Impurity scattering in metallic carbon nanotubes with superconducting pair potentials

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Abstract

Effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes are studied theoretically. The backward scattering of electrons vanishes in the normal state. In the presence of the superconducting pair correlations, the backward scatterings of electron- and hole-like quasiparticles vanish, too. The impurity gives rise to backward scatterings of holes for incident electrons, and it also induces backward scatterings of electrons for incident holes. Negative and positive currents induced by such the scatterings between electrons and holes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur. Relations with experiments are discussed.

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I. Introduction

Recent investigations [1,2] show that the superconducting proximity effect occurs when the carbon nanotubes contact with conventional superconducting metals and wires. The superconducting energy gap appears in the tunneling density of states below the critical temperature $T_c$. On the other hand, the recent theories discuss the nature of the exceptionally ballistic conduction [3] and the absence of backward scattering [4] in metallic carbon nanotubes with impurity potentials at the normal states (not in the superconducting states). Such the peculiar properties might be related with the experimental realization of nanostructures with quantum electronic conductions [5,6]. However, impurity scattering properties in the presence of superconducting pair potential is not investigated theoretically so much. Therefore, it is urgent to study how the peculiar scattering properties in the normal states will change when the superconducting proximity effects occur in the metallic carbon nanotubes.

In this paper, we study the effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes. We use the continuum $k \cdot p$ model for the electronic states in order to consider scattering processes in the normal state and also in the state with the superconducting pair potential. We find that the scattering matrix is diagonal and the off-diagonal matrix elements vanish in the normal state. Such the absence of the backward scattering has been discussed recently [4], too. Next, we consider effects of the superconducting pair correlations. We find the absence of backward scatterings of electron- and hole-like quasiparticles in the presence of superconducting proximity effects. Off-diagonal $2 \times 2$ submatrix has the diagonal matrix elements whose magnitudes are proportional to $\Delta$. Negative and positive currents induced by such the scatterings between electrons and holes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur. This finding is interesting in view of the recent experimental progress of the superconducting proximity effects of carbon nanotubes [1,2].
In the next section, we explain our model and introduce propagators of the normal state and the Nambu representation. In Sec. III, we consider impurity scattering in the normal state. In Sec. IV, we discuss effects of superconducting pair correlations. The summary is given in Sec. V.

II. Model

We will study the metallic carbon nanotubes with the superconducting pair potential. The model is as follows:

\[ H = H_{\text{tube}} + H_{\text{pair}}, \]  

(1)

\( H_{\text{tube}} \) is the electronic states of the carbon nanotubes, and the model based on the \( k \cdot p \) approximation [4,7] represents electronic systems on the continuum medium. The second term \( H_{\text{pair}} \) is the pair potential term owing to the proximity effect.

The hamiltonian by the \( k \cdot p \) approximation [4,7] in the secondly quantized representation has the following form:

\[ H_{\text{tube}} = \sum_{\mathbf{k},\sigma} \Psi_{\mathbf{k},\sigma}^\dagger E_{\mathbf{k}} \Psi_{\mathbf{k},\sigma}, \]  

(2)

where \( E_{\mathbf{k}} \) is an energy matrix:

\[
E_{\mathbf{k}} = \begin{pmatrix}
0 & \gamma(k_x - ik_y) & 0 & 0 \\
\gamma(k_x + ik_y) & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma(k_x + ik_y) \\
0 & 0 & \gamma(k_x - ik_y) & 0
\end{pmatrix},
\]  

(3)

\( \mathbf{k} = (k_x, k_y) \), and \( \Psi_{\mathbf{k},\sigma} \) is an annihilation operator with four components:

\[ \Psi_{\mathbf{k},\sigma}^\dagger = (\psi_{\mathbf{k},\sigma}^{(1)} \uparrow, \psi_{\mathbf{k},\sigma}^{(2)} \uparrow, \psi_{\mathbf{k},\sigma}^{(3)} \uparrow, \psi_{\mathbf{k},\sigma}^{(4)} \uparrow). \]  

Here, the first and second elements indicate an electron at the A and B sublattice points around the Fermi point \( K \) of the graphite, respectively. The third and fourth elements are an electron at the A and B sublattices around the Fermi point \( K' \). The quantity \( \gamma \) is defined as \( \gamma \equiv (\sqrt{3}/2)a\gamma_0 \), where \( a \) is the bond length of the graphite plane and...
\( \gamma_0 \approx 2.7 \text{ eV} \) is the resonance integral between neighboring carbon atoms. When the above matrix is diagonalized, we obtain the dispersion relation \( E_{\pm} = \pm \gamma \sqrt{k_x^2 + \kappa_{\nu \phi}^2(n)} \), where \( k_x \) is parallel with the axis of the nanotube, \( \kappa_{\nu \phi}(n) = (2\pi/L)(n + \phi - \nu/3) \), \( L \) is the circumference length of the nanotube, \( n = (0, \pm 1, \pm 2, ...) \) is the index of bands, \( \phi \) is the magnetic flux in units of the flux quantum, and \( \nu = 0, 1, \text{ or } 2 \) specifies the boundary condition in the \( y \)-direction. The metallic and semiconducting nanotubes are characterized by \( \nu = 0 \) and \( \nu = 1 \) (or 2), respectively. Hereafter, we consider the case \( \phi = 0 \) and the metallic nanotubes \( \nu = 0 \).

The second term in Eq. (1) is the pair potential:

\[
H_{\text{pair}} = \Delta \sum_{\mathbf{k}} (\psi_{1 \uparrow}^{\dagger} \psi_{-1 \downarrow}^{\dagger} + \psi_{-1 \uparrow}^{\dagger} \psi_{1 \downarrow}^{\dagger} + \psi_{2 \uparrow}^{\dagger} \psi_{-2 \downarrow}^{\dagger} + \psi_{-2 \uparrow}^{\dagger} \psi_{2 \downarrow}^{\dagger} + \psi_{3 \uparrow}^{\dagger} \psi_{-3 \downarrow}^{\dagger} + \psi_{-3 \uparrow}^{\dagger} \psi_{3 \downarrow}^{\dagger} + \cdots + h.c.)
\]

where \( \Delta \) is the strength of the superconducting pair correlation. In principle, \( \Delta \) can have spatial dependence. However, we assume the constant \( \Delta \) for simplicity. This corresponds to the case that the spatial extent of the regions where the proximity effect occurs is as long as the superconducting coherence length.

The propagator of the electrons on the nanotube is defined in the matrix form:

\[
G(\mathbf{k}, \tau) = -\langle T_{\tau} \Psi_{\mathbf{k}, \sigma}(\tau) \Psi_{\mathbf{k}, \sigma}^{\dagger}(0) \rangle,
\]

where \( T_{\tau} \) is the time ordering operator with respect to the imaginary time \( \tau \) and \( \Psi_{\mathbf{k}, \sigma}(\tau) = \exp(H\tau) \Psi_{\mathbf{k}, \sigma} \exp(-H\tau) \). The Fourier transform of \( G \) is calculated as:

\[
G^{-1}(\mathbf{k}, i\omega_n) = \begin{pmatrix} G_K^{-1} & 0 \\ 0 & G_{K'}^{-1} \end{pmatrix},
\]

where \( \omega_n = (2n + 1)\pi T \) is the odd Matsubara frequency for fermions. The components of \( G \) are written explicitly:

\[
G_K^{-1}(\mathbf{k}, i\omega_n) = \begin{pmatrix} i\omega_n & -\gamma(k_x - ik_y) \\ -\gamma(k_x + ik_y) & i\omega_n \end{pmatrix},
\]

and

\[
G_{K'}^{-1}(\mathbf{k}, i\omega_n) = \begin{pmatrix} i\omega_n & -\gamma(k_x + ik_y) \\ -\gamma(k_x - ik_y) & i\omega_n \end{pmatrix}.
\]
In order to describe the superconducting pair correlations, it is useful to introduce the Nambu representation:

$$\tilde{\Psi}_K^\dagger(k) = (\psi_{k,\uparrow}^{(1)}, \psi_{k,\uparrow}^{(2)}, \psi_{-k,\downarrow}^{(1)}, \psi_{-k,\downarrow}^{(2)})$$ and $$\tilde{\Psi}_{K'}^\dagger(k) = (\psi_{k,\uparrow}^{(3)}, \psi_{k,\uparrow}^{(4)}, \psi_{-k,\downarrow}^{(3)}, \psi_{-k,\downarrow}^{(4)})$$.

The propagator with the pair correlation is defined in the matrix form:

$$\tilde{G}_K(k, \tau) = -\langle T_\tau \tilde{\Psi}_K(k, \tau) \tilde{\Psi}_K^\dagger(k, 0) \rangle$$, (9)

and

$$\tilde{G}_{K'}(k, \tau) = -\langle T_\tau \tilde{\Psi}_{K'}(k, \tau) \tilde{\Psi}_{K'}^\dagger(k, 0) \rangle$$.

Their Fourier transforms are calculated as:

$$\tilde{G}_K^{-1}(k, i\omega_n) = \begin{pmatrix}
i\omega_n & -\gamma(k_x - ik_y) & -\Delta & 0 \\
-\gamma(k_x + ik_y) & i\omega_n & 0 & -\Delta \\
-\Delta & 0 & i\omega_n & -\gamma(-k_x + ik_y) \\
0 & -\Delta & -\gamma(-k_x - ik_y) & i\omega_n
\end{pmatrix}$$, (11)

and

$$\tilde{G}_{K'}^{-1}(k, i\omega_n) = \begin{pmatrix}
i\omega_n & -\gamma(k_x + ik_y) & -\Delta & 0 \\
-\gamma(k_x - ik_y) & i\omega_n & 0 & -\Delta \\
-\Delta & 0 & i\omega_n & -\gamma(-k_x - ik_y) \\
0 & -\Delta & -\gamma(-k_x + ik_y) & i\omega_n
\end{pmatrix}$$.

The dispersion relation of the quasiparticles becomes $$E = \pm \sqrt{\gamma^2(k_x^2 + k_y^2) + \Delta^2}$$. The dispersions with plus and minus signs are degenerate two fold, respectively.

We note that there are several characteristic parameters of metallic carbon nanotubes. The total carbon number $$N_s$$ is given by $$N_s = A \times L / (\sqrt{3}a^2/2) \times 2 = 4AL/\sqrt{3}a^2$$, where $$A$$ is the length of the nanotube, and $$\sqrt{3}a^2/2$$ is the area of the unit cell. There are two carbons in one unit cell, so the factor 2 is multiplied. The density of states near the Fermi energy $$E = 0$$ is constant, and it is calculated as $$\rho(E) = (A/2\pi) \int_{-\infty}^{\infty} dk_x \delta(E - \gamma k_x) = aN_s/4\pi L\gamma_0$$. Because two sites in the discrete model correspond to one site in the continuum $$k \cdot p$$ model, the density of sites in the continuum model is given by: $$\rho \equiv \rho(E)|_{E=0} = a/2\pi L\gamma_0$$. 

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III. Impurity scattering in normal nanotubes

Now, we consider the impurity scattering in the normal metallic nanotubes. We take into account of the single impurity potential located at the point \( r_0 \):

\[
H_{\text{imp}} = I \sum_{\mathbf{k}, \mathbf{p}, \sigma} e^{i(\mathbf{k} - \mathbf{p}) \cdot r_0} \Psi^\dagger_{\mathbf{k}, \sigma} \Psi_{\mathbf{p}, \sigma},
\]

where \( I \) is the impurity strength.

The scattering \( t \)-matrix at the \( K \) point is

\[
t_K = I [1 - I \frac{2}{N_s} \sum_{\mathbf{k}} G_K(\mathbf{k}, \omega)]^{-1}.
\]

The discussion about the \( t \)-matrix at the \( K' \) point is qualitatively the same, so we only look at the \( t \)-matrix at the \( K \) point. The sum for \( \mathbf{k} = (k, 0) \), which takes account of the band index \( n = 0 \) only, is replaced with an integral:

\[
\frac{2}{N_s} \sum_{\mathbf{k}} G_K(\mathbf{k}, \omega) = \rho \int d\varepsilon \frac{1}{\omega^2 - \varepsilon^2} \begin{pmatrix} \omega & \varepsilon \\ \varepsilon & \omega \end{pmatrix}
\]

\[
\simeq -\rho \pi \text{sgn} \omega \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

Therefore, we obtain

\[
t_K = \frac{I}{1 + I \rho \pi \text{sgn} \omega} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

The scattering matrix \( t_K \) is diagonal, and the off-diagonal matrix elements vanish. This means that only the scattering processes from \( k \) to \( k \) and from \(-k\) to \(-k\) are effective. The scatterings from \( k \) to \(-k\) and from \(-k\) to \( k \) are cancelled. Such the absence of the backward scattering has been discussed recently [4]. They used the rotation properties of wave functions, and have calculated the \( t \)-matrix using the explicit forms of wave functions. Here, we have formulated by the scattering \( t \)-matrix using the propagators, and have shown that the off-diagonal matrix elements becomes zero.
IV. Impurity scattering with superconductivity pair potential

In this section, we consider the single impurity scattering when the superconducting pair potential is present. We look at how the absence of the backward scattering discussed in the previous section changes.

In the Nambu representation, the scattering $t$-matrix at the $K$ point is

$$\tilde{t}_K = \tilde{I}[1 - \frac{2}{N_s} \sum_k \tilde{G}_K(k, \omega)\tilde{I}]^{-1},$$

(17)

where

$$\tilde{I} = I \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$  

(18)

The sign of the scattering potential for holes is reversed from that for electrons, so the minus sign appears at the third and fourth diagonal matrix elements.

The sum over $k$ is performed as in the previous section, and we obtain

$$\frac{2}{N_s} \sum_k \tilde{G}_K(k, \omega) = \rho \int d\varepsilon \begin{pmatrix} G^{(1)} & G^{(1,2)} \\ G^{(2,1)} & G^{(2)} \end{pmatrix}.$$  

(19)

Here, the matrix elements are calculated explicitly, and become as follows:

$$\rho \int d\varepsilon G^{(1)} = \rho \int d\varepsilon G^{(2)} = -\rho \pi i \frac{\omega}{\sqrt{\omega^2 - \Delta^2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$  

(20)

and

$$\rho \int d\varepsilon G^{(1,2)} = \rho \int d\varepsilon G^{(2,1)} = -\rho \pi i \frac{\Delta}{\sqrt{\omega^2 - \Delta^2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$  

(21)
Therefore, we obtain the scattering $t$-matrix:

$$\tilde{t}_K = \frac{(\omega^2 - \Delta^2)I}{[1 + (I\rho\pi)^2]^{\omega^2} - [1 - (I\rho\pi)^2]^{\Delta^2}} \begin{pmatrix} 1 + \alpha\omega & 0 & \alpha\Delta & 0 \\ 0 & 1 + \alpha\omega & 0 & \alpha\Delta \\ \alpha\Delta & 0 & -1 + \alpha\omega & 0 \\ 0 & \alpha\Delta & 0 & -1 + \alpha\omega \end{pmatrix}$$ (22)

where $\alpha = I\rho\pi i/\sqrt{\omega^2 - \Delta^2}$.

Hence, we find that the off-diagonal matrix elements become zero in the diagonal $2 \times 2$ submatrix. This implies that the backward scatterings of electron-line and hole-like quasiparticles vanish in the presence of the proximity effects, too. Off-diagonal $2 \times 2$ submatrix has the diagonal matrix elements whose magnitudes are proportional to $\Delta$. The finite correlation gives rise to backward scatterings of the hole of the wavenumber $-k$ when the electron with $k$ is incident. The back scatterings of the electrons with the wavenumber $-k$ occur for the incident holes with $k$, too. Negative and positive currents induced by such the two scattering processes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur. This effect is interesting in view of the recent experimental progress of the superconducting proximity effects [1,2].

V. Summary

In summary, we have investigated the effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes. We have used the continuum $k \cdot p$ model for the electronic states, and have considered impurity scattering processes in the normal state and also in the state with the superconducting pair potential. The backward scattering of electrons vanishes in the normal state. In the presence of the superconducting pair correlations, the backward scatterings of electron- and hole-like quasiparticles vanish, too. The impurity gives rise to backward scatterings of holes for incident electrons, and it also induces backward scatterings of electrons for incident holes. Negative and positive currents induced by such the
scatterings between electrons and holes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur.

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