Research Article

Applications of Fractional Lower Order Synchrosqueezing Transform Time Frequency Technology to Machine Fault Diagnosis

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1.Introduction

Synchrosqueezing transform is a new time frequency analysis technology for the nonstationary signals. Its principle is to calculate time frequency distribution of the signal, then squeeze the frequency of the signal in time frequency domain, and rearrange its time frequency energy, so as to improve time frequency resolution greatly. Synchrosqueezing transform mainly includes continuous wavelet transform-based synchrosqueezing transform [1], short time Fourier transform-based synchrosqueezing transform [2], and S transform-based synchrosqueezing transform [3]. Synchrosqueezing transform methods have been widely applied in seismic signal analysis [4], biomedical signal processing [5, 6], radar imaging [7], mechanical fault diagnosis, and other fields [8–11].

Daubechies et al. firstly gave synchrosqueezing transform concept based on the continuous wavelet transform and proposed a continuous wavelet transform-based synchrosqueezing transform (WSST) time frequency representation method and its inverse method. The method squeezes the time frequency energy of continuous wavelet transform in a certain frequency range to nearby instantaneous frequency of the signal, and the time frequency resolution was improved effectively [12], whereafter an adaptive wavelet transform-based synchrosqueezing transform based on WSST was brought up by Li et al. who applied a time-varying parameter to control the widths of the time frequency localization window according to the characteristics of signals [13]. The demodulated WSST and FSST methods have been proposed for instantaneous frequency estimation in [14, 15]. To improve the ability of processing...
the nonstationary signals with fast varying instantaneous frequency, a new demodulated high order synchrosqueezing transform method was presented in [11, 16], which can effectively show the time frequency distribution of the fault signal. Fourer et al. proposed a FSST method employing the synchrosqueezing transform and the Levenberg Marquardt reassignment in [17]; the idea of the method is very similar to the WSST method, which is reversible and adjustable. Yu et al. subsequently presented a synchroextracting short time Fourier transform, which is a postprocessing procedure of STFT [18]. Recently, they proposed an improved local maximum synchrosqueezing transform, which can discover more detailed features of the fault signals [19]. To compress and rearrange the S transform time frequency distribution of the signal, Huang et al. proposed a new S transform-based synchrosqueezing transform time frequency method employing synchrosqueezing transform and S transform, which can greatly improve time frequency resolution of S transform [20, 21]. Subsequently, they modified the calculation formula of the instantaneous frequency of the SSST time frequency method by using the second derivative of time frequency spectrum to time and frequency and proposed a new second-order S transform-based synchrosqueezing transform method, which can obtain high time frequency resolution for the nonstationary signals whose instantaneous frequency varies nonlinearly with the time [22]. Recently, an adaptive short time Fourier transform-based synchrosqueezing transform method has been proposed with a time-varying parameter, and the corresponding 2nd-order adaptive FSST was also present in [23, 24].

Recently, it is verified that probability density function (PDF) of the mechanical bearing fault signals has an obvious trail, which is a nonstationary and non-Gaussian distribution and belongs to $\alpha$ stable distribution ($0 < \alpha < 2$); even the noises are also $\alpha$ stable distribution [25–28]. The performance of the above-mentioned methods degenerates under $\alpha$ stable distribution environment, which even fail. Some of the ways they apply are the fractional low order time frequency representation methods to analyze the signals, such as fractional low order short time Fourier transform (FLOSTFT) [29, 30], fractional low order S transform (FLOS) [31, 32], and fractional low order Wigner-Ville distribution [30]. However, the time frequency resolution of the methods is not very ideal and depends jointly on the geometry of the signal and the window function; even false spectral energies would be observed on the time frequency distribution at the locations where no spectral energies should exist. Hence, we propose the improved fractional low order short time Fourier transform-based synchrosqueezing transform (FLOFSST) and fractional low order S transform-based synchrosqueezing transform (FLOSSST) methods inspired by the FSST and SSST methods in this paper, and the derivation procedures of the inverse FLOFSST and inverse FLOSSST are introduced. The simulation results show that the performances of the FLOFSST and FLOSSST time frequency representation methods are superior to the existing ones under $\alpha$ stable distribution noise environment; they have higher time frequency resolution than the existing FLOSTFT and FLOST methods and can better be suitable for the impulse noise environment than the FSST and SST methods. The IFLOFSST and IFLOSSST methods have smaller reconstruction MSEs than the IFSST and ISSST methods under different $\alpha (\alpha < 2)$ and GSNR. Finally, we apply the FLOFSST and FLOSSST time frequency representation methods to analyze the bearing out race fault signal. The simulation results show that the FLOFSST and FLOSSST methods can work in Gaussian noise and $\alpha$ stable distribution noise environment and extract the features of the outer race fault signal, which have some robustness; their performances are better than the existing FSST and SSST methods.

In this paper, the improved FLOFSST and FLOSSST time frequency representation technologies based on fractional lower order statistics and synchrosqueezing transform are presented for the bearing fault diagnosis under Gaussian and $\alpha$ stable distribution environment. The paper is structured in the following manner. $\alpha$ stable distribution and the bearing fault signals are introduced in Section 2. The improved fractional lower order synchrosqueezing transform methods and their inverse transforms are demonstrated in Section 3, and simulation comparisons with the existing time frequency representation methods based on second-order statistics are performed to demonstrate superiority of the improved methods. Applications of the improved methods for the outer race fault signals diagnosis are demonstrated in Section 4. Finally, conclusions and future research are given in Section 5.

2. $\alpha$ Stable Distribution and Bearing Fault Signals

2.1. $\alpha$ Stable Distribution. Probability density function (PDF) of a stable distribution is expressed as

$$\varphi(t) = \exp\left\{jut - |t|^{\alpha} [1 + j\beta \sign(t) \omega(t, \alpha)]\right\},$$

(1)

where

$$\sign(t) = \begin{cases} 1, & t > 0, \\ 0, & t = 0, \\ -1, & t < 0. \end{cases}$$

$$\omega(t, \alpha) = \begin{cases} \tan(\alpha \pi/2), & \alpha \neq 1, \\ (2/\alpha \log |r|), & \alpha = 1, \end{cases}$$

$\varphi(t)$ is its characteristic index, and its variance is infinite. When $\alpha = 2$, which is Gaussian distribution, and when $0 < \alpha < 2$, it is low order stable distribution. $\mu$ is the location parameter and $\gamma$ is the dispersion coefficient, respectively. $\beta$ is the symmetry parameter, when $\beta = 0$, which is called the symmetric $\alpha$ stable distribution ($Sa\delta$). The waveforms of $Sa\delta$ stable distribution are shown in Figure 1 under $\alpha = 0.5, 1.0, 1.5, \text{and} 2.0$, and their PDFs are demonstrated in Figure 2.

2.2. Bearing Fault Signals. The real bearing fault signals data are obtained from the Case Western Reserve University (CWRU) bearing data center [33]. The experimental equipment adopts 6205-2RS JEM SKF type bearing, the outer race diameter is 20.472 inches, and the inner race and the ball diameter are 0.9843 inches and 0.3126 inches,
respectively. The bearing outer race thickness is 0.5906 inches, motor load is 0 HP, and motor speed is 1797 rpm. The bearing faults of inner race, ball, and outer race are set, and the fault diameters are all 0.021 inches. The fault data are collected at 12,000 samples per second, and the outer race position relative to load zone centered at 6:00. The
experiments are conducted with a 2 HP reliance electric motor, and the acceleration data are measured at the proximal and distal of the motor bearings; the points include the drive end accelerometer (DE), fan end accelerometer (FE), and base accelerometer (BA). The normal signal is given in Figure 3(a), and the fault signals of inner race, ball, and outer race are shown in Figures 3(b)–3(d), respectively. We can know that the waveforms of the fault signals have a certain impulse.

In order to further verify the pulse characteristics of bearing failure signals, we use a stable distribution statistical model to estimate the parameters of inner race fault, ball bearing fault signals, we use the normal signal is the drive end accelerometer (DE), fan end accelerometer (FE), and base accelerometer (BA). STFT is a more concise and accurate statistical model for the bearing fault signals.

3. Mathematical Problems in Engineering

3.1. Fractional Lower Order Synchrosqueezing Transform Method

3.1.1. Principle. Short time Fourier transform (STFT) of the fault machinery vibration signal contaminated by SaS distribution noise or Gaussian noise \( v(t) \) can be written as

\[
\text{STFT}_y (t, f) = \int_{-\infty}^{\infty} v(t) h(t - \tau) e^{-j2\pi ft} d\tau,
\]

and its fractional low order short time Fourier transform (FLOFT) is given by [30]

right side of (4) is converted from the time domain to the frequency domain.

Letting \( t = \tau = \eta, \bar{\psi}(\lambda) \) can be written as

\[
\bar{\psi}(\lambda) = \int_{-\infty}^{\infty} h(\tau) e^{j2\pi f \lambda} e^{-j2\pi \eta} d\eta
\]

\[
= \int_{-\infty}^{\infty} h(\eta) e^{j2\pi f (\lambda + \eta)} e^{j2\pi \eta (\lambda + \eta)} d\eta
\]

\[
= e^{-j2\pi (\lambda - f)} \int_{-\infty}^{\infty} h(\eta) e^{j2\pi \eta (f - \lambda)} d\eta
\]

\[
= \bar{\psi}(f - \lambda) e^{-j2\pi (\lambda - f)}.
\]

Substituting (5) into (4), we have

\[
\text{FLOFT}_y (t, f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\psi}(\lambda) \bar{\psi}(f - \lambda) e^{j2\pi (\lambda - f)} d\lambda.
\]
Figure 3: The waveform of the bearing fault signals. (a) The normal signals in DE and FE; (b) the inner race fault signals in BA, DE, and FE; (c) the ball fault signals in BA, DE, and FE; (d) the outer race fault signals in BA, DE, and FE.

Table 1: The parameters of the bearing fault signals based on $\alpha$ stable distribution.

| Parameters        | $\alpha$ | $\beta$  | $\gamma$ | $\mu$  |
|-------------------|----------|----------|----------|--------|
| Normal            |          |          |          |        |
| DE                | 2.000    | -0.2863  | 0.0532   | 0.0121 |
| FE                | 2.000    | 1.000    | 0.0583   | 0.0236 |
| Inner race fault  |          |          |          |        |
| BA                | 1.7682   | 0.0872   | 0.0590   | 0.0062 |
| DE                | 1.4195   | 0.0155   | 0.2407   | 0.0175 |
| FE                | 1.8350   | 0.0322   | 0.1495   | 0.0291 |
| Ball fault        |          |          |          |        |
| BA                | 1.9790   | 0.0592   | 0.0293   | 0.0055 |
| DE                | 1.8697   | 0.1215   | 0.0772   | 0.0193 |
| FE                | 1.998    | -0.0371  | 0.0674   | 0.0321 |
| Outer race fault  |          |          |          |        |
| BA                | 1.6077   | -0.1731  | 0.0530   | 0.0012 |
| DE                | 1.1096   | 0.0433   | 0.1341   | 0.0367 |
| FE                | 1.5435   | -0.0169  | 0.0968   | 0.0296 |
Figure 4: PDFs of the normal and bearing fault signals. (a), (b) PDFs of the normal and inner race fault signals in DE, FE, and BA; (c), (d) PDFs of the normal and ball fault signals in DE, FE, and BA; (e), (f) PDFs of the normal and outer race fault signals in DE, FE, and BA.
Substituting (7) into (6), then

\[
\text{FLOSTFT}_y(\tau, \omega) = \frac{|A|^{-1} \pi^{p-2} \text{sign}(A)}{2} \int_{-\infty}^{\infty} \delta[(\lambda + \omega_0) \mp \delta(\lambda - \omega_0)] \frac{\psi(\omega - \lambda) e^{i\tau(\lambda - \omega)}}{2} e^{i(\omega - \omega_0)} \psi(\omega - \omega_0). 
\] (8)

The instantaneous frequency (IF) formula of FLOSTFT can be written as

\[
F_y(\tau, f) = f + \frac{1}{2\pi \text{FLOSTFT}_y(\tau, f)} \frac{\partial[\text{FLOSTFT}_y(\tau, f)]}{\partial \tau}.
\] (10)

After synchrosqueezing the frequency in (10), the discrete values FLOSTFT, \(f, f_\ell\) can be obtained. Letting the frequency points in FLOSTFT time frequency spectrum, \(f_\ell (\ell = 1, 2, \ldots, K)\) and \(\Delta f_\ell = f_\ell - f_{\ell-1}.\) By centering \(f_\ell\) and letting \(\Delta f = f_\ell - f_{\ell-1},\) the synchrosqueezing calculation is extended to the successive bins \([f_\ell - (1/2)\Delta f, f_\ell + (1/2)\Delta f];\) then, fractional lower order STFT-based synchrosqueezing transform can be defined as

\[
\text{FLOFSST}_y(\tau, f_\ell) = (\Delta f)^{-1} \sum_{f_i \mid F(\tau, f_i) - f_\ell \leq \Delta f/2} \text{FLOSTFT}_y(\tau, f_i) f_i \Delta f_i. 
\] (12)

The FLOFSST can “squeeze” a frequency interval to a frequency point in the time frequency domain; therefore, the process can greatly improve the time frequency resolution. A multicomponent signal can be expressed as

\[
y(t) = \sum_{k=1}^{k} y_k(t) = \sum_{k=1}^{N} A_k(t) \cos(2\pi f_k t), \] (13)

where \(k = 1, 2, \ldots, N.\) Then, the FLOSTFT of \(k\)th component can be expressed as

\[
F_{y_k}(\tau, f_k) = f_k + \frac{1}{2\pi \text{FLOSTFT}_{y_k}(\tau, f_k)} \frac{\partial[\text{FLOSTFT}_{y_k}(\tau, f_k)]}{\partial \tau}. 
\] (16)
The corresponding instantaneous frequency calculation method of \( y(t) \) may be obtained by

\[
F_y(\tau, f) = \sum_{k=1}^{N} F_y(k, f_k) = \sum_{k=1}^{N} \frac{1}{2\pi h(0)} \left| \int_{|f-f_k| \leq \Delta f_k} FOMSTFT_{y_k}(\tau, f_k) df \right|^p \frac{\partial}{\partial \tau} \end{equation}
\]

By substituting (15) and (17) into (12), we can obtain the fractional low order STFT-based synchrosqueezing transform of the multicomponent signal.

According to the definition of inverse STFT-based synchrosqueezing transform in [18, 19], inverse fractional lower order STFT-based synchrosqueezing transform (IFLOFSST) of \( k \)nd signal can be written by

\[
y_k(t) = \left| y_k^{(p)}(t) \right|^{1/p - 1} \text{sign}[\overline{y}_k(t)],
\]

\[
y_k^{(p)}(t) = \Re \left[ \frac{1}{2\pi h(0)} \int_{|f-f_k| \leq \Delta f_k} FLOSTFT_{y_k}(\tau, f_k) df \right].
\]

and the signal \( y(t) \) can be gotten employing

\[
y(t) = \sum_{k=1}^{K} y_k(t).
\]

where \( v(n) \) is SaS distribution noise or Gaussian noise. When the noise \( v(n) \) is SaS distribution noise, generalized signal to noise ratio (GSNR) can be used instead of SNR, which is expressed as

\[
\text{GSNR} = 10 \log_{10} \left( \frac{E[|x(n)|^2]}{\gamma^a} \right) = 10 \log_{10} \left( \frac{1}{N} \sum_{m=0}^{N-1} |x(n)|^2 \right),
\]

where \( \gamma \) is the dispersion coefficient of SaS distribution noise. According to the given GSNR, the amplitude of the signal \( x(n) \) can be written as

\[
A = \left( \frac{10^{(\text{GSNR}/10)} \sum_{m=0}^{N-1} |x(n)|^2}{1/\gamma^a} \right)^{1/2}.
\]

Let \( \text{SNR} = -5 \text{ dB}, \text{GSNR} = 22 \text{ dB}, \) and \( \alpha = 0.8 \). The waveforms of \( x(n) \) and \( y(n) \) in time domain are shown in Figure 5. We apply the fractional lower order STFT transform-based synchrosqueezing transform method, the existing STFT transform-based synchrosqueezing transform method, the fractional lower order STFT method, and traditional STFT method to estimate time frequency distribution of the signal \( x(n) \) under Gaussian distribution noise and SaS stable distribution noise; the simulation results are shown in Figures 6 and 7.

In order to compare the effectiveness of the IFSSST and IFLOFSST methods, letting \( \text{MSE} = (1/K) \sum_{k=1}^{K} \sum_{n=1}^{N} \left[ \tilde{x}(n) - x(n) \right]^2 \), where \( K \) is the number of Monte-Carlo experiment, \( \tilde{x}(n) \) is the reconstructed signal employing the IFSSST method or the IFLOFSST method. Letting \( \text{GSNR} = 20 \), the signal \( x(n) \) is reconstructed employing the IFSSST and IFLOFSST methods under different \( \alpha \); their MSEs are shown in Figure 8(a). We apply the IFSSST and IFLOFSST methods to reconstruct the signal \( x(n) \) when \( \alpha = 1 \); GSNR changes from 14 dB to 24 dB; the simulations are shown in Figure 8(b).

3.1.4. Remarks. The STFT, FLOSTFT, FSST, and FLOFSST time frequency methods in Figure 6 all can estimate out the time frequency representation of the signal \( x(n) \) under...
Figure 5: The waveforms in time domain. (a) The signal $x(n)$; (b) the signal $x(n)$ contaminated by Gaussian noise environment (SNR = −5 dB); (c) the signal $x(n)$ contaminated by SaS noise environment (GSNR = 22 dB and $\alpha = 0.8$).

Figure 6: Time frequency representations of the signal $x(n)$ under Gaussian noise environment (SNR = −5 dB and $p = 1.8$). (a) STFT time frequency representation of the signal $x(n)$; (b) FLOSTFT time frequency representation of the signal $x(n)$; (c) FSST time frequency representation of the signal $x(n)$; (d) FLOFSST time frequency representation of the signal $x(n)$.
Figure 7: Time frequency representations of the signal $x(n)$ under $SaS$ noise environment (GSNR = 22 dB and $\alpha = 0.8, p = 1.2$). (a) STFT time frequency representation of the signal $x(n)$; (b) FLOSTFT time frequency representation of the signal $x(n)$; (c) FSST time frequency representation of the signal $x(n)$; (d) FLOFSST time frequency representation of the signal $x(n)$.

Figure 8: MSE comparisons of signal reconstruction of the IFSST and IFLOFSST algorithms under different $\alpha$ and GSNR ($p = 1.2$). (a) GSNR = 20 and MSE comparison under different $\alpha$; (b) $\alpha = 1$ and MSE comparison under different GSNR.
Gaussian noise environment (SNR = -5 dB), but the synchronosqueezing methods have better performance. The STFT method in Figure 7(a) and FSST time frequency method in Figure 7(c) fail under SaS noise environment (GSNR = 22 dB and $\alpha = 0.8$); the FLOSTFT method in Figure 7(b) can estimate out the time frequency representation of the signal $x(n)$, but its effect is not very ideal. The improved FLOFSST method in Figure 7(d) can better get the time frequency representation of the signal $x(n)$ under SaS noise environment, which has good toughness.

The STFT and FSST are unsuitable for SaS noise environment, and the FLOSTFT method can work under SaS noise environment, but has poor time frequency resolution and is controlled by the window function. The FSST method has better time frequency resolution, but cannot work in SaS noise environment. The improved FLOFSST method can work under SaS noise environment and has high time frequency resolution. As a result, the FSST time frequency method is only suitable to analyze the signals under Gaussian noise environment, but the improved FLOFSST can work under Gaussian and SaS noise environment, which is robust.

Figure 8(a) is MSE comparison under GSNR = 20 dB and different $\alpha$; the experimental results show that MSEs of the IFSST method change from 2 dB to 230 dB when $\alpha$ changes from 0.2 to 2, but MSEs of the IFLOFSST method are 2 dB. Hence, the IFSST method has obvious advantage in reconstructing the signal under different $\alpha$; particularly, the advantage of the IFSST method is more obvious when $\alpha < 1$.

Figure 8(b) is MSE comparison under $\alpha = 1$ and different GSNR; we can know that reconstruction MSEs of the IFSST method have a large variation, which changes from 14 dB to 78 dB; however, MSEs of the IFLOFSST method are stable in 2 dB. Hence, the improved IFLOFSST method has better stability in reconstructing the original signal.

### 3.2. Fractional Lower Order S Transform-Based Synchronosqueezing Transform

#### 3.2.1. Principle

The fault machinery vibration signal contaminated by the noise may be given by

$$y(t) = x(t) + v(t),$$

where $x(t)$ is fault vibration signal and $v(t)$ is SaS distribution noise or Gaussian noise. Its S transform can be written as

$$\text{ST}(\tau, f) = \int_{-\infty}^{\infty} y(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{((t-\tau)f^2)/2}{2}} e^{-j2\pi ft} dt,$$  \hspace{1cm} (23)

and its fractional lower order S transform was defined as [28]

$$\text{FLOST}(\tau, f) = \int_{-\infty}^{\infty} y^{(p)}(t) h(t-\tau) e^{-j2\pi ft} dt,$$  \hspace{1cm} (24)

where $f$ is the frequency parameter and $t$ is the time parameter. $\tau$ denotes the displacement parameter on the time axis. $h(t-\tau)$ is the Gaussian window function related to the frequency.

Equation (24) can be written as

$$\text{FLOST}(\tau, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y^{(p)}(t) e^{-((t-\tau)f^2)/2} e^{-j2\pi ft} dt.$$  \hspace{1cm} (26)

Let $\psi(t) = (1/\sqrt{2\pi}) e^{-((t^2)/2)} e^{j2\pi ft}$, and its complex conjugate function is $\psi(t) = (1/\sqrt{2\pi}) e^{-((t^2)/2)} e^{-j2\pi ft}$. Then, (26) changes as

$$\text{FLOST}(\tau, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y^{(p)}(t) e^{-((t-\tau)f^2)/2} e^{-j2\pi ft} dt,$$

and

$$= |f| \int_{-\infty}^{\infty} y^{(p)}(t) e^{-((t-\tau)^2)/2} e^{-j2\pi ft} dt,$$

and

$$= |f| \int_{-\infty}^{\infty} y^{(p)}(t) \psi(t-t) e^{-j2\pi ft} dt.$$

The right side of (27) is converted from the time domain to the frequency domain based on Plancherel’s theorem and Fourier transform; then we obtain

$$\text{FLOST}(\tau, f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\psi}(\lambda) \psi(A\lambda) e^{j(\lambda-2\pi f)\lambda} d\lambda,$$  \hspace{1cm} (28)

where $\overline{\psi}(\lambda) = \int_{-\infty}^{\infty} y^{(p)}(t) e^{-j2\pi ft} dt$ is fractional lower order Fourier transform (FLOFT) of $y^{(p)}(t)$ and $\lambda$ denotes frequency variant. $\overline{\psi}(\lambda/f)$ is Fourier transform of $\psi(\lambda/f)$, and $\overline{\psi}(\lambda/f)$ is complex conjugate of $\psi(\lambda/f)$.

Letting $y(t) = A\cos(2\pi f_0 t)$ and $\omega_0 = 2\pi f_0$, then

$$y^{(p)}(t) = |A\cos(2\pi f_0 t)|^{p-1} \cdot \text{sign}|A\cos(2\pi f_0 t)|,$$

and its FLOFT can be expressed as

$$\overline{\psi}(\lambda) = \int_{-\infty}^{\infty} y^{(p)}(t) e^{-j2\pi ft} dt = |A|^{p-1} \pi^{p-1} \text{sign}(A) \delta(\lambda + \omega_0) + \delta(\lambda - \omega_0).$$  \hspace{1cm} (29)
By substituting (29) to (28), then

\[
F_{\text{FLOST}}(\tau, f) = \frac{|A|^p |\tau|^{p-2} \text{sign}(A)}{2} \int_{-\infty}^{\infty} \delta(\lambda + \omega_0) + \delta(\lambda - \omega_0) \psi(\lambda+\omega_0) e^{i(\lambda-\omega_0)\tau} d\lambda = \frac{|A|^p |\tau|^{p-2} \text{sign}(A)}{2} e^{-i(\omega_0-\omega_0)\tau} \frac{2\pi \omega_0}{\psi(2\pi \omega_0)}. 
\]  
(30)

Fourier transform of \(\psi(t)\) and \(\check{\psi}(t)\) can assemble \(\lambda\) at around \(\omega_0\), and \(F_{\text{FLOST}}(\tau, f)\) will be concentrated around \(\tau = \omega_0/\omega\). By substituting \(f = \omega/2\pi\) to (30), then

\[
F_{\text{FLOST}}(\tau, f) = \frac{|A|^p |\tau|^{p-2} \text{sign}(A)}{2} e^{-i2\pi(\tau-f_0)} \frac{2\pi f_0}{f}. 
\]  
(31)

By substituting (31) and (33) to (32), we can obtain the squeeved instantaneous frequency. Through synchrosqueezing the frequency with (32), the discrete values \(F_{\text{FLOST}}(\tau, f)\) can be gotten. Letting the frequency points in FLOST time frequency spectrum, \(f_l (l = 1, 2, \ldots, K)\) and

\[
\frac{\partial[F_{\text{FLOST}}(\tau, f)]}{\partial \tau} = -j|A|^p |\tau|^{p-1} \text{sign}(A)(f - f_0)e^{-i2\pi(\tau-f_0)} \frac{2\pi f_0}{f}. 
\]  
(33)

\[\Delta f_l = f_l - f_{l-1}.\] By centering \(f_\tau\) and letting \(\Delta f = f_\tau - f_{\tau-1}\), extend the synchrosqueezing process to the successive bins \([f_{\tau} - (1/2)\Delta f, f_{\tau} + (1/2)\Delta f]\); then, fractional lower order S transform-based synchrosqueezing transform can be written as

\[
F_{\text{FLOSSST}}(\tau, f) = (\Delta f)^{-1} \sum_{f_l | F(\tau, f) - f_l| < \Delta f/2} |F_{\text{FLOST}}(\tau, f_l)| f_l \Delta f_l. 
\]  
(34)

For the calculation of IF and FLOSSST of a multicomponent signal, \(y(t)\), we have

\[
y(t) = \sum_{k=1}^{N} y_k(t) = \sum_{k=1}^{N} A_k(t) \cos(2\pi f_k t), 
\]  
(35)

where \(k = 1, 2, \ldots, N\). Then, the FLOSSST of kth component can be expressed as

\[
F_{\text{LOST}}(\tau, f_k) = |f_k| \int_{-\infty}^{\infty} y_k^{(p)}(t) \psi[f_k(t-\tau)] e^{-i2\pi f_k \tau} d\tau. 
\]  
(36)

FLOSSST is just as linear as ST; then

\[
F_{\text{FLOSSST}}(\tau, f) = \sum_{k=1}^{N} F_{\text{FLOST}}(\tau, f_k) = \sum_{k=1}^{N} \delta(f - f_k) \left\{ f_k + \frac{1}{j2\pi F_{\text{FLOST}}(\tau, f_k)} \frac{\partial[F_{\text{FLOST}}(\tau, f_k)]}{\partial \tau} \right\}. 
\]  
(39)
By substituting (37) and (39) into (34), fractional low order STFT-based synchrosqueezing transform time frequency representation of the multicomponent signal can be obtained.

\[
\int_0^\infty \text{FLOSSST}(t, f) e^{i2\pi f_{\Delta f}} f^{-1} df = \frac{1}{2\pi} \int_{-\infty}^\infty \left[ \int_0^\infty \hat{Y}(\lambda) \overline{\psi(\lambda f^{-1})} e^{i\lambda^1 f^{-1}} d\lambda \right] df. \quad (40)
\]

Let \( \xi = \lambda f^{-1} \); then

\[
\int_0^\infty \text{FLOSSST}(t, f) e^{i2\pi f_{\Delta f}} f^{-1} df = \frac{1}{2} \int_0^\infty \overline{\psi(\xi)} \xi^{-1} d\xi \cdot \left[ \frac{1}{\pi} \int_0^\infty \hat{Y}(\lambda) e^{i\lambda^1 f} d\lambda \right]. \quad (41)
\]

For the real signal \( y(t) \), letting \(-1/2 \int_0^\infty \overline{\psi(\xi)} \xi^{-1} d\xi = 1\), we have

\[
\Re e \left[ \int_0^\infty \text{FLOSSST}(t, f) e^{i2\pi f_{\Delta f}} f^{-1} df \right] = \Re e \left[ \frac{1}{\pi} \int_0^\infty \hat{Y}(\lambda) e^{i\lambda^1 f} d\lambda \right] = y^{(p)} (t). \quad (42)
\]

In the piecewise constant approximation corresponding to the binning in \( f \), we have

\[
\overline{Y}(\tau) = \Re e \left\{ \Gamma^{-1} \left[ \sum_{f_j} \text{FLOSSST}_y(t, f_j) e^{i2\pi f_j^1 f_{\Delta f}} f_j \right] \right\}. \quad (43)
\]

3.2.2. Inverse Fractional Lower Order S Transform-Based Synchrosqueezing Transform. Multiplying \( e^{i2\pi f_{\Delta f}} f_{\Delta f}^{-2} \) on both sides of (28) and taking the integral to \( f \), then

\[
\int_0^\infty \text{FLOSSST}_y(t, f) e^{i2\pi f_{\Delta f}} f_{\Delta f}^{-1} f_{\Delta f}^{-2} = (\Delta f)^{-1} \sum_{f_j: |(r, f_j) - f_i| \leq \Delta f/2} \text{FLOSSST}_y(t, f_j) f_j. \quad (45)
\]

Multiplying \( e^{i2\pi f_{\Delta f}} f_{\Delta f}^{-2} \) on both sides of (45) and letting \( \Phi = e^{i2\pi f_{\Delta f} j[f]} f_{\Delta f}^{-1} \), then

\[
\text{FLOSSST}_y(t, f) e^{i\left[ 2\pi f_{\Delta f} \right] j[f]} f_{\Delta f}^{-2} = (\Delta f)^{-1} \sum_{f_j: |(r, f_j) - f_i| \leq \Delta f/2} \text{FLOSSST}_y(t, f_j) f_j = \text{FLOSSST}_y(t, f) \Phi. \quad (46)
\]

where \( y(t) \) is real signal. According to \( y^{(p)} (t) = |y(t)|^{p-1} \cdot \text{sign}(y(t)) \), inverse fractional lower order S transform-based synchrosqueezing transform (IFLOSSST) of \( y(t) \) can be written as
\[ y(t) = |y^{(p)}(t)|^{1/p-1} \text{sign}[y^{(p)}] \]

We can reconstruct the signal \( y(t) \) in FLOSSST time frequency domain employing (48).

3.2.3. The Steps of the FLOSSST Time Frequency Method

(i) Step 1: compute FLOST \( y_k(t) \) of each component \( y_k(t) \) for the signal \( y(t) \) employing (31).

(ii) Step 2: solve FLOST \( y_k(t) \) by substituting FLOST \( y_k(t) \) of each component \( y_k(t) \) to (37).

(iii) Step 3: compute instantaneous frequency \( F_{y_k}(t, f_k) \) of each component \( y_k(t) \) by substituting FLOST \( y_k(t, f_k) \) to (38).

(iv) Step 4: solve \( F_{y_k}(t, f) \) of the signal \( y_k(t) \) by substituting \( F_{y_k}(t, f_k) \) to (39).

(v) Step 5: compute the discrete values FLOST \( y(t, f) \) employing \( F_{y_k}(t, f) \).

(vi) Step 6: solve FLOSSST of the signal \( y(t) \) by substituting FLOST \( y_k(t, f) \) to (34).

3.2.4. Application Review. In this section, \( x(n) \) in (19) is used as the experiment signal. The proposed fractional lower order S transform-based synchrosqueezing transform method, the existing the S transform-based synchrosqueezing transform method, the fractional lower order S transform method, and traditional S transform method are used to display time frequency distribution of the signal \( x(n) \) under Gaussian distribution noise (SNR = -5 dB) and SaS stable distribution noise (GSNR = 22 dB and \( \alpha = 0.8 \)); the simulation results are shown in Figures 9 and 10.

Letting GSNR = 22 dB and \( \alpha = 1.4 \), the ISSST and IFLOSSST methods are used to reconstruct the original signal; the results are shown in Figure 11. In order to further compare the effectiveness of the ISSST and IFLOSSST methods, letting GSNR = 20 dB, the signal \( x(n) \) is reconstructed employing the ISSST and IFLOSSST methods under different \( \alpha; \) their MSEs are shown in Figure 12(a). We apply the ISSST and IFLOSSST methods to reconstruct the signal \( x(n) \) when \( \alpha = 1 \); GSNR changes from 14 dB to 24 dB; the simulations are shown in Figure 12(b).

3.2.5. Remarks. Figure 9 is the time frequency representations of the signal \( x(n) \) under Gaussian noise environment (SNR = -5 dB) employing the ST, FLOSSST, SSST, and FLOSSST methods, respectively. We can see that all the methods can estimate out the time frequency distribution of the signal \( x(n) \), but the synchrosqueezing transform methods have obvious advantages in time frequency resolution. The time frequency representations of the signal \( x(n) \) employing the ST, FLOSSST, SSST, and FLOSSST methods under SaS noise environment (GSNR = 22 dB and \( \alpha = 0.8 \)) are shown in Figure 10. The results show that the ST method in Figure 10(a) and SSST method in Figure 10(c) fail; the FLOSSST method in Figure 10(b) can estimate out the time frequency distribution of the signal \( x(n) \), but its effect is not very ideal. The improved FLOSSST method in Figure 10(d) can better get the time frequency representation of the signal \( x(n) \), which has higher time frequency resolution.

The reconstructed signal \( \tilde{x}(n) \) employing the ISSST method is shown in Figure 11(b) under SaS noise environment (GSNR = 22 dB and \( \alpha = 1.2 \)); it can be seen that the signal \( x(n) \) is covered by SaS noise; the ISSST method fails. Figure 11(b) is the reconstructed signal \( \tilde{x}(n) \) based on the IFLOSSST method under the same conditions; it shows that the reconstructed signal \( \tilde{x}(n) \) is very similar to the original signal \( x(n) \). Figure 12(a) is reconstruction MSE comparison under GSNR = 20 dB when \( \alpha \) changes from 0.2 to 2; the results show that the reconstruction MSEs of the IFSSST method change from 1 dB to 290 dB, but the reconstruction MSEs of the IFLOSSST method have an obvious low level, which are stable at about 2 dB. Hence, the IFLOSSST method has obvious advantage in reconstructing the signal under different \( \alpha \); particularly, the advantage of the IFLOSSST method is more obvious when \( \alpha < 1 \). Figure 12(b) is the reconstruction MSE comparison under \( \alpha = 1 \) when GSNR changes from 14 dB to 78 dB; it shows that the reconstruction MSEs of the IFSSST method have a large variation, but the reconstruction MSEs of the IFLOSSST method change from -2 dB to 8 dB. Hence, the improved IFLOSSST method has better stability in reconstructing the signal.

As a result, the SSST time frequency method and the ISSST signal reconstruction method are only suitable to analyze and reconstruct the signals under Gaussian noise environment, but the improved FLOSSST and IFLOSSST methods can work in Gaussian and \( \alpha \) stable distribution noise environment, which are robust.

4. Application Simulations

In this simulation, the experiment signal adopts the bearing outer race fault signal (DE) in Section 2. 0.2 seconds’ data is selected as the test signal, which is collected at 12,000 samples per second, and \( N = 2400 \). The improved FLOSSST and FLOSSST methods are applied to analyze time frequency distribution of the outer race fault signal; the simulation results are shown in Figure 13.

Figures 13(a) and 13(b) are the time frequency representations of the outer race fault signal employing the FLOSSST and FLOSSST methods, respectively. It can be seen that two methods have a good lateral resolution, the low-frequency shock pulse mainly includes 0 Hz to 4000 Hz, and the dominant frequency of the vibration components is approximately 600 Hz, 2800 Hz, and 3500 Hz. All the
Figure 9: Time frequency representations of the signal $x(n)$ under Gaussian noise environment (SNR $=-5$ dB and $p=1.8$). (a) ST time frequency representation of the signal $x(n)$; (b) FLOST time frequency representation of the signal $x(n)$; (c) SSST time frequency representation of the signal $x(n)$; (d) FLOSSST time frequency representation of the signal $x(n)$.

Figure 10: Continued.
methods have a good vertical resolution; the gap between the impacts can be clearly seen, which regularly change. The time interval in A, B, C, D, E, and F is about 30 ms; the corresponding characteristic frequency of the outer race fault signal is about 33.333 Hz.

In order to further prove the advantages of the improved FLOFSST and FLOSSST methods, $SaS$ distribution noise ($\alpha = 1$ and GSNR = 22 dB) is added in the $\alpha$ stable distribution outer race fault signal as the background noise of actual working environment. The improved methods and existing methods are applied to demonstrate time frequency representation of the outer race fault signal; the simulations are shown in Figure 14. The results show that the FSST method in Figure 14(a) and SST method in Figure 14(b) fail. However, the FLOFSST method in Figure 14(c) and FLOSSST method in Figure 14(d) can give out time frequency distribution of the fault signal under substandard conditions, which have certain ability in the horizontal and vertical time frequency representation, and we can know the dominant frequency and the time interval in A, B, C, D, E, and F, but the overall resolution is not high and needs to improve. Hence, the improved fractional lower order synchrosqueezing methods have better performance superiority than the existing synchrosqueezing methods, which are more suitable for fault analysis in complex environment and are robust.
Figure 12: MSE comparisons of signal reconstruction of the ISSST and IFLOSSST algorithms under different $\alpha$ and GSNR ($p = 1.2$). (a) GSNR = 20 and MSE comparison under different $\alpha$; (b) $\alpha = 1$ and MSE comparison under different GSNR.

Figure 13: The time frequency representations of the outer race fault signal ($p = 1.8$). (a) The FLOSSST time frequency method; (b) the FLOSSST time frequency method.

Figure 14: Continued.
5. Conclusions

α stable distribution is a more appropriate statistical model for the bearing fault signals. STFT transform-based synchrosqueezing transform and S transform-based synchrosqueezing transform are two new time frequency representation methods for nonstationary signal processing; their time frequency resolution can be greatly improved by rearranging the time frequency energy of the signals. In order to make the FSST and SST methods applicable to Gaussian and α stable distribution noise environment, the improved FLOFSST and FLOSSST time frequency representation methods are proposed by employing fractional low order statistics. The performances of the FLOFST and FLOSSST methods are superior to the existing time frequency analysis methods; they have higher time frequency resolution than the existing FLOSTFT and FLOST methods because of the synchrosqueezing processing and can better suppress the impulse noise than the FSST and SST methods. The IFLOFSST and IFLOSSST methods have smaller reconstruction MSEs than the IFSST and ISSST methods under different α (α < 2) and GSNR. We can apply the improved methods to analyze the α stable distribution bearing fault signal; even α stable distribution noise environment, the fault characteristic frequency, the dominant frequency, and the other fault frequency features of the fault signals can be clearly obtained. In the future, we can also further study time frequency filtering technology based on the proposed IFLOFSST and IFLOSSST methods, and the methods have a good application prospect in the field of the bearing fault analysis and detection.

Data Availability

The data used to support the findings of this study are provided in the Supplementary Materials.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

This section contains the original experimental data of this paper. (Supplementary Materials)

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