A new correlation for estimation of minimum miscibility pressure (MMP) during hydrocarbon gas injection

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Abstract
The satisfactory recovery of the hydrocarbon gases has made them a reliable choice for gas injection-based enhanced oil recovery (EOR) techniques. The minimum miscibility pressure (MMP) is a pivotal parameter governing the recovery factor during gas injection processes. Therefore, the determination of the authentic MMP is of a crucial importance. Due to the drawback of the experimental techniques (time and cost), empirical correlations are valuable tools in MMP determination. In this study, a multi-gene genetic programming and another software known as LINGO as an optimization tool are applied to offer a dependable MMP formula based on a comprehensive MMP dataset (a total of 108 MMP data). The independent parameters of reservoir temperature, pseudocritical temperature of the injection gas, molecular weight of C5+ components of the reservoir fluid and the intermediate (H2S, CO2, C2–C4)-to-volatile (N2 and C1) ratio are considered as input variables. A comprehensive set of experimental data covers wide span of primary parameters. Furthermore, in order to judge the accuracy of the suggested model and assess the precision and compare the predicted MMP by the current model with those estimated by preexisting correlations, the statistical and graphical error analyses have been employed. Based on the results, the proposed model can estimate MMP of the associated gas with an average absolute relative error of 9.86%. Also, the proposed correlation is more trustworthy and precise than the preexisting models in an extensive spectrum of thermodynamic circumstances. Eventually, the relevancy factor has depicted that the pseudocritical temperature of the injected gas has the most severe role in miscibility achievement.

Keywords Minimum miscibility pressure · Hydrocarbon gas · Genetic programming · Constrained multivariable search methods

Introduction
The maturity of crude oil reservoirs has compelled us toward enhanced oil recovery methods to increase recovery efficiency from such reservoirs. Among these EOR techniques, the gas injection is one of the most competent ones attributed to reservoir fluid withdrawal. In fact, during the gas injection, aggregating of the different mechanisms such as reservoir fluid viscosity reduction and tapering the interfacial tension by means of mass transfer of light and intermediate components between the reservoir oil and injected gas leads to ascendance in reservoir fluid production (Taber et al. 1997).

Since during gas injection processes both condensing and vaporization mechanisms contribute in miscibility achievement (Ahmadi and Johns 2011), the injected gas components change dramatically toward that of the associated gas during the gas injection processes. On the other hand, sometimes recycling the associated gas during the crude production is more beneficial than the injection of the CO2 and N2. To be more precise, due to the prerequisite high pressure to reach miscibility and the high cost of the N2 production by cryogenic processes for nitrogen injection, it is not very pragmatic. On the other hand, asphaltene precipitation and its consequence formation damage problems caused by CO2 injection (Fathinasab and Ayatollahi 2016; Hagedorn and Orr Jr 1994) are among the drawbacks of CO2 injections.

Accurate determination of the minimum miscibility pressure is the most decisive factor in associated gas injection operations. There are some techniques that can be employed in MMP determination. Despite of the expensive
and cumbersome experimental techniques like the slim tube (Elsharkawy et al. 1992), rising bubble apparatus (Mihcak 1994) and vanishing interfacial tension (VIT) method (Fathinasab et al. 2018), the MMP-estimating correlations are trustworthy auxiliary tools in feasibility assessment of gas injection techniques. Moreover, these correlations can be exploited in screening gas injection operations before undertaking to the costly and time-consuming experimental techniques. Although there are so many correlations in MMP determination during the carbon dioxide (CO₂) and nitrogen (N₂) injection (Fathinasab and Ayatollahi 2016; Fathinasab et al. 2015), there is no robust correlation in MMP estimation of the associated gas. To be more precise, although there are a few empirical models for MMP prediction of associated gas flooding including Kuo (1985), Maklavani et al. (2010) and Firoozabadi and Khalid (1986), these formulas have been drawn based on the skimp data points that cannot predict reliably the minimum miscibility pressure during hydrocarbon gas injection processes. That is because the independent parameters cover narrow range of affecting thermodynamic parameters. Moreover, due to the fact that these models were engendered based on incomplete data, they cannot precisely estimate the minimum miscibility pressure even in the acclaimed range of the independent factors by their authors.

Nowadays, one of the techniques that have received painstaking attention and have been widely exploited in providing accurate estimation for the various properties of chemical fluids in chemical and petroleum industries is artificial neural network (Shafiei et al. 2013; Shateri et al. 2015; Talebi et al. 2014; Zendehboudi et al. 2013, 2014). Nonetheless, since these smart techniques are black box, they do not afford a vivid interaction among the predicted value by the model as output and impressing parameters as inputs. On the other hand, genetic programming (GP), as an intelligent tool that creates a meaningful relationship (mathematical equation) among the estimated value by these models and influential parameters, has been beneficial in petroleum and chemical engineering to model some crucial properties (Bagheri et al. 2013; Gharagheizi et al. 2012; Kamari et al. 2015). This technique, so far, has not been used for development of a correlation in predicting of MMP during associated gas streams.

This work is targeted to create a more precise MMP model for hydrocarbon gas injection based on a collection of exhaustive experimental data that covers a wide range of thermodynamic conditions (temperature and pressure), injection gas and crude oil compositions that gathered from the literature (Al-Ajmi et al. 2009; Eakin and Mitch 1988; Firoozabadi and Khalid 1986; Jacobson 1972; Jaubert et al. 2002; Zuo et al. 1993). The input variables of the proposed model include reservoir temperature, average critical temperature of injection gas as the property of injection fluid, the ratio of the intermediate to volatile components and the molecular weight of pentane plus fraction of crude oil. For this purpose, GP is utilized to find an acceptable and easily usable mathematical structure for the MMP model. One of the drawbacks of GP is that it is time-consuming. To encounter this shortcoming, some other optimization methods can be coupled with it to accelerate the model development processes. One of the techniques that can be advantageous is the constrained multivariable search methods. There are numerous cases that, previously, have benefitted from the softwares employing this technique, in development of plain and precise models for anticipation of crude oil characteristics such as density and viscosity and some other PVT properties of reservoir fluid (Arabloo et al. 2014; Hemmati-Sarapardeh et al. 2013). For the prominent privileges of constrained multivariable search methods, they are considered as optimization tools that can be merged with GP to boost the precision of the developed prototype by GP. Finally, statistical and graphical error analyses as criterions to assess the precision and credibility of the obtained model and compare it with the existing models have been used. In addition, in order to find which independent parameters have more effect on the MMP, sensitivity analysis was performed. One of the potential usage of the proposed correlation is that it can be applied in any simulation software due to its more reliable predictions over the existing models.

**Model development**

**Genetic programming**

Genetic programming is among the most efficient evolutionary algorithms that, on the base of mathematically rational representation, develops an empirical model of the existing data. In fact, GP by automatically evolving a structure from different parameters constructs the mathematical model representing the system. In order to construct a model, GP firstly generates an initial random population which contains distinct individuals composed of various parameters. Each one of these individuals incorporates some trees (depending on GP parameters) that the weighted linear assemblage of these trees constructs an individual. To better illustrate, Fig. 1 shows the following formula:

\[ Y = a_0 + a_1 \log(x_1 + x_2) + a_2(5x_1^2 - \cos(x_1)) \]  

(1)

Depending on the precision of each individual in anticipating the experimental values in the current population, it will be selected for performing different genetic operations consisting of changing (mutation), merging (crossover) and elitism or replication of best submodels for construction of the next generation. Through an iterative process of
implementation of the genetic operations on the basis of the individual fitness until the construction of an individual with a reasonable accuracy or satisfying the predetermined criterion, the best model will be obtained by GP (Fathinasab and Ayatollahi 2016; Talebi et al. 2014).

**Constrained multivariable search methods**

Since finding an accurate model just by the GP algorithm takes a long time, for hastening the process and boosting the model’s precision, the GP and two other constrained multivariable search methods, namely generalized reduced gradient (GRG) and successive linear programming (SLP), were merged together. GRG utilizes nonlinear constraints manners by developing strategies for linear constraints. In order to know more about GRG, one can be referred to studies (David et al. 1986; Sharma and Glemmestad 2013). SLP method, by using linear programming as a search technique, is extensively used in oil and gas industry (Griffith and Stewart 1961). These algorithms are available in a popular software that is broadly utilized in science and technology known as Linear Interactive and General Optimizer (LINGO).

These algorithms along with the branch-and-bound approaches to discrete the model into different convex parts and the multi-start character of the LINGO that resumes the nonlinear solver from a few of ingeniously generated points (Carvalho et al. 2012) circumvent the obstacle of falling at local optimal solutions. The competency of this software in developing a model for predicting crude oil properties has been tested over the course of time (Fathinasab and Ayatollahi 2016; Fathinasab et al. 2015; Naseri et al. 2012).

To be more precise, the LINGO improves the GP proposed model estimations through optimizing of the model coefficients. The cooperation of GP and LINGO is by the way that after the construction of ten of the satisfactory models by GP, their coefficients will be optimized by LINGO. Finally, the best model based on its ability in MMP approximation has been selected as a model for associated gas MMP determination. Figure 2 manifests the cooperation of GP and LINGO algorithms in model development.

**Development of the new model**

An exhaustive dataset enveloping a broad span of reservoir temperature, oil and gas diversity has been gathered from the literature. The data bank was indiscriminately segregated into two separate subsections of the developing and examining sets. The developing set covers the 80% of the total data. (86 data were made to work for creation of the proposed model.) The rest 20% of the data bank were considered as the testing set. During the model development process, firstly, GP constructs a primary functional format to estimate the best approximation of the MMP. Then, some of the best primary correlations drawn from GP are selected for optimization of their coefficients by LINGO software. In Table 1, the parameters of GP for the best developed model from easiness and precision aspect have been shown.

The final proposed model is as follows:

\[
\text{MMP} = A + B + C 
\]

(2)

where

\[
A = R_{\text{Interm/Vol}} \left( \frac{38.81}{\text{T}_{\text{CM}}} - \frac{84.02 \times 10^2}{\text{T}_{\text{CM}}} \right) 
\]

\[
B = \text{T}_{\text{CM}} \left( \frac{38.31}{\text{MW}_{\text{C5+}}} - 0.5031 \right) + 1.802 \times 10^{-2} \times \text{T} + 165.4 
\]

\[
C = -\left( \frac{1.3166 \times 10^4}{\text{MW}_{\text{C5+}}} + 3.604 \times 10^{-2} \times \text{MW}_{\text{C5+}} \right) 
\]

where \( T \) and \( T_{\text{CM}} \) are the temperature of reservoir and critical temperature of injecting gas, respectively. Moreover, temperatures are in Kelvin (K) and MMP is in Mpa.

**Model assessment**

To evaluate the proficiency of the suggested correlation for MMP, we compare the estimated values by this model to the predicted MMPs by three previously offered famous equations, namely Firoozabadi and Khalid (1986), Kuo (1985) and Maklavani et al. (2010), through statistical and graphical error assessment.
Statistical error assessment

We use some statistical criterions including average relative error [APRE (%)], average absolute relative error [AAPRE (%)], standard deviation of error (SD) and root mean square error (RMSE) to evaluate the efficiency of the new formula and compare it with old ones. These criterions can be calculated based on the following equations.

**Average relative error**

It measures the relative deflection from the experimentally measured base data and is calculated by the following formula:

\[
\text{APRE} = \frac{100}{n} \sum_{i=1}^{n} \left| \frac{y_i - \bar{y}}{y_i} \right|
\]
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\[ E_r \% = \frac{1}{n} \sum_{i=1}^{n} E_i \% \] \hspace{1cm} (3)

where \( E_i \% \) is the relative deviance of an approximated MMP by the correlation from a true MMP and is presented as percent relative error:

\[ E_i \% = \left( \frac{MMP_{exp} - MMP_{pred}}{MMP_{exp}} \right) \times 100 \Rightarrow i = 1, 2, 3, \ldots, n. \] \hspace{1cm} (4)

Average absolute relative error

That is the relative absolute deviance of the experimental data, earned by:

\[ E_a \% = \frac{1}{n} \sum_{i=1}^{n} |E_i| \% \] \hspace{1cm} (5)

Root mean square error

It is expressive of data scattering in vicinity of zero deviance and is described by:

\[ \text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (MMP_{exp} - MMP_{pred})^2} \] \hspace{1cm} (6)

Standard deviation

This parameter is an illustrative of dissipation. The lower its magnitude, the smaller the degree of scattering. However, it is defined as:

\[ \text{SD} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left( \frac{MMP_{exp} - MMP_{pred}}{MMP_{exp}} \right)^2} \] \hspace{1cm} (7)

Graphical error analysis

To better perceive the preciseness and competency of the suggested correlation in MMP estimation, various graphical error analyses such as cross-plot and cumulative frequency have been used.

Cross-plots

These graphs reveal all approximated MMP by the specific model against their corresponding experimentally measured values. Also, a line known as unit slope line \((Y = X)\) among the approximated MMPs and real MMPs is plotted on the cross-plot. The closer the plotted data to \(45^\circ\) line, the more robust the correlation is.

Cumulative frequency

In this plot, a portion of the data points that have absolute percent relative errors below specific absolute percent relative error is plotted in an ascending order against the cumulative number of data for distinct model. To visually compare the results of different models, we sketched the results of cumulative error of various correlations on the same plot.

Results and discussion

The precision of the developed correlation was examined. Table 2 reveals the results of the implemented statistical error assessment for the MMP equations. This table attests most of previously developed formulas that approximate the MMP with huge error and cannot exactly show MMP during associated gas injection. Kuo correlation (Kuo 1985) is the worst model for MMP prediction, while Maklavani et al. (2010) correlation has given the best predictions among the previously offered correlations for the available data bank. Comparison of AARE (%) and ARE (%) for each individual correlation manifests; despite of other two correlations, Kuo (1985) correlation underestimates MMP.

Although the results of predicted MMP by Maklavani et al. (2010) correlation are more reliable among other two correlations, by no means it is sufficiently precise. The depicted results in Table 2 are a justification that the offered model is more reliable for its smallest average absolute percent relative error [AARE (%)], average percent relative error [ARE(%)], root mean square error (RMSE) and standard deviation (SD).

It should be mentioned that the proposed model predicted the developing and examining set data with the error of 9.74% and 10.32%, respectively. The root mean square error and standard deviation of the total data are 3.47 and 0.13, respectively.

| Correlation | AARE (%) | ARE (%) | RMSE | SD  |
|-------------|----------|---------|------|-----|
| Firoozabadi and Khalid (Fathinasab and Ayatollahi 2016) | 21.75 | −12.72 | 12.49 | 0.3595 |
| Maklavani et al. (2010) | 19.85 | −8.49 | 8.47 | 0.2982 |
| Kuo (1985) | 27.76 | 15.98 | 13.87 | 0.432 |
| This work (testing set) | 10.32 | 2.55 | 4.35 | 0.16 |
| This work (training set) | 9.74 | 1.74 | 3.31 | 0.12 |
In order to examine the correctness of different correlations, the experimentally measured MMP data and the estimated values by the offered formula and three existing correlations for MMP approximation are shown in cross-plot. As shown in Fig. 3, most of the predicted values for their corresponding experimental values are in the proximity of 45° line that validates the reliability of the proposed model. On the contrary, there is more scattering for other three rival correlations.

Afterward, comparing the precision and reliability of the offered model and other three models for hydrocarbon MMP estimation [Firoozabadi and Khalid (Fathinasab and Ayatollahi 2016); Kuo 1985 and the Maklavani et al. (Fathinasab and Ayatollahi 2016) models] has been implemented. Figure 4 shows the cumulative frequency for these models. It is vivid that the predicted MMP of developing set has ARE of less than 15%, while the other two well-known correlations predict just 50% of the data points with the ARE of 15%. It is seen that only the 10% of the predicted MMPs by the new correlation have the ARE higher than 20%. It is worth to mention that the Firoozabadi and Khalid (Fathinasab and Ayatollahi 2016) and the Maklavani et al. (2010) models have the relative error of the 25% and 21%, respectively.

Fig. 3 Cross-plot for the proposed MMP correlation and the previously published ones

Fig. 4 Cumulative frequency of the offered model and two previously published ones as a function of absolute percent relative error (%)
the governing parameter in miscibility achievement. Finally, the reservoir temperature has a direct relation with the MMP. It can be interpreted by the way that the more the reservoir temperature, the less the gas components condensation or mass transfer, and therefore the higher the MMP. It should be mentioned that comparison between the proposed model and the preexisting ones has been done only on the data with the values of independent variable in the claimed span by these models.

**Conclusion**

In this article, a comprehensive dataset gathered from the literature and by combination of the GP and LINGO, a new correlation for estimation of the hydrocarbon gas–crude oil MMP was proposed. In this model, MMP is an explicit function of the reservoir temperature, MWC$_{5+}$, pseudocritical temperature of the injection gas and the ratio of intermediate components to volatile ones. The offered model was accredited by experimentally measured data and the well-known correlations. The followings can be concluded from the results:

- Hydrocarbon gas–crude oil MMP is strongly affected by injection gas compositions, and the $T_{CM}$ of the injection gas has the strongest effect on MMP.
- The proposed correlation outperforms the well-known models and has a reasonable precision in MMP estimation with the relative error of the 9.86%.
- The comprehensive data bank guarantees the reliability of the proposed model compared to preexisting models in a wide range of independent parameters such as reservoir temperature.

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