Ultra High Energy Cosmic Rays from Early Decaying Primordial Black Holes

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Abstract

Origin of ultra high energy cosmic rays is an unsolved problem. Several proposals such as Z-burst, decay of super massive matter, susy particles as a primary, neutrino as a primary in extra dimension models exist in the literature which try to address this issue. Many of these proposals solve the problem of propagation of cosmic rays over cosmological distances by introducing new physics. However these do not explain the origin of such high energy cosmic rays. The possible astrophysics sites, such as active galactic nuclei, are highly constrained. Here we determine whether these cosmic rays originated from the decay of some exotic objects, such as primordial black holes (PBHs), present in the early universe. In contrast to the usual Top Down scenario we do not assume that this exotic object necessarily has to decay in our astrophysical neighbourhood since we assume a beyond the standard model scenario, where the propagation problem is absent. We consider the standard 4-dimension PBHs as well as the brane world PBHs. We find that in both cases it is unable to produce the observed ultra high energy cosmic ray flux.
1 Introduction

The observation of cosmic rays with energies in excess of $10^{20} \text{ eV}$ present a major challenge to astro-particle physics. Due to the presence of cosmic microwave background radiation it is predicted that cosmic rays with energies above $10^{20} \text{ eV}$ will not be observed due to the GZK cut-off \cite{1}. The presence of GZK violating events implies new physics unless the source of these events lie within our astrophysical neighbourhood. Some interesting possibilities include topological defects \cite{2,3}, primordial black holes \cite{4} or super heavy particles \cite{5} decaying within a distance of about 100 Mpc in order to evade the GZK bound. However most of such topdown scenarios are severely constrained by existing data. There also exist many proposals which solve the cosmological propagation problem by introducing new physics. Examples include violations of lorentz invariance \cite{6,3}, existence of susy particles \cite{7}, existence of magnetic monopoles \cite{8}, Z-burst \cite{9,10}, a strongly interacting neutrino at ultra high energies(UHE) \cite{11} etc. Many of these proposals are also severely constrained by existing experiments and will be further tested by future planned experiments. The strongly interacting neutrino proposal, for example, will be ruled out by non-observation of UHE neutrinos in experiments by the year 2006 \cite{10}.

The only possible conventional astrophysical sources for UHE neutrinos are Active Galactic Nuclei (AGN) and Gamma Ray Bursts (GRB). These can be considered as possible sources of ultra high energy cosmic rays (UHECR) only if we assume that the propagation problem is solved by some new physics proposal. It is clearly important to consider alternate sources of UHECR, even if they are located at cosmological distances. Here we consider Primordial black holes decaying in the early universe. These objects are interesting because, depending on their masses, they can survive till today. UHECRs from Primordial black holes(PBH) in our astrophysical neighbourhood have been studied in Ref. \cite{4}.

In this paper we consider the production of UHE protons and neutrinos from PBHs decaying today and also PBHs which decay in early epoch of the cosmological evolution of the universe. We calculate UHE fluxes in standard 4D PBHs as well as in 5D braneworld PBHs. In the next two sections we review the 4D PBHs and 5D braneworld PBHs.
2 Standard 4D Primordial Black Holes

It is known that black holes would have formed in very early universe through density fluctuations [12]. These fluctuations may either be primordial or may be formed spontaneously at any epoch. PBHs might have formed at any spontaneous symmetry breaking epoch through the collision of bubbles [13] or through the collapse of cosmic strings [14]. If one assumes the production of black hole with mass of the order of horizon mass at some time $t$ in the evolution of universe then its mass [15],

$$M_{BH}(t) \approx \frac{c^3t}{G} \approx 10^{15} \left( \frac{t}{10^{-23}s} \right) g.$$  \hspace{1cm} (1)

Such a black hole with mass of order $10^{15}g$ would be evaporating now. Masses less than $10^{15}g$ would have evaporated by now. For example, black holes with masses between $10^{10}g$ and $10^{13}g$ would have completed their evaporation between $10^3$ and $10^{12}$ sec.

The emission rate of spin $1/2$ particles from a black hole is given by [16],

$$\frac{d^2N}{dt dE} = \frac{\Gamma_{1/2}(E,T)}{\exp \left( \frac{E}{kT} \right) + 1}$$  \hspace{1cm} (2)

per particle degree of freedom. Here $T$ is the temperature of the hole and $\Gamma_{1/2}(E,T)$ is the absorption coefficient. The above spectrum deviates from the black body spectrum due to the energy dependence of $\Gamma_{1/2}(E,T)$. In the limit $E/(kT) \gg 1$, the spectrum approaches that of a black body with a temperature [16]

$$T \approx 1.06 \times 10^{13} \left[ \frac{1g}{M} \right] ,$$  \hspace{1cm} (3)

where $M$ is the mass of the black hole. For relativistic particles $\Gamma_{1/2}(E,T)$ is given by [16] [17],

$$\Gamma_{1/2}(E,T) \approx \frac{27E^2}{64\pi^2(kT)^2} .$$  \hspace{1cm} (4)

A black hole loses mass at the rate [16] [17]

$$\frac{dM}{dt} = -\frac{\alpha(M)}{M^2} ,$$  \hspace{1cm} (5)

where $\alpha(M)$ depends on the degrees of freedom of the emitted particles and increases with temperature. For standard model

$$\alpha(M) \approx 10^{26} g^3 s^{-1}$$  \hspace{1cm} (6)
above top quark production threshold. From eq. 3 and eq. 5 we find

\[ dt_* = 1.5 \times 10^{-15} \frac{dT_*}{T_*^4}, \]  

(7)

where \( t_* = t/(1 \text{sec}) \) and \( T_* = T/(1 \text{ EeV}) \). Particles with energies above 1 EeV will be produced instantly when the temperature of the black hole is such that \( kT \geq 1 \text{ EeV} \). Characteristic time for the production of EeV energy particles is of the order of \( 10^{-18} \text{ sec} \). Similarly characteristic time required to produce particles with planck energy \( 10^9 \text{ EeV} \) is of the order of \( 10^{-43} \text{ sec} \). Hence duration of the emission of these high energy particles by PBHs is unimportant compared to the evolution time of the universe. After integrating eq. 5, the mass of the black hole, at any time \( t \), is approximately given by

\[ M(t) \simeq (M_i^3 - 3\alpha t)^{1/3} = M_* \left( \frac{M_i}{M_*} \right)^3 - \frac{t}{t_0} \right)^{1/3}, \]

(8)

where \( M_i, t_0, M_* \) are respectively the initial mass of the PBHs, age of the universe and mass of the PBHs evaporating today. The Standard Model provides a lower bound on the high energy value of \( \alpha(M) \), above top production threshold. One can assume same value of \( \alpha(M) \) to hold at higher energies. In next section we review the properties of brane world PBHs.

3 Braneworld Primordial Black Holes

Braneworld cosmological models provide an interesting alternative to the standard cosmology. In this scenario PBHs can be formed in very early universe by density perturbation [19]. In the well known RS2 model our universe is a positive tension brane in a 5d bulk with \( s^1/Z_2 \) type of compactification. It is shown in ref. [20] that in this model PBHs can be easily produced in the radiation dominated universe through spherical collapse.

The Friedmann equation in RS2 model is [19]

\[ H^2 = \frac{8\pi}{3M_4^2} \left( \rho + \frac{\rho^2}{2\lambda} + \rho_{kk} \right) + \frac{\Lambda_4}{3} - \frac{k}{a^2} \]

(9)

where \( H \) is the Hubble constant, \( \rho, p \) are respectively the energy density and pressure of the fluid, \( a \) is the scale factor on the brane, \( M_4 \) is the effective
4D Planck mass, $\Lambda_4$ the 4D cosmological constant and $\rho_{kk}$ the dark radiation density. The energy conservation equation on the brane leads to

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (10)$$

As usual $k = -1, 0, 1$ for open, flat and closed brane universe. From nucleosynthesis constraints one finds that the $\rho_{kk}$ term in equation 9 is negligible. The energy density and scale factor then becomes [19]

$$\rho = \frac{3M_4^2}{32\pi} \frac{1}{t(t + t_c)} \quad (11)$$

and

$$a = a_0 \left( \frac{t(t + t_c)}{t_0(t_0 + t_c)} \right)^{1/4} \quad (12)$$

respectively, where $t_0$ is any nonzero time, $t_c$ is the transition time, $t_c = \frac{l}{2}$, and $l$ is AdS curvature radius. The time $t_c$ separates nonconventional cosmology from the standard cosmology. For time $t \gg t_c$ we recover standard cosmology. For time $t \ll t_c$, which we call high energy regime, the temperature-time relation modifies to [19]

$$\frac{T}{T_4} = \left( \frac{45}{8\pi^3} \right)^{1/4} g^{-1/4} \left( \frac{l}{l_4} \right)^{-1/4} \left( \frac{t}{t_4} \right)^{-1/4}, \quad (13)$$

where $T_4$, $t_4$, and $l_4$ are 4-D Planck temperature, Planck time and Planck length respectively. At transition time $t_c$, transition temperature is [19, 21]

$$T_c = 3 \times 10^{18} \left( \frac{l}{l_4} \right)^{-1/2} \text{GeV} \quad (14)$$

Since current experiments probe gravity to a length scale of 0.2 mm and constrain the size of extra dimension to be $l \leq 10^{31} l_4$, we find that the minimum value of $T_c$ is $10^3$ GeV.

### 3.1 Evaporation rate of Brane World PBHs

In ref. [19] authors have calculated a mass-lifetime relation for black holes formed on the brane due to collapse of matter on the brane. Effect of accretion on the lifetime and mass of brane world PBHs are studied in ref.
We do not consider the effect of accretion in this paper. For a review on brane world PBH, see ref. [25]. If the size of the black holes $r_0 \ll l$ then their geometry is described by 5D Schwarzschild black holes. These black holes will emit Hawking radiation into the brane as well as to the bulk. In this approximation, radius and temperature of the black hole are given by [19],

$$r_0 = \sqrt{\frac{8}{3\pi} \left( \frac{l}{l_4} \right)^{1/2} \left( \frac{M}{M_4} \right)^{1/2} l_4}$$

(15)

and

$$T_{bh} = \frac{1}{2\pi r_0}$$

(16)

respectively. Mass loss rate of these black holes is given by [19],

$$\frac{dM}{dt} \approx -\frac{16\pi}{3} \tilde{g} T^2$$

(17)

where

$$\tilde{g} = \frac{1}{160} g_{brane} + \frac{9\zeta(5)}{32\pi^4} g_{bulk}.$$  

(18)

In our case $g_{brane}$ dominates and most of energy goes to the brane. If we consider only the Standard Model degrees of freedom, $g_{brane} = 100$. From eq. 17 we can derive the lifetime $t_{evap}$ of a black hole of initial mass $M$. Lifetime $t_{evap}$ is

$$t_{evap} \approx \tilde{g}^{-1} \frac{l}{l_4} \left( \frac{M}{M_4} \right)^2.$$  

(19)

eq. 17 gives a relation between time and temperature of the BH as

$$dt_* = \frac{.009\tilde{g}^{-1} dT_*}{512\pi A T_*^5},$$

(20)

where $t_* = t/1\, \text{sec}, \ T_* = T/1 \, \text{EeV}$ and $A = \frac{l}{l_4}$. The parameter $A$ basically determines the length $l$ at which 5D BHs dominate the dynamics and can at most take value $10^{31}$ [22]. This value comes from upper limit on the size of extra dimension constrained by sub-mm gravity experiments. From mass-lifetime relation we get a range of $l$ over which PBH acts as a 5 dimensional black hole. The minimum value of AdS radius [17] is

$$l_{min} = \tilde{g}^{1/3} \left( \frac{t_{evap}}{t_4} \right)^{1/3} l_4$$

(21)
and the maximum value of AdS radius $l_{\text{max}} = 10^{31} l_4$. This implies that the 5-dimensional PBHs lie in the range

$$M_{\text{min}} = \bar{g}^{1/2} \left( \frac{l_{\text{max}}}{l_4} \right)^{-1/2} \left( \frac{t_{\text{evap}}}{t_4} \right)^{1/2} M_4 \quad (22)$$

to

$$M_{\text{max}} = \bar{g}^{1/3} \left( \frac{t_{\text{evap}}}{t_4} \right)^{1/3} M_4 \quad . \quad (23)$$

Similarly temperature of the PBHs ranges from

$$T_{\text{min}} = \sqrt{\frac{3}{32\pi}} \bar{g}^{-1/4} \left( \frac{l_{\text{max}}}{l_{\text{min}}} \right)^{-1/4} \left( \frac{t_{\text{evap}}}{t_4} \right)^{-1/4} T_4 \quad . \quad (24)$$

to

$$T_{\text{max}} = \sqrt{\frac{3}{32\pi}} \bar{g}^{-1/3} \left( \frac{t_{\text{evap}}}{t_4} \right)^{-1/3} T_4 \quad . \quad (25)$$

We have listed a range of mass, size and temperature of brane world PBHs corresponding to their evaporation in Table.

4 Mass distribution of black hole

The mass distribution functions for 4D PBHs formed by the collapse of density perturbations have been derived in ref. [15]. Here we assume that the mass function is dominated by a particular value, as given in [26]. In this case the distribution of the PBHs present throughout the evolution of the universe from their time of formation can be expressed as,

$$n(M) = N(1 + z)^3 \quad . \quad (26)$$

where $z$ is the redshift at time $t$. Here we have neglected the evaporation of PBHs from their formation time to their time of evaporation. We assume similar mass function for 5D brane world PBHs.

5 Flux Calculation for 4D PBHs

We are interested in the flux of particles, specifically neutrinos and protons, at ultra high energies from PBHs. The neutrinos may be emitted directly
by the PBH, which we refer to as the direct flux. Alternatively they may be emitted by hadrons and other particles in the PBH spectrum. We refer to this as the indirect flux.

5.1 Direct Neutrino Flux

Let \( f(E_\nu, T) \) represent the direct neutrino flux at energy \( E_\nu \) given in eq. 2, then the diffuse flux per unit area today is:

\[
\frac{dN_\nu}{dE_\nu} = \frac{1}{4\pi} \times 1.5 \times 10^{-15} \int_{z_{\text{min}}}^{z_{\text{max}}} \int_{kT_i^*}^{kT_{pl}^*} \frac{1}{(1+z)^2} \frac{dn}{dz} f(E_\nu, T) dz ,
\]

(27)

where \( E_\nu = E_{\nu 0} (1 + z) \), \( z \) is the redshift at the time of emission, \( z_{\text{max}} \) corresponds to maximum redshift from which particles of energies 100 EeV can reach us and \( E_{\nu 0} \) is energy of neutrino at redshift \( z_{\text{min}} \). We take \( z_{\text{max}} \leq 10^7 \), which corresponds to the redshift at which particles of initial energy \( 10^{19} \) GeV will be observed at energy of 100 EeV.

5.2 Indirect Neutrino flux

The indirect neutrino flux is obtained dominantly from the decay of hadrons. The hadrons are formed by the fragmentation of quarks and gluons. The hadronic flux is generally dominated by pions which form almost 97% of the flux. The remaining 3% is mostly nucleons. The fragmentation function for quarks into hadrons is given by

\[
x \frac{dN_h}{dx} = \frac{A_h}{x} \exp \left( \frac{(\xi - \xi_\rho)^2}{2\sigma^2} \right)
\]

(28)

where \( \xi_\rho = Y \left( \frac{1}{2} + (c_0/Y)^{(1/2)} - c_0/Y \right) \) and \( 2\sigma^2 = \left( \frac{4Y^3}{36N_c} \right)^{1/2} \) with \( Y = \ln \left( \frac{Q}{\Lambda_{\text{eff}}} \right) \), \( b = \frac{11N_c - 2n_F}{3} \), \( c_0 = \frac{a^2}{16bN_c} \) and \( a = \frac{11N_c}{3} + \frac{2n_f}{3N_c} \). The symbols \( N_c \) and \( n_F \) refer to the number of colors and number of flavours respectively. For \( N_c = 3 \) and \( n_F = 6 \), the parameter \( b \) is equal to 7. We fix the normalization constant \( A_h \) by equating the multiplicity of corresponding hadrons to their experimental value at \( Q = \sqrt{s} = 91 \) GeV and \( \Lambda_{\text{eff}} = 200 \) MeV. For pions we find \( A_h = 4.89 \).

In order to calculate indirect neutrino flux we consider the following processes.
Figure 1: Direct Neutrino flux today from 4d PBHs evaporating at redshift $z = 0$ (solid), $z = 1000$ (short dashed) and $z = 10^6$ (long dashed). Similarly indirect neutrino flux today from 4d PBHs evaporating at redshift $z = 0$ (dotted), $z = 1000$ (small spaced dots) and $z = 10^6$ (large spaced dots).

1. the decays of $\mu^+, \mu^-$ and pions.

2. the fragmentation of quarks into pions and then through the following channel $\pi \rightarrow \mu \rightarrow \nu$.

3. the decays of evaporated W-bosons through the following channel $W \rightarrow e + \nu$ and $W \rightarrow \mu + \nu$.

While calculating the indirect neutrino flux we have incorporated the fragmentation functions by modifying the expression $f(E_\nu, T)$. The final expression for $f(E_\nu, T)$ in this case is given in the Appendix of this paper.
5.3 Proton Flux

The expression for the proton flux is similarly given by

\[
\frac{dN_p}{dE_{p0}} = \frac{1}{4\pi} \times 1.5 \times 10^{-15} \int_{z_{\text{min}}}^{z_{\text{max}}} \int_{kT_{*}(1+z)}^{kT_{\text{exp}}} \frac{d(n(E_q, T))}{dE_p} \frac{1}{(kT_{*})^4 (1+z)^2} dE_p dz f(E_q, T) \left( \frac{dN_{\text{h}}}{dx} \right),
\]

where \( E_p = E_{p0}(1+z) \), \( z \) is the redshift at the time of emission, \( \frac{dN_p}{dx} = .03 \frac{dN_{\text{h}}}{dx} \), \( x = \frac{E_p}{E_q} \) and \( E_{p0} \) is the energy of proton at redshift \( z_{\text{min}} \).
6 Flux Calculation for 5D brane world PBHs

A 5D black hole emits particles with energy in the range \((E, E + dE)\) at a rate
\[
\frac{d^2N}{dt dE} = \frac{1}{4\pi^3 T^2} \frac{E^2}{\exp \left( \frac{E}{kT} \right) + 1}
\]
per particle degree of freedom. Here \(T\) is the temperature of the hole. Let \(f(E_\nu, T)\) represent the total neutrino flux of energy \(E_\nu\) given in eq. (30), then the direct diffuse neutrino flux per unit area today is:
\[
\frac{dN_\nu}{dE_\nu 0} = \frac{.009}{4\pi^4 512 A} \int_{z_{min}}^{z_{max}} \int_{kT_\nu(1+z)}^{kT_{pl}^*} \frac{d(kT_\nu)}{(kT_\nu)^5 (1 + z)^2} \frac{dn}{dz} f(E_\nu, T) dz
\]
where \(E_\nu = E_{\nu 0}(1 + z)\), \(z\) is the redshift at the time of emission, \(z_{max}\) corresponds to max redshift from which particles of energies 100 \(E\) eV can reach us and \(z_{min}\) corresponds to minimum redshift. Similarly we calculate the indirect neutrino flux and proton flux from braneworld PBHs by incorporating fragmentation function in eq. (31).

7 Results and Discussion

The observational constraints on the mass fraction of black holes at evaporation is given by the quantity \(\alpha_{evap} = \frac{\rho_{pbh}(M)}{\rho_{rad}}\). All the constraints with reason are given in table 1 of ref. [27]. We have taken upper limits on the PBH densities corresponding to their evaporation time for our calculation. The results for the neutrino and proton flux from 4d primordial black holes are given in Fig. 1 and Fig. 2 respectively. The neutrino flux from the braneworld PBH is given in Fig. 3. As we see from Figs. 1 and 3, neutrino flux at energy \(10^{20}\) eV from PBHs evaporating today is many orders of magnitude smaller compared to the existing neutrino flux limit. For early decaying PBHs it can be noticed that neutrino flux at \(10^{20}\) eV is even smaller.

For PBHs of mass \(M \approx 5 \times 10^{14}\) g, neutrino flux at \(10^{20}\) eV energy is eight orders of magnitude smaller than the neutrino flux limit at that energy. If these PBHs are clustered in galactic halos then their density may be somewhat higher. For PBHs of mass \(M \approx 10^{13}\) g and \(M \approx 5 \times 10^{10}\) g, neutrino flux at \(10^{20}\) eV is very small compared to neutrino flux limit because the number of UHE particles emitted by low mass black holes decrease rapidly.
Figure 3: Direct Neutrino flux today from 5d BWPBHs evaporating at redshift $z = 0$(solid), $z = 1000$(dotted) and $z = 10^6$ (short dashed). Similarly indirect neutrino flux today from 4d PBHs evaporating at redshift $z = 0$(dotted), $z = 1000$(long dashed) and $z = 10^6$ (large spaced dots).

as their mass decreases. Hence, although the constraints on the lower mass PBHs are much weaker, the final flux produced by the low mass PBHs turns out to be much smaller. For $M \approx 10^{13}g$ the dominant constraint comes from entropy production at nucleosynthesis. While for $M \approx 5 \times 10^{10}g$ the main constraint comes from deuterium destruction.

The astrophysical and cosmological constraints on brane world PBHs are obtained in ref. [22, 28, 29]. In ref. [28, 29] constraints on brane world PBHs are obtained from high energy diffuse gamma ray and from cosmic ray antiproton flux. We observe that neutrino fluxes are much smaller compared to 4D PBHs even considering maximum allowed PBH densities at their corresponding evaporation era. This is understandable because the temperature of 5D PBHs is small compared to the 4D PBHs of same mass. UHE neutrino flux from brane world PBHs is much smaller compared to 4D PBHs of same mass.

We also calculate proton flux from 4D PBHs and brane world 5D PBHs.
It turns out that proton flux at $10^{20}$ eV from 4D PBHs decaying today is roughly ten orders of magnitude smaller than the UHECR flux. Proton flux from early decaying 4D PBHs and brane world PBHs are even smaller.

The main uncertainties in our calculation include: (a) lack of information about the degrees of freedom available at high energies when black hole temperature exceeds the energies currently attained in collider experiments and (b) extrapolation of the fragmentation function to high energies. These unknowns at high energy might change the above picture somewhat but is unlikely to change our results qualitatively.

To conclude, we find that PBHs, decaying in the early universe, contribute negligibly to the ultra high energy cosmic ray flux. However it may be interesting to repeat our calculations for other superheavy particles, decaying in the early universe, which may give a larger contribution.

| $t_{\text{evap}}$ (sec) | $10^{17}$ | $10^{12}$ | $10^{4}$ |
|-------------------------|-----------|---------|---------|
| $l_{\text{min}}$ (cm)   | $6.214 \times 10^{-14}$ | $1.338 \times 10^{-15}$ | $2.885 \times 10^{-18}$ |
| $l_{\text{max}}$ (cm)   | .01       | .01     | .01     |
| $M_{\text{min}}$ (g)    | $1.549 \times 10^{9}$ | $4.898 \times 10^{9} g$ | $4.898 \times 10^{2}$ |
| $M_{\text{max}}$ (g)    | $3.175 \times 10^{14}$ | $1.339 \times 10^{13}$ | $2.885 \times 10^{10}$ |
| $T_{\text{min}}$        | $2.3 \times 10^{-23} T_4$ | $2.45 \times 10^{-22} T_4$ | $2.45 \times 10^{-20} T_4$ |
| $T_{\text{max}}$        | $1.71 \times 10^{-20} T_4$ | $4.053 \times 10^{-19} T_4$ | $1.88 \times 10^{-16} T_4$ |

Table 1: Mass, temperature ranges for BW PBHs at three different epochs.

8 Appendix

Here we give expressions for $f(E_\nu, T)$ for different cases.

1. The indirect neutrino flux due to decay of muons and pions may be expressed as,

$$f(E_\nu, T) = B \int f(E_\mu, T) \frac{dn_\nu(E_\mu, E_\nu)}{dE_\nu} dE_\mu,$$

(32)

where $B$ is the number of degrees of freedom. For the case of muon decay, we can express,

$$\frac{dn_\nu(E_\mu, E_\nu)}{dE_\nu} = \frac{2}{\gamma m_\mu} f(x)$$

(33)
where $x = \frac{2E_{\nu}}{\gamma m_\mu}$, 

\[ f(x) = 2x^2(3 - 2x) \quad (34) \]

for $\nu_\mu$ and

\[ f(x) = 12x^2(1 - x) \quad (35) \]

for $\nu_e$. In the present case $B = 4$ because we have contributions from $\mu^+$ and $\mu^-$. Similar calculation can be done for $\nu_\mu$ spectrum from pion decay.

2. For the fragmentation of quarks $\rightarrow$ pions $\rightarrow$ muons $\rightarrow$ neutrinos, we have

\[
 f(E_\nu, T) = B_q \int f(E_q, T) \frac{dn_\pi(E_q, E_\pi)}{dE_\pi} \frac{dn_\mu(E_\mu, E_\pi)}{dE_\mu} \frac{dn_\nu(E_\mu, E_\nu)}{dE_\nu} dE_\pi dE_\mu dE_\nu \quad (36)
\]

where $\frac{dn_\nu}{dE_\nu}$ is the neutrino spectrum in a $\mu$ decay and $\frac{dn_\mu}{dE_\mu}$ is the muon spectrum in a $\pi$ decay. $B_q$ is degrees of freedom of all six quarks and six antiquarks. Each quark has two degrees of freedom. Hence $B_q = 24$ in this case. We can express,

\[
 \frac{dn_\nu(E_{\mu}, E_\nu)}{dE_\nu} = \frac{dy}{dE_\mu} \left( g_0(y) - Pg_1(y) \right) \quad (37)
\]

in the limit $E_\mu \gg m_\mu, m_e$, where

\[
 g_0(y) = \frac{5}{3} - 3y^2 + \frac{4}{3}y^3 \quad (38)
\]

\[
 g_1(y) = \frac{1}{3} - 3y^2 + \frac{8}{3}y^3 \quad (39)
\]

for $\nu_\mu$ and

\[
 g_0(y) = 2 - 6y^2 + 4y^3 \quad (40)
\]

\[
 g_1(y) = -2 - 12y - 18y^2 + 8y^3 \quad (41)
\]

for $\nu_e$. In eq. $37$ $P$ is the projection of the muon spin in the muon rest frame along the direction of the muon velocity in the laboratory frame

\[
 P = \frac{2E_\pi r}{E_\mu(1-r)} - \frac{1+r}{1-r} \quad (42)
\]

where $r = \left( \frac{m_\mu}{m_\pi} \right)^2$. Similarly muon spectrum from pion decay is

\[
 \frac{dn_\mu(E_\pi, E_\mu)}{dE_\mu} = \frac{1}{1-r} \frac{1}{E_\pi} \quad (43)
\]

3. Similar expressions can be obtained in case of neutrinos from W-boson decays.
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