Particle acceleration efficiencies in astrophysical shear flows

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Abstract. The acceleration of energetic particles in astrophysical shear flows is analyzed. We show that in the presence of a non-relativistic gradual velocity shear, power law particle momentum distributions \( f(p) \propto p^{-(3+\alpha)} \) may be generated, assuming a momentum-dependent scattering time \( \tau \propto p^\alpha \), with \( \alpha > 0 \). We consider possible acceleration sites in astrophysical jets and study the conditions for efficient acceleration. It is shown, for example, that in the presence of a gradual shear flow and a gyro-dependent particle mean free path, synchrotron radiation losses no longer stop the acceleration once it has started to work efficiently. This suggests that shear acceleration may naturally account for a second, non-thermal population of energetic particles in addition to a shock-accelerated one. The possible relevance of shear acceleration is briefly discussed with reference to the relativistic jet in the quasar 3C 273.

INTRODUCTION

Collimated relativistic outflows have been detected in a variety of sources including AGNs, microquasars and \( \gamma \)-ray bursts (e.g., [14, 29]). Today there is strong evidence that many (if not all) of these jets are characterized by a significant shear in their velocity field (cf. [21] for a review). It appears thus very likely that energetic particles can be accelerated efficiently by scattering off magnetic inhomogeneities carried within such shear flows. Possible implications of such an acceleration process have so far been studied by several authors: in the pioneer works on non-relativistic gradual shear flows, Berezhko & Krymskii (1981), for example, showed that under suitable conditions power law particle momentum distributions may be formed (cf. also [3]), whereas Earl, Jokpii & Morfill (1988) derived the corresponding diffusive particle transport equation including shear and inertial effects, assuming the diffusion approximation to be valid. The work by Earl et al. was generalized to the relativistic regime by Webb (cf. [26, 27, 28]) using a mixed-frame approach. Based on those results, Rieger & Mannheim (2002) have recently studied the acceleration of particles in rotating and shearing flows with application to the relativistic jets in AGN. Particle acceleration in non-gradual relativistic shear flows (e.g., in the presence of a relativistic velocity jump), on the other hand, has been studied by Ostrowski ([15, 16, 17]) using Monte Carlo simulations, showing that very flat momentum spectra may be possible.

In the present contribution we will particularly focus on the conditions required for efficient shear acceleration in order to assess whether shear-accelerated particles may indeed significantly contribute to the emission at a certain energy band.
FIGURE 1. Sketch of a two-dimensional, non-relativistic shear velocity profile with the flow directed along the z-axis (left). Qualitatively, a particle may be considered as experiencing a gradual shear profile if its mean free path $\lambda$ is much smaller than the width $\Delta x$ of the shear layer and if the flow velocity changes smoothly within that layer. In the case where the particle becomes so energetic that its mean free path $\lambda = c \tau$ becomes larger than $\Delta x$, it essentially experiences a non-gradual velocity shear, i.e. a discontinuous jump in the flow velocity (right).

**BASIC PRINCIPLES OF SHEAR ACCELERATION**

Shear acceleration is based on the idea that energetic particles may gain energy by scattering off systematically moving small-scale magnetic field irregularities. These irregularities are thought to be embedded in a collisionless shear flow such that their velocities correspond to the local flow velocity, i.e. second-order Fermi effects are neglected. The scattering process is assumed to occur in such a way that the particles are randomized in direction and their energies conserved in the local comoving fluid frame. In the presence of a velocity shear, the momentum of a particle travelling across the shear changes (with respect to the local fluid frame), so that a net increase may occur (e.g., [10]). For illustration consider a non-relativistic continuous shear flow with velocity profile given by (cf. Fig.1)

$$\vec{u} = u_z(x) \hat{e}_z.$$ \hspace{1cm} (1)

Let $\vec{v} = (v_x, v_y, v_z)$ be the velocity vector, $m$ the relativistic mass and $\vec{p}_1$ the initial momentum (relative to local flow frame) of the particle. Within one scattering time $\tau$ (initially assumed to be independent of momentum) the particle travels a distance $\delta x = v_x \tau$ across the shear, so that in the presence of a gradual shear the flow velocity will have changed by $\delta \vec{u} = \delta u \hat{e}_z$, with $\delta u = (\partial u_z / \partial x) \delta x$. Hence the particle’s momentum relative to the flow becomes $\vec{p}_2 = \vec{p}_1 + m \delta \vec{u}$, i.e.

$$p_2^2 = p_1^2 + 2 m \delta u p_{1,z} + m^2 (\delta u)^2 .$$ \hspace{1cm} (2)

As the next scattering event preserves the magnitude of the particle momentum (in the comoving frame) the particle magnitude will have this value in the local flow frame and hence a net increase in momentum may occur with time. Using spherical coordinates and averaging over solid angle (assuming an almost isotropic particle distribution), the average rate of momentum change and the average rate of momentum dispersion can be
written as (cf. [10])

\[
\left\langle \frac{\Delta p}{\Delta t} \right\rangle \equiv \frac{2 < (p_2 - p_1) >}{\tau} = \frac{4}{15} p \left( \frac{\partial u_z}{\partial x} \right)^2 \tau \tag{3}
\]

\[
\left\langle \frac{\Delta p^2}{\Delta t} \right\rangle \equiv \frac{2 < (p_2 - p_1)^2 >}{\tau} = \frac{2}{15} p^2 \left( \frac{\partial u_z}{\partial x} \right)^2 \tau, \tag{4}
\]

i.e. both depending on the square of the flow velocity gradient. Note that it can be shown that a momentum-dependent scattering time obeying a power law of the form \( \tau \propto p^\alpha \) may be accommodated by replacing \( 4 \rightarrow (4 + \alpha) \) in Eq. (3). Using these results we may write down a simple Fokker-Planck transport equation for the phase space particle distribution function \( f(p) \), assuming a mono-energetic injection of particles with momentum \( p_0 \). Solving for the steady-state with \( \alpha > 0 \) one immediately arrives at (see [22])

\[
f(p) \propto p^{-(3+\alpha)} H(p - p_0), \tag{5}
\]

where \( H(p) \) is the Heaviside step function. Hence for a mean scattering time scaling with the gyro-radius (Bohm case), i.e. \( \tau \propto p, \alpha = 1 \), one obtains \( f(p) \propto p^{-4} \), i.e. a power law particle number density \( n(p) \propto p^{-2} \) which translates into a synchrotron emissivity \( j_\nu \propto \nu^{-1/2} \).

**POSSIBLE SHEAR SITES IN ASTROPHYSICAL JETS AND ASSOCIATED EFFICIENCIES**

In general we may distinguish at least three possible shear sites in astrophysical jets, cf. Rieger & Duffy (2004,2005):

1. **Longitudinal gradual shear:** Observationally, there is mounting evidence for an internal velocity stratification parallel to the jet axis. In several sources (including 3C353, M87 and Mkn 501, cf. [7, 18, 25]), for example, at least a two-component velocity structure consisting of a fast velocity spine and a slower moving boundary layer is indicated. Such a velocity structure is also indirectly supported by unification arguments for BL Lacs and FR I ([5]) and results from hydrodynamical jet simulations (e.g., [1]). Moreover, Laing et al. (1999) have argued recently that the intensity and polarization systematics in kpc-scale FR I jets are suggestive of a radially (continuously) decreasing velocity profile \( v_z(r) \). In order to estimate the possible particle acceleration efficiency in the presence of a longitudinal gradual shear, we may consider a simple realization where the flow velocity profile decreases linearly from relativistic to non-relativistic speeds over a scale \( \Delta r \). It can be shown then (see [21]) that the minimum acceleration timescale is of the order of

\[
t_{\text{acc}} \sim \frac{3}{\lambda} \frac{(\Delta r)^2}{c \gamma_b(r)^3}, \tag{6}
\]

where \( \gamma_b(r) > 1 \) is the (position-dependent) bulk Lorentz factor of the flow and \( \lambda \) the particle mean free path. As \( \lambda_{\text{proton}} \gg \lambda_{\text{electron}} \) the acceleration of electrons is in general much more restricted than the acceleration of protons. In the case where the particle
FIGURE 2. Maximum width of velocity shear layer $\Delta r$ (in parsecs) as a function of the bulk Lorentz factor $\gamma_b$, obtained by requiring the acceleration timescale to be smaller or equal to the synchrotron cooling timescale, i.e. $t_{acc} \leq t_{cool}$. The maximum width $\Delta r$ scales with $B^{-3/2}$ and is proportional to the particle's rest mass square. The acceleration mechanism is generally much more favourable to protons than to electrons.

2. Longitudinal non-gradual shear: When a particle becomes so energetic that its mean free path $\lambda$ becomes larger than the width of the velocity transition layer, it will essentially experience a non-gradual velocity shear, i.e. a (discontinuous) jump in the flow velocity. Ostrowski (1990,1998), for example, has convincingly argued that the jet side boundary between the relativistic jet interior and its ambient medium may naturally represent a relativistic realization of such a non-gradual velocity shear. Based on Monte-Carlo simulations the estimated minimum acceleration timescale is of order (cf. [15],[16]).

$$t_{acc} \sim 10 \frac{r_g}{c} \quad \text{provided} \quad r_g > \Delta r,$$

where $r_g$ denotes the gyro-radius of the particle and $\Delta r$ is the width of the transition layer. Due to the condition $r_g > \Delta r$ an efficient acceleration of electrons appears excluded by virtue of their associated rapid radiation losses (e.g., for $B = 0.01$ Gauss and $\Delta r \geq 0.001$ pc, i.e. $t_{acc} > 10^9$ sec, electron Lorentz factors $\gamma \sim 2 \cdot 10^{10}$ would be required in order to
fulfill $r_g > \Delta r$, implying cooling timescales $t_{\text{cool}} \sim 4 \cdot 10^2$ sec well below the acceleration timescale). Efficient acceleration of protons, on the other hand, can be quite possible as long as their particle mean free paths remain smaller than the width of the jet.

3. Transversal gradual shear: Apart from a longitudinal velocity stratification, astrophysical jets are also likely to have a significant velocity shear perpendicular to their jet axis (‘transversal shear’). In particular, several independent arguments suggest that the flow velocity field in these jets is characterized by an additional rotational component. The strong correlation between the disk luminosity and the bulk kinetic power in the jet (e.g., [19]), for example, and the observational evidence for a disk-origin of jets (e.g., [8, 13]), suggest that a significant amount of rotational energy of the disk is channeled into the jet. Such an internal jet rotation is also implied in theoretical MHD models for the origin of jets as magnetized disk winds (e.g., [24]). In order to evaluate the acceleration potential associated with such shear flow, Rieger & Mannheim (2002) have recently analyzed the relativistic particle transport in rotating and shearing jets. Based on an analytical (mixed-frame) approach for the relativistic Boltzmann equation, using a simple (BKG) relaxation scattering term, and assuming validity of the diffusion approximation, they considered the acceleration of particle in a cylindrical jet model with relativistic outflow velocity $v_z$ and different azimuthal rotation profiles. In the case of a simple (non-relativistic) Keplerian shear profile they found that local power law spectra $f(p) \propto p^{-(3+\alpha)}$ are obtained for $\tau \propto p^{\alpha}$, $\alpha > 0$, whereas for more complex rotation profiles (e.g., flat rotation) steeper spectra may be possible.

In order to gain insights into the acceleration efficiency, let us consider a flow velocity field with relativistic $v_z$ and an azimuthal Keplerian rotation profile of the form $\Omega(r) = \Omega_k (r_{in}/r)^{3/2}$, where $\Omega_k$ is a constant. The acceleration timescale then scales as (see [21])

$$t_{\text{acc}}(r) \propto \frac{1}{\lambda} \frac{1}{\Omega_k^2} \left( \frac{r}{r_{in}} \right)^3,$$

assuming the flow to be radially confined to $r_{in} \leq r \leq r_j$, where $r_j$ is the jet radius, indicating that efficient particle acceleration generally requires a region with significant rotation. Note that for a radially decreasing rotation profile the higher energy emission will generally be concentrated closer to the axis (i.e. toward smaller radii). A comparison of acceleration and cooling timescales shows that electron acceleration is usually very restricted (i.e., only possible for $r \sim r_{in}$), whereas proton acceleration appears well possible. Note again that for $\lambda \propto \gamma$ (Bohm case), the acceleration timescales decreases with $\gamma$ in the same way as $t_{\text{cool}}$, suggesting that losses are no longer able to stop the acceleration process once it has started to work efficiently.

APPLICATIONS

Observational and theoretical evidence suggest that astrophysical jets may be characterized by a significant velocity shear. While internal jet rotation, for example, is likely to be present at least in the initial parts of the jet, a significant longitudinal velocity shear (parallel to the jet axis) prevailing all along the jet might be expected for most powerful sources. The acceleration of particles occurring in such shear flows may thus naturally
account for a steady second population of synchrotron-emitting particles, contributing to the observed emission in addition to shock-accelerated ones (cf., [11]). Moreover, shear acceleration may also offer a natural explanation for the extended radio and optical emission observed from several sources. In the case of 3C273, for example, the optical spectral index is found to vary only very smoothly along the (large-scale) jet in contrast to expectations from simple shock scenarios (cf. [9]). The jet seems to be highly relativistic even on kpc scales ([23]) so that longitudinal shear acceleration may perhaps work efficiently nearly all along the jet. Moreover, there is also evidence for helical bulk motion in the large-scale jet ([2]) and internal jet helicity on the VLBI mas-scale and below, suggesting that particle acceleration due to internal jet rotation may contribute on pc-scale and perhaps even on larger scales.

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