Nuclear Mass Dependence of Chaotic Dynamics in Ginocchio Model

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Abstract

The chaotic dynamics in nuclear collective motion is studied in the framework of a schematic shell model which has only monopole and quadrupole degrees of freedom. The model is shown to reproduce the experimentally observed global trend toward less chaotic motion in heavier nuclei. The relation between current approach and the earlier studies with bosonic models is discussed.

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Several recent studies have revealed an intriguing interplay of chaotic and regular motion in the low-lying collective states of the medium-heavy nuclei [1-3]. A close relationship between the breaking of dynamical symmetry and the emergence of chaotic motion has been displayed. All the realistic studies up to now have employed the interacting boson model of Arima and Iachello [4] which is well suited for the study of the dynamical symmetry with its group theoretical formulation. It should not be forgotten, however, that the bosonic models are approximations to the underlying fermionic models, and that despite their success they have their intrinsic limitations. One of the places where bosonic models fail is the physics of higher excitations where the explicit treatment of the broken fermion pairs is called for [5]. Even within the low-lying regime, the effects of the underlying fermionic degrees of freedom might be felt in such subtle question as chaos and order. Moreover, there are certain problems related to the global change of the nuclear characteristics. Usually they are better approached by the microscopic fermionic models because these problems are often related to the shell effect and the Pauli-blocking effect.

One of such global-change problems has recently been brought into light by Shriner, Mitchell and von Egidy [6] who have compiled the spectral statistics of nuclear levels ranging from $^{24}$Na to $^{244}$Am. The fluctuation properties of the nuclear levels are found to vary in different mass region; generally, the level spacing distribution is "chaotic" or Wigner-like among lighter nuclei, and tends to become "regular" or Poisson-like among heavier nuclei. Since large portion of the nuclear levels are of "rotational" nature, it is now clear that we have to depart from the notion that the rotational levels are necessarily regular as suggested by the SU(3) limit of interacting boson model. In fact, no dynamical symmetry is known in the deformed limit of the realistic shell model. We face the problem of examining where and how the regular spectra emerge from the complexity of the fermionic shell model.

There is, however, a long way from the bosonic models to the full-fledged shell model. Several attempts have been made to bridge these two models through some justifiable truncation. One of the more successful among them is the model developed by Ginocchio [7], in which the fermions are allowed to be coupled to monopole and quadrupole pairs only. This
model has an algebraic structure which benefits both the physical intuition and the ease of computation, a feature reminiscent of the interacting boson model. Despite its schematic nature, the Ginocchio model has proven to be a surprisingly good phenomenology for samarium isotopes [8]. The Ginocchio model with its simplicity and flexibility is an ideal tool for the study of the fermionic effects on the chaotic nuclear dynamics.

In this Report, we examine the spectral statistics of the Ginocchio model. We first focus on the transition between vibrational and rotational dynamics. The results are compared to the earlier studies based on the interacting boson model. We then proceed to the problem of global change of the degree of chaos. It is shown that the experimental trend toward the regular motion in heavier nuclei naturally emerges from the Ginocchio model with the different shell degeneracy. We start from a single j-shell model in which a nucleus is described as a system of fermions with spin $i$ filling a shell specified by its orbital angular momentum $\vec{k}$ and total angular momentum $\vec{j} = \vec{k} + \vec{i}$. Ginocchio has conceived a model [7] where the intrinsic spin of the fermion is given by $i = 3/2$ in place of the nucleon’s one-half. We refer to this fictitious spin as the pseudo spin. In the L-S coupling scheme, the two-particle state is specified by its total orbital and spin angular momenta $\vec{K} = \vec{k}_1 + \vec{k}_2$ and $\vec{I} = \vec{i}_1 + \vec{i}_2$. The identity of the two fermions prohibits the odd values for the total spin, namely only $I = 0$ and $I = 2$ are allowed. The unique feature of the Ginocchio model is in the fact that the operators made of two fermions coupled to $K = 0$ form a closed algebra SO(8) whose 28 generators are given by

\begin{align}
S^\dagger &= \sqrt{2k+1} \begin{bmatrix} b_{k,i}^\dagger & b_{k,i} \end{bmatrix}^{(0,0)} \\
S &= \sqrt{2k+1} \begin{bmatrix} \tilde{b}_{k,i} & \tilde{b}_{k,i} \end{bmatrix}^{(0,0)} \\
D^\dagger &= \sqrt{2k+1} \begin{bmatrix} b_{k,i}^\dagger & b_{k,i} \end{bmatrix}^{(0,2)} \\
\tilde{D} &= \sqrt{2k+1} \begin{bmatrix} \tilde{b}_{k,i} & \tilde{b}_{k,i} \end{bmatrix}^{(0,2)}
\end{align}
\[ R^{(L)} = \sqrt{2k + 1} \left[ b_{ki}^+ \tilde{b}_{ki}^{(0,L)} \right] \quad L = 0, 1, 2, 3 \]  

(1e)

where \( b_{ki}^+ \) and \( b_{ki} \) are the fermion creation and annihilation operators. The notation \((0, L)\) means that the pseudo orbital angular momenta are coupled to 0 and the pseudo spins to L. The number operator of fermions \( n \) is given by \( n = 2R^{(0)} \). We also define \( N = n/2 \) which represents the number of pairs for later use.

The SO(8) symmetry of this model has three dynamical sub- symmetries SO(5) \( \times \) SU(2), SO(6) and SO(7), all sharing the common subgroup SO(5) in addition to the rotational symmetry SO(3), namely,

\[
SO(8) \supset \left\{ \begin{array}{c}
SO(5) \times SU(2) \\
SO(6) \\
SO(7)
\end{array} \right\} \supset SO(5) \supset SO(3)
\]  

(2)

The common subgroup SO(5) poses a problem in the actual application of the model, since no such universal symmetry is known among real nuclei. This difficulty has been solved in the extended model developed by Arima, Ginocchio and Yoshida [8] where protons and neutrons are distinguished. In our current work, we adopt a simplified version of this extended model, whose hamiltonian is given by

\[ H = -g_\pi S_\pi^+ S_\pi - g_\nu S_\nu^+ S_\nu - \kappa Q_\nu^{(2)} \cdot Q_\pi^{(2)} \]  

(3)

where quadrupole operator \( Q^{(2)} \) is defined by

\[ Q^{(2)} = R^{(2)} - \frac{\sqrt{7}}{24} \left[ D^\dagger \tilde{D} \right]^{(2)} \]  

(4)

The subscripts \( \pi \) and \( \nu \) in eq. (3) refer to the proton and neutron respectively. The first two terms in eq. (3) represent the paring interactions between like particles, while the last term represents the quadrupole interaction between different particles. Thus this hamiltonian has the essential feature of the “Paring plus QQ” model. The QQ interaction explicitly breaks the unwanted sub- symmetry SO(5). The shell degeneracies \( \Omega_\pi \) and \( \Omega_\nu \) appearing in eq. (4) are defined in terms of the values of orbital angular momenta \( k_\pi \) and \( k_\nu \) as \( \Omega_\pi = 2 \left( 2k_\pi + 1 \right) \) and \( \Omega_\nu = 2 \left( 2k_\nu + 1 \right) \). With this definition the shell degeneracy stands for
half of the maximum number of fermions in each shell. It is known that the Ginocchio model approaches to the interacting boson model when $\Omega_\pi$ and $\Omega_\nu$ approach infinity \cite{7,9}. In physical term, the collectivity realized at $\Omega = \infty$ is hindered by the Pauli-blocking effect when $\Omega$ is small. Without losing the essential feature, we simplify the hamiltonian by introducing a single parameter $x$ and putting the constraints among the three parameters $g_\pi$, $g_\nu$ and $\kappa$ as

$$g_\pi = g_\nu = 1 - x, \quad \kappa = 5 \, x$$  (5)

When the parameter $x$ is varied from 0 to 1, we can simulate nuclei from pairing-vibrational to rotational limits. The factor 5 in the second of eq. (5) is chosen just for convenience, and it can be re-scaled to any value without changing the physical content. The SO(5) dynamical symmetry exists in the pairing-vibrational limit. On the contrary, no dynamical symmetry is present in the rotational limit of the Ginocchio model, except in the limiting case of $\Omega_\pi = \Omega_\nu = \infty$ where the rotational limit becomes the SU(3) limit of the interacting boson model \cite{7,9}.

The exact relation between the level statistics of a quantum system and the dynamics of its classical counterpart is still an open problem in the quantum chaos\cite{10,11}. There exist, however, a fair amount of theoretical and numerical researches supporting the standard assumption of quantum level statistics, the correspondence between the order to chaos transition in the classical system and the Poisson to Wigner statistics in the nearest neighbor level spacing distribution in the quantum counterpart \cite{10-16}. The example of the interacting boson model indicates that this association might be taken seriously to a certain semi-quantitative level \cite{2,3}. On this premise, we examine the nearest level spacing distribution $P(s)$ to probe the degree of chaos in the nuclear levels.

We look at the change in the level statistical of the Ginocchio model along the vibrational and rotational limits as we vary the value of $x$ from zero to one. In Fig. (1), the nearest level spacing distributions $P(s)$ at $x = 0.1, 0.2, 0.4$ and $1$ are displayed. Throughout, the shell degeneracies and the numbers of the pairs are kept to be $\Omega_\pi = 11, \Omega_\nu = 22,$ and $N_\pi = \ldots$
5, \( N_\nu = 4 \), which corresponds to the \(^{150}\text{Nd}\) nucleus. One observes that the \( P(s) \) starts out as Poisson-like around the pairing limit, then becomes Wigner-like at intermediate region \( x = 0.4 \). Further to the rotational limit, it stays Wigner-like up to \( x = 1 \). The result in Fig. (1) can be interpreted as the dynamics of actual rotational nuclei being more chaotic than the previous studies based on the interacting boson model indicate. In hindsight, this is a natural consequence of the absence of dynamical symmetry in the rotational limit of the Ginocchio model.

The Fig. (2) shows how large \( \Omega \) one needs in order to reach the SU(3) limit of the interacting boson model in terms of the level statistics. Here, \( x \) is fixed to be one and \( \Omega = \Omega_\pi = \Omega_\nu \) is varied as 10, 20, 40, 80. \( N_\pi \) and \( N_\nu \) are again fixed to be 5 and 4. One observes that the Poisson statistics as in the boson limit is certainly recovered, but only at very large value of \( \Omega \).

The result shown in Fig. (2) also has an interesting implication to the global trend across the different nuclear mass region. Within the framework of single-j model, larger mass number corresponds to the larger value of \( \Omega \) for the valence shell. Therefore, the transition to Poisson-like statistics in Fig. (2) seems to offer a natural explanation for the experimentally observed global shift toward the regularity in the spectra of heavier nuclei [6]. In order to make a simple estimate, we assume that most nuclei are deformed, and most levels are rotational. We also assume the spectral statistics of the whole range of nuclei in a major shell is represented by a single typical nucleus. These assumptions may appear quite drastic, but we argue that they still keep the underlying physics intact. For the first point, it can be pointed out that even spherical nuclei display rotational bands in their excited states whose number tends to dominate over the sparsely found single particle excitations. For the second point, we may recall that a fixed parameter set has successfully described the entire samarium isotopes [8]. Also, we have checked in the actual calculation that the results are insensitive to the modest variation of the parameters.

In Table (1), the Brody parameters [17] of the level spacing statistics in the various nuclear mass regions are tabulated. Four columns correspond to \( (\Omega_\pi, \Omega_\nu) = (11, 11), (16, \)
(16), (16, 22) and (22, 29), each roughly representing nuclei of the mass regions of $50 < A < 100$, $100 < A < 150$, $150 < A < 210$ and $210 < A$, respectively. In the second line, theoretical estimates of the Brody parameter $\beta$ are shown. They are calculated from the second moment of $P(s)$ as in Ref. [3]. The parameter $x$ is fixed to be one in accordance with our assumption. The numbers of the fermion pairs are set to be $N_\pi = 5$ and $N_\nu = 4$ as a typical example. The number of the levels included in the calculation is 900, of which 80 are of $L = 0$ levels, 250 of $L = 2$ levels, 250 of $L = 3$ and 320 of $L = 4$. In the forth line of the Table (1), the "experimental" values of the Brody parameter for even-even nuclei taken from the paper of Shriner et al. [6] are tabulated. In the fifth line we show the same parameter for all nuclei. The agreement is as good as one can expect from our simplistic estimate and marginal statistics. In fact, we are unable to give, for example, the statistical error of the theoretical value of Brody parameter, since the model hamiltonian employed has very little apparent resemblance to the realistic hamiltonian. However, we stress that the basic properties of the shell model hamiltonian, namely the dominance of pairing and the quadrupole degrees of freedom and the associated dynamical symmetry are nicely captured in the Ginocchio model. This is evidenced both in its bridging role between shell model and the phenomenologically successful interacting boson model [7], and also in its own success as a phenomenology for the samarium isotopes with a simple hamiltonian very similar to the one we have employed in this work [10]. Thus we assert that the experimentally observed trend in the nuclear level statistics is physically understood within a schematic single j-shell model: namely, the chaotic spectrum in lighter nuclei is the result of the incomplete realization of dynamical symmetry due to the Pauli-blocking effect. Concerning the spin effect reported by Abul-Magd and Weidenmüller [18], we are unable to see the difference in the level statistics of $L = 0$, 3 states and $L = 2$, 4 states in the current model calculations. This and other experimental characteristics will be the subjects of the forthcoming publication along with the more detailed description of our calculations.

In summary, we have examined the spectral statistics of a model hamiltonian in the Ginocchio model in order to probe the chaotic dynamics in the nuclear low-lying states
across broad nuclear mass region. The results show several novel features which were not noticed in previous studies with phenomenological bosonic models.

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FIGURES

FIG. 1. The level spacing distribution $P(s)$ of the Ginocchio model with the various values of $x$, with Wigner (solid lines) and Poisson (dotted lines) formulae.

FIG. 2. $P(s)$ for various $\Omega$ at the rotational limit $x = 1$. 
TABLE I. The Brody parameter $\beta$ at various mass region. Here $\beta = 0$ corresponds to the Poisson (or regular) statistics, and $\beta = 1$ to the Wigner (or chaotic) statistics. The $\beta$(calc) means calculated Brody parameter in the present model. The $\beta$(exp)E indicates the experimentally compiled Brody parameter for even-even nuclei, while $\beta$(exp)A, for all nuclei.

| $\Omega_\pi/\Omega_\nu$ | $\beta$(calc) | A          | $\beta$(exp)E | $\beta$(exp)A  |
|-------------------------|--------------|------------|--------------|----------------|
| 11/11                   | 0.74         | 50–100     | –            | 0.88±41        |
| 16/16                   | 0.60         | 100–150    | 0.62±16      | 0.55±11        |
| 22/16                   | 0.51         | 150–180,180-210 | 0.26±11, 0.30±18 | 0.33±07, 0.43±17 |
| 29/22                   | 0.37         | 210–       | 0.27±32      | 0.24±10        |
$P(s)$
