Transits and Lensing by Compact Objects in the Kepler Field: Disrupted Stars Orbiting Blue Stragglers

R. Di Stefano

Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, MA 02138

ABSTRACT

Kepler’s first major discoveries are two hot ($T > 10,000$ K) small-radius objects orbiting stars in its field. A viable hypothesis is that these are the cores of stars that have each been eroded or disrupted by a companion star. The companion, which is the star monitored today, is likely to have gained mass from its now-defunct partner, and can be considered to be a blue straggler. KOI-81 is almost certainly the product of stable mass transfer; KOI-74 may be as well, or it may be the first clear example of a blue straggler created through three-body interactions. We show that mass transfer binaries are common enough that Kepler should discover $\sim 1000$ white dwarfs orbiting main sequence stars. Most, like KOI-74 and KOI-81, will be discovered through transits, but many will be discovered through a combination of gravitational lensing and transits, while lensing will dominate for a subset. In fact, some events caused by white dwarfs will have the appearance of “anti-transits”—i.e., short-lived enhancements in the amount of light received from the monitored star. Lensing and other mass measurements methods provide a way to distinguish white dwarf binaries from planetary systems. This is important for the success of Kepler’s primary mission, in light of the fact that white dwarf radii are similar to the radii of terrestrial planets, and that some white dwarfs will have orbital periods that place them in the habitable zones of their stellar companions. By identifying transiting and/or lensing white dwarfs, Kepler will conduct pioneering studies of white dwarfs and of the end states of mass transfer. It may also identify orbiting neutron stars or black holes. The calculations inspired by the discovery of KOI-74 and KOI-81 have implications for ground-based wide-field surveys as well as for future space-based surveys.

1. Introduction

The Kepler mission was launched on 6 March 2009. Its most startling and potentially important results so far are the discoveries of small, luminous, hot objects orbiting two of
the stars monitored by Kepler (Rowe et al. 2010, hereafter R2010). Each of two “Kepler Objects of Interest” (KOIs) has exhibited repeating transits, and intervening eclipses that are deeper than the transits. In KOI-81, it is estimated that the luminosity and temperature of the compact companion, KOI-81b, are $0.4 L_\odot$ and $\sim 13,000K$, respectively. For KOI-74b the estimates are $L = 0.03 L_\odot$ and $T \sim 12,000K$. These small objects have temperatures ten times higher than would be consistent with heating by the star alone. We explore the possibility that the hot compact objects are the cores of stars whose evolutions were interrupted by binary interactions, focusing on the implications for the Kepler mission and for fundamental science.

In §2 we consider the general evolutionary sequences that may have produced KOI-81 and KOI-74. We turn in §3 to the question of the likelihood of detecting the products of mass transfer in the Kepler data set. We show that many more systems that have experienced mass transfer are likely to be discovered by Kepler, making the mission a unique tool for the study of interacting binaries. Because the eclipse depths and orbital separations of some white-dwarf/blue-straggler binaries are similar to what is expected of terrestrial planets orbiting in the habitable zones of their host stars, it is important to distinguish between the two possibilities. Mass measurements are crucial. An interesting consequence of the higher mass of white dwarfs relative to planets is that white dwarfs can produce detectable lensing of the stars they orbit. The magnification associated with lensing can balance the dimming associated with a normal transit, or it can produce a net magnification (Sahu & Gilliland 2003; Farmer & Agol 2003; Agol 2003). We incorporate the effects of lensing in §4 and summarize our results in §5.

2. Erosion and Disruption

2.1. Evolutionary sequences

When a white dwarf orbits a main-sequence star, the progenitor of the white dwarf must have initially been the most massive star in the binary. Let $M_1(0)$ denote the initial mass of the primary. If the mass of the white dwarf, $M_c$, is smaller than that of the remnant normally produced by a star of this mass, we can infer that the evolution of its progenitor was truncated through interaction with the secondary. The interaction may have been stable, producing a gradual erosion of the primary through mass transfer. Or it may have been unstable, producing an almost explosive disruption of the primary. In either case, the mass $M$ of the main-sequence star we observe today may be larger than its initial mass, $M_2(0)$. Such a star is called a blue straggler, in analogy to stars in clusters that appear to be more massive than the cluster turn-off.
In this section we consider the case in which \( M_c \) is smaller than roughly \( 0.25 \, M_\odot \), because this applies to KOI-81 and KOI-74. (We include more massive cores in the calculations described in sections 3 and 4.) The primary is not a fully evolved giant at the time it fills its Roche lobe. Its mass will be close in value to \( M_1(0) \), and its radius can be expressed as follows.

\[
R_d = 0.85 \, M_*^{0.85} + \frac{3700 \, M_c^4}{1 + M_c^3 + 1.75 \, M_c^4}
\]

For an isolated star, the value of \( M_* \) is simply the star’s initial mass. In the next paragraph we discuss its likely value in systems that undergo two-phase mass transfer. At the time when the primary first fills its Roche lobe, \( q = M_1/M_2 > 1 \). If \( q \) is larger than a critical value \( \eta \), a common envelope will form and the core of the primary will spiral closer to the main-sequence companion. The final separation, \( a_f \), can be expressed in terms of its initial separation, \( a_i \), the value of \( M_c \), the stellar masses \( M_1 \) and \( M_2 \) at the time of Roche-lobe filling, and an efficiency parameter \( \alpha \).

\[
a_f = a_i \left( \frac{M_c}{M_1} \right) \left[ 1 + \left( \frac{2}{\alpha \, f(q)} \right) \left( \frac{M_1 - M_c}{M_2} \right) \right]^{-1}
\]

\( f(q) \) represents the ratio between the radius of the donor and the orbital separation (Eggleton 1983). While the companion’s mass may not change significantly during the common envelope phase, it may have increased during the interval prior to it. This is because mass transfer may have begun in the form of a gravitationally focused wind as the primary expanded to fill its Roche lobe.

There is no single value of the critical mass ratio \( \eta \). This is because mass transfer can be stabilized by a combination of factors that each assume values appropriate to a specific binary. These factors include the magnitude of winds ejected from the system, the specific angular momentum carried by winds, the role of radiation, and the donor’s adiabatic index. When a common envelope is avoided, stars with modest cores will donate mass during two phases. During phase 1, mass transfer occurs at a rate determined by the thermal time scale of the donor, as it attempts to adjust to mass loss. Once the masses have equalized, the donor has a chance to reestablish thermal equilibrium. The donor star may continue to lose mass, but as it comes into equilibrium with a smaller mass than its initial mass, and with a core that is still modest (less than roughly \( 0.25 \, M_\odot \)), it begins to shrink into its Roche lobe.

\(^1\)See, e.g., Webbink (2008) for details. Here we use \( \alpha \) to parameterize the both the binding energy of the primary’s envelope and the efficiency of ejecting the envelope from the system. High values of \( \alpha \) correspond to more efficient ejection and larger final orbits. An alternative approach which uses angular momentum in place of energy considerations can also be applied (Nelemans & Tout 2005). For the purposes of this paper we require only a formulation that parameterizes the resizing of the semimajor axis.
This leads to a quiescent interval. $M_*$ is the mass of the donor during this time of quiescence. The mass of the accretor is approximately equal to $M_*$ as well, although the accretor may be slightly more massive than the donor at this point. Thus, during phase 1, the donor has lost an amount of mass $M_1(0) - M_*$, and the accretor has gained $M_* - M_2(2)$. Generally, $M_* - M_2(2) < M_1(0) - M_*$, since the thermal time scales of the two stars are different.

The continued growth of the core of the first star, and the accompanying stellar expansion, perhaps combined with magnetic braking, eventually cause the donor to fill its Roche lobe again. Mass transfer during the second phase of mass transfer is stable and ends when the donor’s envelope is depleted. Depending on the time elapsed since the start of phase 1, the donor’s core will generally have grown by an amount $\delta$.

### 2.2. Gradual Erosion: KOI-81 and possibly KOI-74

The first question to answer for KOI-81 and KOI-74 is whether each could be an end state of stable mass transfer. If the answer is “yes”, the donor star would have been filling its Roche lobe until the time mass transfer ceased, and the orbital period is:

$$P_{\text{orb}} = \frac{0.372 \text{ days}}{R_d \left( \frac{M_*, M_c}{R_\odot} \right)^{\frac{3}{2}} \left( \frac{M_\odot}{M_c} \right)^{\frac{3}{2}}}.$$ (3)

Given a measured value of the orbital period, Equation (1) and Equation (3) combine to produce a set of possible values for $M_*$, and $M_c$. The value of $M_c$ should correspond to the present-day mass of the core observed today, and it can be measured through radial velocity or other methods (R2010, van Kerkwijk et al. 2010 [vK2010]). The value of $M_*$ represents the mass of the primary, generally as it was during the quiescent interval between phase 1 and phase 2.

The black points in the top two panels of Figure 1 show the results for KOI-81 and KOI-74. The mass of KOI-81b is constrained to lie in a very narrow range: 0.215 ± 0.031 $M_\odot$. (Note however that the systematic uncertainty associated with using the functional forms in Equation (1) is larger than this formal range indicates.) The mass $M_*$ ranges from roughly 1.6 $M_\odot$ and 2.4 $M_\odot$. R2019 interpreted light curve features in terms of tidal effects and found the mass of KOI-81b to be 0.212 ± 0.031 $M_\odot$. This is consistent with our mass transfer model. The R2010 value is shown as a cyan triangle centered on a cyan line which delineates the uncertainty limits. vK2010 used Doppler boosting, and measured the mass to be 0.3 $M_\odot$. vK2010 also used the model of steady mass transfer to predict the mass of KOI-81b, and found a value of 0.25 $M_\odot$. Their model estimate was marginally higher than ours because they used an expression for the orbital period that did not have any dependence on the
donor mass (Rappaport et al. 1995). Such a form is expected to work best for core masses higher than the estimated mass of KOI-81b. Note, however, that both v2010’s expression and Equation (3) are approximate. If Kepler identifies many similar systems and mass measurements are possible for a significant subset, we will learn how to best model the radii of stars with low-mass cores.

There is a benefit in utilizing the significant contribution of the donor mass to the radius by invoking Equation (1). Specifically, the measured value of the orbital period constrains a combination of both $M_*$ and $M_c$. This provides input for binary evolution calculations which can identify all possible initial states for the binary we observe today. We start by computing the amount of mass lost by the donor during phase 2: $\Delta = M_* - M_c$; and the fraction of this mass accreted by the blue straggler: $\beta = (M - M_*)/\Delta$, where $M$ is the present-day mass of the monitored star. We can then derive the initial state by evolving backwards in time. We find the range of possible evolutionary paths by sampling a range of values for (1) $\eta$ and for (2) $\beta$ during phase 1. Typical results are shown in the bottom panels of Figure 1. The initial mass of the primary can be as high as $\sim 3 M_\odot$, or as low as $\sim 1.7 M_\odot$. These differences have detectable consequences, in that more mass is ejected from the binary when $M_1(0)$ is high. Observations can therefore test the predictions of each evolutionary channel, for example the white dwarf’s age and the amount of mass ejected from the binary as a function of time.

The results of KOI-74 are shown in the panels on the right hand side of Figure 1. Using our model, we find that the mass of KOI-74b lies in the range: $0.142 - 0.153 M_\odot$. This is marginally consistent with the results of R2010, based on tidal effects (0.111$^{+0.034}_{-0.038}$). On the other hand, our estimate is smaller than vK2010’s Doppler-boost measurements (0.22$^{+0.03}_{-0.03} M_\odot$), and also lower than predicted by their period/core-mass relationship (0.20$^{+0.03}_{-0.03} M_\odot$). (Note that when the core mass is smaller, the main-sequence contribution to the donor’s radius is more significant.) The same general features of the range of initial stars that we discussed for KOI-81 also apply to KOI-74, but the lower mass of today’s monitored star is consistent with lower initial masses for the components of the binary.

### 2.3. Disruption: KOI-74?

If the masses of KOI-74b and KOI-81b are in the range $0.1 - 0.3 M_\odot$, then the work we sketched above shows that there are binary evolution models in which their progenitors

\footnote{$^2\beta$ during phase 1 is generally smaller than during phase 2 because there is a mismatch between the thermal time scales of the primary and secondary.}
Fig. 1.— Models for the evolution of KOI-74 and KOI-81 that invoke stable mass transfer. **Top panels:** Orbital period versus the mass of the white dwarf. Black dots show the full range of predictions in this paper. Triangles with uncertainty limits show the mass measurements of R2010 (cyan) and vK2010 (red). Green points and uncertainty limits show the mass-transfer model predictions of vK2010. Note that, in addition to the lower limits shown, derived for a tidal-perturbation model, R2010 provide lower limits in the brown dwarf range; these would not be consistent with the formation of the systems through stable mass transfer. **Bottom panels:** Initial mass $M_2(0)$ of the secondary versus the initial mass $M_1(0)$ of the primary for the steady mass transfer model used in this paper (§2). These are the initial states consistent with the present-day binarity parameters of KOI-74 and KOI-81. If the correct models are those with higher mass, the systems we observe today are younger.
filled their Roche lobes until the time their envelopes were totally eroded. This strongly supports the hypothesis that KOI-71 and KOI-81 are end-states of stable mass-transfer. We note here, however, that *Kepler* can also discover binaries in which the mass of the core is too large for the donor to have been filling its Roche lobe while in its current orbit. This would be a signature that the binary was a common-envelope survivor. We could then use the current values of the orbital separation and masses to constrain the parameters of the common envelope evolution [Equation (1)].

For KOI-74b and KOI-81b, however, there may be reason to suggest that the actual masses are *smaller* than the values associated with stable mass transfer. R2010 place lower limits on the masses that extend into the brown dwarf regime. In addition, for KOI-74, there is only marginal overlap between R2010 and the stable mass transfer model presented in the last section. It is therefore worth seriously considering that the mass of one or both of these cores may be in the brown dwarf regime. Here we use KOI-74 as an illustrative example. We will show that a value of the core mass smaller than expected from stable mass transfer may be a signal that there was a common envelope, but that the orbit was eccentric at the time the primary filled its Roche lobe. This is expected in some triple systems. It therefore seems inevitable that, whatever the history of KOI-74 and KOI-81, binaries in which the core mass is too low will eventually be found, and the scenario we present below will apply.

First we note that, even if the core mass of KOI-74 is small, formal solutions to the stable-mass transfer equations exist, predicting $0.005 M_\odot < M_c < 0.018 M_\odot$ and $1.2 M_\odot < M_1 < 2 M_\odot$ at the time of Roche-lobe filling. These solutions are not viable, however, because the equilibrium radius of a low-core-mass primary would decline as mass was donated in a steady, stable manner. The second term in Equation (1) would contribute little, and, although the first term might not apply exactly throughout the evolution, the trend of smaller radius with decreasing mass would be followed during phase 2. The orbital period would have to be much smaller than 5.2 days.

If, therefore, KOI-74b is a low-mass stellar core, the binary likely passed through a common envelope phase. Using Equation (2) we find that $a_i > M_1/M_c$. Furthermore, dynamical instability requires $M_1$ be larger than roughly $(1.3 - 1.5) \times (2.2 M_\odot)$. This predicts an initial separation so large that the primary could not have filled its Roche lobe.

This may indicate that the initial orbit was highly eccentric and that a dynamical instability was triggered during periastron. During the common envelope, the eccentricity was eliminated while the semimajor axis became smaller. Such a high eccentricity is not generally expected in a binary in which one companion is poised to fill its Roche lobe, because tidal effects will tend to circularize the orbit. If, however, KOI-74 is a triple, the Lidov-Kozai (Lidov 1961; Kozai 1962) mechanism could have produced the extreme eccentricity that led
to the disruption of the progenitor of today’s low-mass core. Perets & Fabrycky (2009) have
considered the generation of blue stragglers via this mechanism. They focused on mergers,
but the same general idea could apply to systems like KOI-74. We therefore propose that
mass transfer occurred prior to the approach that triggered the common envelope. During
that earlier epoch, the progenitor of the hot core made close approaches to the star we
monitor today. In a process analogous to what happens in high-mass x-ray binaries, mass
was transferred during these close approaches. Eventually, under the influence of a third
star, the periastron distance became small enough to trigger a dynamical instability.

We used Equation 1 to compute the initial value, $a_i$, of the binary’s semimajor axis
at the time the primary filled its Roche lobe. The secondary mass was considered to be
the mass observed today, and we set $M_c$ to 0.03 $M_\odot$, $a_f = 16 R_\odot$, and assumed efficient
envelope ejection ($\alpha = 100$), so as to compute the minimum value of $a_i$. We found that
the eccentricity was required to be large, generally $> 0.997$. Thus, this scenario predicts
the presence of a third body in the system. Its separation from the 2.2 $M_\odot$ star Kepler is
monitoring today would likely be greater than a few hundred AU (See Figure 2), so that
this star third is potentially detectable today. As shown in Fabrycky & Tremaine (2007) and
references therein, triple stars are common, and a large fraction of close binaries are known
to have another companion in a wide orbit. (See also Eggleton & Kisseleva-Eggleton 2001.)
This triple-star scenario therefore seems plausible. The one feature that seems unlikely is
the extreme value of the eccentricity required.

Finally, we note another, less likely possibility. Eggleton (2002) discusses systems in
which the envelope was so weakly bound to the primary that dynamical instability did not
produce substantial spiral-in. This situation does not seem likely to apply to KOI-74.

The fact that different values of the mass of today’s small-radius core are associated with
different evolutionary scenarios points to the importance of reliable mass measurements for
both KOI-74 and KOI-81. While it is intriguing that the well-sampled Kepler light curves
have allowed both Doppler boosting and tidal methods to be employed, a radial velocity
measurement is required. Preliminary indications are that the rotational velocities of both
KOI-74a and KOI-81a are high, perhaps in the range of $\sim 300$ km s$^{-1}$ (Latham 2010). While
this may complicate radial velocity measurements, it could also be a clue that each has
accreted matter, thereby supporting the mass transfer model. High rotational velocities are,
in fact, often associated with blue stragglers, presumable generated through mass transfer
(see, e.g., De Marco et al. 2005 and Mathieu & Geller 2009). While, therefore, the high
rates of rotation may make mass measurements more difficult, it is also possible that they
are providing a signal that mass transfer did occur.
Fig. 2.— Common envelope evolution for KOI-74. Lower (black) curve shows $a_i$ versus $M_1(0)$ for efficient envelope ejection. If the mass of the hot core detected by R2010 is in the brown-dwarf range, then the orbit may have been eccentric, and the massive donor star may have experienced a dynamical-time-scale instability at periastro. The high eccentricity would likely have been induced by interactions with a third star. Plotted in green are the possible orbital separations of the third star from the star monitored today by *Kepler*.

Fig. 3.— $M_c$ versus log of the current orbital separation. Binaries that have experienced stable mass transfer (top panel); common envelope evolution (bottom panel).
3. Probability

The advent of a new observing capability has almost always led to unanticipated discoveries. The two binaries, KOI-74 and KOI-81, may be examples. Nevertheless, we may question whether the evolutionary models presented above are expected to occur commonly enough that *Kepler* should have been able to discover these interesting binary systems by monitoring only $\sim 150,000$ stars. We therefore conducted a first-principles study of binaries in a stellar population to predict the numbers of transiting systems we expect to be comprised of a main sequence star orbited by a white dwarf that has emerged from an episode of mass transfer. Although these calculations were suggested by the discoveries of KOI-74 and KOI-81, they do not rely on the interpretation of these systems. We note that the evolution of interacting binaries consisting of a main sequence accretor and a subgiant or giant donor is complex and involves a wide range of physical processes. There are uncertainties, including the results of common envelope evolution, the fraction of incoming matter that can be retained by the accretor, and the angular momentum evolution of the system. Nevertheless, by parameterizing the effects of these processes we are able to derive a robust conclusion.

The *Kepler* team selected targets from roughly half a million stars in its field brighter than 16th magnitude. More than 90% of the targets were selected based on signal-to-noise considerations that suggested the possibility of detecting terrestrial-size planets. While a small fraction of the targets are selected to pursue a range of other science opportunities, including, e.g., eclipsing binaries and high-proper-motion stars, the majority of the targets ($\sim 90,000$) are G-type stars on or near the main sequence (Batalha et al. 2010). The presence of KOI-74 and KOI-81 illustrates the presence of more massive main-sequence stars as well.

In the calculations described below, we compute the fraction of monitored stars transited by white dwarfs per year. The number of systems in which transits can be detected is the product of this fraction and the number of monitored stars with high enough signal-to-noise that the transits of white dwarfs can be detected. Massive white dwarfs have radii comparable to the radius of the Earth. Transits of many of the *Kepler* target stars by objects of this size should be detectable. The white dwarf radius increases with decreasing mass (see, e.g., Parsons et al. 2010), so the less massive white dwarfs which are expected to be common among mass-transfer products should also produce detectable transits when they pass in front of 90% of the *Kepler* targets. White dwarfs that have not yet had a chance to cool are even larger. We therefore define a quantity $N_{\text{det}}^{\text{mon}}$, the number of monitored stars against which white dwarf transits can be detected\(^3\). The value of $N_{\text{det}}^{\text{mon}}$ is roughly

\(^3\)Although $N_{\text{det}}^{\text{mon}}$ is a function of the white dwarf mass and age, this is a relatively small effect. The
0.9 \times 150,000 = 135,000. As we will discuss in §4, the decline in received light produced by a transit can be balanced by the gravitational lensing produced by the white dwarf. While this may make it difficult or impossible to detect transits in some systems, lensing produces strong, distinctive signatures, including ati-transits, in others.

3.1. Calculations

We can derive a rough estimate of the fraction of Kepler targets that might be orbited by transiting white dwarfs by generating a population of stars, including both singles and binaries, and then considering the evolution of each system. We identify those that will become main sequence stars that are both (1) in a range of masses likely to be monitored by Kepler, and (2) orbited by white dwarfs. We then compute the probability that the white dwarf will transit during the time of Kepler monitoring. For short orbital periods, the probability is $R_*/a$, where $R_*$ is the radius of the monitored star and $a$ is the final orbital separation. For orbital periods longer than $\tau_{\text{monitor}}$, the duration of monitoring, we multiply by a factor of $\tau_{\text{monitor}}/P_{\text{orb}}$.

We assume that 40% of all stellar systems are singles, and that the remainder are of higher multiplicity. We use a Miller-Scalo IMF (Miller & Scalo 1979) to select the masses of all single stars, and to select the primary mass for all multiples. This has the effect of favoring low-mass stars, even though we generate the full mass spectrum. The only multiple systems we explicitly consider are binaries. We select the secondary mass as follows: $M_2 = M_1 \times u^{0.8}$, where $u$ is a random number chosen from a uniform distribution in the range 0–1. We then generate orbital separations, selecting uniformly in log space from separations as small as three times the radius of the primary, and as large as $10^4$ AU.

Primary stars will fill their Roche lobes at some point in their evolution if the orbital separation lies within roughly an AU. This range of separations encompasses a large fraction of all binaries. Roche-lobe filling will lead either to a common envelope phase or to stable mass transfer. While a large fraction of common envelopes may end in mergers, those binaries that survive will have small orbital separations and therefore high probability of yielding transits. When mass transfer is stable, the final separation may be somewhat larger than the initial separation, but most orbits are smaller than a few AU (see Figure 3).

The paragraph above describes the key reason why transiting white dwarfs must be common: they are produced by a large fraction of all primordial binaries. We used a simple value to which we normalize can therefore be viewed as a sort of average.
model to design a code that would allow us to quantify this and to test a wide range of input assumptions. We conducted 13 simulations. Each was characterized by (a) the value of $\alpha$, (b) the value of $\tau_{\text{monitor}}$, (c) the range of masses of stars monitored by Kepler, (d) the rule used to compute the final core mass for systems experiencing stable mass transfer, (e) the ages of the stars monitored by Kepler. In this way, we were able to take into consideration the uncertainties in the input physics and to identify those results that are robust. The key question is: what fraction of all monitored systems are binaries in which a white dwarf is expected to transit a main-sequence star?

To compute the number of monitored systems we counted members of the following sets of stars, considering only those in the appropriate range of masses: (1) all isolated stars; (2) stars that are the results of mergers; (3) stars with dwarf stellar companions having luminosity less than 0.001 their own luminosity$^4$; (4) stars in binaries so wide that the individual stars could be resolvable$^5$; and (5) stars orbited by white dwarfs. This last category includes all white-dwarf/main-sequence pairs, regardless of the formation history of the white dwarf.

Figure 3 shows the result of one such simulation. In this particular simulation we generated 4 million stellar systems, and used $\alpha = 10$, and $\tau_{\text{monitor}} = 1$ year. We assumed that Kepler is monitoring main-sequence stars with masses in the range $0.9 - 2.9 \, M_\odot$. We assumed that, for systems undergoing stable mass transfer, the final value of the core mass would lie within 10\% of its initial value ($\delta < 0.1 \, M_\odot$). There were approximately 370,000 stars with masses within the selected range that would appear to be single stars. We then identified all binaries in which the primary star fills its Roche lobe. Those with $q > 1.5$ were assumed to undergo common envelope evolution. For smaller values of $q$, we assumed stable mass transfer and selected the final value of the primary’s core mass. The numbers of binaries with the present-day primary having a mass in the monitored range and (1) surviving the common envelope, were $\sim 16,000$; (2) having ended stable mass transfer, were $\sim 35,000$. Most of these will not produce transits. Taking the transit probability into account, we find that $3.6 \times 10^{-3}$ of the monitored stars should exhibit at least one transit per year, in which the transiting object is a white dwarf that survived a common envelope. Many of these will exhibit multiple transits per year. The comparable fraction of systems with with transiting

---

$^4$Depending on the orbital inclination these might provide transits that appear to be planetary. They could also exhibit deep eclipses like those seen in KOI-74 and KOI-81.

$^5$Because we do not attempt to generate the 3-D spatial distribution of stars, it is not clear which binaries will actually be resolvable. We assumed that all binaries with orbital separation greater than $10^{1.5} \, \text{AU}$ were potentially resolvable. By adding to the number of apparently isolated stars, we are lowering the fraction of systems with white dwarfs orbiting main sequence stars. This produces a more conservative estimate.
white dwarfs that emerged from stable mass transfer is $5.0 \times 10^{-3}$.

Table 1 shows a representative sample of the results. In this section we focus on the totals listed in column 6. These totals are normalized to the case in which transits can be detected in 135,000 monitored stars. The first two rows correspond to a single simulation, labeled “1”. Simulation 1 is the one used to produce Figure 3. ($\alpha = 10, \delta < 0.1$, the only criterion imposed on the Kepler target stars is that their masses are between $0.9 M_\odot$ and $2.9 M_\odot$.) If this case applies, more than 1100 transiting systems would be discovered by Kepler. In the next 4 rows, also labeled “1”, we alter only the value of $\alpha$, the common envelope ejection efficiency. We find that, as the ejection efficiency declines, significantly more mergers occur. For example when $\alpha$ declines from 10 to unity, the numbers of mergers increase by 50% and the common-envelope survivors are found in closer orbits than those shown in Figure 3. The transit probability declines by only 15%, however, because transits are more likely for closer orbits. The value of $\alpha$ must decline to $\sim 0.1$ in order for common envelope survivors to produce fewer than 100 transits per year. Thus, the result that a large number of white dwarfs should produce transits is robust with respect to changes in the common-envelope efficiency factor. In any realistic population, the efficiency of envelope ejection will be different for different systems. To determine the numbers of transits expected from common envelope survivors, we should therefore average over a range of efficiencies.

We also tested the effect of changing the rule used to compute the final core mass for the donor stars that contribute mass during stable mass transfer. We used three prescriptions: (1) the maximum value of $\delta$ is 10%; (2) the maximum value of $\delta$ is 20%; (3) the value of $\delta$ can be as large as half the difference between the white dwarf mass expected for a star with initial mass $M_1(0)$ and the core mass $M_c$ at the start of mass transfer. Changing these rules doesn’t change the numbers of stable mass-transfer binaries, but it does affect the transit probability; the larger the core mass at the time of depletion, the wider the orbit, and the smaller the transit probability. With prescription 2, the numbers of transits by white dwarfs that are the end states of stable mass transfer decreased by $\sim 13\%$. With prescription 3, the numbers of transits by the end states of stable mass transfer decreased by roughly 70%.

No single rule for selecting a final core mass can apply to all binaries. Once mass transfer begins, the donor’s envelope is depleted through mass loss, and the envelope is also contributing mass to the growing stellar core. The final core mass depends on the time scales for these two competing processes. These time scales depend on the state of the system and on the evolution of the orbital angular momentum, for example on the amount of angular momentum carried off in winds. For donors that fill their Roche lobes when their core masses are small ($\sim 0.1 - 0.2 M_\odot$), mass transfer may deplete the envelope at a faster rate than the nuclear evolution of the core. Values of $\delta$ may be modest. For donors that fill their
Roche lobes as more evolved stars, the evolution of the core is proceeding more rapidly. The rate of mass loss will also typically be high, however, and can be increased if winds carry significant angular momentum. To incorporate these effects, we could carry out a detailed evolution for each system. The uncertainty would still be large, because the physical effects that determine the binary evolution are not yet well-enough understood. To determine the numbers of transits expected by white dwarfs that are the remnants of Roche-lobe-filling donor stars, we have therefore chosen to average over the three prescriptions described above for the final mass of the core. This averaging process corresponds to considering a range of mass-transfer rates as well as evolutionary states for the donor.

The age of the monitored systems can also influence the results. In particular, the primary needs to have had enough time to evolve and produce at least a modest core. Yet the star that was originally less massive should still be on the main sequence, even after gaining some mass from its companion. To take this into account, in simulations labeled “2”, we considered a constant rate of star formation over an interval of 12 billion years. We randomly generated the time at which each stellar system was formed, and considered further only single stars which would not have had time to evolve during the interval from their formation until the present day. We considered only binaries in which the primary star had time to evolve, but the secondary is still on the main sequence. In this case the fraction of transiting systems derived from common envelope evolution (stable mass transfer evolution) was $4.3 \times 10^{-3}$ ($2.0 \times 10^{-3}$).

We conducted additional simulations, not shown in the table, because the results did not change significantly. For example, changing $\tau_{\text{monitor}}$ from 1 year to 3.5 years changes the results by only a few percent, since most of the transits occurred in binaries with orbital periods smaller than a year. (Multiple transits of the same monitored star will, however, increase the reliability of the detection.) As another example, monitoring stars in a more limited mass range ($0.9 - 1.9 M_\odot$, or $1.9 - 2.9 M_\odot$) changes the results by only $\sim 10\%$.

### 3.2. Results

It is useful to consider the contribution of common-envelope survivors and stable-mass-transfer binaries separately.

$$N_{\text{transit}} = N_{\text{transit}}^{ce} + N_{\text{transit}}^{mt}$$

To estimate the value of each term, we average over the results for the simulations described above. First, we note that simulation 1 is more appropriate for an old population in which the white dwarfs would have long-ago formed from the primary stars in binaries
with secondaries as massive as the \textit{Kepler} targets. Simulation 2 applies to intermediate-age populations. We average the results of these two simulations. In each case we average over the common envelope efficiency factors and also over the prescription for the value of \( \delta \).

\[
N^\text{coe}_{\text{transit}} = \frac{N^\text{detect}_{\text{monitored}}}{135,000} [395 \pm 32] \tag{5}
\]

and

\[
N^\text{mt}_{\text{transit}} = \frac{N^\text{detect}_{\text{monitored}}}{135,000} [338 \pm 143] \tag{6}
\]

Including stellar systems with higher multiplicity would increase the numbers of systems with white dwarfs in close orbits.

### 3.3. Science Returns

The calculations described above suggest that \textit{Kepler} may identify transits in roughly a thousand mass-transfer end states. Each case provides a unique way to measure the white dwarf radius, luminosity and temperature, and even to explore its atmosphere. This is all in addition to the standard “bag of tricks” that can be applied to any white dwarf in a binary. The ability to conduct a large number of such studies in a uniform way will determine the properties of the white dwarfs in terms of the mass lost history of their progenitors.

With regard to mass loss histories, \textit{Kepler} studies will establish the relative frequency of common envelope evolution and provide useful data points for determining the best way to compute common envelope evolution in terms of the properties of the binary at the time the dynamical instability was triggered. The range of systems producing stable mass transfer will also be better understood, including the all-important fraction, \( \beta \), of material that can be retained by a main-sequence accretor. Most of the \textit{Kepler} stars are field stars. Studying the \textit{Kepler} blue stragglers will provide empirical insight into the contribution of ordinary binary evolution to the creation of blue stragglers. This will allow the contribution of stellar interactions in clusters to be better understood, since both primordial binaries and binaries formed or altered through interactions can produce blue stragglers.

Finally, the white-dwarf/main-sequence binaries studied by \textit{Kepler} will experience a second phase of interaction when the main-sequence star of today evolves and begins to transfer matter to the white dwarf. These systems will be novae, some may become double-degnerates that eventually merge, and some may even experience Type Ia supernovae, or accretion-induced collapse, through either the single-degenerate or double-degenerate channel. \textit{Kepler} will identify a set of these systems that can be studied in a unique way.
4. Mass Measurements and Lensing: White Dwarf or Terrestrial Planet?

The primary goal of the *Kepler* mission is to search for terrestrial planets orbiting sun-like stars, especially in the zone of habitability. We do not yet know how common such planets are. The results described in §3 establish, however, that orbiting white dwarfs are well represented in the *Kepler* data. White dwarfs have radii similar to planetary radii and, as Figure 3 shows, many are likely to orbit in their host star’s zone of habitability. It is therefore important to be able to distinguish between planets and white dwarfs.

The discoveries of KO-74b and KOI-81b illustrate that it is possible to identify transiting objects that are candidate white dwarfs. Yet, even though both systems are likely to be products of mass transfer, the natures of the hot compact objects and their evolutionary histories are not yet definitely established. Furthermore, not all white dwarfs will be hot and luminous enough to produce distinctive eclipses. While dwarfs cool with age, and “ultracool” white dwarfs with temperatures below 5000 K have been discovered (Vidrih et al. 2007; Harris et al. 2008).

The key property that is different for planets and white dwarfs, irrespective of age, is the mass. Mass measurements are therefore of central importance, ideally through radial velocity measurements. Interestingly enough, this may be difficult to do for some white-dwarf blue straggler systems, because high rates of stellar rotation can result from mass transfer and complicate the measurements. On the other hand, high rotational velocities may be a signature that supports the mass transfer scenario. As mentioned in §2, there are preliminary indications that the rotation rates of KOI-74a and KOI-81a are high (Latham 2010).

Even if high rates of rotation are rare, the large numbers of interesting systems discovered by *Kepler* may make it challenging to obtain radial velocity measurements for all of them. It is therefore useful that, because of its sensitive photometry, *Kepler* is well-suited to make optimal use of light curve deviations to measure mass. Both tidal effects and, for the first time, Doppler boosting have been used to provide mass estimates of KOI-74b and KOI-81b. If a modest number of systems can be studied through both light-curve and radial-velocity techniques, we may be able to rely more heavily on the former. In addition, when the transiting object is a stellar remnant, its high density may allow us to detect its action as a gravitational lens that deflects light from the star it orbits. Measuring the lensing effects can provide an independent estimate of the gravitational mass of the white dwarf.

The Einstein radius is

\[ R_E = 2.97 \times 10^9 \text{cm} \left[ \frac{M_{WD}}{M_\odot} \frac{a}{\text{AU}} \right]^{1/2} \]  

(7)
In the cases of interest to this investigation, the Einstein radius is comparable to the radius of the white dwarf, which is the lens. Finite-lens effects must therefore be considered; they diminish the magnification and make the signatures of lensing more subtle. This is not the only complication, since the Einstein ring is smaller than the lensed star. Finite-source-size and limb darkening also influence the value of the peak magnification and the shape of the light curve. A set of instructive examples that take these effects into account can be found in the pioneering investigation of Sahu & Gilliland (2003), which studied the detectability of binary self-lensing during transits, as observable by *Kepler*.

While the transiting mass blocks light from the star it orbits, its action as a lens increases the amount of light we receive. Sahu & Gilliland (2003) found that, for low-mass white dwarfs in close orbits, the transit signature dominates. As the mass and/or orbital separation increases, lensing begins to play a larger role and can effectively balance the diminution of light associated with the transit. The transits of some white dwarfs will therefore not be detectable, even though the cancellation is not exact. As the mass and separation continue to increase, the lensing magnification dominates. During transit, and for an interval both before and afterward, there is a highly significant increase in the received light. For systems in which the influence of lensing can be photometrically detected, model fits can provide estimates of the white dwarf’s mass and radius.

Sahu and Gilliland (2003) showed that *Kepler* would be likely to detect lensing by white dwarfs, but did not have a detailed model of white-dwarf/main-sequence binaries and could therefore not predict the rates. The population calculations of the previous section can be used as a basis to compute the expected rates. Ideally, the light curves of each transiting system formed in the population calculation would be computed. This is a large project, because a variety of other effects must also be included; in addition, the age of the white dwarf influences its radius. Here we attempt a simple estimate, with the goal of determining how frequently signals of lensing in the light curve may eliminate confusion with planets. We utilize the value of $R_E/R_{WD}$ to test the detectability of lensing effects, using guidelines derived from the work of Sahu & Gilliland (2003). Note that, at almost the same time as Sahu & Gilliland (2003), Farmer & Agol (2003) took an alternative approach to incorporating the effects of lensing, in the context of a population synthesis analysis. Although they did not classify systems by their previous evolution (common envelope versus stable mass transfer), they did provide numerical results which can be compared with our totals (see §5).

We computed the value of $R_E/R_{WD}$ for every white dwarf in our simulations, using an empirically-derived approximation to estimate the radius of a white dwarf of given mass $M_{WD}$. From Parsons et al. 2010 we use: $R_{WD} = [0.014 – 0.012 (M_{WD} – 0.5)]$. The results are shown in Figure 4 for the same simulation that produced Figure 3. Note that Figure 4
Fig. 4.— $M_c$ versus log of the current orbital separation. Dark blue points represent binaries that have experienced stable mass transfer; light green points represent binaries that have experienced a common envelope. Points in each panel have values of $R_E/R_{WD}$ in the range shown. The two red points shown in the third and fourth panel correspond to KOI-74 (on the left) and KOI-81 (on the right and slightly higher). Both KOI-74b and KOI-81b have larger radii than the mass/radius relationship for white dwarfs would indicate, perhaps indicating that they have not yet cooled. Each has values of $R_E/R_{WD}$ less than or equal to $\sim 0.1$. 
Fig. 5.— Low-mass stellar dwarfs may produce transits that mimic planetary transits. Here we plot the probability of a transit versus the logarithm of the orbital period. The bend at one year corresponds to $P_{\text{orb}} = \tau_{\text{monitor}}$. For shorter orbital periods, the probability falls as $1/a$; for longer orbital periods the ratio between $\tau_{\text{monitor}}/P_{\text{orb}}$ must also be factored in. The presence of low-mass dwarfs with luminosities in the range of $10^{-4} - 10^{-3}$ times the luminosity of the star monitored by *Kepler* may be detectable only because they produce detectable transits, and are themselves detectably eclipsed. We kept track of these systems in our simulations. Considering only those with orbital periods greater than 4 days, we found that they should produce transits at $\sim 5\%$ the rate caused by white dwarfs that emerge from common envelopes. The total contribution is likely to be higher.
Fig. 6.— Log of the probability of detecting a neutron star or black hole lensing event versus the orbital period. As with transits, these events would repeat on a periodic basis as the neutron star or black hole executes an orbit around the star monitored by *Kepler*. *Kepler* could detect a handful of events caused by compact objects more massive than white dwarfs.
provides a guide to the characteristics of binaries in which lensing will be important. It shows, for example, that common envelope systems are more likely to be affected by lensing. We note, however, that for individual systems, the effects must be computed separately. This is because the radius of the white dwarf may larger or smaller; finite source size effects can be important; and, the range of values of $R_E/R_{WD}$ encompassed in a single panel correspond to light curves with a range of properties.

The systems shown in the top panel Figure 4 all have $R_E/R_{WD} > 2$. In these cases we expect the lensing signature to be clear and for the gravitational mass to be potentially measurable. In this regime, many of the events will look like “anti-transits”. For the sake of clarity, we note that we are using the term “event” to refer to the light curve deviation associated with a single passage of the white dwarf in front of its companion star. Just as with ordinary transits, these events repeat periodically, and the ability to detect repetitions increases confidence in the physical interpretation. Figure 15 of Sahu & Gilliland (2003) shows the lensing signature for a white dwarf of $0.6 M_\odot$ located 1 AU from a solar-type star. The light curve exhibits what we have referred to here as an anti-transit, with a significance in the Kepler data greater than 100σ. The significance of some of the events generated by systems in the top panel of Figure 4 will be even higher, and some of these events may therefore be detectable even from the ground.

Figure 14 of Sahu & Gilliland (2003) shows that even with $R_E/R_{WD} = 0.8$, an anti-transit can be clearly detected. The second panel shows systems with $0.8 < R_E/R_{WD} < 2$. Some of these events will be anti-transits as well; lensing should be recognizable in many. Those that are not pure anti-transits are likely to exhibit detectable deviations from baseline that exhibit significant signatures of lensing. From the perspective of studying white dwarfs and the end states of mass transfer, events from systems in these top two panels are important, because lensing provides a unique way to measure the white dwarf mass. From the perspective of Kepler’s primary goal of identifying transits by terrestrial planets, these events are also important, because the degeneracy with transits by planet-mass lenses is clearly broken.

The third panel shows systems with $0.4 < R_E/R_{WD} < 0.8$. Some of these may also produce clear signatures of lensing. Others, particularly when the white dwarf is young and still larger than suggested by the mass/radius relation, may produce events that look like ordinary transits. In addition, a significant subset of the events generated by this group will exhibit some level of cancellation between diminution due to transit and magnification caused by lensing. That is, some systems shown in the third panel generate events (combinations of transit and lensing) may not be detectable by Kepler. This eliminates our ability to identify and study these white dwarfs, but it also eliminates the possibility that these
white dwarfs will be mistaken for transiting planets.

The two red circles shown in this panel correspond to KOI-74 (on the left) and KOI-81 (on the right and slightly higher). Although lensing has not been considered in the light-curve modeling of KOI-74 and KOI-81 carried out by R2010 and vK2010, there was no indication that it was needed. And clearly lensing did not cancel the effect of the transits, since *Kepler* did detect these events. The position of KOI-74 on the graph of $M_{\text{WD}}$ versus $\log(a/\text{AU})$ indicates that $R_E/R_{\text{WD}}$ is expected to be small, so that lensing should not be significant. In fact, the radius of KOI-74b is larger than typical of a white dwarf of this mass (R2010), indicating that it may still be cooling. We find $R_E/R_{\text{WD}}$ is approximately 0.1. The position of KOI-81 on the graph of $M_{\text{WD}}$ versus $\log(a/\text{AU})$ indicates that, if the radius of KOI-81b satisfies the mass/radius relationship for white dwarfs, lensing effects may be important. R2010 find, however, that the radius of KOI-81b is 0.115 $R_\odot$, placing it near the top of the cooling curve. The value of $R_E/R_{\text{WD}}$ is $\sim$ 0.08, indicating that the role of lensing is minimal. The systems in the fourth panel have $R_E/R_{\text{WD}} < 0.4$. They each have a high probability of generating transits, but lensing effects will be more subtle and difficult to detect. Table 1 summarizes the numerical results for these four categories across simulations.

**General Features:** We note that, although the majority of white dwarfs orbiting main-sequence stars lie in larger orbits, and are descended from progenitors that did not interact, white dwarfs in wide orbits contribute little to either transits or lensing events. We expect only roughly 2 *Kepler* lensing events to be generated by white dwarfs that are the remnants of stars that do not fill their Roche lobes. The lensing signal will be dominated by white dwarfs formed in mass transfer binaries. Of these, common envelope survivors seem likely to dominate. This is because, for any given orbital separation, white dwarfs emerging from a common envelope tend to have higher mass. We note that we also kept track of low-luminosity companions to monitored main-sequence. We compute that those with orbital periods larger than $\sim$ 4 days producing transits of *Kepler*-monitored stars may be $\sim$ 20 times less common than the white dwarfs which transit *Kepler* targets (see Figure 5). These will produce transits with no evidence of lensing.

**Magnification without Transits:** The disk of a small object must directly overlap the disk of its stellar companion to decrease the amount of light we receive from it. Detectable lensing can occur, however, even without overlap between the object and its companion. Thus, the cross section for lensing events is higher, and the lensing events caused by transiting white dwarfs will be supplemented by lensing events caused by the dwarfs making more distant approaches. The rate of pure lensing events depends on finite source size effects, so it must be computed in each case. The enhancement of a factor will be modest. For objects more compact and more massive than white dwarfs, lensing is by far the dominant effect, and
finite-lens effects are not important. (See below.)

**Neutron Stars and Black Holes:** We included more massive stars in our simulation, and kept track of those binaries in which a neutron star or black hole orbits a main sequence star within roughly 0.5 AU. The evolutionary pathways that produce such orbits are far less certain than the evolutionary channels producing close white-dwarf/main-sequence binaries. Nevertheless, we know that systems with main sequence stars with mass in the range $0.8 - 2 \, M_\odot$ orbit neutron stars and black holes. When the orbits are close enough (with semimajor axis of a few solar radii), such binaries are detected as low-mass x-ray binaries. In our simulation we considered all binaries with primary mass larger than $8.5 \, M_\odot$ (the lowest mass star producing a neutron star remnant) and secondary mass between $0.8\, M_\odot$ and $2 \, M_\odot$. We then assumed that $\sim 1/6 - 1/5$ of the initial orbital separations could be compatible with evolutionary channels that produce close neutron-star/main-sequence pairs (see, e.g., Kalogera & Webbink 1998; Kiel & Hurley 2006). Although we did not follow the detailed evolution of individual systems, our approach yielded a binary fraction of neutron stars in close orbits with main-sequence stars or their remnants of $(< 0.5 \, AU)$ of $5 \times 10^{-5}$. We found that *Kepler* could detect $1 - 2$ lensing “events” by neutron stars or black holes corresponding to approaches close enough that the compact object transits the companion star. In most cases, the neutron star or black hole would be too dim for eclipses of it to be detected.

5. **Conclusion**

5.1. **Results**

We started by considering evolutionary models for KOI-74 and KOI-81. Our results agree with those of vK2010, which suggest that each system may have evolved to its present state through a process of stable mass transfer. Our approach differs from theirs by using a different prescription for the radius of the donor star prior to envelope exhaustion. Whereas vK2010 used a formula for the radius which depends on only the core mass of the donor, our formula also incorporates the dependence on its equilibrium stellar mass $M_\ast$, prior to the last phase of mass transfer. This has two effects. First our approach tends to predict somewhat smaller values of the white dwarf mass. This will be tested through radial velocity measurements, and Equation (1) can be adjusted if needed, based on the observations. Second, it allows the *Kepler’s* period measurement to constrain both the white dwarf mass and the mass of the donor, thereby providing useful input for detailed evolutionary models.

We also take seriously the possibility that the masses of KOI-74b and KOI-81b are in the brown dwarf range, so that the formalism we and vK2010 have applied is not valid. We
find an interesting new evolutionary pathway. In this case, there is a third body that pumps up the eccentricity of an inner binary. Mass transfer from the primary to the secondary occurs at periastron, in analogy to the mass transfer process in high-mass x-ray binaries. As the orbit evolves, eventually the primary experiences a dynamical time scale instability at periastron, leading to a common envelope phase. During the common envelope, the orbit circularizes and the core of the primary spirals closer to its main-sequence companion, but avoiding a merger. The signature that such an evolution has occurred is a white dwarf in a wider orbit than expected had there been either stable mass transfer of a common envelope initiated from a circular orbit. We do not know if this scenario is needed to explain either KOI-74 or KOI-81, but if it occurs in nature, Kepler may find binaries that have evolved in this way.

Whatever the nature of KOI-74 and KOI-81, their discovery inspires us to compute the frequency of transits by white dwarfs in binaries that have experienced mass transfer. In many of these cases, the companion star has gained mass from the progenitor of the white dwarf and may be considered to be a field blue straggler. We have done a set of preliminary calculations to estimate the number of such white dwarfs that transit stars in the Kepler field. Although our model may be viewed as a “toy model” it allows us to parameterize the uncertainties involved in the complex physics needed to predict the details of the binary evolutions. We find that, under a wide range of reasonable assumptions, roughly $0.0025 - 0.0075$ of systems of the type monitored by Kepler should exhibit white dwarf transits. It would be surprising if fewer than 100 transiting white dwarfs were discovered by Kepler, while it is possible that more than 1000 will be found. Furthermore, evidence of gravitational lensing is expected in $10 - 20\%$ of the white dwarf transits; anti-transits may be observed in $1/3 - 1/2$ of the cases in which there are lensing signatures\footnote{More detailed calculations are needed to provide reliable estimates of the fraction of events exhibiting various lensing signatures. Here we assume that lensing can be deduced from model fits in events with $R_E/R_{WD} > 0.8$, and in roughly half of the events with $0.4 < R_E/R_{WD} < 0.8$, with $\sim 1/2$ of these exhibiting anti-transits.}. The ubiquity of lensing by the white dwarfs in the Kepler sample can be understood in terms of mesolensing, the high probability of lensing associated with certain astrophysical systems, even though the total mass in lenses, as measured by the lensing optical depth, is low. (See DiStefano 2008a, 2008b.)

Our results can be compared with a calculation of the number of white-dwarf transits expected, conducted before the Kepler targets were selected (Farmer & Agol 2003). This previous work had to anticipate the characteristics of the stars that would be monitored and the detectability of transits and lensing. Farmer & Agol (2003) concluded that at least
50 transiting white dwarf binaries could be unambiguously identified. Furthermore, if detectability issues would be less problematic than they assumed for their set of target stars, the number could be increased to 500. Because the Kepler targets have been specifically selected to make planetary (hence white dwarf) transits detectable for $\sim 90\%$ of the monitored stars, the higher number they derived seems more likely. Furthermore, the mass spectrum they assumed for monitored stars was broader, meaning that the efficiency for producing white-dwarf/main-sequence binaries is lower. Taking this into account, their results can be scaled to produce rough agreement with the calculations here. We emphasize that our calculations, as well as those of other population synthesis simulations, have uncertainties that can best be understood by exploring the parameter space, as described in §3, and summarized in Table 1.

5.2. Directions for Future Work

It is possible that KOI-74 and KOI-81 are the first examples of transiting white dwarfs. The fact that they are both in close orbits is expected, since the majority of white dwarfs whose transits will be detected are in close orbits (Table 1). They both appear to produce pure transits, without obvious signs of lensing. (See the discussion in §4.) Their discovery, and the calculations above suggest several productive lines of research.

- There should be several, perhaps dozens of anti-transits in the data already taken. If these can be identified and modeled, we will have the first evidence of binary self-lensing and the first lensing measurements of the gravitational masses of white dwarfs.

- Mass measurements of all transiting objects are crucial. Radial velocity measurements should be obtained whenever possible. Light curve fitting will also be helpful. If enough systems can be subjected to both types of analyses, we may develop more confidence in results based on light curve methods alone. When fitting the light curves, it is important to employ models that include the possible effects of lensing. Although there are potential degeneracies in fits that include a variety of effects (such as lensing, tides, Doppler boosting), mathematical methods similar to those that test for degeneracy in lensing events can be applied to transits and anti-transits (DiStefano & Perna 1997). The result should be to identify those events those that may be associated with compact objects and to derive the radius and mass of the compact object.

- Transiting white dwarfs can be subject to additional observations, using the opportunities presented by transits to study their atmospheres. The radiation from hot white
dwarfs entering or leaving eclipse can also be used to probe the atmospheres of their companions.

- Lensing by white dwarfs will provide unique opportunities to probe the surface of the stars they orbit.

- Although we cannot yet predict the orbital or mass distributions of planets, binary evolution allows us to predict general characteristics of the orbital and mass distribution of white dwarfs in close ($a < a$ few AU) binaries. A set of more detailed calculations is needed, to generate individual binaries for each set of simulations, and to compute the mass transfer history, the time at which mass transfer ended, the radius of the white dwarf, the light curves, and the characteristics of the monitored star (rotation, metallicity) that may have been influenced by accretion.

- By discovering orbiting white dwarfs with lower mass than expected for their measured orbital periods, Kepler can establish the contribution of three-body interactions to the formation of blue stragglers.

- Once a large ensemble of Kepler events exists, we will be able to compare their properties with those predicted by the theoretical work, and learn about a wide variety of outstanding issues, such as the fraction of matter retained during stable mass transfer to a main sequence star and the efficiency of common envelope ejection in a variety of situations.

- Many of the mass transfer products in which a white dwarf orbits a main-sequence star will experience future epochs of mass transfer onto the white dwarf. Some of these white dwarfs are destined to become Type Ia supernovae. Significant uncertainties in our understanding of the progenitors (Di Stefano 2010; Di Stefano & Nelson 1996) will be addressed by the Kepler results. The same is true for other outcomes, such as accretion-induced collapse. The Kepler data on the endpoints of the evolution of the primaries will provide, for the first time, a large number of reliable starting points for us to compute the next phase of evolution in close white-dwarf binaries.

Implications: Kepler was developed as a mission to discover planets. We find that, in addition, it will be a unique resource to study white dwarfs and mass transfer. The results will touch on stellar evolution, binary evolution, and the formation of intriguing systems, such as the progenitors of Type Ia supernovae and accretion-induced collapse.

Acknowledgements: It is a pleasure to thank Alison J. Farmer, Robert J. Harris, David Latham, Hagai Perets, Darin Ragozzine, Jason Rowe, Kailash Sahu, Guillermo Torres, and
participants in CfA’s “exoplanet pizza lunch” for useful discussions. This work was supported in part by NSF under AST-0708924 and AST-0908878, and by a research and development grant from the CfA.

References

Agol, E. 2003, ApJ, 594, 449
Batalha, N. et al. 2010, arXiv:1001.0349v1
De Marco, O., Shara, M. M., Zurek, D., Ouellette, J. A., Lanz, T., Saffer, R. A., & Sepinsky, J. F. 2005, ApJ, 632, 894
Di Stefano, R. 2010, ApJ, in press (arXiv:0912.0757)
Di Stefano, R. 2008a, ApJ, 684, 46
Di Stefano, R. 2008b, ApJ, 684, 59
Di Stefano, R., & Perna, R. 1997, ApJ, 488, 55
Di Stefano, R., & Nelson, L. A. 1996, Supersoft X-Ray Sources, 472, 3
Eggleton, P. P. 2002, ApJ, 575, 1037
Eggleton, P. P. 1986, private communication
Eggleton, P. P. 1983, ApJ, 268, 368
Eggleton, P. P., & Kisseleva-Eggleton, L. 2001, ApJ, 562, 1012
Fabrycky, D., & Tremaine, S. 2007, ApJ, 669, 1298
Farmer, A.J. & Agol, E. 2003, 592, 1151
Harris, H. C., et al. 2008, ApJ, 679, 697
Kalogera, V., & Webbink, R. F. 1998, ApJ, 493, 351
Kiel, P. D., & Hurley, J. R. 2006, MNRAS, 369, 1152
Kozai, Y. 1962, AJ, 67, 591
Landsman, W., Aparicio, J., Bergeron, P., Di Stefano, R., & Stecher, T. P. 1997, ApJ, 481, L93
Latham, D.W. 2010, private communication
Lidov, M.L. 1961, Iskusstvennye Sputniki Zemli, No. 8, p. 5
Mathieu, R. D., & Geller, A. M. 2009, Nature, 462, 1032
Nelemans, G., & Tout, C. A. 2005, MNRAS, 356, 753
Parsons, S. G., Marsh, T. R., Copperwheat, C. M., Dhillon, V. S., Littlefair, S. P., Gänside, B. T., & Hickman, R. 2010, MNRAS, 61 Perets, H. B., & Fabrycky, D. C. 2009, ApJ, 697, 1048
Pyzras, S. et al. 2009, MNRAS, 394, 978
Rappaport, S., Podsiadlowski, P., Joss, P. C., Di Stefano, R., & Han, Z. 1995, MNRAS, 273, 731
Rowe, J.F. et al. 2010, arXiv:1001.3420
Sahu, K. C., & Gilliland, R. L. 2003, ApJ, 584, 1042
Sarna, M. J., Ergma, E., & Gerškevitš-Antipova, J. 2000, MNRAS, 316, 84
van Kerkwijk, M.H. et al. 2010, arXiv:1001.4539
Vidrih, S., et al. 2007, MNRAS, 382, 515
Webbink, R. F. 2008, Astrophysics and Space Science Library, 352, 233
Winget, D. E., Hansen, C. J., Liebert, J., van Horn, H. M., Fontaine, G., Nather, R. E., Kepler, S. O., & Lamb, D. Q. 1987, ApJ, 315, L77
Table 1: Transiting White Dwarfs: Numbers and Lensing Effects

| Method/Type          | $R_{E/WD}$  | $0.8 < R_{E/WD} < 2$ | $0.4 < R_{E/WD} < 0.8$ | $R_{E/WD} < 0.4$ | Total  |
|----------------------|-------------|----------------------|------------------------|-----------------|--------|
| 1:CE ($\alpha = 10$)| 6.3         | 40.0                 | 102.7                  | 343.6           | 492.6  |
| 1:MT ($\delta_{max} = 0.1$) | 3.4         | 26.6                 | 58.5                   | 589.3           | 677.8  |
| 1:CE ($\alpha = 3$)  | 4.3         | 44.0                 | 116.5                  | 311.6           | 476.4  |
| 1:CE ($\alpha = 1$)  | 1.3         | 42.8                 | 139.4                  | 243.8           | 427.3  |
| 1:CE ($\alpha = 0.3$)| 0.2         | 22.9                 | 144.4                  | 159.4           | 326.9  |
| 1:CE ($\alpha = 0.1$)| 0.0         | 0.4                  | 30.3                   | 61.7            | 92.4   |
| 1:MT ($\delta_{max} = 0.2$)| 3.1         | 25.5                 | 55.2                   | 506.1           | 590.0  |
| 1:MT ($\delta_{max} = \delta_{max}(M_c)$)| 3.5         | 26.6                 | 40.1                   | 117.1           | 187.3  |
| 2:CE ($\alpha = 10$) | 4.1         | 36.4                 | 94.9                   | 445.4           | 580.8  |
| 2:MT ($\delta_{max} = 0.1$)| 1.1         | 9.7                  | 21.4                   | 239.1           | 271.3  |
| 2:MT ($\delta_{max} = 0.2$)| 1.0         | 9.3                  | 20.4                   | 204.5           | 235.2  |
| 2:MT ($\delta_{max} = \delta_{max}(M_c)$)| 1.2         | 10.8                 | 16.5                   | 50.2            | 78.7   |