On A Recent Claim Concerning $\tau(D_s)/\tau(D_0)$

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**Abstract**

It has been suggested that the observed $D_s - D^0$ lifetime difference can be ‘understood’ as due to $SU(3)_{FL}$ breaking in the phase spaces for the exclusive $D^0$ and $D_s$ channels ‘removing’ the need for weak annihilation. It is pointed out that several of the statements in this argument are misleading or even inconsistent with a state-of-the-art description of heavy flavour decays based on the operator product expansion.
The authors of Ref. [1] have argued that the observed difference in the lifetimes for $D^0$ and $D_s$ mesons, namely $\tau(D_s)/\tau(D^0) \simeq 1.17$, can be understood as a phase space effect without a need for what is usually referred to as (weak) annihilation contribution. Unfortunately the authors seem to be quite unaware of the extensive theoretical literature on this subject over the last decade. It is thus not surprising that they make several misleading or even wrong statements concerning the theoretical description. I will first recapitulate briefly the main features of the theoretical description before listing the specific criticism of the claims put forward in Ref. [1].

1 Basics of the Heavy Quark Expansion (HQE)

A state-of-the-art description of the weak decays of heavy flavour hadrons $H_Q$ has to be based on the operator product expansion (OPE), which allows to express decay widths – semileptonic, radiative or nonleptonic ones – through an expansion in powers of $1/m_Q$ with $m_Q$ denoting the heavy flavour quark mass [2, 3]:

$$\Gamma(H_Q) \simeq G_F^2 m_Q^5(\mu) \cdot \left[ c_0(\mu) + \frac{c_2(\mu)}{m_Q(\mu)} + \frac{c_3(\mu)}{m_Q^3(\mu)} + \ldots \right]$$  \hspace{1cm} (1)

Some points have to be emphasized for later reference:

- With the leading contribution to the width being proportional to the fifth power of the heavy quark mass it is essential to give an unambiguous and precise definition of the latter. This requires a field theoretical treatment specifying both the renormalization scheme employed and the scale $\mu$ at which the mass is taken \footnote{An observable of course cannot depend on this scale $\mu$; the dependance of $m_Q$ has to be compensated for by a $\mu$ dependance of the radiative corrections, matrix elements etc., as denoted by the coefficients $c_i$ in Eq. (1).}. Such a treatment has been given in terms of the so-called kinetic mass [4] and the large sensitivity to the quark mass does not pose a real problem for the OPE. A quark model treatment without OPE support cannot meet this essential requirement as a matter of principle; it thus suffers from intrinsic ambiguities in the quark mass values, which lead to considerable irreducible uncertainties in its estimates for the weak widths. This is well-known and has been discussed extensively in the literature.

- The colour gauge symmetry forbids contributions of order $1/m_Q$. Hence:
  - Through $O(1/m_Q)$ the contributions to the width are universal for a given flavour; i.e., they do not depend on the specific hadron.
  - In particular, an ansatz where the leading term in the width is proportional to the fifth power of the heavy hadron mass rather than the heavy quark mass is inconsistent with the OPE. A clear-cut violation of quark-hadron duality (duality) would be required for such an ansatz to emerge.
Yet even then it could hold only for some values of the hadron mass, but not parametrically [4].

- The pole mass, though universal for a given flavour, cannot be used here in principle, since it suffers from an irreducible uncertainty $\sim O(\bar{\Lambda})$; it would introduce an uncertainty $\sim 5\bar{\Lambda}/m_Q$ into $\Gamma(H_Q)$, which is of higher order than the contributions one can calculate.

- The contributions of order $1/m_Q^2$ differentiate between baryons and mesons, while treating the latter in a practically universal way.

- In order $1/m_Q^3$ explicitly spectator-dependant contributions enter that are usually referred to as Pauli interference (PT) [3], weak annihilation (WA) and $W$ scattering. Those had been identified in quark model treatments already before the development of the HQE.

- Spectator and (higher order) WA amplitudes can interfere with each other! This property is actually essential for the decay width to be controlled by the large scale $m_Q$, see Eq.(1), rather than a hadronic low energy scale, as has been demonstrated explicitly in Ref. [6]. As discussed in detail in Refs. [7] it is quite possible a priori that the inclusion of WA, when it is not the leading contribution, can actually diminish a width!

- With the OPE-based HQE providing a quantitative description of nonperturbative dynamics in heavy flavour decays, we have now reached a stage where we can go beyond folklore when discussing duality [4].

## 2 Criticism of Ref. [1]

After this review in a nutshell I list specific criticism of various statements in Ref.[1]; the first point is basically conceptual, the others somewhat more technical:

1. The authors suggest that the observed $D^0 - D_s^+$ lifetime difference is due to $SU(3)_{Fl}$ breaking in the phase space of the exclusive $D^0$ and $D_s^+$ channels. For their argument they adopt the following prescription: they take all observed $D^0$ modes, which actually saturate the $D^0$ width, correct them one by one with some phase space factor for the corresponding $D_s$ modes and add up the thus obtained estimates for the partial $D_s$ widths; this procedure yields $\tau(D_s^+)/\tau(D^0) \sim 1.25$ — similar indeed to the observed value.

- Such a description cannot be viewed as a genuine explanation, though. Adding all exclusive modes has to yield the total width, of course; the only nontrivial element here is that one relates $D^0$ and $D_s^+$ channels by simple phase space considerations. It has to be kept in mind, however, that there is no unique prescription for the phase space of multibody final states, since it depends very much on their resonance etc. structure.
• The real challenge is whether one can provide an at least adequate theoretical description of the hadron’s width expressed on the quark-gluon level; that is the very essence of the concept of duality! A success in this task signals that duality applies already at the charm scale for inclusive transitions. It is on the quark-gluon level that \( WA \) is primarily defined.

• In properly addressing duality one has to keep some subtle features in mind. Observable rates depend on the phase space as it applies to hadrons and their masses; it is certainly different from the phase space for quarks (and gluons). Duality cannot be based on some kind of equivalence between hadronic and quark-level phase space alone; it can hold only if quark phase space plus quark dynamics are effectively equivalent to hadronic phase space plus boundstate dynamics. Such equivalence, which at first sight would look miraculous, is enforced through sum rules – as discussed in detail in Ref. [8].

An effect that on the hadronic level represents \( SU(3)_{FI} \) breaking merely in the phase space, has on the quark level to be a combination of \( SU(3)_{FI} \) breaking in the phase space and in quark dynamics. \( SU(3)_{FI} \) breaking in the quark phase space is modest since \( m_c \gg \Lambda \gg m_s, m_d, m_u \) (for current quark masses); likewise for the nonperturbative contributions \( \sim \mathcal{O}(1/m_Q^2) \). In order \( 1/m_Q^3 \) \( PI \) and \( WA \) enter, both of which induce \( SU(3)_{FI} \) breaking in a quark level description.

2. Replacing the quark mass by an effective quark mass reduced by some sort of binding energy \( BE \) is inconsistent with the OPE! For substituting \( m_Q - BE \) for \( m_Q \) in \( \Gamma \propto m_Q^5 \) generates a correction of order \( 5 \cdot BE/m_Q \) which is not allowed due to the absence of an independent colour gauge invariant operator of dimension four. The absence of a \( \mathcal{O}(1/m_Q) \) contribution can be understood in a less abstract way as well: there are indeed \( \mathcal{O}(1/m_Q) \) corrections to the initial state energy and likewise to the final state energies. Both of these effects yield \( \mathcal{O}(1/m_Q) \) contributions to the phase space – yet they compensate each other there up to terms \( \sim \mathcal{O}(1/m_Q^2) \). This compensation is due to colour symmetry, namely that both initial and final state quarks carry the same colour charge.

Replacing the quark mass by an effective mass reduced by some binding energy had been suggested before in the description of \( B_c \) decays where it would have some dramatic consequences: it would shift the bulk of the \( B_c \) width to \( b \) rather than \( c \) quark decays and lead to a relatively long \( B_c \) lifetime in excess of 1 psec. Yet it was pointed out [1, 11] that the HQE derived from the OPE, which does not contain \( 1/m_Q \) terms, predicts the \( B_c \) width to be generated mainly by \( c \) quark decays producing a ‘short’ lifetime below 1 psec – as has subsequently been observed [11].

The authors of Ref. [12] have proposed a different ansatz leading to \( 1/m_Q \)
contributions, namely to replace the heavy quark mass by the heavy flavour hadron mass in the expression for the total width; it might be amusing to note in passing that such an ansatz would reduce $\tau(D_s)/\tau(D^0)$ by 30%. It is authors’ privilege to suggest an ad-hoc ansatz; yet one should be clear on the price such an ansatz entails. In this case it would be a clear-cut breakdown in the OPE. Also the authors of Ref.[1] seem to be quite unaware that questions of duality and its limitations in observables described by an OPE can be addressed in a much more mature way now than five to ten years ago [4].

3. The first footnote of [1] claims that as far as inclusive transitions are concerned the amplitudes for the spectator and WA processes have to be added in modulus squared with no interference since the final states correspond to four- and two-quark states, respectively. As already mentioned, such a statement is incorrect as demonstrated in the literature [6, 7]: the numerically leading WA contributions are provided by non-factorizable terms that are not helicity suppressed; those are of higher order in $\alpha_S$ and interfere with the dominant spectator amplitude (and others in general). A priori, this interference can be destructive as well as constructive.

4. One cannot argue that the WA contribution is much more reduced in $D_s$ than in $D^0$ decays. While this would be true for the factorizable terms, it is not true in general for the non-factorizable ones which are numerically more important due to the chiral suppression of the factorizable terms [7].

5. There are some more detailed tests of the HQE description of $D_s$ decays. The width of Cabibbo suppressed $D_s$ decays leading to final states with the quantum numbers of $u\bar{s}s\bar{s}$ will be reduced by a factor of about two due to $PI$, whereas the latter cannot affect the corresponding $D^0$ transitions; likewise for doubly Cabibbo suppressed modes. This relative suppression is in both cases considerably larger than in the ansatz of Ref. [1]. However, identifying the footprints of WA in subclasses of decays is still an open challenge.

3 Summary

There is nothing wrong with coming up with a simple (or for that matter even a sophisticated) phase space based description for relating exclusive $D^0$ and $D_s$ channels leading to an estimate of $\tau(D_s)/\tau(D^0)$. Yet that does not remove the motivation for analysing whether one can find a ‘dual’ quark based description; furthermore WA is an element of non-perturbative quark rather than hadronic dynamics. Duality could not be realized by simply equating quark phase space with an average over hadronic phase space: it requires quark phase space coupled with quark dynamics to account for hadronic phase space together with boundstate effects on average. The OPE naturally incorporates quark decay, $PI$, WA etc. – and interferences between them.
– in a well-defined way. It is certainly conceivable that the OPE based description fails for charm decays since the charm scale could be too marginal for duality to apply here. In that case – but only in that case – contributions of order \(1/m_Q\), as would be introduced through use of an effective quark mass \(m_Q - BE\), could emerge. But it should be clearly understood and explicitly stated that such an occurrence would constitute a clearcut breakdown of the OPE!

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