GPS/BDS displacement monitoring of railway slopes based on coloured noise analysis

Jun Ma1, 2

1China Railway Siyuan Survey and Design Group Co., LTD, Wuhan 430079, China

2Corresponding author’s e-mail: 794598508@qq.com

Abstract. To analyse the influence of coloured noise in displacement time series on the global navigation satellite system (GNSS) displacement monitoring of railway slopes, and to compare the GPS, BDS and GPS/BDS displacement monitoring results, in the present research the global positioning system (GPS), the BeiDou navigation satellite system (BDS), and GPS/BDS displacement time series data from the GNSS monitoring points on the slopes on both sides of a tunnel entrance of the Changgan High-speed Railway are taken as an example to analyse the displacement of monitoring points before and after removal of coloured noise under the model of white noise + flicker noise. The experimental results show that the flicker noise causes uncertainty around displacement velocity estimation that exceeds the velocity itself, and there is a difference between the GPS and BDS velocity estimation. Eliminating the flicker noise in the displacement time series reduces the uncertainty of the displacement velocity estimation. Most of the uncertainties are less than 0.01 mm/d. Using GPS/BDS technology can help reduce flicker noise and the difference between GPS and BDS speed estimates. That is, the integrated use of GPS/BDS and the colored noise filtering technology in the displacement time series improves the reliability of the slope displacement monitoring results. In addition, the low slope on the side of the railway had a 4 mm displacement in the northerly direction during the selected period. The slope on both sides near the railway has a displacement of 1 to 2 mm in the easterly direction and a rise of about 4 mm in the vertical direction. The displacement of the slope further and higher from the road-bed is not more than 1 mm in both horizontal and vertical directions.

1. Introduction
Railway lines will inevitably pass through many complex geological areas such as mountains and hills. At the same time, they are also facing more serious safety-related risks of slope deformation and instability, which are directly related to the safety of railway construction and operation. Slope deformation and instability are sudden and difficult to judge and predict in advance, therefore, long-term deformation monitoring and analysis is needed, which is of great significance to ensure the safety of railway operations. Global navigation satellite system (GNSS) automatic monitoring characterised by high accuracy, unlimited by climate conditions, a high degree of automation and no need to see between stations, offers real-time dynamic monitoring and other advantages. Therefore, it is often used to monitor the deformation of slopes in real time and provide necessary data for slope design, construction, maintenance, and reinforcement [1-4].

The slope GNSS deformation monitoring actually uses the GNSS displacement time series of the monitoring point to estimate the displacement velocity of the monitoring point, and then obtain the displacement amount. The GNSS displacement time sequence of the monitoring point is composed of
displacement signals and noise. The noise includes white noise and colored noise, and colored noise dominates the noise, which is caused by incompletely eliminated tropospheric delay, satellite ephemeris error, multipath effects, and ignored seasonal signals [5]. Before estimating the displacement velocity, it is necessary to use the prior variance of the noise to construct the covariance matrix of the observation value, and then calculate the displacement velocity of the monitoring point according to the function model of the observation value [6]. The random characteristics of colored noise are different from white noise. However, in the previous GNSS slope deformation monitoring, when estimating deformation parameters, the noise in the displacement time series is treated as white noise and the colored noise is ignored. A covariance matrix that cannot accurately express the random characteristics of the observations is constructed, which is not conducive to obtaining an accurate rate of displacement and total displacement at each monitoring point.

BDS is a high-precision GNSS developed in China. At present, GPS technology is mostly used for GNSS slope deformation monitoring and analysis, while BDS technology is rarely used: under the influence of coloured noise, the monitoring results obtained according to GPS technology may be different from those arising from the use of BDS. Since the comprehensive use of GPS and BDS for deformation monitoring is helpful to reduce the noise in the deformation time series [7], we take the slope at the entrance of a tunnel of Chang-Jiangxi high-speed railway in China as an example and analyses the displacement of slope monitoring points by mainly using GPS/BDS coordinate time series data, taking into account the influence of coloured noise. At the same time, GPS and BDS coordinate time series are used to compare and analyse the influences of different positioning techniques on displacement monitoring results.

2. Experimental data
The slope of Chang-Jiangxi Line is located at the exit from one side of a tunnel in Xiajiang County, Jiangxi Province. The road-bed is in hilly terrain and has a length of about 370 m (Figure 1). The slope on the left-hand side of the tunnel entrance is significantly higher than that on the right. To monitor the slope displacement on both sides of the subgrade in this section in real time, 20 GNSS monitoring stations were installed on the slope, and a solar-powered reference station was established on the roof of a 3-story building with relatively stable geological conditions within 2km [8].

![Figure 1. Slopes on both sides of the tunnel portal.](image)

Due to the limitation of space, the observation data from four monitoring points were selected for analysis herein. Among them, monitoring points P1, P3, and P4 were close to the subgrade, with monitoring point P4 located on the side of the low slope of the subgrade, and monitoring point P2 was located on the side of the high slope of the subgrade (Figure 2, where JZ represents the reference station). All monitoring point receivers receive GPS and BDS satellite signals. Second generation
BDS satellite data were used due to limitations imposed by receiver conditions. Integrate GPS and BDS observation data, and use real-time kinematic (RTK) carrier phase difference technology to obtain GPS/BDS coordinate time series of monitoring points in the east (E), north (N) and vertical (U) directions. The sampling interval of the coordinate time series was five minutes, and the time span was from 0:00 on 28 April to 23:00 on 24 May 2020. Taking the coordinates at 0:00 on 28 April as a reference, the displacement of the monitoring point relative to the reference time point at each sampling time point is obtained, and finally GPS/BDS displacement time series in the three directions are formed. In order to compare with the results of GPS and BDS slope monitoring, this paper uses the same method to obtain GPS and BDS displacement time series (Figure 4).

Figure 2. Distribution of monitoring points and reference stations.

Figure 3. GPS (black), BDS (blue) and GPS/BDS (red) displacement time series data.
3. GPS/BDS displacement monitoring analysis
Since slope monitoring focuses on the displacement of monitoring points, only the linear movement in the displacement time series is considered, and the periodic signals are regarded as coloured noise. It can be seen from Figure 3 that the GPS/BDS displacement time series fluctuates greatly, which makes it impossible to distinguish the displacement trend of the measuring point. Therefore, the displacement speed of the monitoring point must be estimated based on a suitable noise model. Since the best noise combination model of GNSS coordinate time series data is white noise (WN) + flicker noise (FN) [6-9], and flicker noise is colored noise, under the combined model of WN + FN, the commonly used noise estimation software Create and Analyze Time Series (CATS) is used to calculate the noise amplitude (Table 1) and velocity (Table 2) in the displacement time series of each monitoring point [10]. It can be seen (Table 1) that the estimated amplitude of flicker noise is about four to seven times that of white noise, so flicker noise plays a dominant role in the residual of deformation time series data.

### Table 1. Amplitude estimation of white noise (mm) and flicker noise (mm/d^{0.25}) in the displacement time series of the monitoring station.

| Station | E-direction | N-direction | U-direction |
|---------|-------------|-------------|-------------|
|         | WN          | FN          | WN          | FN          | WN          | FN          |
| P1      | 1.79        | 7.78        | 1.46        | 9.15        | 5.04        | 25.00       |
| P2      | 1.09        | 6.49        | 1.46        | 7.01        | 3.39        | 19.63       |
| P3      | 1.29        | 6.52        | 1.48        | 6.90        | 4.05        | 19.77       |
| P4      | 1.56        | 7.54        | 1.77        | 8.70        | 4.48        | 24.31       |

### Table 2. Estimation of velocity considering coloured noise (mm/d).

| Station | E-direction | N-direction | U-direction |
|---------|-------------|-------------|-------------|
|         | Estimation  | Uncertainty | Estimation  | Uncertainty | Estimation  | Uncertainty |
| P1      | -0.01       | 0.14        | -0.04       | 0.13        | 0.17        | 0.46        |
| P2      | -0.03       | 0.12        | 0.05        | 0.13        | 0.04        | 0.37        |
| P3      | 0.06        | 0.12        | 0.03        | 0.13        | 0.15        | 0.36        |
| P4      | 0.07        | 0.14        | 0.17        | 0.16        | 0.36        | 0.45        |

As can be seen from Table 2, the uncertainty in almost all velocity estimates is greater than the velocity itself. This is due to the large amplitude of the coloured noise in the time series [11-13], which almost overwhelms the displacement information, resulting in greater uncertainty and lower reliability of the displacement velocity estimation. Therefore, to improve the accuracy of velocity estimation and reduce the uncertainty therein, the joint wavelet transform and information entropy theory are adopted to eliminate coloured noise in displacement time series data [14]. Figure 4 shows the GPS/BDS displacement time sequence of the monitoring point after the colored noise is removed. It can be seen that, after removing the coloured noise, there is a significant linear trend in the displacement time series, which indicates that the signal-to-noise ratio of the remaining displacement time series is high, and the influence of noise is reduced. To compare and analyse the noise changes in the displacement time series after removing coloured noise, Figure 5 shows the logarithmic power spectrum before and after removing flicker noise from the GPS/BDS displacement time series. The power spectrum of the displacement time series has a certain slope before the flicker noise is eliminated, and the energy is concentrated in the low-frequency region. After removing the coloured noise, the power of the displacement time series decreases, and the decrease in power at low frequencies is significantly greater than that at high frequencies. This is because most of the power of...
the coloured noise is concentrated in the low-frequency range [15], and the low-frequency power decreases the most after it is removed. It also shows that the joint wavelet transform and information entropy theory can eliminate the coloured noise in the displacement time series.

Figure 4. GPS/BDS displacement time series (red) and linear trend (blue) of monitoring points after removing flicker noise.

Figure 5. GPS/BDS displacement time series (red) and linear trend (blue) of monitoring points after removing flicker noise.
The same method was used to calculate the filtered noise amplitude (Table 3) and the velocity estimate (Table 4). By comparing Table 1 and 3, the estimated amplitude of flicker noise after filtering is no more than 0.02 mm/d$^{0.25}$, while the estimated amplitude of white noise also decreases to a certain extent. This indicates that the flicker noise in the displacement time series is almost entirely eliminated, and the white noise is weakened to a certain extent, which is consistent with the changes to the power spectrum before and after filtering in Figure 5. By comparing Table 2 and 4, the uncertainty of velocity estimation after filtering is significantly reduced, and most of the uncertainty is less than 0.01 mm/d. This shows that the flicker noise, which plays a dominant role in noise, is removed, leaving low-amplitude white noise.

Table 3. Amplitude estimation of white noise (mm) and flicker noise (mm/d$^{0.25}$) after removing flicker noise.

| Station | E-direction | N-direction | U-direction |
|---------|-------------|-------------|-------------|
|         | WN | FN | WN | FN | WN | FN |
| P1      | 0.44 | 0.02 | 0.34 | 0.01 | 0.35 | 0.02 |
| P2      | 0.33 | 0.01 | 0.41 | 0.02 | 0.31 | 0.01 |
| P3      | 0.46 | 0.02 | 0.40 | 0.02 | 0.38 | 0.02 |
| P4      | 0.38 | 0.02 | 0.52 | 0.02 | 0.35 | 0.01 |

Table 4. Estimation of velocity considering flicker noise (mm/d) after removing flicker noise.

| Station | E-direction | N-direction | U-direction |
|---------|-------------|-------------|-------------|
|         | Estimation | Uncertainty | Estimation | Uncertainty | Estimation | Uncertainty |
| P1      | 0.05 | <0.01 | -0.01 | <0.01 | 0.17 | <0.01 |
| P2      | 0.01 | <0.01 | 0.03 | <0.01 | 0.07 | <0.01 |
| P3      | 0.07 | <0.01 | 0.04 | <0.01 | 0.15 | <0.01 |
| P4      | 0.07 | <0.01 | 0.15 | <0.01 | 0.16 | <0.01 |

Table 5. Displacement of each monitoring point after flicker noise elimination (mm).

| Station | E-direction | N-direction | U-direction |
|---------|-------------|-------------|-------------|
|         | Displacement | Uncertainty | Displacement | Uncertainty | Displacement | Uncertainty |
| P1      | 1.28 | 0.04 | -0.21 | 0.02 | 4.54 | 0.02 |
| P2      | 0.32 | 0.02 | 0.79 | 0.02 | 1.90 | 0.02 |
| P3      | 1.92 | 0.04 | 0.98 | 0.02 | 3.96 | 0.02 |
| P4      | 1.85 | 0.02 | 4.01 | 0.04 | 4.41 | 0.02 |

Table 5 lists the displacements at monitoring points in three directions within the selected time range obtained based on the GPS/BDS displacement sequence. It can be seen that in the N-direction, the displacement at point P1 to the south is about 0.2 mm; the displacements at points P2 and P3 are less than 1 mm; and the displacement at point P4 is about 4 mm. In the E-direction, the displacements of points P1, P3, and P4 exceed 1 mm, and the displacements at points P3 and P4 are close to 2 mm, while those at point P2 are about 0.3 mm. In the U-direction, the displacement of points P1, P3, and P4 is about 4 mm, and that of point P2 is about 2 mm. The above analysis shows that, in the selected time range, the low slope on the right-hand side of the railway undergoes greater displacement in the
northerly direction compared with the high slope, and the slope on both sides near the railway also undergo a certain displacement in the easterly direction. Moreover, the slope on both sides of the railway has an upwards trend. Figure 6 shows the historical temperature record for the slope in April and May: in April, the highest and lowest local temperatures rose from about 15°C to 30 and 20°C, respectively. The average maximum and minimum temperatures in May remain at 30 and 20°C, therefore, slope heave is likely to be caused by the expansion caused by continuous heating of the surface and metal pier [16]. In addition, the slope further from, and higher than, the road-bed undergoes a smaller displacement in the horizontal direction (albeit no more than 1 mm).

![Figure 6. Logarithmic power spectrum of displacement time series in a certain direction of the monitoring point before (black) and after (red) removing flicker noise.](image)

4. Comparison with GPS and BDS displacement analysis
In order to compare and analyze the GPS and BDS slope displacement monitoring results, the same method is used to eliminate the flicker noise in the GPS and BDS displacement time series. The displacement velocity and noise of the monitoring points before and after filtering are calculated. The results are shown in Table 6 and Table 7, where the symbols "I" and "II" represent the results before and after filtering, respectively. Comparing Table 1 and Table 6, it can be seen that the estimated flicker noise amplitude in GPS deformation time series data exceeds that in BDS, and the estimated noise amplitude in GPS/BDS deformation time series data is the smallest. This may be because the system error of GPS is greater than that of BDS, and the integrated application of GPS and BDS for data processing increases the number of visible satellites, decreases dilution of precision (DOP) values [17], and thus improves positioning accuracy.
### Table 6. The amplitude estimation of white noise (mm) and flicker noise (mm/d\(^{0.25}\)) in the GPS and BDS displacement time series of the monitoring station before and after filtering.

| Station | Time series | Before and after filtering | E-direction | N-direction | U-direction |
|---------|-------------|-----------------------------|-------------|-------------|-------------|
|         |             |                             | WN          | FN          | WN          | FN          |
| P1      | GPS         | I                           | 2.49        | 9.28        | 1.69        | 12.50       | 6.54        | 32.78       |
|         |             | II                          | 0.43        | 0.02        | 0.34        | 0.01        | 0.34        | 0.01        |
|         | BDS         | I                           | 2.75        | 8.83        | 2.31        | 11.39       | 9.56        | 28.27       |
|         |             | II                          | 0.44        | 0.02        | 0.41        | 0.02        | 0.45        | 0.02        |
| P2      | GPS         | I                           | 1.75        | 10.58       | 1.69        | 13.09       | 4.45        | 36.83       |
|         |             | II                          | 0.28        | 0.01        | 0.23        | 0.01        | 0.23        | 0.01        |
|         | BDS         | I                           | 1.61        | 7.20        | 2.3         | 8.19        | 5.51        | 18.99       |
|         |             | II                          | 0.35        | 0.02        | 0.41        | 0.02        | 0.41        | 0.02        |
| P3      | GPS         | I                           | 2.13        | 8.96        | 2.16        | 10.77       | 5.63        | 32.74       |
|         |             | II                          | 0.38        | 0.02        | 0.33        | 0.01        | 0.32        | 0.01        |
|         | BDS         | I                           | 1.64        | 8.36        | 2.21        | 8.24        | 6.54        | 24.14       |
|         |             | II                          | 0.34        | 0.01        | 0.40        | 0.02        | 0.40        | 0.02        |
| P4      | GPS         | I                           | 2.30        | 9.81        | 2.25        | 13.81       | 5.48        | 33.91       |
|         |             | II                          | 0.37        | 0.02        | 0.28        | 0.01        | 0.28        | 0.01        |
|         | BDS         | I                           | -0.03       | 0.12        | 0.12        | 0.12        | -0.13       | 0.34        |
|         |             | II                          | 0.04        | <0.01       | 0.06        | <0.01       | -0.11       | <0.01       |

### Table 7. Estimation of velocity after flicker noise elimination (mm/d).

| Station | Time series | Before and after filtering | E-direction | N-direction | U-direction |
|---------|-------------|-----------------------------|-------------|-------------|-------------|
|         |             |                             | Estimation  | Uncertainty | Estimation  | Uncertainty | Estimation  | Uncertainty |
| P1      | GPS         | I                           | 0.03        | 0.13        | 0.02        | 0.17        | 0.20        | 0.45        |
|         |             | II                          | 0.06        | <0.01       | -0.10       | <0.01       | 0.22        | <0.01       |
|         | BDS         | I                           | -0.03       | 0.13        | 0.02        | 0.17        | 0.12        | 0.41        |
|         |             | II                          | 0.06        | <0.01       | 0.06        | <0.01       | 0.20        | <0.01       |
| P2      | GPS         | I                           | -0.01       | 0.15        | 0.02        | 0.18        | 0.31        | 0.51        |
|         |             | II                          | 0.33        | 0.33        | 0.33        | 0.33        | 0.33        | 0.33        |
|         | BDS         | I                           | 0.01        | 0.10        | 0.05        | 0.12        | -0.13       | 0.28        |
|         |             | II                          | 0.04        | <0.01       | 0.06        | <0.01       | -0.11       | <0.01       |
| P3      | GPS         | I                           | 0.08        | 0.12        | -0.01       | 0.15        | 0.29        | 0.45        |
|         |             | II                          | 0.09        | <0.01       | 0.09        | <0.01       | 0.35        | <0.01       |
|         | BDS         | I                           | -0.03       | 0.12        | 0.12        | 0.12        | -0.11       | 0.34        |
|         |             | II                          | 0.02        | <0.01       | 0.14        | <0.01       | -0.12       | <0.01       |
| P4      | GPS         | I                           | -0.03       | 0.13        | 0.13        | 0.19        | 0.41        | 0.47        |
|         |             | II                          | -0.01       | <0.01       | 0.14        | <0.01       | 0.35        | <0.01       |
|         | BDS         | I                           | 0.10        | 0.13        | 0.13        | 0.13        | 0.31        | 0.50        |
|         |             | II                          | 0.12        | <0.01       | 0.16        | <0.01       | 0.11        | <0.01       |
As can be seen from Table 7 that before filtering, for the same direction of the same monitoring point, there are differences in the displacement velocities of the stations obtained according to GPS, BDS, and GPS/BDS displacement time series data. Among them, 58% of the GPS velocity estimates differ from those made using BDS by more than 0.1 mm, and 66% of the velocities are estimated to have been in the opposite direction, for example, for the E-direction displacement time series at station P1, GPS speed and BDS speed have the same magnitude but opposite direction. BDS speed and GPS/BDS speed have the same direction, but the speed is three times that of GPS/BDS. For example, for the U-direction displacement time series at station P2, the GPS velocity is eight times that of GPS/BDS and twice that of BDS, and the direction is opposite that of BDS. Moreover, after filtering, about 30% of GPS velocity estimates differ from BDS by more than 0.1 mm/d, and about 40% of GPS, BDS, and GPS/BDS velocity directions differ. In the E-direction at station P1 station and the E and N-directions at station P3, the estimated directions of movement become consistent after filtering. In addition, about 75% of GPS/BDS velocity estimates are between those made using GPS and BDS. This shows that the difference between GPS and BDS speeds can be reduced by eliminating flicker noise and using GPS/BDS technology.

5. Conclusion
Taking a tunnel entrance slope of Chang-Jiangxi high-speed railway in China as an example, GPS, BDS, and GPS/BDS coordinate time series data from 28 April and 24 May 2020 are respectively used to analyse the slope displacement under the condition of white noise and flicker noise. The experimental results show that:

1. The amplitude of flicker noise in the displacement time series is four to seven times that of white noise, which leads to an uncertainty in velocity estimation exceeding the velocity itself, the estimated flicker noise amplitude in GPS displacement time series data exceeds that in BDS data, and that in GPS/BDS displacement time series data is the least.

2. There are differences in magnitude and direction among GPS, BDS, and GPS/BDS displacement velocities. By eliminating flicker noise in the displacement time series, the uncertainty of velocity estimation can be significantly reduced, and most of the uncertainties are less than 0.01 mm/d. The difference between GPS and BDS speed can be reduced by eliminating flicker noise and using GPS/BDS technology.

3. Within the selected time range, the monitoring point on the low slope on the right-hand side of the tunnel entrance has a 4 mm displacement in the northerly direction; the monitoring points on both sides of the slope near the railway also undergo a displacement of 1 to 2 mm in the easterly direction, and rise about 4 mm in the vertical direction. The further and higher from the road-bed on the left-hand side of the tunnel entrance, undergoes a displacement of less than 1 mm in both the horizontal and vertical directions.

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