Theoretical prediction of gold vein location in deposits originated by a wall magma intrusion

Abstract. The isotherm time-evolution resulting from the intrusion of a hot dike in a cold rock is analyzed considering the general case of nonvertical walls. This is applied to the theoretical prediction of the gold veins location due to isothermal evolution. As in previous treatments earth surface effects are considered and the gold veins are determined by the envelope of the isotherms. The locations of the gold veins in the Callao mines of Venezuela are now well predicted. The new treatment is now more elaborated and complex that in the case of vertical walls, performed in previous papers, but it is more adequate to the real cases as the one in El Callao, where the wall is not vertical.

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1. Introduction
In a previous paper [1], calculations of temperatures in and around an intrusive dike was performed, assuming that the contact surface between the magma and the cold host rocks, was a vertical plane. The real case in El Callao gold mine in Venezuela is that this wall is inclined in 15° with respect to the vertical. In the present paper a theoretical treatment is presented considering this inclination. In this sense, a little better approximation of the location of the gold veins is given, compared with that given previously [1]. Here, the 400°C and 500°C isotherms are studied in detail, because of theirs importance in the determination of the gold vein sites. As it is usual, we consider that the magma heat propagates normal to the surface separating magma and cold rocks [2]. Our treatment will be 2-D, because the earth magma isotherm is considered, instead of the simplified 1-D diffusion problem considered by other authors [3]. Here it is assumed that the magma fill at the beginning all the semi-infinite space on one side of the wall separating magma and cold rock. This is like in the paper by Aldaz and Martin, but different to a most recent paper on this problem [4].

In this paper the following procedure will be followed:
(i) First, the basis of the theoretical description and the main assumptions will be presented.
(ii) Second, the solution and the mathematical procedures will be described in a general way.
(iii) Numerical calculations and comparison with the real gold vein sites in El Callao, Venezuela, will be followed.
(iv) Finally this work will finish with a short summary and conclusion.
2. Theoretical treatment and main assumptions

The assumptions in this work are: (1) The temperature of the magma at the moment of the intrusion \((t = 0)\) is uniform and equal to \(T_m\); (2) The temperature \(T_0\) of the cold rock is also uniform at time \(t = 0\); (3) The earth surface is a plane of uniform temperature \(T_0\) at any time; (4) The evolution of the cold rock temperature is studied at any time. The magma is brought suddenly in contact with the cold rock at time \(t = 0\), and the changes in temperature are due to cooling and solidification of the magma; (5) A unique melting point temperature \(T_m\) is assumed and the degree of overheating is considered small, assumption that seems well justified [5]. All the temperatures here will be given in centigrades degrees.

Besides our previous considerations, the following quantities will be considered well defined. Latent heat of fusion \(L\); density of the country rocks \(\rho\); its specific heat \(c\), and thermal conductivity \(k\). In the present treatment convection in the magma will not taken into account, neither heat transport due to hydrothermal currents or other fluids.

The starting point in our treatment is the heat conduction equation

\[
\frac{\partial T}{\partial t} = \kappa \nabla^2 T, \tag{1}
\]

where \(T\) is the local temperature of the country rock and \(\kappa\) is the thermal diffusivity, given by \(\kappa = k/\rho c\).

The main equation (1) is derived from the heat flow \(\vec{q}\) per unit area and unit time, due to temperature gradient

\[
\vec{q} = k \nabla T, \tag{2}
\]

and the energy balance, that shows, that the heat changes in the medium must be compensated by flux diverging from the system

\[
\nabla \cdot \vec{q} + c\rho \frac{\partial T}{\partial t} = 0. \tag{3}
\]

Now, dimensionless temperature is introduced as

\[
\theta = \frac{T - T_0}{T_m - T_0}, \tag{4}
\]

given the new equation

\[
\frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta. \tag{5}
\]

Independent dimensionless similarity variables are also defined as

\[
\eta = \frac{x}{\sqrt{2\kappa t}}, \quad \varsigma = \frac{y}{\sqrt{2\kappa t}}, \tag{6}
\]

where \(x\) is measured along the earth surface on the line perpendicular to the intersection line at \(t = 0\), between magma and country rock, and \(y\) is measured along the down vertical line starting on the earth surface. In this way, the diffusion equation takes the form

\[
\eta \frac{\partial \theta}{\partial \eta} + \varsigma \frac{\partial \theta}{\partial \varsigma} = -\frac{1}{2} \left[ \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial^2 \theta}{\partial \varsigma^2} \right]. \tag{7}
\]

The solutions of (7) now have to accomplish with the boundary conditions determined by the dike, which means that \(\theta\) should be one in the magma contact surface. For \(t = 0\), this surface is a plane forming an angle \(\alpha\), with the a vertical plane \((x,y)\). As in previous works [1,3,4] symmetry conditions along the \(z\) axis are assumed. This is the reason that a 2-D treatment is performed.
If this condition is not verified, 3-D treatment has to be performed, which becomes much more complicated.

It is important to point out that this boundary condition is the main one, because the temperatures are mainly determined by the solidification of the dike. There are also the condition that the earth is isotherm also, but the influence of this boundary is no so important and decisive as the previous one.

From here on, the mathematical analysis becomes much more complex and difficult than in previous works [1,4].

3. Mathematical treatment

Several changes of variables have to be performed in order to obtain analytic solution accomplishing the boundary conditions.

First, let us consider a non-orthogonal reference frame, such that the “˜y” axis of coordinates form an angle 2˜α with the vertical line or y-axis. The new coordinates will be ˜x and ˜y, such that

\[
\begin{align*}
\dot{x} &= x - y \tan(2\bar{\alpha}), \quad \dot{y} = y \sec(2\bar{\alpha}) \\
x &= \dot{x} + \dot{y} \sin(2\bar{\alpha}), \quad y = \dot{y} \cos(2\bar{\alpha})
\end{align*}
\]

(8)

(9)

It is important to point out that ˜α is different of α, though its values are nearby.

In similarity variables, the previous change of variables are written as

\[
\begin{align*}
\tilde{\eta} &= \frac{\dot{x}}{\sqrt{2\kappa t}}, \quad \tilde{\varsigma} = \frac{\dot{y}}{\sqrt{2\kappa t}} \\
\eta &= \tilde{\eta} - \varsigma \tan(2\bar{\alpha}), \quad \varsigma = \varsigma \sec(2\bar{\alpha}) \\
\end{align*}
\]

(10)

(11)

(12)

The diffusion equation is now written as

\[
\frac{\partial \dot{\theta}}{\partial \tilde{\eta}} + \frac{\partial \dot{\theta}}{\partial \tilde{\varsigma}} = \frac{1}{2} \left[ \sec^2(2\bar{\alpha}) \frac{\partial^2 \dot{\theta}}{\partial \tilde{\eta}^2} - 2 \tan(2\bar{\alpha}) \sec(2\bar{\alpha}) \frac{\partial \dot{\theta}}{\partial \tilde{\eta} \partial \tilde{\varsigma}} + \sec^2(2\bar{\alpha}) \frac{\partial^2 \dot{\theta}}{\partial \tilde{\varsigma}^2} \right]
\]

(13)

with \(\theta(\eta,\varsigma) = \dot{\theta}(\tilde{\eta},\tilde{\varsigma})\). Unfortunatly in the equation (13), the term \(\frac{\partial^2 \dot{\theta}}{\partial \tilde{\eta} \partial \tilde{\varsigma}}\) hinder the possiblity of making an expansion in eigenfunctions as it was done in previous tratments [1,4]. In order to avoid this term a new change of variables must be done, as follows,

\[
\begin{align*}
u &= \tilde{\eta} \cos(\bar{\alpha}) + \varsigma \sin(\bar{\alpha}), \quad \eta = \tilde{\eta} \sin(\bar{\alpha}) + \varsigma \cos(\bar{\alpha}),
\end{align*}
\]

(14)

and

\[
\begin{align*}
\dot{\eta} &= \frac{\cos(\bar{\alpha})}{\cos(2\bar{\alpha})} u - \frac{\sin(\bar{\alpha})}{\cos(2\bar{\alpha})} v, \quad \dot{\varsigma} = -\frac{\sin(\bar{\alpha})}{\cos(2\bar{\alpha})} u + \frac{\cos(\bar{\alpha})}{\cos(2\bar{\alpha})} v
\end{align*}
\]

(15)

So, with \(\bar{\theta}(\eta,\varsigma) = \Theta(u,v)\), one appropriate partial differential equation appears:

\[
\frac{\partial^2 \Theta}{\partial u^2} + \frac{\partial^2 \Theta}{\partial v^2} = -2u \frac{\partial \Theta}{\partial u} - 2v \frac{\partial \Theta}{\partial v}
\]

(16)

Now, it is possible to proceed to separate variables writting \(\Theta(u,v) = \phi(u)\psi(v)\), and from here it is obtained

\[
\frac{1}{2\phi} \frac{d^2 \phi}{du^2} + \frac{d \phi}{du} = -\left( \frac{1}{2\psi} \frac{d^2 \psi}{dv^2} + \frac{d \psi}{dv} \right) = \frac{\beta}{2},
\]

(17)
where $\beta$ is a constant. The same analysis performed at the infinite in the paper by Aldaz and Martin [1] leads to that $\beta$ must be zero, and the solution for $\Theta$ is

$$\Theta(u, v) = \frac{erf(u) erf(v)}{erf(-\lambda'_1) erf(\lambda'_2)},$$

where the integration constants are now replaced by $\lambda'_1$ and $\lambda'_2$, which will be determined by the following process

$$\lambda'_1 = \cos(\tilde{\alpha}) \tilde{\lambda}_1 + \sin(\tilde{\alpha}) \tilde{\lambda}_2, \quad \lambda'_2 = \sin(\tilde{\alpha}) \tilde{\lambda}_1 + \cos(\tilde{\alpha}) \tilde{\lambda}_2,$$

$$\tilde{\lambda}_1 = \lambda_1 - \lambda_2 \tan(2\tilde{\alpha}), \quad \tilde{\lambda}_2 = \lambda_2 \sec(2\tilde{\alpha}),$$

where $\lambda_1$ and $\lambda_2$ are calculated as it was shown in Aldaz-Martin paper [1], by the equations

$$\frac{L}{c} \sqrt{\frac{\pi}{T_m - T_0}} = \frac{e^{-\lambda_1^2}}{\lambda_1 erf(-\lambda_1)}$$

and

$$\frac{L}{c} \sqrt{\frac{\pi}{T_m - T_0}} = \frac{e^{-\lambda_2^2}}{\lambda_2 erf(\lambda_2)},$$

respectively.

The last unknown is the angle $\tilde{\alpha}$, which has to be determined by the condition that the surface $u = 0$, must be coincident with the magma-cold rock contact of the surface dike at time $t = 0$. This means that $x/y = \tan(\alpha) = \eta/z$. Now, $u = 0$ leads to

$$0 = \tilde{\eta}\cos(\tilde{\alpha}) + \tilde{\zeta}\sin(\tilde{\alpha}) = \eta\cos(\tilde{\alpha}) + [\sec(2\tilde{\alpha})\sin(\tilde{\alpha}) - \tan(2\tilde{\alpha})\cos(\tilde{\alpha})] \zeta.$$ (22)

From here the following equation for $\tilde{\alpha}$ is obtained

$$\tan(2\tilde{\alpha}) - \sec(2\tilde{\alpha})\tan(\tilde{\alpha}) = \tan(\alpha)$$

(23)

If $\alpha$ is measured in radians, then for $\alpha$ small the value of $\tilde{\alpha}$ approaches to $\alpha$ plus terms of order $\alpha^2$. In the case of El Callao mines, the angle $\alpha$ is 15°, and solving the equation (23), $\tilde{\alpha}$ is obtained as $\tilde{\alpha} = 13.72°$, the difference with $\alpha$ is about 10%.

4. Isotherm envelope

Let us consider the envelope of the isotherms for a given temperature $T_1$ (e.g. $T = 400^\circ C$, $500^\circ C$, etc.), the curve tangent to all the isotherms at this temperature will be the envelope of these isotherms. The procedure is similar to that previously followed in the previous works [1,4]. The result is that the isotherm envelope is a straight line defined by the equation:

$$x = m_1 y = \tan(\gamma) y,$$

(24)

where

$$\tan(\gamma) = m_1 = \tan(2\tilde{\alpha}) + \sec(2\tilde{\alpha}) \left[ \frac{v_1 \sin(\tilde{\alpha}) - u_1 \cos(\tilde{\alpha})}{u_1 \sin(\tilde{\alpha}) - v_1 \cos(\tilde{\alpha})} \right],$$

(25)

and $u_1$ and $v_1$ are given by the equations (26) and (27), given below, that is:

$$u_1 e^{-u_1^2} \text{erf}(v_1) = v_1 e^{-v_1^2} \text{erf}(u_1)$$

(26)

and

$$T_1 - T_0 = \alpha' \text{erf}(u_1) \text{erf}(v_1),$$

(27)
with

$$\alpha' = \frac{T_m - T_0}{erfc(-\lambda_1) erf(\lambda_2)}.$$ 

The graph of the isotherms and envelope are show in Figure 1 for the case of $T_1 = 400^\circ C$ and $\alpha = 15^\circ$.

**Figure 1.** In this figure, the cases $T_1 = 400^\circ C$ and $\alpha = 15^\circ$ are presented.

In the next graph it is shown the location of the gold mine galleries in El Callao, Venezuela, compared with the theoretical predictions.

**Figure 2.** In this figure, the location of the gold mine galleries in El Callao, Venezuela, vs. theoretical predictions are presented.
The theoretical results compare very well with the actual location of the gold mine galleries (Colombia mine) in El Callao, Venezuela, operated by Minerven Company. The results are in a little better agreement than previous results [1], and are justified in a much better way, because now there is not need to add the angle $\alpha$ of the dike to the theoretical results, as it was done on this previous work [1]. Here everything is coming out in a natural way, and the results are justified in a much better way. Now, the angle $\gamma$ is 43° compared with 48° = (33° + 15°), which was the actual angle in the Colombia mine in El Callao, Venezuela. The previous angle was 27°, which becomes 42°, once the 15° are added [1]. The difference is only one degree in the right direction, but the important point is that now the result is coming directly from the equations.

5. Conclusion
The analysis here presented of the heat diffusion due to the intrusion of a hot non-vertical dike in a cold rock, leads to a prediction of gold vein location almost in agreement with those of actual Colombia mine in El Callao, Venezuela. The analysis here done now is very general, and it does not need the assumption of vertical dikes. Our results almost coincide with those previously published [1] in the case of vertical dikes. In our treatment the earth is also considered as an isotherm, that is, it is a 2-D analysis. However, our analysis requires more elaborated mathematical tools than all the previous ones. The isotherm considered by the gold deposition, are those between 400°C and 500°C. However, the isotherm does not give the gold vein location, but instead of that, the envelope of the isotherm are the important ones.

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