Multi-task Learning for Gaussian Graphical Regressions with High Dimensional Covariates

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4 p.m., Thursday, October 3, 2024
6011 Donald Bren Hall

Gaussian graphical regression is a powerful means that regresses the precision matrix of a Gaussian graphical model on covariates, permitting the numbers of the response variables and covariates to far exceed the sample size. Model fitting is typically carried out via separate node-wise lasso regressions, ignoring the network-induced structure among these regressions. Consequently, the error rate is high, especially when the number of nodes is large. We propose a multi-task learning estimator for fitting Gaussian graphical regression models; we design a cross-task group sparsity penalty and a within task element-wise sparsity penalty, which govern the sparsity of active covariates and their effects on the graph, respectively. For computation, we consider an efficient augmented Lagrangian algorithm, which solves subproblems with a semi-smooth Newton method. For theory, we show that the error rate of the multi-task learning based estimates has much improvement over that of the separate node-wise lasso estimates, because the cross-task penalty borrows information across tasks. To address the main challenge that the tasks are entangled in a complicated correlation structure, we establish a new tail probability bound for correlated heavy-tailed (sub-exponential) variables with an arbitrary correlation structure, a useful theoretical result in its own right. Finally, the utility of our method is demonstrated through simulations as well as an application to a gene co-expression network study with brain cancer patients.