The merging of the islands of inversion at N=20 and N=28

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I. INTRODUCTION

At the neutron rich edge, the structure of the spherical mean field may be at variance with the usual one at the stability line. The reason is that, at the stability line, the T=0 channel of the nucleon-nucleon interaction has a stronger weight relative to the T=1 channel than it has when the neutron excess is very large. If the spherical mean field gaps get reduced, open shell configurations, usually two neutron excitations across the neutron closure, take advantage of the availability of open shell protons to build highly correlated states that can be more bound than the closed shell configuration. Then the shell closure is said to have vanished. Although it was known since long that the ground state parity of

The N=20 and N=28 "islands of inversion" are described by large scale shell model calculations with an extension of the interaction SDPF-U that makes it possible to mix configurations with different Nhω or equivalently with different number of particles promoted from the sd-shell to the pf-shell. It allows to connect the classical sd-shell calculations below N=18 with the sd(protons)-pf(neutrons) calculations beyond N=24-26, for all the isotopes from Oxygen to Argon, using the same interaction. For some isotopes this range contains all the nuclei between the proton and the neutron drip lines and includes the N=20 and N=28 islands of inversion. We shall pay particular attention to the

The interaction SDPF-U [11] that we proposed some time ago, was aimed to the description of the very neutron rich nuclei around N=28 in a 0hω space, with valence protons in the sd-shell and valence neutrons in the pf-shell. Therefore, it is applicable only to nuclei with 8≤Z≤20 and 20≤Z≤40 and does not describe intruder states. The main asset of SDPF-U was the description of the vanishing of the N=28 shell closure below 48Ca, most notably in 42Si [12] (a result which is now fully verified [13], but which produced initially some heated debates [14]). 42Si was predicted to be oblate deformed and 40Mg prolate deformed, exhibiting perhaps a neutron halo. Since its publication, it has been frequently used and shown to give an excellent description of this region of very neutron rich nuclei [15]. Very recently, these calculations have been repeated in the same valence space with a somewhat different effective interaction, getting (as could be expected) very similar results [16]. As the sd part of SDPF-U is the USD interaction [17] and its pf part a variant of KB3 previous to KB3G, it is appealing to complete SDPF-U with the sd-pf off-diagonal matrix elements and to retune the sd-pf cross shell monopoles in such a way that the SDPF-U results at 0hω are mostly preserved and the sd-pf gaps are in accord with the experiment. This process results in the SDPF-U-MIX inter-
action. More details are given in Appendix A. The calculations are carried out using the codes Antoine and Nathan \[19\] and reach basis dimensions of \(O(10^{10})\). In a (very) loose sense one can pretend that this interaction covers the sector of the Segré chart \(8 \leq Z,N \leq 40\). In this article we shall concentrate in the physics of the \(N=20\) "island of inversion" and its merging in some cases with the neighboring \(N=28\) one.

II. THE PHYSICS AT FIXED \(N\hbar\omega\)

What is the driving force behind the abrupt changes leading to the appearance of these "islands of inversion"? What makes these intruder states special? That they leading to the appearance of these "islands of inversion"? The mechanisms need not to be the same in the different regions. For instance in \(^{11}\)Li the intruder is mostly pairing boosted while in \(^{11}\)Be the quadrupole interaction is more important. In the other three neutron rich regions, \(N=20, N=28\) and \(N=40\), the quadrupole interaction is the main player. Let us concentrate in the \(N=20\) case. Compared to the configurations with closed \(N=20\), the intruders (np-nh) have neutrons in open \(sd\) and \(pf\)-shell orbits and in some cases protons in open \(sd\)-shell orbits. This favors the efficient build up of correlations by the neutron-proton quadrupole interaction when the open orbits are the appropriate ones. And whose are these dictated by the different variants of SU(3). For instance, when valence neutrons or protons occupy quasi-degenerate orbits with \(j_l-j_{l'}=2\) and \(l_l-l_{l'}=2\) the coupling scheme is Quasi-SU(3) \[20\], if they are in quasi-spin doublets the regime is that of Pseudo-SU(3) \[21\]. In the limit of vanishing spin-orbit splitting, all the orbits in a harmonic oscillator shell form an Elliot’s SU3 multiplet \[22\]. To get large coherence the neutrons and the protons must pertain to one or another of these coupling schemes. For example, in the case of the \(N=20\) intruders, the neutrons in the orbits \(0d_{5/2}\) and \(1p_{3/2}\), and the protons in \(0d_{5/2}\) and \(1s_{1/2}\) are in the Quasi-SU(3) regime and the neutrons in \(0d_{3/2}\) and \(1s_{1/2}\) in Pseudo-SU3.

Let’s make these statement quantitative in a few selected cases. In this section all the calculations are performed at fixed \(N\hbar\omega\). We only allow neutron jumps from the \(sd\) to the \(pf\)-shell without any other truncation. We have verified that the effect of the proton excitations to the \(pf\)-shell is negligible. We take care of the (small) center of mass contamination by adding to the effective interaction the center of mass hamiltonian (with \(\hbar\omega=A\)). The expectation value of the center of mass hamiltonian in the physical states is always below 0.001A. The results for the low energy levels of \(^{32}\)Mg are presented in Figure 1. We can follow the evolution from the semimagic \(0p-0h\) result, with a high excited \(2^+\) and a low B(E2) to a rotational-like \(2p-2h\) whose B(E2) corresponds to \(\beta=0.4/0.5\) and finally to a perfect rigid rotor \(4p-4h\) with \(E(4^+)/E(2^+)=3.2\) and a very large B(E2) that corresponds to a super-deformed structure. Most important for our aims is that the gains in energy due to the correlations –defined as the difference between the energy which comes out of the diagonalization and the energy of the lowest \(0^+\) state of seniority zero in the corresponding space– are very different in the \(0p-0h\), \(2p-2h\) and \(4p-4h\) spaces; 1.5 MeV, 12.5 MeV and 21 MeV respectively. These huge correlation energies may eventually overcome the spherical mean field gaps. In fact this is the case in \(^{32}\)Mg. With SDPF-U-MIX the lowest \(4p-4h\) \(0^+\) state is about 250 keV below the lowest \(0^+\) of the \(2p-2h\) space and 1.2 MeV below the \(0^+\) of the \(0p-0h\) configuration. This degeneracy of the \(2p-2h\) and \(4p-4h\) bandheads is not a spurious manifestation of our spherical mean field not producing the right \(sd-pf\) gap, but due to the fact that the the energy gain per particle promoted to the \(pf\)-shell, is the same for both configurations. We want to stress again the fact that, in favorable circumstances like these, the gain in correlation energy of the intruders can beat the spherical mean field. In fact, in the laboratory frame, this is the microscopic mechanism responsible for the shape transitions from spherical to deformed nuclei \[20\]. The lowest negative parity state of \(1p-1h\) nature is a \(3^-\), 4 MeV above the \(2p-2h\) \(0^+\) and the lowest \(3p-3h\) state is a \(2^-\), 2.5 MeV above the \(2p-2h\) \(0^+\). Their respective energy gains are 5 MeV and 16 MeV and the underlying structures correspond to the \(K=3^-\) and \(K=2^-\) band-heads as expected from the Nilsson diagrams for \(\beta=0.15\) and \(\beta=0.4\).

The \(4p-4h\) state of \(^{32}\)Mg has an academic interest in
itself even if the states belonging to its rotational band do not manifest themselves openly in the low energy spectrum, (as do their cousins in the superdeformed bands of $^{36}$Ar and $^{40}$Ca [23, 24]) because of its strong mixing with the 0p-0h and 2p-2h spherical and deformed states. It may well happen that they could become yrast at some higher spin, but the threshold for neutron emission is not very high, and the experiments to find them are probably hopeless. In fact, one can understand semi-quantitatively why this configuration can produce such superdeformed structure in the context of Elliott’s SU3 and its variants. Let’s assume that the four pf-shell neutrons be in Quasi-SU3 and the four neutron holes in sd in Pseudo-SU3; in this case the neutrons will contribute with $24b^2$ (times the effective charge) ($b$ is the harmonic oscillator length parameter) to the intrinsic quadrupole moment. If we go to the SU3 limit in the pf-shell sector this number increases to $26b^2$. The value from the shell model calculation is $24.7b^2$. For the protons, the Quasi-SU3 limit gives $11b^2$ against $9.7b^2$ (times the effective charge) of the shell model calculation. With effective charges 0.46 and 1.31 for neutrons and protons, taken from the work of Dufour and Zuker [26], these values lead to $\beta=0.6/0.7$ depending of the definition of $\beta$.

It follows from the above discussion that the configuration with four neutrons in the pf-shell and two neutron holes in the sd-shell maximizes the quadrupole moment and, a fortiori the quadrupole correlation energy. Therefore one should expect the 2p-2h configuration to be also dominant in $^{34}$Mg. On the contrary one expects the 0p-0h one to begin taking over in $^{36}$Mg. This would establish the limit of the N=20 “island of inversion”. However, as we shall see in the next section, the very large depopulation of the 0f$_{7/2}$ orbit in $^{36}$Mg indicates that before leaving the N=20 “island of inversion” we enter the N=28 “island of inversion”, and that both islands are actually merged in a single one. These arguments apply as well to the 3p-3h excitations in the N=21 isotopes, that we expect very low in energy.

In $^{31}$Mg the configurations 0p-0h, 1p-1h and 2p-2h are nearly degenerate. The lowest one is the 2p-2h which looks like a K=$\frac{1}{2}^+$ band, with an excited $\frac{5}{2}^+$ at $\sim 100$ keV, in agreement with the experimental findings of refs. [4]. The energy gain of the band is 14.5 MeV. The lowest 0p-0h state, a $\frac{3}{2}^+$, gains just 3.5 MeV and is 400 keV less bound than the 2p-2h $\frac{5}{2}^+$. The lowest 1p-1h negative parity state, a $\frac{1}{2}^-$, gains 8.5 MeV and is 400 keV above the 2p-2h $\frac{1}{2}^+$. These results are gathered in Figure 2. The E2 and M1 transition probabilities of the 2p-2h band compare well with the recent experimental values from ref. [27]:

$$B(M1)(\frac{5}{2}^+ \rightarrow \frac{3}{2}^+) = 0.1-0.5 \mu_N^2; \quad \text{th. } 0.35 \mu_N^2$$

$$B(M1)(\frac{3}{2}^+ \rightarrow \frac{1}{2}^+) = 0.019(4) \mu_N^2; \quad \text{th. } 0.03 \mu_N^2$$

$$B(E2)(\frac{5}{2}^+ \rightarrow \frac{3}{2}^+) = 61(7) e^2 fm^4; \quad \text{th. } 84 e^2 fm^4$$

The magnetic moment of the $\frac{1}{2}^+$ (using bare g-factors) is $-0.88 \mu_N$ very close to the experimental value $-0.88355(15) \mu_N$, thus we can expect that its 2p-2h character is rather pure, the more so in view of the absence of nearby $\frac{3}{2}^+$ states to mix with. The intrinsic quadrupole moment of the ground state band is the typical in this region, $Q_0 \approx 70$ fm$^2$.

In $^{33}$Mg the lowest state at fixed configuration is the 3p-3h $\frac{1}{2}^+$, head of a K=$\frac{1}{2}^+$ band. At 150 keV appears the 2p-2h $\frac{3}{2}^-$, head of a K=$\frac{3}{2}^-$ band. The 0p-0h and
1p-1h states lie more than 1.5 MeV higher. These results are gathered in Figure 3. Both structures are highly collective, with B(E2)’s in excess of 100 e²fm⁴. In particular the K=1/2 3p-3h band can be viewed as the addition of two neutrons to the ground state band of ⁴⁰Mg. It turns out that both band heads, in spite of their different spin and parity have negative magnetic moments (−0.49 μₜₙ for the 2p-2h and −0.87 for the 3p-3h). Contrary to the assumption of ref. [37], the magnetic moment of the ⁵/₂ 3p-3h state is positive (+0.62 μₜₙ). The results of the fully mixed calculation will be discussed in Section VI.

Similar analysis can be carried out for the all the remaining isotopes. We want to underline here two important points: i) The configurations at fixed np-nh contain much of the relevant physics, and ii) In the cases in which configurations with different particle hole structures, and hence with very different amounts of energy gains due to the correlations, compete, as in some N=19 and N=21 isotopes, the final balance between monopole energy losses and correlation gains is very delicate and the difficulty in accounting for experimental energy splittings which may be smaller than 100 keV, extreme.

### III. SPHERICAL MEAN FIELD VERSUS CORRELATION ENERGIES: THE MECHANISM OF CONFIGURATION INVERSION

As we have already anticipated, the "islands of inversion" occur when a group of adjacent nuclei have their ground states dominated by intruder configurations. We develop now the case of the N=20 isotopes. We have plotted in Fig. 4 the correlation energies of the lowest states of the 0p-0h and 2p-2h configurations. As the uncorrelated energy we take in each case the lowest diagonal energy (expectation value of the hamiltonian) in a basis of states coupled to good J and with well defined generalized seniority. Because of this choice we incorporate in fact some diagonal pairing energy in our uncorrelated reference, but this is irrelevant for our purpose. As expected for semi-magic nuclei, these correlation energies are small and roughly constant for the 0p-0h configuration. On the contrary, for the 2p-2h intruders, they can be very large and have a rapid variation with Z. The largest values occur at mid proton shell, when the quadrupole collectivity reaches its maximum.

In Fig. 5 we present the differences in energy between the lowest 0p-0h state with well defined Jπ and the lowest 2p-2h state without correlations. It is seen that in all cases the normal filling gives the lowest energy, although between Z=8 and Z=14 there is an almost linear increase from 3 MeV to 12 MeV while from there on the curve is much flatter. This reflect the reduction of the sd-pf gap as we approach the neutron drip line. When we take fully into account the correlations the situation changes dramatically as reflected in the lower curve of the figure. The balance between the correlation gains and the monopole losses of energy defines the borders of the island of inversion at N=20 in ⁴⁰F and ⁴⁴Al. Clearly, ⁴⁰Ne, ⁴¹Na, and ⁴²Mg are bona fide members of the club. Equivalent graphs can be drawn for the other isotonic chains. Roughly speaking the situation is very similar for the N=19, N=21 and N=22 isotonic chains. It is probably not worth to go much more beyond this qualitative definition of the somewhat fuzzy shores of the "island of inversion" around N=20 because the predictions obtained in the analysis at fixed configuration may sometimes change when the full mixing is taken into account, the more so for the nuclei near to the borders. We will be more precise in the chapter dealing with the full scale results of our calculations.

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**FIG. 4.** (Color online) Correlation energies of the 0p-0h (squares) and 2p-2h (circles) configurations at N=20

**FIG. 5.** (Color online) The gap between the 0p-0h and the 2p-2h configurations at N=20. Without correlations (squares) and including correlations (circles). Nuclei close or below the zero line are candidates to belong to the island of inversion.
IV. FROM N=Z TO N=32 IN THE MG, NE, AND SI ISOTOPES

In Fig. 6 we compare the experimental $2^+$ excitation energies of the even Mg isotopes, starting at N=Z, with the shell model calculations with the SDPF-U-MIX interaction. Up to N=16 the results should not differ much from the ones produced by the USD interactions [17]. Beyond N=16 the calculations include (if necessary for convergence) up to 6p-6h excitations from the sd-shell to the full pf. The agreement is excellent and covers all the span of isotopes from $^{20}$Mg (which should be close to the proton drip line) to the neutron drip line. Notice the disappearance of the semi-magic closures at N=20 and N=28 and the presence of a large region of deformation which connects the two islands of inversion, previously though to be split apart. The agreements is really superb. Beyond N=24 the effect of the core excitations is perturbative and produces a small expansion of the spectra which improves slightly the agreement with the experimental data obtained in the $0\hbar\omega$ calculations. The merging of the N=20 and N=28 "islands of inversion" is evident.

![Fig. 6](image_url)

**FIG. 6.** (Color online) Excitation energies of the first $2^+$ state in the Magnesium isotopes; Results of the calculation with the SDPF-U-MIX interaction in the valence space of the sd-shell for the protons and the sd-pf-shells for the neutrons, compared with the available experimental data.

In Fig. 7 we compare the B(E2)'s in the transition region with the experimental data including some unpublished results from Riken [28]. We use effective charges 1.35 and 0.35 for protons and neutrons respectively, which are fully compatible with a recent fit to the sd-shell nuclei with the interaction USD-A [29] and with the results obtained by Dufour and Zuker in ref. [26]. We take $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$. The agreement is very good as well.

![Fig. 7](image_url)

**FIG. 7.** (Color online) B(E2)'s of the Magnesium isotopes compared to the experimental results.

In Figure 8 we have gathered the occupancies of the $pf$-shell orbits in the even-even Magnesium isotopes. We have aggregated the values of the f and p orbits for simplicity. The reference numbers for the total $pf$-shell occupancies are those labeled $0\hbar\omega$ in the figure. The $pf$-shell has more than two neutrons in excess at N=20 and N=22. At N=24 the excess is of about one neutron, and beyond that, the core excitations are much damped.

What is more interesting is that when the sd-shell core excitations become small, the occupancy of the p-orbits (mainly 1p$_3/2$) keeps increasing so that in N=26 and N=28 about two neutrons are in 1p$_3/2$, whereas the expected occupancy if N=28 were a strong closure would have been zero. In this sense we can speak of the merg-
ing of the “islands of inversion” at N=20 and N=28 in the Magnesium isotopes. Notice also that a large occupancy of the \( p \) orbits favors the appearance of a neutron halo when the neutron separation energy becomes close to zero as it might be the case in \( ^{37,39}\text{Mg} \) and \( ^{40}\text{Mg} \). Our occupancies for the \( pf \)-shell orbits in \( ^{32}\text{Mg} \) agree with the experimental results of ref. [30].

The results for the Neon isotopes (Fig. 9) are very similar to the Magnesiums, although in this case the N=28 isotope \( ^{38}\text{Ne} \) is most probably beyond the neutron drip line. The \( 2^+ \) excitation energy of \( ^{32}\text{Ne} \) is from ref. [31]. In Fig. 10 we have collected the occupancies of the \( f \) and \( p \) orbits in the isotopic chain as a function of the neutron numbers. The behavior is very similar to that in the Magnesium chain, except that the \( p \) orbits are even more occupied. We have added the numbers for \( ^{31}\text{Ne} \), because some recent experimental data [32] suggest that it could develop a neutron halo. Indeed, ours results are consistent with this hypothesis because the \( 1p_{3/2} \) orbit has on average more than one neutron.

For the understanding of the rich variety of structural changes which take place in the region. Adding two neutrons to \( ^{30}\text{Mg} \) provokes the inversion of the normal and intruder configurations which are shifted by nearly 3 MeV in \( ^{32}\text{Mg} \). In the isotonic course the transition is even more abrupt as has been recently shown in a GANIL experiment [33]: by removing two protons from \( ^{34}\text{Si} \), the intruder (deformed) state is shifted down by about 4 MeV with respect to the spherical one to become the ground state of \( ^{32}\text{Mg} \).

We compare the experimental data with the shell model results in Figure 12. The calculations include configurations with up to 6 neutrons on the \( pf \)-shell. In

V. LANDING AT THE ISLAND OF INVERSION; \( ^{30}\text{Mg} \rightarrow ^{32}\text{Mg} \) AND \( ^{34}\text{Si} \rightarrow ^{32}\text{Mg} \)

There are two courses to land at the “island of inversion” by the \( ^{34}\text{Mg} \) shore: through the isotopic and the isotonic chains. Both are of paramount importance
$^{30}$Mg and $^{34}$Si the ground states are dominantly (>80%) 0p-0h and the first excited $0^+$ state dominantly 2p-2h. They differ in the structure of the lowest $2^+$ which is 0p-0h in $^{30}$Mg and 2p-2h in $^{34}$Si. More details on this last nucleus can be found in reference 33.

The structure of the $0^+$ states in $^{32}$Mg is extremely singular; the ground state has 9% 0p-0h, 54% 2p-2h, 35% 4p-4h and 1% 6p-6h, while the excited $0^+$ has 33% 0p-0h, 12% 2p-2h, 54% 4p-4h and 1% 6p-6h. The $2^+$ has a structure similar to the ground state. As shown in Fig. 7 its B(E2) agrees with the experimental result. In addition, the calculated spectroscopic quadrupole moments of the $0^+$ and $2^+$ states and the B(E2)'s in the yrast band are compatible with a single intrinsic state with $Q_N\approx 65$ e fm$^2$. The MCSM calculations of ref. 8, which only include the $0f_{7/2}$ and $1p_{3/2}$ orbits of the pf-shell, give results similar to ours except for the excited $0^+$ which is too high by almost 2 MeV. Similarly, in $^{34-40}$Mg the calculated E2 properties are compatible with $Q_N\approx 70$ e fm$^2$, which is another fingerprint of the merging of the N=20 and N=28 islands of inversion/deformation.

Since the early beta decay experiments at Isolde 35 it is known that in $^{32}$Mg there are many states, mostly of negative parity, above the $4^+$. They have been explored more recently via the $^{32}$Na beta decay 36, 37 or in (p,p') experiments 38. Ref. 37 presents also the MCSM predictions for the negative parity states fed in the beta decay. The experimental level at 2.551 MeV is most probably the second $2^+$. MCSM puts it at 3 MeV whereas we get it at nearly the same energy than the $4^+$. According to these references, the lowest experimental negative parity state would appear at 2.858 MeV. The calculated negative parity states are $1^-$ at 3.0 MeV, $2^-$ at 3.1 MeV, $3^-$ at 3.4 MeV, $4^-$ at 3.9 MeV, $0^-$ at 4.0 MeV, $5^-$ at 4.2 MeV. They are mostly of 3p-3h nature. The lowest negative parity states in the MCSM description are of 3p-3h nature as well, and start at 3.8 MeV with four close packed states ($2^-$, $1^-$, $2^-$, $3^-$) followed by a doublet ($4^-$, $5^-$) at about 4.5 MeV.

VI. MISCELLANEOUS RESULTS

$^{31}$Mg and $^{33}$Mg.

The N=19 and 21 isotonic chains are very complex, because of the near degeneracy of configurations with different particle hole structure, as discussed in Section II. In $^{33}$Mg the situation is especially critical. We have not tried to fine-tune the interaction to improve our results which amount to have the $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^-$ states degenerated (see Table I). The magnetic moment of the $\frac{1}{2}^+$ ground state has been measured as $-0.88355(15)\mu_N$. Our predictions show a very strong dependence on the choice of the gyromagnetic factors and marginally agree with the experimental value.

In $^{33}$Mg the fully mixed calculation produces a $\frac{3}{2}^-$ ground state with the $\frac{1}{2}^+$ state just 40 keV higher. The magnetic moment of the $\frac{3}{2}^-$, $-0.49\mu_N\,(b); -0.49\mu_N\,(e)$, is short from the experimental value, $-0.7456(5)\mu_N\,$. on the nature of the ground state of $^{33}$Mg. If the J=3/2 assignment is firm, thus positive parity is excluded by the sign of the magnetic moment. Notice however that our results locate the positive parity states almost degenerated with the ground state. Indeed they should not be taken as the theoretical counterparts of the experimental positive parity states at 546 keV and 705 keV. The magnetic moment of the 4p-4h $\frac{3}{2}^-$ state is $-1.67\mu_N\,(b)$ ($-1.36\mu_N\,(e)$), therefore, a somewhat larger mixing of 4p-4h components than that given by our calculation, may line up the theoretical value with the experimental one.

![FIG. 12. (Color online) Comparison between experiment and theory for the most important low lying states in $^{30}$Mg, $^{32}$Mg and $^{34}$Si](image)

TABLE I. Excitation energies (in MeV) and magnetic moments (in $\mu_N$) for the low lying states of $^{31}$Mg and $^{33}$Mg with (b)are and (e)ffective g-factors.

| $^{31}$Mg | J$^+$ | E(exp) | E(th) | $\mu$(th) (b) | $\mu$(th) (e) |
|---|---|---|---|---|---|
| $^+$ | 0.0 | 0.04 | -0.93 | -0.65 |
| $^+$ | 0.05 | 0.04 | +1.13 | +0.81 |
| $^+$ | 0.221 | 0.0 | -1.24 | -1.07 |

| $^{33}$Mg | J$^+$ | E(exp) | E(th) | $\mu$(th) (b) | $\mu$(th) (e) |
|---|---|---|---|---|---|
| $^+$ | 0.0 | 0.0 | -0.54 | -0.49 |
| $^+$ | 0.484 | 0.33 | -0.07 | -0.09 |
| $^+$ | 0.04 | -0.93 | -0.69 |
| $^+$ | 0.12 | +0.69 | +0.44 |

$^{31}$Na and $^{33}$Na.

$^{31}$Na was for many years the protagonist of the N=20 saga, even if only the properties of its ground state were know (spin-parity, magnetic moment, isotope shift, binding energy). Although other nuclei have taken the relay...
nowadays, it still deserves attention. We have gathered
the available experimental information in Figure 13. The
newest data consists in the excitation energies of
two members of the $K=\frac{3}{2}^+$ ground state rotational band
and the $B(E2)$ of the lowest in-band decay in $^{31}$Na, and
the excitation energies of two levels in $^{33}$Na. Notice the
very nice agreement of the calculation and the data which
extends to the ground state magnetic moment ($2.298 \mu_N$
(exp) vs. the calculated $2.26 \mu_N$ (b) or $1.96 \mu_N$ (e)). As
in the $^{32}$Mg case the calculated E2 properties of $^{31}$Na are
compatible with an intrinsic state with $Q_0 \sim 65$ e fm$^2$. We have also plotted the results for $^{33}$Na in which the
behavior of the calculated excitation energies is closer to
$J(J+1)$ than in the previous case, and $Q_0 \sim 72$ e fm$^2$. The comparison of the new data with the calculated values is
quite good and supports strongly a $\frac{3}{2}^+$ ground state.

$^{30}$Ne, $^{31}$Ne and $^{32}$Ne.

To complete this spectroscopic tour, we visit the Neon
chain. Our results for $^{30}$Ne: $2^+$ at 0.79 MeV and $4^+$ at
2.14 MeV compare very well with the experimental values, $0.792$ MeV and $2.235$ MeV respectively. The $B(E2)$,
$(2^+\rightarrow0^+)$ is predicted at $66$ e$^2$ fm$^4$ compared with the
measured value $90(54)$ e$^2$ fm$^4$. Again a single intrinsic
state with $Q_0\approx60$ e fm$^2$ explains the calculated E2
properties. $^{32}$Ne is more deformed: $Q_0\approx65$ e fm$^2$. The
calculated $2^+$ excitation energy, $0.67$ MeV fits well with
the experimental value, $0.72$ MeV. The $4^+$ is predicted at
1.89 MeV. Finally, for $^{31}$Ne the calculation produces
a $\frac{3}{2}^+$ ground state which is mainly $2p$-$2h$, and the first
excited state at $\approx200$ keV is a $\frac{3}{2}^+$ of $3p$-$3h$ character,
belonging to the $K=\frac{3}{2}^+$ in parallel with what happens
in its isotope $^{33}$Mg. The intrinsic quadrupole moment of
the $K=\frac{3}{2}^+$ band is $Q_0\approx60$ e fm$^2$, and, as we mentioned
before, the occupation of the $p$ orbits exceeds 1.2 neu-
tron.

$^{30}$F and $^{31}$F.

Not very much is known experimentally about $^{29}$F and
$^{31}$F. Our calculations produce a $\frac{5}{2}^+$ ground state in $^{29}$F,
which is $60\%$ of $0\hbar\omega$ with a first excited $\frac{1}{2}^+$ at $0.91$ MeV,
which is $80\%$ intruder and the head of a $K=\frac{1}{2}^+$ band,
as expected from quasi-SU3 and Nilsson diagrams. This
compares fairly well with a recent measure at Riken $^{34}$
which places this state at 1.06 MeV. In our calculation,
the ground state of $^{31}$F is an extremely mixed $\frac{3}{2}^+$ ($66\%$
intruder) and the excited $\frac{1}{2}^+$ (74% intruder) appears at
much lower excitation energy, $0.21$ MeV. Neutron exci-
tations result in binding energy gains of $1.9$ MeV and
$2.5$ MeV respectively, which may help to explain the far
off location of the Fluorine’s neutron drip line.

$^{33}$Al and $^{35}$Al.

$^{33}$Al has its ground states largely dominated by the
"normal" configurations ($\sim80\%$). Thus, it does not be-
long properly to the island of inversion. The calculations
reproduce very well the properties of the $\frac{3}{2}^+$ ground state;
the magnetic moment is $+4.088(5) \mu_N \approx 4.17 \mu_N$ (b)
and $+3.86 \mu_N$ (e); the spectroscopic quadrupole moment
is $+0.12$ eB compared to the calculated $+0.12$ eB. Our
results do not produce a low lying $\frac{5}{2}^+$ as surmised in the
experiment of ref. $^{41}$. This is consistent with the large
excitation energy of the intruder $0^+$ in $^{34}$Si. The lowest
excited state is predicted to be a $\frac{5}{2}^+$ of intruder nature at
$1.70$ MeV followed by two other $2p$-$2h$ states at $1.85$ MeV
($\frac{1}{2}^+$) and $2.28$ MeV ($\frac{3}{2}^+$). Contrary to some compiled
results we do not produce negative parity states in this
range of energies. In the $\frac{3}{2}^+$ ground state of $^{35}$Al the
"normal" configurations still lead, but barely so at $52\%$.
The lowest excited states, $\frac{1}{2}^+$ at $0.63$ MeV and $\frac{3}{2}^+$ at
$0.80$ MeV are intruders.

The limits of the "Big Island of Deformation".

We have argued already that the previously established
$N=20$ and $N=28$ islands of inversion/deformation merge
in the Neon, Sodium and Magnesium isotopic chains, cre-
ating a bigger one (BID). Referring only to the ground
states, their $N=19$ isotopes seem to belong to it as well,
and their $N=18$ ones not (but see below). $N=31$ and the
neutron drip line define the west shore of the BID. Some
heavy Aluminums, Silicons ($N\geq26$), Phosphors and Sul-
furs ($N\geq28$) do belong to the $N=28$ sector of the BID
as well, but their less neutron rich isotopes do not be-
long to the $N=20$ sector. Fluorines are transitional, as
we have just discussed. Of the $N=18$ isotones, only $^{29}$Ne
and $^{30}$Na can pretend to pertain to it with $50\%$ and $40\%$
of intruder components in their ground states. For the
other isotopes, although the ground states are "normal",
quite often intruder states show up at low excitation en-
ergy.
VII. CONCLUSIONS

We have shown that the model space comprising the $sd$-shell for the protons and the $sd - pf$ shell for the neutrons, together with the effective interaction SDPF-U-MIX, make it possible to describe a very large region of nuclei, in particular the very neutron rich nuclei at or around the $N=20$ “islands of inversion”. In many cases the inversion of configurations produces deformed ground state bands, thus the use of the term “islands of deformation” as well. According to our calculations (and also to the meagre experimental data available), the two islands merge in the Magnesium chain. In the calculations they also merge in the Neon and Sodium chains. However this could only be checked experimentally if their neutron drip lines happened to be close enough to $N=28$.

We have studied in detail the mechanisms that lead to the inversion of normal and intruder configurations, paying particular attention to the properties of the states at fixed np-nh configurations and to their correlation energy gains. We have compared the calculations to some selected experimental results. The ubiquitous deformed bands have intrinsic (electric) quadrupole moments in the range $Q_0=60-80$ e fm$^2$. We leave for the future a full scan of the region with the SDPF-U-MIX interaction, as well as the study of the one and two neutron separation energies, which requires some extra monopole work, probably including three body forces.

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Appendix A: The SDPF-U-MIX interaction

The SDPF-U-SI interaction was designed for 0$\hbar\omega$ calculations of very neutron rich $sd$ nuclei around $N=28$ in a valence space comprising the full $sd(pf)$-shell for the protons(neutrons), i. e. this interaction was defined (implicitly) with a core of $^{28}$O. Its single particle energies (SPE’s) and monopoles (neutron-proton $sd-pf$ and neutron-neutron $pf-pf$) were fixed by the spectra of $^{35}$Si, $^{41}$Ca, $^{47}$K and $^{49}$Ca. In order to allow for the mixing among different np-nh neutron configurations across $N=20$, it is necessary to add to SDPF-U-SI the following new ingredients: a) The off-diagonal cross shell $sd-pf$ matrix elements, which we take from the Lee-Kahana-Scott G-matrix [42] scaled as in ref. [25]; b) The SPE’s on a core of $^{16}$O: for the the $sd$-shell orbits we use always the USD values [17], while for the $pf$-shell orbits we have no experimental guidance at all. Nonetheless, for any particular set of $pf$-shell SPE’s, the neutron-neutron $sd-pf$ monopoles must be chosen such as to reproduce the spectrum of $^{35}$Si and the $N=20$ gap. As the solution is not unique, we have anchored our choice to the energy of the first excited $0^+$ state in $^{30}$Mg, because this guarantees that in our isotopic course toward $N=20$ the descent of the intruder states proceeds with the correct slope. Indeed, at 0$\hbar\omega$ SDPF-U-MIX and SDPF-U-SI produce identical results; c) We have incorporated to the isovector pairing of the $sd$-shell the same modifications introduced in ref. [8]. The pairing reduction is needed to avoid double counting when core excitations are taken explicitly into account. A recent study of the effects of the change of the reference valence space in the calculation of the renormalizations of the $sd$ and $pf$ matrix elements, made in ref. [43], gives a robust foundation to these modifications.

The extra bonus of the calculations including core excitations is that SDPF-U-MIX is able to give an unified description of the isotopic chains with $Z \leq 14$ and $Z > 14$. The very large span in neutron number that this interaction has to cope with, brings in some global monopole problems which could be associated to three body effects. We have not yet completed this part of the task that would make it possible to obtain predictions for the neutron separation energies. What we have found necessary is to make a three body like reduction of the global T=1 $pf$-shell monopole to get a smooth connexion between $N=20$ and $N=28$. Therefore the present version of SDPF-U-MIX may not be the final one, although the results shown in this paper should not change appreciably with its possible future evolutions.
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