A robust system for Quantum Computation using Coherent Population Trap states of neutral atoms

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A robust system for use of Quantum Computation using neutral atoms is presented in this communication. This scheme exploits a multilevel atomic system interacting with two lasers in such a way that it leads to formation of two Coherent Population Trap (CPT) states. The atomic population can be pumped alternatively between these two CPT states or any desired superposition of the two can also be formed. These two CPT states can then be mapped to two Qubits for Quantum Computation. Preparation of any desired state and formation of CNOT gates are explained below. A scheme for preparing 1-D lattice of mutually interacting spin systems, which mimic behaviour of magnetic spin systems is also shown. Atoms in CPT states do not interact with the lasers that prepare it and hence are free of perturbation. The states involved here are also made up of only ground states of bare atoms and hence are free of decoherence due to spontaneous emission. Thus, the configuration studied here is a good example of a very robust system.

Quantum Computers solve certain class of problems with lesser number of computational steps than their classical counterparts. This is achieved by exploiting the basic fact that Quantum systems can be prepared in superposition of their eigenstates [1]. Therefore, a basic requirement to realize a quantum computer is the ability to prepare a system in a superposition state which is robust and reliable. Several different realizations such as NMR of large molecules, ions in ion traps and neutral atoms in optical lattices have been proposed and actually implemented, each with its own advantages and disadvantages. For instance, NMR systems tend to be bulky and require cooling upto 3 mK while ion trap systems have to be accounted for Stark shifts due to electric fields used for trapping. Neutral atom systems on the other hand suffer from their dependency on fluctuations in laser intensity, large decoherence rates and critical requirements on pulse shaping etc. One way to overcome these problems is to use Coherent Population Trap (CPT) states of neutral atoms.

Coherent Population Trap states (CPT) are special class of superposition of atomic states such that the transition probability of its constituent states to a common external state cancel each other. Thus, the atom can not make a transition to the external state through absorption of a photon and is hence trapped [2]. Sometimes they are also referred to as ‘dark states’ in the literature. CPT states are eigenstates of the total Hamiltonian which consists both atomic and interaction parts [3]. Thus, they are stationary solutions and hence will not evolve even when interaction is present. In this communication we examine a particular atomic configuration that exhibits two CPT states, both made of ground states. These two can be mapped to Qubits 0 and 1 and one and two-Qubit operations can be performed upon them. A superposition of the two Qubits can also be prepared with any desired phase relations between them. It is shown that the methods of obtaining, manipulating and performing Qubit operations on this system is easy, robust and reliable. The required atomic configuration is readily available in atoms such as $^{87}$Rb and Na.

Therefore this is a very feasible and robust method of doing Quantum Computation.

I. THE CONFIGURATION

The configuration involves two orthogonally polarized beams incident on $^{87}$Rb, both resonant with the transition between hyperfine levels $|5S_{1/2}, F = 1 \rangle$ and $|5P_{1/2}, F = 1 \rangle$. One of them is polarized along the axis of quantization and denoted $E_z = \mathcal{E}_z \exp[i(\omega t - k x)]$. This is referred to as $E_z$ beam. The other beam, referred to as $E_p$ beam, is denoted as $E_p = \mathcal{E}_p \exp[i(\omega t - k z)]$. This beam is a plane polarized beam in the xy plane and can effectively be decomposed into a pair of circularly polarized beams with opposite helicities $\sigma_{\pm}$. Both $E_z$ and $E_p$ beams can be derived from a single laser source by use of a halfwave plate and a polarizing beam splitter as shown in figure 1a. By rotating the HWP, the full intensity of the laser can be distributed between these two beams as desired.

Following the selection rules involved with polarizations [4], these two beams couple transitions between the various Zeeman sublevels as shown in figure 1b. The levels labeled $|g_0\rangle$, $|g_\pm \rangle$ in and $|g_+\rangle$ in the figure denote the Zeeman sublevels of the ground state $|5S_{1/2}, F = 1 \rangle$ with $m_F$ values 0, -1 and +1 respectively. Similarly the levels labeled $|e_0\rangle$, $|e_-\rangle$ and $|e_+\rangle$ are those of the excited state $|5P_{1/2}, F = 1 \rangle$, with $m_F=0$, -1 and +1. $E_z$ beam therefore couples transitions $|g_\pm \rangle \leftrightarrow |e_-\rangle$ which results in $\Delta m_F = 0$. Similarly, the $\sigma_+$ component of $E_p$ couples $|g_\pm \rangle \leftrightarrow |e_0\rangle$ and $|g_0 \rangle \leftrightarrow |e_+\rangle$ while the $\sigma_-$ component couples $|g_0 \rangle \leftrightarrow |e_-\rangle$ and $|g_+ \rangle \leftrightarrow |e_0\rangle$ transitions so that $\Delta m_F = +1$ and -1 respectively. The transition between $|g_\pm \rangle \leftrightarrow |e_0\rangle$ is not coupled by the $E_z$ beam despite $\Delta m_F = 0$ because the Clebsch - Gordon coefficients for this transition vanishes due to symmetry considerations.

If only the $E_p$ beam is present and $E_z = 0$, then the configuration is a classical $\Lambda$ system formed by $|g_- \rangle \leftrightarrow |e_0\rangle \leftrightarrow |g_+\rangle$.  

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Atoms are optically pumped to states. Those that decay into intensity ratio between two laser beams. Due to all of above, the configuration allows a robust and reliable preparation of two fluctuations in the intensity. (vi) Any desired superposition, denoting any Bloch vector can be prepared by simply varying the ratio between the Rabi frequencies. Further, any desired superposition can be prepared as

\[ \psi = \sin(\theta/2)\psi_0 + \exp(i\phi)\cos(\theta/2)\psi_- \],

where states \( \psi_0 \) and \( \psi_- \) are as described above and

\[
\sin(\theta/2) = \frac{\Omega_p}{\sqrt{2|\Omega_c|^2 + |\Omega_p|^2}} \\
\cos(\theta/2) = \frac{\sqrt{2|\Omega_c|^2 + |\Omega_p|^2}}{\sqrt{2|\Omega_c|^2 + |\Omega_p|^2}}
\]

and

\[ \theta/2 = \tan^{-1}\left(\frac{\Omega_p}{\sqrt{2}\Omega_c}\right) \]

This implies that a superposition of the two Qubit states can be prepared as

\[ \psi = \sin(\theta/2)|0\rangle + \exp(i\phi)\cos(\theta/2)|1\rangle \]

by switching on both beams simultaneously. Further, any desired \( \theta \) value in the above superposition can be obtained by varying the ratio between the Rabi frequencies \( \Omega_p/(\sqrt{2}\Omega_c) \). If both beams are derived from a single source as shown in figure 1a, then the ratio is easily manipulated by rotating the Halfwave plate. This divides the intensity between \( E_p \) and \( E_c \) beams in such a way that \( \Omega_p \) is zero when \( \Omega_c \) is maximum and vice versa and hence the angle \( \theta \) can be varied from 0 to \( \pi/2 \). Since Rabi frequencies are normally complex quantities, a phase difference \( \phi \) can be obtained between them.

An even more interesting situation occurs when both \( E_p \) and \( E_c \) beams are simultaneously present. The steady state solution for the atom then is not a simple statistical mixture of the two trap states but a three-component CPT state, given by

\[ |\psi\rangle = \frac{(\Omega_p/\Omega_c)|g_o\rangle - |g_-\rangle + |g_+\rangle}{\sqrt{2 + |(\Omega_p/\Omega_c)|^2}} \]

The above equation can be rewritten in the form

\[ |\psi\rangle = \sin(\theta/2)|\psi_0\rangle + \exp(i\phi)\cos(\theta/2)|\psi_-\rangle \]

where states \( \psi_0 \) and \( \psi_- \) are as described above and

\[
\sin(\theta/2) = \frac{\Omega_p}{\sqrt{2|\Omega_c|^2 + |\Omega_p|^2}} \\
\cos(\theta/2) = \frac{\sqrt{2|\Omega_c|^2 + |\Omega_p|^2}}{\sqrt{2|\Omega_c|^2 + |\Omega_p|^2}}
\]

and

\[ \theta/2 = \tan^{-1}\left(\frac{\Omega_p}{\sqrt{2}\Omega_c}\right) \]

This allows us to prepare the atom in two Qubit states \( |0\rangle \) or \( |1\rangle \) as well as any desired superposition state of the two, with certain distinct advantages. (i) Both these states as well as their superposition are CPT states. Therefore, once prepared, the states do not interact with the preparing laser, which eliminates a need for precise control of the ontime of the laser. (ii) Decoherence due to spontaneous emission is eliminated since all these states are made up of only ground states. (iii) CPT preparation is always the endpoint of the process and therefore the CPT state will be reached even if there are small variations in the intensity and frequency of the lasers involved. The state preparation is therefore robust and certain. (iv) The state preparation involves only cw beams and does not require any complex pulse shaping schemes. (v) Since it does not involve single photon processes, lasers with nominally high intensity can be used. This would allow very precise control of phase \( \phi \) while allowing fluctuations in the intensity. (vi) Any desired superposition, denoting any Bloch vector can be prepared by simply varying the intensity ratio between two laser beams. Due to all of above, the configuration allows a robust and reliable preparation of two Qubit states and its superposition and also the method of state preparation is very easy. In the following sections, methods of performing single Qubit and two-Qubit operations are discussed.
II. QUANTUM COMPUTATIONAL WITH ABOVE QUBITS

A. Single Qubit Operations

The atom prepared in state |0⟩ above can be flipped onto state |1⟩ by simply switching switching on the $E_p$ beam and switching to the $E_z$ beam will prepare flip the atom back to state |0⟩. But this process involves spontaneous emission from states $|e_{0,±}⟩$ and hence is an incoherent process. This technique therefore is not a proper bit-flip operation. However a coherent bit-flip operation can still be operated using a process very similar to the STIRAP process as suggested by Gaubatz and coworkers [7]. This process involves varying parameters of a CPT state, adiabatically slowly such that the atom is always in the CPT state, even as the state itself evolves from an initial state to the final state. In this case, the three component CPT state (1) is made to evolve between $|ψ_0⟩$ and $|ψ_1⟩$ by varying the angle $θ$. If this variation is achieved adiabatically slowly the atom is always in the state $|ψ⟩=cos(θ)|ψ_0⟩+sin(θ)|ψ_1⟩$. The state returns back to original state, but with an additional phase after every period of $T = (h\pi)/V_{dd}$. The phase gate therefore involves bringing the two atoms closer and hold it for a period $T = (h\pi)/V_{dd}$ and taking it apart, by which time, only the state |01⟩ would have acquired a phase.

B. Two Qubit operation; C-Phase gate

Two-Qubit operations can be obtained on this system in three different methods. All the three exploit a dipole-dipole interaction between two atoms in the CPT states, but between magnetic spins instead of electric dipoles, as described below.

In presence of a static magnetic field $\vec{B}_s$, the atomic spin vectors orient themselves such that the projection of the spin vector onto the magnetic field vector is equal to the magnitude of the magnetic quantum number $m_F$. Therefore, the state |0⟩, with is $m_F = 0$ orients perpendicular to this field and the state |1⟩ orients along the field since its constituent components have $m_F = ± 1$.

Further, these spin vectors precess about the magnetic field vector with a Larmor frequency $\omega_L = γ_L |B_s|$, where $γ_L$ is the gyromagnetic ratio and $|B_s|$ the field strength. An additional oscillatory magnetic field $B_f << B_s$, at the frequency $\omega_f$, applied perpendicular to $B_s$ will now cause an energy shift so that the atoms flip from one another $m_F$ state to another [4, 5]. If two such atoms are considered, then the energy of the atom-pair in presence of $B_s$ is as shown in figure 2a, with each spin flip transition separated by an energy $\hbar ω_L$. However, if these atoms are brought sufficiently close together, there exists an interaction between two spin -dipoles of the nature $S_1.S_2$, which will result in an interaction potential [8]

$$V = \frac{\mu_0 \gamma^2}{4\pi r^3} (3\cos^2 θ - 1). \quad (4)$$

Here $r$ is the normal distance between the two spin states and $θ$, the angle between them. $\mu_0$ is the permittivity of free space and the ratio $μ_0/4\pi$ is the scaling factor required to change from CGS to MKS units. This interaction causes a mixing of the pair states |01⟩ and |10⟩ and also causes a shift of the state |01⟩ + |10⟩ as shown in figure 2b. The oscillatory magnetic field oscillating at Larmor frequency will no longer be in resonance with the spin flip operation. This unequal shift is due to the fact that the angle $θ_s$ between the two spins is 0 if both atoms are in same state and is equal to $π/2$ if the two atoms are in different state. The shift is equal to $2Ω$ where $Ω = (\mu_0/(4\pi)) \cdot (γ^2/\epsilon_0)$ which is the case when $θ_s = π/2$ in equation (4).

A controlled-phase operation can now be performed in a manner similar as suggested by Gavin Brennen and coworkers [11], except for using magnetic dipoles instead of electric dipoles. The two atoms are brought close and a short pulse of oscillating magnetic field of frequency $ω_L = 2Ω$ and a McCall-Hahn area of $2π$ is applied. The pulse will not have any effect if the atom pairs are in state |00⟩ or in mixed state |01⟩ − |10⟩. If it is instead in |10⟩ + |01⟩ then the pair gets pumped to state |11⟩ and back, but this cyclic transformation brings about an additional phase $π$ [10]. When pulled apart, |10⟩ would have acquired a $π$ phase. If instead the state was already in |01⟩ − |10⟩, the phase factor is already present.

The controlled phase gate can also be obtained here in the same manner as given by Igor Ryabstev and coworkers for Rydberg atoms[9]. In this method, the two atoms are brought closer. The dipole-dipole interaction causes a time dependent state that periodically moves from |01⟩ and |10⟩.

$$|ψ_{dd}(t)⟩ = \cos(V_{dd}(t)/\hbar)|10⟩ - i \sin(V_{dd}(t)/\hbar)|01⟩.$$ 

The state returns back to original state, but with an additional $π$ phase after every period of $T = (\hbarπ)/V_{dd}$. The phase gate therefore involves bringing the two atoms closer and hold it for a period $T = (\hbarπ)/V_{dd}$ and taking it apart, by which time, only the state |01⟩ would have acquired a phase.
Although both above methods are equally useful, they both suffer from a serious drawback. The phase shift happens with equal certainty whether the atoms are in state $|01\rangle$ or $|10\rangle$. In other words, the above protocols do not distinguish between control and logic Qubit atoms. One suggested workaround for this is to use two different atoms, both of which exhibit similar configurations.

C. heterogeneous atoms

Another atom that shows similar Zeeman sublevel structure as $^{87}$Rb is Sodium, with $3S_{1/2}, F = 1$ and $3P_{1/2}, F = 1$ triplets. That ensures that all the state preparation and Qubit operations described above are valid for Sodium atoms as well. The behaviour of the Sodium atoms under above operations will be identical to that described above for $^{87}$Rb, except for a different Larmor frequency.

The two-Qubit operation can now be adopted using $^{87}$Rb and Sodium atoms together, with one of them as the logic Qubit and the other as the control Qubit. For the present, let Sodium be the control atom and $^{87}$Rb the logic atom, though the other way around is equally appropriate. Since their gyromagnetic ratios are different, the Larmor precession frequencies are also different and therefore, the spin flip resonances are $\omega_1$ and $\omega_2$ respectively as shown in figure 3a. The dipole-dipole interaction between one Sodium and one $^{87}$Rb atom will lead to energy shifts, but not a mixing, with the energy shifts being

$$\Omega = \left(\frac{\mu_0}{4\pi}\right)(\gamma_1 \gamma_2 / r^3)(3\cos^2 \theta_s - 1),$$

where $\gamma_1$ and $\gamma_2$ are their respective gyromagnetic ratios. The shifted energy schematic is shown in figure 3b, where the gray lines show the original position of the energy level.

Since the transitions $|01\rangle \leftrightarrow |11\rangle$ and $|10\rangle \leftrightarrow |11\rangle$ transitions are shifted by different energies, it is easy to distinguish them and operate that $2\pi$ pulse which brings about the desired transition. In other words, if the first atom is the control and the second atom is the logic, then $|10\rangle \rightarrow |11\rangle \rightarrow |10\rangle$ operation can be obtained by using a pulse of the frequency $\omega_1 + \Omega$ or $|01\rangle \rightarrow |11\rangle \rightarrow |01\rangle$ transition can be obtained by using a pulse of the frequency $\omega_2 + \Omega$. Or a combination of the two pulses in a sequence can bring about a Raman transition $|10\rangle \rightarrow |11\rangle \rightarrow |01\rangle$ or vice versa, which is a controlled - swap operation.

III. PRACTICAL CONSIDERATIONS

A few practical considerations requires to be carefully taken into account to use this scheme effectively. Firstly, the state preparation is based on optical pumping and therefore some atoms may be lost to level $|5S_{1/2}, F = 2\rangle$ during this process due to
spontaneous emission. A repumping laser which pumps these atoms back into the main cycle will be needed for this. However, the bit flip operation involves an adiabatic variation of the state of the system from $|\psi_0\rangle$ to $|\psi_1\rangle$ without involving any of the excited states $|e_0,\pm\rangle$. Therefore, this will not require a repump laser.

Secondly, the magnetic spin dipole interactions that are required here for two Qubit operations are much weaker than the electric dipole interactions exploited in reference [11] or [9]. Therefore the shift in energies will also be smaller. But the gyromagnetic ratios and hence the Larmor frequencies are typically a few hundred kHz per Gauss for alkali atoms. The shift due to dipole-dipole interaction will also be in this order of magnitude and precise measurement of such quantities are feasible. Further the shift is inversely proportional to the cube of the distance between the two atoms and hence can be increased by moving them closer.

Thirdly, the method involves optical lattices which can be moved independently so that the atoms are brought closer and taken apart. Although this has been achieved in several laboratories, it is still bogged by technical difficulties. This is probably the biggest disadvantage of this scheme, especially when two different species of atoms such as Sodium and Rubidium have to be independently loaded onto two different lattices and then moved.

IV. CONCLUSION

It is shown that a $^{87}$Rb atom interacting with two lasers that couple its F=1 $\leftrightarrow$ F=1 transition exhibits two trap states. These two trap states can be mapped onto two Qubits $|1\rangle$ and $|0\rangle$ and can be used for Quantum Computation. Both Qubit states are trap states and are made up of only ground states. A three component trap state, which is a coherent superposition of these two trap state is also feasible. This means that the Qubit state preparation is reliable, robust and free of decoherence due to spontaneous emissions. It is also shown that any desired Bloch vector can be prepared out of Qubit states and this is also a robust trap state. Two-Qubit operations such as controlled phase and controlled swap are shown feasible on this system. In short, a robust, reliable and easily available configuration is suggested for use in Quantum Computation. The next question that needs to be examined is the effect of one and two-Qubit operations on the superposition state $|3\rangle$, which will be addressed somewhere else.

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