Calculation of Magnetic Penetration Depth Length $\lambda(T)$ in High Tc Superconductors

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The notion of a finite pairing interaction energy range via Nam, results in the incomplete condensation in which not all states participate in pairings. The states not participating in pairings are shown to yield the low energy states responsible for the linear $T$ dependence of superelectron density at low $T$ in a $s$-wave superconductor. We present extensive quantitative calculations of $\lambda(T)$ for all $T$ ranges, in good agreements with experiments. It is not necessary to have nodes in the order parameter, to account for the linear $T$ dependence of $\lambda(T)$ at low $T$ in high Tc superconductors.

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One of crucial parameters in a superconductor is the magnetic penetration depth length $\lambda(T)$ which reflects the condensation carrier density, superelectron density $\rho_s$, in the London model as

$$\rho_s(T)/\rho_s(0) = [\lambda(0)/\lambda(T)]^2.$$  

(1)

The $\rho_s(T)$ plays an important role for understanding the nature of condensation. In the Gorter and Casimir two fluid model (GC), $\rho_s(T)$ varies as $1 - (T/T_c)^4$. But the BCS-$\rho_s(T)$ has an activation form at low $T$ via the order parameter $\Delta$ which indicates the excitation energy gap. The measurements of $\lambda(T)$ at low $T$ in high $T_c$ superconductors (HTS) are compatible with neither the BCS result nor the GC picture. Data indicate the linear $T$ dependence of $\rho_s(T)$ at low $T$. This linear $T$ dependence of $\lambda(T)$ is in fact taken as providing evidence that the order parameter has nodes, suggesting the d-wave pairing state. On the other hand, one of us has shown that the notion of a finite pairing interaction energy range $T_d$ results in the incomplete condensation and the low energy states responsible for the linear $T$ dependence of $\rho_s(T)$ at low $T$ in a $s$-wave superconductor. Moreover, the incomplete condensation yields the multi-connected superconductors (MS) which can account for the $\pi$-phase shift in Pb-YBCO SQUID and 1/2 fluxoid quantum in the YBCO ring with three grain boundary junctions.

Recently, the oxygen isotope effect, $T_c \propto M^{-\alpha}$ with $\alpha = 0.4 \sim 0.49$ in LSCO single crystal, suggests the electron-phonon interaction would play an important role for understanding superconductivity in cuprate materials. And the BSCCO bicrystal c-axis twist Josephson junction experiment indicates the dominant order parameter contains the $s$-wave and not $d$-wave component. Moreover, no node in the order parameter is observed in the angular dependence of the non-linear transverse magnetic moment of YBCO in the Meissner state. On the other hand, the scanning tunneling microscope imaging the effects of individual zinc impurity atoms on superconductivity in BSCCO shows the four fold symmetric quasiparticle cloud, indicating the d-wave component.

But no four fold is observed in the same system. Perhaps, the observation of may be a reflection of the Fermi surface.

It is highly desirable to carry out quantitative calculations of $\lambda(T)$ for all $T$ ranges to see the accountability of finite $T_d$ picture for $\lambda(T)$ data of HTS. In this letter, we present extensive quantitative calculations of $\lambda(T)$ for all $T$ ranges in good agreements with data of HTS. For this, it is worthy to recapitulate the pertinent results for the notion of a finite $T_d$.

To see the phase transition, the transition temperature $T_c$ should be a finite value, that is, neither zero nor infinite. To have a finite value of $T_c$, the pairing interaction energy range $T_d$ should be finite, since $T_c$ is scaled with $T_d$ within the pairing theory. In other words, the order parameter $\Delta(k, \omega)$ may be written as

$$\Delta(k, \omega) = \begin{cases} \Delta & \text{for } \varepsilon_k < T_d \\ 0 & \text{for } \varepsilon_k > T_d \end{cases}$$

for all frequencies $\omega$. Here $\varepsilon_k$ is the usual normal state excitation energy with the momentum $k$, measured with respect to the Fermi level.

FIG. 1. Schematic diagram showing the order parameter $\Delta$ in the finite pairing interaction energy ranges $T_d$ view.
Here the natural units of $\hbar = c = k_B = 1$ are used. Later we use $\Delta(T)$ with $T$ for $\Delta$ as well. Note that $w$ has no constraint and that in the case of pairings of carriers via the electron-phonon interaction, the $T_d$ corresponds to the Debye temperature. However, the nature of $T_d$ in HTS is still unknown. Our results are not depending on the nature of $T_d$. The order parameter $\Delta$ is the solution of the BCS like equation \[ \Delta = \int_0^{T_d} \frac{d\epsilon}{E} \tanh \frac{\beta E}{2}, \] where $E = (\epsilon^2 + \Delta^2)^{1/2}$, $\beta = 1/T$, and $g$ corresponds to the BCS coupling parameter $N(0)V_{BCS}$. The solution of $\Delta$ for $g$ are shown in the unit of $T_d$ in Fig. 2.

$\Delta(T)$ for $\Delta$ as a function of $g$ and $T$.

The equation for $T_c$ from Eq. (3) via $\Delta(T_c) = 0$ may be written as

$$1/g = (2/\pi) \sum_j (2/j) \tan^{-1}(y/j),$$

where $y = T_d/\pi T_c$, and sum is over the positive odd integers $j$. The factor of arctangent function makes the sum converge. For large $y$, Eq. (4) yields the BCS result $T_c(BCS)$. The quantitative calculations of $T_c$ are given in Fig. 3, together with the BCS $T_c(BCS)$. Unlike the BCS result of $T_c(BCS)$, the $T_c$ from Eq. (3) does not have any upper limit. The fact is that for large $g > 2.32$, $T_c$ increases with increasing $g$ as $T_c = g T_d/2$. One interesting value of $g = 0.657$ yields $T_c = 100$ K with $T_d = 400$ K which is of the order of the Debye temperature in HTS. This value of $g$ may be realized in YBCO by considering the electron-phonon interaction of the order of $\lambda_p = 1.3 \sim 2.3$.

$\Delta(T) = 0$ as a function of $T_c$. As is shown in Fig. 4, $\Delta(T)/T_c$ is a function of $g$ or $T_c/T_d$, and increases with increasing $g$ or $T_c/T_d$. In the range of $g < 0.2$ or $T_c/T_d < 0.0076$, it has a constant BCS value. In fact, this range corresponds to the case of low $T_c$ superconductors(LTS). In the range of $0.5 < g < 1.5$ it increases almost in a linear of $g$, and has a saturated value of 2 for large $g$.

$\Delta(0)/T_c$ versus $g$.

FIG. 4. The BCS parameter $\Delta(0)/T_c$ versus $g$. As is shown in Fig. 4, $\Delta(0)/T_c$ is a function of $g$ or $T_c/T_d$, and increases with increasing $g$ or $T_c/T_d$. In the range of $g < 0.2$ or $T_c/T_d < 0.0076$, it has a constant BCS value. In fact, this range corresponds to the case of low $T_c$ superconductors(LTS). In the range of $0.5 < g < 1.5$ it increases almost in a linear of $g$, and has a saturated value of 2 for large $g$.

In the sprit of Bardeen [20], the normal fluid density $\rho_n(T) = \rho - \rho_s(T)$, within the pairing theory, may be written as

$$\rho_n(T) = \rho_s(T) \Delta(T)/T_c, \quad \Delta(T)/T_c = \frac{\sinh(1/g)}{1/g}. \tag{5}$$
\[ \rho_n(T)/\rho = 2 \int_0^\infty d(\omega/T)n(\omega)f(\omega/T)\left[1 - f(\omega/T)\right], \]  

where \( f(x) \) is the usual Fermi function \( 1/[1+e^{x} \exp(x)] \) and the density of states \( n(\omega) = N(\omega)/N(0) \) is given by \((8)\)

\[ n(\omega) = q(\omega/T_d) + n_{BCS}(\omega)r(\omega/T_d), \tag{7} \]

\( q(\omega/T_d) = (2/\pi) \tan^{-1}(\omega/T_d), \tag{8} \)

\[ r(\omega/T_d) = (2/\pi) \tan^{-1}[n_{BCS}(\omega)T_d/\omega], \tag{9} \]

\[ n_{BCS}(\omega) = \text{Re}\{\omega/(\omega^2 - \Delta^2)^{1/2}\}. \tag{10} \]

Physically, \( \rho_n(T) \) would be resulted from the single particle excitation not pairs. Thus, the factor \( f(x) \) in Eq. \((3)\) is the occupation probability of the state \( |k\uparrow\rangle \) and the factor \( [1 - f(x)] \) is the unoccupation probability of the partner state, say, \( | - k\downarrow\rangle \), and vice versa, respectively. The factor 2 comes from the spin sum. The states of Eq. \((8)\) are reflections of states being not participated in pairings. A word of caution is in order. The \( T \) results in the linear interaction parts. Physically, the sum of spectral weights outside \( T_d < |\epsilon_k| \), result in the states of Eq. \((8)\). Thus, the low energy states are realized. In other words, carriers which do not participate in pairings yield the linear \( T \) dependence of \( \rho_n(T) \) at low \( T \). In fact, these states results in the linear \( T \) dependence of \( \lambda(T) \) at low \( T \). To see this, by inserting Eq. \((3)\) into Eq. \((3)\), one can get the variation of \( \lambda(T) \) at low \( T \) as \((11)\)

\[ \Delta\lambda/\lambda(0) = \frac{1}{2}\rho_n(T)/\rho = (T/T_c)(T_c/T_d)(2/\pi) \ln 2, \tag{11} \]

similar to the result by d-wave picture \((4)\).

\[ [\Delta\lambda/\lambda(0)]_d = (T/T_c)(T_c/\Delta_0) \ln 2 \tag{12} \]

via \( n_d(\omega) = \omega/\Delta_0 \), where \( \Delta_0 \) is the maximum value (antinode) of the order parameter.

For the quantitative calculations of \( \lambda(T) \), we have determined \( T_c/T_d \) or \( g \) [Eq. \((3)\)] via Eq. \((13)\), by taking the slope of \( [\lambda(0)/\lambda(T)]^2 \) near zero temperature. Once \( g \) or \( T_c/T_d \) is set, no adjustable parameter is used in our calculations of Eq. \((3)\).

As is shown in Fig. \((3)\) we have obtained good agreements between calculations and data of BSCCO by Lee et al \((2)\), HBCCO by Panagopoulos et al \((3)\) and LSCO by Panagopoulos et al \((4)\), and Sr214 by Bonalde et al \((5)\), respectively. We picked up not all of data points in the papers for clarity. Bonalde et al \((5)\) reported that their data at low \( T \) vary as \( T^2 \) which are resulted from scatterings by impurities or defects. In a finite \( T_d \) picture, the impurity scatterings make some states at the Fermi level not participate in pairings, and result in the \( T^2 \) term in \( \lambda(T) \) at low \( T \) \((21)\).

\[ R(z) = \int_0^\infty \frac{[n_{BCS}(\omega) - n(\omega)] d\omega}{\Delta}, \tag{13} \]

\[ = \int_0^\Delta n(\omega) d\omega/\Delta \]

FIG. 5. The temperature dependence of \( [\lambda(0)/\lambda(T)]^2 \) (solid lines) compared with the experimental data for HBCCO \( \Delta \), BSCCO \( \square \), LSCO \( \ast \) and Sr214 \( \circ \).

However, as shown in Fig. \((3)\) we have obtained poor agreement near \( T_c \) between calculation and data of YBCO by Hardy et al \((1)\) and anisotropic data of YBCO by Kamal et al \((3)\). The YBCO b case is good.

FIG. 6. \( [\lambda(0)/\lambda(T)]^2 \) (solid line) compared with data for bulk YBCO \( \bigcirc \), and a- and b-axes, respectively \( \triangledown \).
Thus, the linear range stated before, this range corresponds to the case of LTS. For all the quantitative calculations account very well for data observed in LTS.

In Fig. 7 is shown the fraction of states, \( R(z) \), being not participated in pairings versus \( g \).

In Fig. 7, it is shown \( R(z) \) as a function of \( g \). In the range of \( g < 0.2 \) or \( T_c/T_d < 0.0076 \), \( R(z) \) is negligible. As stated before, this range corresponds to the case of LTS. Thus, the linear \( T \) dependence of \( \lambda(T) \) at low \( T \) is hardly observed in LTS.

In summary, even though the model of Eq. (2) is ideal, the quantitative calculations account very well for data for all \( T \) ranges without any adjustable parameter, except for YBCO data near \( T_c \). Perhaps, the Fermi surface effect would play an important role for \( \lambda(T) \) in the case of YBCO. Of course, the retardation and non-local effects should be taken into account as well for improvement. In all, the calculations are quite satisfactory and theoretically sound. We suggest the pairing interaction energy range \( T_d \) in HTS may be of the order of 1 ~ 2 times \( T_c \). The linear \( T \) dependence of \( \lambda(T) \) at low \( T \) does not imply nodes in the order parameter, contrary to general belief.

In the spirit of a finite \( T_d \), it is recently shown \( \Delta(\omega) \) that the spinless impurity scatterings suppress \( T_c \) and destroy superconductivity. Some states at the Fermi level are shown not to participate in pairings when there are scattering centers such as impurities, and result in the linear \( T \) term in the specific heat at low \( T \). The quantitative calculations \( \Delta(\omega) \) account well for the reduction of \( T_c \) and the specific heat data \( \Delta(T) \) in the Zn-doped YBCO, respectively.

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