Scale Relativity and Fractal Space-Time: Theory and Applications

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Abstract  In the first part of this contribution, we review the development of the theory of scale relativity and its geometric framework constructed in terms of a fractal and nondifferentiable continuous space-time. This theory leads (i) to a generalization of possible physically relevant fractal laws, written as partial differential equation acting in the space of scales, and (ii) to a new geometric foundation of quantum mechanics and gauge field theories and their possible generalisations. In the second part, we discuss some examples of application of the theory to various sciences, in particular in cases when the theoretical predictions have been validated by new or updated observational and experimental data. This includes predictions in physics and cosmology (value of the QCD coupling and of the cosmological constant), to astrophysics and gravitational structure formation (distances of extrasolar planets to their stars, of Kuiper belt objects, value of solar and solar-like star cycles), to sciences of life (log-periodic law for species punctuated evolution, human development and society evolution), to Earth sciences (log-periodic deceleration of the rate of California earthquakes and of Sichuan earthquake replicas, critical law for the arctic sea ice extent) and tentative applications to systems biology.

Keywords  Scale-relativity · Fractal space-time · Complex systems · Quantum mechanics

1 Introduction

One of the main concern of the theory of scale relativity is about the foundation of quantum mechanics. As it is now well known, the principle of relativity (of motion) underlies the foundation of most of classical physics. Now, quantum mechanics, though it is harmoniously combined with special relativity in the framework of relativistic quantum mechanics and quantum field theories, seems, up to now, to be founded on different grounds. Actually, its present foundation is mainly axiomatic, i.e., it is based on postulates and rules which are not derived from any underlying more fundamental principle.
The theory of scale relativity (Nottale 1989, 1992, 1993, 1996a, 1998c; Nottale and Célérier 2007) suggests an original solution to this fundamental problem. Namely, in its framework, quantum mechanics may indeed be founded on the principle of relativity itself, provided this principle (applied up to now to position, orientation and motion) be extended to scales. One generalizes the definition of reference systems by including variables characterizing their scale, then one generalizes the possible transformations of these reference systems by adding, to the relative transformations already accounted for (translation, velocity and acceleration of the origin, rotation of the axes), the transformations of these scale variables, namely, their relative dilations and contractions. In the framework of such a newly generalized relativity theory, the laws of physics may be given a general form that transcends and includes both the classical and the quantum laws, allowing in particular to study in a renewed way the poorly understood nature of the classical to quantum transition.

A related important concern of the theory is the question of the geometry of space-time at all scales. In analogy with Einstein’s construction of general relativity of motion, which is based on the generalization of flat space-times to curved Riemannian geometry, it is suggested, in the framework of scale relativity, that a new generalization of the description of space-time is now needed, toward a still continuous but now nondifferentiable and fractal geometry (i.e., explicitly dependent on the scale of observation or measurement). New mathematical and physical tools are therefore developed in order to implement such a generalized description, which goes far beyond the standard view of differentiable manifolds. One writes the equations of motion in such a space-time as geodesics equations, under the constraint of the principle of relativity of all scales in nature. To this purpose, covariant derivatives are constructed that implement the various effects of the nondifferentiable and fractal geometry.

As a first theoretical step, the laws of scale transformation that describe the new dependence on resolutions of physical quantities are obtained as solutions of differential equations acting in the space of scales. This leads to several possible levels of description for these laws, from the simplest scale invariant laws to generalized laws with variable fractal dimensions, including log-periodic laws and log-Lorentz laws of “special scale-relativity”, in which the Planck scale is identified with a minimal, unreachable scale, invariant under scale transformations (in analogy with the special relativity of motion in which the velocity \( c \) is invariant under motion transformations).

The second theoretical step amounts to describe the effects induced by the internal fractal structures of geodesics on motion in standard space (of positions and instants). Their main consequence is the transformation of classical dynamics into a generalized, quantum-like self-organized dynamics. The theory allows one to define and derive from relativistic first principles both the mathematical and physical quantum tools (complex, spinor, bispinor, then multiplet wave functions) and the equations of which these wave functions are solutions: a Schrodinger-type equation (more generally a Pauli equation for spinors) is derived as an integral of the geodesic equation in a fractal space, then Klein–Gordon and Dirac equations in the case of a full fractal space-time. We then briefly recall that gauge fields and gauge charges can also be constructed from a geometric re-interpretation of gauge transformations as scale transformations in fractal space-time.

In a second part of this review, we consider some applications of the theory to various sciences, particularly relevant to the questions of evolution and development. In the realm of physics and cosmology, we compare the various theoretical predictions obtained at the beginning of the 1990s for the QCD coupling constant and for the cosmological constant to their present experimental and observational measurements. In astrophysics, we discuss applications to the formation of gravitational structures over many scales, with a special emphasis on the formation of planetary systems and on the validations, on the new extrasolar