Method for calculating deflections of beams made of rubber concrete

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Abstract. In many bending elements, the defining calculation for ensuring the normal operation of the structure is the calculation of deflections, it is this characteristic of the structure, in particular the beams, that determines the section dimensions of the element, to ensure the necessary rigidity of structures. In view of this, the use of many polymer concrete as a material for the manufacture of bending structures becomes impossible due to the high deformation - as a consequence of the low elastic modulus. Our proposed polymer concrete based on liquid rubber (rubcon) has high compressive and tensile strengths, while the elastic modulus corresponds to ordinary cement concrete class B25. The introduction of fiber filaments into the material structure slightly increases these characteristics, and the rubber concrete obtained in this way with fiber reinforcement is called fiber rubcon. In order to ensure the possibility of calculating by way of determining the deflections of rubcon structures with cracks, we apply the principle of strain averaging of V.I. Murashov, while the uneven distribution of strains in the rubcon was taken into account by the coefficient $\psi_k$, the polymer concrete work between cracks is taken into account by the coefficient $\psi_s$, which are determined similarly to V.I. Murashov, A.S. Zalesov methods.

1. Introduction

Most polymer concrete structures have high strength characteristics, but they are also accompanied by high deformations, due to lower elastic modulus values (compared with cement concrete), and this in turn makes it difficult or impossible to use many polymer concrete as a material for the manufacture of bending elements. In turn, concrete in reinforced concrete constructions, having a sufficient modulus of elasticity, has, as a rule, lower strength characteristics (compared to polymer concrete) and is significantly susceptible to corrosion, which limits its scope to aggressive environments. Thus, there is a need for alternative materials having a complex of positive properties from those listed above. One such material is rubber concrete or abbreviated rubcon. As a result of studies of rubber concrete (rubcon) carried out in [1,2], including rubber concrete with fiber reinforcement (fiber reinforced concrete) [3], the physico-mechanical characteristics of polymer concrete were determined, on the basis of which it can be argued that rubber concrete is structural material. It has also been established that the elastic modules of rubcon and fiber rubcon are similar to the elastic modulus of traditional cement concrete. Many works on the study of structures made of rubber concrete are aimed at studying the strength and crack resistance of bending elements [4-6] or compressed [7]. However, in view of the fact that the elastic modulus has a significant effect on the rigidity of structures, which is also given in the dissertation [4], special attention must be paid to the deformation of structures made
of rubber concrete. The deformation of rubber concrete beams was studied at the center of collective use of VSTU, on experimental samples of rectangular section 60x120 mm with an element length of 1.4 m, these parameters were established on the basis of works [8-10], including works on the study of rubber concrete beams [4-6]. Beams were tested for pure bending - the most characteristic type of loading in the study of such elements [9-14]. Reinforcement of the compressed zone in the zone of pure bending of the experimental beam is absent. The test pattern of the experimental beams and the layout of the deflection meter to determine changes in vertical displacements during loading of the element are shown in figure 1.

![Figure 1. The testing scheme of the bending elements.](image)

Based on the experimental studies performed in SSS studies of the normal cross-sectional [15] and crack resistance [16], the maximum percentage of longitudinal reinforcement is 4.95 due to the most effective increase in the strength of the tensile zone before cracking. As a criterion for the deformation of bent elements from rubber concrete, a deflection was adopted, measured by an electronic type deflection meter in the middle of the span.

2. Determination of deflection of bent elements to crack formation

As a result of experimental studies [17], it was established that at stage I of the SSS (before crack appearance), vertical displacements obey an almost linear relation on the bending moment. When moving to stage II of SSS, i.e. after the appearance of a crack, the deflections continue to develop more intensively. Therefore, we propose to define the determination of vertical displacements in two stages: before crack appearance and after, when it is necessary to take into account the uneven distribution of deformations.

Before crack formation, the deflection value is determined by the formula [18]:

\[ f = \rho_m \frac{M^2}{B}, \]  

where \( B \) – The rigidity of the beam, which is determined by the formula:

\[ B = kE_iJ_{red}, \]  

\( k = 0.65 \) (0.85) – coefficient taking into account the effect on the stiffness of the bending element of inelastic deformations of the rubcon (fiber rubcon) of the tensile zone, obtained from the ratio of the values of elastic deformations to the limiting (at the time of crack formation) of the extreme stretched fiber of the beam without rod reinforcement;

\( J_{red} \) – moment of inertia of the reduced section relative to its center of gravity;

\( \rho_m \) – coefficient taking into account the loads scheme [18].

3. Determination of curvature of flexural elements with cracks

In order to ensure the possibility of calculating by way of determining the deflections of rubcon structures with cracks, we apply the principle of strain averaging of V.I. Murashov [19], according to this principle, the curvature of the bent axis of an element is determined with the assumption that the hypothesis of flat sections is true for a bent element damaged by cracks.

The curvature of the curved axis of the element is determined by the following formula:
where \( \varepsilon_{km(fkm)} \) – average deformation in rubcon (fiber rubcon), determined by the formula:

\[
\varepsilon_{km(fkm)} = \psi_{km(fkm)} \varepsilon_{k(fk)},
\]

(4)

\( \psi_{k(fk)} = 0.85 \) (0.9) - coefficient taking into account the uneven distribution of deformations in the rubcon and in the fiber rubcon, respectively, of the compressed zone between the cracks;

\( \varepsilon_{sm} \) – average strains in the reinforcing bar, determined by the formula:

\[
\varepsilon_{sm} = \psi_s \varepsilon_s,
\]

(5)

\( \psi_s \) – coefficient taking into account the effect of polymer concrete between cracks on tensile forces perceived by stretched reinforcing bar (figure 2), determined by the following formula:

\[
\psi_s = \frac{\sigma_s}{\sigma_s'},
\]

(6)

\( \sigma_s \) – stresses in the reinforcing bar in the section with a crack;

\( \bar{\sigma}_s \) – average values of stresses in the reinforcing bar.

The stresses in the reinforcing bar between the cracks are determined from the condition of equal action of tensile forces in the section with the crack and between them:

\[
\sigma_s A_s + 0.5 R_{k(fk)} \cdot b \cdot x_t = \sigma_s' A_s + 0.5 \sigma_{k(fk)} A_s (h_0 - x) \cdot b,
\]

(7)

where \( \sigma_s' \) – stress in the reinforcing bar between cracks;

\( x_t \) – the height of the tensile zone in the section with a crack;

\( x \) – the height of the compressed zone in the section with a crack;

\( \sigma_{k(fk)} = 0.5 \sigma_{k(fk)} \) – stresses in the rubcon (fiber rubcon) of the tensile zone between the cracks, tending to the ultimate tensile strength, but developing only in the elastic stage of polymer concrete.

After performing the transformations, we obtain the following formula for determining the stresses in the reinforcement between cracks:

\[
\sigma_s' = \sigma_s - 0.5 \frac{R_{k(fk)} (v \cdot (h_0 - x) - x_t) \cdot b}{A_s}.
\]

(8)

These formulas were obtained on the basis of empirical studies and on the basis of analysis of studies of stretched concrete between cracks in reinforced concrete elements [20,21]. Next, through the Mohr integral, we find the value of the deflection.

The calculated and experimental deflections of the experimented rubcon beams are shown in table 1, fiber rubcon beams in table 2.

From tables 1 and 2 it can be seen that the proposed method for determining the deflections provides sufficient convergence of the calculated values with the experimental ones, with the largest difference between them being 14.5%, in rubcon beams, and in fiber rubcon beams, 10.3%. Figures 3...
Table 1. Comparison of calculated and experimental deflections of rubcon beams before crack formation and at the time of fracture.

| Beam Code | \( \mu \), \% | \( M_{cr} \), kNm | \( f_{exp} \), mm | \( f_{cr} \), mm | \( \frac{f_{exp} - f_{cr}}{f_{exp}} \), \% | \( M_u \), kNm | \( f_{exp} \), mm | \( f_{cr} \), mm | \( \frac{f_{exp} - f_{cr}}{f_{exp}} \), \% |
|-----------|---------------|------------------|------------------|-----------------|--------------------------------|---------------|------------------|-----------------|--------------------------------|
| BRR-8     | 0.8           | 1.45             | 1.10             | 1.21            | -10.0                                      | 2.96          | 5.24             | 6.00            | -14.5                                      |
| BRR-10    | 1.25          | 1.48             | 1.24             | 1.24            | 0.4                                       | 4.28          | 7.04             | 6.61            | 6.1                                       |
| BRR-12    | 1.8           | 1.52             | 1.21             | 1.27            | -4.8                                      | 5.60          | 7.99             | 7.45            | 6.8                                       |
| BRR-2x10  | 2.5           | 1.76             | 1.54             | 1.47            | 4.6                                       | 7.82          | 9.10             | 8.29            | 8.9                                       |
| BRR-2x12  | 3.55          | 1.84             | 1.50             | 1.53            | -2.2                                      | 10.30         | 9.77             | 9.85            | -0.8                                      |
| BRR-2x14  | 4.95          | 1.88             | 1.42             | 1.56            | -10.1                                     | 14.00         | 13.25            | 11.83           | 10.7                                      |

*aIn the calculation, the elastic modulus of rubcon is assumed to be 25 \( \times 10^3 \) MPa.

Table 2. Comparison of calculated and experimental deflections of fiber rubcon beams before crack formation and at the time of fracture.

| Beam Code | \( \mu \), \% | \( M_{cr} \), kNm | \( f_{exp} \), mm | \( f_{cr} \), mm | \( \frac{f_{exp} - f_{cr}}{f_{exp}} \), \% | \( M_u \), kNm | \( f_{exp} \), mm | \( f_{cr} \), mm | \( \frac{f_{exp} - f_{cr}}{f_{exp}} \), \% |
|-----------|---------------|------------------|------------------|-----------------|--------------------------------|---------------|------------------|-----------------|--------------------------------|
| BRF-8     | 0.8           | 2.11             | 2.00             | 1.92            | 4.0                                        | 3.78          | 6.21             | 6.18            | -0.5                                        |
| BRF-10    | 1.25          | 2.40             | 2.39             | 2.18            | 8.6                                        | 5.59          | 8.01             | 7.18            | -10.3                                       |
| BRF-12    | 1.8           | 2.45             | 2.41             | 2.23            | 7.5                                        | 6.77          | 8.62             | 8.04            | -6.7                                        |
| BRF-2x10  | 2.5           | 2.50             | 2.49             | 2.27            | 8.7                                        | 8.77          | 9.81             | 9.06            | -7.7                                        |
| BRF-2x12  | 3.55          | 2.51             | 2.20             | 2.27            | -3.3                                       | 11.28         | 11.55            | 10.82           | -6.3                                        |
| BRF-2x14  | 4.95          | 2.70             | 2.26             | 2.45            | -8.4                                       | 15.10         | 13.57            | 12.99           | -4.2                                        |

*aIn the calculation, the elastic modulus of fiber rubcon is assumed to be 30 \( \times 10^3 \) MPa.

Figure 3. The relationship between the calculated and experimental deflections of the rubcon beams and the percentage of longitudinal reinforcement.
4. Discussion

The experimental studies [15-17] and the studies of the author [3] showed that the use of fiber rubcon bending elements is more effective as structures operated under the influence of aggressive environments with increased cracking requirements than rubcon bending elements without fiber reinforcement and reinforced concrete elements from conventional concrete. It should be noted that the deformation of rubcon elements is similar to traditional reinforced concrete elements. Rectangular cross-section rubcon and fiber rubcon elements reinforced with non-tensile longitudinal reinforcing bar can be used as beams in the frame of buildings, overpasses, technological platforms and other structures where aggressive environment are present, including floor and covering beams.

So as the recommended field of application for fiber rubcon bending elements is structures that are operating under the influence of aggressive environment of various nature, it is proposed to use the method for determining deflections before the crack appears as the main calculation method, since when a crack forms in the cross section, the outer surface of the reinforcing cage becomes accessible for exposure to an aggressive environment, and, consequently, as a result of corrosion of reinforcing bar and steel cord fibers, the load-bearing capacity of an element as a whole will decrease. In this case, when determining the curvature of bending elements with cracks, it is possible to apply the principle of strain averaging of V.I. Murashov.

5. Conclusion

1. Calculation of the deflections of the rubcon and fiber rubcon bending elements is recommended to be performed in two stages: the first - until the cracks are formed, taking into account the decrease in structural rigidity under the influence of inelastic deformations of the polymer concrete of the tensile zone with a coefficient of 0.85 for rubcon and a coefficient of 0.65 for fiber rubcon; second, the calculation of the deflections in the element damaged by cracks must be performed taking into account the uneven distribution of deformations in the polymer concrete of the compressed zone and taking into account the operation of the polymer concrete of the extended zone between the cracks with the coefficients $\psi_{ki}$ and $\psi_i$. In this case, the maximum difference between the experimental and calculated values of the deflections is 14.5%.

2. The field of application of rubcon and fiber rubcon bending elements is determined.

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