Amplification and passing through the barrier of the exciton condensed phase pulse in double quantum wells

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The peculiarities and the possibility of a control of exciton condensed pulse movement in semiconductor double quantum well under the slot in the metal electrode are studied. The condensed phase is considered phenomenologically with the free energy in Landau-Ginzburg form taking into account the finite value of the exciton lifetime. It was shown that the exciton condensed phase pulse in the presence of an external linear potential moves along the slot direction with a constant value of a maximum density during exciton lifetime, while the exciton gas phase pulse is blurred. The penetration of the exciton condensed phase pulse through the barrier and its stopping by the barrier are studied. Also, it was shown that the exciton pulse in the condensed phase can be amplified and recovered after damping by imposing an additional laser pulse. Solutions for the system of excitons in double quantum well under the slot in the electrode under steady-state irradiation in the form of bright and dark autosolitons were found.

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I. INTRODUCTION

In recent years, much attention has been paid to both experimental and theoretical study of indirect excitons in semiconductor double quantum wells at low temperatures. An indirect exciton is a bound pair of an electron and a hole which are separated by an electric field to different quantum wells [1]. Consequently, the recombination of the electron and the hole is inhibited, that causes long lifetime of indirect excitons, which is several orders of magnitude higher than the direct exciton lifetime. The study of indirect excitons is promising both in terms of fundamental science, because a high density of excitons can be created and many exciton effects can be studied. Also the system of indirect excitons can be promising for application purposes, since they can travel over large distances, carry energy and information and be used in semiconductor devices based on double quantum wells. Therefore, many experimental and theoretical works were devoted to the study of the indirect excitons’ properties. Under these investigations, the experiments on the base of AlGaAs quantum wells [2–9] revealed new nontrivial effects such as formation of spatially inhomogeneous structures (sometimes periodical) in the distribution of exciton density, and the fact that symmetry of the formed structures is not connected to the symmetry of the external field. In theoretical works [10–22] various models of pattern formation were proposed. However, as the interpretation of the results in different works is versatile, so research in this direction is relevant. The possibility of using the system of indirect excitons in the optoelectronics is studied in several works [23–27]. Namely, the experimental possibility of building an exciton optoelectronic transistor [23], an excitonic integrated circuit [24] and an excitonic conveyor [25] were demonstrated recently. Therefore, the theoretical study of the movement of the interacting system of indirect excitons in external fields is important.

In this work we study the formation, amplification and passing through the barrier of the exciton condensed phase pulse in the semiconductor double quantum well in the system, in which one of the electrodes has a slot, which creates an additional field to uniform field for excitons. The formation of the exciton condensed phase in the double quantum well under the slot excited by a steady-state uniform irradiation was studied in [19]. The approach used to describe the distribution of the exciton density in double quantum wells is based on the assumption of the exciton condensed phase existence due to the attraction between indirect excitons at some distances and the finite value of the
exciton lifetime. The existence of the attractive interaction between excitons is confirmed by the calculations of many-exciton system [28], and by the possibility of biexciton formation [29, 31]. Also, the finite value of the exciton lifetime plays an important role in the formation of the structures in the exciton density distribution. The suggested model explained a series of experimental results (see references in [19]). Particularly, this model has allowed to describe spatial distribution of the excitons on the ring outside the laser spot, observed in [5], spatial structure of the luminescence from the double quantum well under the window in the metal electrode, observed in [7, 8].

Unlike the work [19], where the structure of the exciton density distribution in the quantum well under the slot in the case of uniform in space and time independent pumping was studied, this paper investigates the excitation of the exciton condensed phase by pulse irradiation. This excitation creates the exciton condensed phase pulse, which movement and control are analyzed in this paper. The possibility of the exciton condensed phase pulse amplification by additional laser pulse in the one-dimensional system was investigated in [32]. In comparison to [32], we consider the real physical system (slot in the electrode) with two-dimensional distribution of the exciton density, so the two-dimensional problem for the exciton density is studied. Movement of the pulse takes place in the quantum well under the slot and along it. And along with amplification of the exciton condensed phase pulse, we consider passing of the exciton condensed phase through a barrier. Presence of the barrier allows to control exciton pulse, to stop it in particular. A consideration of the processes with exciton system in the condensed phase means that we are studying the collective effects of exciton system movement and passing through barrier.

II. Model of the system

Let us consider the semiconductor structure with the double quantum well sandwiched between two metal electrodes. There is the slot in the upper electrode (figure 1). The width of the slot is 2b. Let us direct the OX axis along the slot, the axis OY in the transverse direction and the OZ axis is directed along the normal to the electrode. Let the plane of the quantum well has a coordinate z (z < 0). The presence of the slot forms an inhomogeneous electric field that is additional to the homogeneous field of solid electrodes and an additional potential energy for excitons in quantum well.

In [19] the existence of a periodic distribution of the exciton density under a homogeneous stationary irradiation was shown. If the width of the slot is large, then a string of the islands of the exciton condensed phase is divided into two lines with the islands of the exciton condensed phase located along the slot edges and shifted relative to each other in rows along the axis of the slot in the half-period. Periodic structure exists in a certain range of pumping values. Also in [19] the possibility of periodic structure movement in the presence of an external potential (dependent on the coordinates) along the slot was shown. The movement is analogous to exciton Gunn effect in semiconductors. In the considered system, in comparison to the work [19], there are additional elements, that are pointed in the figure and they will be explained later. These elements allow to control exciton pulses.

![Figure 1: Pattern of the system. Semiconductor structure sandwiched between two metal electrodes. There is a slot in the upper electrode. U1 is the potential on the upper electrode, U2 is the potential on the lower electrode and Ug is the gate potential of the strip. Perpendicular metal strip that creates a barrier for excitons. Arrows show laser irradiation of the double quantum well.](image-url)
y, z point of quantum well. This electric field is uniform in the region far from the slot \( E(y, z) = E_0 \). Additional potential for excitons, created by slot, equals \( V(x, y) = V_{\text{tot}} - V_0 \), where \( V_0 = -p_x E_0 \) is the uniform shift of the exciton band under the electric field in the region distant from the slot. Using the solution for the electric field of a grounded conducting plane with a slot in an external electric field problem [33], and making the assumption that the distance between the electrodes is much greater than the width of the slot, for additional potential energy for exciton formed by the slot, we obtain the following equation:

\[
V(y, z) = V_0 - V_0 \left[ \frac{1}{2} + \frac{1}{b} \sqrt{1 + b^2 \xi(y, z)} - \frac{2 \xi(y, z)^2 \sqrt{1 + b^2 \xi(y, z)}}{b^2 y^2 + z^2 + b^2} \right],
\]

where

\[
\xi(y, z) = \sqrt{y^2 + z^2 - b^2 + \sqrt{(y^2 + z^2 - b^2)^2 + 4b^2 z^2}}.
\]

III. Method of solution

The studied system will be described more detailed in the next section, but in this section we will concentrate our attention on the main equations for finding the density dependence of the exciton on time and space. For investigation of the exciton distribution let us consider the conservation law for the exciton density as follows:

\[
\frac{\partial n}{\partial t} = -\text{div} \vec{j} + G(\vec{r}, t) - \frac{n}{\tau},
\]

where \( n \) is the exciton density, \( G(\vec{r}, t) \) is the pumping (the number of excitons created per time unit per space unit of a quantum well), \( \tau \) is the exciton lifetime, \( \vec{j} = -M \nabla \mu \) is the exciton current density, \( M \) is the exciton mobility. For the exciton mobility \( M \) we use Einstein formula \( M = \frac{nD}{kT} \), where \( D \) is a diffusion coefficient, \( k \) is Boltzmann constant, \( T \) is the temperature. Described system is non-equilibrium because the pumping is present, and during phase transitions the system is also inhomogeneous. But the time for establishment of equilibrium in a local area is much less than in the whole system, because the equilibrium in the whole system is formed by slow diffusive processes. That is why the system can be described by free energy that depends on exciton density, which in its turn depends on the space coordinates.

Having expressed the chemical potential as \( \mu = \frac{\delta F}{\delta n} \), we chose the free energy in the Landau model:

\[
F[n] = \int d\vec{r} \left[ \frac{K}{2} (\nabla n)^2 + f(n) + nV \right].
\]

The term \( \frac{K}{2} (\nabla n)^2 \) characterizes the energy of inhomogeneity, \( f(n) \) is a free energy density, \( nV \) accounts for the potential energy. We approximate

![Figure 2: The dependence of the potential created by the slot on the distance from the centre of the slot in the plane of the quantum well for dimensional units that will be introduced in the next section.](image)
the density of the free energy in the form:

\[ f(n) = f_0 + kTn(n/n_0 - 1) + \frac{a}{2}n^2 + \frac{b}{3}n^3 + \frac{c}{4}n^4; \quad (5) \]

where the terms of the power series for the exciton density are important for large \( n \), they are the most important for describing the system, while the term \( kTn(n/n_0 - 1) \) may be essential only if one wants to describe a system in which the density of excitons is small and the interaction between them is not important.

Let introduce dimensionless variables for length, density, energy and time:

\[ l_u = \sqrt{\frac{K}{a}}, \quad n_u = \sqrt{\frac{a}{c}}, \quad V_u = a\nu, \quad t_u = \frac{d_1 l_u^2}{D}. \quad (6) \]

Introducing the dimensionless variables and substituting equation 4 with equation 3, we obtain a nonlinear equation that determines the exciton density:

\[ \frac{\partial n}{\partial t} = d_1 \Delta n + \nabla \left[ n\nabla (-\Delta n + n + b_1 n^2 + n^3 + V) \right] + G - \frac{n}{\tau} - gn^2, \quad (7) \]

where \( b_1 = \frac{b}{\sqrt{ac}} < 0, \quad d_1 = \frac{kT}{V_u} \) and \( V \) is an external potential, dependent on the coordinates.

Exciton-exciton annihilation (Auger process) was taken into account by introducing in equation 7 the term \(-gn^2\).

Equation 7 is non-linear phenomenological equation that describes the distribution of exciton density with regard to the pumping \( G(\vec{r},t) \) and finite lifetime \( \tau \).

IV. CREATION AND MOVEMENT OF THE EXCITON CONDENSED PHASE PULSES

Exciton pulse can be formed under the irradiation of the system by a laser pulse. The laser pulse creates the following pumping for excitons: (in the formula for the laser pulse, the electrodes non-transparency and an additional constraint on the pulse of the y coordinate are taken into account).

\[ P \] is proportional to number of excitons created by the laser pumping per time unit per space unit of a quantum well. The laser pulse causes the temperature increasing. Considering this problem we assume that time of establishment equilibrium state of temperature is much less than the exciton lifetime \( \tau \).

First, let us consider changing of the exciton density in the pulse in the case of the uniform potential along the slot. If the value of the amplitude of the laser pumping (\( P \)) is less than the threshold value, the exciton pulse is formed in the gas phase. In this case, the maximum of the exciton density in the pulse decreases rapidly with time after the end of the laser pulse acting on the system (figure 3), the width of the exciton pulse increases. If the value of the amplitude of the laser pulse exceeds the threshold value, the pulse is formed from excitons in the condensed phase. Maximum value of the exciton density in the pulse formed from the condensed phase remains constant approximately during the exciton lifetime, width of the pulse decreases with time.

In the case of the slot width \( b \) less than some value, the pulse is created in the centre of the slot (equation 8), where the pumping has maximum. For the large value of the width, the exciton pulse will be formed at the edge of the slot.

Exciton-exciton annihilation decrease the way that can be passed by the exciton pulse because the attenuation. The annihilation affects more effectively the duration of the pulse maximum value conservation in the case of the condensed phase (figure 3). This effect is obviously caused by the fact that the exciton density in the condensed phase is greater. During calculations of annihilation effect we have chosen a large annihilation rate such that the lifetime of a relatively annihilation and radiation are the same.

Let us consider the problem of the exciton pulse movement. To do this, let us assume that beside the potential of the slot, there is an additional potential that depends linearly on the coordinates along the slot in the equation for the exciton density:

\[ V_1(x) = -\delta x, \quad (9) \]

where \( \delta \) is constant value. This potential can be created, for example, by using an electric field, as it was done by the authors [31], or by an inhomogeneous stress of the crystal. The exciton pulse is
\[ G(x, y, t) = \begin{cases} 
    P \exp \left( -\frac{(x-x_0)^2 + (y-y_0)^2}{\beta^2} \right) & \text{if } t_1 < t < t_1 + \Delta t, y - b < y < y + b; \\
    0 & \text{in all other cases.}
\end{cases} \]

(8)

Figure 3: Dependence of the exciton maximum on time for different values of the parameters. Squares correspond to the exciton pulse in the condensed phase \((P = 0.64, g = 0)\), triangles correspond to the exciton pulse in the gas phase \((P = 0.1, g = 0)\), circles correspond to the exciton pulse in the condensed phase taking into account Auger recombination of the excitons \((P = 0.64, g = 7 \cdot 10^{-4})\), stars correspond to the exciton pulse in the gas phase taking into account Auger recombination of the excitons \((P = 0.1, g = 7 \cdot 10^{-4})\), \(b = 4, \beta = 10, \tau = 1000, b_1 = -2.23, d_1 = 0.2\)

Figure 5: Dependence of the exciton density on the \(x\) coordinate at different moments of time without the barrier (parameters are the same as in figure 4).

Presented in the figure 4 at different moments of time. And the projection of density distribution on XZ plane is presented in the figure 5. One can see that the exciton condensed phase pulse moves along the slot. The velocity of the exciton pulse movement depends on the potential \((v \approx \frac{\delta D}{kT})\). It is seen from these figures, that the maximum value of the exciton density in the pulse does not change almost during its lifetime, if the exciton pulse is in the condensed phase. But the width of the pulse decreases with time. In the case of the exciton pulse in the gas phase, the exciton pulse blurs.

In the case of the wide slot two exciton pulses are formed and they also move along the slot in parallel to each other.

So one can use the exciton pulse for data transmission in a semiconductor system. If the detector at the output reacts to the amplitude of the exciton density, one can achieve greater distances at which the pulse of excitons is in the condensed phase state.

V. Amplification of the exciton condensed phase pulse by the laser pulse

Exciton condensed phase pulse can be amplified by imposing an additional laser pulse on the system. Let us consider the exciton pulse in the condensed phase. As was shown already, the exciton density is constant during the exciton lifetime and after some time the amplitude of the pulse begins to drop and the system of excitons in the pulse transform to excitons in the gas phase. To recover the exciton condensed phase pulse, let us apply at some fixed moment of time additional laser pulse on the system, amplitude of which can be lower the threshold value for creation of the exciton condensed phase pulse. But imposing the laser pulse on the exciton pulse, when the position by the imposing time matches the position of the first pulse maximum,
leads to creation of the exciton pulse in the condensed phase. Imposing of the two laser pulses arise the exciton pulse in the condensed phase, which exists for longer time and can travel over larger distance (dependence of the maximum value of the exciton density in the first, second and total pulses on time are presented in figure 5). If a detector reacts to the amplitude of the exciton pulse, then the signal being transmitted by the system in the condensed phase can be received on a greater distance.

VI. Control of the exciton condensed phase pulse using the barrier

The exciton condensed phase pulse can be controlled, for example, using a barrier. The barrier can be created by placing a thin metal strip perpendicularly to the slot (see figure 4). The barrier for excitons of different magnitude can be formed by applying a potential to the strip. Let us approximate the additional potential for excitons created by the strip as follows:

\[ V_b(x) = S \exp\left(-\frac{(x - x_0)^2}{2(\Delta x)^2}\right), \]  

(10)

where \( S \) is the magnitude of the barrier and \( \Delta x \) is the width of the barrier. Cumulative potential
for excitons created by the linear potential (equation 9) and the barrier (equation 10) is presented in figure 7, the parameters of these potentials are $\delta = 0.09$, $S = 1$. One can observe that in some cases ($S = 1$) an obstacle is created on the way of the exciton condensed phase pulse movement.

Let us do analysis of an influence of the height of the barrier on the penetration of the pulse through barrier. It is seen from comparison of the projections of the exciton density on the YZ plane for $S = 0$ (figure 5) and $S = 0.2$ (figure 5), that the magnitude of pulse amplitude drops after a barrier passing with increasing the height of the barrier. With increasing $S$ the exciton condensed phase pulse begins to destroy while passing through the barrier (figure 6) and the exciton condensed phase pulse exists beyond barrier only at $S \leq 0.3$. Dependence of the exciton density on spatial coordinates in the case of the presence of the barrier with parameters $S = 1$, $\Delta x = 2.24$ and the position of the maximum $x_0 = 100$ is presented in two-dimensional case in figure 10 and in the projection on YZ plane in figure 11. The position of the pulse does not change as a function of the time after the pulse reaches the barrier. So, the exciton condensed phase pulse stops in the area before the barrier and the exciton condensed phase pulse movement can be controlled by changing the barrier parameters. Simultaneously with stopping, the change of the shape of the pulse occurs (see figure 10).

Thus, the collective penetration of excitons through a barrier is similar to a quantum-mechanical particle tunnelling through the barrier: the probability of the tunnelling decreases with the barrier growth. The situation with the dependence of passing processes on the width of the barrier (on $\Delta x$) is not unambiguous, since that considered by us potential for excitons consist of two parts: a linear part and the potential of the barrier. Depending on $\Delta x$ and $\delta$ the total height of the barrier may be decrease or increase as function of $\Delta x$ (see figure 7) with different effects on the pulse penetration.

Let us do some quantitive estimations. For the quantitative description, we chose the following values of parameters: $\tau = 100$ ns, $T = 3.7$ K, $n_u = 2 \cdot 10^{10}$ cm$^{-2}$, $D = 9$ cm$^2$/s, $a n_u = 1.6$ meV. In this case the width of the slot is $2b = 5.36$ $\mu$m ($b = 4$). Path traversed be the exciton condensed phase pulse during its lifetime is $l = 67$ $\mu$m, and the height of the barrier is $S = a n_u = 1.6$ meV($S = 1$).

VII. EXCITON AUTOSOLITONS

So far we have studied the exciton density pulses localized in space. These pulses are created by laser pulses that act during some time interval. Exciton...
density pulses are damping over time. However, the existence of localized exciton pulses in space (autosolitons) is possible under stationary excitation radiation in some interval of the pumping \[35\].

Let us consider the possibility of formation of the exciton autosolitons in the potential created by the electrode with the slot. As it was shown in \[19\], under the stationary irradiation in some interval of the pumping \( G_{c1} < G < G_{c2} \) in the presence of the slot in the electrode periodical distribution of the exciton density in the quantum well arose. Homogeneous distribution of the exciton density exists outside this interval of pumping.

We showed that at steady-state pumping there is the spatial nonuniform solution of the equation \[7\] at \( G < G_{c1} \) in the form of an isolated peak \[35\]. It may be obtained solving the equation \[7\] at the pumping, which consist of a constant value \( G_0 \) and an additional pulse \( dG \) with the maxima in the some point of the space and in the time moment

\[
dG = s \exp[-w((x-x_0)^2+(y-y_0)^2)] \exp[-p(t-t_0)^2],
\]

where \( s, w, p \) are parameters. The equation \[11\] describes a pulse of the pumping, which acts during some time interval with the maximum in the point \( x_0, y_0 \).

The solution of equation \[7\] for the system of indirect excitons under the slot in the metal electrode obtained in the case of imposing the addition pulse \[\text{(11)}\] is presented in figure \[12\]. The solution exists at \( t \to \infty \), i.e. at the times, when the action of the addition pulse is absent already. The shape of the peak \( n(x, y) \) does not depend on..."
Figure 8: Dependence of the exciton density on the $x$ coordinate at different moments of time in the presence of the barrier at $S = 0.2$, $\Delta x = 2.24$, $\delta = 0.09$, $b = 4$, $P = 0.64$, $\beta = 10$, $\tau = 1000$, $b_1 = -2.23$, $d_1 = 0.2$.

Figure 9: Dependence of the exciton density on the $x$ coordinate at different moments of time in the presence of the barrier, $S = 0.3$, $\Delta x = 2.24$, $\delta = 0.09$, (other parameters are the same as in figure 8).

Figure 11: Dependence of the exciton density on the $x$ coordinate at different moments of time in the presence of the barrier, $S = 1$, $\Delta x = 2.24$, $\delta = 0.09$ (other parameters are the same as in figure 8).

Such localized spatial distribution of the exciton density is called "bright autosoliton". According to classification of [36], it belongs to static soliton. Localized solutions exist also in the some region at the pumping greater the value, at which the periodical structure arises ($G > G_{c2}$). The dependence of the exciton density may be obtained from equation (7) choosing an additional pumping pulse in the form (11), but at $s < 0$. Such solution is presented in figure 13. These structures can be called "dark autosolitons". They exist in some region of pumping ($G = 0.0078 - 0.0081$). If there is an external field in a system, which creates a linear potential (see (9)), the autosoliton moves along the slot. So this autosolitons can be also used for information transmission in semiconductors. In this case the motion occurs under steady-state irradiation, but such pulse does not decay with time.

VIII. Conclusions

We consider the dynamics of the exciton condensed phase pulse in double quantum wells, created by the external irradiation. The model of the system consists of a semiconductor structure sandwiched between two metal electrodes, the top one having a slot that creates an additional potential for excitons. The spinodal decomposition model, generalized to the case of particles with a finite lifetime, was used to describe the exciton density distribu-
Figure 12: Bright autosoliton. Spatial distribution of the exciton density at $G = 0.005 < G_{c1}$, $b = 7$, $b_1 = -2.23$, $x_0 = 60$, $y_0 = 30$.

Figure 13: Dark autosoliton. Spatial distribution of the exciton density at $G = 0.008 > G_{c2}$, $b = 7$, $b_1 = -2.23$, $x_0 = 60$, $y_0 = 30$.

1. Exciton condensed phase pulse arise if the laser pulse is greater than the threshold value dependent on the parameters of the system.

2. In the case of the narrow slot, an exciton condensed phase pulse arise at the centre of the slot, and in the case of the wide slot, an exciton condensed phase pulse splits into two pulses, located near the boundaries of the slot.

3. In the presence of the potential that linearly changes along the slot, the exciton pulse moves with the constant velocity. Maximum density is constant during the lifetime and the pulse does not spread if the exciton system is in the condensed phase.

4. Initial pulse can be amplified by imposing an additional laser pulse thereby increasing the path traversed before attenuation.

5. Exciton pulse can be stopped or its shape can be changed by creating a potential barrier for excitons. With increasing the height of the barrier, passing of the excitons through the barrier decreases and the shape of the pulse changes.

6. Existence of the bright and dark autosolitons in the system of excitons in double quantum well under the slot in the electrode was shown.

The paper analyses peculiarities of collective effects, caused by phases formation, at exciton movement in inhomogeneous external fields. The experimental observation of the effects studied in this paper would allow to determine the parameters of the exciton condensed phase and to investigate the possibility of applying exciton systems in optoelectronics.

REFERENCES

[1] T Fukuzawa, EE Mendez, and JM Hong. Physical review letters, 64(25):3066–3069, 1990.
[2] AV Larionov, VB Timofeev, J Hvam, and K Sørensen. Journal of Experimental and Theoretical Physics, 90(6):1093–1104, 2000.
[3] AA Dremin, AV Larionov, and VB Timofeev. Physics of the Solid State, 46(1):170–172, 2004.
[4] VB Timofeev. Physics-Uspekhi, 48(3):295–306, 2005.
[5] LV Butov, AC Gossard, and DS Chemla. Nature, 418(6899):751–754, 2002.
[6] D Snoke, S Denev, Y Liu, L Pfeiffer, and K West. Nature, 418(6899):754–757, 2002.
[7] AV Gorbunov and VB Timofeev. JETP letters, 84(6):329–334, 2006.
[8] AV Gorbunov and VB Timofeev. Physics-Uspekhi, 49(6):629–634, 2006.
[9] VB Timofeev, AV Gorbunov, and DA Demin. Low Temperature Physics, 37(17):176404, 2005.
[10] LS Levitov, BD Simons, and LV Butov. Physical review letters, 94(17):176404, 2005.
[11] AV Paraskevov and TV Khabarova. Physics Letters A, 368(1):151–154, 2007.
[12] RB Saptsov. JETP letters, 86(10):687–691, 2008.
[13] CS Liu, HG Luo, and WC Wu. Journal of Physics: Condensed Matter, 18(42):9659, 2006.
[14] VK Mukhomorov. Physics of the Solid State, 52(2):241–245, 2010.
[15] VI Sugakov. Solid state communications, 134(1):63–67, 2005. 
[16] AA Chernyuk and VI Sugakov. Physical Review B, 74(8):085303, 2006.
[17] VI Sugakov. Physical Review B, 76(11):115303, 2007.
[18] VI Sugakov and AA Chernyuk. JETP letters, 85(11):579–575, 2007.
[19] VI Sugakov. Journal of Physics: Condensed Matter, 21(27):275803, 2009.
[20] J Wilkes, EA Maljarov, and AL Ivanov. Physical review letters, 109(18):187402, 2012.
[21] SV Andreiev. arXiv preprint arXiv:1212.2748, 2012.
[22] VS Babichenko and I Ya Polishchuk. Pisma Zh. Ekp. Teor. Fiz, 97(11-12), 2013.
[23] AA High, AT Hammack, LV Butov, M Hanson, and AC Gossard. Optics letters, 32(17):2466–2468, 2007.
[24] AA High, EE Novitskaya, LV Butov, M Hanson, and AC Gossard. Science, 321(5886):229–231, 2008.
[25] AG Winbow, JR Leonard, M Remeika, YY Kuznetsova, AA High, AT Hammack, LV Butov, J Wilkes, AA Guenther, AL Ivanov, et al. Physical Review Letters, 106(19):196806, 2011.
[26] M Remeika, JC Graves, AT Hammack, AD Meyerholen, MM Fogler, LV Butov, M Hanson, and AC Gossard. Physical review letters, 102(18):186803, 2009.
[27] YY Kuznetsova, M Remeika, AA High, AT Hammack, LV Butov, M Hanson, and AC Gossard. Optics letters, 35(10):1587–1589, 2010.
[28] Yu E Lozovik and OL Berman. Journal of Experimental and Theoretical Physics Letters, 64(8):573–579, 1996.
[29] MYJ Tan, ND Drummound, and RJ Needs. Physical Review B, 71(3):033303, 2005.
[30] C Schindler and R Zimmermann. Physical Review B, 78(4):045313, 2008.
[31] AD Meyerholen and MM Fogler. Physical Review B, 78(23):235307, 2008.
[32] OI Dmytruk and VI Sugakov. Physics Letters A, 376(44):2804 – 2807, 2012.
[33] LD Landau. Electrodynamics of continuous media: Landau and lifshitz course of theoretical physics, vol. 8, 1998.
[34] JR Leonard, Mikas Remeika, MK Chu, YY Kuznetsova, AA High, LV Butov, J Wilkes, M Hanson, and AC Gossard. Applied Physics Letters, 100(23):231106–231106, 2012.
[35] VI Sugakov. arXiv preprint arXiv:1306.4876, 2013.
[36] BS Kerner and VV Osipov. Physics-Uspekhi, 32(2):101–138, 1989.