A brittleness evaluation method of rock constitutive relationship with Weibull distribution based on double-body system theory

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Abstract
Brittleness of rocks plays an important role in the evaluation of unconventional oil and gas reservoir network fracturing and many rock engineering application fields. However, there is still no commonly accepted definition of the concept of brittleness. In this study, the brittleness differences between rocks are understood as the differences of internal microunit strength cracking characteristics under an external load. Because rock ruptures are regarded as a fracturing problem of a double body system, the region where no cracking occurs during the uniaxial compression loading process is regarded as a body I system with elastic deformation characteristics; the region that breaks first and induces the integral fracture of the rock is regarded as a body II system in which the structural strength of the microunit follows the deformation characteristic of the Weibull distribution. An energy quasi-static equilibrium equation for rock fractures of a double body system under an external load was established. A new index for evaluating brittleness was established based on the energy mutation process of the quasi-static equilibrium equation of double body system. Sandstone, red sandstone, mudstone, sandy mudstone, shale, limestone, and granite were selected as the research objects, while the correlation curve between the new brittleness index, the Weibull distribution characteristic parameters, and the mutation break point was calculated. The brittleness index studied in this paper reveals the evolution process of integral fractures induced by the internal fractures of rocks, which is important for further understanding the essence of brittleness. These results can provide a new method for further studying the formation process of local fracture networks.
1 INTRODUCTION

In recent years, brittleness has become a key parameter in the field of rock mechanics research. Brittleness is a vital component in hydraulic fracturing of unconventional oil and gas reservoirs, enhanced geothermal systems, coal mining, the stability of wellbore walls, and other applications. Understanding the essential characteristics of rock brittleness has important academic value.

However, the definitions of brittleness are still ambiguous. The literature on rock mechanics does not present a unique definition of brittleness. According to existing studies, brittleness characteristics of rocks can be summarized as follows: (a) low deformation has occurred upon load application; (b) brittle rock has obvious fracture surfaces upon fracturing; (c) the smaller the cohesion, the easier it is for rock to form cracks under the load; (d) high resilience; (e) the compressive strength is far greater than the tensile strength, and the tensile-compressive ratio is very small; (f) the greater the internal friction angle, the greater the brittleness of the rock; the greater the Young’s modulus, the greater the brittleness of the rock; the smaller the Poisson’s ratio, the greater the brittleness of rock; (g) the brittleness of rock is largely controlled by its mineral composition; (h) when brittle rock fractures, residual strength appears after peak strength; and (i) when the load increases to the peak strength of the rock specimen, the internal fracture of the rock specimen is localized and the whole specimen collapses. Therefore, the limitations of the existing rock brittleness require the development of a practical and reliable method to evaluate the brittleness of rock materials. This brittleness index should describe the essence of brittleness and reveal the essential connotations of rock fractures. Weibull concludes that the brittleness difference exhibited by the rock fracture process is derived from the difference in the internal microunit strength fracture characteristics. Based on this analysis, the rock is a double body system; the region where no cracks occur during the loading process is regarded as a body I system that is assumed to obey the elastic deformation characteristics of the rock. The part of the rock that first ruptures and induces the overall rupture of the rock is regarded as a body II system, and the structural strength of the microunits in this region is assumed to obey the constitutive characteristics of the Weibull distribution. The dimensionless form of energy release defined by the rupture process of the body II region is used as the brittleness index to evaluate the brittleness characteristics of the rock. We selected 7 types of rocks, including sandstone, mudstone, shale, limestone, granite, red sandstone, and sandy mudstone, as research objects, to evaluate the variations in their brittleness characteristics based on the Weibull distribution with rock distribution parameters.

2 RELATIONSHIP BETWEEN ROCK FRACTURE BEHAVIOR AND BRITTleness

2.1 Rock fracture behavior analysis

The fracturing of rock under an external load is a complicated destructive process. Weibull stated that when the stress at a certain point in the structure reaches the ultimate strength of the unit, the unit is first destroyed and the overall structure is gradually damaged. According to the experimental results of rock loading fractures, the internal microunits of the rock gradually rupture and expand under an external load. These microunits are divided into four stages in the rock, with three specific areas: the first expansion zone (fracture zone), the friction zone (transition zone), and the elastic zone, as illustrated in Figure 1A. During the initial loading stage, the whole rock is compacted and elastic deformation occurs. With the continuous loading, the first expansion zone is formed, which is represented as the fracture zone ①, and the appearance of this zone represents the generation of the initial crack, which corresponds to the stage B in Figure 1A. Crack propagation transition zone ② appears after the formation of the first expansion zone, which corresponds to the stage C in Figure 1A. Elastic deformation zone ③ is always in the stage of elastic deformation, the elastic deformation energy for zone ① and zone ② is provided. With the continuous loading, zone ② expands gradually, zone ② increases gradually, and zone ③ decreases gradually, which shows the process that the rock expands from the internal local microfracture to the overall macro-fracture. For the plastic rock, the microunit block collapses in the fracture zone, and the energy consumed by the microunit crack propagation is large, as illustrated in Figure 1B. Conversely, for the brittle rock, the microunit block does not collapse in the fracture zone during the crack growth process, and the energy consumed in maintaining the microunit crack growth is small, as illustrated in Figure 1C. In the fracture zone, the initial crack is formed, and a series of short tensile cracks are formed at the crack tip and the extension
direction of the crack is determined. The elastic energy accumulated in the rock specimen is sufficient in maintaining the entire fracture process, while the red zone in the figure indicates the process of elastic energy transforming into fracture energy in the first expansion zone.18,36 From the process of rock rupture, the brittleness difference is derived from the different characteristics of internal microunit strength fractures.37,41,42 Weibull38,39 shows that the intact zone in the rock rupture is the body I type (elastic deformation stage), while the fracture zone is the body II type (softening deformation stage), and interactions between the two bodies occur in series. According to Figure 1, body I is always in the stage of elastic deformation, and the control effect of body I is embodied on body II. Body II is in the stage of softening deformation, which reflects the brittle fracture characteristics of the rock, and the constitutive model of the softening of body II determines the difference in brittleness. In this study, we assume that the microunit strength of rocks in body II obeys the Weibull distribution,37,43 and the aim is to carry out a brittleness evaluation of rock satisfying Weibull distribution characteristics.

2.2 | Definition of BI

According to the relationship and mechanical state of body I and body II as shown in Figure 2, body II is in a quasi-static state during the loading deformation process. In this study, the physical meaning of brittleness is characterized as the ability of a body II system to release energy from the quasi-static critical equilibrium state before rupture to the quasi-static critical equilibrium state after rupture. The energy release capacity can be expressed by the energy release (dimensionless) \( \Delta E_{i-j} \) variable of a rock fracture under the quasi-static conditions of a body II system.44,45 Therefore, the brittleness index can be expressed as follows:

\[
BI = -\Delta E_{i-j},
\]

where BI is the brittleness index defined in this study. \( \Delta E_{i-j} \) is the amount of energy released from the quasi-static state before fracture to the quasi-static state after fracture of the body II system due to the amount of energy released, \( \Delta E_{i-j} < 0 \).

According to Equation (1), the brittleness index is mainly determined by \( \Delta E_{i-j} \). The constitutive model of body I and body II systems controls \( \Delta E_{i-j} \). According to different Weibull distribution characteristics, differences in brittleness can be characterized by the brittleness index.

3 | DOUBLE-BODY SYSTEM CONSTITUTIVE RELATIONSHIP AND ENERGY: A WORK MODEL

3.1 | Rock deformation constitutive relationship

3.1.1 | The body I deformation constitutive model

During the loading process, body I does not break and only elastic deformation occurs. Therefore, the constitutive relation of body I deformation can be expressed as follows:

\[
\sigma_1(\varepsilon) = E_1 \varepsilon,
\]
where $\sigma_1 (\varepsilon)$ is the deformation stress of the body I system, $E_1$ is the elastic modulus of the body I system, and $\varepsilon$ is the strain of body I during rock compression.

### 3.1.2 The body II deformation constitutive model

The strength distribution of material (rock) microelements is assumed as satisfying the Weibull distribution; according to the theory of damage mechanics, the constitutive relationship determined by the experiment can be expressed as follows:

$$
\varepsilon = \varepsilon_0 \left( \frac{1}{m} \right)^{\frac{1}{m}},
$$

(5)

and

$$
\begin{cases}
\varepsilon_0 = \varepsilon_c \sqrt{m} \\
m = \frac{1}{\ln \left( \frac{E_c}{\sigma_{c}} \right)}
\end{cases}
$$

(6)

Parameters $m$ and $\varepsilon_0$ can be obtained by testing the stress-strain curve.

### 3.2 Work and strain energy of rock rupture

#### 3.2.1 Elastic energy of body I deformation

In this study, the rock is considered a body II system from the softening deformation stage after the peak point. Therefore, the energy phase of body I for body II should be the deformation phase of the post-peak body II system. Elastic strain energy accumulated in the body II softening deformation can be expressed as follows:

$$
U_{II} = \int_{\varepsilon_c}^{\varepsilon_0} \sigma_{II} (\varepsilon) \, d\varepsilon.
$$

(7)

$U_I$ is the strain energy released by the elastic deformation of body I, $\varepsilon_{II}$ is the strain at the peak point (starting point strain of softening deformation of body II), and $\varepsilon_c (\varepsilon_{II})$ is the strain of the body I system when body II mutates.
3.2.2 | Elastic energy of deformation of body II

The strain energy consumed during the softening deformation phase of body II system can similarly be expressed as follows:

\[ U_\Pi = \int_{\epsilon_{i\Pi}}^{\epsilon_{f\Pi}} \sigma_\Pi (\epsilon) d\epsilon, \quad (8) \]

where \( U_\Pi \) is the release amount of elastic energy for the elastic deformation of body II, and \( \epsilon_p (\epsilon_{f\Pi}) \) is the strain of the body II system when mutation occurs.

3.2.3 | External work model

\[ W_e = \int_{\epsilon_{i\Pi}}^{\epsilon_{f\Pi}} N\epsilon_n d\epsilon, \quad (9) \]

where \( W_e \) is the work done by the external load on the rock in the softening stage of body II, \( N \) is the compressive stress acting on the rock by the testing machine, and \( \epsilon_n (\epsilon_{f\Pi}) \) is the strain deformation length of the unit length rock loading test machine.

According to the loading process of Figure 2, the relationship between the axial strain of body I and body II and the deformation unit length of the loading system of the testing machine is as follows:

\[ \epsilon_p (\epsilon_{f\Pi}) = \epsilon_n (\epsilon_{f\Pi}) - \epsilon_e (\epsilon_{f\Pi}), \quad (10) \]

4 | DOUBLE BODY SYSTEM RUPTURE QUASI-STATIC EQUATION

According to the relationship shown in Figure 2, the body II system is quasi-static on the softening curve segment, and energy is dissipated when the rock in body II breaks and forms a crack. The elastic region without a rupture (body I) correspondingly releases the elastic energy to support the dissipated energy required for the rock fracture in the internal region. If the elastic energy released from the surrounding rock region cannot ensure a rupture of the softening region, then the external load of the test machine must work on the rock (expressed as the continuous rise of the test machine head) to ensure the continuous expansion of body II. Therefore, the specimen is assumed to be a unit volume rock; with body II under the external load, the energy conservation relationship between body I and body II in the testing machine in the quasi-static equilibrium state is expressed as follows:

\[ U_1 + U_\Pi - W_e = 0. \quad (11) \]

This study mainly evaluates the brittleness of rock by studying the deformation characteristics of body II under quasi-static equilibrium. Therefore, Equation (11) is expressed as the quasi-static equilibrium equation of strain \( \epsilon_p (\epsilon_{f\Pi}) \) of body II as follows:

\[
\frac{dU_1}{d\epsilon_p (\epsilon_{f\Pi})} + \frac{dU_\Pi}{d\epsilon_p (\epsilon_{f\Pi})} - \frac{dW_e}{d\epsilon_p (\epsilon_{f\Pi})} = 0. \quad (12)
\]

According to the relationship shown in Figure 2, the relationship between body I and body II and the external force satisfy the following equation:

\[ \sigma_1 (\epsilon_e) = \sigma_\Pi (\epsilon) = N, \quad (13) \]

where \( \epsilon_e \) is the strain at the inflection point (strain of elastic deformation of body I).

According to the mechanical relationship of Equation (13), Equation (12) can also be written as the equilibrium equation of the quasi-static deformation of body II as follows:

\[
\sigma_\Pi (\epsilon) \frac{d\sigma_\Pi (\epsilon)}{E_\Pi d\epsilon_p (\epsilon_{f\Pi})} + \sigma_\Pi (\epsilon) - N \frac{d\epsilon_n (\epsilon_{f\Pi})}{d\epsilon_p (\epsilon_{f\Pi})} = 0. \quad (14)
\]

Due to the nature of the inflection point of the body II deformation curve, the second derivative at the inflection point \( \epsilon_e \) is equal to zero; according to the rock constitutive relationship of Equation (3), the second derivative can be satisfied at the inflection point:

\[
\sigma''_\Pi (\epsilon_e) = -\frac{Em_{\text{exp}} \left( \frac{\epsilon_e}{\epsilon_0} \right)^m \left( \frac{\epsilon_e}{\epsilon_0} \right)^m (m-m \left( \frac{\epsilon_e}{\epsilon_0} \right)^m + 1) \right)}{\epsilon_e} = 0, \quad (15)
\]

because \( \left( \frac{\epsilon_e}{\epsilon_0} \right) \neq 0 \) and \( m \neq 0 \), we can conclude that:

\[ m - m \left( \frac{\epsilon_e}{\epsilon_0} \right)^m + 1 = 0. \quad (16) \]
The relationship between the inflection point $\varepsilon_c$ and the peak intensity $\varepsilon_0$ of the rock can be obtained as follows:

$$\varepsilon_c = \sqrt{\left(1 + \frac{m}{m}\right)} \varepsilon_0 = \sqrt{1 + m \varepsilon_c}. \quad (17)$$

From the research on different Weibull distribution laws, the position of the inflection point is related to the reference coefficient $m$ and the peak point strain value $\varepsilon_c$.

To facilitate the transformation of Equation (14) into a standard form of quasi-static equilibrium equation, Taylor expansion occurs at the inflection point $\varepsilon_c(\sigma''(\varepsilon_c) = 0)$ of the body II deformation curve; according to the deterministic rule, the higher-order terms greater than or equal to $(\varepsilon - \varepsilon_c)^3$ in the equation are eliminated, and Equation (14) can ultimately be expressed as follows:

$$\sigma_{II} (\varepsilon_c) \sigma''_{II} (\varepsilon_c) + \frac{\sigma_{II}'' (\varepsilon_c)}{E_{II}} (\varepsilon - \varepsilon_c)^2 [\frac{\sigma_{II} (\varepsilon_c)}{\varepsilon_c}]^2 \left[\frac{1 + \frac{m}{m}}{\sigma_{II} (\varepsilon_c) \sigma''_{II} (\varepsilon_c)}\right]^2 + \frac{\sigma_{II}'' (\varepsilon_c)}{E_{II}} (\varepsilon - \varepsilon_c) N_{II} \frac{d e_n (\varepsilon_c)}{d e_n (\varepsilon_c)} = 0.$$  \quad (18)

Equation (18) is arranged and written as a nondimensional standard form of the quasi-static equilibrium equation of the double body system:

$$\begin{align*}
-1 + \frac{1}{\varepsilon_c^2} \left[ \frac{1 + \frac{m}{m}}{\sigma_{II} (\varepsilon_c) \sigma''_{II} (\varepsilon_c)} \right]^2 \left[\frac{1 + \frac{m}{m}}{\sigma_{II} (\varepsilon_c) \sigma''_{II} (\varepsilon_c)}\right]^2 \left( \frac{\sigma_{II}'' (\varepsilon_c)}{\sigma_{II} (\varepsilon_c)} \right) \left( \frac{\sigma_{II} (\varepsilon_c)}{\sigma_{II} (\varepsilon_c)} \right) - \frac{2 E_{II}}{\varepsilon_c} \frac{d e_n (\varepsilon_c)}{d e_n (\varepsilon_c)} = 0.
\end{align*}\quad (19)$$

The first term in Equation (19) represents the deformation state variable of the double body system, and the second term represents the deformation control variable.

Where

$$\sigma''_{II} (\varepsilon_c) = -E_{II} \exp \left(-\left(\frac{\varepsilon_c}{\varepsilon_0}\right)^m \right) \left( \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^m - 1 \right), \quad (20a)$$

$$\sigma'''_{II} (\varepsilon_c) = \frac{E_{II} \exp \left(-\left(\frac{\varepsilon_c}{\varepsilon_0}\right)^m \right) \left( \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^m - m^2 \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^m - m^2 \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^{2m} + 1 \right)}{\varepsilon_c^2}. \quad (20b)$$

The following is assumed:

$$x(\varepsilon) = \left[ \frac{(\varepsilon - \varepsilon_c)}{\varepsilon_c} + \frac{\sigma''_{II} (\varepsilon_c)}{\sigma_{II} (\varepsilon_c) \sigma'''_{II} (\varepsilon_c)} \right], \quad (21)$$

With body II deformation strain value $\varepsilon$ before the rupture of the quasi-static equilibrium critical point $i$, the total potential energy is expressed as follows:

$$E_i = \frac{1}{3} m \left( \frac{\varepsilon_i}{\varepsilon_0} \right)^m a_i x (\varepsilon_i). \quad (22)$$

When body II is ruptured, the total potential energy at the $j$ point of the quasi-static equilibrium is expressed as follows:
\[ E_j = \frac{1}{3} x^3 (\varepsilon_j) + a_j x (\varepsilon_j). \] (24)

According to the mutation theory, the dimensionless form of the elastic strain energy release before and after the rupture of body II \( \Delta E_{i-j} \) is expressed as follows:

\[ \Delta E_{i-j} = E_j - E_i = \frac{1}{3} (x^3_j - x^3_i) + (a_j x_j - a_i x_i). \] (25)

The brittleness index can be obtained by substituting the elastic energy release amount calculated by Equation (25) into Equation (1).

### 5 | TESTING PROCEDURE AND RESULT ANALYSIS

#### 5.1 | Test equipment and test scheme

##### 5.1.1 | Introduction of experimental schemes

The triaxial compression experiments for the rock specimens are carried out on a microcomputer-control servo rock test system (TAW-2000, LTD) with a Maximum axial force of 2000 kN and an upper limit of confining pressure of 120 MPa, in Figures 3 and 4. The test force measurement resolution of TAW-2000 is 1/200 000. In the compression deformation test, the extensometer is used to measure the axial and radial deformation of the specimen. Force Sensor Dynamically Measure Axial Force. Due to the brittle failure characteristics of some samples, axial deformation stress control was adopted in the test, and the loading rate is 0.05 kN/min. The test environment temperature is 25°C, and the humidity is 50%. Previous studies have suggested that the diameter of cylindrical specimens in triaxial compression tests should be 20-60 mm and the aspect ratios should be 2-3. The error of unevenness on both ends of the specimen is less than 0.05 mm, and the error along the height of the specimen is less than 0.3 mm, and the maximum deviation of the end face perpendicular to the axis of the specimen is less than 0.25°. The specimen size used in this study conforms to the International Society for Rock Mechanics (ISRM) standards. The compression deformation experiment uses the extension gauge to measure the axial and radial deformation of the specimen. The force sensor dynamically measures the axial force. Due to the brittle failure characteristics of certain specimens, the test is loaded by axial deformation stress control. The loading rate is 0.05 kN/min until the specimen ruptures completely.

#### 5.1.2 | Testing procedures

The purpose of this study is to establish a brittle evaluation method of rock with Weibull distribution characteristics through experimentation. Seven core specimens of sandstone, granite, mudstone, shale, limestone, sandy mudstone, and red sandstone were subjected to a uniaxial compression test. This test was used to determine mechanical parameters such as peak stress and peak strain through experiments, and to determine the rock distribution characteristic parameters \( m \) and \( \varepsilon_0 \) according to Equation (20). Furthermore, the relationship between the characteristic parameters and the brittleness of the rock was calculated and analyzed by using the brittleness evaluation model of the rock based on the double body system proposed in this study; this approach provides a new method for further revealing the brittleness characteristics of rock.

#### 5.2 | Analysis of test results

In the following analysis, Figure 5 shows the stress-strain curves for the seven tested rock types. In the first stage, the stress shows a positive linear correlation with strain, indicating that external stress causes elastic deformation of the rock specimen; however, the stress is not high enough to drive the expansion of the existing microcracks and generate new microcracks. The relatively stable original crack inside the rock specimen and the elastic strain energy accumulate continuously within the specimen. At the plastic deformation stage, the original microcracks inside the rock specimen gradually expand, and new cracks are generated under the axial stress. The second stage is dominated by plastic deformation because of the evolution stage in the stress-strain curve. The higher the plasticity of the rock specimen, the longer the plastic deformation stage in the stress-strain curve. At the postpeak failure stage, the cracks inside the rock specimen begin to expand rapidly, and macroscopic fracture surfaces can be observed on the rock specimens because of the convergence and intersections of the microcracks. The expansion of the macro-cracks results in the structural failure of the rock, while fragments of damaged rock specimens continue to slide along the fractured surface. This indicates that during the postpeak stage the rock undergoes continuous irreversible deformation and rapid structural destruction. The energy release process at this stage is accompanied by the continuous dissipation of strain energy.

According to the broken specimens shown in Figure 5, granite has the highest peak stress at 87.01 MPa, and the minimum peak stress belonged to mudstone at 27.95 MPa. In the case of the rupture, the types of cracks formed in different rocks are unique; the ruptured mudstone, sandy mudstone, and sandstone present a single crack, while granite, limestone, shale, and red sandstone present multicrock extension.
FIGURE 5  Stress-strain relationship and fracture characteristics of different types of cores under uniaxial compression
Figure 5 shows all the stress-strain curves of the sandstone specimens under uniaxial compression. From the calculation results of rock stress, strain, and parameter $m$, the mudstone rock coefficient $m$ is small, while $m$ is large for shale and granite. From the rock fracture results in Figure 5A-G, as the value of $m$ increases, the number of cracks formed by the fractures increases. This result shows that the heterogeneity and discontinuity of the rocks increase with the external load, the weak links of microcracks show instability, and the fractures in rocks increase, increasing the likelihood of multiple cracks forming. Howell$^{49}$ pointed out that the macroscopic fracturing of rock mainly originates from the initiation and propagation of microcracks in the rock, and each initiation and propagation of cracks can be regarded as a brittle failure. Tiankui et al$^{50}$ also demonstrate that a high brittleness of reservoir rocks during fracturing with concordantly increase the $m$ value; additionally, more complex fracture network structures form, which is consistent with the abovementioned theoretical relationship. Therefore, studying the brittleness characteristics of rock based on the internal chain rupture theory of rock proposed by Weibull is a reasonable approach,$^{37,39,46,51}$ and this technique can be used to determine $m$.

As illustrated in Figure 6, there is no correlation between the magnitude of the peak stress and the number of crack formations in the rock. The brittleness index obtained by calculating the proposed brittleness model is shown in Table 1. Figure 7 conveys that as the value of $m$ increases, the mutation point before rupture, the mutation point after rupture, and the difference between the inflection point and the peak point are almost linearly reduced. As the degrees of heterogeneity and discontinuity of rock materials increase, the distance between the critical point of the sudden fracture of the rock and the peak point of the core decreases; the distance from the peak point to break point before mutation also decreases, and the deformation of the inflection point after the mutation becomes smaller. In contrast, from Figure 8, with the increase of $\varepsilon_p$, the differences between the mutation point before rupture, the mutation point after rupture, the inflection point, and the peak point increase approximately exponentially. By comparing the difference between the strain values at the mutation points before and after the rupture, the mutation jump value decreases approximately linearly with the increase of $m$ value. With the increase of $\varepsilon_p$, the mutation rate increases approximately linearly. The results show that the larger the $m$ value, the more Weibull microelements in the rock that are close to the average strength. When the stress reaches the fracture state, the brittle fracture with a short concentration is more likely to occur in the rock. The larger the response characteristic parameter $m$ of rock to the external load, the larger the distance between the critical point of the abrupt rock fracture and the peak point of rock. A large deformation of the rock from the peak point to the rupture point before the mutation and the inflection point after the mutation will increase the likelihood that a prolonged ductile fracture will occur inside the rock.

### 5.3 Correlation between mechanical parameters and characteristic parameters

#### 5.3.1 Relationship between the absolute value of strain and $m$, $\varepsilon_0$

The experimental results shown in Figure 7 convey that as the value of $m$ increases, the mutation point before rupture, the mutation point after rupture, and the difference between the inflection point and the peak point are almost linearly reduced. As the degrees of heterogeneity and discontinuity of rock materials increase, the distance between the critical point of the sudden fracture of the rock and the peak point of the core decreases; the distance from the peak point to break point before mutation also decreases, and the deformation of the inflection point after the mutation becomes smaller. In contrast, from Figure 8, with the increase of $\varepsilon_p$, the differences between the mutation point before rupture, the mutation point after rupture, the inflection point, and the peak point increase approximately exponentially. By comparing the difference between the strain values at the mutation points before and after the rupture, the mutation jump value decreases approximately linearly with the increase of $m$ value. With the increase of $\varepsilon_p$, the mutation rate increases approximately linearly. The results show that the larger the $m$ value, the more Weibull microelements in the rock that are close to the average strength. When the stress reaches the fracture state, the brittle fracture with a short concentration is more likely to occur in the rock. The larger the response characteristic parameter $m$ of rock to the external load, the larger the distance between the critical point of the abrupt rock fracture and the peak point of rock. A large deformation of the rock from the peak point to the rupture point before the mutation and the inflection point after the mutation will increase the likelihood that a prolonged ductile fracture will occur inside the rock.

#### 5.3.2 Slope curves of different characteristic points with $m$ and $\varepsilon_0$ after peaks

Figure 9 shows that the characteristic parameter $m$ has an approximate linear relationship with the average slope of the stress decrease between the point before rupture, the critical rupture point, the inflection point, and the rupture point. Specifically, the change in the average slope of the stress decreases before the rupture has a strong relationship with...
As the rock materials’ degree of heterogeneity and discontinuity increase, a decrease can be observed in the average slope of the stress decrease from the peak point to the mutation point before the rupture, the mutation point for after the rupture, and the inflection point (absolute value increases). A larger m value causes a steeper Weibull microelement strength deformation curve. Hongran et al. also proved that a large m denotes a steep softening curve. Zhang et al. evaluated brittleness by the overall postpeak deformation curve, and observed that a steeper curve after the peak strengthened the brittleness. As illustrated in Figure 10, as ε₀ decreases, the average slope of the stress decreases between the mutation point for before the rupture, the critical mutation point for after the rupture, the inflection point of the curve, and the mutation point of the rupture show an almost linearly negative increase (the absolute value of the slope decreases). As the rock external load response parameter increases, the deformation curve of the microunit in the Weibull rock is

| Type         | Mudstone | Sandy mudstone | Sandstone | Red sandstone | Limestone | Granite | Shale |
|--------------|----------|----------------|-----------|---------------|-----------|---------|-------|
| m            | 2.07070  | 3.95093        | 5.26086   | 5.46807       | 6.01696   | 6.45022 | 7.29878|
| BI           | 0.46564  | 0.91364        | 1.14268   | 1.76133       | 2.09744   | 2.17535 | 2.22817|

**FIGURE 7** The relationship curve between the absolute value of strain and m.

**FIGURE 8** The relationship curve between the absolute value of strain and ε₀.
slower, which is in accordance with37,51 In conclusion, the microunit in the rock is more prone to plastic fractures.

### 5.3.3 Relationship between BI and parameter $m$

According to the experimental results in Figure 11, there is a strong correlation between the characteristic parameter $m$ and this study’s brittleness index (BI). The elastic energy release increases when the rock mutation ruptures, and BI shows an approximate exponential relationship with $m$ ($R^2 = 0.9331$). In contrast, Figure 12’s experimental results indicate that with an increase in $\varepsilon_0$, there is not a strong correlation between the characteristic parameter $\varepsilon_0$ and BI (maximum $R^2 = 0.558$). Therefore, in the evaluation of brittleness of rock with a Weibull distribution, the characteristic parameter $m$ should be considered. Xiao et al51 proposed $m$ as a brittleness evaluation indicator. Based on the double body system proposed in this paper, the brittleness evaluation of the rock satisfying the Weibull distribution is consistent with the results of the macroscopic brittleness evaluations proposed in the existing literature. When evaluating the brittleness of rock, the macroscopic rupture of rock can be regarded as the initiation of internal cracks in rock and parameters such as rock homogeneity and continuity should be considered. The $m$ in the model is easy to obtain for most constitutive rock relationships and distribution characteristics that satisfy the Weibull distribution.46 Therefore, the brittleness index evaluation method proposed in this paper has high practicability and universality. In conclusion, the anisotropy of rock material brittleness should be properly considered when evaluating brittleness.16

### 5.3.4 BI with strain difference curves

From the relationships between BI and the strain difference as well as the average slope of stress decrease in Figures 13 and 14, respectively, the value of BI gradually decreases with...
the increase of strain difference and the slope of the stress decrease. The results in Figure 13 show that during the softening process of body II, the greater the distance between the mutation point before rupture and the starting point (peak point) of the softening stage, the greater the internal deformation of the rock, and the weaker the brittleness. Figure 14 shows that the greater the negative direction of the peak slope of the mutation point of the rock, the smaller the brittleness index. Additionally, the smaller the slope of the postpeak curve obtained from the macro-fracture curve results of the rock, the greater the brittleness. The internal local failures of the samples are consistent with the macro-brittle failures of rock.

These results constitute a macroscopic characterization of internal brittle failure of rock during an overall brittleness failure.

6 | CONCLUSIONS

1. Unlike certain mechanical parameters that represent only a single aspect of rock behavior, such as the elastic modulus and Poisson’s ratio, brittleness is an integrated description of the mechanical behavior of rocks. Therefore, a reasonable and reliable brittleness index can explain the macroscopic fracture behavior of rocks from the perspective of internal fractures. Judging from the whole fracturing process of rocks with Weibull distribution characteristics, the failures of rocks are mainly induced by crack initiation and chain propagation. In this study, a brittleness evaluation method is established for rocks with Weibull function characteristics, and the relationship between the distribution characteristic parameters of Weibull rocks and brittleness is analyzed.

2. Judging from the development of the rock fracturing process, fracturing is characterized by a spatial subarea...
and a temporal subarea. The elastic energy accumulated in a rock is the main source of its rupture and failure. In this paper, this rupture behavior is regarded as a double body system; the initial rupture zone is called body II (postpeak softening characteristics of rock), and the elastic region without a fracture zone is called body I (representing the hardening characteristics of the rock). During the whole fracture process, body I continuously provides energy for body II. The failure of absolute brittle rock is that less energy needs to be dissipated at the fracture tip of rock, and the energy provided by body I is sufficient to drive the fracture behavior of body II, the accumulated elastic energy is sufficient to drive the whole failure process. Therefore, the fracturing behavior of the rock is spontaneous and instantaneous. Rocks with weak brittleness absorb more energy at the crack tip of the rock and require supplemental energy from the outside (work done by the press) to meet cracking requirements.

3. In this study, the postpeak softening stage of the body II fracture is taken as the static parameter, and the quasi-static equilibrium equation of the rock fracturing process is established based on the energy increment relationship of the double body theory. The brittleness of rock is characterized as the energy release capacity from a quasi-static state before fracturing to quasi-static state after fracturing. A new evaluation method of brittleness index is defined, and the brittle characteristics of prefractured rock in rocks are revealed by combining the constitutive relationship of rock with the Weibull function structural characteristics.

4. Based on the experimental results of seven kinds of rock samples, including sandstone, shale, granite, and limestone, the relationship between the brittleness index and the distribution characteristic parameters of rock, \( m \) and \( \varepsilon_0 \), respectively, is established. The calculation results show that there is a positive correlation with the \( m \) value and brittleness index. Conversely, there is a negative correlation with the \( \varepsilon_0 \) and brittleness index.
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NOMENCLATURE

$\Delta E_{ij}$ is the amount of energy released from quasi-static state before fracture to quasi-static state after fracture of body II system, dimensionless

$\sigma_1 (\varepsilon)$ is the deformation stress of body I system

$E_i$ is the elastic modulus of body I system

$\varepsilon$ is the strain of body I during rock compression

$E$ is the elastic modulus of rock

$m$ is the characteristic parameter of rock Weibull distribution, which is characterized by the continuity and homogeneity of the material

$\varepsilon_0$ is the characteristic parameter of rock Weibull distribution, which represents the response characteristic parameter of rock material to external load

$U_i$ is the strain energy released by elastic deformation of body I

$\varepsilon_{P,i}$ is the strain at the peak point (Initial point Strain of Softening Deformation of body II)

$\varepsilon_{e,i}$ is the strain of body I system when body II mutates

$U_{II}$ is the release amount of elastic energy for elastic deformation of body II

$\varepsilon_{p,i}$ is the strain of body II system when mutation occurs

$W_j$ is the work done by the external load on the rock in the softening stage of body II

$N$ is the compressive stress acting on the rock by the testing machine

$\varepsilon_n (\varepsilon_{n,i})$ is the strain deformation length of the unit length rock loading testing machine

$\varepsilon_i$ is the mutation point before rupture

$\varepsilon_j$ is the mutation point after rupture

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