DYNAMIC $k$-STRUVE SUMUDU SOLUTIONS FOR FRACTIONAL KINETIC EQUATIONS

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Abstract. In this present study, we investigate solutions for fractional kinetic equations, involving $k$-Struve functions using Sumudu transform. The methodology and results can be considered and applied to various related fractional problems in mathematical physics.

1. Introduction

The Struve function $H_p(x)$ introduced by Hermann Struve in 1882, defined for $p \in \mathbb{C}$ by

$$H_p(x) := \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(k + 3/2) \Gamma(k + p + 3/2)} \left(\frac{x}{2}\right)^{2k+p+1},$$

(1.1)
is the particular solutions of the non-homogeneous Bessel differential equations, given by,

$$x^2 y''(x) + xy'(x) + (x^2 - p^2) y(x) = \frac{4(\frac{x}{2})^{p+1}}{\sqrt{\pi} \Gamma(p + 1/2)}.$$  (1.2)

The homogeneous version of (1.2) have Bessel functions of the first kind, denoted as $J_p(x)$, for solutions, which are finite at $x = 0$, when $p$ a positive fraction and all integers [6], while tend diverge for negative fractions, $p$. The Struve functions occur in certain areas of physics and applied mathematics, for example, in water-wave and surface-wave problems [11,17], as well as in problems on unsteady aerodynamics [35]. The Struve functions are also important in particle quantum dynamical studies of spin decoherence [34] and nanotubes [38]. For more details about Struve functions, their generalizations and properties, the esteemed reader is invited to consider references, [7,8,18,23,29,33,40]. Recently, Nisar et al. [21] introduced and studied various properties of $k$-Struve function $S_{\nu,\nu}^k(x)$ defined by

$$S_{\nu,\nu}^k(x) := \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma(rk + \nu + \frac{3k}{2}) \Gamma(r + \nu + \frac{3k}{2})} \left(\frac{x}{2}\right)^{2r+\nu+1}.$$  (1.3)

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The sumudu transform of \( k \)-Struve function is given by

\[
S \left[ S_{k\nu,c}(x) \right] = \int_0^\infty e^{-t} S_{k\nu,c}(ut) dt
\]

\[
= \int_0^\infty e^{-t} \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k \left( r + \nu + \frac{3k}{2} \right) \Gamma \left( r + \frac{3}{2} \right)} \left( \frac{ut}{2} \right)^{2r+\frac{1}{2}+1} dt
\]

\[
= \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k \left( r + \nu + \frac{3k}{2} \right) \Gamma \left( r + \frac{3}{2} \right)} \int_0^\infty e^{-t} \left( \frac{ut}{2} \right)^{\frac{1}{2}+2r} dt
\]

\[
= \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k \left( r + \nu + \frac{3k}{2} \right) \Gamma \left( r + \frac{3}{2} \right)} \left( \frac{u}{2} \right)^{\frac{1}{2}+2r}
\]

Now, using

\[
\Gamma_k (\gamma) = k^{\gamma-1} \Gamma \left( \frac{\gamma}{k} \right)
\]

we have the following

\[
S \left[ S_{k\nu,c}(x) \right] = \sum_{r=0}^{\infty} \frac{(-c)^r}{k^{r+\frac{1}{2}+\frac{1}{2}r} \Gamma \left( r + \frac{3k}{2} \right) \Gamma \left( r + \frac{3}{2} \right)} \left( \frac{u}{2} \right)^{\frac{1}{2}+2r}
\]

Denoting the left hand side by \( G(u) \), we have

\[
G(u) = S \left[ S_{k\nu,c}(t); u \right]
\]

\[
= \left( \frac{u}{2} \right)^{\frac{1}{2}+1} k^{-\frac{1}{2}-\frac{1}{2}r} t \Psi_2 \left[ \frac{(\frac{5k}{2}, 2), (1, 1)}{(\frac{3k}{2}, 1), (\frac{3}{2}, 1)} \right] - \frac{cu^2}{4k}
\]

and inverse Sumudu transform of \( k \)-Struve function is given by

\[
S^{-1} \left[ S_{k\nu,c}(x) \right] = S^{-1} \left[ \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k \left( r + \nu + \frac{3k}{2} \right) \Gamma \left( r + \frac{3}{2} \right)} \left( \frac{u}{2} \right)^{\frac{1}{2}+2r} \right]
\]

\[
= \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k \left( r + \nu + \frac{3k}{2} \right) \Gamma \left( r + \frac{3}{2} \right)} \left( \frac{u}{2} \right)^{\frac{1}{2}+2r}
\]

Using (1.4), we get

\[
= \left( \frac{t}{2} \right)^{\frac{1}{2}} k^{\frac{1}{2}+\frac{1}{2}r} \Psi_3 \left[ \frac{(1, 1)}{(\frac{3k}{2}, 1), (\frac{3}{2}, 1), (\frac{3}{2}, 1)} \right] - \frac{ct^2}{4k}
\]

In this paper, we consider (1.3) to obtain the solution of the fractional kinetic equations. Our methodology herein is based on Sumudu transform, [4, 5]. Fractional calculus is developed to large area of mathematics physics and other engineering applications [14, 15, 19, 22, 24, 28, 31] because of its importance and efficiency. The fractional differential
equation between a chemical reaction or a production scheme (such as in birth-death processes) was established and treated by Haubold and Mathai [16], (also see [3,9,14]).

2. Solution of generalized fractional Kinetic equations for $k$-Struve function

Let the arbitrary reaction described by a time-dependent quantity $N = (N_i)$. The rate of change $\frac{dN}{dt}$ to be a balance between the destruction rate $\vartheta$ and the production rate $p$ of $N$, that is, $\frac{dN}{dt} = -\vartheta + p$. Generally, destruction and production depend on the quantity $N$ itself, that is,

$$\frac{dN}{dt} = -\vartheta (N_t) + p (N_t), \quad (2.1)$$

where $N_t$ described by $N_t (t^*) = N (t - t^*), t^* > 0$. Another form of (2.1) is,

$$\frac{dN_i}{dt} = -c_i N_i (t), \quad (2.2)$$

with $N_i (t = 0) = N_0$, which is the number of density of species $i$ at time $t = 0$ and $c_i > 0$. The solution of (2.2) is,

$$N_i (t) = N_0 e^{-c_i t}. \quad (2.3)$$

Integrating (2.2) gives,

$$N (t) - N_0 = -c_0 D_t^{-1} N (t), \quad (2.4)$$

where $D_t^{-1}$ is the special case of the Riemann-Liouville integral operator and $c$ is a constant. The fractional form of (2.4) due to [16] is,

$$N (t) - N_0 = -c_0^\nu D_t^{-\nu} N (t), \quad (2.5)$$

where $D_t^{-\nu}$ defined as

$$D_t^{-\nu} f (t) = \frac{1}{\Gamma (\nu)} \int_0^t (t - s)^{\nu - 1} f (s) ds, \Re (\nu) > 0. \quad (2.6)$$

Suppose that $f(t)$ is a real or complex valued function of the (time) variable $t > 0$ and $s$ is a real or complex parameter. The Laplace transform of $f(t)$ is defined by

$$F (p) = L [f(t) : p] = \int_0^\infty e^{-pt} f (t) dt, \quad \Re (p) > 0 \quad (2.7)$$

The Mittag-Leffler functions $E_{\alpha} (z)$ (see [20]) and $E_{\alpha,\beta} (x)$ [39] is defined respectively as

$$E_{\alpha} (z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma (\alpha n + 1)} \quad (z, \alpha \in \mathbb{C}; |z| < 0, \Re (\alpha) > 0). \quad (2.8)$$
\[ E_{\alpha,\beta}(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(\alpha n + \beta)}. \] (2.9)

**Theorem 1.** If \( d > 0, \nu > 0, \mu, c, t \in \mathbb{C} \) and \( l > -\frac{3}{2}k \) then the solution of generalized fractional kinetic equation

\[ N(t) = N_0 S_{\mu,c}^k (d^\nu t^\nu) - d^\nu 0 D_t^{-\nu} N(t), \] (2.10)

is given by the following formula

\[ N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k (rk + \mu + \frac{3}{2}k)} \left[ 2 \left( \frac{d^\nu (ut)^\nu}{2} \right)^{2r+\frac{3}{2}+1} \right] E_{\nu,\nu(2r+\frac{3}{2}+1)} (-d^\nu t^\nu). \] (2.11)

where \( E_{\nu,\nu(2r+\frac{3}{2}+1)} (-d^\nu t^\nu) \) is given in (2.9).

**Proof.** The Sumudu transform of Riemann-Liouville fractional integral operators is given by

\[ S \{ 0 D_t^{-\nu} f(t); u \} = u^\nu G(u), \] (2.12)

where \( G(u) \) is defined in (1.5). Now applying Sumudu transform both sides of (2.10) and applying the definition of \( k \)-Struve function given in (1.3), we have

\[ N^*(u) = S \{ N(t); u \} \]

\[ = N_0 S \left[ S_{\mu,c}^k (d^\nu t^\nu); u \right] - d^\nu S \left[ 0 D_t^{-\nu} N(t); u \right] \]

\[ = N_0 \left[ \int_0^\infty e^{-pt} \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k (rk + \mu + \frac{3}{2}k)} \left( \frac{d^\nu (ut)^\nu}{2} \right)^{2r+\frac{3}{2}+1} \right] \]

\[ - d^\nu u^\nu N^*(u), \]

where

\[ S \{ t^{\mu-1} \} = u^\mu - 1 \Gamma(\mu). \] (2.13)

By rearranging terms we get,

\[ N^*(u) + d^\nu u^\nu N^*(u) \]

\[ = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r}{\Gamma_k (rk + \mu + \frac{3}{2}k)} \left( \frac{d^\nu}{2} \right)^{2r+\frac{3}{2}+1} \]

\[ \times \int_0^\infty e^{-t} (ut)^{\nu(2r+\frac{3}{2}+1)} \] \[ dt \]
Taking inverse Sumudu transform of (2.14), equation involving classical Struve function as:

\[ N^*(u) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma[2r + \frac{\mu}{k} + 1]}{\Gamma_k(rk + \mu + \frac{3}{2}k) \Gamma\left(r + \frac{3}{2}\right)} \left(\frac{d^\nu}{2}\right)^{2r + \frac{\mu}{k} + 1} \]

Therefore

\[ N^*(u) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma[2r + \frac{\mu}{k} + 1]}{\Gamma_k(rk + \mu + \frac{3}{2}k) \Gamma\left(r + \frac{3}{2}\right)} \left(\frac{d^\nu}{2}\right)^{2r + \frac{\mu}{k} + 1} \]

\[ \times \left\{ u^{\nu(2r + \frac{\mu}{k} + 1)} \sum_{n=0}^{\infty} \left[-(du)^n\right] \right\} \]  

(2.14)

Taking inverse Sumudu transform of (2.14), and by using

\[ S^{-1}\{u^\nu; t\} = \frac{t^{\nu-1}}{\Gamma(\nu)}, \Re(\nu) > 0, \]

we have

\[ S^{-1}\{N^*(u)\} = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma[2r + \frac{\mu}{k} + 1]}{\Gamma_k(rk + \mu + \frac{3}{2}k) \Gamma\left(r + \frac{3}{2}\right)} \left(\frac{d^\nu}{2}\right)^{2r + \frac{\mu}{k} + 1} \]

\[ \times \sum_{n=0}^{\infty} (-1)^n (d)^\nu u^{(2r + \frac{\mu}{k} + n + 1)} \]

which gives,

\[ N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma[2r + \frac{\mu}{k} + 1]}{\Gamma_k(rk + \mu + \frac{3}{2}k) \Gamma\left(r + \frac{3}{2}\right)} \left(\frac{d^\nu}{2}\right)^{2r + \frac{\mu}{k} + 1} \]

\[ \times \left\{ \sum_{n=0}^{\infty} (-1)^n (d)^\nu u^{\nu(2r + \frac{\mu}{k} + n + 1)} \right\} \]

\[ = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma[2r + \frac{\mu}{k} + 1]}{\Gamma_k(rk + \mu + \frac{3}{2}k) \Gamma\left(r + \frac{3}{2}\right)} \left(\frac{d^\nu}{2}\right)^{2r + \frac{\mu}{k} + 1} \]

\[ \times \left\{ \sum_{n=0}^{\infty} (-1)^n (d)^\nu u^{\nu(2r + \frac{\mu}{k} + n + 1)} \right\} \]

\[ = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma[2r + \frac{\mu}{k} + 1]}{\Gamma_k(rk + \mu + \frac{3}{2}k) \Gamma\left(r + \frac{3}{2}\right)} \left(\frac{d^\nu}{2}\right)^{2r + \frac{\mu}{k} + 1} \]

\[ \times \left\{ \sum_{n=0}^{\infty} (-1)^n (d)^\nu u^{\nu(2r + \frac{\mu}{k} + n + 1)} \right\} \]

\[ \times E_{\nu,\nu(2r + \frac{\mu}{k} + 1)} \left(-d^\nu t^\nu\right) \]

which is the desired result.

\[ \square \]

**Corollary 2.1.** If we put \( k = 1 \) in (2.11) then we get the solution of fractional kinetic equation involving classical Struve function as:
If \( d > 0, \nu > 0, \mu, c, t \in \mathbb{C} \) and \( l > -\frac{3}{2} \) then the solution of generalized fractional kinetic equation

\[
N(t) = N_0 S_{\mu,c}^1 (d\nu t^\nu) - d\nu 0D_t^{-\nu} N(t),
\]

is given by the following formula

\[
N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma [\nu (2r + \mu + 1) + 1] 1}{\Gamma (r + \mu + \frac{\nu}{2}) \Gamma (r + \frac{3}{2})} \left( \frac{d\nu t^\nu}{2} \right)^{2r+\mu+1} E_{\nu,\nu(2r+\mu)+1} (-d\nu t^\nu). \tag{2.17}
\]

**Theorem 2.** If \( a > 0, d > 0, \nu > 0, c, \mu, t \in \mathbb{C} \), \( a \neq d \) and \( \mu > -\frac{3}{2}k \), then the equation

\[
N(t) = N_0 S_{\mu,c}^k (a\nu t^\nu) - d\nu 0D_t^{-\nu} N(t),
\]

is given by the following formula

\[
N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma [\nu (2r + \mu + 1) + 1] 1}{\Gamma (r + \mu + \frac{\nu}{2}) \Gamma (r + \frac{3}{2})} \left( \frac{d\nu t^\nu}{2} \right)^{2r+\mu+1} \times E_{\nu,\nu(2r+\mu)+1} (-a\nu t^\nu). \tag{2.19}
\]

where \( E_{\nu,\nu(2r+\mu)+1}(\cdot) \) is given in (2.9).

**Proof.** Theorem 2 can be proved in parallel with the proof of Theorem 1. So the details of proofs are omitted. \( \square \)

**Corollary 2.2.** By putting \( k = 1 \) in Theorem 2, we get the solution of fractional kinetic equation involving classical Struve function: If \( a > 0, d > 0, \nu > 0, c, \mu, t \in \mathbb{C} \), \( a \neq d \) and \( \mu > -\frac{3}{2} \), then the equation

\[
N(t) = N_0 S_{\mu,c}^1 (a\nu t^\nu) - d\nu 0D_t^{-\nu} N(t),
\]

is given by the following formula

\[
N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma [\nu (2r + \mu + 1) + 1] 1}{\Gamma (r + \mu + \frac{\nu}{2}) \Gamma (r + \frac{3}{2})} \left( \frac{d\nu t^\nu}{2} \right)^{2r+\mu+1} \times E_{\nu,\nu(2r+\mu)+1} (-a\nu t^\nu). \tag{2.20}
\]

**Theorem 3.** If \( d > 0, \nu > 0, c, \mu, t \in \mathbb{C} \) and \( \mu > -\frac{3}{2}k \), then the solution of the following equation

\[
N(t) = N_0 S_{\mu,c}^k (t^\nu) - d\nu 0D_t^{-\nu} N(t),
\]

is given by the following formula

\[
N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma [\nu (2r + \mu + 1) + 1] 1}{\Gamma (r + \mu + \frac{\nu}{2}) \Gamma (r + \frac{3}{2})} \left( \frac{t}{2} \right)^{2r+\mu+1} \times E_{\nu,\nu(2r+\mu)+1} (-d\nu t^\nu). \tag{2.23}
\]
where $E_{\nu,\nu(2r+\mu+1)}(\cdot)$ is given in (2.9)

**Corollary 2.3.** If we set $k = 1$ then (2.23) reduced as follows:

If $d > 0, \nu > 0, c, \mu, t \in \mathbb{C}$ and $\mu > -\frac{3}{2}$, then the solution of the following equation

$$N(t) = N_0 \mathcal{S}_{\mu,c}^1(t^{\nu}) - d_0^\nu D_t^{-\nu}N(t),$$

is given by the following formula

$$N(t) = N_0 \sum_{r=0}^{\infty} \frac{(-c)^r \Gamma(\nu(2r+\mu+1)+1)}{\Gamma(r+\mu+\frac{4}{2}) \Gamma(r+\frac{3}{2})} \frac{1}{t^{2r+\mu+1}} \times E_{\nu,\nu(2r+\mu+1)}(-d_0^{\nu}t^{\nu}).$$

(2.25)

3. **Graphical interpretation**

In this section we plot the graphs of our solutions of the fractional kinetic equation, which is established in (2.11). In each graph, we gave three solutions of the results on the basis of assigning different values to the parameters. In figures 1, we take $k = 1$ and $\nu = 0.5, 0.7, 0.9, 1, 1.5$. Similarly figures 2-3 are plotted respectively by taking $k = 2, 3$. Figures 4-6 are plotted by considering the solution given in (2.23) by taking $\nu = 0.5, 0.7, 0.9, 1, 1.5$ and $k = 1, 2, 3$. Other than $\nu$ and $k$ all other parameters are fixed by 1. It is clear from these figures that $N_t > 0$ for $t > 0$ and $N_t > 0$ is monotonic increasing function for $t \in (0, \infty)$. In this study, we choose first 50 terms of Mittag-Leffler function and first 50 terms of our solutions to plot the graphs. $N_t = 0$, when $t > 0$ and $N_t \to \infty$ when $t \to 1$ for all values of the parameters.

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