Cosmic equation of state and advanced LIGO-type gravitational wave experiments

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ABSTRACT

It is hoped that the future generation of interferometric gravitational wave detectors will provide accurate measurements of the final stages of binary in-spirals. The sources probed by such experiments are of extragalactic origin and the observed chirp mass is the intrinsic chirp mass multiplied by $(1 + z)$ where $z$ is the redshift of the source. Moreover the luminosity distance is a direct observable in such experiments. This creates the possibility to establish a new kind of cosmological test, supplementary to more standard ones.

Recent observations of distant type Ia supernovae light curves suggest that the expansion of the Universe has recently begun to accelerate. A popular explanation of the present accelerating expansion of the Universe is to assume that some part $\Omega_Q$ of the matter–energy density is in the form of a dark component called ‘the quintessence’ with the equation of state $p_Q = wQ \rho_Q$ with $w \geq -1$. In this paper we consider the predictions concerning observations of binary in-spirals in future LIGO-type interferometric experiments assuming a ‘quintessence cosmology’. In particular we compute the expected redshift distributions of observed events in the a priori admissible range of parameters describing the equation of state for the quintessence. We find that this distribution has a robust dependence on the cosmic equation of state.

Key words: gravitational waves – cosmological parameters – dark matter.

1 INTRODUCTION

Recent distance measurements from high-redshift type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1999a) suggest that the Universe is presently accelerating its expansion. A popular explanation of this phenomenon is to assume that considerable amount $\Omega_Q \approx 70$ per cent of the matter–energy density is in the form of dark component called ‘the quintessence’ characterized by the equation of state $p_Q = wQ \rho_Q$ with $w \geq -1$ (Caldwell, Dave & Steinhardt 1998; Chiba, Sugiyama & Nakamura 1998; Turner & White 1997). The evidence for a spatially flat universe, reinforced by recent cosmic microwave background experiments BOOMERANG and MAXIMA (de Bernardis et al. 2000; Hanany et al. 2000) calls for an extra unclustered dark component. Within the standard cold dark matter (CDM) scenario only about $0.2 < \Omega_{CDM} < 0.4$ can be clustered in order to be in agreement with galactic rotation curves, abundance of galaxy clusters, gravitational lensing or large-scale velocity fields. Moreover the accelerated expansion of the universe can be achieved with extreme forms of matter. Hence this extra component (quintessence) should be similar to the cosmological constant but is allowed to have its own temporal dynamics. Many current models of dark matter in general and of quintessence in particular (Frieman et al. 1995; Uzan 1999; Chiba 1999), invoke the concepts from particle physics. Particle physics, however, gives little guidance as to concrete models of quintessence. Therefore it has been proposed by Nakamura & Chiba (1999) that future supernova surveys may allow reconstructing the quintessential equation of state. In this paper we shall contemplate the feasibility of constraining the cosmic equation of state from the gravitational wave experiments in a similar vein as proposed by Nakamura & Chiba (1999).

Laser interferometric gravitational wave detectors developed under the projects LIGO, VIRGO and GEO600 are expected to perform a successful direct detection of the gravitational waves. In-spiralling neutron star (NS–NS) binaries are among the most promising astrophysical sources for this class of experiments (Thorne 1996a,b). Besides quite obvious benefits from seeing gravitational waves ‘in the flesh’ and providing valuable information about dynamical processes leading to their generation in-spiralling binaries have one remarkable feature. Namely, the luminosity distance to a merging binary is a direct observable quantity easy to obtain from the waveforms. This circumstance made it possible to contemplate a possibility of accurate measurements of cosmological parameters such like the Hubble...
constant, or deceleration parameter (Finn & Chernoff 1993; Chernoff & Finn 1993; Markovic 1993; Schutz 1986; Krolak & Schutz 1987; Schutz 1989). In particular it was pointed out by Chernoff & Finn (Finn & Chernoff 1993; Chernoff & Finn 1993) how the catalogues of in-spiral events can be utilized to make statistical inferences about the Universe. In the similar spirit we discuss in this paper the possibility to constrain the quintessence equation of state from the statistics of in-spiral gravitational wave events.

2 COSMOLOGICAL MODEL

We shall consider a class of flat quintessential cosmological models. The spatially flat Universe has recently received a considerable observational support (Tegmark & Zaldarriaga 2000) from the measurements of the position of first acoustic peak at \( l = 200 \) in the balloon experiments BOOMERANG (de Bernardis et al. 2000) and MAXIMA (Hanany et al. 2000). This class is parametrized by two quantities: \( \Omega_0 \) and \( \Omega_\Lambda \), where \( \Omega_0 = \rho\rho_{r,0} = (8\pi G\rho_0)/(3H_0^2) \) denotes the current matter density as a fraction of critical density for closing the Universe, \( \Omega_\Lambda \) is analogous fraction of critical density contained in the quintessence and these two sum up to the value one. The equation of state for the quintessence is assumed in a standard form: \( p = \omega \rho \) where \( w \geq -1 \). This form of the equation of state is very general in the sense that it contains the well-known constituents of the universe as special subcases. For example \( w = -1 \) corresponds to the cosmological constant \( \Lambda \), \( w = 0 \) – the dust matter, \( w = -1/3 \) – cosmics strings and \( w = -2/3 \) the domain walls.

Non-Euclidean character of the space–time is reflected in distance measures. For the introduction to observational cosmology and the problems of distances in non-Euclidean spaces (see, e.g., Peebles 1993). In order to fix the notation for further use, let us introduce an auxiliary quantity \( D(z) \):

\[
D(z) = \sqrt{\Omega_0 (1 + z)^3 + \Omega_\Lambda} (1 + z)^{3(1+w)}
\] (1)

As it is well known, one can distinguish three types of distances.

(i) Proper distance

\[
d_M(z) = c \int_0^z \frac{dw}{H} = \frac{c d_L}{h} \frac{dw}{D(w)} = \frac{d_L}{h} \tilde{d}_M(z).
\] (2)

(ii) Angular distance

\[
d_A(z) = \frac{1}{1 + z} d_M(z) = \frac{1}{1 + z} \frac{d_L}{h} \tilde{d}_M(z).
\] (3)

(iii) Luminosity distance

\[
d_L(z) = (1 + z) d_M(z) = (1 + z) \frac{d_L}{h} \tilde{d}_M(z).
\] (4)

As usually \( z \) denotes the redshift, \( h \) denotes the dimensionless Hubble constant i.e. \( H_0 = h \times 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( d_L = 3 \times h^{-1} \text{ Gpc} \) is the Hubble distance (radius of the Hubble horizon). The quantities with an overbar have been defined by factoring out the dependence on the Hubble constant from respective quantities. In the further discussion we will explore the following models:

\[ (\Omega_0, \Omega_\Lambda) = \{(0.2, 0.8); (0.3, 0.7); (0.4, 0.6)\}, \]

with the \( w \) coefficient equal to \( w = \{0, -0.2, -0.4, -0.6, -0.8, -1\} \).

From the observational point of view in the light of constraints from large-scale structure and cosmic microwave background anisotropies, the 95 per cent confidence interval estimates give \( 0.6 \leq \Omega_0 \leq 0.7 \) and \(-1 \leq w < -0.6 \) (Efstathiou 1999; Perlmutter, Turner & White 1999b).

However we retain the full spectrum of a priori possible quintessential equations of state in order to illustrate the discriminating power of the gravitational wave data discussed in this paper.

3 REDSHIFT DISTRIBUTION OF OBSERVED EVENTS

The gravitational wave detector would register only those in-spiral events for which the signal-to-noise ratio exceeded certain threshold value \( \rho_0 \) (Finn & Chernoff 1993; Chernoff & Finn 1993; Finn 1996) which is estimated as \( \rho_0 = 8 \). For LIGO-type detectors. An intrinsic chirp mass \( M_0 = \mu^{165} M_\odot \), where \( \mu \) and \( M_\odot \) denote the reduced and total mass, is the crucial observable quantity describing the in-spiralling binary system. The observed chirp mass \( M(z) = (1 + z)M_0 \) scales with the redshift and therefore can be used to determine the redshift to the source (there is strong evidence that the mass distribution of neutron stars in binary systems is sharply peaked around the value \( 1.4 M_\odot \)). Because the luminosity distance of a merging binary is a direct observable easily read off from the waveforms one has a possibility to determine the precise distance–redshift relation and hence to estimate the Hubble constant (Finn 1996; Biesiada 1996). For a given detector and a source the signal-to-noise ratio reads (Finn 1996)

\[
\rho(z) = 8 \rho_0 \frac{M(z)}{1.2 M_\odot} \frac{\xi(z)}{\xi(f_{\text{max}})}.
\] (5)

where \( \rho_0 \) is a characteristic distance scale, depending on the sensitivity of the detector, \( \rho_0 = 355 \text{ Mpc} \) for advanced LIGO detectors, \( d_L \) is the luminosity distance to the source, \( \xi(f_{\text{max}}) \) is a dimensionless function describing the overlap of the signal with bandwidth of the detector.

The adiabatic in-spiral signal terminates when the binary system reaches the innermost circular orbit (ICO). The corresponding orbital frequency is \( f_{\text{ICO}} \) and \( f_{\text{max}} \) corresponds to observed (i.e. redshifted) \( f_{\text{ICO}} \)

\[
f_{\text{max}} = \frac{f_{\text{ICO}}}{1 + z} = \frac{710 \text{ Hz}}{1 + z} \left( \frac{2.8 M_\odot}{M} \right).
\] (6)

so \( f_{\text{max}} \sim 710 \text{ Hz} \) for neutron star binaries. It is argued that \( \xi(f_{\text{max}}) \approx 1 \) for LIGO/VIRGO interferometers (Finn & Chernoff 1993; Chernoff & Finn 1993; Finn 1996).

Let us denote by \( n_0 \) the local binary coalescing rate per unit comoving volume. One can use ‘the best guess’ for local rate density \( n_0 = 9.9 \times 10^{-8} \text{ Mpc}^{-3} \text{ yr}^{-1} \) as inferred from the three observed binary pulsar systems that will coalesce in less than a Hubble time (Narayan, Piran & Shemi 1991; Phinney 1991).

Source evolution over sample is usually parametrized by multiplying the coalescence rate by a factor \( \eta(z) = (1 + z)^3 \), i.e. \( n = n_0 (1 + z)^3 \eta(z) \) where the \( (1 + z)^3 \) factor accounts for the shrinking of volume with \( z \) and the time dilation of burst rate per unit time. The cosmological origin of gamma-ray bursts (GRBs) has been confirmed since discoveries of optical counterpart of GRB 970228 (Groot et al. 1997) and the measured emission-line redshift of \( z = 0.853 \) in GRB 970508 (Metzger et al. 1997). It has also been known for quite a long time that cosmological time
dilation effects in the BATSE catalogue suggest that the dimmest sources should be located at $z \approx 2$ (Norris et al. 1995). Consequently several authors tackled the question of source evolution in the context of gamma-ray bursts. Early estimates of Dremer (1992) and Piran (1992) indicated that the BATSE data could accommodate quite a large range of source density evolution (from moderate negative to positive one). Later considerations by Horack, Emslie & Hartman (1995) indicated that if $z = 2$ is indeed the limiting redshift then a source population with a comoving rate density $n(z) \sim (1 + z)^{3}$ with $1.5 \leq \beta \leq 2$ is compatible with the BATSE data. Later on Totani (1997) and Horack et al. (1995) considered the source evolution effects and based his calculations on the realistic models of the cosmic star formation history in the context of NS–NS binary mergers. Comparison of the results with the BATSE brightness distribution revealed that the NS–NS merger scenario of GRBs naturally leads to the rate evolution with $2 \leq \beta \leq 2.5$. We shall therefore take the source evolution effects into account in our further considerations. One should stress, however that NS–NS merger scenario is by no means the unique explanation of gamma-ray bursts. Recently the so-called collapsar model became popular (MacFadyen & Woosley 1999; Paczyński 1998; Woosley 1993). The idea that at least some of gamma-ray bursts are related to the deaths of massive stars is supported by the observations of afterglows in GRB 970228 and GRB 980326 (Reichert 1999; Bloom et al. 1999). Therefore we will not prefer any specific value of evolution exponent $D$ but instead we will try to illustrate how strongly and in which direction does the source evolution affect our ability to discriminate between different quintessential equations of state.

The relative orientation of the binary with respect to the detector is described by the factor $\Theta$. This complex quantity cannot be measured nor assumed a priori. However, its probability density averaged over binaries and orientations has been calculated (Finn 1996) and is given by a simple formula:

$$P_{\Theta}(\Theta) = 5\Theta(4 - \Theta)^{3}/256, \quad \text{if } 0 < \Theta < 4$$

$$P_{\Theta}(\Theta) = 0, \quad \text{otherwise.}$$

The rate $[dN(\rho_{0})]/(dz)$ at which we observe the in-spiral events that originate in the redshift interval $[z_{s}, z + dz]$ is given by Nowak & Grossman (1994)

$$\frac{dN(\rho_{0})}{dz} = \frac{n_{0}}{1 + z} \eta(z)4\pi d^{2}_{M}(\frac{d}{dz}) C_{0}(x),$$

where $C_{0}(x) = \int_{\rho}^{\infty} P_{\Theta}(\Theta) d\Theta$ denotes the probability that given detector registers in-spiral event at redshift $z_{s}$ with $\rho > \rho_{0}$. The quantity $C_{0}(x)$ can be calculated as

$$C_{0}(x) = (1 + x)(4 - x)^{3}/256 \quad \text{for } 0 \leq x \leq 4$$

$$= 0 \quad \text{for } x > 4$$

and

$$A = 0.4733 \left(\frac{8}{50}\right) \left(\frac{r_{0}}{355 \text{ Mpc}}\right) \left(\frac{M_{0}}{1.2 \text{ M}_{\odot}}\right)^{5/6}.$$
quintessential equations of state. It has been obtained by numerical integration of the formula (8). The predictions for other realistic proportions of $\Omega_0$ and $\Omega_Q$ are almost indistinguishable at the level of detection rates, so the Fig. 1 is representative for the whole class of models considered. The effect of source evolution on the detection rate is summarized in Fig. 2. For transparency only one member (corresponding to $w_q = -0.8$) of each family of curves (as in Fig. 1) is shown for different values of the evolution exponent $D$.

The method of extracting the cosmological parameters advocated by Finn & Chernoff (Finn 1996) makes use of the redshift distribution of observed events in a catalogue composed of observations with the signal-to-noise ratio greater than the threshold value $r_0$. Therefore it is important to find this distribution function for different quintessence models. The formula for the expected distribution of observed events in the source redshift can be easily obtained from equation (8)

$$P(z > \rho_0) = \frac{1}{N(> \rho_0)} \frac{dN(> \rho_0)}{dr} = \frac{4\pi}{N(> \rho_0)} \left( \frac{d\eta}{dr} \right)^3 \eta_0 \eta^3 \frac{d^2(z)}{D(z)} C_{\rho}(x). \tag{12}$$

The summary of numerical computations for the cosmological quintessence models considered based on the formulae (12) and (8) are given in Figs 3 and 4. Fig. 3 illustrates the $P(z > \rho_0)$ distribution function for the ($\Omega_0 = 0.3, \Omega_Q = 0.7$) cosmological model with different quintessential equations of state. For the purpose of obtaining the Figs 3 and 4 we have assumed the dimensionless Hubble constant equal to $h = 0.65$ as suggested by independent observational evidence [e.g. SNe Ia in the HST project (Ferrarese et al. 2000; Gibson et al. 2000) or multiple image quasar systems (Kundic et al. 1997; Falco et al. 1997; Biggs et al. 1999)].

In Fig. 4 the distribution functions for different cosmological models with the quintessence field with $w_q = -0.8$ have been plotted together. Fig. 5 shows the distribution functions for different evolutionary exponent in the ($\Omega_0 = 0.3, \Omega_Q = 0.7$) model with $w = -0.8$.

4 RESULTS AND DISCUSSION

It is clear from Fig. 1 that different quintessential cosmologies (singled out by $w$ parameter in the equation of state) give different predictions for annual in-spiral event rate to be observed by future interferometric experiments. Unfortunately, this difference is too small to be of observational importance. Moreover, as already pointed out, there exists a degeneracy in terms of cosmological models (labelled by the value of $\Omega_0$ and $\Omega_Q$). There is however a difference between detection rates corresponding to different values of evolutionary exponents as displayed in Fig. 2.

Fig. 3 shows that there is a noticeable difference in predicted event redshift distribution functions $P(z > \rho_0)$ for different values of the cosmic equation of state within given cosmological model (labelled by the values of $\Omega_0$ and $\Omega_Q$). The spread between different cosmological models for a given quintessence equation of state is much smaller as seen from the Fig. 4. This is a reflection of above mentioned effective degeneracy with respect to values of $\Omega$ parameters. Hopefully this degeneracy can be broken by independent estimates of $\Omega_0$ and $\Omega_Q$ parameters in other studies (cluster baryons estimates, Ly$\alpha$ forest surveys, large-scale structure or cosmic microwave background radiation). The spread of redshift distribution functions attributed to evolutionary effects is smaller as shown in Fig. 5 and has a slightly different character – the distribution function is shifted toward increasing redshifts when the evolutionary exponent changes from positive to negative value. This may to some extent mimic the effect of cosmic equation of state, but it should in principle be possible to disentangle – at least
to a certain degree from the complementary information about the detection rates. As can be seen from Fig. 2 the magnitudes of observed event rates for different evolutionary exponents $D$ are clearly distinct, at least for the range of the Hubble constant suggested by independent cosmological evidence (Freedman et al. 2001; Gibson & Brook 2001).

The redshift distribution $P(z > p_0)$ is in fact inferred from observed chirp mass distribution. Therefore it can in principle be distorted by the intrinsic chirp mass distribution. Theoretical studies of the neutron star formation suggest that masses of nascent neutron stars do not vary much with either mass or composition of the progenitor (Finn 1996). Also the mass estimates of observed binary pulsars suggest that there are good reasons to assume a negligible spread of intrinsic chirp mass (as it was done in the

Figure 3. Redshift distribution of observed events in the cosmological model with $\Omega_b = 0.3, \Omega_0 = 0.7$ for different quintessential equations of state.

Figure 4. Redshift distribution of observed events in the cosmological quintessence model with $w = -0.8$ Different cosmological models have been plotted collectively.
present paper). Moreover any intrinsic distribution of mass would be expected as symmetric, whereas the redshift distribution (of cosmological origin) has certain amount of asymmetry.

In conclusion one can hope that the catalogues of inspiral events gathered in future gravitational waves experiments can provide helpful information about the quintessence equation of state complementary to that obtained by other techniques. Even though the most straightforward way of making inference about cosmic equation of state would come from future supernovae surveys it would be good to have in mind alternative ways of reaching the same goal such as the one proposed in the present paper.

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