Wick Rotation in the Light-Front

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We study the electroweak properties of pseudo-scalar mesons in the light and heavy-light sectors. In particular, we address the electromagnetic form factors and decay constants of the pion, kaon and D mesons. The structure of composite systems are given by the Bethe-Salpeter (BS) amplitude of a meson formed by a confined pair of constituent quark and antiquark, which in our work is written in terms of Pauli-Villars regulators. The analytical structure contains single poles in the complex momentum space. The BS amplitude takes into account poles due to the regulator parameters, while the quark-antiquark cut is avoided, implying in confined quarks with the property that the sum of the constituents masses can be larger than the mass of the meson. The one-loop expressions of the electroweak transition amplitudes are conveniently written in terms of light-front momentum. Technically, we introduce a Wick-rotation of he minus component of the momentum (k-minus) in the one-loop amplitudes allowing to avoid the cuts in the complex plane of this momentum variable without crossing them. This is particularly useful as we can study the electroweak properties with several models of the BS amplitude with different powers of Pauli-Villars regulators. The possibility to change the power of the BS amplitude is interesting in order to test the asymptotic behavior of the electromagnetic form factors searching for a suitable form that incorporates the expected QCD decaying power-law form. The results are compared with others models in the literature and with the experimental data.

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1. Introduction

Quantum Chromodynamics is successful in describing subatomic processes for medium and large energies when short distances are probed, and asymptotic freedom justifies the use of perturbative amplitudes. The interesting physics that are investigated in the electroweak structure of hadrons are in part of nonperturbative origin. The understanding of the nonperturbative structure of hadrons with QCD demands a lot of effort and insights from phenomenological models that parameterizes the main properties of confined systems fitted to data can be a helpful guidance. In this respect, relativistic constituent models formulated in the different forms of dynamics proposed by Dirac [1] can be used to describe electroweak transitions. In particular the light-front (LF) form of dynamics, is well known by its maximal number of kinematic boosts generators and the stability of the Fock-state truncation under such boosts. Even this limited covariance property is essential for the calculation electroweak observables and in the definition of the partonic content of the hadrons [2]. Therefore, the description of a covariant transition amplitude that relies on nonperturbative hadron BS amplitudes can be decomposed in terms of matrix elements of operators in the light-front Fock-space. In principle it is possible to rewrite that matrix element in terms of the matrix element of an effective operator acting in the valence sector of the hadron. Then, a model that is able to give a LF valence wave function adequate for describing electroweak observables can be useful phenomenological tool to interpret results from nonperturbative calculations of QCD.

In this work, we study the electroweak properties of pseudo-scalar mesons in the light and heavy-light sectors. In particular, we address the electromagnetic form factors and decay constants of the pion, kaon and D mesons. The composite systems are modeled by Bethe-Salpeter amplitudes written in terms of Pauli-Villars regulators, i.e., the analytical structure contains single poles in the complex momentum space. The mesonic Bethe-Salpeter (BS) model amplitude takes into account poles due to the regulator parameters, while the quark-antiquark cut is avoided, implying in confined quarks with the property that the sum of the constituents masses can be larger than the mass of the meson.

We adopt the light-front momentum as a tool to calculate four-dimensional transition amplitudes involving mesons described as confined systems of a constituent quark and antiquark. Technically, we introduce a Wick-rotation of he minus component of the momentum \((k^− = k^0 - k^3)\) in the one-loop amplitudes allowing to avoid the cuts in the complex plane of this momentum variable without crossing them. This is particularly useful once we can study the electroweak properties of pseudoscalar mesons with several models of the Bethe-Salpeter amplitude with different powers of Pauli-Villars regulators. The possibility to change the power of the BS amplitude is interesting for testing the asymptotic behavior of the electromagnetic form factors and search for a suitable form that incorporates the expected QCD decaying power-law form. The results are compared with others models in the literature and with the experimental data.

2. The Model: Bethe-Salpeter Amplitude

The model of the vertex meson \(-q\bar{q}\) utilized to construct the Bethe-Salpeter amplitude is

\[
\Lambda_M(k,p) = \frac{(k^2 - m_q^2)\Gamma_M((p-k)^2 - m_q^2)}{(k^2 - \lambda^2 + i\epsilon)^n \left( (p-k)^2 - \lambda M + i\epsilon \right)^n},
\]

(2.1)
where, $\lambda_M$ is the scale associated with the meson light-front valence wave function and $n$ is the power of the regulator. $m_1$ and $m_2$ are the quark and anti-quark masses within the meson bound state. The factors $(k^2 - m_1^2)$ and $((p - k)^2 - m_2^2)$ on the numerator of the vertex function, $\Lambda_M(k, p)$, avoids the cuts due the $q\bar{q}$ scattering if $m_1 + m_2$ is smaller than the meson mass. In order to the confine the quarks, $\lambda_M$ obeys the condition $2\lambda_M >$ mass of the meson bound state. The model for $\Lambda_M(k, p)$ is utilized to calculate the electromagnetic observables of pseudoscalar mesons.

3. One-loop electroweak transition amplitudes

The electromagnetic form factors of pseudoscalar mesons are given by the Mandelstam formula:

$$\langle p' | J^\mu_q (q^2) | p \rangle = \frac{N_c}{(2\pi)^4} \int d^4k \, Tr \left[ \Lambda_M (k, p') S_F (k - p') J^\mu_q S(k - p) \Lambda_M (k, p) S_F (k) \right], \quad (3.1)$$

where $S_F(p)$ is the Feynman propagator of the quark with the constituent mass $m_q$ and $\Lambda_M$ is the meson $-q\bar{q}$ vertex function presented in the last section. $N_c = 3$ is the number of quark colors. $p^\mu$ and $p'^\mu$ are the initial and final momenta of the system; $q^\mu$ is the momentum transfer.

The pseudoscalar electromagnetic form factor is calculated with the matrix of the electromagnetic current:

$$\langle p' | J^\mu_q (q^2) | p \rangle = (p + p') F_{PS}^{em}(q^2) \quad (3.2)$$

The weak decay constant of the pseudoscalar mesons is written as

$$\langle 0 | A^\mu (0) | p \rangle = i\sqrt{2} f_{ps} p^\mu, \quad (3.3)$$

where $A^\mu = \bar{q}(x) \gamma^\mu \gamma^5 \frac{i}{2} q(x)$. The final expression for the pseudoscalar decay constant with the plus component of the axial-vector current, is

$$i p^2 f_{p} = \frac{m}{f_{p}} N_c \int \frac{d^4k}{(2\pi)^4} Tr \left[ p \gamma^5 S(k) \gamma^5 S(k - p) \right] \Lambda_M (k, p). \quad (3.4)$$

We use the plus component of the electromagnetic current, $\gamma^+ = \gamma^0 + \gamma^3$, to calculate the form factor of the pseudoscalar mesons. In the next section, the Wick rotation for the light-front approach is explained.

4. Light-Front Wick Rotation

The Wick rotation in the instant form quantum field theory, is realized by the change in the component $k_0$ of the quadri-momentum to $k_0 \rightarrow ik_0$, it is a equivalent change to the Euclidian space. The original idea was proposed by Wick [8], where the relativistic quantum field theory build with Minkowiski spacetime is replaced by the Euclidian space, with the following transformation $\tau = i t$, then, the spacetime metric is written like $d\tau^2 = dx^2 + dy^2 + dz^2$. The Wick rotation is applied to solve the Bethe-Salpeter bound state equation (BS) in the Euclidian space, because, the original BS equation formulated in Minkowski space has singularities making it difficult to solve. The idea is to use the Wick rotation to avoid the singularities in the Bethe-Salpeter equation by performing the calculation in the Euclidian space.
In the light-front quantum field theory, the choice of frame used to calculated Feynman amplitudes is helpful, because the analytical structure of the vertex function of the bound state-\(q\bar{q}\) pole position depends on the energy integration \(k^-\).

The idea of the Wick rotation within the light-front approach comes to avoid the problems related with the poles of the BS amplitudes and the frame used to calculate the Feynman amplitudes. The choice of a frame for the computation of a given amplitude within light-front quantum field theory is sensible because it is related to the breaking of covariance by the valence terms and the contribution non-valence components of the light-front wave function [7, 8, 9].

A common frame used to compute form factors in the light-front is the Breit-frame, characterized by a zero plus component of the momentum transfer, \(q^+ = 0\). In the past, this frame was believed free of the pair terms contributions or zero modes, but is not really true. Not only the frame dependence is important in the light-front approach, but also which component of the electromagnetic current is used to extract the observables. This was evident after the works done in the references [10, 11]. For example for the pion case, the electromagnetic form factor was calculated in the same model for the plus and minus component of the electromagnetic current, and compared with the equal time calculation. The minus component of the electromagnetic current, besides the valence contribution, has also the non-valence contribution, then, the full covariance of the minus component of the electromagnetic current needed this two contributions, valence and nonvalence [10]. However, the plus component of the electromagnetic current do not have other contributions besides the valence one, and the calculated matrix elements of the electromagnetic current is exactly the same one obtained in an equal time calculation or instant form approach [10].

The use of the Wick rotation with light-front formalism is explored in the next section, where the electromagnetic form factors and weak decay constants are calculated.

\[ k^- = k^- e^{i\theta}; \quad 0^0 < \theta < 90^0 \]

![Figure 1: Wick rotation in the light-front coordinates](image)

5. Results and Conclusion

We have calculated with the presented model the electromagnetic form factors and the weak decay constants for the light pseudoscalar mesons, pion, kaon and \(D^+\) with the Wick rotation performed within the light-front approach. The present model, was compared with other light-front models and dispersion relation calculations [3, 10, 12].
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The table 1, shows the results for the electromagnetic radius and weak decay constants for the pion and kaon, compared with the experimental data. The parameters of the model are, the constituents quark masses, the power in the vertex, $n$, and $\lambda_M$. The value of the masses $\lambda_M$ are given by the fit of the experimental value of the weak decay constants $f_{\psi}$. The masses of the constituents quarks used are $m_u = 0.220$ GeV and $m_s = 0.508$ GeV. The value of the electromagnetic radius of the pion is approximately 14% below of the experimental value $(0.675 \pm 0.02)$ \cite{13} with $n=2$ and the mass scale is $\lambda_M = 0.542$ GeV for the fits the pion decay constant is $f_\pi = 92.4$ MeV. The electromagnetic form factor for the pion is presented in figures 1 and 2; and the kaon an $D^+$ electromagnetic form factor are show in figure 3.

In conclusion, the electromagnetic form factors and weak decay constants calculated with the present light-front model were compared with others models and reproduced very well the experimental data.

|       | $r_{\psi}$ (fm) | $f_{\psi}$ (MeV) | $\lambda_M$ (MeV) |
|-------|-----------------|------------------|-----------------|
| Pion  (139 MeV) | 2.076          | 92.4             | 542             |
| $m_u = 220$ (MeV) | 3.049           | 92.4             | 926             |
| $m_d = 220$ (MeV) | 4.045           | 92.4             | 1255            |
| Exp. (Pion) | 0.672           | 92.42            |                 |
| Kaon (494 MeV) | 2.074           | 113              | 648             |
| $m_s = 0.508$ (MeV) | 3.045           | 113              | 933             |
| $m_u = 0.220$ (MeV) | 4.040           | 113              | 1156            |
| Exp.(Kaon) | 0.560           | 113              |                 |

Table 1: Results for the pseudoscalar weak decay constants $f_{\psi}$ and the electromagnetic radius for the pion and kaon compared with the experimental data.

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Figure 2: The left figure shows the pion electromagnetic form factor calculated with the light-front covariant model for different n values and it is compared to the experimental data. In the right figure, the light-front covariant model is compared with other light-front models from references [9, 10] and dispersion relations [12], also with the experimental data [14].

Figure 3: The kaon electromagnetic form factor (left) calculated with light-front covariant model and other models is compared with the experimental data [13, 15]. The electromagnetic form factor for the $D^+$ meson with the covariant light-front model and compared with others models.

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