Quantum turbulence in propagating superfluid vortex front

V.B. Eltsov, A.I. Golov, R. de Graaf, R. Hänninen, M. Krusius, V.S. L’vov, and R.E. Solntsev

1Low Temperature Laboratory, Helsinki University of Technology, P.O.Box 2200, 02015 HUT, Finland
2Kapitza Institute for Physical Problems, Kosygina 2, 119334 Moscow, Russia
3Department of Physics and Astronomy, Manchester University, Manchester, United Kingdom
4Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

(Dated: February 1, 2008)

We present experimental, numerical and theoretical studies of a vortex front propagating into a region of vortex-free flow of rotating superfluid 3He-B. We show that the nature of the front changes from laminar through quasi-classical turbulent to quantum turbulent with decreasing temperature. Our experiment provides the first direct measurement of the dissipation rate in turbulent vortex dynamics of 3He-B and demonstrates that the dissipation is temperature- and mutual friction-independent in the $T \to 0$ limit, and is strongly suppressed when the Kelvin-wave cascade on vortex lines is predicted to be involved in the turbulent energy transfer to smaller length scales.

PACS numbers: 67.57.Fg, 47.32.-y, 67.40.Vs

Turbulent motion of fluids with low viscosity, like water or air is a general phenomenon in Nature and plays an important role in human everyday life. Nevertheless, various features of turbulence are not yet well understood; thus a possibility to study turbulence from another, non-classical viewpoint looks promising. Flows in superfluids (with zero viscosity) can be turbulent; understanding this form of turbulence is crucial for our ability to describe superfluids, including practical applications like cooling superconductor devices. Turbulence in superfluid 4He at relatively high temperatures $T \approx 1$ K has been studied for decades [1]. Fermi-fluid 3He is very different from the Bose-fluid 4He; this allows us to study in 3He some aspects of superfluid turbulence not available in 4He [2,3].

In this Letter we report the first experimental observation and study of propagating laminar and turbulent vortex fronts in rotating 3He which opens a possibility to directly measure the rate of kinetic energy dissipation in a wide temperature range down to 0.18 $T_c$ of the critical temperature $T_c \approx 2.43$ mK (at 29 bar pressure). Theoretical and numerical analysis of the propagating front allows us to identify the mechanisms of energy dissipation and to discover a dissipation anomaly: the saturation of the rate of turbulent kinetic energy dissipation at a nonzero value at temperatures below $0.22 T_c$.

The idea of the experiment is as follows: in rotating 3He (in the B-phase) we can create a vortex-free Landau state, which in the absence of external perturbations persists forever. In this state the superfluid component is at rest in the laboratory frame, while the normal component is in rigid rotation with angular velocity $\Omega$. The Landau state is metastable, having larger free energy than the stable equilibrium vortex state. The latter consists of rigidly rotating normal and superfluid components with a regular array of rectilinear quantized vortices. When we inject a seed vortex into the Landau state, we observe a rapid local evolution of the vorticity toward the equilibrium state. A boundary between the vortex-free and the vortex states propagates with constant velocity $V_f$ toward the metastable region(s), see Fig. 1. In some sense this phenomenon is similar to the propagation of a flame front in premixed fuel. The flame front propagation can also proceed in laminar or in turbulent regimes. In the latter case the effective area of the front increases

![FIG. 1: Experimental setup. (Left) Configuration for vortex injection using AB interface instability in which two vortex fronts propagate independently in the upper and lower B phase sections (as shown by hollow arrows). (Right) Configuration for vortex injection from the orifice when a single vortex front travels first through the bottom and then through the top NMR pick-up coils. Superimposed inside the sample container are snapshots of vortex configurations from numerical simulations of vortex expansion at two different temperatures. The configuration at $0.3T_c$ displays small-scale structure (shown within blow-up on the right) which is absent at $0.5T_c$.](image)
and its propagation speed becomes higher than in the laminar regime. This property finds its practical use in combustion engines but also has been used to describe intensity curves of type Ia supernovae [1]. In all these cases a metastable state of matter is converted to the stable state in the front and $V_I$ is determined by the rate of dissipation of the released energy. In our case (with density $\rho_s$ and velocity $U_s = \Omega r$ of the superfluid component in a sample container of radius $R$) the dissipation rate of total kinetic energy, $E(t)$, is related to $V_I$ as

$$dE/dt = -\pi \rho_s V_I \Omega^2 R^4/4.$$  

(1)

Thus by measuring $V_I$ one determines directly $dE/dt$ in turbulent vortex dynamics. So far measurements in the turbulent regime only concerned the vortex density [2].

Our main experimental observation is that $dE(t)/dt$ does not go to zero in the $T \to 0$ limit. At high enough $T$ the front is laminar and its velocity is determined by mutual friction between the normal and superfluid components $V_I(T) \approx \alpha(T) \Omega R$, where $\alpha(T)$ is a dimensionless mutual-friction coefficient. Although $\alpha(T) \to 0$ when $T \to 0$, the measured velocity $V_I(T)$ saturates at a constant value which corresponds to an effective friction $\alpha_{\text{eff}} \sim 0.1$. We interpret this behavior to be similar to the viscous anomaly in classical turbulence, where the dissipation rate does not vanish when viscosity $\nu$ goes to zero. The viscous anomaly appears due to cascading of energy to smaller length scales. When $\nu \to 0$ the smallest length scale decreases but the global dissipation rate does not change. We believe that a similar mechanism, which we can call mutual friction anomaly, applies to superfluids where mutual friction plays the role of viscosity.

One can say that the viscous and mutual friction anomalies are particular cases of a more general phenomenon, which we call the dissipation anomaly. The nonzero rate of energy dissipation in the limit of a vanishingly small parameter, that governs dissipation.

As mutual friction decreases and turbulent motion reaches progressively smaller length scales, eventually quantized vortex lines become important. The energy cascade on length scales smaller than the intervortex distance and the nature of dissipation on such scales are currently the central questions in research on turbulence in superfluids [1]. At the moment only theoretical speculations exist on the role of the non-linear interaction of Kelvin waves, resulting in a Kelvin-wave cascade [5, 6], terminated by quasiparticle emission, and on the role of vortex reconnections which can immediately redistribute energy over a range of scales and also lead to dissipation [7]. Our experiments show evidence for the importance of the Kelvin-wave cascade: We have observed a rapid decrease of the front velocity with decreasing temperature in the region where the introduction of sub-intervortex scales to energy transfer is expected. We propose an explanation of this effect using a model with a bottleneck crossover between quasi-classical (at super-intervortex scales) and quantum turbulence (at sub-intervortex scales) [8].

**Experiment:** Our measurements are performed in a rotating nuclear demagnetization cryostat at $\Omega \sim 1 \text{rad/s}$. The $^3$He-B sample at 29 bar pressure is contained in a cylindrical cell with radius $R = 3$ mm and length 110 mm, oriented parallel to the rotation axis, Fig. 1. Pick-up coils of two independent NMR spectrometers near the top and bottom of the cell are used to monitor the vortex configuration [3]. To prepare the initial vortex-free state we heat the sample to about 0.75 $T_c$ (for rapid annihilation of all vortices) and then cool it in the vortex-free state in rotation to the target temperature. We can inject seed vortices in the middle of the cell, using the instability of the AB interface in rotation [3], controlled with an applied magnetic field. In this case two vortex fronts propagate independently up and down, arriving to the top and bottom pick-up coils practically simultaneously. A second injection technique uses remnant vortices which are trapped in the vicinity of the orifice on the bottom of the sample, Fig. 1 right. In this case the vortex front propagates upwards along the entire sample through both pick-up coils in succession. We determine the front velocity dividing the flight distance by the flight time, as if the front propagates in steady-state configuration. Although this is not the case due to initial equilibration processes which follow injection, we believe that this simplification is justified here, since the two injection techniques for different propagation lengths give the same result, as seen in Fig. 2.

**Experimental results** on the front velocity are presented in Fig. 2. The temperature range is clearly divided in two regions: At $T \geq 0.4 T_c$ the dimensionless front velocity $V_I = V_I/\Omega R \approx \alpha$. This agrees with previous measurements in this temperature range and can be understood from the dynamics of a single vortex, when inter-vortex interactions are ignored [8]. We call this region the laminar regime. The measured values of $V_I$ are slightly below $\alpha$. We believe that the difference is caused by the twisted vortex state [10] behind the front. It reduces the energy difference across the front and correspondingly the front velocity. Estimating the reduction factor from the uniform twist model we get

$$V_{\text{lamb}} = [2/\log(1 + 1/q^2) - 2/q^2] \alpha,$$

where $q = \alpha/(1 - \alpha')$ and $\alpha'$ is the reactive mutual friction coefficient. This dependence, (the dash-dotted line in Fig. 2) is in good agreement with the experiment.

The new feature is the behavior at $T < 0.4 T_c$. Here $V_I$ rapidly deviates to larger values than $\alpha \Omega R$ and eventually becomes constant in the $T \to 0$ limit with a peculiar transition from one plateau to another at around 0.25 $T_c$. We attribute this behavior to turbulent dynamics and analyze it below in more detail.

**Simulations:** To clarify the vortex front formation and propagation, we simulate vortex dynamics using the vortex filament model with full Bio-Savart equations and an
additional solution of the Laplace equation for solid wall boundary conditions \( b \) in an ideal cylinder with length 40 mm and diameter 3 mm rotating at \( \Omega = 1 \text{rad/s} \). In the initial configuration the equilibrium number of vortices is placed close to one end plate of the sample as quarter-loops between the end plate and the side wall of the sample. During evolution vortices form a front propagating along the sample (for movies see Ref. [12]). The measured value of mutual friction coefficient \( \alpha(T) \) \( \{13\} \), extrapolated below 0.35 \( T_c \), with \( \exp(-\Delta/T) \) law \( \{14\} \). Filled diamonds are results of numerical simulations. Dashed-dotted, thin solid and thick solid lines show model approximations which sequentially account for dissipation in the large-scale motion, turbulent energy transfer and bottleneck effect. (Insert) Value of parameter \( b(T) \) in Eq. \( \{\text{3}\} \) which was used in producing the thick solid line in the main panel.

Our analytical model of the turbulent front is based on the quasi-classical "coarse-grained" equation averaged over the vortex lines \( \{\text{3}\} \),

\[
\dot{U} + (1 - \alpha')(U \cdot \nabla)U + \nabla \mu = -\Gamma U, \quad \Gamma \equiv \alpha \omega_{ef}, \quad \{\text{2}\}
\]

which describes the evolution of the superfluid velocity \( U(r, t) \) at scales exceeding the crossover scale \( \ell \) between hydrodynamic and kinetic regimes of turbulent motions, which is of the order of the intervortex separation. In Eq. \( \{\text{2}\} \) \( \mu \) is the chemical potential and the dissipative term \( \Gamma \) is taken in the simplified form \( \{\text{14}\} \), in which \( \omega_{ef} \) can be understood as an effective vorticity.

To estimate \( V_t \) in the quasi-classical regime, described by Eq. \( \{\text{2}\} \), we consider the total energy dissipation in the front with well developed turbulence which has two contributions. The first one originates from the mutual friction which acts on the global scale. It can be estimated from Eq. \( \{\text{2}\} \) as \( \alpha \omega_{ef} K(z, r) \), where \( K(z, r) = \frac{1}{2} \langle u^2 \rangle \) is the turbulent kinetic energy per unit mass and \( u \) is the turbulent velocity fluctuations (with zero mean). The second contribution is determined by the usual energy flux in classical turbulence \( \varepsilon \sim b K^{3/2}(r)/L(r) \) at an outer scale of turbulence \( L(r) \). Clearly, \( L(r) \approx \Delta(r) \), the thickness of the turbulent front at given radius \( r \) near the centerline of the cell, or, near the surface of the cell, as distance to surface, \( R - r \). In the whole cell, one can use an interpolation formula \( L^{-1}(r) = \Delta(r)^{-1} + (R - r)^{-1} \). The natural assumption is that this energy dissipates on the way to small scales either due to the mutual friction at moderate temperatures or converges into Kelvin waves at the crossover scale \( \ell(r) \). For the classical Kolmogorov-41 regime \( b_{c1} \approx 0.27 \{13\} \). Using Eq. \( \{\text{1}\} \) we can present the overall energy budget as:

\[
V_t \Omega^2 R^4 = 8 \int_0^{R-\ell} r \, dr \, dz \left[ \Gamma K(z, r) + \frac{b K^{3/2}(z, r)}{L(r)} \right]. \quad \{\text{3}\}
\]

Here we accounted for the mutual-friction correction to the nonlinear term in Eq. \( \{\text{2}\} \) \( b \Rightarrow b_{c1} \equiv b(1 - \alpha') \) and used the axial symmetry to perform the integration over the azimuthal angle. The region with \( R - r < \ell \), where Eq. \( \{\text{2}\} \) is not applicable, is excluded from the integration.

In the turbulent boundary layer the kinetic energy is independent of the axial distance to the wall. Therefore qualitatively we can replace \( K(z, r) \) and \( \omega_{ef}(z, r) \) by their mean values across the front \( \overline{K}(r) \) and \( \overline{\omega}_{ef}(r) \). Also dimensional reasoning dictates \( \Delta(r) \overline{\omega}_{ef}(r) \approx \alpha \Omega r \) and \( \overline{K}(r) = c(\Omega r)^2/2 \) with \( c, a \sim 1 \). Now Eq. \( \{\text{3}\} \) gives:

\[
V_t \equiv V_t/\Omega R \approx (2c)^{3/2} b(1 - \alpha') A + 4\alpha c/5 \alpha, \quad \{\text{4}\}
\]

where \( A = 0.2 + d[\ln (R/\ell) - 137/60 + 5\ell/R + \ldots] \approx 1.8 \) for \( \Omega = 1 \text{rad/s} \) which gives \( R/\ell \approx 17 \). We take \( b = b_{cl} \) and choose the parameters \( a = 0.2, c = 0.25 \) and \( d = 2 \) to fit the measurement in the region \( (0.3 - 0.4) T_c \). With these parameters Eq. \( \{\text{4}\} \) gives \( V_t \approx 0.16 \) in the limit \( T \rightarrow 0 \) (when \( \alpha = \alpha' = 0 \)) and a very weak temperature dependence up to \( T \approx 0.45 T_c \). However, both in the region of lower and higher temperatures the experiment shows deviation from this "plateau" (Fig. 2).

The reason for this deviation at \( T > 0.35 T_c \) is that turbulence is not well developed near the cylinder axis where
the shear of the mean velocity, responsible for turbulence excitation, decreases. Therefore in the intermediate temperature region only part of the front volume is turbulent, expanding toward the axis when the temperature decreases. We suggest an interpolating formula between laminar and turbulent regimes: \( V_{\text{f, turb}} = (V_{\text{f, lam}}^2 + \rho_{\text{f, turb}}^2) \), where \( V_{\text{f, turb}} \) is given by Eq. (3). This interpolation is shown in Fig. 2 as a thin line for \( T < 0.3T_c \) and as a thick line for \( T > 0.3T_c \). The agreement with our experimental observation for \( T > 0.3T_c \) is good, but there is a clear deviation below 0.25 \( T_c \), where \( \alpha \lesssim 10^{-2} \). The reason is that we did not account adequately for the quantum character of turbulence, which becomes important in our conditions at \( T < 0.3T_c \), which is close to the measured transition to the lower plateau in Fig. 2.

The mean free path of \(^3\text{He} \) quasiparticles at \( T \approx 0.3T_c \) is close to \( \ell \) while at 0.2 \( T_c \) it exceeds \( R \). This change from the hydrodynamic to the ballistic regime in the normal component may influence the mutual friction force acting on the individual vortices. We neglect this effect because for \( T < 0.3T_c \) the mutual friction is already very small and does not directly affect \( V_{\text{f}} \).

At these temperatures the energy flux toward small scales propagates up to the quantum scale \( \ell \) and vortex discreteness and quantization effects become most important. Even though some part of the energy is lost in intermittent vortex reconnections, the dominant part proceeds to cascade below the scale \( \ell \) by means of nonlinearly interacting Kelvin waves. These waves are generated by both slow vortex filament motions and fast vortex reconnection events. The point is that Kelvin waves are much less efficient in the down-scale energy transfer than classical hydrodynamic turbulence which leads to the bottleneck effect increasing the kinetic energy at crossover scale up to \( \Lambda \approx 12 \) as ratio of the outer and crossover scales and does not directly affect \( V_{\text{f}} \). At these temperatures the energy flux toward small scales propagates up to the quantum scale \( \ell \) and vortex discreteness and quantization effects become most important. Therefore the distortion of the energy spectrum due to the bottleneck reaches the outer scale, which leads to an essential suppression of the energy flux at given turbulent energy or, in other words, to a decrease in the effective parameter \( b \), which relates \( \varepsilon \) and \( K \). This effect is more pronounced at low temperatures when mutual friction is small, thus \( b(T) \) should decrease with temperature. We analyze this effect with the help of the stationary energy balance equation for the energy spectrum \( E_k \) in \( k \)-space

\[
\frac{d\varepsilon(k)}{dk} = -\Gamma(T)E_k, \quad \varepsilon(k) = -(1 - \alpha') \sqrt{k^{11}E_k} \frac{d(E_k/k^2)}{8dk},
\]

in which \( \Gamma(T) \), being the damping \( \alpha' \), and the energy flux over scales \( \varepsilon(k) \) are taken in the Leith differential approximation \( \text{(17)} \). In the calculations we use \( L/\ell = 12 \) as ratio of the outer and crossover scales and characterize the bottleneck with the boundary condition \( E_k/[k^2d(E_k/k^2)/dk] = -4 \cdot 10^5 \) at the crossover scale. The resulting function \( b(T) \), shown in the insert in Fig. 2 decreases from its classical value \( b_0 \approx 0.27 \) down to \( 0.1 \) for \( T < 0.2T_c \). Now, accounting for the temperature dependence of \( b(T) \) in Eq. (4), we get the temperature dependence of the quantum-turbulent front shown in Fig. 2 by the bold-solid green line, which is in good agreement with our experimental data. This significant decrease of the dissipation rate in the quantum regime is a consequence of the relative proximity of the outer and quantum crossover scales in our measurement.

**Conclusions:** We have established that conversion of metastable vortex-free rotating \(^3\text{He}-\text{B} \) to stable state occurs via propagation of a dynamic vortex structure, a vortex front, whose nature depends on the magnitude of mutual friction dissipation. At temperatures below 0.45 \( T_c \) sustained turbulence appears in the front, profoundly affecting the vortex dynamics. Owing to the energy transfer in the turbulent cascade, dissipation becomes temperature and mutual friction independent in the \( T \to 0 \) limit. In this regime we have observed the influence of a quantum cascade, involving individual vortices, on the global dissipation rate.

**Acknowledgements.** This work is supported by ULTI-4 (RITA-CT-2003-505313), Academy of Finland (grants 213496, 211507, 114887), and the US-Israel Binational Science Foundation.

[1] W. F. Vinen and J. J. Niemela, J. Low Temp. Phys. 128, 167 (2002).
[2] D.I. Bradley et al., Phys. Rev. Lett. 96, 035301 (2006).
[3] A.P. Finne et al., Rep. Prog. Phys. 69, 3157 (2006).
[4] S.I. Blinnikov et al., Astron. and Astrophys., 453, 229 (2006).
[5] E.V. Kozik and B.V. Svistunov, Phys. Rev. Lett. 92, 035301 (2004).
[6] W.F. Vinen, Phys. Rev. B. 61, 1410 (2000); W.F. Vinen, M. Tsubota, and A. Mitani, Phys. Rev. Lett. 91, 135301, (2003).
[7] B.V. Svistunov, Phys. Rev. B 52, 3647 (1995).
[8] V.S. L’vov, S. Nazarenko, and O. Rudenko, Phys. Rev. B, 76, 024520 (2007).
[9] A.P. Finne et al., J. Low Temp. Phys. 134, 375 (2004).
[10] V.B. Eltsov et al., Phys. Rev. Lett. 96, 215302 (2006).
[11] R. Hänninen et al., J. Low Temp. Phys. 138, 589 (2005).
[12] http://lt1.tkk.fi/"rhanmine/front
[13] E.B. Sonin, Rev. Mod. Phys. 59, 87 (1987).
[14] V.S. L’vov, S.V. Nazarenko and G.E. Volovik, JETP Lett. 80, 535 (2004).
[15] V.S. L’vov, I. Procaccia and O. Rudenko, JETP Lett. 84, 67 (2006).
[16] G.E. Volovik, JETP Lett. 78, 533 (2003).
[17] C. Leith, Phys. Fluids 10, 1409 (1967).
[18] T.D.C. Bevan et al., Phys. Rev. Lett. 74, 750 (1995).
[19] I.A. Todoshchenko et al., J. Low Temp. Phys. 126, 1449 (2002); we used the superfluid gap \( \Delta = 1.9687T_c \).