Contact Analysis of Cycloidal-pin Gear of RV Reducer Under the Influence of Profile Error

Tianxing Li1,a, Guofeng Wang1,b, Xiaozhong Deng1,c, Xiaotao An1, Chunrong Xing1 and Wensuo Ma2

1Institute of mechanical and electrical engineering, Henan University of Science and Technology, Xiyuan Road, Luoyang 471003, China
2Collaborative Innovation Center of Machinery Equipment Advanced Manufacturing of Henan Province, Luoyang 471003, China

a Corresponding author: ly_litianxing@163.com
844832573@qq.com

Abstract. Because of the influence of machine tool accuracy, the tooth profile error will occur during the processing of cycloidal gear. In order to evaluate the influence of tooth profile error on the transmission accuracy of cycloidal-pin gear pair, the actual tooth surface coordinate points with tooth profile error are calculated based on the measured tooth profile error of cycloidal gear, and the tooth profile error is obtained by non-uniform rational B-spline curve fitting. Based on the meshing principle, the tooth contact analysis (TCA) of the digitized tooth surface is carried out, and the transmission error curve of cycloidal-pin gear pair under the influence of tooth profile error is obtained.

1.Introduction
Compared with the universal decelerator, the RV reducer is demanding for the motion precision, the transmission error cannot exceed 1’, the return error cannot exceed 1-1.5’, and the load cannot exceed 6 [1-2] after the load. The tooth profile error will directly affect the meshing characteristics and contact precision of the transmission system, so it is very important to understand the influence of the tooth profile error on the gear pair meshing quality.

Blanche [3-4] has carried out the error analysis and experimental research of the RV reducer, and revealed the main influencing factors of the transmission precision, and concluded that it is beneficial to improve the transmission precision of the system. He Weidong and other [5] set up the coordinate system which is meshing with the manufacturing error and the installation error. The process of the needle swing meshing transmission is defined from the mathematical point of view, and the error value of the output angle of each moment is obtained. Yuan Xin and other [6] synthetically consider the factors such as machining error, assembly error and clearance and so on. The dynamic transmission precision nonlinear dynamic model of RV system is set up, and the sensitivity analysis is carried out by the numerical differential method, which is of guiding significance for the design and manufacture of high precision RV transmission. However, few studies have been done on the fitting of cycloidal gear surface and the actual tooth surface TCA with tooth profile error.

2.The distribution of the measuring point of the tooth surface
The measuring instrument used in this paper is the JD350 type cycloid gear measuring machine with 90 measuring points. The measurement point is measured according to the equal meshing phase angle. The distribution of the measuring point is shown in Figure 1. The tooth profile error of the point is superimposed on the design tooth profile along the normal direction of the measuring point, and the discrete tooth surface points that can reflect the cycloid tooth profile error are obtained.

![Figure 1. Measurement point of equal meshing phase angle](image)

The homogeneous form of the end face of the cycloid tooth profile equation

\[ r_c(\varphi) = [x_c, y_c, 0, 1] \]  \hspace{1cm} (1)

Unit normal vector representation

\[ n_c(\varphi) = [n_{xc}, n_{yc}, 0] \]  \hspace{1cm} (2)

The tooth profile deviation of cycloid gear at each measuring point is \( \delta_{\varphi}^i \) (i=1,2,3...). From the cycloid gear equation (1) and the unit normal vector (2), the coordinates \((X_c, Y_c)\) of the measuring points on the actual tooth profile of the cycloid gear can be expressed as:

\[
\begin{align*}
X_c &= x_c + n_{xc}(\varphi) \cdot \delta_{\varphi}^i \\
Y_c &= y_c + n_{yc}(\varphi) \cdot \delta_{\varphi}^i 
\end{align*}
\]

3. NURBS representation of cycloid tooth profile with tooth profile error

The vector function form of the three B spline curve can be expressed as follows.

\[ p(u) = \sum_{i=0}^{n} d_i N_{i,3}(u) \]  \hspace{1cm} (4)

The control points of the NURBS curve are calculated based on the measured data, and three B spline curves can be obtained by substituting (4) formula.

According to the above method, taking a RV reducer as an example (the basic parameter is shown in Table 1), the three NURBS curve of the cycloidal gear profile is obtained as shown in Figure 2.

| Name                                | Parameter values |
|-------------------------------------|------------------|
| Cycloidal gear number \(z_c\)       | 11               |
| Pin gear number \(z_p\)             | 12               |
| Pin gear radius \(r_p\)/mm          | 7                |
| Eccentricity/mm                     | 4                |
| Center distribution circle radius of pin gear \(r_s\)/mm | 90              |
Modification of equidistance $\Delta r_p/\text{mm}$ | 0.01
---|---
Modification of moved distance $\Delta r_p/\text{mm}$ | -0.02

Finally, the digitized tooth profile of cycloidal gear with tooth shape error is obtained by interpolating back calculation.

![Figure 2: NURBS curve of cycloid gear](image)

$$R_c = R(u)$$  \hspace{1cm} (5)

As the NURBS curve is a structural curve, all points on the structural curve are controlled by the fitting algorithm except the control points. Therefore, the fitting accuracy of NURBS must be verified. The steps are as follows.

1. According to the equal meshing phase angle, 47 points are obtained on the theoretical tooth profile, and the digital tooth profile of the theoretical tooth profile is fitted.
2. Take another tooth surface point which is different from the original meshing phase angle (take 37 equal meshing phase angles).
3. The fitting deviation is calculated by fitting the tooth profile and the theoretical tooth profile to verify the fitting accuracy.

The deviation between the tooth profile and the theoretical tooth profile after fitting is shown in Figure 3.

![Figure 3: Fitting deviation](image)

It can be seen that the maximum fitting accuracy of cycloidal gear is $1 \times 10^{-8}$ mm, and the fitting accuracy meets the requirement. Therefore, the digital tooth profile obtained by fitting can be used instead of the actual tooth profile for TCA.

4. Contact analysis of cycloid gear with tooth profile error
As shown in Figure 4, the coordinate system of cycloidal gear and needle tooth is represented by coordinate transformation in the fixed coordinate system.

The equation group consists of 3 independent scalar equations and 4 unknown quantities. The solution of the equation group is obtained by the known quantity of the needle tooth corner \( \phi_1 \); the different values corresponds to the different meshing positions, and all contact points are obtained to form the contact trajectory of the tooth profile. The transmission error \( \Delta E \) can be calculated by the next type.

\[
\Delta E = (\phi_2 - \phi_{20}) - \frac{z_c}{z_c} (\phi_1 - \phi_{10}) \quad (6)
\]

\( \phi_{10} \) and \( \phi_{20} \) are the angle of the needle wheel and the cycloid gear when meshing at the reference point respectively. \( \phi_1 \) and \( \phi_2 \) are the angle of the needle wheel and the angle of cycloid gear respectively.

5. Example analysis

As an example of a certain type of RV reducer, the basic parameter, as shown in Table 1, is based on the established tooth profile error, fitting the expression of the cycloid tooth profile containing the tooth profile error, setting up the tooth contact analysis model of the cycloidal-pin gear pair, and analyzing the tooth contact analysis.

| Name | Error value /μm | Name | Error values /μm |
|------|-----------------|------|-----------------|
| 1    | 0.1             | 24   | 4.0             |
| 2    | 0.3             | 25   | 2.9             |
| 3    | 0.7             | 26   | 2.6             |
| 4    | 1.7             | 27   | 2.7             |
| 5    | 2.7             | 28   | 1.8             |
| 6    | 3.7             | 29   | 1.6             |
| 7    | 4.8             | 30   | 1.0             |
| 8    | 5.8             | 31   | 1.3             |
| 9    | 6.6             | 32   | 0.9             |
| 10   | 6.8             | 34   | 0.5             |
| 11   | 7.2             | 35   | 0.1             |
| 12   | 7.7             | 36   | -0.2            |
| 13   | 8.3             | 37   | -0.1            |
| 14   | 5.8             | 38   | -0.7            |
| 15   | 8.2             | 39   | -0.5            |
According to the above method, the actual tooth surface discrete points are synthesized to synthesize the three NURBS curve, and the digitized tooth surface TCA is obtained, and the transmission error of the tooth surface is obtained as shown in Figure 5.

| 16 | 8.2 | 40 | -0.1 |
|----|-----|----|------|
| 17 | 7.2 | 41 | -0.3 |
| 18 | 7.2 | 42 | -0.6 |
| 19 | 7.6 | 43 | -0.7 |
| 20 | 6.7 | 44 | -0.8 |
| 21 | 5.7 | 45 | -0.5 |
| 22 | 4.8 | 46 | -0.2 |
| 23 | 4.6 | 47 | -0.4 |

Figure 5. The transmission error curve that does not take into account the error of the tooth profile.

Figure 6. The transmission error curve of the tooth profile error.

By contrast, when the tooth profile error is not considered, the transmission error curve of the design tooth profile is a flat top parabola, and when the tooth profile error is considered, the overall symmetry of the transmission error curve becomes worse and the transmission performance decreases obviously. The error curve can be seen from the local large map, and the error curve appears in two peak values and in the process of the adjacent two tooth meshing conversion process. There are third teeth engaging in meshing, and the overall transmission error is obviously increased.

6. Conclusion

Based on the coordinate point data of the cycloid tooth point, the digitized tooth profile of cycloid gear with tooth profile error is fitted, and the TCA program of the fitting tooth profile is compiled, and the influence of the tooth shape error on the transmission precision of the cycloidal-pin gear is obtained. It lays a foundation for subsequent high-precision dynamic analysis and stress calculation. An example
of the cycloidal-pin gear pair of a type of RV reducer is taken as an example. The results show that the tooth profile error has a great influence on the transmission performance of the cycloid gear, and the tooth profile error should be reduced in the process of gear tooth processing.

Acknowledgement
Robot This work was financially supported by the National Natural Science Foundation(U1504522) and Major Scientific and Technological project in Henan Province (161100211200). We would like to thank editor and anonymous reviewers for their constructive comments and helpful suggestions for improving the manuscript.

References
[1] Robot Technology and Application Editorial Department. Status quo and development of industrial robots[J]. Robot Technology and Applications, 2013, 1(1): 3-5.
[2] Xu Yangsheng. Intelligent robots leading high-tech development[J]. China Science Daily, August 12, 2010.
[3] D. C. H. Yang, J. G. Blanche. Design and application guidelines for cycloid drives with machining tolerances[J]. Mechanism and Machine Theory, 1990, 25(5): 487-501.
[4] J. G. Blanche, D. C. H. Yang. Cycloid Drivers With Machining Tolerances[J]. Journal of Mechanical Design, 1989, 111(3): 337-344.
[5] He Weidong, Li Lixing, Li Xin. New optimized tooth-profile of cycloidal gear of high precision RV reducer used in robot[J]. Chinese Journal of Mechanical Engineering, 2000, 36(3): 51-55.
[6] Yuan Xin, Wu Lanying, Han Linshan. Sensitivity of error to transmission accuracy of RV reducer. [J]. development and innovation of mechanical and electrical products, 2009, 22(2): 46-48.