Simultaneous Estimation of Rotor Speed and Stator Resistance in Speed-Sensorless Vector Control of IMs

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In recent years, it is required to extend the operation regions of induction motors (IMs) even at low-speed in regenerating while maintaining a high control performance. Conventional rotor speed estimation methods using an adaptive flux observer are unstable at low-speed in regenerating, particularly near the zero frequency. In this paper, we propose a simultaneous estimation method of the rotor speed and the stator resistance to improve the control performance of the speed-sensorless vector control of IMs. In addition, we prove that the proposed estimation method is stable even at low-speed in regenerating. Furthermore, the effectiveness of the proposed method is demonstrated experimentally.

Keywords: Induction Motor (IM), Speed-sensorless control, Estimation of rotor speed and stator resistance, and Adaptive flux observer.

1. Introduction

The vector control is well known as a high-performance control method of AC motors. Rare-earth-free induction motors (IMs) have been re-evaluated these days. Indeed, electric vehicles using IMs have been commercially produced (1)(2). The speed-sensorless vector control has been used widely in industry applications to avoid system bloat, high cost, and low reliability. In recent years, it is required to extend operation regions of IMs even at low speed in regenerating while maintaining high control performance.

Adaptive flux observers (3)(4)(5)(6)(7)(8)(9)(10)(11) are typically used to estimate the rotor speed in the sensorless control systems. In the estimation using the adaptive flux observers, the stability of the system can be easily analyzed due to the independence between estimation and control. However, this method is not stable at low-speed in regenerating, especially near the zero frequency (4)(5)(7)(10). Thus, the adaptive control system becomes unstable due to measurement disturbances and motor parameter variations when IMs are operated by a constant exciting current. There also exist some drawbacks in the existing methods. For example, the stator resistance cannot be estimated under a low-load operation due to the adaptation gain depending on the q-axis stator current (9). In the sliding mode observer with an online stator resistance and a rotor speed estimations, the chattering is inevitable during the sliding mode (9). The adaptive flux observer using a known regressor vector suffers from a heavy computational burden due to its complicated configuration (11).

The principle that the estimated parameters are treated as the actual one is called the certainty equivalent (CE) principle (12)(13). In the CE-based estimation, a fluctuation of the stator resistance enlarges the unstable region of the estimation of the rotor speed, which leads to instability at low speed even in motoring (4). Although the existing estimation of the stator resistance is stable in motoring (9), it is still unstable in regenerating because the error-transfer function of the adaptive flux observer is not strictly positive-real (SPR), which violates the condition in the Popov’s hyper-stability theorem (5)(12)(13). Therefore, it is necessary to establish a stable estimation method for the stator resistance to improve the control performance of the speed-sensorless vector control of IMs.

To this end, we propose an improved simultaneous estimation method for the rotor speed and the stator resistance in the speed-sensorless control of IMs. In the proposed method of the rotor speed, the SPR property in the estimation loop is guaranteed using the dynamic CE (DyCE) principle (12)(14) and a rotation matrix (7) so that the phase characteristic is improved (13). The stator resistance is estimated in the proposed method by introducing the rotation matrix to the conventional CE-based estimation (16). We also prove that the proposed simultaneous estimation is stable even at low speed in regenerating by using the averaging method (8)(17)(18). The effectiveness of the proposed estimation is demonstrated experimentally, focusing on the stability at low speed in regenerating.

This paper is organized as follows: Section 2 reviews

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are denoted by \( I_x \) observer, and the conventional estimation of the speed-sensorless control systems and the adaptive estimation of the rotor speed and the stator resistance. In Section 3, we present an improved estimation method of the rotor speed and the stator resistance that can be used at low speed in regenerating. In Section 4, we prove that the stability region is enhanced by the proposed estimation method using the averaging method. Section 5 shows the effectiveness of the proposed estimation via experiments. Section 6 summarizes this paper.

**Notation** Table 1 lists the notations used in this paper. The symbol “■” over a symbol denotes its estimated value. The unit and the \( \frac{\pi}{2} \) -rotation matrices are denoted by \( I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and \( J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \), respectively. The \( \theta \)-rotation matrix is denoted by \( R_{\theta} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \). The signum function is defined as

\[
\text{sgn}(x) := \begin{cases} 
-1 & \text{if } x < 0, \\
0 & \text{if } x = 0, \\
1 & \text{if } x > 0.
\end{cases}
\]

The notations \( A := B \) and \( A =: B \) mean that “\( A \) is defined as \( B \)” and “\( B \) is defined as \( A \),” respectively.

2. Conventional CE-based estimation

This section describes the adaptive flux observer of IM and the issues in estimations of the rotor speed and the stator resistance. Fig. 1 shows the configuration of the speed sensorless control system of the IM with the adaptive flux observer. In Fig. 1, “\( uvw \)” and “■” attached to a symbol indicate the three-phase and the reference of the symbol, respectively.

2.1 Conventional CE-based adaptive flux observer

The mathematical model of IM can be written as

\[
\begin{bmatrix} i_s \\ i_r \\ \lambda_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & i_s \\ A_{21} & A_{22} & i_r \\ \lambda_r & \lambda_r & \lambda_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \\ \lambda_r \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v_s, \tag{1}
\]

where

\[
A_{11} = -\left( \frac{R_s}{\sigma L_s} + \frac{M^2 R_r}{\sigma L_s L_r} \right) I = -\left( \frac{L_r R_s}{\varepsilon M} + \frac{M R_r}{\varepsilon L_r} \right) I,
\]

\[
A_{12} = \frac{M R_r}{\sigma L_s L_r} I - \frac{\omega_r M}{\sigma L_s L_r} J = -\frac{R_r}{\varepsilon L_r} I - \frac{1}{\varepsilon} \omega_r J,
\]

\[
A_{22} = \frac{R_s}{\sigma L_s} I,
\]

\[
A_{21} = \frac{M R_r}{\sigma L_s L_r} I,
\]

\[
A_{22} = -\frac{R_r}{\varepsilon L_r} I + \omega_r J,
\]

\[
B_1 = \frac{1}{\sigma L_s L_r} I, \quad \sigma = 1 - \frac{M^2}{\sigma L_s L_r}, \quad \varepsilon = \frac{\sigma L_s}{M}.
\]

In the speed-sensorless control, \( \lambda_r \) and \( \omega_r \) are estimated using the adaptive flux observer. The electrical parameters used in the adaptive flux observer are typically regarded as constants even though they vary during operation. In particular, the stator resistance should be estimated in the adaptive flux observer for stabilization since its fluctuation causes instability at low speed in regenerating.

In this paper, the adaptive flux observer to estimate \( \lambda_r \) and the adaptive estimation laws of \( \tilde{\omega}_r \) and \( \tilde{R}_s \) are given, respectively, as follows:

\[
\begin{bmatrix} \dot{i}_s \\ \dot{i}_r \\ \dot{\lambda}_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & i_s \\ A_{21} & A_{22} & i_r \\ \lambda_r & \lambda_r & \lambda_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \\ \lambda_r \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v_s + \hat{L} e_{i_s}. \tag{2}
\]

\[
\tilde{\omega}_r = \frac{K_p \omega_r + K_i \omega_r}{s} (J \lambda_r) e_{i_s} + \frac{K_p \omega_r}{s} + 1 \left( \frac{K_i \omega_r}{s} \right) (J \lambda_r) e_{i_s}, \tag{3}
\]

\[
\tilde{R}_s = \frac{K_{11} \tilde{R}_s - \tilde{I}}{s} e_{i_s}, \tag{4}
\]

where \( e_{i_s} := i_s - i_s \) is the current estimation error, \( K_p \), \( K_i \), \( K_{11} \), \( K_{12} \) are appropriate gains for the estimation, \( \hat{A}_{11}, \hat{A}_{12}, \hat{A}_{21}, \) and \( \hat{A}_{22} \) are obtained by replacing \( R_s \) and \( \omega_r \) with \( \tilde{R}_s \) and \( \tilde{\omega}_r \) in \( \hat{A}_{11}, \hat{A}_{12}, \hat{A}_{21}, \) and \( \hat{A}_{22} \), respectively. \( \hat{L} = \begin{bmatrix} \hat{L}_1^T \\ \hat{L}_2^T \end{bmatrix} \) is the observer gain. Specifically, \( \hat{L} \) in (2) is designed as

\[
\hat{L} = \begin{bmatrix} -\hat{A}_{11} - \hat{A}_{22} + (\alpha_1 + \alpha_2) I \\ \hat{A}_{12} \left( \left( \hat{A}_{11} + \hat{L}_1 \right) \hat{A}_{22} - \alpha_1 \alpha_2 I \right) - \hat{A}_{22} \end{bmatrix}, \tag{5}
\]

where \( \alpha_1 < \alpha_2 < 0 \) are the observer poles.\(^{[20]}\)

Fig. 2 shows the configuration of the adaptive flux observer (2) and the estimation of \( \tilde{\omega}_r \) and \( \tilde{R}_s \) give by (3) and (4), respectively. In Fig. 2, \( \hat{\omega}_r \) and \( \hat{R}_s \) obtained from (3) and (4) are treated as the actual values, and used in (2), which is called the CE principle.\(^{[12],[13]}\)

2.2 Issues in the estimation of \( \tilde{\omega}_r \) and \( \tilde{R}_s \)

We describe issues of the CE-based estimation of \( \tilde{\omega}_r \) and \( \tilde{R}_s \). For a further analysis of \( e_{i_s} \), due to the estimation errors of \( \tilde{\omega}_r \) and \( \tilde{R}_s \), we derive the following error system that is obtained by subtracting (1) from (2):

\[
\begin{bmatrix} \dot{e}_{i_s} \\ \dot{e}_{\lambda_r} \end{bmatrix} = \begin{bmatrix} A_{11} + \hat{L}_1 & A_{12} \\ A_{21} + \hat{L}_2 & A_{22} \end{bmatrix} \begin{bmatrix} e_{i_s} \\ e_{\lambda_r} \end{bmatrix} + \begin{bmatrix} I \\ 1 \end{bmatrix} \left( -J \lambda_r \Delta \omega_r + \frac{1}{\sigma L_s} \right) \begin{bmatrix} -\hat{\omega}_r \Delta \omega_r \end{bmatrix}, \tag{6}
\]

where \( e_{\lambda_r} := \lambda_r - \lambda_r, \Delta \omega_r := \tilde{\omega}_r - \omega_r, \) and \( \Delta \omega_r := \tilde{\omega}_r - R_s. \)
Rewritten using (8) originate from (1) and (2), respectively. The first and second terms of the right-hand sides in (7) and (8) originate from (1) and (2), respectively.

The first term of the right-hand side in (1) can be rewritten using \( \mathbf{u}_{\Delta_{uv}} \) in (7) and \( \mathbf{u}_{\Delta_{Rs}} \) in (8) as:

\[
\mathbf{u}_{\Delta_{uv}} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \lambda_r \end{bmatrix} = \begin{bmatrix} \frac{M_{Rs}}{L_r} \mathbf{I} & \frac{R_{Rs}}{L_r} \mathbf{I} \\ \frac{R_{Rs}}{L_r} \mathbf{I} & \frac{R_{Rs}}{L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma_1 L_r} \mathbf{I} \\ \frac{1}{\sigma_1 L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\Delta_{uv}} \\ \mathbf{u}_{\Delta_{Rs}} \end{bmatrix}
\]

Similarly, the first term of the right-hand side in (2) can be rewritten as:

\[
\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \lambda_r \end{bmatrix} = \begin{bmatrix} \frac{M_{Rs}}{\sigma_1 L_r} \mathbf{I} \\ \frac{R_{Rs}}{\sigma_1 L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma_2 L_r} \mathbf{I} \\ \frac{1}{\sigma_2 L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\Delta_{uv}} \\ \mathbf{u}_{\Delta_{Rs}} \end{bmatrix}.
\]

Fig. 2. Configuration of adaptive flux observer.

Note that (6) is not actually implemented in the controlled system, but used for the performance analysis of (2). Since (6) is derived from (1) and (2), for the later discussion, we divide \( \mathbf{u}_{\Delta_{uv}} \) and \( \mathbf{u}_{\Delta_{Rs}} \) in (6) into two terms originating from (1) and (2), respectively, as:

\[
\mathbf{u}_{\Delta_{uv}} = \begin{bmatrix} \hat{\lambda}_r \hat{\omega}_r \\ \hat{\lambda}_r \end{bmatrix} = \begin{bmatrix} \hat{\lambda}_r \\ \hat{\omega}_r \end{bmatrix} - \begin{bmatrix} \hat{\lambda}_r \\ \hat{\omega}_r \end{bmatrix},
\]

\[
\mathbf{u}_{\Delta_{Rs}} = \begin{bmatrix} \hat{i}_s \hat{R}_s \\ \hat{i}_s \end{bmatrix} = \begin{bmatrix} \hat{i}_s \\ \hat{R}_s \end{bmatrix} - \begin{bmatrix} \hat{i}_s \\ \hat{R}_s \end{bmatrix}.
\]

In (6), the transfer function from \( \mathbf{u}_{\Delta_{uv}} \) to \( \mathbf{e}_{\Delta_{uv}} \) is given by:

\[
e_{\Delta_{uv}} = C_1 (s \mathbf{I} - \hat{A}_1 - \hat{L} \mathbf{C}_1)^{-1} \begin{bmatrix} \frac{1}{\sigma_1 L_r} \mathbf{I} \\ \frac{1}{\sigma_1 L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma_1 L_r} \mathbf{I} \\ \frac{1}{\sigma_1 L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\Delta_{uv}} \\ \mathbf{u}_{\Delta_{Rs}} \end{bmatrix}.
\]

(10)

It is known that the CE-based adaptive control system is stabilized if the Popov’s hyper stability condition is satisfied, which requires the transfer functions of from \( \mathbf{u}_{\Delta_{uv}} \) to \( \mathbf{e}_{\Delta_{uv}} \) and from \( \mathbf{u}_{\Delta_{Rs}} \) to \( \mathbf{e}_{\Delta_{Rs}} \) in (6) be SPR.

2.2.1 Issue in the estimation of \( \hat{\omega}_r \) From (6), the transfer function from \( \mathbf{u}_{\Delta_{uv}} \) to \( \mathbf{e}_{\Delta_{uv}} \) is given by:

\[
e_{\Delta_{uv}} = C_1 (s \mathbf{I} - \hat{A}_1 - \hat{L} \mathbf{C}_1)^{-1} \begin{bmatrix} \frac{1}{\sigma_1 L_r} \mathbf{I} \\ \frac{1}{\sigma_1 L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma_1 L_r} \mathbf{I} \\ \frac{1}{\sigma_1 L_r} \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\Delta_{uv}} \\ \mathbf{u}_{\Delta_{Rs}} \end{bmatrix}.
\]

(11)

where \( C_1 = \begin{bmatrix} 1 & \mathbf{0} \end{bmatrix} \). Since \( \hat{\lambda}_r = [\hat{\lambda}_{\alpha\beta}] \hat{\lambda}_{\beta\alpha} \) in (11) is defined in the \( \alpha\beta \)-axis, the phase of \( \hat{\lambda}_{\beta\alpha} \) is delayed from \( \hat{\lambda}_{\alpha\beta} \) by \( \pi \) at any frequency in the balanced three-phase system. Thus, we can write \( \hat{\lambda}_{\beta\alpha} = -j\hat{\lambda}_{\alpha\beta} \) in the frequency domain. Substituting this property into (11), we obtain

\[
e_{\Delta_{uv}} = \begin{bmatrix} \hat{\lambda}_{\alpha\beta} \\ \hat{\lambda}_{\beta\alpha} \end{bmatrix} = \begin{bmatrix} \hat{\lambda}_{\beta\alpha} \\ \hat{\lambda}_{\alpha\beta} \end{bmatrix} - \begin{bmatrix} \hat{\lambda}_{\beta\alpha} \\ \hat{\lambda}_{\alpha\beta} \end{bmatrix}.
\]

(12)

Similarly, the first term of the right-hand side in (2) can be rewritten as:

\[
\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \lambda_r \end{bmatrix} = \begin{bmatrix} \hat{\lambda}_r \\ \hat{\omega}_r \end{bmatrix} - \begin{bmatrix} \hat{\lambda}_r \\ \hat{\omega}_r \end{bmatrix}.
\]
approaches $90^\circ$ as shown in Fig. 3(b). In addition, the robust performance against unknown disturbances deteriorates due to the low gain of $G_{\hat{\omega}_r}(j\omega)$ at low-speed. In the worst-case scenario, the estimation of $\hat{\omega}_r$ fails.

The adaptive gains $K_P\omega_r$ and $K_I\omega_r$ in (3) in the CE-based estimation must be low for stability. In addition, the gain of $G_{\hat{\omega}_r}(j\omega)$ is small in the high-frequency range as shown in Fig. 3(b). As a result, the estimation speed of $\hat{\omega}_r$ becomes slow. To improve the speed-sensorless control performance, it is necessary

(R1) to improve the phase margin of $G_{\hat{\omega}_r}(j\omega)$ at low speed in regenerating, and

(R2) to improve the gain of $G_{\hat{\omega}_r}(j\omega)$ in the high-frequency range.

### 2.2.2 Issue in the estimation of $\hat{R}_s$

Similarly, the transfer function from $u_{\Delta R_s}$ to $e_{i_s,\Delta R_s}$ is

$$e_{i_s,\Delta R_s} = C_1(sI - A - \hat{L}C_1)^{-1} \left[ \frac{L}{\hat{O}} \right] (-\hat{i}_s\Delta R_s)$$

$$= \frac{s - \hat{A}_{22}}{\sigma L_2 \left( s - \hat{A}_{11} \right)} (-\hat{i}_s\Delta R_s),$$

and thus,

$$e_{i_s,\Delta R_s} = \frac{s - \hat{A}_{22}}{\sigma L_2 \left( s - \hat{A}_{11} \right)} (-\hat{i}_s\Delta R_s).$$

**Fig. 4. Conventional CE-based estimation of $\hat{R}_s$.**

Fig. 4 shows the error system formed by (4) and (13), and the Bode plot of $G_{\hat{R}_s}(j\omega)$ in (13). As shown in Fig. 4(b), $G_{\hat{R}_s}(s)$ is not SPR since $\arg G_{\hat{R}_s}(j\omega) < -90^\circ$ at low speed in regenerating. Therefore, the convergence of the estimation loop of $\hat{R}_s$ cannot be guaranteed in the presence of disturbances since the system shown in Fig. 4(a) cannot satisfy the Popov’s hyper stability condition. For the convergence of the estimation loop of $\hat{R}_s$, it must be satisfied that $-90^\circ < \arg G_{\hat{R}_s}(j\omega) < 90^\circ$ at low speed in regenerating. To improve the speed-sensorless control performance, it is necessary

(R3) to improve the phase margin of $G_{\hat{R}_s}(j\omega)$ at low speed in regenerating.

### 3. Proposed estimation and adaptive flux observer

This section presents a DyCE-based estimation of $\hat{\omega}_r$, and improves the phase margin of the CE-based estimation of $\hat{R}_s$ by introducing $R_{2s}$.

#### 3.1 DyCE-based estimation of $\hat{\omega}_r$ and adaptive flux observer

As shown in Fig. 3(a), in the adaptive scheme in the conventional CE-based estimation $\hat{\omega}_r$ surrounded by the dashed line, (3) can be re-
garded as a series connection of the I-type adaptive law 
\[ K_I \Delta \omega_r \] and \( C(s) \) which behaves as a high-pass filter (HPF). Thus, the PI-type adaptive law in (3) recovers the phase characteristic of the estimation loop of \( \omega_r \) up to 0°. However, \( K_P \Delta \omega_r \) cannot be large enough to in order to avoid the oscillation in \( \omega_r \) resulting from the oscillation in \( e_{\omega_r} \). Therefore, the gain of \( G_{\omega_r}(j\omega) \) must be large in the high-frequency range, and the phase margin of \( G_{\omega_r}(j\omega) \) must be improved at low speed in regenerating without increasing \( K_P \Delta \omega_r \).

We present a DyCE-based estimation of \( \omega_r \) to overcome the disadvantage of the CE-based estimation of \( \omega_r \). In the proposed DyCE-based estimation, the gain and phase margins of the estimation loop in the high-frequency range are enhanced and a rotating matrix is introduced in the estimation loop to improve the phase margin at low speed in regenerating.

Fig. 5 shows the proposed DyCE-based estimation of \( \omega_r \). As shown in Fig. 5(a), an HPF \( C(s) = \frac{1}{s} \) and \( R_0 \) are inserted before and after \( G_{\omega_r}(s) \), respectively. In Fig. 5(a), we consider the following augmented error system:

\[
e_{\omega_r} = G_{\omega_r}(s)C(s) \frac{u_{\omega_r}}{s} + \frac{J\lambda_r}{s^{\alpha_1} \Delta \omega_r} = G_{\omega_r}(s) - \frac{1}{s^{\alpha_1}}. \tag{15}
\]

As discussed in the previous section, we consider the following scalar transfer function of (15):

\[
\tilde{G}_{\omega_r}(s) = \frac{s - \tilde{A}_{22}}{(-\alpha_1)(s - \alpha_2)}.
\tag{16}
\]

Since

\[
R_0 \tilde{e}_{\omega_r} = R_0 \left[ e_{\omega_r, \Delta \omega_r} - j e_{\omega_r, \omega_r} \right]^{\top},
\]

the scalar transfer function of \( R_0 \tilde{G}_{\omega_r}(s) \) is given as \( e^{-j\tilde{\theta}} \tilde{G}_{\omega_r}(s) \). The Bode plot of \( e^{-j\tilde{\theta}} \tilde{G}_{\omega_r}(s) \) is shown in Fig. 5(b). From Fig. 5(b), it can be observed that the gain of \( e^{-j\tilde{\theta}} \tilde{G}_{\omega_r}(j\omega) \) is higher than that in the CE-based estimation in all frequencies. From this improvement of the gain characteristic, the estimation speeds of \( \omega_r \) can be improved in the proposed DyCE-based estimation. In addition, it can be observed that \(-45^\circ < \arg G_{\omega_r}(j\omega) < 45^\circ \) in all frequencies. The phase margin of \( e^{-j\tilde{\theta}} \tilde{G}_{\omega_r}(j\omega) \) is improved at low speed in regenerating. Therefore, the estimation loop of \( \omega_r \) becomes SPR. This improvement of the phase characteristic results from the cancellation of the poles of (2) by \( C(s) \) in the high-frequency range, and the rotation of \( R_0 \) at low speed in regenerating, respectively. Thus, \( R_1 \) and \( R_2 \) can be solved.

Since \( \omega_r = 0 \) during a usual operation of IMs, we have

\[
\frac{u_{\omega_r}}{s} (-\tilde{\xi}_r, \Delta \omega_r) = \tilde{C}(s) \frac{e_{\omega_r}}{s} G_{\omega_r}(s) \frac{u_{\omega_r}}{s} (-\tilde{\xi}_r, \Delta \omega_r).
\]

The second term of the right-hand side in (18) improves the estimation speed of \( \omega_r \) due to \( \omega_r \). In the steady-state, \( e_{\omega_r} \) coincides with \( u_{\omega_r} \) since \( C(0) = 1 \).

We reconsider (3) in accordance with the change of the input to \( G_{\omega_r}(s) \). As shown in Fig. 5(a), one addi-
tional HPF \( C(s) \) is put before \( G_{\omega_r}(s) \) in the DyCE-based estimation loop while another HPF \( C_{\omega_s}(s) \) is included intrinsically in the CE-based estimation loop in (3). To avoid the double HPFs in the DyCE-based estimation loop for the stability, the following DyCE-based adaptive estimation law is adopted:

\[
\dot{\hat{\omega}}_r = \frac{K_I \omega_r e_i^T \hat{\xi}_{\omega_r}}{s},
\]

where \( \Delta \omega_r = \hat{\omega}_r - \omega_r = \dot{\hat{\omega}}_r = K_I \omega_r e_i^T \hat{\xi}_{\omega_r} \), which is valid when IMs are operated normally. To implement (15), (2) is modified as follows:

\[
\begin{bmatrix}
\dot{i}_s \\
\dot{\lambda}_r
\end{bmatrix} =
\begin{bmatrix}
\hat{A}_{11} & \hat{A}_{12} \\
\hat{A}_{21} & \hat{A}_{22}
\end{bmatrix}
\begin{bmatrix}
i_s \\
\lambda_r
\end{bmatrix} +
\begin{bmatrix}
\hat{B}_1 \\
\hat{B}_2
\end{bmatrix} \xi_{\omega_r}
+ \begin{bmatrix} O \\ B_1 \end{bmatrix} v_s + L e_{i_s},
\]

\[
\left(\begin{array}{c}
i_s \\
\lambda_r
\end{array}\right) =
\left(\begin{array}{c}
\hat{A}_{11} \lambda_r \\
\hat{A}_{21} \lambda_r
\end{array}\right) +
\left(\begin{array}{c}
\hat{B}_1 \\
\hat{B}_2
\end{array}\right) \xi_{\omega_r}
+ \begin{bmatrix} O \\ B_1 \end{bmatrix} v_s + L e_{i_s},
\]

\[\dot{\lambda}_r = \frac{1}{\alpha_1} \left( \dot{\hat{\omega}}_r \left[ J \lambda_r \right] |J \lambda_r| - \left( \xi_{\omega_r} \left[ \frac{\lambda_r}{|\lambda_r|} \right] \right) \right) \]

where

\[
\begin{align*}
\hat{A}_{12} &= \frac{R_r}{L_r I} - \frac{1}{\varepsilon} \Omega_r J \hat{\lambda}_r \\
\hat{A}_{22} &= -\frac{R_r}{L_r} I + \Omega_r J, \\
\hat{B}_1 &= \frac{1}{\alpha_1} \hat{\omega}_r I, \\
\hat{B}_2 &= \frac{1}{\alpha_1} \hat{\omega}_r I,
\end{align*}
\]

\[
\Omega_r = \hat{\omega}_r I
\]

\[
\dot{\hat{\omega}}_r = \frac{1}{\alpha_1} \left( \dot{\hat{\omega}}_r \left[ J \lambda_r \right] |J \lambda_r| - \left( \xi_{\omega_r} \left[ \frac{\lambda_r}{|\lambda_r|} \right] \right) \right)
\]

Hereafter, we explain that (21) accomplishes the proposed DyCE-based adaptive flux observer given by (20).

From (21), we obtain

\[
\begin{align*}
\hat{A}_{12} \lambda_r &= \frac{R_r}{L_r} I \dot{\lambda}_r - \frac{1}{\varepsilon} \Omega_r J \dot{\lambda}_r \\
&= \frac{R_r}{L_r} I \dot{\lambda}_r - \frac{1}{\alpha_1} \hat{\omega}_r J \lambda_r \\
&= \frac{1}{\alpha_1} \left( \dot{\hat{\omega}}_r \left[ J \lambda_r \right] |J \lambda_r| - \left( \xi_{\omega_r} \left[ \frac{\lambda_r}{|\lambda_r|} \right] \right) \right) \\
\hat{A}_{22} \lambda_r &= \frac{R_r}{L_r} I \dot{\lambda}_r + \Omega_r J \lambda_r \\
&= \frac{R_r}{L_r} I \dot{\lambda}_r + \hat{\omega}_r J \lambda_r \\
&= \frac{1}{\alpha_1} \left( \dot{\hat{\omega}}_r \left[ J \lambda_r \right] |J \lambda_r| - \left( \xi_{\omega_r} \left[ \frac{\lambda_r}{|\lambda_r|} \right] \right) \right)
\end{align*}
\]

In the derivation of (23) and (24), the formula of orthogonal projection\(^{\dagger}\) is used. From (23) and (24), we can see that (21) is equivalent to (20).

Note that, in the steady-state, \( \hat{\omega}_r = \omega_r \) I, i.e., \( \hat{A}_{12} = A_{12} \) and \( \hat{A}_{22} = A_{22} \), which means that (20) is equivalent to (2) in the steady-state. In other words, the proposed DyCE-based adaptive flux observer improves the transient performance while satisfying the stability condition required in the conventional CE-based adaptive flux observer in the steady-state.

### 3.2 CE-based adaptive estimation of \( \hat{R}_s \) with \( R_s^\Delta \)

From Fig. 4(b), it can be observed that, in the conventional CE-based estimation loop, arg \( G_{\hat{R}_s}(j\omega) < -90° \) at low speed in regenerating. Thus, \( G_{\hat{R}_s}(s) \) can be stabilized by the phase operation only without the DyCE principle. Fig. 6 shows the proposed CE-based estimation of \( \hat{R}_s \) with \( R_s^\Delta \), and the Bode plot of \( e^{j\frac{\pi}{2}} G_{\hat{R}_s}(j\omega) \). From Fig. 6(b), it can be seen that the estimation loop of \( \hat{R}_s \) becomes SPR due to \( R_s^\Delta \). Thus, (R3) can be solved.

### 4. Stability analysis

The speed-sensorless control system addressed in this

\[\dagger(a^T e_1) e_1 + (a^T e_2) e_2 = a, \text{ where } e_1 \text{ and } e_2 \text{ are the orthogonal unit vectors.}\]
paper includes the nonlinear time-varying system such as the estimation of \( \dot{\omega}_t \) and \( \dot{R}_s \). In the previous section, it is assumed that the estimations of \( \dot{\omega}_t \) and \( \dot{R}_s \) are independent of each other. To investigate the detailed stability conditions of the estimation methods, we adopt the averaging method\(^{(9)}\)\(^{(17)}\)\(^{(18)}\). The averaging method enables us to derive the explicit stability condition of periodic systems.

**Definition 1 (Averaging method\(^{(9)}\)\(^{(17)}\)\(^{(18)}\))** Consider the following periodic function:

\[
\dot{x} = \Delta f(x, t),
\]

where \( f(x, t) \) is a periodic function of which period is \( T \), i.e., \( f(x, t) = f(x, t + T) \), and \( \Delta \) is the magnitude of the perturbation. The “average” of \( x \) is denoted by \( \langle x \rangle \) and defined by

\[
\langle \dot{x} \rangle = \Delta f(\langle x \rangle, t), \quad \langle x \rangle(0) = x(0),
\]

where

\[
\frac{d}{dt} f(x, t) = \frac{1}{T} \int_{t}^{t+T} f(x, \tau) d\tau.
\]

For a sufficiently small \( \Delta > 0 \) in (26), \( \langle x \rangle \) can be approximated to \( x \) on the timescale \( \frac{1}{\Delta} \) by the error of \( x - \langle x \rangle = O(\Delta)^{(17)}(18) \).

**4.1 Stability of conventional CE-based estimation of \( \dot{\omega}_t \) and \( \dot{R}_s \)** To analyze the conventional CE-based estimation of \( \dot{\omega}_t \) and \( \dot{R}_s \), we reformulate (6) to apply the averaging method to it since the system (6) has the form of (25) for the small signal \( \Delta \). From (6), \( e_t \) can be expressed as the sum of the functions of \( -J\lambda_t \Delta \omega_t \) and \( -i_s \Delta R_s \). Thus, we can write \( e_t \) as

\[
e_t = g_{\omega_t}(-J\lambda_t \Delta \omega_t) + g_{i_s}(-i_s \Delta R_s).
\]

Substituting (28) into (3) and (4), and considering their derivatives, we can obtain

\[
\begin{align*}
\begin{bmatrix}
\dot{\omega}_t \\
\dot{R}_s
\end{bmatrix}
&= -\frac{d}{dt} \left[ K_{P\omega_t}(J\lambda_t)^T e_t, \Delta \omega_t + K_{P\omega_s}(J\lambda_t)^T e_t, \Delta R_s \right]
\begin{bmatrix}
\dot{\omega}_t \\
\dot{R}_s
\end{bmatrix}
\begin{bmatrix}
O
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
&= \begin{bmatrix}
K_{I\omega_t}(J\lambda_t)^T e_t, \Delta \omega_t + K_{I\omega_s}(J\lambda_t)^T e_t, \Delta R_s \\
K_{I \dot{R}_s i_s} e_t, \Delta \omega_t + K_{I \dot{R}_s i_s} e_t, \Delta R_s
\end{bmatrix}
\end{align*}
\]

Due to the assumption that \( \dot{\omega}_t \) and \( R_s \) vary sufficiently slowly, \( \dot{\omega}_t \) and \( \dot{R}_s \) can be approximated as:

\[
\dot{\omega}_t \approx \dot{\omega}_t, \quad \dot{R}_s \approx \dot{R}_s.
\]

Using (30) and (31), and applying \( K_{P\omega_t} \ll K_{I\omega_t} \), which is required for the stability, to (29), we can derive

\[
\begin{bmatrix}
\Delta \omega_t \\
\Delta R_s
\end{bmatrix}
\]
Since the angle between $J\dot{\lambda}_r$ and $e_{i_2,\Delta r}$ is less than $\frac{\pi}{2}$ as shown in Fig. 5(b), we have $(R_{\text{sgn}(\omega)}\frac{1}{2}J\dot{\lambda}_r)\top g_{\dot{\omega}_r}(-J\lambda_r) > 0$. Similarly, we have $(R_{\text{sgn}(\omega)}\frac{1}{2}i_2)\top g_{\dot{R}_s}(-i_4) > 0$. Therefore, we have $\text{tr} \Phi^1 = \Phi_{11}^1 + \Phi_{22}^1 < 0$. By a similar discussion, we obtain

$$\text{sgn}(\text{det} \Phi^1) = \text{sgn}(\text{det} \Phi^2) > 0,$$

(39)

(see Appendix for the detail). From (39), we can prove that the proposed simultaneous estimation of $\dot{\omega}_r$ and $\dot{R}_s$ is stable irrespective of operating conditions.

### 5. Experiment

This section shows the effectiveness of the proposed speed-sensorless control of IMs using the proposed estimation of $\dot{\omega}_r$ and $\dot{R}_s$. First, we clarify the usefulness of the proposed estimation of $\dot{\omega}_r$ and $\dot{R}_s$ by applying them to the speed control with sensor of IMs in regenerating. Second, we demonstrate the effectiveness of the speed-sensorless control of IMs using the proposed estimation of $\dot{\omega}_r$ and $\dot{R}_s$ in regenerating (The experimental setups used in experiments are not commercial products but are assembled by hand in our laboratory. The ramp response is conducted using IM#1 due to a loose coupling while IM#2 is used for the step response due to a load by the voltage control).

#### 5.1 Experimental condition

Table 2 lists the nominal parameters of the test IMs. All calculation is executed in a DSP (Texas Instruments Incorporated: TMS320C6701) installed in a processing board (MTT Corporation: DSP6367). The carrier frequency of the three-phase voltage-type PWM inverter for the IMs is 5kHz. All controllers and the rotor speed and stator resistance estimation are synchronized with the triangular-wave PWM carrier. The control period is 200 $\mu$s (i.e., 5 kHz). The bandwidths of the ACR and the ASR are 1500 rad/s and 10 rad/s, respectively. The PWM signal is generated by an FPGA (Field Programmable Gate Arrays).

| Parameters | IM#1 | IM#2 |
|------------|------|------|
| $R_c$      | 0.75Ω | 0.6Ω |
| $R_L$      | 0.48Ω | 0.6Ω |
| $L_1$      | 110 mH | 110 mH |
| $L_2$      | 99 mH | 102 mH |
| $M$        | 99 mH | 102 mH |
| $P_m$      | 2     | 2    |
| Rated power| 1.5 kW | 1.5 kW |
| Rated speed| 1745 rpm | 1710 rpm |
| Rated line current| 6.0 A | 6.2 A |

### 5.2 Result of estimation of $\dot{\omega}_r$ and $\dot{R}_s$

Fig. 7 shows stable/unstable regions of the conventional and the proposed estimations of $\dot{\omega}_r$ for IM#1 using the stability analysis described in the previous section. In Fig. 7(a), it can be seen that the conventional CE-based estimation has an unstable region in regenerating. By contrast, no unstable region exists in Fig. 7(b), i.e., the proposed estimation is stable in all operating conditions.

Figs. 8 and 9 show the experimental results of the estimation of $\dot{\omega}_r$ for IM#1 (for convenience, $\dot{\omega}_r = P_n\dot{\omega}_m$) and $\dot{R}_s$ by the conventional and the proposed methods, respectively. In each experiment, IM#1 is driven by the speed control with sensor under a ramp load. Since the estimation of $\dot{\omega}_r$ in the conventional CE-based estimation starts from the unstable region as shown in Fig. 8(a), the estimation of $\dot{R}_s$ diverges as shown in Fig. 8(b), and thus, the estimation of $\dot{\omega}_r$ fails. Note that the final entrance of the estimation curves of $\dot{\omega}_r$ into the stable region shown in Fig. 8(a) does not imply a stable estimation of $\dot{\omega}_r$. If the estimation of $\dot{\omega}_r$ starts from the stable region and remains within the region, it is stable.

By contrast, from Fig. 9, we can observe that both $\dot{\omega}_r$ and $\dot{R}_s$ are effectively estimated without destabilization by the proposed estimations in all rotor speeds.

Fig. 10 shows the experimental results of the estimation of $\dot{\omega}_r$ and $\dot{R}_s$ for IM#2 by the conventional and proposed estimations. In this experiment, IM#2 is driven by the speed control with sensor under a step load. From Fig. 10(a), it can be seen that $\dot{\omega}_r$ drifted due to the estimation error of $\dot{R}_s$. By contrast, from Fig. 10(b), we can see that the estimation of $\dot{\omega}_r$ is successful even in

---

**Table 2. Parameters of IMs.**

| Parameters | IM#1 | IM#2 |
|------------|------|------|
| $R_c$      | 0.75Ω | 0.6Ω |
| $R_L$      | 0.48Ω | 0.6Ω |
| $L_1$      | 110 mH | 110 mH |
| $L_2$      | 99 mH | 102 mH |
| $M$        | 99 mH | 102 mH |
| $P_m$      | 2     | 2    |
| Rated power| 1.5 kW | 1.5 kW |
| Rated speed| 1745 rpm | 1710 rpm |
| Rated line current| 6.0 A | 6.2 A |
regenerating since the stabilization condition is satisfied in the proposed estimation method.

5.3 Result of speed-sensorless control We only show the experimental results of speed-sensorless control with the proposed estimation since the conventional estimation fails and results in instability of the system.

Fig. 11 shows the experimental result of the estimated speed of \( \dot{\omega}_r \) and the actual speed of \( \omega_r \) (for convenience, \( \dot{\omega}_r = P_n \dot{\omega}_m \) and \( \omega_r = P_n \omega_m \)), and the estimation of \( \dot{R}_s \) when IM#1 is driven by the speed-sensorless speed control. Fig. 12 shows the time responses of the results shown in Fig. 11. From Figs. 11 and 12, we can observe that the stable speed-sensorless speed control and \( \dot{R}_s \) can be achieved irrespective of the rotor speed, and the the speed-sensorless speed control can be successfully accomplished. Note that the estimation of \( \dot{\omega}_r \) is also successful since \( \dot{\omega}_r \) is stably controlled in the speed-sensorless speed control. In Fig. 11(a), it can be observed that \( \dot{\omega}_r \) (colored) is slightly larger than \( \dot{\omega}_r \) (black). This discrepancy results mainly from the error between the value of the actual rotor resistance in IM and \( R_s \) used in (2). To see this discrepancy, we show the simulation result of the estimation of \( \dot{\omega}_r \) (\( \dot{\omega}_r = P_n \dot{\omega}_m \)) and the actual speed \( \omega_r \) (\( \dot{\omega}_r = P_n \omega_m \)) in Fig. 13. As shown in Fig. 13,
speed-sensorless speed control. In this case, too, we can estimate the value of the rotor resistance as long as IM#1 is operated normally (unless operated at extremely low or high temperature).

As is well known, the resistance depends on temperature $T$ [°C]. We denote $R_r$ at $T$ by $R_{r,T}$. When $R_{r} = R_{r,20}$ we have $R_{r,T} \in [0.92R_r, 1.52R_r]$ for $0 \leq T \leq 150$ from the generic formula for temperature affects on resistance. Since this range of $R_{r,T}$ ($0 \leq T \leq 150$) is included in the range of the above-mentioned numerical analysis ($R_{r,\text{actual}} \in [0.8R_r, 2.0R_r]$), the proposed speed-sensorless speed control system is not destabilized even if the actual rotor resistance is affected by temperature as long as IM#1 is operated normally (unless operated at extremely low or high temperature).

Fig. 14 shows the experimental results of the torque step response using $\omega_r$ and $R_r$ obtained by the proposed estimation. In this experiment, IM#2 is driven by the speed-sensorless speed control. In this case, too, we can observe that the speed can be successfully controlled by the speed-sensorless speed control due to the successful estimation of $R_r$.

From these experimental results, we can confirm that the proposed estimation of $\omega_r$ and $R_r$ is more effective than the conventional one, and thus, the speed-sensorless speed control can be successfully achieved even in regenerating.

6. Conclusion

In this paper, we proposed a simultaneous estimation...
method for the rotor speed and the stator resistance to improve the control performance of the speed-sensorless vector control of IMs. We also proved that the proposed estimation method was stable irrespective of operating conditions. The experimental results showed that the proposed estimation was more effective than the conventional method, especially at low speed in regenerating.

Future works include an estimation of the rotor resistance for further improvement of the control performance of the speed-sensorless control of IMs.

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Appendix

1. Derivation of phase difference between $G_\omega (j\omega)$ and $G_R (j\omega)$

For stability analysis of (2), we derive the phase difference between $G_\omega (j\omega)$ in (12) and $G_R (j\omega)$ in (14). We consider only the difference in the numerators of $G_\omega (s)$ and $G_R (s)$ for $\arg G_\omega (j\omega) - \arg G_R (j\omega)$ since the denominators of $G_\omega (s)$ and $G_R (s)$ are the same. When in the steady-state $(s = j\omega)$, we obtain

$$
\arg G_\omega (j\omega) - \arg G_R (j\omega) = \arg \left( \frac{j\omega - A_{22}}{\epsilon} - A_{12} \right) - \arg \left( \frac{j\omega - A_{22}}{\sigma L_s} \right)
$$

$$
= \arg \left( \frac{j\omega - R_r}{L_r} - j\omega_r \frac{1}{\epsilon} \frac{R_r}{L_r} \frac{1}{\epsilon} + j\omega_r \right) - \arg \left( \frac{j\omega - R_r}{L_r} - j\omega_r \frac{1}{\sigma L_s} \right)
$$

$$
= \arg \left( \frac{j\omega + \frac{R_r}{\epsilon}}{\frac{\sigma L_s}} \right) - \arg \left( \frac{j\omega + \frac{R_r}{\epsilon}}{\frac{\sigma L_s}} \right)
$$

$$
= \sgn(\omega) \frac{\pi}{2} - \gamma,
$$

where $\omega = \omega_r + \omega_s$. In the above derivation, we used the following deformation:

$$
\arg \left( \frac{j\omega + \frac{R_r}{\epsilon}}{\frac{\sigma L_s}} \right) = \arg \left( \frac{\omega_r}{\frac{R_r}{L_r}} \right)
$$

$$
= \arg \left( \frac{MR_r}{L_r} \frac{i_{qs}}{M_{idr} R_r} \right)
$$

$$
= \arg \left( \frac{MR_r}{L_r} \frac{i_{qs}}{M_{idr}} \right)
$$

$$
= \arg \left( \frac{MR_r}{L_r} \frac{i_{qs}}{\lambda_{dr}} \right)
$$

$$
= \gamma.
$$

2. Derivation of (37) and (39)

2.1 Derivation of (37) The vector diagram used in the proposed estimation of $\dot{\omega}_r$ and $\dot{R}_s$ is shown in app. Fig. 1. For $\Phi$ in (36), $\det \Phi$ is given as:

$$
\det \Phi = K_1 \Delta \omega K_{1,R_s} \times \left( (J\lambda_r) \eta_{g_\omega} (-J\lambda_r) \cdot \dot{i}_s \right) \cdot \dot{g}_{R_s} (-i_s)
$$

$$
- (J\lambda_r) \eta_{g_{R_s}} (-i_s) \cdot \dot{i}_s \cdot \dot{g}_{R_s} (-i_s)
$$

$$
= K_1 \Delta \omega K_{1,R_s} \Delta \omega \Delta R_s \times \left( (J\lambda_r) \eta_{g_{R_s}} (-i_s) \cdot \dot{i}_s \cdot \dot{g}_{R_s} (-i_s)
$$

$$
- (J\lambda_r) \eta_{g_{R_s}} (-i_s) \cdot \dot{i}_s \cdot \dot{g}_{R_s} (-i_s) \right).
$$

Since the average of the inner product of the two vectors in the steady-state is equal to the inner product itself, we have

$$
(J\lambda_r)^T e_{i_{\Delta \omega_r}} = |J\lambda_r| |e_{i_{\Delta \omega_r}}| \cos \theta_{\Delta \omega_r},
$$

$$
\dot{i}_s e_{i_{\Delta R_s}} = |\dot{i}_s| |e_{i_{\Delta R_s}}| \cos \theta_{\Delta R_s},
$$

$$
\dot{i}_s e_{i_{\Delta R_s}} = |\dot{i}_s| |e_{i_{\Delta R_s}}| \cos (\theta + \theta_{\Delta R_s}),
$$

where, as shown in app. Fig. 1, $\theta_{\Delta \omega_r}$ and $\theta_{\Delta R_s}$ are the angles between $J\lambda_r$ and $\dot{i}_s$, $J\lambda_r$ and $e_{i_{\Delta \omega_r}}$, and $\dot{i}_s$ and $e_{i_{\Delta R_s}}$, respectively.

The phase difference between $\lambda_r$ and $\dot{i}_s$ is given by the slip angle $\gamma = \tan^{-1} \left( \frac{M_{idr}}{\lambda_{dr}} \right) \left( -\frac{\pi}{2} \leq \gamma \leq \frac{\pi}{2} \right)$. We have $\theta = \frac{\pi}{2} - \gamma$ from app. Fig. 1, and

$$
\theta_{\Delta \omega_r} - \theta_{\Delta R_s} = \sgn(\omega) \frac{\pi}{2} - \gamma,
$$

from (A1). Using the above relationships, we obtain

$$
\sgn(\det \Phi) = \sgn \left( \cos \theta_{\Delta \omega_r} \cos \theta_{\Delta R_s} - \cos (\theta + \theta_{\Delta R_s}) \cos (\theta + \theta_{\Delta R_s}) \right)
$$

$$
= \sgn \left( \sin (\theta + \theta_{\Delta \omega_r} - \theta_{\Delta R_s}) \sin (\theta + \theta_{\Delta R_s}) \right)
$$

$$
= \sgn \left( \left( \frac{\pi}{2} - \gamma + \theta_{\Delta \omega_r} - \theta_{\Delta R_s} \right) \sin \left( \frac{\pi}{2} - \gamma \right) \right)
$$

$$
= \sgn \left( \left( \frac{\pi}{2} - \gamma + \theta_{\Delta \omega_r} - \theta_{\Delta R_s} \right) \sin \left( \frac{\pi}{2} - \gamma \right) \right)
$$

$$
= \sgn(\omega) \sgn(\sin(2\gamma))
$$

$$
= \sgn(\omega) \sgn(\dot{i}_s).
$$

The deformation in the above is as follows: (A9): Substituting (A4) to (A7) into (A3). (A9) to (A10): By the addition theorem of trigonometric functions. (A10) to (A11): Substituting $\theta = \frac{\pi}{2} - \gamma$ into (A10). (A11) to (A12): Since $\sin \left( \frac{\pi}{2} - \gamma \right) > 0$. (A12) to (A13): From
respectively. From app. Fig. 1, \( J \) and \( R \) are the same in the conventional estimation.

As shown in app. Fig. 1, \( \theta^t \), \( \theta_{\Delta e_r}^t \), and \( \theta_{\Delta r_s}^t \) are the angles between \( R_{-\text{sgn}(\omega)\hat{z}}J\hat{\lambda}_r \) and \( R_{\text{sgn}(\omega)\hat{z}}\hat{i}_s \), \( R_{-\text{sgn}(\omega)\hat{z}}J\hat{\lambda}_r \) and \( e_i, \Delta e_r \), and \( R_{\text{sgn}(\omega)\hat{z}}\hat{i}_s \) and \( e_i, \Delta r_s \), respectively. From app. Fig. 1,

\[
\begin{align*}
\theta^t &= \theta - \text{sgn}(\omega)\frac{\pi}{2} = (1 - \text{sgn}(\omega))\frac{\pi}{2} - \gamma, \quad (A16) \\
\theta_{\Delta e_r}^t - \theta_{\Delta r_s}^t &= \theta_{\Delta e_r} - \theta_{\Delta r_s} - \text{sgn}(\omega)\frac{\pi}{2} = -\gamma. \quad (A17)
\end{align*}
\]

In this case, only we have to do is to replace \( \theta, \theta_{\Delta e_r} \), and \( \theta_{\Delta r_s} \) by \( \theta^t, \theta_{\Delta e_r}^t \), and \( \theta_{\Delta r_s}^t \) in (A9), respectively. Thus, we obtain

\[
\begin{align*}
\text{sgn}(&\text{det } \Phi^t) \\
&= \text{sgn} \left( \sin(\theta^t + \theta_{\Delta e_r}^t - \theta_{\Delta r_s}^t) \sin \theta^t \right) \quad (A18) \\
&= \text{sgn} (\sin(2\gamma) \sin(\gamma)) \quad (A19) \\
&= \text{sgn}(\hat{i}_{qs}) \text{sgn}(\hat{i}_{qs}) > 0. \quad (A20)
\end{align*}
\]

The deformation in the above is as follows: (A19) to (A20): Similar to the derivation of (A10) to (A14). (A20) to (A21): Since \( \text{sgn}(\sin(\gamma)) \) and \( \text{sgn}(\sin(2\gamma)) \) are determined by only \( \text{sgn} (\hat{i}_{qs}) \) when \( \lambda_d \) is constant. Thus, we obtain (39).

2.2 Derivation of (39) In the proposed estimation, we consider \( R_{-\text{sgn}(\omega)\hat{z}}J\hat{\lambda}_r \) and \( R_{\text{sgn}(\omega)\hat{z}}\hat{i}_s \) instead of \( J\hat{\lambda}_r \) and \( \hat{i}_s \), respectively, while \( e_i, \Delta e_r \) and \( e_i, \Delta r_s \) are the same in the conventional estimation.

Then, we obtain

\[
\begin{align*}
\theta^t &= \theta - \text{sgn}(\omega)\frac{\pi}{2} = (1 - \text{sgn}(\omega))\frac{\pi}{2} - \gamma, \quad (A16) \\
\theta_{\Delta e_r}^t - \theta_{\Delta r_s}^t &= \theta_{\Delta e_r} - \theta_{\Delta r_s} - \text{sgn}(\omega)\frac{\pi}{2} = -\gamma. \quad (A17)
\end{align*}
\]

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