Research Article

In-Plane Impact Resistance of a Diamond-Shaped Hierarchical Honeycomb

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1. Introduction

The development of modern engineering has put forward higher requirements for material performance, and the emergence of metamaterials breaks the situation that the performance of materials only depends on the inherent characteristics of materials in engineering applications. Metamaterials have properties such as negative mass density [1] and ultra-high stiffness [2] and are considered as the main candidate materials for the next generation of metamaterials. Hierarchical honeycomb materials, as one of the metamaterials, have been studied and explored by many scholars.

The first-order honeycomb materials are honeycomb materials without hierarchical structures (such as traditional honeycombs). Honeycomb material is a low-density porous solid with good specific stiffness and strength [3, 4]. In addition, honeycomb materials also perform well in heat insulation and energy absorption [5–7]. Honeycomb materials are mainly used in the form of honeycomb sandwich plates and are widely used in vehicle engineering [8, 9], marine engineering [10], and aerospace engineering [11, 12].

Among various first-order honeycomb materials, the first-order hexagonal honeycomb structures have been widely used due to their low cost. People have conducted a series of explorations on first-order hexagonal honeycomb materials, such as its in-plane stiffness [13], the relationship between impact velocity and energy absorption [14], and the influence of wall thickness of honeycombs on deformation [15]. For example, McFarland [16] studied the situation of hexagonal honeycomb subjected to out-of-plane impact and derived the average stress theory. Wierzbicki [17] studied hexagonal honeycomb structure under quasi-static axial load. Through the combination of experiment and finite element (FE) method, Yamashita and Gotoh [18] studied the influence of cell shape and thickness on structural strength of honeycomb under out-of-plane impact. Wang et al. [19] analyzed the mechanical response of the honeycomb structure under high-speed impact. Zhang et al. [20] studied the out-of-plane crushing of aluminum honeycomb structures by experimental and FE simulation, and the effect of the number of cellular components and angle on its impact resistance was obtained. The research studies on other shapes of first-order honeycombs are as follows. Researchers...
used a combination of theoretical analysis and experiments to explore the energy absorption efficiency of square honeycombs [21, 22]. Qiu et al. studied the triangular structure, hexagonal structure, square structure, Kagome structure, and rhombus structure, obtained the relationship between the relative density of each structure and the collapse stress, and concluded that the diamond structure is a membrane-oriented structure. Both hexagonal honeycomb and diamond honeycomb are dominated by bending, and the failure strength of typical hexagonal and diamond structures has the same initial upper and lower bounds. When the inertia effect is considered, the average failure stress of the two structures increases with the increase of impact velocity, substrate density, and lattice relative density. The failure stress of hexagonal honeycomb structure is higher than that of diamond structure under quasi-static compression, but the diamond honeycomb structure is simpler and the utilization rate of space is higher than that of hexagonal structure, and the hierarchical structure of diamond honeycomb is easier to realize [23]. Some first-order honeycombs with negative Poisson’s ratio effect, such as double-arrow structure [24] and concave hexagonal structure [25], have also attracted many scholars to explore.

The second-order honeycomb materials have an additional structural level, that is, the cell edges/walls of the first-order structure are themselves cellular. Compared with first-order honeycomb structures, the research studies on the honeycombs with hierarchical structures are relatively insufficient, and most hierarchical honeycombs are still in the theoretical stage; this is because the current industrial level is difficult to meet the manufacturing requirements of hierarchical structures. In the current research, one school uses the hierarchical honeycombs with hexagonal structure as the macrostructure, which is a hot research direction. The two design concepts in this research school are as follows. The first is to replace the cell wall of the original anisotropic hexagonal honeycomb with triangular substructures [26], Kagome substructures [27], reinforced double-arrow substructures [28], or smaller hexagons [29] to obtain a hierarchical honeycomb. Some scholars use substructures with a negative Poisson’s ratio effect to replace the original anisotropic hexagonal cell wall to design a hierarchical honeycomb [30]. The second is to use self-similar structures to design hierarchical honeycombs, for example, iteratively replace each three-sided vertex of the honeycomb network with a smaller second-order regular hexagon [31] or add subhexagon iteratively on the four corners of the original hexagon [32]. There is another research school that uses other structures as macrostructures to design hierarchical honeycombs, such as square [33] and circular [34]. These programs have also achieved good results.

Honeycomb structure has good heat insulation and bearing capacity, so it is widely used in modern industry. On the other hand, aluminum alloy has excellent mechanical properties and relatively low price. Therefore, the honeycomb structure studied in this paper uses aluminum alloy materials. In this paper, we replace the macrostructure of hierarchical honeycomb studied by Qiao and Chen [26] with diamond structure, design a triangle structure as substructures, and compare the structure with first-order diamond honeycomb (FDH) to explore the advantages of DHH. We use a combination of numerical simulation and theoretical derivation and compare the results. It is concluded that the collapse stress of the DHH is twice that of the FDH and has a better energy absorption effect. This provides a reference for the development of metamaterials in the future.

This article is organized as follows. In Section 2, we introduce the materials used in the DHH and the theory and research methods of this article. In Section 3, first we use FE simulation to obtain the static collapse stress of the DHH in two directions in the plane and compare it with the result of theoretically derived equations. We also discuss the influence of the number of different substructures on the theoretical derivation results. Next, we study the dynamic collapse response of the DHH. Similarly, we use a combination of numerical simulation and theoretical derivation and compare the results. In Section 4, we give conclusions.

2. Materials and Methods

When determining the mechanical model of aluminum alloy, we ignored the influence of strain change rate because aluminum alloy is not sensitive to strain rate. By observing the stress-strain diagram of aluminum alloy, we found that the mechanical model of aluminum alloy is very similar to that of ideal elastoplastic materials, so we set the aluminum alloy material as an ideal elastoplastic material. The material of these honeycombs is aluminum alloy AA6063-T6, and the material properties are as follows: the material density is 2700kg/m³, the elastic modulus is $E_S = 70$ GPa, the yield stress is $\sigma_S = 110$ MPa, and Poisson’s ratio is $\nu_S = 0.3$. The model is shown in Figure 1. The collapse response is studied in two directions, and the $x$ and $z$ directions are shown in Figure 1.

2.1. Theoretical Method of Quasi-Static Collapse Analysis of the DHH in the $x$ and $z$ Directions. According to the FE deformation diagram of the DHH, an analytical model of the collapse stress of the DHH is constructed. A two-scale research method is used in this paper, that is, the destruction analysis of the entire unit is carried out at both macro and microscales, and then the theoretical equations are derived. In the analytical model, the collapse of the whole structure is caused by the destruction of substructures, and the destruction of substructures can be quantified by the collapse stress, and this is the core of the method.

2.2. Theoretical Methods of Dynamic Collapse Analysis of the DHH in the $x$ and $z$ Directions. In this deformation mode, the dynamic collapse stress is obtained by the law of conservation of momentum. This method is verified by DHHs under different impact speeds and with relative densities, and the theoretical calculation results and numerical simulation results are very close.
3. Results and Discussion

3.1. The DHH and FE Modeling. Adding hierarchical structures to a first-order diamond honeycomb can effectively improve the mechanical properties of the honeycomb. This paper proposes a new type of DHH and systematically studies its response. The DHH structure is shown on the left side in Figure 1. The DHH is compressed uniaxially. Enlarge a subunit of the DHH, and its structure is shown on the right side in Figure 1. The cell wall of the subunit is composed of equilateral triangles, \( l \) is the edge length of the equilateral triangle, \( N \times l \) is the macroscopic edge length of the subunit, and \( N \) is the number of substructures on each edge. Assuming that the thickness \( h \) of the cell wall of the substructure is uniformly distributed, set the out-of-plane width of the honeycomb to \( b \). The relative density of the DHH is \( \rho \). Relative density is a very important concept. It refers to the ratio of the density of a substance to the density of a reference substance under their respective specified conditions. As a very important physical quantity of honeycomb materials, it is a key factor to measure the properties of materials. Through calculation, the relative density of the DHH is

\[
\rho = \frac{4(7N - 8)}{\sqrt{3}N^2} \left( \frac{h}{l} \right).
\]

The FE simulation of the DHH was carried out using the commercial software Abaqus. In the FE model, there are \( 6 \times 4 \) elements (that is, there are 6 substructures in the \( x \) direction and 4 substructures in the \( z \) direction) in Abaqus, the honeycomb is constructed with 4-node quadrilateral shell elements. 5 integration points are set on the thickness of the cell wall, and the element type when dividing the mesh is set to S4R (S4R is a universal shell element type in Abaqus). It is a four node curved shell element that can be used to simulate thin or thick shell structures. Set \( l = 0.01m \), \( b = 0.002m \), and \( N = 12 \). We place the DHH between two rigid plates and limit the out-of-plane displacement. Fix the bottom plate, set the upper bottom plate to a constant speed, and perform impact simulation. The overall model uses general contact, the normal behavior is hard contact, and the tangential behavior is frictionless. Before performing the FE simulation, a mesh convergence analysis should be first performed. As shown in Figure 2 (in Figure 2, \( EA \) represents energy absorption and \( S \) represents the distance traveled by the rigid plate), we found that the grid has converged when the grid size is 1.4 mm, so this grid size is used in the following grid generation.

3.2. Validation of the FE Modeling. The FE results are compared with the experimental results to verify its validation. The FE model used is the same as that in experiment [35], and the wall thickness is set to 0.1 mm. The structure is compressed by the rigid plane at a constant speed of 0.5 mm/min in the vertical direction. The results are shown in Figure 3; in the experiment, due to the fact that there are errors in various sizes such as the shape and thickness of the processed aluminum honeycomb unit in the manufacturing of aluminum alloy honeycomb, the current level of processing technology is insufficient, resulting in the irregular shape of the processed honeycomb units, which is different from the ideal shape preset in the finite element analysis. In addition, the overall shape of the aluminum alloy specimen used in the experiment is irregular after cutting, which will also lead to errors and difference between results. Overall, the trend of the finite element results curve is roughly consistent with the experimental curve, and the error is within the allowable range, so that we verify the validation of the FE results of the hierarchical honeycomb.

3.3. Quasi-Static Collapse of the DHH

3.3.1. Uniaxial Compression in the \( x \) Direction. In this section, we simulated the quasi-static collapse response of the DHH under uniaxial compression in the \( x \) direction. Set the impact velocity of the rigid plate to 1m/s, which can approximately simulate the quasi-static collapse of the honeycomb. In order to reflect the superiority of the mechanical properties of the DHH, we set up a set of comparative tests. The relative density of the FDH and the DHH
settto ρ \( \rho \) is set to \( \rho = 5\% \). The FDH has the same size as the DHH (that is, there are 6 substructures in the \( x \) direction and 4 substructures in the \( z \) direction; the macroscopic edge length of the unit is 0.12 m; in Abaqus, these two models have the same element type and the same contact form).

The result is shown in Figure 4. The strain-stress diagram of the honeycombs can be clearly divided into three stages. The first stage is the linear elastic zone. As shown in Figure 4, in the linear elastic zone, the DHH has reached a higher stress level when the strain is small. This is because the stiffness coefficient of the material with hierarchical structure is larger than that of the material without hierarchical structure. After the linear elastic zone, the deformation enters the second stage, which is the plateau zone. The stress value of the platform area can be used as an important indicator of the energy absorption of the honeycomb materials. We define the strain of the first-order cellular platform area as \( \varepsilon_d = 1 - 1.47 \) in the platform area of the FDH is not suitable for that of the DHH but is defined as \( \varepsilon_d = 0.8 (1 - \rho) \) according to [26]. The strain is not only related to the energy absorption capacity of the quasi-static collapse but also closely related to the stress value of the dynamic collapse, which will be introduced in the following sections. In the end, we get that the quasi-static collapse stress of the DHH is twice that of the FDH. The energy absorption of honeycomb material is mainly determined by the stress in the platform area, and the dense area is not the key to measure the energy absorption. Obviously, the DHH has better energy absorption.

1. **Collapse Mode in the \( x \) Direction.** Figure 5 shows the deformation mode of the DHH in the \( x \) direction. With the impact loading of the rigid plate, the honeycomb begins to deform. As shown in Figure 5(a), when the strain \( \varepsilon = 6\% \), a “/” type crushing band is formed in the honeycomb. Continue to load, and when the strain \( \varepsilon = 12\% \), as shown in Figure 5(b), the honeycomb forms a “X” type band, which is similar to the initial deformation mode of regular hexagonal hierarchical honeycombs. Continue to load; as shown in Figure 5(c), when the strain \( \varepsilon = 24\% \), the honeycomb forms a “V” type band. The honeycomb continues to deform, and in the process from \( \varepsilon = 24\% \) to \( \varepsilon = 48\% \), the band transitions from “V” type to “—” type. When the strain \( \varepsilon = 48\% \), as shown in Figure 5(d), a new crushing band is formed due to the local “compaction” formed in the upper part of the honeycomb. Continue to load until the entire honeycomb is destroyed. By comparison, the FDH has the same collapse mode as the DHH.

2. **Collapse Stress in the \( x \) Direction.** According to the deformation mode simulated by FE method, an analytical model of the collapse stress of the DHH can be obtained. For a better description, we enlarge the details of the deformation in Figure 5(a) to obtain Figure 6(a). Then, as shown in Figure 6(b), we constructed a simplified model of the DHH. Here we use a two-scale method to analyze the model. It can be seen from the honeycomb destruction diagram that the collapse of the honeycomb is caused by the deformation of the subunits. To analyze the process, as shown in Figure 6(b), the plastic hinges of the macrostructure are marked with black dots, and the structure in the green box is divided into a unit cell. Consider the deformation of the macromodel first. After the deformation, the red line is rotated by a certain angle and shortened.

According to the deformation diagram in Figure 6(b), the displacement of the unit cell is \( \Delta x = Nl/2 \), and the work done by the external force is
pression is time about an equilateral triangle. Here, the dissipation of the two hinges in the unit cell is

\[ W_{\text{ex}} = \sqrt{3} Nl \sigma_z \Delta x \]

where \( \sigma_z \) is the uniform stress at the distal end.

Consider the rotation of the red line segment. The plastic dissipation of the two hinges in the unit cell is \( W_h = 2M_p \Delta \theta \), the plastic bending moment of the cell wall in the macrostructure is \( M_p = 3bl^2 \sigma^{\text{tri}} / 4 \), and the rotation angle is \( \Delta \theta = \pi / 6 \), so the plastic dissipation of the unit cell is

\[ W_h = \frac{b l^2 \pi \sigma^{\text{tri}}}{4} \]

where \( \sigma^{\text{tri}} \) is the collapse stress of the equilateral triangle substructure. It can be obtained from [26] that

\[ \sigma^{\text{tri}} = C \bar{P}_{\text{tri}} \sigma_z \]

where \( \bar{P}_{\text{tri}} = 7h / \sqrt{3l} \) represents the relative density of equilateral triangles. Here \( C = 0.534 \) is a constant irrelevant to time about an equilateral triangle.

In addition, the dissipation energy of red line compression is

\[ W_d = \frac{Nl^2 b}{2} \sigma_z^{\text{tri}} \]

Finally, work done by external force is equivalent to energy dissipation, namely, \( W_{\text{ex}} = W_h + W_d \). Therefore, the plastic collapse stress of the DHH under uniaxial compression in the \( x \) direction is

\[ \sigma_x = \frac{0.534(2N + \pi)}{2\sqrt{3N^2}} \left( \frac{7h}{\sqrt{3l}} \right)^{2} \sigma_z \]

We obtain the quasi-static collapse response of DHHs with three different relative densities (2.5\%, 5\%, and 7.5\%), and the results are shown in Figure 7. The results show that the theoretical value and the FE simulation value fit well, and equation (6) successfully predicted the static collapse stress of DHHs in the \( x \) direction.

3.3.2. Uniaxial Compression in the \( z \) Direction. In the previous section, we analyzed the uniaxial compression of the DHH in the \( x \) direction. In this section, we will discuss the uniaxial compression of the DHH in the \( z \) direction. The collapse mode of the DHH is shown in Figure 8. Slightly different from the collapse mode in the previous section, we found that the “V” type band starts to form from the beginning of loading. Next, the honeycomb is destroyed layer by layer along the “V” type band until the honeycomb is destroyed. Follow the analysis in the previous section and enlarge the details in Figure 8(a) to get Figure 9(a). It can be seen from Figure 9(a) that the red line is shortened and rotated a certain angle until it is horizontal. This deformation mode is the same as that of the DHH in the \( x \) direction. The macroscopic deformation of the DHH structure in the \( z \) direction is shown in Figure 9(b).

From the deformation mode shown in Figure 9(b), we can derive an analytical model of the collapse stress of the DHH in the \( z \) direction, and the derivation process is similar to that in the previous section. The external work on the unit (marked by the green dashed frame) is

\[ W_{\text{ex}} = \frac{Nl^2 b}{2} \sigma_z \]

where \( \sigma_z \) is the uniform stress at the distal end.

Considering the rotation of the red line segment, the plastic dissipation of the two hinges in the unit structure is \( W_h = 2M_p \Delta \theta \). The plastic bending moment of the cell wall in the macrostructure is \( M_p = 3bl^2 \sigma^{\text{tri}} / 4 \), and the rotation angle is \( \Delta \theta = \pi / 3 \). Therefore, the plastic dissipation of the unit is

\[ W_h = \frac{b l^2 \pi \sigma_z^{\text{tri}}}{2} \]

The dissipation energy of red line compression is

\[ W_d = \frac{\sqrt{3} Nl^2 b}{2} \sigma_z^{\text{tri}} \]

Substitute equations (7)–(9) into the equation \( W_{\text{ex}} = W_h + W_d \) to get

\[ \sigma_z = \frac{0.534(\sqrt{3} N + \pi)}{\sqrt{3N^2}} \left( \frac{7h}{\sqrt{3l}} \right)^{2} \sigma_z \]

Same as the previous section, three sets of FE models with relative densities of \( \bar{p} = 2.5\% \), \( \bar{p} = 5\% \), and \( \bar{p} = 7.5\% \) are also set to verify equation (10). The results are shown in Figure 10. The FE results and the prediction results of equation (10) fit very well.
3.3.3. Effect of the Number of Substructures. In the previous subsections, we fixed the number of substructures \((N = 12)\) of the DHH to start a series of studies on quasi-static collapse. In this section, we change the number of substructures, that is, discuss the accuracy of equations (6) and (10) when \(N\) changes. We set \(N = 8 - 12\) and the relative density of each model is 5%. For each model, the impact is applied in the \(X\) direction and \(Z\) direction, respectively. Figure 11 shows a schematic diagram of the subunits of each structure. As \(N\) becomes larger, the side length of the cell wall also becomes larger. We did 12 sets of simulations, and the results are shown in Tables 1 and 2. We make these two tables into Figure 12. The results show that the

Figure 5: FE prediction of quasi-static collapse of the DHH in the \(x\) direction, \(N = 12\) and \(\rho = 5\%\). (a) \(\epsilon = 6\%\). (b) \(\epsilon = 12\%\). (c) \(\epsilon = 24\%\). (d) \(\epsilon = 48\%\).

Figure 6: (a) Enlarged DHH deformation diagram, \(N = 12, \rho = 5\%\), and \(\epsilon = 6\%\). (b) Schematic of the deformation of the subunits under impact loading in the \(x\) direction.
Figure 7: FE prediction of the quasi-static collapse response of DHHs with different relative densities, $N = 12$. (a) $\bar{\rho} = 2.5\%$. (b) $\bar{\rho} = 5\%$. (c) $\bar{\rho} = 7.5\%$. The direction of impact is $x$.
theoretical value and the FE simulation value fit well, and the error is within the allowable range. As $N$ becomes larger, the error between the theoretical value and the FE simulation value gradually decreases, that is, the larger $N$ is, the better the results fit. This is due to the slight difference in the deformation mode when $N$ changes, but all errors are within the allowable range. The above results and discussion show the great accuracy of equations (6) and (10).

3.4. Dynamic Collapse Response. In this section, we will analyze the dynamic collapse response of the DHH. As in the quasi-static collapse analysis, we set $\bar{\rho} = 5\%$ and $N = 12$ in this section. First look at the collapse mode of the DHH in the $x$ direction. We set up 2 sets of comparative tests. The impact velocity of the first group is 50 m/s, and the second group is 200 m/s. The result is shown in Figure 13, and we find that the DHH presents a layer-by-layer collapse feature in the $x$ direction, and the faster the speed is, the more obvious the layer-by-layer collapse feature is. Under the same setting, the collapse mode of the DHH in the $z$ direction is shown in Figure 14. We see that the collapse mode of the DHH in the $z$ direction is consistent with that in the $x$ direction, and it also shows a layer-by-layer collapse feature, and the faster the speed is, the more obvious the layer-by-layer collapse feature is.
Figure 10: FE prediction of the quasi-static collapse response of DHHs with different relative densities, $N = 12$. (a) $\bar{\rho} = 2.5\%$. (b) $\bar{\rho} = 5\%$. (c) $\bar{\rho} = 7.5\%$. The direction of impact is $z$.

Figure 11: Schematic diagram of DHHs when $N = 8 \sim 12$.

Table 1: The quasi-static collapse stress of DHHs under uniaxial compression in the $x$ direction, $N = 8 \sim 12$ and $\bar{\rho} = 5\%$.

| Number of $N$ | 8    | 9    | 10   | 11   | 12   |
|---------------|------|------|------|------|------|
| Analytical (MPa) | 0.069 | 0.074 | 0.078 | 0.083 | 0.086 |
| FE (MPa)      | 0.095 | 0.090 | 0.092 | 0.089 | 0.085 |

Table 2: The quasi-static collapse stress of DHHs under uniaxial compression in the $z$ direction, $N = 8 \sim 12$ and $\bar{\rho} = 5\%$.

| Number of $N$ | 8    | 9    | 10   | 11   | 12   |
|---------------|------|------|------|------|------|
| Analytical (MPa) | 0.124 | 0.131 | 0.139 | 0.146 | 0.155 |
| FE (MPa)      | 0.150 | 0.155 | 0.160 | 0.165 | 0.170 |
According to the collapse modes shown in Figures 13 and 14, the dynamic collapse stress can be obtained from the law of conservation of momentum [24]. In the process of densification of the DHH, its velocity changes from 0 to a constant speed \( V \). The impulse of the external force is given by the following equation:

\[
I = A \int_0^t (\sigma_d - \sigma_e) dt,
\]

where \( A \) is the cross-sectional area of the subunit, \( \sigma_d \) is the dynamic collapse stress, \( t = \epsilon_d H/V \) is the time from collapse to densification, and \( H \) is the height of the subunit.

\( \sigma(MPa) \)

|   | FE | Analytical |
|---|----|------------|
| 0.00 | 91 | 0.00 |
| 0.05 | 11 | 0.10 |
| 0.10 | 11 | 0.15 |
| 0.15 | 12 | 0.20 |
| 0.20 | 28 | 0.25 |
| 0.25 | 30 | 0.30 |

Figure 12: The quasi-static collapse stress of DHHs under uniaxial compression, \( N = 8 \sim 12 \) and \( \rho = 5\% \). (a) Impact in the \( x \) direction. (b) Impact in the \( z \) direction.

Figure 13: FE prediction of the dynamic collapse of the DHH in the \( x \) direction, \( \rho = 5\% \) and \( N = 12 \). (a) \( V = 50\text{m/s}, \epsilon = 10\% \). (b) \( V = 50\text{m/s}, \epsilon = 17\% \). (c) \( V = 200\text{m/s}, \epsilon = 10\% \). (d) \( V = 200\text{m/s}, \epsilon = 17\% \).
In the previous chapters, we set $\epsilon_d = 0.8 (1 - \overline{p})$. However, $\epsilon_d = 0.8 (1 - \overline{p})$ is not suitable for high-speed impact (that is, speeds up to 200 m/s). Instead, we use the following equation according to [26].

$$
\epsilon_d = \begin{cases} 
0.8 + 0.2V/V_0 & (1 - \overline{p}), \\
(1 - \overline{p}) & V > V_0,
\end{cases} \quad (12)
$$

where $V_0$ is a parameter related to the microstructure and sample size, $V_0$ is a microstructure and specimen size dependent parameter, and its value is obtained through theoretical derivation and experiment [36]. In this study, $V_0 = 200$ m/s.

The momentum change of the unit cell can be described by the following equation:

$$
\Delta P = AH\overline{p}\rho_s V. \quad (13)
$$

According to the law of conservation of momentum, the impulse of the external force is equal to the change of the momentum of the unit cell, namely,

$$
I = \Delta P. \quad (14)
$$

It can be obtained from equations (11)–(14) that

$$
\sigma_d = \sigma_x + \overline{p}\rho_s \frac{V^2}{\epsilon_d}, \quad (15)
$$

where $\sigma_d$ is the dynamic collapse stress in the $x$ direction. To denote the dynamic collapse stress in the $z$ direction, replace $\sigma_x$ with $\sigma_z$.

Figures 13 and 14 show the stress-strain diagram of dynamic collapse of the DHH in the $x$ direction and $z$ direction when the impact velocity is $50$ m/s and $200$ m/s. From Figure 15, we can see that the collapse stress predicted by equation (15) is very good. Next, change the speed and keep the other values unchanged to observe the influence of the speed change on the theoretical value and the FE simulation value. The result is shown in Tables 3 and 4. We make these two tables into Figure 16. It can be clearly seen from the figure that the theoretical value and the FE simulation value of the dynamic collapse stress of the DHH fit well. In addition, it can be seen from the figure that the dynamic collapse stress of the DHH in the $x$ direction and $z$ direction is basically the same. This is because when the speed is high, the inertia effect becomes the main factor of the collapse stress, and when the speed is low, the effect of structure plays a major role.
4. Conclusions

In this article, we analyze the quasi-static and dynamic collapse response of DHH under uniaxial impact in the $x$ and $z$ directions. By analyzing the quasi-static collapse in the $x$ direction, we found that the crushing band in the DHH changed from “/” type to “V” type and finally to “—” type, which finally led to the destruction of the entire honeycomb. The result of quasi-static collapse in the $z$ direction is slightly different. After a uniaxial impact is applied in this direction, only “V” type bands appear in the honeycomb, and finally the whole honeycomb is destroyed. Under the quasi-static

![Graphs and tables showing FE and Analytical results for quasi-static and dynamic collapse in x and z directions.]
impact, we found that the static collapse stress of the DHH is twice that of the FDH, that is, the collapse stress of the DHH is well improved by the hierarchical structure, and the DHH has a better energy absorption effect than the FDH. Then, we analyzed the influence of the change of $N$ on the substructure. The results show that with the increase of $N$, the simulation value is closer to the theoretical value, and the error of all results is within the allowable range.

In the dynamic collapse response analysis, we analyzed the collapse response of the DHH in two directions and found that both the $x$ direction and the $z$ direction showed a layer-by-layer collapse feature, and the faster the speed is, the more obvious the layer-by-layer collapse feature is. Finally, we set different impact velocities and found that the theoretical value and the FE simulation value fit well, and with the increase of velocity, the dynamic collapse stress in the $x$ and $z$ directions is more and more similar. This is because that at the time of quasi-static collapse, the structural design plays a leading role in the collapse stress, and at the time of dynamic collapse, inertia plays a leading role.

Data Availability

All data, models, and codes included in this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

HS was responsible for validation and formal analysis. HS and BM were responsible for investigation and resources. CZ was responsible for data curation. HS and PW were responsible for original draft preparation. CZ and PW were responsible for review and editing. ZW was responsible for supervision, project administration, and funding acquisition. All authors have read and agreed to the published version of the manuscript.

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