Generalized 2d-dilaton models, the true black hole and quantum integrability

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Abstract

All 1 + 1 dimensional diffeomorphism-invariant models can be viewed in a unified manner. This includes also general dilaton theories and especially spherically symmetric gravity (SSG) and Witten’s dilatonic black hole (DBH). A common feature — also in the presence of matter fields of any type — is the appearance of an absolutely conserved quantity C which is determined by the influx of matter. Only for a subclass of generalized dilaton theories the singularity structure vanishes together with C. Such ‘physical’ theories include, of course, SSG and DBH. It seems to have been overlooked until recently that the (classical) ‘black hole’ singularity of the DBH deviates from SSG in a physically nontrivial manner. At the quantum level for all generalized dilaton theories — in the absence of matter — the local quantum effects are shown to disappear. This enables us to compute e.g. the second loop order correction to the Polyakov term. For non-minimal scalar coupling we also believe to have settled the controversial issue of Hawking radiation to infinity with a somewhat puzzling result for the case of SSG.
1 Introduction

The prime motivation for investigating generalized dilaton models \[1, 2, 3, 4\] and especially the dilaton black hole (DBH), always has been the hope to obtain information concerning problems of the 'genuine' Schwarzschild black hole (SBH) in \(d = 4\) General Relativity: the quantum creation of the SBH and its eventual evanescence because of Hawking radiation and the correlated difficulty of information loss by the transformation of pure quantum states into mixed ones, black hole thermodynamics etc. \[5\]. On the other hand, essential differences between DBH and SBH have been known for a long time. We just quote the Hawking temperature \((T_H)\) and specific heat: For the DBH \(T_H\) only depends on the cosmological constant instead of a dependence on the mass parameter as in the SBH. The specific heat is zero for the DBH and negative for the SBH.

It is important for any application of the DBH or its generalizations to compare the respective singularity structure with the ones encountered in General Relativity (GR), e.g. for the (uncharged) spherically symmetric case. However, careful studies of the singularity structure in such theories seem to be scarce. Apart from \[6\] and our recent work \[4\] we are not aware of such a comparison. It always seems to have been assumed that the physical features coincide at least qualitatively in all respects. During our recent work \[4\] we noted that this is not the case: For the ordinary dilaton black hole of \[7\] null extremals are complete at the singularity. Of course, non–null extremals are incomplete, and so at least that property holds for the DBH, but, from a physical point of view, it seems a strange situation that massive test bodies fall into that singularity at a finite proper time whereas it needs an infinite value of the affine parameter of the null extremal (describing the influx e.g. of massless particles) to arrive. This obviously contradicts Penrose’s 1965 theorem \[7\] which is valid in \(d=4\). Problems related to the application of that theorem in \(d=2\) have been voiced also some time ago \[8\] within a particular 2d model. Thus, from the point of view of the SBH care has to be taken within any effort to extract theoretical insight from the usual DBH, whenever the singularity itself is involved. This is certainly the case for the last stages of an evaporating black hole. Thus it is not obvious that other physical questions such as e.g. the information paradox can be dealt with satisfactorily within the DBH model of \[1\]. Of course, effects related to the horizon alone are not affected by our analysis, as long as the horizon is sufficiently far from the related singularity.
In order to pave the way for a more realistic modelling of the SBH we (Section 2) consider a two parameter family of generalized dilaton theories which interpolates between the DBH and other models, several of whom have been suggested already in the literature \[6, 9, 10, 11\]. The Eddington–Finkelstein (EF) form of the line element, appearing naturally in 2d models when they are expressed as 'Poisson–Sigma models' (PSM) \[12\] is very helpful in this context. We indeed find large ranges of parameters for which possibly more satisfactory BH models in \(d = 2\) may be obtained.

Section 3 is devoted to complete quantum integrability, whereas Hawking radiation is treated in Section 4.

In order to be able to compare the family of dilaton theories considered below, the EF metric

\[
(ds)^2 = d\bar{v}(2d\bar{u} + l(\bar{u})d\bar{v})
\]  

is most useful, which explicitly depends on the norm \(l = k^\alpha k_\alpha\) of the Killing vector \(\partial/\partial \bar{v}\).

Eq. (1) is particularly convenient to make contact with the PSM formulation which can be obtained for all covariant 2d theories \[13\]. They may be summarized in a first order Palatini type action

\[
L = \int X^+ T^- + X^- T^+ + X d\omega - e^- \wedge e^+ V(X)
\]  

In our present case only vanishing torsion

\[
T^\pm = (d \pm \omega)e^\pm
\]

as implied by Eq. (2) is expressed in terms of light–cone (LC) components for the zweibein one form \(e^a\) and for the spin connection one form \(\omega^a = e^a \omega\). The ‘potential’ \(V\) determines the dynamics. It is simply related to the Killing norm \(l\) in the EF gauge because (2) can be solved exactly for any integrable \(V\) \[13\] with the solutions (constant curvature is excluded)

\[
e^+ = X^+ df
\]

\[
e^- = \frac{dX}{X^+} + X^- df
\]
A similar equation for $\omega$ will not be needed in the following. The line element immediately yields the EF form (6) with $\bar{u} = X$ and $\bar{v} = f$. The Killing norm

$$l = C - \int^X V(y)dy$$

(6)

follows from a conservation law

$$C = X^+ X^- + \int^X V(y)dy$$

(7)

common to all 2d covariant theories [12] [13] which is related to a global nonlinear symmetry [14]. The usual dilaton models are produced by the introduction of the dilaton field $\phi$ in $X = 2 \exp(-2\phi)$, together with a conformal transformation $e^a = \exp(-\phi)\tilde{e}^a$ of $d\omega$ in (2)

$$\epsilon_{\mu\nu} \partial_\mu \omega_\nu = -\frac{R\sqrt{-g}}{2}$$

(8)

with the components $\omega_\mu$ expressed by the vanishing of the torsion (4) in terms of the zweibein.

The relation

$$\sqrt{-g}R = \sqrt{-\tilde{g}}\tilde{R} + 2\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu \phi)$$

(9)

will be used frequently.

Let us consider the SBH in a little more detail. The starting point is the Schwarzschild solution in EF coordinates [15]

$$ds^2 = 2dvdr + \left(1 - \frac{2M}{r}\right)dv^2 - r^2 d\Omega^2,$$

(10)

whose $r - v$ part is of the type (1). Thus the radial variable may be identified with $\bar{u}$ in (1). Indeed the correct singularity behavior is obtained from the PSM action (2) with
\[ V = -\frac{M}{X^2}. \] (11)

The ‘pure’ PSM model for the SBH with potential (11) is fraught with an important drawback: When matter is added the conserved quantity \( C \) in (27) simply generalizes to a similar conserved one with additional matter contributions. As shown in the second reference of [14], from the equations of motion the conservation law then refers to

\[ C \rightarrow C + C^{(m)} \] (12)

where \( C^{(m)} \) vanishes in the absence of (bosonic as well as fermionic) matter. It should be emphasized that \( C \) coincides with the mass parameter in the ADM as well as Bondi sense [16] for the DBH and for spherically symmetric gravity up to numerical factors, as has been analyzed in detail in [17]. In [14] it was also pointed out that the definition of such a conserved quantity does not require an asymptotically flat space-time. Thus even before a BH is formed by the influx of matter an ‘eternal’ singularity as given e.g. by (11) for the SBH, is present in which the mass \( M \) basically cannot be modified by the additional matter. A general method to produce at the same time a singularity–free ground state with, say, \( C = 0 \) is provided by a Weyl transformation of the original metric. It simply generalizes what is really behind the well-known construction of the DBH theory. Consider the transformation

\[ \tilde{g}_{\mu\nu} = \frac{g_{\mu\nu}}{w(X)} \] (13)

in (1) with (6) together with a transformation of \( X \)

\[ \frac{dX}{d\tilde{X}} = w(X(\tilde{X})). \] (14)

This reproduces the metric \( \tilde{g}_{\mu\nu} \) in EF form

\[ (ds)^2 = 2df \left( d\tilde{X} + \left( \frac{C}{w} - 1 \right) df \right) \] (15)

with a flat ground–state \( C = 0 \). Integrating out \( X^+ \) and \( X^- \) in (2), and using the identity (3) with \( \phi = \frac{1}{2} \ln w \) one arrives at a generalized dilaton theory.
\begin{equation}
L = \sqrt{-\tilde{g}} \left( \frac{X}{2} R + \frac{V_w}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{X} \partial_\nu \tilde{X} - V_w \right) \tag{16}
\end{equation}

where \( X \) is to be re-expressed by \( \tilde{X} \) through the integral of (14).

It should be noted that the (minimal) coupling to matter is invariant under this redefinition of fields. Clearly (16) is the most general action in \( d = 2 \) where the flat ground state corresponds to \( C = 0 \). \( V(X) \) may determine an arbitrarily complicated singularity structure. The DBH is the special case \( V = \lambda^2 = \text{const.} \). Then \( \tilde{X} \) is easily seen to be proportional to the dilaton field. The SBH results from the choice \( V = X^{-1/2} \). Using (14) and comparing (15) with (14) in that case with the interaction constant in \( w \) fixed by \( w = \frac{\lambda^2}{2} \), the conserved quantity \( C \) is identified with the mass \( M \) of the BH and (16) turns into the action of spherically reduced 4D general relativity [9]. Unfortunately such a theory cannot be solved exactly if coupling to matter is introduced.

2 Dilaton Models with Schwarzschild-like Black Holes

In [18] all models with one horizon and power type singularity were analyzed globally. They are described by the action

\begin{equation}
L = \int d^2x \sqrt{-g} e^{-2\phi} (R + 4a(\nabla \phi)^2 + B e^{2(1-a-b)\phi}) \tag{17}
\end{equation}

This action covers e.g. the CGHS model [1] for \( a = 1, b = 0 \), spherically reduced gravity [2] \( a = \frac{1}{2}, b = -\frac{1}{2} \), the Jackiw-Teitelboim model [19] \( a = 0, b = 1 \). Lemos and Sa [3] give the global solutions for \( b = 1 - a \) and all values of \( a \). Mignemi [10] considers \( a = 1 \) and all values of \( b \). The models of [11] correspond to \( b = 0, a \leq 1 \). It turns out [18] that the region leading to Penrose diagrams like the one of the genuine black hole is restricted to the range:

\begin{align}
I) & \quad b < 0; \quad a < 1 \\
II) & \quad b < 1 - a, \quad 1 \leq a < 2,
\end{align}
where region II) is fraught with null-completeness at the singularity. The straight line $b = a - 1$ in I) describes the 'physical' theories with vanishing singularity at $C = 0$. Although $b = 0$ (except for the DBH at $a = 1$!) has vanishing curvature asymptotically, these theories are in that range of Rindler type unfortunately.

3 Quantum theory of 2d-models

3.1 Quantum integrability of matterless theories

Stimulated by the 'dilaton black hole' [1, 2] numerous studies of quantized gravity in the simplified setting of 2D models were performed [20]-[21]. Louis-Martinez et al. [20] treated generic 2D dilaton gravity in the second order formalism

$$\mathcal{L}_{(1)} = \sqrt{-g} \left( -X \frac{R}{2} - \frac{U(X)}{2} (\nabla X)^2 + V(X) \right)$$

using a Dirac quantization scheme. A gauge theoretical formulation for string inspired gravity was developed and quantized by Cangemi and Jackiw [22]. In ref. [23] their solutions were shown to be equivalent to the ones of [20]. A Dirac approach was recently used to quantize string inspired dilatonic gravity [24]. In an alternative approach spherically symmetric gravity was quantized in Ashtekar’s framework by Kastrup [23] and in a geometrodynamical formulation by Kuchař [24]. In particular Strobl [27] has treated a large class of 2D gravity theories within the Poisson-Sigma approach. A common feature of all these studies is that due to the particular structure of the theory the constraints can be solved exactly, yielding a finite dimensional phase space. Then as a consequence of Dirac quantization it is found that quantum effects for only a finite number of variables are observed. Physically this is in agreement with the fact that dilatonic gravity describes no propagating gravitons. Due to the particular structure of the theory the constraints can be solved exactly yielding a finite dimensional reduced phase space. This remarkable property raises hope that in the case of dilatonic gravity one will be able to get insight into the information paradox without being forced to deal with the ultraviolet problems of higher dimensional gravities. Of course, despite of its many appealing features this approach by itself is insufficient to describe Hawking radiation in quantum gravity. In the presence of an additional matter field again an infinite number of modes must be quantized. In
order to tackle that problem the results mentioned above first of all should be translated into the language of (non perturbative) quantum field theory described by the path integral as the most adequate method for dealing with infinite dimensional quantum systems.

Indeed, one–loop quantum corrections to the classical action and renormalization group equations have been also considered perturbatively [28]. Matter fields are easily included in this approach. In this way, however, even pure dilatonic gravity [24] was found to exhibit a highly non–trivial renormalization structure, undermining the main motivation for considering dilatonic gravity as a simple toy model of quantum black hole physics! But even more serious, in our opinion, is the contradiction of these results with the ones from Hamiltonian approaches as mentioned in the last paragraph.

Thus a formalism is required which would combine integrability and simple ultra violet properties of the reduced phase space quantization with the possibility to include matter and obtain the local quantities of the field theoretical quantization.

We remove this contradiction by demonstrating that in pure dilatonic gravity [24] there are no local quantum corrections in the effective action for the path integral approach as well. To this end we generalize [29, 30] and perform an exact non-perturbative path integral quantization of a generic 2D dilaton model containing all the above models. We give the explicit form of the generating functional for connected Green functions. Adding matter fields in general destroys the functional integrability and suffers therefore from the same weaknesses as the first approach. However, the particular case of JT gravity [19] even in the presence of matter fields allows an exact path integral quantization.

Main technical features of our approach are the use of the first order action for Cartan variables in the temporal gauge, corresponding to an Eddington Finkelstein (EF) gauge for the metric [29]. Our analysis is local, meaning that we assume asymptotic fall off conditions for all fields. This is enough for the first step of a path integral quantization, which of course, in a second step should be adapted to take into account global effects familiar from the reduced phase space approach.

We first show the quantum equivalence of the second order form (20) to

\[ \text{For the matterless case this model was quantized exactly already by Henneaux [31].} \]
the first order action

\[ \mathcal{L}_{(2)} = X^+ De^- + X^- De^+ +Xd\omega + \epsilon(V(X) + X^+ X^- U(X)), \]

(21)

where \( De^a = de^a + (\omega \land e)^a \) is the torsion two form, the scalar curvature \( R \) is related to the spin connection \( \omega \) by \(-\frac{R}{2} = *d\omega \) and \( \epsilon \) denotes the volume two form \( \epsilon = \frac{1}{2} \varepsilon_{ab} e^a \land e^b = d^2 x \det e^a_\mu = d^2 x (e) \). Our conventions are determined by \( \eta = \text{diag}(1, -1) \) and \( \varepsilon_{ab} \) by \( \varepsilon^{01} = -\varepsilon^{10} = 1 \). We also have to stress that even with Greek indices \( \varepsilon^{\mu\nu} \) is always understood as the antisymmetric symbol and never as the corresponding tensor. The generating functional for the Green functions is given by

\[ W = \int (\mathcal{D}X)(\mathcal{D}X^+)(\mathcal{D}X^-)(\mathcal{D}e^a_\mu)_{gf}(\mathcal{D}\omega_\mu) \exp \left[ i \int_x \mathcal{L}_{(2)} + \mathcal{L}_s \right], \]

(22)

where \( \mathcal{L}_s \) denotes the Lagrangian containing source terms for the fields. However, since dilaton gravity does not any dependence on \( X^\pm \) and on \( \omega_\mu \) we do not introduce the corresponding sources at this point. A suitable gauge fixing is \( e_0^- = e_1^+ = 1, e_0^+ = 0 \). It is easy to check that in the following no division by \( e_0^+ \) needs to be performed. Also note that \( \det g = \det e = 1 \). Performing the functional integration with respect to \( \omega_0 \) and \( \omega_1 \) results in

\[ W = \int (\mathcal{D}X)(\mathcal{D}X^+)(\mathcal{D}X^-)(\mathcal{D}e^a_\mu)_{gf} \delta_{\omega_0} \delta_{\omega_1} \exp \left[ i \int_x \hat{\mathcal{L}}_{(2)} + \mathcal{L}_s \right], \]

(23)

The path integral measure in our gauge is

\[ (\mathcal{D}e^a_\mu)_{gf} = F_{FP} \mathcal{D}e^-_1 \]

(24)

where \( F_{FP} \) is the Faddeev–Popov factor. We use the abbreviations

\[ \delta_{\omega_0} = \delta \left( \partial_1 X - X^+ e^-_1 + X^- e^+_1 \right), \]

(25)

\[ \delta_{\omega_1} = \delta \left( -\partial_0 X + X^+ e^+_0 - X^- e^-_0 \right), \]

(26)

\[ \hat{\mathcal{L}}_{(2)} = \varepsilon^{\mu\nu} \left[ X^+ \partial_\mu e^-_\nu + X^- \partial_\mu e^+_\nu + e^+_\mu e^-_\nu \left( V(X) + X^+ X^- U(X) \right) \right]. \]

(27)

Integration over \( X^+ \) and \( X^- \) finally yields

\[ W = \int (\mathcal{D}X)(\mathcal{D}g_{\mu\nu})_{gf} \exp \left[ i \int_x \mathcal{L}_{(1)} + \mathcal{L}_s \right]. \]

(28)
In terms of $g_{\mu\nu}$ our gauge condition becomes the Eddington–Finkelstein gauge $g_{00} = 0$, $g_{01} = 1$ with the single unconstrained component $g_{11} = e_1^-$. The path integral measure becomes

$$(\mathcal{D}g_{\mu\nu})_{gf} = F_{FP}\mathcal{D}g_{11}$$

with the same Faddeev–Popov determinant as before [24]. As a result of the commonly used introduction of that determinant in our gauge $F_{FP}$ even turns out to be field independent. $\mathcal{L}_{(1)}$ is exactly given by (20). To obtain (28) we used $(e) \equiv \sqrt{-g}$, $(e)^2 g^{\alpha\beta} = \varepsilon^{\alpha\gamma} \varepsilon^{\delta\beta} g_{\gamma\delta}$ and the relation

$$\tilde{\omega}_\mu = \eta_{ab} \varepsilon^{\alpha\beta}(e) e^a_\mu \partial_\alpha e^b_\beta,$$

the tilde indicating the special case of vanishing torsion such that in (20)

$$\sqrt{-g} R = \varepsilon^{\nu\mu} \varepsilon^{\alpha\beta} \partial_\mu \left( \frac{e^a_\nu}{e} \partial_\alpha e^b_\beta \right).$$

Therefore, (30) will only produce the torsionless part of the scalar curvature as it is given in conventional dilaton theories. Of course, an additional conformal transformation of the zweibein would result in additional kinetic terms in the Lagrangian.

Thus the quantum theory of (21) is indeed equivalent to the one from the action (20).

In our quantization program we use [30] the canonical BVF [21] approach in order to obtain the determinants that appear by fixing the gauge in (21). We will be working in a ‘temporal’ gauge which corresponds to an Eddington Finkelstein gauge for the metric defined by:

$$e_0^+ = \omega_0 = 0 \ , \quad e_0^- = 1$$

After computing the extended Hamiltonian by the introduction of the usual two types of ghosts for a stage 1 Hamiltonian, following the steps of [30] we finally arrive at the generating functional for the Green functions is

$$W = \int (\mathcal{D}X)(\mathcal{D}X^+)(\mathcal{D}X^-)(\mathcal{D}e_1^+)(\mathcal{D}e_1^-)(\mathcal{D}\omega_1) F \exp \left[ i \int_x \mathcal{L}_{(2)} + \mathcal{L}_s \right],$$

where $F$ denotes the determinant.
\[ F = \det(\delta^k_i \partial_0 + C^k_{ij}) = (\det \partial_0)^2 \det(\partial_0 + X^+ U(X)). \] (34)

\[ \mathcal{L}_{(2)} \] is the gauge fixed part of the Lagrangian (21) and \( \mathcal{L}_s \) denotes the contribution of the sources:

\[ \mathcal{L}_s = j^+ e_1^- + j^- e_1^+ + j \omega_1 + J^+ X^- + J^- X^+ + J X \] (35)

Contrary to the standard approach to the path integral we now integrate first the ‘coordinates’ \( e_1^\pm, \omega_1 \) and use the resulting \( \delta \)-functions to perform the \( X \)-integrations yielding the generating functional of connected Green functions

\[ Z = -i \ln W = \int J X + J^- \frac{1}{\partial_0} j^+ + J^+ \frac{1}{\partial_0 + U(X) \frac{1}{\partial_0} j^+} (j^- - V(X)), \] (36)

where \( X \) has to be replaced by

\[ X = \frac{1}{\partial_0} j^+ + \frac{1}{\partial_0} j. \] (37)

It should be stressed that the determinant \( F \) is precisely canceled by the determinants appearing during these last three integrations. Eq. (36) gives the exact non-perturbative generating functional for connected Green functions and it does not contain any divergences, because it clearly describes tree-graphs only. Hence no quantum effects remain.

Now we turn to the ill defined expressions \( (\partial_0)^{-1} \). As shown in [33] a proper (vanishing) asymptotic behavior results from a regularization \( \mu = \tilde{\mu} - i \varepsilon, \lim_{\mu \to 0} := \lim_{\tilde{\mu} \to 0} \lim_{\varepsilon \to 0} \)

\[ \partial_0^{-1} \Rightarrow \begin{cases} 
\lim_{\mu \to 0} (\partial_0 - i \mu)^{-1} = \lim_{\mu \to 0} (\nabla_0^{-1}) \\
\lim_{\mu \to 0} (\partial_0 + i \mu)^{-1} = \lim_{\mu \to 0} (\nabla_0^{-1}) 
\end{cases} \] (38)

where \( \mu^2 \) insures a proper infrared cutoff. Since each partial integration above involved either \( X, X^+ \) or \( X^- \) which in turn all exhibit at least a \( \partial_0^{-1} \) behavior, all our previous steps are justified.

The theory thus produces tree graphs only. It can also be shown that (in our gauge!) the effective action reduces to the classical one — for the local quantization used here. As shown in [32] the Jackiw–Teitelboim model may be even quantized exactly in this case even when matter is present, although this methods fail (in the present version) for more complicated 2d theories.
3.2 Two-loop matter effects

What can be done in the presence of matter is to consider perturbation theory in the matter field, treating the geometrical part still exactly by a nonperturbative path integral \[33\]. Our approach thus differs fundamentally from the conventional ‘semiclassical’ one \[1\] in which (mostly only one loop) effects of matter are added and the resulting effective action subsequently is solved classically.

We take, as our starting point, the action for 1+1 gravity to be the spacetime integral over the Lagrangian

\[
\mathcal{L} = \mathcal{L}^g + \mathcal{L}^m + \mathcal{L}^s ,
\]

which is a sum of the gravitational, the matter and a source contribution.

Our matter contribution is a minimally coupled scalar field whose Lagrangian \(\mathcal{L}^m\) is given by

\[
\mathcal{L}^m = \frac{1}{2} \sqrt{-g} g^{\mu\nu} \partial_\mu S \partial_\nu S = -\frac{1}{2} \frac{\varepsilon^{\alpha\mu} \varepsilon^{\beta\nu}}{e} \eta_{ab} e^a_\mu \partial_\alpha S \partial_\beta S .
\]

The most general dilaton gravity action \[20\] contains the term \(U(X)(\nabla X)^2\). This term can be removed by a dilaton dependent conformal redefinition of the metric. The matter action \[40\] is invariant under such a redefinition. However, the quantum theory is changed: The source terms in eq. \[41\] below acquire field dependent (conformal) factors, destroying straightforward quantum integrability. In addition the path integral measure for the scalar field is changed. Here, for technical reasons, we restrict ourselves to a subclass of 2d models with \(U(X) = 0\). Of course, in this way realistic models like spherically symmetric 4d general relativity \[9\] \(U(X) \propto X^{-1}\) are eliminated.

The Lagrangian \(\mathcal{L}_s\) containing the source terms for our fields is now given by

\[
\mathcal{L}_s = j^+ e^{-1}_e + j^- e^+_e + j^1 \omega_1 + J^+ X^- + J^- X^+ + JX + QS
\]

Using again an Eddington Finkelstein gauge for the metric defined by a temporal (Weyl type) gauge for the Cartan variables \[32\] yields the trivial Faddeev-Popov determinant \[34\]. In this gauge the actions \[2\] and \[10\] are

\[
\mathcal{L}_g^{gf} = X^+ \partial_0 e^{-1}_e + X^- \partial_0 e^+_e + X \partial_0 \omega_1 + X^+ \omega_1 - e^+_1 V(X)
\]

\[
\mathcal{L}_m^{gf} = (e^-_1 (\partial_0 S)^2 - (\partial_0 S)(\partial_1 S)) .
\]
Contrary to the situation in conformal gauge the matter action therefore still contains a coupling to a zweibein component. The generating functional of Green functions is defined by

$$W = \int (D\sqrt{e_1}) (Dx) (DX^+)(DX^-)(De_1^+)(De_1^-)(D\omega_1) F \exp \left[ \frac{i}{\hbar} \int_x L_{gf} \right].$$

(44)

Note that for the scalar field a nontrivial measure must be introduced in order to retain invariance under general coordinate transformations [34]. To compute (44) we first integrate here over $e_1^-, X^-$ and $\omega_1$ to get delta functions which are immediately used to integrate out the remaining variables $X^+, e_1^+$ and $X$. This reduces (44) to

$$W = \int (D\sqrt{e_1} S) e^{\sum_j d^2 x (J^+ - X^+ + J^- - e_1^+) V(X) - (\partial_0 S)(\partial_1 S)},$$

(45)

where $X^+, e_1^+$ and $X$ thus are expressed as

$$X^+ = \frac{1}{\partial_0} J^+ + \frac{1}{\partial_0} (\partial_0 S)^2 = X_0^+ + \frac{1}{\partial_0} (\partial_0 S)^2,$$

$$e_1^+ = -\frac{1}{\partial_0} J^-$$

$$X = \frac{1}{\partial_0} (X^+ + j) = X_0 + \frac{1}{\partial_0} (\partial_0 S)^2.$$  

(46)

$X_0$ and $X_0^+$ represent $X$ and $X^+$ in the absence of matter fields (zero loop order). The nonlocal expressions for the Green functions $\partial_0^{-1}$ and $\partial_0^{-2}$ are regularized as in (38). $V(X)$ is expanded around $X_0$

$$V(X) = V(0) + V(1) + \Delta V$$

(47)

$$V(0) = V(X_0)$$

(48)

$$V(1) = V'(X_0) \frac{1}{\partial_0^2} (\partial_0 S)^2$$

(49)

$$\Delta V = \sum_{n=2}^{\infty} \frac{V^{[n]}(X_0)}{n!} \left( \frac{1}{\partial_0^2} (\partial_0 S)^2 \right)^n$$

(50)

The matter field integration to arbitrary orders is contained in the factor $W_S$ of

$$W = W_S e^{\frac{i}{\hbar} \int d^2 x (J^+ - X_0^+ + J^- - e_1^+ + JX - e_1^+ V(0))},$$

(51)

13
\[ W_S = \int (D\sqrt{e^2 S}) e^{i\pi \int d^2x \left(-e^2 \Delta V + (E^- \Phi_{\partial_b S}) - \Phi_{\partial_b S}) - QS\right)} \quad (52) \]

We introduced
\[ E^- = \frac{1}{\partial^2_0}J - \frac{1}{\partial_0}J^- - \frac{1}{\partial^2_0} \left(\Phi_{\partial_1 S}\right) \quad (53) \]
in order to subsume \( V_{(1)} \) into the propagator term. \( E^- \) clearly is not a zweibein component but will formally play a similar role.

The integration of the term quadratic in \( S \) and thus comprising the full propagator in the geometric background is given by
\[ \int (D\sqrt{e^2 S}) e^{i\pi \int d^2x E^- (\Phi_{\partial_b S} - \Phi_{\partial_b S}) - QS} = e^{i\pi \int d^2x S_F (E^-, e_1^+) e^{\frac{i\pi}{2} \int Q \Theta^{-1} Q}} \quad (54) \]
where \( \Theta^{-1} \) is defined as the inverse of the differential operator
\[ \Theta = \partial_0 \partial_1 - \partial_0 E^- \partial_0 \quad . \quad (55) \]
With a properly regularized \( \partial^{-1}_\mu \) we assume an appropriate definition of \( \Theta \) such that \( \partial_0 \Theta^{-1} = \left(\partial_1 - E^- \partial_0\right)^{-1} \) holds. \( S_F \) denotes the Polyakov-Liouville action
\[ S_F = \sqrt{-g} R \frac{1}{\Box} \quad (56) \]
where, however, \( R \) and \( \Box \) have to be expressed in terms of \( E^- \):
\[ e^{i\pi \int (-e^2 \Delta V) e^{\frac{i\pi}{2} \int Q \Theta^{-1} Q}} = \left(1 - \frac{i}{\hbar} \int \left(e^2 \frac{V''(X_0)}{2} \left(\frac{1}{\partial_0^2} \left(\frac{\partial_0 i \hbar \delta}{\delta Q}\right)^2\right) + ...\right) e^{\frac{i\pi}{2} \int Q \Theta^{-1} Q}|_{Q=0} \right) \]
\[ = 1 + \int \frac{i \hbar e^2}{8} \gamma(z) + O(\hbar^2) \quad (57) \]
where we introduced the abbreviation \( \gamma = \gamma(z) \) in the last line. It is now rather straightforward [33] to show that the two loop contribution \( \gamma \) is independent of the fields and in the generating functional for connected Green functions
\[ Z = \frac{\hbar}{i} \log W \]
\[ = J^- X_0^+ + j^- e_1^+ + J X_0 - e_1^+ V_R(X_0) + \hbar S_F (E^- , e_1^+) + O(\hbar^3) \quad (58) \]
\[ V_R = V - \hbar^2 \gamma V'' \]

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the $\hbar^2$ term expresses a renormalization of the 'potential' $V$. Of course, $X_0, X_0^+, e^+_1$ etc. are expressed in terms of the sources. Let us decompose the potential $V(X)$ in power series of $X$:

$$V(X) = \sum_n \frac{v_n}{n!} X^n$$  \hspace{1cm} (59)

Any coefficient gets infinite renormalization $\delta v_n \propto v_{n-2}$. In general, to fix the potential $V$ one needs an infinite number of normalization conditions. This is not a surprise, however, because even at the classical level an arbitrary function of $X$ is specified by an infinite number of independent parameters. There is an important particular case $V(X) = \alpha \exp(\beta X)$ when the renormalized potential will be automatically exponential, and only one parameter $\alpha$ needs to be renormalized. Note, that this potential gives black hole solutions [35].

As a final remark to this section we may add that in the effective action the Polyakov term does not depend on $E^-_1$ but on $e^-_1$, thus does not acquire 2-loop corrections.

4 Hawking radiation for generalized dilaton theories

This section will return to the more widespread techniques for treating radiation of matter from a background with fixed singularity structure, preferably from the SSG black hole itself.

4.1 Minimally coupled matter

The two most frequently considered theories, the string inspired CGHS [1] and SSG differ drastically in some of their physical properties, e.g. with respect to the completeness of null geodesics for these two models [18]. These differences directly lead one to investigate physical properties of a generalized model of which the two prominent examples are simply particular cases.

An important feature in semiclassical considerations is the behavior of Hawking radiation. In the CGHS model it is just proportional to the cosmological constant whereas the dependence in SRG is inverse to its mass, which implies an accelerated evaporation towards the end of its lifetime. As we will show a generalized theory with minimally coupled matter will exhibit Hawking radiation which is proportional to the black hole mass in terms of positive
or negative powers of the black hole mass, depending on the parameters of the model \[17\].

There are a number of ways of calculating the Hawking radiation \[36\]. One of them consists in comparing vacua before and after the formation of a black hole. In the case of generalized dilaton gravity this way is technically rather involved. We prefer a simpler approach based on an analysis of static black hole solutions.

Consider a generalized Schwarzschild black hole given by

\[
 ds^2 = -L(U)d\tau^2 + L(U)^{-1}dU^2, \tag{60}
\]

where \(L(U)\) has a fixed behavior at the asymptotic region \(I^+\):

\[
 L(U) \to L_0(U) \tag{61}
\]

with \(L_0(U)\) corresponding to the ground state solution. At the horizon we have \(L(U_h) = 0\). We can calculate the geometric Hawking temperature as the normal derivative of the norm of the Killing vector \(\partial/\partial\tau\) at the (nondegenerate) horizon

\[
 T_H = \left|\frac{1}{2}L'(U_h)\right|. \tag{62}
\]

We introduce the coordinate \(z\) by

\[
 dU = dzL(U). \tag{63}
\]

Then the metric takes the conformal trivial form with

\[
 ds^2 = e^{2\rho}(-d\tau^2 + dz^2) \quad \rho = \frac{1}{2} \ln L. \tag{64}
\]

In conformal coordinates the stress energy tensor looks like \[36\]

\[
 T_{\tau\tau} = -\frac{1}{12\pi}((\partial_\tau \rho)^2 - \partial_\rho \partial_\tau \rho) + t_\tau = T_{\tau\tau}[\rho(L)] + t_\tau. \tag{65}
\]

One can choose coordinates such that in the asymptotic region

\[
 T_{\tau\tau}[\rho(L_0)] = 0. \tag{66}
\]

This choice ensures that there is no radiation in the ground state. It means that we measure Hawking radiation of a black hole without any contribution from background Unruh radiation.
The constant $t_-$ is defined by the condition at the horizon

$$T_-|_{\text{hor}} = 0$$

in the spirit of [37]. The corresponding vacuum state is called the Unruh vacuum. In this state there is no energy flux at the black hole horizon. Of course, there cannot be such a thing as an observer at the horizon. However, as we shall demonstrate bellow, predictions of the theory with regard to measurements made at infinity are independent of the choice of coordinates at the horizon. In the case of four dimensional black holes this is well known. Taking into account equations (65), (66) and (67) one obtains a relation between $T_-[\rho]$ at the horizon and the asymptotic value of $T_-:

$$T_-|_{\text{asymp}} = -T_-[\rho]|_{\text{hor}}$$

The following simple identities are useful:

$$\partial_\pm = \frac{1}{2} \partial_z, \quad \partial_z \rho = \frac{1}{2} L', \quad \partial_\pm^2 \rho = \frac{1}{2} L'' L,$$  \hfill (69)

where prime denotes differentiation of $L$ with respect to $U$. By substituting (69) into (68) we obtain the Hawking flux

$$T_-|_{\text{asymp}} = \frac{1}{48\pi} \left( \frac{1}{2} L'(U) \right)^2 |_{\text{hor}}.$$  \hfill (70)

It is easy to demonstrate that our result is independent of a particular choice of conformal coordinates provided the behavior at the asymptotic region is fixed. For the models with the background given by the solutions of (39) we obtain the Hawking flux for the subclass $b = a - 1 \neq 0$ (asymptotic Minkowski-spacetime)

$$T_-|_{\text{asymp}} = \frac{a^2}{384\pi} C^{2(a-1)} \left( \frac{2B}{a} \right)^{\frac{a}{a}} \frac{2-a}{a},$$  \hfill (71)

to be compared with CGHS ($a = 1$)

$$T_-|_{\text{asymp}} = \frac{B}{192\pi} = \frac{\lambda^2}{48\pi},$$  \hfill (72)
4.2 Nonminimally coupled scalars

Somewhat surprisingly until very recent times [38, 39, 40, 41, 42, 43] no computation of the Hawking radiation for that case seems to exist. The purpose of this section is to give a comprehensive and direct answer to that question including all physically interesting models which generalize spherically symmetric gravity (SSG). This also allows us to improve and correct the results of [38, 39, 40, 41, 42] and to show the arbitrariness involved when SSG is generalized.

In the SSG case the (ultralocal) measure for the matter integration is well defined, because

\[ \int d\Omega \sqrt{-g} = e^{-2\phi} \sqrt{-g}, \]

(73)

For the generalized class of models (20), however, this definition is not unique, as well as the one for an eventual nonminimal factor for the possible coupling to matter in (40). Therefore, in that case we have to allow the general replacements \( \Phi \rightarrow \varphi(\Phi) \) in the SSG-factor \( e^{-2\phi} \) for (40) and \( \Phi \rightarrow \psi(\Phi) \) in (73), where \( \varphi \) and \( \psi \) may be general (scalar) functions of the dilaton field. With these replacements and in terms of the field \( \tilde{f} = f e^{-\psi} \) which satisfies the standard normalization condition, \( (40) \) can be rewritten as

\[ S = -\frac{1}{2} \int \sqrt{-g} d^2 x \tilde{f} A \tilde{f} \]

(74)

where

\[ A = -e^{-2\varphi+2\psi} g^{\mu\nu} (\nabla_\mu \nabla_\nu + 2(\psi,_{\mu} - \varphi,_{\mu}) \nabla_\nu + \psi,_{\mu\nu} - 2\varphi,_{\mu} \psi,_{\nu}), \]

(75)

The path integral for \( \tilde{f} \) leads to the effective action

\[ W = \frac{1}{2} \text{Tr} \ln A. \]

(76)

After continuation to the Euclidean domain \( A \) becomes an elliptic second order differential operator. The corresponding one loop effective action \( W \) can be expressed in terms of the zeta function of the operator \( A \).

\footnote{For an extensive discussion of that technique consult [44].}
\[ W = -\frac{1}{2} \zeta_A'(0), \quad \zeta_A(s) = \text{Tr}(A^{-s}) \] (77)

Prime denotes differentiation with respect to \( s \). From \( W \) regularized in this way an infinitesimal conformal transformation \( \delta g_{\mu\nu} = \delta k g_{\mu\nu} \) produces the trace of the (effective) energy momentum tensor

\[ \delta W = \frac{1}{2} \int d^x \sqrt{g} \delta g^{\mu\nu} T_{\mu\nu} = -\frac{1}{2} \int d^x \sqrt{g} \delta k(x) T^\mu_\mu(x) \] (78)

Due to the transformation property \( \delta A = -\delta k A \) of (6) (valid in \( d = 2 \) only) with the definition of a generalized \( \zeta \)-function

\[ \zeta(s|\delta k, A) = \text{Tr}(\delta k A^{-s}) \] (79)

the variation in (78) can be identified with

\[ \delta W = -\frac{1}{2} \zeta(0|\delta k, A) \] (80)

Combining (80) and (78) we get

\[ \zeta(0|\delta k, A) = \int d^x \sqrt{g} \delta k(x) T^\mu_\mu(x). \] (81)

By using the Mellin transformation one can show that \( \zeta(0|\delta k, A) = a_1(\delta k, A) \) [13], where \( a_1 \) is defined as a coefficient in small \( t \) asymptotic expansion of the heat kernel:

\[ \text{Tr}(F \exp(-At)) = \sum_n a_n(F, A)t^{n-1} \] (82)

To evaluate \( a_1 \) we use the standard method [15]. To this end we represent \( A \) as

\[ A = -\tilde{g}^{\mu\nu} D_\mu D_\nu + E, \quad E = \tilde{g}^{\mu\nu}(-\varphi_{,\mu\nu}\varphi_{,\mu} + \varphi_{,\mu\nu}) \] (83)
where \( \hat{g}^{\mu \nu} = e^{-2\varphi + 2\psi} g^{\mu \nu} \), \( D_\mu = \nabla_\mu + \omega_\mu \), \( \omega_\mu = \psi_\mu - \varphi_\mu \). For \( a_1 \) follows \[43\]:

\[
a_1(\delta k, A) = \frac{1}{24\pi} \int d^2x \sqrt{-\hat{g}} \delta k(\hat{R} + 6E). \tag{84}
\]

Returning to the initial metric and comparing with \[78\] we obtain the most general form of the conformal anomaly

\[
T_\mu = \frac{1}{24\pi} (R - 6(\nabla \varphi)^2 + 4\Box \varphi + 2\Box \psi) \tag{85}
\]

It is not difficult to adapt the methods of section 4.1 to obtain the Hawking flux also here. In the CGHS the result is

\[
T_{\text{CGHS}}^{\text{asymp}} = \frac{\lambda^2}{48\pi} \left(1 + \frac{3}{2} \alpha^2 - 2\alpha - \beta\right) \tag{86}
\]

Even for minimal coupling \((\alpha = 0)\) this expression is inherently ambiguous due to the constant \(\beta\) which had its roots in the ambiguous definition of an ultralocal measure. Increasing \(\alpha\) (nonminimal coupling) above \(\alpha = 4/3\) tends to increase \(T_{--}\). Of course, by adjusting \(\beta\) the flux may become zero or even negative as well (‘cold dilaton black hole’). Like the (geometric) Hawking temperature \[59\] in this case does not depend on the mass of the black hole.

The final result for \(a \neq 1\) reads

\[
T^{[a]}_{--} |_{\text{asymp}} = \frac{1}{48\pi} T_H^2 \left(1 - \frac{3\alpha^2}{2(2-a)} - \frac{1}{2-a}(2\alpha + \beta)\right) \tag{87}
\]

which has been expressed in terms of the geometric Hawking temperature for the general models \[16\]

\[
T_H^2 = \frac{a^2}{8} C^{2(a-1)/a} \left(\frac{2B}{a}\right)^{\frac{2-a}{a}} \tag{88}
\]

For SSG all parameters are unambiguously given \((a = \frac{1}{2}, \alpha = \beta = 1)\). Then the bracket in \(25\) yields a factor \(-2\), i.e. a negative flux! For a special case the same qualitative result has been obtained already in \[47\].
The crucial difference of our method is the use of a *local* scale transformation inside the zeta function. Due to the presence of an arbitrary function $\delta k$ all terms there are fixed unambiguously.

Our result for the 'anomaly', (85), is the most general one obtainable in 1+1 dimensional theories. The result for the Hawking flux in SSG, on the other hand, taken literally would mean that an influx of matter is necessary to maintain in a kind of thermodynamical equilibrium the Hawking temperature of a black hole — in complete contradiction to established black hole wisdom. However, to put this result on a sound basis the treatment of Hawking radiation in the asymptotic region in that case certainly requires to go beyond the usual approach adopted also in our present paper. After all, non-minimally coupled scalar fields are strongly coupled in the asymptotic region. Therefore a result like (87) for SSG cannot be the final answer. In fact, probably new methods for extracting the flux towards infinity in such a case have to be invented.

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