ROLE OF NONPERTURBATIVE INPUT IN QCD RESUMMED HEAVY
BOSON $Q_T$ DISTRIBUTION

JIANWEI QIU$^a$ and XIAOFEI ZHANG$^b$

$^a$Department of Physics and Astronomy, Iowa State University
Ames, Iowa 50011, U.S.A.
$^b$Center for Nuclear Research, Department of Physics, Kent State University
Kent, Ohio 44242, U.S.A.

We show that role of nonperturbative input in the $b$-space QCD resummation formalism for heavy boson transverse momentum ($Q_T$) distribution strongly depends on collision energy $\sqrt{S}$. At collider energies, the larger $\sqrt{S}$ is, the weaker role nonperturbative input plays, and better predictive power the $b$-space resummation formalism has.

1 Introduction

With new data coming from Fermilab Run II and from the Large Hadron Collider (LHC) in the near future, we expect to test Quantum Chromodynamics (QCD) to a new level of accuracy, and also expect that a better understanding of QCD will underpin precision tests of the Electroweak interactions and particle searches beyond the Standard Model. In this talk, we will concentrate on Drell-Yan type production of color neutral heavy boson ($W^\pm$, $Z$, and Higgs) of invariant mass $Q$ at small transverse momentum $Q_T$.

When $Q_T \ll Q$, the $Q_T$ distribution of the heavy boson production calculated in the conventional fixed-order perturbation theory receives a large logarithm, $\ln(Q^2/Q_T^2)$. Beyond the leading order, we can get two powers of the logarithm for every power of $\alpha_s$. Therefore, at sufficiently small $Q_T$, convergence of the conventional perturbative expansion in powers of $\alpha_s$ is impaired, and the logarithms must be resummed.

2 The $b$-space resummation formalism

By using the renormalization group equation technique, Collins, Soper, and Sterman (CSS) derived a $b$-space resummation formalism for the $Q_T$ distribution of the heavy boson production.
The formalism has the following generic form for collisions between hadrons $A$ and $B$,

\[
\frac{d\sigma_{A+B\rightarrow V+X}}{dQ^2 dy dQ_T^2} = \frac{d\sigma_{A+B\rightarrow V+X}^{(\text{resum})}}{dQ^2 dy dQ_T^2} + \frac{d\sigma_{A+B\rightarrow V+X}^{(Y)}}{dQ^2 dy dQ_T^2},
\]

where $V$ represents the heavy boson. In Eq. (1), the $\sigma^{(Y)}$ term is negligible for small $Q_T$ and becomes important when $Q_T \sim Q$. The $\sigma^{(\text{resum})}$ includes all orders resummation of the large logarithms and can be expressed as

\[
\frac{d\sigma_{A+B\rightarrow C+X}^{(\text{resum})}}{dQ^2 dy dQ_T^2} = \frac{1}{(2\pi)^2} \int d^2 b e^{iQ_T \cdot b} W(b, Q) = \int \frac{db}{2\pi} J_0(Q_T b) b W(b, Q)
\]

where $J_0$ is Bessel function and $W(b, Q) = \sum_{ij} \sigma_{ij\rightarrow V}^{(0)}(Q) W_{ij}(b, Q)$ is a $b$-space distribution with dependence on rapidity $y$ suppressed. The $\sigma_{ij\rightarrow V}^{(0)}(Q)$ is the lowest order partonic cross section for partons of flavor $i$ and $j$ to produce a heavy boson $V$ of invariant mass $Q$.

Because of initial-state hadrons, $W_{ij}(b, Q)$ depends on momentum scale of hadron wave function ($1/\text{fm} \sim \Lambda_{\text{QCD}}$) and is in principle nonperturbative. However, when $b$ is small ($\ll 1/\Lambda_{\text{QCD}}$), physics associated with momentum scales $1/b$ and $Q$ are perturbative, and large logarithms from $\log(1/b^2)$ to $\log(Q^2)$ can be resummed by solving the following evolution equation.

\[
\frac{\partial}{\partial \ln Q^2} W_{ij}(b, Q) = \left[K(\mu, \alpha_s) + G(Q/\mu, \alpha_s)\right] W_{ij}(b, Q)
\]

where kernels $K$ and $G$ themselves obey renormalization group equations. By solving the linear evolution equation, one derives the resummed $b$-space distribution, $W(b, Q) = e^{-S(b,Q)} W(b, c/b)$, with constant $c = \mathcal{O}(1)$ and $S(b, Q) = \int_{Q^2/\mu^2}^{Q^2} \frac{ds}{s} \left[ A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu})) \right]$. The $A(\alpha_s(\bar{\mu}))$ and $B(\alpha_s(\bar{\mu}))$ are perturbatively calculable in power series of $\alpha_s$. All large logarithms in $W(b, Q)$ are completely resummed into the exponential factor $\exp[-S(b, Q)]$ leaving $W(b, c/b)$ with only one hard scale $1/b$. When $b \leq b_{\text{max}} \ll 1/\Lambda_{\text{QCD}}$, the nonperturbative physics in $W(b, c/b)$ can be factorized into parton distributions, and the resummed $W(b, Q)$ can be factorized as

\[
W^{\text{pert}}(b, Q) = \sum_{ij} \sigma_{ij\rightarrow V}^{(0)} \left[ f_{a/A} \otimes C_{a\rightarrow i} \right] \otimes \left[ f_{b/B} \otimes C_{b\rightarrow j} \right] \times e^{-S(b,Q)}
\]

where $f$ and $C$ are parton distributions and perturbatively calculable coefficient functions, respectively. In Eq. (4), the $\otimes$ represents convolution over parton momentum fraction, and the superscript “pert” indicates that $W^{\text{pert}}(b, Q)$ is perturbatively calculable at small $b$ if parton distributions are known.

When $Q^2$ is large enough, the perturbatively resummed $b$-space distribution $W^{\text{pert}}(b, Q)$ has a generic functional form shown in Fig. 1. The peak and corresponding saddle point ($b_{np}$) depends on values of $Q$ and $\sqrt{Q^2}$. Since the $W^{\text{pert}}(b, Q)$ is only reliable for small $b$ region, an extrapolation to large $b$ is necessary in order to complete the Fourier transform in Eq. (4).

### 3 Extrapolation to large $b$ region

Collins, Soper, and Sterman proposed the following large-$b$ extrapolation,

\[
W^{\text{CSS}}(b, Q) \equiv W(b_*, Q) F^{NP}(b, Q),
\]

where $b_* \equiv b/\sqrt{1 + (b/b_{\text{max}})^2} < b_{\text{max}} \sim 0.5 \text{ GeV}^{-1}$ and $F^{NP}(b, Q) \sim \exp(-\kappa b^2)$ is a Gaussian-like nonperturbative function. The $\kappa$ depends on fitting parameters, $g_i$ with $i = 1, \ldots, n$. By adjusting functional form for $\kappa$ and fitting parameters $g_i$, $Q_T$ distributions derived from $W^{\text{CSS}}(b, Q)$ are not inconsistent with Fermilab data on $Z$ and $W$.\[\]
Although it is successful in interpreting existing data, the $b$-space resummation formalism has been questioned due to two apparent drawbacks. The first is the difficulty of matching the resummed and fixed-order predictions; and the second is to know the quantitative difference between the prediction and the fitting because of the introduction of a nonperturbative $F^{NP}$. Recently, we demonstrated that both apparent drawbacks can be overcome. According to the large-$b$ extrapolation defined in Eq. (5), the nonperturbative function $F^{NP}$ and its fitting parameters can not only affect the large $b$ region, but also significantly change the perturbatively calculated $b$-space distribution at small $b$. In order to quantitatively separate QCD prediction from parameter fitting, we introduce a new large-$b$ extrapolation

$$W(b, Q) = \begin{cases} W^{\text{pert}}(b, Q) & b \leq b_{\text{max}} \\ W^{\text{pert}}(b_{\text{max}}, Q) F^{NP}(b, Q; b_{\text{max}}) & b > b_{\text{max}}. \end{cases}$$

(6)

This new extrapolation preserves the QCD resummed $b$-space distribution at small $b$. For large $b$ region, a new functional form of $F^{NP}$ was derived by adding power corrections to the evolution equations of $W(b, Q)$

$$F^{NP} = \exp \left\{ -\ln \left( \frac{Q^2 b_{\text{max}}^2}{c^2} \right) \left[ g_1 \left( (b^2)^\alpha - (b_{\text{max}}^2)^\alpha \right) + g_2 \left( b^2 - b_{\text{max}}^2 \right) \right] - \bar{g}_2 \left( b^2 - b_{\text{max}}^2 \right) \right\}.$$

(7)

The $(b^2)^\alpha$ term with $\alpha < 1/2$ corresponds to a direct extrapolation of resummed $W^{\text{pert}}(b, Q)$, while the $b^2$ terms correspond to the power corrections to the evolution equation. The $\bar{g}_2$ term corresponds to power correction from soft gluon shower; and the $\bar{g}_2$ is due to parent partons’ nonvanish intrinsic transverse momentum. The parameters, $g_1$ and $\alpha$ are completely fixed by $W^{\text{pert}}$ by requiring the first and second derivatives of $W(b, Q)$ to be continuous at $b = b_{\text{max}}$.

4 Predictive power of the formalism

In order to exam the predictive power, we divide the $b$-integration in Eq. (2) into a perturbative ($b \leq b_{\text{max}}$) and a nonperturbative ($b > b_{\text{max}}$) region. Predictive power of the $b$-space resummation formalism is sensitive to the relative contributions from these two regions.

From the generic $b$-space distribution in Fig. 1, better predictive power requires a smaller $b_{sp}$ for the saddle point. We found that numerical value of $b_{sp}$ has a strong dependence on the $\sqrt{S}$ in addition to its well-known $Q^2$ dependence. The larger $\sqrt{S}$ corresponds to much a smaller $b_{sp}$.

In Fig. 2 we plot the $b$-space distribution for $Z$ production at two different collision energies, $\sqrt{S} = 14$ TeV (solid) and $\sqrt{S} = 2.0$ TeV (dashed) with $W(b, Q)$ at Tevatron energy multiplied by a factor of 50. The $W(b, Q)$ at LHC energy is clearly peaked at a smaller $b_{sp}$.

Precise contribution from large $b$ region depends on the functional form and corresponding parameters of the $F^{NP}$. In Fig. 3, we plot log$(1/F^{NP})$ as a function of $b$ for $Q = M_Z$ and
Figure 3: Nonperturbative $\log(1/F_{NP})$ defined in Eq. (7) as a function of $b$.

$b_{\text{max}} = 0.5 \text{ GeV}^{-1}$. The dotted (dashed) line represents the leading fractional power term at $\sqrt{S} = 14 \text{ TeV}$ ($\sqrt{S} = 2 \text{ TeV}$). Both power correction terms are combined into the solid line. The parameters, $g_2$ and $\bar{g}_2$, are fixed by fitting low energy Drell-Yan data. Since the $b$-integration converges at $b \leq 2 \text{ GeV}^{-1}$ for all $Q_T < Q$, we expect very small power corrections for heavy boson production at collider energies, in particular, at the LHC energy.

Because of its weak role in $F_{NP}$, we can first neglect the power corrections and predict the heavy boson transverse momentum distribution without any free parameter, except the choice of $b_{\text{max}}$. Variation of $b_{\text{max}}$ is a good test of uncertainties of our predictions. In Fig. 4, we compare our prediction with Fermilab data on $Z$ production with $b_{\text{max}} = 0.5 \text{ GeV}^{-1}$ (solid line). We find that the theoretical prediction is insensitive to the choice of $b_{\text{max}}$ within 0.3 to 0.8; and the power corrections in $F_{NP}$ only change the $Q_T$ distribution in Fig. 4 for less than 5% at the lowest $Q_T$ and less than 1% for $Q_T > 5 \text{ GeV}$. As expected from the features shown in Figs. 2 and 3, the power corrections to $Z$ production at the LHC at $\sqrt{S} = 14 \text{ TeV}$ is less than 1% even at the lowest $Q_T$ bin.

5 Conclusions

We conclude that CSS $b$-space resummation formalism with a new large-$b$ extrapolation has an excellent predictive power for heavy boson transverse momentum distribution at collider energies. Larger the collision energy is, better the predictive power is. At collider energies, the large-$b$ nonperturbative contribution is dominated by the extrapolation of $W_{\text{pert}}(b, Q)$, and power corrections plays a very weak role.

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