A hadronic emission model for black hole-disc impacts in the blazar OJ 287

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ABSTRACT
A super-massive black hole (SMBH) binary in the core of the blazar OJ 287 has been invoked in previous works to explain its observed optical flare quasi-periodicity. Following this picture, we investigate a hadronic origin for the X-ray and γ-ray counterparts of the December 2015 major optical flare of this source. An impact outflow must result after the lighter SMBH (the secondary), crosses the accretion disc of the heavier one (the primary). We then consider acceleration of cosmic-ray (CR) protons in the shock driven by the impact outflow as it expands and collides with the active galactic nucleus (AGN) wind of the primary SMBH. We show that the emission of these CRs can reproduce the X-ray and γ-ray flare data self-consistently with the the optical component of the December 2015 major flare. We derive different emission profiles explaining the observed flare data having magnetic fields in the range $B = 0.3-3$ G in the emission region and power-law indices of $q = 2.2-2.4$ for the energy distribution of the emitting CRs. The mechanical luminosity of the AGN wind represents $\lesssim 10\%$ of the mass accretion power of the primary SMBH in all the derived emission profiles.

Key words: accretion – shock waves – astroparticle physics – radiation mechanisms: non-thermal

1 INTRODUCTION

Theoretical arguments as well as indirect observational evidences suggest the presence of super-massive black hole (SMBH) pairs coalescing in the core of certain galaxies. Galaxy mergers (Springel et al. 2005), for instance, might be a natural process leading to the formation of such SMBH binaries. Compelling examples of active galactic nuclei (AGNs) approaching each other can be found in the recent works by, e.g., Pfeifle et al. (2019) and Deane et al. (2014), where the SMBHs of approaching AGNs are localised at distances from tens to hundreds of parsecs between each other.

When the distance among two SMBHs shrinks to sub-parsec scales, the system is theoretically expected to enter its gravitational wave (GW)-driven regime for orbital decay. In such a stage, SMBH binaries are thought to be the most prominent sources of GWs in the cosmos (Begelman et al. 1980; Mingarelli et al. 2017). Current instruments however, are not able to detect either GWs from SMBHs systems (expected in the nHz-µHz domain), or resolve SMBHs binaries at sub-parsec scales. Alternatively, indirect signatures as double line emission (Popović 2012) and quasi-periodical flares in certain AGNs (Komossa & Zensus 2016) are employed to trace the presence of compact, orbiting SMBH pairs. Due to a persistent quasi-periodical feature in optical, the blazar OJ 287 is perhaps the strongest candidate for hosting a sub-parsec SMBH pair (Dey et al. 2018).

OJ 287 (at a red-shift $z = 0.304$) is categorised as a BL Lac object and is known for its regular ∼12 year, double peaked optical variations registered for over 130 years (Sillanpaa et al. 1988; Hudec et al. 2013). These periodic features have motivated a number of possible explanations (e.g., Lehto & Valtonen 1996; Katz 1997; Tanaka 2013; Britzen et al. 2018). Particularly, the SMBH binary scenario proposed by Lehto & Valtonen (1996) (see also Valtonen et al. 2008) appears to predict naturally the timing of the double peaked observed outbursts. Additionally, this model is consistent with the sharp rise of the flare emission and its low polarisation degree, being these aspects not satisfactorily explained by other models (see Dey et al. 2019; Kushwaha 2020, for more details).

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The SMBH binary model of Lehto & Valtonen (1996) explains the periodical outbursts of OJ 287 in terms of thermal bremsstrahlung radiation of the outflows generated by the impacts of the lighter SMBH (the secondary) on the accretion disc of the heavier one (the primary, see also Pihajoki 2016). Within this picture, a general relativistic (GR) approach for the orbit of the secondary BH predicted the starting times of the 1994, 1995, 2005, 2007, and 2015 flares (Valtonen et al. 2008, 2016). With the observed data from the last three outbursts, the BH masses of the binary have been constrained to \( M_1 = 1.83 \times 10^{10} \, M_\odot \) and \( M_2 = 1.5 \times 10^8 \, M_\odot \) for the primary and secondary BHs, respectively (Valtonen et al. 2016).

As expected from BL Lac objects, OJ 287 displays X-ray as well as \( \gamma \)-ray flaring behaviour (Neronov & Vovk 2011; Hodgson et al. 2017; Kushwaha et al. 2013, 2018b, c; Pal et al. 2020). Particularly, Kushwaha et al. (2018b) analyse the multi-wavelength (MW) light curves (LCs) of OJ 287 during and after the December 2015 major optical flare. These authors extracted the corresponding flare SED and investigate a hadronic origin for the X-ray and \( \gamma \)-ray fluxes with emission triggered by proton-proton (\( p-p \)) interactions of cosmic-rays (CRs) with the thermal ions within the impact outflow. In the scenario proposed here, we consider CR shock acceleration driven by the collision of the outflow and the AGN wind of the primary SMBH, as depicted in Figure 1.

This paper is organised as follows. In the next section, we characterise the SMBH-disc impact outflow and its thermal radiation following the considerations of previous works. In Section 3, we describe the non-thermal radiation that results due to \( p-p \) interactions of CRs with the thermal ions of the outflow. In Section 4, we apply the non-thermal emission model to explain the MW SED corresponding to the 2015 major flare of OJ 287. We finally summarise and discuss our results in Section 5.

2 THE OUTFLOW FROM THE SMBH-DISC IMPACT

After the secondary SMBH threads the accretion disc of the primary, two outflows emerge from the accretion disc (one above and the other below) at the location of the impact. This effect was studied analytically by Lehto & Valtonen (1996) and simulated by Ivanov et al. (1998) with a hydrodynamical approach. A cartoon of this event is depicted in Figure 1a. We here estimate the energy released in each outflow as (see Appendix A):

\[
E_0 = \frac{\pi}{10} R_{HL}^3 \rho d v_r^2.
\]

where \( \rho d \) is the gas density of the disc at the location of the impact, \( v_r \) is the velocity of the disc material in the co-moving frame of the travelling BH\(^1\), and

\[
R_{HL} = \frac{2GM_2}{v_r^2},
\]

is the Bondi-Hoyle-Lyttleton radius (Hoyle & Lyttleton 1939; Edgar 2004) of the secondary BH of mass \( M_2 \). Details on the derivation of equation (1) are discribed in Appendix A. With typical values for the parameters of the claimed binary system in OJ 287 (Valtonen et al. 2019), equations (1)-(2) gives

\[
E_0 = 1.3 \times 10^{56} \left( \frac{M_2}{1.5 \times 10^8 \, M_\odot} \right)^3 \left( \frac{n_d}{10^{14} \, \text{cm}^{-3}} \right) \left( \frac{0.075 \, \text{eV}}{v_r} \right)^4 \, \text{erg},
\]

for the energy stored in each outflow.

We consider that only the outflow emerging in the side of the disc pointing toward us contributes to the observed SED. We follow the approach of Lehto & Valtonen (1996, see also Pihajoki 2016) where this emitting outflow is modelled as a spherical expanding bubble (see Figure 1). Within this approach, after the BH-disc impact occurs, the bubble emerges from the disc with an initial radius \( R_0 \), gas density \( \rho_0 \) and temperature \( T_0 \). We here assume that the initial radius of the bubble is a fraction \( f_R \) of \( R_{HL} \), (given by equation 2):

\[
R_0 = f_R R_{HL}: f_R < 1.
\]

In an adiabatic expansion, when the bubble attains a radius \( R_b \) its temperature \( T_b \) and gas density \( \rho_b \) are:

\[
T_b = T_0 (\gamma_\alpha - 1) \frac{M_2}{M_1},
\]

\[
\frac{\rho_b}{\rho_0} = \frac{M_2}{M_1} (\frac{\gamma_\alpha}{\gamma_\alpha - 1})^{-3},
\]

where \( \xi \equiv R_b/R_0 \), and \( \gamma_\alpha = 4/3 \) is the adiabatic index for a radiation dominated mixture.

The thermal bremsstrahlung radiation produced within the outflow bubble is able to escape (and then manifest as an optical flare) when becoming optically thin to Thompson scattering. According to Lehto & Valtonen (1996) this occurs when the expanding radius \( R_b \) meets the condition (see also Pihajoki 2016, for elaborate details):

\[
\xi = \left( \frac{K_a \kappa T R_b^2 n_d v_r^2}{\rho_0 v_r^0} \right)^{1/7}.
\]

In equation (7), \( \kappa_T \equiv \sigma T / (\mu m_p) \) is the Thompson opacity and we use \( \mu = 1 \) (a fully ionised gas, composed hydrogen only), \( K_a = 6.8 \times 10^{22}(1 + 10^7 Z/t_{bf}) \), being \( Z \approx 0.02 \) the fraction of heavy elements in the interstellar medium, and \( t_{bf} = 100 \) a coefficient for bound-free absorption (Valtonen et al. 2019).

The outflow bubble is initially several orders of magnitude denser than the environment where it expands outside the accretion disc and thus we assume that the bubble is still in free expansion by the time it shines in thermal

\(^1\) Considering the toroidal component of the disc velocity \( \omega_r \) and the velocity of the secondary SMBH \( v_{orb} \), at the location of the impact, the velocity of the disc material in the co-moving frame of the secondary BH is \( v_r \approx v_{orb} - \omega_r \) (see Appendix A).
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of the bubble goes into the kinetic energy of the expansion:

\[ V_b = \sqrt{2E_0/M_0} = \left( \frac{3}{20} \frac{1}{f_R} \rho_0 \right)^{1/2} v_r \]  

(9)

where \( M_0 = 4\pi R^3_0\rho_0/3 \) is the mass of the outflow.

Given the values of the initial temperature \( T_0 \) and the fraction \( f_R \) (which we consider as free parameters), the initial gas density \( \rho_0 \) of the bubble can be obtained by solving the following equation

\[ \left( K_{\alpha T} f_R^2 R^2_{HI} \right) \left( \frac{3}{20} \frac{1}{f_R} \rho_0 \right)^{1/2} v_r \Delta t_{b} \]  

(10)

for \( \rho_0 \). Equation (10) is obtained combining equations (1)-(2), and (7) - (9). Once the value of the initial gas density \( \rho_0 \) is found, we compute the radius \( R_b \), temperature \( T_b \), and density \( \rho_b \) of the bubble when it starts shining in thermal bremsstrahlung by first obtaining \( \xi \) from equation (7), and then employing equations (5)-(6).

We chose the appropriate combination of the free parameters \( T_0 \) and \( f_R \) by fitting the implied thermal bremsstrahlung emission of the bubble of radius \( R_b \), temperature \( T_b \), and density \( \rho_b \) (corresponding to an expansion time \( \Delta t_b = 2 \) yr) to the observed SED data of the optical flare (see Section 4). To do this, we calculate the observed flux due to thermal bremsstrahlung radiation as

\[ vF_v = v'\pi \frac{R_b^2}{D_L^2} I_v, \]  

(11)

where \( v' = (1 + z)v \), \( z = 0.304 \) is the redshift of the blazar OJ 287, \( D_L = 1602 \) Mpc its luminosity distance, and \( I_v \) is the specific intensity (calculated for \( v' \)) of the thermal bremsstrahlung radiation at the outer boundary of the outflow, i.e. at \( R = R_b \). To account for thermal bremsstrahlung self-absorption, we compute the specific intensity as

\[ I_v = B_v (1 - \exp(-\tau_v)), \]  

(12)

\[ \tau_v = \int_{R_b}^{R_b} j_v/B_v \, ds \approx B_v j_v/\nu B_v, \]  

(13)

where \( B_v \) is the black-body spectral radiance of temperature \( T_b \)

\[ B_v = \frac{2h\nu^3}{c^2} \left( \exp \left( \frac{h\nu}{kT_b} \right) - 1 \right)^{-1}, \]  

(14)

and \( j_v \) is the thermal bremsstrahlung emission coefficient which here is taken as

\[ 4\pi j_v = 6.8 \times 10^{-38} n_b^2 \nu_b^{-1/2} \exp \left( \frac{h\nu}{kT_b} \right) \text{erg}^c^{-1} \text{cm}^{-3} \text{Hz}^{-1}. \]  

(15)

We consider the photon field density generated by the thermal bremsstrahlung emission of the outflow, as the target photon field for inverse Compton scattering of secondary electrons within the outflow (this cooling process is discussed in Section 3.2). We calculate this target photon field density (number of photons per unit energy, per unit volume) as

\[ n_{ph}(\epsilon, T_b) = \frac{2\pi h\nu (R_b, T_b)}{c\epsilon}, \]  

(16)

where \( \epsilon = h\nu \) is the energy of the thermal bremsstrahlung photons and \( I_v \) is the specific intensity given by equation

\[ \xi = 1 + \frac{V_b \Delta t_{b}}{R_0}, \]  

(8)

where \( \Delta t_{b} \) is the time interval that the outflow takes to shine after it emerged from the accretion disc and \( V_b \) is the velocity of the bubble expansion. For this particular December 2015 flare, we employ the fiducial value of \( \Delta t_{b} = 2 \) yr which is compatible with the value derived in Dey et al. (2018) with a general relativistic, post-Newtonian approach for the orbit of the secondary SMBH. The velocity of the bubble expansion \( V_b \) can be estimated assuming that most of the initial energy

\[ n_{ph} = \frac{2\pi h\nu (R_b, T_b)}{c\epsilon}, \]  

(16)
We also employ the photon field density of equation (16) to calculate the attenuation of the $\gamma$-rays due to photon-photon annihilation, as described in the following subsection.

2.1 The opacity of the impact outflow to gamma-ray photons

The flux of potential $\gamma$-rays produced in the impact outflow is susceptible to be attenuated by internal absorption. We consider the thermal bremsstrahlung radiation discussed in this Section as the dominant source of soft photons for $\gamma$-ray annihilation. If $L_{\gamma}(E_{\gamma})$ is the luminosity of $\gamma$-rays photons of energy $E_{\gamma}$ produced in the impact outflow, the luminosity of $\gamma$-rays that escape the emission region can be calculated as $L(E_{\gamma}) = L_{\gamma}(E_{\gamma}) \exp(-\tau_{\gamma\gamma})$, where $\tau_{\gamma\gamma}$ is the opacity of photon-photon annihilation.

To calculate $\tau_{\gamma\gamma}$ we assume for simplicity that the thermal bremsstrahlung photon field is isotropic and uniform within the outflow volume and non-effective for photon-photon collisions outside this volume. Thus, the opacity due to $\gamma$-ray absorption can be calculated as

$$\tau_{\gamma\gamma}(E_{\gamma}) = \int_{0}^{l_{\gamma\gamma}} ds \int_{E_{\gamma}}^{\infty} dE_{\gamma} \frac{\sigma_{\gamma\gamma}(E_{\gamma}, \epsilon) n_{ph}(\epsilon, T_{b})}{E_{\gamma}},$$

where $n_{ph}$ is the photon field of thermal bremsstrahlung radiation given by equation (16), $l_{\gamma\gamma}$ is the length of the path that $\gamma$-rays photons travel before leaving the outflow volume, and

$$\sigma_{\gamma\gamma}(E_{\gamma}, \epsilon) = \frac{\pi r_{e}^{2}}{2} (1 - \beta^{2}) \ln \left( \frac{1 + \beta}{1 - \beta} \right) + 2 \beta (\beta^{2} - 2).$$

is the total cross section for photon-photon collisions (e. g., Aharonian et al. 1985, Romero et al. 2010), where $r_{e} = 1 - m_{e}^{2}C^{4}/(E_{\gamma}c)$, and $r_{e}$ is the classical electron radius. The main uncertainty of this approach is the size of the length $l_{\gamma\gamma}$, which depends on the direction of the line of sight as well as on the morphology of the emission region (see Figure 1b).

We show in Figure 2 the attenuation factor $\exp(-\tau_{\gamma\gamma})$ calculated for $\gamma\gamma = 0.05R_{b}$ and $2R_{b}$, (blue solid and dashed curves respectively). Clearly, the difference between these two extreme cases is not substantial for defining the cut-off energy of the resulting $\gamma$-ray flux. For simplicity, we will then adopt the intermediate value of $\gamma\gamma = R_{b}$ for the $\gamma$-ray attenuation in all the SED models derived in Section 4.

$\gamma$-rays fluxes can also be attenuated by the extragalactic background light (EBL) on the way to the Earth. For a source of red-shift like that of the blazar OJ 287, photon-photon annihilation by the EBL is significant for $\gamma$-rays with energies $\gtrsim 100$ GeV (Finke et al. 2010). Since the internal absorption in the emission model discussed here let escape photons with energies $\lesssim 50$ GeV (see Figure 2) we neglect attenuation by the EBL in the SED models derived in Section 3.

In the vicinity of the central engine of AGNs with velocities & de Gouveia Dal Pino 2015; Giustini & Proga 2019, and Dey et al. 2018), which is outside the central BH of AGNs, though model dependent, may be of several orders of magnitude higher. Considering the temperature of protons, the speed of sound of an isothermal corona may then be of $\gtrsim 0.095c (T_{\gamma\gamma}/10^{14}K)^{1/2}$ which is comparable to the expansion velocity $V_{s}$ of the impact outflow (see equation 9).
\[ V_n + \cos(\theta)W_n = V_{u-w} \] is the velocity of the AGN wind material in the rest frame of bubble expanding front, \( V_u \) and \( V_w \) are the AGN wind and the bubble expansion velocities (see equation 9) in the rest frame of the primary SMBH, \( \theta \) is the angle between the wind velocity and the vector \(-\mathbf{r}\), being \( \mathbf{r} \) the unit vector normal to the shock surface (see Figure 1b), and \( c_s \) and \( c_A \) are the sonic and Alfvénic speeds, respectively. Using these values, we derive the associated non-thermal, hadronic emission of the AGN wind, the magnetosonic Mach number can be estimated as:

\[
M_{s,A} = V_{s,0} \left( \frac{m_p}{\gamma_n k T_n} \right)^{1/2} \left[ 1 + \frac{B_{w}^2}{4\pi \gamma_n k T_n n_w} \right]^{-1/2} - 1.74 \left( V_{s,0} / 0.25c \right)^{1/2} \left[ 1 + 0.3 \left( \frac{B_{w}}{1G} \right)^2 \left( \frac{10^9 K}{T_w} \right) \left( \frac{10^6 \text{cm}^{-3}}{n_w} \right) \right]^{-1/2},
\]

where we use \( \gamma_n = 5/3 \).

We compare the associated mechanical luminosity of the AGN wind with the mass accretion power of the primary SMBH by defining the wind efficiency parameter

\[
\eta_w = \frac{\pi R_{\text{imp}}^2 \rho v_{\text{imp}} v_{\text{w}}^3}{M_1 c^2} = 0.035 \left( \frac{n_w}{10^6 \text{cm}^{-3}} \right) \left( \frac{v_w}{0.2c} \right)^{3}. \tag{21}
\]

In this ratio, we use \( R_{\text{imp}} = 17566 \text{ AU} \) for the distance of the secondary SMBH-disc impact (corresponding to the 2015 outburst, see Dey et al. 2018) and \( M_1 = 0.1 M_{\text{Edd}} \) for the mass accretion rate of the primary SMBH of OJ 287 (Valtonen et al. 2019).

According to the estimations discussed in this subsection, a strong shock can be formed due to the interaction of the AGN wind and the impact outflow, if the AGN wind has velocity, temperature, magnetic field, and gas density constrained to values \( V_w \gtrsim 0.2c, T_w \lesssim 10^9 K, B_w \lesssim 1G \), and \( n_w \gtrsim 10^6 \text{cm}^{-3} \). In the next subsection, we will assume that such an AGN wind exists above the accretion disc at the location of the secondary SMBH impact and then we derive the associated non-thermal, hadronic emission of the accelerated CR protons.

### 3.2 Emission from proton-proton interactions

To calculate the potential non-thermal radiation produced by the impact outflow, we consider acceleration of CRs in the forward shock \(^3\) formed by the interaction of the expanding bubble outflow with the AGN wind driven by the primary SMBH (see Figure 1).

Assuming diffusive shock acceleration (DSA) in the forward shock, we calculate the acceleration rate of CR protons as

\[
t_{\text{acc}}^{-1} = \frac{dE}{dt} = \frac{V_{s,0}^2}{D(E)} \tag{22}
\]

where \( V_{s,0} \) is the upstream velocity in the rest frame of the shock (see the previous subsection), and \( D \) is the CR diffusion coefficient in the acceleration region. For simplicity, we adopt a spatially uniform, Kolmogorov-like diffusion coefficient of the form (Ptuskin et al. 2006; Celii et al. 2019):

\[
D = D_0 \left( \frac{E}{E_0} \right)^{1/3} \left( \frac{B}{B_0} \right)^{-1/3} \tag{23}
\]

with \( E_0 = m_p c^2 + 2m_\pi c^2 + m_n^2 c^4/(2m_p c^2) = 1.22 \text{ GeV} \) (the threshold energy for the production of \( n^0 \) mesons), \( B_0 = 1 \text{ G} \), and we choose the normalisation constant \( D_0 = 5 \times 10^{25} \text{ cm}^2 \text{ s}^{-1} \). This value for \( D_0 \) is motivated by the condition \( \sqrt{2D\Delta t} > \Delta R \).

In which CRs protons with energies \( E_0 \) diffuse from the forward shock into the bubble in a time \( \Delta t_b \), for the particular case of the December 2015 outburst, \( \Delta t_b \sim 2 \text{ yr} \) (see Section 2), and we use \( \Delta R \sim 0.5 R_b \) for the thickness of the shell of shocked AGN wind material (region ii in Figure 1b). With this normalisation, for \( B = 1 \text{ G} \) and \( E = 10 \text{ GeV} \), equation (23) gives \( D \sim 10^{28} \text{ cm}^2 \text{ s}^{-1} \), which, coincidentally, is of the order of the average diffusion coefficient inferred for our Galaxy.

We estimate the maximum energy \( E_{\text{max}} \) that CRs protons can attain balancing the acceleration rate \( t_{\text{acc}}^{-1} \) with the rates of energy losses of protons in the swept up shell (region ii in Figure 1b):

\[
t_{\text{acc}}^{-1} = \tau_{\text{diff}}^{-1} + \tau_p^{-1} + \tau_{\text{pp}}^{-1} \tag{25}
\]

In this context, we consider the loss energy rates due to CR diffusion \( \tau_{\text{diff}}^{-1} \), proton-interaction rates \( \tau_p^{-1} \) (of CRs with the thermal protons in the swept up shell), and photopion production \( \tau_{\text{pp}}^{-1} \) (due to interactions of CRs with the thermal bremsstrahlung radiation of the expanding bubble, see equation 16). We note that these cooling rates change as the outflow bubble evolves. For the sake of simplicity, we calculate these cooling rates at the time when the outflow bubble allows the thermal bremsstrahlung photons to escape (see Section 2). The expressions that we employ for the rate terms in equation (25) are described in Appendix B. If the maximum energy \( E_{\text{max}} \) obtained from equation (25) corresponds to an acceleration time larger than \( \Delta t_b \), the maximum energy is instead determined by the condition \( t_{\text{acc}} = \Delta t_b \).

In Figure 3a, we plot the characteristic times of the aforementioned acceleration and energy loss rates for protons in the swept up shell as a function of the proton energy,
for a particular configuration of the model parameters (the emission model M2, see Table 1). We find that for all the emission models presented here, the maximum energy $E_{\text{max}}$ is mainly determined by the rate of proton diffusion $r_{\text{diff}}$.

Considering that CRs escape isotropically from their acceleration zone, we note that the material of the outflow bubble (see Figure 1b) is the main target for $p$-$p$ interactions. The accelerated CR protons cool down much more efficiently by $p$-$p$ interactions within the outflow bubble than within the shell of swept up material. For an AGN wind with gas density $n_w \sim 10^6 \text{ cm}^{-3}$, for instance, the $p$-$p$ cooling time within the swept up shell (region ii in Figure 1b) is larger than the expansion time $\Delta t_b \sim 2 \text{ yr}$ of the outflow (corresponding to the 2015 flare), as can be seen in Figure 3a. On the other hand, the $p$-$p$ cooling time within the outflow bubble is of few days (see Figure 3b).

The neutral and charged pions ($\pi^0$ and $\pi^\pm$) produced out of $p$-$p$ interactions decay into $\gamma$-rays and electron-positron pairs $e^\pm$ through the channels:

$$\pi^0 \rightarrow \gamma + \gamma,$$

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu(v_\mu),$$

$$\mu^\pm \rightarrow e^\pm + \bar{\nu}_\mu(v_{\bar{\nu}_\mu}) + \nu_\mu(v_\mu),$$

where $\mu^\pm$ and $\nu_\beta$ represent muons and neutrinos, respectively. For the time scales of the problem discussed here, we can consider that pions and muons decay instantaneously in the primary SMBH rest frame. We note that for energies $< 1 \text{ TeV}$, the secondary $e^\pm$ pairs cool down more efficiently due to IC scattering (of the thermal bremsstrahlung radiation field generated by the outflow bubble) than due to synchrotron radiation, as can be seen in Figure 3b. The IC cooling time is $\lesssim 1 \text{ day}$ for $e^\pm$ with energies in the range $10^7$–$10^9 \text{ eV}$. The cooling time due to synchrotron radiation is $\gtrsim 1 \text{ day}$ for electrons with energies $\lesssim 1 \text{ GeV}$, assuming a magnetic field intensity of 1 G.

To calculate the emission due to $\pi^0$ decay as well as due to the secondary $e^\pm$ pairs generated out of $p$-$p$ interactions, we first assume that a stationary population of CRs (the time that the outflow bubble takes to manifest as a flare). Employing the diffusion coefficient defined in equation (23), the distance that the CRs penetrates within the bubble is $\Delta d \sim \sqrt{2D_E(E)\Delta A - \Delta R}$, where $\Delta R \sim 0.5R_b$ is the thickness of the shell of AGN wind shocked material (region ii in Figure 1). For a background magnetic field of $B = 1 \text{ G}$, $\Delta d \sim 0.2 - 1R_b$ for CR protons of energies $E = 1.22 - 100 \text{ GeV}$. Thus, for the emission models derived in this work where the calculated maximum energies of CR protons are of $< 0.1 - 1 \text{ TeV}$ (see Table 1), the CRs that penetrate the expanding bubble occupy approximately half of the volume of this bubble.

We parameterise the energy distribution of the CR population within the bubble as a power-law (P-L) with exponential cut-off of the form

$$J_p(E_p) = A \left( \frac{E_p}{E_0} \right)^{-q} \exp \left( -\frac{E_p}{E_{\text{max}}} \right) \text{ erg}^{-1}\text{cm}^{-3},$$

where $E_p$ is the energy of the CR protons (rest mass plus kinetic energy), $E_0 = 1.22 \text{ GeV}$ (the threshold energy for $\pi^0$ mesons), $E_{\text{max}}$ is the maximum energy obtained from equation (25), and the normalisation constant $A$ is obtained through the condition

$$0.1E_w = \int_V dV \int_{E_{\text{min}}}^{E_{\text{max}}} dE J_p(E).$$

In this condition, we fix the total energy of the CR population to be one tenth of the kinetic wind energy $E_w$ that crosses the shock formed due to the interaction of the AGN wind and the expanding outflow.

To calculate the energy $E_w$ we consider for simplicity an AGN wind with plane-parallel geometry (locally) of uniform density $\rho_w$ and velocity $V_w$. Considering the surface of the forward shock as nearly spherical, the flux of wind kinetic energy impinging on the shock surface in its rest frame can be estimated as

$$\frac{dE_{\text{w}}}{dtdA} = \frac{1}{2} \rho_w [V_s + \cos(\theta)V_w]^3,$$

where $\theta$ is the angle between the wind velocity and the vector $\hat{n}$, being $\hat{n}$ the unit vector normal to the surface of the shock (see Figure 1b). Thus, the wind kinetic energy that crosses the shock front, when the outflow bubble expands from the radius $R_0$ to $R_b$, can be calculated as

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure3.png}
\caption{Upper: Rates of acceleration and cooling for CR protons in the shell of swept up material (region ii in Figure 1b) driven by the impact outflow (see the text). Lower: Cooling rates for protons and secondary $e^\pm$ pairs within the outflow bubble (region iii in Figure 1b; see the text).}
\end{figure}
\[ E_w = \int_{R_0}^{R_b} \frac{dR}{V_s} \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \sin(\theta) \frac{1}{2} \rho_w \left[ V_s + \cos(\theta) V_w \right]^3 \]

\[ = \frac{1}{2} \rho_w \left( R_b^3 - R_0^3 \right) \left( V_s^2 + \frac{3}{2} V_w V_s + V_w^2 + \frac{V_0^4}{4V_s} \right). \quad (32) \]

\( R_0, R_b \) and \( V_s \) are obtained as described in Section 2. The values of \( \rho_w \) and \( V_0 \) are found by matching the hadronic emission to the observed X-ray and gamma-ray data (see the next Section). In reality the shock formed due to the collision of the outflow bubble and the AGN wind may follow a bow-shock morphology (as depicted in Figure 1b). Therefore, equation (32) slightly underestimates the wind kinetic energy impinging on the shock surface.

The volume integral in the RHS of equation (30) is simplified assuming that the distribution \( J_\theta \) is uniform along the volume of CR emission. The value of the P-L index \( q \) in the distribution (29) is obtained by fitting the calculated SED to the observed X-ray and gamma-ray data (see the next Section).

The thermal bremsstrahlung emission (described in Section 2), whereas we consider losses due to synchrotron radiation, IC pairs, respectively. Further details on the computation of the scattering, relativistic bremsstrahlung and Coulomb collisions, respectively. Further details on the computation of the injection function \( Q_\nu \) as well as on the expressions for the cooling terms in equation (34) can be found in Appendices (C) and (D).

Once the stationary distribution (33) is computed, we apply it to calculate the synchrotron and IC fluxes with the usual prescriptions (e.g., Blumenthal & Gould 1970). For the synchrotron emission, the magnetic field in the hadronic emission region (region iii in Figure 1b) is assumed to have the same value as in the acceleration region (region ii in Figure 1b). To calculate the emission due to IC scattering, the thermal bremsstrahlung radiation of the outflow is employed as the seed photon field (see equation 16).

In the next section, we apply the non-thermal and thermal emission processes described in this and the previous Section, to model the observed MW SED corresponding to the December 2015 flare of OJ 287.

4 SED MODELS FOR THE 2015 MAJOR FLARE OF THE BLAZAR OJ 287

Following its historical ~12 year optical flares, OJ 287 displayed a major optical excess in December 2015 in agreement with the prediction of the SMBH binary model (Valtonen et al. 2016).

In the X-ray and \( \gamma \)-ray bands, flare activity was also reported. Kushwaha et al. (2018b) carried out a MW analysis of the LCs during and after the November 2015 flare finding significant activity most prominently in the NIR, optical UV and X-ray bands, associated with significant change in the polarization angle (PA) and polarization degree (PD) (see also Gupta et al. 2019). Kushwaha et al. (2018b) extracted the MW SED of the flare and interpreted the X-ray and \( \gamma \)-ray components in terms of leptonic jet emission (they find \( \gamma \)-rays consistent with SSC whereas \( \gamma \)-ray data are better explained with EC), and the optical component in terms of multi-temperature disc emission.

Here, we present an alternative model for the SED extracted by Kushwaha et al. (2018b). Motivated by the fact that the LCs in the X-ray and \( \gamma \)-ray bands display flaring simultaneously with the optical excess, we interpret this HE flare state with the hadronic emission model described in Sections 3. To do this, we first determine the values of the parameters \( J_\theta \) and \( T_0 \) of the outflow bubble (which are free parameters according to the model described in Section 2) by matching the associated thermal bremsstrahlung emission with the optical flare data. Then, we calculate the non-thermal emission of the outflow bubble as described in Section 3.2. We display in Figure 4 different SED emission profiles that result from this thermal+hadronic emission model, and the parameters associated to the model profiles are listed in Table 1.

The green data points in the plots of Figure 4 are the flare MW SED extracted from the LCs data corresponding to MJD: 57359-57363 (see Kushwaha et al. 2018b). The data points in magenta represent the SED of what we consider as the pre-burst, quiescent state, for which we take the SED data extracted by Kushwaha et al. (2013) corresponding to the 2009 broadband LCs of the source (their “state 3” data, when no significant variability was displayed).

The blue curve is the calculated total emission that results from the quiescent plus the flare components. The quiescent state is modeled with a SSC + EC jet emission model (black, dashed curve), which here we adapt from Kushwaha et al. (2013). The orange solid curve is the outburst thermal bremsstrahlung emission (described in Section 2), whereas the red curves represent the flare emission of hadronic origin (see Section 3.2). The red solid curve corresponds to \( \pi^0 \) decay emission. The red dotted and dashed curves correspond to synchrotron and IC fluxes of the secondary \( e^\pm \) pairs, respectively. In all the models derived here, the thermal bremsstrahlung radiation is more intense, by two orders of magnitude or more, than the synchrotron radiation generated by the \( e^\pm \) pairs. Because of this, we neglect the SSC contribution, and thus we employ the thermal bremsstrahlung...
Figure 4. Flare and quiescent SEDs of the blazar OJ 287. The flare data (green points, adapted from Kushwaha et al. 2018b) corresponds to the period MJD: 57359-57363 (simultaneous to the November 2015 optical major flare). The data of the quiescent state (purple points, adapted from Kushwaha et al. 2013) corresponds to the period MJD: 55152-55184. The over-plotted curves are emission models for the quiescent (black dashed curve, adapted from Kushwaha et al. 2013) and flare state (blue solid curve). The red and orange curves are the components of the flare thermal+hadronic emission model derived in this paper (see the text). The three panels display the same data points and the same quiescent emission model, whereas a different flare emission profile is displayed in each panel. The model parameters of the flare emission profiles (labelled as Mf) are listed in Table 1.

Table 1. Free and derived parameters for the SEDs profiles of the models M1, M2 and M3 displayed in Figure 3. (See more details in the text.)

| Parameter                  | M1     | M2     | M3     |
|----------------------------|--------|--------|--------|
| $f_r \times 10^{-1}$       | 3.3    | 3.3    | 3.3    |
| $T_0 [10^3 K]$             | 7.5    | 7.5    | 7.5    |
| $n_0 [10^{14} \text{cm}^{-3}]$ | 1.8    | 1.8    | 1.7    |
| $T_w [10^3 K]$             | 1      | 1      | 1      |
| $V_w/c \times 10^{-1}$     | 2.0    | 2.0    | 2.0    |
| $B [G]$                    | 3.0    | 1.0    | 0.3    |
| $q$                        | 2.4    | 2.3    | 2.2    |
| $E_0 [10^{36}\text{erg}]$  | 1.3    | 1.3    | 1.3    |
| $V_\gamma/c \times 10^{-1}$| 0.6    | 0.6    | 0.6    |
| $n_b [10^{14} \text{cm}^{-3}]$ | 6.0    | 6.0    | 6.0    |
| $R_0/R_g$                  | 0.9    | 0.9    | 0.9    |
| $\xi = R_B/R_0$            | 44.9   | 44.9   | 44.9   |
| $n_b [10^{13} \text{cm}^{-3}]$ | 6.5    | 6.5    | 6.5    |
| $T_B [10^4 K]$             | 1.7    | 1.7    | 1.7    |
| $L_\gamma / (M_\gamma c^2) \times 10^{-2}$ | 6.3    | 6.3    | 5.9    |
| $E_{CR} [10^{53}\text{erg}]$ | 1.3    | 1.3    | 1.2    |
| $M_{sA}$                   | 12.4   | 13.0   | 13.1   |
| $E_{max} [\text{TeV}]$     | 1.8    | 0.6    | 0.2    |

The models M1, M2, and M3 are obtained assuming different magnetic fields in the emission region $B = 3.1$, and $0.3$ G, respectively. The derived maximum energy $E_{max}$ for the accelerated CRs protons corresponding to these models is in the range of $\sim 0.1 - 1$ TeV, with larger $E_{max}$ for larger $B$ (see Table 1). As we can see, for all the derived SED profiles the synchrotron emission of the secondary $e^\pm$ pairs does not contribute substantially to any spectral region of the flare data. This is particularly consistent with the low PD initially observed in the optical flare. In all the derived emission profiles of Figure 4, the $\gamma$-ray and X-ray spectral components of the flare data are explained in terms of $\pi^0$ decay together with IC emission of the secondary $e^\pm$ pairs (both produced out of $p - p$ interactions).

According to the derived emission profiles, the high energy flare data is consistent with a CR population (within the outflow bubble) with P-L index of $q = 2.2 - 2.4$. We also find that to reproduce the high energy flare components an AGN wind with efficiency of $\sim 0.06$ is required (according to the definition in equation 21).
5 SUMMARY AND DISCUSSION

The BL Lac blazar OJ 287 displayed a major optical outburst in December 2015, in agreement with its well known ~12 yr optical periodicity. Simultaneous flaring was also reported in the X-ray and γ-ray bands, with hardening in the spectral index compared with previous states seen in this source (Kushwaha et al. 2018b). In the present work, we show that a hadronic emission component compatible with the SMBH-disc impact model of (Lehto & Valtonen 1996) reproduces the broadband flare emission self-consistently including the optical, X-ray and γ-ray spectral components.

The one-zone hadronic emission model presented here is based on the following considerations.

- The impact of the secondary SMBH with the accretion disc of the primary one generates a bipolar outflow which is responsible for the optical flare emission (picture proposed by Lehto & Valtonen 1996). The outflow in the direction of the observer is the dominant source of the observed emission, which is modelled as an expanding sphere.
- CRs protons are accelerated in the forward shock driven by the expanding bubble outflow as it collides with the local AGN wind (see Section 3).
- By the time of the optical outburst (~2 yr after the outflow emerges from the disc), a population of CR protons has been injected within the impact outflow (see Figure 1b). This CR population has a total energy representing one tenth of the AGN wind kinetic energy that impinges the outflow (in the frame of the outflow expanding front) with a P-L energy distribution with exponential cut-off. We then calculate the π⁰ decay emission as well as synchrotron and IC scattering of secondary e⁰ pairs, where the neutral pions and secondary leptons are generated out of p-p interactions (Aharonian & Atoyan 2000; Kelner et al. 2006). The dominant seed photon field for IC scattering is the thermal bremsstrahlung radiation of the outflow (see Section 3).

We present different emission profiles which can explain the observed flare data. These emission models correspond to magnetic fields in the range of $B = 0.3 - 3$ G. For these emission models, the accelerated CR protons can attain maximum energies of $\lesssim 10$ TeV. The P-L index of the CRs population which emission matches the observed data is found in the spectral range of $q = 2.2 - 2.4$. In all the derived models, the mechanical luminosity of the AGN wind represents $\lesssim 10\%$ of the mass accretion power of the primary SMBH (see Table 1).

To interpret the emission data with the one-zone hadronic emission model discussed here, we assumed values for the size of the corona region as well as for the parameters of the AGN wind which appear to be reasonable and according to previous studies of coronae and wind of AGNs (see Section 3.2). However, more realistic models for the corona and the AGN wind (which are beyond the scope of this paper) could, perhaps, modify the results obtained here. For instance, the maximum energy $E_{\text{max}}$ of accelerated protons by DSA depends on the velocity and magnetic field of the wind impinging on the impact outflow. Also, DSA would not be efficient for BH-disc impacts occurring closer (or inside) to the primary SMBH AGN corona (see Section 3.1). This could be the case of the forthcoming BH-disc impacts predicted to occur closer to the primary SMBH (Dey et al. 2018) than the one discussed here (corresponding to the 2015 optical outburst).

The acceleration, propagation and emission of CRs within the source depend on the diffusion of these relativistic particles. For simplicity, here we adopted a spatially uniform, Kolmogorov-like diffusion coefficient. Nevertheless, more elaborate scenarios for CR diffusion could lead to more efficient particle acceleration, implying higher maximum energies for CR protons and detectable fluxes of HE neutrinos.

If the non-thermal emission scenario discussed here is indeed correct, the data of future BH impact outbursts may be applied to test shock acceleration models as well as more realistic multidimensional models of AGN-winds and coronae.

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APPENDIX A: THE ENERGY STORED BY THE BH-DISC COLLISION

Here we estimate the energy injected by the secondary BH after its passage through the disc of the primary one. To do this, we assume that Bondi-Hoyle-Lyttleton (BHL) accretion theory (Hoyle & Lyttleton 1939; Edgar 2004; Zanotti et al. 2011) is accurate to describe the accretion of the disc material onto the travelling BH. This turns out to be the case if the thickness \( \Delta h \) of the accretion disc at the location of the impact is such that \( \Delta h \gtrsim 2R_{\text{HL}} \), being \( R_{\text{HL}} \) the Bondi-Hoyle-Littleton radius of the travelling BH as defined in equation (2).

A wake of shocked material is formed behind the traveling BH, provided that its velocity is supersonic with respect to the sound speed of the disc fluid, which is the case for the situation discussed here. In the co-moving frame of the secondary BH (see Figure A1, lower panel), the fluid material impinging at cylindrical radii \(< R_{\text{HL}}\) is accreted onto the gravitational source. On the other hand, there is an amount of gas impinging at cylindrical radii \(> R_{\text{HL}}\) that is compressed through the shock and have enough kinetic energy to not fall onto the BH as illustrated in see Figure A1 (lower panel).

Here we consider that the kinetic energy of the environment material impinging at radii between \( R_{\text{HL}} \) and \((1+\delta)R_{\text{HL}}\), with \( \delta \ll 1 \), and during a time \( \Delta t = 2R_{\text{HL}}/v_r \) will eventually drive the outflows that emerge from the disc.
We estimate this energy as

\[
E_K = \pi \left[ (1 + \delta) R_{BH}^2 - R_{HL}^2 \right] \frac{1}{2} \rho_d v_T^2 \Delta t \tag{A1}
\]

\[
\simeq 2 \pi \delta R_{BH}^2 \rho_d v_T^2, \tag{A2}
\]

where in the second equality we neglected the term of second order in \(\delta\). We then assume that this energy will be split equally among two outflows that emerge (one above and the other below) the accretion disc. Setting \(\delta = 0.1\), we then estimate the energy of each outflow as

\[
E_0 = \frac{\pi}{10} R_{HL}^3 \rho_d v_T^2. \tag{A3}
\]

**APPENDIX B: ENERGY LOSSES OF CR PROTONS**

The rate of energy losses due to diffusion of CR protons in equation (25) is calculated as

\[
t_{-1}^{-1} = 2D(E_p, B)/R_d^2, \tag{B1}
\]

where \(D\) is the diffusion coefficient defined in equation (23) and \(R_d\) is the radius of the outflow bubble.

The rate of energy losses due to proton-proton interactions is computed as:

\[
t_{pp}^{-1}(E_p, n_0) = K_{pp} n_0 \sigma_{pp}(E_p), \tag{B2}
\]

where \(K_{pp} \sim 0.5\) is the inelasticity factor, \(\sigma_{pp}\) is the total cross section for \(p-p\) interactions taken from Kelner et al. (2006), and \(n_0\) is the local density of thermal ions. Following a strong shock approximation, within the shell of swept up material we set \(n_0 = 4n_w\), where \(n_w\) is the gas density of the impinging AGN wind. For the region within the outflow bubble we set \(n_0 = n_b\) (the density of the bubble, see equation 6).

The rate of energy losses of protons due to photo-pion production is computed as (e.g., Atoyan & Dermer 2003; Romero et al. 2010):

\[
t_{pp}^{-1}(E_p) = \frac{m_p^2 c^5}{2 E_p^2} \int_{2E_p}^{\infty} \frac{d\epsilon}{\epsilon^2} \epsilon^2 \kappa_{pp}(\epsilon) \nu_{pp}(\epsilon), \tag{B3}
\]

where \(\epsilon_{th} = 145\) MeV is the photon energy threshold for pion production in the rest-frame of the incident proton. For the inelasticity and the total cross section of the interaction, \(K_{pp}\) and \(\nu_{pp}\), we follow the approximation given by Atoyan & Dermer (2003).

**APPENDIX C: INJECTION OF GAMMA-RAYS AND SECONDARY ELECTRON-POSITRON PAIRS BY PROTON-PROTON INTERACTIONS**

To calculate the injection of \(\gamma\)-rays and secondary \(e^\pm\) pairs produced by \(p-p\) interactions due to CRs with energies > 100 GeV, we employ the functions derived in Kelner et al. (2006):}

\[
\dot{Q}_\gamma(E_\gamma) = c n_0 \int_{E_p}^{\infty} \sigma_{pp}(E_p) j_\gamma(E_p) F_\gamma(E_p/E_p, 100\text{ GeV}) \frac{dE_p}{E_p} \tag{C1}
\]

\[
\dot{Q}_e(E_e) = c n_0 \int_{E_e}^{\infty} \sigma_{pp}(E_p) j_\gamma(E_p) F_e(E_e/E_p, 100\text{ GeV}) \frac{dE_p}{E_p} \tag{C2}
\]

where \(E_\gamma\) is the energy of \(\gamma\)-ray photons and \(E_e\) the energy of secondary \(e^\pm\) paris, \(n_0\) is the gas number density of the background thermal ions (for the scenario discussed in this paper, this is the gas density of the outflow bubble \(n_0\)), \(\sigma_{pp}\) is the total cross section for \(p-p\) interactions, and \(J_\gamma\) is the energy density distribution of CR protons (see equation 29). The functions \(F_\gamma(E_\gamma/E_p, E_p)\) and \(F_e(E_e/E_p, E_p)\) are taken from equations (58) and (62), respectively, in Kelner et al. (2006). Equations (C1)-(C2) are valid for \(E_p > 100\) GeV and for \(E_{\gamma,\text{peak}}/E_p > 10^{-3}\).

To calculate the injection of \(\gamma\)-rays and \(e^\pm\) pairs produced by protons with energies close to the threshold energy for \(p-p\) interactions (~1.22 GeV), we employ the \(\delta\)-functional approach of Dermer (1986):

\[
\dot{Q}_\gamma = \frac{2c n_0 \bar{n}_\gamma}{K_{pp}} \int_{E_{\text{min}}}^{\infty} \frac{\sigma_{pp}(E_p) j_\gamma(E_p)}{\sqrt{E_p^2 - (m_p c^2)^2}} dE_p, \tag{C3}
\]

\[
\dot{Q}_e = \frac{2c n_0 \bar{n}_e}{K_{pp}} \int_{E_{\text{min}}}^{\infty} \frac{\sigma_{pp}(E_p) j_\gamma(E_p) f_e(E_p)}{E_p} dE_p \tag{C4}
\]

where

\[
y = \frac{E_p}{K_{pp}} + m_p c^2, \tag{C5}
\]

\[
E_{\text{min}} = E_p + (m_\pi^2 c^2)^2/(4E_p), \tag{C6}
\]

\[
E_{\text{min}} = E_p + (m_\pi^2 c^2)^2/(4E_p), \tag{C7}
\]

being \(K_{pp} = 0.17\) the proton inelasticity, \(m_\pi = 0\) and \(m_\pi = \gamma\) the masses of neutral and charged pions, respectively, and the function \(f_e\) in equation (C4) is defined in equation (36) of Kelner et al. (2006). \(\bar{n}_\gamma\) and \(\bar{n}_e\) are numerical constants that have to be adjusted in order to match the injection functions (C3) and (C4) with their higher energy counterparts given by equations (C1) and (C2), respectively.

**APPENDIX D: ELECTRON ENERGY LOSSES**

For the secondary \(e^\pm\) pairs, we consider as the relevant energy losses synchrotron radiation \(P_{\text{syn}}\), inverse Compton scattering \(P_{\text{IC}}\), relativistic bremsstrahlung \(P_{\text{br}}\), and Coulomb collisions.

We calculate the energy losses due to synchrotron cooling as:

\[
P_{\text{syn}} = \frac{4}{3} \sigma_T \frac{B^2}{8\pi} \left( \frac{E_e}{m_e c^2} \right)^2, \tag{D1}
\]

with \(\sigma_T = 6.652\) cm\(^2\) the Thompson cross section, \(m_e\) the electron rest mass and \(B\) the local magnetic field density.

The IC seed photon field considered here (the thermal bremsstrahlung radiation from the outflow bubble; see Section 2) has cut-off at \(\sim 1\) eV. Thus for secondary electrons with energy \(\lesssim 10^{11}\) eV, the IC scattering occurs in the Thompson regime, and their IC energy losses rate is given by:

\[
P_{\text{IC}} = \frac{4}{3} \sigma_T c U_{\text{ph}} \left( \frac{E_e}{m_e c^2} \right)^2. \tag{D2}
\]
where

\[ U_{ph} = \int_{\epsilon_{min}}^{\epsilon_{max}} \epsilon n_{ph}(\epsilon) d\epsilon, \]  

(D3)

is the energy density of the photon field in the emission region of the \( e^\pm \) pairs, and \( n_{ph}(\epsilon) \) is the photon field density generated by the thermal bremsstrahlung radiation of the outflow bubble (see equation 16).

Assuming a fully ionised medium, the cooling of \( e^\pm \) by relativistic bremsstrahlung is calculated as (e.g., Stur erson et al. 1997):

\[ P_{br} = \frac{8e^6R_{B}}{\hbar m_e c^2} (\ln \{\gamma_e\} + 0.36) (\gamma_e + 1), \]  

(D4)

with \( \gamma_e = E_e/(m_e c^2) \).

The energy loss rate due to Coulomb collisions of CR electrons with background thermal electrons is calculated as (e.g., Mannheim & Schlickeiser 1994; Stur erson et al. 1997):

\[ P_{Co} = \frac{2\pi e^4}{m_e c} \frac{n_{p} \lambda_{Co}}{\beta_e} \left[ \psi(x) - \psi'(x) \right], \]  

(D5)

where \( \lambda_{Co} \sim 15 \) is the Coulomb logarithm for the parameters of the problem of this work, \( \beta_e = v_e/c = \sqrt{1 - 1/\gamma_e^2} \) is the velocity of CR electrons in units of the speed of light, and

\[ \psi(x) = \frac{2}{\sqrt{T_e}} \int_0^x y^{1/2} \exp(-y) dy, \]  

(D6)

\[ \psi'(x) = \frac{d\psi}{dx}. \]  

(D7)

where \( x = m_e v_e^2/(2kT_e) \), being \( T_e \) the electron temperature inside the bubble given by equation (5).

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