Photon-bunching measurement after 2×25 km of standard optical fibers

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To show the feasibility of a long distance partial Bell-State measurement, a Hong-Ou-Mandel experiment with coherent photons is reported. Pairs of degenerate photons at telecom wavelength are created by parametric down conversion in a periodically poled lithium niobate waveguide. The photon pairs are separated in a beam-splitter and transmitted via two fibers of 25 km. The wave-packets are relatively delayed and recombined on a second beam-splitter, forming a large Mach-Zehnder interferometer. Coincidence counts between the photons at the two output modes are registered. The main challenge consists in the trade-off between low count rates due to narrow filtering and length fluctuations of the 25 km long arms during the measurement. For balanced paths a Hong-Ou-Mandel dip with a net visibility of 47.3% is observed, which is close to the maximal theoretical value of 50% developed here. This proves the practicability of a long distance Bell state measurement with two independent sources, as e.g. required in an entanglement swapping configuration in the scale of tens of km.

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1. INTRODUCTION

Quantum communication via telecom fibers is still limited to around a hundred of kilometers due to fiber losses and noisy detectors. One way to overcome this limit is the use of entanglement swapping [1, 2], e.g. in a quantum relay configuration as proposed in [2, 3]. Furthermore entanglement swapping is a beautiful manifestation of the oddness of quantum mechanics and deserves thus by itself to be demonstrated over large distances outside the lab. At the heart of the entanglement swapping scheme lies a Bell state measurement (BSM) [4], between two photons, originating from independent sources of entangled photon pairs. By projecting these two independent photons onto one of the four Bell states, the two remaining photons, formerly independent, become entangled, even though they never interacted [1, 2, 4].

A complete BSM requires huge non-linearities [5], which are exceedingly difficult to achieve at the single photon level. Nevertheless, a partial BSM can be realized with a simple beam-splitter (BS). To obtain a successful BSM, the two photons involved must be indistinguishable, i.e. must be in the same spatial, temporal, spectral and polarization mode in order to bunch at the BS. The experimental feasibility of a partial BSM has already been demonstrated for polarization qubits [6] as well as for time-bin qubits [7], but never over tens of kilometers as required for communication under realistic circumstances. However, a realization with distant sources implies additional experimental difficulties, in particular, if the photons are transmitted through optical fibers. A recent experiment [8] showed, that the major problem is to maintain temporal indistinguishability, because of thermally induced fiber length fluctuations and thus differences in the travelling times. One possibility to relax the length stability requirements is to use photons with a long coherence length. However this requires narrow filters which reduce the production rate of photon pairs. Hence longer acquisition times are necessary and the constraints for stability become more severe.

Pairs of entangled photons can be obtained by spontaneous parametric down-conversion (SPDC) in a $\chi^{(2)}$ nonlinear crystal. In this process a pump photon can be converted with a small conversion efficiency, into two entangled photons, according to conservation of energy and momentum. However, photons produced by SPDC show generally a large bandwidth, corresponding to a short coherence length. Different approaches have been considered to increase the coherence length of photons obtained by such a spontaneous process. One possibility is to place the nonlinear crystal in a cavity [9]. Another possibility is to use strongly non-degenerate photons created in long crystals. Bandwidths of 60 GHz have recently been reported for photon pairs at 800 and 1600 nm [10]. But this solution is not suitable if both photons are wanted to be at telecommunication wavelengths. Finally a last and simple approach that we adopt here consists in filtering a small range out of the broad emitted spectrum. Note that this demands a high yield of the photon source, in order to compensate the associated losses, as mentioned above.

In this paper, we present an experiment proving the feasibility of a partial BSM over two times 25 km of optical fibers. We create two degenerate photons at 1550 nm by SPDC in a periodically poled lithium niobate (PPLN) waveguide. In order to increase their coherence length, their initial spectral bandwidth of 80 nm is filtered down to 0.8 nm (100 GHz), corresponding to a fourier limited coherence length of 1.3 mm FWHM in air. As previously shown, this kind of waveguide features a high conversion efficiency [10]. The two photons are sent to a 25 km long balanced Mach-Zehnder interferometer and the necessary indistinguishability is verified by performing a Hong-Ou-Mandel (HOM) experiment [11].
This experiment represents a first step towards the realization of entanglement swapping with sources, which are separated by a long distance. In section 2 we calculate the theoretical predictions for our particular system, showing that beside the HOM interference, a conjunction of one and two photon interference is expected. In section 3, we present the experiment and analyze the results, followed by a discussion on the dispersion cancellation effect in section 4. A conclusion is given in section 5.

2. THEORETICAL EXPECTATIONS

To prove indistinguishability between two photons after travelling 25 km each, we perform a HOM experiment. Thereby the two indistinguishable photons, entering a BS in two different input modes, are superposed. Due to their bosonic nature, they bunch together and exit always in the same output-mode. The coincidence rate between two detectors, one at each output of the BS, is hence dropping to zero. These second order quantum interferences only appear if the two photons are completely indistinguishable in their polarization, spectral, temporal and spatial modes. By shifting now the relative arrival time of one photon in respect to the other, this drop in the coincidence rate can be observed as a dip within their overlapping range, the so-called "HOM-dip".

![FIG. 1: Schematic of the experimental setup. A pair of photons emitted by one source is split by a first beam-splitter (BS1). These photons are passed through a Mach-Zehnder interferometer of 2×25 km and are recombined on a second beam-splitter (BS2). Coincidence counts between the two exit modes are regarded, while the length of one arm is changed.](image)

While a complete demonstration of a partial BSM for independent photons is usually realized with independent sources and a pulsed pump beam, this is not needed here. Since we want to check the temporal indistinguishability due to the transmission in fibers, it suffices to take one source creating pairs of photons, split them, and recombine them, after each has travelled in a different fiber. A CW laser is therefore sufficient. As depicted in Fig. 1, the two photons to be overlapped emerge from the same source completely indistinguishable, and thus only can be separated probabilistically, e.g. by a 50/50 beam-splitter (BS1). The two outputs of BS1 are connected to 25 km of standard optical fiber and recombined at BS2, which forms a balanced Mach-Zehnder interferometer. An important consequence therefore is that both single-photon and two-photon interferences are expected as discussed below.

We label \( a_{in}^\dagger \) the creation operator of input of BS1, evolving as \( a_{in}^\dagger \rightarrow \frac{1}{\sqrt{2}}(a^\dagger + b^\dagger) \), \( a^\dagger \) and \( b^\dagger \) representing the upper and lower path of the interferometer. The evolution of the phase shift is described as \( a^\dagger \rightarrow a^\dagger e^{-i\omega\tau} \), with \( \tau = \Delta L/c \). Suppose that we have a two photon state at input \( a_{in} \) of BS1, described by \( |\psi_{in}\rangle = \int_{-\infty}^{\infty} d\omega d\omega' G(\omega,\omega')a_{in}^\dagger a_{in}^\dagger |0\rangle \). \( a_{in}^\dagger \) and \( a_{in}^\dagger \) stand for the creation operators of respectively one photon at frequency \( \omega \) and one at \( \omega' \), and \( G(\omega,\omega') \) characterizes the spectral distribution of the created photons.

The evolution through the two beam splitters together with the phase shift results in a final state, which is described by

\[
|\psi_{in}\rangle = \int_{-\infty}^{\infty} d\omega d\omega' G(\omega,\omega') \\
\left( c_{in}^\dagger d_{in}^\dagger \sin \omega \frac{\tau}{2} \cos \omega' \frac{\tau}{2} + d_{in}^\dagger c_{in}^\dagger \cos \omega \frac{\tau}{2} \sin \omega' \frac{\tau}{2} \right. \\
\left. + c_{in}^\dagger c_{in}^\dagger \sin \omega \frac{\tau}{2} \sin \omega' \frac{\tau}{2} + d_{in}^\dagger d_{in}^\dagger \cos \omega \frac{\tau}{2} \cos \omega' \frac{\tau}{2} \right) |0\rangle
\]

with \( c^\dagger \) and \( d^\dagger \) being the creation operators of the two output modes of the second beam-splitter (BS2). In order to calculate the coincidence probability \( P_{\text{coinc}} \), we consider \( G(\omega,\omega') \) to be gaussian and integrate over the temporal resolution of our detectors of the order of 1 ns. Taking into account only the terms containing \( c^\dagger d^\dagger \) and \( d^\dagger c^\dagger \), \( P_{\text{coinc}} \) is therefore calculated to be proportional to

\[
P_{\text{coinc}} \propto 2 - e^{-\frac{\delta^2}{4}} - \cos \omega_p\tau
\]

where \( \omega_p \) is the pump frequency and \( \delta \) the bandwidth of the down converted photons. Note that this calculation only holds true for a temporal path difference \( \tau \) much smaller than the time resolution of the detectors.

The calculated probability \( P_{\text{coinc}} \) is plotted in Fig. 2. The form can be explained as a superposition of Franson-type (1−\( \cos \tau \omega_p \)) 2-photon interferences and a Hong-Ou-Mandel dip (1−\( e^{-\tau^2/\delta^2} \)).

The temperature fluctuations are too large to resolve the Franson-type interferences, which are on a mm-scale, but small enough to observe the Hong-Ou-Mandel dip (nm scale). Hence the maximally attainable visibility of the average count-rate in this measurement is 50%.

The width of the dip corresponds to the convolution of the two wave-packets. The solid line represents the average probability of coincidences as it will be measured.

3. THE EXPERIMENT

The experimental setup is shown in Fig. 3. To create the two photons to be superposed, we pump a PPLN
waveguide [18] with 35 μW of a CW diode laser at 783 nm. Pairs of collinear energy-time entangled photons are created by SPDC [13] with a conversion efficiency of the order of $10^{-6}$. The waveguide is temperature stabilized at the degeneracy point, where signal and idler are emitted at 1566 nm and coupled into a standard fiber with an efficiency of 18% for each photon. This coupling efficiency is essentially limited by the mismatch between the guided modes in the waveguide and the collecting fiber. Filtering is achieved by a fiber Bragg grating with 0.8 nm FWHM. This kind of filter reflects more than 99% of the light in the chosen spectral window. A circulator is used to recover the filtered reflected light.

The pump light is sufficiently blocked by the Bragg grating and the circulator, so no further absorbing filters are required. Signal and idler photons are split up probabilistically by a 50/50 beam-splitter (BS1). These two output modes travel through a Mach-Zehnder interferometer configuration, with each arm 25.3 km long. Note that the most demanding point is the stability of the length difference between the two arms. It needs to be kept within the coherence length of the photons during the measurement without any active stabilization, as this is impractical in field applications. The two fibers are on separate spools, thus a local change in temperature affects the whole fiber length. This is not the case in telecom fiber networks, where a local change in temperature only concerns a certain part of the fibers. Hence we can expect to have less fluctuations there, as already measured in former experiments [13] to be of the order of a few mm per day for a 10 km link. Even though the two fibers should behave the same way for temperature fluctuations affecting the two spools, we figured out a stability within 0.1 K/h to be necessary. This is reached by protecting them from air draught in the laboratory and close to the conditions in the field.

To observe a HOM-dip, the arrival time of one photon at BS2 is scanned with respect to the other one. This delay is provided by slightly lengthening one arm mechanically. A maximal length difference of 11 mm is covered by a computer controlled step motor device.

In order to ensure indistinguishability at the beam-splitter (BS2), a fiber optical polarization controller (PC) matches the polarization modes of the two arms. At the two outputs of the beam-splitter, photons are detected by single photon InGaAs avalanche photodiodes (APD) which are Peltier cooled to -30°C. One of them is operated in passive quenching mode [20] with an efficiency of 7% and a dark count rate of 2 kHz. This detector triggers the second one which is optically delayed and operated in gated mode with an efficiency of 8% and a dark count probability of around $10^{-5}$ per ns. Coincidences between these two detectors are registered by a time-to-digital converter (TDC), where only a 2 ns time window is regarded.

In Fig. 4, the obtained coincidence count rate is plotted as a function of the difference between the two arms of the Mach-Zehnder interferometer $\Delta l$. The square points show the raw data, and the lower graph the false de-
tection, due to dark counts and accidental coincidences. The black line represents a fit through these points supposing a gaussian shape. A dip with a raw visibility of 37.6% can be observed when the interferometer is exactly balanced which yields a net visibility of 47.3% when the noise is subtracted. Although dark counts are an important source of error in an entanglement swapping experiment, the use of two pulsed photon pair sources, like intended in the future setup, will allow one to trigger the detectors. Hence the accidental coincidences will be reduce by at least one order of magnitude. In this way, the important figure of merit in this experiment is the net visibility.

The dotted line in Fig. 4 gives the theoretical expected curve of the convolution of two gaussian wave-packets. Their width corresponds to the coherence length of the filtered wave-packets, corrected by the refractive index of a stressed optical standard fiber. In former experiments, this value was found to be 1.8. The width of the obtained dip is slightly broader than the expected theoretical value.

Unfortunately, the first order interference terms cannot be observed experimentally because of phase shifts during the integration time. So just the mean value over several fringes is recorded, leading to a maximal depth of the dip of 50%.

To resolve these fringes, the path difference would required a stability within one wavelength during the acquisition of every data point. But due to low count-rates, integration times of 50s are necessary and temperature fluctuations during this time prevent their resolution. Note that for a temperature length dependence of 4 mm K$^{-1}$ km$^{-1}$, a stability of $10^{-5}$K would be required to resolve these interference fringes.

4. DISPERSION CANCELLATION

As can be seen from the experimental results, even after two times 25 km neither the visibility nor the width of the HOM-dip is significantly altered. In this section we will discuss why chromatic dispersion in optical fibers is cancelled out in our experiment and what it means for further experiments. A detailed discussion can be found in [22], where we also took the schemes from. In the scheme below we give a graphic explanation of the cancellation, followed by an analytical calculation.

Wave packets of a spectral width $\Delta \lambda = 0.8$ nm correspond to a coherence length of $\Delta t = 4.25$ ps. Even though they are broadened to 430 ps after 25 km of standard optical fiber, the form of the obtained 2-photon interference dip still corresponds to the initial coherence length. This can be explained by the following fact. Without loss of generality, we consider the case with a non-zero dispersion coefficient in only one arm.

Light passing through an interferometer is subject to chromatic dispersion like sketched in Fig. 5. Although our spectra are entirely in the infrared light, we will label the two photons "red" (r) and "blue" (b), to point out their difference in frequency. We assume dispersion only in the upper arm A. Interference takes place for two different paths which lead to the same detection scheme and are therefore, even in principle, indistinguishable.

The two alternatives here are either both the photons are transmitted or reflected at BS2, shown as Fig 5a and Fig 5b. In the two cases, the blue photon always arrives with the same delay after the red one. As long as no information about the creation time of the photon pair is available, the two cases remain indistinguishable. But as soon as the centers of the two wave-packets are relatively delayed by more than their coherence length, the two possible detection events become distinguishable and the bunching disappears. This also can be understood by calculating the relative delay of the two photons. Be $\tau(\omega)$ the travel time of a light pulse per length unit. Mathematically, the effects of fiber dispersion are accounted for by expanding $\tau(\omega)$ in a Taylor series around the center frequency $\omega_0$.

$$\tau(\omega) = \tau_0 + \tau_1(\omega - \omega_0) + \frac{1}{2}\tau_2(\omega - \omega_0)^2 + ... \quad (1)$$

Note that $\tau_0 = \frac{1}{v_g}$ with $v_g$ the group velocity of a wave packet. The chromatic dispersion $D$ corresponds to $D = -\frac{\delta \tau_1}{\delta \omega}$. 

FIG. 5: Dispersion cancellation scheme: Even non-zero dispersion in one arm of a Mach-Zehnder interferometer leads to the same firing scheme of the detectors and result in an HOM-dip with a width corresponding to the coherence time of the photons. The dotted line represents the center of the wave-packet. BS1 and BS2 are the two beam-splitters. It can be seen that in both cases detector C clicks with the same time-delay after detector D.

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Be $\omega_0 = \frac{1}{2} \omega_{\text{pump}}$, and $\tau^A(w)$ and $\tau^B(w)$ the different propagation times of fiber $A$ and fiber $B$ at frequency $\omega$. The time difference $\Delta \tau$ of the two cases where the blue photon ($\omega_1$) passes through fiber $A$ ($\tau^A(\omega_1)$) and the red ($\omega_2$) through fiber $B$ ($\tau^B(\omega_2)$), respectively vice versa ($\tau^A(\omega_2)$ and $\tau^B(\omega_2)$) is described by

$$\Delta \tau = (\tau^A(\omega_1) - \tau^B(\omega_2)) - (\tau^B(\omega_1) - \tau^A(\omega_2)) \quad (2)$$

By taking Eq. 1 and Eq. 2, $\Delta \tau$ can be calculated as

$$\Delta \tau = \tau^0_A(\omega_0) - \tau^0_B(\omega_0) + (\tau^1_A - \tau^1_B)(2\omega_0 - \omega_1 - \omega_2) + \frac{1}{2}(\tau^2_A - \tau^2_B)((\omega_0 - \omega_1)^2 + (\omega_0 - \omega_2)^2) \quad (3)$$

The length of the two fibers can be adjusted, so that $\tau^0_A = \tau^0_B$. For energy correlated photons ($\omega_1 + \omega_2 = 2\omega_0$) the terms $\tau_1$ in Eq. 3 cancel out. This effect is known as dispersion cancellation. So just the terms in $\tau_2$ can contribute to $\Delta \tau \neq 0$ and cause a change of the HOM-dip. As we obtain a net visibility of 47.3% which is quite close to the theoretical value of 50%, we can conclude, that $\tau^2_A - \tau^2_B$ should be negligibly small.

For the objective of a Bell-state measurement with photons originating from different sources, the energy-correlation is no longer fulfilled. In this case, $\tau^1_A - \tau^1_B$ has to be small within the coherence time and dispersion cancellation only works for similar dispersion in both paths. For standard telecom fibers (SMF-28) typical values of $D$ between 16.8 and 17.9 $ps \, nm^{-1} \, km^{-1}$ can be found. In our case, that leads to a variation of $\tau(\omega)$ of 0.14 $ps \, km^{-1}$ for a bandwidth of 0.8 nm. This means a limitation to less than 30 km, before this variation becomes larger than the coherence length of the photon of 4.25 ps. Beyond this limit, the fibers have to be chosen to have similar chromatic dispersion or compensated individually in order to still obtain 2-photon interferences. This again leads to a dip width corresponding to the coherence length of the two photons [10]. Equal chromatic dispersion in both arms can, in principle, be compensated by adding a dispersive medium right in front of the detectors with an dispersion index inverse to that of the fibers.

On the other hand, as light is polarized and needs to be in the same mode at BS2, we have to control the polarization. Because of the coherence length of our photons, it doesn’t depolarize and polarization mode dispersion (PMD) doesn’t significantly affect the results over this distance if high quality fibers are used.

5. CONCLUSION

In conclusion, we realized a proof-of-principle experiment of a long distant HOM experiment over two times 25 km. A SPDC source creates signal and idler photons at telecom wavelength with 100GHz bandwidth, corresponding to a coherence length of 1.3 mm in air. They pass through a 25 km Mach-Zehnder interferometer and are recombined on a beam-splitter again. A HOM-dip over this range has been observed, achieving a net visibility of 47.3%, which is quite close to the maximal theoretically value of 50%. This proves the indistinguishability of photon which travel via different fibers. It has been demonstrated that even after $2 \times 25$ km, chromatic dispersion and PMD have no significant impact on the obtained measurements. This represents a first step towards long distance entanglement swapping, as required in real world quantum communication. The following step will be a realization with photons coming from two independent sources.

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