Fast computation approach for parameterized design and simulation of vertical ground heat exchangers and GCHP systems

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Abstract. Vertical ground heat exchanger (GHE) is an expensive part of vertical GCHP systems. Fast and accurate computation of fluid and ground temperatures is desired for the long-term energy performance prediction of GCHP and optimal sizing of bore field. Hourly simulations and iteration of parameterized design in time domain with thermal response factors is quite time-consuming. In this paper, a new thermal response factors, \( \delta \)-function is proposed for computing fluid and ground temperatures of bore field. \( \delta \)-function is a non-dimensional ground temperature response and an accurate analytical solution of finite line source model to a unit rectangular heat pulse. Computation of \( \delta \)-function is quite fast due to its quite small integral range. The validity and accuracy of \( \delta \)-function are examined by the simulation results of \( g \)-function. Through comparing hourly simulation of various periods between \( g \)- and \( \delta \)-functions in both time and spectral domain methods, the combination of \( \delta \)-function with Fast Fourier Transform spends the least computing time and is suitable to quickly and accurately compute fluid and ground temperatures in parameterized design and long-period hourly simulation of vertical heat exchanger and GCHP systems.

1. Introduction

Ground-coupled heat pump (GCHP) system with vertical ground heat exchangers (GHE) (called as vertical GCHP) is one of important applied energy systems and has become the most attractive group of ground source heat pump systems in research field and practical engineering as well, owing to its advantages of less land area requirement and wide range of applicability [1]. In a vertical GCHP system as shown in Figure 1, the GHE configurations may include one, tens, or even hundreds of boreholes, each containing one or double U-tubes through which heat exchange fluid is circulated. Modeling and hourly energy simulation are an important part of design and performance evaluation of vertical GCHP systems [2]. Optimal sizing also requires one to compute iteratively many times the hourly (or another suitable time step) fluid temperature for long periods. Hourly computation or simulations over 10, 20 and even 30 years are often looked for.

In the simulation of GCHP systems with an hour or shorter time-step, thermal response factors, i.e., \( g \)-function [3] are widely used to predict the performance of geothermal bore fields. The thermal response factors of geothermal bore fields are coupled with heat load steps through the spatial and temporal superposition schemes in the hourly computations and simulations. Such schemes are basically the convolution of the thermal response factors of a bore field with hourly heat loads. This computation process is recognized in the literature as quite computationally intensive and very time-consuming [2]. The computation includes both the generation of thermal response factors for a bore field and the convolution product. Hence, both the fastest calculation method for thermal response factors and the most efficient algorithm for convolution are eagerly anticipated to perform the task. Many researchers have made great effort to accelerate the hourly computation and simulation for bore fields [4,5,6]. In order to improve the calculation efficiency, the researchers often take the way by increasing time step
to a day or several days to reduce the computation time. As a result, this greatly reduces calculation accuracy. In order to ensure the computation precision, the simulation with hourly load is essential for the temperature response in a bore field. Most of the current fast computation method is based on the load aggregation scheme which reduces computation by the average load within various periods of time. This scheme may lead to certain calculation error of borehole temperature response. In this study, a new thermal response factors and computation method is developed to ensure the high precision and fast computation speed.

Figure 1. Schematics of ground-coupled heat pump system.

2. Thermal response factors to unit rectangular heat pulse

2.1. Ground temperature response to unit rectangular heat pulse

Cooling/heating load in real operating GCHP system is continuously changing with time. The changing heat transfer rate can be approximately expressed by a set of rectangular heat pulse as shown in Figure 2. The width of rectangular pulse is the time step of simulation, usually 1 hour (or another suitable time step). By employing $g$-function and superposition principle, the temperature response at time $\tau$ and radial distance $r$ from a borehole to any arbitrary variable heat rejection/extraction is the superimposition of temperature response to a set of single step heat pulses and is expressed as:

$$T(r,z,\tau_j)-T_0 = \frac{1}{2\pi\kappa} \sum_{i=1}^{j} (q(\tau_i) - q(\tau_{i-1})) g(\beta,\eta, Fo_j - Fo_{i-1})$$

(1)

where $Fo_j = a \tau_j / H^2$, Fourier time at time $\tau_j$.

As shown in Figure 3, a rectangular heat pulse within any arbitrary time span $[\tau_{i-1}, \tau_i]$ is the superimposition of two step heat pulses. The temperature response to the rectangular heat pulse within the time span $[\tau_{i-1}, \tau_i]$ can be expressed as

$$T(r,z,\tau_j)-T_0 = \frac{1}{2\pi\kappa} q(\tau_j) [g(\beta,\eta, Fo_j - Fo_{i-1}) - g(\beta,\eta, Fo_j - Fo_i)]$$

(2)

Here, the dimensionless temperature response to unit rectangular heat pulse within time step $\Delta \tau = \tau_j - \tau_{i-1}$ is defined as $\delta$-function, which is a function of $\beta, \eta$ and $Fo$ only, and expressed as

$$\delta(\beta,\eta, Fo) = g(\beta,\eta, Fo) - g(\beta,\eta, Fo - \Delta Fo) = \int_{0}^{\Delta Fam} g(\beta,\eta, Fo - \tau)d\tau$$

(3)

where, $\Delta Fam$ is Fourier time step, $\Delta Foa = Foa - Foa_{i-1} = \Delta \tau \cdot a / H^2$. The $\delta$-function for the finite line source model can be obtained through its $g$-functions [7]. Some typical values of Fourier time step $\Delta Foa$ are given in Table 1 for different soil properties. For a 60m borehole length, $\Delta Foa$ is about $4.7 \times 10^{-5}$ and $1.5 \times 10^{-6}$ respectively for wet sand-gravel soil and rocky soil. Thus, the integral range of $\delta$-function is
far smaller than that of the $g$-functions (which is $[0,1]$) and dose change with the number of simulation hours when time step is given as 1 hour.

**Figure 2.** Approximation of heat load with rectangular pulse.

**Figure 3.** Rectangle heat pulse expressed by two step heat pulses.

**Table 1** Fourier time step $\Delta F_0$ of a 60m borehole with 1 hour time step

| Type of soil                  | Conductivity (Wm$^{-1}$K$^{-1}$) | Diffusivity (m$^2$/h) | $\Delta F_0$ |
|------------------------------|----------------------------------|-----------------------|--------------|
| Wet sandy-gravel soil        | 1.745                            | 0.0017                | 4.7×10$^{-7}$|
| Rocky soil (granite)         | 2.908                            | 0.0053                | 1.5×10$^{-6}$|

For the finite line source model, the $\delta$-function at time $\tau_j$, radial distance $r$ and depth $z$ is expressed as

$$\delta(\beta, \eta, F_0) = \int_0^{\infty} g_{\beta,\eta}(\beta, \eta, F_0 - \tau) d\tau = \int_0^{\infty} \frac{\exp(-\beta^2/4(F_0-\tau))}{4(F_0-\tau)} [2-erfc(-\frac{1-\eta}{2\sqrt{F_0-\tau}})+erfc(-\frac{1+\eta}{2\sqrt{F_0-\tau}})-2erfc(-\frac{\eta}{2\sqrt{F_0-\tau}})] d\tau$$

(4)

The middle temperature $\delta$-function on the borehole wall is calculated as

$$\delta_\delta = \int_0^{\infty} g_{\delta,\eta}(\eta, F_0 - \tau) d\tau = \int_0^{\infty} \frac{\exp(-\beta^2/4(F_0-\tau))}{4(F_0-\tau)} [2-3erfc(-\frac{0.5}{2\sqrt{(F_0-\tau)}})+erfc(\frac{1.5}{2\sqrt{(F_0-\tau)}})-A(F_0-\tau))] d\tau$$

(5)

The mean temperature $\delta$-function on the borehole wall is calculated as

$$\overline{\delta} = \int_0^{\infty} \frac{g_{\delta,\eta}(\eta, F_0 - \tau) d\tau}{A(F_0-\tau)}$$

(6)

The temperature response increment at time $\tau_j$, radial distance $r$ and depth $z$ to a rectangular heat pulse is expressed as

$$T(r,z,\tau_j)-T_0 = \frac{1}{2\pi k} q(\tau_j) \delta(\beta, \eta, F_0 - F_{0,-1}) = \frac{1}{2\pi k} q(\tau_j) \delta(\beta, F_{0,-1})$$

(7)

Thus, the temperature response to any arbitrary variable heat rejection/extraction is the superimposition
of temperature response to a set of single rectangular heat pulses and is calculated by

$$ T(r, z, \tau_j) - T_0 = \frac{1}{2\pi k} \sum_{i=1}^{\infty} q(\tau_i) \delta(\beta, \eta, F(\tau_{j+i})) $$

(8)

When $r = r_b$, Eq.(8) gives the borehole wall temperature.

By introducing the relative temperature increment $\theta(\tau_j)$, the ground temperature response is expressed as

$$ T(r, z, \tau_j) = T_0 + \frac{1}{2\pi k} \theta(\tau_j) $$

(9)

where

$$ \theta(\tau_j) = \sum_{i=1}^{\infty} q(\tau_i) \delta(\beta, \eta, F(\tau_{j+i})) $$

(10)

By introducing $\delta$-function, the hourly heat load pulses can directly be employed to calculate the borehole wall temperature and fluid temperature without processing them as the incremental heat load between two successive pulses.

2.2. Fluid temperature in borehole

In long-term simulation, the time step is generally taken as an hour or longer, therefore the heat transfer process in borehole can be regarded as stable one and the fluid temperature response in borehole is expressed as

$$ T_f(\tau_j) = T(r_b, \tau_j) + q(\tau_j)R_b $$

(11)

where $R_b$ is the equivalent thermal resistance in borehole, which is calculated with the average fluid temperature approach proposed by Du & Chen [8]. Incorporating Eq. (9) into Eq.(11), the fluid temperature response in borehole is calculated as

$$ T_f(\tau_j) = q(\tau_j)R_b + T(r_b, \tau_j) = q(\tau_j)R_b + \frac{1}{2\pi k} \theta(\tau_j) + T_0 $$

(12)

For a bore field with $n$ boreholes, the fluid temperature response in the $m$th borehole is calculated as

$$ T_f(m, \tau_j) = T_0 + q_m(\tau_j)R_m + T(r_b, \tau_j) = T_0 + q_m(\tau_j)R_m + \sum_{i=1}^{n} \frac{1}{2\pi k} \{q(\tau_i) \delta(\beta, F_0, \tau_i)\} $$

(13)

where $\beta_i$ is the relative distance between the $i$th and $m$th boreholes, assuming that the radius and depth of all boreholes are $r_b$ and $H$.

3. Comparisons and discussions

3.1. Synthetic loads

We use the simple function described by Marcotte and Pasquier [6] to simulate synthetic loads, which suffices for illustrating the computational efficiency of various approaches. The formula used to generate the synthetic loads is

$$ q(\tau) = A - B \cos\left(\frac{\tau}{8760} - 2\pi\right) - C \cos\left(\frac{\tau}{24} - 2\pi\right) - D \cos\left(\frac{\tau}{24} + 2\pi\right) + \cos\left(\frac{2\tau}{8760} - 2\pi\right) $$

(14)

where, $\tau$ is time in hours, $A$ controls the annual load unbalance, $B$ the half-amplitude of annual load variation, $C$ and $D$ the half-amplitude of daily load fluctuations. The ratio $D/C$ controls the relative importance of the damped component used to simulate larger daily fluctuations in winter and summer. Fig.4 shows synthetic loads per 1m borehole length obtained with parameters $A=-4.25$ W/m, $B=25$ W/m, $C=12.5$ W/m, $D=6.25$ W/m. We use this synthetic load and compute the fluid temperature using two approaches (CTD, FFT) with the $g$-functions and $\delta$-functions.

3.2. Results and discussions

Computing the borehole wall temperature responses and fluid temperatures is the key for the simulation of vertical ground heat exchanger system. The computation of the borehole wall temperature involves two steps: one is computing $g$ or $\delta$-functions, the other is computing the convolution between the sequences of step or rectangular heat pulses $q$ and $g$ or $\delta$-functions in time or spectral domain. Here,
computing the convolution in time-domain with full load is shortened as CTD. Computing the convolution in spectral domain can also be implemented with FFT [9]. We used synthetic loads and two kinds of thermal response factors to calculate the fluid temperatures and compared the differences among the results obtained by two kinds of thermal response factors and computational approaches. Table 2 presents the magnitude order of mean and max absolute differences among the fluid temperatures obtained with the combinations of the thermal response factors of mean temperature and computational approaches. Computation is done over 1 year of a 2×2 bore field with synthetic loads shown in Figure 4. The magnitude order of mean (maximum) difference between the hourly fluid temperatures obtained by two computing approaches with the $g$-function is $10^{-14}$ ($10^{-13}$) °C. The magnitude order of mean (maximum) difference between the hourly fluid temperatures obtained by two computing approaches with the $\delta$-functions is $10^{-15}$ ($10^{-14}$) °C. The magnitude order of mean (maximum) difference between the hourly fluid temperatures obtained by the $g$- and $\delta$-functions is $10^{-4}$ ($10^{-3}$) °C. This indicates that $g$- and $\delta$-functions are the exact results of finite line source model. The results are the same when the thermal response factors for middle temperature are used to calculate the hourly fluid temperatures.

![Figure 4. Synthetic loads per unit borehole length.](image)

| Thermal response factors / Computing approaches | $\mathbf{g}_{b,Zeng}$ (CTD/macro/e) | $\mathbf{\delta}_b$ (CTD/macro/e) | $\mathbf{g}_{b,Zeng}$ (FFT/macro/e) | $\mathbf{\delta}_b$ (FFT/macro/e) |
|-----------------------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| $\mathbf{g}_{b,Zeng}$ CTD                  | $10^{-14}/10^{-13}$               | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 |
| $\mathbf{g}_{b,Zeng}$ FFT                  | $10^{-14}/10^{-13}$               | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 |
| $\mathbf{\delta}_b$ CTD                    | $10^{-15}/10^{-14}$               | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 |
| $\mathbf{\delta}_b$ FFT                    | $10^{-15}/10^{-14}$               | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 | $10^{-4}/10^{-3}$                 |

The total computing time for fluid temperature is the sum of the time taken in the two steps, computing $g$- or $\delta$-functions and computing CTD or FFT. Among the $g$-functions $\mathbf{g}_{b,Zeng}$ [10], $\mathbf{g}_{b,Lam}$ [11] and $\mathbf{g}_{b,2-1}$ [11], it takes the least time for the middle temperature $g$-function $\mathbf{g}_{b,Zeng}$. In order to compare the computation performance of the $\delta$-functions, the borehole wall temperature and fluid temperature are computed by the $g$-functions $\mathbf{g}_{b,Zeng}$ and $\delta$-function $\mathbf{\delta}_b$, $\mathbf{\delta}_b$ with CTD, FFT, respectively. The CPU time of different simulation hour numbers for a 5×8 bore field are listed in Table 3. The computation is performed under Windows XP, on a standard laptop with 2.50 GHz dual-core CPU. Comparisons among computing time show that, the $\delta$-functions are far faster than the $g$-function; and the middle temperature $\delta$-function is faster than the mean temperature $\delta$-function. FFT is greatly faster
than CTD. For 30 year hourly simulation, the computing time by the middle temperature $\delta$-function with FFT is less than 75 seconds for a 5×8 bore field, and the computing time by the mean temperature $\delta$-function with FFT is less than 90 seconds for a 5×8 bore field. However, by Zeng’s middle temperature $g$-function with FFT, the computing time is greater than 2.2 hours for a 5×8 bore field.

| Simulation time | Number of hours | $\delta_{\text{Zeng}}$ | $\overline{\sigma}_0$ | $\overline{\delta}_h$ |
|-----------------|-----------------|------------------------|----------------------|----------------------|
|                 |                 | CTD       | FFT      | CTD     | FFT     | CTD     | FFT     |
| 1 year          | 8760            | 289       | 243      | 56      | 1.86    | 61      | 2.23    |
| 10 years        | 87600           | 10667     | 2656     | 7997    | 18.99   | 8134    | 23.09   |
| 30 years        | 262800          | 89920     | 8201     | 82646   | 74.96   | 83083   | 87.55   |

4. Conclusions

Fast and accurate computation approaches are really desired for design and simulation of vertical ground heat exchanger and GCHP systems in engineering application. The computation speed of hourly simulation depends on the generation speed of thermal response factors and the convolution of thermal response factors with hourly heat loads. A kind of thermal response factors, which is the non-dimensional ground temperature response to unit rectangular heat pulse and referred as $\delta$-function, is proposed. The $\delta$-function is the exact solution of finite line source model as $g$-function, has high accuracy and quite short computation time due to very small integral range. Its accuracy is examined by the hourly simulation results of $g$-function under the same synthetic loads. Through hourly simulation of various periods, the computing speeds of the combination of $\delta$-function with CTD and FFT approaches are compared with the $g$-functions. The results indicate that $\delta$-function is far faster than that $g$-function, and the combination of $\delta$-function with FFT is the fastest and spends only less than 1.5 minutes in 30 year hourly simulation for a 5×8 bore field. The combination of $\delta$-function with FFT is an effective and suitable approach for parameterized design and hourly simulation of vertical ground heat exchanger and GCHP systems.

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