Spatial structure of photons renders them extremely versatile carriers of quantum information [1, 2], as it can be tailored with simple optical elements such as lenses, phase gratings or holograms. Substantial challenges emerge, however, when such spatially-structured photons carrying quantum information need to be stored in quantum memories [3–5] or if advanced quantum information processing capability beyond linear optics is required [6]. These quandaries are usually not shared by material systems, for which strong interaction can be engineered [7], leading to efforts in demonstrating quantum-interferometric properties of atoms [8], phonons [9] or plasmons [10]. Here we harness the full three-dimensional potential of material quasi-particles – collective atomic excitations known as spin waves. We demonstrate that the spatial structure of single spin waves can be manipulated via the off-resonant ac-Stark shift. Through spin-wave diffraction based beam-splitter transformation, we realize the Hanbury Brown-Twiss (HBT) type measurement at the spin-wave level [11], demonstrating nonclassical statistics of atomic excitations. Finally, we observe interference of two spin waves – an analogue of the Hong-Ou-Mandel (HOM) effect for photons [12]. Thanks to the reversible photon-spin wave mapping via the Duan-Lukin-Cirac-Zoller (DLCZ) protocol [13], these techniques enable encoding states from a high-dimensional Hilbert space into the spatial structure of spin waves to facilitate not only new quantum communication schemes [14], but also high data rate classical telecommunication [15, 16].

Collective atomic spin-wave excitations, in addition to being well-interfaced with photons, can interact strongly thanks to dipole-dipole interactions [14–17] if one of the two ground-state components is replaced with a Rydberg state. While hitherto experiments with such excitations have been performed in spatially single-mode regime, extending the capabilities to the continuous-variable two-dimensional or even three-dimensional space could serve as a full photon-coupled platform for quantum information processing – an integral component of a quantum network. Quantum computation and simulation schemes within such a system endowed with the spatial resolution could range from direct nonlinear quantum gates [18], through simulations of effective field theories [19], to more a plenitude of more elaborate scenarios involving formation of topological spin-wave states [20] to perform fault-tolerance computation [21].

Here we use an ultracold gas ensemble of Rubidium atoms to generate, store and process ground-state spin waves. Generation of single spin waves relies on the process of Raman scattering, which forms the basis of the DLCZ protocol [13]. In this process an off-resonant laser illuminates the ensemble of optically pumped atoms. A scattering event, registered as a “write” (Stokes) photon detector heralds creation of a single spin-wave excitation with a wavevector \( \mathbf{k}^w \), using a spatially-resolved single-photon detector heralds creation of a single spin-wave excitation with a wavevector \( \mathbf{K} = \mathbf{k}^W - \mathbf{k}^w \), where \( \mathbf{k}^W \) is the wavevector of photon before the scattering event. Mathematically, a single collective spin-wave excitation with wavevector \( \mathbf{K} \) (a plane-wave) is an effect of acting with a bosonic creation operator as \( \hat{S}^\dagger(\mathbf{K}) = N^{-1/2} \sum_n \exp(i\mathbf{K} \cdot \mathbf{r}_n) |h_n\rangle \langle g_n| \) on the spin-wave vacuum \( |0\rangle = |g_1 \ldots g_N\rangle \) with \( N \approx 10^8 \) atoms in the ground state (see Fig. 1c for atomic level scheme). The wave-like character of the collective atomic excitations emerges due to the spatial phase dependence of the atomic coherence.
Figure 2. Performance of the spin-wave phase modulator. a Light intensity emitted from a spin wave as a function of a pure sine modulation RMS amplitude \(\sqrt{\langle \varphi_S^2 \rangle}\) and the wavevector \(K_y\) component. Panel b presents total intensities in diffraction orders 0 to 2, marked in panel a, along with a theoretical prediction. In panels c and d we change the modulation to include a term with higher frequency (see Methods for details). Depending on the relative phase between the two terms (or equivalently the sign of the skewness of spatial phase imprint profiles depicted in panels e and f) we observe diffraction predominantly in the selected direction. With this scheme we may engineer a broad range of spin-wave splitting schemes. Phase imprint profiles, directly corresponding the averaged ac-Stark beam light intensities, depicted in panels e and f correspond to spin-wave diffraction patterns observed in c and d, respectively.

Through this dependence one atomic ensemble accommodates many independent spin-wave modes.

In this Letter we present the ability to perform beamsplitter transformation with such modes which constitutes a full spin-wave analogue of complex linear-optical networks. A concise demonstration of such transformation is the inherently nonclassical Hong-Ou-Mandel interference [12], in which we realize a three-way splitter to demonstrate that spin-wave always occupy either of the output modes.

To engineer ground-state spin waves residing in a multimode quantum memory (QM) [5] we employ an off-resonant strong laser beam shaped with a spatial light modulator [22] (see Fig. 1). The beam induces a spatially-dependent differential ac-Stark shift \(\Delta_S(r)\) between levels \(|g\rangle\) and \(|h\rangle\), directly proportional to the light intensity \(I_S\). With negligible absorption and a small (\(\sim 300 \mu m\)) transverse size of the ensemble we may assume a constant intensity along the propagation axis \(x\) of the ac-Stark beam and thus write \(\Delta_S(r) = \Delta_S(y, z)\). The ac-Stark shift leads the spin waves to accumulate an additional, spatially-dependent phase \(\varphi_S(y, z) = \Delta_S(y, z)T\) over the interaction (manipulation) time period \(T\). Such a manipulation is equivalent to the following transformation of the spin-wave creation operator within the Heisenberg picture:

\[
\hat{S}_S^\dagger(K) = \mathcal{N}^{-1/2} \sum_n \exp(i(K \cdot r_n + \varphi_S(r_n))|h_n\rangle\langle g_n|
\]

\[
= \int \mathcal{F}[\exp(i\varphi_S(r))](k)\hat{S}_S^\dagger(K + k)dk,
\]

where \(\mathcal{F}\) represents the Fourier transform in the spatial domain. If now the phase dependance is periodic, the above transformation becomes rather a discrete Fourier series, realizing a multi-output spin-wave beamsplitter transformation in two momentum-space dimensions. The combination of the Gradient Echo Memory [23, 24] and the protocol we introduce here projects collective atomic excitations into the full three-dimensional space, where their \(K_z\) component is coupled with the photonic temporal degree of freedom, while the transverse components of spin waves are paired directly with photonic transverse coordinates.

Finally for spin-wave detection we use a laser pulse that converts the spin wave into a “read” (anti-Stokes) photon with wavevector \(k'\). The spatial phase dependance - either modified or intact - of a collective excitation phase entails directional emission of photons at the readout stage from a spin wave determined by the
of phase modulation strength. By integrating the intensity collective excitations.

The presented manipulation can now be used to observe quantum-interferometric properties and interference of spin waves. We select a pair of modes for the write photon (wa and wb) corresponding to spin-wave modes with $K_{y}^{wa}/rb = ±ΔK_y/2 = ±45$ rad mm$^{-1}$ and equal $K_{x}^{ra} = K_{x}^{rb} ≈ 200$ rad mm$^{-1}$ ($ΔK_x = 0$), which we denote ra and rb (see Fig. 4a). By heralding a pair of write photons, we generate a spin-wave pair
Figure 4. Demonstration of quantum interference of single spin waves. Panel a portrays a schematic of the HOM interference experiment. Detection of two write photons in modes \( wa \) and \( wb \) (select through single-mode fibers coupled to avalanche photodiodes) heralds generation of a spin-wave pair in modes \( ra \) and \( rb \). The three-way splitter is then used to interfere the two spin waves. By detecting the spin waves through photons converted to \( rc \) and \( rd \) modes we observe bunching due to their bosonic nature. Panel b presents the input spin-wave modes in the \( (K_x, K_y) \) plane. Photonic detection modes are always set to collect photons emitted from the heralded spin-wave modes. Bunching may be suppressed if the modes \( ra \) and \( rb \) are separated in the \( K_x \) direction of the momentum space (panel c). If we herald only the write photon in the \( wa \) mode, with the three-way splitter we effectively implement a HBT experiment, observing nonclassical statistics of the spin-wave state (panel e). Simultaneously monitoring the simpler second-order correlation between write and read photons (panel d), we validate operation of the three-way splitter. In particular, for \( \Delta K_x = 0 \) output spin-wave modes \( rc \) and \( rd \) are equally correlated with the write photon \( wa \). Vertical errorbars correspond to one standard deviation inferred from Poissonian statistics of photon counts, while horizontal errorbars are due to mechanical precision of the mode selection.

A phase modulation imposing splitting similarly as in Fig. 3b is applied, resulting in each spin wave being equally distributed into three equidistant modes. We select the grating period \( k_g = \Delta K_y = 90 \text{ rad mm}^{-1} \), so that after manipulation we may write operators for resulting modes \( rc \) and \( rd \) as \( \hat{S}_{rc} = (\hat{S}_{ra} + e^{-i\Delta K_y} \hat{S}_{rb})/\sqrt{3} \) and \( \hat{S}_{rd} = (\hat{S}_{rb} + e^{-i\Delta K_y} \hat{S}_{va})/\sqrt{3} \). In consequence, if the modes are well-overlapped, that is \( \Delta K_x = 0 \) and \( \Delta K_y = k_g \) and if we assume modes above and below \( (va \) and \( vb \) with \( K_y/vb = \pm \frac{1}{2} \Delta K_y \) the considered pair reside in vacuum, we will observe the two spin-wave interference effect in the obtained state \( \hat{\rho}_{rc,rd} = 1/9|00\rangle_{rec,rd}\langle 00| + 2/9|01\rangle_{rec,rd}\langle 01| + 2/9|10\rangle_{rec,rd}\langle 10| + 4/9|\psi\rangle\langle \psi| \) with \( |\psi\rangle = (e^{i\varphi}|20\rangle + e^{-i\varphi}|02\rangle)/\sqrt{2} \) at the output after tracing out the unobserved output modes. This “NOON” state possess a quantum-metrological advantage highly desirable in light-matter interfaces [20]. The interference will be observable in the heralded cross-correlation \( g^{(2)}_{rc,rd|wa,wb} = \langle \hat{n}_{rc}\hat{n}_{rd}\hat{n}_{wa}\hat{n}_{wb}\rangle/\langle \hat{n}_{wa}\hat{n}_{wb}\rangle \) which counts coincidences between photons emitted from modes \( rc \) and \( rd \) – these coincidences should vanish due to quantum interference. Furthermore, the number of self coincidences quantified by \( g_{rc,rc|wa,wb}^{(2)} \) (or \( g_{rd,rd|wa,wb}^{(2)} \)), defined in an analogous way as above, should increase. With no interference, obtained for example by setting a large \( \Delta K_x \), we expect \( g_{rc,rd|wa,wb}^{(2)} = 1 \). In Fig. 4c-e we present the results obtained as we change the overlap between shifted modes by changing \( \Delta K_x \). If the modes are nearly perfectly overlapped at \( \Delta K_x = 0 \), we obtain a value of \( g_{rc,rd|wa,wb}^{(2)} = 0.20 \pm 0.06 \), which clearly certifies that two spin-wave quantum interference is observed (see Methods for details). Simultaneously, taking for instance the \( g_{rc,rc|wa,wb}^{(2)} \), auto-correlation we observe more than a two-fold increase from \( 0.3 \pm 0.3 \) to \( 1.3 \pm 0.3 \) compared with the case of non-overlapping modes, showing that the pair of spin wave is bunched and resides in a single mode. This demonstration of
HOM interference of spin-waves not only exposes their bosonic nature, but paves the way towards implementing complex quantum operations, including more spin-wave modes, that are the primitives of the linear-optical quantum computation scheme. We note here that the only hitherto attempt at performing quantum interference of spin waves relied on two different magnetic sublevels coupled through Raman transitions, and thus was fundamentally not viable for much higher-dimensional Hilbert spaces. Here, the inherent possibility to realize multi-input multi-output quantum networks within the presented spin-wave diffraction scheme lends itself to a plethora of quantum protocols such as boson sampling.

A distinct quantum protocol is implemented by post-selecting only write photon detection events in the wa mode. With the spin-wave mode rb being populated by a weak thermal state $\rho_{rb}(n) = 1/(1+n)! \rho_{rb}(0)^n (1+n)^{-1}$, we expect the quantum interference to be mildly affected by the two-excitation term yielding slightly higher $g^{(2)}_{rc,rd|wa} > 1$ than in the pure two-excitation interference (see Methods). With this, we effectively implement a HBT measurement of a single spin-wave excitation without optical beamsplittings. Value of $g^{(2)}_{rc,rd|wa} = 0.34 \pm 0.01 < 1$ clearly confirms the single excitation character. As the modes are decoupled, we observe a single photon statistics with $g^{(2)}_{rc,rd|wa} = 0.67 \pm 0.08 < 1$ for the rc mode and thermal statistics with $g^{(2)}_{rd,rd|wa} = 2.0 \pm 0.4$ for the rd mode.

The presented versatility demonstrates that unique features of our protocol make it a promising candidate for an implementation of a node of a quantum network, with basic processing capability. With the proposed techniques, our spin-wave processor could perform quantum interference in plasmonic circuits. Nature Nanotech. 8, 712–722 (2013).

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This section describes experimental and theoretical methods essential to reproduce the presented results. First, we discuss details of the experimental setup novel to this work. Next, we discuss properties of the ac-Stark modulation scheme in terms of light-atom interaction. Subsequent sections discuss performance of the spin-wave splitter used to observe quantum interference, methods used to evaluate nonclassicality of observed counts statistics and finally experimental and theoretical considerations on phase matching in the system.

**Experimental setup**

Wavevector-resolved detection  A quantum memory based on an ultracold atomic ensemble prepared with a magneto-optical trap (MOT), allowing wavevector-resolved detection has been described in detail in [5]. An essential component is a spatially-resolved single photon detector comprising an image intensifier based on a microchannel plate (Hamamatsu V7090D, ~20% quantum efficiency) coupled with an fast and sensitive scientific complementary metal-oxide semiconductor (sCMOS) sensor (Andor Zyla 5.5 MP). Principles of operation at the single-photon level and localization of single-photon flashes are detailed in Refs. [2, 31]. Here, before the camera write and read photons are separated into two regions to allow HBT measurement (see Fig. a). Using separate regions of the camera effectively allows photon-number resolved detection [32]. Photons emitted from the ultracold atomic ensemble are imaged in the far field onto the image intensifier using a complex multi-lens setup (effective focal length of 50 mm) with angular resolution of 0.6 mrad [31], corresponding to a wavevector of 4.7 rad mm$^{-1}$. Optically-pumped filtering cells heated to 60°C containing Rubidium-87 and Krypton (1 Torr pressure) as buffer gas are used to separate stray laser light from single Raman-scattered photons. Typical measurement comprises $10^7$ camera frames. Note that spatially-insensitive filtering is crucial for this experiment, as typically used filtering cavities would allow us to use only a single spatial mode, negating the prospects of scalability. For the experiment involving highly populated classical spin-wave states we used the same detector operated in the proportional-intensification regime achieved by lowering the electron gain of the image intensifier (see Ref. [33] for details on this operation regime).

For the quantum interference experiment we replace the intensified sCMOS (I-sCMOS) camera with single-mode fibers coupled to single-photon avalanche photodiodes (APD, Perkin Elmer, ~60% quantum efficiency). Fiber detection modes correspond to Gaussian beams with waist radius of 0.15 mm centered in the atomic cloud. The APDs allow for faster experimental repetition rate and provide higher quantum efficiency than the image intensifier. Coherent spatial filtering with single-mode fibers additionally mitigates the requirement of very narrow post-selection of wavevectors (or positions, like in the atomic experiment of Lopes et al. [8] that used a microchannel plate) in the HOM experiment.

Shaping of the ac-Stark beam  Precise shaping of the ac-Stark beam is essential to obtain the desired effect. One reason is the need to obtain a desired pattern, but very importantly a constant intensity profile along the $z$ direction, possibly free of distortions and inhomogeneities [22], is required to read-out spin waves efficiently. Otherwise atoms in different places along the $z$ direction will accumulate random phases, resulting in poor phase matching at the read-out stage. For accurate shaping of the beam we use a spatial light modulator (SLM, Holoeye Pluto) coupled with a charge-coupled device (CCD) camera (Basler Scout scA1400-17fm). The SLM is illuminated with an elliptically shaped beam from a semiconductor taper amplifier (Toptica, BoosTA$^\infty$) seeded with a light from an ECDL (Toptica DL 100) locked using an offset-lock setup [34]. Timing is controlled with an acousto-optic modulator. Both the CCD camera and the atomic ensemble are situated in the same image plane of the SLM ($\times 1.7$ magnification). Importantly, same lenses are used and the only difference between the camera plane and the atomic cloud plane is a flip mirror instead of a vacuum chamber window on the beam path. With this, we achieve best possible representation of light intensity in the vacuum chamber, distorted by a minimal number of optical elements.
Extended Data Figure 1. **Two detection schemes used throughout the experiment.** In panel a a two-dimensional camera sensor (photocathode of an image intensifier, I-sCMOS) situated in the far field with respect to the atomic ensemble detects single write (w) and read (r) photons in four distinct regions. For cross-correlation measurements we add the photon counts from two regions corresponding to either write or read light. In panel b an example setup (one of the two used, for write and read photons) allowing detection of two far-field modes separated in $K_x$ and $K_y$. Single-mode fibers (SMF) collimators are aligned using XY translation stages.

Extended Data Figure 2. **Timing sequence of the experiment.** Panel a portrays the timing sequence used when wavevector-resolved detection using an I-sCMOS camera is performed. Due to camera frame rate limitation only one cycle of QM per one MOT is performed. In b we show the timing sequence for experiment using few-mode detection using APDs. One MOT cycle fits up to 300 QM cycles. In both cases the sequence is repeated at the rate of 420 Hz.

To generate the desired light intensity profile, we first map the camera pixels onto SLM pixels, taking into account possible rotation and distortions. Next, we use an iterative algorithm with feedback from the CCD camera to generate a desired pattern. Due to phase flicker, the experimental sequence, both during the experiment and during the calibration and optimization of the ac-Stark beam profile, is synchronized with the SLM refresh rate.

**Timing** Fig. presents the timing sequence of the experiment, used both in the configurations based on detection with I-sCMOS (panel a) or APDs (panel b). The sequence is repeated at the 420 Hz refresh rate of the SLM in both cases. In a single cycle, the atoms are trapped for 1.8 ms in the case of the I-sCMOS experiment (1.1 ms in the case of the APD experiment). This is followed by polarization gradient cooling in optical molasses (PGC) that allows us to reach a temperature of $22 \mu K$. The magnetic fields are switched off to allow the eddy currents to decay. The atoms are subsequently prepared in the $F = 1, m_F = 1$ state through optical pumping. A single QM cycle consists of a 100 ns long write pulse, a 2 $\mu s$ long ac-Stark spin-wave manipulation pulse, and 300 ns read pulse. In the I-sCMOS experiment the image intensifier gate is open during writing and reading. The sCMOS camera captures photon flashes during both gates, in separate spatial regions of the image intensifier. In the APD experiment the memory cycle is followed by a short 500 ns clear pulse (consisting of read laser pulse, optical pumping and additional pumping of filtering cells to maintain hyperfine polarization). A single QM cycle can be repeated up to 300 times per one MOT.

A faster photon pair generation rate can be achieved using an FPGA-based feedback, in which we only applied ac-Stark manipulation and read-out pulse if a write photon (or a pair of write photons) is detected. While such configuration nearly triples the generation rate, it provides less data as not all correlation functions can be tracked this way. Nevertheless, this is a recommended operation scheme for future experiments with spin-wave pairs.
Extended Data Figure 3. **Generation of seed light.** To generate phase-coherent light detuned from the write laser by exactly the Rubidium-87 hyperfine splitting, we send some of the write laser light into an electro-optic modulator (EOM) producing sidebands at $f_{\text{SHF}} = \pm 6.834$ GHz. A Fabry-Pérot (FP) scanning cavity is used to filter out all sidebands except the desired one. For this, we additionally modulate the $f_{\text{SHF}}$ frequency at $f_{\text{RF}} = 60$ MHz. Light reflected from the cavity is registered using a photodiode (PD) and the signal is mixed with the $f_{\text{RF}} = 60$ MHz modulation, producing a locking signal for the cavity (inset).

**Generation of coherent spin-wave states** To generate a highly-populated coherent spin-wave state we seed the Stokes scattering process. The process is governed by a squeezing Hamiltonian so both the seed light and the spin waves are amplified; however, for strong and coherent seed light, the generated spin-wave state is close to a coherent state. The seed light, detuned by the Rubidium-87 hyperfine splitting from the write laser light, needs to be phase-coherent with the write laser for the process to be efficient. We use an electro-optic modulator (EOM) fed with an SHF signal with central frequency $f_{\text{SHF}} = 6.834$ GHz to generate sidebands (see Extened Data Figure for experimental schematic). The SHF signal is additionally modulated using a 60 MHz sine wave from a direct digital synthesizer (DDS). Modulated light consisting of harmonic frequency components separated by $f_{\text{SHF}}$ is sent to a FP-cavity and its reflected portion is directed onto a fast photodiode. The photodiode registers beat-notes (RF) at 60 MHz, which are then mixed with the 60 MHz local oscillator (LO) signal. After the loop filter, we obtain a locking signal, which thanks to a proper choice of relative phases between LO and RF allows locking only at the desired sideband (the locking signal slopes for positive and negative shifts differs in sign). For seeding purposes we are interested only in the term which is shifted by $-f_{\text{SHF}}$ from the original laser frequency. The cavity reflects the fundamental unmodulated light and other sidebands, resulting in 26 dB net attenuation of all unwanted components. Importantly, this novel setup allows generation of very pure seed light with only one modulator and an uncomplicated cavity system (cf. [28] [29]).

**Phase modulation with the ac-Stark beam**

To theoretically evaluate the performance of the ac-Stark modulation at the single-excitation level, it is crucial to consider both the level of decoherence caused by manipulation and the spurious noise produced. The manipulation should also influence the coherence in a proper way, i.e. the states $|g\rangle (F = 1, m_F = 1)$ and $|h\rangle (F = 2, m_F = -1)$ should be eigenstates of the effective ac-Stark splitting Hamiltonian [36] [37]. Otherwise, the spin wave is transferred to a different combination of magnetic sublevels, which may lead to beat-notes as well as decoherence due to magnetic field inhomogeneities. We found the optimal setting is to red-detune the ac-Stark laser by $\delta_{\text{acS}} = 0.5$–3.0 GHz from the “empty” state $|h\rangle$ (we calculate the detuning from $5^2S_{1/2}F_g = 2 \rightarrow 5^2P_{3/2}$ transition centroid, lying 193.7 MHz below the $F_g = 2 \rightarrow F_e = 3$ transition), so the energy shift of the $|h\rangle$ state is much larger than for the $|g\rangle$ state. While this causes some scattering from the $|h\rangle$ state, we avoid exciting atoms from the $|g\rangle$, which could generate spurious spin-wave excitations that would later be retrieved as noise. Furthermore, by making the ac-Stark light $z$-polarized, we ensure that $|h\rangle$ is an eigenstate of the effective ac-Stark shift Hamiltonian. Note that this setting is very different from the proposal of Sparkes et al. [36] who considered much larger detunings. In their setting, the ac-Stark light is coupled to $|g\rangle$ and $|h\rangle$ with nearly the same strength and the proper phase difference only appears when circular polarizations are used and in only several specific spin-wave magnetic configurations. Such an operation requires multi-watt power levels to obtain reasonable differential ac-Stark shifts. Furthermore, while the scattering rate would be indeed small, the noise generation rate has not been considered and could become a significant problem.

To evaluate the above predictions we model the full behavior of the multi-level atom described by a density matrix $\rho$
Extended Data Figure 4. **Properties of the ac-Stark modulation.** Panel a depicts the differential ac-Stark shift for the Rabi frequency of the ac-Stark beam equal 100 MHz, as a function of ac-Stark laser detuning $\delta_{acS}$ from the $5^2 S_{1/2} F_g = 2 \rightarrow 5^2 P_{3/2}$ transition centroid. Panels b and c portray incoherent scattering rate (decoherence rate) of spin-waves and generation rate of spurious excitations, respectively. The values are not plotted very close to the absorption resonance due to very high incoherent scattering rendering this region useless for the purpose of ac-Stark modulation.

subject to an off-resonant ac-Stark field. We consider a full Hamiltonian $H$ including all ground-state and excited-state sublevels and the following master equation:

$$\frac{d\rho}{dt} = \frac{1}{i\hbar} [\rho, H] - \frac{1}{2} \{\Gamma, \rho\} - \text{Tr}(\rho F) + \Lambda \rho,$$

where $\Gamma$ is the relaxation matrix, $\Lambda$ is the repopulation matrix, and $F$ is the spontaneous emission operator [38, 39]. First we prepare a spin-wave state as $|g\rangle + \epsilon |h\rangle$ and track the behavior of the atomic state in the subspace spanned by $|h\rangle$ and $|g\rangle$. The relative phase is calculated as $\phi = \text{Arg}(\rho_{hg})$ and the ac-Stark shift as $\Delta_S = \frac{d\phi}{dt}|_{t=0}$. The scattering rate $\Gamma_S = -\epsilon^{-2} \frac{d\rho_{hh}}{dt}|_{t=0}$ quantifies decay of the spin-wave population. For the noise rate $\Gamma_n$ we take an atom prepared in a pure $|g\rangle$ state and we again calculate the rate as $\Gamma_n = \frac{d\rho_{hh}(\epsilon=0)}{dt}|_{t=0}$. The results are plotted in Extended Data Figure as a function of ac-Stark laser detuning $\delta_{acS}$. We find the differential splitting $\Delta_S/2\pi = -0.036$ MHz the for the operation point at $\delta_{acS} = 1.43$ GHz and with approx. 40 mW of average power corresponding to Rabi frequency $\Omega_{acS} = 100$ MHz. This setting provides a total phase shift of $\phi_S = \Delta_S T$ of the order of 0.45 rad with the manipulation time $T = 2 \mu s$, while the maximal power available was approx. 150 mW. We find he scattering rate $\Gamma_S = 390$ Hz, which results in destruction of less than 0.1% of the spin waves due to excitation. Finally, we find a very little noise generation rate per atom $\Gamma_n = 1$ mHz (cf. with significantly higher noise rate when ac-Stark laser is tuned closer to the $|g\rangle$ state at $\delta_{acS} \approx -6.8$ GHz in Extended Data Figure). If we assume that photons from these spurious excitations are scattered randomly during read-out to all far-field modes the number of which we estimate as $\sigma_z \sigma_\perp^2/\lambda^3 \approx 7 \times 10^8$ with $\sigma_z = 4$ mm, $\sigma_\perp = 0.3$ mm (longitudinal and transverse size of the ensemble), $\lambda = 795$ nm (wavelength of the read laser) and $N = 10^8$ (number of atoms), we estimate the probability of emitting a noise photon per mode of less than $3 \times 10^{-10}$, which is completely negligible compared with e.g. noise introduced by imperfect optical pumping.

**Phase pattern design and spin-wave splitter performance**

Here we evaluate the performance of spin-wave splitter in the few-mode quantum interference experiment as well as give explicit expressions used to analyze spin-wave diffraction presented in Fig. 2. To explicitly express the
Extended Data Figure 5. **Beam-splitter operation at the single spin-wave level.** Measured second order correlation between the wa write photon mode for two read detection modes rc \((k_y^r + k_y^w = 0)\) and rd \((k_y^r + k_y^w = k_y)\) as a function of phase modulation RMS \(\sqrt{\varphi^2}\). Measurement was performed at a higher spin wave-photon pair generation rate thus the value of unconditional cross-correlation \(g^{(2)}\) is lower than in Fig. e. Curves correspond to a theoretical prediction with a single fit parameter.

Transformation of the spin-wave creation operator \(\hat{S}_S^\dagger(K_y)\) yielding \(\hat{S}_S^\dagger(K_y)\), describing the result of imprinting on atoms a sine-shaped phase pattern \(\varphi_S(y) = \chi \sin(k_y y + \vartheta)\) we may use the well-known Jacobi-Anger identity to expand the modulation term into more convenient form: \(\exp(i \chi \sin(k_y y + \vartheta)) = \sum_{n=-\infty}^{\infty} J_n(\chi) \exp(in(k_y y + \vartheta))\), where \(J_n\) is the \(n\)-th Bessel function of the first kind. With this expansion we easily obtain that:

\[
\hat{S}_S^\dagger(K_y) = \sum_{n=-\infty}^{\infty} J_n(\chi) \exp(in\vartheta)\hat{S}_S^\dagger(K_y + nk_y).
\]

(3)

Using this formula with \(\chi\) chosen so that \(J_0(\chi) = J_1(\chi)\) and neglecting terms with \(n > 1\), we get the transformation used to describe the HOM interference of spin waves (i.e. a 50:50 beamsplitter transformation). Generally, this expression allows us also to predict the unconditional \(g^{(2)}\) function dependence on modulation RMS amplitude. Choosing two modes separated by \(\Delta K_y = k_y\) (i.e. rc and rd) and neglecting the contribution of weak thermal state split into the rc mode (i.e. assuming modes va and vb reside in vacuum) we can write \(g_{wa,rc}^{(2)} = 1 + \alpha J_0(\chi)^2\), \(g_{wa,rd}^{(2)} = 1 + \alpha J_1(\chi)^2\). As \(\chi = \sqrt{2}\varphi^2\), the only fit parameter left is \(\alpha\) which we simply choose to get the measured value of \(g_{wa,rc}^{(2)}\) for \(\chi = 0\) (no modulation).

To generate asymmetric spin-wave patterns, we use different phase modulation. The modified version includes a "second-harmonic" term:

\[
\varphi_S(y) = \chi_1 \sin(k_y y + \vartheta_1) + \chi_2 \sin(2k_y y + \vartheta_2).
\]

(4)

By taking \(\chi_1/\chi_2 = 2.5\), we observe that spin-wave diffraction occurs predominantly in one direction (as in Figs. 2c and 2d). By changing the relative phase between the two terms above, i.e. \(\Delta \vartheta = \vartheta_1 - \vartheta_2\), we can steer the direction of diffraction. In particular, for Fig. 2c we selected \(\Delta \vartheta = 0\) and for Fig. 2d we set \(\Delta \vartheta = \pi\).

**Quantum character certification**

We use appropriate values of the second-order Glauber correlation function to certify nonclassicality of photon-counting statistics [10]. For the wavevector-resolved measurements we utilize multiplexing of many modes and evaluate the averaged correlation function

\[
g_{rw}^{(2)}(k_x^r + k_x^w, k_y^r + k_y^w) = \int \langle \hat{n}_r(k_x^r, k_y^r) \hat{n}_w(k_x^w, k_y^w) \rangle dk_x^r dk_y^r dk_x^w dk_y^w.
\]

(5)

Nonclassicality of the generated state between write and read photons is certified by violating the Cauchy-Schwarz inequality: \([g_{rw}^{(2)}]^2 \leq g_{rr}^{(2)} g_{ww}^{(2)}\). Since we measured auto-correlation values \(g_{rr}^{(2)}\), \(g_{ww}^{(2)} < 2\) (see Extended Data Figure 6), a conservative bound on nonclassicality is also given by \(g_{rw}^{(2)} > 2\), which we have used throughout the article. A high
value of $g_{rw}^{(2)}$ is a good indication that single read photons (or spin waves) is characterized by good purity (or in other words will be close to a single-excitation Fock state). A more direct indicator is the conditional $g_{rc,rc}^{(2)}$, which we found for the interference experiment presented in Fig. 4 as $g_{rc,rc|wa}^{(2)} = 0.67 \pm 0.04 < 1$ and $g_{rd,rd|wb}^{(2)} = 0.69 \pm 0.04 < 1$ for maximum $\Delta K_z$ separation. Finally, the Hong-Ou-Mandel interference is customarily witnessed by the observed depth (visibility $V$) of the dip in coincidences larger than $V = 0.5$. Here we used a normalized conditional correlation function which without interference equals 1 (we measured $0.93 \pm 0.44$ for the largest separation of modes, which is consistent with the Gaussian mode shape). We may thus estimate the visibility as $V = 1 - g_{rc,rd|wa,wb}^{(2)} = 0.80 \pm 0.06 > 0.5$, certifying a non-classical character of interference. Only slightly lower visibility of $V = 0.66 \pm 0.01 > 0.5$ is observed for the interference of a single spin-wave with the weak thermal spin-wave state. The value of conditional $g_{rc,rd|wa}^{(2)} < 1$ for this case also certifies single excitation character of the spin wave in mode $ra$, obtained using the HBT measurement performed at the spin-wave level (as opposed to using optical elements to implement the beam-splitter at the photonic level). Importantly, this figure of merit better witnesses single-excitation character than $g_{rc,rc|wa}^{(2)}$ or $g_{rd,rd|wb}^{(2)}$. All given errors correspond to one standard deviation of the Poissonian counts statistics.

**Phase matching at readout**

To estimate the readout efficiency we resort to a set of equations describing a classical, optical $E_r(k_r,z,t)$ and spin-wave field $S(K,z,t)$ (atomic coherence) at the readout stage. We assume a uniform intensity profile for the Raman pump and include the atom number density as $b(r,z,t) = \sqrt{N}S(r,z,t)$, where $S(r,z,t)$ only includes atoms in a small volume around $r = (r,z) = (x,y,z)$ and $\mathcal{N}(r,z) = N_0 \exp(-r^2/\sigma^2 - z^2/\sigma_z^2)$. Importantly, we include the diffraction term within the wide-angle, slowly-varying envelope approximation [37][41][42] and write the equations in the frame of reference co-moving with the readout pulse:

$$\frac{\partial E_r(k_{r||},z,t)}{\partial z} = gb(k_{r||},z,t) + t \left( \sqrt{k_r^2 + k_{r||}^2 - k_r} \right) E_r(k_{r||},z,t)$$

(6)

$$\frac{\partial b(k_{r||},z,t)}{\partial t} = gE_r(k_{r||},z,t)$$

(7)

with $k_{r||} = k_{R||} + k_{r||}$ and $g$ being the coupling strength. Phase matching arising due to oscillations caused by the $t \left( \sqrt{k_r^2 + k_{r||}^2 - k_r} \right) E_r(k_{r||},z,t)$ term emerges as the most essential factor influencing readout of reshaped
Extended Data Figure 7. **Influence of phase matching on the readout of diffracted spin-waves.** Panel a portrays normalized write-read coincidences in the form of the second order correlation function (obtained from the same data as Fig. b). We compare this experimental result with the calculated normalized readout efficiency of the reshaped spin-waves presented in panel b (corresponding to the same write photons as in a). Strong influence of phase-matching is evident, as read-out is only efficient for values of $k_y'$ around 0.

spin-waves. To quantify this effect it is enough to consider the first-order approximation of the well-studied solution for the above set of equations, for the case of null $E_r$ field at the input [31,32]. Then, we may equivalently write that $\partial_t b(k_{\perp}, z, t) \approx 0$ and easily solve the first equation as a Gaussian integral. By considering the readout efficiency of a plane-wave spin-wave reshaped with a sine grating with $k_g$, we arrive at very good agreement between measured coincidence map presented in Extended Data Figure a and the theoretical prediction (Extended Data Figure b) for phase-matching efficiency (here normalized to unity), indicating that the phase-matching is the most essential wavevector-dependent factor to the net readout efficiency. The main observed effect is the fact that only spin waves with small $K_y$ can be read-out efficiently after modulation. Remaining spin-wave are not lost, but can be retrieved through manipulating their $K_z$ wavevector components to restore the phase-matching. The problem can thus be alleviated through use of the ac-Stark gradient in the $z$-direction. Furthermore, it is worth noting that the region for which the readout is naturally efficient encompasses hundreds of usable modes (in terms of Schmidt decomposition).

Numerical values of the parameters used for the calculation are $k_g = 44$ rad mm$^{-1}$, $\sigma_{\perp} = 0.3$ mm, $\sigma_z = 4$ mm, $k_r = 7899$ rad mm$^{-1}$. Correspondence between the write photon wavevector and the spin-wave wavevector is calculated as $K = k^W - k^w$.

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