Research Article

Hysteretic Energy Demand under Superposition of Bidirectional Ground Motions

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To address the irrationality of making a structure subjected to bidirectional ground motions equivalent to an SDOF system, a new approach method is presented in this paper. The ratio between modal participation factors of the two components of the structure is expressed as $c$, and the superposition of bidirectional ground motions is regarded as one-directional earthquake excitation for the equivalent SDOF system. Based on this, an energy balance equation is established, and a method used to estimate normalized hysteretic energy (NHE) is proposed. Analysis of the ratio between NHE ($c \neq 0$) and NHE ($c = 0$) is suggested in order to analyze the influence of bidirectional ground motions on hysteretic energy demand, and then, $\alpha_1 = NHE (y \neq 0)/NHE (y = 0)$ is defined, and bidirectional ground motion records for different soil sites are selected for establishing superimposed excitations. In addition, the period range of 0–5 s for the energy spectrum is divided into 6 ranges. In each period range, the means of $\alpha_1$ are defined as $\overline{\alpha_1}$. The curves of $\alpha$ of constant ductility factors for different soil sites are established, in which $\alpha$ is the vertical coordinate and $c$ is the horizontal coordinate.

Through nonlinear response history analysis, the influence of soil types at different sites, the ductility factor, the ratio of modal participation factors, and the period on the values of $\alpha$ are analyzed. According to the analytical results, correction coefficient $\alpha_s$ (the simplified value of $\alpha$) is obtained so that the hysteretic energy demand under bidirectional ground motions can be determined.

1. Introduction

Proper design of earthquake-resistant structures is extremely important for seismically active areas because it can be a key instrument that shapes economic and social opportunities. However, designing safe and economically viable structures is a very complex and multidisciplinary process that requires knowledge of a large number of engineering tools, parameters, and skills. Response spectra are also used as a basis for determining artificial earthquake records (e.g., [1, 2]). Response spectra have been recorded in practice since the 1971 San Fernando earthquake [3, 4], are now considered a basic tool for determining cutting forces and dimensioning structures, and are also used in preliminary calculations of important structures [3, 5]. The reason for the good acceptance and frequent application of the response spectrum lies in the simplicity of application, very low requirements for computer resources, and reasonable results for dimensioning structures. Response spectra contain information on the intensity and frequency composition of earthquakes, and their application takes into account the dynamic nature of the problem in structural calculations. However, calculation methods using the response spectrum are primarily applicable and give reasonable results for structures with a pronounced influence of lower forms of oscillation (e.g., [3, 6]). Furthermore, it is common knowledge that normalized response spectra are fabricated using single-degree systems embedded in a rigid substrate (e.g., [7, 8]).

Over the years, energy-based seismic design (EBSD) has been widely developed. Benavent-Climent [9] and Ghodrati Amiri et al. [10] provided an EBSD process for restructuring already built-in structures. Habibi et al. [11] proposed an EBSD process for rearranging structures with passive energy dissipation systems. Wang et al. [12] proposed an earthquake-induced story to estimate hysteretic energy (HE) according to the energy relationship between the SDOF and the original system. Some researchers (Zhang et al. [13] and Greco et al. [14]) use the well-known Park–Ang model to
estimate the damage level of structures in which the HE is an important index. The energy requirement is an important indicator for assessing seismic induction and seismic capabilities [15, 16].

To determine the dissipated earthquake energy of a structure from an energy spectrum, the structure itself should first be made equivalent to an SDOF system, and then, the energy demand of this equivalent SDOF system can be obtained via the energy spectrum to further compute the dissipated earthquake energy of the structure (Bruneau and Wang [17], Decanini and Mollaioli [18], and Erduran [19]). Currently, various energy spectra for the SDOF system have been developed, such as earthquake input energy spectrum [20], hysteretic energy spectrum [21, 22], absorbed energy spectrum [23], momentary energy spectrum [24], and inelastic cyclic demand spectrum [25]. However, for a structure subject to multidimensional ground motions (GMs), rational analysis shows that instead of one-component GM, it is the superposition of multicomponent GMs (GMs), rational analysis shows that instead of one-component GM, it is the superposition of multicomponent GMs (GMs), rational analysis shows that instead of one-component GM, it is the superposition of multicomponent GMs (GMs), and input energy spectra were established based on this equivalent system; however, further research about the energy relationship between this equivalent system and the structure were required for application of this theory. Reyes and Chopra [27] suggested a procedure to estimate structural responses under bidirectional GMs, in which the structural responses were calculated in two orthogonal directions, and the complete quadratic combination (CQC) rule was applied to combine the responses of these two components. This method is easy to use; however, it does not take into account the bidirectional coupling of nonlinear responses of structures. In this paper, a novel approach is proposed, in which the superposition of bidirectional GMs is applied as one-directional excitations of the equivalent SDOF systems, and then used to analyze the HE demand of structures.

2. Principle

2.1. Energy Equation of Modal Equivalent SDOF Systems. Consider an n-story building. The equation of motion governing the response of the n-story building subjected to bidirectional GMs (BGMs) (along x and y components) is expressed as

\[
\mathbf{M}\ddot{u}(t) + \mathbf{C}\dot{u}(t) + \mathbf{F}(t) = -\mathbf{M}_1\ddot{u}_{g_x}(t) - \mathbf{M}_3\ddot{u}_{g_y}(t),
\]

(1)

where \(\mathbf{M}\) and \(\mathbf{C}\) are the diagonal mass and damping matrices of the full structure, \(\mathbf{F}(t)\) and \(\mathbf{u}(t)\) are the resisting forces and displacement response vector, given as

\[
\begin{bmatrix} u_x(t) \\ u_y(t) \end{bmatrix},
\]

in which \(u_x(t)\) and \(u_y(t)\) denote the x, y, and torsion-directional displacement subvector, respectively. The energy balance equation of the building from equation (1) can be

\[
\int_0^{t_f} d\mathbf{u}(t)^T\mathbf{M}\ddot{u}(t) + \int_0^{t_f} d\mathbf{u}(t)^T\mathbf{C}\dot{u}(t) + \int_0^{t_f} d\mathbf{u}(t)^T\mathbf{F}(t) = -\int_0^{t_f} d\mathbf{u}(t)^T(\mathbf{M}_1\ddot{u}_{g_x}(t) + \mathbf{M}_3\ddot{u}_{g_y}(t)).
\]

(2)

\[
\mathbf{u}(t) = \sum_{i=1}^{3n} \varphi_i\eta_i(t),
\]

(3)

where \(\varphi_i\) is the ith mode shape vector which includes three subvectors \(\varphi_{xi}, \varphi_{yi}\) and \(\varphi_{θi}\), and \(\eta_i\) is the ith modal resistive load vector. Equation (3) is substituted into Equation (2); by premultiplying both sides by \(\varphi_{IT}\),

\[
\int_0^{t_f} \varphi_i^T\mathbf{M}\dot{\eta}_i(t)\dot{\eta}_i(t)dt + \int_0^{t_f} \varphi_i^T\mathbf{C}\dot{\eta}_i(t)\dot{\eta}_i(t)dt + \int_0^{t_f} \varphi_i^T\mathbf{F}(t)\dot{\eta}_i(t)dt = -\int_0^{t_f} \varphi_i^T\mathbf{M}_1\ddot{u}_{g_x}(t)\dot{\eta}_i(t)dt - \int_0^{t_f} \varphi_i^T\mathbf{M}_3\ddot{u}_{g_y}(t)\dot{\eta}_i(t)dt, \quad (i = 1, \ldots, 3n),
\]

(4)

where \(\Gamma_{xi}\) and \(\Gamma_{yi}\) are modal participation factors along x and y components, respectively. The ith modal resisting force \(F_i(t)\) can be denoted as \(\varphi_i^T\mathbf{K}_{eq}(t)\varphi_i\dot{\eta}_i(t),\) where \(\mathbf{K}_{eq}(t)\) is the elastic-plastic stiffness matrices (instantaneous).

In previous research studies, the following process is commonly used. Firstly, the 1st modal structure is simplified as two 2D models along x and y directions, respectively, and the responses, \(Q_{xi}\) and \(Q_{yi}\), are solved, respectively, through nonlinear response history analysis. After that, \(Q_{xi}\) and \(Q_{yi}\) are brought into the SRSS rule, and the 1st modal responses can be determined. The above procedure is repeated for different modes. Finally, the CQC rule is used for calculating...
the structural responses based on multimodal responses. In this paper, a new idea is proposed to solve structural responses under BGMs.

\[
\int_0^{t_0} \mathbf{q}^T \mathbf{M} \mathbf{x} \hat{q}_i(t) \hat{q}_i(t) \, dt + \int_0^{t_0} \mathbf{q}^T \mathbf{C} \mathbf{x} \hat{q}_i(t) \hat{q}_i(t) \, dt + \int_0^{t_0} f_i(t) \hat{q}_i(t) \, dt \\
= - \int_0^{t_0} \mathbf{q}^T \mathbf{M} \mathbf{x} \Gamma_y U_{	ext{gr}}(t) \hat{q}_i(t) \, dt,
\]

(5)

Here, \( \hat{U}_{\text{gr}}(t) \) is the superimposed ground acceleration (A), and \( \hat{U}_{\text{gr}}(t) = \hat{u}_{\text{gx}}(t) + \gamma \hat{u}_{\text{gy}}(t) \), in which \( \gamma_i = \Gamma_{yi}/\Gamma_{xi} \), and its range is from -1 to 1. The \( \hat{U}_{\text{gr}}(t) \) can be considered as an earthquake excitation.

For the case of \( |\Gamma_{yi}| \leq |\Gamma_{yi}| \), we set \( \Gamma_{yi} = \Gamma_{y} \), and the superimposed ground “A” is replaced with \( \hat{U}_{\text{gr}}(t) = \gamma_i \hat{u}_{\text{gx}}(t) + \hat{u}_{\text{gy}}(t) \), in which \( \gamma_i = \Gamma_{yi}/\Gamma_{yi} \). The above relationship can be expressed as

\[
\begin{align*}
\text{For} |\Gamma_{yi}| \geq |\Gamma_{yi}|, \quad & \hat{U}_{\text{gr}}(t) = \hat{u}_{\text{gx}}(t) + \gamma \hat{u}_{\text{gy}}(t), \quad \text{where} \quad \gamma = \frac{\Gamma_{yi}}{\Gamma_{yi}} \\
\text{For} |\Gamma_{yi}| \geq |\Gamma_{yi}|, \quad & \hat{U}_{\text{gr}}(t) = \gamma \hat{u}_{\text{gx}}(t) + \hat{u}_{\text{gy}}(t), \quad \text{where} \quad \gamma = \frac{\Gamma_{yi}}{\Gamma_{yi}}. \\
\end{align*}
\]

Here, \( f_i(t) = F_i(t)/\mathbf{q}_i^T \mathbf{M} \mathbf{x} \Gamma_{yi} \), \( \xi \) is damping ratio, and \( \omega_i \) is natural frequency of \( i \)th mode of structure. Equation (7) can be regarded as the energy balance equations of 3n independent modal equivalent SDOF systems with mass equals to 1. The energy terms from left to right of Equation (7) are kinetic energy \( e_{x}(t_0) \), viscous damping energy \( e_{\xi}(t_0) \), HE \( e_{\text{he}}(t_0) \), and input energy \( e_{\text{in}}(t_0) \), respectively, of the \( i \)th modal equivalent SDOF system.

By the above derivation, structural HE due to earthquake can be determined through the following equation:

\[
E_{\text{he}}(t_0) = \sum_{i=1}^{3n} \mathbf{q}_i^T \mathbf{M} \mathbf{x} \dot{x}_i^2 + e_{\text{he}}(t_0)
\]

The relation of deformation and energy dissipation between a structure and its modal equivalent SDOF systems is illustrated in Figure 2, in which the structure subjected to bidirectional GMs and each equivalent SDOF system is subject to their superposition, and the ratio \( \gamma \) in superposition equation depends on the modal participation factors of the structure.

Considering the variation of mode shape in plastic response range of structures, the elastic mode shapes in Equation (8) can be substituted by elastic-plastic mode shapes, which can be obtained by pushover analysis or modal pushover analysis [28, 29].

2.2. Energy Demands under Superposition of Bidirectional Ground Motions. A modal equivalent SDOF system mentioned above is considered. The instantaneous relative displacement is defined as \( \mu(t) \), and \( \mu(t) = d(t)/d_{\text{yie}} \) where \( d_{\text{yie}} \) is yield displacement. The maximum value of \( \mu(t) \) equals to ductility factor \( \mu \). Substituting \( \mu(t) \) into Equation (7), the following equation can be derived:

\[
d_{\text{yie}}^2 \int_0^{t_0} \ddot{\mu}(t) \ddot{\mu}(t) \, dt + 2\xi \omega d_{\text{yie}}^2 \int_0^{t_0} \dot{\mu}(t) \ddot{\mu}(t) \, dt + \omega^2 d_{\text{yie}}^2 \int_0^{t_0} \mu(t) \ddot{\mu}(t) \, dt = -d_{\text{yie}} \int_0^{t_0} \hat{U}_{\text{gr}}(t) \ddot{\mu}(t) \, dt,
\]

(9)
where \( f_{\text{yie}} \) is the yield force of the equivalent SDOF system. The mass of the SDOF system equals to 1; therefore, \( f_{\text{yie}} = \omega^2 d_{\text{yie}} \), and the peak force in the linear elastic stage can be
\[
\max|f(t)| = \beta \cdot \text{PSA},
\]
where \( \beta \) is amplification coefficient spectrum under superposition of BGMs and \( \text{PSA} = \max(|U_g(t)|) \). The strength reduction factors can be expressed as
\[
R = \frac{\max|f(t)|}{f_{\text{yie}}}. \tag{10}
\]
Substituting Equation (10) into (9), the energy equation can be rewritten as
\[
\beta^2 \frac{\text{PSA}^2}{\omega^2 R^2} \int_0^{t_c} \dot{U}_g(t) \dot{\mu}(t) \, dt + \frac{2\xi \beta^2 \text{PSA}^2}{\omega^2 R^2} \int_0^{t_c} \dot{\mu}(t) \, dt + \frac{\beta^2 \text{PSA}^2}{\omega^2 R^2} \int_0^{t_c} \frac{f(t)}{f_{\text{yie}}} \dot{\mu}(t) \, dt = -\frac{\beta \cdot \text{PSA}}{\omega^2 R} \int_0^{t_c} \dot{U}_g(t) \dot{\mu}(t) \, dt. \tag{11}
\]
According to the process of derivation from Equations (7) to (11), the energy terms from left to right of Equation (11) are still kinetic energy \( e_{k,i}(t_0) \), viscous damping energy \( e_{d,i}(t_0) \), HE \( e_{\delta,i}(t_0) \), and input energy \( e_{I,i}(t_0) \), respectively.
The PSA of each pair of GMs are different, so, in order to make mean energy demand which is not affected by PSA, both sides of Equation (11) are divided by PSA squared, and the energy equation is rewritten as

\[
\frac{\beta^2}{\omega^2 R^2} \int_0^{t_s} \ddot{u}(t) \dot{u}(t) \, dt + \frac{2\xi \beta^2}{\omega^2 R^2} \int_0^{t_s} \dot{u}(t) \ddot{u}(t) \, dt + \frac{\beta^2}{\omega^2 R} \int_0^{t_s} f(t) \ddot{u}(t) \, dt = \frac{\beta}{\omega^2 R} \int_0^{t_s} \bar{U}_g(t) \ddot{u}(t) \, dt. \tag{12}
\]

Here, \( \bar{U}_g(t)/PSA \) is normalized superimposed “A,” and each normalized energy term in the equation is the ratio of energy to PSA squared. The 3rd term on the left side of Equation (12) is the normalized HE (NHE).

2.3. Normalized Hysteretic Energy Spectrum under Superposition of Bidirectional Ground Motions. The procedure for establishing NHE spectrum under superposition of BGMs is as follows:

(1) The ratio \( \gamma \) between bidirectional modal participation factors is given, and then, the superimposed “A” time history \( \bar{U}_g(t) \) of the SDOF system is determined using Equation (6).

(2) The damping ratio \( \xi \) is given, and a specific period \( T \) is selected to calculate the corresponding amplification factor spectrum \( \beta \), where the relationship between the period and frequency is \( T = 2\pi/\omega \).

(3) The ductility factor \( \mu_o \) of a target is chosen.

(4) The parameter \( R \) is set to a value smaller than \( \mu_o \), and it is substituted into Equation (12) to calculate the maximum relative displacement \( \max[\ddot{u}(t)] \).

(5) Assume \( \Delta = |\mu_o - \max[\ddot{u}(t)]| \) if \( \Delta \) is within an acceptable range; namely, if \( \max[\ddot{u}(t)] \) is close enough to \( \mu_o \), then pause calculation and determine the NHE demand using Equation (13). Otherwise, continue calculation to the next step:

\[
\text{NHE}(\gamma, \mu, R, T, \xi) = \frac{T^2 \beta^2}{4\pi^2 R^2} \int_0^{t_s} f(t) \ddot{u}(t) \, dt, \tag{13}
\]

s.t. Equation (12): \( \mu = \max[\ddot{u}(t)] \),

where the instantaneous relative displacement \( \ddot{u}(t) \) is obtained by solving Equation (12) and \( t_s \) is the duration of the bidirectional GMs.

(6) Properly raise the value of \( R \). For example, \( R = R + 0.1 \), and then, repeat Steps 4–5 until the requirement of “\( \max[\ddot{u}(t)] = \mu_o \)” is satisfied. After that, determine NHE demand using Equation (13).

(7) Apply a different period \( T \) (e.g., \( T = 0–5 \) s), and then, repeat Steps 2–6 to establish the NHE spectra corresponding to a given \( \gamma \).

3. Selection of Earthquake Records

For analyzing the mean HE demand, 89 pair horizontal BGM records for hard soil site (HSS) \((V_s = 360–750 \text{ m/s})\), ISS (intermediate hard (soft) soil site) \((V_s = 180–360 \text{ m/s})\), and SSS (soft soil site) \((V_s < 180 \text{ m/s})\), corresponding to B, C, and D, respectively, for USGS, are selected (Tables 1–3). The conditions for choosing these records are (1) magnitude equal to 6 to 8, (2) fault distance equal to 15 km to 45 km, (3) and peak “A” for \( x \) or \( y \) directions are greater than or equal to 0.1 g, approximately.

The amplification coefficient spectra \( \beta(T, \xi) \) is illustrated in Figure 3 for three sites with different types of soils based on the traditional spectrum, and \( \gamma = 0 \) and superposition of bidirectional earthquake “A” records (proposed in this paper, and \( \gamma \neq 0 \)). For the latter case, \( \gamma \) is set as 1 and –1 in this figure, respectively. For this reason, the two ratios of modal participation factors may result in a larger deviation of results. As shown in Figure 3, the spectra of superimposed GMs are much closer to the spectra of one-directional GMs, which indicates that the superimposed earthquake “A’s” still have the typical characteristics of the different soil sites.

4. Analysis of Hysteretic Energy Demand

4.1. Analysis of NHE Spectra. According to Equations (5)–(7), NHE(\( \gamma = 0 \)) is the normalized HE under one-directional GM, and NHE(\( \gamma \neq 0 \)) is the normalized HE under the superposition of bidirectional GMs. Figure 4 displays the mean spectra of NHE (\( \gamma = 0 \)) for soil types at different sites. As shown in this figure, the spectrum for each soil base has its own spectral shape features. The spectral curves of the 3 soil bases are all made upward, peak platform, and downward levels, and the time limits for each phase are obviously affected by the ductility factor and the soil type.

Figure 5 illustrates the comparison of NHE spectra between one-directional GMs and superposition of bidirectional GMs (\( \gamma = 0.8 \) and \( \gamma = 0.8 \)). As shown in this figure, the difference between \( \gamma = 0 \) and \( \gamma \neq 0 \) is obvious, and it may be affected by soil type, period, ductility factor, positive and negative of \( \gamma \), and value of \( \gamma \). In this figure, \( \gamma \) is only set as 0.8 and –0.8, but for the other values of \( \gamma \), the system analyses are as follows.

4.2. Analysis of Curves. The mean demand of NHE (\( \gamma \neq 0 \)) is influenced by some parameters. Many researchers have studied the HE spectrum under one-directional GM and achieved various results. In the interest of simplification, the ratios between NHE (\( \gamma \neq 0 \)) and NHE (\( \gamma = 0 \)) are analyzed in this paper to study the HE demand under the superposition of bidirectional GMs. An approach is used to establish the ratio of NHE (\( \gamma \neq 0 \))/NHE (\( \gamma = 0 \)) according to soil types at
different sites, the influence of the soil type, the ductility factor \( \mu \), and the ratio between modal participation factors of two components, \( \gamma \), are analyzed, and the approximate relationships concerning NHE (\( \gamma \neq 0 \))/NHE (\( \gamma = 0 \)) are provided. In addition, the damping ratio \( \xi \) is set as 0.05; according to the process mentioned above, NHE demand can be solved.

Table 1: The ground "A" records for HSS (34 pairs).

| Stations            | Earthquakes | Components | Stations            | Earthquakes | Components |
|---------------------|-------------|------------|---------------------|-------------|------------|
| 1095 Tafe Lin-Coln School | Kern Country (52/7/21,M7.4) | TAF021 | 6604 Cerro Prieto | Victoria, Mexico (80/06/09,M6.4) | CPE045 |
|                     |             | TAF111     |                     |             | CPE315     |
| 1652 Anderson-Dam   | Lorna Prieta (89/10/18,M69) | AND270 | 89324 Rio Dell Overpass-FF | Cape Mendocina (92/04/25,M7.1) | RIO270 |
|                     |             | AND360     |                     |             | RIO360     |
| 24157 LA-Baldwin Hills | Northridge (94/1/17,M6.7) | BLD090 | CHY029         | Chi-Chi (99/09/20,M7.6) | CHY029_N |
|                     |             | BID360     |                     |             | CHY029_W   |
| 14403 LA-116th St School | Northridge (94/01/17,M6.7) | 116090 | ALS             | Chi-Chi (99/09/20,M7.6) | ALS-E       |
|                     |             | 116360     |                     |             | ALS-N       |
| 24605 LA-Univ.Hospital | Northridge (94/01/17,M6.7) | UNI005 | CHY052         | Chi-Chi (99/09/20,M7.6) | CHY029_E   |
|                     |             | UNI055     |                     |             | CHY029_N   |
| 23 Coolwater        | Landers (92/06/28,7.3) | CLW-LN     | 1061 Lamont       | Duzce (99/11/12,M7.1) | ARC000     |
|                     |             | CLW-TR     |                     |             | ARC090     |
| 6604 Cerro Prieta   | Imperial Valley (79/10/15,M6.5) | H-CPE147 | Arcelik         | Kocaeli (99/08/17,M7.4) | ARC000     |
|                     |             | H-CPE237   |                     |             | ARC090     |
| 5704 Coyote Lake Dam | Loma Prieta (89/10/18,M6.9) | CLD195   | 24577 Fort Irwin | Landers (92/06/28,M7.3) | FTI000     |
|                     |             | CLD285     |                     |             | FTI090     |
| Tcu045              | Chi-Chi (99/0920,M7.6) | TCU045-N  | 57064 Fremont-Mission Sen Jose | Loma Prieta (89/10/18,M6.9) | FRE000     |
|                     |             | TCU045-W   |                     |             | FRE090     |
| Tcu047              | Chi-Chi (99/0920,M7.6) | TCU045-N  | 57383 Gilroy ArrayW#6 | Loma Prieta (89/10/18,M6.9) | G06000     |
|                     |             | TCU045-W   |                     |             | G06090     |
| 90021 LA-N Westmoreland | Northridge (94/01/17,M6.7) | WST000   | 58378 APEEL7_pulgas | Loma Prieta (89/10/18,M6.9) | A07000     |
|                     |             | WST270     |                     |             | A07000     |
| CHY086              | Chi-Chi (99/09/20,M6.7) | CHY086-N  | 58498 Hayward-BART Sta | Loma Prieta (89/10/18,M6.9) | HWB220     |
|                     |             | CHY086-W   |                     |             | HWB310     |
| 286 Superstition Mtn Camera | Imperial Valley (79/10/15,M6.5) | H.SUP045 | Gilroy Gavial Coll | Morgan Hill (84/04/24,M6.2) | GIL067     |
|                     |             | H.Sup135   |                     |             | GIL337     |
| 12149 Desert Hot Springs | Landers (92/06/28,M7.3) | DSP000   | 24607 Laka Hughes#12A | Northridge (94/01/17,M6.7) | H12090     |
|                     |             | DSP090     |                     |             | H12180     |
| 24389 LA-Century City CC North | Northridge (94/01/17,M6.7) | CCN090   | 24611 LA-Temple Hope | Northridge (94/01/17,M6.7) | TEM090     |
|                     |             | CCN360     |                     |             | TEM180     |
| 14403 LA-116th School | Whittier Narrows (87/10/01,M6.0) | A-116270 | 90020 LA-W15 *St | Northridge (94/01/17,M6.7) | W15090     |
|                     |             | A-116360   |                     |             | W15180     |
| 9009 N Hollywood Coldwater | Whittier Narrows (87/10/01,M6.0) | A-CWC180 | 900333_LA Cyper Ava | Northridge (94/01/17,M6.7) | CYP053     |
|                     |             | A-CWC270   |                     |             | CYP143     |

According to the range of \( \gamma \), the typical values of \( \gamma \) are selected as \(-1.0, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, \) and 1.0. The period \( T \) is divided into 6-period ranges: 0.5–1 s, 1–2 s, 2–3 s, 3–4 s, and 4–5 s. \( \alpha_t \) is defined as the ratio between NHE (\( \gamma \neq 0 \)) and NHE (\( \gamma = 0 \)), and for the convenience of analysis, \( \alpha \) is defined as the mean of \( \alpha_t \) for each period range. Given \( \gamma \) as horizontal coordinate and \( \alpha \) as vertical...
coordinate, and then, $\alpha$ curves of constant ductility factors can be established (see Figures 6–8).

Figure 6 displays the curves of $\alpha$ of constant ductility factors for hard soil site. From this figure, the following are clear. (1) During the period range of 0–2 s, with the increase of $\gamma$, $\alpha$ shows a tendency to decrease; for the case that the ductility factor is smaller and period is longer, this decreasing tendency becomes more apparent within the range of 0.5–2 s; within the same period range, when $\gamma < 0$, $\alpha > 1$, and the maximum value of 1.12 is obtained. (2) Opposite to the condition shown in the range of 0–2 s, during the period range of 2–5 s, with the increase of $\gamma$, $\alpha$ displays an increasing trend, and this trend is more evident for

| Stations          | Earthquakes          | Components | Stations          | Earthquakes          | Components |
|-------------------|----------------------|------------|-------------------|----------------------|------------|
| 6621 Chihuahua    | Imperial Valley (79/10/15, M6.5) | CHI012 | NST               | Chi-Chi (99/09/20,M7.6) | NST-E      |
|                   |                      | CHI282     |                   |                      |            |
| 1695 Sunnyvale    | Loma Prieta (89/10/18,M6.9) | SVL270     | 6622 Compuertas   | Imperial Valley (79/10/15,M6.5) | H-CMP015 |
| Colton Ava        |                      | SVL360     |                   |                      |            |
| 1028 Holister City Hall | Loma Prieta (89/10/18,M6.9) | HCH090     | 36227 Parkfield Cholame 5W | Coalinga (89/05/02,M6.4) | H-CO5270 |
|                   |                      | HCH180     |                   |                      |            |
| 22074 Yermo Fire Station | Loma Prieta (89/10/18,M6.9) | YER270     | Iznik             | Kocaeli (99/08/17,M7.4) | IZN180     |
|                   |                      | YER360     |                   |                      |            |
| 90063 Glendale Las Palmas | Northridge (94/01/17,M6.7) | GLP177     | 57066AgnewState Hospital | Loma Prieta (89/10/18,M6.9) | AGW000     |
|                   |                      | GLP267     |                   |                      | AGW090     |
| 90016 LA-N Faring RD | Northridge (94/01/17,M6.7) | FAR000     | 57425 Gilroy Array#7 | Loma Prieta (89/10/18,M6.9) | GM0000     |
|                   |                      | FAR090     |                   |                      | GM090      |
| 90091 LA-Saturn St | Northridge (94/01/17,M6.7) | STN020     | 47125 Capitola     | Morgan Hill (84/04/24,M6.2) | CAP042     |
|                   |                      | STN110     |                   |                      |            |
| TCU042            | Chi-Chi (99/09/20,M7.6) | TCU042-N   | 47380 Gilroy Array #2 | Morgan Hill (84/04/24,M6.2) | G02000     |
|                   |                      | TCU042-W   |                   |                      | G02090     |
| TCU107            | Chi-Chi (99/09/20,M7.6) | TCU0107-N  | 24303 LA-Hollywood Stor | Northridge (94/01/17,M6.7) | HOL090     |
|                   |                      | TCU107-N   |                   |                      | HOL360     |
| CHY036            | Chi-Chi (99/0920,M7.6) | CHY036-N   | 24309 Leona Valley#6 | Northridge (94/01/17,M6.7) | L6090      |
|                   |                      | CHY036-N   |                   |                      | L6360      |
| 6605 Delta        | Imperial Valley (79/10/15,M6.5) | H-DLT262  | 90034 LA-Fletcher DR | Northridge (94/01/17,M6.7) | CAP042     |
|                   |                      | H-DLT352   |                   |                      | C042       |
| CHY010            | Chi-Chi (99/0920,M7.6) | CHY010-E   | 90054 LA Centinela ST | Northridge (94/01/17,M6.7) | CEN155     |
|                   |                      | CHY010-N   |                   |                      | CEN245     |
| CHY034            | Chi-Chi (99/0920,M7.6) | CHY034-E   | 12025 Palm Springs Airport | Northridge (94/01/17,M6.7) | PSA000     |
|                   |                      | CHY034-N   |                   |                      | PSA090     |
| CHY046            | Chi-Chi (99/0920,M7.6) | CHY046-N   | 994 Mormon Oso Pump Plant | San Fernando (71/02/09,M6.6) | OPP000     |
|                   |                      | CHY046-W   |                   |                      | OPP270     |
| Atakoy            | Kocaeli (99/08/17,M7.4) | ATK000     | 135 LA-Hollywood Stor Lot | San Fernando (71/02/09,M6.6) | PEL090     |
|                   |                      | ATK090     |                   |                      | PEL090     |
| Bolu              | Duza (99/11/12M7.1)  | BOL000     | 6621 Chihuahua    | Victoria (80/06/09,M6.4) | CHI102     |
|                   |                      | BOL290     |                   |                      | CHI192     |
| El Centro Array #1 | Imperial Valley (79/10/15,M6.5) | H-E01140  | 90003Northridge Saticoy St | Whittier Narrows (87/10/01,M6.0) | A-STC090   |
|                   |                      | H-E01230   |                   |                      | A-STC180   |
| 5061 Calipatria Fire State | Imperial Valley (79/10/15,M6.5) | H-CAL225  |                   |                      |            |
|                   |                      | H-CAL315   |                   |                      |            |
the case that the ductility factor is larger and the period is longer. In this period range, when \( c < 0, \alpha < 1 \), and when \( c > 0, \alpha > 1 \), showing a maximum value of 1.25. Moreover, when \( |c| \) is around 0.6, the \( \alpha \) curve shows peak values in the positive and negative axes, respectively; when \( |c| > 0.6 \), the \( \alpha \) curve displays horizontal or decreasing distribution; when \( |c| < 0.6 \), the \( \alpha \) curve demonstrates an almost linear increasing distribution.

Figure 7 shows the curves of \( \alpha \) of constant ductility factors for ISS. From this figure, it is clear that, in the period range of 0–5 s, for the case that \( y < 0 \), with the decrease of the ductility factor, \( \alpha \) shows a tendency to increase under normal conditions, and the maximum value of \( \alpha \) reaches 1.18, its minimum is 0.84; for the case that \( y > 0 \), the value \( \alpha \) mostly stays below 1, and normally, if the ductility factor is smaller, \( \alpha \) also becomes smaller.

![Graph](image-url)

**Figure 7:** Amplification coefficient spectra under original one-directional earthquake "A"s (\( y = 0 \)) and superposition of bidirectional earthquake "A"s (\( y = 1 \) and \( -1 \)) (damping ratio \( \xi = 0.05 \)). (a) Hard soil site. (b) Intermediate hard (soft) soil site. (c) Soft soil site.

Figure 8 displays the curves of \( \alpha \) of constant ductility factors for SSS. This figure illustrates the following. (1) During the period range of 0–2 s, the distribution of \( \alpha \) along \( y \) shows a concave shape; namely, values of \( \alpha \) are relatively large on the two sides but relatively small in the middle of this figure, and compared to the left side, values of \( \alpha \) on the right side are larger. However, one special condition is found within the range of 1–2 s; the \( \alpha \) curve with a ductility factor of \( \mu = 2 \) shows a gradually increasing trend. (2) Compared to the distribution characteristics shown in the range of 0–2 s, during the period range of 2–5 s, \( \alpha \) decreases with the increase of period and later remains relatively constant on the right side, and the minimum of \( \alpha \) is around 0.9. On the left side of the figure, the value of \( \alpha \) increases with the period, and its maximum value is close to 1.6. All of these

### Table 3: The ground "A" records for SSS (20 pairs).

| Stations   | Earthquakes          | Components | Stations   | Earthquakes          | Components |
|------------|----------------------|------------|------------|----------------------|------------|
| 5057       | EI CentroArray #3    | Imperial Valley (79/10/15,M6.5) | H-E03140   | CHY0002              | CHY002-E   |
| 1002       | APEEL 2-Redwood City | Loma Prieta (89/10/18,M6.9) | A02043     | CHY008               | CHY008-E   |
| 9001       | Montebel-BR          | Loma Prieta (89/10/18,M6.9) | A02133     | CHY025               | CHY025-N   |
| 58117      | Treasure Island      | Loma Prieta (89/10/18,M6.9) | BLF206     | CHY010               | CHY010-N   |
| 0          | Shin-Osaka           | Kobe (95/01/16,M6.9) | SH000      | CHY039               | CHY039-N   |
| 0          | Kakogawa             | Kobe (95/01/16,M6.9) | KAK000     | CHY092               | CHY092-N   |
| Ambaril    |                      | Kocaeli (99/08/17,M7.4) | ATS000     | CHY107               | CHY107-N   |
| TCU117     |                      | Chi-Chi (99/9/20,M7.6) | TCU117-N   | TCU111               | TCU111-N   |
| TCU118     |                      | Chi-Chi (99/9/20,M7.6) | TCU118-N   | TCU118-W             | TCU118-W   |
| CHY104     |                      | Chi-Chi (99/9/20,M7.6) | CHY104-E   | TCU115               | TCU115-N   |
|            |                      |            | TCU117     | TCU111-W             | TCU115-W   |
|            |                      |            |            |                      |            |

**Figure 3:** Amplification coefficient spectra under original one-directional earthquake "A"s (\( y = 0 \)) and superposition of bidirectional earthquake "A"s (\( y = 1 \) and \( -1 \)) (damping ratio \( \xi = 0.05 \)). (a) Hard soil site. (b) Intermediate hard (soft) soil site. (c) Soft soil site.
distribution characteristics are easier to recognize when the ductility factor becomes smaller.

Based on the above analyses, correction coefficient $\alpha_s$ for correcting HE demand of conventional method is listed in Table 4. According to the approach proposed in this paper, once the HE calculated via the conventional method is multiplied by $\alpha_s$, the HE demand under superposition of bidirectional GMs can be obtained. For giving the correction coefficient, the values of $\alpha_s$ in each period range are divided into four or three cases, for example, $\alpha_s \ (−1 \leq \gamma < -0.4)$, $\alpha_s \ (−0.4 \leq \gamma < 0)$, and $\alpha_s \ (0 < \gamma \leq 1)$ for the intermediate hard (soft) soil site. Regarding various $\gamma$ and ductility factors $\mu$, the values of $\alpha_s$ are determined.
Figure 6: The curves of $\alpha$ of constant ductility factors for hard soil site. (a) $T = 0–0.5$ s. (b) $T = 0.5–1.0$ s. (c) $T = 1.0–2.0$ s. (d) $T = 2.0–3.0$ s. (e) $T = 3.0–4.0$ s. (f) $T = 4.0–5.0$ s.

Figure 7: Continued.
which are the maximum values of $\alpha$ for each case of each period range. In this paper, only the enlargement effect of superposition of bidirectional GMs on HE demand is considered. In addition, as a consequence, if all of the $\alpha$ values in a certain zone are smaller than 1; $\alpha_s \equiv 1$ should be adopted for the security reason.
5. Conclusions

Considering the irrationality of the equivalent SDOF system of a structure subjected to bidirectional GMs. The superposition of bidirectional GMs is regarded as the one-directional excitation of the equivalent SDOF system of a structure. An energy balance equation is established based on this system, and the process to determine normalized HE demand is proposed. Since the conventional theory of HE spectrum is rather mature, the ratio between NHE \((\gamma \neq 0)\) and NHE \((\gamma = 0)\) is analyzed to study the influence of superposition of bidirectional ground motions on HE demand. As a consequence, \(\alpha_1 = \text{NHE (}\gamma \neq 0)/\text{NHE (}\gamma = 0)\) is defined, and \(\alpha\) is defined as the mean value of \(\alpha_1\) in each period range. \(\gamma\) is set as the horizontal coordinate and \(\alpha\) is set as the vertical coordinate, and then, the curves of \(\alpha\) of constant ductility factors for soil types at different sites in different period ranges are established. Strong GM records are selected for establishing superimposed excitations to study the influences of soil site types, ductility factor, \(\gamma\), and period on values of \(\alpha\). The following conclusions are achieved through analysis:

(i) For the HSS, during the period range of 0–2 s, \(\alpha\) increases with the decrease of \(\gamma\); especially, for the case that the ductility factor is smaller and the period is longer, this decrease trend is more apparent. Within the same period range (0–2 s), when \(\gamma < 0, \alpha > 1\), and when \(\gamma > 0, \alpha < 1\). During the period range of 2–5 s, \(\alpha\) increases with the increase of \(\gamma\); especially, for the case that the ductility factor becomes larger and the period is longer, this increase trend is more evident. Within the same period range (2–5 s), when \(\gamma < 0, \alpha < 1\), and when \(\gamma > 0, \alpha > 1\).

(ii) For the ISS, during the period range of 0–5 s, for \(\gamma < 0\), normally the smaller the ductility factor, the larger the value of \(\alpha\), and with the increase of the period, the value of \(\alpha\) first increases and then decreases. For \(\gamma > 0\), normally, the smaller the ductility factor, the smaller the value of \(\alpha\), and the value of \(\alpha\) first decreases and then increases with the increase of the period.

(iii) For the SSS, during the period range of 0–2 s, the distribution of \(\alpha\) along \(\gamma\) shows an approximate concave shape, and normally, \(\alpha > 1\). Within the period range of 2–5 s, the distributions of \(\alpha\) on both sides of the curves are almost horizontal; the values of \(\alpha\) are lower with the increase of \(\gamma\) in the middle of the curves. In the same range (2–5 s), when \(\gamma < 0, \alpha > 1\), and \(\alpha\) increases with the increase of the period. This characteristic becomes more apparent with a smaller ductility factor.

Based on the analysis concerning distribution characteristics of \(\alpha\) along with \(\gamma\), the correction coefficient for easier application, namely, \(\alpha_s\) is given, which is the simplified value of \(\alpha\). The value of \(\alpha_s\), combined with conventional HE spectrum, can be used to determine HE demand corresponding to the different \(\gamma\), and then, the HE demand of the structure subjected to bidirectional GMs can be estimated.

Data Availability

All the data can be obtained from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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