On possible $S$-wave bound states for a $N\bar{N}$ system within a constituent quark model

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We try to apply a constituent quark model (a variety chiral constituent quark model) and the resonating group approach for the multi-quark problems to compute the effective potential between the $N\bar{N}$ in $S$-wave (the quarks in the nucleons $N$ and $\bar{N}$, and the two nucleons relatively as well, are in $S$ wave) so as to see the possibility if there may be a tight bound state of six quarks as indicated by a strong enhancement at threshold of $p\bar{p}$ in $J/\psi$ and $B$ decays. The effective potential which we obtain in terms of the model and approach shows if the experimental enhancement is really caused by a tight $S$-wave bound state of six quarks, then the quantum number of the bound state is very likely to be $I = 1, J^{PC} = 0^{-+}$.

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I. INTRODUCTION

Recently the BES collaboration in the radiative decay $J/\psi \rightarrow \gamma p\bar{p}$ observed a sharp enhancement at threshold in the $p\bar{p}$ invariant mass spectrum. They, further based on a multi-quark bound state conjecture, tried to fit the enhancement by means of an $S$-wave Breit-Wigner resonance function, and obtained the resultant mass peak at $M = 1859_{-10}^{+3}(stat)^{+5}_{-25}(sys)$ MeV and width $\Gamma$ less than 30 MeV. In the meanwhile, Belle also reported their observations that in the decays $B^+ \rightarrow K^+ p\bar{p}$ and $B^0 \rightarrow D^0 p\bar{p}$, an enhancement in the $p\bar{p}$ invariant mass distribution near the threshold was also seen clearly.

Theoretical investigations of baryon-antibaryon bound states date back to the proposal of Fermi and Yang, to make the pion with a nucleon-antinucleon pair. The extensive and excellent reviews are given in Ref. 2. In traditional nuclear interaction theories within the potential framework which are based on the single meson exchange mainly, it is shown that $NN$-system is more attractive than $NN$-system due to the fact that in the theories there is a strong $\omega$-exchange. Therefore possible bound states or resonances of a nucleon-antinucleon system (the so-called nuclear baryonia) have been proposed for many years.

Recently being encouraged by the observations and with intuition, several interpretations on the observations near and below the baryon-antibaryon threshold were suggested: (i) Based on the fact that the radiative $J/\psi$ decay may be a gluon-rich process, L. Rosner explained the enhancement is due to the iso-scalar state with $J^{PC} = 0^{++}$ being coupled to a pair of gluons; (ii) B.S. Zou et al with a K-matrix approach summed up the one-pion-exchange final state interaction and showed an important contribution to make the enhancement behavior near the threshold; (iii) Many phenomenological models were used to explain this anomalous (sharp) enhancement as a bound state or a resonance, e.g., A. Datta et al accounted it for a new bound state in a simple potential model with a $\lambda$-$\lambda$ confining interactions; X.-G. He et al also gave their explanation by using linear $\sigma$ model.

The traditional interaction theories to study $N\bar{N}$ system, such as boundary condition model, absorptive potential model, optical model and coupled-channel models etc, specially put forward their own method to handle the short-range part of $N\bar{N}$, although in comparatively long range they are quite similar. For instance, the optical model provides a realistic picture of the scattering process by introducing an imaginary part of the potential to reproduce the effect of other excitation channels, and the spin-isospin dependence of meson-exchange and channel dependence in the annihilation potential as well are also taken into account. The coupled-channel model either describes annihilation in terms of baryon exchange with the same baryon-meson coupling as in the Yukawa potential or uses semi-phenomenological potential adding partial-wave-analysis to study $NN$ interaction. Although the agreement with collision experiments is obtained, from the QCD point of view each of them has shortcoming respectively. For example, it is hard to imagine that a baryon-exchange picture can be applied to such a short-range where quarks are ‘overlapped’ and the color octet configuration must be considered.

In fact, the early studies within the traditional me-
son exchange (mesons exchange between the nucleons as whole) framework found that, if neglecting annihilation channels i.e. taking the real part of the effective potential only, many bound states might be formed, while annihilation effects i.e. the imaginary part of the potential, were included, the binding force decreased and some bound states were washed out [18]. Moreover, in the earlier days experimentally the data there was no clear evidence to imply the existence of strongly bound states. In this paper, to study of the possibility of baryon spectrum to interactions between two baryons, in an interesting tentative. Therefore, we suspect that the bound state should be accordingly and into one gluon as well. In order to keep the model well-described for baryon spectrum and corresponding chiral quark model (with necessary extensions) of QCD. Namely the dynamics should be described by the effective chiral quark Lagrangian (pseud-goldstone particles and light quarks as active degrees of freedom in the theory) [24]:

\[
L = i\bar{\psi}_q(i\partial + V + g_A A_5 - m)\psi_q + \frac{1}{4}f^2 \tau d \tau \bar{\psi} \psi - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \cdots
\]

where

\[
V_\mu = \frac{1}{2}(\xi^+ \partial_\mu \xi + \xi \partial_\mu \xi^+) ;
\]

\[
A_\mu = \frac{1}{2}(\xi^+ \partial_\mu \xi - \xi \partial_\mu \xi^+) ;
\]

\[
\Sigma = e^{2i\Pi/f} , \quad \xi = e^{i\Pi/f} , \quad (\Sigma = \xi) ;
\]

\[
\Pi = \frac{1}{2}\left[\begin{array}{ccc}
\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+
\
\pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0
\
K^- & K^0 & -\sqrt{\frac{1}{2}}\eta
\end{array}\right].
\]

Since at present we consider the \( N\bar{N} \) system, so for the Lagrangian we restrict ourselves to consider two flavors only. As mentioned in Introduction, in order to fit nuclear data some ‘modification’ on the quark chiral model is needed, such as to add a scalar \( \sigma \) ‘meson’ [25] and a ‘survived’ effective gluon into the model etc. Thus with the modification the relevant quark-gluon-meson Lagrangian can be re-written as

\[
L_{ch} = i\bar{\psi}_q(i\partial - m_q)\psi_q + g_{ch}\bar{\psi}_q(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\tau})\psi_q ,
\]

where \( \bar{\psi}_q \) denotes quark field, and \( m_q \) \((q = u, d)\) are the constituent quark masses. Current masses turns to constituent masses accordingly is a consequence of the chiral symmetry breaking. \( g_{ch} \) is the vertex coupling (quarks to mesons) constant and \( D^\mu = \partial^\mu + ig_A \frac{\lambda}{4} A^{\mu} \) \((a \) is an index of color space and \( \lambda^a \) is Gell-Mann matrix for SUc(3) color) \( A^{\mu} \) is gluon field. To study the baryon structure and baryon-baryon interaction, the survived gluon field \( A^{\mu} \) is introduced so as to take care of the necessary non-perturbative contributions in the model. In addition, to provide the non-perturbative QCD effects at long distance, an effective confinement in potential is needed. We will discuss them later.

Here when computing the effective potential between \( N \) and \( \bar{N} \), we only take into account the tree Feynman diagrams: the relevant exchange and annihilation diagrams (see Fig.1-3).
Here we have inserted a form factor between quark and antiquark.

\[ V_{qq}(r) = V_{q̄q}^\sigma(r) = -\frac{g^2_{ch}}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_\sigma^2} \left( \frac{e^{-m_\sigma r}}{r} - \frac{e^{-\Lambda r}}{r} \right). \]

As for the \( \pi \)-exchange, since it has a different sign when taking non-relativistic limit from Dirac spinors to Pauli ones for the effective interactions \( \bar{u}_q \gamma^5 u_q \) and \( \bar{u}_q \gamma^5 \bar{u}_q \gamma^5 v \), so there is an additional minus sign from fermion-fermion into fermion-antifermion in taking the limit.

\[ V_{q̄q}^\pi(r) = -\frac{1}{12} \frac{g^2_{ch}}{4\pi} \frac{m_\pi^2}{m_{q_i} m_{q_j}} e^{-m_\pi r} \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} \left( \frac{e^{-\Lambda r}}{r} \right) \left( \frac{e^{-\Lambda r}}{r} \right). \]

We note that the additional minus may be understood by the G-parity rule easily. Here \( \overline{\sigma}_{ij} \) is a Pauli matrix for spin and \( \overline{\tau}_{ij} \) is a Pauli matrix for isospin.

Since it is not allowed any quark-antiquark exchange between \( N \) and \( \overline{N} \), so the interaction from one-gluon-exchange for \( N \) and \( \overline{N} \) is very different that for the \( NN \) system that one-gluon-exchange between \( N \) and \( \overline{N} \) does not contribute to the effective potential at all. Here we only write down the potential due to one-gluon exchange between two quarks (antiquark) in one baryon (antibaryon) [24].

\[ V_{q̄q}^\pi(r) \propto \left( \frac{\overline{\lambda}_i \cdot \overline{\lambda}_j}{4} - \frac{\pi \delta(r)}{2} \left[ \frac{1}{r^2} + \frac{1}{m_i^2} + 4(\overline{\sigma}_i \cdot \overline{\sigma}_j) \frac{1}{3m_i m_j} \right] \right)^{10} \]

(ii). The effective potential derived from annihilation: Let us take \( \pi \) as an example, and for it the T-matrix of \( \pi \)-annihilation can be written as

\[ -\bar{u}(p'_1, s'_1)\gamma^5 v(p_2, s_2) - \frac{1}{q^2} g_{ch}^2 \bar{u}(p_2, s_2) \gamma^5 u(p, s_1). \]

Using Fierz identities for Dirac matrices, a four-fermion operator can be expressed definitely as a linear superposition of others with a changed sequence of spinors as follows.

\[ \langle \bar{a}O_4 b | \bar{c}O_4 d \rangle = \sum_{k=1,16} C_{4k} \langle \bar{a}O_4 d | \bar{c}O_4 b \rangle. \]

Since we are taking into account the contributions of the lowest order to the ‘full’ \( S \)-wave at this step, i.e., we consider the ‘full’ \( S \) wave cases (all of the quarks and antiquarks are in \( S \)-wave), so under static approximation the contributions from \( \sigma \)-meson \((J^{PC} = 0^{++})\) annihilation \((P\)-wave annihilation\) to the potential in the present
case (for $S$-wave states) are tiny. Thus we omit them here. The contributions from one-gluon annihilation to the potential between quark and anti-quark in momentum representation can be written as

$$V^q_{\text{anni}} = \frac{4\pi\alpha'_q}{s} \frac{1}{4} \left( \frac{16}{9} - \frac{1}{3}(\vec{\lambda}_i \cdot \vec{\lambda}_j) \right) \left( \frac{1}{2} + 1/2(\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) \left( -3/2 + 1/2(\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) .$$

(12)

Here $s = (p_1 + p_2)^2$ (in 4-dimension) in the propagator.

In an annihilation process, $s$ is time-like ($s > 0$), under static approximation so we have $s \simeq (m_1 + m_2)^2$, and when transferring the potential to space-time representation, we have

$$V^q_{\text{anni}} = \frac{4\pi\alpha'_q}{s} \frac{1}{4} \left( \frac{16}{9} - \frac{1}{3}(\vec{\lambda}_i \cdot \vec{\lambda}_j) \right) \left( \frac{1}{2} + 1/2(\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) \left( -3/2 + 1/2(\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) .$$

(13)

For $\pi$ annihilation, we have

$$V^\pi_{\text{anni}} = \frac{g^2_{\text{ch}}}{m^2_\pi - (m_i + m_j)^2} \frac{1}{4} \left( -1/2 - 1/2(\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) \left( 3/2 - 1/2(\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) \left( 1/3 + 1/2(\vec{\lambda}_i \cdot \vec{\lambda}_j) \right) .$$

(14)

B. Hamiltonian and the wave functions of the model

We follow the chiral quark model for multi-quark systems [1], which essentially is an effective theory on exchanges of Goldstone mesons, scalar meson $\sigma$ and gluons as well between quarks, but we extend it to antiquarks involved, so as to study the nucleon-antineucleon system with definite isospin (I) and spin (S).

As the first step and to the conjecture of BES collaboration [1], here only $S$-wave states of the nucleon pair are considered, i.e., the total orbital angular momentum $L = 0$, and we have $J = S$ (the total angular momentum comes from quark spin only). The most parameters are fixed by fitting baryon spectrum of the model [23]: The coupling constant $g^2_{\text{ch}}/4\pi$ is fixed by $g^2_{\text{NN}}/4\pi$, i.e., $g^2_{\text{ch}}/4\pi$ is related to $f^2_{q\bar{q}\pi}$ directly by $\frac{m^2_{\pi}}{4m_mN_m}g^2_{\pi}/4\pi = f^2_{q\bar{q}\pi}$, $(m_\pi, m_N$ are the masses of pion and nucleon, respectively), the one-gluon-exchange coupling constant $\alpha_c$ is determined by mass splitting of $\Delta$ and $N$, the confinement strength $a_c$ is fixed by the stability conditions of $N$, $V_0$ is fixed by the masses of $N$. In summary, the parameters in the model are listed in Table I. The units for $m_q, m_\sigma, m_\pi, V_0, b$ and $a_c$ are MeV, fm and $MeV \cdot f m^{-2}$ respectively.

Table I. The parameters of the model.

| $m_q$  | $b$  | $\alpha_s$ | $a_c$ | $g^2_{\text{ch}}/4\pi$ | $m_\sigma$ | $m_\pi$ | $\Lambda$ | $V_0$  |
|--------|------|------------|-------|-------------------------|------------|--------|--------|--------|
| 313.   | 0.60 | 0.95       | 7.3   | 0.59                    | 570.       | 138.   | 829.   | 72.5   |

To extend this model from $NN$ systems to $N\bar{N}$ (nucleon-antineucleon) systems based on the effective Lagrangian Eq. 4, now the Hamiltonians of the model may be written as

$$H_{p\bar{p}} = \sum_{i=1}^6 (m_i + \frac{V^2_{\text{anni}}}{2m_i}) - T_{CM}$$

+ \sum_{i=1,2,3} j=4,5,6 V(r_{ij}), \quad (15)

where $a_c$ is the confinement strength. $T_{CM}$ is the kinetic energy in the system of center mass. $V_{\text{exch}}^\pi$ and $V_{\text{exch}}^\sigma$ are the effective potential from $\pi$ and $\sigma$ exchanges between the couple of quark and antiquark, respectively. $V_{\text{anni}}^{NN}$ and $V_{\text{anni}}^{\bar{N}\bar{N}}$ are those from the $\pi$ and one-gluon annihilation, respectively.

Each of the wave functions for nucleon and antinucleon can be written as products of three parts respectively:

$$\Phi_p = \Phi^O_p \Phi^\rho_p \Phi^\sigma_p ; \quad \Phi_{\bar{p}} = \Phi^O_{\bar{p}} \Phi^\rho_{\bar{p}} \Phi^\sigma_{\bar{p}} .$$

For the orbital and color parts, the proton and antiproton have the same ‘internal motion’ wave function (here the motion for each center of mass has been removed):

$$\Phi^O_p = (\frac{3/4}{3m_p})^{3/4} (\frac{2}{3\pi})^{3/4} e^{-((\lambda^2)/(3b^2) + (\rho^2)/(4b^2))} ; \quad \Phi^O_{\bar{p}} = (\frac{3/4}{3m_{\bar{p}}})^{3/4} (\frac{2}{3\pi})^{3/4} e^{-((\lambda^2)/(3b^2) + (\rho^2)/(4b^2))} ,$$

here $\lambda, \rho$ are the Jacobi coordinates of the components in each of the two clusters (nucleon, anti-nucleon), respectively. For the color factors:

$$\Phi^\sigma_p = \frac{1}{\sqrt{6}} (r yb - r b y + y b r - y r b - b y r - b r y) ; \quad \Phi^\sigma_{\bar{p}} = \frac{1}{\sqrt{6}} (\bar{r} \bar{y} b - \bar{r} b \bar{y} + \bar{y} \bar{b} r - \bar{y} r \bar{b} - \bar{b} \bar{r} y - \bar{b} \bar{y} r) .$$
For the flavor factors for a $N\bar{N}$ system, there are four possibilities with definite quantum numbers $I$ and $J$. They are precisely (all symbols here have their usual meanings):
\[
\frac{1}{2}(p \uparrow \bar{p} \downarrow + p \downarrow \bar{p} \uparrow - n \uparrow \bar{n} \downarrow - n \downarrow \bar{n} \uparrow),
\text{for } I, J^{PC} = 1, -1; \\
\frac{1}{2}(p \uparrow \bar{p} \downarrow - p \downarrow \bar{p} \uparrow - n \uparrow \bar{n} \downarrow + n \downarrow \bar{n} \uparrow),
\text{for } I, J^{PC} = 0, 0-; \\
\frac{1}{2}(p \uparrow \bar{p} \downarrow + p \downarrow \bar{p} \uparrow + n \uparrow \bar{n} \downarrow + n \downarrow \bar{n} \uparrow),
\text{for } I, J^{PC} = 1, 0--; \\
\frac{1}{2}(p \uparrow \bar{p} \downarrow - p \downarrow \bar{p} \uparrow + n \uparrow \bar{n} \downarrow - n \downarrow \bar{n} \uparrow),
\text{for } I, J^{PC} = 0, 0++. \\
\]

If we relate the states to the observation at BES with the decay $J/\psi \rightarrow p\bar{p}+\gamma$, we are sure that the C-parity of the $p\bar{p}$ pair must be positive i.e. $C = +$. Furthermore if we restrict ourselves to take S-wave into account only, then the states with minimal total angular momentum can be $J^{PC} = 0^{-+} (L = 0, S = 0)$ only. Whereas, here we also consider the states with $J^{PC} = 1^{--} (L = 0, S = 1)$ for comparison. 

C. A outline of the resonating group method

For the the resonating group method [26], first of all, to write down the two-cluster wave function with the conventional ansatz (to factorize out the relative motion of mass centers of the two ‘clusters’) as follows,
\[
|\Psi_{pp}\rangle = |\Phi_p \Phi_p\rangle e^{i|JS}\chi(\vec{R}).
\]
where $|c\rangle = [222]$ gives the total color symmetry. $\chi(\vec{R})$ is relative motion wave function of the two clusters. $\Phi_p$ and $\Phi_p$ are the wave functions of the nucleon and antinucleon clusters in isospin and spin space only.

To the specific problem, Gaussian functions with various reference centers $S_i$ ($i=1...n$) are introduced, which $(S_i)$ play the ‘generating coordinates’ in the formalism,
\[
\chi_i(\vec{R}, S_i) = \left(\frac{3}{2\pi\beta^2}\right)^{3/4} e^{3/4}\exp\{-\frac{3}{4\beta^2}(\vec{R} - S_i)^2\}
\]
and the relative motion wave function of the two clusters of the quarks and antiquarks is expanded into partial waves
\[
\chi(\vec{R}) = \sum_L \chi^L(\vec{R})Y^{LM}(\vec{R})
\]
\[
= \sum_{L=1}^{2}\sum_{i=1}^{N} c_i^L \chi^L_i(\vec{R}, S_i)Y^{LM}(\vec{R})
\]
with
\[
\chi^L_i(\vec{R}, S_i) = \int d\Omega_{S_i} \chi_i(\vec{R}, S_i)Y^{LM}(\vec{S_i})
\]

\[= (\frac{3}{2\pi\beta^2})^{3/4} \int \exp\{-\frac{3}{4\beta^2}(\vec{R} - S_i)^2\}Y^{LM}(\vec{S_i})d\Omega_{S_i},\]

where $i_L$ is the L-th modified spherical Bessel function. For $L = 0$, one has $i_0(x) = sinh(x)/x$ (for a bound state).

According to the ansatz of the RGM and having the center of mass motion
\[
\Phi_{cm}(\vec{R}_{cm}) = \left(\frac{6}{\pi\beta^2}\right)^{2}\exp\{-\frac{3}{4\beta^2}(\vec{R}_{cm})^2\}
\]
included, finally the wave function of six quarks within the two-clusters accordingly can be written as
\[
\Psi_{6q} = A \sum_{k=1}^{6} \sum_{i=1}^{n} C_{k,i} \int d\Omega_{S_i} \prod_{a=1}^{3} \prod_{b=4}^{6} \chi_{a,b}(\vec{S_i})|\Phi_p \Phi_p\rangle e^{i|JS|}, \]

here $\chi_{a,b}(\vec{S_i})$ and $|\Phi_p \Phi_p\rangle$ are the single-particle orbital wave functions with different reference centers
\[
\psi_{a}(|\vec{S_i}\rangle)\psi_{b}(-|\vec{S_i}\rangle)|\Phi_p \Phi_p\rangle e^{i|JS|}, \quad (18)
\]

With the variational principle, one may obtain the RGM equation
\[
\int H(\vec{R}, \vec{R}')\chi(\vec{R}')d\vec{R}' = E \int \chi(\vec{R}')d\vec{R}' \quad (19)
\]
via the variation with respect to the relative motion wave function $\chi(\vec{R})$. With a re-formation, the RGM equation becomes an algebraic eigenvalue equation
\[
\sum_{j,k'} C_{j,k'}H_{j,k'}^{k'} = E \sum_{j} C_{j,k}N_{k,ij}^{k}, \quad (20)
\]

We should note here that for the nucleon-antinucleon system only the direct terms contribute.

III. NUMERICAL RESULTS AND DISCUSSIONS

There are four possible states for an $S$-wave nucleon-antinucleon system with different isospin $I$ and total angular momentum $J$ respectively. The spin, isospin and spin-isospin matrix elements of the interaction for the possible states are listed in Table II.

| $I$ | $J^{PC}$ | $1^{--}$ | $0^{+}$ | $0^{++}$ |
|-----|--------|--------|--------|--------|
| $\langle \sigma_i \cdot \sigma_j \rangle$ | -1/9 | 1/3 | -1/9 | 1/3 |
| $\langle \tau_i \cdot \tau_j \rangle$ | -1/9 | -1/9 | 1/3 | 1/3 |
| $\langle \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j \rangle$ | 25/81 | -75/81 | -75/81 | 25/9 |
For a qualitative analysis, firstly we neglect the nonlocal terms of the potential, so effective nucleon-
antinucleon potential can be obtained by the Born-Oppenheimer approximation. The dependence of the ‘potential’ (minus the ‘rest masses’ of the proton and the antiproton) on the separate distance $S_i$ is defined as the expectation value of the Hamiltonian at a separate distance $S_i$ as follows,

$$
V_{pp}(S_i) = \frac{\langle \Psi_{pp}(S_i) | H | \Psi_{pp}(S_i) \rangle}{\langle \Psi_{pp}(S_i) | \Psi_{pp}(S_i) \rangle} - \langle \Psi_p | H | \Psi_p \rangle. \tag{21}
$$

To see it precisely, we have calculated the nucleon-
antinucleon effective potential with various gluon annihilation coupling constant. For simplifying, only the results in the cases with $I, J = 1, 0$ are drawn in FIG.4. The contributions from $\pi$ and $\sigma$-exchange, $\pi$-annihilation and one-gluon annihilation with different isospin and spin are shown in FIG.5.

In the calculations, all the model parameters are determined by fitting nucleon-nucleon interaction and deuteron properties (for the annihilation contribution, the parameters are related to those for exchanges through the Lagrangian Eq. (3)). From QCD, we know, one-gluon running coupling constant would become small with the increasing of momentum transfer. Here we let gluon annihilation coupling constant $\alpha_s'$ equal to one-gluon exchange coupling constant as that $\alpha_s$ in Table I, firstly, then reduce $\alpha_s'$ appropriately to see the dependence on gluon annihilation coupling constants for the potential of the nucleon-antinucleon systems. For comparison, the case to ignore the contributions from annihilations are also computed.

FIG.4 contains 2 sub-figures. It shows the total nucleon-antinucleon effective potential with $\alpha_s' = \alpha_s = 0.95$ and $\alpha_s' = \frac{1}{4}\alpha_s = 0.316$ for $I, J = 1, 0$ state. FIG.5 contains 3 sub-figures. It shows the contributions from $\pi$ exchange, $\pi$ annihilation, $\sigma$ exchange and one-gluon annihilation for the states $I, J^{PC} = 1, 0^{-}; 0, 1^{-}; 0, 0^{-+}$ respectively. For the other states, we only present their minimum values of the effective ‘potential’ in Table III.

![FIG. 4: The effective potential for an S-wave nucleon-antinucleon system with different gluon annihilation coupling constant with state $I, J = 1, 0$.](image)

Table III. List of the minimum of the effective potential for the possible states (unit for $V_{\text{min}}$ in MeV).

| \(I,J\) | 11 | 10 | 01 | 00 |
|-------|----|----|----|----|
| $\alpha_s' = 0.947$ | -20.1 | -18.1 | -14.1 | -14.9 |
| $\alpha_s' = 0.4735$ | -31.7 | -32.3 | -32.1 | -16.2 |
| $\alpha_s' = 0.3156$ | -37.4 | -39.7 | -43.6 | -16.7 |
| without annihilation | -89.3 | -150.6 | -150.6 | -21.4 |

If the attraction of the effective potential is deep enough for forming bound states, we further do dynamical calculations in the framework of resonating group method exactly, and finally obtain corresponding relative motion wave functions and the mean square roots of the radius as well for the states.
two cases will emerge. The annihilation coupling constant, the difference of these BES collaboration observation. With the increasing of quantum numbers of the $I,J$ states, if it is in S-wave, are $I(J^{PC}) = 0$ or 1 (0$^{-+}$) but not $I(J^{PC}) = 0$ or 1 (1$^{-+}$). When dropping the annihilation contributions, it is clear in Table III that the state with quantum numbers $I,J$ and $\sigma$ has a larger attractive interaction than that with $I,J^{PC} = 1,1^{-+}$, that is consist with the BES collaboration observation. With the increasing of annihilation coupling constant, the difference of these two cases will emerge.

In summary, in this paper we extend to apply the chiral constituent quark model with quark-meson-gluon-degrees of the freedom to the nucleon-antinucleon systems with quantum numbers $(I,J^{PC}) = (1,1^{-+}); (1,0^{-+}); (0,1^{-+}); (0,0^{-+})$. Our results with the original model parameters as we can show the facts as below:

(a) For the $I,J^{PC} = 0,0^{-+}$ (S-wave) system, the repulsion from $\pi$-exchange is so strong that it can cancel the attraction from $\sigma$-exchange. No matter we adopt the annihilation coupling constant for the model in a reasonable range, there is no bound state at all.

(b) The situation for the system with the quantum numbers $I,J^{PC} = 0,1^{-+}$ and $I,J^{PC} = 1,0^{-+}$ for the S-wave nucleon and antinucleon system.

(c) By dynamical calculations we show that the bound states with $I,J^{PC} = 0,1^{-+}; 1,0^{-+}; 1,1^{-+}$ for an S-wave nucleon and antinucleon system may exist if the annihilation coupling constant $\alpha'_s$ is suppressed for certain reason appropriately. For instance, if letting $\alpha'_s = 0.3156$ for $I,J^{PC} = 1,0^{-+}$, one may obtain a ‘tightly bound state’ at $E=1877.80$ MeV and $\sqrt{\langle r^2 \rangle} = 2.06 fm$ (see FIG.6). If we had dropped the annihilation contributions at all, except $I,J^{PC} = 0,0^{-+}$, in the other three cases $(I,J^{PC} = 1,1^{-+}; 1,0^{-+}; 0,1^{-+})$ the nucleon and antinucleon system might be bound tightly within a size not greater than 1.1 fm. Moreover, the binding energy of the $I,J^{PC} = 0,1^{-+}$ and $I,J^{PC} = 1,0^{-+}$ states would be several tens of MeV greater than that of the state $I,J^{PC} = 1,1^{-+}$ and the mean squared root of the radius of the $I,J^{PC} = 1,1^{-+}$ would be a little larger than that of $I,J^{PC} = 1,0^{-+}; 0,1^{-+}$ etc.

In the early 1990s, Dover et al. constructed $\bar{N}N$ potential model from the $N,N$ effective potential by G-parity transformation accordingly, and predicted lower-lying isospin I=0 natural-parity $J^{PC} = 0^{-+},1^{-+}, 2^{++}$ bound states and a few isospin I=1 states, such as $0^{-+},1^{-+}$, also. Their predictions are consistent with

![FIG. 5: The contributions to the effective potential from $\pi$ and $\sigma$-exchange, $\pi$-annihilation and one-gluon annihilation for the states $I,J = 1,0; 0,1; 0,0$. Square points denote the contribution from gluon annihilation, circle ones for pion annihilation, up-triangle ones for pion exchange and down-triangle ones for $\sigma$ exchange.](image)

![FIG. 6: Relative motion wave functions for the states $I,J^{PC} = 0,1^{-+}; 1,0^{-+}$ and $1,1^{-+}$ with $\alpha'_s = 0.3156$ (the left figure), and without annihilation contributions at all (the right figure).](image)
our results, if one compare those of an S-wave nucleon and antinucleon system only. It is interesting to note that there are so many similarities qualitatively, although Dover’s model and ours are based on very different level, especially, to deal with the short-range behavior of $N\bar{N}$ in a very different way in the two approaches (in Dover approach, an arbitrary square-well cutoff is applied for the unknown short-range behavior of $N\bar{N}$ potential).

Finally, we should note here that this work is very preliminary in studying the nucleon-antinucleon interaction in the framework of constituent quark model. There are quite a lot of factors for the nucleon-antinucleon interaction which should be investigated carefully, especially, as a very strong assumption, we ignore the couple channel (such as the multi-pion and other mesons being involved etc) effects at all, although there are a lot of channels which should be considered $^{28}$. Furthermore, the adopted resonating group method also should be tested thoroughly. Even though, we still would like to emphasize that searching for the $N\bar{N}$ bound state with the quantum number $I,J^{PC}=1,0^{-+}$ through various multi-meson decay channels is crucial to confirm the multi-quark conjecture. Even though considering our results and the BES observation, we would like to say that, if there is really an S-wave $p\bar{p}$ bound state, then its quantum number is likely to be $I,J^{PC}=1,0^{-+}$. As a consequence, similar enhancements in the decays, such as $B^+\to n\bar{u}K^+$, $B^+\to p\bar{u}K_S^0$; and $B^0\to n\bar{u}D^0$, $B^0\to p\bar{u}D^+$ etc in Belle and Babar at B-factories, and such as $J/\psi\to n\bar{u}\gamma$ etc in BES at BEPC should be observed, although there are technical difficulties for the observations.

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