CONFINEMENT OF SPIN AND CHARGE IN
HIGH-TEMPERATURE SUPERCONDUCTORS

J. P. Rodriguez\textsuperscript{(a)} and Pascal Lederer\textsuperscript{(b)}

\textsuperscript{(a)} Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545 and Dept. of Physics and Astronomy, California State University, Los Angeles, CA 90032.

\textsuperscript{(b)} Laboratoire de Physique des Solides, Université Paris-Sud, 91405 Orsay Cédex, France. †

Abstract

By exploiting the internal gauge-invariance intrinsic to a spin-charge separated electron, we show that such degrees of freedom must be confined in two-dimensional superconductors experiencing strong inter-electron repulsion. We also demonstrate that incipient confinement in the normal state can prevent chiral spin-fluctuations from destroying the cross-over between strange and pseudo-gap regimes in under-doped high-temperature superconductors. Last, we suggest that the negative Hall anomaly observed in these materials is connected with this confinement effect.

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† Laboratoire associe au CNRS.
The electronic conduction in high-temperature superconductors is characterized by quasi two dimensionality and strong inter-electron repulsion. Motivated by the corresponding situation in one dimension, Anderson proposed early on that the correlated electron in these materials factorizes into independent spin and charge degrees of freedom as a result.\textsuperscript{1} The abelian gauge-field theory formulation of the $t-J$ model for strongly interacting two-dimensional (2D) electrons provides an elegant expression of these ideas.\textsuperscript{2–4}

The electron field in such theories is divided up into fermionic spin and bosonic charge parts, $c_{i\sigma} = f_{i\sigma}^\dagger b_i^\dagger$, where the internal gauge invariance $(f_{i\sigma}, b_i) \rightarrow e^{i\theta_i} (f_{i\sigma}, b_i)$ enforces the constraint

$$\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1 \quad (1)$$

against double occupancy at each site. Here, both the spinon field $f_{i\sigma}$ and the holon field $b_i$ are treated as dynamical quasiparticles that interact with the statistical gauge field $a_\mu$ associated with the latter. Apart from its formal appeal, the gauge-field approach to 2D electron liquids successfully accounts for many of the strange normal-state properties characteristic of high-temperature superconductors; e.g., the paradoxical observation of a large Luttinger Fermi surface in conjunction with a hole-type Hall effect, and the unconventional $T$-linear resistivity.\textsuperscript{4}

Analogous Ginzburg-Landau type gauge-field theories for spin-charge separated superconductivity in two dimensions have also been proposed in order to account for the high-$T_c$ phase diagram (see Fig. 1).\textsuperscript{5–7} Separate superfluid mean-field transitions for the spinon and holon sectors at respective critical temperatures of $T_f$ and $T_b$ provide the common basis for these theories. The Ioffe-Larkin composition formula\textsuperscript{2} $\rho = \rho_b + \rho_f$ for the total resistivity implies a meanfield superconducting transition temperature of $T_{c0} = \min(T_f, T_b)$, while $T_{s0} = \max(T_f, T_b)$ marks the cross-over temperature scale above which strange metallicity (e.g., $\rho \propto T$) sets in. The latter cross-over phenomenon is consistent with a host of normal-state properties generally observed in high-temperature superconductors.\textsuperscript{8,9} It has been recently pointed out,\textsuperscript{10} however, that the entropy generated by the transverse component to the statistical gauge-field, which physically represents chiral spin fluctuations,\textsuperscript{11} completely suppresses the spinon-superfluid crossover in the under-doped regime ($T_b \ll T_f$). If correct, this means that a normal-state pseudo-gap\textsuperscript{12} is not theoretically possible in single-layer oxide superconductors, though one could still exist in multi-layered compounds.\textsuperscript{13}
In general, however, the longitudinal component of the statistical gauge field must also be taken into account in order to properly enforce the constraint (1) against double occupancy. In the case of the strange-metal saddle-point of the $t-J$ model, for example, this component is known to result in a negative entropy contribution,$^{14}$ as well as a slow-zero sound mode near the Mott transition.$^{15}$ In the commensurate flux-phase (CFP) saddle point,$^{16}$ which is believed to be an anyonic superconductor,$^{17,18}$ the longitudinal component leads to confinement of the spinon and the holon into the electron.$^{19}$ On the basis of a phase-only Ginzburg-Landau model for spin-charge separated superconductivity known as the two-component Abelian Higgs (AH$^2$) model,$^7$ and its consistency with the latter, we shall first show in this paper that the spin-charge components of the superfluid electron are confined in the superconducting phase, $T < T_c$. Here, the superfluid transition at $T_c$ is explicitly of the Kosterlitz-Thouless type. We shall then demonstrate that incipient confinement is present in the spin-gap (or Fermi liquid) regime $T < T < T_∗$ if the spinons and the holons couple weakly to the statistical gauge field. This leads to a drastic non-perturbative suppression of gauge-field excitations. (The former can be interpreted as Aslamasov-Larkin type fluctuations$^{20}$ of the confined superconductor within the deconfined normal state.) We conclude that the spin-gap cross-over at $T_∗ \approx T_f$ in the underdoped regime remains intact for weak-coupling saddle-points, of which the CFP anyon superconductor is one.$^{18}$ This means that the unique cross-over phenomenon observed in a variety of normal-state properties of under-doped oxide superconductors$^8$ can in principle be accounted for by the gauge-field theory approach to strongly interacting electrons on the square lattice. In the case of strong coupling to gauge-field excitations, however, we find that the cross-over into the spin-gap (and Fermi liquid) regimes is destroyed.$^7,21$ This is consistent with recent studies of the high-temperature strange-metal phase,$^{10}$ which also couples strongly to gauge-field excitations.

**Spin-charge Separated Superconductivity.** The Ginzburg-Landau energy functional of the AH$^2$ model for spin-charge separated superconductivity on the square lattice reads

$$E = J_b \sum_{r,\mu} \{1 - \cos[\Delta_\mu \phi_b - qa_\mu]\} + J_f \sum_{r,\mu} \{1 - \cos[\Delta_\mu \phi_f - qa_\mu]\} + 
+ \chi_d \sum_{r} \{1 - \cos[\Delta_x a_y - \Delta_y a_x]\}, \quad (2)$$

where $\phi_b(r)$ and $\phi_f(r)$ represent the respective phases of the holon and spinon order
parameters, \( a_\mu(r) = a_\mathcal{F} r + \mathcal{\hat{a}}_\mu \) denotes the statistical gauge-field, and \( \Delta_\mu \) denotes the lattice difference operator \( (\mu = x, y) \). The first two terms above are the stiffness energies for the phase fluctuations, where the local rigidity of each specie \( i = f, b \) is related to the critical temperature of its presumed Kosterlitz-Thouless (KT) transition by \( k_B T_i = \frac{\pi}{2} J_i \). The last term above is the stiffness energy for gauge-field fluctuations, where the corresponding local rigidity is given by the sum \( \chi_d = \chi_f + \chi_b \) of the diamagnetic susceptibilities of each species, \( \chi_f \) and \( \chi_b \). We assume that the superfluidity in both the spinon and the holon subsystems results from Cooper pairing, hence the choice \( q = 2 \). If we integrate out first the gauge field excitations from the corresponding partition function

\[
Z = \int D\phi_b D\phi_f D\alpha_\mu \exp(-E/k_B T) \tag{3}
\]

in the continuum limit, where \( E = \frac{1}{2} \int d^2 r [J_b (\bar{\nabla} \phi_b - q \mathcal{\hat{a}})^2 + J_f (\bar{\nabla} \phi_f - q \mathcal{\hat{a}})^2] \), we obtain an effective free energy \( E_{el} = \frac{1}{2} \bar{J} \int d^2 r [\bar{\nabla} \phi_{el}^{(v)}]^2 \) for configurations of the physical electronic phase \( \phi_{el} = \phi_f - \phi_b \) that carry non-zero vorticity \( (v) \), where \( \bar{J} = (J_f^{-1} + J_b^{-1})^{-1} \). This means that the spin and charge of the superfluid electron are confined while traversing the KT transition at \( T_c = (T_f^{-1} + T_b^{-1})^{-1} \). If, on the other hand, we first integrate out the phase (Higgs) fields from (3), we then obtain the effective free energy

\[
E_{st} = \frac{1}{2} \mu_{st}^{-1} \chi_d \int d^2 r [\partial_x a_y - \partial_y a_x]^2 \tag{4}
\]

for the statistical gauge field in the continuum limit, where each Higgs field contributes a diamagnetic renormalization \( \mu_f^{-1}, \mu_b^{-1} > 0 \) to the total renormalization

\[
\mu_{st}^{-1} = 1 + \mu_f^{-1} + \mu_b^{-1} \tag{5}
\]

of the statistical magnetic field energy. The non-perturbative effects mentioned above are included by making the replacement \( \chi_d \to \chi_d/\mu_{st} \) in all perturbative calculations. Before continuing, let us first identify the statistical London penetration length of the AH\(^2\) model (2),

\[
\lambda_{st} = q^{-1} [\chi_d/(J_f + J_b)]^{1/2}, \tag{6}
\]

here given in units of the lattice constant \( a \). In the weak-coupling limit \( \lambda_{st} \to \infty \), we recover separate superfluid transitions for each species. This implies that \( \mu_{st} = 0 \) at
temperatures $T < T_{s0} = \max(T_f, T_b)$. Below, we will demonstrate that $T_{s0}$ in fact only marks a sharp cross-over\textsuperscript{6,9,21} for finite $\lambda_{st} \gg a$. It will then be demonstrated that the CFP anyon superconductor lies inside the latter weak-coupling regime.

Recall that if one of the two space dimensions of the AH\textsuperscript{2} model is considered as imaginary time, then the pure gauge-field term in (2) describes vacuum electromagnetism in one dimension. Since the latter is trivially confining, the Wilson loop for the AH\textsuperscript{2} model (2) is in general related to the effective diamagnetic renormalization (4) due to the Higgs fields by\textsuperscript{21}

$$\left\langle \exp \left( ip \oint_C a_\mu dx_\mu \right) \right\rangle = \exp \left( -\frac{1}{2} \mu_{st} p^2 g^2 A \right)$$

in the limit $p \to 0$, where $A$ denotes the area contained by a large contour $C$ and $g^2 = k_B T/\chi_d$. Employing the Villain form of the AH\textsuperscript{2} model (2),\textsuperscript{7} it can be shown that the above Wilson loop $W(C)$ is given by the average

$$W(C) = \left\langle \exp \left[ -2\pi i \frac{p}{q} (J_f + J_b)^{-1} \sum_{r \text{ in } C} [J_f q_f(r) + J_b q_b(r)] \right] \right\rangle_{CG}$$

over the corresponding Coulomb gas ensemble

$$Z_{CG} = \sum_{\{q_b, q_f\}} \exp \left\{ -\frac{1}{2} \mathcal{B} \sum_{(r,r')} \left[ G_{lr}(\vec{r} - \vec{r}') q_{el}(r) q_{el}(r') + G_{sr}(\vec{r} - \vec{r}') q_{st}(r) q_{st}(r') \right] \right\},$$

where the physical electronic (el) flux-charge and the statistical (st) flux-charge are given respectively by linear combinations

$$q_{el} = q_f - q_b,$$

$$q_{st} = \left( \frac{J_f}{J_b} \right)^{1/2} q_f + \left( \frac{J_b}{J_f} \right)^{1/2} q_b,$$

of integer fields $q_f(r)$ and $q_b(r)$. Here $(r, r')$ denote combinations of points covering the dual square lattice, while $\mathcal{B} = \bar{J}/k_B T$. Corresponding to these charges are long-range (lr) and short-range (sr) potentials

$$G_{lr} (\vec{r}) = \int_{\text{BZ}} \frac{d^2 k}{(2\pi)^2} e^{i \vec{k} \cdot \vec{r}} \frac{1}{k^2 + \xi_{el}^{-2}},$$

$$G_{sr} (\vec{r}) = \int_{\text{BZ}} \frac{d^2 k}{(2\pi)^2} e^{i \vec{k} \cdot \vec{r}} \frac{1}{k^2 + \lambda_{st}^{-2}},$$

5
where $k^2 = 4 - 2\cos k_x a - 2\cos k_y a$, and where $\xi_{el}$ denotes the correlation length of the physical electronic condensate that diverges in the normal state at $T_c$. It is known that the Wilson loop (8) shows a perimeter law in the superconducting phase.\(^7\) By (7), this implies that $\mu_{st} = 0$ for $T < T_c$; i.e., fluctuations of the statistical magnetic field in spin-charge separated superconductors are entirely suppressed! In the normal state, the correlation length $\xi_{el}$ is exponentially large but finite at temperatures close to $T_c$. Since global vortex charge neutrality is then no longer enforced, the most important configurations per site are $q_f = 0, \pm 1$, but with $q_b = q_f$ so that there be no net electronic vorticity $q_{el}$. Standard manipulations\(^2\(^2\) of expressions (8) and (9) then yield that the diamagnetic renormalization (7) is given by

$$\mu_{st}(T_c+) = 16\pi\beta_c\lambda_{st}^{-2(\beta_c-1)}, \quad (14)$$

where $\beta_c = [(T_b/T_f)^{1/2} + (T_f/T_b)^{1/2}]^2$. At optimum doping, $T_b = T_f$, we therefore have $\max\mu_{st}(T_c+) = \lambda_{st}^6$, which implies a small jump in chiral spin fluctuations by Eq. (4).\(^1\(^9\) Also, Eqs. (4) and (14) indicate that chiral spin fluctuations are suppressed ($\mu_{st} \to 0$) exponentially fast as the system deviates from the optimum $T_c$ along the superconducting phase boundary. This is consistent with the narrow window in composition centered at optimum doping through which strict $T$-linear resistance is observed in the normal state of high-temperature superconductors.\(^8\)

Consider now the spin-gap regime $T_b \ll T_f$ in the same weak-coupling limit $\lambda_{st} \gg a$. Then the diamagnetic renormalization due to the spinon subsystem is approximately given by that of the one component model\(^2\(^1\) \(2\) \(3\) \((J_b = 0)\), which is known to be $1 + \mu_{f}^{-1} = (16\pi)^{-1}(T/T_f)\lambda_{st}^{2(T_f/T)-1}$. On the other hand, at temperatures $T \gg T_b$, a direct high-temperature series analysis of the holon term in (2) yields that\(^2\(^3\) $\mu_{b}^{-1} \simeq \frac{1}{4} q^2 g^2 (J_b/k_B T)^4$. After summing the latter contributions to obtain the total diamagnetic renormalization (5), the relationship $(\partial \ln \mu_{st}/\partial \ln \lambda_{st})|_{T^* = 0}$ then yields $T^* \simeq T_f [1 - 32\pi^{-2}\lambda_{st}^{-2}(T_b/T_f)^4]$ for the cross-over temperature, with

$$\mu_{st} \simeq 16\pi(T_f/T)\lambda_{st}^{-2((T_f/T)-1)}, \quad (15)$$

Last, the application of the previously mentioned high-temperature series results yields that $\mu_{st} \simeq [1 + \frac{1}{4} q^2 g^2 (J_f/k_B T)^4 + \frac{1}{4} q^2 g^2 (J_b/k_B T)^4]^{-1}$ for $T \gg T_{*0}$ in general. The overall picture we obtain in the weak-coupling limit, therefore, is that there is a sharp cross-over\(^6\)\(^,\)^9
into incipient confinement \((\mu_{st}^{-1} \gg 1)\) as temperature falls below \(T_*,\) followed by true confinement \((\mu_{st} = 0)\) as temperature falls below \(T_c.\) In particular, the \(T\)-linear resistivity contribution due to transverse gauge-field fluctuations\(^4\) \((\rho_b \propto \mu_{st} T/\chi_d),\) that is dominant in the strange-metal phase is exponentially suppressed in the spin-gap regime.\(^9\) Also, the entropy at temperatures just above \(T_{*0}\) due to such transverse gauge field excitations is approximately \(S_\perp \sim k_B (k_B T_{*0}/\chi_d)^{2/3} \sim k_B (q \lambda_{st}/a)^{-4/3},\) which is small in the present weak-coupling limit, \(\lambda_{st} \gg a.\) As a result, the cross-over into the spin-gap (or Fermi-liquid) regime remains intact (see Fig. 1). This is contrary to what occurs in the strong-coupling limit,\(^7\) as discussed below.

Let us now analyze the (high-temperature) strong-coupling limit, \(\lambda_{st} \ll a,\) that corresponds to the strange-metal phase,\(^2\) where \(\chi_d \propto T^{-1}.\) It is then more convenient to express the Wilson loop (7) in terms of the original roughening model from which the Coulomb gas ensemble (9) is derived;\(^7\) i.e., \(W(C) = Z[J]/Z[0],\) with

\[
Z[J] = \sum_{\{n_b, n_f\}} \exp \left\{ -\frac{1}{2\beta_b} \sum_{r,\mu} [\Delta_{\mu} n_b]^2 - \frac{1}{2\beta_f} \sum_{r,\mu} [\Delta_{\mu} n_f]^2 - \frac{(qg)^2}{2} \sum_{r} \left[ n_b + n_f + \frac{p}{q} J \right]^2 \right\},
\]

(16)

and with \(J(r) = 0\) unless the point \(r\) lies within the contour \(C,\) in which case \(J(r) = 1.\) Here, \(n_i(r)\) are integer fields and \(\beta_i = J_i/k_B T.\) The present limit \(qg \to \infty\) requires that \(n_b(r) = -n_f(r)\) for \(|p/q| < \frac{1}{2}\). This yields \(Z[0] = \sum_{\{n_f\}} \exp \{ -\frac{1}{2\beta^{-1}} \sum_{r,\mu} [\Delta_{\mu} n_f]^2 \},\) as well as an area law (7) for the Wilson loop with \(\mu_{st} = 1.\) Hence, while the cross-over phenomenon between strange and spin-gap (or Fermi-liquid) regimes is destroyed, we do recover the superfluid transition at \(T_c\) from the former 2D discrete gaussian model (dual to the 2D \(XY\) model). In other words, the entire normal state is “strange” in the strong-coupling limit.\(^7\) This agrees with recent studies, which begin from the strong-coupling strange-metal phase at high temperature, that find that the previously discussed entropy due to transverse gauge-field fluctuations destroys the cross-over into the spin-gap regime.\(^10\) Note that the evolution from the sharp cross-over at \(T_{*0}\) in weak-coupling to its absence in strong-coupling should be smooth since the corresponding normal states have a common high-temperature limit.

**Confinement.** In summary, the AH\(^2\) model for spin-charge separated superconductiv-
ity in two dimensions predicts that chiral spin-fluctuations are completely suppressed in the superfluid phase. Such gap-like behavior cannot, however, result from a Higgs mechanism, since the only auto-correlation function that shows algebraic long-range order in this model is that corresponding to the physical electronic phase \( \phi_{el} = \phi_f - \phi_b \).\(^7\) As a solution to this puzzle, we propose that the complete suppression of fluctuations in the statistical magnetic field is a symptom of confinement.\(^17-19\) In particular, we suggest that the dynamics of the statistical gauge field is described by an effective action for compact quantum electrodynamics in two space dimensions \((x_1, x_2)\), which in the continuum limit reads

\[
S_{st} = \frac{1}{4g_0^2a} \int d^3x (\partial_\mu a_\nu - \partial_\nu a_\mu)^2. \quad (17)
\]

Here the time dimension is rescaled to \( x_0 = c_0 t \) by the velocity of chiral spin-waves, \( c_0 \), while \( \partial_0 = c_0^{-1} \partial_t \). Singular instanton (monopole) configurations in (17) act to confine the statistical electric field into flux tubes and to suppress the corresponding magnetic excitations.\(^19\) However, this occurs only at temperatures below a deconfinement temperature, \( T_c > 0 \), that must necessarily be identified with the critical temperature of the AH\(^2\) model (2). The CFP saddle-point of the \( t - J \) model, which is considered to be an anyon superconductor,\(^17,18\) provides a microscopic realization of this proposal. The statistical gauge-field is in fact described by effective action (17) in such case,\(^19\) with a coupling constant given by \( g_0^{-2} = \frac{2}{\pi}(\Delta_f^{-1} + \Delta_b^{-1})\hbar \omega_0 \). Here \( \Delta_b \sim tx \) and \( \Delta_f \sim Jx^{1/2} \) are the respective charge and spin gaps for the holon and spinon quasi-particles at small concentrations, \( x \), of mobile holes,\(^18\) while \( \omega_0 = c_0/a \) is the Debye frequency cut-off of the theory (17).\(^19\) Employing the strong-coupling limit formula \( k_BT_c \approx \frac{2}{\pi} \hbar \omega_0 g_0^2 \) for the deconfinement temperature then yields \( k_BT_c = (\Delta_f^{-1} + \Delta_b^{-1})^{-1} \), which is of the same form as the \( T_c \) formula for the AH\(^2\) model (2). Also, given that \( g^{-2} = g_0^{-2}(\hbar \omega_0/k_BT) \) by quantum statistical mechanics, we obtain the physically satisfying result \( \lambda_{st} \sim x^{-1/2} \) for the statistical length scale (6). Note that we have here used the result \( (\hbar \omega_0)^2 \sim tJx^{1/2} \) valid for the CFP near half filling, and have identified \( k_BT_i \) with \( \Delta_i \). Hence, we find that the CFP is in the weak-coupling regime, \( \lambda_{st} \gg a \), near half filling, \( x \ll 1 \).

In conclusion, overall consistency of the AH\(^2\) model (2) requires that the spin and charge parts of the superfluid electron be confined in two dimensions. This indicates that the superfluid component, \( \sigma_{xy}^s \), of the in-plane Hall conductance in spin-charge separated
superconductors should have an electron sign. A negative sign anomaly for $\sigma_{xy}$ is in fact observed in high-temperature superconductors [$\sigma_{xy}$ exhibits a (positive) hole sign in the normal state]. Our result therefore justifies the two-fluid explanation for such observations,\textsuperscript{24} wherein the Hall conductance is given by the sum, $\sigma_{xy} = \sigma_{xy}^n + \sigma_{xy}^s$, of a hole-type normal conductance, $\sigma_{xy}^n > 0$, and an electron-type superfluid conductance, $\sigma_{xy}^s < 0$. This proposal of course fails to explain the sign changes of the Hall effect observed in certain low-$T_c$ superconductors.\textsuperscript{25}

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Figure Caption

Fig. 1. Shown is the phase diagram of 2D spin-charge separated superconductors in the weak-coupling limit, $\lambda_{st} \gg a$. The line that encloses the superconducting phase represents a true phase boundary, while the line above it marks the cross-over point $T_\ast$. The monotonically increasing and decreasing dashed lines represent critical temperatures $T_b = (200 \text{ K})x/x_0$ and $T_f = (800 \text{ K})(1 - x/x_0)$, respectively, for the holon and spinon subsystems in isolation, where $x_0$ denotes the mobile hole concentration below which spinon pairing occurs.
