Postural Stabilization of Quadrotor using Extended Kalman Filter and Integral Sliding Mode Control

Hyukwoo Lee¹, Kyunghyun Lee¹, and Kwanho You¹,*

¹Department of Electrical and Computer Engineering, Sungkyunkwan University, Suwon, Korea

*corresponding author: khyou@skku.edu

Abstract. In this paper, we consider the stabilization and the estimate of quadrotor’s posture. Nowadays, as the interest in unmanned aerial vehicle (UAV) has increased, various application fields using UAV have appeared. The quadrotor is the most common type of UAV and many approaches to control the system have been studied. In real control environment, the system of quadrotor is affected by disturbance and measurement noise. Minimizing the effects of disturbance or measurement noise is important part. We propose the extended kalman filter (EKF) to reduce effectively the measurement noise of the nonlinear drones system and the integral sliding mode control (ISMC), a robust controller for the model uncertainty and disturbance. We show the proposed control performance through some simulations.

1. Introduction

The quadrotor is the most general type of UAV, lifted by four motors and propellers. The system of quadrotor has advantages from a variety of applications due to simple structure, vertical take-off and landing (VTOL) capability and payload [1]. Each motor is oriented vertically downward, with two motors rotating in clockwise and the other two motors rotating in counterclockwise. By controlling the four motor speeds, the attitude and altitude of quadrotor can be adjusted. The quadrotor has rolling, pitching and yawing corresponding to the x, y, and z axis, respectively.

Several control techniques such as proportional-integral-differential (PID), backstepping, and optimal control have been studied for controlling the quadrotor system. Garcia [2] proposed a robust PID control strategy via affine parametrization designed for multivariable nonlinear quadrotor. The proposed PID control system is assured by using the $H_\infty$ norm of the weighted complementary sensitivity function. Satici [3] provided the design and implementation of an $L_1$-optimal control of the quadrotor dynamical system that rejects disturbances. The proposed algorithm reduces the magnitude of the errors in $L_1$ -optimal sense.

In this paper, we propose ISMC and EKF to reduce the effects of measurement noise and to design a robust controller for disturbance and model uncertainty. This paper organized as follows. In section II, we obtain the dynamic model of quadrotor and show the overall flow of quadrotor system with EKF and ISMC. In section III, we show the composition and derive the process of EKF. In section IV, we obtain the control input by applying the EKF angle estimates and the dynamic model of the quadrotor to the ISMC. In section V, to prove the performance the stabilization and the estimate, we show simulation results by comparing with a general SMC. Finally, section VI presents the conclusion.
2. Model of quadrotor’s dynamic

Quadrotor is a nonlinear system with many state variables [4]. The main forces acting on the quadrotor are generated by four motors and the propellers. The quadrotor consists of a fixed body frame and the inertial frame \(E\). The four motors are coupled with each propeller, which intersects with each other. To balance the quadrotor, one pair of motors \((w_2, w_4)\) rotate clockwise, and the other motors \((w_1, w_3)\) rotate counterclockwise. We can change the direction and the position of a quadrotor by varying the speed of the four propellers.

The dynamic model of the quadrotor can be derived by Euler-Lagrange [5] as follows:

\[
\begin{align*}
\dot{\phi} &= \left(\frac{i_x-i_y}{i_z}\right)\dot{\theta} + \left(\frac{L}{i_z}\right)u_i, \\
\dot{\theta} &= \left(\frac{i_x-i_z}{i_y}\right)\phi + \left(\frac{L}{i_y}\right)u_i, \\
\dot{\psi} &= \left(\frac{i_y-i_z}{i_x}\right)\phi + \left(\frac{L}{i_x}\right)u_i.
\end{align*}
\]

\(\phi, \theta, \psi\) are the angles of the quadrotor rotation about the \(x, y, z\) axis and are the roll, pitch, yaw angles, respectively. \(i_x, i_y, \) and \(i_z\) are the body inertia of a quadrotor and \(L\) is the lever length. \(u_i\) \((i=1, 2, 3)\) is the control input of a quadrotor system.

\[
\begin{align*}
u_i &= b(\omega^i_x - \omega^i_z), \\
u_z &= b(\omega^i_z - \omega^i_x), \\
u_y &= d(\omega^i_x + \omega^i_z - \omega^i_y - \omega^i_z), \\
\omega &= \omega_x + \omega_y - \omega_z - \omega_z,
\end{align*}
\]

Where \(b\) and \(d\) represent the thrust factor and the drag factor, respectively. \(\omega^i\) \((i=1, 2, 3, 4)\) is the rotational speed of \(i\)-th motor. From equation (1), the model of a quadrotor can be rewritten in a state space form as follows:

\[
\begin{align*}
x_1 &= \phi, \quad x_2 = \dot{\theta}, \quad x_3 = \dot{\psi}, \\
x_4 &= \dot{x}_1 = \dot{\phi}, \quad x_5 = \dot{x}_2 = \dot{\theta}, \quad x_6 = \dot{x}_3 = \dot{\psi}.
\end{align*}
\]

From equations (1) and (3), the quadrotor dynamic is formulated as:

\[
\dot{x} = m(x, u) = \begin{bmatrix} x_2 \\ a_1 x_1 x_3 + b_1 u_1 \\ x_4 \\ a_2 x_2 x_3 + b_2 u_2 \\ x_6 \\ a_3 x_3 x_3 + b_3 u_3 \end{bmatrix}
\]

where

\[
\begin{align*}
a_1 &= \frac{i_x-i_y}{i_z}, \quad a_2 = \frac{i_x-i_z}{i_y}, \quad a_3 = \frac{i_y-i_z}{i_x}, \\
b_1 &= \frac{L}{i_1}, \quad b_2 = \frac{L}{i_2}, \quad b_3 = \frac{L}{i_3},
\end{align*}
\]
3. State estimation using EKF

Kalman filter is a method of estimating the state of a linear system that contains measurement noise or error [6]. Kalman filter basically assumes the linearity of the model, however in practice many models are nonlinear structures. Thus, the continuous-discrete EKF can be applied to estimate states.

The algorithm consists of two steps: time update and measurement update. The nonlinear discrete system with measurement noise is given as follows:

\[
\begin{align*}
\dot{x} &= m(x, u) + \Theta(t)w, \\
z_k &= y[x(t_k), k] + v_i,
\end{align*}
\]

where \(m(x, u)\) is a model of quadrotor system and \(y[x(t_k), k]\) is a measurement model, respectively.

The quadrotor is a time varying system and the sensor in a quadrotor measures in discrete time. In equation (6), \(\Theta(t)\) is the process noise matrix. \(w(t) \approx (0, Q)\) and \(v_i \approx (0, R)\) are the white noises uncorrelated with each other and independent from the system state.

\[
M(x, t) = \frac{\partial m(x, u, t)}{\partial x}, \quad Y(x) = \frac{\partial y(x, k)}{\partial x}.
\]

where \(M(x, t)\) and \(Y(x)\) are Jacobians derived from \(m(x, u, t)\) and \(y[x(t_k), k]\), respectively. The first step of the Kalman filter is time update. To predict the next state, we use the priori measurement update state.

\[
\dot{x} = m(\ddot{x}, u, t),
\]

\[
P = M(\dot{x}, t)P + PM(\dot{x}, t) + \Theta \Theta'.
\]

\(P\) is the error covariance matrix obtained from the prior measurement update. To estimate measurement, the gain \(K_x\) of Kalman filter is obtained as follows:

\[
K_x = P' (t_k) Y^T (\dot{x}_k) Y' (\dot{x}_k) P(t_k) + R]^{-1}.
\]

Using the Kalman filter gain obtained in equation (9) and the error covariance matrix of the previous state, \(P(t_k)\) and measurement estimate are updated as follows:

\[
P(t_k) = [I - K_x Y(\dot{x}_k)] P(t_k) [Y(\dot{x}_k) P(t_k) Y(\dot{x}_k) + R]^{-1}.
\]

\[
\dot{x}_k = \dot{x}_k + K_x [z_k - Y(\dot{x}_k, k)].
\]

where \(I\) is an identity matrix. From equation (10), the update of \(P(t_k)\) and \(\dot{x}_k\) is derived and these values are applied to the covariance error matrix of time update and measurement model. The EKF can be applied to the quadrotor dynamic system, we can estimate the angles of the quadrotor.

4. ISMC for quadrotor stabilization

ISMС is not sensitive to modeling errors and parameter uncertainties with disturbances [7]. To obtain the control input of a quadrotor, we use ISMC with the estimated angles obtained from EKF. The sliding surface is designed as follows:

\[
s_i = (\dot{\phi}_d - \dot{\phi}) + \lambda_i (\phi_d - \phi) + \delta_i \int_0^t (\phi_d - \phi) d\tau.
\]

\[(11)\]
In equation (11), $\lambda_i$ and $\delta_i$ are positive constants of a sliding surface. To satisfy the stability condition of ISMC, we use the Lyapunov function \[ V(s_i) = \frac{1}{2} s_i^2, \quad V(s_i) = s_i \dot{s}_i < 0. \] (12)

To satisfy the Lyapunov conditions (12) and (13), we differentiate the sliding surface using equations (4) and (11) as follows:

\[ \dot{s}_i = -k_i \text{sat}(s_i) - k_2 s_i \\
= \ddot{\phi}_d - \left( a_i \dot{\phi} \dot{\psi} + b_i u_i \right) + \lambda_i \left( \phi_d - \phi \right) + \delta_i \left( \phi_d - \phi \right). \] (13)

To avoid the chattering which happens in ISMC, we replaced the signum function with a saturation function. With equations (14) and (15), we can derive the control input $u_i$ as

\[ u_i = \frac{1}{b_i} \left[ \ddot{\phi}_d - a_i \dot{\phi} \dot{\psi} + \lambda_i \left( \phi_d - \phi \right) + \delta_i \left( \phi_d - \phi \right) + k_i \text{sat}(s_i) + k_2 s_i \right]. \] (14)

Similarly, the control inputs can be derived as:

\[ u_2 = \frac{1}{b_2} \left[ \ddot{\theta}_d - a_2 \dot{\theta} \dot{\phi} + \lambda_2 \left( \theta_d - \theta \right) + \delta_2 \left( \theta_d - \theta \right) + k_3 \text{sat}(s_2) + k_4 s_2 \right], \] (15)

\[ u_3 = \frac{1}{b_3} \left[ \ddot{\psi}_d - d_3 \dot{\psi} \phi + \lambda_3 \left( \psi_d - \psi \right) + \delta_3 \left( \psi_d - \psi \right) + k_5 \text{sat}(s_3) + k_6 s_3 \right]. \]

5. Simulation and analysis

In this section, in order to evaluate the estimate performance of the continuous-discrete EKF and stabilization of ISMC, we analyze the simulation result. In simulations, the measurement state of quadrotor consists of estimated values by EKF. Figure 1 shows the estimate values and the stabilizations of a quadrotor about roll.

![Figure 1. Estimate and measurement of roll angle.](image-url)
Figure 2 shows the control performance comparison between the ISMC and the general SMC. Simulation was carried out to stabilize the roll angle, and the initial values of ISMC and SMC are set as 0.5 radian. The settling times of ISMC and SMC are 0.86 sec and 1.4 sec, respectively.

6. Conclusion
In this paper, we proposed a postural stabilization of a quadrotor using ISMC and EKF. We applied a continuous-discrete EKF to estimate the angle from the discontinuous features of sensor and the continuous system of the quadrotor. We applied the estimate values obtained from EKF to the ISMC for reducing the measurement error of a sensor. The estimation and the control performance of our proposed method have been proved through the simulations.

Acknowledgments
This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (NRF-2019R1A2C1002343).

References
[1] Tayebi A and McGilvray S 2006 Attitude stabilization of a VTOL quadrotor aircraft IEEE Transactions on Control Systems Technology. vol. 14, pp. 562-571
[2] Garcia R and Rubop F 2012 Robust PID control of the quadrotor helicopter Proceeding of 18th World Congress IFAC. vol. 45, pp. 229-234
[3] Satici A, Poonawala H, and Spong M 2013 Robust optimal control of quadrotor UAVs IEEE Access. vol. 1 pp. 79-93
[4] Selfridge J and Tao G 2013 A multivariable adaptive controller for a quadrotor with guaranteed matching conditions Systems Science & Control Engineering. vol. 2, pp. 24-33
[5] Naidoo Y and Stopforth R 2011 Quad-rotor unmanned aerial vehicle helicopter modeling & control International Journal of Advanced Robotic Systems. vol. 8, pp. 139-149
[6] Evensen 2003 The ensemble Kalman filter: theoretical formulation and practical implementation Ocean Dynamics. vol. 53, pp. 343-367
[7] Laghrouche S, Plestan F, and Glumineau A 2007 Higher order sliding mode control based on integral sliding mode Automatica. vol. 43 pp. 531-537
[8] Polyakov A and Poznyak A 2009 Reaching time estimation for “super-twisting” second order sliding mode controller via Lyapunov function designing IEEE Transactions on Automatic Control. vol. 54, pp. 1951-1955