In two-dimensional electron gases, the quantization of the Hall conductance results from the non-trivial topological structure of the quantum states of an electron band. For an infinite system, the Chern number is a global invariant that quantifies the spin projection along the direction of an external magnetic field. The spin coupling amplitudes quantifies the spin projection along the direction of an external magnetic field, such that our system realizes an analog of a quantum Hall ribbon. We characterize the dispersionless bulk and one synthetic dimension encoded in the atomic spin degrees of freedom [16–23]. Our experiments show that the synthetic dimension in the time domain can also give access to higher-dimensional physics [30, 31].

In electronic quantum Hall systems, the topology manifests itself via the spectacular robustness of the Hall conductance quantization to finite-size or disorder effects [26]. Nonetheless, such perturbations typically lead to conducting stripes surrounding insulating domains of localized electrons, making the comparison with simple defect-free models challenging. In topological insulators or fractional quantum Hall systems, topological properties are more fragile, and can only be revealed in very clean samples [4, 5]. Recent experiments with topological quantum systems in photonic or atomic platforms [7, 27] have created new possibilities, from the realization of emblematic models of topological matter [8, 28, 29] to the application of well-controlled edge and disorder potentials. In such systems, internal degrees of freedom can be used to simulate a synthetic dimension of finite size with sharp-edge effects [16–23]. Encoding a synthetic dimension in the time domain can also give access to higher-dimensional physics [30, 31].

In this work, we engineer a topological system with ultracold $^{162}$Dy atoms based on coherent light-induced couplings between the atom’s motion and the electronic spin $J = 8$, with relevant dynamics along two dimensions – one spatial dimension and a synthetic dimension encoded in the discrete set of $2J + 1 = 17$ magnetic sublevels. These couplings give rise to an artificial magnetic field, such that our system realizes an analog of a quantum Hall ribbon. We characterize the dispersionless bulk modes and chiral edge states of the lowest energy band, and study elementary excitations to higher bands. We also measure the Hall drift induced by an external force, and infer the local Hall response of the band via the local Chern marker, which quantifies topological order in real space [32]. Our experiments show that the synthetic dimension is large enough to allow for a meaningful bulk with robust topological properties.

The atom dynamics is induced by two-photon optical transitions involving counter-propagating laser beams along $x$ (see Fig. 1), and coupling successive magnetic sublevels $m$ [33, 34]. Here, the integer $m$ ($-J \leq m \leq J$) quantifies the spin projection along the direction of an external magnetic field. The spin coupling amplitudes then inherit the complex phase $K x$ of the interference between both lasers, where $K = 4\pi/\lambda$ and $\lambda = 626.1 \, \text{nm}$ is the light wavelength (see Fig. 1). Given the Clebsch-Gordan algebra of atom-light interactions for the domi-
Figure 1. Synthetic Hall system. (a) Laser configuration used to couple the magnetic sublevels of a $^{162}$Dy atom (with $-J \leq m \leq J$ and $J = 8$, only a few $m$ values represented). (b) Interpreting the spin projection as a synthetic dimension, the system is mapped to a two-dimensional ribbon of finite width. The photon recoil $p_{\text{rec}} = \hbar K$ imparted upon a spin transition leads to complex-valued hopping amplitudes along $m$, equivalent to the Aharonov-Bohm phase of a charged particle evolving in a magnetic field. The blue area represents a magnetic unit cell pierced by one flux quantum $\phi_0$. (c) Dispersion relation describing the quantum level structure, with flattened energy bands reminiscent of Landau levels. (d) The lowest energy band is explored by applying an external force. We probe the velocity and magnetic projection distributions by imaging the atomic gas after an expansion under a magnetic field gradient. We find three types of behavior: free motion with negative (positive) velocity on the bottom edge $m = -J$ (top edge $m = J$) and zero average velocity in the bulk.

nant optical transition, the atom dynamics is described by the Hamiltonian
\[
\hat{H} = \frac{1}{2} M \hat{v}^2 - \frac{\hbar \Omega}{2} \left( e^{-iK\hat{z}} \hat{\mathbf{J}}_+ + e^{iK\hat{z}} \hat{\mathbf{J}}_- \right) + V(\hat{J}_z) \tag{1}
\]
where $M$ is the atom mass, $\hat{v}$ is its velocity, $\hat{\mathbf{J}}_+ \and \hat{\mathbf{J}}_-$ are spin projection and ladder operators. The coupling $\Omega$ is proportional to both laser electric fields, and the potential $V(\hat{J}_z) = -\hbar \Omega \hat{J}_z^2/(2J + 3)$ stems from rank-2 tensor light shifts (see Methods).

A light-induced spin transition $m \to m + 1$ is accompanied by a momentum kick $-p_{\text{rec}} \equiv -\hbar K$ along $x$, such that the canonical momentum $\hat{p} = M \hat{v} + p_{\text{rec}} \hat{\mathbf{J}}_z$ is a conserved quantity. The dynamics for a given momentum $p$ is then described by the Hamiltonian
\[
\hat{H}_p = \frac{(p - p_{\text{rec}} \hat{J}_z)^2}{2M} - \hbar \Omega \hat{J}_x + V(\hat{J}_z), \tag{2}
\]
which resembles the Landau one,
\[
\hat{H}_{\text{Landau}} = \frac{(\hat{p}_x - eB\hat{y})^2}{2M} + \frac{\hat{p}_y^2}{2M}, \tag{3}
\]
\newpage

describing the dynamics of an electron evolving in 2D under a perpendicular magnetic field $B$. The analogy between both systems can be made upon the identifications $\hat{J}_z \leftrightarrow \hat{y}$ and $eB \leftrightarrow p_{\text{rec}}$. The term $\hat{J}_x$ in (2) plays the role of the kinetic energy along the synthetic dimension, since it couples neighboring $m$ levels with real positive coefficients, similarly to the discrete form of the Laplacian operator $\nabla^2 \equiv \hat{p}_y^2$ in (3) (see Methods). The range of magnetic projections being limited, our system maps onto a Hall system in a ribbon geometry bounded by the edge states $m = \pm J$. The relevance of the analogy is confirmed by the structure of energy bands $E_n(p)$ expected for our system, shown in Fig. 1. The energy dispersion of the first few bands is strongly reduced for $|p| \lesssim J p_{\text{rec}}$, reminiscent of dispersionless Landau levels. A parabolic dispersion is recovered for $|p| \gtrsim J p_{\text{rec}}$ similar to the ballistic edge modes of a quantum Hall ribbon [3].

We first characterize the ground band using quantum states of arbitrary values of momentum $p$. We begin with an atomic gas spin-polarized in $m = -J$, and with a negative mean velocity $\langle \hat{v} \rangle = -6.5(1)v_{\text{rec}}$ (with $v_{\text{rec}} \equiv p_{\text{rec}}/M$), such that it corresponds to the ground state of (2) with $p = -14.5(1)p_{\text{rec}}$. The gas temperature $T = \ldots$
0.55(6) nK is such that the thermal velocity broadening is smaller than the recoil velocity \(v_{\text{rec}}\). We then slowly ramp up the light intensity up to a coupling \(\hbar \Omega = 1.02(6) E_{\text{rec}}\), where \(E_{\text{rec}} = p_{\text{rec}}^2/(2M)\) is the natural energy scale in our system. Subsequently, we apply a weak force \(F_x\) along \(x\), such that the state adiabatically evolves in the ground energy band with \(\hat{p} \equiv F_x\), until the desired momentum is reached. We measure the distribution of velocity \(v\) and spin projection \(m\) by imaging the atomic gas after a free flight in the presence of a magnetic field gradient.

The main features of Landau level physics are visible in the raw images shown in Fig. 2. Depending on the momentum \(p\), the system exhibits three types of behaviors. (i) When spin-polarized in \(m = -J\), the atoms move with a negative mean velocity \(\langle \hat{v} \rangle\), consistent with a left-moving edge mode. (ii) When the velocity approaches zero under the action of the force \(F_x\), the system experiences a series of resonant transitions to higher \(m\) sublevels – in other words a transverse Hall drift along the synthetic dimension. In this regime the atom’s motion is inhibited along \(x\), as expected for a quasi non-dispersive band. (iii) Once the edge \(m = J\) is reached, the velocity \(\langle \hat{v} \rangle\) rises again, corresponding to a right-moving edge mode. Overall, while exploring the entire ground band under the action of a force along \(x\), the atoms are pumped from one edge to the other along the synthetic dimension.

To distinguish between bulk and edge modes, we plot in Fig. 2 the spin projection probabilities \(\Pi_m\) as a function of momentum \(p\). We find that the edge probabilities \(\Pi_{m \pm 1, J}\) exceed 1/2 for \(|p| > 8.0(1) p_{\text{rec}}\), defining the edge mode sectors – with the bulk modes in between. We study the system dynamics via its velocity distribution and mean velocity \(\langle \hat{v} \rangle\), shown in Fig. 2b. We observe that the velocity of bulk modes remains close to zero, which shows via the Hellmann-Feynman relation \(\langle \hat{v} \rangle = \partial E_0 / \partial p\) that the ground band is almost flat. The measured residual mean velocities allow us to infer a dispersion \(\Delta E_0^\text{pk-pk} = 1.2(5) E_{\text{rec}}\) in the bulk mode region – nearly 2% of the free-particle dispersion expected over the same range of momenta. On the contrary, edge modes are characterized by a velocity \(\langle \hat{v} \rangle \simeq (p - p_0)/M\), corresponding to ballistic motion – albeit with the restriction \(\langle \hat{v} \rangle < 0\) for edge modes close to \(m = -J\), and \(\langle \hat{v} \rangle > 0\) at the opposite edge. We also characterize correlations between velocity \(v\) and spin projection \(m\) over the full band, via the local density of states (LDOS) in \((v, m)\) space, integrated over \(p\). We stress here that the LDOS only involves gauge-independent quantities, and could thus be generalized to more complex geometries lacking translational invariance. As shown in Fig. 2c, it also reveals characteristic quantum Hall behavior, namely inhibited dynamics in the bulk and chiral motion on the edges.

The Landau level structure is characterized by a harmonic energy spacing \(\hbar \omega_c\), set by the cyclotron frequency \(\omega_c = eB/M\). We test this behavior in our system by studying elementary excitations above the ground band, via the trajectories of the center of mass following a velocity kick \(v_{\text{kick}} \simeq v_{\text{rec}}\). As shown in Fig. 3 (blue dots), we measure almost closed trajectories in the bulk, consistent with the periodic cyclotron orbits expected for an infinite Hall system. We checked that this behavior remains valid for larger excitation strengths, until one couples to highly dispersive excited bands (for velocity kicks \(v_{\text{kick}} \gtrsim 2v_{\text{rec}}\), see Methods). Close to the edges, the atoms experience an additional drift and their trajectories are similar to classical skipping orbits bouncing on a hard wall. In particular, the drift orientation only depends on the considered edge, irrespective of the kick direction. We report in the inset of Fig. 3 the frequencies of velocity oscillations, which agree well with the expected cyclotron gap to the first excited band. We find that the gap is almost uniform within the bulk mode sector, with a residual variation in the range \(\omega_c = 3.0(1) - 3.8(1) E_{\text{rec}}/\hbar\).

We now investigate the key feature of Landau levels, namely their quantized Hall response, which is intrinsically related to their topological nature. In a ribbon geometry, the Hall response of a particle corresponds to the transverse velocity acquired upon applying a potential difference across the edges (see Fig. 4). In our system, such a potential corresponds to a Zeeman term \(-F_m \hat{J}_z\) added to the Hamiltonian (2), which can now be recast...
as
\[ \hat{H}_p - F_m \hat{J}_z = \hat{H}_{p+Mv'} - v'p, \text{ with } v' = F_m/p_{\text{rec}}, \]

such that the force acts as a momentum shift \( Mv' \) in the reference frame with velocity \( v' \). In the weak force limit, the perturbed state remains in the ground band, and its mean velocity reads
\[ \langle \hat{v} \rangle = \langle \hat{v} \rangle_{p=0} - \mu F_m, \text{ where } \mu = \frac{1}{p_{\text{rec}}} \frac{\partial}{\partial p} (p - M \langle \hat{v} \rangle) \]
is the Hall mobility. We present in Fig. 4 the Hall mobility \( \mu(p) \) deduced from the mean velocity shown in Fig. 2. For bulk modes, it remains close to the value \( \mu = 1/p_{\text{rec}} \), which corresponds to the classical mobility \( \mu = 1/(eB) \) in the equivalent Hall system. The mobility decreases in the edge mode sector, as expected for topologically protected boundary states whose ballistic motion is undisturbed by the magnetic field.

We use the measured drift of individual quantum states to infer the overall Hall response of the ground band. As for any spatially limited sample, our system does not exhibit a gap in the energy spectrum due to the edge mode dispersion. In particular, high-energy edge modes of the ground band are expected to resonantly hybridize with excited bands upon disorder, such that defining the Hall response of the entire ground band is not physically meaningful. We thus only consider the energy branch \( E < E^* \), where \( E^* \) lies in the middle of the first gap at zero momentum (see Methods). We characterize the (inhomogeneous) Hall response of this branch via the local Chern marker
\[ C(m) \equiv 2\pi \text{ Im} \langle m | \hat{P} \hat{x} \hat{P}, \hat{P} \hat{J}_z \hat{P} | m \rangle = \int_{E(p)<E^*} dp \Pi_m(p) \mu(p), \]

where \( \hat{P} \) projects on the chosen branch [32, 33]. This local geometrical marker quantifies the adiabatic transverse response in position space, and matches the integer Chern number \( \mathcal{C} \) in the bulk of a large, defect-free system. Here, it is given by the integrated mobility \( \mu(p) \), weighted by the spin projection probability \( \Pi_m(p) \) (see Methods). As shown in Fig. 4, we identify a plateau in the range \(-5 \leq m \leq 5 \). There, the average value of the Chern marker, \( \mathcal{C} = 0.98(5) \), is consistent with the integer value \( \mathcal{C} = 1 \) – the Chern number of an infinite Landau level. This measurement shows that our system is large enough to reproduce a topological Hall response in its bulk. For positions \( |m| \geq 6 \), we measure a decrease of the Chern marker, that we attribute to non-negligible correlations with the edges.

Such a topological bulk is a prerequisite for the realization of emblematic phases of two-dimensional quantum Hall systems, as we now confirm via numerical simulations of interacting quantum many-body systems. In our system, collisions between atoms \( \text{a priori} \) occur when they are located at the same position \( x \), irrespective of their spin projections \( m, m' \), leading to highly anisotropic interactions. While this feature leads to an interesting phenomenology [36], we propose to control the interaction range by spatially separating the different \( m \) states using a magnetic field gradient, preventing collisions for \( m \neq m' \) (see Fig. 5a). We discuss below the many-body phases expected for bosonic atoms with such short-range interactions, assuming for simplicity repulsive interactions of equal strength for each projection \( m \).

We first consider the case of a large filling fraction \( \nu \equiv N_{\text{at}}/N_\phi \gg 1 \), where \( N_\phi \) is the number of magnetic flux quanta in the area occupied by \( N_{\text{at}} \) atoms – as realized in previous experiments on rapidly rotating gases [37, 38]. In this regime and at low tempera-
We use periodic boundary conditions along acting atoms, where the color encodes the interaction energy. A magnetic field gradient spatially separates the different $m$ states along $z$, such that collisions (represented by the red ellipses) only occur for atoms in the same magnetic sublevel $m$. B. Density distribution of a Bose-Einstein condensate, with a chemical potential at $\pm 2E_{\text{rec}}$ above the single-particle ground state energy. C. Many-body spectrum of a system of $N = 5$ interacting atoms, where the color encodes the interaction energy. We use periodic boundary conditions along $x$, with a circumference $L = 0.6(\lambda/2)$ allowing for $N_{\text{orb}} = 9$ orbitals at low energy, compatible with the Laughlin state. The residual energy dispersion between these orbitals is minimized by using a coupling strength $\Omega = 0.5E_{\text{rec}}/h$.

The realization of a quantum-Hall system based on a large synthetic dimension, as discussed here, is a promising setting for future realizations of topological quantum matter. An important asset of our setup is the large cyclotron energy, measured in the range $\hbar \omega_c \approx k_B \times 1.8 - 2.3 \text{mK}$, much larger than the typical temperatures of quantum degenerate gases, thus enabling the realization of strongly-correlated states at realistic temperatures. The techniques developed here could give access to complex correlation effects, such as flux attachment via cyclotron orbits or charge fractionalization via center-of-mass Hall response.

1. Thouless, D. J., Kohmoto, M., Nightingale, M. P. & den Nijs, M. Quantized Hall Conductance in a Two-Dimensional Periodic Potential. Phys. Rev. Lett. 49, 405–408 (1982).
2. Laughlin, R. B. Quantized Hall conductivity in two dimensions. Phys. Rev. B 23, 5632–5633 (1981).
3. Halperin, B. I. Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential. Phys. Rev. B 25, 2185–2190 (1982).
4. Stormer, H. L., Tsui, D. C. & Gossard, A. C. The fractional quantum Hall effect. Rev. Mod. Phys. 71, S298–S305 (1999).
5. Hasan, M. Z. & Kane, C. L. Colloquium: Topological insulators. Rev. Mod. Phys. 82, 3045–3067 (2010).
6. Ozawa, T. et al. Topological photonics. Rev. Mod. Phys. 91, 015006 (2019).
7. Goldman, N., Budich, J. C. & Zoller, P. Topological quantum matter with ultracold gases in optical lattices. Nat. Phys. 12, 639–645 (2016).
8. Jotzu, G. et al. Experimental realization of the topological Haldane model with ultracold fermions. Nature 515, 237–240 (2014).
9. Aidelsburger, M. et al. Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms. Nat. Phys. 11, 162–166 (2015).
10. Hu, W. et al. Measurement of a Topological Edge Invariant in a Microwave Network. Phys. Rev. X 5, 011012 (2015).
11. Mittal, S., Ganeshan, S., Fan, J., Vaezi, A. & Hafezi, M. Measurement of topological invariants in a 2D photonic system. Nat. Photonics 10, 180–183 (2016).
12. Wu, Z. et al. Realization of two-dimensional spin-orbit coupling for Bose-Einstein condensates. Science 354, 83–88 (2016).
13. Fläschner, N. et al. Experimental reconstruction of the Berry curvature in a Floquet Bloch band. Science 352, 1091–1094 (2016).
14. Ravets, S. et al. Polariton Polaronics in the Integer and Fractional Quantum Hall Regimes. Phys. Rev. Lett. 120, 057401 (2018).
15. Schine, N., Chalupnik, M., Can, T., Gromov, A. & Si-
mon, J. Electromagnetic and gravitational responses of photonic Landau levels. *Nature* **565**, 173 (2019).

[16] Celi, A. *et al.* Synthetic Gauge Fields in Synthetic Dimensions. *Phys. Rev. Lett.* **112**, 2–111 (2019).

[17] Mancini, M. *et al.* Observation of chiral edge states with neutral fermions in synthetic Hall ribbons. *Science* **349**, 1510–1513 (2015).

[18] Stuhl, B. K., Lu, H.-I., Aycock, L. M., Genkina, D. & Spielman, I. B. Visualizing edge states with an atomic Bose gas in the quantum Hall regime. *Science* **349**, 1514–1518 (2015).

[19] Livi, L. F. *et al.* Synthetic Dimensions and Spin-Orbit Coupling with an Optical Clock Transition. *Phys. Rev. Lett.* **117**, 220401 (2016).

[20] Kolkowitz, S. *et al.* Spin–orbit-coupled fermions in an optical lattice clock. *Nature* **542**, 66–70 (2017).

[21] An, F. A., Meier, E. J. & Gadway, B. Direct observation of chiral currents and magnetic reflection in atomic flux lattices. *Sci. Adv.* **3**, e1602685 (2017).

[22] Lustig, E. *et al.* Photonic topological insulator in synthetic dimensions. *Nature* **567**, 356–360 (2019).

[23] Ozawa, T. & Price, H. M. Topological quantum matter in synthetic dimensions. *Nat. Rev. Phys.* **1**, 349 (2019).

[24] Pesin, D. & MacDonald, A. H. Spintronics and pseudospintronics in graphene and topological insulators. *Nature Mater* **11**, 409–416 (2012).

[25] Kitaev, A. Y. Fault-tolerant quantum computation by anyons. *Annals of Physics* **303**, 2–30 (2003).

[26] v. Klitzing, K., Dorda, G. & Pepper, M. New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance. *Phys. Rev. Lett.* **45**, 494–497 (1980).

[27] Lu, L., Joannopoulos, J. D. & Soljačić, M. Topological photonics. *Nat. Photonics* **8**, 821–829 (2014).

[28] Aidelsburger, M. *et al.* Realization of the Hofstadter Hamiltonian with Ultracold Atoms in Optical Lattices. *Phys. Rev. Lett.* **111**, 185301 (2013).

[29] Miyake, H., Siviloglou, G. A., Kennedy, C. J., Burton, W. C. & Ketterle, W. Realizing the Harper Hamiltonian with Laser-Assisted Tunneling in Optical Lattices. *Phys. Rev. Lett.* **111**, 185302 (2013).

[30] Lohse, M., Schweizer, C., Price, H. M., Zilberberg, O. & Bloch, I. Exploring 4D quantum Hall physics with a 2D topological charge pump. *Nature* **553**, 55–58 (2018).

[31] Zilberberg, O. *et al.* Photonic topological boundary pumping as a probe of 4D quantum Hall physics. *Nature* **553**, 59–62 (2018).

[32] Bianco, R. & Resta, R. Mapping topological order in coordinate space. *Phys. Rev. B* **84**, 241106 (2011).

[33] Lin, Y.-J., Jiménez-García, K. & Spielman, I. B. Spin–orbit-coupled Bose–Einstein condensates. *Nature* **471**, 83–86 (2011).

[34] Cui, X., Lian, B., Ho, T.-L., Lev, B. L. & Zhai, H. Synthetic gauge field with highly magnetic lanthanide atoms. *Phys. Rev. A* **88** (2013).

[35] Kitaev, A. Anyons in an exactly solved model and beyond. *Annals of Physics* **321**, 2–111 (2006).

[36] Barbarino, S., Taddea, L., Rossini, D., Mazza, L. & Fazio, R. Magnetic crystals and helical liquids in alkaline-earth fermionic gases. *Nat Commun* **6**, 1–9 (2015).

[37] Schweikhard, V., Coddington, I., Engels, P., Mogendorff, V. P. & Cornell, E. A. Rapidly Rotating Bose–Einstein Condensates in and near the Lowest Landau Level. *Phys. Rev. Lett.* **92**, 040404 (2004).

[38] Breit, V., Stock, S., Seurin, Y. & Dalibard, J. Fast Rotation of a Bose-Einstein Condensate. *Phys. Rev. Lett.* **92** (2004).

[39] Abrikosov, A. A. On the magnetic properties of superconductors of the second group. *Sov. Phys. - J. Exp.* 5, 1174–1182 (1957).

[40] Kane, C. L., Mukhopadhyay, R. & Lubensky, T. C. Fractional Quantum Hall Effect in an Array of Quantum Wires. *Phys. Rev. Lett.* **88**, 036401 (2002).

[41] Goldman, V. J., Su, B. & Jain, J. K. Detection of composite fermions by magnetic focusing. *Phys. Rev. Lett.* **72**, 2065–2068 (1994).

[42] Taddea, L. *et al.* Topological Fractional Pumping with Alkaline-Earth-Like Atoms in Synthetic Lattices. *Phys. Rev. Lett.* **118**, 230402 (2017).

**Acknowledgements**

We thank J. Beugnon, N. Cooper, P. Delpace, N. Goldman, L. Mazza, and H. Price for stimulating discussions. We acknowledge funding by the EU under the ERC projects ‘UQUAM’ and ‘TOPODY’, and PSL research university under the project ‘MAFAG’.

**Author contributions**

All authors contributed to the setup of the experiment, the data acquisition, its analysis and the writing of the manuscript.

**Data availability**

The datasets generated and analyzed during the current study are available from the corresponding author on request.

**Author Information**

Correspondence and requests for materials should be addressed to S.N. (sylvain.nascimbene@lkb.ens.fr).
METHODS

Details on the experimental protocol
Our experiments begin by preparing an ultracold gas of $8(2) 	imes 10^4$ $^{162}$Dy atoms at a temperature $T = 0.55(6)$ µK, and held in an optical dipole trap. The atoms are placed in a magnetic field $B = 172(2)$ mG along the z-axis, corresponding to a Zeeman splitting of frequency $ω_Z = 2π × 298(3)$ kHz, with the electronic spin polarized in the absolute ground state $m = −J$. We then turn off the trap, and turn on the two laser beams shown in Fig. [1] which differ in frequency by $ω_{12} = ω_1 − ω_2$. When $ω_{12}$ is close to the Zeeman splitting $ω_Z$, a spin transition $m → m + 1$ occurs via the absorption of one photon from beam 1 and the stimulated emission of one photon in beam 2. In such processes and in the absence of additional external forces, the canonical momentum $p = Ṁt + hKJ_z$ is conserved. We experimentally verified the conservation of momentum in all our experiments, with an rms deviation $Δp ≃ 0.04$ rec. We checked that the number of atoms remains constant over the timescale of the experiment.

The laser beam frequencies are set close to the optical transition at 626 nm, which couples the electronic ground state $J = 8$ to an excited level $J’ = 9$. The beams are detuned by $Δ = 2π × 22$ GHz with respect to resonance and are linearly polarized along orthogonal directions, each being at 45° with respect to the z-axis. Then, the algebra of Clebsch-Gordan coefficients of $J → J’ = J + 1$ transitions leads to the Hamiltonian [2] at resonance ($ω_{12} = ω_Z$), with

$$\Omega = \frac{2J + 3}{4(J + 1)(2J + 1)} V_0, \quad V_0 = \frac{3πcΓ \sqrt{I_1I_2}}{2ω_0^3} \Delta$$

where $I_{1,2}$ are the laser intensities on the atoms, $Γ ≃ 2π × 135$ kHz is the transition linewidth, and $ω_0$ is its resonant frequency.

The value of the coupling $Ω$ is calibrated using an independent method and remains constant over the experimental sequence since the waists of both laser beams are much larger than the region of atomic motion. The Larmor frequency $ω_Z$ is calibrated from the resonance of the Raman transition between $m = −8$ and $m = −7$. Its shot-to-shot variations are set by transverse magnetic field fluctuations of rms magnitude 0.7 mG. Our imaging setup is such that the 17 magnetic sublevels have different cross-sections. We calibrate the relative cross-sections such that the calculated atom number remains constant for all momentum states, irrespective of their spin composition.

The non-resonant case ($ω_{12} ≠ ω_Z$) can be reduced to the resonant case in a reference frame moving at a velocity $v_{frame} = (ω_Z − ω_{12})/K$. Note that the required change of frame means that fluctuations of $ω_Z$ contribute to the uncertainties of the measured velocities. We apply an external force $F_x$ on the system via the inertial force resulting from a time-dependent frequency difference, with $F_x = (M/K)Ωω_{12}$. The preparation of a state in the lowest band with a given momentum $p$ is performed by adiabatically ramping the frequency difference to $ω_{12} = ω_Z + 2(p/p_{rec} + J)E_{rec}/h$. We use a constant ramp rate $Ωω_{12} ≃ 0.22ω_{min}^2$, where $ω_{min} ≃ 3.06 E_{rec}/h$ is the minimum cyclotron frequency separating the two lowest energy bands for $Ω = E_{rec}/h$.

Depending on the target $p$ state, the preparation takes between 150 µs and 550 µs.

We numerically checked the adiabaticity of the state preparation protocol. While preparing $m = +J$, which requires crossing all momentum states, the squared overlap with the ground band remains greater than 0.96 and the deviation of the mean spin projection $⟨J_z⟩$ from the corresponding ground state value is always less than 0.08. The largest deviations occur near $m ≃ ±l$, where the energy gap to the first excited band is the smallest. This behavior is consistent with our measurements, showing that the adiabatic transfer to $m = J$ after exploring the entire band is above 97%.

Emergence of Landau levels
In Fig. [1] we show the dispersion relation of the Hamiltonian [2] calculated for different couplings $Ω$. In the absence of the light coupling, $Ω = 0$, the Hamiltonian reduces to the kinetic energy term $(p − p_{rec}J_z)^2/(2M)$, leading to 2 $J + 1$ parabolas shifted along $p$. All energy crossings become avoided for $Ω ≠ 0$, leading to flattened energy bands akin to Landau levels. Achieving a flat ground band dispersion in the bulk region requires couplings that are large enough ($hΩ ≥ 0.2E_{rec}$) to reduce short-$p$ oscillations, while still being sufficiently small ($hΩ ≤ E_{rec}$) to minimize longer-scale curvature.

The large spin $J = 8$ allows for a simplified semi-classical description, where the spin is represented by a point on the generalized Bloch sphere, parametrized by its spherical angles $(θ, φ)$. The spin projection is mapped on a continuous variable $m = J cosθ$, with the azimuthal angle $φ$ being its conjugated variable (up to a factor $h$). The semi-classical Hamiltonian corresponding to the quantum Hamiltonian [2] then reads

$$H_p = \frac{(p − hKm)^2}{2M} − hΩ \left( √J^2 − m^2 \cosφ + \frac{m^2}{2J + 3} \right)$$

This energy functional being minimized for $φ = 0$, one can assume $φ ≪ 1$ and obtain a low-energy expansion

$$H_p = \frac{(p − hKm)^2}{2M} + \frac{h^2φ^2}{2M′(m)} + V(m),$$

$$M′(m) = h/ \left( Ω√J^2 − m^2 \right),$$

$$V(m) = −hΩ \left( √J^2 − m^2 + \frac{m^2}{2J + 3} \right),$$

which is exactly the Landau Hamiltonian [3], albeit with a position-dependent mass $M′(m)$ and a confining potential $V(m)$ in the synthetic dimension. The divergence of the mass $M′(m)$ for $|m| → J$ leads to an effective hard-wall condition.
In the middle of the bulk, at $m = 0$, in the semi-classical model, we deduce the cyclotron frequency $\omega_c = \sqrt{2M E_{rec}/\hbar}$, and we expect a value $\omega_c = 4E_{rec}/\hbar$ for $\hbar \Omega = E_{rec}$, which is close to the exact value $\omega_c = 3.84E_{rec}/\hbar$ (see inset of Fig. 3). We also infer the expressions for magnetic lengths

$$\ell_m = \sqrt{\frac{\hbar \Omega}{2E_{rec}}} \quad \text{and} \quad \ell_x = \frac{1}{K} \ell_m$$

in the synthetic and real dimensions respectively. These lengths are the characteristic sizes of the quantum vortices shown in Fig. 4b. For the coupling $\hbar \Omega = E_{rec}$ used in the simulations, we obtain magnetic lengths $\ell_m \approx 1.41$ and $\ell_x \approx 0.11\lambda/2$.

The analogy between the Hamiltonians (2) and (3) can also be inferred using quantum operators, as we explain now assuming $p \simeq 0$ for simplicity. In that case, we expect the system to be polarized in $m = J$ along $x$, such that the commutator

$$[\hat{J}_z, \hat{J}_y] = -i\hat{J}_x \simeq -iJ$$

is a c-number. The operator $-\hat{J}_y/J$ is then canonically conjugated to the spin projection $\hat{J}_z$. We then use the Holstein-Primakoff approximation at second order to express the spin projection $\hat{J}_x$ as

$$\hat{J}_x = J + \frac{1}{2} - \frac{\hat{J}_y^2 + \hat{J}_z^2}{2J},$$

leading to the Hamiltonian

$$\hat{H} = \frac{(p - p_{rec} \hat{J}_z)^2}{2M} + \frac{\hbar \Omega}{2J} \hat{J}_y^2 + \frac{3\hbar \Omega}{2J(2J + 3)} \hat{J}_z^2 - \hbar \Omega \left( J + \frac{1}{2} \right).$$

This Hamiltonian corresponds to the Landau Hamiltonian (3), with an additional $\hat{J}_z^2$ term. This approximation can be generalized to all values of momentum $p$. We show in Fig. S1 the first two energy bands calculated within this approximation, together with the exact spectrum.

**Cyclotron orbits**

In order to probe the excitations of the system we perform a velocity quench, which couples the lowest Landau level to the next higher energy band. The system then responds periodically with a frequency set by the energy difference between the two bands, which for the case of an ideal Hall system would correspond to the cyclotron frequency $\omega_c$. Experimentally, we perform the velocity kick by quenching the detuning $\omega_{12}$, which in practice settles to a steady value after $4\mu s$. We show in Fig. S2a,b an example of coherent oscillations of both magnetization and velocity. We compute the response of the system in real space, $x$, via a numerical integration of the velocity evolution as shown in Fig. S2. The uncertainty on the Larmor frequency leads to a systematic error on the velocity on the order of $0.1v_{rec}$, consistent with the small drift of some cyclotron orbits in the bulk.

The response of the system is probed after a velocity kick $v_{kick} \approx v_{rec}$. This kick ensures a negligible overlap with the second excited band (smaller than $4\%$). Although, in an ideal Hall system, all bulk excitations evolve periodically at the cyclotron frequency $\omega_c = qB/M$ due to the harmonic spacing of successive Landau levels, this is not exactly the case in our system. We test this behavior by varying the strength of the excitation which relates to the magnitude of the velocity kick. As shown in Fig. S2d, we find that the trajectories cease to be closed and start to drift along the kick direction as the excitation strength exceeds $1.5v_{rec}$ (see Fig. S2). This regime corresponds to the onset of significant population of higher energy bands $n \geq 2$, which illustrates the non-harmonic spectrum of our system.

It is important to note that the excitation protocol described so far is inefficient for large values of $p$, where the energy gap is much larger. In that regime, a quench of the coupling amplitude $\Omega$ leads to a more efficient overlap with higher energy bands. This is shown in Fig. S2c, for the case of a sudden branching of the coupling strength to $\hbar \Omega = E_{rec}$. The system initially at $p = -Jp_{rec}$ is then effectively coupled to higher energy bands and the bouncing on the hard wall characteristic of classical skipping orbits is clearly visible.
Transverse drift in a Hall system

Our system is analogous to a Hall system in a ribbon geometry. To understand the role of a sharp edge on the physical quantities measured in the main text, we consider an electronic Hall system in a semi-infinite geometry, described by the Landau Hamiltonian [3], written as

\[
\hat{\mathcal{H}} = \frac{\hat{p}_y^2}{2M} + \frac{1}{2} M \omega_c^2 (\hat{y} - \hat{p}_x \ell^2 / \hbar)^2,
\]

with a hard-wall restricting motion to the half-plane \( y > 0 \). Here, we introduce the cyclotron frequency \( \omega_c = eB/M \) and the magnetic length \( \ell = \sqrt{\hbar/eB} \), assuming a magnetic field \( B \) along \( z \).

We first consider semi-classical trajectories in the absence of external forces, which are either closed cyclotron orbits or skipping orbits bouncing on the edge, parametrized by the rebound angle \( \theta \) (see Fig. S3a). Applying a perturbative force \( F \) along \( y \) leads to a drift of cyclotron orbits of velocity \( v_d = -F/eB \) along \( x \), corresponding to a Hall mobility \( \mu = 1/eB \). For skipping orbits, the Hall drift can be expressed analytically as

\[
\mu = \frac{1}{eB} \left[ 1 - \left( \frac{\sin \theta}{\theta} \right)^2 \right].
\]  

The factor of reduction compared to cyclotron orbits, plotted in Fig. S3b, smoothly interpolates between 1 for almost closed orbits (\( \theta \to \pi \)) and 0 for almost straight orbits (\( \theta \to 0 \)). This behavior provides a simple explanation of the reduced Hall mobility of edge modes (see Fig. S4).

We extend this reasoning to the quantum dynamics in the lowest energy band. In a semi-infinite geometry, the eigenstates of the Hamiltonian [4] can be indexed by the momentum \( p \) along \( x \), and are expressed as

\[
\psi_p(x, y) = \frac{e^{ipx/\hbar}}{\sqrt{2\pi \hbar}} \phi_p(y),
\]

\[
\phi_p(y) \propto D_{\epsilon(p)-1/2} \left[ \sqrt{2}(y - p\ell^2 / \hbar) / \ell \right],
\]

where \( D_{\epsilon(p)}(z) \) is the parabolic cylinder function and \( \epsilon(p) = E_0(p)/\hbar \omega_c \) is the reduced energy determined by the boundary condition \( D_{\epsilon(p)-1/2}(-\sqrt{2}p\ell / \hbar) = 0 \) (see Fig. S3b). By summing over all momentum states of the ground band, we compute the local density of state in \((v_x, y)\) coordinates plotted in Fig. S3c. Far from the edge \( y \gg \ell \), the velocity distribution is a Gaussian centered on \( v_x = 0 \), of rms width \( \delta v_x = \hbar/(M \ell) \). The distribution is shifted to negative velocities when approaching the edge \( y = 0 \), as expected for chiral edge modes.

We now consider the Hall response of the system by studying the perturbative action of a force \( F \) along \( y \), described by the Hamiltonian

\[
\hat{\mathcal{H}}_p = \frac{\hat{p}_y^2}{2M} + \frac{1}{2} M \omega_c^2 \left( \hat{y} - \frac{p\ell^2}{\hbar} - F \hat{y} \right)^2 + \mathcal{E}(p),
\]

\[
\mathcal{E}(p) = -\frac{pF}{M \omega_c} - \frac{F^2}{2M \omega_c^2}.
\]

We identify the perturbed Hamiltonian \( \hat{\mathcal{H}}_p \) as \( \hat{\mathcal{H}}_{p+F/\omega_c} \), with an additional energy shift \( \mathcal{E}(p) \). Assuming the system to remain in the ground band, the group velocity of a localized wavepacket becomes

\[
\langle \dot{\hat{v}} \rangle = v_0(p + F/\omega_c) - \frac{F}{M \omega_c}, \quad v_0(p) = \frac{dE_0(p)}{dp}.
\]
Assuming a small force, we expand the velocity as $\langle \dot{v}(t) \rangle' = v_0(p) - \mu(p) F$, with the mobility

$$\mu(p) = \frac{1}{eB} \left( 1 - M \frac{d}{dp} v_0(p') \right).$$

This formula is analogous to the expression for the Hall mobility in our synthetic system. As shown in Fig. S3, it is close to the classical Hall drift in an infinite plane in the bulk mode region $p \gtrsim \hbar / \ell$, while it decreases towards zero in the edge mode region $p < 0$.

The overall response of an energy branch in the ground band can be obtained by summing the drifts of all populated eigenstates, such that the center of mass drift reads

$$\langle v(t) \rangle' = \langle v(t) \rangle_0 - F \int dp n(p) \mu(p),$$

where we assume the normalization $\int dp n(p) = 1$ for the occupation number $n(p)$. We consider a uniform occupation of the lowest energy band, restricted to the energy branch $E_0(p) < \hbar \omega_c$, i.e. the middle of the bulk gap to the first excited band in the bulk. This condition corresponds to momentum states $p > p' \approx 0.5 \hbar / \ell$ of the ground band. Assuming an upper momentum cutoff $p'$ in the bulk region, we obtain the Hall drift

$$\langle v(t) \rangle' = \langle v(t) \rangle_0 - \frac{F}{eB} \left( 1 - M \frac{v_0(p') - v_0(p)}{p' - p} \right).$$

As long as $p' \gg \hbar / \ell$, the second term can be neglected, and one recovers the Hall drift of a topological band of Chern number $C = 1$.

We finally consider the local Hall response in the ground band, quantified by the local Chern marker $[2]$

$$C(x, y) = 2\pi \text{Im} \langle x, y | [\hat{P} \hat{x} \hat{P}, \hat{P} \hat{y} \hat{P}] | x, y \rangle,$$

where $\hat{P}$ projects on the considered branch of states and $(x, y)$ are localized in $(x, y)$. The calculation of the Chern marker starts by decomposing position states into momentum states as

$$C(x, y) = \frac{2}{\hbar} \text{Im} \left[ \int dp dq e^{i(p-q) x / \hbar} \phi_p(y) \phi_q(y) \tilde{c}(p, q) \right],$$

where $\tilde{c}(p, q) \equiv \langle \psi_p | \hat{x} \hat{P} \hat{y} | \psi_q \rangle$, which can be evaluated using the explicit form (5) for momentum states as

$$\tilde{c}(p, q) \equiv ih \langle y \rangle_q \langle \phi_p | \phi_q \rangle \delta(p - q),$$

where $\langle y \rangle_q$ is the mean $y$ position in the wavefunction $\phi_q$. Using the general formula

$$\int du dv f(u, v)\delta'(u - v) = \frac{1}{2} \int du dv \left[ \partial_u f(u, v) - \partial_v f(u, v) \right] \delta(u - v),$$

we obtain the expression for the Chern marker

$$C(x, y) = \int dp | \phi_p(y) \rangle \frac{d\langle y \rangle_p}{dp}.$$

The relation $p = M v_0 + q B \langle y \rangle_p$ then leads to

$$C(x, y) = \int dp | \phi_p(y) \rangle \frac{d\langle y \rangle_p}{dp},$$

a relation analogous to the local Chern marker expression for our synthetic Hall system. We show in Fig. S3 that the Chern marker calculated for an energy branch $E(p) < \hbar \omega_c$, which is close to 1 for $y \gtrsim \ell$, and decreases towards zero when approaching the edge $y = 0$, similarly to the decrease of the Chern marker close to the edges shown in Fig. 4.

**Local Chern marker in synthetic dimension**

In the synthetic Hall system, the expression of the local Chern marker reads

$$C(x, m) = 2\pi \text{Im} \langle x, m | [\hat{P} \hat{x} \hat{P}, \hat{P} \hat{y} \hat{P}] | x, m \rangle.$$

Translation invariance along $x$ ensures that the Chern marker only depends on the coordinate $m$. In the main text, the notation $| m \rangle$ refers to an arbitrary $| x, m \rangle$ state, the choice of $x$ being irrelevant. The derivation of the Chern marker

$$C(m) = \int dp \Pi_m(p) \mu(p)$$
is obtained following the same procedure as for a standard Hall system, discussed above. So far, we have only
considered one component \( \mu_{xm} \) of the mobility tensor – the one that measures the drift along \( x \) resulting from a
force along \( m \). One can also consider the other component, which quantifies the magnetization drift \( d\langle J_z \rangle/dt \)
which results from a force \( F_x \) along \( x \). In linear response, it is defined as \( \langle J_z \rangle = \langle J_z \rangle_0 - \mu_{mx}(p)F_xt \), where \( \mu_{mx} \)
explicitly designates the mobility component considered here, and \( \langle J_z \rangle_0 \) is the unperturbed magnetization. Its
expression is given by

\[
\mu_{mx} = -\frac{d}{dF_x} \frac{d\langle J_z \rangle}{dt} = -\frac{d\langle J_z(p) \rangle}{dp},
\]

where we used \( F_x = \dot{p} \) and the fact that \( \langle J_z \rangle_0 \) is time-independent. The expression \( \dot{p} = M\dot{\theta} + p_{rec}\dot{J}_z \) allows
to recover the relation \( \mu_{mx} = -\mu_{xm} \) between the two transverse mobilities.

We show in Fig. S4 the measurements of both mobilities as a function of \( p \), and find good agreement between
them. We also present in Fig. S4 the local Chern markers computed using the data of each mobility.

In the main text, the Chern marker is evaluated over a branch of the ground band, below an energy threshold
shown in Fig. S4 (at half the cyclotron gap at \( p = 0 \)). We also show the Chern marker computed using all momen-
tum states (gray points). Compared to the restricted branch, we only find a discrepancy on the edges of the
ribbon. In the region \(-5 \leq m \leq 5 \), the values are nearly identical, showing that the bulk topological response
is insensitive to the momentum cutoff.

**Interactions in the lowest energy band**

Interactions are typically short-ranged in atomic gases. In our system, interactions are thus local in \( x \), but they
can occur between any pair of spin projections \( m_1, m_2 \), corresponding to highly long-range interactions along
the synthetic dimension. To recover short-range interactions, we propose to spatially separate the different \( m \) states
using a magnetic field gradient oriented along another direction \( z \). This separation then prevents collisions be-
tween two atoms in different \( m \) states, leading to local interactions along both \( x \) and \( m \). For a transverse con-
fine of frequency \( \omega_z = 2\pi \times 1\text{kHz}_z \) corresponding to a ground-state extent \( \Delta_z = \sqrt{\hbar/M\omega_z} \approx 250\text{ nm} \),
the interactions become short-ranged for moderate magnetic field gradients \( \nabla B \gtrsim 30\text{ G/cm} \).

The interaction between atoms in a given \( m \) state is then described by a short-range potential \( g_m\delta(x_1 - x_2) \)
with coupling constant \( g_m \), proportional to the s-wave scattering length \( a_m \). At low magnetic field, rotational
symmetry ensures that \( a_m = a_{-m} \), such that interactions are described by \( J + 1 \) independent scattering
lengths. While the \( a_m \) constants are uniform between all nuclear spin levels for two-electron atoms, we do not
expect such a SU(\( N \)) symmetry for lanthanide atoms such as dysprosium, for which only the coupling constant
\( a_8 = 140(20)\text{ a}_0 \) has been measured. All the other \( a_m \) constants remain unknown and we plan to investigate
them in the future. Nonetheless, if all values \( a_m \) are positive, the system will be protected from collapse, making
many-body phases experimentally accessible.

In lanthanide atoms, interactions between magnetic dipoles enrich the situation discussed above. These inter-

---

Figure S4. **a.** Predicted dispersion relation for \( \Omega = E_{rec} \). The branch pictured in blue, chosen as \( E(p) < E^* \) with \( E^* \) at half the
gap, is used for the computation of the local Chern marker. **b.** Measured mobility in \( x \) resulting from the application of a
force along \( m \), as presented in the main text. The points in blue, corresponding to \( |p| < p^* \) (white area), are the ones considered
for the Chern marker presented in the main text (see Fig. 4). **c.** Measured mobility in \( m \) resulting from the application of a
force along \( x \). As for b, the points in red are associated to momentum states lying below \( E^* \). **d.** Chern marker obtained from
the measured mobility, using the whole energy branch \((-\infty < p < \infty, \text{ gray squares, using data in } b) \), or using the branch
defined in a \((-p^* < p < p^*) \). For the latter, the blue dots correspond to the data in b, and are identical to Fig. 4. The red diamonds
correspond to the data in c. Solid lines are theoretical values.
actions offer an additional degree of freedom that could be used to stabilize the system in case of attractive s-wave interaction channels.

For simplicity, we neglect dipolar interactions in the numerical simulations and consider that all scattering lengths are equal and positive, such that the interaction potential reads $g \delta(x_1 - x_2)\delta_{m_1, m_2}$. For such contact interactions, one expects interactions restricted to the lowest Landau level to reduce to a single Haldane pseudo-potential \[ U = \frac{g}{4\pi} \frac{1}{\ell_x \ell_m} = \frac{gK}{4\pi}. \]

Abrikosov vortex lattices

We consider a gas of bosonic atoms with high filling fractions, for which the many-body ground state is well captured by mean-field theory. The system is described by a spinor classical field ($\psi_m(x)$) (with $-J \leq m \leq J$), whose dynamics is governed by the Gross-Pitaevskii equation

\[
\dot{\psi}_m = \frac{\hbar^2}{2M}(i\partial_x + Km)^2\psi_m - \hbar\Omega \left( \frac{\sqrt{J(J+1)-m(m+1)}}{2} \psi_{m+1} + \frac{\sqrt{J(J+1)-m(m-1)}}{2} \psi_{m-1} + \frac{m^2}{2J+3} \psi_m \right) + g|\psi_m|^2\psi_m.
\]

The thermodynamic properties are determined by the coupling $\Omega$ and the interaction energy scale $g\langle n \rangle$, where $\langle n \rangle$ is the mean atom density, or equivalently by the chemical potential $\mu_{\text{chem}}$. Here we explore situations in which the chemical potential lies in the gap between the LLL and the first excited band (see Fig. S5b).

For large enough interactions, we always find ground state configurations in the shape of Abrikosov triangular vortex lattices, such as the ones presented in the main text (see Fig. 5). We give in Figs. S5a and S5, another

Figure S5. a. Ground state density profile and b. associated phase, for $\hbar\Omega = 3E_{\text{rec}}$ and for $\mu_{\text{chem}} \approx 4E_{\text{rec}}$. The local minima of the density exhibit a phase winding around them, and thus correspond to quantum vortices. c. Number of vortex lines as a function of the Raman coupling $\Omega$ and the chemical potential $\mu_{\text{chem}}$. The dots identify the configurations for which a simulation was realized. The color encodes the number of vortex lines that characterizes the low-energy vortex lattice configuration. The phase separation lines are guides to the eye. The dashed line identifies the gap to the first excited band above which the atoms significantly occupy higher Landau levels. d. Momentum $p_0$ associated to the spontaneous breaking of the translational invariance resulting from the appearance of a vortex lattice, as a function of $\Omega$. The points were taken at a chemical potential corresponding to half the gap.
example of such ground state, represented here by both the density profile and the phase associated to the wavefunction. Around each local minimum of the density, the phase profile is reminiscent of the phase winding of a quantum vortex in a continuous 2D system.

The hard walls in the synthetic dimension have a strong impact on the vortex lattice geometry. We distinguish the different configurations by counting the number of vortex lines along $x$. For example, in Fig. S5a we identify a Laughlin-like state with very small interaction energy and at zero momentum, separated in energy from other eigenstates for $L \geq 0.55 \lambda / 2$. For $L = 0.7 \lambda / 2$ we recognize an energy branch of edge excitations of the Laughlin state. The gray area marks the bulk gap of the Laughlin state $\Delta = 0.60(3)U$ expected in dispersionless Landau levels and in the thermodynamic limit [9]. The color scale is identical to the one used in Fig. [8].

Laughlin-like state at low filling

We consider in this section bosonic atoms at low filling fractions, for which one expects strongly-correlated ground states. We calculate the many-body spectrum of this system using exact diagonalization. The stability of Laughlin-like quantum states is based on limited energy dispersion in the ground band, which is improved by considering a coupling $\hbar \Omega = 0.5 E_{\text{rec}}$, i.e. half of the value used in the experiment. We use a cylindrical geometry to avoid edge effects along $x$, and restrict the basis of single-particle levels to an energy $E = 3 E_{\text{rec}}$ above the single-particle ground state, which includes bulk states of the ground and first excited bands, with a few edge modes depending on the circumference $L$ of the cylinder (see Fig. S6g-l). We calculate the energy spectrum of $N_{\text{at}}$ bosonic atoms interacting at short range, with an interaction strength $U = E_{\text{rec}}$. The many-body eigenstates are indexed by the total momentum $p_{\text{tot}}$, a conserved quantity that permits us to subdivide the Hilbert space into independent sectors, limiting the involved matrices to dimensions less than 4000.

We show in Fig. S6a-f six energy spectra calculated for $N_{\text{at}} = 5$ atoms on cylinders, of circumferences $L = 0.45, 0.5, 0.55, 0.6, 0.65, 0.7 \lambda / 2$, respectively. The single-particle states included in the simulations are shown as solid dots in g-l. We identify a Laughlin-like state with very small interaction energy and at zero momentum, separated in energy from other eigenstates for $L \geq 0.55 \lambda / 2$. For $L = 0.7 \lambda / 2$ we recognize an energy branch of edge excitations of the Laughlin state. The gray area marks the bulk gap of the Laughlin state $\Delta = 0.60(3)U$ expected in dispersionless Landau levels and in the thermodynamic limit [9]. The color scale is identical to the one used in Fig. [8].

Figure S6. a-f. Many-body energy spectra calculated for $N_{\text{at}} = 5$ atoms on cylinders, of circumferences $L = 0.45, 0.5, 0.55, 0.6, 0.65, 0.7 \lambda / 2$, respectively. The single-particle states included in the simulations are shown as solid dots in g-l. We identify a Laughlin-like state with very small interaction energy and at zero momentum, separated in energy from other eigenstates for $L \geq 0.55 \lambda / 2$. For $L = 0.7 \lambda / 2$ we recognize an energy branch of edge excitations of the Laughlin state. The gray area marks the bulk gap of the Laughlin state $\Delta = 0.60(3)U$ expected in dispersionless Landau levels and in the thermodynamic limit [9]. The color scale is identical to the one used in Fig. [8].
of distinct orbitals involved in the Laughlin wavefunction \[7, 8\]. For circumferences \( L \geq 0.55 \lambda/2 \), we find a ground state with a very small interaction energy \( E_{\text{int}} \approx 1.5 \times 10^{-4} E_{\text{rec}} \), indicating anti-bunching between atoms, as expected for the Laughlin state. This state is separated from excited levels by an energy gap of maximum value \( 0.27 E_{\text{rec}} \) (reached for \( L = 0.63 \lambda/2 \)), without featuring a low-energy phonon branch. For longer circumferences \( L \geq \lambda/2 \), the low-energy excitations also exhibit very small interaction energy, as expected for edge excitations of the Laughlin state occurring for a number of low-energy orbitals \( N_{\text{orb}} > 2N_{\text{at}} - 1 \) \[9, 10\].

\[1\] De Bièvre, S. & Pulé, J. V. Propagating Edge States for a Magnetic Hamiltonian. In *Mathematical Physics Electronic Journal*, 39–55 (WORLD SCIENTIFIC, 2002).
\[2\] Bianco, R. & Resta, R. Mapping topological order in coordinate space. *Phys. Rev. B* **84**, 241106 (2011).
\[3\] Tang, Y., Sykes, A., Burdick, N. Q., Bohn, J. L. & Lev, B. L. $s$-wave scattering lengths of the strongly dipolar bosons Dy 162 and Dy 164. *Phys. Rev. A* **92** (2015).
\[4\] Haldane, F. D. M. Fractional Quantization of the Hall Effect: A Hierarchy of Incompressible Quantum Fluid States. *Phys. Rev. Lett.* **51**, 605–608 (1983).
\[5\] Cooper, N. R. Rapidly rotating atomic gases. *Adv. Phys.* **57**, 539–616 (2008).
\[6\] Regnault, N. & Jolicoeur, T. Quantum Hall Fractions in Rotating Bose-Einstein Condensates. *Phys. Rev. Lett.* **91**, 030402 (2003).
\[7\] Laughlin, R. B. Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations. *Phys. Rev. Lett.* **50**, 1395–1398 (1983).
\[8\] Rezayi, E. H. & Haldane, F. D. M. Laughlin state on stretched and squeezed cylinders and edge excitations in the quantum Hall effect. *Phys. Rev. B* **50**, 17199–17207 (1994).
\[9\] Wen, X.-G. Theory of the edge states in fractional quantum hall effects. *Int. J. Mod. Phys. B* **06**, 1711–1762 (1992).
\[10\] Soulé, P. & Jolicoeur, T. Edge properties of principal fractional quantum Hall states in the cylinder geometry. *Phys. Rev. B* **86**, 115214 (2012).