Nonexistence of a $\Lambda nn$ bound state

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(Dated: July 31, 2015)

Abstract

It has been recently suggested the existence of a neutral bound state of two neutrons and a $\Lambda$ hyperon, $^3\Lambda n$. We point out that using either simple separable potentials or a full-fledged calculation with realistic baryon-baryon interactions derived from the constituent quark cluster model there is no possibility for the existence of such a $\Lambda nn$ bound state. For this purpose, we performed a full Faddeev calculation of the $\Lambda nn$ system in the $(I, J^P) = (1, 1/2^+)$ channel using the interactions derived from the constituent quark cluster model which describe well the two-body $NN$ and $NY$ data and the $\Lambda np$ hypertriton.

PACS numbers: 21.45.-v,25.10.+s,12.39.Jh

Keywords: baryon-baryon interactions, Faddeev equations

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In a recent Rapid Communication by the experimental HypHI Collaboration [1] it has been suggested the existence of a neutral bound state of two neutrons and a Λ hyperon, $^3\Lambda n$. They analyze the experimental data obtained from the reaction $^6\text{Li} + ^{12}\text{C}$ at 2A GeV to study the invariant mass distribution of $d + \pi^-$ and $t + \pi^-$. The signal observed in the invariant mass distributions of $d + \pi^-$ and $t + \pi^-$ final states was attributed to a strangeness-changing weak process corresponding to the two- and three-body decays of an unknown bound state of two neutrons associated with a Λ, $^3\Lambda n$, via $^3\Lambda n \to t + \pi^-$ and $^3\Lambda n \to t^* + \pi^- \to d + n + \pi^-$. 

This is an intriguing conclusion since one would naively expect the Λnn system to be unbound. In the Λnn system the two neutrons interact in the $^1S_0$ partial wave while in the Λnp system they interact in the $^3S_1$ partial wave. Thus, since the NN interaction in the $^1S_0$ channel is weaker than the $^3S_1$ channel, and the Λnp system is bound by only 0.13 MeV, one may have anticipated that the Λnn system should be unbound. The unbinding of the Λnn system was first demonstrated by Dalitz and Downs [2] using a variational approach.

In a previous, by now somewhat older, paper [3] we concluded the nonexistence of Λnn bound states by solving the Faddeev equations with separable potentials whose parameters were adjusted to reproduce the Λn scattering length and effective range of the two-body channels as obtained from four different versions of the Niemegen model [4–7] as well as the corresponding NN spin-singlet and spin-triplet low-energy parameters. This leads to integral equations in one continuous variable.

As pointed out in Ref. [3], if a system can have at most one bound state then the simplest way to determine if it is bound or not is by looking at the Fredholm determinant $D_F(E)$ at zero energy. If there are no interactions then $D_F(0) = 1$, if the system is attractive then $D_F(0) < 1$, and if a bound state exists then $D_F(0) < 0$. We found in Ref. [3] that $D_F(0)$ lies between 0.46 and 0.59 for the different models constructed by the Niemegen group so that the system is quite far from being bound.

Of course, it can be argued that the use of simple separable potentials is not a realistic assumption. Besides, since our previous work the knowledge of the strangeness −1 two-baryon system has improved and the models to study these systems are more tightly constrained. Therefore, we have now reexamined the Λnn system within a realistic baryon-baryon formalism obtained from the quark model. The baryon-baryon interactions involved in the study of the coupled $\Sigma NN - \Lambda NN$ system are obtained from the constituent quark cluster model [8, 9]. In this model baryons are described as clusters of three interacting massive...
(constituent) quarks, the mass coming from the spontaneous breaking of chiral symmetry. The first ingredient of the quark-quark interaction is a confining potential. Perturbative aspects of QCD are taken into account by means of a one-gluon potential. Spontaneous breaking of chiral symmetry gives rise to boson exchanges between quarks. In particular, there appear pseudoscalar boson exchanges and their corresponding scalar partners \cite{10, 11}. Explicit expressions of all the interacting potentials and a more detailed discussion of the model can be found in Refs. \cite{9, 10}.

In Refs. \cite{10, 11} we established the formalism to study the ΛNN system at threshold using the baryon-baryon interactions obtained from the constituent quark cluster model which leads to integral equations in the two continuous variables \(p\) and \(q\), where \(p\) is the relative momentum of the pair and \(q\) is the relative momentum of the third particle with respect to the pair. In order to solve these equations the two-body \(t\)–matrices are expanded in terms of Legendre polynomials leading to integral equations in only one continuous variable coupling the various Legendre components required for convergence.

This model takes into account the coupling \(NΛ – NΣ\) as well as the tensor force responsible for the coupling between \(S\) and \(D\) waves. In particular, for the \(ΛNN\) channel \((I, J^P) = (1, 1/2^+)\) which corresponds to the conjectured Λnn bound state there is a total of 21

| \(\ell, s, j, i, \lambda, J\) | \(\ell, s, j, i, \lambda, J\) |
|-----------------|-----------------|
| \(000\frac{1}{2}\), \(011\frac{1}{2}\) | \(000\frac{1}{2}\), \(011\frac{1}{2}\) |
| \(011\frac{1}{2}\), \(000\frac{1}{2}\) | \(011\frac{1}{2}\), \(000\frac{1}{2}\) |
| \(211\frac{1}{2}\), \(000\frac{1}{2}\) | \(211\frac{1}{2}\), \(000\frac{1}{2}\) |
| \(000\frac{1}{2}\), \(211\frac{1}{2}\) | \(000\frac{1}{2}\), \(211\frac{1}{2}\) |
| \(011\frac{1}{2}\), \(211\frac{1}{2}\) | \(011\frac{1}{2}\), \(211\frac{1}{2}\) |

TABLE I: Two-body ΣN channels with a nucleon as spectator \((\ell, s, j, i, \lambda, J)\), two-body ΛN channels with a nucleon as spectator \((\ell, s, j, i, \lambda, J)\), two-body NN channels with a Σ as spectator \((\ell, s, j, i, \lambda, J)\) and two-body NN channels with a Λ as spectator \((\ell, s, j, i, \lambda, J)\) that contribute to \((I, J^P) = (1, 1/2^+)\) state. \(\ell, s, j, i, \lambda, J\), are, respectively, the orbital angular momentum, spin, total angular momentum, and isospin of a pair, while \(\lambda\) and \(J\) are the orbital angular momentum of the third particle with respect to the pair and the result of coupling \(\lambda\) with the spin of the third particle.
coupled channels contributing to the state. We give in Table I the quantum numbers of these contributing channels.

In Ref. [11] we showed that if one increases the triplet $N\Lambda$ interaction by increasing the triplet scattering length then the $\Lambda NN$ state with $(I, J^P) = (0, 3/2^+)$ becomes bound and since that state does not exist we are allowed to set an upper limit of 1.58 fm for the $\Lambda N$ spin triplet scattering length. Since, in addition, the fit of the hyperon-nucleon cross sections is worsened [10] when the spin-triplet scattering length is smaller than 1.41 fm we concluded that $1.41 \leq a_{1/2,1} \leq 1.58$ fm. By requiring that the hypertriton binding energy had the experimental value $B = 0.13 \pm 0.05$ MeV we obtained for the $\Lambda N$ spin-singlet scattering length the limits $2.37 \leq a_{1/2,0} \leq 2.48$ fm.

Thus, we constructed twelve different models corresponding to different choices of the spin-singlet and spin-triplet $\Lambda N$ scattering lengths which describe equally well all the available experimental data. We solved the three-body problem taking full account of the $\Lambda NN - \Sigma NN$ coupling as well as the effect of the D waves. We present in Table II the Fredholm determinant at zero energy of the $(I, J^P) = (1, 1/2^+)$ state for these models. The realistic quark model interactions predict a Fredholm determinant at zero energy ranging between 0.38 and 0.42, close to the interval 0.46–0.59 obtained from the separable potentials of the Niemegen group. As one can see, in all cases the Fredholm determinant at zero energy is positive and far from zero, excluding the possibility for binding in this system. From the results of Table II and from the energy dependence of the Fredholm determinant shown in Fig. 2 of Ref. [11] one can infer that the $(I, J^P) = (1, 1/2^+)$ state is unbound by at least 5–10 MeV, which is a large energy in comparison with the 0.13 MeV binding energy of the hypertriton.

To summarize, we have shown that using either simple separable potentials or a full-

| $a_{1/2,0}$ | $a_{1/2,1} = 1.41$ | $a_{1/2,1} = 1.46$ | $a_{1/2,1} = 1.52$ | $a_{1/2,1} = 1.58$ |
|------------|-------------------|-------------------|-------------------|-------------------|
| 2.33       | 0.42              | 0.41              | 0.40              | 0.38              |
| 2.39       | 0.42              | 0.41              | 0.39              | 0.38              |
| 2.48       | 0.42              | 0.41              | 0.40              | 0.38              |
fledged calculation with realistic baryon-baryon interactions derived from the constituent quark cluster model there is no possibility for the existence of a $\Lambda nn$ bound state. Thus, the signal observed in the invariant mass distributions of $d + \pi^-$ and $t + \pi^-$ final states in the analysis of the experimental data obtained from the reaction $^6\text{Li} + ^{12}\text{C}$ at 2A GeV and adduced to the existence of a neutral bound state of two neutrons and a $\Lambda$ hyperon must be due to a different effect.

**Acknowledgments**

This work has been partially funded by COFAA-IPN (México), by Ministerio de Educación y Ciencia and EU FEDER under Contract No. FPA2010-21750-C02-02 and by the Spanish Consolider-Ingenio 2010 Program CPAN (CSD2007-00042).

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