HEAVY QUARKS OR COMPACTIFIED EXTRA DIMENSIONS IN THE CORE OF HYBRID STARS

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Neutron stars with extremely high central energy density are natural laboratories to investigate the appearance and the properties of compactified extra dimensions with small compactification radius, if they exist. Using the same formalism, these exotic hybrid stars can be described as neutron stars with quark core, where the high energy density allows the presence of heavy quarks (c, b, t). We compare the two scenarios for hybrid stars and display their characteristic features.

1. Introduction

Neutron stars are natural laboratories to investigate the overlap of strong, electro-weak and gravitational interaction. Many theoretically determined properties of these astrophysical objects were tested by the observed properties of pulsars, and we have detailed calculations about these stars. However, if new perspectives appear in the description and understanding of the gravitational interaction or in the unification of the above interactions, then revisiting of the models becomes necessary. Such a reinvestigation was triggered by the refreshed attention on compactified extra dimensions. Extra dimensions inside neutron stars were investigated earlier, but the Kaluza-Klein (K-K) excitation modes were not considered in the equation of state (EoS). These modes are important constituents of the recent gravitation theories. Introducing the K-K modes into the EoS of fermion stars at their central core, new features and properties emerged.

Here we display a few of our ideas about these extra dimensions and their possible connection to particle physics. We summarize our numerical results on neutron stars with different interiors (heavy quarks vs. extra dimensions) and discuss the visibility of extra dimensions in these objects.

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2. The Fifth Dimension and the Eötvös Experiment

The introduction of the 5th dimension into the real World has a long history. One interesting attempt is related to the effort of Fishbach et al. in finding the "Fifth Force" in the Eötvös Experiment.

Originally the Eötvös Experiment has proved the Equivalence Principle (the proportionality of inertia and gravitating mass) with high precision. Deviation appeared in the 9th digit, only. This deviation was connected to the "Fifth Force", which may be coupled to the hypercharge \( Y = B + S \) and rather weak. Considering infinite range, the \( g_W \) coupling constant of this interaction is in the order of \( g_W^2/e^2 \sim 10^{-38} - 10^{-41} \). Such a weak force is able to explain simultaneously the CP violation of hyperweak interaction, which has a terrestrial background in this interpretation. This fifth force can be a weak force disturbing the gravity measurement, or the World is at least five-dimensional, mimicking the existence of an extra force.

Earlier papers investigated five-dimensional geodesic motions assuming that metric has a Killing symmetry in the extra direction, \( x^5 \), which is space-like. Since we do not observe a macroscopical 5th dimension, then \( x^5 \) must be compactified on a microscopic scale. If we cannot observe \( u^5 \), then we measure a false \( u^i \). This \( u^i \) satisfies an equation of motion. In lowest order the leading "force" term mimics Coulomb force. The specific "charge" in this force starts as \( K_i u^i \), where \( u^i \) is the true 5-velocity and \( K^i \) is the Killing vector working in the extra direction. One can obtain a constrain:

\[
q^2/Gm_0^2 < 16\pi, \quad \text{where} \quad q \text{ is the charge,} \quad G \text{ is the gravitational constant,} \quad m_0 \text{ is the rest mass.} \quad \text{Charge} \ q \text{ may have sign, following the sign of} \ u^5, \text{so 5th dimension offers a possibility of geometrizing vectorial forces not stronger than gravity. There is a chance that the fifth-dimensional motions of particles is connected to the quantum number hypercharge}^8 \text{ or strangeness}^{11}. \]

Quantization puts a serious constraint on five-dimensional motion. If there is an independence on \( x^5 \), then the particle is freely moving in \( x^5 \). However, being that direction compactified leads to an uncertainty in the position with the size of \( 2\pi R_c \), where \( R_c \) is the compactification radius. An angular momentum-like quantum rule appears on the "charge" connected to the 5th dimensional motion, thus the smallest possible charge is

\[
\hat{q} = n \cdot \frac{2\hbar \sqrt{G}}{c R_c}. \tag{1}
\]

Because of the extra motion into the fifth dimension, an extra mass appears in 4D descriptions. Considering \( R_c \sim 10^{-12} - 10^{-13} \) cm, together with the extra interaction in the range of \( \hat{q} \), this extra "mass" is \( \hat{m} \sim 100 \) MeV.
3. Strange Compact Stars and the Cyg X-3

We do know that pulsars are very compact objects with mass $M \sim 1 \, M_\odot$ in solar mass unit and radius $R \sim 10$ km. Neutron matter in its own gravity can produce such configurations, thus the simplest explanation of the pulsars is connected to neutron stars produced in gravitational collapse$^{1,2}$.

The observed maximal mass for pulsars is $\sim 1.5 \, M_\odot$. Early theoretical calculations$^3$ with non-interacting one component neutron matter have yielded to maximum mass $\approx 0.67 \, M_\odot$. The introduction of nuclear interaction among neutrons$^2$ can increase this mass by a factor of 2, however details may be still crucial, because our knowledge is very much limited about the properties of interactions at 10 times of normal nuclear densities. This leads to the conclusion, that some heavy pulsar may not be a simple neutron star, but a more complicated object with an exotic core$^{12}$. Further astrophysical observations strengthen this expectation.

In 1987 two disjoint neutrino bursts were measured from the supernova (SN) 1987A, separated by several hours. Various SN models predict one neutrino burst when/if the neutron star is formed, but never two. One could understand the double burst with the formation of two possible compact star configurations: hyperon star containing strange hadrons heavier than neutron or quark star containing a deconfined quark matter core.

Between 1981 and 1991 strong muon showers were detected with a 4.8 hours periodicity$^{13}$. The direction of these showers is that of the Cyg X-3 object, which is a close binary object: one component is a normal massive star with $4 \, M_\odot$, the other is a compact star with orbital period of 4.8 hours$^{14}$ and the muons have shown the same periodicity$^{15}$. Their distance from Earth is $\approx 40,000$ ly. Cyg X-3 is a very intensive source in a wide spectral range and surely one important phenomena is the impact of stellar wind on the surface of the compact component generating particle packages, which will hit the Earth later. These packages must consist of neutral particles traversing the 40,000 light years, otherwise galactic magnetic fields would have smeared away the 4.8 hour period. However, photons or neutrinos would generate far too few muons in the terrestrial atmosphere$^{15}$. So dilambdas$^{16}$ or (so far undiscovered) strange nuggets were suggested to be the messengers, which can be stable$^{16,17}$ and Cyg X-3 B was identified to be a hyperon (or a strange quark) star generating these strange particle packages in its surface. Theoretical neutron star calculations have found such a strange quark star configuration$^{18}$, which was stable. Such an object could be the source of these strange messengers.
4. The Tolman-Oppenheimer-Volkov Equation

The formation of neutron stars is based on the existence of a hydrostatic equilibrium between strong and gravitational interaction. In 4D a static spherically symmetric fluid configuration has to satisfy 3 nontrivial components of the Einstein equation. Two of them give pure quadratures, the third constrain leads to the Tolman-Oppenheimer-Volkov equation\(^1,2,19\):

\[
\frac{dp}{dr} = \frac{[p(r) + \varepsilon(r)][GM(r) + 4\pi G r^3 p(r)]}{r[r - 2 GM(r)]},
\]

where \(p(r), \varepsilon(r)\) are the radial distribution of the pressure and the energy density, and \(M(r)\) is the mass of the neutron star within radius \(r\).

Before solving eq.(2) we have to specify the EoS of the interior matter in the form: \(p = p(\varepsilon)\). Since the configuration is static, we can take cold matter in equilibrium and considering the \(n\) density of some conserved particle number (e.g. baryon number for neutron): \(\varepsilon = \varepsilon(n)\). Applying thermodynamic identities the pressure can be extracted: \(p = n\frac{d\varepsilon}{dn} - \varepsilon\).

In eq.(2) the central energy density, \(\varepsilon_{cent}\), can become the initial condition. Starting from a homogenous central core with radius \(R_0 \sim 1\) cm, one can integrate the TOV equation until the surface. Because the static interior solution needs to match the exterior vacuum condition on this surface, the \(p = 0\) condition will fix the radius at \(R \sim 10\) km. Finally, the equilibrium configurations depend on a single parameter, \(\varepsilon_{cent}\).

For stability criteria see Ref.\(^1\), where this question is investigated in details. Practically: stability can change only at extremum points of the curve \(M(\varepsilon_{cent})\), where either the smallest real eigenfrequency of oscillation becomes imaginary through 0, or vice versa. The actual change depends on the sign of \(dR/d\varepsilon_{cent}\) at that point (see Chap. 6 and 7 in Ref.\(^1\)).

In case of quark stars a first order phase transition can appear inside the star. We obtain a critical surface at the critical pressure, \(p_{cr}\), where two different \(\varepsilon\) values can be found, depending on the EoSs. Since \(p\) is continuous, then the sectionwise solutions of the TOV equation can be determined. However, the solution of the TOV equation jumps from \(\varepsilon_1\) to \(\varepsilon_2\). This jump is absent in the case of second order phase transition.

Introducing extra dimensions, in the general case the 5-dimensional Einstein equations cannot be reduced into a single, TOV-type first order differential equation, but a coupled differential equation system is generalized\(^6,7\). Assuming a radially independent compactification circumference the TOV equation reappears with a 5-dimensional interpretation. The details of this correspondence is discussed in the Appendix.
5. Neutron Star in 4 Dimensions

Our reference object is an "ideal" neutron star, which consists of pure neutron matter. We use free fermion gas EoS with multiplicity \( d_N = 2 \). We neglect the "normal matter" constituent, which is a very thin and negligible layer at the surface of neutron stars. The radius is obtained in the range of \( R \sim 5 - 15 \) km for \( \mu_N > 1 \) GeV, reproducing earlier results from Ref.\(^1\). Real neutron star calculations are more complicated (see Refs.\(^2\),\(^18\)), however we want to demonstrate some features and the above simple model is just appropriate for this task.

On Figure 1 the mass and the radius of the neutron star are displayed as the function of central energy density. The first upgoing part of \( M(\varepsilon_{\text{cent}}) \) is stable until the star (\( \ast \)), the next is unstable, and all downslope parts of \( M(\varepsilon_{\text{cent}}) \) are unstable. In parallel, we constructed the \( M(R) \) curve (see right hand side), where the spiraling behaviour can be clearly identified\(^1\).

Introducing a quark core with light (\( u \) and \( d \)) quarks, a first order phase transition occurs inside the neutron star. Considering an interacting EoS for the neutron matter\(^20\) and a free fermion gas EoS for the quark matter with a bag constant \( B = 0.25 \) GeV/fm\(^3\), the quark core appears (\( Q \)) at \( \varepsilon_{\text{cent}} = 1.7 \) GeV/fm\(^3\). No stable configuration was found between turning points \( T_1 \) and \( T_3 \), e.g. light quark star is unstable around \( T_2 \). The region between \( Q \) and \( T_1 \) is promising, but a more sophisticated model is needed.

Figure 1. The mass and the radius of neutron stars containing pure neutron matter or light quark core. The \( M(R) \) function is displayed on the right hand side for both cases. Dotted line between full dot and open dot indicates a discontinuity before the appearance of quark core (\( Q \)). The \( T_1, T_2, T_3 \) display turning points for quark star.
6. Heavy Flavours in Hybrid Stars

One can observe particles heavier than the neutrons, carrying conserved quantum numbers, namely the $S$ strangeness, $C$ charmelessness, $B$ bottomness (and maybe $T$ topness). Weak interaction is allowed to create these flavours inside the core of a neutron star at extremely high energy density.

Let us construct an "ideal heavy hadron star" consisting of neutrons and other heavier hadrons with these heavy flavours. For simplicity we consider the neutral $\Lambda_S(1115)$, $\Lambda_C(2452)$ and $\Lambda_B(5624)$ (we stop at central chemical potential $\mu_{\text{cent}} = 25$ GeV, thus we miss the $\Lambda_T$). The multiplicities are $d_i = 2$ and free fermion EoS is used in the TOV equation as previously.

Figure 2 displays our results for heavy hadron star, which are very close to the pure neutron star case, e.g. the stability and the spiraling feature remain the same. Just before the first peak stable configuration appears with $\Lambda_S$ core and $N$ mantles ("hyperon star"$^2$). In the first maximum ($\star$), far before the appearance of $\Lambda_C$, the stability ceases and never returns.

Introducing a quark core with light ($u, d$) and heavy ($s, c, b$) quarks, a first order phase transition have to be considered, again. Since the $s$ quark is relatively light, then some modifications appear, but the investigated features remain similar. The quark core appears ($Q$) at $\varepsilon_{\text{cent}} = 1.3$ GeV/fm$^3$, when $u, d, s$ quarks are already present. No stable configuration was found between turning points $T_1$ and $T_3$, but close to $Q$ stable states ("strange quark stars"$^{18}$) may appear. However, no charm, bottom or top quark stars are expected to be stable at high central energy densities$^{21}$. 

![Figure 2. The mass and the radius of heavy hadron stars with and without heavy quark core. The $M(R)$ function is displayed in the right hand side for both cases.](image-url)
7. Hybrid Stars in 5 Dimensions

Now let us consider a neutron star with excitations in the 5th dimension, which are compactified. In this case neutrons are moving into the direction of $x^5$. In the special case of $x^5$-independence this motion appears as an extra degrees of freedom with larger masses:

$$ (m_N^{(i)})^2 = (i/R_c)^2 + m_N^2. $$

Here the integer $i$ means the excitation level, $R_c$ is the compactification radius of the 5th dimension and $m_N^{(0)} = m_N(940)$. The EoS is generated as the superposition of the different K-K modes, summing up the free fermion EoS at masses from eq.(3). If we choose $R_c = 0.33$ fm, then the first excitation has the mass identical with that of the $\Lambda$ particle, $m_N^{(1)} = \Lambda(1115)$. The TOV equation remains valid to find the appropriate equilibrium states.

Figure 3 displays the (non-interacting) heavy hadron matter case (full line) and the 5-dimensional cases with two different compactification radii, $R_c = 0.33$ fm (dashed line) and $R_c = 0.66$ fm (doted line). The filled and open triangles indicate the K-K modes.

![Figure 3](image_url)

In the case of $R_c = 0.33$ fm the first K-K mode ($E_1$) appears in a stable configuration indicated until the star (*). Choosing $R_c = 0.66$ fm, we have $F_1$ and $F_2$ modes before the end of stability (the star (+)) on the dotted line. In the case of $R_c > 0.25$ fm, one or more extradimensional modes can fit into the core before losing the stability of the hybrid star.
8. More Extra Dimensions

The introduction of the 5th dimension is a minimal extra-dimensional model. One may assume an $x^6$, or more extra dimensions, roughly on the scale of $x^5$. (The idea of six-dimensional microphysics goes back to 35 years27.) The opening of the 6th dimension leads to similar results as displayed in Figure 3, however now more combinations of extra-dimensional excitations may appear in the stable region. In Ref.7 even two stable regions and two maximums on $M(R)$ appeared, indicating the complexity of the higher dimensional results. It is interesting to mention, that the existence of such a second stable peak can explain the double neutrino burst of SN 1987A. However, detailed calculations are needed to verify this explanation.

9. Conclusions

We have demonstrated in a simple model for compact star that neutron stars with hyperon or extradimensional core are very similar objects. The TOV equation leads to similar structure with well-defined stability region, where the lowest K-K modes may appear in the extradimensional case. The main reason is connected to the size of the compactification dimension: assuming $R_c = 0.33$ fm one obtains the mass of $\Lambda^0$ for the first K-K mode, which dominates the inner structure of the hybrid star in the stable region.

We saw in Section 2 that motion in the fifth dimension results in an apparent new quantum number, similar in structure to strangeness, so a neutron moving into the 5th dimension may be seen as a $\Lambda_S$ particle. (The resulting apparent "violation of equivalence" will appear roughly in the order of Ref.8 as shown in Ref.25.) Now the dilambda explanation of muon bursts triggered by Cyg X-3 for short periods is not utterly hopeless, although rather cataclysmic events are needed to get strange matter to the surface.

The introduction of more extra dimensions leads to the appearance of further stability regions and the existence of more than one stable hybrid star configuration. Such a result is supported by the double neutrino shower arrived from SN 1987A.

The investigation of neutron stars with light and heavy quark core leads to different characteristics and to higher star masses. It remained open the question of stability of these objects, more sophisticated models28,29,30 are needed for interacting neutrons and quarks. Interesting question is if any extra-dimensional setup is able to mimic the features of the heavy quark stars. The connection between strange quark and extra-dimensional propagation of light quark deserves further studies.
Appendix A: 5D Static Equilibrium and the TOV Equation

Let us consider a spherical equilibrium situation in 5 dimensions. The solution of the Einstein equation must be stationary and spherically symmetric. Since we do not have information about the dependence on the microscopic, compact dimension, and cannot observe either, let us assume $x^5$ independence. Thus we have an $U(1) \otimes SO(3) \otimes U(1)$ Killing symmetry, 5 Killing vectors altogether, with 4-dimensional transitivity; $SO(3)$ is transitive in 2 dimensions. Using permitted coordinate transformations, on the analogy of the derivation of Ref. 22 we arrive at the general form of the line element:

$$\text{d}s^2 = g_{00}\text{d}t^2 + g_{11}\text{d}r^2 - r^2\text{d}\Omega^2 + g_{55}\text{d}x^5\text{d}x^5 + 2g_{05}\text{d}t\text{d}x^5.$$  \hspace{1cm} (4)

Here $\text{d}\Omega^2$ is the usual spherical elementary surface. The still unknown components $g_{11}$, $g_{55}$ and $g_{05}$ are independent on $t$, $x^5$ and the angles. Furthermore $g_{11}$, $g_{55} < 0$. However $g_{05}$ cannot be transformed away generally.

If we require static temporal symmetry instead of mere stationarity, then the timelike Killing vector will be hypersurface-orthogonal and even $g_{05}$ can be transformed away. Here we are content with this subclass, since in 4 dimensions all stationary spherical fluid solutions were static too 22,26. The more general stationary class, with some interplay or drag between $\text{d}t$ and $\text{d}x^5$ would deserve further attention in a subsequent paper.

Now we are confronted with a further choice. We are restricted to static fluids, and by definition, the 3-dimensional stress tensor of a static fluid is isotropic (Pascal law). However, we are in 5 dimensions, so the space-like section is of 4 dimensions. What is the proper generalisation of a fluid now?

The problem is discussed but by no means obligatorily solved in Ref. 23. For a macroscopical 5th dimension full equipartition, (thus 4 dimensional isotropy) would have good arguments. However, in our case the 5th dimension is microscopic, when usual interactions in fluids (e.g. van der Waals forces) could not establish full spatial isotropy. Until a convincing ansatz is found, we decouple the 55 component of the Einstein equation. The remaining equations have always two sets of solution: either $dg_{55}/dr = 0$ or not. In the first case the equations can be converted into the TOV equation, and $T_{55}$ can be calculated afterwards. In the second case a system of coupled differential equations appears and the compactification radius has radial dependence7.
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