Determination of basic parameters of the wave gearings with intermediate rolling bodies

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Abstract. In article is described the constructions wave gearings with intermediate rolling bodies. The main relationships between the size of details transmission are proved, the forces exert on intermediate rolling body are determined, and general force laws are given. It is shown that rotational torque from cam to separator or centerwheel, can be transfer by intermediate rolling body only insofar as angles $\gamma$ and $\psi$ are greater 0° and less 90° together.

1. Introduction
Nowadays the wave gearings with intermediate rolling bodies are gain grounded in oil and gas industry, as well as in other industrial settings (aerospace, lift and carry etc.) [1-4]. Introduction of them into constructions make it possible to design machines with improved capability, because they have a wide range of transmission rate, action flexibility and multiple tooth contact gearing [5-7]. However, insufficient information obstructs their production and wide spread occurrence.

2. Theoretical background
Constructions wave gearings with intermediate rolling bodies are right amount, in the paper, a power transmission on the ground of mechanism present on the fig. 1 was considered.

Mechanism works following manner: during the propulsion to eccentric disk of cam 1, the intermediate rolling bodies 4 are received radial motion in mortices of separator 3 and rolled wavelike on toothing of center wheel 2. In the wake of such movement, the intermediate rolling elements 4 can revolve or separator 3, or wheel 2, depending of what detail is fixed in gear box.

As far as the geometric axes of the intermediate rolling bodies change positions in space and output element can be center wheel or separator, as such power transmission fits into differential-planet gears, which has two degrees of freedom.

3. Basic geometrical relationships in power transmission
The majority elements of details of power transmission can be procure base geometrical practice, a different matter stand with crown end of center wheel. We determine the parametric equations of curve, which governs tooth space of crown end of center wheel.

As far as intermediate rolling body undergoes different motion, as expand him on radial and tangential components.
Write S (fig. 2) for the motion in radial direction and determine a dependence between motion of rolling element and rotary motion of cam $\phi_1$. For the purpose let’s use alternative slider-crank mechanism. The intermediate rolling body fulfils the role of ram, which translating in the mortice of motionless separator. The driving crank OB – cam, length whom equals amount of eccentricity of cam (OB=e). The line, bindings geometrical cam centers of intermediate rolling body (BC=R+r), fulfill the function of imaginary connecting rod BC and as it known, such the line runs across of them contact point. For the determination of the motion of point C (center of rolling element) let’s use sine theorem

$$\frac{S}{\sin(-\psi \cdot \phi_1)} = \frac{OB}{\sin \psi} = \frac{BC}{\sin \phi_1}$$

and after some transformational changes we obtain

$$S = BC \cos \psi - OB \cos \phi_1.$$ 

Radial velocity component of point C

$$\bar{V}_{C(R)} = \frac{dS}{dt} = OB (\sin \phi_1 - tg \psi \cos \phi_1).$$
Let us introduce system of axes $Ox'y'$ (fig. 2) with origin, in line with turning point of cam, and let’s write in curves conveying speed of center of intermediate body

$$\vec{V}_{C(R)} = OB(\sin \varphi_1 - \tan \psi \cos \varphi_1) \vec{j}.$$  

The peripheral component of conveyance speed come out of angular motion of separator

$$\vec{V}_{C(t)} = -\frac{S}{z_2} \vec{i},$$

Where $z_2$ is the number of teeth of center wheel

Velocity vector of moving of center of rolling element for center wheel

$$\vec{V}_{rel} = \vec{V}_{C(R)} - \vec{V}_{C(t)} = OB(\sin \varphi_1 - \tan \psi \cos \varphi_1) \vec{j} + \frac{S}{z_2} \vec{i}.$$  

It follows by the gearing theory that line of action of relative velocity vector is perpendicular line of base tangent, as carried out to the joint surfaces in the point of contact.Let us call the normal vector to the joint surfaces $\vec{n}$ (fig. 2), let us call $\gamma$ the angle between $\vec{n}$ and axis $Oy$.

Because vector $\vec{n}$ is perpendicular to vector $\vec{V}_{rel}$, as him coordinates
Angle $\gamma$ is come from following expression

$$
\gamma = \arccos \left( \frac{n_j}{n} \right) = \frac{S}{z_2} = \left( OB \sin \varphi_i - tg \psi \cos \varphi_i \right)^2 + \left( \frac{S}{z_2} \right)^2 .
$$

4. Transmission ratio

Let us derive the formula for determination of transmission ratio such mechanism, deprive of him single degree of freedom using latching control of center wheel. Because the mechanism fall into epicyclic gear train, as for determination the formula, which bind the corner frequencies $\omega_1$ and $\omega_3$, let us use a Willis’s method [8-13], which based on a concept planet carrier shutdown.

We call the cam as planet carrier and draw up a table of the corner frequencies of crew of unreversed and reversed mechanism.

**Table 1. The corner frequencies of crew of unreversed and reversed mechanism**

| Parts      | Keep of mechanism | \(\omega_1\) | \(\omega_2\) | \(\omega_3\) |
|------------|-------------------|---------------|---------------|---------------|
| Unreversed |                   | \(\omega_1\) | \(0\)         | \(\omega_3\) |
| Reversed   |                   | \(0\)         | \(-\omega_1\) | \(\omega_3 - \omega_1\) |

Let us define the transmission ratio from third part to second part for reversed mechanism

$$
i_{32}^{(3)} = \frac{\omega_3 - \omega_1}{- \omega_1},
$$

simplify formula, we obtain

$$
i_{32}^{(3)} = 1 - \frac{\omega_3}{\omega_1}, \quad (1)
$$

where the transmission ratio from separator 3 to cam 1 for unreversed mechanism is

$$
i_{31} = \frac{\omega_3}{\omega_1}, \quad (2)
$$

plugging the (2) in (1), we get

$$
i_{32}^{(3)} = 1 - i_{31},
$$

Transmission ratio from cam 1 to separator 3 is

$$
i_{13} = \frac{1}{1-i_{32}^{(3)}}, \quad (3)
$$

where $i_{32}^{(3)} = \frac{z_2}{z_3}$-transmission ratio from separator 3 to center wheel 2.

Let's insert him in (3) and modify, we get expression for determination of transmission ratio
\[ i_{\alpha} = \frac{z_3}{z_3 - z_2} \]  

Found expression shows, that in case of a positive difference between lot intermediate rolling bodies and number of teeth of center wheel, sense of rotation of cam and separator will be coincide and conversely, in negative difference, how on fig. 1, sense of rotation will be the other way round.

Different sense of rotation of separator will action on part of epicycloid, which circumscribe the tooth space of toothing of center wheel, will be contained in force interaction with intermediate rolling body.

**Figure 3.** Determination of forces, which influences on intermediate rolling body

Let’s consider any one intermediate rolling body and details, with whom it has contact. Further on this combination the body-details will be call the connection joint and compose the calculation model, which featured on fig. 3. Three forces influence on the intermediate rolling body from cam \( P \), separator \( N \) and center wheel \( F \). Let us derive the global axes \( Oxy \) is identical to conventional before on fig. 2, with origin, which same as with center of power transmission. Lines of action of said forces for the used coordinate axes are based following manner: force vector \( F \) generates with off-axis \( Oy \) angle \( \gamma \), intensity of \( \gamma \) was determined before, line of action \( N \) is parallel axes \( Ox \) and locate on distance \( S \) from it, vector of force \( P \) is located on line, which pass through geometrical center of cam and intermediate rolling body. Consequently, given forces are represented the concurrent force system, the lines of action that have line crossing in geometrical center of intermediate rolling body. For this system can be written
\[ F + N + P = 0. \]

Proceeding from this formula, during the known geometry and rotational power \( T_1 \) on the output link, we can determine the forces from following formula:

\[
P = \frac{T_1}{S \cdot \sin \psi},
\]

where \( \psi \) is the angle between line of action \( P \) and axis \( Oy \) (Fig. 3)

\[
N = \frac{T_1}{S \cdot \tan \psi \cdot \cos \gamma},
\]

\[
F = \frac{\tan \psi}{S \cdot \tan \psi} + \frac{1}{S}T_1.
\]

Because the output link can be center wheel or separator then, during separator is fixed, torque output is

\[
T_2 = \frac{\tan \psi}{\tan \psi} T_1,
\]

or during center wheel is fixed

\[
T_2 = \left( \frac{\tan \psi}{\tan \psi} + 1 \right) T_1, \text{ i.e. } T_2 = (T_1 + 1) T_1.
\]

5. Conclusion

Following from the gotten formulas, rotational torque from cam to separator or center wheel, can be transfer by intermediate rolling body only insofar as angles \( \gamma \) and \( \psi \) are greater 0° and less 90° together.

The gotten formulas also illustrate, that during other factors being equal, the most favorable construction is been when output link is separator and center wheel is fixed in case of power transmission, by transmit drive’s eyesight.

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