Modeling of line and continuum spectral emission of hydrogen for recombining plasma conditions

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Abstract. Balmer line and continuum spectral emission of hydrogen are modeled for detached divertor plasmas (electron density, $n_e \sim 10^{20}-10^{21} m^{-3}$ and electron temperature, $T_e \sim 1$ eV) of Tokamaks. The Stark broadened high-n lines are calculated considering electronic broadening using the impact approximation. Continuum emission is calculated using analytical equations for radiative recombination and bremsstrahlung. Discrete to continuum transition is modeled using a dissolution factor approach.

1. Introduction
One of the major challenges confronted by the magnetic fusion community is connected to the handling of the power and particle fluxes escaping from the plasma core of Tokamaks towards the plasma facing components (PCFs). To avoid any damage of the PCFs, the heat and particle loads should not exceed 20 MW/m$^2$ even for the most advanced materials. The most promising scenario to fulfill this requirement is the creation of a radiative mantle in the divertor region, with the plasma “detaching” from the wall. Therefore, a big effort is being devoted to the study of plasma detachment in both L- and H-modes especially in the framework of EUFusion MST1 [1] and JET1 workplans [2] as this scenario is foreseen for future large scale fusion devices like ITER. In support of such studies, the work presented here relies on the modeling of line and continuum emission of hydrogen for conditions relevant to detached divertor plasmas ($n_e \sim 10^{20}-10^{21} m^{-3}$, $T_e \sim 1$ eV) in the aim of their characterization, i.e, providing their main plasma parameters accurately ($n_e$, $T_e$ and $T_i$).

Information obtained from spectral line intensities and profiles are crucial for estimating plasma parameters, e.g. the electron density can be determined from the Stark broadened profiles [3] and the temperature can be inferred from the line intensities as well as from the continuum intensities [3]. In the modeling of the complete spectrum, contributions to hydrogen emission are calculated separately and summed up. High-n line profiles are calculated considering electronic broadening only using the GBK approximation [4]. Radiative recombination and the bremsstrahlung are calculated using simple analytical expressions. In experimental spectra, merging of series lines and the radiative recombination is not a sharp step as calculated by the theory which follows. This discrete to continuum transition is modeled using a dissolution factor approach [5].

2. Modeling of the spectrum
Hydrogen Balmer emission spectra in the detached plasma consists of three contributions from bound-bound, free-bound (radiative recombination) and free-free transitions (bremsstrahlung radiation). The plasma is assumed to be in local thermodynamical equilibrium (LTE). Plasma parameters,
$n_e=10^{21} \text{m}^{-3}$ and $T_e=1 \text{ eV}$ are considered here for the detached plasma as discussed in previous works like [6] for Alcator C-Mod divertor plasmas.

The profiles of the high-$n$ (n is the principal quantum number of the upper level of the transition) lines are dominated by the Stark broadening [5]. The Stark broadening is dominated by the electron contribution and widths are calculated using GBK approximation as discussed in [4]. Intensities are calculated using with LTE theory [7]. Balmer lines (H-5 to H-25) are calculated and shown in figure 1.

The continuum emission of hydrogen in detached plasmas is observed with sufficient intensity for quantitative analysis [6] and is dominated by the radiative recombination process in comparison with the contribution from the free-free transition as we will discuss in the following sections.

The radiative recombination is the inverse process of the photoionization and the transition to a level $n$ of a neutral atom is given by: $H^+ + e \rightarrow H(n) + h\nu$. The radiation power density of the recombination process as a function of frequency is given as: $\rho_n^R(v)dv = h\nu n_i n_e f(\nu)e \sigma_{rec}(n,e)d\nu$ [8] where $h$ is the Planck constant, $n_i$ is the ion density, $v_e$ is the electron velocity, $f(\nu)$ is the energy distribution function of free electrons and $\sigma_{rec}(n,e)$ is the cross section for the recombination process [8]. The cross-section, $\sigma_{rec}(n,e)$ is obtained from its relation with the photoionization cross section for the level $n$, $\sigma_{ion}(n,\nu)$ using Milne’s formula [8]. The corresponding intensity can be rewritten as:

$$\rho_n^R(v) = 2\frac{h^4}{\pi m_\epsilon^2 c^2} 2n^2 \left(\frac{1}{kT_e}\right)^{3/2} \exp\left(-\frac{hv-I_H(p)}{kT_e}\right) \sigma_{ion}(n,\nu) v^3 n_i n_e$$

(1)

with $k$ the Boltzmann constant, $m_\epsilon$ the electron mass, $c$ the speed of light and $I_H(p)$ the ionization potential for level $n$. Electron energy distribution is assumed to obey the Maxwell-Boltzmann distribution. The photoionization cross-section is calculated using Kramer’s formula [7]. Total recombination radiation is calculated by summing up (1) with levels up to $n=10$ as shown in figure 2.

Bremsstrahlung is the emission during deceleration of the electrons in the vicinity of ions and will contribute to the emission spectrum. The intensity of these free-free transitions for an assembly of emitters of charge $Z$ is given as [7]:

$$\rho_B(v) = \left(\frac{32\pi^2 e^6}{3\sqrt{3}}\right) \left(\frac{1}{m_\epsilon e c}\right)^{3/2} Z^2 e^{\frac{\nu}{kT_e}} n_i n_e$$

(2)

The comparison with the photo recombination is also seen in figure 2.

Figure 1. Balmer lines calculated using GBK approximation ($n_e=10^{21} \text{ m}^{-3}$ and $T_e=1 \text{ eV}$)

Figure 2. Comparison of radiative recombination and bremsstrahlung ($n_e=10^{21} \text{ m}^{-3}$ and $T_e=1 \text{ eV}$)

For dense detached plasmas, the transition region from discrete to continuum becomes wide and one needs a specific approach to model this region. The contribution to broadening of the levels near to the series limit is affected by the static behavior of a fraction of the electrons experiencing a strong collision. This leads to a dissolution factor approach as discussed in [3]. High-n lines show a merging beyond the Inglis-Teller limit [9] and series limit shows a shift due to plasma density effect. The considered method also known as occupation probability formalism (OPF) [5] is based on two simple suggestions, i) Potential barrier of the atom is reformed as the electric field is superimposed with the
field of interaction of the external electron with the core of the atom. When the quasistatic microfield exceeds a certain critical value $F_c$ that corresponds to a level above the barrier, energy level $E_n$ disappears. ii) Oscillator strength density is conserved during the replacement of bound-bound transition with a free-bound one. The probability of $E_n$ level realization is given by:

$$W_n = \int_0^{F_c} P(F) dF$$

which is the dissolution factor. $P(F)$ is the microfield distribution function which is given by APEX [10]. The critical field $F_c$ is calculated using the method discussed in [11] as:

$$F_c(n) = \frac{2n-3.5}{6n^3(n+1)^2(n-2)}$$

Dissolution factors of levels corresponding to the first 25 lines of hydrogen Balmer series are calculated. Low-$n$ lines are unperturbed by the plasma microfield (dissolution factor=1). Corresponding line intensities should be multiplied by the dissolution factors. Shift of the series limit is calculated using the method discussed in [6] and it is included in the calculations of $\sigma_{\text{ion}}(\rho, \nu)$ and radiative recombination is extended towards line. According to the second suggestion, the continuum should be multiplied with a factor $(1-W_n)$ in the corresponding positions of the lines. The sum of the lines and recombination continuum which are modified with dissolution factors gives the profile with a smooth transition from line to continuum as shown in Figure 3.

Figure 3. Discrete to continuum transition ($n_e=10^{21} \text{ m}^{-3}$ and $T_e=1 \text{ eV}$)

3. Summary
Line and continuum spectral emissions along with the discrete to continuum transition for hydrogen Balmer spectrum are modeled for detached divertor plasma of Tokamaks. This simple model can be used to fit the whole experimental spectra and characterize the divertor plasma of large Tokamaks like ITER, possibly using a synthetic diagnostic tool in combination with a transport code such as Soledge2d-EIRENE [12] in order to account for the spatial inhomogeneity of the plasma.

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