A hybrid meson \( \bar{q}q \) is a bound state of constituent quark \( q \), anti-quark \( \bar{q} \) and excited gluon \( g \). The existence of hybrids is one of the most important predictions of quantum chromodynamics (QCD). There has been a lot of experimental activity\[1, 2, 3, 4\] in the search for hybrid mesons, for example: PEP-II (Babar), KEKB(Belle), 12 GeV Jefferson Lab upgraded, upgraded CLEO-c detector, and new BES3 detector.

For a conventional meson in the quark model, which is represented by the fermion bilinear \( \bar{q}q \), it can have the \( J^{PC} \) quantum numbers as \( J = |L - S|, |L - S| + 1, \ldots, L + S, \) \( P = (-1)^{L+1} \), and \( C = (-1)^{L+S} \), with \( L \) the relative angular momentum of the quark and antiquark, and \( S \) the intrinsic spin of the meson. For the gluon, the quantum numbers of the color electric field \( E \) and color magnetic field \( B \) are \( 1^{--} \) and \( 1^{++} \) respectively. According to QCD, the operator of a hybrid meson is the gauge-invariant direct product of the fermion bilinear \( \bar{q}q \) and the color electric field \( E^{c_1 c_2} = F^{c_1 c_2}_{c_1 c_2} \) or color magnetic field \( B^{c_1 c_2} = \epsilon_{ijk} F^{c_1 c_2}_{jk} \). Therefore, the quantum numbers of a hybrid meson could be either exotic, with \( J^{PC} = 1^{++}, 0^{+-}, 0^{--}, 2^{+-} \ldots \), inaccessible to conventional mesons, or non-exotic, with \( J^{PC} = 0^{++}, 0^{--}, 1^{--}, 1^{+-}, 2^{++}, 2^{--}, 2^{+-} \ldots \), the same as conventional mesons.

Lattice gauge theory is the most reliable technique for computing hadron spectra. It involves discretization of the continuum theory on a space-time grid, and reduces to QCD when the lattice spacing goes to zero. The implementation of the Symanzik program\[5\] with tadpole improvement\[6\] greatly reduces the discretization errors on very coarse and small lattices. Simulations on anisotropic lattices improve the signal in spectrum computations\[7\].

The \( 1^{--}, 0^{+-}, \) and \( 2^{+-} \) exotic hybrid mesons have been extensively studied on the lattice. Reviews can be found in Refs.\[8, 9\]. Recently, we computed the \( 0^{--} \) exotic hybrid charmonium mass\[10\]. However, the non-exotic hybrid mesons are usually ignored in the literature, simply because their ground states are almost degenerate with the conventional mesons\[11\].

In this letter, we investigate the \( J^{PC} = 0^{++}, 1^{--} \) and \( 1^{+-} \) non-exotic charmed hybrid mesons \( \bar{c}c \), employing quenched lattice QCD with tadpole improved gluon\[12\] and quark\[13\] actions on the anisotropic lattice. We observe, for the first time, very strong gluonic radial excitations in the first excited states.

Our simulation parameters are listed in Tab.\[1\]. We also did simulations on \( 8^3 \times 48 \) and \( 12^3 \times 48 \) at \( \beta = 2.6, 12^3 \times 36 \) at \( \beta = 2.8, \) and \( 16^3 \times 48 \) at \( \beta = 3.0, \) but there and throughout the paper we just list the results from the largest volume, i.e., \( 16^3 \times 48 \) at \( \beta = 2.6 \) and \( 2.8 \) and \( 20^3 \times 60 \) at \( \beta = 3.0. \) At each \( \beta = 6/g^2, \) three hundred independent configurations were generated with the improved gluonic action\[12\]. It is also important to check whether these lattice volumes are large enough. When the spatial extent is greater than 2.2f\( m \), the finite volume effect on the spectrum is less than 0.1% for the ground state, and 0.4% for the first excited state.

We input the bare quark mass \( m_{q0} \) and then computed quark propagators using the improved quark action\[12\], the conventional quarkonium correlation function using the operators \( 0^{++} = \bar{c}c \gamma_5 \bar{c}c, 1^{--} = \bar{c}c \gamma_5 \bar{c}c, \) and \( 1^{+-} = \bar{c}c \gamma_5 \gamma_5 \gamma_5 \bar{c}c, \) and the hybrid meson correlation function using the operators \( 0^{++} = \bar{c}c \gamma_5 \bar{c}c, 1^{--} = \bar{c}c \gamma_5 \bar{c}c, \) and \( 2^{+-} = \bar{c}c \gamma_5 \bar{c}c \) in Ref.\[14\]. Figures\[1\] and\[2\] shows the correlation function \( C(t) \) of the conventional \( 1^{--} \) and hybrid mesons.

The effective masses of the ground and first excited states \( a_{1m_1} \) and \( a_{1m_2} \) are extracted by two different methods: (i) new correlation function method\[15\]; (ii) modified multi-exponential fit\[16\]. The multi-exponential fitting method has been widely used in the literature\[11, 12, 16\] for extracting the charmonium masses, and the results for the ground and first excited states are consistent with experiments; The MILC group\[17\] proposed an improved multi-exponential fitting method, which chooses the best fit according to some criteria. The recently proposed method (i) has been successfully applied to the investigation of the Roper reso-
TABLE I: Simulation parameters at largest volume. We employed the method in Ref.[13] to tune these parameters, $\kappa_t$ and $\kappa_s$ for the quark action. The last two columns are about the spatial lattice spacing and the lattice extent in physical units, determined from the $1P-1S$ charmonium mass splitting[16], with the effective masses extracted by the method of Ref. [16].

| $\beta$ | $\xi = a_s/a_t$ | $L^3 \times T$ | $u_s$ | $u_t$ | $a_t m_{Q0}$ | $c_s$ | $c_t$ | $a_s (1^1 P_1 - 1S)$ [fm] | $L a_s$ [fm] |
|----------|----------------|----------------|-------|-------|---------------|-------|-------|--------------------------|-------------|
| 2.6      | 3              | $16^3 \times 48$ | 0.81921 | 1 | 0.229 | 0.260 | 1.8189 | 2.4414 | 0.1856(84) | 2.970 |
| 2.8      | 3              | $16^3 \times 48$ | 0.83099 | 1 | 0.150 | 0.220 | 1.7427 | 2.4068 | 0.1537(101) | 2.459 |
| 3.0      | 3              | $20^3 \times 60$ | 0.84098 | 1 | 0.020 | 0.100 | 1.6813 | 2.3782 | 0.1128(110) | 2.256 |

Figure 3 shows effective masses for the conventional $1^{--}$ quarkonium, where $a_t m_1$ and $a_t m_1 + a_t m_2$ are extracted respectively from the plateaux of the lower and upper curves, using the new method[15]. Figure 4 shows those for the $1^{--}$ hybrid meson.

The data at two $m_{Q0}$ values were interpolated to the charm quark regime using $M(1S)_{exp} = (m(\eta_c)_{exp} + 3m(J/\psi)_{exp})/4 = 3067.6$ MeV. The results obtained by the method of Ref. [16] are listed in Tabs. II and III respectively for the ground and first excited states.

The first excited state masses for the conventional $0^{++}$, $1^{--}$ and $1^{++}$ charmonium mesons as a function of $a_t^2$ are plotted in Fig. 5 and those for the hybrid charmonium mesons are plotted in Fig. 6. They indicate the linear dependence of the mass on $a_t^2$. By linearly extrapolating the data to $a_t^2 \rightarrow 0$, we obtained the spectrum in the continuum limit, which are listed in Tabs. II and III respectively.

As shown in Tab. III in the continuum limit, the masses of the $0^{-+}$, $1^{--}$ and $1^{++}$ charmonium ground states are consistent with their experimental values 2.9804, 3.0969, and 3.5106 for $\eta_c(1S)$, $J/\psi$ and $\chi_{c1}(1^3 P_1)$. The results in Tab. III also show that the ground state for the non-exotic hybrid charmonium is degenerate with the

Figure 1: Correlation function for the conventional $1^{--}$ quarkonium at $\beta = 2.6$ and $a_t m_{Q0} = 0.229$.

Figure 2: Same as Fig. 1 but for the $1^{--}$ hybrid meson.

FIG. 1: Correlation function for the conventional $1^{--}$ quarkonium at $\beta = 2.6$ and $a_t m_{Q0} = 0.229$.

FIG. 2: Same as Fig. 1 but for the $1^{--}$ hybrid meson.

FIG. 3: Effective masses of the conventional $1^{--}$ quarkonium as a function of $t$ for $\beta = 2.6$ and $a_t m_{Q0} = 0.229$, using the new correlation function method[15]. $a_t m_1 + a_t m_2$ and $a_t m_1$ are extracted respectively from the plateaux of the upper and lower curves, with $[t^*_i, t^*_f] = [1, 10]$ and $[t_i, t_f] = [11, 23]$.

The $\langle C(t) \rangle = \langle C(t+1) \rangle$ in the large time interval $[t_i, t_f]$, and $a_t m_1 + a_t m_2$ from $\langle C(t) \rangle$ in the time interval $[t^*_i, t^*_f] < [t_i, t_f]$, with reasonable $\chi^2/d.o.f.$ and optimal confidence level. Here $C'(t) = C(t+1)C(t-1) - C(t)^2$. Two methods provide a cross-check of the results.
TABLE II: Conventional and hybrid charmonium meson spectrum (GeV) for the ground state from the method of Ref. [16], interpolated to the charm quark sector. The results in the continuum limit ($\beta = \infty$) were obtained by linearly extrapolating the data to $a_s^2 \to 0$. The results in the continuum limit (*), with the effective masses extracted by the method of Ref. [15] are also listed. The last line (**) is the average of the results in the continuum limit from these two methods.

| $\beta$ | $a_s^2$ $(fm^2)$ | $\eta_c$ | $J/\psi$ | $\chi_{c1}$ | $0^{--}$ | $1^{--}$ | $1^{++}$ |
|---------|-----------------|---------|---------|---------|--------|--------|--------|
| 2.6     | 0.0345          | 3.013(3)| 3.080(3)| 3.484(49)| 3.012(43)| 3.133(44)| 3.472(66) |
| 2.8     | 0.0236          | 3.033(1)| 3.079(1)| 3.446(59)| 3.009(51)| 3.112(53)| 3.463(62) |
| 3.0     | 0.0127          | 3.031(2)| 3.080(2)| 3.430(100)| 3.031(87)| 3.078(90)| 3.516(108) |
| $\infty$| 0               | 3.030(2)| 3.080(2)| 3.430(100)| 3.031(87)| 3.078(90)| 3.516(108) |
| $\infty$| 3.053(33)       | 3.107(34)| 3.533(39)| 3.056(34)| 3.120(34)| 3.472(150) |
| $\infty$| 3.042(18)       | 3.094(18)| 3.482(70)| 3.044(61)| 3.099(62)| 3.494(129) |

TABLE III: The same as Tab. II but for the first excited states.

| $\beta$ | $a_s^2$ $(fm^2)$ | $\eta_c$ | $J/\psi$ | $\chi_{c1}$ | $0^{--}$ | $1^{--}$ | $1^{++}$ |
|---------|-----------------|---------|---------|---------|--------|--------|--------|
| 2.6     | 0.0345          | 3.515(50)| 3.614(51)| 4.135(175)| 4.492(64)| 4.525(64)| 7.335(121) |
| 2.8     | 0.0236          | 3.520(60)| 3.625(62)| 4.175(183)| 4.379(98)| 4.494(77)| 7.333(153) |
| 3.0     | 0.0127          | 3.532(66)| 3.624(68)| 4.100(112)| 4.408(382)| 4.40 0(100)| 7.264(150) |
| $\infty$| 0               | 3.540(102)| 3.633(105)| 4.081(205)| 4.335(302)| 4.349(148)| 7.237(237) |
| $\infty$| 3.638(58)       | 3.731(58)| 4.089(67)| 4.368(147)| 4.409(149)| 7.392(276) |
| $\infty$| 3.589(80)       | 3.682(81)| 4.085(136)| 4.352(225)| 4.379(149)| 7.315(257) |

FIG. 4: Same as Fig. 3, but for the $1^{--}$ hybrid meson. $a_t m_1 + a_t m_2$ and $a_t m_1$ are extracted respectively from the plateaux of the upper and lower curves, with $[t^*_i, t^*_f] = [6, 16]$ and $[t_i, t_f] = [17, 23]$.

FIG. 5: Extrapolation of the excited state masses of the conventional $0^{--}$, $1^{--}$, and $1^{++}$ charmonium mesons, with the effective masses extracted by the method of Ref. [15], to the continuum limit.

conventional charmonium with the same quantum numbers. This might mislead people into giving up further study of the non-exotic hybrids.

The last line of Tab. III shows in the continuum limit the first excited state masses of the conventional charmonium and non-exotic hybrid charmonium. The results for the conventional $0^{--}$ and $1^{--}$ charmonium are in good agreement with the experimental data 3.638 and 3.686 for $\eta_c(2S)$ and $\psi(2S)$, which supports the reliability of the methods. Although there has not been experimental input for $\chi_{c1}(2S)$, our result is consistent with earlier lattice calculations.[13, 17]

The minor differences between the data and experiments might be due to the quenched approximation used in the paper.

What new is that the first excited states of non-exotic
fundamental properties of a hadron. Sometimes, the excited states show more carefully study not only the ground state, but also the excited states. This is clearly demonstrated in Figs. 1-6.

Hybrids are completely different from the conventional ones. The results in last line of Tab. III show the masses of the 0^+ and 1^− hybrids to be about 0.7 GeV heavier, and the 1^{++} about 3.2 GeV heavier. These are very strong indications of gluonic excitations. This implies that radial excitations of the charmonium hybrids are completely different from the conventional non-hybrid ones, although their ground states overlap. This is clearly demonstrated in Figs. 1-6.

This also teaches a very important lesson. One should carefully study not only the ground state, but also the excited states. Sometimes, the excited states show more fundamental properties of a hadron.

Finally, we discuss the new state Y(4260), recently observed by the BaBar experiment in the J/ψπ^+π^- channel. It has the quantum numbers J^PC = 1^{−−}.

There have been several phenomenological descriptions of this state: as tetra-quarks, a molecule of two mesons, ψ(4S), or as a hybrid meson. However, most these assumptions were not based on QCD spectrum computations.

If Y(4260) is a hybrid meson, from the last line of Tab. III, it could certainly not be identified as the ground state of the 1^{−−} hybrid meson. However, from our lattice QCD spectrum calculations (the last line of Tab. III), it is most probably the first excited state of the 1^{−−} hybrid charmonium. Further experimental study of the decay modes will clarify this issue.

From the last line of Tab. III, one sees that the first excited state mass of the 0^+ hybrid charmonium is about the same as that of the 1^{−−} hybrid charmonium, but much lighter than the first excited state of the 1^{++} hybrid charmonium. It should not be very difficult to find it in future experiment.

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[1] C. A. Meyer, AIP Conf. Proc. 698, 554 (2004).
[2] K. Peters, Int. J. Mod. Phys. A 20, 570 (2005).
[3] S. L. Olsen, J. Phys. Conf. Ser. 9, 22 (2005).
[4] D. S. Carman, arXiv:hep-ex/0511030.
[5] K. Symanzik, Nucl. Phys. B 226, 187 (1983); Nucl. Phys. B 226, 205 (1983).
[6] G. Lepage and P. Mackenzie, Phys. Rev. D 48, 2250 (1993).
[7] Z. H. Mei and X. Q. Luo, Int. J. Mod. Phys. A 18, 5713 (2003).
[8] C. McNeile, Nucl. Phys. A 711 (2002) 303, and refs. therein.
[9] C. Michael, hep-ph/0308293 and refs. therein.
[10] Y. Liu and X. Q. Luo, arXiv:hep-lat/0511015.
[11] X. Liao and T. Manke, arXiv:hep-lat/0210030.
[12] C. Morningstar and M. Peardon, Phys. Rev. D 56, 4043 (1997); Phys. Rev. D 60, 034509 (1999).
[13] M. Okamoto et al. [CP-PACS Collaboration], Phys. Rev. D 65, 094508 (2002).
[14] C. Bernard et al. [MILC Collaboration], Phys. Rev. D 56, 7039 (1997).
[15] D. Guadagnoli, M. Papinutto and S. Simula, Phys. Lett. B 604, 74 (2004).
[16] C. Bernard et al., [MILC Collaboration], Phys. Rev. D 68, 074505 (2003).
[17] P. Chen, Phys. Rev. D 64, 034509 (2001).
[18] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 95, 142001 (2005).
[19] S. L. Zhu, Phys. Lett. B 625, 212 (2005).
[20] F. J. Llanes-Estrada, Phys. Rev. D 72, 031503 (2005).
[21] E. Kou and O. Pene, Phys. Lett. B 631, 164 (2005).
[22] L. Maiani, V. Riquer, F. Piccinini and A. D. Polosa, Phys. Rev. D 72, 031502 (2005).
[23] X. Liu, X. Q. Zeng and X. Q. Li, Phys. Rev. D 72, 054023 (2005).
[24] F. Close and P. Page, Phys. Lett. B 628, 215 (2005).
[25] C. F. Qiao, arXiv:hep-ph/0510228.
[26] http://physics.utah.edu/~detar/milc/