Quantifying the tension between the Higgs mass and $(g - 2)_\mu$ in the CMSSM

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Supersymmetry has been often invoked as the new physics that might reconcile the experimental muon magnetic anomaly, $a_\mu$, with the theoretical prediction (basing the computation of the hadronic contribution on $e^+e^-$ data). However, in the context of the CMSSM, the required supersymmetric contributions (which grow with decreasing supersymmetric masses) are in potential tension with a possibly large Higgs mass (which requires large stop masses). In the limit of very large $m_h$ supersymmetry gets decoupled, and the CMSSM must show the same discrepancy as the SM with $a_\mu$. But it is much less clear for which size of $m_h$ does the tension start to be unbearable. In this paper, we quantify this tension with the help of Bayesian techniques. We find that for $m_h \geq 125$ GeV the maximum level of discrepancy given current data ($\sim 3.2 \sigma$) is already achieved. Requiring less than $3 \sigma$ discrepancy, implies $m_h \lesssim 120$ GeV. For a larger Higgs mass we should give up either the CMSSM model or the computation of $a_\mu$ based on $e^+e^-$; or accept living with such inconsistency.

I. INTRODUCTION

The magnetic anomaly of the muon, $a_\mu = \frac{1}{2}(g - 2)_\mu$ has been a classical and powerful test for new physics. As it is known, the present experimental value and some of the theoretical determinations of $a_\mu$ show a remarkable discrepancy, suggesting physics beyond the Standard Model (SM) to account it. However, the situation is still uncertain, due essentially to inconsistencies between alternative determinations of the contribution coming from the hadronic vacuum-polarization diagram, say $\delta_{\text{had}}^{\text{SM}} a_\mu$.

This contribution can be expressed in terms of the total hadronic cross section $e^+e^- \rightarrow \text{had}$. Using direct experimental data for the latter, one obtains a final result for $a_\mu$, which is at more than $3 \sigma$ from the current experimental determination [1], namely

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 25.5 \pm 8.0 \times 10^{-10} \quad (1)$$

(the quoted error bars are $1 \sigma$). This discrepancy has been often claimed as a signal of new physics. Obviously, if one accepts this point of view, the discrepancy should be cured by contributions from physics beyond the SM.

Admittedly, such claims are too strong. We are quite aware of past experimental observables in apparent disagreement with the SM prediction, which have eventually converged with it. This has occured due to both experimental and theoretical subtleties that sometimes had not been fully understood or taken into account. As a matter of fact, the experimental $e^+e^- \rightarrow \text{had}$ cross section exhibits some inconsistencies between different groups of experimental data. Using only BABAR data the discrepancy reduces to $2.4 \sigma$, while without it the discrepancy becomes $3.7 \sigma$, [1]. The inconsistency is specially notorious if one considers hadronic $\tau$ decay data, which are theoretically related to the $e^+e^- \rightarrow \text{had}$ cross section. Using just $\tau$-data the disagreement becomes $1.9 \sigma$, [1, 2]. Although the more direct $e^+e^- \rightarrow \text{had}$ data are usually preferred to evaluate $a_\mu^{\text{SM}}$, these inconsistencies are warning us to be cautious about the actual uncertainties involved in the determination of $a_\mu^{\text{SM}}$.

If one takes the discrepancy between theory and experiment shown in eq.(1) as a working hypothesis, one has to consider possible candidates of new physics able to provide the missing contribution to reproduce $a_\mu^{\text{exp}}$. The Minimal Supersymmetric Standard Model (MSSM) is then a natural option. We will consider here the simplest and most extensively analyzed version of the MSSM, namely the so-called constrained MSSM (CMSSM), in which the soft parameters are assumed universal at a high scale ($M_X$), where the supersymmetry (SUSY) breaking is transmitted to the observable sector, as happens e.g. in the gravity-mediated SUSY breaking scenario. Hence, our parameter space is defined by the following parameters:

$$\{\theta\} = \{m, M, A, B, \mu, s\} \quad (2)$$

Here $m$, $M$ and $A$ are the universal scalar mass, gaugino mass and trilinear scalar coupling; $B$ is the bilinear scalar coupling; $\mu$ is the usual Higgs mass term in the superpotential; and $s$ stands for the SM-like parameters of the MSSM, i.e. essentially gauge and Yukawa couplings. All these initial parameters are understood to be defined at $M_X$.

The main supersymmetric (CMSSM) contributions to $a_\mu$ come from 1-loop diagrams with chargino-sneutrino
and neutralino-smuon exchange \[^3\]. In general, these contributions, say \(\delta^{\text{MSSM}}a_\mu\), are larger for smaller supersymmetric masses and can be just of the right magnitude to reconcile theory and experiment (thus constraining the CMSSM parameter space).

In section III we show the potential tension between the requirement of suitable SUSY contributions to the muon anomaly and a possibly large Higgs mass. In section IV we quantify such tension as a function of \(m_h\), with the help of Bayesian techniques. In section IV we show how the probability distributions of the most relevant parameters (universal scalar and gaugino masses, and \(\tan \beta\)) change with increasing \(m_h\). Finally, in section V we present our conclusions.

II. HIGGS MASS VS. G-2

It is well known that in the MSSM the tree-level Higgs mass is bounded from above by \(M_Z\), so radiative corrections (which grow logarithmically with the stop masses) are needed to reconcile the theoretical predictions with the present experimental lower bound, \(m_h > 114.4\) GeV (SM-like Higgs). Roughly speaking, a Higgs mass above 130 GeV requires supersymmetric masses above 1 TeV. In this regime one can expect SUSY to be decoupled, so the prediction for \(a_\mu\) becomes close to \(a_\mu^{\text{SM}}\). Hence, a large Higgs mass in the MSSM would necessarily amounts to a \(>3\) \(\sigma\) discrepancy between the experimental and the theoretical values of \(a_\mu\) (evaluated via \(e^+e^- \rightarrow \text{had}\)).

The main goal of this paper is to quantify the tension between \(m_h\) and \(a_\mu\) in the context of the CMSSM. This is useful since it allows to put an educated upper bound on the Higgs mass, which will depend on the discrepancy one is ready to tolerate. Conversely, it tells us from which minimum value of \(m_h^{\text{exp}}\) we will have to give up either the CMSSM assumption or the theoretical evaluation of \(a_\mu\) via \(e^+e^- \rightarrow \text{had}\) (with the quoted uncertainties).

For the sake of the discussion, we will give now some approximate analytical expressions for \(m_h\) and \(\delta a_\mu^{\text{MSSM}}\). In the MSSM the tree-level squared Higgs mass plus the one-loop leading logarithmic contribution is given by

\[
m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3m_t^4}{2\pi^2 v^2} \left[ \log \frac{m_t^2}{m_t^2} + \frac{X_t^2}{M_S^2} \left(1 - \frac{X_t^2}{12M_S^2}\right) \right] + \ldots
\]

Here \(\tan \beta\) is the ratio of the expectation values of the two MSSM Higgs fields, \(\tan \beta \equiv (H_u)/(H_d)\); \(m_t\) is the (running) top mass and \(X_t\) is the geometrical average of the stop masses. Besides,

\[
X_t \equiv A_t + \mu \cot \beta,
\]

where \(A_t\) is the top trilinear scalar coupling, and \(M_S^2\) is the arithmetical average of the squared stop masses. All the quantities in eqs. (3), (4) are understood at low energy (for more details see e.g. ref. [4][13]). Subdominant terms not written in eq.(3) can be important for a precise determination of \(m_h\) and we have included them in the numerical analysis. The previous equations tell us how \(m_h\) grows with increasing supersymmetric masses and also with increasing \(\tan \beta\). Besides, the contribution associated to the stop mixing (second term within the square brackets in eq.(3)) is maximal at \(X_t = \sqrt{6}M_S\).

On the other hand, as mentioned above, the supersymmetric contribution to the muon anomaly, \(\delta a_\mu^{\text{SUSY}}\), arises mainly from 1-loop diagrams with chargino-sneutrino and neutralino-smuon exchange. This contribution increases with increasing \(\tan \beta\) and decreasing supersymmetric masses. See refs. [11][15].

Although the analytical expressions are complicated, one can get an intuitive idea of the parametric dependence by considering the extreme case where the masses of all supersymmetric particles are degenerate at low energy\(^1\): \(M_1 = M_2 = \mu = m_\tilde{\nu}L = m_\tilde{\nu}R = m_\tilde{\nu} \equiv M_{\text{SUSY}}\). Then \([16]\),

\[
\delta a_\mu^{\text{SUSY}} \simeq \frac{1}{32\pi^2} \frac{m_\mu^2}{M_{\text{SUSY}}} \frac{g_2^2}{\tan \beta} \text{sign}(M_2\mu).
\]

Examining the approximate expressions \([3]\) and \([5]\), it is clear that a large \(m_h\) and a large \(\delta a_\mu^{\text{SUSY}}\) will be more easily obtainable (and thus compatible) for larger \(\tan \beta\). On the contrary, the larger the supersymmetric masses the larger \(m_h\) but the smaller \(\delta a_\mu^{\text{SUSY}}\), and this is the origin of the potential tension.

However, it is difficult from the previous expressions (or the more sophisticated ones) to conclude for which size of \(m_h\) does the tension start to be unbearable. The reason is that a particular value of the Higgs mass, say \(m_h = 120\) GeV, can be achieved through eq.(3) with different combinations of \(\tan \beta\), stop masses and \(X_t\). Besides, there are many ways, i.e. very different regions in the MSSM parameter space, in which these quantities can have similar low-energy values. Still, the corresponding contribution \(\delta a_\mu^{\text{SUSY}}\) can change significatively from one region to another. Unless one performs a complete scan of the parameter space one cannot conclude that the required value of \(\delta a_\mu^{\text{SUSY}}\) is unattainable for \(m_h = 120\) GeV. On the other hand, if it is attainable, but only in an extremely tiny portion of the parameter space, this implies a tension between the two observables since the consistency between \(m_h\) and \(a_\mu\) requires a severe fine-tuning. And it is possible, in principle, to quantify such tension.

In the analysis we have included two-loop leading corrections for the Higgs sector \([17][21]\). \(\delta a_\mu^{\text{SUSY}}\) was computed at full one-loop level adding the logarithmic piece of the quantum electro-dynamics two-loop calculation plus two-loop contributions from both stop-Higgs and chargino-stop/sbottom \([13]\). The effective two-loop effect

\(^1\) This limit is often used because of the simplification of the formulae it implies. However, it is unachievable in the CMSSM.
due to a shift in the muon Yukawa coupling proportional to $\tan^2 \beta$ has been added as well [14].

Next we expound how a systematic analysis of this kind can be done with the help of Bayesian techniques. This will allow us to quantify the tension between $m_h$ and $a_\mu$ as a function of $m_h$.

III. QUANTIFYING THE TENSION BETWEEN $m_h$ AND $a_\mu$

Let us start by recalling some basic notions of Bayesian inference. We refer the reader to [22, 24] for further details. For a model defined by a set of parameters $\theta$, the posterior probability density function (pdf) of a point in parameter space, $\{|\theta\}$, given a certain set of data, is denoted by $p(\theta|\text{data})$ and it is obtained via Bayes theorem as

$$p(\theta|\text{data}) = \frac{p(\text{data}|\theta) \cdot p(\theta)}{p(\text{data})}.$$  \hspace{1cm} (6)

Here $p(\text{data}|\theta)$ is the likelihood function (when considered as a function of $\theta$ for the observed data)$^2$, $p(\theta)$ is the prior, i.e. the probability density that we assign the points in the parameter space before seeing the data (in the context of Bayesian inference, the prior for a new cycle of observations can be taken to be the posterior from previous experiments). Finally, $p(\text{data})$ is a normalization factor, sometimes called the evidence. It is given by

$$p(\text{data}) = \int d\theta \cdot p(\text{data}|\theta) \cdot p(\theta),$$  \hspace{1cm} (7)

i.e. the evidence is the average of the likelihood under the prior, and thus it gives the global probability of measuring the data in the model.

When two different models (or hypotheses) are used to fit the data, the ratio of their evidences gives the relative probability of the two models in the light of the actual data, under the assumption that the model is correct:

$$\frac{p(\mathcal{D} \mid \text{data})}{p(\mathcal{D}_{\text{max}} \mid \text{data})} = \frac{p(\text{data} \mid \mathcal{D})}{p(\text{data} \mid \mathcal{D}_{\text{max}})} = \mathcal{L}(\mathcal{D} \mid \text{data}).$$  \hspace{1cm} (10)

$\mathcal{L}(\mathcal{D} \mid \text{data})$ is analogous to a likelihood ratio in data space, but integrated over all possible values of the parameters of the model. Therefore, it can be used as a test statistics for the likelihood of the data being tested, $\mathcal{D}$, in the context of the model used (the CMSSM). Such test was called $\mathcal{L}$-test in Ref. [23]. Note that, as mentioned above, the $p(D)$ factor cancels out in the expression of $\mathcal{L}(\mathcal{D} \mid \text{data})$, which is simply given by the ratio of the joint evidences.

In our case, the value of $\mathcal{D}_{\text{max}}$ depends on the value of $m_h$ probed. For very large $m_h$, say $m_h \gtrsim 135$ GeV, SUSY must decouple, so $\mathcal{D}_{\text{max}}$ should approach the SM prediction. Hence, in this limit one expects $\mathcal{L}(\mathcal{D} \mid \text{data})$ to show a $3.2 \sigma$ discrepancy; in other words, $-2 \ln \mathcal{L}(\mathcal{D} \mid \text{data}) \rightarrow 3.2^2$. However, the expression (10) allows us to evaluate this likelihood for any intermediate value of $m_h$, and so we can evaluate how quickly this limit is reached as a function of the assumed value for $m_h$.

For the numerical calculation we have used the MultiNest [28, 30] algorithm as implemented in the SuperBayeS code [31, 33]. It is based on the framework of Nested Sampling, recently invented by Skilling [34, 35]. MultiNest has been developed in such a way as to be an

$^2$ Frequentist approaches, which are an alternative to the Bayesian framework, are based on the analysis of the likelihood function in the parameter space; see ref. [24] for a recent frequentist analysis of the MSSM.
extremely efficient sampler even for likelihood functions defined over a parameter space of large dimensionality with a very complex structure as it is the case of the CMSSM. The main purpose of the Multinest is the computation of the Bayesian evidence and its uncertainty but it produces posterior inferences as a by–product at no extra computational cost.

Fig. 1 shows the value of \(-2 \ln \mathcal{L}\) (the analogous of the usual \(\chi^2\)) for different values of the Higgs mass, \(m_h\) (GeV) = 115, 120, 125, 130, 135, and for two different choices of initial priors for the CMSSM parameters, namely log prior (red line) and flat prior (blue line). The precise shape of the log and flat priors used here is the one derived in ref. [27], to which the reader is referred, which take into account the likelihood associated to the electroweak breaking process. The horizontal error bars reflect the uncertainty in the theoretical computation of \(m_h\) in the MSSM, while the vertical error bars come from sources of error in the computation of \(\mathcal{L}\), mainly the numerical accuracy of the evidence returned by MultiNest. Lines of conventional confidence levels thresholds in terms of number of \(\sigma\) are shown as well for comparison.

From the figure we see that the likelihood of the experimental value of \(a_\mu\) approaches asymptotically the expected 3.2 \(\sigma\) discrepancy for large values of \(m_h\), for both types of priors. As mentioned above, this is logical and it represents a nice cross-check of the reliability of the whole procedure. Besides, Fig. 1 tells us how fast this convergence is reached as \(m_h\) increases. And, as a matter of fact, the convergence is very fast. At \(m_h = 125\) GeV the maximum level of discrepancy is already achieved, indicating that SUSY has decoupled, and thus the prediction for \(a_\mu\) coincides with the SM one. If we require less than 3 \(\sigma\) discrepancy, we need \(m_h \lesssim 120\) GeV. This is a prediction of the CMSSM provided we accept the calculation of \(a_\mu\) based on \(e^+e^-\) data. For a larger Higgs mass we should give up either the CMSSM model or the computation of \(a_\mu\) based on \(e^+e^-\); or accept living with such inconsistency. These are the main conclusions of this paper. They stem directly from Fig. 1. Let us also note that, even assuming a Higgs mass as low as it can be, the minimum level of discrepancy is about 2.5 \(\sigma\). However, most of this tension with \(a_\mu\) comes from \(b \to s, \gamma\) data [24], rather from the value of the Higgs mass. This can be checked by repeating the analysis excluding all the experimental information (except \(M_Z\) and the assumed Higgs mass). The resulting plot is similar to that of Fig. 1, except the \(m_h = 115\) GeV point, which shows a \(\sim 1.5\) \(\sigma\) discrepancy.

It is an interesting exercise to compute how our conclusions would change if \(a_\mu\) became more precisely measured in the future (keeping the same central value). If one continued to assume the theoretical evaluation of \(a_\mu\) based on \(e^+e^-\) data, the signal for new physics would obviously become stronger. In this case, the tension between a large Higgs mass and the experimental \(a_\mu\) would get more unbearable. We have done this exercise, by changing (artificially) the experimental uncertainty of \(a_\mu^{\exp}\), so that the discrepancy with the SM result be 5 \(\sigma\), something that could happen in the next years. Now, in the context of the CMSSM, the value of \(-2 \ln \mathcal{L}(\mathcal{G}^{\text{obs}}|D)\) must approach asymptotically such 5 \(\sigma\) discrepancy, and this is indeed what we observe, as shown in Fig. 2. In this hypothetical situation, a Higgs mass above 120 GeV would imply a discrepancy larger than 4 \(\sigma\) with the muon anomaly in the context of the CMSSM. Actually, the present lower bound, \(m_h \geq 114.4\) GeV, would already be inconsistent with the muon anomaly at the 3 \(\sigma\) level.

This gives a fair idea of the tensions within the CMSSM.
to accommodate a value of $a_\mu$ as the measured one (bas-
ing the theoretical calculation on present $e^+e^-$ data).

IV. PROBABILITY DISTRIBUTIONS FOR
SUPERSYMMETRIC PARAMETERS

It is also interesting to investigate the probability distri-
butions of the CMSSM parameters for various assumed
values of the Higgs mass. Figure 3 (upper panels) shows
the marginalized probability distribution functions (pdfs)
of $m$, $M$ assuming a value of $m_\text{h} = 115, 120, 125$ (GeV),
as well as adding in all present-day constraints mentioned
above. The location of the peak in the posterior pdf in-
creases with the assumed Higgs mass since, as mentioned
in section [1] in the MSSM a large $m_\text{h}$ requires large ra-
diative contributions, which grow logarithmically with
the stops masses. This happens even though large values
of $m$ and $M$ are penalized both for a natural electroweak
breaking (see ref. [27, 36]) and by the need of a sizeable
$\delta^{\text{SUSY}} a_\mu$. The model “prefers” to reproduce $m_\text{h}$
at the cost of not reproducing $a_\mu$ rather than vice versa.
Note here that for increasing soft masses the discrepancy of
$a_\mu$ with the experimental value approaches 3.2 σ, but
if the soft masses are not large enough, the discrepancy
associated to $m_\text{h}$ would be much more severe.

Fig. 3 (lower panel) shows the pdf of tan $\beta$ for $m_\text{h} =
115, 120, 125$ (GeV). Its shape is the result two compet-
ing effects. On the one hand, large values of tan $\beta$
are severely penalized for the electroweak breaking [27, 36].
On the other hand, the need of a sizeable $\delta^{\text{SUSY}} a_\mu$
favours large tan $\beta$ (see the approximate expression (5)).
Fig. 3 shows the balance between these two effects.

The Higgs mass increases also with tan $\beta$, but the effect is only
important for small values of tan $\beta$, see eq. (5). Now, since
for larger $m_\text{h}$ the soft masses are larger, with the side-
effect of suppressing $\delta^{\text{SUSY}} a_\mu$, one might expect that the
preferred value of tan $\beta$ increases with $m_\text{h}$, to compensate
this in eq. (5). However, this effect is not very important,
as it is apparent in Fig. 3. To understand this, let us ap-
proximate (for the sake of the argument) $M^{\text{SUSY}} \sim m_{\tilde{e}_{L,R}}$
in eq. (5) and use [37, 38]

$$m_{\tilde{e}_{L}}^2 \simeq m^2 + 0.54 M^2, \quad (11)$$

$$m_{\tilde{e}_{R}}^2 \simeq m^2 + 0.15 M^2,$$

$$m_{\tilde{\tau}}^2 \simeq 3.36 M^2 + 0.49 m^2 - 0.05 A^2 - 0.19 A M + m_\text{h}^2.$$

Since $m_\text{h}$ increases (logarithmically) with $m_\text{h}^2$, while
$\delta^{\text{SUSY}} a_\mu$ is suppressed by $m_{\tilde{e}_{L,R}}^2$, it might seem that the
most efficient way to reproduce both is to increase $M$
rather than $m$ (note the different dependences on $M$ in
eqs. (11)). The problem is that the fine-tuning grows
very fast with $M$: in other words, the number of points
in the parameter space with correct EW breaking de-
creceases very quickly. In consequence this possibility is
statistically penalized. On the contrary, for small $M$ and
large $m$, if tan $\beta > 8$, there is a focus-point region, with
small fine-tuning. This region is statistically favoured,
though this is counteracted by the penalization arising
from the suppression in $\delta^{\text{SUSY}} a_\mu$. This cannot be com-
penated by larger values of tan $\beta$, since in this regime
very big values of tan $\beta$ (as would be needed for such
compensation) start to be forbidden as we increase $m_\text{h}$.

In consequence, a very large tan $\beta$ is hardly favoured by
an increasing $m_\text{h}$.

Finally, let us mention that a lot of effort has been done
in the literature to determine the most probable re-
gion of the parameter space of the CMSSM [24, 27, 30,
32, 36, 39, 40]. This includes both Bayesian approaches
(as the one followed here) and frequentist ones. The lat-
ter (which can be considered as complementary to the
Bayesian ones) are based on the analysis of the likeli-
hood function in the parameter space. Thus they do
not penalize regions from fine-tuning arguments (some-
ting automatic in Bayesian analyses [27, 36]). In con-
sequence, following a frequentist approach it would be
much more hard to show up the tension between $m_\text{h}$
and $g-2$. On the other hand, the present analysis differs from
the previous ones in the fact that several hypothetical fu-
ture scenarios, depending on the value of the Higgs mass,
are considered and compared.

V. CONCLUSIONS

As it is well known, the SM prediction for the mag-
netic anomaly of the muon, $a_\mu$ (basing the computation of
the hadronic contribution on $e^+e^-$ data) shows a $> 3$ σ
discrepancy with the experimental result. It is common
to consider this discrepancy as a signal of new physics
(though, admittedly, the theoretical computation is con-
troversial). In that case, SUSY is a most natural option
for such new physics.

However, as we have discussed in this paper, in the
supersymmetric context there is a potential tension be-
tween the requirement of SUSY contributions to the
muon anomaly, $\delta^{\text{SUSY}} a_\mu$, sufficient to reconcile theory
and experiment, and a possibly large Higgs mass. In
the CMSSM framework a large Higgs mass means $O(10)$
GeV above the present experimental bound, $m_\text{h} \geq 144.4$
GeV (for an SM-like Higgs). The tension arises because
the main contributions to $\delta^{\text{MSSM}} a_\mu$ come from 1-loop
diagrams with chargino-sneutrino and neutralino-smuon
exchange, which grow with decreasing supersymmetric
masses and increasing tan $\beta$. But, on the other hand,
in the MSSM the tree-level Higgs mass is bounded from
above by $M_Z$, so radiative corrections (which grow loga-
 rithmically with the stop masses) are needed to reconcile
the theoretical predictions with the present experi-
mental lower bound. Thus, a large Higgs mass requires large
supersymmetric masses, making impossible the task of
reproducing the experimental value of $a_\mu$.

Although it is clear that in the limit of very large $m_\text{h}$
(say above 135 GeV) the CMSSM must present the same
discrepancy as the SM regarding the prediction for $a_\mu$,
it is much less clear for which size of $m_\text{h}$ does the ten-
FIG. 3: Probability distribution functions of $m$, $M$ (upper panels) and $\tan \beta$ (lower panel) for $m_h = 115$ GeV (blue), 120 GeV (green) and 125 GeV (red).

Our goal has been to quantify such tension, as a function of $m_h$, with the help of Bayesian techniques. As discussed at the end of sec. IV, this is the natural approach if we want to incorporate the statistical penalization of fine-tuned regions of parameter-space. Certainly, if one just assumed a particular supersymmetric model (i.e. a point in the parameter-space, no matter how fine-tuned it were) then the statistical arguments used in this paper would not be appropriate. We have shown that for $m_h \geq 125$ GeV the maximum level of discrepancy ($\sim 3.2\sigma$) is already achieved, indicating that SUSY has decoupled, and thus the prediction for $a_\mu$ coincides with the SM one. Given present-day data, requiring less than a 3 $\sigma$ discrepancy, implies $m_h \lesssim 120$ GeV. This is a prediction of the CMSSM provided we accept the calculation of $a_\mu$ based on $e^+e^-$ data. For a larger Higgs mass we should give up either the CMSSM model (at the $3\sigma$ level or above) or the computation of $a_\mu$ based on $e^+e^-$; or else accept living with such inconsistency. These are the main conclusions of this paper, and can be inferred directly from Fig. 1. It is also important to note that, as discussed in section III, the CMSSM cannot remove the full $3.2\sigma$ discrepancy in $a_\mu$.

We have also examined the possibility that the experimental uncertainty of $a_\mu^{exp}$ will decrease in the future, so that the discrepancy with the SM result be $5\sigma$, something that could happen in the next years. Then,
in the context of the CMSSM, a Higgs mass above 120 GeV would imply a discrepancy larger than 4σ with the muon anomaly. Actually, the present lower bound, $m_h \geq 114.4$ GeV, would already be inconsistent with the muon anomaly at the 3σ level. This illustrates the tensions within the CMSSM to accommodate a value of $a_\mu$ as the measured one (basing the theoretical calculation on present $e^+e^-$ data).

Finally, we have shown how the probability distributions of the most relevant parameters (universal scalar and gaugino masses, and $\tan\beta$) change with increasing $m_h$, which has obvious implications for the detection (or non-detection) of SUSY in the LHC.

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