MOND plus classical neutrinos are not enough for cluster lensing

Priyamvada Natarajan1,2★ and Hongsheng Zhao3,4

1Department of Astronomy, Yale University, P. O. Box 208101, New Haven CT 06511-208101, USA
2Department of Physics, Yale University, P. O. Box 208120, New Haven CT 06520-208120, USA
3School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews KY16 9SS
4Dark Cosmology Centre, Niels Bohr Institute, University of Copenhagen, Juliane Maries Vej 30, 2100 Copenhagen, Denmark

Accepted 2008 June 7. Received 2008 May 17; in original form 2007 November 29

ABSTRACT

Clusters of galaxies offer a robust test bed for probing the nature of dark matter that is insensitive to the assumption of the gravity theories. Both Modified Newtonian Dynamics (MOND) and General Relativity (GR) would require similar amounts of non-baryonic matter in clusters as MOND boosts the gravity only mildly on cluster scales. Gravitational lensing allows us to estimate the enclosed mass in clusters on small (~20–50 kpc) and large (~several 100 kpc) scales independent of the assumptions of equilibrium. Here, we show for the first time that a combination of strong and weak gravitational lensing effects can set interesting limits on the phase-space density of dark matter in the centres of clusters. The phase-space densities derived from lensing observations are inconsistent with neutrino masses ranging from 2–7 eV, and hence do not support the 2 eV-range particles required by MOND. To survive, the most plausible modification for MOND may be an additional degree of dynamical freedom in a covariant incarnation.

Key words: cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

The Newtonian Poisson equation, if sourced by purely ordinary baryonic matter, seriously underpredicts the accelerations seen in a wide range of scales. An extra source term, customarily known as either dark matter or a scalar field source in Modified Newtonian Dynamics (MOND), is needed for consistency (Zhao & Famaey 2006; Famaey, Bruneton & Zhao 2007a). The precise nature of dark matter remains unknown within the Newtonian framework as well. The most viable dark-matter candidate is a fermionic, neutral particle that condensed from the thermal bath of the early Universe (Kolb & Turner 1990). While the detection of neutrinos confirmed the concept of a possible, particulate dark-matter candidate, it did very little in closing the wide gap between observed matter density versus what is required to explain lensing results and motions on galaxy scales. There remains the need for more exotic species of dark matter. In principle, galaxy scale observations ought to be sensitive to the free-streaming scale of these particles.

Another approach that has been followed is to constrain the phase-space densities of dark-matter particles. Studying the phase-space density of galaxy scale haloes derived from observational data, Sellwood (2000) noted that while the peak phase-space density of dark matter in galaxies is far from having a universal value, there does appear to be a favoured scale of a few keV. The same keV mass scale reappears in the recent discussions of the cores and dynamical friction in dwarf spheroidals such as Ursa Minor (Kleyna et al. 2005) and Fornax (Goerdt et al. 2006). The dark matter dominated Fornax dwarf spheroidal has globular clusters orbiting at roughly ~1 kpc from its centre. Goerdt et al. (2006) argue that if the dark-matter halo hosting Fornax has a cuspy density profile, then the globular clusters would sink to the centre from their current positions within a few Gyr, and presenting a puzzle as to why they do indeed survive at the present epoch. They show that this timing problem is alleviated by adopting a cored dark-matter halo. In that instance, using numerical simulations and analytic calculations they argue that the sinking time is many Hubble times; and the globulars would effectively halt at the core radius of the dark-matter halo. Using the current positions of the globulars Goerdt et al. (2006) therefore conclude that the Fornax dwarf spheroidal has a shallow inner-density profile with a finite core radius. This immediately implies that the dark-matter component is warm, with an upper limit to its mass of ~0.5 keV. Such a warm dark-matter candidate would suppress structure formation on small scales alleviating another problem the so-called substructure problem that seems to be endemic to cold dark matter (CDM) models. On the other hand, the flux anomalies of gravitationally lensed quasars argues for the existence of kpc clumps, too dense for keV warm dark-matter particles (Metcalfe & Zhao 2002; Miranda & Maccio 2007).

However, in competing gravity theories like MOND, the interpretation of dwarf galaxy scale rotation curves is very different due to the much larger MONDian gravity than predicted by Newtonian

★E-mail: priya@astro.yale.edu
theory. With a simple boost of gravity below a scale \(a_0 \sim 10^{-8} \text{ m s}^{-2}\), the need for dark matter on dwarf galaxy scales becomes much weaker if we assume MOND (Famaey et al. 2007a and references therein; Sanders & McGaugh 2002). Part of the reason that MOND is able to mimic CDM effectively is that there is a common acceleration scale \(g \sim a_0\) in the dark-matter cusp of the Navarro, Frenk & White profile (Xu, Wu & Zhao 2007), which appears on galaxy-to-galaxy cluster scales. This is exactly the scale on which MOND can supplement ordinary gravity, so it is not surprising that MOND and CDM often give comparable fits to data. However, there are still a few tough challenges for MOND (Famaey et al. 2007b; Klypin & Prada 2007) even on these scales. Additionally, Zhao (2005) noted that MOND would require globular clusters and dwarf galaxies to have the same size tidal radius, which appears to contradict current observations. So while there are ambiguities on galaxy scales, cluster scales are more promising to discriminate between the two theories as the effects of MOND are expected to be mild. This is due to the fact that MOND boosts the gravitational constant up only by a mild factor \(1/\mu(x)\), where \(\mu(x) = x/(1 + x) \sim 0.5\) in clusters of galaxies (Wu et al. 2007). The boosting is much larger in dwarf galaxies, by a factor of \(~11\), where \(x \sim 0.1\). In short, the evidence for dark matter on galaxy scales is weak in the context of alternatives like MOND. However, as we show below, more robust tests definitely derive from lensing in clusters of galaxies, in particular, the potent combination of strong and weak lensing observations.

Clusters of galaxies are the most massive and recently assembled structures in the Universe. In the context of the hierarchical growth of structure in a CDM dominated Universe, clusters are the repository of copious amounts of the dark matter. Gravitational lensing, predicted by Einstein’s theory of General Relativity (GR), is the deflection of light rays from distant sources by foreground mass structures is now detected in over a 100 clusters. Dramatic strong lensing occurs when there is a rare alignment of background sources with the dense central region of a foreground cluster. This produces highly distorted, magnified and multiple images of a single background source (Schneider, Ehlers & Falco 1992). However, more commonly, the observed shapes of background sources viewed via a foreground cluster lens are systematically elongated, in the so-called weak lensing regime. Coupling strong and weak lensing offers the most reliable probe of the distribution of dark matter on various cosmic scales (Blandford & Narayan 1992; Schneider et al. 1992; Mellier 2002). In particular, the combination of data from these two regimes offers unprecendented insight into the detailed mass distribution of clusters (Natarajan et al. 1998; Natarajan, Kneib & Smail 2002; Bradac et al. 2006; Jee et al. 2007). The lensing distortion in the shapes of background galaxies viewed through foreground mass distributions is independent of the dynamical state of the lens, therefore, unlike other methods for mass estimation there are fewer biases in lensing mass determinations.

Strong lensing studies of the inner regions of several clusters indicate that the dark-matter distribution can be represented by the combination of a smoothly distributed, extended component and smaller-scale clumps or subhaloes associated with luminous galaxies (Kneib et al. 1996; Natarajan & Kneib 1997; Natarajan et al. 1998). The smooth component has been detected using weak lensing techniques out to the turn-around radius (typically of the order of several Mpc) in clusters (Kneib et al. 2003; Broadhurst et al. 2005). The lensing derived density profile of the smooth component, and its agreement with profiles computed from high-resolution numerical simulations of structure formation in the Universe is currently well studied (Navarro et al. 1997; Navarro et al. 2004; Sand et al. 2004). In addition, the granularity of the dark-matter distribution associ-ated with individual galactic subhaloes holds important clues to the growth and assembly of clusters (Tyson, Kochanski & dell’Antonio 1998; Broadhurst et al. 2000; Limousin et al. 2007).

We exploit the technique of galaxy–galaxy lensing, which was originally proposed as a method to constrain the masses and spatial extents of field galaxies (Brainerd, Blandford & Smail 1996). The methodology has since been extended and developed to apply inside clusters (Natarajan & Kneib 1997; Natarajan et al. 1998, 2002; Natarajan, De Lucia & Springel 2007). Constraints on the masses of subhaloes associated with galaxies in clusters are also now available for several clusters.

The outline of this paper is as follows. In Section 2, we present the constraints obtained to date on MOND from clusters. In Section 3, we describe our method to derive the central densities of clusters and cluster galaxies, implications thereof for MOND are in Section 4 and we conclude with a discussion in Section 5.

2 WARMING UP TO MOND PREDICTIONS IN CLUSTERS

MOND was proposed by Milgrom (1983) as an alternative to Newtonian gravity, to explain galactic dynamics without the need for dark matter. Although current cosmological observations point to the existence of vast amounts of non-baryonic dark matter in the Universe, it is interesting and important to explore other alternatives. While this component is found to be distributed on a range of length scales, clusters of galaxies seem to be sites that are in fact dominated by dark matter at almost all radii.

In MOND, the gravitational force at large distances and small accelerations is modified when the acceleration is lower than a critical value, defined to be \(g_0 = 1 \times 10^{-8} \text{ cm s}^{-2}\). With one free parameter, namely the mass-to-light ratio, this formulation can explain rather well the rotation curves notably of low-surface brightness galaxies (McGaugh 2005; Gentile et al. 2007; McGaugh et al. 2007). Further tests of the MOND theory are needed in order to understand if there is a fundamental need for such a modulation of gravitational forces.

Recently, Bekenstein (2004) proposed a relativistic formulation of MOND, called TeVeS (Tensor–Vector–Scalar). This enabled the calculation of relativistic phenomena and in particular, the deflection of light rays propagating in a MOND Universe. With such a formulation in hand, the theory can be now be tested using the plethora of gravitational lensing observations currently available. Zhao et al. (2006) have examined the implications for galaxy scale lenses and find that while many candidates in the CASTLES survey are compatible with MOND lensing, there are outliers. Clusters of galaxies offer a more powerful probe as a range of lensing phenomena occurs in them.

The Bullet cluster (1E 0657-56) opened an interesting debate about the nature of dark matter in clusters. The clear separation of the lensing shear signal from the X-ray gas signal implies some form of dark matter in GR (Bradac et al. 2006; Clowe et al. 2006) as well as in MOND. Angus et al. (2007) showed that the data can be reconciled with 2 eV neutrinos, a neutrino mass limit that is allowed by current beta-decay experiments. Thus, they argue that this alleviates the need for more exotic dark matter in MOND and does not offer concrete proof for the existence of CDM as the dark-matter density in these clusters is very low, and can be easily explained by the phase-space density of neutrinos. Nevertheless, several preprints since (Angus, Famaey & Boute 2008) have started to reveal the inadequacy of neutrinos as a plausible constituent for
the Bullet cluster within the MOND framework. And as we show here the central densities for clusters and cluster galaxies estimated by combining strong and weak lensing constraints out to several hundred kpc cannot be explained by 2 eV neutrinos.

In a recent preprint, Takahashi & Chiba (2007) have explored the implications of weak lensing data of clusters for MOND. Using published weak lensing data for 3 Abell clusters and 42 Sloan Digital Sky Survey (SDSS) clusters, they conclude that MOND cannot explain the data unless a dark-matter halo is added. They find that dark matter is required as it cannot be accounted for with neutrinos with masses less than 2 eV.\footnote{Note that Cl 0024+16 one of the clusters studied here was part of the Takahashi & Chiba sample. However, the only constraint they employ is from the weak lensing data that does not provide a calibrated mass distribution.} In earlier work, a massive neutrino with a mass of \(\sim 2\) eV was invoked as dark matter to explain observational data (Sanders 2003; Skordis et al. 2006). In our work, reported here we consider the detailed mass distribution in six clusters spanning a redshift range of \(z = 0.2–0.6\) with very well-calibrated lensing models. Weak lensing observations alone do not give an absolute mass calibration, another mass estimator is needed for the normalization. However, combining strong and weak lensing data using measured spectroscopic redshifts for the multiple images enables us to construct calibrated mass models.

Combining strong and weak lensing data from the Hubble Space Telescope’s (HST) Wide-Field Planetary Camera (WFPC-2), with a large complement of ground based spectroscopy, we have constructed high-resolution mass models for Abell 2218, Abell 2390, AC 114, Cl 2244-02, Cl 0024+16 and Cl 0054-27. This enables us to compute the central density of these clusters and that of the typical subhalo that hosts an early-type \(L^*\) galaxy in these clusters.

Note that while in the following sections we will only construct models with Newtonian–Einsteian gravity, our results are applicable to MOND due to the simple fact that all particles accelerate inside clusters with \(g \sim (0.5\sigma_0–3\sigma_0)\) in general.\footnote{At the Einstein ring radius of \(r_E \sim 50\) kpc, and \(\sigma \sim 500\) km s\(^{-1}\), the acceleration can be estimated by \(g \sim 2\sigma^2/r_E \sim 3\sigma_0\). So in the region of interest we are in moderate or strong gravity, hence MOND effects are mild.} Therefore, MOND effects are always mild, within a factor of 2 at most. Wu et al. (2007) showed that the MOND gravity around a galaxy inside a cluster should have nearly Newtonian or Keplerian behaviour. Hence we will model lensing in the Newtonian–Einsteain framework, but extrapolate our conclusions to MOND.

\section{Construction of Mass Models from Lensing Data}

In this section, we briefly outline the method used to derive constraints on the mass distribution of clusters and galaxies in clusters. The mass distribution in clusters is partitioned into a large-scale smooth component of dark matter and small scale subhaloes that are associated with the locations of bright-cluster galaxies.

To quantify the lensing distortion induced, the large-scale smooth component and the individual galaxy-scale haloes are modelled self-similarly using the Pseudo-Isothermal Elliptical Mass Distribution (PIEMD; Kassiola & Kovner 1993) profile with,

\[
\Sigma(R) = \frac{\Sigma_0}{1 - r_0/r_1} \left( \frac{1}{\sqrt{r_0^2 + R^2}} - \frac{1}{\sqrt{r_1^2 + R^2}} \right),
\]

where \(\Sigma_0\) is the central mass density, \(r_0\) and \(r_1\) are the core and outer radii, respectively.\footnote{In order to relate the light distribution in cluster galaxies to key parameters of the mass model of subhaloes, we adopt a set of physically motivated scaling laws derived from observations (Brainerd et al. 1996; Natarajan & Kneib 1997; Limousin, Kneib & Natarajan 2005):

\[
\sigma_0 = \sigma_0^\star \left( \frac{L/L^\star}{\langle L/L^\star \rangle} \right)^{1/4}, \quad r_0 = r_0^\star \left( \frac{L/L^\star}{\langle L/L^\star \rangle} \right)^{1/2}, \quad r_1 = r_1^\star \left( \frac{L/L^\star}{\langle L/L^\star \rangle} \right)^2.
\]

The total mass \(M\) enclosed within an aperture \(r_a\) and the total mass-to-light ratio \((M/L)\) then scale with the luminosity as follows for the early-type galaxies:

\[
M_d \propto \sigma_0^2 r_d \left( \frac{L/L^\star}{\langle L/L^\star \rangle} \right)^{a+1/2}, \quad M/L \propto \sigma_0^2 r_\star \left( \frac{L/L^\star}{\langle L/L^\star \rangle} \right)^{a-1/2},
\]

where \(a\) tunes the size of the galaxy halo. These scaling laws are empirically motivated by the Faber–Jackson relation for early-type galaxies (Brainerd et al. 1996). For late-type cluster members when the data is available (at the present time only for the cluster Cl 0024+16), we use the analogous Tully–Fisher relation to obtain scalings of \(\sigma_0\) and \(r_0\) with luminosity. The empirical Tully–Fisher relation has significantly higher scatter than the Faber–Jackson relation (see Courteau et al. 2007 and Jorgensen et al. 2006). In this analysis we do not take the scatter into account while employing these scaling relations. We assume these scaling relations and recognize that this could ultimately be a limitation but the evidence at hand supports the fact that mass traces light efficiently both on cluster scales (Kneib et al. 2003) and on galaxy scales (McKay et al. 2001; Wilson, Kaiser & Luppino 2001). The details of the redshift distribution and intrinsic ellipticity distribution assumed for this analysis (and for most lensing analysis in fact) are described in detail in Natarajan et al. (2007). While the core radius of the large-scale smooth components is constrained from observations, the core radii of the individual cluster galaxies cannot be constrained with current data. Therefore, in the modelling, we fix the core radius of a dark-matter subhalo that hosts an \(L^*\) galaxy to be \(0.1\) kpc. This assumption will not be of consequence in the determination of the central density in cluster galaxies as discussed below.

Parameters that characterize both the global components and the perturbers are optimized, using the observed strong lensing features – positions, magnitudes, geometry of multiple images and measured spectroscopic redshifts, along with the smoothed shear field as constraints. With the parameterization presented above, we optimize and extract values for the central velocity dispersion and the aperture scale \((\sigma_0, r_0)\) for a subhalo hosting a fiducial \(L^*\) cluster galaxy. We note here that as argued above MOND is unimportant inside these extremely dense and massive lensing clusters therefore cannot be used as a criterion to question these scaling relations. The...}
scaling relation used in this paper the Faber–Jackson relation is well established observationally in clusters and in lensing clusters. The Faber–Jackson relation is a projection of the Fundamental Plane and this is established observationally in clusters over a range of redshifts (some recent references, Papo, Djorgovski & de Carvalho 1998; Fritz et al. 2005; Holden et al. 2005; Jorgensen et al. 2006). The Tully–Fisher relation is also an empirical relation detected out to these redshifts (see Courteau et al. 2007 for the most recent data).

Maximum-likelihood analysis is used to obtain significance bounds on these fiducial parameters that characterize a typical $L^*$ subhalo in the cluster. The likelihood function of the estimated probability distribution of the source ellipticities is maximized for a set of model parameters, given a functional form of the intrinsic ellipticity distribution measured for faint galaxies. For each ‘faint’ galaxy $j$, with measured shape $\tau_{\text{obs}}$, the intrinsic shape $\tau_{\text{S}}$ is estimated in the weak regime by subtracting the lensing distortion induced by the smooth cluster models and the galaxy subhaloes,

$$\tau_{\text{S}} = \tau_{\text{obs}} - \Sigma_{i}^{N_{\text{f}} \gamma_{p}} - \Sigma_{\text{gal}} \gamma_{s},$$

(7)

where $\Sigma_{i}^{N_{\text{f}} \gamma_{p}}$ are the shear contribution at a given position $j$ from $N_{\text{f}}$ perturbers. This entire inversion procedure is performed numerically using code developed that builds on the ray-tracing routine LENSTOOL written by Kneib (1993).\footnote{This software is publicly available at http://www.oamp.fr/cosmology/lenstool/} This machinery accurately takes into account the non-linearities arising in the strong lensing regime. Using a well-determined ‘strong lensing’ model for the inner regions along with the shear field and assuming a known functional form for $p(\tau_{S})$ the probability distribution for the intrinsic shape distribution of galaxies in the field, the likelihood for a guessed model is given by

$$L(\sigma_{0}, r_{c}) = \prod_{j}^{N_{\text{gal}}} p(\tau_{S}),$$

(8)

where the marginalization is done over $<\sigma_{0}, r_{c}>$. We compute $L$ assigning the median redshift corresponding to the observed source magnitude for each arclet. The best-fitting model parameters are then obtained by maximizing the log-likelihood function $l$ with respect to the parameters $\sigma_{0}$ and $r_{c}$. Note that the parameters that characterize the smooth component are also simultaneously optimized. The results of this analysis for our sample of clusters, that is values of $<\sigma_{0}, r_{c}>$ are presented in earlier papers (Natarajan et al. 2002, 2007).

In summary, the basic steps of our analysis therefore involve lens inversion, modelling and optimization, which are done using the LENSTOOL software utilities (Kneib 1993). These utilities are used to perform the ray tracing from the image plane to the source plane with a specified intervening lens. This is achieved by solving the lens equation iteratively, taking into account the observed strong lensing features, positions, geometry and magnitudes of the multiple images. We also include a constraint on the location of the critical line (between two mirror multiple images) to tighten the optimization. In addition to the likelihood contours, the reduced $\chi^{2}$ for the best-fitting model is also found to be robust.

In addition, our reconstructions enable us to derive the mass function of dark-matter subhaloes inside these clusters. We find very good agreement between the mass functions predicted by the $\Lambda$CDM model derived from high-resolution cosmological simulations and those computed via the above method from lensing observations. More details on this comparison can be found in Natarajan & Springel (2004) and Natarajan et al. (2007). So it is interesting to note that the lensing observations are in consonance with the predictions of the concordance cosmological model and require dark matter.

For the large-scale cluster, we derive the central density from total mass within a few hundred kpc, well outside the Einstein radius. We emphasize here that the lensing analysis also provides an estimate of the total mass enclosed within $r_{c} \sim 20 - 50$ kpc for subhaloes, well outside the typical Einstein radius of $\sim 1-5$ kpc. It is within the Einstein radius that the contribution of baryons dominates. Since our estimates probe the mass well beyond the Einstein radius, the bulk of the mass detected here is dark matter on both cluster and cluster galaxy scales as we demonstrate below.

### 4 ESTIMATING CENTRAL DENSITIES OF CLUSTERS AND GALAXIES IN CLUSTERS FROM LENSLING

The mass models are well calibrated and do not suffer from the ambiguity of the mass-sheet degeneracy as more than two sets of multiple images with measured redshifts are used in constraining the mass of each cluster. The best-fitting mass models enable us to compute the central density for the clusters. For the PIEMD model:

$$\rho(r)_{\text{clus}} = \frac{\rho_{0}}{(1 + r^{2}/r_{0}^{2})(1 + r^{2}/r_{c}^{2})},$$

(9)

where $r_{0}$ is the core radius and $r_{c}$ is the outer truncation radius as before in equations (1)-(4), derived from the best-fitting mass model. The core radius is an additional parameter in these models, which is of consequence in the computation of central densities. For the large-scale cluster component, the core-radius $r_{c}$ is also optimized in the likelihood analysis. The best-fitting values for the core radii for these clusters are listed in Table 1. The error bars in the calculated central density (plotted in Fig. 1) are derived by propagating the errors on the quantities shown in Table 1 and correspond to $3\sigma$ error bars. We note here that due to effects like the anisotropy $\sigma$ can be uncertain by a factor of 0.7–1.4 and the computed density by factor of 2.

For the cluster galaxies, the likelihood method constrains the total mass enclosed ($M_{\text{gal}} \sim \sigma_{0}^{2} r_{c}$) within an aperture radius $r_{c}$. We compute the average smoothed density simply using:

$$\langle \rho \rangle = \frac{3 M_{\text{gal}}}{4\pi r_{c}^{3}}.$$

(10)

These estimates are plotted in Fig. 1. This is a conservative estimate of the central density, as no assumption is made for the detailed density-profile shape, as it cannot be constrained on these scaled from lensing observations. Note that the compact core radius

### Table 1. Parameters that define the mass models of the subhaloes for the lensing clusters. For each cluster the central velocity dispersion of a subhalo that hosts an $L^*$ galaxy ($\sigma_{0}$ in km s$^{-1}$), the aperture radius $r_{c}$ (in kpc), the stellar mass-to-light ratio in the V-band interior to $r_{c}$ in solar units and the density of the large-scale cluster component $\rho_{\text{clus}}(r = r_{0})$ evaluated at the core radius $r_{0}$.

| Cluster | $z$ | $\sigma_{0}$ (km s$^{-1}$) | $r_{c}$ (kpc) | $M/L_{V}$ (M$_{\odot}$/L$_{\odot}$) | $\rho_{\text{clus}}(r = r_{0})$ (10$^{-3}$ M$_{\odot}$ pc$^{-3}$) |
|---------|----|----------------|-------------|-------------------------------|---------------------------------|
| A2218   | 0.17 | 180 ± 10   | 40 ± 12    | 5.8 ± 1.5                   | 3.95 |
| A2390   | 0.23 | 200 ± 15   | 18 ± 5     | 4.2 ± 1.3                   | 16.95 |
| AC114   | 0.31 | 192 ± 35   | 17 ± 5     | 6.2 ± 1.4                   | 9.12  |
| CI2244–02 | 0.33 | 110 ± 7    | 55 ± 12    | 3.2 ± 1.2                   | 3.52 |
| CI0024+16 | 0.39 | 125 ± 7    | 45 ± 5     | 2.5 ± 1.2                   | 3.63 |
| CI0054–27 | 0.57 | 230 ± 18   | 20 ± 7     | 5.2 ± 1.4                   | 15.84 |
mass difference of $\sqrt{\Delta m^2} \sim 1$eV; for example two sterile neutrinos with masses $m_1 = 7$eV and $m_2 = 6.9$eV. Note the cosmic abundance of all neutrino species is constrained by

$$\frac{\Omega_{\gamma} + \Omega_{\mu} + \Omega_{\tau} + \Omega_{\nu}}{0.125(H0/70)^2} = \frac{m_{\nu_e} + m_{\mu} + m_{\tau} + m_{1} + m_{2}}{6eV}. \tag{12}$$

The resultant phase-space densities using Sanders (2003) are plotted as lines in Fig. 1. The dashed line corresponds to a Universe with just three species of $m_{\nu} = 2$eV ordinary (electron, muon and tau) neutrinos and a Universe with 0.08 eV ordinary and two sterile neutrino species with masses of 7 and 6.9 eV, respectively (solid line style).

The total non-baryonic neutrino fraction $\Omega_{\nu_{e}} + \Omega_{\nu_{\mu}} + \Omega_{\nu_{\tau}} + \Omega_{\nu_{\nu}}$ ranges from 0.125 to 0.3 (the second model). These numbers are in broad agreement with the cosmology proposed by Skordis et al. (2006) to account for cosmic microwave background, without violating current limits on electron neutrinos. However, as seen in Fig. 1, both possibilities fall short of explaining the lensing data, in fact to explain the data with neutrinos $\Omega_{\nu} > 0.3$ are required.

### 4.1 The baryonic contribution to cluster and cluster galaxy central densities

The baryonic matter content of galaxy clusters is dominated by the X-ray emitting intracluster gas. The gas mass exceeds the mass of optically luminous material by a factor of $\sim 6$ (White et al. 1993; Fukugita, Hogan & Peebles 1998). As the emissivity of the X-ray emitting gas is proportional to the square of its density, the gas-mass profile can be accurately determined from X-ray data. Measuring the total-mass profile is required to estimate the gas-mass fraction, and is more challenging to determine as it requires the direct measurement of the gas-temperature profile and the assumption of hydrostatic equilibrium for the gas. Observations of nearby and intermediate redshift clusters in the luminosity range $(L_{X,0.1-2.4} \lesssim 5 \times 10^{44} h_{70}^{-2} \text{ erg s}^{-1})$ and the temperature range $kT > 5$ keV, the average mass fraction in stars (in galaxies and intracluster light combined) $f_{\text{star}} \sim 0.16 \sqrt{H_0} f_{\text{gas}}$ (Balogh et al. 2001; Lin & Mohr 2004).

Fitting the results for a total of 68 clusters (Allen et al. 2004, 2008) derive the gas fraction, $f_{\text{gas}} = 0.1104 \pm 0.0016$. These observational determinations of $f_{\text{gas}}$ suggest that the baryonic contribution to the central densities computed above is of the order of $\sim 11$ per cent. This is also in agreement with recent results from lensing and X-ray analysis reported in Takahashi & Chiba (2007). Therefore, our estimates of the central density from weak and strong gravitational lensing reported above do primarily reflect the dark-matter density. The central density plotted in Fig. 1 is now corrected by the estimated factor of 0.89 to reflect that of the dark-matter component alone.

Below we describe the estimation of the baryonic contribution to the central density estimates for cluster galaxies. The results of the maximum likelihood analysis in addition to providing a constraint on the mass enclosed within an aperture also provide a constraint on the total mass-to-light ratio. To separate the contribution of the baryonic component, we estimate the stellar masses and subtract them from the total aperture masses. To do so, the stellar mass-to-light ratios are computed in the $V$ band. Using stellar population synthesis models (Bruzual & Charlot 2003), we estimate the stellar mass.

4 We do not differentiate between the physical mass and the thermal mass since they are nearly the same for eV range sterile neutrinos, which could be produced non-thermally.

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**Figure 1.** The central densities of clusters and cluster galaxies derived from lensing observations. Strong and weak gravitational lensing are combined to obtain constraints on the dark-matter subhaloes associated with cluster galaxies. The cluster data points are shown as solid circles, the solid squares and solid triangle are values derived from the aperture masses derived from galaxy–galaxy lensing studies, for early-type cluster galaxies and late-types, respectively. Note that the baryonic contribution has been subtracted in the points plotted above. Lines show the maximum phase-space density for two species of sterile neutrinos with masses 7eV and 6.9eV + 3 species of ordinary neutrinos with masses of 0.08eV (solid line) and for only three species of ordinary neutrinos (dashed line) with a mass of 2eV each. The dashed line is for $M_\nu^4 = 4.39$ and the solid line is for $M_\nu^4 = 291.73$. The central densities $\rho_0 = \rho(r = 0)$ are plotted for the clusters and the averaged central density $<\rho>$ is plotted for the cluster galaxies.

assumed above for galaxy scale subhaloes, taken to be 0.1 kpc, is not used in the above estimate for the central density.

Considering X-ray clusters in the context of MOND, a massive neutrino with a mass of $\sim 2$eV has been invoked by several authors to explain observational data (Sanders 2003; Skordis et al. 2006). Below, we derive an independent constraint from combined strong and weak lensing data of clusters on the mass of such neutrinos using estimates of phase-space densities. Neutrino oscillation experiments provide limits on the mass differences between the three species ($\nu_e, \nu_\mu, \nu_\tau$) of $\Delta m \sim \sqrt{10^{-3}}$eV$^2$ (Fukuda et al. 1998). Considering massive neutrinos with masses well above $\Delta m$, the maximum density of the neutrino halo can be estimated using phase-space arguments (Tremaine & Gunn 1979). Comparison with the maximum phase-space density of a neutrino halo are evaluated via:

$$\rho_{\nu,\text{max}}^2 \times 10^{-5} \text{M}_\odot \text{pc}^{-3} \left(\frac{\sigma}{400\text{ km s}^{-1}}\right)^{-3} = M_\nu^4;$$

$$M_\nu^4 = \left(\frac{m_{\nu_e}}{2\text{eV}}\right)^4 + \left(\frac{m_{\nu_\mu}}{2\text{eV}}\right)^4 + \left(\frac{m_{\nu_\tau}}{2\text{eV}}\right)^4 + \left(\frac{m_1}{2\text{eV}}\right)^4 + \left(\frac{m_2}{2\text{eV}}\right)^4. \tag{11}$$

In computing the phase-space density, we have used the formula of Sanders (2003) to be conservative instead of Sanders (2008); the latter gives a factor of 3 lower densities for the same neutrinos. Here, we have allowed for two species of sterile neutrinos. This has been invoked to explain the results of neutrino oscillations detected by Liquid Scintillator Neutrino Detector experiment with...
mass-to-light ratios for template early-type galaxies at the redshifts of these clusters. For the clusters studied here the ratio of the total aperture mass-to-light ratio to the stellar mass-to-light ratio is the V band is a factor of 2 and 3. Using these derived stellar mass-to-light ratios and combining with the luminosity, we calculate the total stellar mass within the aperture in these cluster galaxies. We then subtract this from the total mass within the aperture inferred from lensing. Doing so conservatively, we estimate that on average at most 33 per cent of the contribution to the central density derives from baryons. Using the computed stellar mass-to-light ratios, we scale the values plotted in Fig. 1 for cluster galaxies in each cluster accordingly to derive the dark-matter densities. We derive the equivalent temperature (in keV) for clusters and cluster galaxies from their velocity dispersions via the relation:

\[ T = \left( \frac{\sigma}{400 \text{ km s}^{-1}} \right)^2 \text{ keV}. \] (13)

In summary, we find that the central density of massive, lensing clusters with well calibrated mass models precludes the possibility of dark matter as 2 eV neutrinos that are required by MOND. Moreover, the phase-space density calculated for galaxies in clusters also appears to be inconsistent with that of two species of sterile neutrinos. In general, our results are in line with ΛCDM that postulates the existence of more massive dark-matter particles than neutrinos.

5 CURRENT LIMITS ON NEUTRINO MASSES AND WAYS OUT FOR MOND

Have we detected the limiting phase-space density of the dark-matter particle? Is the particle inconsistent with neutrinos? The mass of ordinary neutrinos is still unknown although it must be non-zero. The mass of electron neutrinos is measured in tritium β decay experiments. The decay results in a 3-helium, electron and an electron antineutrino. If neutrinos have non-zero mass, the spectrum of the electrons is deformed at the high-energy part, that is the neutrino mass determines the maximum energy of emitted electrons. To be exact, the experiments measure the neutrino mass squared. Two running experiments, Mainz and Troitsk, constrain the neutrino mass to be above \( m \sim 0.05 \) eV and below 2.2 eV. The upper limit will get tighter once the KATRIN experiment starts in 2009–2010. The KATRIN experiment is expected to push the limit for electron neutrino masses down by an order of magnitude. Our results are barely consistent with neutrinos of such low mass. Therefore, the cluster lensing data clearly rules out the possibility of these low-mass neutrinos constituting the bulk of dark matter as required by MOND.

There are a few ways out for MOND to escape exclusion here.

(i) We have assumed negligible MOND corrections inside clusters. However, there could be regions where the \( \mu \) is much smaller than unity, where classical MOND correction might be important. This is however a priori unlikely because clusters are in the strong or moderate gravity regime with \( g \geq a_0 \), hence \( \mu = g/(g + a_0) \sim 0.5–1 \).

(ii) We have assumed a one-to-one history-independent relation between mass distribution and gravity, as in GR and as in classical MOND. This is not the case in recent covariant incarnations of MOND, which has (almost always) an additional fluid-like vector field in vacuum, hence its stress energy tensor too bends the metric (Bekenstein 2004; Zhao 2007; Zlosnik, Ferreira & Starkman 2007). This fluid is history dependent, except in systems of equilibrium, like-spiral galaxies. With this the Vector-for-Λ model (Zhao 2007) was able to match the ΛCDM cosmology, especially the vacuum field can explain the tiny amplitude of the cosmological constant \( \Lambda \sim c^2/G \). In these covariant models, gravity is determined by the instantaneous distribution of baryons and the (dark) fluid; the latter tracks the former but with a phase-lag, hence resembling the collisionless dark-matter fluid. We note that in the covariant V-Lambda fluid, lensing works as in GR and there are no anisotropic stress corrections. This exit appears plausible for MOND because classical MOND is non-covariant, and unless it is given 3 or 4 vector degrees of freedom, lensing cannot be done properly (i.e. getting the factor of 2 for light deflection, and staying covariant).

(iii) Neutrinos might have right-handed partners, sterile neutrinos, whose mass is still poorly constrained by experiments, for example the latest MiniBooNe experiment. It is foreseeable although not very natural for sterile neutrinos with mass above \( 7 \) eV to be partners to MOND gravity.

(iv) Aside from neutrinos it has been argued recently by Angus et al. (2008) that cluster dark matter could be baryonic in the form of cold gas, analogous to the suggestion by Pfenniger & Combes (1994) on galaxy scales. However, there is no convincing observational evidence to support this claim at the present time.

Examining the detailed feasibility of these options for a safe exit for MOND are beyond the scope of this paper. In conclusion, using combined strong and weak lensing data in clusters we constrain the phase-space density of the dark matter. Our current results rule out neutrinos on eV scales as dark matter from lensing constraints derived from galaxies in clusters on \( \sim 20–50 \) kpc scales.

Our constraints on phase-space densities in the mild-acceleration regime in galaxy cluster environments are much less-dependent on the assumption of gravity in comparison to the case of isolated galaxies, a regime where there could be appreciable MOND effects (Sellwood 2000). Assuming GR, current data unfortunately do not yet suggest a coherent picture of the true phase density of non-baryonic particles that constitute dark matter. While the required finite core density of dwarf spheroidal Galactic satellites favours sub-keV particles, such as sterile neutrino-like warm-dark matter (Gilmore et al. 2008), data from the anomalous flux ratios of gravitational lensed radio quasars (Miranda & Maccio 2007; Metcalf & Zhao 2002) and the flux power-spectrum of SDSS Lyman α systems favour cold dark-matter clumps, which would be erased by the streaming motions of sub-keV sterile neutrinos or warm-dark matter in general. Our results suggest that, in both MOND and in GR ordinary neutrinos and any sterile neutrinos of 2–7 eV are insufficient to explain the gravitational perturbations on scales of \( \sim 20–50 \) kpc that we observe with galaxy cluster lensing data. Nevertheless, what gives neutrino the tiny mass is still an unsolved fundamental problem in physics, and in some theories it is linked to the unsolved problem of dark energy (Mota et al. 2008). Most recently, it has been proposed that MOND effects can come from a mass-varying neutrino with a non-trivial coupling of the neutrino spin with the metric in Einsteinian gravity (Zhao 2008, and references therein). Neutrinos could cluster significantly; there are claims of tantalizing evidence for sterile neutrinos of 11 eV in the Wilkinson Microwave Anisotropy Probe5 data (Angus 2008).

ACKNOWLEDGMENTS

We thank the Dark Cosmology Centre, Niels Bohr Institute, for their hospitality. HSZ acknowledges partial support from UK STFC Advanced Fellowship and National Natural Science Foundation of China (NSFC under grant No. 10428308. We acknowledge...
especially Benoit Famaey, Huanyuan Shan, Martin Feix, Subir Sarkar and Marceau Marleau for helpful discussions on MOND, neutrinos and lensing. Jerry Sellwood, Garry Angus and Benoit Famaey are thanked for comments on the manuscript.

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