Infinite Distances and the Axion Weak Gravity Conjecture

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based on: 1905.00901 with Thomas Grimm
Motivation

\[ \mathcal{L} = -f^2 (\partial_\mu a)^2 + \Lambda^4 \sum_n e^{-nS} (1 - \cos na) \quad (a \sim a + 2\pi) \]

Transplanckian field ranges?
\[ \Rightarrow \text{need large axion decay constant } f \gg M_p \]

Weak Gravity Conjecture for axions [Arkani-Hamed et al. '06]

\[ fS \leq qM_p \]

large \( f \) \[\Rightarrow\] small \( S \)
\[\Rightarrow\text{many instanton corrections become relevant}\]
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Realization in string compactifications

This talk: Type IIA compactified on $Y_3$ (axions: $C_3 = \xi^{I} \gamma_I$)

$\Rightarrow f$ and $S$ vary non-trivially over moduli space of $Y_3$

In limits: power-law behavior of $f$ and $S$ in $y$ ("grow/decrease parametrically")

Focus: infinite distance limits in $\mathcal{M}^\text{cs}(Y_3)$
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Focus: infinite distance limits in $\mathcal{M}^{cs}(Y_3)$
Lessons from limiting mixed Hodge structures
[Deligne] [Schmid '73] [Cattani, Kaplan, Schmid '86]

Main points:

- **splitting** of three-form cohomology:

\[ H^3(Y_3) = \bigoplus V_\ell \quad (\ell = 0, \ldots, 6) \]

- **asymptotic behavior** of metric \[ \langle v, *w \rangle = \int_{Y_3} v \wedge *w \]

  - growth \[ \|v\ell\|^2 \sim y^{\ell-3} \]
  
  - orthogonality \[ \langle v\ell, *\infty v_{\ell'} \rangle = 0 \text{ if } \ell \neq \ell' \]

- **symplectic** property: \[ \langle v\ell, v_{\ell'} \rangle = 0 \text{ unless } \ell + \ell' = 6 \]

\[ \Rightarrow \text{ split into } H^3(Y_3) = V_{\text{heavy}} \oplus V_{\text{light}} \oplus V_{\text{rest}} \]

axions \[ \uparrow \]

\[ \leftarrow \text{ instantons} \]
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Main points:

• **splitting** of three-form cohomology:

\[
H^3(Y_3) = \bigoplus V_\ell \quad (\ell_i = 0, \ldots, 6)
\]

- growth

\[
\|v_\ell\|^2 \sim \left(\frac{y_1}{y^2}\right)^{\ell_1-3} \cdots \left(\frac{y^{n-1}}{y^n}\right)^{\ell_{n-1}-3} (y^n)^{\ell_n-3}
\]

- orthogonality

\[
\langle v_\ell, \ast w \rangle = 0 \text{ if } \ell \neq \ell'
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• **symplectic** property: \(\langle v_\ell, v_{\ell'} \rangle = 0\) unless \(\ell + \ell' = 6\)

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\Rightarrow \text{ split into } H^3(Y_3) = V_{\text{heavy}} \oplus V_{\text{light}} \oplus V_{\text{rest}}
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  \[\text{axions} \quad \leadsto \quad \text{instantons}\]
Axion decay constants

Type IIA on $Y_3$: R-R axions $C_3 = \xi^I \gamma_I$

$$L_{\text{kin}} = G_{IJ} \partial_\mu \xi^I \partial^\mu \xi^J, \quad G_{IJ} = \frac{1}{2} e^{2D} \int_{Y_3} \gamma_I \wedge * \gamma_J$$

LMHS: special choice of three-form basis $\gamma_I \in V_\ell$

$\implies$ kinetic terms **decouple** asymptotically into blocks

$$\implies f_{\ell \ell'} \sim \begin{cases} 0 & \text{if } \ell \neq \ell' \\ e^{D y_{(\ell-3)/2}} & \text{if } \ell = \ell' \end{cases}$$
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$$(f^T \cdot f)_{IJ}$$

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Type IIA on $Y_3$: R-R axions $C_3 = \xi^\mathcal{I} \gamma_\mathcal{I}$

$$L_{\text{kin}} = G_{\mathcal{I}\mathcal{J}} \partial_\mu \xi^\mathcal{I} \partial^\mu \xi^\mathcal{J}, \quad G_{\mathcal{I}\mathcal{J}} = \frac{1}{2} e^{2D} \int_{Y_3} \gamma_\mathcal{I} \wedge * \gamma_\mathcal{J}$$

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$$\implies f_{\ell \ell'} \sim \begin{cases} 0 & \text{if } \ell \neq \ell' \\ e^D \left( \frac{y_1}{y_2} \right)^{\frac{\ell_1-3}{2}} \cdots \left( \frac{y_{n-1}}{y_n} \right)^{\frac{\ell_{n-1}-3}{2}} \left( y_n \right)^{\frac{\ell_n-3}{2}} & \text{if } \ell = \ell' \end{cases}$$
Infinite distances in $\mathcal{M}^{cs}(Y_3)$

Swampland Distance Conjecture [Ooguri, Vafa ’06]

Infinite distance in field space $\Rightarrow$ infinite tower of massless states

- **Type IIB:** Tower of D3-branes wrapping 3-cycles $\Rightarrow$ Particles with masses $M(Q) \to 0$
  
  [Grimm, Palti, Valenzuela ’18] [Grimm, Li, Palti ’18]

- **Type IIA:** Euclidean D2-branes wrapping 3-cycles $\Rightarrow$ Instantons with actions $S(Q) \to 0$

  [Grimm, DH ’19], see also [Marchesano, Wiesner ’19]

\[ S(Q), M(Q) \leq \|Q\| \to 0 \quad \text{(via } S, M = e^{K/2} |\int_{Y_3} \Omega \wedge * Q|) \]

Crucial feature: growth of number of (stable) states $m_{\text{crit}} \sim y$

[Grimm, Palti, Valenzuela ’18], multi-parameter version in [Grimm, DH ’19]
Weak Gravity Conjecture and R-R axions

So far:

- Axions $\xi^\ell \in V_{\text{heavy}}$ with increasing $f \sim y^{(\ell-3)/2}$
- D2-brane instantons $Q \in V_{\text{light}}$ with decreasing $S(Q) \to 0$

Quick look at one-parameter results:

| Singularity Type | $S(Q)$ | $\max(\ell)$ | $fS$ bounded |
|------------------|--------|--------------|--------------|
| II               | $y^{-1/2}$ | 4            | ✓            |
| III              | $y^{-1}$   | 5            | ✓            |
| IV               | $y^{-1/2}$ | 6            | ?            |

Recall $fS \leq qM_p$

$\implies$ **Increase in charge** plays crucial role ($m_{\text{crit}} \sim y$)

Underlying reason: $Q(m) = q_0 + mNq_0$

$\implies$ this axion couples to charge $Nq_0$ that generates tower
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What about multi-parameter limits?

Some two-parameter cases:

| space   | decomposition | dimensions     |
|---------|---------------|----------------|
| $V_{\text{light}}$ | $V_{21} \oplus V_{22} \oplus V_{32}$ | $2 + (b - 2) + r$ |
| $V_{\text{heavy}}$ | $V_{45} \oplus V_{44} \oplus V_{34}$ | $2 + (b - 2) + r$ |
| $V_{\text{rest}}$ | $V_{43} \oplus V_{33} \oplus V_{23}$ | $2 + 2(h^{2,1} - b - r - 1) + 2$ |

| space   | decomposition | dimensions     |
|---------|---------------|----------------|
| $V_{\text{light}}$ | $V_{10} \oplus V_{12} \oplus V_{22} \oplus V_{32}$ | $1 + 1 + c + (r + 1)$ |
| $V_{\text{heavy}}$ | $V_{56} \oplus V_{54} \oplus V_{44} \oplus V_{34}$ | $1 + 1 + c + (r + 1)$ |
| $V_{\text{rest}}$ | $V_{33}$ | $2(h^{2,1} - c - r - 2)$ |

| space   | decomposition | dimensions     |
|---------|---------------|----------------|
| $V_{\text{light}}$ | $V_{30} \oplus V_{22} \oplus V_{32}$ | $1 + a + r$ |
| $V_{\text{heavy}}$ | $V_{36} \oplus V_{44} \oplus V_{34}$ | $1 + a + r$ |
| $V_{\text{rest}}$ | $V_{33}$ | $2(h^{2,1} - a - r)$ |

$\implies$ similar arguments work ✓

More than two parameters?

Short answer: yes ✓
Weak Gravity Conjecture for multiple axions

[Cheung, Remmen '14]

\[ fS \leq qM_p \implies z^I(Q_a) = \frac{f^{IJ}Q_aJ}{S(Q_a)} \]

**Convex Hull Condition**

There exist instantons \( Q_a \) such that the vectors \( \pm z(Q_a) \) span a convex hull that contains the unit ball. (here: ball of finite size)
R-R axions and the Convex Hull Condition

$m_{\text{crit}} = 3$

$m = 2$

$m = 1$

$m = 0$

approach limit point

$m_{\text{crit}} = 9$

$m = 2$

$m = 1$

$m = 0$
Conclusions

Summary:

• Studied infinite distances in $\mathcal{M}^{cs}(Y_3)$ for Type IIA on $Y_3$
  $\implies$ infinite towers of D2-brane instantons

• Studied parametrically growing axion decay constants for the axion Weak Gravity Conjecture
  $\implies$ (growth of) same tower of instantons plays a crucial role

Note: no control over precise coefficients in the WGC
  (i.e. radius of the ball)

Thanks for your attention!
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