Tilting pad gas bearing design for micro gas turbines

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Abstract. This paper presents the results of a dynamic stability investigation of a micro gas turbine supported by two flexible tilting pad bearings. The pad flexibility allows centrifugal and thermal shaft growth of the rotor but can also introduce undesirable rotor instabilities. An eigenvalue analysis on the linearised rotor-bearing dynamics is performed to estimate the required pad stiffness and damping for stability. Results of the eigenvalue analysis are evaluated by fully nonlinear orbit simulations.

1. Introduction

Bearings for micro gas turbines have to operate at extremely high rotational speeds while maintaining dynamic rotor stability over a wide temperature range. Rotational speeds above 200,000 rpm and temperatures of several hundred degrees are not rare for micro gas turbines with power outputs of several kWe. Conventional oil-film or ceramic ball bearings have under these conditions a limited life-span or are causing significant frictional losses.

Aerodynamic gas bearings offer in this context an interesting alternative. They operate virtually wear-free and with relative low frictional losses at high speeds. Recent research at KU Leuven showed stable operation of an experimental 6mm round shaft on aerodynamic gas bearings for speed up to 1.2 million rpm [1]. The developed bearing solution with a fixed inner diameter is however unable to deal with excessive thermal shaft growth. Current research is therefore focused on miniature tilting pad bearings with radial pad flexibility. The flexibility will allow the pads to expand outwards when shaft growth occurs, ensuring an appropriate gap geometry between the shaft and the bearing pads for aerodynamic operation. The main challenge is to obtain the correct amount of radial pad flexibility and damping to accommodate sufficient shaft growth and to ensure dynamic rotor stability over the full speed range.

This paper will show the numerical method followed to investigate the stability of a 200 gram micro gas turbine supported by two flexible tilting pad bearings. The required radial pad stiffness and damping to obtain stability are calculated by deriving first the relation between the pad position and the linear gas-film stiffness and damping coefficients with the use of a numerical perturbation method [2]. The perturbation frequency depending coefficients are then substituted in an extended Laval-Jeffcott rotor model to obtain the overall rotor stability by calculated the eigenvalues of the linearised system. Comparison of the results with fully nonlinear orbit simulations of the rotor dynamics are performed in section 4.
2. Tilting pad geometry

The geometry of the tilting pad bearing under investigation is illustrated in Figure 1. Each individual pad of the bearing and its associated pivot point is flexible mounted in radial direction with stiffness $k_p$, damping $d_p$, and a possible pre-load force $F_p$. When designing such flexible tilting pad bearing it is of interest to know the range for the external pad stiffness $k_p$ and damping $d_p$ which results in dynamic stability. An efficient approach for non-flexible bearings with a fixed bearing geometry is to numerically calculating the linear gas-film stiffness and damping coefficients, followed by an eigenvalue study on the linearised rotor-bearing dynamics [2, 3, 4]. The gas-film coefficients for flexible tilting pad bearings have however a mutual dependency with the initial unknown pad position. This makes the coefficient calculation for flexible bearings a computational intensive task, especially if it is repeated for each stiffness value $k_p$ and damping value $d_p$ of interest.

![Figure 1.a 6-pad bearing example](image1a)

![Figure 1.b Flexible tilting pad geometry](image1b)

A less computational intensive approach is followed in this paper by first calculating the gas-film force $F_g(\epsilon)$ and the gas-film stiffness $K(\nu, \epsilon)$ and damping $D(\nu, \epsilon)$ coefficients for the full range of radial pad position. Results, for the 6-pad bearing, are shown in dimensionless form in Figure 2.

![Figure 2. Gas-film stiffness and damping coefficients for $\nu = 1$, $\Lambda = 0.3$, $\beta_p = 0.7$, $L/D = 1.5$](image2)
3. Stability

Stability of the overall system dynamics can be analysed by substituting the derived gas-film coefficients in the overall rotor-bearing dynamics. The dynamic behavior of the 200 gram turbine under investigation is thereby modeled using an extended Laval-Jeffcott model where the rotor is divided into two end-masses and one center-mass connected by a flexible shaft.

\[ k_s = \frac{6\pi Er^4}{l^3} \]

**Figure 3.** Rotor-Bearing model

The extended Laval-Jeffcott model takes the pads radial degree of freedom into account while the pads pivoting motion is neglected. This effectively assumes that the pads have an infinite moment of inertia. The model will therefore underestimate the stability for pads with finite moment of inertia in return for a more simplified stability analysis method. The stability of the 6-pad bearing can for example be evaluated for a large range of \( k_p \) and \( d_p \) values by first solving equation (1) for the radial pad position \( \epsilon_p \). The gas-film stiffness and damping coefficients, corresponding with this pad position, can then be used in an eigenvalue analysis of the linearised-bearing rotor model. Here, combination of \( k_p \), \( d_p \), and \( F_p \) that lead to undamped eigenfrequency which collide with the perturbation frequency are defined as unstable. That is, eigenvalues that do not satisfy the stability criterion (2).

\[
\epsilon_p = \frac{F_p - F_g(\epsilon_p, \epsilon_s)}{k_p \epsilon} + \epsilon_s \quad (1)
\]

\[
\Re \{\lambda(\nu)\} < 0 \lor |\Im \{\lambda(\nu)\}| \neq \nu \quad (\forall \nu \in (0, \omega)) \quad (2)
\]

Results for the 6-pad bearing are shown in Figure 3 for a pre-tension force of 2N and a shaft growth of 0, 0.2, 0.4 and 0.6% of its original diameter. Notice that combination of pad stiffness \( k_p \) and damping \( d_p \) which are located below the lines will stabilise the rotor dynamics.

**Figure 4.** Design Parameters

**Figure 5.** Stability map for a symmetric rotor (Table 4), supported on two 6-pad bearings with radial pad flexibility

| Rotor speed | \( \omega = 260 \) | krpm |
| Shaft diameter | \( D = 10 \) | mm |
| Bearing length | \( L = 15 \) | mm |
| Pad mass | \( m_p = 0.02 \) | kg |
| Pad pivot stiff. | \( k_t = 0 \) | Nm/r |
| Rotor mid mass | \( m_c = 0.07 \) | kg |
| Rotor end mass | \( m_s = 0.03 \) | kg |
| Rotor stiffness | \( k_s = 5.0 \) | kN/m |
4. Comparison with nonlinear orbit simulation results
The results of the eigenvalue study are validated with a fully nonlinear orbit simulation of the shaft whirl movements [5]. For the orbit simulation, the gas-film forces on the rotating shaft are calculated by solving the general Reynolds equation over time and including all the degrees of freedom for the pads and rotor. The masses and stiffness of the rotor and pads are equal to the linearised case, i.e. as specified in Table 4 with the addition of an unbalance of 0.05gmm.

The calculated orbits trajectories of the shaft center-mass and one of the end point-masses are shown in Figure 6. Transcendent behavior, during the initial face of the simulation, is plotted with a light-colored dashed line while the steady state behavior is plotted with a solid line. The system is said to be stable if the mass center positions convert to a single point, or to a limit cycle in case of shaft unbalance.

![Stability Map](image)

**Figure 6.** Shaft center orbit path

**Figure 7.** Orbit vs. eigenvalue analysis

The eigenvalue method of the previous section is validated by repeating the nonlinear orbit simulation for multiple points in the stability map. An ‘•’ is placed in Figure 7 at each simulated combination of $k_p$ and $d_p$ where the nonlinear orbit simulation converted to a limit cycle or a single stable point. For unstable rotor behavior, an ‘×’ is placed. As can be seen, all the simulated combination of $k_p$ and $d_p$ are in good agreement with the stable region calculated by the eigenvalue method. Interesting to note is that the stability map of Figure 3 was calculated within 10 seconds after the linearised bearing coefficients where obtained. The 40 orbits simulations of Figure 7 where in comparison taking over 8 hours of continues Matlab computing.

Reference
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