The FRB-SGR Connection

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ABSTRACT

The discovery that the Galactic SGR 1935+2154 emitted FRB 200428 simultaneous with a gamma-ray flare demonstrated the common source and association of these phenomena. If FRB radio emission is the result of coherent curvature radiation, the net charge of the radiating “bunches” or waves may be estimated. A statistical argument indicates that the radiating bunches must have a Lorentz factor $\gtrsim 10$. The observed radiation frequencies indicate that their phase velocity (pattern speed) corresponds to Lorentz factors $\gtrsim 100$. Coulomb repulsion implies that the electrons making up these bunches may have yet larger Lorentz factors, limited by their incoherent curvature radiation. These electrons also Compton scatter in the soft gamma-ray field of the SGR. In FRB 200428 the power radiated coherently at radio frequencies exceeded that of Compton scattering, but in more luminous SGR outbursts Compton scattering dominates, precluding the acceleration of energetic electrons. This explains the absence of a FRB associated with the giant 27 December 2004 outburst of SGR 1806−20. SGR with luminosity $\gtrsim 10^{42}$ ergs/s do not emit FRB, while those of lesser luminosity can do so.

Key words: radio continuum: transients, gamma-rays: general, stars: magnetars, stars: neutron

1 INTRODUCTION

Soft Gamma Repeaters (SGR) have long been candidates for the sources of Fast Radio Bursts (FRB). SGR are believed to originate in young neutron stars with extremely high magnetic fields and to be powered by dissipation of their magnetostatic energy (Katz 1982; Thompson & Duncan 1992, 1995), offering an ample source of energy. The energies $\sim 10^{40}$ ergs of even “cosmological” FRB are a tiny fraction of the $\sim 10^{47}$ ergs of magnetostatic energy of a neutron star with a $10^{15}$ gauss field, a value inferred from the spindown rates of some SGR, measured in their quiescent Anomalous X-ray Pulsar (AXP) phases.

SGR also have short characteristic time scales. The most intense parts of their outbursts typically last $\sim 0.1$ s but upper bounds on their rise times are $< 1$ ms. Although the temporal structure of SGR have not been measured on the scale of the fastest temporal structure of FRB ($\sim 10$ µs), the fact that both display extremely short time scales, shorter than any other astronomical time scale except those of pulsar pulses, suggests an association. This hypothesis has been advanced by many authors (Connor, Sievers & Pen 2016; Cordes & Wasserman 2016; Dai et al. 2016; Katz 2016; Zhang 2017; Wang et al. 2018; Wadiasingh & Timokhin 2019); see Katz (2018a) for a review.

2 THE PROBLEM

FRB 200428 was discovered by CHIME/FRB (CHIME/FRB Collaboration 2020) and by STARE2 (Bochenek et al. 2020) during an outburst of the SGR 1935+2154 observed by INTEGRAL (Mereghetti et al. 2020), Insight-HXMT (Li et al. 2020), Konus-Wind (Ridnaia et al. 2020) and AGILE (Tavani et al. 2020) and consistent with the location of the SGR. The ratio of the STARE2 (Bochenek et al. 2020) radio to the Insight-HXMT soft gamma-ray (Li et al. 2020) fluences of SGR 200428 was $\sim 2 \times 10^{12}$ Jy-ms/(erg/cm²), several orders of magnitude greater than the upper limit of $10^7$ Jy-ms/(erg/cm²) set by Tendulkar, Kaspi & Patel (2016) on any FRB associated with the giant 27 December 2004 outburst of SGR 1806−20.

The large observed radio-frequency fluence (Bochenek et al. 2020) of FRB 200428, taking a distance of 6 kpc (a compromise among the 12.5 kpc (Kothes et al. 2018), 9.1 kpc (Zhong et al. 2020) and 6.6 kpc (Zhou et al. 2020) estimated for the embedding SNR G57.2+0.8 and the 2–7 kpc estimated by Mereghetti et al. (2020) from dust-scattered SGR emission), implies an isotropic-equivalent emitted energy $\sim 10^{-6}$ that of a nominal 1 Jy–ms “cosmological” FRB at $z = 1$. Any explanation of FRB as products of SGR must be consistent with “cosmological” FRB whose radio emission

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1 Sometimes referred to as J1935+2154.
is several orders of magnitude more energetic than that of FRB 200428 and with the radio-to-gamma ray fluence ratio of FRB 200428 more than five orders of magnitude greater than that of SGR 1806–20. A number of theoretical interpretations have been suggested (Lu, Kumar & Zhang 2020; Lyutikov & Popov 2020; Margalit et al. 2020; Wang, Xu & Chen 2020).

A past argument (Katz 2020) against a neutron star origin of FRB was the absence of periodicity in repeating FRB, particularly in the well-studied FRB 121102 (Zhang et al. 2018). SGR 1935+2154 has a period of 3.245 s (Israel et al. 2016), which would be expected; to modulate the observatory activity of FRB 200428, whatever its mechanism of emission, unless its magnetic field be a dipole aligned with the spin axis.

3 THE HOST

The characteristic spindown age of SGR 1935+2154 was measured over about 120 days in 2014 to be 3600 y (Israel et al. 2016), several times shorter than the estimated age of SNR G57.2+0.8 (Kothes et al. 2018; Zhou et al. 2020) in which it is embedded. These values of the SNR age were inferred from estimates of its distance; the smaller distance renders the neutron star’s age, measured over about 120 days in 2014 to be 3600 y (Israel et al. 2016), for observed curvature emission, unless its magnetic field be a dipole aligned with the spin axis.

4 CURVATURE RADIATION

FRB emission by a strongly magnetized neutron star has been explained as coherent curvature radiation (Kumar, Lu & Bhattacharya 2017). Its spectrum is the product of the spectrum of radiation emitted by accelerated point charges and the spectrum of the spatial structure of the coherent charge density distribution (Katz 2018b). The spectrum emitted by an accelerated point charge is very smooth and broad, so the observed spectral structure must be attributed to the distribution of charge density. The frequency and spectrum of the emitted radiation is determined by the phase velocity (pattern speed) of the deviations from charge neutrality that radiate. This must be distinguished from the velocities of the individual charges that also radiate incoherently. Describing the phase velocity of the plasma wave that bunches the charge density by its corresponding Lorentz factor $\gamma_w$, its minimum value $\gamma_{\text{min}}$ for observed curvature radiation of angular frequency $\omega$ is

$$\gamma_{\text{min}} \approx \left( \frac{3wR}{c} \right)^{1/3},$$

where $R$ is the radius of curvature of the guiding magnetic field line.

We have no direct evidence that the observed radiation is near this peak of the spectral envelope of curvature radiation (the actual dynamic spectra of FRB are determined by the spatial structure of their charge distribution; (Katz 2018b)), but selection effects favor the detection of the brightest radiation and make that plausible. This is the same argument that justifies the assumption of particle-field equipartition in incoherent synchrotron sources: the most efficient radiators are the most detectable. Taking $R \sim 10^6$ cm, the neutron star radius, because the available energy density decreases rapidly with increasing distance from the neutron star, leads to an estimate $\gamma_{\text{min}} \approx 100$, only weakly dependent on the uncertain parameters. The observed, comparatively narrow but varying, spectral bands of FRB radiation imply that there are comparatively few charge “bunches” radiating at any one time. If there were $\gtrsim \omega/\Delta \omega \sim 10$ such bunches, where $\Delta \omega$ is the width of an individual band, each would have a different peak frequency of radiation corresponding to a peak in the Fourier transform of the spatial distribution of charge. The total spectrum of radiation, a sum over many such peaks, would be smooth and broad, rather than being confined to a few narrower bands as observed.

4.1 Radiating Charges

We model this distribution of charge density as a single charge $Q$, the amplitude of the peak of that Fourier transform; a actual point charge $Q$ would radiate a very broad and smooth spectrum, not seen. The frequency-integrated power received per unit solid angle (Rybicki & Lightman 1979)

$$\frac{dP}{d\Omega} = \frac{4Q^2a^2}{\pi^3} \frac{\gamma^4_w}{(1 + \gamma^2_w)^2} \frac{1 - 2\gamma^2_w \theta^2 \cos 2\phi + \gamma^2_w \theta^4}{1 + \gamma^2_w \theta^4},$$

where $a_\perp \approx c^2/R$ is the magnitude of the acceleration perpendicular to the velocity (and magnetic field line), $\theta$ is the angle between the direction of observation and the velocity vector and $\phi$ is an azimuthal angle. The half-width at half power of the radiation pattern $\theta_{1/2} \approx 0.35/\gamma_w$. For $\gamma_w \theta \gg 1$ the final factor varies $\propto (\gamma_w \theta)^{-8}$, cancelling the factor of $\gamma_w$, leading to a result independent of $\gamma_w$ but $\propto \theta^{-8}$. Taking $\gamma_w \theta \ll 1$ and Eq. 1 for $\gamma_w = \gamma_{\text{min}}$

$$Q \approx 0.2 \frac{\epsilon^2}{R^{1/3} \omega^{4/3}} \frac{dP}{d\Omega} \approx 5 \times 10^{-8} \sqrt{\frac{dP}{d\Omega}},$$

in Gaussian cgs units for L-band radiation. If $\gamma_w \gg \gamma_{\text{min}}$ then the spectral peak and most of the radiated power is at frequencies above the observed L-band. As a result of integrating

$$\int d\Omega \frac{dP}{d\Omega} d\omega \times \int_0^{\omega_{\text{max}}} \omega^{1/3} d\omega \propto \omega_{\text{max}}^{4/3} \propto \gamma_w^4$$

up to $\omega_{\text{max}} \sim c^2/(3R)$, the inferred spectrally integrated $dP/d\Omega$ is multiplied by $(\gamma_w/\gamma_{\text{min}})^4$ and Eq. 2 is replaced by

$$\frac{dP}{d\Omega}_{\text{obs}} = \frac{4Q^2a^2}{\pi^3} \frac{\gamma_{\text{min}}^4}{(1 + \gamma^2_w \theta^4)^2} \frac{1 - 2\gamma^2_w \theta^2 \cos 2\phi + \gamma^2_w \theta^4}{1 + \gamma^2_w \theta^4},$$

where $(dP/d\Omega)_{\text{obs}}$ is the measured power density at the observational frequency, henceforth 1400 MHz, corresponding (Eq. 1) to $\gamma_{\text{min}}$. 

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For FRB 200428 (Bochenek et al. 2020; CHIME/FRB Collaboration 2020), taking a bandwidth of 400 MHz, a distance of 6 kpc and emission lasting 3 ms, and for a nominal “cosmological” FRB with a flux density of 1 Jy at $z = 1$

$$\frac{dP}{d\Omega \obs} \sim \begin{cases} 1 \times 10^{36} \text{erg/sterad-s} & \text{FRB 200428} \\ 2 \times 10^{42} \text{erg/sterad-s} & z = 1 \end{cases}$$

and

$$Q \sim \begin{cases} 5 \times 10^{10} \gamma_w^2 \text{esu} = 15\gamma_w^2 \text{Coulombs} & \text{FRB 200428} \\ 8 \times 10^{13} \gamma_w^2 \text{esu} = 3 \times 10^4 \gamma_w^2 \text{Coulombs} & z = 1, \end{cases}$$

where $\gamma_w \equiv \gamma_w/\gamma_{\text{min}} \approx 100 \geq 1$. These are only the charges whose (collimated) radiation is directly observed. There may be additional charges (much larger in total absolute magnitude) radiating in other directions, either simultaneously with the observed FRB, or at other times, if the FRB is part of a wandering or intermittent beam (Katz 2017).

### 4.2 Empirical Lower Limit on the Lorentz Factor

The upper limits set by Lin et al. (2020) on FRB emission during soft gamma-ray flares of SGR 1935+2154 of $\lesssim 10^{-9}$ of FRB 200428 statistically constrain the Lorentz factor $\gamma_w$ of the emitting charges (or their wave or pattern speed) if the emission is produced by acceleration perpendicular to the velocity. This bound applies to synchrotron radiation as well as to curvature radiation.

For a relativistic particle of Lorentz factor $\gamma$, emission at angles $\theta \gg 1/\gamma$ is $O(\gamma^2) \times$ times that for $\theta \ll 1/\gamma$ (Eq. 2). Brightness selection effects make it likely that FRB 200428 was observed at an angle $\theta \lesssim \theta_{1/2} \approx 0.35/\gamma$. If other observed soft gamma-ray bursts of SGR 1935+2154 produced radio bursts similar to FRB 200428 but beamed in directions statistically uniformly but randomly distributed, then of such bursts the closest to the observer was likely at an angle $\theta \approx \sqrt{N}/\gamma$. Then

$$\gamma_w \gtrsim 0.35 \left( \frac{F_{\text{max}}}{F_{\text{min}}} \right)^{1/8} \sqrt{\frac{N}{4}} \approx 10,$$  

where $N = 29$ is the number of FRB outbursts observed by Lin et al. (2020) and $F_{\text{max}}/F_{\text{min}} \sim 10^8$ is the ratio of the brightest FRB observed (FRB 200428) to the upper limits set on all the other SGR outbursts. The effective (half-width at half-power) beam width $\theta_{1/2} \approx 0.35/\gamma_w \lesssim 2^\circ$. Continuing observation, increasing $N$, will either increase the lower bound of Eq. 8 or find a distribution of observed FRB strengths from which their angular radiation pattern may be inferred.

This method cannot be applied to the numerous observed bursts of FRB 121102 because no corresponding gamma-ray activity is detected. Because of limits on the sensitivity of X- and gamma-ray detectors, it is likely to be feasible only for Galactic FRB.

### 4.3 Particle Energies

The requirement that the electrostatic repulsion of the charge bunches not disrupt them sets a lower bound on the particle energy $E_w$ and Lorentz factor $\gamma_{\text{part}}$; an electron must have sufficient kinetic energy to overcome repulsion by the net bunch charge $Q$. Coherent emission requires that the charge bunch extend over a length $\lesssim \lambda = c/\omega = \lambda 2\pi$ in its direction of motion and radiation in order that fields from its leading and trailing edges, arriving at times separated by $\lesssim \lambda/c$, add coherently. The minimum electron energy is

$$E_w = \gamma_{\text{part}} m_e c^2 \gtrsim \frac{Qe}{\lambda},$$

where $\lambda$ is approximately the largest dimension of the charge cloud. If the cloud is roughly spherical $\lambda \sim 3$ cm for L-band radiation

$$E_w = \begin{cases} 5 \gamma_w^2 \text{ TeV} & \text{FRB 200428} \\ 8 \gamma_w^2 \text{ PeV} & z = 1. \end{cases}$$

If the charge density be spread over a width $\ell \sim R/\gamma_{\text{min}} \sim 10^4$ cm transverse to its direction of motion and radiation (a very oblate shape), the maximum permitted by the condition that the fields add coherently,

$$E_w = \begin{cases} 2 \gamma_w^2 \text{ GeV} & \text{FRB 200428} \\ 3 \gamma_w^2 \text{ TeV} & z = 1. \end{cases}$$

The fact that FRB spectral structure typically consists of bands of width $\Delta \omega \sim 0.1\omega$ indicates that the radiating waves have a minimum of $\sim 10$ periodically spaced charge peaks. Individual regions of uncharged balance may have charges an order of magnitude less than indicated by Eq. 7, with a corresponding reduction in $E_w$. These regions radiate coherently so the effective $Q$ is reduced in Eq. 9 but not in Eqs. 2 and 5. This and the uncertain factor $\gamma_2$ may make it possible to reconcile the values of Eq. 11 with the maximum electron energy $\sim 0.2$ TeV, above which curvature radiation is energetic enough to make pairs in the large magnetic field.

If we set $\gamma_w = \gamma_{\text{part}}$ (so the radiating charges are not a wave or pattern speed but the actual particle speed) and use Eq. 5 to determine $Q$ and Eq. 9 we find

$$\gamma_{\text{part}} = \left( \frac{e}{\ell m_e c^2} \right)^{1/3} \left( \frac{\pi e^2}{4c} \frac{dP}{d\Omega \obs} \right)^{1/6} \gamma_{\text{min}}^{-2/3},$$

Numerically

$$\gamma_{\text{part}} \lesssim \begin{cases} 300(10^4 \text{ cm}/\ell)^{1/3} & \text{FRB 20048} \\ 3500(10^4 \text{ cm}/\ell)^{1/3} & z = 1. \end{cases}$$

The corresponding $\gamma_2$ are $\sim 3(10^4 \text{ cm}/\ell)^{1/3}$ and $\sim 35(10^4 \text{ cm}/\ell)^{1/3}$ respectively. The charges $Q$ may be found from Eq. 7.

### 4.4 Accelerating the Electrons

Can electrons be accelerated to the energies indicated Eqs. 10 and 11? We calculate the required electric fields $E$ by equating the power radiated by an electron in curvature radiation to the power delivered by the electric field $\approx eEc$. There are at least two possible criteria:

(i) The power of the incoherent curvature radiation emitted by electrons with energies Eq. 10 or 11, the energies required for electrons to form bunches with the charges inferred from the observed radiation without being disrupted.

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by electrostatic repulsion, must not exceed the power imparted by the accelerating electric field. Their Lorentz factors \( \gamma_{\text{part}} \) are generally much greater than \( \gamma_{\text{min}} \). The power an electron radiates as incoherent curvature radiation (Rybicki & Lightman 1979)

\[
P_{\text{curve}} = \frac{2 e^2}{3 c^2} a^2 \gamma_{\text{part}}^4 \approx \frac{2 e^2}{3 c^2} \frac{Q^2}{R^2} \gamma_{\text{part}}^4. \tag{14}
\]

For a “bunch” of charge \( Q \) the elementary charge \( e \) is replaced by \( Q \) and \( \gamma_{\text{part}} \) is replaced by \( \gamma_{\text{w}} \) if the “bunch” is a wave or pattern on an underlying particle distribution with different Lorentz factors. Equating \( P_{\text{curve}} = e E c \) (Kumar, Lu & Bhattacharya 2017),

\[
E \gtrsim \frac{2}{3} \frac{e^2}{R^2} \gamma_{\text{part}}^4, \tag{15}
\]

where \( \gamma_{\text{part}} = Q e / e m_{\text{e}} c \) (Eq. 9), is required. The resulting numerical values are shown in Table 1. Faraday’s Law limits the electric fields that can be created by induction to \( E \lesssim B \), and vacuum breakdown (Heisenberg & Euler 1936; Schwinger 1951; Stebbins & You 2015) limits it to \( E \lesssim 2 \times 10^{-2} \text{esu/cm}^2 \). The curvature radiation model can be excluded as an explanation of “cosmological” FRB if \( \ell \sim \lambda \) unless \( \gamma_{\text{w}} \gtrsim 10 \), but smaller values of \( \gamma_{\text{w}} \) are consistent with larger but possible values of \( \ell \).

(ii) The electric field must replenish the coherently radiated energy after the charge bunch has formed. As shown in Sec. 4.6, the kinetic energies of the charge bunches are very small, and must be replenished throughout a burst. This criterion is obtained from Eq. 14, replacing \( e \) by \( Q \), using \( \gamma_{\text{w}} = 100 \) and the power delivered by the electric field \( \approx Q E c \)

\[
E \gtrsim \frac{2}{3} \frac{Q}{R^2} \gamma_{\text{w}}^4. \tag{16}
\]

The numerical results are shown in Table 2, and are independent of \( \ell \) because the relevant Lorentz factor \( \gamma_{\text{w}} \) is determined by the observed frequency, not \( \ell \).

It may not be necessary that work done by the electric field continuously replenish the kinetic energy of the coherently radiating charge bunches (Table 2). Energetic particles may be a sufficient energy reservoir, intermittently producing charge bunches by plasma instability, but if electrons cannot be accelerated to sufficient energy to form the necessary charge bunches (as is the case for spherical bunches with \( \ell \sim \lambda \) and smaller \( \gamma_{\text{w}} \)) then sufficient coherent curvature radiation cannot be emitted.

4.5 Origin of Accelerating Electric Field

Currents in a neutron star magnetosphere flow along closed magnetic loops, anchored in the neutron star in analogy to Solar prominences, as in the “magentar” model of SGR. A plasma instability may introduce a region of large “anomalous” resistivity, much greater than the microscopic plasma resistivity, interrupting the current flow and replacing the conductive region with an effective capacitor. Charge builds up on the boundaries of the newly insulating region.

This is described by an \( LC \) circuit with inductance \( L \sim 4 \pi r^2 c^2 \) (in Gaussian units), where \( r \) is the radius of the current loop (that may be as large as the magnetospheric radius \( R \)) and capacitance \( C \sim A/(4\pi\alpha) \), where \( A \) is the cross-section of the current loop (that may be as large as \( R^2 \) for a distributed current) and \( \alpha \) is the width of the gap that becomes insulating. The charge on the surfaces of the gap

\[
Q_{\text{gap}}(t) = Q_0 \sin \frac{t}{\sqrt{LC}} = \sqrt{LC} J_0 \sin \frac{t}{\sqrt{LC}}, \tag{17}
\]

where \( t \) is the time since the insulating gap opened, \( J_0 \) is the interrupted current, and \( Q_0 = \sqrt{LC} J_0 \). For a distributed current and a wide gap \( A \sim r^2, a \sim r, \) and \( \sqrt{LC} \sim r/c \). Then \( J_0 \sim \Delta B r/c, Q_0 \sim \Delta Br^2 / 4\pi \), the voltage drop \( V \sim Q_0 / C \sim \Delta Br \) and the electric field \( E \sim V/a \sim V/r \sim \Delta B \), where \( \Delta B \) is the change in \( B \) when the current loop is interrupted. The fields indicated in the Tables for \( \ell \sim R/\gamma_{\text{w}} \) can be provided by plausible values of \( \Delta B \). The charges \( Q_{\text{gap}}(t) \) are much larger than the radiating charges inferred from Eq. 7, but are not moving relativistically and do not radiate significantly.

Radiation will be emitted by the changing magnetic field. On dimensional grounds, the expression for the power radiated in the dipole approximation is roughly valid, where the dipole moment \( \mu \sim \Delta Br^3 \), varies on a characteristic time scale \( \sim 1/\omega \sim c/r \) and \( r \) is the radius or characteristic size of the loop:

\[
P \sim \frac{\mu^2 \omega^4}{3c^3} \sim \frac{(\Delta B)^2 r^3}{3\omega^2}. \tag{18}
\]

For the maximum plausible \( \Delta B \sim 10^{15} \) gauss and the observed FRB L-band frequency, \( P \sim 10^{41} \text{ergs/s} \) and would be unbeamed, in contradiction to Sec. 4.2 for FRB 200428. Such unbeamed power would be insufficient to power “cosmological” FRB. Narrow beaming would require highly relativistic motion.

Table 1. Minimum values of electric field (upper) (multiply by 300 to convert to V/cm) required to overcome coherent curvature radiation losses during the radiation of a charge bunch. Because the relevant Lorentz factor is that of the coherent wave the results do not depend on the values of \( \ell \) or of \( \gamma_{\text{w}} \) that determine the minimum particle Lorentz factor.

| \( E \) (esu/cm\(^2\)) | FRB 200428 | \( z = 1 \) |
|-------------------------|-------------|----------------|
| \( \ell = \lambda \)    | 3 \times 10^9 \gamma_{\text{w}}^{-8} | 2 \times 10^{19} \gamma_{\text{w}}^{-8} |
| \( \ell = R/\gamma_{\text{min}} \) | 3 \times 10^{-5} \gamma_{\text{w}}^2 | 2 \times 10^{-7} \gamma_{\text{w}}^{-2} |

| \( \ell \) (s) | FRB 200428 | \( z = 1 \) |
|----------------|-------------|----------------|
| \( \ell = \lambda \)    | 2 \times 10^{-7} \gamma_{\text{w}}^2 | 5 \times 10^{-12} \gamma_{\text{w}}^2 |
| \( \ell = R/\gamma_{\text{min}} \) | 6 \times 10^{3} \gamma_{\text{w}}^{-2} | 1 \times 10^{-6} \gamma_{\text{w}}^{-2} |

Table 2. Minimum values of electric field (multiply by 300 to convert to V/cm) required to overcome coherent curvature radiation losses during the radiation of a charge bunch. Because the relevant Lorentz factor is that of the coherent wave the results do not depend on the values of \( \ell \) or of \( \gamma_{\text{w}} \) that determine the minimum particle Lorentz factor.

| \( E \) (esu/cm\(^2\)) | FRB 200428 | \( z = 1 \) |
|-------------------------|-------------|----------------|
| All \( \ell \) | 3 \times 10^4 \gamma_{\text{w}}^{-8} | 5 \times 10^9 \gamma_{\text{w}}^{-8} |
The achievable value of $E$ may be limited by breakdown creation of electron-positron pairs, either the Schwinger vacuum breakdown that occurs for $E \gtrsim 2 \times 10^{15}$ esu/cm², or the curvature radiation-driven pair production cascade breakdown believed to occur in pulsars. Even if breakdown occurs, it may not necessarily “short out” the electric field and accumulated charges because the region of breakdown may still be resistive as a result of plasma instability. If the current loop is wide ($\ell \sim R/\gamma_w$), $E$ may be large enough to accelerate the electrons to the energies necessary to overcome Coulomb repulsion. Each portion of the area $A$ accumulates charge, limited independently by breakdown in the capacitive gap, so that it may be possible to produce the necessary thin sheet charge distribution.

Faraday’s law

$$\nabla \times \mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$

implies

$$E \sim \frac{1}{c} \frac{\Delta B}{\Delta t}. \quad (19)$$

$$\Delta B \leq B \quad \text{(defining $B$ as its maximum magnitude).} \quad (20)$$

This is a general limit on the electric fields that can be produced in a relaxing current-carrying magnetosphere.

Changing the magnetic field within a loop of area $r^2$ by $\Delta B$ in a time $\tau$ produces an inductive electromotive force (EMF)

$$V_{\text{inductive}} \sim \frac{r^2 \Delta B}{c \tau} \sim 3 \times 10^{10} \frac{\Delta B}{10^8 \text{ gauss}} \frac{r^2}{10^2 \text{ cm}^2} \frac{0.1 \text{ s}}{\tau} \text{ esu/cm} \quad (22)$$

an electron energy

$$E_e = eV_{\text{inductive}} \sim 10^6 \frac{\Delta B}{10^8 \text{ gauss}} \frac{r^2}{10^2 \text{ cm}^2} \frac{0.1 \text{ s}}{\tau} \text{ TeV}. \quad (23)$$

In FRB 200428 the EMF required to accelerate particles to the minimum energy for $\ell = \lambda$ and $\gamma_2 = 1$ (Eq. 10) can be provided by $\Delta B \sim 10^8$ gauss if the loop encompasses much of the magnetosphere ($R \sim 10^6$ cm) if $\tau \sim 0.1$ s, as observed for SGR. If $\ell = R/\gamma_w$ and $\gamma_2 = 1$ (Eq. 11), $\Delta B \sim 2 \times 10^8$ gauss would be sufficient. For the nominal 1 Jy-ms FRB at $z = 1$, $\ell = \lambda$ and $\gamma_2 = 1$ would require $\Delta B \sim 10^{11}$ gauss but $\ell \sim R/\gamma_w$ and $\gamma_2 = 1$ would only require $\Delta B \sim 3 \times 10^7$ gauss. Without a detailed understanding of the magnetohydrodynamics and plasma physics of SGR activity we cannot decide if these values are plausible, but they violate no physical law.

4.6 Energetics

The magnetic energy dissipated is obtained using Eq. 22 and $r \sim R$ to obtain the minimum $\Delta B$ required to accelerate electrons to the energy $E_e = V_{\text{inductive}}c$:

$$E \sim \frac{1}{3} B \Delta B R^3 \sim \frac{B \tau Q \ell}{3 \ell} \sim \begin{cases} 3 \times 10^{39} \text{ ergs} \quad \text{FRB 200428} \\ 3 \times 10^{43} \text{ ergs} \quad \text{for } z = 1, \end{cases} \quad (24)$$

where the numerical values assume $\ell \sim \lambda$ (larger $\ell$ would lead to lesser values), $\gamma_2 = 1$ and the observed width of FRB outbursts $\tau \sim 0.1$ s; for FRB 200428 $B = 2 \times 10^{14}$ gauss (Israel et al. 2016) and for the burst at $z = 1$ $B = 10^{12}$ gauss have been assumed. The value of $E$ for FRB 200428 is consistent with the observed X-ray fluences of SGR 1935+2154. For “cosmological” FRB the value of $E$ is consistent with giant outbursts of Galactic SGR, but the argument of Sec. 4.8 indicates that only less powerful SGR outbursts may produce FRB.

Eq. 10 ($\ell = \lambda$) would permit $\sim 10^8$ bursts in the lifetime of SGR 1934+2154 and $\sim 10^9$ repetitions for the nominal “cosmological” FRB if $B \sim 10^{15}$ gauss. The number of repetitions could be several thousand times greater if $\ell = R/\gamma_w$ (Eq. 11). These values are obtained from the required inductive EMF, not directly from the change in magnetostatic energy. If the magnetic field is regenerated from internal motions, there could be yet more repetitions. Weaker bursts, such as observed from FRB 121102, require smaller $Q$, $E_e$, $V_{\text{inductive}}$, and $\Delta B$, and could repeat many more times during the active lifetime of their source.

The electric fields within the charge bunches

$$E \sim \frac{Q}{\ell^2} \sim \begin{cases} 5 \times 10^5 (R/100) \gamma_2 \text{ esu/cm}^2 \quad \text{ FRB 200428} \\ 8 \times 10^5 (R/100) \gamma_2 \text{ esu/cm}^2 \quad z = 1. \end{cases} \quad (25)$$

If $\ell \sim \lambda$ and $\gamma_2 \sim 1$ the field estimated for the cosmological FRB exceeds the Schwinger pair-production vacuum breakdown field (Heisenberg & Euler 1936; Schwinger 1951; Stebbins & Yoo 2015) several-fold. This paradox is resolved if the charge distribution is oblate, with $\ell \gg \lambda$ or if $\gamma_2 \gg 1$. It might seem unlikely that charge would be concentrated into thin sheets perpendicular to its direction of motion and the magnetic field lines, but there is a strong selection effect favoring the observation of such emitting geometry because for it the fields add coherently, making the radiation stronger and more observable.

The kinetic energies of the motion of the net charges $Q$ (Eqs. 7, 9) are very small, $Q \sim e \ell^2$, much less than the magnetic energy. If the charge distribution is oblate, with $\ell \gg \lambda$ or if $\gamma_2 \gg 1$. It might seem unlikely that charge would be concentrated into thin sheets perpendicular to its direction of motion and the magnetic field lines, but there is a strong selection effect favoring the observation of such emitting geometry because for it the fields add coherently, making the radiation stronger and more observable.

4.7 Curvature Radiation vs. Compton Scattering

The relativistic electrons emitting curvature radiation are moving in the soft gamma-ray radiation field of the SGR. It is necessary to compare the power the emit in curvature radiation to their energy loss by Compton scattering. If the latter were to dominate, then it would be difficult to accelerate a population of electrons to the energies necessary to emit a FRB.

The power the electrons lose to Compton scattering is

$$P_{\text{Compt}} \approx n_e N_e \sigma_{KN} E_e c, \quad (26)$$
where

$$n_\gamma \sim \frac{L_{\text{SGR}}}{4\pi R^2 h\nu_c c}$$

(27)

is the number density of soft gamma-rays, $N_e = Q/e$ is the number of electrons in the charge bunch, $\sigma_{KN} \approx \pi r_e^2 \ln (2h\nu_c E_e/m_e^2 c^4)/(h\nu_c E_e/m_e^2 c^4)$ is the Klein-Nishina cross-section ($r_e = e^2/m_e c^2$ is the classical electron radius) and $E_e$ is the electron energy. In this regime of highly relativistic electrons scattering soft gamma-rays, nearly the entire electron kinetic energy is lost to the photon in a single scattering.

For FRB 200428, using Eqs. 7, 9 and 27, $L_{\text{SGR}} \sim 6 \times 10^{39}$ ergs/s (at 6 kpc distance), $h\nu_c \sim 50$ keV (Mereghetti et al. 2020) and $\gamma_w = 100$, Eqs. 14 and 26 yield

$$\frac{P_{\text{Compt}}}{P_{\text{Curve}}} \sim \frac{8}{3} \frac{Qc(h\nu_c)^2 \gamma_w}{L_{\text{SGR}}}$$

$$\sim 300 \frac{6 \times 10^{39}}{L_{\text{SGR}}} \text{ ergs/s}.$$  

This value is uncertain, but is consistent with the assumption that Compton scattering losses do not exceed the radiated power and therefore the validity of Eq. 16 as a condition on the electric field. The use in Eq. 26 of the lower bound Eq. 9 on $E_e$ is balanced, except for the slowly varying logarithm, by the energy dependence of $\sigma_{KN}$.

Despite the intense soft gamma-ray radiation field, the quadratic dependence of the coherent $P_{\text{Curve}}$ on $Q$ makes it possible for it to exceed $P_{\text{Compt}}$ that is only proportional to one power of $Q = N_e e$. An additional factor of $Q$ enters $P_{\text{Compt}}$ through the minimum electron energy (Eq. 9), but this is nearly cancelled by the inverse energy dependence of the Klein-Nishina cross-section. The number of coherently radiating charges in the bunch or wave $N_e = Q/e \sim 10^{20}$ for FRB 200428 and $\sim 10^{23}$ for the cosmological FRB. These enormous values and the quadratic dependence on $Q$ (or $N_e$) that makes the FRB bright enough to observe also make Compton losses comparatively unimportant.

### 4.8 Why Not SGR 1806–20

The strongest argument against the SGR-AXP hypothesis was empirical: During an unrelated observation, the giant 27 December 2004 outburst of SGR 1806–20 was in a radio telescope sidelobe but no signal was detected from it (Tendulkar, Kaspi & Patel 2016). Although the sidelobe had sensitivity about 70 dB less than that of the main beam, the fact that the SGR was $\sim 3 \times 10^5$ times closer than a typical “cosmological” FRB, as well as the extraordinary brightness of the SGR, led to an upper limit on the ratio of the radio to soft gamma-ray fluences of $< 10^7$ Jy-ms/(erg/cm²).

There are at least two possible explanations.

(i) Eq. 28. The soft gamma-ray luminosity of SGR 1806–20 during its giant outburst (Palmer et al. 2005) was more than seven orders of magnitude greater than that of SGR 1935+2154 during FRB 200428; this was only partially offset by a value of $h\nu_c$ less than or equal to the observed flux ratio $> 2 \times 10^2$ Jy-ms/(erg/cm²) of FRB 200428/SGR 1935+2154.

There are at least two possible explanations.

(ii) The observations of FRB 200428 (Lin et al. 2020) indicate that the observable FRB/SGR ratio may vary from burst to burst by at least eight orders of magnitude, likely because of beaming (Sec. 4.2).

## 5 DISCUSSION

The discovery and identification of FRB 200428 resolved the first question about FRB: What astronomical objects produce them? It took 13 years from their discovery (and 7 years from the time their reality became generally accepted) to answer this question because of the difficulty of accurate localization. The similar difficulty of localizing gamma-ray bursts meant that their identification took 25 years, as did the recognition of extra-Galactic radio sources as the products of Active Galactic Nuclei (AGN).

Identification of FRB with rotating neutron stars predicts that FRB activity should be modulated, at some level, at the rotation rate. Periodicity has not been observed in FRB 121102, the only FRB for which abundant data exist (Zhang et al. 2018); see discussion in Katz (2019). If “cosmological” and Galactic FRB are qualitatively similar phenomena, periodicity should be detectable in any FRB that repeats frequently. Periodicity will be easier to detect in FRB identified with Galactic SGR because their periods would be known a priori from gamma-ray observations of the SGR/AXP.

The magnetospheric densities implied by Eq. 7 and the constraint on the dimensions of a radiating charge bunch $< R/\gamma_w$ exceed the critical plasma density at observed FRB frequencies for the parameters of cosmological FRB. However, this limit on propagation is inapplicable. The plasma is strongly magnetized (so strongly that the electrons’ motion transverse to the field, the direction of the electric vector of a transverse wave propagating along the field, is quantized). In addition, the electrons’ longitudinal motion is highly relativistic (Eq. 9), increasing their effective mass by the factor $\gamma_{\text{part}}$. Finally, the radiating charge bunches may be confined to a shell thinner than the skin depth, like the currents in a metallic antenna radiating radio waves. Propagation and escape of the radiation are beyond the scope of this paper, but are issues that must be faced by any model in which FRB are emitted from a compact region, as required by their narrow temporal structure.

Identification of FRB with SGR does not itself explain their mechanism. Their high brightness temperatures require coherent emission, but there is no understanding of their charge bunching. Even in pulsars, discovered 53 years
ago, the mechanism of charge bunching remains uncertain. Acceleration of relativistic particles is nearly ubiquitous in astrophysics (Katz 1991), and is also required to explain FRB, but is not understood from first principles; if we had not inferred it from observations in AGN, Solar activity, supernova remnants, pulsars, FRB and many other phenomena, we would not have predicted it.

The presence of an intense thermal (X-ray and soft gamma-ray) radiation field interferes with the acceleration and propagation of relativistic electrons. At sufficiently high radiation energy densities, radiative and particle energy thermalizes to a dense equilibrium pair-photon plasma. This predicts that SGR with luminosities $\gtrsim 10^{42}$ ergs/s do not make FRB comparable to FRB 200428.

The issues discussed here of the radiating charges $Q$ and their implied electric fields extend beyond curvature radiation models, and apply however the charges are bunched, whether by plasma instability, maser amplification, or another mechanism. In any model, radiation can only be produced by accelerated charges or changing currents. It is difficult to produce beaming from changing currents because conservation of charge and the assumption of quasi-neutrality imply that current is constant along bundles of field lines; a relativistically moving current front cannot be produced without creating net charge density. The required $Q$ are determined by the very general Eq. 7 and the particle Lorentz factors by Eq. 9 that are not specific to curvature radiation. This does not exclude sources outside an inner neutron star magnetosphere, but Eq. 2 applies and smaller $Q$ imply larger $\gamma_w$, narrower beaming and, if $\gamma_w$ is the Lorentz factor of an actual particle bunch, higher particle energy.

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