$D^0$-$\bar{D}^0$ mixing parameter $y$ in the factorization-assisted topological-amplitude approach

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Abstract

We calculate the $D^0$-$\bar{D}^0$ mixing parameter $y$ in the factorization-assisted topological-amplitude (FAT) approach, considering contributions from $D^0 \to PP$, $PV$, and $VV$ modes, where $P$ ($V$) stands for a pseudoscalar (vector) meson. The $D^0 \to PP$ and $PV$ decay amplitudes are extracted in the FAT approach, and the $D^0 \to VV$ ones with final states in the longitudinal polarization are estimated via the parameter set for $D^0 \to PV$. It is found that the $VV$ contribution to $y$, being of order of $10^{-4}$, is negligible, and that the $PP$ and $PV$ contributions amount only up to $y_{PP+PV} = (0.21 \pm 0.07)\%$, much lower than the experimental data $y_{\text{exp}} = (0.61 \pm 0.08)\%$. We conclude that $D^0$ meson decays into other two-body and multi-particle final states are relevant to the evaluation of $y$, so it is difficult to have its full understanding in an exclusive approach.
I. INTRODUCTION

Studies of neutral meson mixings have marked glorious progress in particle physics: the kaon mixing accounts for the first CP violation observed in the $K_L \rightarrow \pi\pi$ decays [1]; the masses of a charm quark [2] and of a top quark [3, 4] were, before their discoveries, estimated through the GIM mechanism involved in the kaon and $B_d$ meson mixings, respectively. The neutral meson mixings are still a potential regime for searching new physics nowadays, because the relevant flavor-changing amplitudes are loop-suppressed in the Standard Model. To get closer to this goal, it is crucial to make sure that the mixing dynamics is understood to high precision. It has been known that the $B_d(s)$ meson mixing is well described in the heavy quark effective theory [5, 6], indicating that both the power expansion parameter $1/m_b$ and the strong coupling $\alpha_s(m_b)$ at the scale of the bottom quark mass $m_b$ are small enough for justifying a perturbative analysis. However, the understanding of the $D^0-\bar{D}^0$ mixing has remained a challenge since its first observation [7–9]. It is suspected that $1/m_c$ and $\alpha_s(m_c)$ with $m_c$ being the charm quark mass may be too large to allow perturbation expansion.

The products $V_{td}V_{td}^*$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $i = u, c,$ and $t$, which appear in the box diagram responsible for the $B_d$ meson mixing, are of the same order. In the $B_s$ meson mixing, $V_{tb}V_{ts}^*$ and $V_{cb}V_{cs}^*$ are of the same order, and both much larger than $V_{ub}V_{us}^*$. Hence, an intermediate top quark with a much higher mass moderates the GIM cancellation, giving a dominant contribution to the bottom mixing. In the $D^0-\bar{D}^0$ mixing an intermediate bottom quark does not play an important role due to the tiny product $V_{cb}V_{ub}^*$. The charm mixing is then governed by the difference between the other two intermediate quarks $s$ and $d$, namely, by $SU(3)$ symmetry breaking effects, to which nonperturbative contribution is expected to be significant.

The current world averages of the charm mixing parameters are given by [10]

$$x = (0.46^{+0.14}_{-0.15})\%, \quad y = (0.62 \pm 0.08)\%,$$

assuming no CP violation in charm decays [1]. There are two approaches, inclusive and exclusive, in the literature to the evaluation of the charm mixing parameters: the former, with short-distance contributions calculated based on the heavy quark expansion, leads to values of $x$ and $y$ two or three orders of magnitude lower than the data, even after the operators of dimension nine [12, 13].

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1 As CP violation is allowed, the mixing parameters turn into [10, 11]

$$x = (0.32 \pm 0.14)\%, \quad y = (0.69^{+0.06}_{-0.07})\%.$$
or both $\alpha_s$ and subleading $1/m_c$ corrections were taken into account. Obviously, the mass difference between the $s$ and $d$ quarks cannot collect all $SU(3)$ breaking effects in charm decays, which may instead originate mainly from hadronic final states. This speculation is supported by the argument that a modest quark-hadron duality violation of about 20% explains the discrepancy between inclusive predictions and the data. Contributions to the charm mixing from individual intermediate hadronic channels are summed up in an exclusive approach. It was noticed that the $SU(3)$ breaking effects only from the phase space naturally induce $x$ and $y$ at the order of one percent. The two-body $D \to PP$ and $PV$ decays were found to contribute to both $x$ and $y$ at the order of $10^{-3}$ in the topological diagrammatic approach. The above attempts seem to imply that an exclusive approach including the $SU(3)$ breaking effects from two-body hadronic $D$ meson decays can accommodate the charm mixing data qualitatively.

In this paper we will address this issue in the factorization-assisted topological-amplitude (FAT) approach, which provides a more precise treatment of the $SU(3)$ breaking effects from two-body hadronic $D$ meson decays, as indicated by the better global fit to the measured branching ratios than in [17]. The FAT approach is based on the factorization of short-distance and long-distance dynamics in the topological amplitudes for the above decays into Wilson coefficients and hadronic matrix elements of effective operators, respectively. The latter are partly computed in the naive factorization with non-factorizable contributions being parameterized into strong parameters. Through a global fit to the abundant decay-rate data, the strong parameters are determined and then used to make predictions for unmeasured branching ratios and CP asymmetries. This approach has been successfully applied to the studies of the $D \to PP$ and $D \to PV$ modes, as well as the charmed and charmless $B$ meson decays. In particular, the predicted difference of the direct CP asymmetries $\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-0.6 \sim -1.9) \times 10^{-3}$ was confirmed by the LHCb data later. Distinct from the traditional diagrammatic approach, the FAT approach grasps the $SU(3)$ breaking effects in different phase space, decay constants, form factors, and strong phases associated with various final states.

The $D^0 \to PP$ and $PV$ decay amplitudes required for the evaluation of the mixing parameter $y$ are extracted in the FAT approach. The $D^0 \to VV$ ones with final states in the longitudinal polarization are estimated via the parameter set for $D^0 \to PV$, which does yield the corresponding branching ratios in agreement with data. We will show that the $D^0 \to PP$, $PV$ and $VV$ channels contribute $y_{PP} = (0.10 \pm 0.02)\%$, $y_{PV} = (0.11 \pm 0.07)$, and $y_{VV} \sim 10^{-4}$, respectively, to the mixing parameter with small uncertainties. Namely, the above two-body channels alone, which take up about 50% of the total $D^0$ meson decay rate, cannot explain the $D^0$-$\bar{D}^0$ mixing in an exclusive
approach. It is then expected that other two-body and multi-particle hadronic $D$ meson decays are relevant to the calculation of $y$, which are, however, extremely difficult to analyze in an exclusive approach at the current stage. Therefore, a new strategy to understand the charm mixing dynamics is necessary.

In Sec. II we update the determination of the $D^0 \to PP$ and $PV$ amplitudes by performing a global fit to the latest data of the branching ratios in the FAT approach. Their contributions to the charm mixing parameter $y$ are then obtained. The $D^0 \to VV$ amplitudes for the longitudinal polarization are estimated via the parameter set for the $PV$ modes in Sec. III, and found to give a small contribution to $y$. Section IV contains the summary.

II. $y_{PP}$ AND $y_{PV}$

The $D^0$-$D^0$ mixing parameter $y$ is defined by

$$y \equiv \frac{\Gamma_1 - \Gamma_2}{2\Gamma},$$

(3)

where $\Gamma_{1,2}$ represent the widths of the mass eigenstates $D_{1,2}$, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$. In the assumption of CP conservation, the mass eigenstates are identical to the CP eigenstates, i.e., $|D_1\rangle = |D_+\rangle$ and $|D_2\rangle = |D_-\rangle$, with $|D_{\pm}\rangle = (|D^0\rangle \pm |\bar{D}^0\rangle)/\sqrt{2}$. Here we adopt the convention of $\mathcal{CP}|D^0\rangle = +|\bar{D}^0\rangle$. The parameter $y$ can be computed via the formula

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left( |A(D_+ \to n)|^2 - |A(D_- \to n)|^2 \right)$$

$$= \frac{1}{\Gamma} \sum_n \eta_{\mathcal{CP}}(n) \rho_n \Re \left[ A(D^0 \to n) A^*(D^0 \to \bar{n}) \right],$$

(4)

in which $\rho_n$ is the phase-space factor for the $D^0/\bar{D}^0$ decay into the final state $n$, and the transformation $\mathcal{CP}|n\rangle = \eta_{\mathcal{CP}}|\bar{n}\rangle$ has been applied. For the $PP$ and $PV$ modes, $\eta_{\mathcal{CP}} = +1$, and for the $VV$ modes, $\eta_{\mathcal{CP}} = (-1)^L$ with $L$ denoting the orbital angular momentum of the final state. The following expression is also employed in the literature

$$y = \sum_n \eta_{\mathcal{CKM}}(n) \eta_{\mathcal{CP}}(n) \cos \delta_n \sqrt{\mathcal{B}(D^0 \to n) \mathcal{B}(\bar{D}^0 \to \bar{n})},$$

(5)

where $\delta_n$ is the relative strong phase between the $D^0 \to n$ and $D^0 \to \bar{n}$ amplitudes, and $\eta_{\mathcal{CKM}} = (-1)^{n_s}$ with $n_s$ being the number of the $s$ or $\bar{s}$ quarks in the final state.

The explicit parametrizations of the $D \to PP$ and $PV$ amplitudes in the FAT approach can be found in [18] and [19], respectively, which have been extracted from the measured branching
It is noticed that the $W$-exchange diagram $E$ appears only in $D^0$ meson decays, while the $W$-annihilation diagram $A$ contributes only to $D^+$ and $D_s^+$ meson decays. For the study of the $D^0$-$\overline{D}^0$ mixing, we focus on the $D^0$ meson decay modes, so that the irrelevant strong parameters associated with the amplitudes $A$ can be removed from global fits. Below we update the corresponding sets of strong parameters determined by the latest data:

$$\chi_{nf}^C = -0.81 \pm 0.01, \quad \phi_{nf}^C = 0.22 \pm 0.14, \quad S_\pi = -0.92 \pm 0.07,$$

$$\chi_q^E = 0.056 \pm 0.002, \quad \phi_q^E = 5.03 \pm 0.06, \quad \chi_s^E = 0.130 \pm 0.008, \quad \phi_s^E = 4.37 \pm 0.10,$$

for the $D^0 \to PP$ decays, and

$$S_\pi = -1.88 \pm 0.12, \quad \chi_P^C = 0.63 \pm 0.03, \quad \phi_P^C = 1.57 \pm 0.11,$$

$$\chi_V^C = 0.71 \pm 0.03, \quad \phi_V^C = 2.77 \pm 0.10, \quad \chi_q^E = 0.49 \pm 0.03,$$

$$\phi_q^E = 1.61 \pm 0.07, \quad \chi_s^E = 0.54 \pm 0.03, \quad \phi_s^E = 2.23 \pm 0.08,$$

for the $D^0 \to PV$ decays. In both the $PP$ and $PV$ modes, the parameter $\Lambda$ related to the soft scale in $D$ meson decays is fixed to be 0.5 GeV. The decay constants of the vector mesons are from [27], and other theoretical inputs are the same as in [18, 19]. The minimal $\chi^2$s per degree of freedom are 1.1 for the $PP$ modes with 13 data, and 1.8 for the $PV$ modes with 19 data. As observed in Table I, the predictions for the $D^0 \to PP$ and $PV$ branching fractions agree well with the data.

Based on Eqs. (6) and (7), we calculate the $D \to PP$ and $D \to PV$ contributions to $y$ by means of Eq. (4), obtaining

$$y_{PP} = (1.00 \pm 0.19) \times 10^{-3},$$

$$y_{PV} = (1.12 \pm 0.72) \times 10^{-3},$$

respectively. Our results are consistent with those derived in [17]: $y_{PP} = (0.86 \pm 0.41) \times 10^{-3}$, $y_{PV} = (2.69 \pm 2.53) \times 10^{-3}$ ($A, A1$) and $y_{PV} = (1.52 \pm 2.20) \times 10^{-3}$ ($S, S1$) from two different solutions, but with much smaller uncertainties. Actually, the predictions for $y_{PP}$ and $y_{PV}$ in this work are the most precise ones up to now. The uncertainties of the parameters in Eqs. (6) and (7) are basically controlled by those most precisely measured channels, explaining why $y_{PP}$, with the more precise $PP$ data, is more certain than $y_{PV}$. Besides, the branching ratios are correlated to each other by the strong parameters in the FAT approach, so the uncertainties are greatly reduced. Since the $SU(3)$ symmetry is assumed in the topological diagrammatic approach [17], the charm mixing parameter $y$ cannot be extracted in principle. Instead, the data of the branching ratios
TABLE I: Branching ratios for the $D^0 \to PP$ and $PV$ decays in units of $10^{-3}$. Predictions in the FAT approach are compared with the experimental data.

| Modes | $B$(exp) | $B$(FAT) | Modes | $B$(exp) | $B$(FAT) | Modes | $B$(exp) | $B$(FAT) |
|-------|---------|---------|-------|---------|---------|-------|---------|---------|
| $\pi^+ K^0$ | $24.0 \pm 0.8$ | $24.2 \pm 0.8$ | $\pi^0 K^0$ | $37.5 \pm 2.9$ | $35.9 \pm 2.2$ | $K^0 \rho^0$ | $12.8^{+1.4}_{-1.6}$ | $13.5 \pm 1.4$ |
| $\pi^+ K^-$ | $39.3 \pm 0.4$ | $39.2 \pm 0.4$ | $\pi^+ K^*$ | $54.3 \pm 4.4$ | $62.5 \pm 2.7$ | $K^- \rho^+$ | $111.0 \pm 9.0$ | $105.0 \pm 5.2$ |
| $\eta K^0$ | $9.70 \pm 0.6$ | $9.6 \pm 0.6$ | $\eta K^o$ | $9.6 \pm 3.0$ | $6.1 \pm 1.0$ | $K^0 \omega$ | $22.2 \pm 1.2$ | $22.3 \pm 1.1$ |
| $\eta' K^0$ | $19.0 \pm 1.0$ | $19.5 \pm 1.0$ | $\eta' K^o$ | $< 1.10$ | $0.19 \pm 0.01$ | $K^0 \phi$ | $8.47^{+0.66}_{-0.34}$ | $8.2 \pm 0.6$ |
| $\pi^+ \pi^-$ | $1.421 \pm 0.025$ | $1.44 \pm 0.02$ | $\pi^+ \rho^-$ | $5.09 \pm 0.34$ | $4.5 \pm 0.2$ | $\pi^- \rho^+$ | $10.0 \pm 0.6$ | $9.2 \pm 0.3$ |
| $K^+ K^-$ | $4.01 \pm 0.07$ | $4.05 \pm 0.07$ | $K^+ K^*$ | $1.62 \pm 0.15$ | $1.8 \pm 0.1$ | $K^- K^{**}$ | $4.50 \pm 0.30$ | $4.3 \pm 0.2$ |
| $K^0 K^0$ | $0.36 \pm 0.08$ | $0.29 \pm 0.07$ | $K^0 K^*$ | $0.18 \pm 0.04$ | $0.19 \pm 0.03$ | $K^0 K^{**}$ | $0.21 \pm 0.04$ | $0.19 \pm 0.03$ |
| $\pi^0 \eta$ | $0.69 \pm 0.07$ | $0.74 \pm 0.03$ | $\eta \rho^0$ | $1.4 \pm 0.2$ | $\pi^0 \omega$ | $0.117 \pm 0.035$ | $0.10 \pm 0.03$ |
| $\pi^0 \eta'$ | $0.91 \pm 0.14$ | $1.08 \pm 0.05$ | $\eta' \rho^0$ | $0.25 \pm 0.01$ | $\pi^0 \phi$ | $1.35 \pm 0.10$ | $1.4 \pm 0.1$ |
| $\eta$ | $1.70 \pm 0.20$ | $18.6 \pm 0.06$ | $\eta \omega$ | $2.21 \pm 0.23$ | $2.0 \pm 0.1$ | $\eta \phi$ | $0.14 \pm 0.05$ | $0.18 \pm 0.04$ |
| $\eta' \eta'$ | $1.07 \pm 0.26$ | $1.05 \pm 0.08$ | $\eta' \omega$ | $0.044 \pm 0.004$ | $\pi^0 \rho^0$ | $3.82 \pm 0.29$ | $4.1 \pm 0.2$ |
| $\pi^0 K^0$ | $0.826 \pm 0.035$ | $0.78 \pm 0.03$ | $\pi^0 K^*$ | $3.82 \pm 0.29$ | $4.1 \pm 0.2$ | $K^0 \rho^0$ | $0.039 \pm 0.004$ | $K^0 \rho^0$ | $0.144 \pm 0.009$ |
| $\pi^- K^+$ | $0.133 \pm 0.009$ | $0.133 \pm 0.001$ | $\pi^- K^{**}$ | $0.345^{+0.180}_{-0.102}$ | $0.40 \pm 0.02$ | $K^0 \omega$ | $0.064 \pm 0.003$ | $K^0 \phi$ | $0.024 \pm 0.002$ |

were directly input into Eq. (5) by taking $\cos \delta_n = 1$, such that the uncertainties of the data are summed up in the evaluation of $y$. Some other efforts have been devoted to global fits of the $PP$ or $PV$ data recently. However, it is unlikely to make a precise prediction for $y$ without thorough exploration of the $SU(3)$ breaking effects in the relevant $D$ meson decays.

III. $y_{VVV}$

There exist three different polarizations in the final state of a $D \to VV$ channel, whose corresponding amplitudes can be expressed in the transversity basis ($A_0$, $A_{||}$, $A_{\perp}$), or equivalently in the partial-wave basis ($S$, $P$, $D$). The decay amplitudes for different polarizations are independent, and should be described by different sets of strong parameters in the FAT approach. At least six strong parameters are required for the longitudinal amplitude $A_0$ alone, but only one channel has been observed with the longitudinal branching ratio $B_0(D^0 \to \rho^0 \rho^0) = (1.25 \pm 0.10) \times 10^{-3}$. The situation for the transverse amplitudes is even worse. Apparently, it is impossible to extract all the $D \to VV$ amplitudes in the FAT approach due to the lack of experimental data at present.
As a bold attempt, we estimate the $D \to VV$ longitudinal amplitudes by means of the strong parameters in Eq. (7) extracted from the $PV$ data. In detail, the factorizable part in an emission-type amplitude is treated in the naive factorization hypothesis, and the associated non-factorizable amplitude, $\chi_V e^{i\phi_V}$, is assumed to be identical to that of the corresponding $PV$ amplitude. We adopt the definition of the vector meson decay constant $f_V$ via

$$\langle V(q)|\bar{q}\gamma_\mu(1-\gamma_5)q'|0\rangle = f_V m_V \varepsilon^*_\mu(q),$$

and the definition of the $D \to V$ transition form factors $V^{DV}, A_1^{DV}, A_2^{DV},$ and $A_0^{DV}$ via

$$\langle V(k)|\bar{q}\gamma_\mu(1-\gamma_5)c|D(p)\rangle = \frac{2}{m_D + m_V} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho k^\sigma V^{DV}(q^2)$$

$$- i \left( \frac{\epsilon^* \cdot q}{q^2} q_\mu \right) (m_D + m_V) A_1^{DV}(q^2)$$

$$+ i \left( (p + k)_\mu - \frac{m_D^2 - m_V^2}{q^2} q_\mu \right) \frac{\epsilon^* \cdot q}{m_D + m_V} A_2^{DV}(q^2)$$

$$- i \frac{2m_V (\epsilon^* \cdot q)}{q^2} q_\mu A_0^{DV}(q^2),$$

where $\epsilon$ is the polarization vector, $m$'s are the meson masses, and the momentum $q = p - k$. The emission-type amplitudes are then expressed as

$$T(C) = \frac{G_F}{\sqrt{2}} V_{CKM} a_1(\mu) \left( a_2^C(\mu) \right) f_{V1} m_1$$

$$\times \left[ -ix(m_D + m_2) A_1^{DV}(m_1^2) + i \frac{2m_D^2 p^2}{(m_D + m_2)m_1 m_2} A_2^{DV}(m_1^2) \right],$$

in which the Wilson coefficients and the kinetic quantities are given by

$$a_1(\mu) = C_1(\mu) + C_2(\mu), \quad a_2^C(\mu) = C_1(\mu) + C_2(\mu) \left( \frac{1}{N_c} + \chi_V e^{i\phi_V} \right),$$

$$x = \frac{m_D^2 - m_1^2 - m_2^2}{2m_1 m_2}, \quad p^2 = \frac{m_1^2 m_2^2 (x^2 - 1)}{m_D^2},$$

respectively. The values of the form factors $A_{1,2}^{DV}$ are input from [32]. The annihilation-type amplitudes are taken directly from the $PV$ modes with the replacement of the meson masses and decay constants, explicitly written as

$$E = -i \frac{G_F}{\sqrt{2}} V_{CKM} C_2(\mu) \chi_V e^{i\phi_V} f_{V1} f_{V2} m_D^2 \frac{|p_c|}{\sqrt{m_1 m_2}}.$$ 

After estimating the $D \to VV$ longitudinal amplitudes, we can derive the corresponding branching ratios straightforwardly. The comparison of our predictions with the data will tell whether the $PV$-inspired amplitudes are reasonable. The $D^0 \to VV$ longitudinal branching ratios in the FAT approach are listed in Table II, and compared with the data of the total and longitudinal branching
fractions. A general consistency with the data is seen, especially for the single observed longitudinal branching ratio \( B_{\text{long}}(D^0 \to \rho^0 \rho^0) \). For those channels with only measured total branching ratios, most of our predictions for the longitudinal branching ratios do not exceed the data, after considering the uncertainties. Our result for the \( D^0 \to K^{*0} \omega \) mode is larger than the data, but the measurement of this mode was performed in 1992 [33], and should be updated. It is thus a fair claim that our simple estimates for the \( D^0 \to VV \) longitudinal amplitudes are satisfactory. Certainly, more experimental effort toward improved understanding of the \( D \to VV \) decays into final states with different polarizations is encouraged.

A longitudinal amplitude \( A_0 \) is a linear combination of the partial waves \( S \) and \( D \), namely, of the \( L = 0 \) and \( 2 \) final states, leading to \( \eta_{\text{CP}}(n) = +1 \) in Eq. (4). Inserting the amplitudes estimated above to Eq. (4), we obtain the longitudinal \( VV \) contribution

\[
y_{VV} = (0.28 \pm 0.47) \times 10^{-3}. \tag{16}
\]

The central value of \( y_{VV} \) is lower than those of \( y_{PP} \) and \( y_{PV} \) in Eq. (8) and (9), because the \( SU(3) \) breaking effects are much smaller in the \( VV \) modes. Even though Eq. (16) contains a relatively larger uncertainty in our approach, and the contributions from the transverse polarizations have not yet been included, it is reasonable to postulate that \( y_{VV} \) represents a minor contribution to \( y \).

IV. SUMMARY

In this paper we have calculated the \( D^0 \overline{D}^0 \) mixing parameter \( y \) in the FAT approach, considering the \( D^0 \to PP, PV, \) and \( VV \) channels. The \( D^0 \to PP \) and \( PV \) decay amplitudes were extracted in the FAT approach from the latest data, and the \( D^0 \to VV \) ones for the longitudinal polarization were estimated via the parameter set for the \( PV \) modes. It has been confirmed that the \( PV \)-inspired amplitudes work well for explaining the observed \( D^0 \to VV \) branching ratios. We then derived the contribution from the \( PP \) and \( PV \) modes as

\[
y_{PP+PV} = (0.21 \pm 0.07)\% , \tag{17}
\]

which is much more precise than those in the literature, and far below the data \( y_{\text{exp}} = (0.61 \pm 0.08)\% \). It has been also found that the contribution from the longitudinal \( VV \) modes, being of order of \( 10^{-4} \), is negligible. We conjecture that considering the above two-body \( D \) meson decays alone in an exclusive approach cannot account for the charm mixing, and that hadronic channels to other two-body and multi-particle final states are relevant to the evaluation of \( y \). However, it is very
TABLE II: Branching ratios for the $D^0 \rightarrow VV$ decays in units of $10^{-3}$. Estimations of the longitudinal branching ratios in the FAT approach are compared with the data of the total and longitudinal ones [28].

| Modes         | $B_{tot}(exp)$ | $B_{long}(exp)$ | $B_{long}(FAT)$ |
|---------------|----------------|-----------------|-----------------|
| $\rho^0K^0$  | 15.9 ± 3.5     | 14.3 ± 1.6      |                 |
| $\rho^+K^{*-}$ | 65.0 ± 25.0    | 41.8 ± 2.4      |                 |
| $K^*0\omega$ | 11.0 ± 5.0     | 37.7 ± 2.7      |                 |
| $\rho^+\rho^-$ |               | 4.1 ± 0.3       |                 |
| $K^{*-}K^{*-}$ |               | 1.18 ± 0.06     |                 |
| $K^{*0}\bar{K}^{*0}$ | | 0.043 ± 0.006 |                 |
| $\rho^0\rho^0$ | 1.83 ± 0.13    | 1.25 ± 0.13     | 1.4 ± 0.2       |
| $\rho^0\omega$ |               | 1.37 ± 0.08     |                 |
| $\rho^0\phi$  |               | 0.65 ± 0.04     |                 |
| $\omega\omega$ |               | 0.53 ± 0.08     |                 |
| $\omega\phi$  |               | 1.4 ± 0.1       |                 |
| $\rho^0K^{*0}$ |               | 0.041 ± 0.005   |                 |
| $\rho^-K^{*-}$ |               | 0.143 ± 0.008   |                 |
| $K^{*0}\omega$ |               | 0.108 ± 0.008   |                 |

difficult, if not impossible, to gain full control of the $SU(3)$ symmetry breaking effects in all these modes in an exclusive approach currently. As stated in the Introduction, the inclusive approach leads to values of $x$ and $y$ two or three orders of magnitude lower than the data. Therefore, a new strategy has to be proposed for complete understanding of the charm mixing dynamics in the Standard Model. We will leave this subject to a future project.

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