Development of the mathematical model of dynamic interaction in the vibrating technological equipment

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Abstract. The article is devoted to the topical problem of choosing the main parameters of vibrating technological machines used in many technological processes, including the aviation industry. It is shown that the solution of this problem is complicated by the need to create a spatial structure of the vibrating field, which is found, for example, in the vibrating devices designed for oriented placement of rivets in the container (perforated glass) before anodizing, by reporting the last simultaneous vibrations in the vertical and horizontal planes. For the solution of this and similar problems the article offers the method of construction of the mathematical models reflecting features of dynamic interaction of a working environment with vibrating surfaces of technological machines, allowing to choose parameters of a vibrating field by means of structural mathematical modeling.

1. Introduction

Wide use of vibrations in various fields of technology and, above all, in the creation of vibrating machines, equipment and technological processes, on the one hand, and the development of methods and means of vibration protection of modern machines, devices and equipment due to the growth of speeds and loads of actuators, the presence of gaps and backlashes in the interfaces of the links make it necessary to develop research aimed at an in-depth study of the mechanics of vibration interactions of contacting elements [1–5]. Many processes of dynamic interaction are characterized by non-containmental or one-way connections leading to various forms of relative motion, ranging from the preservation of the connection (movement of bodies together with the vibrating surface, slippage on the surface without throwing or unbroken movement of composite bodies), disruptions of the connection (separation of bodies from the surface or from each other, slippage with throwing), free flight or continuous throwing and impacts from solids or media elements with vibrating surfaces and among themselves [6–8]. Such processes may also include processes related to vibration orientation and ordered rivet placement in the perforated glass before anodizing, used in the aviation industry [9]. The actual problem of creation of similar devices is a problem of a choice of the most rational parameters of a vibrating field which now is solved by manufacturing of experimental samples of vibrating devices and carrying out of full-scale tests. In this regard, the use of the method of structural mathematical modeling and the apparatus of transfer functions of interpartial relations, proposed in the works, is a promising way of solving such problems. [10–14]. This article presents the results of research on the development of a mathematical model that reflects the dynamic interaction of the working environment.
with the vibrating surfaces of technological equipment, allowing you to choose the required parameters of the vibrating field by means of structural mathematical modeling, without carrying out field tests.

2. The object and mission of the study
Let us assume that the technological vibrating machine creating a two-dimensional vibrating field, can be presented in the form of a mechanical vibrating system with two levels of freedom with a working body in the form of a long solid body, making a flat vertical vibrating motion, shown in Fig. 1, a. The position of the working body is determined by the following coordinates $y_1$ and $y_2$, connected with the stationary base, and their relation taking into account a sign (plus or minus) at the set geometrical parameters of the working body defines position of a nodal point of oscillations. This figure also shows possible external influences and adjustable parameters of the oscillating system. The structural scheme of this system, obtained on the basis of the technology of structural mathematical modeling, described in [13,14], is shown in Fig. 1, b).

![Figure 1](image_url)

**Figure 1.** Schematic diagram of (a) a vibrating process machine with a working device (M, J) that performs flat movement under complex dynamic loading; structural diagram of (b) the initial mechanical vibrating system

It is assumed that the system is operating under the influence of two vibration exciters. The important factor of dynamic quality of such systems is the possibility of providing vibration fields or structures of amplitudes distribution of working body points vibration with the ability to control the processes of forming the amplitude ratios. In order to evaluate the possibilities of setting up vibration fields it is planned to use the device of interpartial links' transfer functions, which are built taking into account the connectivity coefficient of excitation under the condition $\vec{Q}_2 = \alpha \cdot \vec{Q}_1$. The proposed device of transfer functions allows to receive a convenient way of an estimation of dynamic properties of vibrating system by means of finding of knots of fluctuations on a surface of the working body. The oscillation node is understood as a point of an extended solid body (or machine tool) making flat oscillatory movements formed by the sum of translational vertical and angular movements. The solid body will make a rotational motion relative to the oscillation node, which forms the distribution of amplitudes of the attachment points. The analysis of the positions of these points allows us to determine the possibilities of purposeful formation of vibration field structures.
3. Research and discussion

Since the amplitude of the oscillation of the point coinciding with the oscillation node is zero, the structure of the vibration field will depend significantly on the position of this node. If the oscillation node is located on the working body, the latter will make angular oscillations with different direction of movement of surface points. When the oscillation unit is located outside the working body, it is possible to obtain a working body with linearly distributed amplitudes of oscillations and a vibrating field of a uniform structure.

In order to determine the relationship between the vibration node and the parameters of the mechanical vibrating system, let us first consider a number of fundamental issues related to the adjustment of vibrating fields of the working bodies of technological machines.

1. Transfer functions of the system (see Fig. 1, b) at \( \bar{z}_1 = \bar{z}_2 = 0, \bar{Q}_0 = 0 \), are:

\[
W_1(p) = \frac{\bar{y}_1}{Q} = \frac{(Mb^2 + Jc^2 + L_2)\bar{p}^2 + k_2 + (Jc^2 - Mab)p^2}{A(p)}; \quad (1)
\]

\[
W_2(p) = \frac{\bar{y}_2}{Q} = \frac{(Ma^2 + Jc^2 + L_1)\bar{p}^2 + k_1 + (Jc^2 - Mab)p^2}{A(p)}, \quad (2)
\]

where

\[
A(p) = [(Ma^2 + Jc^2 + L_1)\bar{p}^2 + k_1][(Mb^2 + Jc^2 + L_2)\bar{p}^2 + k_2] - [(Jc^2 - Mab)p^2]^2. \quad (3)
\]

2. Transfer functions, as follows from (1), (2), depend significantly on the peculiarities of external excitation (its nature and forms).

When two force factors are applied simultaneously, the transfer function of interparty relations will be determined by the equation

\[
W_{12}(p) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{[Ma(a-b) + 2Jc^2 + L_1]\bar{p}^2 + k_1}{[Mb(b-a) + 2Jc^2 + L_2]\bar{p}^2 + k_2}. \quad (4)
\]

If we assume that the amplitude ratio of oscillations is \(-1\), i.e.

\[
W_{12}(\bar{Q} = \bar{Q} = 0) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{(Jc^2 - Mab)p^2}{(Mb^2 + Jc^2 + L_2)p^2 + k_2} = -1, \quad (5)
\]

this ratio of amplitudes of vibrations of the working organ is possible at the frequency

\[
\omega_2 = \frac{k_2}{Mb(b-a) + 2Jc^2 + L_2}. \quad (6)
\]

3. If \( \frac{\bar{y}_2}{\bar{y}_1} = -1 \), the distribution of the amplitudes of the vibrations along the length of the implement is quite simple. This point, called the oscillation node, is located in the center of the vibrating machine’s vibrating element, and its position \((E_i)\) will be determined by the length of the arms \(l_{10}\) and \(l_{20}\). If \( |\bar{y}_1| = |\bar{y}_2| \) the equation is complete

\[
l_{10} + l_{20} = l_1 + l_2. \quad (7)
\]

To find the amplitude-frequency characteristics (AFC) of the system by coordinates \( \bar{y}_1 \) and \( \bar{y}_2 \), we will use the transfer functions:

\[
W_1'(p) = \frac{\bar{y}_1}{Q} = \frac{(Mb^2 + Jc^2 + L_2)\bar{p}^2 + k_2}{A(p)}; \quad (8)
\]

\[
W_2'(p) = \frac{\bar{y}_2}{Q} = \frac{(Jc^2 - Mab)p^2}{A(p)}. \quad (9)
\]

4. As an illustration of the possibilities of the method, let’s construct on the basis of equations (8) and (9) AFC for the following values of parameters of the oscillatory system: \( M = 1000 \text{ kg}; J = 200, \ldots \)
400 kg·m$^2$; $k_1 = 1200$ N/m; $k_2 = 1000$ N/m; $L_1 = 100$ kg; $L_2 = 100$ kg; $l_1 = 0.55$ m; $l_2 = 0.45$ m.

Frequency response of the oscillating system arising at different values of parameters of the transmission function of interpartial inertial communication are shown in Fig. 2, a, b. As can be seen from these figures, the implementation of the oscillation node is possible, in particular, when performing

$$Jc^2 - Mab > 0.$$  

also when

$$Jc^2 - Mab < 0,$$

that only change the frequency response.

- Figure 2. Frequency response of the system ($\overline{Q}_1 \neq 0$, $\overline{Q}_2 = 0$, $\overline{Q}_0 = 0$, $\overline{z}_1 = \overline{z}_2 = 0$): a – when $Jc^2 - Mab > 0$; b – when $Jc^2 - Mab < 0$; curve 1 – dependency graph $\frac{\overline{y}_1}{\overline{Q}}(\omega)$; curve 2 – dependency graph $\frac{\overline{y}_2}{\overline{Q}}(\omega)$

5. Assuming that $y_1 \neq y_2$, for example, $y_2 > y_1$, the position of the oscillation node will be determined by the position ($E_i$) – the intersection point of the line connecting the deviations in coordinates $y_2$ and $y_1$, with a horizontal straight line, which is an extension of the working body surface in length ($l_1 + l_2$). As an example, we provide data for a situation where $\overline{Q}_0 \neq 0$ (and $\overline{Q}_1 = 0$, $\overline{Q}_2 = 0$, $\overline{z}_1 = \overline{z}_2 = 0$). The system transfer functions and the interpartial transfer function can be recorded in the form of:
\[ W_1(p) = \frac{\bar{y}_1}{\bar{Q}_0} = \frac{[(a + l_0 c)[(Mb^2 + Jc^2 + L_2)p^2 + k_2] + (b - l_0 c)(Jc^2 - Mab)p^2]}{A(p)}, \quad (10) \]

\[ W_2(p) = \frac{\bar{y}_2}{\bar{Q}_0} = \frac{[(a + l_0 c)[(Ma^2 + Jc^2 + L_4)p^2 + k_1] + (b - l_0 c)(Jc^2 - Mab)p^2]}{A(p)}, \quad (11) \]

\[ W_{12}(p) = \frac{\bar{y}_1}{\bar{y}_2} = \frac{[(a + l_0 c)[(Mb^2 + Jc^2 + L_2)p^2 + k_2] + (b - l_0 c)(Jc^2 - Mab)p^2]}{[(b - l_0 c)[(Ma^2 + Jc^2 + L_4)p^2 + k_1] + (a + l_0 c)(Jc^2 - Mab)p^2]}, \quad (12) \]

If \( y_1 < y_2 \), then length \( l_{10} \) can be found from the equation

\[
\begin{align*}
l_{10}(\omega) &= \frac{\{(l_1 + l_2)(a + l_0 c)\} \cdot \{-(Mb^2 + Jc^2 + L_2)\omega^2 + k_2\} - (b - l_0 c)(Jc^2 - Mab)\omega^2}{-(a + l_0 c)[-(Mb + L_2)\omega^2 + k_2] + (b - l_0 c)[-(Ma + L_4)\omega^2 + k_1]}.
\end{align*}
\]

Assuming that the vibration amplitude ratio is 1, i.e.

\[ W_{12}(p) = \frac{\bar{y}_1}{\bar{y}_2} = \frac{[(a + l_0 c)[(Mb^2 + Jc^2 + L_2)p^2 + k_2] + (b - l_0 c)(Jc^2 - Mab)p^2]}{[(b - l_0 c)[(Ma^2 + Jc^2 + L_4)p^2 + k_1] + (a + l_0 c)(Jc^2 - Mab)p^2]} = 1, \quad (14) \]

then condition (18) is met at frequency

\[ \omega_{11}^2 = \frac{k_2 (l_2 + l_0) - k_1 (l_1 - l_0)}{(l_2 + l_0)(Mb + L_2) - (l_1 - l_0)(Ma + L_4)}. \quad (15) \]

6. On the fig. 3 shows the graph of position dependence \( E_1 \) of the working body from the frequency of external influences. In this fig. the position of points (3) and (3') determine the frequencies of dynamic vibration by coordinate \( \bar{y}_1 \) in case when

\[ Jc^2 - Mab > 0 \]

and

\[ Jc^2 - Mab < 0, \]

respectively.
Figure 3. General graph of position dependence point $E_1$ of the working body from the frequency of external influences

When the length $l_{10}$ is reduced to the value of $l_{10} = \frac{l_1 + l_2}{2}$, the oscillation unit will be located in the centre of the working.

4. Conclusion
The method of construction of dynamic model and structural mathematical modeling of the vibrating technological equipment on the basis of use of the device of transfer functions of interpartial connections, offered in the present work provides possibility of purposeful formation of required structure of a vibrating field. The obtained analytical expressions allow you to build diagrams of the location of vibration nodes depending on the parameters of the tuning means or correction means. The state of the vibrating field, in turn, provides the possibility to regulate the structure of the working space created by the vibrating working environment.

The proposed approach can be used, in particular, to determine the parameters of the vibrating process equipment used in the aircraft industry for orientation and orderly placement of rivets in the perforated glass before anodizing, or to determine the parameters of the vibrating loading devices that feed the measuring bars for drilling riveting machines. Its application will allow to define rational parameters of similar devices without manufacturing of pre-production samples of these devices and carrying out of full-scale tests that is in full conformity with a modern paradigm of digital designing and modeling on the basis of digital counterparts within the limits of strategic programs of the Russian Federation: "National Technological Initiative", "Digital Economy" and "Strategy of Scientific and Technological Development".

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