Creation of 2000-atom Greenberger-Horne-Zeilinger states by entanglement amplification

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We propose a novel entanglement-creation scheme in a multi-atom ensemble, named entanglement amplification, which converts unentangled states into entangled states and amplifies less-entangled ones to maximally-entangled Greenberger-Horne-Zeilinger (GHZ) states. By shifting the energy of a particular Dicke state, we break the Hilbert space of the ensemble into two isolated subspaces to tear the coherent spin state into two components. Afterward, we can utilize the isolated spaces to further enhance the entanglement by coherently separating the two components. By one-particle Rabi drivings on atoms in a high-finesse optical cavity illuminated by a single-frequency light, 2000-atom GHZ states can be created with a fidelity above 80% in an experimentally achievable system, making resources of ensembles at Heisenberg-limit practically available for quantum metrology.

Entanglement plays a central role in quantum mechanics. It is one of the most important topics in fields including quantum information [13], quantum communication [4, 5] and quantum metrology [6–8]. By utilizing different classes of entangled states, one can speed up computations [9–11], secure private communications [12–16], and overcome the standard quantum limit [17–24] to get better precision. Among all the classes of entangled states, the Greenberger-Horne-Zeilinger (GHZ) state [25] is one of the ultimate goals for quantum information and quantum metrology [26–37], displaying the Heisenberg limit [38] with the best precision guaranteed by fundamental principles of quantum mechanics.

However, it is non-trivial or even challenging to create such highly-entangled states in multi-particle ensembles. Usually it requires complex or long-range many-body interactions to entangle different particles which are spatially separated. In the past few years, there are many groups making pioneering contributions in realizing the multi-particle GHZ state at different platforms, and it has been achieved with up to 14 trapped ions [26, 29], 10 photons [30, 32], and 12 superconducting qubits [33, 34]. These outstanding works indeed start a new era in developing scalable quantum computers, advancing quantum metrology, and establishing quantum communication and teleportation. Recently there is a breakthrough where up to 20 qubits [35–37] are entangled with a fidelity above 0.5. Nevertheless, the required precision of the control and technical difficulties increase exponentially as the number of qubit grows, making it difficult to increase the size of GHZ states.

In this Letter, we propose a deterministic scheme to convert non-entangled states into less-entangled states, and further amplify the less-entangled ones to maximally-entangled GHZ states in atomic ensembles. By shifting

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the energy of one particular angular momentum eigenstate of collective atomic spins (Dicke state 39), the Hilbert space is broken into two isolated subspaces separated by this energy-shifted boundary. Any wavefunction in one of the subspaces is not allowed to leak out to or penetrate from the other. When a quantum state is approaching the boundary by Rabi drivings between two levels of each atom, its wavefunction evolves around the boundary, being torn into two separated components, and a cat state is obtained. Furthermore, by carefully choosing the orientation of the wavefunction and its corresponding boundary, one component can be frozen, while the other continues rotating under Rabi drivings. This further stretches the wavefunction separations of the cat state, until the maximally-separated state (GHZ state) is obtained. Estimating with experimentally achievable parameters, the fidelity of the obtained GHZ state for a system containing 2000 rubidium-87 atoms can be above 0.8, which is significantly better than the achievable parameters, the fidelity of the obtained GHZ state of collective atomic spins (Dicke state 39), decribed by the rotation Hamiltonian $H'$. 11, describing the interaction between the cavity field and $N$ two-level atoms:

$$H' = \hbar \omega_s (S_z + S) \hat{c} \hat{c}.$$  

Here, $\omega_s = g^2/\Delta$ is the coupling strength, $S = N/2$ is the total spin magnitude, and $\hat{c} \hat{c}$ is the creation (annihilation) operator of the cavity field.

Each particle in the state $|\uparrow\rangle$ shifts the cavity resonance $\omega_c$ by an amount of $\omega_s$. When the cavity is illuminated by a light beam at frequency $\omega_n = \omega_c + n\omega_s$, the intra-cavity intensity $\langle \hat{c} \hat{c} \rangle_{m,n}$ is negligibly small if $m \neq n$, where $m$ is the number of atoms in the state $|\uparrow\rangle$. Thus, only quantum states with $n$ atoms in $|\uparrow\rangle$ introduce significant intra-cavity intensity with the incident light frequency $\omega_n$. Inspecting atomic AC stark shifts in the Hamiltonian $H'$, the energy of Dicke state $|m = -N/2 + n\rangle$ is shifted by an amount of $n\hbar \omega_s \langle \hat{c} \hat{c} \rangle_{n,n}$, while the energy shifts of other Dicke states are negligible. This achieves the goal of shifting one particular Dicke state away without affecting the others and forms a boundary separating the Hilbert space. In the following context, we choose an incident light beam at frequency $\omega_1 = \omega_c + \omega_s$ to illuminate the cavity and get the Hamiltonian $H' = \text{diag}(0, 0, \ldots, \hbar \delta, 0)$, where $\hbar \delta$ corresponds to the energy shift and each diagonal matrix element corresponds to the coefficient of density matrix $|m\rangle\langle m|$, as $m = N/2$ to $-N/2$ in a descending order. As a result, the boundary separating the Hilbert space is set to the Dicke state $|m = -N/2 + 1\rangle$.

We realize entanglement amplification in the following steps. Step 1: All the atoms are initialized in $|\uparrow\rangle$ and then rotated along $x$ axis by Rabi drivings approaching the Dicke state $|m = -N/2 + 1\rangle$ without turning on the incident light onto the cavity (Fig 2a). This process can be described by the rotation Hamiltonian $\hbar \Omega S_x$, where $\Omega$ is the Rabi frequency of single-particle Rabi drivings. Step 2: Turn on the cavity light to introduce the energy shift at Dicke state $|m = -N/2 + 1\rangle$, and continue the state rotation along $x$ axis (Fig 2b). This process can be described by $\hbar \Omega S_x + H'$. Here we require $\sqrt{\delta \Omega}/|\delta| > 0.5$ to guarantee the off-resonance condition. The wavefunction rotates around $|m = -N/2 + 1\rangle$ and evolves into two separate components. Step 3: Choose a proper time to stop applying Rabi drivings (Fig 2c). The atoms evolve into a cat state where two components of the wavefunction distributions are coherently separated on the Bloch sphere.

Then, we convert the obtained cat state into a GHZ state by applying additional two steps (Fig. 2d and e). Step 4: Turn off the cavity light, apply Rabi drivings to rotate the cat state until one component of this state is aligned into the south pole of the Bloch sphere ($|m = N/2\rangle$). During this process, the separation between two components is unchanged (Fig. 2d). Step 5: Turn on both the cavity light and the Rabi Drivings (Fig. 2e). The component in $|m = -N/2\rangle$ is frozen by
the boundary of Dicke state $|m = -N/2 + 1\rangle$, while the other component is rotated into the state of $|m = N/2\rangle$. A GHZ state with two coherent components each on the north and south pole of the Bloch sphere is thus obtained. With all these steps, we convert a non-entangled CSS into a cat state, and then amplify such an entangled state into a maximally-entangled GHZ state. Here, we could choose different energy shifts $\hbar\delta$ for Step 2 and 5 to optimize the performance.

In a realistic system, we need to consider dissipation processes and finite cavity linewidth to demonstrate the validity of such entanglement amplification processes. We consider rubidium-87 as the candidate atom, with two ground states $|\downarrow\rangle$ and $|\uparrow\rangle$ in different hyperfine manifolds of $5S_{1/2}$ and one excited level $|e\rangle$ in $5P_{3/2}$ with a spontaneous decay rate $\Gamma = 2\pi \times 6$ MHz. Let’s consider the dissipation induced by spontaneous decay. A detuned coupling from $|\uparrow\rangle$ to $|e\rangle$ brings an AC-stark shift $E_s$ to the energy of $|\uparrow\rangle$, and introduces a spontaneous decay rate $\Gamma = E_s / \Delta$ for each atom in the state $|\uparrow\rangle$. Due to the decoherence introduced by spontaneous decay, the final state can be decomposed into two parts, the coherent-evolved part and the incoherent-scattered part. Since the latter is described by a positive-defined density matrix, it should contribute a non-negative number to the fidelity. Without losing generality, we choose to estimate the fidelity contributed by the coherent-evolved part which gives the lower bound of the fidelity. We name this lower bound as fidelity $F$ in the following context for simplicity. Meanwhile, due to finite cavity linewidth, the cavity light at frequency $\omega_c$ also introduces non-negligible AC Stark shifts to Dicke states besides $|\downarrow\rangle$.

Therefore, a non-Hermitian Hamiltonian $H'_{\text{exp}}$ best describes the cavity-assisted energy shift under the dissipation of spontaneous decay and the cavity linewidth broadening:

$$H'_{\text{exp}} = \frac{\hbar\delta}{|T(\omega_s,1)|^2} \times \text{diag}(N)|T(\omega_s,N)|^2,$$

$$(N-1)|T(\omega_s,N-1)|^2, \ldots, 1 \times |T(\omega_s,1)|^2, 0 \times |T(\omega_s,0)|^2).$$

The real part of $H'_{\text{exp}}$ characterizes AC stark shifts for different Dicke states and the imaginary part characterizes the spontaneous-decay-induced decoherence. Here, $T(\xi, n)$ is the amplitude transmission function of the cavity [41]:

$$T(\xi, n) = \frac{1}{1 + i \frac{n\eta}{\xi + i(\xi + \Delta)^2/\Gamma}} - 2i \frac{1}{\frac{\xi}{\kappa} - n\eta \frac{\Delta + \xi}{(\Delta + \xi)^2/\Gamma}}.$$

where $n$ is the atom number in the state $|\uparrow\rangle$, $\eta = 4\Delta^2 / (\Gamma \kappa)$ is the cavity cooperativity, $\kappa$ is the linewidth of the cavity, and $\xi = \omega - \omega_c$ is the light-cavity detuning. The dissipation and finite cavity linewidth lead to the penetration of the wavefunction between two isolated Hilbert subspaces. The realistic system is described by the Hamiltonian $\hbar\Omega S_x + H'_{\text{exp}}$ instead of that in the ideal case $\hbar\Omega S_x + H'$.

We use experimentally achievable parameters to estimate the fidelity of the obtained GHZ state, setting $\eta = 200$, $\kappa = 2\pi \times 0.1$ MHz, $\Delta = -36\Gamma = -2\pi \times 216$ MHz, $\Omega = 2\pi \times 0.2$ MHz, and AC stark shift for $|m = N/2 + 1\rangle$ Dicke State $\delta = -2\pi \times 4$ MHz (or $-2\pi \times 20$ MHz) for Step 2 (or 5). A GHZ state with fidelity of 0.93 is achieved in a 100-atom ensemble. Then we plot the fidelity $F$ of the obtained GHZ state in ensembles with atom numbers ranging from 100 to 2000 in Fig. 3, tuning $\delta$ ranging from $-2\pi \times 4$ MHz to $-2\pi \times 10$ MHz for Step 2 and from $-2\pi \times 20$ MHz to $-2\pi \times 145$ MHz for Step 5 to optimized the fidelity. As the atom number increases, the obtained fidelity $F$ drops slowly. For an ensemble with 2000 atoms, the fidelity drops to 0.83, still much better than the threshold of the fidelity witness [29] ($F \sim 0.5$). The fidelity drops mostly due to spontaneous decay. To understand the dependence of expected fidelity on the cavity cooperativity, we plot fidelity of obtained GHZ states with 100 atoms at different cavity cooperativity $\eta$ with optimized detuning $\Delta$ ranging from $-2\pi \times 108$ MHz to $-2\pi \times 1.13$ GHz (Fig. 3).

The obtained GHZ states can be verified experimentally by inspecting the population distribution and detecting the parity oscillation (Fig. 3a-d) [29]. In our case, we apply a rotation $e^{i\pi S_z/2}$ to GHZ states where $S_\theta = S_x \cos \theta + S_y \sin \theta$, and then measure the mean...
value of the parity operator \( P = \prod_{i=1}^{N} \sigma_z^{(i)} \) where \( \sigma_z^{(i)} \) corresponds to the Pauli \( z \)-matrix for the \( i \)-th atom. The parity \( \langle P \rangle = \cos (N\theta) \) oscillates versus \( \theta \) (see Fig. 3), proving the non-trivial coherence of \( N \)-atoms between the states of \( m = -N/2 \) and \( m = N/2 \), which is crucial for GHZ states. The oscillation amplitude characterizes magnitude of the many-body phase coherence.

To facilitate utilizing the obtained GHZ states for metrological purposes, we plot the Fisher information that characterizes the metrological gain relative to a CSS (Fig. 4). Here we calculate the Fisher information of the obtained GHZ states at different cooperativity \( \eta \) (Fig. 4a) and different atom number \( N \) (Fig. 4b). At a given \( \eta = 200 \), the relative Fisher information reaches 81 for 100 atoms, 335 for 500 atoms, and 1250 for 2000 atoms while the relative Fisher information of a CSS is 1. It confirms that entanglement amplification strongly amplifies the metrological gain in a many-body system, approaching the Heisenberg limit at a given atom number \( N \). We also show the increase of the Fisher information in the atomic ensemble as we go through the entanglement amplification processes from step 3 to step 5 (Fig. 4c). Here we use the spanned angle \( \psi \) between two components of the evolving state as the horizontal axis.

Our method is robust against photon shot noise of the intra cavity light because it mainly relies on an off-resonant suppression of the Rabi driving coupling. For a system with \( N = 100 \) and \( \eta = 200 \), the mean intra-cavity photon number of the state \( m = -N/2 + 1 \) is 29 (or 44) for step 2 (or 5), corresponding to an intra-cavity photon number at Poisson distribution centered at such mean value. We calculate the fidelity of the obtained GHZ state by averaging the weighted density matrix on different photon numbers, and obtain a fidelity 0.91 comparing to the original one of 0.93. This result supports the robustness of our method in a realistic atom-cavity system.

In conclusion, we propose a new scheme, entanglement amplification, for creating entangled states with high metrological gain, including cat states and GHZ states. With realistic experimental parameters, one can obtain a 2000-atom GHZ state with 83% fidelity and approach the Heisenberg limit using this scheme. We believe this scheme simplifies the complexity and enhances the robustness of the creation of many-body entanglement. It may raise a new platform for designing simpler and more robust entanglement-creation schemes for quantum information and quantum metrology. Variations of this method can be generalized to artificial-atom systems such as superconducting qubits, quantum dots, and mechanical oscillators coupled to a resonator.

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