Dynamical spin-spin coupling of quantum dots

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Abstract – We carried out a nested Schrieffer-Wolff transformation of an Anderson two-impurity Hamiltonian to study the spin-spin coupling between two dynamical quantum dots under the influence of rotating transverse magnetic field. As a result of the rotating field, we predict a novel Ising type spin-spin coupling mechanism between quantum dots, whose strength is tunable via the magnitude of the rotating field. Due to its dynamical origin, this new coupling mechanism is qualitatively different from the all existing static couplings such as RKKY, while the strength could be comparable to the strength of the RKKY coupling. The dynamical coupling with the intrinsic RKKY coupling enables to construct a four-level system of maximally entangled Bell states in a controllable manner.

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Introduction. – Quantum control of electron spins in semiconductor nanostructures is a central issue in the emerging fields of spintronics and quantum information processing. Electron spins confined in a semiconductor quantum dot (QD) was proposed [1] as a qubit for the realization of scalable quantum computers. In the context of quantum-computational applications it is necessary to couple qubits which are not nearest neighbors. The long-range type spin-spin interactions include the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction [2], Anderson’s superexchange interaction [3] and couplings mediated by cavity photons [4], etc.

In all of the above-mentioned systems the localized spins in the QDs are assumed to be static. The dynamics of mesoscopic systems have been discussed in context of charge pumping [5]. This mechanism is called quantum pumping and was experimentally realized first by Pothier et al. using QD in 1990 [6]. The pumping mechanism is also suitable for producing spin currents, an essential ingredient for spintronics [7,8]. A pure spin pump was theoretically proposed by Mucciolo et al. [9] in 2001 and was experimentally realized by the Marcus group [10].

The spin-pumping–induced dynamic exchange coupling between ferromagnetic films separated by normal-metal spacers is reported by experiments with sufficiently large normal spacers [11]. The dynamical coupling was first discussed in the context of electron spin resonance by Barnes in 1974 [12], who pointed out its long range nature as compared to static coupling. In the context of ferromagnetic resonance experiments, dynamic exchange coupling has been widely studied by different authors [11–15]. It was shown that in multilayers and superlattices, on top of the equilibrium spin currents that communicate the non-local static exchange coupling, a dynamic exchange interaction with a much longer range becomes important.

Despite the extensive study of dynamical coupling in ferromagnetic nanostructures [11–15], the quantum counterpart of the phenomenon has not been discussed yet. In this paper we study the dynamical coupling of two QDs in Fermi sea. We find a new type of coupling Hamiltonian between the spins in two QDs, which is due to the dynamical cotunneling process induced by the rotating transverse magnetic field applied. The strength of this new dynamical coupling is tunable via the magnitude of the transverse magnetic field, and found to be comparable to the static RKKY coupling. The eigenenergies with this dynamical coupling is different from that of the RKKY coupling, and can be used identifying the dynamical coupling strength. The corresponding eigenstates, a set of Bell states, can be used for quantum computation.

Theoretical formalism. – We consider two singly occupied QDs residing in an electron bath and are exposed to a magnetic field with weak DC component $B_\|$ along the Z-axis (which breaks the spin degeneracy by a Zeeman splitting) and a strong AC component $B_\perp (t)$ in the
XY-plane whose frequency satisfies the resonance condition of the QD [16]. We assume that the effects of the fields $B_{||}$ and $B_{\perp}(t)$ on the conduction electrons in the electron bath are negligible. Such a situation can be achieved by using materials of different $g$-factors for the QDs and the bath [17] or by applying local magnetic fields on the QDs. Since magnetic fields are experimentally hard to localize, alternatives have been introduced using more tunable gate voltages by coupling the spin to a pulsed electric signal mediated through spin orbit or hyperfine interaction [18–20], or by slanting dc magnetic fields [21], or by spin-phonon coupling in the presence of mechanical vibrations of the dot [22]. The Hamiltonian of the 2-QD system can be written as

$$H = H_0 + H_t + H_B(t),$$

where

$$H_0 = \sum_{i,\sigma} \varepsilon_i^\sigma n_i^\sigma + \sum_i U_i n_i^\uparrow n_i^\downarrow + \sum_{i,k,\sigma} \epsilon_k c_{i,k}^\sigma \bar{c}_{i,k}^\sigma,$$

$$H_t = \sum_{i,k,\sigma} \left[ T_{i,k}(R_i) d_{i,k}^\sigma d_{i,k}^\sigma + T_{i,k}^* (R_i) \bar{c}_{i,k}^\sigma \bar{c}_{i,k}^\sigma \right],$$

$$H_B(t) = \sum_i \hbar \omega_{\parallel} \left( \bar{d}_{i}^\dagger \bar{d}_{i} e^{-i\omega t} + \bar{d}_{i}^\dagger \bar{d}_{i} e^{i\omega t} \right).$$

$H_0$ is the energy of noninteracting QDs and conduction electrons where $d_{i}^\dagger (d_{i}^\sigma)$ creates (annihilates) an electron with spin $\sigma = \pm 1 (\uparrow, \downarrow)$ in QD-$i$ ($i = 1, 2$). $\varepsilon_i^\sigma = \varepsilon_i + \sigma \hbar \omega_{\parallel}/2$ is the energy for spin-$\sigma$ in QD-$i$ with $\hbar \omega_{\parallel} = g \mu_B B_{||}$ the Zeeman splitting due to external magnetic field $B_{||}$ and gyromagnetic factor $g$ and Bohr magneton $\mu_B$. $n_i^\sigma \equiv \bar{d}_{i}^\dagger \bar{d}_{i}^\sigma$ is the number operator of QD-$i$. $U_i$ is the Coulomb interaction energy on QD-$i$. The third term in eq. (2a) stands for the kinetic energy for the noninteracting electrons in the bath with $c_{i,k}^\sigma$ being the annihilation operator of a conduction electron with momentum $k$ and spin $\sigma$ with energy $\epsilon_k$. $H_t$ is the tunneling Hamiltonian between the localized electrons in QD-$i$ and the conduction electrons with the tunneling rates at the QD position $R_i$, $T_{i,k}(R_i) = T_{i,k} e^{-i k \cdot R_i}$. [23,24]. The effect of a rotating transverse magnetic field $B_{\perp}(t) = B_{\perp} [i \cos(\omega t) + j \sin(\omega t)]$ is in $H_B(t)$ with $\hbar \omega_{\perp} = g \mu_B B_{\perp}$ and driving frequency $\omega$. We assume that the QDs are in the Kondo regime, i.e. $\varepsilon_i < \epsilon_k < U_i$ and the transfer matrix elements $T_{i,k}(R_i)$ between the dots and the continuum are small compared with $\epsilon_k$ and $U_i$, i.e. $T_{i,k} \ll U_i, U_i - \epsilon_k, \epsilon_k$. Under this conditions the number of electrons on the dot is a well-defined quantity. To eliminate the time dependence of the problem first we make rotating frame transformation with the operator $[-i\omega t (\sum_i S_{i,z} + \sum_k S_{k,z})]$. Schrieffer-Wolff transformation. Following refs. [25–28], we use a two-stage or nested Schrieffer-Wolff (SW) [29] transformation to derive an effective spin Hamiltonian to obtain the low-energy spin interactions of the system. SW transformation [29–32] is based on a canonical transformation of the Hamiltonian $H' = e^{iH}e^{-\Lambda} = H + [\Lambda, H] + (1/2) [\Lambda, [\Lambda, H]] + \cdots$. The perturbative tunneling Hamiltonian $H_{t} [33]$ can be eliminated by choosing a SW transformation operator $\Lambda_{1}$, such that $H_{t} + [\Lambda_{1}, H_{0}] = 0$. Such a transformation leads us to the effective Hamiltonian up to the second order of the tunneling rate

$$H' = H_0 + H_B + \frac{1}{2} [\Lambda_{1}, H_{t}] + \frac{1}{2} [\Lambda_{1}, [\Lambda_{1}, H_B]] + \cdots.$$

In the absence of magnetic field, the above transformation reduces to the two-impurity Anderson model [34], where the third term is the second-order contribution in the tunneling rate and includes the Kondo (also called “s-d”) Hamiltonian plus a potential scattering term [29]. We are interested in the subspace of single occupancy of the QDs, i.e. one requires $n_i = \sum_{\sigma} n_i^\sigma = 1$. This constraint can be established by using the Gutzwiller operator $P = \prod_i \left( n_i^\uparrow - n_i^\downarrow \right)^2 [35]$, and we retain only the contributions in $H'$ that survive under the projection with $PH'P \neq 0$.

The next step is to apply the second SW transformation to study the low-energy spin interactions. The purpose of such nested [25–28] or generalized [23,24,36] transformation is to remove (at least partially) the contribution of second order in the tunneling rate. For the two-impurity Anderson model the transformation should be done with the generator operator $\Lambda_{2}$ fulfilling $[\Lambda_{2}, H_{0}] = -(1/2) [\Lambda_{1}, H_{t}]$. Note that $\Lambda_{2} \propto T_{e,k}^2$. This transformation reveals fourth-order interactions such as RKKY and a correlated Kondo term [23,25]. Using the same idea, we choose a different generator operator $\Lambda_{2}$ and $H'' = e^{\Lambda_{2}} H' e^{-\Lambda_{2}}$ to eliminate the second-order interaction term $(1/2) [\Lambda_{1}, [\Lambda_{1}, H_{B}]] \equiv H_{B}^{(2)}$ to reveal the fourth-order interactions between the two QDs due to the rotating magnetic field. For that we require $[\Lambda_{2}, H_{0}] = -H_{B}^{(2)}$, $\left( \Lambda_{2} \propto T_{e,k}^2 \right)$. The aim of SW transformations is to eliminate perturbatively “old” terms from Hamiltonians in favor of “novel” interactions. The first SW transformation gives rise to the second-order (in tunneling rates) processes, including the standard Kondo interaction, and an analog of the spin pumping and spin torque interaction between the QDs and the continuum. The second SW transformation gives rise to the fourth-order processes resulting from the rotating magnetic field.

The generator operators $\Lambda_{1,2}$ above for realizing the first and second SW transformations can be found by expanding the commutators in the order of tunneling rates $T_{i,k}$. Such a tedious but straightforward algebra can be carried out by using the Mathematica package SNEG [37]. In the rotating frame of reference with frequency $\omega$,

$$\Lambda_{1} = \sum_{i,k,\sigma} \left( \frac{n_i^\sigma}{E_{ki}} - \frac{1 - n_i^\sigma}{E_{ki}} \right) T_{i,k}^* e_i^\sigma d_{i}^\sigma - H.c.,$$

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\[ A_2 = \frac{\hbar \omega}{2} \sum_{i,k,q} P_{i,k,q} [n_{k,q}(S_i^+ + S_i^-) - n_i(S_{k,q}^+ + S_{k,q}^-)] , \]

where spin ladders, \( S_k,q^{\pm(-)} = \epsilon_k^{(1)^{(1)}} d_q^{(1)^{(1)}}, S_i^{\pm(-)} = d_i^{(1)^{(1)^{\dagger}}} d_i^{(1)} \), number operators \( n_{k,q} = \sum_{\sigma} \epsilon_q^{(\sigma)} d_q^{\dagger(\sigma)} \), and \( P_{i,k,q} = T_{i,k q} T_{i q} \left( (E_{q i} - U_i) (E_{q i} - U_i)^{-1} - (E_{q i} E_{q i})^{-1} \right) \) with \( E_{q i} \equiv \epsilon_k - \epsilon_q \). For small Zeeman field the expressions are simplified by assuming that the band energy levels are not spin-dependent \( \epsilon_q^\pm = \epsilon_i \). After the second transformation, the effective Hamiltonian becomes

\[ H'' = H_0 + H_B + \frac{1}{2} [A_1, H_i] + H_{B,1}^{(3)} + H_{B,2}^{(4)} \]

where \( H_{B,1}^{(3)} \equiv \frac{1}{2} [A_2, [A_1, H_i]] \) and \( H_{B,2}^{(4)} \equiv \frac{1}{4} [A_2, [A_1, [A_1, H_i]]] \) are the fourth-order interactions between QDs due to the rotating magnetic field.

**Second-order processes.** – In addition to the standard Kondo Hamiltonian, the first SW transformation yields the spin pumping induced by the rotating field and spin torque due to the absorption of the pumped spins \( (H_B^{(2)}) \). In the rotating frame

\[ H_B^{(2)} = \frac{\hbar \omega}{2} \sum_{i,k,q} P_{i,k,q} e^{i(k-q) \cdot R_i} \times \left[ n_i \left( S_{k,q}^+ + S_{k,q}^- \right) - n_{k,q} \left( S_{i}^+ + S_{i}^- \right) \right] \]

The first term in eq. (7) is responsible for cotunneling-induced spin pumping from QD to the continuum. The second term gives the torque acting the QD by the pumped spins in the continuum. The physics of the precession induced spin pumping from the QD is depicted in fig. 1. Due to the cotunneling processes, one spin↑ electron with momentum \( q \) and energy \( \epsilon_q \) in the continuum tunnels into the dot and occupies energy level \( \epsilon_i \), followed by another spin↓ electron on \( \epsilon_j \) tunneling out to the continuum with momentum \( k \) and energy \( \epsilon_k \). Afterwards, due to the rotating field the electron in the QD absorbs a “photon” and transits to spin↓. With these Kondo-type cotunneling processes, the spin in QD remains unchanged, but one spin down is flipped to spin up in the continuum via QD. Such processes keep transferring electrons in the continuum from spin↓ to spin↑. The reverse process also happens if a spin↓ electron resides in the QD (not shown in the figure): spin↑ electron tunnels into the QD followed by the spin↓ electron tunneling out to the continuum, emits a “photon” and flips to spin↓. The average outcome is that the electrons from continuum flows toward the scattering region, change their spins and flow away from it. It should be noted, that as the average number of pumped electrons with opposite spins are equal and thus no spin or charge current is induced. But we are, in the end, interested in fourth-order processes, when spins pumped from one QD can be absorbed by the second one and thus mediate new coupling mechanism between two dynamic QDs.

**Fourth-order processes.** – After two-step SW transformation, the Hamiltonian in eq. (6) contains two fourth-order interaction terms, i.e. the last two terms. The explicit form of the fourth term is

\[ H_{B,1}^{(4)} = \frac{\hbar \omega}{2} \left[ n_1 \left( S_2^+ + S_2^- \right) + n_2 \left( S_1^+ + S_1^- \right) \right], \]

with the coupling constant

\[ J_1 = \frac{1}{4} \sum_{j,k,q} (\epsilon_k - \epsilon_q) P_{i,k,q} d_{j,q}^{\dagger} e^{i(k-q) \cdot R_{i,j}}, \]

and

\[ J_{i,k,q} = T_{i,k q} T_{i q} \left( \frac{1}{E_{q i} - U_i} - \frac{1}{E_{q i} - U_i} - \frac{1}{E_{q i} - U_i} \right) \]

and \( R_{i,j} = R_i - R_j \). Equation (8) represents a new effective coupling of QDs caused by the spin flips in QDs induced by the rotating magnetic field. It describes the process that an electron flips its spin in one QD and then the flipped spin exchanges with the spin in the second QD. What is more interesting is the last term of eq. (6)

\[ H_{B,2}^{(4)} = \frac{(\hbar \omega)^2}{4} J_2 \left( S_1^+ S_2^- + S_1^- S_2^+ + S_1^+ S_2^- + S_1^- S_2^+ \right), \]

with \( J_2 \equiv \frac{1}{2} \sum_{k,q} (\epsilon_k - \epsilon_q) P_{i,k,q} d_{j,q}^{\dagger} e^{i(k-q) \cdot R_{i,j}} \). The Hamiltonian equation (9) describes the processes when electrons after spin-flipping in one QD tunnel into the second one and flip again.

Since we are interested in the subspace of single occupancy of the QDs (\( \nu = 1 \)) and resonance condition \( (\omega) = \omega \), we can extract the interactions that survive with Gutzwiller projection

\[ H''_B = \sum_i \hbar \omega (2 + J_1) S_i \cdot x + (\hbar \omega)^2 J_2 S_{1,x} S_{2,x}, \]

where the first term is the sum of eq. (8) and the external transverse magnetic field \( H_B = \sum_i \hbar \omega S_i \cdot x \) in rotating reference frame and the second term corresponds to eq. (9) with \( S_{1,x} = \frac{1}{2} (S_1^+ + S_1^-) \). In the laboratory frame eq. (9) becomes

\[ H''_B(t) = \sum_i \hbar \omega S_i \cdot x + \frac{g \mu_B}{i} (2 + J_1) B_\perp (t) \cdot S_i \]

\[ + (g \mu_B)^2 J_2 \left[ B_\perp (t) \cdot S_1 \right] \left[ B_\perp (t) \cdot S_2 \right]. \]
This effective coupling between QDs due to the rotating magnetic field is the main result of this letter. The $J_1$ part in the second term in eq. (11) represents that the spin $S_1$ of the QD-1 feels the magnetic field acting on $S_2$ on QD-2, and vice versa. While the third term of eq. (11) represents the Ising-type coupling between the spins on QD-1 and QD-2 induced by the rotating magnetic field $B_\perp(t)$, and it magnitude is tunable by changing the magnitude of $B_\perp(t)$.

Discussions and conclusions. – In ref. [38] Coqblin and Schrieffer presented their widely used approach [39–42] to the two-impurity Anderson model. After a single SW transformation and treating that Hamiltonian in second-order perturbation theory, they compute a RKKY-like spin-spin interaction [2] of the form $\epsilon_i \equiv (E_i - E_j)$ depending on the strength of the perpendicular magnetic field $B_\perp$. For $B_\perp = 0.5 \text{T}$ and $g = 2$, $\Delta_{1,3} \approx 0.44 J_0$ and $|\Delta_{2,3}| \approx 0.12 J_0$. One should note that electron $g$-factor engineering [17] allows us to have even larger splitting of the Bell states in our system.

In conclusion, using a two-stage SW transformation, we transform a two-impurity Anderson model into an effective spin Hamiltonian. Without the rotating magnetic field the second-order expansion yields the standard Kondo Hamiltonian for two impurities with additional scattering terms [23–25,29,35]. The introduction of the rotating magnetic field gives rise to a magnetic-field–induced spin pumping from QD and the torque that experiences the QD from the continuum via the cotunneling processes. These cotunneling processes yield to two additional QD coupling mechanisms: 1) the QD feels a nonlocal magnetic field acting on the neighboring QD; 2) the QDs are coupled via an Ising-like coupling. Because of its dynamical origin from the rotating field, the new coupling mechanism is intrinsically different from all existing static coupling mechanisms such as RKKY coupling. More importantly, the strength of the new dynamical coupling is tunable via the magnitude of the transverse magnetic field, and found to be comparable to the RKKY coupling strength at reasonably large rotating magnetic field. The interplay of the RKKY and the new dynamical coupling enables to construct a four-level system of the maximally entangled Bell states in a controllable manner.

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Table 1: Eigenenergies and eigenstates for $H_{\text{eff}}$ in eq. (12).

| State | Wave function | Energy |
|-------|--------------|--------|
| $|1\rangle$ | $\Phi^+ + \Psi^+$ | $J_0 + 2\hbar \omega_\perp (2 + J_1) + (\hbar \omega_\perp)^2 J_2$ |
| $|2\rangle$ | $\Phi^+ - \Psi^-$ | $J_0 - 2\hbar \omega_\perp (2 + J_1) + (\hbar \omega_\perp)^2 J_2$ |
| $|3\rangle$ | $\Phi^-$ | $J_0 - (\hbar \omega_\perp)^2 J_2$ |
| $|4\rangle$ | $\Psi^-$ | $-3J_0 - (\hbar \omega_\perp)^2 J_2$ |

Fig. 2: (Color online) The eigenenergies of the eigenstates $|1\rangle$, $|2\rangle$, $|3\rangle$ and $|4\rangle$ as a function of the magnitude of the transverse magnetic field in units of $J_0$. The transverse magnetic field dependence of the eigenenergies in fig. 2 can be measured in experiment, and the coupling strengths $J_1$ and $J_2$ can be inferred from the slope (at zero field) and curvature of the lines, respectively. Finally, the energy splitting of different states, $\Delta_{i,j} \equiv (E_i - E_j)$ depends on the strength of the perpendicular magnetic field $B_\perp$. For $B_\perp = 0.5 \text{T}$ and $g = 2$, $\Delta_{1,3} \approx 0.44 J_0$ and $|\Delta_{2,3}| \approx 0.12 J_0$. One should note that electron $g$-factor engineering [17] allows us to have even larger splitting of the Bell states in our system.

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REFERENCES

[1] Loss D. and DiVincenzo D. P., Phys. Rev. A, 57 (1998) 120.

[2] Ruderman M. A. and Kittel C., Phys. Rev., 96 (1954) 99; Kasuya T., Prog. Theor. Phys., 16 (1956) 45; Yosida K., Phys. Rev., 106 (1957) 893; Van Vleck J. H., Rev. Mod. Phys., 34 (1962) 681.

[3] Anderson P. W., Phys. Rev., 79 (1950) 350; 115 (1959) 2.

[4] Imamoğlu A., Awschalom D. D., Burkard G., DiVincenzo D. P., Loss D., Sherwin M. and Small A., Phys. Rev. Lett., 83 (1999) 4204.

[5] Trouless D. J., Phys. Rev. B, 27 (1983) 6083.

[6] Pothier H., Lefarge P., Urbina C., Estève D. and Devoret M. H., Europhys. Lett., 17 (1992) 249.

[7] Wolf S. A., Awschalom D. D., Buhrman R. A., Daughton J. M., Von Molnár S., Roukes M. L., Chtchelkanova A. Y. and Treger D. M., Science, 294 (2001) 1488.

[8] Zutic I., Fabian J. and Das Sarma S., Rev. Mod. Phys., 76 (2004) 323.

[9] Mucciolo E. R., Chamion C. and Marcus C. M., Phys. Rev. Lett., 89 (2002) 146802; Wang B., Wang J. and Guo H., Phys. Rev. B, 67 (2003) 092408.

[10] Watson S. K., Potok R. M., Marcus C. M. and Umansky V., Phys. Rev. Lett., 91 (2003) 258301.

[11] Heinrich B., Tserkovnyak Y., Woltersdorf G., Brataas A., Urban R. and Bauer G. E. W., Phys. Rev. Lett., 90 (2003) 187601.

[12] Barnes S. E., J. Phys. F: Met. Phys., 4 (1974) 1535.

[13] Hurdequint H. and Malouche M., J. Magn. & Magn. Mater., 93 (1991) 276.

[14] Alzergawi H. and Kekicheff P., J. Magn. & Magn. Mater., 154 (1996) 277.

[15] Lenz K., Tolinski T., Lindner J., Kosubek E. and Baberschke K., Phys. Rev. B, 69 (2004) 144422.

[16] Lenz K., Tolinski T., Lindner J., Kosubek E. and Baberschke K., Phys. Rev. B, 69 (2004) 144422.

[17] Tserkovnyak Y., Brataas A. and Bauer G. E. W., Phys. Rev. Lett., 88 (2002) 117601.

[18] Engel H.-A. and Loss D., Phys. Rev. B, 65 (2002) 195232.

[19] Fieberger R., Keim M., Reuscher G., Ossau W., Schmidt G., Waag A. and Molenkamp L. W., Nature (London), 402 (1999) 787; Ohno Y., Young D. K., Beschoten B., Matsukura F., Ohno H. and Awschalom D. D., Nature (London), 402 (1999) 790; Huang S. M., Tokura Y., Akimoto H., Kono K., Lin J. J., Tarucha S. and Ono K., Phys. Rev. Lett., 104 (2010) 136801; Schiro M. D., Petersson K. D., Jung M. and Petta J. R., Phys. Rev. Lett., 107 (2011) 176811.

[20] Nowack K. C., Koppens F. H. L., Nazarov Yu. V. and Vandersypen L. M. K., Science, 318 (2007) 1430.

[21] Nadj-Perge S., Frolov S. M., Bakkers E. P. A. and Kouwenhoven L. P., Nature, 468 (2010) 1084.

[22] Laird E. A., Barthel C., Rashba E. I., Marcus C. M., Hanson M. P. and Gossard A. C., Phys. Rev. Lett., 99 (2007) 246601.

[23] Pioro-Ladri`ere M., Obata T., Tokura Y., Shin Y.-S., Kubo T., Yoshida K., Taniyama T. and Tarucha S., Nat. Phys., 4 (2008) 776; Tokura Y., Van der Wiel W. G., Obata T. and Tarucha S., Phys. Rev. Lett., 96 (2006) 047202.

[24] Ohm C., Stampfer C., Spllettstoesser J. and Wegewijs M. R., Appl. Phys. Lett., 100 (2012) 143103.

[25] Tzen Ong T. and Jones B. A., EPL, 93 (2011) 57004.

[26] Proetto C. and Lopez Arturo, Phys. Rev. B, 24 (1981) 3031.

[27] Kolley E., Kolley W. and Tietz R., Phys. Status Solidi (b), 204 (1997) 763.

[28] Minn-Tien T., Physica C, 223 (1994) 361.

[29] Braun M., Struck P. R. and Burkard G., Phys. Rev. B, 84 (2011) 15445.

[30] Kolley E., Kolley W. and Tietz R., J. Phys.: Condens. Matter, 4 (1992) 3517.

[31] Schrieffer J. R. and Wolff P. A., Phys. Rev., 149 (1966) 491.

[32] Cornut B. and Coqblin B., Phys. Rev. B, 5 (1972) 4541.

[33] Bruus H. and Flensberg K., The Kondo Problem to Heavy Fermions (Cambridge University Press, Cambridge) 2003.

[34] Hewson A. C., Many-Body Quantum Theory in Condensed Matter Physics, An Introduction (Oxford University Press, Oxford) 2004.

[35] Caroli C., Combescot R., Nozieres P. and Saint-James D., J. Phys. C, 4 (1971) 916.

[36] Ng Tai Kai and Lee Patrick A., Phys. Rev. Lett., 61 (1988) 1768; Meir Yigal, Wingreen Ned S. and Lee Patrick A., Phys. Rev. Lett., 70 (1993) 2601.

[37] Kolley E., Kolley W. and Tietz R., Phys. Status Solidi (b), 186 (1994) 239.

[38] Zhou L.-J. and Zheng Q.-Q., J. Magn. & Magn. Mater., 109 (1992) 237.

[39] Žižko Rok, Comp. Phys. Commun., 39 (2011) 2259.

[40] Coqblin B. and Schrieffer J. R., Phys. Rev., 185 (1969) 847.

[41] Schottmann P., Phys. Rev. B, 62 (2000) 10067.

[42] Bazhanov V. V., Lukyanov S. L. and Tsvelik A. M., Phys. Rev. B, 68 (2003) 094427.

[43] Yang Y. F. and Held K., Phys. Rev. B, 72 (2005) 235308.

[44] Coleman P. and Andrei N., J. Phys. C: Solid State Phys., 19 (1986) 3211.

[45] Goldhaber-Gordon D. et al., Nature (London), 391 (1998) 156; Cronenwett S. M., Oosterkamp T. H. and Kouwenhoven L., Science, 281 (1998) 540; Goldhaber-Gordon D. et al., Phys. Rev. Lett., 81 (1998) 5225; Gerland Ulrich, von Delft Jan, Costi T. A. and Oreg Yuval, Phys. Rev. Lett., 84 (2000) 3710.