Bajc-Melfo Vacua enable YUMGUTs

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Abstract

Bajc-Melfo (BM) two field \((S, \phi)\) superpotentials define metastable F-term supersymmetry breaking vacua suitable as hidden sectors for calculable and realistic unification models. The undetermined vev \(<S>_s>\) of the Polonyi field that breaks Supersymmetry can be fixed either by coupling to N=1 Supergravity or by radiative corrections. BM hidden sectors extend to symmetric multiplets \((S, \phi)_{ab}\) of a gauged \(O(N_g)\) family symmetry, broken at the GUT scale, so that the \(O(N_g)\) charged component vevs \(<\hat{S}_{ab}>\) are also undetermined before accounting for the \(O(N_g)\) D-terms: which fix them by cancellation against D-term contributions from the visible sector. This facilitates Yukawon Ultra Minimal GUTs(YUMGUTs) proposed in [1] by relieving the visible sector from the need to give null D-terms for the family symmetry \(O(N_g)\). We analyze symmetry breaking and and spectra of the hidden sector fields in the Supergravity resolved case when \(N_g = 1, 2, 3\). Besides the Polonyi field \(S_s\), most of the superfields \(\hat{S}_{ab}\) remain light, with fermions getting masses only from loop corrections. Such modes may yield novel dark matter lighter than 100 GeV. Possible Polonyi and moduli problems associated with the the fields \(S_{ab}\) call for detailed investigation of loop effects due to the Yukawa and gauge interactions in the hidden sector and of post-inflationary field relaxation dynamics.

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I. INTRODUCTION

Supersymmetry imposes remarkable restrictions on its own spontaneous violation and thus severely constrains models of BSM physics. Traditionally gravity mediation has been used to generate phenomenologically acceptable soft Susy breaking terms for globally supersymmetric GUTs. This requires supersymmetry breaking in a hidden sector with no superpotential couplings to the ‘visible’ sector in which we live. The GUT Minkowski space vacuum is metastable in the sense of there being a Susy preserving vacuum with lower (negative) energy, but it is thought that it cannot decay to the lower vacuum by tunnelling; or at any rate the timescale for doing so is much larger than the age of the universe. The vacuum in the gravity mediated scenario is defined as the perturbation of a Global Susy preserving minimum by a hidden sector with a global minimum which breaks supersymmetry spontaneously due to specialized superpotentials (Polonyi, O’Raifeartaigh etc) or due to strong dynamics (gaugino condensation etc). After long struggles to tame the hard problem of finding usable Susy breaking global minima it was re-emphasized that metastable i.e local minima could serve just as well as global minima, since the unifying theory would inevitably have vacua at lower energy than the phenomenologically relevant Minkowski vacuum on which the MSSM lived, provided they were stable or long lived. This realization was liberating for phenomenologists because models defined on supersymmetry breaking vacua which are only local minima offer a much larger range of candidate vacua. Various strongly coupled supersymmetric gauge theories were studied to provide the Susy breaking dynamics albeit now considering also metastable vacua. Unfortunately even the metastable dynamical Susy breaking models seem so arbitrarily elaborate as to rival the complexity of the phenomenological theory (GUT, MSSM etc) for which they were supposedly to provide the service of Supersymmetry breaking: making testability of both a moot issue.

Recently Bajc and Melfo (BM) suggested that, from a ‘calculable unification’ viewpoint, it would be more interesting to consider Susy breaking metastable vacua of simple two field hidden sector superpotentials. BM models are related to the classic O’Raifeartaigh susy breaking superpotentials and are variants of a class of models with susy breaking local minima studied systematically earlier in. Such superpotentials typically have flat directions and there is a longstanding vision whereby the determination of such flat directions (running
out of susy breaking minima) by radiative corrections\cite{12} or supergravity\cite{13,14} might profitably fix the undetermined vacuum expectation values (VEVs) at a high Unification scale. Bajc and Melfo took up this challenge and formulated models where the undetermined VEVs were those of $24$ or $75$ irreps of an $SU(5)$ unified theory and the flat directions were determined by radiative corrections. The use of of N=1 Supergravity potentials to lift the flat directions rolling out of the Bajc-Melfo Susy breaking local minimum seems not to have been explicitly analyzed.

From a Grand Unified viewpoint the only credible sign of flavour unification so far is third family Yukawa unification at large $\tan \beta$ in SO(10) at scales of order $M_X > 10^{16}$ GeV. On the other hand parameter counting minimal Supersymmetric SO(10) GUTs\cite{15–17,19–23} have been shown to be completely realistic and even to suppress $d = 5$ proton decay\cite{25} by a novel and generic mechanism based on the necessary emergence of a pair of light doublet Higgs in the effective MSSM. Motivated by these two clues we exploited the defining (i.e $126$) representation of MSGUTs (which gets large vevs yet couples to matter fermions) to design a renormalizable Grand Unified Yukawon/familion model\cite{1} in which we generate hierarchical matter fermion Yukawas from flavour bland parameters of the MSGUT extended by an $O(N_g)$ family symmetry. Family gauge symmetry is broken at the unification scale and the visible sector GUT Higgs fields effloresce into symmetric $(210, 126, T_{26}, 10)$ or anti-symmetric $(120)$ irreps of the family group (as dictated by the properties of bilinears in the $16$-plet matter irreps in SO(10) GUTs), while the matter $16$-plet Yukawa couplings become flavour bland. The neat and exactly soluble\cite{17} gauge symmetry breaking ($SO(10) \rightarrow$ MSSM) of the family singlet MSGUT Higgs system then generalizes straightforwardly for the family variant MSGUT Higgs multiplets. The supersymmetric breaking of family symmetry will be discussed in detail in this paper. Extraction of the light MSSM Higgs from the plethora of MSSM doublets present in the $O(N_g) \times SO(10)$ GUT via calculation of the ‘Higgs fractions’\cite{17,18,20,23} (which specify the make up of the MSSM Higgs) determined by flavour blind GUT parameters generates candidate flavour structures.

In implementing this flavour generation scenario an obstacle arises. Although the solution of the F and D term conditions for the GUT multiplets in the SO(10) sector with flavour non-diagonal VEVs is straightforward, it is very hard or impossible to ensure that the same configurations simultaneously cancel the family symmetry D-terms. It thus becomes necessary to introduce additional fields to soak up the the contribution of the GUT VEVs to
the $O(N_g)$ D terms. Since minimality, simplicity and solubility of the GUT Higgs sector symmetry breaking is a central virtue of MSGUTs it is natural to ask whether the additional fields might not be located in a hidden sector with its own flat directions arising from supersymmetry breaking in a way which did not interfere with the already accomplished MSGUT symmetry breaking\cite{15,17}. Besides making a completely unexpected connection between family symmetry and supersymmetry breaking this idea is straightforward to implement in a context where the BM flat directions are lifted by a combination of supergravity and $O(N_g)$ gauge D term minimization effects.

A notable phenomenological implication of our model is the presence of weakly coupled light fields (‘moduli’). In particular, in the supergravity mediated scenario presented here there is a Polonyi (scalar) field (Planck scale vev, weak scale mass and intermediate scale susy breaking F-term). In addition there are other light fermions and scalars associated with the flat directions of the family non singlet BM fields. These modes may be interesting light (< 50 GeV) Susy WIMP dark matter of the sort perhaps indicated by the DAMA/LIBRA experiment\cite{26} and so far completely missing from MSGUT spectra. On the other hand this clutch of modes may also pose the typical problems associated with ‘moduli’ fields found in other fundamental theories\cite{27}. Recall that the Polonyi problem arises due to late decay of the oscillations of the supersymmetry breaking Polonyi field while it settles into the zero temperature potential’s minimum because of the suppression of its couplings by the Planck scale. The dark matter candidates’ role in cosmogony also needs to be checked for consistency. In view of the new Yukawa and gauge interactions of the hidden sector the cosmological implications require a separate detailed study which should not prematurely prejudice the novel flavour generation aspects of our model. In this Letter we give the relevant details for the symmetry breaking, VEVs and masses in the BM sector for direct use in our ‘yukawonification’\cite{1} models. We will return to a study of cosmological constraints and alternative models elsewhere.

II. THE BAJC MELFO VACUA AND SUPERGRAVITY

The basic superpotential\cite{10,11} has the form

$$W_H(S_s, \phi_s) = W_0 + S_s(\mu_B \phi_s + \lambda_B \phi_s^2)$$

(1)
We have added a constant term $W_0$ for later convenience. It is then easy to see that besides the Susy preserving global minima at $S_s = \phi_s = 0$ and $S_s = 0, \phi_s = -\mu_B/\lambda_B$ with zero vacuum energy, there is also a local minimum at $<\phi_s> = -\mu_B/2\lambda_B$ with $<S_s>$ undetermined provided

$$|<S_s>| \geq \frac{|<\phi_s>|}{\sqrt{2}}$$

(2)

This local minimum breaks supersymmetry since $<F_S> = -\mu_B^2/4\lambda_B \equiv \theta$. Thus the chiral multiplet $S_s$ provides the Goldstino field and its partner scalar, whose VEV remains undetermined, is massless. When supersymmetry is made local by coupling the theory to gravity the goldstino can be gauged away by a local supersymmetry transformation leaving a massive gravitino while the scalar picks up the gravitino mass common to light scalars in gravity mediation.

The generic $N = 1$ supergravity potential for scalar fields $Z_I$ interacting through a superpotential $W(z_I)$ and with canonical Kahler potential

$$V = E(|F^I + Z^* I \kappa^2 W|^2 - 3\kappa^2|W|^2) \quad ; \quad E \equiv e^{\kappa^2\sum_i|Z_i|^2}$$

(3)

Fixing $\phi_s$ at the vev $\bar{\phi}_s = -\mu_B/2\lambda_B$ (vevs to leading order in $m_{3/2}$ are denoted by bars) determined by $F^{\phi_s} = 0$ and allowing for the presence of other background VEVs $\bar{Z}_I(\phi_s, z_i)$ which preserve Susy ($\partial_{\bar{z}_I} W = 0$) gives for the potential to be minimized

$$V(S_s) = e^{\kappa^2 |S_s|^2 + \delta} \left\{ (|\delta \kappa^2 \hat{W}_0 + S_s \theta|^2 + |\theta + \kappa^2 S_s^* (\hat{W}_0 + S_s \theta)|^2 - 3\kappa^2|\hat{W}_0 + S_s \theta|^2 \right\}$$

(4)

Here $\delta = \kappa^2 (|\bar{\phi}_s|^2 + \sum_i |\bar{z}_i|^2)$ includes the contributions of all Susy preserving vevs ($\bar{z}_i$) from the visible sector and the hidden sector($\phi_s$). $\hat{W}_0$ includes the constant $W_0$ as well as the VEV of the visible sector superpotential due to the vevs $\bar{z}_i$. Defining dimension less variables

$$x = \kappa \frac{\hat{W}_0}{\theta} \quad ; \quad y = \kappa S_s$$

$$\varphi_x = Arg[x] \quad ; \quad \varphi_y = Arg[y]$$

(5)

the potential becomes

$$\tilde{V} \equiv \frac{V}{|\theta|^2} = \left\{ (|x|^2 + |y|^2)(\delta - 3) + (1 + |y|^2)^2 + |x|^2|y|^2 + 2\cos(\varphi_y - \varphi_x)|x||y|(\delta - 2) \right\}$$

(6)
The only stable minimum with respect to $S_s$ with zero energy i.e for (field subscripts denote partial derivatives):

$$V = V_{|y|} = V_{\theta_y} = 0 = V_{|y|, \theta_y} ; \quad V_{|y|,|y|, \theta_y, \theta_y} > 0 \quad (7)$$

is achieved as

$$\phi_y = \varphi_x ; \quad x = 2 - \sqrt{3 - \delta} ; \quad y = y_0 = \sqrt{3 - \delta} - 1$$
$$\partial_{|y|} \partial_{|y|} V = 4 \sqrt{3 - \delta} ; \quad \partial_{\varphi_y} \partial_{\varphi_y} V = 4 \delta \sqrt{3 - \delta} - 16 \delta - 32 \sqrt{3 - \delta} + 56 \quad (8)$$

It is clear that the condition for a local minimum (2) is satisfied for

$$\delta < 3 - (1 + \sqrt{\frac{k^2 \theta}{2 \lambda_B}})^2 \simeq 2 \quad (9)$$

With the solutions $\bar{\phi}_s, \bar{z}_i$ these vevs specify a Minkowski vacuum, with the degeneracy in $S_s$ lifted, to leading order in $m_3/2\kappa \sim 10^{-13}$. There is no immediate reason to compute next order shifts by perturbing around these vevs, but it is straightforward to do so iteratively.

The previously undetermined Susy breaking field $S_s$ has obtained a Planck scale VEV. As usual in gravity mediated scenarios, the chiral fermion in the S multiplet is the global Susy breaking Goldstino that furnishes the longitudinal mode of the gravitino which gets the mass

$$m_2 = \kappa^2 |\sqrt{E (\bar{W}_0 + \bar{W}_H})| \quad (10)$$

Since this mass should be $\sim 10^3 - 10^5$ GeV in typical gravity mediated scenarios and typically $\bar{W}_{GUT} \sim M_X^3 > 10^{48}$ GeV, it is clear that $W_0$ must be used to cancel $\bar{W}_{GUT}$ so that

$$|\bar{W}_0 + \bar{W}_H| = M_\mu |\theta| < 10^{39} - 10^{41} GeV^3 \quad (11)$$

The scale of supersymmetry breaking is fixed by the BM superpotential parameters :

$$\sqrt{|F^S|} = \sqrt{|\theta|} = |\frac{\mu_B}{2 \sqrt{\lambda_B}}| \sim 10^{10.5} - 10^{11.5} GeV \quad (12)$$

Provided $|\lambda_B| > 10^{-14}$ one has $|\lambda_B S| >> |\mu_B|$. In any case the VEV $\bar{\phi}_s = -\mu_B/2\lambda_B$ implies that the field $S_s$ (which has no mass term $W = m_s S^2 + ..$ ) does not get a mass by mixing with $\phi_s$ and provides the Goldstino as expected, even though the Supersymmetry is broken by a local rather than global minimum. The fermion masses consist of the $\phi_s$ mode with
mass $2\lambda_{B}\bar{S}$ and the gravitino with longitudinal mode from the goldstino $\tilde{S}$. The scalar mass spectrum consists of two real scalar modes from $\phi_s$ with masses squared $|2\lambda_{B}\bar{S}|^2 \pm |\mu_{B}|^2/2$ and two from $S_s$ with masses squared $m_{3/2}^2((7 + 4\sqrt{3}) \pm (5 + 3\sqrt{3}))$. This completes the discussion of the $N_g = 1$ case apart from working out the effective theory which will be of the usual supergravity type (Global Susy GUT derived MSSM plus soft Susy breaking terms). It bears mention that radiative effects in such systems may be quite significant and have an important bearing on cosmological behaviour [28]. The loop corrected Kahler potential is available up to two loops [29]. We will return to the study of the vacuum and cosmological predictions in the theory with both gravity and radiative effects in a sequel.

III. GAUGED $O(N_g)$ AND BM VACUA

Consider the generic situation where the superpotential $W_H(S, \phi)$ is used as the hidden sector of a supergravity model with total superpotential $W = W_H + W_{GUT}(z_i)$. The fields $z_i$ are the chiral multiplets of a suitable Grand Unified model and are symmetric reps of the family symmetry group $O(N_g)$. Their vevs $\bar{z}_i$ define a globally supersymmetric minimum at which the corresponding (global Susy) F-terms and D terms of the visible sector GUT gauge group $G_{GUT}$ vanish:

$$F_i = \frac{\partial W}{\partial z_i} = 0 ; \quad D_{GUT}^\alpha(\bar{z}_i, \bar{z}_i^*) = 0 \quad (13)$$

We wish to perturb around the metastable minimum defined by (2) and the global minimum equations (13) to fix the remaining undetermined non-singlet components of $\hat{S}$.

The D-term contributions to the supergravity potential are (assuming canonical Kahler potential and gauge Kinetic function are excellent approximations)

$$V_D = \frac{g_2^2}{4} \left[ (z^{*i} + F_i^{*}) (T^{\alpha})^j_i z_j + h.c. \right]$$

We assume that the GUT sector vevs - even though they carry $O(N_g)$ representations - satisfy (13) to leading order in $m_{3/2}$: as indeed one requires from any consistent GUT theory which should be defined by an isolated minimum of the globally supersymmetric potential. So the D-terms reduce to the global supersymmetry form since the extra terms involving the F-terms vanish (only the gauge singlet $S_s$ has $F_{s_s} \neq 0$).

Assuming that the visible sector supersymmetry conditions are satisfied, the vanishing of the family symmetry D terms for the GUT sector vevs alone is by no means guaranteed.
The $O(N_g)$ D terms have the form

\begin{align}
D^a_{O(N_g)} &= \text{Tr}(\hat{\phi}^\dagger [T^a, \hat{\phi}] + \hat{S}^\dagger [T^a, \hat{S}]) + D_X^a \\
D_X^a &= \sum_i \bar{z}_i^\dagger T^a z_i
\end{align}

(14)

where $D_X^a$ is the contribution of the visible sector fields, and $T^a, \mathcal{T}^a$ the generators of $O(N_g)$ in the fundamental and generic representations.

Since we wish to use a flat direction of the hidden sector superpotential to cancel the contribution $D_X^a$ of the visible sector we consider the situation where $\phi_s, S_s$ become the trace modes of $O(N_g)$ symmetric representations $\phi_{ab}, S_{ab}$ so that the BM superpotential becomes

\begin{equation}
W_H = \text{Tr} S (\mu_B \phi + \sqrt{N_g} \lambda_B \phi^2)
\end{equation}

(15)

Generically

\begin{equation}
S = \hat{S} + \frac{1}{\sqrt{N_g}} S_s I_{N_g} \quad ; \quad \text{Tr} \hat{S} = 0
\end{equation}

(16)

$I_{N_g}$ is the $N_g$ dimensional unit matrix. The $O(N_g)$ non-singlet fields from $\hat{S}_{ab}$ (i.e $S_\pm$ for $N_g = 2$ and $S_{0,\pm,\pm,2}$ for $N_g = 3$) being absent from the background superpotential will enter the Supergravity potential only quadratically through $\delta$- like contributions of the visible sector fields. For these fields there is no visible sector type superpotential and hence minimization would set them to zero were it not for the effect of the D-terms of $O(N_g)$: which we regard as a gauged family symmetry under which not only matter fields but also the visible sector Higgs fields as well as the traceless parts of $[\hat{S}_{ab}], [\hat{\phi}_{ab}]$ are non-singlets and hence enter in the $O(N_g)$ D term(s).

A. Determination of $S_\pm$ when $N_g = 2$

For $N_g = 2$ it is convenient to use the isomorphism $O(2) \simeq U(1)$ and define

\begin{equation}
S = \frac{1}{2} \begin{pmatrix}
\sqrt{2} S_s + S_+ + S_- \\
i(S_- - S_+) \\
i(S_+ - S_-) \\
\sqrt{2} S_s - (S_+ + S_-)
\end{pmatrix}
\end{equation}

(17)

and similarly for $\phi = [\phi_{ab}]$ where $S_\pm, S_s$ are properly normalized fields so that $\text{Tr} S^\dagger S = S_+^\dagger S_+ + S_-^\dagger S_- + S_s^\dagger S_s$ and $\text{Tr} S \phi = S_s \phi_s + S_{(+, \phi_-)}$. In this notation an $O(2)$ vector $\psi_a$ has charges $\pm 1/2 : \psi_{\pm 1/2} = (\psi_1 \pm i \psi_2)/\sqrt{2}$. The superpotential becomes

\begin{equation}
W_H = \mu_B (S_s \phi_s + S_{(+, \phi_-)}) + \lambda_B (S_s \phi_s^2 + 2 S_s \phi_+ \phi_- + 2 \phi_s S_{(+, \phi_-)})
\end{equation}

(18)
and it is easy to check that $\bar{\phi}_s = -\mu_B/2\lambda_B, \bar{\phi}_\pm = 0$ solves the $F$-term conditions for $W_H$ (all $F$-terms vanish except that $\partial S_s W = \theta$ as before). The fields $S_{s,\pm}$ remain undetermined at this local minimum and the superpotential reduces to the one for $N_g = 1$ when the fields that are determined are equated to their values at the local minimum.

The calculation for the singlet goes through unchanged fixing $S_s \sim M_p$ as in (S). With this VEV the $O(2)$ charged fermionic components of $\hat{\phi}_\pm$ get masses $\sim \lambda_B S_s$. Along with the GUT scale breaking of the family symmetry the large mass of the modes in $\phi_{ab}$ is responsible for quelling the percolation of the supersymmetry breaking coded in $F_{S_s}$ and ensuring the hidden sector is actually hidden. Since $\hat{\phi}_\pm$ have zero vevs at the Susy breaking minimum they do not mix with the family symmetry gaugino. For scalars there are small splittings $\sim |\mu_B| \ll |\lambda_B S_s|$. Since the charged S fields can at most have vevs of order the GUT scale $M_X \ll M_p$ the potential for their determination to leading order in $m_{3/2}$ is simply the common scalar mass term generated by the Supergravity potential and the $D$ term of the family symmetry. In the $O(2)$ case the leading order potential for the flat family charged $S$ directions is

$$V(S_+, S_-) = m_{3/2}^2(|S_+|^2 + |S_-|^2) + \frac{g_f^2}{2}(|S_+|^2 - |S_-|^2 + \bar{D}_X)^2$$

(19)

here $\bar{D}_X = \sum q_i |\bar{z}_i|^2$ where $q_i, \bar{z}_i$ are the family symmetry charges of the global supersymmetric vevs in the visible sector. Clearly, if $x = \text{Sign}[\bar{D}_X]$ the minimum will occur when

$$S_{-x} = \sqrt{|\bar{D}_X| - x\frac{m_{3/2}^2}{g^2}} ; \quad \bar{S}_x = 0$$

(20)

Notice that the shift in the $S_{-x}$ vev relative to $\sqrt{\bar{D}_X}$ is tiny but the gravity induced mass term does enforce the choice of which component of $\hat{S}$ gets a vev. The vacuum energy after minimization is $\sim m_{3/2}^2|\bar{D}_X|$ and can be tuned to zero by a shift $\delta W_0 \sim m_{3/2}|\bar{D}_X| < m_{3/2}M_p^2$.

With these vevs and due to the lack of any other mass term for $S_{\pm}$ it is easy to check that the fermionic component of $S_{-x}$ mixes due to the family symmetry breaking vevs from the
visible sector to provide a partner Λ for the U(1)_f gaugino in a Dirac mass

\[ m_{\lambda B} = \sqrt{2}g_f \sqrt{|\tilde{\mathcal{S}}_{-s}|^2 + \sum_i q_i^2 |\tilde{z}_i|^2} \]

\[ \mathcal{L} = -m_{\lambda \lambda} \lambda_g + H.c. \]

\[ \Lambda = (\cos[\theta_S]\tilde{Z} - \sin[\theta_S]\tilde{\mathcal{S}}_{-x}) \frac{\sqrt{2}g_f}{m_{\lambda B}} \]

\[ \tilde{Z} = \frac{\sum_i q_i \tilde{z}_i^* \tilde{z}_i}{\sqrt{\sum_i q_i^2 |\tilde{z}_i|^2}} \]

\[ \tan \theta_S = \frac{x|\tilde{\mathcal{S}}_{-x}|}{\sqrt{\sum_i q_i^2 |\tilde{z}_i|^2}} \]  \hspace{1cm} (21)

However due to the absence of other mixing terms for S_{\pm x} the orthogonal combination \tilde{\chi} and S_x remain massless at leading order. Here

\[ \tilde{\chi} = \cos[\theta_S]\tilde{\mathcal{S}}_{-x} + \sin[\theta_S]\tilde{Z} \] \hspace{1cm} (22)

does not mix with the remaining massive modes since it is made of (SM neutral) zero modes \tilde{\mathcal{S}}_{-x}, \Lambda of the chiral mass matrices in the hidden and visible sectors respectively. Shifts due to supergravity are too small to shift the masses \( m_{S_+\phi_-} = m_{\phi_+S_-} = \mu_B + 2\lambda\tilde{\phi}_s = 0 \) appreciably.

So the leading contributions come from loop effects (see Fig.1) (analogous to the one loop corrections to neutrino masses induced by soft susy breaking [30]) and have value

\[ m_{\tilde{S}}^{1-loop} \simeq \frac{|\lambda_B|^2 F_{\tilde{S}}}{16\pi^2 S_{s}} \simeq \frac{|\lambda_B|^2}{16\pi^2 \sqrt{3} - 1} \] \hspace{1cm} (23)

Thus these masses may be as small as a few GeV even for large gravitino mass. Due to their weak interactions with light modes they may be suitable candidates for light Cold Dark matter. The scalar partners have masses

\[ (m^2_{S_x}) = 2m_{3/2}^2 \] \hspace{1cm} ; \hspace{1cm} \[ m_{Re(S_{-x})}^2 = 2g_f^2 |D_X| - 2m_{3/2}^2 \] \hspace{1cm} (24)

Like \( \tilde{\mathcal{S}}_{-x} \) the hidden sector Goldstone scalar component Im[\mathcal{S}_{-x}] will again lead to a nearly massless mode since it too will mix with the O(2) Goldstone contributions from the visible sector and hence the orthogonal mode (partner of the pseudo-Goldstino \tilde{\chi}) will be massless before loop effects. The scalar pseudo-Goldstone mode \chi which remains massless even after symmetry breaking due to doubling of goldstone modes can also obtain loop induced mass from the partner of the graph shown for the fermions, but it will remain light. This is because the supersymmetry breaking F term couples to other particles through \phi_{ab} propagators whose
large masses $\lambda_B < S_s > \sim M_X$ ensure that the effect of one loop corrections is a power of $F/M_X \sim m_{3/2} M_p/M_X$.

Consider the masses of the superfields $\hat{\phi}_{ab}$. The fermions do not couple directly to the Susy violating F-term. Since there is no residual gauge group the chiral mass matrix is just $\bar{\partial}_i \partial_j W$. One easily checks that $W^{SS} = W^{S\phi} = 0$ and it is only $W^{\phi\phi}$ that gives a symmetric $3 \times 3$ chiral mass matrix with rows and columns labelled by $\{ \phi_s, \phi_+, \phi_- \}$:

$$m_{\phi\phi} = \lambda_B \bar{S}_s \begin{pmatrix} 1 & \epsilon & 0 \\ \epsilon & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{25}$$

where $\epsilon = \bar{S}_{-s}/\bar{S}_s \sim M_X/M_p$. This can easily be diagonalized and gives large masses in the GUT scale range for reasonable values $\lambda_B \sim 10^{-3}$ to $10^{-1}$. In the scalar sector the $\phi$ fields also receive mass-squared contributions from $\lambda_B F_{S_s} \sim \mu_B^2$ besides the contributions shown for the fermions. However since $|\mu_B| << M_X << M_p$ these will lead to small splitting of the dominant contributions shared with the fermions.
B. Fixation of $S$ vevs for $N_g = 3$

For $N_g = 3$ we define the components of $S_{ab}$ in terms of $T_3$ eigenfields to be

$$
\begin{pmatrix}
\frac{1}{6} (\sqrt{6} S_0 + 3i S_{-2} - 3i S_{+2} + 2\sqrt{3} S_s) & \frac{1}{2} (S_{+2} + S_{-2}) & \frac{1}{2} (S_+ + S_-) \\
\frac{1}{2} (S_{-2} + S_{+2}) & \frac{1}{2} (\sqrt{6} S_0 - 3i S_{-2} + 3i S_{+2} + 2\sqrt{3} S_s) & \frac{1}{2} i (S_+ - S_-) \\
\frac{1}{2} (S_+ + S_-) & \frac{1}{2} i (S_+ - S_-) & \frac{1}{\sqrt{3}} (S_s - \sqrt{2} S_0)
\end{pmatrix}
$$

and similarly for $\phi = [\phi_{ab}]$. $S_{s,0,\pm,\pm}$ are properly normalized fields so that

$$Tr S\dot{\phi} = S_s \phi_s + S_+ \phi_+ + S_0 \phi_0 + S_{0+} \phi_{0+} + S_{0-} \phi_{0-} + S_{-+} \phi_{-+} + S_{-0} \phi_{-0} + S_{-+} \phi_{-+}$$

(26)

When $N_g = 3$ the D-terms form a vector of $O(3)$ and it is advantageous to change to a basis (denoted by primes) where $\bar{D}_X^a = \delta^a_3 |\bar{D}_X|$. This is easily achieved by a rotation in the 12 plane to rotate the $\bar{D}_X^a$-vector into the 23 plane and then a 23 plane rotation to make it point in the 3 direction:

$$
O = R_{23}[\theta_X].R_{12}[\frac{\pi}{2} - \varphi_X]
$$

$$
\theta_X = ArcTan[\sqrt{\frac{V^2 + V_2^2}{V_3^2}}] ; \quad \varphi_X = ArcTan[\frac{V_2}{V_1}]
$$

(27)

where $V^a = \bar{D}_X^a$. The potential for the flat directions from $S'$ is now

$$
V[S'] = m_{3/2}^2 (|S'_0|^2 + |S'_+|^2 + |S'_{-1}|^2 + |S'_{+2}|^2 + |S'_{-2}|^2)
$$

$$
+ \frac{g_f^2}{4} \left\{ (|S'_+|^2 + 2|S'_{+2}|^2 - |S'_{-1}|^2 - 2|S'_{-2}|^2 + (\bar{D}_X^3)^2) \right. \\
+ 2Tr(S'[T_+,S'])Tr(S'[T_-,S']) \right\}
$$

(28)

This has a solution along the same lines as for the $N_g = 2$ case. Dropping primes one gets:

$$
|S_{-2}| = \sqrt{\frac{\bar{D}_X}{2} - \frac{m_{3/2}^2}{4g_f^2}}
$$

$$
|S_{-0,+,+2}| = 0
$$

(29)

for which all the scalar masses are positive (apart from the Goldstone boson)

$$
M^2_{S_0} = m_{3/2}^2 ; \quad M^2_{S_{-2}} = \frac{1}{2} m_{3/2}^2 \\
M^2_{S_{-2}} = \frac{3}{2} m_{3/2}^2 ; \quad M^2_{S_{+2}} = 2m_{3/2}^2 \\
M^2_{R_{-2}} = 4g_f^2 \left[ \frac{|D_X|}{2} - 2m_{3/2}^2 \right] ; \quad M^2_{R_{-2}} = 0
$$

(30)
In the above—since we are working in the primed basis—we have exhibited only the Goldstone longitudinal component of the \( O(3) \) gauge boson in the 3 direction which comes from the field \( S_{-2} \): of course all three \( O(N_g) \) massive gauge bosons will get longitudinal mode contributions from the visible sector (which we assume breaks the family symmetry completely). Since only a linear combination of \( I_{-2} \) and a visible sector massless mode will be eaten, we would again expect the orthogonal combination to remain massless.

As for the \( N_g = 2 \) case fermionic partners of the \( \hat{S} \) particles (except \( S_{-2} \) but even so the analog of \( \tilde{\chi} \) will remain light via the mechanism explained for the \( N_g = 2 \) case) do not get mass to leading order. The Dirac partners of the family symmetry gauginos in the primed basis are:

\[
\Lambda^\pm = \sum_i \tilde{q}_i \tilde{T}_i \tilde{\bar{z}}_i \left( \sum_i \tilde{q}_i \tilde{T}_i + \tilde{\bar{z}}_i \right)^{-1/2}
\]

As before the fermion mode orthogonal to \( \Lambda^3 \) will remain massless along with \( \tilde{S}_{0,\pm,2} \). Now we will have five light fermionic and one scalar degree of freedom lighter than the gravitino mass and with very weak couplings to ordinary matter. Again they will pick up mass at one loop due to supersymmetry breaking and will remain as the fossils of flavour and super symmetry breaking. Thus they are either candidates for multicomponent relic Dark matter or must be removed somehow.

From the above discussion, it is clear that the BM Susy breaking Minkowski vacuum is characterized by the presence of several light fields coming from the supersymmetry breaking (\( S_s \)) and from the \( O(N_g) \) variant \( \hat{S} \) fields over and above the usual MSSM light fields in the visible sector. The superfields \( (\phi_{ab}) \) are superheavy with masses of order \( \lambda_B \tilde{S}_s \sim M_X \). The effective low energy theory must be written in terms of both sets of light fields by equating the heavy fields to their vevs and separating the light fields out via an expansion in \( \frac{m_{3/2}}{M_p} \).

This calculation proceeds largely like that in [5] but care must be taken due to the additional gauge invariance active in both hidden and gauge sectors.
IV. DISCUSSION

In this letter we have set out how, gravity mediation of Bajc-Melfo type calculable Supersymmetry breaking at metastable vacua in a hidden sector containing fields variant under a family gauge symmetry, ensures the viability of GUT and family symmetry breaking in the visible sector of a “Grand Yukawonification” model. In such models one aims at generating the observed hierarchical fermion flavour structure from a gauged family symmetry model with only generation blind couplings. The special role of the BM supersymmetry breaking is that it provides flat directions in both the $O(N_g)$ singlet and the gauge variant parts of a symmetric chiral supermultiplet $S_{ab} = S_{ba}$. Since it is very difficult to make the contribution of the visible sector GUT fields to the $O(N_g)$ D-terms vanish, the $\hat{S}$ flat direction performs the invaluable function of cancelling this contribution without disturbing the symmetry breaking in the visible sector. The determination of the viability of the “Grand Yukawonification” model then becomes a matter of searching the relatively small remaining parameter space for viable parameter sets that fit the fermion data at $M_X$ while taking account of threshold corrections at low and high scales and while respecting constraints on crucial quantities like the proton lifetime. Note that in this approach not only are the hard parameters of the visible sector superpotential reduced by replacement of the flavoured parameters by bland family symmetric ones but also the soft supersymmetry breaking parameters are determined by the two parameters of the hidden sector superpotential and the Planck scale.

The structure used entails yet further stringent constraints since the moduli multiplets $\hat{S}_{ab}$ entail the existence of $N_g(N_g + 1)/2 - 1$ SM singlet fermions generically lighter than the gravitino mass scale and possibly as light as a few GeV. In addition the Polonyi mode $S_s$ may also lead to difficulties in the cosmological scenario. Thus such modes can be both a boon and a curse for familion GUT models. A boon because generic Susy GUT models are hard put if asked to provide Susy WIMPs of mass below 100 GeV as CDM candidates as suggested by the DAMA/LIBRA experiment. A curse because there are strong constraints on the existence of such light moduli which normally demand that their mass be rather large (> 10 TeV) due to the robust cosmological(‘Polonyi’) problems due to decoupled modes with Planck scale vevs. In contrast to the simple Polonyi model and String moduli, the BM moduli have explicit couplings to light fields through family D-term
mixing and loops. Moreover the MSGUT scenario favours \[23, 25\] large gravitino masses > 10 – 50 TeV. Thus the Polonyi and moduli problems may be evaded. In any case the cosmology need be considered seriously only after we have shown that the fermion spectrum is indeed generated.

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