On equivalence of equations solutions of gravity field and homogenous inertia field

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The metric of a homogenously accelerated system found by Harry Lass is a solution of the Einstein’s equation. The metric of an isotropic homogenous field must satisfy the new gravitational equation.

1. Introduction

In 1907, A. Einstein established the equivalence principle [1]. It reads as follows: “This assumption makes the relativity principle applicable to the case of uniformly accelerated straight-line motion of a reference system. A heuristic value of this assumption consists in the fact that it allows replacing a homogenous gravity field with a uniformly accelerated reference system, which – in some ways – can be considered theoretically.”

Later, Einstein found the following ratio between own time of an accelerated system and time of an inertia system

\[ d\tau = \left(1 - \frac{g}{c^2}\right) dt. \quad (1) \]

Here the potential \( \varphi = -gx \), where \( g \) in its turn is acceleration. In the same paper [1] from symmetry considerations, Einstein obtained the exact ratio for those times

\[ d\tau = \exp \left( \frac{gX}{c^2} \right) dt. \quad (2) \]

From here, one might jump to the conclusion that for the equivalence principle verification in frame of the general relativity theory, sufficient would be just comparison between a homogenous gravity field and a homogenous inertia field by way of their metrics comparison. This is exactly what V A Fok [2] attempted to do. However over the approximate solution of the Einstein’s equation [2], Fok preferred another expression and as a homogenous gravity field with a potential \( \varphi \), he chose one with the following metric

\[ ds^2 = \left(1 - \frac{2\varphi}{c^2}\right)c^2 dt^2 - \left(1 + \frac{2\varphi}{c^2}\right)(dx^2 + dy^2 + dz^2). \quad (3) \]

As an inertia field, the Moeller’s reference system was used [3]

\[ ds^2 = \left(1 + \frac{gX}{c^2}\right)^2 c^2 dt^2 - (dx^2 + dy^2 + dz^2). \quad (4) \]

However neither this nor that metric is homogenous. More to it, the metric (3) has non-zero Riemannian tensor, while the metric (4) describes a plane space.

So the Fok’s speculations turned to be unconvincing. Nevertheless, this point of view has gained widespread. Usually, the following seemingly irrefutable statement is given: in the pseudo-
Euclidean space, bodies’ motion is described by the special relativity theory and hence a reference system connected with с these bodies is plane. Nevertheless, this statement corresponds to our intuitive concept of motion and nothing more.

Anyhow, the Fok’s paper has triggered doubts and discussions. Among other opinion exchanges, there came into the brightest spotlight the discussion held by A A Logunov and his co-authors versus V L Ginzburg and Yu N Yaroshenko [4, 5 and 6]. The issue at stake was the question of irradiation of an accelerated charge. Along with it, Ginzburg and Yaroshenko considered the charge in the Moeller’s system, which was asymptotically homogenous in the limit of small scales. That is why their conclusion on a charge irradiation absence in a homogeneously accelerated system (4) can be considered to be correct.

2. Harry Lass’s solution

Meanwhile, the one-dimensional homogenous accelerated system was described by Harry Lass the mathematician as early as in 1963 [7]. Notably, as a reason for this problem solving, the famous Einstein’s speculation served – the clock (twin) paradox.

Having regard to the light velocity invariance, Lass deducted firstly the Lorentz-Einstein’s transformation (the conversion of one inertia system into another) and then conversion between homogenous reference systems, one of which was an inertia one and second non-inertia one. It is worthy to cite this transformation here as well as the metric of the homogenous system.

\[ x = \frac{c^2}{g} [e^{gX/c^2} \cosh(gT/c) - 1] \]

and

\[ t = \frac{(c/g)}{e^{gX/c^2}} \sinh(gT/c). \]

It should be noted that

\[ x \approx X + \frac{1}{2} gT^2 \]

for \( \frac{gT}{c} \ll 1, \frac{gX}{c^2} \ll 1, \ t \approx T \)

\[ \lim_{T \to \infty} \left( \frac{dx}{dt} \right)_{X=\text{const.}} = c. \]

From the equation (6), it follows that

\[ ds^2 = c^2 t^2 - dx^2 - dy^2 - dz^2 = e^{\frac{2gX}{c^2}} [c^2 T^2 - dX^2] - dy^2 - dZ^2 \]

\[ = g_{\alpha \beta}(X) dX^\alpha dX^\beta \]

and \( y = Y, z = Z \). At a reference system acceleration beginning, its exact time is imposed like \( ds/c = dT \); but in the general case, it is given by the expression \( ds/c = e^{gX/c^2} dT \), which shows that a clock run velocity is a function of a clock location.
It is necessary to be noted that the last Lass’s ratio connecting time in an accelerated system with time in an inertia system coincides with the Einstein’s expression (2) perfectly.

Indeed, the Lass’s metric is homogenous as the acceleration of this system is equal to

$$\frac{d^2X}{dT^2} = c^2 \Gamma_{00} = g = \text{const.}$$

(8)

The Lass’s metric of a homogenously accelerated system satisfies to the Einstein’s equation for empty space ($G_{ik} = 0$). From this, it follows that the metric of non-inertia homogenous system is equivalent to some metric of the general relativity theory, which should be considered by us to be the homogenous field metric.

3. **Isotropic homogenous metric**

The metric (7) is anisotropic. This does not correspond to our ideas on the space-time. Well, it is possible to use the freedom of choosing of the components $g_{yy}$ and $g_{zz}$ of the Lass’s metric tensor. It allows finding out, how the metric would look like of isotropic and homogenous field [8], no matter of inertial one or gravitational one:

$$ds^2 = e^{\frac{2gx}{c^2}}(c^2t^2 - dx^2 - dy^2 - dz^2).$$

(9)

Also the isotropism of this metric is confirmed by the equity (8). This metric is not only isotropic; in addition here, the light velocity is invariant, too (see the Figure). It is excellent that unlike the Lass’s metric (7), its Einstein’s tensor is non-zero. By using the proportionality of the Einstein’s tensor and the energy-momentum tensor, it is possible to calculate the energy-momentum tensor of a homogenous field as follows:

$$\tau_{ik} = \frac{g_{1}^2}{8\pi G} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$ 

It is important that the energy density of the field $\tau_{00}$ in this expression coincides with the density of a gravity field in the Newtonian limit [9].

However, this contradicts to the Einstein’s equation.

It is possible to refuse the Einstein’s equation$^1$. Upon that, it would be possible to solve some problems without recourse to the equation of gravity field [10]. The new solution of gravity field is represented in a form of a metric as follows:

$$ds^2 = \exp^{\frac{2h}{c^2}} \eta_{ik} dx_i dx_k.$$ 

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$^1$ In the course of the Einstein’s equation deduction, the supposition was made about energy smallness of a gravity field (see [10], p. 95. Einstein’s equations).
where $\eta_{ik}$ is a linear element of plane space. Notably, the scalar parameter $\hbar$ is found from the covariant equation

$$\Box \hbar = -4\pi GT. \quad (1)$$

The alternative equation of gravity field [11] gives quite satisfied results notwithstanding that in the area of very strong fields, solutions differ from the solutions of the Einstein’s equation.

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Figure 1. Changes of space-time scales along a geodesic line of a homogeneous isotropic field