Nonlinear Disturbance Observer Backstepping Control for Electric Dynamic Load Simulator

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Abstract—Electric dynamic load simulator (EDLS) is an experimental device that can simulate the load torque of aircraft steering gear under indoor environment. It has the advantages of high loading accuracy, small inertia, quick response and easy operation. There are time-varying parameters and redundant torque disturbance in the EDLS, so the conventional control algorithm cannot get the ideal control effect. In this paper, a controller based on matching uncertain nonlinear system is proposed based on the idea of backstepping control. Lyapunov method is used to prove the asymptotic stability of the closed-loop system. At the same time, the nonlinear disturbance observer is used to reduce the vibration of the controller. Experimental results show the effectiveness of the proposed control strategy.

1. Introduction
Under different flight conditions, the motion mechanism of the aircraft will bear the load torque with instantaneous changes. The load simulator is an important equipment for testing the performance of steering gear, which can reproduce the hinge torque borne by the actuating mechanism in the flight environment in the semi-physical simulation on the ground and test the performance of the specimen under near-real load [1].

Electric dynamic load simulator (EDLS) driven by servo surface permanent magnet synchronous motors (SPMSM) has been widely studied. It has advantages of low torque ripple and wide range of speed regulation. However, friction, tooth gap, sensor measurement accuracy and other nonlinear complex factors inevitably exist in the mechanism, which makes EDLS have high-order nonlinear characteristics. ELDS is a typical passive loading system, the main research difficulty lies in reducing extraneous torque other than loading torque [1]. Existing control strategies include feedforward compensation [2], neural network adaptive compensation [3], and fuzzy adaptive PID control [4]. However, the control strategy based on intelligent algorithms such as neural networks is difficult to implement due to its controllers and has poor real-time performance. In most cases, only simulation verification is performed, which is difficult to apply to engineering.
Figure 1. Structure of electric load simulator.

According to the characteristics of the electric load simulator, this paper proposes a backstepping control strategy for electric load simulation system based on nonlinear disturbance observer. The electric load simulator is divided into three subsystems for control design. Equivalent control and sliding mode adaptive method are used to design the virtual control amount of each subsystem separately. The quadrature axis current control law is obtained recursively, and Lyapunov is applied. The method proves the asymptotic stability of the closed-loop system. Then the non-linear disturbance observer is used to suppress the chattering of the switching control. Under the condition that the control effect does not change, the control effect of the controller is guaranteed.

2. System Structure and Mathematical Model

The structure of EDLS is shown in Figure 1. It consists of the tested servo, optical encoder, torque sensor, and loading motor. As a driving element of the EDLS system, the permanent magnet synchronous motor is connected to the torque sensor and the tested servo through a coupling. The torque sensor measures the load torque on the transmission mechanism in real time, and the encoder feedbacks the position and speed signals in real time to form a feedback loop.

The mathematical model of the permanent magnet synchronous motor used for loading can be expressed as:

\[
\begin{aligned}
\dot{u}_d &= R_m i_d + L_m i_d - n_\omega L_m i_q \\
\dot{u}_q &= R_m i_q + L_m i_q - n_\omega (L_m i_d + \phi_f) \\
\dot{\omega}_r &= \frac{1}{J_m} (T_e - T_L - B_m \omega_r) \\
T_e &= \frac{3}{2} n_p \phi_f i_q
\end{aligned}
\]  

(1)

In the formula, \( i_d, i_q, u_d, u_q \) are used to represent the current and voltage of the d-axis and q-axis of the PMSM, \( \phi_f, R_m, L_m \) are the flux linkage, stator winding resistance, equivalent winding inductance, \( T_e \) is the output electromagnetic torque coefficient, \( T_L \) is the load torque, \( n_p \) is the number of pole pairs, \( B_m \) is the viscosity coefficient, \( J_m \) is the rotor inertia.

According to Hooke's law, the sensor model is shown as

\[
T_L = K_s \left[ \theta - \theta_f \right]
\]

(2)

Where \( \theta_f \) is the angle output of the loaded servo, \( \theta_s \) is the angle output of the load simulation system and \( K_s \) is the equivalent stiffness of the sensor. Derivation of the equation and taking into account the measurement error and unmodeled dynamic available torque change rate:

\[
\dot{T}_L = K_s \left[ \omega - \omega_f \right] + d
\]

(3)
The dynamic characteristics of the current loop are directly related to the electromagnetic torque, which is a key indicator of the performance of the loading system. As the winding temperature changes during the operation of the motor, the electromagnetic characteristics will change, such as resistance, inductance and other parameters. So model errors and unmodeled dynamics are treated as unknown disturbances, Simultaneous equations (1) and (3).

\[
\dot{\xi} = K \left( \omega - \omega_f \right) + d_1
\]
\[
\dot{\omega} = \frac{1}{J_m} \left( k_i x_3 - B_m \omega - T_f \right) + d_2
\]
\[
i_q = \frac{1}{L_m} \left( u_q - R_m i_q - n_p i_d \omega \right) - n_p \phi_f i_d + d_3
\]
\[
i_d = n_p \phi_f i_q - \frac{1}{L_m} \left( u_d - R_m i_q \right) + d_4
\]
\[
y = \omega
\]

Where \( d_i (i=1,2,3,4) \) is the system disturbance.

3. Controller Design and Certification

According to the structure of the torque load simulation system, set \( i_q = x_1, \omega = x_2, T_L = x_3, \) in order to simplify the calculation, let the \( i_d = 0 \), and make the load motor equivalent to be a DC motor. Equation form as:

\[
\begin{align*}
\dot{x}_1 &= K \left( x_2 - \omega_f \right) + d_1 \\
\dot{x}_2 &= \frac{1}{J_m} \left( -x_1 - B_m x_2 + K x_3 \right) + d_2 \\
\dot{x}_3 &= \frac{k_p x_3}{L_m} - \frac{R}{L_m} x_3 + \frac{u_q}{L_m} + d_3 \\
y &= x_1
\end{align*}
\]

Among them, \( x_1, x_2, x_3 \) represent the load output \( T_L \), the load simulator speed \( \omega \) and the quadrature-axis current \( i_q \), \( k_p \) are the back-EMF coefficients, \( K \) is the torque constant, and \( \omega_f \) can be measured in real time by the encoder on the servo side of the test, it can be regarded as a known quantity. \( x_{id} \) is the given signal of the load simulation system.

Assumption 1: The given load signal instruction is sufficiently smooth, and its nth derivative is present and bounded.

Assumption 2: Unknown disturbance \( d \) is bounded, \( d_i < \overline{d}_i (i=1,2,3) \), the upper bound of \( \overline{d}_i \) has already known.

3.1. Controller Design

The main idea of backstepping theory is to disassemble a complex high-order system into several lower-order subsystems, and then design virtual controllers for each subsystem in turn starting from the subsystem where the final control amount is located. The final control signal of the system is obtained recursively through a series of virtual signals. In order to reduce the impact of unknown disturbances on the system, an auxiliary controller is designed to enhance the robustness of the system. And through the use of DOB to reduce the gain of the auxiliary controller, reduce the controller input jitter.

Step 1. Design a virtual control quantity \( x_{ud} \), so that \( x_1 \) can realize the tracking of \( x_{ud} \) without static error within a limited time. Define the error variable \( e_i \) as:

\[
e_i = x_{ud} - x_i
\]
The control goal is to make $e_1$ converge to 0 in a limited time. The virtual control term $x_{2d}$ can be expressed as:

$$
\begin{align*}
\dot{x}_{2d} &= x_{2deq} + x_{2daux} \\
\dot{x}_{2deq} &= \frac{1}{K_s} \left( \dot{x}_{id} + K_s \omega_f + k_1 e_1 \right) \\
x_{2daux} &= -\hat{\delta}_1 \text{sgn}(e_1)
\end{align*}
$$

(7)

Where $x_{id}$ is the equivalent control term without external disturbance, $x_{daux}$ is the auxiliary control term for uncertain disturbance, $\hat{\delta}_1$ is the estimated value of the uncertainty upper bound $\delta_1$, and the estimation error satisfies $i_1 = \delta_1 - \hat{\delta}_1$. The adaptive law of $\hat{\delta}_1$ is

$$
\dot{i}_1 = \gamma_1 |i_1|, \quad \gamma_1 > 0 \quad \text{is the design constant,}
$$

$\text{sgn}(\cdot)$ is the sign function, and $k_i > 0$ is the controller parameter ($i=1,2,3$).

Define the error variable $e_2 = x_{2d} - x_2$, take the first Lyapunov equation as:

$$
V_1 = \frac{1}{2} e_1^2 + \frac{1}{2\gamma_1} \hat{\delta}_1^2
$$

(8)

Differentiate $V_1$ and bring (7) into Equation as

$$
\begin{align*}
\dot{V}_1 &= e_1 \dot{e}_1 = e_1 \left[ \dot{x}_{id} - K_s \left( x_{2d} - e_2 \right) + K_s \omega_f + d_1 \right] - \frac{1}{\gamma} \hat{\delta}_1 \dot{\hat{\delta}}_1 \\
&\leq -k_e e_1^2 + K_s e_1 e_2 + |e_1| |\delta_1 - \hat{\delta}_1| - |e_1| |\delta_1| \\
&= -k_e e_1^2 + K_s e_1 e_2
\end{align*}
$$

(9)

The $K_s e_1 e_2$ items will be eliminated in the next design.

Step 2. Design a virtual control quantity $x_{3d}$, so that $x_3$ can realize the tracking of $x_{2d}$ without static error within a limited time. The virtual control term $x_{3d}$ can be expressed as:

$$
\begin{align*}
\dot{x}_{3d} &= x_{3deq} + x_{3daux} \\
\dot{x}_{3deq} &= \frac{1}{K_f} \left( J_m K_e e_1 + J_m \dot{x}_{2d} + x_1 + B_m x_2 + J_m k_2 e_2 \right) \\
x_{3daux} &= -\hat{\delta}_2 \text{sgn}(e_2)
\end{align*}
$$

(10)

Define the error variable $e_3$ as:

$$
e_3 = x_{3d} - x_3
$$

(11)

Take the second Lyapunov equation as:

$$
V_2 = V_1 + \frac{1}{2} e_2^2 + \frac{1}{2\gamma_2} \hat{\delta}_2^2
$$

(12)

Differentiate $V_2$ and bring (10) into the following equation as:

$$
\begin{align*}
\dot{V}_2 &= \dot{V}_1 + e_3 \dot{e}_3 = -k_e e_1^2 + K_e e_1 e_2 + e_2 \dot{e}_2 - \frac{1}{\gamma} \hat{\delta}_2 \dot{\hat{\delta}}_2 \\
&= -k_e e_1^2 + e_2 \left[ K_s e_1 - \frac{1}{J_m} \left( -x_1 - B_m x_2 + K_s x_3 \right) \right] - \frac{1}{\gamma} \hat{\delta}_2 \dot{\hat{\delta}}_2 \\
&= -k_e e_1^2 + k_2 e_2^2 + \frac{K_s e_1 e_2}{J_m} + e_2 \dot{d}_2 - \hat{\delta}_2 \text{sgn}(e_2) - \frac{1}{\gamma} \hat{\delta}_2 \dot{\hat{\delta}}_2 \\
&= -k_e e_1^2 + k_2 e_2^2 + \frac{K_s e_1 e_2}{J_m} + e_2 \left| \dot{d}_2 \right| - \left| e_2 \right| |\dot{\hat{\delta}}_2| \\
&= -k_e e_1^2 + k_2 e_2^2 + \frac{K_s e_1 e_2}{J_m}
\end{align*}
$$

(13)
The $\frac{K_l}{J_m}e_2 e_3$ items will be eliminated in the next design.

Step 3. Design a real control quantity $u_q$, so that $x_3$ can realize the tracking of $x_{3d}$ without static error within a limited time. The virtual control term $u_q$ can be expressed as:

$$u_q = u_{qeq} + u_{qaux}$$

$$u_{qeq} = L \left( \frac{k}{J_m} e_2 + \dot{x}_{3d} + k_s e_3 \right) + R_m x_3 + K_e x_2$$

$$u_{qaux} = -\delta_s \text{sgn}(e_3)$$  \hfill (14)

3.2. The Proof of System Stability

Take the overall Lyapunov equation of the control system as:

$$V_j = V_j + \frac{1}{2} e_j^2 + \frac{1}{2} \delta_j^2$$  \hfill (15)

Differentiate $V_j$ and bring (14) into

$$\dot{V}_j = \dot{V}_j = \dot{V}_2 + e_j \dot{e}_j - \frac{1}{\gamma} \delta_j \dot{\delta}_j$$

$$= -k_1 e^2 - k_2 e_2^2 + K_l e_2 e_3 + e_j \dot{e}_j - \frac{1}{\gamma} \delta_j \dot{\delta}_j$$

$$= -k_1 e^2 - k_2 e_2^2 + e_j \left( \frac{K_l}{J_m} e_3 + \dot{e}_3 \right) - \frac{1}{\gamma} \delta_j \dot{\delta}_j$$

$$= -k_1 e^2 - k_2 e_2^2 + k_s e_3^2 + e_j \left( \delta_j \dot{\delta}_j - \delta_j \dot{\delta}_j \right)$$

$$\leq -k_1 e^2 - k_2 e_2^2 + k_s e_3^2 + |e_j| \delta_j \delta_j \leq 0$$

Set $a_0 = \min \{2k_1, 2k_2, 2k_3\}$

$$\dot{V}_j \leq a_0 \sum_{i=1}^{n} \frac{1}{2} e_i^2 \leq -a_0 V_j$$  \hfill (17)

All signals in the closed-loop system exponentially converge to 0, and the closed-loop system is exponentially stable.

3.3. Disturbance Observer Design

The basic idea of designing disturbance observer (DOB) is to use the difference between the model output and the actual output to modify the control amount. In order not to lose generality, consider a nonlinear first-order system:

$$\dot{x} = a(x)u + b(x) + d$$  \hfill (18)

Defining the Nonlinear Disturbance Observer Dynamic Equation as:

$$\begin{cases} z = \dot{d} = p(x) \\ \dot{z} = -L(x)z + L(x) \left( -p(x) - a(x)u - b(x) \right) \end{cases}$$  \hfill (19)

Where $p(x)$ satisfies the equation:

$$\dot{p}(x) = L(x)\dot{x}$$  \hfill (20)
Where $\hat{d}$ is the value of DOB, and $L(x)$ is the observer bandwidth. It is assumed that the change of the interference is slow relative to the dynamic characteristics of the observer, that is $\dot{d} = 0$. Let the observation error $\tilde{d}$ as

$$\tilde{d} = d - \hat{d}$$  \hspace{1cm} (21)

The observation error dynamic equation is defined as the observation error is

$$\dot{\tilde{d}} = \tilde{d} - \hat{d} = -L(x)\tilde{d}$$  \hspace{1cm} (22)

If $L(x) > 0$, the observer is globally asymptotically stable, so selecting the appropriate $p(x)$ does not require the information of $\dot{x}$, which brings great convenience to practical applications.

The input of the disturbance observer for each order of the system is

$$\begin{align*}
x_{2dd} &= K_1 \tilde{d}_1 \\
x_{3dd} &= \frac{J_m}{K_1} \tilde{d}_2 \\
u_{qd} &= L_m \tilde{d}_3
\end{align*}$$  \hspace{1cm} (23)

The input of each stage of the controlled system is:

$$\begin{align*}
x_{2d} &= x_{2deq} + x_{2aux} - x_{2dd} \\
x_{3d} &= x_{3deq} + x_{3aux} - x_{3dd} \\
u_q &= u_{qeq} + u_{qaux} - u_{qd}
\end{align*}$$  \hspace{1cm} (24)

After the disturbance observer is added, the interference term changes from $d_i (i = 1, 2, 3)$ into $\hat{d}_i = d_i - \tilde{d}_i$. It can be seen that $\tilde{d}_i \to d_i$ is achieved through the DOB, and the auxiliary control term gain $\hat{\delta}_i$ will be greatly reduced, which will reduce the controller vibration.

The controller designed in this paper has the following characteristics:

In the design of the virtual control quantity, the design of the disturbance observer and auxiliary controller is used to convert the non-matching uncertainty into the matching uncertainty.

The auxiliary controller method is used to design the interference suppression term to make the system more robust.

The finite-time tracking differentiator design method proposed by Lao [5] is used to calculate the high-order derivative of the virtual control variable to obtain a smooth differential estimation signal.

Based on the finite-time convergence of the tracking differentiator, the differential estimation error at $t \geq t_0$ can be considered as Zero, so it can offset the differential expansion in conventional backstepping control.

Figure 2. EDLS experiment system.

Compared with the method of filtering to obtain the differential approximation in dynamic surface control, it has higher control accuracy.
4. Experimental Verification and Analysis

In order to verify the effectiveness of the designed controller, an experimental system is shown in Figure 2.

The controller was selected from NI’s CompactRIO series controller. Because it runs a VxWorks-based real-time system and is equipped with an FPGA chip, it can ensure the real-time performance of the control system. Parameters of EDLS is shown in Table 1.

In order to test the effect of the controller, a typical 3Hz, 8Hz, 15Hz, sinusoidal signal with a maximum load torque of 10Nm is used as the load signal. The tested servo performs a sinusoidal motion with a frequency of 8Hz and an amplitude of 0.5rad. The test results are shown in Figure 3 and Figure 4.

The system has no divergence and vibration. With the increase of the loading frequency, the system output basically overlaps with the desired signal, and the phase delay does not change significantly, so it tracks the desired signal well. The maximum values of excess torque are 4.6%, 6.35%, and 8.1%, respectively.

Compared with the controller signal at the loading frequency of 15hz, as shown in Figure 5, \( u_q \) ’s jitter is significantly reduced when using a nonlinear interference observer.

### Table 1. Parameters of EDLS

| Symbol | Value             |
|--------|-------------------|
| \( J_m \) | \( 1.13 \times 10^{-4} \) kgm |
| \( K_e \) | \( 31.69 \) Nm·A(·rpm)^{-1} |
| \( K_t \) | \( 0.88 \) N·m·A^{-1} |
| \( R_m \) | \( 0.26 \) Ω |
| \( L_m \) | \( 2.81 \) mH |
| \( B_m \) | \( 1.8 \times 10^{-4} \) Nms·rad^{-1} |
| \( K_s \) | \( 7100 \) N·m·rad^{-1} |
5. **Conclusion**

This paper proposes a backstepping control strategy of electric dynamic load simulation system based on nonlinear disturbance observer. The equivalent control plus adaptive control switching method is used to design the virtual control amount of each subsystem separately. The quadrature axis current control law is obtained by recursion. In order to reduce the controller chatter, a nonlinear disturbance observer is used to compensate for unknown disturbances and reduce the controller jitter.

The experimental results verify the effectiveness of the control algorithm. When the loading frequency and the tested servo's movement frequency increase, the system output basically overlaps with the expected signal, and the phase delay does not change significantly, and the expected signal is tracked well. However, the extraneous torque increases with the increase of the loading frequency. Although the experimental results basically meet the requirements, how to further improve the control effect under high-frequency signals will be the main direction of future research.
Figure 4. Redundant torque of EDLS at various loading frequency.

(a) Without DOB

(b) Within DOB

Figure 5. Controller output without/within DOB.

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