I present a brief theory overview of the CHARM-2015 conference.

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}
1 Introduction

CHARM-2015 has been a most lively conference, with more than twenty very interesting theoretical presentations [1–28]. Due to my limitations, as well as for reasons of space, I cannot possibly do justice to all of them, so rather than a full-fledged summary I will only give my personal view of the conference, referring the interested reader to the contributions collected in this volume.

The spectacular experimental progress that we have witnessed in the past few years is leading us in the precision charm physics era, calling for substantial theoretical advances to fully exploit the wealth of available data. Charm physics is now at the forefront of New Physics (NP) searches, allowing us to probe energies as high as $10^4$ TeV’s [29] (see Fig. 1), with ample room for sizable improvements, both from the theoretical and experimental point of view.

In Sec. 2 I quickly report on recent progress in the determination of charm properties: spectroscopy, production, mass, decay constants and form factors. In Sec. 3 I
discuss NP-sensitive, theoretically clean observables, such as CP violation in $D$ mixing and a few rare decays. Finally, in Sec. 4 I mention several potentially NP-sensitive but theoretically challenging processes, such as CP violation in nonleptonic $D$ decays and more rare decays.

2 Charm properties

2.1 Spectroscopy

Twelve years after the $X(3872)$ discovery, the $c\bar{c}$ spectrum has been widely explored: all the states below the open charm threshold have been identified, all the $1^{--}$ states are filled, but the long-standing problem of understanding the structure of exotica is still open [1]. While considering exotica as loosely bound charmed meson molecules gives an economic description of several exotic states very close to threshold, this explanation is challenged by prompt production at the LHC. On the other hand, the description of exotica in terms of compact tetraquarks implies the prediction of (too) many additional states, depending on the details of the diquark interaction (a very interesting subject *per se* [2]), and it is supported by the observation of new charged states. Studying decays in specific channels could discriminate between models [3], and more experimental data will certainly help in finally clarifying this open issue.

The spectroscopy of $c\bar{c}$ states can also be studied on the lattice [4]. While precision results in excellent agreement with experiment have been obtained for the states well below threshold, the situation becomes problematic when the energy raises above threshold. Correlation functions in the Euclidean are always dominated by the state with the lowest energy, preventing the study of interacting multi-meson states above threshold [31]. Finite volume effects allow to overcome this limitation for two-meson states [32], but substantial progress is still needed for three-meson states. In spite of these difficulties, first studies of exotic $X$, $Y$ and $Z$ states have been carried out [5,6,33–35], although no firm conclusion on the nature of these states has been reached yet [36,37]. Lattice studies of charmed baryon spectroscopy are in a similar situation: for ground states there is good agreement between lattice results and experiments, while the study of excited states is really challenging [7].

2.2 Charm production

Let us now briefly review recent progress on charm production, starting with quarkonium production in the vacuum [8–10]. The cross-section for quarkonium production at high-$p_T$ is expected to factorize, order by order in an expansion in the velocity $v$, into the product of the short-distance partonic cross section, convoluted with the pdf, times the long-distance probability for a $Q\bar{Q}$ pair to evolve into a quarkonium
Figure 2: Comparison of recent determinations of the charm quark mass from Lattice QCD (left panel, from ref. [16]) and from QCD sum rules (right panel, from ref. [17]).

2.3 Charm quark mass and Yukawa coupling

The charm quark mass can be determined with nonperturbative methods such as Lattice QCD or QCD sum rules. Remarkable progress has been recently achieved in both approaches. Lattice QCD calculations with three or four active flavours have been performed by several collaborations, using different actions, renormalization procedures and methods; QCD sum rules computations include terms of $O(\alpha_s^3)$ and several tests can be performed on the convergence of the perturbative expansion. A collection of recent results obtained in both approaches is presented in Fig. 2; more details can be found in refs. [16,17].

The determination of quark Yukawa couplings and their relation with quark masses is a crucial test of the validity of the Standard Model. In this respect, a direct determination of the charm Yukawa coupling would be extremely important. Unfor-
fortunately, this is a formidable task, requiring very high integrated luminosity \cite{18,19}. A promising approach to the determination of the charm Yukawa coupling is via \( h \to J/\psi \gamma \), using the interference of direct and indirect production, which is theoretically clean and could give interesting results with 3 \( \text{ab}^{-1} \) \cite{41,42}.

2.4 Charmed meson decay constants and form factors

Considerable progress is also taking place in the precision determination of charmed mesons decay constants and form factors in Lattice QCD. Figures 3 and 4 summarizes the current averages from the Flavour Lattice Averaging Group (FLAG) \cite{43} as well as more recent calculations not yet included in the FLAG averages \cite{20}. The experimental numbers for the decay constants are \( f_{D_s} = 257.5 \pm 4.6 \text{ MeV} \) and \( f_{D_s}/f_D = 1.258 \pm 0.038 \) \cite{44}. The \( n_f = 2 \) FLAG averages and the recent ETMC \( n_f = 2 + 1 + 1 \) results \cite{45} are in fair agreement with data, while some tension is seen comparing the \( n_f = 2 + 1 \) FLAG averages and the recent FNAL/MILC \( n_f = 2 + 1 + 1 \) results \cite{46}.

3 NP-sensitive, theoretically clean processes

CP-violation in \( \Delta F = 2 \) processes is the most sensitive probe of NP, reaching NP scales as high as \( \mathcal{O}(10^5) \text{ TeV} \) for generic flavour structures and \( \mathcal{O}(1) \) couplings (see Fig. 1). Thanks to the recent experimental and theoretical improvements, CP violation in \( D \) mixing is giving the second best constraint on NP; furthermore, combining bounds from \( K \) and \( D \) mixing allows to constrain several NP models much more effectively than considering bounds from individual processes.

From the theoretical point of view, \( D \) mixing is described in terms of the dispersive and absorptive mixing amplitudes \( M_{12} \) and \( \Gamma_{12} \). In the SM, both amplitudes are dominated by long distance contributions and thus not calculable at present \cite{13}. NP contributions to \( \Gamma_{12} \) are expected to be negligible, while NP could give large short-distance contributions to \( M_{12} \), which can be accurately computed using matrix elements computed on the lattice \cite{29}. The observables related to the mixing amplitude are \( |M_{12}|, |\Gamma_{12}| \) and \( \Phi_{12} = \arg(\Gamma_{12}/M_{12}) \). Being Flavour Changing Neutral Current (FCNC) processes, the mixing amplitudes are GIM suppressed, due to the unitarity of the CKM matrix, and in particular to the unitarity relation \( \lambda_d + \lambda_s + \lambda_b = 0 \), where \( \lambda_{di} = V_{cdi}V_{ud}^* \). Unitarity allows to eliminate \( \lambda_d \); furthermore, we can choose \( \lambda_s \) to be real, so that all CPV is generated by terms proportional to \( \lambda_b \) and thus suppressed by \( r = \text{Im}\lambda_b/\lambda_s \sim 6.5 \cdot 10^{-4} \). Denoting by \( f_{d_i d_j} \) the loop amplitude with \( d_i d_j \) intermediate states, both \( M_{12} \) and \( \Gamma_{12} \) have the following structure:

\[
\lambda_s^2(f_{dd} + f_{ss} - 2f_{ds}) + 2\lambda_s\lambda_b(f_{dd} + f_{bs} - f_{bd} - f_{sd}) + \mathcal{O}(\lambda_b^2).
\]
Figure 3: Recent determinations of the $D$ and $D_s$ decay constants from lattice QCD [20].
Figure 4: Recent determinations of the semileptonic form factors (top) and of CKM unitarity (bottom) from lattice QCD \cite{20,43}. 
From eq. (1) it is evident that for the dominant long-distance contributions GIM suppression coincides with SU(3) suppression. Indeed, SU(3) can serve as a guiding principle to estimate the size of the two terms in eq. (1). To this aim, it is useful to rewrite them in terms of U-spin quantum numbers:

\[ \lambda_s^2(\Delta U = 2) + 2\lambda_s\lambda_b(\Delta U = 1 + \Delta U = 2) + \mathcal{O}(\lambda_b^2) \sim \lambda_s^2\epsilon^2 + 2\lambda_b\lambda_s\epsilon. \quad (2) \]

We see that CP violating effects are expected to arise at the level of \( r/\epsilon \sim 2 \cdot 10^{-3} \sim 1/8^\circ \) for nominal SU(3) breaking \( \epsilon = 30\% \). Given the present experimental errors, it is therefore perfectly adequate to assume real \( M_{12} \) and \( \Gamma_{12} \) in the SM as well as real decay amplitudes, allowing to fit all \( D \)-mixing data using the universal parameters \( x, y, \phi \).

A possible NP-induced phase in \( M_{12} \) (we expect NP to give negligible contributions to \( \Gamma_{12} \)) would result in \( |q/p| - 1 \neq 0 \) and in \( \phi \neq 0 \).

The results of a global fit assuming real SM contributions and searching for NP CP-violating effects by the UTfit Collaboration \cite{49} are presented in Fig. 5 and in Table 1; see ref. \cite{50} for the updated HFAG fit. The results show no evidence of CP violation within the current experimental uncertainty, and allow to put severe bounds on the NP scale.

| parameter       | result @ 68% prob.  | 95% prob. range |
|-----------------|---------------------|-----------------|
| \( |M_{12}| \) [ps\(^{-1}\)] | (4.3 \pm 1.8) \cdot 10^{-3} | [0.6, 7.5] \cdot 10^{-3} |
| \( |\Gamma_{12}| \) [ps\(^{-1}\)] | (14.1 \pm 1.4) \cdot 10^{-3} | [11.1, 17.3] \cdot 10^{-3} |
| \( \Phi_{M_{12}} \) [\(^\circ\)] | (0.8 \pm 2.6) | [-5.8, 8.8] |
| \( x \) \quad \sim 2|M_{12}|/\Gamma \ , \quad y \sim |\Gamma_{12}|/\Gamma \ , \quad |q/p| = \sqrt{4|M_{12}|^2 + |\Gamma_{12}|^2 + 2|M_{12}| |\Gamma_{12}| \sin \Phi_{M_{12}} } / \Gamma \sqrt{x^2 + y^2}, \quad \phi = \arg (y + i (1 - |q/p|) x) \ . |
| \( |q/p| - 1 \) | 0.007 \pm 0.018 | [-0.030, 0.045] |
| \( \phi[\circ] \) | -0.21 \pm 0.57 | [-1.53, 1.02] |

Table 1: Results of the fit to \( D \) mixing data. See ref. \cite{49} for details.

The most general effective weak Hamiltonian for \( D \) mixing of dimension six operators is parameterized by Wilson coefficients of the form

\[ C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2}, \quad i = 1, \ldots, 5, \quad (4) \]

where \( F_i \) is the (generally complex) relevant NP flavor coupling, \( L_i \) is a (loop) factor which depends on the interactions that generate \( C_i(\Lambda) \), and \( \Lambda \) is the NP scale, i.e. the
typical mass of new particles mediating $\Delta C = 2$ transitions. For a generic strongly interacting theory with an unconstrained flavor structure, one expects $F_i \sim L_i \sim 1$, so that the phenomenologically allowed range for each of the Wilson coefficients can be immediately translated into a lower bound on $\Lambda$. Specific assumptions on the flavor structure of NP correspond to special choices of the $F_i$ functions. Assuming $F_i = 1$ and $L_i = 1$ and using the matrix elements recently computed in Lattice QCD [29], we obtain the bounds on the NP scale reported in Table 2. See ref. [29] for details.

As anticipated above, the current uncertainty on $\Phi_{12} = (0.8 \pm 2.6) ^\circ$ is certainly compatible with the assumption of real SM amplitudes. However, in view of the expected experimental progress, it is mandatory to understand how one could go beyond this assumption. As discussed in detail in ref. [21], based on the enhancement factor $1/\epsilon$ in eq. [2] which is absent in individual decay amplitudes, the dominant CP violating effect in the SM can be captured by adding a universal phase $\phi_{12}$ and fitting for both $\phi_{M_{12}}$ and $\phi_{\Gamma_{12}}$. With present data we are not sensitive to $\phi_{\Gamma_{12}}$ yet, but extrapolating to the expected experimental accuracies after LHCb upgrade we foresee a determination of $\phi_{\Gamma_{12}}$ with an error of $2^\circ$ and of $\phi_{M_{12}}$ with an error of $1^\circ$.

In addition to searching for CPV in $D$ mixing, another very clean probe of NP is given by lepton number violating $D$ decays such as $D_{(s)}^+ \rightarrow \pi^- \mu^+ \mu^+$. These decay modes are very sensitive to the presence of Majorana neutrinos with mass up to 1.1 GeV, although for masses lower than around 400 MeV Kaon decays provide more
| \( \text{Im} C^D_i \) | 95% upper limit | Lower limit on \( \Lambda \) |
|-----------------|----------------|------------------|
| \( D^1 \)       | \([-1.4, 2.0] \cdot 10^{-14}\) | \(7.1 \cdot 10^3\) |
| \( D^2 \)       | \([-2.5, 1.7] \cdot 10^{-15}\) | \(20.0 \cdot 10^3\) |
| \( D^3 \)       | \([-2.4, 3.5] \cdot 10^{-14}\) | \(5.3 \cdot 10^3\) |
| \( D^4 \)       | \([-5.2, 7.7] \cdot 10^{-16}\) | \(36.0 \cdot 10^3\) |
| \( D^5 \)       | \([-5.3, 7.9] \cdot 10^{-15}\) | \(11.2 \cdot 10^3\) |

Table 2: 95% probability intervals for the imaginary part of the Wilson coefficients, \( \text{Im} C^D_i \), and the corresponding lower bounds on the NP scale, \( \Lambda \), for a generic strongly interacting NP with generic flavor structure (\( L_i = F_i = 1 \)).

Last but not least among the clean probes of NP let me mention \( D \to \mu^+ \mu^- \). Within the SM, this decay is dominated by long-distance contributions, but these can be reliably estimated once a measurement of (or an upper bound on) \( \text{BR}(D \to \gamma \gamma) \) is available. One can then extract tight constraints on NP-induced short-distance \( c \to \mu^+ \mu^- \) transitions. See ref. [22] for more details.

4 NP-sensitive, theoretically challenging observables

Let us close this quick overview with a few potentially NP-sensitive observables that however require substantial theoretical advance to exploit their NP sensitivity. Generally speaking, all \( \Delta C = 1 \) transitions with hadrons in the final state pose serious theoretical challenges. The evaluation of (non-local) matrix elements is problematic since charm is not heavy enough to apply QCD factorization. Thus, waiting for lattice QCD to attack nonleptonic charm decays, we can either look for possible order-of-magnitude NP effects that could emerge over hadronic uncertainties, or try to eliminate hadronic matrix elements using symmetry arguments.

For example, NP could give order-of-magnitude enhancements of the long-distance dominated \( D \to P\ell^+\ell^- \) decays, leading to bounds on NP contributions from the recent LHCb upper bounds on \( D^+ \to \pi^+ \mu^+ \mu^- \) [22][23][51].

CP violation in Singly Cabibbo Suppressed (SCS) \( D \) decays is potentially sensitive to NP contributions, since SM contributions are suppressed by the small CKM ratio \( r = 6.5 \cdot 10^{-4} \). However, a reliable estimate of the relevant hadronic matrix elements is needed to identify possible NP contributions, unless one observes CP asymmetries much larger than \( 10^{-3} \) or is able to get rid of the unknown matrix elements using flavour symmetries. An interesting example of the latter possibility is to study CP
violation in $\Delta I = 3/2$ amplitudes, since no observable CPV is expected in these amplitudes in the SM \[24\].

The situation gets much more complicated if one wants to look for NP in $\Delta I = 1/2$ amplitudes. One could think of using SU(3) to estimate the relevant matrix elements. However, assuming exact SU(3) symmetry one is not able to reproduce the observed branching ratios \[25,52\], and once SU(3) breaking is allowed all possible reduced matrix elements are generated, so no prediction is possible, except for a few sum rules valid to second order in SU(3) breaking \[53,55\]. Thus, while SU(3) can help identifying a hierarchy between the different amplitudes, additional dynamical information is needed to predict CP violation in SCS decays \[25\]. Several interesting attempts have been made in this direction, using factorization \[56-60\], dynamical assumptions about final state interactions \[61\] or $1/N_c$ arguments \[26,62,63\]. However, some degree of model dependence is present in all these approaches, making it difficult to reliably assess the uncertainty of the theoretical predictions. Hopefully, with more experimental data and more theoretical efforts this problem will be overcome in the near future.

5 Conclusions

I hope that this brief summary has stimulated the reader to look into the details of the many interesting theoretical presentations that have made CHARM-2015 such a lively conference. Charm physics is playing a key role in improving our understanding of SM dynamics and of what lies beyond the SM, and this role will be even more important in the near future thanks to the foreseen experimental and theoretical developments. Therefore I am sure that CHARM-2016 will be an even more exciting conference.

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15