$J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ and the structure observed around the $\bar{p}p$ threshold

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We analyze the origin of the structure observed in the reaction $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ for $\eta' \pi^+ \pi^-$ invariant masses close to the antiproton-proton ($\bar{p}p$) threshold, commonly associated with the $X(1835)$ resonance. Specifically, we explore the effect of a possible contribution from the two-step process $J/\psi \rightarrow \gamma \bar{NN} \rightarrow \gamma \eta' \pi^+ \pi^-$. The calculation is performed in distorted-wave Born approximation which allows an appropriate inclusion of the $\bar{NN}$ interaction in the transition amplitude. The $\bar{NN}$ amplitude itself is generated from a corresponding potential recently derived within chiral effective field theory. We are able to reproduce the measured spectra for the reactions $J/\psi \rightarrow \gamma \bar{p}p$ and $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ for invariant masses around the $\bar{p}p$ threshold. The structure seen in the $\eta' \pi^+ \pi^-$ spectrum emerges as a threshold effect due to the opening of the $\bar{p}p$ channel.

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I. INTRODUCTION

The $X(1835)$ resonance, first discovered by the BES Collaboration in 2005 in the decay $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ and subsequently seen in other reactions [2,4], but only faintly by other groups [6,8], has a long and winding history. Initially the resonance was associated with the anomalous near-threshold enhancement in the antiproton-proton ($\bar{p}p$) invariant mass spectrum in the reaction $J/\psi \rightarrow \gamma \bar{p}p$ [2,8] which would point to a baryonium-type state (or $\bar{NN}$ quasi-bound state) as possible explanation for its structure. However, with increasing statistics [9] it became clear that the two phenomena are not necessarily connected, not least due to a striking difference in the width of the respective resonances required for describing the invariant mass spectra of the two reactions in question. Yet another facet was added in the most recent publication of the BESIII Collaboration on the decay $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ [10]. Now the initial peak around 1835 MeV is practically gone but has reappeared as a structure that is located very close to the $\bar{p}p$ threshold, namely around 1870 MeV.

A more detailed coverage of the historical development regarding the $X(1835)$ resonance can be found in recent summary papers [11,12]. These works provide also an overview of the large amount of theoretical investigations performed in the context of the $X(1835)$. Naturally, in many of them an interpretation of the resonance in terms of a baryonium state is the key element. Indeed, some of these studies attempt to establish a direct and quantitative connection of the resonance with predictions of $\bar{NN}$ potentials that were fitted to $\bar{p}p$ scattering data [13,14].

In the present work we aim at a quantitative analysis of the most recent BESIII data on the reaction $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$. The study is based on the hypothesis that the structure seen in the invariant mass spectrum is indeed linked with the opening of the $\bar{p}p$ channel. The incentive for that comes from past studies of $e^+e^-$ annihilation into multipion states. Also in this case, and specifically in the reactions $e^+e^- \rightarrow 3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $\omega \pi^+\pi^-\pi^0$, and $e^+e^- \rightarrow 2(\pi^+\pi^-)\pi^0$, structures were observed in the experiments at energies around the $\bar{p}p$ threshold [13,14]. Calculations by our group [19] and others [20] suggested that two-step processes $e^+e^- \rightarrow \bar{NN} \rightarrow \text{multions}$ could play an important role and their inclusion even allowed one to reproduce the data quantitatively near the $\bar{NN}$ threshold. Accordingly, the structures seen in the experiments found a natural explanation as a threshold effect due to the opening of the $\bar{NN}$ channel, for the majority of the measured channels.

As already indicated above, with the new $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ data [10] the region of interest is now shifted likewise to energies around the $\bar{p}p$ threshold. Accordingly, we investigate the significance of the $\bar{NN}$ channel for the reaction $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$. Since the decay $J/\psi \rightarrow \gamma \bar{p}p$ constitutes one segment of the assumed two-step process (the other being $\bar{p}p \rightarrow \eta' \pi^+ \pi^-$), we reconsider this decay process in the present paper. Indeed, we had already shown in earlier studies that it is possible to describe the large near-threshold enhancement observed in the reaction $J/\psi \rightarrow \gamma \bar{p}p$ by the final-state interaction (FSI) provided by the $\bar{NN}$ interaction [21,22], see also Refs. [13,14,24,26].

A main ingredient of our present calculation is the $\bar{NN}$ interaction. Here we build on our latest $\bar{NN}$ potential, derived in the framework of chiral effective field theory (EFT) up to next-to-next-to-next-to-leading order (N$^3$LO) [27]. That potential reproduces the amplitudes determined in a partial-wave analysis (PWA) of $\bar{p}p$ scattering data [28] from the $\bar{NN}$ threshold up to laboratory energies of $T_{lab} \approx 200 - 250$ MeV [29].

The paper is structured in the following way: In Sect. II an overview of the employed formalism is provided. Sect. III is devoted to the reaction $J/\psi \rightarrow \gamma \bar{p}p$, the first segment of the considered two-step process. In
II. FORMALISM

Our study of the processes \( J/\psi \rightarrow \gamma \bar{p}p \) and \( J/\psi \rightarrow \gamma' \bar{p}p \) is based on the distorted-wave Born approximation (DWBA). It amounts to solving the following set of formally coupled equations:

\[
\begin{align*}
T_{\bar{N}N} & = V_{\bar{N}N} + V_{\bar{N}N}G_0T_{\bar{N}N} \\
T_{\bar{N}N} & = V_{\bar{N}N} + T_{\bar{N}N}G_0V_{\bar{N}N} \\
A_{J/\psi \rightarrow \gamma \bar{N}N} & = A_{J/\psi \rightarrow \gamma \bar{N}N} + A_{J/\psi \rightarrow \gamma \bar{N}N}G_0T_{\bar{N}N} \\
A_{J/\psi \rightarrow \gamma' \bar{N}N} & = A_{J/\psi \rightarrow \gamma' \bar{N}N} + A_{J/\psi \rightarrow \gamma' \bar{N}N}G_0T_{\bar{N}N}.
\end{align*}
\]

The first line in Eq. (1) is the Lippmann-Schwinger equation from which the \( \bar{N}N \) scattering amplitude \( (T_{\bar{N}N}) \) is obtained, for a specific \( \bar{N}N \) potential \( V_{\bar{N}N} \); see Refs. [27, 28] for details. The quantity \( G_0 \) denotes the free \( NN \) Green's function. The second equation defines the amplitude for \( \bar{N}N \) annihilation into the \( \gamma' \bar{N}N \) channel while the third equation contains the \( J/\psi \rightarrow \gamma \bar{N}N \) transition amplitude. Finally, Eq. (2) defines the \( J/\psi \rightarrow \gamma' \bar{NN} \) amplitude. The quantities \( A_{\gamma \bar{N}N} \) denote the elementary (or primary) decay amplitudes for \( J/\psi \) to \( \gamma \bar{NN} \) or \( \gamma' \bar{NN} \).

General selection rules [28] but also direct experimental evidence [23] suggest that the specific (and unique) \( \bar{NN} \) partial wave that plays a role for energies around the \( \bar{p}p \) threshold is the \( ^1S_0 \). For it the equation for the amplitude \( A_{J/\psi \rightarrow \gamma \bar{N}N} \) reads

\[
A = A^0 + \int_0^\infty \frac{dp^2}{(2\pi)^3} A^0 \frac{1}{2E_k - 2E_p + i0^+} T(p, k; E_k),
\]

where \( k \) and \( E_k \) are the momentum and energy of the proton (or antiproton) in the center-of-mass system of the \( \bar{NN} \) pair, i.e. \( E_k = \sqrt{m_p^2 + k^2} \), with \( m_p \) the proton (nucleon) mass. The subscript of \( A \) indicating the channel is omitted in Eq. (3) for simplicity.

The \( \bar{NN} \) \( T \)-matrix that enters Eq. (3) fulfills

\[
T(p', k; E_k) = V(p', p) + \int_0^\infty \frac{dpq^2}{(2\pi)^3} \frac{1}{2E_k - 2E_p + i0^+} T(p, k; E_k),
\]

where \( V \) represents the \( \bar{NN} \) potential in the \( ^1S_0 \) partial wave.

Following the strategy in Refs. [27, 29], the elementary annihilation potential for \( \bar{NN} \rightarrow \gamma' \bar{NN} \) and the transition amplitude \( A_{J/\psi \rightarrow \gamma \bar{NN}} \) are parameterized by

\[
\begin{align*}
V_{\bar{N}N} & = \tilde{C}_{\gamma' \gamma} + C_{\gamma' \gamma} q^2, \\
A_{J/\psi \rightarrow \gamma \bar{NN}} & = \tilde{C}_{\gamma' \gamma} + C_{\gamma' \gamma} q^2, \quad (4)
\end{align*}
\]

i.e. by two contact terms analogous to those that arise up to next-to-next-to-leading order (N^2LO) in the treatment of the \( \bar{NN} \) interaction within chiral EFT [27]. The quantity \( q \) in Eq. (3) is the center-of-mass (c.m.) momentum in the \( \bar{NN} \) system. Note that we multiply the transition potentials in Eqs. (5) and (6) with a regulator (of exponential type) in the actual calculations. This is done consistently with the \( \bar{NN} \) potentials in Ref. [27] where such a regulator is included. We also employ the same cutoff parameter as in the \( \bar{NN} \) sector. Since the threshold for the \( \gamma' \bar{NN} \) channel lies significantly below the one for \( \bar{NN} \), the mesons carry on average already fairly high momenta. Thus, the dependence of the annihilation potential on those momenta should be small for energies around the \( \bar{NN} \) threshold and it is, therefore, neglected [23]. The constants \( \tilde{C}_{\gamma' \gamma} \) and \( C_{\gamma' \gamma} \) can be determined by a fit to the \( \bar{NN} \rightarrow \gamma' \bar{NN} \) event rate.

The term \( A_{J/\psi \rightarrow \gamma' \bar{NN}} \) is likewise parameterized in the form [6], but as a function of the \( \gamma' \bar{NN} \) invariant mass \( Q \),

\[
A_{J/\psi \rightarrow \gamma' \bar{NN}}(Q) = \tilde{C}_{J/\psi \rightarrow \gamma' \bar{NN}} + C_{J/\psi \rightarrow \gamma' \bar{NN}} Q. \quad (7)
\]

The arguments for neglecting the dependence on the individual meson momenta are the same as above and they are valid again, of course, only for energies around the \( \bar{NN} \) threshold. However, since in the \( \gamma' \bar{NN} \) case this term represents a background amplitude rather than a transition potential we allow the corresponding constants to be complex valued, to be fixed by a fit to the \( J/\psi \rightarrow \gamma' \bar{NN} \) event rate.

The explicit form of Eq. (4) reads

\[
A_{\gamma' \bar{NN}, J/\psi}(X; Q) = A_{\gamma' \bar{NN}, J/\psi}(X; Q) + \int_0^\infty \frac{dq^2}{(2\pi)^3} \frac{1}{Q - 2E_q + i0^+} A_{\bar{p}p, J/\psi}(q; Q),
\]

written in matrix notation. The quantity \( X \) stands here symbolically for the momenta in the \( \gamma' \bar{NN} \) system. But
The energies in the reaction $\gamma \eta' \to \pi^+ \pi^-$ are given by
\[
E_N = Q/2, \\
E_{J/\psi} = \frac{m_{\pi}^2 + Q^2}{2Q}, \\
E_\gamma = \frac{m_{\psi}^2 - Q^2}{2Q},
\]
where $Q$ is either the energy in the $\bar{N}N$ system or the invariant mass of the $\bar{p}p$ or $\eta' \pi^+ \pi^-$ systems ($M_{\bar{p}p}$ or $M_{\eta' \pi^+ \pi^-}$), $t_1 = M_{\pi^+ \pi^-}^2$, and $t_2 = M_{\eta' \pi^+ \pi^-}^2$.

In Eq. (9) it is assumed that averaging over the spin states has been already performed. Anyway, in the present manuscript we will consider only individual partial wave amplitudes. The cross section for the reaction $\bar{p}p \to \eta' \pi^+ \pi^-$ is given by
\[
\sigma(\bar{p}p \to \eta' \pi^+ \pi^-) = \int_{t_1}^{t_1^+} dt_1 \int_{t_2}^{t_2^+} dt_2 \frac{|M_{\bar{p}p \to \eta' \pi^+ \pi^-}|^2}{1024\pi^3 Q \sqrt{Q^2 - 4m_{\eta'}^2}},
\]
where
\[
t_1^- = 4m_{\pi^+}^2, \\
t_1^+ = (Q - m_{\eta'}^2)^2, \\
t_2^+ = \frac{1}{4t_1} \left((Q^2 - 4m_{\eta'}^2)^2 - |\lambda(\pi^+)(Q^2, t_1, m_{\eta'}^2)|^2ight), \\
t_2^- = \frac{1}{4t_1} \left((Q^2 - 4m_{\eta'}^2)^2 - |\lambda(\pi^-)(Q^2, t_1, m_{\eta'}^2)|^2ight).
\]

The decay rate for $J/\psi \to \gamma \eta' \pi^+ \pi^-$ is given by
\[
\frac{d\Gamma}{dQ} = \int_{t_1}^{t_1^+} dt_1 \int_{t_2}^{t_2^+} dt_2 \frac{(m_{\psi}^2 - Q^2)^2 |M_{J/\psi \to \gamma \eta' \pi^+ \pi^-}|^2}{6144\pi^3 m_{\psi}^4 Q}.
\]

### III. THE REACTION $J/\psi \to \gamma \bar{p}p$

Due to the unusually large enhancement observed in the near-threshold $\bar{p}p$ invariant mass spectrum in the reaction $J/\psi \to \gamma \bar{p}p$, the topic of many studies and a variety of explanations for the strongly peaked spectrum have been suggested [10, 12]. In scenarios like ours, were FSI effects in the $\bar{N}N$ channel are assumed to be responsible for the enhancement, one faces a challenging task. There are measurements for several other decay channels where the produced $\bar{N}N$ state must be in the very same partial wave, the $^1S_0$, at least near threshold, and accordingly, in principle, the
the isospin $I^2$ PW A \[28\] (circles) at NLO, N $^3$ LO, and N $^4$ LO from Ref. \[27\]. The dashed and solid lines are refits at N $^3$ LO and N $^4$ LO, respectively, utilized in the present work.

FIG. 1. Real and imaginary parts of the $^1S_0$ phase shift in the isospin $I = 1$ channel. The bands represent the fits to the PWA \[28\] (circles) at NLO, N $^3$ LO, and N $^4$ LO from Ref. \[27\]. The dashed and solid lines are refits at N $^3$ LO and N $^4$ LO, respectively, utilized in the present work.

The same FSI effects should arise. This concerns the reactions $J/\psi \to \omega \bar{p}p$ \[33\] and $J/\psi \to \phi \bar{p}p$ \[34\], and also $\psi(2S) \to \gamma pp$ \[8\]. In none of these, enhancements of a comparable magnitude were observed in the experiments. So far, a few suggestions for a way out of this dilemma have been made \[14\] \[23\] \[26\]. In our own work the emphasis was always on the isospin dependence. Already in our initial studies \[21\] \[22\], still based on the Migdal-Watson approximation and on the Jülich meson-exchange $NN$ potential \[35\] \[50\], it was the isospin $I = 1$ amplitude that produced the large enhancement. Then there is no conflict with the rather moderate enhancements observed in the $J/\psi \to \omega p$ and $J/\psi \to \phi pp$ channels, because in those cases the produced $pp$ system has to be in $I = 0$ (assuming that isospin is conserved in this purely hadronic decay).

Indeed, in the decays $J/\psi \to \gamma pp$ and $\psi(2S) \to \gamma pp$ isospin is not conserved and, therefore, in principle, one can have any combination of the $I = 0$ and $I = 1$ amplitudes. This freedom was exploited in a recent and more refined study of $J/\psi$ decays by our group \[23\]. In that work we not only treated the FSI effects within a DWBA approach, but we also employed an $NN$ potential that was derived within the framework of chiral effective field theory up to N$^2$LO \[23\]. Utilizing the “standard” hadronic combination for the $\bar{p}p$ amplitude, namely $T = T_{\bar{p}p} = (T_{I=0}^2 + T_{I=1}^2)/2$, for $J/\psi$ decay and one with a predominant $I = 0$ component, $T = (0.9 T_0^0 + 0.1 T_1^1)$ for $\psi(2S)$ decay allowed us to achieve a consistent description of the $\gamma pp$ spectrum for both decays \[23\].

Nonetheless, it should be said that we had to depart slightly from the $I = 1$ $^1S_0$ $NN$ amplitude as determined in the PWA of Zhou and Timmermans \[28\]. However, already a rather modest modification of the interaction in the $I = 1$ channel – subject to the constraint that the corresponding partial-wave cross sections for $\bar{p}p \to \bar{p}p$ and $pp \to nn$ remain practically unchanged at low energies – allowed us to reproduce the events distribution of the radiative $J/\psi$ decay, and consistently all other decays \[23\].

In the present work we repeat this exercise, employing now the new $NN$ interaction \[27\]. First of all, we want to see whether the same scenario holds for the improved $NN$ potential that is based on a different regularization scheme and that is now calculated up to N$^3$LO. In addition we have to establish the $J/\psi \to \gamma pp$ amplitude in the $I = 0$ channel that enters into the calculation of the 2-step process, see Eq. \[27\]. Results for the $NN$ sector, i.e. the $I = 1$ $^1S_0$ amplitude, are shown in Fig. \[1\]. The parameters of the fit are summarized in Table \[1\]. Corresponding results for the $\bar{p}p$ invariant mass spectrum of the reaction $J/\psi \to \gamma pp$ are displayed in Fig. \[2\]. It is reassuring to see that the results are basically the same as those reported in Ref. \[28\] for the chiral N$^2$LO interaction. The presented results are for the combination $T = (0.4 T_0^0 + 0.6 T_1^1)$ that yields the lowest $\chi^2$ value in the fit. Note, however, that those for weights of the isospin amplitudes differing by, say, ±0.1 are very similar, even on a quantitative level.

Interestingly, the modified potential in Ref. \[23\] gener-
TABLE I. Low-energy constants at N²LO and N³LO, for the \( N\bar{N} \) interaction in the \( I = 1 \, ^1S_0 \) partial wave. Note that all parameters are in units of 10³, see Ref. [27] for details.

| \( \tilde{C}_{31, S_0} \) (GeV⁻²) | N²LO       | N³LO       |
|---------------------------------|------------|------------|
| \( \tilde{C}_{31, S_0} \) (GeV⁻⁴) | 0.1935(14) | 0.3155(15) |
| \( \tilde{C}_{31, S_0} \) (GeV⁻⁶) | -1.8160(52) | -3.5235(101) |
| \( \tilde{D}_{31, S_0} \) (GeV⁻⁶) | -8.0840(627) | -10.0000(286) |
| \( \tilde{C}_{31, S_0} \) (GeV⁻¹) | 0.1733(25) | 0.0230(33) |
| \( \tilde{C}_{31, S_0} \) (GeV⁻³) | -4.1780(21) | -3.1759(100) |

The reaction \( J/\psi \to \gamma \eta' \pi^+ \pi^- \)

As already mentioned in the Introduction, in studies of \( e^+e^- \) annihilation to multipion states structures were observed around the \( N\bar{N} \) threshold for several channels, specifically in \( e^+e^- \to 3(\pi^+\pi^-), e^+e^- \to 2(\pi^+\pi^-\pi^0), \) and \( e^+e^- \to 2(\pi^+\pi^-)(\pi^0) \) [13, 14]. An analysis of those structures performed by us [14] and by others [20] suggested that they could be simply a result of a threshold effect due to the opening of the \( N\bar{N} \) channel. In that work we could estimate the contribution of the two-step process \( e^+e^- \to N\bar{N} \to \) multipions to the total reaction amplitude rather reliably because cross-section measurements for all involved processes were available in the literature. Specifically, the amplitude for \( e^+e^- \to N\bar{N} \) could be constrained from near-threshold data on the \( e^+e^- \to p\bar{p} \) cross section and the one for \( N\bar{N} \to \gamma \eta' \pi^+ \pi^- \) could be fixed from available experimental information on the corresponding annihilation ratios [27]. It turned out that the resulting amplitude for \( e^+e^- \to N\bar{N} \to \) multipions was large enough to play a role for the considered \( e^+e^- \) annihilation channels and that it is possible to reproduce the data quantitatively near the \( N\bar{N} \) threshold in most of the considered reaction channels [19].

In case of \( J/\psi \to \gamma \eta' \pi^+ \pi^- \) we are not in such an advantageous situation. While cross sections (or branching ratios) are available for \( p\bar{p} \to \eta' \pi^+ \pi^- \), so far only event rates have been published for \( J/\psi \to \gamma \eta' \pi^+ \pi^- \) itself and for \( J/\psi \to \gamma p\bar{p} \). Thus, a reliable assessment of the magnitude of the two-step process \( J/\psi \to \gamma p\bar{p} \to \gamma \eta' \pi^+ \pi^- \) cannot be given at present. Nonetheless, in the following we provide a rough order-of-magnitude estimate and plausibility arguments why we believe that the \( N\bar{N} \) intermediate step should play an important role here. The main and most important support comes certainly from the \( \eta' \pi^+ \pi^- \) data itself, where a clear structure is seen at the \( N\bar{N} \) threshold in the latest high-statistics measurement by the BESIII Collaboration [10]. In addition a comparison of the event rates for \( J/\psi \to \gamma p\bar{p} \) and \( J/\psi \to \gamma \eta' \pi^+ \pi^- \) with the cross sections for \( p\bar{p} \to p\bar{p} \) in the \( ^1S_0 \) partial wave and for \( p\bar{p} \to \eta' \pi^+ \pi^- \) suggests that the two-step process in question should be of relevance.

Let us discuss the latter issue in more detail. With the central value of the branching ratio, \( BR(p\bar{p} \to \eta' \pi^+ \pi^-) = 0.626% \) [35], the resulting cross sections at \( p_{lab} = 106 \text{ MeV/c} \) is 2.23 mb, based on the total annihilation cross section given in Ref. [29]. Though the branching ratio is tiny, at first sight, one has to compare the resulting cross section with the relevant quantity, namely the \( p\bar{p} \) elastic cross section in the \( ^1S_0 \) partial wave. The latter is around 20 mb in our \( N\bar{N} \) potential [27], but also in the PWA [28]. Thus, the annihilation cross section for \( p\bar{p} \to \eta' \pi^+ \pi^- \) is roughly a factor 10 smaller than that for \( p\bar{p} \to p\bar{p} \).

When comparing the event rates one has to consider that the number of \( J/\psi \) decay events used in the \( \eta' \pi^+ \pi^- \) analysis [10] is roughly a factor five larger than that in the \( \gamma p\bar{p} \) paper [8]. Moreover, the bin size is different. Combining those two aspects suggests a roughly five times larger rate for \( \gamma p\bar{p} \), based on the data shown in Refs. [8, 11], which mostly compensates for the factor of 10 reduction estimated above.

In the actual calculation we fix the constant \( \tilde{C}_{\eta' \pi^+ \pi^-} \) in the \( N\bar{N} \to \gamma \eta' \pi^+ \pi^- \) transition potential (cf. Eq. (3)) from the corresponding annihilation cross section discussed above. Since there is no experimental information on the energy dependence, we set the constant \( C_{\eta' \pi^+ \pi^-} \) to zero. For the amplitude \( A_{J/\psi \to \gamma p\bar{p}} \) we employ the one described in Sect. III, with \( \tilde{C}_{J/\psi \to \gamma N\bar{N}} \) fixed to the most recent BESIII data [10]. However, we allow for some variations of the overall magnitude because, as said above, only event rates are available in this case. The value for \( C_{J/\psi \to \gamma N\bar{N}} \) obtained in the fit turned out to be very small so that we simply set it to zero.

Finally, the constants in the quantity \( A^0_{J/\psi \to \gamma \eta' \pi^+ \pi^-} \) (cf. Eq. (7)) are adjusted to the event rate for...
$J/\psi \rightarrow \gamma \eta'\pi^+\pi^-$. This term has to account for all other contributions to $J/\psi \rightarrow \gamma \eta'\pi^+\pi^-$, besides the one with an intermediate $\gamma \bar{N}N$ state. Thus, it can have a relative phase as compared to the contribution from the $\bar{N}N$ loop, i.e. the corresponding $C$'s can be complex valued. However, it turns out that optimal results are already achieved for real values of $C_{J/\psi \rightarrow \gamma \eta'\pi^+\pi^-}$ and $C_{J/\psi \rightarrow \gamma \eta'\pi^-\pi^+}$.

In the fit we consider data in the range $1800 \text{ MeV} \leq E \leq 1950 \text{ MeV}$, i.e. in a region that encompasses more or less symmetrically the $\bar{N}N$ threshold.

Our results for the reaction $J/\psi \rightarrow \gamma \eta'\pi^+\pi^-$ are presented in Figs. 3 and 4. They are based on the N$^2$LO and N$^3$LO EFT $\bar{N}N$ interactions with the cutoff $R = 0.9 \text{ fm}$ ($\Lambda = 438 \text{ MeV}$), cf. Ref. 27 for details. Exploratory calculations for the other cutoffs considered in Ref. 27 turned out to be very similar. Like for $\bar{N}N$ scattering itself, much of the cutoff dependence is absorbed by the contact terms ($\tilde{C}_\nu$ and $C_\nu$ in Eqs. (5) and (6)) that are fitted to the data so that the variation of the results for energies of, say, ±50 MeV around the $\bar{N}N$ threshold is rather small. For consistency the momentum-space regulator function as given in Eq. (3.1) (right side) in Ref. 27 is also attached to the transition potentials in Eqs. (5) and (6), i.e. to all quantities that depend on the $\bar{N}N$ momentum $q$.

In Fig. 3 the full results for the $\eta'\pi^+\pi^-$ invariant mass spectrum in the reaction $J/\psi \rightarrow \gamma \eta'\pi^+\pi^-$. Results for the contribution from the $J/\psi \rightarrow \gamma \bar{N}N \rightarrow \gamma \eta'\pi^+\pi^-$ transition (dotted line) and the background term (dashed line) are shown, together with the full results (solid line). The N$^3$LO $\bar{N}N$ potential [27] is employed. Data are from the BESIII Collaboration [10]. The horizontal line indicates the $\bar{p}p$ threshold.

![Fig. 3](image1.png)

**FIG. 3.** The $\eta'\pi^+\pi^-$ invariant mass spectrum in the reaction $J/\psi \rightarrow \gamma \eta'\pi^+\pi^-$. Results for the contribution from the $J/\psi \rightarrow \gamma \bar{N}N \rightarrow \gamma \eta'\pi^+\pi^-$ transition (dotted line) and the background term (dashed line) are shown, together with the full results (solid line). The N$^3$LO $\bar{N}N$ potential [27] is employed. Data are from the BESIII Collaboration [10]. The horizontal line indicates the $\bar{p}p$ threshold.

In Fig. 4 we present the complete results for the N$^2$LO and N$^3$LO interactions, on a scale similar to that in the BESIII publication [10], cf. the inserts in Figs. 3 and 4 of that reference. First we note that the $\eta'\pi^+\pi^-$ invariant mass spectrum based on the two $\bar{N}N$ interactions is very similar around the $\bar{N}N$ threshold. It is also very similar to the fit within the first model considered in Ref. [10] (cf. the corresponding Fig. 3). That model includes explicitly a $X(1835)$ resonance and simulates the effect of the $\bar{N}N$ channel via a Flatté formula [40]. Obviously, in our calculation the data can be described with the same quality, but without such a $X(1835)$ resonance. The more elaborated treatment of the coupling to the $\bar{N}N$ channel via Eq. (8) with the explicit inclusion of the $\bar{N}N$ interaction itself is already sufficient to generate an invariant-mass dependence in line with the data.

For completeness, let us mention that a second resonance has been introduced in Ref. [10] in the invariant-mass region covered by our study, namely an $X(1920)$, in order to reproduce a possible enhancement at the corresponding invariant mass suggested by two data points, cf. Fig. 4. Furthermore, a second model has been considered in Ref. [10] where instead of the coupling to the $\bar{N}N$ channel an additional and rather narrow resonance was included, the $X(1870)$. In that scenario a slightly better description of the data very close to the $\bar{N}N$ threshold could be achieved.

Now the key question is, of course, are those structures seen in the experiment a signal for a $\bar{N}N$ bound state?
We did not find any near-threshold poles for our EFT $\bar{N}N$ interactions in the $^1S_0$ partial wave with $I = 0$, i.e. the one relevant for the $\gamma\eta'\pi^+\pi^-$ channel, neither for the N$^3$LO potential presented in Ref. 22 nor for the new N$^3$LO and N$^5$LO interactions [27] employed in the present calculation. As already discussed in the preceding section, there is only a pole in the $I = 1$ case in the versions established in the study of the reactions $J/\psi \to \gamma\bar{p}p$.

Thus, our results provide a clear indication that bound states are not necessarily required for achieving a quantitative reproduction of the observed structure in the $\eta'\pi^+\pi^-$ invariant-mass spectrum near the $\bar{p}p$ threshold. This is in contrast to other investigations in the literature. For example, bound states in the $I = 0 \ 1S_0$ partial wave are present in the Paris $\bar{N}N$ potential [11] employed in Refs. 13, 24 ($E_B = (-4.8 - i 26)$ MeV) and also in the $\bar{N}N$ interaction constructed in Ref. 14 ($E_B = (22 - i 33)$ MeV). In the latter case, the positive sign of the real part of $E_B$ indicates that the pole found is actually located above the $\bar{N}N$ threshold (in the energy plane). As discussed in Ref. 14, the pole moves below the threshold when the imaginary part of the potential is switched off and that is the reason why it is referred to as bound state.

In this context, it is worth mentioning that no bound states or resonances were found in a study of the $\eta'K\bar{K}$ system [42] in an attempt to explore in how far such states could be generated dynamically as $\eta'f_0(980)$- or $\eta'\sigma_0(980)$-like configurations.

Past studies suggest that there is a distinct difference in the amplitude for $J/\psi \to \gamma + \text{mesons}$ due to the $\bar{N}N$ loop contribution in case of the absence/presence of a bound state. Its modulus exhibits specific features, namely either a genuine cusp at the $\bar{N}N$ threshold (cf. Fig. 3) or a rounded step and a maximum below the threshold. This was discussed in detail in Ref. 13 in the context of the reaction $e^+e^- \to \text{multipions}$ (cf. Fig. 4 in that reference) and also in Ref. 14. However, in both studies the bound states in question belong to the special class discussed above, i.e. they are located above the $\bar{N}N$ threshold.

In order to illustrate what happens for the case of a “regular” bound state we present here an exemplary calculation based on the $I = 1 \ 1S_0$ partial wave of our N$^3$LO potential, where the binding energy is $(50.8 - i 40.9)$ MeV, cf. Sect. IIII. A $J/\psi$ decay reaction where the corresponding $\bar{N}N$ loop could contribute is, for example, $J/\psi \to \gamma \omega \rho^0$. Pertinent predictions are shown in Fig. 5. Obviously, the invariant-mass dependence of the loop (dotted line) is fairly different from the one of the $I = 0$ amplitude, cf. dotted line in Fig. 3. Specifically, there is a clear enhancement in the spectrum around 50 MeV below the $\bar{N}N$ threshold reflecting the presence of the $\bar{N}N$ bound state. Due to the fairly large width ($\Gamma = -2\Im E_B$) the structure is not very pronounced. Of course, the final signal will be strongly influenced and modified by the interference with the background amplitude, as testified by the results presented above for the $\eta'\pi^+\pi^-$ case. For demonstrating this we include also results for two different but arbitrary choices for the background term, see the dashed and solid lines in Fig. 3. Of course, in case that the $\bar{N}N$ bound state is more narrow then the signal will be certainly more pronounced. Note that the decay $J/\psi \to \gamma \omega \rho^0$ has been already measured by the BES Collaboration [13]. However, the statistics is simply too low for drawing any conclusions. It would be definitely interesting to revisit this reaction in a future experiment.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5}
\caption{Predicted $\omega \rho^0$ invariant-mass spectrum for $J/\psi \to \gamma \omega \rho^0$, based on the N$^3$LO $\bar{N}N$ interaction described in Sect. IIII. The contribution from the $J/\psi \to \gamma \bar{N}N \to \gamma \omega \rho^0$ transition alone (dotted line) and with two arbitrary choices for the background term included (dashed and solid lines) are shown. The horizontal line indicates the $\bar{p}p$ threshold.}
\end{figure}

V. CONCLUSIONS

We analyzed the origin of the structure associated with the $X(1835)$ resonance, observed in the reaction $J/\psi \to \gamma \eta'\pi^+\pi^-$. Specific emphasis was put on the $\eta'\pi^+\pi^-$ invariant mass spectrum around the $\bar{p}p$ threshold, where the most recent BESIII measurement [11] provided strong evidence for an interplay of the $\eta'\pi^+\pi^-$ and $\bar{p}p$ channels.

Motivated by this experimental observation, we evaluated the contribution of the two-step process $J/\psi \to \gamma \bar{p}p \to \gamma \eta'\pi^+\pi^-$ to the total reaction amplitude. The amplitude for $J/\psi \to \gamma \bar{p}p$ was constrained from corresponding data by the BESIII Collaboration, while for $\bar{N}N \to \eta'\pi^+\pi^-$ we took available branching ratios for $\bar{p}p \to \eta'\pi^+\pi^-$ as guideline. Combining the contribution of this two-step process with a background amplitude, that simulates other transition processes which do not involve an $\gamma \bar{N}N$ intermediate state, allowed us to achieve a quantitative reproduction of the data near the $\bar{p}p$ threshold. In particular, the structure detected in the experiment emerges as a threshold effect. It results from an in-
terference of the smooth background amplitude with the strongly energy-dependent two-step contribution, which itself exhibits a cusp-like behavior at the $\bar{N}N$ threshold.

The question whether there is an evidence for a $\bar{N}N$ bound state is discussed, but no firm conclusion could be made. While in our own calculation such states are not present, and are also not required for describing the data for the reaction $J/\psi \to \gamma\eta\pi^+\pi^-$, contrary claims have been brought forth in the literature [14, 26]. In any case, it should be said that the possibility that a genuine resonance is ultimately responsible for the structure observed in the invariant mass spectrum cannot be categorically denied, as rather convincing.

Data with improved resolution around the $\bar{p}p$ threshold could possibly help to shed further light on the relation of a possible $X(1835)$ with the $\bar{p}p$ channel. An absolute determination of the relevant invariant-mass spectra would certainly put stronger constraints on the question whether the intermediate $\bar{p}p$ state can play such an important role as suggested by the present study. In addition, we believe that an analogous measurement for channels like $J/\psi \to \gamma\eta\pi^+\pi^-$ could be very instructive. Indeed, this has already been recommended around the time when first evidence for the $X(1835)$ was reported [45]. The branching ratio for $\bar{p}p \to \eta\pi^+\pi^-$ is more than a factor two larger than for $\eta'\pi^+\pi^-$ [30, 44] which would enhance the role played by the $\bar{p}p$ channel. On the other hand, if the count rates for $J/\psi \to \gamma\eta\pi^+\pi^-$ turn out to be much larger than those for $\eta'\pi^+\pi^-$ [30, 44], then the effect of the transition to $\bar{p}p$ should be strongly reduced or even disappear.

Finally, we want to mention that there are data on $J/\psi \to \omega\eta\pi^+\pi^-$ [10] and $J/\psi \to \phi\eta\pi^+\pi^-$ [13]. For the latter, $\eta\pi^+\pi^-$ invariant masses corresponding to the $\bar{p}p$ threshold are already close to boundary of the available phase space and, therefore, no appreciable signal is expected. In case of $J/\psi \to \omega\eta\pi^+\pi^-$ the BES-III Collaboration sees a resonance-like enhancement at $1877.3 \pm 6.3^{+4.7}_{-3.4}$ MeV [10] which coincides almost perfectly with the $\bar{p}p$ threshold. However, the invariant-mass resolution of the present data is only 20 MeV/c$^2$. Moreover, it is our understanding that non-$\omega$ (background) events are not well separated in the data presented in Ref. [40]. These two issues handicap a dedicated analysis for the time being. Clearly, new measurements with higher statistics could be indeed rather useful for providing further information on the role that the (opening of the) $\bar{N}N$ channel plays for the reaction in question.

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