Curriculum Loss: Robust Learning and Generalization against Label Corruption

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Abstract

Generalization is vital important for many deep network models. It becomes more challenging when high robustness is required for learning with noisy labels. The 0-1 loss has monotonic relationship between empirical adversary (reweighted) risk \cite{hu2019generalization}, and it is robust to outliers. However, it is also difficult to optimize. To efficiently optimize 0-1 loss while keeping its robust properties, we propose a very simple and efficient loss, i.e. curriculum loss (CL). Our CL is a tighter upper bound of the 0-1 loss compared with conventional summation based surrogate losses. Moreover, CL can adaptively select samples for training as a curriculum learning. To handle large rate of noisy label corruption, we extend our curriculum loss to a more general form that can automatically prune the estimated noisy samples during training. Experimental results on noisy MNIST, CIFAR10 and CIFAR100 dataset validate the robustness of the proposed loss.

1 Introduction

Noise corruption is a common phenomenon in our daily life. For instance, noisy corrupted (wrong) labels may be resulted from annotating for similar objects \cite{sun2019accurate}, crawling images and labels from websites \cite{chakraborty2018generalization, zhang2019label} and creating training sets by program \cite{sener2018robust, zhang2018spatial}. Learning with noisy labels is thus an promising area, however, it is challenging to train deep networks robustly with noisy labels.

Deep networks have great expressive power (model complexity) to learn challenging tasks. However, they undertake more risk of overfitting to the data. Although many regularization techniques such as adding regularization terms, data augmentation, weight decay, dropout and batch normalization have been proposed, generalization is still vital important for deep learning to fully exploit the super expressive power. It becomes more challenging when the robustness is required for learning with noisy labels. Zhang et al. \cite{zhang2018understanding} show that deep networks can even fully memorize samples with incorrectly corrupted labels. This will significantly degenerate the generalization performance of deep models.

Robustness of 0-1 loss: The problem resulted from label corruption is that test distribution is different from training distribution. Hu et al. \cite{hu2019generalization} analyzed the adversarial risk that the test distribution density is adversarially changed within a limited f-divergence (e.g. KL-divergence) from the training distribution density. They show that there is a monotonic relationship between (empirical) risk and the (empirical) adversarial risk when 0-1 loss function is used. This suggests that minimizing empirical risk with the 0-1 loss function is equivalent to minimize the empirical adversarial risk. Note that we evaluate models on test data in terms of the 0-1 loss (empirical approximation of Bayes risk w.r.t. test distribution). Therefore, without making other assumptions, minimizing the 0-1 loss is the most robust loss against label corruption. Moreover, the 0-1 loss is more robust to outliers compared with an unbounded (convex) loss (e.g. hinge loss) \cite{zhang2019generalization}. This is due to unbounded convex loss put much weight on the outliers (with large loss) when minimizing the loss \cite{zhang2019generalization}. If the unbounded (convex) loss is employed in deep network models, this becomes more prominent. Since training loss of deep
networks can often be minimized to zero, outlier with a large loss has a large impact of the model. The 0-1 loss treats each training sample equally. Thus, each sample does not have too much influence on the model. Therefore, the model is tolerant to a small number of outliers. Although the 0-1 loss has many robust properties, it is difficult to optimize. One possible way to alleviate this problem is to seek a tighter upper bound of the 0-1 loss that is still efficient to optimize. Such a tighter upper bound of the 0-1 loss can reduce the influence of the noisy outliers compared with conventional (convex) losses; while it is easier to optimize compared with the 0-1 loss. When minimizing the upper bound surrogate, we expect that the 0-1 loss objective is also minimized.

Learnability under large noise rate: Even the 0-1 loss cannot deal with large noise rate. When the noise rate becomes large, the systematic error (due to label corruption) grows up and becomes not negligible. As a result, the model’s generalization performance will degenerate due to this systematic error. To reduce the systematic error produced by training with noisy labels, several methods have been proposed. They can be categorized into three kinds: transition matrix based method \([20][17][4]\), regularization based method \([15]\) and sample selection based method \([11][6]\). Among them, sample selection based method is one promising direction that selects samples to reduce noisy ratio for training. These methods are based on the idea of curriculum learning \([3]\) which is one successful method that trains the model gradually with samples ordered in a meaningful sequence. Although they achieve success to some extents, most of these methods are heuristic based.

To efficiently minimize the 0-1 loss while keeping the robust properties, we propose a novel loss that is a tighter upper bound of 0-1 loss compared with conventional surrogate losses. Specifically, giving any base loss function \(l(u) \geq 1(u < 0), u \in \mathbb{R}\), our loss \(Q(u)\) satisfies \(\sum_{i=1}^{n} 1(u_i < 0) \leq Q(u) \leq \sum_{i=1}^{n} l(u_i)\), where \(u = [u_1, \cdots, u_n]\) with \(u_i\) be the classification margin of \(i^{th}\) sample. We name it as Curriculum Loss (CL) because our loss automatically and adaptively select samples for training as a curriculum learning. The selection procedure can be done by a very simple and efficient algorithm in \(O(n \log n)\). Moreover, since our loss is tighter than conventional surrogate losses, it is more robust compared with them. To handle the case of learning with large noise rate, we propose Noise Pruned Curriculum Loss (NPCL) by extending our basic curriculum loss to a more general form. It reduces to our basic CL when the estimated noise rate is zero. Our NPCL automatically prunes estimated noisy samples during training process. Remarkably, our NPCL is very simple and efficient, it can be used as a plug-in for many deep models for robust learning. Our contributions are listed as follows:

- We propose a novel loss (i.e. curriculum loss) that is a tighter upper bound of 0-1 loss compared with conventional summation based surrogate loss. Our curriculum loss can automatically and adaptively select samples for training as a curriculum learning.
- We further propose Noise Pruned Curriculum Loss (NPCL) to address large rate of noise (label corruption) by extending our curriculum loss to a more general form. Our NPCL automatically prune the estimated noisy samples during training. Moreover, our NPCL is very simple and efficient, it can be used as a plug-in in many deep models.

2 Related Literature

Curriculum Learning: Curriculum learning is a general learning methodology that achieves success in many area. The very beginning work of curriculum learning \([3]\) trains a model gradually with samples ordered in a meaningful sequence, which has improved performance on many problems. Since the curriculum in \([3]\) is predetermined by prior knowledge and remained fixed later, which ignores the feedback of learners, Kumar et al. \([13]\) further propose Self-paced learning that selects samples by alternative minimization of an augmented objective. Jiang et al. \([9]\) propose a self-paced learning method to select samples with diversity. After that, Jiang et al. \([10]\) propose a self-paced curriculum strategy that takes different priors into consideration. Although these methods achieve success, the relation between the augmented objective of self-paced learning and the original objective (e.g. cross entropy loss for classification) is not clear. In addition, as stated in \([11]\), the alternative update in self-paced learning is not efficient for training deep networks.

Learning with Noisy Labels: The most related works are the sample selection based methods for robust learning. This kind of works are inspired by curriculum learning \([3]\). Among them, Jiang et al. \([11]\) propose to learn the curriculum from data by a mentor net. They use the mentor net to select samples for training with noisy labels. Co-teaching \([6]\) employs two networks to select samples to
We begin with discussion about robustness of the 0-1 loss. We then show that our CL is a tighter upper bound of the 0-1 loss compared with conventional summation based surrogate loss. A tighter upper bound of the 0-1 loss means that it is less sensitive to the noisy outliers, and it better preserves the robustness of the 0-1 loss against label corruption. Thus, it can deal with noisy samples with small rate of label corruption. When the estimated rate of label corruption is zero. Our CL and NPCL are very simple and efficient, and empirical adversarial risk when 0-1 loss function is used. It means that minimizing (empirical) risk is equivalent to minimize the (empirical) adversarial risk for 0-1 loss. Note that the problem resulted from label corruption is that test distribution is different from training distribution. Without further making other assumptions about the corruption distribution, the 0-1 loss is the most robust loss function because train each other and achieve good generalization performance against large rate of label corruption. Although these methods achieve success to some extent, most of them are heuristic based.

**Construction of tighter bounds of 0-1 loss:** Along the line of construction of tighter bounds of the 0-1 loss, many methods have been proposed. To name a few, Masnadi-Shirazi et al. [14] propose Savage loss, which is a non-convex upper bound of the 0-1 loss function. Bartlett et al. [2] analyze the properties of the truncated loss for conventional convex loss. Wu et al. [23] study the truncated hinge loss for SVM. Although the results are fruitful, these works are mainly focus on loss function at individual data point, they do not have sample selection property. In contrast, our curriculum loss can automatically prune the estimated noisy samples during training process. It reduces to our basic CL for training against label corruption.

In this section, we present our Curriculum Loss (CL) that automatically selects samples for training. We begin with discussion about robustness of the 0-1 loss. We then show that our CL is a tighter upper bound of the 0-1 loss compared with conventional summation based surrogate loss. A tighter upper bound of the 0-1 loss means that it is less sensitive to the noisy outliers, and it better preserves the robustness of the 0-1 loss against label corruption. Thus, it can deal with noisy samples with small rate of label corruption. When the estimated rate of label corruption becomes large, even the 0-1 loss suffers. Thus, we propose Noise Pruned Curriculum Loss (NPCL) to address this issue. Our NPCL can automatically prune the estimated noisy samples during training process. It reduces to our basic CL when the estimated rate of label corruption is zero. Our CL and NPCL are very simple and efficient, which support mini-batch update. They can be used as plug-in for many deep models. A simple multi-class extension and a novel soft multi-hinge loss is included in the supplementary material. All the detailed proofs are also included in the supplemental material.

### 3 Curriculum Loss

In this section, we present our Curriculum Loss (CL) that automatically selects samples for training. We rephrase Theorem 1 in [8] from a different perspective, which motivates us to employ the 0-1 loss for training against label corruption.

**Theorem 1.** (Hu et al. [8]) Let \( p(x, y) \) and \( q(x, y) \) be the training and test density, respectively. Define \( r(x, y) = q(x, y)/p(x, y) \) and \( r_i = r(x_i, y_i) \). Let \( l(\hat{y}, y) = 1(\text{sign}(\hat{y}) \neq y) \) and \( l(\hat{y}, y) = 1(\text{argmax}_k(\hat{y}_k) \neq y) \) be 0-1 loss for binary classification and multi-class classification, respectively. Let \( f(\cdot) \) be convex with \( f(1) = 0 \). Define risk \( R(\theta) \), empirical risk \( \hat{R}(\theta) \), adversarial risk \( R_{adv}(\theta) \) and empirical adversarial risk \( \hat{R}_{adv}(\theta) \) as follows:

\[
\begin{align*}
R(\theta) &= \mathbb{E}_{p(x,y)} [l(g_0(x), y)] \\
\hat{R}(\theta) &= \frac{1}{n} \sum_{i=1}^{n} l(g_0(x_i), y_i) \\
R_{adv}(\theta) &= \sup_{r \in \mathcal{U}_f} \mathbb{E}_{p(x,y)} [r(x, y) l(g_0(x), y)] \\
\hat{R}_{adv}(\theta) &= \sup_{r \in \hat{\mathcal{U}}_f} \frac{1}{n} \sum_{i=1}^{n} r_i l(g_0(x_i), y_i),
\end{align*}
\]

where \( \mathcal{U}_f = \{ r(x, y) | \mathbb{E}_{p(x,y)} [f(r(x, y))] \leq \delta, \mathbb{E}_{p(x,y)} [r(x, y)] = 1, r(x, y) \geq 0, \forall (x, y) \in \mathcal{X} \times \mathcal{Y} \} \) and \( \hat{\mathcal{U}}_f = \{ r | \frac{1}{n} \sum_{i=1}^{n} f(r_i) \leq \delta, \frac{1}{n} \sum_{i=1}^{n} r_i = 1, r \geq 0 \} \). Then we have that

\[
\begin{align*}
\text{If } R_{adv}(\theta_1) < 1, \text{ then } R(\theta_1) < R(\theta_2) \iff R_{adv}(\theta_1) < R_{adv}(\theta_2). \tag{5}
\end{align*}
\]

\[
\begin{align*}
\text{If } R_{adv}(\theta_1) = 1, \text{ then } R(\theta_1) \leq R(\theta_2) \iff R_{adv}(\theta_2) = 1. \tag{6}
\end{align*}
\]

The same monotonic relationship holds between their empirical approximation: \( \hat{R}(\theta) \) and \( \hat{R}_{adv}(\theta) \).

Theorem 1[8] shows that the monotonic relationship between (empirical) risk and the (empirical) adversarial risk when 0-1 loss function is used. It means that minimizing (empirical) risk is equivalent to minimize the (empirical) adversarial risk for 0-1 loss. Note that the problem resulted from label corruption is that test distribution is different from training distribution. Without further making other assumptions about the corruption distribution, the 0-1 loss is the most robust loss function because
minimizing the 0-1 loss is equivalent to minimize the worst case risk, i.e., (empirical) adversarial risk for a changing test distribution within a limited $f$-divergence from the given (empirical) training distribution. For label corruption with a small noise rate, the $f$-divergence between test distribution and corrupted training distribution is small. In this situation, training with 0-1 loss is most robust against adversary changing test distribution without other assumptions. This motivates us to employ 0-1 loss for training against label corruption.

### 3.2 Tighter upper bounds of 0-1 Loss

The 0-1 loss is difficult to optimize, we thus propose a tighter upper bound surrogate loss. We use the classification margin to define the 0-1 loss. For binary classification, classification margin is $u = \hat{y}y$, where $\hat{y}$ and $y \in \{+1, -1\}$ denotes the prediction and ground truth, respectively. (A simple multi-class extension is discussed in supplement.) Let $u_i \in \mathbb{R}$ be the classification margin of the $i^{th}$ sample for $i \in \{1, ..., n\}$. Denote $u = [u_1, ..., u_n]$. The 0-1 loss objective can be defined as follows:

$$J(u) = \sum_{i=1}^{n} 1(u_i < 0). \quad (7)$$

Given a base upper bound function $l(u) \geq 1(u < 0), u \in \mathbb{R}$, the conventional surrogate of the 0-1 loss can be defined as

$$\hat{J}(u) = \sum_{i=1}^{n} l(u_i). \quad (8)$$

Our curriculum loss $Q(u)$ can be defined as Eq. $(9)$. $Q(u)$ is a tighter upper bound of 0-1 loss $J(u)$ compared with the conventional surrogate loss $\hat{J}(u)$, which is summarized in Theorem 2.

**Theorem 2.** Suppose that base loss function $l(u) \geq 1(u < 0), u \in \mathbb{R}$ is an upper bound of the 0-1 loss function. Let $u_i \in \mathbb{R}$ be the classification margin of the $i^{th}$ sample for $i \in \{1, ..., n\}$. Let $\max(\cdot, \cdot)$ denote the maximum between two inputs. Denote $u = [u_1, ..., u_n]$. Define $Q(u)$ as follows:

$$Q(u) = \min_{v \in \{0,1\}^n} \max \left( \sum_{i=1}^{n} v_i l(u_i), n - \sum_{i=1}^{n} v_i + \sum_{i=1}^{n} 1(u_i < 0) \right). \quad (9)$$

Then $J(u) \leq Q(u) \leq \hat{J}(u)$ holds true.

**Remark:** For any fixed $u$, we can obtain an optimum solution $v^*$ of the partial optimization. The index indicator $v^*$ can naturally select samples as a curriculum for training models. The partial optimization w.r.t index indicator $v$ can be solved by a very simple and efficient algorithm (Alg[1]) in $O(n \log n)$. Thus, the loss is very efficient to compute. Moreover, since $Q(u)$ is tighter than conventional surrogate loss $\hat{J}(u)$, it is less sensitive to outliers compared with $\hat{J}(u)$. Furthermore, it better preserves the robust property of the 0-1 loss against label corruption.

Updating with all the samples at once is not efficient for deep models, while training with mini-batch is more efficient and well supported for many deep learning tools. We thus propose a batch based curriculum loss $\hat{Q}(u)$ given as Eq. $(10)$. We show that $\hat{Q}(u)$ is also a tighter upper bound of 0-1 loss objective $J(u)$ compared with conventional loss $\hat{J}(u)$. This property is summarized in Corollary 1.

**Corollary 1.** Suppose that base loss function $l(u) \geq 1(u < 0), u \in \mathbb{R}$ is an upper bound of the 0-1 loss function. Let $b, m$ be the number of batches and batch size, respectively. Let $u_{ij} \in \mathbb{R}$ be the classification margin of the $i^{th}$ sample in batch $j$ for $i \in \{1, ..., m\}$ and $j \in \{1, ..., b\}$. Denote $u = [u_{11}, ..., u_{mb}]$. Let $n = mb$. Define $\hat{Q}(u)$ as follows:

$$\hat{Q}(u) = \sum_{j=1}^{b} \min_{v \in \{0,1\}^n} \max \left( \sum_{i=1}^{m} v_{ij} l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} + \sum_{i=1}^{m} 1(u_{ij} < 0) \right). \quad (10)$$

Then $J(u) \leq Q(u) \leq \hat{Q}(u) \leq \hat{J}(u)$ holds true.

**Remark:** Corollary 1 shows that a batch-based curriculum loss is also a tighter upper bound of 0-1 loss $J(u)$ compared with the conventional surrogate loss $\hat{J}(u)$. This enables us to train deep models with mini-batch update. Note that random shuffle in different epoch results in a different batch-based curriculum loss. Nevertheless, we at least know that all the induced losses are upper bounds of 0-1 loss objective and are tighter than $\hat{J}(u)$. Moreover, all these losses are induced by the same base loss function $l(\cdot)$. Note that, our goal is to minimize the 0-1 loss. Random shuffle leads to a multiple surrogate training scheme. In addition, training deep models without shuffle does not have this issue.
We now present another curriculum loss $E(u)$ which is tighter than $Q(u)$. $E(u)$ is an (scaled) upper bound of 0-1 loss. This property is summarized as Theorem 3.

**Theorem 3.** Suppose that base loss function $l(u) \geq 1(u < 0)$, $u \in \mathbb{R}$ is an upper bound of the 0-1 loss function. Let $u_i \in \mathbb{R}$ be the classification margin of the $i^{th}$ sample for $i \in \{1, ..., n\}$. Denote $u = [u_1, ..., u_n]$. Define $E(u)$ as follows:

$$E(u) = \min_{v \in \{0, 1\}^n} \max \left( \sum_{i=1}^{n} v_i l(u_i), n - \sum_{i=1}^{n} v_i \right).$$  \hspace{1cm} (11)

Then $J(u) \leq 2E(u) \leq 2\tilde{J}(u)$ holds true.

**Remark:** $E(u)$ has similar properties to $Q(u)$ discussed above. Moreover, it is tighter than $Q(u)$, i.e. $E(u) \leq Q(u)$. Thus, it is less sensitive to outliers compared with $Q(u)$. However, $Q(u)$ can construct more adaptive curriculum by taking 0-1 loss into consideration during the training process. Directly optimizing $E(u)$ is not efficient similar to $Q(u)$. We now present a batch loss objective $\tilde{E}(u)$ given as Eq.\[\ref{eq:batch_loss}\]. $\tilde{E}(u)$ is also a tighter upper bound of 0-1 loss objective $J(u)$ compared with conventional surrogate loss $\tilde{J}(u)$.

**Corollary 2.** Suppose that base loss function $l(u) \geq 1(u < 0)$, $u \in \mathbb{R}$ is an upper bound of the 0-1 loss function. Let $b, m$ be the number of batches and batch size, respectively. Let $u_{ij} \in \mathbb{R}$ be the classification margin of the $i^{th}$ sample in batch $j$ for $i \in \{1, ..., n\}$ and $j \in \{1, ..., b\}$. Denote $u = [u_{11}, ..., u_{mb}]$. Let $n = mb$. Define $\tilde{E}(u)$ as follows:

$$\tilde{E}(u) = \sum_{j=1}^{b} \min_{v \in \{0, 1\}^m} \max \left( \sum_{i=1}^{m} v_{ij} l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} \right).$$  \hspace{1cm} (12)

Then $J(u) \leq 2E(u) \leq 2\tilde{E}(u) \leq 2\tilde{J}(u)$ holds true.

All the curriculum losses defined above rely on minimizing a partial optimization problem (Eq.\[\ref{eq:partial_optimization}\]) to find the selection index set $v^*$. We now show that the optimization of $v$ with given classification margin $u_i \in \mathbb{R}, i \in \{1, ..., n\}$ can be done in $O(n \log n)$.

**Theorem 4.** (Partial Optimization) Suppose that base loss function $l(u) \geq 1(u < 0)$, $u \in \mathbb{R}$ is an upper bound of the 0-1 loss function. For fixed $u_i \in \mathbb{R}, i \in \{1, ..., n\}$, an minimum solution $v^*$ of the minimization problem in Eq.\[\ref{eq:partial_optimization}\] can be achieved by Algorithm 1.

$$\min_{v \in \{0, 1\}^n} \max \left( \sum_{i=1}^{n} v_i l(u_i), C - \sum_{i=1}^{n} v_i \right),$$  \hspace{1cm} (13)

where $C$ is the threshold parameter.

**Remark:** The time complexity of Algorithm 1 is $O(n \log n)$. Moreover, it does not involve complex operations, and is very simple and efficient to compute.

Algorithm 1 can adaptively select samples for training. It has some useful properties to help us better understand the objective after partial minimization, we present them in Proposition 5.
Proposition 1. Suppose that base loss function \( l(u) \geq 1(u < 0) \), \( u \in \mathbb{R} \) is an upper bound of the 0-1 loss function. Let \( u_i \in \mathbb{R} \) for \( i \in \{1, \ldots, n\} \) be fixed values. Without loss of generality, assume \( l(u_1) \leq l(u_2) \cdots \leq l(u_n) \). Let \( v^* \) be an optimum solution of the partial optimization problem in \((13)\). Let \( T^* = \sum_{i=1}^{n} v^*_i \) and \( L_{T^*} = \sum_{i=1}^{T^*} l(u_i) \). Then we have

\[
\begin{align*}
L_{T^*} & \leq C + 1 - T^* \\
L_{T^*+1} & > C - T^* \\
L_{T^*+1} & > \max(L_{T^*}, C - T^*) \\
\min_{v \in \{0,1\}^n} \max (\sum_{i=1}^{n} v_i l(u_i), C - \sum_{i=1}^{n} v_i) & = \max(L_{T^*}, C - T^*).
\end{align*}
\]

Remark: When \( C \leq n + \sum_{i=1}^{n} 1(u_i < 0) \), Eq.\((17)\) is tighter than the conventional loss \( \hat{J}(u) \). When \( C \geq n \), Eq. \((17)\) is a scaled upper bound of 0-1 loss \( J(u) \). From Eq.\((17)\), we know the optimum of the partial optimization problem \((13)\) (i.e. our objective) is \( \max(L_{T^*}, C - T^*) \).

When \( L_{T^*} < C - T^* \), we can directly optimize \( L_{T^*} \) with the selected samples for training. When \( L_{T^*} > C - T^* \), note that \( L_{T^*+1} > \max(L_{T^*}, C - T^*) \) from Eq.\((16)\), we can optimize \( L_{T^*+1} \) for training. Note that when \( T^* < n \), we have that \( L_{T^*+1} \leq L_n = \sum_{i=1}^{n} l(u_i) \), which is still tighter than the conventional loss \( \hat{J}(u) \). When \( T^* = n \), for the parameter \( C \leq n + \sum_{i=1}^{n} 1(u_i < 0) \), we have that \( L_{T^*} = \hat{J}(u) > J(u) \geq C - n = C - T^* \). Thus we can safely optimize \( \max(L_{T^*}, C - T^*) = \hat{J}(u) \).

In practice, when training with random mini-batch, we find that optimizing \( L_{T^*} \) in both cases instead of \( L_{T^*+1} \) does not have much influence.

3.3 Noise Pruned Curriculum Loss

The curriculum loss in Eq.\((9)\) and Eq.\((11)\) expect to minimize the upper bound of the 0-1 loss for all the training samples. When model capability (complexity) is high, (deep network) model will still attain small (zero) training loss and overfit to the noisy samples.

The ideal model is that it correctly classifies the clean training samples and misclassify the noisy samples with wrong labels. Suppose that the rate of noisy samples (by label corruption) is \( \epsilon \in [0, 1] \). The ideal model is to correctly classify the \((1 - \epsilon)n\) clean training samples, and misclassify the \(\epsilon n\) noisy training samples. This is because the label is corrupted. Correctly classify the training samples with corrupted (wrong) label means that the model has already overfitted to noisy samples. This will harm the generalization to the unseen data.

Considering all the above reasons, we thus propose the Noise Pruned Curriculum Loss (NPCL) as

\[
\mathcal{L}(u) = \min_{v \in \{0,1\}^n} \max (\sum_{i=1}^{n} v_i l(u_i), C - \sum_{i=1}^{n} v_i),
\]

where \( C = (1 - \epsilon)n \) or \( C = (1 - \epsilon)^2 n + (1 - \epsilon) \sum_{i=1}^{n} 1(u_i < 0) \).

When we know there are \( \epsilon n \) noisy samples in the training set, we can leverage this as our prior. (The impact of misspecification of the prior is included in the supplement.) When \( C = (1 - \epsilon)n \) (assume \( C, \epsilon n \) are integers for simplicity), from the selection procedure in Algorithm\(1\), we know \( \epsilon n \) samples with largest losses \( l(u) \) will be pruned. This is because \( C - \sum_{i=1}^{n} v_i + 1 \leq 0 \) when \( \sum_{i=1}^{n} v_i \geq (1 - \epsilon)n + 1 \). Without loss of generality, assume \( l(u_1) \leq l(u_2) \cdots \leq l(u_n) \). After pruning, we have \( v_{(1-\epsilon)n+1} = \cdots = v_n = 0 \), the pruned loss becomes

\[
\tilde{\mathcal{L}}(u) = \min_{v \in \{0,1\}^{1-\epsilon n}} \max (\sum_{i=1}^{(1-\epsilon)n} v_i l(u_i), (1 - \epsilon)n - \sum_{i=1}^{(1-\epsilon)n} v_i).
\]

It is the basic CL for \((1 - \epsilon)n\) samples and it is the upper bound of \( \sum_{i=1}^{(1-\epsilon)n} 1(u_i < 0) \). If we prune more noisy samples than clean samples, it will reduce the noise ratio. Then the basic CL can handle. Fortunately, this assumption is supported by the "memorization" effect in deep networks\(1\), i.e. deep networks tend to learn clean and easy pattern first. Thus, the loss of noisy or hard data tend to remain high for a period (before being overfitted). Therefore, the pruned samples with largest loss are more likely to be the noisy samples. After the rough pruning, the problem becomes optimizing basic CL.

\footnote{When \( \sum_{i=1}^{(1-\epsilon)n+1} l(u_i) \neq 0 \), \( \epsilon n \) samples will be pruned. Otherwise, \( \epsilon n - 1 \) samples will be pruned.}
We employ the same network architecture and hyperparameters as in Co-teaching [6] for all. When $C$ is very efficient for modern deep learning tools. The training procedure is summarized in Algorithm 2. It uses Algorithm 1 to select a subset of samples from every mini-batch. Then, it uses $\hat{C}$, which is a prior, users can define it based on their domain knowledge.

It has similar properties as choosing $C = (1 - \epsilon)n$. When $C = (1 - \epsilon)2n + (1 - \epsilon)\sum_{i=1}^{m} 1(u_i < 0)$, for a target ideal model (that misclassifies noisy samples only), we know that $E[C] = (1 - \epsilon)^2n + (1 - \epsilon)\sum_{i=1}^{m} 1(u_i < 0) = (1 - \epsilon)^2n + (1 - \epsilon)\epsilon n = (1 - \epsilon)n$. It has similar properties as choosing $C = (1 - \epsilon)n$. Moreover, it is more adaptive by considering 0-1 loss during training at different stages. In this case, the NPCL in Eq. (18) reduces to the CL in Eq. (9) when $\epsilon = 0$. Note that $C$ is a prior, users can define it based on their domain knowledge.

To leverage the benefit of deep learning, we present the batched NPCL as

$$\hat{L}(\mathbf{u}) = \sum_{j=1}^{b} \min_{v \in (0,1)^m} \max \left( \sum_{i=1}^{m} v_{ij}l(u_{ij}), \hat{C}_j - \sum_{i=1}^{m} v_{ij} \right),$$

(20)

where $\hat{C}_j = (1 - \epsilon)m$ or as in Eq. (21):

$$\hat{C}_j = (1 - \epsilon)^2m + (1 - \epsilon)\sum_{i=1}^{m} 1(u_{ij} < 0).$$

(21)

Similar to Corollary 1, we know that $L(\mathbf{u}) \leq \hat{L}(\mathbf{u})$. Thus, optimizing the batched NPCL is indeed minimizing the upper bound of NPCL. This enables us to train the model with mini-batch update, which is very efficient for modern deep learning tools. The training procedure is summarized in Algorithm 3. It uses Algorithm 1 to select a subset of samples from every mini-batch. Then, it uses the selected samples to perform gradient update.

4 Empirical Study

Evaluation of robustness against label corruption: We evaluate our NPCL by comparing Co-teaching [6], MentorNet [11] and standard network training on MNIST, CIFAR10 and CIFAR100 dataset as in [6, 17, 4]. Two types of random label corruption, i.e. Symmetry flipping [22] and Pair flipping [5], are considered in this work. Symmetry flipping is that the corrupted label is uniformly assigned to one of $K - 1$ incorrect classes. Pair flipping is that the corrupted label is assigned to one specific class similar to the ground truth. The noise rate $\epsilon$ of label flipping is chosen from $\{0.2, 0.5, 0.35\}$. We employ same network architecture and network hyperparameters as in Co-teaching [6] for all the methods in comparison. Specifically, the batch size and the number of epochs is set to $m = 128$ and $N = 200$, respectively. The Adam optimizer with same parameter as [6] is employed. For NPCL, we employ hinge loss as the base upper bound function of 0-1 loss. At first a few epochs, we train model using full batch with soft hinge loss (in the supplement) as a burn-in period suggested in [11].
Specifically, we start NPCL at $5^{th}$ epoch on MNIST and $10^{th}$ epoch on CIFAR10 and CIFAR100, respectively. For Co-teaching [6] and MentorNet in [11], we employ the open sourced code provided by the authors of Co-teaching [6]. We implement NPCL by Pytorch. Code of NPCL will be released on GitHub. Experiments are performed five independent runs. The error bar for STD is shaded.

For performance measurements, we employ both test accuracy and label precision as in [6]. Label precision is defined as: \( \frac{\text{number of clean samples}}{\text{number of selected samples}} \), which measures the selection accuracy for sample selection based methods. A higher label precision in the mini-batch after sample selection can lead to a update with less noisy samples, which means that model suffers less influence of noisy samples and thus preforms more robust to label corruption.

The picture of test accuracy and label precision vs. number of epochs on MNIST is presented in Figure 1. It shows that NPCL consistently outperforms Co-teaching in terms of test accuracy on all three cases. Moreover, NPCL achieves higher label precision compared with Co-teaching, which means that NPCL can select more clean samples than Co-teaching. The average of test accuracy over last ten epochs for different methods are reported in Table 1. Again, we can observe that NPCL obtains higher test accuracy than other methods on all three cases, which shows the robustness of our NPCL. More experimental results can be found in the supplemental material. Note that NPCL is a simple plug-in for a single network, while Co-teaching employs two networks to train the model concurrently. Thus, both the space complexity and time complexity of co-teaching is doubled compared with our NPCL.

5 Conclusion

In this work, we proposed a curriculum loss (CL) for robust learning. Theoretically, we analyzed the properties of CL and proofed that it is tighter upper bound of the 0-1 loss compared with conventional summation based surrogate losses. We extended our CL to a more general form (NPCL) to handle large rate of label corruption. Empirically, experimental results on benchmark datasets show the robustness of the proposed loss. As a side product, we proposed a novel soft multi-class hinge loss. Experimental results on CIFAR100 shows that the proposed soft hinge loss is easier to optimize compared with the hard hinge loss.
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A Proof of Theorem 2

Proof. Because \(1(u < 0) \leq l(u)\), we have \(\sum_{i=1}^{n} l(u_i) \geq \sum_{i=1}^{n} 1(u_i < 0)\). Then

\[
Q(u) = \min_{v \in \{0,1\}^n} \max \left( \sum_{i=1}^{n} v_i l(u_i), n - \sum_{i=1}^{n} v_i + \sum_{i=1}^{n} 1(u_i < 0) \right)
\]

\[
\leq \max \left( \sum_{i=1}^{n} l(u_i), n - \sum_{i=1}^{n} 1 + \sum_{i=1}^{n} 1(u_i < 0) \right)
\]

\[
= \max \left( \sum_{i=1}^{n} l(u_i), \sum_{i=1}^{n} 1(u_i < 0) \right)
\]

\[
= \sum_{i=1}^{n} l(u_i)
\]

Since loss \(\tilde{J}(u) = \sum_{i=1}^{n} l(u_i)\), we obtain \(Q(u) \leq \tilde{J}(u)\).

On the other hand, we have that

\[
Q(u) = \min_{v \in \{0,1\}^n} \max \left( \sum_{i=1}^{n} v_i l(u_i), n - \sum_{i=1}^{n} v_i + \sum_{i=1}^{n} 1(u_i < 0) \right)
\]

\[
\geq \min_{v \in \{0,1\}^n} n - \sum_{i=1}^{n} v_i + \sum_{i=1}^{n} 1(u_i < 0)
\]

\[
= \sum_{i=1}^{n} 1(u_i < 0)
\]

Since \(J(u) = \sum_{i=1}^{n} 1(u_i < 0)\), we obtain \(Q(u) \geq J(u)\)

\[\square\]

B Proof of Corollary 1

Proof. Since \(n = mb\), similar to the proof of \(Q(u) \leq \tilde{J}(u)\), we have

\[
\tilde{Q}(u) = \sum_{j=1}^{b} \min_{v \in \{0,1\}^m} \max \left( \sum_{i=1}^{m} v_{ij} l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} + \sum_{i=1}^{m} 1(u_{ij} < 0) \right)
\]

\[
\leq \sum_{j=1}^{b} \max \left( \sum_{i=1}^{m} l(u_{ij}), m - \sum_{i=1}^{m} 1 + \sum_{i=1}^{m} 1(u_{ij} < 0) \right)
\]

\[
= \sum_{j=1}^{b} \max \left( \sum_{i=1}^{m} l(u_{ij}), \sum_{i=1}^{m} 1(u_{ij} < 0) \right)
\]

\[
= \sum_{j=1}^{b} \sum_{i=1}^{m} l(u_{ij}) = \tilde{J}(u)
\]

On the other hand, since the group (batch) separable sum structure, we have that

\[
\tilde{Q}(u) = \sum_{j=1}^{b} \min_{v \in \{0,1\}^m} \max \left( \sum_{i=1}^{m} v_{ij} l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} + \sum_{i=1}^{m} 1(u_{ij} < 0) \right)
\]

\[
= \min_{v \in \{0,1\}^n} \sum_{j=1}^{b} \max \left( \sum_{i=1}^{m} v_{ij} l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} + \sum_{i=1}^{m} 1(u_{ij} < 0) \right)
\]

\[
\geq \min_{v \in \{0,1\}^n} \max \left( \sum_{j=1}^{b} \sum_{i=1}^{m} v_{ij} l(u_{ij}), n - \sum_{j=1}^{b} \sum_{i=1}^{m} v_{ij} + \sum_{j=1}^{b} \sum_{i=1}^{m} 1(u_{ij} < 0) \right)
\]

\[
= Q(u) \geq J(u)
\]

\[\square\]

C Proof of Partial Optimization Theorem (Theorem 4)

Proof. For simplicity, let \(l_i = l(u_i), i \in \{1, \ldots, n\}\). Without loss of generality, assume \(l_1 \leq l_2 \cdots \leq l_n\). Let \(v^*\) be the solution obtained by Algorithm 1 Assume there exits a \(\psi\) such that

\[
\max \left( \sum_{i=1}^{n} v_i l_i, C - \sum_{i=1}^{n} v_i \right) < \max \left( \sum_{i=1}^{n} v_i^* l_i, C - \sum_{i=1}^{n} v_i^* \right).
\]

\[\square\]
Let \( T = \sum_{i=1}^{n} v_i \) and \( T^* = \sum_{i=1}^{n} v_i^* \).

**Case 1:** If \( T = T^* \), then there exists an \( v_k = 1 \) and \( v_k^* = 0 \). From Algorithm 1 we know \( k > T^* \) (\( v_k^* = 0 \Rightarrow k > T^* \)) and \( l_k \geq l_j, j \in \{1, ..., T^*\} \). Then we know \( \sum_{i=1}^{n} v_i^* l_i \leq \sum_{i=1}^{n} v_i l_i \). Thus, we can achieve that

\[
\max \left( \sum_{i=1}^{n} v_i^* l_i, C - \sum_{i=1}^{n} v_i \right) = \max \left( \sum_{i=1}^{n} v_i^* l_i, C - \sum_{i=1}^{n} v_i \right) \\
\leq \max \left( \sum_{i=1}^{n} v_i l_i, C - \sum_{i=1}^{n} v_i \right).
\]

This contradicts the assumption in Eq.(34).

**Case 2:** If \( T > T^* \), then there exists an \( v_k = 1 \) and \( v_k^* = 0 \). Let \( L_{T^*} = \sum_{i=1}^{T^*} l_i \). Since \( l_k \geq 0 \), we have \( L_{T^*} + l_k \geq L_{T^*} \). From Algorithm 1 we know that \( L_{T^*} + l_k > C - T^* \). Thus we obtain that

\[
\max \left( \sum_{i=1}^{n} v_i l_i, C - \sum_{i=1}^{n} v_i \right) \geq L_{T^*} + l_k \\
\geq \max \left( L_{T^*}, C - T^* \right) \\
= \max \left( \sum_{i=1}^{n} v_i^* l_i, C - \sum_{i=1}^{n} v_i^* \right)
\]

This contradicts the assumption in Eq.(34).

**Case 3:** If \( T < T^* \), we obtain \( C - T \geq C - T^* + 1 \). Then we can achieve that

\[
\max \left( \sum_{i=1}^{n} v_i^* l_i, C - \sum_{i=1}^{n} v_i^* \right) = \max \left( L_{T^*}, C - T^* \right) \\
\leq C + 1 - T^* \\
\leq C - T \\
= C - \sum_{i=1}^{n} v_i \\
\leq \max \left( \sum_{i=1}^{n} v_i l_i, C - \sum_{i=1}^{n} v_i \right).
\]

This contradicts the assumption in Eq.(34).

Finally, we conclude that \( v^* \) obtained by Algorithm 1 is the minimum of the optimization problem given in (33). \(\)
E Proof of Theorem

Proof. We first prove that objective (11) is tighter than the loss objective \( \hat{J}(u) \) in Eq.(8). After this, we prove that objective (11) is an upper bound of the 0/1 loss defined in equation (7).

For simplicity, let \( l_i = l(u_i) \), we obtain that

\[
E(u) = \min_{v \in \{0,1\}^n} \max \left( \sum_{i=1}^{n} v_i l(u_i), n - \sum_{i=1}^{n} v_i \right) \leq \max \left( \sum_{i=1}^{n} l(u_i), (n - \sum_{i=1}^{n} 1) \right) \leq \sum_{i=1}^{n} l(u_i). \tag{48}
\]

Note that \( \hat{J}(u) = \sum_{i=1}^{n} l(u_i) \), thus, we have \( E(u) \leq \hat{J}(u) \).

Without loss of generality, assume \( l_1 \leq l_2 \cdots \leq l_n \). Let \( L_i = \sum_{j=1}^{i} l_j, T = \sum_{i=1}^{n} v_i^* \), where \( v^* = [v_1^*, v_2^* \cdots v_n^*]^T \) is the optimum of \( v \) for fixed \( u \). Let \( k = \sum_{i=1}^{n} 1(u_i \geq 0) \). Then we achieve that the 0/1 loss \( J(u) \) is as follows:

\[
J(u) = \sum_{i=1}^{n} 1(u_i < 0) = n - k. \tag{51}
\]

From Algorithm 1 with \( C = n \), we achieve that \( L_T \leq n - T + 1 \) and \( L_{T+1} > n - T \).

Case 1: If \( k \geq T \), we can achieve that

\[
2E(u) - J(u) = 2 \max(L_T, n - T) - (n - k) \geq 2(n - T) - (n - k) = n + k - 2T \geq 0. \tag{52}
\]

Case 2: If \( k < T, n - T \geq L_T \), we can obtain that

\[
2E(u) - J(u) = 2(n - T) - (n - k) = n + k - 2T. \tag{54}
\]

Since \( k < T \), if follows that

\[
L_T = L_k + \sum_{j=k+1}^{T} l_j \geq L_k + \sum_{j=k+1}^{T} [1 - u_j/c]_+ \geq L_k + \sum_{j=k+1}^{T} 1 = L_k + T - k \geq T - k. \tag{55}
\]

Together with \( n - T \geq L_T \), we can obtain that

\[
n - T \geq L_T \geq T - k \Rightarrow n + k - 2T \geq 0. \tag{59}
\]

Thus, we can achieve that

\[
2E(u) - J(u) = n + k - 2T \geq 0. \tag{60}
\]

Case 3: If \( k < T, n - T < L_T \), we can obtain that

\[
2E(u) - J(u) = 2 \max(L_T, n - T) - (n - k) = 2L_T - (n - k) > (n - T) + L_T - n + k. \tag{63}
\]
From (58), we have \( L_T \geq T - k \). Together with (63), it follows that
\[
2E(u) - J(u) > (n - T) + (T - k) - n + k \geq 0.
\]
Finally, we can achieve that \( J(u) \leq 2E(u) \leq 2\hat{J}(u) \).

\[\square\]

**F Proof of Corollary 2**

**Proof.** Since \( n = mb \), similar to the proof of \( \hat{Q}(u) \leq \hat{J}(u) \), we have
\[
\hat{E}(u) = \sum_{j=1}^{b} \min_{v \in \{0,1\}^m} \max \left( \sum_{i=1}^{m} v_{ij}l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} \right)
\]
\[
\leq \sum_{j=1}^{b} \max \left( \sum_{i=1}^{m} l(u_{ij}), m - \sum_{i=1}^{m} 1 \right) \tag{65}
\]
\[
= \sum_{j=1}^{b} \max \left( \sum_{i=1}^{m} l(u_{ij}), 0 \right) \tag{66}
\]
\[
= \sum_{j=1}^{b} \sum_{i=1}^{m} l(u_{ij}) = \hat{J}(u) \tag{67}
\]

On the other hand, since the group (batch) separable sum structure, we have that
\[
\hat{E}(u) = \sum_{j=1}^{b} \min_{v \in \{0,1\}^m} \max \left( \sum_{i=1}^{m} v_{ij}l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} \right)
\]
\[
= \min_{v \in \{0,1\}^m} \sum_{j=1}^{b} \max \left( \sum_{i=1}^{m} v_{ij}l(u_{ij}), m - \sum_{i=1}^{m} v_{ij} \right) \tag{68}
\]
\[
\geq \min_{v \in \{0,1\}^m} \sum_{j=1}^{b} \sum_{i=1}^{m} v_{ij}l(u_{ij}), n - \sum_{j=1}^{b} \sum_{i=1}^{m} v_{ij} \tag{69}
\]
\[
= E(u) \tag{70}
\]

Together with Theorem 3, we obtain that \( J(u) \leq 2E(u) \leq 2\hat{E}(u) \leq 2\hat{J}(u) \).

\[\square\]

**G Experimental Results on CIFAR10 and CIFAR100**

| Flipping-Rate | Standard | MentorNet | Co-teaching | NPCL           |
|---------------|----------|-----------|-------------|----------------|
| symmetry-20%  | 76.62% ± 0.07% | 80.64% ± 0.07% | 82.21% ± 0.06% | 84.30% ± 0.07% |
| symmetry-50%  | 49.92% ± 0.09% | 71.23% ± 0.22% | 74.09% ± 0.06% | 77.66% ± 0.09% |
| pair-35%      | 62.26% ± 0.09% | 71.57% ± 0.06% | 77.82% ± 0.11% | 76.52% ± 0.11% |

| Flipping-Rate | Standard | MentorNet | Co-teaching | NPCL           |
|---------------|----------|-----------|-------------|----------------|
| symmetry-20%  | 47.05% ± 0.11% | 52.28% ± 0.17% | 54.11% ± 0.06% | 55.30% ± 0.09% |
| symmetry-50%  | 25.47% ± 0.07% | 38.97% ± 0.13% | 41.71% ± 0.08% | 42.56% ± 0.06% |
| pair-35%      | 39.91% ± 0.11% | 40.72% ± 0.18% | 43.71% ± 0.12% | 44.43% ± 0.15% |

The average test accuracy over last ten epochs on CIFAR10 and CIFAR100 dataset are reported in Table 2 and Table 3 respectively. We can observe that NPCL obtains higher average test accuracy than Co-teaching on 5 out 6 cases.

The picture of test accuracy and label precision vs. number of epochs is presented in Figure 2 and Figure 3 respectively. It shows that NPCL achieves a slightly better and a competitive performance compared with Co-teaching on CIFAR10 and CIFAR100, respectively. Note that NPCL is a plug-in for a single network, while Co-teaching employs two network to train each other concurrently. Thus, both the time complexity and space complexity of Co-teaching is doubled compared with our NPCL.
H Impact of Misspecified Prior for Noise Rate $\epsilon$

We empirically analyze the impact of misspecified prior for the noise rate $\epsilon$. The average test accuracy over last ten epochs on MNIST for different priors are reported in Table 4. We can observe that NPCL is robust to misspecified prior for small noise cases (symmetry-20%). Moreover, it is moderate robust on large noise case (symmetry-50%) and on the pair flipping case.

Table 4: Average test accuracy of NPCL with different $\epsilon$ on MNIST over last ten epochs

| Flipping, Rate | $0.5\epsilon$ | $0.75\epsilon$ | $1.25\epsilon$ | $1.5\epsilon$ |
|---------------|----------------|----------------|----------------|--------------|
| Symmetry-20%  | 96.31% ± 0.17% | 97.72% ± 0.09% | 99.41% ± 0.01% | 99.55% ± 0.02% |
| Symmetry-50%  | 78.67% ± 0.36% | 87.56% ± 0.29% | 98.53% ± 0.02% | 97.92% ± 0.05% |
| Pair-35%      | 80.59% ± 0.40% | 87.86% ± 0.48% | 97.90% ± 0.04% | 99.33% ± 0.02% |
H.1 Multi-Class Extension

For multi-class classification, denote the groundtruth label as $y \in \{1, \ldots, K\}$. Denote the classification prediction (the last layer output of networks before loss function) as $t_i, i \in \{1, \ldots, K\}$. Then, the classification margin for multi-class classification can be defined as follows:

$$u = t_y - \max_{i \neq y} t_i.$$  \hfill (71)

We can see that $1(u < 0) = 1\left(t_y - \max_{i \neq y} t_i < 0\right)$ is indeed the 0-1 loss for multi-class classification.

With the classification margin $u$, we can compute the base loss $l(u) \geq 1(u < 0)$. In this work, we employ the hinge loss. As we need the upper bound of 0-1 loss, the multi-class hard hinge loss function [16] can be defined as:

$$H(t, y) = \max(1 - u, 0) = \max(1 - t_y + \max_{i \neq y} t_i, 0).$$  \hfill (72)

The multi-class hard hinge loss in Eq. (72) is not easy to optimize for deep networks. We propose a novel soft multi-class hinge loss function as follows:

$$S(t, y) = \begin{cases} 
\max(1 - t_y + \max_{i \neq y} t_i, 0) & t_y - \max_{i \neq y} t_i \geq 0 \\
\max(1 - t_y + \text{LogSumExp}(t), 0) & t_y - \max_{i \neq y} t_i < 0.
\end{cases}$$  \hfill (73)

The soft hinge loss employs the LogSumExp function to approximate the max function when the classification margin is less than zero, i.e., misclassification case. Intuitively, when the sample is misclassified, it is far away from being correctly separate by a positive margin (e.g. margin $u \geq 1$). In this situation, a smooth loss function can help speed up gradient update. Because $\text{LogSumExp}(t) > \max_{i \in \{1, \ldots, K\}} t_i$, we know that the soft hinge loss is an upper bound of the hard hinge loss, i.e., $S(t, y) \geq H(t, y)$. Moreover, we can obtain a new weighted loss $F(t, y; \beta) = \beta S(t, y) + (1 - \beta) H(t, y), \beta \in [0, 1]$ that is also an upper bound of 0-1 loss.

I Evaluation of Efficiency of the Proposed Soft-hinge Loss

We compare our soft multi-class hinge loss with hard multi-class hinge loss [16] on CIFAR100 dataset training with Adam and SGD optimizer, respectively. We keep both the network architecture and hyperparameters same. We employ the default learning rate and momentums of Adam optimizer in PyTorch toolbox, i.e. $lr = 10^{-3}, \beta_1 = 0.9, \beta_2 = 0.999$. For SGD optimizer, the learning rate ($lr$) and momentum ($\rho$) are set to $lr = 10^{-2}$ and $\rho = 0.9$ respectively.

The pictures of training/test accuracy v.s number of epochs are presented in Figure 4. We can observe that both the training accuracy and the test accuracy of our soft hinge loss increase greatly fast as the number of epochs increase. In contrast, the training and test accuracy of hard hinge loss grow very slowly. The training accuracy of soft hinge loss can arrive 100% trained with both optimizers. Both training and test accuracy of soft hinge loss are consistently better than hard hinge loss. In addition, training accuracy of hard hinge loss can also reach 100% when SGD optimizer is used. However, its test accuracy is lower than that of soft hinge loss.

Figure 4: Training/Test accuracy for soft and hard hinge loss with different optimizer on CIFAR100