Dynamic behavior of structurally inhomogeneous multiply connected shell structures considering their viscoelastic properties

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Abstract. The paper presents the statement and methods for solving dynamic problems of multiply connected structurally inhomogeneous shell structures, which make it possible to reduce the problem of calculating a wide class of engineering structures to computer-aided design tasks. On the basis of numerical experiments and multi-parameter analysis of the system as a whole, a number of fundamentally important applied problems have been solved for calculating the dynamic characteristics of oscillations (frequencies, modes, determinant resonant amplitudes and damping coefficients) of special structures depending on the parameters of structural inhomogeneity. The results obtained made it possible to identify some mechanical effects of theoretical and practical importance. The developed methods and algorithms allow, at the stage of inhomogeneous systems design, combining inhomogeneous materials and connections with various rheological properties, to establish the ranges of parameter values for numerical-experimental search of the most rational (in terms of efficiency) dissipative properties and material consumption of engineering structures. It was established that an account for nonlinear strain in the material does not strongly distort the picture of linearly viscoelastic calculation.

1. Introduction
Theoretical and experimental foundations of nonlinear rheological properties manifestation in various elements of structurally inhomogeneous, complex multiply connected shell structures are given in fundamental publications [1-12]. Despite this, the assessment of the stress-strain state of shell structures considering inhomogeneous, viscoelastic properties is carried out only within the framework of linear viscoelasticity. Recently, a number of works have been published [13], which take into account the manifestation of elastic, viscoelastic linear and nonlinear properties of the material of shell structures under dynamic effects. A summary of some of them is given below. In [13], a calculation model of the foundation base deformation was presented based on the layer-by-layer summation method taking into account the components of the deviator and the ball
tensor, the ratio between them being different at different points of the foundation. Nonlinear volume strain of soil over time was considered taking into account the compaction of soil bearing layer. Dynamic reaction of soil dams [13] was studied with account for nonlinear and viscoelastic properties of soil; the dependence of dynamic reactions on load and mechanical properties of soil was established. Based on the results of experiments, local laws of interaction of extended underground pipelines and fragments of the outer surface of underground structures with soils of disturbed and undisturbed structure were constructed [14]. In [15], using the nonlinear rheological models, the stress state of the dam was investigated. The possibility of using the model was demonstrated by comparing the numerical results with the results of laboratory tests. In [16], the generalized rheological models of unsaturated and water-saturated soils were proposed and the corresponding equations were derived, used in quantitative assessment of additional residual strains and stresses in soil. A one-dimensional problem of consolidating a layer of not completely water-saturated soil under cyclic variation in external load was solved. A model and a set of determinant relations for a rheological model of soft soils were proposed in [17]. The possibility of using this model was confirmed by a number of rheological consolidation experiments under different loading velocities. In [18], a tendency was shown to increase the instantaneous strain modulus with an increase in creep. A nonlinear creep model has been introduced for soft soils, in which the creep decay was described by a nonlinear hardening function and viscosity coefficient, and the nonlinear creep curves were in good agreement with experimental data.

The behavior of specific structures using the hereditary theory of viscoelasticity under dynamic loading has not been sufficiently studied though widely considered in literature sources [19, 20, 21, 22]. The overwhelming number of publications related to dynamic problems of hereditary theory of viscoelasticity was addressed to the calculation of (linear and geometrically nonlinear) thin-walled structures — beams, plates, and shells [12, 23]. The scheme for solving dynamic problems of viscoelasticity for thin-walled structures is fairly standard. Choosing a coordinate function that satisfies the boundary conditions, the original problem can be reduced to the problem of oscillations of a system with finite number of degrees of freedom, that is, to a system of linear or nonlinear integro-differential equations with one independent time variable [12, 23]. As a rule, trigonometric or beam functions are used as coordinate functions. Such a choice of coordinate functions limits the class of problems to be solved to the structures of the simplest configurations — beams of constant sections, a rectangular plate, a cylindrical shell [12].

The authors of those publications, assuming a number of inaccuracies in coordinate functions selection, tried to increase the solution accuracy of the system of integro-differential equations. However, for structures with real geometry, it is impossible to choose analytical coordinate functions that satisfy the boundary conditions of the problem. The above review of known works shows the need to assess the stress-strain state and dynamic behavior of structurally inhomogeneous shell designs of earth structures considering not only rheological properties of shell structures, but the heterogeneous structural features and real geometry.

In this paper, we present the methods, algorithm, and results of a study of dynamic behavior of multiply connected structurally inhomogeneous shell structures, taking into account viscoelastic properties of the material under various dynamic effects.

2. Method
When choosing a computational model as a whole, it is necessary to be able to evaluate the effect of one or another structural element on its behavior under real loading, since sometimes small changes in
computational model can have a significant impact on the results of structural analysis. The most complete design scheme for the vast majority of aircraft structures, underground and aboveground structures, structures in shipbuilding and other branches of machine building leads to statically indefinable systems. One of such systems is an arbitrary axisymmetric design of shells of rotation and circular frames. As an example, consider the design scheme shown in Fig. 1. In the general case, these are the multi-connected shell structures representing an arbitrary composition of multilayer shells of rotations and circular frames, and an arbitrary composition of multi-layer cylindrical shells of non-circular cross section and rectilinear stringers.

By analogy with [12], the structure (Figure 1) is presented as an arbitrary composition of nodes interconnected by shell elements. Nodal elements in this case are: end and intermediate frames (nodes 3, 5, and 6), free and supported ends of the shells (node 6), the parallels of direct connection of links or lines along which geometrical and mechanical parameters or components of applied loads are broken (nodes 2 and 4), construction poles in which the shell generatrix intersect with the axis of rotation (node 1). Shell elements are the shells of revolution connecting nodal elements. Elements of “link” type realize the connection between the nodal structural elements or between the nodal elements and the fixed support and are the spring elements with some real rigidity characteristics.

Shell elements are the cylindrical shells connecting the nodal elements. Each shell element of the considered classes of structures can be isotropic, orthotropic, or structurally orthotropic (Figure 2) and possess elastic viscoelastic properties with significantly different hereditary functions of the material in the element structure.

Shell elements in the first-class structures can have variable rigidity and mechanical characteristics of the generatrix, and shell elements in the second-class structures are the variables along the guide. For each shell element the Kirchhoff-Love hypotheses must be fair. No restrictions are imposed on the geometry of the generatrix of the shells of revolution and on the geometry of the guides of the cylindrical shells.

Each circular frame and each stringer can possess elastic or viscoelastic properties with significantly different theological characteristics. Cross sections of frames and stringers are considered non-deformable, i.e. the frames are considered according to the classical scheme of a circular ring, and the stringers - according to the classical scheme of a straight rod. The rigidity characteristics of links can be either elastic or viscoelastic ones, described by the hereditary Boltzmann-Volterra relationships. It is assumed that a system of external dynamic loads acts on the structure.
In the particular case when there are no external mechanical influences, free damped vibrations of the structure are considered, in the presence of periodic influences - the steady-state forced vibrations. Assume that the connection between nodes $i$ and $j$ is realized by $M_{ij}$ viscoelastic hereditary links, the triple index $ijS(1 \leq S \leq M_{ij})$ is assigned to each link and all quantities related to it. The structure has $N_e = \sum_{i=1}^{N_r} \sum_{j=1}^{N_s} M_{ij}$ viscoelastic bonds. To denote the values related to the shell element or viscoelastic link, we will use, where this does not cause any misunderstanding, the order number $p(1 \leq p \leq N_e)$ of the element or the order number $p(1 \leq p \leq N_e)$ of the link.

By analogy with [12], for each shell element, we introduce a local coordinate system $0 \alpha_1 \alpha_2 r$. For this, a surface is defined inside the shell element, called the coordinate surface. The position of the points on this surface is determined by the Gaussian curvilinear coordinates $\alpha_1$ and $\alpha_2$ directed along the lines of principal curvature. In this case, $\alpha_1$ is directed along the guide, and $\alpha_2$ is directed along the generatrix of cylindrical element. The coordinate, which determines the distance from a certain point of the shell element to the coordinate surface is directed so that the coordinate system $O \alpha_1 \alpha_2 z$ forms right-hand orthogonal coordinate system.

Next, consider a thin-walled axisymmetric shell structure. Link the global right-hand rectangular coordinate system $O x_1 x_2 y$ with this structure (Figure 3). The $x_1$ axis is directed along the structure axis of rotation. The structure is presented as an arbitrary composition of $N_r$ annular nodal elements, $N_s$ shells of revolution, and $N_e$ viscoelastic links (Figure 4).

The numbering of nodes, shell elements and links, as well as the indexation of all quantities related to nodes, shell elements and links, is carried out by analogy with prismatic structures. It is known that the internal geometry of the coordinate surface can be characterized by the first quadratic form.

![Figure 3. Design scheme of shell structure](image)

Figure 3. Design scheme of shell structure

![Figure 4. Nodal and shell elements](image)

Figure 4. Nodal and shell elements

If the coordinates $\alpha_1$ and $\alpha_2$ correspond to the lines of principal curvature, then the differentials of the coordinate lines arcs can be expressed in terms of the differentials of curvilinear coordinates

$$dS_1 = A_1 d\alpha_1, \quad dS_2 = A_2 d\alpha_2,$$

where $A_1$ and $A_2$ are the Lame coefficients. The external surface geometry in the selected coordinate system is given by

$$w = f(\alpha_1, \alpha_2).$$
system is characterized by the main radii of curvature \( R_1 \) and \( R_2 \) (or by the main curvatures \( H_1 = \frac{1}{R_1} \) and \( H_2 = \frac{1}{R_2} \)).

The quantities \( A_1(\alpha_1, \alpha_2), A_2(\alpha_1, \alpha_2), H_1(\alpha_1, \alpha_2), H_2(\alpha_1, \alpha_2) \) must satisfy the Gauss-Codazzi relations known from the theory of surfaces

\[
\frac{\partial}{\partial \alpha} (K_2 A_2) = K_1 \frac{\partial A_2}{\partial \alpha}, \quad (1 \parallel z),
\]

\[
\frac{\partial}{\partial \alpha} \left( \frac{1}{A_1} \frac{\partial A_2}{\partial \alpha_1} \right) + \frac{\partial}{\partial \alpha_2} \left( \frac{1}{A_2} \frac{\partial A_1}{\partial \alpha_2} \right) = -K_1 K_2 A_1 A_2
\]

(2)

In the case of axisymmetric shells, the quantities \( A_1, A_2, K_1, K_2 \) do not depend on coordinate \( \alpha_2 \) and depend only on coordinate \( \alpha_1 \) directed along the generatrix. Gauss-Codazzi relations are simplified

\[
\frac{1}{A_1} \frac{\partial}{\partial \alpha_1} = (K_1 - K_2) \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_2},
\]

\[
\frac{1}{A_1} \frac{\partial}{\partial \alpha_1} = \left( \frac{1}{A_1} \frac{\partial A_1}{\partial \alpha_1} \right) = -K_1 K_2 A_2.
\]

(3)

Let us determine the position of points on the coordinate surface of the considered shell element by the Gaussian curvilinear coordinates \( \alpha_1, \alpha_2 \); their positive directions are shown in Fig. 5. Introduce the following notation [12]

\[
(\ldots)' = \frac{1}{A_1} \frac{\partial (\ldots)}{\partial \alpha_1}, \quad (\ldots)' = \frac{1}{A_2} \frac{\partial (\ldots)}{\partial \alpha_2}, \quad \psi = \frac{1}{A_2} \cdot A_2
\]

(4)

The rotations of the normal to coordinate surface of the shell element can be expressed in terms of the displacements of the points of coordinate surface \( u, v, \omega \) and \( \omega \) by formulas

\[
\theta_1 = -\omega + K_1 u, \quad \theta_2 = -\omega + K_2 v
\]

(5)

The components of the bending strain of coordinate surface (changes in curvature and torsion) are related to the displacements \( u, v, \omega \) and rotations \( \theta_1 \) and \( \theta_2 \) by relationships

\[
E_{11} = u' + K_1 \omega, \quad E_{22} = v' + \psi u + K_2 \omega,
\]

\[
E_{12} = E_{21} = v' - \psi v + u
\]

(6)

By direct substitution of relationships (5) into the torsion expressions of coordinate surface \( K_{12} \) and \( K_{21} \) using the Gauss-Codazzi relations (3), it is easy to see that \( K_{12} = K_{21} \). The components of strain at a point of the shell element that at a distance \( z \) from the coordinate surface are related to the components of tangential and bending strain of this surface by relations

\[
\varphi_{z} = -u' - K_2 \omega,
\]

\[
\epsilon_{11} = E_{11} + Z \cdot K_{11} (1 \parallel z), \quad \epsilon_{12} = E_{12} + 2Z \cdot K_{12}
\]

(8)

Consider a circular ring, the cross section of which is assumed to be small in comparison with the distance \( L \) from the axis of rotation to the line of centers of gravity (the midline), as a design scheme of the annular element of axisymmetric shell structure. It is assumed that the plane of the cross section is strain-free [12].

The position of the ring point is determined by the coordinates \( x, \alpha_2 \) and \( z \). The displacements of the point located on the ring midline in directions \( x, \alpha_2 \) and \( z \) are denoted by \( u, v, \omega \) respectively, and the rotation of ring cross section relative to this line - by \( \varphi \). Rotations of the ring cross section relative to the \( z \) and \( x \) axes resulting from the ring strain are expressed through the displacements of the ring midline according to formulas

\[
\varphi_x = -u', \quad \varphi_z = -\omega + K_2 \omega
\]

(9)

where

\[
(\ldots) = \frac{1}{r} \frac{\partial (\ldots)}{\partial \alpha_2}, \quad K_r = \frac{1}{r}
\]

(10)
The extension of the ring midline is expressed through the displacements of this line by formula
\[ \varepsilon = v + K_\gamma \omega \]  
(11)

Changes in curvature and torsion of the ring midline and the rotation of the ring cross section relative to the axis \( \alpha_2 \) can be expressed through the displacements of this line by formulas
\[ x_x = -u - K_\gamma \varphi, \quad x_z = -\omega - K_\gamma \nu, \quad x = -\varphi - K_\gamma \omega \]  
(12)

The strains of the ring point with the coordinates \( x, \alpha_2 \) and \( z \) are determined by formula
\[ \varepsilon(x, \alpha_2, z) = \varepsilon + x \cdot x_x + z \cdot x_z \]  
(13)

As a design scheme of connections between nodal elements, consider the links of spring types [12]. The displacements of the origin of connections in directions \( x, \alpha_2 \) and \( z \), we denote by \( u_0, \nu_0, \omega_0 \) the rotation relative to the axes \( \alpha_2 \) by \( \theta_0 \). Similar quantities for the end of the connection are denoted by \( u_\nu, \nu_\nu, \omega_\nu, \theta_\nu \). Then changes in connections lengths in the directions \( x, \alpha_2 \) and \( z \) are calculated by formulas
\[ \Delta u = u_k - u_\nu, \quad \Delta \nu = \nu_k - \nu_\nu, \quad \Delta \omega = \omega_k - \omega_\nu \]  
(14)

and the change in the angle of rotation according to formula
\[ \Delta \theta = \theta_k - \theta_\nu \]  
(15)

The conditions of continuity of displacements between shell and nodal elements, links and nodal elements are not presented here, because from [12] it is possible and easy to extend all the obtained relations to structurally inhomogeneous shell structures.

Establish just the relationship between vector (16) generalized displacements \( ijs \) of a spring-type element adjacent to the \( i \)-th annular element and the vector \( \Delta_i \) of generalized displacements of this annular element.

\[ V_i = [u_\nu, \nu_\nu, \omega_\nu, \theta_\nu]^T \]

Since the axes of a spring type element (Figure 5) are parallel to the global axes of the structure, this relationship is established by relation \( V_i = [f_i]^T \Delta_i \), in which the matrix \( [f_i] \) is calculated by formula (9), where \( f_i = \frac{\pi}{2} \).

### 3. Results and Discussions

Let each shell element of the considered structure be affected by loads \( q_1^p, q_2^p, q_3^p \) distributed over the coordinate surface. Assume that for each annular element of the structure the external loads reduced to the midline of this element are applied.

To obtain the equilibrium equations of the structure, the variational Lagrange equation is used:
Where $\delta \mathcal{E}_p$ is the variation of the potential strain energy of the $p$-th shell element; $\delta \mathcal{E}_i$ is the variation of the potential strain energy of the $i$-th annular element; $\delta \mathcal{E}_e$ is the variation of the potential strain energy of the $e$-th viscoelastic link; $\delta A_p$ is the elementary work of external loads applied to the $p$-th shell element; $\delta A_i$ is the elementary work of external loads applied to the $i$-th annular element.

Introduce the displacement vector $\vec{U}_p = [u_p, v_p, w_p]$, the component of which is the displacements of the points of coordinate surface of the $p$-th shell element in directions $\alpha_1$, $\alpha_2$, and $z$ respectively, the vector $\vec{U}_i = [u_i \omega \varphi_i v_i \varphi_i_x \varphi_i_z]^T$ of generalized displacements of the midline of annular element and vectors

\[
V_{ne} = [u_{ne} \omega_{ne} \theta_{ne} v_{ne}]^T,
V_{ke} = [u_{ke} \omega_{ke} \theta_{ke} v_{ke}]^T
\]

(18)

generalized displacements at the beginning and end of a viscoelastic link are with order number $e$. Then the elementary work of external loads applied to the $p$-th shell element can be written in the form

\[
\delta A_p = \int_{\alpha_{10}}^{\alpha_{1e}} \phi(q_p, \delta U_p) A_1^p A_2^p d\alpha_1^p d\alpha_2^p
\]

(19)

and the elementary work of external loads applied to the $i$-th annular element in the form $\delta A_i = \phi(f_i, \delta U_i) r_i d\alpha_2$.

The variation of the potential strain energy of the $p$-th shell element can be represented as

\[
\delta \mathcal{E}_p = \int_{\alpha_{10}}^{\alpha_{1e}} \phi(N_p, \delta e_p) A_1^p A_2^p d\alpha_1^p d\alpha_2^p
\]

(20)

The variation of the potential strain energy of the $i$-th annular element is presented in the form $\delta \mathcal{E}_i = \phi(Q_{ki}, \delta e_i) r_i d\alpha_2$.

Variation of the potential strain energy of the $e$-th viscoelastic link can be represented in the form

\[
\delta \mathcal{E}_e = \phi((N_{ce}, \delta V_{ke}) - (N_{ee}, \delta V_{ne})) r_e d\alpha_2
\]

(21)

The expression for $\delta A_i$ is given in the form

\[
\delta A_i = \phi \left( \| \vec{\theta}_i \| f_i, \delta A_i \right) r_i d\alpha_2,
\]

\[
|\vec{\theta}_i| = \begin{pmatrix} 1 & 0 & 0 & 0 & (\ldots) & 0 \\ 0 & 1 & 0 & 0 & 0 & (\ldots) \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_{r_i} \end{pmatrix}
\]

(22)

Using the Green’s formulas and equality

\[
\int_{\alpha_{10}}^{\alpha_{1e}} F_1 \frac{\partial F_2}{\partial \alpha_1} d\alpha_1 = F_1 F_2 \int_{\alpha_{10}}^{\alpha_{1e}} \frac{\partial F_1}{\partial \alpha_1} F_2 d\alpha_1
\]

expression for $\delta \mathcal{E}_p$ is given in the form

\[
\delta \mathcal{E}_p = \phi \int_{\alpha_{10}}^{\alpha_{1e}} \phi(L_p, \delta U_p) A_1^p A_2^p d\alpha_1^p d\alpha_2^p + \phi \left( \frac{Q_p}{\alpha_{1e}^p}, \frac{\delta W_p}{\alpha_{1e}^p} \right) A_2^p (\alpha_{1e}^p) d\alpha_2 - \\
- \phi \left( \frac{Q_p}{\alpha_{10}^p}, \frac{\delta W_p}{\alpha_{10}^p} \right) A_2^p (\alpha_{10}^p) d\alpha_2
\]

(23)

\[
Q_p = \begin{pmatrix} T_{11p} \\ Q_{11p} + H_p \\ M_{11p} \\ S_p + 2K_{2p}, H_p \end{pmatrix}, ~ W_p = \begin{pmatrix} u_p \\ w_p \\ \theta_{1p} \\ v_p \end{pmatrix}
\]
is the vector of generalized forces and the vector of generalized displacements of the boundary contours of shell element, respectively, adjacent to the corresponding annular elements of the structure. The positive directions of these forces and displacements coincide with the positive directions of the corresponding internal forces and displacements in the shell element. The components of the vector \( L_p \) in expression for \( \delta \mathcal{E}_p \) have the following form (for the simplicity index \( p \) is omitted)

\[
L_1 = T_{11}^I + \psi(T_{11} - T_{22}) + S^I + K_1(Q_{11} + H^I),
\]
\[
L_2 = S^I + 2\psi(S + K_1H) + T_{22} + K_2(Q_{22} + H^I),
\]
\[
L_3 = Q_{11}^I + \psi Q_{11} + Q_{22} - K_1T_{11} - K_2T_{22}.
\]

where

\[
Q_{11} = M_{11}^I + \psi(M_{11} - M_{22}) + H^I,
\]
\[
Q_{22} = H^I + 2\psi H + M_{22}.
\]

Using the Green's formula, the expression for \( \delta \mathcal{E}_i \) is presented in the form

\[
\delta \mathcal{E}_i = -\phi(L_i, \delta \Delta_i)\eta_i d\alpha_2.
\]

where (for the simplicity index \( i \) is omitted)

\[
L_{r1} = M_{r1}^I - K_{r1}M^I,
\]
\[
L_{r2} = M_{r2}^I - K_{r2}T^I,
\]
\[
L_{r3} = M^I + K_rM^I,
\]
\[
L_{r4} = T^I + K_rM^I.
\]

Substituting the obtained expressions into the variational Lagrange equation (24) we obtain:

\[
- \sum_{p=1}^{N_p} \int_{\alpha_{1e}^{p}}^{\alpha_{10}^{p}} \phi((L_p, \delta U_p) + (q_p, \delta U_p))A_1^p \delta \mathcal{W}_{1}^p d\alpha_1^p d\alpha_2 - \sum_{p=1}^{N_p} \phi \frac{Q_p}{\alpha_{1e}^p, \alpha_{10}^p} A_2(\alpha_{1e}^p)A_2^p d\alpha_2 - \sum_{p=1}^{N_p} \phi \frac{\delta W_p}{\alpha_{10}^p, \alpha_{11}^p} A_2(\alpha_{10}^p) d\alpha_2 - \sum_{i=1}^{N_1} \phi[(t_1^i, \delta \Delta_i) + (\|\theta_i\|f_1, \delta \Delta_i)]\eta_i d\alpha_2 + \sum_{p=1}^{N_r} \phi[(N_{ce}, \delta V_{1e}) - (N_{ce}, \delta V_{1e})]r_2 d\alpha_2 = 0
\]

In the second and third terms of relationship (26), the summation over \( p \) is replaced by summation over \( i \), the index \( p \) by the indices \( i j s \). For the \( i j s \) -th shell element adjacent to the \( i \)-th annular element, we calculate the integral

\[
J = \phi(Q_i^{ijs}, \delta W_i^{ijs})A_2(\alpha_{1i}^{ijs}) d\alpha_2
\]

It is obvious that \( A_2(\alpha_{1i}^{ijs}) = r_i (1 + \frac{z_i^{ijs}}{r_i}), \) where \( z_i^{ijs} \) - is the coordinate of the contact point of \( ijs \)-th shell element adjacent to the \( i \)-th annular element, related to the vector of generalized displacements \( \Delta_i \) of this annular element by relation \( W_i^{ijs} = [\bar{\phi}_i^{ijs}]^T \Delta_i \).}

Substituting the relationships for \( A_2(\alpha_{1i}^{ijs}) \) and \( W_i^{ijs} \) into expression (27) we obtain:

\[
J = \left(1 + \frac{z_i^{ijs}}{r_i}\right) \phi(Q_i^{ijs}, [\bar{\phi}_i^{ijs}]^T \delta \Delta_i)\eta_i d\alpha_2
\]

It is easy to see that

\[
\phi(Q_i^{ijs}, [\bar{\phi}_i^{ijs}]^T \delta \Delta_i)\eta_i d\alpha_2 = \phi([[\eta_i^{ijs}]Q_i^{ijs}, \delta \Delta_i)\eta_i d\alpha_2
\]

\[
[[\eta_i^{ijs}]] = \begin{bmatrix}
\sin\gamma_i & \cos\gamma_i & 0 & x_i(\ldots) \\
\cos\gamma_i & \sin\gamma_i & 0 & z_i(\ldots) \\
(x_i \sin\gamma_i - x_i \cos\gamma_i) & (-z_i \cos\gamma_i - x_i \sin\gamma_i) & 1 & 0 \\
0 & 0 & 0 & 1 + K_r z_i
\end{bmatrix}
\]

(30)
In deriving equality (30), the Green formula was used. In the fifth term of expression (26), the summation over \( i \) replaced by summing over \( p \), and the index \( p \) by the indices \( ijs \)-th viscoelastic link, by analogy with shell elements

\[
\phi(N^{ijs}, \Delta V^{ijs})r_i da_i = \phi([\hat{\eta}^{ijs}_i]N^{ijs}_c, \Delta \lambda_i) \left( 1 + \frac{z^{ijs}_c}{r_i} \right) r_i da_i \tag{31}
\]

where the matrix \([\hat{\eta}^{ijs}_c] = [\hat{\eta}^{ijs}_c] \) at \( \gamma = \frac{\pi}{2} \)

Introducing to the second and fifth terms of expression (17) the integrals over the middle surface of the annular elements, we obtain the variational Lagrange equation with complex coefficients in the form:

\[
- \sum_{p=1}^{N_e} \int_{a_2^0}^{a_2^e} \phi\left((L_p, \Delta U_p) + (q_p, \delta U_p)\right) A^p_1 A^p_2 da_2 - \sum_{i=1}^{N_r} \left[ \phi\left(L_i^r, \delta \lambda_i\right) + (\|\hat{\theta}_i\| f_i, \delta \lambda_i) \right] + \sum_j \sum_s (\xi^{ijs}_c [\hat{\eta}^{ijs}_c] Q^{ijs}_i, \delta \lambda_i) + \sum_j \sum_s (\xi^{ijs}_c [\hat{\eta}^{ijs}_c] N^{ijs}_c, \delta \lambda_i) r_i da_2 \tag{32}
\]

where \( \xi^{ijs} = (1 + K_{rij} z_i^{ijs} \text{sign}(j - i)(\alpha_i^{ijs} - \alpha_{10}^{ijs})) \),

\( \xi^{ijs}_c = (1 + K_{cij} z_c^{ijs} \text{sign}(j - i)) \)

At independent variations of \( \Delta U_p \) in the surface coordinate of the \( p \)-th shell element and independent variations of \( \delta \lambda_i \) in the midline of the \( i \)-th annular element, from the variational Lagrange equation with complex coefficients a system of interconnected equilibrium equations of structurally inhomogeneous shell structures is obtained

\[
L_p + q_p = 0 \quad (p = 1, 2, ..., N_e),
\]

\[
L_i^r + \|\hat{\theta}_i\| f_i + \sum_j \sum_s (\xi^{ijs}_c [\hat{\eta}^{ijs}_c] Q^{ijs}_i) + \sum_j \sum_s (\xi^{ijs}_c [\hat{\eta}^{ijs}_c] N^{ijs}_c) = 0, \quad (i = 1, 2, ..., N_r) \tag{33}
\]

as well as the conditions of continuity of displacements and relations of linear strains of multiply connected structurally inhomogeneous axisymmetric shell structures. Summation in equations (33) is conducted over all shell elements adjacent to the \( i \)-th annular element.

The equations of motion for an arbitrary change in external loads over time are obtained from the equations of statics (33) using the d'Alembert principle. As a result, the following system of coupled equations of motion of a multiply connected structurally inhomogeneous shell structure is obtained in a complex form:

\[
L_p + q_p = (\tau) - [\bar{\rho}_p] \frac{\partial^2 U_p}{\partial \tau^2} = 0 \quad (p = 0, ..., N_e) \tag{34}
\]

\[
L_i^r + \|\hat{\theta}_i\| f_i (\tau) - [G_{\omega}] \frac{\partial^2 \Delta \lambda_i}{\partial \tau^2} = \sum_j \sum_s (\xi^{ijs}_c [\hat{\eta}^{ijs}_c] Q^{ijs}_i) + \sum_j \sum_s (\xi^{ijs}_c [\hat{\eta}^{ijs}_c] N^{ijs}_c) = 0, \quad (i = 1, ..., N_r) \tag{35}
\]

\[
[G_{\omega}] = \rho_i F_i \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad [\bar{\rho}_p] = \tilde{\rho}_p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};
\]

\[
q_p = q_{p0} \sin \omega_R \tau; \quad f_i = f_{i0} \sin \omega_R \tau
\]
where $\omega_R$ is the circular frequency of external loads. The equations of forced harmonic vibrations of shell structures with complex coefficients have the form

$$L_p + q_{p_0} + \omega_R^2 [\overline{\rho}_p] U_p = 0, \quad (p = 1, \ldots, N_s) \quad (36)$$

$$L'_r + \|\overline{\theta}_i\| f_{i0} + \omega_R^2 [G_{\omega}] \Delta_i + \sum_j \sum_s \xi_{ijs}^{ij} \overline{\eta}_i^{ij} Q_{ij}^{js} + \sum_j \sum_s \xi_{c_t}^{ij} \overline{\eta}_c^{ij} N_{c_t}^{ij} = 0, \quad (i = 1, \ldots, N_r) \quad (37)$$

In the problem of natural vibrations of structures, the solution to equations (34), (35) is sought in the form:

$$U_p = U_p e^{i\tilde{\omega} t}, \quad \Delta_i = \Delta_i e^{i\omega t} \quad (38)$$

Here, $\tilde{\omega} = \omega_R + i \omega_I$ is the complex value of vibration frequency, the real part $\omega_R$ represents the frequency of natural vibrations, and $\omega_I$ is the damping coefficient. Equations of natural vibrations of structures have the form:

$$L_p + \tilde{\omega}^2 [\overline{\rho}_p] U_p = 0, \quad (p = 1, \ldots, N_s) \quad (39)$$

$$L'_r + \tilde{\omega}^2 [G_{\omega}] \Delta_i + \sum_j \sum_s \xi_{ijs}^{ij} \overline{\eta}_i^{ij} Q_{ij}^{js} + \sum_j \sum_s \xi_{c_t}^{ij} \overline{\eta}_c^{ij} N_{c_t}^{ij} = 0, \quad (i = 1, \ldots, N_r) \quad (40)$$

The values of $\tilde{\omega}$ for which the nontrivial solution of the system by complex coefficients (36) are the complex values of eigen frequencies of the considered structurally inhomogeneous shell structures. Let us consider in more detail the mathematical sense of (36), (37). Equations (36) are the contact equations for the motion of multilayer elastic shells and prismatic shells of a non-circular section. Each of the equations describes the behavior of an individual shell element in a wall shell structure. In our case, the difference in elemental equations is fundamental and consists in the fact that the solutions of the equations are complex due to the complexity of the relationships and describes structural heterogeneity. A complete ensemble of equations with complex coefficients (38) - (40) describes the motion of a multiply connected structurally inhomogeneous shell structure, composed in general case from a set of multilayer elastic and viscoelastic shells, hereditary links, frames or stringers with substantially different rheological properties, under joint work of all elements of the structure. No restrictions are imposed on this ensemble of equations, except for the subject of the closed cycle condition for viscoelastic elements and structural links. Based on the developed numerical-analytical algorithms, the calculation is conducted.

As an example, consider a structurally inhomogeneous shell structure - a tank on a special support, representing a torus-cylindrical shell held by a shell of truncated cone type, fixed at the end (Figure 6). The dimensions of the structure shown in the figure are: the torus-cylindrical shell is elastic $E = 2 \cdot 10^{11} N/m^2$; $\nu = 0.3$; $\rho_1 = 7.8 \cdot 10^3 kg/cm^3$ the thickness along the contour is constant and equal to $h = 0.003 m$. The special support (truncated cone) is visco-elastic one, its properties ($E = 2 \cdot 10^{11} N/m^2$; $\nu = 0.3$; $\rho_1 = 7.8 \cdot 10^3 kg/cm^3$) and rheological characteristics are described by a difference kernel with parameters: $A = 0.01$; $\alpha = 0.1$; $\beta = 0.05$. As a parameter of structural inhomogeneity of the support geometry, its thickness varied in the range from 0.001 m to 0.008 m.
Figure 7 shows the results of calculated dependences of three damping coefficients $\omega_{11}, \omega_{12}, \omega_{13}$ of lower modes of vibration on the thickness of special support. The calculation results show that, at the beginning, $\omega_{11}$, and then $\omega_{12}$, act as the determining damping coefficient. At the intersection of the calculated curves, the dissipative properties of the structure manifest themselves most intensively, i.e. the synergetic effect of viscoelastic properties is manifested [1]. An analysis of lower eigen frequencies (Figure 7) shows that in the vicinity of optimal value of the structural inhomogeneity parameter $h$, the frequencies of the corresponding eigen modes ($\omega_{R1}$ and $\omega_{R2}$) tend to converge, which confirms the results obtained for plastic structures and laminated plates [21] This circumstance is of fundamental importance for making recommendations on design of a tank support. Engineering implementation of a tank support of thickness $h$ (see Figure 7) allows us to create the most rational design from the point of view of damping external mechanical impact with frequencies $\omega_{R1}$ and $\omega_{R2}$ specified by the conditions of the product’s operation. Engineering implementation of the design with set determining damping coefficients of the support turned out to be possible.

4. Conclusion
Studies of dynamic behavior of multiply connected structurally inhomogeneous shell structures, considering viscoelastic properties of the material, allow us to draw the following conclusions:
1. It was established that it is possible in principle to significantly intensify dissipative processes in structurally inhomogeneous vibrational systems and to lower the resonance amplitudes of the most dangerous modes of vibration due to convergence of the corresponding eigen frequencies, which makes it possible to develop a general approach to the fundamentals of the synthesis of optimal dissipative properties.
2. Studies have shown that in structurally inhomogeneous systems, the role of rheology is reduced to vibration damping and mutually increasing interactions of different modes of vibrations, which significantly increase the dissipative properties of the system. In this case, the closer the frequencies of vibration of corresponding modes the stronger the interaction, which is promising for the synthesis of structurally inhomogeneous engineering structures that are optimal in terms of dissipative properties and material consumption.

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