Simulation of the tire enveloping properties in case of the unmanned car wheel interaction with a rigid uneven surface

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Abstract. A computational and experimental method is presented for assessing the influence of the roughness of a rigid surface on the stiffness and damping properties of automobile tires. The method is based on the solution of tire-ground contact problem using finite-element analysis. In the paper, the influence of two types of irregularities (peaks and pits), their size and shape and tire air pressure on tire enveloping properties is analyzed.

1. Introduction

The disturbing effect of road irregularities on the sprung masses of the unmanned vehicle essentially depends on the tire enveloping properties, i.e. on the capability of the tire to reduce the displacement of the wheel axis in comparison with the height of the irregularity and to increase the interaction area of the wheel contact patch. To simulate the mechanical properties of a tire, it is necessary to use nonlinear mechanics methods that take into account a set of nonlinear factors (physical and geometric nonlinearities, hyperelastic properties of materials, contact interaction) [1,2,3,4,5,6,7,8].

The current level of the computing technology development allows considering this research problem in a more precise statement, using complex analytical models (taking into account the listed nonlinear factors), reproducing complex reinforced tire design and taking into account the interaction of the tire with the uneven road surface. Therefore, the presented research issue is relevant.

Research objective: creation of new, more accurate compared to the well-known, experimentally-confirmed methods of modeling of the car pneumatic tires, interacting with the uneven road surface; the methods should take into account all the tire general design features, mechanical properties of the materials, loading conditions of car wheels, geometrical parameters of irregularities.

Research tasks:

1. Development of a refined analytical 3D model of car pneumatic tire, taking into account its general dimensional design features (sidewalls and parts of the rubber tread, carcass, tire belt, bead bundle), reflecting nonlinear properties (rubber incompressibility and hyperelasticity, geometrically nonlinear deformation of the tire cord, contact of the tire with the rigid uneven surface), making it possible to estimate tire stiffness properties at the static and dynamic load (for various values of the vertical load on the wheel, different tire pressure values, different shapes of the road irregularities).

2. Development of the method for determination of stiffness and damping characteristics of the car pneumatic tire and for estimation of the tire enveloping properties at the static interaction of the wheel.
with the rigid uneven surface and of the wheel movement along a rough road with irregularities of various sizes and shapes.

3. Determination of Mooney-Rivlin hyperelastic material properties (for rubber modeling in the framework of the method of estimation of the tire enveloping properties of the car pneumatic tires), based on a comparison of the results of calculations and tests of samples cut from the sidewall of the tire, as well as on the basis of a comparison of the results of calculations and tire deformation tests on a flat rigid surface during vertical loading.

4. Calculation of the tire deformation when it interacts with the rigid uneven surface; allowing to reveal non-linear dependencies of the tire stiffness and damping coefficient on its loading characteristics, shape and dimensions of irregularities of the road.

2. Modeling of the car pneumatic tire

In the framework of the presented research, there were analyzed the studies of Russian and foreign scientists, who carried out investigations of external mechanics of tires; papers on modern methods of the tire enveloping properties analysis were studied, as well as general methods of analysis of tire enveloping characteristics (experimental and theoretical) [9,10,11,12,13,14,15,16].

Based on the performed survey, the following conclusions were made:

1. Finite-element analysis (FEA) is the most effective method of assessment of the tire enveloping characteristics.

2. High accuracy of calculation can be provided by solid finite-element models of tires compared to shell (and other) models.

The new analytical tire model is represented by the equations which define mathematical model of interaction of the elastic car wheel with the road surface: 1) nonlinear theory of elasticity, 2) dynamic equilibrium in increments of displacement.

Equation of dynamic equilibrium of the mechanical system at the moment $t$:

$$M \cdot \ddot{U} + C \cdot \dot{U} + (K_T + K_G) \cdot \Delta U = \Delta R + R_{previous}.$$ 

where: $M$, $C$ are the global mass matrix and the global damping matrix of FEA system; $U$ is the vector of nodal displacements; $\dot{U} = \frac{\partial}{\partial t} U$ is the vector of nodal velocities; $\ddot{U} = \frac{\partial^2}{\partial t^2} U$ is the vector of nodal accelerations; $K_T$, $K_G$ are the matrix of tangential stiffness (taking into account nonlinear properties of material) and the matrix of geometric stiffness in reference configuration; $\Delta R$ is the external load increment vector, $R_N$ is the vector of the unbalanced part of the external load corresponding to the previous load step.

Rubber properties are described by the Mooney-Rivlin model of hyperelastic material. The Mooney-Rivlin equation is an adequate model of the behavior of a number of almost incompressible rubber-like materials. Stain energy density function for incompressible material is:

$$W = a_{10}(I_1 - 3) + a_{01}(I_2 - 3)$$

where: $I_1$, $I_2$ are strain tensor invariants; $a_{10}$, $a_{01}$ are the Mooney-Rivlin material constants.

True stress $\sigma$ for sample uniaxial tension is calculated as follows:

$$\sigma = 2 \cdot a_{10} \left( \lambda^2 - \frac{1}{\lambda^2} \right) + 2 \cdot a_{01} \left( \lambda - \frac{1}{\lambda} \right),$$

where $\lambda$ is the elongation in the direction of the load application.
Figure 1. Block diagram of the computational and experimental method for defining stiffness and damping characteristic of the tire.
Mooney-Rivlin material constants were obtained by conducting mechanical tests of rubber samples cut from the object under study.

The contact problem of interaction of the tire and uneven surface is solved by using penalty function method. Friction coefficient as a function of the relative slip velocity $v_{rel}$ is defined as follows:

$$
\mu_c = \mu_s + (\mu_s - \mu_d) e^{-D_c v_{rel}},
$$

where: $\mu_c$ is the friction coefficient; $\mu_s$ is the static friction coefficient; $\mu_d$ is the dynamic friction coefficient; $D_c$ – exponential damping ratio; $v_{rel}$ is the relative slip velocity.

In order to consider the losses due to hysteresis in the rubber, a model of proportional damping is used:

$$
C = \alpha M + \beta (K_f + K_p),
$$

where $\alpha$ and $\beta$ are the constants, which are defined from equivalent damping.

Figure 1 shows the main stages of the developed computational and experimental method for calculation of the stiffness and damping characteristics of the car pneumatic tire and evaluating its enveloping properties.

Tire finite-element simulation has been carried out (Figure 2). Stiffness parameters of the tire equivalent structural elements have been chosen. Solutions for a number of test quasistatic tasks of tire interaction with a flat surface at different values of the tire air pressure and normal load have been found (Figure 3). The authors have calculated tire deflections, which were compared with the results of the experiment at the "Ground path" test rig, and obtained the mechanical characteristics of the rubber (on the testing machine IP 5081-20).

Figure 4 shows finite-element model of the tire (Figure 4, a) and the relative position of the tire and the flat surface before application of the vertical load (Figure 4, b) and after application of the vertical load (Figure 4, c).
load (Figure 4, c). Forming of the contact patch between the tire and the surface and significant change of the tire shape are detectable.

The tire deflection \( h_z \) dependencies on the normal wheel load \( P_z \) at different values of the tire inflation pressure are presented in Figure 5.

The discrepancy between the values of the tire deflections obtained by calculation and experimental methods does not exceed 10%. This confirms the adequacy of the chosen analytic models and the developed principles of modeling, which can be used to study more complex variants of the interaction of the tire with the surface.

\[ h_z = \frac{P_z}{E} \]

3. Shape of irregularities and tire deformation

Rectangular, cylindrical, triangle and rectangular cleats have been studied (Figure 6). All the irregularities had height \( q = 20 \text{ mm} \), which is equivalent to the height of the irregularities of the cobblestone pavement. Length \( s \) was varied from 40 mm to 120 mm for rectangular cleats and to 160 mm for cylindrical cleats. The cleat tests were performed for different vertical loads \( P_z \) and tire inflation air pressures.

\[ h_z = \frac{P_z}{E} \]

\[ q = \frac{P_z}{E} \]

\[ s = \frac{P_z}{E} \]

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Figure 6. Diagram of the wheel loading with the vertical force \( P_z \), internal pressure \( p_w \) and shape of the cleats: a) test model; b) rectangular cleat; c) cylindrical cleat.

Estimated tire deflection \( h_z \) at various load levels for rectangular and cylindrical cleats are
presented in Figure 7 and Figure 8. Inflation tire pressure is constant \((p_w=0.1\,\text{MPa})\).

From the comparison of the dependences presented in Figure 7 it follows that at the same wheel load the tire deflection \(h_z\) on the cylindrical cleats is greater than on the rectangular cleats by \(25 - 27\) % under the same load within the interval of maximum loads from the considered range.

![Figure 7](image)

**Figure 7.** The dependence of the deflection of the tire \(h_z\) on the vertical load \(P_z\) of the wheel interacting with the cleats of length \(s\) and of different shapes: a) rectangular; b) cylindrical; 1 – \(s=40\,\text{mm}\); 2 – \(s=80\,\text{mm}\); 3 – \(s=120\,\text{mm}\); 4 – \(s=40\,\text{mm}\); 5 – \(s=80\,\text{mm}\); 6 – \(s=160\,\text{mm}\).

The dependences of the change in radial stiffness for the two considered cleats depending on their length have been obtained (Figure 8). The tire deformation state is shown in Figure 9. Increase in the load leads to decrease in the tire radial stiffness. Increase in the length \(s\) of the cleats leads to the increase in the tire radial stiffness.

![Figure 8](image)

**Figure 8.** The dependence of the normal tire stiffness \(c\) when interacting with the cleats with lengths \(s\) under different loads: a) rectangular; b) cylindrical; 1 – \(P_z=2000\,\text{N}\); 2 – \(P_z=3000\,\text{N}\); 3 – \(P_z=4000\,\text{N}\); 4 – \(P_z=3000\,\text{N}\); 5 – \(P_z=3500\,\text{N}\); 6 – \(P_z=4000\,\text{N}\).

Analysis of the data, obtained by calculation, demonstrates that the shape of the cleat has significant influence on the tire deflection principles.

The problem of the wheel movement along a firm irregular surface has been studied. The following parameters of irregularities were chosen: \(s\) was assumed to be equal to 80 mm, half height of the irregularities \(q_0=1.1\,\text{mm}\), the overall length of the segment \(l=650\,\text{mm}\). The following boundary conditions were formulated: \(p_w=0.2\,\text{MPa}; P_z=4000\,\text{N}\). Two modes were considered: in the first mode, the wheel axis linear velocity \(v_a=1.6\,\text{m/s}\) was set, in the second mode, angular velocity \(n=5\,\text{cycles/s}\) was set (in
this case the linear velocities of the wheel axis center for both the modes were approximately equal. Calculation results are shown in Figures 10 and 11.

![Figure 9](image9.png)

**Figure 9.** An example of the deformed state of the tire.

![Figure 10](image10.png)

**Figure 10.** Tire deformation state (sinusoidal irregularities).

![Figure 11](image11.png)

**Figure 11.** Change of the vertical displacement of the wheel axis in time: a) with application of linear velocity; b) with application of angular velocity.

Analysis of the influence of the shape of irregularities on the tire damping parameters has been performed. Figure 12 shows calculation results for peaks and pits with rectangular section.
Based on the results of the research conducted, the following conclusions are made:

1. Shape of irregularities has a significant influence on the tire deflection, its stiffness and damping characteristics.

2. Results obtained from finite-element simulations and approximation results for peaks and pits with rectangular section differ no more than by 5%.

3. A tire analytical model has been developed; the model makes it possible to solve (alongside with static contact tasks) dynamical tasks of the wheel rolling on uneven surfaces.

4. The results have been obtained for the wheel displacement caused by different types of loads (linear and angular loads).

5. The shape of irregularities affects tire damping properties.

A series of cleat tests was performed for the case of the tire dynamical loading. The cleat test model is shown in Figure 13. Single cleats of the following shapes were used: square, triangular and rectangular. Models of the cleats are shown in Figure 14.
To verify the adequacy of the developed modeling methods, a computational tire model was developed reflecting the conditions of the experiment. Comparison of the results of the experiment and the calculation confirmed the adequacy of the developed simulation model of the automobile pneumatic tire and proved that it is possible to use the developed methods for the research of the interaction of the automobile wheel with a rigid uneven surface.

Furthermore, testing technique for uniaxial tension of a rubber specimen cut from a tire has been developed in order to obtain Mooney-Rivlin material constants. Flat specimens were used. As a result of the tests, the load – displacement diagram was obtained. The Mooney-Rivlin material constants \( a_{10} \) and \( a_{01} \) for equation (1) were obtained with the use of the least-square method.

4. Conclusions

1. The created physically and geometrically nonlinear spatial model of the car pneumatic tire considers design features (carcass, tire belt, bead bundle), hyperelastic and incompressible rubber properties and makes it possible to research tire contact interaction with road irregularities.

2. With the use of the developed method for experimental determination of the properties of incompressible material (rubber) constants \( a_{10} = 1.285 \text{ MPa} \), \( a_{01} = -0.0004 \text{ MPa} \), of the Mooney-Rivlin material have been obtained. The constants can be used to model the tire interaction with the surface.

3. The research has shown that the tire damping significantly depends on the shape of the road irregularities and loading conditions (vertical load, tire inflation pressure). Difference in the tire deformation when interacting with the cleats of cylindrical and rectangular shapes reaches 20 % at the tire inflation pressure 0.1 MPa (50 % of nominal pressure) and wheel load 3500 N (68 % of nominal load). Study of the influence of the shape of irregularities on the tire radial stiffness showed that the greatest vertical tire deflection occurs in the contact with the triangular cleats with a length of about 20% of the contact patch length. This vertical tire deflection may reach a value of 42 % of the tire deflection on the even surface.

4. Calculation results show that damping ratio is significantly influenced by the tire inflation pressure and shape of the irregularities. For example, decrease in the air pressure by half on a pit results in the decrease of the damping ratio by 18 %. For a peak (of the same length) the damping ratio decreases by 14 %.

5. The developed method for pneumatic tire enveloping properties estimation enables to obtain new values of the tire stiffness and damping properties that can be used in the dynamic analysis of the road – tire – car – driver mechanical system.

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