A Design Method of Generalized Predictive Control Systems in Consideration of Noise

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1. INTRODUCTION

Generalized Predictive Control (GPC) [1] is one of the model-based design methods and the closed-loop system is designed through performance index. The performance index includes the error between the reference signal and the output prediction, and the control input. And the closed-loop system is derived by the design parameters for output prediction, future control input series and weighting factor of inputs. For consideration of designing safe systems, although coprime factorization approach [2] has been used in order to extend the control law in the previous researches, there has been a possibility that the order of the derived control law becomes high. Therefore, this paper extends GPC through newly-defined output prediction and proposes the method to re-design the control law or the disturbance response with keeping the closed-loop transfer function. Numerical example is shown to check the characteristic of the proposed method.

For (1), the following assumptions are hold.

(i) $k_0$ is known.
(ii) The pairs of $(A[z^{-1}], B[z^{-1}])$ and $(A[z^{-1}], C[z^{-1}])$ are coprime.
(iii) $C[z^{-1}]$ is stable polynomial.

The following performance index $J$ is minimized to derive the control law.

$$J = E_x \left[ \sum_{j=1}^{N_x} \left( y(t+j) - w(t+j) \right)^2 + \sum_{j=1}^{N_y} \lambda \left( \Delta u(t+j-1) \right)^2 \right]$$

$E_[]$ means the expected value. The design parameters $[N_x, N_y], [1, N_y]$ and $\lambda$ are prediction horizon, control horizon and weighting factor of control input. Their parameters should be chosen so that the closed-loop system become stable.

In order to derive the control law, the predicted output $\hat{y}(t+j+j)$ for $j=N_x, \ldots, N_y$ is calculated by solving the following Diophantine equation.

$$C[z^{-1}] = \Delta A[z^{-1}] E_j[z^{-1}] + z^{-1} F_j[z^{-1}]$$

$E_j$ and $F_j$ are the poles and zeros of the reference signal.

2. EXTENSION OF GENERALIZED PREDICTIVE CONTROL

2.1. Conventional GPC

A single-input and -output system is considered for $t = 0, 1, 2, \ldots$.

$$A[z^{-1}] y(t) = z^{-k_p} B[z^{-1}] u(t) + C[z^{-1}] \frac{\hat{y}(t)}{\Delta}$$

$y(t)$ and $u(t)$ are output and input respectively. $k_p$ is time delay, $\hat{y}(t)$ is white Gaussian noise and $\Delta = 1 - z^{-1}$. $A[z^{-1}], B[z^{-1}]$ and $C[z^{-1}]$ are the following polynomials with known degrees $n, m$ and $l$.

$$A[z^{-1}] = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$$
$$B[z^{-1}] = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}$$
$$C[z^{-1}] = 1 + c_1 z^{-1} + \cdots + c_l z^{-l}$$

For (2), the following assumptions are hold.

(i) $k_0$ is known.
(ii) The pairs of $(A[z^{-1}], B[z^{-1}])$ and $(A[z^{-1}], C[z^{-1}])$ are coprime.
(iii) $C[z^{-1}]$ is stable polynomial.

The following performance index $J$ is minimized to derive the control law.

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$E_[]$ means the expected value. The design parameters $[N_x, N_y], [1, N_y]$ and $\lambda$ are prediction horizon, control horizon and weighting factor of control input. Their parameters should be chosen so that the closed-loop system become stable.

In order to derive the control law, the predicted output $\hat{y}(t+j+j)$ for $j=N_x, \ldots, N_y$ is calculated by solving the following Diophantine equation.

$$C[z^{-1}] = \Delta A[z^{-1}] E_j[z^{-1}] + z^{-1} F_j[z^{-1}]$$

$E_j$ and $F_j$ are the poles and zeros of the reference signal.
$E_j[z^{-1}]$ and $F_j[z^{-1}]$ are expressed as follows.

$$E_j[z^{-1}] = 1 + a_1 z^{-1} + \cdots + e_{j-1} z^{-j+1}$$  \hspace{1cm} (7)

$$F_j[z^{-1}] = f_{j1}^1 + f_{j2}^1 z^{-1} + \cdots + f_{jn}^j z^{-n}$$  \hspace{1cm} (8)

Multiplying $\Delta E[z^{-1}]$ to (1) and substituting (6) into it, the following equation is obtained.

$$C[z^{-1}]y(t+j) = E_j[z^{-1}]B[z^{-1}]\Delta u(t+j-k_m) + F_j[z^{-1}]y(t) + E_j[z^{-1}]C[z^{-1}]\xi(t+j)$$  \hspace{1cm} (9)

2.2. Proposed Method

In conventional GPC, the prediction is defined without including the future noise term $E_j[z^{-1}]C[z^{-1}]\xi(t+j)$. On the other hand, this paper proposes the use of the noise term to date, which can be calculated by (1). Concretely, the following output prediction is newly defined by introducing constant parameter $s_c$.

$$\hat{y}(t+j|t) = \frac{1}{C[z^{-1}]}(E_j[z^{-1}]B[z^{-1}]\Delta u(t+j-k_m)$$

$$+ F_j[z^{-1}]y(t) + s_c C[z^{-1}]\xi(t))$$  \hspace{1cm} (10)

where

$$C[z^{-1}]\xi(t) = \Delta A[z^{-1}]y(t) - z^{-k_m}B[z^{-1}]\Delta u(t)$$  \hspace{1cm} (11)

The following equations are considered.

$$E_j[z^{-1}]B[z^{-1}] = C[z^{-1}]R_j[z^{-1}] + z^{-j}S_j[z^{-1}]$$  \hspace{1cm} (12)

$$R_j[z^{-1}] = r_0 + r_1 z^{-1} + \cdots + r_{j-1} z^{-(j-1)}$$  \hspace{1cm} (13)

$$S_j[z^{-1}] = s_1^j + s_2^j z^{-1} + \cdots + s_{n_j}^j z^{-n_j}$$  \hspace{1cm} (14)

where $n_j = \max(m_l) - 1$. Then the output prediction (10) can be re-expressed as follows.

$$\hat{y}(t+j|t) = \frac{1}{C[z^{-1}]}(C[z^{-1}]R_j[z^{-1}]\Delta u(t+j-k_m)$$

$$+ (F_j[z^{-1}] + s_c \Delta A[z^{-1}])y(t)$$

$$+ (S_j[z^{-1}] - s_c B[z^{-1}])\Delta u(t-k_m))$$  \hspace{1cm} (15)

Next, the following equations are defined.

$$F_j[z^{-1}] = F_j[z^{-1}] + s_c \Delta A[z^{-1}]$$  \hspace{1cm} (16)

$$S_j[z^{-1}] = S_j[z^{-1}] - s_c B[z^{-1}]$$  \hspace{1cm} (17)

The past and present signals with output and input are also defined by $h_j(t)$.

$$C[z^{-1}]h_j(t) = F_j[z^{-1}]y(t) + S_j[z^{-1}]\Delta u(t-k_m)$$  \hspace{1cm} (18)

From these equations, the output prediction can be expressed as the following equation, which separates the input term $R_j[z^{-1}]\Delta u(t+j-k_m)$ in the current and future time from the other signal $h_j(t)$.

$$\hat{y}(t+j|t) = R_j[z^{-1}]\Delta u(t+j-k_m) + h_j(t)$$  \hspace{1cm} (19)

The following vectors and coefficients at time $t$ are defined for $j = N_1, \ldots, N_c$.

$$\hat{Y} = [\hat{y}(t+N_1|t) \cdots \hat{y}(t+N_c|t)]^T$$  \hspace{1cm} (20)

$$U = [\Delta u(t) \cdots \Delta u(t+N_u-1)]^T$$  \hspace{1cm} (21)

$$H = [h_{N_1}(t) \cdots h_{N_c}(t)]^T$$  \hspace{1cm} (22)

$$W = [w(t+N_1) \cdots w(t+N_c)]^T$$  \hspace{1cm} (23)

$$R = \begin{bmatrix} r_{N_1-k_m} & \cdots & r_0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ r_{N_c-k_m} & \cdots & r_{N_c-N_c-k_m-1} & \end{bmatrix}$$  \hspace{1cm} (24)

Then the output prediction (15) can be expressed as following vector form.

$$\hat{Y} = RU + H$$  \hspace{1cm} (25)

By using the above equation, the performance index $J$ can be described as follows.

$$J = (RU + H - W)^T (RU + H - W) + \lambda U^TU$$  \hspace{1cm} (26)

Minimizing the performance index $J$ for the input vector $U$, the control input can be given.

$$\Delta u(t) = [10 \cdots 0] (R^T R + \lambda I)^{-1} R^T (W - H)$$  \hspace{1cm} (27)

The following vector and polynomials are defined.

$$[P_{N_1} \cdots P_{N_c}] = [10 \cdots 0] (R^T R + \lambda I)^{-1} R^T$$  \hspace{1cm} (28)

$$P_{N_1} (z^{-1}) = P_{N_1} + P_{N_1-1} z^{-1} + \cdots + P_{N_1-N_1} z^{-(N_1-N_1)}$$  \hspace{1cm} (29)

$$F_{N_1} (z^{-1}) = F_{N_1} + F_{N_1} z^{-1} + \cdots + P_{N_1} F_{N_1} z^{-1}$$  \hspace{1cm} (30)

$$S_{N_1} (z^{-1}) = S_{N_1} + S_{N_1} z^{-1} + \cdots + P_{N_1} S_{N_1} z^{-1}$$  \hspace{1cm} (31)

Then the transfer function of control law is expressed as follows.

$$\Delta u(t) = \frac{C[z^{-1}]P[z^{-1}]}{C[z^{-1}] + z^{-k_m} S[z^{-1}]} w(t+N_2)$$

$$- \frac{F_p[z^{-1}]}{C[z^{-1}] + z^{-k_m} S_p[z^{-1}]} y(t)$$  \hspace{1cm} (32)
Moreover, the following polynomials are defined.

\[ D_p[z^{-1}] = \Delta A[z^{-1}] S_p[z^{-1}] + B[z^{-1}] F_p[z^{-1}] \]  
(33)

\[ T[z^{-1}] = \Delta A[z^{-1}] C[z^{-1}] + z^{-k_w} D_p[z^{-1}] \]  
(34)

Then the closed-loop system can be derived as follows.

\[ y(t) = \frac{z^{-k_w} B[z^{-1}] C[z^{-1}] P[z^{-1}]}{T[z^{-1}]} w(t + N_2) \]
\[ + \frac{C[z^{-1}](C[z^{-1}] + z^{-k_w} S_p[z^{-1}])}{T[z^{-1}]} \xi(t) \]  
(35)

It means that the output response is the same as the conventional GPC. On the other hand, it can find that the disturbance response can be tuned through the design polynomial \( S_p[z^{-1}] \) including \( s \).

### 3. NUMERICAL EXAMPLE

The following controlled plant is considered as Okazaki et al. [5].

\[ A[z^{-1}] = 1 - 0.998775z^{-1}, \quad B[z^{-1}] = 14.4 \]
\[ C[z^{-1}] = 1, \quad k_m = 1 \]

The number of simulation steps is 500, the initial values of input and output are set to be 0, and the variance of \( \xi(t) \) is \( \sigma^2 = 0.001 \) (each data of noise \( \xi(t) \) is the same for the conventional and the proposed method). The reference signal \( w(t) \) is 0.1. The design parameters of GPC are given as follows.

\[ N_1 = 1, N_2 = 6, N_u = 6, \lambda = 8500 \]

In this example, the design parameter of the proposed method is given as \( s = -1.8 \).

Figures 1 and 2 show the conventional output and input. In Figure 1, the dashed line and solid line mean the reference signal \( w(t) \) and the output signal \( y(t) \) respectively. In Figure 2, the upper figure shows the control input \( u(t) \) and the lower one shows the input increment \( \Delta u(t) \). Figures 3 and 4 show the proposed output and input. Their lines are the same meanings as conventional method. From these figures, it can find that the proposed method can change the noise influence on output.

### 4. CONCLUSION

This paper proposed a directly extended method of conventional GPC through newly-defined output prediction. The derived control law can re-design the characteristic from noise to output with keeping the closed-loop transfer function. Numerical example was given to check the characteristic of the proposed method.
As future work, the selection method of design parameter $s_j$ should be considered.

**CONFLICTS OF INTEREST**

The author declares no conflicts of interest.

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**AUTHOR INTRODUCTION**

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He received his PhD in Engineering from Okayama University, Japan in 2001. He worked with School of Engineering, Kinki University from 2002 to 2008, and Graduate School of Natural Science and Technology, Okayama University from 2009 to 2016. He joined Department of Radiological Technology, Kawasaki College of Allied Health Professions in 2016, and Faculty of Health Science and Technology, Kawasaki University of Medical Welfare in 2017, as an Associate Professor. His research interests include adaptive control, strong stability systems and estimation.