Thermal fluctuations and disorder effects in vortex lattices

Dingping Li∗

National Center for Theoretical Sciences
P.O.Box 2-131, Hsinchu, Taiwan, R. O. C.

Baruch Rosenstein†

National Center for Theoretical Sciences and
Electrophysics Department, National Chiao Tung University
Hsinchu 30050, Taiwan, R. O. C.

(November 21, 2018)

Abstract

We calculate using loop expansion the effect of fluctuations on the structure function and magnetization of the vortex lattice and compare it with existing MC results. In addition to renormalization of the height of the Bragg peaks of the structure function, there appears a characteristic saddle shape ”halos” around the peaks. The effect of disorder on magnetization is also calculated. All the infrared divergencies related to soft shear cancel.

PACS numbers: 74.60.-w, 74.40.+k, 74.25.Ha, 74.25.Dw

∗e-mail: lidp@phys.nthu.edu.tw
†e-mail: baruch@phys.nthu.edu.tw
I. INTRODUCTION

Decoration [1], neutron scattering [2] and STM [3] clearly demonstrated the Abrikosov flux line lattice in low and high $T_c$ type II superconductors. There are however important differences between the two classes of materials. Ginzburg parameter $G_i$ characterizing importance of thermal fluctuations is much larger in high $T_c$ superconductors than in the low temperature ones. Moreover in the presence of magnetic field the importance of fluctuations in high $T_c$ superconductors is further enhanced. The lattice melts and becomes vortex liquid over large portions of the phase diagram [4–6]. In ”strongly fluctuating” superconductors, even far below the melting line, corrections to various physical quantities like magnetization or specific heat are not negligible. The vortex lattice becomes distorted. It is quite straightforward to systematically account for the fluctuations effect on magnetization, specific heat or conductivity perturbatively above the mean field transition line using Ginzburg - Landau (GL) description [7]. However in the interesting region below this line it turned out to be extremely difficult to develop a quantitative theory. A direct approach to the low temperature fluctuations physics is to start from the mean field solution and then take fluctuations around this inhomogeneous solution into account perturbatively. Experimentally it is reasonable since, for example, specific heat at low temperatures is a smooth function and the fluctuations contribution is quite small. For some time this was in disagreement with theoretical expectations. Eilenberger calculated spectrum of harmonic excitations of the triangular vortex lattice [8] and noted that the gapless mode is softer than the usual Goldstone mode expected as a result of spontaneous breaking of translational invariance. The inverse propagator for the ”phase” excitations behaves as $k_z^2 + const(k_x^2 + k_y^2)^2$. It was shown [9,10] that this behavior is directly related to the nondispersive nature of the shear modulus $c_{66}$ and is in agreement with numerous experiments. The influence of this additional ”softness” goes beyond enhancement of the contribution of fluctuations at leading order. It apparently leads to disastrous infrared divergencies at
higher orders rendering the perturbation theory around the vortex state doubtful. One therefore tends to think that nonperturbative effects are so important that such a perturbation theory should be abandoned [11]. However it was shown in [12] that a closer look at the diagrams reveals that in fact one encounters actually only logarithmic divergencies. This makes the divergencies similar to so called "spurious" divergencies in the theory of critical phenomena with broken continuous symmetry and they exactly cancel at each order provided we are calculating a symmetric quantity. One can effectively use properly modified perturbation theory to quantitatively study various properties of the vortex liquid phase. Magnetization calculated using this perturbative approach agrees very well with the direct Monte Carlo simulation of [13]. The method was then extended beyond the lowest Landau level (LLL) [14].

In this paper we calculate the effect of fluctuations on the magnetic field distribution, structure function of the vortex lattice and compare with existing MC results. Fluctuations cause the spread of the peaks in diffraction pattern in a very specific way, while height of the peaks is slightly corrected. Effects of fluctuation and disorder on magnetization and specific heat are computed. The paper is organized as follows. In section II the model and the fluctuation spectrum approximation are briefly reviewed. In section III the calculation of the structure function is presented. Section IV contains analysis of the result, comparison with MC simulation and some generalizations. In section V the distribution of magnetic field is calculated, while effects of weak disorder on magnetization and specific heat are treated in section VI. Summary is given in section VII.

II. MODEL, MEAN FIELD SOLUTION AND THE PERTURBATION THEORY

Our starting point is the GL free energy:

\[
F = \int d^3x \frac{\hbar^2}{2m_{ab}} \left| \left( \nabla - \frac{ie^*}{\hbar c} A \right) \psi \right|^2 + \frac{\hbar^2}{2m_c} |\partial_z \psi|^2 + a |\psi|^2 + \frac{b'}{2} |\psi|^4
\]  

(1)

Here \( A = (-By, 0) \) describes a nonfluctuating constant magnetic field. For strongly type II superconductors (\( \kappa \sim 100 \)) far from \( H_{c1} \)(this is the range of interest in this paper)
magnetic field is homogeneous to a high degree due to superposition from many vortices. For simplicity we assume $a = \alpha (1 - t)$, $t \equiv T / T_c$ although this dependence can be easily modified to better describe the experimental coherence length.

Throughout most of the paper will use the following units. Unit of length is

$$\xi = \sqrt{\frac{\hbar^2}{2m_\alpha \alpha T_c}}$$

and unit of magnetic field is $H_{c2}$, so that dimensionless magnetic field is $b \equiv B / H_{c2}$. The dimensionless free energy in these units is (the order parameter field is rescaled as $\psi^2 \rightarrow 2\alpha T_c \psi^2$):

$$F = \frac{1}{\omega} \int d^3x \left[ \frac{1}{2} |D\psi|^2 + \frac{1}{2} |\partial_z \psi|^2 - \frac{1-t}{2} |\psi|^2 + \frac{1}{2} |\psi|^4 \right],$$

(2)

The dimensionless coefficient is

$$\omega = \sqrt{2Gi \pi^2 t}. \quad (3)$$

where the Ginzburg number is defined by $Gi \equiv \frac{1}{2} \left( \frac{32\pi^2 \kappa^2 \xi T_c \gamma^{1/2}}{c^3 \hbar^2} \right)^2$ and $\gamma \equiv m_c / m_{ab}$ is an anisotropy parameter. This coefficient determines the strength of fluctuations, but is irrelevant as far as mean field solutions are concerned.

The second expansion parameter is (see [9,14] for details):

$$a_h \equiv \frac{1 - t - b}{2}. \quad (4)$$

If $a_h$ is sufficiently small GL equations can be solved perturbatively:

$$\psi = \Phi = (a_h)^{1/2} \left[ \Phi_0 + a_h \Phi_1 + ... \right]$$

(5)

It is convenient to represent $\Phi_0, \Phi_1, ...$ in the basis of eigenfunctions of operator $\mathcal{H} \equiv \frac{1}{2}(-D^2 - b)$, $\mathcal{H} \varphi^n = nb \varphi^n$, normalized to unit "Cooper pairs density"

$$< |\varphi^n|^2 > \equiv \int_{\text{cell}} d^2x |\varphi^n|^2 \frac{b}{2\pi} = 1,$$

where "cell" is a primitive cell of the vortex lattice. Assuming hexagonal lattice symmetry one explicitly has:

$$\varphi^n = \sqrt{\frac{2\pi}{\sqrt{\pi^2 2^n n!}}} \sum_{l=-\infty}^{\infty} \mathcal{H}_n(y\sqrt{b} - \frac{2\pi}{a} l)$$

$$\times \exp \left\{ i \left[ \frac{\pi(l-1)}{2} + \frac{2\pi \sqrt{b}}{a} lx \right] - \frac{1}{2} \left( y\sqrt{b} - \frac{2\pi}{a} l \right)^2 \right\}$$

(6)
where \( a = \sqrt{\frac{4\pi}{\sqrt{3}b}} \) is the lattice spacing. One finds
\[
\Phi_0 = \frac{1}{\sqrt{\beta A}} \varphi.
\] (7)

To order \( a_i \), we expand
\[
\Phi_i = g_i \varphi + \sum_{n=1}^{\infty} g_i^n \varphi^n.
\] (8)

These coefficients can be found in [14].

To find an excitation spectrum one expands free energy functional around the solution. The fluctuating order parameter field \( \psi \) is divided into a nonfluctuating (mean field) part and a small fluctuation
\[
\psi(x) = \Phi(x) + \chi(x).
\] (9)

Field \( \chi \) can be expanded in a basis of quasimomentum eigenfunctions:
\[
\varphi_k^n = \sqrt{\frac{2\pi}{\sqrt{\pi^2 n! a}}} \sum_{l=-\infty}^{\infty} H_n(y\sqrt{b} + \frac{k_x}{\sqrt{b}} - \frac{2\pi}{a}) l)
\times \exp \left\{ i \left[ \frac{\pi l(l-1)}{2} + \frac{2\pi(y\sqrt{b} - \frac{k_x}{\sqrt{b}})}{a} - \frac{1}{2} \frac{2\pi}{a} \right] \right\} - \frac{1}{2} (y\sqrt{b} + \frac{k_x}{\sqrt{b}} - \frac{2\pi}{a})^2
\] (10)

Instead of complex field \( \chi_k^n \) we will use two ”real” fields \( O_k^n \) and \( A_k^n \) satisfying
\[
O_k^n = O_{-k}^n, A_k^n = A_{-k}^n:
\]
\[
\chi(x) = \frac{1}{\sqrt{2}} \int_k d_k e^{-ik_3x_3} \sum_{n=0}^{\infty} \frac{\varphi_k^n(x)}{\sqrt{2\pi}} (O_k^n + iA_k^n)
\] (11)
\[
\chi^*(x) = \frac{1}{\sqrt{2}} \int_k d_k e^{ik_3x_3} \sum_{n=0}^{\infty} \frac{\varphi_k^n(x)}{\sqrt{2\pi}} (O_{-k}^n - iA_{-k}^n)
\]

where \( d_k = \exp[-i\theta_k/2] \) where \( \gamma_k = |\gamma_k| \exp[i\theta_k] \). Within the LLL, the eigenstates are \( A_k^n, O_k^n \), while the eigenvalues (in two dimensions; in three dimensions simply plus \( k_3^2 \)) are
\[
\epsilon_A = a_h \epsilon_A^1 = a_h \left( -1 + \frac{2}{\beta} \beta_k - \frac{1}{\beta} |\gamma_k| \right)
\] (12)
\[
\epsilon_O = a_h \epsilon_O^1 = a_h \left( -1 + \frac{2}{\beta} \beta_k + \frac{1}{\beta} |\gamma_k| \right).
\]
where $\epsilon_A, \epsilon_O$ are dependent on two dimensional vector $k$. Higher order corrections and higher Landau levels eigenstates and eigenvalues can be found in [14]. With spectrum of excitations and expansion of solutions of GL equations in $a_h$ one can start the calculation of correlators to any order in $\omega$.

### III. STRUCTURE FUNCTION OF THE VORTEX LATTICE

In this section the structure function is calculated to order $\omega$ (harmonic approximation) within the LLL, namely neglecting higher $a_h$ corrections. We discuss these corrections in the next section. First we calculate the density correlator defined by

$$\tilde{S}(z, z_3) = \langle \rho(x, x_3) \rho(x + z, x_3 + z_3) \rangle_x = \langle \rho(x) \rho(y) \rangle_x$$

(13)

where $<>_x$ indicates average over $x$ (which means here over the unit cell) and $\rho \equiv |\psi|^2$.

The correlator is calculated using Wick expansion:

$$\tilde{S} = \tilde{S}_{mf} + \omega \tilde{S}_{fluct}.$$  

(14)

The first term is the mean field part, while the second term is the fluctuation part. The mean field part is simply

$$\tilde{S}_{mf} = \langle |\Phi(x)|^2 |\Phi(y)|^2 \rangle_x.$$  

(15)

The structure function is the Fourier transform $S(q, 0) = \int dz e^{i q \cdot z} \tilde{S}(z, z_3 = 0)$. Within the LLL, $\Phi(x) = (\frac{a_h}{\beta A})^{1/2} \varphi(x)$ and the mean field part of the structure function becomes:

$$S_{mf}(q, 0) \equiv \int dz e^{i q \cdot z} < |\Phi(x)|^2 |\Phi(y)|^2 >_x$$

$$= (\frac{a_h}{\beta A})^2 \frac{b}{2\pi} \int_{cell} |\varphi(x)|^2 e^{-i q \cdot x} \int_z |\varphi(z)|^2 e^{i q \cdot z}$$

(16)

$$= (\frac{a_h}{\beta A})^2 4\pi^2 \delta_n(q) \exp \left[ -\frac{q^2}{2b} \right],$$

where we made use of formulas .and function $\delta_n(q)$ defined in Appendix. This is just sum of delta functions of various heights at reciprocal lattice points.
The fluctuation part contains four terms (diagrams) \( \tilde{S}_1, \ldots \tilde{S}_4 \). The first term is

\[
\tilde{S}_1(z, z_3) = \frac{1}{4\pi \cdot 2\pi \omega} \sum_{k,l} \left< \Phi(x) \Phi(y) \right| \int d_{k,l}^* d_{l,k}^* \varphi_k^m(x) \varphi_l^m(y) \left| \Phi(x) \Phi(y) \right> \\
x \times \left< O_k^n O_l^n - A_k^n A_l^n \right> e^{i k_3 (y-x)_3} + c.c.
\]

(17)

\(< O_k^n O_l^n > \) and \(< A_k^n A_l^n > \) are propagators:

\[
< O_k^n O_l^n > = \frac{\omega}{\epsilon_O^m(k) + k^2/2}
\]

(18)

To calculate structure functions we will need only the \( z_3 = 0 \) correlator:

\[
\tilde{S}_1(z, 0) = \frac{1}{4} \sum_{k,l} \frac{a_h}{(2\pi)^2 \beta_A} \left< \varphi(x) \varphi(y) \right| \int d_{k,l}^* \varphi_k^m(x) \varphi_l^m(y) \left| \varphi(x) \varphi(y) \right>
\\
x \times \left[ \sqrt{\frac{2}{\epsilon_O^m(k)}} - \sqrt{\frac{2}{\epsilon_A^m(k)}} \right] + c.c.
\]

(19)

Within the LLL approximation it simplified to:

\[
\tilde{S}_1(z, 0) = \frac{1}{4} \sum_{k,l} \frac{a_h}{(2\pi)^2 \beta_A} \left< \varphi(x) \varphi(y) \right| \int d_{k,l}^* \varphi_k^m(x) \varphi_l^m(y) \left| \varphi(x) \varphi(y) \right>
\\
x \times \left[ \sqrt{\frac{2}{\epsilon_O^m(k)}} - \sqrt{\frac{2}{\epsilon_A^m(k)}} \right] + c.c.
\]

(20)

The first fluctuation correction term to structure function can be evaluated as follows:

\[
S_1(q, 0) = \frac{1}{4} \sum_{k,l} \frac{a_h}{(2\pi)^2 \beta_A} \int \left( \frac{b}{\epsilon_O^m(k)} - \frac{2}{\epsilon_A^m(k)} \right) + c.c.
\]

(21)

where formulas of Appendix were used. \( Q \) is the integer part of \( q \). \( k \) is the fractional part of \( q \): \( q = k + n_1 \tilde{d}_1 + n_2 \tilde{d}_2 = k + Q \) (see Appendix for the definitions of \( \tilde{d}_1, \tilde{d}_2 \)). The second fluctuation correction term is

\[
\tilde{S}_2(z, z_3) = \frac{1}{4\pi \cdot 2\pi \omega} \sum_{k,l} \left< \Phi(x) \Phi^*(y) \right| \int d_{k,l}^* d_{l,k}^* \varphi_k^m(x) \varphi_l^m(y) \left| \Phi(x) \Phi^*(y) \right>
\\
x \times \left< O_k^n O_l^n + A_k^n A_l^n \right> e^{i k_3 (y-x)_3} + c.c.
\]

(22)
\( \tilde{S}_2(z, z_3 = 0) \) is equal to (in the LLL approximation):

\[
\tilde{S}_2(z, 0) = \frac{1}{4} \frac{a_h}{(2\pi)^2} \frac{1}{\beta A} \int_\varphi(x) \varphi^*(y) \int_k \varphi_k^*(x) \varphi_k(y) \left\langle \sqrt{\frac{2}{\epsilon_O(k)}} + \sqrt{\frac{2}{\epsilon_A(k)}} \right\rangle + c.c. \tag{23}
\]

and

\[
S_2(q, 0) = \frac{1}{4} \frac{a_h}{(2\pi)^2} \frac{1}{\beta A} \int_k \frac{b}{2\pi} \int_{cell} \varphi(x) \varphi_k^*(x) e^{-iq \cdot x} \int_z \varphi^*(z) \varphi_k(z) e^{iq \cdot z} \times (d_k^*)^2 \left[ \sqrt{\frac{2}{\epsilon_O(k)}} - \sqrt{\frac{2}{\epsilon_A(k)}} \right] + c.c. \tag{24}
\]

\[
= \frac{a_h}{2\beta A} \exp \left[ \frac{-q^2}{2b} \right] \left[ \sqrt{\frac{2}{\epsilon_O(k)}} + \sqrt{\frac{2}{\epsilon_A(k)}} \right] + (q \rightarrow -q) \tag{25}
\]

The third term is

\[
\tilde{S}_3(z, z_3) = \frac{1}{4\pi \cdot (2\pi)^2} \omega \left\langle |\Phi(x)|^2 \int_k d_k d_l \varphi_k^*(y) \varphi_l^*(y) \right\rangle \times (\langle O_k^{mn} O_l^{mn} + A_k^{mn} A_l^{mn} \rangle + \text{x} \leftarrow \text{y}) \tag{26}
\]

and within the LLL at \( z_3 = 0 \) is equal to:

\[
\tilde{S}_3(z, 0) = \frac{1}{4(2\pi)^2} \left\langle |\Phi(x)|^2 \int_k \varphi_k^*(y) \varphi_k^*(y) \right\rangle \times \left[ \sqrt{\frac{2}{\epsilon_O(k)}} + \sqrt{\frac{2}{\epsilon_A(k)}} \right] + (\text{x} \leftarrow \text{y}). \tag{27}
\]

Consequently the correction to the structure function is:

\[
S_3(q, 0) = \frac{a_h}{4\beta A} \int_k \frac{b}{2\pi} \int_{cell} |\varphi(x)|^2 e^{-iq \cdot x} \int_z |\varphi_k(z)|^2 e^{iq \cdot z} \times \left[ \sqrt{\frac{2}{\epsilon_O(k)}} + \sqrt{\frac{2}{\epsilon_A(k)}} \right] + (q \rightarrow -q)
\]

\[
= \frac{a_h}{2\beta A} \delta_n(q) \exp \left[ \frac{-q^2}{2b} \right] \int_k \cos \left( \frac{k \times Q}{b} \right) \left[ \sqrt{\frac{2}{\epsilon_O(k)}} + \sqrt{\frac{2}{\epsilon_A(k)}} \right] \tag{28}
\]

The final term is from the vacuum renormalization contribution. The shift \( \nu \) in \( \psi(x) = \nu \phi(x) + \chi(x) \) is renormalized, that is, to one loop order, \( \nu^2 = \nu_o^2 + \omega \nu_1^2 \), where \( \nu_o^2 = \frac{a_h}{\beta A} \). One can find \( \nu_1^2 \) by minimizing the effective one loop free energy:

\[
\frac{1}{\omega} \left[ -a_h \nu^2 + \frac{1}{2} \beta \nu^4 \right] + \frac{1}{2} \left\{ Tr \ln \left[ 2 \epsilon_O(k) + k_0^2 \right] + Tr \ln \left[ 2 \epsilon_A(k) + k_0^2 \right] \right\}, \tag{28}
\]
where we write
\[
\epsilon_A(k) = -a_h + 2v^2\beta_k - v^2|\gamma_k| \tag{29}
\]
\[
\epsilon_O(k) = -a_h + 2v^2\beta_k + v^2|\gamma_k|.
\]

Straightforward calculation gives:
\[
v_1^2 = -\frac{1}{16\pi^2} \int \left[ \left( \frac{2\beta_k + |\gamma_k|}{\beta} \right) \sqrt{\frac{2}{\epsilon_O(k)}} + \left( \frac{2\beta_k - |\gamma_k|}{\beta} \right) \sqrt{\frac{2}{\epsilon_A(k)}} \right]
\]
\[
(30)
\]

The last contribution to the one loop correction to the correlator is therefore:
\[
S_4(z, z_3) = 2\frac{a_h}{\beta_A} \left( \int_{\text{cell}} |\varphi(x)|^2 |\varphi(y)|^2 \right)_{x_1} (v_1)^2
\]
\[
S_4(q, 0) = 2\frac{a_h}{2\beta_A} \int_{\text{cell}} |\varphi(x)|^2 e^{-iq \cdot x} \int_{z} |\varphi(z)|^2 e^{iq \cdot z} v_1^2
\]
\[
= -\frac{a_h}{2\beta_A} \delta_n(q) \exp \left[ -\frac{q^2}{2b} \right] \int_{k} \left[ \left( \frac{2\beta_k + |\gamma_k|}{\beta} \right) \sqrt{\frac{2}{\epsilon_O(k)}} + \left( \frac{2\beta_k - |\gamma_k|}{\beta} \right) \sqrt{\frac{2}{\epsilon_A(k)}} \right]
\]
\[
(31)
\]

The sum of all the four terms can be cast in the following form:
\[
S(q, 0) = \left( \frac{a_h}{\beta_A} \right)^2 4\pi^2 \delta_n(q) \exp \left[ -\frac{q^2}{2b} \right] + \frac{\omega a_{1/2}}{2} \exp \left[ -\frac{q^2}{2b} \right]
\]
\[
\times \left[ f_1(q) + \delta_n(q)f_2(Q) + \delta_n(q)f_3 \right]
\]
\[
f_1(q) = \left[ 1 + \cos \left( \frac{k_x k_y + Q}{b} + \theta_k \right) \right] \sqrt{\frac{2}{\epsilon_O(k)}}
\]
\[
+ \left[ 1 - \cos \left( \frac{k_x k_y + Q}{b} + \theta_k \right) \right] \sqrt{\frac{2}{\epsilon_A(k)}} \tag{32}
\]
\[
f_2(Q) = \int_{k} \left[ -1 + \cos \left( \frac{k \times Q}{b} \right) \right] \left[ \sqrt{\frac{2}{\epsilon_O(k)}} + \sqrt{\frac{2}{\epsilon_A(k)}} \right]
\]
\[
f_3 = -\int_{k} \left[ \sqrt{2\epsilon_O(k)} + \sqrt{2\epsilon_A(k)} \right] = -28.5275 b.
\]

The reason we write the sum in such a form will be explained in the next section.

**IV. ANALYSIS OF THE RESULT, COMPARISON WITH MC SIMULATIONS**

Although all of the four terms \( S_1, S_2, S_3 \) and \( S_4 \) are divergent as any of the peaks is approached, \( k \to 0 \), the sums \( S_1, S_2 \) and \( S_3, S_4 \) are not. We start with the first two:
\[ S_1(q, 0) + S_2(q, 0) = \frac{\omega a_h}{2 \beta_A} \exp \left[ -\frac{q^2}{2b} \right] f_1(q), \] (33)

where \( f_1(q) \) defined in eq.(32) contains a function \( \frac{1}{b}(k_x k_y + k \times Q) + \theta_k \). When \( k \to 0 \) it can be shown that \( \frac{k_x k_y}{b} + \theta_k = O(k^2) \), thus \( \frac{1}{b}(k_x k_y + k \times Q) + \theta_k \to k \times Q \), and \( 1 - \cos(\frac{k_x k_y + k \times Q}{b} + \theta_k) \to (k \times Q)^2 \). Hence it will cancel the \( 1/k^2 \) singularity coming from \( \sqrt{\epsilon_1 A(k)} \). Thus \( f_1(q) \) approaches \( \text{const.} + \text{const.} \cdot (k \times Q)^2 \) when \( Q \neq 0 \), and approaches \( \text{const.} + \text{const.} \cdot k^6 \) when \( Q = 0 \).

Similarly the sum of \( S_4(q, 0) \) and \( S_3(q, 0) \) is not divergent, although separately they are. Their sum is.

\[ S_3(q, 0) + S_4(q, 0) = \frac{\omega a_h}{2 \beta_A} \delta_n(q) \exp \left[ -\frac{q^2}{2b} \right] [f_2(Q) + f_3] \] (34)

Now we compare our results with numerical simulation of the LLL system in [13]. The general shape of the structure function in the vicinity of a peak (see Fig. 1b) and the data near the origin according to a MC simulation of the same system within the same LLL approximation in ref. [13] (Fig. 1a) are qualitatively same pattern. It is easier to compare using rescaled quasimomenta: \( q \to q \sqrt{b}, k \to k \sqrt{b} \). We get

\[ S(q, 0) = \left( \frac{a_h}{\beta_A} \right)^2 \frac{4\pi^2}{b} \delta_n(q) \exp \left[ -\frac{q^2}{2b} \right] \] (35)

\[ + \omega a_h \exp \left[ -\frac{q^2}{2} \right] [f_1(q) + \delta_n(q)f_2(Q) + \delta_n(q)f_3] \]

where \( f_1(q), f_2(Q) \) and \( f_3 \) are defined in eq.(32), but with \( b = 1 \) (for example, \( f_3 = -28.5275 \)) and the region of integration in the formula rescaled to the cell with \( \tilde{d}_1, \tilde{d}_2 \) being the reciprocal lattice basis vectors

\[ \tilde{d}_1 = \frac{2\pi}{a} \left( 1, -\frac{1}{\sqrt{3}} \right); \quad \tilde{d}_2 = \left( 0, \frac{4\pi}{a \sqrt{3}} \right) \]

Furthermore we define \( s(q) \) which is used also in [13]:

\[ S(q, 0) = \left( \frac{a_h}{\beta_A} \right)^2 \frac{4\pi^2}{b} \exp \left[ -\frac{q^2}{2b} \right] s(q) \] (36)

\[ s(q) \equiv \delta_n(q) + \frac{\beta_A b \omega}{8\pi^2} (a_h)^{-3/2} \left[ f_1(q) + \delta_n(q)f_2(Q) + \delta_n(q)f_3 \right] \]

For reciprocal lattice vectors close to origin the value of \( f_2(Q) \) are:
Table 1.

| $n_1, n_2$ | 0,1 | 1,0 | 1,1 | 1,2 | 0,2 | 2,2 | 1,3 |
|-----------|-----|-----|-----|-----|-----|-----|-----|
| $f_2(Q)$  | -46.19 | -46.19 | -46.19 | -63.165 | -64.35 | -64.35 | -73.825 |

In ref. [13], the material parameters describe YBCO: $T_c = 93$ K, $dH_c2(T)/dT = -1.8 \times 10^4$ Oe/K, $\gamma = 5$, and $\kappa = 52$. At $T = 82.8$ K, $H = 50$ kOe. This leads to following dimensionless parameters $Gi = 2.04 \times 10^{-5}$, $\omega = 0.056$, $a_h = 0.039904$. However as discussed in [14] effective expansion parameters are $\frac{a_h}{\omega_0} = 0.22268$ and $\frac{\omega(a_h)^{-3/2}}{2\sqrt{2}\pi} = 0.79328$, both less than one. $\frac{a_h}{\omega_0}$ is the parameter for the expansion of the classical solution. The factor 6 comes from a fact that due to hexagonal, thus only 6th, 12th, etc. Landau levels appears in perturbation expansion. $\frac{\omega(a_h)^{-3/2}}{2\sqrt{2}\pi}$ is the parameter for the fluctuations (loop) expansion and is just barely less than one here. It justifies the quantum correction of the formula using perturbation expansion. The numerical factor in front of the fluctuation correction in this case is

$$c_1 = \frac{\beta_{Ab}\omega (a_h)^{-3/2}}{8\pi^2} = 3.0932 \times 10^{-3} \quad (37)$$

In a finite size sample, $\delta_n(q)$ is equal to $\frac{L_xL_y}{(2\pi)^2}$ when $q$ lies on the reciprocal lattice $m_1d_1 + m_2d_2$, otherwise it is zero. Because $L_xL_y = N\phi 2\pi$ ($N\phi$ is the number of vortices of order 100 only in the MC simulation), thus it is equal to $\frac{N\phi}{2\pi}$. The normalized structure function ($s_n(0) = 1$, as it was used in [13]) is:

$$s_n(q) = \Delta(q) + \frac{1}{(1 + c_1f_3)}[c_2f_1(q) + c_1\Delta(q)f_2(Q)] \quad (38)$$

and

$$c_2 = c_1\frac{2\pi}{N\phi} = 1.9435 \times 10^{-4} \quad (39)$$

$$1 + c_1f_3 = .91315$$

The correction to the height of the peak at $Q$, $\frac{c_1\Delta(q)}{(1 + c_1f_3)}f_2(Q)$, is quite small. We find the height of peak away from origin found in the MC simulation [13] are typically smaller than
ours, while around the peaks are larger than analytical. It may be due to finite size effect or finite samplings of MC calculation. In the MC calculation part of the peak might "belong" to a neighboring pixel. We plot the correction to the non-peak region on Fig. 1b and find that the theoretical prediction has roughly the same characteristic saddle shape "halos" around the peaks as in ref. [13], Fig. 1a on which all the peaks were removed (so it is different from Fig.2a in [13] on which only the central peak was removed).

We can extend our formula to higher orders which will include also the HLLs (higher Landau levels). To next order of $a_h$, we should include $\Phi_1$ in $\Phi$, $\Phi = (a_h)^{1/2} [\Phi_0 + a_h \Phi_1]$, consider $\epsilon_O(k), \epsilon_A(k)$ to next order $a_h^2$, and $\epsilon_O^n(k), \epsilon_A^n(k)$ to order $a_h$. It is straightforward to do it.

V. FLUCTUATION OF MAGNETIC FIELD

Another quantity which can be measured is the magnetic field distribution. In addition to constant magnetic field background there are $1/\kappa$ magnetization corrections due to field produced by supercurrent. To leading order in $a_h$ it is given by $m(x) \propto \frac{<\rho(x)>}{\kappa}$ (for example, see ref. [13]). $\langle\rho(x)\rangle$ can be calculated using the following equation,

$$\langle\rho(x)\rangle = \langle |\Phi(x) + \chi(x)|^2 \rangle$$
$$\quad = |\Phi(x)|^2 + \langle \Phi^*(x)\chi(x) \rangle + \langle \Phi(x)\chi^*(x) \rangle + \langle \chi(x)\chi^*(x) \rangle$$
$$\quad = \frac{a_h}{\beta A} |\phi(x)|^2 + \langle \chi(x)\chi^*(x) \rangle.$$  \hspace{1cm} (40)

Using eq.(18) and eq. (11), and considering only $x_3 = 0$, one obtains:

$$\langle\chi(x)\chi^*(x)\rangle = \frac{\omega}{16\pi^2} \int |\varphi_k(x)|^2 \left[ \frac{1}{\epsilon_O(k) + \frac{k^2}{2}} + \frac{1}{\epsilon_A(k) + \frac{k^2}{2}} \right]$$
$$\quad = \frac{\omega}{16\pi^2} \int |\varphi_k(x)|^2 \left[ \frac{2}{\sqrt{\epsilon_O(k)}} + \frac{2}{\sqrt{\epsilon_A(k)}} \right]$$

However, as pointed out in Sec. III the coefficient $\nu$ in $\psi(x) = v\phi(x) + \chi(x)$ is renormalized to one loop order, $v^2 = v_0^2 + \omega v_1^2$, with $v_1$ given in eq.(30). Thus we need to add a term, $\omega v_1^2 |\phi(x)|^2$ to eq.(30).

12
\[
\langle \rho(x, 0) \rangle = \frac{a_h}{\beta_A} |\phi(x)|^2 + \frac{\omega}{16\pi^2} \int_k |\varphi_k(x)|^2 \left[ \sqrt{\frac{2}{\epsilon_O(k)}} + \sqrt{\frac{2}{\epsilon_A(k)}} \right]
- \frac{\omega |\phi(x)|^2}{16\pi^2} \int_k \left[ \left( \frac{2\beta_k + |\gamma_k|}{\beta} \right) \sqrt{\frac{2}{\epsilon_O(k)}} + \left( \frac{2\beta_k - |\gamma_k|}{\beta} \right) \sqrt{\frac{2}{\epsilon_A(k)}} \right]
\]

(41)

Its Fourier transform \( \rho(q) \equiv \int dze^{iq\cdot z} \langle \rho(x, 0) \rangle \) can be easily calculated:

\[
\rho(q) = 4\pi^2 \delta_n(q) \exp \left[ -\frac{q^2}{4b} + \frac{iq_x q_y}{2b} + \frac{\pi i}{2} (n_1^2 - n_1) \right]
\left\{ \frac{a_h}{\beta_A} + \frac{\omega}{16\pi^2} \int_k \left[ \left( \exp \left( \frac{ik \times q}{b} \right) - \left( \frac{2\beta_k + |\gamma_k|}{\beta} \right) \right) \right] \frac{2}{\epsilon_O(k)}
+ \left( \exp \left( \frac{ik \times q}{b} \right) - \left( \frac{2\beta_k - |\gamma_k|}{\beta} \right) \right) \frac{2}{\epsilon_A(k)} \right\}
\]

(42)

Performing integrals and rescaling the quasimomenta again, one obtains:

\[
\rho(q) = 4\pi^2 \delta_n(q) \exp \left[ -\frac{q^2}{4b} + \frac{iq_x q_y}{2b} + \frac{\pi i}{2} (n_1^2 - n_1) \right]
\left\{ \frac{a_h}{\beta_A} + \frac{\omega ba_h^{-1/2}}{16\pi^2} [-28.5275 + f_2(Q)] \right\}
\]

(43)

The function \( f_2(Q) \) appeared in eq.(42).

**VI. DISORDER EFFECT ON MAGNETIZATION AND SPECIFIC HEAT**

One can introduce weak disorder by adding a quadratic term in eq. (2) [6],

\[
\Delta f \equiv \int d^3x \alpha(x)|\psi|^2.
\]

(44)

Loosely speaking it represents local variation of temperature. For pointlike defects one can assume that the correlation of \( \alpha(x) \) is \( \langle \alpha(x) \alpha(y) \rangle = W \delta(x - y) \), \( \langle \alpha(x) \rangle = 0 \).

Before the disorder average we calculate the free energy \(-T \ln Z\) with

\[
Z = \int \mathcal{D}\psi^* \mathcal{D}\psi \exp \left\{ -\frac{1}{\omega} \left[ f[\psi^*, \psi] - \int d^3x \alpha(x)|\psi|^2 \right] \right\}.
\]

(45)

If \( W \) is very small, we can calculate \( Z \) by perturbation theory in \( W \). To the second order \( Z \) is given as

\[
Z = Z_0 \left[ 1 - \frac{1}{\omega} \int_x \alpha(x) < \rho(x) > + \frac{1}{2\omega^2} \int_x \int_y < \rho(x) \rho(y) > \alpha(x) \alpha(y) \right],
\]

(46)

13
where $Z_0$ is the free energy without disorder and it had been obtained in ref. [14]. Thus the free energy with disorder is

$$F = -T \ln Z = F_0 + \Delta F$$

$$= F_0 + T \int \frac{\alpha(x)}{\omega} \langle \rho(x) \rangle - \frac{T}{2\omega^2} \int [\langle \rho(x) \rangle \rho(y) > - \langle \rho(x) \rangle < \rho(y) >] \alpha(x) \alpha(y),$$

where $F_0 = -T \ln Z_0$. Averaging free energy over disorder one obtains:

$$F = F_0 - \frac{TWV}{2\omega^2} \int \left[ \langle \rho(x) \rho(x) > - \langle \rho(x) \rangle \langle \rho(x) > \right]$$

$$= F_0 - \frac{TWV}{2\omega^2} \omega \tilde{S}_{fluct}(0).$$

From eq.(32), one finds that $\tilde{S}_{fluct}(0) = -0.18619 \frac{a_{1/2}}{h b}$. Hence the energy density difference due to disorder is $\mathcal{F} = \frac{\Delta F}{V} = -0.0931 \frac{TWa_{1/2}}{\omega b^2}$. Since $\omega = \sqrt{2GI} \pi^2 t \mathcal{F} = ca_{1/2} b$ with $c = -\frac{0.0931TW}{\sqrt{2GI} \pi^2 b a}$. The disorder effect on magnetization and specific heat are

$$\Delta m = -\frac{\partial \Delta f}{\partial b} = -c \left( a_{1/2} - b \frac{1}{4a_{-1/2}} \right)$$

$$\Delta c = -t \frac{\partial^2}{\partial t^2} \Delta f = \frac{c}{16} t b a_{-3/2}$$

respectively.

**VII. CONCLUSIONS**

To conclude, we have calculated the effect of fluctuations on the structure function of the vortex lattice and compared it to existing MC results. In addition to renormalization of the height of the Bragg peaks, there appears a characteristic saddle shape "halos" around the peaks as found in ref. [13]. The calculated fluctuations contribution to the magnetic field can be more easily observed in low temperature strongly type II superconductors. Finally the predicted dependence of magnetization and specific heat on disorder via fluctuations also can be experimentally studied.

Correlations in flux lattices can be experimentally measured using neutron scattering as well as some other more exotic methods like muon spin relaxation, electron tomography,
scanning SQUID microscopy etc. [1–3,16,17]. It would be interesting to detect the effect of fluctuations given in the present paper directly from experiments by subtracting the "background" of the well known mean field correlator. The calculations show that infrared divergencies naively expected in all of the physical quantities calculated above due to "supersoft" shear modes in the large $\kappa$ limit cancel. This strengthens the view that the loop expansion is a reliable theoretical tool to study the fluctuations effects in vortex lattice below the melting point.

ACKNOWLEDGMENTS

We are grateful to our colleagues A. Knigavko, B. Bako, V. Yang. One of us (B.R.) is specially grateful to R. Sasik and D. Stroud for providing raw numerical data which was essential for the present comparison with the MC data. The work is part of the NCTS topical program on vortices in high Tc and was supported by NSC of Taiwan.
APPENDIX A:

In this appendix, we present some basic formulas used in the calculations. The basic matrix element is:

\[
\frac{b}{2}\pi \int_{c\ell t} dx \varphi(x) \varphi^*_k(x) \exp[-ix \cdot q] = \Delta_{q,k} \exp \left[ \frac{\pi i}{2} (n_1^2 - n_1) - \frac{q^2}{4b} - \frac{i q_x q_y}{2b} + \frac{i k_x q_y}{b} \right] \quad (A1)
\]

The Kronecker delta is defined by:

\[
\Delta_{q,k} = \Delta(q - k) = \begin{cases} 
1, & \text{if } q = k + n_1 \tilde{d}_1 + n_2 \tilde{d}_2 \\
0, & \text{otherwise}
\end{cases} \quad (A2)
\]

where integers \(n_1 = \frac{1}{2\pi} d_1 \cdot (q - k)\) and \(n_2 = \frac{1}{2\pi} d_2 \cdot (q - k)\). Here \(\tilde{d}_1, \tilde{d}_2\) are the reciprocal lattice basis vectors

\[
\tilde{d}_1 = \frac{2\pi \sqrt{b}}{a} \left(1, -\frac{1}{\sqrt{3}} \right) ; \quad \tilde{d}_2 = \left(0, \frac{4\pi \sqrt{b}}{a\sqrt{3}} \right), \quad (A3)
\]

which are dual to \(d_1 = (a/\sqrt{b}, 0), d_2 = \left(\frac{a}{2\sqrt{b}}, \frac{a\sqrt{3}}{2\sqrt{b}} \right)\) and \(a = \frac{\sqrt{2} \pi}{\sqrt{3}}\). Integrating over the sample area \(A\), one obtains:

\[
\int_A dx \varphi(x) \varphi^*_k(x) \exp[-ix \cdot q] = 4\pi^2 \delta_n(q - k) \exp \left[ \frac{\pi i}{2} (n_1^2 - n_1) \right] \\
\times \exp \left[ -\frac{q^2}{4b} - \frac{i q_x q_y}{2b} + \frac{i k_x q_y}{b} \right] \quad (A4)
\]

where \(\delta_n(q - k)\) is defined as \(\delta_n(q - k) = \sum_{m_1, m_2} \delta(q - k - m_1 \tilde{d}_1 - m_2 \tilde{d}_2)\).
REFERENCES

[1] P.L. Gammel et al, Phys. Rev. Lett. 68, 3343 (1992).

[2] B. Keimer et al, Phys. Rev. Lett. 73, 3459 (1994).

[3] I. Maggio-Aprile et al, Phys. Rev. Lett. 75, 2754 (1995).

[4] E. Zeldov, D. Majer, M. Konczykowski, V.B. Geshkenbein, V.M. Vinokur and H. Shtrikman, Nature 375, 373 (1995).

[5] D.R. Nelson, Phys. Rev. Lett. 60, 1973 (1988).

[6] G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A. I. Larkin, V. M. Vinokur, Rev. Mod. Phys. 66, 1125 (1994).

[7] M. Tinkham, Introduction to Superconductivity, (McGraw - Hill, New York, 1996).

[8] G. Eilenberger, Phys. Rev. 164, 628 (1967); K. Maki and H. Takayama, Prog. Theor. Phys. 46, 1651 (1971).

[9] A. Ikeda, T. Ohmi and T. Tsuneto, J. Phys. Soc. Jpn. 59, 1740 (1990); 61, 254(1992); A. Ikeda, J.Phys. Soc. Jpn. 64, 1683 (1994); 64, 3925 (1995).

[10] M.A. Moore, Phys. Rev. B39, 136 (1989); Phys. Rev. B31, 7336 (1992).

[11] G.J. Ruggeri, Phys. Rev. B20, 3626 (1978).

[12] B. Rosenstein, unpublished.

[13] R. Sasik and D. Stroud, Phys. Rev. Lett. 75, 2582 (1995).

[14] D. Li and B. Rosenstein, cond-mat/9902294.

[15] I. Affleck and E. Brezin, Nucl. Phys B257, 451 (1985).

[16] R. Cubitt et al, Nature 365, 407 (1993).

[17] L.N. Vu, M.S.Vistrom and D.J. Van Harlingen, Appl. Phys. Lett. 63, 1693 (1993).
Figure captions

Fig. 1a
Structure factor from the MC simulation of ref. [13]. The peaks at reciprocal lattice points are removed.

Fig. 1b
Fluctuation correction to structure factor of the Abrikosov vortex lattice, eq.(38). The peaks at reciprocal lattice points are removed.