disk in front of the hole is more strongly polarized but less intense. Thus, the observed light at small Doppler shifts is dominated by the back part of the disk and is weakly polarized. At larger blueshifts, the light is dominated by the approaching part of the disk. This light is strongly polarized because it suffers less gravitational deflection and therefore emerges from the atmosphere at a large angle to the normal. These points are illustrated in Figure 5, which shows the received flux and polarization for an annulus that emits at a single radius $r_*=10.52$ and wavelength $\lambda = 912$ Å. The received emission from the near and far sides of the ring are shown separately. The flux is much stronger at the blue end than at the red end of the observed range of wavelength shifts because of Doppler boosting. Intense but weakly polarized flux is received from the back side of the disk with relatively little shift from the emitted wavelength. Thus, the Lyman edge in the total flux will ramp up or down progressively from the red to the blue side of the nominal wavelength, whereas the polarization will rise only at shorter wavelengths.

The blueshift of the polarization rise in the toy model depends on $\mu_*$ as well as $a_*$. Figure 6 shows results for a maximally rotating hole with $a_*=0.9982$. Again, $\dot{M}$ and $M_\text{f}$ are adjusted in each case to fit the energy distribution. The blueshift of the polarization rise increases with decreasing $\mu_*$, and we estimate by inspection a best fit $\mu_* \approx 0.33$. However, in this case, large values of the intrinsic polarization $a_p \approx 19$ are required to match the observed degree of polarization of PG 1630+377. This corresponds to $p(\mu) = 76\%$ at $\mu = 0.33$ and $p(\mu) > 100\%$ at $\mu < 0.2$. The unknown mechanism for producing the postulated Lyman-
continuum polarization could have a limb-darkening law quite different from $p_\lambda (\mu)$, but polarization of roughly 76% at $\mu \approx 0.33$ will likely be required in any case and may be difficult to achieve. If one rejects such a large intrinsic polarization, then $a_*$ must be less than 0.9982 in the context of this toy model.

The cases $\mu_0 = 0.25$ and 0.50 are shown in Figures 7 and 8. For $\mu_0 = 0.25$, the blueshift is best-fitted for $a_* \approx 0.7$. For $\mu_0 = 0.5$, the blueshift depends little on $a_*$ (even for $a_* = 0.9982$; see Fig. 6), and it is too small to fit PG 1630 + 377. Figures 4 and 6–8 show that the wavelength of the polarization rise provides a constraint in the $(a_*, \mu)$-plane. Interpolating, we find $\mu_0 = (0.10, 0.25, 0.30)$ for $a_* = (0.50, 0.75, 0.90)$. From the fit to the energy distribution, we find $M_0 = (27, 25, 20)$, $M_\alpha = (5.0, 9.0, 15)$, and $L/L_E = (0.22, 0.14, 0.099)$, respectively. Note that $M$ decreases with increasing

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**Fig. 6.—** Toy model results for $a_* = 0.9982$ for several viewing angles, compared with observations of PG 1630 + 377. The models have $p/p_\lambda = (9.0, 14.9, 30.8)$ for $\mu_0 = (0.1, 0.25, 0.5)$. 

**Fig. 7.—** Toy model results for $\mu_0 = 0.25$ and several values of $a_*$, compared with observations of PG 1630 + 377. The models have $p/p_\lambda = (6.4, 7.1, 9.6)$ for $a_* = (0.0, 0.5, 0.9)$. 

$a_*$ because of the increasing efficiency of energy production. Also, $M/M_\text{H}$ decreases with increasing $a_*$ because of stronger Doppler boosting (requiring lower $T_{\text{eff}}$) and a smaller radiating area in units of $(GM/c^2)^2$, which gives higher $T_{\text{eff}}$ for a given $M/M_\text{H}$. All of the values are comfortably below the Eddington limit, consistent with a thin disk.

This toy model was also used to fit PG 1222+228. The blueshift of the observed polarization rise is somewhat less than for PG 1630+377 (Impey et al. 1995; Koratkar et al. 1995), and the rise is possibly more abrupt, but the uncertainties are large. Figure 9 shows results for $\mu_*=0.25$. The fit for $a_*=0.5$ is fairly good, requiring $a_p=1.7$, $M_0=43$, $M_*=6.2$, and $L/L_E=0.26$. Reference to Figure 7 suggests that $a_*=0.5$ would fit PG 1222+228 fairly well for appropriate $a_p$, independent of $a_*$, although the blueshift may be slightly less than observed.

Fig. 8.—Toy model results for $\mu_*=0.5$ and several values of $a_*$, compared with observations of PG 1630+377. The models have $p/p_*=(14.0, 17.5, 24.0)$ for $a_*(0.0, 0.5, 0.9)$.

Fig. 9.—Toy model results for $\mu_*=0.25$ and several values of $a_*$, compared with observations of PG 1222+228. Values of $a_p$ are one-fourth as large as for the corresponding models applied to PG 1630+377 (Fig. 7).
Figures 6–8 indicate that the wavelength at which the polarization rises to one-half of its full value, $\lambda_{1/2}$, will be between about 640 and 830 Å, at least for inclinations $\mu_{0} \leq 0.5$, which are most likely to give strong polarization. This is consistent with the two objects discussed above and with PG 1338 + 416 (Koratkar et al. 1995).

The observed degree of polarization for the toy model scales with $a_{\ast}$ and results can be derived from the figures. For example, if we fix $a_{\ast} = 0.5$ and $a_{\ast} = 4.4$ (above discussion of PG 1630 + 377), we may ask what will be the plateau value of the polarization $p_{\text{max}}$ for this disk viewed at various angles. From Figures 4, 7, and 8, we find $p_{\text{max}} = (20\%, 13\%, 5.0\%)$ and $\lambda_{1/2} = (750, 770, 810)$ Å for $\mu_{0} = (0.1, 0.25, 0.5)$.

### 4. DISCUSSION

We have shown that the relativistic transfer function precludes an explanation of the observed polarization rises in PG 1222 + 228 and PG 1630 + 377 in terms of the stellar atmosphere effect proposed by Blaes & Agol (1996). Relativistic effects add an additional blueshift to the polarization rise that makes the predicted rise occur at wavelengths shorter than observed. The model also cannot explain the strong degree of polarization observed in PG 1630 + 377.

As a step toward a more successful model, we have shown that the observed polarization rises are consistent with a simple model in which the polarization rises abruptly at $\lambda_{912}$ in the rest frame of the orbiting gas. The observed polarization rise is blueshifted by an amount that depends on $a_{\ast}$ and $\mu_{0}$. For PG 1630 + 377, the observations are consistent with $a_{\ast}$ of about 0.5 or greater but probably less than 0.9982.

What could be the origin of the polarized Lyman emission? In PG 1630 + 377, the polarized flux $F_{p_{\ast}} = p_{\ast} F_{\ast}$ rises steeply with decreasing wavelength (Koratkar et al. 1995). An abrupt increase in polarized flux at the Lyman edge in the rest frame of the gas suggests free-bound emission from ionized hydrogen that undergoes electron scattering, either in the emitting gas or in a separate scattering location (see Impey et al. 1995; Koratkar et al. 1995). In view of the evidence of irradiation of AGN disks by a hard incident continuum, at least in lower luminosity objects (e.g., Clavel et al. 1992; Mushotzky, Done, & Pounds 1993 and references therein), one naturally thinks of a photoionized layer on the disk surface (e.g., Sincell & Krolik 1997). A simple picture would be a uniform slab of ionized gas in which recombination of H I produces a fairly uniform emissivity in Lyman continuum photons that scatter off electrons in the slab on their way out of the slab. Such a surface layer, overlying a photosphere with a Lyman edge in absorption in its total flux, might resemble our toy model. Results by Phillips & Mészáros (1986) show polarizations of 15% or more for $\mu \approx 0.1$ in a scattering slab with uniformly distributed emitters and the possibility of even larger polarizations for emitters concentrated near the surface. The strongest polarization occurs for $\tau_{\text{es}} \approx 0.2$–0.3. However, it is not clear from their results that strong enough polarization for PG 1630 + 377 will be possible, and the limb-darkening law is very different from $P_{\text{es}}(\mu)$.

The temperature of such a slab can be constrained by the fact that the polarized flux in PG 1630 + 377 jumps by an order of magnitude across the polarization rise. If the polarized flux has the same energy distribution as the radiation feeding into the scattering process, then the emissivity of the gas must have a similar contrast. The emissivity ratio $f_{\ast}(\lambda_{912} - )/f_{\ast}(\lambda_{912} + )$ decreases with increasing temperature because of free-free and Balmer continuum emission. The observed polarized flux requires $T \leq 80,000$ K for PG 1630 + 377, based on expressions by Brown & Mathews (1970).

A layer that is mechanically heated rather than photoionized may be an alternative source of the polarized Lyman continuum. Yet another possible source involves the fact that for luminous QSOs, a standard (Shakura & Sunyaev 1973) disk may become optically thin in the inner radial zone.

Further work is needed to establish whether the continuum and line emission in such a model is consistent with all known observations for QSOs with Lyman polarization rises. Koratkar et al. (1995) find a polarization rise at the expected position of Ly$\alpha$, weaker than the Lyman continuum polarization and not blueshifted. This may come from larger disk radii with lower orbital velocities. Simple estimates, based on the blackbody limit for the Ly$\alpha$ surface brightness, suggest that the disk radii considered here will not be a significant source of Ly$\alpha$ emission.

The spin rates of black holes in AGN have received increasing attention in recent years. An interesting “spin paradigm,” in which radio-loud objects have rapidly rotating holes (Blandford 1990) has been developed by Wilson & Colbert (1995). Values $a_{\ast} \approx 0.5$ could arise in several ways. (1) If two black holes coalesce, the resulting hole has $a_{\ast} \approx 0.5$ for a mass ratio $\geq 0.2$, if roughly half the angular momentum of the binary at the last stable circular orbit is radiated as gravitational waves during the coalescence (Wilson & Colbert 1995). (2) Successive accretion events at random inclinations could cause the hole’s angular momentum to wander around a value $a_{\ast} \approx 0.5$, if the mass accreted in one event is not too small as a fraction of the black hole mass (Moderski, Sikora, & Lasota 1997). (3) Angular momentum extraction by the Blandford-Znajek (1977) mechanism may balance angular momentum supplied by accretion to give an equilibrium value $a_{\ast, \text{eq}}$. Results by Moderski & Sikora (1996) indicate $a_{\ast, \text{eq}} \approx 0.5$ for $\alpha \approx 0.01$, where $\alpha$ is the viscosity parameter (Shakura & Sunyaev 1973) and $\dot{m} \equiv c^{2}M/L_{\text{edd}}$. Our model for PG 1630 + 377 has $\dot{m} \approx 1$, and values at $\alpha \approx 10^{-2}$ are not precluded in active galactic nuclei. However, a value of $a_{\ast}$ as large as 0.5 might correspond to an object with substantial radio luminosity in the “spin paradigm,” whereas PG 1630 + 377 and PG 1222 + 228 are radio-quiet objects.

The toy model discussed here essentially addresses the question, what must the polarization be in the rest frame of the gas in order for the observed polarization rise to match the observations? The remarkably simple answer is that it should be an abrupt rise in polarization and polarized flux at the Lyman edge. This may provide a significant clue to the source of the polarized radiation.

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