Research on Computer Modeling of Fractional Differential Equation Applied Mathematics

Linlin Su*
University College London, UK

*Corresponding author e-mail: sulinlin@london.ac.uk

Abstract. This paper qualitatively analyzes the stability of the equilibrium solution of a class of fractional chaotic financial systems and the conditions for the occurrence of Hopf bifurcation, and uses the Adams-Bashford-Melton predictive-correction finite difference method to pass the analysis Bifurcation diagrams, phase diagrams, and time series diagrams are used to simulate the complex evolution behavior of the system.

Keywords: fractional differential equation; financial system; computer modeling; financial model; chaos.

1. Introduction
The FarolBar problem and the minority game model and the application of some important theories and methods of physics in economic and financial research have epoch-making academic significance. They have formed a strong cohesive force, which has contributed to the establishment and promotion of economic physics [1]. In many scientific fields or engineering technologies, there are many problems that can be described by partial differential equations. However, the inverse problem of partial differential equations is a new research field, which is proposed from actual needs. For example: in geological prospecting, underground target reconstruction, oil extraction, structural fault diagnosis, non-destructive testing, aircraft airfoil design, medical imaging, weather forecasting, etc., there are inverse problems of partial differential equations [2]. The inverse problem of partial differential equations is also called the inverse problem of mathematical physics equations. It is different from the traditional partial differential equation problem that is the positive problem (known mathematical physics equations and corresponding definite solution conditions to solve the definite solution problem). Partial differential equations the inverse problem of equation is to reverse the "cause" from the "effect", that is, to reverse the unknown part of the delimitation problem from the known or partly known information of the solution. We use the language of system theory to describe the difference between the two: the positive problem is that in a given system, the corresponding output results are obtained from the known input conditions, and these output results contain parts of the given system [3]. Known information; and the inverse problem is to reverse some unknown structural features in the system from part of the known information of the output result. Therefore, in practice, the inverse problem corresponds to solving the structural problem inside the medium through the measurable indirect information outside the medium.
2. Preliminary knowledge of mathematics
At present, there are many discussions on the basic problems of the solution of fractional differential equations, mainly studying the existence and uniqueness of the solutions of fractional differential equations and the continuous dependence of the solutions on initial values and orders [4]. ("Analysis of Solutions to Fractional Differential Equations") For example: Diathermal studied nonlinear fractional differential equations with initial value conditions

\[
\begin{align*}
D^\alpha(y - T_{m-1}[y])(x) &= f(x, y(x)), \\
y^{(k)}(0) &= y^{(0)}(k),
\end{align*}
\] (1)

He used the fixed-point theorem to prove the local existence of understanding, the uniqueness of the solution, and the dependence of the solution on the initial value and order; Diathermal extended Diathermia's research conclusions to fractional differential systems. El-Sayed used Banach contraction mapping principle and Sahauder fixed point theorem to prove linear fractional differential equations

\[
\frac{d^{\alpha_i}x(t)}{dt^{\alpha_i}} = f(t, x, \frac{d^{\alpha_i}x(t)}{dt^{\alpha_i}}, \ldots, \frac{d^{\alpha_i}x(t)}{dt^{\alpha_i}})
\] (2)

\(\alpha, \alpha_i\) is a real number, \((i=1,2,\ldots,n)\), \(0 < \alpha_1 < \alpha_2 < \ldots < \alpha_n\) the existence and uniqueness of the solution and some other properties of the solution.

Assuming that \(K : X \rightarrow Y\) is a compact linear operator, we consider the stability of the solution to the ill-posed problem corresponding to equation \(Kx = y\). The right end data \(y^\delta\) with error satisfies

\[\|y^\delta - y\| \leq \delta\] (3)

How to find the approximate value \(x^\delta\) of \(x\) from \(y^\delta\)? Generally speaking, it cannot be obtained directly from the corresponding equation \(Kx^\delta = y^\delta\). There are two reasons: 1) When \(y^\delta \notin K(x)\), the equation has no solution; 2) Even if \(y^\delta \in K(x)\), because \(K^{-1}\) is unbounded, when \(\delta \rightarrow 0\), the solution \(x^\delta \rightarrow x\) obtained by equation (11) cannot be guaranteed. In order to make \(y^\delta \notin K(x)\) also find a suitable method to construct the approximate solution \(x^\delta\) and ensure the continuous dependence of \(x^\delta\) on \(y^\delta\), the bounded approximation operator \(R : Y \rightarrow X\) of \(K^{-1} : K(X) \rightarrow X\) must be constructed.

3. Spatial Fractional Reverse Time Diffusion Problem
In this paper, the following one-dimensional reverse heat conduction problem is established and studied:

\[
\begin{align*}
u(t, x) &= D_0^\alpha u(x, t), (0 < \alpha < 1) \\
u(x, T) &= f(x), x \in R, t \in (0, T)
\end{align*}
\] (4)

Our goal is to invert the solution \(u(x, t)\) of the problem from the final value data \(u(x, T) = f(x)\), where \(0 < t < T\) is satisfied. It is a serious ill-posed problem, because a small disturbance in the final value data \(f(x)\) will cause a huge change in the solution \(u(x, t)\). Therefore, this paper will use the quasi-inverse regularization method to solve the ill-posed problem and give an error estimate [5]. In practice, the original data can only be measured by physical instruments, and there must be errors. We set the final value at the time \(t = T\). The accurate data \(f(x)\) and the measured data \(f^\delta(x)\) meet the following relationship.
Where $\delta$ is the error level. In order to get faster convergence, we also need to assume that function $u(x,0)$ satisfies the prior bound

$$\left\| u(x,0) \right\| \leq E$$

(6)

Here $E$ is a finite normal number. After the above preparations, we make Fourier changes to $x$ on both sides of the original equation (1) to obtain:

$$\begin{cases}
\hat{u}(\omega,t) = (i \omega)^\alpha \hat{u}(\omega,t), (0 < \alpha < 1) \\
\hat{u}(\omega,T) = \hat{f}(\omega)
\end{cases}$$

(7)

After Fourier change, the original problem is transformed from partial differential equation to ordinary differential equation, so that the Fourier transform solution of the original problem can be easily obtained:

$$\hat{u}(\omega,t) = e^{(i \omega)^\alpha (T-t)} \cdot \hat{f}(\omega)$$

(8)

And the initial time, namely $t = 0$, the Fourier transform solution of the original problem:

$$\hat{u}(\omega,0) = e^{(i \omega)^\alpha T} \cdot \hat{f}(\omega)$$

(9)

Due to the ill-posed nature of the reverse heat conduction problem, we will choose an appropriate regularization method to solve this problem [6]. Through the analysis of the nature of this one-dimensional reverse heat conduction problem and the previous literature search, we use the quasi-inverse regularization method to construct a stable regularization solution and realize the inversion of the initial distribution of the problem.

$$\begin{cases}
\hat{v}(x,t) = D_\alpha v(x,t) + \mu \hat{v}(x,t), (0 < \alpha < 1) \\
\hat{v}(x,T) = \hat{f}^\delta(x)
\end{cases}$$

(10)

Similarly, the Fourier change of $x$ on both sides of equation (4) gives:

$$\begin{cases}
\hat{\nu}(\omega,t) = (1 + \mu \omega^2) \cdot (i \omega)^\alpha \hat{v}(\omega,t), (0 < \alpha < 1) \\
\hat{\nu}(\omega,T) = \hat{f}^\delta(\omega)
\end{cases}$$

(11)

Get the regularized solution of the Fourier transform:

$$\hat{v}(\omega,t) = e^{\frac{(i \omega)^\alpha (T-t)}{1 + \mu \omega^2}} \cdot \hat{f}^\delta(\omega)$$

(12)

Among them, $\mu$ is the regularization parameter, and $\delta$ is the error level [7]. In this article, we adopt a posteriori strategy to select the regularization parameters.

4. Fractional chaotic financial system complexity evolution simulation

This part uses the Adams-Bashford-Melton predictor-corrected finite difference method to simulate the impact of changes in savings $a$ and differential order $q_1$ on the evolution of system (2) complexity. We set $q_1=0.88$, $Q_2=0.98$ and $q_3=0.92$, $b=0.1$ and $c=1$, and another initial value $(x_0, y_0, z_0) = (2, 3, 2)$. When we adjust the amount of savings $a \in [0, 10]$ When, we can draw the bifurcation diagram of the interest rate $x$ of the system (2), as shown in Figure 1.
Figure 1. The bifurcation diagram of the interest rate $x$ of the system (2) changing with the amount of savings $a$

Bifurcation Figure 1 shows that the complexity of system (2) changes accordingly with our adjustment to the amount of savings $a \in [0, 10]$. We can take the amount of savings $a=4$, then the relative value of system (2) The graph is shown in Figure 2, and the time series graph of interest rate $x$ is shown in Figure 3.

Figure 2. The phase diagram of the system (2) when the amount of savings $a=4$

Figure 3. Time series diagram of the interest rate $x$ of the system (2) when the amount of savings $a=4$

Combining Figures 2 and 3 with bifurcation diagram 1, we can find that the economic and financial system has changed drastically and is in a state of chaos [8]. We can take the amount of savings $a=9$, 
then the phase diagram of the system (2) is shown in Figure 4, and the time series diagram of interest rate \( x \) as shown in Figure 5, combined with the bifurcation diagram 1, it can be found that the Hopf bifurcation occurs in the system at \( a=9 \), and the whole system is in a state of regular periodic movement.

![Figure 4](image1.png)

**Figure 4.** The phase diagram of the system (2) when the amount of savings \( a=9 \)

![Figure 5](image2.png)

**Figure 5.** Time series diagram of the interest rate \( x \) of the system (2) when the amount of savings \( a=9 \)

We can set the amount of savings to \( a=9.01 \), then the phase diagram of system (2) is shown in Figure 6, and the time series diagram of interest rate \( x \) is shown in Figure 7. Combined with bifurcation diagram 1, we can find that when \( a>9 \) o'clock the system tends to a stable equilibrium state.

![Figure 6](image3.png)

**Figure 6.** The phase diagram of system (2) when the amount of savings \( a=9.01 \)

In short, when \( 0\leq a<9 \), the system (2) is in an unstable chaotic state, and the fluctuation is relatively severe, which will have a greater destructive effect on the stable operation of the economy. It needs
timely regulation to prevent long-term violent fluctuations in the economic and financial system; When \( a=9 \), the system (2) will have Hopf bifurcation, and the whole system is in a state of periodic movement, but it is also an unstable state; when \( a>9 \), the system (2) is in an asymptotically stable equilibrium state, and the whole system is in an asymptotically stable equilibrium state. The economic and financial system can operate stably and orderly, and long-term predictions can be made on the future state of the system.

5. Conclusion
This paper studies a qualitatively the stability of a fractional-order chaotic financial system and the conditions for the occurrence of Hopf bifurcations, and the Adams-Bashford-Melton prediction-correction finite difference method, through bifurcation diagrams, phase diagrams and time series diagrams, the influence of the amount of savings and the order of differentiation on the evolution of the complexity of the fractional financial system is numerically simulated.

References
[1] Rao, R. Global stability of a Markovian jumping chaotic financial system with partially unknown transition rates under impulsive control involved in the positive interest rate. Mathematics, vol. 7, pp.579-588,July 2019.
[2] Rao, R. & Zhong, S. Input-to-state stability and no-inputs stabilization of delayed feedback chaotic financial system involved in open and closed economy. Discrete & Continuous Dynamical Systems-S, vol. 14, pp. 1375-1389 April 2021.
[3] Bryant, A. Liquid uncertainty, chaos and complexity: The gig economy and the open source movement. Thesis Eleven, vol. 156, pp. 45-66, January 2020.
[4] Kasianova, N. Tarasova, E. & Kravchuk, N. Enterprise development management through managed chaos. Independent Journal of Management & Production, vol. 10, pp. 1626-1644, May 2019.
[5] Xin, B. Peng, W. Kwon, Y. & Liu, Y. Modeling, discretization, and hyperchaos detection of conformable derivative approach to a financial system with market confidence and ethics risk. Advances in Difference Equations, vol.19, pp. 1-14, January 2019.
[6] Gomes, O. & Gubareva, M. Complex Systems in Economics and Where to Find Them. Journal of Systems Science and Complexity, vol.34, pp.314-338, January 2019.
[7] Mosteau, N. R. Intelligent tool to prevent Economic Crisis–Fractals. A possible solution to assess the Management of Financial Risk. Calitatea, vol.20, pp. 13-17, January 2019.