Explanation of an Invisible Common Constraint of Mind, Mathematics and Computational Complexity

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Abstract:

There is a cognitive limit in Human Mind. This cognitive limit has played a decisive role in almost all fields including computer sciences. The cognitive limit replicated in computer sciences is responsible for inherent Computational Complexity. The complexity starts decreasing if certain conditions are met, even sometime it doesn’t appear at all. Very simple Mechanical computing systems are designed and implemented to demonstrate this idea and it is further supported by Electrical systems. These verifiable and consistent systems demonstrate the idea of computational complexity reduction. This work explains a very important but invisible connection from Mind to Mathematical axioms (Peano Axioms etc.) and Mathematical axioms to computational complexity. This study gives a completely new perspective that goes well beyond Cognitive Science, Mathematics, Physics, Computer Sciences and Philosophy. Based on this new insight some important predictions are made.

Keywords: cognitive, comparator, computational complexity, converges, observability

1. Introduction:

History of struggle to find answers of difficult questions is as old as humans itself e.g. history of traveling Salesman problem. Unreasonable effectiveness of Mathematics was once a sufficient reason to believe that one day we will get answers of many difficult questions. Struggle to formalize Mathematics by David Hilbert started in 1900 and ended with Kurt Gödel's Incompleteness Theorems in 1931[1]. In 1956 Gödel argued why some problem are difficult to solve using a hypothetical machine in a letter to John von Neumann. Importance to understand computational complexity grew significantly with arrival of Computers because this complexity was a bottle neck to decide the kind of problems that can be solved by a computer. Formal definition of questions that are hard to solve (NP-Complete) by a Computer is credited to Stephen A. Cook [2]. Implications of Computational complexity are not limited to only Computer Sciences anymore. Finding a DNA sequence, Nash equilibriums, Ground state in the Ising model phase transitions, Feynman diagrams, optimal Protein threading procedures etc. all these problems involve computational complexity that is beyond the reach of today’s computers [3]. For sure Riemann Hypothesis is much Older Math problem with significant consequences, but once P vs. NP problems is settled, it will have far more decisive impact on how we will perceive this World holistically.

In this paper, simple but consistent and verifiable experiments are designed to demonstrate the cognitive limitations and ways to defy them. Complex expression are avoided for plain and comprehensive discussion, but reason behind computational complexity remains same, so explanation can be extended to any other computing problem. Some important results and conclusions are drawn using Deductive Logic. Possible outcomes of this new insight go well beyond
Mathematics, Physics, Cognitive Science, Philosophy and Computer Sciences. Predictions and possibilities in a few subjects are described briefly but comprehensive understanding and extended discussion is required on this new insight.

2. Tools and Techniques

2.1 Binary Observability (O₀=2)

Human mind has capacity to interact at most two objects (or numbers) at a time, this binary observability (O₀=2) requires certain time T₀₂ to reach a conclusion. This limit to observe only two objects (or interact with two numbers) has played a decisive role on how humans collect, interpret store and use information. Historical inventions such as Balance Scale and recent inventions such as Electrical Comparator are also based on binary observability. If the task is find a lighter apple from two apples, hold them in your hands, or put them in balance scale or produce electrical signal proportional to mass and pass it through comparator, by these methods you will get required answer.

![Binary Observability systems](image1)

Fig. 1. Binary Observability systems, Hands, Balance Scale and Electrical Comparator. A system of observability Order two cannot interact with more than two objects or numbers at an instant.

What if the task is to find lighter apple from three apples? No third hand, no scale to compare three apples or comparator to compare three states at a single instant. The idea of time complexity appears when number of objects (three apples) are higher than observability order, in this case O₀=2 (two hand, two sides of balance scale and two input of comparator). If there is system of observability order three then we will be able to find lighter apple from three apples in a single instant i.e. as long as number of apple is same as observability order (O₀) time complexity should not appear. If such systems exist that can compare more than two objects in a single instant, then it is cognitive limit in Humans, as we cannot observe, interact and compare more than two objects or numbers at an instant, and we have not developed any such systems or techniques that can compare more than two objects in a single instant. Is there any possibility to develop system with O₀ higher than two? Yes.

2.2 Observability Order three (O₀=3)

Binary observability systems (O₀=2) can observe and compare only two objects simultaneously in time T₀₂. Similar system is shown in Fig.2 (Binary Balance Scale), it has two arms 180° apart. If we have three apples and want to find lighter apple, for binary observability system O(n)>1 for
n=3 (O(n) denotes number of steps required to find answer involving n objects). A system of observability order three (O_o=3) is shown in Fig. 2 (Balance Scale of Observability Order Three), it has three arms each 120° apart. For this system O(n)=1 for n=3 and T_{o2}=T_{o3} (Time to compare two objects is same to that of three objects). O_o=3 system will compare three apples simultaneously to find lighter apple. So, time complexity does not appear as long as observability order is same as objects involved. A task to find smaller apple from three apples will be performed by O_o=3 in single step and it will be performed by O_o=2 in two steps. System of O_o=3 is 200% more efficient with respect to system of O_o=2. If we have ten apple, nine of them have same size and tenth apple has smaller size, and task is to find smaller apple with system of O_o=10 (now ten arms each 36° apart), even in this scenario O(n)=1 for n=10 and T_{o2}=T_{o10} (Time to compare two apples is same as time to compare 10 apples).

Fig. 2. System in Left side has Observability Order 2. System in middle has Observability Order 3. System in the right side is prototype of Observability Order 3.

2.3 Observability Order Ten (O_o=10)

Now the task is to find a lighter and heavier apple from ten apple, normally sorting is necessary step to perform this task and sorting is executed by binary comparison. As shown in fig. 3, a new system of O_o=10 is designed where binary comparison is redundant. This scheme looks like parallel computing but it is not; because in parallel computing tasks are divided and no redundant steps appear, here binary comparisons is completely redundant. The system will find heavier and lighter apples simultaneously, independent to each other. As long as K (number of apples) is less than or equal to O_o (observability order), O(n)=1 and T_{o2}=T_{o3}=T_{o4}=......=T_{o9}=T_{o10} or time would remain same up to ten apples. If we want to find smaller apple using this system for n=20 with O_o=10 then O(n)=3 with total time 3T_{o2}. Even if task is to find lighter apple from one billion apples with system of O_o=1billion, for this task also O(n)=1 and time T_{1 billion}=T_{o2}. When observability order starts increasing computational time start decreasing. If observability order is same or higher than objects involved then O(n)=1 and time to perform the task will be T_{o2}, same time as it involve only two objects.
2.3.1 Sorting Charges ($O_0=10$)

Sorting is very important task in computing ($O(n)=n\log(n)$ in insert sort for objects $n$). Another electrical system is designed to sort charges in ascending (or descending) order. Charges of mass $m$ moving with speed $V$ pass through uniform electric field $H$. All charges will be sorted in ascending order from bottom to top, but can be sorted by descending order by just changing the polarity of electric field $H$. Time complexity will not appear as long as observability order is equal to the number of charges involved in sorting and best case, average case, worst case execution times would remain same because $O(n)=1$ for all cases. Charges can be sorted based on their mass or velocity (Lorentz Forces). Similarly many experiments can be designed (based on mass, volume, density or other electric properties) to show how time complexity starts decreasing when observability order increases and eventually disappear when observability order is equal to the number of objects involved. So the systems of $O_0=3$ and higher exist that can compare more than two objects simultaneously without increasing computational complexity and are easy to design. A system of $O_0=10$ has capacity to interact with ten objects at a time in same way humans interact with only two objects or numbers.
2.3.2 Collecting Charges (O_0=10, Exponential Functions)

NP-Complete problems involve exponential complexity. Exponential functions behave quite differently in higher order of observabilities. Suppose the task is to collect positive charges in confined space G. Suppose it require time T_{E1} and 50 eV of energy to bring one charge into space G. Every incoming charge requires additional 30% of total energy spent previously. So, energy spent on any charge can be calculated by function 50(1.3)^n, in this exponential function n is charge number. In a task to confine ten charges in space G, first charge will require 50 eV whereas 10th charge will require 689.29 eV of energy and total required energy will be 2820.26 eV. If this task is performed by O_0=10, all charges put together simultaneously, total energy consumed by each observability will 282.02 eV, complexity of task will be distributed equally in all ten observabilities and it will be performed in time T_{E10} with O(n)=1, and T_{E1}=T_{E10}. So, it can be deduced that in higher order observabilities, function (previously exponential) behave quite differently; even some time exponential function transforms to non-exponential function if observability orders are higher enough, as describe in this task.

Fig. 4. An Electrical System to sort Charges. The system has Observability Order 10.

Fig. 5. Observability Order 10 system to collect charges of same polarity in confined Space G.
3. Discussion

There is no computer architecture or algorithm that can compare more than two numbers/values at a time in single step. Time complexity appears because of lower observabilities, in case of humans or man made machine only binary observability/comparison is possible. When observability order start increasing some steps in $O(n)$ start becoming redundant. The rate of redundancies depends on both observability order and nature of problem. Here $P$ and $NP$ denote deterministic and non-deterministic polynomials respectively. By applying Deductive Logic on previous experiments the Principal of Observability can be deduced and it states that

If NP problem involves $K$ objects whereas $O$ is the order of observability then

\[
\text{if } O \geq K \text{ then } O(n) = 1 \Rightarrow P = NP
\]

\[
\text{if } O < K \text{ then } O(n) > 1 \Rightarrow P \neq NP
\]

There would not be any memory required in first postulate of Principal because $O(n)=1$. It can be inferred from the Principal of observability that $P \neq NP$ converges toward $P = NP$ as the order of observabilities increases. Regardless of the complexity involved in a problem, answer of any question appears at the moment of completion of question; how much obvious the answer is and how quickly it would be found? It depends on order of observability. Intelligence of any observability can be considered as product of $T_0$(observability times) and $O_0$(observability order). This fresh insight gives radically new dimensions of understanding of Cognitive Science, Mathematics, Physics, Philosophy and other disciplines.

3.1 Cognitive Science

Humans (Homo sapiens) are on the mountaintop of binary observability but binary observability is the simplest possible pattern among all other possibilities; at any instant human can interact with only two objects/numbers. Any extraterrestrial life of $O_0=3$ (with same time span of evolution) will be at least 200% more intelligent (see section 2.2, $O_0=3$) with two times stronger senses. Binary Observability was one of the most important driving factor in physical evolution because in the presence of binary observability one hand (in comparison) would be useless and third hand would be redundant, so only two hands were compatible with binary observability. Even though high observability patterns are possible in evolution, why it was restricted to binary observability only and how higher order observability life will look like? It requires comprehensive extended discussion. Similarly, binary observability also has played a decisive role on how we receive, process, and store and use information, and how we learn different cognitive skills. By and large most consequential impact of binary observability has been on Mathematics.

3.2 Mathematics and Physics

When we interact with numbers why we interact with only two numbers at a time? Why we cannot interact with more than two numbers at an instant? Humans cannot interact and manipulate with more than two numbers simultaneously. Axioms of Mathematics (Peano Axioms etc.) are based on binary observability e.g. $a.0 = 0$, $a+0=a$, $b.1=b$, $a+b=b+a$ etc. as they involves only two numbers. Third order observability will have set of its own axioms that would be reducible to binary observability but opposite would not be possible without involving time complexity i.e. $a+b+c$ would
be single step addition in third order observability whereas this addition will be performed in two steps in binary observability and number of steps determines time complexity. For third order observability there will be different and complex but time efficient method for math operations (prime factoring, Division etc.). Binary observability is proper subset of higher of observabilities, so some proves will only exist in higher order of observabilities (Gödel's first Incompleteness theorem). Gödel second Incompleteness theorem will also be valid in all observabilities; Inconsistencies in Mathematics appear because of boundaries imposed by observability orders. Binary observability might be the reason of Human's inability to understand counterintuitive ideas and phenomenon's (infinities in Mathematics, quantum superposition and entanglement in Physics etc.). Invisible constraint of binary observability imposed by mind on Mathematics is responsible for inherent computational complexity.

3.3 Computer Sciences

A computer (or algorithm) of $O_0=3$ will be 200% more efficient as compared to conventional computers (or algorithm) of $O_0=2$ (see section 2.2, $O_0=3$). Developing a comparator of $O_0=3$ or higher observability will not be impossible. Quantum computers can provide higher observability because of their many possible superposition states, developing additional hardware of higher observability can be gateway toward future computing. Currently additional circuitry for quantum computers being investigated has binary observability, and will not reduce $O(n)$, that will result $P \neq NP$ for quantum computes as well. Higher observability will also reduce space complexity (memory) but the nature of space will be different (not queues binary state only). In designing higher observabilities computing systems, there will be tradeoff between time/space complexity and size of computing system. Development of algorithms and computer architectures of higher observability will require understanding of Mathematics for higher observabilities.

3.4 Philosophy

It was the binary observability that restricted Humans to think about possibility to compare/observe more than two objects (or numbers) simultaneously. There are numerous examples to prove that Nature works in higher observabilities because $O(n)=1$ and no space complexity (memory) is involved, possibly this is why nature is consistent unlike to Mathematics. For natural phenomena's there is no memory and $O(n)=1$, but when same tasks of Nature are performed by humans or man-made machines they require a lot of memory and time complexity. In other words, any system working in lower observability (binary in case of humans) will have involve time complexity and memory to perform some tasks. In third observability ($O_0=3$), third logical state will exist, in this third state the nature of Schrödinger's Cat paradox, Gödel's Liar Paradox etc. will be explained. A paradox in $O_0=n$ appears real phenomenon in $O_0=n+1$ and paradoxes can be linked between consecutive observabilities i.e. $O_0=3$ will have its own paradox that will be solved in $O_0=4$ and so on. There are two logic states in binary observability and number of logic states will increase along with observability orders and it will eventually become Fuzzy Logic in higher observabilities.

Safely it can be stated explicitly that computational complexity appears because of Mathematical Axioms that are constrained by and based on human cognitive limits as human cannot interact with more than two numbers at an instant. This explanation can be verified by numerous other approaches and methods, and some of which are described in this paper. Very simple explanations are carried out in this paper but this new insight require comprehensive understanding and discussion that would be extended into many fields in many folds.
4. Conclusion

Humans cannot interact and observe more than two numbers (or objects.) at a given instant. Limitation of binary observability shared by Mind, Mathematics and Computational complexity has played a decisive role in how humans receive, interpret, manipulate, accumulate and use information. Computational complexity appears because of axioms of Mathematics (Peano Axioms etc.) that are constrained by human cognitive limit to interact with only two numbers at a time. If the limit on observability order is removed, time complexity starts reducing, and even sometime it does not appears at all. This fresh insight provides a path to explore many new possibilities in the fields of Mathematics, Physics, Cognitive Sciences, Computer Sciences and numerous others. Apparently mathematical inconsistencies appear because of limitations imposed by observability orders. There are plenty of examples to show that nature works in higher observability because it does not require any memory and $O(n)=1$, this is why nature is consistent unlike to mathematics. Many other Laws and Principals would be deduced based on this new insight. Binary observability restricts our intelligence level and plays pivotal role on how we perceive reality in which we live. Understanding of higher order observabilities will help us to develop new fields and will stretch our limitations to comprehend the world we inhabit.

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