THE INTERPRETATION OF ROTATION MEASURES IN THE PRESENCE OF INHOMOGENEOUS FOREGROUND SCREENS

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ABSTRACT

We analyze the redshift evolution of the rotation measure (RM) in the Taylor et al. data set, based on NVSS radio data at 21 cm, and compare with results from our previous work based on RMs determined at lower wavelengths, e.g., 6 cm. We find that, in spite of the same analysis, Taylor et al.’s data set produces neither an increase of the RM dispersion with redshift nor the correlation of RM strength with Mg II absorption lines that we found previously. We develop a simple model to understand this discrepancy. The model assumes that the Faraday rotators, namely the QSO’s host galaxy and the intervening Mg II host galaxies along the line of sight, contain partially inhomogeneous RM screens. We find that this leads to an increasing depolarization toward longer wavelengths and to wavelength-dependent RM values. In particular, due to cosmological redshift, observations at fixed wavelength of sources at different redshift are affected differently by depolarization and are sensitive to different Faraday active components. For example, at 21 cm the polarized signal is averaged out by inhomogeneous Faraday screens and the measured RM mostly reflects the Milky Way contributions for low-redshift QSOs whereas polarization is relatively unaffected for high-redshift QSOs. Similar effects are produced by intervening galaxies acting as inhomogeneous screens. Finally, we assess the performance of RM synthesis on our synthetic models and conclude that the study of magnetic fields in galaxies as a function of cosmic time will benefit considerably from the application of such a technique, provided there is sufficient instrumental bandwidth. For this purpose, high-frequency channels appear preferable but not strictly necessary.

Key words: galaxies: evolution – galaxies: high-redshift – galaxies: magnetic fields – quasars: absorption lines

1. INTRODUCTION

The Faraday rotation measure (RM) is one of the very few methods to probe extragalactic magnetic fields. The RM is given by the change in observed polarization angle, $\Delta \chi_0$, over a change in the observed wavelength squared, $\Delta \lambda_0^2$. For a polarized radio source at cosmological redshift $z$, it is defined as

$$\text{RM}(z) = \frac{\Delta \chi_0}{\Delta \lambda_0^2} = 8.1 \times 10^5 \int_{z_i}^{0} n_e(z)B_\parallel(z) \frac{dl}{(1+z)^2} dz, \quad (1)$$

where $RM$ is in units of rad m$^{-2}$, the free electron number density, $n_e$, is in cm$^{-3}$, the magnetic field component along the line of sight, $B_\parallel$, is in Gauss, and the comoving path increment per unit redshift, $dl/dz$, is in parsecs. Equation (1) assumes a uniform RM screen across the source and a spatial separation of the linearly polarized source and the Faraday rotating plasma.

In Kronberg et al. (2008, hereafter K08), we used a sample of 268 RM values of extragalactic radio sources to assess the redshift evolution in the RM dispersion. We found an increase in the RM dispersion with redshift, which became statistically significant above $z \sim 1$. We postulated that the increase in the RM dispersion is produced by magnetic fields in intervening galaxies. To test this hypothesis spectra of 71 QSOs at UVES/VLT were taken. In Bernet et al. (2008, hereafter B08), we showed that sight lines with intervening strong Mg II absorption systems indeed have significantly higher RM values than those without. The findings in that work implies that magnetic fields of order 10 $\mu$G exist in galaxies out to $z \sim 1.3$.

Recently, Taylor et al. (2009, hereafter TSS09) determined RM values of 37,543 sources, based on polarization observations from the NRAO VLA Sky Survey (NVSS). After the release of the RM catalog we compared our RM values with those of TSS09 and found major differences between the two data sets. As we show in Section 2.2 using the RM values of TSS09, the results of K08 and B08 cannot be reproduced. In this work, we present an analysis of the differences found between the two data sets. We then develop a toy model to show that such differences can be accounted for by inhomogeneities in the RM screens of the QSOs’ host galaxies and intervening galaxies. In view of this model, due to the strong depolarization effects, RM data based on low-frequency observations and interpreted according to Equation (1) are inadequate to probe inhomogeneous RM screens produced by intervening galaxies.

Recently, increased attention has been paid to the effects of inhomogeneous RM screens on the observed degree of polarization and polarization angle as functions of wavelength. Rossetti et al. (2008), and Mantovani et al. (2009) used multi-wavelength radio observations to model depolarization of their sources. They showed that the depolarization as a function of wavelength can be better described including a covering factor for the inhomogeneous screens. Farnsworth et al. (2011) used Westerbork Synthesis Radio Telescope (WSRT) observations at $\sim 1$ m combined with NVSS observations at 21 cm to test different depolarization models and RM determination methods. They considered the traditional Faraday dispersion screen (Burn 1966), a two-component model for Faraday rotation which produces oscillation in the degree of polarization, and the RM synthesis method. The comparison of the different RM values obtained from these methods revealed that if Faraday structure is present, the different methods may lead to different RMs. This further stresses the importance of considering both the polarization angle and amplitude for the correct determination of the RMs.

While several effects may be at work, as briefly discussed in Section 2.3, in this paper we keep the level of sophistication of the model to a minimum and focus on the role of the
Faraday screen, particularly on simple ideas associated with inhomogeneity which can account for important observational effects and test these ideas for consistency with available data. The rest of this paper is organized as follows. In Section 2, the RM data sets of B08 and K08 are compared and the differences are presented. Previous work on depolarization of extragalactic sources is briefly presented in Section 3. In Section 4 a depolarization toy model is presented which can account for the differences in the RM values of the two data sets. A discussion of the nature of the partial inhomogeneous RM screens and computed Faraday spectra as comparisons for RM surveys is presented in Section 5. In Section 6 we give a summary of our findings.

2. ROTATION MEASURE DATA

2.1. Data Sets

The RM values used in K08, B08, and Bernet et al. (2010) were collected by P. Kronberg and collaborators over the past decades using polarization observations at various telescopes, including the Very Large Array (VLA) and Effelsberg. The sample consists of 901 sources with determined RM and redshifts, partly taken by Kronberg and collaborators and partly taken from the literature (Simard-Normandin et al. 1981). Until the work of TSS09 it had been the only existing large RM catalog. Generally at least three wavelengths were used for the RM determination (P. Kronberg 2008, private communication) using polarization data between 37 and 0.9 GHz, with the bulk of the data between ~10.5 and 1.6 GHz (Simard-Normandin et al. 1981).

In the following sections, we will assume that the determinations of the RM values of K08 were typically done at 6 cm and designate the RM values as RMs. Of course the wavelength range and typical values over which the RMs are determined might vary due to the heterogeneity of the sample. While this is undesirable, its main effect should be the introduction of noise in the relations between measured quantities predicted by our model. There might be additional issues associated with the heterogeneous character of the polarization data used for the RM determinations which, however, are not addressed here.

The sample of K08 consists of 268 sources at Galactic latitudes |b| > 45° (exact definition given in K08). In B08, we obtained high-resolution spectra of 71 relatively bright QSOs and relaxed the selection to |b| > 30°. Here we use the subset that was employed in the study of B08 and Bernet et al. (2010) (the RM data are still proprietary and will be published elsewhere by P. Kronberg), except when we look for differences in the redshift evolution where we use all sources at |b| > 30° which were also in the Taylor et al. (2009) catalog.

The RM values of TSS09 are based on the NVSS from Condon et al. (1998), which covered the sky at declinations |δ| > 40° in Stokes I, Q, and U. The survey imaged the sky at 21 cm with a resolution of ~45 arcsec and produced a catalog of 2 × 10⁶ discrete sources. TSS09 chose a subsample of this catalog and derived RM values of these sources based on determination of the polarization angles at 1364.9 and 1435.1 MHz. The subsample was selected by requesting a source intensity I > 5 mJy and an 8σ detection in polarized intensity. To ensure that the polarized intensity was not dominated by instrumental effects, they only considered sources with a fractional polarization greater than 0.5%, which yielded an RM catalog of 37,543 objects. In the following sections we will designate the RM values of TSS09 as RM21.

![Figure 1. Comparison of the RM values of K08 and TSS09, RMs and RM21 respectively, split according to whether the degree of polarization is above (black crosses) or below (red filled circles) p21 = 3.2%.

The RM data sets of K08, B08, and Bernet et al. (2010) were compared and the differences are presented. Previous work on depolarization of extragalactic sources is briefly presented in Section 3. In Section 4 a depolarization toy model is presented which can account for the differences in the RM values of the two data sets. A discussion of the nature of the partial inhomogeneous RM screens and computed Faraday spectra as comparisons for RM surveys is presented in Section 5. In Section 6 we give a summary of our findings.
Figure 2. Comparison of the cumulative distributions of RM values measured typically at 6 cm (left panel) and at 21 cm (right panel). The samples are split according to the polarized fraction $p_{21}$ and the redshifts of the sources. Significant redshift evolution is seen in the $|RM_6|$ values but not the $|RM_{21}|$ values. For sources with $0.5% < p_{21} < 2.5%$ the RM distributions for sources above $z = 1$ are broader than for sources below $z = 1$, at a significance level of 99.75%. Red histograms are for NVSS sources not included in the Taylor et al. (2009) catalog. This is because they are below the inclusion limits of the catalog, which are 8σ detection in polarized intensity and $p_{21} > 0.5%$. For these sources also a significant redshift evolution can be seen with a significance level of 99.64%.

3. The result of B08 that the RM distributions for QSOs with strong Mg II systems along their sight lines are broader (at ~95% significance level) than for QSOs free of absorbers completely disappears using the RM values of TSS09. Figure 3 shows the cumulative distribution (dashed histograms) of $|RM_{21}|$ values from TSS09 for sight lines with $(N_{\text{MgII}} > 0, \text{black})$ and without Mg II $(N_{\text{MgII}} = 0, \text{red})$ absorption systems. Clearly, there is no difference between the two RM distributions. For comparison the solid histograms show the distributions of the $|RM_6|$ values of B08.

Figure 3. Comparison of the cumulative RM distributions of B08 (solid) and TSS09 (dashed) for QSO sight lines with no Mg II absorption systems (black lines) or with one or two, respectively (red lines). RM data in TSS09 were available for 54 out of the 71 QSOs in B08.

2.3. Caveats

In the following sections we present a simple model to describe the impact of inhomogeneous, redshift-dependent Faraday screens on the polarization and RM of distant QSOs as a function of wavelength.

However, additional effects may be at work when comparing quantities measured at different frequencies. In particular, besides potential large uncertainties measured with both data sets, the low- and high-frequency emissions may originate from separate components in the source (e.g., the radio lobes and the compact core in a radio galaxy), inducing all sorts of frequency-dependent effects, e.g., location, angular size, spectral indices, and fractional polarization. For example, in some radio sources...
the fractional polarization appears to increase and then decrease with wavelength (Conway et al. 1974).

Also, while our model could be validated (or ruled out) by comparison of its predictions with observational data, this would require a high-quality data set, where the effects discussed above (and possibly others) are kept under strict control. Unfortunately, such a data set is not yet available to us, which forces us to refrain from such a comparison. A high-quality data set might, however, be available in the near future, with the delivery of new radio data from polarization surveys, either planned or just underway (see Section 5.2).

3. DEPOLARIZATION BY INHOMOGENEOUS FARADAY SCREENS

3.1. Previous Work

In this section, we develop a simple model to understand the effects arising from inhomogeneous Faraday screens and show that these can produce important depolarization effects that reproduce the differences between the RM data sets of K08 and TSS09.

The observed polarization \( p \) can be written as a complex number as

\[ p = p e^{2i\chi}, \]  
(2)

where \( p = P/I \) is the fractional degree of polarization given by the ratio of the polarized and total intensities, \( P \) and \( I \), respectively, and \( \chi \) is the polarization angle determined by the other Stokes parameters, \( U \) and \( Q \), as \( \chi = 1/2 \arctan(U/Q) \).

The linear dependence between the polarization angle \( \chi \) and \( \lambda^2 \) in Equation (1) is only valid for the case of a uniform foreground screen. The presence of unresolved inhomogeneities in the Faraday screen and/or sources of polarized radiation embedded within the Faraday active region produce increasing depolarization of the source at longer wavelength (Burn 1966; Tribble 1991; Sokoloff et al. 1998). This leads to nonlinear dependencies of the polarization angle on \( \lambda^2 \). In such cases, detailed modeling is necessary for the correct interpretation of the RMs and for their use in measurements of magnetic fields.

Several mechanisms can reduce the fractional degree of polarization either in the radio source itself or in its foreground.

1. Burn (1966) and Sokoloff et al. (1998) showed that random fluctuations in the magnetic field within the source lead to a wavelength-independent reduction of the degree of polarization. The latter is given by the ratio of the regular-to-total magnetic field energy, \( p_0 = \rho_{\gamma}(\gamma)B^2_0/B^2 \), where \( \rho_{\gamma}(\gamma) = (3\gamma + 3)/(3\gamma + 7) \) is the theoretical value of the degree of polarization, which depends on the spectral index \( \gamma \) of the emitting relativistic electrons (~0.74 for \( \gamma \sim 2.8 \)), and \( p_0 \) is the degree of polarization as \( \lambda \to 0 \).

2. A further effect which reduces the degree of polarization is differential Faraday rotation (Sokoloff et al. 1998). This effect occurs if the synchrotron emission and the medium producing the Faraday rotation are not spatially separated. In such a case, polarization angles from different depths within the source are rotated differently. The line-of-sight integrated emission suffers increasing depolarization with increasing wavelength. This effect seems not to be important for extragalactic sources (Tribble 1991).

3. Inhomogeneous Faraday rotation screens within the radio beam lead to depolarization of the sources. If the RM screen is modeled by many independent RM cells, this effect is called depolarization by external Faraday dispersion (Burn 1966; Sokoloff et al. 1998). If the RM screen varies systematically within the radio beam, this effect is called beam depolarization.

Burn (1966) gives the formula

\[ p = p_0 \exp\left(-2\sigma^2_{\text{RM}} \lambda^4\right) \]  
(3)

to describe the depolarization induced by an inhomogeneous Faraday screen with an RM dispersion \( \sigma_{\text{RM}} \). Assuming for simplicity that each RM cell is a cube of linear size \( l_0 \), then each cell contributes a dispersion \( \sigma = 0.81 Bn_0 l_0 \) and \( \sigma_{\text{RM}} = \sigma / \eta \), where \( \eta = L/l_0 \) is the number of cells along the path length \( L \) traversed by the radio waves.

The size of the cell is determined by the scale above which the RM contributions are uncorrelated and can be determined by computing the structure function in high-resolution RM maps.\(^5\)

Rossetti et al. (2008) and Mantovani et al. (2009) modeled polarization observations of ~65 compact steep spectrum sources between 2.8 and 21 cm done with the WSRT, VLA, and Effelsberg telescope. They observed that the fractional degree of polarization at large wavelengths is too large to be explained by Burn’s depolarization law (Equation (3)). They observed that for a large fraction of sources \( p \) remains approximately constant above 6–13 cm. To account for these observations, they suggested that just a fraction of the polarized source is covered by a depolarizing inhomogeneous RM screen and modified Equation (3) to

\[ p = p_0(f_c \exp\left(-2\sigma^2_{\text{RM}} \lambda^4\right) + (1 - f_c)), \]  
(4)

where \( f_c \) is the covering factor of the source.

Here we emphasize that partial coverage of the polarized source by inhomogeneous RM screens is the key for explaining the differences in the RM values of K08 and TSS09.

4. MODELING

4.1. Extension to Cosmological Screens

In Equation (4), the \((1 + z)^{-2}\) correction for cosmological distances is not included. Assuming that the RM dispersion causing the depolarization, \( \sigma_{\text{RM}} \), is constant with \( z \), we modify Equation (4) for cosmological sources as

\[ p = p_0(f_c \exp\left(-2\sigma^2_{\text{RM}}(1 + z)^{-4}\lambda^4\right) + (1 - f_c)). \]  
(5)

Thus, the width of \( p(\lambda) \) changes as a function of \( z \) as

\[ \sigma_p = \frac{(1 + z)^2}{2\sigma_{\text{RM}}}. \]  
(6)

This shows that depolarization by a non-evolving rest-frame Faraday dispersion screen is expected to decrease for screens at higher redshift. Below, we explicitly compute the depolarization by typical RM screens as a function of redshift of the sources.

4.2. A Simple Model

In our model, the Faraday screens consist of three components: (1) an inhomogeneous foreground screen local to the source with covering factor \( f_c_{\text{QSO}} \), (2) an inhomogeneous foreground screen in intervening galaxies with covering factor

\(^5\) The structure function \( D \) at scale \( s \) is defined as
\[ D(s) = \langle (\text{RM}(x + s) - \text{RM}(x))^2 \rangle \] and \( \langle \rangle \) stands for ensemble averaging.
$f_{c,Mg\,\text{II}}$ and (3) a homogeneous screen due to the Milky Way, which is assumed to be uniform across the extension on the sky of the polarized source.

The uniform Faraday screen in the Milky Way is set to a constant value, $RM_{MW} = -10$ rad m$^2$. This value is consistent with Schnitzeler (2010), who determined the Milky Way contribution of RMs at $|b| > 20\degree$ using the Taylor et al. (2009) data.

On the other hand, each inhomogeneous foreground Faraday screen is characterized by an ensemble of cells of size $l_0$ with uncorrelated RM values. The RM values have a Gaussian distribution with dispersion $\sigma_{RM}/(1 + z_s)^2$ and zero mean, where $\sigma_\text{RM}$ is the rest-frame RM dispersion and $z_s$ is the screen’s redshift, with $x = \text{QSO, Mg\,\text{II}}$ labeling the different screen types for which the above parameters may differ.

If the RM screen is at the source redshift and the source has linear size $s$, then it is covered by $N = s^2/l_0^2$ cells. Of these, only a fraction $f_{c,x}$ will be Faraday active for each screen. If the RM screen is at the redshift of the Mg\,\text{II} system, $s$ is the projected linear size of the source viewed from the Earth at the distance to the Mg\,\text{II} system.

Each screen is then fully described by a realization of $N$ RM values, a fraction $f_{c,x}$ of which are extracted from the associated Gaussian distribution to characterize the active cells while the reminder are set to zero to represent the inactive cells. Assuming a uniform flux of normalized intensity ($I = 1$) and uniform (zero) intrinsic polarization angle across the source surface, we can write the following relations for the remaining non-zero Stokes parameters $U$ and $Q$, the angle, and the degree of polarization, respectively:

$$U(\lambda^2) = \frac{1}{N} \sum_{i=1}^{N} \sin(2RM_i \lambda^2)$$

$$Q(\lambda^2) = \frac{1}{N} \sum_{i=1}^{N} \cos(2RM_i \lambda^2)$$

$$\chi(\lambda^2) = \frac{1}{2} \arctan \left( \frac{U}{Q} \right)$$

$$p(\lambda^2) = \sqrt{U^2 + Q^2},$$

where

$$RM_i = RM_{\text{QSO},i} + RM_{\text{Mg\,\text{II}},i} + RM_{MW}$$

are random variables determined by a Monte Carlo realization.

In Figure 4, we present generic results for three representative cases, namely a low-redshift ($z_{\text{QSO}} < 1$) and a high-redshift ($z_{\text{QSO}} > 1$) QSO with no intervening absorber in the top and middle panel, respectively, and a high-redshift QSO with intervening absorber in the bottom panel. More details are specified below. In all cases, the inhomogeneous screen comprises eight cells, $N f_{c,x} = 8$.

In each panel we plot the polarization angle, $\chi$ (black solid line, left $y$-axis) and theoretical polarization fraction, $p$ (red solid line, right $y$-axis) as a function of $\lambda^2$ for three models of inhomogeneous Faraday screens: a low-redshift (top) and a high-redshift (middle) QSO with no intervening absorbing system, and a high-redshift QSO with intervening absorber (bottom). The dashed line indicates the rotation in polarization angle due to the average RM within the beam, $\chi = RM_{avg} \lambda^2$, and the dash-dotted line that due to the Milky Way, $\chi = RM_{MW} \lambda^2$. The vertical blue dashed lines from left to right indicate the typical wavelengths of observations at 1.5 cm, at 6 cm (K08), at 20.89 and 21.96 cm (NVSS survey; Condon et al. 1998) for the calculated RM values of Taylor et al. (2009). In the top and middle panels the Faraday screens include an inhomogeneous screen local to the source with a covering factor $f_c = 0.5$ and a screen in the Milky Way that is uniform across the source extension. In the top panel, the source QSO is at $z < 1.0$ and has $\sigma_{RM,QSO}/(1 + z)^2 = 28$ rad m$^{-2}$; in the middle panel, the QSO is at $z > 1.0$ and has $\sigma_{RM,QSO}/(1 + z)^2 = 8$ rad m$^{-2}$. The bottom panel is the same as the middle panel with an additional Faraday screen due to an intervening system with $\sigma_{RM,Mg\,\text{II}}/(1 + z)^2 = 230$ rad m$^{-2}$ and $f_c = 0.5$. 

Figure 4. Angle of polarization, $\chi$ (black solid line, left $y$-axis) and theoretical polarization fraction, $p$ (red solid line, right $y$-axis) as a function of $\lambda^2$ for three models of inhomogeneous Faraday screens: a low-redshift (top) and a high-redshift (middle) QSO with no intervening absorbing system, and a high-redshift QSO with intervening absorber (bottom). The dashed line indicates the rotation in polarization angle due to the average RM within the beam, $\chi = RM_{avg} \lambda^2$, and the dash-dotted line that due to the Milky Way, $\chi = RM_{MW} \lambda^2$. The vertical blue dashed lines from left to right indicate the typical wavelengths of observations at 1.5 cm, at 6 cm (K08), at 20.89 and 21.96 cm (NVSS survey; Condon et al. 1998) for the calculated RM values of Taylor et al. (2009). In the top and middle panels the Faraday screens include an inhomogeneous screen local to the source with a covering factor $f_c = 0.5$ and a screen in the Milky Way that is uniform across the source extension. In the top panel, the source QSO is at $z < 1.0$ and has $\sigma_{RM,QSO}/(1 + z)^2 = 28$ rad m$^{-2}$; in the middle panel, the QSO is at $z > 1.0$ and has $\sigma_{RM,QSO}/(1 + z)^2 = 8$ rad m$^{-2}$. The bottom panel is the same as the middle panel with an additional Faraday screen due to an intervening system with $\sigma_{RM,Mg\,\text{II}}/(1 + z)^2 = 230$ rad m$^{-2}$ and $f_c = 0.5$. 

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where

\[
RM_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} RM_i \tag{12}
\]

\[
= RM_{\text{MW}} + \frac{1}{N} \sum_{i=1}^{N} RM_{\text{QSO},i} + RM_{\text{Mg}^\text{ii}}, \tag{13}
\]

and (2) by the Milky Way (dash-dotted line),

\[
\chi = RM_{\text{MW}} \lambda^2.
\]

Note that RM\text{avg} is, in general, non-zero and different from RM\text{MW}. This is because for a fixed fraction of active cells the second term in Equation (13) is a random number of amplitude \(1/\sqrt{N}\) with null probability of being either zero or \(-RM_{\text{MW}}\). Finally, from left to right, the vertical blue dashed lines indicate the typical observational wavelengths of 20 GHz (Jackson et al. 2010; Murphy et al. 2010), 5 GHz for K08, and 1.435 GHz and 1.365 GHz for the NVSS survey (Condon et al. 1998) and Taylor et al. (2009).

The top panel illustrates the case of a low-redshift source, \(z < 1.0\), with observed RM dispersion of \(\sigma_{\text{RM,QSO}}/(1 + z)^2 = 28 \text{ rad m}^{-2}\), \(f_{z,\text{QSO}} = 0.5\), and no intervening Mg\text{ii} absorber \((f_{z,\text{Mg}^\text{ii}} = 0)\). It can be seen that at short wavelengths, \(\lambda^2 < 0.014\), application of the simple \(\lambda^2\) relation in Equation (1) yields the average RM within the beam from which the RM distributions in the foreground screens can be inferred (Bernet et al. 2008). However, at longer wavelengths the flux through the inhomogeneous screen is depolarized so that the change in polarization angle is dominated by the contribution of the Milky Way. Correspondingly, the degree of polarization is given by \(P \sim 1 - f_{z,\text{QSO}}\).

The middle panel illustrates the case for a high-redshift source, \(z > 1.0\), with observed RM dispersion of \(\sigma_{\text{RM,QSO}}/(1 + z)^2 = 8 \text{ rad m}^{-2}\) and \(f_{z,\text{QSO}} = 0.5\). In this case, depolarization is negligible up to \(\lambda^2 = 0.05\) or at 1.4 GHz. This means that across most of the wavelength range considered application of the \(\lambda^2\) relation in Equation (1) would yield the average RM contributed by both the QSO screen and the Milky Way.

Finally, the bottom panel illustrates the same case as the middle panel but with an additional Faraday screen due to an absorber, with \(\sigma_{\text{RM,Mg}^\text{ii}}/(1 + z)^2 \sim 230 \text{ rad m}^{-2}\) and \(f_{z,\text{Mg}^\text{ii}} = 0.5\).

This case has a direct connection to the study of B08, who showed that intervening galaxies traced by Mg\text{ii} absorption lines contribute an additional Faraday screen with an observed rest-frame RM dispersion \(\sigma_{\text{Mg}^\text{ii}} = 140 \text{ rad m}^{-2}\). In the context of our simple model, this implies an RM dispersion of the individual cells of the screen, responsible for the depolarization, \(\sigma_{\text{RM,Mg}^\text{ii}} = \sigma_{\text{Mg}^\text{ii}}/\sqrt{N}/f_{z,\text{Mg}^\text{ii}}\). The value for \(\sigma_{\text{RM,Mg}^\text{ii}}/(1 + z)^2\) assumed above is based on this relation and the choice of \(N f_{z,\text{Mg}^\text{ii}} = 8\). The bottom panel of Figure 4 shows that with this choice of parameters the intervening screen causes significant depolarization above \(\lambda^2 \sim 0.003 \text{ m}^2\). In this case, the application of the \(\lambda^2\) relation at longer wavelengths yields the Milky Way RM value.

The approach taken here is valid in the limit of a large number of random cells. Using the total-flux–angular-diameter relation given by Windhorst et al. (1984), we can estimate the extent of the region of polarized emission. The median flux of our sources which are in the Taylor et al. (2009) catalog (371 sources) is 1.2 Jy. This translates to a median source size in the total flux of \(\sim 10\) arcsec. Since the polarized emission is likely to be dominated by some hot spots, the effective size in polarized emission is expected to be smaller than that. Given that the extent of the polarized emission is a few arcsec together with what we know about the scale of fluctuations of the RM (see next section), we expect a significant number of cells within the beam. In addition, using Equation (5) and future multiwavelength radio polarization observations one can determine \(\sigma_{\text{RM,Mg}^\text{ii}}\) and \(f_{z,\text{Mg}^\text{ii}}\) directly and thus calculate \(N\).

Finally, we have assumed a homogeneous RM screen in the Milky Way. Gaensler et al. (2005), however, showed that arcsec-scale fluctuation exists in the RM screen of the Large Magellanic Cloud and Havercorn et al. (2008) showed that there is RM fluctuation in the local interstellar medium. The inhomogeneous Galactic RM contribution, which might vary for different sight lines, would lead to additional depolarization and add scatter to the predicted trends (see Section 4.3) of our simple model, but cannot mimic them.

### 4.3. Model Predictions

Based on this toy model with the free parameters \(f_{z,\text{Mg}^\text{ii}}\) and \(\sigma_{\text{RM,Mg}^\text{ii}}\), we can predict simple trends in the observed angle and degree of polarization measured at fixed frequency as a function of the radio source’s redshift. For simplicity, we will assume there is no redshift evolution in the model parameters. We must also distinguish between the two cases with and without the Faraday screen provided by intervening galaxies.

For the case without intervening galaxies the model predicts the following.

1. \(p_{21}/p_{21 \to 0} \sim 1 - f_{z,\text{QSO}}\) toward low redshifts because the flux through the inhomogeneous Faraday screen is depolarized.

2. \(p_{21}/p_{21 \to 0} \approx 1\) toward high redshifts because the depolarization effects are suppressed. However, \(p_{21}/p_{21 \to 0}\) anti-correlates with \(\sigma_{\text{RM,QSO}}\) due to residual depolarization effects associated with the local screen.

3. The discrepancy \(\mid \text{RM}_{z \to 0} - \text{RM}_{21}\) increases toward low redshifts because, due to depolarization effects, \(\text{RM}_{z \to 0}\) measures \(\text{RM}_{\text{avg}}\) while \(\text{RM}_{21}\) is dominated by the Milky Way contribution.

4. The discrepancy \(\mid \text{RM}_{z \to 0} - \text{RM}_{21}\) decreases toward high redshifts because both \(\text{RM}_{z \to 0}\) and \(\text{RM}_{21}\) tend to measure the average RM\text{avg} (Equation (12)).

For the case with intervening galaxies the model predicts the following.

1. The discrepancy \(\mid \text{RM}_{z \to 0} - \text{RM}_{21}\) is large at all redshifts because, due to depolarization effects, \(\text{RM}_{z \to 0}\) measures \(\text{RM}_{\text{avg}}\) and \(\text{RM}_{21}\) is dominated by the Milky Way contribution.

2. \(p_{21}/p_{21 \to 0} \lesssim 1 - f_{z,\text{Mg}^\text{ii}}\) at all redshifts because the sources will be significantly depolarized.

### 5. DISCUSSION

#### 5.1. Nature of the Partial Inhomogeneous RM Screen

It is instructive to look at high-resolution Very Long Baseline Array (VLBA) radio polarization observations of some of our objects in K08. Examples of sources where VLBA...
polarization observations exist are 3C43 (Cotton et al. 2003), 3C118 (Mantovani et al. 2009), and B1524-026 (Mantovani et al. 2002). The typical resolutions of these observations are \( \sim 8 \times 8 \text{ mas}^2 \), which corresponds to a resolution of 64 pc \( \times 64 \text{ pc} \) at \( z \sim 1 \). Complex structures in polarization angles and RM maps are revealed at such a high resolution. Often it can be seen, e.g., B1524-026 (Mantovani et al. 2002), that there is a dominant compact component in the polarized flux and a more extended diffuse polarized component. Further, it can be seen that the diffuse component consist of many independent RM cells. That means that for unresolved observations at large wavelengths the diffuse component will cancel and the more compact component will dominate the observations.

For some sources it is possible that the number of cells \( f_c N \) in the RM screen is very low. In this case no depolarization is observed but \( p \) and \( \chi \) oscillate. This situation was observed for the source 3C27 by Goldstein & Reed (1984). Rossetti et al. (2008) fitted a two-component model to the data to describe \( p \) and \( \chi \) for the source B3 0110+401. Farnsworth et al. (2011) also used a two-component model to describe radio polarization observations at large wavelengths, \( \lambda \sim 1 \text{ m} \).

For sight lines with intervening galaxies, it is very natural to assume that the magnetic fields within them will lead to depolarization, an effect studied in the LMC by Gaensler et al. (2005). Using the Milky Way as a typical galaxy, we would expect depolarization of the background QSOs. This effect was observed by Fromalont et al. (1989) and Schulman & Fomalont (1992) for NGC 1310. One possibility is that the source covers both the spiral arms and interarm regions. For the spiral arms the coherence lengths of the magnetic fields are much shorter, as suggested by the observations of Haverkorn et al. (2008). These authors gave an outer scale of turbulence of 0.1 kpc for interarm regions and 10 pc for the spiral arms in the Milky Way.\(^4\) In this way, the area covered by the spiral arms would be depolarized and the interarms would produce a coherent screen. Other interpretations are again as for the QSO itself: the polarized emission is dominated by one compact component which competes with a more extended diffuse emission. If the size of the compact component is small enough, this component could get a coherent screen whereas the extended source gets different RM screens which will lead to depolarization of this component.

5.2. Rotation Measure Synthesis

There are currently several polarization surveys underway or planned, e.g., GALFACTS (Taylor & Salter 2010), LOFAR (Anderson et al. 2012), or POSSUM (Gaensler et al. 2010) within ASKAP. These polarization surveys should pave the way for the planned polarization survey for SKA (Gaensler et al. 2004; Beck & Gaensler 2004; Gaensler 2009). In particular, one wishes to study the evolution of magnetic fields in galaxies and in the intergalactic medium as a function of cosmic time using large RM data sets.

With the large number (e.g., a few thousands) of available spectral channels in these surveys, Faraday RM synthesis (Brentjens & de Bruyn 2005) can be performed. Using this technique one performs a Fourier transformation of the observed complex polarization \( p(\lambda^2) \) to obtain a Faraday depth spectrum \( F(\phi) \). Here the case of a single uniform Faraday screen corresponds to a delta function in Faraday depth space and in this case the Faraday depth is equal to the traditional RM (Equation (1)).

Basic parameters for a Faraday survey are the maximum observable Faraday depth, \( \Phi_{\text{max}} = \sqrt{3}/8\Delta \lambda^2 \), and the largest scale in Faraday depth that one is sensitive to, \( \Phi_{\text{max}} \approx 2\sqrt{3}/\Delta \lambda^2 \). Here \( \Delta \lambda^2 \) is the channel width, \( \Delta \lambda^2 \) is the total bandwidth, and \( \lambda_{\text{min}} \) is the smallest wavelength (squared) of the observations. Regions which produce only Faraday rotation but do not emit polarized radiation can be described by Dirac \( \delta \) functions in \( \phi \)-space and are Faraday-thin sources (Brentjens & de Bruyn 2005). Below we discuss our model of the depolarization of QSOs in terms of Faraday RM synthesis. This is to provide some basic comparisons for ongoing or future RM surveys. See, e.g., O’Sullivan et al. (2012) for current observations of Faraday depth spectra.

Next, with the help of Figures 5–7, we give a simple example of the application of RM synthesis to the same cases studied in Figure 4; this illustrates the advantages of this emerging technique. In Figure 5, the original Faraday spectrum for a low-redshift QSO without intervening galaxies is shown. Here the Monte Carlo realization is the same as in Figure 4 (upper panel) with adopted parameters \( \sigma_{\text{RM, QSO}}/(1+z)^2 = 28 \text{ rad m}^{-2} \) and \( f_c, \text{QSO} = 0.5 \). In particular, all of the 16 cells have a contribution from the Milky Way and eight cells have an additional intrinsic low-\( z \) QSO contribution. The Faraday spectrum is only real-valued, which means that the intrinsic polarization angles are zero. The adopted minimum and maximum wavelengths are \( \lambda_{\text{min}} = 0.03 \text{ m} \) and \( \lambda_{\text{max}} = 0.27 \text{ m} \) and correspond to a Faraday depth resolution of \( \delta \phi \approx 48.1 \text{ rad m}^{-2} \). The chosen wavelength range corresponds to the wavelength range offered at the Australia Telescope Compact Array, which is similar to the EVLA (Beck et al. 2012). The recovered Faraday spectrum with this resolution is shown in the lower left panel.

There are several methods to recover the information lost due to the incomplete wavelength coverage (Heald et al. 2009; Li et al. 2011). We use here the compressive sampling (CS) method of Li et al. (2011). The best results are obtained if prior information about the Faraday spectrum is present, e.g., if the sources are Faraday thin or thick. The Faraday spectrum obtained after applying the CS–RM–thin method of Li et al. (2011) to the \( Q \) and \( U \) vectors (upper middle panel) is shown in the lower right panel. It can be seen that while the homogeneous RM component at \( -10 \text{ rad m}^{-2} \) can be approximately recovered, this is not necessarily true for the inhomogeneous RM components.

In Figure 6, the Faraday spectrum for a high-redshift QSO with an intervening galaxy is shown. Here the Faraday spectrum is identical to the one used in Figure 5 in the lower panel with the parameters \( \sigma_{\text{RM, MgII}}/(1+z)^2 \approx 230 \text{ rad m}^{-2} \) and \( f_c, \text{MgII} = 0.5, \sigma_{\text{RM, QSO}}/(1+z)^2 \approx 8 \text{ rad m}^{-2} \) and \( f_c, \text{QSO} = 0.5 \). In the Monte Carlo realization shown in Figure 6 all of the 16 cells have a contribution from the Milky Way, 8 (random) cells have an additional intrinsic high-\( z \) QSO contribution, and 8 (random) cells have an additional contribution from the intervening galaxy. With the observational parameters \( \lambda_{\text{min}} = 0.03 \text{ m} \) and \( \lambda_{\text{max}} = 0.27 \text{ m} \) and \( \delta \phi \approx 48.1 \text{ rad m}^{-2} \) the MW+QSO components cannot be resolved but lead to a distinct peak at \( \phi \sim -10 \text{ rad m}^{-2} \) with an amplitude \( \sim 0.45 \text{ m}^2 \text{ rad}^{-1} \) in the recovered Faraday spectrum.

\(^4\) The small coherence lengths of the magnetic fields are probably connected with our point of view in the disk of the Milky Way. We know, of course, from observations of nearby spiral galaxies that there is a magnetic field component which is coherent over several kpc.
On the other hand, the eight RM components with contributions from intervening galaxies can be well resolved with the assumed wavelength range covered.

For comparison, in Figure 7 we show the recovered Faraday spectrum for observations using longer wavelengths, $\lambda_{\text{min}} = 0.27 \, \text{m}$ and $\lambda_{\text{max}} = 0.38 \, \text{m}$ but with the same $\Delta \lambda^2$. This shows that the qualitative features of the Faraday spectrum are also recovered, although with a lower quality than in Figure 6.

In summary, from Section 4.3 using polarization angle $\chi$ versus $\lambda^2$ observations one might conclude that it is best to perform observations at short wavelengths (below the exponential falloff) in order to measure the average RM. However, as we have illustrated above using RM synthesis this is not necessary. In order to be able to resolve the individual RM components, a large $\Delta \lambda^2$ is required. (See Beck et al. (2012) for a summary of the Faraday depth resolution $\delta \Phi$ of current and future radio telescopes.)

The proposed wavelength range for SKA with $\lambda \sim 0.03-4.3 \, \text{m}$, $\Delta \lambda^2 \sim 18 \, \text{m}^2$, and $\delta \Phi \sim 0.2 \, \text{rad} \, \text{m}^{-2}$ will offer a superb Faraday resolution to resolve individual RM components in intervening galaxies.
6. SUMMARY AND CONCLUSIONS

In previous work we have used RM values from K08, typically determined at 6 cm, to study magnetic fields in normal galaxies at high redshift. When the analysis was repeated using the recently released RM data of TSS09, determined at 21 cm, we could find neither a correlation between |RM| and absorbing systems as in B08 nor an increase in the RM dispersion with \( z \) as found in K08. Motivated by the above results, we have attempted to understand those differences in terms of a simple model based on inhomogeneous Faraday screens associated both with the QSO and the Mg \( \text{II} \) host galaxies, and for both low and high redshifts.

We find that the presence of inhomogeneous screens leads to important departures from the classical \( \lambda^2 \) dependence of the rotation of the polarization angle. Related to this are depolarization effects which become stronger toward higher wavelengths. As a result, due to the cosmological redshift, observations at fixed wavelength are affected differently by depolarization and are sensitive to different Faraday-active components. In particular, depolarization effects become stronger for lower redshift QSOs. We find that the depolarization saturates to a value given by the intrinsic polarization times the complement of the covering factor of the inhomogeneous screen, \((1 - f_c)\).

Actual predictions depend on the assumed values for the model parameters. The following results apply for the choices made in Section 4, which are relevant for the current investigation. When the line of sight to the QSO is free of absorption systems, application of the \( \lambda^2 \) analysis to extract RM values from radio observations then has the following consequences.

1. For low-redshift QSOs, \( R_{M21} \) is dominated by the Milky Way contribution, while \( R_{M_{\lambda \rightarrow 0}} \) measures \( R_{M\text{avg}} \) (see Equation (12)).
2. For high-redshift QSOs, in general the RM value reflects \( R_{M\text{avg}} \).

The presence of absorption systems contributes an additional Faraday screen which further depolarizes the low-frequency radiation. This has the following consequence.

3. While \( R_{M_{\lambda \rightarrow 0}} \) is given by \( R_{M\text{avg}} \), \( R_{M21} \) is dominated by the Milky Way contribution and the discrepancy \( |R_{M_{\lambda \rightarrow 0}} - R_{M21}| \) is large at all redshifts.

In conclusion, while the model is admittedly simple, it seems plausible to consider that the discrepancy between results based on K08 and TSS09 RMs are due to the severe depolarization induced by an inhomogeneous Faraday screen on high-wavelength radiation. This, however, does not exclude the importance of other effects.

Finally, the study of magnetic fields in galaxies as a function of cosmic time will benefit considerably from the application of RM synthesis, which has the power to disentangle the contribution from inhomogeneous magnetoactive components. For this purpose, instrumental bandwidth is most important, although higher frequency channels appear to deliver higher quality. These conclusions may be refined with future investigations.

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