Ghost Tachyon Condensation, Brane-like States and Extra Time Dimension

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Abstract

NSR superstring theory contains a tower of physical vertex operators (brane-like states) which exist at non-zero pictures only, i.e. are essentially mixed with superconformal ghosts. Some of these states are massless, they are responsible for creating D-branes. Other states are tachyonic (called ghost tachyons) creating a problem for the vacuum stability of the NSR model. In this paper we explore the role played by these tachyonic states in string dynamics. We show that the ghost tachyons condense on D-branes, created by massless brane-like states. Thus the vacuum stability is achieved dynamically, as the effective ghost tachyon potential exactly cancels the D-brane tension, in full analogy with Sen’s mechanism. As a result, from perturbative NSR model point of view, massless and tachyonic brane-like states appear to live in a parallel world, as the brane is screened by the tachyonic veil. We extend the analysis to the brane-antibrane pair in AdS space and show that in this case due to the effect of the ghost tachyon condensation one can construct extra time-dimensional phenomenological models without tachyons and antibranes.

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1. Introduction

Recently it has been conjectured from string field theoretic arguments that effective potential of the tachyonic mode of non-BPS Dp-branes in type II string theory has an important property of universality, that is, the form of the tachyon potential, bounded from below, remains the same for different Dp-brane backgrounds and independent on specifics of boundary conformal field theory \[1, 2\]. An important argument in favour of such a conjecture comes from open string field theory (OSFT); Namely, the off-shell computation of the effective tachyon potential in OSFT (truncated at the lowest massive level) shows that the OSFT tachyon potential is bounded from below; its value at its minimum is negative and precisely cancels the non-BPS Dp-brane tension (or the tension of brane-antibrane pair). As a result of such a screening of D-branes, caused by the condensation of tachyon at minimum point of the potential, one effectively obtains the type II flat vacuum without branes, with full ten-dimensional Lorentz symmetry restored \[1, 2\]. In standard evaluations of the tachyon potential in open superstring field theory one usually imposes strict constraints on the form of possible off-shell CFT vertex operators that contribute to the effective OSFT Wess-Zumino type action. In particular, one fixes the picture-changing gauge symmetry by restricting the vertices to have picture 0 (with the picture/ghost number assignment adopted in \[3\]); such a gauge fixing is needed to avoid summing over physically identical OSFT modes (which differ by a picture changing transformation). So one starts with the tachyonic state at the lowest level of OSFT; consequently, the higher level states are constructed by acting on the tachyon vertex operator with various combinations of stress tensor, supercurrent and ghost fields \[3\]. Such a consistent truncation of the OSFT is sufficient to determine the effective potential of tachyonic mode in the Dirichlet strings in flat background. At the same time, this derivation does not take into account the OSFT modes which exist at non-zero pictures only, but which do play a significant role in the open string dynamics. In NSR critical string theory these vertex operators correspond to a specific class of NS physical states (BRST invariant and BRST nontrivial vertex operators) which are not associated with any perturbative particle emission but appear to play an important role in the non-perturbative physics. For example in the previous works it has been shown that inserting this new class of operators to NSR string theory is equivalent to introducing branes; In particular, some of these operators dynamically deform flat ten-dimensional space time to $AdS_5 \times S^5$ background \[4\].
Generally, these states are described by vertex operators that exist at nonzero ghost pictures only, not admitting ghost number zero representation. This crucially distinguishes them from usual perturbative open or closed string states, such as photon, graviton or dilaton, which in principle are allowed to exist at any arbitrary ghost picture. This new class of states appears in both closed and open superstring theories and includes both massless and tachyonic states. In open string theory the massless ones are represented by two-form and five-form vertex operators; they are given by:

\[
V_{m_1...m_5}(z, k) = e^{-3\phi} \psi_{m_1}...\psi_{m_5} e^{ikX}
\]

\[
V_{m_1...m_5}(k) = e^{\phi} \psi_{m_1}...\psi_{m_5} e^{ikX}(z) + \text{ghosts}
\]

\[
V_{m_1m_2} = e^{-2\phi} \psi_{m_1} \psi_{m_2} e^{ikX}
\]

These dimension 1 primary fields must also be integrated over the worldsheet boundary (or multiplied by the fermionic ghost field \(c\)). Apart from the massless (on shell) 5-form vertex operator one can also construct the tachyonic 4-form state in the \(-3\)-picture given by

\[
V_{m_1...m_4} = e^{-3\phi} \psi_{m_1}...\psi_{m_4} e^{ikX}(z)
\]

as well as the two-form:

\[
V_{ab} = e^{-3\phi} \partial X_a \partial X_b e^{ikX}
\]

As is easy to see, the on-shell condition for this state is given by \(k^2 = -1\) Continuing this way, one may also construct tachyonic states of bigger negative \(k^2\) in the \(-3\)-picture, i.e. the \(1\)-form, the \(2\)-form and the \(3\)-form, corresponding to the on-shell masses \(k^2 = -4, -3\) and \(-2\). Many important questions naturally arise with regard to these new open string massless and tachyonic states. As was stressed above, the 5-form vertex does not correspond to any perturbative open string mode, but is rather related to dynamics of branes. However, doesn’t this additional physical state in the NSR superstring spectrum lead to violation of unitarity of open string amplitudes? On the other hand, as for the \(-3\)-picture tachyonic states, their appearance clearly may create a problem for the vacuum stability. At the same time, in principle there is no reason to exclude these extra open string states from consideration, as they all appear to be physical - i.e. BRST invariant and BRST non-trivial, and the only reason why they were overlooked before is because of their mixing with ghosts - there is no picture zero representation for these operators. So clearly it is of interest to investigate the non-perturbative dynamics related to the brane-like massless and tachyonic states and in particular to understand how to reconcile their
appearance with unitarity and vacuum stability in string theory. In the present paper we shall attempt to address these questions showing that in fact the massless and tachyonic brane-like states “compensate” each other due to mechanism similar to the one proposed in the Sen’s conjecture. Namely, as has been mentioned above, introducing the 5-form vertex operator is equivalent introducing a D-brane with a certain mass (tension) to the theory; on the other hand, the tachyonic brane-like states (2) condense on the brane created by the 5-form, just like in the context of Sen’s conjecture usual tachyon condenses on a D-brane. Then we show, through open string field theory calculations, that the negative energy density of the condensate of the brane-like tachyon (2) (which we also refer to as the ghost tachyon) at the minimum of the effective potential precisely cancels the positive tension of the brane created by the 5-form vertex operator (1). This phenomenon is exactly analogous to what happens in the framework of Sen’s mechanism and we call it “parallel tachyon condensation”. The phenomenon of parallel tachyon condensation actually resolves both stability and unitarity problems at the same time: as the tachyon gets localized on the brane (related to the 5-form), the energy density at the potential minimum screens the positive brane tension and therefore the brane becomes “invisible” from the usual perturbative string theory point of view - and such a screening effectively resolves the unitarity issue. In fact, it appears as if the 5-form brane-like states live in a parallel world, invisible from perturbative string theory point of view.

This paper is organized as follows. In the section 2, we review the on – shell BRST properties of the brane-like states to show their invariance and non-triviality under BRST transformations - to stress their importance to open string and D-brane dynamics (for detailed analysis see [5]). In the sections 3,4,5, we develop the string field theory network to compute the effective ghost tachyon potential and also calculate the brane tension associated with massless brane-like vertex operators. In the section 6 we calculate the effective ghost tachyon potential and analyze its minimum. We show, by comparing the brane-tension and the minimum value of the potential, that the analogue of the the Sen’s conjecture does emerge in this case and the potential minimum cancels the brane mass. In the section 7 we will discuss the relation between the ghost tachyon vertex operators and tachyonic solutions for the equations of motion in theories with time-like extra dimension. We attempt to interpret the ghost tachyonic vertex operators (2), (3) as emission vertices for tachyonic KK modes which appear as the e.o.m. solutions in the above theories and analyze the effective potential of these tachyonic modes in the context of brane world
scenario with extra time-like dimension. Usually brane world scenarios are known to involve branes with negative tensions - which are actually fictitious and unnatural objects because of their instability. We argue, however, that tachyonic solutions in theories with extra time-like dimension can cure the antibrane instability as the effective potential for “tachyonic gravitons” is bounded from below and the tachyonic gravitons condense on the antibrane (with positive energy density at the potential minimum) and cancels the negative tension of the antibrane insuring its stability. As a result, due to such an “anti-Sen” scenario, one can consider phenomenological theories with extra time-like dimensions involving stable brane configurations: negative tension of a hidden brane is screened by the condensate of the tachyonic KK modes - the e.o.m. solutions (in the meantime the visible non-BPS brane with positive tension is to be stabilized by the usual tachyon condensation scenario). Finally, in the concluding section 8 we discuss some other possible implications of our results.

2. Brane-like states - review of BRST properties

The five-form picture \(-3\) operator \(V_{m_1\ldots m_5}^{(-3)}\) is BRST-invariant at any momentum, as is easy to see by simple and straightforward computations involving the BRST charge; however it is BRST non-trivial only if its momentum \(k\) is polarized along the 5 out of 10 directions orthogonal to \(m_1, \ldots, m_5\) (for any given polarization choice of \(m_i\)). Indeed, it is easy to see that the only BRST triviality threat for the 5-form operator may appear from the expression \(\{Q_{\text{brst}}, S_{m_1\ldots m_5}\}\) where \(S_{m_1\ldots m_5} = e^{\chi - 4\phi} \partial \chi (\psi \partial X) \psi_{m_1} \ldots \psi_{m_5} e^{ikX}\) - indeed, its commutator with the \(\oint \gamma (\psi \partial X)\) in the BRST charge does give the \(V_5\) operator, while the \(\oint \gamma^2 b\) of \(Q_{\text{brst}}\) commutes with \(S\). However, it is clear that if the momentum \(k\) is orthogonal to \(m_1, \ldots, m_5\) directions the \(S_{m_1\ldots m_5}\) is not a primary field: the \((\psi \partial X)\) part of it always has internal O.P.E. singularities with either \(\psi_{m_1} \ldots \psi_{m_5}\) or \(e^{ikX}\). As a result, whenever the momentum is orthogonal to \(m_1, \ldots, m_5\) directions, the O.P.E. of the stress-energy tensor with \(S_{m_1\ldots m_5}\) always has a cubic singularity. Therefore \(S_{m_1\ldots m_5}\) does not commute with the \(\oint cT\) term of \(Q_{\text{brst}}\) and \(\{Q_{\text{brst}}, S_{m_1\ldots m_5}\}\) does not reproduce the 5-form vertex operator \(V_{m_1\ldots m_5}^{(-3)}\). However, in case if the momentum \(k\) of \(V_{m_1\ldots m_5}^{(-3)}\) is longitudinal, i.e. is polarized along \(m_1\ldots m_5\) directions it is easy to see that the vertex operator becomes BRST trivial: indeed, it can be written as a BRST commutator with the primary field: \(\{Q_{\text{brst}}, C_{m_1\ldots m_5}\}\) where \(C_{m_1\ldots m_5} = e^{\chi - 4\phi} \partial \chi (\psi \partial X)^\dagger \psi_{m_1} \ldots \psi_{m_5} e^{ikX}\) with the supercurrent part \((\psi \partial X)^\dagger\) now polarized orthogonally to \(m_1, \ldots, m_5\), i.e. both to \(e^{ikX}\) and other worldsheet fermions of the 5-form. So we see that BRST non-triviality condition imposes significant constraints on
the propagation of the 5-form: namely, it is allowed to propagate in the 5-dimensional subspace transverse to its own polarization. This also is a remarkable distinction of this vertex operator from usual vertices we encounter in perturbative string theory; it is well known that those are able to propagate in entire ten-dimensional space-time. The two-form is also BRST-invariant at any k; although it is BRST-trivial at zero momentum as it can be represented as a commutator \( \{ Q_{\text{brst}}, e^{\chi - \frac{3}{4}\phi} \psi_{[m_1} \partial X_{m_2]} + e^{\chi - \frac{3}{4}\phi} (\psi \partial X) \psi_{m_1} \psi_{m_2} \} \), it becomes BRST non-trivial at non-zero momenta and again, in complete analogy with the 5-form case, its momentum must be orthogonal to the \( m_1, m_2 \) two-dimensional subspace, i.e. the two-form propagates in eight transverse dimensions.

Constructing the BRST-invariant version of the five-form at picture +1 is a bit more tricky since the straightforward generalization given by \( e^{\phi} \psi_{m_1}...\psi_{m_5} \) does not commute with two terms in the BRST current given by \( b\gamma^2 \) and \( \gamma \psi_m \partial X^m \). To compensate for this non-invariance one has to add two counterterms, one proportional to the fermionic ghost number 1 field \( c \) and another to the ghost number −1 field \( b \).

To construct these ghost counterterms one has to take the fourth power of picture-changing operator \( \Gamma^4 \sim e^{4\phi} G \partial G \partial^2 G \partial^3 G \) : with G being the full matter + ghost worldsheet supercurrent and calculate its full O.P.E. (i.e. including all the non-singular terms) with the picture −3 five-form operator. If the −3-picture vertex operator is at the point 0 then

\[
V_{5}^{(+1)}(0) = e^{\phi} \psi_{m_1}...\psi_{m_5} - \frac{1}{2} \lim_{z \to 0} \{ z^2 be^{2\phi - \chi} \psi_{m_1}...m_5 \\
- \frac{1}{2} z^2 ce^{\chi} \psi_{m_1}...\psi_{m_5} (\psi_{m_5} (\psi_n \partial X^n) + \partial X_{m_5} (\partial \phi - \partial \chi)) + O(z^3) \}
\]

This operator is BRST invariant by construction since both \( \Gamma^4 \) and picture −3 5-form operator are BRST invariant.

The BRST commutator with counterterms must be computed at a point \( z \) and then the limit \( z \to 0 \) is to be taken. Fortunately, due to the condition of fermionic ghost number conservation this unpleasant non-local ghost part is unimportant in computations of correlation functions and can be dropped at least in cases when not more than one picture +1-operator is involved. Also, since in open string field theory all the vertices are off-shell, the BRST invariance is not important and therefore we shall consider only the local part of the +1-picture 5-form in all our OSFT computations.

3. Effective potential of the ghost tachyon

In this section we will consider the contribution of the massless tachyonic brane-
like vertices (2),(3) to the OSFT effective action. Let us start with reviewing the basic framework of open superstring field theory (for full details see [6], [3], [7]). The off-shell superstring field theory is described by the following Wess-Zumino type action

\[ S = \frac{1}{2g^2} \ll (e^{-\Phi} Q_{\text{brst}} e^\Phi)(e^{-\Phi} \eta_0 e^\Phi) - \int_0^1 dt (e^{-t\Phi} \partial_t e^{t\Phi}) \{ (e^{-t\Phi} Q_{\text{brst}} e^{t\Phi})(e^{-t\Phi} \eta_0 e^{t\Phi}) \} \rr \]  

where \( b, c, \beta, \gamma \) are fermionic and superconformal ghosts with bosonization formulae

\[ c = e^{\sigma}, b = e^{-\sigma} \]

\[ \gamma = e^{\phi - \chi}, \beta = e^{\chi - \phi} \partial \chi \]

\[ <\sigma(z)\sigma(w)> = <\chi(z)\chi(w)> = - <\phi(z)\phi(w)> = \log(z - w) \]

\[ \eta(z) \equiv e^{-\chi}(z), \xi(z) \equiv e^\chi(z) \]

and BRST operator is given by

\[ Q_{\text{BRST}} = \oint \frac{dz}{2i\pi} (c(T_m + T_{gh}) + \frac{1}{2} e^{\phi - \chi} \psi_m \partial X^m + \frac{1}{4} e^{2\phi - 2\chi} b - b : c\partial c) \]

The time ordered correlators \( \ll ... \rr \) are defined by:

\[ \ll V_1...V_N \rr = \ll f_1^{(N)} \circ V_1(0)...f_N^{(N)} \circ V_N(0) \rr \]

\[ f_k^{(N)}(z) = e^{\frac{2\pi i (k-1)}{N}} \frac{1 + iz}{1 - iz} \]

where the function \( f(z) \) defines the conformal transform of \( V \) by \( f \), for instance, if \( V \) is a primary field of dimension \( h \), one has \( f \circ V(0) = (\frac{df}{dz})^h V(f(0)) \) This action is invariant under the local gauge transformation given by

\[ \delta e^\Phi = (Q_{\text{brst}} \epsilon) e^\Phi + e^\Phi (\eta_0 \lambda) \]

where \( \epsilon, \lambda \) are Grassmann odd. Using this gauge symmetry it is convenient to choose the gauge

\[ b_0 \Phi = 0, \xi_0 \Phi = 0 \]

String fields satisfying these conditions are related to local NS operators by

\[ \Phi =: \xi V : \]

For example, if \( V = ce^{-\phi} \) is tachyon vertex operator in the \(-1\)-picture, the corresponding string field will be given by the following picture zero operator: \( \Phi = ce^{\chi - \phi} \) Also, it should
be noted that, since all the GSO(-) states are Grassmann, odd, they should be accompanied by the appropriate $2 \times 2$ internal Chan-Paton factor, i.e. the Pauli matrix $\sigma_1$. At the same time, the GSO(+) fields must be multiplied by the $2 \times 2$ identity matrix so that the complete string field may be written as

$$\hat{\Phi} = \Phi_+ \otimes \sigma_1 + \Phi_- \otimes I$$

(12)

Also, $\hat{Q}_{\text{brst}} = Q_{\text{brst}} \otimes \sigma_3$ and $\hat{\eta}_0 = \eta_0 \otimes \sigma_3$.

It is convenient to expand the exponents of the OSFT action (5) in power series, so that the action is expressed as

$$S = \frac{1}{2g^2} \sum_{M,N=0}^{\infty} \frac{(-1)^N(M + N)!}{M!N!(M + N + 2)!} \langle\langle (\hat{Q}_{\text{brst}}\hat{\Phi})^M (\hat{\eta}_0\hat{\Phi})^N \rangle\rangle$$

(13)

To compute the effective potential of the tachyonic modes (2), (3) one has to specify the consistent truncation [1] of the string field $\Phi$ which takes into account the brane-like vertices. Let $H_{m,n}$ be a subspace of operators of ghost number $m$ and picture $n$. First of all, as in [3] the truncation we are looking for will include all the operators in the $H_{0,0}$-subspace of total picture zero and total ghost number zero; As has been shown in [3], the $Z_2$ twist-even string field, satisfying the gauge condition (11) and restricted to $H_{0,0}$ up to level $3/2$ is given by

$$\hat{\Phi}_{0,0} = t\hat{T} + a\hat{A} + e\hat{E} + f\hat{F}$$

$$\hat{T} = \xi ce^{-\phi} \otimes \sigma_1$$
$$\hat{A} = c\partial^2 c\xi \partial \xi e^{-2\phi} \otimes I$$
$$\hat{E} = \xi \eta \otimes I$$
$$\hat{F} = -\frac{1}{2}\xi(\psi_m \partial X^m) \otimes I$$

(14)

However, in our case, to account for the brane-like states, the consistent truncation also must include operators of the subspaces $H_{0,2}$ and $H_{0,-2}$ which do not admit a picture zero representation; let us denote the appropriate subspaces as $\tilde{H}_{0,2}$ and $\tilde{H}_{0,-2}$; of course, $\tilde{H}_{0,2} \subset H_{0,2}$ and $\tilde{H}_{0,-2} \subset H_{0,-2}$. At the lowest levels, the subspace $\tilde{H}_{0,-2}$ consists of the following vertex operators: $c e^{x-3\phi}, c e^{x-3\phi} \psi_{m_1} \ldots \psi_{m_p} (p = 1, \ldots, 5), c e^{x-3\phi} \partial X_m, c e^{x-3\phi} \partial X_m \partial X_n$ and the same for $\tilde{H}_{0,2}$ with $e^{x-3\phi} \rightarrow e^{x+\phi}$ At the
same time, the operators

\[
\begin{align*}
\hat{P}^{(-3)} &= c e^{x - 3\phi} (\psi_m \partial X_m) \otimes I \\
\hat{Q}_{np}^{(-3)} &= c e^{x - 3\phi} (\psi_m \partial X^m) \psi_n \psi_p \otimes I \\
\hat{P}^{(+1)} &= c e^{x + \phi} (\psi_m \partial X_m) \otimes I \\
\hat{Q}_{np}^{(+1)} &= c e^{x + \phi} (\psi_m \partial X^m) \psi_n \psi_p \otimes I
\end{align*}
\]

(15)

do not belong to \(\tilde{H}_{0,-2}\) (and also \(\tilde{H}_{0,2}\)) since \(\hat{P}^{(-3)}\) and \(\hat{Q}_{np}^{(-3)}\) can be transformed to higher picture representation by the picture changing transformation and \(\hat{P}^{(+1)}\) and \(\hat{Q}_{np}^{(+1)}\) are nothing but unity operator and Lorentz generator in the +1-picture (multiplied by \(c\xi\)) So we will be looking for the truncation of the OSFT that includes the vertices from the above subspaces \(H_{0,0}, H_{0,2}\) and \(H_{0,-2}\) and which is consistent up to level \(\frac{3}{2}\) of the open superstring field theory (i.e. the same maximum level considered in [3]). Moreover, not all of the operators of \(\tilde{H}_{0,2}\) and \(\tilde{H}_{0,-2}\) must be included in the mode expansion of the open string field but only those which, in a group with the vertices of \(H_{0,0}\) define a consistent truncation of the open string field theory (so that the effective OSFT equations of motion could be obtained by merely varying the OSFT action with respect to the vertices belonging to the truncation set). The consistency of the OSFT truncation is equivalent to the following condition. Suppose \(\mathcal{H}\) is a truncation. Then, for the truncation to be consistent, the OSFT action must always be quadratic or higher order in the components of \(\Phi\) orthogonal to \(\mathcal{H}\). Then, if one takes \(\Phi\) to be entirely inside \(\mathcal{H}\), the OSFT equations of motion, obtained by varying \(S\) with respect to all the states of \(\{\Phi\}_{\mathcal{H}}\) are satisfied automatically and the truncation is consistent [2]. It is now not difficult to check that the truncation consistency condition (up to level \(\frac{3}{2}\) excludes the following tachyonic \(\tilde{H}_{0,-2}\) operators: \(\tilde{R} = c e^{x - 3\phi} \otimes \sigma_1\), \(\tilde{R}_{np} = c e^{x - 3\phi} \psi_n \psi_p \otimes \sigma_1\) and \(\tilde{R}_m = c e^{x - 3\phi} \partial X_m \otimes \sigma_1\) from from \(\mathcal{H}\) because the following OSFT correlators:

\[
\begin{align*}
\langle\langle (\hat{Q}_{\text{brst}}, \hat{T}) \hat{R}(\hat{\eta}_0 \hat{P}^{(-3)}) \hat{F} \rangle\rangle, \\
\langle\langle (\hat{Q}_{\text{brst}}, \hat{T}) \hat{R}_{np}(\hat{\eta}_0 \hat{P}_{np}^{(-3)}) \hat{F} \rangle\rangle, \\
\langle\langle (\hat{Q}_{\text{brst}}, \hat{T}) \hat{R}_m(\hat{\eta}_0 \hat{P}_m^{(-3)}) \hat{F} \rangle\rangle
\end{align*}
\]

(16)
do not vanish and therefore the OSFT action has the terms linear in \(\hat{P}^{(-3)}\), \(\hat{P}_m^{(-3)}\) and \(\hat{P}_{np}^{(-3)}\) unless we exclude \(\tilde{R}, \tilde{R}_m\) and \(\tilde{R}_{np}\) from the truncation scheme. The totally similar argument excludes also the analogues of \(\tilde{R}, \tilde{R}_m\) and \(\tilde{R}_{np}\) carrying the ghost \(\phi\) number +1. Next, as in [3] the OSFT action truncated on \(\mathcal{H}\) has a twist \(Z_2\)-symmetry under which
string fields associated with vertices of dimension \( h \) carry charge \((-1)^{h+1}\) for even \( 2h \) and 
\((-1)^{h+\frac{1}{2}}\) for odd \( 2h \). Using this twist symmetry we can further truncate the string field components from the \( \tilde{H}_{0,2} \) and \( \tilde{H}_{0,-2} \) sectors to exclude the brane-like tachyonic states:

\[
\begin{align*}
&c e^{x-3\phi} \psi_m, \\
&c e^{x-3\phi} \psi_m \psi_{m_2} \psi_{m_3}, \\
&c e^{x-3\phi} \psi_{m_1} \cdots \psi_{m_5}, \\
&c e^{x+\phi} \psi_m \\
&c e^{x+\phi} \psi_{m_1} \psi_{m_2} \psi_{m_3}, \\
&c e^{x+\phi} \psi_{m_1} \cdots \psi_{m_5},
\end{align*}
\]

so that only the tachyonic 4-form brane-like string field is left in the expansion. In the rest of the paper, for the sake of certainty, we will be analyzing the case of tachyon condensation on the D3-brane. Finally we are ready to write the relevant expansion for consistently truncated OSFT needed to evaluate the ghost tachyon potential:

\[
\hat{\Phi} = t \hat{T} + a \hat{A} + e \hat{E} + f \hat{F} + \tau \hat{\Theta} + \lambda_{ab} \hat{\Lambda}_{ab}
\]

\[
\hat{T} = \xi e^{-\phi} \otimes \sigma_1
\]

\[
\hat{A} = c \partial^2 e \xi \partial e^{-2\phi} \otimes I
\]

\[
\hat{E} = \xi \eta \otimes I
\]

\[
\hat{F} = \frac{-1}{2} \xi (\psi_m \partial X^m) \otimes I
\]

\[
\hat{\Lambda}_{ab} = c \xi (e^\phi + e^{-3\phi}) \partial X_a \partial X_b
\]

\[
\hat{\Theta} = c \xi (e^\phi + e^{-3\phi}) \psi_{a_1} \cdots \psi_{a_4} e^{a_1 \cdots a_4}
\]

\[
a, b, a_i = 0, \ldots, 3
\]

In the last formula we have chosen the polarizations of the ghost tachyon so that the 4-form spans the 4-dimensional \( D3 \)-brane worldvolume; the tachyonic graviton is localized in the \( D3 \)-brane worldvolume as well. Therefore in our construction the ghost tachyonic modes consist of the tachyonic scalar (corresponding to the ghost tachyonic 4-form) and the 4-dimensional tachyonic 2-form (graviton) Also, to determine the contribution of brane-like states to the modified \( D3 \)-brane tension (which is to be compared with the energy density at the minimum of the ghost tachyon potential), we shall need to properly polarize the massless 5-form:

\[
\Lambda_j = c \xi (e^\phi + e^{-3\phi}) \psi_0 \psi_1 \psi_2 \psi_3 \psi_j (j = 4, \ldots, 9)
\]

i.e. 4 out of five fermions are directed along the \( D3 \)-brane worldvolume while the fifth is the
“orthogonal” one, accounting for the transverse oscillation of the $D3$ -brane, associated with the 5-form brane-like state.

4. Brane-like states and D3-brane tension

In this section we shall calculate the tension of the brane associated with the massless 5-form vertex operator and show that it is equal to the tension value for the usual $D3$-brane, i.e. $\frac{1}{2\pi^2 g^2}$. To compute the tension, corresponding to the massless 5-form, we use the strategy totally similar to [1], [3]. Namely,

$$\hat{V}_5 = \lambda_j(k) c e^x (e^\phi + e^{-3\phi}) \psi_0 \psi_3 \psi_j e^{i k_i X^i}(\tau)$$

one has to consider the $k$-dependent quadratic term involving the $\lambda_j$ mode in the OSFT action (5), given by $\frac{1}{2g^2} << (Q_{brst} \hat{V}_5)(\eta_0 \hat{V}_5)>>$. The only term in the BRST charge contributing to the $k$-dependent part of the quadratic term is given by $\oint d\tau 2i\pi c T_{\text{matter}}(\tau)$. Evaluating its commutator with $\hat{V}_5$ we find the kinetic part of the quadratic term in the effective Lagrangian to be given by

$$L^{(\lambda)}_{\text{kin}} = \frac{1}{g^2} \sum_k (k)^2 \lambda_j(k) \lambda_j(-k)$$

or, after the Fourier transform,

$$L^{(\lambda)}_{\text{kin}} = \frac{1}{g^2} \partial_t \lambda_j \partial_t \lambda^j$$

Now, to find a tension of the brane one has to find the relation between $\lambda_j$ and transverse collective coordinates $\Lambda_j$ of the D-brane associated with $V_5$. As in [3], $\lambda_j = \alpha \Lambda_j$ up to normalization factor. Once we find the value of this normalization constant, we can elucidate the brane tension, as the overall normalization of the kinetic term, written in terms of the $\Lambda_j$ collective coordinates is equal to the half of the square of the brane mass. In order to find the normalization constant $\alpha$ one has to consider the brane-antibrane pair (created by the $V_5$ vertices with appropriate external Chan-Paton’s factors) at a distance $b^t$ along $X^t$ transverse direction, and an open string stretched between them which has the tension given by $\frac{1}{2\pi} |b^t|$. If one moves one of the branes by an amount $Y^t$ along $X^t$, the tension (or the mass)$^2$ change, up to the first order in $Y$ is given by

$$\frac{1}{(2\pi)^2} (b^t Y^t)$$
On the other hand, just as in the standard D-brane case, shifting one of the branes along the $X^t$ direction corresponds to turning on the string field background given by $ce^{\chi - \phi} \psi^t \otimes I \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Therefore in order to find the normalization constant $\alpha$ one has to compare this tension change with the one that follows from relevant three-point OSFT correlation functions involving one photon vertex operator (corresponding to the open string between the brane and the antibrane) and two massless five-form vertices (with necessary external CP factors) that account for the brane and the antibrane. The relevant string field expansion is given by:

$$\hat{\Phi} = \xi_t c \hat{P}^t + \lambda_t(k) \hat{U}^t_5(k) + \lambda^*_t(k) \hat{W}^t_5(k)$$

$$\hat{P}^t = ce^{\chi - \phi} \psi^t \otimes I \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \hat{P}^t(k) = ce^{\chi - \phi} \psi^t \otimes I \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\hat{W}^t(k) = ce^{\chi - \phi} \psi^t \otimes I \otimes \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

(23)

Our aim now is to establish connection between the $\lambda^t$ polarization vector of the 5-form and the transverse collective coordinates $\Lambda^t$ of the D-brane created by the 5-form vertex. Finding the normalization constant relating $\lambda$ and $\Lambda$ will allow us to determine the tension of the brane. This normalization must be determined from the $(\bar{b}\xi)(\bar{\lambda}\lambda^*)$ coupling originating from relevant three-point correlators in string field theory involving $\hat{P}$, $\hat{U}$ and $\hat{W}$. The three point OSFT vertex is given by

$$\frac{1}{12g^2}(<\langle (\hat{Q}_{\text{brst}}, \hat{\Phi})(\hat{\eta}_0 \hat{\Phi}) \rangle> - <\langle (\hat{Q}_{\text{brst}} \hat{\Phi})(\hat{\eta}_0 \hat{\Phi}) \hat{\Phi} \rangle>) \quad (24)$$

It is easy to see that this vertex involves two different classes of three-point correlators, those involving the commutator of BRST charge with the photon $\hat{P}$ and those with the commutators of $\hat{Q}_{\text{brst}}$ with the 5-forms $\hat{U}$ and $\hat{W}$. Using the expression (7) for $Q_{\text{brst}}$ we find that these BRST commutators are given by:

$$\{\hat{Q}_{\text{brst}}, \hat{P}^t\} = c\partial X^t \otimes \sigma_1 \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

(25)

As for the BRST commutators with the five-forms $\hat{U}$ and $\hat{W}$, we are only interested in the commutators of $\int \frac{dz}{2i\pi} (\gamma G_{\text{matter}})$ of $\hat{Q}_{\text{brst}}$ with $\hat{U}$ and $\hat{W}$ as these are the only commutators contributing to three-point correlation functions (since these are the only ones proportional
to the ghost field $c$). So we have

$$-\frac{1}{2} \left\{ \int \frac{dz}{2i\pi} e^{\phi - \chi} \psi_m \partial X^m \otimes \sigma_1 \otimes I, \lambda_t(k) \hat{U}^t(k) \right\}$$

$$= \lambda_t(k) \left[ \{ ce^{2\phi} \psi_0 \ldots \psi_3 \psi^t \{ G_{\text{matter}} P^{(1)}_{\phi - \chi} + \partial G_{\text{matter}} \} e^{ikX}$$

$$- \frac{1}{2} \lambda_t(k) ce^{2\phi} [\psi_0 \ldots \psi_2 \partial X_3] P^{(2)}_{\phi - \chi}$$

$$+ \psi_0 \ldots \psi_2 \partial^2 X_3] P^{(1)}_{\phi - \chi} + \frac{1}{2} \psi_0 \ldots \psi_2 \partial^3 X_3] \psi^t e^{ikX}$$

$$- \frac{1}{2} \lambda_t(k) ce^{2\phi} [\psi_0 \ldots \psi_3 e^{ikX} (\partial X^t + i(k \psi) \psi^t) P^{(2)}_{\phi - \chi}$$

$$(\partial^2 X^t + i(k \partial \psi) \psi^t) P^{(1)}_{\phi - \chi} + \frac{1}{2} (\partial^3 X^t + i(k \partial^2 \psi) \psi^t)] + \ldots \otimes I \otimes \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(26)

and the same formula for $\{ \hat{Q}_{\text{brst}}, \hat{W} \}$, with only the external CP factor changed. In the formula (26) the square brackets encompassing the longitudinal indices 0, 1, 2, 3 denote the antisymmetrization over these indices; the $P^{(1),(2)}_{\phi - \chi}$ are the polynomials in derivatives of free $\phi$ and $\chi$ fields (of conformal weights 1 and 2 respectively) that emerge as a result of the differentiation of the $\gamma(z) = e^{\phi - \chi}(z)$ ghost field with respect to $z$. Also, we have dropped all the terms that are irrelevant (i.e. not contributing) to the $(\tilde{\xi} \bar{b})(\tilde{\lambda} \bar{\lambda}^*)$ coupling that we seek to elucidate from three-point string field correlation functions. For instance, we are not interested in couplings involving the scalar product $\lambda$ and $\xi$ and all the terms in three-point correlators giving rise to these couplings will be dropped for the sake of shortness. Finally, note that a half of all the three-point correlation functions vanish due to the matrix identity

$$\text{Tr} \left( \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right) = 0$$

(27)

Now, using the relations (25), (27), evaluating all the Chan-Paton’s traces and denoting for convenience

$$\tau_1 \equiv 1$$

$$\tau_2 \equiv e^{\frac{2\pi}{3}}$$

$$\tau_3 \equiv e^{\frac{4\pi}{3}}$$

$$\vec{p} = \vec{k} + \frac{\vec{b}}{2\pi}$$

(28)

we easily calculate the first type of string field theory contribution to the $(\tilde{\xi} \bar{b})(\tilde{\lambda} \bar{\lambda}^*)$ coupling.
determined by the vertex

\[ C_3^{(1)} = \frac{1}{12g^2} \ll (\hat{Q}_{\text{brst}} \hat{P}) \hat{U} (\hat{\eta}_0 \hat{W}) \gg \ll (\hat{Q}_{\text{brst}} \hat{P}) (\hat{\eta}_0 \hat{U}) \hat{W} \gg \]

\[ = \ll \xi_q(0) \lambda_s(\vec{k}) \lambda_t^* (-\vec{k}) \ll : c \partial X^q : (\tau_1) : c(e^{\phi} + e^{-3\phi})(1 + e^{\chi})\psi_0 ... \psi_3 \psi^s e^{i\vec{p} \vec{X}} : (\tau_2) : c(e^{\phi} + e^{-3\phi})(1 + e^{\chi})\psi_0 ... \psi_3 \psi^s e^{-i\vec{p} \vec{X}} : (\tau_3) >\]

\[ = \frac{1}{24\pi} (\vec{b}, \vec{\xi})(\lambda(\vec{k}) \lambda^*) (\tau_1 - \tau_2)(\tau_1 - \tau_3)(\tau_2 - \tau_3)(\tau_1 - \tau_2 - \tau_3)^{-5} (\frac{2}{\tau_1 - \tau_2} - \frac{2}{\tau_1 - \tau_3}) + \{\text{permut.} \tau_2 \leftrightarrow \tau_3\} \]

\[ = \frac{1}{6\pi g^2} (\vec{b}, \vec{\xi})(\lambda(\vec{k}) \lambda^*) (\frac{\tau_1 - \tau_3}{\tau_2 - \tau_3} - \frac{\tau_1 - \tau_2}{\tau_2 - \tau_3}) = \frac{1}{6\pi g^2} (\vec{b}, \vec{\xi})(\lambda(\vec{k}) \lambda^*) \]

(29)

The second group of three point correlators contributing to the \((\vec{b}, \vec{\xi})(\lambda(\vec{k}) \lambda^*)\) coupling, that involves the BRST commutators (26) with the five-form requires much more work to calculate; the relevant non-vanishing correlators contribute to the three-point vertex given by

\[ C_3^{(2)} = \frac{1}{12g^2} \ll (\hat{Q}_{\text{brst}} \hat{U}) \hat{W} (\hat{\eta}_0 \hat{P}) \gg \ll (\hat{Q}_{\text{brst}} \hat{U}) (\hat{\eta}_0 \hat{W}) \hat{P} \gg \]

\[ + \ll (\hat{Q}_{\text{brst}} \hat{W}) \hat{P} (\hat{\eta}_0 \hat{U}) \gg \ll (\hat{Q}_{\text{brst}} \hat{W}) (\hat{\eta}_0 \hat{P}) \hat{U} \gg \]

Substituting the string field expansion (23), using the expressions for BRST commutators derived in (26) and evaluating the three-point functions at \(\tau_1 = 1, \tau_2 = e^{\frac{2\pi}{3}}, \tau_3 = e^{\frac{4\pi}{3}}\) as previously, we find that the second type three-point vertex gives

\[ C_3^{(2)} = \frac{8.83272}{g^2} (\vec{b}, \vec{\xi})(\lambda(\vec{k}) \lambda^*) \]

(31)

Adding together these two types of vertices, we obtain the answer for the relevant coupling:

\[ C_3 = C_3^{(1)} + C_3^{(2)} = \frac{8.88577}{g^2} (\vec{b}, \vec{\xi})(\lambda(\vec{k}) \lambda^*) \]

(32)

Now, to normalize \(\lambda\) in terms of collective coordinates of the brane we have to compare it with the tension change given by \(\frac{1}{2\pi}(\vec{b}, \vec{Y})\) of (22) where \(\vec{Y}\) is a collective coordinate associated with massless photonic excitation of an open string stretched between the branes. As has been pointed out in [3], the relation between this collective coordinate and polarization vector \(\vec{\xi}\) of the photon is given by

\[ \vec{\xi} = -\frac{\vec{Y}}{\pi \sqrt{2}} \]

(33)

Substituting into (32) we get

\[ C_3 = 2(\vec{b}, \vec{Y})(\lambda \lambda^*) \]

(34)
On the other hand, it follows from (22) that $C_3$ should be given by

$$C_3 = \frac{1}{2\pi^2}(\vec{b}\vec{Y})(\vec{\Lambda}\vec{\Lambda}^*)$$

(35)

with $\vec{\Lambda}$ and $\vec{\Lambda}^*$ being collective coordinates of the brane and the anti-brane associated with the five-forms. Therefore we obtain

$$\vec{\lambda} = \frac{1}{2\pi}\vec{\Lambda}$$

(36)

Substituting this to the kinetic term (20), (21) of string field theory Lagrangian, associated with the five-form, we express it in terms of the brane collective coordinates and get

$$L_{kin}^{(\lambda)} = \frac{1}{4\pi g^2} \sum_k (k^2)(\vec{\Lambda}\vec{\Lambda}^*)$$

(37)

where the coefficient before the sum over $k$ gives us a half of the brane tension. Therefore the D-brane tension, associated with the massless five-form, is given by

$$T = \frac{1}{2\pi g^2}$$

(38)

It is quite remarkable that the massless 5-form reproduces the well-known tension value of a standard $D$-brane. In the next sections we will perform the computation of the ghost tachyon potential and compare this tension with the energy density at the minimum of the potential to demonstrate the phenomenon of cancellation.

5. BRST Commutators of String Fields

To start the computation of the potential in the OSFT we will need expressions for commutators of the BRST charge with string fields in the twist-odd sector, including tachyonic and ghost tachyonic modes. The commutators are straightforwardly evaluated
and are given by:

\[
\{\hat{Q}_{brst}, \hat{T}\} = (-\frac{1}{2} : c\partial c : e^{x-\phi} + \frac{1}{4}\gamma) \otimes I
\]

\[
\{\hat{Q}_{brst}, ce^{x-3\phi} \psi_0...\psi_3 \} \otimes \sigma_1 = (-\frac{1}{2} : c\partial c : e^{x-3\phi} \psi_0...\psi_3) \otimes I
\]

\[
\{Q_{brst}, \hat{A}\} = (-2e^{3\sigma + 2\chi - 2\phi} + \frac{1}{2}[c(2\partial^2 \phi - 2\partial^2 \chi + 4(\partial \phi - \partial \chi)^2) + 2\partial c(2\partial \phi - 2\partial \chi)]) \otimes \sigma_1
\]

\[
\{\hat{Q}_{brst}, \hat{E}\} = (\partial(c \partial \chi) - \frac{1}{2}\gamma(\psi_m \partial X^m) - \frac{1}{2}\gamma^2b) \otimes \sigma_1
\]

\[
\{Q_{brst}, \hat{F}\} = -\frac{1}{2} : c\partial c : e^{x- \phi}(\psi_m \partial X_m) - \frac{1}{8}\gamma(\psi_m \partial X^m) + \frac{1}{2}c(\partial X_m \partial X^m + \psi_m \partial \psi^m)
\]

\[
+ (2\partial^2 \phi - 2\partial^2 \chi + 4(\partial \phi - \partial \chi)^2) \otimes \sigma_1
\]

\[
\{\hat{Q}_{brst}, ce^{x+\phi} \psi_0...\psi_3 \otimes \sigma_1\} = (-\frac{1}{2} : c\partial c : e^{x+\phi} \psi_0...\psi_3
\]

\[
-\frac{1}{2}e^{2\phi}[\psi_0...\psi_3((\psi_m \partial X^m)(\partial \phi - \partial \chi) + \partial(\psi_m \partial X^m))
\]

\[
+ \psi[0...\psi_2 \partial X_3](\partial^2 \phi - \partial^2 \chi + (\partial \phi - \partial \chi)^2)
\]

\[
+ \psi[0...\psi_2 \partial X_3](\partial \phi - \partial \chi)
\]

\[
- \frac{1}{4}e^{3\phi-\chi} \psi_0...\psi_3 P_{2\phi-2\chi-\sigma}^{(4)} \otimes I
\]

\[
\{\hat{Q}_{brst}, ce^{x-3\phi} \partial X_a \partial X_b \otimes \sigma_1\} = -\frac{1}{2} : c\partial c : e^{x-3\phi} \partial X_a \partial X_b \otimes I
\]

\[
\{\hat{Q}_{brst}, ce^{x+\phi} \partial X_a \partial X_b \otimes \sigma_1\} = (-\frac{1}{2} : c\partial c : e^{x+\phi} \partial X_a \partial X_b
\]

\[
- \frac{1}{2}e^{2\phi}\{\partial X_a \partial X_b((\partial \phi - \partial \chi)(\psi_m \partial X^m) + \partial(\psi_m \partial X^m))
\]

\[
+ (\psi_a \partial X_b + \psi_b \partial X_a) P_{\phi-\chi}^{(2)}(\partial X_a \partial \psi_b + \partial \psi_a \partial X_b) + \partial^2 X_a \psi_b + \partial^2 X_b \psi_a) P_{\phi-\chi}^{(2)}
\]

\[
+ \frac{1}{4}e^{3\phi-\chi} \partial X_a \partial X_b P_{2\phi-2\chi-\sigma}^{(4)} \otimes I
\]

where \( P_{f(\phi,\chi,\sigma)}^{(n)} \) is the polynomial of conformal weight \( n \) given by the pre-exponential factor that appears as a result of taking the \( n \)th derivative (with respect to \( z \)) of the operator \( e^{f(\phi,\chi,\sigma)} \) multiplied by \( \frac{1}{n!} \) (with \( f \) being a linear function of derivatives of the scalar fields). In particular,

\[
P_{2\phi-2\chi-\sigma}^{(4)} = \frac{1}{24}(2\partial^4 \phi - 2\partial^4 \chi - \partial^4 \sigma) + \frac{1}{6}(2\partial^3 \phi - 2\partial^3 \chi - \partial^3 \sigma)(2\partial \phi - 2\partial \chi - \partial \sigma)
\]

\[
+ \frac{1}{4}(2\partial^2 \phi - 2\partial^2 \chi - \partial^2 \sigma)(2\partial \phi - 2\partial \chi - \partial \sigma)^2 + \frac{1}{8}(2\partial^2 \phi - 2\partial^2 \chi - \partial^2 \sigma)^2
\]

\[
+ \frac{1}{24}(2\partial \phi - 2\partial \chi - \partial \sigma)^4
\]
In the next section we shall calculate the ghost tachyon potential using the string field truncated expansion (17), the modified tension of the D3-brane (associated with the 5-form) and to verify that they cancel each other. We will perform the computations up to the level $\frac{3}{2}$ for the OSFT modes belonging to $H_{0,0}$ but only up to the level $\frac{1}{2}$ for $\Phi \subset \tilde{H}_{0,-2} \oplus \tilde{H}_{0,-2}$, i.e. for string field modes associated with the brane-like states.

### 6. Computation of the ghost tachyon potential

In this paragraph we will demonstrate the computation of the quartic ghost tachyonic term $\sim \tau^4$ and present the answer to the tachyon potential that we have found (calculations involving the ghost tachyonic modes and the level $3/2$ $H_{0,0}$ modes are in fact quite cumbersome and one has to use Mathematica to get numerical answers as well as to analyze the extremal points of the potential). The relevant 4-point functions involve ghost tachyonic 4-forms with both $+1$ and $-3$ bosonic ghost number. However, as all the vertex operators are multiplied by the c ghost field, it is clear that only the correlators involving \( \{ \tilde{Q}_{brst}, c e^{\chi+\phi} \psi_0 ... \psi_3 \otimes \sigma_1 \} \) contribute to the potential, since, as one can see from the table of BRST commutators (39), this is the only commutator containing a term without a c-ghost given by: \( P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 \). Using $\eta_0 e^{\chi-3\phi} \sim e^{-3\phi}$, $\eta_0 e^{\chi+\phi} \sim e^{\phi}$ and evaluating the CP factor trace we find that the relevant contribution is given by the following piece of the OSFT action (5):

$$A_{\tau^4} = \frac{\tau^4}{48g^2}$$

\[\begin{align*}
&\{<<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{-3\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 : >>
\}
+ <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{-3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 : >>
+ <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{\chi+\phi} \psi_0 ... \psi_3 :: c e^{3\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 : >>
+ <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{3\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 : >>
+ <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 : >>
+ <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{3\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 : >>
+ <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 : >>
+ <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{3\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 : >>
- 2 <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{\chi+\phi} \psi_0 ... \psi_3 :: c e^{-3\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 : >>
- 2 <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{-3\phi} \psi_0 ... \psi_3 :: c e^{\chi+\phi} \psi_0 ... \psi_3 : >>
- 2 <<: P^{(4)}_{2\phi-2\chi-\sigma} e^{3\phi-\chi} \psi_0 ... \psi_3 : c e^{\chi-3\phi} \psi_0 ... \psi_3 :: c e^{\phi} \psi_0 ... \psi_3 :: c e^{\chi-3\phi} \psi_0 ... \psi_3 : >>\}
\]

Let us demonstrate, for example, the calculation of the first correlator,
\[ <:< P_{2\phi-2\chi-\sigma}^{(4)} e^{3\phi - x} \psi_0 \ldots \psi_3 : c e^{-3\phi} \psi_0 \ldots \psi_3 :: c e^{\chi - 3\phi} \psi_0 \ldots \psi_3 : : c e^{\chi + \phi} \psi_0 \ldots \psi_3 : : > \], the evaluation of all others is totally analogous. Denoting for convenience

\[ x \equiv 1, \]
\[ y \equiv e^{\frac{4\pi}{3}} = i, \]
\[ z \equiv e^{i\pi} = -1, \]
\[ a \equiv e^{\frac{3\pi}{4}} = -i \]

and using the fact that for primary any dimension \( h \) primary operator \( V \)

\[ f_i^{(N)}(Q_{brst}, V(0)) = \{ Q_{brst}, f_i^{(N)}(V(0)) \} = \left( \frac{d_i^{f_{(N)}(z)}}{dz} \right)^h \{ Q_{brst}, V(f_i^{(N)}(0)) \} \]

we obtain:

\[ <:< P_{2\phi-2\chi-\sigma}^{(4)} e^{3\phi - x} \psi_0 \ldots \psi_3 : c e^{-3\phi} \psi_0 \ldots \psi_3 :: c e^{\chi - 3\phi} \psi_0 \ldots \psi_3 : : c e^{\chi + \phi} \psi_0 \ldots \psi_3 : : > \]
\[ \equiv < f_1^{(4)} \circ P_{2\phi-2\chi-\sigma}^{(4)} e^{3\phi - x} \psi_0 \ldots \psi_3(0) f_2^{(4)} \circ c e^{-3\phi} \psi_0 \ldots \psi_3(0) \]
\[ \times f_3^{(4)} \circ c e^{\chi - 3\phi} \psi_0 \ldots \psi_3(0) f_4^{(4)} \circ c e^{\chi + \phi} \psi_0 \ldots \psi_3(0) > \]
\[ = \prod_{k=1}^{14} (f_k^{(4)}(0)) f < P_{2\phi-2\chi-\sigma}^{(4)} e^{3\phi - x} \psi_0 \ldots \psi_3(x) \]
\[ \times c e^{-3\phi} \psi_0 \ldots \psi_3(y) c e^{\chi - 3\phi} \psi_0 \ldots \psi_3(z) c e^{\chi + \phi} \psi_0 \ldots \psi_3(a) > \]

Firstly, evaluating the matter part of the correlator and substituting numerical values of \( x, y, z \) and \( a \) we get:

\[ A_{matter} = <: \psi_0 \ldots \psi_3 : (x) : \psi_0 \ldots \psi_3 : (y) : \psi_0 \ldots \psi_3 : (z) : \psi_0 \ldots \psi_3 : (a) > \]
\[ = (((x - y)^{-4} (z - a)^{-4} + (x - z)^{-4} (y - a)^{-4} + (x - a)^{-4} (y - z)^{-4}) \]
\[ -4((x - y)^{-3} (z - a)^{-3}((y - z)^{-1} (x - a)^{-1} + (y - a)^{-1} (x - z)^{-1}) \]
\[ + (x - z)^{-3} (y - a)^{-3}((x - y)^{-1} (z - a)^{-1} + (x - a)^{-1} (y - z)^{-1}) \]
\[ + (x - a)^{-3} (y - z)^{-3}((x - z)^{-1} (y - a)^{-1} + (x - y)^{-1} (z - a)^{-1})) \]
\[ + 12((x - y)^{-2} (y - z)^{-2} (z - a)^{-2} (x - a)^{-2} + (x - z)^{-2} (x - a)^{-2} (y - z)^{-2} (y - a)^{-2} \]
\[ + (x - y)^{-2} (x - a)^{-2} (z - a)^{-2} (y - z)^{-2}) \]
\[ - 24((x - y)^{-2} (z - a)^{-2} (y - z)^{-1} (x - a)^{-1} (y - a)^{-1} \]
\[ + (x - z)^{-2} (y - a)^{-2} (y - y)^{-1} (x - a)^{-1} (y - z)^{-1} (z - a)^{-1} \]
\[ + (x - a)^{-2} (y - z)^{-2} (x - y)^{-1} (x - z)^{-1} (y - a)^{-1} (z - a)^{-1})) \]
\[ = \frac{-159}{64} \]
\[ (45) \]
To evaluate the ghost correlator involving the exponents and the $P^{(4)}_{2\phi-2\chi-\sigma}$ polynomial, it is convenient to use the O.P.E.’s:

\[
: 2\partial\phi - 2\partial\chi - \partial\sigma : (z) : ce^{-3\phi}(w) \sim \frac{5}{z-w} e^{-3\phi(w)}
\]
\[
: 2\partial\phi - 2\partial\chi - \partial\sigma : (z) : ce^{\chi-3\phi}(w) \sim \frac{3}{z-w} e^{\chi-3\phi(w)}
\]
\[
: 2\partial\phi - 2\partial\chi - \partial\sigma : (z) : ce^{\chi+\phi}(w) \sim -\frac{5}{z-w} e^{\chi+\phi(w)}
\]

(46)

Then, using the O.P.E. (26) and the polynomial definition (20) it is straightforward to compute the ghost part of the correlator (24) and the answer is

\[
< P^{(4)}_{2\phi-2\chi-\sigma}e^{3\phi-\chi}(x)ce^{-3\phi(y)}ce^{\chi-3\phi(z)}ce^{\chi+\phi(a)} > \\
= [(1/24)(18/((x-z)^4) - 30/((x-y)^4) + 30/((x-a)^4)) + (1/6)(10/((x-y)^3)
\]
+6/((x-z)^3) - 10/((x-a)^3))(5/(x-y) + 3/(x-z) - 5/(x-a)) + (1/4)(-3/((x-z)^2)
\]
-5/((x-y)^2) + 5/((x-a)^2))(5/(x-y) + 3/(x-z) - 5/(x-a))^2 + (1/8)(-3/((x-z)^2)
\]
-5/((x-y)^2) + 5/((x-a)^2))^2 + (1/24)(5/(x-y) + 3/(x-z) - 5/(x-a))^4]
\]
\times (x-y)^9(x-z)^8(x-a)^{-4}(y-z)^{-8}(y-z)^{-8}(y-a)^4(z-a)^5
\]

(47)

Note that all the correlators in (21) differ only in their ghost parts while their matter part is always the same, given by $A_{\text{matter}}^{(4)}$ of (25). Computing all the ghost correlators of (21) similarly to (27), summing them up and multiplying by the we get the following answer for the $\tau^4$ contribution:

\[
A_{\tau^4} = \prod_{k=1}^{4} (f^{(4)}_{k}(0))^{i} \times \frac{g^2}{3} \left\{ \frac{256}{3} \left( \frac{239}{8} - \frac{77}{2} \right) + \frac{145}{2048} \right\} \times \tau^4
\]
\[
= \prod_{k=1}^{4} (f^{(4)}_{k}(0))^{i} \times \left( -\frac{735.929i}{g^2} \right) \tau^4
\]

(48)

Finally, evaluating the conformal transformation factor we get

\[
A_{\tau^4} = \frac{735.929i}{g^2} \tau^4
\]

(49)

This concludes the computation of the $\tau^4$ contribution. The strategy for computing all other OSFT contributions to the tachyon potential is totally similar; our final result
for the potential is given by:

\[ V(t, a, e, f, \tau, \lambda, ab) = -2ae - 5f^2 + at^2 + 0.25et^2 \]

\[-4.96aet^2 + 0.6654e^2t^2 + 5.476ef^2 - 5.82f^2t^2 \]

\[+0.518et^4 - 0.27777e^2t^4 + 0.0826f^2Tr(\lambda^2) - 1583.73et^2Tr(\lambda^2) + 47022.1325eTr(\lambda^4) \]

\[+0.0113118f^2\tau^2 + 44.598428eTr(\lambda^2)x^2 + \]

\[10482.76792e^2Tr(\lambda^2)\tau^2 + 0.350694t^2(2Tr(\lambda^2) + \tau^2) + 36.3293et^2(2Tr(\lambda^2) + x^2) \]

\[ -97.2121e^2t^2(2Tr(\lambda^2) + \tau^2) + 0.25(t^2 + 2Tr(\lambda^2) + \tau^2) \]

\[-735.92919921875 \times (0.515625Tr(\lambda^2)^2 + 1.5Tr(\lambda^4) + 0.1015625Tr(\lambda^2)\tau^2 - \tau^4) \]

\[+0.8(-63639.61875eTr(\lambda^2)^2 - 127279.2375eTr(\lambda^4) - 79718.8925et\tau^4) \]

\[-0.1 \times (-6.7155426 \times 10^6e^2u^2 - 4.623888 \times 10^6e^2Tr(\lambda^4) \]

\[-6a(\tau^2 + 2Tr(\lambda^2)) + 1.4522886 \times 10^6e^2\tau^4) \]

(50)

The minimum of this potential is attained at:

\[ a = 9.5307, f = 0.0315, t = 0.9148, e = 0.136, \tau = 0.0115, Tr(\lambda^2) = 0.017, Tr(\lambda^4) = 0.00059 \]

(51)

and the value of the potential at this point is

\[ V_{min} = -\frac{1.157 \times 10^{-1}}{g^2} \]

(52)

On the other hand, the D3-brane tension associated with massless 5-form brane-like state, calculated in the appendix, is given by \( \frac{1}{2\pi g^2} \) so we see that already at the level 3/2 of the OSFT about 99.7 percent of the brane tension is cancelled by the ghost tachyon potential! In particular this may explain why the usual perturbative string theory does not feel the brane-like states and why the latter do not cause any difficulties with unitarity: it appears that these states are simply “screened” by the tachyonic veil, associated with brane-like tachyonic states (2), (3).

### 7. Extra Time Dimensions, Brane phenomenology and Tachyon Condensation

In this section we shall discuss possible implications of the above results for phenomenological brane models with extra dimensions [9]. The great disadvantage of known brane world scenarios [10] is that they essentially include objects with negative tensions, antibranes which in reality are unstable and do not exist. In a separate development, it
has been shown recently that when one places the brane-antibrane pair in a de Sitter-type background with time-like extra dimension, tachyonic modes appear among solutions to effective gravity equations of motion in such a background [11]. In this section we shall attempt to show that these two instabilities, i.e. the negative tension antibrane cancel each other by the mechanism very similar to the ghost tachyon condensation discussed above. Namely, we shall argue that, in terms of string theory, the tachyonic modes corresponding to the e.o.m. solutions correspond precisely to the ghost tachyonic vertex operators described above. As a result, one can again use the OSFT formalism to show that these tachyonic modes again have an effective potential bounded from below and the energy density at the potential minimum precisely compensates the antibrane tension. The analysis is quite similar to the one performed in the case of single $D$-brane; the only difference is that now the usual matter tachyonic field needs to be switched off, i.e. the ghost tachyon potential (30) must be analyzed at $t = 0$.

The tachyonic potential (30) with the extra condition $t = 0$ reaches its minimum at

$$f = 0.0549, e = 0.258,$$

$$\tau = 1.0035 \times 10^{-7}, Tr(\lambda^2) = 0.0141, Tr(\lambda^4) = 0.00087$$

and the potential value at the extremum is given by

$$V_{min} = \frac{1.139 \times 10^{-1}}{g^2}$$

(54)

On the other hand the antibrane tension is given by $-\frac{1}{2\pi g^2} \tau$, i.e. in this case the ghost tachyon potential compensates about 97 percent of the antibrane tension.

8. Discussion

It appears that there is a deep physical reason behind relation of the tachyonic modes in the de Sitter-type backgrounds and ghost tachyonic vertex operators. Some time ago in an unrelated development [4] it has been shown that the backgrounds of such a type can be understood as a result of dynamical compactification caused by the massless 5-form brane-like state. To explore the mechanism of the dynamical compactification of flat ten-dimensional space-time on $AdS_5 \times S^5$ due to presence of the $V_5$ vertex in the sigma-model one has to study the modification of the dilaton’s beta-function in the $V_5$-background. Such an analysis has been carried out in [4]. The analysis of the dilaton’s beta-function shows that the compactification on $AdS_5 \times S^5$ occurs as a result of certain
very special non-Markovian stochastic process. Namely, the $V_5$ background in the sigma-model has a meaning of a “random force” term with the $V_5$-operator playing the role of a non-Markovian stochastic noise, which correlations are determined by the worldsheet beta-function associated with the $V_5$ vertex. The straightforward computation shows that the dilaton’s beta-function equation in the presence of the $V_5$-term has the form of the non-Markovian Langevin equation:

$$\frac{d\varphi(p)}{d(\log \Lambda)} = -\int d^{10}q C_\varphi(q) \varphi\left(\frac{p-q}{2}\right) \varphi\left(\frac{p+q}{2}\right) + \eta_5(p||, \Lambda)$$

(55)

where

$$\eta_5(p||, \Lambda) \equiv -\lambda_0^2 \int \frac{d^4k}{(2\pi)^4} \int_0^{2\pi} d\alpha \int_0^\infty drr V_5(r + \Lambda, \alpha, k||))$$

(56)

In this equation the role of the stochastic noise term being played by the truncated world-sheet integral of the $V_5$-vertex. The logarithm of the worldsheet cutoff parameter plays the role of the stochastic time in the Langevin equation for stochastic quantization [12], [13], [14]. The noise is non-Markovian and it is generated by the $V_5$ operator, as was already noted above.

The noise correlations in stochastic time are given by the worldsheet correlators of the $V_5$ vertices (one has to take their worldsheet integrals at different cutoff values and to compute and to evaluate the cutoff dependence). Knowing the $V_5$-noise correlators it is then straightforward to derive the corresponding non-Markovian Fokker-Planck equation for this stochastic process and to show that the Fokker-Planck distribution solving this equation is given by the exponent of the ADM-type $AdS_5$ gravity Hamiltonian (computed from the $AdS_5$ gravity action at a constant radial $AdS$ “time” slice using the Verlinde’s prescription [13], [16]. Such a mechanism naturally relates the radial $AdS$ coordinate, stochastic time and the worldsheet cutoff, pointing out an intriguing relation between holography principle, AdS/CFT correspondence [17], [18], [19], [20] and non-Markovian stochastic processes. Therefore from space-time point of view the $V_5$ insertion leads to non-Markovian stochastic process which deforms flat space-time geometry the one of $AdS$ with the role of the radial $AdS$ coordinate played by the stochastic time parameter. At the same time, if one studies the gravity equations of motion in a background in which the radial $AdS$ dimension is replaced by the time-like coordinate one finds a set of tachyonic solutions to these equations, including tachyonic scalars and tachyonic gravitons. Given these tachyons in the and observing that the AdS background can be represented by insertion related to the 5-form vertex operators (1), the appearance of tachyons in higher
ghost number cohomologies [21] is not incidental. It appears that they merely reflect the $AdS$ structure (with the time-like radial direction) of the new space-time geometry created by the massless 5-form. On the other hand, due to the demonstrated analogue of Sen’s mechanism of ghost tachyon condensation, the massless 5-form (1) and the tachyons (2),(3) “compensate” each other and this is why the conventional perturbative NSR string theory does not “see” neither these states nor the $AdS$ space-time geometry.

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