Does $m^* g^*$ diverge at a finite electron density in silicon inversion layers?

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For the two-dimensional electron system in silicon MOSFET’s, the scaled magnetoconductivity has been shown to exhibit critical behavior at finite density $n_0$. Analysis of these magnetotransport experiments yields a product $g^* m^*$ that diverges at this density (here $g^*$ is the interaction-enhanced Landé $g$-factor and $m^*$ is the effective mass). This claim has been disputed based on direct determinations of the $g^* m^*$ obtained from Shubnikov-de Haas measurements. We briefly review these experiments, and possible sources of the discrepancies.

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There is considerable current interest in the unusual behavior of dilute two-dimensional systems of electrons (or holes) [1]. Contrary to the long-held expectation that non-interacting [2] or weakly interacting [3] electrons become localized (i. e. are insulators) in two dimensions in the limit of zero temperature, experiments within the past decade have shown that the conductivity exhibits metallic temperature dependence within a range of low electron densities (where electron interactions are actually quite strong). Unexpected metallic behavior has been observed in this low-density regime: for electron ($n_e$) or hole ($p_h$) densities above some critical density $n_c$ (or $p_c$), the conductivity $\sigma$ increases with decreasing temperature; $\sigma$ is approximately independent of temperature near the quantum unit of conductance $(e^2/h \approx 3.9 \times 10^{-5}\Omega^{-1})$ at $n_c$ and exhibits insulating behavior below this critical density [4]. In silicon MOSFETs, the conductivity has been shown to increase down to a temperature $\approx 35\text{ mK}$ for electron densities just above $n_c$ [5]. This suggests there exists a metallic phase and a true metal-insulator transition in dilute strongly interacting electron (hole) systems.

The response of these systems to external magnetic field is unusual and dramatic. An external magnetic field of the order of a few Tesla applied parallel to the plane of the electrons gives rise to an enormous positive magnetoconductance [5][6][7][8][9] on both sides of the transition: the longitudinal resistivity increases (conductivity decreases) dramatically as a function of magnetic field $H$ applied parallel to the plane of the electrons and saturates to a value that is approximately constant for magnetic fields $H > H_{sat}$ with the value of $H_{sat}$ depending on the electron (hole) density ($n_e$). Interestingly, a parallel magnetic field has been shown to suppress the metallic behavior [10][11]. Moreover, recent experiments have yielded evidence that the magnetotransport exhibits critical behavior at a finite density $n_c$ [12]. It is not clear whether and how the metallic temperature-dependence and the magnetic response are related.

These developments have elicited intense interest, and have fueled a lively debate. The issue is whether the novel effects found in dilute two-dimensional materials represent fundamentally new physics or whether they can be explained by an extension of physics that is already understood. A view held by some is that these features signal a true zero-temperature quantum phase transition to a novel ground state at $T=0$ (such as a “perfect” metal, a superconductor, a ferromagnet, a spin liquid, a Wigner glass, etc. [1]). Others believe that the metallic temperature dependence can be understood within the framework of Fermi liquid theory and is due, for example, to temperature-dependent screening, percolation, interband scattering, or scattering at charged traps. No consensus has been reached; the enigmatic behavior of dilute two-dimensional systems continues to be one of the most interesting unresolved issues in Condensed Matter Physics.

I. DOES $G^* M^*$ DIVERGE IN SILICON MOSFET’S?

For the 2D electron system in silicon MOSFET’s, several groups have determined renormalized values of the effective mass $m^*$ and the Landé $g$-factor $g^*$ based on measurements of the magnetoconductivity and of Shubnikov-de Haas oscillations as a function of temperature and electron density. There is currently disagreement whether either or both of these renormalized quantities diverge at a finite electron density in this system [13]. This is indeed an important question: such a divergence would signal critical behavior of the spin susceptibility in silicon MOSFET’s. In this paper we briefly review and examine this controversy.

A quantitative characterization of the system’s response to parallel magnetic field has been difficult to obtain. Attempts to scale the experimentally measured magnetoconductivity yield a match at low fields or at high magnetic fields, but none provide satisfactory results over the entire range, particularly in the low-density regime. By considering the magnetoconductance instead of the magnetoconductance, we have recently succeeded in obtaining an excellent data collapse over a
broad range of electron densities and temperatures using a single scaling parameter $H_\sigma$. (Possible reasons for the difference between resistivity and conductivity are discussed below). We showed that the scaling parameter $H_\sigma$ is consistent with the empirical relation: $H_\sigma(n_s, T) = A(n_s)[|\Delta(n_s)|^2 + T^2]^{1/2}$. Fig. 1 (a) shows $H_\sigma$ versus $T$ for different densities $n_s$ approaching the transition. Fits to this expression yield values of $\Delta$ shown in Fig. 1 (b) as a function of electron density $n_s$.

The parameter $\Delta$ represents an energy $k_B\Delta$: for high densities and low temperature, $T < \Delta \sim \hbar/\tau_H$, $H_\sigma$ is determined by $\Delta$ and the system is in the zero temperature limit; at lower densities the measurement temperature $T > \Delta \sim \hbar/\tau_H$, the field $H_\sigma$ is dominated by

FIG. 1. (a) $H_\sigma$ versus temperature for different electron densities; the solid lines are fits to the empirical form $H_\sigma(n_s, T) = A(n_s)[|\Delta(n_s)|^2 + T^2]^{1/2}$. (b) The parameter $\Delta$ as a function of electron density; the solid line is a fit to the expression $\Delta = \Delta_0(n_s-n_0)^\alpha$.

FIG. 2. (a) The conductivity (left axis, and resistivity (right axis) as a function of magnetic field applied parallel to the electron plane; the electron density $n_s = 0.94 \times 10^{11} \text{ cm}^{-2}$; $H_\sigma$ and $H_\rho$ denote the saturation fields deduced from plotting the conductivity and resistivity, respectively. (b) The (normalized) difference between $H_\sigma$ and $H_\rho$ versus electron density. (Different symbols denote two different samples.)
thermal effects. The energy scale $k_B \Delta$ depends on electron density and extrapolates to zero at a finite density $n_0 \approx 0.85 \times 10^{11} \text{ cm}^{-2}$. That we have identified a critical regime and an approach to $\Delta = 0$ is further supported by our finding that for a density $n_s \approx n_0$ the magnetoconductivity scales with $H/T$ down to our lowest measuring temperature (0.25 K), indicating that $\Delta = 0$ in the vicinity of $n_0$. The observed critical behavior of the magnetoconductivity suggests there is a (zero-temperature) quantum phase transition at a density $n_0$.

The origin of the critical behavior of $H_c$ is currently under investigation. Based on early observations at high electron densities [14,15,16] that the parameter $H_c$ is close to the field required for complete spin polarization of the 2D carriers, we suggested that the critical behavior of the magnetoconductivity is associated with the spin degrees of freedom of the system [11]. In the remainder of this paper, we compare values of $g^*m^*$ deduced from the magnetoconductivity with those obtained from Shubnikov-de Haas measurements.

For high electron densities, Shubnikov-de Haas measurements [14,15,16,17] have shown directly that the saturation of the magnetoresistance at field $H_{\text{sat}}$ is associated with full polarization of the electron spins by the in-plane magnetic field. It is now generally recognized that for lower densities, the relation between full polarization and resistivity saturation is considerably more complicated. As shown in Fig. 2 (a), we have recently noted that examination of the resistivity and of the conductivity (its inverse), do not yield the same apparent saturation point. As illustrated in Fig. 3, we have suggested [18] that the field $H_s$ where the conductivity appears to saturate corresponds to full polarization of the highly mobile band states ($g^*\mu_B H_s = 2E_F$), while the higher magnetic field $H_p$ required for resistivity saturation corresponds to full alignment of ALL the electrons, including those in tail states below the bottom of the band ($g^*\mu_B H_p = 2E_F + \delta$). As shown in Fig. 2 (b), the difference between $H_s$ and $H_p$ becomes larger as the density is decreased toward $n_0$, indicating that disorder plays an increasingly important role.

We suggest that the $H_{\text{sat}}$ deduced from our magnetoconductance measurements is associated with the response of the highly mobile band states (not the localized states in the band tails) so that $H_s = H_{\text{sat}}$. If one sets $g^*\mu_B H_{\text{sat}} = 2E_F$, one can obtain zero-temperature values of the susceptibility $\chi^* \propto g^*m^*$. The open circles plotted in Fig. 4 denote the inverse of the normalized susceptibility $\chi_0/\chi^*$ plotted as a function of electron density $n_s$.
arguments given above. Shashkin et al.’s findings differ from ours in some respects: for example, they find that \( H_s \) extrapolates linearly to zero at a finite density; for the sample used in their experiment they found \( n_0 = n_c \). Despite these differences, however, their results for \( \chi_0/\chi^* \), designated by the crosses in Fig. 4, are quite similar to ours. The values deduced by both groups from transport measurements indicate that \((g^*m^*)^{-1}\) extrapolates to zero \((g^*m^* \text{ diverges})\) in silicon MOSFET’s at a finite density.

Pudalov et al. \cite{19} dispute this claim based on direct determinations of \((g^*m^*)\) from Shubnikov-de Haas (SdH) measurements in crossed magnetic fields down to very low densities. The susceptibility \( \chi_0/\chi^* \) determined from these measurements is shown by the closed circles in Fig. 4. The susceptibility obtained by all three groups, two using different analyses of magnetotransport and the third from SdH measurements, are remarkably similar. However, Pudalov et al. report that detailed examination of their data reveals a sharp increase for the value of \((g^*m^*)\) (by a factor of \( \approx 4.6 \)) as the electron density decreases toward \( n_c \approx 0.85 \times 10^{11} \text{ cm}^{-2} \), but no divergence at this density. These conflicting claims must be resolved through further experimentation, and a better understanding of both transport and Shubnikov-de Haas data.

Our procedure for scaling the magnetoconductance yields a parameter \( H_s \) from which we have deduced the renormalized susceptibility \((g^*m^* \propto \chi^*)\), and the zero-temperature limit of this quantity appears to diverge. The \( H_s \) that results from this scaling ansatz may not provide a reliable measure of the susceptibility, particularly at very low densities where disorder and band-tail states play an important role. By the same token, it is important to take additional, very precise measurements of the Shubnikov-de Haas oscillations. Fig. 1(a) shows that \( H_s \) depends on temperature. However, if one restricts one’s attention to the region below 1 K, \( H_s \) depends on temperature only for low densities near the apparent divergence at \( n_0 \). The importance of examining the temperature dependence of \( \chi^* \) and obtaining a reliable extrapolation to \( T = 0 \) is further illustrated in Fig. 5, where the normalized susceptibility \( \chi_0/\chi^* \) obtained from our data is plotted at different temperatures between 0.25 K and 1 K. If it occurs, the divergence of \( \chi \) is apparent only in the limit of zero temperature. This possibility is further supported by recent experiments of Reznikov and Sivan \cite{20}, who have determined the magnetization by measuring the change in chemical potential with applied magnetic field applying the thermodynamic relation \( d\mu/dH = -dM/dn \) (here \( \mu \) is the chemical potential, \( H \) is the magnetic field, \( M \) is the magnetization, and \( n \) is the electron density). Their preliminary results show that the susceptibility depends on temperature, and the \( T \)-dependence appears to extend to lower temperatures at low electron densities. This is consistent with the temperature dependence of \( \chi^* \) deduced from our transport measurements shown in Fig. 5. On the other hand, Pudalov et al. \cite{19} report that the \((g^*m^* \propto \chi^*)\) obtained from their Shubnikov-de Haas measurements is almost independent of temperature over the range 0.3–1 K. Additional, detailed studies of Shubnikov-de Haas are needed at low electron densities near \( n_0 \) to determine whether \((g^*m^*)\) is indeed independent of temperature at very low temperatures, or whether the behavior is closer to that shown in Fig. 1 (a) and Fig. 5. We note that low density is precisely the region where SdH measurements are most difficult to perform and interpret.

![Graph](image)

**FIG. 5.** The normalized inverse susceptibility \( \chi_0/\chi^* \) versus electron density at 0.25 K, 0.7 K, and 1 K. The closed circles denote extrapolations to \( T = 0 \).

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