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Application of GM(1, N) model in groundwater mineralization in Cheng'an County

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Abstract: In order to make full use of the salt water resources in Cheng'an County, Handan City, improve the availability of water resources in the region, and alleviate the shortage of water resources, so we simulated the change of groundwater salinity in this area. Based on the grey theory, a mathematical model was established for the groundwater salinity of the region from 2010 to 2015, and the actual depth was studied. The GM(1,N) model was established by analyzing the groundwater level, actual pumping amount and regional rainfall and other related factors, the GM(1,1) model was used to simulate the area, and the simulation results of the two were compared. The result is that GM (1,N) model considering the factors is closer to the actual value of the mineralization degree fitting, and the simulation accuracy is higher. At the same time, the groundwater mineralization degree of the region from 2016 to 2020 is predicted. The results show that the groundwater salinity will change significantly in the next five years. Except for the decreasing trend of Da Zhai monitoring wells, the others are obviously rising.

1. Introduction
Located in the southwestern part of Hebei Province, the city of Handan has always had a shortage of water resources. In recent years, with the population growth and the development of social economy, the extreme shortage of water resources has led to severe constraints on the social and economic development of Handan City[1]. Cheng'an County, a typical agricultural county with water shortage in the eastern part of Handan City, has a per capita water resource of only 197m³, which is less than 1/10 of the national average[2]. Due to the shortage of water resources, water shortages in all aspects, the water sector is over-exploited despite the consequences, causing obvious subsidence of the ground, and the balance of groundwater is unconsciously broken, as a result, the salinity of groundwater affecting the degree of water use is changed. Groundwater salinity is an important indicator of the determination of water chemical composition and is essential for the quality of groundwater and the total amount of salt in groundwater. Most importantly, the degree of mineralization directly determines the applicability of groundwater, which is critical for water resources throughout the study area[3].
For the study of groundwater mineralization, there are many research methods in the predecessors, but the use of gray model theory is rare. Moreover, the GM (1,1) model was mainly used for simple operation simulation, and complicated factors were not considered. Therefore, this paper establishes a GM (1,N) model considering several factors to further study the groundwater mineralization.

The biggest advantage of the GM (1, N) model is that it does not require long-term data sequences. It still has high precision in the case of data shortage, and the calculation process is simple and feasible. This advantage is applicable to the simulation prediction of Cheng an County, which only has the measured data of groundwater salinity from 2010 to 2015. This paper uses the GM (1,N) model to simulate and predict the salinity of Cheng an County. By comparing with the simulation results of GM (1,1) model, the model with higher precision and better reference value for mineralization prediction is determined. Through the study of mineralization degree, it is possible to better develop and utilize underground brackish water and salt water resources, which has certain practical significance for solving the problem of water shortage in the region.

2. Introduction to the gray model
The grey theory was proposed by Professor Julong Deng in the 1980s. It is very suitable for some problems where the sample is small, the information is insufficient, and the surface is clear but the connotation is vague. The grey system, as the name implies, is the middle of white and black, covering both known and unknown information. The Grey Model (GM model) is a model for predicting gray systems [4].

2.1 GM (1,1) Model
GM(1,1) model: The grey GM(1,1) model is a univariate first-order model, a first-order univariate constant-coefficient differential equation, which is established by the gray differential fitting method for the cumulative generation of a given gray time series. It is very suitable for generalized systems, so it is a gray model suitable for prediction, and it is also the basic model of gray prediction [5].

Let the non-negative original equidistant time series be \( X^{(0)}=(X^{(0)}(1),X^{(0)}(2),\ldots,X^{(0)}(n)) \), \( n \) is the length of the sequence. Perform a first-order accumulation of \( X^{(0)} \) to generate a data sequence (1-AGO). Then get a generated sequence \( X^{(1)}=(X^{(1)}(1),X^{(1)}(2),\ldots,X^{(1)}(n)) \), among them,

\[
X^{(1)}(K) = \sum_{i=1}^{K} X^{(0)}(t)(k=1,2,\ldots,n), \quad \text{determine the data matrix}
\]

\[
B = \begin{bmatrix}
-Z^{(1)}(2) & 1 \\
-Z^{(1)}(3) & 1 \\
\vdots & \vdots \\
-Z^{(1)}(n) & 1
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
X^{(0)}(2) \\
X^{(0)}(3) \\
\vdots \\
X^{(0)}(n)
\end{bmatrix}
\]

Among them, \( Z^{(1)}(t+1) = 0.5X^{(1)}(t) + 0.5X^{(1)}(t+1) \), \( r=1,2,\ldots,n-1 \). The parameter column \( a=(B^T B)^{-1}B^T Y \) is obtained from the least squares estimation, and the generated data sequence model (1) and the time response formula (2) are established.

\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = b 
\]

(1)

\[
x^{(1)}(t+1) = (x^{(0)}(1) - b / a)e^{-at} + b / a
\]

(2)

The above is the principle of the GM (1,1) model [6].
2.2 GM(1, N) Model

The GM (1, N) model is a first-order differential equation for N models. This model contains a main factor and multiple related factors. The operating mechanism is to analyze the effects of multiple influencing factors on the main system factors. The trend of factors is obvious, and the main system factors are predicted [5].

Since the gray theory was put forward, various experts and scholars have carried out research and application on various aspects of the GM (1, N) model, including economy, society, politics and so on. It can be said that the application of this model is very extensive. For example, in the literature [4] Zhen Fan et al. obtained the influencing factors through the gray correlation analysis method. Through the improved GM (1, N) model, the soybean price was simulated and predicted, which provided the basis for the soybean price market. Literature [7] Bin Wang et al. used the five-year energy consumption data in Beijing as an example to establish a GM (1, N) model to simulate and predict energy consumption. The prediction results were accurate and suitable for promotion; Literature [9] Yu Yang et al. used GM (1, N) model to simulate the production capacity of 15 gas wells in Chang Qing gas field, which was very good for high-yield wells; Literature [8] Kai Cao et al. combined the GM (1,1) model and GM (1, N) to form a joint model to predict settlement of buildings; In [10], Shao dong Chen proposed an improved method to correct the forecasting error of the GM (1, N) model and simulate the economy.

It can be seen that the GM (1, N) model is widely used, but there are not many studies on groundwater salinity. Therefore, the GM (1, N) model is applied to the simulation prediction of groundwater salinity, which has certain significance for the study of enriched salinity.

Principle of GM(1,N) model: Based on the accumulation of n groups of data sequences, n groups of new data sequences with obvious trends are formed, and models are established according to the growth trend of the new data sequences, and then predicted. Then the inverse calculation is performed by the subtractive method to restore the original data sequence, and finally the prediction result is obtained [10]

Let $X_i^{(0)} = (X_i^0(1), X_i^0(2), \cdots, X_i^0(n))$ as the feature data sequence of grey system,

$$X_i^{(0)} = (X_i^0(1), X_i^0(2), \cdots, X_i^0(n))$$

(3)

$$X_2^{(0)} = (X_2^0(1), X_2^0(2), \cdots, X_2^0(n))$$

(4)

$$X_3^{(0)} = (X_3^0(1), X_3^0(2), \cdots, X_3^0(n))$$

(5)

The equations (1)、(2)and (3) are sequences of related factors. We can call $X_i^{(1)}$ is $X_i^{(0)}$ 1-AGO sequence($X=1,2,\ldots,N$), $Z_i^{(1)}$ is the mean of adjacent sequences of $X_i^{(1)}$, where

$$Z_i^{(1)}(k) = \frac{1}{2} X_i^{(1)}(k) + \frac{1}{2} X_i^{(1)}(k-1)$$

(7)

$$X_i^{(0)}(k) + a Z_i^{(1)}(k) = \sum_{i=2}^{N} b_i X_i^{(1)}(k)$$

(8)

The equation (6) is called GM(1,N)[11]
3. Example application of GM(1,N) model

3.1 Study area overview and data collection

Cheng an County is located in the southeast of Handan City, Hebei Province. It is located at the junction of Shanxi, Hebei, Shandong and Henan provinces. The geographical coordinates are 36°18'-36°30' north latitude and 114°29'-114°53' east. The area is 481.5 square kilometers. Cheng an County is situated at the Jinan Plain, slightly higher in the southwest, slightly lower in the northeast, and flat. The north and south of the west are the sands of the old road of the Zhang he River. Cheng an County is a temperate continental monsoon climate. It is dry and windy in winter and spring, and rainy in summer and autumn. It is located in the eastern plain of Handan City and is a region with serious water scarcity. Due to the rapid economic development, the various water departments have been forced to over-exploit groundwater, which has greatly affected the local groundwater environment and brought many problems. The detailed location map is shown in Figure 1 below.

![Figure 1 Location map of administrative divisions and monitoring wells in Cheng an County](image)

This paper will select Zhang Xin Zhuang, Zhang Zhuang and Da Zhai, which have sufficient data in recent years in Cheng an County, as the monitoring points. The location of the specific monitoring points is shown in Figure 1. For the degree of mineralization, there are many factors affecting its change. From the perspective of the regional characteristics of Cheng an County, and through the gray correlation analysis, we finally selected the three most influential factors, namely the well water level, rainfall and pumping amount. This paper selects the statistical data from 2010-2015, which is from the Handan Water Resources Bulletin (2010-2015), as shown in Table 1.

| name            | related indicators | 2010    | 2011    | 2012    | 2013    | 2014    | 2015    |
|-----------------|--------------------|---------|---------|---------|---------|---------|---------|
| Zhang Xin Zhuang| salinity (mg/L)    | 465.00  | 437.00  | 443.50  | 931.50  | 539.50  | 887.00  |
|                 | water level (m)    | 26.76   | 27.63   | 28.06   | 28.59   | 29.48   | 30.75   |
|                 | pumping amount (10^8 m^3) | 1.21 | 0.97 | 1.14 | 0.98 | 0.78 | 0.91 |
|                 | rainfall (mm)      | 417.20  | 485.90  | 506.10  | 507.10  | 451.70  | 371.30  |
| Zhang Zhuang    | salinity (mg/L)    | 458.50  | 592.00  | 835.50  | 470.00  | 797.00  | 862.50  |
|                 | water level (m)    | 41.33   | 40.20   | 40.40   | 40.47   | 43.09   | 54.83   |
|                 | pumping amount (10^8 m^3) | 1.21 | 0.97 | 1.14 | 0.98 | 0.78 | 0.91 |
|                 | rainfall (mm)      | 417.20  | 485.90  | 506.10  | 507.10  | 451.70  | 371.30  |
| Da Zhai         | salinity (mg/L)    | 629.5   | 720     | 968.5   | 780.5   | 1265    | 516.0   |
|                 | water level (m)    | 34.58   | 36.96   | 36.21   | 37.58   | 38.24   | 42.16   |
|                 | pumping amount (10^8 m^3) | 1.21 | 0.97 | 1.14 | 0.98 | 0.78 | 0.91 |
|                 | rainfall (mm)      | 417.20  | 485.90  | 506.10  | 507.10  | 451.70  | 371.30  |

3.2 Establishment of GM(1, N) model for groundwater salinity in Cheng an County

The GM(1,N) model is a gray model that reflects the linear relationship between various factors, and
which based on the volatility and gray between the indicators of single cumulative addition and weakened [12]. The mineralization is the variable $X_1^{(0)}$ in studying this paper, select three indicators such as groundwater level, pumping amount and regional precipitation as the relevant factor sequence $X_2^{(0)}$, $X_3^{(0)}$, $X_4^{(0)}$ and $X_i^{(0)}$ ($i=1,2,3$) The steps of mineralization establishing the GM(1,3) model of the input variable ($i=1,2,3$) in the groundwater of Cheng an County are as follows:

Step 1: Analyze the targets that need to be simulated and predict, determine the relevant factor sequences, and collect sample data [11]. The mineralization degree is used as the predictive index, and the variables such as water level, pumping amount and regional precipitation of the study site are related factors in this paper.

Step 2: In view of four variables $X_1$, $X_2$, $X_3$, $X_4$, taking the total sum for $X_1^{(0)}$, that is 1-AGO. Set up an albinism differential equation based on the gray model of mineralization in Cheng an County, also known as the shadow equation:

$$\frac{dx_i^{(0)}}{dt} + ax_i^{(0)} = b_2x_2^{(0)} + b_3x_3^{(0)} + b_4x_4^{(0)}$$

Of which: $x_i$ is the mineralization, $x_2$ is the water level, $x_3$ is pumping amount, $x_4$ is precipitation.

Step 3: calculate the parameter vector:

$$\hat{a} = [a, b_2, b_3, b_4]^T$$

The least squares estimate of (10) satisfies:

$$\hat{a} = (B^T B)^{-1} B^T Y$$

Step 4: Calculating matrix $B$ and matrix $Y$ seen from the above.

$$B = \begin{bmatrix}
-\frac{1}{2} (x_1^{(0)}(1) + x_1^{(0)}(2)) & x_2^{(0)}(2) & x_3^{(0)}(2) & x_4^{(0)}(2) \\
-\frac{1}{2} (x_1^{(0)}(2) + x_1^{(0)}(3)) & x_2^{(0)}(3) & x_3^{(0)}(3) & x_4^{(0)}(3) \\
-\frac{1}{2} (x_1^{(0)}(3) + x_1^{(0)}(4)) & x_2^{(0)}(4) & x_3^{(0)}(4) & x_4^{(0)}(4) \\
-\frac{1}{2} (x_1^{(0)}(4) + x_1^{(0)}(5)) & x_2^{(0)}(5) & x_3^{(0)}(5) & x_4^{(0)}(5) \\
-\frac{1}{2} (x_1^{(0)}(5) + x_1^{(0)}(6)) & x_2^{(0)}(6) & x_3^{(0)}(6) & x_4^{(0)}(6)
\end{bmatrix}$$

$$Y = \begin{bmatrix}
x_1^{(0)}(2) \\
x_1^{(0)}(3) \\
x_1^{(0)}(4) \\
x_1^{(0)}(5) \\
x_1^{(0)}(6)
\end{bmatrix}$$

Step 5: Calculate the simulated and predicted values by the above principles according to the programmed MATLAB program.

Step 6: It compared the relative error of the actual measured value and the simulated value of the model with the variance, and verifying the accuracy of the model simulation result [13].

3.3 GM (1, N) model operation results

We used the MATLAB to program and imported the data of Table 1 into the established GM (1, N) model.
model to obtain the simulated value of groundwater salinity in Cheng an County from 2010 to 2015. The simulated values of the three monitoring points are listed in Table 2 below. At the same time, the measured and simulated curves of salinity were drawn respectively, as shown in Figures 2–4, so that the fitting accuracy can be seen clearly and intuitively.

### Table 2    GM (1, N) model simulation results table (mg/L)

| year | Zhang Xin Zhuang | Zhang Zhuang | Da Zhai |
|------|-----------------|-------------|--------|
| 2010 | 460.00          | 458.50      | 629.50 |
| 2011 | 460.00          | 637.30      | 727.20 |
| 2012 | 270.00          | 707.50      | 836.50 |
| 2013 | 1000.00         | 600.40      | 922.10 |
| 2014 | 550.00          | 734.10      | 1185.10|
| 2015 | 870.00          | 876.60      | 538.10 |

Based on the principle of the GM(1,1) model above, we simulated the groundwater salinity of three monitoring points selected in Cheng an County, and the GM(1,N) simulation results were compared with it to further illustrate the reliability of the GM(1,N) model for mineralization prediction. The relative errors of the simulated and measured values of the three monitoring points of Zhang Xin Zhuang, Zhang Zhuang and Da Zhai in the two different models are shown in Table 3-5 below.

4. Precision analysis

4.1 Comparison of fitting accuracy

Based on the principle of the GM(1,1) model above, we simulated the groundwater salinity of three monitoring points selected in Cheng an County, and the GM(1,N) simulation results were compared with it to further illustrate the reliability of the GM(1,N) model for mineralization prediction. The relative errors of the simulated and measured values of the three monitoring points of Zhang Xin Zhuang, Zhang Zhuang and Da Zhai in the two different models are shown in Table 3-5 below.
Table 3 Comparison table of measured values and fitting results of two models of Zhang Xin Zhuang

| year | measured value (mg/L) | GM(1,1) model | fitted value (mg/L) | relative error (%) | GM(1,N) model | fitted value (mg/L) | relative error (%) |
|------|----------------------|---------------|---------------------|--------------------|---------------|---------------------|--------------------|
| 2010 | 465.00               | 465.00        | 0                   | 460.00             | -1.08         |
| 2011 | 437.00               | 471.50        | 7.89                | 600.00             | 37.30         |
| 2012 | 443.50               | 546.70        | 23.27               | 270.00             | -39.12        |
| 2013 | 931.50               | 634.00        | -31.94              | 1000.00            | 7.35          |
| 2014 | 539.50               | 735.20        | 36.27               | 550.00             | 2.87          |
| 2015 | 887.00               | 852.50        | -3.89               | 870.00             | -1.92         |
|      | average error (%)    | 5.27          |                     |                    | 1.02          |

Table 4 Comparison of measured values and fitting results of two models of Zhang Zhuang

| year | measured value (mg/L) | GM(1,1) model | fitted value (mg/L) | relative error (%) | GM(1,N) model | fitted value (mg/L) | relative error (%) |
|------|----------------------|---------------|---------------------|--------------------|---------------|---------------------|--------------------|
| 2010 | 458.50               | 458.00        | 0                   | 458.00             | 0             |
| 2011 | 592.00               | 609.60        | 2.97                | 637.30             | 7.65          |
| 2012 | 835.50               | 656.40        | -21.44              | 707.00             | -15.38        |
| 2013 | 470.00               | 706.00        | 50.37               | 600.40             | 27.75         |
| 2014 | 797.00               | 761.00        | -4.52               | 734.10             | -7.89         |
| 2015 | 862.50               | 819.40        | -5.00               | 876.60             | 1.64          |
|      | average error (%)    | 3.73          |                     |                    | 2.30          |

Table 5 Comparison of measured values and fitting results of two models of Da Zhai

| year | measured value (mg/L) | GM(1,1) model | fitted value (mg/L) | relative error (%) | GM(1,N) model | fitted value (mg/L) | relative error (%) |
|------|----------------------|---------------|---------------------|--------------------|---------------|---------------------|--------------------|
| 2010 | 629.50               | 629.50        | 0.00                | 629.50             | 0.00          |
| 2011 | 720.00               | 809.24        | 20.72               | 727.20             | 1.00          |
| 2012 | 968.50               | 859.58        | -11.25              | 863.60             | -10.78        |
| 2013 | 780.50               | 850.02        | 8.91                | 922.10             | 18.14         |
| 2014 | 1265.00              | 840.57        | -33.55              | 1185.10            | -6.32         |
| 2015 | 516.50               | 831.23        | 60.93               | 538.10             | 4.18          |
|      | average error (%)    | 7.63          |                     |                    | 1.04          |

According to the comparison table of the above three monitoring points, the average error of Zhang Xin Zhuang monitoring point under GM (1,1) model is 5.27%, and the average error under GM (1,N) model is 1.02%; The average error of the Zhang Zhuang monitoring point under the GM(1,1) model is 3.73%, and the average error under the GM(1,N) model is 2.30%; The average error of the Da Zhai monitoring point under the GM(1,1) model is 7.63%, and the average error of the GM(1,N) model after operation is 1.04%. Obviously, the average error of the GM(1,N) model in the three research sites of Zhang Xin Zhuang, Zhang Zhuang and Da Zhai is smaller than the average error of the GM(1,1)
model, which is close to zero and has higher precision.

In order to further prove the simulation accuracy of the GM(1,N) model and to reliably predict the mineralization degree, it is necessary to calculate the root mean square error of the GM(1,N) model and the GM(1,1) model separately. See Table 6 for details.

| Monitoring points     | GM(1,1) model | GM(1,N) model |
|-----------------------|---------------|---------------|
| Zhang Xin Zhuang      | 152.66        | 101.48        |
| Zhang Zhuang          | 123.31        | 81.37         |
| Da Zhai               | 230.28        | 79.54         |

Table 6 comparison of root mean square errors of GM(1,1) and GM(1,N) models

From table 3-6, the GM(1,N) model has better simulation accuracy for the three monitoring wells than the GM(1,1) model, both from the average error and the root mean square error. The main reason is the GM(1,1) model, which only uses relatively simple and simple time series data. The mutual influence, common development and constraints of multiple variables cannot be reflected. However, the GM(1,N) model is not a simple superposition of the GM(1,1) model. It considers the influence between variables and is a kind of correlation prediction [12]. Therefore, the GM(1,N) model considering the water level, pumping amount and rainfall related factors is more accurate than the GM(1,1) model considering only the single factor.

4.2 Groundwater mineralization prediction

At the same time as the simulation of groundwater salinity in Chang County from 2010 to 2015, it's also a prediction of the mineralization of 2016 to 2020. Due to the high precision of the GM(1,N) model in the region, the model has great reliability for future predictions. It provides a reference for the change of groundwater mineralization in Chang County in the next few years. The forecast results are shown in the following Table 7.

| year   | Zhang Xin Zhuang | Zhang Zhuang | Da Zhai |
|--------|------------------|-------------|--------|
| 2016   | 389.85           | 913.60      | 888.70 |
| 2017   | 370.21           | 1007.90     | 723.90 |
| 2018   | 434.15           | 1065.20     | 729.40 |
| 2019   | 450.09           | 1134.10     | 671.40 |
| 2020   | 475.58           | 1192.60     | 643.30 |

Table 7 GM(1, N) model prediction analysis table of each monitoring well (mg / L)

5. Conclusions

In this paper, it used GM (1, N) model to simulate that salinity of groundwater at three monitoring point of Zhang Xin Zhuang, Zhang Zhuang and Da Zhai by using water level, pumping volume, regional rainfall and salinity data as input of the model. Studied the change of groundwater salinity and got the following conclusions:

(1) The gray model is very suitable for Chang County with insufficient salinity data, which further promotes the development of groundwater mineralization;

(2) Comparing the GM(1,N) model with the running results of the GM(1,1) model, the results show that the simulation accuracy of the GM(1,N) model considering the influence of multiple factors is higher. It shows that the GM(1,N) model has high reliability for the simulation and prediction of groundwater salinity in Chang County.;

(3) Through prediction, the mineralization degree of Zhang Xin Zhuang and Zhang Zhuang monitoring points will show an upward trend in the next few years, while Da Zhai has a downward trend.

This study not only has important significance for the prevention and control of land salinization in
the study area, but also has important reference value for rational development and utilization of groundwater resources in the region and solving the problem of groundwater resources.

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References:
[1] Long, Q.B. Study on Sustainable Water Resources Management in the Eastern Plains of Handan City[D]. Hebei University of Engineering
[2] Wang, Y. M. Comprehensively promote the comprehensive management of groundwater over-exploitation in Cheng an County [J]. Water and soil side,2016.
[3] Lu, Q., Pan, W.B., Environmental monitoring tutorial Version 2: South China University of Technology Press, 2014.
[4] Fan, Z., Ma K.P., Jang, S.J., Shi, B., Analysis and Prediction of Influencing Factors of Soybean Price in China Based on Improved GM(1,N) Model [J]. Soybean science 2016, 35(05):847-849.
[5] Li, J.F., Chen, J., Du, L.Y., Comparison of GM(1,1) and GM(1,N) Models in GDP Forecasting [J]. SCIENCE & TECHNOLOGY INFORMATION, 2008, 20, 498-499.
[6] Lv, Y.Q., He, Y., Prediction Model of the Development Trend of the Third Industry Based on Gray Neural Network -《Statistics and Decision》- 2011,04.
[7] Wang, B., Zhu, S.D., Application of GM(1,N) Grey Model in Energy Consumption Forecasting [J]. Business research,2006,483:29-30.
[8] Yang, Y., Zeng, X.B., Application of GM(1,N) Grey Model in Gas Well Production Capacity Prediction [J]. Journal of Guilin University of Technology 2004, 24(03): 286-288.
[9] Cao, K., Xu, C., Application of GM(1,1) and GM(1,N) joint model in building settlement prediction [J]. Water Science and Engineering Technology, 2007, 06: 54-57.
[10] Chen, S.D., Prediction and application of improved grey GM (1, N) model in economy [J]. Journal of Yi Chun University, 2010, 32(04).
[11] Bi, X.J., Wu, X.A., Research and Application of Multi-level Grey GM(1,N) Model with Feedback Mechanism [J]. Journal of Harbin Engineering University, 2007; 28(5): 577-580.
[12] Zhou, W., Fang, Z.G., Nonlinear Optimization GM(1,N) Model and Its Application [J]. Systems Engineering and Electronics, 2010, 32(02): 317-319.
[13] Zhang, Z.Y., Application of Grey Model in Prediction of Groundwater Mineralization in Da Ming County of Handan City [D]. Hebei University of Engineering,2018.