Baryon Spin and Magnetic Moments in Relativistic Chiral Quark Models

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Abstract

The spin and flavor fractions of constituent quarks in the baryon octet are obtained from their lowest order chiral fluctuations involving Goldstone bosons. SU(3) breaking suggested by the mass difference between the strange and up, down quarks is included, as are relativistic effects by means of a light-cone quark model for the proton. Magnetic moments are analyzed and compared with the Karl-Sehgal formulas.

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I. INTRODUCTION

The nonrelativistic quark model (NQM) explains at least qualitatively many of the strong, electromagnetic and weak properties of the nucleon and other baryons in terms of three valence quarks whose dynamics is motivated by quantum chromodynamics (QCD), the gauge field theory of the strong interaction. The effective degrees of freedom at low energies are dressed or constituent quarks along with Goldstone bosons which are expected to emerge in the spontaneous chiral symmetry breakdown of QCD.

At energy-momentum scales below $\Lambda_{QCD}$ chiral perturbation theory \cite{1} allows incorporating systematically the chiral dynamics of QCD. Chiral quark models \cite{2}, which explicitly include valence quarks as effective degrees of freedom and their chiral fluctuations, apply to scales from $\Lambda_{QCD}$ up to a (presumed) chiral symmetry breaking scale $4\pi f_\pi \approx 1169$ MeV for $f_\pi = 93$ MeV. Other degrees of freedom, such as gluons, are integrated out. Clearly, such chiral quark models are a drastic truncation of the full gauge field theory with additional assumptions about confining potentials. While the NQM description of baryon states has not been derived from first principles of QCD, solutions of Schwinger-Dyson equations for light quarks with approximations for confinement lead to a momentum dependent quark mass $m_q(p^2)$. Such dynamical quarks become constituent quarks when $m_q(p^2)$ is approximated by the constant $m_q(0)$. The nonrelativistic broken SU(6) spin-flavor symmetry of the NQM has been linked with the large $N_c$ limit, \cite{3} where $N_c$ is the number of colors. The symmetry structure of the baryon sector of QCD is constrained by the condition that pion-baryon scattering amplitudes remain finite as $N_c \to \infty$, and the approximate NQM spin-flavor structure of the S-wave baryons with small total spin in the [56] multiplet of SU(6) can be understood as a consequence of large $N_c$.

Chiral (off-mass-shell, space-like) fluctuations of valence quarks inside hadrons, $q_{\uparrow,\downarrow} \to q_{\uparrow,\downarrow} + (q\bar{q})_0$, into pseudoscalar mesons, $(q\bar{q})_0$, of the SU(3) flavor octet of $0^-$ Goldstone bosons, were first applied to the spin problem of the proton in ref. \cite{4}. It was shown that chiral dynamics can help one understand not only the reduction of the proton spin carried
by the valence quarks from $\Delta \Sigma = 1$ in the NQM to the experimental value of about $1/3$, but also the reduction of the axial vector coupling constant $g_A^{(3)}$ from the NQM value $5/3$ to about $5/4$. In addition, the violation of the Gottfried sum rule [5] which signals an isospin asymmetric quark sea in the proton became plausible. The SU(3) symmetric chiral quark model explains several spin and sea quark observables of the proton, but not all of them. The data [6,7] call for SU(3) breaking because some of the spin fractions such as $\Delta_3/\Delta_8 = 5/3$ and the weak axial vector coupling constant of the nucleon, $g_A^{(3)} = F + D$, still disagree with experiments in the SU(3) symmetric case. In [8,9] the effects of SU(3) breaking were built into chiral quark models and shown to lead to a remarkable improvement of the spin and quark sea observables in comparison with the data. It was also shown [8] that the $\eta'$ meson gives a negligible contribution to the spin fractions of the nucleon not only because of its large mass but also due to the small singlet chiral coupling constant. We therefore ignore it in the following. In Sect. II we describe some of the SU(3) breaking formalism.

A reduction of the axial charge $g_A^{(3)}$ is known to come from relativistic effects as well. Therefore, it is an objective here to include relativistic effects along with chiral fluctuations for the proton. Since chiral fluctuations in [4,8,9] are based on the NQM spin fractions of the proton which they improve, they still contain nonrelativistic aspects which we replace here by their relativistic analogs obtained from light cone quark models. Such relativistic quark models significantly improve many predictions of the NQM of which the nucleon weak axial charge is the best known example. The relevant formalism is described in Sect. III.

We also calculate the spin and flavor fractions of several hyperons including chiral fluctuations and relativistic effects in Sect. II because from their spin fractions one can immediately obtain estimates for their magnetic moments. In Sect. IV we compare these magnetic moment values that include chiral fluctuations with the Karl-Sehgal formulas involving the corresponding proton spin fractions. Such a comparison sheds light on the latter’s validity. In Sect. V we discuss numerical results more comprehensively. The paper concludes with a summary in Sect. VI.
II. SPIN FRACTIONS OF BARYONS

If the spontaneous chiral symmetry breakdown in the infrared regime of QCD is governed by chiral $SU(3)_L \times SU(3)_R$ transformations then the effective interaction between the octet of Goldstone boson fields $\Phi_i$ and quarks is a pseudoscalar flavor scalar

$$L_{int} = -\frac{g_A}{2f_\pi} \sum_{i=1}^{8} \bar{q}\gamma_\mu \gamma_5 \lambda_i \Phi_i q$$

which flips the polarization of quarks: $q_\downarrow \rightarrow q_\uparrow + GB$, etc. In Eq. (1), $\lambda_i$, ($i = 1, 2, ..., 8$) are the Gell-Mann SU(3) flavor matrices, and $g_A$ is the dimensionless axial vector quark coupling constant that is taken to be 1 here, while

$$g_A^{(3)} = \Delta u - \Delta d = \Delta_3 = F + D = \left(\frac{G_A}{G_V}\right)_{n\rightarrow p},$$

is the isotriplet axial vector coupling constant of the weak decay of the neutron, and $\Delta u$, $\Delta d$ and $\Delta s$ stand for the fraction of proton spin carried by the u, d and s quarks, respectively. They are defined by the matrix elements of the singlet, triplet, octet axial vector currents, $A_\mu^{(i)}$ for $i=0,3,8$ of the nucleon state at zero momentum transfer. Similar axial vector matrix elements for the hyperons define their axial charges. It is also common to define the hypercharge spin fraction $\Delta_8$ and the total proton spin $2S_z = \Delta \Sigma$ in the infinite momentum frame as

$$\Delta_8 = \Delta u + \Delta d - 2\Delta s = 3F - D,$$

$$\Delta \Sigma = \Delta u + \Delta d + \Delta s.$$  

The success of hadronic mass relations suggests that a chiral interaction which breaks the SU(3) flavor symmetry also be governed by the flavor generator $\lambda_8$, as it is expected to originate from the mass difference between the strange and up and down quarks (and the corresponding mass differences of the Goldstone bosons).

Writing only the flavor dependence of these interactions we therefore extend the SU(3) symmetric Eq. (1) to the standard form in [8],

$$L_{int} = \frac{g_8}{\sqrt{2}} \sum_{i=1}^{8} \bar{q}(1 + \epsilon \lambda_8) \lambda_i \Phi_i q,$$
\[
\frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i \Phi_i = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\
-\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^- & K^0 \\
K^- & K^0 & -\frac{2}{\sqrt{6}} \eta
\end{pmatrix}.
\] (5)

Here \( g_8^2 := a \sim f_{\pi NN}^2/4\pi \approx 0.08 \) where \( f_{\pi NN} := g_{\pi NN} m_\pi / 2m_N \) denotes the pseudovector \( \pi N \) coupling constant and \( g_{\pi NN} \) the pseudoscalar one. The latter can be related to Eq. (4) via the Goldberger-Treiman relation \( g_{\pi NN} f_\pi = g_A^{(3)} m_N \). Despite the nonperturbative nature of the chiral symmetry breakdown the interaction between quarks and Goldstone bosons is small enough for a perturbative expansion in \( g_8 \) to apply. Note also that \( \epsilon \) is the SU(3) breaking parameter which was found to be small, \( \approx 0.2 \) \( \bar{8} \), in line with the small constituent quark mass ratio \( m_q/m_s \approx 0.5 \) to 0.6.

From Eq. (4) the following transition probabilities \( P(u_\uparrow \rightarrow \pi^+ + d_\downarrow),... \) for chiral fluctuations of quarks can be organized as coefficients in the symbolic reactions:

\[
\begin{align*}
 u_\uparrow & \rightarrow a(1 + \frac{\epsilon}{\sqrt{3}})^2(\pi^+ + d_\downarrow) + a(1 + \frac{\epsilon}{\sqrt{3}})^2\frac{1}{6}(\eta + u_\downarrow) + a(1 + \frac{\epsilon}{\sqrt{3}})^2\frac{1}{2}(\pi^0 + u_\downarrow) + a(1 - 2\epsilon)^2(K^+ + s_\downarrow), \\
d_\uparrow & \rightarrow a(-1 - \frac{\epsilon}{\sqrt{3}})^2(\pi^- + u_\downarrow) + a(1 + \frac{\epsilon}{\sqrt{3}})^2\frac{1}{6}(\eta + d_\downarrow) + a(-1 - \frac{\epsilon}{\sqrt{3}})^2\frac{1}{2}(\pi^0 + d_\downarrow) + a(1 - 2\epsilon)^2(K^0 + s_\downarrow), \\
s_\uparrow & \rightarrow a(1 - 2\epsilon)^2\frac{2}{3}(\eta + s_\downarrow) + a(-1 + 2\epsilon)^2(K^- + u_\downarrow) + a(-1 + 2\epsilon)^2(K^0 + d_\downarrow),
\end{align*}
\] (6)

and similar ones for the other quark polarization.

From the \( u \) and \( d \) quark lines in Eq. (6) the total meson emission probability \( P \) of the proton is given to first order in the Goldstone fluctuations by

\[
P = a\left[\frac{5}{3}(1 + \frac{\epsilon}{\sqrt{3}})^2 + (1 - 2\epsilon)^2\right],
\] (7)

while the total strange quark probability

\[
P_s = \frac{8}{3} a(1 - \frac{2\epsilon}{\sqrt{3}})^2
\] (8)
can be read off the s quark line in Eq. 6.

One of the main tools for incorporating chiral fluctuations in the NQM’s valence quark spin fractions \( \Delta u_v = 4/3, \quad \Delta d_v = -1/3, \quad \Delta s_v = 0 \) is the proton’s probability composition expression [4,8,9]

\[
(1 - P)\left( \frac{5}{3} \hat{u}_\uparrow + \frac{1}{3} \hat{u}_\downarrow + \frac{2}{3} \hat{d}_\uparrow + \frac{5}{3} P(u_\uparrow) + \frac{1}{3} P(u_\downarrow) + \frac{1}{3} P(d_\uparrow) + \frac{2}{3} P(d_\downarrow) \right). \tag{9}
\]

It follows from its SU(6) spin-flavor wave function. For those other baryon NQM spin-flavor wave functions that can be obtained from the nucleon by permutations of quarks, such as

\[|n\rangle = -|p(u \leftrightarrow d)\rangle, \quad |\Sigma^+\rangle = |p(d \rightarrow s)\rangle, \quad |\Sigma^-\rangle = -|n(u \rightarrow s)\rangle, \quad |\Xi^-\rangle = -|p(u \rightarrow s)\rangle, \quad |\Xi^0\rangle = |n(d \rightarrow s)\rangle, \tag{10}\]

the corresponding composition law becomes

\[
(1 - P)(\frac{5}{3} \hat{u}_\uparrow + \frac{1}{3} \hat{u}_\downarrow + (1 - P_s)(\frac{1}{3} \hat{s}_\uparrow + \frac{2}{3} \hat{s}_\downarrow) + \frac{5}{3} P(u_\uparrow) + \frac{1}{3} P(u_\downarrow) + \frac{1}{3} P(s_\uparrow) + \frac{2}{3} P(s_\downarrow)) \tag{11}
\]

for the \( \Sigma^+ \),

\[
(1 - P)(\frac{2}{3} \hat{d}_\uparrow + (1 - P_s)(\frac{5}{3} \hat{s}_\uparrow + \frac{1}{3} \hat{s}_\downarrow) + \frac{1}{3} P(d_\uparrow) + \frac{1}{3} P(d_\downarrow) + \frac{5}{3} P(s_\uparrow) + \frac{1}{3} P(s_\downarrow)) \tag{12}
\]

for the \( \Xi^- \), etc. Moreover, since the **antiquarks from Goldstone bosons are unpolarized**, which is a major consequence of chiral dynamics and supported by the data, we use \( \bar{u}_\uparrow = \bar{u}_\downarrow \) in the spin fractions \( \Delta u = u_\uparrow - u_\downarrow + \bar{u}_\uparrow - \bar{u}_\downarrow \), etc. and \( \Delta s = \Delta s_{\text{sea}} \), \( \Delta \bar{u} = \Delta \bar{d} = \Delta \bar{s} = 0 \). Small antiquark polarizations are consistent with the most recent SMC data [7]. The NQM valence quark spin fractions in conjunction with the probabilities displayed in Eq. 8 and Eq. 9 to first order in the chiral fluctuations then yield the following spin fractions for the proton [8]

\[
\Delta u_P = u_\uparrow - u_\downarrow = \frac{4}{3}(1 - P) - \frac{5}{9} a(1 + \frac{\epsilon}{\sqrt{3}})^2, \tag{13}
\]

\[
\Delta d_P = -\frac{1}{3}(1 - P) - \frac{10}{9} a(1 + \frac{\epsilon}{\sqrt{3}})^2, \tag{14}
\]
\[
\Delta s_P = -a(1 - \frac{2\epsilon}{\sqrt{3}})^2.
\] (15)

and the relevant hyperons

\[
\Delta u_{\Sigma^+} = \frac{4}{3}(1 - P) - \frac{8}{9}a(1 + \frac{\epsilon}{\sqrt{3}})^2 + \frac{1}{3}a(1 - \frac{2\epsilon}{\sqrt{3}})^2,
\] (16)

\[
\Delta d_{\Sigma^+} = -\frac{4}{3}a(1 + \frac{\epsilon}{\sqrt{3}})^2 + \frac{1}{3}a(1 - \frac{2\epsilon}{\sqrt{3}})^2,
\] (17)

\[
\Delta s_{\Sigma^+} = \frac{1}{3}(1 - P_s) - \frac{10}{9}a(1 - \frac{2\epsilon}{\sqrt{3}})^2,
\] (18)

\[
\Delta u_{\Xi^-} = -\frac{4}{3}a(1 + \frac{\epsilon}{\sqrt{3}})^2 + \frac{1}{3}a(1 - \frac{2\epsilon}{\sqrt{3}})^2,
\] (19)

\[
\Delta d_{\Xi^-} = \frac{1}{3}(1 - P) + \frac{2}{9}a(1 + \frac{\epsilon}{\sqrt{3}})^2 - \frac{4}{3}a(1 - \frac{2\epsilon}{\sqrt{3}})^2,
\] (20)

\[
\Delta s_{\Xi^-} = \frac{4}{3}(1 - P_s) - \frac{5}{9}a(1 - \frac{2\epsilon}{\sqrt{3}})^2.
\] (21)

The identities \(\Delta u_{\Sigma^-} = \Delta d_{\Sigma^+}, \Delta d_{\Sigma^-} = \Delta u_{\Sigma^+}\) and \(\Delta u_{\Xi^0} = \Delta d_{\Xi^-}, \Delta d_{\Xi^0} = \Delta u_{\Xi^-}\) follow from quark permutations \(u \leftrightarrow d\), while the \(\Delta s_B\) stay the same. Although these hyperon spin fractions are not measured they will serve us as a diagnostic tool in a study of the Karl-Sehgal formulas for magnetic moments in Sect. IV. We now turn our attention to another major ingredient that is still missing in the spin fractions, viz. relativistic effects.

III. SPIN-FLAVOR FRACTIONS WITH RELATIVISTIC EFFECTS

Since quarks were first detected as pointlike particles at SLAC late in the 1960s, spin fractions are measured in deep inelastic scattering (DIS) from the proton at high energy and momentum. In the Bjorken limit, where \(0 \leq x = -q^2/2pq \leq 1\), the relevant hadronic tensor is dominated by contributions from the tangent plane to the light cone. Thus, the appropriate form of relativistic dynamics is Dirac’s light cone or front form rather than
the more usual instant form \([\text{[11]}]\). The front form is obtained from the instant form in the infinite momentum limit, and this amounts to a change of momentum variables to
\[
p^+ = p_0 + p_z, \quad p^- = p_0 - p_z \quad \text{[11]}
\]
The longitudinal quark fractions are defined as
\[
x_i = \frac{p_i^+}{P^+}
\]
with the total proton momentum \(P^+ = \sum_i p_i^+\) so that \(\sum_i x_i = 1\). The constituent quark model has been formulated on the light cone in these variables by many authors \([12–15]\).

Such LCQM’s include kinematic boosts at the expense of interaction dependent angular momentum operators. Free Melosh rotations are of central importance in the LCQM’s for the construction of relativistic many-body spin-flavor wave functions for hadrons from those of the NQM. In contrast, chiral bag models treat only the interacting quark relativistically, thereby violating translation and Lorentz invariance.

For a three-quark bound state the relative four-momentum variables are the space-like Jacobi momenta in which the kinematic invariants \(x_i\) play the role of masses. For example, \(q_3\) is the relative quark momentum between the up quarks of the proton in the uds-basis and \(Q_3\) between the down quark and the up quark pair, so that for the + and \(\perp = (x, y)\) components
\[
q_3 = \frac{x_1 p_2 - x_2 p_1}{x_1 + x_2}, \quad Q_3 = (x_1 + x_2)p_3 - x_3(p_1 + p_2), \quad (22)
\]
etc. In light front dynamics the total momentum motion rigorously separates from the internal motion. Therefore, the internal baryon wave function \(\psi(x_i, q_3, Q_3, \lambda_i)\) does not change under kinematic Lorentz transformations or translations. Thus, if the wave function is known in the baryon rest frame it is known everywhere.

In these circumstances the proton composition law of Eq. \([9]\) will be modified as follows in a relativistic quark model based on light-front dynamics
\[
(1 - P)(u_\uparrow^0 \hat{u}_\uparrow + u_\downarrow^0 \hat{u}_\downarrow + d_\uparrow^0 \hat{d}_\uparrow + d_\downarrow^0 \hat{d}_\downarrow) + u_\uparrow^0 P(u_\uparrow) + u_\downarrow^0 P(u_\downarrow) + d_\uparrow^0 P(d_\uparrow) + d_\downarrow^0 P(d_\downarrow), \quad (23)
\]
involving the polarized quark-parton probabilities \(q_\lambda^0\) in the LCQM \([13]\) that are obtained from the standard quark-parton probability densities
\[
q_\lambda(x) = \sum_{\lambda_j \neq \lambda_i} \int \frac{dx_1 dx_2 d^2 \vec{q}_{3\perp} d^2 \vec{Q}_{3\perp}}{(16\pi^3)^2} \delta(x_i - x) |\psi_N(x_j, \vec{q}_{3\perp}, \vec{Q}_{3\perp}, \lambda_j)|^2 \quad (24)
\]
by integrating over Bjorken $x$. Proceeding as in Sect. II, Eqs. 1, 23 yield the proton spin fractions

$$\Delta u_P = \Delta u^0 (1 - P) - \left( \frac{2}{3} \Delta d^0 + \Delta u^0 \right) a(1 + \frac{\epsilon}{\sqrt{3}})^2, \quad (25)$$

$$\Delta d_P = \Delta d^0 (1 - P) - \left( \frac{2}{3} \Delta d^0 + \Delta u^0 \right) a(1 + \frac{\epsilon}{\sqrt{3}})^2, \quad (26)$$

$$\Delta s_P = -(\Delta u^0 + \Delta d^0) a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad (27)$$

where the $\Delta q^0 = q_i^0 - q_s^0$ contain the probabilities of Eq.23. When $\Delta u^0, \Delta d^0$ are replaced by the NQM spin fractions, Eqs. 25 to 27 reduce to Eqs. 13 to 15. If one ignores the difference between relativistic effects for the u,d quarks and the s quark, spin fractions for hyperons can be obtained by the relevant quark permutations from the proton or neutron,

$$\Delta u_{\Sigma^+} = \Delta u^0 (1 - P) - \frac{2}{3} \Delta u^0 a(1 + \frac{\epsilon}{\sqrt{3}})^2 - \Delta d^0 a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad (28)$$

$$\Delta d_{\Sigma^+} = -\Delta u^0 a(1 + \frac{\epsilon}{\sqrt{3}})^2 - \Delta d^0 a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad (29)$$

$$\Delta s_{\Sigma^+} = \Delta d^0 (1 - P_s) - \left( \frac{2}{3} \Delta d^0 + \Delta u^0 \right) a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad (30)$$

$$\Delta u_{\Xi^-} = -\Delta d^0 a(1 + \frac{\epsilon}{\sqrt{3}})^2 - \Delta u^0 a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad (31)$$

$$\Delta d_{\Xi^-} = \Delta d^0 (1 - P) - \frac{2}{3} \Delta d^0 a(1 + \frac{\epsilon}{\sqrt{3}})^2 - \Delta u^0 a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad (32)$$

$$\Delta s_{\Xi^-} = \Delta u^0 (1 - P_s) - \left( \frac{2}{3} \Delta u^0 + \Delta d^0 \right) a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad (33)$$

and for the other hyperons by the appropriate quark permutation. The antiquark fractions in 8 stay unchanged for the proton; for the hyperons they are given by

$$\bar{u}_{\Sigma^+} = \frac{2a}{9} [4(1 + \frac{\epsilon}{\sqrt{3}})^2 + 5(1 - \frac{2\epsilon}{\sqrt{3}})^2], \quad \bar{d}_{\Sigma^+} = \frac{10a}{9} [2(1 + \frac{\epsilon}{\sqrt{3}})^2 + (1 - \frac{2\epsilon}{\sqrt{3}})^2],$$
\[ \bar{s}_{\Sigma^+} = \frac{2a}{9}(1 - \frac{\epsilon}{\sqrt{3}})^2 + \frac{22}{9}a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad \bar{s}_{\Xi^-} = \frac{a}{9}(1 + \frac{\epsilon}{\sqrt{3}})^2 + \frac{17}{9}a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \]
\[ \bar{u}_{\Xi^-} = \frac{10}{9}a(1 + \frac{\epsilon}{\sqrt{3}})^2 + \frac{20}{9}(1 - \frac{2\epsilon}{\sqrt{3}})^2, \quad \bar{d}_{\Xi^-} = \frac{4}{9}a(1 + \frac{\epsilon}{\sqrt{3}})^2 + \frac{20}{9}a(1 - \frac{2\epsilon}{\sqrt{3}})^2, \]

(34)

and similar expressions for the other hyperons by applying the appropriate quark permutations.

Finally, let us turn to the $\eta$ meson case and its recent problems. The $\eta$ meson arises as the octet Goldstone boson when the chiral $SU(3)_L \times SU(3)_R$ symmetry is spontaneously broken. Predictions from PCAC are not in good agreement with experiments, e. g. its octet Goldberger-Treiman relation is violated because it implies a fairly large $\eta NN$ coupling constant which disagrees with the much smaller value extracted from analyses of both $p\bar{p}$ collisions [16] and recent precision data from MAMI [17] on $\eta$ photoproduction off the proton at threshold. Corrections from chiral perturbation theory are of order 30% [18] and much too small to help one understand the problem of the suppressed $\eta NN$ coupling better. This conflict with the data can be avoided if the $\eta$ meson couples only to the strange, but not the up and down, quarks and takes on features of a strange Goldstone boson [19]. This amounts to striking the $\eta$ meson from the up and down reactions in Eq. 6 so that the factor 5/3 in P changes to 3/2, \( \frac{2}{3}\Delta u^0 + \Delta d^0 \) becomes \( \frac{1}{2}\Delta u^0 + \Delta d^0 \), and \( \frac{2}{3}\Delta d^0 + \Delta u^0 \) goes to \( \frac{1}{2}\Delta d^0 + \Delta u^0 \) in the expressions for the spin fractions. This case is labeled $\eta^{(s)}$ in the Tables 1 to 7, while the standard octet case is labeled $\eta^{(8)}$; it is included here not to obtain a better fit but because it may turn out to be more realistic, as it avoids a conflict with the $g_{\eta NN}$ data. In this case, $\bar{u}/\bar{d}$ is lowered from the SU(3) symmetric value 3/4 to 7/11. Note that the experimental value 0.51±0.04(stat.)±0.05(syst.) is at x=0.18 [20] and not summed over Bjorken x.

IV. MAGNETIC MOMENTS OF BARYONS

In the nonrelativistic quark model with three-valence quark wave functions for the baryon octet the magnetic moments contain the NQM spin fractions $\Delta u =4/3$, $\Delta d =-1/3$, $\Delta s =0$, $\Delta l =0$, $\Delta s =0$. The magnetic moments are given by:

\[ m_1 = \frac{1}{\sqrt{2}} \left( \frac{1}{3} \Delta u + \frac{1}{3} \Delta d + \Delta s \right), \quad m_2 = \frac{1}{\sqrt{6}} \left( \frac{1}{3} \Delta u - \frac{1}{3} \Delta d + \Delta s \right), \quad m_3 = \frac{1}{\sqrt{3}} \Delta d, \]

(35)

Finally, the magnetic moments of the hyperons are calculated using the appropriate quark permutations.
e.g. $\mu(p) = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d$, etc. For other octet baryon flavor wave functions that follow from the proton or neutron by the quark permutations of Eq. [10], the simple NQM expressions for the nucleon’s magnetic moments suggest the validity of the Karl-Sehgal equations [21], called K-S below,

$$
\mu(p) = \mu_u \Delta u + \mu_d \Delta d + \mu_s \Delta s, \quad \mu(n) = \mu_u \Delta d + \mu_d \Delta u + \mu_s \Delta s,
$$

$$
\mu(\Sigma^+) = \mu_u \Delta u + \mu_d \Delta s + \mu_s \Delta d, \quad \mu(\Sigma^-) = \mu_u \Delta s + \mu_d \Delta u + \mu_s \Delta d,
$$

$$
\mu(\Xi^-) = \mu_s \Delta s + \mu_d \Delta d + \mu_u \Delta u, \quad \mu(\Xi^0) = \mu_u \Delta d + \mu_d \Delta s + \mu_s \Delta u,
$$

which are SU(3) symmetric when effective quark magnetic moments are chosen in accord with the quark charge ratios, $-2\mu_d = \mu_u$, $\mu_d = \mu_s$. Let us study their validity when chiral fluctuations are included by comparing with the appropriate additive baryon magnetic moments

$$
\mu(B) = \mu_u \Delta u_B + \mu_d \Delta d_B + \mu_s \Delta s_B
$$

with $\Delta q_B$ from Eqs. [16]...[21]. To this end we write linear expressions for $\Delta q_B = c_1 \Delta u_p + c_2 \Delta d_p + c_3 \Delta s_p$ in terms of the $\Delta q_B$ of the proton in Eqs. [13][14][15] and obtain

$$
\Delta u_{\Sigma^+} = \frac{16}{15} \Delta u_p + \frac{4}{15} \Delta d_p - \frac{1}{3} \Delta s_p, \quad \Delta d_{\Sigma^+} = \frac{4}{15} \Delta u_p + \frac{16}{15} \Delta d_p - \frac{1}{3} \Delta s_p,
$$

$$
\Delta s_{\Sigma^+} = -\frac{8}{9} \Delta u_p - \frac{8}{45} \Delta d_p + \frac{9}{9} \Delta s_p, \quad \Delta s_{\Sigma^-} = -\frac{8}{9} \Delta u_p + \frac{8}{45} \Delta d_p + \frac{9}{9} \Delta s_p,
$$

$$
\Delta u_{\Xi^-} = \frac{4}{15} \Delta u_p + \frac{16}{15} \Delta d_p - \frac{4}{3} \Delta s_p, \quad \Delta d_{\Xi^-} = \frac{16}{15} \Delta u_p + \frac{4}{15} \Delta d_p - \frac{4}{3} \Delta s_p,
$$

$$
\Delta s_{\Xi^-} = \frac{4}{9} \Delta u_p - \frac{4}{15} \Delta d_p + \frac{25}{9} \Delta s_p, \quad \Delta s_{\Xi^0} = \frac{4}{9} \Delta u_p - \frac{20}{15} \Delta d_p + \frac{25}{9} \Delta s_p,
$$

$$
\Delta u_{\Xi^0} = -\frac{4}{15} \Delta u_p - \frac{1}{15} \Delta d_p + \frac{4}{3} \Delta s_p, \quad \Delta d_{\Xi^0} = -\frac{1}{15} \Delta u_p - \frac{4}{15} \Delta d_p + \frac{4}{3} \Delta s_p.
$$

Clearly, all coefficients are independent of $a$ and $\epsilon$, and the K-S relations are not satisfied because chiral fluctuations break the SU(3) flavor symmetry so that, e.g., the total chiral probabilities $P \neq P_s$, etc. However, setting

$$
\Delta u_{\Sigma^+} = \Delta u_p, \quad \Delta d_{\Sigma^+} = \Delta s_p, \quad \Delta s_{\Sigma^+} = \Delta d_p,
$$
\[ \Delta u_{\Sigma^-} = \Delta s_P, \quad \Delta d_{\Sigma^-} = \Delta u_P, \quad \Delta s_{\Sigma^-} = \Delta d_P, \]
\[ \Delta d_{\Xi^-} = \Delta d_P, \quad \Delta u_{\Xi^-} = \Delta s_P, \quad \Delta u_{\Xi^-} = \Delta s_P, \]
\[ \Delta u_{\Xi^0} = \Delta d_P, \quad \Delta d_{\Xi^0} = \Delta s_P, \quad \Delta s_{\Xi^0} = \Delta u_P, \]

(38)

According to the K-S relations in conjunction with Eq. 36, and solving for \( \Delta s_P \) yields just one constraint
\[ \Delta s_P = \frac{[\Delta u_P + 4\Delta d_P]}{5} \]
which is small, \( O(a) \), but too large by a factor
\[ \frac{(1 + \epsilon/\sqrt{3})^2}{(1 - 2\epsilon/\sqrt{3})^2} \approx \frac{15}{4} \]
for \( \epsilon = 1/3 \) compared to the correct \( \Delta s_P \).

These linear relations can be generalized to relativistic quark models. The coefficients of such generalized linear relations then depend only on \( \Delta u^{0}, \Delta d^{0} \) of the LCQM. Again, they all lead to a single constraint for \( \Delta s_P \) which differs from the correct \( \Delta s_P \) by the same \( \epsilon \) dependent factor.

Next we show that the K-S relations are no longer valid when relativistic effects are included which break the SU(3) flavor symmetry also, but in different ways. In [14] the nucleon magnetic moments \( \mu(p) = 2.80 \) n.m., \( \mu(n) = -1.73 \) n.m. for quark mass \( m_u = m_d = m_q = 0.33 \) GeV and harmonic oscillator parameter \( \alpha = 0.32 \) GeV were obtained in the LCQM using Dirac magnetic moments for the quarks. The valence quark spin fractions of the LCQM are \( \Delta u = 0.96 \) and \( \Delta d = -0.24 \) for this case so that \( g_{A}^{(3)} = 1.2 \). The K-S formulas yield instead
\[ \mu(p) = (2\Delta u - \Delta d) \frac{m_N}{3m_q} = 2.05 \text{ n.m.}, \quad \mu(n) = (2\Delta d - \Delta u) \frac{m_N}{3m_q} = -1.36 \text{ n.m.} \]

(39)

The discrepancy is smaller for the lower quark mass \( m_q = 0.263 \) GeV adopted in the LCQM [15] where \( \mu(p) = 2.81 \) n.m. and \( \mu(n) = -1.66 \) n.m. are calculated, while \( \Delta u = 1 \) and \( \Delta d = -1/4 \) of this LCQM yield the K-S magnetic moments \( \mu(p) = 2.68 \) n.m., \( \mu(n) = -1.78 \) n.m. The small difference between these values and the magnetic moments [14] above illustrates that in quark models which include relativistic effects to all orders in \( p/m_q \) or \( v/c \), magnetic moments of baryons are much less sensitive to the quark mass than the \( 1/m_q \) dependence of the K-S formulas would suggest, so that additive formulas like Eq. 36 become invalid also. Therefore, it is easy to reach misleading conclusions from fits of K-S formulas [22] to the data. Finally, let us mention that electromagnetic gauge invariance requires many-body quark currents to be present [23] which invalidate additive quark model
results for magnetic moments also. Pion pair and exchange currents have been studied \[24\] and found to be not negligible, but there are significant cancellations of such pion loop contributions to the nucleon magnetic moments so that their net effect is just a 1 to 2% correction.

V. NUMERICAL RESULTS

The simplest relativistic quark models depend on two parameters, the common u, d quark mass \(m_q\) and the harmonic oscillator constant \(\alpha\) of the confinement potential. The proton size determines \(\alpha\) so that \(1/\alpha \sim \langle r^2 \rangle^{1/2}\) up to relativistic corrections. Since the proton magnetic moment is not changed much by chiral fluctuations, in contrast to the axial charge \(g_A^{(3)}\), we determine the range of the parameter \(\alpha\) from a wave function independent relation for \(\mu_p\) as a function of proton radius \[15\].

From Fig.1 in \[15\] we find \(\alpha \approx 3.6/m_N \sim 0.26\text{ GeV} \sim (0.76\text{fm})^{-1}\). For \(\alpha = 0.25\text{ GeV}\), in Table 1 proton spin and flavor fractions for the NQM are compared with those of the chiral NQM and chiral LCQM of \[14\]. The spin fractions of the LCQM are \(\Delta u^0 = 1.1, \Delta d^0 = -0.275, \Delta s^0 = 0\), so that \(g_A^{(3)} = 1.375\), for \(m_q = 0.33\text{ GeV}\). Typically, \(\Delta s\) values are very small, and \(\Delta u, \Delta \Sigma\) and \(g_A^{(3)}\) are substantially reduced from the NQM and LCQM values by chiral fluctuations. A comparison of the relativistic and nonrelativistic results in Table 2 shows that relativistic effects lead to a lower chiral strength \(a\), but the SU(3) flavor breaking parametrized in terms of \(\epsilon\) stays unchanged in the octet \(\eta\) cases, while \(\epsilon\) is reduced for the nonrelativistic \(\eta^{(s)}\) case and enhanced for the relativistic \(\eta^{(s)}\) case. Thus, the effective SU(3) breaking brought about by relativistic effects is sensitive to the role of the \(\eta\) meson in the chiral dynamics.

These spin and flavor fractions are the chiral quark model results of Sects. III and IV that are independent of momentum \(Q^2\) and valid at long distances below the chiral scale \(\Lambda_\chi = 4\pi f_\pi\). In the chiral limit, the axial charges of the nucleon (and the baryon octet), i.e. \(\Delta_3 = g_A^{(3)}\) and \(\Delta_8\), are constants independent of \(Q^2\) because conserved currents have
vanishing anomalous dimension. The singlet axial charge, $\Delta \Sigma$, is not conserved because of the axial anomaly, but its $Q^2$ dependence is rather weak to lowest orders in pQCD. Due to the U(1) anomaly of the singlet axial vector current the axial charge contains a gluon contribution. In the Adler-Bardeen renormalization scheme [26] the $Q^2$ dependent axial charges are given by $a_i(Q^2) = \Delta q_i - \frac{\alpha_s(Q^2)}{2\pi}\Delta g(Q^2)$ for $i=u,d,s$ and $a_0(Q^2) = \Delta \Sigma - n_f\frac{\alpha_s(Q^2)}{2\pi}\Delta g(Q^2)$ for $n_f=3$ flavors. At $Q^2 = 5 \text{ GeV}^2$, the SMC [7] experiment has determined $\Delta g = 1.7 \pm 1.1$ so that $\frac{\alpha_s(Q^2)}{2\pi}\Delta g(Q^2) \approx 0.084 \pm 0.054$. These $Q^2$ dependent axial charges $a_i$ are compared with the data in Table 2. Clearly, the nonrelativistic and relativistic quark model results are in fair agreement with the data, especially for the $\eta^s$ case. Typically $g_A^{(3)}$ for the NQM cases is above the data and for the LCQM below the data, but not by much. Without the momentum dependent gluon contribution the relativistic fits become worse, and nonrelativistic fits better, comparable to those in Table 4. In this sense the gluon contribution supports the relativistic chiral quark models that are expected to contain more of the relevant physics than the chiral NQM. However, the results in Table 2 are not the best fits. In Table 3 results are presented for the LCQM for a smaller value of $\alpha$ that produce the better fits in Table 4. This parameter range is driven by the proton’s axial charge $g_A^{(3)}$. Fitting the axial charge with relativistic effects and chiral fluctuations is more difficult than in the nonrelativistic case, especially without the singlet axial gluon contribution, requiring a low value of $\alpha$ so as to minimize the reduction of $g_A^{(3)}$ by relativistic effects. Thus, even though $g_A^{(3)}$ has the large value 1.375 for the relativistic valence quark model in Tables 1, 2 despite relativistic effects, the axial charge still falls below the observed value when chiral fluctuations are included. The preference of LCQMs for an unrealistically large proton size $\geq 1 \text{ fm}$ should not be taken too literally. Given the simplicity of the models one should not expect better fits than those in Table 2, and it is not surprising, therefore, that much better fits

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1Quark models that do not include chiral dynamics typically lead to lower $m_q$ and smaller proton size, but are misleading because they are inconsistent with chiral perturbation theory.
fits like those in Table 4 push the parameter $\alpha$ to extreme values.

We have also examined a light-cone quark model [13] that includes two harmonic oscillator constants $\alpha^2(1 \pm D)$ in the Gaussian momentum wave function which, for $D>0$, amounts to smaller u-d quark pairs in the proton simulating an attractive spin force between the u, d quark pairs. The value $D=0.37$ generates the full $\Delta(1232)$-N mass splitting. The spin and flavor fractions of the proton do not change by much, nor do the SU(3) flavor breaking parameters $\epsilon$ and $a$, when this quark clustering is included, nor does it help with the axial charge. Therefore, no detailed results are presented.

In Table 5 the magnetic moments are obtained from the additive relation, Eq. [36], by fitting the magnetic moment $\mu_u$ of the up quark while maintaining fixed ratios $\mu_d = -\frac{1}{2}\mu_u$ suggested by the quark charge ratio and $\mu_s = \frac{3}{5}\mu_d$ by the quark mass ratio. As is shown in Table 3, the magnetic moments from Eq. [36] give better fits, mainly due to the $\Xi$, than the K-S relations which the additive formula of Eq. [36] improves by properly taking SU(3) breaking into account. As expected, the relativistic version of the one-body relation in Eq. [36] does not improve the fits of magnetic moments of the NQM version noticeably, so that only one case, the LCQM results, are presented. The fits are better than expected on theoretical grounds discussed in Sect.IV.

The spin fractions for the hyperons in Tables 6 and 7 show changes similar to the proton with chiral fluctuations. In a comparison with the spin fractions at the one-loop level of chiral perturbation theory [18] we note that relativistic effects typically improve the dominant $\Delta q$, but not all three.

VI. SUMMARY AND CONCLUSION

When relativistic effects are included along with chiral dynamics to lowest order, it becomes more difficult for chiral quark models to reproduce the measured value of the axial charge $g_A^{(3)}$ of the nucleon. The range of relativistic quark model parameters giving acceptable fits is pushed towards higher u,d quark mass and larger proton size when chiral
fluctuations are included. Chiral fluctuations that imply unpolarized antiquark fractions give a remarkably successful description of the spin fractions of the proton. The SU(3) breaking is significant. The NQM is able to simulate the missing relativistic effects and the axial anomaly by adjusting the chiral strength $a$ and the SU(3) breaking $\epsilon$, the latter being too small and misleading.

The extension of spin fractions to hyperons allows us to calculate their magnetic moments and improve the Karl-Sehgal formulas. We find that SU(3) breaking via chiral dynamics and relativistic effects invalidates the K-S formulas; and this breakdown casts doubt on them as a useful tool for probing the spin fractions of the proton via hyperon magnetic moments.

Despite being more sophisticated than the NQM, relativistic chiral quark models are not yet able to predict the Bjorken-x dependence of the flavor and spin distributions of baryons. Nonetheless, the successful description of most spin and flavor fractions with just a few parameters is encouraging and suggests further development and study of these models.

\section*{VII. ACKNOWLEDGEMENT}

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Table 1 Quark Spin and Sea Observables of the Proton; $m_q = 0.33$ GeV, $\alpha = 0.25$ GeV

|       | NQM | $a = 0.09$ | $a = 0.1$ | $a = 0.07$ | $a = 0.07$ |
|-------|-----|------------|------------|------------|------------|
|       |     | $\eta^{(8)}$ | $\eta^{(8)}$ | $\eta^{(8)}$ | $\eta^{(8)}$ |
|       | NQM | NQM | LCQM | LCQM | NQM | NQM | LCQM | LCQM |
|       | $\epsilon = 0.32$ | $\epsilon = 0.3$ | $\epsilon = 0.32$ | $\epsilon = 0.34$ | $\epsilon = 0.32$ | $\epsilon = 0.34$ |
| $\Delta u$ | 4/3 | 0.94 | 0.96 | 0.84 | 0.88 |
| $\Delta d$ | -1/3 | -0.39 | -0.41 | -0.31 | -0.32 |
| $\Delta s$ | 0 | -0.04 | -0.04 | -0.02 | -0.02 |
| $\Delta \Sigma$ | 1 | 0.51 | 0.50 | 0.51 | 0.53 |
| $\Delta_3/\Delta_8$ | 5/3 | 2.16 | 2.17 | 2.00 | 2.01 |
| $g_\lambda^{(3)}$ | 5/3 | 1.33 | 1.37 | 1.16 | 1.2 |
| $\mathcal{F}/\mathcal{D}$ | 2/3 | 0.58 | 0.58 | 0.60 | 0.60 |
| $\bar{u}/\bar{d}$ | - | 3/4 | 7/11 | 3/4 | 7/11 |
| $f_3/f_8$ | 1/3 | 0.22 | 0.19 | 0.24 | 0.22 |
| $I_G$ | 1/3 | 0.28 | 0.24 | 0.29 | 0.27 |
Table 2 Quark Spin Observables of the Proton; \(m_q=0.33\) GeV, \(\alpha=0.25\) GeV, with singlet axial gluon contribution

|                | Data   | \(a=0.09\) | \(a=0.10\) | \(a=0.07\) | \(a=0.07\) |
|----------------|--------|------------|------------|------------|------------|
|                | \(\eta^{(8)}\) | \(\eta^{(s)}\) | \(\eta^{(8)}\) | \(\eta^{(s)}\) |
| \(E143 \)  | NQM    | \(\epsilon=0.32\) | NQM        | LCQM       | LCQM       |
| at 3 GeV\(^2\) |        |            |            |            |            |
| \(SMC \)  | \(\epsilon=0.32\) | \(\epsilon=0.3\) | \(\epsilon=0.32\) | \(\epsilon=0.34\) |
| at 5 GeV\(^2\) |        |            |            |            |            |
| \(a_u\)   | 0.84±0.05 | 0.85       | 0.87       | 0.76       | 0.80       |
| \(a_d\)   | -0.43±0.05 | -0.48      | -0.50      | -0.40      | -0.41      |
| \(a_s\)   | -0.08±0.05 | -0.12      | -0.13      | -0.11      | -0.11      |
| \(a_0\)   | 0.30±0.06  | 0.26       | 0.25       | 0.26       | 0.28       |
| \(a_3/a_8\)| 2.09±0.13  | 2.16       | 2.17       | 2.00       | 2.01       |
| \(g_A^{(3)}\) | 1.2573±0.0028 | 1.33    | 1.37       | 1.16       | 1.20       |
| \(\mathcal{F}/\mathcal{D}\) | 0.575±0.016 | 0.58    | 0.58       | 0.60       | 0.60       |
| \(\bar{u}/\bar{d}\) | 0.51±0.09 | 3/4       | 7/11       | 3/4        | 7/11       |
| \(f_3/f_8\) | 0.23±0.05 | 0.22      | 0.19       | 0.24       | 0.22       |
| \(I_G\)   | 0.235±0.026 | 0.28      | 0.24       | 0.29       | 0.27       |
Table 3 Quark Spin and Sea Observables of the Proton; $m_q = 0.33$ GeV, $\alpha = 0.20$ GeV

|       | NQM       | LCQM       | $a = 0.07$ | $a = 0.08$ |
|-------|-----------|------------|------------|------------|
| $\Delta u$ | 4/3       | 1.18       | 0.90       | 0.90       |
| $\Delta d$ | -1/3      | -0.29      | -0.34      | -0.36      |
| $\Delta s$ | 0         | 0          | -0.02      | -0.02      |
| $\Delta \Sigma$ | 1       | 0.88       | 0.54       | 0.52       |
| $\Delta_3/\Delta_8$ | 5/3   | 5/3        | 2.07       | 2.13       |
| $g_A^{(3)}$ | 5/3      | 1.47       | 1.24       | 1.26       |
| $F/D$ | 2/3       | 2/3        | 0.59       | 0.58       |
| $\bar{u}/\bar{d}$ | 3/4 | 7/11       |            |            |
| $f_3/f_8$ | 1/3       | 0.23       | 0.20       |            |
| $I_G$ | 1/3       | 0.29       | 0.25       |            |
Table 4 Quark Spin and Sea Observables of the Proton with singlet axial gluon contribution, $m_q=0.33$ GeV, $\alpha =0.2$ GeV

|               | Data | Data             | $a =0.07$  | $a =0.08$ |
|---------------|------|------------------|------------|-----------|
|               | E143 | SMC              | $\eta^{(8)}$ | $\eta^{(s)}$ |
|               | at 3 GeV$^2$ | at 5 GeV$^2$ | $\epsilon =0.38$ | $\epsilon =0.38$ |
| $a_u$         | 0.84±0.05 | 0.82±0.06 | 0.81 | 0.82 |
| $a_d$         | -0.43±0.05 | -0.44±0.06 | -0.42 | -0.44 |
| $a_s$         | -0.08±0.05 | -0.10±0.06 | -0.10 | -0.11 |
| $a_0$         | 0.30±0.06 | 0.28±0.17 | 0.29 | 0.27 |
| $a_3/a_8$     | 2.09±0.13 | 2.17±0.16 | 2.07 | 2.13 |
| $g_A^{(3)}$   | 1.2573±0.0028 | 1.24 | 1.26 |
| $\mathcal{F}/\mathcal{D}$ | 0.575±0.016 | 0.59 | 0.58 |
| $\bar{u}/\bar{d}$ | 0.51±0.09 | 3/4 | 7/11 |
| $f_3/f_8$     | 0.23±0.05 | 0.23 | 0.20 |
| $I_G$         | 0.235±0.026 | 0.29 | 0.25 |
Table 5  Magnetic Moments of Baryons, $\alpha = 0.25$ GeV, $m_q = 0.33$ GeV

| magnetic moments | Data     | K-S | $\alpha = 0.07$ | $\alpha = 0.07$ |
|------------------|----------|-----|----------------|----------------|
| $\mu_n$ [n.m.]   | PDG [25] |     | $\eta^{(8)}$   | $\eta^{(a)}$   |
|                  |          |     | $\epsilon = 0.32$ | $\epsilon = 0.34$ |
| p                | 2.793    | 2.720 | 2.720          | 2.617          |
| n                | -1.913   | -1.965 | -1.965        | -1.890         |
| $\Sigma^+$       | 2.458±0.01 | 2.563 | 2.590          | 2.484          |
| $\Sigma^-$       | -1.160±0.025 | -0.949 | -0.949        | -0.909         |
| $\Xi^-$          | -0.6507±0.0025 | -0.324 | -0.507        | -0.460         |
| $\Xi^0$          | -1.250±0.014 | -1.496 | -1.444        | -1.361         |
| $\mu_u$ [n.m.]   | input    | 2.7  | 2.7           | 2.5            |
Table 6  Quark Spin-Sea Observables of the $\Sigma^+$; $m_q = 0.33$ GeV, $\alpha = 0.25$ GeV

| Pert. Theory | Chiral | $a = 0.09$ | $a = 0.10$ | $a = 0.07$ | $a = 0.07$ |
|--------------|--------|-----------|-----------|-----------|-----------|
| NQM          | $\eta^{(8)}$ | $\eta^{(s)}$ | $\eta^{(8)}$ | $\eta^{(s)}$ |
| $\epsilon$ | $\epsilon = 0.32$ | $\epsilon = 0.30$ | $\epsilon = 0.32$ | $\epsilon = 0.34$ |
| $\Delta u$ | 0.68$\pm$0.12 | 0.91 | 0.92 | 0.83 | 0.86 |
| $\Delta d$ | 0.05$\pm$0.12 | -0.16 | -0.17 | -0.10 | -0.10 |
| $\Delta s$ | -0.49$\pm$0.09 | -0.34 | -0.34 | -0.28 | -0.28 |
| $\bar{u}$ | - | 0.17 | 0.12 | 0.13 | 0.08 |
| $\bar{d}$ | - | 0.32 | 0.39 | 0.25 | 0.28 |
| $\bar{s}$ | - | 0.03 | 0.10 | 0.02 | 0.06 |

Table 7  Quark Spin-Sea Observables of the $\Xi^-$; $m_q = 0.33$ GeV, $\alpha = 0.25$ GeV

| Pert. Theory | Chiral | $a = 0.09$ | $a = 0.10$ | $a = 0.07$ | $a = 0.07$ |
|--------------|--------|-----------|-----------|-----------|-----------|
| NQM          | $\eta^{(8)}$ | $\eta^{(s)}$ | $\eta^{(8)}$ | $\eta^{(s)}$ |
| $\epsilon$ | $\epsilon = 0.32$ | $\epsilon = 0.30$ | $\epsilon = 0.32$ | $\epsilon = 0.34$ |
| $\Delta u$ | -0.18$\pm$0.14 | -0.006 | -0.01 | -0.004 | -0.001 |
| $\Delta d$ | -0.50$\pm$0.10 | -0.27 | -0.28 | -0.24 | -0.24 |
| $\Delta s$ | 0.83$\pm$0.12 | 1.19 | 1.16 | 1.01 | 1.01 |
| $\bar{u}$ | - | 0.22 | 0.27 | 0.17 | 0.18 |
| $\bar{d}$ | - | 0.08 | 0.13 | 0.06 | 0.08 |
| $\bar{s}$ | - | 0.07 | 0.08 | 0.05 | 0.05 |
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