Exclusive $\pi^0$ production at EIC of China within GPD approach

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June 15, 2022

Abstract

The exclusive $\pi^0$ electroproduction is analyzed within the handbag model based on Generalized Parton Distributions (GPDs) approach. We consider the leading-twist contribution together with the transversity effects. It is shown that the transversity, $H_T$ and $\bar{E}_T$ GPDs are essential in the description of $\pi^0$ cross section. Predictions for future Electron-Ion Collider of China (EiC) energy range are done. It is found that transversity dominance $\sigma_T \gg \sigma_L$, observed at low energies is valid up to EiC energy range.

1 Introduction

Study of hadron structure is one of the key problems of the modern physics. Some time ago, in analyzing of exclusive processes there was proposed new object, Generalized Parton Distributions (GPDs) $[1, 2, 3]$. It was found that the exclusive processes at large photon virtuality $Q^2$ such as the deeply virtual compton scattering
(DVCS) \cite{1 5 6}, deeply virtual meson production (DVMP) \cite{7 8} factorizes into the hard subprocess that can be calculated perturbatively and the GPDs \cite{4 5 6}.

GPDs are complicated nonperturbative objects which depend on 3 variables \(x\) - the momentum fraction of proton carried by parton, \(\xi\) - skewness and \(t\) - momentum transfer. GPDs contain the information about longitudinal and transverse distributions of the partons inside the hadron. It gives information on its 3D structure see e.g. \cite{9}.

In the forward limit (\(\xi = 0, t = 0\)), GPDs become equally to the corresponding parton distribution functions (PDFs). The form factors of hadron can be calculated from GPDs through the integration over \(x\) \cite{11}. Using Ji sum rules \cite{11}, the parton angular momentum can be extracted. More information on GPDs can be found e.g. in \cite{7 10 11}.

Study of exclusive meson electroproduction is one of the effective way to access GPDs. Experimental study of \(\pi^0\) production was performed by CLAS \cite{12} and COMPASS \cite{13}. These experimental data can be adopted to constrain the models of GPDs. Electron-Ion Colliders (EICs) are the next generation collider to investigate of nucleon structure in the future. USA and China both plan to build the EICs at next 20 years \cite{14 15}. The GPDs are one of the most important aspects to study for the EICs \cite{15}.

Theoretical study of DVMP in terms of GPDs is based often on the handbag approach where, as mentioned before, the amplitudes factorize into the hard subprocess and GPDs \cite{2 3 4 5} see Fig. 1. This amplitude contains another non-perturbative object Distribution Amplitudes, which probe the two-quark component of the meson wave function. One of the popular way to construct GPDs is using so called Double Distribution (DD) \cite{17} which construct \(\xi\) dependencies of GPDs and connect them with PDFs, modified by \(t\)-dependent term. The handbag approach with DD form of GPDs was successfully applied to the light vector mesons (VM) leptoproduction at high photon virtualities \(Q^2\) \cite{18} and the pseudoscalar mesons (PM) leptoproduction \cite{19}. In this work, We compute \(\pi^0\) production applying the handbag approach at the kinematics for EIC in China (EicC). Our prediction for \(\pi^0\) production is helpful to estimate the meson cross section at EicC in the future.

In the leading twist approximation the amplitudes of the pseudoscalar mesons leptoproduction are sensitive to the GPDs \(\tilde{H}\) and \(\tilde{E}\). It was found that these contributions to the longitudinal cross section \(\sigma_L\) are not sufficient to describe physical observables in the \(\pi^0\) production at sufficiently low \(Q^2\) \cite{19}. The essential contributions from the transversity GPDs \(\tilde{H}_T, \tilde{E}_T\) are needed \cite{20} to be consistent with experiment. Within the handbag approach the transversity GPDs together with the twist-3 meson wave function \cite{20} contribute to the amplitudes with transversely polarized photons which produce transverse cross section \(\sigma_T\) that is much larger with respect to the leading twist \(\sigma_L\).

We discuss the handbag approach and properties of meson production amplitudes in section 2. We show that the transversity GPDs contribution which have
the twist-3 nature lead to a large transverse cross section.

In beginning of section 3 we investigate the role of transversity GPDs in the cross sections of the $\pi^0$ leptonproduction at CLAS and COMPASS energies and show that our results are in good agreement with experiment. Later on we perform predictions for $\pi^0$ cross section at EicC energies.

2 Handbag approach. Properties of meson production amplitudes

Within the handbag approach the meson production amplitude is factorized at sufficiently high $Q^2$ \[2, 3\] into a hard subprocess amplitude $H$ and GPDs $F$ which contain information on the hadron structure see, Fig. 1.

The subprocess amplitude is calculated within the modified perturbative approach (MPA) \[21\]. We consider the power $k_\perp/Q^2$ corrections in the propagators of the hard subprocess $H$ together with the nonperturbative $k_\perp$-dependent meson wave function \[22\]. The power corrections can be regarded as an effective consideration of the higher twist effects. The gluonic corrections are treated in the form of the Sudakov factors whose resummation can be done in the impact parameter space \[21\].

The unpolarized $ep \rightarrow e\pi^0p$ cross section can be decomposed into a number of partial cross sections which are observables of the process $\gamma^*p \rightarrow \pi^0p$

$$\frac{d^2\sigma}{dt d\phi} = \frac{1}{2\pi} \left( \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2}(1 + \epsilon) \cos \phi \frac{d\sigma_{LT}}{dt} \right). \quad (1)$$

The partial cross sections are expressed in terms of the $\gamma^*p \rightarrow \pi^0p$ helicity amplitudes. When we omit small $M_{0-0+}$ amplitude, they can be written as follows

$$\frac{d\sigma_L}{dt} = \frac{1}{\kappa} \left| M_{0+0+} \right|^2 + \left| M_{0-0+} \right|^2,$$

$$\frac{d\sigma_T}{dt} = \frac{1}{2\kappa} \left( \left| M_{0-++} \right|^2 + 2 \left| M_{0+,+} \right|^2 \right),$$
\[
\frac{d\sigma_{LT}}{dt} = -\frac{1}{\sqrt{2}\kappa} \text{Re} \left[ M_{0-,0+}M_{0-,++} \right],
\]
\[
\frac{d\sigma_{TT}}{dt} = -\frac{1}{\kappa} |M_{0+,++}|^2.
\] (2)

With
\[
\kappa = 16\pi(W^2 - m^2)\sqrt{\Lambda(W^2, -Q^2, m^2)}.
\] (3)

Here \(\Lambda(x, y, z)\) is defined as \(\Lambda(x, y, z) = (x^2 + y^2 + z^2) - 2xy - 2xz - 2yz\).

The amplitudes can be written as
\[
M_{0-,0+} = \frac{e_0 \sqrt{-t'}}{Q \cdot 2m} \langle \tilde{E} \rangle,
\]
\[
M_{0+,0+} = \sqrt{1 - \xi^2} \frac{e_0}{Q} [\langle \tilde{H} \rangle - \frac{\xi^2}{1 - \xi^2} \langle \tilde{E} \rangle],
\]
\[
M_{0-,++} = \frac{e_0}{Q} \sqrt{1 - \xi^2} \langle H_T \rangle,
\]
\[
M_{0+,++} = -\frac{e_0 \sqrt{-t'}}{Q \cdot 4m} \langle \tilde{E}_T \rangle,
\] (4)

where \(e_0 = \sqrt{4\pi\alpha}\) with \(\alpha = \frac{1}{137}\) is the fine structure constant.

\[
x_B = \frac{x_B}{2 - x_B}(1 + \frac{m_P^2}{Q^2}), \quad t' = t - t_0, \quad t_0 = -\frac{4m^2\xi^2}{1 - \xi^2}.
\] (5)

\(x_B\) is the Bjorken variable with \(x_B = Q^2/(W^2 + Q^2 - m^2)\). \(m\) is the proton mass and \(m_P\) is the meson mass.

The \(\langle F \rangle\) in Eq. (4) are the convolutions of the hard scattering amplitude \(H_{\mu'\mu+}\) and GPDs \(\tilde{F}\)
\[
\langle F \rangle = \int_{-1}^{1} dx H_{\mu'\mu+}F(x, \xi, t).
\] (6)

At the leading-twist accuracy the PM production is only sensitive to the polarized GPDs \(\tilde{H}\) and \(\tilde{E}\) which contribute to the amplitudes for longitudinally polarized virtual photons [19].

The hard part is calculated employing the \(k\)-dependent wave function [22], describing the longitudinally polarized mesons. The amplitude \(H\) is represented as the contraction of the hard part \(M\), which can be computed perturbatively, and the non-perturbatively meson wave function \(\phi_M\) which can be found in Ref. [19].

\[
H_{\mu'\mu+} = \frac{2\pi\alpha_s(\mu_R)}{\sqrt{2N_c}} \int_{0}^{1} d\tau \int \frac{d^2k_\perp}{16\pi^3} \phi_M(\tau, k_\perp^2) M_{\mu'\mu}. \] (7)

The GPDs are estimated using the double distribution representation [17]
\[
F(x, \xi, t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\gamma \delta(\rho + \xi \gamma - x) \omega(\rho, \gamma, t),
\] (8)
which connects GPDs $F$ with PDFs $h$ via the double distribution function $\omega$. For the valence quark contribution, it looks like

$$ \omega(\rho, \gamma, t) = h(\rho, t) \frac{3}{4} \frac{(1 - |\rho|^2 - \gamma^2)}{(1 - |\rho|^3).} \quad (9) $$

The $t$-dependence in PDFs $h$ is considered in the Regge form

$$ h(\rho, t) = N e^{(b - \alpha' \ln \rho)} (1 - \rho)^\beta, \quad (10) $$

and $\alpha(t) = \alpha(0) + \alpha' t$ is the corresponding Regge trajectory. The parameters in Eq. (10) are obtained from the known information about PDFs [23] or from the nucleon form factor analysis [24].

It was found that at low $Q^2$ data on the PM leptoproduction also require the contributions from the transversity GPDs $H_T$ and $\bar{E}_T = 2H_T + E_T$ which determine the amplitudes $M_{0-+,+}$ and $M_{0-++}$ respectively. Within the handbag approach the transversity GPDs are accompanied by a twist-3 meson wave function in the hard amplitude $\mathcal{H}$ [20] which is the same for both the $M_{0-++}$ amplitudes in Eq. (4). For corresponding transversity convolutions we have forms similar to (6) as follow:

$$ \langle H_T \rangle = \int_{-1}^{1} d\tau \mathcal{H}_{0-+,+}(\tau, ...) H_T; \quad \langle \bar{E}_T \rangle = \int_{-1}^{1} d\tau \mathcal{H}_{0-++}(\tau, ...) \bar{E}_T. \quad (11) $$

There is a parameter $\mu_P$ in twist-3 meson wave function which is large and enhanced by the chiral condensate. In our calculation, we adopt $\mu_P = 2$ GeV at scale of 2 GeV.

The $H_T$ GPDs are connected with transversity PDFs as following

$$ h_T(\rho, 0) = \delta(\rho); \quad \text{and} \quad \delta(\rho) = N_T \rho^{1/2} (1 - \rho) [q(\rho) + \Delta q(\rho)], \quad (12) $$

by employing the model [25]. We define $t$-dependence of $h_T$ as in Eq. (10).

The information on $\bar{E}_T$ can be obtained now only in the lattice QCD [26]. The lower moments of $\bar{E}_T^u$ and $\bar{E}_T^d$ were found to be quite large, have the same sign and a similar size. As a result, we have large $\bar{E}_T$ contributions to the $\pi^0$ production. It is parameterized by the form as Eq. (10).

### 3 Transversity effects in $\pi^0$ meson leptoproduction

In this section, we present our results on the $\pi^0$ leptoproduction based on the handbag approach. In the calculation, we adopt the leading contribution Eq. (2) together with the transversity effects described in Eq. (11) which are essential at low $Q^2$. The amplitudes are calculated based on the PARTONS collaboration code [27] that was modified to Fortran employing results of GK model for GPDs [20].

In Fig. 2, we present the model results for $\pi^0$ production cross section comparing the CLAS experimental data [12]. The transverse cross section, where the $\bar{E}_T$ and
Figure 2: Cross section of $\pi^0$ production in the CLAS energy range together with the data [12]. Black lines describe $\sigma = \sigma_T + \epsilon\sigma_L$, red lines represent $\sigma_{LT}$, blue lines depict $\sigma_{TT}$.

Table 1: Regge parameters and normalizations of the GPDs, at a scale of 2 GeV. Model I.

| GPD | $\alpha(0)$ | $\beta^u$ | $\beta^d$ | $\alpha'[\text{GeV}^{-2}]$ | $b[\text{GeV}^{-2}]$ | $N^u$ | $N^d$ |
|-----|-------------|-----------|-----------|-----------------------------|---------------------|-------|-------|
| $\bar{E}$ | 0.48 | 5 | 5 | 0.45 | 0.9 | 14.0 | 4.0 |
| $\bar{E}_T$ | 0.3 | 4 | 5 | 0.45 | 0.5 | 6.83 | 5.05 |
| $H_T$ | - | - | - | 0.45 | 0.3 | 1.1 | -0.3 |

$H_T$ contributions are important [20] and dominates at low $Q^2$. At small momentum transfer the $H_T$ effects are visible and provide a nonzero cross section. At $|t'| \sim 0.3\text{GeV}^2$ the $\bar{E}_T$ contribution becomes essential in $\sigma_T$ and gives a maximum in the cross section. A similar contribution from $\bar{E}_T$ is observed in the interference cross section $\sigma_{TT}$ [20]. For calculations we use parameters in Table 1. Details for the parameterization can be found in [20]. The fact that we describe well both unseparated $\sigma = \sigma_T + \epsilon\sigma_L$ and $\sigma_{TT}$ cross sections indicates that the transversity $H_T$ and $E_T$ effects were observed at CLAS [12]. Note that in this experiment there was not possibility to separate $\sigma_L$ and $\sigma_T$. Model produces at CLAS kinematics the leading twist $\frac{\text{d}\sigma}{\text{d}t}(|t| = 0.3\text{GeV}^2) \sim \text{few nb/GeV}^2$. This is about in two order of magnitude smaller with respect to $\sigma$. Thus we see that $\sigma_T$ determined by twist 3 effects give predominated contribution to unseparated $\sigma$. This prediction of the model [20] was confirmed by JLab Hall A collaboration [28] by using the Rosenbluth separation of the $\pi^0$ electroproduction cross section.

Our results for COMPASS kinematics are shown in Fig. 3. It can be seen that Model I give results about two times larger with respect to COMPASS data.
Figure 3: Models results at COMPASS kinematics. Experimental data are from [13], solid curve is the prediction of Model I and dashed line presents the results of Model II.

That was the reason to change model parameters that permit to describe both CLAS and COMPASS data. New parameters for Model II are exhibited at Table. 2 [29]. Because $E_T$ contribution is essential in $\sigma_T$ and $\sigma_{TT}$ cross section, parameterization change mainly energy dependence of this GPD. Other GPDs are slightly changed to be consistent with experiments see Fig. 2 and Fig. 3 where both model results are shown.

| GPD     | $\alpha(0)$ | $\alpha'[\text{GeV}^{-2}]$ | $b[\text{GeV}^{-2}]$ | $N^u$  | $N^d$  |
|---------|-------------|-----------------------------|----------------------|--------|--------|
| $\tilde{E}_n$ | 0.32       | 0.45                        | 0.6                  | 18.2   | 5.2    |
| $\tilde{E}_T$ | -0.1       | 0.45                        | 0.67                 | 29.23  | 21.61  |
| $H_T$   | -           | 0.45                        | 0.04                 | 0.68   | -0.186 |

Table 2: Regge parameters and normalizations of the GPDs at a scale of 2 GeV. Model II.

For average COMPASS kinematics results for the cross sections are [13]

$$\langle \frac{d\sigma_{TT}}{dt} \rangle = -(6.1 \pm 1.3 \pm 0.7) \text{nb/GeV}^2$$

$$\langle \frac{d\sigma_{LT}}{dt} \rangle = (1.5 \pm 0.5 \pm 0.3) \text{nb/GeV}^2$$

Model II give the following results at the same kinematics

$$\langle \frac{d\sigma_{TT}}{dt} \rangle = -6.4 \text{nb/GeV}^2$$
that is closed to COMPASS results as Eq. (13). The Model I give cross sections that are about two times larger with respect to Model II. This is the same effect as we see in Fig. 3. This means that COMPASS provide an essential constraints on $E_T$ contribution.

Using new GPDs parameterization may be important at EicC because its energy range lies not far from COMPASS. In future analyzes we will give predictions for both GPDs models I and II since at higher energies the detailed study of transversity GPDs can be done.

\[
\left\langle \frac{d\sigma_{LT}}{dt} \right\rangle = 0.1 \text{nb/GeV}^2, \tag{14}
\]

Figure 4: Models results for $\sigma = \sigma_T + \epsilon \sigma_L$ and $\sigma_{TT}$ cross section at EicC kinematics. $W$ dependencies at fixed $Q^2$ are shown. The curves above X-axis are predictions of $\sigma$ and curves below X-axis are predictions of $\sigma_{TT}$.

In Fig. 4 and Fig. 5 we show $W$ and $Q^2$ dependencies of $\sigma$ and $\sigma_{TT}$ cross sections at EicC energy range. We show results for $W = 8, 12, 16 \text{ GeV}$ and $Q^2 = 2, 5, 7 \text{ GeV}^2$ that are typical for EicC kinematics. Cross sections $\sigma_{LT}$ are rather small and difficult distinguished on these figures. Thus we separate them into individual...
Figure 5: Models results for $\sigma = \sigma_T + \epsilon \sigma_L$ and $\sigma_{TT}$ cross sections at EicC kinematics. $Q^2$ dependencies at fixed $W$ are shown. The curves above X-axis are predictions of $\sigma$ and curves below X-axis are predictions of $\sigma_{TT}$.

Fig. 6 and Fig. 7, where $W$ and $Q^2$ dependencies of $\sigma_{LT}$ are shown in pb/GeV$^2$. We use the same $W$ and $Q^2$ values as for Fig. 4 and Fig. 5. One can see that all cross section decreases with $W$ and $Q^2$ growing. Model II gives typically smaller results with respect to Model I. At EicC kinematics we get rather small leading twist cross section $\sigma_L$ which is about in order of magnitude smaller with respect to $\sigma_T$. This means that observed at low energy dominance of twist-3 transversity effects [20, 28] is valid up to high EicC energies. Our predictions on $\pi^0$ production give possibility to perform a more detail test of energy dependencies of transversity GPDs in future EicC experiments.

Now we shall briefly discuss is it really possible to analyze energy dependencies of transvesity GPDs $H_T$ and $\bar{E}_T$ from experimental data on cross sections. In experiments (see e.g. [12]) usually unseparated cross section $\sigma = \epsilon \sigma_L + \sigma_T$, $\sigma_{LT}$ and $\sigma_{TT}$ are measured. $\sigma_L$ is determined by twist-2 contribution. It is rather small and can be omitted in our estimations. Thus $\sigma \propto \sigma_T$ here. We will not discuss
Figure 6: Models predictions on $\sigma_{LT}$ cross sections (in pb/GeV$^2$) at EicC kinematics as a function of $W$ at fixed $Q^2$.

here $\sigma_{LT}$.

We see that if

$$\frac{d\sigma_T}{dt} \sim -\frac{d\sigma_{TT}}{dt},$$

this means that in this range the essential contribution comes from $M_{0^{++}}$ amplitude (see (2)). At CLAS and COMPASS energies it approximately happened at $|t'| = 0.3$ GeV$^2$. This means, that at this momentum transfer $\bar{E}_T$ contribution dominates. At $|t'| = 0$ GeV$^2$ the $\bar{E}_T$ is equal to zero. This means that at this point $H_T$ contribution essential.

Thus using Eqs. (2-4) we can determine two quantities

$$<H_T> \propto \sqrt{\kappa \frac{d\sigma_T}{dt}(|t'| = 0 \text{ GeV}^2)},$$

$$<\bar{E}_T> \propto \sqrt{\kappa \frac{d\sigma_{TT}}{dt}(|t'| = 0.3 \text{ GeV}^2)},$$

(15)
Figure 7: Models results $\sigma_{LT}$ cross sections (in pb/GeV$^2$) at EicC kinematics as a function of $Q^2$ at fixed $W$.

and once more in addition

$$< \tilde{E}_T(TT) > \propto \sqrt{\kappa} \frac{d\sigma_{TT}}{dt}(|t'| = 0.3 \text{ GeV}^2)].$$

(16)

Eq. (15) is a some approximation based on $\tilde{E}_T$ dominance near $|t'| \sim 0.3 \text{ GeV}^2$. Eq. (16) gives direct information on $\tilde{E}_T$, but $\frac{d\sigma_{TT}}{dt}$ is more difficult to study.

Thus one can try to analyze $W$ dependencies of cross section at $|t'| \sim 0 \text{ GeV}^2$ and $|t'| \sim 0.3 \text{ GeV}^2$ to determine energy dependencies of $H_T$ and $\tilde{E}_T$.

Result of model calculations for quantities Eqs. (15) for GPDs Model I and II can be parameterized as follow:

$$< H > \sim AW^n.$$  \hspace{1cm} (17)

We shall estimate $n$ power using results from (15) and $n_{H^}$ directly from energy dependencies of GPDs in the $W = 3 \sim 15 \text{ GeV}$ interval. Results are

$$< \tilde{E}_T^{Model-II} >: \quad n = 0.53, \quad n_{H} = 0.5;$$  \hspace{1cm} (18)
We see that energy dependencies for model II and I are rather different. From (16) we find the same power as in Eq. (19).

Thus we find very close powers \( n \) from cross section analyzes and directly from GPDs. This mean that we really can estimate energy \( (x_B) \) dependencies of GPDs from experimental data.

4 Conclusion

The exclusive electroproduction of \( \pi^0 \) mesons was analyzed here within the handbag approach where the amplitude factorized in two parts. The first one is the subprocess amplitudes which are calculated using the \( k_T \) factorization [21]. The other essential ingredients are the GPDs which contain information about the hadron structure. The results based on this approach on the cross sections were found to be in good agreement with data at HERMES, COMPASS energies at high \( Q^2 \) [20].

The leading-twist accuracy is not sufficient to describe \( \pi^0 \) leptoproduction at not high \( Q^2 \). It was confirmed [20] that rather strong transversity twist-3 contributions are required by experiment. In the handbag approach they are determined by the transversity GPDs \( H_T \) and \( E_T \) in convolution with a twist-3 pion wave function. The transversity GPDs lead to a large transverse cross section for \( \pi^0 \) production.

Here we consider two GPDs parameterization. Model I was proposed in [20] to obtain good description of CLAS collaboration [12]. Later on COMPASS experiment produced \( \pi^0 \) data at higher energies [13]. Model I predictions at COMPASS energies are higher with respect to experiment by factor of the order 2. The energy dependencies of transversity GPDs were modified in Model II [29] which describes properly both CLAS and COMPASS data.

In this analysis we perform predictions for unseparated \( \sigma, \sigma_{LT} \) and \( \sigma_{TT} \) cross section for EicC kinematics for both model I and II. We confirm that transversity dominance \( \sigma_T \gg \sigma_L \), observed at low CLAS energies is valid up to EicC energies range. Our results can be applied in future EicC experiments to give additional essential constraints on transversity GPDs at EicC energies range.

Acknowledgment

S.G. expresses his gratitude to P.Kroll for long-time collaboration on GPDs study. The work is partially supported by the CAS president’s international fellowship initiative (Grant No. 2021VMA0005) and Strategic Priority Research Program of Chinese Academy of Sciences (Grant NO. XDB34030301) .
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