Resilient guaranteed cost control of a power system

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ABSTRACT

With the development of power system interconnection, the low-frequency oscillation is becoming more and more prominent which may cause system separation and loss of energy to consumers. This paper presents an innovative robust control for power systems in which the operating conditions are changing continuously due to load changes. However, practical implementation of robust control can be fragile due to controller inaccuracies (tolerance of resistors used with operational amplifiers). A new design of resilient (non-fragile) robust control is given that takes into consideration both model and controller uncertainties by an iterative solution of a set of linear matrix inequalities (LMI). Both uncertainties are cast into a norm-bounded structure. A sufficient condition is derived to achieve the desired settling time for damping power system oscillations in face of plant and controller uncertainties. Furthermore, an improved controller design, resilient guaranteed cost controller, is derived to achieve oscillations damping in a guaranteed cost manner. The effectiveness of the algorithm is shown for a single machine infinite bus system, and then, it is extended to multi-area power system.

Introduction

Power system stability is the property of a power system that describes its ability to remain in a state of equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after a disturbance. However, it is observed, all around the world, that power system stability margins decrease. This feature is due to many reasons among which we point out the following three main ones [1]:

1. The inhibition of further transmission or generation constructions by economic and environmental restrictions. Consequently, power systems must be operated with smaller security margins.
2. The restructuring of the electric power industry. Such a process decreases the stability margins due to the fact that power systems are not operated in a cooperative way anymore.
3. The multiplication of pathological characteristics when power system complexity increases. These include the following: large scale oscillations originating from nonlinear phenomena, frequency differences between weakly tied power system areas, interactions with saturated devices, and interactions among power system controls.

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Beyond a certain level, the decrease in power system stability margins can lead to unacceptable operating conditions and/or to frequent power system. One way to avoid this phenomenon and to increase power system stability margins is to control power systems more efficiently.

Synchronous generators are normally equipped with power system stabilizers (PSSs), which provide supplementary feedback stabilizing signals through the excitation system. The stability limit of power systems can be extended by PSS, which enhances system damping at low-frequency oscillations associated with electromechanical modes [2]. The conventional PSS (CPSS) is designed as outlined in Kundur [1]. The problem of PSS design has been addressed in the literature using many techniques including, but not limited to, fuzzy control, adaptive control, robust control, pole placement, H∞ design, and variable structure control [3–8]. The method of Jabr et al. [9] is implemented through a sequence of conic programming runs that define a multivariable root locus along which the eigenvalues move. The powerful optimization tool of linear matrix inequalities is also used to enhance PSS robustness through state and output feedback [2,8–11]. The availability of phasors measurement units was recently exploited [12] for the design of an improved stabilizing control based on decentralized and/or hierarchical approach. Furthermore, the application of multiagent systems to the development of a new defense system, which enabled assessing power system vulnerability, monitoring hidden failures of protection devices, and providing adaptive control actions to prevent catastrophic failures and cascading sequences of events was previously proposed [13]. Attempts to enhance power system stabilization in case of controllers’ failure are given in the literatures [14,15].

None of the above references tackled the problem of controller inaccuracies. Continuous-time control is implemented using operational amplifiers and resistors that are characterized by tolerances. So, the uncertainties exist not only in the plant, due to the continuous load variations, but also in the controller. It can be shown that the controllers designed using robust synthesis techniques can be very sensitive or fragile with respect to errors in the controller coefficients, which might lead even to system instability. Therefore, it is required that there exists a nonzero (possibly small) margin of tolerance around the controller parameters, within which the closed loop system stability is maintained. A control synthesis ensuring this property is known in the literature as resilient control [16].

Electric power systems are composed of new power stations, equipped with discrete-time digital PSSs, and old ones with continuous-time PSSs. Although digital PSS is precise, still it has uncertainties. Some sources of uncertainties are finite word length, impressin in analog to digital and digital to analog conversions, finite resolution measurements, and round-off errors in numerical computations. In the present manuscript, we consider the worst-case, old power stations equipped with continuous-time PSS.

The present work proposes a design methodology of resilient excitation controller for a single machine infinite bus power system. The system is comprised of state feedback power system stabilizer (PSS) through the excitation system of the generator. Generally, it is acceptable for system operators to achieve a damping of the transient oscillations following small disturbances within a settling time of 10–15 s [17]. Expressing the settling time as a desired degree of stability, the proposed design methodology optimizes the controller parameters using an iterative LMI technique such that the degree of stability is kept within the desired range under both controller parameter inaccuracies and plant uncertainties.

The developed controller is tested under extreme load conditions and controller uncertainties. The results indicate evident effectiveness of the proposed design in maintaining robust stability with the desired settling time. Extension to multi-area power system is also given.

The paper is organized as follows: Section 2 briefly describes the power system under study and formulates the problems. In section 3, a sufficient LMI condition is derived for the design of a resilient PSS that achieves robust stability with prescribed degree of stability, under controller and plant perturbation. Adding the constraint of guaranteed cost, a better controller design is developed. Section 4 provides numerical simulation to verify the results. Finally, conclusions are made in Section 5.

**Notations and a fact [16]**

In this paper, $W^+, W^−$, and $\|W\| ≤ 1$ will denote, respectively, the transpose, the inverse, and the induced norm of any square matrix $W$. $W > 0$ ($W < 0$) will denote a symmetric positive (negative)-definite matrix $W$, and $I$ will denote the identity matrix of appropriate dimension.

The symbol $\bullet$ is as an ellips is in matrix expressions that are induced by symmetry, e.g.,

\[
\begin{bmatrix}
L + (W + N + W') & N \\
N' & R
\end{bmatrix} = \begin{bmatrix}
L + (W + N + \bullet) & N \\
\bullet & R
\end{bmatrix}
\]

**Fact**

For any real matrices $W_1$, $W_2$, and $\Delta(t)$ with appropriate dimensions and $\Delta \leq I$, $\|\Delta\| ≤ 1$, it follows that

\[
W_1^T \Delta W_2 + W_2^T \Delta^T W_1' \leq \varepsilon \|W_1\| W_2^T + \varepsilon W_2^T W_2, \quad \varepsilon > 0
\]

where $\Delta(t)$ represents system bounded norm uncertainty. The usefulness of this fact lies in bounding the uncertainties.

**Methodology**

The system under study consists of a single machine connected to an infinite bus through a tie-line as shown in the block diagram of Fig. 1. It should be emphasized that the infinite bus could be representing the Thévenin equivalent of a large interconnected power system. The machine is equipped with a solid-state exciter.

![Fig. 1 Basic components of a single machine infinite bus power system.](image)
Modeling of single machine infinite bus system (SM1B)

The nonlinear model of the system is given through the following differential equations [1].

$$\dot{\delta} = \omega, \dot{\omega} = \left( T_m - T_e \right) / M$$

$$E_q = \frac{1}{T_{do}} \left( E_{qf} - \frac{x_d + x_e}{x_d + x_e} E_q^* + \frac{x_d + x_e}{x_d + x_e} V \cos \delta \right)$$

$$E_{qf} = \frac{1}{T_{ek}} \left( K_e \left( V_{sdf} - V_i + u \right) - E_{qf} \right)$$

where the symbols have their usual meaning [1]. Typical data for the system under consideration are given as follows:

- Synchronous machine parameters: $x_d = 1.6$, $x_e = 0.32$, $x_q = 1.55$, $f = 50$ Hz, $T_{do} = 6$ sec, $M = 10$ s.
- Exciter-amplifier parameters: $K_e = 25$, $T_e = 0.05$ s.
- Transmission line reactance: $x_c = 0.4$.

For PSS design purposes, the linearized forth order state space model around an equilibrium point is usually employed [1]. The parameters of the model have to be computed at each operating point since they are load dependent. Analytical expressions for the parameters ($k_1$–$k_6$) were derived from Solomon et al. [5]. The parameters are functions of the loading condition, real and reactive powers, $P$ and $Q$, respectively. The operating points considered vary over the intervals (0.4, 1.0) and (0.1, 0.5), respectively.

For small perturbation around an operating point, the linearized state equation of the system under study is given by Kundur [1] as,

$$\dot{x} = Ax + Bu$$

where

$$x = [\Delta \delta \Delta \omega \Delta E_q \Delta E_{qf}]$$

$$A = \begin{bmatrix} 0 & c_0 & 0 & 0 \\ -k_1 c_0 & 0 & -k_2 c_0 & 0 \\ -k_1 c_0 & 0 & -k_2 c_0 & -k_3 c_0 \\ -k_1 c_0 & 0 & -k_2 c_0 & -k_3 c_0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_{do} \end{bmatrix}$$

Table 1 gives the extreme operating range of interest, heavy and light loads, as well as the nominal load.

The corresponding system matrices are given in Appendix A.

To represent system dynamics at continuously changing loads, system (2) can be cast in the following norm-bounded form

$$\dot{x} = (A_o + \Delta A)x + Bu$$

where $A_o$ is the state matrix at the nominal load and the uncertainty in $A$ is

$$\Delta A = M \cdot \Delta t(t) \cdot N$$

The matrices $M$ and $N$ being known constant real matrices, and $\Delta t(t)$ is the uncertain parameter matrix. The matrix $\Delta A$ has bounded norm given by $||\Delta A|| \leq 1$, Appendix A. It is worth mentioning that $\Delta t(t)$ can represent power system uncertainties, unmodelled dynamics, and/or nonlinearities. It is worth mentioning that other representations for uncertainties exist: the polytopic structure [11], and the weighting functions in the $H_\infty$ approach. Among them, the norm-bounded structure is the easiest.

Our objective now is to study two main problems

1. The first problem is to design a robust PSS that for different loads, it preserves the settling time, $t_s$, following any small disturbance within the range of 10–15 s. This is equivalent to finding a controller which achieves a closed loop system with a prescribed degree of stability $\lambda$. That is, for some prescribed $\lambda > 0$, the states $x(t)$ approach zero at least as fast as $e^{-\lambda t}$. We will focus on the time-invariant case where the controller is constant and achieves closed loop eigenvalues with real parts less than $-\lambda$. Of course, the larger is $\lambda$, the more stable is the closed loop system [18]. Since $t_s = 4/\lambda$, selecting $\lambda$ around 0.5 guarantees that the desired settling time is satisfied. 

2. The second problem deals with the design of a resilient PSS that in addition to achieving robust stabilization with a degree of stability in face of load variations, it takes into consideration the controller inaccuracies as well. That is, the resilient controller accommodates both plant parametric uncertainties and controller gain perturbations. For state feedback PSS, $u = Kv$, $K = [k_1 \ldots k_d]$, these $k$’s are implemented using operational amplifiers with resistors as shown in Fig. 2.

Remark 1. The tolerance of resistors is in practice ±5%, ±10%, and ±20%. When resistors having the best precision, ±5%, are used with operational amplifiers; its errors are reflected on the controller gains. So, there are inherent errors in the controller gains.

Any $k$ is $-R_f / R$, assuming the resistors used has inherent uncertainty (tolerance) ±5%, this is reflected on the $k$ as ±10% of its nominal value. In Mahmoud [16], $\Delta K$ is given then $K_o$ is calculated. Our objective here is different: what is $K_o$ that it tolerates $\Delta K \leq \pm 10\% K_o$?
For a given state feedback PSS, the actual controller implemented is thus assumed to be inaccurate of the form
\[ u = Kx = (K_o + \Delta K)x \]  \hspace{1cm} (6)
where \( K_o \) is the nominal controller gain and \( \Delta K \) represents the gain perturbations. Here, the perturbations are assumed of the norm-bounded form
\[ \Delta K = H \cdot \Delta x(t) \cdot E, \| \Delta x \| \leq 1. \]  \hspace{1cm} (7)
where \( H \) and \( E \) being known constant matrices and \( \Delta x(t) \) is the uncertain parameter matrix. We thus have the following two design problems

**Design case 1:** Resilient PSS + robust stability with degree \( \varepsilon \)

Design \( K_o \), with tolerance \( \Delta K \leq \pm 10\% \ K_o \), such that the poles of the closed loop
\[ \dot{x} = \{ (A_o + \Delta A) + B(K_o + \Delta K) \}x = A_\Delta x \]  \hspace{1cm} (8)
lie to the left of the vertical line \( -\varepsilon \) in the complex plane with the presence of admissible uncertainties in plant and controller, (5) and (7), respectively.

**Design case 2:** Resilient PSS + robust stability with degree \( \varepsilon \) + guaranteed cost

Although pole placement in a region, left to \( -\varepsilon \), puts an interesting practical constraint on system oscillation settling time, in practice, it might be desirable that the controller be chosen to minimize a cost function as well.

The cost function associated with the uncertain system (1) is
\[ J = \int_0^\infty (x^TQx + u^TRu)dt \]  \hspace{1cm} (9)
where \( Q = Q^T > 0 \) and \( R = R^T > 0 \) are given weighting matrices. With the state feedback (6), the cost function of the closed loop is
\[ J = \int_0^\infty \dot{x}^T(Q + K_o \cdot R \cdot K)x dt \]  \hspace{1cm} (10)
The guaranteed cost control problem is to find \( K \) such that cost function \( J \) exists and to have an upper bound \( J' \), i.e., satisfying \( J < J' \), Mahmoud [16].

**Problem solution**

Design case 1 is considered with the following theorem

**Theorem 1.** Consider the uncertain system (4), there exist a resilient state feedback gain \( K_o \) (6), with a prescribed degree of stability \( \varepsilon \) if the following LMIs have a feasible solution.

\[
\begin{bmatrix}
(A_o + B + \bullet) + 2\varepsilon X + \varepsilon MM' + \rho BH(BH)' & \bullet & \bullet \\
NX & -\varepsilon I & \bullet \\
EX & 0 & -\rho I
\end{bmatrix} < 0
\]  \hspace{1cm} (11)

\[ X > 0, \varepsilon > 0, \rho > 0 \]  \hspace{1cm} (12)
Moreover, the controller gain matrix is given by \( K_o = XY^{-1} \).

**Proof.** Selecting a Lyapunov function \( V = \dot{x}'Px, dV/dt < 0 \) or equivalently the closed loop system (8) is robustly stabilized with a degree of stability \( \varepsilon \) if and only if, Anderson and Moore [18]
\[ P(A_o + \Delta A) + (A_o + \Delta A)'P < 0, P > 0 \]  \hspace{1cm} (13)
where the closed loop uncertain matrix is \( A_o + \Delta A = A_o - \Delta A + B(K_o + \Delta K) \). Eq. (13) is equivalent to
\[
P(A_o + BK_o) + (A_o + BK_o)'P + P \cdot \Delta A + \Delta A' \cdot P + P \cdot B \cdot \Delta K + \Delta K' \cdot B' \cdot P + 2\varepsilon P < 0
\]  \hspace{1cm} (14)
Using the aforementioned fact, Eq. (14) is satisfied if
\[
P(A_o + BK_o) + (A_o + BK_o)'P + \varepsilon PM(PM)' + \varepsilon^{-1}N'N + \rho BH(BH)' + \rho^{-1}EE'EX + 2\varepsilon P < 0
\]  \hspace{1cm} (15)
For \( \varepsilon, \rho > 0 \).

Using the congruence transformation [16], by pre- and post-multiply (15) by \( P^{-1} \) and let \( P^{-1} = X, K_oP^{-1} = Y \); we get
\[ (A_oX + BY) + (A_oX + BY)' + \varepsilon MM' + \varepsilon^{-1}N'N + \rho B(H'BH)' + \rho^{-1}X'EX + 2\varepsilon X < 0 \]  \hspace{1cm} (16)
Eq. (16) can be rewritten as:
\[
(A_oX + BY + \bullet) + 2\varepsilon X + \varepsilon MM' + \rho^{-1}B'H'BH'X
+ [X N' X'] \begin{bmatrix}
\varepsilon^{-1}I & 0 \\
0 & \rho^{-1}I
\end{bmatrix}[N X] < 0
\]
Applying Schur complement [16], to linearize the above nonlinear matrix inequality, we have (11, 12). Note that all the terms are linear in the variables \( X, Y, \varepsilon, \rho \). The controller can be calculated by:
\[ K_o = XY^{-1} \]
This completes the proof. \( \square \)

A general framework for algorithm based on (11, 12) can be specified as follows:

**Algorithm.** Given starting feedback matrix \( K_o \), e.g., by solving (11, 12) with no uncertainty in \( K \). Calculate \( \Delta K = \pm 10\% K_o \). From (7), select \( H = 1 \) and calculate \( E \).

For \( i = 0, 1, 2, \ldots \)

Given \( H \) and \( E \), solve (11, 12) and get \( K_{o,i} \) and terminating when \( \| K_{o,i} - K_{o,i+1} \| < tol \); final convergence test is satisfied. STOP and obtain the approximate solution \( K_{o,i} \).

Choose new starting matrix \( K_{o,i+1} = K_{o,i} \). Calculate \( \Delta K = \pm 10\% K_{o,i+1} \). Select \( H = 1 \) and calculate \( E \)

End (for)

Design case 2 is considered with the following theorem

**Theorem 2.** Consider the uncertain system (8) and the cost function (9), if the following LMIs hold for all possible uncertainties satisfying (5, 7),
\[
\begin{bmatrix}
AX + BY + \bullet + 2xX + xM^\Gamma + \rho BH[BH]^\Gamma \\
x + HX \\
y + \rho HFB \\
\end{bmatrix}
\begin{bmatrix}
Q^{-1} \\
0 \\
0 \\
\end{bmatrix} < 0
\]

Then, the resilient PSS providing robust stability with degree \( z \) and guaranteed cost is

\[ K_o = YX^{-1} \]

Moreover, the cost function has an upper bound

\[ J' = x'Px \]

where initial condition \( x_o = x(0) \).

**Proof.** The resilient PSS achieving robust stabilization + a degree of stability \( z \) is given by (13). We impose a bound on the cost function \( J', (9) \), by the following design requirement:

\[ \dot{V} < -(x'Qx + u'R_\alpha) \]

The constraint (20) is added to (13) to get

\[ (P[A\Delta + zI] + \bullet) + Q + K^\Gamma RK < 0 \]

It is clear that if (21) is satisfied, it implies that (13) is fulfilled as well, \( (Q > 0, R > 0) \). Substituting for \( A\Delta \) and \( K = K_o + \Delta K \), inequality (21) is equivalent to

\[ \left[ \begin{bmatrix} P[A_o + zI] + PBK_o + \bullet + Q & \bullet \\ K_o & -R^{-1} \end{bmatrix} + \begin{bmatrix} PA_o + PB\Delta K & \bullet \\ \Delta K & 0 \end{bmatrix} \right] < 0 \]

Substituting for \( \Delta A, \Delta K \) from (5), (7), Eq. (22) is equivalent to

\[ \left[ \begin{bmatrix} P[A_o + zI] + PBK_o + \bullet + Q & \bullet \\ K_o & -R^{-1} \end{bmatrix} + \begin{bmatrix} PM & \bullet \\ 0 & 0 \end{bmatrix} \Delta [N 0] + \bullet \right] + \left[ \begin{bmatrix} PBH^\Gamma H & \bullet \end{bmatrix} \Delta [E 0] + \bullet \right] < 0 \]

To eliminate the uncertainties, the aforementioned fact is applied and (23) is satisfied if

\[ \left[ \begin{bmatrix} P[A_o + zI] + PBK_o + \bullet + Q & \bullet \\ K_o & -R^{-1} \end{bmatrix} + \varepsilon \begin{bmatrix} PM & \bullet \\ 0 & 0 \end{bmatrix} \right] \begin{bmatrix} PM & \bullet \\ 0 & 0 \end{bmatrix}^\Gamma + \varepsilon^{-1} \begin{bmatrix} N & \bullet \\ 0 & 0 \end{bmatrix} + \rho \begin{bmatrix} PBH^\Gamma H & \bullet \end{bmatrix} \begin{bmatrix} PBH^\Gamma H & \bullet \end{bmatrix} + \rho^{-1} \begin{bmatrix} E & \bullet \end{bmatrix} \begin{bmatrix} E & \bullet \end{bmatrix} < 0 \]

The last equation is equivalent to

\[ \left[ \begin{bmatrix} P[A_o + zI + BK_o] + \bullet + Q + \rho PM(PM)^\Gamma + \rho PBH[BH]^\Gamma + \varepsilon^{-1} N \times \rho^{-1} E \times E \times \bullet \end{bmatrix} K_o + \rho HFB \right] < 0 \]

Linearizing (24) by pre- and post-multiply by diag\([P, I]\) and letting \( P^{-1} = X, KP^{-1} = Y \), we get (17). This completes the proof. \( \square \)

This shows that the obtained resilient controller achieves robust stabilization with degree \( z \).

To show that the controller provides an upper bound of the cost function, consider a Lyapunov function

\[ V = x'Px, P = P' > 0 \]

Notice that (20, 21) is equivalent to

\[ PA_o + \bullet < -(Q + K^\Gamma RK) \]

Differentiating \( V(x(t)) \) with respect to time and using (20), we obtain

\[ \dot{V} = x'(PA_o + \bullet)x \leq -x'(Q + K^\Gamma RK)x \]
Therefore, integrating both sides of the above inequality from \(0 \to \infty\) gives
\[
\int_0^\infty x'(Q + KR)Kx\,dt \leq V(x_0) - V(x(\infty))
\]

Since the stability of the system has already been established, \(x(t) \to 0\) as \(t \to \infty\), it can be concluded that \(V(x(t)) \to 0\) as \(t \to \infty\). This completes the proof.

**Results and discussion**

The linear matrix inequalities (17, 18), with \(Q\) and \(R\) matrices chosen to be unity, are solved using the matlab LMI control toolbox, Gahenit et al. [19], to get the feedback matrix. The results are summarized in Table 2.

The dominant closed loop eigenvalues of the system without control and with the proposed GC-PSS for different loads and controller’s uncertainties are shown in Fig. 3a and b, respectively.

**Remark 2.** With no control, the system has poor degree of stability, even it can become unstable, Fig. 3a. Resilient guaranteed cost PSS provides robust stability with degree \(a\) for different loads and controller inaccuracies, Fig. 3b.

**Testing the proposed resilient GC-PSS at extreme cases**

Two extremities are considered: (1) heavy machine load with PSS inaccuracy \(+0.1\,K_0\) and (2) light machine load with PSS inaccuracy \(-0.1\,K_0\). To check rotor angle stability, a three phase fault is applied at the machine terminal which causes 0.1 rad angle deviation. The response with and without the proposed PSS is shown in Fig. 4.
Testing system performance by another input, 0.1 step increases in the reference field voltage, the frequency stability is checked in Figs. 5 and 6. The time response of the system under all operations confirms that the settling time of the system is within the desired range.

Comparison with conventional PSS (CPSS)

Many existing generators are commissioned with a PSS of this form.

\[ u = K_o \frac{T_w}{1 + T_w s} \frac{(1 + T_1 s)(1 + T_3 s)}{(1 + T_{25} s)(1 + T_{45} s)} \Delta \omega \tag{26} \]

where \( T_w \) is the time constant of a washout circuit that eliminates the controller action in steady state. Typical ranges of the CPSS parameters are [0.001–50] for \( K_o \) and [0.06–1.0] for \( T_1 \) and \( T_3 \). The time constants \( T_w, T_2, \) and \( T_4 \) are set as 5, 0.05, and 0.05 s, respectively [1]. The parameters of the CPSS designed based on the nominal load and achieving the same control task as before, \( \alpha = 0.5 \), are found to be as follows: \( K_o = 25, T_1 = T_3 = 0.76 \). Assuming an error \( \pm 10\% \) in the CPSS gain, the responses at the same extremities considered before are shown in Figs. 6 and 7.

Since \( \max |\Delta \omega| \) are in the order of 2–3 \( \times 10^{-6} \) p.u. and 5–6 \( \times 10^{-6} \) p.u. in the case of the proposed PSS and the CPSS, respectively Figs. 4–7, it is evident the superiority of the proposed design. (see Fig. 8)

Next, the proposed resilient guaranteed cost PSS is applied to a multi-area power system.

Multi-area load frequency control (LFC)

The operation objectives of the LFC are to maintain system frequency and the tie-line power as near as possible to the scheduled values, Saadat [20]. The data and the linearized state space model for a two-area system are given in Appendix B. Due to the continuous tie-line power changes, the uncertainty in the synchronizing power coefficient \( P_s \) is assumed to be \( \pm 50\% \) of its nominal value. Given next the resilient controller with errors \( \pm 10\% \), which is obtained by solving (17), it achieves guaranteed cost performance as well as a desired settling time of less than 4 s (\( \alpha = 1 \)).

\[
K = [2.139, 5.8087, 329.06, -6.8248, -8.5834, -229.5, 811.07; -3.354, -6.7315, -286.87, 5.9457, 8.9864, 324.09, -911.62] 
\]

The closed loop poles with plant and controller uncertainties \( \pm 50\%, \pm 10\% \), respectively are shown in Fig. 9. It is evident that the design objectives are indeed satisfied.

**Remark 3.** The states used with the proposed controllers can be easily measured or calculated. So, no need to use observers which add more dynamics to the original system.

At the extremes of 150% \( P_s \) and controller error \( \pm 10\% \), the frequency deviation for 200 MW step load increase in area 1 is shown in Fig. 10, while for 50% \( P_s \) and \( -10\% \), control error is shown in Fig. 11.
Conclusions

Sufficient conditions for the design of resilient state feedback PSS are presented. The proposed design accommodates both plant uncertainties, due different loads, and controller uncertainties faced in practical implementation. The proposed design is very effective in damping system oscillation within the desired settling time for any plant/controller uncertainties. Design of resilient digital PSS is currently under investigation.

Conflict of interest

The authors have declared no conflict of interest.

Appendix A. Plant model uncertainties as norm-bounded structure

The state space equations for the three operating conditions are given as follows:

Nominal load:

\[ A = \begin{bmatrix} 0 & 314 & 0 & 0 \\ -0.1186 & 0 & -0.0906 & 0 \\ -0.1934 & 0 & -0.4633 & 0.1667 \\ -5.9319 & 0 & -255.7990 & -20 \end{bmatrix} \]

Heavy load:

\[ A = \begin{bmatrix} 0 & 314 & 0 & 0 \\ -0.1445 & 0 & -0.0976 & 0 \\ -0.2082 & 0 & -0.4633 & 0.1667 \\ 16.0486 & 0 & -262.7201 & -20 \end{bmatrix} \]

Light load:

\[ A = \begin{bmatrix} 0 & 314 & 0 & 0 \\ -0.0875 & 0 & -0.0759 & 0 \\ -0.162 & 0 & -0.4633 & 0.1667 \\ -27.1071 & 0 & -255.97 & -20 \end{bmatrix} \]

Neglecting small deviations in \( A \), the uncertainty in \( A \) over the different loads can be approximated by a norm-bounded structure \([A_0(t)]N\),

\[ M = [0, 0, 0, 4.69]^T, \quad N = [4.69, 0, -1.5, 0]. \]

Appendix B. Two-area power system

The data for a two-area connected by a tie-line are as follows, with \( P_s = 2 \) p.u. [20].

| Area | 1 | 2 |
|------|---|---|
| Speed regulation, \( R \) | 0.05 | 0.0625 |
| Freq. sens. load coeff., \( D \) | 0.6 | 0.9 |
| Inertia constant, \( H \) | 5 | 4 |
| Base power, MVA | 1000 | 1000 |
| Governor time constant, \( T_g \) | 0.2 | 0.3 |
| Turbine time constant, \( T_t \) | 0.5 | 0.6 |

Selecting the state vector as \( x = [\Delta P_s, \Delta P_m, \Delta w_1, \Delta P_2, \Delta P_m, \Delta w_2, \Delta P_{12}]^T \), the linearized state space equation can be derived as

\[
A = \begin{bmatrix}
\frac{-1}{T_s} & 0 & -\frac{1}{\tau_s} & 0 & 0 & 0 & 0 \\
\frac{-1}{T_s} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\tau_H} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\tau_s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\tau_H} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_s} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_H} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\tau_H} \end{bmatrix}
\]

With the usual meaning of the variables [20]. For tie-line load variations which causes \( \pm 50\% \) uncertainty in the synchronizing power coefficient \( P_s \), the norm-bounded model for the uncertainty can be found as

\[ M = [0, 0, 0, 0, 0, 0, 1]^T, \quad N = [0, 0, 0.5P_s, 0, 0, -0.5P_s, 0] \]

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