Modelling transverse dunes

Veit Schwämmle(1) and Hans J. Herrmann(1)

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Abstract

Transverse dunes appear in regions of mainly unidirectional wind and high sand availability. A dune model is extended to two dimensional calculation of the shear stress. It is applied to simulate dynamics and morphology of transverse dunes which seem to reach translational invariance and do not stop growing. Hence, simulations of two dimensional dune fields have been performed. Characteristic laws were found for the time evolution of transverse dunes. Bagnold’s law of the dune velocity is modified and reproduced. The interaction between transverse dunes led to interesting results which conclude that small dunes can pass through bigger ones.

1 Introduction

Transverse dunes have been investigated with constant interest recently. Field measurements have been carried out in order to obtain more knowledge about airflow and sediment transport over transverse dunes (Wilson 1972; Lancaster 1982; Lancaster 1983; Mulligan 1988; Burkinshaw and Rust 1993; McKenna Neuman and Nickling 2000). Several numerical models have been proposed to model dune formation (Wippermann and Gross 1986; Zeman and Jensen 1988; Fisher and Galdies 1988; Stam 1997; Nishimori, Yamasaki, and Andersen 1999; van Boxel, Arens, and van Dijk 1999; van Dijk, Arens, and van Boxel 1999; Herrmann and Sauermann 2000; Herrmann and Sauermann 2000; Moniji and Warren 2000; Sauermann, Kroy, and Herrmann 2001; Kroy, Sauermann, and Herrmann 2002). But still many questions are not answered and some efforts are needed to better understand dune morphology and dynamics or even to anticipate dune formation. The simulations in this article will try to answer some questions and justify the importance of further field measurements.

First some general aspects will be introduced. A model for dunes in three dimensions will show why translational invariance appears in transverse dune formation. The time evolution and the dune velocity in a two dimensional model with constant sand influx will be presented. The time evolution of a model of two dimensional dunes with periodic boundary conditions will give some more insight into dune field dynamics. A final discussion of the statement that dunes behave like solitons will close this article.
2 General aspects of transverse dunes

About 40% of all terrestrial sand seas are covered by transverse dunes. They are mostly located in sand seas where sand is available. Thus in larger sand seas one mainly finds ensembles of many transverse dunes which interact with respect to their dynamics. The crest to crest spacing ranges from a few meters to over 3 km [Breed and Grow 1979]. They are common in the Northern Hemisphere, for example in China, and along coasts. On Mars transverse dunes dominate the sand seas. An example of a field of transverse dunes in China is shown in Figure 1.

![Figure 1: An aerial photo (STS047–153A–263) of transverse dunes in Lop Nur, China. Image courtesy of Earth Sciences and Image Analysis Laboratory, NASA Johnson Space Center](http://eol.jsc.nasa.gov)

Normally transverse dunes have more irregular patterns and even smaller hierarchies of smaller transverse dunes can be found in a field of big transverse dunes. Strong winds coming essentially from the same direction are the main environment where this type of dune can be found. Cooke, Warren, and Goudie (1993) proposed that in highly mobile environments cross winds distort the dune shape.

In Figure 2 a dune of a height of about 27 m is depicted which is part of a
two dimensional calculation of our model. This dune is situated between other similar dunes which together result in a simulation of a dune field of a length of four kilometers. The dashed curve shows the sand flux which is set to zero at the slip face where the shear stress is not strong enough to entrain sand. This occurs in the region of flow separation, defining the boundary between quasi-laminar flow and the turbulent layer of the eddy after the brink. The brink separates windward side and slip face. In this case the brink and the crest do not coincide which means in this case that this dune has not yet reached a stationary state.

3 The dune model

The model described here can be seen as a minimal model including the main processes of dune morphology. As predecessor of the model described here the work of [Sauermann (2001)] revealed interesting new insights into dynamics and formation. The model is now extended to a two dimensional shear stress calculation (longitudinal and lateral direction), a full sand bed and different boundary conditions. Nevertheless the wind is restricted to be constant and unidirectional in time. In this article the model is used to simulate transverse dune fields. In every iteration the horizontal shear stress $\tau$ of the wind, the saltation flux $q$ and the flux due to avalanches are calculated. The time scale of these processes is much smaller than the time scale of changes in the dune surface so that they are treated to be instantaneous. In the calculation of the
surface evolution a time step of 3–5 hours is used. In the following the different steps at every iteration are explained.

The air shear stress $\tau$ at the ground: The shear stress perturbation over a single dune or over a dune field is calculated using the algorithm of Weng, Hunt, Carruthers, Warren, Wiggs, Livingstone, and Castro (1991). The $\tau_x$-component points in wind direction and the $\tau_y$-component denotes the lateral direction. The calculation is made in Fourier space, where $k_x$ and $k_y$ denote the wave numbers,

$$
\hat{\tau}_x(k_x, k_y) = \frac{h(k_x, k_y)k_x^2}{|k|} \frac{2}{U^2(l)} \left( 1 + \frac{2 \ln L |k_x| + 4 \gamma + 1 + i \text{sign}(k_x)\pi}{\ln l/z_0} \right),
$$

and

$$
\hat{\tau}_y(k_x, k_y) = \frac{h(k_x, k_y)k_xk_y}{|k|} \frac{2}{U^2(l)},
$$

where $|k| = \sqrt{k_x^2 + k_y^2}$ and $\gamma = 0.577216$ (Euler’s constant). $U(l)$ is the normalized velocity of the undisturbed logarithmic profile at the height of the inner region $l$ (Sauermann 2001) defined in Weng, Hunt, Carruthers, Warren, Wiggs, Livingstone, and Castro (1991). The roughness length $z_0$ is set to 0.0025 m and the so called characteristic length $L$ to 10 m.

Equations (1) and (2) are calculated in Fourier space and have to be multiplied with the logarithmic velocity profile in real space in order to obtain the total shear stress. The surface is assumed to be instantaneously rigid and the effect of sediment transport is incorporated in the roughness length $z_0$. For slices in wind direction the separation streamlines in the lee zone of the dunes are fitted by a polynomial of third order going through the point of the brink $x_0$. The length of the separation streamlines is determined by allowing a maximum slope of $14^0$ (Sauermann 2001). Where the separation streamlines cross the surface at the reattachment point $x_1$, a new streamline is calculated as third polynomial attaching $x_0$ and $x_1$. The separation bubble guarantees a smooth surface and the shear stress in the area of the separation bubble is set equal to zero. Problems can arise due to numerical fluctuations in the value of the slope of the brink where the separation bubble begins and its influence on the calculation of a separation streamline for each slice. To get rid of this numerical error the surface is Fourier-filtered by cutting the small frequencies. Figure 3 depicts the separation bubble and an interesting similarity of the separation bubble profile to a sine function.

The saltation flux $q$: The time to reach the steady state of sand flux over a new surface is several orders of magnitude smaller than the time scale of the surface evolution. Hence, the steady state is assumed to be reached instantaneously. The length scale of the model is too large to include sand ripples. Nevertheless
Figure 3: The transverse dune profile $h(x)$, its separation bubble $s(x)$ and a sine function. The separation bubble ensures a smooth surface. The wind is coming from the left.

The kinetics and the characteristic length scale of saltation influence the calculation by breaking the scale invariance of dunes and by determining the minimal size of a barchan dune (Sauermann, Kroy, and Herrmann 2001). A calculation of the saltation transport by the well known flux relations (Bagnold 1941; Lettau and Lettau 1978; Sørensen 1991) would restrict the model to saturated sand flux which is not the case for example at the foot of the windward side of a barchan dune due to little sand supply or at the end of the separation bubble in the interdune region between transverse dunes due to the vanishing shear stress in the separation bubble. The sand density $\rho(x, y)$ and the grain velocity $u(x, y)$ are integrated in vertical direction and calculated from mass and momentum conservation, respectively. We simplified the closed model of (Sauermann, Kroy, and Herrmann 2001) by neglecting the time dependent terms and the convective term of the grain velocity $u(x, y)$. The expansion to two dimensions yields Equations (3) and (5) where $\rho$ and $u$ are determined from the before obtained shear stress and the gradient of the actual surface,

$$\text{div}(\rho u) = \frac{1}{T_s} \rho \left( 1 - \frac{\rho}{\rho_s} \right) \begin{cases} \Theta(h) & \rho < \rho_s \\ 1 & \rho \geq \rho_s \end{cases},$$

with

$$\rho_s = \frac{2\alpha}{g} (|\tau| - \tau_t) \quad T_s = \frac{2\alpha|u|}{g} \frac{\tau_t}{\gamma(|\tau| - \tau_t)}.$$
and
\[ \frac{3}{4} C_d \frac{\rho_{\text{air}}}{\rho_{\text{quartz}}} d^{-1} (v_{\text{eff}} - u) |v_{\text{eff}} - u| \frac{u}{2\alpha |u|} - g \nabla h = 0, \]
where \( v_{\text{eff}} \) is the velocity of the grains in the saturated state,
\[ v_{\text{eff}} = \frac{2u_a}{\kappa |u_a|} \left( \frac{z_1}{z_m} u_a^2 + \left( 1 - \frac{z_1}{z_m} \right) u_{st}^2 + \left( \ln \frac{z_1}{z_0} - 2 \right) \frac{u_{st}}{\kappa} \right), \]
and
\[ u_a = \sqrt{\tau/\rho_{\text{air}}} \]
The constants and model parameters have been taken from (Sauermann, Kroy, and Herrmann 2001) and are summarized here: \( g = 9.81 \text{ m s}^{-2}, \kappa = 0.4, \rho_{\text{air}} = 1.225 \text{ kg m}^{-3}, \rho_{\text{quartz}} = 2650 \text{ kg m}^{-3}, z_m = 0.04 \text{ m}, z_0 = 2.5 \times 10^{-5} \text{ m}, D = d = 250 \mu\text{m}, C_d = 3, u_{st} = 0.28 \text{ m s}^{-1}, \gamma = 0.4, \alpha = 0.35 \) and \( z_1 = 0.005 \text{ m}. \) The sand density and the sand velocity define the sand flux over a surface element \( q(x, y) = u(x, y) \rho(x, y). \)

**Avalanches:** Surfaces with slopes which exceed the maximal stable angle of a sand surface, the called angle of repose \( \Theta \approx 34^\circ, \) produce avalanches which slide down in the direction of the steepest descent. The unstable surface relaxes to a somewhat smaller angle. For the study of dune formation two global properties are of interest. These are the sand transport downhill due to gravity and the maintenance of the angle of repose. To determine the new surface after the relaxation by avalanches the model proposed by Bouchaud, Cates, Ravi Prakash, and Edwards (1994) is used,
\[ \frac{\partial h}{\partial t} = -C_a R (|\nabla h| - \tan \Theta) \]
\[ \frac{\partial R}{\partial t} + \nabla (R u_a) = C_a R (|\nabla h| - \tan \Theta), \]
where \( h \) denotes the height of the sand bed, \( R \) the height of the moving layer, \( C_a \) is a model parameter and the velocity \( u_a \) of the sand grains in the moving layer is obtained by,
\[ u_a = -u_a \frac{\nabla h}{\tan \Theta}, \]
where \( u_a \) is the velocity at the angle of repose. Like in the calculation of the sand flux the steady state of the avalanche model is assumed to be reached instantaneously. In the dune model a certain amount of sand is transported over the brink to the slip face and in every iteration the sand grains are relaxed over the slip face by this avalanche model determining the steady state.
The time evolution of the surface  The calculation of the sand flux over a not stationary dune surface leads to changes by erosion and deposition of sand grains. The change of the surface profile can be expressed using the conservation of mass,

$$\frac{\partial h}{\partial t} + \nabla \Phi = 0,$$

where $\rho$ is the sand density and $\Phi$ the sand flux per time unit and area. Both $\rho$ and $\Phi$ are now integrated over the vertical coordinate assuming that the dune has a constant density of $\rho_{\text{sand}}$,

$$h = \frac{1}{\rho_{\text{sand}}} \int \rho dz, \quad q = \int \Phi dz.$$

Thus Equation (11) can be rewritten, as

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho_{\text{sand}}} \nabla q.$$

Finally, it is noted that Equation (13) is the only remaining time dependent equation and thus defines the characteristic time scale of the model which is normally between 3–5 hours for every iteration.

The initial surface and boundary conditions: As initial surface we take a plain sand bed of arbitrary sand height over the solid ground. The surface can additionally be disturbed by small Gaussian hills. An initial surface has to be smooth (at least under consideration of the separation bubble) and have slopes not larger than the angle of repose. The boundary conditions influence the surface height $h$ with its separation bubble, the sand flux $q$ and the height $R$ of the moving layer of the avalanche model. The boundary in both directions $x$ and $y$ with respect to the direction of the incoming wind are open or periodic. At open boundary in $x$–direction an additional parameter controls the sand influx $q_in$ into the simulated dune field. It is set constant along the lateral direction at $x = 0$.

All calculations presented in the following sections are made with the conditions of a completely filled sand bed and unidirectional wind. All simulations model dune fields instead of single dunes.

4 The model of three dimensional dunes and translational invariance

The main aim in this section is to justify why models of two dimensional dunes can be used in the following sections. The advantage to omit the lateral dimension makes it possible to look at larger dune fields within a still tolerable cost of computational time. A plain initial surface would lead to no change of the height profile. This is because the system needs at least one small fluctuation to begin dune growth.
Therefore as initial surface a large number of low Gaussian hills is introduced. The simulation models dune dynamics for a dune field of a length and width of 400 m and 200 m, respectively. The boundary conditions are periodic in wind direction and open in lateral direction and the shear velocity is $u_\ast = 0.45\,\text{m}\,\text{s}^{-1}$. First the Gaussian hills lead to a growth at their corresponding positions on the dune field. After some time they build a slip face which extends its size in lateral direction.

We assume that transverse dunes try to reach a state of translational invariance. For an illustration of this dynamics see Figure 4. The slip faces become wider until they reach the lateral boundary and extend over the entire width of the simulated fields. Thus the slip face traps all the sand going over the brink. The trapped sand relaxes there through avalanching and maintains the angle of repose constant. Shear stresses of the wind field transport more sand over the brink than that which is transported down at the slip face by avalanches. Hence, transverse dunes are growing wherever there is a part of their lee zone in which saltation transport can be neglected (Section 5). When dunes grow the length of their separation bubble increases. But for a dune field with a limited length due to the periodic boundary condition and to a certain number of dunes which increase their mutual distance there is a state where the number of dunes must decrease by one. This leads to a displacement of the dune with the lowest
height which looses its sand to the next dune situated upwind. In this state the
system breaks the symmetry of translational invariance and a part of the slip
face disappears. There sand is transported to the following dune by saltation.
When the dune has vanished once again the system approaches translational
invariance. Hence, the effect of converging dunes perturbs the steady growth of
transverse dunes in a field with a periodic boundary. To get more information
how a system of transverse dunes shows translational invariance a simulation
of a dune fiel with periodic boundary in both horizontal directions is made.
The field extends over 400 m in length and width. As initial surface also small
Gaussian hills are used. Figure 6 shows the height profile after 5,000 iterations
which corresponds to a time of 1.19 years. The results of this simulation yield
the same conclusions as the simulation with open lateral boundary. Also each
merging of two dunes leads to a breaking of the symmetry. Figure 6 shows
the system at a state close to translational invariance. The final state of the
calculation is reached when only one dune is left. A simulation with periodic
boundaries in both directions shows no state where the system has a struc-
ture or oscillations in lateral direction. The conclusion from these calculations
for three dimensional dunes would be that an open system of transverse sand
dunes reaches translational invariance under ideal unidirectional wind. In the
precedent simulations the periodic boundary inhibited the system to break the
invariance. A calculation with open boundary conditions in both horizontal di-
rections which is closer to real dune fields would give more information about
this assumption but is rather complicated due to the fact that the dunes move
out of the simulated area. More insight is given in the following section. Ass-
suming translational invariance simulations of fields of two dimensional dunes
are much more effective. They consume much less computational time and give
the opportunity to simulate larger dune fields. In the following two sections we
consider two situations of two dimensional dunes, a model with constant sand
influx and a model with periodic boundary. Both models lead to new interesting
conclusions.

5 The model of two dimensional dunes with con-
stant sand influx

The free parameters for this simulation are the sand influx $q_{in}$ and the shear
velocity $u_\ast$. As initial surface we choose a plain ground filled with sand because
the sand influx differs at least a little bit from the saturation flux on the dune
field. Thus dune formation is initiated at the beginning of the dune field, i.e.
where wind comes in.

5.1 Time evolution

The height profile of a dune field with a length of 4 km is presented at different
times. The sand influx is set $q_{in} = 0.017$ kg m$^1$s$^{-1}$ and the shear velocity
$u_\ast = 0.5$ m s$^{-1}$. A sand influx $q_{in}$ which is not equal to the sand flux of
Figure 5: Surface of a transverse dune field with periodic boundary conditions in both directions 1.19 years after initiation. The shear velocity is $u_* = 0.4$ m s$^{-1}$. Height units are in meters, length and width units in two meters.

Figure 6: Surface of a transverse dune field with periodic boundary conditions in both directions 9.5 years after initiation. The system reached a state close to translational invariance. The shear velocity is $u_* = 0.4$ m s$^{-1}$. One unit corresponds to the length of 2 m.
saltation transport over a plain surface lowers or raises the inlet of the dune field constantly. This initiates a small oscillatory structure which begins to move in wind direction (Figure 7) and generates more and more small dunes. The initiating dunes at the inlet of the dune field have increasing size in length.

Figure 7: Surface of a two dimensional simulation with constant sand influx after 0.23 years in the left figure and 1.14 years on the right. The shear velocity is \( u_* = 0.5 \text{ m s}^{-1} \) and sand influx is \( q_{in} = 0.017 \text{ kg m}^{-1}\text{s}^{-1} \). In the left figure we see that initiation of dune formation began at \( x = 0 \). Dune height decreases with the distance to the inlet of the dune field.

Figure 8: Surface of a two dimensional simulation with constant sand influx after 4.56 years in the left figure and 45.58 years on the right. The shear velocity is \( u_* = 0.5 \text{ m s}^{-1} \) and sand influx is \( q_{in} = 0.017 \text{ kg m}^{-1}\text{s}^{-1} \). On the left the first dune does not have a slip face. A final stationary surface is not reached.

and height. The increasing difference between the starting points and the first crest leads to the creation of bigger dunes. With increasing size the dunes have a lower velocity \( v_{dune} \) (Section 5.2). Hence, dune spacing, the distance between adjacent crests, increases with time and no dune collides or converges with another. So real dune fields can maintain a structure of translational invariance without the effect of the breaking of symmetry which was found in Section 4.

Dune fields where the sand influx stays rather constant in time can have a more regular structure than dune fields where the sand influx varies strongly with respect to time.
The slip face of the first dune is missing or is short due to the short evolution time (Figure 8). An observation of this absence for example at a coast where the sea provides a certain amount of sand supply was not found in the literature. The simulation depicted here and any other simulation of dune fields with lengths of 1 km to 4 km do not show a system that reaches a stationary state. According to Cooke, Warren, and Goudie (1993) older dune fields and less climate changes produce larger transverse dunes. The smaller slope at the brink found in this modeling agrees also with qualitative observations. In the model a dune height of 100 m is reached in roughly 50 years. Estimates have predicted some 10,000 years to develop a 100 m dune. Probably this large difference can be explained by a smaller average wind velocity, changes of wind direction over longer periods and changes in sand supply and climate acting on real dunes. The so called memory (the time to build a dune beginning with a plain sand bed) is related to the ratio of height of the dune and annual rate of sand flux \( H/Q_{ann} \) (Cooke, Warren, and Goudie 1993). The memory can vary by about four orders of magnitude in time.

The results of the numerical calculations with different sand influxes \( q_{in} \) at the same shear velocities lead to the conclusion that there is a direct dependency between influx and height growth of the first dune (Figure 9). The nearer the sand influx gets to the saturation flux of saltation transport the slower increases the height of the first dune. Hence, dune fields where the sand influx varies strongly in time around the saturation flux of saltation initiate first dunes with different heights. So there can be smaller dunes moving faster into bigger ones and the symmetry breaking explained in the Section 4 will occur. In the following some relations found for the time evolution of transverse dunes are presented. Figure 9 indicates also that height versus time increases with a power law as was also found in the model of Momiji (2001),

\[
h(t) \propto \sqrt{a \cdot t},
\]

where \( a \) is a parameter which is dependent on shear velocity and sand influx. \( a \) is a measure for the growth rate. The growth rate seems to be smaller for a larger distance from the beginning of the dune field. There the dunes contain longer slip faces because of their higher age. We observe from Figure 9 that the growth rate should converge to a constant value for very large distances. The same relation is found for the spacing \( d_{ij} \) of the dunes \( i \) and \( j \) (Figure 10),

\[
d_{ij}(t) \propto \sqrt{b \cdot t},
\]

where \( b \) denotes a parameter which measures the spacing rate. This spacing rate approaches the same value for all dunes far away from the influx region. The values fitting well to these rates are approximately \( b = 8.53 \cdot 10^{-5} \text{ m}^2\text{s}^{-1} \) and \( b = 1.56 \cdot 10^{-4} \text{ m}^2\text{s}^{-1} \) for a shear velocity \( u_* = 0.4 \text{ m s}^{-1} \) and \( u_* = 0.5 \text{ m s}^{-1} \), respectively. Figure 11 shows the spacing height relationship which seems to be linear. This agrees with the data of measurements of transverse dunes in the Namib Sand Sea in Namibia by Lancaster (1983). For different shear stresses the slope stays constant whereas the axis intercept of \( h \) increases with higher
shear velocities. Hence, the dunes in a transverse dune field are located closer to each other for higher shear velocities.

5.2 Dune velocity

The validity of Bagnold’s law,

$$v_{dune} = \frac{\Phi_{dune}}{h},$$  \hspace{1cm} (16)

where $\Phi_{dune}$ is the bulk flux of sand blown over the brink has been shown by observations of real dunes. According to Sauermann (2001) a better fit is given by using instead of the height $h$ the characteristic length $l$, i.e. the length of the envelope comprising the height profile and the separation bubble. Bagnold’s law already fits quite well. Nevertheless, the Equation (16) is generalized. The length of the envelope can be expressed as a function of the height. The function is developed into a Taylor series and orders higher than the linear order are neglected. This finally yields,

$$v_{dune} = \frac{\Phi_{dune}}{h + C},$$  \hspace{1cm} (17)
where $C$ denotes a constant. In Figure 12, the dune velocities with respect to the height and their fits to Equation (17) are compared for the shear velocities $u_* = 0.4 \text{ m s}^{-1}$ and $u_* = 0.5 \text{ m s}^{-1}$. The observed bulk fluxes are $\Phi_{dune} = 454.5 \text{ m}^2\text{s}^{-1}$ and $\Phi_{dune} = 833.3 \text{ m}^2\text{s}^{-1}$ with the corresponding constants $C = 0.45 \text{ m}$ and $C = 1.08 \text{ m}$, respectively. The values are smaller than the bulk fluxes of isolated 2-dimensional dunes calculated by Sauermann (2001) on plain ground without sand. Thus the velocities are also smaller than those observed for isolated transverse dunes on plain ground without sand. Lancaster (1985) found less speed-up of the wind velocity over continuous sand dunes than over isolated transverse dunes. This agrees with the results in this model.

6 The model of two dimensional dunes with periodic boundary

The simulations in this section have a periodic boundary condition in wind direction. So the parameter of sand influx that was used additionally before is no more available. Sand influx is set equal to the outflux. The avalanches flow down even if the slip face is divided by the boundary. Also the separation bubble follows the periodic boundary conditions. The calculations are made with dune fields of a length of two kilometers. The ground is completely filled up with sand. The initial surface consists of small Gaussian hills which disturb
Figure 11: Relationship between spacing and height of the dunes for the shear velocities $u_\ast = 0.4 \text{ ms}^{-1}$ and $u_\ast = 0.5 \text{ ms}^{-1}$. The dependency seems to be linear.

6.1 Time evolution

The situation is similar to the simulations of three dimensional transverse dune fields. In the beginning many dunes of different size grow in the system but after some time the dunes approach same heights. As in the three dimensional case all the dunes keep growing so that the number of dunes has to decrease. A more detailed description of the process of merging dunes is given in the following section. The periodic boundary forces a decrease of the number of dunes and this process makes the evolution rather complex. Figures 13, 14 and 15 show the height profile of a dune field of a length of two kilometers at three time steps. Figure 13 depicts a surface with dunes of different sizes which seem to interact strongly with each other. The regular state of the system in Figure 14 is disturbed in Figure 15. The number of dunes decreases quite regularly versus time (Figure 16). This process is very slow and the dunes grow much slower than in the model with open boundary.
Figure 12: Dune velocity $v_{\text{dune}}$ versus height $h$ for the shear stresses $u_* = 0.4 \text{ m s}^{-1}$ and $u_* = 0.5 \text{ m s}^{-1}$. The velocity decreases proportional to the reciprocal height.

### 6.2 Do transversal dunes behave like solitons?

Besler (1997) proposed that barchan dunes behave like solitons. Solitons are self-stabilizing wave packs which do not change their shape during their propagation not even after collision with other waves. They are found in non-linear systems like for example in shallow water waves. Observations of barchan dunes seem to give evidence that small dunes can migrate over bigger ones without being absorbed completely. A closer examination of the merging of two or more dunes from calculations of a two dimensional dune field with periodic boundary let to some interesting observations. In this case the dune field cannot break its translational scale invariance to reach a faster colliding of two adjacent dunes. Thus the supposition would be that a smaller dune due to its higher velocity collides with the next bigger one in wind direction without passing over it. That is not the case. As an example see the Figure 17. A small dune climbs up the windward side of the following bigger dune. As it reaches the same height as the following one it seems to hand over the state of being the smaller dune. The dune that was bigger before then wanders towards the following dune of the dune field. This process can be observed several times. The fact that the volume of the smaller dune decreases after every time it passes a dune let us conclude that there will be a final dune which will absorb the small one. Hence, these dunes do not behave exactly like solitons because of their loss of volume.
Figure 13: Surface of a dune field with a length of two kilometers. The shear velocity is $u_\ast = 0.45 \text{ m s}^{-1}$ after 31.7 years, the boundary conditions are periodic. The structure is quite irregular.

The suggestion of [Wilson (1972)] that dunes of different hierarchies (for example dunes and mega dunes) could exist with each other without interactions is not valid in our case. Nevertheless the fact that small dunes can migrate over others leads to the conclusion that different hierarchies of dune sizes can coexist in a field of transverse dunes. The loss of volume demonstrates that there is some interaction between different hierarchies of dune sizes.

Finally Figure [17] shows a part of the same simulation where a very small dune finally swallowed the bigger one and the number of dunes decreases by one.
Figure 14: Surface of a dune field with a length of two kilometers. The shear velocity is $u^* = 0.45 \text{ m s}^{-1}$ after 95.1 years, the boundary conditions are periodic. There are three dunes left with similar heights.

7 Conclusions

It was shown how transverse dune fields can develop with respect to time. None of the simulations gave evidence that a final stage would be reached, neither the model with periodic nor the model with open boundaries. More knowledge about the still not very well understood trapping efficiency in the lee side of the dunes and the inclusion of this effect as done by [Momiji and Warren (2000)] could lead to stable final states. The use of translational invariance, shown in the three dimensional model, made it possible to restrict to the model of two dimensional dunes. All simulations also showed that the evolution of a slip face leads to a continuous growth of transversal dunes, and no final state will be reached. In the model of two dimensional dunes with constant influx the height of the dunes increases proportional with respect to square root of time. The same power law is found for the crest-to-crest spacing in the dune field. The difference of influx and saturation flux seems to play a crucial role in dune size. The same relation between dune velocity and height is found for barchan dunes. Two dimensional modeling with periodic boundary shows that different hierarchies of dunes with different sizes can exist but they interact with each other.
Figure 15: Surface of a dune field with a length of two kilometers. The shear velocity is $u^* = 0.45 \text{ m s}^{-1}$ after 190.2 years, the boundary conditions are periodic. The number of dunes will decrease by one after coalescence of two dunes.

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Figure 16: The number of dunes decreases quite regularly in time.

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Figure 17: Left: Wandering of a small dune over a big one. Right: Coalescence of small dune in a big one. These results are part of a simulation of a transverse dune field with a length of 2km. The shear velocity is $u_* = 0.5\text{ m s}^{-1}$ and boundary conditions are periodic.