PHASE DIFFERENCE AND COHERENCE AS DIAGNOSTICS OF ACCRETING COMPACT SOURCES

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ABSTRACT

We present calculations of the time lags and the coherence function of X-ray photons for a novel model of radiation emission from accretion-powered high-energy sources. Our model involves only Comptonization of soft photons injected near the compact object in an extended but nonuniform atmosphere around the compact object. Our results show that this model produces time lags between the hard and soft bands of the X-ray spectrum that increase with Fourier period, in agreement with recent observations; it also produces a coherence function equal to one over a wide range of frequencies if the system parameters do not change significantly during the observation, also in agreement with the limited existing observations. We explore various conditions that could affect coherence functions. We indicate that measurements of these statistical quantities could provide diagnostics of the radial variation of the density of this class of sources.

Subject headings: accretion, accretion disks — black hole physics — radiation mechanisms: thermal — stars: neutron — X-rays: general

1. INTRODUCTION

It is believed that bright Galactic X-ray sources are compact objects (black holes and neutron stars) powered by thermalization of the accretion kinetic energy on the surface of the neutron star or near the black hole horizon. The X-ray emission is, then, naturally accounted for as the result of Comptonization of soft photons by the hot electrons that are expected to be present in the deep gravitational potential of a compact object. In fact, the spectra of these sources have been modeled successfully with this process, which has been analyzed in great depth theoretically (see, e.g., Sunyaev & Titarchuk 1980; Titarchuk 1994; Hua & Titarchuk 1995); much information about the conditions of the accreting gas can be derived from fitting the high-energy spectra to observations.

It is well known, however, that the Comptonization spectra alone cannot provide any clues about the dynamics of accretion of the hot gas onto the compact object. One needs time variation information as well. However, since it is tacitly accepted that the X-ray emission originates at the smallest radii of the accreting flow, one expects that such information should reflect the dynamical timescales or the electron-scattering timescales associated with that region. In fact, the observed energy spectra indicate Thomson depths of greater than one and thus guarantee that these two timescales are of roughly the same order of magnitude. The recent Rossi X-Ray Timing Explorer (RXTE) observations (W. Focke, 1997 private communication; Meekins et al. 1984), which resolve shots of duration ~1 msec, appear to provide a validation for this simple expectation.

With these comments in mind, it appears strange that the X-ray fluctuation power spectral densities (PSD) of accreting compact sources contain most of their power at frequencies \( \omega \lesssim 1 \text{ Hz} \), far removed from the kHz frequencies expected on the basis of the arguments given above. This fact hints that one may have to modify the notion that the entire X-ray emission in this class of sources derives from a region a few Schwarzschild radii in size. Alternatively, the lack of high-frequency power in the PSD could be attributed to the viscous timescales of accretion disks, much longer than a millisecond near compact objects or to an overall modulation of the accretion rate with an (otherwise undetermined) power spectrum similar to the observed PSD.

While the above arguments could provide explanations for the observed PSD form, they have difficulty in addressing observations associated with more involved tests of Comptonization-induced variability and, in particular, the correlated variability in different energy bands. Specifically, Miyamoto et al. (1988, 1991) studied the time lag between soft and hard photons in the X-ray light curves of Cyg X-1 using Ginga data. It was shown in these references that the hard time lag increases roughly linearly with the Fourier period \( P \) from \( P \leq 0.1 \) to ~10 s and is of order 0.01\( P \) across this range of \( P \). These long lags are very hard to understand in a model where the X-ray emission arises from soft photon Comptonization in the vicinity of the compact object. In such a model, the lags should simply reflect the photon scattering time in the specific region (on the order of milliseconds). The authors found the lag dependence on the Fourier period disturbing enough to question whether the process of Comptonization is indeed responsible for the formation of the X-ray spectrum of Cyg X-1. In fact, the dependence of the time lag on \( P \) also rules out scattering in an extended, uniform, very low density X-ray corona, a process often invoked in models of accreting sources, since this process also produces time lags independent of the Fourier period.

More recently, Nowak & Vaughan (1996) and Vaughan & Nowak (1997, hereafter VN97) have brought attention to another statistic of importance in understanding the spatio-temporal structure of accreting sources, namely the coherence of the X-ray light curves. The coherence is the normalized linear correlation function of the light curves at two different energy bands obtained from an ensemble of measurements. These authors computed the coherence function from the Ginga data for Cyg X-1 and GX 339–4 and found it equal to one for both sources over the frequency range from below 0.1
Hz to \(\sim 10\) Hz. The coherence function for Cyg X-1 has also been computed with the more recent, higher quality data of RXTE (Cui et al. 1997b) and was found to be equal to one up to frequency \(\approx 20\) Hz. Vaughan and Nowak considered this fact quite surprising, since most of the models they produced had coherence functions substantially smaller than those obtained from observations.

Motivated by the discrepancy between the expected and observed variability behaviors of accreting compact sources, Kazanas, Hua, & Titarchuk (1997, hereafter KHT) proposed that the Comptonization process responsible for the formation of the spectra of accreting sources takes place in an nonuniform “atmosphere” that extends over several decades in radius. It was shown that this model can account for the form of the observed PSDs and the energy spectra and, at the same time, predicts a correlation between the slopes of the PSD and of the energy spectra.

In the present paper, we test this model further by studying its hard X-ray time lags and coherence function and examine under what conditions agreement with observation can be obtained. In § 2 we briefly review the model of KHT and compute the associated time lags as a function of Fourier frequency. In § 3 the coherence function is computed for the same model, while in § 4 the results are reviewed and conclusions are drawn concerning the possibility of uncovering the structure of accreting sources from spatio-temporal measurements.

2. TIME AND PHASE DIFFERENCES

The model of KHT considers Comptonization in a cloud at constant temperature but with a nonuniform density with a profile \(n(r) \propto r^{-1}\) that extends over several decades in the spherical radial coordinate \(r\) measured from the compact source. In order to explore the Comptonization in clouds with density configurations such as these, we developed a Monte Carlo method that can treat photon propagation and Compton scattering in inhomogeneous media. The method was described in detail by Hua (1997) and its first calculations were displayed in KHT.

The parameters of the calculations carried out in the present study were chosen so as to provide qualitative agreement of the resulting spectra with the spectra of Cyg X-1 recently obtained by BATSE aboard the Compton Gamma-Ray Observatory (CGRO). The CGRO/BATSE data given by Ling et al. (1997) show that Cyg X-1, in its soft (\(\gamma_0\)) state, has a spectrum consistent with Comptonization in an electron cloud of temperature \(\sim 110\) keV and Thomson optical depth \(\sim 0.45\). With these facts in mind, we employ a spherical model similar to that described in KHT but with temperature \(kT_e = 100\) keV, in order to account for the excess emission at \(\gtrsim 80\) keV.

In fact, this spectrum extrapolates at low energies to the spectrum of Cyg X-1 obtained by RXTE (Cui et al. 1997a) at a different epoch but with the source in a similar spectral state. The entire source has a radius \(r_s \approx 1.5\) lt-s and consists of a central core of radius \(r_1\) and an extended “atmosphere” with density profile \(n(r) = n_s/r^2\) for \(r < r_1\). For the region \(r < r_1\) we assume that the source has a uniform density \(n = n_s\) and that a soft photon source of blackbody spectrum at temperature \(0.2\) keV (Cui et al. 1997a) is located at its center (note that the density jump at \(r = r_1\) used in KHT is absent in the present profile).

As was shown in KHT, the density gradient of the extended atmosphere can significantly affect the resulting photon spectrum relative to that resulting from a uniform configuration of the same temperature and total Thomson depth. However, with a redefinition of the total Thomson depth, to account for the more efficient photon escape in this nonuniform configuration in comparison with the uniform one, the resulting spectra can be made similar. Thus, the total optical depth of the source in our investigation should be larger than the uniform source value (\(\tau_0 = 0.45\)) used in Ling et al. (1997) to fit the BATSE data. We found that our nonuniform configuration, assuming a total optical depth \(\tau_0 = 1\), produces as good a fit to the BATSE data as the one used by Ling et al. (1997).

Furthermore, we tested three different values for the radius of the central core of the cloud, \(r_1 = 2.4 \times 10^{-3}, 2.4 \times 10^{-4}\), and \(2.4 \times 10^{-5}\) lt-s. These conditions suffice to determine the density \(n_1\), which is given by

\[
\frac{1}{r_1} = \frac{\tau_0}{\sigma_T \left[ 1 + \ln \left( \frac{n_1}{n_s} \right) \right]}. \tag{1}
\]

With the above configuration, we have calculated, using the Monte Carlo code, the energies and arrival times of the photons emerging from the cloud and arriving at a distant observer. The photons are collected in the energy bands 2–6.5 and 13.1–60 keV in order to simulate precisely the recent RXTE observation (Cui et al. 1997b). In time, the photons are collected in 4096 bins, each with a width 6/4096 s. Based on the light curves so obtained, we calculated the phase and time lags of the higher (13.1–60 keV) energy band with respect to the lower one (2–6.5 keV) for clouds with the three different values of \(r_1\) given above. The resulting time lags (solid curves) as well as the corresponding phase lags (dotted curves) are shown in Figure 1 as a function of the Fourier frequency. It is apparent that these quantities have quite different forms from the forms associated with Comptonization in uniform electron clouds (Hua & Titarchuk 1996). For the latter, the phase lag functions have maxima at a characteristic frequency roughly equal to the reciprocal mean escape time of the photons from the cloud. The phase lags obtained from the present model, however, are almost constant, extending from a low frequency of \(\sim 0.1\) Hz, characteristic of the electron scattering time in the largest, least dense part of the atmosphere, to a cutoff frequency.
frequency at ~100 Hz determined by the time resolution of our calculations. Consequently, the corresponding time lags are power-laws of indices \(\approx 1\), as large as \(\approx 0.1\) s at the lowest frequencies, in rough agreement with the observations of Cui et al. (1997b) and Miyamoto et al. (1988, 1991); the slight increase in the lags at high frequencies arises from the effects of scattering in the uniform part of the configuration at \(r < r_i\).

It is apparent, that the relation of the time lags to the Fourier frequency obtained in these simulations depends on the specific form of the radial dependence of density of the extended “atmosphere”. To indicate the effect of such a dependence, we also display, in the same figure, the phase and extended “atmosphere”. To indicate the effect of such a dependence, we also display, in the same figure, the phase and extended “atmosphere”.

The difference between the curves corresponding to the different density profiles is evident. The lags associated with the steeper profile have a much weaker dependence on the Fourier frequency, because most of the photon scatterings, which give rise to the lags, take place near the radius at which the Thomson depth is highest, i.e., at the smallest radii. Once the photon leaves that region, it suffers very little additional scatterings and hence little additional lag occurs at lower Fourier frequencies. In this sense, the density profile used in KHT and in the present study, is special in obtaining agreement with observation, a fact whose importance has not escaped the authors. Similarly, for density profiles flatter than the one used herein, which cuts off beyond a certain radius to ensure finite Thomson depth, most of the lags are produced by scattering at the largest radii and would therefore be representative of the scattering time at that radius. It becomes apparent, therefore, that the observations of frequency-dependent hard X-ray lags not only argue in favor of the presence of the extended atmosphere used herein, but also point to study of the lags as a means for probing density profiles in detail. Furthermore, the range of the Fourier frequencies over which the phase lags are constant is indicative of the range in radii over which the specific power-law profile of the atmosphere extends. We plan to provide a more extensive investigation of these issues in future publications.

3. THE COHERENCE FUNCTIONS

In search of more probing tests of the variability of accreting sources, VN97 have examined the coherence function, which indicates to what extent the light curves at two different energy bands track each other linearly during the period over which the observations are made. These authors indicated that while most models of accreting compact sources have not been tested against this diagnostic, the models generally tend to produce incoherent sources (Nowak 1994). Their conclusion was that it is very easy to destroy coherence and difficult to produce it. Nevertheless, when analyzing Ginga data associated with Cyg X-1 and GX 339—4 they found (VN97) that the coherence function of these sources was equal to one for Fourier frequencies below 10 Hz!

Motivated by these considerations, we computed the coherence functions for the model outlined in the previous section. In order to examine what conditions would lead to loss of coherence in our model, we assumed that the configuration of our system changes during an observation. We used light curves in the 2–6.5 and 13.1–60 keV energy bands resulting from two configurations represented by different parameters of our model to simulate this evolution. In order to estimate the noise due to the statistical nature of the Monte Carlo calculations, we produced a group of eight light curves for each energy band of each configuration, obtained by following the history of \(10^5\) photons with different random number seeds. The difference between each light curve and the average of the eight curves in the same group is taken as the noise. For the two energy bands given above, labeled 1 and 2 respectively, we used the 16 pairs of light curves, eight from each of two distinct configurations, and their respective noises to compute the coherence function defined in equation (2) of VN97. The power spectra \(S_f^2\) in this equation should be understood as being noise-corrected, that is, \(S_f^2 = P_i - |N_i|^2 (i = 1, 2)\), where \(P_i\) and \(|N_i|^2\) are power spectra obtained from the calculated light curve and from the corresponding noise, respectively. (We are indebted to B. Vaughan for his insistence on this point.) The averages in the equation are taken over the 16 pairs of “measurements”.

For the purpose of verifying the above procedure, we first computed the coherence function of our model by averaging over 16 pairs of light curves from two identical configurations. We found that the coherence function of our model was 1 across the entire frequency range, as expected. We then computed coherence functions for a variety of configuration evolutions; the results are presented in Figure 2. It is seen that changes in the parameters of our model do result in loss of coherence over the frequency range (1/6)–(2048/6) Hz. The four thick curves represent the evolution of the configuration starting from the configuration with \(r_1 = 2.4 \times 10^{-3}\) lt-s and evolving to one of the following configurations: (1) one with \(n_i\) and \(\tau_0\) increased by a factor of 5 with the density profile unchanged (solid curve). In this case, the physical sizes \(r_1\) and \(r_2\) remain the same and the coherence reduces to less than 0.6 over the entire frequency range, with large statistical fluctuations at high frequencies. (2) One with electron temperature decreased from the initial value of 100 keV to 50 keV (dotted curve). In this case, the coherence is \(\approx 0.85\) below \(\approx 100\) Hz and drops at higher frequencies. (3) Configurations with different energy of initial soft photons. While in the initial configuration the source photons have a blackbody distribution at temperature \(kT_0 = 0.2\) keV, the dashed curve is obtained with a source spectrum between the blackbody spectrum and a spectrum with source photons of a single energy \(E_0 = 13.1\) keV. It is seen that in this case the coherence is reduced virtually to zero. On the other hand, if the final configuration has source photons at the blackbody temperature \(kT_0 = 4\) keV, the coherence becomes \(\approx 0.8\) over the entire

![Figure 2](image-url)
frequency range, as the dash-dotted curve indicates. (The coherence is slightly greater than 1 at high frequencies, probably because of noise overcorrection.)

Special attention should be paid to the thin curve in Figure 2, which represents an evolution from the configuration with $r_1 = 2.4 \times 10^{-2}$ It-s to that with $r_1 = 2.4 \times 10^{-4}$ It-s (or vice versa). It is seen that the coherence is virtually unity over the entire range of frequencies under consideration, although the size of the uniform part of the configuration changes by 2 orders of magnitude. The cause of this outcome becomes apparent by taking a closer look at the PSD and phase lag of these two configurations. Since the light curves are obtained from two independent time series, say $q$ and $r$ for the two energy bands 1 and 2. One can then use equation (10) in VN97 to compute their coherence. Using the same notation, we rewrite the equation in terms of the ratios of PSD in the two energy bands $\alpha_1 = |Q_1|/|R_1|$ and $\alpha_2 = |Q_2|/|R_2|$, so that

$$
\gamma_1^2 = \frac{1 + \alpha_1^2 + 2\alpha_1 \alpha_2 \cos (\delta \theta_1 - \delta \theta_2)}{(1 + \alpha_1^2)(1 + \alpha_2^2)}.
$$

(2)

It is found that at low frequencies, the PSDs in the two energy bands are in the proportion $\alpha_1 \approx \alpha_2 \approx 1$. From Figure 1, we see that the difference in the phase lag between these energy bands is small, with $\delta \theta_1 - \delta \theta_2 \leq 10^3$; equation (2) then suggests that the coherence should be $\gamma_1^2 \approx 1$. At higher frequencies, however, we found $\alpha_1 \neq \alpha_2$ and $\delta \theta_1 - \delta \theta_2$ as great as $50^\circ$ (see Fig. 1). We might then expect the coherence to be less than one at these frequencies. But in this frequency range the values of $\alpha_1$ and $\alpha_2$ are found to be much smaller than 1, yielding $\gamma_1^2 \approx 1$. Therefore the coherence at high frequencies is only superficial, since neither of the conditions outlined in VN97 are satisfied, and occurs because the values of $\alpha_1$ and $\alpha_2$ are much smaller than one. In a similar way, one can examine the true causes for the coherences or lack thereof displayed in Figure 2. For example, when we examine the case corresponding to the dashed curve in Figure 2, it is found that the near zero coherence over the entire frequency range occurs because $\alpha_2 \ll \alpha_1$, so that $\gamma_1^2 \approx 1/\alpha_1^2 \approx 0.01$.

In view of these results, one should be cautious in drawing strong conclusions from measurements of coherence close to one, like the measurements of Cyg X-1 and GX339−4 given in VN97. Such data may indicate the constant state of the responsible mechanism over the observation time. On the other hand, it may also indicate changes in the system to which the coherence statistic is insensitive.

4. DISCUSSION AND CONCLUSIONS

We have presented above two statistics associated with a model for the time variability of the X-ray radiation emitted by accreting compact objects. In particular, using a Monte Carlo code, we have computed the time lags between soft and hard photons, as well as the coherence of the X-ray light curves between the same energy bands resulting from Compton scattering in a constant temperature but nonuniform density “atmosphere” that extends over several decades in radius. In our simulations we have assumed that the sole source of phase differences is Compton scattering of soft photons by the hot electrons of the atmosphere and that the sole source of loss of coherence is change of the system during observation. In addition, we have assumed that the source of photons resides near the center of the configuration.

Our results indicate that:

1. The hard photons lag behind the soft ones by amounts which increase with the Fourier period of the variability. This behavior is distinctly different from that of Compton scattering by an electron cloud of uniform density, and is in agreement with both the GINGA and the RXTE data.

2. The hard photon phase and time lags as a function of the Fourier frequency depend on the density profile of the extended scattering atmosphere. The observed lags are consistent with the density profile $n(r) \propto r^{-2}$ used in this study, with the range of frequencies over which the phase lags remain constant reflecting the range of radii over which this density profile holds. Accurate measurements of the lag dependence on frequency could be used in the deconvolution of the density profile of the atmosphere from observations.

3. Our model generally produces coherence close to one. This fact, in view of the results of VN97 on the coherence of Cyg X-1, would indicate that the parameters of Cyg X-1 remain constant over very long timescales (hours). The coherence function between two energy bands, within our model, can be reduced to less than one by changing the macroscopic parameters of the Comptonization cloud and/or the energy of the source photons. However, the converse of this statement is not true; coherence equal to one is, for practical purposes, only a sufficient condition for the parameters of the configuration remaining constant.

We believe that the simplicity and the ability of our model to correctly reproduce these variability statistics argues for the correctness of its basic premises, namely the presence of a nonuniform atmosphere and the effects of Compton scattering on the variability of accreting compact sources. We also believe that further scrutiny of this model by detailed comparisons of all the information available, including the PSD and energy spectra, could determine the temperature and density structure of these accretion flows.

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REFERENCES

Cui, W., Heindl, W. A., Rothschild, R. E., Zhang, S. N., Jahoda, K., & Focke, W., 1997a, ApJ, 474, L57 (http://xxx.lanl.gov/abs/astro-ph/9610072)

Cui, W., Zhang, S. N., Focke, W., & Swank, J. H., 1997b, ApJ, in press

Hua, X.-M., 1997, Comput. in Phys., submitted

Hua, X.-M., & Titarchuk, L., 1995, ApJ, 449, 188

———, 1996, ApJ, 469, 280

Kazanas, D., Hua, X.-M., & Titarchuk, L., 1997, ApJ, 480, 735 (KHT)

Ling, J. C., et al. 1997, ApJ, in press

Meekins, J. F., Wood, K. S., Heidler, R. L., Byram, E. T., Yentis, D. T., Chubb, T. A., & Friedman, H. 1984, ApJ, 278, 288

Miyamoto, S., Kimura, K., Kitamoto, S., Dotani, T., & Ebisawa, K. 1988, Nature, 336, 450

Miyamoto, S., Kitamoto, S., Mitsuda, K., & Dotani, T. 1991, ApJ, 383, 784

Nowak, M. A., 1994, ApJ, 422, 688

Nowak, M. A., & Vaughan, B. A. 1996, MNRAS, 280, 227

Sunyaev, R. A., & Titarchuk, L. G. 1980, A&A, 86, 121

Titarchuk, L. G. 1994, ApJ, 434, 570

Vaughan, B. A., & Nowak, M. A. 1997, ApJ, 474, L43 (VN97)