The Muon \( (g - 2) \) Theory Value: Present and Future

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Abstract

This White Paper briefly reviews the present status of the muon \( (g - 2) \) Standard-Model prediction. This value results in a 3 – 4 standard-deviation difference with the experimental result from Brookhaven E821. The present experimental uncertainty is \( \pm 63 \times 10^{-11} \) (0.54 ppm), and the Standard-Model uncertainty is \( \simeq \pm 49 \times 10^{-11} \). Fermilab experiment E989 has the goal to reduce the experimental error to \( \pm 16 \times 10^{-11} \). Improvements in the Standard-Model value, which should be achieved between now and when the first results from Fermilab E989 could be available, should lead to a Standard-Model uncertainty of \( \sim \pm 35 \times 10^{-11} \). These improvements would halve the uncertainty on the difference between experiment and theory, and should clarify whether the current difference points toward New Physics, or to a statistical fluctuation. At present, the \( (g - 2) \) result is arguably the most compelling indicator of physics beyond the Standard Model and, at the very least, it represents a major constraint for speculative new theories such as supersymmetry, dark gauge bosons or extra dimensions.
1 Introduction

The Standard-Model (SM) value of the muon anomaly can be calculated with sub-parts-per-million precision. The comparison between the measured and the SM prediction provides a test of the completeness of the Standard Model. At present, there appears to be a three- to four-standard deviation between these two values, which has motivated extensive theoretical and experimental work on the hadronic contributions to the muon anomaly.

A lepton ($\ell = e, \mu, \tau$) has a magnetic moment which is along its spin, given by the relationship

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}, \quad g_\ell = 2(1 + a_\ell), \quad a_\ell = \frac{g_\ell - 2}{2}$$

where $Q = \pm 1$, $e > 0$ and $m_\ell$ is the lepton mass. Dirac theory predicts that $g \equiv 2$, but experimentally, it is known to be greater than 2. The small number $a$, the anomaly, arises from quantum fluctuations, with the largest contribution coming from the mass-independent single-loop diagram in Fig. 1(a). With his famous calculation that obtained $a = (\alpha/2\pi) = 0.00116 \ldots$, Schwinger [1] started an “industry”, which required Aoyama, Hayakawa, Kinoshita and Nio to calculate more than 12,000 diagrams to evaluate the tenth-order (five loop) contribution [2].

![Feynman graphs](image)

Figure 1: The Feynman graphs for: (a) The lowest-order (Schwinger) contribution to the lepton anomaly; (b) The vacuum polarization contribution, which is one of five fourth-order, $(\alpha/\pi)^2$, terms; (c) The schematic contribution of new particles $X$ and $Y$ that couple to the muon.

The interaction shown in Fig. 1(a) is a chiral-changing, flavor-conserving process, which gives it a special sensitivity to possible new physics [3 4]. Of course heavier particles can also contribute, as indicated by the diagram in Fig. 1(c). For example, $X = W^\pm$ and $Y = \nu_\mu$, along with $X = \mu$ and $Y = Z^0$, are the lowest-order weak contributions. In the Standard-Model, $a_\mu$ gets measurable contributions from QED, the strong interaction, and from the electroweak interaction,

$$a^{SM} = a^{QED} + a^{Had} + a^{Weak}.$$  \hspace{1cm} (2)

In this document we present the latest evaluations of the SM value of $a_\mu$, and then discuss expected improvements that will become available over the next five to seven years. The uncertainty in this evaluation is dominated by the contribution of virtual hadrons in loops. A worldwide effort is under way to improve on these hadronic contributions. By the time that the Fermilab muon $(g - 2)$ experiment, E989, reports a result later in this decade, the uncertainty should be significantly reduced. We emphasize that the existence of E821
at Brookhaven motivated significant work over the past thirty years that permitted more
than an order of magnitude improvement in the knowledge of the hadronic contribution.
Motivated by Fermilab E989 this work continues, and another factor of two improvement
could be possible.

Both the electron \([5]\) and muon \([6]\) anomalies have been measured very precisely:

\[
\begin{align*}
a_e^{\text{exp}} &= 1159652180.73(28) \times 10^{-12} \pm 0.24 \text{ ppb} \quad (3) \\
a_\mu^{\text{exp}} &= 116592089(63) \times 10^{-11} \pm 0.54 \text{ ppm} \quad (4)
\end{align*}
\]

While the electron anomaly has been measured to \(\simeq 0.3 \text{ ppb} \) (parts per billion) \([5]\), it is
significantly less sensitive to heavier physics, because the relative contribution of heavier
virtual particles to the muon anomaly goes as \((m_\mu/m_e)^2 \simeq 43000\). Thus the lowest-order
hadronic contribution to \(a_e\) is \([7]\): \(a_e^{\text{had.}LO} = (1.875 \pm 0.017) \times 10^{-12}\), 1.5 ppb of \(a_e\). For the
muon the hadronic contribution is \(\simeq 60 \text{ ppm} \) (parts per million). So with much less precision,
when compared with the electron, the measured muon anomaly is sensitive to mass scales
in the several hundred GeV region. This not only includes the contribution of the \(W\) and
\(Z\) bosons, but perhaps contributions from new, as yet undiscovered, particles such as the
supersymmetric partners of the electroweak gauge bosons (see Fig. 1(c)).

2 Summary of the Standard-Model Value of \(a_\mu\)

2.1 QED Contribution

The QED contribution to \(a_\mu\) is well understood. Recently the four-loop QED contribution
has been updated and the full five-loop contribution has been calculated \([2]\). The present
QED value is

\[
a_\mu^{\text{QED}} = 116584718.951 \times (0.009)(0.019)(0.007)(.077) \times 10^{-11} \quad (5)
\]

where the uncertainties are from the lepton mass ratios, the eight-order term, the tenth-
order term, and the value of \(\alpha\) taken from the \(^{87}\text{Rb}\) atom \(\alpha^{-1}(Rb) = 137.035999049(90)
[0.66 \text{ ppb}]\). \([8]\).
2.2 Weak contributions

The electroweak contribution (shown in Fig. 2) is now calculated through two loops \[9, 10, 11, 12, 13, 14\]. The one loop result

\[
a_{\mu}^{\text{EW}(1)} = \frac{G_F m_{\mu}^2}{\sqrt{2} 8\pi^2} \left\{ \frac{10}{3} \frac{m_{\mu}^2}{m_{W}^2} + \frac{1}{3} (1 - 4 \sin^2 \theta_W)^2 - \frac{5}{3} \right\} + \mathcal{O} \left( \frac{m_{\mu}^2}{M_{Z}^2} \log \frac{M_{Z}^2}{m_{\mu}^2} \right) + \frac{m_{\mu}^2}{M_{H}^2} \int_0^1 dx \frac{2x^2(2-x)}{1 - x + \frac{m_{\mu}^2}{M_{H}^2} x^2} \right\} = 194.8 \times 10^{-11},
\]

was calculated by five separate groups \[15\] shortly after the Glashow-Salam-Weinberg theory was shown by ’t Hooft to be renormalizable. Due to the small Yukawa coupling of the Higgs boson to the muon, only the $W$ and $Z$ bosons contribute at a measurable level in the lowest-order electroweak term.

![Figure 2: Weak contributions to the muon anomalous magnetic moment. Single-loop contributions from (a) virtual $W$ and (b) virtual $Z$ gauge bosons. These two contributions enter with opposite sign, and there is a partial cancellation. The two-loop contributions fall into three categories: (c) fermionic loops which involve the coupling of the gauge bosons to quarks, (d) bosonic loops which appear as corrections to the one-loop diagrams, and (e) a new class of diagrams involving the Higgs where $G$ is the longitudinal component of the gauge bosons. See Ref. \[16\] for details. The $\times$ indicates the photon from the magnetic field.](image)

The two-loop electroweak contribution (see Figs. 2(c-e)), which is negative \[11, 10, 9, 12\], has been re-evaluated using the LHC value of the Higgs mass \[14\]. The total electroweak contribution is

\[
a_{\mu}^{\text{EW}} = (153.6 \pm 1.0) \times 10^{-11}
\]

where the error comes from hadronic effects in the second-order electroweak diagrams with quark triangle loops, along with unknown three-loop contributions\[12, 17, 18, 19\]. The leading logs for the next-order term have been shown to be small \[12, 14\]. The weak contribution is about 1.3 ppm of the anomaly, so the experimental uncertainty on $a_{\mu}$ of $\pm 0.54$ ppm now probes the weak scale of the Standard Model.
### 2.2.1 Hadronic contribution

The hadronic contribution to $a_\mu$ is about 60 ppm of the total value. The lowest-order diagram shown in Fig. 3(a) dominates this contribution and its error, but the hadronic light-by-light contribution Fig. 3(e) is also important. We discuss both of these contributions below.

![Diagrams](image)

*Figure 3: The hadronic contribution to the muon anomaly, where the dominant contribution comes from the lowest-order diagram (a). The hadronic light-by-light contribution is shown in (e).*

![Diagrams](image)

*Figure 4: (a) The “cut” hadronic vacuum polarization diagram; (b) The $e^+e^-$ annihilation into hadrons; (c) Initial state radiation accompanied by the production of hadrons.*

The energy scale for the virtual hadrons is of order $m_\mu c^2$, well below the perturbative region of QCD. However it can be calculated from the dispersion relation shown pictorially in Fig. 4:

$$a_\mu^{\text{had};\text{LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\mu^2}^{\infty} \frac{dR(s)}{s^2} K(s) \sigma(s), \quad \text{where} \quad R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (8)$$

using the measured cross sections for $e^+e^- \rightarrow \text{hadrons}$ as input, where $K(s)$ is a kinematic factor ranging from 0.4 at $s = m_\mu^2$ to 0 at $s = \infty$ (see Ref. [16]). This dispersion relation relates the bare cross section for $e^+e^-$ annihilation into hadrons to the hadronic vacuum polarization contribution to $a_\mu$. Because the integrand contains a factor of $s^{-2}$, the values of $R(s)$ at low energies (the $\rho$ resonance) dominate the determination of $a_\mu^{\text{had};\text{LO}}$, however at the level of precision needed, the data up to 2 GeV are very important. This is shown in Fig. 5 where the left-hand chart gives the relative contribution to the integral for the different energy regions, and the right-hand gives the contribution to the error squared on the integral. The contribution is dominated by the two-pion final state, but other low-energy
multi-hadron cross sections are also important. These data for $e^+e^-$ annihilation to hadrons are also important as input into the determination of $\alpha_{QED}(M_Z)$ and other electroweak precision measurements.

Two recent analyses \cite{20,21} using the $e^+e^- \rightarrow \text{hadrons}$ data obtained:

\begin{align}
    a_{\mu}^{\text{had;LO}} &= (6.923 \pm 42) \times 10^{-11}, \\
    a_{\mu}^{\text{had;LO}} &= (6.949 \pm 43) \times 10^{-11},
\end{align}

respectively. Important earlier global analyses include those of Hagiwara et al. \cite{22}, Davier, et al., \cite{23}, Jegerlehner and Nyffler \cite{24}.

In the past, hadronic $\tau$ spectral functions and CVC, together with isospin breaking corrections have been used to calculate the hadronic contribution \cite{25,20}. While the original predictions showed a discrepancy between $e^+e^-$ and $\tau$ based evaluations, it has been shown that after $\gamma$-$\rho$ mixing is taken into account, the two are compatible \cite{26}. Recent evaluations based on a combined $e^+e^-$ and $\tau$ data fit using the Hidden Local Symmetry (HLS) model have come to similar conclusions and result in values for $a_{\mu}^{\text{HVP}}$ that are smaller than the direct evaluation without the HLS fit \cite{27,28}.

The most recent evaluation of the next-to-leading order hadronic contribution shown in Fig. 3(b-d), which can also be determined from a dispersion relation, is \cite{21}

\begin{equation}
    a_{\mu}^{\text{had;NLO}} = (-98.4 \pm 0.6_{\text{exp}} \pm 0.4_{\text{rad}}) \times 10^{-11}.
\end{equation}

### 2.2.2 Hadronic light-by-light contribution

The hadronic light-by-light contribution (HLbL) cannot at present be determined from data, but rather must be calculated using hadronic models that correctly reproduce properties of QCD. This contribution is shown below in Fig. 6(a). It is dominated by the long-distance contribution shown in Fig. 6(b). In fact, in the so called chiral limit where the mass gap between the pseudoscalars (Goldstone-like) particles and the other hadronic particles (the $\rho$ being the lowest vector state in Nature) is considered to be large, and to leading order
in the $1/N_c$–expansion ($N_c$ the number of colors), this contribution has been calculated analytically \[29\] and provides a long-distance constraint to model calculations. There is also a short-distance constraint from the operator product expansion (OPE) of two electromagnetic currents which, in specific kinematic conditions, relates the light-by-light scattering amplitude to an Axial-Vector-Vector triangle amplitude for which one has a good theoretical understanding \[30\].

Unfortunately, the two asymptotic QCD constraints mentioned above are not sufficient for a full model independent evaluation of the HLbL contribution. Most of the last decade calculations found in the literature are compatible with the QCD chiral and large-$N_c$ limits. They all incorporate the $\pi^0$-exchange contribution modulated by $\pi^0\gamma^*\gamma^*$ form factors correctly normalized to the Adler, Bell-Jackiw point-like coupling. They differ, however, on whether or not they satisfy the particular OPE constraint mentioned above, and in the shape of the vertex form factors which follow from the different models.

\[\begin{align*}
X^{\mu} + \text{Permutations} \\
q_{\pm} k_{\pm} \leftrightarrow \text{Permutations} \\
p_{\pm} \mu \\
H
\end{align*}\]

(a) Figure 6: (a) The Hadronic Light-by contribution. (b) The pseudoscalar meson contribution.

A synthesis of the model contributions, which was agreed to by authors from each of the leading groups that have been working in this field, can be found in ref. \[31\]. They obtained

\[a^{\text{HLbL}}_{\mu} = (105 \pm 26) \times 10^{-11}. \quad (12)\]

An alternate evaluation \[24, 32\] obtained, \[a^{\text{HLbL}}_{\mu} = (116 \pm 40) \times 10^{-11}\], which agrees well with the Glasgow Consensus \[31\]. Additional work on this contribution is underway on a number of fronts, including on the lattice. A workshop was held in March 2011 at the Institute for Nuclear Theory in Seattle \[33\] which brought together almost all of the interested experts. This will be followed by a workshop at the Mainz Institute for Theoretical Physics in April 2014.

One important point should be stressed here. The main physics of the hadronic light-by-light scattering contribution is well understood. In fact, but for the sign error unraveled in 2002, the theoretical predictions for \[a^{\text{HLbL}}_{\mu}\] have been relatively stable for more than ten years\[2\].

\[1\] This compilation is generally referred to as the “Glasgow Consensus” since it grew out of a workshop in Glasgow in 2007. [http://www.ippp.dur.ac.uk/old/MuonMDM/]

\[2\] A calculation using a Dyson-Schwinger approach \[34\] initially reported a much larger value for the HLbL...
2.3 Summary of the Standard-Model Value and Comparison with Experiment

We determine the SM value using the new QED calculation from Aoyama [2]; the electroweak from Ref. [3], the hadronic light-by-light contribution from the “Glasgow Consensus” [31]; and lowest-order hadronic contribution from Davier, et al., [20], or Hagiwara et al., [21], and the higher-order hadronic contribution from Ref. [21]. A summary of these values is given in Table 1.

Table 1: Summary of the Standard-Model contributions to the muon anomaly. Two values are quoted because of the two recent evaluations of the lowest-order hadronic vacuum polarization.

| Value                  | Value $\times 10^{-11}$ units |
|------------------------|-------------------------------|
| QED $(\gamma + \ell)$  | $116\,584\,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077\alpha$ |
| HVP(lo) [20]           | $6\,923 \pm 42$              |
| HVP(lo) [21]           | $6\,949 \pm 43$              |
| HVP(ho) [21]           | $-98.4 \pm 0.7$              |
| HLbL                   | $105 \pm 26$                 |
| EW                     | $154 \pm 1$                  |
| Total SM [20]          | $116\,591\,802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}} (\pm 49_{\text{tot}}$ |
| Total SM [21]          | $116\,591\,828 \pm 43_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}} (\pm 50_{\text{tot}}$ |

This SM value is to be compared with the combined $a^+_{\mu}$ and $a^-_{\mu}$ values from E821 [6] corrected for the revised value of $\lambda = \mu^2 / \mu^p$ from Ref. [35],

$$a^\text{E821}_{\mu} = (116\,592\,089 \pm 63) \times 10^{-11} \quad (0.54 \text{ ppm}), \quad (13)$$

which give a difference of

$$\Delta a_{\mu}(\text{E821} - \text{SM}) = (287 \pm 80) \times 10^{-11} \quad [20]$$

$$= (261 \pm 78) \times 10^{-11} \quad [21]$$

depending on which evaluation of the lowest-order hadronic contribution that is used [20, 21].

This comparison between the experimental values and the present Standard-Model value is shown graphically in Fig. 7. The lowest-order hadronic evaluation of Ref. [28] using the hidden local symmetry model results in a difference between experiment and theory that ranges between 4.1 to 4.7σ.

This difference of 3.3 to 3.6 standard deviations is tantalizing, but we emphasize that whatever the final agreement between the measured and SM value turns out to be, it will have significant implications on the interpretation of new phenomena that might be found at the LHC and elsewhere. Because of the power of $a_{\mu}$ to constrain, or point to, speculative models

contribution. Subsequently a numerical mistake was found. These authors are continuing this work, but the calculation is still incomplete.
of New Physics, the E821 results have been highly cited, with more than 2450 citations to date.

3 Expected Improvements in the Standard-Model Value

The present uncertainty on the theoretical value is dominated by the hadronic contributions \[20, 21\] (see Table \[1\]). The lowest-order contribution determined from \(e^+e^- \rightarrow \text{hadrons}\) data using a dispersion relation is theoretically relatively straightforward. It does require the combination of data sets from different experiments. The only significant theoretical uncertainty comes from radiative corrections, such as vacuum polarization (running \(\alpha\)), along with initial and final state radiation effects, which are needed to obtain the correct hadronic cross section at the required level of precision. This was a problem for the older data sets. In the analysis of the data collected over the past 15 years, which now dominate the determination of the hadronic contribution, the treatment of radiative corrections has been significantly improved. Nevertheless, an additional uncertainty due to the treatment of these radiative corrections in the older data sets has been estimated to be of the order of \(20 \times 10^{-11}\) \[21\]. As more data become available, this uncertainty will be significantly reduced.

There are two methods that have been used to measure the hadronic cross sections: The energy scan (see Fig. \[4\](b)), and using initial state radiation with a fixed beam energy to measure the cross section for energies below the total center-of-mass energy of the colliding beams (see Fig. \[3\](c)). Both are being employed in the next round of measurements. The data from the new experiments that are now underway at VEPP-2000 in Novosibirsk and BESIII in Beijing, when combined with the analysis of existing multi-hadron final-state data from
BaBar and Belle, should significantly reduce the uncertainty on the lowest-order hadronic contribution.  

The hadronic-light-by-light contribution does not lend itself to determination by a dispersion relation. Nevertheless there are some experimental data that can help to pin down related amplitudes and to constrain form factors used in the model calculations.

### 3.1 Lowest-order Hadronic Contribution

Much experimental and theoretical work is going on worldwide to refine the hadronic contribution. The theory of \((g - 2)\), relevant experiments to determine the hadronic contribution, including work on the lattice, have featured prominently in the series of tau-lepton workshops and PHIPSI workshops which are held in alternate years. Over the development period of Fermilab E989, we expect further improvements in the SM-theory evaluation. This projection is based on the following developments:

#### 3.1.1 Novosibirsk

The VEPP2M machine has been upgraded to VEPP-2000. The maximum energy has been increased from \(\sqrt{s} = 1.4 \text{ GeV}\) to 2.0 GeV. Additionally, the SND detector has been upgraded and the CMD2 detector was replaced by the much-improved CMD3 detector. The cross section will be measured from threshold to 2.0 GeV using an energy scan, filling in the energy region between 1.4 GeV, where the previous scan ended, up to 2.0 GeV, the lowest energy point reached by the BES collaboration in their measurements. See Fig. 5 for the present contribution to the overall error from this region. Engineering runs began in 2009, and data collection started in 2011. So far two independent energy scans between 1.0 and 2.0 GeV were performed in 2011 and 2012. The peak luminosity of \(3 \times 10^{31} \text{cm}^{-2}\text{s}^{-1}\) was achieved, which was limited by the positron production rate. The new injection facility, scheduled to be commissioned during the 2013-2014 upgrade, should permit the luminosity to reach \(10^{32} \text{cm}^{-2}\text{s}^{-1}\). Data collection resumed in late 2012 with a new energy scan over energies below 1.0 GeV. The goal of experiments at VEPP-2000 is to achieve a systematic error 0.3-0.5% in the \(\pi^+\pi^-\) channel, with negligible statistical error in the integral. The high statistics, expected at VEPP-2000, should allow a detailed comparison of the measured cross-sections with ISR results at BaBar and DAΦNE. After the upgrade, experiments at VEPP-2000 plan to take a large amount of data at 1.8-2 GeV, around the \(N\bar{N}\) threshold. This will permit ISR data with the beam energy of 2 GeV, which is between the PEP2 energy at the \(\Upsilon(4S)\) and the 1 GeV \(\phi\) energy at the DAΦNE facility in Frascati. The dual ISR and scan approach will provide an important cross check on the two central methods used to determine the HVP.

#### 3.1.2 The BESIII Experiment

The BESIII experiment at the Beijing tau-charm factory BEPC-II has already collected several femtobarns of integrated luminosity at various centre-of-mass energies in the range 3 - 4.5 GeV. The ISR program includes cross section measurements of:  
\(e^+e^-\rightarrow \pi^+\pi^-\),  
\(e^+e^-\rightarrow \pi^+\pi^-\pi^0\),  
\(e^+e^-\rightarrow \pi^+\pi^-\pi^0\pi^0\) – the final states most relevant to \((g - 2)_\mu\). Presently,
a data sample of 2.9 fb$^{-1}$ at $\sqrt{s} = 3.77$ GeV is being analyzed, but new data at $\sqrt{s} > 4$ GeV can be used for ISR physics as well and will double the statistics. Using these data, hadronic invariant masses from threshold up to approximately 3.5 GeV can be accessed at BESIII. Although the integrated luminosities are orders of magnitude lower compared to the $B$-factory experiments BaBar and BELLE, the ISR method at BESIII still provides competitive statistics. This is due to the fact that the most interesting mass range for the HVP contribution of $(g - 2)_\mu$, which is below approximately 3 GeV, is very close to the centre-of-mass energy of the collider BEPC-II and hence leads to a configuration where only relatively low-energetic ISR photons need to be emitted, providing a high ISR cross section. Furthermore, in contrast to the $B$ factories, small angle ISR photons can be included in the event selection for kinematic reasons which leads to a very high overall geometrical acceptance. Compared to the KLOE experiment, background from final state radiation (FSR) is reduced significantly as this background decreases with increasing center of mass energies of the collider. BESIII is aiming for a precision measurement of the ISR $R$-ratio $R_{\text{ISR}} = N(\pi\pi\gamma)/N(\mu\mu\gamma)$ with a precision of about 1%. This requires an excellent pion-muon separation, which is achieved by training a multi-variate neural network. As a preliminary result, an absolute cross section measurement of the reaction $e^+e^- \rightarrow \mu^+\mu^-\gamma$ has been achieved, which agrees with the QED prediction within 1% precision.

Moreover, at BESIII a new energy scan campaign is planned to measure the inclusive $R$ ratio in the energy range between 2.0 and 4.6 GeV. Thanks to the good performance of the BEPC-II accelerator and the BESIII detector a significant improvement upon the existing BESII measurement can be expected. The goal is to arrive at an inclusive $R$ ratio measurement with about 1% statistical and 3% systematic precision per scan point.

### 3.1.3 Summary of the Lowest-Order Improvements from Data

A substantial amount of new $e^+e^-$ cross section data will become available over the next few years. These data have the potential to significantly reduce the error on the lowest-order hadronic contribution. These improvements can be obtained by reducing the uncertainties of the hadronic cross-sections from 0.7% to 0.4% in the region below 1 GeV and from 6% to 2% in the region between 1 and 2 GeV as shown in Table 2.

| $\sqrt{s}$ (GeV) | $\delta(\sigma)/\sigma$ present | $\delta a_\mu$ present | $\delta(\sigma)/\sigma$ future | $\delta a_\mu$ future |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\sqrt{s} < 1$  | 0.7%            | 33              | 0.4%            | 19              |
| $1 < \sqrt{s} < 2$ | 6%             | 39              | 2%              | 13              |
| $\sqrt{s} > 2$  | 12              | 12              | 12              |                 |
| total           | 53              |                 | 26              |                 |

Table 2: Overall uncertainty of the cross-section measurement required to get the reduction of uncertainty on $a_\mu$ in units $10^{-11}$ for three regions of $\sqrt{s}$ (from Ref. [40]).
3.1.4 Lattice calculation of the Lowest-Order HVP:

With computer power presently available, it is possible for lattice QCD calculations to make important contributions to our knowledge of the lowest-order hadronic contribution. Using several different discretizations for QCD, lattice groups around the world are computing the HVP [41, 42, 43, 44, 45] (see also several recent talks at Lattice 2013 (Mainz). The varied techniques have different systematic errors, but in the continuum limit \( a \to 0 \) they should all agree. Many independent calculations provide a powerful check on the lattice results, and ultimately the dispersive ones too.

Several groups are now performing simulations with physical light quark masses on large boxes, eliminating significant systematic errors. So called quark-disconnected diagrams are also being calculated, and several recent theory advances will help to reduce systematic errors associated with fitting and the small \( q^2 \) regime [46, 42, 47, 48, 49, 50]. While the HVP systematic errors are well understood, significant computational resources are needed to control them at the \( \sim 1\% \) level, or better. Taking into account current resources and those expected in the next few years, the lattice-QCD uncertainty on \( a_\mu \) (HVP), currently at the \( \sim 5\% \)-level, can be reduced to 1 or 2\% within the next few years. This is already interesting as a wholly independent check of the dispersive results for \( a_\mu \) (HVP). With increasing experience and computer power, it should be possible to compete with the \( e^+e^- \) determination of \( a_\mu \) (HVP) by the end of the decade, perhaps sooner with additional technical advances.

3.2 The Hadronic Light-by-Light contribution

There are two approaches to improving the HLbL contribution: Measurements of \( \gamma^* \) physics at BESIII and KLOE; Calculations on the lattice. In addition to the theoretical work on the HLbL, the KLOE detector at DAΦNE has been upgraded with a tagging system to detect the final leptons in the reaction \( e^+e^- \to e^+e^-\gamma^* \). Thus a coincidence between the scattered electrons and a \( \pi^0 \) would provide information on \( \gamma^*\gamma^* \to \pi^0 \) [51], and will provide experimental constraints on the models used to calculate the hadronic light-by-light contribution [52].

Any experimental information on the neutral pion lifetime and the transition form factor is important in order to constrain the models used for calculating the pion-exchange contribution (see Fig. 6(b)). However, having a good description, e.g. for the transition form factor, is only necessary, not sufficient, in order to uniquely determine \( a_\mu^{\text{HLbL};\pi^0} \). As stressed in Ref. [53], what enters in the calculation of \( a_\mu^{\text{HLbL};\pi^0} \) is the fully off-shell form factor \( F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) \) (vertex function), where also the pion is off-shell with 4-momentum \((q_1 + q_2)\). Such a (model dependent) form factor can for instance be defined via the QCD Green’s function \( \langle VVP \rangle \), see Ref. [32] for details. The form factor with on-shell pions is then given by \( F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) \equiv F_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2) \). Measurements of the transition form factor \( F_{\pi^0\gamma^*\gamma^*}(Q^2) \equiv F_{\pi^0\gamma^*\gamma^*}(m_\pi^2, -Q^2, 0) \) are in general only sensitive to a subset of the model parameters and do not permit the reconstruction the full off-shell form factor.

For different models, the effects of the off-shell pion can vary a lot. In Ref. [32] the off-shell lowest meson dominance (LMD) plus vector meson dominance (LMD+V) form factor was proposed and the estimate \( a_\mu^{\text{HLbL};\pi^0} = (72\pm12) \times 10^{-11} \) was obtained (see also Ref. [53]). The error estimate comes from the variation of all model parameters, where the uncertainty
of the parameters related to the off-shellness of the pion completely dominates the total error. In contrast to the off-shell LMD+V model, many other models, e.g. the VMD model or constituent quark models, do not have these additional sources of uncertainty related to the off-shellness of the pion. These models often have only very few parameters, which can all be fixed by measurements of the transition form factor or from other observables. Therefore, for such models, the precision of the KLOE-2 measurement can dominate the total precision of $a_{\mu}^{\text{HLL};\pi^0}$.

Essentially all evaluations of the pion-exchange contribution use for the normalization of the form factor, $F_{\pi^0\gamma\gamma}(m_{\pi}^2,0,0) = 1/(4\pi^2 F_\pi)$, as derived from the Wess-Zumino-Witten (WZW) term. Then the value $F_\pi = 92.4$ MeV is used without any error attached to it, i.e. a value close to $F_\pi = (92.2 \pm 0.14)$ MeV, obtained from $\pi^+ \to \mu^+\nu_\mu(\gamma)$ \[55\]. If one uses the decay width $\Gamma_{\pi^0\to\gamma\gamma}$ for the normalization of the form factor, an additional source of uncertainty enters, which has not been taken into account in most evaluations \[56\]. Until recently, the experimental world average of $\Gamma_{\pi^0\to\gamma\gamma}^{\text{PDG}} = 7.74 \pm 0.48$ eV \[55\] was only known to 6.2% precision. Due to the poor agreement between the existing data, the PDG error of the width average is inflated (scale factor of 2.6) and it gives an additional motivation for new precise measurements. The PrimEx Collaboration, using a Primakoff effect experiment at JLab, has achieved 2.8% fractional precision \[57\]. There are plans to further reduce the uncertainty to the percent level. Though theory and experiment are in a fair agreement, a better experimental precision is needed to really test the theory predictions.

### 3.2.1 Impact of KLOE-2 measurements on $a_{\mu}^{\text{HLL};\pi^0}$

For the new data taking of the KLOE-2 detector, which is expected to start by the end of 2013, new small angle tagging detectors have been installed along DAΦNE beam line. These “High Energy Tagger” detectors \[58\] offer the possibility to study a program of $\gamma\gamma$ physics through the process $e^+e^- \to e^+\gamma\gamma e^-\gamma^* \to e^+e^-X$. In Ref. \[59\] it was shown that planned measurements at KLOE-2 could determine the $\pi^0 \to \gamma\gamma$ decay width to 1% statistical precision and the $\gamma^*\gamma \to \pi^0$ transition form factor $F_{\pi^0\gamma\gamma}(Q^2)$ for small space-like momenta, $0.01 \text{ GeV}^2 \leq Q^2 \leq 0.1 \text{ GeV}^2$, to 6% statistical precision in each bin. The simulations have been performed with the Monte-Carlo program EKHARA \[60\] for the process $e^+e^- \to e^+e^-\gamma\gamma\gamma^* \to e^+e^-\pi^0$, followed by the decay $\pi^0 \to \gamma\gamma$ and combined with a detailed detector simulation. The results of the simulations are shown in Figure 8. The KLOE-2 measurements will allow to almost directly measure the slope of the form factor at the origin and check the consistency of models which have been used to extrapolate the data from larger values of $Q^2$ down to the origin. With the decay width $\Gamma_{\pi^0\to\gamma\gamma}^{\text{PDG}} [\Gamma_{\pi^0\to\gamma\gamma}^{\text{PrimEx}}]$ and current data for the transition form factor $F_{\pi^0\gamma\gamma}(Q^2)$, the error on $a_{\mu}^{\text{HLL};\pi^0}$ is $\pm 4 \times 10^{-11} [\pm 2 \times 10^{-11}]$, not taking into account the uncertainty related to the off-shellness of the pion. Including the simulated KLOE-2 data reduces the error to $\pm (0.7 - 1.1) \times 10^{-11}$.

### 3.2.2 BESIII Hadronic light-by-light contribution

Presently, data taken at $\sqrt{s} = 3.77$ GeV are being analyzed to measure the form factors of the reactions $\gamma^*\gamma \to X$, where $X = \pi^0, \eta, \eta', 2\pi$. 

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BESIII has launched a program of two-photon interactions with the primary goal to measure the transition form factors (TFF) of pseudoscalar mesons as well as of the two-pion system in the spacelike domain. These measurements are carried out in the single-tag mode, i.e. by tagging one of the two beam leptons at large polar angles and by requiring that the second lepton is scattered at small polar angles. With these kinematics the form factor, which in general depends on the virtualities of the two photons, reduces to $F(Q^2)$, where $Q^2$ is the negative momentum transfer of the tagged lepton. At BESIII, the process $\gamma^*\gamma \to \pi^0$, which is known to play a leading contribution in the HLbL correction to $(g - 2)$, can be measured with unprecedented precision in the $Q^2$ range between 0.3 GeV$^2$ and 4 GeV$^2$. In the future BESIII will also embark on untagged as well as double-tag measurements, in which either both photons are quasi-real or feature a high virtuality. The goal is to carry out this program for the final states $\pi^0, \eta, \eta', \pi\pi$. It still needs to be proven that the small angle detector, which recently has been installed close to the BESIII beamline, can be used for the two-photon program.

3.2.3 Lattice calculation of Hadronic Light-by-Light Scattering:

Model calculations show that the hadronic light-by-light (HLbL) contribution is roughly $(105 \pm 26) \times 10^{-11}$, $\sim 1$ ppm of $a_\mu$. Since the error attributed to this estimate is difficult to reduce, a modest, but first principles calculation on the lattice would have a large impact. Recent progress towards this goal has been reported [43], where a non-zero signal (statistically speaking) for a part of the amplitude emerged in the same ball-park as the model estimate. The result was computed at non-physical quark mass, with other systematic errors mostly uncontrolled. Work on this method, which treats both QED and QCD interactions non-
perturbatively, is continuing. The next step is to repeat the calculation on an ensemble of gauge configurations that has been generated with electrically charged sea quarks (see the poster by Blum presented at Lattice 2013). The charged sea quarks automatically include the quark disconnected diagrams that were omitted in the original calculation and yield the complete amplitude. As for the HVP, the computation of the HlbL contribution requires significant resources which are becoming available. While only one group has so far attempted the calculation, given the recent interest in the HVP contribution computed in lattice QCD and electromagnetic corrections to hadronic observables in general, it seems likely that others will soon enter the game. And while the ultimate goal is to compute the HlbL contribution to 10% accuracy, or better, we emphasize that a lattice calculation with even a solid 30% error would already be very interesting. Such a result, while not guaranteed, is not out of the question during the next 3–5 years.

4 Summary

The muon and electron anomalous magnetic moments are among, if not the most precisely measured and calculated quantities in all of physics. The theoretical uncertainty on the Standard-Model contribution to $a_\mu$ is $\approx 0.4$ ppm, slightly smaller than the experimental error from BNL821. The new Fermilab experiment, E989, will achieve a precision of 0.14 ppm. While the hadronic corrections will most likely not reach that level of precision, their uncertainty will be significantly decreased. The lowest-order contribution will be improved by new data from Novosibirsk and BESIII. On the timescale of the first results from E989, the lattice will also become relevant.

The hadronic light-by-light contribution will also see significant improvement. The measurements at Frascati and at BESIII will provide valuable experimental input to constrain the model calculations. There is hope that the lattice could produce a meaningful result by 2018.

We summarize possible near-future improvements in the table below. Since it is difficult to project the improvements in the hadronic light-by-light contribution, we assume a conservative improvement: That the large amount of work that is underway to understand this contribution, both experimentally and on the lattice, will support the level of uncertainty assigned in the “Glasgow Consensus”. With these improvements, the overall uncertainty on $\Delta a_\mu$ could be reduced by a factor 2. In case the central value would remain the same, the statistical significance would become 7–8 standard deviations, as it can be seen in Fig. 9.

Thus the prognosis is excellent that the results from E989 will clarify whether the measured value of $a_\mu$ contains contributions from outside of the Standard Model. Even if there is no improvement on the hadronic error, but the central theory and experimental values remain the same, the significance of the difference would be over $5\sigma$. However, with the worldwide effort to improve on the Standard-Model value, it is most likely that the comparison will be even more convincing.
Figure 9: Estimated uncertainties $\delta a_\mu$ in units of $10^{-11}$ according to Refs. [20, 21] and (last column) prospects for improved precision in the $e^+e^-$ hadronic cross-section measurements. The final row projects the uncertainty on the difference with the Standard Model, $\Delta a_\mu$. The figure gives the comparison between $a_\mu^{SM}$ and $a_\mu^{EXP}$. DHMZ is Ref. [20], HLMNT is Ref. [21]; “SMXX” is the same central value with a reduced error as expected by the improvement on the hadronic cross section measurement (see text); “BNL-E821 04 ave.” is the current experimental value of $a_\mu$; “New (g-2) exp.” is the same central value with a fourfold improved precision as planned by the future (g-2) experiments at Fermilab and J-PARC.

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