Neutrino Masses and Mixing in Supersymmetric Models without $R$ Parity

Francesca M. Borzumati, Yuval Grossman, Enrico Nardi and Yosef Nir

Department of Particle Physics
Weizmann Institute of Science, Rehovot 76100, Israel

We study neutrino masses and mixing in Supersymmetric Models without $R$ parity and with generic soft Supersymmetry breaking terms. Neutrinos acquire mass from various sources: tree level neutrino–neutralino mixing, loop effects and non–renormalizable operators. Abelian horizontal symmetries (invoked to explain the smallness and hierarchy in quark parameters) replace $R$ parity in suppressing neutrino masses. We find lower bounds on the mixing angles: $\sin \theta_{ij} \gtrsim m(\ell_i^\tau)/m(\ell_j^-) \ (i < j)$ and unusual order of magnitude predictions for neutrino mass ratios: $m(\nu_e)/m(\nu_\mu) \sim \sin^2 \theta_{12}$; $m(\nu_i)/m(\nu_\tau) \sim 10^{-7} \sin^2 \theta_{i3} \ (i = 1, 2)$. Bounds from laboratory experiments exclude $m_{\nu_\tau} \gtrsim 3 \text{ MeV}$ and cosmological constraints exclude $m_{\nu_e} \gtrsim 100 \text{ eV}$. Neither the solar nor the atmospheric neutrino problems are likely to be solved by $\nu_\mu - \nu_e$ oscillations. These conclusions can be evaded if holomorphy plays an important role in the lepton Yukawa couplings.
1. Introduction

The search for neutrino masses is one of the most promising directions to find evidence for the incompleteness of the Standard Model. Theoretical input is required in order to direct experiments to the most plausible values of neutrino masses and mixing angles. In particular, an understanding of the neutrino sector by the same means that explain the quark and charged lepton parameters would be desirable. Supersymmetry combined with horizontal symmetries can provide this understanding \[1-6\].

In Supersymmetric models with the MSSM particle content and with $R$ parity ($R_p$), lepton number is violated by non–renormalizable terms only. Terms of the form $\frac{1}{M} L \phi_u L \phi_u$ ($M$ is a high energy scale, $L$ is the lepton doublet and $\phi_u$ is the $Y = +1/2$ Higgs doublet) lead to neutrino masses of the see–saw type, $m_\nu \sim \frac{\langle \phi_u \rangle^2}{M}$. The consequences of Abelian horizontal symmetries in this framework were investigated in ref. \[3\]. A number of interesting order of magnitude relations among the lepton parameters were found to hold in a large class of models:

$$\frac{m_{\nu_i}}{m_{\nu_j}} \sim \sin^2 \theta_{ij},$$

$$\sin \theta_{ij} \gtrsim \frac{m_{\ell_i}}{m_{\ell_j}},$$

$$\frac{m_{\nu_i}}{m_{\nu_j}} \gtrsim \left( \frac{m_{\ell_i}}{m_{\ell_j}} \right)^2,$$

$$m_{\nu_e} \lesssim m_{\nu_\mu} \lesssim m_{\nu_\tau},$$

where $i < j$ and $\nu_e, \nu_\mu, \nu_\tau$ denote the mass eigenstates with mixing of $O(1)$ with $e, \mu, \tau$, respectively. Interestingly, predictions analogous to \[1.2\] and \[1.4\] apply to the quark sector (namely, $V_{ij} \gtrsim \frac{m_{(u)}}{m_{(u)}}$, $\frac{m_{(d)}}{m_{(d)}}$ and $V_{\text{CKM}} \sim 1$) and are experimentally valid \[3\].

The same horizontal symmetries that explain the smallness and hierarchy in fermion parameters can naturally solve the problems related to lepton flavor and lepton number violation that arise in Supersymmetric models without $R_p$ \[7-9,5\]. In this case, lepton

1 While horizontal symmetries can rather easily take the role of $R_p$ in suppressing lepton number violation, it is much more difficult to do so for baryon number violation \[10\]. Therefore, as in ref. \[8\], we simply assume that baryon number is a symmetry of Nature.
number is violated by renormalizable terms, and the resulting phenomenology is strikingly different from the one predicted by Supersymmetric $R_p$-symmetric models. In this work we examine the question of lepton masses and mixing angles in models of Supersymmetry without $R_p$ but with a horizontal symmetry.

2. The Theoretical Framework

We work in the framework of the Abelian horizontal symmetry $\mathcal{H}$ that has been introduced in refs. \cite{11-13}. $\mathcal{H}$ is explicitly broken by a small parameter $\lambda$ to which we attribute charge $-1$ (and a numerical value of $O(0.2)$, to explain the Cabibbo angle). This can be viewed as the effective low energy theory that comes from a Supersymmetric extension of the Froggatt–Nielsen mechanism at a high scale \cite{14}. Then, the following selection rules apply:

(a) Terms in the superpotential that carry charge $n \geq 0$ under $\mathcal{H}$ are suppressed by $O(\lambda^n)$, while those with $n < 0$ are forbidden by holomorphy;

(b) Terms in the Kähler potential that carry charge $n$ under $\mathcal{H}$ are suppressed by $O(\lambda|n|)$.

Without $R_p$ (or lepton number), there is a–priori no distinction between the $Y=-1/2$ Higgs doublet $\phi_d$ and the three lepton doublets $L_i$. (Wherever convenient, we denote the four doublets by $L_\alpha$, $\alpha = 0, 1, 2, 3$.) The four doublets, however, carry in general different horizontal charges $H$. We identify the Higgs doublet with the doublet field that carries the smallest (positive) charge, which we choose to be $L_0$ (we use interchangeably $L_0 \equiv \phi_d$), and we order the remaining doublets according to their charges:

$$H(L_1) \geq H(L_2) \geq H(L_3) \geq H(\phi_d) \geq 0.$$

(2.1)

A similar ordering is made for the three generations of $\bar{\ell}_i$ (charged lepton singlets), $Q_i$ (quark doublets), and $\bar{d}_i$ (down quark singlets). The model is phenomenologically viable if (for $\tan \beta \sim 1$) the following condition holds \cite{8}:

$$H(L_i) - H(\phi_d) \geq 3.$$

(2.2)

Our methods of analyzing lepton and neutralino mass matrices are described in detail in refs. \cite{3} and \cite{8}, respectively. Specifically, we use the following selection rules to estimate...
the magnitude of the various contributions. For the quadratic terms in the superpotential, 
\[ \mu_\alpha L_\alpha \phi_u, \]
it is:
\[ \mu_\alpha \sim \begin{cases} 
\tilde{\mu} \lambda^{H(L_\alpha)+H(\phi_u)} & H(L_\alpha) + H(\phi_u) \geq 0, \\
\tilde{m} \lambda^{H(L_\alpha)+H(\phi_u)} & H(L_\alpha) + H(\phi_u) < 0.
\end{cases} \tag{2.3} \]
Here \( \tilde{\mu} \) is the natural scale for the \( \mu \) terms and \( \tilde{m} \) is the Supersymmetry breaking scale. For simplicity we assume that \( \tilde{\mu} \) is \( \mathcal{O}(\tilde{m}) \). As \( \mu_0 \) is phenomenologically required to be of \( \mathcal{O}(\tilde{m}) \), we take \( H(\phi_d) + H(\phi_u) \sim 0 \). Modifications to the case where the natural scale for \( \mu \) is, say, \( M_{\text{Planck}} \) and it is suppressed down to \( \tilde{m} \) by the horizontal symmetry, as in the models of [13] and [8], are straightforward. The selection rule for the coupling of the quadratic soft Supersymmetry breaking terms, \( B_\alpha L_\alpha \phi_u \) (here \( L_\alpha \) stand for the scalar components) is:
\[ B_\alpha \sim \tilde{m}^2 \lambda^{H(L_\alpha)+H(\phi_u)}. \tag{2.4} \]
Finally, the selection rules for the trilinear terms \( \lambda'_{\alpha jk} L_\alpha Q_j \bar{d}_k \) and \( \lambda_{\alpha \beta k} L_\alpha L_\beta \bar{\ell}_k \) are
\[ \lambda'_{\alpha jk} \sim \begin{cases} 
\lambda^{H(L_\alpha)+H(Q_j)+H(\bar{d}_k)} & H(L_\alpha) + H(Q_j) + H(\bar{d}_k) \geq 0, \\
0 & H(L_\alpha) + H(Q_j) + H(\bar{d}_k) < 0.
\end{cases} \tag{2.5} \]
\[ \lambda_{\alpha \beta k} \sim \begin{cases} 
\lambda^{H(L_\alpha)+H(L_\beta)+H(\bar{\ell}_k)} & H(L_\alpha) + H(L_\beta) + H(\bar{\ell}_k) \geq 0, \\
0 & H(L_\alpha) + H(L_\beta) + H(\bar{\ell}_k) < 0.
\end{cases} \tag{2.6} \]
Note that \( \lambda'_{0 jk} \) and \( \lambda_{0 jk} \) are practically the Yukawa couplings for the down sector and for the charged lepton sector.

The order of magnitude relations (1.1)–(1.4) were derived in a large class of models where all entries in the lepton mass matrices carry positive charges. (They are actually applicable in a larger class of models, where the holomorphy–induced zero entries, if any, do not affect the physical parameters.) In this work we restrict ourselves to this class of models.

### 3. Neutrino Masses and Mixing

There are several important sources for neutrino masses in this framework, each giving a different scale: renormalizable tree–level mixing with neutralinos [15–24]; quark–squark and lepton–slepton loop corrections [14,25–32]; and non–renormalizable see–saw contributions [33–34]. We now discuss each contribution in turn. In our various estimates we take \( \tan \beta \sim 1 \).
(i) **Renormalizable tree–level contributions.**

These contributions arise when the $\mu$–terms in the superpotential and the Supersymmetry breaking $B$ terms in the scalar potential are misaligned, $B_\alpha \neq B\mu_\alpha$, or the Supersymmetry breaking scalar masses do not satisfy the eigenvalue condition $m^{2}_{\alpha\beta}\mu_\beta = \tilde{m}^{2}\mu_\alpha$. This yields misalignment between the VEVs $v_\alpha \equiv \langle L_\alpha \rangle$ and the $\mu_\alpha$ terms,

$$
\sin^2 \xi = \frac{1}{2} \sum_{\alpha,\beta} (\mu_\alpha v_\beta - \mu_\beta v_\alpha)^2 \mu^2 v^2_d, \quad (\mu^2 \equiv \mu_\alpha \mu_\alpha, \ v^2_d \equiv v_\alpha v_\alpha)
$$

which induces neutrino mixing with the neutralinos. Only one neutrino acquires a mass from this effect: $m_\nu \sim m_Z \sin^2 \xi$ (here $m_Z$ stands for the electroweak or Supersymmetry breaking scale). In the absence of any symmetry reason for alignment, we expect $\sin \xi \sim 1$ and the natural scale for $m_\nu$ is the electroweak scale. The further required suppression comes from $H$-violation, $v_i/v_0 \sim \lambda^2[H(L_i)-H(\phi_d)]$. Thus, (3.1) together with (2.1) gives

$$
m_{\nu_\tau} \sim \lambda^2[H(L_3)-H(\phi_d)]m_Z.
$$

The massive neutrino is then $\nu_\tau$, which is close to the interaction eigenstate with the smallest horizontal charge among the $L_i$. The experimental upper bound $m_{\nu_\tau} \leq 24$ MeV [33], when confronted with (3.2), is the source of the constraint (2.2).

(ii) **Quark–squark loop contributions.**

Loops with down quark and squarks contribute

$$
\frac{1}{2} M^\nu_{ij} \sim \frac{3\lambda^l_{ikl} \lambda^l_{jmn} \langle M^d \rangle_{lm} \langle \tilde{M}^d_{LR} \rangle_{kn}}{16\pi^2 \tilde{m}^2},
$$

where $M^d$ is the $d$–quark mass matrix, and $\tilde{M}^d_{LR}$ is the left–right sector in the $\tilde{d}$-squark mass-squared matrix. The experimental value of $V_{cb} \sim m_s/m_b$ strongly suggests that $H(\bar{d}_2) = H(\bar{d}_3)$ and consequently $(M^d)_{32} \sim (M^d)_{33} \sim m_b$ and $(\tilde{M}^d_{LR})_{32} \sim (\tilde{M}^d_{LR})_{33} \sim \tilde{m}m_b$. From this, together with (2.3), we learn that the largest contributions to (3.3) come from $k = m = 3$, and $l, n = 2, 3$. This, in general, gives mass to the two light neutrinos.

Taking into account that $\lambda^l_{033} \sim m_b/m_Z$, one obtains:

$$
\frac{m_{\nu_\mu}}{m_{\nu_\tau}} \sim \epsilon^{\text{loop}} \lambda^2[HL_2-HL_3],
$$

$$
\frac{m_{\nu_e}}{m_{\nu_\tau}} \sim \epsilon^{\text{loop}} \lambda^2[HL_1-HL_3],
$$

$$
\epsilon^{\text{loop}} = \frac{3m^4_b}{8\pi^2 m^2_Z} \sim 10^{-7},
$$
which implies the upper limit for the loop–induced masses \( m_{\nu_i} \lesssim 10^{-7} m_{\nu_e} \lesssim 1 \text{eV}, \ i = 1, 2 \).

Note that if the Supersymmetry breaking trilinear scalar couplings are proportional to the Yukawa couplings (\( \tilde{M}_{LR}^{d2} = \tilde{m} M^d \)), the dominant quark–squark loop contributions to (3.3) yield a degeneracy in the mass matrix, and only one neutrino in (3.4) acquires mass. In the presence of a tree–level mass (3.2) for \( \nu_\tau \), this mass eigenstate is \( \nu_\mu \) (and is close to the interaction eigenstate \( L_2 \)). The same result applies also to the case that \((M^d)_{32} \ll (M^d)_{33}\) and \((\tilde{M}_{LR}^{d2})_{32} \ll (\tilde{M}_{LR}^{d2})_{33}\). Then a contribution to \( m_{\nu_e} \) arises from quark-squark loops with e.g. \( k=n=2, \ l=m=3 \), giving \( m_{\nu_e}/m_{\nu_\tau} \sim \frac{3m_u^2 m_e^2}{8\pi^2 m_Z^2} \lambda^2 [H(L_1) - H(L_3)] \), about two to three orders of magnitude below (3.4). This is somewhat smaller than the contribution from lepton–slepton loops discussed below.

(iii) Lepton–slepton loop contributions.

Loops with charged leptons and sleptons contribute

\[
\frac{1}{2} M^\nu_{ij} \text{loop} \sim \frac{\lambda_{ijkl} \lambda_{jmn}}{16\pi^2} \frac{(M^\ell)_{lm} (\tilde{M}_{LR}^{d2})_{kn}}{\tilde{m}^2}.
\]

(3.5)

Using (2.6), we learn that the largest contribution from (3.5) has

\[
\epsilon_{\text{loop}} = \frac{m_\tau^4}{8\pi^2 m_Z^2} \sim 10^{-9},
\]

(3.6)

about two orders of magnitude lower than the dominant quark–squark contributions. As already mentioned, this contribution plays a significant role only when \( \tilde{M}_{LR}^{d2} \simeq \tilde{m} M^d \) holds to a good approximation.

(iv) Non–renormalizable contributions.

The dimension–5 terms \( \frac{1}{M^\nu_{ij} \phi_u \phi_u} \) give [9]:

\[
\frac{1}{2} M^\nu_{ij} \text{nr} \sim \lambda [H(L_1)+H(L_3)+2H(\phi_u)] \frac{m_Z^2}{M}.
\]

(3.7)

This, in general, contributes to both light neutrinos:

\[
\frac{m^\nu_{\mu}}{m_{\nu_e}} \sim \epsilon_{\text{nr}} \lambda^2 [H(L_2) - H(L_3)],
\]

\[
\frac{m^\nu_{\tau}}{m_{\nu_e}} \sim \epsilon_{\text{nr}} \lambda^2 [H(L_1) - H(L_3)],
\]

(3.8)

\[
\epsilon_{\text{nr}} = \lambda^2 [H(\phi_u) + H(\phi_d)] \frac{m_Z^2}{M} \sim 10^{-7} \left( \frac{10^9 \text{ GeV}}{M} \right).
\]
The relative importance of the non–renormalizable and loop contributions to $m_{\nu_{\mu}}$ and $m_{\nu_{e}}$ depends on the scale $M$ (which is, roughly speaking, the natural scale for the masses of right–handed neutrinos). For $M > \sim 10^{9}$ $GeV$, the leading contributions come from loops, while for $M < \sim 10^{9}$ $GeV$, the non–renormalizable contributions dominate.

Adding up the various contributions, and defining

$$\epsilon = \max(\epsilon^{nr}, \epsilon^{loop}),$$

leads to the following order of magnitude estimates for the neutrino masses and mixing angles:

$$m_{\nu_{\tau}}/m_Z \sim \lambda^2[H(L_3) - H(\phi_d)],$$  \hspace{1cm} (3.10)

$$m_{\nu_i}/m_{\nu_{\tau}} \sim \epsilon \lambda^2[H(L_i) - H(L_3)] \quad (i = 1, 2),$$

$$\sin \theta_{ij} \sim \lambda H(L_i) - H(L_j) \quad (i < j).$$  \hspace{1cm} (3.11)

The charged lepton mass ratios are estimated to be

$$m_{\ell_i}/m_{\ell_j} \sim \lambda^{H(L_i) + H(\bar{\ell}_i) - H(L_j) - H(\bar{\ell}_j)}.$$  \hspace{1cm} (3.12)

The charged current mixing matrix mixes not only the leptons among themselves, but also leptons with higgsinos and gauginos:

$$\sin \theta_{\nu_i\tilde{\phi}_d} \sim \lambda^{H(L_i) - H(\phi_d)},$$

$$\sin \theta_{\nu_i\tilde{\omega}} \sim \lambda^{H(L_i) + H(\phi_u)}.$$  \hspace{1cm} (3.13)

The fact that the neutrino mass eigenstates have an is triplet $\tilde{\omega}_3$ component in them, leads to flavor changing couplings of the $Z$-boson to neutrinos $\sim g \Omega_{ij} Z\nu_i\bar{\nu}_j$:

$$\Omega_{ij} \sim \lambda^{H(L_i) + H(L_j) + 2H(\phi_u)}.$$  \hspace{1cm} (3.14)

The estimates (3.10) and (3.11) give the following relations between neutrino masses and mixing angles:

$$\frac{m_{\nu_{\mu}}}{m_{\nu_{\tau}}} \sim \epsilon \sin^2 \theta_{23},$$

$$\frac{m_{\nu_e}}{m_{\nu_{\tau}}} \sim \epsilon \sin^2 \theta_{13},$$

$$\frac{m_{\nu_e}}{m_{\nu_{\mu}}} \sim \sin^2 \theta_{12}.$$  \hspace{1cm} (3.15)
There are two order of magnitude relations that are independent of $\epsilon$:

$$\sqrt{\frac{m_{\nu_e}}{m_{\nu_\mu}}} \sim \sin \theta_{12} \sim \frac{\sin \theta_{13}}{\sin \theta_{23}}, \quad (3.16)$$

so that for the two light neutrinos (1.1) still holds. The order of magnitude inequality (1.2) is maintained:

$$\sin \theta_{ij} \gtrsim \frac{m_{\ell_i}}{m_{\ell_j}} \quad (i < j). \quad (3.17)$$

The relation (1.4) is also maintained,

$$m_{\nu_e} \lesssim m_{\nu_\mu} \lesssim m_{\nu_\tau} . \quad (3.18)$$

However, unlike (1.3), the light neutrinos are much lighter than $\nu_\tau$:

$$\frac{m_{\nu_i}}{m_{\nu_\tau}} \lesssim \epsilon \quad (i = 1, 2). \quad (3.19)$$

For a scale $M \gtrsim 10^9 \text{ GeV}$, this gives $\frac{m_{\nu_i}}{m_{\nu_\tau}} \lesssim 10^{-7}$. It is interesting that these models can naturally give mixing angles of $\mathcal{O}(1)$ with the third generation [24] while the corresponding mass ratios are very small. (For different mechanisms that give such a result, see [36-37].)

4. Theory Confronts Experiment

In this and the next sections we show that the order of magnitude relations derived in the last section, when combined with various experimental and cosmological constraints, exclude large regions of the mass–mixing parameter space. Most of our discussion in these two sections is independent of the question of $R_p$ violation.

As the charged lepton masses are known, eq. (3.17) provides significant lower bounds on the lepton mixing angles. With $m_e/m_\mu \sim \lambda^3$ and $m_\mu/m_\tau \sim \lambda^2$, we get

$$\sin \theta_{23} \gtrsim \lambda^2, \quad \sin \theta_{13} \gtrsim \lambda^5, \quad \sin \theta_{12} \gtrsim \lambda^3. \quad (4.1)$$

The lower bound on $\sin \theta_{23}$ is particularly significant. First, if $m_{\nu_\tau}$ is in the appropriate range, $\nu_\mu - \nu_\tau$ oscillations will be observed in the CHORUS, NOMAD and E803 experiments. Second, combining it with the upper bound $\sin^2 \theta_{23} = BR(\pi \to \mu \nu_\tau) \leq 6 \times 10^{-5}$ for $m_{\nu_\tau} \gtrsim 3 \text{ MeV}$ [38] results in

$$m_{\nu_\tau} \lesssim 3 \text{ MeV}. \quad (4.2)$$
Third, in combination with the bound on $\nu_\mu - \nu_\tau$ oscillations, $\sin^2 2\theta_{23} \leq 0.004$ for $\Delta m^2 \gtrsim 100 \text{ eV}^2$ \cite{39}, it gives\footnote{The interpretation of oscillation experiments might change in the framework of Supersymmetry without $R_p$ because new neutrino interactions are introduced \cite{10}. In our case, however, the horizontal suppression makes these new interactions practically negligible.}

$$\sin \theta_{23} \sim \lambda^2 \quad \text{for} \quad 10 \text{ eV} \lesssim m_{\nu_\tau} \lesssim 3 \text{ MeV}. \quad (4.3)$$

As we predict $\sin \theta_{13} \lesssim \sin \theta_{23}$, $(4.3)$ implies also

$$\sin \theta_{13} \lesssim \lambda^2 \quad \text{for} \quad 10 \text{ eV} \lesssim m_{\nu_\tau} \lesssim 3 \text{ MeV}. \quad (4.4)$$

This bound is stronger than the bound from $\nu_e - \nu_\tau$ oscillations, $\sin^2 2\theta_{13} \leq 0.12$ for $\Delta m^2 \gtrsim 100 \text{ eV}^2$ \cite{39}, which, in this range, gives $\sin \theta_{13} \lesssim \lambda$. The latter bound, however, holds independently of whether holomorphy plays a role in determining the mixing angles.

Eqs. $(4.2)$, $(4.3)$ and $(4.4)$ are applicable also in models with $R_p$ because they result from $(3.17)$ which holds independently of $R_p$ violation.

5. Theory Confronts Cosmology

Cosmological considerations related to the age and the present energy density of the Universe provide a constraint on the mass and lifetime of neutrinos. For masses in the range $100 \text{ eV} - \text{ a few MeV}$, the constraint reads (see e.g. \cite{41})

$$m^2_{\nu_\tau} \lesssim 2 \times 10^{20} \text{ eV}^2 \text{ sec.} \quad (5.1)$$

The framework of Abelian horizontal symmetries allows an estimate of the neutrino decay rates. It is interesting to find whether $\nu_\tau$ could have a fast enough decay mode to fulfill $(5.1)$ and have its mass above $100 \text{ eV}$.

The dominant decay modes are most likely those which proceed via gauge interactions.

The bound $(4.2)$ leaves only a very small window where the $W$–mediated tree level $\nu_\tau \to e^+ e^- \nu_e$ is allowed. The rate can be estimated to be:

$$\frac{\Gamma(\nu_\tau \to e^+ e^- \nu_e)}{\Gamma(\tau \to e \bar{\nu}_e \nu_\tau)} = \frac{m_{\nu_\tau}^5}{m_{\tau}^5} \sin^2 \theta_{13}$$

$$\implies m_{\nu_\tau}^5 \sim \left(\frac{\lambda^5}{\sin \theta_{13}}\right)^2 3 \times 10^{41} \text{ eV}^5 \text{ sec.} \quad (5.2)$$
Together with (4.2), we find that (5.1) is satisfied only for \( \sin \theta_{13} \gtrsim \lambda^4 \). Since there are charged particles in the final state, however, a stronger bound (from considerations of the cosmic microwave background radiation) applies, \( \tau_{\nu_e} \lesssim 10^4 \text{sec} \). This cannot be satisfied for \( m_{\nu_e} \lesssim 3 \text{MeV} \) and \( \sin \theta_{13} \lesssim \lambda^2 \). Furthermore, detailed studies of the effects of a massive \( \nu_\tau \) during the Nucleosynthesis era \([12-14]\) suggest that for \( m_{\nu_\tau} \gtrsim 0.5 \text{MeV} \), and independently of the decay modes, \( \tau_{\nu_\tau} \lesssim 10^2 \text{sec} \) is required, which closes the window even more firmly. Therefore we conclude that \( \nu_\tau \to e^+ e^- \nu_e \) does not open any window for a heavy \( \nu_\tau \). Again, this conclusion holds also for models with \( R_p \).

All other decay modes are flavor changing neutral current processes. There are three types of contributions to such processes:

(a) Loop diagrams with gauge particles, suppressed by the charged current mixing angles;
(b) Tree level \( Z \)–mediated decays suppressed by the \( \Omega_{ij} \) mixing angles;
(c) Tree level slepton–mediated decays suppressed by the selection rules for the \( \lambda_{ijk} \) couplings.

The first class is common to models with and without \( R_p \), but the other two are present only in \( R_p \)–violating models. In any case, we found that none of these channels is fast enough to allow \( m_{\nu_\tau} \gtrsim 100 \text{eV} \). For example, the rate for the \( Z \)–mediated \( \nu_\tau \to 3\nu_\mu \) can be estimated to be:

\[
\frac{\Gamma(\nu_\tau \to 3\nu_\mu)}{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)} \sim \frac{m_{\nu_\tau}^5}{m_\tau^5} \Omega_{23}^2
\]

\[
\implies m_{\nu_\tau}, \tau_{\nu_\tau} \sim \left( \frac{\lambda^6}{\Omega_{23}} \right)^2 \times 10^{43} \text{eV}^5 \text{sec}.
\]  

This is significantly suppressed compared to (5.2) and does not satisfy (5.1).

As \( \nu_\tau \) is predicted to be the heaviest among the neutrinos, we conclude that in the framework of Supersymmetry and Abelian horizontal symmetry with or without \( R_p \) (and assuming that holomorphy does not play a role in determining \( \sin \theta_{23} \))

\[
m_{\nu_\tau} \lesssim 100 \text{eV}
\]  

holds for all neutrino masses. We note that in the framework of a single \( U(1) \) or \( Z_n \) broken by \( \lambda \sim 0.2 \), this requires \( H(L_i) - H(\phi_d) \gtrsim 6 \) which may be too large for reasonable models.
In some models of ref. [11], however, where the symmetry breaking parameters are much smaller, this can be achieved with charge differences $\leq 2$.

6. Solar and Atmospheric Neutrinos

The upper bound $m_{\nu_e} \lesssim 100 \text{ eV}$ leads to even stronger bounds on $m_{\nu_\mu}$ and $m_{\nu_e}$. These bounds, however, depend on $M$ and on $\tan \beta$. We believe that the most likely situation is (a) $M \gtrsim 10^9 \text{ GeV}$ (which, for example, applies in all models where the Supersymmetric extension of the Standard Model is valid up to some GUT scale) and (b) $\tan \beta \sim 1$ (which is the natural value [45-46]). Then in (3.3) $\epsilon \sim 10^{-7}$ leads to

$$m_{\nu_e} \lesssim 100 \text{ eV}, \quad m_{\nu_e} \lesssim m_{\nu_\mu} \lesssim 10^{-5} \text{ eV}. \quad (6.1)$$

This bears important consequences for the solar and atmospheric neutrino problems. The value $m_{\nu_\mu} \lesssim 10^{-5} \text{ eV}$ is inconsistent with

$$m_{\nu_\mu}^2 - m_{\nu_e}^2 \sim 6 \times 10^{-6} \text{ eV}^2 \quad (6.2)$$

that is required to solve the solar neutrino problem through the MSW mechanism (see e.g. [17]), but is consistent with $m_{\nu_\mu}^2 - m_{\nu_e}^2 \sim 10^{-11} \text{ eV}^2$ that could solve it through vacuum oscillations (see e.g. [18]). It is also inconsistent with

$$m_{\nu_\mu}^2 - m_{\nu_e}^2 \sim 10^{-2} \text{ eV}^2 \quad (6.3)$$

that is required to solve the atmospheric neutrino problem through $\nu_\mu - \nu_e$ oscillations (see e.g. [19]).

The solar neutrino problem can still be solved by the MSW mechanism with $\nu_\tau - \nu_e$ oscillations (if $m_{\nu_\tau} \sim 10^{-3} \text{ eV}$) or the atmospheric neutrino problem can be explained by $\nu_\tau - \nu_\mu$ oscillations (if $m_{\nu_\tau} \sim 10^{-1} \text{ eV}$). However, we find that:

a. The two problems cannot be solved simultaneously;

b. The required horizontal charges are inconveniently large: $H(L_3) - H(\phi_d) \sim 10$ and $H(L_1) - H(\phi_d) \sim 12$ to get the small angle MSW solution for the solar neutrino problem, and $H(L_{2,3}) - H(\phi_d) \sim 8$ to solve the atmospheric neutrino problem;
c. Such a light $\nu_\tau$ does not contribute to the dark matter and cannot play any role in structure formation. This would require

$$m_{\nu_\tau} \sim 10 \text{ eV}. \quad (6.4)$$

This situation is very different from the Supersymmetric models with $R_p$, where (6.4) and (6.2) can be simultaneously accommodated [3].

Things are different if we relax either of our two extra assumptions. As the tree level contribution to $m_{\nu_\tau}$ is suppressed by $\tan^2 \beta$ and the loop contributions are enhanced by $\tan^3 \beta$, a large $\tan \beta$ would give a large $\epsilon$ (e.g. $\tan \beta \sim 20$ gives $\epsilon \sim 10^{-2}$). Then we can easily accommodate the dark matter (6.4) and solar neutrino (6.2) constraints ($H(L_2) = H(L_3) + 1 = H(\phi_d) + 6$). Alternatively, the solar and atmospheric neutrino problems can be solved simultaneously ($H(L_2) = H(L_3) = H(\phi_d) + 6$). It is non–trivial that, in this scenario, $H(L_2)$ and $H(L_3)$ which are fixed by the requirements on $m_{\nu_\mu}$ and $m_{\nu_\tau}$ give, at the same time, $\sin \theta_{23} = \mathcal{O}(1)$ as required to solve the atmospheric neutrino problem.

For $M < 10^9 \text{ GeV}$, the non–renormalizable contributions to $m_{\nu_\mu}$ dominate over the loop corrections. In this case $m_{\nu_\tau}$ can account for the dark matter, while $m_{\nu_\mu}$ can accommodate the solar neutrino constraint (6.2) (however, this requires $M \lesssim 10^6 \text{ GeV}$).

Finally, if holomorphy does play an important role in the physical parameters, then even the conclusions of sections 3–5 can be evaded. For example, we can construct models where $\sin \theta_{23} \ll \lambda^2$, which would allow for $m_{\nu_\tau}$ above the 3 $\text{MeV}$ bound of section 4. Then, with $\sin \theta_{13} \sim \lambda^4$ (which is marginally compatible with the experimental limits [50-51]), the decay $\nu_\tau \rightarrow e^+e^-\nu_e$ can still open a window for $m_{\nu_\tau}$ close to its experimental bound. Explicit examples of models where holomorphy induces approximate zeros in the mass matrices and affects physical parameters can be found in refs. [12,13,3].

### 7. Discussion

Models of Supersymmetry with Abelian horizontal symmetries have interesting implications for neutrino masses and mixing. We distinguish three cases:
(i) Models with $R_p$.

(ii) Models without $R_p$ and with generic soft Supersymmetry breaking terms (up to the selection rules from the horizontal symmetry).

(iii) Models without $R_p$ but with universal Supersymmetry breaking terms implying alignment at a high scale (namely $B_\alpha = B_{\mu \alpha}$ and $m^2_{\alpha \beta} = \tilde{m}^2 \delta_{\alpha \beta}$).

Class (i) was analyzed in [3]. Class (ii) has been studied in this work. Class (iii), which yields a scenario quite different from the one investigated here, will be discussed in a forthcoming paper. We now compare the predictions of class (ii) with those of (i).

In a large class of models, where holomorphy does not introduce zeros in the mass matrices (or, if there are such zeros, they do not affect the order of magnitude of the physical parameters), we find the following order of magnitude relations among the mass ratios and mixing angles:

\[
\begin{align*}
\frac{m_{\nu_e}}{m_{\nu_\tau}} &\quad \frac{m_{\nu_e}}{m_{\nu_\tau}} &\quad \frac{m_{\nu_e}}{m_{\nu_\mu}} \\
(i) &\quad \sin^2 \theta_{23} &\quad \sin^2 \theta_{13} &\quad \sin^2 \theta_{12} \\
(ii) &\quad 10^{-7} \sin^2 \theta_{23} &\quad 10^{-7} \sin^2 \theta_{13} &\quad \sin^2 \theta_{12}
\end{align*}
\]

(7.1)

Note that the following relation among the mixing angles holds in both classes:

\[
(i), (ii) : \quad \sin \theta_{13} \sim \sin \theta_{12} \sin \theta_{23}.
\]

(7.2)

Furthermore, since $\sin \theta_{ij} \gtrsim m_{\ell_i}/m_{\ell_j}$ holds independently of $R_p$,

\[
(i), (ii) : \quad \sin^2 \theta_{23} \gtrsim 10^{-3}, \quad \sin^2 \theta_{13} \gtrsim 10^{-7}, \quad \sin^2 \theta_{12} \gtrsim 10^{-4}.
\]

(7.3)

This leads to the following ranges for the mass ratios:

\[
\begin{align*}
\frac{m_{\nu_e}}{m_{\nu_\tau}} &\quad \frac{m_{\nu_e}}{m_{\nu_\tau}} &\quad \frac{m_{\nu_e}}{m_{\nu_\mu}} \\
(i) &\quad 10^{-3} - 1 &\quad 10^{-7} - 1 &\quad 10^{-4} - 1 \\
(ii) &\quad 10^{-10} - 10^{-7} &\quad 10^{-14} - 10^{-7} &\quad 10^{-4} - 1
\end{align*}
\]

(7.4)

We conclude that measurements of the lepton mixing angles would test the Supersymmetric Abelian horizontal symmetry framework while measurements of neutrino mass ratios will serve to distinguish between models with or without $R_p$. 

12
In ref. [41], it was shown that \( m_{\nu_{\mu}}/m_{\nu_{\tau}} \gtrsim (m_{\mu}/m_{\tau})^2 \) together with cosmological considerations, strongly suggests that all neutrinos are lighter than \( \mathcal{O}(100 \text{ eV}) \). Models without \( R_p \) predict \( m_{\nu_{\mu}}/m_{\nu_{\tau}} \ll (m_{\mu}/m_{\tau})^2 \) but we still find that all neutrinos are lighter than \( \mathcal{O}(100 \text{ eV}) \). This is a consequence of the fact that there is no decay mode large enough to fulfill the cosmological constraints on massive neutrinos.

In models with \( R_p \), one can accommodate \( m_{\nu_{\tau}} \) to contribute sizeably to the cosmological dark matter, as well as \( m_{\nu_{\mu}} \) in the correct range required by the MSW solution of the solar neutrino problem. In models without \( R_p \) (and without any alignment condition) we find that, unless the scale of New Physics \( M \) is surprisingly low (\( \lesssim 10^6 \text{ GeV} \)), \( \nu_{\mu} \) is too light to play any role for matter enhanced oscillations of the solar \( \nu_e \)’s.

Finally, we emphasize that our various predictions are not entirely generic to models of Abelian horizontal symmetries. As described briefly in section 6, a large \( \tan \beta \) and/or a small scale \( M \) would modify our discussion of the solar (and atmospheric) neutrino problem. But more important, one can construct models where holomorphy plays an important role and circumvents the otherwise model–independent predictions of eqs. (3.16), (3.17) and (3.18).

**Acknowledgments:** FMB acknowledges discussions with R. Hempfling. YN is supported in part by the United States – Israel Binational Science Foundation (BSF), by the Israel Commission for Basic Research, and by the Minerva Foundation (Munich).
References

[1] H. Dreiner et al., Nucl. Phys. B436 (1995) 461.
[2] A. Rasin and J.P. Silva, Phys. Rev. D49 (1994) 20.
[3] Y. Grossman and Y. Nir, Nucl. Phys. B448 (1995) 30.
[4] G.K. Leontaris, S. Lola, C. Scheich, and J.D. Vergados, hep-ph/9509351.
[5] P. Binetruy, S. Lavignac, and P. Ramond, hep-ph/9601243.
[6] E.J. Chun and A. Lukas, hep-ph/9605377.
[7] I. Hinchliffe and T. Kaeding, Phys. Rev. D47 (1993) 279.
[8] T. Banks, Y. Grossman, E. Nardi, and Y. Nir, Phys. Rev. D52 (1995) 5319.
[9] C.D. Carone, L.J. Hall, and H. Murayama, hep-ph/9512399, hep-ph/9602364.
[10] V. Ben-Hamo and Y. Nir, Phys. Lett. B339 (1994) 77.
[11] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. B398 (1993) 319.
[12] Y. Nir and N. Seiberg, Phys. Lett. B309 (1993) 337.
[13] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. B420 (1994) 468.
[14] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147 (1979) 277.
[15] C.S. Aulakh and R.N. Mohapatra, Phys. Lett. 119B (1982) 136.
[16] L.J. Hall and M. Suzuki, Nucl. Phys. B231 (1984) 419.
[17] L.-H. Lee, Phys. Lett. 138B (1984) 121; Nucl. Phys. B246 (1984) 120.
[18] G.G. Ross and J.W.F. Valle, Phys. Lett. 151B (1985) 375.
[19] J. Ellis et al., Phys. Lett. 150B (1985) 142.
[20] S. Dawson, Nucl. Phys. B261 (1985) 297.
[21] A. Santamaria and J.W.F. Valle, Phys. Lett. B195 (1987) 423.
[22] D.E. Brahm, L.J. Hall, and S. Hsu, Phys. Rev. D42 (1990) 1860.
[23] J.C. Romao and J.W.F. Valle, Nucl. Phys. B381 (1992) 87.
[24] A.S. Joshipura and M. Nowakowski, Phys. Rev. D51 (1995) 2421.
[25] S. Dimopoulos and L.J. Hall, Phys. Lett. B207 (1987) 210.
[26] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 64 (1990) 1705.
[27] R. Barbieri, M.M. Guzzo, A. Masiero, and D. Tommasini, Phys. Lett. B252 (1990) 251.
[28] E. Roulet and D. Tommasini, Phys. Lett. B256 (1991) 218.
[29] K. Enqvist, A. Masiero, and A. Riotto, Nucl. Phys. B373 (1992) 95.
[30] R.M. Godbole, P. Roy, and X. Tata, Nucl. Phys. B401 (1993) 67.
[31] R. Hempfling, hep-ph/9511288.
[32] B. de Carlos and P.L. White, hep-ph/9602381.
[33] T. Yanagida, in Proc. Workshop on Unified theory and baryon number in the universe, eds. O. Sawada and A. Sugamoto (KEK, 1979).
[34] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuiizen and D. Freedman (North-Holland, 1980).
[35] D. Buskulic et al., ALEPH Collaboration, Phys. Lett. B349 (1995) 585.
[36] E. Nardi, E. Roulet, and D. Tommasini, Phys. Lett. B327 (1994) 319.
[37] K.S. Babu and S.M. Barr, hep-ph/9511446.
[38] M. Daum et al., Phys. Rev. D36 (1987) 2624.
[39] N. Ushida et al., Fermilab E531 Collaboration, Phys. Rev. Lett. 57 (1986) 2897.
[40] Y. Grossman, Phys. Lett. B359 (1995) 141.
[41] H. Harari and Y. Nir, Nucl. Phys. B292 (1987) 251.
[42] E. Kolb, M.S. Turner, A. Chakravorty, and D.N. Schramm, Phys. Rev. Lett. 67 (1991) 533.
[43] A.D. Dolgov and I.Z. Rothstein, Phys. Rev. Lett. 71 (1993) 476.
[44] M. Kawasaki et al., Nucl. Phys. B 419 (1994) 105.
[45] A.E. Nelson and L. Randall, Phys. Lett. B316 (1993) 516.
[46] R. Rattazzi and U. Sarid, Phys. Rev. D53 (1996) 1553.
[47] N. Hata and P. Langacker, Phys. Rev. D50 (1994) 632.
[48] E. Calabrese, N. Ferrari, G. Fiorentini, and M. Lissia, Astropart. Phys. 4 (1995) 159.
[49] G.L. Fogli and E. Lisi, Phys. Rev. D52 (1995) 2775.
[50] De Leener-Rosier et al., Phys. Rev. D43 (1991) 3611.
[51] K. Zuber, hep-ph/9605403.