Charged Pseudospin Textures in Double-Layer Quantum Hall Systems with Spontaneous Interlayer Coherence

Kun Yang and A.H. MacDonald

Physics Department, Indiana University, Bloomington IN 47405

(November 1, 2018)

Abstract

We report on a Hartree-Fock approximation study of the meron pseudospin-texture excitations of the broken-symmetry incompressible ground states of double-layer quantum Hall states at $\nu = 1$. We have obtained results for meron core energies and core sizes. We use these to estimate the charge gap which determines the activation energy for dissipation in the quantum Hall effect.

75.10.-b, 73.20.Dx, 64.60.Cn
Recent work from several different points of view has led to the realization that double-layer two-dimensional electron systems (2DES) in a strong magnetic field can, under appropriate circumstances, exhibit an unusual broken symmetry in which interlayer phase coherence develops in the absence of interlayer tunneling. This broken symmetry has useful analogies with the spin-ferromagnetism which occurs in a single-layer 2DES in the quantum Hall regime, for example, at odd integral Landau level filling factors, \( \nu \). In the pseudospin representation commonly used for the (spin-polarized) double-layer system, spontaneous interlayer phase coherence is equivalent to the spontaneously broken \( U(1) \) symmetry of an easy-plane ferromagnet. In the limit of zero separation \( d \) between the layers, the (spin-polarized) double-layer system with no tunneling is equivalent to the single-layer system with no Zeeman coupling; in this limit the ferromagnetism of the pseudospin representation for the double-layer 2DES becomes isotropic. Interesting physical consequences follow from spontaneous interlayer phase coherence, including the occurrence of a finite temperature Kosterlitz-Thouless phase transition, superfluid-like behavior in the pseudospin channel, and a commensurate-incommensurate phase transition driven by an in-plane component of the external magnetic field in the presence of weak interlayer tunneling. Arguably, the most remarkable property associated with the broken symmetry is the occurrence of highly collective, topologically nontrivial pseudospin texture excitations which carry a physical charge equal (in units of the electron charge) to the topological charge of the texture. These charged pseudospin textures are analogous to the skyrmion charged spin textures of single-layer systems whose presence has recently been detected experimentally. In this paper we report on a Hartree-Fock calculation which estimates the energy of these excitations for double-layer 2DES’s at \( \nu = 1 \).

The fractional quantum Hall effect, an anomaly in the transport properties of 2DES’s in a strong magnetic field, occurs when the system has a charge gap, \( i.e., \) a discontinuity in the chemical potential, at a critical density which depends on magnetic field. (It is the magnetic field dependence of the critical density which leads to the charge carried by the spin and pseudospin textures.) At the critical density the dissipative conductivity, \( \sigma_{xx} \),
has an activated temperature dependence with an activation energy which, for the case of interest here, is half the charge gap. In single-layer systems, it is known that the lowest energy charged excitations at \( \nu = 1 \) (but not at larger odd integral values of \( \nu \)) are skyrmion spin-textures. It follows from continuity that in spin-polarized double-layer systems the charge gap must also be given by the energy of its charged spin textures, at least if the layer separation, \( d \), is not too large. In the double-layer case, where the broken symmetry is planar, vortices might be expected to play an important role. Indeed, it has been proposed that the charged pseudospin textures of double-layer systems consist of meron-antimeron pairs. Merons are vortex-like pseudospin textures where the pseudospin tilts out of the easy plane in the vortex core; neither the core size nor the core energy are fixed by the parameters which characterize the system at long length scales, and both must be determined by microscopic calculations. Merons have an infinite energy and carry charge \( \pm e/2 \), but meron pairs with like charges and opposite vorticities carry charge \( \pm e \) and have a finite energy. In this paper we evaluate the meron-core energy and size using a Hartree-Fock approximation and use the results to estimate the energy of the charge \( \pm e \) excitations of double-layer systems and to limit the range of validity of the meron pair description of these excitations.

In a recent paper, Moon et al. introduced single-Slater-determinant microscopic wave functions for both skyrmion and meron states near \( \nu = 1 \), for which the spin and pseudospin textures have the appropriate topological charges. The form of these wave functions provides an understanding of the connection between topological charge and physical charge from a microscopic physics point of view, as we discuss below, but the wavefunctions were not energetically optimized. For the single-layer case, Fertig et al. were able to find the optimal single-Slater-determinant wavefunctions for a skyrmion spin texture as a function of the strength of the Zeeman coupling by solving a set of Hartree-Fock equations. In this paper we generalize the Hartree-Fock calculations to the case of double-layer systems and find the optimal single-Slater-determinant approximation to the meron state as a function of \( d \). The technical details of our calculation are very similar to those of Fertig et al. However, meron states are quite different from skyrmion states and have richer physics as we demonstrate.
We assume that the Landau level spacing is so large that all electron stay in the lowest Landau level (LLL), hence we only need to keep one body states in the LLL. To take advantage of the circular symmetry of the meron state, we use the symmetric gauge in which the one body wave functions in the LLL have the form
\[ \phi_m(r) = (2\pi 2^m m!)^{-1/2} z^m \exp(-|z|^2/4), \]
where \(z = x + iy\) is the complex coordinate, \(m\) is the angular momentum quantum number and lengths are expressed in units of the magnetic length. \((\ell \equiv \hbar c/eB \text{ where } B \text{ is the magnetic field strength})\). In the limit \(d = 0\) (which is equivalent to the real spin case), inter- and intralayer Coulomb interaction have the same form and the system is invariant under pseudospin rotation.\cite{7,8} In the Hartree-Fock approximation, the ground state\cite{1,5} at \(\nu = 1\) in the standard pseudospin representation is completely polarized along an arbitrary direction in the \(\hat{x} - \hat{y}\) plane:
\[ |\Psi_0\rangle = \prod_{m=0}^{N-1} \left( \frac{1}{\sqrt{2}} c^\dagger_m \uparrow + e^{i\phi} \frac{1}{\sqrt{2}} c^\dagger_m \downarrow \right) |0\rangle. \tag{1} \]
Here \(|0\rangle\) is the vacuum state, \(c^\dagger_m \uparrow\) creates an electron in the \(\phi_m\) state in the upper (lower) layer, \(N\) is the number of electrons, and \(\phi\) is an arbitrary phase. (In the following we choose \(\phi = 0\) so that the pseudospin is oriented in the \(\hat{x}\) direction in the ground state.) Note that the orbitals are peaked farther from the origin as the angular momentum increases. For \(d = 0\), the state in Eq. (1) is the exact ground state\cite{8} and we expect it to remain accurate as long as \(d\) is not too large.

The single-Slater-determinant states for merons centered at the origin with vorticity +1 and with charge \(-e/2\) and \(+e/2\) have the form\cite{8}
\[ |\Psi_{+1,-\frac{1}{2}}\rangle = \prod_{m=0}^{N-1} (u_m c^\dagger_m \uparrow + v_m c^\dagger_{m+1} \downarrow) |0\rangle, \tag{2} \]
\[ |\Psi_{+1,\frac{1}{2}}\rangle = c^\dagger_{0\uparrow} \prod_{m=0}^{N-2} (-v_m c^\dagger_m \uparrow + u_m c^\dagger_{m+1} \downarrow) |0\rangle. \tag{3} \]
Here \(u_m\) and \(v_m\) are variational parameters which, for \(N \to \infty\), have the same values in both wavefunctions because of the particle-hole symmetry of the Hamiltonian\cite{8}, and which satisfy
the constraint $|u_m|^2 + |v_m|^2 = 1$ so that the wavefunction is normalized. The easy-plane anisotropy of the double-layer system requires that

$$\lim_{m \to \infty} \frac{|u_m|}{|v_m|} = 1,$$

so that at the charge density is the same for both layers far enough from the meron center and the electrostatic energy cost of the meron texture is finite. The charges of the two states are $\pm e/2$ as can be verified by evaluating the differences in mean occupation numbers between the meron states and the ground state.

In this paper we assume, without loss of generality, that $u_m$ and $v_m$ are real. Far from the origin, single-particle orbitals with adjacent angular momenta have nearly identical magnitudes near their peaks where they satisfy $\phi_{m+1}(r)/\phi_m(r) \approx \exp(i\phi)$ where $\phi$ is the angular coordinate. It follows that for large $m$, the single-particle state in the Slater determinant is effectively proportional to the pseudospinor whose azimuthal angle is equal to the angular coordinate:

$$\chi(\phi) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \cos(\theta/2) \\ \sin(\theta/2) \exp(i\phi) \end{array} \right).$$

(We have imposed the normalization constraint by setting $u_m = \cos(\theta_m/2)$ and $v_m = \sin(\theta_m/2)$. $\theta$ is the polar angle for the pseudospin contribution from orbital $m$. At large $m$, $\theta = \pi/2$, the pseudospinor is confined to the $\hat{x} - \hat{y}$ plane and has vorticity $+1$. At the center of the meron only $\phi_{m=0}(r) \neq 0$ so that the pseudospinor points up for the charge $-e/2$ meron and down for the charge $e/2$ meron. Meron states with opposite pseudospin vorticity can be generated by interchanging the roles of the two layers.

The Hartree-Fock equations we solve determine the $u_m$ and $v_m$ which minimize the expectation value of the Hamiltonian in states of the form of Eq. (3) and Eq. (4). We perform our calculation at finite $N$ (up to $N = 60$) and to mitigate finite-size effects we include in the Hamiltonian the external potential from a fixed background which neutralizes the ground state charge density. The full Hamiltonian of the system is
\[ H = \frac{1}{2} \sum_{m_1m_2m_3m_4} \sum_{\sigma_1\sigma_2} V_{m_1m_2m_3m_4}^{\sigma_1\sigma_2} c_{m_1\sigma_1}^{\dagger} c_{m_2\sigma_2} c_{m_4\sigma_2} c_{m_3\sigma_1} \]
\[ - \sum_{m} \sum_{\sigma} c_{m\sigma}^{\dagger} c_{m\sigma} \left[ \sum_{n=0}^{N-1} \frac{1}{2} (V_{mnmn}^A + V_{mnmn}^E) \right], \] (6)

where \( \sigma_i = \uparrow \) or \( \downarrow \); \( V_{\uparrow\uparrow} = V_{\downarrow\downarrow} = V^A \) is the intralayer interaction and \( V_{\uparrow\downarrow} = V_{\downarrow\uparrow} \) is the interlayer interaction. For the sake of definiteness we have taken the electron layers to have negligible thickness so that

\[ V_{m_1m_2m_3m_4}^A = \int d\mathbf{r}_1 d\mathbf{r}_2 \phi_{m_1}^{\ast} (\mathbf{r}_1) \phi_{m_2}^{\ast} (\mathbf{r}_2) \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \phi_{m_3} (\mathbf{r}_1) \phi_{m_4} (\mathbf{r}_2), \] (7)

and

\[ V_{m_1m_2m_3m_4}^E = \int d\mathbf{r}_1 d\mathbf{r}_2 \phi_{m_1}^{\ast} (\mathbf{r}_1) \phi_{m_2}^{\ast} (\mathbf{r}_2) \frac{e^2}{\sqrt{|\mathbf{r}_1 - \mathbf{r}_2|^2 + d^2}} \phi_{m_3} (\mathbf{r}_1) \phi_{m_4} (\mathbf{r}_2). \] (8)

The one body term in (6) is due to the interaction of the electrons with the neutralizing background charge; it reduces to an irrelevant constant for \( N \to \infty \) and, although we include it in our numerical calculations, we omit it to free the discussion below from unnecessary clutter.

An elementary but somewhat tedious calculation gives the following result for the expectation value of the Hamiltonian in the charge \(-e/2\) meron state:

\[ \langle \Psi_{+1,-\frac{1}{2}} | H | \Psi_{+1,-\frac{1}{2}} \rangle = \frac{1}{2} \sum_{m,n=0}^{N-1} \left[ u_m \left( V_{mnmn}^A - V_{mnmn}^A \right) \right. \]
\[ + u_n \left( V_{m+1,n+1,m+1,n+1}^A - V_{m+1,n+1,m+1,n+1}^A \right) \]
\[ + 2u_m \left( V_{m,n+1,n+1,m+1}^E - V_{m+1,n+1,m+1,n+1}^E \right). \] (9)

Minimizing Eq. (3) with respect to \( \theta_m \) gives the following equation which must be solved self-consistently for the pseudospinor polar angles,

\[ \tan(\theta_m) = A_m / B_m, \] (10)

where

\[ A_m = -2 \sum_{n=0}^{N-1} V_{m,n+1,n,m+1}^E \sin(\theta_n) \] (11)
\[ B_m = \sum_{n=0}^{N-1} \left[ V_{mnmn}^A - V_{mnmm}^A - V_{m+1,n+1,m+1,n+1}^A + V_{m+1,n+1,m+1,n+1}^A + V_{m,n+1,m+1,n+1}^E - V_{m,n+1,m+1,n+1}^E - V_{n,m+1,n+1,m+1}^E + \cos(\theta_n)(V_{mnmn}^A + V_{m+1,n+1,m+1,n+1}^A) \right]. \] (12)

Similar equations can be obtained for merons with charge \( +e/2 \). The various terms in \( A_m \) and \( B_m \) can be understood rather simply. For large \( m \) the matrix elements are insensitive to unit shifts in the angular momentum indices so that the constant terms in the denominator vanish rather quickly. These unit angular momentum shifts are important in determining the the structure of the meron core but we will temporarily ignore them for the following qualitative discussion. The term proportional to \( \cos(\theta_n) \) in the denominator is the sum of intralayer exchange matrix elements and the difference of direct matrix elements for intralayer and interlayer interactions. If \( V^A \) were equal to \( V^E \) Eq. (10) would be satisfied for any constant \( \theta_n \), corresponding to the pseudospin isotropy at \( d = 0 \). The difference between intralayer and interlayer matrix elements which appears in the denominator of Eq. (10) gradually forces \( \theta_m \) to \( \pi/2 \) as we move away from the meron core.

The most important result of our calculation is shown in Fig. [1] where we plot our estimates of the chemical potentials at densities just larger and just smaller than the critical density at which the incompressible state occurs, \( \mu^+_{MP} \) and \( \mu^-_{MP} \) respectively. These chemical potentials were obtained from the optimized meron state energies by the following procedure. For each value of \( d \) the difference between the meron state energy and the ground state energy, \( E_m^\pm \) was evaluated at a series of \( N \) values. The meron core energies, \( E_{mc}^\pm \) were extracted by fitting our results to the expected form

\[ E_m^\pm = E_{mc}^\pm + \pi \rho_E \ln(R/R_{mc}), \] (13)

where \( R = \sqrt{2(N-1)\ell} \) is the radius of the system, \( \rho_E \) is the Hartree-Fock approximation for the in-plane pseudospin stiffness,

\[ \rho_E = \frac{1}{32\pi^2} \int_0^\infty dk k^3 V^E(k) e^{-k^2/2}, \] (14)
$V_E(k)$ is the Fourier transform of the interlayer interaction. The meron core radius $R_{mc}$ (which is identical for merons with charges $\pm e/2$) was defined by $m^z(R_{mc}) = 4\pi\ell^2\langle S^z(R_{mc}) \rangle = 0.1$, where $S^z(\mathbf{r})$ is the $z$ component of the pseudo spin density operator. The energy of an individual meron is infinite because of the large distance contribution to the gradient energy, but the energy of a meron pair of opposite vorticity is finite. The optimal separation $R^*$, for the meron pair is determined by minimizing the sum of the Coulomb energy and the gradient energy. For $R \gg R_{mc}$ we find that $R^* = e^2/8\pi\rho_E$ and $E_{MP}^\pm = 2E_{mc}^\pm + 2\pi\rho_E(1 + \ln(R^*/R_{mc}))$. In Fig. 2 we plot $R^*$ and $R_{mc}$ as a function of $d$; the meron pair picture of the charge excitations is valid only when $R > R_{mc}$ which is satisfied for $d/\ell > 0.6$. Results at smaller values of $d/\ell$ in Fig. 1 and Fig. 2 are plotted as dotted lines. $E_{MP}^\pm$ is the energy to make a meron pair at fixed total electron number by adding or removing charge from the edge of the system to compensate for the charge of a meron; to add or remove an electron it is necessary to change the electron number in the incompressible state. It follows that

$$
\mu_{MP}^\pm = \pm E_{MP}^\pm + \epsilon_0(d)
$$

where $\epsilon_0(d)$ is the energy per electron in the incompressible state. We remark that as a consequence of a particle-hole symmetry which applies in the thermodynamic limit $\mu_{MP}^+ + \mu_{MP}^- = 2\epsilon_0(0)$. We have used this identity to check the reliability of our numerical procedures, particularly those involved in the extraction of the meron core energy from the size dependence of the meron energy. For $d/\ell > \approx 1.2$ we do not believe that our estimates are reliable; results in this range of $d/\ell$ are also shown as dotted lines in Fig. 1 and Fig. 2. The problem at these values of $d/\ell$ is connected with the fact that the Hartree-Fock ground state at $\nu = 1$ is a pseudospin-density-wave for $d/\ell > \approx 1.2$. Within the range of $d$ that the meron pair picture of charge excitations is valid, the meron-pair states are clearly energetically more favorable than naive Hartree-Fock single particle states (see Fig. 1) and our results agree with exact diagonalization studies qualitatively.

We thank L. Brey, R. Côté, H.A. Fertig, S.M. Girvin, F.D.M. Haldane, K. Moon, N.
Read, and S. Sondhi for helpful discussions. This work was supported by NSF grants DMR-9416906 and DMR-9224077.
REFERENCES

* Current address: Department of Electrical Engineering, Princeton University, Princeton, NJ 08544.

1 H.A. Fertig, Phys. Rev. B 40, 1087 (1989).

2 X.G. Wen and A. Zee, Phys. Rev. Lett. 69, 1811 (1992); X.G. Wen and A. Zee, Phys. Rev. B 47, 2265 (1993).

3 Z. F. Ezawa and A. Iwazaki, Int. J. Mod. Phys. B 19, 3205 (1992).

4 A.H. MacDonald, P.M. Platzman, and G.S. Boebinger, Phys. Rev. Lett. 65, 775 (1990); Luis Brey, Phys. Rev. Lett. 65, 903 (1990); O. Narikiyo and D. Yoshioka, J. Phys. Soc. Jpn. 62, 1612 (1993).

5 X.M. Chen and J.J. Quinn, Phys. Rev. B 45, 11054 (1992); R. Côté, L. Brey, and A.H. MacDonald, Phys. Rev. B 46, 10239 (1992).

6 S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B 47, 16419 (1993); X.G. Wu and S.L. Sondhi, preprint (1995); E.H. Rezayi, Phys. Rev. B 36, 5454 (1987).

7 Kun Yang, K. Moon, L. Zheng, A. H. MacDonald, S. M. Girvin, D. Yoshioka and Shou-Cheng Zhang, Phys. Rev. Lett. 72, 732 (1994).

8 K. Moon, H. Mori, Kun Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka and Shou-Cheng Zhang, Phys. Rev. B 51, 5138 (1995).

9 S. Q. Murphy et al., Phys. Rev. Lett. 72, 728 (1994).

10 K. Moon, H. Mori, Kun Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka and Shou-Cheng Zhang, in preparation. To be submitted to Phys. Rev. B (1995).

11 D.-H. Lee and C. L. Kane, Phys. Rev. Lett. 64, 1313 (1990).
12 H. A. Fertig, L. Brey, R. Côté and A. H. MacDonald, Phys. Rev. B 50, 11018 (1994).

13 S.E. Barrett, G. Dabbagh, L.N. Pfeiffer, K.W. West, and R. Tycko, preprint (1995).

14 See for example A.H. MacDonald, *Introduction to the Quantum Hall Effect* in the Proceedings of the Les Houches Summer School on Mesoscopic Physics, edited by Eric Akkermans, Gilles Montambaux and Jean-Louis Pichard, to appear.

15 A.H. MacDonald and S.M. Girvin, Phys. Rev. B 34, 5639 (1986).

16 Kun Yang and A.H. MacDonald, unpublished.

17 Although the Hartree-Fock approximation correctly points out that there is a phase transition happening at $d = d^*$, it is believed that when $d > d^*$ the system is a compressible phase with no broken symmetry instead of a SDW state implied by Hartree-Fock. We are only interested in meron excitations in the broken U(1) symmetry phase, where the Hartree-Fock approximation is adequate unless $d$ is very close to $d^*$. 
FIGURES

FIG. 1. Estimates of the chemical potentials (in units of $e^2/\epsilon \ell$) just above ($\mu^+_{MP}$) and below ($\mu^-_{MP}$) $\nu = 1$ as a function of $d/\ell$, assuming that the meron pair state is a good description of the charged excitations. The dotted portions of the lines are for the ranges of $d$ were the meron pair picture is invalid and our estimate is unreliable. The difference between $\mu^+_{MP}$ and $\mu^-_{MP}$ is the charge gap of the system at $\nu = 1$. For comparison we also plot the chemical potentials for the Hartree-Fock single particle excitations (dashed lines) which cost more energy than the meron pairs.

FIG. 2. Estimates of the optimal meron core size $R_{mc}$ and optimal separation $R^*$ between merons in a charged meron pair state as a function of $d/\ell$. The dotted portions of these curves are in ranges of layer separation where the meron pair picture is not valid.