Phenomenological models for unified dark matter with fast transition

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Accepted 2013 February 28. Received 2013 January 29; in original form 2012 November 29

ABSTRACT

A fast transition between a standard matter-like era and a late Λ cold dark matter (ΛCDM)-like epoch (or more in general, a CDM+DE era), generated by a single unified dark matter (UDM) component, can provide a new interesting paradigm in the context of general relativity, alternative to ΛCDM itself or other forms of dark energy (DE) or modified gravity theories invoked to explain the observed acceleration of the Universe. UDM models with a fast transition have interesting features, leading to measurable predictions, thus they should be clearly distinguishable from ΛCDM (and alternatives) through observations. Here, we look at different ways of prescribing phenomenological UDM models with fast transition, then focusing on a particularly simple model. We analyse the viability of this model by studying features of the background model and properties of the adiabatic UDM perturbations, which depend on the effective speed of sound and the functional form of the Jeans scale. As a result, theoretical constraints on the parameters of the models are found that allow for a behaviour compatible with observations.

Key words: gravitation – cosmology: theory – dark energy – dark matter.

1 INTRODUCTION

The acceleration of the expansion of the Universe is, within the homogeneous and isotropic paradigm of cosmology, a well-accepted and observationally well-supported reality. The simplest possible framework explaining this acceleration is provided by the concordance Λ cold dark matter (ΛCDM) model (Komatsu et al. 2011), where a cosmological constant Λ in Einstein equations sources the acceleration, while CDM is the main component for structure formation. Alternatives to Λ are various forms of dark energy (DE) (Amendola & Tsujikawa 2010) or a modified gravity theory (Tsujikawa 2010; Nojiri & Odintsov 2011; Clifton et al. 2012). A different approach is to consider models of unified dark matter (UDM), where a single matter component is supposed to source the acceleration and structure formation at the same time (see e.g. Kamenshchik, Moschella & Pasquier 2001; Bento, Bertolami & Sen 2002; Bubic, Tupper & Viollier 2002; Carturan & Finelli 2003; Sandvik et al. 2004; Scherrer 2004; Giannakis & Hu 2005; Bertacca, Matarrese & Pietroini 2007; Bertacca & Bartolo 2007; Balbi, Bruni & Quercellini 2007; Quercellini, Bruni & Balbi 2007; Bertacca, Bartolo & Matarrese 2008; Bertacca et al. 2008; Pietrobon et al. 2008; Bubic, Tupper & Viollier 2009; Camera et al. 2009; Li & Barrow 2009; Camera et al. 2010; Gao et al. 2010; Lim, Sawicki & Vikman 2010; Piattella et al. 2010).

In a previous paper (Piattella et al. 2010), the concept of UDM models with fast transition was introduced (see Bassett et al. 2002 for DE models with a sharp transition in their equation of state). In essence, these models are based on the idea that the Universe may have undergone a transition between a standard matter-like era, well described by an Einstein–de Sitter (EdS) model, and a ΛCDM-like epoch. If the transition is slow the differences with ΛCDM are negligible while if the transition is fast, these models show interesting features1 (Piattella et al. 2010; Bertacca et al. 2011). The transition can be quantified as fast by looking both at parameters that govern the evolution of the background as well as at quantities that dictate the dynamics of perturbations. This analysis has been carried out in Piattella et al. (2010) for a specific barotropic model and will be generalized in this paper to a new class of models, but in essence the transition needs to be fast because:

(a) we are especially interested in background models that, at least in principle, can be clearly distinguished from a standard ΛCDM;

(b) as shown in Piattella et al. (2010), otherwise the evolution of perturbations is such that observational constraints are violated, in particular causing a strong deviation from the Integrated Sachs–Wolfe (ISW) effect occurring in ΛCDM models.

1 In a ΛCDM model of course there is an early matter era when Λ is negligible, with a smooth slow transition to the epoch when Λ plays a role. We are instead interested in models where there is a longer matter era that almost suddenly ends into a ΛCDM-like late behaviour; in other words, models where the UDM component evolves for long time in a CDM-like fashion, and suddenly a Λ-like term is switched on in the dynamics.
The UDM models with fast transitions may be an interesting alternative to other explanations of the observed acceleration of the Universe, providing a good fit to standard observables such as the cosmic microwave background (CMB) and the matter distribution (Piattella et al. 2010) that require consideration of the perturbations, while at the same time showing interesting new features, leading to measurable predictions. So at least in principle, UDM models with a fast transition are able to avoid the fate of some UDM models such as the generalized Chaplygin gas, which need to become indistinguishable from ΛCDM in order to survive observational tests, which spells their end (Sandvik et al. 2004), cf. (Gorini et al. 2008; Piattella 2010). On the other hand, UDM models based on scalar fields, such as the one introduced in Bertacca et al. (2008), can be compatible with observations. See also Bertacca, Bartolo & Matarrese (2010) for a recent review on UDM models.

In contrast with more standard ΛCDM+DE models, where the CDM component is perturbed and leads to structure formation while the DE component takes care of the acceleration of the background, with small or negligible effects on perturbations, the single UDM component must accelerate the Universe and provide acceptable perturbations. In particular, while CDM density perturbations evolve in a scale-independent fashion, this is not the case for UDM. In view of testing models against observations, e.g. with Markov Chain Monte Carlo methods and likelihood analysis, these differences may become computationally expensive. In particular, modifying CAMB (Lewis, Challinor & Lasenby 2000) to treat fast transition UDM models implies to switch off CDM while introducing a rapidly varying single inhomogeneous component with scale-dependent evolution. It is then a non-trivial task to obtain a working code that it is efficient enough for likelihood analysis (Piattella et al. 2010), given that the running time of a code like CAMB (Lewis et al. 2000), when dealing with a non-standard model like a fast transition UDM, increases enormously when the accuracy is increased in order to retain convergence of the results.

In view of this, and lacking a fundamental model, it is therefore essential to consider simple phenomenological models of the fast transition paradigm for which as much theoretical progress as possible can be made from analytical calculations. This can then be used to increase the efficiency of numerical codes such as CAMB (Lewis et al. 2000) and CLASS (Lesgourgues 2011) in dealing with these models.

Our first goal here is therefore to look at simple phenomenological recipes, such that the most important variables required for the numerical problem can be expressed analytically. It then turns out that, unlike the fast transition UDM model introduced in Piattella et al. (2010), where the prescription for the fast transition is introduced in the equation of state, the best recipe to proceed analytically is to prescribe the evolution of the energy density ρ and pressure p, with uμ its four-velocity: T_{μν} = (ρ + p)u_μ u_ν + pg_{μν}. Starting from these assumptions, and choosing units such that 8πG = c = 1 and signature \{-, +, +, +\}, Einstein equations imply the Friedmann and Raychaudhuri equations:

\[
H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3},
\]

\[
\frac{\dot{a}}{a} = -\frac{1}{6} (\rho + 3p),
\]

where H = \dot{a}/a is the Hubble expansion scalar and the dot denotes derivative with respect to the cosmic time. Assuming that the energy density of the radiation is negligible at the times of interest, and disregarding also the small baryonic component, ρ and p represent the energy density and the pressure of the UDM component.

Independently from Einstein equations, projecting the conservation equations \(T^{μν}_{;;ν} = 0\) along \(u^μ\), one obtains the energy conservation equation

\[
\dot{ρ} = -3H(ρ + p) = -3Hρ(1 + w),
\]

where \(w = p/ρ\) represents the equation of state (hereafter EoS) that is needed to close the system and is the quantity that characterizes the background of our UDM model.

When needed, we shall introduce different components, each with energy density \(ρ_i\). From this, we can define the dimensionless function

\[
Ω_ρ(a) = \frac{ρ(a)}{ρ_c(a)},
\]

where \(ρ_c = 3H^2\) is the critical density; values today will be denoted by the parameter Ω_ρ0.

2.2 Perturbations

Assuming a perfect fluid, perturbations of the FLRW metric in the longitudinal gauge read

\[
d\tau^2 = -a^2(η) \left[ (1 + 2Φ) dη^2 - (1 - 2Φ) δ_{ij} dx^i dx^j, x^i \right],
\]

where Φ represents the analogues of the Newtonian gravitational potential and we now are using conformal time \(η\). Defining,

\[
u = \frac{2Φ}{\sqrt{ρ + p}},
\]

and linearizing the 0–0 and 0–i components of Einstein equations, one obtains the following second-order differential equation for the Fourier component of \(u\) (Mukhanov, Feldman & Brandenberger...
1992; Giannakis & Hu 2005; Bertacca & Bartolo 2007; Piattella et al. 2010):
\[
\frac{d^2u}{d\eta^2} + k^2 c_s^2 u - \frac{1}{a} \frac{d^2}{da^2} u = 0 ,
\]
(7)
where
\[
\theta = \sqrt{\frac{\rho}{3(\rho + p)} (1 + z)} ,
\]
(8)
with \(z\) the redshift, \(1 + z = a^{-1}\). The quantity \(c_s^2\) in (7) is the effective speed of sound and characterizes the perturbative dynamics of our UDM model, being also crucially involved in the growth of the overdensities \(\delta \rho\). Assuming adiabatic perturbations this is the same as the adiabatic speed of sound:
\[
c_s^2 = c_s^2 = \frac{dp}{d\rho} = \frac{d\rho}{a \, d\eta} .
\]
(9)

Starting from equation (7), let us define the squared Jeans wavenumber:
\[
k_J^2 = \left| \frac{1}{c_s^2 \frac{d\theta}{d\eta}} \right| ;
\]
(10)
its reciprocal defines the squared Jeans length: \(k_J^2 = a^2/k_J^2\).

An important aspect of UDM models is the possible manifestation of an effective sound speed significantly different from zero at late times: this generally corresponds to the appearance of a Jeans length (or sound horizon) below which the dark fluid does not cluster (e.g. see Hu 1998; Pietrobon et al. 2008; Piattella et al. 2010). This causes a strong evolution in time of the gravitational potential, which at small scales starts to oscillate and decay, with effects on structure formation. In general, UDM models may also exhibit a strong ISW effect (Bertacca & Bartolo 2007).

Thus, the squared Jeans wavenumber plays a crucial role in determining the viability of a UDM model, because of its effect on perturbations, which is then revealed in observables such as the CMB and matter power spectrum (Pietrobon et al. 2008; Piattella et al. 2010). As discussed in Piattella et al. (2010) there are two different regimes of evolution, respectively for scales much smaller and much larger than the Jeans length. In practice, any viable UDM model should satisfy the condition \(k_J^2 \gg k^2\) for all the scales of cosmological interest, in turn giving an evolution for the gravitational potential \(\Phi\) in Fourier’s space of the following type (we are dealing with the gravitational potential after recombination so there is no more speed of sound due to radiation):
\[
\Phi(\eta, k) \simeq A_k \left[ 1 - \frac{H(\eta)}{a(\eta)} \int a(\eta)^2 \, d\eta \right] .
\]
(11)
The integration constant \(A_k = \Phi(0, k) T_m(k)\) is fixed during inflation by the primordial potential \(\Phi(0, k)\) at large scales; \(T_m(k)\) is the matter transfer function, describing the evolution of perturbations through the epochs of horizon crossing and radiation–matter transition, see e.g. Dodelson (2003).

The explicit form of the Jeans wavenumber is
\[
k_J^2 = \frac{3}{2} \rho a^2 \left[ 1 + \frac{1}{2} \left( c_s^2 - w \right) - \frac{d c_s^2}{d\rho} \right]
\]
\[\quad + \frac{3(c_s^2 - w)^2 - 2(c_s^2 - w) + 1}{6(1 + w)} \right] .
\]
(12)
It is therefore clear from this expression that, if we want an analytic expression for \(k_J^2\) in order to obtain some insight on the behaviour of perturbations in a given UDM model, we need to be able to obtain analytic expressions for \(\rho, p, w\) and \(c_s^2\). Unfortunately, it is not possible to find such expressions as functions of \(\eta\) (or \(t\)), simply because this requires the knowledge of an analytic expression for the scale factor as function of time, i.e. to solve the Friedmann equation (1), which in general is only possible for very special cases, as is well known.

### 3 THREE POSSIBLE PRESCRIPTIONS

A way out of this problem is to disentangle the evolution of the quantities of interest \((\rho, p, w\) and \(c_s^2\) from Einstein equations, noticing that we can obtain these quantities as functions of the scale factor \(a\) if we use only the conservation equation (3). This has also the advantage that the expressions so obtained will be the same in any theory of gravity that satisfies the conservation equations. Equation (3) becomes, using \(a\) as time variable:
\[
\rho' = -\frac{3}{a} (\rho + p) ,
\]
(13)
where a prime derivative with respect to \(a\).

We now briefly indicate three different possible ways to prescribe the dynamics of the UDM component and to derive analytic expressions for the needed variables.

#### 3.1 Starting from \(w(a)\)

Suppose that \(p/\rho = w\) is pre-assigned as a function of the scale factor: \(w = w(a)\). For instance, \(w(a) = w_0 + w_1(1 - a)\) is a typical phenomenological assumption well motivated when setting observational constraints on dark energy models (Chevallier & Polarski 2001; Linder 2003). In principle, this is a convenient practical prescription to model UDM, because we have a good idea about what type of \(w(a)\) we should have. Then the adiabatic speed of sound
\[
c_s^2 = \frac{dp}{d\rho} = \frac{d\rho}{a \, dw} = \frac{p'}{\rho'} .
\]
(14)
can be computed from the conservation equation (13). Indeed, from \(p = \rho w\) we can compute \(p'\), then substituting the latter and \(\rho'\) from (13) to obtain
\[
c_s^2 = w - \frac{aw'}{3(1 + w)} .
\]
(15)
This prescription however doesn’t lead to analytic expressions for \(\rho(a)\) and \(p(a)\) in general, unless \(\int \frac{1 + w(a)}{a} \, da\) is integrable.

#### 3.2 Starting from \(p(a)\)

Prescribing the pressure as a function of the scale factor, \(p = p(a)\), can be useful, e.g. if one is dealing with a scalar field, in which case this is equivalent to prescribing a Lagrangian, see e.g. Bertacca, Matarrese & Pietroni 2007; Quercellini, Bruni & Balbi 2007; Bertacca, Bartolo & Matarrese 2010 and references therein. Again, we have a good idea about the functional form that \(p(a)\) should have, so that this also seems a good starting point. Let us rewrite the energy conservation (13) as
\[
\rho' + \frac{3}{a} \rho = -\frac{3}{a} p(a) .
\]
(16)
The homogeneous solution is \(\rho_m \propto a^{-3}\), i.e. standard matter (dust), and for a given \(p(a)\) an analytic expression for \(\rho(a)\) can be found if
$E = 3 \int a^2 p(a) \, da = \int p \, dV$ is integrable, giving $\rho = E/V$. With this prescription $c_s^2$ is immediately found, given $p(a)$ and (16), but an analytic expression for $w(a)$ can only be found if that for $\rho(a)$ is found.

### 3.3 Starting from $\rho(a)$

It is perhaps less obvious what the functional form for $\rho(a)$ should be, but some guess can be made in view of constructing UDM models with fast transition. In this case, we want to recover a CDM-like behaviour, i.e. an EdS model, at early times (before the transition), i.e. $\rho \simeq \rho_m = \rho_{3d} a^{-3}$, and a CDM+DE behaviour after the transition. If we want to recover the simplest case of a $\Lambda$CDM at late times, we would have $\rho \simeq \rho_\Lambda + \rho_{k,0} a^{-3}$.

Assuming that $\rho(a)$ is given, we can compute $\rho'(a)$ and then use (16) to obtain $p(a)$ and then $\rho(p)$. In this case, therefore any choice of $\rho(a)$ guarantees analytic expressions for the EoS $w(a)$ and the adiabatic speed of sound $c_s^2(a)$.

To summarize, given a function (at least of class $C^3$) $\rho = \rho(a)$ for the energy density, we have the following expressions for the quantities that enter into the Jeans wavenumber (12):

\[
w = \frac{a \rho'}{3 \rho} - 1, \quad (17)
\]

\[
c_s^2 = -\frac{a \rho''}{3 \rho'} + \frac{4}{3}, \quad (18)
\]

\[
\frac{dc_s^2}{d\rho} = -\frac{1}{3 \rho'^2} \left[ a \rho'' + \rho'' - d \rho'^2 \right]. \quad (19)
\]

Substituting in equation (12) from (17) to (19), we can obtain an analytic expression for the function $k^2_{\text{J}}(a)$. Armed with this, we can obtain some insight about the behaviour of adiabatic perturbations in a model with a specified energy density $\rho = \rho(a)$.

### 4 PHENOMENOLOGICAL UDM MODELS WITH FAST TRANSITION

#### 4.1 An overidealized model

Before looking at a possible model for a UDM with fast transition, we now introduce an overidealized model, using a Heaviside function to describe an instantaneous transition. This model cannot serve our purposes, because it is clear from the expressions above that we need the function $\rho(a)$ to be at least of class $C^3$, but it is useful to get an idea of what we want to obtain. We assume that the Universe is well described by an EdS model before the transition, while for generality we describe the post-transition era with an ‘affine’ model (Ananda & Bruni 2006a,b; Balbi et al. 2007; Quercellini et al. 2007; Pietrobon et al. 2008):

\[
\rho = \begin{cases} 
\rho_i \left( \frac{a}{a_i} \right)^3 & \text{if } a < a_i \\
\rho_\Lambda + (\rho_i - \rho_\Lambda) \left( \frac{a}{a_i} \right)^{3(1+\alpha)} & \text{if } a > a_i 
\end{cases} \quad (20)
\]

Of course, any other post-transition model could be chosen. Here, $\rho_i$ is the energy scale at the transition and $\rho_\Lambda$ is the effective cosmological constant: $1 + \alpha > 0$ and the energy density at late times tends to $\rho_\Lambda$, so that the late time evolution in these models is ala de Sitter, even if there is no cosmological constant in the Friedmann and Raychaudhuri equations, see (Ananda & Bruni 2006a,b; Balbi et al. 2007; Pietrobon et al. 2008). On the other hand, $\alpha_i$ is not an independent parameter: using the Friedmann equation (1) and the second of equation (20), and neglecting radiation, we obtain

\[
\alpha_i = \left[ \frac{1 - \Omega_{\Lambda,0}}{\Omega_{\Lambda,0} \left( \frac{\Omega_{\Lambda,0}}{\Omega_{\Lambda,0}} - 1 \right) ^{1/3}} \right]^{1/7}, \quad (21)
\]

where $\Omega_{\Lambda,0} / \Omega_{\Lambda,0} = \rho_i / \rho_\Lambda$; this gives a redshift for the transition $z_i = a_i^{-1} - 1$. We can then interpret our UDM model after the transition as made up of the effective cosmological constant $\rho_\Lambda$ and an evolving part, with energy density at the transition $\rho_m \equiv \rho_i - \rho_\Lambda$, that decreases after the transition.

In the affine model, the energy density of UDM is given by the second of equation (20). If one assumes this model all the way deep into the radiation era, the evolving part must have $w_m \to 0$ and $c_s^2 \to 0$ in the past in order to recover standard matter domination at early times, thus the evolving part can be interpreted as the ‘dark matter’ component today. Indeed, the value of the parameter $\alpha$ is extremely constrained if one assumes an affine model with adiabatic perturbations all the way back to recombination and beyond, $\alpha \approx 10^{-3}$ (Pietrobon et al. 2008) (cf. also Muller 2005), making this model indistinguishable from $\Lambda$CDM.\footnote{\textsuperscript{3}} However, such a strong bound mainly comes from the matter power spectrum at small scales, $k \gtrsim k_i$, where the growth of perturbations is affected by a non-vanishing $k_i$. Since this is an integrated effect, it is reasonable to expect that, if the affine-like evolution only starts below a transition redshift $z_i$, the bound will be much weaker.

It is useful to explicitly incorporate a Heaviside function $H(a - a_i)$ in equation (20), so that

\[
\rho = \rho_i \left( \frac{a}{a_i} \right)^3 + \left[ \rho_\Lambda + (\rho_i - \rho_\Lambda) \left( \frac{a}{a_i} \right)^{3(1+\alpha)} - \rho_i \left( \frac{a}{a_i} \right)^3 \right] \times H(a - a_i). \quad (22)
\]

For $\alpha = 0$ this reduces to

\[
\rho = \rho_i \left( \frac{a}{a_i} \right)^3 + \rho_\Lambda \left[ 1 - \left( \frac{a}{a_i} \right)^3 \right] H(a - a_i), \quad (23)
\]

representing a sudden transition to $\Lambda$CDM. In the following, we shall restrict our attention to this sub-class of models.

It is now clear that, simply replacing $H(a - a_i)$ with a smoother transition function $H_i(a - a_i)$, we can obtain simple UDM models with a fast transition.

#### 4.2 A simple model for the background

In this paper, among the many known continuous approximations to the Heaviside function (Bracewell 2000), we shall consider the only one that we found compatible with having $c_s^2 > 0$:

\[
H_i(a - a_i) = \frac{1}{2} + \frac{1}{\pi} \arctan(\beta(a - a_i)), \quad (24)
\]

\footnote{\textsuperscript{3}} In this two-component interpretation, the two energy densities satisfy their own conservation equations and therefore there is no coupling between them.\footnote{\textsuperscript{4}} The $\Lambda$CDM can be obtained as a sub-case of the UDM affine model for $\alpha = 0$ (Ananda & Bruni 2006a,b; Balbi et al. 2007); the constraint is weaker if the perturbations are not adiabatic, see Pietrobon et al. (2008).}
where the parameter $\beta$ represents the rapidity of the transition. In addition to $a_i$ and $\beta$, there is a third parameter in the model, which is $\rho_\Lambda$, or, equivalently, the corresponding density parameter $\Omega_{\Lambda,0}$. From inserting equation (24) in equation (23), we see that asymptotically in time, in the limit $a \to \infty$, $\rho \to \rho_\Lambda$, which implies $p \to -\rho_\Lambda$. As already mentioned above, $\rho_\Lambda$ plays the role of an effective cosmological constant, i.e. it is an attractor for equation (13).

The Universe necessarily evolves towards an asymptotic de Sitter phase, i.e. a sort of cosmic no-hair theorem holds (see Bruni, Mena & Tavakol 2002; Bruni, Matarrese & Pantano 1995 and references therein and Ananda & Bruni 2006a,b; Balbi et al. 2007). In Fig. 1, we show a parametric plot of $p$ versus $\rho$, normalized to $\rho_\Lambda$, where we have assumed, as an example, that the transition takes place at $z_1 = 5.34$, for a representative choice of values of $\beta$. It can be seen that all models gradually approach the effective cosmological constant $\rho_\Lambda$.

In order to make our UDM model close to $\Lambda$CDM at late times, the transition must occur at relatively high redshifts, such that $\rho_i$ is quite larger than $\rho_\Lambda$, which corresponds to a minimum value of the redshift $z_i$. Otherwise, it could be difficult to have a good fit of supernovae and ISW effect data (Piattella et al. 2010). For instance, we need $z_i \gtrsim 2.65$ if we want to have $\rho_i \gtrsim 20\rho_\Lambda$. In Fig. 2, the evolution of the EoS $w$ is depicted as a function of the scale factor for different values of $\rho_i/\rho_\Lambda$ and $\beta$. As shown, models with a smaller rapidity $\beta$ have a background evolution more similar to that of the $\Lambda$CDM model at all times. On the other hand, a larger $\beta$ implies a sharper transition between the CDM-like phase and the $\Lambda$CDM phase. Likewise, it clearly illustrates that the transition has to take place far enough in the past, i.e. $\rho_i$ is larger than $\rho_\Lambda$, in order for the late time evolution of $w$ to be close to that of the $\Lambda$CDM model.

Finally, let us consider the evolution of the three components $\rho_i$ (radiation) $\rho_m$ and $\rho_\Lambda$, represented by the dimensionless functions $\Omega_i(a)$, equation (4). The evolution of these functions is shown in Fig. 3 for UDM models with fast transition ($\beta = 500$) occurring at different times, contrasted with the pure CDM (EdS) and $\Lambda$CDM models. For UDM and $\Lambda$CDM, it is assumed that today $\Omega_{\Lambda,0} = 0.72$. In a flat Universe, $\Sigma_0 = 1$, and when a component $j$ dominates $\Omega_j \approx 1$, with the other $\Omega_i \approx 0$. It can be seen from these figures that if the transition is too late (today in the extreme case $z_i = 0$) then the matter–radiation equality of the UDM model is basically the same as in a pure CDM model, i.e. much earlier than in $\Lambda$CDM. In addition, the effective cosmological constant of the UDM becomes dominant at a later time than in $\Lambda$CDM. Since the matter–transition equality dictates when matter perturbations inside the horizon start to grow, while the late dominance of the cosmological constant slows down this growth, the matter power spectrum in fast transition UDM models with too late a transition will be at odds with the observed one, with too much power on small scales.

Conversely, we see from Fig. 3 that if the transition is at $z_i \sim 1$ the matter–radiation equality is closer to that of $\Lambda$CDM, and it essentially coincides with the latter for an even earlier transition, $z_i \sim 2$. Basically, as long as the transition is at a redshift $z_i$ higher than the one at which $\Omega_\Lambda = \Omega_m$ in $\Lambda$CDM, the late time evolution of $\Omega_i$ and $\Omega_m$ in the UDM models is the same as in $\Lambda$CDM.

### 4.3 Angular diameter distance

The angular diameter distance is an important quantity that comes into play in current observations of weak lensing, Baryon Acoustic Oscillations (BAO) and galaxy clustering and will become even more important for comparing models against the new data that will become available from surveys such as Dark Energy Survey (DES), Planck and Euclid. It is also relevant to the measurements of CMB anisotropies. In view of this, we now briefly comment on the deviation of the angular diameter distance in our UDM model from that of...
Figure 3. Density dependence of the various components for UDM models with a fast transition ($\beta = 500$) occurring at different times, contrasted with the pure CDM (EdS) and $\Lambda$CDM models, assuming $\Omega_{\Lambda,0} = 0.72$ today. Top panel: $z_t = 0$; medium panel: $z_t = 1$; bottom panel: $z_t = 2$.

$\Lambda$CDM. The angular diameter distance, $d_A$, is defined as the ratio of the actual size $\Delta x$ of an object and the angle $\Delta \theta$ this object subtends orthogonal to the line of sight and can also be expressed as:

$$d_A = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')} ,$$

where $H(z)$ is the Hubble function. In Fig. 4, we have shown the angular diameter distance normalized with the Hubble distance,

$$\frac{d_A}{d_H}$$

Figure 4. The dimensionless angular diameter distance $d_A/d_H$ for the $\Lambda$CDM and the UDM model with a fast transition at $\beta = 500$ for different transition redshifts.

Figure 5. Difference between $d_A/d_H$ for the $\Lambda$CDM and for the UDM model with a fast transition at $\beta = 500$.

d_H \equiv H_0^{-1}$, for the $\Lambda$CDM model and our UDM model with a fast transition with $\beta = 500$. As it can be seen, the larger the transition redshift, the smaller the departure of $d_A$ for the UDM with respect to the $\Lambda$CDM is. Notice that $d_A$ does not increase indefinitely as $z \to \infty$; it turns over at $z \sim 1.5$ and thereafter more distant objects actually appear larger in angular size. See also Fig. 5.

5 THE JEANS SCALE AND THE GRAVITATIONAL POTENTIAL

5.1 The Jeans wavenumber

We now focus on the Jeans wavenumber for our UDM model and investigate its behaviour as a function of the speed of sound, in particular around $\rho = \rho_c$, which corresponds to the middle of the transition where the speed of sound is at its peak.

By inspection of equation (12), we see that a large $k_J^2$ can be obtained not only when $c_s^2 \to 0$, but when $c_s^2$ changes rapidly as well. In other words, when equation (12) is dominated by the $\rho \, dc_s^2/d\rho$ term we may say that the EoS is characterized by a fast transition. These two quantities, $c_s^2$ and $\rho \, dc_s^2/d\rho$, are depicted in Fig. 6 as functions of $\rho/\rho_\Lambda$. Thus, viable adiabatic UDM models can be constructed which do not require $c_s^2 \ll 1$ at all times if the speed of sound goes through a rapid change, a fast transition period during which $k_J^2$ can remain large, in the sense that $k_J^2 \ll k_1^2$ for all scales of cosmological interest to which the linear perturbation theory of equation (7) applies.
When we consider a fast transition, it is interesting to compare the term $\rho \frac{dc_s^2}{d\rho}$ with the remaining ones contained in the squared brackets of equation (12) for the Jeans wavenumber, that is:

$$B = \frac{1}{2}(c_s^2 - w) + \frac{3(c_s^2 - w)^2 - 2(c_s^2 - w)}{6(1 + w)} + \frac{1}{3}.$$  \hspace{1cm} (26)

In Fig. 7, we plot $\frac{dc_s^2}{d\rho}$, $B$ and the Jeans wavenumber $k_j$ as functions of $\rho/\rho_\Lambda$. From Fig. 7, we learn that $\rho \frac{dc_s^2}{d\rho}$ is negative for $\rho > \rho_\Lambda$, then for $\rho \approx \rho_\Lambda$, it increases becoming positive and intersecting the $B$ curve first time for $\rho = \rho_\Lambda$. For smaller values of the energy density, $\rho \frac{dc_s^2}{d\rho}$ decreases again to zero, again intersecting the $B$ curve. Since the difference between the two curves is very large, we have depicted them in the right top panel of Fig. 7 choosing a logarithmic scale. For this reason, the negative part of $\rho \frac{dc_s^2}{d\rho}$ has been omitted. It can be also seen that the relative maximum of the Jeans wavenumber between the two zeros of the curve approximately corresponds to the point where $\frac{dc_s^2}{d\rho}$ reaches its maximum value.

The place where the curves $\rho \frac{dc_s^2}{d\rho}$ and $B$ intersect corresponds to the vanishing points of the Jeans wavenumber $k_j$, as it can be seen in the bottom panel of Fig. 7. In general, around these points the corresponding Jeans length becomes very large, possibly causing all sort of problems to perturbations, with effects on structure formation in the UDM model. Defining $c_s^2$ as in equation (18), in Fig. 8 we plot the Jeans wavenumber as a function of the redshift for various values of $\beta$ and $\rho/\rho_\Lambda$. In this figure, for sufficiently high $\beta$ we note that (i) in general the Jeans wavenumber becomes larger, with a vanishingly small Jeans length before and after the transition, and (ii) it becomes vanishingly small for extremely short times, so that the effects caused by its vanishing are sufficiently negligible, as we are going to show in the next subsection when we analyse the gravitational potential $\Phi$.

To conclude, we shall make some comments on building phenomenological UDM (or DE) fluid models intended to represent the homogeneous FLRW background and its linear perturbations. A fast transition in a fluid model could be characterized by a value of $c_s^2 > 1$ during the transition. Notice that it is standard to refer to the parameter $c_s^2$ as the speed of sound because this is what it would be if equation (7) was a simple wave equation. In reality, $c_s^2$ is not the speed of signal propagation because in equation (7) we also have a potential term $\theta''/\theta$ and, if there is any signal propagation, this would only happen on scales smaller than the Jeans length, and with a speed given by the group velocity (Brillouin 1960). Therefore, having $c_s^2 > 1$ does not raise per se any issue with respect to causality, see Brillouin (1960), Babichev, Mukhanov & Vikman (2008). More specifically on our model, equation (7) is the Fourier component of a wave equation with potential $\theta''/\theta$, and the latter does not allow propagation for $k \ll k_\Lambda$. Therefore, we note in addition that we can always build our fluid model in such a way that all scales smaller than the Jeans length $\lambda \ll \lambda_\Lambda$ correspond to those in the non-linear regime, for which this model may not apply. In order to study the behaviour of the perturbations of a UDM model at these scales, we would have to go beyond the perturbative regime investigated here. That would possibly imply to build a more refined fluid model that could maintain causality and at the same time be able to deal with the increased complexity of small scale non-linear physics.

### 5.2 The gravitational potential

The differential equation that governs the behaviour of the gravitational potential $\Phi$ in our model in terms of the scale factor $a$ is

\begin{equation}
\frac{\ddot{\Phi}}{a^2} + \frac{2}{a} \dot{\Phi} = \rho \left( \frac{dc_s^2}{d\rho} - B \right).
\end{equation}
obtained from equation (7):

$$\frac{d^2 \Phi(k, a)}{da^2} + \left( \frac{d}{\mathcal{H}} \frac{da}{a} + \frac{4}{a} + 3 c_s^2 \right) \frac{d \Phi(k, a)}{da} + \left[ \frac{2}{a^2 \mathcal{H}} \frac{da}{a} + \frac{1}{a^2} (1 + 3 c_s^2) + \frac{c_s^2 k^2}{a^2 H^2} \right] \Phi(k, a) = 0,$$

(27)

where $\mathcal{H} = \frac{da}{dt}/a$ is the conformal time Hubble function. Also, $\mathcal{H} = a H$, $c_s^2$ is given in equation (18) and we have assumed plane-wave perturbations $\Phi(x, a) \propto \Phi(k, a) \exp(ik \cdot x)$ of comoving wavenumber $k \equiv |k|$. On the other hand, the $\Lambda$CDM gravitational potential, $\Phi_\Lambda$, solves equation (27) for $c_s = 0$:

$$\frac{d^2 \Phi_\Lambda}{da^2} + \left( \frac{d}{\mathcal{H}} \frac{da}{a} + \frac{4}{a} \right) \frac{d \Phi_\Lambda}{da} + \left[ \frac{2}{a^2 \mathcal{H}} \frac{da}{a} + \frac{1}{a^2} \right] \Phi_\Lambda = 0,$$

(28)

where

$$\mathcal{H}^2 = H_0^2 \left( \Omega_{\Lambda, 0}a^2 + \Omega_{m, 0}a^{-3} \right),$$

(29)

if we assume that the energy density of radiation is negligible and ignore the contribution of the baryon matter.

Comparing equations (27) and (28) we see that the gravitational potential in our UDM model has the same evolution as in a $\Lambda$CDM universe in the limit when $c_s \to 0$. In particular, both models behave as an EdS model at early times, so that for UDM $c_s \to 0$ and in both models $\mathcal{H}^2 \sim a^{-1}$, and $\Phi$ is constant in this regime, as in an EdS universe, as is well known.

The normalized initial conditions are $\Phi_\Lambda(k; a_{\text{rec}}) = 1$ and $d \Phi_\Lambda / da|_{a_{\text{rec}}} = 0$, where $a_{\text{rec}}$ stands for the scale factor at recombination time. Since the class of UDM models we consider here is constructed to behave as the $\Lambda$CDM model in the early Universe, we thus set the same initial conditions for both the UDM and the $\Lambda$CDM gravitational potentials.

In $\Lambda$CDM, the background evolution causes a gradual time evolution of the gravitational potential when the cosmological constant starts to dominate (Hu 1995); this causes an ISW effect. On the other hand, in our UDM model the evolution of the gravitational potential is determined by the background and the perturbative evolution of the single dark fluid and, crucially, by the adiabatic speed of sound $c_s^2$. The gravitational potential stays constant before the transition, during which a sudden rapid evolution of $\Phi$ is induced. The subsequent evolution is in general scale dependent: for scales $k > k_1$, $\Phi$ oscillates and decays; for larger scales, $k < k_1$, the evolution of $\Phi$ becomes scale independent and is governed by the evolution of the background, mainly by $H$ and to a small extent by $c_s^2$, and $\Phi$ approaches its $\Lambda$CDM behaviour in a way that depends mainly on the rapidity $\beta$ and from the epoch of the transition, $z_t$. In general, we expect an ISW effect starting from the transition which can be very different from the $\Lambda$CDM one, cf. Piattella et al. (2010). We show the behaviour of $\Phi$ in Fig. 9, where we explore its dependence on the background parameters $\beta$ and $z_t$ (or, equivalently, $a_0$).

We already know from the evolution of $w$ in Fig. 2 that for $\beta < 200$ the transition is not that fast. This is also apparent in Fig. 9, where we have plotted the normalized gravitational potential $\Phi_\Lambda(z) = \Phi_\Lambda(k; z)/\Phi_\Lambda(0; 10^3)$ as a function of the redshift $z$ for $k = 0.2 h \text{ Mpc}^{-1}$ and different values of $\beta$ and $z_t$. For wavenumbers $k > k_{\text{cl}} \simeq 0.2 h \text{ Mpc}^{-1}$, we expect non-linear matter overdensities contributions to the evolution of the gravitational potential to become important.

As expected, for large enough values of $\beta$ the gravitational potential is practically constant in time, corresponding to a pure matter (EdS) background evolution, until $z \sim z_t$. From $z \leq z_t$ onwards, its value decreases very fast until it finally approaches the gravitational potential of the $\Lambda$CDM model. The expected strong ISW effect caused by this behaviour would be mostly due to the particular expansion history and can be very different from the $\Lambda$CDM one. As a matter of fact, these differences are smaller for $z_t > 2$ and become smaller and smaller at higher transition redshifts, until for $z_t \sim 100$ and larger the fast transition UDM models practically become indistinguishable from $\Lambda$CDM cf. Piattella et al. (2010). As we have already pointed out, our UDM model allows the value $w = -1$ for $a \to \infty$, i.e. it admits an effective cosmological constant energy density $\rho_\Lambda$ at late times. Hence, if we wanted to compare the predictions of our UDM model with observational data, we would follow the prescription given in Piattella et al. 2010, where the density contrast is $\delta \equiv \delta \rho / \rho_\Lambda$ and $\rho_\Lambda = \rho - \rho_\Lambda$, is the clustering ‘aether’ part of the UDM component (Ananda & Bruni 2006b; Linder & Scherrer 2009). In UDM models gravity is described by general relativity, but to link the density contrast with the gravitational potential at scales much smaller than the cosmological horizon we only need the Newtonian Poisson equation. For $z < z_{\text{rec}}$, where $z_{\text{rec}}$ is the recombination redshift ($z_{\text{rec}} \approx 10^3$) we then have

$$\delta(k; z) = \frac{-2k^2 \Phi(k; z)(1 + z)^2}{\rho_\Lambda}.$$

(30)

To conclude, we have argued that for an early enough fast transition with $\beta > 500$ and $z_t > 2$ our UDM model should be compatible with observations. On the other hand, a study of the matter and CMB power spectra is needed to study the viability of models with $10 \leq \beta < 500$, and those with $\beta > 500$ and $z_t < 2$. We shall undertake this work in the future.

6 CONCLUSIONS

UDM models, when compared with the standard DM $+$ DE scenario, specifically $\Lambda$CDM, are in principle interesting because the dynamics of the Universe can be described with a single component in the matter sector which triggers the accelerated expansion at late times and is also able to cluster and produce a satisfactory structure formation. The challenge for UDM models is however
to satisfy observational constraints while maintaining features that can make them distinguishable from $\Lambda$CDM, otherwise they lose interest (Sandvik et al. 2004). In this paper, we have introduced and examined a new class of UDM models with a fast transition between an early matter era and a late $\Lambda$CDM-like phase, building on previous work (Piattella et al. 2010; Bertacca et al. 2011).

First, in Section 2, we have introduced some generalities of UDM models. In Section 3, we have considered three possible prescriptions for building phenomenological UDM models, with the aim of obtaining models in which all the variables of interest can be expressed analytically, so that in principle they could be implemented into numerical codes such as CAMB (Lewis et al. 2000) and CLASS (Lesgourgues 2011) while maintaining the code efficiency. Indeed, in comparing models with observational data this is of crucial importance in view of likelihood analysis, and a major motivation for the UDM model presented here: modifying these numerical codes to deal with fast transition UDM models while maintaining their efficiency is in general a non-trivial task (Piattella et al. 2010), thus having as many variables as possible expressed analytically simplifies the task considerably. While in Piattella et al. (2010) the fast transition was introduced in the EoS, we have shown in Section 3 that the best prescription to proceed as much as possible analytically is to assume a specific evolution of the energy density of UDM.

A general feature of UDM models is in the possible difference of the expansion history with that of $\Lambda$CDM, causing, among other features, a strong ISW effect incompatible with observations. In addition, in UDM models the effective speed of sound may become significantly different from zero. This corresponds, in general, to the appearance of a Jeans length (or sound horizon) below which the dark fluid cannot cluster and which, if large enough, can cause a strong evolution in time of the gravitational potential, preventing structure formation at small scales. In building satisfactory UDM parametric models it is therefore crucial to find the region in parameter space where the Jeans length remains small enough, well beyond the linear regime that we explore here.

UDM models with a fast transition, first introduced in Piattella et al. (2010); Bertacca et al. (2011) are a viable and interesting alternative to $\Lambda$CDM because they seem to survive observational tests while maintaining interesting features.

The new general phenomenological UDM models we have introduced in Section 4 (following the prescription obtained in Section 3) are characterized by a fast transition between a standard matter era and a post-transition epoch described by an ‘affine model’ (Ananda & Bruni 2006a,b; Balbi et al. 2007; Quercellini et al. 2007; Pietrobon et al. 2008) with affine parameter $\alpha$. We have then focused on the $\alpha = 0$ case, which represents a sudden transition to a $\Lambda$CDM-like late evolution. In constructing these models in practice, we have to choose a step-like function representing the fast transition. In doing this, for physical reasons we want to maintain the condition $c_s^2 > 0$ at all times: after carrying out an extensive study over many possibilities (Bracewell 2000), we have chosen the function in equation (24) as the only one we found that complied with this condition. In Section 4, we have also compared the angular diameter distance between $\Lambda$CDM and our UDM with fast transition, finding small differences of the order of percent when the transition is fast enough.

Finally, in Section 5, in order to study the viability of our UDM model, we have carried out a study of the functional form of the Jeans scale in adiabatic UDM perturbations. In doing so, we have found analytical expressions for the quantities involved in the Jeans wavenumber and have shown that our model presents a small Jeans length even when a non-negligible sound speed is present. Subsequently, we have analysed the properties of perturbations in our model, focusing on the evolution of the effective speed of sound, the Jeans scale and the gravitational potential. In general, in building a phenomenological model, we have chosen its parameter values in order to always satisfy the condition $k < \kappa$ for all $k$ of cosmological interests to which linear theory applies. In this way, we have been able to set theoretical constraints on the parameters of the model, predicting sufficient conditions for the model to be viable. We have argued that for large enough values of the rapidity $\beta$ of the transition and $\zeta$, our model should be compatible with observations. Overall, we have found results for our new UDM model similar to those in Piattella et al. (2010) but, given that we have started by prescribing a specific energy density evolution for the UDM component rather than from a fast transition in the EoS, this is a non-trivial outcome.

Computing the CMB and the matter power spectra for our model, for a wide range of parameters values, as well as a full likelihood analysis for this model and its parameters, including $\alpha \neq 0$, will be the subject of a forthcoming work. Other possible extensions of the work presented here could aim at including isocurvature (entropy) perturbations, following the prescription of Pietrobon et al. (2008) for the ‘affine model’, an ‘affine’ post-transition era (a possibility that we have considered in Section 4.1), as well as formulating our model in terms of a non-standard scalar field, along the lines of Bertacca et al. (2011).

ACKNOWLEDGEMENTS

The authors thank Robert Crittenden, Marc Manera and Francesco Pace for useful discussions. MB is supported by the STFC (grant no. ST/H002774/1), RL by the Spanish Ministry of Economy and Competitiveness through research projects FIS2010-15492 and Consolider EPI CSD2010-00064, the University of the Basque Country UPV/EHU under program UFI 11/55 and also by the ETORKOSMO special research action. ARF is supported by the ‘Fundación Ramón Areces’.

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