Calculation and Experimental Determination of Inductance Parameters of Brushless Double Fed Machine for Magnetic Barrier Rotor

Xiuping Wang*, Yanwei Ji, Chunyu Qu and Yan Li  
School of Shenyang Institute of Engineering, ShenYang 110136, China  
*Corresponding author e-mail: wangxpmail@163.com

Abstract. Brushless doubly fed machine (BDFM) is a new type of AC motor, which has the advantages of brushless reliability and low cost. The inductance parameters of this kind of motor are relatively complex, and the inductance parameters are the basis for motor performance analysis and high precision control. In this paper, the inductance parameters of the motor are calculated by finite element analysis method, along with the experimental prototype is developed to measure the inductance parameters. The correctness of the theoretical analysis is proved, which provides useful reference for the high precision control of BDFM.

1. Introduction  
BDFM has the advantages of simple structure, reliable operation, small frequency converter capacity, etc., and has a wide application prospects in the field of frequency conversion speed regulation. In recent years, scholars at home and abroad have made great progress in the industrialization process [1-3].  
The stator of BDFM is embedded with two sets of windings with different poles, namely power winding and control winding. Power winding is directly connected to the power grid at run time, and Control winding is connected to the power grid through frequency converter. The two sets of BDFM stator windings are coupled by the magnetic field modulation of the special rotor. The rotor plays the role of "pole number converter", and its performance directly affects the power density and efficiency.  
In this paper, a new type of BDFM for magnetic barrier rotor is studied, Inductance parameters are analyzed, the calculation method of inductance parameters is provided, and the experimental determination is carried out. The two methods are compared and the important research conclusions are obtained. It will lay the foundation for further theoretical analysis of the BDFM.

2. Magnetic barrier rotor structure of BDFM  
The radial laminated rotor structure is shown in Figure 1. The rotor structure adopts radial laminated structure, which reduces eddy current loss and simplifies the processing process. The kind of structure makes it easier to process and suitable to the large BDFM.
3. Calculation of inductance parameters

The energy perturbation method is an effective method for calculating inductance with finite element analysis. It has high generality and accuracy, and is not limited by the complexity of winding and air gap. It can be used to calculate the inductance parameters of various Machines.

The machine is a coupling system composed of multiple windings. Assuming that the motor has \( n \) windings, the end voltage of the \( j \) windings is \( u_j \), and the corresponding magnetic chain is \( \psi_j \). From the circuit theory, the voltage equation of the \( j \) windings is

\[
 u_j = r_j i_j + \frac{d\psi_j}{dr} \tag{1}
\]

Because \( \psi_j \) is a function of \( j \) winding currents, it can be written as

\[
 \psi_j = \sum_{k=1}^{n} \psi_{jk} = \sum_{k=1}^{n} L_{jk}^{app} i_k \tag{2}
\]

The equation \( L_{jk}^{app} \) is the apparent inductance coefficient between windings \( j \) and \( k \).

Because the magnetic chain \( \psi_j \) is both a function of the winding current and a function of the rotor position angle \( \theta_r \):

\[
 u_j = r_j i_j + \sum_{k=1}^{n} \frac{\partial \psi_j}{\partial i_k} \frac{di_k}{dr} + \frac{\partial \psi_j}{\partial \theta_r} \frac{d\theta_r}{dr} \tag{3}
\]

In order to facilitate the explanation of the principle of finding inductive parameters by the magnetic field energy perturbation method, the perturbation of \( \theta_r \) is not considered. Only the changes in magnetic field energy caused by the current perturbation are considered, and the formula (2) is available in equation (3):

\[
 u_j = r_j i_j + \sum_{k=1}^{n} \left[ L_{jk}^{app} \frac{di_k}{dt} + \frac{\partial L_{jk}^{app}}{\partial i_k} \frac{di_k}{dt} \right] = r_j i_j + \sum_{k=1}^{n} \frac{di_k}{dt} \left[ L_{jk}^{app} + \sum_{s=1}^{n} \frac{\partial L_{jk}^{app}}{\partial i_s} i_s \right] \tag{4}
\]

In which:

\[
 L_{jk}^{dif} = L_{jk}^{app} + \sum_{s=1}^{n} \frac{\partial L_{jk}^{app}}{\partial i_s} i_s, \quad k=1,2,\ldots,n \tag{5}
\]

When \( j = k \), \( L_{jj}^{dif} \) is called the differential self-inductance coefficient of the \( j \) windings; When \( j \neq k \), \( L_{jk}^{dif} \) is called the differential mutual sensing coefficient between the \( i \) and \( j \) windings.

The product of the voltage \( u_j \) and the current \( i_j \) represents the instantaneous power of the \( j \) windings, so:

\[
 P_j = r_j i_j^2 + \sum_{k=1}^{n} i_j L_{jk}^{dif} \frac{di_k}{dt} \tag{6}
\]
The magnetic field energy of the j winding can be expressed as

\[ W_j = \int_0^t (P_j - r_j i_j^2)\, dt = \sum_{k=1}^n \int_{i_k(0)}^{i_k(t)} i_j L_{jk}^{\text{diff}}\, di_k \]  

(7)

The magnetic field storage energy of the entire system:

\[ W = \sum_{j=1}^n W_j = \sum_{j=1}^n \sum_{k=1}^n \int_{i_k(0)}^{i_k(t)} i_j L_{jk}^{\text{diff}}\, di_k \]  

(8)

If the k windings current \( i_k \) has a slight variation \( \Delta i_k \), because \( \Delta i_k \) is small, it can be considered that the magnetic field is a linear change and the differential inductance \( L_{jk}^{\text{diff}} \) is a constant. Therefore, the perturbation increment of the system's magnetic field energy is

\[
\Delta W = \sum_{j=1}^n \int_{i_j(0)}^{i_j(t)} i_j d i_j + \sum_{j=1}^n \sum_{k=1, k \neq j}^n \int_{i_k(0)}^{i_k(t)} i_j d i_k
= \sum_{j=1}^n \int_{i_j(0)}^{i_j(t)} i_j \Delta i_j + \frac{1}{2} \Delta i_j^2 + \sum_{j=1}^n \sum_{k=1, k \neq j}^n \int_{i_k(0)}^{i_k(t)} i_j \Delta i_k + \frac{1}{2} \Delta i_k \Delta i_j
\]  

(9)

Because the total magnetic field energy in before and after the current perturbation of the motor winding is \( W \) and \( W^* \), so:

\[ W^* = W + \Delta W \]  

(10)

The secondary partial differentiation of \( \Delta i_j \) by the changed total magnetic field energy \( W^* \) gives the self-perception of the j windings:

\[ L_{jj}^{\text{diff}} = \frac{\partial^2 W^*}{\partial (\Delta i_j)^2} \]  

(11)

After the change of the total magnetic field energy \( W^* \), partial differentiation of \( \Delta i_j \), and partial differentiation of \( \Delta i_k \), the mutual inductance of j and k windings can be obtained:

\[ L_{jk}^{\text{diff}} = \frac{\partial^2 W^*}{\partial \Delta i_j \partial \Delta i_k} \]  

(12)

Changing expressions (10) and (11) to discrete forms suitable for finite element processing. If W represents the magnetic field energy when the motor’s n windings currents are \( i_1, i_2, \ldots, i_n \) at static working point, while \( W(i_j \pm \Delta i_j) \) represents the \( i_j \) perturbation of the current \( \pm \Delta i_j \) of the j windings and the other winding currents. The magnetic field energy when the constant current, the corresponding magnetic field energy is \( \Delta W^*(i_j \pm \Delta i_j) \), then:

\[
L_{jj}^{\text{diff}} = \frac{\partial^2 W^*}{\partial (\Delta i_j)^2} \approx \frac{W(i_j + \Delta i_j) + W(i_j - \Delta i_j) - 2W}{(\Delta i_j)^2} = \frac{W(i_j + \Delta i_j) + W(i_j - \Delta i_j)}{(\Delta i_j)^2} - 2W
\]  

(13)

\[
L_{jk}^{\text{diff}} = \frac{\partial^2 W^*}{\partial \Delta i_j \partial \Delta i_k} \approx \frac{W(i_j + \Delta i_j, i_k + \Delta i_k) + W(i_j - \Delta i_j, i_k - \Delta i_k)}{4(\Delta i_j)(\Delta i_k)} - \frac{W(i_j - \Delta i_j, i_k + \Delta i_k) + W(i_j + \Delta i_j, i_k - \Delta i_k)}{4(\Delta i_j)(\Delta i_k)}
\]  

(14)
The Vissible inductance parameter can be expressed as:

\[ L_{jj}^{app} = \frac{2W_{j}^{app}(i_j)}{i_j} \]  
\[ L_{jk}^{app} = \frac{W_{j}^{app}(i_j,i_k) - W_{j}^{app}(i_j) - W_{k}^{app}(i_k)}{i_j i_k} \]  

4. Development of prototype and determination of inductance parameters

The experimental prototype of BDFM for magnetic barrier rotor is designed and developed, as shown in Figure 2. Figure 2 (a), (b) and (c) are the stator, rotor and experimental system of BDFM.

According to energy perturbation method, finite element analysis model is established. The inductance parameters of BDFM with magnetic barrier rotor are calculated by magnetic field analysis method, so the calculation and measurement value of inductance parameters are shown in figure 3-5.

![Prototype of brushless doubly fed machine.](image)

(a) stator  (b) rotor  (c) BDFM system

**Figure 2.** Prototype of brushless doubly fed machine.

The inductance parameters of BDFM are measured by static measurement method: First, the sinusoidal alternating current with an angular frequency of \( \omega \) is introduced into the \( j \) windings, while the other windings is open and the measurement records \( U_j, I_j, U_k \). Then the self-inductance of the \( j \) windings and the functional relationship between the mutual inductance of the \( j \) windings and other windings and the rotor position angle \( \theta_r \) are obtained by the following formula.

\[ L_{j}^{app}(\theta_r) = \frac{1}{\omega} \sqrt{\left( \frac{U_j(\theta_r)}{I_j(\theta_r)} \right)^2 - (R_j)^2} \]  
\[ L_{kj}^{app}(\theta_r) = \frac{1}{\omega} \frac{U_j(\theta_r)}{I_j(\theta_r)} \]  

The rotor position angle \( \theta_r \) is measured by a fractional disk fixed to the shaft. In order to overcome the measurement error caused by factors such as the eccentricity of the air gap of the motor, the measurement range is 360°mechanical angle of the entire circumference. According to energy perturbation method, parameters are calculated using equation (13-19).

(1) The self-inductance and interphase mutual inductance of power winding are approximately a constant value. They generate pulsations related to the rotor position, this pulsation period is related to the winding pole number and rotor pole number. The pulse is as small as expected.

(2) The self-inductance and mutual inductance parameters of the control winding are similar to those of the power winding, except that the pulsation period is inversely proportional to the pole number of the winding, which is two times of the pulse of the power winding inductance.
(3) The interaction between the power windings and the control windings is approximately sinusoidal with the rotor position angle. The period of change is the equivalent pole $2(p_r + p_c)$ pole. For electromechanical energy conversion, mutual inductance should be greater.

**Figure 3.** Calculation and measurement of self and mutual inductance of power winding.

**Figure 4.** Self-inductance and mutual inductance of control windings.

**Figure 5.** Mutual inductance parameters between power and control windings.
5. Conclusion
In this paper, the inductance parameters of BDFM are calculated by means of energy perturbation method. The experimental prototype was developed and the inductance parameters were measured. The experimental results further verified the correctness of the theoretical analysis.

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