Tidal interaction in binary black hole inspiral

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Abstract

In rotating viscous fluid stars, tidal torque leads to an exchange of spin and orbital angular momentum. The horizon of a black hole has an effective viscosity that is large compared to that of stellar fluids, and an effective tidal torque may lead to important effects in the strong field interaction at the endpoint of the inspiral of two rapidly rotating holes. In the most interesting case both holes are maximally rotating and all angular momenta (orbital and spins) are aligned. We point out here that in such a case (i) the transfer of angular momentum may have an important effect in modifying the gravitational wave “chirp” at the endpoint of inspiral. (ii) The tidal transfer of spin energy to orbital energy may increase the amount of energy being radiated. (iii) Tidal transfer in such systems may provide a mechanism for shedding excess angular momentum. We argue that numerical relativity, the only tool for determining the importance of tidal torque, should be more specifically focused on binary configurations with aligned, large, angular momenta.

Introduction

The problem of the dynamics of a binary pair of comparable mass black holes (BHs) has been of great interest in connection with sources of gravitational waves [1]. In the early stages of such a binary, when the separation of the two holes is large compared to the size of the holes, the interaction is weak and post-Newtonian approximations suffice [2]. At the latest stage, the “close-limit” approximation provides the waveform [3]. Speculations about the strong field interaction have been based on extrapolating both methods [4,5]. These methods are useful, but cannot be trusted to show the effects of phenomena that are unique to the strong field stage of the binary interaction. Here we point out that one such effect that has been overlooked may play an important role in systems with high angular momentum.

The effect can be thought of as the torque exerted on a rapidly rotating BH due to the tidal distortion created by the other BH. When the separation \( r \) of the BHs is large, and the holes are interacting weakly, the results of a perturbative analysis, as well as an analogy with tidal torquing in fluid stars, shows that this tidal torque falls off as \( r^{-6} \) and (as will be shown below) is unimportant. When \( r \) is on the order of the size of the BHs, on the other hand, this torque may be comparable to the torque exerted by radiation reaction, and may therefore be of great importance to the waveform of gravitational radiation for just that phase of inspiral that generates the highest gravitational wave power.

The late-stage strong-field stage of inspiral, in which tidal torque may play a crucial role, is not amenable to any presently available approximation. Reliable answers appear only to be possible with numerical relativity, solution of Einstein’s equations on supercomputers. The application of numerical relativity to binary inspiral has proved to be enormously difficult, but there has been considerable progress recently [6] and an analysis of tidal torque appears now to be a challenging but not impossible goal. A major purpose of the present paper is to bring to the attention of the numerical relativity community the importance of investigating the phenomenon of late-stage tidal torque.

It is useful to point out that the phrase “tidal torque” must not be taken too literally. The distinction between the spin angular momentum of orbiting objects, and the orbital angular momentum of those objects, is meaningful only when the objects are well separated, or if the individual objects are rigid. These criteria are certainly not valid at the stage of inspiral at which two disjoint horizons are merging into a single horizon. Nevertheless, the imagery and vocabulary of tidal torque, used cautiously, are useful as a background to approximations and descriptions, and to point to interesting directions for exploration with numerical relativity.

When two bodies are in binary orbit, each raises tidal bulges in the shape of the other. If the objects are rotating, their fluid flows through this distorted pattern undergoing shear. Viscosity in the rotating fluid shifts the position of the tidal bulges, and gravitational forces on the bulges produce a torque. Bildsten and Cutler [7] showed that tidal torque could not be important in inspiral involving neutron star (NS) systems for two reasons.
First, the tidal torque becomes significant only when separation is so small that a NS would have been tidally disrupted. Their conclusion that small separation is required is completely consistent with our findings. Tidal disruption, of course, has no meaning in the BH case (although the merger of the horizons is somewhat analogous).

The second argument given by Bildsten and Cutler is that tidal torque for a NS would require unrealistically large coefficient of viscosity, of order \( c \times (\text{NS radius}) \). On the basis of perturbative computations, Hartle [8] was the first to suggest that it is useful to treat horizons as having effective surface shear viscosity and to be subject to tidal torques in the same manner as fluid stars. The formalism for ascribing fluid mechanical properties more generally to highly distorted horizons was subsequently developed [9,10]. In that formalism the surface shear viscosity has a value of \( 1/16\pi \), in the c = G = 1 units that we use. If converted to an effective volume viscosity, this meets the Bildsten-Cutler criterion.

Weak field approximation

For quantitative, though imperfect, insights into the tidal torquing process, we present an analysis here of the effect of tidal torquing in a BH-BH binary, in the limit of slow rotation and large separation. In this approximation, the effect of the tidal field \( E \) on the spin angular momentum \( S \) of a hole has been shown by Teukolsky to be [1]

\[
\frac{dS}{dt} = -\frac{2}{5} \epsilon^2 S \mu^3 \left[ 1 + 3 \frac{S^2}{\mu^2} \right],
\]

where \( \mu \) is the mass of the hole. The tidal field at one of the holes, due to the other hole, also of mass \( \mu \), at the the binary separation \( r \), is [2]

\[
E = -\frac{2 \mu}{r^3}.
\]

These combine to give the tidal torque spindown rate

\[
\left. \frac{dS}{dt} \right|_{\text{Tidal}} = -\frac{8}{5} \frac{\mu^5}{r^6} S \left[ 1 + 3 \frac{S^2}{\mu^2} \right].
\]

This result must be compared with the rate of radiation of angular momentum in gravitational waves. For a quasi-Newtonian circular orbit of two BHs, each of mass \( \mu \), degraded by quadrupole radiation, this rate is [13]

\[
\frac{dJ}{dt} \bigg|_{\text{GW}} = -\frac{256}{5\sqrt{2}} \frac{\mu^{9/2}}{r^{7/2}}.
\]

For two extreme (\( S = \mu^2 \)) Kerr holes, the above two equations predict equal influence of tidal torque and radiation reaction on the orbital angular momentum \( L = J - 2S \) for separation \( r \) slightly less than \( \mu \). This helps to confirm that tidal torque in BH-BH binaries, as in NS binaries, can only be important at very small separations.

The approximations leading to Eqs. [1] and [3], cannot be trusted to better than rough order of magnitude at small separations, and hence cannot be used to rule out strong, even dominant, tidal effects for \( r \) several times \( \mu \). Reliable answers, presumably from numerical relativity, are not yet available. Lacking those we must make do here with simple illustrative toy models that show how the importance of tidal torque depends on the unknown strong-field details. In particular, we show orbital frequency \( \omega \) as a function of time \( t \), for several assumptions about tidal torque at small \( r \). All models assume that the binary starts at large separation with \( S = \mu^2 \), and in all models the Newtonian expression for orbital angular momentum \( L = r\omega^2\mu/2 \) is equal to \( J - 2S \). The mass \( \mu \) of the individual holes is taken to increase due to tidal dissipation by a law \( d\mu/dt = -\omega dS/dt \) based on perturbative models [14]. The factors shown in the figure are inserted into the right hand side of Eq. [1] to give different models of strong-field tidal torque in comparison with the Newtonian expression. Each model then consists of the relationship of (i) \( r \), \( \mu \) and \( \omega \) given by Newtonian theory (ii) the Newtonian angular momentum relationship \( r\omega^2\mu/2 = J - 2S \), (iii) the models for \( dJ/dt \), \( dS/dt \) and \( d\mu/dt \) discussed above.

The results in Fig. 1 show what should be expected. The highest (solid) curve in Fig. 1 is the evolution of the orbital frequency in the absence of tidal torque. This curve is unbounded, since the Newtonian computation on which it is based has no reference to a special radius at which the inspiral changes its nature. If tidal torque is included according to Eq. [1], no change can be seen in the result until a yet higher frequency is reached. When \( dS/dt \) is enhanced with a factor of \( 1 - 6\mu/r \), or its square, the result is to stall the inspiral near an orbital frequency \( \omega \) equal to
the Newtonian value for \( r = 6\mu \). An analogous inspiral stall occurs at a lower frequency for enhancement at \( r = 8\mu \). These models illustrate that plausibly strong tidal torque can have profound effects on the waveform. The period of nearly constant frequency corresponding to the inspiral stall has a dramatically different character than the frequency sweep of the “chirp” usually associated with gravitational waves from binary inspiral. Such a constant frequency signal should be more easily detectable than a chirp signal. Furthermore, the stall of the inspiral means that the binary will be radiating for a prolonged time, and hence radiating more energy than would be inferred from models that omit tidal torque. The additional energy radiated is in a sense the rotational energy of the holes which becomes “radiatable” through conversion to orbital motion.

**Late stage approximation**

The challenges of numerical relativity mean that we must temporarily be satisfied with calculations that are less than definitive. The models of the previous section show features of direct importance to the question of tidal torque, but require guesses about strong-field interactions. We next turn to a calculation that correctly represents strong-field interaction, but is related to tidal torque somewhat indirectly. Figure 2 pictures three different initial configurations of two equal mass holes. In Fig. 2a, two nonspinning holes, at separation \( L \), start from rest; in Fig. 2b, two nonspinning holes represent orbital motion by having initial transverse momentum \( P \); Fig. 2c shows two holes, each having spin \( S \), starting with no momentum. A comparison of Fig. 2c with the other two cases is intended to show the following: (i) Since the holes are spinning, the infall in configuration (c) is no longer “radial,” as in Fig. 2a, and the radiation emitted contains much the same multipole structure as for the initial configuration in (b). (ii) The addition of spin increases the radiated energy in (c) much the same as transverse motion does in (b).

The radiation computation for the configuration in Figs. 2a and 2b can be computed using the close-limit approximation [3] for the late stages of inspiral. In that method the spacetime evolving from the initial data is treated as a perturbation of the spacetime of the final hole, with the initial separation \( L \) taken as the perturbation parameter. The actual initial data, i.e., solution of the initial value problem for Einstein’s equations, follow the Bowen-York [15] prescription. This close-limit calculation was remarkably successful in finding the radiation for the case of head-on collisions starting with holes at rest, as in Fig. 2a, or initially moving towards each other. The method has also been applied [3] with Bowen-York initial data for holes with transverse, or orbital, momentum \( P \) as in Fig. 2b. In that application it was necessary to treat the momentum \( P \), as well as the initial separation \( L \), as a perturbation parameter. The radiated energy found with this multiparameter perturbation calculation is:

\[
\frac{\text{Energy}}{M} = 9.81 \times 10^{-5} \left( \frac{L}{M} \right)^4 + 1.61 \times 10^{-2} \left( \frac{J}{M^2} \right)^2.
\]

Here the ADM angular momentum \( J \) is given by \( J = PL \), the same expression as in Newtonian theory. When this result is extrapolated to conditions approximately representing the “ISCO,” the innermost stable pre-merger circular orbit, the results are in reasonable agreement (factor of 5 or so) with recent results from numerical relativity [16].

In principle, the configuration in Fig. 2c, with Bowen-York [15] initial data, can be analyzed with \( S \) and \( L \) treated as perturbations, but a new complication arises in the details: the analysis turns out to require higher order perturbation theory. This analysis is being carried out at present by Gleiser and Dominguez [17], but some features of the result do not require the completion of that work. In particular, the radiated energy must have the form

\[
\frac{\text{Energy}}{M} = 9.81 \times 10^{-5} \left( \frac{L}{M} \right)^4 + \alpha \left( \frac{J}{M^2} \right)^2 \left( \frac{L}{M} \right)^4,
\]

where \( J = 2S \). Preliminary numerical relativity results by Baker, Campanelli, Lousto and Takahashi [18] suggest that \( \alpha \) is around \( 2 \times 10^{-4} \). (This is in accord with our own order-of-magnitude estimates of \( \alpha \).)

For definiteness in a discussion of multipole structures for Fig. 2, we let the page be the \( xy \) plane, so that the angular momenta in Figs. 2b,c are in the \( z \) direction. The axisymmetric configuration in Fig. 2a can radiate only in even-parity even-\( \ell \) multipoles. Since the symmetry axis in Fig. 2a is not the \( z \) axis, there will be \( m \neq 0 \) multipoles of the radiation, but the axisymmetry imposes constraints on the ratio of energy in multipoles of different \( m \). The less stringent symmetry arguments for the configurations in Figs. 2b,c require only that the radiation be in even-parity even-\( \ell \) modes or odd-parity odd-\( \ell \) modes. The multipolar distributions in both Fig. 2b and Fig. 2c greatly differ from the radial symmetry pattern of Fig. 2a. As an example, much of the radiated energy in the close-limit calculation for Fig. 2c is in the odd-parity \( \ell = 3 \) mode (although details require the completion of the higher order perturbation theory calculation). This demonstrates that adding spin to the black holes changes the pattern of the radiated energy, just as orbital motion does.
Discussion

We have used the term “tidal torque” to refer rather generally to the processes related to BH spin in the late stage of BH binary inspiral. We have argued, with oversimplified models, that such processes may produce crucially important effects that cannot be seen in post-Newtonian computations, and that have not yet been studied with numerical relativity. Using a close-limit slow rotation viewpoint, we have demonstrated that spin angular momentum and orbital angular momentum play somewhat the same role in generating gravitational radiation, and hence that tidal torque (in our generalized sense) definitely does play an important role if the holes are close together.

The role that tidal torques play in binary inspiral will depend on whether the holes are ever close enough together. Towards the end of inspiral the BH-BH system will reach a point at which gradual inspiral of two bodies is replaced either by a plunge or by the formation of a single final rotating hole. If the plunge occurs at relatively large separation, tidal effects may never be significant, and may have no effect on the chirp waveform. If the two horizons merge to a single final horizon at sufficiently large separation, the very meaning of tidal effects is obscured.

The role of tidal effects is closely linked to a long standing question about binary inspiral. A Kerr black hole, the stationary solution that is the final state of the inspiral, is constrained to have its angular momentum and mass related by $J \leq M^2$. The angular momentum per unit energy radiated in gravitational waves is dependent on geometry and frequency. Plausible but nonrigorous arguments suggest [1] that a highly dynamical final hole will radiate at frequencies too high to reduce $J/M^2$. Similar arguments make it seem unlikely that $J/M^2$ could be reduced during a plunge. An awkward question then arises in the case (as in Fig. 1) that the individual holes are maximally rotating, and all angular momentum is parallel. A BH binary just before merger may have excess angular momentum. This situation may arise through gradual inspiral, or may be an initial value solution chosen expressly to have excess angular momentum. Tidal torque provides a way of viewing how the system can meet the requirements to form a final hole: the transfer from spin to orbital motion can keep the frequency low (as in Fig. 1) until $J/M^2$ is sufficiently reduced.

We hope that the points made in this article are sufficiently persuasive that some effort will be directed to numerically evolving configurations that would clarify the role of tidal torque. In particular it would be useful to have studies comparing the evolution of initial data with and without large spin angular momentum.

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[1] É. É Flanagan and S. A. Hughes, Phys.Rev. D57 4535 (1998). Many other references can be found in this paper.
[2] An introduction to the method can be found in L. Blanchet, in Gravitational Waves, Proceedings of the Como School on Gravitational Waves in Astrophysics, edited by I. Ciufolini, V. Gorini, U. Moschella and P. Fré (Institute of Physics Publishing, London, 2001); very recent work is reported in T. Damour, P. Jaranowski, and G. Schäfer, submitted to Phys. Lett. B, preprint gr-qc/0105038 and L. Blanchet, G. Faye, B. Iyer, and B. Joguet, submitted to Phys. Rev. Lett, preprint gr-qc/0105099.
[3] R. H. Price and J. Pullin, Phys. Rev. Lett. 72, 3297 (1994).
[4] A. Buonanno and T. Damour, Phys. Rev. D62, 064015 (2000).
[5] R. Gleiser, G. Khanna, R. Price and J. Pullin, New Jour. Phys. 2, 3 (2000). See http://njp.org/.
[6] Recent reports of progress in numerical relativity applied to black hole binary inspiral can be found in S. Brandt, et al., Phys.Rev.Lett. 85 (2000) 5496-5499; J. Baker, B. Brügmann, M. Campanelli, C. O. Lousto, R. Takahashi, submitted to Phys. Rev. Lett., preprint gr-qc/0102037; M. Alcubierre et al., preprint gr-qc/0012074.
[7] L. Bildsten and C. Cutler, Astroph. J. 400, 175 (1992).
[8] Phys. Rev. D8, 1010.1973; Phys Rev D9, 2749, 1974.
[9] R. Price and K. S. Thorne, Phys. Rev. D33, 915 (1986).
[10] K. S. Thorne, R. H. Price, and D. A. Macdonald, Black Holes: The Membrane Paradigm (Yale University Press, New Haven, 1986).

[11] S. A. Teukolsky, Ph.D. thesis, California Institute of Technology, 1973; Ref. [10], Eq. 7.51.

[12] J. M. Bardeen, W. H. Press, and S. A. Teukolsky Astrophys. J. 178, 347 (1972); Ref. [10], Eqs. 6.9 and 6.10.

[13] P. C. Peters and J. Mathews, Phys. Rev. 131, 435 (1963); C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation (Freeman, San Francisco, 1973), Sec. 36.6.

[14] See, for example, Ref. [10], Sec. VII B1. The mass change in the models, in any case, turns out to be unimportant.

[15] J. Bowen and J. W. York, Jr., Phys. Rev. D 21, 2047 (1980).

[16] J. Baker, B. Brügmann, M. Campanelli, C. O. Lousto, and R. Takahashi, preprint gr-qc/0102037.

[17] R. Gleiser and E. Dominguez (private communication).

[18] J. Baker, M. Campanelli, C. O. Lousto and R. Takahashi (private communication).

FIG. 1. Results from simple models of tidal torque. The Newtonian tidal torque $dS/dt$ is multiplied by the factors shown.

FIG. 2. Initial conditions for close limit calculations. (a) Nonspinning holes at rest. (b) Nonspinning holes moving transversely. (c) Unmoving spinning holes.