Giant monopole resonance and nuclear compression modulus for

$^{40}$Ca and $^{16}$O

D. Galetti

Instituto de Física Teórica, Universidade Estadual Paulista -
UNESP
Rua Pamplona 145
01405 - 900 São Paulo, S.P.; Brazil

A.F.R. de Toledo Piza

Instituto de Física, Universidade de São Paulo
C.P. 66318, 05315-970, São Paulo, S.P.; Brazil

Abstract

Using a collective potential derived on the basis of the Generator Coordinate Method with Skyrme interactions we obtain values for the compression modulus of $^{40}$Ca which are in good agreement with a recently obtained experimental value. Calculated values for the compression modulus for $^{16}$O are also given. The procedure involved in the derivation of the collective potential is briefly reviewed and discussed.

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I. INTRODUCTION.

Among the collective vibrational modes of nuclei, the isoscalar giant monopole resonance (GMR), also called breathing mode, has received considerable attention due to the fact that its energy is related to the nuclear compression modulus. This property corresponds, in the limit of an infinite number of nucleons, to the nuclear matter incompressibility, which is of great interest in areas such as heavy ion collisions and nuclear astrophysics. The states corresponding to this type of collective motion have long been experimentally identified [1] and various microscopic theoretical approaches have been developed in order to account for and improve the deduced values of the compression modulus [2–5]. In recent years, more accurate experimental determination of the GMR energies [6] led to renewed interest in the determination of the nuclear matter incompressibility. Widely ranged theoretical approaches have been employed to obtain the compression modulus of finite nuclei and of nuclear matter, including microscopic calculations with the Gogny interaction [7] and with generalized Skyrme forces [8], self-consistent Hartree-Fock plus Random Phase Approximation treatments with Skyrme interactions [9], relativistic mean field calculations [10], a Thomas-Fermi [11] and a Fermi liquid drop calculations [12].

Motivated by the recent experimental results [6], and by the discussion of the compression modulus in finite nuclei presented in [13], we report on a microscopic calculation of the compression modulus for $^{40}$Ca and $^{16}$O based on the Generator Coordinate Method (GCM) [14–16]. This method allows for the microscopic determination of a collective potential $V(q)$, $q$ being the appropriate collective coordinate for the breathing mode, from which the compression modulus can be calculated directly as

$$K = \frac{1}{A} \frac{d^2}{dq^2} V(q) \bigg|_{q=q_0},$$

where $A$ is the nuclear mass number and $q_0$ corresponds to the minimum of $V(q)$. This approach has been developed in its general aspects many years ago [17] and can be directly used in the present context. We show in continuation that the collective potential does not
coincide with the diagonal part of the GCM energy kernel due to non-negligible off-diagonal contributions; moreover, the correlations embodied in the choice of the generator coordinate introduce anharmonicities in the collective potential, with the result that the deduced compression modulus comes out higher than that obtained from the standard variational approach.

In our calculations we have used Skyrme interactions without Coulomb forces. Values for the compression modulus were obtained for $^{40}$Ca and for $^{16}$O. In the case of the heavier nucleus there is good agreement with the available experimental data especially when the Skyrme interactions SII and SIII are used. The GCM results also give, in the case of the lighter nucleus, values for the compression modulus which are only moderately lower than the values obtained for the heavier nucleus. Skyrme forces without density dependence such as SV lead to values for the compression modulus which are appreciably lower in both cases.

In section II the GCM and its connection with collective Hamiltonians is briefly reviewed. In section III the collective potential for the breathing mode and the microscopically deduced expression for the compression modulus are derived. Values of the incompressibility are calculated for five sets of Skyrme parameters. Finally, in section IV we present our conclusions.

II. GCM KERNELS AND COLLECTIVE HAMILTONIANS.

The well known Generator Coordinate Method is based on an ansatz for the collective wave function which is set up as

$$|\Psi(\tilde{r})\rangle = \int f(\alpha) \Phi(\tilde{r}, \alpha) \, d\alpha = \int f(\alpha) |\alpha\rangle \, d\alpha,$$

where the $\{\Phi(\tilde{r}, \alpha)\}$ (or, for short, the $\{|\alpha\rangle\}$) constitute a set of auxiliary nuclear many-body wave functions parametrized by $\alpha$ (the generator coordinate), related to the nuclear degree of freedom one wishes to describe, $f(\alpha)$ is a weight function to be determined variationally and $\tilde{r}$ is a shorthand notation for all the nucleon spatial coordinates. One way of constructing the
\( \Phi (\vec{r}, \alpha) \) is through the use of Slater determinants of the lowest single particle eigenfunctions \( \varphi (\vec{r}; \alpha) \) of an ad hoc \( \alpha \)-dependent auxiliary potential. The choice of the \( \alpha \)-dependence of the auxiliary potential should therefore suit the particular collective degree of freedom under consideration (e.g. an overall scale parameter in the case of the breathing mode).

For a given nucleon-nucleon interaction, we can write the nuclear many-body Hamiltonian \( H \) and use the Rayleigh-Ritz variational principle to obtain the so-called Griffin-Wheeler equation \( [15] \) for the weight function \( f(\alpha) \)

\[
\int \left[ \langle \alpha | H | \alpha' \rangle - E \langle \alpha | \alpha' \rangle \right] f(\alpha') d\alpha' = 0, \tag{1}
\]

where \( E \) is introduced as a Lagrange multiplier to account for wave function normalization. In this equation the energy kernel \( \langle \alpha | H | \alpha' \rangle \) and the overlap kernel \( \langle \alpha | \alpha' \rangle \) are functions of \( \alpha \) and \( \alpha' \) only, the nucleon coordinates having been integrated out.

In principle, the solutions of the integral eigenvalue equation (1) give a variational ground state and a set of collective excited states expressed in terms of weight functions \( f_i(\alpha) \) and the corresponding eigenvalues \( E_i \). Technical problems related to the overlap properties of the auxiliary many-body functions \( |\alpha\rangle \) in many cases prevent the implementation of such a direct approach, however. A discussion of these difficulties and of general methods to circumvent them can be found in [18,19], where it is shown that by applying general transformations to the Griffin-Wheeler equation the determination of the collective spectrum can be reduced to a standard eigenvalue problem.

A convenient choice for the nucleon-nucleon interaction is the Skyrme force [20], which contains two body and three body terms. The two-body term is

\[
v_{1,2} (\vec{r}_1, \vec{r}_2) = t_0 (1 + x_0 P_{\sigma}) \delta (\vec{r}_1 - \vec{r}_2)
+ \frac{t_1}{2} \left[ \delta (\vec{r}_1 - \vec{r}_2) \vec{k}^2 + \vec{k'}^2 \delta (\vec{r}_1 - \vec{r}_2) \right]
+ t_2 \vec{k'} \cdot \delta (\vec{r}_1 - \vec{r}_2) \vec{k}
+ i W_0 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k'} \times \delta (\vec{r}_1 - \vec{r}_2) \vec{k},
\]

where \( t_0, x_0, t_1, t_2 \) and \( W_0 \) are adjusted parameters, and \( P_{\sigma} \) is the spin exchange operator.
The three-body part of the Skyrme force is simply taken as a zero range Wigner force

\[ v_{1,2,3} (\vec{r}_1, \vec{r}_2, \vec{r}_3) = t_3 \delta (\vec{r}_1 - \vec{r}_2) \delta (\vec{r}_2 - \vec{r}_3), \]

where \( t_3 \) is an additional parameter. For even nuclei this contribution can be rewritten as the density dependent two-body interaction

\[ v_{1,2,3} (\vec{r}_1, \vec{r}_2) = \frac{t_3}{6} (1 + P_\sigma) \delta (\vec{r}_1 - \vec{r}_2) \rho \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right). \]

Since we are interested in the monopole giant resonance spectrum of spherical nuclei, we choose an oscillator potential with the oscillator parameter \( \beta \) as generator coordinate, which is thus directly related to the nuclear radius. The auxiliary many-body functions \(|\beta\rangle\) will therefore be Slater determinants built from single particle oscillator states \( \{\varphi_\lambda (\vec{r}; \beta)\} \) with oscillator parameter \( \beta \). The corresponding overlap kernel then reads

\[
\langle \beta | \beta' \rangle = \left( \frac{2 \beta \beta'}{\beta + \beta'} \right)^T,
\]

where \( T \) is 6, 36 and 120 for \(^4\text{He}, ^{16}\text{O}\) and \(^{40}\text{Ca}\) respectively. In order to handle this overlap kernel it is convenient to perform a change of variable by writing \( \beta = \beta_0 \exp (\alpha) \), where \( \beta_0 \) defines a reference length scale and \( \alpha \) is a new generator coordinate. With this choice the overlap kernel appears as

\[
\langle \alpha | \alpha' \rangle = \text{sech}^T (\alpha - \alpha')
\]

which, for the heavier nuclei, can be approximated by the Gaussian

\[
\langle \alpha | \alpha' \rangle \simeq \exp \left[ -\frac{T}{2} (\alpha - \alpha')^2 \right]. \quad (2)
\]

The energy kernel can be put in the form

\[
\langle \alpha | H | \alpha' \rangle = \langle \alpha | \alpha' \rangle \times \left[ C_1 \text{sech} (\alpha - \alpha') \exp (\alpha + \alpha') \right] \\
+ C_2 \cosh^{3/2} (\alpha - \alpha') \exp \left[ \frac{3}{2} (\alpha + \alpha') \right] \\
+ C_3 \cosh^{5/2} (\alpha - \alpha') \exp \left[ \frac{5}{2} (\alpha + \alpha') \right] \\
+ C_4 \cosh^3 (\alpha - \alpha') \exp [3 (\alpha + \alpha')], \quad (3)
\]

The oscillator parameter \( \beta \) is taken as: \( 0.75 \text{fm} \) for \(^4\text{He}\), \( 1.18 \text{fm} \) for \(^{16}\text{O}\) and \( 1.40 \text{fm} \) for \(^{40}\text{Ca}\).
where the coefficients $C_i$, $i = 1$ to $4$ are given by

$$C_1 = \frac{h^2}{2m} a_1, \quad C_2 = \frac{3}{2} \frac{t_0}{(2\pi)^{3/2}} a_2,$$

$$C_3 = \frac{3t_1 + 5t_2}{8(2\pi)^{3/2}} a_3 + \frac{9t_1 - 5t_2}{16(2\pi)^{3/2}} a_4, \quad C_4 = 4 \frac{t_3}{(\pi \sqrt{3})^{3/2}} a_5,$$

and $a_1$, $a_2$, $a_3$, $a_4$, and $a_5$ are constants resulting from the integrations over the densities, having therefore values specific to each nucleus.

Attempts to solve directly the resulting Griffin-Wheeler equation run into instability problems in the determination of the weight functions $f(\alpha)$. They can however be circumvented by means of an analytical procedure discussed by Piza and Passos [18]. It consists in transforming the Griffin-Wheeler equation by introducing the new set of states

$$|k\rangle = \int \frac{U_k(\alpha)}{\Lambda^{1/2}(k)} |\alpha\rangle d\alpha,$$

where $U_k(\alpha)$ and $\Lambda(k)$ are defined through the diagonalization process for the overlap kernel

$$\int U_k^\dagger(\alpha) \langle\alpha|\alpha\prime\rangle U_{k'}(\alpha') d\alpha d\alpha' = \Lambda(k) \delta(k-k'). \quad (4)$$

Since the states $|k\rangle$ are orthonormal and complete [18] we are led to the new equation

$$\int [H(k,k') - E] g(k') dk' = 0,$$

where the new energy kernel is written as

$$H(k,k') = \langle k|H|k'\rangle = \int \frac{U_k^\dagger(\alpha)}{\Lambda^{1/2}(k)} \langle\alpha|H|\alpha\prime\rangle \frac{U_{k'}(\alpha')}{\Lambda^{1/2}(k')} d\alpha d\alpha', \quad (5)$$

and

$$g(k) = \Lambda^{1/2}(k) \int U_k^\dagger(\alpha) f(\alpha) d\alpha.$$

For translationally invariant kernels (i.e., depending on $\alpha - \alpha'$ only) such as (3), the diagonalization (4) is performed simply by a Fourier transform, so that $\Lambda(k)$ is also a Gaussian, and
there are no longer harmful divergences. The transformations can be directly performed on Eq.(5) giving the new energy kernel in terms of the Fourier variable $k$.

At this stage, numerical calculations can be performed and the energies and wave functions of the breathing mode can be obtained [4,17]. The resulting collective spectrum, obtained using the Skyrme interaction without the Coulomb interaction, is known [4,17] and needs no further discussion.

III. COLLECTIVE POTENTIAL AND COMPRESSION MODULUS.

The point we want to stress in this note is that a value for the nuclear compression modulus can be deduced from the energy kernel given by Eq.(5). Although at first sight one is tempted to associate a collective potential to the diagonal part of the GCM energy kernel $\langle \alpha | H | \alpha' \rangle$, we will show that off-diagonal elements also contribute significantly to the properly defined collective potential. Since the energy kernel has the overlap kernel $\langle \alpha | \alpha' \rangle$ as a global factor (see Eq.(3)) and the overlap kernel is narrower for heavier nuclei, the contributions due to off-diagonal elements will be relatively more significant for lighter nuclei.

In order to extract a collective potential from the transformed energy kernel $H(k, k')$, Eq.(3), we follow a method presented many years ago [17]. The first step consists of performing a double Fourier transform on $H(k, k')$

$$H(k, k') \rightarrow \mathcal{F} H(x, x').$$

The resulting non-local energy function is then subjected to a Weyl-Wigner transformation

$$h(q, p) = \int \langle q - \frac{\sigma}{2} | H | q + \frac{\sigma}{2} \rangle \exp \left( \frac{i p \sigma}{\hbar} \right) d\sigma,$$

where we have introduced the new variables

$$q = \frac{x + x'}{2}, \quad \sigma = x' - x.$$

It is then clear that the nonlocality of $H(x, x')$ gives rise to the momentum dependence of the collective Hamiltonian. If we expand in the nonlocality parameter, $\sigma$, we obtain the series
\[ H (q, \sigma) = \langle q - \frac{\sigma}{2} | H | q + \frac{\sigma}{2} \rangle = \sum_{n=0}^{\infty} H^{(n)} (q) \delta^{(n)} (\sigma), \]

where the \( H^{(n)} \) coefficients are \( n \)-th moments of the energy kernel

\[ H^{(n)} (q) = \frac{(-1)^n}{n!} \int H (q, \sigma) \sigma^n d\sigma. \]

The resulting Weyl-Wigner energy function

\[ h (q, p) = \sum_{n=0}^{\infty} H^{(n)} \int \delta^{(n)} (\sigma) \exp \left( ip \frac{\sigma}{\hbar} \right) d\sigma \]

is associated to the collective Hamiltonian operator

\[ H (\hat{q}, \hat{p}) = H^{(0)} (\hat{q}) + \sum_{n=1}^{\infty} \left\{ \left\{ ... \left\{ H^{(n)} (\hat{q}), \hat{p} \right\} ... \right\} \right\} \]

in which the \( p \)-independent term \( H^{(0)} (\hat{q}) \) is interpreted as the collective potential, namely

\[ V (\hat{q}) \equiv H^{(0)} (\hat{q}) = \int H (q, \sigma') d\sigma'. \]

We will not discuss the inertia associated to the breathing mode, since this has been already before [17]. In what follows, we will concentrate instead on the collective potential.

In order to evaluate \( V (\hat{q}) \) let us return to the GCM expressions and expand the reduced kernel \( H (\alpha, \alpha') / N (\alpha, \alpha') \) (which can be read directly from Eq. (3)) as follows. Introducing the variables \( \eta = \alpha - \alpha' \), and \( \gamma = \frac{\alpha + \alpha'}{2} \), the general term of the series expansion of this object around the minimum \( \gamma_0 \) of its the diagonal part is

\[ C_{mn} \eta^m (\gamma - \gamma_0)^n = \frac{1}{n! m!} \frac{\partial^{n+m}}{\partial \eta^m \partial \gamma^n} \left[ \frac{H (\alpha, \alpha')}{N (\alpha, \alpha')} \right]_{\gamma=\gamma_0, \eta=0} \eta^m (\gamma - \gamma_0)^n. \]

As a result of the symmetry of the GCM kernels only even values of \( m \) and \( n \) appear in this expansion. The collective potential \( V (q) \) is correspondingly also given as a sum of terms \( G_{mn} (q - \gamma_0) \) which can be expressed as

\[ G_{mn} (q - \gamma_0) = \frac{C_{mn}}{2\pi} \int \exp \left( -\frac{T}{2} \eta^2 \right) \eta^m d\eta \int \frac{\exp \left[ ik \left( \gamma - q \right) \right]}{\Lambda \left( k \right)} (\gamma - \gamma_0)^n dk d\gamma. \quad (6) \]

This is a somewhat symbolic expression, the meaning of which is specified by the prescription that the \( \gamma \) integration is to be performed first in terms of derivatives of the delta function.
\(\delta(k)\), so that the integral over \(k\) gives derivatives of the \(k\)-dependent part of the remaining integrand evaluated at \(k = 0\). Following this procedure one may collect the powers of \((q - \gamma_0)\) and write the collective potential as

\[
V(q) = \sum_{\nu} V_{\nu} \times (q - \gamma_0)^\nu.
\]

Numerical results indicate sufficient convergence when this sum is truncated at \(\nu = 6\) in the two cases studied here. The relevant coefficients \(V_{\nu}\) are given by

\[
V_0 = C_{00} - \frac{1}{4T}C_{02} + \frac{1}{T}C_{20} + \frac{3}{16T^2}C_{04} + \frac{3}{T^2}C_{40} - \frac{1}{4T^2}C_{22} - \frac{15}{64T^3}C_{06} + \frac{15}{T^3}C_{60}
\]

\[
+ \frac{3}{16T^3}C_{24} - \frac{3}{4T^3}C_{42}
\]

\[
V_1 = -\frac{3}{4T}C_{03} + \frac{1}{T}C_{21} + \frac{5}{16T^2}C_{05} - \frac{3}{4T^2}C_{23} + \frac{3}{T^2}C_{41}
\]

\[
V_2 = C_{02} - \frac{3}{2T}C_{04} + \frac{1}{T}C_{22} + \frac{45}{16T^2}C_{06} - \frac{3}{2T^2}C_{24} + \frac{3}{T^2}C_{42}
\]

\[
V_3 = C_{03} - \frac{5}{2T}C_{05} + \frac{1}{T}C_{23}
\]

\[
V_4 = C_{04} - \frac{15}{4T}C_{06} + \frac{1}{T}C_{24}
\]

\[
V_5 = C_{05}
\]

\[
V_6 = C_{06}.
\]

The above results show explicitly that the collective potential does in fact contain contributions from the off-diagonal terms of the GCM energy kernel, as mentioned before. In fact, the diagonal part generates only the terms \(C_{0n}\). Furthermore, it is also evident that the minimum of the collective potential does not coincide with that of the diagonal part.
of the GCM energy kernel due to the off-diagonal contributions, which therefore introduce corrections to the equilibrium radius of the nucleus. Furthermore, we obtain the main result that the compression modulus, defined by

\[ K = \frac{1}{A} \left. \frac{d^2}{dq^2} V(q) \right|_{q=q_0}, \]

where \( q_0 \) is the minimum of the collective potential, does not coincide with the value obtained when the standard variational principle (based on the use of the diagonal part only of the GCM energy kernel) when the oscillator parameter is taken as the variational parameter. In fact, we obtain from the collective potential

\[ K = \frac{2}{A} \left[ V_2 + 3V_3 (q - \gamma_0) + 6V_4 (q - \gamma_0)^2 
+ 10V_5 (q - \gamma_0)^3 + 15V_6 (q - \gamma_0)^4 \right]_{q=q_0}, \tag{8} \]

whereas on the basis of the simpler variational method we would have

\[ K_v = \frac{2}{A} C_{02}. \tag{9} \]

Comparison with Eq.(9) reveals the off-diagonal contributions to the predicted value of the compression modulus.

Numerical results for \( ^{40}\text{Ca} \) using Eq.(8) are shown in Table I using five different sets of Skyrme parameters, labelled as SI to SV. The values corresponding to the standard variational procedure (9) are also shown for comparison.

| Table I - Compression modulus for \( ^{40}\text{Ca} \) |
|-----------------|---|---|---|---|---|
| Compression \ Interaction | SI  | SII | SIII | SIV | SV  |
| \( K \) (MeV)   | 253.84 | 229.08 | 239.04 | 217.86 | 205.37 |
| \( K_v \) (MeV) | 245.87 | 222.28 | 231.73 | 211.57 | 199.64 |

It can be seen that the compression modulus obtained from the collective potential is higher than that calculated through the simple variational method. In fact, it is interesting to observe that the results obtained with interactions SII and SIII are in good agreement with the experimental value presented by Youngblood [13].
Using the sixth-order approximation to the collective potential we can also calculate the compression modulus for $^{16}$O. The results are shown in Table II below.

| Compression \Interaction | SI  | SII | SIII | SIV | SV  |
|-------------------------|-----|-----|------|-----|-----|
| $K$ (MeV)               | 242.02 | 213.78 | 223.93 | 202.60 | 190.48 |
| $K_v$ (MeV)             | 216.15 | 192.17 | 200.56 | 182.71 | 172.47 |

In this case the value of the compression modulus obtained from the collective potential is about 10% higher than that obtained from the simple variational calculation. This indicates a theoretical value for the compression modulus of $^{16}$O which is not much lower than the observed experimental values for heavier nuclei [13]. It is also worth noticing that for SV, which does not include the three-body force and therefore does not present a density dependent term, the compressibility modulus is lower in both cases.

**IV. CONCLUDING REMARKS.**

Values for the compression moduli of finite nuclei have been obtained from a collective potential associated to a description of the breathing modes based on the Generator Coordinate Method. The procedure involved in the derivation of the collective potential from the GCM kernels has been reviewed. It has been shown that this procedure leads to a collective potential that embodies contributions coming from off-diagonal elements of the GCM kernels. They are associated to correlations introduced by the choice of generator coordinate and by the use of the Griffin-Wheeler ansatz, which lead to results which differ from those obtained from simpler variational procedures. In particular, values of nuclear radii and nuclear densities are different in both cases [22].

This approach has been applied to the calculation of the nuclear incompressibility for $^{40}$Ca and $^{16}$O, using Skyrme effective interactions without Coulomb forces. The results are, as expected, higher than those obtained from simple variational approaches and, for interactions SII and SIII, are in good agreement with the experimental results given by
Youngblood et al. [13] for $^{40}$Ca. Our results for the lighter nucleus $^{16}$O give values which are not too low when compared to the available experimental values for heavier nuclei and to the mean value of $231 \pm 5$ MeV assigned to nuclear matter by Youngblood.

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