Harmless R-parity violation from $Z_{12-I}$
compactification of $E_8 \times E_8'$ heterotic string

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Abstract

In a recent $Z_{12-I}$ orbifold model, an approximate $Z_2$ symmetry which forbids the baryon number violating operators up to sufficiently high orders is found. The dimension-4 $\Delta B \neq 0$ operators of the MSSM fields occur at dimension 10. The effective dimension-5 $\Delta B \neq 0$ operators derived from these are harmless if some VEVs of neutral singlets are $O(\frac{1}{10})$ times the string scale. The main reason for forbidding these $\Delta B \neq 0$ operators up to such a high order is the large order $N = 12$ of $Z_N$ since the $H$-momentum rule is $(-1,1,1)$ mod $(12,3,12)$. For a lower order $N < 12$, the $\Delta B \neq 0$ operators would appear at lower dimensions.

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I. INTRODUCTION

The main reason for imposing the R-parity in the minimal supersymmetric standard model (MSSM) is to forbid the dangerous $\Delta B \neq 0$ superpotential terms. As a bonus, the exact R-parity ensures an absolutely stable lightest supersymmetric particle (LSP) as a candidate for cold dark matter. The R-parity is a simple discrete symmetry in the MSSM to forbid dangerous (renormalizable) $\Delta B \neq 0$ operators. However, as for the condition on proton longevity, other discrete symmetries in addition to the R-parity can be possible. All possible candidates are classified in Refs. [1, 2]. In this paper, we search for a scheme to obtain such a discrete symmetry in compactifications of $E_8 \times E'_8$ heterotic string.

The well known R-parity in SO(10) grand unified theory (GUT) is by assigning $-1$ for the spinor $16$ and $+1$ for the vector $10$. This kind of spinor-vector disparity can be adopted in the untwisted sector of heterotic string also, in particular in the phenomenologically attractive $E_8 \times E'_8$ heterotic string [3]. Let us consider only the $E_8$ part for an illustration. The untwisted sector massless matter spectrum in $E_8$ can be $P^2 = 2$ weights distinguished by the spinor or the vector property

$$S : ([+ + + + + + + +]) \quad V : (\pm 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

where $\pm$ represents $\pm \frac{1}{2}$, the notation $[\ ]$ means including even number of sign flips inside the bracket, and the underline means permutations of the entries on the underline. Since two spinors in the group space can transform as a tensor in the group space, cubic Yukawa couplings arising from the untwisted sector involving two spinors are of the form $SSV$, which can be used to assign a kind of matter parity. For this scheme to work, 48 fermions of the three families (including the singlet neutrinos to generate neutrino masses) must belong to the untwisted sector $S$ and Higgs doublets must belong to the untwisted sector $V$. This requirement is nontrivial as one finds that the standard-like models of [4] do not satisfy this condition. On the other hand, a part of the spectrum of the $Z_{12}$ model of [5] satisfies this condition. But this spinor-vector disparity condition alone is not enough to guarantee a harmless discrete symmetry in supersymmetric standard models because there are twisted matter, and Yukawa couplings are more constrained than this simple spinor–vector disparity.

The twisted matter is more complicated because of the form (internal momenta) plus (shift vectors), $P + kV$ ($k = 1, 2, \cdots, N - 1$ in a $Z_N$ orbifold). Except in the $Z_2$ orbifold,
there appear fractional numbers such as \( \frac{1}{3}, \frac{1}{4}, \cdots \). So, in general it is very much involved to find a discrete parity if one includes interactions of twisted matter.

GUTs allow harmless proton decay via gauge interactions if the GUT scale \( M_{\text{GUT}} \) is greater than \( 10^{15} \) GeV. String models in addition contain proton decay operators through Yukawa couplings which are measured by the string scale \( M_S \) being considered to be \( O(10) \) larger than the GUT scale. If these proton decay operators occur at dimension 4 and 5, they can dominate over the proton decay operators via gauge interactions \[6\]. The R-parity forbids dimension-4 \( \Delta B \neq 0 \) operators, but does not forbid dimension-5 \( \Delta B \neq 0 \) operators. So, in some supergravity GUT models dimension-5 \( \Delta B \neq 0 \) operators are considered to be the dominant ones of proton decay, predicting \( p \to K + (\text{antilepton}) \).\footnote{One of simple ways to forbid the dimension-4 and -5 \( \Delta B \neq 0 \) operators in supergravity is to introduce a \( U(1) \) R-symmetry. However, it should be broken to a discrete symmetry in orbifold string compactifications. It is known that in a supergravity model \[2\], a \( Z_6 \) symmetry also forbids such \( \Delta B \neq 0 \) operators. In Ref. \[7\], for instance, an anomalous \( U(1)_{\text{an}} \) is employed for the R-parity and also for the proton longevity.}

In string compactifications, however, the dimension-5 operators are considered to be dangerous because the coefficients are considered to be \( O(1) \) in general \[1\]. So, if we introduce a kind of matter parity, it must work very ingeniously to forbid the dimension-4 and dimension-5 \( \Delta B \neq 0 \) operators.

If a parity is introduced, it is better to be a discrete gauge symmetry \[8\], otherwise large gravitational corrections such as through wormhole processes may violate it. Even if the discrete symmetry is broken as we consider in this paper, it is better to be a discrete gauge symmetry to free us from gravitational corrections. The reason for anticipating a broken parity in string compactifications is that we will embed the parity in a global \( U(1) \) which is not an exact symmetry in string compactifications. But the breaking of the parity will be considered to be \textit{harmless} if the \( \Delta B \neq 0 \) operators derived from breaking that parity is sufficiently suppressed or suppressed by masses greater than \( 10^{17} \) GeV since anyway \( \Delta B \neq 0 \) operators are present in the gauge sector of string GUTs.

Suppose an approximate global symmetry \( U(1)_\Gamma \) and its discrete \( Z_2 \) subgroup \( P_\Gamma \). It is a kind generalizing the R-parity. In this paper, we use the word ‘R-parity’ even though we restrict the discussion to the matter parity.

The parity we consider cannot be put in a general form but must be discussed based on specific models. Restricting to specific models is obvious because the approximate global
TABLE I: Relevant visible sector chiral fields. The multiplicity is shown as the coefficients of representations. + and – represent $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. The $\Gamma$ values of $-2$ and $-1$ for $10_{-1}^L(T6)$ are those for the vectorlike ones and for the $t$-quark family, respectively.

symmetry $U(1)_\Gamma$ must be given in a specific model. This leads us to the discussion of the specific model, Ref. [5].

II. CONTINUOUS U(1) SYMMETRIES

We observe that a discrete subgroup of the anomalous $U(1)_{\text{an}}$ of Ref. [3] is not good for the $R$-parity because neutral singlets carry even and odd $U(1)_{\text{an}}$ charges. A good candidate for housing a part of $R$-parity is $U(1)_X$ of flipped SU(5). In Table II we list $X$ charges of the non-exotic fields as subscripts. We do not list exotic fields and $X=0$ singlet fields.

The key representations of the flipped SU(5), i.e. $SU(5) \times U(1)_X$, are

\[
\text{matter : } \overline{10}_{-1}, \ 5_3, \ 1_{-5} \\
\text{Higgs : } \begin{cases} 
\overline{5}_2, \ 5_{-2}, & \text{electroweak scale} \\
10_H^H, \overline{10}_{-1}^H, & \text{GUT scale}
\end{cases}
\]

If we restrict to matter and the electroweak scale Higgs fields only, the $Z_2$ subgroup of $U(1)_X$ is a good candidate for the prime source of the $R$-parity since matter fields, Eq. (1), carry odd $X$ quantum numbers and the electroweak scale Higgs $\overline{5}$ and $\overline{5}$ (the first line of Eq. (2))
carry even $X$ quantum numbers. Singlets with $X = 0$ ($\pm 5$) are neutral ($Q_{em} = \pm 1$) singlets. To break the flipped SU(5) down to the standard model, the GUT scale Higgs $10^H$ and $\overline{10}^H$ develop GUT scale VEVs.

In fact, the $Z_2$ subgroup of U(1)$_X$ distinguishes the spinor or the vector origin of our spectrum since we assign

$$X = (-2, -2, -2, -2, -2, 0, 0, 0).$$

(3)

For example, $\overline{10}$ from spinor of the form $(- - + + + \cdots)$ has an odd $X$, while 5 from vector of the form $(1 0 0 0 0 \cdots)$ has an even $X$. This shows that both $10^H$ and $\overline{10}^H$ having odd $X$s. Therefore, for the light fields we have a perfect definition for the R-parity, but including heavy fields $10^H$ and $\overline{10}^H$ make us rethink on the R-parity.

The basic difference between Higgs $\overline{10}^H$ and matter $\overline{10}$ in T6 is that the former forms a vectorlike representation with $10^H$ in T6 and is removed at the GUT scale, while the latter remains as a chiral one. From Table I, there appear four $\overline{10}^L_1$ s and three $10^L_i$ s in the twisted sector T6. One unmatched $\overline{10}^L_1$ is interpreted as belonging to the t-quark family. The gauge sector of four $\overline{10}^L_1$s and three $10^L_i$s has the symmetry $U(4) \times U(3)$. We factor out one U(1) belonging to the t-quark family. The remaining symmetry of three $\overline{10}^H$ and three $10^H$ is $U(1)_{V_{10}} \times SU(3)_L \times SU(3)_R \times U(1)_A$. The $U(1)_A$ symmetry is broken by the anomaly and hence we do not consider it. We are interested only in U(1)s because we will assign the parity as a subgroup of a U(1). Thus, we do not consider the nonabelian symmetry $SU(3)_L \times SU(3)_R$. Then, we are left with the global symmetry $U(1)_{V_{10}}$. This choice of anomaly free $U(1)_{V_{10}}$ is consistent with the discrete gauge symmetry [8]. We define $V_{10}$ charges of $10^H$ and $\overline{10}^H$ are +1 and –1, respectively. Matter $\overline{10}$ corresponding to the t-quark family carries the vanishing $V_{10}$. Of course, this global symmetry $U(1)_{V_{10}}$, not protected in string models, is broken by Yukawa couplings.

From T6 sector in Table I we also find $5^L_j$ s and $\overline{5}^L_3$ s carrying odd-$X$ quantum numbers. Again, we call these two vectorlike pairs as $5^H$ and $\overline{5}^H$, for which we define another vector global symmetry $U(1)_{V_5}$. Let $V_5$ charges of the vectorlike $5^H$ and $\overline{5}^H$ be +1 and –1, respectively. On the other hand, matter $5$s which are chiral, carry the vanishing $V_5$ charge. Again, this global symmetry $U(1)_{V_5}$ is broken by Yukawa couplings in our string compactification. For two vectorlike pairs of $5$ and $\overline{5}$ of T4, carrying even-$X$ quantum numbers, we can consider such a U(1) symmetry but the $X$ charges are already even and we do not need
another manipulation for these even-\(X\) vectorlike pairs.

Before continuing discussion on \(V_{10}\) and \(V_{5}\) charges, let us briefly comment on vectorlike representations of exotics. There are two kinds of exotics. One kind is E-exotics which carry \(X = \pm \frac{5}{2}\) charges, and hence they are \(Q_{em} = \pm \frac{1}{2}\) exotics. The other kind is G-exotics which carry SU(5) charges also: \(5_{+\frac{1}{2}}\) and \(\overline{5}_{-\frac{1}{2}}\). Being vectorlike, the E-exotics and G-exotics can be assigned respective \(U(1)_V\) quantum numbers as discussed in the preceding paragraph. But they are more restricted than the global charges we discussed for \(U(1)_{V_{10}}\) and \(U(1)_{V_{5}}\). The \(U(1)_V\) symmetry for exotics is identical to \(U(1)_{em}\) and it is not broken. The electromagnetic charges of G-exotics are \(\pm \frac{1}{6}\) for colored ones in \(\overline{5}_{-\frac{1}{2}}\) and \(5_{+\frac{1}{2}}\) and \(\pm \frac{1}{2}\) for the doublet members in \(\overline{5}_{-\frac{1}{2}}\) and \(5_{+\frac{1}{2}}\). The color singlet bound states composed of colored ones of G-exotics carry \(Q_{em} = \pm \frac{1}{2}\). Therefore, all color singlet exotics, elementary and composite, carry \(Q_{em} = \pm \frac{1}{2}\). Because of the exact \(U(1)_{em}\), the lightest \(Q_{em} = \pm \frac{1}{2}\) exotics are absolutely stable. Therefore, we separate out the exotics from the rest integer charged sector and do not consider the vectorlike exotics anymore.

Going back to the vectorlike representations of \(10^H\), \(\overline{10}^H\), \(5^H\), and \(\overline{5}^H\), let us define a global charge \(\Gamma\) as

\[
\Gamma = X + V_{10} + V_{5}.
\]  (4)

These \(\Gamma\) charges are shown in Table II. The \(U(1)_X\) symmetry is a gauge symmetry and exact, and \(U(1)_{V_{10}}\) and \(U(1)_{V_{5}}\) are global symmetries and approximate. When we break these gauge and global symmetries spontaneously by one direction in the \(U(1)\) spaces, the gauge symmetry is considered to be broken and a global symmetry remains. The surviving global symmetry is \(U(1)_\Gamma\). This is the so-called 't Hooft mechanism \(\mathbb{G}_\Gamma\). In our case, the gauge symmetry \(U(1)_X\) is broken by the VEVs \(10^H\) and \(\overline{10}^H\). Because \(10^H\) carries \(\Gamma = 2\) and \(\overline{10}^H\) carries \(\Gamma = -2\), VEVs of \(10^H\) and \(\overline{10}^H\) do not break the \(Z_2\) subgroup of \(U(1)_\Gamma\). At this level, the continuous symmetry is broken

\[
U(1)_\Gamma \xrightarrow{\langle 10^H \rangle} P_\Gamma.
\]  (5)

III. HARMLESS R-PARITY AS A DISCRETE SUBGROUP OF U(1)\(_\Gamma\)

Of course, the continuous global symmetry \(U(1)_\Gamma\) is not exact, being broken by superpotential terms. But, “Is the discrete subgroup \(P_\Gamma\) of \(U(1)_\Gamma\) respected by all superpotential
terms?” It is not so. However, this is harmless in the proton decay problem. To show this, let us note possible superpotential terms in the MSSM, generating $\Delta B \neq 0$ operators,

$$D = 4 : \ u^c d^c d^c,$$  \hfill (6)

$$D = 5 : \ qqql, \ u^c u^c d^c e^+$$ \hfill (7)

where $q$ and $l$ are quark and lepton doublets, respectively. The dimension-4 operator of Eq. (6) alone does not lead to proton decay, but that term together with the $\Delta L \neq 0$ superpotential $qd^c l$ leads to a very fast proton decay and the product of their couplings must satisfy a very stringent constraint, $< 10^{-26}$. The $D = 5$ operators in (7) are not that much dangerous, but still the couplings must satisfy constraints, $< 10^{-8}$ [1, 6].

Therefore, we must forbid the $D = 4$ operator. So let us look for possibilities of generating the $u^c d^c d^c$ superpotential term. In the flipped SU(5), it is contained in $5_3 \overline{10}_{-1} \overline{10}_{-1}$, which is however forbidden by the SU(5)$\times$U(1)$_X$ gauge symmetry. In the flipped SU(5), the $D = 4, 5$ operators are generated by

$$D = 4 : \ [d^c d^c u^c]_F, \ [qd^c l]_F \leftrightarrow \langle \overline{10}^H \rangle \ 10 \ 10 \ 5,$$ \hfill (8)

and

$$D = 5 :$$

$$O_1 = [qqql]_F \leftrightarrow \overline{10} \ 10 \ 10 \ 5,$$  \hfill (9)

$$O_2 = [u^c u^c d^c e^+]_F \leftrightarrow \overline{5} \ 5 \ 10 \ 1$$

$$O_3 = [qqqH_d]_F \leftrightarrow \langle \overline{10}^H \rangle \ 10 \ 10 \ 5_2,$$  \hfill (9)

$$O_4 = [qu^c e^+ H_d]_F \leftrightarrow \langle 10^H \rangle \overline{10} \ 5 \ 1 \ 5_2$$

$$O_5 = [llH_u H_u]_F \leftrightarrow \langle \overline{10}^H \rangle \langle \overline{10}^H \rangle \ 5 \ 5 \ 5_{-2} \ 5_{-2},$$  \hfill (9)

$$O_6 = [lH_d H_u]_F \leftrightarrow \langle \overline{10}^H \rangle \ 5 \ 5 \ 5_{-2} \ 5_{-2} \ 5_{-2}.$$

In our model, even with including the possibilities of multiplying a number of neutral singlets, we find that some of the above operators are forbidden up to very high orders.

Since there are numerous neutral singlets which can acquire GUT or string scale VEVs, we must consider all higher order terms also. $u^c$ appears from $5_3$ in $U_3, U_1$ and $T2$. $d^c$ appears from $\overline{10}_{-1}$ in $U_3, U_1$ and $T6$. Note that matter $\overline{10}_{-1}$ in $T6$ is the chiral one and does not carry a $V_{10}$ charge, and it is the $\Gamma = -1$ part out of four $\overline{10}$s in $T6$. At $D = 3$ level, there does not appear $u^c d^c d^c$ due to the flipped SU(5) gauge symmetry before considering any superstring property. One may consider $u^c d^c d^c$ (neutral singlets). Neutral singlets carry
$X = 0$ and hence the possibility $u^c d^c d^c$-(neutral singlets) is forbidden. On the other hand, $5_3 \overline{10} - 1 \overline{10} - 1 \overline{10} - 1$ is allowed by the gauge symmetry. Giving a GUT scale VEV to one of $\overline{10} - 1$s, we obtain the term $u^c d^c d^c$. It is a very dangerous R-parity violating term. The $\overline{10} - 1$ obtaining a GUT scale VEV is $\overline{10}^{\prime}$. So, effectively, we need a coupling $5 \cdot \overline{10} \cdot \overline{10} \cdot \overline{10}^{\prime}$ which breaks the R-parity $P_{\Gamma}$. $5$ and $\overline{10}$ carry $P_{\Gamma} = 1$ while $\overline{10}^{\prime}$ carries $P_{\Gamma} = 0$ according to (4) and (5)). Thus the flipped SU(5) model at the supergravity level is in jeopardy if the couplings of the form $5_3 \overline{10} - 1 \overline{10} - 1 \overline{10} - 1$-(neutral singlets) are allowed. The only cure of this problem at the supergravity level is imposing the R-parity by assumption. But superstring models are free from such an assumption except for choosing the vacuum. It must be shown that such dangerous terms are effectively forbidden in a superstring model, since the prime motivation toward supersymmetry and superstring was to understand the hierarchy of order $10^{-26}$.

We checked the coupling $5_3 \overline{10} - 1 \overline{10} - 1 \overline{10} - 1$ attached with singlets, and we do not find any such term up to $D = 9$ with the application of the program of Ref. [10]. This invites for a careful check of the individual $H$-momenta of the fields so that the computing time is drastically reduced. The needed $H$-momenta are

$$U_1 : (-1, 0, 0), \quad U_2 : (0, 1, 0), \quad U_3 : (0, 0, 1),$$

$$T_2 : (\frac{1}{6}, \frac{4}{6}, \frac{1}{6}), \quad T_4 : (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), \quad T_6 : (\frac{1}{2}, 0, \frac{1}{2})$$

(10)

Since $5_3$ has three possible locations $U_1, U_3$ and $T_2$, and $\overline{10} - 1$ has three possible locations $U_1, U_3$ and $T_6$, the possible combinations to be considered are 18 if we require one $\overline{10} - 1$ must be in $T_6$. These are tabulated in Table III. Now a function of neutral singlets must be found so that the total $H$-momentum together with those of Table III becomes $(-1, 1, 1)$ mod $(12, 3, 12)$.

The neutral singlets appear from the sectors of $U_2, T_2, T_4$, and $T_6$. The numbers of neutral singlets from $U_2, T_2, T_4$, and $T_6$ are nonnegative $w_2, t = \sum t_i, f = \sum f_i$, and $s = \sum s_i$, respectively. The subscripts in the twisted sector distinguishes the neutral singlets due to the oscillator contributions. All neutral singlets in $T_6$ carry oscillators and there are four kinds of oscillators in neutral singlets in $T_6$ [5]; thus we consider $s_i (i = 1, \cdots, 4)$. In $T_2$, neutral singlets appear with or without oscillators. There are seven different $H$-momenta due to their oscillator contributions; thus $t_i (i = 1, \cdots, 7)$. In $T_4$, neutral singlets have four different oscillator contributions; thus $f_i (i = 1, \cdots, 4)$. Thus, in $T_2, T_4$, and $T_6$ sectors, in
Thus, we require three equations from the entries of $H$-momenta

\[
T2 : (0, 0, 0)t_1, \ (1, 0, 0)t_2, \ (0, 1, 0)t_3, \ (0, 0, -1)t_4, \\
\quad (2, 0, 0)t_5, \ (0, 0, -2)t_6, \ (1, 0, -1)t_7
\]

\[
T4 : (0, 0, 0)f_1, \ (1, 0, 0)f_2, \ (0, -1, 0)f_3, \ (0, 0, -1)f_4, \\
T6 : (1, 0, 0)s_1, \ (-1, 0, 0)s_2, \ (0, 0, 1)s_3, \ (0, 0, -1)s_4.
\]

Thus, we require three equations from the entries of $H$-momenta

\[
(0,1,0)u_2 + (-\frac{1}{6}, \frac{2}{3}, \frac{1}{6})t + (-\frac{1}{3}, \frac{1}{3}, \frac{1}{3})f + (-\frac{1}{2}, 0, \frac{1}{2})s \\
+ (0,0,0)t_1 + (1,0,0)t_2 + (0,1,0)t_3 + (0,0,-1)t_4 \\
+ (2,0,0)t_5 + (0,0,-2)t_6 + (1,0,-1)t_7 \\
+ (0,0,0)f_1 + (1,0,0)f_2 + (0,-1,0)f_3 + (0,0,-1)f_4 \\
+ (1,0,0)s_1 + (-1,0,0)s_2 + (0,0,1)s_3 + (0,0,-1)s_4 \\
+ (\text{the entries of H momenta}) = (-1,1,1) \mod (12,3,12).
\]

The first entry equation is

\[
-\frac{1}{6}(t_1 - 5t_2 + t_3 + t_4 - 11t_5 + t_6 - 5t_7) - \frac{1}{3}(f_1 - 2f_2 + f_3 + f_4) \\
- \frac{1}{2}(-s_1 + 3s_2 + s_3 + s_4) + (\text{the first entry of H momenta}) = -1, \mod 12
\]
The third entry equation is
\[ \frac{1}{6}(t_1 + t_2 + t_3 - 5t_4 + t_5 - 11t_6 - 5t_7) + \frac{1}{3}(f_1 + f_2 + f_3 - 2f_4) \]
\[ + \frac{1}{2}(s_1 + s_2 + 3s_3 - s_4) + \text{(the third entry of H momenta)} = +1, \text{ mod. 12.} \quad (16) \]

The second entry equation is
\[ u_2 + \frac{2}{3}(t_1 + t_2 + 5t_3 + t_4 + t_5 + t_6 + t_7) + \frac{1}{3}(f_1 + f_2 - 2f_3 + f_4) \]
\[ + \text{(the second entry of H momenta)} = +1, \text{ mod. 3.} \quad (17) \]

Adding Eqs. (15) and (16), we obtain
\[ t_2 - t_4 + 2t_5 - 2t_6 + f_2 - f_4 + s_1 - s_2 + s_3 - s_4 \]
\[ + \text{(sum of the 1st and 3rd entries of H momenta)} = 0, \text{ mod. 12.} \quad (18) \]

Consider the first case \((U_1U_1U_1T6)\) of Table II. We find a minimum order solution as \(u_2 = 2, t_2 = 3\), which would give a dimension-9 operator. By the computer program, we checked that there is no operators up to dimension-9. So, this solution must be forbidden by the gauge symmetry. To check this, let us note that the gauge \(U(1)\) charges of these singlets are \([5]\)

\[ u_2 : (0^8)(1, 1; 0^6)', \]
\[ t_2 : (1_6; -\frac{2}{3}, \frac{1}{2}, \frac{1}{2})(\frac{1}{3}; \frac{1}{3}; 0^6)' \]

The gauge charge \((0^8)(2, 2; 0^6)\)' of two \(u_2\)s, i.e. two \(s^u\)s of \([5]\), cannot be canceled by three \(t_2\)s. Thus, even if the \(H\)-momentum rule allows it, the gauge invariance forbids it. Another \(H\)-momentum solution \(u_2 = 2, t_1 = 1, t_2 = 1, t_5 = 1\) does not satisfy the gauge invariance either.

We looked for gauge invariant solutions satisfying Eqs. (15, 17, 16). The restriction from the \(H\)-momentum rule saved computing time and we find that the lowest order \(\Delta B \neq 0\)
operators occur at $D = 10$. There are thirty-four operators at $D = 10$.

\[
W = T_{25}U_{10}U_{10}T_{6_{10}} \begin{cases}
C_1^0 C_5^0 C_6^0 C_1^0 s_6 s_8 + C_3^0 C_5^0 C_6^0 c_1 s_6 s_8 \\
+ C_2^0 C_5^0 C_6^0 c_1^+ s_+ s_8 \\
C_6^0 C_6^0 C_1^0 s_6 s_7 + C_3^0 C_6^0 C_1^+ s_7 s_8 \\
+ C_2^0 C_6^0 C_1^0 s_2 s_3 + C_3^0 C_6^0 C_1^+ s_2 s_3 \\
+ C_7^0 C_5^0 C_2^0 s_6 s_8 + C_3^0 C_5^0 C_2^+ s_6 s_8 \\
+ C_5^0 C_6^0 C_2^0 s_3 s_2 + C_3^0 C_6^0 C_2^+ s_3 s_2 \\
+ C_3^0 C_6^0 D_2^+ d_2^+ s_3 + C_7^0 C_5^0 C_6^0 c_6 s_5 s_3 \\
+ C_7^0 C_5^0 C_6^0 s_8 s_1 \\
+ T_{25}U_{10}U_{10}T_{6_{10}} \begin{cases}
C_7^0 C_5^0 C_6^0 s_5 s_7 + C_3^0 C_5^0 C_6^0 c_5 s_5 s_7 \\
+ C_7^0 C_5^0 C_6^0 s_2 s_3 + C_3^0 C_5^0 C_6^0 c_2 s_3 s_8 \\
+ C_7^0 C_5^0 C_6^0 s_1 c_6 s_5 s_7 \\
+ T_{25}T_{6_{10}}T_{6_{10}} \begin{cases}
C_7^0 C_5^0 C_6^0 s_3 s_2 + C_3^0 C_5^0 C_6^0 c_3 s_2 s_8 \\
+ C_7^0 C_5^0 C_6^0 s_1 c_2 s_3 s_8 \\
+ C_7^0 C_5^0 C_6^0 s_1 c_2 s_3 s_8 \\
\end{cases}
\end{cases}
\end{cases}
\tag{19}
\]

Thus, for Table II the couplings $5_{210_{-1}10_{-1}10_{-1}}$ are of the form

\[
W \sim \left( \frac{\langle S^0 \rangle}{M_S} \right)^6 \frac{\langle 10H \rangle}{M_S} u^c d^c d^e
\tag{20}
\]

where $M_S$ is the string scale close to $O(10)$ times the GUT scale. The upper bound of the coefficient is the squared scale of $10^{-26}$ since $10^{-26}$ is on the product of coefficients of two effective operators $[d^c d^c u^c]_F$ and $[g d l]_F$ out of the same coupling $\langle 10^H \rangle$ in flipped SU(5). Thus a coefficient of $u^c d^c d^e$ less than $10^{-13}$ is easily achievable for $\langle S^0 \rangle \sim (\frac{1}{100}) M_S$ with $\langle 10^H \rangle \sim M_{GUT}$. The number $\frac{1}{100}$ is understood as an average number if we choose the overall coefficient as 1. However, it should be noted that some singlet VEVs can be much smaller than $10^{-2}$ and the overall coefficient can be a relatively small number, in which case

\footnote{If the nonrenormalizable couplings appear as connected cubic diagrams, dimension-10 couplings would...}
other singlet VEVs can be closer to $M_S$. This shows that the dangerous $D = 4$ operators of Eq. (10) are not harmful in some vacua, $\langle S^0 \rangle \sim (1/100)M_S$, of the $\mathbb{Z}_{12-1}$ compactification of the heterotic string [5]. This proof also shows that the operators $O_1$ of Eq. (9) is perfectly harmless since one requires the coefficient of $O_1$ being less than $10^{-8}$. The operator $O_3$ is not dangerous since the $\Delta B \neq 0$ process with $O_3$ also needs the second operator of Eq. (8).

Thus, for dangerous $\Delta B \neq 0$ processes we are left with $O_2$ of Eq. (9). The phenomenological bound on the coefficient of dimension-5 operator $O_2$ is taken as order $10^{-8}$. The relevant flipped $SU(5)$ term $\mathbf{5} \otimes \mathbf{10} \otimes \mathbf{1}$ needs $U_1, U_3, T_2$ (for matter $\mathbf{5}$), $U_1, U_3, T_6$ (for matter $\mathbf{10}$), and $U_1, U_3, T_2$ (for matter $\mathbf{1}$). Possible cases are $54$, which are listed in Table [III].

We analyzed also $\Gamma$ violating or R-parity violating $O_2$ terms. In this case, we have the lowest order R-parity violating $O_2$ terms at dimension 8. They are

$$W = T_2 s T_2 \tau T_6 \sigma T_2 \left\{ C_7^0 C_4^0 s_5^0 s_8^0 + C_3^+ D_2^+ d_2^+ s_2^- \right. + C_3^- D_2^- d_2^- s_2^- + \left. C_6^0 C_5^0 s_5^0 s_3^0 \right\} .$$

(21)

As commented earlier, the phenomenological constraint on the coefficient of $O_2$ is rather mild, $< 10^{-8}$, and dimension-8 operator can be easily below this bound satisfying the bound on $O_1$. Note that all terms in Eqs. (19) and (21) include $C_7^0$ or $C_3^-$. So relatively small values of $\langle C_7^0 \rangle$ and $\langle C_3^- \rangle$ are helpful to fulfill the constraints.

Therefore, the R-parity violating terms of [5] can be made harmless.

Other possible $\Delta B \neq 0$ operators may be present. For example, we may consider a $P_\Gamma$ breaking $\langle 10^H \rangle \langle 10^H \rangle 10 \mathbf{10} \otimes \mathbf{5} \otimes \mathbf{10} \otimes \mathbf{5} \otimes \mathbf{5} \otimes \mathbf{5}$, $\langle 10^H \rangle \langle 10^H \rangle 10 \mathbf{10} \otimes \mathbf{5} \otimes \mathbf{10} \otimes \mathbf{5} \otimes \mathbf{5}$, etc. The former one leads to dimension-6 operators suppressed by $M_S^4$, and the second one leads to dimension-5 operators with heavy particle attached. Here, if any R-parity violating operator occurs, it must involve sufficient suppression by $M_S$ powers or inclusion of heavy fields. The baryon number violations with such operators involving the heavy field $5^H$ are safe phenomenologically. It is not any more dangerous than the standard baryon number violation in GUTs.

In sum, the parity $P_\Gamma$ which is a subgroup of an (approximate) continuous global symmetry is an exact symmetry for operators involving the MSSM fields only, but is not an exact symmetry of the full theory. Nevertheless, it is good enough to forbid dangerous proton

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involve an extra factor of (cubic couplings)$^7$ power. For cubic couplings of $O(\frac{1}{10})$, the average VEV ratio can be made small to $O(\frac{1}{10})$. 

12
\[
\begin{array}{cccccc|cc|cccccc|cc}
5_3 & 5_3 & \overline{10}_{ - 1} & 1_{ - 5} & H \text{-mom.} & N & 5_3 & 5_3 & \overline{10}_{ - 1} & 1_{ - 5} & H \text{-mom.} & N & 5_3 & 5_3 & \overline{10}_{ - 1} & 1_{ - 5} & H \text{-mom.} & N \\
U_1 & U_1 & U_3 & (-4, 0, 0) & 12 & U_1 & T2 & U_1 & U_1 & (-\frac{13}{6}, \frac{2}{3}, \frac{7}{6}) & 11 & U_3 & T2 & U_1 & U_1 & (-\frac{13}{6}, \frac{2}{3}, \frac{7}{6}) & 11 \\
U_1 & U_1 & U_3 & (-3, 0, 1) & 12 & U_1 & T2 & U_1 & U_3 & (-\frac{13}{6}, \frac{2}{3}, \frac{7}{6}) & 11 & U_3 & T2 & U_1 & U_3 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 11 \\
U_1 & U_1 & U_1 & T2 & (-\frac{13}{6}, \frac{2}{3}, \frac{7}{6}) & 11 & U_1 & T2 & U_1 & T2 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 10 & U_3 & T2 & U_1 & T2 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 10 \\
U_1 & U_3 & U_1 & (-3, 0, 1) & 12 & U_1 & T2 & U_3 & U_1 & (-\frac{13}{6}, \frac{2}{3}, \frac{7}{6}) & 11 & U_3 & T2 & U_3 & U_1 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 11 \\
U_1 & U_3 & U_3 & (-3, 0, 1) & 12 & U_1 & T2 & U_3 & U_3 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 11 & U_3 & T2 & U_3 & U_3 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 11 \\
U_1 & U_3 & T2 & (-\frac{13}{6}, \frac{2}{3}, \frac{7}{6}) & 11 & U_1 & T2 & U_3 & T2 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 10 & U_3 & T2 & U_3 & T2 & (-\frac{7}{6}, \frac{2}{3}, \frac{13}{6}) & 10 \\
U_1 & T6 & U_1 & (-\frac{7}{6}, \frac{2}{3}, \frac{7}{6}) & 11 & U_1 & T6 & T6 & U_1 & (-\frac{7}{6}, \frac{2}{3}, \frac{7}{6}) & 10 & U_3 & T6 & T6 & U_1 & (-\frac{7}{6}, \frac{2}{3}, \frac{7}{6}) & 10 \\
U_1 & T6 & U_3 & (-\frac{5}{3}, 0, \frac{2}{3}) & 11 & U_1 & T6 & T6 & U_3 & (-\frac{5}{3}, 0, \frac{2}{3}) & 10 & U_3 & T6 & T6 & U_3 & (-\frac{5}{3}, 0, \frac{2}{3}) & 10 \\
U_1 & T6 & T2 & (-\frac{5}{3}, 0, \frac{2}{3}) & 10 & U_1 & T6 & T2 & T2 & (-\frac{5}{3}, 0, \frac{2}{3}) & 9 & U_3 & T6 & T2 & T2 & (-\frac{5}{3}, 0, \frac{2}{3}) & 9 \\
U_3 & U_1 & U_1 & (-3, 0, 1) & 12 & U_3 & U_3 & U_1 & U_1 & (-2, 0, 2) & 12 & T2 & T2 & U_1 & U_1 & (-2, 0, 2) & 12 \\
U_3 & U_3 & U_1 & (-3, 0, 1) & 12 & U_3 & U_3 & U_1 & U_3 & (-2, 0, 2) & 12 & T2 & T2 & U_1 & U_3 & (-2, 0, 2) & 12 \\
U_3 & U_3 & T2 & (-\frac{13}{6}, \frac{2}{3}, \frac{13}{6}) & 11 & U_3 & U_3 & U_1 & T2 & (-\frac{13}{6}, \frac{2}{3}, \frac{13}{6}) & 11 & T2 & T2 & U_3 & U_1 & (-\frac{13}{6}, \frac{2}{3}, \frac{13}{6}) & 11 \\
U_3 & U_3 & U_1 & (-3, 0, 1) & 12 & U_3 & U_3 & U_3 & U_1 & (-3, 0, 1) & 12 & T2 & T2 & U_3 & U_3 & (-3, 0, 1) & 12 \\
U_3 & U_3 & T2 & (-\frac{13}{6}, \frac{2}{3}, \frac{13}{6}) & 11 & U_3 & U_3 & U_3 & T2 & (-\frac{13}{6}, \frac{2}{3}, \frac{13}{6}) & 11 & T2 & T2 & U_3 & T2 & (-\frac{13}{6}, \frac{2}{3}, \frac{13}{6}) & 11 \\
U_3 & T6 & U_1 & (-\frac{5}{3}, 0, \frac{2}{3}) & 11 & U_3 & T6 & T6 & U_1 & (-\frac{5}{3}, 0, \frac{2}{3}) & 11 & T2 & T2 & T6 & U_1 & (-\frac{5}{3}, 0, \frac{2}{3}) & 11 \\
U_3 & T6 & U_3 & (-\frac{5}{3}, 0, \frac{2}{3}) & 11 & U_3 & T6 & T6 & U_3 & (-\frac{5}{3}, 0, \frac{2}{3}) & 11 & T2 & T2 & T6 & U_3 & (-\frac{5}{3}, 0, \frac{2}{3}) & 11 \\
U_3 & T6 & T2 & (-\frac{5}{3}, 0, \frac{2}{3}) & 10 & U_3 & T6 & T6 & T2 & (-\frac{5}{3}, 0, \frac{2}{3}) & 10 & T2 & T2 & T6 & T2 & (-1, 2, 1) & 8
\end{array}
\]

TABLE III: Possible assignments for \(O_2\) of Eq. (\text{[1]}). \(N\) denotes the dimension of the lowest dimensional \(\Gamma\) breaking operators.

decay operators.

There may be \(\Delta B = 0\) and still \(P_\Gamma\) violating operators. One well known example is the lepton number violating operators. Since there is no quadratic superpotential term, \(P_\Gamma\) violation with \(\Delta B = 0\) must be looked where superpotential terms have \(D \geq 3\). The phenomenological constraints on \(\Delta B = 0\) and still \(P_\Gamma\) violating operators are not so serious [11]. Indeed, the lepton number violating operator \(l l e^+\) in the MSSM is generated also through \((\overline{10}^H) 551\) in the flipped SU(5), which is discussed above. Thus, \(l l e^+\) is also strongly suppressed.
IV. CONCLUSION

In the $Z_{12-I}$ model of Ref. [5], a discrete parity $P_T$ which is a discrete subgroup of $U(1)_T$ is found. $U(1)_T$ contains the $U(1)_X$ symmetry of the flipped $SU(5)$. If the $SU(5)$ breaking components $\overline{10}^H$ and $10^H$ are found to carry even $X$ quantum numbers, then we might have achieved an exact discrete parity as a subgroup of $U(1)_X$ of flipped $SU(5)$. But in our model these $\overline{10}^H$ and $10^H$ carry odd $X$ quantum numbers and hence we must resort to an approximate $U(1)_T$ and the resulting approximate $P_T$. Nevertheless, the dangerous dimension-4 and dimension-5 $\Delta B \neq 0$ operators are forbidden up to sufficiently high dimensions, and the discrete R-parity $P_T$ is harmless. We also noted that this very high order constraint occurs from the high order 12 of $Z_{12}$.

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