Theoretical and numerical study for unsteady tube expansion using hemispherical dies

Hazim Khaleel Khalaf1,*, Azal Rifaat Ismail2 and Yusra Abdullah Jasim1
1Department of Mechanical Engineering, Tikrit University, Tikrit, Iraq
2Department of Mechanical Engineering, Kirkuk University, Kirkuk, Iraq

* Corresponding author: hazimkhalil@tu.edu.iq

Abstract: unsteady hemispherical die expansion process was investigated theoretically and numerically, using a 48 mm base diameter die. Brass specimens of 32 mm inner diameter, and 1.5, 2, and 3 mm initial wall thickness were simulated using Coulomb’s friction concept with a coefficient of 0.1. The governing equation was derived, and solved for the perfect plastic condition. Reasonable agreement was found between the theoretical and the finite-element results for the relation between forming stress vs. expansion ratio which was noticed to be less affected by the wall thickness. The wall plastic bending at the die entrance was developed to wall curling and waving for all cases. The three components of strains were also studied using the finite-element method. It was noticed that the circumferential strain is always tensile, whilst the radial strain is compressive at the expanded end but turns to tensile at the die entrance. The complexity of expansion process was reflected clearly through the longitudinal strain distribution, it begins as a compressive strain at the expanded end with increasing compressive strain until it reaches its maximum compressive value at a distance about 5 mm from the expanded end, then vanishes at the die entrance.

Keywords: tube expansion, hemispherical die, unsteady expansion, curved die

1. Introduction
Tube expansion is an important metal forming process, it is widely used in military industries, space ships, oil and gas wells, heat exchangers and many other applications [1]. In the expansion process, a deformation tool is pushed inside the tube. The external diameter of the deformation tool has to be greater than the internal diameter of the tubes to ensure the permanent deformation required [2]. This process may be performed hot or cold depending on many factors. The tube expansion process may be steady, which is when the expansion extends beyond the die base, or unsteady when the expansion is made up to the die base, figure 1. The metal flow and deformation mechanisms in the unsteady...
expansion process are complex and influenced by several factors, including the geometry of the die and the specimen, material properties, friction and lubrication conditions, and the process speed.

There are many studies that have dealt with this process. Karrech A. and Seibi A. [4], applied the theory of thin plates to analyze the stresses in conical dies expansion. Jialing Y. et al [5] studied, experimentally, the absorbed energy through the expansion process. Four conical dies with different cone angle and different wall thickness were used. AA5060 aluminum alloy specimens were tested using MoS$_2$ lubricant. They found good agreement with the results those gained by the FEM analysis.

![Figure 1. (a) Steady expansion. (b) Unsteady expansion [3].](image)

Fischer F. D., et al [6] studied the unsteady expansion for thin tubes. They derived theoretical expressions those involving stresses, strains, and the load required for a certain expansion ratio. A good agreement with those obtained from the FEM analysis was found.

Liu, Y and Qiu, X [7] investigated the conical die expansion by proposing a theoretical model, the proposed model starts from the energy conservation principle. That is, in the steady stage deformation of an expansion tube, the external work done by the compressional force should be dissipated by expansion in circumferential direction, bending in meridian direction, and friction. The steady compressional force predicted by the theoretical analysis was found to be accurate, compared with existing experimental data, and also the corresponding finite element simulations.

Zhu, J. et al [8] characterized the cyclic loading in the tube expansion process. They provide detailed information on the plastic behavior of the offshore pipe material under reversed loading. The study presented a comprehensive investigation for the tube expansion process, an analytical solution is developed to compute the contact force. Numerical simulation of the expansion process for a deep water pipe is carried out. The prediction of the analytical model agrees well with the FE simulation results in regards to the steady-state contact force. Furthermore, the stress/strain evolutions of the tube during the expansion process were investigated.

Yuwono B. P. et al [9] studied the combination of an expansion tube and a deformable rigid tube with axial splitting which was developed as a new mechanism for use as an impact energy absorber. The new formula of analytical calculations has proven to be implemented in impact energy absorber, with a maximum percentage error of 10.52%.

The present study aims to investigate the effect of the initial wall thickness, for a cold expansion process using a circular dies, on:

1- The relation between forming stress and expansion ratio.
2- The distribution of strains in the expanded specimen.
3- The deformation mechanism in specimen wall starting from the die entrance up to the die base.

The study aims also to investigate the relation between the forming stress and the expansion ratio, theoretically and numerically using the FEM analysis.
2. Governing equations

The unsteady cold expansion process is a complicated process due to the many factors that involved in it, such as the specimen geometry, die profile, and friction conditions. Fortunately, many of the important factors can be related to the relation between forming stress, $P/A_0$ and the expansion ratio, $R = (r_f - r_0)/r_0$. The present analysis is based on the penetration of a circular section die into a tubular specimen of an initial mean radius $r_0$, under the effect of a driving force, $P$ up to a depth $\bar{S}$. The expanded end has a final mean radius $r_f$, slightly less than die diameter, $r_d$ as shown in figure 2. The described setting condition is assumed to be under a quasi–static condition. This assumption is accepted since any part of the expanded end has a static equilibrium when it is isolated. Hill [10] and Johnson, W. and Mellor, P. B. [11] used a similar analysis in tube sinking. To perform the present analysis, the following assumptions were made. The wall thickness of the specimen is assumed to be constant, the friction obeys Coulomb’s law with constant coefficient of friction along the contact area, and the distribution of the radial stresses is uniform through the wall thickness, so the plastic bending stresses and shearing stresses through the thickness could be neglected.

Referring to figure 2, an element of radius $r$ from the die longitudinal axis was cut, where $r_0 \leq r \leq r_f$. The element can be assumed as a part of a right inverted cone with base radius $r$, and half cone angle $\alpha$. The cone vertex was along the common longitudinal axis of the die and specimen, and the cone is tangent to the die surface at $r$.

The above imaginary section assumes that the curved part of the element is plane and it is a part of the cone side. Finally a part of the element is cut at a distance $r$ from the longitudinal axis and bounded by an angle $d\theta$ in the vertical plane, as shown in figure 3.

The shown enlarged element of thickness $t$ is affected by the longitudinal stress component, $\sigma_\alpha$, the circumferential stress component, $\sigma_\theta$, the radial stress component, $\sigma_t$, and the shear stress component, $\mu \sigma_t$.

Forces equilibrium in $\sigma_t$ - direction gives:

$$
\sigma_t r \frac{d\theta}{\sin \alpha} - 2\sigma_\theta \frac{d\theta}{2} \cos \frac{\theta}{2} \cos \alpha \ t \ \frac{dr}{\sin \alpha} = 0
$$

Figure 2. The geometry of the die. Figure 3. The stresses on an enlarged element.

Neglecting small quantities, multiplying both sides by $\sin \alpha / r \ dr \ d\theta$ and rearranging yields:

$$
\sigma_t = \frac{t}{r} \cdot \sigma_\alpha \cdot \cos \alpha
$$

Similarly, summation of the forces in $\sigma_\alpha$ - direction gives:

$$
\sigma_\alpha \cdot r \ d\theta \cdot t - (\sigma_\alpha + \sigma_\alpha) (r + dr) d\theta (t + dt) - \mu \cdot \sigma_t \cdot r \ d\theta \cdot \frac{dr}{\sin \alpha} - 2\sigma_\theta \frac{d\theta}{2} \cos \alpha \ t \ \frac{dr}{\sin \alpha} = 0
$$
Dividing both sides of equation (3) by d\(d\theta\), opening brackets, neglecting small terms and treating similar terms, yields:

\[
\frac{d(\sigma_\alpha r)}{dr} + \sigma_\theta t + \mu \sigma_\theta \frac{r}{\sin \alpha} = 0
\]  

(4)

substituting \(\sigma_t\) from equation (2) into equation (4) and treating \(t\) as a constant, equation (4) becomes:

\[
\frac{d(\sigma_\alpha r)}{dr} + \sigma_\theta (1 + \mu \cot \alpha) = 0
\]  

(5)

The variable \(\cot \alpha\) in equation (5) can be expressed in terms of \(r\) as, \(\cot \alpha = \frac{r}{\sqrt{r^2 - r_1^2}}\).

substituting this into equation (5) gives:

\[
\frac{d(\sigma_\alpha r)}{dr} + \sigma_\theta \left(1 + \frac{\mu r}{\sqrt{r^2 - r_1^2}}\right) = 0
\]  

(6)

Equation 6 is the governing equation for the stresses in the curved die tube expansion process. The used analysis is similar to that of Sachs G. and Baldwin, W. M. [11] in analyzing the tube sinking process using conical dies. The solution of equation 6 requires a yield criterion to relate the circumferential stress, \(\sigma_\theta\), to the longitudinal stress, \(\sigma_\alpha\). Von Mises theory was adopted by Sachs and Baldwin [11]. This theory can be formulated for the tube expansion with three normal stresses \(\sigma_\alpha, \sigma_\theta,\) and \(\sigma_t\) and one shear stress \(\mu \sigma_t\) to take the form:

\[
(\sigma_\alpha - \sigma_\theta)^2 + (\sigma_\theta - \sigma_t)^2 + (\sigma_t - \sigma_\alpha)^2 + 6(\mu \sigma_t)^2 = 2Y^2
\]  

(7)

From equation 2 it could be concluded that, for small thickness compared to the instantaneous radius \(r\), the radial stress, \(\sigma_t\) reaches zero. Setting \(\sigma_t = \mu \sigma_t = 0\), equation 7 can be simplified to:

\[
\sigma_\alpha^2 - \sigma_\alpha \sigma_\theta + \sigma_\theta^2 = Y^2
\]  

(8)

Figure 4 shows the yield criterion that identified in equation 8 for both tube sinking and tube expansion. Note that \(\sigma_\theta\) is compressive in tube sinking. Applying Johnson method [10] to simplify equation 8 by replacing von Mises curve in the second quarter by a straight line to give a best approximation, the yield criterion equation will be:

\[
\sigma_\theta - \sigma_\alpha = 1.1 Y \text{ For tube expansion}
\]  

(9)

Substituting equation (9) into the governing equation (6), gives

\[
\frac{d(\sigma_\alpha r)}{dr} + (1.1 Y + \sigma_\alpha) \left(1 + \frac{\mu r}{\sqrt{(r_1^2 - r^2)}}\right) = 0
\]  

(10)

![Figure 4. The relation between the longitudinal and circumferential relative stresses at yielding.](image-url)

Differentiating \(\sigma_\alpha r\) with respect to \(r\) and dividing the result upon \(r\), yields:

\[
\frac{d(\sigma_\alpha)}{dr} + \frac{1}{r} \left(2 + \frac{\mu r}{\sqrt{(r_1^2 - r^2)}}\right) \sigma_\alpha = - \frac{1.1 Y}{r} \left(1 + \frac{\mu r}{\sqrt{(r_1^2 - r^2)}}\right)
\]  

(11)
3. Solving of the differential equation

Equation 11 may be classified as a linear first order differential equation can be solved using the integrating factor [13].

\[ \sigma_\alpha(r) = e^h \int e^{-h} Q(r) \, dr \] (12)

The integrating factor \( h \) and \( Q(r) \) can be expressed as:

\[ h = \int \left( \frac{\mu}{r} + \frac{\mu r}{\sqrt{(r_d^2 - r^2)}} \right) \, dr \quad \text{and} \quad Q(r) = \frac{1}{r} \left( 1.1Y + \frac{1.1Y \mu r}{\sqrt{(r_d^2 - r^2)}} \right) \]

Performing the integration yields:

\[ h = \ln r^2 + \mu \sin^{-1} \frac{r}{r_d} \] (13)

Substituting \( h \) and \( Q(r) \) into equation (12) and integrating, results in the longitudinal stress \( \sigma_\alpha \) for ideal plasticity:

\[ \sigma_\alpha = \frac{1.1Y_o}{r_o^2} \left[ B \sin 2\psi + \frac{\mu B}{2} \cos 2\psi - \frac{r^2}{2} - \frac{r_d^2}{4} \right] + C e^{-\mu \psi} \] (14)

Where \( B = \frac{\mu r_d^2}{2(2^2 + \mu^2)} \) and \( \psi = \sin^{-1} \frac{r}{r_d} \)

The integration constant \( C \) can be determined from the boundary condition that is the longitudinal stress \( \sigma_\alpha \) equals zero at \((r = r_f)\) so:

\[ C = 1.1Y_o e^\mu \psi_f \left[ \frac{r_f^2}{2} + \frac{r_d^2}{4} - B \sin 2\psi_f - \frac{\mu B}{2} \cos 2\psi_f \right] \] (15)

Substituting \( C \) from equation (15) into equation (14) and applying the second boundary condition that is the longitudinal stress reaches its maximum value \( (\sigma_\alpha)|_{\max} \) at \((r = r_o)\) gives:

\[ (\sigma_\alpha)|_{\max} = \frac{1.1Y_o}{r_o^2} \left[ \left( B \sin 2\psi_o + \frac{\mu B}{2} \cos 2\psi_o - \frac{r_o^2}{2} - \frac{r_d^2}{4} \right) + e^{\mu (\psi_f - \psi_o)} \left( \frac{r_f^2}{2} + \frac{r_d^2}{4} - B \sin 2\psi_f - \frac{\mu B}{2} \cos 2\psi_f \right) \right] \] (16)

Where \( \psi_f = \sin^{-1} \frac{r_f}{r_d} \) and \( \psi_o = \sin^{-1} \frac{r_o}{r_d} \)

The required load to perform the expansion process could be determined from the equilibrium state between the resultant of the longitudinal stress at the die entrance and the load \( P \), as in figure 5.
Equilibrium equation is:

\[ P = A_o \cos \phi (\sigma_\alpha)|_{\max} \] (17)

Where \( \cos \phi = r/r_d \) then, by dividing both sides of equation (17) by \( A_o \), forming stress can be written as:

\[ S_y = \frac{P}{A_o} = \cos \phi (\sigma_\alpha)|_{\max} \] (18)
4. Material properties

In order to predict the required load for performing the expansion process, the material adopted for the specimens is 30:70 brass, which was annealed at 650 °C. The mechanical properties for this material are presented in Table 1. The friction factor was adopted to be 0.1, as was suggested by Cockcroft, M. G. and Male, A. T. [14]. For the present study a die of 48 mm base diameter and 20 mm die entrance diameter was selected, as shown in Figure 6. In order to make sure that the simulation of the expansion process would be in the unsteady region, the final diameter $2r_f$ should be less than the die base diameter, $2r_d \leq 48$ mm, so the maximum expansion ratio should be within the range of 30-40%. The internal diameter of the tube is selected to be 32 mm.

| Modulus of elasticity N/mm² | Poisson’s ratio, $\nu$ | Yield stress N/mm² | $\rho$ kg/m³ | $K$ N/mm² |
|-----------------------------|------------------------|--------------------|--------------|-----------|
| $117 \times 10^3$           | 0.34                   | 135                | 8930         | 835       |

5. FEM simulation

The simulation of the expansion process requires the die information, specimen geometry, material properties, and coefficient of friction, in addition to the selection of a suitable element for the deformable material (the specimen), the solid material (the die) and the contact area. Targe 169 element was used to simulate the die. It consists of three nodes which are compatible with those of Contac 172 element. Contac 172 is being used whenever a sliding contact between any two surfaces occurs in 2D geometries. The Plane 183 element was used to simulate the specimen. It consists of four surfaces and eight nodes, each node has two degrees of freedom in the directions x and y. This element was used for axisymmetric geometries in plasticity problems with large deformations. After determining the die and specimen geometries, the material, the friction factor, and the proper elements have been determined, the whole geometry was introduced to ANSYS. Because of the process is axisymmetric, the process was treated as 2D to save time and effort. This treatment was adopted by many previous researchers for conical expansion [1, 15, 16].

6. Simulation of the curved die expansion using ANSYS

The die was pressed inside the tubular specimen to produce an expansion ratio of about 30 to 40%. The expansion process simulation was divided into six stages for more clarification. The forming stress and expansion ratio was measured and the deformation sketch was captured after each stage. Measuring of the longitudinal, circumferential, and radial strains each 5 mm of the last stage (with expansion ratio of 30%), starting from the expanded free end up to an axial distance of 40 mm.
7. Results and discussion

7.1. The expansion mechanism during metal flow

The flow of metal during unsteady expansion is complex and influenced by many factors, such as die geometry, friction conditions, specimen material, and the specimen thickness. Understanding the metal flow mechanism has a major role to analyze and compare the results. Expansion process has three stages which can be summarized as follows:

1- Plastic bending: This stage starts at the die entrance region. The tube wall bends, due to the sudden change in the metal flow direction. This phenomenon is continuous during the expansion process as shown in figure 7. A plastic deformation happens for the free end of the wall, which is due to the die pressure on the internal corners, which causes compressive deformation of the wall end, but this diminishes as the metal slides on the die face.

2- Wall curling: The specimen wall tends to wrap out due to the plastic bending as shown in figure 8.

3- Flow of the tube on the die face: At this stage the wall slides on the external face of the die producing a circumferential strain. This strain was affected by the die geometry where the inner wall in a case of complete contact with the die face as shown in figure 9.

The deformation mechanism of the expansion process is a combination of these stages. The contribution of each stage depends on the die geometry and the tube thickness.

7.2. Deformation mechanism in a hemispherical die

It can be seen in figures 10-12, that all the three different thickness specimens undergo plastic bending at the die entrance region during all stages, in addition to wall wrapping continues to the fourth stage for the 1.5 mm wall specimen, to the fifth stage for the 2 mm wall specimen, and to the sixth stage for the 3 mm wall specimen. Curling that is accompanying to the wrapping is also noticed for all cases.

Figure 7. The plastic bending at the die entrance region.
Figure 8. The two plastic bending and curling stages.
Figure 9. Waving phenomenon.

Figure 10. Deformation mechanism for the specimen wall \( t_0 = 1.5 \) mm.
7.3. The relation between the deformation stress and the expansion ratio

The expansion ratio, \( R = (r_f - r_o) / r_o \), is the most important variable in the expansion process. It determines the expansion in the tubular end. The theoretical analysis for the present study depends on \( R \), which may take other forms depending on the geometrical relations that imposed by die geometry, such as the penetration angle, \( \alpha \). The forming stress, \( P/A_o \), is the only variable that could be measured experimentally through measuring the driving force. Figure 13 shows the theoretical relation between the forming stress and the expansion ratio for three values of wall thickness. It can be seen that all the relationships have a direct proportion and have a straight form as the expansion ratio increases, while the wall thickness has slight effect. The same behavior is also noticed by using the FEM analysis as shown in figure 14.

In addition to the theoretical results, figures 15 show the results that obtained from FEM analysis for a perfect plastic material, \( K = 0 \). It could be seen that the theoretical results, generally speaking, are less than those acquired from FEM analysis, or it can be inferred that there is a good agreement between the two results at the beginning but this agreement deteriorates as the wall thickness increases.
The differences among these results may be attributed to the assumptions that those used in the theoretical analysis and the nature of the equivalent stress, in addition to the theoretical analysis limitations for describing the plastic bending, wrapping, and crimping.

Figure 13. Forming stress vs. expansion ratio for different wall thickness.

Figure 14. Forming stress vs. expansion ratio for different wall thickness.

7.4. Strain distribution
Figures 16 show the nature and manner of distribution of the radial, $\varepsilon_t$, circumferential, $\varepsilon_\theta$, and longitudinal, $\varepsilon_\alpha$, strains. The strains were measured for the last stage of expansion for each 5 mm increment, starting from the expanded end towards the specimen base along the longitudinal axis. Generally, it can be noticed that the circumferential strain, $\varepsilon_\theta$, is positive due to the die profile. The positive values seem to be larger at the expanded end.

Conversely, the radial strain $\varepsilon_t$ is negative near the expanded end and begins to decrease towards the die entrance region changing to positive at a certain distance from the die’s entrance. It is expected that the effect of the plastic bending at the die entrance continues up to the beginning of the die sliding.

The complexity of the plastic deformation in the expansion process is reflected in the longitudinal strain, $\varepsilon_\alpha$, which, generally, has a negative sign, starting with a specific value, depends on wall thickness, at the expanded end the strain starts increasing to reach a maximum compressive value at a distance about 5 mm from the expanded end. The effect of wall thickness on the strain at the expanded end, $\varepsilon_\alpha f$, and maximum longitudinal strain, $\varepsilon_{am}$, are shown in figure 17. It can be noticed that the longitudinal strain decreases with the increase of the wall thickness. The circumferential strain $\varepsilon_\theta$ is always positive regardless of the radius of curvature of the die, as was expected.
8. Conclusions
A mathematical model describing the expansion process was developed and validated through finite element analysis. The following conclusions were recorded.
1. The flow stress increases with the increasing of the expansion ratio with the tendency of the relation to be linear, whilst the relation seems to be not affected by the tube wall thickness.
2. The governing equation for the curved die expansion was differentiated, but the solution is limited to expansion of circular dies and perfect plastic material.
3. All specimens exhibited the plastic bending at the die entrance region.
4. Increasing the plastic bending at the die entrance, followed by wall curling and waving.
5. The circumferential strain $\varepsilon_\theta$ is always positive regardless of the radius of curvature of the die.
6. The radial strain $\varepsilon_r$ is negative near the expanded free end and begins to decrease toward the die entrance direction then it changes to positive at a certain distance from the die entrance.
7. The longitudinal strain, $\varepsilon_\alpha$, generally, has a negative sign. It decreases near the expanded end, while the maximum longitudinal strain decreases with the increase of the wall thickness.

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