Evolution of low-mass binaries with black-hole components

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Abstract

We consider evolutionary models for the population of short-period ($P_{\text{orb}} \lesssim 10$ hr) low-mass black-hole binaries (LMBHB) and compare them with observations of soft X-ray transients (SXT). Evolution of LMBHB is determined by nuclear evolution of the donors and/or orbital angular momentum loss due to magnetic braking by the stellar wind of the donors and gravitational wave radiation. We show that the absence of observed stable luminous LMBHB implies that upon RLOF by the low-mass donor angular momentum losses are substantially reduced with respect to the Verbunt & Zwaan "standard" prescription for magnetic braking. Under this assumption masses and effective temperatures of the model secondaries of LMBHB are in a satisfactory agreement with the masses and effective temperatures (as inferred from their spectra) of the observed donors in LMBHB. Theoretical mass-transfer rates in SXTs are consistent with the observed ones if one assumes that accretion discs in these systems are truncated ("leaky"). We find that the population of short-period SXT is formed mainly by systems which had unevolved or slightly evolved ($X_c \gtrsim 0.35$) donors at the Roche-lobe overflow. Longer period ($P_{\text{orb}} \simeq (0.5 - 1)$ day) SXT might descend from systems with initial donor mass about 1 M\textsubscript{\odot} and $X_c \lesssim 0.35$. It is unnecessary to invoke donors with almost hydrogen-depleted cores to explain the origin of LMBHB. Our models suggest that a very high efficiency of common-envelopes ejection is necessary to form LMBHB, unless currently commonly accepted empirical estimates of mass-loss rates by winds for pre-WR and WR-stars are significantly over-evaluated.

Key words: Binaries: close, stars: evolution, X-rays: binaries
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1. Introduction

Out of twenty dynamically-confirmed black-hole candidate X-ray binaries ten have K/M spectral type secondaries and orbital periods $\lesssim 1$ day (Remillard and McClintock, 2006). All these systems are transient in X-ray. Their quiescent X-ray luminosity is $\simeq (3 \times 10^{30} - 3 \times 10^{33})$ erg s\textsuperscript{-1}, while in the outburst luminosity may be by a factor $\sim 10^5 - 10^8$ higher. These low-mass black-hole binary (LMBHB) X-ray systems belong to the class of “Soft X-ray transients” (SXT). Recent reviews of observational data on SXT may be found e.g. in Charles and Coe (2006), Remillard and McClintock (2006). From the evolutionary stand-point SXT are considered as semidetached binaries in which matter is transferred to the compact object via accretion disc. The commonly accepted model for the variability of SXT is based on the thermal-viscous instability of irradiated accretion discs (see Lasota, 2001 and references therein). The estimated number of SXT in the Galaxy ranges from several hundred (Chen et al, 1997) to several thousand (Romani, 1998). Below, we will describe the models...
of a population of short-period LMBHB, discuss their observational properties, and consider possible existence of a population of faint LMBHB with quasi-stable, cold accretion discs (Menou et al., 1999). This review is based mainly on the studies by Yungelson et al. (2006) (henceforth, Paper I) and Yungelson & Lasota (in prep., Paper II).

2. The model

2.1. Formation of black holes with low-mass companions

Modeling of the Galactic population of LMBHB includes two major steps: (a) modeling of the population of zero-age binaries containing a black hole accompanied by a low-mass companion and (b) following the evolution of the system to Hubble time.

The population of zero-age LMBHB was computed (Paper I) using population synthesis code SEBA (Portegies Zwart and Verbunt, 1996, Portegies Zwart and Yungelson, 1998, Nelemans et al., 2001, Nelemans et al., 2004) with 250000 initial binaries with $M_{10} \geq 25M_\odot$. Assumed time- and position-dependent Galactic star formation history is based on the model of Boissier and Prantzos (1999), but for the inner 3 kpc of the Galaxy we doubled the star formation rate given in the latter study to mimic Galactic bulge. We assumed a 50% binarity rate (2/3 of stars in binaries), an IMF after Kroupa et al. (1993), an initial distribution of separations in binaries (a) flat in log $a$ between contact and $10^6 R_\odot$, a flat mass ratio distribution, and initial distribution of eccentricities of orbits $\Xi(e) = 2e$. A summary of the assumptions on transformations of stellar masses in the course of the evolution is shown in Fig. 1. Black hole progenitors have $M_b = 25 - 100M_\odot$. The algorithm for formation of black holes in the code follows the fall-back scenario (Fryer and Kalogera, 2001) under assumption that explosion energy is fixed at $10^{56}$ ergs, which is within the expected range but favours formation of rather massive black holes (up to $\sim 15M_\odot$). Nascent black holes receive an asymmetric kick at formation following Paczyński (1990) velocity distribution with $\sigma_v = 300$ km s$^{-1}$, but scaled down with the ratio of the black hole mass to a neutron star mass. The population of LMBHB is not sensitive to the assumed kick distribution, since scaled down with $M_{bh}/M_{ns}$-ratio kick velocities are too small to disrupt close binaries in SN explosions. Test run for Maxwellian kick distribution for pulsars with $\sigma = 265$ km/s (Hobbs et al., 2005) confirmed this assumption. For the common-envelope phase we used the standard prescription (Webbink, 1984; de Kool et al., 1987), with the efficiency and structure parameters $\alpha_{CE}$ and $\lambda$ product $\alpha_{CE} \times \lambda = 2$ (see discussion of the issue of $\alpha_{CE} \times \lambda$ in [5]). For more detailed description of input parameters we refer the reader to Paper I.

2.2. Angular momentum loss

Since low-mass components in LMBHB have KV/MV spectral types, it is usually assumed that evolution of LMBHB is governed by angular momentum losses (AML), like for cataclysmic variables. In our computations we assumed that AML via magnetic stellar wind (MSW) follows the model suggested by Verbunt and Zwaan (1981):

$$\frac{d\ln J}{dt} = -\frac{1}{2} \times 10^{-28} k^2 \left(\frac{2\pi}{F}\right)^{10/3} \frac{1}{G^{2/3}} \frac{M_t^{1/3} R_t^4}{M_1 f^2}$$  \hspace{1cm} (1)$$

where $M_t$ is mass of the primary, $M_t - \text{total mass of the system}$, $P - \text{orbital period}$, $R - \text{radius of the secondary}$, $k^2 \sim 0.1 - \text{its gyration radius}$, $f \sim 1 - \text{a parameter derived from observations}$. Note, that this model extrapolates via almost two orders in magnitude stellar rotation braking law derived by Skumanich (1972) for single field stars to rapidly rotating components of close binaries and assumes efficient spin-orbital coupling.

Angular momentum loss via gravitational waves radiation (GWR) is described by the standard Landau and Lifshitz (1971) formula.
Fig. 2. Fractions of low-mass bh+ms binaries formed at different epochs in the lifetime of the Galaxy (solid line) and the shape of star formation rate (SFR) adopted in present study (dashed line). At maximum, the SFR is \( \approx 15.6 \, M_\odot \, yr^{-1} \). Current SFR is \( \approx 1.9 \, M_\odot \, yr^{-1} \).

\[
\frac{d\ln J}{dt} = -\frac{32}{5} \frac{G^{5/3}}{c^5} \left( \frac{2\pi}{P} \right)^{8/3} \frac{M_1 M_2}{(M_1 + M_2)^{1/3}}.
\]

(2)

Below, we will refer to the model in which semidetached systems obey Eqs. (1) and (2) as to a “standard” model.

2.3. Population of unevolved LMBHB

With our assumptions we find that, within a Hubble time (13.5 Gyr), \( \approx 49,000 \) binaries that have orbital periods below 2.0 day and contain black holes accompanied by main-sequence stars with \( M_{20} < 1.6 \, M_\odot \) (for which magnetic braking is supposed to operate) were formed in the Galaxy. Out of them, \( \approx 17,000 \) were brought into contact by AML via MSW and/or GWR and \( \approx 14,000 \) evolved to shorter periods under “standard” assumptions on MSW (see § 2.2).

Because black holes form in the first \( \approx 3 \) Myr after their progenitor formation, formation history of bh+ms binaries strictly follows the star formation history (Fig. 2). Figure 3 shows relations between initial masses of components in model systems and their post-circularization orbital period. Solid lines in the upper panel of the Figure outline the mass and period range from which systems may be brought in the contact via action of AML, however, in “standard” model only systems that overflow Roche lobes at periods less than \( \approx 15 \) hr evolve to shorter periods (the rest have small helium cores and evolve to longer periods, forming at the end helium white dwarfs, if time span between contact and Hubble time is large enough; otherwise, at present their donors are subgiants).

3. Evolution of LMBHB

For simulation of the current Galactic population of semidetached LMBHB we convolved the above-described “underlying” population of binaries born at different epochs in the history of the Galaxy with the grids of evolutionary tracks for low-mass components in the binaries with different combinations of masses of components and post-circularization (initial) orbital periods. For
evolutionary computations, appropriately modified TWIN version (September 2003) of Eggleton (1971) evolutionary code was used. Every “initial” system was “evolved” through 13.5 Gyr − T0, where T0 is the moment of formation. In the Figures presented below, we show only systems that have at present \( q = M_2/M_1 \geq 0.02 \). At \( q \lesssim 0.02 \) the circularization radius of accretion stream becomes greater than the outer radius of the accretion disc, resonance phenomena in the disc become important and, though computations may be continued in the same fashion as before, until models cease to converge due to very low mass, actually it is unclear how mass transfer occurs (see Paper I for more details). The systems with \( q < 0.02 \) have \( P_{\text{orb}} \lesssim 2 \) hr and mass-transfer rates \((<10^{-10} \text{ M}_\odot \text{ yr}^{-1})\). In the “standard” model currently more than 75% of all low-mass ms+bh systems have \( q < 0.02 \) and have yet to be observed if mass-transfer occurs in them.

### 3.1. Luminous persistent LMBHB?

Figure 4 shows the distribution of systems with \( q > 0.02 \) in the “standard” model over mass-transfer rates. The upper two panels show the break-down of the binaries that have stable or unstable accretion discs (370 and 2900 objects, respectively). Irradiated discs are hot and stable if accretion rate (in \( \text{M}_\odot \text{ yr}^{-1} \)) exceeds (Dubus et al., 1999)

\[
\dot{M}_{\text{crit}}^+ \approx 2.4 \times 10^{-11} \text{M}_\odot^{-0.4} \text{R}_{d,10}^{-1.5} \left( \frac{C}{5 \times 10^{-4}} \right)^{-0.5},
\]

where \( M_{\text{BH}} \) is the mass of the accretor in \( \text{M}_\odot \), \( R_{d,10} \) is the disc outer radius in \( 10^{10} \text{ cm} \), and \( C \) is a measure of heating of the disc by X-rays (Shakura and Sunyaev, 1973); for a given disc radius \( R \) the irradiation temperature \( T_{\text{irr}} \) is

\[
T_{\text{irr}}^4 = C \frac{\dot{M} \text{c}^2}{4\pi\sigma R^2},
\]

where \( \sigma \) is the Stefan-Boltzmann constant. We use \( C = 5 \times 10^{-4} \), following (Dubus et al., 1999) who have found that this value is consistent with properties of persistent low-mass X-ray sources, but actually \( C \) might e.g. vary in time (Esin et al., 2000).

However, no persistent LMBHB with \( P_{\text{orb}} \lesssim 4 \) hr are observed\(^3\). Stable model systems have an average luminosity \( L = 0.1\dot{M}\text{c}^2 \approx 5 \times 10^{36} \text{ erg s}^{-1} \). The accretion luminosity may be assumed to be emitted as that of a 1 keV blackbody, corresponding to the “high/soft” X-ray spectral state of LMBHB. Given their location in the Galaxy provided by the population synthesis code, we may roughly estimate intervening X-ray absorption. We found that the number of persistent systems expected above the completeness limit for the X-ray sources seen by the RXTE All Sky Monitor (about 3 mCrab or 0.2 counts/s) in the 2-10 keV band is 85. Setting the threshold ten times higher still yields 13 bright, persistent LMBHB with ASM fluxes comparable to e.g. SMC X-1. Noticeable number of stable “observed” persistent model sources remain in the model also under another assumptions on the spectrum of their emission (see for details Paper I). If the disc were not irradiated, no systems would be stable. However, both persistent neutron-star low-mass X-ray binaries and outbursting black hole transients show clear signatures of irradiation. Note that with parameter \( C = \)

\[\text{Fig. 4. Mass transfer rates vs. orbital period for the model population of semidetached low-mass binaries with assumed black hole accretors. Upper panel – stable systems in the model for a “standard” Verbunt-Zwaan assumption on AML via MSW. Middle panel – population of unstable systems in the same model. Lower panel – population for the model in which MSW does not operate in semidetached systems. In this model all binaries have unstable discs. The arrows mark approximate upper limits to the average mass-transfer rates for SXT under assumption of “leaky” discs, while plusses show estimates of rates based on assumed recurrence period of 30 years, except for V616 Mon and IL Lup whose recurrence times are known to be \sim 50 and 10 years, respectively (see § 3.3.2 for details).}\]
5 \times 10^{-4}, only a 0.5% of the accretion luminosity (efficiency of 10%) is reprocessed in the disc.

Reduction of $d\ln J/dt$ by factor 2 compared to Eq. (1) does not change the results significantly (see Paper I) and points to the necessity of stronger re-duction of AML for semidetached systems.

### 3.2. Reduced angular momentum loss?

The problem of production of unobserved persistent low-mass black-hole X-ray binaries under “standard” assumptions was noted before e.g. by King et al. (1996); Ergma and Fedorova (1998); Menou et al. (1999); Ivanova and Kalogera (2006). Skumanich (1972) “law” on which Verbunt and Zwaan (1981) model is based, is apparently in conflict with observational data on rotation velocities in young open clusters (Collier Cameron, 2002; Andronov et al., 2003). According to the latter study, the time-scale of rotation braking is two orders of magnitude longer than the one based on Skumanich law. Necessity for “weaker” than Verbunt & Zwaan AML by MSW was suggested e.g. by Hameury et al. (1988); Ivanova and Taam (2003) for cataclysmic variables and by van Paradijs (1996) for SXT.

In Paper I we suggested, in line with observational evidence mentioned above, that magnetic braking operates on a much reduced scale (as compared with Verbunt and Zwaan (1981) prescription) or that it does not operate at all in the semidetached systems with black-hole accretors. As a test of this hypothesis, we computed a population of LMBHB under assumption that MSW is not operating once the RLOF occurs.

Since the mechanism that brings the systems to the RLOF is the same both in “standard” and in “no-MSW” cases, the number of semidetached systems is the same in both models ($\approx 14000$), but in the latter case, because of a weaker AML, there are currently in the Galaxy about 5000 LMBHB that evolve to shorter periods and have $q \geq 0.02$ (compared to $\approx 3200$ in the former case). All these systems are transient according to the criterion (3).

Table 1

| Object          | $P_{\text{orb}}$, hr | $Sp$ | $q$ | $M_{\text{bh}}$, $M_{\odot}$ |
|-----------------|----------------------|------|-----|-------------------------------|
| 1 XTE J1118+480 | 4.10                 | K5V-M1V | 0.044 - 0.035 | 6.5 - 7.2 |
| 2 GRO J0422+32  | 5.09                 | M2 ± 2V | 0.313 - 0.008 | 3.7 - 5.0 |
| 3 GRS 1009-45   | 6.84                 | G5V-M0V | 0.159 - 0.125 | 3.6 - 4.7 |
| 4 XTE J1650-500 | 7.68                 | K4 V   | 0.1   | 3.4 - 7.9 |
| 5 A0620-00      | 7.75                 | K3V-K7V | 0.075 - 0.055 | 8.7 - 12.9 |
| 6 GS 2000+25    | 8.28                 | K3V-K6V | 0.053 - 0.035 | 7.2 - 7.8 |
| 7 XTE J1859+226 | 9.12                 | G5V-K0V |
| 8 GRS 1124-68   | 10.39                | K3V-K7V | 0.208 - 0.114 | 6.5 - 8.2 |
| 9 H 1705-25     | 12.50                | K3V-K7V | $< 0.053$ | 5.6 - 8.3 |

In the “no-MSW” case, donor-stars in the LMBHB have $X_c \approx 0.35$ at RLOF, while in the “standard” case all systems which did not exhaust hydrogen in the core to the instant of the RLOF evolve to the shorter $P_{\text{orb}}$. This also means that the range of initial periods of systems in “no MSW” case is smaller: it extends to $\approx 35$ hr.

### 3.3. Observational parameters of short-period LMBHB

In Table 1 we summarized the main parameters of short orbital period SXT that are important from evolutionary point of view. The range of the spectral types assigned to secondaries is based on published data, see for references Paper II; the values of mass ratios of components $q = M_2/M_{\text{bh}}$ and the estimates of black-hole masses are taken from Orosz (2003); Orosz et al. (2004). The population is plotted in the lower panel of Fig 3.

Our estimate of the number of SXT apparently exceeds the estimates based on observations. Assuming that their outbursts peak on average at $0.1 L_{\text{edd}}$ in X-rays, all of these transients should be seen in outburst by the RXTE ASM, regardless of their location in the Galaxy. Matching the observed discovery rate of SXT would require the outbursts to be sub-Eddington at maximum and/or recurrence times to be $\approx 100$ years for most sources.

In the “no-MSW” case, donor-stars in the LMBHB have $X_c \approx 0.35$ at RLOF, while in the “standard” case all systems which did not exhaust hydrogen in the core to the instant of the RLOF evolve to the shorter $P_{\text{orb}}$. This also means that the range of initial periods of systems in “no MSW” case is smaller: it extends to $\approx 35$ hr.

#### 3.3.1. Effective temperatures and masses of donors

The essential information available on the companion-stars in SXT is their spectral types. Determination of the latter is a challenging task since the emission of the cool star is contaminated by the radiation from the accretion disc and the hot spot where the accretion stream hits the disc’s edge (Charles and Cox, 2006). Different methods of $Sp$-determination result in a scatter of assigned spectra that may amount to several subtypes (Table 1). In the absence of direct determinations of such pa-
adopted for the present study and tracks that turn to longer periods immediately evolve upon RLOF continuously to shorter periods. We should note the following. In order to avoid interpolating between pre-computed tracks that produce MSW operating after RLOF and (1+4) \( M_\odot \) system with MSW operating after RLOF (see §3.3.1 for data on tracks and discussion). \( S_p - T_{\text{eff}} \) relation used in the paper is shown at the right border of the coordinate box.

In our model we reduced AML by MSW to zero. In reality some AML by MSW can be still operating. We plot in Figs. 5 and 6 two “limiting” tracks: for (1+12) \( M_\odot \), \( P_0 \approx 9 \) hr and for (1.1+4) \( M_\odot \), \( P_0 = 45.6 \) hr system with MSW operating after RLOF (see §3.3.1 for data on tracks and discussion). \( S_p - T_{\text{eff}} \) relation used in the paper is shown at the right border of the coordinate box.

Within uncertainties of the \( S_p \)-determinations and conversion \( S_p - T_{\text{eff}} \) the model satisfactorily reproduces \( T_{\text{eff}} \) and \( M_2 \) of the donors in the LMBHB with \( P_{\text{orb}} \approx 9 \) hr. Several systems – XTE 1650-500, A0620-00, and GS 2000+25 – seem to be located below the “populated” areas of the plots. However, we should note the following. In order to avoid interpolation between pre-computed tracks that evolve upon RLOF continuously to shorter periods and tracks that turn to longer periods immediately or change the direction of evolution in \( P_{\text{orb}} \), we restricted ourselves by binaries that evolve to short \( P_{\text{orb}} \). For a system with given \( M_{10} \) and \( M_{20} \) the direction of evolution changes quite abruptly, over a narrow range of initial \( \Delta P_{\text{orb}} \approx 0.1 \) day. For a given combination of \( M_{10} \) and \( M_{20} \) a “gap” between tracks evolving in different directions forms. This is seen from Fig. 5 where we plot evolutionary tracks for \( (1+4) M_\odot \), \( P_0 = 1.4 \) and 1.5 day and for \( (1.1+4) M_\odot \), \( P_0 = 1.3 \), 1.4, and 1.45 day systems. But since there is a continuity in the initial parameters of the systems, the “gap” has to be filled by the systems that start RLOF in the well populated area in the \( P_0 - M_{20} \) diagram (Fig. 3). Thus it is at least qualitatively clear that the origin of short-period LMBHB may be explained within paradigm of strong reduction of magnetic braking in systems with donors overflowing Roche lobes.

In our model we reduced AML by MSW to zero. In reality some AML by MSW can be still operating. We plot in Figs. 5 and 6 two “limiting” tracks: for (1+12) \( M_\odot \), \( P_{\text{orb},0} = 9.6 \) hr in which the donor overflows Roche lobe almost unevolved and MSW is absent after RLOF and for (1+4) \( M_\odot \), \( P_{\text{orb},0} = 45.6 \) hr in which donor has \( X_c \approx 10^{-4} \) at RLOF and MSW continues to operate. Crudeely, model populations with MSW and without MSW that evolve to short \( P_{\text{orb}} \) have to be located between these two limiting curves. There is a contribution of lower and higher mass systems of course as we plotted tracks.
for $1M_\odot$ donors for simplicity. Adding some AML to our model will shift the population to the right, giving a better agreement with observations while still not producing stable luminous X-ray sources. From Fig. 6 in Paper I it is seen that such an adding of AML will influence mainly the long-period systems.

Our computations for the “no-MSW” model show that LMBHB evolve to longer periods if $X_c \lesssim 0.35$ at RLOF. For instance a $1.1M_\odot$ donor with $P_{\text{orb}} = 1.4$ (middle curve) spends in the RLOF-state almost 10 Gyr, out of which about 5 Gyr it evolves to longer periods. This provides a possibility of explaining SXTs with periods $> 9$-10 hr.

3.3.2. Mass-transfer rates

Our model predicts the distribution of SXT over mass-transfer rates $\dot{M}$ that can also be compared to $\dot{M}$ deduced from observations of SXT sources. Usually, $\dot{M}$ for SXT are evaluated by dividing the mass accreted during outburst by the recurrence time (see e.g. White and van Paradijs 1996). Among short-period SXT the recurrence time is known only for A0620-00 (about 60 years) and 4U 1543-47 (about 10 years). For the other systems one can only obtain upper limits of $\dot{M}$ since only one outburst has been seen in the X-ray observations epoch (30 years are usually assumed). These estimates of $\dot{M}$, derived from the data on outburst parameters from Chen et al. (1997) and for KV UMa and V406 Vul based of observations presented in Chaty et al. (2003) and Hynes et al. (2002) respectively, are shown as plusses in Figs. 4 and 7.

For the “standard” model all above-described $\dot{M}$-estimates are inconsistent with the model (two upper panels of Fig. 4). Even if only GWR acts, $\dot{M} \sim 10^{-12} M_\odot$ yr$^{-1}$ cannot be explained if one assumes that they represent secular values (lower panel of Fig. 4). It would take post-minimum period systems several Hubble times to reach $P_{\text{orb}} \gtrsim 4$ hr and have a secular $\dot{M} \sim 10^{-12} M_\odot$ yr$^{-1}$. If the extremely low $\dot{M}$ deduced for short period SXTs were due to downward fluctuations from an intrinsically high secular $\dot{M}$, one would still have to explain why the bright counterparts are not observed ($\S$ 3.1). Furthermore, some systems would then necessarily have higher rates than their secular value, hence there should be even more persistent systems.

In Paper I we have shown that in the “standard” model systems with $P_{\text{orb}} \approx 8 - 10$ hr may be consistent with systems where donors overflow Roche lobes extremely close to TAMS, but then one needs to assume some very special initial distribution of binaries over orbital separations which will after several evolutionary stages with mass transfer and mass and momentum loss from the system and supernova explosion, result in concentration of zero-age black-hole and main-sequence-star systems just at the desired very narrow range of separations. On the other hand, if a small amount of AML is added to the alternative “no-MSW” model, the latter will become consistent with recurrence time based estimates of $\dot{M}$ for observed $P_{\text{orb}} \approx 8 - 10$ hr systems, as shows comparison of two lower panels of Fig. 4.

The longest period SXT of the “short-period” group, 4U1543-47 ($P_{\text{orb}} = 27$ hr; it is not shown in the Figures) is, in principle, consistent with the evolution of “standard” model systems with donors overflowing Roche-lobes after formation of He-cores. But again, these binaries evolve through the Hertzsprung gap extremely fast, and the probability of observing such systems is very low (see Kolb 1998). In the “no-MSW” model, formation of the systems similar to 4U1543-47 is much easier explained, since to the range of periods about 13 hr slowly enough evolve binaries with $M_2 \sim 1 M_\odot$ donors which overfilled Roche lobes having $X_c \sim 0.35$.

3.3.3. Truncated discs

It is, however, plausible that the $\dot{M}$ in short-period SXTs are much higher than $\dot{M}$ derived above. These $\dot{M}$ were obtained treating accretion disc as a reservoir which during quiescence is filled up to a critical value at which the outburst is triggered and the disc emptied. The implicit assumption is that the reservoir is not leaky, i.e. that the accretion rate at the disc’s inner edge is much smaller than $\dot{M}$. However, the inner truncation of the discs is required to explain observed quiescent luminosities (e.g. Lasota 1996, Lasota et al. 1996). The same may be true for non-quiescent discs too (see e.g. Done and Gierliński 2000). Dubus et al. (2001) showed that the disc instability model can reproduce the observed X-ray light-curves only if discs in SXTs are truncated and irradiated. The truncated (or “leaky”) disc paradigm is now commonly used in describing the observed timing and spectral properties of X-ray LMBHB (McClintock and Remillard 2006 and references therein).

For systems with non-stationary quiescent accretion discs $\dot{M}$ can be estimated as (Lasota 2001)
\[ M \approx \frac{\epsilon M_{\text{D, max}}}{t_{\text{recc}}} + \dot{M}_{\text{in}} \]  

(5)

where

\[ M_{\text{D, max}} = 2.7 \times 10^{21} \alpha^{-0.83} \left( \frac{M_1}{M_\odot} \right)^{-0.38} R_{d,10}^{3.14}, \]  

(6)

is the maximum quiescent-disc mass (in g) and \( \epsilon = \Delta M_D/M_{\text{D, max}} \) – the fraction of the disc’s mass lost during outburst; \( \alpha \) is the kinematic viscosity parameter, \( R_{d,10} \) is the disc’s outer radius in \( 10^{10} \) cm and \( \dot{M}_{\text{in}} \) is the accretion rate at the disc’s inner edge. The usual, non-leaky disc estimates, neglect \( \dot{M}_{\text{in}} \). However, in SXTs \( \dot{M}_{\text{in}} \) is the dominant term as argued by Menou et al. (1999) (henceforth, MNL).

According to the DIM, the disc is in cold thermal equilibrium if accretion rate at all annuli satisfies the inequality \( \dot{M}(r) < \dot{M}_{\text{crit}} \)

\[ \approx 2.64 \times 10^{15} \alpha_{0.1} R_{10}^{2.58} \left( \frac{M_1}{M_\odot} \right)^{-0.85} \dot{M}_{\text{in}} \]  

(7)

where \( \alpha_{0.1} \) – the viscosity parameter in 0.1. Therefore, the mass accretion rate at the truncation radius \( (\dot{M}_{\text{in}}) \) must be smaller than \( \dot{M}_{\text{crit}}(\dot{r}_{\text{in}}) \). Parameterizing the truncation radius as a fraction of the circularization radius \( r_{\text{in}} = f_t r_{\text{circ}} \) (where \( f_t < 0.48 \), see Menou et al. (1999) one gets (in \( M_\odot \) yr\(^{-1}\))

\[ M_{\text{max}} \lesssim 2.5 \times 10^{-7} \left[ (1 + q)^{1/3} (0.5 - 0.227 \log q) \right]^{10.32} P_d^{1.72} f_t^{2.58}, \]  

(8)

where period is in days. Inserting in Eq. (8) the lower limits on \( q \) from Table I and assuming \( f_t = 0.48 \), one obtains the upper limits to the mass-transfer rates shown by arrows in Figs. 4 and 7. One can therefore conclude that a substantially reduced MSW AML would be consistent with both the stability properties and \( M \) of SXTs.

4. Cold, stable systems

Possible problem of excess of SXT in our model mentioned in §3.1 may be resolved in the model suggested by Menou et al. (1999) (henceforth, MNL). MNL pointed out that if the truncation radii were slightly larger than estimated to fit observations of quiescent SXTs the discs would be globally stable, since their accretion rates are lower than the critical value given by Eq. (7) corresponding to cold stable equilibria.

Figure 7 shows the distribution of stars in two population models with respect to stability criterion suggested by MNL under assumption the truncation occurs at \( r_{\text{in}} = 0.48 r_{\text{circ}} \). In the “standard” case with MSW AML all (\( \simeq 2900 \)), but very few (\( \simeq 60 \)), systems are unstable (Fig. 7 upper panel) vs. MNL-criterion. On the other hand, in the model without MSW after RLOF there are almost equal numbers (\( \simeq 2500 \)) of stable and unstable vs. MNL-criterion. The on the other hand, in the model without MSW after RLOF there are almost equal numbers (\( \simeq 2500 \)) of stable and unstable vs. MNL-criterion systems (Fig. 7 two lower panels). This is because \( M \) in the GWR-only scenario are lower than in the MSW+GWR scenario, thereby enabling the criterion for a cold disc to be met more easily. Truncating the disc at \( r_{\text{circ}} \) would make all transients “MNL-stable” for both models. Therefore, it is possible that most or even all LMBHB are secularly cold/stable systems, erupting from time to time due to randomly acting factors like variability of truncation radii or mass transfer rate. The predicted population size of these systems is too small to contribute significantly to the large number of faint \( (L_X \sim 10^{31} \text{ erg s}^{-1}) \) hard X-ray sources seen.
in the deep Chandra exposures toward the Galactic Center (Muno et al., 2003).

5. Conclusion

1. We have modeled populations of short-period semidetached close binary systems containing black holes and low-mass stars under various assumptions about the AML mechanism. We have found that the "standard" model (based on Skumanich (1972) stellar rotation braking law prescription for AML via magnetic braking) produces unobserved persistent LMBHB, while a model with pure GWR AML produces only transient systems. In the latter model it is possible to reproduce, within uncertainty of observations, the number of LMBHB in the Galaxy, the effective temperatures and masses of the donors in these systems (as inferred from the spectra of the latter). Mass-transfer rates in this model are consistent with the upper limits on $\dot{M}$, if quiescent accretion discs are truncated and, hence, leaky. All this suggests that the strength of MSW AML should be strongly reduced compared to the "standard" model.

In the model without strong MSW it is unnecessary to invoke systems with the donors that are almost at TAMS or already left the latter ($X_c \lesssim 0.01$) in order to explain observed orbital periods of SXT and their mass-exchange rates. More, population of short-period ($P_{\text{orb}} \lesssim 8 - 9$ hr) is formed by systems that overflow Roche lobes unevolved or slightly evolved ($X_c > 0.35$), while evolution of systems with more evolved donors with masses $\simeq 1 M_\odot$ opens possibility to explain the origin of LMBHB with longer $P_{\text{orb}}$.

2. An alternative model suggests that if accretion discs are truncated in quiescence they could be cold and stable. Then truncation of discs at radii close to $R_{\text{crit}}$ can make discs in virtually all LMBHB cold and stable. In this case SXT outbursts perhaps are not related to DIM and are random events that result from "external factors", like episodes of enhanced mass transfer or variability in truncation radius.

3. Population synthesis models presented above are obtained under assumption of a high value of the product of binding energy parameter of stellar envelopes and expulsion efficiency of common envelopes – $\alpha_{\text{CE}} \times \lambda = 2$. Model with $\alpha_{\text{CE}} \times \lambda = 0.5$ gives similar results (but a reduced total number of systems). Further decrease of $\alpha_{\text{CE}} \times \lambda$ results in models that are not consistent with observed SXT. The issue of $\alpha_{\text{CE}} \times \lambda$ is still controversial (see, e.g. Tauris and Dewi, 2001; Podsiadlowski et al., 2003) in the sense that there are no strict criteria for defining binding energy of stellar envelopes and there is no clear understanding whether sources other than gravitational energy may contribute to unbinding common envelopes. Apart of common envelopes, crucial role in determining the possibility of formation of LMBHB is played by stellar wind mass-loss in all stages of stellar evolution, since it defines parameters of the system at the beginning of common envelope stage and immediately before supernova explosion that gives birth to black hole. If general notions on mass-loss will not be revised, our results suggest that existence of short-period low-mass X-ray binaries demands high values of $\alpha_{\text{CE}} \times \lambda$ parameter.

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