The Inertial Polarization Principle: The Mechanism Underlying Sonoluminescence?

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Abstract

In this paper we put forward a mechanism in which imploding shock waves emit electromagnetic radiation in the spectral region $\lambda_0 \cong 2\pi R_0$, where $R_0$ is the radius of the shock by the time it is first formed. The mechanism relies on three different pieces of Physics: Maxwell’s equations, the existence of corrugation instabilities of imploding shock waves and, last but not least, the Inertial Polarization Principle. The principle is extensively discussed: how it emerges from very elementary physics and finds experimental support in shock waves propagating in water. The spectrum of the emitted light is obtained and depends upon two free parameters, the amplitude of the instabilities and the cut-off $R_{\text{max}}$, the shocks’ spatial extension. The spectral intensity is determined by the former, but its shape turns out to have only a mild dependence on the latter, in the region of physical interest. The matching with the observed spectrum requires a fine tuning of the perturbation amplitude $\varepsilon \sim 10^{-14}$, indicating a quantum mechanical origin. Indeed, we support this conjecture with an order of magnitude estimative. The Inertial Polarization Principle clues the resolution of the noble gas puzzle in SL.

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The Inertial Polarization Principle

In this paper we put forward a mechanism responsible for transducing the kinetic energy stored in an imploding spherical shock wave into electromagnetic radiation, which is based solely upon Maxwell’s equations, the existence of very small instabilities away from the spherically symmetric flow and the inertial polarization paradigm. Based on these premisses we obtained the spectral intensity of the outgoing radiation. The mechanism turns out to be so efficient that the observed energy emission rate of $P(\lambda) \sim 10^{-10}$ Watt/nm calls for perturbation amplitudes no larger than $\varepsilon = 10^{-14}$! Maxwell’s equations are a pillar of theoretical physics while inertial polarization is a consequence of very elementary physics: an atom that undergoes an acceleration, say $a$, develops in its interior polarized electromagnetic fields. The issue is made clear for an observer sitting in the frame of the molecule, where he sees inertial forces acting both upon the nucleus $F_N = M_N a$ and on the electronic cloud $F_e = M_e a$. The gradient between these forces tends to sag the cloud away from the nucleus, and the atom develops internal polarization fields, say $E_0$, to compensate this gradient $e E_0 \sim (M_N - M_e) a$. The role of inertial polarization remained hitherto unnoticed only because detectable polarization fields call for tremendous accelerations, say, $E_0 \sim 1 V/m$ would require $a \sim (e/M_p) E_0 \sim 10^{-2} (e V/ (M_p c^2)) (c^2/cm) \sim 10^{10}$ cm/sec$^2$ which are absent in every day life experiments. Nevertheless, there are two instances where such large accelerations manifest: i) in the realm of very strong gravitational fields where inertial polarization was shown to be the working mechanism that rescues the second law of thermodynamics from bankruptcy (otherwise super-luminal motion of black-holes inside dielectric media would entail a violation of the generalized second law [1],[2]); ii) in the realm of shock waves, because shocks are powerful accelerators of fluid molecules: a fluid molecule that crosses the shock undergoes a macroscopic velocity change (of the order of the fluid velocity itself) within a microscopic distance – the shock width (of the order of the mean free path for the atomic collisions [3]).

The inertial polarization principle is the single non-very-well-established piece of physics in our recipe and we proceed by making our case for it. Consider a planar strong shock wave propagating within a perfect gas. Let $v_2$ and $v_1$ represent the fluid velocity in the back and in front the shock, respectively ( likewise, the index 2 (1) refer to physical quantities behind (in front) the shock ). As the fluid molecules cross the shock they experience a mean acceleration $\bar{a}$ ( likewise, the index 2 (1) refer to physical quantities behind (in front) the shock ). The mean free path for the atomic collisions [3]).

The shock width $\delta$ is known to be of order of the mean free path for collisions of atoms in the fluid, $\delta \approx (n \sigma)^{-1}$, where $n$ stands for the number density of atoms and $\sigma$ for the collision’s cross section. Bearing in mind that $nV = A/\mu$ where $A$ is Avogadro’s number and $\mu$ is the molecular weight of the gas, we obtain the colossal figure for the mean acceleration atoms experience as they cross the shock:

$$\bar{a} \approx \frac{\gamma}{\gamma - 1} 6 \times 10^{13} \left( \frac{p_2}{\text{atm}} \right) \left( \frac{\sigma}{10^{-16} \text{cm}^2} \right) \left( \frac{\text{gram}}{\mu} \right) \text{ cm/sec}^2.$$  \(4\)

The mean electric polarization developed across the shock $E_0 \approx (M_p/e) \bar{a}$ is also sizeable

$$E_0 \approx 6 \times 10^3 \frac{V}{\text{meter}} \frac{\gamma}{\gamma - 1} \left( \frac{p_2}{\text{atm}} \right) \left( \frac{\sigma}{10^{-16} \text{cm}^2} \right) \left( \frac{\text{gram}}{\mu} \right).$$  \(5\)
Unfortunately, the shock is so thin that the voltage developed across its ends is very small

\[ V \sim \frac{\bar{a}M_p \delta}{e} \sim 1.2 \times 10^{-6} V_{olt} \frac{\gamma}{\gamma - 1} \left( \frac{p}{\text{atm}} \right) \left( \frac{\text{cm}^3}{	ext{g s}} \right) \]  

(6)

Shock Polarization was first observed in the early sixties \cite{4} for shock waves propagating inside water. Since then, both quality and range of the measurements improved considerably \cite{4, 3}. Harris \cite{4, 3} credits the effect to the fact that large pressure gradients inside the shock results in a torque field acting upon the water molecule causing the molecule’s dipole to align. We reproduce his results via the table:

| \( p(\text{kbar}) \) | 98 | 75 | 74.5 | 58 | 54 | 45 | 36 | 20 |
|-----------------|---|---|-----|---|---|---|---|---|
| \( V(\text{mV})/p(\text{kbar}) \) | 1.97 | 1.33 | 0.97 | 0.77 | 0.43 | 0.89 | 0.680 | 0.8 |

The underlying Physics for a shock propagating in water is the very same as for a gas and we infer the averaged electric potential across the shock from eq. \( (6) \) bearing in mind that: i.) the compression rate for water is of order one, therefore we go one step back in this equation by replacing \( \gamma/(\gamma - 1) \rightarrow \gamma/(\gamma + 1) \approx 1/2 \); ii.) the equation was obtained for a gas and for liquids it should be regarded as the linear expansion of the function \( V(p) \). Then it follows that \( V(\text{mV})/p(\text{kbar}) \approx 0.6 \), in agreement with the lower pressure region of the experimental data. Detection of shock polarization for non-polar fluids would vindicate the Inertial Polarization Principle.

The acceleration field inside planar shocks is space and time independent (and so the corresponding polarized electromagnetic fields) . Nevertheless, planar shocks are known to develop corrugation instabilities \cite{3}, small deformations of the planar geometry that detach from the shock and propagate throughout the fluid. They correspond to the spontaneous emission of sound from the shock. These instabilities will cause a space time dependent acceleration field inside the shock, and by the Inertial Polarization Principle a wiggling \( 6 \times 10^3 V/m \) electric field vector that is radiated away: sound and light are emitted simultaneously, provided the Inertial Polarization relaxation time is small enough. This brings to one’s mind the famous and intriguing sonoluminescence effect \cite{5, 6} in which under heavy bombarding of ultra-sound waves, a little ( \( 5 \mu m \) ) bubble of air cavitating within a flask of water undergoes a spectacular collapse, attains the supersonic regime and glows (mainly) violet light. The effect has been around for sixty years or so ( \( \left( 5 \mu m, \right) \) ) and proper understanding of the problem remains elusive. The most popular mechanism is the Bremsstrahlung from free electrons in the gas where the ionization is caused by two successive heating processes: first the adiabatic collapse of the bubble which is then followed by the motion of a shock wall inside the bubble (the shock’s Mach number controls the temperature rate \( T_2/T_1 \sim M^2 \) \cite{10}).

The formation of a shock wall, a collapsing spherical front of radius \( R(t) = A(-t)^\alpha \) (\( \alpha < 1 \)), happens by the time supersonic regime is attained inside the bubble \cite{3}. The acceleration of the shock front surface \( a(t) \sim A(-t)^{\alpha - 2} \), becomes very large at focusing (\( t \to 0 \)) engendering very large space and time dependent Inertial-Polarization fields. Nevertheless, the spherical symmetric geometry of the problem prevents these fields to be radiated away: pursuing the present avenue seems to require some supplementary mechanism to account for the radiation flash (a sparking mechanism was proposed \cite{3, 10}). Fortunately, no supplementary mechanism is needed: numerical calculations \( (\text{I}3) \) have shown the existence of unstable perturbations of the collapsing shock which provide the multipole time-dependent inertial-polarization fields that are radiated away. The purpose of this paper is to calculate the spectral distribution of the emitted light.

The paper is organized as follows. The following section reviews the dynamics of imploding shocks, and the existence of unstable multipole perturbation modes is rigorously proved. As a bonus, we obtain the energy and the power carried away by the sound waves that detach from the shock (corrugation instabilities). A novel semi-analytical procedure for solving the differential equations for the perturbations is developed, which nevertheless, is displayed in the appendix in order prevent the disruption of the main argument line with technicalities. In section II , we obtain the polarization fields engendered by the corrugation instabilities and show that they act as a source term in Maxwell’s equations. Then we calculate the spectrum of the outgoing radiation. The spectrum depends on the dynamics of the corrugation instabilities, but fortunately it is possible to obtain the main structure of the spectrum without having to delve too deeply into the dynamics. The intensity of the outgoing radiation turned
out to be proportional to $pc^2$ where $\varepsilon$ is the corrugation instability amplitude and $p = E_p^2/(2\hbar)$, is Inertial-Polarization power-constant ($E_p$ stands for the proton’s rest energy and $\alpha$ for the fine structure constant). This constant is of the order $\simeq 1.47 \times 10^{16}$ Watt (!): collapsing shock waves are the most efficient power-stations in nature, with the sole possible exception of astrophysical objects! Agreement with the experimental data calls for amplitudes of the order $\varepsilon \sim 10^{-13}$ or $\delta r \sim 10^{-19}$m! These tiny perturbations must have a quantum mechanical origin, and we support this conjecture by an order of magnitude estimative. Finally we suggest the resolution of the noble gas puzzle in SL.

1 Dynamics of Imploding Shocks

The non-viscous implosion of a spherical shock cannot be characterized by any dimensional parameter. Consequently the flow admits a self-similar symmetry. Let $R(t) = A_t(-t)^\alpha$ represent the radius of the shock front, where $A_t$ and $\alpha$ are two constants and $v_{\text{shock}} = \alpha R(t)/t$, its implosion velocity. The self-similar parameter here is $\xi = r/R(t)$; the surface of the shock is given by $\xi = 1$. Self-similarity constrains the form of the speed of sound, radial flow velocity and density [14]:

\[ \alpha^2 = \left(\frac{\alpha r}{t}\right)^2 Z(\xi) \]  
\[ v_2 = \left(\frac{\alpha r}{t}\right) V(\xi) \]  
\[ \rho_2 = \rho_0 G(\xi) \] (7) (8) (9)

When expressed in terms of the self similar quantities $Z, V$ and $G$, the boundary conditions for a strong shock $\vec{n} \cdot \vec{v}_{\text{shock}} \gg c$ read,

\[ G(1) = \frac{\gamma - 1}{\gamma + 1}, V(1) = \frac{2}{\gamma + 1}, Z(1) = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} \] (10)

The equations that govern the flow are the entropy and mass conservation laws and Euler’s equation. They provide a set of non-linear coupled equations for $G(\xi), V(\xi)$ and $Z(\xi)$, which when solved for $Z(V)$ and $\xi(V)$, yield the pair of equations [3]

\[ \frac{dZ}{dV} = \frac{Z}{1-V} \left[ \frac{(Z - (1-V)^2)(2/\alpha - (3\gamma - 1)V)}{(3V - \kappa)Z - V(1-V)(1/\alpha - V) + \gamma - 1} \right] \] (11)

and

\[ \frac{d\ln \xi}{dV} = -\frac{Z - (1-V)^2}{(3V - \kappa)Z - V(1-V)(1/\alpha - V)} \] (12)

where $\kappa = 2(1-\alpha)/(\alpha\gamma)$. Inspection of these equations reveals the existence of a singular point at $Z = (1-V)^2$ ( $dV/d\xi \rightarrow \infty$). Clearly, all physical quantities, and their derivatives must be finite across the singular point, meaning that the conditions $(3V - \kappa)Z - V(1-V)(1/\alpha - V) = 0$ and $Z = (1-V)^2$ are simultaneous to each other at this point, such as to keep their ratio finite. Call $V_c(\alpha), Z_c(\alpha)$ the solution of this pair of algebraic equations. The parameter $\alpha$ is obtained by numerically integrating $Z(V)$ from $V = V(1)$ to $V_c$ for different values of $\alpha$ until the matching $Z(V_c(\alpha)) = Z_c(\alpha)$ is obtained. The good values for $\alpha$ are 0.688376/0.71717 for a monatomic/diatomic gas. The limit $t \rightarrow 0_-$ corresponds to the shock’s focusing time, after which the shock reflects and reexpands. For latter reference, we mention the asymptotic behavior $V \sim \xi^{-1/\alpha}$ as $\xi \rightarrow \infty$ [3].

We are seeking now perturbations away from this flow. Let $\delta = \delta \rho/\rho$ be the contrast function and $\delta \vec{v}$ the velocity fluctuation. The latter can be decomposed into its normal and perpendicular components $\delta v_n = \vec{n} \cdot \delta \vec{v}, \delta v_\perp = \delta \vec{v} - \delta v_n \vec{n}$.

The linearized mass and entropy conservation equations read

\[ \left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial r} \right) \delta + \delta v_n \frac{\partial \ln \rho}{\partial r} + \vec{\nabla} \cdot \delta \vec{v} = 0 \] (13)
while perturbing Euler’s equation yields
\[
\left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial r} \right) \delta \vec{v} + \delta v_n \frac{\partial}{\partial r} \vec{n} + v_p \delta \vec{v}_\perp = \frac{\delta \vec{v} \cdot \delta \vec{p}}{\rho}
\]  

Next, we introduce the self-similar ansatz
\[
\delta v_n = \varepsilon \frac{\alpha r}{t_0} \left( \frac{t}{t_0} \right)^{\alpha \beta - 1} (1 - V) \Phi(\xi) Y_{lm}(\theta, \phi)
\]
\[
\delta \vec{v}_\perp = \varepsilon \frac{\alpha r}{t_0} \left( \frac{t}{t_0} \right)^{\alpha \beta - 1} r \vec{v} (\xi) Y_{lm}(\theta, \phi)
\]
\[
\delta = \varepsilon \left( \frac{t}{t_0} \right)^{\alpha \beta} \Delta(\xi) Y_{lm}(\theta, \phi)
\]
\[
\delta s = \varepsilon c_p \left( \frac{t}{t_0} \right)^{\alpha \beta} \sigma(\xi) Y_{lm}(\theta, \phi)
\]

where \( c_p \) is the specific heat of the gas, \( t_0 \) is shock formation time and \( \varepsilon \) the amplitude of the perturbation at this moment. After some tedious algebra we translate the previous equations in terms of the self-similar quantities. The mass and entropy conservation yield
\[
(1 - V) \xi (\Delta' - \Phi') = \beta \Delta + 3 \Phi - l(l + 1) \tau
\]
\[
(1 - V) \xi \sigma' = \beta \sigma - \kappa \Phi
\]
where \( \kappa = 2(1 - \alpha) / (\alpha \gamma) \). The projection of Euler’s equation into the perpendicular direction yields a compact form
\[
(1 - V) \xi \tau' = (2V + \beta - \frac{1}{\alpha}) \tau + Z(\Delta + \sigma).
\]

but the normal projection gives a more cumbersome expression
\[
\frac{d}{dV} |Y(V)\rangle = \mathcal{M}(V)|Y(V)\rangle \quad ; \quad 0 \leq V \leq V(1) \equiv V_1
\]
where \|X(V)\| = (\phi(V), \tau(V), \pi(V), \sigma(V)), ; |X(V)\rangle = \exp[\beta \int_{V_1}^V m(V)dV] |Y(V)\rangle and, furthermore
\[
\mathcal{M}(V) = m(V) \left( \begin{array}{cccc}
Z & P(V) \phi_2(V) & P(V) \phi_3(V) & P(V) \phi_4(V) \\
2V - \frac{1}{\alpha} & Z & 0 & 0 \\
3 + \beta & -l(l + 1) & 0 & 0 \\
-\kappa & 0 & 0 & 0
\end{array} \right)
\]

with
\[
m(V) = \frac{1}{1 - V} \frac{d \ln \xi}{dV} \quad ; \quad P(V) = \frac{1}{(1 - V)^2 - Z}
\]

\[
\phi_1(V) = Z[5 - 2/\alpha + 2\beta + (\gamma - 1)(3V + dV)] + (1 - V)^2[-1/\alpha + 2V + 2dV]
\]
\[
\phi_2(V) = -Z(l(l + 1))
\]
\[
\phi_3(V) = Z[(\gamma - 1)(3V + dV - \kappa) + \beta]
\]
\[
\phi_4(V) = Z[\gamma(3V + dV - \kappa) + \beta]
\]
where \( dV(V) \equiv \xi V' \). Clearly this set of differential equations possess a regular singular point when \( Z - (1 - V)^2 = 0 \), that is to say, at \( V_c \). The limit \( V \to 0 \) \( (\xi \to \infty \), \( m(V) \to -\alpha/V, P(V) \to 1; \phi_1 \to -1/\alpha; \phi_{2,3,4} \to 0 \), reveals an additional singularity

\[
\frac{d}{dV} |Y(V)| \approx \frac{1}{V} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-\alpha(3 + \beta) & \alpha(l + 1) & 0 & 0 \\
\alpha \kappa & 0 & 0 & 0
\end{pmatrix} |Y(V)| \tag{26}
\]

The matrix on the right-hand-side of this equation defines an eigenvalue problem whose solution

\[
\lambda_{1,2} = 0 \rightarrow \left\{ \begin{array}{l}
\theta_1 \rightarrow (0, 0, 1, 0) \\
\theta_2 \rightarrow (0, 0, 0, 1)
\end{array} \right. \\
\lambda_{3,4} = 1 \rightarrow \left\{ \begin{array}{l}
\theta_3 \rightarrow (l(l + 1), 3 + \beta, 0, \alpha(l + 1)) \\
\theta_4 \rightarrow (0, 1, \alpha(l + 1), 0)
\end{array} \right. \tag{27}
\]

yields the asymptotic form

\[
|X(V)| \approx V^{-\alpha \beta} [(a_1 \theta_1 > + a_2 \theta_2 >) + V(a_3 \theta_3 > + a_4 \theta_4 >)] \quad V \to 0, \tag{28}
\]

where \( a_n \) are integration constants. Asymptotically regular fields require \( \Re(\beta) \leq 0 \) (except for the the particular mode \( a_1 = a_2 = 0 \) which calls for a less stringent condition \( \Re(\beta) \leq 1/\alpha \)). A further constraint on \( \beta \) arises from energetic considerations. The energy of a polytropic gas is

\[
E = \int \rho [v^2 + \frac{c^2}{\gamma(\gamma - 1)}] dV. \tag{29}
\]

The lowest order contribution (in the perturbation parameter \( \varepsilon \)) to the energy stored in the perturbed-shock is the second order expression

\[
\delta E_l(t) = \int \delta \rho [v \delta v_n + \frac{\delta c^2}{\gamma(\gamma - 1)}] 4\pi r^2 dr, \tag{30}
\]

or after some algebra

\[
\delta E_l(t) = 4\pi \alpha^2 \varepsilon^2 \rho_0 R_0^5 \frac{\ell_{2\alpha\beta + 5\alpha - 2}}{\ell_0^{(2\beta + 5)}} C_l, \tag{31}
\]

where

\[
C_l = \int_1^{\xi_c} G(\xi) [\Phi(\xi) + \Pi(\xi)][V(1 - V) \Phi + \frac{Z}{\gamma(\gamma - 1)} (\gamma \sigma(\xi) + (\gamma - 1)(\Phi(\xi) + \Pi(\xi))] \xi^4 d\xi \tag{32}
\]

and \( R_0 \) stands for the radius of the shock by the time it is first formed \( t_0 \). Note that for \( \xi > > 1 \), \( \Phi(\xi) + \Pi(\xi) \sim V^{-\alpha \beta} \sim \xi^{\beta}; G(\xi) \sim \text{const} \); the integral diverges as \( \xi^{5 + 2\beta} \), vindicating the introduction of the cut off \( \xi_c \), which represents the boundary of the self-similarity solution. Clearly, this energy has to remain finite at any time and at focusing it requires that \( 1/\alpha - 2.5 \leq \Re(\beta) \leq 0 \). In the appendix we develop a semi-analytical method for solving eq.\(21\) and obtaining the correspondent spectrum for \( \beta_{1,2} \).

In consonance with previous numerical calculations (\[21\]) we confirm that \( \beta \) lies in this interval. By the way, the most unstable modes are shown to lie in the interval \( 1 + 1/\alpha < \Re(\beta) < -2.5 + 3/(2\alpha) \), even for very large values of \( l \). For these modes, the energy emission rate

\[
P_1(t) = 4\pi (2\alpha + 5\alpha - 2) \alpha^2 \varepsilon^2 \rho_0 R_0^5 \frac{\ell_{2\alpha\beta + 5\alpha - 3}}{\ell_0^{(2\beta + 5)}} C_l \tag{33}
\]

diverges. This means that, in analogy with the corrugation instabilities in planar shocks, a burst of sound is emitted at the focusing. The total energy carried away during the shock-collapse is

\[
E_{\text{sound}} = \sum_{l=1}^\infty \delta E_l(t_0) = \frac{4\pi \alpha^2 \varepsilon^2 \rho_0 R_0^5}{\ell_0^{(2\beta + 5)}} C \tag{34}
\]

where we defined \( C = \Re[\sum_{l=1,2} C_l(\beta)] \).
2 Inertial Polarization At Work

As discussed already, electromagnetic bounded systems whose constituents have sizeable mass differences, say \( \Delta M \), and which are subjected to a strong acceleration field \( \frac{d\vec{v}}{dt} \) engender polarization fields \( \vec{E}_0, \vec{B}_0 \) that tend to restore the balance between electromagnetic and inertial forces. Clearly, these polarization fields satisfy

\[
\Delta M \frac{d\vec{v}}{dt} = Ze \left( \vec{E}_0 + \frac{\vec{v}}{c} \times \vec{B}_0 \right)
\]

(35)

where \( A \) and \( Z \) correspond to the atomic and proton numbers and \( e \) is the electronic charge. Clearly, \( \Delta M \approx AM_p \), where \( M_p \) is the proton mass. Defining a polarized potential-vector \((\Phi_0, \vec{A}_0)\) in the usual way, allows us to write the balance equation in the form

\[
\left[ \frac{\partial \vec{v}}{\partial t} - \vec{v} \times (\nabla \times \vec{v}) + \nabla v^2 \right] = -\frac{Ze}{AM_p c} \left\{ \frac{\partial \vec{A}_0}{\partial t} - \vec{v} \times (\nabla \times \vec{A}_0) + (c\Phi_0) \right\}
\]

(36)

that suggests the identification \( \vec{A}_0 \rightarrow -\frac{AM_p c}{Ze} \vec{v} \) and \( \Phi_0 \rightarrow -\frac{AM_p Ze}{2} v^2 \). Other possible identifications exist, but they are gauge equivalent. The corresponding polarization fields are

\[
\vec{E}_0 = \frac{AM_p}{Ze} \left[ \frac{\partial \vec{v}}{\partial t} + \nabla \frac{v^2}{2} \right] ; \quad \vec{B}_0 = -\frac{AM_p c}{Ze} \nabla \times \vec{v}
\]

(37)

The time varying inertial-polarization fields engender the radiation fields \( \vec{E}, \vec{B} \) and their superposition must satisfy the sourceless Maxwell’s equations:

\[
\begin{align*}
\nabla \cdot (\vec{E} + \vec{E}_0) &= 0 \quad \rightarrow \quad \nabla \cdot \vec{E} = 4\pi \varrho_{eff} \\
\nabla \cdot (\vec{B} + \vec{B}_0) &= 0 \quad \rightarrow \quad \nabla \cdot \vec{B} = 0 \\
\n\nabla \times (\vec{E} + \vec{E}_0) + \frac{1}{c} \frac{\partial}{\partial t} (\vec{B} + \vec{B}_0) &= 0 \quad \rightarrow \quad \nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0 \\
\n\nabla \times (\vec{B} + \vec{B}_0) - \frac{1}{c} \frac{\partial}{\partial t} (\vec{E} + \vec{E}_0) &= 0 \quad \rightarrow \quad \nabla \times \vec{B} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c} (\vec{J}_{eff} + \vec{J}_{eff})
\end{align*}
\]

(38)

with

\[
\varrho_{eff} = -\frac{AM_p}{4\pi Ze} \left[ \frac{\partial \nabla \cdot \vec{v}}{\partial t} + \nabla^2 \frac{v^2}{2} \right]
\]

\[
\vec{J}_{eff} = -\frac{AM_p c^2}{4\pi Ze} \left( \frac{\partial^2 \vec{v}}{\partial t^2} + \frac{1}{2} \frac{\partial}{\partial t} (\nabla v^2) \right)
\]

clearly satisfying the conservation equation \( \partial \varrho_{eff}/\partial t + \nabla \cdot \vec{J}_{eff} = 0 \) and

\[
\vec{J} = \frac{AM_p c^2}{4\pi e} \nabla \times \nabla \times \vec{v}
\]

For non-relativistic flows \( |j^\mu|/|J^\mu| \sim (L/T)^2/c^2 \sim v^2/c^2 \), and the field equations reduce to

\[
\begin{align*}
\nabla \cdot \vec{E} &= 0 \\
\nabla \cdot \vec{B} &= 0 \\
\n\nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} &= 0 \\
\n\nabla \times \vec{B} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} &= \eta \nabla \times \nabla \times \vec{v}
\end{align*}
\]

(39)
where $\eta = AM_e/cZe$. Next we expand \( \left( \frac{\vec{E}}{\vec{B}} \right) = \sum_{n=1}^{3} \left( \frac{E_n}{B_n} \right) \vec{e}_n \) where $\vec{e}_n$ is the familiar vector basis [16]:

$$\mathcal{E} = (\vec{e}_1, \vec{e}_2, \vec{e}_3) = \left( \vec{n}Y_{lm}(\theta, \phi), (r \vec{\nabla})Y_{lm}(\theta, \phi), \ (r \vec{\nabla})Y_{lm}(\theta, \phi) \right)$$

(40)

For latter reference we mention the following identities:

$$\vec{n} \cdot \mathcal{E} = \frac{Y_{lm}}{r}(2, -l(l+1), 0); \vec{n} \times \mathcal{E} = \frac{1}{r}(-\vec{e}_3, \vec{e}_3, -\vec{e}_2 - l(l+1)\vec{e}_1)$$

(41)

The unperturbed flow is rotation free and the leading contribution to Maxwell’s equations [eq. (39)] comes from the perturbed flow $\delta \vec{v} = \delta v_n \vec{e}_1 + \delta v_\perp \vec{e}_2$.

$$\frac{\partial (r^2B_1)}{\partial r} - \frac{l(l+1)B_2r}{l} = 0$$

(42)

$$\frac{\partial (r^2E_1)}{\partial r} - \frac{l(l+1)E_2r}{l} = 0$$

(43)

$$\frac{r}{c} \dot{B}_1 - \frac{l(l+1)E_3}{l} = 0$$

(44)

$$\frac{r}{c} \dot{B}_2 - \frac{\partial (rE_3)}{\partial r} = 0$$

(45)

$$\frac{r}{c} \dot{B}_3 - \frac{E_1}{l} + \frac{\partial (rE_2)}{\partial r} = 0$$

(46)

$$\frac{r}{c} \dot{E}_1 + \frac{l(l+1)B_3}{l} = \eta l(l+1)f(r,t)$$

(47)

$$\frac{r}{c} \dot{E}_2 + \frac{\partial (rB_3)}{\partial r} = \eta \frac{\partial (r f)}{\partial r}$$

(48)

$$\frac{r}{c} \dot{E}_3 - B_1 + \frac{\partial (rB_2)}{\partial r} = 0$$

(49)

where $f(r,t) = \frac{\partial \delta v_\perp}{\partial r} + \frac{\delta v_n - \delta v_n}{r}$. Notice that $B_2, B_1$ and $E_3$ are independent of the source term, and are taken to vanish identically. The other mode is

$$\vec{E} = E_1 \vec{e}_1 + E_2 \vec{e}_2; \vec{B} = B_3 \vec{e}_3.$$  

(50)

Averaging the Poynting vector

$$\vec{S} = \frac{c}{8\pi} (\vec{E} \times \vec{B}^*) = \frac{cB_3^*}{8\pi} \left[ -rE_1(\vec{\nabla} Y^*) + r^2E_2(\vec{\nabla} \cdot \vec{\nabla} Y^*) \vec{n} \right],$$

(51)

over all directions gives the radial energy flux

$$S_r = \frac{cl(l+1)}{8\pi} \Re (E_2B_3^*).$$

(52)

The corresponding spectral intensity is

$$I_l(\omega) = \frac{cl^2l(l+1)}{2} \left| E_2(\omega)B_3^*(\omega) \right|$$

(53)

We obtain the wave equation for $\Lambda \equiv E_1(\omega)r$ by combining eqs. (42)-(49)

$$\left( \nabla_r^2 + k^2 \right) \Lambda(\omega) = ik\eta(l+1)f(\omega, r)$$

(54)
and in terms of \( \Lambda \), the spectral intensity reads

\[
I_\ell(\omega) = \frac{\omega}{2\ell(l+1)} |r\Lambda(\omega)\frac{\partial (r\Lambda^*(\omega))}{\partial r}|^2
\]

The wave equation is solved through the Green’s function method in the region away from the near zone:

\[
\Lambda(\omega) = \frac{\omega}{2\ell(l+1)} \int [-ik\eta l(l+1)f(\omega, r\ell)] j_j(kr\ell)r^2 dr. \]

In the radiation zone, \( \Lambda(\omega) \) reduces to:

\[
\Lambda(\omega) \approx -e^{ikr}(-i)^l+1k\eta l(l+1) \int f(\omega, r\ell)j_j(kr\ell)r^2 dr.
\]

Putting these pieces together,

\[
I_\ell(\omega) = \frac{1}{2} c^2 \eta^2 k^4 l(l+1) |A_\ell(k)|^2
\]

with

\[
A_\ell(k) = \int \int f(r, t)e^{-i\omega t} j_j(kr\ell)r^2 drdt
\]

The function \( f(r, t) \) can be expressed in terms of the fluctuation functions [eqs. (14)],

\[
f(r, t) = \frac{\alpha \xi}{t_0} \left( \frac{t}{t_0} \right)^{\alpha \beta - 1} \frac{\xi r'(\xi) + 2\tau(\xi) - (1 - V(\xi))\Phi(\xi)}{\xi \tau'(\xi) + 2\tau(\xi) - (1 - V(\xi))\Phi(\xi)}.
\]

Calling \( x = kr \) and performing a change of integration variables we obtain radiation emission rate per wave-length \( \lambda \):

\[
I_\ell(\lambda) = \frac{P_\ell}{\lambda} c^2 \alpha^2 l(l+1) |W_\ell(k)|^2
\]

with

\[
W_\ell(k) = \int_0^\infty j_j(x) \ x^2 dx \int_0^1 [\xi \tau'(\xi) + 2\tau(\xi) - (1 - V(\xi))\Phi]y^{\alpha \beta - 1} \ exp[-iQy]dy,
\]

where \( Q \equiv kR_0(t_0/R_0c) \) and \( p \equiv c^4 \eta^2/2 \). According to Barber ([14]) the ratio \( \alpha R_0/t_0 = c_0 \), the speed of sound, and \( Q = \alpha kR_0(c_0/c) \sim 10^{-5}(kR_0) \). The asymptotic behavior given by eq. (28) and the fact that \( V \propto \xi^{-1/\alpha} \) suggests the expansion:

\[
[\xi \tau' + 2\tau - (1 - V)\Phi] = \sum_{n=1} b_n \xi^{\beta-n/\alpha} = \sum_{n=1} b_n \left( \frac{x}{kR_0} \right)^{\beta-n/\alpha} y^{n-\alpha \beta}
\]

where the coefficients \( b_n \) are determined by the dynamics of perturbations. Note that the sum does not contain the \( n = 0 \) term because the leading term of the series [see again eq. (28)] for the velocity components \( \Phi, \tau \) is \( V^{1-\alpha \beta} \). Therefore,

\[
W_\ell(k) = \sum_{n=1} b_n (kR_0)^{n/\alpha - \beta} \int_0^1 y^{n-1} \ exp[-iQy]dy \int_0^{kR_{\text{max}}} j_j(x) \ x^{2+\beta-n/\alpha} dx.
\]

The cutoff \( kR_{\text{max}} \) in the \( x \)-integral was introduced because the shock does not extend beyond \( R_{\text{max}} \), the ambient radius of the bubble. For \( Q \ll 1 \) we might transform this expression into

\[
W_\ell(k) = (kR_0)^{-\beta} \sum_{n=1} \frac{b_n}{n} \left[ (kR_0)^{n/\alpha} \int_0^{kR_{\text{max}}} j_j(x) \ x^{2+\beta-n/\alpha} dx + \int_0^{kR_{\text{max}}} j_j(x) \ x^{2+\beta} dx \right]
\]

The detailed form of the spectrum requires a full knowledge of \( b_n \), that is to say, dynamics of the fluctuations must be specified (this can be done analytically by using the method developed in the appendix).
Fortunately, the major features of the spectrum can be obtained without delving into the differential equations. For instance, in the region where \( kR_{\text{max}} < 1 \) we can approximate \( j_\ell(x) \approx (2\ell)!/(2\ell + 1)! \) and then

\[
W_\ell(k) \approx \frac{2\ell!}{(2\ell + 1)!} (kR_0)^{\ell+3} \sum_{n=1}^{\beta} b_n \left\{ \frac{1}{l + 3 + \beta - n/\alpha} \left[ \left( \frac{R_{\text{max}}}{R_0} \right)^{\ell+3+\beta-n/\alpha} - 1 \right] - \frac{1}{l + 3 + \beta} \left( \frac{R_{\text{max}}}{R_0} \right)^{\ell+\beta+3} \right\}
\]

In the other end of the spectrum \( kR_0 > 1 \), taking the asymptotic expression \( j_\ell(x) \approx 1/x \sin(x-l\pi/2) \) is justified, either because in the first integral the integration variable \( x > 1 \) or because in the second integral the measure \( x^{2+\beta} \) (with \( 2 + \beta > 1 \)) ensures that important contributions to the integral comes from the large arguments. Thus,

\[
W_\ell(k) \approx (kR_0) \left( \frac{R_{\text{max}}}{R_0} \right)^{\beta+1} \sum_{n=1}^{\beta} b_n \left\{ f(\beta; kR_{\text{max}}) + \left( \frac{R_0}{R_{\text{max}}} \right)^{n/\alpha} f(\beta - n/\alpha; kR_{\text{max}}) - f(\beta - n/\alpha; kR_0) \right\}
\]

where

\[
f(\beta; x) = \text{Im} \left[ e^{-i\pi/2} \sum_{m=0}^{\infty} \frac{(ix)^{m+1}}{(m+\beta+2-n/\alpha)m!} \right].
\]

The dominant power low contribution to \( W_\ell(k) \) in the region \( kR_0 > 1 \) comes from the linear term \( (kR_0) \) because the series \( f(\beta; x) \) behaves nearly like \( \sin(x) \), for \( x > 1 \). Taking the following figures \( R_{\text{max}} \sim 5\mu \text{m} \), the ambient radius of the bubble and \( R_0 \sim 0.15\mu \text{m} \), we shall explain in a moment) and defining \( \lambda_0 = 2\pi R_0 \), we display our asymptotic expressions in the form

\[
P_\ell(\lambda) \sim p e^{2} \left\{ A_\ell \lambda^{-1} (\lambda_0/\lambda)^{2\ell+6} : \lambda \gg \lambda_0 \right\}
\]

where

\[
A_\ell = \alpha \frac{2\ell!}{(2l + 1)!} \sum_{n=1}^{\beta} b_n \left\{ \frac{1}{l + 3 + \beta - n/\alpha} \left[ \left( \frac{R_{\text{max}}}{R_0} \right)^{\ell+3+\beta-n/\alpha} - 1 \right] - \frac{1}{l + 3 + \beta} \left( \frac{R_{\text{max}}}{R_0} \right)^{\ell+\beta+3} \right\}
\]

and

\[
g(\lambda) = l(l+1) \left\{ \alpha \left( \frac{R_{\text{max}}}{R_0} \right)^{\beta+1} h_\ell(k) \right\}
\]

with

\[
h_\ell(k) = \sum_{n=1}^{\beta} b_n \left\{ f(\beta; kR_{\text{max}}) + \left( \frac{R_0}{R_{\text{max}}} \right)^{n/\alpha} f(\beta - n/\alpha; kR_{\text{max}}) - f(\beta - n/\alpha; kR_0) \right\}
\]

The apparent divergence of \( g_\ell(\lambda) \) at large angular momenta [see eq. (72)] seems to endanger the present results. This worry is removed studying the asymptotic behavior \( g_\ell(\lambda) \), bearing in mind that in this limit \( \beta \approx \pm i\sqrt{\gamma - 1}/(\gamma + 1) \). This yields that \( g_\ell(\lambda) \to 0 \) as \( l \to \infty \), regardless of the specific form of the dynamical coefficients \( b_n \) may take.

### 3 Assessment of the Results

The present SL mechanism relies on very basic pieces of physics, the existence of corrugation instabilities in spherical shocks, whose existence is well known, Maxwell’s equations and the inertial polarization
paradigm. As we had the opportunity to explain, this paradigm stems from very elementary physics and it has remained hitherto unnoticed only because huge accelerations are required for sizeable polarizations. The detection of shock polarization in non-polar liquids would lend an undisputable status to the inertial polarization principle. In the transduction of sound into radiation, the flash of light must be coincident with a burst of sound since the emission of radiation is caused by corrugation instabilities. According to eq.(50), only one field-mode is related to the sonoluminescent light. This mode has a longitudinal electric field component $E_1$, and some experiment must be devised to detect it. The transversal component $E_2$ points into the direction of the vector 

$$\mathbf{e}_2 = \sqrt{\frac{2l + 1}{4\pi (l + m)!}} e^{ilm} \sin(\theta) (imP^n_1(cos(\theta)) \mathbf{e}_\varphi - P^m_1(cos(\theta)) \mathbf{e}_\theta)$$

and this (weird) polarization should be observed in sonoluminescent light.

Physics is seldom controlled by cut-off parameters, and we expect the cut-off parameter $R_{max}$ (the bubble’s ambient radius) to play a marginal role in defining the frequency band where light is emitted. The main features of the spectrum should be controlled by the remaining parameters: $R_0$, the radius of the shock-wave when it is first formed and the perturbation amplitude $\varepsilon$. Thus, $R_0$ should characterize the typical wave-length of the emitted light $\lambda \approx \lambda_0 = 2\pi R_0$. Our asymptotic results [eq.(63)] confirm this feeling. Numerical and theoretical studies of the dynamics of imploding shocks support the picture that the bubble collapses at the speed of sound by the time it passes through its ambient radius as the right criterion both for shock formation and the existence of SL ([13]-[17]). According to these investigations, at 100μs before the bubble reaches its minimum size, a shock wave of initial radius $R_0 = 0.15\mu m$ develops: by this time the interface is imploding with 4 to 5 times the ambient speed of sound. With these figures, we predict the emitted light to lie in $\lambda \approx \lambda_0 = 900nm$ spectral region, regardless the kind of gas present in the bubble; in SL experiments light is observed in the $200nm \lesssim \lambda \lesssim 800nm$ interval. According to this result, it is legitimate to infer the spectrum in this wave-length interval through the asymptotic formula for $\lambda \lesssim \lambda_0$[see eq.(69)]. How does the particular kind of gas present in the bubble impact on the emitted power? The dependence of the emitted light upon the particular type of gas present in the bubble stems from two different factors:

i.) different values of the adiabatic index $\gamma$ leads to a different shock-wave and corrugation instability dynamics; ii.) different gases have different dielectric permeability $\varepsilon$.

The dielectric nature of the gas is implemented through the replacement $E \rightarrow D$ in the Poynting vector, which corresponds to the replacement of $|W_i(k)|^2$ by $\bar{\varepsilon}(k) |W_i(k)|^2$, or $g_i(\lambda) \rightarrow \bar{\varepsilon}(k) g_i(\lambda)$. Different adiabatic indexes would cause $h_i(k)$ to change because both the spectrum of $\beta$ and the dynamical coefficients $b_n$ depend upon $\gamma$. These two conditions will cause a change on the shape of the function $g_i(\lambda)$.

Assuming that after taking these corrections into account, the function $g_i(\lambda)$ still remains marginally dependent upon the wave-length (non power law), the overall change produced by different gases in the shape on the logarithmic representation of the spectrum $\ln P \sim -3\ln \lambda + \ln g_i(\lambda) + const$ for $\lambda \lesssim \lambda_0$, is a displacement of the nearly parallel lines of inclination $m \approx -3$. This behaviour is changed as we approach the $\lambda \ll \lambda_0$ region because then the dielectric constant being governed by the plasma frequency of the gas, causes the function $\ln g_i(\lambda)$ to strongly depend upon $\lambda$.

Inferring the uncorrected spectra for transmission by the surrounding medium observed by Hiller in SL experiments for bubbles trapping pure noble gases bubbles at $0^\circ C$ ([15]) we inferred $m \approx -2.7$. For pure He, $m \approx -2.5$. Inspection of the spectra shows the nearly linear dependence for $Ar, He, He^3$, and $Ne$. The agreement is less accurate for $Xe$ and $Kr$, for reasons which are presently unclear: it might well be that heavier noble gases cannot be handled with the naive classical Inertial Polarization picture, they have too much internal structure and must be handled with a full quantum mechanical approach. The spectrum for a mixture of 1% of $He$ and $N_2$ closely resemble the behavior of pure $He_2$([15]). Differences might be credited to the superimposition of the Bremsstrahlung spectrum of free electrons of the ionized
$N_2$ gas in the mixture to the original spectrum, or even the effect of the Inertial-Polarization fields upon these electrons.

Regarding now the intensity of the outgoing radiation, it is governed by the product $p e^2$. A small $p$ would require large corrugation instabilities, invalidating the linear regime approximations. Surprisingly, $p = E_2^2/(2\hbar\alpha) \simeq 1.47 \times 10^{16}\text{Watt}$, imploding shocks are fantastic power stations! Actually, we have to worry to have sufficiently small perturbations to fit the experimental data! Typical power emissions are of the order of $10^{-11}\text{Watt}/\text{nm}$ in the $\lambda_0$ region [8], calling for an amplitude $\varepsilon \sim 10^{-12}$ or $\delta r = \varepsilon R_0 \sim 10^{-19}m$, which being much smaller then the nuclear dimensions can have only a quantum mechanical origin. Now, the radius of the shock at the moment it is formed $R_0$ is governed by the radius of the bubble wall $R_b$, by the time it is collapsing at 4-5 times the ambient speed of sound. The dependence of the former on the latter is linear. In a semi-classical approach, it is to be expected that the fluctuations on the shape of the imploding shock are also governed by bubble wall fluctuations, $\epsilon = \delta R_0/R_0 = \delta R_b/R_b$.

The fluctuations of the bubble interface should be of the order of $10^{-19}$ nm, calling for an amplitude $\varepsilon \sim 10^{-12}$ or $\delta r = \varepsilon R_0 \sim 10^{-19}m$, which being much smaller then the nuclear dimensions can have only a quantum mechanical origin. Now, the radius of the shock at the moment it is formed $R_0$ is governed by the radius of the bubble wall $R_b$, by the time it is collapsing at 4-5 times the ambient speed of sound. The dependence of the former on the latter is linear. In a semi-classical approach, it is to be expected that the fluctuations on the shape of the imploding shock are also governed by bubble wall fluctuations, $\epsilon = \delta R_0/R_0 = \delta R_b/R_b$.

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Appendix – A semi-analytical solution of the differential equations for the perturbed flow

In order to solve the set of differential equations we split the matrix into its regular and divergent parts

$$
\mathcal{M}(V) = \frac{A}{(V-V_c)} + \mathcal{B}(V)
$$

with

$$
A = \frac{m(V_c)}{(dP/dV)_V} \begin{pmatrix}
\phi_1 & \phi_2 & \phi_3 & \phi_4 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} ;
\mathcal{B} = m(V) \begin{pmatrix}
\phi_1(V) & \phi_2(V) & \phi_3(V) & \phi_4(V) \\
Z & 2V - \frac{1}{4} & Z & Z \\
3 + \beta & -l(l+1) & 0 & 0 \\
-\kappa & 0 & 0 & 0
\end{pmatrix}
$$

where $\phi_\alpha$ is a short notation for $\phi_\alpha(V_c)$ and $\phi_\alpha(V) = \phi_\alpha(V)/P(V) - m(V_c)/(m(V)(dP/dV)_V(V - V_c))$. Assuming $B(V)$ and $|Y(V)|$ regular functions at the critical point $V_c$ permits the expansions $B(V) = \sum_n B_n (V - V_c)^n$; $|Y(V)| = \sum_k Y_k (V - V_c)^k$. Substitution into the differential equation yields the recurrence formulae:
\[ \mathcal{A} Y_0 = 0 \]  
\[ Y_{n+1} = [(n + 1) I - \mathcal{A}]^{-1} \sum_{m \leq n} B_{n-m} Y_m \]  

The matrix \( \mathcal{A} \) possess three distinct null-eigenvectors:

\[ Y_0^{(2)} = (-\phi_2, \phi_1, 0, 0) \]
\[ Y_0^{(3)} = (-\phi_3, 0, \phi_1, 0) \]
\[ Y_0^{(4)} = (-\phi_4, 0, 0, \phi_1). \]  

Associated to each one of these eigenvectors we can construct through the recurrence relations \( Y^0(V) \). The solution of the differential equation is the linear combination \( |Y(V)\rangle = \sum_{i=2,4} c_i Y^{(i)}(V)\rangle \). The fulfillment of the boundary requires that

\[ |X(V_i)\rangle = |Y(V_i)\rangle = \sum_k [c_2 Y_k^{(2)} + c_3 Y_k^{(3)} + c_4 Y_k^{(4)}](V_1 - V_c)^k. \]  

This equation constitutes a set of four equations for the unknown \((c_i, \beta)\), which can be solved in a perturbational approach in powers \((V_1 - V_c)\), once the state \(|X(V_i)\rangle\) is known. The only missing piece of information is the set of boundary conditions for the perturbed fields.

The boundary conditions for the perturbed flow

Supersonic motion produces a discontinuity in the fluid flow known as a shock wave or simply shock. Let us call \( \vec{v}_2 \) and \( \rho_2 \) the fluid velocity and density, and \( c_2 \) the speed of sound behind the shock, as measured in the laboratory frame (likewise, the subscript 1 refers to the same quantities in the front of the shock). The normal to the shock is \( \vec{n} \) and its velocity in the lab frame is \( \vec{v}_{\text{shock}} \). The discontinuities have to fulfill the following conditions at the shock surface \([3]\)

\[ \vec{n} \times [\vec{v}_1 - \vec{v}_{\text{shock}}] = \vec{n} \times [\vec{v}_2 - \vec{v}_{\text{shock}}] \]  
\[ \frac{\vec{n} \cdot [\vec{v}_2 - \vec{v}_{\text{shock}}]}{\vec{n} \cdot [\vec{v}_1 - \vec{v}_{\text{shock}}]} = \frac{\rho_1}{\rho_2} = \frac{\gamma - 1}{\gamma + 1} \]  
\[ c_2^2 = \frac{\gamma - 1}{\gamma + 1} \left[ c_1^2 + \frac{2\gamma}{\gamma + 1} (\vec{n} \cdot \vec{v}_{\text{shock}})^2 \right] \]  

In the perturbed flow the shock front is displaced from \( \Sigma_0 : r - R(t) = 0 \) to \((\Sigma_0 + \delta \Sigma) : r - R(t) = \delta r(t, \theta, \phi) = 0 \). The corresponding perturbed normal is \( \delta \vec{n} = -\vec{n} \delta r \), the perturbed shock velocity is \( \delta \vec{v} \) while the location of the shock itself in self-similar coordinate is \( 1 + \delta \xi, \delta \xi = \delta r/R(t) \). Accordingly,

\[ \delta \xi = \varepsilon \left( \frac{t}{t_0} \right)^{\alpha \beta} Y_{lm}(\theta, \phi) \]
\[ \delta r = \varepsilon R(t) \left( \frac{t}{t_0} \right)^{\alpha \beta} Y_{lm}(\theta, \phi) \]
\[ \delta \vec{n} = -\varepsilon \left( \frac{t}{t_0} \right)^{\alpha \beta} (R(t) \vec{n}) Y_{lm}(\theta, \phi) \]
\[ \delta \vec{v}_s = \varepsilon \alpha (1 + \beta) R(t) \left( \frac{t}{t_0} \right)^{\alpha \beta - 1} Y_{lm}(\theta, \phi) \vec{n} \]  

13
The first order corrections to the boundary conditions [eqs. (79)-(81)] are:

\[
\vec{n} \times \left[ \left( \frac{\partial v}{\partial \xi} + \delta v - \delta v_s \right) \vec{n} + \delta \vec{v}_\perp \right] + \delta \vec{n} \times \vec{n} \left( v - v_s \right) = -\vec{n} \times \delta \vec{v}_s - \delta \vec{v}_s \\
\delta \rho_2(1) + \frac{\partial \rho_2}{\partial \xi} \delta \xi = 0 \tag{83}
\]

\[
\vec{n} \cdot \left[ \left( \frac{\partial v}{\partial \xi} + \delta v - \delta v_s \right) \vec{n} + \delta \vec{v}_\perp \right] + \delta \vec{n} \cdot \vec{n} \left( v - v_s \right) = -\frac{\gamma - 1}{\gamma + 1} (\delta v_s + \vec{n} \cdot \delta \vec{v}_s) \\
\delta c^2 + \frac{\partial c^2}{\partial \xi} \delta \xi = 2Z(1) v_s (\delta \vec{n} \cdot \vec{v}_s + \vec{n} \cdot \delta \vec{v}_s) \tag{84}
\]

Inserting eqs.(82)-(16) into these boundary conditions, yields

\[
\Phi_1 = \frac{\beta V_1 - V'_1}{1 - V_1} ; \\
\tau_1 = -V_1 ; \\
\Delta_1 = \frac{G'_1}{G_1} = \frac{\delta Z_1}{Z_1} = (2\beta - \frac{Z'_1}{Z_1}) \tag{85}
\]

or, equivalently

\[
|X(V_1)) = \varepsilon \begin{pmatrix}
\frac{(\beta V_1 - V'_1)/(1 - V_1)}{1 - V_1} \\
-(\beta + 3)V_1/(1 - V_1) \\
2(\beta + (1/\alpha - V_1)/(1 - V_1))/\gamma
\end{pmatrix}
\]

**Numerical Procedure**

Our procedure for resolving the spectrum of $\beta$ consists of first fitting the unperturbed flow $(Z(V), dV(V))$ by a polynomial in $V$, from which we extract the matrices $A$ and $B(V)$ as power series in $V$. Then through the recurrence formulae (8?) we obtain the expansion coefficients $Y_n(\beta)$ up to a given order and insert then into eq.(78), in conjunction with the above boundary condition $|X(V_1))$ [eq.(82)] . This procedure yields a polynomial equation for $\beta$, which is solved numerically. We display the results for $\gamma = 7/5, l = 1, 2, 3, 4$.

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