Neutrino mean free path in neutron stars
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The Landau parameters of nuclear matter and neutron matter are extracted from the Brueckner theory including three-body forces. The dynamical response function to weak neutrino current is calculated in terms of the Landau parameters in the RPA limit. Then, the neutrino mean free path in neutron stars is calculated for different conditions of density and temperature.

1. INTRODUCTION

The interaction of neutrinos with baryons has been mostly studied in connection with the stability of nuclei. But it also plays a crucial role in the thermal evolution of supernovae and neutron stars, where the nuclear medium is far from stability. A large effort has been devoted to study the production of neutrinos via direct or modified URCA processes \cite{1} and, more recently, via the bremsstrahlung from nucleons in the strong magnetic field of neutron stars\cite{2}. The propagation of neutrinos in neutron matter has received less attention\cite{3, 4}. Being the experimental information on this topic still insufficient, the theoretical predictions have to be developed further, extending the microscopic description of many-body systems to a wide range of density, temperature and isospin asymmetry. This means extending the study of the nuclear equation of state far away from the saturation point.

In this note we report on the calculation of the response function to weak interaction coupling in neutron matter. It is based on the equation of state (EOS) predicted by the Brueckner theory, including relativistic and finite-size effects (nucleon resonances). Recently it has made a step forward as to reproducing the empirical saturation properties of nuclear matter\cite{5}. The effective NN interaction is extracted from the Brueckner theory and cast in terms of the Landau parameters. Then, the response function is calculated in RPA along with the neutrino mean free path in neutron matter for various conditions of neutron density and temperature.
2. NUCLEAR MATTER EOS FROM THE BRUECKNER THEORY

The EOS of nuclear matter has been investigated from several microscopic approaches. Their common feature is that of adopting as $V_{NN}$ interaction a realistic potential, i.e. fitted on the experimental phase shifts of the nucleon-nucleon scattering in the vacuum. Among them the Brueckner theory has proved to be very powerful for its capability of incorporating both relativistic and nucleon finite-size effects [6]. This feature is illustrated in Fig. 1, where a calculation with the interaction Argonne $V_{18}$ is plotted. In the first order of the hole-line expansion (BHF approximation), the saturation density turns out to be too large, when the NN interaction is just a two-body force (2BF). Extending the BHF calculation to include a three-body force (3BF) effects one improves the agreement with the empirical saturation properties of nuclear matter. The origin of 3BF is twofold: the $NN$ excitations missing from the non relativistic Brueckner theory, amount to a special 3BF as shown by G.E. Brown et al.[7]; low-lying nucleonic excitations as intermediate states in the interaction between two nucleons are pure 3BF. As shown in Fig.1 both effects play a important role for the reproduction of the empirical saturation properties of nuclear matter.

The predictions of the EOS can be extended far beyond the saturation density of cold and symmetric nuclear matter. A variety of physical situations can be studied, including neutron-rich and spin-polarized matter at finite temperature (here we limit ourselves to zero temperature). For such cases there are less physical constraints: the isospin-symmetry energy at the saturation point is constrained by the empirical value of $\approx 30\,\text{MeV}$ extracted from the nuclear mass table: finally, the spin-symmetry energy can be probed by spin-flip excitations of nuclei.

A set of Brueckner calculations have been performed in different states of spin and
isospin polarization of nuclear matter, defined by
\[
\delta = \frac{A_\uparrow - A_\downarrow}{A}, \quad \beta = \frac{N - Z}{A},
\]
respectively. Here \(A_\uparrow(A_\downarrow)\) are the number of nucleons with spin polarized upward (downward). In both cases the Brueckner calculations predict a quadratic law so that the isospin and spin symmetry energies describe completely the properties of isospin and spin polarized nuclear matter, respectively. In Fig. 2 the isospin symmetry energy and in Fig. 3 the spin symmetry energy are plotted as a function of density.

3. LANDAU PARAMETERS FROM THE BRUECKNER THEORY

The Brueckner theory basically provides us with the \(G\)-matrix, which is nothing else than the in-medium NN interaction. The latter is not suitable for calculations, but it can be cast in the form of effective interaction within the quasiparticle description of Fermi systems \([8,9]\). Alternatively, for the present purpose one first determines from the EOS of nuclear matter fundamental equilibrium properties such as the compression modulus, symmetry energy and spin susceptibility and then calculates the corresponding Landau parameters.

In the Landau theory of Fermi liquids one writes the particle-hole residual interaction as
\[
N(0)V_{NN} = F + F' \vec{\tau} \cdot \vec{\tau}' + G\vec{\sigma} \cdot \vec{\sigma}' + G'\vec{\tau} \cdot \vec{\tau}' \vec{\sigma} \cdot \vec{\sigma}',
\]
where \(F, F', G, G'\) are the Landau parameters. Each of them is expanded in a Legendre series, for instance \(F = \sum F_l P_l(\cos \theta)\) where \(\theta\) is the Landau angle. The physical quantities related to the static response functions can be expressed in a simple way in terms of the
Table 1
Landau parameters of neutron matter from Brueckner (BRU) calculations with Argonne V18 and microscopic 3BF described in the text. Compression modulus and magnetic susceptibility are also reported in comparison with Monte Carlo (MC) calculations using V18 plus phenomenological UIX 3BF \[10\] and Hartree-Fock calculations (SLy230b) with Skyrme-like force \[11\].

| $\rho$ (fm$^{-3}$) | $F_0$ (BRU) | $G_0$ (BRU) | $K/K_F$ (BRU, MC, SLy) | $\chi/\chi_F$ (BRU, MC, SLy) |
|-----------------|-------------|-------------|-----------------------|-------------------------------|
| 0.20            | 0.615       | 1.170       | 1.84                  | 0.41                         |
|                 | 3.395       | 1.461       | 5.24                  | 0.37                         |

Landau parameters. The response to density fluctuation, i.e. the compression modulus is directly related to $F_0$; the response to charge variation, i.e. the symmetry energy, is related to $F_0'$; the response to spin variation, i.e. the magnetic susceptibility, is related to $G_0$; finally, the response to spin-isospin variation, which is the isospin susceptibility, is related to $G_0'$.

From the Brueckner calculations we found that the energy per nucleon of nuclear matter as a function of $\delta_n$ and $\delta_p$ (Eq. (1) for neutron and proton, separately) can be fairly approximated by the quadratic form

$$E_A(\delta_n, \delta_p) - E_A(0, 0) = \Delta_{nn}\delta_n^2 + \Delta_{pp}\delta_p^2 + 2\Delta_{np}\delta_n\delta_p. \quad (3)$$

This equation is the basis of our calculation of the zero-order Landau parameters, which are plotted in Fig. 4 and Fig. 5. Notice that $F_0$ becomes less than -1 (negative compression modulus) at low density which corresponds to the unphysical spinodal region of the EOS, signaled by the multifragmentation events of heavy ion collisions.
4. NEUTRINO MEAN FREE PATH

Let us consider the propagation of neutrinos in pure neutron matter. The neutrino mean free path $\lambda$ is derived from the transport equation. One obtains

$$\frac{c}{\lambda_k} = \sum W_{fi}(k - k')(1 - n_{k'}) + W_{fi}(k' - k)n_{k'},$$

(4)

where $n_k$ is the occupation number of neutrinos and $W_{fi}$ is the neutrino-neutron scattering probability via the weak neutral current[3]. The medium effects are incorporated into $W_{fi}$ via the dynamical form factors, which, in turn, are related to the imaginary part of the dynamical response function. In the low-frequency limit $\omega/(kv_F) \ll 1$ the latter is expressed in a simple way in terms of the Landau parameters[3], which is the approximation adopted in the present calculation. The Landau parameters were extracted from the Brueckner prediction of the EOS of neutron matter. They are listed in Table 1 in comparison with other predictions. In Fig. 6 (left panel) our prediction for $\lambda_\nu$ in interacting neutron matter is reported as a function of the density in comparison with that in a free neutron gas. One can see that the large medium effects are due to the large value taken by both Landau parameters $F_0$ and $G_0$ in the interacting system. For densities in the range of neutron-star density $\lambda_\nu$ is by one order of magnitude larger than in a free Fermi gas. In the right panel of Fig. 6 the Hartree-Fock predictions from two Skyrme forces and Gogny force taken from Ref.[4] are reported for comparison. In our case $\lambda_\nu$ is increasing much faster with density due to the more repulsive character of our 3BF. Fig. 7 shows the dependence of $\lambda_\nu$ on the temperature $T$. The expected $T$ dependence of the Landau parameters is neglected. The relative weight of each elementary process giving rise to the neutrino propagation is plotted in Fig. 8. The role played by the Landau zero mode appearing as a singularity of the response function, is negligible.

In summary, we have presented a prediction of mean free path of neutrinos in neutron matter. Since the strong nuclear correlations enhance the propagation of neutrino one
may predict a faster cooling of a neutron stars via neutrino escape than in a degenerate neutron gas. The neutron star temperature plays also important role in the neutrino propagation. Due to the large value of $T$ involved in proton-neutron stars, the present predictions should be improved by introducing $T$ dependent Landau parameters.

This work has been supported in part by the Chinese Academy of Science, within the **one Hundred Person Project**, the Knowledge Innovation Project of CAS under No. KJCX2-SW-N02, and the Important Pre-research Project of the Ministry of Science and technology under No.2002CCB00200, China.

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