Color Transparency at Low Energies: Predictions for JLAB

B.Z. Kopeliovich\textsuperscript{1,2}, J. Nemchik\textsuperscript{3} and Ivan Schmidt\textsuperscript{1}

\textsuperscript{1}Departamento de Física y Centro de Estudios Subatómicos, Universidad Técnica Federico Santa María, Valparaíso, Chile
\textsuperscript{2}Joint Institute for Nuclear Research, Dubna, Russia
\textsuperscript{3}Institute of Experimental Physics SAS, Watsonova 47, 04001 Kosice, Slovakia

Abstract

The study of color transparency (CT) in elastic electroproduction of vector mesons off nuclei encounters the problem of the onset of coherence length (CL) effects. The problem of CT-CL separation arises especially at medium energies, corresponding to HERMES experiment, when the coherence length is of the order of the nuclear radius $R_A$. Only at asymptotic large energies, corresponding to large CL, $l_c \gg R_A$, the CT-CL mixing can be eliminated. On the other hand, the net CT effects can be studied in the kinematic range accessible by the CLAS experiment, since in this case the CL is much smaller than the nuclear radius. Using light-cone quantum chromodynamics (QCD) dipole formalism we investigate manifestations of CT effects in electroproduction of vector mesons. Motivated by expected data from the CLAS experiment at JLab, we predict the $A$ and $Q^2$ dependence of nuclear transparency for $\rho^0$ mesons produced incoherently off nuclei. We demonstrate that in the CLAS kinematic region the CL effects are weak enough to keep the photon energy at such values as to obtain maximal photon virtualities keeping optimal statistics of the data. This has a clear advantage in comparison with a standard investigation of net CT effects fixing CL.

1 Introduction

One of the fundamental phenomena coming from quantum chromodynamics (QCD) is color transparency (CT), studied intensively for more than two decades. CT was predicted to occur already in 1981 [1,2], in diffractive interaction with nuclei. The CT phenomenon is manifested as a vanishing interaction cross section $\sigma_{\bar{q}q}(r) \propto r^2$ [1] for vanishing hadron (quark configuration) transverse size $r$. As a result the nuclear medium is more transparent for smaller transverse size of the hadron (quark configurations). The history of CT investigation was continued further in Refs. [3,4], where CT was predicted to be manifested also in quasielastic high-$p_T$ scattering of electrons and hadrons off nuclei with high momentum transfer. Later, CT was predicted to appear also in quasi-free charge-exchange scattering [5,6]. It was also...
proposed to search for strong CT effects in reaction of back-to-back dijet production in coherent diffractive dissociation on nuclei.

Searches of a CT signal in quasielastic electron scattering \( A(e, e'p)A^* \) have not been successful so far. The first E18 experiment at SLAC [9] and later measurements at Jefferson Lab [10, 11] failed to detect any signal of CT, which was expected to be a deviation from the Glauber model calculations (see the recent overview in [12]). The energy of recoil protons was too low, and the formation length was too short in these experiments. A promising way of detecting a signal of CT at these energies would be a measurement of an asymmetric dependence of nuclear transparency on missed energy [13].

Searches of CT effects in quasielastic proton scattering, \( A(p, 2p)A^* \), were not conclusive either. Although a very interesting variation of nuclear transparency was observed [14], no unambiguous interpretation has been proposed so far. The observed effect maybe related either to the energy threshold for charm production [15], or to interplay between hard and soft components of the production mechanism [16] (see, however, finite energy corrections in [6]). One can find more details in the review [17].

Only a few experiments were able to confirm the CT phenomenon. A very strong signal was observed in the PROZA experiment at Serpukhov [18], in quasi-free charge-exchange pion scattering of nuclei, at an energy of 40 GeV. A sizable CT effect was detected in the E791 experiment [19] in diffractive coherent dissociation of pions on nuclei.

An observation of the onset of CT in virtual diffractive photoproduction of \( \rho \) mesons was claimed by the E665 collaboration [20], measuring the \( Q^2 \)-dependence of nuclear transparency defined as:

\[
T_{\gamma A}^{inc} = \frac{\sigma_{\gamma^* A \rightarrow VX}}{\sigma_{\gamma^* N \rightarrow VX}}
\]

for the diffractive incoherent (quasielastic) production of vector mesons, \( \gamma^* A \rightarrow V X \).

The observed signal of CT by the E665 collaboration [20] had been predicted in [21, 22] as a rising nuclear transparency (vanishing final state interaction) with increasing hardness of the reaction \( Q^2 \). Although electroproduction of vector mesons off nuclei is a very effective tool for the study of CT, available experimental setups do not allow to reach such kinematic region where a strong signal of CT is expected. In these cases one has to deal only with an onset of this phenomenon. Therefore for a detailed study of CT other effects which compete with the phenomenon of CT should be considered. This was first done in Refs. [23, 24] within the Glauber model. There it was shown that the main competition to CT effects at moderate energies is the appearance of the so called coherence length (CL) effects, leading to an analogous \( Q^2 \)-dependence of nuclear transparency. Later, in 1997, a multichannel evolution equation for the density matrix was developed [25], describing the propagation of a hadronic wave packet through nuclear matter. This approach, applied to electroproduction of vector mesons off nuclei in the hadronic basis, incorporates both CT and CL effects. It was also suggested in this reference to separate the net CT signal by investigating the \( Q^2 \) dependence of nuclear transparency at values of the photon energy \( \nu \) which keep the CL constant.

An alternative description of CT in electroproduction of vector mesons can be realized

\[1\] An analogous definition of nuclear transparency can also be done for the coherent or elastic process, \( \gamma^* A \rightarrow VA \).
also within the quark-gluon representation \cite{26, 21, 22, 27, 28, 29, 30}. Here a photon of high virtuality $Q^2$ is expected to produce a pair with a small $\sim 1/Q^2$ transverse separation. Then CT manifests itself as a vanishing absorption of the small size colorless $\bar{q}q$ wave packet during propagation through the nucleus. The dynamical evolution of this small size $\bar{q}q$ pair to a normal size vector meson is controlled by the time scale called formation time. Due to uncertainty principle, one needs a time interval to resolve different levels $V$ (the ground state) or $V'$ (the next excited state) in the final state. In the rest frame of the nucleus this formation time is Lorentz dilated,

$$t_f = \frac{2\nu}{m_V^2 - m_{V'}^2}.$$  

A rigorous quantum-mechanical description of the pair evolution was suggested in \cite{26}, based on the nonrelativistic light-cone (LC) Green function technique. The same LC Green function formalism has been already applied also for Drell-Yan production in proton-nucleus and nucleus-nucleus interactions \cite{31}, for nuclear shadowing in DIS \cite{32, 33}, and for coherent and incoherent electroproduction of vector mesons off nuclei \cite{28, 29, 30}.

Another phenomenon known to cause nuclear suppression is quantum coherence. It results from destructive interference of the amplitudes for which the interaction takes place on different bound nucleons. It is characterized by the coherence length (CL), which is related to the longitudinal momentum transfer by $q_c = 1/l_c$. It also corresponds to the coherence time ($l_c = t_c$), given by

$$t_c = \frac{2\nu}{Q^2 + m_V^2}.$$  

In electroproduction of vector mesons off nuclei one needs to disentangle CT (absorption) and CL (shadowing) as the two sources of nuclear suppression. Conventionally, one can associate these effects with final and initial state interactions, respectively. Detailed analysis of the CT and CL effects in electroproduction of light vector mesons off nuclei showed \cite{28} that the coherence length is larger or comparable with the formation one, $l_c \gtrsim l_f$, starting from the photoproduction limit up to $Q^2 \sim (1 \div 2)$ GeV$^2$. This does not happen in charmonium production, where there is a strong inequality $l_c < l_f$, independent of $Q^2$ and $\nu$ \cite{29, 30}, and which leads to a different scenario of CT-CL mixing in comparison with the production of light vector mesons.

Recently new HERMES data \cite{34, 35} have been gradually obtained for diffractive exclusive electroproduction of $\rho^0$ mesons on nitrogen target. At the beginning the data were presented as a dependence of nuclear transparency \cite{1} on coherence length \cite{3}. The data for incoherent $\rho^0$ production decrease with $l_c$, as expected from the effects of initial state interactions. On the other hand, the nuclear transparency for coherent $\rho^0$ production increases with coherence length, as expected from the effects of the nuclear form factor \cite{28}. However, each $l_c$-bin of the data contains different values of $\nu$ and $Q^2$, i.e. there are different contributions of both effects, CT and CL. For this reason, the $l_c$-behavior of nuclear transparency does not allow to study separately CT and CL effects. Therefore it was proposed in \cite{25, 28} that CT can

\footnote{In fact, the situation is somewhat more complicated. For very asymmetric pairs the $q$ or $\bar{q}$ carry almost the whole photon momentum, and then the pair can have a large separation, see for example Ref. \cite{28}.}
be separately studied eliminating the effect of CL from the data on the $Q^2$ dependence of nuclear transparency, in a way which keeps $l_c = \text{const}$. According to this prescription, later the HERMES data [35] were presented as $Q^2$ dependence of nuclear transparency at different fixed values of $l_c$. Then the rise of $T_{RA}$ with $Q^2$ represents a signature of CT. The HERMES data [35] are in a good agreement with the predictions coming from Ref. [28]. New HERMES data on neon and krypton target should be available soon. This will allow to verify further the predictions for CT from Ref. [28].

Moreover, another investigation of CT has been carried out by the CLAS collaboration at JLab [12], studying incoherent electroproduction of $\rho$ mesons off nuclei at small photon energies $2.2 < \nu < 4.5$ GeV. At such values of $\nu$ the corresponding coherence length $l_c \ll R_A$, where $R_A$ is the nuclear radius. For this reason CL effects are expected to be much weaker than CT and CL-CT mixing does not play an important role. Therefore the study of vector meson electroproduction at small energies represents an alternative way for investigating a clear signal of CT. Because new data from the CLAS collaboration will appear soon we will present here the corresponding predictions for CT. The main emphasis will be devoted to discussing the advantages in the investigation of CT effects in the CLAS energy range in comparison with the HERMES experiment.

The paper is organized as follows. In Sect. 2 we present a brief summary of the light-cone dipole approach to diffractive electroproduction of vector mesons. In the next Sect. 3 we calculate the predictions for the $Q^2$ dependence of nuclear transparency, for various nuclear targets, at the small energies corresponding to the CLAS experiment at JLab. We analyze also the strength of CL effects and their influence on the CT signal. We show that CL-CT mixing at small energies does not spoil the clear onset of CT effects and permits to treat the CLAS experimental values at different small $l_c \ll R_A$, allowing to process the data at maximal possible statistics. We discuss the fact that the weak CL effects in the CLAS energy region lead also to the possibility of studying the CT phenomenon at much larger values of $Q^2$ than those which are presently available, using a prescription to fix CL, in particular for the HERMES experiment. The results of the paper are summarized and discussed in Sect. 4.

2 A Short Review of the Color Dipole Phenomenology

The light-cone (LC) dipole approach for the process $\gamma^* N \rightarrow V N$ was already used in Ref. [26, 36] to study the exclusive photo- and electroproduction of charmonia, and in Ref. [28] for elastic virtual photoproduction of light vector mesons $\rho^0$ and $\Phi^0$ (for a review see also [37]). Therefore, we present only a short review of this approach. Here a diffractive process is treated as elastic scattering of a $\bar{q}q$ fluctuation of the incident particle. The elastic amplitude is given by a convolution of the universal flavor independent dipole cross section for the $\bar{q}q$ interaction with a nucleon, $\sigma_{\bar{q}q}$ [1], and the initial and final wave functions. Then the forward production amplitude for the process $\gamma^* N \rightarrow V N$ can be represented in the quantum-mechanical form

$$M_{\gamma^* N \rightarrow V N}(s, Q^2) = \langle V | \sigma_{\bar{q}q}(\vec{r}, s) | \gamma^* \rangle = \int_0^1 da \int d^2r \Psi_V^*(\vec{r}, \alpha) \sigma_{\bar{q}q}(\vec{r}, s) \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2)$$ (4)
with the normalization
\[
\left. \frac{d\sigma(\gamma^*N \to VN)}{dt} \right|_{t=0} = \frac{|M_{\gamma^*N \to VN(s, Q^2)}|^2}{16\pi}.
\] (5)

There are three ingredients contributing to the amplitude (4):

(i) The dipole cross section \(\sigma_{\bar{q}q}(\vec{r}, s)\) which depends on the \(\bar{q}q\) transverse separation \(\vec{r}\) and the c.m. energy squared \(s\).

(ii) The LC wave function of the photon, \(\Psi_{\gamma^*}(\vec{r}, \alpha, Q^2)\), which besides the \(\vec{r}\) dependence, depends also on the photon virtuality \(Q^2\) and the relative share \(\alpha\) of the photon momentum carried by the quark.

(iii) The LC wave function \(\Psi_V(\vec{r}, \alpha)\) of the vector meson.

Detailed description of these ingredients can be found in Refs. [28, 29].

Notice that in the LC formalism the photon and meson wave functions contain also higher Fock states \(|\bar{q}q\rangle, |\bar{q}qG\rangle, |\bar{q}q^2G\rangle\), etc. The effect of these higher Fock states is implicitly incorporated into the energy dependence of the dipole cross section \(\sigma_{\bar{q}q}(\vec{r}, s)\), as is given in Eq. (4).

Since our main interest is electroproduction of light vector mesons, we explicitly consider the nonperturbative interaction effects between the \(q\) and \(\bar{q}\), which can be included in the LC wave function of the photon, \(\Psi_{\gamma^*}(\vec{r}, \alpha, Q^2)\). For this purpose we use the corresponding phenomenology based on the LC Green function approach, developed in Ref. [26, 38]. The Green function \(G_{\bar{q}q}(z_1, \vec{r}_1; z_2, \vec{r}_2)\) describes the propagation of an interacting \(\bar{q}q\) pair between points with longitudinal coordinates \(z_1\) and \(z_2\) and with initial and final transverse separations \(\vec{r}_1\) and \(\vec{r}_2\). This Green function satisfies the two-dimensional Schrödinger equation,

\[
i \frac{d}{dz_2} G_{\bar{q}q}(z_1, \vec{r}_1; z_2, \vec{r}_2) = \left\{ \frac{\epsilon^2 - \Delta r^2}{2\nu \alpha (1 - \alpha)} + V_{\bar{q}q}(z_2, \vec{r}_2, \alpha) \right\} G_{\bar{q}q}(z_1, \vec{r}_1; z_2, \vec{r}_2).
\] (6)

Here the Laplacian \(\Delta r\) acts on the coordinate \(r\).

The imaginary part of the LC potential \(V_{\bar{q}q}(z_2, \vec{r}_2, \alpha)\) in (6) is responsible for the attenuation of the \(\bar{q}q\) pair in the medium, while the real part represents the interaction between the \(q\) and \(\bar{q}\). This potential is supposed to provide the correct LC wave functions of vector mesons. For the sake of simplicity we use the oscillator form of the potential,

\[
\text{Re} V_{\bar{q}q}(z_2, \vec{r}_2, \alpha) = \frac{a^4(\alpha) \vec{r}_2^2}{2\nu \alpha (1 - \alpha)},
\] (7)

which leads to a Gaussian \(r\)-dependence of the LC wave function of the meson ground state. The shape of the function \(a(\alpha)\) can be found in Ref. [38].

In this case equation (6) has an analytical solution leading to an explicit form of the harmonic oscillator Green function [39],

\[
G_{\bar{q}q}(z_1, \vec{r}_1; z_2, \vec{r}_2) = \frac{a^2(\alpha)}{2\pi i \sin(\omega \Delta z)} \exp \left\{ \frac{i a^2(\alpha)}{\sin(\omega \Delta z)} \left[ (\vec{r}_1^2 + \vec{r}_2^2) \cos(\omega \Delta z) - 2 \vec{r}_1 \cdot \vec{r}_2 \right] \right\} \times \exp \left( -\frac{i \epsilon^2 \Delta z}{2\nu \alpha (1 - \alpha)} \right),
\] (8)
where $\Delta z = z_2 - z_1$ and
\[
\omega = \frac{a^2(\alpha)}{\nu \alpha(1 - \alpha)}.
\] (9)

The boundary condition is $G_{\bar{q}q}(z_1, \vec{r}_1; z_2, \vec{r}_2) |_{z_2 = z_1} = \delta^2(\vec{r}_1 - \vec{r}_2)$.

Using concrete forms of all the above ingredients (specified in Ref. [28], for example) we can calculate the forward production amplitude for the process $\gamma^* N \to V N$, separately for transverse and longitudinal photons and vector mesons. Assuming $s$-channel helicity conservation (SCHC), the forward scattering amplitude reads,

\[
\mathcal{M}^{T}_{\gamma^* N \to V N}(s, Q^2) \big|_{t=0} = N_C Z_q \sqrt{2 \alpha_{em}} \int d^2r \sigma_{\bar{q}q}(\vec{r}, s) \int_0^1 d\alpha \big\{ m_q^2 \Phi_0(\epsilon, \vec{r}, \lambda) \Phi^T_V(\vec{r}, \alpha) - [\alpha^2 + (1 - \alpha)^2] \vec{\Phi}_1(\epsilon, \vec{r}, \lambda) \cdot \vec{\nabla}_r \Phi^T_V(\vec{r}, \alpha) \big\};
\] (10)

\[
\mathcal{M}^{L}_{\gamma^* N \to V N}(s, Q^2) \big|_{t=0} = 4 N_C Z_q \sqrt{2 \alpha_{em}} m_V Q \int d^2r \sigma_{\bar{q}q}(\vec{r}, s) \int_0^1 d\alpha \alpha^2 (1 - \alpha)^2 \Phi_0(\epsilon, \vec{r}, \lambda) \Phi^T_V(\vec{r}, \alpha),
\] (11)

with the normalization given by Eq. (5) for both T and L polarizations. The functions $\Phi_{0,1}$ in Eqs. (10) and (11) include nonperturbative interaction effects between $q$ and $\bar{q}$, and $\Phi_V$ represents the vector meson wave function, whose explicit form will be taken from Ref. [28]. The real part of the amplitude is included according to the prescription described in Ref. [28].

In Eqs. (10) and (11) the terms proportional to $[\Phi_0(\epsilon, \vec{r}, \lambda) \Phi^T_V(\vec{r}, \alpha)]$ and to $[\vec{\Phi}_1(\epsilon, \vec{r}, \lambda) \cdot \vec{\nabla}_r \Phi^T_V(\vec{r}, \alpha)]$ correspond to the helicity conserving and helicity-flip transitions in the $\gamma^* \to \bar{q}q$ and $V \to \bar{q}q$ vertices, respectively. The helicity flip transitions represent the relativistic corrections. For heavy quarkonium these corrections become important only at large $Q^2 \gg m_V^2$. For production of light vector mesons, however, they are non-negligible even in the photoproduction limit, $Q^2 = 0$.

Usually the data are presented in the form of the production cross section $\sigma = \sigma^T + \epsilon' \sigma^L$, at fixed photon polarization $\epsilon'$. Here the transverse and longitudinal cross sections, integrated over $t$, read:

\[
\sigma^{T,L}(\gamma^* N \to V N) = \frac{\left| \mathcal{M}^{T,L}_{\gamma^* N \to V N} \right|^2}{16 \pi B_{\gamma^* N}},
\] (12)

where $B_{\gamma^* N}$ is the $t$-slope parameter in the differential cross section for reaction $\gamma^* p \to V p$, Eq. (5). The absolute value of the production cross section has already been checked in Ref. [28], by comparing with data for elastic $\rho^0$ and $\Phi^0$ electroproduction, and for charmonium exclusive electroproduction $\gamma^* p \to J/\Psi p$ in Refs. [29, 30].

The generalization of the LC dipole approach to nuclear targets is straightforward. We focus in the present paper on diffractive incoherent (quasielastic) production of vector mesons off nuclei, $\gamma^* A \to V X$, where the observable usually studied experimentally is nuclear transparency, defined by Eq. (1). Since the $t$-slope of the differential quasielastic cross section is
the same as on a nucleon target, instead of the integrated cross sections one can also use the forward differential cross sections, Eq. (5), to write

\[
T_{\gamma A}^{inc} = \frac{1}{A} \left| \frac{M_{\gamma^* A \to V X(s, Q^2)}}{M_{\gamma^* N \to V N(s, Q^2)}} \right|^2.
\]

(13)

The nuclear forward production amplitude \(M_{\gamma^* A \to V X(s, Q^2)}\) in Eq. (13) is calculated using the LC Green function approach \[28\]. In this approach the physical photon \(|\gamma^*\rangle\) is decomposed into different Fock states, namely, the bare photon \(|\gamma^*\rangle_0\), plus \(|\bar{q}q\rangle, |\bar{q}qG\rangle\), etc. As we mentioned above the higher Fock states containing gluons describe the energy dependence of the photo-production reaction on a nucleon. Besides, these Fock components also lead to gluon shadowing as far as nuclear effects are concerned. Detailed description and calculation of gluon shadowing for the case of vector meson production off nuclei is presented in Refs. \[28, 40\]. However, in the CLAS and HERMES kinematic range studied in the present paper the gluon shadowing is negligible and therefore is not included in our calculations.

The propagation of an interacting \(\bar{q}q\) pair in a nuclear medium is also described by the Green function satisfying the evolution Eq. (6). However, the potential in this case acquires an imaginary part which represents absorption in the medium,

\[
\text{Im}V_{\bar{q}q}(z^2, \vec{r}^2, \alpha) = -\frac{\sigma_{\bar{q}q}(\vec{r}, s)}{2} \rho_A(b, z^2),
\]

(14)

where \(\rho_A(b, z)\) is the nuclear density function defined at given impact parameter \(b\) and longitudinal coordinate \(z\).

The analytical solution of Eq. (6) is only known for the harmonic oscillator potential \(V(r) \propto r^2\). To keep the calculations reasonably simple we use the dipole cross section approximation,

\[
\sigma_{\bar{q}q}(r, s) = C(s) r^2,
\]

(15)

which allows to obtain the Green function in an analytical form (see Eq. (8)). The energy dependent factor \(C(s)\) was adjusted by the procedure described in ref. \[28\].

With the potential that follows from Eqs. (14) – (15) the solution of Eq. (6) has the same form as Eq. (8), except that one should replace \(\omega \Rightarrow \Omega\), where

\[
\Omega = \sqrt{\frac{a^4(\alpha) - i \rho_A(b, z) \nu \alpha (1 - \alpha) C(s)}{\nu \alpha (1 - \alpha)}}.
\]

(16)

The evolution equation (6) was recently solved numerically for the first time in Ref. \[33\], with the potential \(V_{\bar{q}q}(z^2, \vec{r}^2, \alpha)\) containing the imaginary part (14) and with the realistic dipole cross section given in \[38\].

As is discussed in \[28\] the value of \(l_c\) can distinguish different regimes of vector meson production:

(i) The region where CL is much shorter than the mean nucleon spacing in a nucleus (\(l_c \to 0\)). In this case \(G(z_2, \vec{r}_2; z_1, \vec{r}_1) \to \delta(z_2 - z_1)\). Correspondingly, the formation time of the meson wave function is very short and is given by Eq. (2). For light vector mesons \(l_f \sim l_c\), and since both the formation and coherence lengths are proportional to the photon energy both must
be short. Consequently, nuclear transparency is given by the simple formula corresponding to the Glauber approximation:

\[
T_{\gamma A}^{inc} \big|_{l_c < R_A} = \frac{\sigma_{\gamma A}^\ast}{A \sigma_{VN}^\ast} = \frac{1}{A} \int d^2 b \int_{-\infty}^{\infty} dz \ \rho_A(b, z) \exp \left[ -\sigma_{VN}^{in} \int_z^{\infty} dz' \ \rho_A(b, z') \right] \\
= \frac{1}{A \sigma_{VN}^{in}} \int d^2 b \left\{ 1 - \exp \left[ -\sigma_{VN}^{in} T_A(b) \right] \right\} = \frac{\sigma_{\gamma A}^{in}}{A \sigma_{VN}^{in}},
\]

where \(\sigma_{VN}^{in}\) is the inelastic \(VN\) cross section.

(ii) The production of charmonia and other heavy flavors corresponds to the intermediate case where as before \(l_c \to 0\), but where \(l_f \sim R_A\) can be realized. Then the formation of the meson wave function is described by the Green function, and the numerator of the nuclear transparency ratio Eq. (13) has the form [26],

\[
|M_{\gamma^* A \rightarrow VX}(s, Q^2)|_{l_c \to 0; l_f \sim R_A}^2 = \int d^2 b \int_{-\infty}^{\infty} dz \ \rho_A(b, z) \left| F_1(b, z) \right|^2,
\]

where

\[
F_1(b, z) = \int_0^1 d\alpha \int d^2 r_1 d^2 r_2 \Psi_V^\ast(\vec{r}_2, \alpha) G(z', \vec{r}_2; z, \vec{r}_1) \sigma_{\bar{q}q}(r_1, s) \Psi_{\gamma^*}(\vec{r}_1, \alpha) \mid_{z' \to \infty}\]

(iii) In the high energy limit \(l_c \gg R_A\). In this case \(G(z_2, \vec{r}_2; z_1, \vec{r}_1) \to \delta(\vec{r}_2 - \vec{r}_1)\), i.e. all fluctuations of the transverse \(\bar{q}q\) separation are “frozen” by Lorentz time dilation. Then the numerator on the right-hand-side (r.h.s.) of Eq. (13) takes the form [26],

\[
|M_{\gamma^* A \rightarrow VX}(s, Q^2)|_{l_c \gg R_A}^2 = \int d^2 b T_A(b) \times \int d^2 r \int_0^1 d\alpha \ \Psi_V^\ast(\vec{r}, \alpha) \sigma_{\bar{q}q}(r, s) \times \exp \left[ -\frac{1}{2} \sigma_{\bar{q}q}(r, s) T_A(b) \right] \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2) \},
\]

and the \(\bar{q}q\) attenuates with a constant absorption cross section as in the Glauber model, except that the whole exponential is averaged rather than just the cross section in the exponent. The difference between the results of the two prescriptions are the well known inelastic corrections of Gribov [1].

(iv) This regime reflects the general case when there are no restrictions for either \(l_c\) or \(l_f\). The corresponding theoretical tools have been developed only recently and for the first time in [28]. In this general case the incoherent nuclear production amplitude squared is represented as a sum of two terms [41],

\[
|M_{\gamma^* A \rightarrow VX}(s, Q^2)|^2 = \int d^2 b \int_{-\infty}^{\infty} dz \ \rho_A(b, z) \left| F_1(b, z) - F_2(b, z) \right|^2.
\]

The first term \(F_1(b, z)\), introduced above in Eq. (19), corresponds to the short \(l_c\) limit (ii). The second term \(F_2(b, z)\) in (21) corresponds to the situation when the incident photon produces a \(\bar{q}q\) pair diffractively and coherently at the point \(z_1\), prior to an incoherent quasielastic
scattering at point $z$. The LC Green function describes the evolution of the $\bar{q}q$ over the distance from $z_1$ to $z$ and further on, up to the formation of the meson wave function. Correspondingly, this term has the form,

$$F_2(b, z) = \frac{1}{2} \int dz_1 \rho_A(b, z_1) \int d\alpha \int d^2r_1 d^2r_2 d^2r \times \Psi^\dagger(r_2, \alpha) G(z' \rightarrow \infty, r_2; z, r) \sigma_{qq}(r, s) G(z, r; z_1, r_1) \sigma_{\bar{q}q}(r_1, s) \Psi^\dagger_\gamma(r_1, \alpha). \quad (22)$$

Finally, we would like to emphasize that Eq. (21) correctly reproduces the limits (i) - (iii) at small and large energies, as was already analyzed in Ref. [28].

### 3 Signal of CT Expected at Low Energies

Exclusive incoherent electroproduction of vector mesons off nuclei has been shown in [22, 28] to be a very effective tool for the investigation of CT. Increasing the photon virtuality $Q^2$ has the effect of squeezing the produced $\bar{q}q$ wave packet. Such a small colorless system propagates through the nucleus with little attenuation, provided that the energy is sufficiently high ($l_f \gg R_A$), so the fluctuations of the $\bar{q}q$ separation are “frozen” during propagation. Consequently, a rise of the nuclear transparency $T_{\text{inc}}^A(Q^2)$ with $Q^2$ should give a signal for CT. Indeed, such a rise was observed in the E665 experiment [20] at Fermilab for exclusive production of $\rho^0$ mesons off nuclei, and this has been claimed as a manifestation of CT.

However, the effect of coherence length [23, 24] leads also to a rise of $T_{\text{inc}}^A(Q^2)$ with $Q^2$ and so it can imitate the CT effects. Both effects work in the same direction and this produces the problem of CT-CL separation, which was solved in Refs. [25, 28], where a simple prescription for the elimination of CL effects from the data on the $Q^2$ dependence of nuclear transparency was presented. One should bin the data in a way which keeps $l_c = \text{const}$. It means that $\nu$ and $Q^2$ should be correlated,

$$\nu = \frac{1}{2} l_c (Q^2 + m_V^2).$$

(23)

In this case any rise with $Q^2$ of the nuclear transparency ratio is a clear signal of CT [25, 28].

In the present paper we investigate a manifestation of CT effects in the production of vector mesons at small energies, corresponding to CLAS experiment at JLab. Motivated by expected new data from the CLAS collaboration we concentrate on the production of $\rho^0$ vector mesons. Besides this experimental motivation, there is also theoretical interest because the coherence and formation effects are much more visible for the light than for the heavy vector mesons, as was discussed and analyzed in Refs. [28, 29, 30].

We start, however, with a discussion about the first theoretical analysis of CT phenomenon, related with the E665 experiment [20]. The corresponding predictions [22] were based on the assumption that the energy corresponding to the E665 experiment is high enough, so that $l_f, l_c \gg R_A$, allowing to use the “frozen” approximation [20]. Correspondingly one can neglect a variation of CL with $Q^2$. However, this is true only at $Q^2 \lesssim 3 \div 5 \text{ GeV}^2$, as was shown later in Ref. [28]. It means that fluctuations of the size of the $\bar{q}q$ pair become important at larger $Q^2 \gtrsim 5 \text{ GeV}^2$, and therefore at such values of $Q^2$ one should use the general expression
for the calculation of nuclear transparency. Thus the observed variation of $T_{A}^{\text{inc}}(Q^2)$ is a net manifestation of CT only at smaller $Q^2$. Although the model predictions, within the color dipole approach [22], were in agreement with the E665 data, small corrections for CL effects, done at larger $5 < Q^2 < 10$ GeV$^2$ in Ref. [28], did not spoil this agreement.

Figure 1: $Q^2$ dependence of the nuclear transparency $T_{A}^{\text{inc}}$ for exclusive electroproduction of $\rho$ mesons on $^{207}$Pb target. The dotted lines represent the CL fixed at $l_c = 1.0$ and 5.0 fm. The solid lines represent predictions at the fixed mean photon energy $\langle \nu \rangle = 13$ GeV corresponding to HERMES experiment.

On the other hand, CT effects are also under investigation at lower energies in both the HERMES experiment at HERA and the CLAS experiment at JLab, measuring the same process of incoherent electroproduction of $\rho$ mesons. Because CL effects are also important one should use the LC Green function formalism as a very effective tool for such study, because then both CT and CL effects are naturally incorporated. Due to a strong CL-CT mixing one should eliminate the effect of CL according to the prescription mentioned above (see Eq. (23)). The corresponding predictions for CT [28] are in a good agreement with the HERMES data [35] on the $Q^2$ dependence of nuclear transparency at different fixed values of $l_c$. Expected new HERMES data on neon and krypton targets should allow to verify further the predictions for CT from [28].
Following the relation Eq. (23), CT effects are found to be much stronger at low (HERMES, CLAS) than at high (E665) energies [28]. At high energies the CL $l_c$ is long. Consequently, the formation length is long too, $l_f \gtrsim l_c \gg R_A$, and nuclear transparency rises with $Q^2$ only because the mean transverse size of the $q\bar{q}$ photon fluctuations decreases. It is not so at lower energies when $l_c \lesssim R_A$. Then, according to the prescription (23), the photon energy rises with $Q^2$ and consequently the formation length, Eq. (2), rises as well. Thus, these two effects add up leading to a steeper growth of $Tr_A^{inc}(Q^2)$ for short $l_c$.

The prescription (23) for studying net signals of CT fixing the values of $l_c$, has one hidden disadvantage. Increasing $Q^2$ one needs to increase also the photon energy, and this can be done only up to the kinematic limit given by the HERMES experiment, $\nu_{\text{max}}^2 = 24$ GeV. In turn this leads then to a restriction for the maximal values of the photon virtualities $Q_{\text{max}}^2$. The situation gets a bit more complicated because an effort to obtain high statistics data leads to a $\nu - Q^2$ correlation and then to stronger restrictions on $Q_{\text{max}}^2$ at different fixed values of $l_c$. For example, at $\langle l_c \rangle = 1.35$ fm the value of $Q_{\text{max}}^2 = 3.5$ GeV$^2$; and at $\langle l_c \rangle = 2.45$ fm the value is $Q_{\text{max}}^2 = 1.5$ GeV$^2$, as follows from HERMES experiment [35, 42].

Therefore, we present in Fig. 1 the $Q^2$ dependence of the nuclear transparency $Tr_A^{inc}$, for exclusive electroproduction of $\rho$ mesons on $^{207}\text{Pb}$ target at different fixed $l_c$ values. The top and bottom dotted lines represent the predictions at fixed $l_c = 1.0$ and 5.0 fm, respectively. These two values limit approximately the range of $l_c$-values corresponding to a mean $\langle \nu \rangle = 13$ GeV and $0.5 < Q^2 < 5.0$ GeV$^2$, which follow from the HERMES kinematics [12]. Because $l_c \lesssim R_{\text{Pb}}$, one can expect a strong onset of CL effects, as one can see in Fig. 1 as the difference between the top and bottom dotted lines.

Different fixed $l_c$-values determine different maximal photon virtualities $Q_{\text{max}}^2$ allowed for the experimental analysis of the CT phenomenon. In order to reach the maximal possible values of $Q^2$ one should fix $l_c$ at its minimal possible values, keeping reasonable statistics of the data. Such a situation is depicted by the top dotted line in Fig. 1 and allows to study the onset of CT effects up to $Q_{\text{max}}^2 = 5.0$ GeV$^2$ within the HERMES kinematics. The higher fixed values of $l_c$ mean a stronger restriction for $Q_{\text{max}}^2$, and consequently a weaker onset of CT effects. For example, at the second fixed value of $l_c = 5.0$ fm one can study CT effects only up to $Q_{\text{max}}^2 = 1.3$ GeV$^2$, although the model calculations shown by the bottom dotted line in Fig. 1 formally reach the value $Q_{\text{max}}^2 = 5.0$ GeV$^2$.

As a demonstration of the importance of CL effects in the HERMES kinematic range we also present, by the solid line in Fig. 1, predictions for nuclear transparency at fixed mean photon energy $\langle \nu \rangle = 13$ GeV. This corresponds to a change of CL from 5.0 to 1.0 fm. Because both CL and CT effects affect the rise of $Tr_A$ with $Q^2$, the net observable effect is much more visible than that when the effect of CL is eliminated by the prescription (23) (compare each dotted line with the solid one in Fig. 1).

Now we switch on to the low energy region corresponding to the CLAS collaboration at JLab. Because the new data will appear soon we provide predictions for the expected onset of CT effects for different nuclei. The model calculations for nuclear transparency as a function of $Q^2$ are depicted in Fig. 2 for several nuclear targets $^{12}\text{C}$, $^{56}\text{Fe}$ and $^{207}\text{Pb}$. According to the CLAS kinematics [12], the values of CL changes from $l_c = 0.9$ to 0.5 fm when the photon virtuality rises from $Q^2 = 0.9$ to 2.5 GeV$^2$. Eliminating the CL effects by fixing $l_c = 0.9$ and 0.5 fm, one can obtain the predictions shown in Fig. 2 by the difference between the dotted top
and bottom lines with respect to each solid line, for different nuclear targets. One can see that in the CLAS kinematic range the CL effects are rather weak, much weaker than the observed signal of CT. This comes from the rather small photon energies for values of CL, $l_c \ll R_A$. A weak onset of CL effects can be seen in Fig. 2 as the difference between the top and bottom dotted lines, for each nuclear target.

Therefore in contrast with the HERMES kinematics one can study the variation of nuclear transparency with $Q^2$ at approximately fixed photon energy, $\nu = 3.5 - 3.6$ GeV, using maximal statistics of the data. Such a situation is depicted in Fig. 2 by the solid lines, for different nuclear targets. It allows to reach much higher values of $Q^2$ than those reached in the HERMES experiment, using the CL-elimination procedure described above.

![Figure 2: $Q^2$ dependence of the nuclear transparency $T_r^{inc}$ for exclusive electroproduction of $\rho$ mesons on nuclear targets $^{12}C$, $^{56}Fe$ and $^{207}Pb$ (from top to bottom). The top and bottom dotted line with respect to each solid line represents predictions fixing CL at $l_c = 0.9$ and 0.5 fm, respectively. The solid lines represent predictions at the fixed mean photon energy $\langle \nu \rangle = 3.5$ GeV corresponding to CLAS experiment at JLab.](image)

As a consequence of a small onset of CL effects the corresponding maximal CLAS value of photon virtuality used for studying CT effects $Q^2_{max} \sim 2.5 - 3.0$ GeV\(^2\) does not differ much from the HERMES value $Q^2_{max} \sim 3.5 - 4.0$ GeV\(^2\), although the mean photon energy is four
times lower than that in the HERMES experiment. It means that the CLAS kinematic range leads to a more effective investigation of CT phenomenon than in a case when we are forced to use the CL-elimination procedure, i.e. keeping \( l_c = \text{constant} \) at different values of \( Q^2 \), which is typical for the HERMES experiment.

Such an advantage in the investigation of CT effects in the CLAS experiment should be maintained also after its future upgrade to a beam energy of 12 GeV \([43]\). As a consequence of larger photon energies, larger values of the photon virtualities \( Q^2 \) will be reached, keeping the condition of a weak onset of CL effects \( l_c \ll R_A \) for heavy nuclear targets. Thus it allows to study a stronger onset of CT effects investigating nuclear transparency at larger values of \( Q^2 \).

### 4 Summary and conclusions

Electroproduction of vector mesons off nuclei is a very effective tool for studying the interplay between coherence (shadowing) and formation (color transparency) effects. In the present paper we investigated how those effects manifest themselves in the production of vector mesons off nuclei, at different energies corresponding to the E665, HERMES and CLAS experiments. We used, from \([28]\), a rigorous quantum-mechanical approach based on the light-cone QCD Green function formalism, which naturally incorporates these interference effects. Motivated by expected new data from the CLAS collaboration we studied the onset of CT effects at small energies, predicting a rising nuclear transparency as a function of \( Q^2 \).

As the first step we discussed CT effects at large energies corresponding to E665 experiment. It was already shown in Ref. \([28]\) that the high energy limit \( (l_f \sim l_c \gg R_A) \) can be applied only at small and medium values of photon virtualities, \( Q^2 \lesssim 3 \div 5 \text{ GeV}^2 \). Here the expressions for nuclear production cross sections are sufficiently simplified. This so called “frozen” approximation includes only CT, because there are no fluctuations of the transverse size of the \( \bar{q}q \) pair. At larger values of \( Q^2 \gtrsim 5 \text{ GeV}^2 \) both the CL \( l_c \lesssim R_A \) and the onset of CL effects start to be important. The “frozen” approximation cannot be applied anymore for the study of a signal of CT effects. Therefore one should use the general expression Eq. \([21]\), which incorporates in addition CL effects and therefore interpolates between the previously known low and high energy limits for incoherent production of vector mesons.

In the incoherent electroproduction of vector mesons at low and medium energies the onset of coherence effects (shadowing) can mimic the expected signal of CT. Both effects, CT and CL, work in the same direction. In order to single out the formation effect the experimental data should be binned in \( l_c \) and \( Q^2 \) \([28]\). The observation of a rising nuclear transparency as function of \( Q^2 \) for fixed \( l_c \) would signal CT. Such a procedure for the elimination of CL effects must be applied at medium energies, corresponding to the HERMES kinematic range, when \( l_c \lesssim R_A \). We showed that a \( Q^2 \)-variation of CL affects a strong onset of CL effects as one can see in Fig. 1 as a difference between the top and bottom dotted lines.

At still smaller photon energies, corresponding to the CLAS experiment, both \( l_c \) and \( l_f \) are short enough compared to the mean spacing of the bound nucleons. Consequently, the CL effects are rather weak, and in fact much weaker than the observed effect of CT (see the difference between the top and bottom dotted lines for each nuclear target in Fig. 2). This provides an advantage in the investigation of the onset of CT when one can study the rise
of nuclear transparency with $Q^2$ at fixed photon energies, processing the data at maximal statistics (see the solid lines in Fig. 2). We showed that such a procedure allows to reach much higher values of $Q^2$ effective for studying the CT phenomenon, than those used in the HERMES experiment applying the CL-elimination prescription of fixing $l_c$ (see Eq. 23).

Concluding, the exploratory study of CT effects at different energies in electroproduction of light vector mesons off nuclei opens new possibilities for the investigation of different manifestations of CL effects. It gives a good basis for the further effective studies of the onset of CT effects, after an upgrade of the CLAS experiment at JLab to a 12 GeV energy electron beam.

Acknowledgments: This work was supported in part by Fondecyt (Chile) grant 1050519, by DFG (Germany) grant PI182/3-1, and by the Slovak Funding Agency, Grant No. 2/7058/27.

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