Study on the shape change of crack growth rate curve under different forms of additional stress field

Wang Jing\textsuperscript{1,a}, Song Chenxi\textsuperscript{2,b}, Shao Xiaolin\textsuperscript{3,c}

\textsuperscript{1}Beijing University of Technology, Beijing 100022 China
\textsuperscript{2}Beijing University of Technology, Beijing 100022 China
\textsuperscript{3}Beijing University of Technology, Beijing 100022 China
\textsuperscript{a}wjing@bjut.edu.cn, \textsuperscript{b}2234986505@qq.com, \textsuperscript{c}1183355938@qq.com

Abstract. In this paper, FRANC3D is used to simulate the fatigue crack growth process under different additional stress fields to explore the key mechanical parameters influencing the fatigue crack growth curve. For finite large plate model with penetrating crack, the crack growth rate is calculated under different forms and different values of applied load and additional stress coupling, the fatigue life curves, as well as crack growth rate curves and Paris formulas under different applied loads are obtained. Through the analysis, the relationship between the additional stress field and the parameters in Paris formula of crack growth rate is obtained. Because of the existence of the additional stress field, the stress intensity factor amplitude of crack tip changes, which is the root cause of the change of crack growth rate curve shape.

1. Introduction

The actual stress field of the crack tip is the result of the interaction between the stress field generated by the applied load and the additional stress field. Establishing a numerical model of fatigue crack growth and simulating the process of fatigue crack propagation with additional stress is the theoretical basis for safety evaluation.

The main factors affecting fatigue crack growth are stress ratio $R$, maximum load $P_{\text{max}}$ and loading frequency $f$. As for the numerical simulation of fatigue crack propagation, Wang Yongwei \textsuperscript{[1]} successfully simulated the two-dimensional plane crack propagation path based on the Paris formula and the maximum energy release rate criterion. Yu Tiantang \textsuperscript{[2]} used XFEM to give a model and basic principle for simulating three-dimensional crack growth. Jia Xueming \textsuperscript{[3]} developed an adaptive system for crack growth simulation based on the FRANC3D.

In this paper, FRANC3D is used to simulate the fatigue crack growth process with additional stress field to obtain the key mechanical parameters $C$ and $n$. Through the data processing method of GB/T 6398-2017 Standard test method for fatigue crack growth rates of metallic materials \textsuperscript{[4]}, the crack growth rates under different applied stress and additional stress filed are obtained, and the curve shape of fatigue crack growth rate is analyzed.

2. Establishment of numerical model

2.1. Introduction to mechanical model

The model studied in this section is a plate with prefabricated unilateral penetrating crack, whose size
is 100mm long, 50mm wide and 10mm thick, and the length of the prefabricated crack is 15mm, elastic modulus \( E \) of the plate is 200GPa, and poisson's ratio is 0.3. Alternating loads are applied on the upper side of the plate, and fixed constraint is applied on the bottom. Three different loading modes are shown in Fig. 1.

There are three types of the additional stress conditions. The first type of additional stress is shown in Table 1; The second type is a finite uniform stress field, stress direction is consistent with the applied load, the range starts from the crack tip, and the length along the crack direction is 15mm, the maximum values are 300MPa, 600MPa and 900MPa respectively; The third type of additional stress is a finite large exponential distributed stress field, direction and range are the same as the second type, the values are \( \sigma = 300 \times e^{-\frac{x}{20}} \), \( \sigma = 600 \times e^{-\frac{x}{20}} \), \( \sigma = 900 \times e^{-\frac{x}{20}} \) respectively.

The simulation process is as follows: different applied stress fields are set in the software to obtain the stress intensity factor curve along the direction path of crack, and the crack growth \( a-N \) curve.

| Number | Applied uniform load \( P_{\text{max}} \) (MPa) | Stress ratio of applied uniform load \( R \) | Applied uniform load amplitude \( \sigma_{\text{applied}} \) (MPa) | Additional stress \( \sigma_{\text{additional}} \) (MPa) | Additional stress ratio \( R \) | Additional stress amplitude \( \Delta \sigma \) (MPa) | Actual stress amplitude \( \Delta \sigma' \) (MPa) |
|--------|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Case 1 | 100                             | 0.5             | 50              | 0               | 1               | 0               | 50              |
| Case 2 | 100                             | 0.5             | 50              | 100             | 1               | 0               | 50              |
| Case 3 | 100                             | 0.5             | 50              | 100             | 0.5             | 50              | 100             |

2.2. Results of numerical simulation

Through calculation, the crack growth life \( a-N \) curve of the first type of loading modes are shown in Fig. 2, and the logarithmic form of crack growth rate are shown in Fig. 3.
The a-N curve of crack growth life with uniformed additional stress field are shown in Fig. 4. Using the numerical method of crack growth rate property with additional stress, the crack growth rate curves are obtained, as shown in Fig. 5.

The a-N curve of crack growth life with the exponential distributed additional stress field are shown in Fig. 6. Using the numerical method of crack growth rate property with additional stress, the crack growth rate curves are obtained, as shown in Fig. 7.

### Table 2 Crack propagation rate parameter

| Loading mode | Maximum additional stress (MPa) | Stress ratio | C (10⁻¹⁰) | n |
|--------------|--------------------------------|--------------|-----------|---|
| Case 1       | 0                              | no           | 7.932     | 1.634 |
| Case 2       | 100                            | 1            | 7.978     | 1.632 |
| Case 3       | 100                            | 0.5          | 20.75     | 1.672 |
| Case 4       | 300                            | 0.5          | 0.946     | 2.502 |
| Case 5       | 600                            | 0.5          | 0.488     | 2.855 |
| Case 6       | 900                            | 0.5          | 0.303     | 3.116 |
| Case 7       | 300                            | 0.5          | 1.741     | 2.301 |
| Case 8       | 600                            | 0.5          | 1.097     | 2.578 |
3. Analysis of simulation results

3.1. Result analysis of different additional stress fields
Numerical simulation of fatigue crack growth behavior under different applied stress fields was conducted, and the following results were obtained: As additional internal stress field to a constant value, the crack growth life of a - N curve and crack growth rate does not change, this is because the actual the range of stress intensity factor of the crack tip at this time is generated only by applied load, which is the same as the range of stress intensity factor without additional stress. When the internal additional stress also fluctuates with the applied load, the actual stress intensity factor range at the crack tip is generated by the interaction of applied load and internal additional stress field. For type I through cracks, $\Delta K$ has a linear relationship with the stress amplitude $\Delta \sigma$, $\Delta K = \Delta \sigma \sqrt{\pi a}$ so when $\Delta \sigma$ changes, the crack growth rate curve will only move in parallel.

3.2. Analysis of the result of the uniformly distributed additional stress field

3.2.1. Discussion on a-N curve of crack growth life
It can be seen from Fig. 4 that crack tip stress field has a certain influence on the crack growth life a-N curve. With the same stress ratio, when the stress fields of crack tip are different, the shape of a-N curve is similar, but the crack growth life is significantly different. With the same length of crack growth, the crack growth life of the three cases is different from that without additional stress. It can be seen that the additional stress will reduce the crack growth life, the larger the additional stress, the smaller the crack growth life.

3.2.2. Discussion on crack growth rate
It can be clearly seen from Fig. 5 that with the increase of additional internal stress, the crack growth rate curve gradually moves up, indicating that the crack growth amount in each cycle gradually increases under the same the range of stress intensity factor.

When the range of the additional internal stress changes, the shape of the crack growth rate curve also changes significantly: Not only the intercept but also the slope of the crack growth rate curve in the double log space changed obviously. This is because the expression of the stress intensity factor caused by the additional internal force also changes, namely:

$$K = f(a)\sigma S I D \sqrt{\pi a}$$

In the above formula, $f(a)$ is the correction coefficient of the stress intensity factor, and $f(a)$ is obtained by consulting the manual of stress intensity factor, that $f(a)$ is $1.12 - 0.23a/h + 10.6(a/h)^2 - 21.7(a/h)^3 + 30.4(a/h)^4$. Where $a$ is the crack length and $h$ is the half length of the plate.

As can be seen from table 2, with the increase of additional load, parameter $n$ in the Paris formula keeps increasing, which means that the acceleration trend of crack growth rate keeps increasing. In the form of the curve, the slope changes most obviously at the initial stage of the crack growth, but the slope of the crack growth rate curve becomes stable gradually with the continuous growth of the crack.

3.3. Analysis of the results of the exponentially distributed additional stress field

3.3.1. Discussion on a-N curve of crack growth life
It can be seen from figure 6 that the additional stress field has a certain influence on the a-N curve of crack growth life. For the same stress ratio and different additional stress fields, the shape of a-N curve is similar, but the crack growth life is significantly different. It can be seen that the additional stress...
field reduces the crack growth life, the larger the additional stress field is, the smaller the crack growth life will be.

3.3.2. Discussion on crack growth rate
When the crack begins to expand, the additional stress has a certain influence on the crack growth rate. It can be seen from figure 7 that under the same stress intensity factor amplitude, different examples have different crack growth rates. Therefore, the additional stress field has a certain influence on the crack growth rate.

3.4. Comparison of fatigue crack growth behavior under different additional stress fields
Comparison of crack propagation life curves among three cases are shown in Fig.8 and Fig.9. It can be seen from the comparison of two curves with additional stress fields that when the maximum additional stress is the same, a-N curve and da/dN-ΔK curve of uniform additional stress field (case 5) are similar to those of the exponential distributed stress field (case 8), the first half of the curve almost completely coincides, only the second half of the curve has certain differences: The a-N curve of exponential additional stress field (case 8) is slightly shorter than that of uniform additional stress field (case 5), and the crack growth rate curve is also slightly lower. With the crack growth, the stress of exponential additional stress field gradually reduce, the proportion of K caused by the additional stress in Kact becomes smaller and smaller, so the slope of the crack growth rate curve of this case is smaller than that of the uniform distribution type.

4. Conclusion
The following conclusions are obtained by simulating the fatigue crack growth behavior under different forms and sizes of applied loads and additional stress fields:

(1) When the additional stress field is constant and does not fluctuate with the fluctuation of applied load, the simulated crack growth rate curve is only the translation of the crack growth rate curve without additional stress under the dual log coordinates.

(2) When the additional stress field is a finite large field located at crack tip, and the stress field fluctuates synchronously with the applied load, both the uniform distributed stress field and the exponential distributed stress field can change the slope and intercept of the curve in the double log space. With the increase of the value of additional stress, the crack growth rate increases gradually.

(3) When the form of additional stress field differently, the crack growth rate will be affected. With the crack propagation, crack growth rate of exponential additional stress field will gradually be slower than that of uniform distributed type, this is because the proportion of stress intensity factor generated by the additional stress is smaller in the actual stress intensity factor.

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