Possible Negative Pressure States in the Evolution of the Universe

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Abstract

Hydrodynamic derivation of the entrainment of matter induced by a surface elastic wave propagating along the flexible vacuum-matter interface is conducted by considering the nonlinear coupling between the interface and the rarefaction effect. The critical reflux values associated with the product of the second-order (unit) body forcing and the Reynolds number (representing the viscous dissipations) decrease as the Knudsen number (representing the rarefaction measure) increases from zero to 0.1. We obtained the critical bounds for matter-freeze or zero-volume-flow-rate states corresponding to specific Reynolds numbers (ratio of wave inertia and viscous dissipation effects) and wave numbers which might be linked to the dissipative evolution of the Universe. Our results also show that for certain time-averaged evolution of the matter (gas) there might be existence of negative pressure.

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I. INTRODUCTION

The mean cosmic density of dark matter (plus baryons) is now pinned down to be only ca. 30% of the so-called critical density corresponding to a 'flat'-Universe. However, other recent evidence—microwave background anisotropies, complemented by data on distant supernovae—reveals that our Universe actually is 'flat', but that its dominant ingredient (ca. 70% of the total mass energy) is something quite unexpected: 'dark energy' pervading all space, with negative pressure. We do know that this material is very dark and that it dominates the internal kinematics, clustering properties and motions of galactic systems. Dark matter is commonly associated to weakly interacting particles (WIMPs), and can be described as a fluid with vanishing pressure. It plays a crucial role in the formation and evolution of structure in the universe and it is unlikely that galaxies could have formed without its presence [1-2]. Analysis of cosmological mixed dark matter models in spatially flat Friedmann Universe with zero $\Lambda$ term have been presented before. For example, we can start from the Einstein action describing the gravitational forces in the presence of the cosmological constant [3]

\[ S = \frac{1}{2k_N^2} \int \sqrt{-g} \, R - \int \sqrt{-g} \, \Lambda + S_m, \]

where $S_m$ is the contribution of the matter and radiation, $k_N^2 = 8\pi G_N = 8\pi M_P^2$, $G_N$ is the Newton’s constant and $M_P = 1.22 \times 10^{19}\text{GeV}/c^2$ is the Planck mass. The set of equations governing the evolution of the universe is completed by the Friedman equations for the scale factor ($a(t)$)

\[ (\frac{\dot{a}}{a})^2 = \frac{8\pi G_N}{3}(\rho + \Lambda) - \frac{k}{a^2}, \]
\[ (\frac{\ddot{a}}{a}) = -\frac{4\pi G_N}{3}(\rho + \Lambda + 3(p - \Lambda)), \]

where $k = 0$ is for the flat-Universe, $\rho$ and $p$ is the density and pressure [4-5]. A large majority of dark energy models describes dark energy in terms of the equation of state (EOS) $p_d = \omega \rho_d$, where $\omega$ is the parameter of the EOS, while $p_d$ and $\rho_d$ denote the pressure and the energy density of dark energy, respectively. The value $\omega = -1$ is characteristic of the cosmological constant, while the dynamical models of dark energy generally have $\omega \geq -1$. The case of the growing cosmological term $\Lambda$ and its implications for the asymptotic expansion of the universe and the destiny of the bound systems have been studied in [5] using above system of equations. Their results showed that even for very slow growth of $\Lambda$ (which satisfies
all the conditions on the variation of $G_N$), in the distant future the gravitationally bound systems become unbound, while the nongravitationally bound systems remain bound. Meanwhile, it is convenient to express the mean densities $\rho_i$ of various quantities in the Universe in terms of their fractions relative to the critical density: $\Omega_i = \rho_i/\rho_{\text{crit}}$. The theory of cosmological inflation strongly suggests that the total density should be very close to the critical one ($\Omega_{\text{tot}} \sim 1$), and this is supported by the available data on the cosmic microwave background (CMB) radiation. The fluctuations observed in the CMB at a level $\sim 10^{-5}$ in amplitude exhibit a peak at a partial wave $l \sim 200$, as would be produced by acoustic oscillations in a flat Universe with $\Omega_{\text{tot}} \sim 1$. At lower partial waves, $l \gg 200$, the CMB fluctuations are believed to be dominated by the Sachs-Wolfe effect due to the gravitational potential, and more acoustic oscillations are expected at $l > 200$, whose relative heights depend on the baryon density $\Omega_b$. At even larger values, $l \geq 1000$, these oscillations should be progressively damped away [1-2,3,6].

Influential only over the largest of scales—the cosmological horizon—is the outermost species of invisible matter: the vacuum energy (also known by such names as dark energy, quintessence, $x$-matter, the zero-point field, and the cosmological constant $\Lambda$). If there is no exchange of energy between vacuum and matter components, the requirement of general covariance implies the time dependence of the gravitational constant $G$. Thus, it is interesting to look at the interacting behavior between the vacuum (energy) and the matter from the macroscopic point of view. One related issue, say, is about the dissipative matter of the flat Universe immersed in vacua [7] and the other one is the macroscopic Casimir effect with the deformed boundaries [8].

Theoretical (using the Boltzmann equation) and experimental studies of interphase nonlocal transport phenomena which appear as a result of a different type of nonequilibrium representing propagation of a surface elastic wave have been performed since late 1980s [9-10]. These are relevant to rarefied gases (RG) flowing along deformable elastic slabs with the dominated parameter being the Knudsen number ($\text{Kn} = \text{mean-free-path}/L_d$, mean-free-path (mfp) is the mean free path of the gas, $L_d$ is proportional to the distance between two slabs) [11-13]. The role of the Knudsen number is similar to that of the Navier slip parameter $N_s$ [14]; here, $N_s = \mu S/d$ is the dimensionless Navier slip parameter; $S$ is a proportionality constant as $u_s = S\tau$, $\tau$ : the shear stress of the bulk velocity; $u_s$ : the dimensional slip velocity; for a no-slip case, $S = 0$, but for a no-stress condition. $S = \infty$, $\mu$ is the fluid
viscosity, \( d \) is one half of the distance between upper and lower slabs).

Here, the transport driven by the wavy elastic vacuum-matter interface will be presented. The flat-Universe is presumed and the corresponding matter is immersed in vacua with the interface being flat-plane like. We adopt the macroscopic or hydrodynamical approach and simplify the original system of equations (related to the momentum and mass transport) to one single higher-order quasi-linear partial differential equation in terms of the unknown stream function. In this study, we shall assume that the Mach number \( Ma \ll 1 \), and the governing equations are the incompressible Navier-Stokes equations which are associated with the relaxed slip velocity boundary conditions along the interfaces [11-14]. We then introduce the perturbation technique so that we can solve the related boundary value problem approximately. To consider the originally quiescent gas for simplicity, due to the difficulty in solving a fourth-order quasi-linear complex ordinary differential equation (when the wavy boundary condition are imposed), we can finally get an analytically perturbed solution and calculate those physical quantities we have interests, like, time-averaged transport or entrainment, perturbed velocity functions, critical unit body forcing corresponding to the freezed or zero-volume-flow-rate states. These results might be closely linked to the vacuum-matter interactions (say, macroscopic Casimir effects) and the evolution of the Universe (as mentioned above: the critical density [1-2]). Our results also show that for certain time-averaged evolution of the matter (the maximum speed of the matter (gas) appears at the center-line) there might be existence of negative-pressure states.

II. FORMULATIONS

We consider a two-dimensional matter-region of uniform thickness which is approximated by a homogeneous rarefied gas (Newtonian viscous fluid). The equation of motion is

\[
(\lambda_L + \mu) \text{grad div } \mathbf{u} + \mu \nabla \mathbf{u} + \rho \mathbf{p} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2},
\]

where \( \lambda_L \) and \( \mu \) are Lamé constants, \( \mathbf{u} \) is the displacement field (vector), \( \rho \) is the mass density and \( \mathbf{p} \) is the body force for unit mass. The Navier-Stokes equations, valid for Newtonian fluids (both gases and liquids), has been a mixture of continuum fluid mechanics ever since 1845 following the seemingly definitive work of Stokes and others [15], who proposed the following rheological constitutive expression for the fluid deviatoric or viscous stress (tensor)
\[ T : T = 2\mu \nabla u + \lambda I \nabla \cdot u. \]

The flat-plane boundaries of this matter-region or the vacuum-matter interfaces are rather flexible and presumed to be elastic, on which are imposed traveling sinusoidal waves of small amplitude \( a \) (possibly due to vacuum fluctuations). The vertical displacements of the upper and lower interfaces \( y = h \) and \( -h \) are thus presumed to be \( \eta \) and \( -\eta \), respectively, where \( \eta = a \cos[2\pi(x - ct)/\lambda] \), \( \lambda \) is the wave length, and \( c \) the wave speed. \( x \) and \( y \) are Cartesian coordinates, with \( x \) measured in the direction of wave propagation and \( y \) measured in the direction normal to the mean position of the vacuum-matter interfaces. It would be expedient to simplify these equations by introducing dimensionless variables. We have a characteristic velocity \( c \) and three characteristic lengths \( a, \lambda, \) and \( h \). The following variables based on \( c \) and \( h \) could thus be introduced:

\[
\begin{align*}
x' &= \frac{x}{h}, & y' &= \frac{y}{h}, & u' &= \frac{u}{c}, & v' &= \frac{v}{c}, & \eta' &= \frac{\eta}{h}, & \psi' &= \frac{\psi}{ch}, & t' &= \frac{ct}{h}, & p' &= \frac{p}{\rho c^2},
\end{align*}
\]

where \( \psi \) is the dimensional stream function, \( u \) and \( v \) are the velocities along the \( x \)- and \( y \)-directions; \( \rho \) is the density, \( p \) (its gradient) is related to the (unit) body forcing. The primes could be dropped in the following. The amplitude ratio \( \epsilon \), the wave number \( \alpha \), and the Reynolds number (ratio of wave inertia and viscous dissipation effects) \( Re \) are defined by

\[
\begin{align*}
\epsilon &= \frac{a}{h}, & \alpha &= \frac{2\pi h}{\lambda}, & Re &= \frac{c h}{\nu}.
\end{align*}
\]

We shall seek a solution in the form of a series in the parameter \( \epsilon \):

\[
\psi = \psi_0 + \epsilon \psi_1 + \epsilon^2 \psi_2 + \cdots,
\]

\[
\frac{\partial p}{\partial x} = (\frac{\partial p}{\partial x})_0 + \epsilon(\frac{\partial p}{\partial x})_1 + \epsilon^2(\frac{\partial p}{\partial x})_2 + \cdots,
\]

with \( u = \partial \psi / \partial y, \quad v = -\partial \psi / \partial x \). The 2D (\( x \)- and \( y \)-) momentum equations and the equation of continuity could be in terms of the stream function \( \psi \) if the \( p \)-term (the specific body force density, assumed to be conservative and hence expressed as the gradient of a time-independent potential energy function) is eliminated. The final governing equation is

\[
\frac{\partial}{\partial t} \nabla^2 \psi + \psi_y \nabla^2 \psi_x - \psi_x \nabla^2 \psi_y = \frac{1}{Re} \nabla^4 \psi, \quad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},
\]

and subscripts indicate the partial differentiation. Thus, we have

\[
\frac{\partial}{\partial t} \nabla^2 \psi_0 + \psi_{0y} \nabla^2 \psi_{0x} - \psi_{0x} \nabla^2 \psi_{0y} = \frac{1}{Re} \nabla^4 \psi_0,
\]

(3)
\[
\frac{\partial}{\partial t} \nabla^2 \psi_1 + \psi_{0y} \nabla^2 \psi_{1x} + \psi_{1y} \nabla^2 \psi_{0x} - \psi_{0x} \nabla^2 \psi_{1y} - \psi_{1x} \nabla^2 \psi_{0y} = \frac{1}{Re} \nabla^4 \psi_1, \tag{4}
\]

\[
\frac{\partial}{\partial t} \nabla^2 \psi_2 + \psi_{0y} \nabla^2 \psi_{2x} + \psi_{1y} \nabla^2 \psi_{1x} + \psi_{2y} \nabla^2 \psi_{0x} - \psi_{0x} \nabla^2 \psi_{2y} - \psi_{1x} \nabla^2 \psi_{1y} - \psi_{2x} \nabla^2 \psi_{0y} = \frac{1}{Re} \nabla^4 \psi_2, \tag{5}
\]

and other higher order terms. The (matter) gas is subjected to boundary conditions imposed by the symmetric motion of the vacuum-matter interfaces and the non-zero slip velocity: \(u = \pm \text{Kn} \, du/dy \) [11-13], \(v = \pm \partial \eta / \partial t \) at \(y = \pm (1 + \eta)\), here \(\text{Kn} = mfp/(2h)\). The boundary conditions may be expanded in powers of \(\eta\) and then \(\epsilon\):

\[
\psi_{0y}|_1 + \epsilon \cos \alpha (x-t) \psi_{0yy}|_1 + \psi_{1y}|_1 + \epsilon^2 \left[ \psi_{0yy}|_1 \right] \cos^2 \alpha (x-t) + \psi_{2y}|_1 + \rho \cos \alpha (x-t) \psi_{1yy}|_1 + \psi_{2yy}|_1 + \cdots,
\]

\[
\psi_{0x}|_1 + \epsilon \cos \alpha (x-t) \psi_{0xy}|_1 + \psi_{1x}|_1 + \epsilon^2 \left[ \psi_{0xy}|_1 \right] \cos^2 \alpha (x-t) + \rho \cos \alpha (x-t) \psi_{1xy}|_1 + \psi_{2x}|_1 + \cdots = -\epsilon \alpha \sin \alpha (x-t).
\]

Equations above, together with the condition of symmetry and a uniform \((\partial p/\partial x)_0\), yield:

\[
\psi_0 = K_0 [(1 + 2 \text{Kn}) y - \frac{y^3}{3}], \quad K_0 = \frac{Re}{2} \left( -\frac{\partial p}{\partial x} \right)_0, \tag{8}
\]

\[
\psi_1 = \frac{1}{2} \{ \phi(y) e^{i\alpha (x-t)} + \phi^*(y) e^{-i\alpha (x-t)} \}, \tag{9}
\]

where the asterisk denotes the complex conjugate. A substitution of \(\psi_1\) into Eqn. (3) yields

\[
\left\{ \frac{d^2}{dy^2} - \alpha^2 + i \alpha Re[1 - K_0(1 - y^2 + 2 \text{Kn})] \right\} \left( \frac{d^2}{dy^2} - \alpha^2 \right) \phi - 2i \alpha K_0 Re \phi = 0
\]

or if originally the (matter) gas is quiescent: \(K_0 = 0\) (this corresponds to a free (vacuum) pumping case)

\[
\left( \frac{d^2}{dy^2} - \alpha^2 \right) \left( \frac{d^2}{dy^2} - \alpha^2 \right) \phi = 0, \quad \alpha^2 = \alpha^2 - i \alpha Re. \tag{10}
\]

The boundary conditions are

\[
\phi_y(\pm 1) \pm \phi_{yy}(\pm 1) \text{Kn} = 2K_0 (1 \pm \text{Kn}) = 0, \quad \phi(\pm 1) = \pm 1. \tag{11}
\]

Similarly, with

\[
\psi_2 = \frac{1}{2} \{ D(y) + \frac{E(y) e^{i2\alpha (x-t)} + E^*(y) e^{-i2\alpha (x-t)}}{2} \}, \tag{12}
\]
we have
\[ D_{yyyy} = -\frac{i\alpha Re}{2} (\phi\phi^*_y - \phi^*\phi_y)_y, \]  
(13)
\[
\left(\frac{d^2}{dy^2} - (4\alpha^2 + 2i\alpha Re)\right)(\frac{d^2}{dy^2} - 4\alpha^2)E - i2\alpha ReK_0(1 - y^2 + 2Kn) \\
(\frac{d^2}{dy^2} - 4\alpha^2)E + i4\alpha K_0 ReE + \frac{i\alpha Re}{2} (\phi_y\phi_{yy} - \phi\phi_{yyy}) = 0; \]  
(14)
and the boundary conditions
\[ D_y(\pm 1) + \frac{1}{2} [\phi_{yy}(\pm 1) + \phi^*_{yy}(\pm 1)] - 2K_0 = \mp Kn\left\{\frac{1}{2} [\phi_{yyy}(\pm 1) + \phi^*_{yyy}(\pm 1)] + D_{yy}(\pm 1)\right\}, \]  
(15)
\[ E_y(\pm 1) + \frac{1}{2} \phi_{yy}(\pm 1) - \frac{K_0}{2} = \mp Kn\left[\frac{1}{2} \phi_{yyy}(\pm 1) + E_{yy}(\pm 1)\right], \]  
(16)
\[ E(\pm 1) + \frac{1}{4} \phi_y(\pm 1) = 0 \]  
(17)
where \(K_0\) is zero in Eqns. (13-16). After lengthy algebraic manipulations, we obtain
\[ \phi = c_0 e^{\alpha y} + c_1 e^{-\alpha y} + c_2 e^{\bar{\alpha} y} + c_3 e^{-\bar{\alpha} y}, \]
where \(c_0 = (A + A_0)/Det, c_1 = -(B + B_0)/Det, c_2 = (C + C_0)/Det, c_3 = -(T + T_0)/Det; \)
\[ Det = Ae^\alpha - Be^{-\alpha} + Ce^{\bar{\alpha}} - Te^{-\bar{\alpha}}, \]
\[ A = e^{\alpha} \bar{\alpha}^2 (r e^{-2\bar{\alpha}} - s e^{2\bar{\alpha}}) - 2\alpha \bar{\alpha} e^{-\alpha} w + \alpha \bar{\alpha} e^\alpha z (e^{-2\bar{\alpha} r} + e^{2\bar{\alpha} s}), \]
\[ A_0 = e^{-\alpha} \bar{\alpha}^2 (r^2 e^{-2\bar{\alpha}} - s^2 e^{2\bar{\alpha}}) + 2\alpha \bar{\alpha} e^\alpha z - \alpha \bar{\alpha} e^{-\alpha} w (e^{2\bar{\alpha} s} + e^{-2\bar{\alpha} r}), \]
\[ B = e^{-\alpha} \bar{\alpha}^2 (r e^{-2\bar{\alpha}} - s e^{2\bar{\alpha}}) + 2\alpha \bar{\alpha} e^\alpha z - \alpha \bar{\alpha} e^{-\alpha} w (e^{-2\bar{\alpha} r} + e^{2\bar{\alpha} s}), \]
\[ B_0 = e^{\alpha} \bar{\alpha}^2 (r^2 e^{-2\bar{\alpha}} - s^2 e^{2\bar{\alpha}}) - 2\alpha \bar{\alpha} e^{-\alpha} w + \alpha \bar{\alpha} e^\alpha z (e^{-2\bar{\alpha} r} + e^{2\bar{\alpha} s}), \]
\[ C = e^{-\alpha} \bar{\alpha} \alpha (w e^{\bar{\alpha} - \alpha} - r e^{\alpha - \bar{\alpha}}) - \alpha e^{2\bar{\alpha} + \alpha} z (z e^{2\bar{\alpha} - \bar{\alpha}} + \alpha e^{\alpha - \alpha} w - \alpha e^{\bar{\alpha} - \alpha} r), \]
\[ C_0 = e^{\alpha} \bar{\alpha} \alpha (w e^{\bar{\alpha} - \alpha} - r e^{\alpha - \bar{\alpha}}) - \alpha z e^{2\bar{\alpha} - \bar{\alpha}} + \alpha e^{\alpha - \alpha} w (\alpha e^{\bar{\alpha} - \alpha} w - \bar{\alpha} e^{(2\alpha - \bar{\alpha}) r}), \]
\[ T = e^{-\alpha} \bar{\alpha} \alpha (z e^{\bar{\alpha} + \alpha} - r w e^{-(\alpha + \bar{\alpha})}) - \bar{\alpha} \alpha (e^{2\bar{\alpha} - \bar{\alpha}} z r - e^\alpha w s) + \alpha^2 e^{-\alpha} (-e^{2\alpha} z^2 + e^{-2\alpha} w^2), \]
\[ T_0 = e^{\alpha} \bar{\alpha} \alpha (z e^{\bar{\alpha} + \alpha} - r w e^{-(\alpha + \bar{\alpha})}) - \bar{\alpha} \alpha (e^{-\alpha} z r - e^{-\bar{\alpha}} w s) + \alpha^2 e^\alpha (-e^{2\alpha} z^2 + e^{-2\alpha} w^2), \]
with \(r = (1 - \bar{\alpha} Kn), s = (1 + \bar{\alpha} Kn), w = (1 - \alpha Kn), z = (1 + \alpha Kn). \)
To obtain a simple solution which relates to the mean transport so long as only terms of \(O(\epsilon^2)\) are concerned, we see that if every term in the x-momentum equation is averaged over
an interval of time equal to the period of oscillation, we obtain for our solution as given by above equations the time-averaged (unit) body forcing

\[
\frac{\partial \rho}{\partial x} = \epsilon^2 \left( \frac{\partial \rho}{\partial x} \right)_2 = \epsilon^2 \left[ \frac{D_{yy} \phi y}{2Re} + \frac{iRe}{4} (\phi_y^* \phi_y^* - \phi_y^* \phi_y) \right] + O(\epsilon^3) = \epsilon^2 \frac{\Pi_0}{Re} + O(\epsilon^3),
\]

where \( \Pi_0 \) is the integration constant for the integration of equation (12) and could be fixed indirectly in the coming equation (22). Now, from Eqn. (14), we have

\[
D_y(\pm1) \pm KnD_{yy}(\pm1) = -\frac{1}{2} [\phi_{yy}(\pm1) + \phi_{yy}^*(\pm1)] \mp Kn \left\{ \frac{1}{2} [\phi_{yy}(\pm1) + \phi_{yy}^*(\pm1)] \right\},
\]

where \( D_y(y) = \Pi_0 y^2 + a_1 y + a_2 + C(y) \), and together from equation (12), we obtain

\[
C(y) = \frac{\alpha^2 Re^2}{2} \left[ \frac{c_0 c_2^*}{g_1^2} e^{(\alpha + \bar{\alpha}^*)y} + \frac{c_0 c_3^*}{g_2^2} e^{(\alpha + \bar{\alpha})y} + \frac{c_0 c_3^*}{g_3^2} e^{(\alpha - \bar{\alpha})y} + \frac{c_0 c_3^*}{g_4^2} e^{(\alpha - \bar{\alpha})y} + \frac{c_0 c_3^*}{g_5^2} e^{(\alpha - \bar{\alpha})y} + \frac{c_0 c_3^*}{g_6^2} e^{(\alpha - \bar{\alpha})y} + \frac{c_0 c_3^*}{g_7^2} e^{(\alpha - \bar{\alpha})y} + \frac{c_0 c_3^*}{g_8^2} e^{(\alpha - \bar{\alpha})y} + \frac{c_0 c_3^*}{g_9^2} e^{(\alpha - \bar{\alpha})y} \right],
\]

with \( g_1 = \alpha + \bar{\alpha}^* \), \( g_2 = \alpha + \bar{\alpha} \), \( g_3 = \alpha - \bar{\alpha}^* \), \( g_4 = \alpha - \bar{\alpha} \), \( g_5 = \bar{\alpha} - \bar{\alpha}^* \), \( g_6 = \bar{\alpha} + \bar{\alpha}^* \). In realistic applications we must determine \( \Pi_0 \) from considerations of conditions at the ends of the matter-region. \( a_1 \) equals to zero because of the symmetry of boundary conditions.

Once \( \Pi_0 \) is specified, our solution for the mean speed (\( u \) averaged over time) of matter-flow is

\[
U = \epsilon^2 \frac{D_y}{2} = \frac{\epsilon^2}{2} \left\{ C(y) - C(1) + R_0 - Kn C_y(1) + \Pi_0 \left[ y^2 - (1 + 2Kn) \right] \right\}
\]

where \( R_0 = -\left\{ [\phi_{yy}(1) + \phi_{yy}^*(1)] - Kn [\phi_{yy}^*(1) + \phi_{yy}^*(1)] \right\} / 2 \), which has a numerical value about 3 for a wide range of \( \alpha \) and \( Re \) (playing the role of viscous dissipations) when \( Kn = 0 \).

To illustrate our results clearly, we adopt \( U(Y) = u(y) \) for the time-averaged results with \( y = Y \) in the following.

III. RESULTS AND DISCUSSION

We check our approach firstly by examining \( R_0 \) with that of no-slip (\( Kn = 0 \)) approach. This can be done easily once we consider terms of \( D_y(y) \) and \( C(y) \) because to evaluate \( R_0 \) we shall at most take into account the higher derivatives of \( \phi(y) \), like \( \phi_y(y) \), \( \phi_{yy}^*(y) \) instead of \( \phi_g(y) \) and escape from the prescribing of \( a_2 \).
Our numerical calculations confirm that the mean streamwise velocity distribution (averaged over time) due to the induced motion by the wavy elastic vacuum-matter interface in the case of free (vacuum) pumping is dominated by $R_0$ (or $Kn$) and the parabolic distribution $-\Pi_0(1-y^2)$. $R_0$ which defines the boundary value of $D_y$ has its origin in the $y$-gradient of the first-order streamwise velocity distribution, as can be seen in Eqn. (14).

In addition to the terms mentioned above, there is a perturbation term which varies across the channel: $C(y) - C(1)$. Let us define it to be

$$ F(y) = \frac{-200}{\alpha^2 Re^2}[C(y) - C(1)] $$

(22)

To compare with no-slip ($Kn=0$) results, we plot three cases, $\alpha = 0.1, 0.4,$ and 0.8 for the same Reynolds number $Re = 1$ of our results: $Kn=0.1$ with those $Kn=0$ into Fig. 1. We remind the readers that the Reynolds number here is based on the wave speed. This figure confirms our approaches since we can recover no-slip results by checking curves of $Kn=0$ and finding them being almost completely matched in-between. The physical trend herein is also the same as those reported in Refs. [12-13] for the slip-flow effects. The slip produces decoupling with the inertia of the wavy interface.

Now, let us define a critical reflux condition as one for which the mean velocity $U(Y)$ equals to zero at the center-line $Y = 0$ (cf. Fig. 1). With equations (12,20-21), we have

$$ \Pi_{0,cr} = Re\left(\frac{\partial p}{\partial x}\right)_2 = \frac{[\alpha^2 Re^2 F(0)/200 + Kn C’(1) - R_0]}{-(1 + 2Kn)} $$

(23)

which means the critical reflux condition is reached when $\Pi_0$ has above value. Pumping against a positive (unit) body forcing greater than the critical value would result in a backward transport (reflux) in the central region of the stream. This critical value depends on $\alpha$, $Re$, and $Kn$. There will be no reflux if the (unit) body forcing or pressure gradient is smaller than this $\Pi_0$. Thus, for some $\Pi_0$ values less than $\Pi_{0,cr}$, the matter (flow) will keep moving or evolving forward. On the contrary, parts of the matter (flow) will move or evolve backward if $\Pi_0 > \Pi_{0,cr}$. This result could be similar to that in [16] using different approach or qualitatively related to that of [5]: even for very slow growth of $\Lambda$, the gravitationally bound systems become unbound while the nongravitationally bound systems remain bound for certain parameters defined in [5] (e.g., $\eta$).

We present some of the values of $\Pi_0(\alpha, Re; Kn = 0, 0.1)$ corresponding to frozen or zero-volume-flow-rate states ($\int_{-1}^1 U(Y) dY = 0$) in Table 1 where the wave number ($\alpha$) has the
range between 0.20 and 0.80; the Reynolds number \( (Re) = 0.1, 1, 10, 100 \). We observe that as Kn increases from zero to 0.1, the critical \( \Pi_0 \) or time-averaged (unit) body forcing decreases significantly. For the same Kn, once Re is larger than 10, critical reflux values \( \Pi_0 \) drop rapidly and the wave-modulation effect (due to \( \alpha \)) appears. The latter observation might be interpreted as the strong coupling between the vacuum-matter interface and the inertia of the streaming matter-flow. The illustration of the velocity fields for those zero-flux (zero-volume-flow-rate) or freezed states are shown in Figure 2. There are three wave numbers: \( \alpha = 0.2, 0.5, 0.8 \). The Reynolds number is 10. Both no-slip and slip (Kn=0.1) cases are presented. The arrows for slip cases are schematic and represent the direction of positive and negative velocity fields.

Some remarks could be made about these states: the matter or universe being freezed in the time-averaged sense for specific dissipations (in terms of Reynolds number which is the ratio of wave-inertia and viscous effects) and wave numbers (due to the wavy vacuum-interface or vacuum fluctuations) for either no-slip and slip cases. This particular result might also be related to a changing cosmological term (growing or decaying slowly) or the critical density mentioned in Refs. [1-2]. If we treat the (unit) body forcing as the pressure gradient, then for the same transport direction (say, positive x-direction), the negative pressure (either downstream or upstream) will, at least, occur once the time-averaged flow (the maximum speed of the matter (gas) appears at the center-line) is moving forward! For example, for \( \Pi_0 = Re (\frac{\partial \Pi}{\partial x})_2 = -10 < 0 \) (\( Re = 1, \alpha = 0.5 \)), the velocity field (profile) is shown in Fig. 3. One possible \( p \)-pair for uniform (negative) gradient (mean value theorem): \( p_{\text{downstream}} - p_{\text{upstream}} < 0 \), with \( x_{\text{downstream}} - x_{\text{upstream}} > 0 \): \( p_{\text{downstream}} < 0, p_{\text{downstream}} < p_{\text{upstream}} < 0 \).

Meanwhile, the time-averaged transport induced by the wavy interface is proportional to the square of the amplitude ratio (although the small amplitude waves being presumed), as can be seen in Eqn. (12) or (20), which is qualitatively the same as that presented in [9] for analogous interfacial problems. In brief summary, the entrained transport (pattern, either positive or negative and there is possibility: freezing) due to the wavy vacuum-matter interface is mainly tuned by the (unit) body forcing or \( \Pi_0 \) for fixed Re. Meanwhile, \( \Pi_{0,c} \) depends strongly on the Knudsen number (Kn, a rarefaction measure) instead of Re or \( \alpha \). We hope that in the future we can investigate other issues like the role of phase transition.
and that of cyclic universes [16-18] using the present or more advanced approach.

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Fig. 1 Demonstration of Kn and $\alpha$ effects on the time-averaged velocity (fields) profiles. $\Pi_0 = \Pi_{0cr}$. The Reynolds number (the ratio of the wave-inertia and viscous dissipation effects) is 1. $U(Y) = 0$ at $Y = 0$. Kn is the Knudsen number (a rarefaction measure).
Fig. 2 Demonstration of the zero-flux states: the mean velocity field \( U(Y) \) for wave numbers \( \alpha = 0.2, 0.5, 0.8 \). The Reynolds number is 10. Kn is the rarefaction measure (the mean free path of the particles divided by the characteristic length). The arrows are schematic and illustrate the directions of positive and negative \( U(Y) \).

The integration of \( U(Y) \) w.r.t. \( Y \) for these velocity fields gives zero volume flow rate.

Fig. 3 Demonstration of the negative-\( p \) states: the mean velocity field \( U(Y) \) for the wave number \( \alpha = 0.5 \) and the Reynolds number \( Re = 1 \).
**TABLE I:** Zero-flux or freezed states values ($\Pi_0$) for a flat vacuum-matter interface.

| Kn | α  | 0.1 | 1   | 10  | 100 |
|----|----|-----|-----|-----|-----|
| 0  | 0.2| 4.5269 | 4.5269 | 4.5231 | 4.3275 |
| 0  | 0.5| 4.6586 | 4.6584 | 4.6359 | 4.2682 |
| 0  | 0.8| 4.9238 | 4.9234 | 4.8708 | 4.4488 |
| 0.1| 0.2| 2.4003 | 2.4000 | 2.3774 | 1.2217 |
| 0  | 0.5| 2.4149 | 2.4132 | 2.2731 | -0.9054 |
| 0  | 0.8| 2.4422 | 2.4379 | 2.0718 | -3.4151 |
\( \eta(X,t) = a \cos \left[ \frac{2\pi(X - ct)}{\lambda} \right] \)

\( a = \varepsilon h \)

wave speed: \( c \)