The Holographic Principle for General Backgrounds

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Abstract.
We aim to establish the holographic principle as a universal law, rather than a property only of static systems and special space-times. Our covariant formalism yields an upper bound on entropy which applies to both open and closed surfaces, independently of shape or location. It reduces to the Bekenstein bound whenever the latter is expected to hold, but complements it with novel bounds when gravity dominates. In particular, it remains valid in closed FRW cosmologies and in the interior of black holes. We give an explicit construction for obtaining holographic screens in arbitrary space-times (which need not have a boundary). This may aid the search for non-perturbative definitions of quantum gravity in space-times other than AdS.

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1. Entropy Bounds, Holographic Principle, and Quantum Gravity

How many degrees of freedom are there in nature, at the most fundamental level? In other words, how much information is required to specify a physical system completely? The world is usually described in terms of quantum fields living on some curved background. A Planck scale cutoff divides space into a grid of Planck cubes, each containing a few degrees of freedom. Applied to a spatially bounded system of volume \( V \), this reasoning would seem to imply that the number of degrees of freedom, \( N_{\text{dof}} \), is of the order of the volume, in Planck units:

\[
N_{\text{dof}}(V) \sim V. \tag{1.1}
\]

Manifestly, the thermodynamic entropy of a system cannot exceed the number of degrees of freedom: \( S \leq N_{\text{dof}} \). One might expect maximally disordered systems to saturate the inequality, whence

\[
S_{\text{max}}(V) \sim V. \tag{1.2}
\]

This conclusion, however, will have to be rejected when gravity is taken into account. The overwhelming majority of states have so much energy that the system

\[ Strictly, we define \( N_{\text{dof}} \) to be the logarithm of the number of independent basis elements of the quantum Hilbert space.\]
will be inside its own Schwarzschild radius. If gravitational stability is demanded, these states will be excluded and the maximal entropy will be much lower than in Eq. (1.2). Guided by the second law of thermodynamics, Bekenstein [3] obtained a quantitative result in 1981. For spherically symmetric thermodynamic systems it implies that

\[ S(V) \leq A/4, \]  

(1.3)

where \( A \) is the surface area of the system in Planck units. This result does not depend on the detailed properties of the system and can thus be applied to any spherical volume \( V \) of space in which gravity is not dominant. The bound is saturated by the Bekenstein-Hawking entropy associated with a black hole horizon. In other words, no stable spherical system can have a higher entropy than a black hole of equal size.

The Bekenstein bound does not refer to the degrees of freedom underlying the entropy, so one might think that it leaves the earlier conclusion, Eq. (1.1), intact. All expected degrees of freedom, a few per Planck volume, may well be there; the only problem is that if too many of them are used for generating entropy (or storing information) the system collapses under its own gravity.

However, a stronger interpretation was proposed by ’t Hooft [4] and Susskind [5]: Degrees of freedom that cannot be utilized should not be considered to exist. Thus the number of independent quantum degrees of freedom contained in a given spatial volume \( V \) is bounded from above by the surface area of the region:

\[ N_{\text{dof}}(V) \leq A/4 \]  

(1.4)

A physical system can be completely specified by data stored on its boundary without exceeding a density of one bit per Planck area. In this sense the world is two-dimensional and not three-dimensional as in Eq. (1.1). For this reason the conjecture is called the holographic principle.

The holographic principle constitutes an enormous reduction in the complexity of physical systems and has dramatic conceptual implications. It is far from obvious in a description of nature in terms of quantum field theories on curved space. There ought to be a formulation of the laws of physics in which Eq. (1.4) is manifest: a holographic theory. Indeed, it is widely believed that quantum gravity must be formulated as a holographic theory. This view has received strong support from the AdS/CFT duality [6–8], which defines quantum gravity non-perturbatively in a certain class of space-times and involves only the physical degrees admitted by Eq. (1.4) [9].

2. Limitations of the original formulation

The holographic principle, Eq. (1.4), was proposed as a universal property of theories that include gravity. It is believed to be necessary for (and perhaps to hold the key to) the formulation of quantum gravity. In particular, of course, it should apply to common classical solutions of Einstein’s equation.

The validity of the Bekenstein bound, however, is restricted to systems of “limited self-gravity” [3], i.e., systems in which gravity is not the dominant force. Indeed, it is
easy to violate the bound in regions where gravity is dominant, such as a collapsing star. The surface area becomes arbitrarily small, while the enclosed entropy cannot decrease. Thus Eq. (1.3) will not hold. Another example is a super-horizon size region in a flat Friedmann-Robertson-Walker (FRW) universe [10]. In this system gravity is dominant in the sense that the overall dynamics is dictated by the cosmological expansion. The entropy density is constant, and volume grows faster than area. Therefore, the entropy contained in a large enough sphere will exceed the bound.

Since $S \leq N_{\text{dof}}$, the violation of the entropy bound implies that the holographic principle does not hold in these examples. This means that the principle, as it was stated above, is not a universal law.

3. Light-sheets

The limitations of the original proposals motivate us to seek a new, more general formulation of the holographic principle and the associated entropy bound. We shall retain the formula $S \leq A/4$ and start by specifying an arbitrary boundary surface $A$. The task is to find a rule that tells us which entropy we mean by $S$.

3.1. Follow the light

We shall be guided by a demand for covariance. Given a closed surface of area $A$, the traditional formulation bounds the entropy on the enclosed volume, or spacelike hypersurface. But the hypersurface we select will depend on the choice of time coordinate (Fig. 1a). One way to make the statement covariant would be to demand that the bound hold for all spacelike hypersurfaces enclosed by $A$. But this possibility is already excluded by the counterexamples given above. Therefore we must use a null hypersurface bounded by $A$, following Fischler and Susskind (FS) [10]. (Several concepts crucial to a light-like formulation were recognized earlier in Ref. [16].)

Generalizing from the simple example shown in Fig. 1b, it is easy to see that in fact every 2-dimensional surface, regardless of shape or size, bounds exactly four null hypersurfaces. They may be constructed by following the past- and future-directed orthogonal light-rays leaving the surface on either side. But which of the four should be selected for the entropy bound? And how far should the light-rays be followed?

3.2. Go inside

FS [10] considered spherical areas and proposed that the past-directed ingoing null hypersurface be used. The resulting entropy bound works well for large regions in flat spaces. This differs from a more conservative approach taken in Refs. [11–15] which aimed to establish the range of validity of the Bekenstein bound in certain cosmological solutions. Our formalism is able to resolve this interesting question as a special case (see Sec. 5).

By a “surface” we mean a surface at some fixed time, i.e., a 2-dimensional spatial submanifold. We do not mean the history of a surface (which would be $2 + 1$-dimensional).
The spatial volume $V$ enclosed by a surface $A$ depends on time slicing (a). Thus the original formulation of the holographic principle was not covariant. However, $A$ is the 2D boundary of four 2+1D light-like hypersurfaces (b). They are covariantly generated by the past- and future-directed light-rays going to either side of $A$. E.g., for a normal spherical surface they are given by two cones and two “skirts” (b).

In a Penrose diagram, where spheres are represented by points, the associated null hypersurfaces show up as the 4 legs of an X (c). Null hypersurfaces with decreasing cross-sectional area, such as the two cones in (b), are called light-sheets. The entropy passing through them cannot exceed $A/4$ (covariant entropy bound).

The light-sheets for normal (d1), trapped (d2), and anti-trapped (d3) spherical surfaces are shown. If gravity is weak, as in (b), the light-sheet directions agree with our intuitive notion of “inside” (d1). For surfaces in a black hole interior, both of the future-directed hypersurfaces collapse (d2). Near the big bang, the cosmological expansion means that the area decreases on both past-directed hypersurfaces (d3).

Figure 1. [In (a,b) we have suppressed one spatial dimension (surface → line). In (c,d) we have suppressed two (surface → point). A fixed light-like angle separates spacelike and timelike directions.]

FRW universes, to which the Bekenstein bound could not be applied. In closed or collapsing space-times, however, the FS bound is not valid [10, 14]. At the root of this difficulty lies the ambiguity of the concept of “inside” in curved, dynamic space-times. For a closed surface in asyptotically flat space, the definition seems obvious: “Inside” is the side on which infinity is not. But what if space is closed? For example, which side is inside the equatorial surface (an $S^2$) of a three-sphere?

We propose a different definition which is unambiguous, local, and covariant. *Inside is where the cross-sectional area decreases.* Consider a two-sphere in flat space (Fig. 2a). Let us pretend that we do not know which side is inside. We can make an experiment to find out. We measure the area of the surface. Now, we move every point of the surface by some fixed infinitesimal distance along surface-orthogonal rays (radial rays in this case), to one particular side. If this increases the area, it was the outside. If the area
has decreased, we have gone inside.

![Figure 2.](image)

**Figure 2.** [Time and one spatial dimension are suppressed.] We define the “inside” of a 2D surface $A$ to be a light-like direction along which the cross-sectional area decreases (a): $A' \leq A$, or equivalently, $\theta \leq 0$. Such light-rays generate 2+1D light-sheets, the entropy on which is bounded by $A/4$.

This definition can be applied to open surfaces as well (b). Light-sheets end on caustics, as $\theta$ becomes positive there. If one stops earlier, the bound can be strengthened to $(A - A')/4$ [17].

It is useful to introduce a slightly more formal language. The expansion $\theta$ of a family of light-rays is defined as the logarithmic derivative of the infinitesimal cross-sectional area spanned by a bunch of neighbouring light-rays [18, 19]. Thus $\theta$ is positive (negative) if the cross-sectional area is locally increasing (decreasing). So the “inside” condition is simply:

$$\theta \leq 0.$$  \hspace{1cm} (3.1)

In other words, those light-rays leaving $A$ with negative or zero expansion generate an “inside” null hypersurface bounded by $A$, which we call a light-sheet. The entropy on a light-sheet will not exceed $A/4$.

If the expansion is positive for the future-directed light-rays to one side, it will be negative for the past-directed light-rays to the other side and vice-versa. Therefore at least two of the four directions will be allowed. If the expansion is zero in some directions, three or even all four null hypersurfaces will be light-sheets.

It turns out that our covariant definition of “inside” has unexpectedly paved the way for a vast generalization of the formalism of holography and entropy bounds: In contrast to the naive definition, there is no need for the surface $A$ to be closed. It works just as well for open surfaces, selecting at least two of the four null hypersurfaces bounded by $A$. This is illustrated in Fig. 2b. In fact the definition is local, so that we can split a surface into infinitesimal area elements and construct allowed hypersurfaces piece by piece. (This permits us to assume, without loss of generality, that the inside directions are continuous on $A$; if they flip, we split $A$ into suitable domains.)

### 3.3. Know when to stop

How far do we follow the light-rays? We need a rule telling us, for example, to stop at the tip of each of the cones in Fig. 1b. Otherwise, the light-rays would go on to generate another cone which could become arbitrarily large. This is clearly undesirable.
An elegant and economical feature of the decreasing area rule is that it has already resolved this question. We simply insist that \( \theta \leq 0 \) everywhere on the null hypersurface, not only in the vicinity of \( A \). When neighbouring light-rays intersect, they form a “caustic” or “focal point” (Fig. 2b). Before the caustic, the cross-sectional area is decreasing (\( \theta < 0 \)); afterwards, it increases and one has \( \theta > 0 \). This forces us to stop whenever a light-ray reaches a caustic, such as the tip of a light-cone.

3.4. Summary

Let \( A \) be an arbitrary surface. We define a light-sheet \( L(A) \) as a null hypersurface that is bounded by \( A \) and constructed by following a family of light-rays orthogonally away from \( A \), such that the cross-sectional area is everywhere decreasing or constant (\( \theta \leq 0 \)).

Since the expansion \( \theta \) is a local quantity, the entire construction is local. The decreasing area rule enters in two ways: We use it to determine the “inside directions;” as we stressed above, there will be at least two, and each will yield a distinct light-sheet. Having picked one such direction, we then follow each light-ray no further than to a caustic, where it intersects with neighbouring light-rays and the area starts to increase.

4. Covariant Entropy Bound and Holographic Principle

We are now ready to formulate a covariant entropy bound: Let \( A \) be an arbitrary surface area in a physical\( ^\dagger \) space-time, and let \( S \) be the entropy contained on any one of its light-sheets. Then \( S \leq A/4 \).

For spherical surfaces, our conjecture can be viewed as a modification of the FS bound [10]. It differs in that it considers all four light-like directions without prejudice and selects some of them by the criterion of decreasing cross-sectional area. Unlike previous proposals, the covariant bound associates entropy-containing regions with any surface area in any space-time in a precise way. Gravity need not be low. The area can have any shape and need not be closed.

The covariant bound makes no explicit reference to past and future. It is manifestly invariant under time-reversal. In a law about thermodynamic entropy, this is a mysterious feature. It cannot be understood unless we interpret the covariant bound not only as an entropy bound, but more strongly as a bound on the number of degrees of freedom that constitute the statistical origin of entropy: \( N_{\text{dof}} \leq A/4 \), where \( N_{\text{dof}} \) is the number of degrees of freedom present on the light-sheet of \( A \). Since no assumptions about the microscopic properties of matter were made, the limit is fundamental [1]. There simply cannot be more independent degrees of freedom on \( L \) than \( A/4 \), in Planck units. Thus we have obtained a holographic principle for general space-times\( ^\ddagger \).

\( ^\dagger \) This requirement is further discussed in Ref. [1], and in the Appendix.

\( ^\ddagger \) Logically, of course, the entropy bound follows from the holographic principle, since \( S \leq N_{\text{dof}} \). We have found it interesting to consider the entropy bound first, because the mystery of its T-invariance leads naturally to the holographic conjecture. The analogous step had to be considered bold when it was taken by ’t Hooft [4] and Susskind [5], since more conservative interpretations of the Bekenstein
We summarize our conjectures:

- **Holographic principle:** \( N_{\text{dof}}(\text{light-sheet}) \leq A/4 \) (4.1)
- **Covariant entropy bound:** \( S(\text{light-sheet}) \leq A/4 \) (4.2)

Their non-trivial content lies in the construction of light-sheets given above.

The bound takes its strongest form when the light-sheet is made as large as possible, i.e., if we stop only at caustics. It remains correct, but becomes less powerful if we choose to stop earlier, as this will decrease the entropy on the light-sheet, but not the boundary area. Flanagan, Marolf and Wald (FMW) [17] have pointed out, however, that the bound can be strengthened to \((A - A')/4\) in this case. Here \(A'\) is the surface area spanned by the endpoints of the light-rays (Fig. 2b), which goes to 0 as a caustic is approached. This expression is particularly pleasing because it makes the bound additive over all directions on the light-sheet, including the transverse (null) direction. In this form the covariant entropy bound implies the generalized second law of thermodynamics [17].

5. Evidence for the conjecture

5.1. The Bekenstein bound as a special case

The first important test is whether the covariant bound implies the Bekenstein bound. The covariant formulation uses null hypersurfaces, so how can it bound the entropy on spatial volumes, as the Bekenstein bound does? The corresponding argument is presented in Fig. 3. It relies on assumptions which can be taken as a definition of bound were available (see Sec. 1). Given the covariant bound, on the other hand, the necessity for a holographic interpretation is much more obvious.
Bekenstein’s “limited self-gravity” condition.

As an immediate application, we are able to settle a controversial question which has received much attention [11–15]: In cosmological solutions, what is the largest surface to which Bekenstein’s bound can be reliably applied? Various types of horizons were suggested and counter-examples found. We have established that the Bekenstein bound holds if the surface permits a complete, future-directed, ingoing light-sheet. This singles out the apparent horizon, which usually separates a normal from an anti-trapped region. Nevertheless, the claim that the Bekenstein bound holds for the apparent horizon [13] is not always valid [14], since the completeness condition must also be satisfied [1].

While it yields a precise formulation of the Bekenstein bound as a special case, the covariant entropy bound is more general. We have already stressed that it applies also to open surfaces. Moreover, it is valid in strongly gravitating regions for which no entropy bounds were previously available (or even hoped for). As an illustration of this claim, let us test the bound for surfaces deep inside a black hole.

5.2. Trapped surfaces and a new type of entropy bound

One can set up a worst-case scenario in which one considers the horizon surface $A$ of a small black hole at some moment of time. It possesses a future-directed light-sheet $L$ which will coincide with the horizon as long as no additional matter falls in. Far outside the black hole, we may set up a highly entropic shell of arbitrary mass. In order to avoid angular caustics [1], which would terminate parts of the light-sheet, we assume exact spherical symmetry. Entropy will be carried in radial modes and can be arbitrarily large. The shell is allowed to collapse around the small black hole. When it reaches the light-sheet, the configuration is already deep inside a much larger black hole of the shell mass, and no conventional entropy bound applies. One might expect a violation of the bound, since all of the shell will necessarily squeeze through the radius of the small black hole and reach the singularity at the center. It is not difficult to verify, however, that the light-sheet $L$ ends before any entropy in excess of $A/4$ passes though it [1]. If the shell entropy exceeds $A/4$, the light-sheet will reach $r = 0$ before all of the shell does. The bound can be saturated, but not exceeded. Thus we see that for trapped surfaces, the covariant formalism implies genuinely new entropy bounds, which could not have been anticipated from the original formulation.

5.3. The closed universe

It is particularly instructive to verify the covariant bound for a closed, matter-dominated FRW universe, to which the FS bound [10] could not be applied. Consider a small two-sphere near the turn-around time (Fig. 4a). The FS bound would consider the entropy on a light-cone which traverses the large part of the $S^3$ space (Fig. 4b). This would be almost the entire entropy in the universe and would exceed the arbitrarily small area of the two-sphere. The covariant bound, on the other hand, considers the entropy on light-sheets which are directed towards the smaller part (the polar cap). This entropy vanishes
as the two-sphere area goes to zero [1]. This illustrates the power of the decreasing area rule.

**Figure 4.** The closed FRW universe. A small two-sphere divides the $S^3$ spacelike sections into two parts (a). The covariant bound will select the small part, as indicated by the normal wedges (see Fig. 1d) near the poles in the Penrose diagram (b). After slicing the space-time into a stack of light-cones, shown as thin lines (c), all information can be holographically projected towards the tips of wedges, onto an embedded screen hypersurface (bold line).

5.4. Questions of proof

More details and additional tests are found in Ref. [1]. No physical counterexample to the covariant entropy bound is known (see the Appendix). But can the conjecture be proven? In contrast with the Bekenstein bound, the covariant bound remains valid for unstable systems, for example in the interior of a black hole. This precludes any attempt to derive it purely from the second law. Quite conversely, the covariant bound can be formulated so as to imply the generalized second law [17].

FMW [17] have been able to derive the covariant bound from either one of two sets of physically reasonable hypotheses about entropy flux. In effect, their proof rules out a huge class of conceivable counterexamples. Because of the hypothetical nature of the FMW axioms and their phenomenological description of entropy, however, the FMW proof does not mean that one can consider the covariant bound to follow strictly from currently established laws of physics [17]. In view of the evidence we suggest that the covariant holographic principle itself should be regarded as fundamental.

6. Where is the boundary?

Is the world really a hologram [5]? The light-sheet formalism has taught us how to associate entropy with arbitrary 2D surfaces located anywhere in any spacetime. But to call a space-time a hologram, we would like to know whether, and how, *all* of its information (in the entire, global 3+1-dimensional space-time) can be stored on some surfaces. For example, an anti-de Sitter “world” is known to be a hologram [6, 9]. By this we mean that there is a one-parameter family of spatial surfaces (in this case, the
two-sphere at spatial infinity, times time), on which all bulk information can be stored at a density not exceeding one bit per Planck area. Such surfaces will be called screens of a spacetime. Can analogous screens be found in other space-times?

The trick is to slice the 3+1 space-time into a one parameter family of 2+1 null hypersurfaces $H(\tau)$. On each null hypersurface $H(\tau)$, locate the 2D spatial surface $A_{\text{max}}(\tau)$ of maximum area. Unless it lies on a space-time boundary, this surface divides $H(\tau)$ into two parts. But the cross-sectional area decreases in both directions away from $A_{\text{max}}(\tau)$ by construction. Thus the entropy on the entire hypersurface $H(\tau)$ cannot exceed $A_{\text{max}}(\tau)/2$. In other words, all the physics on $H(\tau)$ can be described by data stored on $A_{\text{max}}(\tau)$ at a density not exceeding 1 bit per Planck area.

By repeating this construction for all values of the parameter $\tau$, one obtains a one parameter family of 2D screens, $A_{\text{max}}(\tau)$. It forms a 2+1 hypersurface $A$, which is embedded in the bulk space-time and on which the entire space-time information can be stored holographically. As an example, Fig. 4c shows the construction of a holographic screen hypersurface in a closed, matter-dominated FRW universe.

Other examples can be found in Ref. [2]. Note that $A$ will depend on the choice of slicing, and will not always be connected. In Minkowski space, one finds that either one of the two null infinities can play the role of a global screen. In global de Sitter, one must use both past and future infinity; however, the observable part of de Sitter can be encoded on the event horizon, which forms a null hypersurface of finite area.

7. Dreams of a holographic theory

The stunning success of the AdS/CFT duality [6] has led to speculations that a non-perturbative definition of quantum gravity would involve theories on holographic screens in other space-times as well. Such hypotheses were vague, however, because no general definition of holographic screens was available. In particular, it was completely unclear how holography could be compatible with space-times that had no boundary, such as a closed FRW universe.

What we have shown is that embedded holographic screens exist in any space-time, and how to construct them. This result lends strong support to the holographic hypothesis. We hope that it will be of use in the search for a fully general, manifestly holographic unified theory.

Indeed, the structure of screens in cosmological solutions provides interesting constraints on the general formulation of holographic theories. Generically, the screen area (and thus the number of degrees of freedom of a manifestly holographic theory defined on screens) will be time-dependent. Moreover, the causal character of the screen hypersurface $A$ can change repeatedly between Euclidean and Lorentzian.

These pathologies appear to exclude the possibility of formulating a well-defined, conventional theory on the screens. Moreover, in generalizing a screen theory approach à la AdS one would encounter the basic drawback that the screen itself must be put in by hand, for example by constraining oneself to asymptotically AdS spaces. Ideally, all
geometric features should come out of the theory.

A background-independent holographic theory is likely to require a more radical approach. We suspect that quantum gravity is a pre-geometric theory containing a sector from which classical general relativity can be constructed. The recovery of local physics will likely be highly non-trivial, as can be gleaned from the example of AdS/CFT duality. One would need to build the geometry from a pre-geometric set of degrees of freedom in such a way that the covariant holographic principle is manifestly satisfied. This would be the fundamental origin of the principle.

Appendix

Here we add some comments on the generality of the covariant entropy bound. We also discuss energy conditions and respond to recent criticism.

Classical gravity. The covariant entropy bound applies to all physical solutions of classical general relativity. This constitutes an enormous generalization compared to previous formulations of entropy bounds [3, 10]. We obviously do not mean to assume that \( \hbar \) is exactly zero. Rather, the term “classical” refers to space-times in which quantum gravitational effects are negligible. This holds for most known solutions, in particular for cosmology on scales larger than the Planck scale, for ordinary thermodynamic systems, and for black holes on time-scales less than the evaporation time. The covariant entropy bound (and the Bekenstein bound, where applicable) provides a simple, yet highly non-trivial and powerful relation between area and entropy in such space-times. This renders nugatory the observation by Lowe [20] that the bound, \( A/4G\hbar \), diverges for \( \hbar \rightarrow 0 \).

Quantum and semi-classical gravity. The holographic principle may hold a key to the non-perturbative formulation of string theory and quantum gravity. This does not mean that we should expect it to remain useful in the strong quantum regime, where an approximation to classical geometry need not exist. Indeed, the very concept of area might become meaningless. As we argued above, the fundamental role of the holographic principle would then be in the recovery of classical relativity from a suitable sector of a pre-geometric unified theory. In semi-classical gravity, however, there ought to be a place for the holographic principle. Indeed, we are unaware of any semi-classical counter-examples to the covariant entropy bound. However, the situation is more subtle than in classical gravity. It is easy to construct apparent problems, but they turn out to stem from limitations of the semi-classical description, not of the holographic principle.

Consider, for example, a sphere just outside a black hole horizon. This surface possesses a future-directed light-sheet which crosses the horizon and sweeps the entire interior of the black hole. How much entropy is on the light-sheet? Naively, we might count first the Bekenstein-Hawking entropy of the horizon (\( A/4 \), which by itself saturates the bound). In addition, we might count the ordinary thermodynamic entropy of the matter which formed the black hole and passes through the interior part of the
light-sheet. This would lead to a violation. The breakdown is not in the holographic principle, however, but in our naive interpretation of the semi-classical picture. We have overcounted. The horizon entropy represents the potential information content of the system to an observer outside the black hole. The matter entropy represents the actual information to an observer falling in. The viewpoints are complementary, and it is operationally meaningless to take a global stance and count both entropies.

**Resolving Lowe’s objection.** Lowe [20] has argued that the covariant entropy bound [1] fails for a semi-classical black hole in thermal equilibrium with a surrounding heatbath. He considers a surface $A$ on the black hole horizon, which possesses a future-directed outgoing light-sheet $L$. Two assumptions are made about this set-up [20]. 1) $L$ will coincide with the black hole horizon forever. 2) The configuration is stable and thus eternal. By the second assumption, an infinite amount of entropy crosses the black hole horizon. By the first assumption, this entropy crosses the light-sheet $L$ of a finite area $A$. Thus, it would seem, the bound is violated.

Because black holes have negative specific heat, however, the configuration is in fact unstable [21]. The black hole decays in a runaway process of accretion or evaporation. This invalidates Lowe’s second assumption and thus his conclusion. The static approximation used in Ref. [20] breaks down completely within a time of the order of the black hole evaporation time-scale, $M^3$. During this time, the bound will be satisfied, since no more than the black hole entropy, $A/4$, gets exchanged in an evaporation time. (We thank L. Susskind for a discussion.)

**Can Lowe’s criticism be strengthened?** Stable semi-classical black holes actually do exist in AdS space. However, they do not provide a counter-example either, because the global structure of AdS does not permit us to pipe information through a black hole as would be required in order to violate the bound. Instead, the Hawking radiation forms a thermal atmosphere around the black hole. No new entropy can be introduced without increasing the area of the black hole.

It is interesting to note that Lowe’s first assumption is also problematic. It does hold, of course, for a classical black hole in vacuum. In a semi-classical equilibrium, however, the ingoing radiation necessarily possesses microscopic fluctuations. They will get imprinted on the expansion $\theta$ of the light-sheet generators. Thus, $\theta$ will not vanish exactly at all times. The $\theta^2$-term in Raychauduri’s equation effectively provides a bias towards negative values of $\theta$: 

$$\dot{\theta} = -\frac{1}{2} \theta^2 + \text{ terms which average to zero} \quad (7.1)$$

We see that the effects of in- and out-going radiation on the expansion will not cancel out. Eq. (7.1) leads to a runaway process in which the light-sheet $L$ departs from the apparent horizon and collapses.

It is not clear whether, in a baroque twist, one could set up a demon to keep the black hole at constant size. The demon would have to maintain a position close enough to the black hole to react to temperature changes by increasing or decreasing
the matter influx, and it would have to perform this task for a time longer than $M^3$ without violating the self-consistency of the solution. But even if we assume that this was possible, and that the black hole could be kept at a constant size, the light-sheet would collapse due to small fluctuations. The black hole horizon might last forever, but a light-sheet with $\theta \leq 0$ will not. Thus one might be able to transfer an infinite amount of entropy through the horizon, but not through the light-sheet.

**Further objections.** Lowe also points out that angular caustics [1] cannot protect the bound if only radial modes carry entropy. Because we shared this concern, however, we addressed the issue explicitly in Sec. 6.2 of the criticized paper [1], where it was shown that the bound can be saturated, but not exceeded, in the worst-case scenario of exactly-spherical collapse of an arbitrarily massive shell. The various objections raised by Lowe against the Bekenstein bound have previously been answered by Bekenstein and others (see, e.g., Refs. [22–25]).

**Conditions on matter.** In particular, we stress that the covariant bound, like the Bekenstein bound, is conjectured to hold only for matter that actually exists in nature. It thus predicts that the fundamental theory will not contain an exponentially large number of light non-interacting particles [25] or permit the kind of negative energy densities that would be needed to break the bound. These requirements are met both empirically and by current theoretical models. In this sense we conjecture the bound to be generally valid. We cannot spell out precise conditions on matter until the fundamental theory is fully known. Its matter content is unlikely to satisfy any of the usual energy conditions on all scales. However, at the price of making the conjecture slightly less general than necessary, we can obtain a testable prediction for a huge class of space-times by demanding the dominant energy condition [1].

**A proposed modification.** Instead of using the $\theta \leq 0$ rule to determine where to end a light-sheet, Tavakol and Ellis (TE) [26] have suggested an interesting modification*. They propose to terminate when generating light-rays depart from the boundary of the causal past or future of $A$. Basically, this amounts to the following difference. In our prescription [1], one stops only when *neighbouring* light-rays intersect locally, i.e., at caustics. The TE proposal would be to stop also when *non-neighbouring* light-rays intersect. This makes no difference in spherically symmetric situations, but generically it leads to smaller light-sheets and thus to a weaker bound. In the absence of any counter-examples to our formulation [1], and in view of the FMW results [17], this relaxation does not appear to be necessary.

Moreover, the TE formulation is not self-consistent unless one gives up one of the most attractive features of the light-sheet formalism: its locality in the area. This is most easily seen by considering (in a nearly empty 2+1D world) a 1D oval “surface” $A$ consisting of two half-circles, $A_1$ and $A_2$, joined by long, parallel line segments $A_3$ and $A_4$. Non-neighbouring light-rays intersect on a line midway between $A_3$ and $A_4$. If

* We thank Ted Jacobson for discussions of related ideas.
one stopped there, the 1+1D light-sheet would take the shape of a roof. Now consider only one of the parallel line segments, $A_3$, by itself. If we construct its light-sheet individually, the light-rays will continue indefinitely unless they are bent into caustics by some matter. The entropy on this large light-sheet will be bounded by $A_3/4$. Thus, if we applied the alternative rule [26] to each area element separately, and added up the resulting bounds, we would recover the covariant entropy bound in our formulation [1].

Tavakol and Ellis [26] correctly point out that light-sheets can be extremely complicated structures in inhomogeneous space-times. But this should not motivate us to change the $\theta \leq 0$ rule, just as we would not discard the standard model merely because its application becomes impractically complicated when one is describing an elephant. Of course, it may often be practical to consider the smaller light-sheets suggested by TE. But we are interested mostly in the fundamental theoretical role of the holographic principle. Therefore we advocate its strongest and most general formulation [1, 2, 17].

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