ON THE LUMINOSITY FUNCTION OF EARLY–TYPE GALAXIES

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ABSTRACT

In a recent paper Loveday et al. (1992) have presented new results on the luminosity function for a sample of galaxies with $b_J \leq 17.15$. After having morphologically classified each galaxy (early–type, late–type, merged or uncertain), they have estimated the parameters of a Schechter luminosity function for early– and late–type galaxies. However, in their sample there is a bias against identifying early–type galaxies at large distances and/or faint magnitudes: in fact, many of the early–type galaxies at faint magnitudes have probably been classified as “uncertain”. As discussed in Loveday et al., the existence of such a bias is indicated by the fact that for these galaxies $<V/V_{max}> = 0.32$.

In this paper we show, both theoretically and through the use of simulated samples, that this incompleteness strongly biases the derived parameters of the luminosity function for early–type galaxies. If no correction for such incompleteness is applied to the data (as done by Loveday et al.), one obtains a flatter slope $\alpha$ and a brighter $M^*$ with respect to the real parameters.

Key words: Galaxies: luminosity function

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1. Introduction

An accurate knowledge of the optical luminosity function of galaxies is required for many applications in cosmology. For instance, it is essential in interpreting galaxy number counts and in analyzing the spatial distribution of galaxies from redshift surveys; in addition, the shape of this function is of theoretical interest as it may provide constraints on models of galaxy formation.

An interesting question about the luminosity function of galaxies concerns its universality: indeed, Binggeli, Sandage & Tammann (1988) have shown that the luminosity function depends on the morphological type, particularly at the faint end. On the other hand, it has been demonstrated that the mix of morphological types is closely related to the local matter density (Dressler 1980). The accurate knowledge of the luminosity function for each morphological type is of great interest also for the models of number–magnitude counts. These models strongly depend on the morphological mix and therefore need the knowledge not only of the fraction of the various galaxy types but also of their K–corrections and luminosity functions.

Loveday et al. (1992) have recently presented new results on the luminosity function for a sample of galaxies with $b_J \leq 17.15$. After having morphologically classified each galaxy (early–type, late–type, merged or uncertain), they have estimated the parameters of a Schechter luminosity function for early– and late–type galaxies, using the STY parametric maximum likelihood method (Sandage, Tammann & Yahil 1979). While for late–type galaxies their parameters are in reasonable agreement with those derived from other samples (see f.i. Efstathiou, Ellis & Peterson 1988), the parameters for early–type galaxies are not consistent with previous determinations.

As mentioned by Loveday et al., in their sample there is a bias against identifying early–type galaxies at large distances and/or faint magnitudes: in fact, many of the early–type galaxies at faint magnitudes have probably been classified as “uncertain”, and therefore have not been used in computing the luminosity function. The existence of such a bias is demonstrated by the fact that for these galaxies $< V/V_{\text{max}} > = 0.32$. The same bias appears not to be present in the classification of the late–type galaxies, for which $< V/V_{\text{max}} > = 0.47$ (see Table 1 in Loveday et al. 1992). In this paper we show, both theoretically and through the use of simulated samples, that this incompleteness strongly biases the derived parameters of the luminosity function for early–type galaxies.
In Sect. 2 we demonstrate that the classification incompleteness biases the results of the STY method and in Sect. 3 we estimate the amount of this bias through simulations.

2. The luminosity function of galaxies

The luminosity function of galaxies is well represented by a Schechter (1976) form

\[ \phi(L) dL = \phi^* \left( \frac{L}{L^*} \right)^{\alpha} e^{-L/L^*} \frac{dL}{L^*} \]  

(1)

where \( \alpha \) and \( L^* \) are parameters referring to the shape of the function and \( \phi^* \) contains the information about the normalization; these parameters have to be determined from the data.

Many different methods have been used in the past years to compute the parameters of the galaxy luminosity function. Recently, however, the STY method (Sandage et al. 1979) has been the most widely used, and it has been shown that this estimator is unbiased with respect to density inhomogeneities (see f.i. Efstathiou et al. 1988). The basic idea of this method is to compute the estimator of the quantity \( \tilde{\phi} = \phi \), where \( \Phi \) is the integrated luminosity function. Under the assumption that \( \phi(L) \) is not a function of position \( \text{i.e.} \phi(L, x, y, z) \) \( dL \) \( dV = \rho(x, y, z) dV \psi(L) dL \), the probability of seeing a galaxy of luminosity \( L_i \) at redshift \( z_i \) is

\[ p_i = \frac{\psi(L_i)}{\int_{L_{\text{min}}(z_i)}^\infty \psi(L) dL} \]  

(2)

where \( L_{\text{min}}(z_i) \) is the minimum luminosity observable at redshift \( z_i \) in a magnitude–limited sample.

The best parameters \( \alpha \) and \( L^* \) of the luminosity function are then determined by maximizing the likelihood function \( L(\alpha, L^*) \), which is the product over all the galaxies of the individual probabilities \( p_i \). This corresponds to minimize the function

\[ S = -2 \ln L = \]

\[ = -2 \left[ \alpha \sum_{i=1}^{N} \ln L_i - N(\alpha + 1) \ln L^* - \frac{1}{L^*} \sum_{i=1}^{N} L_i - \sum_{i=1}^{N} \ln \Gamma \left( \alpha + 1, \frac{L_{\text{min}}(z_i)}{L^*} \right) \right] \]  

(3)

where \( \Gamma \) is the incomplete Euler gamma function and \( N \) is the total number of galaxies in the sample.

This formula is correct only for a complete, unbiased sample in which all galaxies with \( m < m_{\text{lim}} \) are members of the sample or all galaxies with \( m < m_{\text{lim}} \) have the same probability
of being members of the sample (as, for example, in a redshift survey with $\frac{1}{n}$ sampling). In other cases in which each galaxy of the sample has a different weight $w_i$, which may be a function of an intrinsic property of the galaxy (f.i. the distance, the absolute or apparent magnitude, the diameter, etc.), eq. (2) is not valid anymore. If we define the weight $w_i$ as the inverse of the probability that the $i^{th}$ galaxy has of being included in the sample, eqs. (2) and (3) have to be modified in:

$$p_i = \left( \frac{\psi(L_i)}{\int_{L_{min}(z_i)}^{\infty} \psi(L)dL} \right)^{w_i} \tag{4}$$

and

$$S = -2 \left[ \alpha \sum_{i=1}^{N} w_i \ln L_i - (\alpha + 1) \ln L^* \sum_{i=1}^{N} w_i - \frac{1}{L^*} \sum_{i=1}^{N} w_i L_i - \sum_{i=1}^{N} w_i \ln \Gamma \left( \alpha + 1, \frac{L_{min}(z_i)}{L^*} \right) \right] \tag{5}$$

Loveday et al. (1992) have used eq. (2) to compute the luminosity function for the galaxies in their sample, both when they considered all galaxies and galaxies divided in sub–groups, as a function of the morphological type. Since, as mentioned in the Introduction, their morphological classification of early–type galaxies is biased at faint apparent magnitudes, a weight $w_i(m_i)$ should be associated to each galaxy and the use of eq. (2) for determining the luminosity function of early–type galaxies is not correct anymore. In the following section we will quantify, through simulated samples, the differences between the results obtained through the use of eq. (3) and (5).

3. Results

In order to estimate in a quantitative way the error introduced applying eq. (3) instead of eq. (5) to an incomplete sample, we have used two types of random simulations.

1) We have randomly distributed 5 millions of points following a Schechter luminosity function with parameters $\alpha = -1.10$ and $M^* = -19.50$ (which are typical values for the galaxy luminosity function), obtaining a sample of $\sim 11000$ galaxies with $m_{lim} = 17.15$. Then we have introduced in this sample an incompleteness function, depending on the apparent magnitude of the galaxies in the form of

$$f(m) = a(m - m_o) \tag{8}$$
and for each $m$ we have randomly eliminated from the sample a fraction $f(m)$ of galaxies. We assumed that $f(m) = 0$ for $m < m_o$, i.e. for galaxies brighter than $m_o$ the sample is complete. We have chosen $m_o = 16$, assuming that for galaxies brighter than $16^{th}$ magnitude the morphological classification is relatively easy, and we have used various values for $a$, corresponding to different values of $<V/V_{max}>$ and, therefore, to different levels of incompleteness. Then we have computed the parameters of the luminosity function for these samples using both eq. (3) and eq. (5).

Table 1 lists the derived parameters for three representative cases: column (1) gives the adopted incompleteness function, column (2) the $<V/V_{max}>$ of the sample and column (3) the number of galaxies; the parameters $\alpha$ and $M^*$ derived from eq. (3) and (5) are listed in columns (4) and (5), (6) and (7), respectively. From this table it is clear that, as $a$ increases, the use of eq. (3) produces flatter slopes and brighter $M^*$ with respect to the real parameters. In all these cases the derived parameters are not compatible with the real ones, as shown by the confidence ellipses in Fig. 1. But if we use eq. (5), which takes into account the incompleteness function, we obtain “corrected” parameters in very good agreement with the real ones.

2) In the previous case we have used a simulation with a large number of points in order to minimize the effects of statistical fluctuations in the test of eq. (3) and (5). Now we try to reproduce as much as possible the characteristics of the early–type galaxy sample of Loveday et al. ($<V/V_{max}> = 0.32$). We have generated 100 random catalogues, distributed following a Schechter function with the parameters derived by Efstathiou et al. (1988) for early–type galaxies ($\alpha = -0.48$ and $M^* = -19.37$). Note that these values are not consistent with the values $\alpha = +0.2$ and $M^* = -19.71$ found by Loveday et al. (see below in Fig. 2). Then we have applied the incompleteness function $f(m) = 0.8(m-16)$, which gives $<V/V_{max}> = 0.33$ (this value has been computed as the mean of the values of the $<V/V_{max}>$ derived for each catalogue). The number of object in each incomplete catalogue is of the same order as the number of galaxies in the Loveday et al. sample (311 objects). Finally, we have computed $\alpha$ and $M^*$ applying both eq. (3) and (5). The results are given in Table 2, whose columns have the same meaning as in Table 1, except that in this case all the parameters are the mean of the values derived for each catalogue. In Fig. 2 we show in the $M^* - \alpha$ plane the parameter pairs found for each incomplete catalogue: open and solid circles refer to parameters derived with eq. (3) and (5) respectively, while the star indicates the location of the input parameters.
and the cross represents the Loveday et al. parameters. The dashed and dotted curves are the $1\sigma$ confidence ellipses of the parameters as derived by Efstathiou et al. and Loveday et al., respectively. The figure shows that there is a clear separation between the two sets of points: while the solid circles are very well consistent with the input parameters (star), the open circles are displaced toward a flatter slope $\alpha$ and a brighter $M^*$. 

In conclusion, our simulations suggest that, qualitatively, the correction for incompleteness should move the $(\alpha, M^*)$ parameters of Loveday et al. toward those determined by Efstathiou et al.. Taken at face value, the shift between uncorrected and corrected values as resulting from the simulations is only about half of what is needed to obtain a perfect agreement between the results for the two samples. With such a shift, however, the two confidence ellipses would overlap somewhat at the $1\sigma$ level, thus becoming reasonably consistent with each other. Moreover, it is important to stress that in our simulations we were forced to use arbitrary incompleteness function. A detailed knowledge of the functional form of the incompleteness of the Loveday et al. sample should allow to fully assess the consistency between the luminosity functions which can be derived from the two samples.
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Figure Captions

Figure 1):
Confidence ellipses at 1σ level for the parameters listed in Table 1, referred to the complete and the three incomplete samples of case 1). The parameters derived for the incomplete samples are clearly not consistent with the real ones.

Figure 2):
Parameter pairs ($M^* - \alpha$) found for each incomplete catalogue of case 2): open and solid circles refer to parameters derived with eq. (3) and (5) respectively. There is a clear separation between the two sets of points, being the solid circles very well consistent with the input parameters (star). The star (and the dashed ellipse) and the cross (and the dotted ellipse) refer to the parameters derived by Efstathiou et al. and Loveday et al., respectively.
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