Modal Analysis of Variable Structure of Gate Rotor of CP-type Single-screw Structure

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Abstract. Single screw technology, due to its superiority of structure, has been widely used in many fields of gas compression, refrigeration, power and so on at present. However, specialized research on the dynamics of single screw structure has not been systematic or in-depth yet. As for single screw compressors and expanders, the dynamics of the gate rotor are the key factors influencing its performance, and the variable-structure modal analysis of the gate rotor is an important task to study its dynamic characteristics. In this paper, the modal parameters of the CP-type single-screw structure gate rotor are compared by modal simulation and modal experiment, and the accuracy of the finite element method is verified. And mainly carried out the simulation modal analysis of the gate rotor variable structure. The results show that the change of structural parameters is an important factor affecting the natural frequency of the gate rotor mode. The variable structure modal analysis can be used to provide reference for the structural optimization of the gate rotor.

1. Introduction

At present, under the background of severe energy and environmental problems in China, the research and application of various energy-saving technologies have attracted extensive attention of the government and society [1]. In the field of medium-low temperature waste heat utilization, small and medium-sized Organic Rankine Cycle (ORC) is considered as one of the most potential technologies [2], and a lot of relevant research work has been carried out. Kang et al.[3] used R245fa working medium. The evaporating temperature and condensing temperature were 80 ℃ and 30 ℃ respectively. The expander was a radial turbine with an efficiency of 78.7%, an expansion ratio of 2.65 and a system cycle thermal efficiency of 5.22%. Li et al. [4] used R123 working medium, of which the evaporation temperature and condensation temperature were 90.7 ℃ and 39.4 ℃, respectively. The expander was a radial turbine, the efficiency was 58.53%, the expansion ratio was 4.19, and the system cycle thermal efficiency was 6.15%. Lei Biao et al.[5] used R123 working medium, and the expander was a single screw type. The evaporation temperature and condensation temperature were 126.1 ℃ and 67.9 ℃, respectively, the efficiency was 56%, the expansion ratio was 8.5, and the net efficiency of the system was 7.98% (excluding the consumption of auxiliary machines). Through the above comparison, it can be seen that the small and medium capacity expander has the problems of small expansion ratio and low internal efficiency at the same time. Based on the existing research methods and achievements of compressor, some scholars have carried out theoretical analysis and experimental research on various
irreversible factors of expander, including leakage, heat transfer, mechanical friction, etc. However, the thermal performance can only reflect the actual performance of the expander, and can not provide a direct technical basis for improving the performance and optimizing the structure of the expander. In order to further improve the performance of the expander, the systematic study of the expander dynamics needs to be carried out.

Because of the simple structure of the single screw, the basic structure is that the screw and the gate rotor form a spatial meshing relationship, and the movement form of the meshing pair is similar to sliding, and the soft and hard meshing of the metal rotor and the organic material gate rotor is usually used, and it is the best type of vibration, noise and seal in all kinds of positive displacement power machinery. Therefore, since F. B. Zimmern proposed the original structure of single screw compressor in 1965 [6], the single screw technology has developed rapidly, and has been widely used in refrigeration compressor, process gas compressor, extruder and other fields. Beijing University of technology is the first unit to apply the single screw technology to the field of expander in China. Through relevant research, it has been proved that the single screw expander is one of the most potential types of small and medium capacity expander. How to further improve the performance of expander is the focus of technical research.

Single screw has the advantages of simple structure, and the key parts are only screw and gate rotor. However, this leads to a more complex structure of key parts, which is difficult to process, especially the gate rotor. In order to ensure a good meshing relationship, the gate rotor structure and materials are required to have good mechanical properties (certain elasticity, strength and good wear resistance), accurate processing and precise spatial positioning, and the determination of the gate rotor structure parameters is mainly based on the spatial geometric relationship, and the mechanical aspects are more dependent on engineering experience, which makes the adjustment and optimization of single screw structure lack of theoretical basis. It is an important part of dynamic analysis to obtain the modal natural frequency of the gate rotor structure through the variable parameter modal analysis, which can provide the basic data for the vibration characteristic analysis and dynamic optimization design of the variable structure system. At present, the research on the variable parameter mode of gate rotor is not systematic, which can not provide technical guidance for the application of new materials, structural parameter adjustment and optimization of gate rotor. In this paper, the variable structure simulation modal analysis of the gate rotor is carried out. Through the simulation analysis, the reasonable structural parameters of the gate rotor are explored to provide the basic data of dynamics and technical support for the optimization of the gate rotor.

2. Finite element analysis verification
As an important part of single screw structure, gate rotor plays a key role in the performance of single screw expander. The single screw structure is composed of one screw and several gate rotor pieces. It is generally divided into four types: CP, PC, PP and CC. Because the latter three types are complex, the single screw structure used in this paper is CP type, which is a conventional structure. According to different work requirements, the gate rotor pieces with a certain number of teeth are processed.

In order to verify the accuracy of the finite element method and provide technical support for the modal analysis of the variable structure of the gate rotor, the modal analysis method of the combination of simulation and experiment is used to verify the accuracy of the finite element results. The modal analysis module of ANSYS Workbench software is used for the finite element simulation, and the vibration test points of DASP V11 provided by Beijing Oriental Institute of vibration and noise are used for the experiment. Analysis instrument to carry out the experimental test.

2.1 Modal analysis theory
Modal analysis is the basic content of dynamic analysis. It is a dynamic analysis method used to analyze modal characteristics such as structural modal parameters and modal shapes [7]. In practice, the system where the main structure is located is a continuous medium system, whose modal parameters can be calculated by discrete equation. Assuming that the gate rotor system is divided into a N dimensional
degree of freedom system, and the vibration displacement of each position of the gate rotor is $u$, its basic motion equation [8] can be expressed as:

$$\begin{bmatrix} M \end{bmatrix}\ddot{u} + \begin{bmatrix} C \end{bmatrix}\dot{u} + \begin{bmatrix} K \end{bmatrix}u = {F(t)}$$

(1)

In the formula, $F(t)$ is N dimensional excitation force vector; $u$, $\dot{u}$, $\ddot{u}$ are respectively $N$ dimensional displacement, velocity and acceleration; $M$, $C$ and $K$ are the mass, stiffness and damping matrices of the main structure, usually n-order real symmetric matrix.

In the modal analysis of a continuum, the change of external load does not affect the modal characteristics of the continuum itself, and the damping of the continuum has little effect on the modal analysis. Therefore, the vibration equation of the continuum can be simplified as:

$$\begin{bmatrix} M \end{bmatrix}\ddot{u} + \begin{bmatrix} C \end{bmatrix}\dot{u} = \begin{bmatrix} 0 \end{bmatrix}$$

(2)

When simple harmonic vibration occurs, equally to $u = Usin(\omega t)$, the equation is:

$$(\begin{bmatrix} K \end{bmatrix} - \omega^2 \begin{bmatrix} M \end{bmatrix})\{\varphi\} = 0$$

(3)

For the modal analysis of a main structure, the $r$ order natural frequency $\omega_r$ and the corresponding modal shape $\{\varphi_r\}$ can be obtained from the above matrix equation.

According to the calculated modal shape, the transfer function between two positions in the continuum can be obtained, that is, the frequency response function between the excitation point and the response point in the experimental mode, which can be expressed by the following formula:

$$H_{ab}(\omega) = \sum_{r=1}^{N} \frac{\varphi_{ra}\varphi_{rb}}{m_r(\omega^2 - \omega^2_r) + j2\xi_r\omega_r\omega}$$

(4)

Where $H_{ab}(\omega)$ is the transfer function between the excitation point and the response point in the system, $N$ is the total order of the measured mode, and $\varphi_{ra}$ and $\varphi_{rb}$ are the $r$ order modes of the $a$ point and the $b$ point respectively; and $m_r$, $\xi_r$ and $\omega_r$ are the $r$ order mode mass, damping ratio and natural frequency respectively.

### 2.2 Finite element modal analysis

At present, in order to obtain a better meshing performance of CP structure, the structure of 6 screw slots and 11 teeth are usually used. Table 1 shows the structural parameters of the standard CP type gate rotor currently used for working conditions. The diameter of the gate rotor is determined by a certain working condition, the tooth length is determined by the depth of the gate rotor engaged in the screw, the tooth width is determined by the engagement depth, the constraint of the screw rib and experience, while the thickness of the gate rotor is completely determined by experience, and the diameter of the inner hole is determined by the diameter of the gate rotor shaft.

| Diameter/mm | Tooth length/mm | Tooth width/mm | Thickness/mm | Inner hole diameter /mm |
|-------------|-----------------|----------------|--------------|------------------------|
| 117         | 24.9            | 17.23          | 6            | 26                     |

Table 1. Structure parameter of gate rotor.

The three-dimensional solid model of the gate rotor is drawn by the three-dimensional modeling software Pro/E, and the generated model is imported into the finite element software. First, the actual material properties of the gate rotor are defined. In this paper, the gate rotor is formed by the injection molding of Peek (Poly Ether Ether Ketone ) which is a new self-lubricating material, and its material properties are shown in table 2.

| Diameter/mm | Material | Density/(g/cm³) | Modulus of Elasticity/GPa | Poisson ratio |
|-------------|----------|-----------------|---------------------------|--------------|
| 117         | PEEK     | 1.4             | 10.87                     | 0.44         |

Table 2. Material properties of gate rotor.
Secondly, the finite element analysis and solution of the gate rotor model are carried out. Combined with the modal truncation theory, the first six modal frequencies are mainly studied to obtain the first six modal natural frequencies of the gate rotor as shown in table 3.

| Order number | Frequency/Hz |
|--------------|--------------|
| 1            | 966.42       |
| 2            | 966.44       |
| 3            | 1775.0       |
| 4            | 1775.7       |
| 5            | 1863.7       |
| 6            | 2278.0       |

### 2.3 Experimental modal analysis

In order to verify the results of finite element modal simulation analysis, experimental modal analysis is used to test and analyze the actual gate rotor, as shown in figure 1 is the actual structure diagram of the CP type gate rotor and screw.

![Figure 1. Actual structure diagram of CP type gate rotor and screw.](image)

The experimental modal analysis method is to acquire the input and output signals of the system through the experiment, and obtain the modal frequency of the gate rotor through the parameter identification. That is to say, it can be seen from the reciprocity assumption that when two or more sensors are fixed at the same position, the moving hammer is used to measure the excitation force at each point of the gate rotor, and the complete modal analysis results of the gate rotor can be obtained by each sensor. Therefore, in this experiment, the moving knock point excitation is used, and the response signal is collected by two sound pressure sensors, and the more input and more output (MIMO) modal analysis experiment is carried out. The flow chart of this experiment is shown in figure 2.
In order to effectively compare with the results of the finite element analysis, the first six modal natural frequencies are also extracted. Table 4 shows the first six experimental modal natural frequencies of the gate rotor.

Table 4. Natural frequency measured by gate rotor experiment.

| Order number | Frequency/Hz |
|--------------|--------------|
| 1            | 981.80       |
| 2            | 983.90       |
| 3            | 1822.2       |
| 4            | 1823.5       |
| 5            | 1925.4       |
| 6            | 2396.1       |

2.4 Comparative analysis and discussion

It can be seen from the comparison between the above simulation and experimental frequencies that the natural frequencies of simulation and experimental modes have high consistency, with the maximum error of 4.7% and the minimum error of 1.5%, meeting the error range. The reason for the error may not be the same as the simulation due to the boundary conditions and other constraints in the experiment, and may be due to the rubber band suspension and gate rotor materials (special engineering plastics) injection molding process, etc. This shows that the finite element software ANSYS Workbench can effectively simulate the modal characteristics of the actual gate rotor, which provides an effective analysis method for the later variable structure modal analysis of the gate rotor.

3. Analysis of the influencing factors of structural parameters

Through the optimization analysis of the structural parameters of the gate rotor, the volume of the screw groove of the single screw expander can be increased, and a higher content product ratio can be obtained, so as to improve the efficiency of the expander. The existing literature research has proved that the main structural parameters affecting the gate rotor include: the thickness, diameter, tooth width, unequal diameter ratio. In this section, the variables of these four structural parameters are analyzed to optimize the structure of the gate rotor.

3.1 Effect of thickness

At present, the structural parameters of the thickness of the gate rotor used in the single screw expander are completely determined by experience, without any data reference. Although increasing the tooth thickness can increase the mechanical strength of the gate rotor in the meshing process, the corresponding cost will also increase, and the friction between the gate rotor and the screw groove will
also increase, and the gate rotor is too thin and easy to break. Therefore, when the diameter of the gate rotor is defined as a fixed value of 350mm and the fundamental frequency is 50Hz, figure 3 shows the change curve of the natural frequency of the gate rotor mode with the increase of the thickness.

It can be seen from figure 3 that the tooth thickness is in direct proportion to the frequency. When the diameter of the gate rotor remains unchanged, increasing the thickness of the gate rotor will help to improve the natural frequency of the gate rotor mode. When the thickness of the gate rotor is 2 ~ 3mm, the natural frequency of the gate rotor mode is close to the fundamental frequency of 50Hz.

![Figure 3. The change curve of the natural frequency of the mode with the tooth thickness of the gate rotor.](image)

3.2 Effect of diameter
The diameter of the gate rotor has a great influence on the volume of the screw groove. When the thickness of the gate rotor is fixed, increasing the diameter of the gate rotor can effectively improve the volume of the screw groove, reduce the weight of the screw, reduce the centrifugal force of the screw, and reduce the overall volume. However, the disadvantage is that when running in a large displacement machine, due to the increase of the diameter of the gate rotor, the length of the gate rotor teeth is too long, which leads to the excessive force on the gear and is easy to be broken. Therefore, when the thickness of gate rotor is defined as fixed value of 3mm and the fundamental frequency is 50Hz, the change curve of natural frequency of the gate rotor mode with the increase of diameter is shown in figure 4.
Figure 4. The change curve of the natural frequency of the mode with the diameter of the gate rotor.

It can be seen from figure 4 that the diameter is inversely proportional to the frequency. When the thickness of the gate rotor tooth remains unchanged, the natural frequency of the mode decreases in a parabola trend with the increase of the diameter of the gate rotor. When the diameter increases to 350mm, the natural frequency of the gate rotor mode is close to the fundamental frequency region.

3.3 Effect of width of gear

The width of gate rotor is generally equal width, which is mainly determined by center distance, diameter and screw rib thickness. Increasing the tooth width of the gate rotor can increase the volume of the screw groove of the single screw expander, but the thickness of the screw rib is correspondingly reduced. When the thickness of the screw rib is too small, the tooth top of the screw groove becomes sharp and the strength is not enough, and the stress concentration at the root of the gate rotor increases because the fillet radius is too small, which is easy to produce cracks and cause damage. Therefore, the width of the gate rotor cannot be increased infinitely. When the thickness of the screw rib is required, the increase of the width of the gate rotor will increase the length of the screw and the length of the whole casing, thus increasing the cost. When the thickness of gate rotor is defined as fixed value of 3mm and the fundamental frequency is 50Hz, the change curve of natural frequency of the gate rotor mode with the increase of tooth width is shown in figure 5.

Figure 5. The change curve of the natural frequency of the mode with the tooth width of the gate rotor.
It can be seen from figure 5 that the influence of tooth width on the modal natural frequency of the gate rotor is the same as the diameter. When the tooth width is increased to 55mm, the modal natural frequency is close to the fundamental frequency of 50Hz.

3.4 Effect of unequal diameter ratio

The existing CP structure is equal diameter structure, which is, the diameter of screw and gate rotor is equal, and the unequal diameter structure is put forward on this basis, aiming to improve the efficiency by changing the unequal diameter of screw and gate rotor. Unequal diameter structure means that there is a certain proportional relationship between the diameter of the gate rotor and screw, that is, unequal diameter ratio. Previous studies have shown that increasing the unequal diameter ratio can greatly increase the volume of the screw groove. When the screw diameter and the thickness of the screw rib are fixed, the increase of the unequal diameter ratio will reduce the width of the gate rotor teeth, so that the gate rotor is easy to break. To analyze the unequal diameter ratio, it is necessary to define the screw diameter as 117mm and the thickness of the gate rotor tooth as 2mm. Figure 6 shows the change curve of the natural frequency of the gate rotor mode with the increase of the unequal radius rate.

![Figure 6](image_url)

**Figure 6.** The change curve of the natural frequency of the mode with the unequal diameter ratio of the gate rotor.

It can be seen from figure 6 that the unequal diameter ratio is inversely proportional to the frequency. When the screw diameter and the thickness of the gate rotor remain unchanged, the increase of the unequal diameter ratio reduces the modal natural frequency. However, from the perspective of the decreasing trend, the current analysis of the unequal diameter ratio will not be close to the fundamental frequency of 50Hz.

3.5 Analysis and discussion

(1) From the above figure, it can be seen that there is a phenomenon of multilevel modal overlap. This is because the number of gate rotor teeth is 11, which is generally defined as an approximate center symmetric structure. Therefore, it can be seen from the figure that there is a phenomenon of multilevel multiple order roots, that is, the two-order frequencies are approximately equal.

(2) From the appeal figure, it can be seen that increasing the thickness of the gate rotor helps to improve the modal natural frequency while keeping certain structural parameters unchanged, while increasing the diameter, tooth width and unequal diameter ratio of the gate rotor, the modal natural frequency decreases gradually.

(3) In the process of optimizing the structural parameters of the gate rotor, all areas within the fundamental frequency 50Hz should be avoided, so as to avoid causing resonance of the gate rotor and damaging the normal operation of the machine.
4. Conclusion
In this paper, the influence of structural parameters on the natural frequency of the gate rotor is analyzed. These structural parameters mainly include: tooth thickness, diameter, tooth width and unequal diameter ratio. Through the simulation analysis, the following conclusions are obtained:

(1) According to the modal data of different structures, the structural parameters have a great influence on the natural frequency of the gate rotor. The general trend is that the natural frequency increases with the increase of the thickness of the gate rotor teeth, and decreases with the increase of the diameter, tooth width and unequal diameter ratio of the gate rotor.

(2) Minimizing vibration is a good method to design the structure of the gate rotor successfully, and it is also helpful to reduce the cost and improve the stability. However, in order to optimize the structure of the gate rotor and minimize the vibration, it is necessary to avoid the area within 50Hz of the fundamental frequency, and also to consider the separation from harmonic vibrations of other components to prevent resonance.

(3) Through the variable structure modal analysis of the gate rotor, it provides the basic data for the study of the mechanical characteristics of the gate rotor, and provides the reference for the design, verification and optimization of the subsequent gate rotor structure from the perspective of dynamics, as well as the experience accumulation for the variable structure modal analysis of similar products.

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