Influence of the Long-Range Forces in Non-Gaussian Random-Packing Dynamics

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Abstract

Studies on random packing of polydispersive particles have shown that such systems can capture the underlying behavior of more complex phenomena found in physics and materials engineering. In industry, polydispersive particles are used to allow the increase of density and fluidity of the formed compounds. The understanding of the dynamics of these processes is therefore of great theoretical and practical interest. In this paper, we perform molecular dynamics (MD) simulations to study the packing process of spheres in different particle size distributions. In particular, we deal with non-Gaussian distributions by means of the Lévy and uniform distributions. The initial positions as well as the radii of five thousand non-overlapping particles are assigned inside a confining rectangular box. After that, the system is allowed to settle under gravity towards the bottom of the box. Both the translational and rotational movements of each particle are considered in the simulations. In order to deal with interacting particles, we take into account both the contact and long-range cohesive forces. The normal viscoelastic force is calculated according to the nonlinear Hertz model, whereas the tangential force is calculated through an accurate nonlinear-spring model. Assuming a molecular approach, we account for the long-range cohesive forces using a Lennard-Jones(LJ)-like potential. The packing processes are studied assuming different long-range interaction strengths. We carry out statistical calculations of the different quantities studied including packing density, mean coordination number and kinetic energy during the time evolution of the system.

Keywords: Molecular Dynamics Simulations, Random Packing, Polydispersive particles, Lennard-Jones Potential

1. Introduction

Over the years, scientific research has revealed that systems such as molecular liquids, colloids, and granular media possess, in certain conditions, a similar phenomenological behavior with respect to their glassy phase transitions. Colloidal systems composed of hard spheres display a fluid-like phase with density $\phi$ from 0 to intermediate values, a freezing crystallization at $\phi \approx 0.494$, and a melting transition at $\phi \approx 0.545$. Above this melting point, the colloidal system can be compressed until the close-packing point reaches $\phi \approx 0.74$, where the equilibrium state is crystalline. Remarkably, a small amount of polydispersity (i.e., particles with slightly different sizes) in the system can effectively prevent crystallization. As a consequence, the system can be easily “super-compressed” above the freezing transition without nucleation or crystal growth. It has also been observed that such systems exhibit relaxation time scales that increase rapidly with increasing $\phi$. Likewise, a polydisperse granular fluid, at high packing density, displays a relaxation (or diffusion) time that rises rapidly with increasing density $\phi$, without an evident change in its structural properties. This universal characteristic has been referred to as the jamming state [1]. Typically for colloids and granular media [2], a “jammed” phase could be obtained either by increasing the packing density or by decreasing the external drive (e.g., shearing and tapping).

The random packing of spherical particle, in particular, has been an interesting tool used to capture the underlying behavior of more complex phenomena for applications in physics and materials engineering such as modeling ideal liquids [3, 4], amorphous materials [5, 6], granular media [7], emulsions [8].
gases [9], jamming [10], living cells [11], ceramic compounds [12, 13] and sintering processes [14, 15]. Understanding the structure of random close-packed particles is important because its physical properties may depend on the packing features such as packing density and porosity. The packing density (i.e., the volume ratio occupied by particles to the total aggregate) is affected by the particle size distribution, particle shape and long-range cohesive forces. In general, random packing structures possess packing densities that increase with increasing width of the size distribution, increasing sphericity, and decreasing long-range cohesive forces. For micro-sized particles, or smaller, both van der Waals and electrostatic forces play an important role in particle rearrangements as they dominate the dynamical packing process [16, 17], forming local particle clusters [18–21] that can eventuate into large percolation clusters [22] depending on the nature of the particles involved.

There have been few prior experimental and computational studies concerning the micro-sized particles packing in which long-range cohesive forces have to be taken into account to describe the adequate behavior of the colliding particles involved in these dynamical processes. Forsyth et al. [23] experimentally investigated the influence of van der Waals forces in hard-sphere packing; however, they did not take into account neither the impacts caused by electrostatic force nor polydispersity. Liu et al. [24] performed computational simulations to address the centripetal packing of mono-sized spheres. Yen and Chaki [18], Cheng et al. [20] and Yang et al. [19] each applied a simplified version of the so-called distinct element method [25] to study the effects of both van der Waals and frictional forces present in hard-sphere packing processes but also did not consider particle size distributions in their investigations. More recently, a computational study [21] has considered particle packing dynamics using Gaussian size distribution where the van der Waals forces were calculated using the standard Hamaker form [26] without the inclusion of the electrostatic forces between particle pairs or non-Gaussian effects in particle size distributions. Electrostatic interactions are, however, quite important because of their fundamental role in governing the properties of many systems, including soft matter, colloidal suspensions, electrolyte solutions and various biological systems [27, 28].

Alternatively, one can represent the long-range cohesive forces present in many molecular systems by coarse-grained intermolecular potentials, notably by avoiding the full atomic representation of molecules or macromolecules, to find a description for the interaction at either long or coarse length scales. This approach, despite its simplicity, has been successfully used to model systems such as liquid crystal [29], proteins [30] and water molecules [31]. In most of these studies, a modified version of the Lennard-Jones (LJ) potential has been employed. Hence, one begins to wonder whether such an approach could also be applied to modeling micro-sized particles and their long-range interactions. This “molecular” approach, particularly for large particulate systems, can be guaranteed as long as we realize that when two microspheres (with radius $R$) are separated by a certain distance $D >> R$, the effective potential $(\Phi)$ is analogous to that between two molecules, i.e., falling off as $\Phi(D) \propto -1/D^6$ [17, 32]. Assuming the validity of this modified LJ approximation, we therefore are able to account for the long-range forces involved in the packing process. This approximation will allow us to study a variety of different packing cases by considering LJ particles with different potential well depths, which play a dominant role in the strength of these long-range forces.

In this paper, we perform molecular dynamics (MD) simulations to study the packing process of spheres using different particle size distributions. Here, while we deal with non-Gaussian distributions through the rescaled Lévy profiles and uniform distributions, we also account for particle packing utilizing Gaussian distributions in order to compare the different packing features. Both the translational and rotational movements of each particle are also considered in the simulations. In order to deal with interacting particles, we take into account both the contact and long-range cohesive forces. The contact forces result from the deformation of the colliding particles, which can be decomposed into two main types: the normal viscoelastic force and the tangential force. The normal viscoelastic force is calculated according to the nonlinear Hertz model [33, 34], whereas the tangential force is calculated through a nonlinear-spring model that is derived from the Mindlin–Deresiewicz theory [35]. By assuming a molecular approach, we account for the long-range forces using a modified LJ potential. The packing processes were studied by applying different long-range interaction strengths. We performed statistical calculations of the different quantities studied including packing density, mean coordination number, kinetic energy and radial distribution function (RDF) as the system evolved over time.

The content of the manuscript is organized as follows. In section 2, we describe in detail, the model and MD simulations. In section 3, we present and discuss the results. Finally, in section 4, we draw the conclusions and
give some perspective on possible future developments.

2. Model and Molecular Dynamics Simulation

The time evolution of the random packing of spheres was simulated by using the MD method. The equations of motion of an \( i \)-th particle of mass \( m_i \) and radius \( R_i \) are:

\[
m_i \frac{d^2 \vec{r}_i}{dt^2} = \sum_j (\vec{F}^n_{ij} + \vec{F}^t_{ij} + \vec{F}^{LJ}_{ij}) + m_i \vec{g}
\]

and

\[
I_i \frac{d\vec{\omega}_i}{dt} = \sum_j R_i \hat{n}_{ij} \times \vec{F}_{ij} - \gamma_i R_i \vec{v}_{nij} \hat{n}_{ij},
\]

where \( \vec{r}_i \) is the position, \( \hat{n}_{ij} \) is the angular velocity, \( \hat{n}_{ij} \) is the unity vector in the direction \( j \to i \), \( \gamma_i \) is the rolling friction coefficient and \( I_i = 2/5 m_i R_i^2 \) is the moment of inertia of the particle.

In the above equations, \( \vec{F}^n_{ij} \) is the normal viscoelastic force, \( \vec{F}^t_{ij} \) is the tangential friction force, \( \vec{F}^{LJ}_{ij} \) is the LJ force between the \( i \)-th and \( j \)-th particle, and \( \vec{g} \) is the gravity acceleration. The normal viscoelastic force \( \vec{F}^n_{ij} \) is derived from the nonlinear Hertz theory, and it can be written as

\[
\vec{F}^n_{ij} = \frac{2}{3} E \sqrt{R} \delta_n^{3/2} - \gamma_n E \sqrt{R} \sqrt{\delta_n (\vec{v}_{nij} \cdot \hat{n}_{ij})} \hat{n}_{ij},
\]

where \( E \) is the elastic modulus of the two particles, \( \bar{R} = R_i R_j (R_i + R_j) \) is the effective radius, \( \delta_n \) is the deformation which is expressed by

\[
\delta_n = (R_i + R_j) - (\vec{r}_i(t) - \vec{r}_j(t)),
\]

\( \vec{v}_{nij} \) is the relative velocity between \( i \)-th and \( j \)-th particle, and \( \gamma_n \) is the normal damping coefficient.

The tangential friction force \( \vec{F}^t_{ij} \) is calculated according to the Mindlin-Deresiewicz theory as

\[
\vec{F}^t_{ij} = \gamma_t |\vec{F}^t_{ij}| \left| 1 - \left( \frac{|\delta_t|}{\delta_{max}} \right)^{3/2} \right| \hat{t}_{ij},
\]

where \( \gamma_t \) is the friction coefficient, \( \hat{t}_{ij} \) is the unit vector perpendicular to \( \hat{n}_{ij} \), \( \delta_t \) is the tangential displacement which is determined as

\[
\delta_t = \int_0^{t_c} (\vec{v}_{nij} \cdot \hat{t}_{ij} + R_i \vec{h}_{ij} \times \hat{w}_i + R_j \vec{h}_{ij} \times \hat{w}_j) dt,
\]

where the above integral is calculated during the contact time \( t_c \) (see below) between the particles. The \( \delta_{max} \) is the maximum tangential displacement and in the condition that \( \delta_t > \delta_{max} \), the sliding friction takes place between the particles. In Eqs. 3 and 5 \( E \) and \( \delta_{max} \) are given, respectively, by

\[
E = Y/(1 - \xi^2)
\]

and

\[
\delta_{max} = \gamma_t \frac{2 - \xi}{2(1 - \xi)} \delta_n,
\]

being \( Y \) the Young’s modulus and \( \xi \) the Poisson’ ratio. For damped collision [34], the contact time is given by

\[
t_c = 2.94 \Omega^{-2/5} |\vec{v}_{ij}|^{-1/5} (1 + \frac{1}{10} \zeta \Omega^{2/5} |\vec{v}_{ij}|^{1/5}),
\]

where \( \Omega = 2/3 E (R/M) \) (\( R \) and \( M \) are, respectively, the effective radius and mass) and \( \zeta = \zeta (\gamma_n, Y, \xi) \) is a material-dependent constant. For undamped collision [57], one just takes \( \zeta = 0 \) in above equation.

The LJ force between the particles \( i \) and \( j \) can be evaluated as

\[
\vec{F}^{LJ}_{ij} = \frac{24 \varepsilon}{\sigma} \left[ \left( \frac{\sigma}{r_{ij}} \right)^{13} - \left( \frac{\sigma}{r_{ij}} \right)^{7} \right] \hat{n}_{ij},
\]

where \( r_{ij} \) is the distance between the particles, \( \varepsilon \) is the well depth of the LJ potential, which rules the strength of the interaction, and \( |\sigma| = 2^{-1/6} (R_i + R_j) \) defines the hard core of the potential. Here, it is important to say that the LJ force is only activated when \( r_{ij} > R_i + R_j \). For \( r_{ij} \leq R_i + R_j \), the contact forces given by Eqs. 3 and 5 take over control of the particles’ driving. In addition, we have also used a cutoff at \( r_{ij} = 3 (R_i + R_j) \) for saving time during the simulations.
Because of the good accuracy, low computational cost and symplectic feature, a leapfrog scheme \cite{38} was used to integrate numerically the Eqs. (1) and (2). The physical parameters used in the simulations were obtained by trial tests and are given in Table 1. In order to avoid the complicating effects of the pouring rate, the particles were suspended along the box at the beginning of the simulation. Owing to frictional forces, stable simulations were already achieved by taking a time-step \( \delta t = 10^{-6} \text{s} \). The average CPU time to update the state of one particle was approximately 0.72 \( \mu \text{s} \) on one 3.70 GHz Intel Xeon microprocessor.

3. Results and Discussion

In this work, the particle packing processes were investigated using different size distributions and assuming different long-range interaction strengths \( \varepsilon \). The initial positions, as well as the radii of 5000 non-overlapping particles ranged from 1.0 to 7.0 \( \mu \text{m} \) were defined inside a confining (80 \( \times \) 80 \( \times \) 100) \( \mu \text{m} \) box by using a random number generator \cite{39}. The particles thereafter were pulled down by gravity and started to collide each other. Here, no periodic boundary conditions were assumed and, hence, the particle-wall interactions were also taken into account. The non-Gaussian distributions were represented by the Lévy and uniform distributions. The former is given by

\[
    f(r) = \frac{c}{2\pi(r-\mu)} \exp \left[ -\frac{c}{2(r-\mu)} \right],
\]

where \( \mu \) is the location parameter, \( c \) is the scale parameter and \( k \) is a rescaling factor, which was used to set this distribution within the value range from 0 to 1. The latter distribution attributes a probability of 1/2 to the particle radius inside the box. While the Gaussian distribution is determined by the well-known form

\[
    f(r) = \frac{1}{\sqrt{2\pi \Delta r}} \exp \left[ -\frac{(r-\bar{r})^2}{2\Delta r^2} \right],
\]

being \( \bar{r} \) the mean \( r \) value and \( \Delta r \) the standard deviation. Fig. 1 displays the described distributions above and are defined in the range of 1.0 to 7.0 \( \mu \text{m} \). For the Lévy distribution, two different parameter sets were considered. We termed it with parameters \((\mu = 0.05, c = 0.50)\) as type I distribution and with parameters \((\mu = 0, c = 2.0)\) as type II distribution. As can be seen from Fig. 1, the type I distribution generates packs that contain more small particles \((r < 0.4)\) than large ones \((r \geq 0.4)\). Conversely, the type II distribution gives preference to larger particles rather than smaller ones during the particle radius assignment. While the Gaussian distribution is centered at \( \bar{r} = 0.40 \) with a deviation of \( \Delta r = 0.15 \). It is worth mentioning that the only constraint during the particle radius assignment was that the particles are initially non-overlapping inside the box. Note also that each of these distributions is nonzero at the corresponding end points.

The packing process is depicted in Fig. 2 for polydispersive particles with Gaussian distribution. Snapshots at the instants \( t = 0.0 \text{ms} \), \( t = 2.0 \text{ms} \), \( t = 4.0 \text{ms} \) and \( t = 10.0 \text{ms} \) are shown in these figures. The parameters used in this simulation are given in Table 1 for \( \varepsilon = 10.0 \mu \text{J} \). This figure was rendered with a color gradient along the vertical direction (z axis) to display the different particle layers fall towards the bottom base of the box. We performed statistical calculations of the different quantities such as packing density, mean coordination number, kinetic energy, and RDF as the system evolved over time. To determine the average value of these quantities and estimate their statistical error, we averaged over 20 independent realizations. Furthermore, a smaller virtual box with an offset distance from each actual wall measuring 5 \( \mu \text{m} \) and centered in the bulk region of the aggregate was also used to eliminate wall effects \cite{40} in these calculations.

The packing densities \( \phi \) for different probability distributions, considering several long-range interaction strengths \( \varepsilon \), are shown as a function of time in Fig. 3. In this figure the \( \phi \) values are given at short time intervals of 1.0 \( \mu \text{m} \) up to 10.0 ms. At 10.0 ms, the ultimate \( \phi \) values were obtained for each case. Fig. 4 shows the time evolution of the average kinetic energy per particle in the Gaussian case for four different \( \varepsilon \) values. One can see that the stationary state was already achieved before 3.0 ms for all \( \varepsilon \) values considered. Similar energy curves

\[
\text{Table 1: Physical parameters used in the simulations.}
\]

| Parameter | Value |
|-----------|-------|
| Number of particles \( (N) \) | 5000 |
| Particle size \( (R) \) | 1.0 – 7.0 \( \mu \text{m} \) |
| Particle density \( (\rho) \) | 2500/\( \pi \) \( \text{Kg/m}^3 \) |
| Minimum potential energy \( (\varepsilon) \) | 0 – 25.0 \( \mu \text{J} \) |
| Young’s modulus \( (Y) \) | \( 10^8 \) \( \text{N/m}^2 \) |
| Poisson’s ratio \( (\mu) \) | 0.30 |
| Normal damping coefficient \( (\gamma_n) \) | 0.05 \( \text{s} \) |
| Tangential damping coefficient \( (\gamma_t) \) | 0.30 |
| Rolling friction coefficient \( (\gamma_r) \) | 0.002 |

* It is assumed that both particles and walls have the same physical parameters.
were found for all remaining cases. In Fig. 3, the initial packing densities were 0.36 for the Gaussian case, 0.33 for the type I case, 0.43 for the type II case, and 0.32 for the uniform case. The packing density minimum around 2.0 ms was due to the first particles bouncing after hitting the bottom of the box. In most cases, the ultimate $\phi$ values were below $\frac{\pi}{\sqrt{18}} \approx 0.74$, which corresponds to closest-packing crystal structures, namely, face-centered cubic and hexagonal close-packed structures. For every case, the $\phi$ value was seen to decrease with increasing interaction strength $\epsilon$. This behavior is in accordance with the experimental results obtained by Forsyth et al. [23] for monosized particles. Moreover, one can see from Fig. 3 that the packing dynamics was also quite sensitive to the distribution type used to generate the particle packs. In fact, packs with an uniform distribution displayed a broader range in the final $\phi$ values compared with other distributions. While packs with a type II distribution displayed a narrower and higher range in the final $\phi$ values. These higher $\phi$ values, particularly for the type II case, can be attributed to the existence of a great number of large particles that either are wrapped around by smaller particles or create voids that are filled by smaller ones or both. The narrow range in the final $\phi$ values found for this case can also explained by the existence of a greater number of large particles in the aggregate. The larger the particle, the larger its effective action distance of the long-range forces ($r_{eff} \approx 3\sigma$). As a consequence, the $\phi$ values, in this case, become less sensitive with a rise in $\epsilon$. Thus, it is more difficult to compress packs with a greater number of large particles than those with small ones. On the
other hand, for both the uniform (mainly due to the initial non-overlapping condition) and type I cases (where more small particles are present), a broader range in the \( \phi \) values was obtained when different \( \epsilon \) values were considered.

Fig. 3(a) shows the time evolution of \( \phi \) for the Gaussian case using several \( \epsilon \) values. Here, we obtain an ultimate density of \( 0.718 \pm 0.001 \) when \( \epsilon = 0 \) (i.e., absence of long-range forces) and \( 0.60 \pm 0.01 \) when \( \epsilon = 25 \mu J \). Similarly, Figs. 3(b), 3(c) and 3(d) show the time evolutions of \( \phi \) for types I and II as well as the uniform case. For the type I case, we obtained \( 0.705 \pm 0.002 \) when \( \epsilon = 0 \) and \( 0.58 \pm 0.01 \) when \( \epsilon = 25 \mu J \). For the type II case, we obtained \( 0.742 \pm 0.001 \) when \( \epsilon = 0 \). This is in good agreement with the experimental result of 0.746 achieved by Ref. [42] for the random packings of millimeter-sized particles. When \( \epsilon = 25 \mu J \), we got a density of \( 0.58 \pm 0.01 \). For the uniform case, we obtain \( 0.704 \pm 0.004 \) when \( \epsilon = 0 \) and \( 0.535 \pm 0.005 \) when \( \epsilon = 25 \mu J \). The ultimate \( \phi \) values found in Fig. 3 were then plotted in Fig. 5 as a function of the interaction strength \( \epsilon \) together with non-linear curve fits to the data. In each case, data trend seems to gradually decay with increasing \( \epsilon \) values. Using the following expression

\[
\phi = \phi_{\text{max}} - A \exp(B \epsilon),
\]

where \( A \) and \( B \) are fitting parameters, one can observe a good fitting of the data. Error bars at each data points were calculated using 20 independent realizations. The fitting parameters are: \( A \approx 0.003 \) and \( B \approx 0.13 \) for the type II case; and \( A \approx 0.02 \) and \( B \approx 0.10 \) for all remaining cases. This behavior of \( \phi \) as the long-range force strength increases has been corroborated both by the experimental work [23] and by simulations of the random close packing of disks [2]. For an overall analysis of the mean coordination number \( z \) of the particles that compose the packs for all cases studied, we divide these particles by size into six different bins. The first bin \( (\Delta_1) \) is composed of particles with radii ranging from 1.0 \( \mu m \) to 2.0 \( \mu m \), the second bin \( (\Delta_2) \) is composed of particles

\[\text{Figure 3: Plot of the packing densities } \phi \text{ for different probability distributions as a function of time. Several interaction strengths } \epsilon \text{ are considered for each case.}\]
with radii ranging from from 2.0 $\mu$m to 3.0 $\mu$m and so on. Fig. 6 displays the frequency inside the confining box against the mean coordination number $z$ at each bin defined for all cases when $\varepsilon = 0$. Bins with increasing particle size are showed from left to right in Fig. 6. One can see that packs with the uniform and type I distributions possess both a greater number of smaller particles ($\Delta_1$) and higher $z$ values at almost every bin. Conversely, packs with type II and Gaussian distributions possess both a greater number of bigger particles($\Delta_3$ and $\Delta_4$) and lower $z$ values at almost every bin.

Following the same behavior as the $\phi$ values, it can be seen that the mean coordination number $z$ also decays gradually as the $\varepsilon$ value increases for all distributions considered. Fig. 7 shows the mean coordination number $z$ as a function of $\varepsilon$. Remarkably, the $z$ value decreases more steeply as the $\varepsilon$ value increases for the uniform case. While for the type II case it decreases more smoothly with increasing $\varepsilon$ values. The $z$ values for the Gaussian and type I cases are located in an intermediate region between the two other cases mentioned.

The RDF has been widely used to characterize random structures of spherical particles [18, 21], where it can be understood as the probability of finding one particle at a given distance from the center of a reference particle. Here we define RDF as

$$g(r_i) = \frac{n(r_i)}{4\pi r_i^2 Z}$$  \hspace{1cm} (14)$$

where $n(r_i)$ the number of particle centers within the $i$-
Figure 8: RDFs versus the radial distance for spheres packing structures with uniform distribution for different $\varepsilon$ values. a) $\varepsilon = 0$ (absence of long-range forces), b) $\varepsilon = 5$, c) $\varepsilon = 15$ and d) $\varepsilon = 25$.

Figure 9: RDFs versus the radial distance for spheres packing structures with Gaussian distribution for different $\varepsilon$ values. a) $\varepsilon = 0$, b) $\varepsilon = 5$, c) $\varepsilon = 15$ and d) $\varepsilon = 25$.

$\text{th spherical shell of radius } r_i \text{ and thickness } \delta r_i$. In the above equation, $Z$ is the normalization factor given by

$$Z = \sum_{j=1}^{N} \frac{n(r_j)}{4\pi r_j \delta r_j}. \quad (15)$$
where \( N_r \) is the total number of spherical shells considered. In the above equations, we set \( \delta r_i = 0.1 \mu m \) and \( N_r = 150 \). Figs. 8-11 show the RDFs versus the radial distance for spheres packing structures with different size distributions for several \( \varepsilon \) values. From these figures, we can see that the general shape of the RDFs reflects the random distribution of the particles, where it is practically unchanged by the long-range interaction.
forces, even though they strongly influence the transient state of the packing process. The peaks are also observed slightly decrease as the $\phi$ values increase. While, the typical plateaus become, in general, a little more tilted as the $\epsilon$ values increase.

4. Conclusions

In this study, MD simulations were performed to study the random packing process of spherical particles at micrometer scales. Both contact forces and long-range dispersive forces were taken into account in these simulations. Several size distributions were considered along with different physical quantities, including the packing density, mean coordination number, kinetic energy, and RDF. The later were computed to study the packing process dynamics so as to characterize the particle structures formed over different values of the long-range interaction strength $\epsilon$. It was found that the packing dynamics is quite sensitive to both the distribution type and the long-range interaction strength. The simulation results showed that both the packing density $\phi$ and mean coordination number $z$ gradually decayed as the $\epsilon$ value increased for all distributions considered. Remarkably, both $\phi$ and $z$ values decreased more steeply for the uniform distribution and more smoothly for the Lévy type II distribution as the $\epsilon$ value increased, whereas the same values for the Gaussian and Lévy type I distributions were found to be in an intermediate region between the values of other distributions studied. The general shape of the RDFs obtained reflected the random particle distribution, where it was practically unchanged by the long-range interaction forces.

Future investigations will involve the study of more complex systems such as the random close packing of pairs and triplets of particles.

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