Gyrotropic-nihility state in a composite ferrite-semiconductor structure

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Abstract
Characteristics of the gyrotropic-nihility state are studied in a finely-stratified ferrite-semiconductor structure, which is under an action of an external static magnetic field. Investigations are carried out with the assistance of the effective medium theory, according to which the studied structure is approximated as a uniform gyroelectromagnetic medium. The theory of the gyrotropic-nihility state is developed in terms of the eigenwaves propagation in such gyroelectromagnetic medium. The frequency and angular dependencies of the transmittance, reflectance and absorption coefficient are presented. It turns out that in the frequency band around the frequency of gyrotropic-nihility state the studied structure appears to be matched to free space with both the refractive index and the wave impedance which results in its high transmittance almost in the entire range of angles of the electromagnetic wave incidence.

Keywords: electromagnetic theory, magneto-optical materials, effective medium theory, metamaterials

(Some figures may appear in colour only in the online journal)

1. Introduction
The conception of ‘nihility’ media [1] determines a distinctive exotic state of a hypothetical lossless medium whose material parameters happen to be zero quantities. It corresponds to the fulfillment of the constitutive relations where the electric and magnetic flux densities are both equal to zero, \( \vec{D} = 0 \) and \( \vec{B} = 0 \). Thereby, the nihility medium exhibits a zero refractive index, so, there are \( \nabla \times \vec{E} = 0 \) and \( \nabla \times \vec{H} = 0 \), and the wave propagation in such extreme-parameter medium becomes forbidden in the absence of sources therein. It means that the directionality of the phase velocity in relation to the wave-vector is a non-issue for this medium. Although the nihility media determined in such a way are unachievable in nature, they can be approximately simulated at a specified frequency.

As such approximation the theory of ‘epsilon-near-zero’ and ‘mu-near-zero’ materials [2–4] can be considered since it is fundamentally identical with the conception of nihility media. In this theory the Drude medium model is consistently used to simulate a certain dispersive medium in which real parts of both complex permittivity and complex permeability simultaneously acquire zero at a specified frequency, and the refractive index of the resulting medium appears to be zero value, too. It means that while the zero refractive index is not matched to free space, the wave impedance is matched one. The most intriguing properties of such medium is the nearly zero phase progression of propagating waves, which may be of interest in the problems of transformation optics [5], and can be employed for the efficient energy tunneling [6] and for the design of the next generation of waveguides [7].

However, further mixing together electric and magnetic responses of the medium, for example, by utilizing magneto-optic (gyrotropic) and/or bi-anisotropic effects, allows one to drastically change the peculiarities of the wave propagation in nihility media. For instance, in [8] it has been proposed a possible way to achieve a nihility state in a bi-isotropic medium using canonical chiral wire particles. It is found that in such a medium, at a specified frequency, the real parts of both complex effective permittivity and complex effective permeability happen to be close to zero while the chirality...
parameter is maintained at a finite value. This distinct extreme-parameter state is referred to as ‘chiral-nihility’, and it turns out that, in contrast to the conventional nihility medium, in the chiral-nihility both the refractive index and the wave impedance appear to be matched to free space, and the directionality of the phase velocity relative to the wavevector would not be a non-issue any more. So, the wave propagation is allowed in chiral-nihility media, and, in particular, this wave propagation can be described in the term of two orthogonal circularly polarized eigenstates. It means that in this medium there are two eigenwaves with right- and left-circularly polarized states, and, noteworthy, one of these eigenwaves experiences a backward propagation. It results in some exotic characteristics (e.g. wave tunneling and rejection) in the waves interaction with a single layer and a multilayer system consisting of such chiral-nihility media.

More recently, the extreme-parameter nihility state has been also found for the other fundamental class of reciprocal bi-anisotropic media, namely, for omega materials. In contrast to chiral media where the chirality breaks the symmetry of the propagation constants of the circularly polarized eigenwaves while the wave impedances are not affected, in omega materials the properties are dual: the magneto–electric coupling breaks the symmetry of the wave impedances while the propagation constants remain symmetric. In this case, too, the extreme ‘omega-nihility’ state of the omega material appears at a specified frequency, when real parts of both complex permittivity and complex permeability of the medium happen to be close to zero, and the magnetoelectric parameter alone defines the material response. Among other effects, the omega-nihility medium provides an extreme asymmetry in reflection from a material slab: the reflection coefficients from the two opposite sides differ by sign, while the transmission coefficient is symmetric as in any conventional reciprocal material slab.

It is a common knowledge that, besides chiral media, the circularly polarized eigenstates are also inherent to circularly birefringent (gyrotropic) media. These media are characterized by the permittivity or permeability tensor with non-zero off-diagonal elements (gyrotropic parameters). It is no wonder, that properly combining together gyroelectric and gyromagnetic materials into a uniform gyroelectromagnetic structure allows one to reach a ‘gyrotropic-nihility’ state at a specified frequency. In particular, as it has been shown in [17], in a composite finely-stratified ferrite-semiconductor structure the conditions of gyroscopic-nihility state are valid in the microwave band near the frequencies of ferromagnetic and plasma resonances. In this case real parts of diagonal elements of both complex effective permittivity and complex effective permeability tensors of such artificial medium simultaneously acquire zero, while the off-diagonal elements appear to be non-zero quantities. It is revealed that in this structure, in the certain frequency band, the backward propagation takes place for one of the circularly polarized eigenwaves which can lead to some unusual features of the system and provides an enhanced polarization rotation, impedance matching and complete light transmission.

The objective of this paper is a formalization of the theory of the gyrotropic-nihility state in terms of the eigenwaves propagation in such composite finely-stratified ferrite-semiconductor structure.

2. Constituents of a composite ferrite-semiconductor structure

The essence of any nihility medium is extreme characteristics of its constitutive parameters. These extreme characteristics appear in the small region near singular points of dispersion curves where real parts of both complex permittivity and complex permeability simultaneously make transitions from negative to positive values or vice versa. It is worth noting that media for which these transitions separately exist for permittivity or permeability may be directly found in nature. A well-known example is an electron gas, in which, due to the conduction current created by the drift of free electric charges, may efficiently interact with radiation as a continuous medium characterized by a Drude dispersion model, near the plasma frequency the real part of permittivity appears to be close to zero. At infrared and optical frequencies some low loss noble metals (e.g. silver, gold), semiconductors (e.g. indium antimonide), and some polar dielectrics (e.g. silicon carbide) may behave an epsilon-near-zero state near their plasmas frequencies. At the same time, a mu-near-zero state is inherent to magnetic materials (e.g. ferromagnets, ferri-magnets) in the vicinity of a gyromagnetic resonance. Thereby, although any nihility medium does not exist in nature in its pure form, there is a possibility of obtaining nihility artificially by mixing together materials which manifest epsilon-near-zero and mu-near-zero states in the same frequency range.

In particular, such an opportunity exists in the microwave part of spectrum where a low temperature magnetized plasma can be obtained whose dispersion properties are simultaneously defined by tensors of effective permittivity and effective permeability. Its constituents are micron ferrite grains (e.g. yttrium iron garnet) admixed to the magnetized electron–ion plasma. The dispersion of permeability is caused by the high frequency magnetization of the grain subsystem and is important in the vicinity of the frequency of a ferromagnetic resonance, which coincides with the electron cyclotron frequency. Since the resulting medium is under an action of the static magnetic field, an effect of gyrotropy appears. We predict that through careful adjustment of the ratio of the magnetic grain fraction in the plasma one can reach epsilon-near-zero and mu-near-zero states simultaneously at the same frequency, and, thus, achieve a gyrotropic-nihility.

As an alternative, a layered heterostructure can be considered. It is constructed by periodically...
arranging into a certain uniform system of gyreoelectric (semiconductor) and gyromagnetic (ferrite) layers, provided that these layers are optically thin and their underlying materials correspondingly have epsilon-near-zero and mu-near-zero states in the same frequency range. If the constitutive layers as well as the period of the final structure are optically thin (i.e. it is a finely-stratified one) the effective medium theory can be consistently applied in order to identify its homogenized material parameters. It results in the consideration of the finely-stratified structure as a homogeneous gyreoelectromagnetic medium described by tensors of effective permittivity \( \hat{\varepsilon}_{\text{eff}} \) and effective permeability \( \hat{\mu}_{\text{eff}} \) which possess some dispersion characteristics.

Thereby, further in this paper we consider a stack of \( N \) identical double-layer slabs (unit cells) periodically arranged along the \( z \)-axis (figure 1). Each unit cell is constructed by juxtaposition together of ferrite (with constitutive parameters \( \varepsilon_1, \; \hat{\mu}_1 \)) and semiconductor (with constitutive parameters \( \hat{\varepsilon}_2, \; \mu_2 \)) layers with thicknesses \( d_1 \) and \( d_2 \), respectively. The structure’s period is \( L = d_1 + d_2 \), and along the \( x \) and \( y \) directions the system is infinite. We suppose that the structure is a finely-stratified one, i.e. its characteristic dimensions \( d_1, \; d_2 \) and \( L \) are all much smaller than the wavelength in the corresponding layer \( d_1, d_2 \ll \lambda \), and period \( L \ll \lambda \) (the long-wavelength limit). An external static magnetic field \( \vec{M} \) is directed along the \( z \)-axis. The input \( z \leq 0 \) and output \( z \geq NL \) half-spaces are homogeneous, isotropic and have constitutive parameters \( \varepsilon_0, \; \mu_0 \).

We use common expressions for underlying constitutive parameters of normally magnetized ferrite and semiconductor layers with taking into account the losses. For ferrite layers the permittivity and permeability are defined in the form [29, 30]:

\[
\varepsilon_1 = \varepsilon_f, \quad \hat{\mu}_1 = \left( \begin{array}{cc} \mu_1^T & \text{i} \alpha \varepsilon_0 \\ -\text{i} \alpha & \mu_1^R \end{array} \right),
\]

where

\[
\mu_1^T = 1 + \chi' + \text{i} \chi'', \quad \chi' = \omega_0 \omega_n [\omega_0^2 - \omega^2 (1 - b^2)]D^{-1},
\]

\[
\chi'' = \omega_0 \omega_n b [\omega_0^2 + \omega^2 (1 + b^2)]D^{-1}, \quad \alpha = \Omega' + \text{i} \Omega'',
\]

\[
\Omega' = \omega_0 \omega_n [\omega_0^2 - \omega^2 (1 + b^2)]D^{-1}, \quad \Omega'' = 2 \omega_0^2 \omega_0 \omega_n b D^{-1},
\]

\[
D = [\omega_0^2 - \omega^2 (1 + b^2)]^2 + 4 \omega_0^2 \omega_0^2 b^2, \quad \omega_0 \text{ is the Larmor frequency and } b \text{ is a dimensionless damping constant.}
\]

For semiconductor layers the permittivity and permeability are defined as follows [31]:

\[
\hat{\varepsilon}_2 = \left( \begin{array}{cc} \varepsilon_2^T & \text{i} \beta \\ -\text{i} \beta & \varepsilon_2^R \end{array} \right), \quad \mu_2 = \mu_s,
\]

where

\[
\varepsilon_2^T = \varepsilon_f \left[ 1 - \omega_0^2 \omega_n \left( \omega + \text{i} \omega_n \right)^2 \left( \omega_0 + \text{i} \omega_n \right)^2 \right],
\]

\[
\varepsilon_2^R = \varepsilon_f \left[ 1 - \omega_0^2 \omega_n \left( \omega + \text{i} \omega_n \right)^2 \right],
\]

\[
\beta = \varepsilon_f \omega_0 \omega_n \omega \left( \omega + \text{i} \omega_n \right)^2 - \omega_0^2 \right], \quad \varepsilon_f \text{ is the part of permittivity attributed to the lattice, } \omega_0 \text{ is the plasma frequency, } \omega_n \text{ is the cyclotron frequency and } \nu \text{ is the electron collision frequency in plasma.}
\]

The frequency dependencies of the real and imaginary parts of the underlying permittivity and permeability calculated using equations (1) and (2) are presented in figure 2. Note that imaginary parts of \( \mu_1^T \), \( \alpha \) and \( \varepsilon_2^T \), \( \beta \) are so close to each other that the curves of their frequency dependencies coincide in the corresponding figures.

### 3. Homogenized material parameters of a gyroelectromagnetic medium

In the long-wavelength limit, in order to reveal the gyrotropic-nihilth state conditions which are attainable in the studied structure, the effective medium theory can be engaged [17, 32]. From the viewpoint of this theory, the periodic structure is approximately represented as an anisotropic (gyroelectromagnetic) uniform medium whose optical axis is directed along the structure periodicity, and this medium is described with some tensors of effective permittivity \( \hat{\varepsilon}_{\text{eff}} \) and effective permeability \( \hat{\mu}_{\text{eff}} \) which should be retrieved.

Let us consider a unit cell of the studied structure. It is made of two layers \( 0 < z < d_1 \) and \( d_1 < z < L \) of dissimilar materials whose constitutive relations are as follows:

\[
\begin{align*}
\hat{D} &= \varepsilon_1 \hat{E}, & d_1 < z < L, \\
\hat{B} &= \hat{\mu}_1 \hat{H}, & 0 < z < d_1, \\
\hat{D} &= \varepsilon_2 \hat{E}, & \text{in general form, in Cartesian coordinates}, \; \text{the system of}
\end{align*}
\]

\[
\hat{B} = \hat{\mu}_2 \hat{H}
\]

\[
\begin{align*}
&d_1 < z < L.
\end{align*}
\]
Maxwell’s equations for each layer has a form:

\[ \begin{align*}
ike_z H_z - \partial_i H_i &= -ik_0\left(\hat{e}_z E_z\right), i k_z E_z \\
\partial_z H_z - ik_z H_z &= -ik_0\left(\hat{e}_z E_z\right), \partial_z E_z \\
ike_z H_y - ik_z H_y &= -ik_0\left(\hat{e}_y E_y\right), i k_z E_y \\
ike_z H_y - ik_z H_y &= -ik_0\left(\hat{e}_y E_y\right), \partial_z E_z \\
ike_z H_z - ik_z H_z &= -ik_0\left(\hat{e}_z E_z\right), \partial_z E_z \\
\end{align*} \]

where \( \partial_i = \partial/\partial x_i \); \( k_z = k_0 \sin \theta_0 \cos \phi_0 \) and \( k_x = k_0 \sin \theta_0 \sin \phi_0 \) are tangential components of the wavevector \( \vec{k} \); \( \theta_0 \) and \( \phi_0 \) are polar and azimuthal angles, respectively, which define the electromagnetic wave propagation direction; \( k_0 = \omega/c \) is the free-space wavenumber; \( j = 1, 2 \); \( \hat{e}_i \) and \( \hat{\mu}_i \) are the tensors with diagonal and zeros elsewhere (i.e., \( \hat{e}_i = e_i \hat{I} \), \( \hat{\mu}_i = \mu_i \hat{I} \), where \( \hat{I} \) is the identity tensor). From six components of the electromagnetic field \( \vec{E} \) and \( \vec{H} \), only four are independent. Thus the components \( E_z \) and \( H_z \) can be eliminated from system (4), and derived a set of four first-order linear differential equations related to the transversal field components inside a layer of the structure [33]. The obtained sets of equations can be abbreviated by using a matrix notation:

\[ \partial_z \vec{\Psi}(z) = ik_0 \vec{A}(z) \vec{\Psi}(z), \quad 0 < z < L. \]  

In this equation, \( \vec{\Psi} = \{E_z, E_y, H_z, H_y\} \) is a four-component column vector (here the prime symbol denotes the matrix transpose operator), while the \( 4 \times 4 \) matrix function

\[ \vec{A}(z) \] is piecewise uniform as

\[ \vec{A}(z) = \begin{cases} \vec{A}_1, & 0 < z < d_1, \\ \vec{A}_2, & d_1 < z < L, \end{cases} \]  

where the matrices \( \vec{A}_1 \) and \( \vec{A}_2 \) correspond to ferrite and semiconductor layers, respectively.

Since the vector \( \vec{\Psi} \) is known in the plane \( z = 0 \), equation (5) is related to the Cauchy problem [34] whose solution is straightforward, because the matrix \( \vec{A}(z) \) is piecewise uniform. Thus, the field components referred to boundaries of the double-layer period of the structure are related as:

\[ \vec{\Psi}(L) = e^{i2\pi f(L)} e^{i2\pi f(0)} = \exp\left[ik_0 \vec{A}_{2d_2} \right] \exp\left[ik_0 \vec{A}_{d_1} \right] \vec{\Psi}(0), \]

where \( \vec{M}_j \) and \( \Re \) are the transfer matrices of the corresponding layer (\( j = 1, 2 \)) and the period, respectively.

Suppose that \( \gamma_j \) is the eigenvalue of the corresponding matrix \( k_0 \vec{A}_j \) (\( \det[k_0 \vec{A}_j - \gamma \vec{I}] = 0 \)), and \( \vec{I} \) is the \( 4 \times 4 \) identity matrix. When \( |\gamma_j|d_j \ll 1 \) (i.e., both layers in the period are optically thin), the next long-wave approximation can be used [32]

\[ \exp\left[ik_0 \vec{A}_{2d_2} \right] \exp\left[ik_0 \vec{A}_{d_1} \right] \cong \vec{I} + ik_0 \vec{A}_{d_1} + ik_0 \vec{A}_{2d_2}. \]

Let us now consider a single layer of effective permittivity \( \varepsilon_{\text{eff}} \), effective permeability \( \mu_{\text{eff}} \) and thickness \( L \). Quantity \( \vec{A}_{\text{eff}} \) can be defined in a way similar to (7):

\[ \vec{\Psi}(L) = M_{\text{eff}} \vec{\Psi}(0) = \exp\left[ik_0 \vec{A}_{\text{eff}}L \right] \vec{\Psi}(0). \]

Provided that \( \varepsilon_{\text{eff}} \) is the eigenvalue of the matrix \( k_0 \vec{A}_{\text{eff}} \) (\( \det[k_0 \vec{A}_{\text{eff}} - \varepsilon_{\text{eff}} \vec{I}] = 0 \)) and \( |\varepsilon_{\text{eff}}|L \ll 1 \) (i.e., the entire composite layer is optically thin as well), the next

\[ \sum_{n=0}^{\infty} \frac{1}{n!} X^n \text{ converges for square matrices } X, \text{ i.e. function } \exp(X) \text{ is defined for all square matrices} \]
approximation follows
\[ \exp [ik_0A_{\text{eff}}L] \approx I + ik_0A_{\text{eff}}L. \] (10)

Equations (8) and (10) permit us to establish the following equivalence between bilayer and single layer:
\[ A_{\text{eff}} = f_1 A_1 + f_2 A_2, \] (11)
where \( f_1 = d_1/L \), and the elements of the matrices \( A_1 \), \( A_2 \) and \( A_{\text{eff}} \) one can find in appendix.

From equation (11), in the particular case of coincidence of the directions of both the wave propagation and the static magnetic field bias (\( \theta_0 = 0 \)), the following simple expressions for the effective constitutive parameters of the homogenized medium can be obtained:
\[ \mu^T = f_1 \mu_1^T + f_2 \mu_2^T, \quad \alpha_{\text{eff}} = f_1 \alpha, \quad \epsilon_{\text{eff}} = f_1 \epsilon_1 + f_2 \epsilon_2, \quad \beta_{\text{eff}} = f_2 \beta. \] (12)

The effective constitutive parameters calculated according to equalities (12) are given in figure 3. One can see that there is a frequency range where real parts of both \( \mu^T \) and \( \epsilon_{\text{eff}} \) reach zero. It is significant that, by special adjusting ferrite and semiconductor types, the external static magnetic field strength and the ratio of the layers’ thicknesses, it is possible to obtain the condition where real parts of both \( \mu^T \) and \( \epsilon_{\text{eff}} \) simultaneously acquire zero at the same frequency. Note well that at this frequency, the parameters \( \alpha_{\text{eff}} \) and \( \beta_{\text{eff}} \) are non-zero and the medium losses are relatively small (see, figures 3(b) and (c)). We define exactly this situation as a gyrotropic-nihility state and distinguish it in figure 3(a) by an arrow. In the structure under consideration such gyrotropic-nihility state appears at a particular frequency \( f_{\text{no}} \approx 4.94 \text{GHz} \) which corresponds to the following geometric factors: \( d_1/L = 0.2; \ d_2/L = 0.8; \ L/\lambda \approx 4 \times 10^{-3} \).

It is expectable that in the general case when the directions of the static magnetic field bias and the wave propagation do not coincide (\( \theta_0 \neq 0 \)), the gyrotropic-nihility state can acquire some changes, but nevertheless it should stay distinguishable. We discuss these changes further through the eigenwaves and structure’s transmittance analysis.

4. Eigenwaves and gyrotropic-nihility state

Evidently, the formulation of the eigenvalue problem on the matrix \( A_{\text{eff}} \) (det\([A_{\text{eff}} - \eta I] = 0\)), whose coefficients are defined as (11), yields us the characteristic equation on the effective refractive index of the medium. Thus, we arrive to the following biquadratic equation with respect to \( \eta^2 \) [18, 20]:
\[ \Lambda \eta^4 + B \eta^2 + C = 0, \] (13)
where
\[ \Lambda = 1; \quad B = -2(\epsilon_{\text{eff}}^T \mu_{\text{eff}}^T + \alpha_{\text{eff}} \beta_{\text{eff}}) + (\epsilon_{\text{eff}}^T \epsilon_{\text{eff}} + \mu_{\text{eff}}^T \mu_{\text{eff}})(k_2^2 + k_0^2), \]
\[ C = (\mu_{\text{eff}}^T \mu_{\text{eff}} - \alpha_{\text{eff}}^2 - (k_2^2 + k_0^2)\mu_{\text{eff}}^T k_0^2 \epsilon_{\text{eff}}) \]
\[ = (\eta_{\text{eff}}^2 - \beta_{\text{eff}}^2 - (k_2^2 + k_0^2)\epsilon_{\text{eff}}^T k_0^2 \mu_{\text{eff}}^T) \] are known functions of frequency.

One can see that regarding the eigenwaves propagation in an unbounded gyroelectromagnetic medium, the problem acquires the cylindrical symmetry which results in the independence of solutions of biquadratic equation (13) on the azimuthal angle \( \theta_0 \), since there is \((k_2^2 + k_0^2)k_0^2 = \sin^2 \theta_0 \). Thereby these solutions are
\[ \eta^2 \pm (\omega, \theta_0) = -\frac{B \pm \sqrt{B^2 - 4AC}}{2A} = \epsilon_{\text{eff}} \mu_{\text{eff}}. \] (14)

Obtained root branches describe two effective refractive indexes \( \eta^2 \) and \( \eta^2 \) related to two distinct eigenwaves with propagation constants \( \gamma_1 \) and \( \gamma_2 \). Those are the ordinary (+) and extraordinary (−) waves, respectively, as it is accepted in the optics community, and in the general case of \( \theta_0 \neq 0 \), these eigenwaves are elliptically polarized ones. Remarkably that each eigenwave propagates through its own medium with distinctive constitutive parameters \( e_+, \mu_+ \) and \( e_-, \mu_- \) for ordinary and extraordinary eigenwave, respectively.

For lossless medium the electromagnetic waves can propagate through a medium only if the condition \( \eta^2 (\omega, \theta_0) > 0 \) holds. Nevertheless, equation (14) does not specify a sign of the refractive index. So the sign of \( \eta^2 \) must be chosen providing that the energy carrying by the wave goes away from the source. Here the situation is possible when directions of the Poynting vector \( \vec{S} = (c/8\pi) \text{Re} (\vec{E} \times \vec{H}^*) \) and the wavevector \( \vec{k} \) do not coincide and so-called backward propagation appears. This problem has been discussed before in many papers related to the double-negative (left-handed) materials including those addressed on the circularly birefringent (gyrotropic) media (e.g., see [9, 20, 35–37]), and so we omit the details here. Howbeit, in our case the medium losses should be taken into consideration, and the signs of the real and imaginary parts of the complex refractive index \( \eta^2 = \eta^2_+ + i\eta^2_- \) can be determined from the solution of the next equation [38]
\[ (\eta^2_+ + i\eta^2_-)^2 = \left(\epsilon_{\text{eff}}^+ + i\epsilon_{\text{eff}}^-ight)(\mu_{\text{eff}}^+ + i\mu_{\text{eff}}^-), \] (15)
which is further reduced to the set of two equations obtained by deriving the real and imaginary parts from (15)
\[ \left(\eta^2_+\right)^2 - \left(\eta^2_-\right)^2 = e_{\text{eff}}^+ \mu_{\text{eff}}^- - e_{\text{eff}}^- \mu_{\text{eff}}^+, \quad 2n^2\eta^2_+ \eta^2_- = e_{\text{eff}}^+ \mu_{\text{eff}}^+ + e_{\text{eff}}^- \mu_{\text{eff}}^-. \] (16)

Thus, in order to ensure an electromagnetic wave damping as the wave propagates through the medium, the imaginary parts of permittivity, permeability and refractive index all must be positive quantities (\( e_{\text{eff}}^+ > 0, \mu_{\text{eff}}^- > 0, \) and \( \eta^2_- > 0 \)). From these conditions it follows that according to the second equation in (16) the sign of \( \eta^2_+ \) is defined by the signs and absolute values of \( e_{\text{eff}}^+ \) and \( \mu_{\text{eff}}^- \), and in the particular case of the double-negative medium (\( e_{\text{eff}}^- < 0, \mu_{\text{eff}}^+ < 0 \)) the real part of refractive index \( \eta^2_+ \) must be determined as a negative quantity [35].

Keeping this in mind, the root branches (14) of equation (13) are properly chosen and plotted in figure 4 for two different values of the polar angle \( \theta_0 \). Each plot consists of four dispersion curves, from which we distinguish a pair of
effective refractive indexes $\eta_{+}$ and $\eta_{-}$ having positive imaginary parts (see, figures 4(b) and (c)). It can be seen that the curves within this pair demonstrate drastically distinctive features. While the dispersion curve of the refractive index $\eta_{-}$ related to the extraordinary eigenwave undergoes a small monotonic increasing with $\eta'_{-} > -1$ and $\eta''_{-} \approx 0$, the dispersion curve of the refractive index $\eta_{+}$ related to the ordinary eigenwave experiences considerable changing in which $\eta'_{+}$ has consistently gone through negative values to positive ones and $\eta''_{+}$ acquires some maximum. Thus, based on characteristics of $\eta_{+}$, the whole frequency range of interest can be divided into three specific bands. The first frequency band is located between 2 and 4 GHz where $\eta'_{+}$ is a positive quantity and $\eta''_{+}$ is very significant. In the second band from 4 to 6 GHz, $\eta'_{+}$ is a negative value which eventually acquires a transition to positive one on the band boundaries while $\eta''_{+}$ decreases as the frequency rises. And finally, in the third frequency band which starts from 6 GHz, $\eta_{+}$ and $\eta_{-}$ become comparable quantities.

Figure 3. (a) Two surfaces depict behaviors of real parts of effective permeability (purple surface) and effective permittivity (orange surface) versus frequency and the ratio of the layers’ thicknesses. The blue and red curves plotted on the surfaces show the mu-near-zero and epsilon-near-zero states, respectively. Curves at the bottom are just projections and are given for an illustrative purpose. Dispersion curves of the tensor components of (b) effective permeability and (c) effective permittivity at particular structure parameters ($d_1 = 0.05$ mm, $d_2 = 0.2$ mm) for which the gyrotropic-nihility state exists. Other parameters of the ferrite and semiconductor layers are the same as in figure 2; $L = 0.25$ mm.
equation (13) has simple analytical solutions
\[
\eta_\pm(\omega, 0) = (\mu_{\text{eff}}^T \pm \alpha_{\text{eff}})(\epsilon_{\text{eff}}^T \pm \beta_{\text{eff}}),
\]
that allows us to analyze in more details the conditions at which the gyrotropic-nihility state occurs. At the same time, we recall that when \(\eta_{\pm}(\omega, 0) > 0\), \(\eta_{\pm}^+\) and \(\eta_{\pm}^-\) in equation (17) are related to two eigenwaves which propagate along the positive direction of the \(z\)-axis with right- and left-circular polarizations, respectively.\(^6\)

From figures 3(b) and (c) one can conclude that in the whole considered frequency band the imaginary parts of diagonal and off-diagonal components of both effective permeability and effective permittivity tensors are close values (\(\mu_{\text{eff}}^T \approx \alpha_{\text{eff}}, \epsilon_{\text{eff}}^T \approx \beta_{\text{eff}}\)), while the real parts of diagonal elements of both tensors are greater than the real parts of the corresponding gyrotropic parameters (\(\mu_{\text{eff}}^T \beta_{\text{eff}}^T > \alpha_{\text{eff}}, \alpha_{\text{eff}}, \beta_{\text{eff}}^T\)). Since all imaginary parts of the constitutive parameters must be positive quantities, the sign and absolute values of their real parts obviously define the sign of the refractive indexes related to the ordinary and extraordinary eigenwaves.

Thus, in accordance with the mentioned above characteristics of the complex effective constitutive parameters, it is appreciated that in the whole considered frequency band the refractive index \(\eta_{\pm}\) related to the extraordinary eigenwave is a positive quantity with the vanishingly small imaginary part. At once, the refractive index \(\eta_{\pm}\) related to the ordinary eigenwave can be either positive or negative quantity in the corresponding frequency bands. In particular it becomes a negative quantity in the frequency bands where the next conditions hold: (i) \((\mu_{\text{eff}}^T) < 0\), \((\epsilon_{\text{eff}}^T) < 0\); (ii) \(\alpha_{\text{eff}}^T < 0\), \((\epsilon_{\text{eff}}^T) < 0\); (iii) \((\mu_{\text{eff}}^T) < 0\), \((\epsilon_{\text{eff}}^T) < 0\) or \((\mu_{\text{eff}}^T) < 0\), \((\epsilon_{\text{eff}}^T) < 0\), and it is evident that in all these bands the imaginary part of \(\eta_{\pm}\) cannot be zero. From our estimations the figure of merit \(\Phi_{\pm} = |\eta_{\pm}^+|^2/|\eta_{\pm}^-|^2\) is the order of tens and hundreds for ordinary and extraordinary eigenwaves, respectively. It means that there is a different damping of the ordinary and extraordinary eigenwaves as they propagate through the medium, which is a known manifestation of the circular dichroism.

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\(^6\) We use here the optical community convention on the handedness of the circular polarization. According to this convention right-circularly polarized light is defined as a clockwise rotation of the electric vector when the observer is looking against the direction the wave is traveling.
Remarkably, from this set of conditions, the second one is unambiguously satisfied at the particular frequency of the gyrotropic-nihility state \( f_{gn} \), where \((\mu_{eff}^f)\) and \((\epsilon_{eff})\) simultaneously appear to be close to zero. In this case the imaginary parts of \(\mu_{eff}^f\), \(\alpha_{eff}\) and \(\epsilon_{eff}\), \(\beta_{eff}\) are relatively small, so we can write \(|\epsilon_{eff}\mu_{eff}^f| \approx |\alpha_{eff}\beta_{eff}|\), which leads to the following approximate equalities related to the eigenwaves propagation constants

\[
\gamma_\pm \approx -\gamma_e \approx k_0 \sqrt{\alpha_{eff}\beta_{eff}}. \tag{18}
\]

Thus, the propagation constants of the ordinary and extraordinary eigenwaves are equal in the magnitude but opposite in sign to each other, and thus a backward propagation appears for the ordinary eigenwave while for the extraordinary eigenwaves are equal in the magnitude but opposite. Thus, the propagation constants of the ordinary and extraordinary eigenwaves appear to be close to zero. In this case the imaginary parts of \(\mu_{eff}^f\), \(\alpha_{eff}\) and \(\epsilon_{eff}\), \(\beta_{eff}\) are relatively small, so we can write \(|\epsilon_{eff}\mu_{eff}^f| \approx |\alpha_{eff}\beta_{eff}|\), which leads to the following approximate equalities related to the eigenwaves propagation constants

\[
Z_\pm \approx Z_e \approx \sqrt{|\alpha_{eff}|/|\beta_{eff}|}. \tag{19}
\]

Therefore it turns out that this simultaneous matching of both the refractive index and the wave impedance to free space should inevitably result in the reflectionless interaction of electromagnetic waves when they impinge on the studied structure having a finite number of periods.

5. Wave transmission through a gyrotropic-nihility layer

In order to find the transmittance and reflectance of the studied structure, we use the results of [33] and write the solution of equation (5) in the form

\[
\Psi(0) = (\mathfrak{M}_N^{N\times N})^{-1} \Psi(NL) = \mathfrak{T}^\dagger \Psi(NL), \tag{20}
\]

where the field vector \(\Psi\) at the input and output structure’s surfaces consists of the incident, reflected and transmitted wave contributions as

\[
\Psi(0) = \Psi_{inc} + \Psi_{ref}, \quad \Psi(NL) = \Psi_{tr}. \tag{21}
\]

The vectors \(\Psi_{inc}\), \(\Psi_{ref}\) and \(\Psi_{tr}\) are composed from tangential components of the electromagnetic field, and in turn these components are determined by their complex amplitudes. In the general case, the numerical solution of the Cauchy problem (5) for particular structure’s layers results in the matrices \(\mathfrak{M}_1\) and \(\mathfrak{M}_2\), and then subsequent rising of their product \(\mathfrak{M} = \mathfrak{M}_2 \mathfrak{M}_1\) to the power \(N\) allows us to find both the coefficients of the transfer matrix \(\mathfrak{T}\) and the complex amplitudes of the transmitted and reflected fields (we refer the reader to [33] here for further details on the calculation procedure). The ratios between the amplitudes of the transmitted field and the incident field, and between the amplitudes of the reflected field and the incident field establish the complex transmission
and reflection coefficients, respectively, which can be calculated as functions of the frequency and angle of wave incidence, $T(\omega, \theta_0)$ and $R(\omega, \theta_0)$. Accordingly, the quantities $|T|^2$, $|R|^2$ and $W = 1 - |T|^2 - |R|^2$ are defined as the transmittance, reflectance and absorption coefficient.

In figure 6(a) the transmittance calculated as a function of the frequency and angle of incidence is shown in the form of a surface plot. One can see that this surface is quite smooth on the distance from the ferromagnetic and plasma resonances. At once, in the frequency band of interest (4.5–5.5 GHz) there is a specific ridge on the surface where the transmittance reaches high values. This ridge runs through almost the entire range of angles, and maximum of the transmittance appears near the frequency of the gyrotropic-nihility state $f_{gn} = 4.94$ GHz, which is distinguished on the bottom contour by an arrow. Such high transmittance obviously appears due to the mentioned peculiarities of the refractive index and the wave impedance that are both matched to free space.

This feature is also confirmed by the curves plotted in the bottom planes of figure 6, where the transmittance, reflectance and absorption coefficient are presented for two different values of the frequency and polar angle. Here again, the frequency of the gyrotropic-nihility state is marked by an arrow. From curves plotted in figure 6(b) one can conclude that despite the fact that the angle of incidence rises the minimum of reflectance remains to be nearly the frequency of the gyrotropic-nihility state wherein a certain absorption in the medium exists.

Besides, in figure 6(c) the first frequency is chosen at the gyrotropic-nihility state while the second one is selected to be far from the frequencies of the gyrotropic-nihility state and the ferromagnetic and plasma resonances. At the frequency of $f = 10$ GHz, the curves have typical form where the transmittance monotonically decreases and the reflectance...
monotonically increases as the angle of incidence rises. On the other hand, at the frequency of the gyrotropic-nihility state, the curves of the transmittance and reflectance are different drastically from those of the discussed case. Thus, the level of the transmittance/reflectance remains to be invariable almost down to the glancing angles. At the same time, the reflectance is small down to the glancing angles because at this frequency the medium is matched to free space.

6. Conclusions

To conclude, in this paper we study characteristics of the gyrotropic-nihility state in a finely-stratified ferrite-semiconductor structure which is under an action of an external static magnetic field applied in the Faraday configuration. In the long-wavelength limit, when the structure’s layers as well as its period are optically thin, with an assistance of the effective medium theory, the studied structure is approximated as a uniform gyromagnetic medium defined with effective permittivity and effective permeability tensors. In general, the investigations of the eigenwaves propagation in such gyroelectromagnetic medium were carried out on the basis of numerical calculations. At the same time, in the case, when directions of the electromagnetic waves propagation and the static magnetic field bias are coincident, the components of effective permittivity and effective permeability tensors, effective refractive indexes and normalized wave impedances are obtained analytically.

The gyrotropic-nihility phenomenon is considered as some extreme-parameter state that appears in a small region near singular points of dispersion curves where real parts of the diagonal components of both complex effective permittivity and complex effective permeability tensors simultaneously make transitions from negative to positive values while the off-diagonal components of the corresponding tensors remain to be non-zero quantities. On the basis of these constitutive parameters the peculiarities of the ordinary and extraordinary eigenwaves are equal in the magnitude but opposite in sign to each other, and thus a backward propagation appears for the ordinary eigenwave while for the extraordinary eigenwave it is a forward one. Therefore, at the frequency of the gyrotropic-nihility state, both ordinary and extraordinary eigenwaves appear to be left-circularly polarized because the wavevector of the ordinary eigenwave reverses its direction and the handedness changes, accordingly, from right to left.

The frequency and angular dependencies of the transmittance, reflectance and absorption coefficient are presented. It turns out that near the gyrotropic-nihility state the studied structure is matched to free space with both the refractive index and the wave impedance which results in its high transmittance almost in the entire range of angles of the electromagnetic waves incidence. We believe that this outcome can be of great interest, particularly, in the problem of transformation optics.

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Appendix

The matrix \( A \) in equation (5) can be written in the 2 \( \times \) 2 block representation [40]

\[
A = \begin{pmatrix} 0 & A^t \\ A^e & 0 \end{pmatrix},
\]  
(A.1)

where \( 0 \) is the 2 \( \times \) 2 matrix with all its entries being zero, and for the ferrite (\( A_i \)), semiconductor (\( A_s \)) and entire composite (\( A_{\text{eff}} \)) layers corresponding matrices \( A^e \) are [17]:

\[
A_i^+ = \begin{pmatrix} k_i k_s / k_0^2 \varepsilon_i + i \alpha & \mu_i^e - k_i^2 / k_0^2 \varepsilon_i \\ -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i & k_i k_s / k_0^2 \mu_i^e + i \alpha \end{pmatrix},
\]  
(A.2)

\[
A_i^- = \begin{pmatrix} k_i k_s / k_0^2 \varepsilon_i + k_i^2 / k_0^2 \mu_i^e & -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i \\ -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i & k_i k_s / k_0^2 \mu_i^e - i \beta \end{pmatrix},
\]  
(A.3)

\[
A_s^+ = \begin{pmatrix} k_i k_s / k_0^2 \varepsilon_i + k_i^2 / k_0^2 \mu_i^e & -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i \\ -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i & k_i k_s / k_0^2 \mu_i^e - k_i^2 / k_0^2 \mu_i^e + i \alpha \end{pmatrix},
\]  
(A.4)

\[
A_s^- = \begin{pmatrix} k_i k_s / k_0^2 \varepsilon_i + i \beta & \mu_i^e - k_i^2 / k_0^2 \varepsilon_i \\ -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i & k_i k_s / k_0^2 \mu_i^e - i \beta \end{pmatrix},
\]  
(A.5)

\[
A_{\text{eff}}^+ = \begin{pmatrix} k_i k_s / k_0^2 \varepsilon_i + i \alpha & \mu_i^e - k_i^2 / k_0^2 \varepsilon_i \\ -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i & k_i k_s / k_0^2 \mu_i^e + i \alpha \end{pmatrix},
\]  
(A.6)

\[
A_{\text{eff}}^- = \begin{pmatrix} k_i k_s / k_0^2 \varepsilon_i + i \beta & \mu_i^e - k_i^2 / k_0^2 \varepsilon_i \\ -\mu_i^e + k_i^2 / k_0^2 \varepsilon_i & k_i k_s / k_0^2 \mu_i^e - i \beta \end{pmatrix},
\]  
(A.7)

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