Finite density effects on chiral symmetry breaking in a magnetic field in $2+1$ dimensions from holography

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Abstract

In this work we study finite density effects in spontaneous chiral symmetry breaking as well as chiral phase transition under the influence of a constant magnetic field in $2+1$ dimensions. For this purpose, we use an improved holographic softwall model based on an interpolated dilaton profile. We find inverse magnetic catalysis (IMC) at finite density. We observe that the chiral condensate decreases as the density increases, and the two effects (addition of magnetic field and chemical potential) sum up decreasing even more the chiral condensate.
I. INTRODUCTION

Systems with finite chemical potential for fermions are a challenging and actual subject. The main reason for this interest is that in many physical systems we have to take into account a fermionic density, such as, in heavy ion collisions, neutron stars, condensed matter theories, among others. For a review, one can see, e. g., Refs. [1, 2].

In particular, nuclear matter can be treated within a very well established theory, which is based on first-principles and a non-perturbative approach, called lattice QCD (LQCD). The calculations in LQCD are numerical and usually via Monte Carlo simulations. However, at finite chemical potential LQCD seems to crash due to the sign problem, meaning that action of the theory becomes complex [3, 4].

Some proposals have been made to overcome this difficulty. For instance, in Refs. [5, 6] a complex chemical potential was used. In Refs. [7, 8] the authors have used reweighting approaches, in Ref. [9] it was used non-relativistic expansions, and in Ref. [10] the author have dealt with the reconstruction of the partition function. Very recently in Ref. [11] the authors proposed a solution to this problem and provided extremely accurate results for the QCD transition, extrapolating from imaginary chemical potential up to real baryonic potential \( \mu_B = 300 \) MeV.

There is an alternative approach to non-perturbative QCD, or even LQCD, based on the AdS/CFT correspondence [12, 13]. Presented in 1997, this correspondence, generically referred to as holography, relates a strong coupling theory, without gravity, in a four-dimensional space to a weak coupling theory, including gravity, in a curved higher dimensional space. The theoretical framework to deal with nuclear matter in the presence of a finite chemical potential within the AdS/CFT correspondence was put forward in many important works. See for instance, Refs. [14–17], and more recently Refs. [18–28].

Despite real QCD is a \((3+1)\)-dimensional gauge theory the numerical calculations in a such a background (in the presence of a magnetic field) are extremely hard, and only reliable for low values of the magnetic field, as can be seen in Refs. [29–35] in the holographic context as well as in the non-holographic approach [36, 37].

Here, in this work, our focus is to study the finite density effects on chiral symmetry breaking in the presence of an external constant magnetic field \( B \) in \( 2+1 \) dimensions based on holographic studies done at zero density in Refs. [38–41]. Our choice for a dimensional
reduction comes from the fact that in 2 + 1 dimensions, our model has a computational task easier than in real QCD even when considering both non-zero chemical potentials and external magnetic fields. This approach is very useful since we can learn from this model and try to extrapolate it to real QCD. Some previous non-holographic works in 2+1 dimensions with magnetic fields can be seen, for instance, in [42–47].

This work is organized as follows. In section II we describe our holographic model. In subsection II A we detail the background geometry which is an AdS$_4$-Reissner-Nordstrom black hole and present all relevant quantities for our further calculations. In the subsection II B we show the holographic description of our effective action for a complex field. Such a field will be related to the chiral condensate at the boundary theory. In section III we present our numerical results where we observe inverse magnetic catalysis (IMC) at finite density as well as the decreasing of the chiral condensate when the density increases. In section IV we make our conclusions.

II. HOLOGRAPHIC MODEL

A. Background geometry

In this section we will establish the holographic description of our background geometry. Considering the Einstein-Maxwell action on AdS$_4$ (for more details see [38, 39] and references therein):

\[ S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left( -\frac{6}{L^2} - L^2 F_{MN} F^{MN} \right), \]  

(1)

where \( \kappa_4^2 \) is the 4-dimensional coupling constant (with the relation \( 2\kappa_4^2 \equiv 16\pi G_4 \) with 4D Newton’s constant \( G_4 \), \( x^M = (t, z, x, y) \), with \( z \) being the holographic coordinate, and \( F_{MN} = \partial_M A_N - \partial_N A_M \) is the field strength for the \( U(1) \) gauge field \( A_M \). Throughout the text we will work in units such that \( 2\kappa_4^2 = L = 1 \).

The field equations from (1) are

\[ R_{MN} = 2 \left( F_M^P F_{NP} - \frac{1}{4} g_{MN} F^2 \right) - 3g_{MN}, \]  

(2)

\[ \nabla_M F^{MN} = 0, \]  

(3)

where \( R_{MN} \) is the Ricci tensor and \( g_{MN} \) is the metric tensor. Since we want to include a nonzero chemical potential \( \mu \) and a constant magnetic field \( B \), the ansatz we are going
to consider is the dyonic AdS/Reissner-Nordstrom black hole solution [50, 51], with both electric and magnetic charge, given by:

\[
\begin{align*}
    ds^2 &= \frac{1}{z^2} \left( -f(z) dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2 \right), \\
    f(z) &= 1 - (1 + \mu^2 z_h^2 + B^2 z_h^4) \left( \frac{z}{z_h} \right)^3 + (\mu^2 z_h^2 + B^2 z_h^4) \left( \frac{z}{z_h} \right)^4, \\
    A &= \mu \left( 1 - \frac{z}{z_h} \right) dt + B \left( x dy - y dx \right).
\end{align*}
\]

The temperature of the black hole solution can be obtained through the Hawking formula,

\[
    T = -\frac{f'(z_h)}{4\pi},
\]

and is given by

\[
    T(z_h, \mu, B) = \frac{1}{4\pi z_h} \left( 3 - B^2 z_h^4 - \mu^2 z_h^2 \right).
\]

1. IR near-horizon geometry: Emergence of AdS\(_2\times\mathbb{R}^2\)

Here, we briefly discuss an important feature of the near-horizon \((z \to z_h)\) geometry of extremal \((T = 0)\) charged black holes in asymptotically AdS space-time, which is the emergence of the AdS\(_2\times\mathbb{R}^2\) space [52–54]. An extremal black hole is characterized by the fact that its temperature \(T\), Eq. (8), is zero. In this case the horizon function, Eq. (5), becomes

\[
    f(z) \bigg|_{T=0} = 1 - 4 \left( \frac{z}{z_h} \right)^3 + 3 \left( \frac{z}{z_h} \right)^4,
\]

where it has a double zero at the horizon and can be Taylor expanded as

\[
    f(z) \approx \frac{6}{z_h^2} (z - z_h)^2.
\]

Now, defining a dimensionless coordinate \(w\) through the rescaling \(w := z/z_h\) and changing variables according to \(w = 1 + z_h \eta\), we find that the geometry in the near-horizon region \((z \to z_h)\) becomes

\[
    ds^2 \approx \left( -\frac{\eta^2}{L_{\text{eff}}^2} dt^2 + \frac{L_{\text{eff}}^2}{\eta^2} d\eta^2 \right) + \frac{1}{z_h^2} (dx^2 + dy^2),
\]

which is the AdS\(_2\times\mathbb{R}^2\), with AdS\(_2\) curvature radius \(L_{\text{eff}} \equiv 1/\sqrt{6}\), in units of \(L = 1\). Thus, in the near-horizon regime (IR), the AdS\(_2\times\mathbb{R}^2\) space controls the low-energy physics of the dual gauge theory on the boundary. Therefore, it seems that supergravity on AdS\(_4\) flows in
the IR to a gravity theory on AdS$_2$ which, in turn, is dual to a (0 + 1)-dimensional effective conformal quantum theory, which is referred to an IR CFT$_1$. For a more extensive discussion on this topic, we refer the reader to [53, 54].

B. Effective action for chiral symmetry breaking

The effective action we consider to describe the chiral symmetry breaking is given by (we refer the reader to [40, 41] and references therein)

$$S = \frac{1}{2\kappa_4^2} \int d^3 x \, dz \sqrt{-\bar{g}} \, e^{-\Phi(z)} \text{Tr} \left( D_M X^\dagger D^M X - V(X) - G^2 \right),$$

(12)

where $X$ is a complex scalar field with mass squared $M_4^2 = -2$ dual to the chiral condensate $\sigma \equiv \langle \bar{\psi} \psi \rangle$ in 3 spacetime dimensions, whose conformal dimension is $\Delta = 2$. $D_M$ is the covariant derivative defined as $D_M \equiv \partial_M + iA_M$, with $A_M$ being a non-abelian gauge field, and its field strength $G_{MN}$ defined as $G_{MN} \equiv \partial_M A_N - \partial_N A_M - i[A_M, A_N]$. $V(X)$ is the potential for the complex scalar field $X$ given by $V(X) = -2X^2 + \lambda X^4$, where $\lambda$ is the quartic coupling, which we will fix as $\lambda = 1$ from now on. This coupling allows the spontaneous and explicit chiral symmetry breakings to occur independently as pointed in [55].

Concerning the dilaton profile $\Phi(z)$ appearing in (12) we will consider [31, 33, 34]

$$\Phi(z) = -\phi_0 z^2 + (\phi_0 + \phi_\infty) z^2 \tanh(\phi_2 z^2),$$

(13)

having three parameters, which captures both IR and UV behaviours. It interpolates between the positive quadratic dilaton profile in the IR, $\Phi(z \to \infty) = \phi_\infty z^2$, and the negative quadratic dilaton profile in the UV, $\Phi(z \to 0) = -\phi_0 z^2$. This dilaton field plays the role of a soft IR cutoff promoting the breaking of the conformal invariance. Note that in Ref. [48] the authors proposed a quadratic dilaton profile, however, for the positive sign for this quadratic dilaton it was shown in Refs. [33, 34, 49] that the spontaneous chiral symmetry breaking cannot be reproduced.

Assuming that the expectation value of the complex scalar field $X$ takes a diagonal form $\langle X \rangle = \frac{1}{2} \chi(z) I_2$ for the SU(2) case [29, 31], where $I_2$ is the $2 \times 2$ identity matrix, the field equations for $\chi(z)$, derived from (12), are given by

$$\chi''(z) + \left( -\frac{2}{z} - \Phi'(z) + \frac{f'(z)}{f(z)} \right) \chi'(z) - \frac{1}{z^2 f(z)} \partial_\chi V(\chi) = 0,$$

(14)
where $'$ means derivative with respect to $z$, $f(z)$ is given by (5), and the potential becomes $V(\chi) \equiv \text{Tr} \, V(X) = -\chi^2 + \chi^4$.

The boundary conditions used to solve (14) are [40, 41]:

$$\chi(z) = m_f \, z + \sigma z^2 + O(z^3), \quad z \to 0, \tag{15}$$

$$\chi(z) = c_0 + \frac{c_0 (4 \, c_0^2 - 2)}{z_h (B^2 \, z_h^4 + \mu^2 \, z_h^2 - 3)} (z - z_h) + O((z - z_h)^2), \quad z \to z_h, \tag{16}$$

where $m_f$ is the source (fermion mass), and $\sigma$ is the chiral condensate. Moreover, $c_0$ is a coefficient obtained from evaluating (14) as a series expansion. Since we want to study spontaneous symmetry breaking, most of the results in this work will be derived with the source turned off, i.e., $m_f = 0$. Then, in both UV and IR side, we have one undetermined coefficient, $\sigma$ and $c_0$ respectively. With certain values of the two coefficients, one can obtain the solutions $\chi_{\text{UV}}$ and $\chi_{\text{IR}}$ from both sides. Requiring $\chi_{\text{UV}} = \chi_{\text{IR}}$ and $\chi'_{\text{UV}} = \chi'_{\text{IR}}$, one obtains two equations and could solve out $\sigma$ and $c_0$.

In the next section we will present our results concerning the chiral symmetry breaking at finite density and magnetic field as well as the phase diagram in the $\mu - T$-plane. For convenience, we will express the dilaton parameters in units of the mass scale $\sqrt{\phi_\infty}$ as well as all our results. To be more clear, note that one can define a dimensionless variable by rescaling the $z$ coordinate as $u := \sqrt{\phi_\infty} \, z$, so that the dilaton profile (13) takes the form

$$\Phi(u) = -\tilde{\phi}_0 \, u^2 + (1 + \tilde{\phi}_0) \, u^2 \, \text{tanh}(\tilde{\phi}_2 \, u^2), \tag{17}$$

where $\tilde{\phi}_0 := \frac{\phi_0}{\phi_\infty}$ and $\tilde{\phi}_2 := \frac{\phi_2}{\phi_\infty}$ are the dimensionless parameters.

Furthermore, one can check that the tachyon equation (14) can also be put in the dimensionless form by redefining all the dimensional quantities, for instance, $(z_h, \mu, B)$, in units of $\phi_\infty$. In this way, we have a two-parameter dilaton profile controlled by the dimensionless parameters $(\tilde{\phi}_0, \tilde{\phi}_2)$. Also, note that $\tilde{\phi}_0$ is just the ratio between the dilaton parameter in the UV, $\phi_0$, and the dilaton parameter in the IR, $\phi_\infty$. Finally, for reference in the next section, we fix our parameters as $\tilde{\phi}_0 = 4.675$, $\tilde{\phi}_2 = 0.0375$.

**III. RESULTS**

In this section we present our results concerning the chiral phase transition in $2 + 1$ dimensions at finite temperature and density, in the presence of an external magnetic field.
All the physical quantities presented in this section have a tilde, meaning that they are in units of the mass scale $\sqrt{\phi_\infty}$. To be more precise:

$$(\tilde{T}, \tilde{\mu}) \equiv \left( \frac{T}{\sqrt{\phi_\infty}}, \frac{\mu}{\sqrt{\phi_\infty}} \right) \quad \text{and} \quad (\tilde{B}, \tilde{\sigma}) \equiv \left( \frac{B}{\phi_\infty}, \frac{\sigma}{\phi_\infty} \right).$$

(18)

Figure 1. The chiral condensate $\tilde{\sigma}$ versus the temperature $\tilde{T}$. Left panel: $\tilde{\sigma}$ versus $\tilde{T}$ for $\tilde{\mu} = 0$ and $\tilde{B} = 0$. In the chiral limit, i.e, $\tilde{m}_f = 0$, one sees a second order phase transition, while for $\tilde{m}_f \neq 0$ there is a crossover. Right panel: $\tilde{\sigma}$ versus $\tilde{T}$ in the chiral limit $\tilde{m}_f = 0$ for different values of magnetic field and chemical potential. All quantities are in units of $\sqrt{\phi_\infty}$, in both panels.

In Figure 1 the behavior of the chiral condensate $\tilde{\sigma}$ as a function of temperature $\tilde{T}$ is presented. In the Left panel, for zero magnetic field and density, one can see that the chiral phase transition is second-order in the chiral limit ($\tilde{m}_f = 0$), while for finite fermion mass ($\tilde{m}_f \neq 0$) the phase transition turns to a crossover. We have checked numerically that there is always spontaneous chiral symmetry breaking whenever the parameter $\tilde{\phi}_0 \neq 0$. However, in the limit $\tilde{\phi}_0 \to 0$ the chiral condensate vanishes. This limit corresponds exactly to the situation where the positive quadratic dilaton profile ($\phi(z) \sim \phi_\infty z^2$) dominates, and in this case it is known that the chiral symmetry breaking cannot be reproduced [33, 34, 49].

In the Right panel of Figure 1, the chiral condensate $\tilde{\sigma}$ as a function of the temperature $\tilde{T}$ for different values of magnetic field and density is presented. At zero magnetic field and finite density, the value of the chiral condensate is reduced from its value at both zero magnetic field and density. At zero density and finite magnetic field, this reduction is observed, in agreement with previous works [40, 41], signaling an inverse magnetic catalysis (IMC) effect. At finite density, with or without magnetic field, we observe a reduction of
the condensate with respect to the condensate at zero density and magnetic field. This is expected to happen since the introduction of a chemical potential generates an asymmetry \[56, 57\] between the fermions \( (\Psi) \) and anti-fermions \( (\bar{\Psi}) \) which renders difficult the pairing \( \bar{\Psi}\Psi \), i.e., the formation of a chiral condensate. At finite density and magnetic field there is also an IMC due to a summation of the effects such that the net effect is a reduction of the chiral condensate.\(^1\)

Figure 2. The chiral condensate \( \tilde{\sigma} \) versus the chemical potential \( \tilde{\mu} \) in the chiral limit \( \tilde{m}_f = 0 \). **Left panel:** \( \tilde{\sigma} \) versus \( \tilde{\mu} \) at zero temperature and different values of magnetic field. **Right panel:** \( \tilde{\sigma} \) versus \( \tilde{\mu} \) at small finite temperatures and different values of magnetic field. All quantities are in units of \( \sqrt{\phi_\infty} \), in both panels.

In Figure 2, the chiral condensate as function of the chemical potential in the chiral limit \( \tilde{m}_f = 0 \) is shown. In the **Left panel** this behavior is shown at zero temperature. One can see that the finite density affects the chiral condensate destructively, i.e., causing a decreasing, until a critical chemical potential from which the chiral condensate starts to increase slowly again at large chemical potential. This behavior does not agree with QCD expectations, since at large chemical potentials the chiral symmetry is expected to be restored. However, it is worthy to point out that at extremely high densities (\( \mu >> T \)) chiral symmetry can be broken through the formation of a condensate of quark Cooper pairs in the color-flavor-locked (CFL) phase, via a different mechanism \[61, 62\].

\(^{1}\) This reduction in the chiral condensate at finite density also appears in more sophisticated holographic approaches in higher dimensions, see for instance \[58, 59\]. Furthermore, for an alternative interpretation of IMC at vanishing chemical potential, based on the anisotropy caused by a magnetic field, see \[60\].
In the Right panel of Figure 2, the chiral condensate as function of the chemical potential in the chiral limit \( \bar{m}_f = 0 \) for zero and finite magnetic fields is shown at finite small temperatures. One can observe in this case that the thermal effects affect the chiral condensate substantially, which is expected since thermal fluctuations have a huge impact on the chiral condensation, especially in \( 2 + 1 \) dimensions [63]. Moreover, with a finite magnetic field turned on, the decrease of the chiral condensate is much more pronounced, even at low temperatures, characterizing an IMC. Note however that, at low temperatures, IMC is not expected to happen in QCD, since it is known that a magnetic field is a strong catalyst of chiral symmetry breaking, and therefore magnetic catalysis is expected to dominate in this low temperature regime and, in particular, is universal behavior at zero temperature [44–47].

![Graph](image1)

Figure 3. Left panel: critical temperature \( \tilde{T}_c \) versus the chemical potential \( \tilde{\mu} \) in the chiral limit for different values of the magnetic field. The critical temperature is defined as the temperature where the chiral condensate vanishes for fixed \( \mu \) and \( B \). Right panel: critical chemical potential \( \tilde{\mu}_c \) versus \( \tilde{B} \) in the chiral limit for different values of temperature. As for the critical temperature, the critical chemical potential is defined as the chemical potential where the chiral condensate vanishes for fixed \( T \) and \( B \). All quantities are in units of \( \sqrt{\phi_\infty} \), in both panels.

Finally, in Figure 3 is presented the critical temperature \( \tilde{T}_c \) as a function of the chemical potential \( \tilde{\mu} \) in the chiral limit for different values of magnetic field (Left panel) and the critical chemical potential \( \tilde{\mu}_c \) versus \( \tilde{B} \) in the chiral limit for different values of temperature (Right panel). These critical quantities \( (T_c, \mu_c) \) are defined as follows. The critical temperature \( T_c \) is defined as the temperature where the chiral condensate vanishes for fixed \( \mu \) and \( B \).
Analogously, the critical chemical potential $\mu_c$ is defined as the chemical potential where the chiral condensate vanishes for fixed $T$ and $B$. These findings give additional support to the fact that our holographic model captures the IMC effect at zero and finite densities as well as a decrease on the chiral condensate with increasing chemical potential with or without magnetic fields. We also find that the two effects related to the presence of chemical potential and magnetic fields on the chiral condensate sum up decreasing the chiral condensate even more.

IV. CONCLUSIONS

In this work we have described holographically finite density effects on the spontaneous chiral symmetry breaking and chiral phase transition of a system in $2 + 1$ dimensions in the presence of magnetic fields. We observe inverse magnetic catalysis (IMC), which is the reduction of the chiral condensate with increasing magnetic field, at zero or at finite density. We also observe a decreasing of the chiral condensate with increasing chemical potential, with or without magnetic fields. Furthermore, the reduction of the chiral condensate is even more pronounced when one takes both finite densities and magnetic fields simultaneously, as shown in Figures 1 and 2. Moreover, we have also find that the critical temperature $T_c$ diminishes with increasing chemical potential and that the critical chemical potential $\tilde{\mu}_c$ decreases with increasing magnetic field, as pictured in Figure 3. These results are in good agreement with other higher dimensional holographic studies, as the one presented in Refs. [58–60].

As a possible extension to our holographic model, it would be interesting to include a Dirac-Born-Infeld (DBI) action in which the magnetic field and the tachyon are coupled. In this setup one might reproduce magnetic catalysis (MC) in our holographic model along with the standard inverse magnetic catalysis (IMC) which comes from contribution of the magnetic field introduced via the metric. A possible clue in this direction is given by the recent higher dimensional holographic analysis presented in Ref. [64] at zero density where there is MC, as would be expected from QCD.
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