Texture Properties of a-2D Granular Mixture of an Angular Particle under Bi-axial Compression

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Abstract
Utilizing Discrete Element Modeling (DEM), we investigate the shear strength properties and microstructure of 2D granular mixture composed of various regular polygonal particles (from triangle to icosagon and disk) under biaxial compression. Three cases were considered: 1) increase of proportion and angularity started from disk (S1); 2) increase of proportion by decreasing the angularity started from triangle (S2); 3) increase of proportion by random angularity started from disk (S3) and polygons (S4). The results show that the shear strength changes with the mean angularity. On the other hand, the packing fraction slightly varies with the mean angularity. At the microscopic scale, the granular texture in term of the force, the contacts transmitted the force higher than the average force (strong force) are oriented in the direction of externally applied loading but they represent only 40% of the total number of contacts. Besides, the tangential contact forces direction tend to be consistent with the maximum shear strength direction.

Keywords: Granular materials, Granular packing, Granular rheology, Particle shape effects Discrete Element Method

1. Introduction
Granular materials in nature are composed of a mixture of particles with different sizes, shapes and properties (such as density, friction, and elasticity). For example, sand, rock, plastic bead, a particle of pharmaceutical[1–3], etc. Exploring the effect of granular composition or polydispersity, the physical properties of granular media can help to analyze civil engineering structure and optimized industrial processes[4,5]. A well-known example is the particle size distribution of rock aggregates in concretes, which performances and workability were optimized by proportion[4]. In addition, the particle shape distribution of the granular mixture was studied, such as mechanical behavior and packing properties.
Many kinds of particle shapes were found in everyday life and industry, for instance, angularity, roundedness, elongation or platy-ness and nonconvex shapes. These properties are used to characterize the particle shapes[6–11]. Binary mixtures of two shapes have been very investigation[12,13], such as segregation behavior or the strength properties, while the effect of multiple particle shapes remains still poorly investigated.

The aim of this paper is exploring/to explore the effect of angular particle shape polydispersity under biaxial compression by mean of the contact dynamic method simulation whereas analyzed in the steady-
state in term of shear strength and packing fraction at the macroscopic scale, and force transmission at the microscopic scale.

2. Numerical Model

2.1. Contact dynamics methods.
In this study, our simulations were implemented by the contact dynamics (CD) method, which is appropriate with simulating mixture of rigid particles\[14\]. This method integrates the motion equation of collection of rigid objects with contact interactions. These interactions are defined by friction coefficient, normal restitution coefficient, and tangential restitution coefficient. A parallelized iterative research algorithm based on nonlinear Gauss-Seidel method. These method used to determine velocities and forces at the end of the time step, concurrently. The contact dynamics method is widely used both in 2D and 3D simulation\[11,15\]. LMGCC90 was implemented by this method and is multipurpose software developed by the University of Montpellier, France.

2.2. Packing construction and bi-axial test
The Granular mixture is characterized by three parameters: 1) the number of species $N_s$, 2) shape properties of each species 3) the proportion of each species $k_i$. In this samples, we assume the particles in packing are generate with uniform distribution by particle number, $k_i = 1/N_s$. The total number of particles $N_p = 10,000$. Particles used to prepare the granular mixture are regular polygon with the number of sides $n_s \in \{3,4,5,6,7,8,10,15,20\}$ and disk. Shape properties were determined by angularity $\alpha$ is given by $2\pi/n_s$, where $n_s$ is the number of sides of particle. The proportion of particle in a mixture is defined by the number of species, which varied from 1 to 10. There are three case considered 1) increase of angularity with $N_s$ ($S_1$) 2) decrease of angularity with $N_s$ ($S_2$) 3) random angularity with $N_s$ ($S_3$) and ($S_4$); see Table 1. All samples considered in this paper are prepared according to the same protocol. First, Particles were geometrically placed in to a rectangular box using a sequential layer by layer deposition\[16\]. In order to avoid long-range ordering, we introduced small size polydispersity by taking diameter in the range $[d_{min}, d_{max}]$ where $d_{max} = 1.25d_{min}$ with a uniform distribution in particle volume fractions.

| $N_s$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|------|------|------|------|------|
| 1    | {disk} | {3}  | {disk} | {7}  |
| 2    | {disk,20} | {3,4} | {disk,3} | {7,5} |
| 3    | {disk,20,15} | {3,4,5} | {disk,3,20} | {7,5,15} |
| 4    | {disk,20,15,10} | {3,4,5,6} | {disk,3,20,4} | {7,5,15,10} |
| 5    | {disk,20,15,10,8} | {3,4,5,6,7} | {disk,3,20,4,15} | {7,5,15,10} |
| 6    | {disk,20,15,10,8,7} | {3,4,5,6,7,8} | {disk,3,20,4,15,5} | {7,5,15,10,disk,8} |
| 7    | {disk,20,15,10,8,7,6} | {3,4,5,6,7,8,10} | {disk,3,20,4,15,5,10} | {7,5,15,10,disk,8} |
| 8    | {disk,20,15,10,8,7,6,5} | {3,4,5,6,7,8,10,15} | {disk,3,20,4,15,5,10,6} | {7,5,15,10,disk,8,6} |
| 9    | {disk,20,15,10,8,7,6,5,4} | {3,4,5,6,7,8,10,15,20} | {disk,3,20,4,15,5,10,6,8} | {7,5,15,10,disk,8,6,2} |
| 1    | {disk,20,15,10,8,7,6,5,4,3} | {3,4,5,6,7,8,10,15,20, disk} | {disk,3,20,4,15,5,10,6,8,7} | {7,5,15,10,disk,8,6,2,0,3} |

All packings are compacted by isotropic compression in a container (rectangle frame) which the right and top walls are applied by compressive stress $\sigma$, and the left and bottom walls are fixed. During the isotropic compression, the gravity acceleration $g$, the friction coefficients $\mu$ between particles, and the friction coefficients $\mu$ between particles and walls are set to zero. This procedure leads to a random close
packing state. Dense samples obtained at the end of isotropic compression are then subject to biaxial compression by applied constant velocity \( v_0 \) on the top wall and constant compressive stress \( \sigma_0 \) acting on the side walls. The friction coefficients \( \mu \) between particles are set to 0.4 but wall friction kept to zero. Quasi-static (rate-independent) behavior was interested in this study which can be defined by the inertia number \( I = \dot{\varepsilon}/m/\sigma_0 \) where \( \dot{\varepsilon} = \dot{y}/y \) is the strain rate, \( m \) are the mass of the particles[17]. In this simulation, \( I \) is set to 10^{-4}.

3. Numerical Result

3.1. Macroscopic behaviour

In the numerical simulation, the stress tensor can be determined from the geometrical configuration of packing and contact force. The stress tensor \( \sigma \) in a given volume \( V \) defined by[18]:

\[
\sigma = \frac{1}{V} \sum_{\omega \in \omega} M^{\omega}_{ij}\]

(1)

Where \( M^{\omega}_{ij} \) is a tensorial moment given by \( M^{\omega}_{ij} = \sum_{c \in \omega} f^c_i r^c_j \) which \( f^c_i \) is the force component on the particle \( \omega \) at the contact \( c \), \( r^c_j \) is the \( j \) the position vector component of the same contact \( c \). In dry granular materials, the shear strength can be quantified by the macroscopic friction angle \( \varphi \) following:

\[
\sin \varphi = \frac{q}{p}
\]

(2)

Where \( q = (\sigma_1 - \sigma_2)/2 \) (deviator stress), and \( p = (\sigma_1 + \sigma_2)/2 \) (mean stress) which \( \sigma_1 \) and \( \sigma_2 \) are principal stress. For the bi-axial compression, the major principal direction is assumed to be vertical. The packing fraction is defined by \( \rho = V_p/V \), where \( V_p \) is the volume occupied by particles, and \( V \) is the container volume.

![Figure 1](image_url)

**Figure 1.** Averaged macroscopic friction angle \( \sin \varphi^* \) and (inset) the packing fraction \( \rho^* \) in steady-state as a function of mean angularity \( < \alpha > \).

For the macroscopic friction angle \( \sin \varphi^* \) and the packing fraction \( \rho^* \), both determined from the averaged of \( \sin \varphi \) and \( \rho \), respectively, in the steady-state. Figure 1 show \( \sin \varphi^* \) and \( \rho^* \) as a function of mean angularity \( < \alpha > \) of each sample. We observe the shear strength increase with the mean angularity, whereas the packing fraction slightly decrease. In other words, the packing becomes a little loose but with higher a shear strength. Furthermore, in random mixtures S3 and S4, at the identical value of mean angularity, the value of \( \sin \varphi^* \) and \( \rho^* \) , no collapse data is observed.
3.2. Texture properties
In this section, texture properties are described by force networks and forces distributions at a microscopic scale. Figure 2 shows a map of normal in the steady-state for \( N_s = 4 \) in S1, and S2. The thickness of the segment joining the particle centers were represented the normal forces. The long force chains are observed from the upper wall to lower wall of sample s and especially oriented near the compression direction which is 90 degrees. Although the thin forces chains distribute similar to the higher forces, it tends to be disconnected. Comparing with the polar representation of normal contact forces direction; see Figure 3(a). Mostly the probability of normal forces direction is in 90 degrees which be consistent with force network and direction of compression. Furthermore, the tangential contact forces tend to be aligned with the maximum shear strength angle (45 degrees) as shown in Figure 3(b). A large number of contacts were transmitted weak forces, whereas a small fraction of contacts carries strong forces which about 40% of active contacts; see Figure 4. Besides, the proportion of several strong force contacts with the total active contact nearly constant with mean angularity.

![Figure 2](image1.png)

**Figure 2.** The force network in S1 (a), and S2 (b) for \( N_s = 4 \)

![Figure 3](image2.png)

**Figure 3.** Polar representation of the probability of normal contact forces direction (a), and tangential contact forces direction (b) in the steady-state for \( N_s = 4 \) in the steady-state.

Besides, the force transmission has been investigated by the probability distribution function (pdf) which separated into two networks following the weak network and the strong network. The strong network is a network of the contact carrying the force more than average force, while the weak network is a network of the contact transmitting the force less than average force. Figure 5 shows the probability
distribution function (pdf) of normal contact force $f_n$ normalized by average normal contact force in strong networks, we observed the normal force distribution is exponential decrease, whereas the normal force distribution in the weak networks is very flat with values close to 1 as shown in Figure 6, thus can be characterized by power-law $P(f) \propto (f_n/\langle f_n \rangle)^\beta$ where $\beta$ is close to zero[16]. These distributions were found in all cases.

Figure 4. The Proportion of the number of strong force contacts $C_s$ with a number of active contacts $C_\alpha$ in steady-state as a function of mean angularity $<\alpha>$

Figure 5. The Probability distribution function of normal force $f_n$ normalized by average normal force $\langle f_n \rangle$ for the strong networks: (a) S1 and S3 (inset), and (b) S2 and S4 (inset), both represented in semi-logarithmic axes.

Figure 6. The Probability distribution function of normal force $f_n$ normalized by average normal force $\langle f_n \rangle$ for the weak networks: (a) S1 and S3 (inset), and (b) S2 and S4 (inset), both plotted on semi-logarithmic axes.
Conclusion

The strength and texture properties of a 2D granular mixture of angular particles were investigated by the contact dynamic simulation. Each packing in this studied consists of 10000 particles which contained in rectangle frame under biaxial compression. The number of species is varied from 1 to 10 with various shapes (regular polygon and disk). The samples were built with three cases 1) increase of proportion of particle shapes and angularity by starting from disk, 2) increase of the proportion of particle shapes by decreasing the angularity started from the triangle; 3) increase of the proportion of particle shapes by random angularity started from disk and polygons.

At the macroscale, the mechanical behavior depends on geometrical properties of the granular mixture. The shear strength increases when mean angularity in packings increase, otherwise the shears strength decrease. On the other hand, the packing fraction slightly decreases, which nearly constant. These results represent that packing density does not affect the mechanical strength of the granular mixture.

At the microscopic scale, the forces were transmitted in direction of compression and the tangential contact force remain on maximum shear stress direction, which not depend on the mean angularity of packing. In all cases, only 40% of the number of contacts carry force higher than the average force whereas these forces are sorted to be a network from the upper boundary to lower boundary to support the system. Furthermore, the probability distribution function of normal force does not depend on mean angularity (particle shape). The force distribution of the strong network is regressed by exponential drop, while the weak network can be approximated by a power law.

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