Tidally–induced warps in protostellar discs

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Abstract. We review results on the dynamics of warped gaseous discs. We consider tidal perturbation of a Keplerian disc by a companion star orbiting in a plane inclined to the disc. The perturbation induces the precession of the disc, and thus of any jet it could drive. In some conditions the precession rate is uniform, and as a result the disc settles into a warp mode. The tidal torque also leads to the truncation of the disc, to the evolution of the inclination angle (not necessarily towards alignment of the disc and orbital planes) and to a transport of angular momentum in the disc. We note that the spectral energy distribution of such a warped disc is different from that of a flat disc. We conclude by listing observational effects of warps in protostellar discs.

1. Introduction

1.1. Observations of pre–main sequence binary systems

In the course of the past few years, accretion discs around low–mass pre–main sequence stars known as T Tauri have been directly imaged by the HST. As expected, they appear to be common (Stauffer et al. 1994).

Furthermore, most T Tauri stars are observed to be in multiple systems (see Mathieu 1994 and references therein). In some of these systems, one or even two circumstellar discs are resolved or inferred from the spectral energy distribution of the stars. This is the case for UY Aurigae (Duvert et al. 1998, Close et al. 1998), T Tau (Akeson et al. 1998), HK Tau (Stapelfeldt et al. 1998, Koresko 1998), although in that case it is not certain whether the two stars are bound or not, and GG Tau (Roddier et al. 1996). Very recently, it has been announced that two circumstellar discs made of dust have been imaged by the VLA in a binary located in Taurus (L. F. Rodriguez et al. 1998, press release). The binary separation is about 45 AU, and the radius of each disc is on the order of 10 AU.

There are indications that the plane of at least one of the circumstellar discs and that of the orbit may not necessarily be aligned. The most striking evidence for such non–coplanarity is given by HST and adaptive optics images
of HK Tau (Stapelfeldt et al. 1998, Koresko 1998). The binary system T Tau is also very likely to be non–coplanar, because two bipolar outflows of very different orientations originate from this system (Böhm & Solf 1994). Since it is unlikely that they are both ejected by the same star, each of them is more probably associated with a different member of the binary. Furthermore, jets are usually thought to be ejected perpendicularly to the disc from which they originate. These observations then suggest that discs and orbit are not coplanar in this system. More generally, observations of molecular outflows in star forming regions commonly show several jets of different orientations emanating from an unresolved region the extent of which can be as small as a few hundreds astronomical units (Davis et al. 1994).

1.2. Tidal interactions in binary systems

There is evidence that the sizes of circumstellar discs contained within binary systems are correlated with the binary separation (Osterloh & Beckwith 1995, Jensen et al. 1996). This suggests that binary companions are responsible for limiting the sizes of the discs through tidal truncation (see, e.g., Papaloizou & Pringle 1977, Paczyński 1977, Artymowicz & Lubow 1994).

The tidal effect of an orbiting body on a differentially rotating disc has been well studied in the context of planetary rings (Goldreich & Tremaine 1978), planet formation and interacting binary stars (see Lin & Papaloizou 1993 and references therein). In these studies, disc and orbit are usually taken to be coplanar, so that the only waves excited in the disc by the perturbing companion are density waves.

However, as mentioned above, the plane of at least one of the circumstellar discs may not be aligned with that of the orbit. If this is the case, the companion’s perturbation excites both density and bending waves in the disc and induces it to warp. Here we will focus on the effects of this warping and the propagation of bending waves. We note that when the perturbation is linear, the effects of density waves may be superimposed on those of bending waves.

The discs we consider here are gaseous. The work we present should then be contrasted with studies of warping of purely viscous discs in which pressure and self–gravity are ignored (Bardeen & Petterson 1975, Katz 1980, Steiman–Cameron & Durisen 1988, Pringle 1996). In a purely viscous disc, evolution occurs only through viscous diffusion, whereas in a gaseous disc pressure effects manifesting themselves through bending waves can control the disc evolution (see also the chapter by Nelson et al. in this book). When these waves propagate, they do so on a timescale comparable to the sound crossing time (Papaloizou & Lin 1995). Since this is much shorter than the viscous diffusion timescale, communication through the disc occurs as a result of the propagation of these waves. Even when the waves are damped before reaching the disc boundary, a diffusion coefficient for warps that is much larger than that produced by the kinematic viscosity may occur because of pressure effects (Papaloizou & Pringle 1983).

2. Perturbing potential

We consider a binary system in which the primary has a mass \( M_p \) and the secondary has a mass \( M_s \). The binary orbit is assumed circular with radius \( D \).
Figure 1. Sketch of the binary system.

We suppose that the primary is surrounded by a disc of radius $R \ll D$ with negligible mass so that precession of the orbital plane can be neglected. The secondary describes a prograde Keplerian orbit about the primary with angular velocity $\omega$. We define a Cartesian coordinate system $(x, y, z)$ centered on the primary star, where the $z$-axis is normal to the initial disc mid-plane. We also define the associated cylindrical polar coordinates $(r, \varphi, z)$. The orbit of the secondary star is in a plane which has an initial inclination angle $\delta$ with respect to the $(x, y)$ plane. This situation is illustrated in Figure 1.

We are interested in disc warps which are excited by terms in the perturbing potential which are odd in $z$ and have azimuthal mode number $m = 1$ when a Fourier analysis in $\varphi$ is carried out. There are three terms in the perturbing potential with the required form. One of them is secular (time independent), while the two others, with frequency $2\omega$ and $-2\omega$, are prograde and retrograde, respectively (see Papaloizou & Terquem 1995 for more details).

3. Precession of warped discs
3.1. Theory

Hunter & Toomre (1969) showed that an isolated self–gravitating disc subject to a vertical displacement generally precesses differentially, and thus cannot sustain a warped configuration. However, differential precession is prevented by gravitational torques from the distorted disc itself if the disc has a sharp edge. A similar process can occur if the disc orbits in the external potential due to a companion (Hunter & Toomre 1969) or a flattened halo whose equatorial plane is misaligned with the disc plane (Toomre 1983, Sparke 1984, Sparke & Casertano 1988).

Papaloizou & Terquem (1995) have shown that radial pressure forces are also able to smooth out differential precession in a non self–gravitating inviscid accretion disc. This process can be effective if the sound crossing time through the disc is much smaller than the precession period. The condition for rigid body precession is then $H/r > |\omega_p|/\Omega_o$, where $\omega_p$ is the (uniform) precession frequency, $H$ is the disc semi–thickness and $\Omega_o$ is the angular velocity at the disc outer edge. When this condition is satisfied, bending waves are able to propagate through the disc sufficiently fast so that the different parts of the disc can “communicate” with each other and adjust their precession rate to a constant value. This also happens when viscosity is present, but the communication becomes diffusive rather than wave–like when the Shakura & Sunyaev (1973) viscosity parameter $\alpha$ significantly exceeds $H/r$ (Papaloizou & Pringle 1983). However, in protostellar discs, we expect $\alpha < H/r$ so that, assuming the effective viscous stress tensor is isotropic, disc communication is governed by bending waves.

The disc precesses because, in a non–rotating frame, the component of the secular torque along the direction of the line of nodes is non–zero. The precession frequency $\omega_p$ can then be calculated from the condition that, in a rotating frame, this component of the torque is balanced by the Coriolis torque. This leads to (Papaloizou & Terquem 1995; see also Kuijken 1991 in the context of galactic discs):

\[
\frac{\omega_p}{\Omega_o} = -\frac{3}{4} \frac{M_s}{M_p} \left( \frac{\omega}{\Omega_o} \right)^2 \cos \delta \int_{r_{in}}^{R} \frac{\Sigma}{(\Omega/\Omega_o)^2} dr \int_{r_{in}}^{R} \frac{\Sigma}{\Omega/\Omega_o} dr ,
\]

where $r_{in}$ is the disc inner radius, $\Omega$ is the angular velocity in the disc, and $\Sigma$ is the disc surface mass density.

Although the above assumes a circular orbit, an eccentric binary orbit can be considered by replacing $D$ by the semi-major axis and multiplying the precession frequency by $(1 - e^2)^{-3/2}$, with $e$ being the eccentricity.

The analysis used to derive equation (1) is accurate only when $|\omega_p|/\Omega_o$ is smaller than the maximum of $H^2/r^2$ and $\alpha$.

We can approximate $\omega_p$ by assuming that $\Sigma$ is constant throughout the disc and that the rotation is Keplerian, so that $\Omega = \sqrt{GM_p/r^3}$. Equation (1) can then be written in the form:

\[
\omega_p = \frac{15}{32} \frac{M_s}{M_p} \left( \frac{R}{D} \right)^3 \cos \delta \sqrt{\frac{GM_p}{R^3}}.
\]
We note that even though $\omega_p \propto \cos \delta$, the amplitude of the precessional motion vanishes when $\delta = 0$.

These results are completely supported by the non-linear, three-dimensional hydrodynamic simulations of the tidal perturbation of accretion discs in non-coplanar binary systems performed by Larwood et al. (1996), using a SPH code (see also the chapter by Nelson et al. in this book). They found that the disc tends to precess approximately as a rigid body if it is not too thin, and the precession period they derived agree well with the linear estimate given above. These simulations also show that extremely thin discs are severely disrupted by differential precession. For a binary mass ratio around unity, $D/R$ in the range 3 to 4, and $\alpha \sim 0.03$ corresponding to the dissipation in the code used here, the crossover between obtaining a warped disc structure and disc disruption appears to occur for values of $H/r \sim 0.033$.

We also point out that, according to these simulations, tidal truncation operates effectively when the disc and the binary orbit are not coplanar, being only marginally affected by the lack of coplanarity. This is consistent with the observations of HK Tau (Stapelfeldt et al. 1998, Koresko 1998).

3.2. Observational tests: precessing jets

Observations of molecular outflows in star-forming regions show in some cases knots forming a helical pattern or 'wiggling' which can be interpreted as being the result of the precession of the jet (see, e.g., Bally & Devine 1994, Eislöffel et al. 1996, Davis et al. 1997, Mundt & Eislöffel 1998). Assuming that such precession is caused by tidal interaction between the disc from which the outflow originates and a companion star in a non-coplanar orbit, Terquem et al. (1998) have estimated the separation of the binary for several systems. The numbers they found are characteristic of those expected for pre-main sequence stars.

Also, as mentioned in the introduction, jets with very different orientations are commonly observed in star-forming regions, and it is generally assumed that they originate from a binary where the discs are misaligned. For some systems, a binary has actually been resolved. It is the case for T Tau, which is a binary where a circumstellar disc has been resolved (Akeson et al. 1998) and from which two almost perpendicular well collimated outflows originate (Böhm & Solf 1994). It is also the case for HH 24, which is a hierarchical system with 4 or even 5 stars and several outflows (Eislöffel, personnel communication). For these systems, Terquem et al. (1998) have evaluated the precession frequency and given an estimate of the length-scale over which the outflows should 'wiggle' as a result of this precessional motion.

4. Warp mode and disc deformation

When the disc can precess as a rigid body under the action of the tidal force, it settles into a discrete bending mode (representing a warp) which is referred to as the modified tilt mode because in the limit that the external potential is spherically symmetric it reduces to the trivial rigid tilt mode. The difference in shape and frequency between the rigid and modified tilt modes is due to the fact that, in a non-spherically symmetric potential, the disc has to bend to alter the precession frequency at each radius so that the rate is everywhere
the same (Sparke & Casertano 1988). This asymptotic state is possible because bending waves transport away the energy associated with the transient response (Toomre 1983). The timescale for settling to the warp mode is then given by the characteristic time for bending waves to propagate through the disc but the process may be slowed down in the low surface density regions near the outer edge.

When pressure is present, the waves may be reflected from the edge before they attain arbitrary short wavelength there. Some dissipation is then needed for the disc to settle into a state of near rigid precession. This can be provided by disc viscosity which is least effective on a global disturbance such as near rigid body precession. Other transient disturbances would have shorter wavelengths and accordingly would be expected to dissipate significantly more rapidly. In this context note that a warp mode which deviates only slightly from a rigid tilt has a wavelength which is much larger than the disc. In the simulations of Larwood et al. (1996), of non self–gravitating inclined discs with pressure and viscosity, the disc was seen to quickly settle into a state of near rigid body precession. The time for this to happen was consistent with about $\sim 1/\alpha$ orbital periods, this being the decay time of short wavelength bending waves without self–gravity (see Papaloizou & Lin 1995).

Since the analysis we have performed and we report in this chapter is linear, its domain of validity is limited (see Terquem 1998 for a summary). We are interested in protostellar discs in which we expect $\alpha < H/r$ (we assume here that the effective viscous stress tensor is isotropic). In these conditions, the secular perturbation produces a tilt the variation of which across the disc can be up to $H^2/r^2$ in the direction perpendicular to the line of nodes and $\alpha$ (or $H^2/r^2$ if the disc is inviscid) along the line of nodes. Superimposed on this tilt, there is another tilt produced by the finite frequency perturbations, the variation of which across the disc can be up to $H/r$. Such deformations were observed in the SPH simulations performed by Larwood & Papaloizou (1997), in which $\alpha > H^2/r^2$.

We note that since in protostellar discs $H/r \sim 0.1$, the variation of the vertical displacement across the disc due to the finite frequency perturbations can be as large as about a tenth of the disc radius while the perturbation remains linear.

5. Evolution of the inclination angle

In section 3 we have described the effect of the component of the secular tidal torque along the direction of the line of nodes. We now discuss the effect of the component of this torque along the axis perpendicular to the line of nodes in the disc plane. It induces the evolution of the inclination angle of the disc plane with respect to that of the orbit.

Papaloizou & Terquem (1995) have found that this evolution does not necessarily tend to align the disc plane with that of the orbit. The situation were these planes are aligned may indeed be unstable.

They have also shown that, in a non–self gravitating disc, this evolution occurs on a timescale related to the rate at which angular momentum is transferred from the disc rotation to the orbital motion. This conclusion might be expected
since the disc inclination as a whole can change only if angular momentum is transferred between different parts of the disc and then between the disc and the companion’s orbit. In the case we consider here, angular momentum exchange between the disc rotation and the orbital motion takes place as a result of the viscous shear stress, which induces a lag between the response of the disc and the perturbing potential (see next section). Therefore we expect the timescale for the evolution of the inclination angle to be at least the disc viscous timescale.

This was seen in the numerical simulations of Larwood et al. (1996), in which the disc inclination evolved much more slowly than the precession rate. We then conclude that the probability of observing a warped disc, if binary systems are not all coplanar when they form, may be significant.

We emphasize that these results only hold if the disc is able to find a state in which its precesses as a rigid body. Settling toward a preferred orientation is indeed much faster if differential precession is so important that it cannot be controlled by pressure (or self–gravitating) forces (Steiman–Cameron & Durisen 1988).

6. Angular momentum transport associated with the warp

So far we have discussed the effect of the components of the secular torque in the disc plane. We now consider the effect of the component of the torque (secular or not) parallel to the disc rotation axis $z$. It does not modify the direction of the disc angular momentum vector but its modulus, and thus changes the angular momentum content of the disc. As a result of this torque, angular momentum is exchanged between the disc rotation and the orbital motion.

If the disc is inviscid and does not contain any corotation resonance, the nature of the boundaries determines whether such an exchange takes place or not, i.e. whether the $z$–component of the tidal torque is finite or zero (see Lin & Papaloizou 1993 for example). Because of the conservation of wave action in an inviscid disc, the tidal waves excited at the outer boundary propagate through the disc with an increasing amplitude if the disc surface density increases inwards or is uniform. It is usually assumed that they become non linear before reaching the center and are dissipated through interaction with the background flow. Thus, the inner boundary can be taken to be dissipative. This introduces a phase lag between the perturber and the disc response, enabling a net torque to be exerted by the perturber. This torque is transferred to the disc through dissipation of the waves at the boundary. Because of the conservation of angular momentum, the net torque is equal to the difference of angular momentum flux through the disc boundaries. This flux is constant (independent of $r$) inside the disc, since there is no dissipation there. We note that in these conditions only the finite frequency terms (not the secular term) produce a torque. Whenever the perturber rotates outside the disc, the torque exerted on the disc is negative. Through dissipation of the waves, the disc then loses angular momentum. Papaloizou & Terquem (1995) have calculated this torque in the context of pre–main sequence binaries. They found that $m = 1$ bending waves can lead to the accretion of the disc on a timescale which, if it were written as a viscous timescale, would correspond to an $\alpha$ parameter smaller than $10^{-4}$. This upper limit was actually reached only in extreme cases, and the equivalent $\alpha$ is more
likely to be at least one order of magnitude smaller. We note that of course transport of angular momentum by waves does not produce an \( \alpha \)–type disc, since energy is not dissipated locally but transported by the waves away from the location where it has been extracted from the disc.

We remark that bending waves are more efficient at transporting angular momentum in the disc than density waves, because they have a longer wavelength (Papaloizou & Lin 1995). For the same reason they can also affect the disc at smaller radii than density waves.

The situation is different from that described above when a corotation resonance is present in the disc, since this singularity provides a location where angular momentum can be absorbed or emitted (Goldreich & Tremaine 1979). However, there is no such resonance in the cases we consider here.

When the disc is viscous, its response is not in phase with the perturber. A net tidal torque is then exerted on the disc even if the boundaries are reflective, and the angular momentum flux inside the disc is not constant, since the perturbed velocities are viscously dissipated. In that case, both the secular and the finite frequency perturbing terms produce a torque. Terquem (1998) has calculated the tidal torque in this context and has found that it can be comparable to the horizontal viscous stress acting on the background flow when the perturbed velocities in the disc are on the order of the sound speed. If these velocities remain subsonic, the tidal torque can exceed the horizontal viscous stress only if the viscous stress tensor is anisotropic with the parameter \( \alpha \) which couples to the vertical shear being larger than that coupled to the horizontal shear. We note that, so far, there is no indication on whether these two parameters should be the same or not. When the perturbed velocities become supersonic, shocks reduce the amplitude of the perturbation such that the disc moves back to a state where these velocities are smaller than the sound speed (Nelson & Papaloizou 1998, see also the chapter by Nelson et al. in this book). When shocks occur, the tidal torque exerted on the disc may become larger than the horizontal viscous stress. Terquem (1998) also found that if the waves are reflected at the center, resonances occur when the frequency of the tidal waves is equal to that of some free normal global bending mode of the disc. If such resonances exist, tidal interactions may then be important even when the binary separation is large. Out of resonance, the torque associated with the secular perturbation is generally much larger than that associated with the finite frequency perturbations. As long as the waves are damped before they reach the center, the torque associated with the finite frequency perturbations does not depend on the viscosity, in agreement with theoretical expectation (Goldreich & Tremaine 1982).

7. Effect of the warp on the spectral energy distribution of the disc

The spectral energy distribution of a circumstellar disc can be significantly affected by a warp because reprocessing of radiation from the central star by a disc depends crucially on the disc geometry.

Terquem & Bertout (1993) have shown that T Tauri stars with tidally warped circumstellar discs may display far–infrared and submillimetric flux well in excess of that expected from flat circumstellar discs. The excess oc-
curs at relatively long wavelength because the warp affects mostly the outer, low-temperature regions of the disc, at least when the perturbation is not too severe.

Terquem & Bertout (1996) produced a broad variety of synthetic spectral energy distributions of warped discs, and compared them with actual observations. They found that they could reproduce the spectral energy distribution of a T Tauri star with infrared excess (Class II source), that similar to GW Ori with a double-peak or even that of a Class I source with not unrealistic, though a bit extreme, warp and disc parameters. The claim is of course not to provide an explanation for all these non-standard spectral energy distributions in terms of a warped disc. However, it appears that tidal interaction in T Tauri binary systems with intermediate separations may play a role in shaping the spectral energy distributions of these stars.

8. Observational effects of warps in protostellar discs

We have already mentioned above that the precession of jets and a non-standard spectral energy distribution would be an observational effect of warped discs. As far as precessing jets are concerned, we hope to be able to compare some predictions of our models with observations in the near future.

Also, because the variation of the tilt angle along the line of nodes and along the perpendicular to the line of nodes depend on $\alpha$ and $H/r$, respectively, observations of warped protostellar discs have the potential to give important information about the physics of these discs.

Protostellar discs are believed to be rather thick, i.e. $H/r \sim 0.1$. In addition, $\alpha$ is believed to be at most on the order of $10^{-3} - 10^{-2}$. We then expect bending waves to propagate on a relatively short timescale across the disc. It may then be possible to observe some time-dependent phenomena with a frequency equal to that of these waves, i.e. twice the orbital frequency. Such a phenomenon may actually have been detected in the X-ray binary SS 433, under the form of a “nodding motion” (Margon 1984).

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