Mathematical Modeling of the Image Processing in Television Systems

D A Bezuglov¹ and V V Voronin²

¹ Russian customs academy, Budennovskiy 20, Rostov-on-Don 344002, Russia
² Don State Technical University, Gagarin Square 1, Rostov-on-Don 344000, Russia

E-mail: bezuglovda@mail.ru

Abstract. Modern vision systems include a television image processing system. Improving the technical characteristics and expanding the capabilities of modern devices for recording television images makes it necessary to develop and create highly efficient algorithms for processing television images. Such systems process large amounts of information, therefore, the requirements for the reliability of processing large amounts of data at limited time intervals are constantly increasing. The rapid development of high-performance computing systems has recently created the conditions for expanding the range of tasks that can be solved with the help of vision systems. The research for improving the efficiency of methods and algorithms for digital image processing significant. Currently, the processing of television images is divided into several steps. The first step is the formation of a digital representation of the image (discretization, quantization and input of the image into the computer memory). The second step is the pre-processing of images (restoration and filtering). The third step is the formation of a graphic preparation of the image (segmentation and contour detection). One of the most complex and relevant is the scientific and technical task of the contour detections of objects in television images on the background with additive noise. The purpose of this study is the development of the contour detection method on the image with impulse noise using mathematical modeling.

1. Introduction

Modern complexes of technical vision of autonomous robots include television image processing systems. Improving the technical characteristics and expanding the capabilities of modern devices for recording television images makes it necessary to develop and create highly efficient algorithms for processing television images. Such systems process large flows of information, and the requirements for the reliability of processing large amounts of data at limited time intervals are constantly increasing. The fast development of high-performance computing systems in recent years has created the conditions for expanding the range of tasks solved with the help of vision systems. This indicates the relevance of research in the field of improving the efficiency of methods and algorithms for digital image processing [1-4].

Currently, television image processing is divided into stages. The first stage is the formation of a digital representation of the image: discretization, quantization, and save of the image into the computer memory. The second stage is the pre-processing of images: recovery and filtering. The third stage is the formation of a graphic preparation of the image: segmentation and contouring. In the future, analysis, classification, image recognition and the using of the information obtained in modern systems of technical vision are carried out [5-10].
One of the most complex and relevant is the scientific and technical task of contour extraction of objects in television images on the background of additive interference.

**Objective:** to study the efficiency of contours extraction of the vision system under the influence of impulse noise by reducing the standard deviation (RMS) of the error using mathematical modeling methods.

2. **Methods**

Mathematical modeling - is the simulate physical process with a system of mathematical relationships, the solution of which allows getting an answer to the question about the behavior of an object without creating a physical model. But often it’s expensive and inefficient.

Mathematical modeling is a way of studying a real system by replacing its a mathematical model that is more convenient for experimental research using a computer. The mathematical model is an approximate representation of real objects, processes or systems, expressed in mathematical terms and preserving the essential features of the original. In our case, we will consider a digital representation of a grayscale image as a mathematical model.

The expression for the parabolic B-spline is written as follows [16-19]:

$$F_i(x) = \sum_{k=0}^{n+1} b_k B_k^1(x_{i3}), \quad x_{i3} = \frac{1}{n}(x - x_{i+n+1})$$  \hspace{1cm} (1)

where $b_k$ - spline coefficients; $x_{i+n+1}$ - coordinates of the middle of the B-spline carrier; $n$ - degree of spline; $h$ - grid pitch.

We get the expression for the spline approximation of the derivative of the one-dimensional signal:

$$F_{cv}(x) = \sum_{i=1}^{N-1} \left( \frac{x^2}{2h^2} (b_{i+2} - 3b_{i+1} + 3b_i - b_{i-1}) + \frac{x}{h} (b_{i+1} - 2b_i + b_{i-1}) + \frac{1}{2h} (b_{i+1} - b_{i-1}) \right)$$  \hspace{1cm} (2)

Let a matrix $S(i,j)$ of a black-and-white image of size $N \times N$ be given. To solve the problem of contours extraction, we divide the image matrix into rows and columns, differentiate by the proposed method, and then calculate the operator for selecting contours. In contrast to the known approaches [1-4], the differentiation will take into account information about the intensity in the entire line of the image.

The expression for the intensity gradient $G(S(i,j))$ can be written as follows:

$$G(S(i,j)) = \sqrt{\left[ \sum_{i=1}^{N-1} \left( \frac{x^2}{2h^2} (b_{i+2}^n - 3b_{i+1}^n + 3b_i^n - b_{i-1}^n) + \frac{x}{h} (b_{i+1}^n - 2b_i^n + b_{i-1}^n) + \frac{1}{2h} (b_{i+1}^n - b_{i-1}^n) \right)^2 + \frac{1}{2} \right]}$$  \hspace{1cm} (3)

Thus, the proposed new noise-resistant method of gradient selection of the contours of objects in digital halftone television images, involves the following operations [16-19]:

1. Calculation of coefficients of spline approximation by normalized smoothing one-dimensional cubic B-splines for all rows and columns of digital grayscale image $S$.
2. Calculation of two matrices of components of the brightness gradient at each point in the image by the inverse transform. Where analytical functions describing the derivatives of the inverse transform spline functions used in the corresponding coordinates are used as the transformation kernel.
3. The calculation of the module of the brightness gradient at each point in the image by calculating the square root of the sum of squares and the components of the brightness gradient of the image is an expression (3).
4. Performing threshold conversion of the brightness gradient module, during which the contours of objects are formed by highlighting the elements in black on a new white matrix. The gradient modulus of which in the corresponding coordinates exceeds the transformation threshold.

To obtain quantitative estimates of the proposed algorithm, we will use mathematical modeling methods. Afterward, 2 types of noise were superimposed on the original image $S$: impulse noise
“broken pixels”; impulse noise "salt and pepper". Moreover, in the course of mathematical modeling, the influence of the spline smoothing coefficient on the accuracy characteristics was studied.

As quantitative criteria for the accuracy characteristics of the contouring algorithm, we use the following:

1. The standard deviation $e_{RMS}$:

$$e_{RMS} = \sqrt{\frac{1}{MN} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} (MK_{ij} - \overline{MK}_{ij})^2}$$

(4)

where: $MK$ - test image, $\overline{MK}$ - image obtained by the proposed method.

2. Peak SNR:

$$SNR = \frac{255 - \mu}{\sigma_{background}}$$

$$\mu = \frac{1}{MN} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \left( \overline{MK}_{ij} - \mu \right)$$

$$\sigma_{background} = \frac{1}{(N_{background})^2} \sum_{i=n_1}^{n_1+N_{background}} \sum_{j=m_1}^{m_1+N_{background}} (\overline{MK}_{ij} - \mu_{background})^2$$

$$\mu_{background} = \frac{1}{(N_{background})^2} \sum_{i=n_1}^{n_1+N_{background}} \sum_{j=m_1}^{m_1+N_{background}} \overline{MK}_{ij}$$

(5)

where: $\mu$ is the average value $\overline{MK}_{ij}$.

3. Peak SNRF (using background RMS calculations):

$$SNRF = \frac{255 - \mu}{\sigma_{background}}$$

(6)

where $\sigma_{background}$ - standard deviation of the background; $\mu_{background}$ - is the average value of the background; $n_1$, $m_1$ - coordinates of the selected background area with the size $N_{background} \times N_{background}$ in the image under study $\overline{MK}_{ij}$.

3. Experiments and Results

The essence of the computational experiment in mathematical modeling was as follows. The original Lena image shown in Figure 1a was processed by the Canny border detector from the Mathcad 14 package (Figure 1b). In the future, all received images will be compared with this image $MK$. 
Figure 1. Test image: a) the original image; b) the result of processing the test image with a Canny border detector.

Then the test image was exposed to 2 types of noise: impulse noise “broken pixels” (Figure 2a); pulse noise “salt-pepper” (Figure 2b).

Figure 2. Test image exposed to pulsed noise: a) “broken pixels”; b) "salt and pepper".

Figure 3 shows the results of processing by the Sobel operator of the images presented in Figure 2, respectively.

Figure 3. Results of processing by a Sobel operator of a test image exposed to noise: a) “broken pixels”; b) "salt and pepper".

Figure 4 shows the results of processing by the spline differentiation method for the images shown in Figure 2, respectively.
The results of mathematical modeling of the proposed method for selecting contours are given in tables 1-2.

**Table 1.** $e_{RMS}$, SNR and SNRF, broken pixel noise.

| Noise probability | Spline function smoothing factor |
|-------------------|----------------------------------|
|                   | 1      | 5      | 10     | 50     | 100    | 200    | 300    | 500    |
| 0,3               | 0,82   | 1,52   | 1,63   | 1,73   | 1,73   | 1,72   | 1,71   | 1,71   |
| 0,5               | 0,95   | 1,75   | 1,90   | 2,03   | 2,04   | 2,05   | 2,04   | 2,04   |

**Table 2.** $e_{RMS}$, SNR and SNRF, noise "salt and pepper".

| Noise probability | Spline function smoothing factor |
|-------------------|----------------------------------|
|                   | 1      | 5      | 10     | 50     | 100    | 200    | 300    | 500    |
| 0,3               | 0,99   | 1,48   | 1,54   | 1,55   | 1,55   | 1,54   | 1,52   | 1,51   |
| 0,5               | 1,44   | 2,26   | 2,41   | 2,54   | 2,56   | 2,56   | 2,56   | 2,55   |

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**Figure 4.** Results of processing by spline differentiation of a test image exposed to impulse noise: a) “broken pixels”; b) "salt and pepper".

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4. Conclusions
The conducted experimental studies using special test images against the background of impulse noise showed the efficiency and high efficiency of the developed method for isolating the vision system under the influence of impulse noise by reducing the standard deviation (RMS) of the error using mathematical modeling methods.

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