Bottom quark production cross section at fixed-target $pp$ experiments

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Abstract

The cross section for bottom quark production at fixed-target energies is calculated for a wide range of beam momenta. A detailed analysis is given for the HERA-B experiment. We consider both the order $\alpha_s^3$ cross section and the resummation of soft gluon corrections in all orders of QCD perturbation theory. The inclusive transverse momentum and rapidity distributions, including resummation, for bottom quark production at HERA-B are also presented.
1 Introduction

The calculation of production cross sections for heavy particles in QCD is made by invoking the factorization theorem [1] and expanding the contributions to the amplitude in powers of the coupling constant $\alpha_s(\mu^2)$. Recent investigations have shown that near threshold there can be large logarithms in the perturbation expansion which have to be resummed to make more reliable theoretical predictions. The application of these ideas to fixed-target Drell-Yan production has been the subject of many papers over the past few years [2]. The same ideas on resummation were applied to the calculation of the top-quark cross section at the Fermilab Tevatron in [3] and [4]. What is relevant in these reactions is the existence of a class of logarithms of the type $(\ln(1-z))^i/(1-z)$, where $i$ is the order of the perturbation expansion, and where one must integrate over the variable $z$ up to a limit $z = 1$. These terms are not actually singular at $z = 1$ due to the presence of terms in $\delta(1-z)$. However the remainder can be quite large. In general one writes such terms as “plus” distributions, which are then convoluted with regular test functions (the parton densities).

In this paper we examine the production of $b$-quarks in a situation where the presence of these large logarithms is of importance, namely in a fixed-target experiment to be performed in the HERA ring at DESY. This actual experiment has the name HERA-B [5, 6] and involves colliding the circulating proton beam against a stationary copper wire in the beam pipe. The nominal beam energy of the protons is 820 GeV, so that the square root of the center-of-mass (c.m.) energy is $\sqrt{S} = 39.2$ GeV. Taking the $b$-quark mass as $m_b = 4.75\text{GeV}/c^2$ then the ratio of $m_b/\sqrt{S} \approx 1/8$. If we choose the renormalization scale in the running coupling constant as $m_b$ then $\alpha_s(m_b^2) \approx 0.2$ so $\alpha_s(m_b^2) \ln(\sqrt{S}/m_b) \approx 0.4$. This number is small enough that we expect a reasonably convergent perturbation series.

In perturbation theory with a hard scale we can use the standard expression for the order-by order cross section in QCD, namely

$$\sigma(S, m^2) = \int_{4m^2}^1 dx_1 \int_{4m^2}^1 dx_2 \sum_{ij} f_i(x_1, \mu^2) f_j(x_2, \mu^2) \sigma_{ij}(s = x_1 x_2 S, m^2, \mu^2),$$

(1.1)

where the $f_i(x, \mu^2)$ are the parton densities at the factorization scale $\mu^2$ and the $\sigma_{ij}$ are the partonic cross sections. The numerical results for the hadronic
cross sections depend on the choice of the parton densities, which involves the mass factorization scale $\mu^2$; the choice of the running coupling constant, which involves the renormalization scale (also normally chosen to be $\mu^2$); and the choice for the actual mass of the $b$-quark. In lowest order or Born approximation the actual numbers for the cross section show a large sensitivity to these parameters. In section 3 we will show plots of the production cross section in leading order (LO), i.e. $O(\alpha_s^2)$, and next-to-leading order (NLO), i.e. $O(\alpha_s^3)$. The NLO results follow from the work of the two groups [7] and [8, 9]. However even including the NLO corrections does not completely fix the cross section. The sensitivity to our lack of knowledge of even higher terms in the QCD expansion is usually demonstrated by varying the scale choice up and down by factors of two. In general it is impossible to make more precise predictions given the absence of a calculation in next-to-next-to-leading order (NNLO). However in specific kinematical regions we can do so.

The threshold region is one of these regions. In this region one finds that there are large logarithms which arise from an imperfect cancellation of the soft-plus-virtual (S+V) terms in the perturbation expansion. These logarithms are exactly of the same type mentioned above. We will see in section 3 that the gluon-gluon channel is the dominant channel for the production of $b$-quarks near threshold in a fixed-target $pp$ experiment. This is not the case for the production of the top quark at the Fermilab Tevatron, which is a proton-antiproton collider, and where the dominant channel is the quark-antiquark one. That was fortunate as the exponentiation of the soft-plus-virtual terms in [3] is on a much more solid footing in the $q\bar{q}$ channel, due to all the past work which has been done on the Drell-Yan reaction [2]. Since the gluon-gluon channel is now the most important one we are forced to reexamine all “large” corrections near threshold, including both Coulomb-like and large constant terms. We will do that in section 2 where we will present all the relevant formulae at the partonic level and will also discuss the exponentiation of these terms. In addition we will present sub-leading S+V terms and discuss their contribution to the total S+V cross section. Section 3 contains the analysis of the hadron-hadron cross section which is relevant for the HERA-B experiment as well as for fixed-target $pp$ experiments in general. We give results in LO, in NLO and after resummation. Finally in Section 4 we give our conclusions and discuss where more work should be done in the future.
2 Results for parton-parton reactions

The partonic processes that we examine are

\[ i(k_1) + j(k_2) \rightarrow Q(p_1) + \bar{Q}(p_2), \]  

(2.1)

where \( i, j = g, g \) or \( i, j = q, \bar{q} \) and \( Q, \bar{Q} \) are heavy quarks \( (c, b, t) \). The square of the parton-parton c.m. energy is \( s = (k_1 + k_2)^2 \).

We begin with an analysis of heavy quark production in the \( q\bar{q} \) channel. The Born cross section in this channel is given by

\[ \sigma^{(0)}_{q\bar{q}}(s, m^2) = \frac{2\pi}{3} \alpha_s^2(\mu^2) K_{q\bar{q}} N C_F \frac{1}{s} \beta \left( 1 + \frac{2m^2}{s} \right), \]

(2.2)

where \( C_F = (N^2 - 1)/(2N) \) is the Casimir invariant for the fundamental representation of \( SU(N) \), \( K_{q\bar{q}} = N^{-2} \) is a color average factor, \( m \) is the heavy quark mass, \( \mu \) denotes the renormalization scale, and \( \beta = \sqrt{1 - 4m^2/s} \). Also \( N = 3 \) for the \( SU(3) \) color group in QCD. The threshold behavior \( (s \rightarrow 4m^2) \) of this expression is given by

\[ \sigma^{(0)}_{q\bar{q}, \text{thres}}(s, m^2) = \pi \alpha_s^2(\mu^2) K_{q\bar{q}} N C_F \frac{1}{s} \beta. \]

(2.3)

Complete analytic results are not available for the NLO cross section as some integrals are too complicated to do by hand. However in [9] analytic results are given for the soft-plus-virtual contributions to the cross section, and for the approximation to the cross section near threshold. Simple formulae which yield reasonable approximations to the exact \( O(\alpha_s^3) \) results have been constructed in [10]. From these results one can derive that the Coulomb terms to first order in the \( q\bar{q} \) channel are given by

\[ \sigma^{(\pi^2)}_{q\bar{q}}(s, m^2) = \sigma^{(0)}_{q\bar{q}}(s, m^2) \frac{\pi \alpha_s(\mu^2)}{2\beta} \left( C_F - \frac{C_A}{2} \right) \]

(2.4)

in the \( \overline{\text{MS}} \) scheme, where \( C_A = N \) is the Casimir invariant for the adjoint representation of \( SU(N) \). These terms are distinguished by their typical \( \beta^{-1} \) behaviour near threshold which, after multiplication by the Born cross section, yield finite cross sections at threshold in NLO. We note that \( C_F - C_A/2 = -1/6 \) is a negative quantity for \( SU(3) \) and that the first-order
Coulomb correction is negative (the interaction is repulsive). From the original work of Schwinger \[11\] we know that the Coulomb terms exponentiate. Therefore we resum them by writing

\[
\sigma^{(\pi^2)}_{\bar{q}q}, \text{res} (s, m^2) = \sigma^{(0)}_{\bar{q}q} (s, m^2) \exp \left[ \frac{\pi \alpha_s(\mu^2)}{2\beta} \left( C_F - \frac{C_A}{2} \right) \right]. \tag{2.5}
\]

Since the exponent is negative, the exponentiation of the Coulomb terms actually suppresses the cross section, and the resummed result goes to zero at threshold. In a previous treatment of threshold effects \[12\] the exponentiation was done in a different way, namely by writing

\[
\sigma^{(\pi^2), \text{res}}_{\bar{q}q} (s, m^2) = \sigma^{(0)}_{\bar{q}q} (s, m^2) \frac{X_{(8)}}{\exp X_{(8)} - 1}, \tag{2.6}
\]

where

\[
X_{(8)} = -\frac{\pi \alpha_s(\mu^2)}{\beta} \left( C_F - \frac{C_A}{2} \right) = \frac{1}{6} \frac{\pi \alpha_s}{\beta}. \tag{2.7}
\]

The subscript in \(X_{(8)}\) indicates that in the \(q\bar{q}\) channel, where the process has to go via an intermediate \(s\)-channel gluon, the heavy quark pairs are produced exclusively in the color octet state. We have checked that the difference between the two methods of resummation is numerically negligible.

In the DIS scheme in addition to the Coulomb terms we also have a large constant contribution so that the first order result near threshold is

\[
\sigma^{(\pi^2)}_{\bar{q}q} (s, m^2) = \sigma^{(0)}_{\bar{q}q} (s, m^2) \left[ \frac{\pi \alpha_s(\mu^2)}{2\beta} \left( C_F - \frac{C_A}{2} \right) + \frac{\alpha_s(\mu^2)}{\pi} C_F \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \right]. \tag{2.8}
\]

We know that large constants exponentiate in the Drell-Yan reaction \[2\] so we assume that the same holds here and write the result in the DIS scheme as

\[
\sigma^{(\pi^2), \text{res}}_{\bar{q}q} (s, m^2) = \sigma^{(0)}_{\bar{q}q} (s, m^2) \exp \left[ \frac{\pi \alpha_s(\mu^2)}{2\beta} \left( C_F - \frac{C_A}{2} \right) \right] \\
\times \exp \left[ \frac{\alpha_s(\mu^2)}{\pi} C_F \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \right]. \tag{2.9}
\]

This expression also goes to zero at threshold. We have included the constant terms to see their effect at larger values of \(\beta\).
Since the total parton-parton cross sections only depend on the variables $s$ and $m^2$ they can be expressed in terms of scaling functions as follows

$$\sigma_{ij}(s, m^2) = \sum_{k=0}^{\infty} \sigma_{ij}^{(k)}(s, m^2) = \frac{\alpha_s^2(\mu^2)}{m^2} \sum_{k=0}^{\infty} (4\pi \alpha_s(\mu^2))^k \sum_{l=0}^{k} f_{ij}^{(k,l)}(\eta) \ln^l \frac{\mu^2}{m^2},$$

where we denote by $\sigma^{(k)}$ the $O(\alpha_s^{k+2})$ contribution to the cross section. The scaling functions $f_{ij}^{(k,l)}(\eta)$ depend on the scaling variable $\eta = s/4m^2 - 1 = s\beta^2/4m^2$.

In fig. 1 we plot $f_{ij}^{(k,0)}(\eta)$ for $k = 0, 1$ for the exact and threshold expressions (from [9]) in the $\overline{\text{MS}}$ scheme. We also plot $f_{ij}^{(\pi^2),\text{res}}(\eta, \alpha_s)$ which we define by

$$\sigma_{qq}^{(\pi^2),\text{res}}(s, m^2) = \frac{\alpha_s^2(\mu^2)}{m^2} f_{qq}^{(\pi^2),\text{res}}(\eta, \alpha_s).$$

We see that the threshold Born approximation is excellent for small $\eta$ and reasonable for the entire range of $\eta$ shown. As expected the resummed Coulomb terms suppress the Born result throughout the entire range of $\eta$ and go to zero at threshold. We also note that the threshold first-order approximation is good only very near to threshold. In fig. 2 we plot the corresponding functions for the DIS scheme. Here the first-order corrections are smaller than in the $\overline{\text{MS}}$ scheme. Again the threshold first-order approximation is good only very close to threshold. The resummed result differs greatly from that in the $\overline{\text{MS}}$ scheme. Here the additional exponentiation of large constants cancels the negative contribution of the Coulomb terms and produces a large enhancement of the Born term in the region $0.1 < \eta < 1$ which, as we will show in the next section, is the most important region kinematically for the production of $b$-quarks at HERA-B. For small $\eta$, however, the total resummed result is dominated by the pure Coulomb terms and thus suppresses the Born term. Finally, we also show the resummed result where the only constant that we exponentiate is the $\pi^2/3$ term in (2.9). In this case the enhancement of the Born term in our region of interest is much smaller.

The analysis of the contributions to the gluon-gluon channel in NLO is much more complicated. First of all there are three Born diagrams each with a different color structure. Therefore only few terms near threshold
are proportional to the Born cross section. The exact Born term in the $gg$ channel is
\[
\sigma^{(0)}_{gg}(s, m^2) = 4\pi\alpha_s^2(\mu^2) K_{gg} NC_F \frac{1}{s} \left\{ C_F \left[ -\left( 1 + \frac{4m^2}{s} \right) \beta \right. \right.
\]
\[
+ \left. \left( 1 + \frac{4m^2}{s} - \frac{8m^4}{s^2} \right) \ln \frac{1 + \beta}{1 - \beta} \right] \right.
\]
\[
+ \left. C_A \left[ -\left( \frac{1}{3} + \frac{5m^2}{3s} \right) \beta + \frac{4m^4}{s^2} \ln \frac{1 + \beta}{1 - \beta} \right] \right\}, \quad (2.12)
\]
where $K_{gg} = (N^2 - 1)^{-2}$ is a color average factor. The threshold behavior $(s \to 4m^2)$ of this expression is given by
\[
\sigma^{(0)}_{gg, \text{thres}}(s, m^2) = \pi\alpha_s^2(\mu^2) K_{gg} \frac{1}{s} NC_F [4C_F - CA]\beta. \quad (2.13)
\]
Again, the complete NLO expression for the cross section in the $gg$ channel is unavailable but analytic results are given for the S+V terms in [8]. These were used in [10] to analyze the magnitude of the cross section near threshold. From the approximate expressions given in [10] one can extract the $\pi^2$ terms to first order in the $gg$ channel. These are
\[
\sigma^{(\pi^2)}_{gg}(s, m^2) = \alpha_s^3(\mu^2) NC_K K_{gg} \frac{\pi^2}{s} \left[ \frac{5}{8} + \frac{1}{24} \beta^2 + \frac{16}{s^3} m^6 \right.
\]
\[
+ \left( \frac{32}{s^4} \frac{m^6}{s^4} - \frac{10}{s^2} \frac{m^4}{s^4} \right) \frac{1}{\beta} \ln \frac{1 + \beta}{1 - \beta} \right]
\]
\[
+ \alpha_s^3 C_{\text{QED}} K_{gg} \frac{\pi^2}{s} \left[ -\frac{1}{4} - \frac{16}{s^3} m^6 \right.
\]
\[
+ \left. \left( -32 \frac{m^8}{s^4} + 8 \frac{m^4}{s^2} \right) \frac{1}{\beta} \ln \frac{1 + \beta}{1 - \beta} \right], \quad (2.14)
\]
where $C_K = (N^2 - 1)/N = 2NC_F C_A - 4NC_F$, and $C_{\text{QED}} = (N^4 - 1)/N^2 = -4C_F^2 + 4C_A C_F$. These are not proportional to the Born term so that it is not clear how to resum them. The threshold behavior of (2.14) is given by
\[
\sigma^{(\pi^2)}_{gg, \text{thres}}(s, m^2) = \alpha_s^3(\mu^2) K_{gg} \frac{\pi^2}{s} \frac{1}{4} \left[ -\frac{NC_K}{2} + C_{\text{QED}} \right], \quad (2.15)
\]
which is proportional to the threshold Born term. Therefore the threshold approximation for the $\pi^2$ terms in NLO can be written as

$$\sigma^{(0)+\pi^2}_{gg,\text{thres}}(s, m^2) = \sigma^{(0)}_{gg,\text{thres}}(s, m^2) \left[ 1 + \frac{\pi \alpha_s(\mu^2)}{4\beta} \left( \frac{-NC_K/2 + C_{QED}}{(4C_F - CA)NC_F} \right) \right],$$

(2.16)

or, writing the color factors in terms of $N$, as

$$\sigma^{(0)+\pi^2}_{gg,\text{thres}}(s, m^2) = \sigma^{(0)}_{gg,\text{thres}}(s, m^2) \left[ 1 + \frac{\pi \alpha_s(\mu^2)}{4\beta} \frac{N^2 + 2}{N(N^2 - 2)} \right].$$

(2.17)

The correction is positive indicating a Coulomb attraction in this channel. Then we proceed to resum these terms by the Schwinger method

$$\sigma^{(\pi^2),\text{res}}_{gg,\text{thres}}(s, m^2) = \sigma^{(0)}_{gg,\text{thres}}(s, m^2) \frac{X}{1 - \exp(-X)},$$

(2.18)

where

$$X = \frac{\pi \alpha_s(\mu^2)}{2\beta} \frac{N^2 + 2}{N(N^2 - 2)} = \frac{11\pi \alpha_s}{42\beta} \text{ for } SU(3).$$

(2.19)

The reason that we have exponentiated in this way is that $X$ is positive and tends to infinity when $\beta \to 0$. We expect that the Coulomb terms are only important very close to threshold. In [11] the Coulomb singlet (attractive interaction) and octet (repulsive interaction) contributions to the $gg$ channel were exponentiated separately giving

$$\sigma^{(s)}_{gg} = \frac{2}{7} \sigma^{(0)}_{gg} \frac{X(s)}{1 - \exp(-X(s))}, \quad X(s) = \frac{4}{3} \frac{\pi \alpha_s}{\beta},$$

(2.20)

and

$$\sigma^{(8)}_{gg} = \frac{5}{7} \sigma^{(0)}_{gg} \frac{X(8)}{\exp(X(8)) - 1}, \quad X(8) = \frac{1}{6} \frac{\pi \alpha_s}{\beta},$$

(2.21)

respectively. Again we have checked that the difference between the two methods of exponentiation is not significant numerically.

In fig.3 we plot the scaling functions $f_{gg}^{(k,0)}(\eta)$ with $k = 0, 1$ in the $\overline{\text{MS}}$ scheme for the exact and threshold expressions (from [8]). We also show
\[ f_g^{(\pi^2), \text{res}} \] which is defined in analogy to (2.11). We see that the Born and first-order threshold approximations are good only very close to threshold. The resummed result enhances the Born cross section and tends to a positive constant at threshold. The approximation is not good in the region \( 0.1 < \eta < 1 \) so we turn now to a discussion of the important terms in this region.

In [3] an approximation was given for the NLO soft-plus-virtual (S+V) contributions and the analogy with the Drell-Yan process was exploited to resum them to all orders of perturbation theory. The S+V approximation is adequate in the kinematical region of interest \( 0.1 < \eta < 1 \) for the \( q\bar{q} \) channel, but not as good for the \( gg \) channel in the MS scheme. Therefore we reexamined the approximate formulae given in [10] for the initial state gluon bremsstrahlung (ISGB) mechanism to see if there are subleading terms that will improve the S+V approximation. Let us see the structure of these terms.

We are discussing partonic reactions of the type

\[ i(k_1) + j(k_2) \to Q(p_1) + \bar{Q}(p_2) + g(k_3), \]

and we introduce the kinematic variables

\[ t_1 = (k_2 - p_2)^2 - m^2, \]
\[ u_1 = (k_1 - p_2)^2 - m^2, \]
\[ s_4 = s + t_1 + u_1. \]

The variable \( s_4 \) depends on the four-momentum of the extra partons emitted in the reaction. The first-order S+V result for the \( q\bar{q} \) channel in the MS scheme is

\[
s^2 d^2\frac{d\sigma^{(1)}_{q\bar{q}}(s, t_1, u_1)}{dt_1 du_1} = \sigma_{q\bar{q}}^B(s, t_1, u_1) \frac{2C_F}{\pi} \alpha_s(\mu^2) \times \left\{ \left[ \frac{1}{s_4} \left( 2 \ln \frac{s_4}{m^2} + \ln \frac{m^2}{\mu^2} \right) \theta(s_4 - \Delta) \right. \right.
\]
\[ + \left( \ln^2 \frac{\Delta}{m^2} + \ln \frac{\Delta}{m^2} \ln \frac{m^2}{\mu^2} \right) \delta(s_4) \bigg] \]
\[ + \left[ -\frac{C_A}{2C_F} \frac{1}{s_4} \theta(s_4 - \Delta) - \frac{C_A}{2C_F} \ln \frac{\Delta}{m^2} \delta(s_4) \bigg] \right\} (2.22) \]

where

\[
\sigma_{q\bar{q}}^B(s, t_1, u_1) = \pi \alpha_s^2(\mu^2) K_{q\bar{q}} NC_F \left[ \frac{t_1^2 + u_1^2}{s^2} + \frac{2m^2}{s} \right]. \quad (2.23)
\]

Here \( \Delta \) is a small parameter used to allow us to distinguish between the soft \( (s_4 < \Delta) \) and the hard \( (s_4 > \Delta) \) regions in phase space. The terms in the first pair of square brackets in (2.22) are the leading S+V terms given in [3] and those in the second pair of square brackets are subleading terms that we want to examine. In fig. 4 we plot the scaling functions \( f_{q\bar{q}}^{(1,0)} \) for the exact result, the leading S+V result, and the S+V result with both leading and subleading...
terms. The leading S+V result is a reasonable approximation to the exact result in our region of interest $0.1 < \eta < 1$. The addition of Coulomb terms worsens the leading S+V result. We also see that when we include the subleading terms our approximation does not improve much in the region of interest. However, when we add both the first order Coulomb term and subleading terms to the leading S+V result we get a very good agreement with the exact result. Nevertheless, this is still not a major improvement over the simple leading S+V result. In the DIS scheme the analogous results are

$$s^2 \frac{d^2 \sigma^{(1)}_{q\bar{q}}(s, t_1, u_1)}{dt_1 du_1} = \sigma^{B}_{q\bar{q}}(s, t_1, u_1) \frac{2C_F}{\pi} \alpha_s(\mu^2) \times \left\{ \left[ \frac{1}{s_4} \left( \ln \frac{s_4}{m^2} + \ln \frac{m^2}{\mu^2} \right) \theta(s_4 - \Delta) \right. \right.$$

$$+ \left( \frac{1}{2} \ln^2 \frac{\Delta}{m^2} + \ln \frac{\Delta}{m^2} \ln \frac{m^2}{\mu^2} \right) \delta(s_4) \left. \right\} + \left[ \left( \frac{3}{4} + \ln 2 - \frac{C_A}{2C_F} \right) \frac{1}{s_4} \theta(s_4 - \Delta) \right.$$

$$+ \left( \frac{3}{4} + \ln 2 - \frac{C_A}{2C_F} \right) \ln \frac{\Delta}{m^2} \delta(s_4) \right\} . \quad (2.24)$$

In fig. 5 we plot the corresponding scaling functions. Here the addition of subleading terms worsens the leading S+V approximation. The addition of Coulomb terms and large constants enhances the first-order approximate results considerably. We also show, for comparison, the results of the addition of the Coulomb terms and the $\pi^2/3$ constant term only to the approximate results. These last curves are the best fits to the exact result in the region $0.1 < \eta < 1$.

The resummation of the leading S+V terms has been given in [3]. The result is

$$s^2 \frac{d^2 \sigma_{q\bar{q}}^{res}(s, t_1, u_1)}{dt_1 du_1} = \sigma^{B}_{q\bar{q}}(s, t_1, u_1) \left[ \frac{df(s_4/m^2, m^2/\mu^2)}{ds_4} \theta(s_4 - \Delta) \right.$$

$$+ f(\frac{\Delta}{m^2}, \frac{m^2}{\mu^2}) \delta(s_4) \right], \quad (2.25)$$
where

\[
f \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) = \exp \left[ A \frac{C_F}{\pi} \bar{\alpha}_s \left( \frac{s_4}{m^2}, m^2 \right) \ln^2 \frac{s_4}{m^2} \right] \frac{s_4/m^2}{\Gamma(1+\eta)} \exp(-\eta \gamma_E). \tag{2.26}
\]

Expressions for \(A, \bar{\alpha}_s, \eta,\) and \(\gamma_E\) are given in [3]. As the NNLO cross section is not known exactly we do not how to resum the subleading terms. A reasonable guess would be (2.25) with the function \(f\) given now by

\[
f \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) = f_{\text{Leading}} \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) \exp \left[ -C_A \frac{\pi}{\alpha_s(\mu^2)} \ln \frac{s_4}{m^2} \right] \tag{2.27}
\]

in the \(\overline{\text{MS}}\) scheme, and

\[
f \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) = f_{\text{Leading}} \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) \exp \left[ \frac{C_F}{2\pi} \left( 3 + 4 \ln 2 - \frac{2C_A}{C_F} \right) \alpha_s(\mu^2) \ln \frac{s_4}{m^2} \right] \tag{2.28}
\]

in the DIS scheme, where now we call \(f_{\text{Leading}}\) the expression in (2.26).

Now let us see the analogous results for the \(gg\) channel in the \(\overline{\text{MS}}\) scheme.

We have

\[
s^2 \frac{d^2 \sigma_{gg}^{(1)}(s,t_1,u_1)}{dt_1 du_1} = \sigma_{gg}^B(s,t_1,u_1) \frac{2C_A}{\pi} \alpha_s(\mu^2) \times \left\{ \left[ \frac{1}{s_4} \left( 2 \ln \frac{s_4}{m^2} + \ln \frac{m^2}{\mu^2} \right) \theta(s_4 - \Delta) \right.ight.
\]
\[
+ \delta(s_4) \left( \ln^2 \frac{\Delta}{m^2} + \ln \frac{\Delta}{m^2} \ln \frac{m^2}{\mu^2} \right) \left. \right\}
\]
\[
+ \left[ \frac{3C_A - 8C_F}{-2C_A + 8C_F} \left( \frac{1}{s_4} \theta(s_4 - \Delta) + \ln \frac{\Delta}{m^2} \delta(s_4) \right) \right]. \tag{2.29}
\]

where

\[
\sigma_{gg}^B(s,t_1,u_1) = 2\pi \alpha_s^2(\mu^2) K_{gg} N C_F \left[ C_F - C_A \frac{t_1 u_1}{s^2} \right]
\]
\[
\times \left\{ \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s}{t_1 u_1} \left( 1 - \frac{m^2 s}{t_1 u_1} \right) \right\}. \tag{2.30}
\]

Again, the terms in the first pair of square brackets in (2.29) are the leading \(S+V\) terms and those in the second pair of square brackets are subleading.
terms. In fig. 6 we plot the scaling functions $f_{gg}^{(1,0)}$ for the exact result, the leading S+V result, and the S+V result with both leading and subleading terms. We note that the leading S+V approximate result is significantly smaller than the exact result and that the addition of subleading terms improves the approximation considerably. This is important since, as we will see in the next section, the $gg$ channel is dominant for the production of $b$-quarks at HERA-B. Also the addition of Coulomb terms further improves the approximation. The resummation of the leading S+V terms for the $gg$ channel has also been given in [3]. The result is

$$s^2 \frac{d^2 \sigma_{gg}^{res}(s,t_1,u_1)}{dt_1 du_1} = \sigma_{gg}^{B}(s,t_1,u_1) \left[ \frac{df(s_4/m^2, m^2/\mu^2)}{ds_4} \theta(s_4 - \Delta) + f \left( \frac{\Delta}{m^2}, -\frac{m^2}{\mu^2} \right) \delta(s_4) \right],$$

(2.31)

where

$$f \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) = \exp \left[ \frac{2C_A}{\pi} \tilde{\alpha}_s \left( \frac{s_4}{m^2}, m^2 \right) \ln^2 \frac{s_4}{m^2} \frac{[s_4/m^2]^{\eta}}{\Gamma(1 + \eta)} \exp(-\eta \gamma_E) \right].$$

(2.32)

Again, as the NNLO cross section is not known exactly we do not know how to resum the subleading terms. A reasonable guess would be (2.31) with the function $f$ given now by

$$f \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) = f_{Leading} \left( \frac{s_4}{m^2}, \frac{m^2}{\mu^2} \right) \exp \left[ \frac{2C_A}{\pi} \frac{3C_A - 8C_F}{-2C_A + 8C_F} \tilde{\alpha}_s(\mu^2) \ln \frac{s_4}{m^2} \right],$$

(2.33)

where now we call $f_{Leading}$ the expression in (2.32).

## 3 Results for bottom quark production at fixed-target $pp$ experiments and HERA-B

In this section we discuss $b$-quark production at HERA-B and also at fixed-target $pp$ experiments in general, and we examine the effects of the various resummation procedures that were discussed in the previous section. Following the notation in [3] the total hadron-hadron cross section in order $\alpha_s^k$
\[
\sigma_H^{(k)}(S, m^2) = \sum_{ij} \int_{4m^2/S}^{1} d\tau \Phi_{ij}(\tau, \mu^2) \sigma_{ij}^{(k)}(\tau S, m^2, \mu^2),
\]

where \(S\) is the square of the hadron-hadron c.m. energy and \(i, j\) run over \(q, \bar{q}\) and \(g\). The parton flux \(\Phi_{ij}(\tau, \mu^2)\) is defined via

\[
\Phi_{ij}(\tau, \mu^2) = \int_{\tau}^{1} dx \frac{1}{x} H_{ij}(x, x, \mu^2),
\]

and \(H_{ij}\) is a product of the scale-dependent parton distribution functions \(f_{h}^{i}(x, \mu^2)\), where \(h\) stands for the hadron which is the source of the parton \(i\)

\[
H_{ij}(x_1, x_2, \mu^2) = f_{h}^{i}(x_1, \mu^2)f_{j}^{h}(x_2, \mu^2).
\]

The mass factorization scale \(\mu\) is chosen to be identical with the renormalization scale in the running coupling constant.

In the case of the all-order resummed expression the lower boundary in (3.1) has to be modified according to the condition \(s_0 < s - 2m_s^{1/2}\), where \(s_0\) is defined below (see [3]). Resumming the soft gluon contributions to all orders we obtain

\[
\sigma_H^{\text{res}}(S, m^2) = \sum_{ij} \int_{\tau_0}^{1} d\tau \Phi_{ij}(\tau, \mu^2) \sigma_{ij}(\tau S, m^2, \mu^2),
\]

where \(\sigma_{ij}\) is given in (3.24) of [3] and

\[
\tau_0 = \frac{[m + (m^2 + s_0)^{1/2}]^2}{S},
\]

with \(s_0 = m^2(\mu_0^2/\mu^2)^{3/2}\) (\(\overline{\text{MS}}\) scheme) or \(s_0 = m^2(\mu_0^2/\mu^2)^2\) (DIS scheme). Here \(\mu_0\) is the non-perturbative parameter used in [3]. It is used to cut off the resummation since the resummed corrections diverge for small \(\mu_0\).

We now specialize to bottom quark production at HERA-B where \(\sqrt{S} = 39.2\) GeV. In the presentation of our results for the exact, approximate, and resummed hadronic cross sections we use the MRSD\(') parametrization for the parton distributions [13]. Note that the hadronic results only involve partonic distribution functions at moderate and large \(x\), where there is little difference between the various sets of parton densities. We have used the MRSD\(') set 34 as given in PDFLIB [14] in the DIS scheme with the number
of active light flavors $n_f = 4$ and the QCD scale $\Lambda_5 = 0.1559 \text{ GeV}$. We have used the two-loop corrected running coupling constant as given by PDFLIB.

First, we discuss the NLO contributions to bottom quark production at HERA-B using the results in [7-9]. Except when explicitly stated otherwise we will take the factorization scale $\mu = m_b$ where $m_b$ is the $b$-quark mass. Also, throughout the rest of this paper, we will use $m$ and $m_b$ interchangeably.

In fig. 7 we show the relative contributions of the $q\bar{q}$ channel in the DIS scheme and the $gg$ channel in the $\overline{\text{MS}}$ scheme as a function of the bottom quark mass. We see that the $gg$ contribution is the dominant one, lying between 70% and 80% of the total NLO cross section for the range of bottom mass values given. The $q\bar{q}$ contribution is smaller and makes up most of the remaining cross section. The relative contributions of the $gq$ and the $g\bar{q}$ channels in the DIS scheme are negative and very small and they are also shown in the plot. The situation here is the reverse of what is known about top quark production at the Fermilab Tevatron where $q\bar{q}$ is the dominant channel with $gg$ making up the remainder of the cross section, and $gq$ and $g\bar{q}$ making an even smaller relative contribution than is the case for bottom quark production at HERA-B. The reason for this difference between top quark and bottom quark production is that the Tevatron is a $p\bar{p}$ collider while HERA-B is a fixed-target $pp$ experiment. Thus, the parton densities involved are different and since sea quark densities are much smaller than valence quark densities, the $q\bar{q}$ contribution to the hadronic cross section diminishes for a fixed-target $pp$ experiment relative to a $p\bar{p}$ collider for the same partonic cross section.

In fig. 8 we show the $K$ factors for the $q\bar{q}$ and $gg$ channels and for their sum as a function of bottom quark mass. The $K$ factor is defined by $K = (\sigma^{(0)} + \sigma^{(1)}|_{\text{exact}})/\sigma^{(0)}$, where $\sigma^{(0)}$ is the Born term and $\sigma^{(1)}|_{\text{exact}}$ is the exact first order correction. We notice that the $K$ factor is quite large for the $gg$ channel, which means that higher order effects are more important for this channel than for $q\bar{q}$. Since $gg$ is the more important channel numerically, the $K$ factor for the sum of the two channels is also quite large. We also show the $K$ factor for the total which is slightly lower since we are also taking into account the negative contributions of the $qg$ and $\bar{q}g$ channels.

These large corrections come predominantly from the threshold region for bottom quark production where it has been shown that initial state gluon bremsstrahlung (ISGB) is responsible for the large corrections at NLO \cite{10}. This can easily be seen in fig. 9 where the Born term and the $O(\alpha_s^3)$ cross
section are plotted as a function of $\eta_{\text{cut}}$ for the $q\bar{q}$ and $gg$ channels, where $\eta = (s - 4m^2)/4m^2$ is the variable into which we have incorporated the cut in our programs for the cross sections. As we increase $\eta_{\text{cut}}$ the cross sections increase. The cross sections rise sharply for values of $\eta_{\text{cut}}$ between 0.1 and 1 and they reach a plateau at higher values of $\eta_{\text{cut}}$ indicating that the threshold region is very important and that the region where $s > 4m^2$ only makes a small contribution to the cross sections. This is the reason why we stressed in section 2 that our region of interest for comparison of the various approximations at the partonic level was $0.1 < \eta < 1$. In fig. 10 we plot as a function of $\eta_{\text{cut}}$ the Born term and the NLO cross section for the sum of the $q\bar{q}$ and $gg$ channels and also the NLO cross section for the sum of all channels, including the small negative contribution of the $qg$ and $\bar{q}g$ channels. We thus see that the $qg$ and $\bar{q}g$ channels contribute a small negative contribution to the total exact NLO cross section. Note that in the last two figures as well as throughout the rest of this paper we are assuming that the bottom quark mass is $m_b = 4.75$ GeV/c$^2$.

Next, we discuss the scale dependence of our NLO results. In figs. 11 and 12 we show the Born term, the exact first-order correction, and the total $O(\alpha_s^3)$ cross section as a function of the factorization scale for the $q\bar{q}$ and $gg$ channels. We see that as the scale decreases, the Born cross section increases without bound but the exact first order correction decreases faster so that the NLO cross section peaks at a scale close to half the mass of the bottom quark and then decreases for smaller values of the scale. For the $q\bar{q}$ channel the NLO cross section is relatively flat. The situation is much worse for the $gg$ channel, however, since the peak is very sharp and the scale dependence is much greater. Since the $gg$ channel dominates, this large scale dependence is also reflected in the total cross section. Thus the variation in the NLO cross section for scales between $m/2$ and $2m$ is large. For comparison we note that the scale dependence for top quark production at the Fermilab Tevatron for $m_{\text{top}} = 175$ GeV/c$^2$ is much smaller.

In fig. 13 we plot the Born contribution for $\mu = m$ and the NLO cross section for $\mu = m/2$, $m$, and $2m$, as a function of the beam momentum for $b$-quark production at fixed-target $pp$ experiments. The big width of the band reflects the large scale dependence that we discussed above. We see that the NLO cross section is almost twice as big as the Born term for the whole range of beam momenta that we are showing, and in particular for 820 GeV/c which is the value of the beam momentum at HERA-B. The total NLO cross
section for $b$-quark production at HERA-B is 28.8 nb for $\mu = m/2$; 9.6 nb for $\mu = m$; and 4.2 nb for $\mu = 2m$. We also give the NLO results for the individual channels in fig. 14 for $\mu = m$.

In figs. 15 and 16 we examine the $\mu_0$ dependence of the resummed cross section for $b$-quark production at HERA-B. We also show, for comparison, the $\mu_0$ dependence of $\sigma^{(0)} + \sigma^{(1)}|_{\text{app}} + \sigma^{(2)}|_{\text{app}}$ where we have imposed the same cut on the phase space of $s_4 (s_4 > s_0)$ as for the resummed cross section. Here $\sigma^{(1)}|_{\text{app}}$ and $\sigma^{(2)}|_{\text{app}}$ denote the approximate first and second order corrections, respectively, where only soft gluon contributions are taken into account. The effect of the resummation shows in the difference between the two curves. At small $\mu_0$, $\sigma^{\text{res}}$ diverges, signalling the presence of the infrared renormalon. There is a region where the higher-order terms are numerically important, for instance $0.5 < \mu_0 < 1$ in fig. 15. At high values of $\mu_0$ the two lines are practically the same. For the $q\bar{q}$ channel in the DIS scheme the resummation is successful in the sense that there is a relatively large region of $\mu_0$ where resummation is well behaved before we encounter the divergence. For the $gg$ channel, however, the situation is not as good. From these curves we choose what we think are reasonable values for $\mu_0$. We choose $\mu_0 = 0.6$ GeV for the $q\bar{q}$ channel and $\mu_0 = 1.7$ GeV for the $gg$ channel. The value we chose for the $gg$ channel is such that the resummed cross section is a little bit higher than the sum $\sigma^{(0)} + \sigma^{(1)}|_{\text{app}} + \sigma^{(2)}|_{\text{app}}$.

Using the values of $\mu_0$ that we chose from the previous graphs, we proceed to plot the resummed cross section for $b$-quark production at fixed-target $pp$ experiments versus beam momentum. We present the results in fig. 17 for the $q\bar{q}$ and $gg$ channels. We also show the results we have if in addition we resum subleading terms, and also if we resum both subleading terms and Coulomb and constant terms. For comparison the exact NLO results are shown as well. The resummed cross sections were calculated with the cut $s_4 > s_0$ while no such cut was imposed on the NLO result. Since we know the exact $O(\alpha_s^4)$ result, we can make an even better estimate by calculating the perturbation theory improved cross sections defined by

$$\sigma^{\text{imp}}_H = \sigma^{\text{res}}_H + \sigma^{(1)}_H|_{\text{exact}} - \sigma^{(1)}_H|_{\text{app}},$$

(3.6)

to exploit the fact that $\sigma^{(1)}_H|_{\text{exact}}$ is known and $\sigma^{(1)}_H|_{\text{app}}$ is included in $\sigma^{\text{res}}_H$. Then, in fig. 18 we plot the improved total cross section versus beam momentum (where we have also taken into account the small negative contributions.
of the $qg$ and $\bar{q}g$ channels) and, for comparison, the total exact NLO cross section for the three choices $\mu = m/2$, $m$, and $2m$. We also show the improved total cross section including resummation of subleading terms, and of both subleading terms and Coulomb and constant terms. The last curve lies above the NLO result for $\mu = m/2$ for most values of the beam momenta shown (including the value at HERA-B) so that the effect of resummation exceeds the scale dependence of the NLO cross section. The improved total cross section for $b$-quark production at HERA-B is 19.4 nb, if we resum leading terms only; 23.8 nb, if we resum leading and subleading terms; and 31.4 nb, if we resum leading, subleading, Coulomb, and constant terms.

Finally, we present some results on the inclusive transverse momentum ($p_t$) and rapidity ($Y$) distributions of the bottom quark at HERA-B. Heavy quark differential distributions are known in NLO [9, 15]. Some of the relevant formulae for this part have been given already in [16], where the $p_t$ and $Y$ distributions, including resummation, were presented for top-quark production at the Fermilab Tevatron. The heavy-quark inclusive differential distribution in $p_t^2$ is given by

$$\frac{d\sigma_{H}^{(k)}(S, m^2, p_t^2)}{dp_t^2} = \sum_{ij} \int_{4m_t^2/S}^1 d\tau \frac{\Phi_{ij}(\tau, \mu^2) d\sigma_{ij}^{(k)}(\tau S, m^2, p_t^2, \mu^2)}{dp_t^2}, \quad (3.7)$$

with $m_t^2 = m^2 + p_t^2$. In the case of the all-order resummed expression the lower boundary in (3.7) has to be modified according to the condition $s_0 < s - 2m_t s_1^{1/2}$. Resumming the soft gluon contributions to all orders we obtain

$$\frac{d\sigma_{H}^{\text{res}}(S, m^2, p_t^2)}{dp_t^2} = \sum_{ij} \int_{\tau_0}^1 d\tau \frac{\Phi_{ij}(\tau, \mu^2) d\sigma_{ij}^{(k)}(\tau S, m^2, p_t^2, \mu^2)}{dp_t^2}, \quad (3.8)$$

with $d\sigma_{ij}/dp_t^2$ given in (3.6) of [16] and

$$\tau_0 = \frac{m_t + (m_t^2 + s_0)^{1/2}}{S}. \quad (3.9)$$

The corresponding formula to (3.7) for the heavy quark inclusive differential distribution in $Y$ is

$$\frac{d\sigma_{H}^{(k)}(S, m^2, y)}{dy} = \sum_{ij} \int_{4m_t^2 \cosh^2 y/S}^1 d\tau \frac{\Phi_{ij}(\tau, \mu^2) d\sigma_{ij}^{(k)}(\tau S, m^2, y, \mu^2)}{dy}. \quad (3.10)$$
The all-order resummed differential distribution in $Y$ is given by
\[
\frac{d\sigma_{H}^{\text{res}}(S, m^2, Y)}{dY} = \sum_{ij} \int_{\tau_0}^{1} d\tau \Phi_{ij}(\tau, \mu^2) \frac{d\sigma_{ij}(\tau S, m^2, y, \mu^2)}{dy},
\] (3.11)
with $d\sigma_{ij}/dy$ given in (3.9) of \cite{16} and
\[
\tau_0 = \frac{m \cosh y + (m^2 \cosh^2 y + s_0)^{1/2}}{S}.
\] (3.12)

The hadronic heavy quark rapidity $Y$ is related to the partonic heavy quark rapidity $y$ by
\[
Y = y + \frac{1}{2} \ln \frac{x_1}{x_2}.
\] (3.13)

We begin with the $pt$ distributions. For these plots the mass factorization scale is not everywhere equal to $m$. We chose $\mu = m$ in $s_0$, $f_k(s_4/m^2, m^2/\mu^2)$ and $\bar{\alpha}_s$, but $\mu = m_t$ in the MRSD parton distribution functions and the running coupling constant $\alpha_s(\mu)$. In fig. 19, we give the results for the $q\bar{q}$ channel in the DIS scheme. We plot the Born term $d\sigma_{H}^{(0)}/dpt$, the first order exact result $d\sigma_{H}^{(1)}/dpt|_{\text{exact}}$, the first order approximation $d\sigma_{H}^{(1)}/dpt|_{\text{app}}$, the second order approximation $d\sigma_{H}^{(2)}/dpt|_{\text{app}}$, and the resummed result $d\sigma_{H}^{\text{res}}/dpt$ for $\mu_0 = 0.6$ GeV. This is the same value for $\mu_0$ that was used above for the total cross section. We also show resummed results with the inclusion of subleading terms, and with both subleading terms and Coulomb and constant terms. If we decrease $\mu_0$ the differential cross sections will increase. The resummed distributions were calculated with the cut $s_4 > s_0$ while no such cut was imposed on the phase space for the individual terms in the perturbation series. We continue with the results for the $gg$ channel in the MS scheme. The corresponding plot is given in fig. 20. We note that the corrections in this channel are large. In fact the exact first-order correction is larger than the Born term and the approximate second-order correction is larger than the approximate first-order correction. In this case the value of $\mu_0$ has been chosen to be $\mu_0 = 1.7$ GeV as above. We define the improved $pt$ distribution by
\[
\frac{d\sigma_{H}^{\text{imp}}}{dpt} = \frac{d\sigma_{H}^{\text{res}}}{dpt} + \frac{d\sigma_{H}^{(1)}}{dpt}|_{\text{exact}} - \frac{d\sigma_{H}^{(1)}}{dpt}|_{\text{app}}.
\] (3.14)

In fig. 21 we plot the improved $pt$ distributions for the sum of all channels, where we have included the small negative contributions of the $qg$ and $\bar{q}g$
channels. For comparison we also show the total exact NLO results for \( \mu = m_t/2, m_t, \) and \( 2m_t. \) The improved \( p_t \) distributions are uniformly above the exact \( O(\alpha_s^3) \) results. We see that the effect of the resummation exceeds the uncertainty due to scale dependence.

We finish with a discussion of the \( Y \) distributions. In this case we set the factorization mass scale equal to \( m \) everywhere. We begin with the \( q\bar{q} \) channel. In fig. 22 we show the Born term \( d\sigma^{(0)}_{H}/dY \), the first order exact result \( d\sigma^{(1)}_H/dY \mid_{\text{exact}} \), the first order approximation \( d\sigma^{(1)}_H/dY \mid_{\text{app}} \), the second order approximation \( d\sigma^{(2)}_H/dY \mid_{\text{app}} \), and the resummed result \( d\sigma_{H}^{\text{res}}/dY \) for \( \mu_0 = 0.6 \) GeV. We also show resummed results with the inclusion of subleading terms, and with both subleading terms and Coulomb and constant terms. Again, the resummed distributions were calculated with the cut \( s_4 > s_0 \) while no such cut was imposed on the phase space for the individual terms in the perturbation series. We continue with the results for the \( gg \) channel in the \( \overline{\text{MS}} \) scheme. The corresponding plot is given in fig. 23. Here, the value of \( \mu_0 \) is \( \mu_0 = 1.7 \) GeV. The corrections in this channel are large as was the case for the \( p_t \) distributions. We define the improved \( Y \) distribution by

\[
\frac{d\sigma_{H}^{\text{imp}}}{dY} = \frac{d\sigma_{H}^{\text{res}}}{dY} + \frac{d\sigma_{H}^{(1)}_H}{dY} \mid_{\text{exact}} - \frac{d\sigma_{H}^{(1)}_H}{dY} \mid_{\text{app}}.
\]

(3.15)

In fig. 24 we plot the improved \( Y \) distributions for the sum of all channels, where we have included the small negative contributions of the \( qg \) and \( \bar{q}g \) channels. For comparison we also show the total exact NLO results for \( \mu = m/2, m, \) and \( 2m. \) The improved \( Y \) distributions are uniformly above the \( O(\alpha_s^3) \) results. Again, we see that the effect of the resummation exceeds the uncertainty due to scale dependence.

4 Conclusions

We have presented NLO and resummed results for the cross section and differential distributions for bottom quark production at HERA-B. Results for the cross section as a function of beam momentum have also been given for fixed-target \( pp \) experiments in general. It has been shown that the \( gg \) channel is dominant and that the threshold region gives the main contribution to the NLO cross section. Approximations for the soft gluon contributions in that region have been compared with the exact results. The resummation of the
leading S+V logarithms produces an enhancement of the NLO results. The leading S+V approximation is not very good in the $gg$ channel in the $\overline{\text{MS}}$ scheme in the kinematic region that is important for bottom quark production at HERA-B. The addition of subleading S+V terms and Coulomb terms improves the approximation considerably. The resummation of these additional terms further enhances the cross section. We must stress, however, that our formula for the exponentiation of subleading terms is not based on any rigorous analysis and more work in this area will have to be done in the future.

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Fig. 1. The scaling functions $f_{qar{q}}^{(k,0)}$ in the $\overline{\text{MS}}$ scheme. Plotted are $f_{qar{q}}^{(0,0)}$ (exact, upper solid line at large $\eta$; threshold approximation, upper dotted line at large $\eta$); $f_{qar{q}}^{(1,0)}$ (exact, lower solid line at large $\eta$; threshold approximation, lower dotted line at large $\eta$), and $f_{qar{q}}^{(\pi^2),\text{res}}$ (dashed line).

Fig. 2. Same as fig. 1 but now for the DIS scheme. Also shown is $f_{qar{q}}^{(\pi^2),\text{res}}$ where the only constant that we exponentiate is the $\pi^2/3$ term (dash-dotted line).

Fig. 3. Same as fig. 1 but now for $f_{gg}^{(k,0)}$ in the $\overline{\text{MS}}$ scheme.

Fig. 4. The scaling functions $f_{gg}^{(1,0)}$ in the $\overline{\text{MS}}$ scheme. Plotted are the exact result (solid line), the leading S+V approximation (dotted line), the leading S+V approximation plus Coulomb terms (short-dashed line), the S+V approximation with leading plus subleading terms (long dashed line), and the S+V approximation with leading plus subleading terms plus Coulomb terms (dash-dotted line).

Fig. 5. Same as fig. 4 but now for the DIS scheme. Also shown are the leading S+V approximation plus Coulomb terms and the $\pi^2/3$ constant term only (lower short-dashed line), and the S+V approximation with leading plus subleading terms plus Coulomb terms and the $\pi^2/3$ constant term only (lower dash-dotted line).

Fig. 6. Same as fig. 4 but now for $f_{gg}^{(1,0)}$ in the $\overline{\text{MS}}$ scheme.

Fig. 7. Fractional contributions of the $gg$ ($\overline{\text{MS}}$ scheme, short-dashed line), $qar{q}$ (DIS scheme, long-dashed line), $qg$ (DIS scheme, lower dotted line), and $\bar{q}g$ (DIS scheme, upper dotted line) channels to the total $O(\alpha_s^3)$ $b$-quark production cross section at HERA-B as a function of $b$-quark mass.

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Fig. 10. Cross sections for $b$-quark production at HERA-B versus $\eta_{\text{cut}}$
with $m_b = 4.75 \text{ GeV}/c^2$. Plotted are the total Born term (solid line), the total $O(\alpha_s^3)$ cross section (dashed line), and the $O(\alpha_s^3)$ cross section for the sum of the $q\bar{q}$ and $gg$ channels (dotted line).

Fig. 11. The scale dependence of the cross section for $b$-quark production at HERA-B with $m_b = 4.75 \text{ GeV}/c^2$ for the $q\bar{q}$ channel in the DIS scheme. Plotted are the Born term (solid line), the exact first-order correction (dotted line), and their sum (dashed line).

Fig. 12. Same as fig. 11 but now for the $gg$ channel in the $\overline{\text{MS}}$ scheme.

Fig. 13. The total Born (dotted line) and $O(\alpha_s^3)$ ($\mu = m$/2 upper dashed line, and $\mu = 2m$ lower dashed line) $b$-quark production cross sections at fixed-target $pp$ experiments versus beam momentum for $m_b = 4.75 \text{ GeV}/c^2$.

Fig. 14. Contributions of individual channels to the total $O(\alpha_s^3)$ $b$-quark production cross section at fixed-target $pp$ experiments versus beam momentum for $m_b = 4.75 \text{ GeV}/c^2$. Plotted are the contributions of the $gg$ ($\overline{\text{MS}}$ scheme, short-dashed line) and $q\bar{q}$ (DIS scheme, long-dashed line) channels, and the absolute value of the contributions of the $gg$ (DIS scheme, upper dotted line) and $\bar{q}g$ (DIS scheme, lower dotted line) channels.

Fig. 15. The $\mu_0$ dependence of the resummed cross section for $b$-quark production at HERA-B with $m_b = 4.75 \text{ GeV}/c^2$ for the $q\bar{q}$ channel in the DIS scheme. Plotted are $\sigma_{q\bar{q}}^{\text{res}}$ (solid line) and the sum $\sigma^{(0)} + \sigma^{(1)} |_{\text{app}} + \sigma^{(2)} |_{\text{app}}$ (dotted line).

Fig. 16. Same as fig. 15 but now for the $gg$ channel in the $\overline{\text{MS}}$ scheme.

Fig. 17. Resummed and NLO cross sections versus beam momentum for $b$-quark production at fixed-target $pp$ experiments for $m_b = 4.75 \text{ GeV}/c^2$. Plotted are the resummed cross sections for the $q\bar{q}$ channel in the DIS scheme for $\mu_0 = 0.6 \text{ GeV}$ (leading terms only, short-dashed line; with subleading terms, lower short-dash-dotted line; with both subleading terms and Coulomb and constant terms, upper short-dash-dotted line) and for the $gg$ channel in the $\overline{\text{MS}}$ scheme for $\mu_0 = 1.7 \text{ GeV}$ (leading terms only, long-dashed line; with subleading terms, lower long-dash-dotted line; with both subleading terms and Coulomb terms, upper long-dash-dotted line); and the $O(\alpha_s^3)$ cross sections for the $gg$ channel in the $\overline{\text{MS}}$ scheme and the $q\bar{q}$ channel in the DIS scheme (upper and lower solid lines, respectively).

Fig. 18. Improved and NLO cross sections versus beam momentum for $b$-quark production at fixed-target $pp$ experiments for $m_b = 4.75 \text{ GeV}/c^2$. Plotted are the total improved cross section (leading terms only, short-dashed
line; with subleading terms, lower long-dashed line; with both subleading terms and Coulomb and constant terms, upper long-dashed line) and the total $O(\alpha_s^3)$ result (\(\mu = m\) solid line, \(\mu = m/2\) upper dotted line, \(\mu = 2m\) lower dotted line).

Fig. 19. The bottom quark $p_t$ distributions $d\sigma^{(k)}_H/ dp_t$ at HERA-B for the $q\bar{q}$ channel in the DIS scheme for $m_b = 4.75$ GeV/c$^2$. Plotted are $d\sigma^{(0)}_H/ dp_t$ (solid line), $d\sigma^{(1)}_H/ dp_t |_{\text{exact}}$ (dotted line), $d\sigma^{(1)}_H/ dp_t |_{\text{app}}$ (short-dashed line), $d\sigma^{(2)}_H/ dp_t |_{\text{app}}$ (long-dashed line), and $d\sigma^{\text{res}}_H/ dp_t$ for $\mu_0 = 0.6$ GeV (leading terms only, short-dash-dotted line; with subleading terms, lower long-dash-dotted line; with both subleading terms and Coulomb and constant terms, upper long-dash-dotted line).

Fig. 20. Same as fig. 19 but now for the $gg$ channel in the $\overline{\text{MS}}$ scheme and with $\mu_0 = 1.7$ GeV.

Fig. 21. The bottom quark $p_t$ distributions $d\sigma_H/ dp_t$ at HERA-B for the sum of all channels for $m_b = 4.75$ GeV/c$^2$. Plotted are $d\sigma^{(0)}_H/ dp_t + d\sigma^{(1)}_H/ dp_t |_{\text{exact}}$ (\(\mu = m_t\) solid line, \(\mu = m_t/2\) upper dotted line, \(\mu = 2m_t\) lower dotted line) and $d\sigma^{\text{imp}}_H/ dp_t$ (leading terms only, short-dash-dotted line; with subleading terms, lower long-dashed line; with both subleading terms and Coulomb and constant terms, upper long-dash-dotted line).

Fig. 22. The bottom quark $Y$ distributions $d\sigma^{(k)}_H/ dY$ at HERA-B for the $q\bar{q}$ channel in the DIS scheme for $m_b = 4.75$ GeV/c$^2$. Plotted are $d\sigma^{(0)}_H/ dY$ (solid line), $d\sigma^{(1)}_H/ dY |_{\text{exact}}$ (dotted line), $d\sigma^{(1)}_H/ dY |_{\text{app}}$ (short-dashed line), $d\sigma^{(2)}_H/ dY |_{\text{app}}$ (long-dashed line), and $d\sigma^{\text{res}}_H/ dY$ for $\mu_0 = 0.6$ GeV (leading terms only, short-dash-dotted line; with subleading terms, lower long-dash-dotted line; with both subleading terms and Coulomb and constant terms, upper long-dash-dotted line).

Fig. 23. Same as fig. 22 but now for the $gg$ channel in the $\overline{\text{MS}}$ scheme and with $\mu_0 = 1.7$ GeV.

Fig. 24. The bottom quark $Y$ distributions $d\sigma_H/ dY$ at HERA-B for the sum of all channels for $m_b = 4.75$ GeV/c$^2$. Plotted are $d\sigma^{(0)}_H/ dY + d\sigma^{(1)}_H/ dY |_{\text{exact}}$ (\(\mu = m\) solid line, \(\mu = m/2\) upper dotted line, \(\mu = 2m\) lower dotted line) and $d\sigma^{\text{imp}}_H/ dY$ (leading terms only, short-dashed line; with subleading terms, lower long-dashed line; with both subleading terms and Coulomb and constant terms, upper long-dash-dotted line).
