Nonequilibrium transport through coupled quantum dots with electron–phonon interaction

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Abstract. We theoretically study transport properties of coupled quantum dots in parallel in the presence of electron–phonon (e–ph) interaction. Nonequilibrium transport under finite bias is calculated using the Keldysh Green function method. Firstly, we examine a double-dot interferometer with a penetrating magnetic flux (Aharonov–Bohm phase \(\phi\)) between the two quantum dots. The differential conductance shows a sharp dip between double resonant peaks, as a function of energy levels in the quantum dots, when the two dots are equivalently coupled to external leads and \(0 < \phi < \pi\). The e–ph interaction significantly decreases the dip, reflecting an emission of phonons from one of the quantum dots. This dephasing effect is more prominent under larger bias voltage. Secondly, we study a T-shaped double-dot system in which one of the dots is connected to the external leads (dot 1) and the other is disconnected (dot 2). The differential conductance shows a dip between two resonant peaks, as in the double-dot interferometer. The dip is weakly reduced by an emission of phonons from dot 2. Phonon emission from dot 1 does not result in dephasing and hence does not influence the dip. Therefore the dip of the conductance is more robust against the e–ph interaction in the T-shaped double-dot system than in the double-dot interferometer.
1. Introduction

In semiconductor quantum dots, preservation of quantum coherence is an important issue for application to quantum information processing. To examine the coherence of electrons, the transport measurements have been reported using an Aharonov–Bohm (AB) ring with an embedded quantum dot, as an interferometer [1]–[3]. The AB oscillation of the current, as a function of magnetic flux penetrating the ring, indicates the coherence kept in the transport through the dot as well as the reference arm of the ring.

When the energy levels in the quantum dot are changed using a gate voltage, the conductance of the AB interferometer shows an asymmetric resonant shape with peak and dip [3]. This is ascribable to the Fano resonance which is caused by the interference between a discrete level in the quantum dot and the continuum of states in the reference arm [4]: the phase shift differs by $\pi$ on both sides of resonant tunnelling through a quantum dot (Breit–Wigner resonance). The electron waves which pass by the quantum dot interfere positively or negatively with the electron waves which pass through the reference arm. This results in a resonant structure with a peak and dip.

The AB oscillation and Fano resonance in the AB interferometer have been theoretically studied by several groups [5]–[20]. Taking into account the electron–electron interaction, the ‘Fano–Kondo effect’ has been proposed [7, 10]. The dephasing effect on the AB oscillation by spin-flip processes has been studied when an odd number of electrons are localized in the quantum dot [5, 8].

The Fano resonance is observable only when high coherence is kept in the whole system. Indeed the asymmetric shape of the conductance becomes symmetric when the bias voltage increases [3]. This implies that the finite bias significantly reduces the coherence. To explain the experimental result, we have examined the effect of electron–phonon (e–ph) interaction on the Fano resonance [20] because the e–ph interaction with acoustic phonons should be a major decoherence mechanism under finite bias. By considering the e–ph interaction only in the quantum dot, we have obtained the following results.
1. There are two decoherence processes by the e–ph interaction. One is an elastic process in which electrons emit and absorb phonons virtually. The other is an inelastic process in which the electrons actually emit phonons. (We assume that the temperature is $T = 0$.) In the Keldysh Green function formalism [21]–[23], the elastic process is involved in the time-ordered ($\Sigma^\prime$) and antitime-ordered ($\Sigma^\prime\prime$) self-energies for the e–ph interaction, whereas the inelastic process is expressed by the lesser ($\Sigma^<$) and greater ($\Sigma^>$) self-energies. In the inelastic process, a phonon emission from the quantum dot breaks the interference with the wave through the reference arm (so-called dephasing) [16].

2. Within the second-order perturbation with respect to the e–ph interaction, the elastic process decreases the amplitude of the resonant peak owing to the zero-point fluctuation of phonons. This has been also discussed in terms of the Debye–Waller factor in [18]. The inelastic process, on the other hand, broadens the peak width since the real emission of phonons decreases the lifetime of the electron state.

3. In the case of optical phonons with energy $\omega_0$, the elastic process always exists and diminishes the peak height. The inelastic process takes place only when $eV > \omega_0$. Then the differential conductance $dI/dV$ as a function of the gate voltage shows subpeaks corresponding to real emission of phonons. This bias-voltage dependence of $dI/dV$ is difficult to obtain using the canonical transformation method for the e–ph interaction [24] if the renormalization of the tunnel coupling is disregarded [23, 25, 26].

4. For acoustic phonons, we treat the e–ph interaction in an extended self-consistent Born approximation using the Keldysh Green function method [27, 28]. With increasing bias voltage, more phonons participate in the decoherence processes and in consequence the decoherence is more prominent. The dip of the Fano resonance becomes almost invisible and as a result, the asymmetric shape of the resonance grows like a symmetric one. This change of the Fano resonance with the bias voltage qualitatively agrees with the experimental result [3]. However, consideration of the e–ph interaction in the reference arm as well as in the quantum dot is required for quantitative explanation of the experimental result.

In the present paper, we examine double quantum dots connected in parallel to study decoherence effects due to the e–ph interaction further. We take into account the e–ph interaction with acoustic phonons in both the quantum dots using the extended self-consistent Born approximation. We study two models. Firstly, we consider a double-dot interferometer with a penetrating magnetic flux between the two quantum dots. The magnetic flux is considered as an AB phase $\phi$ in figure 1(a). When the two quantum dots are equivalently coupled to external leads, the differential conductance shows a sharp dip between double resonant peaks, as a function of energy levels in the quantum dots. The differential conductance becomes zero at the dip in the absence of e–ph interaction, which stems from the perfectly destructive interference between the waves through the two quantum dots. We show that the e–ph interaction significantly decreases the dip. This should be due to the dephasing effect on the destructive interference by real emission of phonons from one of the quantum dots. The dephasing is more prominent when the e–ph is present in both the quantum dots than in just one of the dots. The dephasing is stronger under larger bias voltage.

Secondly, we study a T-shaped double-dot system in which one of the dots is connected to the external leads (dot 1) and the other is disconnected (dot 2) (figure 1(b)). This situation is analogous to a quantum dot side-coupled to a lead in which a conductance dip has been observed in the Kondo region [29]. The differential conductance shows a dip between two resonant peaks,
Figure 1. (a) A model for double-dot interferometer. Both the quantum dots are connected to leads L and R by the tunnel couplings $t_L$, $t_R$ or $w_L$, $w_R$. The phase $\varphi$ represents the magnetic flux penetrating the interferometer. (b) A model for T-shaped double-dot system. Dot 1 is connected to the external leads by $t_L$ and $t_R$, whereas dot 2 is disconnected from the leads. The tunnel coupling between the quantum dots is given by $t_C$. In both the models, we assume a single energy level $\varepsilon_1$ ($\varepsilon_2$) in quantum dot 1 (2).

as in the first model. The conductance is exactly zero at the dip without the e–ph interaction [30]. We find that the dip is weakly reduced by the phonon emission from dot 2. The phonon emission from dot 1 does not result in dephasing and hence has no influence on the dip. We conclude that the dip is more robust against the e–ph interaction in the T-shaped double-dot than in the double-dot interferometer.

For both the models, we consider a realistic situation in which phonons are not confined in the quantum dots but are extended in a bulk semiconductor containing the quantum dots. Hence the electrons in the quantum dots are coupled to a common bath of phonons. In consequence, there are ‘interdot decoherence processes’ besides the decoherence processes in one of the quantum dots. In the elastic process involved in $\Sigma_{12}$ and $\Sigma_{12}'$ in section 3.2 (subscripts indicate quantum dot 1 or 2), a phonon emitted from a quantum dot is absorbed by the other quantum dot. An inelastic process involved in $\Sigma_{12}'$ and $\Sigma_{12}$ represents an interference of phonon emissions from both the quantum dots. We find that these processes may be important only when the dot–dot distance $L$ is comparable to the dot size.

The organization of the present paper is as follows. In section 2, we present our models for the double-dot interferometer and T-shaped double-dot system. Section 3 is devoted to the calculation method; expression of the current under finite bias using Keldysh Green functions and how to treat the e–ph interaction in the self-consistent Born approximation. The calculated results are given in section 4. The conclusions are presented in section 5.

2. Model

A model for the double-dot interferometer is shown in figure 1(a). Both the quantum dots are connected to leads L and R by the tunnel couplings $t_L$, $t_R$ or $w_L$, $w_R$. The phase $\varphi$ represents the magnetic flux penetrating the interferometer. The bias voltage between the leads is given by $eV = \mu_L - \mu_R$, where $\mu_L$ and $\mu_R$ are the chemical potentials in leads L and R, respectively.
We fix $\mu_L = eV$ and $\mu_R = 0$. Omitting the spin indices, the Hamiltonian for electrons is written as

$$H_{el} = H_{\text{leads}} + H_T + H_D, \tag{1}$$

$$H_{\text{leads}} = \sum_k \varepsilon_k c_{L,k}^\dagger c_{L,k} + \sum_k \varepsilon_k c_{R,k}^\dagger c_{R,k}, \tag{2}$$

$$H_D = \varepsilon_1 d_1^\dagger d_1 + \varepsilon_2 d_2^\dagger d_2, \tag{3}$$

$$H_T = \sum_k (t_L c_{L,k}^\dagger d_1 + \text{h.c.}) + \sum_k (t_R c_{R,k}^\dagger d_1 + \text{h.c.}) + \sum_k (w_L e^{i\omega_2/2} c_{L,k}^\dagger c_{L,k} + \text{h.c.}) + \sum_k (w_R e^{i\omega_2/2} c_{R,k}^\dagger d_2 + \text{h.c.}), \tag{4}$$

where $c_{L(R),k}^\dagger$ and $c_{L(R)}$ denote the creation and annihilation operators of an electron with momentum $k$ in lead L (R), respectively. We assume a single energy level $\varepsilon_1$ ($\varepsilon_2$) in quantum dot 1 (2), with the creation and annihilation operators being $d_1^\dagger$ ($d_2^\dagger$) and $d_1$ ($d_2$). The electron–electron interaction is neglected.

We consider acoustic phonons. The Hamiltonian is given by

$$H_{\text{ph}} = \sum_q \omega_q a_q^\dagger a_q, \tag{5}$$

where $a_q^\dagger$ ($a_q$) creates (annihilates) a phonon with momentum $q$. Unless otherwise noted, we will be setting $\hbar$ to unity. The dispersion relation of the phonons is

$$\omega_q = c_s |q|. \tag{6}$$

The sound velocity is $c_s = 5000$ m s$^{-1}$ in GaAs.

The e–ph interaction by piezoelectric coupling is considered in both the quantum dots

$$H_{e-\text{ph}} = \sum_q M_{q,1}(a_q + a_{-q}^\dagger)d_1^\dagger d_1 + \sum_q M_{q,2}(a_q + a_{-q}^\dagger)d_2^\dagger d_2, \tag{7}$$

where the coupling coefficient $M_{q,i}$ is written as

$$M_{q,i} = \lambda_q \langle d_i | e^{iqr} | d_i \rangle. \tag{8}$$

$\lambda_q$ is the amplitude of the e–ph interaction in the bulk semiconductor

$$|\lambda_q|^2 = \frac{g^2 \pi^2 c_s^2}{V|q|}, \tag{9}$$

with a coupling constant $g$ (we set $g = 0.1$) [31, 32]. $|d_i\rangle$ in equation 8 represents the envelope function of electrons in quantum dot $i$. There is no direct e–ph interaction between different dots, e.g. $(a_q + a_{-q}^\dagger)d_i^\dagger d_2$, assuming $\langle d_1 | e^{iqr} | d_2 \rangle = 0$. This assumption is justified when the overlap between envelope functions $|d_1\rangle$ and $|d_2\rangle$ is negligibly small.
It should be noted that the e–ph interaction is negligibly small for \(|q| \gtrsim 1/\sigma_{i}\), where \(\sigma_{i}\) is the radius of quantum dot \(i\), owing to an oscillating factor of \(\langle d_{i}|e^{iq\cdot\mathbf{r}}|d_{j}\rangle\). For the calculation of the self-energies by e–ph interaction in section 3.2, we assume that

\[
|M_{q,i}|^2 = |\lambda_q|^2 \langle d_i|e^{iq\cdot\mathbf{r}}|d_i\rangle^2 = \frac{\sqrt{2}}{\pi^{1/2}\sigma_i^2} |\lambda_q|^2 \frac{1}{|q|^2 + (1/\sigma_i)^2},
\]

where \(\sigma^2 = (\sigma_1^2 + \sigma_2^2)/2\) and \(\mathbf{L}\) is a vector connecting the centres of quantum dots 1 and 2. We fix \(\sigma_1 = \sigma_2 = c_s/\Gamma\), where \(\Gamma\) is introduced later as a level broadened by the tunnel couplings. When \(\Gamma = 0.3\) meV, the dot size is \(2\sigma_1 = 0.1\) \(\mu\)m.

The second model we adopt is a T-shaped double-dot system in figure 1(b). Dot 1 is connected to the external leads by tunnel couplings \(t_L\) and \(t_R\), whereas dot 2 is disconnected from the leads. The tunnel coupling between the quantum dots is denoted by \(t_C\). \(H_T\) in equation (4) is replaced by

\[
H_T = \sum_k (t_L c_{L\uparrow}^\dagger d_1 + \text{h.c.}) + \sum_k (t Rc_{R\uparrow}^\dagger d_1 + \text{h.c.}) + t_C(d_1^\dagger d_2 + \text{h.c.}).
\]

The remaining parts of the Hamiltonian are the same as those for the double-dot interferometer.

3. Calculation method using Keldysh Green function method

Nonequilibrium transport under finite bias is calculated using the Keldysh Green function formalism [21]–[23]. There are four kinds of the Green function

\[
G_{ij}(t-t') = -i \langle \mathcal{T} d_i(t) d_j^\dagger(t') \rangle,
\]

(13)

\[
\hat{G}_{ij}(t-t') = -i \langle \hat{\mathcal{T}} d_i(t) d_j^\dagger(t') \rangle,
\]

(14)

\[
G_{ij}^\langle(t-t') = i \langle d_j^\dagger(t')d_i(t) \rangle,
\]

(15)

\[
G_{ij}^\rangle(t-t') = -i \langle d_i(t) d_j^\dagger(t') \rangle,
\]

(16)

with \(\mathcal{T} (\hat{\mathcal{T}})\) being the time-ordering (antitime-ordering) symbol. They are called time-ordered, antitime-ordered, lesser and greater Green functions, respectively. The subscripts \(i, j\) indicate quantum dot 1 or 2. We represent the Fourier transform of the Green functions in a matrix form

\[
G^{\gamma}(\omega) = \begin{pmatrix}
G_{11}^{\gamma}(\omega) & G_{12}^{\gamma}(\omega) \\
G_{21}^{\gamma}(\omega) & G_{22}^{\gamma}(\omega)
\end{pmatrix}
\]

(17)

for \(\gamma = t, \bar{t}, <, \text{ and } >\). We also use the retarded Green function in the expression of the current, which is defined by \(G_{ij}^{\gamma}(t-t') = -i\theta(t-t')\langle [d_i(t), d_j^\dagger(t')] \rangle\). It is related to the Keldysh Green functions as

\[
G^{\gamma}(\omega) = G^{\gamma}(\omega) - G^{\langle}(\omega).
\]

(18)

The unperturbed Green functions, \(G^{\gamma(0)}(\omega)\), are defined in the same way.
3.1. Expression of current

First, we express the current in terms of the Green functions for the double-dot interferometer. The current operator from lead L to the quantum dots is given by the time derivative of the number operator of electrons in the lead, \( \hat{N}_L = \sum_k \hat{c}^{\dagger}_L \hat{c}_L \);

\[
I_L = -e \langle \dot{N}_L \rangle = -ie \langle [H, N_L] \rangle. \tag{19}
\]

It is written as

\[
I_L = 2e \text{Re} \left[ t_L \sum_k G_{1L,k}^{\leq}(t, t) + w_L e^{-i\phi/2} \sum_k G_{2L,k}^{\leq}(t, t) \right], \tag{20}
\]

\[
= \frac{2e}{\hbar} \text{Re} \int d\omega \left[ t_L \sum_k G_{1L,k}^{\leq}(\omega) + w_L e^{-i\phi/2} \sum_k G_{2L,k}^{\leq}(\omega) \right], \tag{21}
\]

where \( G_{1(2),L,k}^{\leq}(t - t') = i \langle \hat{c}^{\dagger}_L(t') \hat{d}_{L,k}(t) \rangle \) and \( G_{1(2),L,k}^{\leq}(\omega) \) is its Fourier transform. The current from lead R to the quantum dots, \( I_R \), is obtained in the same way.

Using the equation-of-motion method and analytic continuation rules \cite{23}, the current from lead L (R) is rewritten using the lesser and retarded Green functions for the quantum dots. For the steady state, in which \( I_L + I_R = 0 \) holds

\[
I = \frac{1}{2} (I_L - I_R)
\]

\[
= \frac{e}{\hbar} \int d\omega \left\{ - \left[ \frac{\Gamma_{1L}}{2} - \frac{\Gamma_{1R}}{2} \right] \text{Im} G_{11}^{\leq}(\omega) - \left[ \frac{\Gamma_{2L}}{2} - \frac{\Gamma_{2R}}{2} \right] \text{Im} G_{22}^{\leq}(\omega)
\]

\[
+ \left[ \frac{\sqrt{\Gamma_{1L} \Gamma_{2L}}}{2} + \frac{\sqrt{\Gamma_{1R} \Gamma_{2R}}}{2} \right] \sin \frac{\phi}{2} \text{Re} \left[ G_{21}^{\leq}(\omega) - G_{12}^{\leq}(\omega) \right]
\]

\[
- \left[ \frac{\sqrt{\Gamma_{1L} \Gamma_{2L}}}{2} - \frac{\sqrt{\Gamma_{1R} \Gamma_{2R}}}{2} \right] \cos \frac{\phi}{2} \text{Im} \left[ G_{21}^{\leq}(\omega) + G_{12}^{\leq}(\omega) \right]
\]

\[
- \left[ \Gamma_{1L} f_L(\omega) - \Gamma_{1R} f_R(\omega) \right] \text{Im} G_{11}^{\leq}(\omega) - \left[ \Gamma_{2L} f_L(\omega) - \Gamma_{2R} f_R(\omega) \right] \text{Im} G_{22}^{\leq}(\omega)
\]

\[
+ \left[ \sqrt{\Gamma_{1L} \Gamma_{2L}} f_L(\omega) + \sqrt{\Gamma_{1R} \Gamma_{2R}} f_R(\omega) \right] \sin \frac{\phi}{2} \text{Re} \left[ G_{21}^{\leq}(\omega) - G_{12}^{\leq}(\omega) \right]
\]

\[
- \left[ \sqrt{\Gamma_{1L} \Gamma_{2L}} f_L(\omega) - \sqrt{\Gamma_{1R} \Gamma_{2R}} f_R(\omega) \right] \cos \frac{\phi}{2} \text{Im} \left[ G_{21}^{\leq}(\omega) + G_{12}^{\leq}(\omega) \right] \right\}. \tag{22}
\]

Here, the level broadenings by the tunnel coupling between the quantum dots and lead L (R) are \( \Gamma_{1L(R)} = 2\pi v\nu_{L(R)}^2 \) and \( \Gamma_{2L(R)} = 2\pi v\nu_{L(R)}^2 \), where \( v \) is the density of states in the leads. We fix \( \Gamma_{1L} = \Gamma_{2L} = \Gamma_{1R} = \Gamma_{2R} = \Gamma/2 \) and \( \epsilon_1 = \epsilon_2 \); energy levels are matched in the two quantum dots, which are coupled to the leads equivalently.

For a T-shaped double-dot system, the current is obtained in a similar way. We express the current from lead L (R) in terms of the lesser and retarded Green functions for quantum dot 1;

\[
I_{L(R)} = \frac{2e}{\hbar} \int d\omega \left[ - \frac{\Gamma_{L(R)}}{2} \text{Im} G_{11}^{\leq}(\omega) - \Gamma_{L(R)} f_L(\omega) \text{Im} G_{11}^{\prime}(\omega) \right], \tag{23}
\]
where $\Gamma_{L(R)} = 2\pi v_0^2 L_{L(R)}$ is the level broadening by the tunnel coupling between quantum dot 1 and lead L (R). In this case, we can eliminate $G_{11}^<(\omega)$ by taking a linear combination of the currents

$$I = x I_L - (1-x) I_R,$$

with $x = \Gamma_R / (\Gamma_L + \Gamma_R)$. Then the current is rewritten as

$$I = \frac{2e}{h} \int d\omega \left( -\frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \right) [f_L(\omega) - f_R(\omega)] \text{Im} G_{11}^r(\omega).$$

(25)

We choose $\Gamma_L = \Gamma_R \equiv \Gamma/2$ and $\epsilon_1 = \epsilon_2$.

3.2. Electron–phonon interaction

We need the retarded Green function $G^r_{ij}(\omega)$ and the lesser Green function $G^<(\omega)$ to calculate the current. $G^r(\omega)$ and $G^< (\omega)$ follow the Dyson equations

$$G^r(\omega) = G^r(0) + G^r(0)(\omega) \Sigma'(\omega) G^r(\omega),$$

(26)

$$G^<(\omega) = [1 + G^r(\omega) \Sigma'(\omega)]G^<(0)(\omega) [1 + \Sigma'(\omega) G^r(\omega)] + G^r(\omega) \Sigma^<(\omega) G^a(\omega),$$

(27)

where the self-energy of the retarded Green function is given by

$$\Sigma'(\omega) = \Sigma'(\omega) - \Sigma^<(\omega).$$

(28)

If we know $\Sigma'(\omega)$ and $\Sigma^<(\omega)$, we obtain $G^r(\omega)$ and $G^<(\omega)$ using equations (26) and (27).

We treat the e–ph interaction by the self-consistent Born approximation [27, 28]. Then the self-energies by the e–ph interaction are written as

$$\Sigma_{ij} = \frac{i}{2\pi} \sum_q M_{q,i} M^*_{q,j} \int d\omega' G^r_{ij}(\omega - \omega') D'(q, \omega'),$$

(29)

and

$$\Sigma^<_{ij} = \frac{i}{2\pi} \sum_q M_{q,i} M^*_{q,j} \int d\omega' G^<_{ij}(\omega - \omega') D^<(q, \omega'),$$

(30)

where the Fourier transforms of the phonon Green functions are

$$D'(q, \omega) = -2\pi i [N_q \delta(\omega + \omega_q) + N_q \delta(\omega - \omega_q)] + \frac{1}{\omega - \omega_q + i\delta} - \frac{1}{\omega + \omega_q - i\delta},$$

(31)

$$D^<(q, \omega) = -2\pi i [(N_q + 1) \delta(\omega + \omega_q) + N_q \delta(\omega - \omega_q)].$$

(32)

$N_q$ denotes the phonon occupation number, $1/[\exp(\omega_q/k_B T) - 1]$. The summation over $q$ in equations (29) and (30) is performed using equations (10) and (11). The self-energies by the e–ph interaction are written as

$$G^r_{ij}(\omega)$$ and $G^<(\omega)$ are expressed in terms of $G^r_{ij}(\omega)$ and $G^<(\omega)$, using equation (18). We determine $\Sigma^r_{ij}(\omega)$ and $\Sigma^<_{ij}$ by solving equations (26), (27), (29) and (30) self-consistently. The obtained $G^r(\omega)$ and $G^< (\omega)$ yield the current in equations (22) or (25). We fix the temperature at $T = 0$.
Figure 2. Differential conductance as a function of the dot level $\varepsilon_1$ ($\varepsilon_1 = \varepsilon_2$) in the absence of e–ph interaction, for (a) double-dot interferometer and (b) T-shaped double-dot system. In (a), the phase of magnetic flux is $\varphi = \pi/6$ (dotted line), $\pi/2$ (broken line) and $5\pi/6$ (solid line).

4. Calculated results

4.1. Double-dot interferometer

Now we present the calculated results for the double-dot interferometer shown in figure 1(a). In the absence of e–ph interaction, the differential conductance is expressed as

$$\frac{dI}{dV} = \frac{2e^2}{h} \frac{(1 + \cos \varphi)\tilde{E}^2/2}{\tilde{E}^4 + (3 + \cos \varphi)\tilde{E}^2/4 + (\cos \varphi - 1)^2/64}, \tag{33}$$

where $\tilde{E} = (\varepsilon_1 - \mu_L)/\Gamma$. Note that we choose $\varepsilon_1 = \varepsilon_2$. Generally, $dI/dV$ is independent of the bias voltage in the absence of e–ph interaction. Unless $\varphi = 0$ or $\pi$, $dI/dV$ as a function of $\varepsilon_1$ shows a double-peak structure with a sharp dip between the peaks, as shown in figure 2(a). The differential conductance becomes precisely zero at $\tilde{E} = 0$ in equation (33). This should be ascribable to perfectly destructive interference between the electron waves passing by the two quantum dots. (This is not a simple interference of the AB effect. The dip is located at $\tilde{E} = 0$, irrespective of the magnetic flux $\varphi$.) When $\varphi = 0$, $dI/dV$ shows a single Lorentzian peak with the centre of $\tilde{E} = 0$. The height and width of the peak are given by $2e^2/h$ and $\Delta\tilde{E} = 1$, respectively (not shown in figure 2(a)). When $\varphi = \pi$, $dI/dV = 0$ for all $\tilde{E}$. When increasing the magnetic flux from $\varphi = 0$ to $\pi$, the height of the double peaks diminishes.

We take into account the e–ph interaction in the case of $\varphi = \pi/2$, where $dI/dV = e^2/h$ at the double peaks and $dI/dV = 0$ at the dip without the interaction. In figure 3, we plot the differential conductance $dI/dV$ as a function of the energy level $\varepsilon_1$. The e–ph interaction is present in (a) both the quantum dots or (b) one of the quantum dots. Assuming that the dot–dot distance $L$ is much larger than the dot size, the interdot decoherence is neglected. The bias voltage is $eV = \Gamma$. 

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Figure 3. Differential conductance as a function of the dot level $\varepsilon_1 (\varepsilon_1 = \varepsilon_2)$ for the double-dot interferometer. The bias voltage is $eV = \Gamma$. The dot–dot distance is $L = \infty$ (interdot decoherence is not taken into account). (a) The e–ph interaction is present in both the quantum dots (solid line). The magnetic phase is $\varphi = \pi/2$. (b) The e–ph interaction is present only in quantum dot 1. The magnetic phase is $\varphi = \pi/2$ (broken line) or $-\pi/2$ (solid line). In (a) and (b), the dotted line indicates $dI/dV$ in the absence of e–ph interaction, with $\varphi = \pi/2$.

In figure 3, the e–ph interaction significantly decreases the dip of $dI/dV$, indicating that the destructive interference is reduced by an inelastic process of e–ph interaction. In our previous paper [20], we have discussed two kinds of decoherence processes, one is an elastic process in which electrons virtually emit and absorb phonons and the other is an inelastic process in which the electrons actually emit phonons from one of the dots. The reduction of the dip should be due to the latter process since the former process does not break the destructive interference to make the dip. The dephasing effect by the inelastic process is stronger when the e–ph interaction exists in both the quantum dots than when it is present only in one of the dots.

The two peaks of $dI/dV$ slightly increase in height and become broadened by the e–ph interaction in figure 3(a). The broadening of the peaks is due to the inelastic process, as discussed in our previous paper [20]. The behaviour of the peak height is complicated since it is determined by a competition between the resonant tunnelling through the dot levels and destructive interference which makes the above-mentioned dip. Note that the peak height is $e^2/h$ in the absence of e–ph interaction, which is a half of the conductance by the usual resonant tunnelling. The peak by the resonant tunnelling should be decreased by the elastic process (zero-point fluctuation of phonons) [20], whereas the destructive interference should be weakened by the inelastic process (dephasing). The competition between the processes results in a slight increase of the peak height.

The double-peak structure of $dI/dV$ is slightly asymmetric when the e–ph interaction is present in both the quantum dots, whereas it is largely asymmetric when the interaction is present in one of the quantum dots. This is because the energy levels in the quantum dots are differently renormalized by the e–ph interaction under finite bias, which breaks the symmetry of the model. The asymmetry is enhanced by the presence of e–ph interaction only in one of the dots. Interestingly, $dI/dV$ at $\varphi = -\pi/2$ is not identical to that at $\varphi = \pi/2$ when the e–ph interaction exists only in one of the dots (figure 3(b)). This does not contradict Onsager’s relation.
Figure 4. Differential conductance as a function of the dot level $\varepsilon_1$ ($\varepsilon_1 = \varepsilon_2$) for the double-dot interferometer. The bias voltage is $eV = 2\Gamma$. The dot–dot distance is $L = \infty$ (interdot decoherence is not taken into account). (a) The e–ph interaction is present in both the quantum dots (solid line). The magnetic phase is $\varphi = \pi/2$. (b) The e–ph interaction is present only in quantum dot 1. The magnetic phase is $\varphi = \pi/2$ (broken line) or $-\pi/2$ (solid line). In (a) and (b), dotted line indicates $dI/dV$ in the absence of e–ph interaction, with $\varphi = \pi/2$.

for the conductance in the linear response region, $G(\varphi) = G(-\varphi)$, because we are discussing the nonequilibrium region with finite bias. We observe that $dI/dV$ at $\varphi = -\pi/2$ with e–ph interaction in one dot coincides with $dI/dV$ at $\varphi = \pi/2$ with e–ph interaction in the other dot, as expected from the symmetry of the model. We obtain the identical $dI/dV$ at $\varphi = \pm \pi/2$ when the e–ph interaction is present in both the quantum dots.

The decoherence by the e–ph interaction becomes stronger if the bias voltage is increased. Figure 4 shows $dI/dV$ when $eV = 2\Gamma$. The other conditions are the same as in figure 3. We observe a larger change of the dip than in figure 3, indicating a larger dephasing effect by the real emission of phonons. The double-peak structure is more asymmetric in figure 4(a) than in figure 3(a).

4.2. T-shaped double-dot system

Next, we discuss the transport properties of the T-shaped double-dot system shown in figure 1(b). The differential conductance in the absence of e–ph interaction is expressed as

$$\frac{dI}{dV} = \frac{2e^2}{h} \frac{1}{4} \frac{\tilde{E}^2}{\tilde{E}^4 + (1/4 - 2\tilde{t}_C^2) \tilde{E}^2 + \tilde{t}_C^4},$$

(34)

where $\tilde{E} = (\varepsilon_1 - \mu_L)/\Gamma$ with $\varepsilon_1 = \varepsilon_2$ and $\tilde{t}_C = t_C/\Gamma$. $dI/dV$ shows double peaks and a dip with changing $\varepsilon_1$, as shown in figure 2(b). Although the shape of $dI/dV$ is analogous to that in the previous model with $0 < \varphi < \pi$, the resonant peaks reach $2e^2/h$ in this model. This means that the resonant tunnelling takes place through the bonding state ($\tilde{E} = -\tilde{t}_C$) or anti-bonding state ($\tilde{E} = \tilde{t}_C$) between the two levels in the dots. $dI/dV$ becomes exactly zero at the midpoint between
Figure 5. Differential conductance as a function of the dot level $\varepsilon_1$ ($\varepsilon_1 = \varepsilon_2$) for the T-shaped double-dot system. The bias voltage is $eV = 2\Gamma$. The dot–dot distance is $L = \infty$ (interdot decoherence is not taken into account). (a) The e–ph interaction is present in both the quantum dots (solid line) or absent (dotted line). (b) The e–ph interaction is present only in quantum dot 1 (solid line) or in quantum dot 2 (dotted line).

In figure 5, we show the differential conductance in the presence of e–ph interaction in (a) both the quantum dots, or (b) one of the dots. The dot–dot distance $L$ is much larger than the dot size and hence the interdot decoherence is neglected. The bias voltage is $eV = 2\Gamma$, which is the same as in figure 4 for the double-dot interferometer. The two peaks of $dI/dV$ are decreased in height and broadened in width. The change of the height should be due to the elastic process while that of the width should be due to the inelastic process [20]. These changes are more prominent when the e–ph interaction is present in both the quantum dots than when the interaction is present only in one of the dots. The interaction in quantum dot 1 and that in quantum dot 2 equivalently influence the height and width of the peaks, as seen in figure 5(b). This is because the resonant tunnelling through bonding or anti-bonding states is equally affected by the e–ph interaction in either of the quantum dots.

The dip of $dI/dV$ is weakly decreased by the e–ph interaction in figure 5(a). The change of the dip is much smaller, compared with the double-dot interferometer in figure 4(a). In figure 5(b), we observe that the e–ph interaction in dot 2 reduces the dip by almost the same amount as in figure 5(a), whereas the interaction in dot 1 does not affect the dip at all. The reason is as follows. Since the dip reflects the destructive interference between electron waves through the two quantum dots, the dip should be influenced by the dephasing process accompanied by the real emission of phonons (inelastic process). When the e–ph interaction is present in quantum dot 2, some waves pass by the dot and emit a phonon while the others do not pass by the dot. The interference between the waves is broken. When the e–ph interaction exists in quantum dot 1, on the other hand, all the waves pass by the dot. Then the emission of a phonon does not break the interference between the waves and hence does not change the dip of $dI/dV$. 

the peaks ($\tilde{E} = 0$), which is due to the destructive interference between the tunnelling through the bonding and anti-bonding states [30].
4.3. Interdot decoherence

Up to now, we have assumed that the dot–dot distance $L$ is much larger than the dot size and disregarded the interdot decoherence, $\Sigma_{12}$ and $\Sigma_{12}^\dagger$ for the elastic process and $\Sigma_{12}^<$ and $\Sigma_{12}^>$ for the inelastic process. In this subsection, we take into account the processes by setting $L = 2\sigma$.

In figure 6, we show the differential conductance for (a) the double-dot interferometer and (b) the T-shape double-dot system. Solid lines indicate the case of $L = 2\sigma$ and dotted lines indicate the case of $L = \infty$ where the interdot decoherence is absent. For both the models, the dip is hardly influenced by the interdot decoherence, whereas the height of the resonant peaks is changed. No change of the dip implies that the inelastic process involved in $\Sigma_{12}^<$ and $\Sigma_{12}^>$ is not important for the dephasing mechanism. The change of the peak height is due to the elastic process by $\Sigma_{12}$ and $\Sigma_{12}^\dagger$, in which a phonon emitted from a quantum dot is absorbed in the other quantum dot virtually.

5. Conclusions

We have studied the decoherence effects by the e–ph interaction in coupled quantum dots in parallel. The e–ph interaction with acoustic phonons is taken into account under finite bias, using the Keldysh Green function method. We have mainly examined a dip of the differential conductance in the double-dot interferometer and T-shaped double-dot system since the dip is created by the destructive interference between the electron waves through two quantum dots. In the double-dot interferometer, the e–ph interaction significantly decreases the dip, owing to the dephasing by real emission of phonons from one of the quantum dots. In the T-shaped double-dot system, the dip is weakly reduced by a real emission of phonons from dot 2 in figure 1(b). The phonon emission from dot 1 does not result in dephasing and hence does not influence the dip. Therefore, the dip is more robust against the e–ph interaction in the T-shaped double-dot system than in the double-dot interferometer. When the dot–dot distance is comparable to the dot size,
the interdot decoherence may be important, in which a phonon emitted from a quantum dot is absorbed in the other quantum dot virtually.

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