TOWARDS THE UNIFICATION OF FUNDAMENTAL INTERACTIONS

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Abstract

Recent observations of distant supernovae show that the universe, contrary to popular belief, is expanding for ever. Similarly, recent experiments at the Superkamiokande facility demonstrate that the Neutrino has a non-vanishing mass. Both these discoveries necessitate, at the very least, a re-examination of conventional theories—the Big Bang Theory and the Standard Model. On the other hand, neither has quantum gravity yielded the desired results, nor can String Theory be taken as the last word.

We briefly examine this scenario, and in this light consider the recent description of Fermions in terms of the Kerr-Newman metric and the related model of Fluctuational Cosmology. All this is not only consistent with known physics including the latest results alluded to, but also explains hitherto inexplicable empirical facts like the handedness of the Neutrino or the large number relations of Cosmology and so on. It is shown that this scheme leads to a unified description of the fundamental interactions. At the same time, pleasingly, we recover the usual quark picture at the Compton wavelength scale and the Big Bang scenario at the Planck scale.

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1 Introduction

"....the aim is to see complete nature as different aspects of one set of phenomena...". This is a quotation not from the Up-anishads but from the celebrated twentieth century Physicist Richard P. Feynman[1]. This has been the goal pursued over the millennia in man’s quest for an understanding of the universe. Today looking back we can see the logic of Occam’s razor or an economy of hypothesis– a far cry from the times of pre history when one God was designated for each phenomenon of nature.

In the words of F.J. Dyson[2], ".... the very greatest scientists in each discipline are unifiers. This is especially true in Physics. Newton and Einstein were supreme as unifiers. The great triumphs of Physics have been triumphs of unification. We almost take it for granted that the road of progress in Physics will be a wider and wider unification...".

However at this time when we seem to be on the threshold of the Theory of Everything (TOE), there are a few cautionary tales which I would like to recount, and whose significance we will encounter shortly.

The first is a sense of perfection and aesthetics, so much a part of scientific philosophy from the Greeks, for example Plato to twentieth century Physicists, for example Dirac. Thus to the Greeks the circle or sphere were perfect, because of their all round symmetry and so all planetary orbits had to be circular. The very word orbit is derived from "orb" which is Greek for circle. This lead to the very complicated epicyclic model of the Greeks which dominated science for nearly 2000 years, till almost the 17th century. As observations improved and revealed
discrepancies in the Greek model, more and more epicycles were added to bring theory in tune with observation. It was only when Kepler around 1608 finally introduced his elliptical orbits, that the Greek umbilical cord had finally been cut and modern science emerged. Clearly the Greeks were trying to approximate a single elliptical motion by a series of complicated circular motions.

The second incident also involves Kepler who thought that he had found the answer to the question, ”why are there only five planets besides the earth?” Undoubtedly the creator had used the five perfect polyhedra as celestial scaffolding, in building up the universe. This idea was not just attractive, but to Kepler it was breathtaking.

The third story comes from India and is recounted in the famous English poem, ”The Five Blind Men of Hindustan”. Each of the blind men felt an elephant, and ofcourse immediately understood what the elephant was like: It was like a rope, it was like the trunk of a tree and so on.

Let us now come back to modern science. Sir Isaac Newton was the first great unifier. He discovered the Universal Law of Gravitation: The force which kept the moon going round the earth, or the earth round the sun and the force which kept binary stars going around each other and so on were all basically the same force of gravitation which brought apples down from a tree. This apart his Laws of Motion were also universal.

In the 19th Century the work of the likes of Faraday and Ampere show the close connection between the apparently totally dissimilar forces of electricity and magnetism. These studies culminated in the work of Maxwell who unified not just electricity and magnetism but optics as well.
Another great unification of the last Century was that of Thermodynamics and the kinetic theory of gases.[4]

In the early part of this Century Einstein fused space and time, giving them an inseparable identity. He went on to unify space-time with gravitation in his General Theory of Relativity. However he and subsequent scientists failed in what may be called Einstein’s IDEE FIXE, namely the unification of electromagnetism and gravitation.

Yet another unification in this century, which often is not recognised as such is the fusion of Quantum Mechanics and Special Relativity by Dirac, through his celebrated equation of the electron.

Some thirty years ago, another unification took place due to the work of Salam, Weinberg, Glashow and others – the unification of electromagnetism with the weak forces. This is the story to which we now come.

The weak force is one of two forces, the other being the strong force, discovered during this Century itself. Studies and work earlier this Century revealed that there seem to be three basic particles in the universe, the protons, the neutrons and the electrons.

While the proton and the electron interact via the electromagnetic force, in the absence of this force the proton and the neutron appear to be a pair or a doublet, because now the distinction between the electrically charged proton and the neutral neutron disappears. However the proton and the neutral neutron interact via ”strong forces”, forces which are stronger than the electromagnetic but have a much shorter range of just about $10^{-13} \text{cms}$.

The existence of the neutrino was postulated by Pauli in 1930 to
explain the decay of the neutron, and it was discovered by Reines and Cowan in 1955. The weak force is associated with neutrino type particles and has an even shorter range, $10^{-16}$ cms. The neutrino itself has turned out to be one of the most enigmatic of particles, with peculiar characteristics, the most important of which is its handedness. This handedness property appears to be crucial for weak forces.

Later work revealed that while particles like the electron and neutrino, namely the leptons may be truly elementary, other particles like protons and neutrons may be composite, infact made up of still smaller objects called quarks – six in all[5]. Today it is believed that the quarks interact via the strong forces. It must be mentioned that all these ”particles”, or loosely material particles are fermions, that is they have half integral spin. It was realized that the forces or interactions on the other hand are mediated by messengers like photons which are bosons, that is, have integral spin.

According to contemporary thinking, the unification of the various interactions relies on the fact that the messengers have spin one, that is are gauge forces, where as in this sense gravitation is not a gauge force as it is thought to be mediated by spin 2 particles.

To picturize the above let us consider the interaction between a proton and an electron.
A proton could be imagined to emit a photon which is then absorbed by the electron.
Instead of a single mediating particle we could think of multiplets, all having equal masses. With group theoretical inputs, one could shortlist, singlets with one particle like the photon triplets, octets and so on as possible candidates.
Motivated by the analogy of electromagnetism mediated by the spin one photon, it was realized in the 1950s that the $W^+$ and $W^-$ bosons could be possible candidates for the mediation of the weak force. However there had to be one more messenger so that there would be an allowable triplet alluded to earlier.
It was suggested by Ward and Salam that the third candidate could be the photon itself, which would then provide not only a description of the weak force but would also unify it with electromagnetism. However while the $W$ particles were massive, the photon was massless so that they could not form a triplet. So a heavy photon or $Z^0$ was postulated to make up a triplet, while the photon was also used for the purpose of unification, and moreover a mixing of $Z^0$ and the photon was required for what has been called renormalization, that is the removal of infinities.
The question was how could the photon be massless while the $W$ and $Z$ particles would be massive? It was suggested that this could be achieved through the spontaneous breaking of symmetry. For example a bar magnet when heated, looses its magnetism, that is it becomes symmetric because one cannot distinguish between the North and South poles. However as the magnet cools down, the magnetism or the polarity or asymmetry is regained spontaneously. In other words there would be a phase transition (from symmetry to asymmetry).
In our case, before the spontaneous breaking of symmetry or
the phase transition, the $W$s, $Z$s and the photon would all be massless. After the phase transition, while the photons remain massless, the others would acquire mass. This phase transition would occur at temperatures $\sim 10^{15}$° Centigrade. At higher temperatures there would be a single electroweak force. As the temperature falls to the above level electromagnetism and weak forces would separate out. Clearly the direction to proceed appeared to be to identify the gauge character of the strong force—mediated by spin one particles, the gluons. This force binds the different quarks to produce the different elementary particles, other than the leptons. This is the standard model. However we have not yet conclusively achieved a unification of the electroweak force and the strong force\[6\].

One proceeds by analogy with the electroweak unification to obtain a new gauge force that has been called by Jogesh Pati and Abdus Salam as the electro nuclear force, or in a similar scheme the Grand Unified Force by Glashow and Georgi. It must be mentioned that one of the predictions is that of the proton would decay with a long lifetime of about $10^{32}$ years, very much more than the age of the universe itself. However we are near a situation where this should be observable.

The recent super Kamiokande determination of neutrino mass is the first evidence of Physics beyond the standard model. Interestingly in this theory we would also require a right handed neutrino in this case.

Meanwhile extended particles had come into vogue from about twenty years with string theory. Starting off with objects of the size of the Compton wavelength, the theory of superstrings now deals with the Planck length of about $10^{-33} cms$. We have noted that all interactions except gravitation are generalizations
of the electromagnetic gauge theory. String theory combines Special Relativity, Quantum Mechanics and General Relativity - we need ten, \((9 + 1)\), dimensions for quantizing strings, and we also get a mass less particle of spin two which is the mediator of the gravitational force. This way there is the possibility of unifying all interactions including gravitation. Further, in the above ten dimensions there are no divergences – this is because the spatial extension of the string fudges the singularities. However, we require, for verification of the string model, energies \(\sim 10^{18} m_P\), as against the presently available \(10^3 m_P\).

Another interesting feature of string theory is duality. There are five different solutions (compactifications) leading to the same physical picture reminiscent of the ”Five Blind Men of Hindustan”. These five theories are but different descriptions of a single theory.

The exotic dimensions, the large number of solutions and the non verifiable nature of the theory are some of the unsatisfactory features of this development, just as the 18 arbitrary parameters[7], and the unseen monopoles are some of the unsatisfactory features of the standard model.

2 A New Model Towards Unification

In previous communications[8, 9, 10] a model for leptons as Kerr-Newman type Black Holes was developed, and it was pointed out that several hitherto inexplicable features turned out to have a natural explanation, for example the quantum of charge, the electromagnetic-gravitational interaction ratio, the handedness of the neutrino and so on. We briefly recapitulate some relevant
facts.
It must be mentioned that the possibility that a particle could be a Schwarzschild black hole has been examined earlier by Marko, Motz and others\cite{11, 12} and leads to a high particle mass of $10^{-5}gm$, the Planck mass without much insight into other properties.

So let us approach the problem from a different angle. We consider a charged Dirac (spin half) particle. If we treat this as a spinning black hole, there is an immediate problem: The horizon of the Kerr-Newman black hole becomes in this case, complex\cite{9},

$$r_+ = \frac{GM}{c^2} + ib, b \equiv \left( \frac{G^2 Q^2}{c^8} + a^2 - \frac{G^2 M^2}{c^4} \right)^{1/2} \tag{1}$$

where $G$ is the gravitational constant, $M$ the mass and $a \equiv L/Mc, L$ being the angular momentum. That is, we have a naked singularity apparently contradicting the cosmic censorship conjecture. However, in the Quantum Mechanical domain, (1) can be seen to be meaningful.

Infact, the position coordinate for a Dirac particle is given by Dirac\cite{13}

$$x = (c^2 p_1 H^{-1} t + a_1) + \frac{i}{2} c\hbar (\alpha_1 - c p_1 H^{-1}) H^{-1}, \tag{2}$$

and similar equations for $y$ and $z$ where $a_1$ is an arbitrary constant and $c\alpha_1$ is the velocity operator with eigen values $\pm c$. The real part in (2) is the usual position while the imaginary part arises from zitterbewegung. Interestingly, in both (1) and (2), the imaginary part is of the order of $\frac{\hbar}{mc}$, the Compton wavelength, and leads to an immediate identification of these two equations. We must remember that our physical measurements are gross as indeed is required by the Uncertainty Principle
- they are really measurements averaged over a width of the order \( \frac{\hbar}{mc} \). Similarly, time measurements are imprecise to the tune \( \sim \frac{\hbar}{mc^2} \). Very precise measurements if possible, would imply that all Dirac particles would have the velocity of light, or in the Quantum Field Theory at least of fermions, would lead to divergences. (This is closely related to the non-Hermiticity of position operators in relativistic theory as can be seen from equation (2) itself. Physics as pointed out begins after an averaging over the above unphysical space-time intervals. In the process (cf.ref. [9]), the imaginary or non-Hermitian part of the position operator in (2) disappears. That is the above naked singularity is fudged or shielded by a Quantum Mechanical censor, namely the minimum Compton scales.

We observe that if we adhoc treat a Dirac particle as a Kerr-Newman black hole of mass \( m \), charge \( e \) and spin \( \frac{\hbar}{2} \). The gravitational and electromagnetic fields at a distance are given by (cf.ref.[14]),

\[
\Phi(r) = -\frac{Gm}{r} + 0(\frac{1}{r^3}) \quad E^\hat{r} = \frac{e}{r^2} + 0(\frac{1}{r^3}) \quad E^\hat{\theta} = 0(\frac{1}{r^4}) \quad E^\hat{\phi} = 0 \quad B^\hat{r} = \frac{2ea}{r^3} \cos \theta + 0(\frac{1}{r^4}) \quad B^\hat{\theta} = \frac{easin \theta}{r^3} + 0(\frac{1}{r^4}) \quad B^\hat{\phi} = 0, \quad (3)
\]

which is correct. Infact, (3) also leads to the electron’s anomalous gyromagnetic ratio \( g = 2 \).

We next examine more closely, this identification of a Dirac particle with a Kerr-Newman black hole. We reverse the arguments after equation (2) which lead from the complex or non-Hermitian coordinate operators to Hermitian ones: We consider instead the displacement,

\[
x^\mu \rightarrow x^\mu + u^\mu \quad (4)
\]
and first consider the temporal part, \( t \rightarrow t + \imath a^0 \), where \( a^0 \approx \frac{\hbar}{2mc^2} \), as before. That is, we probe into the QMBH or the zitterbewegung region inside the Compton wavelength as suggested by (1) and (2). Remembering that \( |a^{\mu}| \ll 1 \), we have, for the wave function,

\[
\psi(t) \rightarrow \psi(t + \imath a^0) = \frac{a^0}{\hbar} [\imath \hbar \frac{\partial}{\partial t} + \frac{\hbar}{a^0}] \psi(t)
\]

As \( \imath \hbar \frac{\partial}{\partial t} \equiv p^0 \), the usual fourth component of the energy momentum operator, we identify, by comparison with the well known electromagnetism-momentum coupling, \( p^0 - e\Phi \), the usual electrostatic charge as,

\[
\Phi e = \frac{\hbar}{a^0} = mc^2
\]

In the case of the electron, we can verify that the equality (3) is satisfied:

We follow the classical picture of a particle as a rotating shell with velocity \( c \), as indeed is suggested by the Compton wavelength cut off and which will be further justified in the sequel. The electrostatic potential inside a spherical shell of radius 'a' is,

\[
\Phi = \frac{e}{a}
\]

As is well known, the balance of the centrifugal and Coulomb forces gives, for an electron orbiting another at the distance \( a \),

\[
a = \frac{e^2}{mc^2},
\]

which is the classical electron radius.

So, (3) now gives,

\[
e\Phi = mc^2,
\]
which is \((\text{3})\).

If we now use the usual value of \(a\) viz., \(2.8 \times 10^{-13} \text{cm.}\), while rewriting \(mc^2\) as \(\hbar c/(\hbar/mc)\) and substitute the value of the electron Compton wavelength, \(\frac{\hbar}{mc} = 3.8 \times 10^{-11} \text{cm.}\), we gets from the above

\[ \hbar c \approx 136e^2 \]

That is, we get the rationale for this fundamental relation, which no longer turns out to be accidental. In any case, we can now see the connection between the charge, mass and the velocity of light.

(It may be noted in passing that in the usual displacement operator theory \((\text{13})\) the operators like \(\frac{d}{dx}\) or \(\frac{d}{dt}\) are indeterminate to the extent of a purely imaginary additive constant which is adjusted against the Hermiticity of the operators concerned).

We next consider the spatial part of (4), viz.,

\[ \vec{x} \to \vec{x} + i\vec{a}, \text{where} |\vec{a}| = \frac{\hbar}{2mc}, \]

given the fact that the particle is now seen to have the charge \(e\) (and mass \(m\)). As is well known\([\text{15}]\), this leads in General Relativity from the static Kerr metric to the Kerr-Newman metric where the gravitational and electromagnetic field of the particle is given by \((\text{3})\), including the anomalous factor \(g = 2\). In General Relativity, the complex transformation (4) and the subsequent emergence of the Kerr-Newman metric has no clear explanation. Nor the fact that, as noted by Newman\([\text{16}]\) spin is the orbital angular momentum with an imaginary shift of origin. But in the Quantum Mechanical context, We can see the rationale: the origin of (4) lies in the QMBH.

More specifically, the temporal part of the transformation (4)
leads to the appearance of charge in (9). The space part then, as is known leads to the Kerr-Newman metric.

Ever since Einstein put forward his theory of Gravitation in 1915, a problem that has vexed physicists including Einstein himself is the incorporation of electromagnetism into the theory of gravitation. Einstein himself said it all in his Stafford Little Lectures delivered in May 1921 at Princeton University[17], ”... if we introduce the energy tensor of the electromagnetic field into the right hand side of (the gravitational field equation) we obtain (the first of Maxwell’s systems of equations in tensor density form), for the special case \((\sqrt{-g} \rho \frac{dx_\nu}{ds} = \tau^\mu = 0,...\). This inclusion of the theory of electricity in the scheme of General Relativity has been considered arbitrary and unsatisfactory.... a theory in which the gravitational field and the electromagnetic field do not enter as logically distinct structures would be much preferable...”

The very early attempt of Hermann Weyl to which we will return very briefly failed for this reason. As Einstein put it[17], ”H. Weyl, and recently Th.Kaluza, have put forward ingeneous ideas along with this direction; but concerning them, I am convinced that they do not bring us nearer to the true solution of the fundamental problem...”. Fruitless attempts over the years lead to Pauli’s famous remark that one should not put together what God had intended to be separate viz., electromagnetism and gravitation (cf. also ref.[18]).

The basic problem is that General Relativity belongs to the domain of classical physics whereas as Einstein observed in the above quotation, electromagnetism belongs to the domain of ”elementary electrically charged particles”, that is Quantum Theory, more specifically the theory of the electron. And, as J.A.
Wheeler\textsuperscript{[14]} put it, "the most evident shortcoming of the geometrodynamical model as it stands is this, that it fails to supply any completely natural place for spin $\frac{1}{2}$ in general and for the neutrino in particular", while "it is impossible to accept any description of elementary particles that does not have a place for spin half." This apart it should be remembered that the space time we speak of in general relativity is not only deterministic, but we also speak in terms of definite points of space time. This as seen above is forbidden in Quantum Theory by the Uncertainty Principle. Infact four dimensional space time exists only as a classical approximation \textsuperscript{[19, 20]}. On the other hand, Quantum Gravity wherein one invokes phenomena at the scale of the Planck length, that is $10^{-33} cms$, has not been satisfactorily concluded. The above Planck length considerations as is well known and briefly alluded to earlier, lead to what have been sometimes termed as, "maximons" particles which are essentially Schwarzchild black holes with the Planck length radius, and with mass $\sim 10^{-5} gm$. Such particles are too massive to be termed "elementary", are transient, with life times $\sim 10^{-42} sec$ and moreover they do not exhibit the all important spin.

However the following question is to be clarified: How can an electron described by the Quantum Mechanical Dirac spinor be identified with the geometrodynamical Kerr-Newman black hole characterized by curved space time? The answer is as follows: As we approach the Compton wavelength the "negative energy" two rowed component $\chi$ of the full four rowed Dirac spinor ($\theta_\chi$), where $\theta$ denotes the positive energy two spinor\textsuperscript{[10]}, begins to dominate. Further under reflections, while $\theta$ goes to $\theta, \chi$ be-
haves like a psuedo-spinor (cf.ref.[21]),

\[ \chi \to -\chi \]

Hence the operator \( \frac{\partial}{\partial x^\mu} \) acting on \( \chi \), a density of weight \( N = 1 \), has the following behaviour,

\[ \frac{\partial \chi}{\partial x^\mu} \to \frac{1}{\hbar} [\hbar \frac{\partial}{\partial x^\mu} - N A^\mu] \chi \]

where,

\( A^\mu = \hbar \Gamma_\sigma^\mu = \hbar \frac{\partial}{\partial x^\mu} \log(\sqrt{|g|}) \equiv \nabla^\mu \Omega \)

Equation (7) is apparently identical to Weyl’s formulation for the unification of electromagnetism and gravity. But there is a fine print. Weyl’s Christofell symbol contains two independent entities - the metric tensor \( g^{\mu\nu} \) and the electromagnetic potential \( \phi \). In effect there is no unification of electromagnetism and gravity, which prompted Einstein’s remark quoted earlier. In our formulation this follows from the Quantum Mechanical pseudo spinor property.

We can easily identify \( NA^\mu \) in (7) with the electromagnetic four potential. The fact that \( N = 1 \) explains why the charge is discrete.

In this formulation, electromagnetism is the result of the covariant derivative that arises due to the Quantum Mechanical behaviour of the negative energy components within the Compton wavelength. Thus we have endowed the electron with curvature and have introduced the double connectivity of spin half into a geometrodynamic formulation. This may be called a Quantum Mechanical Black Hole (QMBH).
gravitational field and the electromagnetic field, both emerge from the above formulation. We will first demonstrate the emergence of the mass and the charge.

We start with the metric of a relativistic "fluid" in the linearized theory\[14\] (in units $G = 1$, $c = 1$, $\hbar = 1$),

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} = \int \frac{4T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'$$  \hspace{1cm} (8)

In the expression (8) there is no restriction on the velocity, while the stresses $T^{jk}$ and momentum densities $T^{oj}$ can be comparable to the energy momentum density $T^{oo}$.

As is well known when $|\vec{x}'| << 1$, where $r \equiv |\vec{x}|$, and in a frame of reference with origin at the centre of mass at rest with respect to the particle we have

$$m = \int T^{oo} d^3x$$

$$S_k = \int \epsilon_{klm} x^l T^{mo} d^3x$$

where $m$ is the mass and $S_k$ is the angular momentum, and

$$T^{\mu\nu} = \rho u^\mu u^\nu$$  \hspace{1cm} (9)

If we now work in the Compton wavelength region, we have\[9\],

$$|u^l| = 1$$  \hspace{1cm} (10)

(This is the Quantum Mechanical input) Substitution of (9) and (10) in (8) gives on using the Mean Value Theorem,

$$S_k = \langle x^l \rangle = \int \rho d^3x$$

As $\langle x^l \rangle \sim \frac{1}{2m}$, using (9), we get, $S_k \approx \frac{1}{2}$, as required for a spin half particle. This is reminiscent of and bears a superficial
resemblance to Dirac’s membrane model [23].

The gravitational potential can similarly be obtained from (8) and (9) (cf. ref. [14]).

\[ \Phi = -\frac{1}{2}(g^{oo} - \eta^{oo}) = -\frac{Gm}{r} + O\left(\frac{1}{r^3}\right) \]

Now using the expression for the Christoffel symbols, in (7) we have,

\[ A_\sigma = \frac{1}{2}(\eta^{\mu\nu} h_{\mu\nu})_\sigma \]

so that, from (8),

\[ A_0 = 2 \int \eta^{\mu\nu} \frac{\partial}{\partial t} \left[ \frac{T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} \right] d^3x' \]

Remembering that \(|\vec{x} - \vec{x}'| \approx r\) for the distant region we are considering, we have,

\[ A_0 \approx \frac{2}{r} \int \eta^{\mu\nu} \left[ \frac{\partial}{\partial \tau} T_{\mu\nu}(\tau, \vec{x}') \cdot \frac{d}{dt} (t - |\vec{x} - \vec{x}'|) \right] d^3x' \]

or finally

\[ A_0 \approx \frac{2}{r} \int \eta^{\mu\nu} \frac{d}{d\tau} T_{\mu\nu} d^3x' \]  

(11)

where we have used the fact that in the Compton wavelength region, \(u_\nu = 1 (= c)\)

It has already been observed that QMBH can be treated as a rotating shell distribution with radius \(R \equiv \frac{1}{2m}\). So we have,

\[ \left| \frac{du_\nu}{dt} \right| = |u_\nu| \omega \]

(12)

where \(\omega\), the angular velocity is given by,

\[ \omega = \frac{|u_\nu|}{R} = 2m \]  

(13)
Finally, on using (12), (13) in (11), we get, on restoring usual units,
\[ \frac{e'e}{r} = A_o \sim \frac{\hbar c^3}{r} \int \rho \omega d^3x' \sim (Gmc^3) \frac{mc^2}{r} \]  \hspace{1cm} (14)
where \( e' = 1 \text{esu} \) corresponds to the charge \( N = 1 \) and \( e \) is the test charge.

We now test the validity of (14): Because of the approximations taken in deducing (14), a dimensional constant \( (L^T)^5 \) has to be multiplied on the left side, which then becomes, in units, \( c = G = 1 \),

\[ e'e.(\text{dimensional constant}) \approx 1.6 \times 10^{-111}\text{cm}^2 \]

The right side is,
\[ Gm^2c^5 \approx 4.5 \times 10^{-111}\text{cm}^2, \]
in broad agreement with the left side.

Alternatively, using the values of \( G, m \) and \( c \) in (14), we get,
\[ e \sim 10^{-10} \text{esu}, \]
which is correct.

Yet another way of looking at (14) is, that we get, as \( e' = 1 \text{esu} \sim 10^{10}e \)
\[ \frac{e^2}{Gm^2} \sim 10^{40}, \]  \hspace{1cm} (15)
that is, we have theoretically deduced a result which is well known empirically.

Finally given the mass and charge we take the following short cut to the full Kerr-Newman metric by considering the imaginary displacement (4). As is well known, in General Relativity
this leads us inexplicably and as if by accident[13, 16], from the static Kerr-Metric to the full Kerr-Newman metric. Newman himself observed, ”...one does not understand why it works. After many years of study I have come to the conclusion that it works simply by accident”.

However in the preceding setting of covariant derivatives in a Quantum Mechanical context one can see the rationale for the transformation (1): We take \( a^o \approx \frac{\hbar}{2mc^2} \), and \( a^i \approx \frac{\hbar}{2mc} \). In other words we are probing the Zitterbewegung region of the Quantum Mechanical Black Hole. This then leads to (7). This again indicates how electromagnetism or the Kerr-Newman metric for the electron emerge from the Zitterbewegung region.

In any case remembering that the Kerr-Newman field is a stationary field, and to this extent we are dealing with the approximation of an isolated electron at rest, it can be shown shown that the electromagnetic mass of the Kerr-Newman field coincides with the mass itself, in this theory. This was not possible in the Classical Theory of the electron[24].

It is worth mentioning that spin can be considered to be the angular momentum within the Zitterbewegung region[25], while the electron (and its massless version, the neutrino) have been considered to be the primary elementary particle[26]. Hestenes too has argued that the inertial energy could be considered as due to Zitterbewegung processes within the Compton wavelength though the general relativistic scheme is absent in this case. Finally the emergence of the inertial mass from within the Compton wavelength can be demonstrated without recourse to General Relativity[9, 27].

In an early paper attempting to describe elementary particles within the framework of the gravitational field equations specif-
ically, the Schwarzchild and Kerr solutions,\cite{28}. Einstein and Rosen wondered to what extent Quantum Mechanics had been included in their scheme. After that, much work has been done in attempts to unify General Relativity and Quantum Theory, notwithstanding the difficulties mentioned in earlier. However the fact remains that not only are Planck scale particles with mass of about $10^{-5}gms$ too massive to be elementary, but as pointed out earlier, they do not exhibit the all important spin half: they are essentially Schwarzchild Black Holes. On the other hand spinorial behaviour is a very Quantum Mechanical characteristic - it is fermions that represent the material content of the universe.

Neutrinos are also described by the above model. In this case as pointed out\cite{9}, because they have vanishingly small mass, they have a very large Compton wavelength so that at our usual spatial scales we encounter predominantly the negative energy components with the peculiar handedness (cf.ref.\cite{9} for details). It was shown in ref.\cite{9} that if on the other hand instead of considering distances $\gg$ the electron Compton wavelength we consider the distances of the order of the Compton wavelength itself \cite{8} leads to a QCD type potential,

$$4 \int \frac{T_{\mu\nu}(t, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' + (\text{terms independent of } \vec{x}),$$

$$+ 2 \int \frac{d^2}{dt^2} T_{\mu\nu}(t, \vec{x}') \cdot |\vec{x} - \vec{x}'| d^3x' + 0(|\vec{x} - \vec{x}'|^2) \propto -\frac{\alpha}{r} + \beta r \ (16)$$

The above considerations immediately lead us to consider the possibility of describing weak interactions on the one hand and quarks on the other in the above model.

We will now indicate how it is possible to do so \cite{29, 30}. \hfill 20
Let us start with the electrostatic potential given in equations (7) and (11). We will first show how the characteristic and puzzling $\frac{1}{3}$ and $\frac{2}{3}$ charges of the quarks emerge. For this we first note that the electron’s spin half which is correctly described in the above model of the Kerr-Newman Black Hole, outside the Compton wavelength automatically implies three spatial dimensions\[14\]. This is no longer true as we approach the Compton wavelength in which case we deal with low space dimensionality. So for the Kerr-Newman Fermions spatially confined to distances of the order of their Compton wavelength or less, we consider two and one spatial dimensionality. Using now the well known fact\[9\] that each of the $T_{ij}$ in is given by $\frac{1}{3}\epsilon$, $\epsilon$ being the energy density, it follows from (11) that the particle would have the charge $\frac{2}{3}e$ or $\frac{1}{3}e$, as in the case of quarks. Moreover, as noted earlier (cf.ref.\[8, 9\] also), because we are at the Compton wavelength scale, we encounter predominantly the components $\chi$ of the Dirac wave function, with opposite parity. So, as with neutrinos, this would mean that the quarks would display helicity, which indeed is true: As is well known, in the $V - A$ theory, the neutrinos and relativistic quarks are left-handed while the corresponding anti-particles are right handed (brought out by the small Cabibo angle). This also automatically implies that these fractionally charged particles cannot be observed individually because they are by their very nature spatially confined. This is also expressed by the confining part of the QCD potential (10). We come briefly to this aspect now. Let us consider the QCD type potential (10). To facilitate comparison with the standard literature\[4\], we multiply the left hand
expression by $\frac{1}{m}$ (owing to the usual factor $\frac{\hbar^2}{2m}$) and also go over to natural units $c = \hbar = 1$ momentarily. The potential then becomes,

$$\frac{4}{m} \int \frac{T_{\mu\nu}}{r} d^3x + 2m \int T_{\mu\nu} r d^3x \equiv -\frac{\propto}{r} + \beta r \quad (17)$$

Owing to (9), $\propto \sim O(1)$ and $\beta \sim O(m^2)$, where $m$ is the mass of the quark. This is indeed the case for the QCD potential (cf.ref.[5]). Interestingly, as a check, one can verify that, as the Compton wavelength distance $r \sim \frac{1}{m}$ (in natural units), the energy given by (17) $\sim O(m)$, as it should be. Thus both the fractional quark charges (and handedness) and their masses are seen to arise from this formulation.

To proceed further we consider (11) (still remaining in natural units):

$$\frac{e^2}{r} = 2Gm_e \int \frac{\eta^{\mu\nu} T_{\mu\nu}}{r} d^3x \quad (18)$$

where at scales greater than the electron Compton wavelength, $m_e$ is the electron mass. At the scale of quarks we have the fractional charge and $e^2$ goes over to $\frac{e^2}{10} \approx \frac{1}{1370} \sim 10^{-3}$. So we get from (18)

$$\frac{10^{-3}}{r} = 2Gm_e \int \frac{\eta^{\mu\nu} T_{\mu\nu}}{r} d^3x$$

or,

$$\frac{\propto}{r} \sim \frac{1}{r} \approx 2G.10^3m_e \int \frac{\eta^{\mu\nu} T_{\mu\nu}}{r} d^3x$$

Comparison with the QCD potential and (18) shows that the now fractionally charged Kerr-Newman fermion, viz the quark has a mass $\sim 10^3m_e$, which is correct.
If the scale is such that we do not go into fractional charges, we get from (18), instead, the mass of the intermediary particle as $274m_e$, which is the pion mass. All this is of course completely consistent with the physics of strong interactions.

It has already been noted that in the formulation of leptons as Kerr-Newman Black Holes, for Neutrinos, which have vanishingly small mass, if at all, so that their Compton wavelength is infinite or very large, we encounter predominantly the negative energy components of the Dirac spinor. It was shown in, for example, ref. [9], that this explains their characteristic helicity and two component character. One should expect that from the above considerations, we should be able to explain weak interactions also.

Even in the early days leading to the electro-weak theory, [31], it was realized, that with the weak coupling constant set equal to the electromagnetic coupling constant and with a massive intermediary particle $m_w \sim 100m_p$, where $m_p$ is the proton mass, we get the Fermi local weak coupling constant,

$$G_w = g^2/m_w^2 \approx \frac{10^{-5}}{m_p^2}gm^{-2}$$

(19)

This is also the content of our argument: We propose to show now that (19) is consistent with (13). However, it should be borne in mind that now (13) is not an adhoc experimental result, but rather follows from our model as seen earlier.

Our starting point is equation (13), which we rewrite, as

$$\frac{e^2 \times 10^{19} \times 10^8}{10^4 m_p^2} \approx \frac{10^{40} \times 10^{19}}{10^4} \approx 10^{55},$$
remembering that $G \sim 10^{-8}$ and $e^2 \sim 10^{-19}$. From here we get,

$$\frac{g^2}{m_w^2} \approx 10^{43} g m^{-2}$$

with $g^2 \approx 10^{-1}$ and $m_w \approx 100 m_p$ which is consistent with (19).

### 3 Discussion

The model of leptons as Kerr-Newman Black Holes, as discussed above leads to a cosmology\cite{8, 32} in which the universe continues to expand and accelerate with ever decreasing density. Interestingly this has been confirmed by several independent recent observations\cite{33}.

On the other hand the model also predicts that near Compton wavelength scales electrons would display a neutrino type bosonization or low dimensionality\cite{34}. This type of behaviour has been noticed, already at nano scales in recent experiments with nano tubes \cite{35, 36}.

What we have shown above is that the Kerr-Newman Black Hole description of the electron leads to a unification of gravitation and electromagnetism as expressed by e.g. (13). It also gives a clue to the peculiar fractional charges and also masses of the quarks and the QCD interaction. Finally the model explains the handedness of the neutrino and gives a clue to the origin of weak interactions.

So at the heart of the matter is the description of the electron as a Kerr-Newman Black Hole, valid at length scales greater than the Compton wavelength. The other phenomena appear from here at different length (or energy) scales.
References

[1] R.P. Feynman, The Feynman Lectures on Physics, 2, Addison-Wesley, Mass., 1965.

[2] F.J. Dyson, ”Infinite in all directions”, Harper and Row, New York, 1988.

[3] A. Koestler ”Sleep Walkers”, Hutchinson, London.

[4] L.N. Cooper, ”An Introduction to the Meaning and Structure of Physics”, Harper International, New York, 1968.

[5] T.D. Lee, ”Particle Physics and Introduction to Field Theory”, Harwood Academic Publishers, New York, 1981, p.586.

[6] A. Salam, ”Unification of Fundamental Forces”, Cambridge University Press, Cambridge, 1990.

[7] R.M. Barnett et al Rev.Mod.Phys., 68(3), 1996, pp.611-732.

[8] B.G. Sidharth, Intl.J.Mod.Phys.A, 12(27), 1997.

[9] Sidharth, B.G., Ind.J. of Pure and App.Phys., 35, 1997, p.456ff. (Also xxx. lanl.gov quant-ph 9808020).

[10] Sidharth, B.G., Gravitation and Cosmology, 4 (2) (14), 1998, p.158ff, and references therein.

[11] M.A. Markov, Soviet Phys. JETP 24, 584 1967.

[12] L. Motz, Nuovo Cim... 12B, 1972.

[13] P.A.M. Dirac, ”The Principles of Quantum Mechanics”, Clarendon Press, Oxford, 1958.
[14] C.W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation, Freeman (San Francisco), 1973.

[15] E.T. Newman, J.Math.Phys. 14 (1), 1973, p102.

[16] E.T. Newman, Enrico Fermi International School of Physics Proceedings 1975, p557.

[17] A. Einstein, "The Meaning of Relativity", Oxford & IBH, New Delhi, 1965, pp.93-94.

[18] D. Pandres Jr., Lett. Nuovo Cim, 8, 9, 1973.

[19] S. Sonego, Phys Letts A 208, 1995.

[20] C. Wolfe, Hadronic Journal 15, 1992, 321-332.

[21] J.D. Bjorken and S.D. Drell, "Relativistic Quantum Fields, McGraw-Hill Inc., New York, 1965.

[22] P.G., Bergmann, "Introduction to the Theory of Relativity", Prentice-Hall (New Delhi), 1969, p248ff.

[23] P.A.M. Dirac, Proc. Roy. Soc., A 133, 1931, pp.60 ff. 1931.

[24] F. Rohrlich, "Classical Charged Particles", Addison-Wesley, Redwood, California, 1965.

[25] A.O. Barut, and A.J. Bracken, Phys. Rev. D 23, (10), 1981.

[26] D. Hestenes, Found. Phys. 15 (1), 1985.

[27] B.G. Sidharth, Non Linear World, 1, 1994, pp.403-408.

[28] A. Einstein and N. Rosen, Phys Rev. 48, 1935.
[29] B.G. Sidharth, Mod. Phys. Lett. A., 14, 5, 1999, pp.387-389.

[30] B.G. Sidharth, ”Consequences of the Quantum Mechanical Black Hole Model” to appear in Non Linear Analysis, USA.

[31] J. Bernstein, ”Elementary Particles and their currents”, Freeman, San Francisco, 1968.

[32] B.G. Sidharth, Int.J.Th.Phys. 37(4),1998 pp1317-1322.

[33] Perlmutter, S., et.al. Nature391 (6662), 1998, p51-54.

[34] Sidharth, B.G., ”Quantum Mechanical Black Holes: An Alternative Perspective”, in ”Frontiers of Quantum Physics”, Eds., Lim, S.C., et al, Springer- Verlag, 1998.

[35] M.S. Dresselhaus, Nature, 391, 1998, 19-20.

[36] W.G. Wildoer, C. Liesbeth, Venema, Andrew, G. Rinzler, Richard E. Smalley & Cees Dekker, Nature, 391, 1998, 59-62.