Temporal step fluctuations on a conductor surface: electromigration force, surface resistivity and low-frequency noise

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Abstract. Scattering of charge carriers from surface structures will become an increasing factor in the resistivity as the structure decreases in size to the nanoscale. The effects of scattering at the most basic surface defect, a kink in a step edge, are here analyzed using the continuum step model. Using a Langevin analysis, it has been shown that the electromigration force on the atoms at the step edge causes changes in the temporal evolution of the step-edge. For an electromigration force acting perpendicular to the average step edge and mass-transport dominated by step-edge diffusion, significant deviations from the usual $t^{1/4}$ scaling of the displacement correlation function occur dependent on a critical time $\tau$ and the direction of the force relative to the step edge (i.e. uphill or downhill). Experimental observations of step fluctuations on Ag(111) show the predicted changes among step fluctuations without current, and with current in the up- and down-hill directions for a current density of order $10^5 \text{ A cm}^{-2}$. The results yield the magnitude of the electromigration force acting on kinked sites at the step-edge. This in turn yields the contribution of the fluctuating steps to the surface resistivity, which exceeds 1% of the bulk resistivity as wire diameters decrease below 10s of nanometres. The temporal fluctuations of kink density can thus also be related to resistivity noise. Relating the known fluctuation spectrum of the step displacements to fluctuations in their lengths, the corresponding

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resistivity noise is predicted to show spectral signatures of \( \sim f^{-1/2} \) for step fluctuations governed by random attachment/detachment, and \( \sim f^{-3/4} \) for step fluctuations governed by step-edge diffusion.

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1. Introduction

A large surface-to-volume ratio, and non-trivial fluctuations are both fundamental consequences of shrinking structures to nanoscale dimensions. As a result, structural fluctuations at surfaces have the potential for significantly impacting the desired nanofunctional behavior. The impact may be observed in the properties of quantum dots [1, 2], the stability of nanostructures [3]–[8], the properties of interfacial contacts [9]–[12], electrical noise [13]–[15] and electromigration [16]–[18]. These problems are amenable to fundamental understanding, and thus prediction, because the statistical mechanics of the fundamental unit of surface structural fluctuations, the surface step, has been the subject of intense investigation [19, 20].

Both continuum models and discrete lattice simulations have been applied to understanding experimental observations of step fluctuations in which the step position is monitored as a function of time [21]–[24]. While the mechanisms of atomic motion near steps can be extremely complex [25]–[28], the continuum step model, in which the dynamics are reduced to those of a governing rate limiting step, has proven highly successful in predicting the thermal properties in experimental systems. Particularly useful in developing clear physical understanding have been two classes of models, where the temporal evolution of step edge fluctuations are driven by the exchange of atoms between the step and the adjacent terrace and/or by motion of adatoms along the step edge itself [29]–[31]. In these cases, the Langevin formalism for describing step motion is straightforward, and the experimentally measurable temporal correlation function of the step displacements has a power-law time dependence, varying as \( t^{1/n} \). In cases where mass transport at the step is dominated by diffusion of atoms along the step edge, \( n = 4 \). When mass transport proceeds via exchange of atoms between the step edge and the terrace \( n = 2 \). While there are many more complex combinations of surface diffusion processes that can be addressed within the continuum step model [32]–[37], we will limit our discussion to the step-edge diffusion and attachment/detachment cases for clarity of discussion.

In this paper, we investigate how the scaling of the step edge fluctuations is changed by the presence of an electromigration force [38] acting upon atoms diffusing along the step edges, and how in turn the step fluctuation properties can affect electrical transport. One motivation for this study is the possibility of using measurements of these changes to obtain information about the electromigration force itself. In conducting materials, a surface electromigration force can
be generated by passing an electrical current through the sample [17, 18], [39]–[41]. In terms of a simple discrete model, the presence of the electromigration force introduces a small bias in the diffusion of atoms on the surface, parallel to the current (and field). By convention, this bias can be expressed as an energy difference between atoms diffusing parallel or anti-parallel to the current \( \Delta U = Z^* e E a \) where \( E \) is the electric field applied to the sample, \( Z^* \) is the effective valence of a diffusing atom and \( a \) is the atomic lattice spacing. A characteristic property of surface electromigration is that the electromigration bias, \( \Delta U \), is several orders of magnitude smaller than the other energy scales in the problem: \( \tilde{\beta} a \), where \( \tilde{\beta} \) is the step stiffness, and the energy associated with thermal fluctuations \( k_B T \). A typical estimate of the bias is \( 10^{-5} \text{eV} \text{a}^{-1} \) at a current density of \( 10^7 \text{A cm}^{-2} \) [42]. This suggests that thermal fluctuations will completely dominate the short-time behavior of the step fluctuations, with the effect of the electromigration bias emerging only on much longer timescales. Nevertheless, such time scales (of the order of seconds) are accessible to experiment, offering the possibility that the observation of step fluctuations under electromigration conditions may allow us to obtain quantitative information about the magnitude of the force itself under conditions well before the large-scale mass displacement has occurred.

A second motivation for investigating the properties of step fluctuations under an electromigration force is to understand the reactive properties of the transport itself. The electromigration force acting on diffusing adatoms is matched by an equal and opposite effect on the scattered charge carriers [40]. Thus, step structure and step fluctuations at surfaces will cause changes in the surface resistivity and in the resistivity noise. Here, we will first review the effects of the electromigration force on fluctuating steps [43], and discuss an experimental observation of the effect [44]. The magnitude of the electromigration force determined from the experimental analysis will be discussed in terms of the kink structure at the step edge, and related to the surface resistivity in small structures. Finally, we will develop an analysis of the relationship of the temporal fluctuations of the steps to resistivity noise, showing that the noise spectrum will be distinctly different than normal \( 1/f \) noise. The magnitude of the resistivity noise, evaluated using experimental parameters, is shown to be of similar magnitude to bulk resistance noise for small structures.

2. Analysis of biased step fluctuations

Here, we synopsize an analysis [43] of the effects of a very small electromigration bias (small with respect to the other energy scales governing step behavior) on step fluctuations. In this analysis, we consider the limiting case where the step motion occurs most easily by diffusion along the step edge itself, as is frequently observed for metal surfaces near room temperature [9, 20, 23, 24]. The free energy functional is determined by the step stiffness, and the electromigration force, \( F \), felt by atoms at the step edge. If the force acts perpendicular to the step edge (i.e. when \( F > 0 \), the force acts in the +x, or stepdown, direction) then, for small fluctuations, one can linearize the stochastic equation of motion for the step edge [45]–[48], yielding a tractable Langevin equation for the step dynamics,

\[
\frac{\partial}{\partial t} - \frac{\Gamma_4 \tilde{\beta}}{k_B T} \frac{\partial^4}{\partial y^4} \mp \frac{\Gamma_4 F}{k_B T a_1 a} \frac{\partial^2}{\partial y^2} x(y, t) = \eta(y, t),
\]

(1)

where \( a_1 \) and \( a \) are the lattice parameters parallel and perpendicular to the step edge respectively, and \( \Gamma_4 \) is the step mobility, defined below. The position of the step edge is described by its edge
profile \(x(y,t)\) where the \(y\)-axis is oriented parallel to the step edge. The noise term, \(\eta\), describes the thermal fluctuations and is correlated for the case of step-edge diffusion,

\[
\langle \eta(y, t)\eta(y', t') \rangle = 2\Gamma_4 \delta(t-t') \frac{\partial^2}{\partial x^2}\delta(y-y'). \tag{2}
\]

To first-order, the electromigration force does not alter the correlation properties of the noise so that the step diffusivity \(\Gamma_4\) retains its equilibrium value,

\[
\Gamma_4 = \frac{a_2^4 a_2^2}{\tau_4}, \tag{3}
\]

where \(\tau_4\) is the average time between hopping events [19, 49]. The parameter space for the problem is reduced by introducing a scaled mobility \(\alpha\) and a critical wavevector \(q_c\),

\[
\left(\frac{\partial}{\partial t} - \alpha \frac{\partial^4}{\partial y^4} \mp \alpha q_c^2 \frac{\partial^2}{\partial y^2}\right) x(y,t) = \eta(y,t), \tag{4}
\]

where

\[
\alpha \equiv \frac{\Gamma_4 \bar{\beta}}{k_B T}, \tag{5}
\]

\[
q_c \equiv \frac{1}{\sqrt{a_2 d}} \sqrt{\frac{|F|}{\bar{\beta}}} \tag{6}
\]

The critical wavevector \(q_c\)-depends only on the ratio of the electromigration force to the step stiffness, and in equation (1) the \(\mp\) denotes the force acting in the step-down (−) or step-up (+) direction. To determine the statistical properties of the solutions of the biased step fluctuations, the Green’s function for the problem is derived using standard Fourier transform methods,

\[
G(y,t|y',t') = i \frac{2\pi}{\alpha} \int_{-\infty}^{+\infty} e^{-\alpha(k^4+q_c^2k^2)t}e^{ik(y-y')}dk. \tag{7}
\]

In terms of the Green’s function, the displacement of the step at time \(t\) is:

\[
x(y,t') = x(y,t) + \int_{-\infty}^{t'} \int_{-\infty}^{t} G(y,t'|y',t)\eta(y',t')dy'dt. \tag{8}
\]

By substituting the Green’s function, equation (7) into (8) we find, after some calculation, the time correlation function of the step displacements, \(g(t)\), after time \(t = t' - t'\) has elapsed,

\[
g_{\pm} \equiv \left\langle \left( x(y,t') - x(y,t) \right)^2 \right\rangle \tag{9}
\]

\[
= \frac{\Gamma_4}{2\pi \alpha} \int_{-\infty}^{\infty} \frac{1}{(k^2 \pm q_c^2)^4} \left( 1 - e^{-2\alpha(k^4+q_c^2k^2)t} \right)dk. \tag{10}
\]

Using substitution with equations (5) and (6) yields

\[
g_{\pm} (t) = t^{1/4} \left( \frac{(k_B T)^3 \Gamma_4}{\bar{\beta}} \right)^{1/4} \int_{0}^{\infty} \frac{1}{(u^2 \pm \sqrt{\alpha t q_c^2})^{3/2}} \left( 1 - e^{-2\alpha u^2 \sqrt{2a_1}t} \right)du. \tag{11}
\]
When there is no electromigration force (i.e. $|F| = 0, q_c = 0$), the integral in equation (11) is time independent and reduces to the well-known result [49] where the time correlation function $g(t)$ evolves as $t^{1/4}$ for step-edge limited diffusion:

$$\lim_{F\to 0} g_{\pm}(t) \equiv g_{0}(t) = a^2 \left( \frac{t}{\tau_0} \right)^{1/4},$$



(12)

where the parameter $\tau_0$ encompasses the characteristic time constant $\tau_4$ and stiffness $\tilde{\beta}$ of the equilibrium step,

$$\tau_0 \equiv \tau_4 \left( \frac{\tilde{\beta} a^2}{2k_BT a_\parallel} \right)^{3} \left( \frac{\pi}{2\Gamma(3/4)} \right)^{4}.$$



(13)

When the electromigration force is finite (i.e. $|F| > 0$), it is apparent from equation (11) that the scaling behavior is modified by the appearance of an explicit time dependence in the integral. This can be seen more clearly by defining a critical time, $\tau$, and a rescaled time, $\xi = t/\tau$, where,

$$\tau = \frac{1}{\alpha q_c^4} = 2\tau_4 \left( \frac{kT}{Fa} \right)^2 \left( \frac{\tilde{\beta} a^2}{2k_BT a_\parallel} \right).$$



(14)

Then, equation (12) can be rewritten as

$$g_{\pm}(t) = a^2 \left( \frac{t}{\tau_0} \right)^{1/4} I_{\pm} \left( \frac{t}{\tau} \right),$$



(15)

where

$$I_{\pm}(\xi) \equiv \frac{1}{\sqrt[4]{2^{1/4} \Gamma(3/4)}} \int_{0}^{\infty} \frac{1}{(u^2 \pm \sqrt{\xi})} \left( 1 - e^{-2u^2} e^{\pm 2u^2 \sqrt{\xi}} \right) du,$$



(16)

where $I_{\pm}$ is a universal function of the rescaled time, $\xi$ and is normalized such that $I_{\pm}(t \to 0) = 1$. The integral appearing in equation (16) is easily evaluated numerically and is shown in figure 1, where $I_{\pm}$ is shown as a function of the rescaled time (i.e. time is expressed in units of $\tau$). The solid curves show $I_{\pm}$ obtained for $F > 0$ (step-up), $F < 0$ (step-down) electromigration forces (for $F = 0$, $I_{\pm}(\xi) = 1$). From figure 1, it is apparent that $g_{\pm}(t)$ (equation (17)) deviates significantly from the $t^{1/4}$ scaling behavior of $g_0(t)$ (equation (15)) as $t$ approaches $\tau_0(\xi \to 1)$. In particular, the correlation function no longer follows a power law in time, e.g. will not follow a straight line on a log-log plot. These deviations are easily distinguishable at earlier times, $t \sim 0.1\tau$, as shown in figure 2, which displays the correlation function of a step plotted as a function of time for $\tau_0 = 10^{-3}s$ and $\tau = 10^3$s. Deviations from $t^{1/4}$ scaling are qualitatively visible when $t \sim 0.1\tau = 1000$s, and the predicted behavior has been demonstrated quantitatively using Monte Carlo simulations [43].

It is useful to perform a power series expansion of the integral about $\xi = 0$ such that equation (15) becomes

$$g_{\pm}(t) = a^2 \left( \frac{t}{\tau_0} \right)^{1/4} \left( 1 \mp a_{1/2} \left( \frac{t}{\tau} \right)^{1/2} + a_1 \left( \frac{t}{\tau} \right) + O \left( \frac{t}{\tau} \right)^{3/2} \right).$$



(17)
Figure 1. The integral function $I(\zeta)$ (equation (16)) plotted as a function of the rescaled time $\zeta = t/\tau$ for no electromigration force ($F = 0$), the electromigration force in the step-down direction ($F > 0$) and in the step-up direction ($F < 0$). The dashed lines show the approximation of equation (17).

Figure 2. The time correlation function, $g(t)$, of the step edge displacements predicted by the continuum Langevin model (equations (15) and (17)), plotted as a function of time for $\tau_0 = 10^{-3} \text{s}$ and $\tau = 10^3 \text{s}$.
The expansion coefficients can be obtained analytically,
\[ a_{1/2} = \frac{\pi}{6\Gamma^2(3/4)^2} = 0.3487. \]  
\[ a_1 = \frac{3}{20} = 0.1500. \]  
(18)  
(19)

The results of this series approximation for \( I_{\pm}(\zeta) \) (equation (16)), are shown as the dashed lines in figure 1, evaluated up to, and including the terms linear in time. The truncated expression is clearly a useful approximation for use in data analysis for \( t \leq 0.4\tau \).

While the effects of a vertical electric field on atomic motion are strong and readily observed, as in scanning tunneling microscopy (STM) [50] and field ion microscopy (FIM) [51], the field strengths due to current biasing are smaller, and the effects are much harder to observe. Observation and analysis of the predicted bias in the step time correlation function is a far more direct, and cleanly interpretable, probe of the magnitude of the surface electromigration force than analysis of the observations of large-scale structural rearrangements [17, 48], [52]–[57] that are the signature of electromigration effects.

3. Experimental methods and results

The critical time needed to see substantial effects of an electromigration force on the step correlation function can be estimated from previous calculations of the effective charge for Ag surfaces. The predicted effective valence of an isolated Ag adatom on Ag(111) is \( Z^* = -19 \) [58]. For atoms in a close-packed site along a step edge, with a perpendicular current direction, the direct force per step atom may be two times higher than the force on an adatom [41, 59], which would yield a predicted valence of \( Z^* \sim 38 \). To estimate the corresponding critical time, we use a reasonable value of \( \tilde{\beta} \sim 4.5 \text{ eV nm}^{-1} \) and fundamental hopping time constant \( \tau_4 \) (equations (3), (13), (14)) of \( 1.4 \times 10^{-8} \text{ s} \) [60]. The estimate yields the dependence of the critical time as \( \tau(s) \sim 1 \times 10^{14}/(J(\text{A cm}^{-2}))^2 \). For an observable change in the correlation function, which is typically measurable over \( \sim 10 \text{ s} \), critical times less than hundreds of seconds are needed. To allow the corresponding large current density \((>10^5 \text{ A cm}^{-2})\) needed to observe biased fluctuation effects, while minimizing Joule heating, very thin films were used. Well-ordered single crystalline Ag films with the (111) orientation exposed were prepared on freshly cleaved mica substrates [62]–[65] under UHV conditions (base pressure <10^{-10} \text{ Torr}) and imaged with STM without exposure to air. Ag was deposited at 20–40 Å s^{-1}, with the substrate held at 330–350°C [65, 66]. The films used in the measurements were 100–200 nm thick and 1–2 mm wide. All the films were single crystalline, as confirmed using low-energy electron diffraction, and had flat surface areas of dimension 1 µm or larger, with variable densities of steps. However, with decreasing thickness, the films showed increasing void areas, as illustrated in figure 3. Current densities are reported as nominal values without correction for the voids in the films. The cleanliness of the films was confirmed using Auger electron spectroscopy, and high-resolution STM.

4 In previous work [60] the step stiffness was estimated using a kink energy of 0.1 eV. There now exists a calculated value [61] of 0.117 eV, which results in higher values of the step stiffness. The hopping time constants previously estimated [60], can be corrected for the more accurate stiffness value by multiplying by the third power of the ratio of the old stiffness value to the new value.
Figure 3. Large area STM images (differential mode) for film thickness (a) 100 nm (image areas 250 × 500 nm) and (b) 350 nm, (image area 500 × 1000 nm).

The mica substrate was mounted in a heater assembly that allows independent indirect heating, as well as direct application of current across the thin film sample. To allow electrical contact to the sample area, and current stressing, wide (4 mm) and thick (∼3500 Å) silver leads were deposited onto the mica through a mask in situ. The sample area was then evaporated across the leads. Sample clips on the STM were used to make independent contact to the direct-current contact leads and the indirect heater current leads. Films prepared in this way could withstand an electron current up to 1 A (nominally ∼10⁶ A cm⁻² in the sample area). Imaging was performed using tunneling conditions of 0.6–0.8 nA and 1 V and at scan rate of ∼9 pixels ms⁻¹. Under these conditions, we [60, 66] and others [20, 67] have shown that the tip–sample interactions cause no measurable changes in the step fluctuation properties. The temperature–current relationship for the indirect heater was measured using an alumel-chromel thermocouple, which was spot-welded to a tiny Ta tab (strip) and brought into direct contact with the film surface [60]. Under direct current, the samples also experienced Joule heating. The magnitude of the correlation function (equations (9), (13), (17)) for each measurement under current stress was used to determine the temperature for analysis of the data by comparison with measurements without current stress [60].

Measurements of the step fluctuations were carried out by repeatedly scanning the same path across a step edge, with a scan time of 56 ms per line. The step position x(t) was extracted from each line scan by flattening the image, and then identifying the point at which surface height was midway between the heights of the upper and lower terraces. The individual x(t) data sets are used to calculate individual correlation functions, and the reported correlation functions are averages over multiple measurements on the same step. Measurements with large
Figure 4. (a) 150 × 150 nm STM image (shown in differential mode) showing single layer steps originating at a screw dislocation. Sequential scan directions are illustrated by the red arrows for electron current perpendicular to the step edge and electron current near parallel to the step edge. (b) Correlation functions measured for each of the two step positions indicated, data indicated by solid symbols, and the lines show the results of uncritical fits to the functional form \( At^w \). Data under current stress at \( T = 55 \)°C is shown along with data measured at room temperature (RT) with no current stress.

Current densities both parallel and perpendicular to the step edge yielded correlation functions for the step fluctuations with the dominant \( t^{1/4} \) behavior characteristic of thermal Ag step fluctuations [20, 60, 66, 67], as shown in figure 4. To evaluate the effects of the current in relationship to the theory presented in section 2, careful measurements of step fluctuations with the current direction perpendicular to the step edges in both the up-hill and down-hill directions were performed. The data have been presented elsewhere [44]. Here, we will present a more detailed description of the results and conclusions. In one measurement in which two steps with opposite orientation with respect to the applied current were both in the same measurement frame, a clear
Table 1. Analysis of correlation function for steps with current bias perpendicular to the step edge via a fit to equation (11).

| Current direction | $T$ (K) | Film thickness (nm) | Nom. current density $J$ (A cm$^{-2}$) | Critical time $\tau$ (s) | Relaxation time $\tau_0$ (s) | Step stiffness, $\tilde{\beta}$ (eV nm$^{-1}$) |
|-------------------|--------|--------------------|----------------------------------------|--------------------------|-------------------------------|---------------------------------|
| Uphill            | 325    | 100                | $4 \times 10^5$                        | 90                       | 0.063                         | 5.6                             |
| Uphill            | 350    | 100                | $4 \times 10^5$                        | 33                       | 0.0015                        | 4.5                             |
| Uphill            | 370    | 200                | $1 \times 10^5$                        | 52                       | 0.00034                       | 3.9                             |
| Downhill          | 370    | 200                | $1 \times 10^5$                        | 16                       | 0.00026                       | 3.9                             |

difference in the step correlation functions depending on the direction of the applied current was visible. The effects correspond to the force acting in the opposite direction to the current, indicating that the results are due to an electron wind force (momentum transfer from electron scattering). Two additional measurements at a higher current density were also performed, although it was not possible to find other matched pairs of uphill/downhill steps. In the latter cases, the deviations from a simple power could not be judged easily by eye (see for instance figure 4).

To test the qualitative observation for the uphill/downhill step pair, the data were fit to the full integral form of the correlation function (equation (11)) with three free parameters, $\tau$, $\tau_0$, and a zero-offset for $g(t)$. The zero-offset was found to be necessary due to small amounts of sporadic measurement noise that become significant in the averaging used to calculate the time correlation function. The results of the fit were found to be insensitive to the upper limit on the integral, and fits were performed well above the limit at which a dependence became detectable. The analysis of the experimental data confirmed the predicted deviations from a pure power law behavior with high statistical significance, based on a chi-squared analysis. In all cases in which current was applied, a distinct minimum was observed in the chi-squared value as a function of the critical time $\tau$, with the other fit parameters set at the optimum value. Tests of full functional fits to correlation functions measured without an applied current showed a monotonic decay with increasing fit values of $\tau$, with no minimum in the chi-squared. For the current-biased samples, the data fitting provided values of the critical time $\tau$, and the relaxation time $\tau_0$ that defines the prefactor of the fit to equation (11). Note that both $\tau$ and $\tau_0$ are temperature dependent, so values cannot be compared directly between different temperatures. The results are shown in table 1.

The electromigration force can be determined from the values of the critical time and relaxation time using equations (13) and (14), given the temperature dependent values of the step stiffness. Using previous results, we evaluate the temperature dependence of the stiffness using an effective kink energy of $\varepsilon = 0.117$ eV [61, 68], and the approximate expression [69, 70]:

$$\left(\frac{\tilde{\beta}a_x}{k_B T}\right) \left(\frac{a_x}{a_y}\right) = \left(\frac{2}{3}\right) \exp\left(\frac{\varepsilon}{k_B T}\right),$$

(20)
giving the values shown in the last column of table 1. With these values of the stiffness, the four measurements of the relaxation time constant $\tau$ yield average values of the force per step-edge atom of $-2.7 \times 10^{-5}$ eV nm$^{-1}$ for $J_{\text{nom}} = 4 \times 10^5$ A cm$^{-2}$ and $-9.7 \times 10^{-6}$ eV nm$^{-1}$ for $J_{\text{nom}} = 1 \times 10^5$ A cm$^2$. Thus, within the cumulative uncertainties of ±50% in the measured forces, the force increases in proportion to the current density, as expected for the electron wind force.
The measured forces, \( F = \Delta U/a \), can be related to the corresponding effective charge given the local electric field \( \Delta U = Z^* e E a \). An estimate of the local field can be obtained using the bulk current density and the resistivity, \( E = \rho J \). The temperature dependent bulk resistivities of silver are approximately \( 1.8 \times 10^{-6} \, \text{Ω cm} \) at 325 K and \( 2.2 \times 10^{-6} \, \text{Ω cm} \) at 370 K [62, 71]. The resulting values of the effective charge, calculated using the nominal current densities in table 1, are \( Z^* = -360 \) for \( J_{\text{nom}} = 4 \times 10^5 \, \text{A cm}^{-2} \), and \( Z^* = -430 \) for \( J_{\text{nom}} = 1 \times 10^5 \, \text{A cm}^{-2} \). The values are essentially identical for the two current densities, consistent with the predicted effects of variable current density. However, the magnitude of the values is substantially higher than our original expectation that \( |Z^*| \sim 38 \). Experimental factors that could contribute to the discrepancy include strain in the thin film and overestimation of the film cross-sectional area. Epitaxial films on mica are under large compressive stress [63], however the related changes in resistivity should be only at the percent level [62]. The cross-section effect is larger. As described earlier (see figure 3), at 100 nm film thickness there are vacancies in the film up to 50% of the surface area. Therefore, the true current densities may be as much as 2x higher than the nominal values, and as a result the effective valence could be half as large, i.e. our results set a lower limit on the magnitude of the effective charge of \( |Z^*| > (2 \pm 1) \times 10^2 \).

The measured value of the effective charge is unexpectedly large, based on simple arguments of classical transport [18], and on full calculations of the wind force on adatoms and straight step edges on surfaces [59]. The origins of the large value are likely to rest in the kink structure of the step edge, schematically illustrated in figure 5. Because the electromigration force is orthogonal to the step edge, while mass transport occurs along the step edge, only the tangential component of the force will be effective in biasing diffusion. Where the step edge is locally flat (no kinks), the tangential component will be zero. Only where there are thermally excited kinks will there be a tangential component of the force. The presence of kink structure in turn represents a significantly different environment than that of the straight step edges, for which previous calculations of the effective charge have been performed. Firstly, the complex geometry will enhance the ‘shadowing’ effect of the kinked step, analogous to current crowding in classical problems [72–74]. Secondly, the local electronic structure at a kink is distinctly different than at a straight step-edge segment [27, 28], with enhanced charge density due to the small local coordination at the kink site. The unique character of a kink site is also evidenced in the prediction [74, 75] and observation [77] of kink Ehrlich Schwoebel barriers. Confirmation of the mechanism for enhanced electromigration force will require expanded theoretical evaluation including multiple scattering effects [41] to the highly confined geometry of a kink site, which is likely to display the type of inhomogeneous current flow predicted for nanoelectronic junctions [12, 78].

The stronger-than-expected electromigration force observed at kinks will result in more rapid electromigration-induced structural rearrangements [79], with the correlated potential for failure of current-carrying elements [80–83]. The wind force acting on the diffusing atom is related to an equal and opposite force on the charge carriers, which translates into changes in the surface resistivity [40, 58]. To evaluate the magnitude of the contribution to the surface resistivity, it is first necessary to know the density of the sites contributing to the scattering. Our results suggest that strong scattering (\( Z^* \sim 100–300 \)) occurs at low-symmetry kink sites. The surface density of such sites will be the product of the step density, and the density of kinks per step. Thus, we estimate the overall strong-scattering site density \( n_k = (d_k d_{\text{step}})^{-1} \), where \( d_k \) is the average distance between kink sites along the step edge and \( d_{\text{step}} \) is the average distance...
between steps. The change in the surface resistivity $\rho_s$ resulting from a wind force $F_w$ acting on a scattering site density $n_k$ is [58]:

$$\mu_t \frac{\partial \rho_s}{\partial n_k} = -\frac{1}{en_c J} \left( F^k + \sum_j \delta F^j \right),$$

(21)

where $t_f$ is the film thickness, $n_c$ is the local carrier density, which we estimate as $n_{Ag} = 58.5 \text{ nm}^{-3}$, bulk carrier density for Ag, $F^k$ is the wind acting per kink site (our measured value) and $F^j$ are additional changes in force on the carriers due to the perturbation of atomic structure in the immediate vicinity of the step-edge atoms. Using the measured force per atom and the upper limit of the current density yields the component of the surface resistivity due to scattering at the kink sites of the step-edge atoms alone, $t_f(\partial \rho_s/\partial n_k) \geq -(F^k/en_{Ag}J_{\text{max}}) \sim (3 \pm 1.5 \text{ nm}^3) \rho_0$, where $\rho_0$ is the bulk value of the Ag resistivity.

The perturbed lattice terms ($F^j$ in equation (21)) may contribute as much as 2/3 of the total resistivity change [58], thus it is reasonable to expect that scattering at the kink sites of the step-edge will contribute a total change in the surface resistivity $t_f(\partial \rho_s/\partial n_k) \geq (9 \pm 5 \text{ nm}^3) \rho_0$.

As an example, we can consider the Ag nanowire shown in figure 6 [84]. The surface of the wire, particularly near the junctions of different facets, is populated with single-layer height steps. The average step spacing is $d_{\text{step}} \sim 0.5$–1 nm over >25% of the surface. To determine the kink density, we use the fact that the steps show the same type of structural fluctuations that are observed on steps on nominally flat Ag surfaces [20, 60, 67], and estimate the kink density based on the kink energy $\varepsilon$ and the temperature as $P_k \sim \exp (-\varepsilon/k_BT)$ [20]. The kink energy is largest on the steps along high symmetry directions, with a maximum value of 0.117 eV for Ag(111) [61]. This yields the most conservative estimate of the kink density $P_k$ ranging from 2% at 300 K to 10% at 450 K, or values of $d_{\text{kink}} = 130 \text{ nm}$ to 30 nm respectively. For a cylindrical symmetry, the film thickness $t_f$ is replaced by the wire radius in equation (21), yielding a conservative value for the kink-induced surface resistivity of 1% at room temperature and 5% at 450 K.

In addition to kinks, now identified as anomalously stronger scatters, there will also be effects on the surface resistivity due to the straight components of the step edge and to adatoms diffusing on the terraces. The total surface resistivity will be:

$$\rho_s = \left[ \frac{P_k}{ad_{\text{step}}} z^*_{\text{kink}} + \frac{1}{ad_{\text{step}}} z^*_{\text{step}} + c_a z^*_{\text{a}} \right] \frac{\rho_0}{t_f n_c},$$

(22)

where the number density of the adatoms is $c_a$. As discussed above, $z^*_{\text{step}} \sim z^*_{\text{kink}}/5$ and $z^*_{\text{a}} \sim z^*_{\text{kink}}/10$. Since the kink probability is only a few percent, the static step contribution will generally be larger than the kink contribution. The effects of both steps and kinks will also increase more rapidly than $1/r$ with decreasing wire radius because the step density will increase with decreasing radius. The effects of the electron wind force will thus be significant in nanoscale wires carrying large current densities [85, 86].

4. Step fluctuations and transport noise

The experimental results show that the interactions of kinks at surface steps with electrical current will give a larger contribution to the surface resistivity than would have been expected based on calculations and measurements of the effects of free adatoms on the terraces [58, 87, 88]. This will yield larger electromigration effects and, due to the dynamic nature of the kinks,
will also contribute to transport noise [10, 11, 13, 14, 89]. The 1/\(f\) spectrum characteristic of resistance fluctuations, once hoped to signify some universal physical property, is now widely recognized as the result of independent activated sources, which have different relaxation time constants and may represent different physical processes [90]. Each individual source is likely to have a frequency spectrum different from 1/\(f\), but the collective effect is a power law in frequency with an exponent close to −1. The interaction of thermally fluctuating steps with electrical current is a well-defined physical example of such a local source. Moreover it is an example for which the corresponding frequency spectrum can be directly calculated. In this section, we will present the outline of such a calculation, and illustrate with some simple approximations how the spectral signature will vary with the mechanism of step fluctuation.

As a basis for estimating the step-induced resistance noise, we recognize that the excitation of kinks equivalently causes changes in the length of a step, as illustrated in figure 5. Therefore, with the simple assumption that all kinks affect surface resistivity equivalently, changes in surface resistivity will be proportional to changes in the kink density (see equation (21)). This in turn can be captured by evaluating the kink density in terms of changes, \(\delta L\), in the length of a single step as \(\delta n_k = \delta L/(a L_o)\), where \(a\) is the kink depth and \(L_o\) is the projected step length as shown in figure 5, and \(d_s\) is the average separation between steps. Then, equation (21) can be rewritten as:

\[
t_i \delta \rho_s = - \frac{1}{en_c J} \left( Z^* e \rho_o J + \sum_j \delta f_{\psi,j} \right) \Delta n_k \sim - \frac{3Z^* \rho_o}{n_c} \frac{\delta L}{a d_{step} L_o},
\]

where the factor of three results from the rule of thumb that the lattice distortion effects \(\sum_j \delta f_{\psi,j}\) contribute approximately 2/3 of the total force on the scattered charge carriers [58].

The auto-correlation function \(\psi_{\rho}(t)\) of the changes in surface resistivity \(\delta \rho_s(t) = \rho_s(t) - \rho_s\), which will be related below to the resistivity noise [91], can then be reduced to a Fourier modes of the auto-correlation function for the changes in length \(\delta L = L - \langle L \rangle\) for a single step of projected length \(L_o\). If the resistivity noise arises from the independent fluctuations of \(N_{step}\) such steps, we have:

\[
\psi_{\rho_s(N_{step}, t)} = N_{step} \int_{-\infty}^{\infty} \delta \rho_s(t + t') \delta \rho_s(t') \, dt' \simeq N_{step} \left( \frac{3Z^* \rho_o}{n_c a d_{step} L_o} \right)^2 \int_{-\infty}^{\infty} \delta L(t + t') \delta L(t') \, dt'.
\]

Estimating the noise due to step fluctuations is now reduced to finding an expression for the auto-correlation function of the step length (or alternatively, the Fourier modes of the auto-correlation function for the step length). To our knowledge, this problem, which is analogous to finding the autocorrelation function of the changes in length \(\delta L = L - \langle L \rangle\) for a single step of projected length \(L_o\). If the resistivity noise arises from the independent fluctuations of \(N_{step}\) such steps, we have:

\[
\psi_{\rho_s(N_{step}, t)} = N_{step} \int_{-\infty}^{\infty} \delta \rho_s(t + t') \delta \rho_s(t') \, dt' \simeq N_{step} \left( \frac{3Z^* \rho_o}{n_c a d_{step} L_o} \right)^2 \int_{-\infty}^{\infty} \delta L(t + t') \delta L(t') \, dt'.
\]

As illustrated in figure 5, instead of addressing the density of kinks directly, the continuum step model treats the step edge as a smoothly varying line. The step displacements, \(x(y, t)\), can be written as a sum of periodic waves with wavelengths ranging from a single lattice constant to the maximum projected step length \(L_o\),

\[
x(y, t) = \frac{1}{\sqrt{L_o}} \sum_k x_k(t) e^{iky}.
\]
**Figure 5.** Schematic illustration of current flow perpendicular to average orientation of steps on the surface. Enhanced scattering from step sites at the surface is suggested by the arrows. The inset illustrates lattice model of kinked (thermally roughened) step edge segment of horizontal extent $L_o = 13a$, with 6 single and 1 double height kinks. The total length of the step edge is $21a$, and the length $\Delta L = 8a$ is the difference between the total length and the horizontal extent.

**Figure 6.** STM image of a $\sim 20$ nm diameter Ag nanowire [84]. Single layer steps with spacing $\sim 0.5–1$ nm occupy regions between facets, covering more than 25% of the surface area.

where $k = 2\pi p/\lambda$. Similarly, to analyze fluctuations in the lengths of the steps, we will evaluate the overall change in the length in terms of the different wavelength components:

$$
\delta L(t) = L(t) - \langle L \rangle \approx \left\{ \sum_k \frac{k}{k_0} (L_k - \langle L_k \rangle) e^{i\omega_k t} \right\},
$$

(26)
where $L_k$ is the length of one cycle (one wavelength) of each component of fluctuation, and is multiplied by $L_o/\lambda(=k/k_o)$ to give the total contribution over the projected step length $L_o$. The value of $L_k$ is simply the length of a sinusoid of wavevector $k$ and amplitude $A_k$. It is evaluated as:

$$L_k = \frac{2\pi}{k} \int_0^{2\pi/k} \sqrt{1 + (\text{d}x/\text{d}y)^2} \text{d}x = \int_0^{2\pi/k} \frac{1}{k} \sqrt{1 + A_k^2 k^2 \sin^2 x} \text{d}x,$$

which has the useful property that $kL_k$ depends on the product of $k$ and $A_k$, but not the wavevector and amplitude independently. Equation (27) can easily be evaluated numerically, as shown in figure 7. We will primarily be interested in the region of small fluctuations, i.e. small values of $kA_k$. Using a Taylor series expansion, the small amplitude limit of equation (27) has the simple form:

$$kL_k = 2\pi + \frac{\pi}{2} (kA_k)^2 .$$

if we assume $\langle L_k \rangle = 2\pi/k$, the projected step length, equation (26) simplifies to $\delta L(t) \approx L_o \sum_k (kA_k)^2 e^{\beta k t}$. This neglects the contribution of the thermal fluctuations to the average length [93], leading to an overestimate of $\delta L$ (see below). To estimate the length correlation function, we will use an estimate for the amplitude components $A_k$ based on the known behavior of the time correlation function (equations (9), (15)), which can also be described in terms of wavevector components [19]:

$$G_k(t) = \frac{2k_B T}{\beta k^2} (1 - e^{-t/\tau_k}) .$$

A reasonable approximation for the wavelength-specific displacements $x_k$ (equation (25)) is to approximate their amplitude as the square root of the prefactor of equation (29). This is
Figure 8. Calculated temperature dependence of resistivity noise of equations (34) and (35), for a Ag nanowire of \(Z^* = 200\), diameter = 5 nm and step spacing = 1 nm. The energy term is \((k_B T / k_B T 2\hat{\beta})^{(2+1/n)}\), calculated using equation (20). The mobility term is \((2\pi \Gamma_n)^{-1/n}\), calculated using previously measured values for the step mobility. The prefactor to the frequency term is \((z^*/z^s)^2(d_{step}/8\pi n A_{surf})(k_B T / \hat{\beta})^{(2+1/n)} (2\pi \Gamma_n)^{-1/n}\).

An underestimate, since the prefactor represents an rms value. The magnitude of the error roughly compensates the error introduced above by setting the average step length equal to the wavelength. The resulting equation for each mode of the fluctuating steps then is:

\[
x_k(t) = A_k \sin(\omega_k t) \approx \frac{1}{\sqrt{L_0}} \sqrt{\frac{2k_B T}{\beta k^2}} \sin(\omega_k t).
\]

An additional reasonable approximation defines the frequencies \(\omega_k\) in terms of the time constant for relaxation of each mode \(\tau_k\), \(\omega_k \approx (2\pi / \tau_k) = (2\pi \hat{\beta} k^n \Gamma_n / k_B T)\), where \(n = 2\) or 4 depending on the mechanisms of step fluctuation and \(\Gamma_n\) is the mobility for the step, which has units (length\(^{n+1}\))/time [19] (see equation (3) for \(\Gamma_n\)). For steps fluctuating via step edge-diffusion, as discussed in sections 2 and 3, the value of \(n = 4\). The simplest alternative case is fluctuation due to attachment and detachment of atoms from the step edges, for which \(n = 2\).

The wavevector dependence of the approximate amplitude \(A_k = \frac{1}{\sqrt{2k_B T}} \sqrt{L_0/\beta}\) of equation (30) goes as \(1/k\). As a result, shown by inserting the approximate amplitude in equation (28), the relative length variation, \(kL_k = L_k / \lambda_k\), is independent of wavevector. This result is related to the fact that the local kink-induced change in length is proportional to the slope of the step edge [93], and greatly simplifies the estimate of the spectral density of the resistivity noise, as follows.

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The spectral density of the noise, \( S_\rho(\omega) \), is the Fourier transform of the correlation function \( \psi_\rho \) of equation (24). As shown in equation (24), the correlation functions of the excess surface resistivity and excess length are simply related. Therefore, we can calculate the Fourier transform for the length correlation, which is easier to visualize physically, and then use equation (24) to obtain the resistivity-noise. Using the properties of correlation, the Fourier transform of the length auto-correlation function is equal to the product of the Fourier transforms of the lengths:

\[
S_L(\omega) = F(\psi_L(t)) = \{F(\delta L(t))\} \{F^*(\delta L(t))\}. \tag{31}
\]

Using equations (26) and (28) ff for the fluctuation length gives

\[
F(\delta L(t)) = F \left( \frac{L_o}{4} \sum_k (k A_k)^2 e^{i o t} \right) = \frac{(k A_k)^2}{4} F \left\{ \sum_k e^{i o t} \right\}, \tag{32}
\]

where \( k A_k = \sqrt{2k_B T / (L_o \beta)} \) is a unitless parameter independent of \( k \) (equation (30) ff). The Fourier transform yields a sum, over the accessible values of the wavevector \( k \), of delta functions \( \delta(\omega - \omega_k) \). The product of the sums obtained when subsequently evaluation equation (31) also remains nonzero only at \( \omega = \omega_k \), yielding:

\[
S_L(\omega) = \left\{ \frac{(k A_k)^2 L_o}{4} \right\}^2 \sum_k \delta(\omega - \omega_k) \approx \left\{ \frac{(k A_k)^2 L_o}{4} \right\}^2 \frac{1}{k_o} \int_{k_o}^{k_{\text{max}}} \delta(\omega - \omega_k) \frac{dk}{d\omega} d\omega, \tag{33}
\]

where the conversion of the sum to an integral is accomplished using the assumption that the dispersion will be the same for length fluctuations as for displacement fluctuations, and thus \( \omega_k = (2\pi / \tau_k) = (2\pi \beta k^n \Gamma_n / k_B T) \) (equation (30) ff). Inserting the derivative in equation (33) yields the noise spectrum in units of m\(^2\) Hz\(^{-1}\):

\[
S_L(\omega) = \frac{1}{k_o} \left\{ \frac{(k A_k)^2 L_o}{4} \right\}^2 \left[ \frac{k_B T}{2\pi \beta \Gamma_n} \right]^{1/n} \omega^{(1-n)/n} = \frac{L_o}{8\pi n} \left( \frac{k_B T}{2\beta} \right)^{(2+1/n)} \left( 2\pi \Gamma_n \right)^{-1/n} \omega^{(1-n)/n}, \tag{34}
\]

Thus the two different step fluctuations will generate noise spectra that go as \( \omega^{-1/2} \) for attachment/detachment \( (n = 2) \), and as \( \omega^{-3/4} \) for step edge diffusion \( (n = 4) \), consistent with the Fourier transform of the analytical expression for the real-space autocorrelation function [93].

To obtain the spectral density of the surface resistivity noise from the spectral density of the length-noise, the relationships shown in equations (24) and (31) are combined:

\[
S_\rho_s(N_{\text{step}}, \omega) \cong N_{\text{step}} \left[ \frac{3Z^*_s}{n c a t d_{\text{step}} L_o} \right]^2 S_L(\omega). \tag{35}
\]

Here, we explicitly use the assumption that \( N \) steps are fluctuating independently to give the total noise. Determining the reduced noise requires estimating the total step resistivity \( N \rho_s \) using equation (20):

\[
S_\rho_s(N_{\text{step}}, \omega) \frac{1}{N_{\text{step}} \rho_s^2} = \frac{S_\rho_s(N_{\text{step}}, \omega)}{\left( N_{\text{step}} \rho_s^2 \right)} = \left( \frac{3Z^*_s}{n c a t d_{\text{step}} L_o} \right)^2 \left( \frac{\rho_s}{n c} \right)^2. \tag{36}
\]
The number of steps is related to the total surface area $A_{\text{surf}}$ of the sample through $Nd_{\text{step}}L_o = A_{\text{surf}}$. Using this, and equation (35) in equation (36), and assuming that the resistivity due to the static step edges dominates, yields

$$S_{\rho}\left(\frac{N_{\text{step}}}{N_{\text{step}}^2}\right) \approx \left(\frac{z_k^4}{z_s^2}\right)^2 \frac{d_{\text{step}}}{A_{\text{surf}}L_o} S_L(\omega). \quad (37)$$

Equation (37) shows that the reduced noise due to the fluctuating steps is inversely proportional to the surface area, as expected for surface-generated noise.

The step-fluctuations on Ag, discussed in section 3, will be used as an example to demonstrate the temperature dependence and approximate magnitude of the resistance noise. We will consider a small nanowire of radius $\sim t_f = 5$ nm, step spacing $d_{\text{step}} = 1$ nm, and length $L = 10$ nm and thus surface area $\sim 300$ nm$^2$. The temperature dependence of the step stiffness (equation (20)) and mobility have been well characterized [60] (See footnote 4). The energy term in equation (34), $(k_B T/\beta)^{(2+1/n)}$ shown as the red squares in figure 8, is the dominant factor and is a gradually increasing function of temperature. The smaller mobility term $(2\pi \Gamma_n)^{-1/n}$, shown as blue diamonds, is a decreasing function of temperature. The overall prefactor of the spectral density of the relative resistivity noise is a slowly increasing function of temperature shown by the yellow triangles. The magnitude of the calculated noise prefactor falls in the range of $10^{-9}$–$10^{-10}$ Hz$^{-1/4}$. To compare with the bulk resistance noise in metallic conductors, we can use the empirical Hooge relationship, $S_R/R^2 = (\alpha H/N) f^{-1}$, where $N$ is the total number of carriers in the system and Hooge’s constant $\alpha$ is typically in the range of $10^{-3}$–$10^{-5}$. Considering a small wire, of diameter and length both about 10 nm, the total number of carriers is $\sim 4.6 \times 10^4$. Thus at $f = 1$ Hz, the relative spectral noise due to the bulk would be $\sim 2 \times 10^{-8}$ Hz$^{-1/2} \times 10^{-10}$ Hz$^{-1}$. For the step-fluctuation induced noise at $f = 1$ Hz, assuming kink densities similar to those of the wire shown in figure 6, the surface resistance spectral noise of equation (35), will be $\sim 10^{-10}$ Hz$^{-1}$. The surface resistance noise and the bulk noise will both be inversely proportional to the wire length at fixed wire radius. Thus for wire diameters around 10 nm, the relative resistivity noise due to step fluctuations will be comparable in magnitude to the relative bulk resistance noise.

5. Summary and conclusions

Experimental observation of biased step-edge fluctuations on Ag(111) under electromigration conditions has confirmed the predictions of a continuum Langevin model for the case where diffusion is limited to mass transport along the step edge. The electromigration force due to current densities greater than $10^5$ A cm$^{-2}$ causes deviations from the equilibrium $t^{1/4}$ scaling of the terrace-width distribution driven by thermal fluctuations alone. The predicted step correlation function increases more rapidly than $t^{1/4}$ for a down-hill force, and less rapidly than $t^{1/4}$ for an uphill force, a behavior analogous to the Bales-Zangwill instability induced by an asymmetric diffusion barrier at a step edge [94].

The observed bias of the time correlation function for step displacements on Ag(111) under applied current was used to determine the electromigration force. It is found to be a wind force, with substantially (5–15 times) larger magnitude than the force on a Ag adatom diffusing on a terrace. The origin of the force enhancement is possibly due to the constrained geometry and enhanced charge density at kink sites. The magnitude of the electron wind force has significant impact for the stability of nano-electronic conductors and contacts. It also will
cause a responsive effect of increased surface resistivity. The magnitude of the surface resistivity due to steps/kinks will be material dependent. For Ag, we show that the surface contribution to the resistivity will be about 1% at room temperature for a 20 nm diameter wire, but will increase rapidly with temperature or decreasing wire diameter.

Finally, the potential contribution of the temporal fluctuations of the steps to transport noise has been evaluated, using the simplest assumption that the changes in surface resistivity will be directly proportional to changes in the number of kink sites at the step edge, and thus to the step length. The known spectral behavior of step fluctuations then was used to estimate the spectral noise response as $1/f^{1/2}$ for steps fluctuating via attachment/detachment, and $1/f^{3/4}$ for steps fluctuating by diffusion along the step edge. The step-fluctuation contribution to the resistivity noise is a slowly increasing function of temperature, with a magnitude comparable to the bulk resistance noise in wires of 10 nm dimensions.

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References

[1] Prado S J, Trallero-Giner C, Alcade A M, Lopez-Richard V and Marques G E 2004 Influence of quantum dots shape on the Lande g-factor determination Phys. Rev. B 69 201310
[2] Hens Z, Vanmaekelbergh D, Stoffels E J A J and Van Kempen H 2002 Effects of crystal shape on the energy levels of zero-dimensional PbS quantum dots Phys. Rev. Lett. 88 236803
[3] Ichimiya A, Hayashi K, Williams E D, Einstein T L, Uwaha M and Watanabe K 2000 Decay of silicon mounds: scaling laws and description with continuum step parameters Phys. Rev. Lett. 84 3662–5
[4] Mullins W W and Rohrer G S 2000 Nucleation barrier for volume conserving shape changes of faceted crystals J. Am. Ceram. Soc. 83 214
[5] Zhang C-H, Kassubek F and Stafford C A 2003 Surface fluctuations and the stability of metal nanowires Phys. Rev. B 68 165414
[6] Ross F M, Tersoff J and Reuter M C 2005 Sawtooth faceting in silicon nanowires Phys. Rev. Lett. 95 146104
[7] Bid A, Bora A and Raychaudhuri A K 2005 Observation of large low-frequency resistance fluctuations in metallic nanowires: implications on its stability Phys. Rev. B 72 113415
[8] Degawa M, Thürmer K and Williams E D 2006 Constrained evolution of nano-crystallites Phys. Rev. B 74 155432
[9] Zhirkov V V and Cavin R K 2006 Chemistry of molecules or physics of contacts Nat. Mater. 5 11–12
[10] Basch H, Cohen R and Ratner M A 2005 Interface geometry and molecular junction conductance: geometric fluctuation and stochastic switching Nano Lett. 5 1668–5
[11] Tao C, Stasevich T J, Cullen W G, Einstein T L and Williams E D 2007 Metal-molecule interface fluctuations Nano Lett. 7 1495–9
[12] Sai N, Bushong N, Hatcher R and Di Ventra M 2007 Microscopic current dynamics in nanoscale junctions Preprint cond-mat/0701634
[13] Pelz J and Clarke J 1985 Dependence of $1/f$ noise on defects induced in copper films by electron irradiation Phys. Rev. B 55 738
[14] Bid A, Bora A and Raychaudhuri A K 2006 $1/f$ noise in nanowires Nanotechnology 17 152–6
[15] Ishigami M, Chen J H, Williams E D, Tobias D, Chen Y F and Fuhrer M S 2006 Hooge’s constant for carbon nanotube field effect transistors Appl. Phys. Lett. 88 203116

New Journal of Physics 9 (2007) 387 (http://www.njp.org/)
[16] Chu C S and Sorbello R S 1988 Local-field method for resistivity and electromigration in metallic microstructures: application to thin films Phys. Rev. B 38 7260
[17] Yasunaga H and Natori A 1992 Electromigration on semiconductor surfaces Surf. Sci. Rep. 15 205–80
[18] Rous P J, Einstein T L and Williams E D 1994 Theory of surface electromigration on metals Surf. Sci. 315 L995–1002
[19] Jeong H-C and Williams E D 1999 Steps on surfaces: experiment and theory Surf. Sci. Rep. 34 171–294
[20] Giesen M 2001 Step and island dynamics at solid/vacuum and solid/liquid interfaces Prog. Surf. Sci. 68 1–153
[21] Bartelt N C, Goldberg J L, Einstein T L, Williams E D, Heyraud J C and Métois J J 1993 The Brownian motion of steps on Si(111) Phys. Rev. B 48 15453–6
[22] Pimpinelli A, Villain J, Wolf D E, Métois J J, Heyraud J C, Elkinani I and Uimin G 1993 Step dynamics on vicinal surfaces Surf. Sci. 295 143
[23] Kuipers L, Hoogeman M S and Frenken J W M 1993 Step dynamics on Au(110) studied with a high-temperature, high speed STM Phys. Rev. Lett. 71 3517–20
[24] Giesen-Seibert M, Jentjens R, Poensgen M and Ibach H 1993 Time dependence of step fluctuations on vicinal Cu(119) surfaces investigated by tunneling microscopy Phys. Rev. Lett. 71 3521–4
[25] Stumpf R and Scheffler M 1996 Ab initio calculations of energies and self diffusion on flat and stepped surfaces of Al and their implications on crystal growth Phys. Rev. B 53 4958–73
[26] Feibelman P J 1992 Systematics of adsorption near a step Phys. Rev. Lett. 69 1568
[27] Feibelman P J 2001 Accelerated mound decay at adjacent kinks on Cu(111) Surf. Sci. 478 L349–54
[28] Mehmood F, Kara A and Rahman T S 2006 First principles study of the electronic and geometric structure of Cu(532) Surf. Sci. 600 4501–7
[29] Bartelt N C, Goldberg J L, Einstein T L and Williams E D 1992 The equilibration of terrace width distributions on stepped surfaces Surf. Sci. 273 252–60
[30] Jeong H-C and Weeks J D 1999 Effects of step–step interactions on the fluctuations of an individual step on a vicinal surface and its wavelength dependence Surf. Sci. 432 101–14
[31] Le Goff E, Barbier L and Salanon B 2003 Time–space height correlations of thermally fluctuating 2d systems. Application to vicinal surfaces and analysis of STM images of Cu(115) Surf. Sci. 531 337–58
[32] Pimpinelli A, Elkinani I, Karma A, Misbah C and Villain J 1994 Step motions on high-temperature vicinal surfaces J. Phys: Condens. Matter 6 2661–80
[33] Khare S V and Einstein T L 1998 A unified view of step-edge kinetics and fluctuations Phys. Rev. B 57 4782–97
[34] Ihle T, Misbah C and Pierre-Louis O 1998 Equilibrium step dynamics on vicinal surfaces revisited Phys. Rev. B 58 2289
[35] Flynn C P 2002 Step-edge fluctuations on crystal surfaces Phys. Rev. B 66 155405
[36] Blagovejevic B and Duxbury P M 1999 Atomic diffusion, step relaxation, and step fluctuations Phys. Rev. E 60 1279
[37] Bisani M and Selke W 1999 Step fluctuations and random walks Surf. Sci. 437 137
[38] Sorbello R S 1999 Solid State Physics ed H Ehrenreich and F Spaepen (New York: Academic) p 159
[39] Ishida H 1994 Microscopic theory of the wind force for adsorbates on simple metal surfaces Phys. Rev. B 49 14610–8
[40] Ishida H 1999 Semiclassical derivation of the surface resistivity formula Phys. Rev. B 60 4532–4
[41] Rous P J 2000 Multiple-scattering theory of the surface resistivity of stepped Al surfaces Phys. Rev. B 61 8484–8
[42] Rusanen M, Kuhn P and Krug J 2006 Kinetic Monte Carlo simulations of oscillatory shape evolution for electromigration-driven islands Phys. Rev. B 74 245423
[43] Rous P J and Bole T 2007 Temporal evolution of step-edge fluctuations under electromigration conditions Preprint 0704.0624
[44] Bondarchuk O, Williams E D, Bole T and Rous P J 2007 Biased structural fluctuations due to electron wind force submitted Phys. Rev. Lett. at press
[45] Stoyanov S 1991 Electromigration induced step bunching on Si Surfaces—how does it depend on the temperature and heating current Japan J. Appl. Phys. 30 1–6
[46] Williams E D, Fu E, Yang Y-N, Kandel D and Weeks J D 1995 Measurement of the anisotropy ratio during current-induced step bunching Surf. Sci. 336 L746–52
[47] Fu E, Johnson M D, Williams E D, Liu D and Weeks J D 1996 Size-scaling in the decay of metastable structures Phys. Rev. Lett. 77 1095
[48] Thürmer K, Liu D-J, Weeks J D and Williams E D 1999 Onset of step anti-banding instability due to surface electromigration Phys. Rev. Lett. 83 5531–4
[49] Bartelt N C, Einstein T L and Williams E D 1994 Measuring surface mass diffusion coefficients by observing step fluctuations Surf. Sci. 312 411–22
[50] Braun K-F, Soe W-H, Flipse C F J and Rieder K-H 2007 Electromigration of single metal atoms observed by scanning tunneling microscopy Appl. Phys. Lett. 90 023118
[51] Ehrlich G 1994 Diffusion of individual adatoms Surf. Sci. 299/300 628
[52] Visikovkiy A, Mizuno S and Tochihara H 2006 Reversible electromigration of thallium adatoms on the Si(111) surface Surf. Sci. 600 L189
[53] Stahlmecke B, Meyer zu Heringsdorf F-J, Chelaru L I, Horn-von Hoegen M, Dämigich G and Roos K R 2006 Electromigration in self-organized single-crystalline silver nanowires Appl. Phys. Lett. 88 053122
[54] Minoda H 2003 Direct current heating effects on Si(111) vicinal surfaces J. Phys.: Condens. Matter 15 S3255–80
[55] Barinov A, Gregoratti L, Kaulich B and Kiskinova M 2002 Surface electromigration patterns in a confined adsorbed metal film: Ga on GaN Chem. Phys. Chem. 12 1019
[56] Latyshev A V, Minoda H and Yagi K 1998 Electromigration and gold-induced step bunching on the Si(111) surface Surf. Sci. 401 22
[57] Métois J-J and Audriffen M 1997 An experimental study of step dynamics under the influence of electromigration: Si(111) Int. J. Mod. Phys. B 11 3691
[58] Hedouin M F G and Rous P J 2000 Relationship between adatom-induced surface resistivity and the wind force for adatom electromigration: a layer Korringa–Kohn–Rostoker study Phys. Rev. B 62 8473
[59] Rous P J 1999 Electromigration wind force at stepped Al surfaces Phys. Rev. B 59 7719–23
[60] Bondarchuk O, Dougherty D B, Degawa M, Constantin M, Dasgupta C and Das Sarma S 2005 Correlation time for step structural fluctuations Phys. Rev. B 71 045426
[61] Stasevich T J, Gebremariam H, Einstein T L, Giesen M, Steimer C and Ibach H 2005 Low-temperature orientation dependence of step stiffness on {111} surfaces Phys. Rev. B 71 245414
[62] Verma B S and Juretschke H J 1970 Strain dependence of the resistivity of silver films J. Appl. Phys. 41 4732–5
[63] Koch R, Winau D and Rieder K H 1993 Intrinsic stress of epitaxial thin-films Phys. Scr. T 49B 539–43
[64] Baski A A and Fuchs H 1994 Epitaxial growth of silver on mica as studied by AFM and STM Surf. Sci. 313 275
[65] Levin M and Laakso A 2001 Evaporation of silver thin films on mica Appl. Surf. Sci. 171 257
[66] Tao C and Williams E D 2007 Generalized survival in step fluctuations Phys. Rev. E at press
[67] Mugele F, Rettenberger A, Boneberg J and Leiderer P 1998 The influence of tip-sample interaction on step fluctuations on Ag(111) Surf. Sci. 400 80–86
[68] Steimer C, Giesen M, Verheij L and Ibach H 2001 Experimental determination of step energies from island shape fluctuations: a comparison to the equilibrium shape method for Cu(100), Cu(111), and Ag(111) Phys. Rev. B 64 085416
[69] Akutsu N and Akustsu Y 1999 Statistical mechanical calculation of anisotropic step stiffness of a two-dimensional hexagonal lattice-gas model with next-nearest-neighbour interactions: application to Si(111) surface J. Phys.: Condens. Matter 11 6635
[70] Nowicki M, Bombis C, Emmundts A and Bonzel H P 2003 Absolute step and kink formation energies of Pb derived from step roughening of two-dimensional islands and facets Phys. Rev. B 67
[71] Matula R A 1979 Resitivity of Noble Metals J. Phys. Chem. Ref. Data 8 1260
[72] Gungor M R and Maroudas D 1999 Electromigration with current crowding J. Appl. Phys. 85 2233
[73] Rous P J, Yongsonthun R, Stanishevsky A and Williams E D 2004 Real-space imaging of current distributions at the sub-micron scale using magnetic force microscopy: inversion methodology J. Appl. Phys. 95 2477–86
[74] Wang H, Bruynseraede C and Maex K 2004 Impact of current crowding on electromigration-induced mass transport Appl. Phys. Lett. 84 517–9
[75] Kallunki J and Krug J 2003 Effect of kink-rounding barriers on step edge fluctuations Surf. Sci. 523 L53–8
[76] Pierre-Louis O, D’Orsogna M and Einstein T L 1999 Edge diffusion during growth: the Kink Ehrlich–Schwoebel effect and resulting instabilities Phys. Rev. Lett. 82 3661
[77] Ikonomov J, Starbova K and Giesen M 2007 Island coalescence and diffusion along kinked steps on Cu(001): evidence for a large kink Ehrlich–Schwoebel barrier Surf. Sci. 601 1403–8
[78] Todorov T N 1999 Spatial distribution of the electric current and field in atomic scale conductors Phil. Mag. B 79 1577–90
[79] Li D-J and Weeks J D 1998 A quantitative theory of current-induced step bunching on Si(111) Phys. Rev. B 57 14891–900
[80] Cespedes O, Jan G, Viret M, Bari M and Coey J M D 2003 Random telegraph noise in a nickel nanoconstriction J. Appl. Phys. 93 8433–5
[81] Smir R H M, Untiedt C and van Ruitenbeek J M 2004 The high-bias stability of monatomic chains Nanotechnology 15 S472–8
[82] Gardinowski G, Schmeidel J, Pfirnur H, Block T and Tegenkamp C 2006 Switchable nanometer contacts: ultrathin Ag nanostructures on Si(100) Appl. Phys. Lett. 89 063120
[83] Yang Z, Chisiev M, Zwolak M, Chen Y-C and Di Ventra M 2005 Role of heating and current-induced forces in the stability of atomic wires Phys. Rev. B 71 041402
[84] Tao C G, Cullen W G, Williams E D, Hunyadi S E and Murphy C J 2007 Surface morphology and step fluctuations on Ag nanowires, unpublished
[85] Durkan C and Welland M E 2000 Size effects in the electrical resistivity of polycrystalline nanowires Phys. Rev. B 61 14215
[86] Wu W, Brongersma S H, Van Hove M and Maex K 2004 Influence of surface and grain-boundary scattering on the resistivity of copper in reduced dimensions Appl. Phys. Lett. 84 2838
[87] Chavineau J P 1980 Diffusion superficielle des adatomes observee a basse temperature par la variation de resistance electrique de films minces d’or et de bismuth Surf. Sci. 93 471–86
[88] Schumacher D 1995 Surface Scattering Experiments with Conduction Electrons Springer Tracts in Modern Physics vol 128 (Dusseldorf: Springer)
[89] Ralph D C and Buhrman R A 1994 Effects of electron heating on conductance fluctuations in mesoscopic wires Phys. Rev. B 49 2257–60
[90] Weissman M B 1988 1/f noise and other slow, nonexponential kinetics in condensed matter Rev. Mod. Phys. 60 537–71
[91] Kogan S 1996 Electronic Noise and Fluctuations in Solids (Cambridge: Cambridge University Press)
[92] Beijren H V 2006 private communication
[93] Pierre-Louis O 2007 Fluctuation dynamics of macroscopic quantities, in preparation
[94] Bales G S and Zangwill A 1990 Morphological instability of a terrace edge during step-flow growth Phys. Rev. B 41 5500–8