Rotating Hayward’s regular black hole as particle accelerator

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Recently, Banados, Silk and West (BSW) [1] have demonstrated that the collision of two particles falling from rest at infinity into the Kerr black hole [2], can have an infinitely large center-of-mass energy ($E_{CM}$) when the collision takes place near the horizon. The rotating Hayward’s regular black hole, apart from Mass ($M$) and angular momentum ($a$), has a new parameter $g$ ($g > 0$ is a constant) that provides a deviation from the Kerr black hole. We demonstrate that for each $g$, with $M = 1$, there exist critical $a_E$ and $r_H^g$, which corresponds to a regular extremal black hole with degenerate horizon, and $a_E$ decreases and $r_H^g$ increases with increase in $g$. While $a < a_E$ describe a regular non-extremal black hole with outer and inner horizons. We apply BSW process to the rotating Hayward’s regular black hole, for different $g$, and demonstrate numerically that $E_{CM}$ diverges in the vicinity of the horizon for the extremal cases, thereby suggesting that a rotating regular black hole can also act as a particle accelerator and thus in turn may provide a suitable framework for Plank-scale physics. For a non-extremal case, there always exist a finite upper bound of $E_{CM}$, which increases with deviation parameter $g$.

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I. INTRODUCTION

Recently, Banados, Silk and West (BSW) [1] have demonstrated that the collision of two particles falling from rest at infinity into the Kerr black hole [2], can have an infinitely large center-of-mass energy ($E_{CM}$) when the collision takes place near the horizon. The rotating Hayward’s regular black hole, apart from Mass ($M$) and angular momentum ($a$), has a new parameter $g$ ($g > 0$ is a constant) that provides a deviation from the Kerr black hole. We demonstrate that for each $g$, with $M = 1$, there exist critical $a_E$ and $r_H^g$, which corresponds to a regular extremal black hole with degenerate horizon, and $a_E$ decreases and $r_H^g$ increases with increase in $g$. While $a < a_E$ describe a regular non-extremal black hole with outer and inner horizons. We apply BSW process to the rotating Hayward’s regular black hole, for different $g$, and demonstrate numerically that $E_{CM}$ diverges in the vicinity of the horizon for the extremal cases, thereby suggesting that a rotating regular black hole can also act as a particle accelerator and thus in turn may provide a suitable framework for Plank-scale physics. For a non-extremal case, there always exist a finite upper bound of $E_{CM}$, which increases with deviation parameter $g$.

On the other hand a spacetime singularity or a naked singularity is the final fate of continual gravitational collapse [27], and it is widely believed that a singularity must be removed by quantum gravity effects. However, we are yet far away from well defined quantum gravity, and hence much attention is devoted to study the research on the properties and implications of classical black holes with regular or non-singular properties. In particular, an interesting proposal was made by Hayward [28] for the formation and evaporation of a regular black hole, based on the idea of Bardeen [29], who proposed first regular black hole. These black holes are solution of modified Einstein’s equation, yielding alteration to classical black holes, but near the center they behave like de Sitter [28, 29]. Over the past few years there has been an increasing interest in the study of rotating regular black holes, e.g., rotating Hayward’s regular black hole [32], rotating Ayón-Beato-García black hole [33] etc., which
are axisymmetric, asymptotically flat and depend on the mass and spin of the black hole as well as on a deviation parameter that measure potential deviations from the Kerr metric, and includes the Kerr metric as the special case if this deviation parameter vanishes. Further, these regular black holes are very important as astrophysical black holes, like Cygnus X-1, although suppose to be Kerr black hole \cite{32,31}, but the actual nature of astrophysical black hole has still to be tested \cite{32,31}, and they may be a non-Kerr black holes. More recently, the BSW mechanism when applied to extremal regular black holes \cite{34,35}, also lead to divergence of $E_{CM}$. The main purpose of this paper is to study the collision of two particles with equal rest masses in the background of the rotating Hayward’s regular black hole and to see what effect the deviation parameter $g$ makes on the $E_{CM}$ in near horizon collision. It may be mentioned that rotating Hayward’s regular black hole is a prototype of non-Kerr black hole with additional parameter $g$ apart from $M$ and $a$, which looks like Kerr black hole with different spin \cite{32}, may be a suitable candidate for astrophysical black hole. Further, if observation demands vanishing deviation parameter, the compact object may be taken as Kerr black hole or non-Kerr black hole otherwise. It turns out that observation may allow both cases. We also study of horizon structure of rotating Hayward’s regular black hole, explicitly show the effect of parameter $g$. Further, our results go over to Kerr black hole when parameter $g$ vanish, and to nonrotating Hayward’s regular black hole when $a = 0$.

Further, there are several questions that motivate our analysis: how does the deviation term $g$ affect the BSW mechanism? What is the horizon structure in the presence of term $g$? Whether such solutions lead to some important outcome? Do the calculation $E_{CM}$ has departure from usual Kerr black hole? As we will see, these regular solutions do have several interesting features and consequences on BSW mechanism. The paper is organized as follows. In the next section, we review the rotating Hayward’s regular black hole, and discuss in detail its horizon structure. The calculation of $E_{CM}$ for two colliding particles coming from rest at infinity in the background of rotating Hayward’s regular black hole is the subject of sec. IV. The required equation of motion and study of effective potential and calculating the range of angular momentum are done in sec. III. We conclude the paper in sec. V. We have used units which fix the speed of light and the gravitational constant via $8\pi G = c^4 = 1$.

II. ROTATING HAYWARD’S REGULAR BLACK HOLE

The aim of this paper is to show that rotating Hayward’s black hole can act as particle accelerators. In the Boyer-Lindquist coordinates the metric of the rotating Hayward’s solution, which is equivalent to the Kerr metric \cite{32}, reads

$$ds^2 = -(1 - \frac{2mr}{\Sigma})dt^2 - \frac{4amr\sin^2\theta}{\Sigma}dtd\phi + \frac{\Sigma}{\Delta}d\phi^2$$

$$+ \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2a^2m\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi d\phi,$$

(1)

where

$$\Sigma = r^2 + a^2 \cos^2\theta,$$

$$\Delta = r^2 - 2mr + a^2,$$

(2)

with mass function is replaced by $m_{\alpha,\beta}(r, \theta)$ \cite{32}, given by

$$m \rightarrow m_{\alpha,\beta}(r, \theta) = \frac{M}{r^{3+\alpha}\Sigma^{-\alpha/2} + g^2r^\beta\Sigma^{-\beta/2}}.$$  

(3)

The constants $\alpha$, $\beta$ are two real numbers, $g$ is a real positive constant, $a = J/M$ is angular momentum per unit mass, and $M$ is the mass of the black hole. Thus the rotating Hayward’s regular metric can be seen as the prototype of a large class of non-Kerr black hole metrics, in which the metric tensor, in Boyer-Lindquist coordinates, has the same expression of the Kerr one with $m$ replaced by a mass function $m_{\alpha,\beta}(r, \theta)$ with $g$ gives deviation from the standard Kerr solution \cite{2} and one recover Kerr solution in the limit $g \rightarrow 0$. Further, in addition, if the rotational parameter $a = 0$, we get Schwarzschild solution. The metric (1) is regular everywhere, including at $r = 0$ for $g \neq 0$, which can be checked by the behavior of Ricci scalar and Kretschman scalar. In fact, the curvature invariants are regular everywhere, including at $r = 0$, where they remarkably become zero \cite{32}.

The rotating Hayward’s spacetime is stationary and axisymmetric with Killing vector $\partial_t$ and $\partial_\phi$. However, like the Kerr metric, the Hayward’s rotating metric (1) is also singular at $\Delta = 0$. The metric (1) generically must have two horizons, viz., the Cauchy horizon and the event horizon. This surface of no return is known as the event horizon. The zeros of $\Delta = 0$ gives the horizons of the black hole, i.e., the roots of the following equation

$$r^2(r^{3+\alpha}\Sigma^{-\alpha/2} + g^2r^\beta\Sigma^{-\beta/2}) - 2Mr^{4+\alpha}\Sigma^{-\alpha/2}$$

$$+ a^2(r^{3+\alpha}\Sigma^{-\alpha/2} + g^2r^\beta\Sigma^{-\beta/2}) = 0,$$

(4)

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{g} & \textbf{a} & \textbf{r_+}^H & \textbf{r_-}^H & \textbf{r_+}^{-1/2}^H & \textbf{r_-}^{-1/2}^H \\
\hline
0.3 & 0.58 & 1.80480 & 0.31720 & 1.47860 & 0.40222 \\
0.3 & 0.68 & 1.72142 & 0.38345 & 1.33707 & 0.47299 \\
0.3 & 0.78 & 1.61034 & 0.47826 & 1.13208 & 0.57087 \\
0.3 & 0.88 & 1.45062 & 0.62166 & 0.82896 & 0.68705 \\
\hline
0.4 & 1.02901 & 0.62166 & 0.31720 & 1.06144 & 0.61444 \\
\hline
\end{tabular}
\caption{The event horizon ($r^H_+$) and the Cauchy horizon ($r^H_-$) of the black hole and their difference $\delta^H = r^H_+ - r^H_-$ for $\alpha = 1, \beta = 2$ and different values of spin $a$, and constant $g$ (with $M = 1$ and $\theta = \pi/6$).}
\end{table}
which depends on \( a, g, \) and \( \theta, \) and which is different from the Kerr black hole. The largest possible root of Eq. (1) gives the location of the event horizon. We have studied the horizon properties for nonzero values of \( a \) and \( g \) (cf. Fig. [1,3] and Table III) by solving Eq. (1) numerically. We have demonstrated that for a given value of \( g, \) there exist extremal value of \( a = a_E \) and \( r = r_H^E \) such that for \( a < a_E, \) Eq. (1) admits two positive roots and no root at \( a > a_E \) (see Fig. [1,3]). It turns out that for \( \alpha = 1, \beta = 2, g = 0.3, a_E = 0.980078951651 \) and \( r_H^E = 1.02901 \) similarly for \( \alpha = \beta = 0, g = 0.3, a_E = 0.9745094360075 \) and \( r_H^E = 1.04466 \) (see Table II). It can be seen from the Table II and Table III that when the value of \( a \) increases then the event horizon \( (r_H^E) \) decreases and Cauchy horizon \( (r_H^C) \) increases. The difference \( (\delta^g) \) of both the horizons are also listed in Table II and Table III which decreases with increases \( a \) and equals to 0 for the case of extremal black hole, and when \( g \) increases we can see from the Table II and Table III that \( \delta^g \) also decreases. The Fig. [1] and Fig. [2] shows that there exist a set of values of parameters for which black hole \( (\beta = 0) \) has two horizons or we have a regular black hole with both Cauchy and event horizons. Further, one can find values of parameters for which these two horizons coincide and we get extremal black holes.

Next, we investigate the structure and location of the ergo-surface or static limit surface \( (r_H^{\text{sls}}) \) which requires coefficient of \( dt^2 \) to be zero. Then it follows from Eq. (1) that static limit surface satisfies

\[
\begin{align*}
\rho^2 &\left(r^{3+\alpha}\Sigma^{-\alpha/2} + g^2 r^{\beta}\Sigma^{-\beta/2}\right) - 2M r^{4+\alpha}\Sigma^{-\alpha/2} \\
&+a^2\left(r^{3+\alpha}\Sigma^{-\alpha/2} + g^2 r^{\beta}\Sigma^{-\beta/2}\right)\cos^2 \theta = 0.
\end{align*}
\]

The location of static limit surface is shown in Fig. [4] for different values of \( a \) and \( g. \) It is seen that when \( g = 0, \)

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**TABLE II:** The event horizon \((r_H^E)\) and the Cauchy horizon \((r_H^C)\) of the black hole and their difference \(\delta^g = r_H^E - r_H^C\) for \(\alpha = 0\) and different values of spin \(a, \) and constant \(g\) (with \(M = 1\) and \(\theta = \pi/6)).

| \(g = 0.3\) | \(g = 0.4\) |
|---|---|
| \(a\) | \(r_H^E\) | \(r_H^C\) | \(\delta^E\) | \(r_H^E\) | \(r_H^C\) | \(\delta^E\) |
| 0.58 | 1.80442 | 0.36513 | 1.43929 | 1.78999 | 0.45591 | 1.33408 |
| 0.68 | 1.72073 | 0.43301 | 1.28772 | 1.70287 | 0.53124 | 1.17163 |
| 0.78 | 1.69000 | 0.52439 | 1.08461 | 1.58430 | 0.63159 | 0.95271 |
| 0.88 | 1.44727 | 0.66341 | 0.78386 | 1.40249 | 0.79004 | 0.61245 |
| \(a_E\) | 1.04466 | 1.04466 | | 1.08796 | 1.08796 | |

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**FIG. 1:** Plots showing the behavior of \(\Delta\) vs \(r\) for \(\alpha = 1, \beta = 2, \theta = \pi/6\) and different values of \(a. \) Top: For \(g = 0.3\) (left), for \(g = 0.4\) (right). Bottom: For \(g = 0.6\) (left), for \(g = 0.7\) (right).
FIG. 2: Plots showing the behavior of $\Delta$ vs $r$ for $\alpha = \beta = 0, \theta = \pi/6$ and different values of $a$. Top: For $g = 0.3$ (left), for $g = 0.4$ (right). Bottom: For $g = 0.6$ (left), for $g = 0.7$ (right).

FIG. 3: Plots showing the behavior of $\Delta$ vs $r$ with $\theta = \pi/6$ and $M = 1$. (Left) For $\alpha = 1, \beta = 2, a = a_E = 0.980078951651$ and different values of $g$. (Right) For $\alpha = 0, \beta = 0, a = a_E = 0.9745094360075$ and different values of $g$.

Eq. (4) and (5) exactly the same as Kerr black hole [2]. The ergo-sphere is the region between the static limit surface and event horizon. It lies outside the black hole and it is possible to enter an ergo-sphere and leave again, a object moves in the direction of spin of the black hole. Interestingly the area of ergo-region decreases with increases in $a$ as well as same result in increase in $g$. Thus the event horizon of rotating Hayward’s regular metric is located at $r = r_E^{EH}$, where $\Delta = 0$, and it is rotating with angular velocity $\Omega_{EH}$. Whereas the static limit surface is located at $r = r_{sls}^{EH}$, when $\alpha = 0$. Further, one has extremal black holes, when $\Delta = 0$ has double roots in which case the two horizons coincides. The ergo-region is given by $r_E^{EH} < r < r_{sls}^{EH}$, where the Killing vector $\partial_{\tau}$ is spacelike. When $\Delta = 0$ has no root, i.e., no horizon exists, one gets naked singularity or no black hole (cf.
III. PARTICLES ORBITS

In this section, we would like to study the equations of motion for a particle with rest mass $m_0$ falling in the background of a rotating Hayward’s regular black hole. The geodesic motion of a particle in the rotating Hayward’s spacetime can be analysed by Hamilton-Jacobi equation. The Hamilton for the geodesic motion is given by

$$H = \frac{1}{2} g^{\mu\nu} P_\mu P_\nu, \quad (6)$$
where $P_\mu$ is the momentum. If $S = S(\lambda, x^\alpha)$ be the action which is a function of the parameter $\lambda$ and coordinate $x^\alpha$. Then the corresponding Hamilton-Jacobi equation

$$\frac{\partial S}{\partial \tau} = -\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu},$$

(7)

where $\tau$ is an affine parameter along the geodesics and $S$ is Jacobi action which is given by

$$S = \frac{1}{2} m_0^2 \tau - E t + L \phi + S_r(r) + S_\theta(\theta),$$

(8)

where $S_r$ and $S_\theta$ are function of $r$ and $\theta$ respectively. The constants $m_0, E,$ and $L$ correspond to rest mass, conserved energy and angular momentum of the particle through $m_0^2 = -\mu\nu^\mu$, $E = -p_t$, and $L = p_\phi$. In addition to $E$ and $L$, there is another conserved quantity, namely Carter constant $K$, which is related to total angular momentum. Inserting Eq. (8) into the Eq. (7), we get the following equations

$$\frac{\Sigma dt}{d\tau} = -a(aE \sin^2 \theta - L) + \frac{T}{\Delta},$$

(9)

$$\frac{\Sigma d\theta}{d\tau} = \pm \sqrt{K - \cos^2 \theta \left[ a^2(m_0^2 - E^2) + \frac{L^2}{\sin^2 \theta} \right]},$$

(10)

$$\frac{\Sigma d\phi}{d\tau} = -\left( aE - \frac{L}{\sin^2 \theta} \right) + \frac{aT}{\Delta},$$

(11)

$$\frac{\Sigma dr}{d\tau} = \pm \sqrt{T^2 - \Delta \left[ m_0^2 r^2 + (L - aE)^2 + K \right]},$$

(12)

where $T = E(r^2 + a^2) - La$ and $K$ is the Carter constant. For a particle moving in the equatorial plane and to remain in the equatorial plane, the Carter constant $K = 0$ \[30\]. In the limit $g = 0$, one recovers the equations of motion of the Kerr black hole \[1\].

To determine the range of the angular momentum of the particles, we need to calculate the effective potential. The radial equation for the timelike particle moving along the geodesic in the equatorial plane ($\theta = \pi/2$) is described by

$$\frac{1}{2} \dot{r}^2 + V_{eff} = 0,$$

(13)
TABLE III: The range for angular momentum \( L \), and value of \( a = a_E, r = r_H^E \) for different \( g \) for extremal rotating Hayward’s regular black hole.

| g   | a      | \( r_H^E \) | \( L_1(\text{min}) \) | \( L_1(\text{max}) \) |
|-----|--------|------------|----------------|----------------|
| 0.2 | 0.992150355200 | 1.01501   | -4.82270  | 2.03055       |
| 0.3 | 0.974509436007 | 1.04466   | -4.80977  | 2.09438       |
| 0.4 | 0.942997092861 | 1.08796   | -4.78643  | 2.19822       |
| 0.5 | 0.896105795904 | 1.13802   | -4.75113  | 2.34136       |
| 0.6 | 0.832765393050 | 1.18888   | -4.70236  | 2.52988       |

TABLE IV: The range for angular momentum \( L \), and value of \( a = a_E, r = r_H^E \) for different \( g \) for non-extremal rotating Hayward’s regular black hole.

| g   | a      | \( r_H^E \) | \( L_1(\text{min}) \) | \( L_1(\text{max}) \) |
|-----|--------|------------|----------------|----------------|
| 0.2 | 0.96   | 0.77195    | 1.26151   | -4.79983       | 2.37942       |
| 0.3 | 0.88   | 0.66341    | 1.44727   | -4.71464       | 2.66755       |
| 0.4 | 0.78   | 0.63159    | 1.58430   | -4.66671       | 2.90664       |
| 0.5 | 0.72   | 0.68597    | 1.62411   | -4.61962       | 3.01039       |
| 0.6 | 0.68   | 0.77888    | 1.61674   | -4.58625       | 3.05693       |

with the effective potential

\[
V_{\text{eff}} = -\frac{[E(r^2 + a^2) - La]^2 - \Delta m_0^2 r^2 + (L - aE)^2}{2r^4},
\]

The condition of circular orbit of the particles is given by

\[
V_{\text{eff}} = 0, \quad \text{and} \quad \frac{dV_{\text{eff}}}{dr} = 0.
\]

Since geodesics are timelike, i.e., \( dt/d\tau \geq 0 \), then Eq. (15) leads to

\[
\frac{1}{r^2} \left[ -a(aE - L) + (r^2 + a^2) \frac{T}{\Delta} \right] \geq 0,
\]

the above condition, as \( r \to r_H^E \), reduces to

\[
E - \Omega_H L \geq 0,
\]

\[
\Omega_H = \frac{a}{2mr_H^E} = \frac{a}{(r_H^E)^2 + a^2},
\]

where \( \Omega_H \) is the angular velocity of the black hole on the horizon. The limiting values of angular momentum of freely falling particles are calculated from Eq. (15) for both extremal and non-extremal rotating Hayward’s regular black hole which are listed in Table III and Table IV. The critical angular momentum of the particle is given by \( L_c = E/\Omega_H \) and from (17), \( L \leq L_c \). The values of critical angular momentum are listed in Table III for different combinations of spin \( a \) and constant \( g \). In Fig. 6 we plot \( \dot{r} \) vs \( r \) for different values of \( L, a \) and \( g \). It is shown that if the angular momentum of the particle is larger than the critical angular momentum, i.e., \( L > L_c \), then the geodesics never fall into the black hole. On the otherhand, if the angular momentum is smaller than the critical angular momentum \( L < L_c \), then the geodesics always fall into the black hole and if both are equal \( L = L_c \), then the geodesics fall into the black hole exactly at the event horizon. We plot the effective potential in Fig. 7 choosing different values of angular momentum \( L \). If the angular momentum of the particles is in the range, then effective potential is negative and particles are bounded. If the angular momentum of particle is outside the range, then the effective potential is always positive.

IV. CENTER-OF-MASS ENERGY IN THE ROTATING HAYWARD’S REGULAR BLACK HOLE

In the last section, we have found a range for angular momentum for which a particle can reach the horizon, i.e., if the angular momentum is in the range, the collision is possible near horizon of black hole. Next, we study the \( E_{\text{CM}} \) of two colliding particles moving in the equatorial plane of rotating Hayward’s regular black hole. Let us consider colliding particles have same rest mass \( m_1 = m_2 = m_0 \), and they are coming from rest at infinity with \( E_1/m_0 = E_2/m_0 = 1 \), approaching the black hole with different angular momenta \( L_1, L_2 \) and collide at some radius \( r \). We want to compute the collision energy of the particles in center-of-mass frame and explicitly bring out the effect of the parameter \( g \) on BSW mechanism. We observe that two particles with the four-momentum

\[
P_i^\mu = m_i u_i^\mu,
\]

where \( u_i^\mu \) the four-velocity of the particles \( i (i = 1, 2) \). The \( E_{\text{CM}} \) of two particles is given by

\[
E_{\text{CM}}^2 = -P_i^\mu P_i^\mu.
\]

Inserting, Eq. (18) into Eq. (19), with \( m_1 = m_2 = m_0 \), we obtain

\[
\frac{E_{\text{CM}}^2}{2m_0^2} = 1 - g_{\mu\nu} u_i^{\mu(1)} u_i^{\nu(2)},
\]

On substituting the values of \( g_{\mu\nu}, u_i^{\mu(1)} \) and \( u_i^{\mu(2)} \), the expression for \( E_{\text{CM}} \) of two colliding particles has the following simple form:

\[
\frac{E_{\text{CM}}^2}{2m_0^2} = \frac{1}{r(r^2 - 2mr + a^2)}[2a^2(m + r)
- 2am(L_1 + L_2)
- L_1L_2(-2m + r) + 2(-m + r)r^2
- \sqrt{2m(a - L_1)^2 - L_1^2 r + 2mr^2}
+ \sqrt{2m(a - L_2)^2 - L_2^2 r + 2mr^2}],
\]

where \( m = m_{\alpha,\beta}(r, \theta) \) is given by Eq. (3). Obviously the above result confirms that the parameter \( g \) indeed has
We apply l’Hospital’s rule twice, the value of \( n \)ominator of Eq. (21) is zero and so is the numerator.

FIG. 8: Plots showing the behaviour of \( E \) for non-extremal black hole. (Left) For \( a = a_E = 0.9745094360075 \) and \( g = 0.3 \). (Right) For \( a = a_E = 0.9429970792861 \) and \( g = 0.4 \) where the vertical line denotes the event horizon.

FIG. 9: Plots showing the behaviour of \( E_{CM} \) vs \( r \) for extremal black hole. (Left) For \( a = a_E = 0.9745094360075 \) and \( g = 0.3 \). (Right) For \( a = a_E = 0.9429970792861 \) and \( g = 0.4 \) where the vertical line denotes the event horizon.

influence on the \( E_{CM} \), and when the deviation parameter vanish \( g = 0 \), the above equation reduces to

\[
\frac{E_{CM}^2}{2m_0^2} = \frac{1}{r(r^2 - 2Mr + a^2)} \left[ 2a^2(M + r) - 2aM(L_1 + L_2) - L_1L_2(-2M + r) + 2(-M + r)r^2 - \sqrt{2M(a - L_1)^2 - L_1^2r} + 2Mr^2 - \sqrt{2M(a - L_2)^2 - L_2^2r} + 2Mr^2 \right],
\]

which exactly same as obtained for the Kerr black hole [1]. We are interested to investigate the properties of \( E_{CM} \) as \( r \to r_{H}^E \). We observe that at \( r \to r_{H}^E \) the denominator of Eq. (21) is zero and so is the numerator. We apply l’Hospital’s rule twice, the value of \( E_{CM} \), as

\[
E_{CM}^2(r \to r_{H}^E) = 3.7942 + 0.0812(L_1 + L_2) - 0.0819L_1L_2 + \frac{G_1^2(L_2 - L_c)}{8(L_1 - L_c)^3} + \frac{G_2^2(L_1 - L_c)}{8(L_2 - L_c)^3} - \frac{G_1G_2}{4(L_1 - L_c)(L_2 - L_c)} - \frac{H_1(L_2 - L_c)}{4(L_1 - L_c)} - \frac{H_2(L_1 - L_c)}{4(L_2 - L_c)},
\]

where \( a = a_E = 0.9745094360075 \), \( r_{H}^E = 1.04466 \), \( g = 0.3 \), \( M = 1 \), \( G_i = 4.1189 - 0.0883L_i - 0.9554L_i^2, \)
\( H_i = 3.8008 + 0.3443L_i - 0.1735L_i^2 \) (\( i = 1, 2 \)), and \( L_c = E/\Omega_H = 2.09438 \). Eq. (23), gives the limiting values of the \( E_{CM} \) at the event horizon with critical angular momentum \( L_c \). Obviously, the \( E_{CM} \) diverges when \( L_1 = L_c \) or \( L_2 = L_c \) and \( E_{CM} \) is finite for other generic values of \( L_1 \) and \( L_2 \). Hence, we can say that an extremal rotating Hayward’s regular black hole can act as particle accelerator to an infinitely high energy and may
provide an effective framework for Planck scale physics. However, to get this infinite $E_{CM}$, the particles should be approaches the black hole with angular momentum in the range of angular momentum which is mentioned in Table [11]. Further, Fig. [8] depicts the variation of $E_{CM}$ vs $r$ for different values of $L_1$ and $L_2$ with fixed values of $a$ and $g$, it is clear that $E_{CM}$ blows up at the horizon when either $L_1$ or $L_2 = L_c$. It is clear that $L_c$ is in the range for which particle can reach the horizon of the black hole.

a. Particle collision near non-extremal black hole

Next, we want to study the properties of the $E_{CM}$ as $r$ tends to the event horizon $r_H^+$ in the case of non-extremal rotating Hayward’s regular black hole. At $r \rightarrow r_H^+$ both numerator and denominator of Eq. (24) vanish. So, we apply l’Hospital’s rule to find the near horizon $E_{CM}$ for non-extremal black hole. The $E_{CM}$ as $r \rightarrow r_H^+$ is given as

$$E_{CM}^2 = \frac{m_H^2}{2} \left[ \frac{1}{0.6657(L_3 - L_4)(L_4 - L'_4)} \left( 14.1523 + L_4(1.2332L_4 - 4.3409) + L_3(1.2332L_3 - 1.1349L_4 - 4.3409) \right) \right],$$

(24)

where $L'_4 = E/\Omega_H = 3.2602$. From Eq. (24), it seems that one can get infinite $E_{CM}$ if either $L_3$ or $L_4 = L'_4$. However, we have to guarantee that the particles with angular momentum $L'_4$ reaches the horizon or in other words $L'_4$ should be in the range of angular momentum with which the particles can reach the horizon and collision is possible. It can be seen from Table [10] the range of the angular momentum of the particles, for $g = 0.3$, and $a = 0.88$ is $L_4 < L < L_3$. It turns out that the value of $L'_4$ does not lie in the range and $L_c > L_3$, which means that in non-extremal rotating Hayward’s regular black hole, thus the particles with angular momentum $L = L'_4$ could not fall into the black hole. Hence, the $E_{CM}$ for non-extremal rotating Hayward’s regular black hole has a finite upper limit. For non-extremal black hole, the behaviour of the $E_{CM}$ vs $r$ can be seen from Fig. [9] for different values of parameters $a$ and $g$. We can also see from the Fig. [10] the $E_{CM}$ is increases with $g$.

V. CONCLUSION

The celebrated singularity theorems predicts formation of singularities in classical general relativity. However, it is widely believed that spacetime singularities do not exist in Nature, but that they represent a limitation or creation of the classical theory. As we do not yet have any well defined theory of quantum gravity, hence recently more attention is given for phenomenological approaches to somehow solve these singularity problem in classical general relativity and study possible implications. So an important line of research to understand the inside of a black hole is to investigate classical black holes and their consequences, with regular, i.e., nonsingular, properties. In view of this, we have discussed the features of horizons by stationary, rotating Hayward’s regular black hole and explicitly bring out effect of the deviation parameter $g$. It turns out that for each $g$, for proper choice of parameters $a$, $\beta$, $M$, and $\theta$, we can find critical value $a = a_E$, which corresponds to an extremal black hole with degenerate horizons, i.e., where two horizons coincides. However, when $a < a_E$, we have a regular black hole with outer and inner horizon. It turns out that horizon structure is complicated as compared to usual Kerr black hole. Thus extremal regular black hole depends on the value of $g$. Further, we have modified the original BSW mechanism suitable for rotating Hayward’s regular black hole, which has a complicated horizon structure as compared to usual Kerr black hole, but the main qualitative features of BSW mechanism still holds. Then we study the collision of two particles of equal rest masses falling freely from rest at infinity into the equatorial plane of extremal rotating Hayward’s regular black hole to calculate $E_{CM}$, for various values of $g$, which are infinite if one of the colliding particles has the critical angular momentum.
On the other hand $E_{CM}$ has always finite upper limit for non-extremal black hole. Thus BSW mechanism depends both on rotation parameter $a$ as well as on deviation parameter $g$. For non-extremal black hole, we have also seen the effect of $g$ on the $E_{CM}$ which shows an increase in the value of $E_{CM}$ with an increase in the value of $g$. In particular, our results in the limit $g \to 0$, reduced exactly to vis-à-vis Kerr black hole results.

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