Optical Tamm state and extraordinary light transmission through nanoaperture

Ilya V. Treshin and Vasily V. Klimov

P.N. Lebedev Physical Institute, Russian Academy of Sciences,
53 Leninsky Prospekt, Moscow 119991, Russia

Pavel N. Melentiev and Victor I. Balykin

Institute for Spectroscopy Russian Academy of Sciences,
5 Phizicheskaya str., Troitsk, Moscow 142190, Russia

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Abstract

We investigate the light transmission through a nanoaperture in a metal film deposited on a planar metamaterial. An effect of an anomalously high light transmission through the nanoaperture is revealed, which we associate with the enhancement of the field at the interface of the planar structure “metamaterial-metal film” due to the appearance of an optical Tamm state. In this structure, we also observe an “optical diode” effect: the light transmission radically changes as the direction of irradiation of the structure is reversed. Our numerical results agree well with experimental data.

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*Electronic address: vklim@sci.lebedev.ru
I. INTRODUCTION

The effect of an extraordinary transmission of light through an array of nanoapertures in a metal film has been observed for the first time by Ebbesen et al. [1]. The effect describes the light transmission through a nanoaperture array, which is considerably higher in magnitude than predicted according to the Bethe-Bouwkamp theory [2, 3]. At present an extraordinary light transmission has been observed both for a periodic nanoaperture array in a metal film and for single nanoapertures in it. In the latter case, various techniques are used to increase the light transmission. For example, in [4, 5], a method of structuring of the surface near the aperture is applied in order to match the wave vectors of the incident electromagnetic field and plasmons on the film surface. In general, there exists a large variety of physical mechanisms and methods of their realization that lead to similar effects. It seems that, in all these cases, the field intensity in the region of the aperture is enhanced due to the excitation of propagating plasmons, and precisely this enhancement of the field causes the intensity of the passing light to increase [6]. At the same time, the field enhancement can also occur in ideally conducting geometries, where there are no plasmons but an anomalously high transmission also arises [7, 8].

Recently, an extraordinary increase in the light transmission has been experimentally detected in the case of a gold film with nanoapertures on the surface of a multilayer periodic dielectric structure, a planar metamaterial [9, 10]. At a certain frequency of the incident light, an optical Tamm state of the electromagnetic field arises [11, 12], in which the intensity of the magnetic field from the side of the metamaterial on the surface of the metal film has a maximum [13]. (In what follows, we will term the light frequency at which the optical Tamm state arise the resonance frequency.) The light transmission through nanoapertures increases by several orders of magnitude compared to the case in which there is no metamaterial.

In view of the importance of the observed effect, the question of its theoretical explanation arises, to which this work is devoted. The plan of the remaining part of the paper is as follows. Section II presents the descriptions of the geometry and model of the numerical experiment. Results of the numerical simulation of a gold film on the surface of a metamaterial with and without apertures are presented in sections III and IV respectively. Section V is devoted to the discussion of the results.
II. DISCUSSION OF THE MODEL OF THE NUMERICAL EXPERIMENT

The theoretical calculation of this work is based on the geometry of experiment of [9]. The values of the geometric parameters and the optical properties of materials were taken from that paper, unless otherwise specified.

The experiment of [9] is extremely subtle, because it is very difficult to control the manufacturing process (the shape of apertures, the thicknesses of layers) with a nanometer accuracy. Figure 1(a) shows the images of the array and one of nanoapertures used in the experiment of [9]. We can assume from this figure that the nanoaperture does not have an ideal cylindrical shape. Therefore, in the numerical model, the aperture was approximated by a generalized truncated cone.

A cross section of one period of the structure “metamaterial-metal film” that was used in the numerical experiment is shown in Fig. 1(b).

A 12-layer periodic dielectric structure (metamaterial) is deposited on a quartz substrate (which is shown at the top in Fig. (b)). The dielectric structure consists of an $\text{Al}_2\text{O}_3$ layer (with a thickness of 125 nm) and alternating $\text{TiO}_2$ and $\text{MgF}_2$ layers (with their thicknesses being 82 and 125 nm, respectively). A gold film with a thickness of 220 nm is deposited on the surface of the metamaterial (at the “bottom” in Fig. 1(b)). Since the thickness of this film considerably exceeds the thickness of the skin layer ($\sim 80$ nm), the film is opaque in the considered wavelength range of the incident light (from 575 to 875 nm). Nanoapertures in the metal film form a square lattice with a period of 2 $\mu$m. In the numerical model, the aperture is approximated by a truncated cone, with the diameters of its bases being $d_1$ (on the $\text{TiO}_2$ layer) and $d_2$ (on the opposite side). The whole space inside the aperture and behind the gold film is filled with an immersion oil. In [9], the thickness of the SiO$_2$ substrate was 2 mm, and the immersion oil was used to reduce the reflection. In our numerical experiment, the quartz layer and immersion oil are assumed to be infinitely thick. The refractive indices $n$ of all the used dielectric materials were taken from the experimental data of [9]: SiO$_2$ ($n_q = 1.443$), Al$_2$O$_3$ ($n = 1.63$), TiO$_2$ ($n = 2.23$), MgF$_2$ ($n = 1.38$), and immersion oil ($n_{io} = 1.51$). In calculations, the dispersion of the dielectrics was neglected. The dispersion dependence of the dielectric permittivity of gold was taken from [14].

The light transmission through nanoapertures was simulated for the normal incidence of light on the structure. In this case, the wave that was incident on the structure from the
FIG. 1: (a) Images of the array and one of the nanoapertures in the experiment of [9] obtained by the scanning electron microscopy method; (b) schematic image of the cross section of one period of the nanoaperture array in the gold film on the metamaterial used in the numerical experiment.

side of the quartz substrate (from above) was expressed as $E_{up} = E_0 \exp(i k_0 n q z)$, while the wave incident from the side of the immersion oil was given by $E_{down} = E_0 \exp(i k_0 n_{io} z)$.

III. OPTICAL PROPERTIES OF A STRUCTURE “METAMATERIAL-METAL FILM”

Initially, we will consider the optical properties of a structure “metamaterial-metal film” with no apertures. To do this, we will use the analytical method of [15–17]. In each separate layer, the solution is represented as a sum of the “incident” and “reflected” waves. The amplitudes of these waves can be found from the condition of continuity for the tangential
components of the electric and magnetic fields at the boundary between the layers. As a result, we obtain a system of linear algebraic equations, the numerical solution of which was found using MATLAB®. Let us consider initially the case in which the light is incident on the structure “metamaterial-metal film” with no apertures from the side of the quartz [from the top, see Fig. 1(b)].

Figures 2(a) and 2(b) show the calculated and experimental [9] dependences of the energy reflection coefficient on the wavelength and on the angle of incidence of light for the metamaterial and the gold film on it, respectively. The experimental curves are shown in black. It can be seen from Fig. 2 that there is a qualitative agreement between the theory and experiment. In the numerical experiment, we also slightly varied the thicknesses of the layers, as a result of which the agreement became much better. This is indicative of possible errors in specifying the optical constants of the materials or of the imperfect preparation of experimental specimens.

A characteristic feature of the dependence of the reflection coefficient of the optical structure “metamaterial-metal film” is the occurrence of a narrow resonance dip. The corresponding resonant wavelength can be estimated from the condition [13]:

\[ r_M r_{MM} = 1, \]  

where \( r_M \) is the amplitude reflection coefficient at Au-TiO\(_2\) interface, while \( r_{MM} \) is the amplitude reflection coefficient of the wave incident from TiO\(_2\) half-space on the metamaterial starting with a TiO\(_2\) layer.

In order to elucidate the physical nature of this narrow resonance peak, we calculated the distributions of the electric and magnetic fields and of the energy flux in this structure, Fig. 3. The field and the energy flux distributions were normalized to the values of \((E_0, H_0)\) in the incident wave.

From Fig. 3(a), it can be clearly seen that, in the nonresonance case \((\lambda = 700 \text{ nm})\), almost all the energy is reflected on the surface of the metamaterial, and the amplitudes of the electric and magnetic fields exponentially decay as the gold film is approached. The energy flux inside the metamaterial is also close to zero. Conversely, in the resonance case, i.e., in the case of the appearance of the optical Tamm state, \((\lambda = 796 \text{ nm}, \text{Fig. 3(b)})\), the reflection is small, while the intensities of the electric and magnetic fields near the film are high. The energy flux is directed inward the metamaterial and is absorbed nearly completely in the
FIG. 2: Dependences of the reflection coefficient on the wavelength and on the angle of incidence of light for (a) the metamaterial and (b) the optical structure “metamaterial-metal film”. In the two cases, the light is incident from the side of the SiO$_2$ layer.

gold film. In this case, the transmission coefficient is very small ($\sim 10^{-6}$). An increase in the amplitudes of the electric and magnetic fields near the metamaterial surface is caused by the occurrence of the optical Tamm state of the electromagnetic field $^{[11-13]}$. It is especially important to note that the magnetic field is maximal at the interface metamaterial-metal, and it is the increase in this field that is responsible for the considerable increase in the light transmission through the nanoaperture (see section $^{[V]}$.

If the structure “metamaterial-metal film” is illuminated from the opposite side, the
FIG. 3: Relative electric (black) and magnetic (blue) field strengths and Poynting vector flux (red) in the optical structure “metamaterial-metal film” in relation to the $z$ coordinate inside the structure: (a) nonresonance case, $\lambda = 700$ nm; (b) resonance case, $\lambda = 796$ nm. The arrows show the direction of incidence of the light on the structure. The gold film is shown in yellow, and the metamaterial is shown by alternating white and gray bars. The dashed lines correspond to $1, \pm 2$ guidelines.

situation radically changes. In this case, the reflection coefficient does not have any resonance peak similar to the peak in Fig. 2(b), since the light does not penetrate into the metamaterial due to the absorption in the gold film. Therefore, for the reflection from the structure “metamaterial-metal film”, we have an asymmetric situation with respect to the direction
of incidence of the light onto the structure; namely, the *reflection coefficients for the light that is incident from opposite sides do not coincide with each other.*

Conversely, in accordance with the reciprocity principle for planar structures \[18\], the situation for the light transmission coefficient [see Fig. 4] is always *symmetric* with respect to the direction of incidence of the light; i.e., the transmission coefficients for the light incident *from opposite sides completely coincide with each other.*

Investigations of the behavior of the properties of the structure “metamaterial-metal film” upon variation of the film thickness from 100 nm to 220 nm showed that the shape of the reflection curve [see Fig. 2(b)] changes insignificantly, whereas the transmission coefficient [see Fig. 4(b)] changes by more than three orders of magnitude.

**IV. OPTICAL PROPERTIES OF AN OPTICAL STRUCTURE “METAMATERIAL-METAL FILM” WITH NANOAPERTURES**

Having clarified the behavior of the optical structure “metamaterial-metal film” with no apertures, we consider now how apertures in the gold layer affect the light transmission through the structure. To do this, we used the finite element method, implemented in the program COMSOL\(^\text{®}\) (with a relative calculation accuracy of no worse than \(10^{-3}\) at resonance). Further, under the transmission coefficient of light we understand the energy flux through the area of one period of the optical lattice structure normalized to the flux incident onto the same area. The light transmission coefficient defined in this way is always smaller than unity. Figure 5 shows the light transmission coefficient through a gold film with different apertures but with no metamaterial.

It can be seen from this figure that there is qualitative coincidence between the experiment of \[9\] and the numerical calculation for cylindrical apertures with a diameter of 100 nm. Furthermore, there is no resonance transmission related to the periodicity of the lattice, as that observed in the experiment by Ebbesen \[1, 6\].

In the case of a gold film on the surface of a metamaterial (the optical structure “metamaterial-metal film” with nanoapertures), the situation radically changes. Figure 6 shows the coefficient of the light transmission through this structure [see Fig. 1(b)].

It can be seen from Fig. 6 that, in this structure, there arises an anomalously high light transmission at the resonance frequency (\(\lambda = 796\) nm). It is also seen that the shape of the
FIG. 4: Dependences of the light transmission coefficients through the optical structure “metamaterial-metal film”: (a) on the wavelength and on the angle of incidence of light; (b) on the wavelength and on the depth of Au film ($d_{Au}$), normal incidence of light. Solid curves correspond to the incidence of the light from the side of SiO$_2$; dots show the reflection from the side of the immersion oil.

The obtained transmission curve coincides well with the measurement data from [9]. The shift of the resonance of the transmission coefficient can evidently be a consequence of factors such as (i) the use of the oblique incidence in the experiment, (ii) deviations in the values of the optical constants that were used for the description of gold, and (iii) errors of preparation of experimental specimens. In addition, possible reasons can also be the distinction of the shape
FIG. 5: Dependences of the transmission coefficient through the gold film with apertures but with no metamaterial on the wavelength of the incident light for different geometries of nanoapertures: (a) the light is incident from the side of the SiO$_2$ layer; (b) the light is incident from the side of the immersion oil. Curves: (1) the case with no apertures ($d_1 = d_2 = 0$ nm); (2) apertures in the shape of a cylinder ($d_1 = d_2 = 60$ nm); (3) apertures in the shape of a truncated cone ($d_1 = 60$ nm, $d_2 = 100$ nm); (4) apertures in the shape of a truncated cone ($d_1 = 100$ nm, $d_2 = 60$ nm); (5) apertures in the shape of a cylinder ($d_1 = d_2 = 100$ nm); (6) experimental data from [9].

of the real aperture from the shape of the aperture used in our model, and the occurrence of a shell from gallium atoms on the aperture walls, which appears because apertures were formed with the gallium ion beam. The thickness of the gold film strongly affects the
FIG. 6: Dependences of the transmission coefficient through the optical structure “metamaterial-metal film” with apertures on the wavelength of the incident light for different geometries of apertures ((a) the light is incident from the side of the metamaterial; (b) the light is incident from the side of the gold film): (1) the case with no apertures \(d_1 = d_2 = 0\) nm; (2) apertures in the shape of a cylinder \(d_1 = d_2 = 60\) nm; (3) apertures in the shape of a truncated cone \(d_1 = 60\) nm, \(d_2 = 100\) nm; (4) apertures in the shape of a truncated cone \(d_1 = 100\) nm, \(d_2 = 60\) nm; (5) apertures in the shape of a cylinder \(d_1 = d_2 = 100\) nm; (6) experimental data from [9].

transmission coefficient. It is interesting to note that the transmission curves for the two types of apertures in the shape of a truncated cone, which differ only in the orientation, are almost coincide [see Figs. 5 and 6]. We can conclude from this that the transmission is
mainly determined by the effective volume of the aperture rather than by its shape.

Also, we note that the numerically simulated curves of the transmission coefficient in Figs. 5 and 6 have multiple maxima and minima, whereas the experimentally determined curves do not show such features [9, 10]. This difference is caused by interference effects in the lattice of nanoapertures, since we used periodic boundary conditions at the cell edges [see Fig. 1(b)]. At the same time, the experiments of [9, 10] were performed on a single nanoaperture, and no interference phenomena were observed in this case. Results of investigation of these interference effects and their possible applications will be considered elsewhere [19].

Comparison of Figs. 6(a) and 6(b) shows that the transmission coefficient considerably depends on the direction of incidence of light. If the light is incident on the structure from the side of the metamaterial, the light transmission considerably increases (by two orders of magnitude) compared to the case of an aperture in a metal film alone, whereas, if the light is incident from the side of the metal film, no increase in the light transmission is actually observed. Therefore, the structure “metamaterial-metal film” can serve as a basis for the creation of an optical diode, a device that transmits light in one direction and is opaque in the opposite one. A detailed investigation of this optical diode will be described in [20].

The picture of the light transmission through the aperture in the structure under study becomes more clear from the consideration of Figs. 7 and 8 which show the distributions of the electromagnetic field ($|E|^2/|E_0|^2$) in the cases of the normal incidence from the top and from the bottom at resonance ($\lambda = 796$ nm).

As can be seen from Fig. 7(a), when the light is incident from the side of the metamaterial, the amplitude of the field behind the aperture is comparable with the amplitude of the incident wave.

Far away from the aperture, the field decreases as a spherical wave according to a law $\sim 1/r$. In addition, the formation of a dipole directivity pattern of the transmitted wave is clearly seen in Fig. 7(a). Figure 7(b) shows the section of the two-dimensional distribution along the dashed line. The red solid line shows the intensity distribution along the structure for the case with no aperture. The black and blue dashed lines show the dependences of the squared electric and magnetic field components along the structure for the case with no aperture. The corresponding distributions for the case with the aperture are shown by the black and blue solid lines. Figure 7(b) shows how the magnitude of the electromagnetic
FIG. 7: (a) Distribution of the electromagnetic field ($|\mathbf{E}|^2/|\mathbf{E}_0|^2$) of a plane light wave propagating through the optical structure “metamaterial-metal film” with apertures in the case where the light wave is incident on the structure from the side of the metamaterial at resonance ($\lambda = 796$ nm). (b) The section of the two-dimensional distribution (a) along the dashed line; the red solid line corresponds to the case with no aperture. A logarithmic scale is presented.

Field increases in the direction toward the gold film and an optical Tamm state is formed on the surface of this film due to the constructive interference of the light propagating in the metamaterial. It is also seen that the magnitude of the field on the inner surface of gold at the center of the aperture is higher than in the absence of the aperture.
If the light is incident from the side of the metal film, Fig. 8(b), no enhancement of the field in front of the metal film occurs, and the amplitude of the field behind the aperture is considerably smaller than the amplitude of the incident wave. Therefore, in the structure “metamaterial-metal film” with nanoapertures, the reciprocity principle does not hold [20].

FIG. 8: (a) Distribution of the electromagnetic field \(|E|^2/|E_0|^2\) of a plane light wave propagating through the optical structure “metamaterial-metal film” with apertures in the case where the light wave is incident on the structure from the side of the metal film and at resonance (\(\lambda = 796\) nm). (b) The section of the two-dimensional distribution (a) along the dashed line; the red solid line corresponds to the case with no aperture. A logarithmic scale is presented.
V. DISCUSSION OF RESULTS

In this work, using the finite element method, we have numerically simulated the light transmission through a periodic lattice of nanoapertures in a gold film deposited on the surface of a metamaterial. An effect of an anomalously high light transmission has been revealed, which we associate with the enhancement of the field at the interface “metamaterial-metal film” due to the appearance of an optical Tamm state.

We also have shown that the optical structure “metamaterial-metal film” with nanoapertures is an optical diode: the light transmission radically changes (by two orders of magnitude) as the direction of irradiation of the structure is reversed. Our numerical results agree well with the experimental results of [9] for a cylindrical nano aperture with a diameter of 100 nm.

Qualitatively, an increase in the light transmission through the nanoaperture can be estimated in the case of interest by applying the Bethe-Bouwkamp theory simultaneously with considering the enhancement of the local magnetic field due to the occurrence of an optical Tamm state. Indeed, according to the Bethe-Bouwkamp theory, the transmission coefficient of light at a wavelength of $\lambda = 796$ nm through a cylindrical nanoaperture with a diameter of $d = 100$ nm is

$$T_{\text{Bethe}} = \left(\frac{64\pi^2}{27}\right) \cdot \left(\frac{d}{\lambda}\right)^4 \approx 5.8 \cdot 10^{-3}$$

(here, we use the normalization to the light flux incident on the aperture cross section). The coefficient of transmission through the film with apertures alone at a wavelength of $\lambda = 796$ nm found by numerical simulations is $1.7 \cdot 10^{-3}$ (see Fig. 5; with the normalization to the flux incident on the aperture cross section). Qualitatively, these quantities coincide. The smaller value obtained upon the simulation is seemingly related to a greater thickness of the gold film (220 nm).

In the case of the film with apertures on the metamaterial, the transmission coefficient at a wavelength of $\lambda = 796$ nm is $1.4 \cdot 10^{-1}$ (see Fig. 6; here, the normalization to the flux incident on the aperture cross section is also used). That is, the use of the metamaterial leads to a 90-fold increase in the transmission coefficient. At the same time, the intensity of the magnetic field in the metamaterial, which excites the magnetic dipole moment of the nanoaperture (which, in turn, forms the transmitted field), is 50 times higher than the intensity of the magnetic field on the surface of the gold film. That is, there is a qualitative coincidence between these quantities, which allows us to state that the extraordinary light transmission in the case under consideration is related to an increase in the intensity of the
magnetic (as well as, electric) field. Thus, the transmission coefficient of our structure can be estimated with the following simple formula indeed:

\[ T = T_{\text{Bethe}} G, \]  

where \( T_{\text{Bethe}} = (64\pi^2/27) \cdot (d/\lambda)^4 \) is the Bethe-Bouwkamp transmission coefficient, and \( G = |H|^2/|H_0|^2 \) is the enhancement of the magnetic field intensity at metal surface due to appearance of Tamm state.

Therefore, in this work, it has been theoretically shown that the enhancement of the field related to the occurrence of the optical Tamm state of the electromagnetic field leads to a corresponding increase in the light transmission through the nanoaperture.

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