Sea-quark and gluonic contributions to strange baryons and its properties

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Abstract. We treat the hadrons as an ensemble of quark-gluon Fock states where contributions from sea-quarks and gluons can be studied in detail for the properties of low lying baryons. Statistical model is applied to calculate individual probabilities from various scalar, vector and tensor sea components in flavour, spin and colour subspaces for each quark-gluon Fock state. The mass of strange quark is imposed in terms of constraints to the availability of strange quarks in the sea and free energy of gluons in conformity with experimental indications. We calculate multiplicities for different sets of Fock states to compute the role of strange sea for cascade doublet \( \Xi^+ \) and \( \Xi^0 \). The low energy properties like magnetic moment, weak decay matrix elements and axial coupling constant ratios have been studied. The incorporation of strange quark gluon sea is discussed and they are found to affect the results by almost 46%. The results provide deeper understanding for baryon structure thereby motivating experiments for further inspection, especially the spin distribution among the quarks and gluons.

1. Introduction
In the low energy limit, QCD has a challenging behaviour. The non-perturbative and relativistic nature of quarks and gluons makes the hadronic structures difficult to understand. The non-perturbative mechanism can be made to understand via several models. Although the constituent quark model tends to explain the static properties of a baryonic system, still there is a need of some new degrees of freedom. The extension of valence quarks to sea with quark-antiquark pair is a remarkable step in hadron spectroscopy. A variety of approaches have been developed to understand the behaviour of baryons with three valence quarks and sea consisting of gluons and quark-antiquark pair. These models help to make a deeper understanding to the various properties like weak decay coupling ratios, magnetic moments, masses of hadrons and flavour asymmetry. Latest studies, dedicated towards the study of quark-antiquark pairs and gluon condensates includes statistical balance model [1], lattice QCD [3], effective QCD approaches [2] and meson-cloud based approaches [4–6]. The static properties of hadrons had also been studied through a large variety of experiments [7–9]. Recent experimental studies show that strange quark also make a significant contribution to nucleonic spin which can be studied by using polarised deep-inelastic scattering experiments of electrons or muons from nucleons. The aim of present work is to analyse the probabilities of various properties are analysed by determining the multiplicities in colour, flavour and spin states. The multiplicities are used to obtain the probabilities for each quark-gluon Fock states in the ground state baryons. In section II, we briefly present the theoretical frame. Numerical results are given in section III, putting forward predictions for the probabilities of all Fock states for Cascade baryonic system.
2. Theory

A wave-function \( \psi = \Phi(\phi \mid \psi \mid \chi \mid \xi) \) including 3-quark wave-function and suitable sea contents is framed keeping in mind the total anti-symmetry of the wave-function where, \( | \phi \rangle, | \psi \rangle, | \chi \rangle, | \xi \rangle \) is for flavour, spin, colour, and space-time wave-function respectively [13]. Let \( \Phi_{1/2} \) is the standard SU(3) \( q^3 \) wave-function transforming as 56 of SU(6) wave-function and \( H_{0,1,2} \) and \( G_{0,1,2} \) denotes the spin and colour possibilities of sea with spin and colour combinations of 0, 1, 2 and 1, 8, 10 respectively. The total flavour-spin-colour wave function of a spin up baryon which consists of three-valence quarks and sea components can be written as given in [13].

\[
| \phi \rangle = \frac{1}{N}(|\phi_1 \rangle^{(1)} H_0 G_1 + a_8 |\phi_2 \rangle^{(1)} H_0 G_8 + b_{10} |\phi_3 \rangle^{(1)} H_0 G_{10} + b_1 |\phi_1 \rangle^{(1)} H_1 + b_8 (\phi_2 \otimes H_1) \rangle G_1 + b_8 (\phi_3 \otimes H_1) \rangle G_8 + c_8 (\phi_3 \otimes H_1) \rangle G_8 + d_8 (\phi_2 \otimes H_2) \rangle G_8 \]

where \( N^2 = 1 + a_2^2 + a_{10}^2 + b_2^2 + b_{12}^2 + b_8^2 + c_8^2 + d_8^2 \). All the terms have to be written properly with appropriate CG coefficients and by taking into account the symmetry property of the component wave function. To obtain all the coefficients in the wave-function, a general parameterisation method is employed to calculate the different observables in the form of eigen values for baryonic system. Assuming the expansion of hadrons in terms of quark-gluon Fock states where Fock states avail the presence of quark-antiquark pairs multi-connected to gluons. Mathematically, it can be expressed as:

\[
| h \rangle = \sum_{i,j,k} c_{i,j,k} | \{ q \}, \{ i, j, l \}, \{ k \} \rangle,
\]

where \( \{ q \} \) represents the valence quarks of the baryon, \( i, j, l \) represent the number of \( q \bar{q}(u\bar{d}, d\bar{s}, s\bar{s}) \) and \( k \) is the number of gluons. The principle states that the transition between any two processes either via splitting or recombination involving any two kind of partons should balance each other. The splitting and recombination procedure involves three kinds of processes and their transition probabilities involve number of gluons and quark-antiquark pairs [10–12].

(i) When \( q = qg \) is considered. The general expression of probability for these sub-processes can be described as

\[
\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{1}{k}
\]

(ii) When both the processes \( q = qg \) and \( g = gg \) are included. We can write

\[
\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{3 + 2i + 2j + 2l + k - 1}{(3 + 2i + 2j + 2l)k + \frac{k(k-1)}{2}}
\]

(iii) When the processes \( g = q\bar{q} \) are involved: The transition probabilities involving \( g = q\bar{q} \) depend upon the valence quark content and differ in all baryons. The generation of \( s\bar{s} \) pair from gluons is restricted due to non-negligible mass of s-quark. The constraint on the free energy of gluon comes in the form of factor \((1-C_l)^{n-1}[12]\) where \( k \) is the number of gluons and \( n \) represents the total number of partons present in that state. Therefore, the splitting and recombination for the processes involving \( g = q\bar{q} \) undergoes SU(3) symmetric breaking in sea where \( q \) is for some heavier quark flavour. In general, \( n = 3 + 2i + 2j + 2l + k \) and \( C_{l-1} = \frac{2M_s}{M_B - sM_s - 2(l-1)M_s} \), \( M_s \) is the mass of s-quark and \( M_B \) is the mass of the baryon.

\[
\frac{\rho_{i,j,l,k}}{\rho_{i,j,l+k,0}} = \frac{(k)(k-1) - 1(1 - C_0)n+k-2}{(l+1)(l+2) - (-l+k)(l+k+1)}
\]

The above expression produces all the probabilities in the form of ratios. All the probabilities in flavour space can be obtained from the table given below for individual particle. Complete expressions for cascade doublet can be written as:
Table 1. Probability Expressions

| Baryon Expression | \( \Xi^0(uss) \) | \( \Xi^- (dss) \) |
|-------------------|-----------------|-----------------|
| \( \rho_{i,j+l,k,0} \) | \( \frac{2}{2} \) | \( \frac{2}{2} \) |
| \( \rho_{0,0,0,0} \) | \( (i)!((i+1)!((j)!((l+k)!((l+k+2))! \) | \( (i)!((i+1)!((j)!((l+k)!((l+k+2))! \) |

These probabilities in flavour space is further needed to calculate probabilities in spin and colour space [14, 15]. The decomposition of baryonic states in various quark-gluon Fock states with relevant operators acting on the valence and sea part is used to find probabilities for all possible spin and colour sub-states so as to produce spin \( \frac{1}{2} \) and colour singlet state of the baryon. The procedure can be applied to states like \( | uu \rangle, | dd \rangle, | ss \rangle, | uu \rangle dd \rangle, | gg \rangle \) etc. More favourable situation occurs when the higher multiplicities are suppressed with the fact that the larger multiplicity leads to higher possibility of interaction and less survival probability. Moreover, non-zero mass of s-quark limits the free energy of gluon and hence states with strange quark-antiquark pairs are assumed to be less probable. The suppression of states with higher multiplicities can be made to archive on phenomenological grounds by assuming the probability of a system being in a particular spin and colour state is inversely proportional to multiplicity of the spin and colour of that state. The probability in terms of multiplicity for light and strange quarks are mentioned in tables 2 and 3. The sum of all probabilities in individual cases provide a suitable normalisation constant and the coefficients in the wave-function for baryon. Static properties of ground state strange baryon octets can be determined from these coefficients.

3. Results and discussions

We aim to choose one set of the strange baryon to compute probabilities of several Fock states in spin, flavour and colour space and to check the validity of the statistical model to study their low energy properties. The study has been divided into three cases to study the dominant role of strange sea with proper assumptions. Case I comes with the assumption that gluon has a possibility to create \( qq \) pair (including strange quark) which carries the quantum numbers of gluon, hence it suggests the suppressed multiplicity of \( qq \) and g to have spin 1. In case II we rule out the assumption for pair to have the same quantum numbers as that of gluon and g to have spin 1. In case II we rule out the assumption for pair to have the same quantum numbers as that of gluon and it can be in spin state 0, 1, or 2. Third case is similar to the first case where we modify the detailed balance principle by ruling out the sub-processes \( g \leftrightarrow q\bar{q} \). Fock states in all three cases are mentioned in the table given below. The table 2 differentiates the impact of presence of strange quark-anti-quark pair in sea in each case. The table 3 shows the probability contribution to sea in terms of different number of strange quark and gluon Fock states. Case I, II and III all shows the lower likelihood for the occupancy of Fock states where \( s\bar{s} \) pair is present than those without \( s\bar{s} \). In table 2, the probability enhances in all cases by almost 0 to 85% as we exclude strange quark-antiquark pairs. In table 3, where we already have \( s\bar{s} \) pair in addition to gluons, the relative probability for a single \( s\bar{s} \) pair is large as compared to other combinations and it is least for \( | s\bar{s} \) possibly with a reason where a larger multiplicity leads to the reduced possibility. The results are shown in table 4, where magnetic moment and weak decay couplings are given on the basis of different assumption. Case I with strange sea favours the experimental values, there by suggesting the gluons in the sea to be generating strange quark condensates with similar quantum numbers. The data shifts by 7 to 46% approaching the experimental value favouring the strangeness in the sea. Here vector strange sea dominates and flavours the results. Present framework suggests a stronger base to choose the model with suggested cases to verify the experimental and other theoretical values and hence provide a deeper understanding to the strange baryon structure.
Table 2. Fock States with Probabilities [with (a) and without (b) s̅s]

| State | Case-I (a) | Case-II (a) | Case-III (a) | Case-I (b) | Case-II (b) | Case-III (b) |
|-------|------------|-------------|--------------|------------|-------------|--------------|
| -0.012 | 0.029 | 0.165 | 0.165 | 0.111 | 0.103 |
| -0.027 | 0.002 | 0.083 | 0.003 | 0.086 | 0.059 |
| -0.049 | 0.009 | 0.02 | 0.006 | 0.03 | 0.021 | 0.015 |
| -0.080 | 0.165 | 0.148 | 0.148 | 0.135 | 0.184 |
| -0.080 | 0.082 | 0.073 | 0.073 | 0.068 | 0.092 |
| -0.080 | 0.023 | 0.072 | 0.023 | 0.072 | 0.149 | 0.203 |
| -0.080 | 0.012 | 0.036 | 0.011 | 0.036 | 0.075 | 0.102 |
| -0.080 | -0.225 | -0.225 | -0.225 | -0.115 | -0.073 | -0.052 |

Table 3. Strange Sea Fock State with Probabilities

| Fock State | Case-I | Case-II | Case-III |
|------------|--------|---------|----------|
| s̅s         | 0.055  | 0.049   | 0.033    |
| s̅sg        | 0.002  | 0.002   | 0.050    |
| s̅sgg       | 0.042  | 0.028   | 0.027    |

Table 4. Static Properties at Low Energy [with (a) and without (b) s̅s]

| Property                  | Case-I (a) | Case-II (a) | Case-III (a) | Expt. Value [16] |
|---------------------------|------------|-------------|--------------|------------------|
| \( \mu_{\Xi^-} \)        | -0.51      | -0.35       | -0.48        | -0.48 -0.46      | -0.65 ± 0.025    |
| \( \frac{g_A}{g_V}(\Xi^- \rightarrow \Sigma^0) \) | 0.89       | 0.80        | 0.82         | 0.79 0.72        | 1.29 [17]        |
| \( \frac{g_A}{g_V}(\Xi^0 \rightarrow \Lambda^0) \) | -0.25      | 0.19        | 0.24         | 0.23 0.21        | 0.25 ± 0.5       |
| \( \frac{g_A}{g_V}(\Xi^- \rightarrow \Xi^0) \) | -0.25      | -0.27       | -0.22        | -0.21 -0.19      | -0.31 [18]       |

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