Neural Network with Semiclassical Excitation

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We have constructed a simple semiclassical model of a neural network where the neurons have quantum links and affect one another in a fashion analogous to action potentials. However, there are significant differences with the classical models in the nonlinear functional dependence of the probability of firing of a neuron on the superposed signal from the neighbors and also the non-deterministic fundamental stochasticity of the quantum process. Remarkably, average periodicity is nevertheless observed in simulation, which agrees quite well with a formula derived from the nonlinear relationship. Short-term retentivity of input memory is observed to be subtle but perceptible. This suggests the use of such a network where short-term dynamic memory may be a desirable objective.

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I. INTRODUCTION

The classical integrate-and-fire neural network has been studied both in the simpler zero-width action potential and the more involved finite-width cases. In these works every neuron integrates the current coming from neighboring neurons and when the threshold for firing is exceeded, it too sends off an action potential to its neighbors. Hopfield and Herz had found that there is a simple relation between the contributions $A$ from the neighbors and an external current $I$, with the time period of the firing of the network when phase-lock is established:

$$\tau = \frac{(1 - A)}{I} \quad (1)$$

It has been shown that there is some modification of this formula when the action potential is not exactly a delta function but is spread over time, which is, of course, a more realistic assumption, in biological as well as physical contexts.

In view of the recent great interest in quantum computing we think it is worthwhile to investigate the changes if any that would result from converting such networks to a quantum model. As a first step, we have here tried to see the changes when the action potential acts like a quantum radiation to the neighbors, instead of a current, and the firing of the neuron is replaced by induced radiation from it to the neighbors. The quantity integrated therefore gives not a classical state with deterministic firing on reaching the threshold, but the superposed quantum amplitude, the square of whose magnitude is related to the probability of firing.

Hence, instead of the deterministic approach of the classical case, we have a quantum stochastic interpretation of the dynamics of the network. It is obvious that if the neurons are allowed to emit according to time dependent perturbation theory at random, there will be no exact phase-locking, but it may still be possible to find average periods of firing of a typical neuron which may depend on the strengths of the various parameters of the model, as in the classical case. We study this important possibility in this paper, because it may throw light on some of the most fundamental similarities and differences between classical and quantum networks.

This model is not a fully quantized network where the states of all the neurons are entangled and the operations are all unitary, as in quantum computing models. Our interest is not in the exploitation of the enlargement of the capacity of the net in the quantum form, or of the speed of information processing in such a system, but to make an investigation of the significant changes that the nonlinearity and the stochasticity may be expected to bring about. The problems of measurement make the attractive possibilities of a fully quantized networked system of gates and other devices (see e.g.) unreachable in the near future. Hence, as a first step it is possible that quantum computers will make a beginning as semiclassical devices with all links not fully entangled by a unitary operation. How dissipation destroys the memory of the initial state and how quickly, and what periodicity, if any, emerges, depending on which parameters, are important questions that need to be studied.
II. THE MODEL

We have simulated a square lattice of such neurons which can communicate with neighbors with quantum signals and have varied their durations, in analogy with the action potential curvets of varying widths in the classical case. We have also retained a constant potential background in analogy with the constant external current.

We know

\[ |t > = U(t, t_0)|t_0 > \tag{2} \]

with

\[ U(t, t_0) = \exp[i \int dt V] \approx 1 + i \int dt V \tag{3} \]

So that the transition rate

\[ \Gamma \approx |\int dt V|^2 \tag{4} \]

In our case we have retained a constant \( V_0 \) background and have added the contributions \( \int dt f(t_i)v \) from the \( i-th \) neighbor. The integration is spread over the duration of the quantum pulse since the last triggering.

We might expect in this case

\[ \tau = 1/|\int dt (V_0 + \Sigma f_i(v))|^2 \tag{5} \]

For small perturbations this would look like

\[ \tau = k' (1 - |\Sigma \int dt f_i(v)|^2)/|\int dt V|^2 \tag{6} \]

which is analogous to the classical formula \[1, 2\].

However, we must realize that in the classical case it is possible to have \( 1 = A \) (in the standardized units used there) and get a zero period. This is impossible in this quantum case because the more exact expression (1) cannot give a zero.

If we make the further assumption that the pulse width is a duration greater than the average period, then we have to take a proportionate amount of contribution from the neighbors. In this case we get the relation, for a square lattice network:

\[ 1/\tau = k'(V_0 + 4v\tau/w)^2 \tag{7} \]

which is a cubic equation in the period and can be solved in terms of the system parameters \( V_0 \), width \( w \) of the pulse and the pulse size \( v \). The nonlinearity of the quantum version of the network mentioned earlier is responsible for this form.

Unlike the classical case we have a randomness in the contribution from the neighbors. We may try to account for this by introducing a parameter \( q \) with the interaction \( v \) between neighbors. This gives:

\[ 1/\tau = k(V_0 + 4qv/w)^{(2/3)} \tag{8} \]

III. INPUT DEPENDENCE

A system that does not respond to the environment is useless. So while the system period found above has its intrinsic interest, we need also study the behavior of the system to different inputs, i.e. initial states of pre-assigned nodes to serve as the interface with the environment. Obviously the system behavior will depend on the geometry and connectivity of the net and the pattern of the input. However, in this semi-quantum model we foresee a loss of memory as the nodes fire and collapse from their original states. This may be a desirable characteristic if we are interested in a system with only a short-term memory, because the automatic erasure of memory after a time may save us the software steps or the hardware needed to erase.

We shall principally study simulation experiments, as the details of memory loss are expected to depend in a complicated way on the system used and any simple closed analytic expression is unlikely. However, we shall also try to see if a simple formula such as Eq. 8 can give agreements with the simulation for the average periods of oscillation after the initial memory is effaced.

IV. RESULTS OF SIMULATION

Our simulations show that:

1. For a 40X40 lattice (as taken previously [1] in the classical case) with periodic boundary conditions - to make it look like an infinite lattice - for about 40,000 simulations for a given set of parameters but different initial states, we get a constant average for the neurons in the lattice. So in an average sense we do have a period (Fig. 1). The system is not chaotic. For a single neuron, if we follow its history we see a fairly linear relation between the cumulative number of firings and time (Fig. 2). If we consider the pooled sum of all nodes of the lattice, the stochasticity almost disappears and we see a practically straight line (Fig. 3).

2. Indeed, as the strength of the signal from the neighbor increases, the time period decreases (Table II), but it does not appear to go to zero. Unlike the classical case, here we can make the signals arbitrarily strong without worrying about running into a singularity or negative periods. The parameter \( q \) was fitted to the first value and then Eq. 8 seems capable of predicting all the other periods with remarkable accuracy.
FIG. 1: \( v_0 = 0.2, \) width = 0.2, \( k = 0.2 \): typical pattern of the triggering of the neurons in the semiclassical neural network. There is apparently no phase locking.

FIG. 2: Cumulative number of triggering against time. One can see a fairly regular linear behavior despite quantum stochasticity. This is for a single chosen neuron.

FIG. 3: Same as Fig 2, but for the whole system.

(3) The width of the pulse is important here too. If the pulse is spread out, the average period increases (Table II). Again we get excellent agreement of Eq. 8 with the simulation results for the periods, with the same value of \( q \) used in Table I.

(4) The input dependence is observed by averaging over 100 simulation runs. The following four types of inputs were used:

(a) All peripheral nodes in state \( |1\rangle \); all body nodes in state \( |0\rangle \). In this case we see (Fig. 4) a smooth transition from a state with an initial firing rate proportional to the number of initially excited nodes dying down quickly as the system forgets the input and lets the system parameters take over with a noisy pattern, despite the averaging over the runs. It is remarkable that the initial few cycles with the memory show virtually no noise.

(b) Peripheral nodes alternately in states \( |1\rangle \) and \( |0\rangle \); body nodes in state \( |0\rangle \) (Fig. 5). In this case we begin with a smaller number of firings on account of halving the initially excited nodes, and there are a few kinks in the initial cycles, possibly due to the conversion of the spatial lack of symmetry to temporal. As expected the system moves to the common noisy asymptotic behavior after forgetting the input.

(c) Peripheral nodes in random states between \( |0\rangle \) and \( |1\rangle \). Here too we see fairly prominent kinks (slightly less than the oscillating \( |1\rangle \leftrightarrow |0\rangle \) pattern in the previous case) from the interaction of the noncoherent randomness of input in the neighboring peripheral nodes until the short-term memory disappears (Fig. 6).

(d) In this case we take the whole lattice to be initially in state \( |0\rangle \). However because of the external driving potential the system soon develops into a noisy final state

### Table I: Strength of Quantum Potential and Average Period of Neurons

| \( k \) | \( q \) | \( V_0 \) | \( T_{pred} \) | \( T_{sim} \) |
|---|---|---|---|---|
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 |
| 0.050 | 0.035 | 0.027 | 0.023 | 0.020 | 0.013 |
| 0.0028 | 0.028 |

### Table II: Variation of Period with Duration of Quantum Potential

| \( k \) | \( q \) | \( width \) | \( T_{pred} \) | \( T_{sim} \) |
|---|---|---|---|---|
| 0.1 | 0.2 | 0.3 | 0.5 | 1.0 |
| 0.023 | 0.035 | 0.044 | 0.056 | 0.074 |
| 0.023 | 0.035 | 0.043 | 0.054 | 0.065 |
from a smooth initial state with no excitation.

V. DISCUSSION

We have seen that this semiclassical neural network which is designed to mimic in some ways the neural network with an integrate-and-fire model with classical action potentials indicates the presence of an average time period reminiscent of the classical model. The following differences stand out: 1) We cannot have a system with a zero time period (infinitely fast). 2) We can have arbitrarily high potentials linking the neurons.

We have seen that the average system period depends in a simple way on the system parameters as given in Eq. 8. A single parameter choice predicts a wide range of potential pulse strengths and durations.

An interesting possible extension of the ideas presented here would be to use multi-state neurons (multibits) with no classical analogue and of course also to use coherent complex quantum action to investigate the possibility of phase-locking from coherence.

We have not considered Hebbian learning by introducing dissipation. Altaisky [4] and Zak [5] have investigated the effects of dissipation in quantum neural networks. In our model also dissipation is inevitable as the neurons are all allowed to fire independently, with no coherence or entanglement. However, unlike these authors, we have considered large systems where the role of adaptivity in the quantum context becomes too complex for a preliminary investigation. The system, nevertheless, can serve as short-term memory with a built-in mechanism for facing all input dependence in the long run, which may be a desirable characteristic where learning and unlearning must go together. As the system we have considered is very basic, it is possible that more complex systems may be designed for greater functionality.

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