A detection of the level of non-Gaussianity in the CMB data is essential to discriminate among inflationary models and also to test alternative primordial scenarios. However, the extraction of primordial non-Gaussianity is a difficult endeavor since several effects of non-primordial nature can produce non-Gaussianity. On the other hand, different statistical tools can in principle provide information about distinct forms of non-Gaussianity.

Thus, any single statistical estimator cannot be sensitive to all possible forms of non-Gaussianity. In this context, to shed some light in the potential sources of deviation from Gaussianity in CMB data it is important to use different statistical indicators. In a recent paper we proposed two new large-angle non-Gaussianity indicators which provide measures of the departure from Gaussianity on large angular scales. We used these indicators to carry out analyses of non-Gaussianity of the bands and of the foreground-reduced WMAP maps with and without the $KQ75$ mask. Here we briefly review the formulation of the non-Gaussianity indicators, and discuss the analyses made by using our indicators.

1. Introduction

A detection of the level of primordial non-Gaussianity in the CMB data is crucial to discriminate inflationary models and also to test alternative scenarios for the physics of the early universe. Clearly the study of detectable non-Gaussiansities in the WMAP data ought to take into account that they may have non-cosmological origins as, for example, unsubtracted foreground contamination, unconsidered point sources emission and systematic errors. Deviation from Gaussianity may also have a cosmic topology origin (see, e.g., the review Refs. [1] and related Refs. [2]). If, on the one hand, different statistical tools can in principle provide information about distinct forms of non-Gaussianity, on the other hand one does not expect that a single statistical estimator can be sensitive to all possible forms of non-Gaussianity in CMB data. In view of this, a great deal of effort has recently gone into verifying the existence of non-Gaussianity by employing several statistical estimators (an incomplete list of references is given, e.g., in Refs. [3] and references therein).

Recently we have proposed two new large-angle non-Gaussianity indicators, based on skewness and kurtosis of large-angle patches of CMB maps, which provide measures of the departure from Gaussianity on large angular scales. We used these indicators to search for the large-angle deviation from Gaussianity in both band and foreground-reduced maps with and without a $KQ75$ mask (see also the related
Here we briefly summarize the main results of Refs. 4 and 5.

2. Results and Concluding Remarks

A constructive way of formulating our non-Gaussianity indicators $S$ and $K$ from CMB data is through the following steps:

(a) Take a finite set of points $\{j = 1, \ldots, N_c\}$ homogeneously distributed on the CMB celestial sphere $S^2$ as the centers of spherical caps of a given aperture $\gamma$; and calculate for each cap $j$ the skewness ($S_j$) and kurtosis ($K_j$) by using that

$$S_j \equiv \frac{1}{N_p \sigma_j^3} \sum_{i=1}^{N_p} (T_i - \bar{T}_j)^3$$

and

$$K_j \equiv \frac{1}{N_p \sigma_j^4} \sum_{i=1}^{N_p} (T_i - \bar{T}_j)^4 - 3,$$

where $N_p$ is the number of pixels in the $j$th cap, $T_i$ is the temperature at the $i$th pixel, $\bar{T}_j$ is the CMB mean temperature of the $j$th cap, and $\sigma$ is the standard deviation. The numbers $S_j$ and $K_j$ obtained in this way for each cap can be seen as a measure of non-Gaussianity in the direction of the center ($\theta_j, \phi_j$) of the $j$th cap.

(b) Patching together the $S_j$ and $K_j$ values for each spherical cap, one obtains two discrete functions $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ defined on the sphere $S^2$, which can be used as statistical indicators to measure the deviation from Gaussianity as a function of the angular coordinates ($\theta, \phi$). The Mollweide projection of skewness and kurtosis functions $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ are nothing but skewness and kurtosis maps (hereafter $S$-map and $K$-map).

Clearly, the discrete functions $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ can be expanded into their spherical harmonics in order to determine their power spectra $S_\ell$ and $K_\ell$. Thus, for example, for the skewness one has

$$S_\ell = (2\ell + 1)^{-1} \sum_m |b_{\ell m}|^2.$$

In the remainder of this work we shall report the results of our Gaussianity analysis performed with $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ indicators calculated from single frequency and foreground reduced maps with and without a $KQ75$ mask. To minimize the statistical noise, in the calculations of $S$-map and $K$-map from the input maps, we have scanned the celestial sphere with spherical caps of aperture $\gamma = 90^\circ$, centered at 12 288 points homogeneously distributed on the two-sphere.

For the sake of brevity we do not show examples of $S$ and $K$ maps, which provide only qualitative information on large-angle deviation from Gaussianity (see figures in Refs. 3–5 for examples). To obtain quantitative information about the large angular scale (low $\ell$) distributions for the non-Gaussianity $S$ and $K$ maps obtained from the CMB input maps used, we have calculated the (low $\ell$) power spectra $S_\ell$ and $K_\ell$ for $S$ and $K$ maps. The deviation from Gaussianity and the statistical significance were estimated by comparing these power spectra with the corresponding averaged power spectra $\overline{S}_\ell$ and $\overline{K}_\ell$ calculated from $S$ and $K$ maps obtained
by averaging over 1000 Monte-Carlo-generated statistically Gaussian CMB maps. To have an overall assessment of low $\ell$ power spectra $S_\ell$ and $K_\ell$ calculated from each CMB input, we have performed a $\chi^2$ test to find out the goodness of fit for $S_\ell$ and $K_\ell$ multipole values as compared to the expected multipole values from the MC Gaussian maps. In this way, we obtained one number for each map that collectively (‘globally’) quantifies the deviation from Gaussianity. For the power spectra $S_\ell$ calculated from $S-$maps obtained from the five-years maps with a $KQ75$ mask we found that the ratio $\chi^2$/dof (dof stands for degree of freedom) from the $K$, $K_a$, $Q$, $V$, and $W$ maps are given, respectively, by 21.5, 4.9, 6.0, 5.2, and 3.9, while for the kurtosis power spectra $K_\ell$ of these maps the values of $\chi^2$/dof are, respectively, 35.652, 135.0, 6.4, and 5.6. Clearly a good fit occurs when $\chi^2$/dof $\sim 1$. Moreover, the greater are the $\chi^2$/dof values the smaller are the $\chi^2$ probabilities, that is the probability that the multipole values $S_\ell$ and $K_\ell$ and the expected MC multipole values agree. For $S_\ell$ and $K_\ell$ obtained from the full-sky foreground-reduced five years ILC input maps we found that $\chi^2$/dof are, respectively, 35.7 and 2368. These results reduce to 1.2 and 0.4 when the $KQ75$ mask is employed. In brief, our analyses show that the unmasked band maps are significantly non-Gaussian but the deviation from Gaussianity is substantially reduced to a level compatible with Gaussianity for $Q$ and $V$ maps, whereas the full-sky foreground reduced five years ILC mask is again significantly non-Gaussian but the level of non-Gaussianity drops to a level that is consistent with Gaussianity when the $KQ75$ mask is used.

Acknowledgments

A. Bernui and M.J. Rebouças thank CNPq for the grants under which this work was carried out. Some of the results in this work were calculated by using the HEALPix.

References

1. G. F. R. Ellis, Gen. Rel. Grav. 2, 7 (1971); M. Lachièze-Rey and J.-P. Luminet, Phys. Rep. 254, 135 (1995); J. Levin, Phys. Rep. 365, 251 (2002); M. J. Rebouças and G. I. Gomero, Braz. J. Phys. 34, 1358 (2004).
2. G. I. Gomero, M. J. Rebouças and R. Tavakol, Class. Quantum Grav. 18, 4461 (2001); G. I. Gomero, M. J. Rebouças and R. Tavakol, Class. Quantum Grav. 18, L145 (2001); B. Mota, M. J. Rebouças and R. Tavakol, Class. Quantum Grav. 20, 4837 (2003); B. Mota, G. I. Gomero, M. J. Rebouças and R. Tavakol, Class. Quantum Grav. 21, 3361 (2004); B. Mota, M. J. Rebouças and R. Tavakol, Phys. Rev. D 78, 083521 (2008); B. Mota, M. J. Rebouças and R. Tavakol, arXiv:1002.0834 [astro-ph.CO].
3. E. Komatsu et al., Astrophys. J. Suppl. 180, 330 (2009); A. Bernui, B. Mota, M. J. Rebouças and R. Tavakol, Astron. Astrophys. 464, 479 (2007); A. Bernui, B. Mota, M. J. Rebouças and R. Tavakol, Int. J. Mod. Phys. D 16, 411 (2007).
4. A. Bernui and M. J. Rebouças, Phys. Rev. D 79, 063528 (2009).
5. A. Bernui and M. J. Rebouças, Phys. Rev. D 81, 063533 (2010).
6. A. Bernui and M. J. Rebouças, arXiv:1004.0974 [astro-ph.CO]; A. Bernui, M. J. Rebouças and A. F. F. Teixeira, arXiv:1004.0207 [astro-ph.CO]; A. Bernui and M. J. Rebouças, Int. J. Mod. Phys. A 24, 1664 (2009).
7. K. M. Górski, E. Hivon, A. J. Banday, B. D. Wandelt, F. K. Hansen, M. Reinecke and M. Bartelmann, Astrophys. J. 622, 759 (2005).