OVERCOMING THE CIRCULAR PROBLEM FOR GAMMA-RAY BURSTS IN COSMOLOGICAL GLOBAL-FITTING ANALYSIS

HONG LI,1 JUN-QING XIA,2 JIE LIU,2 GONG-BO ZHAO,3 ZU-HUI FAN,1 AND XINMIN ZHANG2

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ABSTRACT

Due to the lack of low-redshift long gamma-ray bursts (GRBs), the circular problem has been a severe obstacle for using GRBs as cosmological candles. In this paper, we present a new method to deal with such a problem in Markov chain Monte Carlo (MCMC) global fitting analysis. Assuming a certain type of correlation, for the parameters involved in the correlation relation, we treat them as free parameters and determine them simultaneously with cosmological parameters through MCMC analysis on GRB data together with other observational data. Then the circular problem is naturally eliminated in this procedure. To demonstrate the feasibility of our method, we take the Ghirlanda relation \( E_{\text{peak}} \propto C_{13} \) as an example, while keeping in mind the debate about its physical validity. Together with SN Ia, WMAP, and SDSS data, we include 27 GRBs with the reported Ghirlanda relation in our study and perform MCMC global fitting. We consider the \( \Lambda \)CDM model and dynamical dark energy models, respectively. We also include the curvature of the universe in our analysis. In each case, in addition to the constraints on the relevant cosmological parameters, we obtain the best-fit values as well as the distributions of the correlation parameters \( A \) and \( C \). With CMB+LSS+SNe+GRB data included in the analysis, the results on \( A \) and \( C \) for different cosmological models are in agreement well within a 1 \( \sigma \) range. It is also noted that the distributions of \( A \) and \( C \) are generally broader than the priors used in many studies in the literature. Our method can be readily applied to other GRB relations, which might be better physically motivated.

Subject headings: cosmological parameters — cosmology: observations — gamma rays: bursts

Online material: color figures

1. INTRODUCTION

Searching for the nature of dark energy has been one of the most challenging tasks in cosmological studies. Because of the existence of degeneracies between dark energy parameters and the other cosmological parameters in different observables, multi-probe analyses are essential in constraining tightly the properties of dark energy. In this regard, exploring new probes has great importance. On the other hand, thorough investigations on different probes both observationally and theoretically are equally important, so that we can understand their validity and limitations in cosmological applications.

Gamma-ray bursts (GRBs) are the most powerful events observed in the cosmos and can potentially be used to probe the high-redshift universe. Recently, several empirical correlations between GRB observables were reported, and these reports have triggered intensive studies on the possibility of using GRBs as cosmological known candles (Norris et al. 2000; Lloyd-Ronning & Ramirez-Ruiz 2002; Ghirlanda et al. 2004a, 2004b; Dai et al. 2004; Xu et al. 2005; Firmani et al. 2005, 2006; Friedman & Bloom 2005; Schaefer 2007). Constraints on cosmological parameters from GRBs alone and in conjunction with other geometrical probes, including SNe Ia, the shift parameter from cosmic microwave background (CMB) measurements (Wang & Mukherjee 2006), and the \( A \) parameter for the signals of baryon acoustic oscillation (BAO) from galaxy redshift surveys (Eisenstein et al. 2005), have been analyzed (Su et al. 2006; Wright 2007; Wang et al. 2007). Li et al. (2008), for the first time, performed global fitting on the GRB data with the Markov chain Monte Carlo (MCMC) technique together with SNe Ia data (Riess et al. 2007) and data from WMAP (Spergel et al. 2007), SDSS (Tegmark et al. 2004), and 2dFGRS (Cole et al. 2005). On the other hand, the physics behind the empirical correlations are poorly understood. There are also observational indications that some of the reported correlations may have potential problems. Thus there is an ongoing debate for the validity of using GRBs as cosmological candles.

The circular problem has been recognized as another obstacle in the cosmological applications of GRBs. Up to now, there have been about 100 GRBs with measured redshifts, and few are at low redshifts with known distances. Thus, there is a lack of observational data to calibrate, in a cosmology-independent way, the correlation relations. The reported relations are often given assuming an input cosmology. Applying such relations to constrain cosmological parameters leads to the circular problem. Different methods have been put forward to avoid this problem (Firmani et al. 2005; Schaefer 2007). All of them are discussed in the context of using geometrical constraints only.

In this paper, we present a method to deal with the circular problem in the MCMC global fitting. It is known that for constraining cosmological parameters, the most reliable way is to perform global fitting from observational data directly. Li et al. (2008) made a first effort to integrate GRBs in the MCMC chains. However, the GRB data they used were released by Schaefer (2007), where the distance moduli of GRB samples are not independent of the input cosmology model and are still subject to the circular problem. Due to this reason, here we aim at introducing a new method to get rid of the circular problem of GRBs in order to avoid biases arising from it, so that the advantage of the MCMC global fitting can be fully realized. We are aware of the current debate regarding the cosmological applicability...
of GRBs (Bloom et al. 2003; Friedman & Bloom 2005). On the other hand, with both observational and theoretical advances, reliable correlation relations from subclasses of GRBs may eventually emerge. With these considerations in mind, we have made our method as general as possible. It is not limited to any specific correlations. However, to demonstrate its feasibility, we have to work on a concrete example. We choose the Ghirlanda relation (Firmani et al. 2006) in our present study. Our method can be applied to other correlations. The cosmological models to be analyzed include the $\Lambda$CDM model and dynamical dark energy models with the equation of state (EOS) following $w_{\text{DE}} = w_0 + w_1 (1 - a)$ and $w_{\text{DE}} = w_0 + w_1 \sin(w_2 \ln(a))$, respectively. The dark energy perturbations are fully taken into account.

2. METHODOLOGY

For the paper to be self-contained, in this section, we firstly describe briefly the Ghirlanda relation, and then we present our analyzing method dealing with the circular problem of GRBs in the MCMC fitting procedure. The general global-fitting procedure will be explained in § 2.2.

2.1. The Ghirlanda Relation and the Method

Keeping in mind its general applicability, we have to choose a specific correlation for GRBs as an example to show quantitatively the feasibility of our method in the MCMC global-fitting analysis. Among the reported correlations, the Ghirlanda relation is the one that has been used most extensively in constraining cosmology due to its relatively small scatters and the relatively large number of data points available. Recently, there have been intensive arguments questioning this relation, largely because of the observed complexities of the X-ray light curves, which lead to the difficulties in identifying jet break features. On the other hand, it has been pointed out that the X-ray and optical emissions of GRB afterglows may have different origins, and thus can behave differently. In the recent study of Ghirlanda et al. (2007), they emphasize that the jet break features should be considered only if they appear in optical light curves. With the awareness of these debates, we here adopt the Ghirlanda relation in our analysis, and focus on the method for the circular problem, instead of on the cosmological constraints from GRB data.

The Ghirlanda relation, or the $E_{\text{peak}}$-$E_\gamma$ correlation, relies on the jet break feature to calculate the jet opening angle $\theta_{\text{jet}}$, which, in turn, is crucial in correcting the GRB prompt emission energy for the collimation effects. Here we adopt the homogeneous medium model with

$$
\theta_{\text{jet}} = 0.161 \left( \frac{t_{\text{jet,day}}}{1+z} \right)^{3/8} \left( \frac{n_0 \eta_0}{E_{\text{iso,52}}} \right)^{1/8},
$$

where $z$ is the redshift, $\eta_0$ is the radiative efficiency, $t_{\text{jet,day}}$ is the break time in days with $t_{\text{jet,day}} = t_j/1$ day, $n_0$ is the number density of particles in the surrounding interstellar medium with $n_0 = n_1 \text{ cm}^{-3}$, and $E_{\text{iso}}$ is the isotropic-equivalent energy of GRBs with $E_{\text{iso,52}} = E_{\text{iso}}/10^{52}$ ergs. The quantity $E_{\text{iso}}$ is related to the observed fluence $S_\gamma$ in units of erg cm$^{-2}$ as follows:

$$
E_{\text{iso}} = \frac{4\pi d_L^2 S_\gamma k}{1+z},
$$

where $d_L$ is the luminosity distance at redshift $z$ and $k$ is a multiplicative correction related the observed bandpass to a standard rest-frame bandpass (1–10$^4$ keV in this paper; Bloom et al. 2001). The collimation-corrected energy $E_\gamma$ is

$$
E_\gamma = (1 - \cos \theta_{\text{jet}}) E_{\text{iso}}.
$$

Following Xu et al. (2005), we write the $E_{\text{peak}}$-$E_\gamma$ correlation in the following form:

$$
\frac{E_\gamma}{10^{50} \text{ ergs}} = C \left( \frac{E_{\text{peak}}}{100 \text{ keV}} \right)^A,
$$

where the parameters $A$ and $C$ are assumed to be constant, and $E_{\text{peak}} = E_{\text{peak,obs}} (1+z)$. It is seen that besides the direct observables, the luminosity distance $d_L$ comes in through equations (1)–(2). If, for a set of GRBs, their distances can be determined independently, one can calibrate the correlation relation (4) and find the values of $A$ and $C$ from observations directly. Then the cosmology-independent correlation can be used to estimate $d_L$ for other GRBs and further to constrain cosmology. Unfortunately, we lack GRBs with known distances. Therefore, in order to obtain values of $A$ and $C$, one has to assume a cosmological model to calculate $d_L$. The circular problem arises when the luminosity distances derived from such a cosmology-dependent correlation relation are used as cosmological candles. Different methods have been discussed to avoid the circular problem in grid-based $\chi^2$ analyses involving only geometrical probes. In the following we describe our method to deal with the circular problem in MCMC global-fitting procedures.

The essential aspect of our method is that for an assumed functional form of a correlation relation such as in equation (4), we set the correlation parameters free in our analyzing process instead of using the values reported in other studies. Simultaneously with the cosmological parameters, their values are determined through global fittings with GRB data and other data sets, including CMB, large-scale structure (LSS), and SNe. Specifically, for each element on MCMC chains with a set of parameters $(x_i, A, C)$, where $x_i$, $i = 1, \ldots, n$ are cosmological parameters we are interested in, the “observed” luminosity distance for each GRB is obtained through equations (1)–(4):

$$
d_L = 7.575 \left( \frac{1+z}{C^2} \right)^{2/3} \left[ \frac{E_{\text{peak,obs}} (1+z)/100 \text{ keV}}{k S_\gamma t_{\text{jet,day}}^{1/2} (n_0 \eta_0)^{1/6}} \right]^{24/3} \text{ Mpc},
$$

where the small angle approximation with $\theta_{\text{jet}} \ll 1$ has been applied. With the assumption that all the GRB observables are independent of each other with Gaussian distributed errors, the uncertainty for each of these “data” points is estimated as follows (Friedman & Bloom 2005):

$$
\frac{\sigma_{d_L}}{d_L}^2 = \frac{1}{4} \left[ \frac{\sigma_S}{S_\gamma} \right]^2 + \frac{1}{4} \left( \frac{\sigma_k}{k} \right)^2 + \frac{1}{4} \left( \frac{C_0}{1 - \sqrt{C_0}} \right)^2 \left[ \frac{A E_{\text{peak,obs}}}{E_{\text{peak}}} \right]^2
$$

$$
\quad + \frac{1}{4} \frac{C_0}{(1 - \sqrt{C_0})^2} \left[ \left( \frac{3 t_{\text{jet,day}}}{t_{\text{jet,day}}} \right)^2 + \left( \frac{\sigma_n}{n_0} \right)^2 + \left( \frac{\sigma_\eta}{\eta_0} \right)^2 \right],
$$

where $\sigma_S$ is the uncertainty in the observed flux, $\sigma_k$ is the uncertainty in the jet opening angle, $\sigma_n$ and $\sigma_\eta$ are the uncertainties in the number density and radiative efficiency, respectively.
Additional scatters besides the uncertainties for the parameters \(2003; (65) \text{Tiengo et al. 2003.}

(41) \text{Galassi et al. 2004; (42) Hullinger et al. 2005; (43) Cummings et al. 2005; (44) Sakamoto et al. 2006b; (45) Markwardt et al. 2006; (46) Frail et al. 2003; (13) Vreeswijk et al. 2003; (14) Greiner et al. 2003a; (15) Rol et al. 2003; (16) Greiner et al. 2003b; (17) Jakobsson et al. 2004; (18) Fugazza et al. 2004; (19) Fynbo et al. 2005b; (20) Enck et al. 2005; (21) Berger et al. 2005; (22) Foley et al. 2005; (23) Prochaska et al. 2005; (24) Cucchiara et al. 2006a; (25) Berger & Gladders 2006; (26) Ghirlanda et al. 2007; (27) Jimenez et al. 2001; (28) Amati et al. 2002; (29) Amati et al. 2003; (30) Barraud et al. 2003; (31) Sakamoto et al. 2005; (32) Crew et al. 2003; (33) Vanderspek et al. 2004; (34) HETE 2006, http://space.mit.edu/HETE/Bursts/; (35) Krimm et al. 2006; (36) Sakamoto et al. 2006a; (37) Krimm et al. 2005; (38) Schaefer et al. 2005; (39) Bloom et al. 2003; (40) Frontera et al. 2001; (41) Galassi et al. 2004; (42) Hullinger et al. 2005; (43) Cummings et al. 2005; (44) Sakamoto et al. 2006b; (45) Markwardt et al. 2006; (46) Frontera et al. 2003; (47) Stanek et al. 1999; (48) Masetti et al. 2000; (49) Bjornsson et al. 2000; (50) Halpern et al. 2000; (51) Jakobsson et al. 2003; (52) Berger et al. 2002; (53) Holland et al. 2003; (54) Holland et al. 2004; (55) Klose et al. 2006; (56) Andersen et al. 2005; (57) Price et al. 2003b; (58) Stanek et al. 2005; (59) Hakkert et al. 2006; (60) Blustin et al. 2006; (61) Dai & Stanek 2006; (62) Moretti et al. 2006; (63) Panaitescu & Kumar 2002; (64) Schaefer et al. 2003; (65) Tiengo et al. 2003.

\[
C = \left[ \theta \sin \theta / (8 - 8 \cos \theta) \right]^2. 
\]

(7)

We take \(\theta = 0.2\) and \(d_{\gamma} = 0\) throughout this paper (Frai et al. 2001). It is noted that when equation (6) is used to calculate \(d_{\gamma}\), it is implicitly assumed that the correlation equation (4) has no additional scatters besides the uncertainties for the parameters \(A\) and \(C\). Considering the distance modulus, we have

\[
\mu_{\text{obs}} = 5 \log d_L + 25, 
\]

(8)

\[
\sigma_{\mu_{\text{obs}}} = \frac{5}{\ln 10} \frac{\sigma_{d_L}}{d_L}. 
\]

(9)

In order to constrain the cosmological parameters \(\chi_i\), we have marginalized the free parameters \(A\) and \(C\), and finally we get the probability for a certain cosmological parameter \(\chi_i\):

\[
P(\chi_i) = \int P(\chi_i | x_i, \ldots, A, C) P(x_i) \ldots P(A) P(C) \, d\chi_i \ldots dA \, dC, 
\]

(10)

which is related to \(\chi^2\) given by the statistical results of the observational data via \(P \propto e^{-\frac{\chi^2}{2}}\). And the \(\chi^2\) contributed by GRB "data" at the point \((x_i, A, C)\) is then computed as

\[
\chi^2(x_i, A, C) = \sum \left[ \frac{\mu_{\text{obs}}(x_i) - \mu_{\text{obs}}(x_i, A, C)}{\sigma_{\mu_{\text{obs}}}} \right]^2, 
\]

(11)

where the summation is over the number of GRB data points. We use 27 GRBs, which are reported to satisfy the \(E_\text{peak}\) vs. \(E_\gamma\) relation, in our study. The relevant data are listed in Table 1. In Table 1, the data are mostly from Ghirlanda et al. (2007) except for GRB 050505 and GRB 060210. For these two GRBs, we take the data from Schaefer (2007; G. Ghirlanda 2007, private communication).

### 2.2. Global-Fitting Program

Different observations play complementary roles in the determination of cosmological parameters. Their combination can effectively break out the degeneracies between different parameters, and therefore can deliver much better constraints on cosmology.
than any single probe can. For different observables, it is important to understand the main factors that affect the determination of interested parameters. The information on these elements extracted from observational data is very useful. Under certain conditions, the extracted values of these factors can be used to probe cosmology without invoking complicated observational data, which could greatly simplify the analyzing procedures. The two important examples are the shift parameter from CMB observations and the BAO parameter from galaxy redshift surveys, and they have been widely used in constraining properties of dark energy. On the other hand, however, careful attention must be paid to the conditions under which the extracted information is obtained. Inappropriate use of these pieces of information can lead to biased conclusions about the values of cosmological parameters. Therefore, the most reliable way in determining cosmology is to perform global-fitting analyses using observational data directly. Our global-fitting analyses are based on the publicly available MCMC package CosmoMC (Lewis & Bridle 2002).4 We have made modifications according to our own research purposes. Besides the modifications described in § 2.1, which are specific for the circular problem of GRBs, we include dark energy perturbations in our general analyzing program.

For dark energy models with equation of state \( w \neq -1 \), the perturbations inevitably exist. While the effects of dark energy perturbations are yet to be fully explored, it is generally believed that they may only show their influences near-horizon scales. For CMB anisotropy, the power spectrum at low \( l \) (large angular scale) is affected by the late integrated Sachs-Wolfe effect (ISW), whose strength depends on the properties of dark energy in a flat universe. Thus, large-scale anisotropy is important, and the perturbations can play roles in dark energy studies. It is well recognized that in the fluid approach, there is a divergence problem at \( w = -1 \) when dark energy perturbations are included. On the other hand, there are observational indications that the equation of state of dark energy may cross \(-1\) during the evolutionary history of the universe (Huterer & Cooray 2005; Feng et al. 2005; Xia et al. 2006; Zhao et al. 2007a). Thus for dark energy perturbations, the divergence problem must be carefully dealt with. From our analysis on two-field quintom models (Feng et al. 2005; Zhang et al. 2006), in which \( w \) crosses \(-1\) can be realized, we find that the dark energy perturbations are well behaved at \( w = -1 \). Thus, the divergence in the fluid treatment should be a mathematical one instead of a physical one. Along this line of thinking, we develop a scheme in the fluid approach to avoid the divergence problem (Zhao et al. 2005; Xia et al. 2006).

In the conformal Newtonian gauge, the perturbation equations of dark energy are

\[
\dot{\delta} = -(1 + w)(\theta - 3H\dot{\delta}) - 3H(c_s^2 - w)\delta, \tag{12}
\]

\[
\dot{\theta} = -H(1 - 3w)\theta - \frac{\dot{w}}{1 + w}\theta + k^2\left(\frac{c_s^2\delta}{1 + w} + \Psi\right), \tag{13}
\]

where \( \delta \) and \( \theta \) are the energy density and velocity perturbations, respectively. The divergence at \( w = -1 \) can be seen from the second equation. To handle this problem, we introduce a small constant \( \epsilon \) and divide \( w \) into three parts with (1) \( w > -1 + \epsilon \), (2) \( -1 + \epsilon \geq w \geq -1 - \epsilon \), and (3) \( w < -1 - \epsilon \), respectively. For regions 1 and 3, the perturbations are analyzed following the equations. For region 2, we need a special treatment. We match the perturbation quantities of region 2 to regions 1 and 3 at the corresponding boundaries, and set

\[
\delta = 0, \quad \dot{\theta} = 0. \tag{14}
\]

within the region. Thus there are discontinuities in the derivatives in region 2. But with small enough \( \epsilon \), the discontinuities have negligible effects. We compare the results from this analysis with those of two-field quintom models and find that with \( \epsilon \leq 10^{-5} \), the perturbations of the quintom models can be well reproduced by this fluid approach. For more details of this method we refer the readers to our previous companion papers (Zhao et al. 2005; Xia et al. 2006). Thus, we set \( \epsilon = 10^{-5} \) in our studies.

We consider three cosmological models, the \( \Lambda \)CDM model including the curvature term, and dynamical dark energy models with the equation of state parameterized respectively as

\[
(1) \ w_{\text{DE}}(a) = w_0 + w_1(1 - a), \tag{15}
\]

\[
(II) \ w_{\text{DE}}(a) = w_0 + w_1 \sin[w_2 \ln(a)], \tag{16}
\]

where \( a = 1/(1 + z) \) is the scale factor and \( w_1 \) characterizes the “running” of the EOS. For parameterization I (Para I), we include the curvature term. In parameterization II (Para II), we are limited in the flat universe and fix \( w_2 = 3\pi/2 \), so that we are not introducing too many parameters during the fitting process. Our most general parameter space is then

\[
P = (\omega_b, \omega_c, \Omega_k, \Theta, \tau, w_0, w_1, n_s, \ln(10^{10}A_s), A, C), \tag{17}
\]

where \( \omega_b \equiv \Omega_b h^2 \), \( \omega_c \equiv \Omega_c h^2 \), and \( \Omega_k \) represent the contribution of the curvature term to the total energy budget, \( \Theta \) is the ratio (multiplied by 100) of the sound horizon at decoupling to the angular distance to the last scattering surface, \( \tau \) is the optical depth due to reionization, \( w_0 \) and \( w_1 \) are the parameters of the EOS of dark energy, \( A \) and \( n_s \) characterize the power spectrum of primordial scalar perturbations, and \( A \) and \( C \) are the free parameters related to the \( E_{\text{peak}}/E_{\gamma} \) correlation. For the \( \Lambda \)CDM models, \( w_0 = -1 \) and \( w_1 = 0 \).

We vary the above parameters and fit to the observational data with the MCMC method. For the pivot of the primordial spectrum we set \( k_{\text{pivot}} = 0.05 \text{ Mpc}^{-1} \). The following weak priors are taken: \( \tau < 0.8, 0.5 < n_s < 1.5, -3 < w_0 < 3, -5 < w_1 < 5, 0.5 < A < 2.5, \) and 0.01 < \( C < 2.5 \). We impose a top-hat prior on the cosmic age as 10 Gyr < \( t_0 < 20 \) Gyr. Furthermore, we make use of the Hubble Space Telescope (HST) measurement of the Hubble parameter \( H_0 \equiv 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) by multiplying the likelihood by a Gaussian likelihood function centered around \( h = 0.72 \) with a standard deviation \( \sigma = 0.08 \) (Freedman et al. 2001). We also adopt a Gaussian prior on the baryon density \( \Omega_b h^2 = 0.022 \pm 0.002 (1 \sigma) \) from big bang nucleosynthesis (Burles et al. 2001).

In our calculations, we take the total likelihood to be the product of the separate likelihoods (\( L \)) of CMB, LSS, SNe Ia, and GRBs. For CMB, we include the three-year WMAP (WMAP3) data and compute the likelihood with the routine supplied by the WMAP team (Spergel et al. 2007). For the large-scale structure information, we have used the Sloan Digital Sky Survey (SDSS) luminous red galaxy (LRG) sample (Tegmark et al. 2006). To minimize the nonlinear effects, we have only used the first 15 bins, 0.0120 < \( k_{\text{qG}} < 0.0998 \), which are supposed to be well within the linear regime. For SNe Ia, we mainly present the results with the recently released ESSENCE 192 sample supernovae published in Miknaitis et al. (2007) and Davis et al. (2007). In the

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4 See also the CosmoMC Web site at http://cosmologist.info.
calculation of the likelihood from SNe Ia data, we marginalize over the nuisance parameter (Goliath et al. 2001; Di Pietro & Claeskens 2003).

For each regular calculation, we run eight independent chains comprising 150,000–300,000 chain elements, and spend thousands of CPU hours on a supercomputer. The average acceptance rate is about 40%. We test the convergence of the chains by Gelman and Rubin criteria (Gelman & Rubin 1992) and find that $R - 1$ is on the order of 0.01, which is much more conservative than the recommended value $R - 1 < 0.1$.

3. RESULTS AND DISCUSSION

In this section, we present our main results. Firstly, we consider the constraint on the nonflat $\Lambda$CDM model using GRB data only. In order to study the dependence of $A$ and $C$ on the cosmology models, we combine the GRB data with other cosmological observational data, such as CMB, LSS, and SNe, to constrain different dark energy models, namely the $\Lambda$CDM model and the dynamical dark energy models, Para I and Para II. Considering the degeneracy between the dark energy parameters and the curvature of the universe (Zhao et al. 2007b; Clarkson et al. 2007), we also consider the nonflat case in the dynamical dark energy model Para I. During these calculations, we focus on discussions of constraints on $A$ and $C$, and the effects of GRB data on the determination of cosmological parameters.

In Figure 1, we show the one-point likelihood function for $A$ and $C$, respectively. The black solid line in each panel represents the result for the nonflat $\Lambda$CDM model using GRB data only. The other lines are the results from combined analysis of CMB+LSS+SNe+GRB for different cosmological models. The gray solid, dashed, dash-dotted, and dotted lines are for the $\Lambda$CDM, flat model with dark energy Para I, nonflat model with dark energy Para I, and flat model with dark energy Para II, respectively.

On the other hand, given the observational data sets, the dependence of $A$ and $C$ on cosmological models is rather weak, as seen from the results shown by the gray and nonsolid lines in Figure 1. The mean value of $A$ in all four cosmological models is $A \sim 1.53$ with $1 \sigma$ error about 0.08. For the parameter $C$, its variation for different cosmological models is slightly larger than that of $A$, with the average value of $C = 0.94, 0.96, 0.98$, and 0.94, for the four models, respectively. They are in agreement with each other well within a $1 \sigma$ range with $\sigma \simeq 0.1$. Keeping in mind the hot debate regarding GRBs as cosmological candles due to the lack of thorough understanding of GRB physics and the quality of observational data, the consistency in $A$ and $C$ for different cosmological models seen from our global-fitting analysis may hint that the Ghirlanda relation could be intrinsic to a subsample of GRBs.

Because we determine $A$ and $C$ simultaneously with the cosmological parameters, their likelihood distributions shown in Figure 1 have included the effects of uncertain cosmology constrained by the current observational data. Therefore our global-fitting analyses give a consistent evaluation on the contribution of GRBs to the determination of cosmological parameters. Both the distributions in $A$ and $C$ are broader than those estimated with a fixed cosmological model. Thus, inappropriate use of the narrow uncertainties in $A$ and $C$ resulting from a given cosmology can lead to an overestimate of the power of GRBs in cosmological studies.

The results on the cosmological parameters constrained from our analysis are listed in Table 2. In Figure 2, we show, respectively, the constraints on the dark energy parameters $w_0 - w_1$ for the flat model with dark energy Para I (top left), nonflat model with dark energy Para I (top right), and flat model with dark energy Para II (bottom), respectively. The black solid and the dashed lines represent the results with and without the 27 GRBs included. For all the cases considered, the flat $\Lambda$CDM model is very consistent with the observational data with or without GRBs. For the case of flat model with Para I for the dark energy EOS, quintom A type of models with $w_0 < -1$ and $w_1 > 0$ are mildly favored by the data. Including the GRB data in the analysis, the $1 \sigma$ contour shifts more toward the quintom A region. Relaxing the strong prior on the flatness of the universe, the $\Lambda$CDM gives

![Figure 1](https://example.com/figure1.jpg)

**Figure 1.** One-dimensional posterior constraints for the parameters $A$ and $C$ obtained via MCMC methods. The black solid lines are for the nonflat $\Lambda$CDM model using GRB data only. The other lines are the results from joint analysis of CMB+LSS+SNe+GRB. The gray solid, dashed, dash-dotted, and dotted lines are for the $\Lambda$CDM, flat model with dark energy Para I, nonflat model with dark energy Para I, and flat model with dark energy Para II, respectively. [See the electronic edition of the Journal for a color version of this figure.]
a better fit to the data than the above case. Considering the oscillating dark energy model, it is seen that the ΛCDM model with \( w_0 = -1 \) and \( w_1 = 0 \) is again in excellent agreement with the observational data. Comparing the black solid lines with the dashed lines in Figure 2, we see some shrinkage of the error contours when including GRB data in the analysis, which is largely attributed to the high-redshift range of GRBs. This indicates the possible potential in using GRBs as high-redshift cosmological candles. The contribution from the current GRB data is however not greatly significant. Nevertheless, given the apparent

![Diagram](image)

**Fig. 2.** Two-dimensional joint 68% and 95% confidence regions for the parameters \( w_0 \) and \( w_1 \) of flat model with dark energy Para I (top right), and flat model with dark energy Para II (bottom), respectively. The black solid lines are given by using WMAP3+SNe Ia+LSS+GRBs while the dashed lines come from WMAP3+SNe Ia+LSS without GRBs. For both cases we considered the dark energy perturbation. [See the electronic edition of the Journal for a color version of this figure.]

### TABLE 2

| Parameter | \( \Lambda \)CDM | Para I | Para II |
|-----------|-----------------|--------|---------|
| \( w_0 \) | -1               | -1     | -1      |
| \( w_1 \) | 0                | 0      | 0       |
| \( \Omega_{DE} \) | 0.748 ± 0.046    | 0.761 ± 0.017 | 0.762 ± 0.020 |
| \( \Omega_{k} \) | -0.235 ± 0.047   | 0      | -0.002 ± 0.014 |
| \( A \)   | 1.51 ± 0.0836    | 1.54 ± 0.0764 | 1.53 ± 0.0761 |
| \( C \)   | 0.912 ± 0.048    | 0.943 ± 0.106 | 0.983 ± 0.139 |

**Note.**—Para I and Para II represent \( w_{DE}(a) = w_0 + w_1 (1 - a) \) and \( w_{DE}(a) = w_0 + w_1 \sin \frac{3\pi}{2} \ln(a) \), respectively. For the current constraints we have shown the mean values 1 σ (Mean).
advantage of GRBs as the tracers of the high-redshift universe, it is important to perform detailed analysis as we did here to investigate their usefulness in cosmological studies.

4. SUMMARY

In this paper, we present a new method in dealing with the circular problem for GRBs in the determination of cosmological parameters. This method is implemented in our MCMC global-fitting program. The methodology is to treat the parameters involved in a GRB correlation relation as free parameters when performing global-fitting analysis. Their values are then simultaneously estimated together with the cosmological parameters we are interested in, and therefore the circular problem is naturally eliminated. Furthermore, our analysis can give the likelihood distributions of the correlation parameters with the uncertainties in the cosmological parameters being taken into account.

From the distributions of $A$ and $C$, we can see that the dependence of $A$ and $C$ on the cosmology model is rather weak, and the constraints on $A$ and $C$ for different cosmological models are in agreement well within a $1\sigma$ range. However, the distributions of $A$ and $C$ are generally broader than the priors used in many studies in the literature, which will lead to the overestimate of the power of GRBs in cosmological studies. With the combined data sets CMB+LSS+GRB+SNe, our global-fitting results show that in different dynamical dark energy models the constraints on dark energy parameters become stringent by taking into account high-redshift GRBs, which show the potential of GRBs in the cosmology studies.

We emphasize that our method can be readily applied to different correlation relations of GRBs, although we take the Ghirlanda relation as a concrete example in this paper. In fact, the applicability of our method is even not limited to GRB studies. Any cosmological probe involving parameters other than cosmological ones can be analyzed with our method. Thus, our implemented MCMC program presented in this paper can be a platform with wide applications. For example, in using the abundance of clusters of galaxies to constrain cosmology, the relations between direct observables, such as the X-ray brightness (temperature), the Sunyaev-Zel’dovich effect and the richness of galaxies, and the total mass of a cluster have to be involved. Applying the relations derived based on simplified assumptions regarding the physical state of clusters may lead to biased cosmological conclusions. It has been proposed to analyze such relations simultaneously with cosmological parameters to be studied. Our program is then perfectly suitable for such analysis.

It is noted that all our investigations and implementations are carried under the framework of MCMC global fitting using observational data directly. Therefore, we can give more reliable estimates on the considered parameters than those of Fisher matrix analysis or the constraints derived from some extracted parameters, such as the CMB shift parameter and the BAO parameter of large-scale structures of the universe.

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