On existence and semi-analytical results to fractional order mathematical model of COVID-19

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ABSTRACT
Corona virus disease 2019 (Covid-19) is an illness caused by a natural corona virus called severe acute respiratory syndromes corona virus 2 (SARS-COV-2 formally called Covid-19) which is respiratory illness and has been declared the Covid-19 outbreak a global pandemic by World Health Organization (WHO). Presently, Covid-19 becomes a health concern around the globe. In present article, we investigated the dynamics of Covid-19 infectious disease under the fractional order derivative from both theoretical as well as analytical aspects. Using fixed-point theory results, we developed the indispensable conditions for the existence of the solution of the proposed model. Further we have used the techniques of Laplace transform coupled with the Adomian’s decomposition method to obtain the semi analytical solution for the model under consideration. Finally we have provided some graphical presentation corresponding to different fractional order derivatives via Matlab for the desired solutions.

1. Introduction
The world community, despite of having modern technologies in every walk of life, is striving to fight an unknown enemy termed as corona virus, which affected the developed world more intensively as compared to the developing countries. At the closing of the year 2019, the world was strike by a virus named as novel corona virus in short (Covid-19). The Corona virus was first identified in 2019, in the city of Wuhan, China (Jasper, Kin, & Zheng, 2020). Despite the fact that there is no scientific authentication, the scientists and the researchers are still believe that the virus has been transferred from animals to beings (Ji, Wang, Zhao, Zai, & Li, 2020). Instantaneously, it is also perceptible that the transfer of the virus occurs from person to person (Chen & Guo, 2016). The World Health Organization (WHO) on October 10th, 2020, reported that the virus has hit 210 countries around the globe. The total number of deaths reported as 1,072,711 out of 37,110,933 people have been affected. The death ratio in countries like Italy, Spain and the USA were higher as compared to the other countries. The symptoms of the virus include difficulties in respiratory procedure, likewise breath spar, mild temperature, cold and other indications alike. However, other symptoms such as gastroenteritis and acoustic illness of conflicting extremity have also been reported (Wang et al., 2006).

The main cause of the eruption of the virus is, whenever the affected individuals cough or sneeze which produced small droplets and people who comes in contact with the droplets are attacked by the virus. To stop the escalation of virus, several authorities of the world community adopted the policy of a full-fledged lock down and completely closed all the activities to ensure safety for their respective citizens. At this crucial time, the world health communities and law enforcement authorities put forward their level best to prevent the escalation of the virus in the world community. At the first instance, the scientists and researchers found that the main causes of the Covid-19 are bats where these viruses are identi cal to the virus spread in China in 2003 termed as Severe Acute Respiratory Syndromes (SARS) (Ge et al., 2013; Lu, 2020). Later on, the newly born virus (Covid-19) has been compared with the virus SARS and MERS (Middle East Respiratory Syndromes) to classify the virus and presented a reliable strategy against Covid-19. For the said purpose, the researchers used to the previous studies dealing with SARS and MERS. Lu (Zhou et al., 2020) claimed that Covid-19 relates SARS-Cov and MERS-Cov. For further study, the readers are referred to (Benvenuto et al., 2020; Kumar,
Initially, it was believed that bats are the primary sources of the COvid-19. Therefore the disease was compared with Severe Acute Respiratory Syndromes (SARS) which was originated from China and then covered almost the rest of the world in 2003 (Ge et al., 2013; Lu, 2020). Later on, this virus was also compared to Middle East Respiratory Syndromes (MERS) to examine the suitable class for the virus. Beside this several other researchers argued that this Covid-19 virus belongs to the Beta-Corona genus (Benvenuto et al., 2020; Kumar, Chauhan, et al., 2020; Rambaut, 2020; Shen et al., 2021; Tian et al., 2020). It is important to note that the main source of spreading this infectious disease is due to the droplets of infected individuals excreted through nose or mouth. For instance, if infected individual is in contact with any susceptible vector through his droplet (sneeze or cough), then the susceptible has a great risk to be attacked by the virus. For precautionary measures, various authorities around the globe adopted the policies including complete lock down and smart lock down.

Since the mathematical models and biological models are presented through differential equations with integer order derivatives which are local and not able to interpret in between two integral values. Therefore the researchers are focusing on differential equations of fractional order (FODES) which can precisely explain the universal laws. Due to this outstanding property, the FODES are widely used in numerous applied fields like control systems, elasticity, bio mathematics, bio medicines, heat transfer, fluid mechanics, circuit systems and bio engineering (Chen, Jahanshahi, et al., 2020; Kilbas, Srivastava, & Trujillo, 2006; Kumar, Chauhan, et al., 2020; 2020; Kumar, Kumar, Samet, & Dutta, 2021; Miller, Kenneth, & Ross, 1993; Rahimy, 2010; Rossikhin, Yuriy, & Shitikova, 1997; Sabatier, Agrawal, & Machado, 2007).

Unlike the integer order derivatives, the fractional order derivatives are free from the restrictions of locality, having memory effect and carries hereditary properties which attracts the researchers around the globe. For further applications of the fractional differential operators the authors suggest to read (Baleanu, Machado, & Luo, 2011; Shah, Fahd, & A, 2020; Biazar, 2006; Rafei, Ganji, & Danieli, 2007; Chen, Jahanshahi, et al., 2020; Chen, Jahanshahi, et al., 2020; El-Sayad, 1998; Khan & Shah, 2015; Zhou & Hu, 2008; Kumar, Chauhan, et al., 2020; Kumar, Kumar, Samet, et al., 2021; Khan, Ullah, & Kumar, 2021; Yang, Xiao, & Su, 2010).

In the present paper, we presented a mathematical approach to Covid-19 infectious disease. We develop the numerical scheme for the underlying Covid-19 model. For qualitative analysis, we used the tools of fixed-point theory such as Layer-Schauder and Arzila-Ascoli theorem, in order to obtain the desired results. To investigate the approximate solution of the underlying model, we utilize the techniques of Laplace transform coupled with Adomian decomposition (1). The concerned techniques work on the combination of Laplace and Adomian. With the help of Adomian, the problem is decomposed into polynomial and Laplace convert that differential equation into simple algebraic equations, which then solved and converted back to subsidiary equations. At the end of the work, we acquired the approximate solution as infinite series for different fractional value of $\mu$ and provided it plots. For the aforementioned computational work, we utilized the tools of Matlab.

There are various numerical techniques to obtain the concerned solutions for different biological models, such as Homotopy Perturbation Method (HPM), Variation Iteration Method (VIM), Adomian Decomposition Method (ADM), Differential Transform (DTM) and Generalization Differential Transform (GDTM), for details (Odibat & Momani, 2008; Hu, Luo, & Lu, 2008; Dehghan, Yousefi, & Lotfi, 2011; Kumar, Singh, & Kumar, 2015; Anjum, He, & He, 2021; Ain, He, Anjum, & Ali, 2020; Anjum & He, 2019; Anjum, Suleman, Lu, He, & Ramzan, 2020; Adomian, 1994).

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Probably, Laplace Adomain Decomposition Method (LADM) is one the reliable and efficient techniques to obtained numerical approximation of different problems. Therefore, the concerned technique has been adopted for the numerical computation of our proposed problem. George Adomian (1923—1996) introduced a method to solve nonlinear differential equation, known as Adomian Decomposition Method (Koonin & Cetron, 2009), which we utilized by replacing the terms in the series form and then each term is calculated through a polynomial generated by power series. The series is considered in the manner adopted in (16) and (17). The method adopted in (17) is the representation of nonlinear terms involved in the system, while (16) has been used for the representation of linear terms that are involved in the system under consideration.

Note that modeling is a powerful tool to describe a real world situation in mathematical concepts. In recent decade, the concerned technique has attained considerable attention of researchers, due to verity of applications in all discipline of sciences. Mathematical modeling plays a key role in investigation of dynamical behavior of infected diseases and its control at the earlier stages. The area of research involving the study of infectious diseases is an area of interest for the researchers nowadays. In the
In the first section of this work, the authors presented the existence of the solutions for the concerned model. We utilize the tools of fixed point to obtain the conditions for existence of the solutions. The deserted results are obtained via Layer-Schauder and Arzila á-Ascoli theorems. Since, for the problem involving non-linearity the study and development of the criteria for the existence of the solution of the problem is very tough, therefore the researchers paid more attention to obtain approximate solutions for such a problem. The second part of this work, is committed to the general produced for obtain analytical solutions of the aforementioned Covid-19 viral disease model. In order to obtain the analytical solution, we used the techniques based on the combination of Adomian decomposition and Laplace transform known as LADM. With help of aforesaid techniques, we obtained the solution in the form of infinite series. In Section 3, the authors have provided the numerical solution and discussion of the model under consideration. Using Matlab, we obtained the deserted results in the form of infinite series up to three terms for different fractional order derivatives and presented through plots.

### 2. Preliminaries

In this section, we presented some basic definitions and results from fractional calculus, for further study, we recommend (Ali, Shah, & Khan, 2018; Chen, Jahanshahi, et al., 2020; Eberhard, 2013; Kilbas et al., 2006).

**Definition 2.1** (Chen, Jahanshahi, et al., 2020; Kilbas et al., 2006). “The fractional integral of Riemann-Liouville type of order \( \mu \in (0,1) \) of a function \( \psi \in L^1([0,\infty),\mathbb{R}) \) is defined as

\[
\mathcal{I}_0^\mu \psi(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} \psi(s) \, ds,
\]

“provided that integral on the right is point wise defined on \((0,\infty)\).”

**Definition 2.2** (Chen, Jahanshahi, et al., 2020; Kilbas et al., 2006). “The Caputo fractional order derivative of a function \( \psi \) on the interval \([0,\infty)\) is defined by

\[
\mathcal{D}_0^\mu \psi(t) = \frac{1}{\Gamma(n-\mu)} \int_0^t (t-s)^{n-\mu-1} \psi^{(n)}(s) \, ds,
\]

where \( n = [\mu] + 1 \) and \([\mu]\) represents the integer part of \(\mu\).

**Lemma 2.3** (Ali et al., 2018; Eberhard, 2013). The following result holds for fractional differential equations

\[
\mathcal{I}_0^{\mu} \left[ \mathcal{D}_0^\mu \psi(t) \right] = \psi(t) + C_0 + C_1 t + C_2 t^2 + \ldots + C_{n-1} t^{n-1},
\]

for arbitrary \( C_i \in \mathbb{R}, i = 0,1,2, \ldots, n-1 \), where \( n = [\mu] + 1 \) and \([\mu]\) represents the integer part of \(\mu\).
Definition 2.4 (Ali et al., 2018; Eberhard, 2013). We recall the definition of Laplace transform of Caputo derivative as follows:

\[ \mathcal{L}\{D^\mu \psi(t)\} = s^\mu \psi(s) - \sum_{k=0}^{n-1} s^{\mu-k-1} \psi^{(k)}(0), \]

where \( n-1 < \mu < n, n \in \mathbb{N} \).

Theorem 2.5 (Godefroy, 1991). Consider \( \mathcal{D} : \mathcal{B} \rightarrow \mathcal{B} \) be a compact and continuous mapping corresponding to the bounded set

\[ \mathcal{D} = \{ \mathcal{Y}^* \in \mathcal{B} : \mathcal{Y}^* = \Lambda \mathcal{Y}^* \text{ with } \Lambda \in [0,1] \} \]

then it assures one fixed point of the operator \( \mathcal{D} \).

2.1 Existence results for the proposed model

We are interested to investigate that whether the solution of dynamical problem/model exists or not. To answer this query we impose the mechanisms of fixed-point theory to inspect the existence of the solution of the considered problem. In addition, we discard the right hand side of the problem (1) in the form of

\[
\begin{align*}
\Phi_1(t, S, E, I, A, R, M) &= n-mS(t)-bS(t)I(t), \\
\Phi_2(t, S, E, I, A, R, M) &= bS(t)I(t) + bkS(t)A(t), \\
\Phi_3(t, S, E, I, A, R, M) &= (1-a)d_1E(t) - (r_1 + m)I(t), \\
\Phi_4(t, S, E, I, A, R, M) &= ad_2E(t) - (r_2 + m)A(t), \\
\Phi_5(t, S, E, I, A, R, M) &= r_3I(t) + r_3A(t) - mR(t), \\
\Phi_6(t, S, E, I, A, R, M) &= \gamma I(t) + \beta A(t) - \lambda M(t).
\end{align*}
\]

Assume that \( \mathcal{B} = \mathcal{C}([0, T], \mathbb{R}^6) \) be the Banach spaces, with

\[ \|\Psi\|_{\mathcal{B}} = \sup_{t \in [0, T]} [\|S(t)\| + \|E(t)\| + \|I(t)\| + \|A(t)\| + \|R(t)\| + \|M(t)\|], \]

where

Table 3. Numerical values for the parameters used in model (1).

| Notation | Numerical value | Source | Notation | Numerical Value | Source |
|----------|----------------|--------|----------|----------------|--------|
| \( S_0 \) | 8055518 | (Khan & Atangana, 2020) | \( c \) | 0.00000123 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( E_0 \) | 20000 | (Khan & Atangana, 2020) | \( a \) | 0.5 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( I_0 \) | 282 | (Khan & Atangana, 2020) | \( k \) | 0.5 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( A_0 \) | 200 | (Khan & Atangana, 2020) | \( d_1 \) | 0.1923 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( R_0 \) | 0 | (Khan & Atangana, 2020) | \( r_1 \) | 0.1724 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( M_0 \) | 5000 | (Khan & Atangana, 2020) | \( r_2 \) | 0.005 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( n \) | 0.0018 | (Ali, 2019; Chen, Jahanshahi et al., 2020) | \( \gamma \) | 0.003981 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( m \) | 0.0018 | (Ali, 2019; Chen, Jahanshahi et al., 2020) | \( \beta \) | 0.001 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
| \( b \) | 0.0000000068 | (Ali, 2019; Chen, Jahanshahi et al., 2020) | \( \lambda \) | 0.01 | (Ali, 2019; Chen, Jahanshahi et al., 2020) |
Also defined an operator
\[ \mathcal{T} \in \mathcal{D} \quad \forall \; \Psi \in \mathcal{D} \quad \text{and} \quad |\Psi_0| = \nu_0. \]

Let
\[
|\mathcal{T}\Psi(t)| = \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} Y^{\mu}(s, \Psi(s)) ds \right|
\leq |\Psi_0| + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} Y^{\mu}(s, \Psi(x)) ds.
\]

|\mathcal{T}\Psi(t)| \leq \nu_0 + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} |Y^{\mu}(s, \Psi(s))| ds,

\[ = \nu_0 + C \Gamma(-\xi + \nu M_{\gamma^n} \leq \xi, \]

\[ \Rightarrow ||\mathcal{T}(\Psi)|| \leq \xi. \]

Hence \( \mathcal{T}(\mathcal{D}) \subseteq \mathcal{D} \) and hence the operator \( \mathcal{T} \) is continuous.

Now assume that \( t_1 < t_2 \leq [0, T] \), we need to prove that the operator \( \mathcal{T} \) is completely continuous. For this let,
\[
|\mathcal{T}\Psi(t_2) - \mathcal{T}\Psi(t_1)| = \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^{t_2} (t_2-s)^{\mu-1} Y^{\mu}(s, \Psi(s)) ds \right|
\leq \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^{t_1} (t_1-s)^{\mu-1} Y^{\mu}(s, \Psi(s)) ds \right|
\]
\[ = \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^{t_1} (t_1-s)^{\mu-1} Y^{\mu}(s, \Psi(s)) ds \right|
\]
\[ \leq \xi. \]

If \( t_2 \rightarrow t_1 \), then right side of (6) approaches to Zero,
\[ \Rightarrow ||\mathcal{T}\Psi(t_2) - \mathcal{T}\Psi(t_1)|| \rightarrow 0 \quad \text{as} \quad t_2 \rightarrow t_1. \]

Hence the operator \( \mathcal{T} \) is equi-continuous and bounded. Therefore by Arzila-Ascoli theorem \( \mathcal{T} \) is relatively compact which implies that \( \mathcal{T} \) is completely continuous. By making use of the Schauder fixed-point theorem we can claim that the discussed system (1) must have at least one solution.

**Theorem 1.2.** If the supposition (H2) holds true and \( t^\mu L_{\gamma^n} \leq \Gamma(\mu + 1) \), then the considered problem has unique solution.

**Proof.** If \( \Psi, \; \Psi^* \in \mathcal{B} \), and the operator \( \mathcal{T} \mathcal{B} \rightarrow \mathcal{B} \) defined above. Now consider the following
\[ ||\mathcal{T}\Psi - \mathcal{T}\Psi^*||_B = \max_{t \in [0, T]} \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} Y^{\mu}(s, \Psi(s)) ds \right| \]
\[ \leq \max_{t \in [0, T]} \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} Y^{\mu}(s, \Psi^*(s)) ds \right| \]
\[ \leq \max_{t \in [0, T]} \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} Y^{\mu}(s, \Psi(s)) ds \right| \]
\[ \leq \max_{t \in [0, T]} \left| \Psi_0 + \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} Y^{\mu}(s, \Psi^*(s)) ds \right| \]
\[ \leq \frac{t^\mu}{\Gamma(\mu + 1)} L_{\gamma^n} ||\Psi^* - \Psi||_B. \]

Thus, \( \mathcal{T} \) is continuous. Therefore the system (1) has unique solution.

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### 2.2. General procedure of LADM for Caputo derivative

To derive the general procedure of LADM for the Caputo’s fractional derivative, we consider the following differential equation
\[ \begin{aligned}
\left\{ D^\mu u(t) + N(u(t) + R(t) &= f(t), \quad 0 < \mu \leq 1, \\
u(0) &= u_0, \quad u_0 \in \mathbb{R}. \end{aligned} \]

where \( c \) stands for Caputo fractional derivative, \( N \) represents the nonlinear term, \( R \) represent the linear term involved in the given equation and \( f(t) \) is a source term. Taking Laplace on both sides of (7) and rearranging the terms yields the following.
\[ \begin{aligned}
\mathcal{L}\{ D^\mu \alpha(t) \} &= \mathcal{L}\{ f(t) - N\alpha(t) - R\alpha(t) \} \end{aligned} \]

By applying the definition of Laplace transform for Caputo’s fractional derivative we get the following
\[ \begin{aligned}
\mathcal{L}\{ \alpha(t) \} &= \frac{\alpha(0)}{\mu} + \frac{1}{\mu^\mu} \mathcal{L}\{ f(t) - N\alpha(t) - R\alpha(t) \}. \end{aligned} \]

Let us the required solution may be expressed in the form of infinity series as
\[ u(t) = \sum_{n=0}^{\infty} u_n(t). \]

Further the nonlinear term is decomposed as
\[ \begin{aligned}
Nu(t) &= \sum_{n=0}^{\infty} P_n(t), \quad \text{where}, \\
P_n(t) &= \frac{1}{\mu(n+1) \Gamma^n} \left[ \sum_{i=0}^{n} N(\mu^i u_r) \right]_{\mu=0}. \end{aligned} \]

By plugging (9) and (10) in (8), we get the following
\[ \begin{aligned}
\mathcal{L}\{ \sum_{n=0}^{\infty} \alpha_n(\ell) \} &= \frac{\alpha(0)}{\mu} \\
+ \frac{1}{\mu^\mu} \mathcal{L}\{ f(t) - \sum_{n=0}^{\infty} P_n(t) - R \sum_{n=0}^{\infty} \alpha_n \} \end{aligned} \]

By comparison the terms on both sides of (11), we have
\[ \begin{aligned}
\mathcal{L}(\alpha_0) &= \frac{u(0)}{s}, \\
\mathcal{L}(\alpha_1) &= \frac{1}{s^\mu} \mathcal{L}\{ f(t) - \mathcal{P}_0 - \mathcal{R}_0 \}, \\
\mathcal{L}(\alpha_2) &= \frac{1}{s^\mu} \mathcal{L}\{ f(t) - \mathcal{P}_1 - \mathcal{R}_1 \} \\
& \vdots \\
\mathcal{L}(\alpha_{n+1}) &= \frac{1}{s^\mu} \mathcal{L}\{ f(t) - \mathcal{P}_n - \mathcal{R}_n \} \quad \text{for} \; n \geq 0. \end{aligned} \]
After evaluating the inverse Laplace transforms, we get the required solution as
\[ u(t) = u_0 + u_1(t) + u_2(t) + u_3(t) + \ldots \]

### 2.3. General procedure for approximate solution

In the segment, we developed the scheme for approximate solution of our proposed problem (1). Taking Laplace transform on (1), we have

\[
\begin{align*}
\mathcal{L}\{S'(t)\} &= \mathcal{L}\{u - mS(t) - bS(t)\beta(t) - cS(t)\eta(t) - dS(t)A(t) - eS(t)M(t)\}, \\
\mathcal{L}\{E'(t)\} &= \mathcal{L}\{\beta S(t)\beta(t) + bS(t)\eta(t) + cS(t)\eta(t) - (1 - a)d_1 + a d_2 + m\}E(t), \\
\mathcal{L}\{I'(t)\} &= \mathcal{L}\{(1 - a)d_1 E(t) - (t_1 + m)I(t)\}, \\
\mathcal{L}\{A'(t)\} &= \mathcal{L}\{ad_2 E(t) - (t_2 + m)A(t)\}, \\
\mathcal{L}\{R'(t)\} &= \mathcal{L}\{r_1 I(t) + t_2 A(t) - mR(t)\}, \\
\mathcal{L}\{M'(t)\} &= \mathcal{L}\{r I(t) + \beta A(t) - \lambda M(t)\}.
\end{align*}
\]

Applying the Laplace transform in the sense of Caputo fractional derivative on (13), we get

\[
\begin{align*}
\mathcal{L}\{S(t)\} &= \frac{S(0)}{s} + \frac{1}{s^2} \mathcal{L}\{n - mS(t) - bS(t)\beta(t) - cS(t)\eta(t) - dS(t)A(t) - eS(t)M(t)\}, \\
\mathcal{L}\{E(t)\} &= \frac{E(0)}{s} + \frac{1}{s^2} \mathcal{L}\{\beta S(t)\beta(t) + bS(t)\eta(t) + cS(t)\eta(t) - (1 - a)d_1 + a d_2 + m\}E(t), \\
\mathcal{L}\{I(t)\} &= \frac{I(0)}{s} + \frac{1}{s^2} \mathcal{L}\{(1 - a)d_1 E(t) - (t_1 + m)I(t)\}, \\
\mathcal{L}\{A(t)\} &= \frac{A(0)}{s} + \frac{1}{s^2} \mathcal{L}\{ad_2 E(t) - (t_2 + m)A(t)\}, \\
\mathcal{L}\{R(t)\} &= \frac{R(0)}{s} + \frac{1}{s^2} \mathcal{L}\{r_1 I(t) + t_2 A(t) - mR(t)\}, \\
\mathcal{L}\{M(t)\} &= \frac{M(0)}{s} + \frac{1}{s^2} \mathcal{L}\{r I(t) + \beta A(t) - \lambda M(t)\}.
\end{align*}
\]

Applying inverse Laplace and plugging the initial conditions on (14), we have

\[
\begin{align*}
S(t) &= S_0 + \mathcal{L}^{-1}\left[ \frac{1}{s^2} \mathcal{L}\left\{ n - mS(t) - bS(t)\beta(t) - cS(t)\eta(t) - dS(t)A(t) - eS(t)M(t) \right\} \right], \\
E(t) &= E_0 + \mathcal{L}^{-1}\left[ \frac{1}{s^2} \mathcal{L}\left\{ \beta S(t)\beta(t) + bS(t)\eta(t) + cS(t)\eta(t) - (1 - a)d_1 + a d_2 + m \right\}E(t) \right], \\
I(t) &= I_0 + \mathcal{L}^{-1}\left[ \frac{1}{s^2} \mathcal{L}\left\{ (1 - a)d_1 E(t) - (t_1 + m)I(t) \right\} \right], \\
A(t) &= A_0 + \mathcal{L}^{-1}\left[ \frac{1}{s^2} \mathcal{L}\left\{ ad_2 E(t) - (t_2 + m)A(t) \right\} \right], \\
R(t) &= R_0 + \mathcal{L}^{-1}\left[ \frac{1}{s^2} \mathcal{L}\left\{ r_1 I(t) + t_2 A(t) - mR(t) \right\} \right], \\
M(t) &= M_0 + \mathcal{L}^{-1}\left[ \frac{1}{s^2} \mathcal{L}\left\{ r I(t) + \beta A(t) - \lambda M(t) \right\} \right].
\end{align*}
\]

Assuming the solutions \( S(t), E(t), I(t), A(t), R(t) \) and \( M(t) \) in the form of infinite series give by

\[
\begin{align*}
S(t) &= \sum_{n=0}^{\infty} S_n(t), \quad E(t) = \sum_{n=0}^{\infty} E_n(t), \quad I(t) = \sum_{n=0}^{\infty} I_n(t), \\
A(t) &= \sum_{n=0}^{\infty} A_n(t), \quad R(t) = \sum_{n=0}^{\infty} R_n(t), \quad M(t) = \sum_{n=0}^{\infty} M_n(t).
\end{align*}
\]

The non-linear terms are represented as

\[
\begin{align*}
S(t)I(t) &= \sum_{n=0}^{\infty} Y_n(t), \quad S(t)A(t) = \sum_{n=0}^{\infty} Z_n(t), \quad S(t)M(t) = \sum_{n=0}^{\infty} X_n(t).
\end{align*}
\]

where \( Y_n(t), Z_n(t) \) and \( X_n(t) \) are Adomian’s polynomials defined as
\[
Y_n(t) = \frac{1}{\Gamma(n+1)} \frac{d^n}{dt^n} \left[ \sum_{j=0}^{n} \theta^j \phi_j(t) \sum_{j=0}^{n} \theta^j \phi_j(t) \right] \bigg|_{t=0^+},
\]
\[
Z_n(t) = \frac{1}{\Gamma(n+1)} \frac{d^n}{dt^n} \left[ \sum_{j=0}^{n} \theta^j \phi_j(t) \sum_{j=0}^{n} \theta^j \phi_j(t) \right] \bigg|_{t=0^+},
\]
\[
Y_n(t) = \frac{1}{\Gamma(n+1)} \frac{d^n}{dt^n} \left[ \sum_{j=0}^{n} \theta^j \phi_j(t) \sum_{j=0}^{n} \theta^j \phi_j(t) \right] \bigg|_{t=0^+}.
\]

Using (16) and (17) in (15), we get

\[
\begin{align*}
\mathcal{L}(S_0) &= \frac{S_0}{\alpha} \frac{\mathcal{E}_0}{\alpha} \frac{\mathcal{I}_0}{\alpha} \frac{\mathcal{A}_0}{\alpha} \frac{\mathcal{R}_0}{\alpha} \frac{\mathcal{M}_0}{\alpha} \\
\mathcal{L}(S_1) &= \frac{1}{\alpha} \left[ \alpha - \alpha S_0 - \alpha Y_0 - \alpha X_0 - \alpha Z_0 \right] \\
\mathcal{L}(E_1) &= \frac{1}{\alpha} \left[ \alpha - \alpha Y_0 + \alpha X_0 - (1-\alpha) \alpha Z_0 \right] \\
\mathcal{L}(I_1) &= \frac{1}{\alpha} \left[ \alpha - (1-\alpha) \alpha Y_0 + \alpha Z_0 \right] \\
\mathcal{L}(A_1) &= \frac{1}{\alpha} \left[ \alpha - \alpha Y_0 - (1-\alpha) \alpha Z_0 \right] \\
\mathcal{L}(R_1) &= \frac{1}{\alpha} \left[ \alpha - \alpha Y_0 + \alpha Z_0 \right] \\
\mathcal{L}(M_1) &= \frac{1}{\alpha} \left[ \alpha - \alpha Y_0 + \alpha Z_0 \right] \\
\mathcal{L}(S_2) &= \frac{1}{\alpha} \left[ \alpha - \alpha S_0 + \alpha X_0 - \alpha Z_0 \right] \\
\mathcal{L}(E_2) &= \frac{1}{\alpha} \left[ \alpha - \alpha Y_0 + \alpha X_0 - (1-\alpha) \alpha Z_0 \right] \\
\mathcal{L}(I_2) &= \frac{1}{\alpha} \left[ \alpha - (1-\alpha) \alpha Y_0 + \alpha Z_0 \right] \\
\mathcal{L}(A_2) &= \frac{1}{\alpha} \left[ \alpha - (1-\alpha) \alpha Y_0 + \alpha Z_0 \right] \\
\mathcal{L}(R_2) &= \frac{1}{\alpha} \left[ \alpha - \alpha Y_0 + \alpha Z_0 \right] \\
\mathcal{L}(M_2) &= \frac{1}{\alpha} \left[ \alpha - \alpha Y_0 + \alpha Z_0 \right] \\
\vdots
\end{align*}
\]
Now, applying inverse Laplace on (18), we obtained

\[
S(t) = S_0, \quad E(t) = E_0, \quad I(t) = I_0, \quad A(t) = A_0, \quad R(t) = R_0, \quad M(t) = M_0,
\]

\[
S_1 = \frac{t^\mu}{\mu!} \left[ n - mS_0 - bY_0 - bkZ_0 - cX_0 \right], \quad E_1 = \frac{t^\mu}{\mu!} \left[ bY_0 + bkZ_0 + cX_0 - (1 - a) d_1 + ad_2 + m \right]E_0,
\]

\[
I_1 = \frac{t^\mu}{\mu!} \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right], \quad A_1 = \frac{t^\mu}{\mu!} \left[ ad_2 E_0 - (r_2 + m) A_0 \right],
\]

\[
R_1 = \frac{t^\mu}{\mu!} \left[ r_1 I_0 + r_2 A_0 - m R_0 \right], \quad M_1 = \frac{t^\mu}{\mu!} \left[ \gamma I_0 + \beta A_0 - \lambda M_0 \right],
\]

\[
S_2 = \frac{t^\mu}{\mu!} \left[ \frac{n}{2\mu!} \right] \left[ (m + bI_0 + bkA_0 + cb_0) \left( n - mS_0 - bY_0 - bkZ_0 - cX_0 \right) + bS_0 \left( (1 - a) d_1 E_0 - (r_1 + m) I_0 \right) \right]
\]

\[
+ bkS_0 \left[ ar_2 E_0 - (r_2 + m) A_0 \right] + cS_0 \left[ \gamma I_0 + \beta A_0 - \lambda M_0 \right] - (1 - a) d_1 + ad_2 + m \left[ bY_0 + bkZ_0 + cX_0 \right],
\]

\[
I_2 = \frac{t^\mu}{\mu!} \left[ \frac{1}{2\mu!} \right] \left[ d_1 (1 - a) \left[ bY_0 + bkZ_0 + cX_0 - (1 - a) d_1 + ad_2 + m \right] E_0 \right] - (r_1 + m) \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right],
\]

\[
A_2 = \frac{t^\mu}{\mu!} \left[ \frac{1}{2\mu!} \right] \left[ ad_2 \left[ bY_0 + bkZ_0 + cX_0 - (1 - a) d_1 + ad_2 + m \right] E_0 \right] - (r_2 + m) \left[ ad_2 E_0 - (r_2 + m) A_0 \right],
\]

\[
M_2 = \frac{t^\mu}{\mu!} \left[ \frac{1}{2\mu!} \right] \left[ \gamma \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right] + \beta \left[ ad_2 E_0 - (r_2 + m) A_0 \right] - \lambda \left[ \gamma I_0 + \beta A_0 - \lambda M_0 \right] \right] + \ldots
\]

Proceeding on the way, the computation can obtain for other terms too and finally the solution in term of infinite series up-to three term, as given by

\[
S_n = S_0 + \frac{t^\mu}{\mu!} \left[ (n - mS_0 - bY_0 - bkZ_0) + \frac{t^\mu}{\mu!} \left[ (n - mS_0 - bY_0 - bkZ_0 - cX_0) + \frac{t^\mu}{\mu!} \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right] + \lambda \left[ \gamma I_0 + \beta A_0 - \lambda M_0 \right] \right] \right] + \ldots
\]

\[
E_n = E_0 + \frac{t^\mu}{\mu!} \left[ bY_0 + bkZ_0 + cX_0 - (1 - a) d_1 + ad_2 + m \right] E_0 \right] + \ldots
\]

\[
I_n = I_0 + \frac{t^\mu}{\mu!} \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right] + \frac{t^\mu}{\mu!} \left[ d_1 (1 - a) \left[ bY_0 + bkZ_0 + cX_0 - (1 - a) d_1 + ad_2 + m \right] E_0 \right]
\]

\[
- (r_1 + m) \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right], + \ldots
\]

\[
A_n = A_0 + \frac{t^\mu}{\mu!} \left[ ar_2 E_0 - (r_2 + m) A_0 \right] + \frac{t^\mu}{\mu!} \left[ ar_2 \left[ bY_0 + bkZ_0 + cX_0 - (1 - a) d_1 + ad_2 + m \right] E_0 \right]
\]

\[
- (r_2 + m) \left[ ar_2 E_0 - (r_2 + m) A_0 \right], + \ldots
\]

\[
R_n = R_0 + \frac{t^\mu}{\mu!} \left[ r_1 I_0 + r_2 A_0 - m R_0 \right] + \frac{t^\mu}{\mu!} \left[ r_1 \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right] + r_2 \left[ ar_2 E_0 - (r_2 + m) A_0 \right]
\]

\[
- m \left[ r_1 I_0 + r_2 A_0 - m R_0 \right], + \ldots
\]

\[
M_n = M_0 + \frac{t^\mu}{\mu!} \left[ \gamma I_0 + \beta A_0 - \lambda M_0 \right] + \frac{t^\mu}{\mu!} \left[ \gamma \left[ (1 - a) d_1 E_0 - (r_1 + m) I_0 \right] + \beta \left[ ar_2 E_0 - (r_2 + m) A_0 \right]
\]

\[
- \lambda \left[ \gamma I_0 + \beta A_0 - \lambda M_0 \right], + \ldots
\]

2.4. Numerical simulation and discussion

This part of the research work, is committed to the numerical discussion of proposed problem. We assigned the following values to the parameters as shown in Table 2. And obtained the series solution for the proposed model up-to three terms for different values of $\mu$. 

Putting the above values of parameters and $\mu = 0.99$ in (20), we have

$$S_n(t) = \frac{8065518 - 64411.48965958619t^{0.99}}{C_0} + 451.34870260107544t^{1.98} + \ldots,$$

$$E_n(t) = 2000 - 389.832t^{0.99} + 37.8332t^{1.98} + \ldots,$$

$$I_n(t) = 282 + 1881.75t^{0.99} + 2044.39t^{1.98} + \ldots,$$

$$A_n(t) = 200 + 48.8445t^{0.99} + 57.2517t^{1.98} + \ldots,$$

$$R_n(t) = 0 + 49.8254t^{0.99} + 141.871t^{1.98} + \ldots,$$

$$M_n(t) = 5000 - 48.882t^{0.99} + 4.01446t^{1.98} + \ldots.$$

Similarly, for same values parameters and $\mu = 0.98$ in (20), we have

$$S_n(t) = \frac{8065518 - 64411.48965958619t^{0.99}}{C_0} + 453.217t^{1.96} + \ldots,$$

$$E_n(t) = 2000 - 391.446t^{0.98} + 37.9898t^{1.96} + \ldots,$$

$$I_n(t) = 282 + 1889.54t^{0.98} + 2052.85t^{1.96} + \ldots,$$

$$A_n(t) = 200 + 49.0467t^{0.98} + 57.4887t^{1.96} + \ldots,$$

$$R_n(t) = 0 + 50.0316t^{0.98} + 142.459t^{1.96} + \ldots,$$

$$M_n(t) = 5000 - 49.0843t^{0.98} + 4.03108t^{1.96} + \ldots.$$
The dynamical behavior of infected population.

Figure 3. The dynamical behavior of exposed population.

Figure 2. The dynamical behavior of susceptible population.

Figure 5. The dynamical behavior of asymptotically infected population.

Similarly, putting the values parameters and \( \mu = 0.96 \) in (20), we have

\[
\begin{align*}
S_n(t) &= 8065518 - 65201.8t^{0.96} + 456.887t^{1.92} + \ldots, \\
E_n(t) &= 2000 - 394.615t^{0.96} + 38.2974t^{1.92} + \ldots, \\
I_n(t) &= 282 + 1904.84t^{0.96} + 2069.47t^{1.92} + \ldots, \\
A_n(t) &= 200 + 49.4438t^{0.96} + 57.9542t^{1.92} + \ldots, \\
R_n(t) &= 0 + 50.4368t^{0.96} + 143.612t^{1.92} + \ldots, \\
M_n(t) &= 5000 - 49.4838t^{0.96} + 4.06372t^{1.92} + \ldots
\end{align*}
\]

At then, the infinite series at \( \mu = 0.95 \), for the system (20), as given by

\[
\begin{align*}
S_n(t) &= 8065518 - 65458.8t^{0.95} + 458.688t^{1.90} + \ldots, \\
E_n(t) &= 2000 - 396.171t^{0.95} + 38.4484t^{1.90} + \ldots, \\
I_n(t) &= 282 + 1912.35t^{0.95} + 2077.63t^{1.90} + \ldots, \\
A_n(t) &= 200 + 49.6387t^{0.95} + 58.1826t^{1.90} + \ldots, \\
R_n(t) &= 0 + 50.6356t^{0.95} + 144.1787t^{1.90} + \ldots, \\
M_n(t) &= 5000 - 49.6768t^{0.95} + 4.07974t^{1.90} + \ldots
\end{align*}
\]

By assigning the values to the terms defined in system (1) from (Gao et al., 2020; Khan & Atangana, 2020), we get the following graphs.

From Figures 2–7, it can be observe that the rise of the population of infected individuals is related to the rise in the population of exposed individuals which is clearly related to the up rise of the class of the susceptible population. Therefore infections is going up with asymptotic infection that goes up. Therefore, the rate of recovery is increasing as infection and so more death cases will occur. Hence the densuseness of the class consist of removed population is raising up. The rate of increase is expressed by taking different values of the arbitrary order of the fractional derivative. It is evident that having smaller order of the fractional derivative the rate is quickly moving up and by making the order greater the rate is becoming slow.
2.5. Conclusion

At present, Covid-19 becomes the pandemic and a health concern around the globe. The world social and economic sectors have been smashed by this infectious disease. Several authorities like medical professionals, scientists and other researchers are trying their level best to put forward a reliable strategy for stopping the escalation of the virus in the community. The main aim of this article is to present and appropriate mathematical approach to Covid 19 infectious disease. For instance Using fixed-point theory and nonlinear analysis, we have proved the existence of the proposed Covid-19 model. By Laplace Adomian decomposition method, we have derived semi-analytical results. Graphical presentations are given to illustrate the dynamics of the model.

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We all authors have read and approved the final version.

Author’s contribution

All authors have equally contributed this work.

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References

Adomian, G. (1994). Solving frontier problems of physics: The decomposition method. Fund. Theories of Phy, 60.
Ain, Q. T., He, J. H., Anjum, N., & Ali, M. (2020). The Fractional complex transform: A novel approach to the time-fractional Schrodinger equation. Fractals, 28(07), 2050141. doi:10.1142/S0218348X20501418
Ali, A. (2019). Numerical solution of fractional order immunology and AIDS model via Laplace transform Adomain decomposition method. Journal of Fractional Calculus and Application, 10(1), 242–252.
Ali, A., Shah, K., & Khan, R.A. (2018). Numerical treatment for traveling wave solutions of fractional Whitham-Broer-Kaup equations. Alexandria Engineering Journal, 57(3), 1991–1998. doi:10.1016/j.aej.2017.04.012
Anjum, N., & He, J. H. (2019). Laplace transform: Making the variational iteration method easier. Applied Mathematics Letters, 92, 134–138. doi:10.1016/j.aml.2019.01.016
Anjum, N., He, C., & He, J. (2021). Two scales fractal theory for population dynamics. Fractals, 29(07), 2150182. doi:10.1142/S0218348X21501826
Anjum, N., Suleman, M., Lu, D., He, J. H., & Ramzan, M. (2020). Numerical iteration for nonlinear oscillators by Elzaki transform. Journal of Low Frequency Noise, Vibration and Active Control, 39(4), 879–884. doi:10.1177/1461348419873470
Baleanu, D., Machado, J.A.T., & Luo, A.C.J. (2011). Fractional Dynamics and Control: Springer Sci. Bus. Med. New York, Dordrecht, Heidelberg, London: Springer.
Benvenuto, D., Giovannetti, M., Ciccozzi, A., Spoto, S., Angeletti, S., & Ciccozzi, M. (2020). The 2019-new coronavirus epidemic: Evidence for virus evolution. Journal of Medical Virology, 92(202), 455–459.
Biazar, J. (2006). Solution of the epidemic model by Adomian decomposition method. Journal of Applied Mathematics and Computing., 173(2), 1101–1106. doi:10.1016/j.amc.2005.04.036
Chen, S. B., Jahanshahi, H., Alhadjii Abba, O., Solis-Perez, J., Bekiros, S., Gomz-Aguilar, J., … Chu, Y. M. (2020). The effect of market confidence on a financial system from the perspective of fractional calculus: Numerical investigation and circuit realization. Chaos, Solitons & Fractals, 140, 110223. doi:10.1016/j.chaos.2020.110223
Chen, S. B., Rashid, S., Aslam Noor, M., Ashraf, R., & Chu, Y. M. (2020). A new approach on fractional calculus and probability density function. AIMS Mathematics, 5(6), 7041–7054. doi:10.3934/math.2020451
Chen, S. B., Rashid, S., Noor, M. A., Hammouch, Z., & Chu, Y. M. (2020). New fractional approaches for n-polynomial P-convexity with applications in special function theory. Advances in Difference Equations, 2020(1), 1–31. doi:10.1186/s13662-020-03000-5
Chen, T. M., Rui, J., Wang, Q. P., Zhao, Z. Y., Cui, J. A., & Yin, L. (2020). A mathematical model for simulating the phase-based transmissibility of a novel coronavirus. Infectious Diseases of Poverty, 9(1), 24. doi:10.1186/s40249-020-00640-3
Chen, Y., & Guo, D. (2016). Molecular mechanisms of coronavirus RNA capping and methylation. Virolologia Sinica, 31(1), 3–11. doi:10.1007/s12250-016-3726-4

Dehghan, M., Yousefi, S. A., & Lotfi, A. (2011). The use of He's variational iteration method for solving the telegraph and fractional telegraph equations. International Journal for Numerical Methods in Biomedical Engineering, 27(2), 219–231. doi:10.1002/cnm.1293

Eberhard, Z. (2013). Nonlinear functional analysis and its applications: IV: Applications to mathematical physics. Springer Science & Business Medicine.

El-Sayad, A. M. A. (1998). Non-linear functional differential equation of ordinary orders. Nonlinear Anal, 33, 181–186.

Gao, W., Veeresha, P., Prakasha, D. G., & Baskonus, H. M. (2020). Novel dynamic structures of 2019-nCoV with nonlocal operator via powerful computational technique. Biology, 9(5), 107. doi:10.3390/biology9050107

Ge, X.-Y., Li, J.-L., Yang, X.-L., Chmura, A. A., Zhu, G., Epstein, J. H., … Shi, Z.-L. (2013). Isolation and characterization of a bat SARS-like coronavirus that uses the ACE2 receptor. Nature, 503(7477), 535–538. doi:10.1038/nature12711

Godefroy, M. (1901). La fonction Gamma; Theorie, Histoire. Gauthier-Villars. Imprimeur Libraire Paris

Hu, Y., Luo, Y., & Lu, Z. (2008). Analytical solution of the linear fractional differential equation by Adomian decomposition method. The Journal of Computational and Applied Mathematics, 215(1), 220–229. doi:10.1016/j.cam.2007.04.005

Jasper, F.W.C., Kin, H.K., & Zheng, Z. (2020). Genomic characterization of a bat SARS-like coronavirus that uses the ACE2 receptor. Nature, 503(7477), 535–538. doi:10.1038/nature12711

Khan, M. A., & Atangana, A. (2020). Modeling the dynamics of novel coronavirus (2019-nCoV) with fractional derivative. Alexandria Engineering Journal, 59, 2379–2389.

Kumar, S., Chauhan, R. P., Momanli, S., & Hadid, S. (2020). Numerical investigations on COVID-19 model through singular and non-singular fractional operators. Numerical Methods for Partial Differential Equations, 1–27. doi:10.1002/num.22707

Kumar, S., Ghosh, S., Samet, B., & Guofo, E. F. D. (2020). An analysis for heat equations arises in diffusion process using new Yang-Abdel-Aty-Cattani fractional operator. Mathematical Methods in the Applied Sciences, 43(9), 6062–6080. doi:10.1002/mma.6347

Kumar, S., Kumar, A., Samet, B., & Dutta, H. (2021). A study on fractional host-parasitoid population dynamical model to describe insect species. Numerical Methods for Partial Differential Equations, 37(2), 1673–1692. doi:10.1002/num.22603

Kumar, S., Kumar, R., Agarwal, R. P., & Samet, B. (2020). A study of fractional Lotka-Volterra population model using Haar wavelet and Adams-Bashforth-Moulton methods. Mathematical Methods in the Applied Sciences, 43(8), 5564–5578. doi:10.1002/mma.6297

Kumar, S., Kumar, R., Cattani, C., & Samet, B. (2020). Chaotic behaviour of fractional predator-prey dynamical system. Chaos, Solitons & Fractals, 135, 109811. doi:10.1016/j.chaos.2020.109811

Kumar, S., Kumar, R., Momanli, S., & Hadid, S. (2021). A study on fractional COVID-19 disease model by using Hermite wavelets. Mathematical Methods in the Applied Sciences, 1–17. doi:10.1002/mma.7065

Lu, R. (2020). Genomic characterisation and epidemiology of 2019 novel coronavirus: Implications for virus origins and receptor binding. The Lancet, 6736(20), 1–10.

Lu, H., Stratton, C.W., & Tang, Y.W. (2020). Outbreak of pneumonia of unknown etiology in Wuhan China: The mystery and the miracle. Journal of Medical Virology, 401–402.

Miller, Kenneth, S., & Ross, B. (1993). An introduction to the fractional calculus and fractional differential equations. Wiley.

Odibat, Z., & Momanli, S. (2008). A generalized differential transform method for linear partial differential equations of fractional order. Applied Mathematics Letters, 21(2), 194–199. doi:10.1016/j.aml.2007.02.022

Rafei, M., Ganji, D.D., & Daniali, H. (2007). Solution of the nonlinear hereditary mechanics of solids. Applied Mathematics and Computation, 187(2), 1056–1062. doi:10.1016/j.amc.2006.09.019

Rahimi, M. (2010). Applications of fractional differential equations. Applied Mathematical Sciences, 4, 2453–2461.

Rambaut, A. (2020). Phylogenetic analysis of 23 nCoV-2019 genomes.

Rossikhin, A., Yuriy., & Shitikova, V.M. (1997). Applications of fractional calculus in mechanics of solids. Applied Mechanics Reviews, 50(1), 15–67. doi:10.1115/1.3106862

Sabatier, A.J., Agraval, O.P., & Machado, J.A.T. (2007). Advances in fractional calculus (Vol. 4). Dordrecht: Springer.

Shah, K., Fahd, J., & A, T. (2020). On a nonlinear fractional order model of dengue fever disease under Caputo-Fabrizio derivative. Alexandria Engineering Journal, 59, 2305-2313.

Shen, Z. H., Chu, Y. M., Khan, M. A., Muhammad, S., Al-Hartomy, O. A., & Higazy, M. (2021). Mathematical modeling and optimal control of the COVID-19 dynamics.
Tian, X., Li, C., Huang, A., Xia, S., Lu, S., Shi, Z., … Ying, T. (2020). Potent binding of 2019 novel coronavirus spike protein by a SARS coronavirus-specific human monoclonal antibody. *Emerging Microbes & Infections*, 9(1), 382–385. doi:10.1080/22221751.2020.1729069

Wang, L.-F., Shi, Z., Zhang, S., Field, H., Daszak, P., & Eaton, B. T. (2006). Review of bats and SARS. *Emerging Infectious Diseases*, 12(12), 1834–1840. doi:10.3201/eid1212.060401

Yang, S., Xiao, A., & Su, H. (2010). Convergence of the variational iteration method for solving multi-order fractional differential equations. *Computers & Mathematics With Applications.*, 60(10), 2871–2879. doi:10.1016/j.camwa.2010.09.044

Zhou, L., & Hu, G. (2008). Global exponential periodicity and stability of cellular neural networks with variable and distributed delays. *Journal of Applied Mathematics and Computing.*, 195(2), 402–411. doi:10.1016/j.amc.2007.04.114

Zhou, P., Yang, X.-L., Wang, X.-G., Hu, B., Zhang, L., Zhang, W., … Shi, Z.-L. (2020). A pneumonia outbreak associated with a new coronavirus of probable bat origin. *Nature*, 579(7798), 270–273. doi:10.1038/s41586-020-2012-7