Exploiting optical degrees of freedom for information multiplexing in diffractive neural networks

Chao Zuo and Qian Chen

Abstract
Exploiting internal degrees of freedom of light, such as polarization, provides efficient ways to scale the capacity of optical diffractive computing, which may ultimately lead to high-throughput, multifunctional all-optical diffractive processors that can execute a diverse range of tasks in parallel.

In the last decades, artificial intelligence (AI) technologies, especially artificial neural networks (ANNs), have led to a revolution in a range of applications, including autonomous driving, remote sensing, medical diagnosis, natural language processing, and the Internet of Things. However, the rapid progress of AI and the increasing scale of ANNs are actually accompanied with a tremendous amount of computational resources and energy costs. The main reason behind this is that the dominant computational algorithm for ANNs consists of a large number of matrix-vector multiplications, which are typically the most computationally-intensive operations with the computing cost scales as the square of the input dimension.

Optical neural networks (ONNs) built using optical matrix-vector multipliers are promising candidates for next-generation neuromorphic computation, because they offer a potential solution to the energy consumption problem faced by their electrical counterparts. In addition, the constituent scalar multiplication operations can be performed in parallel completely in the optical domain, at the speed of light, and with zero energy consumption in principle.

Optical computing, or more specifically ONNs, where people seek to perform neuromorphic computation with optics, is, in fact, not a new idea. In 1987, Mostafa and Psaltis, for the first time, focused on the need and practical implementation of optical neural computers. Taking inspiration from the distributed topology of the brain, they created a physical implementation of neural networks by arranging optical components in the way neurons are arranged in the human brain. Since then, research in optical neuromorphic computing has flourished, spanning decades of development efforts on various novel optical implementations of neural networks. But until recently experimental implementations of large-scale, highly parallel, high-speed, and trainable ONNs have been made with the breakthroughs in deep learning, optoelectronics, and photonic material engineering, leading to a resurgence of interest in this area.

ONNs are usually built based on an optical architecture that is mathematically described as an input-output function, i.e., a scattering matrix relating the input to the output electric field. And this naturally implements a matrix-vector multiplication, which can be realized by a diverse set of optical architectures, including integrated silicon photonic neuromorphic circuits, fiber-optic sensor arrays, and convolutional networks through diffractive optics. Introduced by Ozcan Research Group...
ONNs formed through the integration of successive spatially engineered transmissive diffractive layers, i.e., diffractive neural networks, have been demonstrated to enable both statistical inference and optical information processing, such as image classification\(^9\), single-pixel image reconstruction\(^12\), quantitative phase imaging\(^13\), and imaging through random diffusers\(^14\).

The diffractive neural network has its roots in Fourier optics, wherein a simple positive lens applies a physical two-dimensional Fourier transform to the wave field, and the prevalent wave propagation is described by Kirchhoff's diffraction integral that amounts to a convolution of the field with the impulse response of free space. These operations provide basic building blocks of convolutional neural networks (CNNs), making diffractive neural networks well-suited for most vision computing applications. By leveraging the light-matter interaction as an implementation of element-wise multiplication, the “pixels” on the diffractive surfaces embody the “neurons” on the network layers, which are interconnected by the physics of optical diffraction. As an analogy to standard neural networks, the complex-valued transmission coefficient (including amplitude and phase) of each pixel is a learnable network parameter, which is iteratively optimized based on error back-propagation algorithms, using standard deep learning tools implemented in a computer. After this training stage, the resulting transmissive layers are fabricated with 3D printing or lithography to construct a task-specific physical network that computes based on the diffraction of the light passing through these trained diffractive layers.

Though most of the current diffractive neural networks are constructed based on linear optical materials, “deep” diffractive neural networks show evident “depth” advantages: an increase in the number of diffractive layers and neurons improves its statistical inference accuracy and information processing capability\(^9,15\). More specifically, adding more trainable diffractive layers into a given network increases the dimensionality of the solution space that can be all-optically processed by the network. It has been recently demonstrated that a diffractive neural network can be trained to perform an arbitrary complex-valued linear transformation between its input and output fields with negligible error, provided that the total number of engineered pixels in the network is sufficient\(^16\). In a more general sense, a diffractive neural network can be regarded as a special, task-specific optical system, which performs specific computational tasks with the use of light information carriers. The object field can be viewed as a source of information flow characterized by various fundamental properties, which can all be ingeniously manipulated to extend the information processing capacity of diffractive networks.

In a recent issue of *Light: Science & Applications*, the UCLA group introduced polarization division multiplexing (PDM), a long-established technique of enhancing the transmission capacity in telecommunications, to all-optically perform multiple, arbitrarily-selected linear

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**Fig. 1 Information multiplexing in diffractive neural networks.** (a) Polarization-multiplexed diffractive neural networks utilizing a series of structured diffractive surfaces and a simple polarizer array. By enabling the trainable diffractive layers to communicate with the polarization elements embedded in the diffractive volume, a single network can create multiple computing channels that can be accessed using specific combinations of input and output polarization states. (b) Exploiting the internal degrees of freedom of light provide new possibilities for information multiplexing to enhance the performance and capacity of optical diffractive networks.
transformations through a single diffractive network
(Fig. 1a). Instead of using birefringent, anisotropic, or
polarization-sensitive materials for trainable diffractive
layers, their polarization-multiplexed diffractive networks
are still built based on standard (isotropic) diffractive
surfaces where the trainable coefficients are independent
of the polarization state of the input light. To gain addi-
tional sensitivity to different polarization states and pola-
ization mode diversity, a non-trainable, pre-selected linear
polarizer array (at 0°, 45°, 90°, and 135°) is inserted within
the trainable diffractive surfaces, and different target lin-
ear transformations are uniquely assigned to different
combinations of input and output polarization states.
They demonstrated that a single well-trained polariza-
tion-multiplexed diffractive network could successfully
perform multiple (2 or 4) arbitrarily-selected linear
transformations, which had not yet been implemented by
using metasurfaces or metamaterials-based designs.
Such a polarization-multiplexed diffractive computing
framework is poised to be used to build all-optical, passive
processors that can execute multiple inference and optical
information processing tasks in parallel.

Harnessing the intrinsic high-dimensionality of light
brings new insights into the diffractive neural network
design by providing additional degrees of freedom to both
optical signals and systems. The concept of degrees of
freedom was first introduced in optics by Laue in 1914 as
the decisive property in determining the information
capacity of optical signals and systems, even before Shan-
non’s information theory for communication systems was
established. According to Laue, and later Gabor, Francia,
and Lukosz, the number of degrees of freedom in optics is most often understood to be the number of
independent parameters needed to represent an optical
signal or system, which is closely related to the number of
independent communication channels available for the
information transfer in the field of electrical communica-
tion. However, unlike communication systems, an optical
system transmits many kinds of information, which can be
divided into two groups: (1) "dimensional" information which is related to spatial intervals by coordinates x, y, and
z, as well as to temporal intervals t; (2) "internal" infor-
mation which is related to physical properties of light,
including amplitude, phase, wavelength, polarization,
coherence, and angular momentum. The total number of
degrees of freedom can be expressed through the product of
freedom degree numbers related to all these different
kinds of information. Though optical systems are often
expected to transmit as much information as possible, the
number of degrees of freedom is a fundamental invariant
of an optical system, as noted by Gabor and Lukosz.
Within this limit, it is possible to increase the degrees of
freedom for one kind of information at the expense of that
of another kind.

The concept of degrees of freedom can be straightforwardly extended to the diffractive neural network, as a
special kind of optical signal processing system. For example, the information content of the input or output
signal, which is often an image formed through a pupil of
finite size, can be quantified by the definite number of
resolvable regions in which the signals can be indepen-
dent (defined as \( N_i \) and \( N_j \) for the input or output signal
in ref. 15), taking both the diffraction limit and sampling
theorem into account. Diffractive neural networks
manipulate light by reshaping the spatial profile of an
input beam into a desired output beam. If the diffractive
neural network is designed to perform arbitrary linear
transformations from the input beam to the output beam,
as demonstrated in refs. 15,16, the entire optical system
can be described by a single \( N_j \times N_i \) matrix, mapping \( N_i \)
input degrees of freedom to \( N_j \) output degrees of free-
don. In optical communications, the concept of "modes"
or "eigenfunctions", is commonly used to provide an
"economical" description of degrees of freedom of the
optical signal, reducing complicated wave functions to a
small number of mode amplitudes, as in propagating
fiber modes and ideal laser beams. In such a sense, the
linear transformation function realized by the diffraction
neural network is similar to that of an optical mode
converter. In contrast, diffractive neural networks can
be built in a "deep" manner, consisting of several dif-
fractive surfaces containing a large number of trainable
neurons. Such a multi-layer design presents additional
spatial optical degrees of freedom, significantly enhancing
the information capacities and processing capability of
the network compared with a single diffractive layer, as
demonstrated by the UCLA group. In particular, any
linear transformation matrix from \( N_i \) to spatial degrees of
freedom has \( N_j \times N_i \) free parameters. When the degrees of
freedom of the diffractive neural network, i.e., the
number of controllable parameters, is no less than \( N_j \times N_i \),
the network has in theory the capability to perform
arbitrary linear transformations between the input and
output signals perfectly. In their recent work, two
additional degrees of freedom of polarization are intro-
duced to the input signal for simultaneously carrying the
different information through the network. The two
orthogonal polarization states carried by the beam pre-
sent an attractive avenue to enhance the maximum
information capacity of the diffractive neural network by
a factor of \( N_p \) (from \( N_i \times N_j \) to \( N_p \times N_i \times N_j \)), where \( N_p \) is
the number of unique linear transformations assigned to
different input/output states of polarization combina-
tions. It should be mentioned that the use of polarization
freedom as high-dimensional information carriers has
been reported by Lohmann et al. for optical super-
resolution imaging and Chen et al. in optical data
communications.
The study of the UCLA group published in *Light Science & Application* is part of a larger movement to scale the capacity of optical diffractive computing by exploiting the *internal* degrees of freedom of light, such as polarization, spectrum, coherence, and orbital angular momentum, in addition to the *spatial* degrees of freedom (Fig. 1b). With such a multidimensional multi-link upgrade, diffractive neural networks can transmit optical signals over more independent channels, which could lead to all-optical multiplexed diffractive processors that can execute multiple tasks in parallel. Another benefit of polarization multiplexing is that the effective bandwidth can be reduced to the half of that of single-polarization transmission. That makes a high-dimensional diffractive neural network possible by using lower numerical-aperture optics, which has been proved to be extremely important for reducing the physical size of diffractive neural networks and relaxing the stringent requirements on the interlayer distances\(^1\)\(^-\)\(^6\) Finally, in most current diffractive network designs, the input field is assumed to be monochromatic, spatially coherent, and forward-propagating. A variety of computational imaging techniques that exploit partial coherence and evanescent waves for improving imaging performance (especially spatial resolution) prompted us to consider the possibility of their adaptation to diffraction neural networks. We believe that significant progress in developing high-performance optical diffractive computing schemes could be made if it became common practice to consider explicitly the internal degrees of freedom of light as the physical source of information gain.

**Author details**

1. Zuo and Chen *Light: Science & Applications* (2022)11:208

**Conflict of interest**

The authors declare no competing interests.

Published online: 06 July 2022

**References**

1. Sze, V. et al. Hardware for machine learning: challenges and opportunities. In: 2017 IEEE Custom Integrated Circuits Conference (CICC) 1–8 (IEEE, 2017). https://doi.org/10.1109/CICC.2017.7993626.

2. LeCun, Y., Bengio, Y. & Hinton, G. Deep learning. *Nature* 521, 436–444 (2015).

3. Wetzstein, G. et al. Inference in artificial intelligence with deep optics and photonics. *Nature* 588, 39–47 (2020).

4. Brunner, D. et al. Parallel photonic information processing at gigabyte per second data rates using transient states. *Nat. Commun.* 4, 1364 (2013).

5. Abu-Mostafa, Y. & Psaltis, D. Optical neural computers. *Sci. Am.* 256, 88–95 (1987).

6. Denz, C. Optical Neural Networks. (Springer Science & Business Media, Wiesbaden, 2013).

7. Shen, Y. C. et al. Deep learning with coherent nanophotonic circuits. *Nat. Photonics* 11, 441–446 (2017).

8. Tejin, U. et al. Scalable optical learning operator. *Nat. Comput.* 5, 542–549 (2021).

9. Lin, X. et al. All-optical machine learning using diffractive deep neural networks. *Science* 361, 1004–1008 (2018).

10. Chang, J. L. et al. Hybrid optical-electronic convolutional neural networks with optimized diffractive optics for image classification. *Sci. Rep.* 8, 12324 (2018).

11. Liu, C. et al. A programmable diffractive deep neural network based on a digital-coding metasurface array. *Nat. Electron.* 5, 113–122 (2022).

12. Li, J. X. et al. Spectrally encoded single-pixel machine vision using diffractive networks. *Sci. Adv.* 7, eabe7690 (2021).

13. Meng, D. & Ozcan, A. All-optical phase recovery: diffractive computing for quantitative phase imaging. *Adv. Opt. Mater.* https://doi.org/10.1002/adom.202002081 (2022).

14. Luo, Y. et al. Computational imaging without a computer: seeing through random diffusers at the speed of light. *eLight* 2, 4 (2022).

15. Kulce, O. et al. All-optical information-processing capacity of diffractive surfaces. *Light.: Sci. Appl.* 10, 25 (2021).

16. Kulce, O. et al. All-optical synthesis of an arbitrary linear transformation using diffractive surfaces. *Light.: Sci. Appl.* 10, 196 (2021).

17. Li, J. X. et al. Polarization multiplexed diffractive computing: all-optical implementation of a group of linear transformations through a polarization-encoded diffractive network. *Light.: Sci. Appl.* 11, 153 (2022).

18. Lauve, M. V. Die Freiheitsgrade von strahlenbündeln. *Ann. Phys.* 349, 1197–1212 (1914).

19. Shannon, C. E. A mathematical theory of communication. *Bell Syst. Tech. J.* 27, 379–423 (1948).

20. Gabor, D. in Progress in Optics (ed Wolf, E.) 109–153 (Elsevier, 1961).

21. Di Francia, G. T. Degrees of freedom of an image. *J. Opt. Soc. Am.* 59, 799–804 (1969).

22. Lukosz, W. Optical systems with resolving powers exceeding the classical limit. *J. Opt. Soc. Am.* 56, 1463–1471 (1966).

23. Lukosz, W. Optical systems with resolving powers exceeding the classical limit. *J. Opt. Soc. Am.* 57, 952–941 (1967).

24. Miller, D. A. B. Waves, modes, communications, and optics: a tutorial. *Adv. Opt. Photonics* 11, 679–825 (2019).

25. Miller, D. A. B. All linear optical devices are mode converters. *Opt. Express* 20, 23985–23993 (2012).

26. Lohmann, A. W. & Paris, D. P. Superresolution for nonbirefringent objects. *Appl. Opt.* 3, 1037–1043 (1964).

27. Chen, Z. Y. et al. Use of polarization freedom beyond polarization-division multiplexing to support high-speed and spectral-efficient data transmission. *Light.: Sci. Appl.* 6, e16207 (2017).