The limiting sheet diameter prediction model for cup-shaped part drawing process with diverse mould assemblage based on RSM

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Abstract. In order to ensure unbroken production in cup-shaped part deep drawing process with diverse mould assemblage, a prediction model of limiting sheet diameter based on response surface method (RSM) was presented. The simulated samples were used to fit the sheet diameter RSM prediction model, and the multiple determination coefficient of RSM was reached to 0.9. Setting the minimum sheet thickness to the critical value, the RSM will become a limiting sheet diameter prediction model. It shows a close agreement with FE simulated results through the test of examples. It can be concluded that presented RSM not only has higher efficiency, but also has higher accuracy. It will provide an early quality forecasting before actual deep drawing production.

1. Introduction
Sheet metal deep drawing is a widely used in manufacturing process, it is very suitable for mass production. Efficacious parameters for the defects of different types, which may appear in the forming parts can be divided into three main categories [1]: (1) material properties such as yield stress, work hardening coefficient, anisotropic coefficient, blank dimensions, and thickness of blank; (2) tool properties such as punch radius, die radius, and clearance; (3) process parameters such as blank holder force, friction coefficient, type and position of lubricant, strain rate, and pressure.

To achieve a desirable quality with a minimum cost, traditional process design has been performed in a time-consuming and costly way [2], usually involving a series of design modifications and test. As an efficient engineering tool, finite element (FE) simulation has become more and more prevalent, it allows us to precisely predict a forming process by detecting defects such as wrinkling and fracture in an early stage, thereby, reducing design and test costs to a considerable extent [3]. Utilization surrogate approximate model to map the relation between analysis results and input variables to replace the FE analysis is an efficient and convenient method, it cost less analysis times than direct simulation. Many kinds of surrogate model can achieve the approximation for the deep drawing process, such as artificial neural network (ANN) [4], radial basis function (RBF) [5] and response surface method (RSM) [6], et al.

In deep drawing process, the blank holder plays a key role in adjustment of metal flow in to the die cavity. Moreover, the quality of drawn parts is extremely affected by this flow [7]. Srirat, et al. [8] optimized the variable blank holder force (VBHF) trajectory and tools motion simultaneous by a sequential approximate optimization with RBF network. Ehsan Karajibani, et al. [9] investigated the formability of two layer (aluminum-st12 steel) sheets in the deep drawing process through numerical simulations and experiments. Jae-Jun Lee and Gyung-Jin Park [10] optimized the structural
parameters and process parameters to avoid defects in sheet metal forming production. To observe above articles, it can be found that the research is insufficient for limiting sheet diameter in cup-shaped part deep drawing process.

At present, there are some moulds with diverse arc radius used in cup-shaped part drawing process. The diverse arc radius is the only difference during the same type moulds. Using the diverse exchange and assemblage of moulds, diverse cup-shaped parts can be produced. As we all know, in the deep drawing process, the required process conditions for different assemblage of mould are not the same. Users need to carry out a large number of repeated testing to determine the best drawing process for the eligible product, the efficiency is lower. Thus, it is necessary to develop a speedy and accurate prediction model to evaluate the drawing quality. In this paper, the precise FE simulation result was used to obtain the initial training samples, and then, based on the RSM surrogate model, the sheet diameter prediction model was established. Setting the minimum sheet thickness to the critical value, the RSM will become a limiting sheet diameter prediction model. Through the test examples, the RSM shows higher prediction accuracy. This prediction model can provide an early forecasting before actual deep drawing production, reducing the rate of flawed products and labor consumption of repeated testing, it is very helpful to improve the production efficiency and develop the numerical control system.

2. Limiting sheet diameter, RSM and material

2.1 Limiting sheet diameter
The limiting sheet diameter is a diameter of just before an edge crack occurs. In a drawing process, if the sheet diameter is more than the limiting sheet diameter, the drawing part will be occurred fracture defect. The assembly method of cup-shaped part in a deep drawing process is shown as figure 1.

![Figure 1. Schematic diagram of cup-shaped part deep drawing process.](image)

The punch and die are installed on the bases respectively. In the deep drawing process, change the punch with diverse radius and initial sheet with diverse diameter, will obtain diverse cup-shaped part; change the die with diverse radius and blank holder force, will adjust the material flow condition, avoiding wrinkling and fracture. The combination of blank holder force (BHF), sheet diameter (D), punch radius (Rp) and die radius (Rd) in every group is corresponding to a drawing process; different drawing process will produce the parts with different quality. Thus, to develop a prediction model which can speedily and accurately evaluate the limiting sheet diameter, will reduce the repeated testing before drawing, it is significant to improve the production efficiency. In this paper, the BHF, D, Rp and Rd are considered as the research parameter, and their ranges and levels are shown in Table 1.

| Table 1. Variable parameters with their levels. |
|----------|----------|----------|-----------|-----------|
| levels   | BHF(kN)  | D(mm)    | Rp(mm)    | Rd(mm)    |
| 1        | 1        | 50       | 3         | 3         |
| 2        | 4        | 53       | 4         | 4         |
| 3        | 7        | 56       | 5         | 5         |
| 4        | 10       | ---      | ---       | ---       |
2.2 RSM
RSM provides an approximate relationship between the input variables x and their response y. In this paper, the approximate function of the response y is considered as second order polynomial:

\[ y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{i<j} \beta_{ij} x_i x_j + \varepsilon \]

In this paper, the accuracy of RSM was evaluated by multiple determination coefficients \((R^2)\):

\[ R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \]

where \( n \) is the number of the samples, \( y \) is the response of RSM model, \( \bar{y} \) is the actual value of the sample outputs, and the \( \bar{y} \) is the mean value of these actual outputs.

The more close to 1 the \( R^2 \) is, the higher the RSM prediction accuracy is.

2.3 Material and fracture criterion
In this paper, the low carton steel DC06 with 0.6 mm thickness was used to deep draw, the mechanical properties of material are obtained from a series of tensile tests and shown in Table 2, where \( E \) and \( \nu \) are young’s modulus and poisson’s ratio respectively. In general, the true stress - true strain curve is fitted to exponential function type, in this paper, it was considered to the formula \( \sigma = k (\varepsilon_0 + \varepsilon)^n \) \((k_0 = 0.02)\). Material anisotropic property uses the barlat’89 criterion \([11,12]\).

| Table 2. Mechanical properties of low carton steel DC06. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( E \) (GPa)   | \( \nu \)       | \( k \) (MPa)   | \( n \)         | \( r_{45} \)    | \( r_{90} \)     |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 207             | 0.28            | 530.0           | 0.24            | 2.21            | 1.77            | 2.76            |

In the deep drawing process, the fracture is a very important failure type. It is used to measure the drawing quality as the criterion. In this paper, the maximum permit thinning was used to judge and measure the fracture. Usually, a maximum thinning of 25% thickness is used as the critical fracture criterion \([4]\). Because the initial blank thickness is 0.6 mm, the maximum allowed thinning is 0.15 mm.

3. FE model of deep drawing process

3.1 FE model
In this paper, the deep drawing process was simulated by the FE software Dynaform. The drawing depth was set up to 35 mm, in this drawing depth, if the fracture don’t occurred, the final cup-shaped part will have not any flange section. The element type was chosen as a quadrilateral shell element (Belytschko-Tsay). The punch, die and binder were considered as rigid body, the drawing speed of the punch was set to 50 mm/s, friction coefficient was set to 0.1 as an ordinary scale of metal contact, and
the mesh size of the parts was set between 0.5 and 1, with the mesh density in the arc region and planar region more dense and sparse, respectively. The mesh of blank and moulds was shown in figure 2, and the rest of the model parameters were set to the default values of the software.

3.2 Verification of FE model
For the verification of FE model, it is unnecessary to verify each FE model in the variable space of Table 1. In this paper, a set of process parameters within the variable space was chosen randomly to implement the FE simulation. When the BHF=10 kN, Rp=3 mm, Rd=3 mm, the FE simulated results for D=54.6 mm and D=54.3 mm were respectively shown as figure 3 (a) and (b).

It can be found that the minimum sheet thicknesses of D=54.6 mm and D=54.3 mm are 0.419 mm and 0.453 mm respectively. Therefore, the limiting sheet diameter must be a value between 54.6 mm and 54.3 mm, take the average value 54.45 mm as the FE simulated limiting sheet diameter of this drawing process. In the article of Zhang, et al[13], the drawing experiments with same process condition as the FE simulation have been done. The experimental limiting sheet diameter is 55.3 mm. The deviation between the EF simulated and experimental limiting sheet diameter is only 0.85 mm, the relative error is 1.5% at the same time. Thus, it is an indication that the FE model is capable to precisely simulate the drawing process under diverse process conditions.

![Figure 3. FE simulated results for (a) D=54.6mm; (b) D=54.3mm.](image)

4. RSM prediction model for limiting sheet diameter

4.1 Training samples
To further reduce the numbers of simulation, we implemented an efficient experimental design, which includes an orthogonal experimental design with 16 numbers and extra 24 random experiments, here only 40 samples need to simulate, and they are needful and sufficient to generate the required data for fitting RSM. In Table 3, the last column is the simulated minimum section thickness under each FE variable samples. If minimum thickness is less than 0.45 mm, the fracture has been occurred in deep drawing process.

| Num | BHF(KN) | D(mm) | Rp(mm) | Rd(mm) | t(mm) |
|-----|---------|-------|--------|--------|-------|
| 1   | 1       | 50    | 3      | 3      | 0.522 |
| 2   | 1       | 53    | 4      | 4      | 0.523 |
| 3   | 1       | 56    | 5      | 5      | 0.501 |
| 4   | 1       | 53    | 3      | 3      | 0.505 |
| --  | --      | --    | --     | --     | --    |
| 39  | 10      | 56    | 4      | 4      | 0.277 |
| 40  | 10      | 56    | 5      | 3      | 0.047 |

4.2 RSM prediction model
Taking the BHF, Rp, Rd and final minimum sheet thickness t as the inputs, the initial sheet diameter D as the output respectively, the sheet diameter RSM prediction model was established by the samples of
Table 3. Then, the RSM is an approximate model for predicting the sheet diameter $D$. Fitted RSM function was shown as equation (1), the compared results between initial and predicted sheet diameter were shown as figure 4, and the statistical histogram of deviation was shown as figure 5 respectively.

$$D(BHF, Rp, Rd, t) = 88.58 - 3.95 \times BHF + 2.61 \times Rp + 0.46 \times Rd - 60.03 \times t + 0.015 \times BHF^2 - 0.278 \times Rp^2 + 0.023 \times Rd^2 - 52.56 \times t^2 - 0.098 \times BHF \times Rp - 0.031 \times BHF \times Rd + 7.956 \times BHF \times t + 0.196 \times Rp \times Rd + 0.214 \times Rp \times t - 1.05 \times Rd \times t$$

**Figure 4.** Initial and predicted sheet diameter.

**Figure 5.** Statistical histogram of deviation between initial and predicted sheet diameter.

From the figure 4, it can be found that the maximum diameter deviation is 1.57 mm at the sample serial number 15, still in the acceptable range. From the deviation statistical histogram of figure 5, it be clearly shown that there is only one diameter deviation more than 1.5 mm. Therefore, it can be declared that fitted RSM prediction model can provide a better approximate sheet diameter value.

$R^2$ of RSM was reached 0.9. The RSM is capable to map the complex and non-linear relation between sheet diameter and multi-parameter inputs. For the limiting drawing state, the sheet thickness has been reached to the crack edge. In this case, the minimum sheet thickness was equal to 0.45 mm. Therefore, setting the value of $t$ in equation (1) to 0.45, the equation (1) will become a limiting sheet diameter prediction model. From this model, the user needs not to simulate the complex deep drawing process anymore by FE, the limiting sheet diameter can be obtained immediately when others input parameters are entered.

### 4.3 Test examples and discussion

In order to test the applicability and accuracy of limiting sheet diameter RSM, four groups of drawing process parameters were selected randomly, and their minimum sheet thickness, simulated and predicted sheet diameter were shown in Table 4.

| Num | BHF(KN) | Rp(mm) | Rd(mm) | $D$(mm)/t(mm) |
|-----|--------|--------|--------|---------------|
|     |        |        |        | RSM | Unbroken | Broken |
| 1   | 10     | 3      | 4      | 52.8/0.45 | 53.3/0.49 | 53.6/0.44 |
| 2   | 8      | 3      | 4      | 53.86/0.45 | 53.6/0.46 | 53.8/0.448 |
| 3   | 4      | 3      | 3      | 55.7/0.45 | 55.2/0.48 | 55.6/0.38 |
| 4   | 7      | 5      | 3      | 54.66/0.45 | 55.2/0.49 | 55.6/0.23 |

The procedure for obtaining the limiting sheet diameter on FE simulation as the following:

For one test example, FE was used to simulate two different deep drawing processes respectively, whose sheet diameter sizes must be different and very close. If one of the two sheet diameters is
broken, and another one is unbroken, then, the limiting sheet diameter on FE simulation must be a value between these two sheet diameters. The comparisons were shown in figure 6.

From the figure 6, it can be found that the maximum limiting sheet diameter deviation is at the serial number 4, because the limiting sheet diameter is a value between the unbroken and broken sheet diameter, here, the average value 55.4mm was treated as the limiting sheet diameter, the deviation between predicted by RSM and simulated by FE is 0.74mm, the relative error is only 1.4% at this time. Thus, the limiting sheet diameter prediction RSM model has higher prediction accuracy. Because the RSM model has fast calculation speed and higher accuracy, it could provide a guidance to prevent fracture at the early stage of deep drawing process.

![Image](Image)

**Figure 6.** Drawing quality of initial and predicted sheet diameter.

5. Conclusions
In this paper, an efficient and accurate approximate model based on RSM was developed to predict the limiting sheet diameter in the cup-shaped part deep drawing process with diverse mould assemblage. It can be found that the limiting sheet diameter RSM prediction model in this paper not only has higher efficiency, but also has higher accuracy. It is very suitable to provide a guide for mould assemblage and parameter setting in the early stage of deep drawing process, and is also very suitable to be integrated in the numerical control system. In the future, we will use RSM model to predict the actual drawing production to improve the production efficiency.

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References
[1] Gantar G and Kuzman K 2002 *J Mater Process Tech.* 125 p 302-308
[2] Yi K, Choi K K, Kim N H and Botkin M E 2007 *Int J Numer Meth Eng.* 71 p 1483-1511
[3] Sun G Y, Li G Y and Li Q 2012 *Finite Elem Anal Des.* 59 p 76-90
[4] Manoochehri M and Kolahan F 2014 *Int J Adv Manuf Tech.* 73 p 241-249
[5] Kitayama S, Kita K and Yamazaki K 2012 *Int J Adv Manuf Tech.* 61 p 1067-1083
[6] Song J H, Huh H and Kim S H 2007 *J Eng Mater-T ASME.* 129 p 397-406
[7] Hosseini A and Kadkhodayan M 2014 *Int J Adv Manuf Tech.* 71 p 337-355
[8] Srirat J, Kitayama S and Yamazaki K 2012 *J Adv Mech Des Syst.* 6 p 1081-1092
[9] Karajibani E, Fazli A and Hashemi R 2015 *Int J Adv Manuf Tech.* 80 p 113-121
[10] Lee J J and Park G J 2014 *J Mech Sci Technol.* 28 p 605-619
[11] Barlat F and Lian K 1989 *Int J Plasticity.* 5 p 51-66
[12] Zhang D J, Cui Z S, Ruan X Y and Li Y Q 2006 *Comp Mater Sci.* 38 p 256-262
[13] Zhang W R, Cao Y G, Yang S L, Cheng L X, Deng P R and Yong X C 2014 *Forming and Stamping Technology.* 39 p 41-45 (in Chinese)