Drained shear behavior of an unsaturated soil during cyclic triaxial loadings

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ABSTRACT

Many research works have been conducted on saturated soils under cyclic loadings. Most embankments such as fill dams, road and railway embankments are constructed by compacted saturated geo-materials. Nowadays, it has been more important to study the dynamic properties of unsaturated soils in geotechnical engineering. However, there have been few research works on the unsaturated soils under cyclic loadings. The main purposes of this paper are to investigate cyclic deformation properties of an unsaturated silt named DL clay, and to obtain stress-dilatancy relationships under unsaturated conditions. The stress-dilatancy relationships are expressed as the relationships between plastic strain increment ratio: \( \frac{\Delta e_p}{\Delta \gamma} \) and stress ratio: \( \frac{q}{p'} \). Triaxial cyclic loading tests were conducted for isotropically consolidated and unsaturated-state silt specimens under a constant net confining pressure and different suctions with three different types of cyclic shear loading applied. The stiffness of the soil increased with an increase in suctions and cyclic times. Total amount of volume reductions of the specimens decreased with an increase in suction. As the numbers of cyclic loadings and suction values increased, the amount of dilations also increased. The stress-dilatancy relationships could be formulated as linear lines and the relationships were valid under loading and unloading processes for all specimens.

Keywords: unsaturated soil, cyclic sheared loadings, drained conditions, triaxial apparatus, stress-dilatancy relations

1 INTRODUCTION

When we consider safety of fill dams and their foundations against cyclic motions of level 2 earthquakes, it is important to estimate permanent deformations of the soil structures. The level 2 earthquakes are defined as motions with large magnitude but low frequency to occur in service. These soil structures are generally constructed by compacting geo-materials (soils and rocks). The materials usually exist under unsaturated conditions. Therefore, it should be clarified the deformation properties of unsaturated soil under cyclic loading in order to estimate the permanent deformations of the soils in saturated zones and vadose zones.

When a soil was subjected to monotonic loading under a drained condition, Rowe et al. (1962) found the stress-dilatancy relationship that meant a unique relationship between the stress ratio and strain increment ratio, which is often called “dilatancy ratio.” Many researchers have conducted cyclic loading tests for saturated soils and obtained the stress-dilatancy relationships. Pradhan et al. (1989) performed cyclic triaxial, cyclic torsional shear and cyclic torsional simple shear tests using saturated Toyoura sand under drained condition to investigate general tendencies of the stress-dilatancy characteristics under cyclic loading. A unique stress-dilatancy relation, which was rather independent of the specimen density, the type of stress path, the stress history and the pressure level, was obtained in terms of the ratio of the plastic shear strain increment \( \Delta \gamma_p \) and the plastic volumetric strain increment \( \Delta e_p \). The relationship discontinuously changed on the reverse of loading direction. Pradhan and Tatsuoka (1989) modified the stress-dilatancy relationships on the basis of some theories. De Silva et al. (2014) experimentally investigated the stress-dilatancy relationships during cyclic torsional shear loading for saturated Toyoura sand under drained conditions and then proposed a bi-linear non-unique stress-dilatancy model for the stress-controlled and drained cyclic torsional shear loading. Moreover, many studies have been conducted on the cyclic loading tests for saturated soils to obtain stress-dilatancy relations (e.g. Yasuhara et al., 1982; Tatsuoka and Ishihara, 1974; Shahnazari and Towhata, 2002).

For unsaturated soils, the potential for liquefaction of unsaturated soils has been investigated (e.g. Yoshimi et al., 1989; Tsukamoto et al.; 2002, Unno et al., 2008). Kimoto et al. (2011) conducted cyclic triaxial tests for
an unsaturated sandy soil under drained and undrained conditions to investigate the effects of the initial suction, confining pressure and degree of compaction. The higher initial suction provided the larger deviator stress, smaller volumetric strain and axial strain for the cyclic loading under drained condition. However, there has been no clear detail report of stress-dilatancy relations for unsaturated soil under cyclic loading conditions. Though Kohgo et al. (1993b, 2007) proposed elastoplastic models for unsaturated soils based on the cyclic plasticity theory, the stress-dilatancy relationships are needed to accomplish more accurate estimations especially for the cyclic hysteresis loops. So, in this study, several series of cyclic triaxial shear loading tests were conducted to investigate stress-dilatancy relationships under cyclic loadings for an unsaturated silt.

2 APPARATUS, SPECIMEN AND TESTING PROCEDURE

A cyclic triaxial apparatus for unsaturated soils was used. It has a ceramic disc, whose air entry value = 100 kPa, in the pedestal and a porous stone with a water repellent filter in the cap. Then suction can be applied to triaxial specimens by using the axis translation technique. The pore water pressure $u_w$ and pore air pressure $u_a$ can be measured or applied through the bottom and top of the specimens, respectively. This apparatus has double cells system: inner and outer cells, to measure volume changes accurately. These cells are filled with de-aired water. The volume change of each specimen during the whole test were measured by monitoring water level in the inner cell with a gap sensor. An axial load cell is mounted inside the triaxial cell above the top cap.

A silty soil named DL clay was used as the test material under a series of drained cyclic triaxial shear loading tests. DL clay consists of sand, silt and clay, and the percentage of them are 0.1, 90.4 and 9.5 %, respectively. It is also non-plastic. The soil particle density is 2.650 g/cm$^3$, maximum dry density is 1.538 g/cm$^3$, optimum water content is 21.2 % and saturated coefficient of permeability is 6.7x10$^{-7}$ m/s.

Soil water retention curve (SWRC) of the soil at the end of consolidation, which was conducted under prescribed constant confining pressures and suction values, is shown in Figure 1 (Kohgo et al. 2002). The mean dry densities were 1.316 and 1.525 g/cm$^3$ for loose and dense specimens, respectively. The SWRC obtained from dense specimens is slightly higher than that done from low specimens. That means the SWRC is affected by the density. The tangential model (Kohgo et al. 1995, 2008) was fitted to these SWRCs. The air entry suction of the soil was around 10 kPa.

The tests were performed under the single net confining pressure $\sigma_{3}^{\text{net}}$ = 100 kPa and five different values of constant suction $s$ = 0, 10, 30, 60, and 90 kPa.

Fig. 1. Soil water retention curve of DL clay after consolidation

Table 1. Experimental Conditions

| Test Types | Stress Conditions | Loading conditions | Cyclic No. |
|------------|------------------|--------------------|------------|
| CS Series  | Constant stress ratio $(q/p')$ amplitude $q/p' = 0.7 \sim 1.0$ | 10 |
| CS00       | 0                | 17.22 1.321 1.006 45.38 |
| CS10       | 10               | 16.94 1.322 1.004 44.70 |
| CS30       | 30               | 17.79 1.318 1.010 46.66 |
| CS60       | 60               | 16.46 1.331 0.992 43.98 |
| CS90       | 90               | 16.80 1.322 1.005 44.31 |
| CA Series  | Constant axial strain $(\varepsilon_x)$ amplitude $\varepsilon_x = \pm 1.5\%$ | 4 |
| CA00       | 0                | 16.78 1.324 1.001 44.41 |
| CA10       | 10               | 16.98 1.314 1.016 44.28 |
| CA30       | 30               | 16.80 1.318 1.011 44.01 |
| CA60       | 60               | 16.94 1.316 1.013 44.30 |
| CA90       | 90               | 17.16 1.323 1.003 45.35 |
| SS Series  | Shear strain $(\gamma)$ amplitude increased $\gamma = \pm (0.5, 1, 2, 4\%)$ | 4 |
| SS00       | 0                | 16.78 1.324 1.001 44.41 |
| SS10       | 10               | 16.98 1.314 1.016 44.28 |
| SS30       | 30               | 16.99 1.320 1.008 44.68 |
| SS60       | 60               | 16.65 1.324 1.001 44.09 |
| SS90       | 90               | 17.04 1.319 1.008 44.78 |

Three different stress conditions, constant stress ratio (CS series), constant axial strain (CA series) and increased shear strain (SS series), were carried out. The details of experimental conditions are shown in Table 1. The soil specimen was compacted with $\rho_d = 1.3$ g/cm$^3$ and water content $w = 17\%$. The dimensions of the specimens are 5 cm in diameter and 10 cm in height. The initial conditions of the specimens are shown in Table 2. Back pressure 200 kPa were only applied to the saturated specimens. In all the tests, the specimens...
were isotropically consolidated up to the prescribed pressure and then cyclic shear loadings were applied to the specified stress or strain amplitude under a constant shear axial strain rate of 0.05%/min. After the cyclic shear loadings finished, processes of shearing were continued until 15% axial strain in all the tests. During the shearing process, the net confining pressure and suction were kept constant.

3 CALCULATION OF STRESSES AND STRAINS

It is necessary to consider two suction effects to estimate the mechanical properties of unsaturated soils. The suction effects are that (1) an increase in suction increases yield stresses and affects the resistance to plastic deformation (Kohgo et al. 1993a, 2007). The effective stresses for unsaturated soils, namely the first suction effect, are assumed to evaluate by the following equations.

\[ \sigma' = \sigma - u_{\text{eq}} \]

\[ u_{\text{eq}} = u_a - s \quad (s \leq s_c) \]

\[ u_{\text{eq}} = u_a - \left( s_c + \frac{a_e s^*}{s^* + a_e} \right) \quad (s > s_c) \]

\[ s^* = (s - s_c) \]

\[ s = u_s - u_u \]

where \( \sigma' \) is effective stress, \( \sigma \) is total stress, \( u_{\text{eq}} \) is equivalent pore pressure, \( a_e \) is a material parameter, \( s^* \)
is the effective suction, \( s_c \) is air entry suction, \( s \) is soil suction, \( u_a \) is pore-air pressure, \( u_u \) is pore-water pressure and the brackets \( \langle \cdot \rangle \) denote the operation \( \langle z \rangle = 0 \) at \( z < 0 \) and \( \langle z \rangle = z \) at \( z \geq 0 \).

The stresses and strains were calculated at the mid-height of the specimen. Stress invariants are mean effective principal stress \( p' = (\sigma'_x + 2\sigma'_z)/3 \) and deviator stress \( q = (\sigma'_x - \sigma'_z) \), where \( \sigma'_x \) and \( \sigma'_z \) are effective axial stress and effective radial stress, respectively. The effective stresses are evaluated by Eqs. (1) – (5), where the only one parameter \( a_e \) is identified to be 33.3 kPa (Kohgo et al. 2002). The volumetric and shear strains were evaluated by \( \varepsilon_v = \varepsilon_a + 2\varepsilon_r \) and \( \gamma = \varepsilon_a - \varepsilon_r \), where \( \varepsilon_a \) and \( \varepsilon_r \) are axial and radial strain, respectively.

The dilatancy ratio is defined as the ratio of plastic volumetric strain increment to plastic shear strain increment in this study. The volumetric strain increment \( d\varepsilon_v \) consists of plastic volumetric strain increment \( d\varepsilon_v^p \) and elastic volumetric strain increment \( d\varepsilon_v^e \). The plastic volumetric strain increment can be evaluated as follows,

\[ d\varepsilon_v^p = d\varepsilon_v - d\varepsilon_v^e \]

\[ d\varepsilon_v^e = \frac{\kappa \cdot dp'}{2.3(1+\varepsilon_0) p'} \]

where \( \kappa \) is the slope of \( e - \ln p' \) curve at unloading, \( \varepsilon_0 \) is the void ratio of the specimen after consolidation. The plastic shear strain increment was evaluated by

\[ d\gamma^p = d\gamma - d\gamma^e \]

![Fig. 2. stress-strain relationship for constant stress ratio amplitude](image-url)
where $d\gamma^e$ is elastic shear strain increment that can be evaluated by choosing the appropriate shear stress increment and each tangent shear modulus at the beginning of each loading and unloading.

4 TEST RESULTS

4.1 Stress-strain behavior of the unsaturated soil

We commence to mention the test results of the CS series. In this series, the shearing loads with the constant stress ratio amplitude 0.7 for compression and 1.0 for extension were applied. The number of shearing cycles was ten and the shearing was conducted under drained condition. The experiment results of relationships between stress ratio and shear strain are shown in Figure 2. The relationships in CS00 and CS10 specimens largely moved leftward at the first unloading. After a few cyclic times, the hysteresis loops approached to almost unique ones. The hysteresis loops in the specimens with high suction values (see CS60 and CS90 specimens) formed almost unique loops at the third cycle. The slopes of the hysteresis loops

![Stress-strain relationship for constant strain amplitude](image1)

![Stress-strain relationship for increasing shear strain amplitude](image2)
became steeper with an increase in suction. The shear strains induced by 10 loading cycles decreased with an increase in suction. Inhibited plastic deformations by applying a higher suction can induce these behaviors. It is considered as one of the two suction effects pointed out by Kohgo et al. (1993a).

In CA series, the constant axial strain amplitude 1.5 % for both compression and extension were applied. The number of shearing cycles was four and the shearing was conducted under drained condition. Figure 3 shows the relationships between stress ratio and axial strain. It can be seen from these figures that the stress ratios increased with cyclic times. The inclines of the reloading lines increased with cyclic times. The cyclic time at which the hysteresis loops become unique decreased with an increase in suction.

In SS series, shear strain amplitudes were sequentially increased up to 0.5, 1, 2 and 4 % for the cyclic loadings. The relationships of stress ratio and shear strain obtained from this series of tests are shown in Figure 4. The stress ratios increased with the cyclic times in all the cases. It can be seen in these figures that the slopes of the reloading lines become steeper with the cyclic loadings in all the specimens. The hysteresis slopes became also steeper with an increase in suction at the first few cyclic times.

4.2 Volume change behavior of the unsaturated soil

The relationships between plastic volumetric strain and shear strain obtained from three series of tests are shown in Figures 5, 6 and 7. Figure 5 shows the relationships of CS series. The plastic volumetric strains were largely accumulated for CS00 and CS10 specimens. In CS00 and CS10 cases, volume reductions occurred at the first loading and unloading but as subsequent progresses of loadings, the behavior changed from compression to expansion (dilation). In CS30, 60 and 90 cases, expansion appeared even at the first unloading. The amounts of volume reductions decreased with an increase in suction. In all the cases, volume reductions occurred in loadings, but the amounts of reductions decreased with an increase in cyclic times. The total volumetric strains after 10 cyclic shearing loadings reached 6.6, 6.0, 3.0, 1.6, and 1.2 % for cases CS00, CS10, CS30, CS60 and CS90, respectively.

Figure 6 shows the same relationships in CA series as those plotted in Figure 5. The volumetric strains (compression) were accumulated in all the cases. In loading processes, the volumetric strains expressed compression in CA00 and CA10 specimens while in CA30, CA60 and CA90 specimens, after a few cyclic times, they showed dilation. In unloading processes, the volume changes for CA00 and CA10 specimens expressed compression at first and after the second cycle changed to dilation. However, in all the unloading processes those for CA30, CA60 and CA90 specimens expressed dilation. The amounts of dilation increased with cyclic times in all the cases. The specimens with high suction values (CA60 and CA90) behaved more dilative than those of specimens with low suction values (CA00 and CA10) in all loading cycles. The total amounts of volume reductions for CA00, 10, 30, 60 and 90 after four cyclic times were 5.4, 5.2, 3.6, 2.8 and 2.3 %, respectively. The volume change behaviors of both CA00 and CA10 including their total amounts were quite similar because the suction values were

![Fig. 5. Volume change behavior for constant stress ratio amplitude](image-url)
smaller than the air entry value $s_e$, which was about 10 kPa for DL clay as shown in Figure 1, and the conditions of them were both insular saturations.

In order to understand the suction effects in unsaturated soils more clearly during the cyclic shear loadings, cyclic loading tests with the shear strain amplitude increment of 0.5, 1, 2, and 4 \% for compression and extension (SS series tests) were conducted. The results are shown in Figure 7. Contractive volume change behaviors could be seen at the first loading and unloading in all the specimens.

The behaviors progressively changed to dilative with the cyclic times. The amounts of compression in each loading and unloading step became larger and larger in all the cases as the shear strain amplitudes were increased. The total volume reductions for SS0, SS10, SS30, SS60 and SS90 specimens were 5.3, 5.4, 3.5, 3.0 and 2.4 \%, respectively. The smallest total volume reduction was found in SS90 specimen with the highest suction. The dilative behavior at the unloading stages started from the third unloading stage for SS00 and SS10 specimens, while it started at the second

Fig. 6. Volume change behavior for constant strain amplitude

Fig. 7. Volume change behavior for increasing shear strain amplitude
unloading stage for SS30, SS60 and SS90 specimens.

4.3 Stress-dilatancy relationship

It is very important to know the stress-dilatancy relationships of the unsaturated soils to evaluate permanent deformations due to cyclic loadings. Figures 8, 9 and 10 show the relationships of stress ratio and dilatancy ratio, i.e., ratio of plastic volumetric and shear strain increments, for CS, CA and SS series tests, respectively. The black and yellow open circles are the relationships obtained from the first loading and unloading cycle, and the blue and red closed circles are those obtained from the following loading and unloading cycles, respectively. In CS series, the stress-dilatancy relationship on the first cycle was different from others. The stress-dilatancy relationships at both loading and unloading, except the first cycle one, appeared to be almost linear in all the cases. The best fitting lines are shown as the solid lines, and the dashed lines are expressed as the lines with the incline = 2, which derives the similar plastic potential function of Cam clay model. The dashed lines also might express the relationships of the same as Cam clay model. Both
lines might be relatively well coincident.

In CA and SS series, shown in Figures 9 and 10, the both solid and dashed lines might be associated with each other and the stress-dilatancy relationships for unsaturated soils might be represented as the dashed lines in the figures. Thus, each unique linear relationship for both loading and unloading could be obtained in all the specimens with different suction values. Namely the relationships might be considered to be almost similar to those obtained for saturated soils.

5 CONCLUSIONS

Cyclic triaxial compression shear loading tests on an unsaturated artificial silt, named DL clay, with constant five different suction levels and a net confining stress were conducted under drained conditions. The following major conclusions were obtained from the tests with three different cyclic loading conditions.

The unsaturated effects (the suction effects) affected the cyclic behavior. The slopes of the hysteresis loops of stress-strain relationships became steeper with an increase in suction. The shear strains and volume changes induced by the cyclic loadings decreased with an increase in suction. The shear behaviors were similar each other when the applied suction values were smaller than the air entry value. The amount of dilation due to cyclic loading tended to increase with an increase in suction.

Each linear stress-dilatancy relationship at both loading and unloading could be obtained in all the cases except for the first loading cycle. The stress-dilatancy relationships obtained from the unsaturated soil were almost similar to those of saturated soils. Then, the first suction effect was dominant on the stress-dilatancy relationships.

The deformation behavior of an unsaturated silt under cyclic loadings could be interpreted by the two suction effects concept, Kohgo et al. (1993).

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