Determination of friction angle of soil using double-punch test approach: An experimental study

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Abstract: Estimation of soil strength indices is prerequisite for the design of foundations, pavements, retaining walls and many other engineering interventions. Friction angle, $\phi$, is a shear strength parameter which is very crucial in assessing the stability of slopes and soil. There are a variety of options computing friction angle of soil. This paper renders the experimental study of determination of friction angle of soil applying latterly developed double-punch test method. Sandy loam soil was taken as sample soil for the study. Some factors which may affect the double-punch test...

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PUBLIC INTEREST STATEMENT

Prior to undertaking a structural project, it is essential to know the mechanical characteristics of the soil. The safety of any geotechnical structure is principally depends on the strength of the soil. Soils generally fail in shear. The shear strength of a soil mass may be broadly defined as the ability of the soil mass to resist failure and sliding along any plane inside it. It is very crucial to understand the nature of shearing resistance with the aim of analyzing the stability of soil masses. Friction angle of soil is a shear strength parameter. There are a range of computing approaches exists for the determination of friction angle of soil. Due to several inconsistencies of the prevalent soil friction angle determining method, this current study was carried out using double-punch test approach to perform better estimation of the angles. This newly developed testing approach appears promising for practical use.
were also studied thoroughly. These factors include sample-punch size, punch diameter, moisture content etc. The formulation of punch equation is based on plasticity theory of the upper bound limit theorem. The calculation of soil strength and its related parameters require only the simple linear analysis. The study revealed that the friction angle of soil obtained for different moisture contents were realistic and the double punch test may be treated as the most useful technique of determining tensile strength of both rigid and loose materials.

Subjects: Engineering & Technology; Civil, Environmental and Geotechnical Engineering; Soil Mechanics

Keywords: friction angle; soil mechanics; double-punch test; direct shear test; unconfined compressive strength

1. Introduction
Existing engineering practices use ultimate compressive strength and tensile strength as the characteristic properties of any artificially made rigid block like compacted soil or sand cement block etc. In fact, these strengths are the aggregated effects of strength properties such as, cohesion $c$, friction angle $\phi$ and unit weight $\gamma$ of the material of rigid block. A wide variety of techniques for instance, direct pull test or Briquette test, split cylinder test or Brazilian test are used to measure the tensile strength of concrete, mortar and soil block.

In the case of direct pull test, the specimen of fixed shape known as the Briquette is pulled by the gripping devices called Briquette apparatus. The major drawbacks of this test are the development of stress concentration near the gripping devices at which the premature failure occurs and difficulty to eliminate eccentricity of the line of action of the applied load (Evans, 1946; Humphreys, 1957). Besides, the direct pull test is accomplished after 3 to 7 days of curing. However, it is crucial to know the tensile strength of concrete after 28 days of curing specially for design purposes. Owing to the complication involved in applying direct tension to concrete, the split-cylinder test has gained wide acceptance (Bazant, Kazemi, Hasegawa, & Mazars, 1991) although, no authentic concept of the strength of the concrete under combined stress has developed for the fruitful practical application (Övünç, 1969). In the Brazilian or split-cylinder test, a cylindrical specimen laid horizontally is loaded along two opposite generators until the specimen splits across the vertical diametric plane. Its physical characteristics are also influenced by the alteration of temperature and moisture content (Hanson, 1968). However, results obtained from the split test are more or less representative of the exact figure determined.

In consequence of several discrepancies of the aforementioned methods, a plasticity solution for tensile problem of both the soft and rigid materials has been introduced. The output resulting from the theory of perfect plasticity using the limit analysis technique is found to be identical to that of derived from the theory of linear plasticity (Chen, 1975). The achievement in applying the theory of perfect plasticity to the problem of split tensile test suggests an alternative testing technique for the determination of the tensile strength of concrete, sand, stone or soil. The punch test is likely to be one of the accurate procedures in this regard. In this test, generally compressive load is used to determine the tensile strength.

The double-punch test is an indirect tensile test originally proposed by Chen (1970, 1975) aimed at measuring the tensile strength of concrete and recently reviewed by Chao, Karki, Cho, and Waweru (2011). In this method, a compressive load is applied to cylindrical or cubical specimen along two opposite faces. This condition sets up an almost uniform tensile stress over the vertical plane comprising applied load. The specimen splits similar to the splitting test. The double-punch test also known as the Barcelona test is believed to be more attractive than the conventional split cylinder test or Brazilian test because: (1) the testing method is much easier to perform than that of the
splitting test; (2) a lighter applied load is required to crack the specimen and (3) the estimated tensile strength in a double-punch test gives an average of strength on several fractured diametric planes, compared to one crack plane in the Brazilian test (Wei & Chau, 2000).

The objectives of the work explored here are to determine the friction angle $\varphi$, cohesion $c$ of soil and to formulate punch test by means of limit analysis. Influences of several variables such as punch size factor, punch diameter, moisture content on ultimate compressive strength are also examined. In addition to that, the comparison of friction angle of soil by direct shear test and double-punch test are also presented in this study.

2. Theoretical considerations

The precise resolution of soil strength requires undisturbed soil samples extracted from the field, carefully placement in the testing equipment and loading under controlled conditions (Freitag, 1985). Soil strength at failure is important because the reliability and economy of constructions depend on its measurement (Trofimenkov, 1977). The volume change behavior of soil affects the stability of the structures involving compacted soils (Kai, Peng, & Jian-qing, 2011). The movement of soil particles while failing depends on the structural arrangement of particles in coarse-grained soils and the degree of bonding between adjacent particles in fine-grained soils (Harris, 1971).

2.1. Punch test

A punch is a narrow or smaller diameter loading plate through which the external load is applied to a rigid block of rock or soil having large exposed surface. The cross-sectional area of a punch must be less than the exposed area of the rigid block. The method of determining the failure load of a block loaded by a punch is called by the punch test.

2.2. Limit analysis

The limit theorem revealed by Drucker, Prager, and Greenberg (1952) is a metal theory. Currently, the hypothesis is used in investigating the stability problems of soils (Chen, 1975). Punch test is principally performed for breaking a rigid block to analyze the stability problem. The limit theorem, therefore, can be applied in this case.

The externally applied load at which the rigid block of a material collapses is called the limit load. The limit load is analogous to the ultimate failure load. The method of determining the limit load is known as the limit analysis. The limit analysis can be classified into two categories such as the (i) lower bound limit analysis and (ii) upper bound limit analysis.

The theory of upper bound limit analysis expresses that the loaded block will fail if the rate of work done by the applied load exceeds the internal rate of dissipation for any assumed failure mechanism (Chen, 1975). Equating internal and external energies for any failure mechanism, therefore, gives an upper bound value of the collapse load.

In mathematical form, \[ \sum W = \sum D_i \] (1)

Where, $W$ and $D_i$ are the work and energy dissipation respectively.

2.3. Work

Work may be defined as the product of a force and a displacement in the direction of the force (Leet, Uang, & Gilbert, 2002).

Generally the work, $W$ can be expressed as $W = F \times \delta$ (2)

Here, $F$ is the force and $\delta$ is the component of displacement covered in a definite time $t$ in the direction of the force. The work done per unit time or the rate of work done can be expressed symbolically as $\frac{W}{t} = F\delta/t = FV$ (3)
where, $V$ is the velocity and the rate and capacity of doing work is the power and energy respectively.

2.4. Energy dissipation

The dissipation of energy along a failure plane depends on the nature of yielding along it. The failure along a plane may occur due to simple shearing, frictional sliding, and tensile separation or a combined effect of the above causes. In simple shearing or sliding failure, the effect of tensile separation is zero.

2.5. Double-punch test

Double-punch test is an indirect tensile test that is based on the compressive load applied on the two opposite faces of the rigid block (Figure 1). This condition sets up almost uniform tensile stress over the vertical plane containing the applied load, and the specimen splits across the plane similar to the splitting test.

The upper bound technique of limit analysis determines the load bearing capacity of punches. The boundary condition for this problem is that the contact area of the punch and the base of sample block would remain plane at any load level. Theoretical prediction of forces always associates with the modes of soil failure incurred while loading by a punch. When the stress imposed in the soil exceeds its available strength, the soil fails. The soil deformation associating with the failure under compression appears either in a compressive way or by brittle fissuring depending on the spherical pressure. The analysis of punch problem frequently uses plasticity theory for its relevance to soil failure.
2.6. Yield criterion

In the field of soil mechanics, a simple theory of soil failure originally expressed by Coulomb (1776) and later generalized by Mohr (1900) has satisfied the needs to deal with stresses of soil at failure. Coulomb defined as a linear function of the total normal stress and put forth the strength equation as

\[ \tau = c + \sigma \tan \phi \]  

This is a straight line relating \( \tau \) to \( \sigma \) acting on the failure plane shown in Figure 2. It measures soil cohesion \( c \) as the intercept, \( \tan \phi \) as the slope of line and \( \phi \) as the internal friction angle of the soil. When dealing with saturated soils, the total normal stress

\[ \sigma = \sigma' + u \]  

where, \( \sigma' \) = effective stress and \( u \) = pore pressure.

The Mohr-Coulomb failure criterion, expressed in terms of effective stress, will be of the form

\[ \tau = c' + \sigma' \tan \phi' \]  

where, \( c' \) = cohesion and \( \phi' \) = friction angle, based on effective stress. The effective stress \( \sigma' \) allows the component of total stress carried by the pore fluid pressure. The actual values of \( c \) and \( \phi \) depend both on the drainage condition and consolidation history of the soil specimen being tested.

3. Methodology

3.1. Materials and testing procedures

Moderately arid sandy loam soil was collected and subsequently grind for the study. The samples were then sieved by the US Standard Sieve No. 4. The sieved soils were moistened with water and mixed thoroughly. The moistened soil piled on a flat metallic pan and was kept under polyethylene cover for 24 hrs. for the maturation of soil moisture content. Then the samples were ready for the preparation of cylindrical soil block. The circular timber punches of 5.08, 10.16, 12.7 and 15.24 cm in diameter and approximately 2.5 cm in thickness were made in pairs from rain-tree timber. The peripheral edges of each punch were made as sharp and vertical as possible. The cylindrical mold of 15 cm diameter and 15 cm height was made of iron. A top collar of 5 cm height was fastened to the mold. Initially, the cylindrical metallic molds were placed on a base plate with two stands threaded at top ends and then tightened with a screw to keep the mold in position. The prepared moistened soil poured into the mold up to its one-third height and was compacted by 25 blows with the ham-mer of modified Proctor test. The blows were applied to the entire soil surface to ensure uniform compaction. The compaction procedure was repeated in three equal layers. After compaction, the top collar of mold was removed and the exposed surface of cylindrical soil sample was labeled. It was then weighed and bulk density was calculated. All tests were performed at very low water content (degree of saturation, \( S_r \leq 8 \)) and all samples were considered as dry soil.

The soil sample was extruded from compaction mold using Universal Compression Testing Machine capable of 2 kg load as the minimal value. Two punches were placed vertically, one on the
top and another is vertically at the bottom of the soil sample. The punches were placed painstakingly on the opposite faces of the soil sample for the avoidance of eccentricity in loading and sample was laden slowly to elude sudden failure. The average values of three samples were taken as the compressive load for a specific punch. The soil moisture content of crushed soil was figured from the average value of six samples. In total, 11 Nos. of cylindrical soil specimens were tested. Among those, nine specimens with replication of three samples for each of three punches and the rest two replications for full loading were cast. The test was conducted with the same soil of four different moisture content. Moreover, it is noted here that entire tests were performed considering total stress only.

3.2. Soil strength and parameters

The load carrying capacity of circular punches depends on soil density as a function of its water content and the dimensions of punch and soil sample. The solution to punch equation requires the values of wedge angle $\alpha$, soil friction $\varphi$ and shape parameter, $bh/a^2$.

Mathematically,

$$
\frac{bh}{a^2} = \tan \alpha \left[ \frac{1 + \sin \varphi}{1 - \sin \varphi} + \frac{1}{m} \left( \frac{q}{q_u} - 1 \right) \right]
$$

Alternatively,

$$
\frac{bh}{a^2} = \tan \alpha \left[ \tan^2 \left( 45^\circ + \frac{1}{2} \varphi \right) + \frac{1}{m} \left( \frac{q}{q_u} - 1 \right) \right]
$$

Or, $\frac{bh}{a^2} = K + M \left( \frac{q}{q_u} - 1 \right)$

(7)

Here, $M = 1/m$ = slope and $m$ = stress ratio

The soil strength, $q$ and $q_u$ are experimental parameter and the term $(q/q_u - 1)$ is the gain in strength over the unconfined compressive strength, $q_u$. The linearity of $bh/a^2$ with $(q/q_u - 1)$ carried out by the least square method given the value of $M$ as the slope that in turn resulted the value of $m = q_u/q_u$.

The intercept, $K$ arrived from linearity analysis apparently equals to $\tan^2(45^\circ + \frac{1}{2} \varphi)$. In fact, the value of $\varphi$ cannot be calculated from $K$ until the angle $\alpha$ is known. However, the angle could be calculated using the linearity of equation rewritten as

$$
\frac{bh}{a^2} = \tan \alpha \tan^2 \left( 45^\circ + \frac{1}{2} \varphi \right) + \tan \alpha \left( \frac{q}{q_u} - 1 \right) m
$$

Or, $\frac{bh}{a^2} = K' + M' \left( \frac{q}{q_u} - 1 \right)$

(9)

Where, $M' = \tan \alpha$ or, $\alpha = \tan^{-1}(M')$ and $K' = \tan \alpha \tan^2 \left( 45^\circ + \frac{1}{2} \varphi \right)$

Or, $\tan^2 \left( 45^\circ + \frac{1}{2} \varphi \right) = K'/M'$

(10)

The solution to Equation (9) yields the values of $\alpha$ and $\varphi$. Once $\varphi$ is known, the cohesion of soil as obtained from the following equation.

$$
c = \frac{1}{2} q_u \tan \left( 45^\circ - \frac{1}{2} \varphi \right)
$$

(11)
4. Result and discussions

4.1. Experimental outcomes of friction angle of soil
This piece of research work was conducted to measure friction angle of soil and its related parameters particularly in a tensile state adopting the double-punch test that based on the ultimate compressive strength of soil when loaded by punches. The mode of failure of a soil specimen is given herein (Figure 3).

It was found from the experimental study that the friction angle ($\phi$) of soil using double-punch test approach was 32.55° (See Appendix A) while friction angle ($\phi$) of soil sample through Direct shear Test was achieved 33° (See Appendix A). In consequence of unstable reading given by the dial gauge, the test result was varied to some extent.

4.2. Effect of soil moisture content on bulk unit weight and unconfined compressive strength
Bulk unit weight with a moisture content of different soil samples are tabulated in Table 1. The relationship between bulk unit weight with soil moisture content (Figure 4) shows that bulk unit weight

| Punch diameter (cm) | Bulk unit weight (kN/m$^3$) | Moisture content (%) |
|---------------------|-----------------------------|----------------------|
| 15.24               | 13.4                        | 2.5                  |
| 12.7                | 13.6                        | 3.1                  |
| 10.16               | 13.5                        | 2.5                  |
| 5.08                | 13.7                        | 4.1                  |

Figure 4. Changes in bulk unit weight and ultimate compressive strength with soil moisture content.
gradually increases with the soil moisture content. This is attributable to the almost equal weight of cylindrical soil blocks.

The relationship of unconfined compressive strength, $q_u$, with soil moisture content, is given in Figure 4 shows that $q_u$ reaches the maximum value at a specific soil moisture content and above or below of which there exist two specific soil moisture content and above or below of which there exist two values of unconfined compressive strength for the same soil. This is the indication of Proctor results. In this test, relative density curve looks like an inverted letter “V” in which, there must be two intersecting points for a finite density other than optimum density. The maximum observed $q_u$ was noticed at about 4.1% soil moisture content. It may therefore be included that the soil moisture content has the dominant role on the unconfined compressive strength.

### 4.3. Effect of punch size on ultimate compressive strength

Punch diameter, $d$ comparing to the diameter, $D$ of cylindrical soil block was expressed by the dimensionless punch size factor $d/D$ ratio. The relationship of coefficient of confinement ($q_u/f_c'$) soil with punch size factor $d/D$ is given in Figure 5 while the effect of punch diameter, $d$ on the ultimate compressive strength is shown in Figure 6.

![Figure 5. Effect of punch size factor on co-efficient of confinement.](image1)

![Figure 6. Changes in ultimate compressive strength with punch diameter.](image2)
Punch size factor and punch diameter significantly affects ultimate compressive strength as shown in Figures 5 and 6 respectively. Ultimate compressive strength decreases gradually with increasing of punch size. At smaller punch size, the outer region provides wide peripheral confinement aiding lateral movement of soil and hence ultimate compressive strength is high. Conversely, with the increases of punch size, the peripheral confinement also shrinks which lessens lateral movement of soil and eventually the failure strength is reduced.

5. Conclusion
The tensile strength parameter of soil for instance, friction angle is fairly impossible to get by the conventional split cylinder test though it is essentially required for the design of earth retaining structure in the active state of the soil. In this regard, the developed double-punch test is the unique technique for the determination of tensile strength parameter of both the rigid and loose materials. The major advantage of double-punch test over the conventional split cylinder test is that the soil is loaded with double punches always fails along the weakest plane which is inconvenient to identify in the split cylinder test. The formulation of punch equation is self-explanatory and the calculation of soil strength and its related parameters require only the simple linear analysis.

The present study was conducted neglecting the mechanical properties which were influential parameters of soil strength. Therefore, there are a number of areas in which further study could be conducted. The effect of sample height on soil strength and the influences of cohesion, c and friction angle, \( \varphi \) on failure strength still necessitate deeper understanding.

Notations

- \( \alpha \): Wedge angle
- \( \delta U, \delta V, \delta W \): Velocity vectors
- \( \theta \): Angle of velocity vector
- \( a \): Punch radius
- \( b, h \): Radius and height of cylindrical soil sample
- \( M, K \): Linear Parameters
- \( Q \): Applied vertical load
- \( q_u, q_t \): Unconfined compressive and tensile strength of soil
- \( f' \): Uniaxial compressive strength for full loading
- \( MPa \): Mega Pascal
- \( KN/m^3 \): Kilo Newton per cubic meter

Supplementary material
Supplementary material for this article can be accessed here [https://doi.org/10.1080/23311916.2017.1419415](https://doi.org/10.1080/23311916.2017.1419415).

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Cover image
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Appendix A

Calculation of friction angle (φ)

From Equation (10) we know,
\[
\tan^2 \left( 45^\circ + \frac{1}{2} \varphi \right) = \frac{K}{M}
\]
\[
\tan^2 \left( 45^\circ + \frac{1}{2} \varphi \right) = \frac{3.29}{0.988}
\]
\[
\varphi = 32.55^\circ
\]

Friction angle of soil by Direct Shear Test

From Equation (4), we know,
\[
\tau = \sigma + c \tan \varphi
\]
Or, \[
\varphi = \tan^{-1} \left( \frac{\tau - c}{\sigma} \right)
\]
\[
\varphi = \tan^{-1} \left( \frac{0.49 - 0.129}{0.556} \right) = 33^\circ
\]

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