Effect of Crack on Dynamic Characteristics of Simple Supported Beam

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Abstract. Simple supported beam is extensively applied in practical engineering structure. Crack is a common damage in simple supported beam and has greatly influences on the dynamic characteristics of the system. In this study, effect of crack on dynamic characteristics of simple supported beam is studied. The model of simple supported beam with crack damage is established by finite element (FE) method firstly. Then, effect of crack location and crack depth on dynamic characteristics of the simple supported beam is studied by comparison with the case when no crack occurs. Results of simulation studies conclude that crack with different location and depth will change natural frequencies differently and the effects of crack location on natural frequencies are related to the vibration model of the simple supported beam. This conclusion provides a good reference for simple supported beam structure performance optimization, damage detection and localization.

1. Introduction

In engineering practice, many mechanical and civil structural systems, such as frame of sugarcane harvester [1], Bridge [2], and pipeline [3], can be represented by simple supported beam. All such structural systems are prone to suffering fatigue crack damage due to long service time, improper use, or hostile working environments, which can cause the whole machine system abnormally. Therefore, it is very important to study the dynamic characteristics and failures of the simple supported beam.

The fatigue crack damage will decrease the local stiffness of the system and changes the system's natural frequency, vibration mode and other system parameters, resulting in deterioration of system performance. Therefore, many studies are concerned with effect of crack on system dynamic characteristics. For example, extensive researchers derived the relationship of crack parameters and modal parameters by analytical method. Guo [4] derived the modal frequency and vibration mode calculation formula of the beam by using the first-order perturbation theory, and analyzed the influence of crack depth and position on the characteristic modal parameters of multiple open cracked beams. The results show that the frequency changes caused by the cracks have obvious patterns. Zhang [5] derived the free vibration equation of the cracked beam structure, analyzed the influence of the change of crack position and depth on the natural frequencies, and then developed characteristic curve intersection method. Ma [6] derived the transfer matrix of the whole beam with the temperature and geometric parameters of cracks, and found that the natural frequencies of the simply supported steel beam were mostly affected by the temperature load and geometric parameters.

Besides, numerical simulation method is also used to study effects of crack on beam system dynamic characteristics. Song [7] established a finite element model of simply supported beam with vertical internal cracks, and proved that the natural frequency of simply supported beams decreases
with the increase of crack length, and the difference between simply supported beams and healthy simply supported beams increases gradually. The vertical internal crack will cause the local stiffness of the simply supported beam to change in the section where the crack is located, and the affected area increases with the increase of the crack length. Zou [8] carried out theoretical analysis, numerical simulation and experimental study on the crack damage of simply supported beams. The results show that the modal frequencies of the simply supported beams decrease with the increase of crack depth. Sun [9] studied the bending deformation of simply supported beam under the uniform load and the concentrated force and force couple at the location of crack. The results show that there is a sharp point in the deflection of the beam and a jump in the corner distribution. The response of the beam deflection to the load is generally in the form of a double-fold line, which corresponds to the open and closed state of the crack. Bakhtiari-Nejad [10] predicted the first natural frequency of simply supported fractured beams by the analysis estimates of Rayleigh method. Vigneshwaran [11] calculated the stiffness and natural frequency of multi-cracked beams, and obtained the modal shape and FRF of the cantilever beam with fracture and respiratory cracks, indicating that the shape of the model changes significantly due to the presence of cracks.

Most of above researches are mainly concerned with one crack parameter, location or depth, but both location and depth should be considered at the same time. Therefore, the effects of both location and depth of crack on the dynamic characteristics of the simple supported beam are investigated systematically in this paper. The paper is organized as follows. After this introduction, finite element model of a simple supported beam with a crack is established in Section 2. Effect of crack location and crack depth on dynamic characteristics of simple supported beam is studied by comparison with the case when no crack occurs in Section 3. Finally, the conclusions are presented in Section 4.

2. Finite element model of simple supported beam with a crack
In this paper, a traditional supported beam with a straight crack is investigated, as shown in Figure 1. In the figure, one end of the simple supported beam is full fixed, namely, all freedom of this end is fixed, the other end of the simple supported beam is semi-fixed end, namely, only freedoms in y-direction and z-direction are fixed. L, B and H denote the beam length, width and thickness, respectively; l denotes the distance between the crack tip and the full fixed end in x-direction; h is the crack depth. In order to present the crack location and depth more clearly, relative crack location and depth are exploited. The dimensionless relative crack location Lr is defined as Lr = l/L and dimensionless relative crack depth is defined as Hr = h/H.

![Figure 1. Schematic of a simple supported beam with a straight crack](image)

2.1. Finite element model of crack
The ANSYS/Workbench software is used to establish the Finite Element (FE) model of the cracked simple supported beam. In order to simplify the modeling process and reduce the element number, two assumptions about the crack are made as follows:

1) Crack is assumed to be straight crack and non-propagating, and the crack surfaces are assumed to be smooth and planar;
(2) Crack is assumed to be breathing crack, which is considered to be a frictionless contact problem where the contact elements are located on the semi-fixed end and the target elements on the fixed end.

V-crack is one of the most common crack types in engineering practice and is employed in this paper. And the crack geometric model is obtained by removing a small portion of the material in the position of the crack and then forming the V-gap, whose main parameters are crack location, crack depth, crack width and crack vertex angle. Crack location indicates the position where crack occurs. The crack depth is the depth of the V-gap, the crack width is equal to the length of the bottom of the V-gap, and the crack tip angle is the angle between the two sides of the V-gap. It must be pointed out that the crack gap should be as small as possible so as to make the crack model closer to the actual situation.

The main body of the crack is meshed by tetrahedral meshes, and the mesh refinement is conducted in a local sphere region near the middle of the crack. As is displayed in Figure 2, the radius of the sphere is 6mm, and the grid size inside the sphere is 1mm.

\[ \text{Figure 2. Finite element model of crack} \]

2.2. Finite element model of the simple supported beam

In order to simplify the modeling process, the material properties of the cracked simple supported beam are assumed to be linearly elastic. The structural material of the simple supported beam is stainless steel, young’s modulus 207 GPa, poisson's ratio 0.3 and density 7850 kg·m\(^{-3}\). The size parameter of the cracked beam is assumed to be: B=10mm, H=5mm, L=100mm.

In finite element model of the simple supported beam, all freedom of full fixed end is constrained and is shown in Figure 3 (a), and the freedom on y-direction and z-direction is constrained and is shown in Figure 3 (b). The finite element model of the cracked simple supported beam is meshed by the Quad shell element with the smallest unit size 0.5 mm. When the relative position of the crack is 0.5 and the relative depth is 0.5, finite element model of the simple supported beam is shown in Figure 3 (c). The total number of elements and nodes are 74405 and 108914, respectively, and the minimum Jacobian of the grid element is 1.

\[ \text{Figure 3. FE model of the simple supported beam} \]

3. Simulation analyses
In this section, the effects of crack parameters, including crack location and crack depth, on the beam dynamic characteristics are analyzed. The crack is assumed to locate on \( L_r = 0, 0.2, 0.4, 0.5, 0.6, 0.8 \) and 1, respectively, and the crack depth is assumed to \( H_r = 0, 0.1, 0.2, 0.3, 0.4 \) and 0.5, respectively. \( H_r = 0, L_r = 0 \) and 1 means there is no crack. Therefore, 26 cases are selected to analyze the effects of different parameters on the beam dynamic characteristics, and the corresponding crack parameters are shown in Table 1. In ANSYS/Workbench, there are 5 types of modal analysis methods including Direct Method, Iterative Method, Supernode Method, Superspace Method, and Unsymmetric Method. The Space Method and Iterative Method are suitable for solving large symmetric matrices, and the Subspace Method and the Subnode Method have higher accuracy and are applicable for solving the higher order model. The Asymmetric Method is mainly applied to acoustic or fluid solid coupling analysis, while the Direct Method is faster and it can also meet the requirement of solving precision. Therefore, Direct Method is chosen to conduct modal analysis.

| Relative location | Relative depth \( H_r \) | Case 1 (no crack) | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 |
|-------------------|--------------------------|-------------------|--------|--------|--------|--------|--------|
| 0 and 1           | 0                        | Case 1 (no crack) |        |        |        |        |        |
| 0.2               | 0.1                      | Case 2            | 7      | 12     | 17     | 22     |        |
|                   | 0.2                      | Case 3            | 3      | 8      | 13     | 18     | 23     |
| 0.4               | 0.3                      | Case 4            | 4      | 9      | 14     | 19     | 24     |
| (no crack)        | 0.4                      | Case 5            | 5      | 10     | 15     | 20     | 25     |
| 0.5               | 0.5                      | Case 6            | 6      | 11     | 16     | 21     | 26     |

3.1. Modal analysis of the simple supported beam without crack

In order to understand the dynamic characteristics of the simple supported beam in the normal working status, the modal analysis of the simple supported beam without crack is studied in this section. The first 6 order natural frequencies and the vibration mode are shown in Table 2 and Figure 4. It can be seen from Table 2 that the first 6 order natural frequencies are between 1758.8 Hz and 11511 Hz, and vibration mode covers 1st-order bending, 1st-order torsion, 2nd-order bending, 3rd-order bending.

Figures 4 (a) and (b) indicate that the first and second vibration modes are 1st-order bending in \( y \)-direction and \( z \)-direction, respectively, and maximum vibration amplitude is located on the 3/5 of the whole length from the full fixed end. Figures 4 (c) and (d) indicate that the third and fourth vibration modes are 2nd-order bending in \( z \)-direction and \( y \)-direction, respectively, and maximum vibration amplitude is located on the 1/3 and 4/5 of the whole length from the full fixed end. Figure 4 (e) indicate that the fifth vibration mode is 3rd-order bending in \( z \)-direction, and maximum vibration amplitude is located on the 1/7, 1/2 and 4/5 of the whole length from the full fixed end. The sixth vibration mode is 1st-order torsion, and maximum torsion amplitude is located in the middle of the whole length as shown in Figure 4(f).
Table 2. The first 6-order modes of beam without crack

| Order | Natural frequency (Hz) | Location of maximum amplitude | Vibration mode       | Order | Natural frequency (Hz) | Location of maximum amplitude | Vibration mode       |
|-------|------------------------|------------------------------|----------------------|-------|------------------------|------------------------------|----------------------|
| 1     | 1758.8                 | 3/5                          | 1st-order bending in y-direction | 4     | 10435                  | 1/3 and 4/5                  | 2nd-order bending in y-direction |
| 2     | 3414.3                 | 3/5                          | 1st-order bending in z-direction | 5     | 11438                  | 1/7, 1/2 and 4/5            | 3rd-order bending in z-direction |
| 3     | 5607.9                 | 1/3 and 4/5                  | 2nd-order bending in z-direction | 6     | 11551                  | 1/2                          | 1st-order torsion       |

(a) First order  (b) Second order  (c) Third order
(d) Fourth order  (e) Fifth order  (f) Sixth order

Figure 4. The first 6-order vibration modes of beam without crack

3.2. Modal analysis of the simple supported beam with crack

In order to study the influence of the crack parameters on the dynamic characteristics of the beam, the modal analyses of the simple supported beam with cracks in different positions and depths are carried out. The first 6-order natural frequencies in 26 different cases are shown in Figure 5 by taking the crack location as variable.
Figure 5. Effect of crack depth and crack location on the first 4-order natural frequencies

It can be seen from Figure 5 that when cracks occur, the natural frequencies of the first 6 orders become smaller, whatever the crack depth or the crack position changes; and at the same crack location, deeper cracks causes smaller natural frequencies. But when cracks occur at different locations, the effect of crack depth on the natural frequencies is quite different.

Figure 5(a) and (b) indicate that the changes of first and second natural frequencies are most obvious when the crack is located on the 0.6 of the whole length from the full fixed end where maximum vibration amplitude happens in both cases. Figures 5(c) and (d) indicate that the changes of third and fourth natural frequencies are most obvious when the crack is located on the 0.4 and 0.8 of the whole length from the full fixed end near which maximum vibration amplitudes happen in both cases. Figure 5(e) demonstrates that the changes of fifth order natural frequencies are most obvious when the crack is located on the 0.2 and 0.5 of the whole length from the full fixed end near which maximum vibration amplitudes happen. Figure 5(f) demonstrates that the changes of sixth order natural frequencies are most obvious when the crack is located on the 0.2 and 0.8 of the whole length from the full fixed end and least obvious when the crack is located in the middle of the whole length where maximum vibration amplitudes happen. To sum up, the most obvious decreases of the natural frequencies always happen where maximum vibration amplitudes happen in bending mode and the least obvious decreases of the natural frequencies happen where maximum torsion amplitudes happen in torsion model. Therefore, the effects of crack location on natural frequencies are related to the vibration model of the simple supported beam.

4. Conclusions
Both crack model and the model of the simple supported beam are investigated, and corresponding finite element model of the simple supported beam with crack damage is established in
ANSYS/Workbench software. Modal analysis of 26 cases with different crack location and crack depth are conducted so as to study effect of crack location and crack depth on dynamic characteristics of the simple supported beam. It is found that when cracks occur, the natural frequencies of the simple supported beam become smaller. The effects of crack location on natural frequencies are related to the vibration model of the simple supported beam.

Acknowledgements
The authors would like to express thanks to the Beijing Natural Science Foundation (No: 3184053) and the National Natural Science Foundation of China (No: 11702318) for their financial support of this research.

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