Reliability analysis of doubly Reinforced Concrete Beams retrofitted by plate bonding

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Abstract: This paper concerns the safety of the simply supported reinforced concrete beams subjected to only bending loads: so that the reliability index and probability of failure are to be calculated. Reliability index was carried out by using Hasofer-Lind method for the doubly reinforced beam with and without retrofitted beams having externally bonded plates (porous and solid) with variable thickness for the limit state in bending. Reliability index can be calculated by using MS-EXCEL. The reliability index for control beam is 0.4022 and the FR-A4 beam is 3.9791. So, the reliability index is high for retrofitted beam only. The Probability of failure (3.5E-05) for FR-C4 beam is very low under the combination of porous plate and 3mm plate thickness.

Keywords: Reinforced concrete beams, Retrofitting, Plate bonding, Bending, Reliability analysis, Limit state function, AFOSM, Reliability Index, Probability of failure.

1. Introduction
1.1 General
Reliability-based techniques have been used broadly in structural engineering. This approach allows designers to realistically assess the possibility of structural failure. It involves the use of probability and definition of a safety index to achieve a balance between safety and cost. Evaluation of structural safety related to the design procedure for reinforced concrete beams in pure bending. The so-called reliability index method was also used to design reinforced concrete beams and structural steel members. The performance of a structure is evaluated by its safety, serviceability, and economy. The information about input variables is never certain, precise, and complete. The sources of uncertainties may be (i) inherent uncertainty, i.e. physical uncertainty, (ii) limited data, i.e. statistical uncertainty, (iii) imperfect knowledge, i.e., model uncertainty, and (iv) gross errors. In the presence of uncertainties, the absolute safety of a structure is difficult due to (i) the unpredictability of (a) loads on a structure during its life, (b) in-place material strengths, and (c) human mistakes, (ii) structural inventions in forming the mathematical model of the structure to calculate its response or behaviour, and (iii) the limitations in numerical methods. Therefore, some risk of unacceptable performance must be tolerated. For the risk of life, structural safety is important. In the conventional deterministic analysis and design methods, it is assumed that all parameters (loads, strengths of materials, etc.) are not subjected to probabilistic variations. The safety factors provided in the existing codes and standards, primarily based on practice, decision, and understanding, may not be satisfactory and economical.

The concept of reliability has been applied to many fields and has been understood in many ways the most common meaning, and accepted by all, of reliability is that reliability is the probability of an item performing its intended function over a given period under the operating conditions encountered. It is
important to note that the above definition stresses the four significant elements, viz. (i) probability, (ii) intended function, (iii) time, and (iv) operating conditions. Because of the uncertainties, the reliability is a probability which is the first element in the definition. The second point, intended function, signifies that the reliability is a performance characteristic. For a structure to be reliable, it must perform a certain function or functions reasonably for which it has been designed, i.e. safety against shear or flexure or torsion, etc. The reliability is always related to time. In the case of structure, it is related to the lifetime of the structure. During this specified life of the structure, it must perform the assigned function satisfactorily. The last point is the operating conditions. This establishes the actions or stresses that will be imposed on the structure. These may be loads, temperature, shock, vibrations, corrosive atmosphere, etc. Reliability also changes for quality control, workmanship, production procedure, inspection, etc.

In structural reliability analyses, a performance function or failure function, $G$, can be defined as the resistance minus the loading (Thoft-Christensen & Baker, 1982). The probability of failure refers to the probability that an undesired performance occurs. Besides, both the resistance ($R$) and the loading ($S$) are regarded as continuous random variables. Therefore, each has a probability density function (PDF) as shown in Figure 1. Furthermore, the failure function, $G = R - S$, can also be observed as a random variable with its own probability density function. The probability of failure corresponds to the shaded area which is shown in Figure 1 and Figure 2. In this study, the resistance was characterised by the moment carrying capacity and the loading was signified by the ultimate dead and live load moment that could be maintained by the beam. When the failure action, $G$, was greater than zero, a safe structure state existed. Besides, when the failure function $G$ was less than or equal to zero, a failure state existed as shown in Figure 2 (Steinberg, 1997).

![PDFs of Load Effect and Resistance and Failure Function](image1)

**Figure 1. PDFs of Load Effect and Resistance and Failure Function**

![Distribution of Failure Function $G = R - S$](image2)

**Figure 2. Distribution of Failure Function $G = R - S$**

1.2 Objective and Scope of work
1) The objective of this study is to compare the reliability index and probability of failure for RC beams and those strengthened by externally bonded plates of varying thickness.
1.3 Scope
1) The thickness of Plates considered: 1mm and 3mm.
2) Types of plates considered - Solid plate and perforated plate.
3) Size of beam considered is doubly reinforced beams for 120×210 mm² size beam.

2. Literature review
Nikolaos et al. (1995) [1] opined that the Strengthening of concrete structures in flexure with superficially epoxy-bonded carbon-fibre-reinforced plastic (CFRP) laminates is becoming a progressively popular retrofit technique among scholars and engineers worldwide. Strength reduction factors are derived, to attain a uniform reliability index of about three over a broad range of design situations. A general strength reduction factor $\varnothing = 0.80$ is offered. Carol Shield (2009) [2] says that the structural reliability of concrete flexural members reinforced with fibre reinforced polymer (FRP) reinforcement is examined. Flexural designs using either ACI 440.1R-03 or ACI 440.1R-06 provide satisfactory reliability, with reliability indices between 3.5 and 4.8, with the older versions of ACI 440.1R yielding higher reliability. Hany and Antonio (2013) [3] explained the concept of reliability index, an interim index of “comparative reliability” is proposed that bypasses the loading variables and weighs the resistances of two structural fundamentals with the same ultimate limit state directly against each other. This concept is put into practice by calculating flexural and shear-strength reduction factors for fibre-reinforced polymer (FRP) reinforced concrete (RC) members by comparison with conventional steel-reinforced concrete beams possessing the same ultimate capacity. Salma et al. [4] (2017) observed that the prevailing approach of designing rectangular bolstered concrete beam is based on limit state plan philosophy which makes use of partial safety essentials for fabric electricity and cargo. A strengthened concrete body is modelled in ETABS software program, the moment and vicinity of anxiety reinforcement values are extracted from the equal software program for statistical analysis. Probability of failure is acquired by Monte Carlo Simulation method. The complete reliability analysis turned into applied thru developing software in MATLAB software program.

3. Experiment work
3.1 Details of beam:
A total of 5 beams, with 120 mm × 210 mm rectangular cross-section with an overall length of 1500 mm and a clear span of 1440 mm were cast. A clear cover of 20 mm is adopted. The beams are designed and cast as flexure deficit beams with M20 grade of concrete, Fe500 grade of steel. The beams are tested under a two-point loading condition. Out of 5 beams, 1 beam is taken as control beam and the remaining 4 beams were retrofitted with steel plates having different thickness and the plates with perforations. To describe the specimens conducted in the study, the following notation is adopted:
CB: Control beam; FR: Flexure retrofitted beam

| Name of specimen | Shear reinforcement | Asc | Thickness of plate (mm) | Type of plate | Ast | Thickness of epoxy layer (mm) |
|------------------|--------------------|-----|-------------------------|----------------|-----|-----------------------------|
| CB               | 8mm Dia. 2 legged. vertical stirrups with spacing 120 mm | 10 mm, Dia. 2 bars | - | - | 12 mm Dia. 2 bars | - |
| FR-1             |                    |     | 1 | Solid porous | 1 |
| FR-2             |                    |     | 3 | Solid Porous | 1 |
| FR-3             |                    |     | 3 |                       | 1 |
| FR-4             |                    |     | 3 |                       | 1 |

3.2 Beam reinforcement details
The beam reinforcement details and details of plates bonded to the beams are presented in table 2 and figure 3, 4, 5 and 6.
Table 2 Beam reinforcement details

| Beam designation | Width of beam (mm) | Depth of beam (mm) | Longitudinal Reinforcement (mm) | Thickness of plate (mm) | Depth of beam after retrofitting with a 1mm thickness of an epoxy layer (mm) |
|------------------|-------------------|-------------------|-------------------------------|------------------------|---------------------------------------------------------------|
| CB               | 120               | 210               | 10 12                         |                        | 210                                                           |
| FR-1             | 120               | 210               | 10 12                         |                        | 212                                                           |
| FR-2             | 120               | 210               | 10 12                         |                        | 212                                                           |
| FR-3             | 120               | 210               | 10 12                         | 3                      | 214                                                           |
| FR-4             | 120               | 210               | 10 12                         | 3                      | 214                                                           |

Figure 3. Detailed drawing of Beam reinforcement

Figure 4. Detailed drawing of a solid plate

Figure 5. Detailed drawing of the porous plate

Figure 6. Longitudinal sectional details for all beams
Table 3 Practical values of strengths & deflections and ultimate load value:

| Beam designation | fck (N/mm²) | 1st flexure crack Load (kN) | Deflection at First flexure crack (mm) | 1st shear crack Load (kN) | Deflection at 1st shear crack (mm) | Ultimate failure load (kN) | Deflection at Ultimate load (mm) |
|------------------|-------------|-----------------------------|----------------------------------------|--------------------------|-----------------------------------|-----------------------------|---------------------------------|
| CB               | 36.46       | 91.096                      | 8.638                                  | 89.098                   | 7.739                             | 94.72                       | 11.709                          |
| FR-1             | 32.70       | 94.719                      | 4.33                                   | 104.58                   | 4.891                             | 117.248                     | 6.523                           |
| FR-2             | 37.81       | 84.868                      | 8.678                                  | -                        | -                                 | 90.209                      | 18.267                          |
| FR-3             | 29.40       | 108.99                      | 17.826                                 | 108.98                   | 19.44                             | 110.658                     | 22.089                          |
| FR-4             | 35.50       | 73.926                      | 3.45                                   | -                        | -                                 | 84.96                       | 4.588                           |

4. Reliability analysis of reinforced concrete control beam

The step by step procedure for Reliability analysis is presented as follows.

1. Limit state equation in terms of basic variables is

   \[ g(x) = 0.133 f_{ck} b d^2 + 0.87 f_y A_{st2} (d-d^1) \frac{WL}{6} \]  

   \[ \frac{W}{2} \quad \frac{W}{2} \]

   ![Figure 7. Bending moment diagram of a simply supported beam](image)

2. The basic variables are normalized as follows.

   \[ Z_1 = \frac{f_{ck} - \mu f_{ck}}{\sigma f_{ck}}; \quad Z_2 = \frac{b - \mu b}{\sigma b}; \quad Z_3 = \frac{d - \mu d}{\sigma d} \]

   \[ Z_4 = \frac{f_y - \mu f_y}{\sigma f_y}; \quad Z_5 = \frac{A_{st2} - \mu A_{st2}}{\sigma A_{st2}}; \quad Z_6 = \frac{w - \mu w}{\sigma w}; \quad Z_7 = \frac{l - \mu l}{\sigma l} \]
3. The mean and standard deviations are calculated by using the below formula.

Table 4 Mean and Standard deviations for random variables

| Random variable | Mean of random variable $\mu_{fy} = \frac{x_{fy}}{1 - 1.645\text{cov}_{fy}}$ | Standard deviation of random variable $\sigma_{fy} = \mu_{fy} \times \text{cov}_{fy}$ |
|-----------------|-------------------------------------------------|-------------------------------------------------|
| $f_y$           | 598.444                                         | 59.844                                          |
| $A_{st2}$       | 69.073                                          | 2.762                                           |
| $D$             | 181.987                                         | 3.639                                           |
| $fck$           | 26.551                                          | 3.9823                                          |
| $W$             | 124.082                                         | 2.481                                           |
| $l$             | 1488.987                                        | 29.779                                          |

4. Substituting mean and standard deviations values in normal distributions. Basic variables in terms of normal distributions are calculated.

$$f_{ck} = 3.9823Z_1 + 26.551$$
$$b = 2.481Z_2 + 124.082$$
$$d = 3.639Z_3 + 181.987$$
$$f_y = 59.844Z_4 + 598.444$$
$$A_{st2} = 2.762Z_5 + 69.073$$

5. Substituting the above values in equation (1) we obtain

$$g(x) = 0.133\{ (3.982Z_1 + 26.551)(2.481Z_2 + 124.082)(3.639Z_3 + 181.987)^2 \} + 0.87\{ (59.844Z_4 + 598.444)(2.762Z_5 + 69.073) - \{ (70.7499 \times 10^3Z_6 + 3.537585 \times 10^6Z_4 + 353.7546 \times 10^3Z_7 + 17.6881729 \times 10^6) \} \}

6. At the design point adopting $Z_1 = \alpha_1 \beta_1$, $g(z)$ in terms of $\alpha_1, \beta_1$ is re-written by using the Initial iteration-1: $\beta = 3, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7 = 0.377$

$$\alpha_6 \alpha_7 = 0.377$$

Then the equation becomes

$$\beta_{\text{new}} = \frac{-3.326959918 \times 10^6}{17.3996 \times (\beta^3)\alpha_1\alpha_2\alpha_3^2 + 43.5168 \times (10^3)\beta\alpha_1\alpha_2 + 1.7403 \times (10^3)\beta^2\alpha_1\alpha_2\alpha_3 + 870.2120\beta^2\alpha_1\alpha_3^2 + 2.176413937 \times 10^6\alpha_1 + 87.0389 \times 10^5\beta^3\alpha_1\alpha_3 + 116.0172 \times \beta^2\alpha_2 + 290.1609 \times 10^5\alpha_2 + 11.6040 \times 10^3\beta^2\alpha_1\alpha_3 + 5.8023 \times 10^3\beta^3\alpha_2^2 + 580.3541 \times 10^4\beta\alpha_1\alpha_3 + 523.9290\beta^2\alpha_2\alpha_4\alpha_5 + 26.1699 \times 10^5\beta^3\alpha_4\alpha_5 + 13.0866 \times 10^5\beta^2\alpha_4\alpha_5 + 654.4626 \times 10^4\alpha_4 + 5.2329 \times 10^5\beta^3\alpha_4\alpha_5 + 261.7017 \times 10^3\alpha_5 + 130.8678 \times 10^4\alpha_6 - 2.8760 \times 10^3\beta^4\alpha_4\alpha_5 - 71.9247 \times 10^2\alpha_4 + 1.6529 \times 10^3\alpha_5 - 70.7499 \times 10^3\beta\alpha_4\alpha_6 + 7.5755849 \times 10^5\alpha_6 - 353.7546 \times 10^3\alpha_7}

$$\beta_{\text{new}} = 0.747523$$

7. A derivative of $g(x)$ with respect to $Z_1, Z_2, Z_3, Z_4, Z_5, Z_6$ and $Z_7$ will result in

$$\frac{\partial g(x)}{\partial z_1} = 17.3996Z_2Z_3^2 + 43.5168 \times 10^3Z_2 + 1.7403 \times 10^3Z_2Z_3 + 870.2120 \times Z_3^2 + 2.176413937 \times 10^6 + 87.0389 \times 10^5Z_3$$

Substituting $Z_1 = \alpha_1\beta_1$, $Z_2 = \alpha_2\beta_2$, $Z_3 = \alpha_3\beta_3$, $Z_4 = \alpha_4\beta_4$, $Z_5 = \alpha_5\beta_5$, $Z_6 = \alpha_6\beta_6$, $Z_7 = \alpha_7\beta_7$.

Like this similarly, the derivatives of $g(x)$ with respect to $Z_2, Z_3, Z_4, Z_5, Z_6$ and $Z_7$ and then substitute $Z_2 = \alpha_2\beta_2$, $Z_3 = \alpha_3\beta_3$, $Z_4 = \alpha_4\beta_4$, $Z_5 = \alpha_5\beta_5$, $Z_6 = \alpha_6\beta_6$, $Z_7 = \alpha_7\beta_7$. 
Then we get the 
\[ \frac{\partial g(x)}{\partial z_2} = 230168.6, \quad \frac{\partial g(x)}{\partial z_3} = 581048.2, \quad \frac{\partial g(x)}{\partial z_4} = 542060.9, \quad \frac{\partial g(x)}{\partial z_5} = 228454.4, \]
\[ \frac{\partial g(x)}{\partial z_6} = -7617603, \quad \frac{\partial g(x)}{\partial z_7} = -433773 \]

8. Calculating the k value:

\[ k = \sum_{i=1}^{n} \left( \frac{\delta g_1}{\partial g_2} \right)^2 = 794241 \]

The new values of \( \alpha_i \) are determined as follows.

**Table 5 Design points for different random variables**

| Design point \( \alpha_i \) | \( \alpha_i = \frac{1}{k} \left( \frac{\delta g_1}{\delta g_2} \right) \) |
|-----------------------------|---------------------------------|
| \( \alpha_1 \)              | -0.25585                        |
| \( \alpha_2 \)              | -0.02898                        |
| \( \alpha_3 \)              | -0.07316                        |
| \( \alpha_4 \)              | -0.06825                        |
| \( \alpha_5 \)              | -0.02876                        |
| \( \alpha_6 \)              | 0.95910                         |
| \( \alpha_7 \)              | 0.054615                        |

**Figure 8: Iterations for control beam (CB)**
## Table 6 Results

| Sl. No | Beam designation | Reliability Index (β) | Probability of failure (Pf) |
|--------|------------------|-----------------------|-----------------------------|
| 1      | CB               | 0.4022                | 3.4E-01                     |
| 2      | FR-1             | 1.6029                | 5.4E-02                     |
| 3      | FR-2             | 3.0045                | 1.3E-03                     |
| 4      | FR-3             | 2.5332                | 5.7E-03                     |
| 5      | FR-4             | 3.9791                | 3.5E-05                     |

### 5. Results discussion

5.1 *Comparison of reliability index for control beam and flexure retrofitted beam having different thickness of the plate*

![Figure 9. Reliability index for control beam and flexure retrofitted beam](image)

From figure 9, it can be observed that the reliability index is more for flexure retrofitted beam when compared to the control beam. Further, if the thickness of the plate is increased then the reliability index also increases and the probability of failure is decreased.

5.2 *Comparison of reliability index for control beam and flexure retrofitted beam having different thickness of plate with different types of plates*

![Figure 10. Reliability index for control beam and flexure retrofitted beam having different thickness with different types of plates](image)

From figure 10, it can be observed that the reliability index is relatively high for 3mm porous plate thickness when compared to the solid plate having 1mm and 3mm plate thickness. The Lowest probability of failure corresponds to PPT3 beam, Pf = 3.5× 10⁻⁵ (β=3.9791) i.e. which is having more thickness of the plate and the porous plate is retrofitted to beam. The highest probability of failure corresponds to CB beam is Pf = 3.4× 10⁻¹ (β=0.4022).
6. Conclusion

Based on the results obtained from the analytical investigation of the present study on Reliability Analysis of Retrofitted Reinforced Concrete Beams, the following specific conclusions can be made:

1. The reliability index for control beam and flexure retrofitted beam is different. For the failure mode (flexure) the reliability index against CBA is lower than the other types of beams in this research.

2. It is observed that the reliability index varies from 0.4022 to 3.9791 which corresponds to a probability of failure of $3.4 \times 10^{-1}$ to $3.5 \times 10^{-5}$ for the size of beam (120 x120) mm having 2 No’s 12 mm dia of bars in the bottom for the control and flexure retrofitted beam for two different types of plates (solid and porous) with varying thickness of the plate. It can be seen that as the thickness of plate increases and the type of plate change from solid to porous there is an increment in reliability index which leads to lesser value in the probability of failure.

3. The Probability of failure ($3.5 \times 10^{-5}$) for FR-C4 beam is very low under the combination of porous plate and 3mm plate thickness.

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Appendix

$b$ = width of cross section; 
\( \alpha \) = design points; 
\( d \) = effective depth; 
\( \mu \) = mean of random variable; 
\( R \) = resistance; 
\( \sigma \) = standard deviation of random variable; 
\( S \) = loading; 
\( F_y \) = steel tensile strength; 
\( G \) = limit state function; 
\( A_{st} \) = area of steel in tension zone; 
\( CB \) = control beam; 
\( A_{sc} \) = area of steel in compression; 
\( FR \) = flexure retrofitted; 
\( COV \) = coefficient of variation; 
\( \beta \) = reliability index; 
\( W \) = ultimate load; 
\( Pf \) = probability of failure; 
PDF = probability density function; 
FR-A1= retrofitted beam with solid plate having 1mm plate thickness; 
FR-A2= retrofitted beam with porous plate having 1mm plate thickness; 
FR-A3= retrofitted beam with solid plate having 3mm plate thickness; 
FR-A4= retrofitted beam with porous plate having 3mm plate thickness.

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