Does the Vacuum Gravitate? Rydberg Atoms Say “Probably Not!”

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One of the least understood features of our Universe is the existence of a small cosmological constant. The initial impetus was to ascribe the cosmological constant to the gravitating effects of the quantum energy of the vacuum. This seems to be wrong. We need a more detailed investigation of the quantum vacuum and its gravitating properties. Excited Rydberg atoms provide just such a laboratory. The exquisite precision attainable and the relatively large size of excited Rydberg atoms enable us to measure the gravitating properties of the vacuum. It does not gravitate as naively expected. We find an upper limit to the energy scale of gravitating vacuum energy to be \((7 \text{ GeV})^4\) considerably lower than the scale of the Standard Model \((100 \text{ GeV})^4\). This provides important limitations on how to incorporate gravity into our understanding of Quantum Field theory.

I. INTRODUCTION

One of the most pressing and perplexing problems facing theoretical physics is the explanation of the observed cosmological constant (CC) \[1–4\]. A suggestive, back-of-the-envelope calculation leads to an estimate off by 120 orders of magnitude! The calculation is motivated by the most basic features of quantum mechanics and general relativity. Quantum mechanics tells us that there is a zero-point energy while quantum field theory tells us vacuum energy is pervasive and huge. Energy is important in physics, but only energy differences enter most physical situations. This changes in general relativity, where the absolute value of energy participates in gravity. Since zero-point energy \(\hbar \omega/2\) is ubiquitous in quantum field theory, we have to sum this contribution over all modes of the field (wave numbers). The contribution to the vacuum energy density from a field of mass \(m\), incorporating a momentum cut-off \(\Lambda_{\text{UV}}\) is (Unless otherwise stated we use units with \(c = \hbar = 1\))

\[
\rho_v = \int_{\Lambda_{\text{UV}}} \frac{d^3 p}{(2\pi)^3} \frac{1}{2} \sqrt{(p^2 + m^2)} = \frac{\Lambda_{\text{UV}}^4}{16\pi^2} \tag{1}
\]

Thus, the vacuum acquires energy, and this energy should be a source of gravity. The gravitational interaction of the vacuum energy in turn gives rise to a CC. Since we do not have a quantum theory of gravity, the Planck scale \((M_P)\) represents the limit of our knowledge and is the natural candidate for the cutoff in eqn. \(1\), giving \(\rho_v\) of order \((M_P)^4\), off by the record 120 orders of magnitude from the measured value of \(\rho_v\), \((0.002 \text{ eV})^4\) \[5\].

If we are careful about introducing the cutoff, \(\Lambda_{\text{UV}}\) is replaced by the largest mass scale in the theory \[6,7\]. The Standard Model, outstandingly successful in describing physics from radioactive decay to the Higgs particle, has a typical scale of 100 GeV. Barring incredible fine-tuning and/or theoretical gymnastics, the tiny value of the CC is evidence that the vacuum does not gravitate, on cosmological scales. Explaining this is a challenge for theoretical physics. Crucial to this picture is the conviction that the vacuum energy gravitates. But is this true?

Since this problem was first noticed by Pauli almost a century ago \[8\], hundreds, if not thousands, of articles have appeared looking for a solution (For review, see \[9–11\]). Some were clever, some ingenious, some promising, but none compelling. Given our current understanding, or lack thereof, of Quantum Gravity and the generation of a CC it is logically possible that the vacuum, quantum or classical manifests itself differently on large (cosmic) scales and at meso and microscopic scales. There are numerous models embracing such dichotomy, accepting the reality of gravitating vacuum energy while providing a mechanism to vitiate its effects on the scale of our current cosmos. Some models involve the incoherent addition \[12–15\], and attendant dilution, over large distances or time scales of patches of basic vacuum energy. Others rely on damping from large times and distances (see Brax \[12\] for review and further references). All imply the existence of vacuum gravity on scales smaller than cosmic.

Experimental probes of the CC most naturally involve astronomical studies. Amazingly, precision atomic-physics experiments can be added to this list. They can determine if the quantum vacuum gravitates and offer the best, most informative, non-astronomical, measurements of the CC. An atom immersed in a background vacuum energy experiences a perturbative gravitational interaction proportional to \(r^2\) times \(\rho_v\), the vacuum energy density. Highly excited Rydberg atoms, with \(r \propto n^2\), thus feel a perturbation proportional to \(n^4\). The large values of \(n\) available \((100\ or\ more)\) coupled with the incredible accuracy of the order of \(10^{-10}\ eV\ attainable in modern Rydberg experiments \[20,21\] enable us to put a limit on thegravitating component of \(\rho_v\) of \((7\ \text{GeV})^4\). This sets an upper limit on the vacuum energy scale at 7 GeV. Although not remotely competitive to the cosmological determination of the CC this is interesting because it
eliminates most of the putative contributions from particle physics which have a scale of at least 100 GeV. This has interesting implications for cosmology and nascent theories of quantum gravity.

We find it remarkable that an atom in a terrestrial laboratory can teach us something about cosmology. More significant is that Rydberg experiments provide important information about a hitherto inaccessible question. Rydberg atoms allow us to probe non-cosmological scales at which the vacuum energy does or does not gravitate.

Failure to see evidence of the expected gravitational effect of the vacuum at scales greatly different from cosmic scales indicates the problem lies in the assumption that the vacuum gravitates.

II. WHAT ATOMS KNOW ABOUT EXPANSION OF THE UNIVERSE AND VACUUM ENERGY

Consider a Hydrogen atom in an expanding universe. We adopt the Newtonian approach of Price and Romano since this keeps the physics simple, and can be justified with a full General Relativistic treatment (see Carrera and Giuliani for a review and references). The physical position \( r \) of the electron relative to its (heavy) nucleus is,

\[
r = a(t)R
\]

where \( R \) is the comoving coordinate and \( a(t) \) the scale factor of the Universe. If the Universe expands exponentially because of a nonzero CC the scale factor is,

\[
a(t) \propto e^{V_{eff}/2} \]

where

\[
\Lambda_{CC} = \frac{8\pi}{M_P^2} \rho_v
\]

The electron, due to expansion, is subjected to a repulsive force,

\[
F_{exp} = m_e \Lambda_{CC} r
\]

where \( m_e \) is the mass of the electron. The equation of motion of the electron, including contributions from the Coulomb, centrifugal, and repulsive force is

\[
m_e \frac{d^2r}{dt^2} - \frac{L^2}{m_p r^3} = -\frac{\alpha}{r^2} + \frac{m_e \Lambda_{CC} r}{3}
\]

where \( \alpha \) is the fine structure constant and \( L \) is the angular momentum (we are using the \( m_e \) explicitly as opposed to \( \rho_v \)). From eqn. 6 we identify an effective potential,

\[
V_{eff} = \frac{L^2}{2m_e r^2} - \frac{\alpha}{r} + \frac{m_e \Lambda_{CC} r}{6} r^2
\]

Price and Romano introduce two characteristic time scales, \( T_{atom} \) the typical atomic orbital period and \( T_{exp} \) the “Hubble” expansion time. They study \( V_{eff} \) for different ratios of \( T_{atom}/T_{exp} \). For sufficiently small values of this ratio the atom remains bound and resists the expansion of the universe while for larger values (> 1/4) it escapes and shares in the expansion. This accords with an intuitive feeling that a tightly bound system decouples from expansion. The electron, however, “remembers” the expansion since it now sits in a modified potential. Rather than the purely classical \( V_{eff} \) we treat the electron quantum mechanically and use as our modified potential in Schrodinger’s equation

\[
V = \frac{\alpha}{r} - m_e \Lambda_{CC} \frac{r^2}{6}
\]

Rydberg atoms are capable of noticing this modification as a perturbation to the Coulomb potential and thus can measure or place a limit on the CC!

We rewrite \( G \) in terms of the Planck Mass \( M_P \sim 10^{19} \) GeV. If, as we see in section [14], \( \rho_v/M_P^2 \) is small \( \delta V \) leads to a perturbation to the electron energy, \( \delta E \).

As a check on the consistency of this viewpoint, namely that the electron decouples from expansion due to a local \( \rho_v \) but retains a memory with a shifted potential, we compare \( T_{atom} \) \( \sim 10^{-16} \) s for the ground state of a H atom to \( T_{exp} \) \( \sim 10^{-6} \) s, for \( \rho_v = (100 \text{ GeV})^4 \). Furthermore even after the electron decouples and is excited to a high state (say \( n = 100 \)) it is still well within the de Sitter horizon \( \sqrt{3}/\Lambda_{CC} \sim 10 \) cm and safely below the orbits (with \( n > 160 \)) which are too close to the turnover of the modified potential.

III. BRIEF OVERVIEW OF RYDBERG EXPERIMENTS

Rydberg atoms are the template of some of the most precise experiments in physics. We quote Deiglmayr et al. “With the rapid development of methods to generate cold samples of molecules and the extension of frequency combs to shorter wavelengths, measurements of molecular ....energies with sub-MHz precision are becoming possible by Rydberg-state spectroscopy ....Alkaline metal atoms offer distinct advantages for precision measurements of ....energies: their Rydberg states ....can be
reached by single-photon UV excitation from the ground state, i.e. in a range where modern frequency-metrology tools can be fully exploited. Alkali-metals can be easily laser cooled to sub-mK temperatures so that Doppler and transition time broadening become almost negligible. Finally the closed-shell nature of the ion core implies that Rydberg series of alkali-metal atoms can be accurately treated as single ionization channels with "...Ritz formula".

Experiments on Rydberg atoms are now capable of exciting the atoms to energy levels of order \( n = 100 \) while measuring the energy levels to a precision of \( 10^{-10} \) eV. Peper et al. [21] performed such a precision experiment to measure the absolute frequency of transitions from the ground state to large \( n \) Rydberg states. They first prepare sub-Doppler-cooled \(^{39}\text{K}\) samples in the \( 4s_{1/2} \) ground state by confining them inside magneto-optic traps. They excite the atoms with pulses of frequency tune-able light from the \( 4s_{1/2} \) state to over 20 different \( np_{1/2} \) and \( np_{3/2} \) Rydberg states. In these experiments \( n \) ranging from 22 to 100 are achieved. They record these Rydberg states by millimeter-wave spectroscopy with extraordinary precision. We employ these high precision energy measurements for \(^{39}\text{K}\) atoms [21] to place a limit on the magnitude of the vacuum energy density. The data is accurately described, to a few parts to \( 10^{-6} \) cm, by the modified Ritz formula (In the following formulae we use \( h = 1 \) to convert wave numbers to energies)

\[
E_{nlj} = E_I - \frac{R_K}{(n - \delta_{lj}(n))^2} \tag{10}
\]

where

\[
\delta_{lj}(n) = \delta_{0,lj} + \frac{\delta_{2,lj}}{(n - \delta_{0,lj})^2} + \frac{\delta_{4,lj}}{(n - \delta_{0,lj})^4} + \frac{\delta_{6,lj}}{(n - \delta_{0,lj})^6} + \ldots \tag{11}
\]

are the energy dependent quantum defects for the respective series. \( E_I \) is the ionization energy and \( R_K \) is the reduced Rydberg constant for \(^{39}\text{K}\).

The quantum defects \( \delta \) were originally introduced by Ritz as purely phenomenological factors which provided an accurate representation of his data. Subsequently Sommerfeld, in old quantum theory [32], and then Hartree, using quantum mechanics [33], derived these quantum defects to account for the effects of the short range corrections (due to the inner electrons) to the Coulomb potential experienced by the outer electron. Hence they are experimentally and theoretically well founded.

## IV. VACUUM INDUCED ENERGY SHIFTS

Consider an excited Rydberg atom immersed in a vacuum teeming with energy described by a constant vacuum energy density \( \rho_v \). If the vacuum gravitates, the outer electron experiences a gravitational potential given by eqn. [9] For a Hydrogen like atom the shift in energy due to this potential is given by

\[
\delta E = -4\pi m_e \rho_v \langle r^2 \rangle \frac{1}{3M_P^2} \tag{12}
\]

\[
= -4\pi m_e \rho_v \frac{n^2}{3M_P^2} \frac{[5m^2 - 3(l + 1) + 1]}{\alpha^2 m_e^2} \tag{13}
\]

where \( \langle r^2 \rangle \) is the expectation value of \( r^2 \) in the Hydrogen like atom, \( n \) is the principal quantum number, \( l \) is the angular momentum quantum number, and \( \alpha \) is the fine structure constant. For \( n >> l \), the leading order energy perturbation in \( n \) is

\[
\delta E(n) \sim -\frac{10\pi}{3} \frac{\rho_v}{\alpha^2 M_P^2 m_e^4} n^4 \tag{14}
\]

Since the perturbation \( \delta E \) (eqn [14]), shifts the energy levels, the shifts are more relevant for us than the absolute values of \( E_I \). Therefore we chose to fit the energy differences between the various \( n > 22 \) levels and \( n = 22 \). We use the \( np_{1/2} \) data from Peper et al. [21] for the fits. Using the \( np_{1/2} \) data leads to the same results. We chose \( n = 22 \) since the most precise data start from \( n = 22 \) and all the \( n \) are sufficiently high that the Ritz formula is accurate. Since the fits are relatively insensitive to defects beyond second order in eqn. [11] we fit the energy differences to the expression

\[
E_{nlj} - E_{22lj} = R_K \left[ \frac{1}{22 - \delta_{0,lj} - \frac{\delta_{2,lj}}{(22 - \delta_{0,lj})^2}} \right] \tag{15}
\]

\[
- \frac{1}{n - \delta_{0,lj} - \frac{\delta_{2,lj}}{(n - \delta_{0,lj})^2}} + (\delta E(n) - \delta E(22))
\]

We first fit the energy level differences without the perturbation to check that our fit was equivalent in quality to that in [21], where they utilize the full Ritz formula eqn. [10] including defects up to \( \delta_6 \) in eqn. [11]. After verifying this equivalence we re-did the fits including the perturbation \( \delta E \). For \( \rho_v \lesssim (7 \text{ GeV})^4 \) the fits with the \( \delta E \) are almost as good as the original Ritz formula but rapidly deteriorate with increasing \( \rho_v \). Table [1] summarizes the fit statistics. We judge the quality of the fits by comparing the \( \chi^2/\text{dof} \) (degrees of freedom) in Fig. [2] and the size and randomness of the residues of fits with \( \delta E \) to the unperturbed fits in Figure [2].

Using \( n_s = n - \delta_{lj}(n) \) instead of \( n \) in eq. [14] makes no material difference in our results and gives an energy scale that is little changed.

This is our chief result, an upper limit to the scale of a gravitating vacuum \( \rho_{vac} \lesssim (7 \text{ GeV})^4 \) (The 7 GeV limit is valid independent of the sign of \( \rho_{vac} \)). Since this scale is well below the established scales of the Standard Model it raises the question of whether vacuum energy
gravitates at all. There are claims that zero point energy does not contribute in the usual manner \[3, 37\], but vacuum energy is a broader concept than zero point energy. The Higgs potential contributes to the vacuum energy and should gravitate. The absolute value of the Higgs potential is unknown but we expect its value should be of the same order as the difference between the local maximum of the potential and its symmetry breaking vacuum (the difference between the top of the “Mexican hat and its brim”) which is again of order 100 GeV. There are also QCD contributions to vacuum energy from chiral and gluon condensates. Inflationary models depend on the gravitating of the inflaton potential. The absence of gravitating vacuum energy poses serious questions to our understanding of quantum field theory and semiclassical gravity.

V. VALIDITY OF THE EFT DESCRIPTION

A recent flurry of activity, known as the UV/IR connection \[36–38\], explores how a consistent quantum theory of gravity will manifest itself at scales well below the Planck scale \[39, 40\]. In string theory this is the attempt to avoid getting stuck in the swampland but there are also attempts to understand how gravity may restrict effective field theories (EFT). It is instructive to consider our results from the point of the EFT approach. Consider an EFT with a high scale \(\Lambda_{\text{UV}}\), either the cut-off or another relevant scale beyond which the theory no longer applies. We apply the EFT to a space-time patch, of radius \(R\), with a high energy density. The maximum energy density is \(\Lambda_{\text{UV}}^4\) in this EFT. If this patch, is large enough, however, it will collapse into a Black Hole, a state which does not appear in the EFT. Thus the EFT must break down or change. The critical size \(R_c\) for this to happen is when \(R_c\) equals the Schwarzschild radius \(R_S\)

\[
R_c = R_S = \frac{2G\Lambda_{\text{UV}}^4}{3} \frac{4\pi}{3} R_c^3 \sim \Lambda_{\text{UV}}^4 R_c^3 \frac{8}{M_P} \quad (16)
\]

The criterion for this NOT to happen is that its radius \(R\), be less than \(R_c\), i.e.

\[
R < M_P/\sqrt{8\Lambda_{\text{UV}}^4} \quad (17)
\]

The above equation imposes an IR limit to our theory and implies limitations on physical scales well below cosmic scales. Alternatively, if we restrict our attention to a very large region \((R = 1/\Lambda_{\text{IR}} \gg 1)\) then fixing \(R\) restricts \(\Lambda_{\text{UV}}\) to be small. If we investigate the universe and choose \(R\) as today’s Hubble radius, we find \(\rho_v \sim (0.002 \text{ eV})^4\), intriguingly close to the observed CC. This interconnection is very different from the expectations of the swampland that its radius gravitates at all.

VI. CONCLUSION

Rydberg atoms have a long history of utility in studies of atomic physics, chemistry and recently in quantum computing. We claim that fundamental physics and cosmology can be added to the list. The idea that atomic physics experiments are capable of giving any information about the expansion of the Universe is amazing and a celebration of the unity of physics. While Rydberg measurements of the CC will never replace conventional astronomical measurements they provide further evidence for the need to better understand the origin of the CC. What is remarkable is not that Rydberg atoms are a good way to measure the CC, and that they can do it at all!

Where Rydberg atoms add unprecedented information is in their ability to measure local effects of vacuum energy. We find precision studies of Rydberg atoms put an upper limit for gravitating vacuum energy of \(\rho_{\text{vac}} \lesssim (7 \text{ GeV})^4\). This is well below the typical scale of the Standard Model and so contradicts our naive expectations. It points to a shutting off of gravitating vacuum energy at all scales as the path to understanding the smallness of the CC on cosmological scales.

The limit to the accuracy of the vacuum energy scale attainable by measuring sufficiently precisely (so that the limit is the line width \(\Delta\)) is a factor of \(\sim 10\) for \(n = 100\) and improves as \((n/100)^{1.6}\) as we go to higher \(n\). As the precision of Rydberg atoms continues to increase and as higher excited states are studied we may someday probe QCD scales \((\sim 1 \text{ GeV})\) leading to a better understanding of how gravity can be incorporated into our quantum field theoretic descriptions of nature.
| $\rho_v$   | $(0 \text{ GeV})^4$ | $(5 \text{ GeV})^4$ | $(7 \text{ GeV})^4$ | $(8 \text{ GeV})^4$ |
|-----------|---------------------|---------------------|---------------------|---------------------|
| $\delta_0$ | 1.7108778(1)        | 1.7108779(1)        | 1.7108781(1)        | 1.7108782(1)        |
| $\delta_2$ | 0.2330963(4)        | 0.2330741(4)        | 0.2330110(7)        | 0.2329508(5)        |
| $\chi^2/dof$ | 0.7731793           | 0.7093683           | 0.8110845           | 1.2986782           |

TABLE I. Fit parameters for the best fit to the energy difference (w.r.t $n = 22$) vs $n$ data \[21\] for selected values of the vacuum energy density $\rho_v$.

**FIG. 2.** Fit residuals for the best fit to the energy difference (w.r.t $n = 22$) vs $n$ data \[21\] for selected values of the vacuum energy density $\rho_v$ (top legend of each panel).

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