Transient Current-Current Correlations and Noise Spectra

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In this paper, we present an exact formalism for transient current-current correlations and transient noise spectra. The exact solution of transient current correlations in both the time domain and the frequency domain are obtained. Without taking the wide band limit, we investigate transient current-current correlations with different bias voltages and different finite temperatures. Transient noise spectra over the whole frequency range are calculated and that in the steady-state limit are also reproduced. From transient current-current correlations and noise spectra, we analyze the frequency-dependence of electron transport for the system evolving far away from equilibrium to the steady state. Various time scales associated with the energy structure of the nanosystem are also obtained from the transient current-current correlations and transient noise spectra.

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I. INTRODUCTION

Noise spectra provide the information of temporal correlations between individual electron transport events. It has been shown that noise spectra can be a powerful tool to reveal different possible mechanisms which are not accessible by the mean current measurement. Examples include the information on electron kinetics, quantum statistics of charge carriers, correlations of electronic wave functions, and effective quasiparticles charges. Noise spectra can also be used to reconstruct quantum states via a series of measurements known as quantum state tomography. Conventionally, evaluations of noise spectra are largely limited to the rather low frequency ($\hbar \omega \ll k_B T$), where the noise spectrum is symmetric at zero bias. However, experimental measurements of high frequency noise spectra inspired the exploration of the frequency-resolved noise spectrum both in symmetric and asymmetric form. The asymmetric noise spectrum, which is directly proportional to the emission-absorption spectrum of the system, has been demonstrated experimentally. In recent years, the higher order current-correlations in a nonequilibrium steady state are also explored both in experimental and theoretical studies.

The above investigations were focused on the steady-state transport regime. Whereas the theoretic development on transient quantum transport dynamics there are of considerable interests of the transient current fluctuation and noise in the time domain. Recently, the transient current fluctuation (correlation at equal time) of a two-probe transport junction in response to the sharply turning off the bias voltage is analyzed by Feng et al. The transient evolution of finite-frequency current noise after abruptly switching on the tunneling coupling in the resonant level model and the Majorana resonant level model has also been studied by Joho et al. In this paper, we shall investigate the transient current-current correlations of a biased quantum dot system in the nonlinear transient transport regime. Using the exact master equation we developed recently, a general formalism for transient current-current correlations and transient noise spectra are presented for an arbitrary spectral density of nanostructures. This enables one to closely look at electron correlations during transport in the time domain for the system not only in the steady state but also when it is far away from the equilibrium. In particular, it can unveil how the electron correlation changes in the system when it evolves far away from the equilibrium to the steady state, and the time-scale the system reaches the steady state. These results should be useful for understanding the role of quantum coherence and non-Markovian dynamics in quantum transport. They are also essential for reconstructing quantum states of electrons in nanostructures for further applications in nanotechnology, such as the controlling of quantum information processing and quantum metrology on quantum states, etc.

The rest of paper is organized as follows. In the next section, transient current-current correlations in nanoelectronic systems are formulated, and a general solution is presented using the master equation formalism associated with the quantum Langevin equation. To justify the correctness of our formalism, we examine the steady-state current-current correlation of a single-level nanostructure over the whole frequency range in Sec. III in comparison with the results obtained recently by Rothstein et al. In Sec. IV the transient current-current correlations of the same system are analyzed in details, and the energy structures of the noise spectra are also explored. Conclusion is given in Sec. V.

II. TRANSIENT CURRENT-CURRENT CORRELATIONS

To study the transient electronic transport and transient current-current correlations in mesoscopic systems, we begin with Anderson impurity model. The Hamiltonian of the total system, including the central dot, the leads and the coupling between them can be expressed
as
\[ H = \sum_{ij} \varepsilon_{ij} a_i^\dagger a_j + \sum_{\alpha k} \epsilon_{\alpha k} c_{\alpha k}^\dagger c_{\alpha k} + \sum_{i\alpha k} [V_{i\alpha k} a_i^\dagger c_{\alpha k} + H.c.] \]
and the electron-electron interaction is not considered. Here \( \varepsilon_{ij} \) and \( \epsilon_{\alpha k} \) are the corresponding energy levels of the dot and the lead \( \alpha \), which are experimentally tunable through the bias and gate voltages, \( V_{i\alpha k} \) is the tunneling amplitude between the orbital state \( i \) of the dot and the orbital state \( k \) of the lead \( \alpha \), which can also be tuned by changing tunneling barriers via external gate voltages, \( a_i^\dagger \) and \( c_{\alpha k}^\dagger \) (\( c_{\alpha k} \)) are creation (annihilation) operators of electrons in the dot and the lead \( \alpha \), respectively.

The current-current auto-correlation \( \alpha = \alpha' \) and cross-correlation \( \alpha \neq \alpha' \) functions are defined as follows,
\[ S_{\alpha\alpha'}(t + \tau, t) \equiv \langle \delta I_\alpha(t + \tau)\delta I_{\alpha'}(t) \rangle \]  
(2)

\[ S_{\alpha\alpha'}(t + \tau, t) = \frac{\hbar}{2} \sum_{ijkk'} \left\{ - V_{i\alpha k} V_{j\alpha' k'} [\langle a_i^\dagger(t + \tau)c_{\alpha k}(t + \tau) a_j^\dagger(t)c_{\alpha' k'}(t) \rangle - \langle a_i^\dagger(t + \tau)c_{\alpha k}(t + \tau) \rangle \langle a_j^\dagger(t)c_{\alpha' k'}(t) \rangle] \right. \\
- V_{i\alpha k}^* V_{j\alpha' k'}^* [\langle c_{\alpha k}(t + \tau) a_i(t + \tau)c_{\alpha' k'}(t) a_j(t) \rangle - \langle c_{\alpha k}(t + \tau) \rangle \langle a_i(t)c_{\alpha' k'}(t) a_j(t) \rangle] \\
+ V_{i\alpha k} V_{j\alpha' k'} [\langle a_i^\dagger(t + \tau)c_{\alpha k}(t + \tau) c_{\alpha' k'}(t) a_j(t) \rangle - \langle a_i^\dagger(t + \tau)c_{\alpha k}(t + \tau) \rangle \langle c_{\alpha' k'}(t) a_j(t) \rangle] \\
\left. + V_{i\alpha k}^* V_{j\alpha' k'}^* [\langle c_{\alpha k}(t + \tau) a_i(t + \tau)c_{\alpha' k'}(t) a_j(t) \rangle - \langle c_{\alpha k}(t + \tau) \rangle \langle a_i(t)c_{\alpha' k'}(t) a_j(t) \rangle] \right\}. \]  
(4)

We may simply denote \( S = S^{(1)} + S^{(2)} + S^{(3)} + S^{(4)} \), corresponding to the four terms respectively in Eq. (4). We will explore the contribution of each term to the transient noise spectra later.

Current-current correlations are in general complex, the physical observables are related to its real or imaginary parts,

\[ S_{\alpha\alpha'}(t + \tau, t) = S'_{\alpha\alpha'}(t + \tau, t) + i S''_{\alpha\alpha'}(t + \tau, t), \]  
(5)

where
\[ S'_{\alpha\alpha'}(t + \tau, t) = \frac{1}{2} \langle [\delta I_\alpha(t + \tau), \delta I_{\alpha'}(t)] \rangle \]
(6a)

\[ S''_{\alpha\alpha'}(t + \tau, t) = \frac{1}{2i} \langle [\delta I_\alpha(t + \tau), \delta I_{\alpha'}(t)] \rangle \]  
(6b)

are directly proportional to the fluctuation function and the response function, respectively, in the linear response theory. On the other hand, we may introduce the total current-current correlation defined by
\[ S(t + \tau, t) = \langle \delta I(t + \tau)\delta I(t) \rangle, \]  
(7)

where \( \delta I_\alpha(t) \equiv I_\alpha(t) - \langle I_\alpha(t) \rangle \) is the fluctuation of the current in the lead \( \alpha \) at time \( t \). \( I_\alpha(t) \) is the current operator of electrons flowing from the lead \( \alpha \) into the central dot. It is determined by
\[ I_\alpha(t) = -\frac{e}{\hbar} \frac{d}{dt} N_\alpha(t) = i \frac{e}{\hbar} [N_\alpha(t), H(t)] \]
\[ = -\frac{e}{\hbar} \sum_{ik} [V_{i\alpha k} a_i^\dagger(t)c_{\alpha k}(t) - V_{i\alpha k}^* c_{\alpha k}^\dagger(t)a_i(t)], \]  
(3)

where \( e \) is the electron charge, \( N_\alpha(t) = \sum_k c_{\alpha k}^\dagger(t)c_{\alpha k}(t) \) is the particle number operator of the lead \( \alpha \). The angle brackets in Eq. (2) takes the mean value of the operator over the whole system, which is defined as \( \langle O(t) \rangle = \text{tr}[O(t)\rho_{\text{tot}}(t_0)] \). Here \( \rho_{\text{tot}}(t_0) \) is the initial state of the total system. Current-current correlations measure the correlations between currents flowing in different time. If we take Fourier transform of the current-current correlation in Eq. (2) with \( \tau \), an asymmetric noise spectrum of the electronic transport at time \( t \) is obtained. Explicitly,

\[ I(t) = aI_L(t) - bI_R(t), \]  
(8)

and the coefficients satisfy the relation \( a + b = 1 \), associated to the symmetry of the transport setup (e.g., junction capacitances). Then Eq. (7) can be written as

\[ S(t + \tau, t) = a^2 S_{LL}(t + \tau, t) + b^2 S_{RR}(t + \tau, t) - ab[S_{LR}(t + \tau, t) + S_{RL}(t + \tau, t)]. \]  
(9)

The usual total current-current correlation corresponds to \( a = b = 1/2 \). Taking different values of \( a \) and \( b \) can also give other current-current correlations, such as the auto-correlation \( (a = 1, b = 0) \) or \( (a = 0, b = 1) \), etc.

Now, we shall calculate exactly these correlation functions in terms of the exact master equation we developed recently for the investigation of transient quantum electron transports in nanostructures. By consider the central dot as an open system and the leads as its environment, the exact master equation to describe the ele-
tron dynamics in the dot system is given by
\[
\frac{d\rho(t)}{dt} = \frac{1}{i}[H_S(t), \rho(t)] + \sum_{ij} (\gamma_{ij}(t) |2a_j, \rho(t) a_i^\dagger - a_i^\dagger a_j \rho(t) - \rho(t) a_i^\dagger a_j| + \tilde{\gamma}_{ij}(t) |a_i \rho(t) a_j - a_j \rho(t) a_i^\dagger + a_i^\dagger \rho(t) - \rho(t) a_i^\dagger a_j|),
\]
(10)
where \(\rho(t) \equiv \text{tr}_B[\rho_{\text{tot}}(t)]\) is the reduced density matrix of the central dot. The initial state of the dot system is assumed to be uncorrelated with the leads before the tunneling couplings are turned on, namely, \(\rho_{\text{tot}}(t_0) = \rho(t_0) \otimes \rho_E(t_0)\). Here the dot can be in any arbitrary initial state \(\rho(t_0)\) but the leads are initially at equilibrium: \(\rho_E(t_0) = \frac{1}{Z} e^{-\sum_{\alpha} \beta_\alpha H_{\alpha,0} - \sum_{\alpha} N_{\alpha}}\), and \(\beta_\alpha = (1/k_B T_\alpha)\) is the initial inverse temperature of the lead \(\alpha\). The first term in the master equation describes the unitary evolution of electrons in the dot system, where the renormalization effect after integrated out all the lead degrees of freedom has been fully taken into account. The resulting renormalized Hamiltonian is \(H_S(t) = \sum_{ij} \epsilon'_{ij}(t) a_i^\dagger a_j\).

The remaining terms give the nonunitary dissipation and fluctuations induced by backactions of electrons from the leads, and are described by the dissipation and fluctuation coefficients \(\gamma(t)\) and \(\tilde{\gamma}(t)\), respectively. All those time-dependent coefficients in Eq. (10) are given explicitly by
\[
\gamma'(t) = \frac{i}{2} [\mathbf{u}(t, t_0) \mathbf{u}^{-1}(t, t_0) - \text{H.c.}],
\]
(11a)
\[
\gamma(t) = \frac{1}{2} [\mathbf{u}(t, t_0) \mathbf{u}^{-1}(t, t_0) + \text{H.c.}],
\]
(11b)
\[
\tilde{\gamma}(t) = \mathbf{v}(t, t) - [\mathbf{u}(t, t_0) \mathbf{u}^{-1}(t, t_0) \mathbf{v}(t, t) + \text{H.c.}],
\]
(11c)
where \(\mathbf{u}(t, t_0)\) and \(\mathbf{v}(t, t_0)\) are related to the nonequilibrium Green’s function of the dot system in the Schwinger-Keldysh nonequilibrium theory. These Green’s functions obey the following integro-differential Dyson equations,
\[
\frac{d}{d\tau} \mathbf{u}(\tau, t_0) + i \mathbf{u}(\tau, t_0) = \sum_\alpha \int_{t_0}^{\tau} d\tau' g_\alpha(\tau, \tau') \mathbf{u}(\tau', t_0) = 0,
\]
(12a)
\[
\frac{d}{d\tau} \mathbf{v}(\tau, t) + i \mathbf{v}(\tau, t) + \sum_\alpha \int_{t_0}^{\tau} d\tau' g_\alpha(\tau, \tau') \mathbf{v}(\tau', t) = \sum_\alpha \int_{t_0}^{\tau} d\tau' \tilde{g}_\alpha(\tau, \tau') \mathbf{u}(\tau', t_0),
\]
(12b)
subject to the boundary conditions \(\mathbf{u}(t_0, t_0) = 1\) and \(\mathbf{v}(t_0, t) = 0\) with \(t_0 \leq \tau \leq t\). Here, the self-energy correlations from the lead to the central dot, \(g_\alpha(\tau, \tau')\) and \(\tilde{g}_\alpha(\tau, \tau')\), are found to be
\[
g_\alpha(\tau, \tau') = \int \frac{d\omega'}{2\pi} J_\alpha(\omega') e^{-i\omega'(\tau - \tau')},
\]
(13a)
\[
\tilde{g}_\alpha(\tau, \tau') = \int \frac{d\omega'}{2\pi} J_\alpha(\omega') f_\alpha(\omega') e^{-i\omega'(\tau - \tau')}.
\]
(13b)
In Eq. (13), the function \(J_\alpha(\omega) = 2\pi \int_k V_{\text{tot}} V_{\text{tot}}^* \delta(\omega - \epsilon_{\text{tot}})\) is an arbitrary spectral density of the environment (the leads), and \(f_\alpha(\omega) = [e^{\beta_\alpha (\omega - \mu_\alpha)} + 1]^{-1}\) is the Fermi-Dirac distribution of lead \(\alpha\) at initial time \(t_0\).

The above exact master equation can be connected to the exact quantum Langevin equation for the dot operator. The later can be derived formally from the Heisenberg equation of motion
\[
\frac{d}{dt} a_\alpha(t) = -i \sum_j \epsilon_{\alpha j} a_j(t) - \sum_{\alpha j} \int_{t_0}^{t} d\tau g_{\alpha j}(t, \tau) a_j(\tau) - i \sum_{\alpha k} V_{\text{tot}} c_{\text{tot}}(t_0) e^{-i\epsilon_{\alpha k}(t-t_0)}.
\]
(14)
In the above quantum Langevin equation, the first term is determined by the evolution of the dot system itself, the second term is the dissipation risen from the coupling to the leads, and the last term is the fluctuation induced by the environment (the leads), and \(c_{\text{tot}}(t_0)\) is the electron annihilation operator of the lead \(\alpha\) at initial time \(t_0\). The time non-local correlation function \(g_{\alpha j}(t, \tau)\) in Eq. (11b) is also given by Eq. (13b), which characterizes backactions between the dot system and the leads. Because the quantum Langevin equation (14) is linear to \(a_\alpha\), its general solution can be written as
\[
a_\alpha(t) = \sum_j u_{\alpha j}(t, t_0) a_j(t_0) + F_{\alpha}(t),
\]
(15)
where \(u_{\alpha j}(t, t_0)\) is the same nonequilibrium Green’s function of Eq. (12a) that determines the energy level renormalization and dissipation in the dot system, as described by the master equation. The noise operator \(F_{\alpha}(t)\) obeys the following equation,
\[
\frac{d}{dt} F_{\alpha}(t) = -i \sum_j \epsilon_{\alpha j} F_{j}(t) - \sum_{\alpha j} \int_{t_0}^{t} d\tau g_{\alpha j}(t, \tau) F_{j}(\tau) - i \sum_{\alpha k} V_{\text{tot}} c_{\text{tot}}(t_0) e^{-i\epsilon_{\alpha k}(t-t_0)}
\]
(16)
with the initial condition \(F_{\alpha}(t_0) = 0\). Since the system and the leads are initially decoupled to each other, and the leads are initially in equilibrium, it can be shown that the solution of Eq. (10) gives
\[
\langle F_{\alpha}(t) F_i(\tau) \rangle = \langle v_{\alpha j}(\tau, t) \rangle = \sum_{\alpha} \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \int_{t_0}^{t_2} \langle [\mathbf{u}(\tau, t_2) \mathbf{g}_\alpha(t_1, t_2) \mathbf{u}^\dagger(\tau, t_2)]_{ij},
\]
(17)
which is indeed the solution of Eq. (12b). Thus the connection of the solution of the quantum Langevin equation to the fluctuation dynamics in the master equation is explicitly established. Furthermore, the time-dependent operator \(c_{\text{tot}}(t)\) of the lead \(\alpha\) can also be obtained from
its equation of motion:

\[ c_{ak}(t) = c_{ak}(t_0) e^{-i \epsilon_{ak}(t-t_0)} - i \sum_i \int_{t_0}^t d\tau V_{ak}^* a_i(\tau) e^{-i \epsilon_{ak}(t-\tau)}. \]  

(18)

Using the solutions of Eq. (15) and (18), we can calculate explicitly and exactly the current-current correlation function [11]. The explicit expression is still very complicated so we take the situation that the dot has no initial occupation. Then, the four terms in Eq. (4) is given respectively by

\[
S^{(1)}_{\alpha\alpha'}(t + \tau, t) = -\frac{e^2}{\hbar^2} \text{Tr}\{ \left[ \int_{t_0}^{t+\tau} ds g_{\alpha}(t+\tau, s) \tilde{v}(s, t) - \int_{t_0}^t ds \tilde{g}_\alpha(t+\tau, s) u^\dagger(t, s) \right] \\
\times \left[ \int_{t_0}^{t+\tau} ds' \tilde{g}_{\alpha'}(t, s') u^\dagger(t + \tau, s') - \int_{t_0}^t ds' g_{\alpha'}(t, s') v(s', t + \tau) \right] \},
\]

(19a)

\[
S^{(2)}_{\alpha\alpha'}(t + \tau, t) = -\frac{e^2}{\hbar^2} \text{Tr}\{ \left[ \int_{t_0}^{t+\tau} ds v(t, s) g_{\alpha}(s, t + \tau) - \int_{t_0}^t ds u(t, s) \tilde{g}_\alpha(s, t + \tau) \right] \\
\times \left[ \int_{t_0}^{t+\tau} ds' u(t + \tau, s') \tilde{g}_{\alpha'}(s', t) - \int_{t_0}^t ds' v(t + \tau, s') g_{\alpha'}(s', t) \right] \},
\]

(19b)

\[
S^{(3)}_{\alpha\alpha'}(t + \tau, t) = \frac{e^2}{\hbar^2} \text{Tr}\{ \left[ \tilde{g}_\alpha(t + \tau, t) \delta_{\alpha\alpha'} + \int_{t_0}^{t+\tau} ds g_{\alpha}(t+\tau, s) \tilde{v}(s, t) \right] \\
\times \left[ \int_{t_0}^{t+\tau} ds' \tilde{g}_{\alpha'}(s, t') u^\dagger(s', t) - \int_{t_0}^t ds' g_{\alpha'}(s, t') v(s', t + \tau) \right] \},
\]

(19c)

\[
S^{(4)}_{\alpha\alpha'}(t + \tau, t) = \frac{e^2}{\hbar^2} \text{Tr}\{ \left[ \tilde{v}(t + \tau, t) [\tilde{g}_\alpha(t + \tau, t) \delta_{\alpha\alpha'} + \int_{t_0}^{t+\tau} ds g_{\alpha'}(t, s') v(s', t) \right] \\
\times \left[ \int_{t_0}^{t+\tau} ds' \tilde{g}_{\alpha'}(s', t) u^\dagger(s, t') - \int_{t_0}^t ds' g_{\alpha'}(s, t') v(s', t + \tau) \right] \}.
\]

(19d)

Here, \( \tilde{g}_\alpha(t, t) = \langle a_i(t)a_j^\dagger(t) \rangle \) is related to the greater Green’s function in Keldysh’s nonequilibrium approach. Its general solution is given by

\[
\tilde{v}(t, t) = \theta(t - T) u(t, t) + \theta(t - \tau) u(t, \tau) - v(t, \tau).
\]  

(20)

The function \( \tilde{g}_\alpha(t, \tau') = \int \frac{d\omega}{2\pi} J_\alpha(\omega) [1 - f_\alpha(\omega')] e^{-i\omega'(\tau-\tau')} \) is a self-energy correlation of electron holes. As one see, the transient current-current correlations have been expressed explicitly in terms of our nonequilibrium Green’s functions \( u(t, t_0) \) and \( v(t, t) \) that determine the dissipation and fluctuation coefficients in the exact master equation [10].

III. NOISE SPECTRA OF A SINGLE-LEVEL NANOSTRUCTURE

To justify the correctness of the above formalism, we first calculate the noise spectra of a single level quantum dot coupled to two leads over the whole frequency range, which has been recently investigated in the literatures [16,18]. The noise spectra can be obtained by taking the steady-state limit, namely, setting \( t_0 \to -\infty \) and let \( t \to \infty \), and then making a Fourier transformation to the total correlation [7]. This gives the asymmetric noise spectrum as follows:

\[
S(\omega) = \lim_{t \to \infty} \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \langle \delta I(t + \tau) \delta I(t) \rangle,
\]  

(21)
and it can be expressed by

\[ S(\omega) = a^2 S_{LL}(\omega) + b^2 S_{RR}(\omega) - ab[S_{LR}(\omega) + S_{RL}(\omega)]. \]  

(22)

\[ S_{\alpha\alpha'}(\omega) \] in Eq. (22) denotes the auto-correlation noise (\(\alpha = \alpha'\)) and the cross-correlation one (\(\alpha \neq \alpha'\)):

\[ S_{\alpha\alpha'}(\omega) \equiv \lim_{t \to \infty} \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \delta I_{\alpha}(t + \tau) \delta I_{\alpha'}(t) \rangle. \]  

(23)

Since the entire system is in the steady-state limit, the current correlations only depend on the time difference between measurements, it is clear that \(S_{\alpha\alpha'}(\omega) = S_{\alpha\alpha}^* (\omega)\). This relation makes the auto-correlation noise be real. We may define the average cross-correlation noise \(\bar{S}_{LR}(\omega) = (S_{LR}(\omega) + S_{RL}(\omega))/2\), which is also real. As one can see from Eq. (22), the total noise spectrum is also real.

In the literature, one also assumes that the tunneling couplings between the leads and the dot as well as the densities of states of the leads are energy independent, i.e. the so-called wide band limit (WBL). In our formalism, this corresponds to \(J_{\alpha}(\omega') = \Gamma_{\alpha}\), and the self-energy correlations are reduced to:

\[ g_{\alpha}(\tau, \tau') = \Gamma_{\alpha} \delta(\tau - \tau'), \]

\[ g_{\alpha}^* (\tau, \tau') = \Gamma_{\alpha} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} f_{\alpha}(\omega') e^{-i\omega' (\tau - \tau')}. \]  

(24)

The corresponding nonequilibrium Green’s function are

\[ u(\tau, t) = e^{i(\tau + \frac{\omega}{2})(\tau - t)}, \]

\[ v(\tau, t) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\Gamma_L f_L(\omega') + \Gamma_R f_R(\omega')}{(\epsilon - \omega')^2 + (\Gamma/2)^2} \times \left[ e^{i\omega'(\tau - t_0)} - e^{i(\tau + \frac{\omega}{2})(\tau - t_0)} \right] \]

\[ \times \left[ e^{i\omega'(\tau - t_0)} - e^{i(\tau + \frac{\omega}{2})(\tau - t_0)} \right]. \]  

(25)

Substituting Eqs. (24) and (25) into Eq. (19) and letting \(t \to \infty\) (the steady-state limit), and then taking a Fourier transform, we obtain the auto-correlation noise and the cross-correlation noise for a single-level quantum dot system.

The above two equations provide the exact noise spectra at finite temperature and finite bias over the entire frequency range in the WBL. The noise spectrum is proportional to the emission-absorption spectrum of the system, so \(S(\omega)\) can be viewed as the probability of a quantum energy \(\hbar \omega\) being transferred from the system to a measurement apparatus.

In Fig. 1 we plot the noise spectra at several different initial temperatures under a fixed bias. In all the plots, we set \(\mu_L + \mu_R = 0\) so that \(\mu_L = -\mu_R = eV/2\). The noise spectra clearly shows an asymmetric structure. For auto-correlation noises (see Fig. 1(a) and (b)), there are step structures (except for frequency near zero) as a function of frequency, with the step edges located roughly at the resonant tunneling frequencies \(\omega_{\alpha} = |\mu_{\alpha} - \varepsilon| = \pm 5\Gamma \) (\(\pm 15\Gamma\)) for the left (right) lead at temperature \(k_B T = 0.1\Gamma\), as evidences of sequential tunnelings. The step height saturates at \(S_{\alpha\alpha}(\omega) = \epsilon^2 \Gamma_{\alpha}/\hbar^2\). The plateaus in the step structure come from the WBL approach (the result goes beyond the WBL will be presented in the next section, combining with the analysis of the transient noise spectra). They correspond to those events in which electron transport processes are independent to each other (uncorrelated). As a result, the tunneling rate and the level population can be extracted from the step heights and the ratio of step heights, respectively. When the initial temperature increases, the thermal broadening effect near the fermi surface of the leads smears the reso-
nance effects, which causes the step structure to vanish. The Lorentzian dips in the two auto-correlation noises around \( \omega = 0 \) are associated with negative correlations between the currents. Since here the electron-electron interaction inside the dot is not considered, the dips are purely due to the Pauli exclusion principle. The average cross-correlation noise (see Fig. 1(c)) has a much smaller scale, in comparison with the auto-correlation noise. This is reasonable because the cross-correlation is established between the two leads through the central dot. In Fig. 1(c), there are two peaks at the resonant tunneling frequencies \( \omega_{L,R} = |\mu_{L,R} - \epsilon| = 5\Gamma \) and \( 15\Gamma \) when the initial temperature is low. Again, when the temperature gets higher, the effect is smeared. The peaks are associated with a positive correlation of the currents between the initially uncorrelated left and right leads. There is also a Lorentzian dip in the cross-correlation noise at zero frequency with the same reason as the auto-correlation noises, so that the scale of the dips are the same. For a symmetric transport setup \( (a = b = 1/2) \), the total net current noise (see Fig. 1(d)) has four steps, corresponding to the two resonances in each frequency side, but there is no Lorentzian dip around zero frequency. This is because the dips in the auto-correlation noises and in the cross-correlation noises are canceled each other in the symmetric setup. These results coincide with the results obtained recently by Rothstein et al.\cite{Rothstein} using the scattering matrix approach.

Fig. 2 concerns noise spectra under different biases applying to the leads. To keep away with the thermal noise, we set the temperature of the two leads to be zero, that is, the whole system is in the shot noise regime. We can see that the noise spectra would exactly be zero when the detected frequency is larger than the bias voltage. This means that no tunneling process can happen in this regime. For auto-correlation noises (See Fig. 2(a) and Fig. 2(b)), we still have step structure with the step edges locating at the resonant tunneling frequencies \( \omega_{\alpha} = |\mu_{\alpha} - \epsilon| \) for both the positive and negative frequency axes except for zero bias \( (eV = 0\Gamma) \). At zero bias, the noise spectrum only has one step edged at negative resonant tunneling frequency. For the cross-correlation noise (see Fig. 2(c)), the dip is shifted to the resonant frequency without a bias \( (eV = 0\Gamma) \), and the scale of the dip is smaller than that in biased cases. When we increase the bias, saying \( eV = 10\Gamma \), in addition to the two peaks at the resonant frequency, we find a small dip around the bias voltage in the negative frequency axis. We speculate that this is an evidence of cotunneling processes. The cotunneling dip is less obvious because the cotunneling events should have a much smaller probability, in comparison with the sequential tunnelings.

\[ S(\omega) = S_L(\omega) + S_R(\omega) - 2S_{LR}(\omega) \]

\[ \text{FIG. 1: Steady-state noise spectra (in unit of } e^2\Gamma/2 \text{) of the current transport through a single-level quantum dot as a function of the detected frequency } \omega, \text{ where } \epsilon = 5\Gamma \text{ with } \Gamma_L = \Gamma_R = 0.5\Gamma \text{ at the bias } eV = 20\Gamma. \text{ Difference curves correspond to different initial temperatures of the leads as shown in the figures.} \]

\[ \text{FIG. 2: Steady-state noise spectra (in unit of } e^2\Gamma/2 \text{) of the current transport through a single-level quantum dot as function of detected frequency } \omega \text{ at zero temperature with different bias voltages, where } \epsilon = 3\Gamma, \text{ with } \Gamma_L = \Gamma_R = 0.5\Gamma. \]
relations come to only depend on the time difference allows one to monitor the transient processes until the approach the steady-state values at $t \approx 5\Gamma$. The real part of the auto-correlation has a maximal value at $\tau = 0$ (namely when it is measured in the same time), this gives the current-fluctuation, and we find that this current-fluctuation is independent of the observing time $t$ (less transient). When the time difference $\tau$ gets larger, the auto-correlation decays rather faster, and it reaches to zero after $\tau > 2/\Gamma$, namely the correlation vanishes. With the observing time goes on, the real part of auto-correlation becomes more and more symmetric, and the imaginary part gets more antisymmetric, and eventually they becomes fully symmetric and antisymmetric function of $\tau$, respectively, in the steady-state limit, as one expected. We also find that the cross-correlation is rather small (about of one order of magnitude smaller in comparison with the auto-correlation) so that it is not presented in Fig. 3.

To have a more general picture how the system reaches the steady state, we present a contour plot of the real part of the total-correlation in the 2-D time domain in Fig. 4. As one see it is symmetric in the diagonal line ($\tau = 0$), as a consequence of the identity: $S_{\alpha\alpha'}(t + \tau, t) = S_{\alpha\alpha'}^*(t, t + \tau)$. The contour-plot clearly shows an oscillating profile of the correlation in the region $t < 3/\Gamma$. The oscillation quickly decays for the time $3/\Gamma < t < 5/\Gamma$. The correlation reaches a steady-state value after $t \approx 5/\Gamma$. The imaginary part has much the same behavior, except that it has an antisymmetric profile in terms of $t$ and $t + \tau$. This gives the whole picture of the transient current-current correlation.

To see the energy structure in electron transports through the transient current-current correlations, we use the fast Fourier transform (FFT) to convert the correlation functions from the time domain ($\tau$) into the frequency domain for different observing time $t$. The result gives the standard definition of the transient noise.

![Fig. 3: Auto-correlation function $S_{RR}$ in terms of their real and imaginary parts (in units of $e^2\Gamma^2/\hbar^2$) in a single-level nanostructure for different $t$ as a function of $\tau$. Where $\varepsilon = \Gamma$, with $\Gamma_L = \Gamma_R = 0.5\Gamma$, $W_L = W_R = 5\Gamma$, $eV = 10\Gamma$, at $k_B T = 0.5\Gamma$ for both two leads.](image)

![Fig. 4: The contour plot of the real part of the total current-current correlation, $S'(t + \tau, t)$ (in units of $e^2\Gamma^2/\hbar^2$), in the single-level nanostructure in the two-time plane (scaled by $\Gamma$). Here the parameter $\varepsilon = \Gamma$, with $\Gamma_L = \Gamma_R = 0.5\Gamma$, $W_L = W_R = 5\Gamma$, $eV = 10\Gamma$, at $k_B T = 0.5\Gamma$ for both two leads.](image)
spectra. In Fig. 5 we plot the FFT amplitude of auto-correlation $S_{RR}(t + \tau, t)$ and total-correlation $S(t + \tau, t)$. From Fig. 5 one can analyze the electron transport properties through the noise spectra not only just in the steady state, but also in the entire transient regime. To make the energy structure manifest in the transient noise spectra, we let the initial temperature approaches to zero ($k_B T = 0.1 \Gamma$). The right auto-correlation shows only one single peak at $\omega = -\omega_R = |\mu_R - \varepsilon|$ in the beginning. This is because the dot is initially empty so that electrons tunneling from the Fermi surface of the right lead to the dot have a maximum probability. This peak corresponds to the energy absorption of the electron tunnelings. On the other hand, we also observed that the tunneling process for $\omega > eV$, which is forbidden in the steady state near zero temperature, can happen in the transient regime. As the time $t$ varies, the second peak shows up. This comes from backward electron tunnelings (i.e. emission processes) from dot to the right lead, with the peak edge locating at the resonance frequency $\omega = \omega_R = |\mu_R - \varepsilon|$. Note that with a finite band-width spectral density, the spectrum decays when the frequency passes over the resonant frequencies, which is different from the WBL where spectrum is flat. The noise spectrum still has a dip at zero frequency in both the transient and steady-state regimes. The FFT amplitude of the total-correlation has the same properties as the right auto-correlation, with two more peaks coming from the left auto-correlation functions as effects of the emission and absorption processes between the left lead and the central dot. By calculating the individual contribution of the four terms in the auto-correlation expression (Eq. 19a-19d), we find that $S^{(3)}$ and $S^{(4)}$ dominate the tunnelings from the dot to the leads and via versus. The contributions from $S^{(1)}$ and $S^{(2)}$ are much smaller because they describe the correlations of electron tunneling in the same direction, and mostly contribute to the noise around zero frequency due to the Pauli exclusion principle.

In the end, we investigate how the current-current correlation changes for different band widths of the spectral density. We first examine such changes in the time domain (see Fig. 6). For the real part of the auto-correlation, the notable difference is manifested in the current fluctuation ($\tau = 0$). The current-fluctuation is increased with the increase of the band width, and goes to infinity at WBL. For the imaginary part, the amplitude of oscillation increases with the increase of the band width only for $|\tau| < 0.5 \Gamma$. For the transient noise spectra (see Fig. 7), we see that the tails of the correlation decay more and more slowly with increasing the band width at $t = 0$. As the time $t$ varying, besides the occurrence of another peak mentioned before, we notice that the dip at zero frequency gradually vanishes as the band width is decreased. When the system reaches to the steady state,
V. CONCLUSION

In this paper, we present an exact formulation of transient current-current correlations and transient noise spectra, and also carry out the exact solution to Anderson impurity model, but the electron-electron interaction has not been included. By inspecting the transient current-current correlations, we obtain the information of electron transport processes before the system reaching the steady state. We find that current-current correlations have stronger signals in transient regime. It provides interesting results on nonequilibrium two-time correlation functions in nanostructures that could be measured in experiments. Indeed, two-time correlation functions have been proposed and experimentally measured in optical measurements, which provided further information on microstructure of materials and the interplay between the material’s structure and transport properties. An more recent application is in photosynthetic systems, in which one measured the two-dimensional electronic spectroscopy through the Fourier transform of two-time correlation functions, to understand the high efficiency of light-harvesting associated with possible non-Markovian coherence energy transfer. We expect that similar measurements on two-time current correlation functions could be done in nanostructured systems, which should provide more useful information for the controlling of nanosystems, in particular, for applications on quantum information processing and quantum metrology of quantum states.

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FIG. 7: The FFT Amplitude of \(S_{RR}\) (in arbitrary unit) in the single-level nanostructure for different band width \(W = W_L = W_R\), at three different observing time \(t\). Where \(\varepsilon = 5\Gamma\), \(\Gamma_L = \Gamma_R = 0.5\Gamma\), \(eV = 20\Gamma\), and \(k_B T = 0.1\Gamma\) for both two leads.

the band-width dependence of the noise spectra over the whole frequency range is clearly shown up. These results could be examined easily in experiments.

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