Interfacial flow dynamic micro-response and spatiotemporal evolution of flow pattern for thermocapillary–buoyancy convection in a liquid bridge

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Abstract

Compared with the former studies, the perturbation behavior of thermocapillary–buoyancy convection caused by the simultaneous coupling response of the microscale surface flow, free surface deformation and spatiotemporal evolution of flow patterns is revealed by the combination of experimental and numerical methods for the first time. The free surface morphology transforms from the ‘S’-shape into the twisted ‘M’-shape in the corresponding balanced stage of thermocapillary–buoyancy convection (at \( t = 975, \text{Bo}_d = 251.5 \)), and eventually becomes ‘S’-shape in the corresponding third stage (\( \text{Bo}_d = 229.9 \)). Meanwhile, there is a weak response of the free surface flow during each transition stage accompanied by periodic hydrothermal waves. The perturbation characteristics of the velocity, the temperature and the transverse location of surface flow are the most prominent at the intermediate height of liquid bridge (\( y = 0.2 \)). The characteristic of longitudinal velocity mainly presents as the pulsation, while there is also the pulsation inside the oscillation of transverse velocity with the large amplitude (the oscillating period of \( 2f_u = 7.2 \text{ s} \) and the amplitude of \( A_u = 0.0057 \)). The periodic characteristic of temperature oscillation is obvious (\( 2f_\theta = 0.2 \text{ s}, A_\theta = 0.015 \)).

1. Introduction

The surface tension plays a very important role under natural conditions. That may be affected by the multiple physical fields, such as temperature difference, solute concentration difference, acoustic flow coupling field [1–3], light pressure [4, 5], magnetic field [6] and electric potential [7, 8], etc. One of the most common phenomena is thermocapillary convection caused by non-uniform surface tension in the temperature difference. Therefore, this phenomenon is universal and important in the semiconductor industry, national defense science and biomedical engineering, especially the growth of crystal in the space [9], microfluidic control system development [10], and protein crystallization in extreme environments [11]. However, the buoyancy effect induced by temperature difference seriously interferes thermocapillary phenomenon in liquid bridge under the Earth’s surface gravity, and it becomes the main reason for the instability of thermocapillary convection [12, 13] or thermosolutal capillary convection [14] under gravity. The researchers have used microgravity condition to counteract buoyancy effect for highlight thermocapillary convection [15–17] with the development of space technology. However, space experiments show that it is difficult to obtain ideal microgravity experimental condition. The residual acceleration cannot be averted in the manned spacecraft [18, 19] due to the atmospheric drag, solar radiation pressure, and flight induction force. Therefore, many researchers have devoted themselves to the numerical and experimental studies of thermocapillary–buoyancy convection in the past few decades [20, 21]. The research on the flow pattern and oscillatory mechanism of thermocapillary–buoyancy convection have already become a hotspot [22]. Most scholars analyzed the oscillation mechanism of...
thermocapillary–buoyancy convection mainly by exploring the external influencing factors of the excited instability, including the geometrical morphology (interface shape [23] and aspect ratio [24]) and interface ambient environment (temperature difference [25, 26], evaporation on the interface [27], phase-change [28], adulteration [29, 30], relative motion [31, 32], and interface coating [33], etc). Hu [13] investigated the mechanism of the onset of oscillatory thermocapillary convection for a liquid bridge with small Bond number. It can be found that the oscillatory state will be excited if the instability condition of convection driven by the buoyant force is satisfied, although the thermocapillary convection is primarily driven by the gradient of surface tension. Meanwhile, for smaller Bond numbers such as experiments on the space shuttle, the critical Marangoni number will be larger. This result is very beneficial to the space material preparation processes for the delay of oscillatory state appearance. Watanabe et al [34] performed an experimental investigation to study a critical flow driven by the combined effects of buoyancy and thermocapillary forces in a non-isothermal liquid bridge. The chaos analysis was also performed on the obtained data, and different regimes of the supercritical flow were identified. Oztop et al [20] adopted evaluation of entropy generation to the numerical investigation of three-dimensional buoyancy and thermocapillary convection in an enclosure for the first time. It is found that Marangoni number becomes more effective parameter on total entropy generation for lower values of Rayleigh numbers. Kuhlmann et al [22] analyzed the linear stability of incompressible axisymmetric flow in a buoyant–thermocapillary liquid pool. They found that the centrifugal instability of the toroidal vortex flow is assisted by buoyancy in the low Prandtl number, and a stable thermal stratification can suppress the hydrothermal-wave instabilities of moderately high Prandtl number. Li et al [21] indicated that the critical Marangoni number increases with the increasing depth of the liquid pool. The hydrothermal waves can be suppressed with the increase of Prandtl number and the liquid pool depth. Furthermore, the radial waves near the edge of the crystal were observed in deep liquid pools with the large Marangoni numbers. Kolsi et al [29] investigated the heat transfer, fluid flow and entropy generation due to combined buoyancy and thermocapillary forces in a 3D differentially heated enclosure containing Al$_2$O$_3$ nanofluid of the different Marangoni numbers and nanoparticles concentrations. For all Marangoni numbers, the results showed that flow enhancement, heat transfer and total entropy generation increased with the increasing the nanoparticle volume fraction. Simanovskii et al [33] investigated the influence of an interfacial heat release and heat consumption on nonlinear convective flows in a laterally heated two-layer system under the periodic boundary conditions. It is found that a rather intensive heat sinks at the interface would cause changes in the direction of wave’s propagation under the joint action of buoyant and thermocapillary effects.

As mentioned above, the coupling law of the dynamic free surface deformation, the flow pattern and the surface flow are of great significance for revealing oscillatory characteristic of thermocapillary–buoyancy convection. At present, the study on the oscillatory capillary convection in liquid bridge has been mainly based on the stability analysis of nonlinear systems, however, there are few direct experimental observations on the onset of coupling oscillation relationship between the micro-disturbance behavior of surface flow and the cellular flow for the thermocapillary–buoyancy convection. In this paper, the response relationship of the flow pattern, the dynamic free surface deformation, and the perturbational surface flow (the coupled micro-oscillation of velocity, temperature and transverse location) was investigated for the thermocapillary–buoyancy convection with a large Prandtl number (Pr) fluid by the experimental and numerical coupling methods.

2. Experimental platform and experimental procedures

The experimental platform includes a control system and measurement feedback system for liquid bridge, see figures 1(a) and (b). The volume and feed rate of liquid bridge is control via the flow meter under the environment temperature $T_0 = 21 \, ^\circ\text{C}(\pm 1 \, ^\circ\text{C})$. $T_i$ and $T_b$ are the temperatures of upper (or hot) and lower (or cold) disks, respectively. The diameter of upper and lower disks is $D = 2R = 6 \, \text{mm}$. The positional information of monitoring points on the dynamic free surface of mathematical model are shown in figure 1(c), and installation altitude of thermocouple is $y = 1.2 \, \text{mm}$.

2.1. Experimental research procedure

(a) Initially, the liquid bridge generator is fixed on a horizontal test bench. Use deionized water to clean up the upper and lower disks after the platform is assembled.

(b) Inject the base solution (1–20 cm$^3$) from the lower micro-channel. As the fluid enters, use upper disk micrometer pulling device to adjust the height of the liquid bridge to a prescribed value ($H = 3 \, \text{mm}$).

(c) After the establishment of liquid bridge system. Set up a white background screen and open the laser source (with a wave length of 532 nm), color high-speed camera (Phantom VEO 410L), bolt electric
Figure 1. (a) Experimental platform of liquid bridge. (b) Temperature recorder with a micro thermocouple. (c) The locations of dynamic monitoring points in mathematical model of liquid bridge, for extracting the surface flow velocity and the transverse location of free surface (the eight monitoring points on the dynamic free surface is \( y = 0.1^\circ\), \( y = 0.2^\circ\), \( y = 0.15^\circ\), \( y = 0.25^\circ\), \( y = 0.3^\circ\), \( y = 0.35^\circ\), \( y = 0.4^\circ\), and \( y = 0.45^\circ\), respectively).

Figure 2. (a) Schematic diagram of the experimental light path and device. (b) Numerical calculation region and the corresponding relationship with experimental result.

Figure 3. Evolution of cellular flow pattern with the dimensionless time.

heating rod, and temperature recorder (equipped with a micro thermocouple, the diameter of 0.2 mm, measurement range from \(-200^\circ\text{C}\) to \(200^\circ\text{C}\), the temperature measurement error is \(\pm 0.1^\circ\text{C}\)) to record the free surface migration, the internal flow pattern and the temperature field perturbation, respectively.
Since the flow pattern needs to be observed in this experiment, the tracer particles are premixed in the base solution. Commonly used tracer particles include hollow glass cenosphere (18–24 μm), silver plated hollow glass cenosphere (8–12 μm) and aqueous fluorescent tracers (420–520 nm). According to the previous research, the aluminate powder with a diameter of 10–15 μm is selected as the tracer particles in this experiment. A better imaging effect would be obtained if the aluminum powder in the flow field is scanned by the laser plane with a wavelength of 532 nm. In addition, the Stokes number (St) is used to estimate the tacking characteristic of tracer particle, the expression is as follows.

\[ St = \frac{\rho_p D_p^2 U_{\text{max}}}{18 \mu \ell H} \]  

where, \( \rho_p \) is the density of the aluminum powder, \( D_p \) is the particle diameter of the aluminum powder, \( U_{\text{max}} \) is the maximum flow rate in the liquid bridge. The \( \mu \) is the dynamic viscosity of the solution and the \( H \) is the height of liquid bridge. According to the calculation, the St number of 10 μm aluminum powder is \( 8.03 \times 10^{-7} \) (far less than 1) in 10 cSt silicon oil which satisfies the requirement of tacking characteristic.

(d) Since there is a certain refractive index of the base solution in the experiment, the shooting position of the color high-speed camera must be at an angle to the laser plane and also cannot be vertical to the area light source. The optimal angle of the tracer particle reflection can be obtained by adjusting the position of the camera and the laser plane (see figure 2(a)).

(e) The clear motion trajectory of the particles in the flow field and the speed of the target particles can be obtained via the use of photo fastcam analysis software (PFA). The experimental images of research objects in this paper are shown in figure 3.

PFA (Photo Fastcam Analysis) software is used to track and capture the tracer particles in the flow field of thermocapillary–buoyancy convection to obtain the particle trajectory, flow pattern, and velocity in the two-dimensional plane flow field. However, due to the requirement of high-resolution imaging effect for liquid bridge interface morphology and flow pattern, the original image must be processed by image post-processing relying on the MATLAB compilation platform to make up for the limitations of traditional experimental research. This paper adopts image processing technology shown as follows.

(a) The spatial domain enhancement method is used for gray processing in this experiment, which is shown as follows,

\[ g(x, y) = f(x, y) \cdot h(x, y) \]  

where \( f(x, y) \) is the original image, \( g(x, y) \) is the new image obtained after enhancement, \( h(x, y) \) is a spatial operation function.

(b) A weighted average method is conducted to convert colorful RGB color model into a grayscale image, and the extraction process of interface configuration from the experimental image is shown in figure 4.

\[ R = G = B = \frac{(W_r \times R + W_g \times G + W_b \times B) \div 3} \]
where $W_r$, $W_g$ and $W_b$ are weighting coefficients of $R$, $G$ and $B$ respectively in the formula, meanwhile $W_r + W_g + W_b = 1$. As experiments and theory proved, a high-precision grayscale image can be obtained when $W_r = 0.30$, $W_g = 0.59$, $W_b = 0.11$.

c) Median filtering is used to filter high or low frequency components, pulses and particle noise in digital images to ensure edge information complete.

d) The experiment uses the global threshold method to divide the liquid bridge image into several parts to accurate extraction of the liquid bridge area from background zone,

$$G = G [x, y, f (x, y)]$$  \(\text{(4)}\)

where $f (x, y)$ represents the gray value of the point $(x, y)$, the image $G$ after global threshold segmentation is defined as,

$$G' = \begin{cases} 
\text{obj}, f (x, y) \leq T \\
\text{bkg}, f (x, y) > T.
\end{cases}$$  \(\text{(5)}\)

Under normal circumstances, $\text{obj} = 0$ represents the target area after the segmentation process, and $\text{bkg} = 1$ represents the background area after the segmentation process.

e) The grayscale image is smoothed by the first derivative of the two-dimensional Gaussian function, set the approximation function as,

$$G (x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$  \(\text{(6)}\)

$$\nabla G = \begin{bmatrix} \frac{\partial G}{\partial x} \\ \frac{\partial G}{\partial y} \end{bmatrix}.$$  \(\text{(7)}\)

Decompose the two convolution integrals of $\nabla G$ into two one-dimensional row and column filters,

$$\frac{\partial G}{\partial x} = k x \exp \left( -\frac{x^2}{2\sigma^2} \right) \exp \left( -\frac{y^2}{2\sigma^2} \right) = h_1 (x) h_2 (y)$$  \(\text{(8)}\)

$$\frac{\partial G}{\partial y} = ky \exp \left( -\frac{x^2}{2\sigma^2} \right) \exp \left( -\frac{y^2}{2\sigma^2} \right) = h_1 (x) h_2 (y)$$  \(\text{(9)}\)

where $k$ is a constant, $\sigma$ is a Gaussian filter parameter that controls the degree of smoothness.

f) In order to smooth the boundary curve and make up for the zigzag defect of boundary curve caused by the canny method, the fourth-order fitting of the liquid bridge boundary coordinates is performed by MATLAB least squares method.

Via the discrete data points given by the experiment are $\{(x_i, y_i) \mid i = 1, 2, \ldots, m\}$, and the weighting coefficient $a_i$ of each point, to get the corresponding function relationship of $y = s (x; a_0, a_1, \ldots, a_n) \ (n < m)$ between independent variable $x$ and the dependent variable $y$ and the minimum residual error is required either. Set the approximation function as,

$$s (x) = a_0 \varphi_0 (x) + a_1 \varphi_1 (x) + \cdots + a_n \varphi_n (x).$$  \(\text{(10)}\)

Given a set of data $\{(x_i, y_i) \mid i = 0, 1, 2, \ldots, m\}$, and a corresponding set of weighting coefficients $\{\rho_i \mid i = 0, 1, 2, \ldots, m\}$, make $y = s (x; a_0, a_1, \ldots, a_n) \ (n < m)$ meet the minimum value of

$$||\delta||^2_2 = p \left( a_0, a_1, \ldots, a_n \right) = \sum_{i=0}^{m} \rho_i \left[ s (x_i) - y_i \right]^2.$$  \(\text{(11)}\)

Then, $(a'_0, a'_1, \ldots, a'_n)$ is the point where the multivariate function takes the minimum value. According to the necessary conditions for the extreme value,\n
$$\frac{\partial p}{\partial a_k} = \sum_{i=0}^{m} \rho_i \left[ a_0 \varphi_0 (x_i) + a_1 \varphi_1 (x_i) + \cdots + a_n \varphi_n (x_i) - y_i \right] \varphi_k (x_i), \quad k = 0, 1, \ldots, n.$$  \(\text{(11)}\)

The corresponding weighted inner product sign is introduced according to the inner product definition:

$$\begin{Bmatrix} \langle \varphi_j, \varphi_k \rangle = \sum_{i=0}^{m} \rho_i \varphi_j (x_i) \varphi_k (x_i) \\ \langle y, \varphi_k \rangle = \sum_{i=0}^{m} \rho_i y \varphi_k (x_i) \end{Bmatrix}.$$  \(\text{(12)}\)

Equation (11) can be rewritten as,

$$\langle \varphi_0, \varphi_k \rangle a_0 + \langle \varphi_1, \varphi_k \rangle a_1 + \cdots + \langle \varphi_n, \varphi_k \rangle a_n = \langle y, \varphi_k \rangle, \quad k = 0, 1, \ldots, n.$$  \(\text{(13)}\)
3. Results and discussion

In this paper, we use the 10 cSt silicone oil as the research object, the initial temperature difference between the top and bottom disks is $\Delta T_0 = 15$ K, and the temperature of bottom disk is $T_0 = 25$ °C, and the heating rate is $DT' = 0.1$ °C s$^{-1}$ under the initial conditions. The local gravity level is $g = 9.8035 \pm 0.0986$ m s$^{-2}$ (Shenyang, Liaoning Province, China) and the other adopted initial conditions are shown in Table 1.

| Physical properties of 10 cSt silicone oil. |
|-------------------------------------------|
| Reference temperature (or initial temperature) | Silicone oil |
| Density $\rho$ | (25 °C) | [kg m$^{-3}$] | 935 |
| Dynamic viscosity $\mu$ | (25 °C) | [N s m$^{-2}$] | $9.35 \times 10^{-3}$ |
| Kinematic viscosity $\nu$ | (25 °C) | [m$^2$ s$^{-1}$] | 10$^{-3}$ |
| Thermal diffusivity $\alpha$ | (25 °C) | [m$^2$ s$^{-1}$] | $8.96 \times 10^{-8}$ |
| Thermal conductivity $\kappa$ | (25 °C) | [W (m$^{-1}$ K$^{-1}$)] | 0.14 |
| Thermal expansion coefficient $\beta$ | (25 °C) | [1/K] | 1.08 $\times 10^{-3}$ |
| Surface tension $\sigma$ | | [N m$^{-1}$] | 20.1 $\times 10^{-3}$ |
| Temperature coefficient of surface tension $\sigma'_T$ | | [N (m$^{-1}$ K$^{-1}$)] | $-6.83 \times 10^{-5}$ |
| Temperature coefficient of viscosity $\nu'_T$ | | | $-0.55$ |
| Specific heat capacity $C_p$ | | | 1672 |
| Prandtl number $Pr$ | | | 111.6 |
| Static Bond number $Bo$ | | | 16.40 |
| Capillary number $Ca$ | | | 0.05 |

2.2. Physical properties of experimental medium

In this paper, we use the 10 cSt silicone oil as the research object, the initial temperature difference between the top and bottom disks is $\Delta T_0 = 15$ K, and the temperature of bottom disk is $T_0 = 25$ °C, and the heating rate is $DT' = 0.1$ °C s$^{-1}$ under the initial conditions. The local gravity level is $g = 9.8035 \pm 0.0986$ m s$^{-2}$ (Shenyang, Liaoning Province, China) and the other adopted initial conditions are shown in Table 1.

The general governing equations of the problem are given by the following non-dimensional mass, Navier–Stokes, energy and level set conservation equations.

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
$$

$$
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
$$

$$
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{Bo_d}{Re} + \frac{Gr}{Re^2 \theta}
$$

$$
\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Ma} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)
$$

$$
\phi_t + (u \phi)_x + (v \phi)_y = 0
$$

where $u$ and $v$ are the fluid velocity ($u = u_{ref}/U, v = v_{ref}/U$). $t$ is dimensionless time, $t = t_{ref}/(L/U)$. $U$ is characteristic velocity, $U = |\sigma'T| DT/\mu \sigma$ is the surface tension, $\sigma = \sigma_0 + \sigma_T'(T - T_0)$, and the $\sigma_T'$ is defined as $\sigma_T' = \partial \sigma/\partial T$. $L$ is characteristic length, $L = 2R$. $\mu$ is dynamic viscosity. $P$ is the pressure, $\theta$ is the excess temperature, $\theta = (T - T_0)/(T_s - T_b)$. In addition, the coordinate scale is normalized with respect to the diameter of liquid bridge. The $x$ and $y$ are dimensionless coordinates. We expressed $x$ and $y$ as $X/ID$ and $Y/ID$. The aspect ratio is $\Gamma = H/ID$. Re is the ratio of inertia force and viscous force, $Re = |\sigma'T| DT/\nu^2$. $\nu$ is kinematic viscosity. Ma is Marangoni number and represents the intensity of thermocapillary force, $Ma = |\sigma'T| DT/\rho \omega \alpha$. Pr is Prandtl number, $Pr = \nu/\alpha$. $\alpha$ is thermal diffusivity. $Gr (Gr = g \beta DT^2/\nu^2)$ is the ratio of buoyancy and viscous force caused by temperature changing, and reflects the intensity of buoyancy convection. $\beta$ is thermal expansion coefficient. $Bo_d$ is the ratio of body force and surface tension force, $Bo_d = \rho g L^2 / |\sigma'T| DT$. $Bo$ is the static Bond number, $Bo = \rho g L^2 / \sigma$. 

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**Table 1.** Physical properties of 10 cSt silicone oil.

| Physical properties | Reference temperature (or initial temperature) | Silicone oil |
|---------------------|-----------------------------------------------|-------------|
| Density $\rho$ | (25 °C) | [kg m$^{-3}$] | 935 |
| Dynamic viscosity $\mu$ | (25 °C) | [N s m$^{-2}$] | $9.35 \times 10^{-3}$ |
| Kinematic viscosity $\nu$ | (25 °C) | [m$^2$ s$^{-1}$] | 10$^{-3}$ |
| Thermal diffusivity $\alpha$ | (25 °C) | [m$^2$ s$^{-1}$] | $8.96 \times 10^{-8}$ |
| Thermal conductivity $\kappa$ | (25 °C) | [W (m$^{-1}$ K$^{-1}$)] | 0.14 |
| Thermal expansion coefficient $\beta$ | (25 °C) | [1/K] | 1.08 $\times 10^{-3}$ |
| Surface tension $\sigma$ | | [N m$^{-1}$] | 20.1 $\times 10^{-3}$ |
| Temperature coefficient of surface tension $\sigma'_T$ | | [N (m$^{-1}$ K$^{-1}$)] | $-6.83 \times 10^{-5}$ |
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| Specific heat capacity $C_p$ | | | 1672 |
| Prandtl number $Pr$ | | | 111.6 |
| Static Bond number $Bo$ | | | 16.40 |
| Capillary number $Ca$ | | | 0.05 |
In this paper, the contact conditions of free surface with the upper and lower disks is a fixed contact point. Boundary and initial conditions are shown in follows,

\[
\begin{align*}
\theta = 0, u = 0, & \frac{\partial \phi}{\partial y} = 0 \quad \text{at } y = 0.0 \\
\theta = 1, u = 0, & \frac{\partial \phi}{\partial y} = 0 \quad \text{at } y = 0.5 \\
\frac{\partial \theta}{\partial x} = 0, u = 0, & \frac{\partial \phi}{\partial x} = 0, \frac{\partial \phi}{\partial x} = 0 \quad \text{at } x = 0.5 \text{ or } x = 1.5 \\
\theta = 0, u = 0, P = 0, & \phi(0.5, y) = 0 \quad \text{or } \phi(1.5, y) = 0 \quad \text{at } t = 0.0.
\end{align*}
\]

The level set method is a mainstream method to describe the evolution of curve or surface and to deal with the problem of tracking motion interface. This method can successfully calculate and analyze the free surface motion which depends on the time, the location, the interface structure, and external physical characteristics under the action of velocity field, such as interface integration and break up, etc. The zero isosurface of the function \( \varphi(x, t) \) is regarded as the moving interface with time in this method. As long as the function \( \varphi(x, t) \) is determined, the position of its zero isosurface can be tracked. Therefore, the zero isosurface \( \Gamma(t) \) of the function \( \varphi(x, t) \) can be defined as the following mathematical form of the moving interface,

\[
\Gamma(t) = \{ x \in \Omega : \varphi(x, t) = 0 \}.
\]

Near the interface \( \Gamma(0) \), the function of \( \varphi \) should satisfy the normal monotonicity and is zero at the moving interface \( \Gamma(t) \). In general, the \( \varphi(x, 0) \) is defined as the symbolic distance between the \( x \) point and free surface,

\[
\varphi(x, 0) = \begin{cases} 
\rho(x, \Gamma(0)) x \in \Omega_1 \\
0 \quad x \in \Gamma(0) \\
-\rho(x, \Gamma(0)) x \in \Omega_2 
\end{cases}
\]

where the two media regions and interface are represented as \( \Omega_1, \Omega_2 \) and \( \Gamma(0) \), respectively. The function of \( \rho(x, \Gamma(0)) \) is the distance between the \( x \) point and free surface \( \Gamma(0) \). For any times \( t \), when the \( \varphi \) moves at an appropriate speed, the function of \( \varphi \) is required to clarify the position of motion interface at a certain time, so as to avoid explicit solution of the motion interface and improve the computing capability of tracking complex interface. In order to ensure that the zero isosurface of function \( \varphi \) is the motion interface at any time \( t \), the function \( \varphi \) must satisfy certain governing equation,

\[
\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t} + u \cdot \nabla \varphi = 0
\]

where \( u = (u, v) \) is the velocity. The curvature function of free surface also has motion meanings in free surface tracking and two-phase flow problems and itseffect on the nearby flow field cannot be ignored (the curvature function of free surface can be expressed as \( \kappa = \nabla \cdot \mathbf{n} \)). In the motion interface problem, the curvature effect of the free surface is added to the stress term of the momentum equation. Under the eulerian correction of Navier–Stokes equations of incompressible viscous fluid, the normal vector of level set function and the velocity in the normal direction of moving interface are respectively in the following,

\[
F = u_n = \frac{dx}{dt} \cdot \mathbf{n} = \frac{dx}{dt} \frac{\nabla \varphi}{|\nabla \varphi|} = u \cdot \frac{\nabla \varphi}{|\nabla \varphi|}, \mathbf{n} = \frac{\nabla \varphi}{|\nabla \varphi|}.
\]

If the equation (23) is substituted into equation (22), the governing equation can be obtained as,

\[
\nabla \cdot u = 0
\]

\[
\frac{\partial u}{\partial t} = -(u \cdot \nabla)u + \eta \nabla^2 u - \nabla p / \rho + \mathbf{f}
\]

\[
\frac{\partial \varphi}{\partial t} + F |\nabla \varphi| = 0
\]

where \( \eta \) is the viscous coefficient, \( \rho \) is the density, \( P \) is the pressure, and \( \mathbf{f} \) is the external force. The dimensionless form of equations (24)–(26) are given above, see equations (14)–(18).

Considering the numerical instability at the interface, especially the density ratio of two phases is large, the physical parameters near the interface are smoothed [35]. As shown in the figure 5, the free interface is treated as a thin layer with a virtual thickness of \( 2 \varepsilon (\varepsilon \equiv O(h)) \), the physical parameters near the interface
layer are continuously distributed without saltation. Therefore, Dirac delta function and Heaviside function are introduced,

$$\delta_\varepsilon(\varphi) \equiv \begin{cases} \frac{1}{2} \left( 1 + \cos \left( \frac{\pi \varphi}{\varepsilon} \right) \right) / \varepsilon, & \text{if } |\varphi| < \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

$$H_\varepsilon(\varphi) = \begin{cases} \frac{1}{2} \left( 1 + \frac{\varphi}{\varepsilon} + \sin \left( \frac{\pi \varphi}{\varepsilon} \right) / \pi \right), & \text{if } |\varphi| \leq \varepsilon \\ 1, & \text{if } \varphi > \varepsilon. \end{cases}$$

Then, the physical parameters $\rho$ and $\mu$ can be calculated in the following,

$$\rho(\varphi) = \rho_{\text{in}} + (\rho_{\text{out}} - \rho_{\text{in}}) H_\varepsilon(\varphi)$$

$$\mu(\varphi) = \mu_{\text{in}} + (\mu_{\text{out}} - \mu_{\text{in}}) H_\varepsilon(\varphi).$$

The relevant algorithm can be utilized to program according to the different boundary conditions of the liquid-bridge problem by using the above series of governing equations.

The discrete treatment of surface tension adopts the continuum surface force method. The surface tension only exists on the free surface (see figure 6) in the numerical calculation. It is assumed that the surface tension term ($F_{sv}$) is continuously distributed on the free interface of liquid bridge,

$$\frac{1}{\text{We}} \kappa(\varphi) \delta(d) \mathbf{n} = \frac{1}{\text{We}} \kappa(\varphi) \delta(\varphi) \nabla \varphi$$

where the Weber number ($\text{We} = \rho \rho^2 L / \sigma$) presents the ratio of inertial forces to surface tension forces, and the interface curvature $\kappa(\varphi)$ is determined by the following two equations,

$$\kappa(\varphi) = -\nabla \cdot (\mathbf{n})$$

$$\nabla \cdot \mathbf{n} = \nabla \cdot \left( \frac{\nabla \varphi}{|\nabla \varphi|} \right).$$

The accuracy of interface curvature is crucial to calculate the surface tension, according to the dimensionless form of $\kappa(\varphi) = \nabla \cdot \varphi = -\nabla \cdot \left( \frac{\nabla \varphi}{|\nabla \varphi|} \right)$, the general expression of $\kappa(\varphi)$ can be derived as follows.

$$\kappa(\varphi) = -\frac{\partial}{\partial x} \left( \frac{\partial \varphi / \partial x}{\sqrt{\left( \partial \varphi / \partial x \right)^2 + \left( \partial \varphi / \partial y \right)^2}} \right) - \frac{\partial}{\partial y} \left( \frac{\partial \varphi / \partial y}{\sqrt{\left( \partial \varphi / \partial x \right)^2 + \left( \partial \varphi / \partial y \right)^2}} \right).$$

Figure 5. The schematic diagram of the interface.
\( \kappa (\varphi) = - \left( \frac{\partial^2 \varphi}{\partial x^2} \left( \frac{\partial \varphi}{\partial y} \right)^2 - \frac{\partial^2 \varphi}{\partial x \partial y} \frac{\partial \varphi}{\partial x} \frac{\partial \varphi}{\partial y} + \frac{\partial^2 \varphi}{\partial y^2} \left( \frac{\partial \varphi}{\partial x} \right)^2 \right) \bigg/ \left( \left( \frac{\partial \varphi}{\partial x} \right)^2 + \left( \frac{\partial \varphi}{\partial y} \right)^2 \right)^{3/2} \right). \]  

Moreover, in order to make the total mass completely satisfy the mass conservation at any time, the mass conservation procedure proposed by Liang et al [36, 37] is used. Meanwhile, this method has been adopted by other scholar and its feasibility has been verified in the reference [38].

In order to ensure better numerical stability when solving hyperbolic partial differential equations, the level set function must always maintain the symbolic distance from point \( x \) to the interface in the whole solution region which is represented as \( \varphi(x,t) \). The symbolic distance function needs the level set function \( \varphi(x,t) \) to satisfy equation (21) at any time. In general, the \( \varphi(x,t) \) will no longer satisfy the definition of equation (21) after several time steps. In order to keep its good properties, it is necessary to re-initialize the symbolic distance function to compensate the area. Therefore, \( \varphi(x,t) \) is reconstructed to satisfy the following two conditions.

\[
\begin{aligned}
\frac{\partial \varphi}{\partial t} &= \text{sign} (\varphi_0) \left( 1 - |\nabla \varphi| \right) \\
\varphi(x,0) &= \varphi_0.
\end{aligned}
\]  

In order to facilitate the solution, the symbol function of \( \text{sign} (\varphi_0) \) is smoothed as follows,

\[
\text{sign}(\varphi_0) = \frac{\varphi_0}{\sqrt{\varphi_0^2 + \varepsilon^2}}.
\]  

We ignore the volume change due to the thermal expansion or evaporation. However, the area dissipation is finally caused in practical calculation due to the limitations of Euler method and the lack of area compensation in level set method. In 1996, the improved method is proposed by Chang [39] et al. The main idea is to track the area loss in time and modify the level set function. In the operation, a varying ratio is used to make a moderate scale correction to the lost area for improving the conservation of mass,

\[
\frac{\partial \varphi}{\partial t} + (A_0 - A(t)) f (\kappa (\varphi)) |\nabla \varphi| = 0
\]  

where \( A_0 \) is the total area at the initial time of \( t = 0 \). \( A(t) \) is the total area corresponding to the \( \varphi(x,t) \) at the time \( t \). \( f (\kappa(\varphi)) \) is a curvature function.

The area compensation equation is similar to the reinitialization equation of \( \varphi(x,t) \) in form. Therefore, the area compensation equation has the same discrete mode as the reinitialization equation. The third order TVD-Runge–Kutta format is used to discretize in the time direction. Therefore, equation (38) can be
object, the movement of larger cellular flow is controlled by buoyancy convection which is in the right-half of liquid bridge. By taking the right-half of liquid bridge as the research area, the experimental results are all dimensionless, see figure 2(b). As shown in figures 8(a) and (b), thermocapillary convection begins to develop with the increase of temperature difference. Till then, thermocapillary–buoyancy convection system has already been established inside liquid bridge. From the view of influence scope, buoyancy convection still dominates inside flow and there is no intense competition between the two sides, therefore, it is predictable that the internal flow state is also stable. The development of thermocapillary convection will bring novel influence on the free surface deformation and the flow pattern.

Thermocapillary–buoyancy convection under gravity condition can be divided into three modes [43, 44]: steady convection, instability hydrothermal waves and oscillatory convection. Furthermore, the steady

\[
\phi^{(n+1)}(i,j) = \phi^{(n)}(i,j) - \Delta \tau \left( A_0^{(n)} - A_i^{(n)} \right) f(\kappa(\phi^{(n)}(i,j))) G(\phi^{(n)}(i,j))
\]

where \(G(\phi^{(n)}(i,j))\) is a spatial discrete operator, \(f(\kappa(\phi^{(n)}(i,j)))\) \(\equiv 1\).

In order to verify the present numerical model of liquid bridge, a comparison between the present results and available numerical results is performed by prescribing a liquid bridge of 10 cSt silicone oil. The results match those of Shvetsova et al [40] (1998) well in the reference [41], moreover, the preciseness of our present calculation model has been fully verified by other scholars, as shown in figure 7(a) (or see the figure 3 in reference Zhang et al [42]). Furthermore, figure 7(b) shows the radial velocity distribution along the right axial direction at the position \(x = 0.5\) with or without dynamic surface deformation. The difference in the radial velocity distribution along the axial direction with or without dynamic surface deformation can be found, which indicate that it is necessary to consider the dynamic free surface deformation to investigate the thermocapillary–buoyancy convection in the liquid bridge.

For the typical symmetric flow field in this paper, the results only present the flow characteristics in the right half of liquid bridge. In order to comparative analysis, the coordinates of numerical results and experimental results are all dimensionless, see figure 2(b). As shown in figures 8(a) and (b), thermocapillary convection has not yet formed at the hot corner. By taking the right-half of liquid bridge as the research object, the movement of larger cellular flow is controlled by buoyancy convection which is in the lower-middle part of liquid bridge and near the free surface during the early stage \((t_1 = 50 \sim t_2 = 75)\). As shown the blue arrow in figure 8, after a period of time, the structure of cellular flow begins to expand, and the influence scope of buoyancy convection extends to the internal and upper part of liquid bridge. Between \(t_3 = 125\) to \(t_4 = 150\), the cellular flow is accelerated by buoyancy convection rather than by early thermocapillary convection, furthermore, a certain scale of the backflow occurs. In addition, it generates the small-scale cellular flow above the entire convection by the effect of buoyancy (see the triangle region in figures 8(c) or (d)), which is opposite to the bulk backflow. There are two cellular flows inside right-half of liquid bridge at present, the small one is near the upper disk and the large one is located at the lower part of liquid bridge. Furthermore, vortex centers of the two cellular flows are both near the free surface, and their flow directions are opposite. Therefore, they both affect by the characteristics of surface flow and free surface deformation.

From figure 9, the thermocapillary convection originates from the hot corner at \(t = 200\), and it replaces the reverse small-scale cellular flow driven by buoyancy convection (reference the figure 8). The effect of thermocapillary convection on internal flow and surface deformation are more active and obvious, although they have the same flow direction. As a consequence, figure 9 shows that the shape of larger cellular flow generated by buoyancy is squeezed to deform at the lower part of liquid bridge. Comparing with figures 8(c) and (d), thermocapillary convection begins to develop with the increase of temperature difference. Till then, thermocapillary–buoyancy convection system has already been established inside liquid bridge. From the view of influence scope, buoyancy convection still dominates inside flow and there is no intense competition between the two sides, therefore, it is predictable that the internal flow state is also stable. The development of thermocapillary convection will bring novel influence on the free surface deformation and the flow pattern.
convection mode can be subdivided into steady single cellular flow and multiple cellular flows. When other parameters maintain unchanged and it exceeds critical temperature difference $\Delta T_{cr}$, the flow mode will develop from steady convection to instability hydrothermal waves (corresponding to the smaller $Bo_d$) or oscillatory convection (corresponding to the bigger $Bo_d$).

From figure 10, the influence scope driven by thermocapillary force become increasingly greater with the temperature difference increases. Overall, thermocapillary convection initiatively squeezes buoyancy convection. In addition, the outermost bulk backflow is pulled out from the buoyancy convection by the thermocapillary convection (see the figures 10(a) and (b)). It is entrained toward the upper disk and quickly supplementary to the hot corner and free surface. Furthermore, the traction is more obvious with the developing of thermocapillary convection. Therefore, the scope of influence of buoyancy convection shrinks gradually as shown in figure 10(b).
Figure 10. Flow pattern of cellular flow at different times in the competitive process of thermocapillary convection and buoyancy convection. ((a) $t_1 = 975$, (b) $t_2 = 1000$, (c) $t_3 = 1550$, (d) $t_4 = 2000$). ($Bo = 251.5$ corresponding to the dimensionless time of $t = 975$, $Bo = 16.4$, $\Delta T = 19.2^\circ C$).

Figure 10(a) presents that the cellular flow pattern respectively dominated by thermocapillary force and buoyancy force is very similarly at $t = 975$, and the competition of two patterns is evenly matched. In other words, before the time ($t = 975$), the thermocapillary convection is weaker than the buoyancy convection, but after the time ($t = 975$) the situation becomes opposite. When the time is $t = 1000$, thermocapillary convection encroaches on buoyancy convection, and it begins to dominate entire flow inside liquid bridge. The buoyancy convection has been further engulfed by the thermocapillary convection during the time of $t = 1500 \sim 2000$, as shown in figures 10(c) and (d). Moreover, the competition of two patterns also brings disturbance for entire convection and dynamic free surface, and the analysis of this phenomenon will be illustrated below.

Figure 11 shows the complex deformation of dynamic free surface during the transition from dominant buoyancy convection to dominant thermocapillary convection (the following analysis refers to the right free surface of liquid bridge). There are different characteristics for the free surface deformation at each stage. Specifically, from $t = 500$ to $t = 975$, the buoyancy convection drives entire flow with gradually developing thermocapillary convection. Therefore, the upper semi-interface is convex and the lower semi-interface is concave (free surface presents twisted 'S'-shape) at the time of $t = 500$, $t = 600$, and $t = 700$. Meanwhile, the twisted direction of 'S'-shape free surface is labeled by the blue arrow in figure 11(a). When the time is $t = 800$ and $t = 900$, the two cellular flows tend to be equilibrium and the free surface is twisted to 'M'-shaped. The free surface is the 'M'-shape when the cellular flows reach equilibrium at the time of $t = 975$, and evolution direction of the free surface deformation is shown by using the labeled black arrow in figure 11(a). Furthermore, the thermocapillary convection drives the entire flow after the time of $t = 975$. As a consequence, the free surface transforms to the twisted 'M'-shape at the time of $t = 975$, $t = 1000$, and $t = 1100$, the evolution direction of the free surface deformation is shown by using the labeled blue arrow in figure 11(b). When the time is $t = 1200$ and $t = 1300$, the upper semi-interface is concave and the lower semi-interface is convex (the free surface presents the 'Z'-shape). It is not hard to analyze that the time of $t = 975$ is an important turning point of free surface morphology and flow pattern transition.

The cellular flow dominated by buoyancy convection is near the lower disk (see the figure 8). Therefore, when the time is $t = 50$ and $t = 150$ in the figure 12(a), $v > 0$ can be observed from $y = 0$ to $y = 0.38$ and $v < 0$ can be observed from $y = 0.38$ to $y = 0.5$ (this height is correspond to the position of small-scale cellular flow generated by buoyancy convection in figures 8(c) and (d)). The positive peak increases and its position moves up with the time in figure 12(a). When the time is $t = 1750$ and $t = 2000$, the thermocapillary convection engulfs buoyancy convection, and there is only one cellular flow inside the liquid bridge, therefore, the direction of free surface longitudinal velocity is downward ($v < 0$). In figure 12(b), the structures of two cellular flows driven by thermocapillary–buoyancy convection become approximately the same size at the time of $t = 850$, $t = 900$ and $t = 975$ (corresponding to the transitional stage in figure 10(a)). Two kinds of convection are opposite to each other on the free surface (see figure 13). When the two kinds of surface flows meet near the intermediate height of free surface ($y = 0.27$), there is a
velocity stagnation point ($v = 0$) on dynamic free surface. Therefore, longitudinal velocity curve gradually close to the line of $v = 0$, as shown in the yellow enlarged region in figure 12(b). In figure 12(c), $v > 0$ can be observed from $y = 0.2$ to $y = 0.5$ and $v < 0$ can be observed from $y = 0$ to $y = 0.2$, because the thermocapillary convection has already engulfed buoyancy convection at $t = 1000$ and $t = 1550$. Therefore, the positive or negative change of longitudinal velocity along the $Y$ axial direction corresponds to the change of flow pattern.

In the figure 14, $u > 0$ means transverse velocity to the right. On the contrary, $u < 0$ shows the surface flow moves toward left. At $t = 50$, there is only one cellular flow dominated by the buoyancy convection. Therefore, the positive and negative peaks of transverse velocity appear on the position which is near the cold disk. On the other hand, when the time is $t = 150$, a couple of positive and negative peaks appear near the hot disk due to the reverse small-scale cellular flow. Then, thermocapillary–buoyancy convection affects flow state together. The surface flow is enhanced with increasing peak velocity at the time of $t = 850$ (see the blue and yellow region labeled in figure 14(a)). Figure 14(b) shows the transverse velocity change of surface flow around at the turning point ($t = 975$) in which thermocapillary convection and buoyancy

Figure 11. Free surface deformation of the liquid bridge at different times by taking the right-half of liquid bridge as the research object. Deformation trend of dynamic free surface is shown by using the labeled black or blue arrow.

Figure 12. (a)–(c) Variation of longitudinal velocity on the right free surface at different times. (d) Distribution of peak longitudinal velocity on the dynamic surface. (Corresponding numerical calculation results under experimental temperature difference $\Delta T$. $Bo = 229.9$ corresponding to the dimensionless time of $t = 1550$, $Bo = 16.40$, $\Delta T = 21^\circ C$).
convection steady at equilibrium. During this period, the outermost streamlines of two cellular flows progressively contact with each other (see figure 13). At this time, the span between negative peaks reduces gradually (see the height difference labeled by yellow, purple and green respectively in figure 14(b)). Therefore, the wave trough between the negative peaks is closer to the line \((u = 0)\) and moves down to the intermediate height. The positive and negative maximum of transverse velocity are respectively near hot and cold disks when thermocapillary convection engulfs buoyancy convection, shown as the labeled yellow and blue circle in figure 14(c). Then, the span increases between the two peaks over time. In general, it has a direct relationship between the number of cellular flows and the number of the peaks (reference the table 2 and 3).

Figure 15 shows the micro-disturbance of free surface deformation during the competition process between the thermocapillary convection and buoyancy convection (the locations of numerical monitoring points are shown in figure 1(c)). The first disturbance period is from \(t = 150\) to \(t = 275\) (see the labeled blue mark region in figure 15). On the one hand, this micro-disturbance derives from the generation of reversed small-scale cellular flow at hot corner when the buoyancy convection dominates entire flow at the time of \(t = 150\), as shown in the figure 8(d). On the other hand, due to the formation of thermocapillary convection at the hot corner (see figure 9), the free surface morphology changes rapidly, which also explains the disturbance from the transverse location of the free surface at \(t = 200 \sim t = 275\). The second obvious disturbance period is because of the directly competition between the buoyancy convection and thermocapillary convection at the time of \(t = 975\) (see the labeled yellow region in figure 15). The third disturbance change occurs at \(t = 1550\) (see the labeled green region in figure 15), corresponding to the stage
Table 2. Corresponding relation between flow pattern and transverse velocity peak by taking the right-half of liquid bridge as the research object.

| Time (−) | 50 | 75 | 150 | 850 | 900 |
|----------|----|----|-----|-----|-----|
| Flow pattern of thermocapillary–buoyancy convection | ![Diagram](image1) | ![Diagram](image2) | ![Diagram](image3) | ![Diagram](image4) | ![Diagram](image5) |
| Number of cellular flows | 1 | 1 | 1 + 1 | 2 + 1 | 2 + 1 |
| Number of positive perks of surface transverse velocity | 1 | 1 | 1 + 1 | 2 | 2 |
| Number of negative perks of surface transverse velocity | 1 | 1 | 1 + 1 | 2 + 1 | 2 + 1 |

Note: (1) The number of cellular flows include: the cellular flow is driven by thermocapillary convection, the cellular flow is driven by buoyancy convection, and the cellular flow is caused by the backflow. (2) ‘+1’: the enhanced thermocapillary or buoyancy convection excites backflow at narrow edge region of the hot or cold corner. Thus, the additional number of cellular flows and transverse velocity peaks can be observed.

Table 3. Corresponding relation between flow pattern and transverse velocity peak by taking the right-half of liquid bridge as the research object.

| Time (−) | 975 | 1000 | 1550 | 1750 | 2000 |
|----------|-----|------|------|------|------|
| Flow pattern of thermocapillary–buoyancy convection | ![Diagram](image6) | ![Diagram](image7) | ![Diagram](image8) | ![Diagram](image9) | ![Diagram](image10) |
| Number of cellular flows | 2 + 1 | 2 + 1 | 1 | 1 | 1 |
| Number of positive perks of surface transverse velocity | 2 | 2 | 1 | 1 | 1 |
| Number of negative perks of surface transverse velocity | 2 + 1 | 2 + 1 | 1 | 1 | 1 |

Note: (1) The number of cellular flows include: the cellular flow is driven by thermocapillary convection, the cellular flow is driven by buoyancy convection, and the cellular flow is caused by the backflow. (2) ‘+1’: the enhanced thermocapillary or buoyancy convection excites backflow at narrow edge region of the hot or cold corner. Thus, the additional number of cellular flows and transverse velocity peaks can be observed.

Figure 15. Disturbance of free surface on the different dynamic monitoring points with time by taking the right-half of liquid bridge as the research object. We define $f = \frac{|F - F_0|}{F_0} + 1.5$, where $F_0$ is initial position of the liquid bridge interface, $F$ is the position of free surface after a certain time. (Corresponding numerical calculation results under experimental temperature difference $\Delta T$).

From figure 15, the transverse position of free surface at the height of $y = 0.25$ ~ $y = 0.2$ is the most sensitive to the change of thermocapillary–buoyancy convection compared to the other heights. Therefore, in which the thermocapillary convection engulfs the buoyancy convection. The disturbance degree of the interface transverse position is enhanced with time. The height of the stronger disturbance position on the free surface is gradually decreased, which is consistent with the height tendency for the multiple cellular flows contact position in competition process.
the surface flow disturbance behavior at the height of $y = 0.2$ is studied around a particular time of $t = 975$. The surface flow presents special characteristic in the competitive process of buoyancy convection and thermocapillary convection, and the experimental images show that the circumferential hydrothermal waves exist on the free surface of the liquid bridge with alternating motion directions (see figure 16(a)). The buoyancy convection directly contacts with thermocapillary convection near the intermediate height of free surface ($y = 0.2$). The longitudinal component of surface flow velocity shows the characteristics of small amplitude pulsation. The transverse component of surface flow velocity is characterized by a large period oscillation, and large amplitude pulsation exists within the period (the oscillation period and amplitude of transversal velocity component are $2f_u = 7.2$ s and $A_u = 0.0057$ respectively). Meanwhile, the periodicity of temperature oscillation is obvious in experimental observation. The period and amplitude of oscillatory temperature are $2f_\theta = 0.2$ s and $A_\theta = 0.015$, respectively. The oscillation of transverse position on the free surface is influenced by the hydrothermal waves, and its amplitude increases with the spatiotemporal evolution of thermocapillary–buoyancy convection (see figure 16(b)).

4. Conclusions

In this paper, a new cognition has been proposed about the relationship among the flow pattern, the free surface dynamic deformation and the surface flow perturbation. The flowing state of thermocapillary–buoyancy convection can be divided into three stages. In the first stage, the entire flow is dominated by buoyancy convection. The curvature of free surface gradually increases with the development of buoyancy convection, and the free surface morphology is ‘$S$’-shape (upper convex and down concave shape) in the initial stage. A couple of cellular flows are located on the upper and lower disks respectively, in which one of cellular flows is small-scale near the hot corner by the entrainment of buoyancy convection. In the second stage, thermocapillary convection and buoyancy convection reach equilibrium in this transitional stage. Thermocapillary convection supersedes the small-scale reversed cellular flow driven by buoyancy convection. Dynamic free surface transforms from the ‘$S$’-shape to the twisted ‘$M$’-shape. With the increasing of temperature difference, thermocapillary convection develops, but buoyancy convection gradually shrinks. Meanwhile, there is a velocity stagnation point ($u = 0, v = 0$) near the intermediate height of free surface ($y = 0.27$). In the final stage, thermocapillary convection engulfs buoyancy convection, and the free surface transforms from twisted ‘$M$’-shape to ‘$\mathcal{C}$’-shape. Therefore, the morphology and number of cellular flows directly affect free surface deformation and surface flow characteristic, and each transformations of flow pattern brings the disturbance of free surface. Specifically, the periodically hydrothermal waves occur on the free surface around the time of $t = 975$ due to the competition of buoyancy convection and thermocapillary convection. The disturbance of surface flow is most intense near the intermediate height of free surface ($y = 0.2$). It is found that there are coupling oscillations of temperature, velocity and transverse position, and it exists significant periodic oscillation in the micro-perturbation of temperature and transverse velocity component. This kind of oscillation characteristic should be studied in detail in the following research.
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