Theoretical Study on One-Dimensional Consolidation of Underconsolidated Marine Clay Induced by Ramp Loading

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Abstract. The position of maximum excess pore water pressure is found to be a critical indication in the design of horizontal drainage systems for saturated soil, controlled by the drainage capacity at the boundaries and the self-weight of soil both of which are often ignored. In sea reclamation works, however, the dredged clay used is mostly under consolidated and will undergo the self-weight consolidation. In this paper, an exponentially time-growing drainage boundary condition is adopted, based on which the analytical solution of one-dimensional consolidation of underconsolidated soil under a ramp loading is obtained by using the eigenfunction expansion technique. The obtained solution is verified and a parameter analysis is later carried out. It turns out that when considering the self-weight of the soil subjected to the ramp loading, the variation of the position of maximum excess pore water pressure over time depends heavily on the symmetry of the boundary conditions. The smaller the over consolidation ratio ($OCR \leq 1$), the shorter the distance of the plane of maximum excess pore water pressure to the bottom surface of the soil layer. Moreover, the larger the loading rate, the higher the plane of maximum excess pore water pressure at the loading stage.

Keywords. One-dimensional consolidation, exponentially time-growing drainage boundary, underconsolidated soil, ramp loading, horizontal drain.

1. Introduction

Consolidation theory has been attracting great attention by its uses for ground improvement and settlement prediction in sea reclamation. Former researches on the one-dimensional consolidation theory of saturated soil focus more on constitutive relation [1-2] and seepage characteristics [3-4] and less on boundary condition [5-6]. However, the boundary condition in the conventional Terzaghi’s consolidation theory conflicts with the initial condition, so that the consequent consolidation solutions are discontinuous. Given this, Mei and Chen [7] proposed an exponentially time-growing drainage boundary (hereafter referred to as ETDB) condition and the corresponding analytical consolidation solution. It turned out that such drainage boundaries can lead to consolidation solutions with better continuity and cover various cases, including the fully permeable and impermeable boundaries as well as the ones with different asymmetric drainage capacity. The boundary conditions, especially those asymmetric ones, and the self-weight of soil will both significantly influence the position of the plane of maximum excess pore water pressure (hereafter referred to as PMEP) which...
can guide the layout of horizontal drainage blanket for the soil layer to improve the consolidation efficiency, according to a numerical study [8]. Feng et al. [9] later derived an analytical solution for the one-dimensional self-weight consolidation problem under the ETDB and discussed the use of the PMEP in the acceleration of consolidation under the instantaneous load.

Nonetheless, Schiffman [10] believed that the external loading should be applied gradually with time rather than instantly, such as the ramp loading. Following the spirit of time-dependent loading, Sun et al. [11] and Tian et al. [12] presented the consolidation solutions under the ramp and multi-stage loading, respectively, but without concerning the impact of the self-weight of soil. Land reclamation projects in coastal areas produce a growing amount of underconsolidated dredged marine clay whose self-weight cannot be neglected [13]. Thus, it is necessary to take account of all three factors into the consolidation analysis, that is, the boundary condition, the time-dependent loading, and the self-weight of soil.

In this paper, an analytical solution is derived for one-dimensional consolidation of under-consolidated marine clays with ETDB induced by the ramp loading, and then the change in the PMEP is analyzed in detail for different situations.

2. Basic Equations of Consolidation and Its Solutions

2.1. Calculation Model of One-Dimensional Consolidation

Figure 1 shows the calculation model of one-dimensional consolidation of the underconsolidated marine clay with ETDB at the top and bottom surfaces. h is the thickness of the soil layer. A time-dependent external loading, q(t), is uniformly exerted at the top surface of the soil layer. With the use of the assumptions from the Terzaghi’s consolidation theory except for the external loading and the boundary conditions, the consolidation equation can be expressed as,

\[
\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} + \frac{dq}{d\tau} \quad (0 \leq z \leq h)
\]

where \( u \) is the excess pore water pressure; \( t \) is the time; \( z \) is the depth; \( C_v \) is the vertical consolidation coefficient of soil; \( C_v = k/\gamma_w m_v \); \( k \) and \( m_v \) are the permeability coefficient and the volume compressibility coefficient of soil, respectively; \( \gamma_w \) is the unit weight of pore water.

The initial condition is given by

\[
u(z,0) = q(0) + \gamma'z - \sigma'_0(z) \quad (2)
\]

where \( \gamma' \) is the submerged unit weight of the soil; \( \sigma'_0(z) \) is the initial effective stress of the under-consolidated soil and is assumed to be a linear function of the depth, as expressed by \( \sigma'_0(z) = \xi \gamma' z \) \( (\xi \leq 1) \). According to the definition of the over consolidation ratio (OCR), one can obtain OCR=\( \xi \), so that equation (2) can be rewritten as

\[
u(z,0) = q(0) + \eta \gamma' z \quad (3)
\]

where \( \eta \) is defined as the coefficient of initial stress and \( \eta = 1 - \text{OCR} \). \( \eta \) can reflect the initial stress state (ie, the stress history) of the soil layer before the preloading process as well as the consolidation behavior induced by the self-weight of soil. For normally consolidated soil, \( \eta = 0 \).

Consequently, the ETDB conditions considering the self-weight of soil are expressed as

\[
u(0,t) = q(t)e^{-\alpha t} \quad (4)
\]

\[
u(h,t) = [q(t) + \eta \gamma'h]e^{-\beta t} \quad (5)
\]
where $\alpha$ and $\beta$ are the top and bottom interface parameters that can reflect the drainage capacity at the two boundaries of the soil layer, respectively. And the interface parameters can be estimated by the interpolation technique through indoor experiments or field tests.

Figure 1. Diagram of calculation model.

Figure 2. Diagram of ramp loading.

As shown in figure 2, the ramp loading is adopted in this study to model the construction process realistically, and it is expressed by the following equations

$$q(t) = \begin{cases} \frac{m t}{t_c} q_c & (0 \leq t < t_c) \\ q_c & (t \geq t_c) \end{cases}$$

(6)

where $t_c$ is the construction period of the ramp loading; $m$ is the loading rate during the loading stage; $q_c$ is the final load.

2.2. Analytical Solutions of Excess Pore Water Pressure and Consolidation Degree

By using the eigenfunction expansion technique, equation (1) can be solved and thus one can obtain the analytical solution of excess pore water pressure of the underconsolidated marine clay during the loading stage ($0 \leq t < t_c$):
\[
\begin{align*}
\psi(z,t) &= \frac{(h-z)mt e^{-\alpha t}}{h} + \frac{z(mt + \eta' h)e^{-\beta t}}{h} - \sum_{n=1}^{\infty} (-1)^n \frac{2\beta \eta' h(e^{\beta t} - e^{-\alpha t})}{n\pi(r-\beta)} \sin \frac{n\pi z}{h} + \\
\sum_{n=1}^{\infty} \frac{2m}{n\pi} \left[ \frac{(\alpha r - \alpha t - r)e^{\alpha t} + r e^{-\alpha t}}{(r-\alpha)^2} - (-1)^n \frac{(\beta r - \beta t - r)e^{\beta t} + r e^{-\beta t}}{(r-\beta)^2} \right] + \left[ \frac{1}{r} \right] \sin \frac{n\pi z}{h} 
\end{align*}
\]

(7)

and during the constant load stage \((t \geq t_c)\):

\[
\begin{align*}
\psi(z,t) &= \frac{q_r(h-z)e^{\alpha t}}{h} + \frac{(q_e + \eta' h)z e^{\beta t}}{h} - \sum_{n=1}^{\infty} (-1)^n \frac{2\beta \eta' h(e^{\beta t} - e^{-\alpha t})}{n\pi(r-\beta)} \sin \frac{n\pi z}{h} + \\
\sum_{n=1}^{\infty} \frac{2q_r}{n\pi} \left[ \frac{\alpha e^{\alpha t}}{r-\alpha} - (-1)^n \frac{\beta e^{\beta t}}{r-\beta} + A_n e^{-\alpha t} \right] \sin \frac{n\pi z}{h}
\end{align*}
\]

(8)

where

\[
\begin{align*}
r &= C_v \left( \frac{n\pi}{h} \right)^2 \\
A_n &= \frac{r[1 - e^{(r-\alpha) t_c}]}{(r-\alpha)^2 t_c} - (-1)^n \frac{r[1 - e^{(r-\beta) t_c}]}{(r-\beta)^2 t_c} + \frac{1}{r} \frac{[1 - (-1)^n][e^{\eta t_c} - 1]}{t_c}
\end{align*}
\]

(10)

The average consolidation degree of a single-layered soil in terms of the excess pore water pressure is given by

\[
U_p = \frac{\int_0^b [\psi(z,t) - \psi(z,t)]dz}{\int_0^b \psi(z,\infty)dz}
\]

(11)

Substituting equations (7) and (8) into equation (11), respectively, the average consolidation degree \((U_p)\) can be derived and during the loading stage \(0 \leq t < t_c\), it can be expressed as,

\[
U_p = \frac{2mt + \eta' h - mt e^{-\alpha t} - (mt + \eta' h)e^{-\beta t}}{2q_r + \eta' h} - \frac{8\beta \eta' h}{2q_r + \eta' h} \sum_{n=1,3,5,\ldots}^{\infty} \frac{e^{\beta t} - e^{-\alpha t}}{(n\pi)^2(r-\beta)} - \\
\frac{8q_r}{2q_r + \eta' h} \sum_{n=1,3,5,\ldots}^{\infty} \left[ \frac{2(1-e^{-\alpha t})}{(n\pi)^2 rt_c} + \frac{(\alpha r - \alpha t - r)e^{\alpha t} + r e^{-\alpha t}}{(n\pi)^2(r-\alpha)^2 t_c} + \frac{\beta r - \beta t - r)e^{\beta t} + r e^{-\beta t}}{(n\pi)^2(r-\beta)^2 t_c} \right]
\]

(12)

During the constant load stage \((t \geq t_c)\), one can obtain,

\[
U_p = 1 - \frac{q_e e^{-\alpha t}}{2q_r + \eta' h} - \frac{(q_e + \eta' h)e^{\beta t}}{2q_r + \eta' h} - \frac{8q_r}{2q_r + \eta' h} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{(n\pi)^2} \left( \frac{\alpha e^{\alpha t}}{r-\alpha} + \frac{\beta e^{\beta t}}{r-\beta} + A_n e^{-\alpha t} \right) - \\
\frac{8\beta \eta' h}{2q_r + \eta' h} \sum_{n=1,3,5,\ldots}^{\infty} \frac{(e^{\beta t} - e^{-\alpha t})}{(n\pi)^2(r-\beta)}
\]

(13)

From this moment on, both the excess pore water pressure and the average consolidation degree of the underconsolidated marine clay with ETDB subjected to the ramp loading can be calculated by equations (7)–(8) and (12)–(13), respectively.

3. Evaluation of the Solutions

3.1. Simplification of the ETDB to the Permeable Boundary

Letting \(\alpha, \beta \to \infty\) and \(\eta = 0\), equations (7) and (8) are simplified as follows
\[
 u(z,t) = \begin{cases} 
 \frac{16q}{\pi T_{cv}} \sum_{n=1,3,5...} \frac{1 - \exp \left( - \frac{n^2 \pi^2 T_v}{4} \right)}{n^3} \sin \left( \frac{n \pi z}{h} \right) & (0 \leq t < t_c) \\
 \frac{16q}{\pi T_{cv}} \sum_{n=1,3,5...} \frac{\exp \left( - \frac{n^2 \pi^2 T_v}{4} \right) - 1}{n^3} \exp \left( - \frac{n^2 \pi^2 T_v}{4} \right) \sin \left( \frac{n \pi z}{h} \right) & (t \geq t_c) 
\end{cases}
\]

where \( T_v \) is the time factor; \( T_v = 4C_v t / h^2 \); \( T_{cv} \) is the construction time factor; \( T_{cv} = 4C_v t_c / h^2 \).

It can be observed that equation (14) is fully equivalent to Schiffman’s solution [10] with the perfectly permeable boundary, which verifies the proposed solutions.

### 3.2. Comparison with Existing Analytical Solution

In order to validate the above proposed solution in this paper, figure 3 compares the curves of excess pore water pressure computed by the proposed solution and the existing solution of Schiffman [10], respectively. From figure 3, a good agreement can be observed between the results of Schiffman’s solution and the proposed solution (\( \alpha = \beta = 10000, \eta = 0 \)). This suggests that the ETDB can be considered the perfectly permeable boundaries when \( \alpha \) and \( \beta \) are large enough, and the self-weight of soil can be neglected when \( \eta = 0 \). Meanwhile, when the time factor is very small (\( T_v = 0.01 \)), the proposed solution for the case of small values of interface parameters (\( \alpha = \beta = 1 \)) presents better continuity than Schiffman’s solution near the top and bottom boundaries. This is because the excess pore water pressure has not fully dissipated even at the boundaries yet and only the ETDB have the ability to describe such a phenomenon. This advantage of ETDB was also mentioned in former studies [6-7, 11-12].

![Figure 3](image-url)

**Figure 3.** Comparison of excess pore water pressure of the proposed solution with that for Schiffman’s solution [10].

### 3.3. Analysis of the Solutions

From equations (7) and (8), it can be seen that the excess pore water pressure of the under-consolidated marine clay is related to not only the consolidation coefficient (\( C_v \)), but also the interface parameters (\( \alpha, \beta \)), the coefficient of initial stress (\( \eta \)), and the loading rate (\( m \)) of the ramp loading.
4. Results and Discussion

According to some previous studies [8-9], it is suggested to lay a horizontal drain at the plane of maximum excess pore water pressure (PMEP) to improve the consolidation efficiency of soft soils such as dredged marine clay. That plane is also called the undrained plane where the velocity of seepage flow equals to zero. Thus, it is essential to investigate the position of PMEP for the layout of horizontal drain in order to accelerate the consolidation in practical engineering.

For convenience, in the following parametric study, the thickness, permeability coefficient, Young’s modulus, Poisson ratio and submerged unit weight of the soil take 10 m, $7.4 \times 10^3$ m/d, 10 MPa, 0.3 and 10 kN/m$^3$, respectively. And the final load $q_c$ of the ramp loading takes 100 kPa.

4.1. Influence of Interface Parameter on Position of PMEP

Figure 4 shows the position of the PMEP for cases with different drainage boundaries (a constant top interface parameter, $\alpha = 0.5$, and two bottom interface parameters, $\beta = 0.5$ and $\beta = 1.0$) during the loading stage, where the coefficient of initial stress is $\eta = 0.5$ and a ramp load with $T_{vc}=1$ is applied onto the top surface of the soil. It can be seen that at the same time factor, the excess pore water pressure for $\beta = 0.5$ peaks deeper than that for $\beta = 1.0$ does. In other words, as the interface parameter at one boundary increases, the PMEP moves towards the other boundary of the soil. This phenomenon accords with the conclusion from previous studies [9, 12]. Besides, at all time factors, even though the top and bottom boundaries are equal in interface parameters ($\alpha = \beta = 0.5$), the excess pore water pressure is not symmetrically distributed along the depth and does not peak at the mid-height of the soil layer. It is because of the self-weight of the soil ($\eta = 0.5$). And with the increase of the time factor, the PMEP tends to move upwards gradually from the bottom surface of the soil. The possible reason is that the dissipation of excess pore water pressure near the bottom boundary is faster than that near the middle of the soil layer.

Likewise, the variation in the position of the PMEP with different interface parameters at the constant load stage is depicted in figure 5. With the increase of the time factor, the PMEP continues to move upwards slowly for the case of $\alpha = 0.5$ and $\beta = 1.0$, as the drainage capacity at the top boundary is poorer than that at the bottom boundary. Nevertheless, for the case of symmetric boundary condition, $\alpha = \beta = 0.5$, the PMEP tends to keep at a certain depth below the mid-height of the soil layer, though the distribution of excess pore water pressure is still asymmetric. This indicates the excess pore water pressure induced by the self-weight of soil has completely dissipated so that the impact of the self-weight on maximum excess pore water pressure can be neglected at the constant loading stage. Furthermore, it is worth noting from figure 6 that when the boundary condition is symmetric ($\alpha = \beta$), the larger the interface parameters, the higher the PMEP and its ultimate position.

![Figure 4](image-url)
4.2. Influence of Initial Stress State on Position of PMEP
Figure 7 presents the influence of the coefficient of initial stress ($\eta$) on the PMEP. It can be observed that the PMEP for $\eta = 0.5$ is always lower than that for $\eta = 0.2$. This makes sense as the smaller the coefficient of initial stress (i.e., the greater the value of OCR), the smaller the difference in excess pore water pressure between the upper and lower parts of the soil layer (i.e., the more obvious the symmetry of the distribution of excess pore water pressure) and the shorter the distance of the PMEP to the mid-height of the soil layer.

4.3. Influence of Loading Rate on Position of PMEP
Figure 8 illustrates the change in the position of PMEP for soils with symmetric boundary conditions ($\alpha = \beta$) under the ramp loading with different loading rates. From figure 8, it can be observed that the PMEP becomes higher with the decrease of the construction time factor $T_{vc}$ at the loading stage (i.e., A2 and B2 are located below A1 and B1, respectively), while it occurs at the same depth at the constant load stage (i.e., all depths of B1, C1–D2 are the same, namely, the ultimate depth). For the ramp loading with an invariant final load $q_c$, the smaller the construction time factor $T_{vc}$, the larger the loading rate. Therefore, if all properties of a soil layer are identified, such as the interface parameters, the coefficient of initial stress and the coefficient of consolidation, the ultimate depth of peak excess pore water pressure can be determined and reached faster by increasing the loading rate.

In practice, the horizontal drain is advised to be set at the PMEP, but it can only be placed at a fixed depth, so it might be beneficial to keep that plane at a constant depth as long as possible. In other words, it is profitable to shorten the loading time for the design of the horizontal drainage
system as the maximum excess pore water pressure ranges vastly along the depth from the bottom boundary of the soil layer to the ultimate depth at the loading stage. Thus, such ultimate depth is worth consideration for placing a horizontal drain when the drainage capacity at the top and bottom boundaries is similar and the loading rate of ramp loading is large enough.

\[ \eta = 0.5, \quad \alpha = \beta = 0.5, \quad T_{vc} = 1.0 \]

**Figure 7.** Influence of initial stress state on PMEP.

\[ \eta = 0.5, \quad \alpha = \beta = 0.5, \quad T_{vc} = 1.0, \quad T_{vc} = 1.5 \]

**Figure 8.** Influence of loading rate on PMEP.

5. **Conclusion**

This paper presented an analytical solution for one-dimensional consolidation of underconsolidated marine clay with the ETDB under the ramp loading. The proposed solution was then evaluated through a comparison with the existing analytical solution. At last, a parameter study was conducted and the following conclusions were drawn.

1. For different combinations of interface parameters, the trends of PMEP with time are the same at the loading stage of the ramp loading, while they differs vastly at the constant load stage, when considering the self-weight of the soil.

2. With the increase of the coefficient of initial stress (ie, the decrease of value of OCR), the PMEP is further away from the mid-height of the soil layer.

3. When the boundaries are symmetric, the loading rate controls the time when the PMEP reaches the ultimate position but not the depth of such position.

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