Associate $J/\psi + \gamma$ Production:
A Clean Probe of Unpolarized and Polarized Gluon Densities

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Abstract

Color-octet contributions to the associate $J/\psi + \gamma$ are found to be negligible, compared to the ordinary color-singlet contribution. Within the color-singlet model the $J/\psi + \gamma$ production in the leading order is possible only through gluon-gluon fusion process. Therefore, the associate $J/\psi + \gamma$ production remains to be useful as a clean channel to probe the unpolarized and polarized gluon distribution inside proton and to study heavy quarkonia production mechanism.

We discuss in detail the associate production of $J/\psi + \gamma$ at $p\bar{p}$ (or $pp$) and $ep$ colliders. By requiring the $J/\psi$ to decay into an $e^+e^-$ or $\mu^+\mu^-$ pair, we end up with an exceptionally clean final state. This process can therefore serve as a very clean probe of the gluon densities inside the proton as well as the photon. Numerical results are presented for the TEVATRON $p\bar{p}$ and HERA $ep$ colliders. This same mechanism can be used to probe the polarized gluon content of the proton in polarized $p + p(\bar{p})$ collisions. We study in detail $J/\psi + \gamma$ production at both polarized fixed target and polarized collider energies for RHIC.

1 Introduction

The production of heavy $Q\bar{Q}$ bound states (quarkonia) offers a good testing ground for perturbative QCD, since here one combines relatively large cross sections with rather clean final states. The large cross sections are, of course, due to the strong (colour) interactions of the quarks $Q$; clean signals emerge when the $s$-wave vector states ($J/\psi$ or $\Upsilon$) decay into a pair of charged leptons ($e^+e^-$ or $\mu^+\mu^-$). Unfortunately the theoretical analysis of the inclusive production of quarkonia in hadronic collisions

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is complicated, due to the large number of contributing processes. For instance, the following processes contribute [1] to hadro-production of $J/\psi$:

$$
\begin{align*}
\text{g} + \text{g} & \rightarrow J/\psi + \text{g}; \\
\text{g} + \text{g} & \rightarrow \chi_i(\rightarrow J/\psi + \gamma) + \text{g}; \\
\text{g} + \text{q} & \rightarrow \chi_i(\rightarrow J/\psi + \gamma) + \text{q}; \\
\text{g} + \text{q}, \text{g} + \overline{\text{q}} & \rightarrow b(\rightarrow J/\psi + X) + \overline{b}.
\end{align*}
$$

(1.1)

For the photo- (or lepto-) production of $J/\psi$, one has to consider the processes [2]

$$
\begin{align*}
\text{\gamma} + \text{g} & \rightarrow J/\psi + \text{g}; \\
\text{\gamma} + \text{g} & \rightarrow \text{b}(\rightarrow J/\psi + X) + \overline{\text{b}}.
\end{align*}
$$

(1.2)

At the relatively low energies of fixed target experiments, only the first reaction in (1.2) leads to a sizeable cross section. Indeed, recently the NMC collaboration has shown [3] that this reaction can be used to determine the large-$x$ behaviour of the gluon distribution inside the proton. However, at the much higher energies that can be achieved at the HERA collider the situation is considerably more complicated; here one does not only have to include [4] $J/\psi$ production from $b$ decays, but also all processes of (1.1), since at these energies the quark and gluon content [5] of (quasi-)real photons can no longer be ignored. Indeed, $J/\psi$ production at HERA has been suggested [6, 7] as a probe of the gluon content of the photon. At the same time, one hopes to constrain [8] the small-$x$ behaviour of the gluon content of the proton using reactions (1.2). In order to achieve both goals, one will clearly have to discriminate between the different mechanisms of $J/\psi$ production, and various methods to do this have been suggested [9].

Another staple of perturbative QCD is the production of direct photons in hadron-hadron collisions [9]; more recently, these calculations have also been extended to $ep$ colliders [10]. In leading order, direct photons can be produced by $gq$ scattering or $q\overline{q}$ annihilation, so the analysis of such events at high energy $p\overline{p}$ or $ep$ colliders is again quite complicated. The situation is simplified when one requires [11] a heavy $c$ or $b$ quark to be produced together with the photon, but these reactions are not well suited to measure gluon structure functions, since the cross sections also depend on the poorly understood heavy quark distribution functions. This difficulty is avoided if we move the second heavy quark from the initial to the final state; if we in addition require the two heavy quarks to form a bound state, we can expect a very clean final state, as discussed above. In this paper we therefore study the associate production of a $Q\overline{Q}$ bound state and a hard, isolated photon. We focus on the production of $J/\psi$ states, which offer the largest rates, but the generalization to other $s$-wave $J = 1$ $c\overline{c}$ or $b\overline{b}$ states is straightforward. In leading order the $J/\psi + \gamma$ final state can only be produced in $gg$ fusion [12]:

$$
\begin{align*}
\text{g} + \text{g} & \rightarrow J/\psi + \gamma; \\
\text{g} + \text{g} & \rightarrow \chi_c \rightarrow J/\psi + \gamma.
\end{align*}
$$

(1.3, 1.4)
The reaction (1.4) can only produce photons with small transverse momentum $p_T$; since we are interested in the production of isolated photons with high $p_T$, we only need to consider reaction (1.3). (Note that $g + g \rightarrow \chi_c + \gamma$ is not possible.) The cross section is then given by

$$d\sigma(A + B \rightarrow J/\psi + \gamma + X) = \int dx_1 dx_2 f_{g|A}(x_1) f_{g|B}(x_2) d\sigma(g + g \rightarrow J/\psi + \gamma),$$

where $A$ and $B$ can be a hadron, photon, or electron. We will use the so-called colour singlet model to estimate the hard subprocess cross section $\hat{\sigma}$. In this model the $J/\psi$ is treated as a nonrelativistic $c\bar{c}$ bound state. One then has

$$d\hat{\sigma}(g + g \rightarrow J/\psi + \gamma) dt = \frac{16\pi\alpha^2 m_{\psi} |R(0)|^2}{2s^2} \left[ \hat{s}^2 \left( \hat{t} - m_{\psi}^2 \right) \left( \hat{u} - m_{\psi}^2 \right) + \frac{\hat{t}^2}{\left( \hat{u} - m_{\psi}^2 \right) \left( \hat{s} - m_{\psi}^2 \right) \left( \hat{t} - m_{\psi}^2 \right)} \right];$$

(1.6)

here, $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the Mandelstam variables of the parton-parton collision, $m_{\psi} = 3.1$ GeV is the mass of the $J/\psi$ meson, and the $c\bar{c}$ wave function at the origin $|R(0)|^2$ can be determined from the leptonic decay width of $J/\psi$, in leading order expression:

$$\Gamma(J/\psi \rightarrow e^+e^-) = \frac{16\alpha^2}{9m_{\psi}^2} |R(0)|^2 = 4.72 \text{ keV} \Rightarrow |R(0)|^2 = 0.48 \text{ GeV}^3.$$  

(1.7)

## 2 Color-Singlet and Color-Octet Contribution in $J/\psi + \gamma$ Production

[For the details of this Section, see Ref. (14).] Up to very recently the conventional color-singlet model (13) had been used as only possible mechanism to describe the production and decay of the heavy quarkonium such as $J/\psi$ and $\Upsilon$. An artificial $K$-factor ($\sim 2$–5) was first introduced to compensate the gap between the theoretical prediction and experimental data. The uncertainties related to this large $K$-factor are the following: We don’t know precisely the correct normalization of the bound state wave function, the possible next-to-leading order contributions, the mass of the heavy quark inside the bound state, etc. However, even with this large $K$-factor some experimental data are found to be difficult to describe. For the direct $J/\psi$ production with large $p_T$ in $p\bar{p}$ collisions, the dominant mechanism has been found to be through final parton fragmentation (15). As applications of this fragmentation mechanism, various studies of prompt charmonium production at the Tevatron collider (16, 17, 18) have been carried out, and the CDF data on the prompt $J/\psi$ production (19) qualitatively meet these theoretical predictions. Nevertheless, the $\psi'$ production rate at CDF is about 30 times larger than the theoretical predictions, which is the so-called $\psi'$ anomaly, even after considering the fragmentation mechanisms of $g \rightarrow \psi'$ and $c \rightarrow \psi'$. As a scenario to resolve this $\psi'$ anomaly, the color-octet production mechanism was proposed (20). By using this idea, heavy quarkonium (charmonium)
hadroproduction through the color-octet \((\mathcal{C})_8\) pair in various partial wave states \((^{2S+1}L_J)\) has been studied in addition to the color-octet gluon fragmentation approach \cite{21, 22}. We note that the inclusive \(\Upsilon\) production at the Tevatron also shows an excess of the data over theoretical estimates based on perturbative QCD and the color-singlet model \cite{23}. In this case, the \(p_T\) of the \(\Upsilon\) is not so high, so that the gluon fragmentation picture may not be a good approximation any more.

Since it has been proposed that the color-octet mechanism can resolve the \(\psi'\) anomaly at the Tevatron collider, it is quite important to test this mechanism at other high energy heavy quarkonium production processes. Up to now, the following processes have been theoretically considered: inclusive \(J/\psi\) production at the Tevatron collider and at fixed target experiments \cite{21, 22, 24}, spin alignment of leptons to the decayed \(J/\psi\) \cite{23}, the polar angle distribution of the \(J/\psi\) in \(e^+e^- \rightarrow J/\psi + X\) \cite{20}, inclusive \(J/\psi\) productions in \(B\) meson decays \cite{27}, \(Z^0\) decays at LEP \cite{29, 30, 31}, \(J/\psi\) photoproduction \cite{28, 32, 33, 34}, and color-octet \(J/\psi\) production in \(\Upsilon\) decays \cite{35}. For more details on recent progress, see Ref. \cite{36}, which summarizes the theoretical developments on the quarkonium production. Recently, the helicity decomposition method in NRQCD factorization formalism was developed by Braaten and Chen \cite{37}. With this method, polarized \(J/\psi\) production in \(B\) decay was considered in Ref. \cite{38}. In Ref. \cite{34, 39}, it was pointed out that there are interference terms of different \(^{3}P_J\) contributions in polarized \(J/\psi\) production.

In this Section, we investigate the color-octet mechanism for associate \(J/\psi + \gamma\) production in hadronic collisions. The associate \(J/\psi + \gamma\) production has been first proposed as a clean channel to probe the gluon distribution inside the proton or photon \cite{12}, and then to study the heavy quarkonia production mechanism \cite{10, 11} as well as to investigate the proton’s spin structure \cite{12}. If the color-octet mechanism gives a significant contribution to this process, the merit of this process to probe the gluon distribution etc. would be decreased or one should find suitable cuts to get rid of this new contribution. Associate \(J/\psi + \gamma\) production has also been studied as a significant QCD background to the decay of heavier \(P\)-wave charmonia \((\chi_{cJ}(P))\) in fixed target experiments \cite{13, 14} within the color-singlet model \cite{15}. In Ref. \cite{16}, the fragmentation contribution from \(p + \bar{p} \rightarrow (c \text{ or } g) + \gamma + X \rightarrow J/\psi + \gamma + X\) at Tevatron energies \((\sqrt{s}=1.8\text{ TeV})\) is studied, and these fragmentation channels are found to be significantly suppressed compared to the conventional color-singlet gluon-gluon fusion process.

### 2.1 General Discussions

Now we consider the associate production of a \(J/\psi\) and a photon at hadronic colliders and fixed target experiments, including the new contribution from the color-octet mechanism in addition to the usual color-singlet contribution. The only possible subprocess in the framework of the color-singlet model in the leading order is through gluon-gluon fusion,

\[
g + g \rightarrow \gamma + (\mathcal{C})(^{3}S_1^{(1)}) (\rightarrow J/\psi). \quad \text{(See Fig. 1)} \quad (2.8)
\]
This gluon-gluon fusion process for the associate $J/\psi + \gamma$ production is again possible within the color-octet mechanism. (See Fig. 1). However, in the leading order color-octet contributions there also exist many more subprocesses, which are

\[ g + g \rightarrow \gamma + (c\bar{c})(1S_0^{(8)})(\rightarrow J/\psi), \]

\[ g + g \rightarrow \gamma + (c\bar{c})(3P_j^{(8)})(\rightarrow J/\psi), \]

\[ q + \bar{q} \rightarrow \gamma + (c\bar{c})(1S_0^{(8)})(\rightarrow J/\psi), \]

\[ q + \bar{q} \rightarrow \gamma + (c\bar{c})(3P_j^{(8)})(\rightarrow J/\psi), \]

\[ q + \bar{q} \rightarrow \gamma + (c\bar{c})(3S_1^{(8)})(\rightarrow J/\psi). \]

These subprocesses are possible through the effective vertices of

\[ \gamma + g \rightarrow (c\bar{c})(1S_0^{(8)} \text{ or } 3P_j^{(8)}), \]

which is shown in Fig. 2, and

\[ q + \bar{q} \rightarrow (c\bar{c})(3S_1^{(8)}), \]

which is shown in Fig. 3. We note that the $(3S_1^{(8)})$ channel in Eq. (2.14) is vanishing, even though the gluon is not on its mass shell. The Feynman diagram for the gluon initiated subprocesses, Eqs. (2.9, 2.10), is shown in Fig. 4 (a). For the case of quark initiated subprocesses, there are two kinds of diagrams; The Feynman diagram for quark initiated subprocesses Eqs. (2.11, 2.12), due to the effective vertex Eq. (2.14), is shown in Fig. 4 (b). The other diagrams, which are possible through the effective vertex, Eq. (2.13), are shown in Fig. 5.

We first perform a naive power counting analysis of those various subprocesses. Free particle amplitude-squared for the color-singlet gluon-gluon fusion process (2.8) is $[O(\alpha_s^2)]$. In the NRQCD framework [17], the color-octet $(Q\bar{Q})$ states can also form a physical $J/\psi$ state with dynamical gluons inside the quarkonium with wavelengths much larger than the characteristic size of the bound state ($\sim 1/(M_Qv_Q)$). In Coulomb gauge, which is a natural gauge for analyzing heavy quarkonium, these dynamical gluons enter into the Fock state decomposition of physical state of $J/\psi$ as

\[ |J/\psi \rangle = (Q\bar{Q})|3S_1^{(1)}\rangle + O(v_Q)|3P_j^{(8)}\rangle + O(v_Q^2)|3S_1^{(1,8)}\rangle + O(v_Q^2)|1S_0^{(8)}\rangle + \ldots \]

In general, a state $(Q\bar{Q})^{(2S+1)L_J}$ can make a transition to $(Q\bar{Q})^{(2S+1)(L \pm 1)_J}$, or more specifically $(Q\bar{Q})^{(3P_j^{(8)})} \rightarrow (Q\bar{Q})^{(3S_1^{(1)})}$ through the emission of a soft gluon (chromo-electric dipole transition), and it is an order of $v_Q$ suppressed compared to the color-singlet hadronization. For the case of chromo-magnetic dipole transition, such as $(Q\bar{Q})^{(1S_0^{(8)})} \rightarrow (Q\bar{Q})^{(3S_1^{(1)})}$, it is suppressed by $v_Q$ at the amplitude level. If we also consider the fact that the $P$ wave state is $M_Qv_Q$ order higher than $S$ wave state in the amplitude level, one can naively estimate that the transitions

\[ (Q\bar{Q})^{(3P_j^{(8)})} \rightarrow J/\psi + X \quad \text{and} \quad (Q\bar{Q})^{(1S_0^{(8)})} \rightarrow J/\psi + X \]

(2.17)
are commonly $v_{Q}^{4}$ order suppressed at the amplitude-squared level, compared to the transition

$$(Q\bar{Q})(^{3}S_{1}^{(1)}) \rightarrow J/\psi.$$ (2.18)

Since these color-octet processes have the same order $O(\alpha\alpha_{s}^{2})$ as that of the color-singlet gluon-gluon fusion process in free particle scattering amplitude, the color-octet subprocesses are order $v_{Q}^{4}$ suppressed compared to the color-singlet gluon-gluon fusion subprocess.

However, such analyses are only naive power counting, and do not guarantee that the color-octet contributions are suppressed compared to the color-singlet gluon-gluon fusion contribution all over the allowed kinematical region. If we consider inclusive $J/\psi$ photoproduction via $2 \rightarrow 2$ subprocesses, as shown in Refs. [28, 32], color-octet contributions dominate in some kinematical region, even if the naive power counting predicts the suppression of color-octet contributions compared to the color-singlet one. In this respect, it is worthwhile to investigate in detail how much the color-octet mechanism contributes to the $J/\psi + \gamma$ hadroproduction.

In order to predict numerically the physical production rate, we need to know a few nonperturbative parameters characterizing the fragmentation of the color-octet objects into the physical color-singlet $J/\psi$. These nonperturbative matrix elements for the color-octet operators have not been determined completely yet. After fitting the inclusive $J/\psi$ production at the Tevatron collider, using the usual color-singlet $J/\psi$ production, the cascades production from $\chi_{c}(1P)$, and the new color-octet contributions, the authors of Ref. [22] have determined

$$\frac{\langle 0|O_{S}^{\psi}(^{3}P_{0})|0 \rangle}{M_{c}^{2}} = (2.2 \pm 0.5) \times 10^{-2} \text{GeV}^{3}, \quad (2.20)$$

$$\frac{\langle 0|O_{S}^{\psi}(^{1}S_{0})|0 \rangle}{3} = (6.6 \pm 2.1) \times 10^{-3} \text{GeV}^{3}, \quad (2.19)$$

with $M_{c} = 1.48$ GeV. Although the numerical values of the above two matrix elements, $(0|O_{S}^{\psi}(^{3}P_{0})|0)$ and $(0|O_{S}^{\psi}(^{1}S_{0})|0)$, are not separately known in Eq. (2.20), one can still extract some useful information from them. Assuming both of the color-octet matrix elements in Eq. (2.20) are positive definite, then one has[28]

$$0 < \langle 0|O_{S}^{\psi}(^{3}P_{0})|0 \rangle < (6.6 \pm 1.5) \times 10^{-2} \text{GeV}^{3}, \quad (2.21)$$

$$0 < \frac{\langle 0|O_{S}^{\psi}(^{1}S_{0})|0 \rangle}{M_{c}^{2}} < (2.2 \pm 0.5) \times 10^{-2} \text{GeV}^{3}. \quad (2.22)$$

These inequalities could provide us with a few predictions on various quantities related to inclusive $J/\psi$ productions in other processes, and also enable us to test the idea of color-octet mechanism in the associate $J/\psi + \gamma$ production process.

\[3\]The matrix element $\langle 0|O_{S}^{\psi}(^{3}P_{0})|0 \rangle$ can be negative whereas $\langle 0|O_{S}^{\psi}(^{1}S_{0})|0 \rangle$ is always positive definite [28]. Here we choose these ranges just for simplicity.
2.2 Kinematics

Let us consider the process

\[ a(p_1)/p(p_1) + b(p_2)/p(p_2) \rightarrow J/\psi(P) + \gamma(k). \]  (2.23)

in \( p\bar{p} \) (or alternatively \( pp \)) collisions. We can express the momenta of the incident hadrons \( (p, \bar{p}) \) and partons \((a, b)\) in the \( p\bar{p} \) CM frame as

\[
P_1 = \frac{\sqrt{s}}{2}(1, +1, 0), \quad p_1 = \frac{\sqrt{s}}{2}(x_1, +x_1, 0), \quad (2.24)
\]

\[
P_2 = \frac{\sqrt{s}}{2}(1, -1, 0), \quad p_2 = \frac{\sqrt{s}}{2}(x_2, -x_2, 0), \quad (2.25)
\]

where the first component is the energy, the second is longitudinal momentum, and the third is the transverse component of the particle’s momentum. The variables \( x_1 \) and \( x_2 \) are the momentum fractions of the partons. The momenta of the outgoing particles are given by,

\[
P = (E^\psi, P_L^\psi, +P_T) = (M_T \cosh y^\psi, M_T \sinh y^\psi, +P_T), \quad (2.26)
\]

\[
k = (E^\gamma, P_L^\gamma, -P_T) = (P_T \cosh y^\gamma, P_T \sinh y^\gamma, -P_T), \quad (2.27)
\]

where \( P_T \) is the common transverse momentum of the outgoing particles, \( M_T \) is the transverse mass of the outgoing \( J/\psi \), and \( y^\psi \) (or \( y^\gamma \)) is the rapidity of \( J/\psi \) (or \( \gamma \)). In order to get the distributions in the invariant mass \( M_{J/\psi+\gamma} \) (\( = \sqrt{s} \)) and transverse momentum \( P_T \) for the process \( g+g \rightarrow J/\psi+\gamma \) process, after introducing dimensionless variables,

\[
x_T = 2P_T/\sqrt{s}, \quad \tau_T = 2M_T/\sqrt{s} \quad \text{and} \quad \tau = M_{\psi}^2/s, \quad (2.28)
\]

we express the differential cross section as,

\[
d\sigma = f_{g/p}(x_1, Q^2)f_{g/\bar{p}}(x_2, Q^2)\frac{d\hat{\sigma}}{dt}dx_1dx_2d\hat{t} = f_{g/p}(x_1, Q^2)f_{g/\bar{p}}(x_2, Q^2)\frac{d\hat{\sigma}}{dt}J \left( \frac{x_1x_2}{x_1x_TM_{J/\psi+\gamma}} \right) dx_1dx_TdM_{J/\psi+\gamma} = f_{g/p}(x_1, Q^2)f_{g/\bar{p}}(x_2, Q^2)\frac{d\hat{\sigma}}{dt}J \left( \frac{x_1x_2}{x_1y^\psi P_T} \right) dx_1dy^\psi dP_T, \quad (2.29)
\]

where the corresponding \( Jacobians \) are given by

\[
J \left( \frac{x_1x_2}{x_1x_TM_{J/\psi+\gamma}} \right) = \frac{2x_2x_TM_{J/\psi+\gamma}}{\tau_T(x_2e^{+y^\psi} - x_1e^{-y^\psi})} \quad \text{and} \quad J \left( \frac{x_1x_2}{x_1y^\psi P_T} \right) = \frac{4x_1x_2P_T}{2x_1 - \tau_T e^{+y^\psi}}. \quad (2.30)
\]

Then the distributions are expressed as

\[
\frac{d\sigma}{dM_{J/\psi+\gamma}} = \int dx_1 \int dx_TJ \left( \frac{x_1x_2}{x_1x_TM_{J/\psi+\gamma}} \right) \frac{d\hat{\sigma}}{dx_1dx_Tdt}, \quad (2.31)
\]

\[
\frac{d\sigma}{dP_T} = \int dx_1 \int dy^\psi J \left( \frac{x_1x_2}{x_1y^\psi P_T} \right) \frac{d\hat{\sigma}}{dx_1dx_Tdt}. \quad (2.32)
\]
And the allowed regions of the variables are given by

\[
\frac{M^2_{J/\psi+\gamma}}{s} \leq x_1 \leq 1,
\]
\[
\hat{s} = M^2_{J/\psi+\gamma} = x_1x_2s \geq M^2_{J/\psi},
\]
\[
0 \leq x_T \leq \frac{(x_1x_2 - \tau)}{\sqrt{x_1x_2}}.
\]

(2.33)

2.3 The Nonrelativistic-QCD (NRQCD) Factorization Formalism

First we consider the general method to get the NRQCD cross section for the process \(a+b \rightarrow (Q\overline{Q})(^{2S+1}L_j^{(1,8)})(\rightarrow H) + c\), where \(H\) is the final state heavy quarkonium and \((Q\overline{Q})(^{2S+1}L_j^{(1,8)})\) is the intermediate \((Q\overline{Q})\) pair which has the corresponding spectroscopic state. From now on, we use the subscript \(n\) to represent the spectroscopic \((Q\overline{Q})\) state of \((^{2S+1}L_j^{(n=1,8)})\), for simplicity. Once the on-shell scattering amplitude of the process \(A(a+b \rightarrow Q+\overline{Q}+c)\) is given, we can expand the amplitude in terms of relative momentum \(q\) of the quarks inside the bound state because the quarks, which make up the bound state, are heavy. For more details of the method to deal with the heavy quarkonium production following the BBL formalism [47], we refer to Refs. [21, 22, 28]. The heavy quarkonium \(H\) production cross section

\[
d\hat{\sigma} = \frac{1}{16\pi s^2} \sum |\mathcal{M}'(a(p_1) + b(p_2) \rightarrow (Q\overline{Q})_n(P) + c(p_3))|^2,
\]

(2.35)

where

\[
d\hat{\sigma}'_n = \frac{1}{16\pi s^2} \sum |\mathcal{M}'(a(p_1) + b(p_2) \rightarrow (Q\overline{Q})_n(P) + c(p_3))|^2.
\]

(2.36)

Here, \(\mathcal{M}'\) is the amplitude of the process

\[
a(p_1) + b(P_2) \rightarrow (Q\overline{Q})_n(P) + c(p_3),
\]

(2.37)

which can be obtained by integrating the free particle amplitude over the relative momentum of the quark inside the intermediate state \((Q\overline{Q})_n(P)\), after projecting appropriate spectroscopic state Clebsch-Gordon coefficients. The parameter \(C_n\) is defined by

\[
C_n = \begin{cases} 
2N_c & \text{(color – singlet)}, \\
N_c^2 - 1 & \text{(color – octet)}.
\end{cases}
\]

(2.38)

And \(\langle 0|O^H_n|0 \rangle\) is the non-perturbative matrix element representing the transition

\[
(Q\overline{Q})(^{2S+1}L_j^{(1,8)}) \rightarrow H.
\]

(2.39)

Finally, \(J\) denotes the angular momentum of the intermediate state \((Q\overline{Q})(^{2S+1}L_j^{(1,8)})\), not of the physical state \(H\). For the case of color-singlet intermediate state, which
has the same spectroscopic configuration with \( H \), we can relate the matrix elements to the radial wave-function of the bound state as

\[
\frac{\langle 0 | O^H | 0 \rangle}{C_n \times (2J + 1)} = \begin{cases} 
\frac{1}{4\pi} |R_S(0)|^2 & (S - \text{wave}), \\
\frac{3}{4\pi} |R_P(0)|^2 & (P - \text{wave}).
\end{cases}
\] (2.39)

For example, if we consider the \( \psi \) production via \((^3S_1^{(1)}), (^1S_0^{(8)}), (^3S_1^{(8)}), (^3P_0^{(8)}), (^3P_1^{(8)})\) and \((^3P_2^{(8)})\) intermediate states, then the partonic subprocess cross sections are given by

\[
\begin{align*}
\frac{d\hat{\sigma}}{dt}^{\text{(octet)}} &= \frac{1}{8M_c} \left( \frac{d\hat{\sigma}'}{dt}(^1S_0^{(8)}) \times \langle 0 | O^\psi (^1S_0^{(8)}) | 0 \rangle + \frac{d\hat{\sigma}'}{dt}(^3S_1^{(8)}) \times \frac{\langle 0 | O^\psi (^3S_1^{(8)}) | 0 \rangle}{3} + \langle 0 | O^\psi (^3P_0^{(8)}) | 0 \rangle \times \sum_J \frac{d\hat{\sigma}'}{dt}(^3P_J^{(8)}) \right), \\
\frac{d\hat{\sigma}}{dt}^{\text{(singlet)}} &= \frac{1}{M_c} \frac{\hat{R}_S(0)}{4\pi} \frac{d\hat{\sigma}'}{dt}(^3S_1^{(1)}),
\end{align*}
\] (2.40)

after imposing the heavy quark spin symmetry

\[
\langle 0 | O^{J/\psi} (^3P_J^{(8)}) | 0 \rangle = (2J + 1) \langle 0 | O^{J/\psi} (^3P_0^{(8)}) | 0 \rangle.
\] (2.41)

The subprocess cross section for the color-singlet gluon-gluon fusion is well known:

\[
\frac{d\hat{\sigma}}{dt}^{\text{(singlet)}} = \frac{N_1}{16\pi s^2} \left[ \frac{s^2 (s - 4M_c^2)^2 + \ell^2 (\ell - 4M_c^2)^2 + \tilde{u}^2 (\tilde{u} - 4M_c^2)^2}{(s - 4M_c^2)^2 (\ell - 4M_c^2)^2 (\tilde{u} - 4M_c^2)^2} \right],
\] (2.42)

where the overall normalization \( N_1 \) is defined as

\[
N_1 = \frac{4}{9} (4\pi \alpha_s)^2 (4\pi \alpha) e_c^2 M_c^3 G_1(J/\psi).
\] (2.43)

The parameter \( G_1(J/\psi) \), which is defined in NRQCD as

\[
G_1(J/\psi) = \frac{\langle 0 | O (^3S_1^{(1)}) | 0 \rangle}{3M_c^2} = \frac{3}{2\pi M_c^2} |R_S(0)|^2,
\] (2.44)

is proportional to the probability of a color-singlet \((c\bar{c})\) pair in the \((^3S_1^{(1)})\) partial wave state to form a physical \( J/\psi \) state. It is related to the leptonic decay width

\[
\Gamma(J/\psi \to l^+l^-) = \frac{2}{3} \pi e_c^2 \alpha^2 G_1(J/\psi),
\] (2.45)

where \( e_c = 2/3 \). From the measured leptonic decay rate of \( J/\psi \), one can extract

\[ G_1(J/\psi) \approx 106 \text{ MeV}. \] (2.46)
After including the radiative corrections of $O(\alpha_s)$ with $\alpha_s(M_c) = 0.27$, this value is increased to $\approx 184$ MeV. Relativistic corrections tend to increase $G_1(J/\psi)$ further to $\sim 195$ MeV [27].

For the case of color-octet subprocesses, we can use the average-squared amplitude of the processes $\gamma + g$ (or $q$) $\to (c\bar{c})(^{2S+1}L_J^{(8)}) + g$ (or $q$) in Ref. [28], after crossing ($k \to -k$ and $q_2 \to -q_2$)

$$\sum |M'|^2 (g + g) \to (c\bar{c})(^{2S+1}L_J^{(8)}) + \gamma)(\hat{s}, \hat{t}, \hat{u}) = \frac{1}{8} |M'|^2 (\gamma + g \to (c\bar{c})(^{2S+1}L_J^{(8)}) + g)(\hat{t}, \hat{s}, \hat{u}), \quad (2.47)$$

$$\sum |M'|^2 (q + q) \to (c\bar{c})(^{2S+1}L_J^{(8)}) + \gamma)(\hat{s}, \hat{t}, \hat{u}) = \frac{1}{3} |M'|^2 (\gamma + q \to (c\bar{c})(^{2S+1}L_J^{(8)}) + q)(\hat{t}, \hat{s}, \hat{u}). \quad (2.48)$$

As previously explained, there is only one color-singlet subprocess, but 9 color-octet subprocesses are contributing to the process

$$p + p (\bar{p}) \to J/\psi + \gamma + X. \quad (2.49)$$

Whereas the initial partons for the color-singlet process are only the gluons, quarks can also be the initial partons for the color-octet subprocesses. For the $^{3S_1(8)}$ channel, the gluon contribution corresponding to Fig. 4 (a) is absent because the effective vertex corresponding to Fig. 2 is vanishing. Using the parton level differential cross sections for various channels, in the next section we discuss in detail the $p_T$ and $M_{J/\psi + \gamma}$ distributions in the hadronic $J/\psi + \gamma$ production. We also compare the color-octet contributions with the color-singlet gluon-gluon fusion contribution.

2.4 NRQCD Results

[For all the numerical results including the comparison of the color-singlet contribution with the color-octet contributions for $J/\psi + \gamma$ productions at the fixed target energy and Tevatron energy, see Ref. [14]]. To conclude of this Section, we have considered the color-octet contributions to the associate $J/\psi + \gamma$ production in the hadronic collisions, also compared to the conventional color-singlet gluon-gluon fusion contribution. Within the color-singlet model the $J/\psi + \gamma$ production in the leading order is possible only through gluon-gluon fusion process. As we expected according to the naive power counting, we found that the color-octet contributions are significantly suppressed compared to the color-singlet gluon-gluon fusion process. For the case of the hadroproduction at the Tevatron energies, we found that there is a crossover point at high $p_T$ where the color-octet contributions become dominant, and it is still larger than the fragmentation contributions shown in Ref. [16]. This suppression of the color-octet contributions in the associated production is qualitatively consistent with the recent result of Cacciari and Krämer [34]. They investigated the $J/\psi + \gamma$ production via resolved photon collisions in HERA, and found the strong suppression on the color-octet contributions compared to the color-singlet one as at
\( \sqrt{s_{\gamma p}} = 100 \) GeV for \( p_T > 1 \) GeV \[34\].

\[
\frac{\sigma(1S_0^{(8)}) + \sigma(3P_j^{(8)})}{\sigma(3S_1^{(1)})} \approx 15\%.
\]

(2.50)

Though most events at the Tevatron collider are in the region where the color-singlet gluon-gluon fusion contribution dominates, one could require an additional cut, for example: \( p_T < 6 \) GeV, to guarantee that the color-singlet gluon-gluon fusion process remains one of the cleanest channels to probe the unpolarized \[12\] and polarized gluon distribution inside proton \[42\], and to study heavy quarkonia production mechanism \[40, 41\].

In next two sections, strictly within the color-singlet model we consider the productions of \( J/\psi + \gamma \) at \( e + p \) and \( p + p(\bar{p}) \) colliders to probe the unpolarized gluon densities (Section 3), as well as at the polarized fixed target experiment and polarized \( p + p \) collider to probe the polarized gluon densities of proton (Section 4).

### 3 Probing Unpolarized Gluon Densities from \( e + p \) and \( p + p(\bar{p}) \) Colliders

In this Section, we use the color-singlet model and leading order expressions everywhere; we will also stick to leading order structure functions when evaluating \( (1.5) \). The NMC collaboration found \[3\] that this leading order formalism describes the shape of all kinematical distributions quite well, while the prediction for the normalization of the signal was too small by a factor of 2.4, even though the QCD corrected version of \( (1.7) \) was used, which increases the cross section by roughly 45%. This large “k-factor” is probably mostly due to the nonrelativistic treatment of the \( J/\psi \), in which all information about the wave function is contained in \( |R(0)|^2 \). One can therefore expect a similar k-factor for our reaction, so that our results for total cross sections should be considered as conservative estimates. In accord with our philosophy, we use the one-loop expression for \( \alpha_s \) with \( N_f = 4 \) active flavors and \( \Lambda_{QCD} = 200 \) MeV, and we take \( Q^2 = m_{\psi}^2 + p_T^2 \) as momentum scale both in \( \alpha_s \) and in the structure functions.

In a hadronic environment, a \( J/\psi \) can probably only be identified when it decays into an \( e^+e^- \) or \( \mu^+\mu^- \) pair; all our results therefore include a factor of 0.14, which is \[18\] the combined branching ratio for these decays. Our final state thus consists of a hard, isolated photon in one hemisphere, balanced by an \( e^+e^- \) or \( \mu^+\mu^- \) pair in the opposite hemisphere, without any hadronic activity (other than the usual spectator jets resulting from the break-up of the incoming hadrons). This signal should be virtually free of any physical or instrumental backgrounds. We are now in a position to present predictions for \( J/\psi + \gamma \) production at \( p\bar{p} \) and \( ep \) colliders.

We begin with a discussion of \( J/\psi + \gamma \) production in \( p\bar{p} \) collisions. We focus on the Tevatron, which offers the largest cross sections of all existing colliders, and is expected to eventually accumulate an integrated luminosity of at least several hundred \( pb^{-1} \). As discussed above, we require the \( J/\psi \) meson to decay into a pair of charged
leptons. We then apply the following cuts, which should guarantee that the events are contained in the detectors and can be triggered upon:

\[ p_T^{\gamma} = p_T^{\psi} > 5 \text{ GeV}; \quad (3.51) \]
\[ |y^{\gamma,e,\mu}| < 3.5. \quad (3.52) \]

At present, the CDF detector can only detect muons with \(|y^\mu| < 0.7\), but future upgrades, as well as the upcoming D0 detector, should provide better coverage; (3.52) roughly describes the coverage of the electromagnetic calorimeters at CDF.

In Figs. 6a,b we present the transverse momentum and energy spectra of the photon and the two leptons after the cuts (3.51,3.52) have been applied, where we have used EHLQ1 [49] structure functions. In both figures we show the spectrum of the “harder” (denoted by ‘b’) and “softer” (denoted by ‘s’) lepton separately, where the “hardness” is defined by the quantity plotted. Notice that the cut (3.51) implies that at least one lepton satisfies \(p_T^l > 2.5 \text{ GeV}, E_l^s > 2.9 \text{ GeV}\); together with the hard photon, this harder lepton can therefore be used to construct a trigger for this reaction. The transverse momentum and energy of the other lepton can in principle be arbitrarily small; however, Figs. 6 show that the additional cuts \(p_T^l, E_l > m_\psi/2\) would not reduce the signal very much. On the other hand, it is necessary to include events where at least one lepton has less than 5 GeV transverse momentum. Due to the relatively mild cut (3.52) on the rapidities of the final state particles, the energy distributions are substantially harder than the \(p_T\) spectra. Events where the whole \(J/\psi + \gamma\) system undergoes a strong boost are interesting since they can yield information about the gluon densities at very small \(x\), down to a few times \(10^{-4}\).

In Fig. 7a we show the rapidity distribution of the produced photon. As discussed in Sec. 1, the overall normalization of the cross section is quite uncertain. We therefore normalize these distributions by dividing by the total cross section after cuts. In order to demonstrate the sensitivity of this distribution to the shape of the gluon density function \(f_{g|p}\), we show results for the EHLQ1 [49] and DO2 [50] parametrizations. These two parametrizations make quite different assumptions about the large \(x\) behaviour of \(f_{g|p}\):

\[ f_{g|p}(x, Q^2_0 = 5 \text{ GeV}^2) \propto x^{-1}(1 + 3.5x)(1 - x)^{5.9} \quad (EHLQ1); \quad (3.53) \]
\[ f_{g|p}(x, Q^2_0 = 4 \text{ GeV}^2) \propto x^{-1}(1 + 9x)(1 - x)^4 \quad (DO2). \quad (3.54) \]

The harder gluon distribution function of the DO2 parametrization leads to a significantly broader rapidity distribution; when going from \(y^\gamma = 0\) to \(|y^\gamma| = 3\) the cross section only falls by a factor of 2.2, while the EHLQ1 gluon predicts a reduction by a factor of 2.9.

The results of Fig. 7a have been obtained by integrating over \(p_T^\gamma\) and the rapidity of the \(J/\psi\). It is conceivable that two parametrizations of \(f_{g|p}\) which differ both in the large \(x\) and small \(x\) regions lead to similar results for the single differential cross section shown in this figure, since large photon rapidities correspond to very

\[^4\text{Using the recently available structure functions from MRS and CTEQ will not change the essence of physics given here, although it will certainly give different numerical results.}\]
asymmetric initial states. Once a sufficiently large number of events has been accumulated, such ambiguities can be resolved by studying the triple differential cross section \( d\sigma/(dp_T^\gamma dy^\gamma dy^\psi) \). As an example, we show in Fig. 7b this triple differential cross section as a function of \( p_T^\gamma \) at the symmetric point \( y^\gamma = y^\psi = 0 \). This cross section is directly proportional to 

\[
\left. \frac{f_g}{p^\gamma} \left( x \approx \sqrt{\frac{(2m_{\psi}^2 + 4(p_T^\gamma)^2)\gamma}{s}} \right) \right|^2,
\]

where \( s \) is the \( p\bar{p} \) centre-of-mass energy; the cut \( p_T^\gamma > 5 \text{ GeV} \) then implies \( x > 6 \cdot 10^{-3} \), while the cross section becomes too small to be useful if \( p_T^\gamma > 15 \text{ GeV} \), i.e. \( x > 0.02 \), even if some \( 10^3 \text{pb}^{-1} \) of data can be accumulated. As mentioned above, the range of \( x \) values that can be probed can be extended by studying more asymmetric configurations; it should therefore be quite easy to distinguish between parametrizations whose small \( x \) behaviour is governed by a different (negative) power of \( x \), like the HMRS+-parametrizations of Ref. [51], which assume \( x \cdot f_g \left( x \right) \propto x^{\pm 0.5} \).

We now turn to a discussion of \( J/\psi + \gamma \) production at the upcoming \( ep \) collider HERA. Since in leading order this final state can only originate from a \( gg \) initial state, the observation of a sizeable signal would be an unambiguous proof for a nonvanishing gluon content of the photon. (The same is true [52] for inclusive \( J/\psi \) production in \( \gamma\gamma \) collisions.) In order to use (1.5) for the cross section calculation, one has to convolute the gluon content of the photon with the photon content of the electron:

\[
f_{g|e}(x, Q^2) = f_x \frac{d}{dx} f_{\gamma|e}(z, Q^2) f_{g|\gamma}(z, Q^2),
\]

where

\[
f_{\gamma|e}(z, Q^2) = \frac{\alpha}{\pi z} \left[ 1 + (1 - z)^2 \right] \ln \frac{Q^2}{m_e^2}.
\]

We use the following cuts:

\[
p_T^\gamma = p_T^\psi > 1.5 \text{ GeV}; \quad -3.5 < y^{\gamma,e,\mu} < 3,
\]

where negative rapidities correspond to the proton beam direction. Notice that the cut (3.57) is still sufficient to remove the contribution from reaction (1.4); the cut (3.58) roughly describes the acceptance of the ZEUS detector.

Notice that \( f_{g|\gamma} \) is not constrained by a momentum sum rule; both the shape and the normalization of that function are unknown. It has recently been shown [53] that even a global fit to all existing data on \( F_2^\gamma \) does not yield much information on \( f_{g|\gamma} \). Our results should therefore be taken as examples, not QCD predictions.

In Figs. 8a,b we show the transverse momentum and energy spectrum that result when the LAC1 parametrization [53] is used for \( f_{g|\gamma} \). We see that the signal will only be useful if one lepton is allowed to have \( p_T \) below 2 GeV; on the other hand the cut (3.57) and the Jacobian peak in the \( p_T \) distribution of the leptons relative to the axis of the \( J/\psi \) imply that one lepton usually does have more than 2 GeV transverse momentum. Also, due to the asymmetric nature of \( ep \) colliders, the difference between the energy distributions of Fig. 8a and the \( p_T \) distributions of Fig. 8b is even more pronounced than for \( p\bar{p} \) colliders; e.g., one could impose a cut \( E^\gamma > 5 \text{ GeV} \) without losing much signal. Such a cut might be helpful since the
resolution of electromagnetic calorimeters increases, and hence the relative errors on $E\gamma$ and $p_T^\gamma$ shrink, with $\sqrt{E\gamma}$.

In Fig. 9 we again compare the shape of the photon rapidity distributions as predicted by different parametrizations of photon structure functions. The present lack of data constraining $f_{g\gamma}$ is reflected by the large differences between the three curves, corresponding to sets 1 and 3 of Ref. [53] and the older DG parametrization of Ref. [54], even though we have used a logarithmic scale for the $y$-axis. In Ref. [54] it was assumed that all gluons inside photons originate from radiation off quarks; in contrast, the analysis of Ref. [53] includes a truly “intrinsic” gluon inside the photon. At the input scale, the three parametrizations for $f_{g\gamma}$ have the following functional forms:

$$f_{g\gamma}(x, Q_0^2 = 4 \text{ GeV}^2) \propto x^{-1.34}(1 - x)^{12.6} \quad (\text{LAC1});$$

$$f_{g\gamma}(x, Q_0^2 = 1 \text{ GeV}^2) \propto x^{5.9}(1 - x)^{0.56} \quad (\text{LAC3});$$

$$f_{g\gamma}(x, Q_0^2 = 1 \text{ GeV}^2) \propto x^{-1.41}(1 - x)^{4.5} \quad (\text{DG}).$$

Clearly, the LAC3 parametrization is rather “pathological”; however, even this extremely hard gluon distribution cannot be ruled out by data on $F_2^\gamma$ alone. Fig. 9 shows that it would lead to a rapidity distribution that is much more symmetric around $y^\gamma = 0$ than the predictions of the other two parametrizations. Notice that the LAC2 gluon distribution falls even more rapidly at large $x$ than is assumed in the LAC1 parametrization; it thus predicts even fewer events with $y^\gamma > 0$. We therefore conclude that a few dozen well measured $J/\psi + \gamma$ events at HERA would suffice to distinguish between the DG and the three LAC parametrizations, using only the shape of the rapidity distribution. Of course, there is no guarantee that any of these parametrizations will describe the data.

4 Probing Polarized Gluon Densities of Proton

Interest in high energy spin physics has been recently revived with the result from (and interpretations thereof) the EMC collaboration [55] on polarized $\mu - p$ scattering. Processes in polarized $pp$ collisions (such as achievable at an upgraded Fermilab fixed target facility or at a polarized collider [56]) sensitive to the polarized gluon content of the proton, such as jets [57, 58, 59], direct photons [59, 60, 61], and heavy quark production [62], have been discussed. Another intriguing suggestion, due to Cortes and Pire [53], is to consider $\chi_2(c\overline{c})$ production where the dominant lowest-order subprocess would be $gg \rightarrow \chi_2$. The partonic level asymmetries for $\chi_2/\chi_0$ production have been calculated in the context of potential models [63] and are large. Low transverse momentum quarkonium production in polarized $pp$ collisions using other methods has also been considered [62, 60] as has high $p_T$ $\psi$ production [64].

In all cases of charmonium production, the experimental signal is $\ell^+\ell^-$ ($\ell = e$ or $\mu$) with the lepton-lepton invariant mass giving the $J/\psi$ mass, since $\chi_J$ can decay radiatively to $J/\psi + \gamma$, and the $J/\psi$ signature is quite clean. As has been noted [58], the question of extracting the gluon distribution is made less clean by the multitude
of contributing processes, e.g.:

\[
\begin{align*}
    g + g & \rightarrow \chi_{0,2} \\
    g + g & \rightarrow \chi_J + g \\
    q + g & \rightarrow \chi_{0,2} + q \\
    q + \bar{q} & \rightarrow \chi_{0,2} + g \\
    g + g & \rightarrow J/\psi + g \\
    g + g & \rightarrow b(\rightarrow J/\psi + X) + \bar{b} \\
    q + \bar{q} & \rightarrow b(\rightarrow J/\psi + X) + \bar{b}.
\end{align*}
\] 

The simplicity of the Cortes and Pire idea is now gone. A full \(\mathcal{O}(\alpha_s^3)\) calculation of the spin-dependent production of \(\chi_J\) is necessary. At low \(p_T\), \(\chi_J\) production will also involve \(q + g\) and \(q + \bar{q}\) initial states, while at high \(p_T\) in addition the \(2 \rightarrow 2\) kinematics make the extraction of parton distribution functions less direct. Furthermore, a very careful calculation is required because even processes with small cross section can have a large effect on the asymmetry. The extraction of \(\Delta g(x, Q^2)\) using inclusive \(J/\psi\) will be a challenge.

Recently, \(J/\psi\) produced in association with a \(\gamma\) has been proposed as a clean channel to study the gluon distribution at hadron colliders [12]. The radiative \(\chi_J\) decays can produce \(J/\psi\) at both low and high \(p_T\), but the photon produced will be soft \(\langle E \sim \mathcal{O}(400 \text{ MeV})\rangle\). If we insist that the experimental signature consist of a \(J/\psi\) and \(\gamma\), with large but equal and opposite \(p_T\), there is only one production mechanism within the color-singlet model [12]:

\[
g + g \rightarrow J/\psi + \gamma. \tag{4.63}
\]

Following Ref. [12], this mechanism has been proposed in Ref. [69] to study the polarized gluon distribution in polarized fixed target experiments; we perform a more detailed analysis, including the analysis of this mechanism at the Brookhaven Relativistic Heavy Ion Collider (RHIC) at both 50 GeV and 500 GeV center of mass energy and at the Large Hadron Collider (LHC). Polarized proton-proton operation is being considered for RHIC, for at least several months data collection, while the tunnel design of the LHC has been modified for the possible future inclusion of the Siberian Snakes needed for polarized proton-proton mode. Also, we list the full set of helicity amplitudes for this process, explicitly stating the Lorentz frame in which the \(J/\psi\) helicities are given.

The full helicity amplitudes for \(g + g \rightarrow J/\psi + \gamma\) can be calculated following the approach of Gastmans, Troost and Wu [70], with the addition of explicit helicity polarization vectors for the \(J/\psi\). A convenient set of polarization vectors can be found in Böhm and Sack [71]. These polarization vectors reduce to the usual massive vector boson \((+,−,0)\) polarization vectors in the parton center of mass frame, and so, although the expressions for the helicity amplitudes have Lorentz invariant form, the \((+,−,0)\) only refer to the \(J/\psi\) helicity in this one particular frame. We find only one independent helicity amplitude \((M(++,++),\text{ where the ‘++,’++’ refer to the)}\)

15
helicity of $g_1g_2, \gamma J/\psi$ respectively), and the remaining 5 non-zero helicity amplitudes can be found by crossing and parity symmetries:

\[
M(++,++) = M(+-,--) = C \frac{\hat{s}(\hat{s} - M^2)}{(\hat{s} - M^2)(\hat{t} - M^2)(\hat{u} - M^2)} \\
M(+-,++) = M(--,+-) = C \frac{\hat{u}(\hat{u} - M^2)}{(\hat{s} - M^2)(\hat{t} - M^2)(\hat{u} - M^2)} \\
M(--,++) = M(+-,+-) = C \frac{\hat{t}(\hat{t} - M^2)}{(\hat{s} - M^2)(\hat{t} - M^2)(\hat{u} - M^2)}
\]

(4.64)

where $C = \frac{4e_cg_2^2R(0)M\delta^{ab}}{\sqrt{3\pi M}}$. Here, $M$ is the $J/\psi$ mass, $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables, $R(0)$ is the radial wavefunction at the origin of the $c\bar{c}$ in the $J/\psi$ and $a,b$ are the color indices of the incident gluons. Thus, the (spin and color) summed and averaged matrix element squared can be found [13]:

\[
|M(g + g \rightarrow J/\psi + \gamma)|^2 = \frac{(16\pi^2\alpha_s^2M|R(0)|^2}{27} \left[ \frac{\hat{s}^2}{(\hat{t} - M^2)^2(\hat{u} - M^2)^2} \right] \\
+ \frac{\hat{t}^2}{(\hat{u} - M^2)^2(\hat{s} - M^2)^2} \\
+ \frac{\hat{u}^2}{(\hat{s} - M^2)^2(\hat{t} - M^2)^2}
\]

(4.65)

$|R(0)|^2$ can be related to the leptonic width of the $J/\psi$:

\[
\Gamma(J/\psi \rightarrow e^+e^-) = \frac{16\alpha_c^2}{9M^2}|R(0)|^2 = 4.72 \text{ keV}
\]

\[
|R(0)|^2 = 0.48 \text{ GeV}^3.
\]

(4.66)

We are interested in the longitudinal spin-spin asymmetry, defined as:

\[
A_{LL} = \frac{\sigma(++) - \sigma(+-)}{\sigma(++) + \sigma(+-)}
\]

(4.67)

where $\sigma(++) (\sigma(+-))$ is the cross section for the collision of 2 protons with the same (opposite) helicities. This can be calculated in the parton model,

\[
A_{LL}\sigma = \int dx_1\, dx_2\, \hat{a}_{LL}\hat{\sigma}\, \Delta g(x_1,Q^2)\, \Delta g(x_2,Q^2)
\]

(4.68)

where $\hat{\sigma}$ is the parton level cross section (related to $|M|^2$ given earlier), $\Delta g(x,Q^2)$ is the polarized gluon distribution in the proton ($= (g^+(x,Q^2) - g^-(x,Q^2))$ where $g^+(x,Q^2)$ ($g^-(x,Q^2)$) is the distribution for gluons with the same (opposite) helicity as that of the proton) and $\hat{a}_{LL}$ is the parton level asymmetry

\[
\hat{a}_{LL} = \frac{\hat{\sigma}(++) - \hat{\sigma}(+-)}{\hat{\sigma}(++) + \hat{\sigma}(+-)}.
\]

(4.69)

Given the known helicity amplitudes for this process, the parton level asymmetry is simply

\[
\hat{a}_{LL} = \frac{\hat{s}^2(\hat{s} - M^2)^2 - \hat{t}^2(\hat{t} - M^2)^2 - \hat{u}^2(\hat{u} - M^2)^2}{\hat{s}^2(\hat{s} - M^2)^2 + \hat{t}^2(\hat{t} - M^2)^2 + \hat{u}^2(\hat{u} - M^2)^2}.
\]

(4.70)
Measurable quantities of interest are the $p_T$ distribution and the joint $p_T-y_1-\gamma$ distribution with $y_1 = y_2 = 0$, where $y_{1(2)}$ is the rapidity of the $\gamma$ ($J/\psi$). In the latter case, both partons have the same Bjorken-$x$ (which is a function of $p_T$ only). The corresponding asymmetries are given by:

$$A_{LL}^1 = \frac{\sigma(++) - \sigma(+-)}{\sigma(++) + \sigma(+-)}$$

$$A_{LL}^2 = \frac{d\sigma(++)}{dp_T} - \frac{d\sigma(+-)}{dp_T}$$

$$A_{LL}^3 = \frac{d\sigma(++)}{dp_T dy_1 dy_2|_{y_1=y_2=0}} - \frac{d\sigma(+-)}{dp_T dy_1 dy_2|_{y_1=y_2=0}} + \frac{d\sigma(++)}{dp_T dy_1 dy_2|_{y_1=y_2=0}} + \frac{d\sigma(+-)}{dp_T dy_1 dy_2|_{y_1=y_2=0}}.$$ (4.71)

Note that $A_{LL}^3$ is proportional to $[\Delta g(x(p_T), Q^2)]^2$. Another interesting theoretical concept (though not measurable experimentally) is the average $\hat{a}_{LL}$, or ‘resolving power’. It is defined in the following way:

$$\langle \hat{a}_{LL} \rangle = \frac{\int dx_1 dx_2 \hat{a}_{LL} \sigma g(x_1, Q^2) g(x_2, Q^2)}{N}. \quad (4.72)$$

As we wish to determine if a given experimental scenario can shed light on the size of the polarized gluon in the proton, we need, in addition to calculating the asymmetry, to estimate the experimental uncertainty in the asymmetry. We will approximate the uncertainty by the statistical uncertainty, since ratios of cross sections should be relatively free of systematic uncertainties. The statistical uncertainty in the measurement of an asymmetry is given by $\delta A$, where

$$\delta A = \frac{\sqrt{1 - A^2}}{\sqrt{N}}$$ (4.73)

and $N$ is the number of events.

We examine this process in several different experimental settings. First, we consider an hypothetical fixed target experiment and to be specific, take the proton beam energy to be 800 GeV (such as would exist at the upgraded Fermilab fixed target facility). In order to estimate the luminosity possible at such an experiment, we must make some assumptions. First, the Main Injector at Fermilab can provide $\sim 10^{14}$ (unpolarized protons)/sec, with a 65% duty cycle \[\text{[2]}\]. We’ll assume a one month run, at a much reduced proton rate (say, a factor of 100), combined with a small polarized gas ($H_2$) jet target (approximately 1 cm long). This will give, we think, a very conservative estimate of $\int L dt = 50$ pb$^{-1}$. We place no cuts on the rapidity of the photon or $J/\psi$, nor on the $p_T$ of the photon or leptons. We find a cross section of approximately 200 pb, most of which is at low $p_T$. The resolving power (or average $\hat{a}_{LL}$) is found to be about 28%.

We use the polarized distributions of Bourrely, Guillet and Chiappetta \[\text{[3]}\]. They provide 2 sets of distributions, one with a large polarized
gluon distribution and small polarized strange quark distribution (we’ll refer to it as the set BGC0) and one with a moderately large polarized gluon and moderately large polarized strange quark distribution (we’ll refer to this set as BGC1). The \( p_T \) distribution is shown in Figure 10a (in cross section) and in Figure 10b (in \( A_{LL}^2 \)). We were also interested the asymmetry \( A_{LL}^3 \), (technically, instead of taking \( y_1 \) and \( y_2 \) derivatives, we bin the events in the usual way, displaying the contents of the bin with \(-0.1 \leq y_1, y_2 \leq 0.1\)). The results are shown in Figure 12a (distribution in cross section) and 3b (\( A_{LL}^3 \) vs. \( p_T \)). We present in Table 1 the total number of events expected (at all \( p_T \) and \( y_1, y_2 \) consistent with our cuts) as well as the ‘resolving power’ and asymmetry \( A_{LL}^1 \) and an estimate of the statistical uncertainty, \( \delta A_{LL}^1 \). We also list the number of events in a single \( p_T \) bin (\( p_T \) given in the table caption), and \( A_{LL}^2 \) and \( \delta A_{LL}^2 \) for that particular \( p_T \) bin. Finally, we present the the number of events in the same \( p_T \) bin, further restricting the events to lie within \( |y_1, y_2| \leq 0.1 \), and the value \( A_{LL}^3 \) and \( \delta A_{LL}^3 \) in the particular \( p_T \) bin. These are representative results. Higher statistics can be obtained by the inclusion of all \( p_T \) bins.

At this point, we would like to further address the work of Ref. [69]. The large asymmetries shown are surprising, and in our opinion not correct. The parton level asymmetry, making the following replacements for \( \hat{t} \) and \( \hat{u} \) (i.e. working in the parton center of mass frame):

\[
\hat{t} = -\frac{1}{2}(\hat{s} - M^2)(1 - \cos \theta)
\]
\[
\hat{u} = -\frac{1}{2}(\hat{s} - M^2)(1 + \cos \theta)
\]

reduces to

\[
\hat{a}_{LL} = \frac{1 - \frac{1}{8}(1 + 6 \cos^2 \theta + \cos^4 \theta) + \frac{2M^2}{s}(1 - \cos^4 \theta) + \frac{M^4}{s^2}(1 - \cos^2 \theta)^2}{1 + \frac{1}{8}(1 + 6 \cos^2 \theta + \cos^4 \theta) + \frac{2M^2}{s}(1 - \cos^4 \theta) + \frac{M^4}{s^2}(1 - \cos^2 \theta)^2}. \tag{4.75}
\]

Here \( \cos \theta \) is measured in the parton center of mass frame. It is obvious that for \( \cos \theta = \pm 1 \), \( \hat{a}_{LL} \) is a minimum (actually zero), and so, for any \( \hat{s} \), the maximum of \( \hat{a}_{LL} \) should be at \( \cos \theta = 0 \). In this limit, the asymmetry reduces to

\[
\hat{a}_{LL}(\cos \theta = 0) = \frac{1 - \frac{1}{8}\left(\frac{\hat{s} + M^2}{s}\right)^2}{1 + \frac{1}{8}\left(\frac{\hat{s} + M^2}{s}\right)^2}. \tag{4.76}
\]

Two further limiting cases are possible, namely production at threshold (\( \hat{s} = M^2 \)) which gives \( \hat{a}_{LL} = \frac{1}{8} \) and production at very high energy (\( \hat{s} \to \infty \)) which gives \( \hat{a}_{LL} = \frac{7}{9} \). For \( \sqrt{s} = \sqrt{\hat{s}} = 38.75 \) GeV (the fixed target energy considered both here and in Ref. [69]), the parton level asymmetry is near its maximum value. Since \( \Delta g(x, Q^2)/g(x, Q^2) \leq 1 \) generally, the maximum observable asymmetry is bounded by the maximum parton level asymmetry. Thus we are unable to understand the prediction, in Ref. [69], that the observable asymmetry can be as large as 85%.

Next, we consider collider experiments at RHIC. RHIC is a high luminosity (\( \mathcal{L} = 2 \times 10^{32} \text{ cm}^{-2}\text{sec}^{-1} = 6000 \text{ pb}^{-1}/\text{yr} \)) collider capable of producing proton on
proton collisions for center of mass energies between 50 and 500 GeV. A program of polarized proton on proton collisions, at full energy and luminosity, is being discussed. We will assume a nominal running time of 2 months, at full luminosity, for 50 GeV and 500 GeV each. In order to be somewhat conservative, we will estimate event numbers based on 300 pb\(^{-1}\) integrated luminosity. We will assume a generic collider type detector, and in order to simulate the acceptance we will require the photon and electrons observed to lie in the rapidity range \(|y| \leq 2\) (this simulates the acceptance of the proposed STAR detector at RHIC, level 2 for photons and electrons. We will not consider the possibility of the detection of the \(\mu^+\mu^-\) final state at RHIC). Furthermore, we will (rather arbitrarily) require the \(p_T\) of the photon larger than 1 GeV in the following discussion. We present our results for the \(p_T\) distribution in Figure 12a, and \(A_{LL}^2\) in Figures 12b (\(\sqrt{s} = 50\) GeV) and 12c (\(\sqrt{s} = 500\) GeV). See Figure 13a for \(\frac{d\sigma}{dp_Tdy_1dy_2}\) vs. \(p_T\) and Figures 13b (\(\sqrt{s} = 50\) GeV) and 13c (\(\sqrt{s} = 500\) GeV) for \(A_{LL}^3\) vs. \(p_T\). The 'resolving power' increases with energy (actually \(p_T\)), even though the observed asymmetry decreases. This is simply a consequence of the behavior of the polarized gluon distribution. Please refer to Table 1 for some representative results.

In conclusion of this Section, we have studied the process \(p + p \rightarrow J/\psi + \gamma + X\) in polarized proton-proton collisions. We first presented the necessary helicity amplitudes and discussed the calculation. Then we studied this process at polarized fixed target and in colliders, at polarized RHIC (50 and 500 GeV center of mass energy). Our results indicate that a polarized (double spin) fixed target program can be very useful in the determination of the polarized gluon distribution. It is unfortunate that no such experiment is planned. RHIC (especially at lower energies) is an excellent probe of the polarized gluon distribution. Since \(A_{LL}^3\) is directly proportional to \(\Delta g(x(p_T), Q^2)/g(x(p_T), Q^2))^2\), this distribution provides an easy determination of the polarized gluon distribution at various \(x\) values. It will prove especially useful to measure this distribution at several center of mass energies. Even a measurement of \(A_{LL}^2\) can provide much useful information (though it is not clear whether the higher statistics involved in this measurement will outweight the cleanliness of the extraction of the polarized gluon distribution in a measurement of \(A_{LL}^3\)). The LHC can probe a much lower \(x\) in this process, and since \(\Delta g(x, Q^2)/g(x, Q^2) \ll 1\) there is no measurable asymmetry. However, the 'resolving power' at LHC can be still very large, so the smallness of the asymmetry is purely a consequence of the small-\(x\) behavior of \(\Delta g(x, Q^2)\). Polarized LHC can still be a useful tool for the study of high energy spin properties of the proton by utilizing a subprocess that will probe larger \(x\) (e.g. heavy Higgs production). We should also point out that we have considered only the color singlet model of heavy quarkonium production in this paper. A similar analysis can be performed using local duality, if it is determined at HERA that this mechanism contributes to \(J/\psi + \gamma\) production. Some slight modifications will be required, namely the inclusion of charm in the proton (this effect should be small) and light quark\(\bar{q}\) fusion, and in addition the modification of the parton level asymmetries.
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\[ \langle \hat{a}_{LL} \rangle, \ A_{iLL}^1(\delta A_{iLL}^1), \ N_{p_T}, \ A_{iLL}^2(\delta A_{iLL}^2), \ N_{p_T} |y_{1,2} \leq 0.1 \]

|                  | \( N_{TOT} \) | \( \langle \hat{a}_{LL} \rangle \) | \( A_{iLL}^1(\delta A_{iLL}^1) \) | \( N_{p_T} \) | \( A_{iLL}^2(\delta A_{iLL}^2) \) |
|------------------|----------------|----------------|-----------------|----------------|----------------|
| Fixed Target     | 10500          | 28.4%          | 12.5% (1%)      | 5000           | 16% (1.4%)     |
| RHIC 50 GeV      | 11430          | 43.3%          | 19.1% (1%)      | 4500           | 26% (1.5%)     |
| RHIC 500 GeV     | 86400          | 44.7%          | .4% (0.3%)      | 4500           | 1.7% (1.5%)    |

Table 1: Summary of representative predictions for \( J/\psi + \gamma \) production in polarized proton-proton interactions. \( N_{TOT} \) is the total number of events above some minimum \( p_T \) (= 0 GeV for fixed target, 1 GeV for RHIC). \( \langle \hat{a}_{LL} \rangle \) is the ‘resolving power’ as defined in the text (this is independent of the polarized parton distributions). \( A_{iLL}^1 \) and \( \delta A_{iLL}^1 \) are defined in the text; the upper entry corresponds to the large \( \Delta g(x, Q^2) \) (set BGC0) and the lower entry corresponds to the moderately large \( \Delta g(x, Q^2) \) (set BGC1). \( N_{p_T} \) is the number of events in the particular \( p_T \) bin (0.5-1.5 GeV for fixed target, 1-2 GeV for RHIC at 50 GeV, 3-5 GeV for RHIC at 500 GeV).

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Figure Captions

Figure 1 - Feynman diagrams for the color-singlet subprocess for $g + g \rightarrow (c\bar{c})(^3S^1_1) + \gamma$ and the color-octet subprocess for $g + g \rightarrow (c\bar{c})(^1S^0_0)$ or $^3P^j_{J^P} + \gamma$.

Figure 2 - Feynman diagrams for the effective $\gamma - g - (c\bar{c})(^1S^0_0)$, or $^3P^j_{J^P}$ vertex.

Figure 3 - Feynman diagram for the effective $q - \bar{q} - (c\bar{c})(^3S^0_1)$ vertex.

Figure 4 - Feynman diagrams for the color-octet contributions to the subprocesses (a) $g + g \rightarrow (c\bar{c})(^1S^0_0)$, or $^3P^j_{J^P} + \gamma$ and (b) $q + \bar{q} \rightarrow (c\bar{c})(^1S^0_0)$, or $^3P^j_{J^P} + \gamma$.

Figure 5 - Feynman diagrams for the color-octet contributions to the subprocess $q + \bar{q} \rightarrow (c\bar{c})(^3S^0_1) + \gamma$.

Figure 6 - The transverse momentum (a) and energy (b) spectrum of the photon and the two leptons from $J/\psi + \gamma$ production at the Tevatron collider with subsequent $J/\psi \rightarrow e^+e^-, \mu^+\mu^-$ decay, after the cuts of (3.51,3.52) have been applied. The subscripts “b” and “s” refer to the lepton with the bigger and smaller $p_T$ (in a) or energy (in b), respectively.

Figure 7 - The normalized rapidity distribution (a) and transverse momentum spectrum at the point $y_{\gamma} = y_{\psi} = 0$ (b) of the photon from $J/\psi + \gamma$ production at the Tevatron collider. Results for different parametrizations of the gluon content of the proton are compared: solid curves - set 1 of Ref. [51]; dashed curves - set 2 of Ref. [50].

Figure 8 - The transverse momentum (a) and energy (b) spectrum of the photon and the two leptons from $J/\psi + \gamma$ production at HERA with subsequent $J/\psi \rightarrow e^+e^-, \mu^+\mu^-$ decay, after the cuts of (3.57,3.58) have been applied. The meaning of the subscripts “b” and “s” is as in Fig. 2.

Figure 9 - The normalized rapidity distribution of the photon from $J/\psi + \gamma$ production at HERA. The solid, short dashed and long dashed curves have been obtained using the parametrizations of Ref. [54] and sets 1 and 3 of Ref. [53], respectively, for the gluon content of the photon.
Figure 10 - $p_T$ distribution, $\frac{d\sigma}{dp_T}$ vs. $p_T$ (a) and $A_{LL}^2$ vs. $p_T$ (b) for large $\Delta g(x, Q^2)$ (solid line) and for moderately large $\Delta g(x, Q^2)$ (dashed line) at fixed target.

Figure 11 - $\frac{d\sigma}{dp_T | y_1 = y_2 = 0}$ vs. $p_T$ (a) and $A_{LL}^2$ vs. $p_T$ (b) for large $\Delta g(x, Q^2)$ (solid line) and moderately large $\Delta g(x, Q^2)$ (dashed line) at fixed target.

Figure 12 - $p_T$ distribution, $\frac{d\sigma}{dp_T}$ vs. $p_T$ (a) for RHIC at $\sqrt{s} = 500$ GeV (solid line) and at $\sqrt{s} = 50$ GeV (dot-dashed line), and $A_{LL}^2$ vs. $p_T$ for RHIC at $\sqrt{s} = 50$ GeV (b) and at $\sqrt{s} = 500$ GeV (c) for large $\Delta g(x, Q^2)$ (solid line) and for moderately large $\Delta g(x, Q^2)$ (dashed line).

Figure 13 - $\frac{d\sigma}{dp_T | y_1 = y_2 = 0}$ vs. $p_T$ (a) for RHIC at $\sqrt{s} = 500$ GeV (solid line) and at $\sqrt{s} = 50$ GeV (dot-dashed line), and $A_{LL}^2$ vs. $p_T$ for RHIC at $\sqrt{s} = 50$ GeV (b) and at $\sqrt{s} = 500$ GeV (c) for large $\Delta g(x, Q^2)$ (solid line) and moderately large $\Delta g(x, Q^2)$ (dashed line).

(*) For Figures 1 – 5, please look Figs. 1 – 5 of hep-ph/9610294.

(*) For Figures 6 – 9, please look Figs. 2 – 5 of Z. Phys. C53 (1992) 673.

(*) For Figures 10 – 13, please look Figs. 1 – 4 of Phys. Rev. D49 (1994) 4463.