Probing the neutron skin with ultrarelativistic isobaric collisions

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Particle production in ultrarelativistic heavy ion collisions depends on the details of the nucleon density distributions in the colliding nuclei. We demonstrate that the charged hadron multiplicity distributions in isobaric collisions at ultrarelativistic energies provide a novel approach to determine the poorly known neutron density distributions and thus the neutron skin thickness in finite nuclei, which can in turn put stringent constraints on the nuclear symmetry energy.

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Introduction. Nuclei are bound states of protons and neutrons by the attractive nuclear force. The nuclear force is short ranged, and is surpassed by Coulomb repulsion between protons in heavy nuclei. This is compensated by more neutrons to keep heavy nuclei bound. With more neutrons comes the penalty symmetry energy associated with the asymmetry between the proton and neutron numbers. The symmetry energy influences the proton and neutron density distributions, and in particular, the neutron skin thickness (difference between the rms radii of the neutron and proton distributions, $\Delta r_{np} \equiv r_n - r_p$) [1]. The symmetry energy and its density dependence are critical for our understanding of the masses and drip lines of neutron-rich nuclei and the equation of state (EOS) of nuclear and neutron star matter [2, 10].

Measurements of the neutron density and the $\Delta r_{np}$, complemented by state-of-the-art theoretical calculations [11–14], can yield valuable information on the symmetry energy [15–18]. Exact knowledge of nucleon density distributions is also crucial to new physics search beyond the standard model [19]. Because protons are charged, its density distributions are well measured by electron scattering off nuclei [20, 21]. The neutron density distributions are not as well measured [17]. For example, the $\Delta r_{np}$ measurements of the benchmark, closed shell and spherical $^{208}$Pb nucleus fall in the range of 0.15–0.22 fm with a typical precision of 20–50% [16, 17, 22]. One limitation is the inevitable uncertainties in modeling the strong interaction of the reaction mechanisms [22]. A promising way to measure neutron densities is through electroweak parity-violating electron scattering, exploiting the large weak charge of the neutron compared to the diminishing one of the proton [24, 25]. Such measurements, although much cleaner to interpret, require large luminosities [18]. The current measurement by PREX (Parity Radius Experiment) on the $^{208}$Pb $\Delta r_{np}$ is $0.33^{+0.16}_{-0.18}$ fm [26]. In addition, the coherent elastic neutrino-nucleus scattering [27] also provides a clean way to extract the neutron densities, but the current uncertainty is too large [13].

The symmetry energy has been shown to affect observables in low to intermediate energy heavy ion collisions, such as the isospin diffusion [28, 29], the neutron-proton flow difference [30], the isospin dependent pion production [31], and light cluster formation [32]. Heavy ion collisions at relativistic energies are generally considered insensitive to nuclear structures and the symmetry energy. Recent studies of isobaric $^{44}$Ru+$^{44}$Ru and $^{96}$Zr+$^{96}$Zr collisions indicate, however, that nuclear density distributions have a noticeable effect on the total charged hadron multiplicity ($N_{ch}$) [33]. This can be understood because the numbers of participants ($N_{part}$) and binary nucleon-nucleon collisions ($N_{bin}$) differ slightly for different nuclear densities and because $N_{ch}$ depends on $N_{part}$ and $N_{bin}$ in relativistic collisions. In fact, this can be readily used to distinguish simplistic Woods-Saxon nuclear density parameterizations from more sophisticated calculations by energy density functional theory (DFT) [34]. Since $N_{ch}$ can be measured very precisely, we demonstrate in this work that the $N_{ch}$ distributions in isobaric collisions may be used to determine the $\Delta r_{np}$ (and hence the symmetry energy) to a precision that may exceed those achieved by traditional low energy nuclear experiments.

The symmetry energy and the neutron skin. The nuclear matter EOS is conventionally defined as the binding energy per nucleon and can be approximately expressed as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho) \delta^2 + O(\delta^4),$$

where $\rho = \rho_n + \rho_p$ is the nucleon number density and $\delta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry with $\rho_p$ ($\rho_n$) denoting the proton (neutron) density. $E_0(\rho) \equiv E(\rho, \delta = 0)$ and the symmetry energy is defined by $E_{\text{sym}}(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} |_{\delta=0}$. At the saturation density $\rho_0$, the $E_0(\rho)$ can be expanded in $\chi = (\rho - \rho_0)/3\rho_0$ as
TABLE I: Effective nuclear rms radii, $\sqrt{\langle r^2 \rangle} = \sqrt{\int \rho(r) r^4 dr / \int \rho(r) r^2 dr}$, for neutron ($r_n$) and proton ($r_p$) distributions, and the neutron skin thickness ($\Delta r_{np} = r_n - r_p$) of the $^{96}$Ru and $^{96}$Zr nuclei, for four sets of the symmetry energy slope parameters $L(p_c)$, $L(\rho_0)$. The $^{208}$Pb $\Delta r_{np}$ values are also listed. The unit for the radii is fm and for the slope parameters is MeV.

|            | $L(p_c)$ | $L(\rho_0)$ | $r_n$ | $r_p$ | $\Delta r_{np}$ | $r_n$ | $r_p$ | $\Delta r_{np}$ | $\Delta r_{np}$ |
|------------|----------|--------------|-------|-------|-----------------|-------|-------|-----------------|-----------------|
| Lc20       | 20       | 13.1         | 4.386 | 4.27  | 0.115           | 4.327 | 4.316 | 0.011           | 0.109           |
| Lc47       | 47.3     | 55.7         | 4.449 | 4.267 | 0.183           | 4.360 | 4.319 | 0.042           | 0.190           |
| Lc70       | 70       | 90.0         | 4.494 | 4.262 | 0.232           | 4.385 | 4.326 | 0.066           | 0.264           |
| SLy4       | 42.7     | 46.0         | 4.432 | 4.271 | 0.161           | 4.356 | 4.327 | 0.030           | 0.160           |

$E_0(\rho) = E_0(\rho_0) + \frac{1}{2} K_0 \chi^2 + \frac{1}{12} J_0 \chi^3 + O(\chi^4)$, where $K_0$ is the incompressibility coefficient and $J_0$ is the skewness coefficient. Similarly, at a reference density $\rho_r$, $E_{sym}(\rho) = E_{sym}(\rho_r) + L(\rho_r) \chi_r + \frac{1}{2} K_{sym}(\rho_r) \chi_r^2 + O(\chi_r^3)$, where $\chi_r = (\rho - \rho_r)/3\rho_r$ with the slope parameter $L(\rho_r) = 3\rho_r \frac{dE_{sym}(\rho)}{d\rho} \bigg|_{\rho=\rho_r}$ and the curvature parameter $K_{sym}(\rho_r) = 9\rho_r^2 \frac{d^2E_{sym}(\rho)}{d\rho^2} \bigg|_{\rho=\rho_r}$ . The $L \equiv L(\rho_0)$ and $K_{sym} \equiv K_{sym}(\rho_0)$ characterize the density dependence of the $E_{sym}(\rho)$ around $\rho_0$.

In the present work, we use two different nuclear energy density functionals to describe the properties of finite nuclei, namely, the standard Skyrme-Hartree-Fock (SHF) model (see, e.g., Ref. [33]) and the extended SHF (eSHF) model [34, 37]. These two models have been shown to be very successful in describing the structures of finite nuclei, especially global properties such as binding energies and charge radii. Compared to SHF, the eSHF model contains additional momentum and density-dependent two-body forces to effectively simulate the momentum dependence of the three-body forces [37]. Fitting to data using the strategy in Ref. [38], we first obtain a parameter set (denoted as Lc47) within eSHF by fixing $E_{sym}(\rho_c) = 26.65$ MeV and $L(\rho_c) = 47.3$ MeV at the subsaturation density $\rho_c = 0.11\rho_0/0.16$. We also construct two more parameter sets denoted as Lc20 and Lc70 with $L(\rho_c) = 20$ MeV and 70 MeV, respectively, keeping $E_{sym}(\rho_c) = 26.65$ MeV [38], to cover the current range of uncertainty on the symmetry energy.

Table I lists the nuclear radii of $^{96}$Zr and $^{96}$Ru, assuming spherical symmetry, from the eSHF calculations using Lc20, Lc47 and Lc70, together with the $L(\rho_0)$ and $L(\rho_c)$ parameters. Also included are the corresponding results from the SHF calculations with the famous SLy4 interaction [40, 41]. It is seen that the four interactions give similar proton rms radius $r_p$ for $^{96}$Zr and $^{96}$Ru since they are experimentally well constrained, but the neutron radius $r_n$ increases with $L(\rho_c)$ and $L$, leading to a positive correlation between $\Delta r_{np}$ and $L(\rho_c)$ (and $L$) as expected. The $\Delta r_{np}$ of $^{208}$Pb nucleus from our calculations are also listed in Tab. I We note that those values essentially cover the current uncertainty in the $^{208}$Pb measurements.

![FIG. 1: (Color online). Proton and neutron density distributions of (a) $^{96}$Ru and (b) $^{96}$Zr nuclei from Lc20, Lc47, Lc70 and SLy4.](image-url)

Heavy ion collision models. We use four typical, commonly used models for relativistic heavy ion collisions. The Hijing (Heavy ion jet interaction generator, v1.411) model [42, 43] simulates heavy ion collisions by binary nucleon-nucleon (NN) collisions based on the Glauber theory, incorporating nuclear shadowing effect and partonic energy loss in medium. Each NN collision is described by multiple mini-jet production inspired by perturbative Quantum Chromodynamics, with the LUND [44] string fragmentation. The default version of AMPT (A Multi-Phase Transport, AMPT-def, v1.26) model [45] uses Hijing but subjects the mini-jet partons to partonic scatterings via ZPC [46] and, after fragmentation, hadronic scatterings via ART [47]. The string melting version of AMPT (AMPT-sm, v2.26) [48] converts all hadrons from Hijing to partons under partonic scatterings, and uses a simple coalescence to hadronize, followed by hadronic rescatterings. The UrQMD (Ultra relativistic Quantum Molecular Dynamics, v3.4) model [49, 50] is a microscopic transport model with covariant propagation of hadrons on classical trajectories, combined with stochastic binary scatterings, color string formation and resonance decays. Except for the input nuclear density distributions, all parameters are set to default. About 30 million events within the impact parameter range [0, 20] fm are simulated in each model for each set of nuclear densities for Ru+Ru and Zr+Zr collisions at $\sqrt{s_{NN}} = 200$ GeV.

Model results of $N_{ch}$ distributions. Charged hadrons are counted with transverse momentum $p_T > 0.2$ GeV/c.
and pseudo-rapidity $|\eta| < 0.5$. Figure 2(a) shows the $N_{ch}$ distributions in Zr+Zr collisions calculated by the four models using the nuclear density set Lc47. The distributions are similar except at large $N_{ch}$. The absolute $N_{ch}$ values are subject to large model dependence because particle production in heavy ion collisions is generally hard to model precisely. The shape of the $N_{ch}$ distribution is, on the other hand, more robust. It is determined by the interaction cross-section as a function of the impact parameter ($b$). While the tail fall-off shapes are similar among AMPT-sm, AMPT-def, and UrQMD, that of Hijing is distinct. To quantify the shape, we fit the tail distributions by

$$dP/dN_{ch} \propto -\text{Erf}(-(N_{ch}/N_{1/2} - 1)/w) + 1,$$

where $N_{1/2}$ is the $N_{ch}$ at half height and $w$ is the width of the tail relative to $N_{1/2}$. The fitted curves are superimposed in Fig. 2(a). Figure 2(b) depicts the fit $w$ values. The Hijing model has a factor of $\sim 2$ narrower tail than the other three, transport models which are similar. This feature can be used to readily distinguish models once data are available, though not the main goal of this work.

The main goal of this work is to identify which density set best describes data and hence to determine the neutron skin thickness and the symmetry energy. In a given model, at a given $b$, the $N_{part}$ and $N_{bin}$ slightly differ for different nuclear densities. Since $N_{ch}$ generally depends on $N_{part}$ and $N_{bin}$, those differences can produce an effect on $N_{ch}$. The effect is understandably small, hardly observable in a plot of the $N_{ch}$ distributions themselves, but can be magnified by the ratio of the $N_{ch}$ distribution in Ru+Ru to that in Zr+Zr [34]. These ratios using the four sets of densities, in AMPT-sm as an example, are shown in Fig. 3. The splittings of the $N_{ch}$ tails are clear.

The ratios in Fig. 3 are illustrative to highlight the differences but are cumbersome to quantify. As seen from Fig. 2(b), the tail widths are equal among the densities in a given model, so the splittings are mostly due to the slight shifts in $N_{1/2}$, or differences in the average $N_{ch}$ values. The $N_{1/2}$ value is sensitive to the chosen fit range. We thus use the relative $\langle N_{ch} \rangle$ difference between Ru+Ru and Zr+Zr,

$$R = 2\frac{\langle N_{ch} \rangle_{\text{RuRu}} - \langle N_{ch} \rangle_{\text{ZrZr}}}{\langle N_{ch} \rangle_{\text{RuRu}} + \langle N_{ch} \rangle_{\text{ZrZr}}},$$

(3)

to quantify the splitting of the $N_{ch}$ tails. Experimental measurements of $N_{ch}$ is affected by tracking efficiency, usually multiplicity dependent. While this effect is mostly canceled in $R$, it is better to use only central collisions, say top 5% centrality, where the tracking efficiency is constant to a good degree. To experimentally determine the centrality percentage, the peripheral collisions that are not recorded because of online trigger inefficiency should be taken into account. This trigger inefficiency can be experimentally corrected. Again, since $R$ is a relative measure between Ru+Ru and Zr+Zr collisions, much of the experimental effects are cancelled.

The $R$ in each model must depend on how much the Ru and Zr nuclear density distributions differ, which can be characterized by the neutron skin thickness of the Zr (or Ru) nucleus. We therefore plot in Fig. 3 the $R$ in the top 5% centrality against $\Delta r_{np}$ of the Zr nucleus from the eSHF (SHF) calculations of Lc20, Lc47 and Lc70 (SLy4). It is found that $R$ monotonically increases with $\Delta r_{np}$. This is because, with increasing $\Delta r_{np}$, the difference between Ru and Zr densities increases. This results in an increasing difference in $N_{ch}$ between Ru+Ru and Zr+Zr collisions.

Figure 4 further shows that the value of $R$ has a relatively small model dependence. Experimentally, the $N_{ch}$ distributions can be measured very precisely. The relative $\langle N_{ch} \rangle$ difference in central collisions is immune to many experimental uncertainties. Figure 4 thus strongly
suggests that the isobar data may determine $\Delta r_{np}$ relatively precisely, a conclusion that is especially strong when considering that the large difference in the $N_{ch}$ tails between Hijing and the other models can first be distinguished by data. The $^{208}$Pb $\Delta r_{np}$ calculated by the eSHF (SHF) are written on the top of Fig. 4. The band indicates the experimental range of the $^{208}$Pb $\Delta r_{np}$, which covers the entire parameter range of our calculations. Our results in Fig. 4 indicate that with a given measurement of $R$, the precision in the derived $\Delta r_{np}$ of $^{96}$Zr can be as good as 0.03 fm (or about 15%), as illustrated by the lower band (taking hypothetically $R = 0.06$). This would be an improvement of a factor of several over the current constraint from $^{208}$Pb. This shall provide a significant input to help constrain the symmetry energy, bearing important implications to nuclear matter and neutron star EOS.

We have assumed spherical nuclei in our calculations. The main idea of our work is still valid with deformed nuclei. There are a number of promising ways to determine neutron skin thickness from heavy ion collisions [51–54].

**Conclusions.** The neutron density distribution and the neutron skin thickness are not well measured experimentally but are crucial for our understanding of several important physics. In the present work, we calculated nuclear densities by energy density functional theory using several symmetry energy parameters. We show, using four heavy ion collision models, that the charged hadron multiplicity difference between isobar $^{96}$Ru+$^{96}$Ru and $^{96}$Zr+$^{96}$Zr collisions has an exquisite sensitivity to the neutron skin and symmetry energy, with small model dependence. Because the charged hadron multiplicity can be precisely measured and because the systematic uncertainties are largely canceled between the isobar collisions conducted at RHIC in 2018, our findings suggest potentially significant improvement to neutron skin and symmetry energy determination using relativistic heavy ion collision data.

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FIG. 4: (Color online). The relative $\langle N_{ch} \rangle$ ratio $R$ as a function of the Zr neutron skin thickness. The four sets of data points in order from left to right are from Lc20, SLY4, Lc47, Lc70 densities.
[21] L. Lapikas, Nucl. Phys. A553, 297e (1993).
[22] C. M. Tarbert et al., Phys. Rev. Lett. 112, 242502 (2014), 1311.0168.
[23] L. Ray, G. W. Hoffmann, and W. R. Coker, Phys. Rept. 212, 223 (1992).
[24] T. W. Donnelly, J. Dubach, and I. Sick, Nucl. Phys. A503, 589 (1990).
[25] C. J. Horowitz, G. W. Hoffmann, and W. R. Coker, Phys. Rept. 212, 223 (1992).
[26] T. W. Donnelly, J. Dubach, and I. Sick, Nucl. Phys. A503, 589 (1990).
[27] S. Abrahamyan et al., Phys. Rev. Lett. 108, 112502 (2012), 1201.2568.
[28] D. Akimov et al. (COHERENT), Science 357, 1123 (2017), 1708.01294.
[29] L.-W. Chen, C. M. Ko, and B.-A. Li, Phys. Rev. Lett. 94, 032701 (2005), nucl-th/0407032.
[30] M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch, and A. W. Steiner, Phys. Rev. Lett. 105, 122701 (2005), 0811.3107.
[31] B.-A. Li, Phys. Rev. Lett. 85, 4221 (2000), nucl-th/0009069.
[32] B.-A. Li, Phys. Rev. Lett. 91, 192701 (2003), nucl-th/0205002.
[33] L.-W. Chen, C. M. Ko, and B.-A. Li, Phys. Rev. C68, 017601 (2003), nucl-th/0302068.
[34] X. B. Wang, J. L. Friar, and A. C. Hayes, Phys. Rev. C94, 034907 (2016), 1607.02149.
[35] L.-W. Chen, C. M. Ko, and B.-A. Li, Phys. Rev. C68, 017601 (2003), nucl-th/0302068.
[36] H.-j. Xu, X. Wang, H. Li, J. Zhao, Z.-W. Lin, C. Shen, and F. Wang, Phys. Rev. Lett. 121, 022301 (2018), 1710.03086.
[37] H. Li, H.-j. Xu, J. Zhao, Z.-W. Lin, H. Zhang, X. Wang, C. Shen, and F. Wang, Phys. Rev. C98, 054907 (2018), 1808.06711.
[38] E. Chabanat, J. Meyer, P. Bonche, R. Schaeffer, and P. Haensel, Nucl. Phys. A627, 710 (1997).
[39] N. Chamel, S. Goriely, and J. M. Pearson, Phys. Rev. C80, 054304 (2009), 0911.3346.
[40] Z. Zhang and L.-W. Chen, Phys. Rev. C94, 064326 (2016), 1510.06459.
[41] Y. Zhou, L.-W. Chen, and Z. Zhang, Phys. Rev. D99, 121301 (2019), 1901.11364.
[42] Z. Zhang and L.-W. Chen, Phys. Lett. B726, 234 (2013), 1302.5327.
[43] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A635, 231 (1998), [Erratum: Nucl. Phys.A643,441(1998)].
[44] X. B. Wang, J. L. Friar, and A. C. Hayes, Phys. Rev. C94, 034314 (2016), 1607.02149.
[45] X.-N. Wang and M. Gyulassy, Phys. Rev. D44, 3501 (1991).
[46] B. Zhang, C. Ko, and Z.-W. Lin, Phys. Rev. C61, 067901 (2000), nucl-th/9907017.
[47] B. Zhang, Comput. Phys. Commun. 109, 193 (1998), nucl-th/9709009.
[48] B.-A. Li and C. M. Ko, Phys. Rev. C52, 2037 (1995), nucl-th/9505016.
[49] Z.-W. Lin and C. Ko, Phys. Rev. C65, 034904 (2002), nucl-th/0108039.
[50] S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998), nucl-th/9803035.
[51] M. Bleicher et al., J. Phys. G25, 1859 (1999), hep-ph/9904007.
[52] A. Goldschmidt, Z. Qi, C. Shen, and U. Heinz, Phys. Rev. C92, 044903 (2015), 1507.03910.
[53] G. Giacalone, Phys. Rev. C99, 024910 (2019), 1811.03959.
[54] L.-G. Pang, K. Zhou, and X.-N. Wang (2019), 1906.06429.
[55] G. Giacalone (2019), 1910.04673.