HOW CAN ONE PROBE PODOLSKY ELECTRODYNAMICS?

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We investigate the possibility of detecting the Podolsky generalized electrodynamics constant $a$. First we analyze an ion interferometry apparatus proposed by B. Neyenhuis et al. (Phys. Rev. Lett. 99, 200401 (2007)), who looked for deviations from Coulomb’s inverse-square law in the context of Proca model. Our results show that this experiment has not enough precision for measurements of $a$. In order to set up bounds for $a$, we investigate the influence of Podolsky’s electrostatic potential on the ground state of the Hydrogen atom. The value of the ground state energy of the Hydrogen atom requires Podolsky’s constant to be smaller than 5.6 fm, or in energy scales larger than 35.51 MeV.

Keywords: Podolsky electrodynamics; ion interferometry; hydrogen atom.

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1. Introduction

The inference of the mass of the particles is a key problem in physics. The Higgs mechanism is the most simple and popular way to generate massive particles from an originally gauge invariant massless theory. From the theoretical point of view the
existence of a massive photon, usually considered in the context of Proca model, has many implications. One of the most important is the fact that interactions between particles are commonly described in terms of gauge theories and, as it is well known, the gauge field is supposed to be massless.\textsuperscript{1} Since the electromagnetic interactions are described in terms of the $U(1)$ symmetry group, all Quantum Electrodynamics, which is constructed on a gauge framework, should be reviewed if a mass for the photon was verified. The same occurs for instance in atomic physics, where the energy spectrum is supposed to be different if a non-Coulomb potential is considered.

Although it is widely accepted by physicists (especially by the theoreticians) that the photon is a massless particle, this is not an affirmation that can be easily done from the experimental point of view since all experiments are subject to uncertainties — the experimentalists basically establish upper limits for the photon mass.

Many experiments have been proposed to measure the mass of the photon\textsuperscript{2–6} and among them, several try to accomplish this by using the fact that the electric field produced by a point charge is not the one predicted by Coulomb law if the photon is supposed to be massive. They try to verify the existence of a photon mass by looking for small deviations from the Coulomb law\textsuperscript{7} — usually a potential $1/r^{1+\delta}$ is tested, and $\delta$ is evaluated. However, as mentioned in Ref. 8, the problem with this type of potential is that it does not come from any underlying theory and usually many assumptions regarding the measurement of $\delta$ are done, so that its evaluation is strongly dependent on these hypothesis. In order to avoid these problems, the authors of Ref. 8 proposed an experiment where an ion interferometry is used to measure the photon mass. The idea of the experiment consists, roughly speaking, in using interferometry of an ion beam that passes through a tube where different voltages are applied — if the mass of the photon is nonnull then a difference in the interferometer phase is expected. According to the authors of Ref. 8, the experiment will be very accurate, predicting a sensitivity to the (Proca) mass of $9 \times 10^{-50}$ g, “2 orders of magnitude smaller than the limit in Ref. 9.” In their case the underlying theory is the Proca model.

However, if instead of using the Proca model, the Podolsky Generalized Electrodynamics\textsuperscript{10–12} is taken into account, it is still possible to find a mass for the (massive mode of the) photon and preserve gauge symmetry. In a recent paper,\textsuperscript{13} a gauge theory for systems depending on the second order derivative of the gauge field was developed and it was verified that the gauge Lagrangian should depend on the usual field strength, $F_{\mu\nu}$, and on its covariant derivative, $G_{\mu\nu}^a = D_{\mu\nu}F_{\mu\nu}^a$. In particular, for the $U(1)$ group it was verified that the Podolsky Lagrangian,$^a$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2} \partial_\rho F^{\rho\sigma} \partial_\sigma F_{\mu\nu}^\sigma, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

\textsuperscript{a}To preserve the correct units of the Lagrangian, the constant $a$, henceforth referred as the Podolsky constant, has unit $1/l$, where $l$ stands for length; the metric signature $(+−−−)$ is considered.
fulfills all the requirements of a second order gauge theory with an important feature: all Lagrangians of the type $G^2$ for the $U(1)$ group differ from Podolsky Lagrangian only by a total divergence. The (fourth-order) field equations obtained from this Lagrangian are

$$(1 + a^2 \Box) \partial_\mu F^{\nu\mu} = 0,$$

and under a generalized Lorenz condition,\(^{14}\) $(1 + a^2 \Box) \partial_\mu A^\mu(x) = 0$, massive and massless modes for $A_\mu$ are identified:

$$p^2 (1 - a^2 p^2) A_\mu(p) = 0 \Rightarrow \begin{cases} p^2 = 0, \\ p^2 - \frac{1}{a^2} = 0. \end{cases}$$

The massless mode should be understood as the usual photon, while the massive mode was tentatively interpreted by Podolsky as being a neutrino. This interpretation is of course outdated. Since its original formulation, several aspects of this theory have been analyzed, including its canonical structure,\(^{14-16}\) quantization,\(^{17,18}\) and others.\(^{19-24}\) Problems with Podolsky electrodynamics have been pointed out, such as unitarity violation and the presence of ghosts. These states with negative norm are typical of theories with higher derivatives.\(^{25}\) In spite of their drawbacks, these theories display interesting properties (see Refs. 17 and 18 for the Podolsky’s case), a fact that motivates their careful study as effective field theories (EFT). Podolsky electrodynamics itself should be interpreted as an EFT in which the parameter $a$ sets the length scale where the theory is valid. We emphasize that only classical aspects of Podolsky electrodynamics will be considered. Therefore, typical problems of the quantization procedure should not be a concern here.

Since Podolsky electrodynamics predicts the existence of a massive mode for the photon, if the experiment proposed in Ref. 8 finds a deviation in the interferometer phase, then this could be either an indicative of the existence of the photon mass in the context of the Proca model or of the existence of a nonnull value for Podolsky constant, giving support to the Podolsky theory. One of the purposes of the present work is to analyze how the Podolsky constant can be determined or constrained by the ion interferometry experiment proposed in Ref. 8. This is discussed in Sec. 2, where the analytical solution for the problem will be analyzed and numerical estimations for Podolsky constant will be made.

On the other hand, if Podolsky theory is to be verified, then many implications in other known results are expected. As an example, the energy spectrum of the Hydrogen atom as described by quantum mechanics is to be altered, since the Coulomb potential should be substituted by the potential predicted by Podolsky electrodynamics. This is the second point to be studied here. A perturbative solution for the quantum mechanics wave function of the electron will be obtained — see Sec. 3 — and another constraint on $a$ will be made. Section 4 presents our conclusions.
2. Ion Interferometry Experiment

In the experiment proposed in Ref. 8 a time-varying voltage is applied to a conducting cylinder that is nested inside a grounded second cylinder. A beam of ions pass through the inner conductor through three gratings, forming a Mach–Zehnder interferometer — for more details see the original paper. If there is an electric field inside the cylinder, i.e. if the ions go through different potentials, then an interferometer phase shift is expected. Notice that this is not what is predicted by Maxwell equations for a conducting shell, according to which the potential inside the apparatus should be constant.

After passing through the first grating the ion beam is split in two arms: one travels horizontally (parallel to the cylinder axis), while the second goes diagonally. When the two arms reach the second grating, the one that was advancing horizontally begins to travel diagonally while the second starts to go horizontally, until they reach the third grating, where they start to travel under the same conditions. Since the distance between the gratings is the same, the diagonal segments of each arm travel through the same potentials and they induce the same phase shift. However, the segments of the arms that go horizontally pass through different potentials; if there is a phase shift in the interferometer it is caused by the difference of potentials between the horizontal segments (see Fig. 1). We consider that the distances of the horizontal segments from the center of the cylinder are \( r_0 \) and \( r_0 + s \). This way, what the interferometer actually does is to measure a phase shift induced by the potential difference between these horizontal segments of the arms the split beam.

The first information required is the equation for the potential inside the cylinder as predicted by the theory. In Ref. 8 the authors considered the Proca model. Here we will analyze Podolsky electrodynamics,\(^{10-12}\) where the equation for the electrostatic potential is given by

\[
(1 - a^2 \nabla^2) \nabla^2 \phi = 0.
\]

To solve this equation, let us define

\[
U \equiv \nabla^2 \phi.
\]
First we solve the homogeneous equation for $U$

$$(1 - a^2 \nabla^2) U = 0,$$  

and then consider the nonhomogeneous equation for $\phi$,

$$\nabla^2 \phi = U_h,$$  

where $U_h$ is a solution of (1). In view of the symmetry of the problem, cylindrical coordinates are considered and no angular dependence is expected. Also, since the inner cylinder has an elongated geometry, the infinite tube approximation can be done and no longitudinal dependence exists. The solution for (1) is found under these assumptions, and Eq. (2) becomes

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{d\phi}{dr} \right) = A I_0 \left( \frac{r}{a} \right) + B K_0 \left( \frac{r}{a} \right),$$  

where $I_0$ and $K_0$ are the modified Bessel functions of the first and second kind.

The integration of Eq. (3) gives us

$$\phi \left( \frac{r}{a} \right) = a^2 A I_0 \left( \frac{r}{a} \right) + a^2 B K_0 \left( \frac{r}{a} \right) + D \ln \frac{r}{a} + C.$$  

This solution carries a desirable feature: the homogeneous part is the usual Maxwell term and the particular solution is the Podolsky contribution. In fact, this split always occurs in the electrostatic case of Podolsky theory when vacuum is assumed.

Four integration constants appear in the solution (4), as expected from a fourth-order equation, and boundary conditions are used to fix them. First we consider that the potential in the limit $r \to 0$ should be finite. Using the asymptotic form for $I_0$ and $K_0$, we conclude that $D = a^2 B$. Another boundary condition that is used is the value of the potential at $r = R$, where $R$ is the radius of the inner tube. If $V_0$ is the voltage applied to the inner tube relative to the outer tube, whose unknown (ground) potential is $V_g$, then

$$V_0 + V_g = a^2 A \left[ I_0 \left( \frac{R}{a} \right) + g(a) \left[ K_0 \left( \frac{R}{a} \right) + \ln \frac{R}{a} \right] + f(a) \right],$$

where $B$ and $C$ were redefined as $B = g(a) A$ and $C = f(a) a^2 A$, and $A$ is supposed to be nonnull. This expression is used to determine $A$ in terms of the other constants.

Yet another expected boundary condition is that the electric field $E$ at $r = 0$ is null (otherwise it would be discontinuous without a physical reason). Actually with the redefinitions of $B$ and $C$ above, it is verified that this condition is already satisfied, so that no other constant is fixed with this condition. However, if we claim that the divergence of the electric field is finite at $r = 0$, then we must set $g(a) = 0$. At last, in order to fix $f(a)$ we assume that the potential at $r = 0$ can be measured — this is an additional step in the experimental procedure proposed in Ref. 8 where no measurement of the potential at $r = 0$ is suggested; in our case

bWhat makes the electric field flux finite at the origin.
this is essential for determining the last integration constant. We suppose that the measured $\phi(0)$ is expressed as $\phi(0) = (V_0 + V_g)\epsilon$, where $\epsilon$ is a parameter to be determined experimentally and is likely to be $\approx 1$, since we do not expect large deviations from Maxwell’s theory. This fixes $f(a)$ as

$$f(a) = \frac{\epsilon I_0 \left(\frac{r_0}{a}\right)}{1 - \epsilon}.$$ 

Finally the potential is written as

$$\phi \left(\frac{r}{a}\right) = (V_0 + V_g) \left[ \frac{I_0 \left(\frac{r_0 + s}{a}\right)}{I_0 \left(\frac{r_0}{a}\right)} \right] (1 - \epsilon) + \Phi_0.$$ 

Notice that if no Podolsky term is supposed to exist, then the potential inside the inner tube will be the same everywhere, i.e. $V_0 + V_g$, which means that $\epsilon = 1$.

Now the potential difference between the horizontal segments of the arms of the split beam can be evaluated as

$$\Delta \phi = \phi \left(\frac{r_0 + s}{a}\right) - \phi \left(\frac{r_0}{a}\right) = (V_0 + V_g) \left[ \frac{I_0 \left(\frac{r_0 + s}{a}\right) - I_0 \left(\frac{r_0}{a}\right)}{I_0 \left(\frac{r_0}{a}\right)} \right] (1 - \epsilon).$$

The interferometer phase is given by

$$\Phi = \frac{e\tau}{\hbar} \Delta \phi + \Phi_0 = \frac{e\tau}{\hbar} (V_0 + V_g) \left[ \frac{I_0 \left(\frac{r_0 + s}{a}\right) - I_0 \left(\frac{r_0}{a}\right)}{I_0 \left(\frac{r_0}{a}\right)} \right] (1 - \epsilon) + \Phi_0,$$

where $e$ is the charge of the ion (in the present case $e$ is the electron charge), $\tau$ is the time that the ion takes to travel lengths of the horizontal segments and $\Phi_0$ is the phase indicated by the interferometer when $V_0 + V_g = 0$. In order to eliminate the two unknown constants $\Phi_0$ and $V_g$, two potential differences $V_0$ and $V_0 + \Delta V$ can be applied to the inner tube. The difference in the phases due to this change will be

$$\Delta \Phi = \frac{e\tau}{\hbar} \Delta V \left[ \frac{I_0 \left(\frac{r_0 + s}{a}\right) - I_0 \left(\frac{r_0}{a}\right)}{I_0 \left(\frac{r_0}{a}\right)} \right] (1 - \epsilon).$$

This expression is inverted in order to obtain the Podolsky constant $a$ as a function of the experimental parameters. This will be done under some assumptions. First we expect that the value of Podolsky constant is small, so that only small differences from Maxwell equations can be detected. If this is the case, then the asymptotic limit for $I_0$ can be used,

$$I_0(x) \sim \frac{1}{\sqrt{2\pi x}} e^x.$$ 

This allows us to estimate the Podolsky constant as

$$a = \frac{R - (r_0 + s)}{\ln(1 - \epsilon) - \ln \left( \frac{\hbar \Delta \Phi}{e\tau \Delta V \sqrt{\frac{r_0 + s}{R}}} \right)}.$$ 

Notice that $\lim_{\epsilon \to 1} a = 0$, which means that Podolsky electrodynamics reduces to the Maxwell one. Besides, in order to have a positive massive mode, the approximation above shows that $\epsilon$ must be less than 1.
How Can One Probe Podolsky Electrodynamics?

We shall obtain numerical estimations for \( a \) considering ion beams composed by \(^1\)H\(^+\) and \(^{133}\)Cs\(^+\). According to Ref. 8, these ions can travel at a speed \( v \) of 311 m/s and 27 m/s respectively; the length of the horizontal segments are fixed in 1 m so that \( \tau = L/v \) is determined for both cases. The potential difference \( \Delta V \) can be fixed as 400 kV and the values of \( R \), \( r_0 \) and \( s \) are set to \( R = 27 \text{ cm} \), \( r_0(\text{H}^+) = 24.4 \text{ cm} \), \( r_0(\text{Cs}^+) = 24.9 \text{ cm} \) and \( s(\text{H}^+) = 6.4 \text{ mm} \), \( s(\text{Cs}^+) = 0.56 \text{ mm} \). Figure 2 shows the numerical estimations for \( a \) for different values of \( \epsilon \) and \( \Delta \Phi \) for \(^1\)H\(^+\).

The value of \( \epsilon \) can be determined by measuring two potential differences \( \Delta \phi_0 \) and \( \Delta \phi_1 \). The potential difference \( \Delta \phi_0 \) is measured when a voltage \( V_0 \) is applied to the inner tube; \( \Delta \phi_1 \) corresponds to a voltage \( V_1 = V_0 + \Delta V \) applied to the inner tube. One has,

\[
1 - \epsilon = \frac{\Delta \phi_1 - \Delta \phi_0}{\Delta V}.
\]

The domain of values assumed by \( \epsilon \) is established by considering that a precision of \( 10^{-8} \) could be achieved with the available commercial multimeters (in the best case). Concerning \( \Delta \Phi \), it was considered that phase shifts as small as \( 10^{-4} \text{ rad} \) can be detected.\(^8\)

According to these numerical evaluations, the experiment would be able to detect values of the Podolsky constant \( a_{\text{Cs}^+} \geq 0.091 \text{ cm} \) in the case of \(^{133}\)Cs\(^+\).

**Fig. 2.** Values of \( a \) (cm) for different values of \( \epsilon \) and \( \Delta \Phi \) (rad) using \(^1\)H\(^+\) ion beam. The graphic for the \(^{133}\)Cs\(^+\) ion beam is very similar.
ion beam and \( a_{H^+} \geq 0.098 \) cm if the \(^1\text{H}^+\) ion beam is used. These values seem consistent with the asymptotic limit taken for \( I_0 \); indeed, they are small when compared to the values of \( R \) and \( r_0 \) and therefore the ratios that appear in \( I_0 \) — namely, 
\[
\frac{R}{a_{H^+}} = 276, \quad \frac{R}{a_{Cs^+}} = 297, \quad \frac{r_0}{a_{H^+}} = 249, \quad \frac{r_0}{a_{Cs^+}} = 274
\]
are all of order of \( 10^2 \).

The mass of the photon is evaluated using these values for \( a \) and the expression 
\[
m_\gamma = \frac{\hbar}{ac}.
\]

As the mass scales with the inverse of the Podolsky constant, the smallest value of \( a \) that can be measured will give the greatest measurable value for the photon mass. Each ion beam will predict a different upper limit: 
\[
m_{133\text{Cs}^+} = 3.9 \times 10^{-40} \text{ kg} = 2.2 \times 10^{-8} \text{ eV} \quad \text{and} \quad m_{^1\text{H}^+} = 3.6 \times 10^{-40} \text{ kg} = 2.0 \times 10^{-8} \text{ eV}.
\]

Although Proca and Podolsky approaches predict a massive mode for the photon, there are some important differences between them. First, Podolsky electrodynamics is a gauge theory, while Proca model explicitly breaks such symmetry, what could have implications for the charge conservation. Second, in the Proca context it is expected that the photon mass, if it exists, should be very small. Conversely, the Podolsky’s massive model would be very large once it is the inverse of the scale of length where the generalized theory is effective, cf. Eq. (6). That is, Proca (Podolsky) model predicts deviations from Maxwell electrodynamics in very low (high) energy regimes.

It is important to emphasize that the photon mass in independent of the nature of the ion composing the beam in the experiment. The different values for \( m_\gamma \) for \(^{133}\text{Cs}^+\) and \(^1\text{H}^+\) express only the different values of \( a \) accessed by the experiment.

One could argue that the values of \( a \) that can be measured by the ion interferometer are very high in absolute terms. In fact, one would say that if \( a \) were of order of \( 10^{-2} \) cm as indicated here, the deviations from the Maxwellian electromagnetism would have been detected long ago. In face of this, the conclusion would be that the experiment proposed in Ref. 8 is not appropriate for measuring the Podolsky constant and therefore the photon mass in this theory. This is indeed a strong argument, but we would like to give a quantitative justificative for ruling out the ion beam apparatus as an appropriate set to find the Podolsky mass.

In the next section we will make the hypothesis that Podolsky electrodynamics hold at the atomic scale\(^c\) and see the implications for the elementary physics of the Hydrogen atom; in particular, we will analyze the energy of the fundamental state.

### 3. Hydrogen Atom

Now we turn to the problem of considering the Hydrogen atom, as treated by quantum mechanics, where the electromagnetic potential is the one described by Podolsky electrodynamics. The goal of this section is to analyze the effects of a

\(^c\)This is not mandatory once Podolsky’s theory for the electromagnetism is an effective theory.
nonnull Podolsky constant and verify what are the implications of the values found for $a$. We consider $\hbar = 1$ to simplify the notation, but the units are restored when numerical evaluations are done.

The electrostatic potential is given by

$$\phi(r) = -\frac{e}{r} \left(1 - e^{-\frac{r}{a}}\right),$$

and the Hamiltonian operator reads

$$\hat{H} = \frac{\hat{p}^2}{2m} + e\phi(r).$$

The variational method will be employed so that a perturbative solution for the wave function of the ground state, $\psi(r)$ may be found. The tentative solution is

$$\psi(r) = Ne^{-\gamma r},$$

where $N$ is a normalization constant set as $N = \sqrt{\frac{\pi}{3}}$. $\gamma$ is a parameter that will be determined by the variational method, according to which the energy, given by

$$E = \int dV \psi^*(r) \hat{H} \psi(r) = \frac{\gamma^2}{2m} - e^2\gamma + e^2\left(\frac{4\gamma^3}{(2\gamma + \frac{1}{a})^2}\right),$$

should be minimized:

$$\frac{\partial E}{\partial \gamma} = \frac{8a^3}{m} \gamma^4 + \frac{12a^2}{m} \gamma^3 + \frac{6a}{m} \gamma^2 - 6ae^2\gamma + \frac{\gamma}{m} - e^2 = 0. \quad (7)$$

Now suppose that the value of the Podolsky constant is actually small, then Eq. $(7)$ can be solved considering only terms up to first order in $a$. The solution found for $\gamma$ is $\gamma_+ = \frac{me^2}{\alpha}$ and $\gamma_- = -\frac{1}{6\alpha}$. The energies evaluated with these solutions are

$$E(\gamma_+) = -\frac{me^2}{2}e^2\left(1 - 2(2mae^2)^2\right) + O(a^3), \quad E(\gamma_-) = \frac{9ame^2 + 1}{72a^2m}.$$ 

The value of $E(\gamma_-)$ gives a positive energy and is not suitable to describe a bound state, therefore this result should be excluded. $E(\gamma_+)$ can only be calculated with a given value of $a$, but for small $a$ it is only a perturbation on the known result given by quantum mechanics, $E = -\frac{mae^2}{2}\alpha^2$. If we want to find a value for $a$ that is compatible with the known results given in the literature we should expect the perturbation $2(2mae^2)^2$ to be smaller than the relative experimental uncertainty of the energy of the ground state. Proceeding this way it follows

$$a \leq \frac{r_B}{2} \sqrt{\frac{\sigma E_0}{2|E_0|}},$$

where $r_B = \frac{1}{me^2}$ is the Bohr radius. Restoring the units and using values given in the literature$^{28}$ we should expect

$$a \leq 5.56 \text{ fm} \quad \text{or} \quad m_{\gamma} \geq 35.51 \text{ MeV}. \quad (8)$$

Clearly these values for $a$ and $m_{\gamma}$ are not compatible with the possible values that can be found in the interferometry experiment.
4. Conclusions

We have discussed how the ion interferometry experiment proposed in Ref. 8 could be used to measure the value of Podolsky constant $a$ and the massive mode of the photon in the context of Podolsky electrodynamics. The minimum value of $a$ that could be detected — $a = 0.091$ cm with the $^{133}$Cs$^+$ ion beam — is too large as an admissible effective scale, and would lead to a mass $m_{\gamma} \leq 3.9 \times 10^{-40}$ kg $= 2.2 \times 10^{-8}$ eV for the photon which is excluded by current experimental data. 28

We might think of improving the accuracy of the measurements of the phase shift and/or of the potential at $r = 0$ (for instance, using some better technology in the apparatus). But the logarithmic behavior of (5) in terms of these quantities makes this possibility unlikely: great improvements in the detection of $\Delta \Phi$ and $\Delta V$ would lead to small changes in $a$ [see Eq. (5)]. Therefore, this rules out the interferometric ion beam experiment as a suitable one for testing Podolsky electrodynamics.

Besides gauge invariance, Podolsky electrodynamics has another peculiar feature that distinguishes it from the Proca field: the smaller the characteristic constant $a$ the greater the mass associated to the photon. Hence we are strongly constrained: the Maxwellian electromagnetism must hold until small scales of length, and therefore $a$ has to be small, otherwise the additional Podolsky term in the Lagrangian for the electromagnetic field would be significant and the resulting modifications in the ordinary theory would be easily detected. These scales of length are set, for instance, by the spectroscopy of Hydrogen atom. So, we tested Podolsky’s theory calculating the value of $a$ that would be consistent with the experimental error in the energy of the fundamental level of the Hydrogen. The result, $a \leq 5.56$ fm, clearly shows that the ion interferometer experiment does not have enough precision to measure a Podolsky constant that is this small.

The constraint $a \leq 5.56$ fm coming from quantum mechanics considerations set a high energy scale for the photon mass: $m_{\gamma} > 35.51$ MeV. This way, if Podolsky model is correct, it is expected to engender deviations from Maxwell electrodynamics only in high energy scales, which are accessible by particle accelerators. Therefore, the next necessary step is to investigate in more detail which kind of effects appear in QED$_4$ due to the Podolsky term.

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