How concrete operational student generalize the pattern?: use semiotic perspective

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Abstract. This study aimed to explore the generalization of patterns based on semiotic perspectives on concrete operational student. Semiotic perspectives included gestures, speeches and writings. This explorative-qualitative research was conducted in a Junior High School in Tuban, East Java, Indonesia. The subject of this research, selected using Group of Assessment of Logical Thinking (GALT) was in 7th Grade with skill of logical thinking at the concrete operational stage. The procedure of this qualitative study was giving tasks of pattern generalization and interviews. The results showed that the concrete operational student passed through three stages of the generalization process, namely discovering of regularity, confirming of regularity and producing of the expression of $n$. The concrete operational student cannot justify of the expression of $n$. He declared the generalization products in the form of simple sentences based on the image context viewed.

1. Introduction
Patterns are important topics underlying learning and mathematical thinking. Mathematics is often called as “the science of patterns”[1-2]. This perspective highlights the existence of patterns in some areas of mathematics. In particular, patterns are seen by some researchers as a way to develop algebraic thinking as it is a fundamental step to build a generalization which is the essence of mathematics [3-4].

Generalization is a competency concerned in mathematics learning at all levels. For example, in the field of arithmetic, students can generalize that the multiplication of every integer with number 5 will produce an integer with the last digit of 0 or 5. In the field of geometry, the theorems can be derived as a product of generalization [5]. Meanwhile, generalization and formalization are intrinsic to mathematical activities and thinking [6]. On another side, the generalization of geometric pictorial patterns can help to develop students’ ability to visualize, analyze and argue [7-8].

In completing a pattern generalization task, different student has different strategies [9]. Each of the strategies used by students is accompanied by interesting signs or symbols to be examined, both mentally (thinking process) and physically (gestures, words, written symbols, drawings and so on). A theory that learns about signs, sign functions, and sign meanings is a semiotic theory. Symbols or signs formed by students can be studied through semiotic theories with interpretations according to the context being studied. Mathematics is a science relating to activity-based signs. Semiotics is highly appropriate to be applied in mathematics [10]. It is impossible to comprehend abstract mathematical operations and contexts without certain semiotic activities [11].
There are three pattern generalization strategies viewed from the aspect of semiotic, namely factual, contextual, and symbolic strategies [12-13]. Some semiotic components that appear in the pattern generalization activity include gestures, words, and symbols [14]. The semiotic on the mathematics learning activity is a conceptual way that focuses on the symbol and its use as a way to understand the function and mental structures [10] of semiotic as an interpretation of mathematical expression [15]. In this research, the semiotic aspect studied in the process of pattern generalization was focused on students’ gestures, speeches and writings.

The developmental stages of pattern abilities in children are in accordance with the hierarchical classification system. The hierarchical framework corresponds to cognitive domains in general [3]. Cognitive domains are generally constructed based on Piaget’s cognitive developmental theory. Meanwhile, The answers to questions between adults and children about quadratic patterns and linear patterns can be classified into several stages, starting from answers in the form of concrete numbers to algebraic generalizations [3].

Relating to individual cognitive development, logical thinking is the most appropriate way to identify the level of individual cognitive development, on whether an individual is in concrete operational stage or formal operational stage. Logical thinking is also considered as a character existing in humans in general and a high-level cognitive skill. Roadrangka [16] established three stages of individual cognitive development by measuring the level of logical thinking skills, namely: concrete operational stage, transitional stage, and formal operational stage.

However, we found limited resources that examined the generalization process based on semiotic perspectives in terms of the different levels of students ‘logical thinking abilities, especially students' logical thinking about the level of concrete operational. Whereas previous studies, as we found, only examined the process of generalizing patterns based on semiotic perspectives, with the subject of research of all students without regard to their level of logical thinking ability. Thus, in this study, we will explore the profile of generalization based on semiotic perspectives especially concrete operational students. Generalization with symbolic algebraic is easily accessible and understood by students who have reached the formal operational stage. Therefore, we are interested in examining the generalization profile of students who have not yet reached the formal operational stage. We propose a hypothesis that concrete operational students can reach and understand generalizations with the help of semiotic resources, which include gestures, speeches and writings.

2. Method
This qualitative research was conducted in a Junior High School in Tuban, East Java, Indonesia. The subject of this research was a 7th grade student with the skill of logical thinking at the concrete operational stage. The selection of research subjects uses Group of Assessment of Logical Thinking (GALT) that has been developed by Roadrangka [16].

Data collection techniques in this research use pattern generalization tasks and interviews. Figure 1 describe pattern generalization tasks used in this study.

![Pattern generalization tasks](image-url)

*Figure 1. Pattern generalization tasks (PGT)*
The answer sheets of the pattern generalization tasks and the interview results were analyzed based on the generalization process indicators adopted from Fadiana [17]. Analysis of research data used Miles and Huberman [18] model that was reduction of data, presentation of data, and conclusion.

3. Result and Finding
Data analysis of this research was focused on the generalization process of the research subject based on the generalization stages developed by Fadiana [17] as follows: discovering of regularity, confirming of regularity, producing of the expression of \( n \) and justifying of the expression of \( n \).

3.1 The Stage of Discovering of Regularity
The concrete operational student observed the matchstick configurations presented in the PGT. His observation was focused on the matchstick configurations forming triangles. He also counted the number of triangles in the two configurations mediated by pointing gestures.

Figure 2 shows the pointing gesture from the concrete operational student. He pointed to triangular shapes using a pencil. He identified that 1 triangle was made up of 3 matchsticks. Then, he also identified the number of matchsticks that had been counted. In identifying the counted number of matchsticks, the concrete operational student was mediated by representational gestures, speeches, and writings. He said, “this is done, this is also done, done, done...”. His speeches were followed by representational gestures. There was a correspondence among gestures, speeches and writings of the concrete operational student in identifying the matchsticks that he had counted.

Figure 2. The pointing gesture from the concrete operational student in discovering of regularity

The concrete operational student counted how many matchsticks had not been counted mediated by pointing gestures. They are vertical matchsticks that did not make up the triangles. Once convinced that all matchsticks had been identified and counted, then he added up the number of matchsticks that made up the triangles and the number of matchsticks in the center (matchsticks that did not make up the triangles).

3.2 The Stage of Confirming of Regularity
The concrete operational student drew a matchstick configuration consisting of 6 squares. He counted the number of triangles (determining how many triangles were in the matchstick configuration consisting of 6 squares) mediated by pointing gestures. Then he counted the number of matchsticks making up the triangles and proceeded with identifying the matchsticks that had been counted. After that, he counted the matchsticks that had not been counted. Lastly, he counted the total number of matchsticks.

Figure 3 illustrates strategy visualization used by the concrete operational student in determining the number of matchsticks in the configuration consisting of 6 squares.
The concrete operational strategy in determining the number of matchsticks in the configuration consisting of 6 squares.

From Figure 3, it appears that the concrete operational student applied the principle of regularity he had discovered earlier from observing the two matchstick configurations. Thus, it is indicated that SK confirmed regularity.

Task c was to determine the number of matchsticks required for 50 squares. According to Stacey nomenclature [19], task c belongs to far generalization. Therefore, in answering task c, the concrete operational student did not draw completely the matchstick configuration consisting of 50 squares. Instead, he just sketched or visualized.

Figure 4 illustrates the concrete operational student’s sketch in answering task about far generalization.

The concrete operational student counted the number of triangles by analyzing the quantitative relationship mediated by representational gestures, that was by making a horizontal line trajectory by using a pencil in the sketch of the matchstick configuration consisting of 50 squares representing the number of upper triangles, lower triangles, right side triangle, and left side triangle. After that, he counted the number of matchsticks making up the triangles. Then, he counted how many vertical matchsticks had not been counted previously by analyzing the quantitative relationship with the mediation of representational gestures, that was by making an abstract horizontal line trajectory in the air using his index finger on the vertical matchstick in the center (the ones that were not made up the triangles). Lastly, the concrete operational student counted the total number of matchsticks.

3.3 The Stage of Producing the Expression of n

In producing the expression for n-configuration, the concrete operational student did not write down the general formula in the form of variable “n”. He knew that the given matchstick configuration formed a pattern and could be generalized using the same strategy that he used in the previous. Thus, with the mediation of a written symbol, the concrete operational student wrote as shown in Figure 5.
In addition to being mediated by a written symbol (the upward arrow), the pattern generalization was also mediated by speeches. The general formula for determining the number of matchsticks to construct “n” squares as in the figure was expressed in the form of spoken words (not using the variable n), as stated by the concrete operational student as follows:

“The number of upper or lower triangles is equal to the number of squares, plus one right side triangle and one left side triangle. The number of triangles is then multiplied by 3; proceed with counting the number of midlines. The number of midlines is equal to the number of squares minus 1. The last is totalizing all the result.”

3.4 The Stage of Justifying of the Expression of n

The concrete operational student did not provide a written proof of the general formula of the number of matchsticks used to construct “n” squares. He just provided justification for the generalization results with the mediation of spoken words as follows:

“I do not really understand what it means, Ma’am. But, I am already confident that the answer is d because the proof is just the same as the way I use for question b and question c. But, yaa...I do not really know, Ma’am. I am confused.”

Based on the research results above, the concrete operational student stated the generalization according to the context contained explicitly on the pattern (e.g. the next image, upper triangles, lower triangles, left side triangle, right side triangle and so forth). In generating the general formula, SK was mediated by written words and symbols.

The product generalization was expressed in sentences or words (not expressing the general formula in the form of symbolic algebra). Variables lacked roles at this stage. The concrete operational student did not understand the meaning of variable n, so he constructed a rule or formula to determine the number of matchsticks making up “n” squares using simple sentences which are the basic pattern rules. The general formula given was “The number of upper or lower triangles is equal to the number of squares, plus one right side triangle and one left side triangle. The number of triangles is then multiplied by 3, proceed with counting the number of midlines. The number of midlines is equal to the number of squares minus 1. The last is totalizing all the result.”

The product generalization was expressed by the concrete operational student is a contextual type of generalization [20]. Radford [20] defined contextual generalization as a generalization constructed based on the context contained explicitly in the pattern.

The concrete operational student used pointing and representational gestures in the generalization process. He pointed to the triangles by using a pencil. When pointing to the triangles, he also spoke some words. In addition to being mediated by pointing gestures, the generalization process of the concrete operational student was also mediated by representational gestures. The representational gesture used by the concrete operational student was painting or drawing a horizontal circle surrounding the triangles in the configuration to represent how many upper triangles were. Pointing gestures have a role as the sign to point or indicate the mathematical object or reference being discussed. Pointing
gestures integrated with words will make listeners easier to understand the speaker’s intent. In addition, pointing gestures also have a role to reduce cognitive tension. Meanwhile, representational gestures are beneficial for communication between a speaker and the audience as well as helping the audience to stimulate the actions and perceptions presented by the speaker's body movements.

The words spoken by the concrete operational student indicate the generalization meaning. He said, "the method is the same as before". The generalization generated by the concrete operational student was constructed based on the basis of inductive reasoning. Moreover, he also said the word "always". Radford [21] stated that adverbs such as "always" can support the generative function of language, that is a function that allows describing reiterative procedures and actions by imagining.

4. Conclusion
Based on the research results that have been explained above, it can be concluded that the concrete operational student makes a generalization by going through three stages, namely discovering of regularity, confirming of regularity and producing the expression of n. The concrete operational student is not able to justify of the expression of n. At the stage of discovering of regularity, he focuses his observation on the arrangement of matchsticks forming triangles and vertical lines with the mediation of pointing gestures. Then, he confirms regularity by applying the principles of regularity that have been found. At this stage, the generalization process of the concrete operational student is mediated by gestures, spoken words, and written symbols. The role of images is highly important for him that he always draws first before answering task b and c. At the stage of producing the expression of n, he generalizes the pattern based on the context contained explicitly on the pattern. The generalization product is expressed in the form of words.

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