Increasing Security Degree of Freedom in Multi-user and Multi-eve Systems

Kun Xie and Wen Chen, Senior Member, IEEE and Lili Wei, Member, IEEE

Abstract—Secure communication in the Multi-user and Multi-eavesdropper (MUME) scenario is considered in this paper. It has been shown that secrecy can be improved when the transmitter simultaneously transmits information-bearing signal to the intended receivers and artificial noise to confuse the eavesdroppers. Several processing schemes have been proposed to limit the co-channel interference (CCI). In this paper, we propose the increasing security degree of freedom (ISDF) method, which takes idea from the dirty-paper coding (DPC) and ZF beam-forming. By means of known interference pre-cancelation at the transmitter, we design each precoder according to the previously designed precoding matrices, rather than other users’ channels, which in return provides extra freedom for the design of precoders. Simulations demonstrate that the proposed method achieves the better performance and relatively low complexity.

Index Terms—MUME-MIMO, Block Diagonalization, ZF beam-forming, ISDF, Secrecy Capacity,

I. INTRODUCTION

The growing interest in security at the physical layer of wireless communications has sparked a resurgence of research in secure communication. In the early works on information theoretic security, Wyner introduces the wiretap channel model, in which the eavesdropper’s channel is defined to be a degraded version of the legitimate receiver’s channel [1]. It is shown that a non-zero secrecy capacity can be obtained only if the eavesdropper’s channel is of lower quality than that of the intended recipient. Csiszár and Körner extend this problem to a general non-degraded channel condition in which a common message is transmitted to the two receivers and the confidential message to only one of them [2]. Another main assumption in the aforementioned works is that the eavesdropper’s channel is known at the transmitter [3], [4], and then the generalized singular value decomposition (GSVD) [5] method can be used to transmit the signal to the null space of the channel from transmitter to eavesdropper. Clearly, these assumptions are usually impractical and unreasonable, particularly for passive eavesdroppers. In this paper, we overcome this problem and propose a scheme without using any CSI of eavesdroppers.

In order to achieve secure communication, even when the receiver’s channel is worse than the eavesdropper’s channel, or the absence of eavesdroppers’ channel state information (CSI), various physical-layer techniques have been proposed. One of the most common techniques is the use of cooperative interference or artificial noise to confuse the eavesdropper. The cooperative interference method can be divided into two categories: (i) the trust-friend model, in which two base stations connected by a high-capacity backbone such as optical fiber, and one base station can continuously transmit an interfering signal to secure the uplink communication for the other base station [5], [6]; (ii) the helper-relay model, where the secrecy level can be increased by having the cooperative interferer [7] or relay [8] to send codewords independent to the source message, which can be canceled at the intended receiver.

Another major techniques for secure communication is the use of multiple antennas. When multiple antennas are equipped at the transmitter, it is possible for the transmitter to simultaneously transmit both the information-bearing signal and artificial noise to achieve secrecy in a fading environment [9], [11], which may replace the role of the cooperative interference method in [5], [8]. In the design of secure communication with artificial noise, the transmit power allocation between the information signal and the artificial noise is an important issue, which has not been discussed in [9], [10], [11]. A suboptimal power allocation strategy is considered in [12], which aims to meet an ideal signal-to-interference-and-noise ratio (SINR) at the intended receiver to satisfy a quality of service requirement. The secure communication with artificial noise is also discussed in [11], in which the closed-form expression of achievable rate and the optimal power allocation has been obtained, however only single-receiver and single-antenna at receiver was considered.

Most of the previous papers focus on the single-user systems. However most practical communication systems have more than one user and the eavesdroppers may not appear alone as well, and they may choose to cooperate or not [11]. In addition, each terminal may be equipped with multiple antennas, which is representative, for example, of downlink transmission in LTE systems and wireless local area networks. This is the so called Multi-User and Multi-Eve (MUME) MIMO systems, which have been seldom investigated before. In this paper, we will focus on investigating the MUME systems.

It’s also worth noting that the achievable secrecy rate of MUME systems is different from that of single-user and single-antenna systems studied before, which must make sure...
that any legitimate user will not be wiretapped by any eavesdropper. The authors in [13] put forward an MUME model in which all other users are viewed as potential eavesdroppers by the targeted user. They also give a definition of the achievable secrecy rate of multiuser wiretap model in terms of secrecy sum rate. The authors in [14] give another definition of achievable secrecy rate in Gaussian MIMO multi-receiver Wiretap channel, which is named secrecy capacity region. Besides, the authors in [15], [16] considers the compound wire-tap channel, which is based on the classical wire-tap channel with channel from the source to the destination and the channel from the source to the wiretapper taking a number of states respectively. It can be viewed as the multicast wire-tap channel with multiple destinations and multiple wire-tappers with the same message transmitted to different destinations, which is slightly different from the broadcast wire-tap channels in this paper. They also give another significant definition of the achievable secrecy rate in terms of absolute secrecy rate, which idea is to take the security of the poorest-performance receive-wiretap pair into consideration. If the poorest-performance pair can meet the quality of service requirement, then all other pairs can do. Therefore this definition of achievable secrecy rate may be more reasonable and constructive in the practical secure communication systems. The authors in [16], [17] discussed the realization of the achievable secrecy rate of multiple users (multiple eavesdroppers) with artificial noise separately, in which, however, the system model is compound wiretap channel but broadcast MUME wiretap channel [13].

Since the transmitter needs to transmit different message to different receivers in the broadcast MUME wiretap model, there must be considerable co-channel interference (CCI) in the system. In order to limit the CCI from the signals transmitted to other users and mask receivers’ own message signal simultaneously, two practical linear transmission schemes were often used in the early works: (i) the SVD method discussed in [9], [11], which conducts an SVD decomposition on each user’s channel matrix to get a maximum channel gain for their own message but can not suppress the interference from other user’s message; (ii) the ZF beamforming method [19] and its promotion—the BD method [20], [21], in which all the information is transmitted in the null space of all other receivers’ channels. The SVD method and ZF beamforming method are simple, but of little ideal performance. While the BD method is of somewhat ideal performance but more complicated than the formers.

In view of the drawbacks of the previous schemes, we propose an alternative approach, which takes idea from dirty-paper coding (DPC) [22], [23] and ZF beam-forming [19]. It can directly increase the degree of freedom when designing the transmission precoders, which in return make obvious improvement not only at the achievable secrecy rate, but also at the antenna constraints at transmitter compared with the BD method. What’s more, in our proposed scheme, we choose to map the artificial noise into the null space of co-precoder matrix instead of the null space of legitimate receivers’ channels in the existing schemes, which may offer extra improvement on the secrecy rate in the low SNR region. The performance will be further improved when the water-filling (WF) method is used. Besides, the power allocation between the information signal and the artificial noise are also discussed.

In this paper, $(\cdot)^H$ and $\text{tr}(\cdot)$ denote the Hermitian transpose and trace of a matrix. $E(\cdot)$ denotes expectation, and $\mathbf{I}$ denotes an identity matrix. $I(\cdot, \cdot)$ denotes mutual information. $|x|^+ = \max\{0, x\}$.

II. SYSTEM MODEL

In this paper, we consider the broadcast MUME wiretap model as shown in Fig. 1, in which there is one transmitter named Alice, $J$ legitimate users named Bobs and $K$ passive eavesdroppers named Eves. Alice tries to send independent messages to all the legitimate receivers while keeping each of the eavesdropper ignorant of all the messages. All of the terminals are equipped with multiple antennas. $N_{Bj}$ antennas are equipped at the $j$-th Bob, $N_{Ek}$ antennas at the $k$-th Eve, and $N_A$ antennas at the single Alice. This scenario is representative, for example, of downlink transmission in the LTE systems and wireless local area networks.

Let the transmit signal $\mathbf{X} = \sum_{j=1}^{J} \mathbf{U}_j + \mathbf{V}$, where $\mathbf{U}_j$ is the information bearing signal vector for user $j$, and $\mathbf{V}$ is the artificial noise signal vector to interference Eves. Then the received signals at Bobs and Eves are respectively:

$$
\text{Bob } j: \quad \mathbf{Y}_j = \mathbf{H}_j \mathbf{X} + \mathbf{N}_j^B, \quad \text{for } j = 1, ..., J,
$$

$$
\text{Eve } k: \quad \mathbf{Z}_k = \mathbf{G}_k \mathbf{X} + \mathbf{N}_k^E, \quad \text{for } k = 1, ..., K,
$$

where $\mathbf{H}_j$ is the $N_{Bj} \times N_A$ channel matrix between the transmitter and Bob $j$, $\mathbf{G}_k$ is the $N_{Ek} \times N_A$ channel matrix between the transmitter and eavesdropper $k$, $\mathbf{N}_j^B$ and $\mathbf{N}_k^E$ are respectively the additive white Gaussian noise vectors observed at the $j$-th Bob and $k$-th Eve, which covariance matrices satisfy $E[\mathbf{N}_j^B \mathbf{N}_j^B^H] = \sigma^2_{Bj} \mathbf{I}$, and $E[\mathbf{N}_k^E \mathbf{N}_k^E^H] = \sigma^2_{Ek} \mathbf{I}$ respectively.

We assume that the channel matrix $\mathbf{H}_j$ and $\mathbf{G}_k$ are block-fading, whose entries are complex Gaussian variables with zero-mean and unit-variance. We also assume that perfect channel state information (CSI) of the receiver, i.e., the channel matrices $\mathbf{H}_j$, $j = 1, 2, \ldots, J$, are available at Alice, e.g., either through reverse channel estimation.

![Fig. 1: The MUME-MIMO wiretap system model.](image-url)
in time-division-duplex (TDD) or feedback in frequency-division-duplex (FDD). While the channel matrices \( G_k \), \( k = 1, 2, \ldots, K \), are unavailable at Alice due to the passive nature of eavesdroppers.

Our objective is to transmit different secret messages to the corresponding Bob. We try to reduce the CCI from the others, and make sure that the underlying Eves can not wiretap any communication between Alice and Bob. In the following, we provide lower and upper bounds on the achievable secrecy rate of the generalized broadcast MUME wiretap channel.

Let \( R^B_j \) denote the mutual information rate between Alice and Bob \( j \), and \( R^E_k \) denote that between Alice and Eve \( k \) for Bob \( j \). Then

\[
R^B_j = \max \{ I(U_j; Y_j) \}, \quad \text{for } j = 1, \ldots, J,
\]

\[
R^E_{jk} = \max \{ I(U_j; Z_k) \}, \quad \text{for } k = 1, \ldots, K.
\]  

(2)

In the sequel, the achievable secrecy rate of the receive-wiretap pair \( (j, k) \) (for Bob \( j \) and Eve \( k \)) can be denoted by

\[
R_{jk} = [R^B_j - R^E_{jk}]^+.
\]  

(3)

The achievable secrecy rate of MUME wiretap model is usually noted by secrecy sum rate \( (R_{ssr}) \) or secrecy rate region \([14]\). However, in the practical broadcast MUME wireless communication systems, the massages transmitted vary with different users, we should take each user into consideration, once it is chosen according to some criterion. Therefore we must make sure that any user’s communication can not be wiretapped by any eavesdropper. Hence the secrecy rate of the system is determined neither by the best transmission pair nor the total rate gap between Bobs and Eves, but by the poorest-performance transmission pair in reality. Then we propose an alternative definition of the secrecy rate for MUME wiretap channel, which is called absolute secrecy rate \( (R_{asr}) \).

\[
R_{asr} = \min_{j, k} \{ R_{jk} \} = \min_{j, k} \{ [R^B_j - R^E_{jk}]^+ \} = \min_{j} \{ R^B_j \} - \max_{k} \{ R^E_{jk} \}.
\]

(4)

The emphasis of this paper is to design the precoders \( W_\ell \) for \( \ell = 1, 2, \ldots, J \) and \( W \). In the SVD method all the data is transmitted in their own channel image space, so that each Bob can get the maximum channel gain for the corresponding message. Another one is the block diagonalization (BD) method, in which all the data is transmitted in the null space of all other Bobs, which is the promotion of the ZF beamforming method and can reduce the interference from the other users’ message signal. Here, we propose an alternative approach—the increasing security degrees of freedom (ISDF) method, which is based on dirty-paper coding (DPC) \([22, 23]\) and ZF beam-forming \([19]\). In this paper, a single data stream is to be sent to each receiver when \( d_j = 1 \), \( \forall j \), and multiple data streams are sent when \( d_j > 1 \), \( \forall j \). Note that, only a maximum of \( N_{Bj} \) streams can be transmitted simultaneously for user \( j \), else the message will not be decoded. Criterion of judging the design is whether the secrecy rate is sufficiently good under the given power constraints, which will be discussed in detail with the design of precoders in the following part.

In MUME scenarios, several co-channel Bobs with multiple antennas aim to communicate with Alice in the same frequency and time slots. In this case, it is necessary to design a transmission scheme that is able to suppress the CCI at Bobs. In multi-user wireless security communications with artificial noise, the precoding matrix is usually designed in the null space of channel matrix \( H_j \). We may called it precoding selection space as well. Obviously, the smaller the rank of matrix \( H_j \) is, the larger the dimension of its corresponding precoding selection space will be. In return, the design of the corresponding precoder has more freedom and the secrecy performance will be better. Therefore we define the dimension of the precoding selection space as the security degrees of freedom (SDF).

### III. THE DESIGN OF PRECODERS IN MUME-MIMO NETWORK BASED ON ISDF

At Alice, the data for each user is processed before transmission. Then it is launched into the MIMO channel with the random artificial noise. Let \( W_j \) be an \( N_A \times d_j \) linear precoder, \( u_j \) be a \( d_j \times 1 \) symbol vector for Bob \( j \), and \( d_j \) be the number of parallel data symbols transmitted simultaneously for Bob \( j \). Satisfying \( 1 \leq d_j \leq N_{Bj} \). Let \( V \) be the artificial noise signal vector, \( W \) be the transmission preprocessing matrix, and \( v \) be the symbol vector. They are both used for the artificial noise. Then the transmission signal is

\[
X = \sum_{j=1}^{J} U_j + V = \sum_{j=1}^{J} W_j u_j + W v.
\]  

(6)

The received signal at Bobs and Eves are respectively

\[
Y_j = H_j \sum_{\ell=1}^{J} W_\ell u_\ell + H_j W v + N_j^B, \quad j = 1, \ldots, J,
\]

\[
Z_k = G_k \sum_{\ell=1}^{J} W_\ell u_\ell + G_k W v + N_k^E, \quad k = 1, \ldots, K.
\]  

(7)

The emphasis of this paper is to design the precoders \( W_\ell \) for \( \ell = 1, 2, \ldots, J \) and \( W \). In the SVD method all the data is transmitted in their own channel image space, so that each Bob can get the maximum channel gain for the corresponding message. Another one is the block diagonalization (BD) method, in which all the data is transmitted in the null space of all other Bobs, which is the promotion of the ZF beamforming method and can reduce the interference from the other users’ message signal. Here, we propose an alternative approach—the increasing security degrees of freedom (ISDF) method, which is based on dirty-paper coding (DPC) \((22, 23)\) and ZF beam-forming \((19)\). In this paper, a single data stream is to be sent to each receiver when \( d_j = 1 \), \( \forall j \), and multiple data streams are sent when \( d_j > 1 \), \( \forall j \). Note that, only a maximum of \( N_{Bj} \) streams can be transmitted simultaneously for user \( j \), else the message will not be decoded. Criterion of judging the design is whether the secrecy rate is sufficiently good under the given power constraints, which will be discussed in detail with the design of precoders in the following part.

In MUME scenarios, several co-channel Bobs with multiple antennas aim to communicate with Alice in the same frequency and time slots. In this case, it is necessary to design a transmission scheme that is able to suppress the CCI at Bobs. In multi-user wireless security communications with artificial noise, the precoding matrix is usually designed in the null space of channel matrix \( H_j \). We may called it precoding selection space as well. Obviously, the smaller the rank of matrix \( H_j \) is, the larger the dimension of its corresponding precoding selection space will be. In return, the design of the corresponding precoder has more freedom and the secrecy performance will be better. Therefore we define the dimension of the precoding selection space as the security degrees of freedom (SDF).

### A. Design of Precoders for Bobs Based on ISDF

In the existing schemes e.g., the SVD method, ZF beam-forming method and BD method, the precoder is designed based on their corresponding user’s own channel matrix or the other users’ channel matrices, which means the SDF will be largely limited by the rank of their corresponding channel matrices and the performance will be inevitably affected. To solve this problem, here we propose a new method, which are
designed based on the previously designed precoders instead. Because the new method can directly increase the SDF when designing each precoder, we just name it as the increasing security degree of freedom (ISDF) method. This method is similar to the idea of DPC method \(23\) in some sense. For example, Alice first picks a precoder for Bob 1 and then chooses a precoder for Bob 2 with full (noncausal) knowledge of the precoder for Bob 1. Therefore, Bob 1 does not see the signal intended for Bob 2 as interference. Similarly, the precoder for Bob 3 is chosen such that Bob 1 and Bob 2 do not see the signals intended for Bob 3 as interference. This process continues for all Bobs. Bob J subsequently sees the signals intended for all other users as interference, Bob 2 sees the signals intended for Bob 1 as interference, etc.

Suppose that the \(J\) Bobs has been sorted as Bob 1, Bob 2, \ldots, Bob \(J\) according to some criterion, which will be discussed in the simulation section. We first design \(W_1\) for Bob 1 without loss of generality.

\[
W_1 \propto \max d_1 \text{eigenvectors of } (H_1^H H_1),
\]

where Eq. \([8]\) means that \(W_1\) is composed by the \(d_1\) eigenvectors corresponding to the largest \(d_1\) eigenvalues of \(H_1^H H_1\).

Then the following precoders are designed to satisfy an basic condition that each of them must be located in the null space of all previously designed precoders.

\[
\begin{align*}
W_2 & \subset \ker(W_1), \\
W_3 & \subset \ker(W_1) \cap \ker(W_2), \\
& \vdots \\
W_j & \subset \bigcap_{i=1,i<j} \ker(W_i), \\
& \vdots \\
W_J & \subset \bigcap_{i=1}^{J-1} \ker(W_i),
\end{align*}
\]

where \(\ker(\cdot)\) denotes the null space (the kernel) of some matrix, and \(\bigcap\) represents the intersection of subspaces. Here, we define the co-designed-precoders matrix for the Bob \(j\) as

\[
\hat{W}_j = [W_1, W_2, \ldots, W_{j-1}].
\]

From Eq. \([9]\), we know that the design of each precoder should satisfy \(W_j^T W_j = [0, \ldots, 0]\).

Let \(L_j\) be the dimension of \(\ker(\hat{W}_j)\). Then the precoder \(W_j\) can be composed by

\[
W_j = T_{null,j} T_{stream,j},
\]

where \(T_{null,j}\) is used to suppress the interference, which is an \(N_A \times L_j\) matrix. \(T_{stream,j}\) is an \(L_j \times d_j\) matrix used for streams selection, which can make better use of the space resource and therefore improve the capacity.

Note that the precoding matrix \(W_j\) should be a nonzero matrix, otherwise, no signal is transmitted. To guarantee the existence of a nonzero precoding matrix, a sufficient condition is that the number of the transmit antennas is larger than the previous \(j - 1\) users’ total data streams, i.e.,

\[
N_A > \max_{j=1,2,\ldots,J} \sum_{i=1}^{j-1} d_i.
\]
B. Precoder Design for Eves Based on ISDF

Since the CSI of all receivers (except for the eavesdroppers) is available at the transmitter, in order to guarantee that it does not impact the desired receivers, the artificial noise is often mapped into the subspace orthogonal to the effective downlink co-channel matrix $\hat{H}$ [11][17], where

$$\hat{H} = [H_1^T, H_2^T, \ldots, H_J^T].$$

Then we can get the precoder $W \subset ker(\hat{H})$, i.e., the kernel of $\hat{H}$. Note that the precoding matrix $W$ should also be a non-zero matrix. To guarantee the existence of a non-zero power of artificial noise, a sufficient condition is that the number of the transmit antennas is larger than the rank of matrix $\hat{H}$. Because the practical channel matrix is usually assumed to be full-rank, $N_A$ must satisfy $N_A > \sum_{j=1}^J N_{Bj}$, which is a very tight constraint.

However, we actually don’t have to make $W$ orthogonal to each user’s channel matrix $H_j$. We can achieve the goal by transmitting the artificial noise into the null space of all users’ precoder matrices instead. Define the effective co-precoder matrix as

$$\hat{W} = [W_1, W_2, \ldots, W_J].$$

To transmit the artificial noise more effectively, it may be mapped into the subspace orthogonal to the effective co-precoder matrix $\hat{W}$. Then we can get $W \subset ker(\hat{W})$, i.e., $W$ lies in the null space of $\hat{W}$. Because inequality $d_j \leq N_{Bj}$ is always valid, the rank of $\hat{W}$ is usually smaller than that of $\hat{H}$. Therefore, we have more freedom to transmit the artificial noise, and the constraint on $N_A$ can therefore be relaxed as $N_A > \sum_{j=1}^J d_j$.

To distinguish the two schemes of transmitting artificial noise, we category them as the ISDF1 scheme and the ISDF2 scheme. If the noise is mapped into the subspace orthogonal to the effective downlink co-channel matrix $\hat{H}$, it is called ISDF1; if the noise is mapped into the subspace orthogonal to the effective co-precoder matrix $\hat{W}$, it is called ISDF2. In the simulation section, we will compare the two schemes.

C. The analysis of complexity

In this section, we will make a discussion on the computational complexity of the proposed approach versus the other three methods. As introduced in section II, The SVD method and ZF beamforming method are simple, but of little ideal performance. The BD method get somewhat better performance on secrecy rate, but its computational complexity becomes higher compared with the former.

Then, the emphasis is the comparison of complexity between the proposed ISDF method and the BD method. Essentially, the main difference between the ISDF and BD method lies on solving the precoding selection matrix (PSM). In the ISDF method, the PSM is obtained by implementing an SVD decomposition on the previously designed precoders $W_j$, which is a $N_A \times \sum_{n=1}^{J-1} d_n$ matrix. The complexity of this SVD decomposition is $O((\sum_{n=1}^{J-1} d_n)^3)$. While the PSM in the BD method is obtained by an SVD decomposition on all others’ channel matrices $H_j$, where $H_j = [H_{1j}^T, H_{2j}^T, \ldots, H_{(J-1)j}^T, H_{(J+1)j}^T, \ldots, H_{Jj}^T]$, which is a $N_A \times \sum_{n \neq j} N_{Bn}$ matrix. And the complexity of this SVD decomposition is $O((\sum_{n \neq j} N_{Bn})^3)$, which is higher than the former. Since both of the two methods have J precoders to design, there are J PSMs need to be solved accordingly. Therefore, the complexity of the ISDF method is $O(\sum_{j=1}^J (\sum_{n=1}^{J-1} d_n)^3)$, on the other hand, that of the BD method is $O(\sum_{j=1}^J (\sum_{n \neq j} N_{Bn})^3)$. Obviously, the complexity of the our proposed approach is lower.

IV. THE SECRECY RATE OF MUME-MIMO SYSTEM

In this section, we will analyze the secrecy rate of ISDF1 and ISDF2. Suppose that the variance of the transmit symbol vector $u$ is $\sigma_v^2$, and the complex Gaussian random elements of $v$ are i.i.d whose variance is $\sigma_v^2$. It is assumed that Alice has a total amount of transmit power budget $P$. Due to the normalization of the noise variance at Bob, we can also refer to $P$ as the transmission SNR. One important parameter should be designed is the power ratio, denoted by $\rho_j\ (0 < \rho_j < 1)$, allocated for the user $j$‘s information transmission. We define the power ratio for transmitting artificial noise as $\alpha\ (0 < \alpha < 1)$. Let

$$Q_j = E(u_j u_j^H)\ , \ Q_v = E(vv^H).$$

Then we have

$$tr(Q_j) = P_j = \rho_j P, \quad tr(Q_v) = \alpha P,$$

and

$$P = \sum_{j=1}^J \rho_j P + \alpha P = \sum_{j=1}^J d_j \sigma_{u_j}^2 + (N_A - \sum_{j=1}^J d_j) \sigma_v^2,$$

in which, we have used the following facts

$$\alpha = 1 - \sum_{j=1}^J \rho_j,$$

$$N_A = \sum_{j=1}^J d_j + 1,$$

$$\sigma_{u_j}^2 = \frac{P_j}{d_j} - \rho_j P,$$

$$\sigma_v^2 = \frac{(1 - \sum_{j=1}^J \rho_j) P}{N_A - \sum_{j=1}^J d_j}.$$
In order to analyze the secrecy rate concisely, Eq. (7) can be rewritten as:

\[ Y_j = H_j \sum_{i=1}^{J} W_i u_i + H_j Wv + N_j^P \]
\[ = H_j W_j u_j + H_j \sum_{i, i \neq j}^{J} W_i u_i + H_j Wv + N_j^P \]
\[ = \hat{H}_{jj} u_j + \sum_{i, i \neq j}^{J} \hat{H}_{ji} + \hat{H}_j v + N_j^P \]  

(24)

\[ Z_k = G_k \sum_{\ell=1}^{K} W_{\ell} u_{\ell} + G_k Wv + N_k^E \]
\[ = G_k W_k u_k + G_k \sum_{\ell=1, \ell \neq j}^{J} W_{\ell} u_{\ell} + G_k Wv + N_k^E \]
\[ = \hat{G}_{kk} u_k + \sum_{\ell=1, \ell \neq j}^{J} \hat{G}_{k\ell} + \hat{G}_k v + N_k^E \]

where we have defined

\[ \hat{H}_{jj} \triangleq H_j W_j, \quad \hat{H}_j \triangleq H_j W, \]  
\[ \hat{G}_{k\ell} \triangleq G_k W_{\ell}, \quad \hat{G}_k \triangleq G_k W, \]  

(25)

(26)

for \( j, i, \ell = 1, 2, \ldots, J \), \( k = 1, 2, \ldots, K \).

The secrecy rate is the maximum transmission rate at which the intended receiver can decode the data with arbitrarily small error, which is bounded by the difference in the capacity between Alice and Bob and that between Alice and Eve [2]. In the following part, the secrecy rate will be given in terms of secrecy sum rate [13] and absolute secrecy rate, where the secrecy sum rate is noted by \( R_{\text{ssr}} \) and the absolute secrecy rate is noted by \( R_{\text{asr}} \).

A. The secrecy rate of ISDF1

As in [11], we can normalize the distance of each Bob to make the variance of the elements of \( H_j \) equal to unity without loss of generality, and the noise vector \( N_j^P \) is of unit variance. Since the artificial noise is transmitted in the null space of all legitimate users’ matrixes, it will be nulled in any user’s received signal. Then the capacity between Alice and Bob \( j \) is

\[ R_j^B = E_{\hat{H}} \left\{ \log_2 \left| I + \sigma^2_{uj} \hat{H}_{jj} \hat{H}_j^H \right|^{-1} \right\} \]
\[ = E_{\hat{H}} \left\{ \log_2 \left| I + \rho P_{dj} \hat{H}_{jj} \hat{H}_j^H \left( I + \sum_{i \neq j}^{J} \rho P_{di} \hat{H}_{ij} \hat{H}_{ij}^H \right)^{-1} \right| \} \]  

(27)

where we used the fact \( \hat{H}_j = 0 \).

Next, we study the capacity between Alice and the multiple colluding or non-concluding Eves. When multiple Eves are allocated at different places, the noise at each Eve may be different. In addition, the receiver noise levels at Eves may not be known by Alice or Bobs. To guarantee secure communication, it is therefore reasonable to consider the worst-case scenario where the noises at Eves are arbitrarily small. Note that this approach has been also taken in [9] and [11]. In this case, the noiseless eavesdropper assumption gives an upper bound on the rate between Alice’s message for the user \( j \) and the eavesdropper \( k \) as

\[ R_{jk}^E = E_{\hat{H}, \hat{G}_k} \left\{ \log_2 \left| I + \sigma^2_{uj} \hat{G}_{kj} \hat{G}_{kj}^H \left( \sum_{\ell=1, \ell \neq j}^{J} \sigma^2_{uk} \hat{G}_{k\ell} \hat{G}_{k\ell}^H \right)^{-1} \right| \right\} \]
\[ = E_{\hat{H}, \hat{G}_k} \left\{ \log_2 \left| I + \rho_{j} P_{dj} \hat{G}_{kj} \hat{G}_{kj}^H \left( \sum_{\ell=1, \ell \neq j}^{J} \rho_{j} P_{d\ell} \hat{G}_{k\ell} \hat{G}_{k\ell}^H \right)^{-1} \right| \right\} \]
\[ + \frac{\alpha P}{N_{A} - \sum_{i=1}^{J} N_{Bi}} \hat{G}_k \hat{G}_k^H \]  

(28)

After deriving the expressions of \( R_j^B \) and \( R_{jk}^E \), the ergodic secrecy rate can now be obtained as \( R_{jk} = [R_j^B - R_{jk}^E]^+ \).

1) Secrecy Sum-Rate : As proposed in [13], the secrecy rate for Bob \( j \) is

\[ R_{\text{ssr}} = \min_{ 1 \leq k \leq K } \left\{ R_{jk} \right\}. \]

Then the secrecy sum rate is

\[ R_{\text{asr}} = \sum_{j=1}^{J} R_j^B = \sum_{j=1}^{J} \min_{ 1 \leq k \leq K } \left\{ R_{jk} \right\}. \]
\[ = \sum_{j=1}^{J} \min_{k} E_{\hat{H}, \hat{G}_k} \left\{ \log_2 \left| I + \frac{\rho_j P_{dj}}{d_j} \hat{H}_{jj} \hat{H}_j^H \right|^{-1} \right\} \]
\[ - \log_2 \left| I + \frac{\rho_j P_{dj}}{d_j} \hat{G}_{kj} \hat{G}_{kj}^H \left( \sum_{\ell=1, \ell \neq j}^{J} \frac{\rho_j P_{d\ell}}{d_{\ell}} \hat{G}_{k\ell} \hat{G}_{k\ell}^H \right)^{-1} \right| \]
\[ + \frac{\alpha P}{N_{A} - \sum_{i=1}^{J} N_{Bi}} \hat{G}_k \hat{G}_k^H \]  

(29)

2) Absolute secrecy rate : As we have discussed, for the broadcast MUME-MIMO wiretap system, the absolute secrecy capacity is the lower bound of the ergodic secrecy rate, which is given by

\[ R_{\text{asr}} = \min_{ 1 \leq j, k \leq K } \left\{ R_{jk} \right\} = \min_{j,k} E_{\hat{H}, \hat{G}_k} \left\{ \right\}, \]
\[ \log_2 \left| I + \frac{\rho_j P_{dj}}{d_j} \hat{H}_{jj} \hat{H}_j^H \left( I + \sum_{i \neq j}^{J} \frac{\rho_i P_{di}}{d_i} \hat{H}_{ij} \hat{H}_{ij}^H \right)^{-1} \right| \]
\[ - \log_2 \left| I + \frac{\rho_j P_{dj}}{d_j} \hat{G}_{kj} \hat{G}_{kj}^H \left( \sum_{\ell=1, \ell \neq j}^{J} \frac{\rho_j P_{d\ell}}{d_{\ell}} \hat{G}_{k\ell} \hat{G}_{k\ell}^H \right)^{-1} \right| \]
\[ + \frac{\alpha P}{N_{A} - \sum_{i=1}^{J} N_{Bi}} \hat{G}_k \hat{G}_k^H \]  

(30)
B. The secrecy rate of ISDF2

In the ISDF2 method, the rate between Alice and Bob $j$ is

$$R_j^B = E_{\mathbf{H}} \left\{ \log_2 \left| I + \sigma_{uj}^2 \hat{H}_{jj} \hat{H}_{ij}^H \left( I + \sum_{i \neq j} \sigma_{ui}^2 \hat{H}_{ij} \hat{H}_{ij}^H \right)^{-1} \right| \right\}$$

$$= E_{\mathbf{H}} \left\{ \log_2 \left| I + \frac{\rho_j P}{d_j} \hat{H}_{jj} \hat{H}_{ij}^H \left( I + \sum_{i \neq j} \frac{\rho_i P}{d_i} \hat{H}_{ij} \hat{H}_{ij}^H \right)^{-1} \right| + \frac{\alpha P}{N_A - \sum_{i=1}^d d_i} \hat{H}_{jj} \hat{H}_{ij}^H \right\}.$$ (31)

Similar to Eq. (28), the rate between Alice’s message for the user $j$ and the eavesdropper $k$ can be rewritten as:

$$R_{kj}^E = E_{\mathbf{H}, \mathbf{G}_k} \left\{ \log_2 \left| I + \sigma_{uj}^2 \hat{G}_{kj} \hat{G}_{kj}^H \left( I + \sigma_{uk}^2 \hat{G}_{kl} \hat{G}_{kl}^H \right)^{-1} \right| \right\}$$

$$= E_{\mathbf{H}, \mathbf{G}_k} \left\{ \log_2 \left| I + \frac{\rho_j P}{d_j} \hat{G}_{kj} \hat{G}_{kj}^H \left( I + \sum_{l=1, l \neq j}^{J} \frac{\rho_l P}{d_l} \hat{G}_{kl} \hat{G}_{kl}^H \right)^{-1} \right| + \frac{\alpha P}{N_A - \sum_{i=1}^d d_i} \hat{G}_{kj} \hat{G}_{kj}^H \right\}. \tag{32}$$

1) Secrecy Sum-Rate : Accordingly, the secrecy sum-rate is

$$R_{ssr} = \sum_{j=1}^J R_j^B = \frac{1}{N_c} \sum_{j=1}^J \min_{1 \leq k \leq K} \{ R_{jk} \}.$$ 

$$= \sum_{j=1}^J \min_k E_{\mathbf{H}, \mathbf{G}_k} \left\{ \left| \log_2 \left| I + \frac{\rho_j P}{d_j} \hat{H}_{jj} \hat{H}_{ij}^H \right| \right| \right\}$$

$$\left( I + \sum_{i \neq j} \frac{\rho_i P}{d_i} \hat{H}_{ij} \hat{H}_{ij}^H \right)^{-1} \right| + \frac{\alpha P}{N_A - \sum_{i=1}^d d_i} \hat{H}_{jj} \hat{H}_{ij}^H \right\}.$$ (33)

2) Absolute secrecy rate: we can also get the lower bound of ergodic secrecy rate for ISDF2 method,

$$R_{asr} = \min_{1 \leq j \leq J, 1 \leq k \leq K} E_{\mathbf{H}, \mathbf{G}_k} \left\{ \left| \log_2 \left| I + \frac{\rho_j P}{d_j} \hat{H}_{jj} \hat{H}_{jj}^H \right| \right| \right\}$$

$$\left( I + \sum_{i \neq j} \frac{\rho_i P}{d_i} \hat{H}_{ij} \hat{H}_{ij}^H \right)^{-1} \right| + \frac{\alpha P}{N_A - \sum_{i=1}^d d_i} \hat{H}_{jj} \hat{H}_{jj}^H \right\}.$$ (34)

C. The secrecy rate of ISDF1 with Water Filling method

This section focuses on improving the secrecy rate with water-filling method based on ISDF1 method introduced in section III. As mentioned in section III-A, when the users have multiple antennas and $d_j$ ($d_j > 1$) data streams are transmitted to user $j$ simultaneously, the water-filling method may be employed together with ISDF1 to further improve the performance of achievable secrecy rate. Our goal is to implement single-user water-filling on each legitimate user to maximize their rate under the given power ratio constraint.

The single-user optimization problem has a well-known water-filling solution. The water-filling algorithm takes advantage of the problem structure by decomposing the channel into orthogonal modes, which greatly reduces the optimization complexity. Here, without loss of generality, we just take user $j$ for example with the constraint of the given total power $P_j$. As derived in Eq. (22), the signal received at user $j$ can be written as:

$$Y_j = \hat{H}_{jj} u_j + K_j \tag{35}$$

where $K_j = \sum_{i=1, i \neq j} \hat{H}_{ij} u_i + \hat{H}_j v + N_j^B$ is the interference towards user $j$ from the message signal transmitted to other users. We first implement an Whiting processing on the interference vector $K_j$ in Eq. (35), before conducting the water-filling algorithm. Let $M = E(K_j K_j^H)$, which unitary decomposition is $M = E \Lambda E^H$. The we can whiten $K_j$ by

$$\Lambda^{-1/2} E^H Y_j = \Lambda^{-1/2} E^H \hat{H}_{jj} u_j + \Lambda^{-1/2} E^H K_j. \tag{36}$$

Then the corresponding mutual information rate $R_j^B$ is

$$R_j^B = E_{\mathbf{H}} \left\{ \log_2 \left| I + \sigma_{uj}^2 \Lambda^{-1/2} E^H \hat{H}_{jj} \hat{H}_{jj}^H E \Lambda^{-1/2} \right| \right\}. \tag{37}$$

Suppose that $R_{ji}^B$ is the rate from the $n$-th subcarrier to Bob $j$, and $h_{ji}$, the corresponding effective sub-channel after whitening and orthogonalizing. Since only $d_j$ streams are transmitted to user $j$, the optimal problem on the dynamic power allocation aiming to maximize the rate $R_j^B$ between transmitter and user $j$ can be expressed as following:
\[ R_j^* = \max d_j \sum_{n=1}^{d_j} \log_2 (1 + P_{jn} h_{jn}), \]  
\[ s.t. \sum_{n=1}^{d_j} P_{jn} \leq P_j = \rho_j P, \text{ and } P_{jn} > 0, \forall n, \]
where \( P_{jn} \) denotes the power allocated for the \( n \)-th subcarrier of user \( j \). Based on Eq. \( [35] \), we use the classical Lagrange algorithm to construct an Lagrange function,
\[ L = \sum_{n=1}^{d_j} \log_2 (1 + P_{jn} h_{jn}) - \lambda \left( \sum_{n=1}^{d_j} P_{jn} - \rho_j P \right). \]

Let
\[ \frac{\partial L}{\partial P_{jn}} = \frac{1}{\ln 2} \frac{h_{jn}}{1 + h_{jn} P_{jn}} - \lambda = 0, \]
and \( \beta = \lambda \ln 2 \). Then we can obtain
\[ \beta = \frac{h_{jn}}{1 + P_{jn} h_{jn}}, \quad P_{jn} = \left[ \frac{1}{\beta} - \frac{1}{h_{jn}} \right]^+. \]

Then solving the optimal problem depends on the computation of \( \beta \) and \( P_{jn} \). The paper [26] put forward a fast iterative algorithm, which give the preliminary value of \( \beta \) and its update method. That is
\[ \beta_0 = \frac{1}{d_j} \left( \left| P_j \right| + \sum_{n=1}^{d_j} \frac{1}{h_{jn}} \right), \]  
\[ \beta_{k+1} = \beta_k \left( \left| P_j \right| + \sum_{n=1}^{d_j} P_{jn} \right). \]

In this paper, we will employ this fast iterative algorithm to perform simulations.

V. SIMULATION RESULTS

In this section, we will carry out some simulations to show the achievable secrecy rate. In all simulations, the entries of all channel matrices are assumed to be independent, zero-mean Gaussian random variables with unit variance. All results are based on an average of 1000 independent trials. The background noise power is the same for all Bobs with a variance \( I \). To guarantee the secure communication, it is therefore reasonable to consider the worst case scenario, where the noises variance at Eves are arbitrarily small (approaching zero). The desired rate for Bobs and Eves will be measured by the ergodic capacity rather than the outage capacity.

A. Secrecy Rate and Power Efficiency

Fig. 2, Fig. 3 and Fig. 4, are corresponding to the cases \( d_j = 1 \), which exhibits the comparison of secrecy rate and information power ratio for information signal among the 5 methods. Fig. 2 and Fig. 3 demonstrate that the ISDF1 method offers the best performance compared with the other methods, in term of secrecy sum rate and absolute secrecy rate respectively. From Fig. 2 and Fig. 3, we can see that ISDF2 performs best in the low SNR region but performs worse in the high SNR region. This is because that at the high SNR region, the artificial noise is not well canceled at receive terminal by using the null space of the co-precoder matrix to design \( W \). Since ISF1 uses the null space of the co-channel matrix to design \( W \), which can well cancel the artificial noise. From Fig. 4, we can see that both ISDF1 and ISDF2 methods have less power efficiency (information power over total power) than the ZF Beamforming method. While in secure communications, the secrecy rate is the major concern. Therefore the ISDF scheme provides a good candidate for secure communications.
B. Secrecy Rate and Power Efficiency for Eves’ Colluding and Non-Colluding

Fig. 5 and Fig. 6 show the absolute secrecy rate and information power ratio of ISDF1 for the Eves’ colluding and non-colluding scenarios. If the Eves choose to wiretap the message jointly, we may think they are colluding, else non-colluding. As shown in Fig. 5, it will be more difficult to achieve secure communication if the Eves choose to cooperate, which is as we expected. We are interesting in whether we need to allocate more power to transmit information signal or artificial noise when the Eves choose to cooperate. Fig. 6 shows that more power needs to be allocated to artificial noise if Eves choose to cooperate. So does the case with more Eves. These demonstrate that when power has been optimized already and the eavesdroppers’ condition is getting better, the power allocating towards artificial noise can make more contribution for the secrecy rate than allocating towards users’ information signal.

C. Power Ratio for different Ordering and Power Allocation

As mentioned in section III-A, since the ordering of the users will affect the performance, we wish to study how the power is allocated between the information signal and the artificial noise for giving different user priority of precoding. Fig. 7 illustrates that the the system performs better in term of absolute secrecy capacity when we give the user with least antennas or largest noise variance the priority of precoding. Fig. 7 shows that the absolute secrecy capacity is mainly determined by the poorest-performance receive-wiretap pair to a large extent. In order to get a large secrecy rate, we should
make the secrecy rate for each user equivalently. Therefor should give the weaker user (less antennas or larger noise) more SDF by rendering them the priority of precoding. Fig. 8 shows that the secrecy capacity can be further increased as introduced in IV-C, if the water-filling (WF) algorithm is used.

VI. CONCLUSIONS

This paper proposes the precoding strategy based on the ISDF method for providing secure communication at the physical layer in broadcast MUME-MIMO wiretap channels combined with artificial noise. We derive both the secrecy sum rate and absolute secrecy rate for the proposed ISDF1 and ISDF2 method. Simulations show that the ISDF2 perform best in low SNR region and ISDF1 outperforms other four methods in high SNR region in terms of achievable secrecy rate. Furthermore, we find that more power should be allocated to artificial noise instead of information signal when the eavesdroppers’ condition is better than the intended users, and we should first precode the user with bad condition (least antennas or largest noise variance).

REFERENCES

[1] A. Wyner, "The wire-tap channel," *Bell System Technical Journal*, vol. 54, no. 8, pp. 1355-1387, Oct. 1975.
[2] I. Csiszár and J. Körner, "Broadcast channels with confidential messages," *IEEE Trans. Information Theory*, vol. 24, no. 3, pp. 339-348, May 1978.
[3] J. Huang, A. L. Swindlehurst, "Cooperative Jamming for Secure Communications in MIMO Relay Networks" *IEEE Trans. on Signal Processing*, No. 59, no. 10, pp. 2011 4871-4884, 2011.
[4] S. A. Fakoorian, A. L. Swindlehurst, "Optimal Power Allocation for GSYD-Based Beamforming in the MIMO Wiretap Channel," *IEEE International Symposium on Information Theory (ISIT)*, Cambridge, MA, August 2012, pp. 2321-2325.
[5] M. L. Jorgensen, B. R. Yanakiev, F. E. Krikelund, P. Popovski, H. Yomo, and T. Larsen, "Shout to secure: Physical-layer wireless security with known interference," in *Proc. IEEE GLOBECOM*, Washington, DC, Nov. 2007, pp. 33-38.
[6] O. Simeone and P. Popovski, "Secure communications via cooperating base stations," *IEEE Communications Letters*, vol. 12, no. 3, pp. 188-190, Mar. 2008.
[7] X. Tang, R. Liu, P. Spasojevic, and H. V. Poor, "Interference-assisted secret communication," in *Proc. IEEE ITW*, Porto, Portugal, May 2008, pp. 164-168.
[8] L. Lai and H. E. Gamal, "The relay-eavesdropper channel: Cooperation for secrecy," *IEEE Trans. Information Theory*, vol. 54, no. 9, pp. 4005-4019, 2008.
[9] R. Negi and S. Goel, "Secret communications using artificial noise," in *Proc. IEEE VTC*, Dallas, TX, Sep. 2005, pp. 1906-1910.
[10] S. Goel and R. Negi, "Guaranteeing secrecy using artificial noise," *IEEE Trans. Wireless Communications*, vol. 7, no. 6, pp. 2180-2189, 2008.
[11] X. Zhou, M. McKay "Secure transmission with artificial noise over fading channels: achievable rate and optimal power allocation," *IEEE Trans. on Vehicular Technology*, vol. 59, no. 8, pp. 3831-3842, 2010.
[12] A. L. Swindlehurst, "Fixed SINR solutions for the MIMO wiretap channel," in *Proc. IEEE ICASSP*, Taipei, Taiwan, Apr. 2009, pp. 2437-2440.
[13] G. Geraci, J. Yuan, A. Razi and I. B. Collings, "Secrecy Sum-Rates for Multi-User MIMO Linear Precoding," 8th International Symposium on Wireless Communication Systems (ISWCS), 2011.
[14] E. Ekrem, and S. Ulukus "The Secrecy Capacity Region of the Gaussian Multi-Receiver Wiretap Channel," *IEEE Trans. on Information Theory*, vol. 57, no. 4, pp. 2083-2114, April 2011.
[15] Y. Liang, H. V. Poor, and S. Shamai, "Secure communication over fading channels," *IEEE Trans. on Information Theory*, vol. 54, no. 6, pp. 2470-2492, Jun. 2008.
[16] Y. Liang, G. Kramer, H. V. Poor, and S. Shamai (Shitz), "Compound wire-tap channels," in *Proc. 45th Annu. Allerton Conf. Communication, Control and Computing*, Monticello, IL, USA, 2007.
[17] A. Mukherjee, S. A. A. Fakoorian, J. Huang, A. L. Swindlehurst, "Principles of Physical Layer Security in Multiuser Wireless Networks: A Survey"; available online: [http://arxiv.org/abs/1011.3754](http://arxiv.org/abs/1011.3754)
[18] K. Xie, and W. Chen, "Precoding Strategy Based on SLR for Secure Communication in MUME Wiretap Systems," *IEEE Global communications (GC)*, 2012.
[19] A. Mukherjee and A. L. Swindlehurst, "Utility of Beamforming Strategies for Secrecy in Multiuser MIMO Wiretap Channels," in *Proc. 47th Allerton Conf. on Communication, Control and Computing*, October, 2009.
[20] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. on Signal Processing*, vol. 52, no. 2, pp. 461471, 2004.
[21] Q. Spencer and A. Swindlehurst, "A hybrid approach to spatial multiplexing in multi-user MIMO downlinks," *EURASIP Journal on Wireless Communications and Networking*, pp. 225-247, Dec. 2004.
[22] D. Gesbert, M. Kountouris, R. W. Heath Jr, C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm: From single user to multiuser communications," *IEEE Signal Processing Magazine*, vol. 24, no. 10, pp. 36C46, 2007.
[23] M. H. M. Costa, Writing on dirty paper, *IEEE Trans. Information Theory*, vol. IT-29, no. 5, pp. 439C411, 1983.
[24] F. Ogger and B. Hassibi, "The secrecy capacity of the MIMO wiretap channel," in *Proc. IEEE ISIT*, Toronto, ON, Canada, Jul. 2008, pp. 524-528.
[25] L. Choi and R.D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Trans. on Wireless Communications*, vol. 2, no. 4, pp. 775C786, 2003.
[26] Jang J, Lee K B, and Lee Y H . Transmit power and bit allocations for OFDM systems in a fading channel. in *Proc. IEEE Global Communications Conference*, San Francisco, CA, Dec. 2003: 283C288.