The Kondo-lattice state in the presence of Van Hove singularities: a next-leading order scaling description

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A renormalization group treatment of the Kondo model with a logarithmic Van Hove singularity in the electron density of states is performed within next-leading order scaling, different magnetic phases being considered. The effective coupling constant remains small, renormalized magnetic moment and spin-fluctuation frequency decreasing by several orders of magnitude. Thus wide non-Fermi-liquid behavior regions are found from the scaling trajectories in a broad interval of the bare coupling parameter. Applications to physics of itinerant magnetism are discussed.

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I. INTRODUCTION

Anomalous rare-earth and actinide compounds (Kondo lattices and heavy-fermion systems) are studied extensively starting from the middle of 1980s [1]. Besides heavy-fermion features (huge electronic specific heat), they exhibit the non-Fermi-liquid (NFL) behavior: logarithmic or anomalous power-law temperature dependences of magnetic susceptibility and specific heat [2]. The magnetism of these systems is also highly interesting and has both localized and itinerant features; in particular, magnetic moment can be strongly reduced or unstable [3–5]. Thus a combined treatment of Kondo and itinerant-electron systems becomes usual [6, 7]. NFL behavior and strongly enhanced electronic specific heat are observed also in some d-systems [2], including nearly ferromagnetic ruthenates [8–10].

Main role in the physics of the Kondo lattices belongs to interplay of on-site Kondo screening of magnetic moments and intersite exchange interactions inducing magnetic order. This idea was developed in a series of the papers [3, 12] treating the mutual renormalization of two energy scales: the Kondo temperature $T_K$ and characteristic spin-fluctuation frequency $\omega$. The corresponding scaling consideration of this renormalization process in the $s-f$ exchange model [3, 13] yields, depending on the values of bare parameters, both the “usual” states (a non-magnetic Kondo lattice or a magnetic state with weak Kondo corrections) and the peculiar magnetic Kondo-lattice state, including the NFL behavior. However, the region of the latter behavior depends strongly on the approximations and concrete models of electron and magnetic structure [13–15]. The NFL behavior can be related to peculiar features of electron and spin fluctuation spectrum, as well as in the case of itinerant systems (which are usually described by the Hubbard model).

It is well known that peculiarities of bare electron structure play an important role in the formation of magnetism. The case of singular density of states (DOS) for the Kondo lattices was considered in Ref.[14] within the lowest-order scaling. Although a considerable increase of NFL region was obtained, the situation did not qualitatively change in comparison with the smooth DOS case.

At the same time, the NFL behavior occurs naturally in the one-impurity $M$-channel Kondo model [11, 16, 17]. This model, which assumes existence of degenerate electron bands, explains power-law or logarithmic behavior of electronic specific heat and magnetic susceptibility [17]. Physically, such a behavior is connected with overscreening of impurity spin by conduction electrons in many channels. The model permits a consistent scaling investigation in the next-leading approximation. A characteristic feature of this approximation is occurrence of an intermediate fixed point. This is reasonable for $M > 2$ since the fixed point is within the weak-coupling region (however, the marginal case $M = 2$ requires a more accurate consideration). On the other hand, for $M = 1$ the fixed point is unphysical. The situation for the lattice is more complicated, especially in the case of singular DOS, since the singularities change the structure of perturbation theory.

In the present paper we treat the Kondo lattice model with the electron spectrum containing a logarithmic DOS singularity to compare the results of leading and next-leading order scaling. Such a singularity is typical, in particular, for the two-dimensional case. The scaling equations are discussed in Sect.2. The numerical results are presented in Sect.3 for the cases of a paramagnet with smooth spin spectral function and a ferromagnet (singular spin spectral function).

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II. SCALING EQUATIONS

We use the degenerate-band (multichannel) Kondo lattice model

$$H = \sum_{km\sigma} t_{km\sigma} c_{km\sigma}^\dagger c_{km\sigma} - I \sum_{im\sigma'} S_i \sigma_{\sigma\sigma'} c_{im\sigma'}^\dagger c_{im\sigma'} + H_f$$

(1)

where $t_{km}$ is the band energy, $S_i$ are spin-1/2 operators, $I$ is the $s-f$ exchange parameter, $\sigma$ are the Pauli matrices, $m = 1...M$ is the orbital degeneracy index. For the sake of convenient constructing perturbation theory, we explicitly include the Heisenberg $f-f$ exchange interaction

$$H_f = \sum_{ij} J_{ij} S_i S_j$$

(2)

in the Hamiltonian, although in fact this interaction is usually the indirect RKKY coupling. This interaction competes with the Kondo effect and results in occurrence of cutoffs for the corresponding infrared divergences. Depending on character of $f-f$ interactions, we can treat paramagnetic and various magnetically ordered (ferro- or antiferromagnetic) phases with reduced moments.

The density of states corresponding to the spectrum $t_{km}$ is supposed to contain a Van Hove singularity (VHS) near the Fermi level. The simplest example is the square lattice with the spectrum $t_{km} = 2t(\cos k_x + \cos k_y)$ where we have the density of states

$$\rho(E) = \frac{2}{\pi^2 D} K\left(\sqrt{1 - \frac{E^2}{D^2}}\right)$$

$$\simeq \rho F(E), \quad F(E) = \ln \frac{D}{|E|} + 2 \ln 2, \quad \rho = \frac{2}{\pi^2 D}$$

(3)

where $K(E)$ is the complete elliptic integral of the first kind, the bandwidth is determined by $|E| < D = 4|t|$. However, considerable singularities can occur for three-dimensional lattices too [18].

At $M > 2$ the fixed point lies in a weak coupling region, which makes possible successful application of perturbation theory and renormalization group approaches. We apply the “poor man scaling” approach [19]. This considers the dependence of effective (renormalized) model parameters (effective $s-f$ coupling and spin-fluctuation frequencies) on the flow cutoff parameter $C \to -0$.

To find the equation for the renormalized coupling parameter $I_{cf}(C)$ we pick out in the sums for the Kondo corrections the contribution of intermediate electron states near the Fermi level with $C < t_{k+q} < C + \delta C$. Bearing in mind a NFL-type behavior, we write down the scaling equations to next-leading order by taking into account the corrections of order of $MT^3$ (see details in Refs. [3, 11, 15]):

$$\delta I_{cf}(C) = 2\rho I^2 F(C) + I\rho MF(C/2)F(-C/2)|\eta(-\frac{\omega}{C})\delta C/C$$

(4)

where $\omega$ is a characteristic spin-fluctuation energy (the ratio $\omega/|C|$ is initially assumed to be small within perturbation theory, but can become arbitrary depending on scaling behavior during renormalization process), $\eta(x)$ is a scaling function taking into account spin dynamics and satisfying the condition $\eta(0) = 1$. Strictly speaking, the next-to-leading order contribution is exact in the large-$M$ limit only, but the finite-$M$ case can be treated in agreement with the one-impurity limit (see Sect.3).

For the paramagnetic (PM), ferromagnetic (FM) and antiferromagnetic (AFM) phases we have

$$\eta^{PM}(\frac{\omega}{C}) = \text{Re} \int_{-\infty}^{\infty} d\omega \langle \mathcal{J}_{k-k'}(\omega) \rangle_{t_k = t_{k'} = E_f} \frac{1}{1 - (\omega + i0)/C}$$

(5)

$$\eta^{FM,AFM}(\frac{\omega_{cf}}{|C|}, \delta) = \text{Re} \left\langle \left(1 - (\omega_{k-k'} + i\delta)^2/C^2\right)^{-1} \right\rangle_{t_k = t_{k'} = E_f}$$

(6)

Here $\omega_{q}$ is the magnon frequency, angle brackets stand for the average over the wavevector $k$ on the Fermi surface,

$$\langle A(k) \rangle_{t_k = E_F} = \sum_k A(k) \delta(t_k - E_F)$$

(7)
\( J_0(\omega) \) is the spectral density of the spin Green’s function for the Hamiltonian \( H_f \), which is normalized to unity, \( \delta \) is a cutoff owing to damping.

In the simple spin-diffusion approximation for a paramagnet and spin-wave approximation for the magnetic phases we have \[ \mathbf{3, 15} \]

\[
\eta^{PM}(x) = \arctan \frac{x}{x} \\
\eta^{FM}(x) = \frac{1}{4x} \ln \left\{ \frac{(1 + x)^2 + \delta^2}{(1 - x)^2 + \delta^2} \right\} \\
\eta^{AFM}(x) = -(2x^2)^{-1} \ln [(1 - x^2)^2 + 4\delta^2]
\]

The scaling functions for the ordered phases contain Van Hove singularities at \( x = 1 \).

The renormalizations of magnetic moment and spin fluctuation frequency are also obtained from perturbation theory and are given by \[ \mathbf{3, 14, 15} \]

\[
\delta \omega_{ef}(C)/\omega = a \delta \omega_{ef}(C)/S = 2a_\rho^2 I^2 F(C/2) F(-C/2) \eta(-\omega_C/|C|) \delta C/C
\]

The latter result holds for all magnetic phases with \( a = 1 - \alpha \) for the paramagnetic (PM) phase, \( a = 2(1 - \alpha) \) for the ferromagnetic (FM) phase, \( a = 1 - \alpha' \) for the antiferromagnetic (AFM) phase. Here \( \alpha \) and \( \alpha' \) are some averages over the Fermi surface (see Ref. \[ \mathbf{3} \]). In the approximation of nearest neighbors at the distance \( |R| = d \) one obtains

\[
\alpha = |\langle \exp(i k R_2) \rangle|_{t_k = E_F} |^2 \sim \left( \frac{\sin k_F d}{k_F d} \right)^2
\]

For the staggered AFM ordering we have

\[
\alpha' \sim b J_2 J_1 |\langle \exp(i k R_2) \rangle|_{t_k = E_F} |^2
\]

where \( b = 2 \) and \( b = 4 \) for the square and simple cubic lattices, \( J_1 \) and \( J_2 \) are the Heisenberg exchange integrals between nearest and next-nearest neighbors (\( |J_1| > |J_2| \)) in \( H_f \), \( R_2 \) runs over the next-nearest neighbors. Although \( \alpha' = 0 \) in the nearest-neighbor approximation, this parameter enters physical properties in the NFL regime \[ \mathbf{13} \].

Defining the renormalized and bare dimensionless coupling constants

\[
ge_{ef}(C) = -2g_{ef}(C), \quad g = -2I \rho
\]

and the function

\[
\psi(\xi) = \ln(\omega/\omega_{ef}(\xi))
\]

which determines renormalization of spin dynamics we obtain

\[
\partial g_{ef}(\xi)/\partial \xi = [\xi - \gamma(\xi + \ln 2)]^2 g_{ef}(\xi)\Psi(\lambda + \psi - \xi),
\]

\[
\partial \psi(\xi)/\partial \xi = a \gamma g_{ef}^2(\xi)(\xi + \ln 2)^2 \Psi(\lambda + \psi - \xi)
\]

where \( \gamma = M/2 \), we put in spirit of scaling consideration \( \xi = \ln |D/C| + 2 \ln 2 \sim \ln |D/C| \) (note that a constant DOS contribution is absorbed by the replacement \( \xi \rightarrow \xi + c \)),

\[
\Psi(\xi) = \eta(e^{-\xi}), \quad \lambda = \ln(D/\omega) \gg 1.
\]

First we discuss briefly the one-impurity case (\( \Psi = 1 \)). The solution of the lowest-order (one-loop) scaling equation according to \[ \mathbf{11} \] yields

\[
1/g_{ef}(\xi) = 1/g - \xi^2/2
\]

The divergence of \( g_{ef}(C) \) occurs at the Kondo temperature

\[
T_K \propto D \exp \left[-\frac{\pi^2 D}{2I} \right]^{1/2}
\]
This result is in agreement with perturbation theory and numerical renormalization group (NRG) results for the singular DOS case, unlike the parquet approach of Ref. 20 (see discussion in Refs. 14, 21).

For comparison with the standard Kondo problem, it is instructive to introduce the function

\[ G_{ef}(\xi) = g_{ef}(\xi)\xi. \]

Owing to the structure of perturbation theory, this quantity is an effective coupling parameter in the singular DOS situation. In particular, \( G_{ef}(\xi) \) (with the replacement \( |C| \to T \)) enters corrections for electronic properties like magnetic susceptibility and specific heat, cf. Ref. 14. When neglecting \( \ln 2 \) in comparison with \( \xi \), the scaling equation for this function takes the form

\[ \partial G_{ef}(\xi)/\partial \xi = G_{ef}(\xi)/\xi + [1 - \gamma G_{ef}(\xi)]G_{ef}^2(\xi) \]  (20)

Apart from the first term (which is small at large \( \xi \), i.e. at low energies), the right-hand side does not depend explicitly on \( \xi \). Thus Eq. (20) has the structure of a standard Gell-Mann-Low equation and is similar to the scaling equation for \( g_{ef}(\xi) \) in the smooth DOS case. In the latter case the two-loop equation corresponding to (10) gives a finite fixed point \( g_{ef}(\xi \to \infty) = 2/M \). It is known that this point is unphysical (unreachable) for \( M = 1 \), but for \( M > 2 \) the scaling consideration gives a qualitatively correct description 17 (the case \( M = 2 \) is marginal, so that additional logarithmic factors occur in the physical properties).

Unlike the smooth DOS case, the equation (10) cannot be solved analytically even in the one-impurity case, but an asymptotic solution at large \( \xi \) can be obtained:

\[ g_{ef}(\xi) = 2M/\xi + \left(1 - 4\ln 2/M\right)1/\xi^2 \]  (21)

The second term in brackets can change sign, being positive for large \( M \) and negative for small \( M \), so that occurrence of a maximum in the dependence of \( g_{ef}(\xi) \) is possible. Besides that, the factor in (21) is well determined only within the \( 1/M \)-expansion. Thus the solution is rather sensitive to details of approximations.

The above results demonstrate existence of the “fixed point”, which is similar to the fixed point in the flat-band case,

\[ G_{ef}(\xi) = G^* = 2/M \]  (22)

The scaling trajectories approach this according the law

\[ G^* - G_{ef}(C) \propto 1/|C|, \]  (23)

unlike the power law in the case of smooth DOS 13, 16. Note that corresponding \( 1/\ln T \)-dependences are obtained in NRG calculations of impurity magnetic susceptibility and specific heat 21, 22.

Writing down the Kondo correction to magnetic susceptibility by analogy with Refs. 3, 14, 16, we obtain the scaling equation for the effective magnetic moment:

\[ \partial \ln S_{ef}(\xi)/\partial \xi = -(M/2)G_{ef}^2(\xi) \]  (24)

so that to leading order for \( |C| < T_K \)

\[ S_{ef}(C) \simeq (|C|/T_K)^\Delta, \Delta = 1/\gamma = G^* \]  (25)

It should be noted that in the non-singular DOS case the power-law critical behavior like 25 takes place in a wide region, including \( |C| > T_K \) and \( |C| < T_K \) 16. Thus, unlike the situation of total screening (where the strong-coupling region cannot be described by simple methods) we have an interpolation description.

On taking into account higher orders in \( 1/M \) one has in the flat-band one-impurity case 16

\[ \Delta = 2M \left(1 - 2/M\right) \simeq 2/M + 2 \]  (26)

which agrees with the Bethe ansatz solution, see Ref. 17.
FIG. 1: The scaling trajectories for a paramagnet, $G_{ef}(\xi)$ (a) and $\psi(\xi)$ (b). The parameter values are $\lambda = 6$, $a = 0.7$, $g = 0.03, 0.05, 0.08, 0.15$, $M = 1$ ($\gamma = 3/2$, solid lines) and $g = 0.05, 0.15$, $M = 3$ ($\gamma = 5/2$, dashed lines) – for the curves from below to above (when considering the left-hand part of the figure) respectively.

III. SCALING BEHAVIOR IN THE KONDO LATTICE

As discussed above, the structure of perturbation theory is similar to the non-singular case with the replacement $g_{ef}(\xi) \rightarrow G_{ef}(\xi)$. Now we pass to the lattice case, main point being inclusion of spin dynamics into scaling equations. To establish properly the correspondence with the one-impurity case (26), we may put $\gamma = M/2 + 1 \equiv 1/\Delta$. This yields at $M > 2$ correct critical exponents for magnetic susceptibility, specific heat and resistivity. The important case $M = 2$ is more difficult from the theoretical point of view: additional logarithmic factors occur in electronic specific heat and magnetic susceptibility, although the resistivity is still described by the $1/\Delta$ expansion, see [11, 17].

We present below numerical results for $M = 1$ ($\gamma = 3/2$) and for $M = 3$ ($\gamma = 5/2$); the latter case may be relevant for Ce$^{3+}$ ion [17].

A. Paramagnetic case

The dependences $G_{ef}(\xi)$ and $\psi(\xi)$ from solution of the full scaling equations (16) and (17) in the paramagnetic phase are shown in Fig.1. During the scaling process $\psi(\xi)$ increases according to (17). One can see that $G_{ef}(\xi)$ demonstrates a plateau at $G^* \approx 1/\gamma$, which can be named a “quasi-fixed point”.

Provided that the bare coupling parameter $g$ is not too small, at intermediate $\xi$ (near the plateau) we can put for rough estimations $G_{ef}(\xi) \approx G^* = 1/\gamma$ to obtain

$$\psi(\xi) \approx a\gamma G_{ef}^2(\xi) - a/\gamma g \approx (a/\gamma)(\xi - 1/g)$$

($\Psi(\xi > 1) \approx 1$). Thus a power-law behavior occurs

$$\varpi_{ef}(C) \approx \varpi(|C|/T_K)^\beta, \quad \varpi_{eff}(C) \approx (|C|/T_K)^\Delta,$$

$$\beta = a/\gamma = a\Delta,$$

which corresponds to the one-impurity NFL behavior [25]. The dependence [27] takes place up to the point

$$\xi_1 \approx (\lambda - \beta/g)/(1 - \beta).$$

For $\xi > \xi_1$, $\psi(\xi) \approx \psi(\xi_1) \approx \lambda\beta/(1 - \beta)$ is practically constant since $\Psi(\lambda + \chi - \xi)$ becomes small.

When the DOS singularity is shifted from the Fermi level by the distance $v$, its influence on the scaling behavior becomes weaker. A similar effect occurs when the logarithmic singularity is smeared (e.g., small electron damping is introduced, $\ln E \rightarrow (1/2)\ln(E^2 + \Gamma^2)$). At small $v$ the scaling behavior is determined by a combined action of the Kondo and Van Hove singularities. The influence of the shift is described by the replacement $C \rightarrow C - v$ in the singular factors in the scaling equations. The scaling trajectories for $v = 0.005$ are shown in Fig.2. The influence of the shift on the dependence $G_{ef}(\xi)$ is more pronounced than that on $\psi(\xi)$ (for the latter, we have some quantitative difference only).
FIG. 2: The scaling trajectories for a paramagnet, $G_{ef}(\xi)$ (a) and $\psi(\xi)$ (b). The singularity is shifted from the Fermi level by $v = 0.005$. Other parameter values are the same as in Fig. 1.

On the other hand, at not too small $v$ and $g$ the DOS singularity becomes unimportant, so that with increasing $\xi$ we come rapidly to a non-physical fixed point, $g_{ef}(\xi) \rightarrow g^*$ with large $g^*$.

Since exact divergence of DOS is not required, we can suppose that not only strong logarithmic singularities, but also weaker VHS (e.g., those in 3D cubic lattices) may change considerably the scaling behavior. Note that the increase of $\psi(\xi)$ in paramagnetic phase is much more stronger than for a smooth DOS (cf. Ref. [15]).

The interval of bare coupling constant, where a NFL-like behavior occurs, is very wide: we come to a quasi-fixed point independently of $g$, although this point becomes unstable with increasing $\xi$. On the other hand, the one-loop scaling yields for finite $M$ the NFL behavior in a narrow interval of the bare coupling constant $g$ only, since with increasing $g$ we come rapidly to strong-coupling regime where $g_{ef}(\xi > \lambda) \rightarrow \infty$, the critical value $g_c$ being rather small [14]. On the contrary, in the two-loop scaling there is no such a critical $g$ value at all: $g_{ef}(\xi)$ remains finite for any $g$ in the paramagnetic case. Thus the lowest-order scaling cannot describe properly the case of not too small $g$.

In the region of the plateau ($G_{ef}(\xi) = \text{const}$), $g_{ef}(\xi)$ decreases with increasing $\xi$. Therefore the scaling curves $G_{ef}(\xi)$ can intersect each other for different $g$. However, this feature disappears when introducing small shift of the singularity from the Fermi level, the size of the plateau decreasing (Fig. 2).

From (28) we obtain the power-law dependence of magnetic susceptibility

$$
\chi(T) \propto S_{ef}^2(T)/T \propto (T/T_K)^{2\Delta-1}
$$

As demonstrate NRG calculations for the one-impurity Kondo model with VHS [22], the local magnetic susceptibility

$$
\chi_{\text{loc}}(T) = \int_0^{1/T} d\tau \langle S_z(\tau)S_z \rangle
$$

(which just determines spin correlation functions and therefore corresponds to present calculations) has a slight maximum tending to a constant value with lowering temperature for $M = 1$ and demonstrates a power-law NFL behavior for $M = 2$ (unlike logarithmic behavior in the flat-band case). Thus the tendency to NFL behavior increases in the presence of VHS.

B. Ferromagnetic case

Now we come to the situation of magnetic ordering. Of especial interest is ferromagnetic state: its realization is connected with large density of states at the Fermi level, so that peculiarities of the NFL state in the presence of VHS should be also treated.

In magnetically ordered phases, the behavior for $\xi < \xi_1$ is similar to that in a paramagnet, but the situation for $\xi > \xi_1$ changes since the Van Hove singularity of $\Psi(\xi)$ at $\xi = 0$ plays an important role. Instead of decreasing, $\Psi(\lambda + \psi - \xi)$ starts to increase at approaching $\xi_1$.

For sufficiently large $g$ (needed to reach the appreciable $G_{ef}$ value during the scaling increase at small $\xi$), provided that

$$
a\gamma G_{ef}^2(\xi \simeq \xi_1)\Psi^{\text{max}} \simeq a\gamma g^*2\Psi^{\text{max}} \simeq (a/\gamma)\Psi^{\text{max}} > 1,
$$

(31)
FIG. 3: The scaling trajectories for a ferromagnet, $G_{ef}(\xi)$ (a) and $\psi(\xi)$ (b). The parameter values are $\lambda = 6$, $a = 1$, $g = 0.025, 0.04, 0.06, 0.1$, $M = 1$ ($\gamma = 3/2$, solid lines) and $g = 0.02, 0.03, 0.08$, $M = 3$ ($\gamma = 5/2$, dashed lines) – for the curves from below to above respectively, the damping parameter is $\delta = 10^{-2}$.

FIG. 4: The scaling trajectories for a ferromagnet, $G_{ef}(\xi)$ (a) and $\psi(\xi)$ (b). The parameter values are $M = 3$ ($\gamma = 5/2$), $\lambda = 6$, $a = 1$, $g = 0.03, 0.04, 0.06, 0.1$ for the curves from below to above respectively, the damping parameter is $\delta = 0.02$.

at $\xi > \xi_1$ the argument of the function $\Psi$ in (17) becomes almost constant (fixed), $\psi(\xi) \simeq \xi - \lambda$. Thus further behavior is determined by the singularity of the scaling function and is similar to that for smooth DOS [15]. We have for the frequency and magnetic moment

$$\omega_{ef}(C) \simeq |C|, \quad S_{ef}(C)/S \simeq (|C|/\omega)^{1/a}. \quad (32)$$

The scaling curves for a ferromagnet are shown in Fig.3 (In the antiferromagnetic case the picture is qualitatively the same, cf. Ref. [15]).

Thus the scaling behavior is changed at some critical value $g_c$. Above the critical value $g_c$, the qualitative picture of the nearly linear scaling trajectories $\psi(\xi)$ does not depend on $g$: they are almost parallel and slowly come together.

In turn, there is a critical value of the damping parameter $\delta$ in the scaling function (10) (which is a cutoff for the singularity determining $\Psi^{\max}$), so that for $\delta > \delta_c$ the infinite linear behavior does not occur. This value, $\delta_c$, is determined by the values of $a$ and $M$. For large $\delta$ and $M > 1$ the linear dependence of the type (32) can take place in a restricted region changing the behavior of the type (27), so that two NFL-like regions are observed (see Fig.4, $\delta_c$ is about 0.015 for $M = 3$).

For $M = 1$ the critical damping is not small: $\delta_c$ is about 0.1. This is favorable for occurrence of the NFL regime (32): it can take place in the presence of a somewhat pronounced (even not too sharp) peak in the scaling function $\eta$. Of course, the simple model with constant damping can be generalized. So, the increase of the damping with growing of $g_{ef}(\xi)$ was considered in the scaling versions of Refs. [13, 14]. As well as in the paramagnetic case, a small shift of the singularity from the Fermi level results in a change of $G_{ef}(\xi)$ behavior at large $\xi$, but influences weakly the behavior $\psi(\xi)$.

Remember again that the quantity $\psi(\xi)$ determines the temperature dependences of magnetic moment and thermodynamic characteristics (for the corresponding discussion in the antiferromagnetic case, see Refs. [13, 15]).
IV. CONCLUSIONS

We have shown that the Kondo lattice with Van Hove singularities in electron spectrum demonstrates non-Fermi-liquid behavior in a wide parameter region. As follows from (19), the value of the Kondo temperature is rather high. Thus the system is characterized by moderate specific heat, but large magnetic susceptibility. The renormalization of magnetic moment is much stronger than in the case of smooth DOS. Although the heavy-fermion behavior is not expressed, a tendency to magnetic ordering occurs, which is characteristic for weak itinerant magnets with VHS like ZrZn$_2$ too.

In this connection, we can also mention some experimental examples of NFL features for ruthenate d-systems. An enhancement of the electronic specific heat and magnetic susceptibility was observed in the layered system Sr$_{2−x}$La$_x$RuO$_4$ with increasing x, the Fermi-liquid behavior being violated near the critical value x = 0.2. Such a tendency is explained by the elevation of the Fermi energy toward VHS of the thermodynamically dominant Fermi-surface sheet. The NFL behavior is attributed to two-dimensional ferromagnetic fluctuations with short-range correlations at VHS [8]. The bilayered ruthenate Sr$_2$Ru$_2$O$_7$ is a paramagnetic Fermi liquid with strongly enhanced quasiparticle masses [9]. The Fermi-liquid region of the phase diagram is suppressed by magnetic field, and NFL behavior extends up to very low temperatures upon approaching the critical metamagnetic field $B = 7.8$ T [10].

Occurrence of giant Van Hove singularities (which are important, e.g., for ferromagnetism of iron) is intimately connected with intersection of more weak singularities, i.e. with degeneracy of electron bands [18]. Already in the classical textbook on magnetism [23] such a degeneracy is considered as a key to itinerant ferromagnetism. This statement is in spirit of a multichannel model too.

The results obtained for the scaling behavior are qualitatively reliable for $M > 2$. On the other hand, they, generally speaking, should be verified by more strict analytical and numerical methods, including two-loop field-theoretical or functional renormalization group (fRG) (see, e.g., investigations in Refs. [24–26] performed for the Hubbard model). It should be noted that a scaling treatment in the presence of logarithmic singularities meets with difficulties [27]. The corresponding problems of higher-order scaling are also discussed in recent works [28, 29]. Probably, the nesting problem (see, e.g., [30]) can be considered in a similar way, but strong $k$-dependence requires a more sophisticated treatment without averaging over the Fermi surface. The case of the Kondo lattice seems to be more simple since the scaling equations are obtained from those for the one-impurity model by inclusion of spin dynamics.

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