Robust Nonlinear Control Scheme for Electro-Hydraulic Force Tracking Control with Time-Varying Output Constraint

Wanshun Zang¹, Qiang Zhang¹,*, Jinpeng Su¹,² and Long Feng¹

¹ College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, Qingdao 266590, China; zangws@sdu.edu.cn (W.Z.); suchen@alumni.sjtu.edu.cn (J.S.); fenglong12634@sdu.edu.cn (L.F.)
² State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Dongchuan Road 800, Shanghai 200240, China
* Correspondence: zhangqiangskd@sdu.edu.cn

Abstract: This paper presents a robust nonlinear control scheme with time-varying output constraint for the electro-hydraulic force control system (EHFCS). Two typical double-rod symmetrical hydraulic cylinders are employed to simulate force environments in the EHFCS. Therefore, in order to improve the performance of the EHFCS, firstly, the model of the EHFCS is established with taking external disturbances, parameter uncertainties as well as structural vibrations into consideration. Secondly, in order to estimate external disturbances, parameter uncertainties and structural vibrations in the EHFCS and compensate them in the following robust controller design, two disturbance observers (DOs) are designed according to the nonlinear system model. Thirdly, with two estimation values from two DOs, a time-varying constraint-based robust controller (TVCRC) is presented in detail. Moreover, the stability of the proposed controller is analyzed by defining a proper Lyapunov functions. Finally, in order to validate the performance of the proposed controller, a series of simulation studies are conducted using the MATLAB/Simulink software. These simulation results give a fine proof of the efficiency of the proposed controller. What’s more, an experimental setup of the EHFCS is established to further validate the performance. Comparative experimental results show that the proposed controller exhibits better performance than the TVCRC without two DOs and a conventional proportional integral (PI) controller.

Keywords: electro-hydraulic force control system; time-varying output constraint; disturbance observer; Lyapunov stability

1. Introduction

Electro-hydraulic force control systems (EHFCSs) have been widely applied in industry, due to their great advantages such as small size-to-power ratios, being capable of handling large inertia and heavy loads, high accuracy and high dynamic response [1]. In some load simulators or some similar applications, the EHFCS is supposed to apply a desired force on a specimen to simulate the complicated mechanics environment, and one can consequently obtain some desired characteristics of the specimen. Therefore, many researchers have paid attention to improving the performance of the EHFCS [2].

A conventional proportional integral (PI) controller without considering the model of the EHFCS, can make the EHFCS produce a desired force to some extent [3–5]. However, the performance of the PI controller is always unsatisfactory due to some nonlinear factors such as external disturbances, uncertain parameters as well as unmodeled characteristics like structural vibrations in the EHFCS. Therefore, some control strategies based on the model of the EHFCS, such as robust controllers [5–8], controllers based on feedback linearization technique [9–12], adaptive controllers [13–16], sliding mode controllers [17–20] etc., are developed to improve the force tracking performance. Robust controllers employ the LMI (linear matrix inEquation) tool in the MATLAB software or some similar software to...
design control strategies to make controllers robust to these nonlinear factors. Feedback linearization techniques utilize some linearization tools to linearize these nonlinear factors to linear ones, thus the performance of the controllers can be improved. If uncertain parameters in the model of the EHFCS are taken into special consideration, adaptive controllers can be designed according to the system model of the EHFCS and the force tracking performance can be improved through the designed online parameter adaptive control law. Backstepping controllers can almost perfectly handle these nonlinear factors based on some disturbance observers, extended state observers, or some online parameter adaptive estimation approaches as long as proper Lyapunov functions are chosen [21]. By employing disturbance observers (DOs) or extended state observers, these nonlinear factors can be compensated in the backstepping controller design, which will improve the performance of the controller. Yao [21] proposed an extended state observer based backstepping controller for electro-hydraulic servo systems. Tri [22] presented an iterative backstepping nonlinear control scheme for trajectory control of an electro hydraulic actuator. Guo [23], considering unknown plant dynamics in electro-hydraulic servo systems, proposed an extended state observer-based control scheme. High gain DO [24] and low gain DO [25] are both proved their efficiency in estimating external disturbances.

The controllers mentioned above are all designed to make the piston rod of a hydraulic cylinder in the EHFCS apply a desired force as the reference force, which is certainly the control goal. However, there must be a tracking error between the desired reference force and the system feedback output commonly due to the mathematical model established being different from the real physical system. What is more, the tracking error can never be eliminated however advanced controllers are designed. Therefore, when a desired force is executed on the EHFCS, one needs to reserve a margin for the tracking error of the system during designing controllers, which is namely the system tracking error constraint [26]. By defining a proper barrier Lyapunov function, one can achieve an arbitrary accuracy output constraint on the system output ideally [27,28]. Niu [29] focused on the output-constraint tracking control problem of the nonlinear switched systems based on barrier Lyapunov functions. Zhang [30] paid attention on the multi-constraint control problem of strict-feedback nonlinear systems. Based on the static output constraint method, controllers with a time-varying output constraint are consequently proposed to make the design of controllers less conservative [31]. Time-varying output constraint-based controllers are designed by utilize the desired trajectory to make the output constraint vary with time, providing designers with new concepts to design different control strategies for nonlinear systems. Yang [32] proposed a dual extended state observer-based backstepping controller for the position tracking control of electro-hydraulic systems with time-varying output constraints and some simulation results verify the efficiency of the proposed controller. Liu [33] presented a time-varying output constraint-based neural network controller for an uncertain robot system.

Therefore, in this paper, we present a time-varying constraint-based robust controller (TVCRC) for the EHFCS with two disturbance observers (DOs). Two DOs are employed to handle external disturbances, parameter uncertainties as well as unmodeled characteristics like structure vibrations in the EHFCS. With observation values from two DOs, the TVCRC for the EHFCS is presented in detail. The stability of the proposed controller can be guaranteed by properly choosing positive control gains.

The reminder of the paper is organized as follows: In Section 2, problem formulation and preliminaries in the EHFCS are analyzed. Section 3 presents the design of the TVCRC in detail. Section 4 presents the simulation and experimental study to validate the performance of the proposed controller. The main conclusions are summarized in Section 5.
2. Problem Formulation and Preliminaries

As is shown in Figure 1, if we let $k_f$ denote the stiffness of the force detector, then the output force $F_L$ of the EHFCS can be expressed as:

$$F_L = k_f (x_f + x_p)$$  \hspace{1cm} (1)

where $x_f$ denotes the displacement of the force loading hydraulic cylinder [m] and $x_p$ denotes the displacement of the position hydraulic cylinder [m].

![Figure 1. Electro-hydraulic cylinder configuration.](image)

A typical double-rod symmetrical hydraulic cylinder is employed to execute a force on the specimen. Therefore, by applying the continuity law to two actuator chambers of the force loading hydraulic cylinder, the load flow $Q_L$ yields:

$$Q_L = \frac{Q_1 + Q_2}{2} = A_p \frac{dx_f}{dt} + C_{tp} P_L + \frac{V_i}{4\beta_e} \frac{dP_L}{dt}$$  \hspace{1cm} (2)

where, $m_p$ denotes the mass of the load [kg], $C_{tp}$ denotes the total leakage coefficient of the hydraulic cylinder $[\text{m}^3/(\text{N} \cdot \text{s})]$, $A_p$ denotes the effective action area of the piston rod $[\text{m}^2]$, $V_i$ denotes the total volume of two hydraulic chambers $[\text{m}^3]$, $\beta_e$ denotes the effective bulk modulus of the hydraulic oil $[\text{N} \cdot \text{m}^2]$, $P_L$ denotes the load pressure of the hydraulic actuator $[\text{Pa}]$.

By applying the Newton second law, the force balance Equation on the piston rod of the force loading hydraulic cylinder yields:

$$m_p \frac{d^2x_f}{dt^2} = P_L A_p - B_p \frac{dx_f}{dt} - F_L - F_f$$  \hspace{1cm} (3)

where, $B_p$ denotes the viscous damping coefficient of the hydraulic oil $[\text{N} \cdot \text{m}/\text{s}]$, $F_f$ denotes the Coulomb friction $[\text{N}]$.

Thus, if state variables are defined as $x = [x_1, x_2, x_3]^T = [F_L, x_f, P_L]^T$, we can obtain the system state representation yields

$$
\begin{align*}
\dot{x}_1 &= \frac{k_f}{g_1} x_2 + \frac{k_f}{f_1} x_p \\
\dot{x}_2 &= \frac{A_p}{m_p} x_3 - \frac{B_p}{m_p} x_2 - \frac{1}{m_p} \Delta \frac{x_1}{m_p} x_2 - \frac{F_L}{m_p} + \frac{F_f}{m_p} + \mu \\
\dot{x}_3 &= -\frac{4\beta_e A_p}{V_i} x_2 - \frac{4\beta_e C_{td}}{V_i} x_3 + \frac{4\beta_e}{V_i} \mu + \frac{\Delta a_1 x_2 + \Delta a_1 x_3}{d_2}
\end{align*}
$$  \hspace{1cm} (4)
where:

\[
\begin{align*}
    f_1 &= k_f \dot{x}_p, \quad f_2 = -\frac{B_p}{m_p} x_2 - \frac{1}{m_p} x_1, \quad f_3 = -\frac{4\lambda C_d}{V_i} x_2 - \frac{4\lambda C_d}{V_i} x_3, \\
    g_1 &= k_f, \quad g_2 = \frac{A_p}{m_p}, \quad g_3 = \frac{4\lambda}{V_i}, \quad u = Q_L,
\end{align*}
\]

(5)

\[
\begin{align*}
    d_1 &= -\frac{\Delta a_1}{m_p} x_2 - \frac{F_f}{m_p} + \mu d_2 = \Delta a_1 x_2 + \Delta a_1 x_3, \quad a_1 = -\frac{4\lambda A_p}{V_i}, \quad a_2 = -\frac{4\lambda C_d}{V_i}.
\end{align*}
\]

\[\Delta a_1 \text{ and } \Delta a_2 \text{ are the uncertainties of parameters } a_1 \text{ and } a_2 \text{ respectively, and } \mu \text{ denotes the structural vibrations in the EHFCs. With the above results, the system state representation can be simplified to yield:}\]

\[
\begin{align*}
    \dot{x}_1 &= g_1 x_2 + f_1 \\
    \dot{x}_2 &= g_2 x_3 + f_2 + d_1 \\
    \dot{x}_3 &= g_3 \dot{Q}_L + f_3 + d_2
\end{align*}
\]

(6)

**Assumption 1.** All state variables are measurable and bounded.

**Assumption 2.** It is assumed that \(d_1\) and \(d_2\) are both bounded and varying slowly, i.e., \(|d_1| \leq d_{1_{\text{max}}}\), \(|d_2| \leq d_{2_{\text{max}}}\) and \(d_1 = d_2 = 0\).

**Assumption 3.** The desired executed force for the hydraulic actuator, \(y_d\), its velocity \(\dot{y}_d\), acceleration \(\ddot{y}_d\), and \(\dot{\beta}_d\) are all bounded.

**Remark 1.** By taking nonlinear factors in the EHFCs into consideration, the nonlinear model of the EHFCs is established. These nonlinear factors mainly consist of external disturbances, nonlinear friction force, some parameter uncertainties and structural vibrations, which are classified into \(d_1\) and \(d_2\) in the velocity dynamics and the load pressure dynamics, respectively. In the following controller design, one should design an explicit control strategy to estimate and compensate them.

### 3. Dos-Based TVCRC Design

#### 3.1. DOs Design

With the state representation, we can obtain the following equations:

\[
\begin{align*}
    d_1 &= \dot{x}_2 - g_2 x_3 - f_2 \\
    d_2 &= \dot{x}_3 - g_3 u - f_3
\end{align*}
\]

(7)

If we define estimation values of disturbances \(d_1\) and \(d_2\) as \(\hat{d}_1\) and \(\hat{d}_2\), thus estimation errors can be obtained as:

\[
\begin{align*}
    \hat{d}_1 &= \hat{d}_1 - d_1, \\
    \hat{d}_2 &= \hat{d}_2 - d_2
\end{align*}
\]

(8)

Therefore, the dynamics of \(\hat{d}_1\) and \(\hat{d}_2\) can be designed as [24]:

\[
\begin{align*}
    \hat{d}_1 &= \frac{1}{\lambda_1} \left( \dot{x}_2 - g_2 x_3 - f_2 - \hat{d}_1 \right) \\
    \hat{d}_2 &= \frac{1}{\lambda_2} \left( \dot{x}_3 - g_3 u - f_3 - \hat{d}_2 \right)
\end{align*}
\]

(9)

where, \(\lambda_1\) and \(\lambda_2\) are positive control gains of two DOs. If two auxiliary state variables are defined as:

\[
\begin{align*}
    \xi_1 &= \hat{d}_1 - \frac{1}{\lambda_1} x_2, \quad \xi_2 = \hat{d}_2 - \frac{1}{\lambda_2} x_3
\end{align*}
\]

(10)

then, the dynamics of two auxiliary state variables can be expressed as:

\[
\begin{align*}
    \dot{\xi}_1 &= -\frac{1}{\lambda_1} \left[ \xi_1 + \frac{1}{\lambda_1} x_2 \right] + \frac{1}{\lambda_1} \left( -g_2 x_3 - f_2 \right) \\
    \dot{\xi}_2 &= -\frac{1}{\lambda_2} \left[ \xi_2 + \frac{1}{\lambda_2} x_3 \right] + \frac{1}{\lambda_2} \left( -g_3 u - f_3 \right)
\end{align*}
\]

(11)
Finally, one can obtain the ultimate equation form of two DOs as follows:
The DO for estimating \( d_1 \) can be expressed as:
\[
\begin{align*}
\dot{d}_1 &= \xi_1 + \frac{1}{\lambda_1} x_2 \\
\dot{\xi}_1 &= -\frac{1}{\lambda_1} [\xi_1 + \frac{1}{\lambda_1} x_2] + \frac{1}{\lambda_1} (-g_2 x_3 - f_2)
\end{align*}
\] (12)

The DO for estimating \( d_2 \) can be expressed as:
\[
\begin{align*}
\dot{d}_2 &= \xi_2 + \frac{1}{\lambda_2} x_3 \\
\dot{\xi}_2 &= -\frac{1}{\lambda_2} [\xi_2 + \frac{1}{\lambda_2} x_3] + \frac{1}{\lambda_2} (-g_3 u - f_3)
\end{align*}
\] (13)

**Remark 2.** According to the velocity dynamics equation and the pressure dynamics equation, two DOs are consequently designed to cope with nonlinear factors. What is more, with two observation values from two DOs, these nonlinear factors can be compensated to improve the performance of controllers.

### 3.2. The TVCRC Design

Let us define the real force tracking error as \( z_1 = x_1 - y_d \), thus we can obtain the system tracking error vector as:
\[
\mathbf{z} = [z_1, z_2, z_3, \tilde{d}_1, \tilde{d}_2]^T = [x_1 - y_d, x_2, x_3 - \alpha_2, \tilde{d}_1 - d_1, \tilde{d}_2 - d_2]^T
\] (14)

Therefore, with the system state representation (5) and, two DO (12) and (13), the TVCRC can be designed as follows:

**Step 1:** Consider the following candidate Lyapunov function as:
\[
V_1 = \frac{1}{2} \log \frac{k_b^2(t)}{k_b^2(t) - z_1^2}
\] (15)

where, \( k_b(t) = k_c(t) - \overline{y}_d(t) \) is the real force tracking error constraint, namely, \( |z_1| < k_b \), \( k_c(t) \) and \( \overline{y}_d(t) \) will be defined in follows. What’s more, \( k_c(t) \) is generally symmetrical.

In order to eliminate time’s influence on function \( V_1 \), we define an auxiliary variable as:
\[
\xi = \frac{z_1}{k_b}
\] (16)

Thus, \( V_1 \) can be rewritten as:
\[
V_1 = \frac{1}{2} \log \frac{1}{1 - \xi^2}
\] (17)

Therefore, one can obtain the time derivative of \( V_1 \) to yield:
\[
\dot{V}_1 = \frac{\xi \dot{\xi}}{1 - \xi^2}
\] (18)

with \( \xi = \frac{z_1}{k_b} \), one can further obtain:
\[
\dot{\xi} = \frac{\partial \xi}{\partial z_1} \dot{z}_1 + \frac{\partial \xi}{\partial k_b} \dot{k}_b
\] (19)

where \( \dot{z}_1 = \dot{x}_1 - \dot{y}_d \), and with the system state representation \( x_1 = g_1 x_2 + f_1, \dot{z}_1 \) can be shown as:
\[
\dot{z}_1 = g_1 x_2 + f_1 - y_d
\] (20)
One can consequently obtain the following equations:

\[ \begin{align*}
\frac{\partial \xi}{\partial t} &= \frac{1}{k_b} \xi - \frac{z_1}{k_b} \eta \\
\frac{\partial \eta}{\partial t} &= -\frac{z_1}{k_b} \eta
\end{align*} \]  

(21)

With results of Equations (20) and (21), Equation (19) can be rewritten as:

\[ \dot{\xi} = \frac{1}{k_b} \xi_1 - \frac{z_1}{k_b} \xi_2 = \frac{1}{k_b} \left( g_1 \xi_2 + f_1 - \dot{y}_d - \frac{z_1}{k_b} \frac{\dot{\xi}}{k_b} \right) = \frac{1}{k_b} \left( g_1 (\xi_2 + \alpha_1) + f_1 - \dot{y}_d - \frac{z_1}{k_b} \frac{\dot{\xi}}{k_b} \right) \]  

(22)

Therefore, substituting Equation (22) into Equation (18) yields:

\[ \dot{V}_1 = \frac{\xi}{k_b (1 - \xi^2)} \left( f_1 + g_1 (\xi_2 + \alpha_1) - \dot{y}_d - \xi \frac{\dot{\xi}}{k_b} \right) \]  

(23)

Thus, one can obtain the virtual control law \( \alpha_1 \) to stabilize the controller:

\[ \alpha_1 = \frac{1}{g_1} \left[ -f_1 - (k_b \kappa_1 - \dot{k_b}) \xi + \dot{y}_d \right] \]  

(24)

where \( \kappa_1 \) is a positive control gain of the TVCRC.

**Remark 3.** It can be seen that the virtual control law \( \alpha_1 \) contains the time derivative of the desired trajectory \( y_d \), the variable \( f_1 \), the positive control gain \( \kappa_1 \) and some variables containing the time-varying output constraint. Actually, the virtual control law \( \alpha_1 \) is the desired velocity. As a reference velocity in \( \alpha_1, \dot{y}_d \) will provide ideal velocity and the rest will revamp it by utilizing real time system feedbacks.

Substituting the result of \( \alpha_1 \) into Equation (23), one can obtain:

\[ \dot{V}_1 = -\frac{\kappa_1 \xi^2}{1 - \xi^2} + \frac{g_1 \xi_2 \xi}{k_b (1 - \xi^2)} \]  

(25)

In Equation (25), it can be seen that \( \dot{V}_1 \) still contains a cross product term \( \xi \) and \( \xi_2 \), therefore, in Step 2, in order to eliminate this term, we can define the following Lyapunov function as:

\[ V_2 = V_1 + \frac{1}{2} \dot{z}_2^2 + \frac{1}{2} \dot{d}_1^2 \]  

(26)

Thus, the time derivative of \( V_2 \) yields:

\[ V_2 = \dot{V}_1 + \dot{z}_2 \dot{z}_2 + \dot{d}_1 \dot{d}_1 \]  

(27)

where \( \dot{z}_2 \) can be obtained by the following equation:

\[ \dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = g_2 \dot{x}_3 + f_2 + \dot{d}_1 - \dot{\alpha}_1 = g_2 \dot{x}_3 + f_2 + \dot{d}_1 - \dot{\alpha}_1 - \dot{d}_1 \]  

(28)

Thus Equation (27) can be rewritten as:

\[ V_2 = -\frac{\kappa_1 \xi^2}{1 - \xi^2} + \frac{g_1 \xi_2 \xi}{k_b (1 - \xi^2)} + \frac{2}{g_1} \left( g_2 \dot{x}_3 + f_2 + \dot{d}_1 - \dot{\alpha}_1 - \dot{\alpha}_1 - \dot{d}_1 \right) + \dot{\alpha}_1 \dot{d}_1 \]  

(29)
and the time derivative of \( \alpha_1 \) can be expressed as:

\[
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial t} f_1 + \frac{\partial \alpha_1}{\partial k_b} k_b + \frac{\partial \alpha_1}{\partial \alpha} \dot{\alpha} + \frac{\partial \alpha_1}{\partial y_d} \dot{y}_d
\]

\[
= -\frac{1}{\delta_1} f_1 - \frac{1}{\delta_1} \epsilon_1 \tilde{k}_b + \frac{1}{\delta_1} \tilde{\xi} + \frac{1}{\delta_1} \tilde{\zeta} - \frac{1}{\delta_1} \left( k_b \kappa_1 - \tilde{k}_b \right) \dot{\tilde{k}} - \frac{1}{\delta_1} \dot{y}_d
\]

where \( \dot{\tilde{k}} = \frac{\partial}{\partial t} \left( \frac{z_1}{k_b} \right) = \frac{1}{k_b} \frac{z_1 k_b}{k_b^2} \) and \( f_1 = k_f \tilde{x}_p \). Thus, one can further obtain the following equation:

\[
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial t} f_1 + \frac{\partial \alpha_1}{\partial k_b} k_b + \frac{\partial \alpha_1}{\partial \alpha} \dot{\alpha} + \frac{\partial \alpha_1}{\partial y_d} \dot{y}_d
\]

\[
= -\frac{1}{\delta_1} f_1 - \frac{1}{\delta_1} \epsilon_1 \tilde{k}_b + \frac{1}{\delta_1} \tilde{\xi} + \frac{1}{\delta_1} \tilde{\zeta} - \frac{1}{\delta_1} \left( k_b \kappa_1 - \tilde{k}_b \right) \dot{\tilde{k}} - \frac{1}{\delta_1} \dot{y}_d
\]

Consequently, the time derivative of \( \alpha_1 \) can be obtained as:

\[
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial t} f_1 + \frac{\partial \alpha_1}{\partial k_b} k_b + \frac{\partial \alpha_1}{\partial \alpha} \dot{\alpha} + \frac{\partial \alpha_1}{\partial y_d} \dot{y}_d
\]

\[
= -\frac{1}{\delta_1} (k_f \tilde{x}_p - \tilde{y}_d) \kappa_1 \kappa_2 - \frac{1}{\delta_1} \tilde{k}_b + \frac{1}{\delta_1} \tilde{\xi} + \frac{1}{\delta_1} \tilde{\zeta} - \frac{1}{\delta_1} \left( k_b \kappa_1 - \tilde{k}_b \right) \dot{\tilde{k}} - \frac{1}{\delta_1} \dot{y}_d
\]

Consequently, one can choose the virtual control law \( \alpha_2 \) in Equation (29):

\[
\alpha_2 = \frac{1}{\delta_2} \left( -f_2 - \kappa \alpha_2 - \dot{\alpha}_1 - \frac{1}{\kappa v} \alpha_1 \right)
\]

where \( \kappa_2 \) is a positive control gain of the TVCRC.
Remark 4. Actually, the virtual control law $\alpha_2$ is the desired load pressure. It can be seen that $\alpha_2$ contains the real time tracking error with its positive control gain $\kappa_2$, the estimation value $\hat{d}_1$, the time vary output constraint $k_b$ with some relative variables, the ideal acceleration $\dot{\alpha}_1$ and $f_2$ containing the displacement of the driving hydraulic cylinder. The ideal acceleration $\dot{\alpha}_1$ will provide a reference and the rest variables in $\alpha_2$ will revamp it to tracking the real time load pressure.

By substituting Equation (35) into Equation (29), one can obtain:

$$V_2 = -\frac{\kappa_1 \xi^2}{1 - \xi^2} + g_2 z_2 z_3 - \kappa_2 z_2^2 - z_2 \hat{d}_1 + \hat{d}_1 \dot{\hat{d}}_1$$  \hspace{1cm} (36)

and it can be seen that $V_2$ still contains the cross-product term $g_2 z_2 z_3$, therefore in the following step one should define a proper Lyapunov function to eliminate it.

In the final step we consider the following candidate Lyapunov function as:

$$V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2} \hat{d}_2^2$$  \hspace{1cm} (37)

Thus, the time derivative of $V_3$ yields:

$$\dot{V}_3 = V_2 + z_3 \dot{z}_3 + \hat{d}_2 \dot{\hat{d}}_2$$  \hspace{1cm} (38)

By substituting $\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = g_3 u + f_3 + \dot{d}_2 - \dot{\alpha}_2 - \ddot{d}_2$, one can consequently obtain:

$$\dot{V}_3 = \dot{V}_2 + z_3 (g_3 u + f_3 + \dot{d}_2 - \dot{\alpha}_2 - \ddot{d}_2) + \dot{\hat{d}}_2 \dot{\hat{d}}_2$$  \hspace{1cm} (39)

and $\dot{\alpha}_2$ can be obtained from the following equation:

$$\ddot{\alpha}_2 = \frac{\partial V_2}{\partial \dot{x}_2} + \frac{\partial V_2}{\partial \dot{z}_2} \ddot{z}_2 + \frac{\partial V_2}{\partial \dot{\alpha}_1} \dot{\alpha}_1 + \frac{\partial V_2}{\partial \hat{d}_1} \hat{d}_1$$

$$= -\frac{1}{g_2} f_2 - \frac{\kappa_2}{g_2} \ddot{z}_2 + \frac{1}{g_2} \ddot{\alpha}_1 - \frac{1}{g_2} \ddot{\alpha}_1 \dot{\alpha}_1 - \frac{g_1}{g_2 k_b (1 - \xi^2)} \frac{2 \xi^2}{g_2 k_b (1 - \xi^2)} \left[ \frac{1}{k_b} (g_1 x_2 + f_1 - y_d) - \frac{z_1 k_b}{k_b} \right] - \frac{1}{g_2} \ddot{d}_1$$

$$= \frac{1}{g_2} \left[ \frac{\partial}{\partial \alpha_2} \left( g_2 x_3 + f_2 + \dot{d}_1 - \ddot{\alpha}_1 \right) + \frac{1}{m} (g_1 x_2 + f_1) \right] + \frac{\partial}{\partial \alpha_2} \left( g_2 x_3 + f_2 + \dot{d}_1 - \ddot{\alpha}_1 \right) + \frac{1}{g_2} \ddot{\alpha}_1 \dot{\alpha}_1 - \left[ \frac{g_1}{g_2 k_b (1 - \xi^2)} + \frac{2 \xi^2}{g_2 k_b (1 - \xi^2)} \right] \left[ \frac{1}{k_b} (g_1 x_2 + f_1 - y_d) - \frac{z_1 k_b}{k_b} \right] - \frac{1}{g_2} \ddot{d}_1$$

$$+ \frac{1}{g_2} \ddot{\alpha}_1 \dot{\alpha}_1 - \left[ \frac{g_1}{g_2 k_b (1 - \xi^2)} + \frac{2 \xi^2}{g_2 k_b (1 - \xi^2)} \right] \left[ \frac{1}{k_b} (g_1 x_2 + f_1 - y_d) - \frac{z_1 k_b}{k_b} \right] - \frac{1}{g_2} \ddot{d}_1$$ \hspace{1cm} (40)

where $\ddot{f}_2$ can be presented as:

$$\ddot{f}_2 = -\frac{B_f}{m} \dot{x}_2 - \frac{1}{m} \dot{x}_1 + \dot{d}_1 = -\frac{B_f}{m} (g_2 x_3 + f_2 + \dot{d}_1) - \frac{1}{m} (g_1 x_2 + f_1) + \dot{d}_1$$

$$= -\frac{B_f}{m} (g_2 x_3 + f_2 + \dot{d}_1) - \frac{1}{m} (g_1 x_2 + f_1) + \frac{B_f}{m} \dot{d}_1 + \dot{d}_1 - \ddot{d}_1$$ \hspace{1cm} (41)

With the result of $\ddot{f}_2$ in (41), $\dot{\alpha}_2$ can be finally expressed as:
\[ \dot{a}_2 = \frac{1}{g_2^2} \left[ \frac{g_2}{g_2 k_1 (1 - \xi^2)} + \frac{g_2^2 \dot{\alpha}_1}{g_2 k_2 (1 - \xi^2)^2} \right] \left[ \frac{1}{k_1^2} (g_1 x_2 + f_1 - \hat{y}_d) - \frac{2 \dot{\alpha}_1}{k_1} \right] \]
\[ = \frac{1}{g_2^2} \left[ \frac{g_2}{g_2 k_1 (1 - \xi^2)} + \frac{g_2^2 \dot{\alpha}_1}{g_2 k_2 (1 - \xi^2)^2} \right] \left[ \frac{1}{k_1^2} (g_1 x_2 + f_1 - \hat{y}_d) - \frac{2 \dot{\alpha}_1}{k_1} \right] - \frac{1}{g_2^2} \hat{z}_1 + \frac{2 \dot{\alpha}_1}{k_1} \]
\[ = \hat{a}_{2c} + \hat{a}_{2u} \]

where the certain term \( \hat{a}_{2c} \) and the uncertain term \( \hat{a}_{2u} \) are shown as follows:

\[ \hat{a}_{2c} = \frac{1}{g_2^2} \left[ \frac{g_2}{g_2 k_1 (1 - \xi^2)} + \frac{g_2^2 \dot{\alpha}_1}{g_2 k_2 (1 - \xi^2)^2} \right] \left[ \frac{1}{k_1^2} (g_1 x_2 + f_1 - \hat{y}_d) - \frac{2 \dot{\alpha}_1}{k_1} \right] - \frac{1}{g_2^2} \hat{z}_1 + \frac{2 \dot{\alpha}_1}{k_1} \]
\[ \hat{a}_{2u} = -\frac{1}{g_2^2} \left( \frac{k_1 - \frac{k_b}{k_2}}{k_2} \right) \hat{d}_1 - \frac{1}{g_2^2} \hat{d}_1 - \frac{k_2}{g_2} \hat{d}_1 \]

Remark 5. Although the expression of \( \dot{a}_2 \) is complicated, \( \dot{a}_2 \) can be divided into a certain term \( \dot{a}_{2c} \) and an uncertain term \( \dot{a}_{2u} \), which are clearly show the constitution of \( \dot{a}_2 \).

Thus, with the result of Equation (42), Equation (39) can be rewritten to yield:

\[ \dot{V}_3 = \dot{V}_2 + z_3 \left( g_3 u + f_3 + \hat{d}_2 - \dot{a}_{2c} - \dot{\tilde{d}}_2 \right) - z_3 \dot{a}_{2u} + \tilde{d}_2 \]
\[ = -\frac{k_1^2}{1 - \xi^2} + g_2 z_2 z_3 - \xi s \hat{z}_2^2 - z_2 \hat{d}_2 \hat{d}_1 + \hat{d}_1 \hat{d}_2 + \hat{d}_2 \hat{d}_2 + z_3 \left( g_3 u + f_3 + \hat{d}_2 - \dot{a}_{2c} \right) - z_3 \hat{d}_2 - z_3 \dot{a}_{2u} \]

Consequently, the real control law \( u \) of the TVCRC can be chosen as:

\[ u = \frac{1}{g_3} \left( -f_3 - \hat{z}_3 \hat{z}_3 - \hat{d}_2 - \dot{a}_{2c} - \hat{a}_{2u} \right) \]

where \( \kappa_3 \) is a positive control gain of the TVCRC.

Remark 6. As is shown in Equation (46), the real control law \( u \) contains the desired load pressure \( \dot{a}_{2c} \), the state variable \( f_3 \), the load pressure tracking error \( z_3 \) with the positive control gain \( \kappa_3 \), the piston rod velocity \( z_2 \) with the constant system parameter \( g_2 \), and the estimation value \( \hat{d}_2 \). Terms \( \kappa_3 \hat{z}_3 \), \( \hat{d}_2 \) and \( g_2 z_2 \) are employed to revamp the desired load pressure \( \dot{a}_{2c} \) to track the real system load pressure. Thus, the executed force of the piston rod will impose a force on the specimen as the desired force.

By substituting Equation (46) into Equation (45), one can obtain:

\[ \dot{V}_3 = \frac{\kappa_1^2}{1 - \xi^2} - \kappa_2 \hat{z}_2^2 - \kappa_3 \hat{z}_3^2 - z_2 \hat{d}_1 \hat{d}_1 + \hat{d}_1 \hat{d}_2 + z_3 \hat{d}_2 - z_3 \dot{a}_{2u} + \tilde{d}_2 \]

\[ (47) \]
With \( \hat{a}_{2u} \) in Equation (44), Equation (47) can be rewritten as:

\[
V_3 = -\frac{\kappa_1 \xi^2}{1 - \xi^2} - \kappa_2 z^2 - \kappa_3 z_3^2 - z_2 \hat{d}_1 + \hat{d}_1 d_1 - z_3 \hat{d}_2 + \hat{d}_2 \hat{d}_2 - \frac{1}{\lambda_1} \left( \kappa_1 - \frac{\xi}{\lambda_1} \right) \hat{d}_1 - \frac{1}{\lambda_2} \hat{d}_2 - \frac{\kappa_2 \xi}{\lambda_2} \hat{d}_2
\]  

(48)

With Equation (8), \( \hat{d}_1 \) and \( \hat{d}_2 \) can be expressed with the following equation:

\[
\hat{d}_1 = \hat{d}_1 - \hat{d}_1, \quad \hat{d}_2 = \hat{d}_2 - \hat{d}_2
\]  

(49)

According to Assumption 2, \( d_1 \) and \( d_2 \) are both varying slowly, thus, \( \hat{d}_1 = 0 \) and \( \hat{d}_2 = 0 \). Consequently, Equation (49) can be rewritten as:

\[
\hat{d}_1 = \hat{d}_1, \quad \hat{d}_2 = \hat{d}_2
\]  

(50)

and, with Equations (10) and (11), the following equations can be obtained:

\[
\hat{d}_1 = \dot{\xi}_1 + \frac{1}{\lambda_2} \hat{x}_2 = -\frac{1}{\lambda_1} \left[ \xi_1 + \frac{1}{\lambda_2} \hat{x}_2 \right] + \frac{1}{\lambda_1} (-g_2 x_3 - f_2) + \frac{1}{\lambda_1} \hat{x}_2 = -\frac{1}{\lambda_1} \xi_1 + \frac{1}{\lambda_1} \hat{d}_1
\]  

(51)

\[
\hat{d}_2 = \dot{\xi}_2 + \frac{1}{\lambda_2} \hat{x}_3 = -\frac{1}{\lambda_2} \left[ \xi_2 + \frac{1}{\lambda_2} \hat{x}_3 \right] + \frac{1}{\lambda_2} (-g_3 u - f_3) + \frac{1}{\lambda_2} \hat{x}_3 = -\frac{1}{\lambda_2} \xi_2 + \frac{1}{\lambda_2} \hat{d}_2
\]  

(52)

Furthermore, with the results of Equations (51) and (52), Equation (48) can be rewritten as:

\[
\dot{V}_3 = -\frac{\kappa_1 \xi^2}{1 - \xi^2} - \kappa_2 \hat{d}_1^2 - \kappa_3 \hat{d}_2^2 - \frac{1}{\lambda_1} \hat{d}_1^2 + \frac{1}{\lambda_2} \hat{d}_2^2 - z_3 \hat{d}_2 + \delta z_3 \hat{d}_2
\]  

(53)

where, \( \delta = \frac{1}{\lambda_2} \left( \kappa_1 - \frac{\xi}{\lambda_1} \right) + \frac{1}{\lambda_2} + \frac{\kappa_3}{\lambda_2} \).

Therefore:

\[
\dot{V}_3 = -\frac{\kappa_1 \xi^2}{1 - \xi^2} - \kappa_2 \hat{d}_1^2 - \frac{\kappa_3}{\lambda_2} \hat{d}_2^2 - \frac{1}{\lambda_1} \hat{d}_1^2 - \frac{1}{\lambda_2} \hat{d}_2^2 - \frac{\kappa_3}{\lambda_2} \hat{d}_2^2 - \frac{\kappa_3}{\lambda_2} \hat{d}_2^2 - \frac{\kappa_3}{\lambda_2} \hat{d}_2^2 - \frac{\kappa_3}{\lambda_2} \hat{d}_2^2 - \delta z_3 \hat{d}_2
\]  

(54)

In Equations (15) and (16), on can obtain \( \xi = \frac{z_1}{\kappa_1} \) and \( |z_1| < \kappa_1 \), therefore, \( \left| \frac{z_1}{|z_1|} \right| < 1 \), and:

\[
1 - \xi^2 = 1 - \left( \frac{|z_1|}{\kappa_1} \right)^2 > 0
\]  

(55)

Therefore, one can obtain \( -\frac{\xi^2}{1 - \xi^2} \leq 0 \). Due to control gains \( \kappa_1, \kappa_2 \) and \( \kappa_3 \) are all positive, thus, if positive control gains of two DOFs are both selected as:

\[
\frac{1}{\lambda_1} > \frac{1}{4\kappa_2} + \frac{\delta^2}{\kappa_3} \quad \text{and} \quad \frac{1}{\lambda_2} > \frac{2}{\kappa_2}
\]  

(56)
which will guarantee $\dot{V}_3 < 0$ and the proposed controller is stable. Thus, the overall architecture of the closed-loop can be summarized as seen in Figure 2.

![Figure 2. The overall architecture of the closed-loop.](image)

### 3.3. Stability of the Closed Loop

In order to prove the stability of the closed loop, we have the following proposition:

**Proposition 1.** If the control gains $\lambda_1, \lambda_2, \kappa_1, \kappa_2$ and $\kappa_3$ are properly chosen, then the stability of the closed loop can be guaranteed by the proposed control law.

**Proof.** We define a Lyapunov candidate function $V$ as:

$$V = \frac{1}{2} \tilde{d}_1^2 + \frac{1}{2} \tilde{d}_2^2 + \frac{1}{2} \tilde{z}_1^2 + \frac{1}{2} \tilde{z}_2^2 + \frac{1}{2} \tilde{z}_3^2$$

(57)

With results from (51)–(53), the derivative of Equation (57) with respect to time yields:

$$\dot{V} = -\frac{\kappa_1 \tilde{d}_1^2}{1 - \kappa_3^2} - \kappa_2 \tilde{z}_1^2 - \kappa_3 \tilde{z}_3^2 - \tilde{z}_2 \tilde{d}_1 - \frac{1}{\kappa_1} \tilde{d}_1^2 - \frac{1}{\kappa_2} \tilde{d}_2^2 - \tilde{z}_3 \tilde{d}_2 + \delta z_3 \tilde{d}_1$$

(58)

$$\dot{V} = -\frac{\kappa_1 \tilde{d}_1^2}{1 - \kappa_3^2} - \kappa_2 \tilde{z}_1^2 - \kappa_3 \tilde{z}_3^2 - \tilde{z}_2 \tilde{d}_1 - \frac{1}{\kappa_1} \tilde{d}_1^2 - \frac{1}{\kappa_2} \tilde{d}_2^2 - \tilde{z}_3 \tilde{d}_2 + \delta z_3 \tilde{d}_1$$

(59)

If control gains are properly chosen as $\lambda_1 > \frac{1}{4\kappa_3^2}, \lambda_2 > \frac{2}{\kappa_3^2}, \kappa_1 > 0, \kappa_2 > 0$ and $\kappa_3 > 0$, the stability of the closed-loop of the EHFCS control system can be guaranteed. □

### 4. Simulation and Experimental Study

#### 4.1. Simulation Study

In order to validate the performance of controllers, a series of simulation study are conducted using MATLAB/Simulink. The desired reference force is selected as a sine wave with an amplitude 3000 N and a frequency 1 Hz. Control gains are selected as $\kappa_1 = 190, \kappa_2 = 100, \kappa_3 = 190, \lambda_1 = 0.001$ and $\lambda_2 = 1/700$ in all simulation study.

1. The TVCRC without two DOs: with $\tilde{d}_1 = 0$ and $\tilde{d}_2 = 0$, the simulation study is conducted on the software MATLAB/Simulink to validate the efficiency of the TVCRC without two DOs.
2. The TVCRC with two DOs: in order to further improve the force tracking performance, two DOs based TVCRC are employed. Based on [32], the time-varying output constraint for the EHFCS can be selected as $k_c(t) = 3000 + 300 \cos(t)$, and as a function of $y_d$, $\tilde{y}_d(t)$ is presented in Equation (57).
A possible selection of $\bar{Y}_d(t)$ can be presented as [32]:

$$\bar{Y}_d(t) = \begin{cases} \frac{-2\lambda}{\pi} \cos \frac{\pi y(t)}{2\lambda} + \lambda, & y(t) \leq \lambda, \\ |y(t)|, & y(t) > \lambda. \end{cases}$$  \hspace{1cm} (60)$$

where, $\lambda=1000$. The three following cases are considered:

1. Case 1: constant disturbances: $d_1 = 800$, $d_2 = 50,000,000$. The power of $d_1$ and $d_2$ is 640,000 and $2.5 \times 10^{15}$ respectively. The simulation results are presented in Figures 3–5.

2. Case 2: sinusoidal disturbances: $d_1 = 2000\sin(4\pi t)$, $d_2 = 50,000,000\sin(8\pi t)$. The power of $d_1$ and $d_2$ is $1.9997 \times 10^6$ and $1.2498 \times 10^{15}$ respectively. The simulation results are presented in Figures 6–8.

3. Case 3: uniform random disturbances: $d_1$ is a uniform random number with amplitude from $-200$ to $200$ and a bandpass filter from $4$ Hz to $20$ Hz, $d_2$ is a uniform random number with amplitude from $-50,000,000$ to $50,000,000$ and a bandpass filter from $2$ Hz to $20$ Hz. The power of $d_1$ and $d_2$ is $373.5790$ and $2.6454 \times 10^{13}$ respectively. The simulation results are presented in Figures 9–11.

![Figure 3. The force tracking performance with the TVCRC without two DOs in Case 1. (a) The tracking force with the TVCRC without two DOs. (b) The force tracking error with the TVCRC without two DOs.](image-url)
Figure 3. The force tracking performance with the TVCRC without two DOs in Case 1. (a) The tracking force with the TVCRC without two DOs. (b) The force tracking error with the TVCRC without two DOs.

Figure 4. The force tracking performance with the TVCRC with two DOs in Case 1. (a) The tracking force with the TVCRC with two DOs. (b) The force tracking error with the TVCRC with two DOs. (c) The external disturbance $d_1$ and its estimation value. (d) The estimation error of $d_1$. (e) The external disturbance $d_2$ and its estimation value. (f) The estimation error of $d_2$. 

Figure 5. The force tracking error comparison of two controllers in Case 1.
Figure 4. The force tracking performance with the TVCRC with two DOs in Case 1. (a) The tracking force with the TVCRC with two DOs. (b) The force tracking error with the TVCRC with two DOs. (c) The external disturbance \(d_1\) and its estimation value. (d) The estimation error of \(d_1\). (e) The external disturbance \(d_2\) and its estimation value. (f) The estimation error of \(d_2\).

Figure 5. The force tracking error comparison of two controllers in Case 1.
Figure 6. The force tracking performance with the TVCRC without two DOs in Case 2. (a) The tracking force with the TVCRC without two DOs. (b) The force tracking error with the TVCRC without two DOs.

Figure 7. Cont.
Figure 7. The force tracking performance with the TVCRC with two DOs in Case 2. (a) The tracking force with the TVCRC with two DOs. (b) The force tracking error with the TVCRC with two DOs. (c) The external disturbance $d_1$ and its estimation value. (d) The estimation error of $d_1$. (e) The external disturbance $d_2$ and its estimation value. (f) The estimation error of $d_2$. 
Figure 8. The force tracking error comparison of two controllers in Case 2.

Figure 9. The force tracking performance with the TVCRC without two DOs in Case 3. (a) The tracking force with the TVCRC without two DOs. (b) The force tracking error with the TVCRC without two DOs.
Figure 9. The force tracking performance with the TVCRC without two DOs in case 3. (a) The tracking force with the TVCRC without two DOs. (b) The force tracking error with the TVCRC without two DOs.

Figure 10. The force tracking performance with the TVCRC with two DOs in Case 3. (a) The tracking force with the TVCRC with two DOs. (b) The force tracking error with the TVCRC with two DOs. (c) The external disturbance $d_1$ and its estimation value. (d) The estimation error of $d_1$. (e) The external disturbance $d_2$ and its estimation value. (f) The estimation error of $d_2$. 

Figure 11. The force tracking error comparison of two controllers in Case 3.
Figure 10. The force tracking performance with the TVCRC with two DOs in Case 3. (a) The tracking force with the TVCRC with two DOs. (b) The force tracking error with the TVCRC with two DOs. (c) The external disturbance $d_1$ and its estimation value. (d) The estimation error of $d_1$. (e) The external disturbance $d_2$ and its estimation value. (f) The estimation error of $d_2$.

Figure 11. The force tracking error comparison of two controllers in Case 3.

Table 1 shows key parameters of the EHFC.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $A_p$      | $1.88 \times 10^{-3}$ m$^2$ | $m_p$ | 500 kg |
| $\beta_p$ | $6.9 \times 10^8$ Pa | $V_t$ | $0.38 \times 10^{-3}$ m$^3$ |
| $\Delta P_r$ | $6 \times 10^6$ Pa | $u_{\text{max}}$ | 10 V |
| $P_e$      | $8 \times 10^6$ Pa | $Q_0$ | 38 L/min |
| $B_p$      | 7500 N/(m/s) | $C_{H}$ | $4.6 \times 10^{-17}$ m$^3$/s/Pa |
Simulation results of Case 1:

As is shown in Figure 3, it can be seen that without two DOs, the TVCRC can stabilize the EHFCS in the presence of two external disturbances being \( d_1 = 800 \) and \( d_2 = 50,000,000 \). As time goes on, the tracking error of the TVCRC in Figure 5 tends to increase gradually, which indicates that two constant disturbances have a considerable influence on the performance of the TVCRC. Therefore, in order to eliminate the influence, two DOs are employed to online estimate and compensate them. In Figure 4, it can be seen that with observation values from two DOs, the TVCRC can substantially improve the force tracking performance. From Figure 4, two DOs can estimate two disturbances in real time. Although two estimation errors of \( d_1 \) and \( d_2 \) are considerable, but errors become small gradually. Figure 5 presents force tracking errors of the TVCRC with and without two DOs. As is shown in Figure 5, the TVCRC with two DOs can substantially reduce the force tracking error. Simulation results of the case 1 prove the efficiency of the TVCRC with two DOs.

Simulation results of Case 2:

As shown in Figure 6, two sinusoidal external disturbances are exerted on the simulation model of the EHFCS. It can be seen that two sinusoidal external disturbances have a considerable influence on the performance of the TVCRC, whereas the TVCRC with two DOs can almost perfectly handle disturbances and improve the force tracking performance. The proposed disturbance observers can online estimate and compensate two disturbances, which is shown in Figure 7. Figure 8 presents the force tracking error of the TVCRC with and without two DOs. As is shown in Figure 8, the force tracking error reaches 982.1252 N in the presence of two sinusoidal external disturbances, whereas the max force tracking error is 17.0257 N with observation value from two DOs. The simulation results of Case 2 prove the efficiency of the TVCRC with two DOs.

As is shown in Figures 9–11, likewise, the force tracking error of the TVCRC is obviously bigger than that of the TVCRC with two DOs in the presence of two uniform random external disturbances, which is the proof of the efficiency of the TVCRC with two DOs.

Simulation results of Case 3:

The peak error (PE) of the force tracking error yields:

\[
\text{PE} = \max_{i=1,\ldots,N} |e(i)| \tag{61}
\]

where, \( e(i) \) is the force tracking error. The root mean square error (RMSE) is employed to illustrate the performance of two controllers yields:

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{j=1}^{n} (R_{\text{in},j} - R_{\text{out},j})^2} \tag{62}
\]

where \( R_{\text{in},j} \) denotes the reference signal, \( R_{\text{out},j} \) denotes the feedback signal from the displacement sensor, \( n \) denotes the length of the signal. Results of the RMSE is presented in Table 2.

Table 2 presents the peak error and the RMSE of the TVCRC with and without two DOs in three cases simulation study. In the Case 1 simulation study the PE is 876.1 N and 779.938 N respectively, and the RMSEs are 225.39 N and 15.8003 N respectively. The PEs are 876.1 N and 779.938 N, respectively, and the RMSEs are 225.39 N and 15.8003 N, respectively in the Case 2 simulation study. In the Case 3 simulation study, the PEs are 25.1118 N and 9.5499 N, respectively, and the RMSEs are 5.8018 N and 0.6556 N, respectively. These results give a good proof for that the TVCRC with two DOs > the TVCRC without two DOs.
Table 2. RMSE of the controllers in the simulation study.

| Controllers                      | The PE/N | The RMSE/N |
|----------------------------------|----------|------------|
| The TVCRC without two DOs        | 876.1    | 225.39     |
| The TVCRC with two DOs           | 779.938  | 15.8003    |
| The TVCRC without two DOs        | 982.1252 | 375.4193   |
| The TVCRC with two DOs           | 17.0257  | 5.2238     |
| The TVCRC without two DOs        | 25.1118  | 5.8018     |
| The TVCRC with two DOs           | 9.5499   | 0.6556     |

Remark 7. In order to validate the efficiency of the TVCRC with two DOs, a series of simulation study are conducted by the software MATLAB/Simulink, which are classified to three cases, i.e., under two constant disturbances, under two sinusoidal disturbances and under two uniform random disturbances. From the results of the simulation study, one can conclude that: (1) Since $y_d$ is a sine wave signal, it is symmetrical, which results in $k_c$ being also symmetrical. (2) although the TVCRC without two DOs can still stabilize the EHFCS in the presence of three types of disturbances, the force tracking performance is considerably affected by these disturbances. (3) The proposed two DOs can handle the constant, the sinusoidal and the uniform random disturbances. (4) the TVCRC with two DOs can improve the force tracking performance with observation values from two DOs.

4.2. The EHFCS Experimental Setup

The block schematic of the EHFCS real-time control system is presented in Figure 12. The host computer communicates with the target computer via Ethernet. A PCI-1716 AD board and an ACL-6126 DA board are both installed in the target computer. The force, the displacement and two pressures of two chambers of the hydraulic cylinder, which will be transferred from 4–20 mA to 2–10 V by the signal modulator, are acquired by the PCI-1716 board. The control voltage from –10 V–10 V will be transferred to –40 mA–40 mA to drive the servo valve, and then the piston rod of the hydraulic cylinder will impose the desired force on the specimen.

Figure 12. The block schematic of the EHFCS real-time control system.
computer, is downloaded to the target computer for real-time execution. The sampling rate for the experimental real-time control system is selected as 1000 Hz. Table 3 presents the main hardware of the EHFCS real-time control system.

![Image of experimental Simulink model]

**Figure 13.** The experimental Simulink model.

| Hardware                  | Quantity | Type                  |
|---------------------------|----------|-----------------------|
| The servo valve           | 2        | G762/Moog             |
| PCI-1716                  | 1        | Advantech             |
| ACL-6126                  | 1        | Linghua               |
| The displacement sensor   | 1        | 18 Series/Germanjet   |
| The pressure sensor       | 2        | NS-P-I/Tianmu         |
| The force detector        | 1        | NS-WL2/Tianmu         |

### 4.3 Comparative Experimental Results

In the experimental study, a sine waves reference signal with a 3000 N amplitude and a 1 Hz frequency are employed to verify the performance of the TVCRC with two DOs. Experimental results are presented in Figures 14–16.

![Graph of force tracking performance]

**Figure 14.** Cont.
Figure 14. The experimental force tracking performance with the PI controller. (a) The tracking force with the PI controller. (b) The force tracking error with the PI controller. (c) The displacement of the force loading hydraulic cylinder. (d) The displacement of the position hydraulic cylinder.
Figure 14. The experimental force tracking performance with the PI controller. (a) The tracking force with the PI controller. (b) The force tracking error with the PI controller. (c) The displacement of the force loading hydraulic cylinder. (d) The displacement of the position hydraulic cylinder.

Figure 15. The experimental force tracking performance of the TVCRC without two DOs. (a) The tracking force with the TVCRC without two DOs. (b) The force tracking error with the TVCRC without two DOs. (c) The displacement of the force loading hydraulic cylinder. (d) The displacement of the position hydraulic cylinder. It can be seen that the TVCRC without two DOs can stabilize the EHFCS and the proposed time-varying output constraint can constraint the EHFCS's force output. As a model based robust controller, the TVCRC without two DOs can improve the force tracking performance comparing with the PI controller.

Figure 16 presents the force tracking performance of the TVCRC with two DOs. Estimation values of $d_1$ and $d_2$ are presented in Figure 16c,d and Figure 16e,f show the displacement of the force loading hydraulic cylinder and the position hydraulic cylinder, respectively.
Figure 16. Cont.
The TVCRC without two DOs

Peak error of the TVCRC with two DOs

The force tracking error with the TVCRC with two DOs. (1) The TVCRC with two DOs: in order to further improve the force tracking performance, the tracking performance is best with control gains being selected as $K_p = 0.0012$ and $K_I = 0.03$. The corresponding experimental results are shown in Figure 14; (2) The TVCRC without two DOs: with $\hat{d}_1 = 0$ and $\hat{d}_2 = 0$, when control gains are chosen as $\kappa_1 = 420$, $\kappa_2 = 421$, $\kappa_3 = 365$, the tracking performance are the best. The corresponding experimental results are presented in Figure 15; (3) The TVCRC with two DOs: in order to further improve the force tracking performance, the TVCRC with two DOs are employed. Control gains of two DOs and the TVCRC are selected as $\lambda_1 = 0.05$, $\lambda_2 = 0.025$, $\kappa_1 = 435$, $\kappa_2 = 426$, $\kappa_3 = 375$. The performance of the TVCRC is the best among three controllers with estimated values from two DOs. The corresponding experimental results under a normal condition are presented in Figure 16. In order to further validate the robustness of the TVCRC with two DOs, a sine waves reference signal with a 0.006 m amplitude and a 0.5 Hz frequency is conducted at the position hydraulic cylinder. The corresponding experimental results under a sinusoidal position disturbance are presented in Figure 16.

Comparative experimental results under normal conditions:

Figure 14 presents the force tracking performance of the PI controller. As is shown in the figure, the max force tracking error with the PI controller is 563.7954 N. Figure 14c,d show the displacement of the force loading hydraulic cylinder and the position hydraulic cylinder respectively. The force tracking performance of the TVCRC without two DOs is presented in Figure 15.

It can be seen that the TVCRC without two DOs can stabilize the EHFCs and the proposed time-varying output constraint can constraint the EHFCs’s force output. As a model based robust controller, the TVCRC without two DOs can improve the force tracking
performance comparing with the PI controller. Figure 15c,d show the displacement of the force loading hydraulic cylinder and the position hydraulic cylinder respectively. Figure 16 presents the force tracking performance of the TVCRC with two DOs. Estimation values of $d_1$ and $d_2$ are presented in Figure 16c,d and Figure 16e,f show the displacement of the force loading hydraulic cylinder and the position hydraulic cylinder, respectively.

Its force tracking performance is quite similar to that of the TVCRC without two DOs. On the surface, it is difficult to distinguish performances of the TVCRC with and without two DOs. The force tracking error comparison of two controllers (the TVCRC with and without two DOs) are presented in Figure 17. It can be seen that the max force tracking error of the TVCRC without two DOs (246.8023 N) is apparently bigger than that of the TVCRC with two DOs (185.9212 N), which indicates that the TVCRC with two DOs > the TVCRC without two DOs.

Table 4 presents the PE and the RMSE of three controllers. As is shown in Table 4, the PE 563.7953 N > 246.8023 N > 185.9212 N and 198.1104 N > 47.9127 N > 36.7162 N. From all above, it can be concluded that the TVCRC with two DOs > the TVCRC without two DOs > the PI controller.

Table 4. RMSE of the controllers.

| Controllers                  | The PE/N | The RMSE/N |
|------------------------------|----------|------------|
| The PI controller            | 563.7953 | 198.1104   |
| The TVCRC without two DOs    | 246.8023 | 47.9127    |
| The TVCRC with two DOs       | 185.9212 | 36.7162    |

Remark 8. Actually, due to external disturbances, parameter uncertainties and structural vibrations (these can be called by a joint name, i.e., external disturbances.), a mathematical model of the EHFCs can never be same as the real physical system of the EHFCs. Therefore, estimation values of $d_1$ and $d_2$ in Figure 16c,d are observation values of external disturbances, which exist in the real physical system of the EHFCs themselves. From comparative experimental results from the experimental study under a normal condition, one can conclude that: (1) the TVCRC without two DOs can robustly stabilize the EHFCs in the presence of external disturbances. (2) the proposed two DOs can estimate and compensate these external disturbances in the real time EHFCs control system. (3) the TVCRC with two DOs can improve the force tracking performance than the TVCRC without two DOs and the PI controller.

The experimental results under a sinusoidal position disturbance:

As is shown in Figure 18, the TVCRC with two DOs can still stabilized the EHFCS under a sinusoidal position disturbance which is conducted on the position hydraulic cylinder. Due to the sinusoidal position disturbance, the PE is larger than both of the TVCRC without and with two DOs under a normal condition (250.8398 N > 246.8023 N > 185.9212 N).
all above, it can be concluded that the TVCRC with two DOs > the TVCRC without two DOs > the PI controller.

### Table 4. RMSE of the controllers.

| Controllers         | The PE/N | The RMSE/N |
|----------------------|----------|------------|
| The PI controller    | 563.7953 | 198.1104   |
| The TVCRC without two DOs | 246.8023 | 47.9127    |
| The TVCRC with two DOs | 185.9212 | 36.7162    |

### Remark 8.

Actually, due to external disturbances, parameter uncertainties and structural vibrations (these can be called by a joint name, i.e., external disturbances), a mathematical model of the EHFCS can never be same as the real physical system of the EHFCS. Therefore, estimation values of $d_1$ and $d_2$ in Figure 16c,d are observation values of external disturbances, which exist in the real physical system of the EHFCS themselves. From comparative experimental results from the experimental study under a normal condition, one can conclude that:

1. The TVCRC without two DOs can robustly stabilize the EHFCS in the presence of external disturbances.
2. The proposed two DOs can estimate and compensate these external disturbances in the real time EHFCS control system.
3. The TVCRC with two DOs can improve the force tracking performance than the TVCRC without two DOs and the PI controller.

### Figure 18. The experimental force tracking performance with the TVCRC with two DOs under a sinusoidal position disturbance.

- **Figure 18(a)**: The tracking force with the TVCRC with two DOs.
- **Figure 18(b)**: The force tracking error with the TVCRC with two DOs.
- **Figure 18(c)**: The estimation value of $d_1$.
- **Figure 18(d)**: The estimation value of $d_2$.
- **Figure 18(e)**: The displacement of the force loading hydraulic cylinder.
- **Figure 18(f)**: The displacement of the position hydraulic cylinder.
Table 5 presents the PE and the RMSE of the TVCRC with two DOs under a sinusoidal position disturbance. The RMSE of the TVCRC with two DOs under a sinusoidal position disturbance is 56.6911 N. Obviously, 56.6911 N > 47.9127 N > 36.7162 N. It is larger than the RMSE of the TVCRC without and with two DOs due to the sinusoidal position disturbances. These comparative experimental results all prove the efficiency of the TVCRC with two DOs.

Table 5. RMSE of the controllers under a sinusoidal position disturbance.

| Controllers | The PE/N | The RMSE/N |
|-------------|----------|------------|
| The TVCRC with two DOs under a sinusoidal position disturbance | 250.8398 | 56.6911 |

Remark 9. Actually, whether a desired reference position is conducted on the position hydraulic cylinder or not, there must be a slight displacement on the position hydraulic cylinder when a reference force is exerted on the force loading hydraulic cylinder. Therefore, when a reference position is conducted on the position hydraulic cylinder, external disturbances are enlarged by the position hydraulic cylinder, which has a considerable influence on the force tracking performance of the TVCRC with two DOs. Results from the experimental study under a sinusoidal position disturbance prove the above statement. What’s more, the force loading and the position are actually a coupled system. There are some nonlinear coupling factors can never be eliminated. Therefore, it can be seen
that the feedback displacement signal of the position hydraulic cylinder has a considerable difference with the reference displacement signal, which is shown in Figure 18f.

5. Conclusions

In this paper, in order to improve the force tracking performance of the EHFCS, a TVCRC with two DOs is presented. Two DOs are proposed to handle these nonlinear factors such as external disturbances, parameter uncertainties and structural vibrations et al. With two DOs, the TVCRC with backstepping design scheme is designed in detail. Moreover, simulation and experimental results show that the proposed controller exhibits a better performance than the TVCRC without two DOs and the conventional PI controller. Therefore, the full paper can be summarized as follows:

(1) Consider nonlinear factors like the external disturbance, parameter uncertainties as well as unmodeled characteristics in the EHFCS, the state space representation of the EHFCS is presented.

(2) Based on the state representation, two DOs for the EHFCS is presented and its stability is proved by defining proper Lyapunov functions. Consequently, the TVCRC with backstepping design scheme is presented in detail.

(3) Results from simulation and experimental study show that the proposed controller exhibits better performance than the TVCRC without two DOs and the conventional PI controller.

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