Quantum light transport in a nanophotonic waveguide coupled to an atomic ensemble

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Realizing efficient interactions between photons and multiple quantum emitters is an important ingredient in quantum optics and quantum information processing. Here we theoretically study the optical properties of an ensemble of two-level atoms coupled to a one-dimensional waveguide. In our model, the atoms are randomly located in the lattice sites along the one-dimensional waveguide. The results reveal that the optical transport properties of the atomic ensemble are influenced by the lattice constant and the filling factor of the lattice sites. We also focus on the atomic mirror configuration and quantify the effect of the inhomogeneous broadening in atomic resonant transition on the scattering spectrum. Since atoms are randomly placed in the lattice sites, we analyze the influence of the atomic spatial distributions on the transmission spectrum. Furthermore, we find that initial bunching and quantum beats appear in photon-photon correlation function of the transmitted field, which are significantly changed by filling factor of the lattice sites. With great progress to interface quantum emitters with nanophotonics, our results should be experimentally realizable in the near future.

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I. INTRODUCTION

In the past decades, waveguide quantum electrodynamics (QED) has raised great interest owing to its promising applications in quantum devices and quantum information technologies [1–17]. Waveguide QED describes interaction phenomena between electromagnetic fields confined to a one-dimensional (1D) waveguide and nearby quantum emitters. In practice, the waveguide can be realized with a number of physical systems such as surface plasmon nanowire [18–21], diamond waveguide [22,23], optical nanofiber [26–45], photonic crystal [46–74], and superconducting microwave transmission line [63,74]. Recently, photon transport in a 1D waveguide coupled to quantum emitters has been widely studied both in theory [1,3,5,13,53,60,66,69] and experiment [12,23,26,27,33,36,51,54,64,65].

With the real space description of the Hamiltonian and the Bethe-ansatz approach, Shen and Fan studied the transport properties of single photon scattered by a single emitter coupled to a 1D waveguide [1,68]. They found that, a photon with frequency resonant to the two-level emitter can be completely reflected, which arises from destructive quantum interference. Then, some other methods were used to study the photon transport in a 1D waveguide, such as the input-output theory [3], the time-dependent theory [75] and Lippmann-Schwinger scattering approach [76]. Besides the single-emitter case, single-photon scattering by multiple emitters has also been studied, which gives rise to much richer optical behaviors due to multiple scattering effects. Kien et al. calculated the spontaneous emission from a pair of two-level atoms near a nanofiber, where a substantial radiative exchange between distant atoms was demonstrated [77]. Later, Tsoi and Law studied the interaction between a single photon and a chain of N equally spaced two-level atoms in a 1D waveguide [78]. In contrast to the single-atom case, they found that a photon can be perfectly transmitted near the resonance atomic frequency, and the positions of transmission peaks and their widths are sensitive to the relative position between atoms. Moreover, Chang et al. showed that an ensemble of periodically arranged two-level atoms with a specific lattice constant can form an effective cavity within the nanofiber [79].

Motivated by these important works, we here focus on the optical properties of an ensemble of two-level atoms coupled to a 1D waveguide. Due to atomic collisions during the loading process, each lattice trap site surrounding a 1D waveguide contains at most a single atom in current experiments [31,32]. Thus, we assume that the atoms in our system are randomly trapped in the lattice along the 1D waveguide, which is different from the above cases.

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where the atoms are arranged periodically. In this paper, we calculate the scattering properties of a weak coherent input field through an atomic chain and average over a large sample of atomic distributions.

In the present work, provided that the input field is monochromatic, we first study the scattering properties of a two-level atomic chain coupled to a 1D nanophotonic waveguide. The results show that the transport properties are influenced by the lattice constant and the filling factor of the lattice sites. We calculate the optical depth as a function of the lattice constant, concluding that different choices of the lattice constant do not qualitatively change the optical depth, excluding those close to $m\pi/k_n$ ($m$ is an integer and $k_n$ is the associated wave vector). We then focus on the atomic mirror configuration and give the reflection spectra of the incident field with different choices of the filling factors of the lattice sites. Besides, we analyze the effect of the inhomogeneous broadening in atomic resonant transition on the scattering spectrum of the input field. Moreover, since atoms are randomly located in the lattice sites, we analyze the influence of the atomic spatial distributions on the transmission of the input field. We find that the influence of atomic spatial distributions on the transmission varies with the frequency detuning. Finally, we calculate the second-order correlation function of the transmitted field with different choices of the filling factors of the lattice sites. We find that quantum beats (oscillations) appear in photon-photon correlation function of the transmitted field. Moreover, when we increase the filling factor of the lattice sites, quantum beat lasts longer. Therefore, the filling factor of the lattice sites provides an efficient way to modify the quantum beats in the second-order correlation function of the transmitted field.

We have organized this article as follows: In Sec. II, we introduce the physics for an atomic chain coupled to a 1D waveguide, and present the derivation of an effective Hamiltonian for the system. In Sec. III, we study the scattering properties of a weak coherent input field through the atomic chain. Moreover, we analyze the transmission variance caused by atomic positions and compute the photon-photon correlation function of the transmitted field in the resonant case. Finally, we discuss the feasibility of our model with current experimental technology, and summarize the results in Sec. IV.

II. MODEL SYSTEM

In this section, we consider a system comprising an ensemble of two-level atoms spaced along a 1D waveguide, as shown in Fig. 1. Each atom has two electronic levels, i.e., the ground state $|g\rangle$ and the excited state $|e\rangle$. We assume that the transition with the resonance frequency $\omega_n$ between states $|g\rangle$ and $|e\rangle$ is coupled to the guided modes of the 1D dielectric waveguide. By generating an optical lattice external to the waveguide, the atoms can be trapped in fixed positions [22, 23]. Here, the frequency $\omega_n$ is assumed to be away from the waveguide cut-off frequency so that the left- and right-propagating fields can be treated as completely separate quantum fields [11, 12]. Under rotating wave approximation, the Hamiltonian of the system in real space is given by ($\hbar=1$) [11]

$$H = \sum_{j=1}^{n} \omega_n \sigma_{ee}^j + iv_g \int dz \left[ a_L^\dagger(z) \frac{\partial a_L(z)}{\partial z} - a_R^\dagger(z) \frac{\partial a_R(z)}{\partial z} \right]$$

$$-\delta \int dz \sum_{j=1}^{n} \delta(z-z_j) \left\{ \sigma_{ee}^j [a_R(z)+a_L(z)] + H.c. \right\},$$

(1)

where $v_g$ is the group velocity of the field, $z_j$ represents the position of the atom $j$, and $a_L$ ($a_R$) denotes the annihilation operator of left (right) propagating field. The coupling constant $\delta = \sqrt{2\pi g}$ is assumed to be identical for all modes, where $g$ denotes the single-atom coupling strength to waveguide modes. The atomic operators $\sigma_{\alpha\beta} = |\alpha_j\rangle\langle\beta_j|$, where $\alpha, \beta = g, e$ are energy eigenstates of the $j$th atom. $n$ is the number of the atoms trapped along the waveguide.

The Heisenberg equation of the motion for the atomic operator is

$$\dot{\sigma}_{ge}^j = -i\omega_n \sigma_{ge}^j + i\delta(\sigma_{ee}^j - \sigma_{ee}^j) [a_R(z_j)+a_L(z_j)].$$

(2)

Likewise, we can also obtain the Heisenberg equations of motions for left and right propagating fields in the...
waveguide
\[
\left(\frac{1}{v_g}\frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right) a_L(z) = \frac{i\dot{g}}{v_g} \sum_{j=1}^{n} \delta(z - z_j)\sigma_j^L e_i,
\]
\[
\left(\frac{1}{v_g}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right) a_R(z) = \frac{i\dot{g}}{v_g} \sum_{j=1}^{n} \delta(z - z_j)\sigma_j^R e_i.
\]

(3)

Then, we transform them to a co-moving frame with coordinates \(z' = z, t' = t - z/v_g\), and get the equation of motion for \(a_R\)
\[
\frac{\partial}{\partial z'} a_R(z') = \frac{i\dot{g}}{v_g} \sum_{j=1}^{n} \delta(z' - z_j)\sigma_j^R(t').
\]

(4)

Integrating over \(z' \in [z - v_g t, z]\), we obtain
\[
a_R(z, t) - a_R(z - v_g t) = \frac{i\dot{g}}{v_g} \sum_{j=1}^{n} \int dz' \delta(z' - z_j)\sigma_j^R(t').
\]

(5)

Since the contribution from a time earlier than \(z - v_g t\) is zero, the lower limit of the integral on the right hand side of Eq. (5) can be extended to \(-\infty\). We then get
\[
a_R(z, t) = a_{R, in}(z - v_g t) + \frac{i\dot{g}}{v_g} \sum_{j=1}^{n} \theta(z_j - z)\sigma_j^R(t - z_j/z/v_g).
\]

(6)

Here \(\theta\) represents the Heaviside step function, and \(a_{R, in}(z - v_g t)\) denotes the input field which evolves from the initial time to the present without interacting with the atoms. Similarly, the operator \(a_L(z, t)\) for the left-moving field is written as
\[
a_L(z, t) = a_{L, in}(z + v_g t) + \frac{i\dot{g}}{v_g} \sum_{j=1}^{n} \theta(z_j - z)\sigma_j^L(t - z_j/z/v_g).
\]

(7)

Then, we insert Eq. (6) and Eq. (7) into Eq. (2) and get the evolution of the atomic coherence
\[
\dot{\sigma}_j^{ge} = -i\omega_a \sigma_j^{ge} - \frac{g^2}{v_g} (\sigma_j^{ge} - \sigma_j^{ee}) \sum_i \sigma_i^{L} (t - |z_j - z|/v_g).
\]

(8)

By defining new operators \(S_j^{ge}\) via \(\sigma_j^{ge} = S_j^{ge} e^{-i\omega_a t}\), we transform Eq. (5) into a slow-varying frame. Here \(\omega_a\) denotes the frequency of an external driving field, which is close to the atomic resonance frequency \(\omega_a\) with wave vector \(k_a\). Thus, we find the equation of motion
\[
\dot{S}_j^{ge}(t) = i\Delta S_j^{ge}(t) - \frac{\Gamma_0}{2} [S_j^{ss}(t) - S_j^{ee}(t)]
\]
\[
\times \sum_{l} S_l^{ge}(t - |z_j - z_l|/v_g) e^{ik_{in}|z_j - z_l|},
\]

(9)

where \(\Delta = \omega_{in} - \omega_a\), and \(k_{in} = \omega_{in}/v_g\). \(\Gamma_0 = 4\pi g^2/v_g\) denotes the single-atom spontaneous emission rate into waveguide modes. In fact, the operator on the right hand side of Eq. (9) can be expanded as
\[
S_j^{ge}(t - |z_j - z_l|/v_g) = S_j^{ge}(t) - \frac{1}{2} (|z_j - z_l|/v_g)^2
\]
\[
\times \ddot{S}_j^{ge}(t) + \cdots.
\]

(10)

For small separations, i.e., \(|z_j - z_l| \ll v_g\), the system is Markovian [50, 53]. In this case, the causal propagation time of the field between two atoms \(j\) and \(l\) can be neglected. Omitting higher order terms of Eq. (9), we get
\[
\dot{S}_j^{ge}(t) = -\frac{\Gamma_0}{2} [S_j^{ss}(t) - S_j^{ee}(t)] \sum_l S_l^{ge}(t) e^{i k_{in}|z_j - z_l|}
\]
\[
\times i\Delta S_j^{ge}(t).
\]

(11)

From the above equation, we can extract an effective Hamiltonian for our system
\[
H_{eff} = -\Delta \sum_j S_j^{ee} - i\frac{\Gamma_0}{2} \sum_{j,k=1}^{n} e^{ik_{in}|z_j - z_k|} S_j^{ge} S_k^{ge}.
\]

(12)

Considering the spontaneous emission of the excited state into free space, we can add an imaginary part \(-i\Gamma_{e}'/2\) to the energy of the excited state [83]. Thus, the atomic chain mediated by the 1D waveguide can be described by a non-Hermitian effective Hamiltonian [70, 83]
\[
H_{1} = -\Delta \sum_j (S_j^{ee} - i\frac{\Gamma_0}{2} \sum_{j,k=1}^{n} e^{ik_{in}|z_j - z_k|} S_j^{ge} S_k^{ge} e^{i\Delta z_{in}}),
\]

(13)

where \(\Gamma_{e}'\) denotes the decay rate of the state \(|e\rangle\) into free space.

In this work, we mainly study the scattering properties of a weak coherent input field. Then, the driving part is given by \(H_d = \sqrt{\frac{v_g}{\omega_{in}}} \mathcal{E} \sum_j (S_j^{ge} e^{ik_{in} z_j} + H.c.)\), with \(\mathcal{E}\) being the amplitude of the weak input field (Rabi frequency \(\sqrt{\frac{v_g}{\omega_{in}}} \mathcal{E}\)). Finally, the dynamics of the atomic ensemble is described by the Hamiltonian \(H = H_1 + H_d\).

Since the incident field is assumed to be sufficiently weak (\(\sqrt{\frac{v_g}{\omega_{in}}} \mathcal{E} \ll \Gamma_{e}'\)), we can neglect quantum jumps [55]. Initially, all atoms are prepared in the ground state \(|g\rangle\), and the weak coherent field is input from the left. Using input-output method [83], we obtain the transmitted \((t)\) and reflected \((r)\) fields
\[
a_t(z) = \mathcal{E} e^{ik_{in} z} + \sqrt{\frac{\Gamma_0}{2v_g}} \sum_{j=1}^{n} S_j^{ge} e^{ik_{in}(z-z_j)},
\]

(14)

\[
a_r(z) = i\sqrt{\frac{\Gamma_0}{2v_g}} \sum_{j=1}^{n} S_j^{ge} e^{-ik_{in}(z-z_j)}.
\]
weak input field are given by

\[ 0.8 \text{ (green dashed-dotted line), 1.0 (blue dotted line). Parameters: (a)-(d)} \]

Thus, the transmittance \( T \) and reflection \( R \) of the weak input field are given by

\[ T = \frac{\langle \psi | a_d^\dagger a_d | \psi \rangle}{E^2}, \quad R = \frac{\langle \psi | a_d^\dagger a_r | \psi \rangle}{E^2}, \quad (15) \]

where \( | \psi \rangle \) denotes the steady state of the atomic ensemble.

III. NUMERICAL RESULTS

A. Scattering properties of the input field

Here, provided that the incident field is monochromatic, we study the scattering properties of the weak input field with \( N = 100 \) equally spaced lattice sites along the 1D waveguide. We assume that either zero or one atom is trapped in each lattice site, and all sites are identical with a filling factor \( p \). In other words, \( n \) atoms are placed randomly over \( N \) sites with a filling factor \( p = n/N \) for each site. In Fig. 2 we show transmission spectra of the incident field as a function of the detuning \( \Delta/\Gamma_e \) with different values of lattice constant \( d \). For each lattice constant, we present the transmission spectra with four different filling factors, i.e., \( p = 0.4, 0.6, 0.8, 1.0 \). We find that, the transmission spectrum for \( k_0d = 0 \) is identical to that for \( k_0d = \pi \), and the transmission spectra for \( k_0d = 1 \) and \( k_0d = \pi/2 \) are the same. Moreover, after calculating many transmission spectra with different choices of lattice constant \( d \) (not shown), we conclude that different values of \( k_0d \) do not qualitatively influence the transmission properties, excluding those very close to \( m\pi \) (\( m \) is an integer). Besides, for any given lattice constant \( d \), when we increase the filling factor, the lineshapes of the transmission spectra exhibit significant broadening, as shown in Fig. 2. This is because, the collective decay rates of the atoms into the waveguide modes become enhanced when the filling factor of the lattice sites rises. Different from the Lorentzian line shape in the transmission spectrum of the single two-level atom case [1], we find that the transmission for an atomic array is approximately zero in a window centered at \( \Delta = 0 \) for the cases \( k_0d = \pi/2 \) and \( k_0d = 1.0 \), as shown in Figs. 2b, 2c.

The opacity of a medium is described by the optical depth \( D \), where \( T(\Delta = 0) = e^{-D} \). In Fig. 3a, we calculate the optical depth as a function of \( k_0d \) with a filling factor \( p = 0.5 \). We find that different choices of \( k_0d \) do not qualitatively change the optical depth, excluding those close to \( m\pi \). This is consistent with the conclusion about the influence of the lattice constant on
a filling factor $p = 0.5$. We find that, the reflection of the input field in the case $k_a d = m \pi$ is much larger than those for $k_a d \neq m \pi$. Especially, for the case $k_a d = m \pi$, we find that an array of $N$ atoms is equivalent to an effective ‘superatom’ with $N$ times the coupling strength to the 1D waveguide (not shown). Thus it is possible to use such an atomic ensemble to compensate for the fact that the ratio $\Gamma_\theta / \Gamma_e'$ is small for an individual atom and then the input field is strongly reflected. Moreover, in Fig. 4(c), we calculate the optical depth $D$ as a function of the filling factor $p$ for three choices of the lattice constant, i.e., $k_a d = 1, \pi/2, \pi$. The results show that, for the case $k_a d \neq m \pi$, such as $k_a d = 1$ and $k_a d = \pi/2$, the optical depth is almost not influenced by the choices of the lattice constant $d$. We also observe that the optical depth scales linearly with the filling factor $p$ for the case $k_a d \neq m \pi$. While for the case $k_a d = m \pi$, such as $k_a d = \pi$, the optical depth changes slowly with the filling factor of the lattice sites.

To proceed, we focus on the atomic mirror configuration, e.g., the case $k_a d = \pi$. In Fig. 4(a), we give reflection spectra of the incident field as a function of the detuning $\Delta / \Gamma_e'$ with four choices of filling factor. We find that a high filling factor results in an increase in the reflection of the input field. In fact, for a resonant input field, a large number of periodically arranged atoms with a lattice constant $d = m \pi / k_a$ can be regarded as an atomic Bragg mirror \cite{79, 85}. This effect is not sensitive to the filling imperfections and depends mainly on the choice of the lattice constant. Moreover, in the resonant case $\Delta = 0$, as shown in Fig. 4(b), the reflection of the atomic Bragg mirror is enhanced by large number $N$ of lattice sites. In fact, for a modest filling factor, when the number of the lattice sites is sufficiently large, the reflection of the atomic Bragg mirror will approach to 100%. For example, with the filling factor being 0.6, the reflection of the atomic chain in the resonant case $\Delta = 0$ is 98.6% when the number of the lattice sites is 1000 \cite{35, 36}. In particular, two sets of such atomic Bragg mirrors can form a cavity for an atom located between them, which is shown in Fig. 4(d). Here, the distance $d_0$ between the central atom and the nearest neighbors in the atomic mirrors satisfies the condition $k_a d_0 = 1.5 \pi$, such that the central atom is located at the atomic cavity anti-node to maximize the coupling. As shown in Fig. 4(c), we calculate the population $p_e$ of an initially excited atom inside an atomic cavity with four choices of the filling factors. The results reveal that vacuum Rabi oscillations occur between the excited central atom and the atomic cavity. Moreover, the higher the filling factor of the lattice sites is, the stronger the vacuum Rabi oscillation becomes. This is because the reflection of the atomic cavity rises when we increase the filling factor, which is shown in Fig. 4(a).

In the above discussions, we assume that these two-level atoms trapped in the lattice are the same. While, due to strong trap light fields, inhomogeneous broadening of atomic transitions exists in practical experi-
for a fixed filling factor, when the standard deviation \( \sigma \) of the input field for the case of the transmission transition. Besides, in Fig. 6, we also calculate transmission spectra of the incident field as a function of the detuning \( \Delta / \Gamma_e \) for \( \sigma_{ih} = 0 \) (black solid line), \( \sigma_{ih} = 1.0 \Gamma_e \) (blue dashed line), \( \sigma_{ih} = 2.0 \Gamma_e \) (green dashed-dotted line), \( \sigma_{ih} = 3.0 \Gamma_e \) (red dotted line) with filling factors (a) \( p = 0.4 \), (b) \( p = 0.6 \), (c) \( p = 0.8 \), (d) \( p = 1.0 \). Parameters: (a)-(d) \( \mathcal{E} = 10^{-4} \sqrt{\frac{1}{2 \pi \sigma}} \), \( k_0 d = \pi, \Gamma_0 = 0.3 \Gamma_e, N = 100 \).

Here, we consider the inhomogeneous broadening by assigning a random Gaussian distributed detuning \( \Delta_{ih} \) with a constant deviation \( \sigma_{ih} \) to each atom. The probability density is \( \rho_{ih}(\Delta_{ih}) = \frac{1}{\sigma_{ih} \sqrt{2 \pi}} \exp(-\frac{\Delta_{ih}^2}{2 \sigma_{ih}^2}) \). As shown in Fig. 5, we show the influence of Gaussian inhomogeneous broadening of atomic transitions on the reflection of atomic Bragg mirror, e.g., the case \( k_0 d = \pi \). We find that, for a fixed filling factor, when the standard deviation \( \sigma_{ih} \) increases, the reflection in a region of the frequency detuning around \( \Delta = 0 \) decreases. That is, the reflection of the atomic cavity shown in Fig. 4(d) is weakened by the inhomogeneous broadening of atomic transitions. The results reveal that, for \( |\Delta| \gg \Gamma_e \), the reflection of the atomic Bragg mirror is almost robust to the effect of the inhomogeneous broadening of the atomic transition. Besides, in Fig. 5(b) also we calculate transmission spectra of the input field for the case \( k_0 d = \pi / 2 \) with four filling factors for \( \sigma_{ih} = 0, 2 \Gamma_e, 3 \Gamma_e \). We find that, for a fixed filling factor, when the standard deviation \( \sigma_{ih} \) is changed from 0 to 3.0 \( \Gamma_e \), lineshapes of the transmission spectra exhibit significant broadening. Moreover, for a fixed \( \sigma_{ih} \), the higher the filling factor of the lattice sites is, the broader the line shape of the transmission spectrum becomes. In particular, with \( p = 0.4 \), when the standard deviation \( \sigma_{ih} \) is sufficiently large, e.g., \( \sigma_{ih} = 3.0 \Gamma_e \), a window occurs centered on \( \Delta = 0 \) where the transmittance is no longer approximately zero.

### B. Transmission variance caused by atomic positions

In the present work, atoms are randomly located at these lattice sites along the 1D waveguide. We thus study the mean optical properties, averaging transmissions and reflections of the input field over many random spatial configurations. To calculate the influence of the atomic spatial distributions on the transmission of the input field, we define the variance \( s^2 \) as

\[
s^2 = \frac{1}{q} \sum_{i=1}^{q} (T_i - \bar{T})^2.
\]

Here, \( q \) represents the sample size of the atomic spatial configurations, \( T_i \) denotes the transmission of the input field for the \( i \)th sample, and \( \bar{T} \) is the average transmission for all samples.

In Fig. 6 we give the variance \( s^2 \) of the transmission in two cases, i.e., \( k_0 d = m \pi \) (e.g., \( k_0 d = \pi \)) and \( k_0 d \neq m \pi \) (e.g., \( k_0 d = \pi / 2 \)), where \( m \) is an integer. For each case, we average over 1000 samples of atomic spatial distributions with four filling factors, i.e., \( p = 0.2, 0.4, 0.6, 0.8 \). For the two cases \( k_0 d = m \pi \) and \( k_0 d \neq m \pi \), the similarities of the variance \( s^2 \) are: (i) the plot is symmetric for the frequency detuning, which is consistent with the transmission spectra shown in Fig. 2 (ii) there are two peaks around the resonance frequency \( \Delta = 0 \). When the detuning is at the positions of the peaks, the effect of the atomic spatial configurations on the transmission is the most obvious; (iii) for a large detuning, the variance \( s^2 \) approaches zero. In other words, the input photon transmits the atomic chain with almost no interaction for any random spatial configuration when \( \Delta \gg \Gamma_e \). Different, in the case \( k_0 d \neq m \pi \), the results show that the variance \( s^2 \) is approximately zero in a region of the frequency detuning around \( \Delta = 0 \). This phenomenon is consistent with the transmission spectra shown in Figs. 2(b) and 2(c), where the transmission of the input field is approximately zero in a window centred on \( \Delta = 0 \). That
is, for any given atomic spatial configuration in the case $k_a d \neq m \pi$, no transmission occurs in the region around $\Delta = 0$. Besides, we also observe that the positions of the two peaks in the case $k_a d = m \pi$ are more sensitive to the filling factor of the lattice sites than that in the case $k_a d \neq m \pi$.

### C. Two-photon correlation

Correlation between photons is a main feature of non-classical light, which are characterized by photon-photon correlation function (second-order correlation function) $g^{(2)}(t)$ [86]. For a weak coherent state in our system, the photon-photon correlation function $g^{(2)}$ of the output field is defined as

$$g^{(2)}_{\alpha}(t)=\frac{\langle |\psi\rangle|a^\dagger_\alpha(z)e^{iHt}a^\dagger_\alpha(z)a_\alpha(z)\rangle e^{-iHt}a_\alpha(z)\langle |\psi\rangle|}{\langle |\psi\rangle|a^\dagger_\alpha(z)a_\alpha(z)\rangle^2}. \quad (17)$$

Here, $|\psi\rangle$ is the steady-state wave vector and $\alpha = T, R$.

Now, with a weak input field ($\sqrt{\frac{P_{\text{input}}}{v^2}} \ll \Gamma'$), we discuss the photon correlation function of the output field in the resonant case $\Delta = 0$. In Fig. 8 we give the photon correlation function of the transmitted field with four choices of the filling factor, i.e., $p = 0.1, 0.2, 0.3, 0.4$. The results show that strong initial bunching appears in the transmitted field in each case, i.e., $g^{(2)}_{\gamma}(t = 0) \gg 1$.

When we increase the filling factor, the initial bunching becomes much stronger. Furthermore, we find quantum beats in the photon-photon correlation function of the transmitted field. Evidently, the higher the filling factor of the lattice sites is, the more visible the quantum beat becomes. By comparing these four cases shown in Fig. 8 we find that quantum beat in the photon-photon correlation function $g^{(2)}_{\gamma}$ lasts longer when we increase the filling factor. The phenomena mentioned above reveal that many-body quantum systems significantly modify nonclassical property of light in the waveguide.

### IV. DISCUSSION AND SUMMARY

Experimentally, our model may be realized in the current nanofiber system. In Refs. [33, 36], arrays of cesium atoms are trapped in the evanescent field of a tapered optical fiber. For a cesium atom, the ground and excited states are chosen as $|g\rangle = \{6S_1/2, F = 4\}$ and $|e\rangle = \{6P_3/2, F = 5\}$, respectively. The optical lattice for trapping atoms can be constructed by a pair of horizontally polarized red-detuned counterpropagating beams (wavelength $\lambda_{\text{trap}} = 1057$ nm and power $P_{\text{trap}} \approx 2 \times 1.3$ mW) and a vertically polarized blue-detuned beam (wavelength $\lambda_{\text{blue}} = 780$ nm and power $P_{\text{blue}} = 14$ mW). In their experiments, cesium atoms are first loaded from a background vapor to a 6-beam magneto-optical trap, and then they are loaded into an optical lattice via sub-Doppler cooling. Because of atomic collisions during the loading process [51], each trap site in their device hosts at most a single atom, which is consistent with the assumption in the present work. In Ref. [46] to avoid saturation, experimentalists adopt an extremely weak probe field with a power of $P_{\text{input}} \approx 150$ pW. Finally, with the techniques mentioned above, it is able to trap thousands of atoms in the lattice sites along the 1D waveguide as in Refs. [33, 36].
In conclusion, in this work we study scattering properties of an ensemble of two-level atoms coupled to a 1D waveguide. Since the precise control of the atomic positions is still challenging in nanophotonic waveguide system, we assume that the atoms in this work are randomly placed in the lattice along the 1D waveguide. With the effective non-Hermitian Hamiltonian, we calculate the transmission spectrum of a weak coherent input field, concluding that the optical transport properties are influenced by lattice constant and the filling factor of the lattice sites. We compute the optical depth as a function of the lattice constant, and the results reveal that the optical depth is reduced when lattice constant is close to \( n \pi/k_a \). We then focus on the atomic mirror configuration and give the reflection spectra of the incident field with different filling factors of the lattice sites. We also quantify the influence of the inhomogeneous broadening in atomic resonant transition on the transmission, and find that the lineshape of the transmission spectrum exhibits significant broadening when the standard deviation \( \sigma_n \) becomes larger. Besides, we find that the transmission variance, caused by atomic spatial distributions, for the atom mirror configuration \( (k_ad = m\pi) \) is distinct from other cases. Finally, we analyze the role of filling factor played in photon-photon correlation of the transmitted field, and find that initial bunching and quantum beats are quite sensitive to the filling factor. Since great progress has been made to interface quantum emitters with nanophotonic waveguide [5], our results in this work should be realizable in the near future.

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