Condition for the formation of micron-sized dust grains in dense molecular cloud cores

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ABSTRACT

We investigate the condition for the formation of micron-sized grains in dense cores of molecular clouds. This is motivated by the detection of the mid-infrared emission from deep inside a number of dense cores, the so-called ‘coreshine,’ which is thought to come from scattering by micron ($\mu$m)-sized grains. Based on numerical calculations of coagulation starting from the typical grain size distribution in the diffuse interstellar medium, we obtain a conservative lower limit to the time $t$ to form $\mu$m-sized grains: $t/t_{ff} > 3(5/S)(n_{H}/10^{5}$ \text{cm}^{-3})^{-1/4}$ (where $t_{ff}$ is the free-fall time at hydrogen number density $n_{H}$ in the core, and $S$ the enhancement factor to the grain-grain collision cross-section to account for non-compact aggregates). At the typical core density $n_{H} = 10^{5}$ \text{cm}^{-3}, it takes at least a few free-fall times to form the $\mu$m-sized grains responsible for coreshine. The implication is that those dense cores observed in coreshine are relatively long-lived entities in molecular clouds, rather than dynamically transient objects that last for one free-fall time or less.

Key words: dust, extinction — infrared: ISM — ISM: clouds — ISM: evolution — turbulence

1 INTRODUCTION

Dense cores of molecular clouds are the basic units for the formation of Sun-like, low-mass stars. A fundamental question about these cores that has not been answered conclusively is: are they long-lived entities or simply transient objects that disappear in one free-fall time or less?

The core lifetime is important to determine, because it affects the rate of star formation as well as the time available for chemical reactions, which in turn affect the chemical structure of not only the cores themselves but also the disks (and perhaps even objects such as comets) that form out of them (Caselli & Ceccarelli 2012). It also has implications on how the cores are formed (Ward-Thompson et al. 2007). If the cores are relatively long lived, it would favor those formation scenarios that involve persistent support against gravity from, for example, magnetic fields (Shu et al. 1987; Mouschovias & Ciolek 1999), or long mass-accumulation time (e.g. Gong & Ostriker 2011). If the core lifetime turns out comparable to the free-fall time or less, then rapid formation and collapse, through for example turbulent compression, would be preferred (Mac Low & Klessen 2004).

One way to constrain the core lifetime is to compare the number of starless cores to that of young stellar objects (YSOs), whose lifetimes can be independently estimated (Ward-Thompson et al. 2007; Evans et al. 2009). Ward-Thompson et al. (2007) found that cores of $10^{4}$–$10^{5}$ \text{cm}^{-3} typically last for $\sim 2$–5 free-fall times. Such estimates depend, however, on the lifetimes of YSOs, which are uncertain. Here, we explore another, completely independent, way of constraining the core lifetime, through the grain growth implied by the recently discovered phenomenon of ‘coreshine.’

The so-called ‘coreshine’ refers to the emission at mid-infrared [especially the 3.6 \text{\mu m} \text{Spitzer Infrared Array Camera (IRAC) band}] from deep inside dense cores of molecular clouds (Pagani et al. 2010; Steinacker et al. 2010). It is found in about half of the cores where the emission is searched for (Pagani et al. 2010). The emission is thought to come from light scattered by dust grains up to 1 $\mu$m in size. Such grains are much larger than those in the diffuse interstellar medium (e.g. Mathis, Rumpl, & Nordsieck 1977, hereafter MRN). Since it takes time for small MRN-type grains to grow to $\mu$m-size, the observed coreshine should provide a constraint on the core lifetime. The goal of our investigation is to quantify this constraint. Specifically, we want to answer the question: how long does it take for the grains in a dense core to grow to $\mu$m-size at a given density?

Grain growth through coagulation has been studied for a long time (e.g. Chokshi, Tielens, & Hollenbach 1993; Dominik & Tielens 1997). Even before the discovery of coreshine, Ormel et al. (2009) was able to demonstrate that coagulation can in principle produce $\mu$m-sized grains in dense cores, provided that the grains are coated by ‘sticky’ materials such as water ice and that the cores are relatively long-lived (see also Ormel et al. 2011). In this paper, we aim to strengthen Ormel et al. (2009)’s results.
by deriving a robust lower limit to the lifetimes for those cores with μm-sized grains inferred from coreshine through a simple framework that isolates the essential physics of coagulation. We find that cores of typical density $10^5$ cm$^{-3}$ must last for at least a few free-fall times in order to produce μm-sized grains. Our coagulation models are explained in Section 2 and the results are described in Section 3. We discuss the robustness and implication of the results in Section 4 and conclude in Section 5.

2 MODELS

2.1 Coagulation

We consider the time evolution of grain size distribution by coagulation in a dense core. We adopt the formulation used in our previous paper (Hirashita 2012) (see also Hirashita & Yar 2009), with some changes to make it suitable for our purpose. We briefly summarize the formulation here, and refer to Hirashita (2012) for further details.

We assume that the grains are spherical with a constant material density $\rho_{gr}$. We define the grain size distribution such that $n(a, t) da$ is the number density of grains whose radii are between $a$ and $a + da$ at time $t$. For numerical calculation, we consider $N = 128$ discrete logarithmic bins for the grain radius (or mass), and solve the discretized coagulation equation. In considering the grain–grain collision rate between two grains with radii $a_1$ and $a_2$, we estimate the relative velocity by

$$v_{12} = \sqrt{v(a_1)^2 + v(a_2)^2 - 2v(a_1)v(a_2)\mu}, \hspace{1cm} (1)$$

where the grain velocity as a function of grain radius, $v(a)$, is given below in equation (3), and $\mu \equiv \cos \theta$ ($\theta$ is an angle between the two grain velocities) is randomly chosen between $-1$ and $1$ in each time-step and the cross-section by

$$\sigma_{12} = S\pi(a_1 + a_2)^2, \hspace{1cm} (2)$$

where $S$ is the enhanced factor of cross-section, which represents the increase of cross-section by non-compact aggregates. Note that we always define the grain radius $a$ and the grain material density $\rho_{gr}$ for the compact geometry, even if $S > 1$, to avoid the extra uncertainty caused by the grain geometry [see also the comment in the item (ii) in Section 2.2]. We adopt the turbulence-driven grain velocity derived by Ormel et al. (2009), who assume that the driving scales of turbulence is given by the Jeans length and that the typical velocity of the largest eddies ($\sim$ the Jeans length) is given by the sound speed (see also Hirashita 2012).

$$v(a) = 1.1 \times 10^3 \left(\frac{T_{gas}}{10 \text{ K}}\right)^{1/4} \left(\frac{a}{0.1 \text{ μm}}\right)^{1/2}$$
$$\times \left(\frac{n_H}{10^5 \text{ cm}^{-3}}\right)^{-1/4} \left(\frac{\rho_{gr}}{3.3 \text{ g cm}^{-3}}\right)^{1/2} \text{ cm s}^{-1}, \hspace{1cm} (3)$$

where $T_{gas}$ is the gas temperature assumed to be 10 K in this paper. Thermal velocities are small enough to be neglected. The robustness of our conclusion in terms of the grain velocity is further discussed in Section 4.1.

The form of equation (1) suggests that the motions of dust particles are random. This treatment is not valid in general, since turbulent motions are correlated. However, we do not include the full treatment of the probability distribution function of the true relative particle velocity in turbulence for the following three reasons: (i) Our simple formulation is sufficient to give a lower limit for the coagulation time-scale (Section 4.1). (ii) The probability distribution function of the true relative particle velocity in turbulence is unknown, and has only recently been investigated (Hubbard 2013, Fan & Padoan 2014). (iii) In the environments of interest to this paper, the forcing of turbulent eddies can be represented by a model where the particle motions experience ‘random kicks’: in this so-called ‘intermediate regime’ (Ormel & Cuzzi 2007), the prescription given by equation (1) is applicable. Indeed, we can confirm that the condition for the intermediate regime is satisfied as follows. The intermediate regime is defined by $Re^{-1/2} < St < 1$, where $Re$ is the Reynolds number and $St$ is the Stokess number (Ormel & Cuzzi 2007). This condition is translated into $11(a/1 \mu m)^2(T_{gas}/10 K)^{-1} \leq n_H \leq 2.9 \times 10^{-12}(a/1 \mu m)^4(T_{gas}/10 K)^{-1} \text{ cm}^{-3}$. Since we are interested in the range of grain radius, $0.1 \mu m \lesssim a \lesssim 1 \mu m$, the intermediate regime is applicable to the density range considered in this paper.

We adopt the following coagulation threshold velocity, $v_{coag}^k$, given by (Chokshi, Tielens, & Hollenbach 1993; Dominik & Tielens 1997, Yan, Lazarian & Draine 2004).

$$v_{coag}^k = 21.4 \left[\frac{a_1^3 + a_2^3}{a_k + a_i}^{1/2} \frac{\gamma_{5/6}}{\xi_{1/3} (R_{ki}^1)^{1/2} \rho_{gr}}\right], \hspace{1cm} (4)$$

where $\gamma$ is the surface energy per unit area, $R_{ki} \equiv a_k a_i/(a_k + a_i)$ is the reduced radius of the grains, $\xi$ is the the relevant elastic modulus. This coagulation threshold is valid for collision between two homogeneous spheres and would not be applicable to collisions between aggregates. At low velocities, grains stick with each other and develop a non-compact or fluffy aggregates. These aggregates stick with each other at low relative velocities, and start to deform or bounce as the relative velocities increases. Because the deformation absorbs the collision energy, the aggregates can stick with each other at a velocity larger than the above coagulation threshold. At very high velocities, cratering and catastrophic destruction will halt the growth (Paszun & Dominik 2009; Wada et al. 2011; Seizinger & Kley 2013). In this paper, we only limit the application of this threshold to compact spherical grains [i.e. cases (i) and (ii) in Section 2.2 see Ormel et al. (2009) and references therein for a detailed treatment of coagulation of aggregates.]

2.2 Initial condition and selection of parameters

For the initial grain size distribution, we adopt the following powerlaw distribution, which is typical in the diffuse ISM (MRN):

$$n(a) = \mathcal{C} a^{-3.5} \left(a_{\min} \leq a \leq a_{\max}\right), \hspace{1cm} (5)$$

where $\mathcal{C}$ is the normalizing constant, with $a_{\min} = 0.001 \mu m$ and $a_{\max} = 0.25 \mu m$. The normalization factor $\mathcal{C}$ is determined according to the mass density of the grains in the ISM:

$$\mathcal{D} \mu m_1 n_H = \int_{a_{\min}}^{a_{\max}} 4\pi \frac{a^3}{3} \rho_{gr} \mathcal{C} a^{-3.5} \, da, \hspace{1cm} (6)$$

where $n_H$ is the hydrogen number density, $m_1$ is the hydrogen atom mass, $\mu$ is the atomic weight per hydrogen (assumed to be 1.4) and $D$ (0.01; Ormel et al. 2009) is the dust-to-gas mass ratio.

We adopt $n_H = 10^5 \text{ cm}^{-3}$ for the typical density of dense cores emitting coreshine (Steinacker et al. 2010), but also survey a
wide range in $n_H$. We normalize the time to the free-fall time, $t_{ff}$:

$$t_{ff} = \sqrt{\frac{3\pi}{52G\mu m_Hn_H}} = 1.38 \times 10^5 \left( \frac{n_H}{10^5 \text{ cm}^{-3}} \right)^{-1/2} \text{ yr.} \quad (7)$$

To isolate the key pieces of physics that determine the rate of coagulation, we examine the following three models:

(i) **Standard silicate model**: We adopt coagulation threshold given by equation (4) with silicate material parameters ($\rho_{gr} = 3.3 \text{ g cm}^{-3}$, $\gamma = 25 \text{ cm}^2$, and $\delta^* = 2.8 \times 10^{11} \text{ dyn cm}^{-2}$; Chokshi, Tielens, & Hollenbach 1993). We estimate the cross-section by the compact spherical case (i.e. $S_0(2009)$, the volume filling factor of the grains after coagulation is ice have a large coagulation threshold velocity (Ormel et al. 2009). Section 4.1 for discussion). Note that $\rho_{gr}$ is given by the condition for the radius at the peak, $a_{\text{peak}}$, reaches or exceeds 1 $\mu$m. We also examine a more conservative criterion by using $a_{\text{peak}} = 0.5 \mu$m instead of 1 $\mu$m, motivated in part by the fact that $a \sim 0.5 \mu$m is the grain radius at which scattering is comparable to absorption at $\lambda = 3.6 \mu$m (Steinacker et al. 2010). We will concentrate on the maximal coagulation model with an enhancement factor for cross-section $S = 5$; the result from the sticky coagulation model with $S = 1$ can be obtained through a simple scaling. In Fig. 2 we show a grid of models with different core densities and times (in units of the free-fall time at the core density). The solid line marks roughly the critical time $t_{grow}$ at a given density $n_H$ above which $\mu$m-sized grains are produced. It is given by

$$\frac{t_{grow}}{t_{ff}} = A \left( \frac{5}{S} \right) \left( \frac{n_H}{10^5 \text{ cm}^{-3}} \right)^{-1/4},$$

where $A = 5.5$ and 3.0, respectively, if we adopt $a_{\text{peak}} = 1 \mu$m and 0.5 $\mu$m for the criterion of micron-sized grain formation. The condition for forming $\mu$m-sized grains is therefore $t > t_{grow}$. The same condition applies to the sticky coagulation model (with $S = 1$) as well, since coagulation time is inversely proportional to the cross-section for grain-grain collision.

Equation (8) can be understood in the following way. Since coagulation is a collisional process, $t_{grow}$ should be given by the collision time-scale, $t_{coll} = (vS\pi a^2 n_{dust})^{-1} = 4\pi \rho_{gr} / (3 D \mu m_H n_H v S)$, where $v$ and $n_{dust}$ are the velocity and the number density of grains, respectively (Ormel et al. 2009). The growth time-scale in terms of grain radius is $t_{grow} \simeq 3 t_{coll}$ (note that $t_{coll}$ is the time-scale of grain volume being doubled by coagulation). Then, $t_{coll} / t_{ff}$ is evaluated by using equations (8) and (7) as $t_{grow} / t_{ff} \simeq 7.4(a/1 \mu$m)$^{5/2} (n_H/10^5 \text{ cm}^{-3})^{-5/2} (S/5)^{-1} (T_{gas}/10 \text{ K})^{-1/4}$, $(\rho_{gr}/3.3 \text{ g cm}^{-3})^{-1/2}$; that is, $A = 7.4(5.2)$ for $a = 1 \mu$m (0.5 $\mu$m), in a fair agreement with the above numerical estimate. Thus, $t_{grow}$ can be understood in terms of collision time-scale, which strengthens our numerical results.

Note that, to form $\mu$m-sized grains in one free-fall time, the density $n_H$ must be of order $10^6 \text{ cm}^{-3}$ or higher, even in the maximal coagulation model. In the sticky coagulation model, the required density would be higher still. Such densities are much higher than the typical core value (of order $10^5 \text{ cm}^{-3}$). At $10^5 \text{ cm}^{-3}$, Fig. 2 and equation (8) indicate that, under reasonable conditions, it takes at least several free-fall times for the grains to grow to $\mu$m-size (see Section 4.2 for more discussion). The implication is that those dense cores detected in coreshine should be rather long-lived entities rather than transient objects that disappear in one free-fall.
Figure 1. Evolution of grain size distribution. The solid, dashed, and dot-dashed lines show the grain size distributions at $t = 1t_{ff}$, $3t_{ff}$, and $10t_{ff}$, respectively, for (a) the standard silicate model, (b) the sticky coagulation model, and (c) the maximal coagulation model. The dotted line presents the initial condition. The upper and lower panels show the cases with $n_H = 10^3 \text{cm}^{-3}$ and $10^7 \text{cm}^{-3}$, respectively.

Figure 2. The condition for the formation of $\mu$m-sized grains. The success and failure of the formation of $a > 1$ $\mu$m grains in the maximal coagulation model are shown by ‘o’ and ‘x’, respectively. The solid and dashed lines show the boundary of those two cases in the maximal coagulation model and the sticky coagulation model, respectively, if we adopt $a_{\text{peak}} = 1$ $\mu$m for the criterion for coreshine. The dot-dashed line marks the boundary for the maximal coagulation model for a more conservative criterion: $a_{\text{peak}} = 0.5$ $\mu$m.

4 DISCUSSION

4.1 A lower limit to $\mu$m-sized grain formation time

One may argue that coagulation would be faster if the grains were to collide at higher speeds than adopted in our model. However, it will be difficult for this to happen because of the existence of a coagulation threshold. As mentioned earlier, bare silicate grains already acquire velocities larger than the threshold at a rather small size $a \sim 0.1$ $\mu$m; they do not grow beyond $0.1$ $\mu$m under reasonable conditions. To grow to larger sizes, the grains must be ‘more sticky’ than silicate, as is the case when the grains are coated with water ice (Ormel et al. 2009). For such coated grains, we can estimate the coagulation threshold for equal-sized grains from equation (4) using $\rho_{\text{ice}} = 3.3$ $\text{g cm}^{-3}$, $\gamma = 370$ $\text{erg cm}^{-2}$, $\mathcal{E}^* = 3.7 \times 10^{10}$ dyn cm $^{-2}$. The result is $v_{\text{coag}} = 9.4 \times 10^3 (a/1$ $\mu$m)$^{-5/6}$ cm s$^{-1}$. For the micron-sized grains that we aim to form, this threshold is already smaller than the typical grain velocity $v \sim 3.5 \times 10^3 (a/1$ $\mu$m)$^{1/2}$ that was used in our model. In other words, our model is already generous with the grain-grain collision speed. (Collisions at the relatively high speed that we adopted may lead to the compaction of aggregates, which should reduce the enhancement factor $S$ for grain-grain collision cross-section and hence the rate of grain growth.) Increasing the collision speed further should not lead to faster growth to $\mu$m-size. For this reason, we believe that the critical time $t_{\text{grow}}$ for the formation of $\mu$m-sized grains estimated in equation (8) is a robust lower limit.

4.2 The case for long-lived dense cores

Equation (8) indicates that it takes more than $\sim 5$ free-fall times to form $1$ $\mu$m-sized grains at the typical core density $n_H = 10^7 \text{cm}^{-3}$ if the enhancement factor for cross-section is $S = 5$. If the enhancement factor is larger, the coagulation would be faster. In particular, if $S = 25$, the formation of micron-sized grains may occur in a single, rather than 5, free-fall time. However, $S = 25$ requires the grain volume filling factor to be $25^{-3/2} \sim 1$ per cent, which is extreme. For example, to form such a grain of $a = 1$ $\mu$m with compact spherical grains with $a = 0.1$ $\mu$m, one needs to connect 1,000 grains linearly, which is unlikely. We doubt that there is much room to increase $S$ well beyond 5, which corresponds aggregates of rather low volume filling factor ($\sim 0.1$) already. If the cross-section enhancement factor $S$ is not much larger than 5, it would take several free-fall times (or more) to form $\mu$m-sized grains at typical core densities. The long formation time would indicate that those dense cores with observed coreshine are relatively long-lived entities, rather than transient objects that form and disappear in one free-
fall time. This estimate of core lifetime based on grain growth is consistent with that inferred from the number of starless cores (relative to YSOs) [Ward-Thompson et al. 2007]. It is also consistent with the observational results that only a small fraction of dense cores show any detectable sign of gravitational collapse and that even those collapsing cores tend to have infall speeds less than half the sound speed (Di Francesco et al. 2007). Such slowly-evolving, relatively long-lived cores can form, for example, as a result of ambipolar diffusion in magnetically supported clouds [Shu et al. 1987; Mouschovias & Ciolek 1999], even in the presence of a strong, supersonic turbulence [Nakamura & Li 2005]. They are less compatible with transient cores that are formed rapidly through fast compression by supersonic turbulence without any magnetic cushion [Mac Low & Klessen 2004], unless the core material is slowly accumulated in the post-shock region over several free-fall times (e.g. Gong & Ostriker 2011).

4.3 Source of large grains

Large grains ($a \geq 0.1 \mu$m), once they are injected into the diffuse ISM, are rapidly shattered into smaller grains [Hirashita & Yan 2009; Asano et al. 2013]. Thus, there should be a continuous supplying mechanism of large grains [Hirashita & Nozawa 2013]. If dense molecular cores has lifetimes long enough to produce μm-sized grains, they can be an important source of large grains. Including the supply of large grains from dense cores will be an interesting topic in modeling the evolution of dust in galaxies.

5 CONCLUSION

Motivated by recent coreshine observations, we have examined the condition for the formation of μm-sized grains by coagulation in dense molecular cloud cores. We obtained a simple, conservative lower limit to the core lifetime $t$ for the formation of 0.5 μm-sized grains: $t/t_{ff} > 3(5/S)(n_{H}/10^5 \text{ cm}^{-3})^{-1/4}$, where $t_{ff}$ is the free-fall time at the core density $n_{H}$ and $S$ the enhancement factor for grain-grain collision that accounts for aggregates. The formation time for 1 μm-sized grains is roughly a factor of 2 longer. Since $S$ is unlikely much larger than 5, we conclude that dense cores of typical density $n_{H} = 10^5 \text{ cm}^{-3}$ must last for at least several free-fall times in order to produce the μm-sized grains thought to be responsible for the observed coreshine. Such cores are therefore relatively long-lived entities in molecular clouds, rather than dynamically transient objects.

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