Numerical analysis of concrete temperature stress at different loading ages

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Abstract. This paper investigates the thermal stresses of low-heat cement concrete under different loading ages, and thermal stresses were calculated with different compensation cycles by the Boltzmann superposition principle. An application of the Boltzmann superposition principle on the temperature stress test shows how the formulation may be used to evaluate the mechanical properties of low-heat cement concrete and how the accuracy of calculated stresses applies to measured stresses. The comparison between measured stresses and the simulated results shows good agreement under different compensation cycles, which indicates that the thermal stresses can be well estimated via the line superposition. Furthermore, these experiment results indicate that the maximum compression stresses increase rapidly with the increment of curing ages while the cracking stresses have subtle differences.

1. Introduction

Thermal stresses occur in young concrete due to the restrained deformations, and mechanical properties have some differences with different curing conditions. Furthermore, the longer the curing age, the higher preformation of concrete. A linear one-dimensional constitutive model was adopted to calculate the thermal stresses. With a minimal set of required input parameters, the thermal stresses could have a good prediction[1]. The geometry size of the concrete structure and ambient also have influences on the thermal stress. The thermal stress of concrete is the result of multiple factors, and a long concrete wall was analysed to investigated the thermal stresses and crack resistance of concrete[2].

The restrained deformation of early age concrete will create stress due to the thermal or shrinkage, so a coating shrinkage reducing admixture and a curing compound were coated onto the concrete surface to reduce the thermal stress. This curing method can significantly improve the chloride-penetration resistance and reduce water absorption of concrete[3]. The thermal stresses often reach considerable values and may lead to cracking of concrete structures, which is of high interest from durability point of view. In general, it can be said that the cracks occur when tensile stress in a most dangerous position in the structure exceeds the tensile strength of concrete[4-6]. Different initial curing conditions has been investigated on the mechanical properties and durability of early age concrete. Steam curing and standard curing conditions have different influences on the compressive strength, splitting tensile strength, elastic modulus, shrinkage, freeze-thaw resistance, carbonation, and sulfate resistance of concrete. The standard-curing concrete shows higher performance compared with the steam-curing concrete[7].

Thermal stress is affected by various material parameters including cement type, w/b ratio,
aggregate size and type, temperature, moisture, etc.\cite{8-15}. For a given concrete in practical structures, creep and temperature development were extremely important to influence thermal stress. In this paper, the early age restrained stress behaviours due to autogenous shrinkage of low heat Portland cement concrete were investigated at sealed conditions, and the effects of compensation cycles on stress characteristics were taken into account.

2. Experiment

Based on the working principle of the temperature stress testing machine (TSTM) developed by R. Springenschmid, Israel have developed the similar test devices\cite{16}. Since Lin zhihai developed the first TSTM from the building materials research institute of tsinghua university in domestic in 2002. To improve the accuracy of temperature control and displacement test, the closed-loop computer control system was proposed. As shown in Figure 1, the heating pipe and compressor controlled by the temperature-control system can simulate the temperature development of concrete structures in temperature range from $-20^\circ\text{C}$ to $80^\circ\text{C}$. The experiment results show that the accuracy of temperature can reach $\pm 0.01^\circ\text{C}$ and the precision of displacement-control can reach $\pm 0.1\mu\text{e}$ through the optimized design.

The length deformation and temperature of two concrete specimens were measured during the temperature stress test, and the distance between the two gauge points is 1.0 m. One end of the concrete specimen is connected with the displacement gauge and the concrete specimen can deform freely with the temperature history. The other end of the restrained concrete specimen is connected with the stepping motor, when the length of the specimen changed the stepping motor would drive the specimen to keep its original length. The temperature will arise when the hydration reaction of water and cement starts, while the temperature starts to decrease if the hydration rate slow down gradually. A restrained concrete specimen will, on the other hand, rapidly experience both high compressive stresses as the concrete expands during the hydration process and high tensile stresses as it contracts due to the decreasing hydration rate.

![Figure 1. Schematic diagram of temperature control system.](image_url)

2.1. Materials and mixture proportions

The physical and chemical properties of low-heat Portland cement and fly ash (FA) used in this study are presented in Table 1, and the strength characteristics of cement are given in Table 2. The experiments were performed with two 0.5 w/b mixtures, including low heat Portland cement (P-LH 42.5) dam concrete. Among them, 35% of cement was replaced by FA. The concrete mix proportion is presented in Table 3.
Table 1. Physical and chemical properties of P-LH 42.5 and FA (%)

| Item       | CaO | SiO₂ | Al₂O₃ | Fe₂O₃ | MgO | SO₃ | Na₂O | K₂O | Specific gravity |
|------------|-----|------|-------|-------|-----|-----|------|-----|-----------------|
| P-LH 42.5  | 58.7| 22.8 | 4.3   | 4.3   | 4.2 | 3.0 | 0.1  | 0.3 | 3.21            |
| FA         | 3.2 | 59   | 21.6  | 9.6   | 1.2 | 0.1 | 0.53 | 0.45| 2.28            |

Table 2. Strength characteristics of cement

| Item       | Setting time (min) | Flexural strength (MPa) | Compressive strength (MPa) |
|------------|--------------------|-------------------------|-----------------------------|
|            | initial final 3 days | 7 days 28 days | 3 days 7 days 28 days       |
| P-LH 42.5  | 233 343 | 4.3 5.2 | 8.2 14.9 23.6 | 44.5 |

Table 3. Concrete mixture proportions

| Cement | Water | FA | Coarse | Fine | Superplasticizer | Air entraining agent |
|--------|-------|----|--------|------|------------------|----------------------|
| 216    | 166   | 116| 844    | 1118 | 1.992            | 0.053                |

2.1.1. Temperature history
The temperature history curves are shown in Figure 2. The adiabatic temperature rise of P-LH 42.5 about 32°C. To investigate the effect of compensation cycle on the temperature stress, the shrinkage strain and accumulative strain of restrained specimen, free shrinkage strain were measured.

![Figure 2. Temperature history of low heat cement concrete](image)

2.2. Experiment method

In this study, the restraint degree is controlled by displacement, i.e. the restraint degree is linearly proportional to the deformation. The restraint degree can be defined as:

\[
\gamma_R = \frac{\varepsilon^0(t) - \varepsilon(t)}{\varepsilon^0(t)}
\]

(1)

where \(\varepsilon^0(t)\) is the specimen deformation without restraint, in this paper, the free shrinkage deformation is just the thermal dilation; \(\varepsilon(t)\) is the actual deformation after the specimen is loaded, for a simpler calculation, \(\gamma_R = 100\%\), i.e. \(\varepsilon(t) = 0\). To maintain the deformation zero, the load was applied when the shrinkage strain exceeded the compensation cycles (1.5, 2, 3). The shrinkage strain and stress were recorded by the computer system every 20 seconds, one for free deformation test and the other for 100% restraint test. Each stage of stress can be calculated by the Eq. 2:

\[
(\Delta \sigma)_j = \frac{E(t_j)}{1 + \varphi(t_{i+1/2}, t_j)} \left\{ \varepsilon(t_{i+1/2}) - \varepsilon^0(t_{i+1/2}, t_j) - \sum_{j=1}^{i} [(\Delta \sigma)_j, 1 + \varphi(t_{i+1/2}, t_j)] \right\} \frac{1}{E(t_j)}
\]

(2)

\[
\sigma = \sum_{j=1}^{i} (\Delta \sigma)_j
\]

(3)
where $\Delta$ denotes an increment during the actual time step; $(\Delta \sigma)_i$ is a stress increment during the time step $t_i$, MPa; $E(t_i)$ is elastic modulus at the time $t_i$, GPa; $\varphi(t_{i+1/2}, t_i)$ is the average creep strain coefficient during the time step in question; $\varepsilon(t_{i+1/2})$ is the average strain of $\varepsilon(t_i)$ and $\varepsilon(t_{i+1})$ which are the actual strain when the load is applied at $t_i$ and $t_{i+1}$, respectively; $\delta^p(t_{i+1/2}, t_0)$ is the average strain between $\delta^p(t_i, t_0)$ and $\delta^p(t_{i+1}, t_0)$ which are the free shrinkage strain from $t_0$ to $t_i$, $t_0$ to $t_{i+1}$, respectively.

### 3. Results and discussion

A series of tests including different ages ($\tau$=1,3,7,20) were conducted to investigate the stress process under the same temperature history in this experimental study. In the uniaxial restrained shrinkage test, the specimens were restrained with 1.5µm compensation cycle. The free shrinkage tests were carried out with the same temperature history at the same time.

All specimens were cast immediately into the molds after mixing and sealed for 12 h. Two layers of plastic film were fitted around the molds before casting, then the specimens were wrapped with the rest of the plastic film. Each specimen was cured and tested in the same laboratory at constant temperature (20°C). Therefore, all specimens were in the same external environment condition.

The analysis and comparisons with other experiments and theory calculation are presented. The error of the studied linear stress-strain calculations and non-linear stress-strain calculations is compared with the measured data for each data set characterized by a coefficient of variation $\omega_{\sigma}$, which is defined as

$$\bar{\omega}_{\sigma} = \frac{1}{N} \sum_{i=1}^{N} \omega_{\sigma}.$$  

$$\omega_{\sigma} = \left[ \frac{1}{M} \cdot \sum_{i=1}^{M} \left( \frac{\sigma_{\text{meas}} - \sigma_{\text{cal}}}{\sigma_{\text{meas}}} \right)^2 \right]^{1/2}$$

where $N$ = number of loading ages for the data set in question; $M$ = number of data points; $\sigma_{\text{meas}}$ = measured stresses in points with spacing in time; $\sigma_{\text{cal}}$ = calculated stresses based on linear or non-linear stress-strain method.

Variation coefficients describing how the curing age at $\tau$=1 day, $\tau$=3 days, $\tau$=7 days and $\tau$=20 days influence the calculated temperature stress development compared to the measured stress is summarized in Table 4. The overall error $\bar{\omega}_{\sigma}$ for each age of evaluation in the bottom of the table is calculated as the algebraic average of $\omega_{\sigma}$ from all data sets.

The temperature stress was measured with 1.5 µm compensation cycle while the rest curves was calculated with $\Delta \varepsilon = 3\mu$m, 7µm and 11µm, as shown in Fig. 2. It can be seen that there is a large deviation between measured and calculated stress while the concrete specimens have broken when the calculated stress still increase. During the compressive phase, the stress will earlier achieve the maximum compressive stress with smaller compensation cycle. As the compressive stresses starts to decrease to tensile stresses, these calculated stress curves are basically parallel.

The variation coefficients for all tested concretes at different ages ($\tau$=1 day, $\tau$=3 days, $\tau$=7 days and $\tau$=20 days) of evaluation are summarized in Table 7, where the overall error $\bar{\omega}_{\sigma}$ for each compensation cycle of evaluation in the bottom of the table is calculated as the algebraic average of $\omega_{\sigma}$ from all data sets.

As can be seen in Table 4, the average variation coefficient under the compensation cycle $\Delta \varepsilon = 7$ for the compressive phase will be $\bar{\omega}_{\sigma} = 5.6\%$ of the measured compressive stress. The main reason to the variation is that the damage of the specimens is very close to the calculation error. As we know, the linear superposition equation will have a very small loss at each stack. At the same time, the compensation cycle is smaller. The calculation precision will be higher, but the measured stress will be influenced by the constant pressure or tension. While the compensation cycle increases to $\Delta \varepsilon = 11$, the average variation coefficient will be a small growth during the compressive phase and the average variation coefficient will be a constant decrease during the tensile phase. This also indicates that the damage of the concrete specimen raises with the increase of the tensile stress.
A somewhat peculiar result in Table 4 is obtained for the P-LH 42.5 cement concrete, as the average variation coefficients for $\Delta \varepsilon = 7$ to $\Delta \varepsilon = 11$ are almost the same, about 5.6 to 7 percent of measured compressive stress and 13.5 to 12 percent of measured tensile stress. This shows that the greater the compensation cycle, experimental results have better compatibility with the calculation.

![Temperature stress curves under different compensation cycles at different ages](image)

**Figure 3.** Temperature stress curves under different compensation cycles at different ages: (a) $\tau=1$ day; (b) $\tau=3$ days; (c) $\tau=7$ days; (d) $\tau=20$ days.

| Compensation cycles | Loading age (day) | $M$ | Compressive stress (MPa) | Tensile stress (MPa) |
|---------------------|------------------|-----|--------------------------|---------------------|
| $\Delta \varepsilon = 3$ | 1                | 59  | 0.15                     | 0.18                |
|                     | 3                | 65  | 0.28                     | 0.19                |
| $\bar{\omega}_\sigma$ | -                | -   | 0.13                     | 0.19                |
|                     | 1                | 59  | 0.054                    | 0.049               |
| $\Delta \varepsilon = 7$ | 3                | 65  | 0.036                    | 0.10                |
|                     | 7                | 87  | 0.08                     | 0.19                |
|                     | 20               | 46  | 0.06                     | 0.20                |
| $\bar{\omega}_\sigma$ | -                | -   | 0.056                    | 0.135               |
|                     | 1                | 59  | 0.061                    | 0.043               |
| $\Delta \varepsilon = 11$ | 3                | 65  | 0.045                    | 0.10                |
|                     | 7                | 87  | 0.12                     | 0.13                |
|                     | 20               | 46  | 0.07                     | 0.20                |
| $\bar{\omega}_\sigma$ | -                | -   | 0.07                     | 0.12                |
4. Conclusion
By virtue of TSTM, the linear stress-strain behavior of concrete was studied by means of Boltzmann superposition principle and the thermal stress is calculated at different compensation cycles to estimate the early stress of concrete, which is in good agreement with the measured results.

35% of cement are substituted by FA in this study. The results show that fly ash can reduce self-shrinkage and self-generation restrained stress, and the early creep ability of fly ash concrete with a large amount of fly ash is higher than that of ordinary concrete.

This study adopts the 100% uniaxial restraint method to simplify the restraint stress problem. However, this situation is much more complex in practical engineering. The restraint degree can’t keep the same but generally decrease with the increment of curing ages. The experiment results show that an appropriate compensation cycle or increasing curing ages in the linear superposition way will be closer to the measured value.

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