Review of the multilayer coating model

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Abstract

The recent theoretical study on the multilayer-coating model published in Applied Physics Letters [1] is reviewed. Magnetic-field attenuation behavior in a multilayer coating model is different from a semi-infinite superconductor and a superconducting thin film. This difference causes that of the vortex-penetration field at which the Bean-Livingston surface barrier disappears. A material with smaller penetration depth, such as a pure Nb, is preferable as the substrate. Appropriate thicknesses of layers can be extracted from contour plots of the field limit of the multilayer coating model given in Ref. [1].

INTRODUCTION

The multilayer coating is one of approaches for pushing up the field limit of superconducting (SC) accelerating cavities [4], which consists of alternating layers of SC layers (S) and insulator layers (I) formed on a bulk-SC substrate. A theoretical understanding on this topic showed progresses last year [1, 2, 3]. The magnetic-field distribution in multilayered SC was derived by solving the Maxwell and the London equations with correct boundary conditions [1, 2]; and then appropriate choices of layer thicknesses and materials to enhance the field limit were revealed [1]. The above results were then reproduced by an other group [5].

In this paper, we review Ref. [1]. Especially the Bean-Livingston surface barrier is explained in detail by comparing those of a semi-infinite SC, an SC thin-film and a multilayer SC. Their analytical expressions are given by

\[ B_{[\text{Fig. } S]}(S) = B_0 \frac{\cosh \left( \frac{\lambda S}{\lambda_1} \right) - \frac{dS}{\lambda_1}}{\cosh \left( \frac{dS}{\lambda_1} \right)}, \quad \lambda = \lambda_2 + \lambda_3 \sinh \frac{dS}{\lambda_1}. \]  

Note that Eq. (3) represents the magnetic field in the S layer \( 0 \leq x \leq d_S \). Fields in other regions \( x > d_S \) are found in Ref. [1, 2]. As shown in Fig [1(a)-(c)] and Fig. [1(d)], a behavior of magnetic-field attenuation depends on a system, which is essential for understanding difference of surface barrier among different systems.

Surface Barriers

Suppose there exist a vortex with the flux quantum \( \phi_0 = 2.07 \times 10^{-15} \text{Wb} \) parallel to \( \hat{z} \) at a surface of SC. This vortex feels two distinct forces \( f_1 \) and \( f_2 \), where \( f_1 \) is a force from an image current due to an image antivortex, and \( f_M \) is that from a Meissner current due to an applied magnetic-field. The force \( f_1 \) is common in all configurations of Fig. [1] if \( \xi_1 \ll d_S \). The derivation is reviewed in detail in Ref. [3], in which \( f_1 \) is given by

\[ f_1 = -\frac{\phi_0^2}{4\pi \mu_0 \lambda_1^2 \xi_1} \hat{\lambda}, \]

where \( \mu_0 = 4\pi \times 10^{-7} \text{H/m} \) is the magnetic permeability of vacuum. Thus the image antivortex attracts the vortex to the surface and prevents the vortex penetration. The other force, \( f_M \), is obtained by evaluating the product \( f_M = J_M \times \hat{\phi}_0 \hat{\lambda} \), where \( J_M = (0, -\mu_0^{-1} dB/dx, 0) \) is a Meissner-current density at the vortex position \( x \approx 0 \). By using Eqs. (1), (2) and (3), we find

\[ f_M[\text{Fig. } S] = \frac{B_0 \phi_0}{\mu_0 \lambda_1} \hat{\lambda}, \]

\[ f_M[\text{Fig. } I] = \frac{B_0 \phi_0}{\mu_0 \lambda_1} \frac{dS}{\lambda_1} \hat{\lambda} \]

by which the vortex is attracted to the inside of each SC. The total force acting on the vortex is given by \( f_{\text{tot}} = f_1 + f_M \).
balancing the two forces:

\[
B_v[\text{Fig. 1}(a)] = \frac{\phi_0}{4\pi\lambda_1 \xi_1} \quad (\equiv B_{v0}),
\]

\[
B_v[\text{Fig. 1}(b)] = \frac{\phi_0}{2\pi d_S \xi_1} = \frac{2\lambda_1}{d_S} B_{v0},
\]

\[
B_{v(S)}^{(S)} = \frac{\cosh \frac{d_L}{\lambda_1} + (\frac{d_S}{\lambda_1} + \frac{d_L}{\lambda_1}) \sinh \frac{d_S}{\lambda_1}}{\sinh \frac{d_S}{\lambda_1} + (\frac{d_S}{\lambda_1} + \frac{d_L}{\lambda_1}) \cosh \frac{d_S}{\lambda_1}} B_{v0} = B_v^{(S)}.
\]

Eq. (4) is the well-known result for the semi-infinite SC [6, 7], Eq. (5) corresponds to the result shown in Ref. [8], and Eq. (6) is the vortex-penetration field of the top S layer of the multilayer SC [11, 13].

Differences among Eq. (4), (5) and (6) are due to those of slopes of magnetic-field attenuation at the surfaces, because the force pushing a vortex into SC is given by \( |\mathbf{B}_v| \propto |\mathbf{J}_3| \propto |d\mathbf{B}/dx| \). A smaller \( |d\mathbf{B}/dx| \) at \( x = 0 \) induces a larger \( B_v \). In fact \( |d\mathbf{B}/dx| \) at \( x = 0 \) of the SC thin film is smaller than that of the semi-infinite SC as shown in Fig. 1(a) and (b), and Eq. (5) is larger than Eq. (4) by a factor \( 2\lambda_1/d_S \). Similarly, when \( |d\mathbf{B}/dx| \) at \( x = 0 \) of the S layer of multilayer SC is smaller than that of the semi-infinite SC, Eq. (6) can be larger than Eq. (4).

**TOWARD EXPERIMENTS**

**Surface Barrier of the S Layer**

Figure 2 shows enhancement factor \( B_v^{(S)}/B_{v0} \) as functions of \( d_S/\lambda_1 \). A combination of small \( d_S/\lambda_1 \) and \( d_L/\lambda_1 \) yields a large enhancement. Substituting \( d_S/\lambda_1 \ll 1 \) and \( d_L/\lambda_1 \ll 1 \) into Eq. (6), we find

\[
B_v^{(S)} \left| \frac{d_S}{\lambda_1} \frac{d_L}{\lambda_1} \ll 1 \right. \simeq \left( \frac{\lambda_1}{\lambda_2} \right) B_{v0}.
\]

Eq. (7) tells the importance of a choice of bulk-SC substrate, a material with smaller \( \lambda_2 \), such as a pure Nb with a long mean free path, should be chosen for an enhancement of \( B_v^{(S)} \).

**Field Limit of Multilayer SC**

The field limit of the whole structure of the multilayer SC, \( B_v^{(ML)} \), is limited not only by \( B_v^{(S)} \) but also by that of the bulk-SC substrate, \( B_v^{(bulk)} \), because the magnetic-field is not completely shielded by the S layer alone and that on the interface of the bulk-SC substrate \( B_1 \) is finite. \( B_1 \) is given by \( B_1 = \alpha B_0 \), where \( \alpha = |\cosh \frac{d_S}{\lambda_1} + (\frac{d_S}{\lambda_1} + \frac{d_L}{\lambda_1}) \sinh \frac{d_S}{\lambda_1}|^{-1} \). If \( B_1 \) is larger than \( B_v^{(bulk)} \), the bulk-SC substrate can also suffer a vortex penetration. Thus \( B_v^{(ML)} \) is given by

\[
B_v^{(ML)} = \begin{cases} 
B_v^{(S)} & (\alpha B_v^{(S)} < B_v^{(bulk)}) \\
\alpha^{-1} B_v^{(bulk)} & (\alpha B_v^{(S)} \geq B_v^{(bulk)})
\end{cases}.
\]

Note that Eq. (8) ceases to be valid at \( d_S \approx \xi_1 \) and \( d_L \sim \xi_2 \) at a few nm, at which the model should be reevaluated by more accurate theories.
Figure 2: Enhancement factors $B_v(S)/B_v(0)$ as functions of $d_S/\lambda_1$, where a penetration depth of the bulk-SC substrate is assumed to be $\lambda_2 = 0.2\lambda_1$.

The derived formula and detailed discussions will be presented elsewhere [9].

**SUMMARY**

In this paper, we have reviewed the multilayer coating model [1, 2, 3].

- Magnetic-field attenuation behavior in a multilayer SC is different from a semi-infinite SC and an SC thin film. This difference causes a difference of the vortex-penetration field at which the Bean-Livingston surface barrier disappears.

- A material with smaller penetration depth is preferable as the bulk-SC substrate for pushing up the vortex-penetration field of the $S$ layer, $B_v(S)$.

- The field limit of the whole structure of the multilayer SC, $B_v^{(ML)}$, is limited not only by $B_v(S)$, but also by that of the bulk-SC substrate, $B_v^{(bulk)}$. Appropriate thicknesses of $S$ and $I$ layers can be extracted from contour plots of $B_v^{(ML)}$ given in Ref. [1].

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