Noise fluctuations and drive dependence of the skyrmion Hall effect in disordered systems

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Abstract

Using a particle-based simulation model, we show that quenched disorder creates a drive-dependent skyrmion Hall effect as measured by the change in the ratio \( R = \frac{V_{\perp}}{V_{\parallel}} \) of the skyrmion velocity perpendicular \( (V_{\perp}) \) and parallel \( (V_{\parallel}) \) to an external drive. \( R \) is zero at depinning and increases linearly with increasing drive, in agreement with recent experimental observations. At sufficiently high drives where the skyrmions enter a free flow regime, \( R \) saturates to the disorder-free limit. This behavior is robust for a wide range of disorder strengths and intrinsic Hall angle values, and occurs whenever plastic flow is present. For systems with small intrinsic Hall angles, we find that the Hall angle increases linearly with external drive, as also observed in experiment. In the weak pinning regime where the skyrmion lattice depins elastically, \( R \) is nonlinear and the net direction of the skyrmion lattice motion can rotate as a function of external drive. We show that the changes in the skyrmion Hall effect correlate with changes in the power spectrum of the skyrmion velocity noise fluctuations. The plastic flow regime is associated with \( 1/f \) noise, while in the regime in which \( R \) has saturated, the noise is white with a weak narrow band signal, and the noise power drops by several orders of magnitude. At low drives, the velocity noise in the perpendicular and parallel directions is of the same order of magnitude, while at intermediate drives the perpendicular noise fluctuations are much larger.

1. Introduction

Skyrmions in magnetic systems are particle-like objects predicted to occur in materials with chiral interactions [1]. The existence of a hexagonal skyrmion lattice in chiral magnets was subsequently confirmed in neutron scattering experiments [2] and in direct imaging experiments [3]. Since then, skyrmion states have been found in an increasing number of compounds [4–8], including materials in which skyrmions are stable at room temperature [9–14]. Skyrmions can be set into motion by applying an external current [15, 16], and effective skyrmion velocity versus driving force curves can be calculated from changes in the Hall resistance [17, 18] or by direct imaging of the skyrmion motion [9, 14]. Additionally, transport curves can be studied numerically with continuum and particle based models [19–23]. Both experiments and simulations show that there is a finite depinning threshold for skyrmion motion similar to that found for the depinning of current-driven vortex lattices in type-II superconductors [24–26]. Since skyrmions have particle like properties and can be moved with very low driving currents, they are promising candidates for spintronic applications [27, 28], so an understanding of skyrmion motion and depinning is of paramount importance. Additionally, skyrmions represent an interesting dynamical system to study due to the strong non-dissipative effect of the Magnus force they experience, which is generally very weak or absent altogether in other systems where depinning and sliding phenomena occur.

For particle-based representations of the motion of objects such as superconducting vortices, a damping term of strength \( \alpha_m \) aligns the particle velocity in the direction of the net force acting on the particle, while a Magnus term of strength \( \alpha_m \) rotates the velocity component in the direction perpendicular to the net force. In most systems studied to date, the Magnus term is very weak compared to the damping term, but in skyrmion...
systems the ratio of the Magnus and damping terms can be as large as $\alpha_m / \alpha_d \sim 10$ [17, 19, 21, 29]. One consequence of the dominance of the Magnus term is that under an external driving force, skyrmions develop velocity components both parallel ($V_{||}$) and perpendicular ($V_{\perp}$) to the external drive, producing a skyrmion Hall angle of $\theta_h = \tan^{-1}(R)$, where $R = |V_{||}/V_{||}^2|$. In a completely pin-free system, the intrinsic skyrmion Hall angle has a constant value $\theta_h^{int} = \tan^{-1}(\alpha_m / \alpha_d)$; however, in the presence of pinning a moving skyrmion exhibits a side jump phenomenon in the direction of the drive so that the measured Hall angle is smaller than the clean value [22, 23, 30]. In studies of these side jumps using both continuum and particle based models for a skyrmion interacting with a single pinning site [22] and a periodic array of pinning sites [30], $R$ increases with increasing external drive until the skyrmions are moving fast enough that the pinning becomes ineffective and the side jump effect is minimized.

Particle-based studies of skyrmions with an intrinsic Hall angle of $\theta_h^{int} = 84^\circ$ moving through random pinning arrays show that $\theta_h = 40^\circ$ at small drives and that $\theta_h$ increases with increasing drive until saturating at $\theta_h = \theta_h^{int}$ for high drives [23]. In recent imaging experiments performed in the single skyrmion limit [31] it was shown that $R = 0$ and $\theta_h = 0$ at depinning and that both quantities increase linearly with increasing drive; however, the range of accessible driving forces was too low to permit observation of a saturation effect. These experiments were performed in a regime of relatively strong pinning, where upper limits of $R \sim 0.4$ and $\theta_h = 20^\circ$ are expected. A natural question is how universal the linear behavior of $R$ and $\theta_h$ is as a function of drive, and whether the results remain robust for larger intrinsic values of $\theta_h$. It is also interesting to ask what happens in the weak pinning limit where the skyrmions form a hexagonal lattice and depin elastically. In studies of overdamped systems such as superconducting vortices, it is known that the strong and weak pinning limits are separated by a transition from elastic to plastic depinning and have very different transport curve characteristics [24, 26], so a similar phenomenon could arise in the skyrmion Hall effect. Noise fluctuations have also been used as another method to study the dynamics of magnetic systems [32]. In superconducting vortex systems, the plastic flow regime is associated with large voltage noise fluctuations of $1/f$ form [33–36], while when the system dynamically orders at higher drives, narrow band noise features appear and the noise power is strongly reduced [26, 37–39]. Here we show that changes in the skyrmion Hall angle are correlated with changes in the skyrmion velocity fluctuations and the shape of the velocity noise spectrum. In the plastic flow region where $R$ increases linearly with drive, there is a $1/f$ velocity noise signal with $\alpha = 1.0$, while when $R$ reaches its saturation value, there is a crossover to white noise or weak narrow band noise, indicating that noise measures could provide another way to probe skyrmion dynamics. In general, we find that the narrow band noise features are much weaker in the skyrmion case than in the superconducting vortex case due to the Magnus effect.

**Simulation and system**— We consider a 2D simulation with periodic boundary conditions in the $x$ and $y$-directions using a particle-based model of a modified Thiele equation recently developed for skyrmions interacting with random [21, 23] and periodic [30, 40] pinning substrates. The simulated region contains $N$ skyrmions, and the time evolution of a single skyrmion at position $\mathbf{r}_i$ is governed by the following equation:

$$\alpha_d \dot{\mathbf{v}}_i + \alpha_m \mathbf{z} \times \mathbf{v}_i = \mathbf{F}_i^{\text{ss}} + \mathbf{F}_i^{\text{p}} + \mathbf{F}_D.$$  \hspace{1cm} (1)

Here, the skyrmion velocity is $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$, $\alpha_d$ is the damping term, and $\alpha_m$ is the Magnus term. We impose the condition $\alpha_d^2 + \alpha_m^2 = 1$ to maintain a constant magnitude of the skyrmion velocity for varied $\alpha_m/\alpha_d$. The repulsive skyrmion–skyrmion interaction force is given by $\mathbf{F}_i^{\text{ss}} = \sum_{j=1}^{N} \mathbf{K}_i(\mathbf{r}_i - \mathbf{r}_j)$, where $\mathbf{K}_i = (\mathbf{r}_i - \mathbf{r}_j)$, $\mathbf{r}_i = (x_i, y_i)$, and $\mathbf{K}_i$ is a modified Bessel function that falls off exponentially for large $r_i$. The pinning force $\mathbf{F}_i^{\text{p}}$ arises from non-overlapping randomly placed pinning sites modeled as harmonic traps with an amplitude of $F_p$, and a radius of $R_p = 0.3$ as used in previous studies [23]. The driving force $\mathbf{F}_D = F_D \hat{x}$ is from an applied current interacting with the emergent magnetic flux carried by the skyrmion [17, 29]. We increase $F_D$ slowly to avoid transient effects. In order to match the experiments, we take the driving force to be in the positive $x$-direction so that the Hall effect is in the negative $y$-direction. We measure the average skyrmion velocity $V_{||} = \left( N \sum_{i=1}^{N} v_{ix} \cdot \mathbf{z} \right) / \left( N \sum_{i=1}^{N} v_{ix} \cdot v_{ix} \right)$ in the direction parallel (perpendicular) to the applied drive, and we characterize the Hall effect by measuring $R = |V_{||}/V_{\perp}|$ for varied $F_D$. The skyrmion Hall angle is $\theta_h = \tan^{-1} R$. We consider a system of size $L = 36$ with a fixed skyrmion density of $\rho_{sk} = 0.16$ and pinning densities ranging from $n_p = 0.006$ to $n_p = 0.2$.

### 2. Results and discussion

In figures 1(a) and (b) we plot $|V_{||}|, |V_{\perp}|$, and $R$ versus $F_D$ for a system with $F_p = 1.0, n_p = 0.1$, and $\alpha_m/\alpha_d = 5.708$. In this regime, plastic depinning occurs, meaning that at the depinning threshold some skyrmions can be temporarily trapped at pinning sites while other skyrmions move around them. The velocity–force curves are nonlinear, and $|V_{||}|$ increases more rapidly with increasing $F_D$ than $|V_{\perp}|$. The inset of figure 1(a) shows that $|V_{||}| > |V_{\perp}|$ for $F_D < 0.1$, indicating that just above the depinning transition the skyrmions are moving predominantly in the direction of the driving force. In figure 1(b), $R$ increases linearly with increasing $F_D$. 


for 0.04 < \( F_D \) < 0.74, as indicated by the linear fit, while for \( F_D > 0.74 \) \( R \) saturates to the intrinsic value of \( R = 5.708 \) marked with a dashed line. The inset of figure 1(b) shows the corresponding \( \theta_{bk} \) versus \( F_D \). From an initial value of 0°, \( \theta_{bk} \) increases with increasing \( F_D \) before saturating at the clean limit value of \( \theta_{bk} = 80.06° \).

Although the linear increase in \( R \) with \( F_D \) is similar to the behavior observed in the experiments of [31], \( \theta_{bk} \) does not show the same linear behavior as in the experiments; however, we show later that when the intrinsic skyrmion Hall angle is small, \( \theta_{bk} \) varies linearly with drive. We note that the experiments in [31] were performed in the single skyrmion limit rather than in the many skyrmion plastic flow limit we study. This could impact the behavior of the Hall angle, making it difficult to directly compare our results with these experiments.

In figure 2 we illustrate the skyrmion positions and trajectories obtained during a fixed period of time at different drives for the system in figure 1. At \( F_D = 0.02 \) in figure 2(a), \( R = 0.15 \) and the average drift is predominantly along the x-direction parallel to the drive, taking the form of riverlike channels along which individual skyrmions intermittently switch between pinned and moving states. In figure 2(b), for \( F_D = 0.05 \) we find \( R \approx 0.6 \), and observe wider channels that begin to tilt along the negative y-direction. At \( F_D = 0.2 \) in figure 2(c), \( R = 1.64 \) and \( \theta_{bk} = 58.6° \). The skyrmion trajectories are more strongly tilted along the −y direction, and there are still regions of temporarily pinned skyrmions coexisting with moving skyrmions. As the drive increases, individual skyrmions spend less time in the pinned state. Figure 2(d) shows a snapshot of the trajectories over a shorter time scale at \( F_D = 1.05 \) where \( R = 5.59 \). Here the plastic motion is lost and the skyrmions form a moving crystal translating at an angle of −79.8° with respect to the external driving direction, which is close to the clean value limit of \( \theta_{bk} \). In general, the deviations from linear behavior that appear as \( R \) reaches its saturation value in figure 1(b) coincide with the loss of coexisting pinned and moving skyrmions, and are thus correlated with the end of plastic flow.

In figure 3(a) we show \( R \) versus \( F_{Dp} \) for the system from figure 1 at varied \( \alpha_m/\alpha_d \). In all cases, between the depinning transition and the free flowing phase there is a plastic flow phase in which \( R \) increases linearly with \( F_{Dp} \) with a slope that increases with increasing \( \alpha_m/\alpha_d \). In contrast to the nonlinear dependence of \( \theta_{bk} \) on \( F_D \), \( \alpha_m/\alpha_d = 5.71 \) illustrated in the inset of figure 1(b), figure 3(b) shows that for \( \alpha_m/\alpha_d = 0.3737 \), \( \theta_{bk} \) increases linearly with \( F_{Dp} \) and \( \theta_{bk}^{Dp} = 20.5° \). To understand the linear behavior, consider the expansion of \( \tan^{-1}(x) = x - x^3/3 + x^5/5 \ldots \). For small \( \alpha_m/\alpha_d \) as in the experiments, \( \tan^{-1}(R) \sim R \), and since \( R \) increases linearly with \( F_{Dp} \), \( \theta_{bk} \) also increases linearly with \( F_{Dp} \). In general, for \( \alpha_m/\alpha_d < 1 \) we find an extended region over which \( \theta_{bk} \) grows linearly with \( F_{Dp} \) while for \( \alpha_m/\alpha_d > 1 \), the dependence of \( \theta_{bk} \) on \( F_{Dp} \) has nonlinear features similar to those shown in the inset of figure 1(b). In figure 3(c) we plot \( R \) versus \( F_D \) for a system with \( \alpha_m/\alpha_d = 5.708 \) for varied \( F_D \). In all cases \( R \) increases linearly with \( F_D \) before saturating; however, for increasing \( F_D \), the slope of \( R \) decreases while the saturation of \( R \) shifts to higher values of \( F_D \). In general, the linear behavior in \( R \) is present whenever \( F_{Dp} \) is strong enough to produce plastic flow. In figure 3(d) we show \( R \) versus \( F_{Dp} \) at \( \alpha_m/\alpha_d = 5.708 \) for varied pinning densities \( n_p \). In each case, there is a region in which \( R \) increases linearly with
FD, with a slope that increases with increasing np. As np becomes small, the nonlinear region just above depinning where R increases very rapidly with drive becomes more prominent.

For weak pinning, the skyrmions form a triangular lattice and exhibit elastic depinning, in which each skyrmion maintains the same neighbors over time. In figure 4 (a) we plot the critical depinning force Fc and the fraction P6 of sixfold-coordinated skyrmions versus Fp for a system with np = 0.1 and αm/αd = 5.708. For 0 < Fp < 0.04, the skyrmions depin elastically. In this regime, Bp = 1.0 and Fc increases as Fc ∝ Fp as expected for the collective depinning of elastic lattices [25]. For Fp ≈ 0.04, P6 drops due to the appearance of topological defects in the lattice, and the system depins plastically, with Fc ∝ Fp as expected for single particle depinning or plastic flow.

In figure 4 (b) we plot R versus Fp in samples with np = 0.01 and np = 0.1 in the elastic depinning regime for varied αm/αd. We highlight the nonlinear behavior for the αm/αd = 5.708 case by a fit of the form R ∝ (Fp - Fc)β with β = 0.26 and Fc = 0.000 184. The dotted line indicates the corresponding clean limit value of R = 5.708. We find that R is always nonlinear within the elastic flow regime, but that there is no universal value of β, which ranges from 0.15 to 0.5 with varying αm/αd. The change in the Hall angle with drive is most pronounced just above the depinning threshold, as indicated by the rapid change in R at small Fp. This results from the elastic stiffness of the skyrmion lattice which prevents individual skyrmions from occupying the most favorable substrate locations. In contrast, R changes more slowly at small Fp in the plastic

Figure 2. Skyrmion positions (dots) and trajectories (lines) obtained over a fixed time period from the system in figure 1 (a). The drive is in the positive x-direction. (a) At Fp = 0.02, R = 0.15 and the motion is mostly along the x direction. (b) At Fp = 0.05, R = 0.6 and the flow channels begin tilting into the −y direction. (c) At Fp = 0.2, R = 1.64 and the channels tilt further toward the −y direction. (d) Trajectories obtained over a shorter time period at Fp = 1.05 where R = 3.59. The skyrmions are dynamically ordered and move at an angle of −79.8° to the drive.
Figure 3. (a) $R$ versus $F_D$ for samples with $F_p = 1.0$ and $n_p = 0.1$ at $\alpha_m/\alpha_d = 9.962, 7.7367, 5.708, 3.042, 1.00,$ and $0.3737$, from left to right. The line indicates a linear fit. (b) $\theta_{sk} = \tan^{-1}(R)$ for $\alpha_m/\alpha_d = 0.3737$ from panel (a). The solid line is a linear fit and the dashed line indicates the clean limit value of $\theta_{sk} = 20.5^\circ$. (c) $R$ versus $F_D$ for $F_p = 5.708$ at $F_p = 0.06125, 0.125, 0.25, 0.5, 0.75,$ and $1.0$, from left to right. (d) $R$ versus $F_D$ for $F_p = 1.0$ at $\alpha_m/\alpha_d = 5.708$ for $n_p = 0.00617, 0.01234, 0.02469, 0.04938, 0.1,$ and $0.2$, from left to right. The clean limit value of $R$ is indicated by the dashed line.

Figure 4. (a) Depinning force $F_c$ (circles) and fraction $P_6$ of six-fold coordinated particles (squares) versus $F_p$ for a system with $\alpha_m/\alpha_d = 5.708$ and $n_p = 0.1$, showing a crossover from elastic depinning for $F_p < 0.04$ to plastic depinning for $F_p \geq 0.04$. (b) $R$ versus $F_D$ for a system in the elastic depinning regime with $F_p = 0.01$ and $n_p = 0.1$ at $\alpha_m/\alpha_d = 9.962, 7.7367, 5.708, 3.042,$ and $1.00$, from top to bottom. Circles indicate the case $\alpha_m/\alpha_d = 5.708$, for which the dashed line is a fit to $R \propto (F_D - F_p)^{\beta}$ with $\beta = 0.26$ and the dotted line indicates the pin-free value of $R = 5.708$. (c) $R$ versus $F_D$ for samples with $\alpha_m/\alpha_d = 5.708$ and $n_p = 0.1$ at $F_p = 0.005, 0.01, 0.02, 0.03, 0.04,$ and $0.05$, from left to right. The solid symbols correspond to values of $F_c$ for which plastic flow occurs, while open symbols indicate elastic flow. The line shows a linear dependence of $R$ on $F_D$ for $F_p = 0.04$. 

flow regime, where the softer skyrmion lattice can adapt to the disordered pinning sites. In figure 4(c) we plot \( R \) versus \( F_D \) at \( \alpha_m/\alpha_d = 5.708 \) and \( n_p = 0.1 \) for varied \( F_p \), showing a reduction in \( R \) with increasing \( F_p \). A fit of the \( F_p = 0.04 \) curve in the plastic depinning regime shows a linear increase of \( R \) with \( F_D \), while for \( F_p < 0.04 \) in the elastic regime, the dependence of \( R \) on \( F_D \) is nonlinear. Just above depinning in the elastic regime, the skyrmion flow direction rotates with increasing drive.

**Correlations between noise fluctuations and the skyrmion Hall effect.**

The power spectrum of the velocity noise fluctuations at different applied drives represents another method that can be used to probe the dynamics of driven condensed matter systems. In the superconducting vortex case, the total noise power over a particular frequency range or the overall shape of the noise power spectrum can be determined by measuring the voltage time series at a particular current. Both experiments and simulations have shown that in the plastic flow regime, where the vortex flow is disordered and consists of a combination of pinned and flowing particles, the low frequency noise power is large and the voltage noise spectrum has a \( 1/f^\alpha \) character with \( 1.0 \leq \alpha \leq 2.0 \). In contrast, the low frequency noise power is considerably reduced in the elastic or ordered flow regime, where the noise is either white with \( \alpha = 0 \) or exhibits a characteristic washboard frequency associated with narrow band noise. Based on these changes in the noise characteristics, it is possible to map out a dynamical phase diagram for the vortex system.

In the skyrmion system, the Hall resistance can be used to detect the motion of skyrmions, and therefore, in analogy with the voltage response in a superconductor, fluctuations in the Hall resistance at a specific applied current should reflect fluctuations in the skyrmion velocity. Since the recent experiments of [31] used imaging techniques to measure \( R \), it is desirable to understand whether changes in \( R \) are correlated with changes in the fluctuations of other quantities. We measure the time series of \( V_\parallel(t) \) and \( V_\perp(t) \) in samples with \( \alpha_m/\alpha_d = 5.71 \), \( F_p = 1.0 \), and \( n_p = 0.1 \), the same parameters used in figures 1 and 2. For these values, \( R \) increases linearly with increasing \( F_D \) over the range \( 0.01 < F_D < 0.8 \) before saturating close to the clean value. For each value of \( F_D \), we then construct the power spectrum

\[
S(\omega) = \left| \int V_{\parallel,\perp}(t)e^{-i\omega t}dt \right|^2, \tag{2}
\]

where \( F_D \) is held constant during an interval of \( 1.7 \times 10^5 \) simulation time steps.

In figure 5(a) we plot \( S(\omega) \) for \( V_\parallel(t) \) and \( V_\perp(t) \) at \( F_D = 0.05 \) in the plastic flow regime. Here, the spectral shape is very similar in each case, while the noise power at low frequencies is slightly higher for \( V_\parallel \) than for \( V_\perp \). At \( F_D = 0.4 \) in figure 5(b), deep in the plastic flow phase, the noise power is much higher for \( V_\perp \) than for \( V_\parallel \) and can be fit reasonably well to a \( \omega^{-1} \) form, while \( V_\parallel \) also has an \( \omega^{-1} \) shape over a less extended region. The two spectra have equal power only for high \( \omega \). Figure 5(c) shows \( S(\omega) \) at \( F_D = 1.05 \), which corresponds to the saturation region of \( R \). The \( V_\perp \) signal still has the highest spectral power, but both spectra now exhibit a white or \( \omega^0 \) shape.
There is a small bump at low frequency which is more prominent in $V$ that may correspond to a narrow band noise feature. We note that in the overdamped limit of $a_a = 0$ at this same drive, where the particles have formed a moving lattice, there is a strong narrow band noise feature, suggesting that the Magnus term is responsible for the lack of a strong narrow band noise peak in figure 5(c). In general, we find that the power spectrum for the skyrmions shows $1/\omega$ noise in the plastic flow regime and white noise in the saturation regime.

Using the power spectrum, we can calculate the noise power $S_0$ at a specific value of $\omega$. In figure 6 we plot $S_0 = S(\omega = 50)$ for $V_\parallel$ and $V_\perp$ versus $F_D$, along with the corresponding $R$ curve from figure 1. At low $F_D$, the value of $S_0$ is nearly the same for both $V_\parallel$ and $V_\perp$. The noise power for $V_\perp$ increases more rapidly with increasing $F_D$ and both $S_0$ curves reach a maximum near $F_D = 0.5$ before decreasing as $R$ reaches its saturation value. In general $S_0$ is large whenever the spectrum has a $1/\omega$ shape. This result shows that noise power fluctuations could be used to probe changes in the skyrmion Hall effect and even dynamical transitions from plastic to elastic skyrmion flow.

We note that in real skyrmion systems, the skyrmions can also have internal modes of motion that could affect the noise power. Such internal modes are not captured by the particle model, and would likely occur at much higher frequencies than those of the skyrmion center of mass motion that we analyze here. It would be interesting to see if such modes arise in experiment and to determine whether they can also modify the skyrmion Hall angle.

3. Summary

We have investigated the skyrmion Hall effect by measuring the ratio $R$ of the skyrmion velocity perpendicular and parallel to an applied driving force. In the disorder-free limit, $R$ and the skyrmion Hall angle take constant values independent of the applied drive; however, in the presence of pinning these quantities become drive-dependent, and in the strong pinning regime $R$ increases linearly from zero with increasing drive, in agreement with recent experiments. For large intrinsic Hall angles, the current-dependent Hall angle increases nonlinearly with increasing drive; however, for small intrinsic Hall angles such as in recent experiments, both the current-dependent Hall angle and $R$ increase linearly with drive as found experimentally. The linear dependence of $R$ on drive is robust for a wide range of intrinsic Hall angle values, pinning strengths, and pinning densities, and appears whenever the system exhibits plastic flow. For weaker pinning where the skyrmions depin elastically, $R$ has a nonlinear drive dependence and increases very rapidly just above depinning. We observe a crossover from nonlinear to linear drive dependence of $R$ as a function of the pinning strength, which coincides with the transition from elastic to plastic depinning. We also show how $R$ correlates with changes in the power spectra of the velocity noise fluctuations both parallel and perpendicular to the drive. In the plastic flow regime where $R$ increases linearly with increasing $F_D$, we find $1/f$ noise that crosses over to white noise at higher drives. The noise power drops dramatically as $R$ saturates at high drives.
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References

[1] Rößler U K, Bogdanov A N and Pfeifferer C 2006 Nature 442 797
[2] Mühlbauer S, Binz B, Jonietz F, Pfeifferer C, Rosch A, Neubauer A, Georgii R and Boni P 2009 Science 323 915
[3] Yu X Z, Onose Y, Kanazawa N, Park J H, Han J H, Matsu Y, Nagaoa N and Tokura Y 2010 Nature 465 901
[4] Heinze S, von Bergmann K, Menzel M, Brede J, Kubetzka A, Wiesendanger R, Ihlmayer G and Blugel S 2011 Nat. Phys. 7 713
[5] Yu X Z, Kanazawa N, Onose Y, Kimoto K, Zhang W Z, Ishiwata S, Matsu Y and Tokura Y 2011 Nat. Mater. 10 106
[6] Seiki S, Yu X Z, Ishiwata Sand Fukura Y 2012 Science 336 198
[7] Shibata K, Yu X Z, Har A T, Morikawa D, Kanazawa N, Kimoto K, Ishiwata S, Matsu Y and Tokura Y 2013 Nat. Nanotechnol. 8 723
[8] Kézsmárki I 2015 Nat. Mater. 14 1116
[9] Jiang W et al 2015 Science 349 283
[10] Chen G, Mascaraque A, N’Diaye A T and Schmid A K 2015 Appl. Phys. Lett. 106 242404
[11] Tokunaga Y, Yu X Z, White J S, Remnow H M, Morikawa D, Taguchi Y and Tokura Y 2015 Nat. Commun. 6 7658
[12] Moreau-Lachaire C et al 2016 Nat. Nanotechnol. 11 444
[13] Boulle O et al 2016 Nat. Nanotechnol. 11 449
[14] Woo S et al 2016 Nat. Mater. 15 501
[15] Jonietz F et al 2010 Science 330 1618
[16] Yu X Z, Kanazawa N, Zhang W Z, Nagai T, Har A T, Kimoto K, Matsu Y, Onose Y and Tokura Y 2012 Nat. Commun. 3 988
[17] Schulz T, Ritz B, Bauer A, Halder M, Wagner M, Franz C, Pfeifferer C, Everschor K, Garst M and Rosch A 2012 Nat. Phys. 8 301
[18] Liang D, DeGrave J P, Stolt M J, Tokura Y and Jin S 2015 Nat. Commun. 6 8217
[19] Iwasaki J, Mochizuki M and Nagaosa N 2013 Nat. Commun. 4 1465
[20] Iwasaki J, Mochizuki M and Nagaosa N 2013 Nat. Nanotechnol. 8 742
[21] Lin S Z, Reichhardt C, Batista C D and Saxena A 2013 Phys. Rev. B 87 214419
[22] Müller J and Rosch A 2015 Phys. Rev. B 91 054410
[23] Reichhardt C, Ray D and Reichhardt C J O 2015 Phys. Rev. Lett. 114 217202
[24] Bhattarcharya S and Higgins M J 1993 Phys. Rev. Lett. 70 2617
[25] Blatter G, Feigelman M V, Geshkenbein V B, Larkin A I and Vinokur V M 1994 Rev. Mod. Phys. 66 1125
[26] Olson C J, Reichhardt C and Nori F 1998 Phys. Rev. Lett. 81 3757
[27] Yafet A, Cross V and Sampaio J 2013 Nat. Nanotechnol. 8 152
[28] Tomassello R, Martinez E, Zavieri R, Torres L, Carpentieri M and Finocchio G 2014 Sci. Rep. 4 6784
[29] Nagaosa N and Tokura Y 2013 Nat. Nanotechnol. 8 899
[30] Reichhardt C, Ray D and Reichhardt C J O 2015 Phys. Rev. B 91 104426
[31] Jiang W 2016 arXiv:1603.07393 (unpublished)
[32] Weissman M B 1988 Rev. Mod. Phys. 60 537
[33] Marley A C, Higgins M J and Bhattacharya S 1995 Phys. Rev. Lett. 74 3029
[34] Rabin M, Merhiweh R, Weissman M, Higgins M J and Bhattacharya S 1998 Phys. Rev. B 57 R720
[35] Olson C J, Reichhardt C and Nori F 1998 Phys. Rev. Lett. 80 2197
[36] Holthom A, Dominguez D and Cronbech-Jensen N 1999 Phys. Rev. Lett. 83 3061
[37] Maeda A, Tsuibo T, Abiru R, Togawa Y, Kitano H, Iwata K and Hanaguri T 2002 Phys. Rev. B 65 054506
[38] Okuma S, Inoue I and Kokubo N 2007 Phys. Rev. B 76 172503
[39] Mangan N, Reichhardt C and Reichhardt C J O 2008 Phys. Rev. Lett. 100 187002
[40] Reichhardt C and Reichhardt C J O 2015 Phys. Rev. B 92 224432