Abstract We present models of temperature distribution in the crust of a neutron star in the presence of a strong toroidal component superposed to the poloidal component of the magnetic field. The presence of such a toroidal field hinders heat flow toward the surface in a large part of the crust. As a result, the neutron star surface presents two warm regions surrounded by extended cold regions and has a thermal luminosity much lower than in the case the magnetic field is purely poloidal. We apply these models to calculate the thermal evolution of such neutron stars and show that the lowered photon luminosity naturally extends their life-time as detectable thermal X-ray sources.

Keywords Neutron star · Magnetic field

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1 Introduction

It is generally considered that, within a few decades after its birth in a core collapse supernova, a neutron star reaches a state of “isothermality” characterized by a uniform high interior temperature. The stellar surface is much colder and protected from the hot interior by a thin (∼100 meters) layer, the envelope, which acts as a heat blanket. As was argued by [Greenstein & Hartke (1983)], in the presence of a sufficiently strong magnetic field, > 10^{10} G, the surface temperature of the neutron star will not be uniform as is expected in the unmagnetized case since the magnetic field severely limits the ability of electrons to transport heat in directions perpendicular to itself. As a result, the regions around the magnetic poles, where the magnetic field is almost radial, are expected to be significantly warmer than the regions around the magnetic equator, where the field is almost tangent to the surface. Since then much work has been dedicated to study the effects of the magnetic field on the properties of the neutron star envelope and crust (see Potekhin & Yakovlev [2001] and Potekhin et al. [2003] for recent works). In the presence of a sufficiently strong magnetic field, ≥ 10^{12} − 10^{13} G, the anisotropy of heat transport actually extends to much higher densities and can even be present within the whole crust. Recently, we have shown (Geppert et al. [2004, 2006]) that, in cases where the field geometry in the crust is such that the meridional component of the field dominates over its radial component in a large part of the crust, the non-uniformity of the temperature, previously considered to be restricted to the envelope, may actually extend to the whole crust. The largest effect was found when a strong toroidal component was included in the crust, superposed to the poloidal component. This result, that the geometry of the magnetic field in the interior of the neutron star leaves an observable imprint at the surface, potentially allows us to study the internal structure of the magnetic field through modelling of the spectra and pulse profiles of thermally emitting neutron stars.

There exist growing observational evidence that the anisotropy of heat transport in the envelope alone, assuming an otherwise isothermal crust, can not explain the surface temperature distributions of some observed neutron stars. In the case of several of the “Magnificent Seven” (see, e.g., Haberl [2004, 2007]) optical broad band photometric detections can be interpreted as being due to the Rayleigh-Jeans tail of a blackbody. However, these optical data are well above the Rayleigh-Jeans tail of the blackbody detected in the X-ray (“optical excess”) and indicate the presence of an extended cold component of much larger area than the warm component observed in X-ray, the latter having an emitting radius (∼3 − 5 km) much smaller than the usually assumed radius.
of a neutron star (~10–15 km), Schwop et al. (2005) tried to fit the light curve of RBS 1223 and concluded that only a surface temperature profile with relatively small, about 4–5 kms across, hot polar regions may explain the observations. Pons et al. (2002) and Trümper et al. (2004) had arrived qualitatively at the same conclusion when they fitted the combined X-ray and optical spectrum of RX J1856.5–3754. In both cases, the smallness of the hot region is much below what can be reached by considering anisotropic heat transport limited to only a thin envelope.

Little is known about the magnetic field structure in neutron stars which is very likely determined by processes during the proto-neutron star phase and/or in a relatively short period after that epoch. A proto-neutron star dynamo (Thompson & Duncan 1993) is unlikely to generate purely poloidal fields while differential rotation will easily wrap any poloidal field and generate strong toroidal components (Klütz & Ruderman 1998, Wheeler et al. 2002). The magneto-rotational instability (Balbus & Hawley 1991) also most certainly acts in proto-neutron stars (Akiyama et al. 2003) and results in toroidal fields from differential rotation (Balbus & Hawley 1998). Thus, it seems realistic to consider the effect of magnetic field configurations which consist of poloidal and toroidal components.

Besides their possible relevance for modelling the observed thermal radiation of the “Magnificent Seven”, and probably other strong field isolated neutron stars, toroidal magnetic fields may have a strong effect on the thermal evolution of the stars. The highly non-uniform surface temperatures they can induce result in reduced thermal luminosities and hence reduced energy losses during the late photon cooling era. It is our purpose in this work to explore this issue and we will present preliminary models of cooling of neutron stars with strong toroidal fields.

The structure of this paper is the following. In §2 we briefly review the basic ingredients and concepts involved in modelling the cooling of a neutron star and in §3 we present in some detail the simple mathematical formalism which describes dipolar fields, both poloidal and toroidal. The next section, §4, presents our results on the effect of strong magnetic fields on the neutron star crust temperature distribution, and these results are applied to model the cooling of the star in §5. Finally, we discuss our results in §6 and offer some tentative conclusions and prospects for future work.

2 Basic mechanisms of neutron star cooling

The source term \( H \) includes all possible “heating mechanisms” which will be neglected in the present work; however the decay of a strong magnetic field can result in significant heating (Kaminker et al. 2006, 2007). Only the basic points will be presented here and we refer the reader to the reviews Yakovlev & Pethick (2004) and Page et al. (2006), for more details.

The specific heat \( C_v \) receives its dominant contribution from the baryons in the core of the star and about 10% contribution from the leptons. The crustal lattice, free neutrons in the inner crust and electrons also provide a small contribution. When nucleons become superfluid (neutrons) or superconducting (protons) their contribution to \( C_v \) is severely reduced and may even practically vanish.

The neutron luminosity \( L_{\gamma} \) is usually dominated by the core. We consider the slow emission processes of the modified Urca family and the similar bremsstrahlung ones. Once nuclear burning turns on, neutrino emission from the formation and breaking of Cooper pairs is also included.

2.1 Neutron star envelopes (without magnetic fields)

Our main interest here is the photon luminosity \( L_{\gamma} \). In the absence of a magnetic field one expects the surface to have a uniform temperature \( T_e \), the effective temperature, and can express \( L_{\gamma} \) as

\[
L_{\gamma} = 4 \pi R_e^2 \sigma_{SB} T_e^4 \quad \text{or} \quad L_{\gamma,\infty} = 4 \pi R_e^2 \sigma_{SB} T_{e,\infty}^4
\]  

(2)

where \( R_e \) is the star’s radius and \( \sigma_{SB} \) the Stefan-Boltzmann constant (the quantities “at infinity” are defined as \( L_{\gamma,\infty} = e^{\gamma} L_{\gamma} \), \( R_{\infty} = e^{\gamma} R_e \) and \( T_{e,\infty} = e^{\gamma} T_e \), where \( e^{\gamma} \approx (1 - 2GM/Rc^2)^{1/2} \) is the redshift factor). In order to integrate Eq. 1 one needs a relationship between \( T_e \) and \( T_c \). The assumption of uniform interior temperature \( T \) is reasonable for most of the interior, given the huge thermal conductivity of degenerate matter and once the star is old enough to have relaxed from the initially complicated temperature structure produced at its birth, but is certainly not possible in the upper layers of the star where density is low enough for the electrons not to be fully degenerate. One traditionally separates out these upper layers from the cooling calculation and treats them as an envelope. A typical cut density is \( 10^{10} \text{ g cm}^{-3} \) and the resulting envelope, with a depth of the order of 100 meters, can be studied separately in a plane parallel approximation. Gundmundsson et al. (1982) and Gundmundsson et al. (1983) presented detailed models of neutron star envelopes and showed that \( T_e \) is related to the temperature at the bottom of the envelope, \( T_b \) at density \( \rho_b = 10^{10} \text{ g cm}^{-3} \), through the simple relation

\[
T_e = 0.87 \times 10^6 \text{ K} \, s_{14}^{1/4} \, r_{b}^{0.55}
\]  

(3)

where \( s_{14} \) is the surface gravity in units of \( 10^{14} \text{ cm s}^{-2} \). A relation such as Eq. 3 is usually called a “\( T_b - T_e \) relationship”. The models leading to Eq. 3 assumed the envelope is formed of iron-like nuclei, and it was shown by Chabrier et al. (1997) (see also Potekhin et al. 1997) that if
light elements, such as $H$, $He$, $C$, $O$, are present deep enough in the envelope, the increase in thermal conductivity (which is roughly proportional to $Z^{-1}$ in liquid matter, $Z$ being the element’s charge) results in much higher luminosities, by up to one order of magnitude. Since the temperature gradient penetrates deeper in hotter stars, larger amounts of light elements are necessary to alter heat transport at high $T_b$ than at lower $T_b$.

\[ \kappa \propto c v T \]

2.2 Heat transport with magnetic fields

In the absence of a magnetic field the thermal conductivity $\kappa$ is conveniently written as

\[ \kappa_0 = \frac{1}{3} c v \nabla^2 \tau = \frac{\pi^2 k_B^2 T n_e}{3 m_e^*} \tau \]  

(4)

where $v$ is the mean velocity of the heat carriers, $c$, their specific heat per unit volume, and $\tau$ their collisional time; the second expression is particularized to relativistic electrons, the dominant heat carriers in neutron star crusts (Yakovlev & Urpin 1980). In the presence of a magnetic field, due to the classical Larmor rotation of electrons, heat flow may be anisotropic and $\kappa$ becomes a tensor

\[ \kappa = \begin{pmatrix} \kappa_\parallel & \kappa_\perp & 0 \\ -\kappa_\perp & \kappa_\parallel & 0 \\ 0 & 0 & \kappa_\perp \end{pmatrix} \]  

(5)

assuming the field $B$ oriented along the $z$-axis) whose components have the form

\[ \kappa_\parallel = \kappa_0 \]  

\[ \kappa_\perp = \frac{\kappa_0}{1 + (\omega_B^2 \tau)^2} \]  

\[ \kappa_\perp = \frac{\kappa_0 \omega_B^2 \tau}{1 + (\omega_B^2 \tau)^2} \]  

(6)

where $\omega_B = eB/m_e^*c$ is the electron cyclotron frequency. The condition $\omega_B \tau \gg 1$, which implies strong anisotropy, is easily realized in a neutron star envelope, and also possibly in the whole crust (Geppert et al. 2004). Values of the magnetization parameter $\omega_B \tau$ are plotted in Fig. 1. Notice that $\tau$ receives contribution from both electron-phonon and electron-impurity scattering, with frequencies $\nu_{\text{e-ph}}$ and $\nu_{\text{e-imp}}$ respectively, and is given by $\tau = (\nu_{\text{e-ph}} + \nu_{\text{e-imp}})^{-1}$. Electron-impurity scattering dominates at low temperatures, and/or high densities, and is $T$-independent while $\nu_{\text{e-ph}}$ goes roughly as $T^2$: combination of these two different $T$-dependencies is the reason for the complex behavior of $\omega_B \tau$ seen in Fig. 1.

In case the field is strong enough to be quantizing, the expressions have to be modified (in particular $\tau$ also becomes anisotropic) but the essential result that $\kappa_\perp < \kappa_\parallel$, when $\omega_B \tau \gg 1$, remains (Potekhin 1999).

2.3 Magnetized neutron star envelopes

In considering magnetic field effects on heat transport in a neutron star the simplest case to handle is the envelope: since its thickness, $\sim 100$ m, is much smaller than the length scale over which the field is expected to vary significantly, $\sim$ km, one can consider heat transport on a given small patch on the surface to be independent of the rest of the surface. Moreover, the field strength $B$ and its angle with respect to the radial direction, $\Theta_B$, can be considered as uniform in the patch and the heat flux considered as essentially radial. The thermal conductivity in a direction making an angle $\Theta$ with the field, considering the $\kappa$ tensor of Eq. 5 is then given by

\[ \kappa(\Theta_B) = \cos^2 \Theta_B \kappa_\parallel + \sin^2 \Theta_B \kappa_\perp \]  

(7)

With this form of $\kappa(\Theta_B)$ and a radial flux, heat transport in the envelope at this surface patch is a one dimensional problem. The solution is a “$T_b - T_s$” relationship which depends on $B$ and $\Theta_B$, and the obtained effective temperature $T_s$ is a local one which we will write as $T_s(\Theta, \phi)$ in the sense that the flux emerging from this patch is $\sigma_B T_s^4$. Given the form of $\kappa(\Theta_B)$, Eq. 7 Greenstein & Hartke (1983) proposed the simple interpolation formula

\[ T_s(\Theta, \phi)^4 = T_s(T_b; B, \Theta_B = 0)^4 \cos^2(\Theta_B) + T_s(T_b; B, \Theta_B = 90^\circ)^4 \sin^2(\Theta_B) \]  

(8)

for arbitrary angle $\Theta_B$ in terms of the two cases of radial ($\Theta_B = 0$) and tangential ($\Theta_B = 90^\circ$) field. Recently, Potekhin & Yakovlev (2001) and Potekhin et al. (2003) have presented detailed calculations and fitted their results by an expression similar to Eq. 8.

For a given geometry of the magnetic field (not necessarily dipolar) and assuming that the temperature at the bottom of the envelope, $T_b$, is not affected by the magnetic field and hence is uniform around the star, one can generate the expected surface temperature distribution $T_s(\Theta, \phi)$ by piecing together envelope models through applying relationships of
coordinate directions, with the convenient to separate the magnetic field in two components once a 3D transport code is available. In this case it is general ones will hopefully be considered in the future. We will only consider here axisymmetric configurations (metry is maintained throughout the underlying thin envelope).

3 The internal magnetic field

We will only consider here axisymmetric configurations (more general ones will hopefully be considered in the future once a 3D transport code is available). In this case it is convenient to separate the magnetic field in two components

\[ \mathbf{B} = \mathbf{B}^{\text{pol}} + \mathbf{B}^{\text{tor}} , \]

the poloidal and toroidal components, respectively, where \( \mathbf{B}^{\text{pol}} \) only has \( \mathbf{e}_r \) and \( \mathbf{e}_\theta \) components and \( \mathbf{B}^{\text{tor}} \) only an \( \mathbf{e}_\phi \) component\(^1\) (the \( \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi \) being units vectors in the spherical coordinate directions, with the \( \theta = 0 \) axis coinciding with the field symmetry axis). Thus, the magnetic field lines of \( \mathbf{B}^{\text{pol}} \) are simply circles centered on the symmetry axis but the field lines of \( \mathbf{B}^{\text{tor}} \) are more complicated. Let us expand \( B_r \) in Legendre polynomials as \( B_r = \sum F_l(r) P_l(\cos \theta) \). We will only consider the first \((l = 1)\) term, the dipole, and write \( F_1 \) as \( \frac{S(r)}{r^2} \) so that

\[ B_r = \frac{S(r)}{r^2} \cos \theta . \]

\(^1\) Such decomposition is also possible for non-axisymmetric fields but is more involved (Radler 2000).

\[ \int \frac{d\Omega}{4\pi} \sigma B_\theta(\theta, \phi) = 4\pi R^2 \cdot \sigma B \]

where \( T_0 \) is again defined as in Eq. 2.

By this method one can easily generate surface temperature distributions corresponding to the geometry of the magnetic field at the surface (assuming that the same field geometry is maintained throughout the underlying thin envelope). However, the results of Fig. 1 show that the anisotropy in heat transport is likely to extend much deeper into the crust than just the envelope. To handle such cases one is hence forced to define the field geometry in the whole crust, which is what we do in the next section.

3 The internal magnetic field

The maps cover the whole neutron star surface in an area preserving projection. For better viewing, the magnetic field symmetry axis is located in the equatorial plane, oriented from \( \phi = 0^\circ \) to \( \phi = 270^\circ \). The left panel has a dipolar field only while the right panel also contains a quadrupolar field superposed to the same dipole. (Figure from Page et al. 2006.)

\[ L_\gamma = 4\pi R^2 \cdot \int \frac{d\Omega}{4\pi} \sigma B_\theta(\theta, \phi)^4 = 4\pi R^2 \cdot \sigma B \]

Then Maxwell’s equation \( \nabla \cdot \mathbf{B} = 0 \) implies

\[ B_\theta = -\frac{1}{2r} \frac{\partial S}{\partial \theta} \sin \theta . \]

We also need the three boundary conditions

\[ S(r = 0) = 0 \]

\[ S(r = R) = B_0 R^2 \quad \text{and} \quad \frac{\partial S}{\partial r} \bigg|_{r = R} = -\frac{S(R)}{R} = -B_0 R \]

which ensure regularity at the star’s center and smooth matching with an external vacuum dipolar field, of strength \( B_0 \) at the magnetic poles, at the stellar surface. Besides these boundary conditions, the Stoke function \( S \) is totally arbitrary but a choice of it is equivalent to choosing the location of the currents sustaining the poloidal field since the latter, having only a \( j_\phi \) component, are given by

\[ j_\phi = \frac{c}{8\pi} \frac{\sin \theta}{r} \left( \frac{\partial^2 S}{\partial r^2} - \frac{2S}{r^2} \right) \]

Notice also a simple physical interpretation of the Stoke function: the magnetic flux through the star’s equatorial plane in a circle of radius \( r \) is simply given by \( \pi S(r) \), and the boundary condition gives a total flux \( \Phi = \pi R^2 B_0 \), as it should be. In vacuum, \( S \) simply reduces to \( B_0 R^2 / r \).

The locations of the currents in the stellar interior is unknown but it is natural to separate them into two components, located in the core and in the crust and accordingly separate \( \mathbf{B}^{\text{pol}} \) as

\[ \mathbf{B}^{\text{pol}} = \mathbf{B}^{\text{core}} + \mathbf{B}^{\text{crust}} . \]
The core protons are expected to become a superconductor, with critical temperatures \( T_c \approx 10^8 \) K (see, e.g., [Page et al. 2004]), soon after the star’s birth: the magnetic field is then confined into flux tubes and maintained by proton supercurrents. Since, by definition, \( \mathbf{B}^\text{core} \) corresponds to currents located in the core, within the crust it is described by a vacuum dipolar poloidal field, and we will parametrize it by its surface strength at the magnetic pole \( B^\text{pol}_{0} \). We do not need to specify the distribution of the supercurrents sustaining \( \mathbf{B}^\text{core} \) and the geometry of this component in the core since the field, being confined to fluxoids which occupy only a very small volume, is not expected to alter heat transport in the core.

For the \( \mathbf{B}^\text{crust} \) component, we need to specify its geometry in the crust, i.e., the corresponding Stokes function \( s(x) \). The arbitrariness involved in such specification can be somewhat relieved by considering models of the time evolution of this component: currents spontaneously migrate toward the highest density region of the crust, where the electrical resistivity is smallest, until they reach the crust-core boundary where their migration is stopped by the proton superconductor (see, e.g., Fig. 4 in [Page et al. 2000]). We use an \( S^\text{crust} \) function resulting from such evolutionary calculations and scale it to vary the overall strength of \( \mathbf{B}^\text{crust} \), which we parametrize by \( B^\text{crust}_{0} \) defined as its strength at the magnetic pole.

For the toroidal component \( \mathbf{B}^\text{tor} \), we also expand it in Legendre polinomials \( P_l(\sin \theta) \) and keep only the \( l = 1 \) dipolar term

\[
B^\text{tor} = T(r) \sin \theta. \tag{16}
\]

We do not have to consider the part of \( \mathbf{B}^\text{tor} \) confined to the core, because of proton superconductivity, and specify only the part confined to the crust. The only restrictions are the boundary conditions

\[
T(r = R_{\text{core}}) = 0 \quad \text{and} \quad T(r = R) = 0. \tag{17}
\]

We are not aware of evolutionary calculations of toroidal field in neutron star crust and hence are left with a guess about the possible shape and size of the \( T \) function: following [Geppert et al. 2006], we see it as a free parameter and consider several choices. We parametrize the strength of \( \mathbf{B}^\text{tor} \), for each choice of \( T \), by \( B^\text{tor}_{0} \) defined as the maximum value of \( ||\mathbf{B}^\text{tor}|| \) in the crust.

Pérez-Azorín et al. (2006) have presented similar models of field structure and the resulting temperature distributions in the crust and chose force-free field configurations to specify the field geometry. It is reasonable to assume the field could evolve into a force-free configuration during the early neutron star life, but there is no physical reason why the later evolution, driven by Ohmic decay and Hall interactions, will conserve the force-free condition and this motivates our considering the function \( T \) as rather arbitrary. However, our choice of \( S^\text{crust} \) is despite of being based on evolutionary calculations of purely poloidal fields, may not be too realistic because of the coupling between poloidal and toroidal components from the Hall term. We show in Fig. 5 a sketch of the field geometry we consider and in Fig. 4 our choices of the functions \( S \) and \( T \).

4 Magnetic field effects in the crust

Given the magnetic field geometries described in the previous sections, Geppert et al. (2006) calculated the resulting temperature distributions in the neutron star crust, in a sta-

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**Fig. 4** Continuous lines: normalized Stokes function \( s(x) = S / R^2 B^\text{crust}_{0} \) and its derivative (scaled by a factor 10) used in this work, which generate \( B_t \) (Eq. 11) and \( B_0 \) (Eq. 12) respectively. Discontinuous lines: the three different normalized functions \( T(x) = T / B^\text{tor}_{0} \) we consider for the toroidal field, \( B_\theta \) (Eq. 16), which we label as T1, T2, and T3. On the horizontal axis \( x = r / R \). (Figure from [Geppert et al. 2006].)

**Fig. 5** Temperature distribution in a strongly magnetized neutron star crust (whose thickness has been stretched by a factor five for easier reading). The chosen field scale parameters are \( B^\text{core}_{0} = 7.5 \times 10^{13} \) G, \( B^\text{crust}_{0} = 2.5 \times 10^{13} \) G, \( B^\text{tor}_{0} = 3 \times 10^{15} \) G, and the toroidal component’s generating functions \( T \) is the model “T1” of Fig. 4. The color code maps the relative temperature, i.e., \( T(r, \theta) / T_{\text{core}} \), with a core temperature \( T_{\text{core}} = 6 \times 10^8 \) K. White lines show field lines of \( \mathbf{B}^\text{pol} \), the field lines of \( \mathbf{B}^\text{tor} \) being perpendicular to the plane of the figure. The heat blanketing effect of the toroidal component is clearly visible. (From [Geppert et al. 2006].)
tionary state. In these calculations, heat transport is solved in the crust for densities from $\rho = \rho_{\text{core}} = 1.6 \times 10^{14}$ g cm$^{-3}$ down to $\rho = \rho_b = 10^{10}$ g cm$^{-3}$. In the core, at $\rho > \rho_{\text{core}}$, the temperature $T_{\text{core}}$ is assumed to be constant and uniform while at the outer zone, at $\rho = \rho_b$, the models are matched with magnetized envelopes (treated as the outer boundary condition). Since the 2D transport code we use does not yet include correct specific heat it does not have the capability to perform realistic time dependent calculations and only stationary results can be obtained, i.e., the thermal evolution is followed until the temperature profile would not change anymore with time (this stationary limit is independent of the specific heat). Moreover, neutrino energy losses are also not included (neutrino emission, for high temperature, and also strong magnetic fields, is not negligible and should be included for accurate calculations, see, e.g., Potekhin et al. 2007). Fig. 5 shows an example of the resulting temperature distribution in the crust and Fig. 6 shows the resulting surface temperature distribution: this latter figure should be compared with the surface temperature distribution shown in Fig. 2 where isothermal crusts were considered and magnetic field affected heat transport only within the thin envelope.

For a fixed magnetic field configuration and several values of $T_{\text{core}}$, we calculate the crustal temperature distribution and the resulting surface temperature $T_s(\theta)$, which is $\theta$-independent because of axial symmetry, and obtain $T_s$ from Eq. 9. A set of results is displayed in Fig. 7. We show models which, within our selection of field configurations, maximize the magnetic field effects, with 75% of the poloidal flux entering the crust and the toroidal component localised in the middle of the crust. With a purely poloidal field, i.e., $B_{\text{tor}} = 0$, one sees little effect and the crust stays close to isothermal, except at high temperature, $T_{\text{core}} = 10^9$ K, where the envelope’s temperature gradient extends deeper into the crust than the $\rho = 10^{10}$ g cm$^{-3}$ arbitrary cut. Significant effects appear when $B_{\text{tor}} \gg 10^{14}$ G and when $B_{\text{tor}} \gg 10^{15}$ G the crust is highly non-isothermal even at the highest temperature $T_{\text{core}} = 10^9$ since then $\omega_b \tau \gg 1$ in the whole crust. There is an intriguing change in the shape of the $T_s$-surface when going from $T_{\text{core}} > 10^8$ K to $T_{\text{core}} < 10^8$ K, which is most evident at the highest value $B_{\text{tor}}^0 = 3 \times 10^{15}$ G: it is most probably due to the shift in the dominant scattering process from electron-phonon, at high $T$ where $\tau$ is $T$-independent, to electron-impurity, at low $T$ where $\tau$ is $T$-dependent, as mentioned in Section 4 (We hope to study this effect in more detail in a future work.).

5 Cooling of strongly magnetized neutron stars

In order to perform (time dependent) cooling calculations we will here use the models described in the previous section to produce a set of outer boundary conditions for our 1D cooling code (see, e.g., Page et al. 2004). We fix the outer boundary at a density $\rho_b = 10^{14}$ g cm$^{-3}$, instead of $10^{10}$ g cm$^{-3}$ when envelope models are used as boundary condi-

![Fig. 6 Surface temperature distribution corresponding to the crust temperature shown in Fig. 5. For better viewing the magnetic symmetry axis is in the equator ($\theta = 90^\circ$) pointing at $\phi = 90^\circ$. (From Geppert et al. 2008.)](image)

tions. As is obvious from Fig. 6 compared to Fig. 2 the photon luminosity, and hence $T_s$, is much lower in presence of a strong toroidal field than when the crust is considered as isothermal and Fig. 8 shows the resulting $T_b - T_s$ relationships obtained for nine different magnetic field configurations. For the most extreme case, $T_s$ can be reduced by a factor of 2.5, and hence $L_p$ by a factor 40, compared to the isothermal crust case. Notice that we chose $\rho_b = 10^{14}$ g cm$^{-3}$ instead of our crustal models starting at $\rho_{\text{core}} = 1.6 \times 10^{14}$ g cm$^{-3}$ because we found a very small temperature gradient in the range $10^{-1} \times 10^{-14}$ g cm$^{-3}$ and hence prefer to treat this density range with the cooling code. The 1D cooling code solves the energy balance and heat transport equations in their full general relativistic forms.

The cooling models we will consider here are within the minimal cooling paradigm of Page et al. (2004). In short, this means neutrino emission in the stellar core is from the modified Urca and the similar nucleon bremsstrahlung processes, neutron and proton pairing is taken into account with the resulting neutrino emission from the Cooper pair breaking and formation process and the alteration of the specific heat. We use pairing critical temperatures for proton $1S_0$ from Takatsuka (1973), for neutron $1S_0$ from Schwenk et al. (2003), and for neutron $3P_2$ the model “a” from Page et al. (2004). The star model is a $1.4 M_\odot$ neutron star built with the equation of state of Akmal et al. (1998) and hence, as part of the minimal cooling paradigm, charged meson condensate, hyperons, and deconfined quark matter are not present in the star.

First we assess the effect of the approximation of truncating the star at $\rho_b = 10^{14}$ g cm$^{-3}$, and treating most of the crust through our stationary solutions as part of the outer boundary condition, instead of using the traditional outer boundary at $10^{10}$ g cm$^{-3}$. A comparison of cooling trajectories with these two different boundary densities, and in the absence of a toroidal field component, is shown in Fig. 9. The major difference appears at early times during the crust relaxation era: a “full” (i.e., using $\rho_b = 10^{14}$ g cm$^{-3}$) model shows strong radial temperature radiants in the crust at this stage (see, e.g., Gnedin et al. 2001) while the truncated (i.e., using $\rho_b = 10^{14}$ g cm$^{-3}$) model assumes an isothermal crust.

2 The APR equation of state shows the presence of a $\pi^0$ condensate, but it has no noticeable effect on the cooling.
Another way of seeing it is that our 2D crustal models only considered stationary states and are hence only applicable when the cooling time scale is much longer than the thermal relaxation time of the crust, a condition which is certainly not fulfilled during this early cooling phase. Later, during the neutrino cooling era, the truncated model is slightly warmer than the “full” model because neutrino emission from its truncated crust is missing. During the photon cooling era the difference between the two models becomes larger, now due to the smaller specific heat of the truncated model which consequently cools faster. Overall, moving the outer boundary from $10^{10}$ g cm$^{-3}$ to $10^{14}$ g cm$^{-3}$ only has a small, and quite negligible, effect except at early ages which we will hence not show in our next results.

Having now the confidence that stripping the star of most of its crust, and properly including the stripped part into the outer boundary condition, introduces an acceptably small error, we can proceed with models having strong toroidal crustal fields. We show in Fig. 10 our results for the nine field configurations of the $T_b - T_e$ relationships presented in the Fig. 8. During the neutrino cooling era, the star’s core evolution is driven by $L_\nu$ and the surface temperature, and $L_\nu$, simply follows the evolution of the core. So during this phase all nine models have exactly the same evolution but they look very different at the surface: models with a higher

\[
\begin{align*}
B_0^{tor} &= 0 \text{ G} \\
T_{core} &= 3 \times 10^{14} \text{G} \\
10^9 \text{K} \\
10^8 \text{K} \\
10^7 \text{K} \\
10^6 \text{K}
\end{align*}
\]

**Fig. 7** 3D plots of the temperature distribution in a strongly magnetized neutron star crust. The horizontal axes show radial coordinate $x = r/R$ and polar angle $\theta$ while the vertical scale is $T/T_{core}$. Four values of $T_{core}$ are considered and the chosen field scale parameters for the poloidal field are $B_0^{core} = 2.5 \times 10^{13}$ G and $B_0^{cusp} = 7.5 \times 10^{13}$ G in all cases while the toroidal component scale $B_0^{tor}$ is varied, the overall shape of the $T$ function, Eq. 16 being the model “T2” of Fig. 8.
Fig. 8 Possible $T_e - T_c$ relationships for a neutron star with a strong toroidal magnetic field. The various sets of curves correspond to two different values of $B_0^{tor}$ and two different locations of the toroidal field, labelled as “T1” and “T2” as in Fig. 4. For each case we also consider two different splittings of the poloidal field: $B_0^{pol} = 7.5 \times 10^{12}$ G and $B_0^{pol} = 2.5 \times 10^{12}$ G, or $B_0^{pol} = 2.5 \times 10^{12}$ G and $B_0^{pol} = 7.5 \times 10^{12}$ G, the cases with the larger $B_0^{pol}$ resulting in higher $T_c$. The continuous curve shows the $T_e - T_c$ relationships for an isothermal crust with the same poloidal field $B_0^{pol} = 10^{13}$ G and magnetic field effects included only in the envelope (Potekhin & Yakovlev 2001).

$T_e$ for a given $T_b$, as shown in Fig. 8, have a higher photon luminosity. During the photon cooling era the results are inverted since $L_{\gamma}$ drives the cooling and models with a higher $T_e$ for a given $T_b$ result in a larger $L_{\gamma}$ and consequently they undergo faster cooling.

We have preferred to plot the cooling curves as $L_{\gamma}$ vs $t$ instead of $T_e$ vs $t$: given the highly non-uniform $T_e(\theta, \phi)$ the effective temperature $T_e$ loses any observational meaning. More detailed models taking this into account will be presented in a future work (Page et al. 2007).

6 Discussion and Conclusions

We have extended our previous results about temperature distribution in a strongly magnetized neutron star crust (Geppert et al. 2004, 2006) to a broader range of temperatures and applied them to study the cooling of neutron stars with strong toroidal magnetic fields. Geppert et al. (2004) had shown that with a purely poloidal field entirely confined to the crust strongly non-uniform temperature distributions develop in the crust. Then Geppert et al. (2006) showed that allowing part of this poloidal field to permeate the core significantly reduced the temperature non-uniformity but that the inclusion of a strong toroidal field component can result in crustal temperature gradients even stronger than in the first case (similar results have been obtained by Pérez-Azorín et al. 2006). We have shown here that, due to the strength of the field, the magnetization parameter $e_B \rho$ is very large in most of the crust even at core temperatures as high as $10^{12}$ K and the large crustal temperature gradients are still present in such hot stars. Considering these results we have been able to perform cooling calculations of such neutron stars with a strong toroidal crustal magnetic field, with some approximations which, as we have shown, introduce only very small errors in the results.

Our final results are displayed in Fig. 10 and compared with the estimated luminosity range of the “Magnificent Seven” (Haberl 2004). This comparison indicates most probable ages between 0.5 and 1.5 Myrs for these stars, depending on the magnetic field structure, but this range could be extended down to 0.05 Myrs for the brightest ones and up to 3 Myrs for the dimest ones if the most extreme field geometry is considered. Besides the uncertainty in the field structure, which in itself results in the range of predicted ages shown...
in Fig. [10] there are two other key ingredients in cooling models which can have similar effects: nucleon pairing in the core and the chemical composition of the envelope. The former significantly affects the specific heat during the photon cooling era and different assumptions about the values of $T_c$ can introduce a factor of a few in the predicted cooling ages while the latter can significantly affect the photon luminosity (see discussion in §3.1 and introduce another uncertainty of a few (we refer the reader to the detailed presentation of Page et al. [2004]). Considering these three theoretical uncertainties, magnetic field geometry, nucleon pairing, and envelope chemical composition, it is certainly possible to extend the theoretical cooling ages, for photon luminosities between $\sim 3 \times 10^{30}$ and $\sim 3 \times 10^{31}$, to cover the range from $10^7$ to almost $10^9$ years!

Can we reasonably expect to reduce this enormous theoretical age uncertainty? Precession of some neutron stars (Link [2007], Pons [2007]) may imply that neutron pairing critical temperatures in the core are very low (Link 2003), a possibility which moreover has a strong theoretical fundament (Schwenk & Friman [2004]): this would imply a large specific heat and favor long cooling times (within the minimal cooling paradigm). Interpretation of the absorption lines detected in most of the “Magnificent Seven” (see, e.g., Haberl 2007 and van Kerkwijk & Kaplan [2007]) may provide crucial information about the surface chemical composition. However, atomic and molecular physics in strong field still has many secrets to be unveiled (Turbiner [2007]) and, moreover, what is needed is the chemical composition of the deeper layers of the envelope, i.e., several tens of meters below the surface. Finally, the structure and evolution of the magnetic field with toroidal components, in the neutron star context as considered in this paper, is an almost uncharted territory and much progress can be expected.

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