Locality problem in quantum theory∗

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Abstract

We discuss the locality problem in relativistic and nonrelativistic quantum theory. We show that there exists a formulation of quantum theory that, on one hand, preserves the mathematical apparatus of the standard quantum mechanics and, on the other hand, ensures the satisfaction of the locality condition for each individual event including the measurement procedure. As an example, we consider the scattering from two slits.

Keywords: locality of measurement, algebra of local observables, double-slit experiment, delayed choice.

1 Introduction

Since the famous debates between Einstein [1, 2, 3] and Bohr [4, 5], specialists have argued about locality in quantum mechanics. Einstein claimed that the locality principle is violated in quantum mechanics, which is therefore inconsistent or at least incomplete. Bohr’s main counterargument was that physical reality and therefore locality in quantum theory cannot be interpreted as Einstein did. The contradiction between Einstein and Bohr was mainly due to their different views of the basic purpose of physics. Einstein considered that the basic purpose is to understand and describe the structure of the material world. Bohr thought that the basic purpose is to formulate rules that one specialist can use to communicate to another specialist what must be done to repeat an experiment and obtain the same result.

In the years following the debates, Einstein made no progress on his way to quantum theory. In contrast, Bohr’s followers were realizing his program with much success.

As a result, the scientific community was overwhelmingly convinced that Bohr was right in the argument. More precisely, the vast majority of specialists in the field of quantum mechanics prefer not to go into the depths of this argument, considering it a purely philosophical problem not deserving attention. But because they successfully apply rules developed by Bohr’s supporters, they are automatically included in Bohr’s camp. On the other hand, most of them consider themselves solving the problem of describing reality. Such a dual position cannot but affect quantum physics. The relation between relativistic quantum field theory and nonrelativistic quantum mechanics is vague as regards the locality problem.

The locality property occupies the central place in quantum field theory. The so-called Einstein locality is one of the basic postulates in the Wightman approach [6]. In Bogoliubov’s approach [7], the causality condition is the principal constructive element. This condition

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is closely related to locality and states that any excitation in one domain of the Minkowski space does not affect physical processes in another spacelike separated domain. Finally, the algebraic approach attributed mainly to Haag [8] and Araki [9] is completely based on algebras of local observables, i.e., observables that can be measured in a bounded domain of the Minkowski space.

The locality property is not denied in nonrelativistic quantum mechanics when considering interactions of quantum objects. But the situation is drastically changed when discussing the problem of interaction between quantum objects and classical measuring devices. The main constructive tool describing such an interaction is the projection principle [10]. In the most cases, the projection principle had indeed proved its usefulness when describing the influence of a measuring device on a quantum object in the framework of the standard mathematical apparatus of quantum mechanics. But the physical mechanism for realizing this principle as well as its consistency with the locality condition is still missing.

Instead, people vaguely reason that the human brain has an experience of describing only classical objects and therefore cannot provide a concrete image of how a quantum object interacts with a classical device. But if the human brain can provide a more or less explicit picture of interactions between quantum objects, why should it fail when providing a picture of interactions of quantum objects with a classical device?

The locality problem has become more and more relevant in recent years. This is because it passes more and more from the domain of theoretical reasonings and Gedankenexperiments into the domain of actual experiments. Moreover, the first attempts are being made to construct prototypes of engineering constructions in the domain of so-called quantum telecommunication. The quantum locality problem plays a key role in this domain.

We mention here that the results of modern experiments are interpreted in most cases as evidence that a "local physical reality" does not exist in quantum physics. The locality of the quantum theory then acquires a somewhat metaphysical status removed from material reality.

Here, we attempt to demonstrate a quantum theory formulation that, on one hand, preserves the mathematical apparatus of the standard quantum mechanics and, on the other hand, admits an explicit interpretation of the locality property.

2 Basic notions and postulates

The basic notions of the proposed approach to quantum mechanics were presented in [11] and described in more detail in [12]. We only briefly review these notions here. We chose the inductive method of theory construction in [11]. We first considered a physical phenomenon, singled out its basic characteristics, and then described them mathematically.

Here, we choose the deductive method, i.e., we formulate mathematical postulates from the very beginning. Those who are interested in their phenomenological justification can turn to [11] and [12]. The entire construction is performed in the framework of the algebraic approach. We therefore do not assume that a physical system state is described by a vector of a Hilbert space (or by a density matrix) and that observables are described by operators in this space.

We take the following postulates as basic.

**Postulate 1.** Observables of a physical system are described by Hermitian elements of
some $C^*$-algebra $\mathfrak{A}$.

Elements of the algebra $\mathfrak{A}$ are called dynamical variables. We let $\mathfrak{A}_+$ denote the set of observables. We let $\mathfrak{Q}_\xi$ denote maximum commutative subalgebras of the algebra $\mathfrak{A}$ belonging to $\mathfrak{A}_+$. We use the subscript $\xi \in \Xi$ to distinguish among these subalgebras.

**Postulate 2.** For observables $\hat{A}$ and $\hat{B}$ to be compatible (simultaneously measurable), it is necessary and sufficient for them both to belong to some subalgebra $\mathfrak{Q}_\xi$.

We let $\varphi_\xi(\cdot)$ denote a character of the subalgebra $\mathfrak{Q}_\xi$, i.e., $\hat{A} \xrightarrow{\varphi_\xi} \varphi_\xi(\hat{A})$ is a homomorphic map of the algebra $\mathfrak{Q}_\xi$ ($\hat{A} \in \mathfrak{Q}_\xi$) into the algebra of real numbers.

We call the collection $\varphi = [\varphi_\xi]$ ($\xi \in \Xi$) of functionals $\varphi_\xi(\cdot)$, each of which is a character of the corresponding subalgebra $\mathfrak{Q}_\xi$, an elementary state of a physical system.

**Postulate 3.** The result of any individual experiment on measuring physical system observables is determined by an elementary state of this system.

The same observable may simultaneously belong to several subalgebras $\mathfrak{Q}_\xi$. We say that an elementary state $\varphi$ is stable on the observable $\hat{A}$ if for all subalgebras $\mathfrak{Q}_\xi$, $\mathfrak{Q}_{\xi'}$, containing $\hat{A}$, we have the equality

\begin{equation}
\varphi_\xi(\hat{A}) = \varphi_{\xi'}(\hat{A}), \quad \hat{A} \in \mathfrak{Q}_\xi \cap \mathfrak{Q}_{\xi'}, \quad \varphi_\xi(\cdot) \in \varphi, \quad \varphi_{\xi'}(\cdot) \in \varphi.
\end{equation}

Condition (1) does not hold in the general case. This means that the measurement result can depend not only on the elementary state $\varphi$ but also on the index $\xi \in \Xi$. We say that a device used for measuring the observable $\xi \in \Xi$ is of type $\xi$ if using this device, we obtain $A_\xi = \varphi_\xi(\hat{A})$ as the measurement result. We thus set a definite type of measuring device into correspondence with each subalgebra $\mathfrak{Q}_\xi$. We must use the devices to perform the compatible measurements of observables belonging to the same subalgebra $\mathfrak{Q}_\xi$.

We say that a measurement of an observable $\hat{A}$ is reproducible if a repeat measurement of the same observable (by a device not necessarily of the same type) produces a result coinciding with the initial result. Obviously, a reproducible measurement acts on a physical system by transforming it into an elementary state that is stable with respect to the observable $\hat{A}$.

Because compatible measurements are possible only for compatible observables, we cannot unambiguously fix the elementary state $\varphi$ in an experiment. The maximum that we can do is to determine the reduction of the elementary state to a subalgebra $\mathfrak{Q}_\xi$, i.e., we can fix the functional $\varphi_\xi(\cdot)$. We say that elementary states $\varphi$ are $\varphi_\xi$-equivalent if they have the same reduction $\varphi_\xi(\cdot)$ on the subalgebra $\mathfrak{Q}_\xi$.

We define the purely quantum state $\Psi_{\varphi_\xi}$ to be the class $\{\varphi_\xi(\cdot)\}$ of $\varphi_\xi$-equivalent elementary states that are stable with respect to the subalgebra $\mathfrak{Q}_\xi$. We can therefore experimentally determine only whether a system under investigation belongs to a definite quantum state.

**Remark.** The above definition of the quantum state is applicable only to physical systems in which there are no identical particles. If such particles are present, then we must replace the equivalence with weak equivalence in the quantum state definition (see [12]).
The collection of physical systems whose elementary states constitute the equivalence class \( \{ \varphi \} _{\varphi \xi} \) is called a pure quantum ensemble.

**Postulate 4.** A quantum ensemble admits the structure of the probability space.

We recall that the probability space is the fundamental object in the classic probability theory (see, e.g., [13, 14]). The probability space is a triple \((\Omega, \mathcal{F}, P)\). The first term in the triple, \(\Omega\), is a set of elementary events. In our case, the elementary state \(\varphi\) plays the role of an elementary event. The second term of the triple, \(\mathcal{F}\), is the Boolean \(\sigma\)-algebra of the set \(\Omega\). Elements of the algebra \(\mathcal{F}\) are subsets of the set \(\Omega\). These subsets are called (probability) events. Among them, there must be the set \(\Omega\) itself and the empty set \(\emptyset\). The algebraic operations in \(\mathcal{F}\) are the operations of taking the union of subsets, their intersection, and the complement with respect to the set \(\Omega\). The algebra \(\mathcal{F}\) must be closed (invariant) under the operation of taking the complement and a denumerable number of the union and intersection operations. The third term in the triple is a probability measure \(P\). This is a map of the set \(\mathcal{F}\) into the set of real numbers: each \(F \in \mathcal{F}\) is sent to a number \(P(F)\). This mapping must satisfy the conditions \(0 \leq P(F) \leq 1\) for all \(F \in \mathcal{F}\), \(P(\Omega) = 1\), and \(P(\sum_j F_j) = \sum_j P(F_j)\) for any denumerable union \(\sum_j F_j\) of nonintersecting subsets \(F_j \in \mathcal{F}\).

We recall that the probability measure is defined only for the events \(F \in \mathcal{F}\). For elementary events, the probability measure may not exist in general. This is because before considering a possibility of this or that event, we must ensure that this event can be realized (at least, in principle). Because the resolution of a measuring device is finite, we cannot single out an elementary event. It can be demonstrated (see, e.g., [12]) that the assumption that a probability measure necessarily exists for elementary events results in a contradiction in some cases.

In this situation, quantum theory imposes more severe restrictions than the classical theory. This is because in order to single out an event, we must have the possibility to perform simultaneous (or at least compatible) measurements of a collection of observables. In the quantum case, only observables belonging to the same subalgebra \(\mathcal{Q}_\xi\) are compatible. Hence, with each such subalgebra \(\mathcal{Q}_\xi\), we must associate its own \(\sigma\)-algebra \(\mathcal{F}\) and its own system of probability measures \(P_\xi(F)\), where \(F \in \mathcal{F}_\xi\).

On the other hand, there are events that can be singled out by measuring observables belonging to different subalgebras \(\mathcal{Q}_\xi\). An example of such an event is the event \(F_A\) stating that we register a value not exceeding \(A\) in the experiment for an observable \(\hat{A}\). If \(\hat{A} \in \mathcal{Q}_\xi \cap \mathcal{Q}_{\xi'}\), then this event can be registered by both type-\(\xi\) and type-\(\xi'\) devices. From our experience, we can conclude that the probability of such the event is independent of the type of the device we use. We must therefore introduce one more postulate.

**Postulate 5.** If \(\hat{A} \in \mathcal{Q}_\xi \cap \mathcal{Q}_{\xi'}\), then for the system in a quantum state \(\Psi\), the probability of the event \(F_A\) is independent of the type of device used, i.e., \(P(\varphi : \varphi_\xi(\hat{A}) \leq A) = P(\varphi : \varphi_{\xi'}(\hat{A}) \leq A)\).

The mathematical representation of a physical system is the algebra of its dynamical variables; vice versa, the physical representation of the algebra of dynamical variables is some physical system. We can therefore consider the physical representation of a subalgebra to be the corresponding physical subsystem. This subsystem is by no means isolated from the rest of the system, i.e., it can be an open system and not have its own dynamics. But in most cases, the conclusions of the probability theory are not related to the dynamics.
In particular, we can treat the subalgebra $Q_\xi$ as an algebra of observables of a classical subsystem of the quantum system under investigation. Because we can confine ourself to the measurements compatible with the measurements of observables from the subalgebra $Q_\xi$ in order to find the mean $\langle \hat{A} \rangle$ of an observable $\hat{A} \in Q_\xi$, the classical probability theory suffices for calculating such a mean. The formula

$$\langle \hat{A} \rangle = \int_{\varphi \in \Psi} P_\hat{A}(d\varphi) A(\varphi) \equiv \int_{\varphi \in \Psi} P_\hat{A}(d\varphi) \varphi(\hat{A}).$$

then holds. Here,

$$P_\hat{A}(d\varphi) = P(\varphi : \varphi(\hat{A}) \leq A + dA) - P(\varphi : \varphi(\hat{A}) \leq A), \quad A_\xi(\varphi) \equiv \varphi(\hat{A}).$$

Having in mind Postulate 5, we can omit the index $\xi$ of the functionals $A_\xi(\varphi)$ and $\varphi_\xi(\hat{A})$ in formulas (2) and (3).

Formula (2) determines the mean of the observable $\hat{A}$ with respect to the quantum ensemble. We can define the quantum mean $\Psi(\hat{A})$ experimentally as the arithmetic mean of the results of measurements for the observable $\hat{A}$. The relation between the quantities $\langle \hat{A} \rangle$ and $\Psi(\hat{A})$ is established by the Khinchin theorem (the large number law; see, e.g., [14]), which can be formulated as follows in the terms used in this paper.

$$A_{\xi_j} = \varphi_{\xi_j}(\hat{A})$$

be the result of measuring an observable $\hat{A}$ in the experiment with the number $j$. Let $A_{\xi_j}$ be random mutually independent quantities having the same probability distribution $P_\hat{A}$ with the finite expectation $\langle \hat{A} \rangle$. The quantity $n^{-1}(A_{\xi_1} + \ldots + A_{\xi_n})$ then converges to $\langle \hat{A} \rangle$ in the probability sense as $n \to \infty$. Therefore,

$$\Psi(\hat{A}) \equiv \lim_{n \to \infty} P \left[ n^{-1} \left( \varphi_{\xi_1}(\hat{A}) + \ldots + \varphi_{\xi_n}(\hat{A}) \right) \right] = \langle \hat{A} \rangle.$$  

Experiment proves that the following statement holds.

**Postulate 6.** The quantity $\Psi(\hat{A})$ is a linear functional of observables, i.e.,

$$\Psi(\hat{A}) + \Psi(\hat{B}) = \Psi(\hat{A} + \hat{B}) \quad \text{for all} \quad \hat{A}, \hat{B} \in \mathfrak{A}_+$.$$

This functional can be unambiguously extended to the algebra $\mathfrak{A}$ using the formula $\Psi(\hat{A} + i\hat{B}) = \Psi(\hat{A}) + i\Psi(\hat{B})$, where $\hat{A}, \hat{B} \in \mathfrak{A}_+$.

Every $C^*$-algebra $\mathfrak{A}$ is isometrically isomorphic to a subalgebra $\mathfrak{B}(\mathfrak{H})$ of bounded linear functionals in a Hilbert space $\mathfrak{H}$ (see, e.g., [15]), i.e.,

$$\hat{A} \leftrightarrow \Pi(\hat{A}), \quad \hat{A} \in \mathfrak{A}, \quad \Pi(\hat{A}) \in \mathfrak{B}(\mathfrak{H}).$$

It can be shown (see [11, 12]) that the mean $\langle \hat{A} \rangle$ of the observable $\hat{A}$ with respect to the quantum ensemble $\Psi$ defined by formula (2) can be represented as the expectation of the operator $\Pi(\hat{A})$:

$$\langle \hat{A} \rangle = \langle \Psi | \Pi(\hat{A}) | \Psi \rangle,$$

where $|\Psi\rangle \in \mathfrak{H}$ is the corresponding vector in the Hilbert space.
3 Locality of observables and nonlocality of states

Formulas (4) and (5) indicate that, on one hand, we can use the mathematical tools of the standard quantum mechanics to calculate quantum means $\Psi(\hat{A})$ and, on the other hand, we can interpret a quantum state as an equivalence class of elementary states. An equivalence class is a mathematical notion existing out of time and space. Speaking about a localization of a quantum state is therefore absurd.

The equivalence class corresponding to a definite quantum state can be composed using features common to all the elements from this class. Such common features can be the same values of observables obtained with a definite measurement procedure. Neither coordinate nor time are observed quantities. They are parameters of the Minkowski space in which physical objects dwell. But an observable called the "coordinate" is very often used in nonrelativistic quantum mechanics.

To find a way out of this mess, we consider the procedure for measuring this "coordinate" in more detail. For simplicity, we assume the physical object under study to be pointlike. To measure the "coordinate" of a physical object, we use a measuring device, a ruler with graduations. To each graduation interval, we associate the observable $\hat{p}_i$, where $i$ is the number of the corresponding interval. We ascribe the value 1 to the observable $\hat{p}_i$ if the physical object under study is inside the $i$-th interval and the value 0 to the observable $\hat{p}_i$ if it is outside this interval. The observable $\hat{p}_i$ thus defined has the properties of the projector and is an element of the algebra of dynamical quantities.

Freely speaking, we call the index $i$ the coordinate and state that the object under study has the coordinate $i$ if the value of the corresponding observable $\hat{p}_i$ is equal to 1. This "coordinate" $i$ is very distantly related to the genuine coordinate, which is a parameter of the Minkowski space. We can repeat our experiment in another domain of the Minkowski space. We must then carry both the object under study and the ruler to this domain. If the object under study again turns out to be in the interval with the number $i$, then we say that the second experiment produces the same value of the "coordinate." We can organize the equivalence class with respect to just this "coordinate" (in fact, with respect to the value of the observable $\hat{p}_i$) and say that we have constructed a quantum state concentrated in the vicinity of the "coordinate" $i$. This "coordinate" has no relation to a localization in the Minkowski space.

To discuss the locality problem in more detail, we consider how we can describe particle scattering by two slits $a$ and $b$ using the idea of the elementary state. An interference pattern is vividly observed in this experiment. This picture is clearly determined by the probability distribution of particle momenta after scattering. Three events are essential in the experiment under consideration: the event $F_a$, which means that the particle hits a domain of slit $a$, the event $F_b$, which means that the particle hits a domain of slit $b$, and the event $F_k$, which means that the scattered particle momentum falls into a fixed small solid angle around the direction $K$.

The problem under consideration can be formulated in these terms as a typical problem of calculating conditional probability. We must calculate the probability of the event $P(F_k)$ under the condition of realization of either event $F_a$ or event $F_b$. Classical probability theory provides a standard formula, but we cannot apply it directly in the quantum case because it involves the probability of simultaneous realization of the events $P(F_k)$ and $F_a + F_b$. But a probability measure does not exist for this event because the events $P(F_k)$ and $F_a + F_b$ are incompatible because of the incompatibility of simultaneous measurements of the coordinate
and the momentum.

But we can propose a detour for calculating such a conditional probability. For this, it suffices to consider the first stage of scattering in which the particle hits either the domain of slit \( a \) or the domain of slit \( b \) as the preparation of a quantum state. When using this quantum state as the new probability space, we can consider the event \( F_k \) as an unconditional one.

We can set the observable \( \hat{p}_a \), which takes the value \( p_a = 1 \) if the particle hits the domain of slit \( a \) and value \( p_a = 0 \) if the particle misses this domain, into correspondence with the event \( F_a \). We set the analogous observable \( \hat{p}_b \) into correspondence with the event \( F_b \). Only those particles whose elementary states correspond to the value of the observable \( \hat{p}_a + \hat{p}_b \) equal to one contribute to the interference pattern. Such elementary states constitute an equivalence class, denoted by \( \Psi_{a+b} \). Because the observable \( \hat{p}_a + \hat{p}_b \) is not the only independent generator of the maximum subalgebra of compatible observables in the general case, the quantum state corresponding to the equivalence class \( \Psi_{a+b} \) can be mixed. But even in this case, the functional describing the means of observables with respect to this quantum state is positive definite, linear, and normalized to unity. We let \( \Psi_{a+b}(\cdot) \) denote this functional. It has the property

\[
\Psi_{a+b}(\hat{I}) = 1,
\]

where \( \hat{I} \) is the unit element of the algebra \( \mathfrak{A} \). Moreover, for all the elementary states in this quantum ensemble, we have \( p_a + p_b = 1 \), and the functional \( \Psi_{a+b}(\cdot) \) by virtue of formulas \( \Psi_{a+b}(\cdot) \) (2) and (1) satisfies the condition

\[
\Psi_{a+b}(\hat{p}_a + \hat{p}_b) = 1.
\]

Because the functional \( \Psi_{a+b}(\cdot) \) is positive definite, the Cauchy-Buniakowski-Schwarz inequality holds for it,

\[
\left| \Psi_{a+b}(\hat{A}(\hat{I} - \hat{p}_a - \hat{p}_b)) \right|^2 \leq \Psi_{a+b}(\hat{A}^* \hat{A}) \Psi_{a+b}(\hat{I} - \hat{p}_a - \hat{p}_b).
\]

By virtue of equalities (6) and (7), the right-hand side of inequality (5) is zero. Therefore,

\[
\Psi_{a+b}(\hat{A}) = \Psi_{a+b}(\hat{A}(\hat{p}_a + \hat{p}_b)).
\]

Analogously,

\[
\Psi_{a+b}(\hat{A}) = \Psi_{a+b}(\hat{(p}_a + \hat{p}_b) \hat{A})
\]

Replacing \( \hat{A} \rightarrow \hat{A}(\hat{p}_a + \hat{p}_b) \) in (10) and taking (9) into account, we obtain

\[
\Psi_{a+b}(\hat{A}) = \Psi_{a+b}(\hat{(p}_a + \hat{p}_b)) \hat{A}(\hat{p}_a + \hat{p}_b).
\]

We set the observable \( \hat{K} \) into correspondence with the event \( F_k \). Using formula (11), we obtain the expression for the mean of this observable

\[
\langle \hat{K} \rangle = \Psi_{a+b}((\hat{K}) = \Psi_{a+b}(\hat{p}_a \hat{K} \hat{p}_a) + \Psi_{a+b}(\hat{p}_b \hat{K} \hat{p}_b) + \Psi_{a+b}(\hat{p}_a \hat{K} \hat{p}_b + \hat{p}_b \hat{K} \hat{p}_a).
\]

The first and second terms in the right-hand side of (12) describe the scattering from the respective slits \( a \) and \( b \). The third term describes the interference. Because \( \hat{p}_a \hat{p}_b = \hat{p}_b \hat{p}_a = 0 \), in the case where \( [\hat{p}_a, \hat{K}] = 0 \) or \( [\hat{p}_b, \hat{K}] = 0 \), the interference term disappears.
The interference pattern is purely determined by the structure of the abstract equivalence class $\Psi_{a+b}$ would therefore organize our experiment as follows. We can prepare many copies of the same experimental device and distribute it over the globe. At each device, we perform one scattering act at random time instants. We then put together all the screens on which we have spots from hits of the scattered particles and put all these screens in one stack. For a sufficiently large number of screens, we must obtain a pattern close to that described by formula (12).

We note that in contrast to considering the same experiment in the standard quantum mechanics, we consider that the scattered particle hit either the domain of slit $a$ or the domain of slit $b$ in each separate case, not passing in a mysterious way through both slits simultaneously. This means that we consider a particle well localized in each separate act. The interference pattern appears because the functional $\Psi_{a+b}(\cdot)$ cannot be represented as a sum of the functionals $\Psi_a(\cdot)$ and $\Psi_b(\cdot)$ corresponding to the respective acts of separate scattering on the slits $a$ and $b$. Physically, this means that the scattering on one slit depends on the presence or absence of the other slit, i.e., a nonlocality is present here.

We see how this nonlocality can be explained in the framework of a local field theory. For clarity, we here discuss the example of the process of scattering of an electron on a nucleus, well studied both theoretically and experimentally. Because the electron is much lighter than the nucleus, this process is well approximated by the electron scattering on a classical source. In what follows, we discuss exactly this process in the framework of the perturbation theory in the standard quantum electrodynamics (see, e.g., [16]).

In the first order of the perturbation theory in the electron charge, this process is described by the Feynman diagram (a) shown in Fig. 1. In this figure, straight lines correspond to the electron, wavy lines correspond to the photon, and the crossed circle corresponds to the source of the classical electromagnetic field. Calculating the differential scattering cross section when taking diagram (a) into account causes no troubles and results in the celebrated Rutherford formula corrected by taking the electron spin into account. The obtained formula describes the experimental situation well. But both theory and experiment have now gone far beyond the accuracy level ensured by the first correction to the perturbation theory.

The next order of the perturbation expansion that contributes to the process under study is the third order. There, we must take contributions coming from diagrams (b) and (c) in Fig. 1 into account. Taking these diagrams into account results in substantial theoretical difficulties. First, the so-called ultraviolet divergences appear because the intermediate (virtual) particles can carry arbitrarily large energies and momenta. In quantum field theory, a well-defined algorithm (the renormalization theory) was developed to overcome this difficulty. We do not discuss this problem in what follows. Second, diagram (b) results in the so-called infrared divergences caused by the presence of massless particles in the complete particle set. In the example under consideration, such particles are the photons. Quantum field theory also provides an algorithm for overcoming this difficulty. We discuss it in more detail.

The algorithm is based on the following experimental fact. The elastic scattering process described by diagrams (a-c) cannot be experimentally separated from the process of bremsstrahlung depicted in diagrams (d-f) in Fig. 2. In this process, electron scattering is accompanied by emitting one (diagrams (d) and (e)) or more (diagram (f)) photons. The contribution of diagrams of such type to the scattering cross section cannot be experimentally separated from the contributions of diagrams (a-c) if the total energy of photons emitted in the bremsstrahlung is below the sensitivity threshold of the measuring device.
Calculations show that if we take diagrams (d) and (e) into account together with diagrams (a-c), then infrared singularities are compensated. But the scattering cross section then becomes dependent on a parameter characterizing the sensitivity of the measuring device. We can take the total energy $E_{\text{max}}$ of additionally emitted photons as such a parameter. This is an absolutely physical parameter, and the dependence of the measured scattering cross section on this parameter should therefore not cause any principal objection. But taking diagrams (a-e) alone into account results in one more difficulty. The dependence of the scattering cross section on the parameter $E_{\text{max}}$ is singular, and the cross section can become negative at sufficiently small $E_{\text{max}}$.

In modern theory, this difficulty is attributed to an artifact related to using the perturbation theory. Indeed, if higher orders of the perturbation theory are considered taking contributions from diagrams of type (f) with an infinitely increasing number of emitted photons into account and summing all these contributions, then the cross section dependence on the parameter $E_{\text{max}}$ becomes regular. Moreover, this cross section tends to zero as $E_{\text{max}} \to 0$. This does not cause objections from the physical standpoint. The purely elastic scattering in which no bremsstrahlung photons are emitted is just one among infinitely many channels along which this process may proceed. It is therefore not amazing that each of these channels contributes infinitesimally to the total cross section.

Here we need a new insight into the process called elastic scattering. In reality, this process is never purely elastic. Electron scattering is always accompanied by the bremsstrahlung, which cannot be registered even by a measuring device with very high sensitivity. Moreover, the result of the experiment becomes strongly dependent on the device sensitivity if the latter is too high. When the sensitivity becomes infinitely high, the registered scattering cross section must tend to zero because we study the measuring device in this case and not the physical object (electron) under investigation.

The above example teaches us several useful lessons.
Lesson 1. Separating characteristics of a physical object under study is somewhat conditional. These characteristics cannot be separated completely from the characteristics of the measuring device with which this object interacts.

Lesson 2. A physical object (electron in the above example) with which we associate definite physical characteristics (differential cross section of elastic scattering) is accompanied by a field (bremsstrahlung photons) that is not registered by the measuring device but affects the result of the measurement of the characteristics under study.

Lesson 3. The presence of the accompanying field does not contradict locality axioms of quantum field theory. In the above example, both the electron and the bremsstrahlung photons propagate in the future light cone with the vertex at the scattering point.

Lesson 4. The result of measuring the characteristics (scattering cross section) ascribed to a well-localized object under study (electron) may depend on the characteristics of physical objects (bremsstrahlung photons) that are located in the domain that is spacelike with respect to the localization domain of the object under study. This may be interpreted as a nonlocality of the object under study.

The above lessons result in the following conclusion. We can split the physical problem under study into two parts as regards the measurement process. The first part, called the kernel in what follows, is registered by a measuring device. The second part, called the dark field in what follows, is not directly registered by the measuring device, but the instrument reading can depend on characteristics of the dark field. The separation into these two parts is not absolute and depends on the measurement procedure. This mobility of the boundary finds its partial realization in the renormalization group formalism in the mathematical apparatus of quantum field theory.

In the framework of the algebraic approach to quantum theory described in this paper, such a division of a physical system into two parts can be related to two elements of the mathematical apparatus: the algebra of dynamical quantities and the elementary state. Each
quantum particle reveals itself through the corresponding observable quantities or, more precisely, through local observables whose values can be found by performing measurements in a bounded domain $O$ of the Minkowski space. We can therefore consider the local algebra $\mathfrak{A}(O)$ to be the mathematical representation of the quantum particle. In the standard algebraic approach to quantum field theory, this algebra is customarily called the algebra of local observables. This is not completely correct because the set of observables constitutes not an algebra but rather a subset of the local algebra of the corresponding dynamical quantities.

The domain $O$ can be naturally considered to be a localization domain for the quantum particle under consideration. In any case, the domain $O$ must contain the particle localization domain. More precisely, the localization domain of a particle must be associated with its kernel. Indeed, as explained above, the registered values of observables may depend on the characteristics of the dark field. This field is not necessarily localized in the domain $O$. On the other hand, in the approach of this paper, observable values are determined by the elementary state. The matter carrier of the elementary state is therefore not only the kernel of the quantum object under study but also the associated dark field. The elementary state therefore cannot be regarded as being localized in the domain $O$. But in contrast to the quantum state, which has no localization in the Minkowski space, the elementary state has a localization: it is the dark field localization.

We note that not every dark field affects the result of measuring observables (the scattering cross section in the above example); only the one that is created together with the kernel does, i.e., only the dark field coherent to the kernel is essential.

We now return to discussing the experiment on the scattering from two slits. We regard the electron as the scattered particle. We obtained formula (12), which describes the interference phenomenon using the fact that the quantum ensemble $\Psi_{a+b}$ has a definite structure. First, at the instant of the ensemble creation, the electron is localized either in the domain of slit $a$ or in the domain of slit $b$ (it is better to speak about the electron kernel, not the electron itself). Second, as a result of interaction between the electron and the slits $a$ and $b$, an ensemble is created whose structure is such that the corresponding functional $\Psi_{a+b}(\cdot)$ is linear.

We in fact replaced a real description of interaction between the electron and slits by the appropriate boundary conditions. These boundary conditions suffice for the mathematical description of the phenomenon under investigation. But it would be desirable to find out, at least on a qualitative level, what the physical processes underlying these boundary conditions are.

The first condition for electron localization is self-evident and does not need additional comments. We only note that it will certainly provoke frantic objections from orthodox followers of the standard quantum mechanics, who will insist on that we cannot speak about electron localization before performing a measurement. Why not? Only because they cannot say anything meaningful on this subject?

To explain the second condition qualitatively, we can propose the following model of how the electron interacts with the slits or, more precisely, with the screen in which these slits are cut. In the scattering process, not only the electron kernel but also the companion dark field that is coherent with the kernel approach the screen. Because this field is massless, it reaches the screen even before the kernel. This field generates collective oscillations of the screen that are also coherent with the kernel. The arising oscillations are very weak, but because of the coherence, they may interact resonantly with the kernel. At least, they may
play the role of a random force participating in creating the probability distribution of the scattered electron momentum. In contrast to the electron kernel, the dark field reaches both slits, and the character of the random force depends essentially on whether only one slit is open or both slits are open simultaneously. This can be a physical reason for the appearance of the interference pattern.

The dark field reveals itself in the experiment when measuring values of the observables describing the kernel coherent with this field. Separated from its kernel, the dark field becomes explicitly experimentally unobservable. It is therefore a good candidate for the role of a constituent of dark matter. Of course, in addition to the electromagnetic field, other massless fields such as gluon and gravitational fields may contribute to the dark field.

As stated above, to remove infrared divergences self-consistently, we must assume, for example, that in electromagnetic interactions, an infinite number of photons with a finite total energy is emitted. Such a system behaves as a classical electromagnetic field, and massless observable quantum fields must therefore feature classical "tails." This opens an interesting perspective for a gravitational field. A consistent quantum model of a gravitational field is still missing despite numerous attempts to construct it. The quanta of a gravitational field, the gravitons, have never been observed. Can it be that the gravitational field consists only of the classical "tail?"

The dark field mechanism provides a very clear explanation of the result of the experiment with the so-called delayed choice. Wheeler proposed the idea of this experiment 30 years ago [17]. Wheeler’s idea was recently realized almost ideally [18]. The actual experiment completely confirmed Wheeler’s predictions.

The principal scheme of the experimental setup is depicted in Fig. 3. In this figure, \( M_2 \) and \( M_3 \) are two totally reflecting mirrors, and \( M_1 \) and \( M_4 \) are two half-silvered mirrors. Mirror \( M_4 \) is removable. By the experimenter’s choice, it can be either absent (the device is then in position (a)) or present (the device is then in position (b)). Single photons are emitted toward mirror \( M_1 \) at time intervals such that no more than one photon can be in the device at each given instant. After passing through the device, the photon reaches either detector \( D_A \) or detector \( D_B \).
If the device is in position \((a)\) and photon behaves as a particle, then after passing through mirror \(M_1\) it chooses the path \(A\) or \(B\) with equal probabilities. As a result, it reaches either detector \(D_A\) or detector \(D_B\).

If the device is in position \((b)\) and photon behaves as a wave, then the process of passing through the installation can be described as follows. The photon-wave reaches mirror \(M_1\). The wave here splits into two coherent parts. One part propagates along path \(A\), and the other part propagates along path \(B\). The coherence of the parts is preserved. The wave phase is changed by \(\pi/2\) when the wave is reflected by any of the mirrors, and the phase remains unchanged when the wave passes through a mirror. The coherent addition of the two waves occurs at mirror \(M_4\). The balance of the phase changes is such that the wave propagates only in the direction of detector \(D_B\) after reaching mirror \(M_4\).

The result of the actual experiment is as follows. If the device is in position \((a)\), then detector \(D_A\) responds with probability 0.5, and detector \(D_B\) responds with the same probability. If the device is in position \((b)\), then detector \(D_B\) responds with probability 1. Therefore, in accordance with the device position, the photon behaves either as a particle or as a wave. Such photon behavior agrees with Bohr’s context principle \([5]\). According to this principle, the result of a quantum experiment depends on the general context of the experiment.

But Wheeler proposed complicating the choice problem for the photon. He proposed installing or removing mirror \(M_4\) after the photon has passed through the mirror \(M_1\), i.e., the photon must predict the subsequent acts of the experimenter. The actual experiment demonstrated that a photon handles this task successfully and behaves properly in every situation: either as a particle or as a wave.

It seems that a time nonlocality is manifested in this experiment: the future action (the manipulation of mirror \(M_4\)) affects the preceding action (the photon’s choice to behave as a wave or as a particle). Wheeler himself interpreted the result of this experiment confirming the principle that “no registration of the experimental result means that no physical phenomenon exists.”

Explaining the experimental result is much simpler in the framework of the dark field mechanism. When the photon interacts with mirror \(M_1\), in addition to the scattering (reflection or passage) of the photon, bremsstrahlung photons are created. The scattered photon kernel propagates along either path \(A\) or path \(B\). The bremsstrahlung photons (the dark field) propagate along both paths. Both parts of the dark field reach mirror \(M_4\) (if it is present) where they are added coherently and generate small collective oscillations in mirror \(M_4\). These small oscillations are coherent with the photon kernel, interact with it resonantly, and play the role of a random force directing the kernel toward detector \(D_B\). If mirror \(M_4\) is absent, then the photon kernel propagates along one of the paths it takes in mirror \(M_1\) and reaches either detector \(D_A\) or detector \(D_B\). No time nonlocality arises in this case.

The quantum correlation problem is closely related to the nonlocality problem. This is because these correlations often look like a distant action. A typical example is the Einstein-Podolsky-Rosen paradox \([1]\). For example, in the variant proposed by Bohm \([19]\), the result of measuring the projection of a spin of one particle from the singlet pair of particles with the spins \(1/2\) on one direction instantly and unambiguously predicts the result of measuring the projection of the other particle spin on the same direction even if the particles are separated by a large distance in space. It seems that this result contradicts the locality principle. But this contradiction arises only if we assume that the correlation results from the interaction between the particles at the instant of the measurement.

The notion that a correlation between separate elements of a physical system is always
due to interaction between these elements is a deeply rooted delusion. It is even reflected in the terminology used in quantum mechanics. We can often hear the terms "exchange interaction," "nonforce quantum action," or reasonings about "strong quantum correlations."

In fact, quantum correlations are not caused by features specific to quantum interactions. For example, in the Einstein-Podolsky-Rosen paradox, the correlations between spin projections of two particles arise because these particles were created as a singlet pair for which the law of conservation of the proper angular momentum is satisfied. But the angular momentum conservation law also holds in classical physics.

In most cases, quantum correlations are due to the structure of the physical system ensemble participating in the quantum experiments. This structure is fixed by the procedure for preparing the ensemble under consideration, and the preparation procedure is in turn determined by the properties of the classical device used. As a rule, quantum correlations are therefore caused by the interaction between each separate constituent of the quantum ensemble and the classical device (or devices) preparing this quantum ensemble, not by the interaction between quantum objects. This interaction can be smeared both in time and space for separate constituents of the ensemble. It is therefore not amazing that it often seems that correlations contradict the principle of the locality of interaction. In fact, the locality principle is always satisfied for correlations. But this correlation must be verified not from the standpoint of interaction between different constituents of the quantum ensemble but from the standpoint of interaction of separate constituents of this ensemble with the devices preparing this ensemble.

4 Conclusion

Summarizing, we can draw the following conclusions.

Quantum theory, both relativistic and nonrelativistic, can be formulated such that it does not contradict the locality condition accepted in quantum field theory. The measurement process also does not contradict this condition.

The incompleteness of quantum mechanics noted by Einstein can be removed by introducing a new notion of the "elementary state," which is to be attributed to an individual physical system.

From the measurement standpoint, a physical system under study can be separated into two parts: the so-called kernel and the accompanying dark field. The kernel is the material carrier of corpuscular properties of the physical system. The kernel is localized in the Minkowski space. The algebra of local observables is the mathematical representation of the kernel. The structure of the dark field does not contradict the relativistic condition of locality, but the dark field has a localization worse than that of the kernel. The elementary state of a physical system is determined by both the kernel and the dark field structure. The elementary state is the mathematical representation of the material carrier of the wave properties of the physical system.

A quantum state is an equivalence class in the set of elementary states and plays the role of the mathematical representation of the (quantum) ensemble of physical systems under investigation. The quantum state does not have the property of locality in the Minkowski space.
References

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev., Ser. 2, 47, 777-780 (1935).
[2] A. Einstein, J. Franklin Inst., 221, 313-347 (1936).
[3] A. Einstein, Science, 91, 487-492 (1940).
[4] N. Bohr, Phys. Rev., II. Ser., 48, 696-702 (1935).
[5] N. Bohr, "Discussion with Einstein on epistemological problems in atomic physics," in: Albert Einstein: Philosopher-Scientist (P. A. Schilpp, ed.), Library of Living Philosophers, Evanston, 111. (1949), pp. 200-241.
[6] R. F. Streater and A. S. Wightman, PCT, Spin and Statistics, and All That, Benjamin, New York (1964).
[7] N. N. Bogoliubov and D. V. Shirkov, Introduction to the Theory of Quantized Fields [in Russian], Nauka, Moscow (1984); English transl. prev. ed., Wiley, New York (1980).
[8] R. Haag and D. Kastler, J. Math. Phys., 5, 848-861 (1964).
[9] H. Araki, Progr. Theoret. Phys., 32, 844-854 (1964).
[10] J. von Neumann, Mathematical Foundations of Quantum Mechanics, Princeton Univ. Press, Princeton, N. J. (1996).
[11] D. A. Slavnov, Theor. Math. Phys., 149, 1690-1701 (2006).
[12] D. A. Slavnov, Phys. Part. Nucl, 38, 147-176 (2007).
[13] A. N. Kolmogorov, Foundations of the Theory of Probability [in Russian], Nauka, Moscow (1974); English transl. prev. ed., Chelsea, New York (1956).
[14] J. Neveu, Mathematical Foundations of the Calculus of Probability (Transl. by A. Feinstein), Holden-Day, San Francisco, Calif. (1965).
[15] J. Dixmier, Les C*-algébres et leurs représentations, Gauthier-Villars, Paris (1969).
[16] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory, Addison-Wesley, Reading, Mass. (1995).
[17] J. A. Wheeler, "The 'past' and the 'delayed-choice' double-slit experiment," in: Mathematical Foundations of Quantum Theory (Presentations to a Conference at Loyola University, New Orleans, La., June 2-4, 1977, A. R. Marlow, ed.), Acad. Press, New York (1978), pp. 9-48.
[18] V. Jacques et al., "Experimental realization of Wheeler's delayed-choice GedankenExperiment," arXiv:quant-ph/0610241v1 (2006).
[19] D. Bohm, Quantum Theory, Constable, London (1952).