General Covariance and Free Fields in Two Dimensions.

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Abstract

We investigate the canonical equivalence of a matter-coupled 2D dilaton gravity theory defined by the action functional

\[ S = \int d^2x \sqrt{-g} (R\phi + V(\phi) - \frac{1}{2}H(\phi)(\nabla f)^2), \]

and a free field theory. When the scalar field \( f \) is minimally coupled to the metric field (\( H(\phi) = 1 \)) the theory is equivalent, up to a boundary contribution, to a theory of three free scalar fields with indefinite kinetic terms, irrespective of the particular form of the potential \( V(\phi) \). If the potential is an exponential function of the dilaton one recovers a generalized form of the classical canonical transformation of Liouville theory. When \( f \) is a dilaton coupled scalar (\( H(\phi) = \phi \)) and the potential is an arbitrary power of the dilaton the theory is also canonically equivalent to a theory of three free fields with a Minkowskian target space. In the simplest case (\( V(\phi) = 0 \)) we provide an explicit free field realization of the Einstein-Rosen midisuperspace. The Virasoro anomaly and the consistence of the Dirac operator quantization play a central role in our approach.

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1 Introduction

One of the main features of the Liouville field theory is that a canonical transformation maps the theory into a free field theory [1, 2, 3]. The free field realization of the interacting theory allows to carry out an operator quantization of the model although a complete understanding of the full quantum theory remains elusive [4]. The conformal invariance of the theory can be understood, in a natural way, in terms of a generally covariant action [5] which provides an improved, traceless energy momentum tensor. In conformal gauge the generally covariant action describes a Liouville field together with a free field which, due to general covariance, possesses the same improvement term as the Liouville field but with opposite contribution to the hamiltonian constraint. At the quantum level the hamiltonian and momentum constraints generate two copies of the Virasoro algebra but there exists an ambiguity to determine the value of the central charge as it has been pointed out recently [6] in the context of the CGHS theory [7]. The string-type quantization is insensitive to the signature of the target space and produces a non-vanishing central charge. However, in the functional Schrödinger quantization, the total value of the Virasoro anomaly vanishes and one can consistently impose all the Virasoro quantum constraints [8]. Remarkably, it has been possible to explicitly solve all the Virasoro quantum constraints for a generic model of two-dimensional dilaton gravity in the functional Schrödinger representation [8]. Turning things round one could expect that the possibility of finding solutions to the quantum constraints could reflect the fact of an anomaly cancellation between two free fields. Therefore one could guess that the above discussion on the Liouville theory could be extended to a generic theory of 2D dilaton gravity. In fact the canonical transformation mapping the CGHS model into a free field theory, both on-shell [9] and off-shell [6, 10], can be seen as a limiting case of a canonical transformation converting the generally covariant (Liouville) theory into a free field theory with a Minkowskian target space. The aim of this paper is to investigate the validity of this conjecture for a large class of generally covariant theories involving a two-dimensional metric, a dilaton and a single massless scalar field.

In section 2 we shall analyze the covariant theory associated with the Liouville action showing how it can be transformed into a theory of two free
fields without any improvement term. This way of presenting the canonical structure of the theory suggests that the underlying reason to transform it into a free field theory is general covariance, thus supporting the motivation of this work. In section 3 we shall demonstrate, completing the construction sketched in Ref. [11], that a first order generic theory of 2D dilaton gravity minimally coupled to a massless scalar field can be mapped into a theory of free fields with a Minkowskian target space. We shall also address the problem of the comparison of the Schrödinger quantization in geometrical variables and in terms of free field variables. It is well-known that the quantization procedure does not commute with the canonical transformations and it has to be checked on a case-by-case basis whether the quantum wave functionals obtained with one set of variables are related by unitarity with the quantum wave functionals derived with another set of canonical variables. This question has been analyzed for the matter free CGHS model in [12, 13] with a positive answer and we shall prove that for a generic pure gravity model the quantum wave functionals in geometrical variables are equivalent to those obtained with the free field variables. This result reenforces the idea that the free field representation provides an adequate quantization of two-dimensional dilaton gravity theories.

In the remaining part of this paper we shall investigate whether the result of sections 2 and 3 can be extended to theories of 2D dilaton gravity with a non-minimal coupling to a matter field. In section 4 we study the simplest situation: the CGHS model coupled to a Liouville field. Using previous results it is clear that the dilaton-gravity sector is equivalent to two free fields and the remaining Liouville-gravity sector is also equivalent to two free fields. However it is not clear that the full theory can be transformed into a theory with three free fields. We shall show that this is the case and, in contrast with minimally coupled matter field models, it is necessary to mix in a non-trivial way the metric-dilaton and matter fields to produce a theory with three free fields. In section 5 we consider a more realistic family of models with a dilaton coupled scalar. This way one consider important cases of dimensionally reduced general relativity, which has been recently studied from a path integral approach with different results [14]. We shall start our study analyzing with detail the midisuperspace model of cylindrically symmetric gravitational waves with one polarization [15]. The canonical analysis of this system was initiated in Ref. [16] and it is mostly studied in the framework
of the reduced phase-space quantization [7]. In this approach, first solve the constraints and then quantize, the possible anomalies of the constraint (Virasoro) algebra do not arise and it could imply the scheme be inequivalent to the standard Dirac quantization. We shall prove that the theory can also be mapped into a theory of three free fields with a Minkowskian target space, thus generalizing the results of the reduced phase-space approach. Moreover we shall show that the free field realization of the Einstein-Rosen midisuperspace can be extended to models with a potential of the form $V(\phi) \propto \phi^a$, where $a$ is an arbitrary real parameter. This includes the important case of spherically reduced Einstein gravity. Finally, in section 6 we shall state our conclusions.

2 Canonical Structure of the generally covariant Liouville Action

The Liouville action can be obtained from the generally covariant action

$$\int d^2x \sqrt{-g} \left( \beta (\nabla \phi)^2 + 4\lambda^2 e^{2\beta \phi} + R \phi \right).$$

The equation of motion $R = 0$ allow us to fix the gauge $\rho = 0$ in conformal coordinates ($ds^2 = -e^{2\rho} dx^+ dx^-\). The equation of motion of the field $\varphi = 2\beta \phi$ is then the Liouville equation

$$\partial_+ \partial_- \varphi + 2\lambda^2 \beta e^\varphi = 0.$$ (2)

To study the generally covariant theory it is convenient to parametrize the metric in the form

$$ds^2 = e^{2\rho} \left[ -u^2 dt^2 + (v dt + dx)^2 \right],$$ (3)

where the functions $u$ and $v$ are related to the shift and lapse functions. In terms of the fields $\varphi = 2\rho + 2\beta \phi$ and $\eta = 2\rho$ the hamiltonian form of the action is

$$S = \int d^2x (\pi_\eta \dot{\eta} + \pi_\varphi \dot{\varphi} - uH - vP),$$ (4)

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where the Hamiltonian and momentum constraints read as
\[
H = - (\beta \pi^2 + \frac{1}{4 \beta} \varphi'^2 - 4 \lambda^2 \varphi^2 - \frac{1}{\beta} \varphi''') + \beta \pi^2 + \frac{1}{4 \beta} \eta'^2 - \frac{1}{\beta} \eta''',
\]
\[
P = \pi \varphi' + \pi \eta' - 2 \pi \varphi' - 2 \pi \eta' .
\]
It is well-known that one maps the Liouville field \( \varphi \) into a free field \( \psi \) by exploiting the form of the classical solution of the Liouville equation
\[
\varphi = \log \frac{\partial A_+ (x^+) \partial A_- (x^-)}{(1 + \lambda^2 \beta A_+ A_-)^2} ,
\]
where \( A_+ \) and \( A_- \) are two arbitrary functions. The free field \( \psi \) can be constructed as a linear combination of chiral functions but the explicit form is not unambiguous. We choose a definition closely related to the one given in [2]
\[
\varphi = \psi - 2 \log (1 + \lambda^2 \beta A_+ A_-) \]
\[
\pi \varphi = \pi \psi - \lambda^2 \frac{A_+ A_- - A_+ A_-'}{(1 + \lambda^2 \beta A_+ A_-)} ,
\]
with
\[
(\log A_+')' = \frac{\psi'}{2} \pm \beta \pi \psi .
\]
The above expressions define a canonical transformation and a straightforward way to check this is to compute the canonical 2-form of the Liouville theory in terms of \( A_+ \) and \( A_- \)
\[
\omega_L = \int dx \, \delta \varphi \wedge \delta \pi \varphi ,
\]
where \( \delta \) stands for the exterior derivative on phase-space. After same computation one obtains (from now on we shall omit the exterior product)
\[
\omega_L = \frac{1}{\beta} \int \delta (\log A_+ A_-) \delta (\log \frac{A_+}{A_-})' + \omega_b ,
\]
where \( \omega_b \) is a total derivative term\[\]
\[
\omega_b = -2 \lambda^2 \beta \int d \left[ \frac{\delta (A_+ A_-) \delta (\log \frac{A_+}{A_-}) + \delta A_+ \delta A_-}{1 + \lambda^2 \beta A_+ A_-} \right] .
\]
\[\]
\[\text{This term vanishes when the functions } A_\pm \text{ have a diagonal monodromy.}\]
In terms of $\psi$ and $\pi_\psi$, and up to a total derivative, the symplectic 2-form of the theory adopts the canonical form

$$\omega_L = \int dx \, \delta \psi \delta \pi_\psi. \quad (14)$$

Going back to the generally covariant theory and, in terms of the canonical variables $(\psi, \pi_\psi; \eta, \pi_\eta)$, the constraints become

$$H = -(\beta \pi_\psi^2 + \frac{1}{4\beta} \psi'^2 - \frac{1}{\beta} \psi''') + \beta \pi_\eta^2 + \frac{1}{4\beta} \eta'^2 - \frac{1}{\beta} \eta'' \quad , \quad (15)$$

$$P = \pi_\psi \psi' + \pi_\eta \eta' - 2\pi'_\psi - 2\pi'_\eta. \quad (16)$$

Therefore, the theory is described by two improved free fields with opposite contributions to the hamiltonian constraint. The gauge fixing $\eta = 0 = \pi_\eta$ recovers the standard results but due to the presence of the field $\eta$ one can simplify further the constraints using new canonical transformations. First we shall introduce the canonical variables $(\bar{X}^\pm, \bar{\Pi}^\pm)$ defined as

$$2\bar{\Pi}^\pm = -\sqrt{2} \beta (\pi_\eta + \pi_\psi) + \frac{1}{\sqrt{2} \beta} (\eta' - \psi'), \quad (17)$$

$$2\bar{X}^{\pm'} = \pm \sqrt{2} \beta (\pi_\eta - \pi_\psi) - \frac{1}{\sqrt{2} \beta} (\eta' + \psi'), \quad (18)$$

in terms of which the constraints $C^\pm = \pm \frac{1}{2} (H \pm P)$ become

$$C^\pm = \bar{X}^{\pm'} \bar{\Pi}^\pm + \sqrt{\frac{2}{\beta}} \bar{\Pi}^\pm', \quad (19)$$

and a further canonical transformation similar to the one introduced in \cite{6}

$$\bar{X}^\pm = -\sqrt{\frac{2}{\beta}} \log X^{\pm'}, \quad (20)$$

$$\bar{\Pi}^\pm = -\sqrt{\frac{2}{\beta}} \frac{X^{\pm'}}{(X^{\pm'})^2} \bar{\Pi}^\pm + \sqrt{\frac{2}{\beta}} \bar{\Pi}^\pm', \quad (21)$$

removes the “improvement” terms of the energy-momentum tensor \cite{3},\cite{3}

$$C^\pm = X^{\pm'} \bar{\Pi}^\pm. \quad (22)$$
Finally, the transformation

\begin{align}
2\Pi_\pm &= - (\pi_0 + \pi_1) \mp (r^0 - r^1), \\
2X^{\pm'} &= \mp (\pi_0 - \pi_1) - (r^0 + r^1),
\end{align}

brings the constraints into those of two scalar free fields of opposite signature

\begin{align}
H &= \frac{1}{2} \left( \pi_0^2 + (r^0)^2 \right) - \frac{1}{2} \left( \pi_1^2 + (r^1)^2 \right), \\
P &= \pi_0 r^0 + \pi_1 r^1'.
\end{align}

In the standard quantization of conformal field theory each scalar field contributes with $c = 1$ to the total value of the Virasoro anomaly. Therefore, the constraints have a non-trivial center and the physical states cannot be annihilated by all the Virasoro quantum operators. A proper quantization requires the introduction of ghost fields and background charges to achieve a total zero center [19]. However, explicit solutions to the quantum constraints were constructed in Ref. [8] and the way out to the apparent contradiction was provided, in the context of the CGHS theory, in Ref. [3]. In the functional Schrödinger representation, where the states have manifestly positive norms, the scalar with negative kinetic energy has a opposite definition of creation and annihilation operators and contributes with $c = -1$ to the Virasoro anomaly. So, the contributions of the two scalar fields cancel and one can consistently impose all the Virasoro constraints. A similar result can be obtained by evaluating the contribution to the central charge of the fields $\varphi$ and $\eta$. Due to the presence of "background charges" in (15),(14) we have $c_\varphi = 1 + 3Q^2$ and $c_\eta = -1 - 3Q^2$, in the Schrödinger representation while $c_\eta = 1 - 3Q^2$ in the standard conformal field quantization. In the Schrödinger representation one gets a zero center $c = c_\varphi + c_\eta = 0$, while one obtains $c = 2$ in the standard quantization approach. In general, the BRST and the functional Schrödinger quantization give inequivalent physical spectrum [20].

The fact that the solutions for the quantum constraints given in [8] are valid for a generic theory of pure dilaton-gravity suggests that the above mechanism for getting a vanishing central charge could also work for a generic theory. This question will be considered in the next section.
3 Canonical Structure of a Generic Model of 2D dilaton gravity minimally coupled to a scalar field

In the previous section we have seen that the generally covariant Liouville theory can be converted, by means of a canonical transformation, into a free field theory with a target space of indefinite signature. We have obtained this result by composing the classical canonical transformation of Liouville theory with some additional canonical transformations. One can also get this result in a different way by using the form of the classical solutions of the covariant theory expressed in terms of four arbitrary chiral functions [21]. This way the generally covariant Liouville theory and the Jackiw-Teitelboim model [22], which can be regarded as particular cases of a large family of 2D dilaton gravity models [23] can be explicitly transformed into a theory of free fields. It is well-known that by a conformal reparametrization of the fields, one can get rid of the kinetic term of the dilaton and bring the action of a generic model of 2D dilaton gravity into the form

\begin{equation}
S = \int d^2 x \sqrt{-g} \left( R\phi + \lambda^2 V(\phi) - \frac{1}{2} (\nabla f)^2 \right),
\end{equation}

where we have added, for convenience, a minimally coupled massless scalar field \( f \), \( \lambda^2 \) is a dimensionful parameter and \( V(\phi) \) is a dimensionless arbitrary function of the dilaton. For the exponential (Liouville) model we have \( V(\phi) = 4e^{\beta\phi} \), while \( V(\phi) = 4 \) for the CGHS model and \( V(\phi) = 4\phi \) for the Jackiw-Teitelboim model. The spherically reduced Einstein gravity can also be seen as a 2D dilaton gravity model with \( V(\phi) = \frac{2}{\sqrt{\phi}} \). Restricting the 4D metric to be spherically symmetric

\begin{equation}
\text{d}s^2(4) = g_{\mu\nu}dx^\mu dx^\nu + \frac{\phi^2}{\lambda^2} d\Omega^2,
\end{equation}

where \( x^\mu \) are coordinates on a two-dimensional spacetime with metric \( g_{\mu\nu} \), \( \frac{\phi}{\lambda} \) is the radial coordinate and \( d\Omega^2 \) is the line element of the 2-sphere with area \( 4\pi \), the dimensionally reduced Hilbert-Einstein action is

\begin{equation}
S = \int d^2 x \sqrt{-g} \left( R\phi + 2 (\nabla \phi)^2 + 2\lambda^2 \right),
\end{equation}
and a conformal reparametrization of the 2D metric leads to an action of the form

\[ S = \int d^2x \sqrt{-g} \left( R\phi + \frac{2\lambda^2}{\sqrt{\phi}} \right). \quad (30) \]

It is interesting to point out now that, for the Liouville model, the bulk part of the symplectic two-form (12) and the constraints (15), (16) are independent of the coupling parameter \( \lambda^2 \). It only appears in the boundary term \( \omega_b \) of the symplectic form. This also happens for the Jackiw-Teitelboim model [21] and suggests that the free field behaviour of \( \rho \) and \( \phi \) in the theory could be transplanted to the fields \( \rho(x^\pm, \lambda^2 = 0) \) and \( \phi(x^\pm, \lambda^2 = 0) \) for a generic model and that the exact expressions of \( \rho \) and \( \phi \), in terms of chiral functions, could define a proper canonical transformation. To elaborate this idea we shall start our analysis parametrizing the two-dimensional metric as in (3). The canonical form of the action (27) then reads as

\[ S = \int d^2x \left( \pi_\rho \dot{\rho} + \pi_\phi \dot{\phi} + \pi_f \dot{f} - uH - vP \right), \quad (32) \]

where

\[ H = -\frac{1}{2} \pi_\rho \pi_\phi + 2 (\phi'' - \phi' \rho') - e^{2\rho} \lambda^2 V(\phi) + \frac{1}{2} (\pi_f^2 + f'^2), \quad (33) \]

\[ P = \rho' \pi_\rho - \pi'_\rho + \phi' \pi_\phi + \pi_f f'. \quad (34) \]

Mimicking the idea of Liouville theory we want to use the expression of the classical solutions in terms of chiral functions to promote them into a canonical transformation. In conformal gauge the system of equations of motion is

\[ 8e^{-2\rho} \partial_+ \partial_- \rho = -\lambda^2 V'(\phi), \quad (35) \]

\[ -4e^{-2\rho} \partial_+ \partial_- \phi = \lambda^2 V(\phi), \quad (36) \]

\[ -\partial_\rho^2 \phi + 2 \partial_\pm \phi \partial_\pm \rho = T^f_{\pm \pm} = \frac{1}{2} (\partial_\pm f)^2, \quad (37) \]

\[ \partial_+ \partial_- f = 0, \quad (38) \]
but the problem is that, up to specific models, the general solution is unknown. To bypass this situation one can first restrict the theory imposing chirality to the scalar matter field. Assuming that $T^f_{-} = 0$ one can identify the two independents chiral functions of the nontrivial sector of the theory. It is easy to see that equation (37) implies
\begin{equation}
-2\rho \partial_\phi = a, \tag{39}
\end{equation}
where $a$ is an arbitrary function of the $x^+$ coordinate. Inserting this equation into (36) we get
\begin{equation}
\partial_+ \partial_- \phi + \frac{\lambda^2}{4a} V(\phi) \partial_- \phi = 0, \tag{40}
\end{equation}
which has the solution
\begin{equation}
\partial_+ \phi = A - \frac{\lambda^2}{4a} J(\phi), \tag{41}
\end{equation}
with $A$ another arbitrary function of the $x^+$ coordinate and $J'(\phi) = V(\phi)$. Equations (37), (39), (41) we can evaluate the non-trivial component of the energy momentum tensor in terms of the fields $A$ and $a$
\begin{equation}
C_+ = 2 \left( \partial_A + \frac{A}{a} \partial_a \right). \tag{42}
\end{equation}
For arbitrary $J(\phi)$ it is not possible to integrate explicitly equation (41). However, the implicit solution for $\phi$ in terms of $A, a$ and a constant of integration $\beta$, which is in this case an arbitrary function of the $x^-$ coordinate, contains enough information to work out the symplectic 2-form of the theory
\begin{equation}
\omega = \int dx \delta \phi \delta \pi_\phi + \delta \rho \delta \pi_\rho + \delta f \delta \pi_f, \tag{43}
\end{equation}
where the canonical momenta are given by
\begin{equation}
\pi_\phi = -2A = \frac{\lambda^2}{4a} V(\phi) + \frac{a'}{a} - \left( \frac{\partial_\phi^2}{\partial_\phi} \right), \tag{44}
\end{equation}
\begin{equation}
\pi_\rho = -2A = -2A + \frac{\lambda^2}{2a} J(\phi) - 2\partial_- \phi. \tag{45}
\end{equation}
Inserting this into (43) we get

\[ \omega = -\int \! dx \left[ -\delta \phi \frac{\delta a'}{a} + \frac{a'}{a^2} \delta \phi \delta a + \delta \phi \frac{\partial^2 \delta \phi}{\partial \phi} - \frac{\partial^2 \phi}{(\partial \phi)^2} \delta \phi \partial \delta \phi + \frac{\partial \delta \phi}{\partial \phi} \partial \delta \phi \partial \delta \phi a \right. 
\]

\[ + \frac{\partial \delta \phi}{\partial \phi} \partial \delta \phi \partial A + \lambda^2 \frac{J(\phi)}{4a^2} - \frac{\lambda^2}{4a^2} V(\phi) \delta \phi \delta a - \frac{\delta a}{a} - \frac{\delta a}{a} \delta A \]

\[ - \delta a \partial \delta \phi + \delta f \delta \pi_f \right] \]

(46)

Taking into account the following identities

\[ - \frac{\delta \phi}{a} \delta a' = - \left( \frac{\delta \phi \delta a}{a} \right)' + \delta A \frac{\delta a}{a} - \frac{\lambda^2}{4a^2} V(\phi) \delta \phi \delta a - \partial \delta \phi \frac{\delta a}{a} - \frac{a'}{a^2} \delta \phi \delta a \]

(47)

\[ \delta \phi \frac{\partial^2 \delta \phi}{\partial \phi} = - \left( \frac{\delta \phi \partial \delta \phi}{\partial \phi} \right)' + \delta \phi \frac{\partial \delta \phi}{\partial \phi} + \frac{\delta \phi \partial \delta \phi}{\partial \phi} \]

\[ - \frac{\partial \phi \partial \delta \phi}{\partial \phi} + \frac{\partial^2 \phi}{(\partial \phi)^2} \delta \phi \partial \delta \phi \]

(48)

which can be checked by using the relation (41), the infinite-dimensional part of the symplectic form turns out to be independent of the coupling parameter \( \lambda^2 \) and can be expressed in terms of the fields \( A, a, f \) and \( \pi_f \) only

\[ \omega = \int \! dx \left[ 2 \frac{\delta a}{a} \delta A + \delta f \delta \pi_f \right] + \int \! d \left( \delta \phi \frac{\delta a}{a} + \delta \phi \frac{\delta \phi}{\partial \phi} \right) \]

(49)

The parameter \( \lambda^2 \) only appears in the boundary term in an implicit way through the relation \( \phi = \phi(A, a; \lambda^2) \) defined by the equation (41). It is then clear that defining the canonical variables

\[ X^+ = \log \frac{a A}{A}, \quad \Pi_+ = 2A \]

(50)

the symplectic form takes, up to a boundary term, the form

\[ \omega = \int \! dx \delta X^+ \delta \Pi_+ + \delta f \delta \pi_f \]

(51)
and the non-trivial constraint $C_+ = \frac{1}{2} (H + P)$ takes the free field form

$$C_+ = \Pi_+ X^{+'} + \frac{1}{4} (\pi_f + f')^2 .$$  \hspace{1cm} (52)

At this point we must stress that in the above discussion we have made use of the unconstrained equations of motion (35),(36). To define an off-shell canonical transformation we have to check that the above derivation is still valid if the functions $A, a, \beta$ are arbitrary functions of the space time coordinates. To this end we introduce the following definition. The symbol \(\tilde{\cdot}\) affecting any functional of the previous chiral functions $A, a, \beta$ means that they are considered as arbitrary (not chiral) functions \(\tilde{A}, \tilde{a}, \tilde{\beta}\) and that the possible derivatives or integrations have been replaced according to the rule \(\partial_{\pm} \rightarrow \pm \partial_x \left( \partial_{\pm}^{-1} \rightarrow \pm \partial_x^{-1} \right)\) as in passing from the solution (8) of Liouville theory into the canonical transformation (8),(9). Therefore, we define now a transformation to the new set of variables $\tilde{A}, \tilde{a}, \tilde{\beta}$

$$\phi = \tilde{\phi} , \hspace{1cm} (53)$$

$$\pi_\phi = -2 \tilde{\rho} = \frac{1}{4a} V \left( \tilde{\phi} \right) + \tilde{\beta} ' + \left( \frac{\partial^2 \phi}{\partial_{-} \phi} \right) , \hspace{1cm} (54)$$

$$\rho = \frac{1}{2} \log \frac{\partial_{-} \phi}{\tilde{a}} , \hspace{1cm} (55)$$

$$\pi_\rho = -2 \tilde{\phi} = -2 \tilde{A} + \frac{1}{2a} J \left( \phi \right) - 2 \tilde{\partial}_{-} \phi , \hspace{1cm} (56)$$

where from now on we absorb the $\lambda^2$ parameter in the potential function \(V(\phi)\). Since the dependence of $\phi$ on $\beta$ is ultralocal we can prove immediately the following identities

$$\left( \tilde{\phi} \right)' = \partial_{+} \phi - \partial_{-} \phi , \hspace{1cm} (57)$$

$$\left( \tilde{\partial}_{-} \phi \right)' = \partial_{+} \tilde{\partial}_{-} \phi - \tilde{\partial}_{-} \phi . \hspace{1cm} (58)$$

To prove (57) we expand $\left( \tilde{\phi} \right)'$ in terms of $\tilde{A}, \tilde{a}, \tilde{\beta}$

$$\left( \tilde{\phi} \right)' = \sum_i \frac{\partial \tilde{\phi}}{\partial \tilde{A}^{(i)}} \left( \tilde{A} \right)^{(i+1)} + \frac{\partial \tilde{\phi}}{\partial \beta} \left( \tilde{\beta} \right)' = \sum_i \frac{\partial \tilde{\phi}}{\partial \tilde{\beta} \partial_{+} \beta} \left( \tilde{A} \right)^{(i+1)} A - \frac{\partial \tilde{\phi}}{\partial \beta} \partial_{-} \beta$$

$$= \partial_{+} \phi - \partial_{-} \phi ,$$
where the superindex \( (i) \) indicates the order of derivation. Equation (58) can be checked in an analogous way. These two identities imply that the computation of the constraints and the symplectic form in terms of \( A, a \) can be extended to the fields \( \tilde{A}, \tilde{a} \) and therefore all the above results are valid when \( A \) and \( a \) are replaced by \( \tilde{A} \) and \( \tilde{a} \).

We shall now return to the general situation. Without assuming chirality the solution to the equations of motion is parametrized by four arbitrary chiral functions \( A(x^+), a(x^+), B(x^-), b(x^-) \). They have a very simple interpretation. Two of them are the two arbitrary functions associated with the residual conformal coordinate transformations while the other two account for the two components of the energy-momentum tensor. It is convenient to choose \( A, a, B, b \) in such a way that when \( T^{ff}_{--} = 0 \) the equations of motion are equivalent to (39), (41) and when \( T^{ff}_{++} = 0 \) they are equivalent to

\[
\partial_\phi B - \frac{1}{4\phi} J(\phi),
\]

\[
e^{-2\rho} \partial_\phi \phi = b.
\]

The classical solution \( \phi = \phi(A, a; B, b), \rho = \rho(A, a; B, b) \) can be employed to construct a transformation to the new variables \( A, a, B, b \)

\[
\phi = \tilde{\phi}, \quad \pi_\phi = -2\tilde{\rho}, \quad \rho = \tilde{\rho}, \quad \pi_\rho = -2\tilde{\phi}.
\]

This transformation reduces to (53-56) when \( T^{ff}_{--} = 0 \) and to the analogous chiral transformation when \( T^{ff}_{++} = 0 \). Due to general covariance the constraints \( C_\pm = \pm\frac{1}{2}(H \pm P) \) are chiral functions on-shell and therefore \( C_+ (C_-) \) must take the same form as when we impose the condition \( C_- = 0 (C_+ = 0) \). This is so because restriction to chirality (e.g. \( T^{ff}_{--} = 0 \)) gives a condition over the fields \( X^-, \Pi_- \) and keeps free \( X^+, \Pi_+ \). Therefore any correction to the constraint \( C_+ \) should depend only on the fields \( X^-, \Pi_- \) in order to get the adequate chiral limit. But over solutions the fields \( X^\pm, \Pi_\pm \) must depend only on the \( x^\pm \) coordinate respectively so such a correction would not be consistent. So in accordance with our previous result we have

\[
C_\pm = \Pi_\pm X^{\pm f} \pm \frac{1}{4}(\pi_f \pm f^n)^2,
\]

where the superindex \( (i) \) indicates the order of derivation. Equation (58) can be checked in an analogous way. These two identities imply that the computation of the constraints and the symplectic form in terms of \( A, a \) can be extended to the fields \( \tilde{A}, \tilde{a} \) and therefore all the above results are valid when \( A \) and \( a \) are replaced by \( \tilde{A} \) and \( \tilde{a} \).
where $X^+, \Pi_+$ are given by (50) and
\[ X^- = \log \tilde{b} \tilde{B}, \quad \Pi_- = 2\tilde{B}. \] (63)

Now the question to be considered is the canonicity of the transformation $(\phi, \pi_\phi, \rho, \pi_\rho) \mapsto (X^\pm, \Pi^\pm)$ (up to a boundary term). In doing so let us write the most general expression for the symplectic 2-form $\omega$ in terms of $X^\pm, \Pi^\pm$
\[ \omega = \int dxdy \left[ F \delta X^+(x) \delta \Pi_+(y) + G \delta X^-(x) \delta \Pi_-(y) + L \delta X^+(x) \delta X^-(y) + M \delta X^+(x) \delta \Pi_-(y) + N \delta X^-(x) \delta \Pi_+(y) + Q \delta \Pi_+(x) \delta \Pi_-(y) \right]. \] (64)

In order to reproduce the hamiltonian equations of motion in conformal gauge $\partial_\pm X^\pm = \partial_\pm \Pi^\pm = 0$, it is clear that the hamiltonian vector field $X_H$ must be
\[ X_H = \int dx \left[ X^+(x) \frac{\delta}{\delta X^+(x)} - X^-(x) \frac{\delta}{\delta X^-(x)} + \Pi'_+(x) \frac{\delta}{\delta \Pi_+(x)} - \Pi'_-(x) \frac{\delta}{\delta \Pi_-(x)} \right]. \] (65)

The condition
\[ i_{X_H} \omega = -\delta H, \] (66)

together with the requirement that the symplectic form be invariant under spatial diffeomorphisms (i.e., the integral of a scalar density), implies that the transformation is canonical, up to a boundary term $\omega_b$
\[ \omega = \int dx \delta X^+ \delta \Pi_+ + \delta X^- \delta \Pi_- + \omega_b. \] (67)

Therefore the canonical form of the action is of the form
\[ S = \int d^2x \left[ \Pi_+ \dot{X}^+ + \Pi_- \dot{X}^- + \pi_\rho \dot{f} 
- u \left( \Pi_+ X'^+ - \Pi_- X'^- + \frac{1}{2} \left( \pi_\rho^2 + f'^2 \right) \right) 
- v \left( \Pi_+ X'^+ + \Pi_- X'^- + \pi_\rho f' \right) \right] + \text{boundary terms}. \] (68)

An indirect way of checking the consistence of the above results can be carried out by evaluating the lagrangian density on the space of solutions of the
hamiltonian equations of motion. It is easy to see from the free field form of the theory \((68)\) that it should be a total derivative. For models with a potential of the form \(V(\phi) \propto \phi^a\), where \(a\) is an arbitrary real parameter, one can get the same results by using equations \((35),(36),(38)\) to write \((27)\) in the desired way. However for the Liouville model one has to explicitly solve the hamiltonian equations of motion to obtain the total derivative. Only for the models with \(V \propto \phi^a\) it is possible to get the total derivative by direct manipulation of the hamiltonian equations.

To finish this section we would like to analyze the problem of the equivalence of the quantum theory defined in the geometrical variables and the one which can be defined in terms of the free fields. As is well-known in classical mechanics one can transform, via Hamilton-Jacobi theory, an arbitrary interacting theory into a trivial one. However, the canonical transformation which do that cannot be promoted into to a unitary transformation relating the quantum wave functionals. This question has been analyzed in detail for the CGHS theory \([12]\) in the absence of matter fields. Moreover, in Ref. \([13]\) it was shown that the wave functionals obtained in terms of the CJZ variables \(\parallel\) for a generic 2D dilaton gravity \([24]\) are equivalent to the wave functionals given in terms of the geometrical variables. Here we shall extend the analysis of \([13]\) to show the equivalence of the quantum theory based on geometrical variables and the quantization obtained in the free field representation. The basic idea to show the quantum equivalence is that in both representations the constraints can be written in a form which is linear in momenta. In the geometrical variables \((\rho, \phi)\), the constraints can be brought to the form \((8)\)

\[
\pi_\rho = Q [C; \rho, \phi] , \quad (69)
\]

\[
\pi_\phi = \frac{g [C; \rho, \phi]}{Q [C; \rho, \phi]} , \quad (70)
\]

where

\[
Q [C; \rho, \phi] = 2\sqrt{(\phi'')^2 + [C - J(\phi)] e^{2\rho}} , \quad (71)
\]

\[
g [C; \rho, \phi] = 4\phi'' - 4\phi' \rho' - 2V(\phi)e^{2\rho} , \quad (72)
\]

\(\parallel\)The CJZ variables are only free field variables for the CGHS theory.
being the constant $C$ the ADM energy of the system. In the functional
Schrödinger representation the quantum version of the above constraints have
the solution
\[ \psi [C; \rho, \phi] = \exp \left\{ i \int dx \left[ Q + \phi' \log \left( \frac{2\phi' - Q}{2\phi' + Q} \right) \right] \right\} . \] (73)

On the other hand, the constraints in the free field variables become
\[ \pi_0 = \pm r^1 \quad \pi_1 = \pm r^0 . \] (74)

We have to point now that, on the constrained surface, the symplectic form
of the theory turns out to be equal to the boundary term $\omega_b$ only. This is
consistent with the fact that, in the absence of matter, the theory is topologi-
al and it is described by a pair of canonically conjugated global variables
$(C, P)$. If we quantize the theory in the Schrödinger representation with wave
functional $\Psi [r^a, P]$, the physical eigenstates of the hamiltonian operator are
\[ \Psi [r^a, P] = \exp \left\{ \pm \frac{i}{2} \int dx \left[ r^0 (r^{1'}) - r^1 (r^{0'}) \right] \right\} \exp iCP . \] (75)

The point is to show that both wave functionals (73) and (75) are equivalent.
To do that let us consider the generating functional $F [\rho, \phi, r^0, r^1, P]$ of the
canonical transformation relating geometrical and free field variables
\[ \pi_\rho = \frac{\delta F}{\delta \rho} \quad \pi_\phi = \frac{\delta F}{\delta \phi} , \] (76)
\[ \pi_0 = -\frac{\delta F}{\delta r^0} \quad \pi_1 = -\frac{\delta F}{\delta r^1} . \] (77)

The wave functional $\psi [C; \rho, \phi]$ is determined by the equations
\[ -i \frac{\delta}{\delta \rho} \psi [C; \rho, \phi] = Q [C; \rho, \phi] \psi [C; \rho, \phi] . \] (78)
\[ -i \frac{\delta}{\delta \phi} \psi [C; \rho, \phi] = g [C; \rho, \phi] \frac{Q [C; \rho, \phi]}{Q [C; \rho, \phi]} \psi [C; \rho, \phi] , \] (79)
\[ \hat{M} \psi = C \psi , \] (80)
and $\Psi [C; r^a, P]$ by

$$-i \frac{\delta \Psi}{\delta r^0} = \pm r^1 \Psi ,$$

(81)

$$-i \frac{\delta \Psi}{\delta r^1} = \pm r^0 \Psi ,$$

(82)

$$\hat{M} \Psi = C \Psi ,$$

(83)

We can use the constraint equations (69),(70) together with (76) to find a relation $\rho = \rho(r^0, r^1)$ and $\phi = \phi(r^0, r^1)$ between the two sets of coordinates

$$\frac{\delta F}{\delta \rho} = Q [C; \rho, \phi] ,$$

(84)

$$\frac{\delta F}{\delta \phi} = \frac{g [C; \rho, \phi]}{Q [C; \rho, \phi]} .$$

(85)

Because of the canonical transformation converts the constraints (69),(71) into (74) this relation is exactly the same that

$$-\frac{\delta F}{\delta r^0} = \pm r^1 ,$$

(86)

$$-\frac{\delta F}{\delta r^1} = \pm r^0 .$$

(87)

Now it is easy to see that the relation between the quantum wave functionals is given for a generic model by

$$\Psi [C; r^a, P] = [\exp (-iF [\rho, \phi, r^a, P]) \psi [C; \rho, \phi]] |_{\rho = \rho(r^0, r^1), \phi = \phi(r^0, r^1)} .$$

(88)

It can be easily checked that $\Psi$ satisfies equations (81),(82),(83) whenever $\psi$ satisfies (78),(79),(80). It is interesting to remark that the validity of the argument is based on the fact that in both representations the constraints can be written in a form which is linear in momenta and therefore leads to first order differential functional equations in the Schrödinger representation.

4 The CGHS model coupled to a Liouville Field

One of the drawbacks of considering dilaton gravity models minimally coupled to a scalar field in two-dimensions is that, unlike the realistic situation,
the matter field $f$ does not "feel" the gravitational field. The purpose of the remaining part of this paper is to investigate theories with a non-minimal coupling to the matter field. If we consider the dimensional reduction of the action of a scalar field minimally coupled to a 3D or 4D metric

$$S_m \propto \int d^2x \sqrt{-g} \phi (\nabla f)^2 , \quad (89)$$

we see that matter couples to the dilaton, which plays the role of the radial coordinate. In this section we shall consider an intermediate situation in which the matter couples to the metric in a non-minimal way but the coupling is independent of the dilaton field. The simplest example is provided by the CGHS model coupled to a Liouville field $f$

$$S = \int d^2x \sqrt{-g} \left[ R\phi + 4\lambda^2 - \beta (\nabla f)^2 - \gamma e^{2\beta f} - Rf \right] . \quad (90)$$

The pure gravity sector of this theory is canonically equivalent to two free fields and the matter sector can also be mapped into a theory of two free fields. We shall show that the full theory can be transformed into a theory of three free fields by combining the metric-dilaton and the matter field in proper way. This example provides a further evidence to the existence of a wide class of dilaton gravity models coupled to a scalar field which are essentially equivalent to a theory of three free fields with a Minkowskian target space.

If we parametrize the metric as in (3), the action becomes (32) in hamiltonian form where now the constraint functions are given by

$$H = -\frac{1}{2} \pi_\rho \pi_\phi + 2 (\phi'' - \rho'\phi') - 4\lambda^2 e^{2\rho} + \beta (f')^2$$

$$-2 (f'' - f' \rho') + \gamma e^{2(\rho + \beta f)} + \frac{1}{4\beta} (\pi_\phi + \pi_f)^2 , \quad (91)$$

$$P = \pi_\phi \phi' + \pi_\rho \rho' - \pi'_\rho + \pi_ff' . \quad (92)$$

The equations of motion derived from the action (90) are

$$\partial_+ \partial_- \rho = 0 , \quad (93)$$

$$\partial_+ \partial_-(\phi - f) = -\lambda^2 e^{2\rho} + \frac{1}{4} \gamma e^{2(\rho + \beta f)} , \quad (94)$$
\[ \partial_+ \partial_- (\rho + \beta f) = -\frac{1}{4} \gamma \beta e^{2(\rho + \beta f)}, \]

\[ \partial^2 \phi - 2 \partial_+ \phi \partial_+ \rho = \partial^2 f - \beta (\partial_\pm f)^2 - 2 \partial_\pm f \partial_\pm \rho. \]

The general solution to the Hamiltonian equations (93), (94), (95) can be expressed in terms of six arbitrary chiral functions (which are independent only before imposing the constraints) \( A(x^+), a(x^+), P(x^+), B(x^-), b(x^-), Q(x^-) \) as

\[ \rho = \frac{1}{2} \log \partial_+ A \partial_- B, \]

\[ \phi = -\lambda^2 AB + a + b, \]

\[ f = \frac{1}{2\beta} \log \frac{\partial_+ P \partial_- Q}{(1 + \frac{1}{4} \gamma \beta PQ)^2} - \frac{1}{2\beta} \log \partial_+ A \partial_- B. \]

Guided by the solution (97), (98), (99) we can write the following transformation to the new variables

\[ \phi = -\lambda^2 AB + a + b, \]

\[ \pi_\phi = (\log A')' + (\log B')', \]

\[ \rho = \frac{1}{2} \log -A'B', \]

\[ \pi_\rho = 2\lambda^2 (A'B - AB') - 2a' + 2b' + \frac{1}{\beta} (\log P')' - \frac{1}{\beta} (\log Q')' \]

\[ -\frac{1}{\beta} (\log A')' + \frac{1}{\beta} (\log B')' - \frac{\gamma P'Q - PQ'}{2 + \frac{1}{4} \gamma \beta PQ}, \]

\[ f = \frac{1}{2\beta} \log \frac{-P'Q'}{(1 + \frac{1}{4} \gamma \beta PQ)^2} - \frac{1}{2\beta} \log -A'B', \]

\[ \pi_f = (\log P')' - (\log Q')' - \frac{\gamma \beta P'Q - PQ'}{2 + \frac{1}{4} \gamma \beta PQ}. \]

The two-form (43) can be written in terms of the new variables as

\[ \omega = \int dx \left[ -2\delta A \delta \left( \frac{a' - m'}{A'} \right)' + 2\delta B \delta \left( \frac{b' - n'}{B'} \right)' + 2\beta (\delta m + \delta n) (\delta m' - \delta n') \right], \]
where
\[ m = \frac{1}{2\beta} (\log P' - \log A') \] (107)
\[ n = \frac{1}{2\beta} (\log Q' - \log B') \] (108)
and the constraints \( C_\pm \) are expressed
\[ C_+ = -2a' (\log A')' - \frac{1}{2\beta} [(\log A')']^2 + 2a'' - \frac{1}{\beta} (\log P'') \]
\[ + \frac{1}{\beta} (\log A')'' + \frac{1}{2\beta} [(\log P')']^2, \] (109)
\[ C_- = 2b' (\log B')' + \frac{1}{2\beta} [(\log B')']^2 - 2b'' + \frac{1}{\beta} (\log Q'') \]
\[ - \frac{1}{\beta} (\log B')'' - \frac{1}{2\beta} [(\log Q')']^2. \] (110)

If we now define
\[ X^+ = -2A, \quad \Pi_+ = \left( a' - \frac{m'}{A'} \right)' \] (111)
\[ X^- = 2B, \quad \Pi_- = \left( b' - \frac{n'}{B'} \right)' \] (112)
\[ X^f = -\sqrt{2\beta} (m + n), \quad \Pi_f = \sqrt{2\beta} (m' - n') \] (113)
the 2-form (106) becomes
\[ \omega = \int dx \left[ \delta X^+ \delta \Pi_+ + \delta X^- \delta \Pi_- + \delta X^f \delta \Pi_f \right], \] (114)
indicating that the transformation \( \rho, \pi_\rho, \phi, \pi_\phi, f, \pi_f \longrightarrow X^\pm, \Pi_\pm, X^f, \Pi_f \) is canonical. It can be also checked that
\[ C_\pm = X^\pm' \Pi_\pm \pm \frac{1}{4} (X^f' \pm \Pi_f)^2, \] (115)
and therefore the canonical transformation converts the theory into a theory of free fields. Thus we have seen that, even when the coupling to the matter
is not conformal, it is possible to capture the canonical structure of the full theory in terms of a set of free fields. We must note that, although the free field variables \((X^f, \Pi_f)\) are the standard ones of Liouville theory, the remaining pair of free fields \((X^\pm, \Pi_\pm)\) are not made out of purely dilaton gravity variables. In the definition (111), (112) the fields \(\Pi_\pm\) are functions of the Liouville field \(f\) in addition to the \((\phi, \rho)\) fields. So the canonical transformation mixes intrinsically the three fields \((\phi, \rho; f)\) to produce three free fields.

5 Free field representation of dilaton-coupled scalar models

In this section we continue our analysis of the canonical structure of matter-coupled 2D dilaton gravity. When a massless scalar field is coupled to gravity in higher dimension and the theory is reduced to two dimensions by symmetry reduction one derives an effective 2D dilaton gravity model with a dilaton-dependent coupling. The simplest model can be obtained from 3-dimensional gravity minimally coupled to a massless Klein-Gordon field under the assumption of axi-symmetry

\[
\left(3\right) \quad ds^2 = g_{\mu \nu}(t,r) dx^\mu dx^\nu + \phi^2(t,r) d\psi^2,
\]

with \(x^0 = t\) and \(x^1 = r\). After dimensional reduction the matter-coupled 2D dilaton gravity theory is described by the action

\[
S = \int d^2x \sqrt{-g} \left( R\phi - \frac{1}{2} \phi \left( \nabla f \right)^2 \right),
\]

where \(f\) is the Klein-Gordon field. Remarkably the above model is also equivalent to the 4-dimensional Einstein-Rosen wave sector of general relativity. Imposing the cylindrical symmetry to the source-free Einstein theory

\[
\left(116\right) \quad ds^2 = g_{\mu \nu}(t,r) dx^\mu dx^\nu + \phi^2 g_{ab}(t,r) dx^a dx^b \quad a, b = 2, 3,
\]

with \(x^2 = z\), \(x^3 = \phi\) and \(det g_{ab} = 1\), the reduced theory is a 2D dilaton gravity theory coupled to a \(SL(2,R)/SO(2)\) coset space \(\sigma\) model (see for instance [25]). If the matrix \(g_{ab}\) is assumed to be diagonal

\[
\left(119\right) \quad g_{ab} = \begin{pmatrix} e^{-f} & 0 \\ 0 & e^{f} \end{pmatrix},
\]
the solutions are the Einstein-Rosen gravitational waves and the 2D theory is
given by (117). If the 4D theory is general relativity minimally coupled to a scalar field \( f \) and we consider spherically symmetric fields like (12) the spherical coordinates can be integrated out and, after a conformal reparametrization of the 2D metric, the action becomes

\[
S = \int d^2x \sqrt{-g} \left( R\phi + \frac{2\lambda^2}{\sqrt{\phi}} - \frac{1}{2} \phi (\nabla f)^2 \right).
\]

(120)

Note that this class of models are different from those considered in \cite{25} (and references therein), for which the matter sector is given by a non-linear sigma model but the potential is trivial. By contrast we consider models with a matter sector described by a single scalar field but allows for a non-trivial potential.

An important ingredient in our analysis of minimally coupled models of section 3 is that the free field behaviour of the fields \( \rho \) and \( \phi \) of the theory \cite{27} with a trivial potential can be extended to the models with an arbitrary potential. Following the same argument we shall show in this section that the free field representation of the Einstein-Rosen midisuperspace model, which we shall explicitly construct in the next subsection, can be extended to a wide family of models which includes the very important case of spherically symmetric gravity \cite{20}.

### 5.1 Free field representation of the Einstein-Rosen midisuperspace

In conformal gauge the equations of motion derived from the action (117) are

\[
\partial_+ \partial_- \phi = 0 , \tag{121}
\]

\[
4 \partial_+ \partial_- \rho + \partial_+ f \partial_- f = 0 , \tag{122}
\]

\[
2\phi \partial_+ \partial_- f + \partial_+ \phi \partial_- f + \partial_- \phi \partial_+ f = 0 , \tag{123}
\]

\[
C_\pm = \pm 2 \partial_\pm^2 \phi \mp 4 \partial_\pm \phi \partial_\pm \rho \pm \phi (\partial_\pm f)^2 = 0 . \tag{124}
\]

Equation (121) is a free field equation with solution

\[
\phi = A + B , \tag{125}
\]
where \( A = A(x^+) \) and \( B = B(x^-) \) are arbitrary chiral functions. Inserting this into (123) we get
\[
2(A + B)\partial_+\partial_- f + \partial_+ A\partial_- f + \partial_- B\partial_+ f = 0 ,
\]
which can also be written
\[
2(A + B) \frac{\partial^2 f}{\partial A \partial B} + \frac{\partial f}{\partial B} + \frac{\partial f}{\partial A} = 0 ,
\]
or
\[
- \frac{\partial^2 f}{\partial T^2} + \frac{\partial^2 f}{\partial X^2} + \frac{1}{X} \frac{\partial f}{\partial X} = 0 ,
\]
where
\[
T = \frac{1}{2}(A - B), \quad X = \frac{1}{2}(A + B) .
\]
This is a 2D Laplace equation in polar coordinates. Its solution is known to be
\[
f = \frac{1}{2} \int_{-\infty}^{\infty} d\lambda \ J_0 \left( \frac{\lambda}{2}(A + B) \right) \left[ A_+(\lambda)e^{i\frac{\lambda}{2}(A-B)} + A_-(\lambda)e^{-i\frac{\lambda}{2}(A-B)} \right] ,
\]
where \( J_0 \) is the zero order Bessel function and \( A_+(\lambda) = A_-(\lambda) \), \( A_+(-\lambda) = A_-(\lambda) \). Finally we can use (122) to calculate \( \rho \) as
\[
\partial_+\rho = -\frac{1}{4} \int_{-\infty}^{x^-} \partial_+ f \partial_- f + \partial_+ a ,
\]
\[
\partial_-\rho = \frac{1}{4} \int_{x^+}^{+\infty} \partial_+ f \partial_- f + \partial_- b ,
\]
where we have introduced two new chiral functions \( a = a(x^+) \), \( b = b(x^-) \). The constraint equations can be written in the form
\[
C_+ = 2\partial_+^2 A - 4\partial_+ A\partial_+ a - 2P ,
\]
\[
C_- = -2\partial_-^2 B + 4\partial_- B\partial_- b + 2Q ,
\]
where the functions \( P \) and \( Q \) are given by
\[
P = -\frac{\partial_+ A}{2} \int_{-\infty}^{x^-} \partial_+ f \partial_- f - \frac{1}{2}(A + B)(\partial_+ f)^2 ,
\]
\[
Q = \frac{1}{2}(A + B)(\partial_+ f)^2 .
\]
\[ Q = \frac{\partial_- B}{2} \int_{x^+}^{x^+} \partial_+ f \partial_- f - \frac{1}{2} (A + B) (\partial_- f)^2. \]  

(136)

Observe that in the gauge \( A = x^+ \), \( B = -x^- \) and on the constraint surface \( C_\pm = 0 \) we recover the standard expression for the conformal factor

\[ \partial_- \rho = \pm \frac{1}{4} (x^+ - x^-) (\partial_\pm f)^2. \]

(137)

Due to Bianchi identities \( P \) must depend only on the coordinate \( x^+ \), so we can calculate its value doing \( x^- = -\infty \)

\[ P = \lim_{x^- \to -\infty} -\frac{1}{2} (A + B) (\partial_+ f)^2, \]  

(138)

and an analogous argument leads to

\[ Q = \lim_{x^+ \to \infty} -\frac{1}{2} (A + B) (\partial_- f)^2. \]

(139)

To explicitly calculate \( P \), let us write

\[
\partial_+ f = \frac{1}{4} \int_{-\infty}^{\infty} d\lambda \lambda \partial_+ A \left\{ A_+(\lambda) e^{i\frac{\lambda}{2}(A-B)} \left[ iJ_0 \left( \frac{\lambda}{2} (A + B) \right) - J_1 \left( \frac{\lambda}{2} (A + B) \right) \right] - A_-(\lambda) e^{-i\frac{\lambda}{2}(A-B)} \left[ iJ_0 \left( \frac{\lambda}{2} (A + B) \right) + J_1 \left( \frac{\lambda}{2} (A + B) \right) \right] \right\}. 
\]

(140)

In order to take the limit \( x^- \to -\infty \) we shall assume that \( B(x^-) \) is a monotonic decreasing function which goes as \( B(x^-) \sim -x^- \) when \( x^- \to -\infty \). This requirement is necessary to preserve the meaning of \( \phi \) as the radial coordinate. If we substitute in (140) the leading terms of the asymptotic expansions of the Bessel functions when the argument goes to infinity

\[ J_0(x) \sim \left( \frac{2}{\pi x} \right)^{\frac{1}{2}} \cos \left( x - \frac{\pi}{4} \right) \quad |x| \to \infty, \]

(141)

\[ J_1(x) \sim \left( \frac{2}{\pi x} \right)^{\frac{1}{2}} \sin \left( x - \frac{\pi}{4} \right) \quad |x| \to \infty, \]

(142)
we get
\[
\partial_+ f \sim \frac{1}{2} \int_{-\infty}^{\infty} d\lambda \left( \frac{\lambda}{\pi (A + B)} \right)^{\frac{3}{2}} \partial_+ A [A_+ (\lambda) e^{i\lambda A} e^{-i\frac{\pi}{4}} - A_- (\lambda) e^{-i\lambda A} e^{i\frac{\pi}{4}}],
\]
and therefore
\[
P = -\frac{1}{2} \left( \frac{i}{2} \int_{-\infty}^{\infty} d\lambda \left( \frac{\lambda}{\pi} \right)^{\frac{3}{2}} \partial_+ A [A_+ (\lambda) e^{i\lambda A} e^{-i\frac{\pi}{4}} - A_- (\lambda) e^{-i\lambda A} e^{i\frac{\pi}{4}}] \right)^2.
\]
In a similar way, and assuming that \( A(x^+) \) is a monotonic increasing function which goes as \( A(x^+) \sim x^+ \) when \( x^+ \to +\infty \), it can be proven that
\[
Q = -\frac{1}{2} \left( \frac{i}{2} \int_{-\infty}^{\infty} d\lambda \left( \frac{\lambda}{\pi} \right)^{\frac{3}{2}} \partial_- B [A_+ (\lambda) e^{-i\lambda B} e^{-i\frac{\pi}{4}} - A_- (\lambda) e^{i\lambda B} e^{i\frac{\pi}{4}}] \right)^2.
\]
The above expressions for \( P \) and \( Q \) can be rewritten as follows
\[
P = -\frac{1}{2} (\partial_+ F)^2, \quad Q = -\frac{1}{2} (\partial_- F)^2,
\]
where the field \( F \) is given by
\[
F = \frac{1}{2} \int_{-\infty}^{\infty} d\lambda (\lambda \pi)^{-\frac{1}{2}} [A_+ (\lambda) e^{i\lambda A} e^{-i\frac{\pi}{4}} + A_- (\lambda) e^{-i\lambda A} e^{i\frac{\pi}{4}} + A_+ (\lambda) e^{-i\lambda B} e^{-i\frac{\pi}{4}} + A_- e^{i\lambda B} e^{i\frac{\pi}{4}}],
\]
and we must note that the two chiral functions of the field \( F = F_+ (A (x^+)) + F_- (B (x^-)) \) are the same
\[
F_+ = F_-,
\]
in other words, the field \( F \) is reflected at the boundary line \( \phi = 0 \).
For our problem it is convenient to define the symplectic form using the light-cone $x^+ = x_0^+$ and $x^- = x_0^-$ as the initial data surface

$$\omega = \omega_+ + \omega_- = \int_{x^- = x_0^-} dx^+ \delta j^- + \int_{x^+ = x_0^+} dx^- \delta j^+, \quad (149)$$

where the light-cone components of the current $j^\mu$ can be easily calculated from the action

$$j^+ = -4\phi \partial_+ \delta \rho - \phi \partial_+ \delta f, \quad (150)$$

$$j^- = 4\partial_+ \phi \delta \rho - \phi \partial_+ \delta f. \quad (151)$$

To evaluate $\omega$ explicitly we choose $x_0^+ = \infty$, $x_0^- = -\infty$. Taking into account (150), (151) and (125), (131), (132) we arrive at

$$\omega = \omega_+ + \omega_- = \int_{x^- = -\infty} dx^+ - 4\delta A \delta \partial_+ a + \delta f \delta ((A + B)\partial_+ f)$$

$$+ \int_{x^+ = \infty} dx^- - 4\delta B \delta \partial_- b + \delta f \delta ((A + B)\partial_- f). \quad (152)$$

Let us concentrate ourselves in the calculation of $\omega_+$. We shall work out first the differential of $f$

$$\delta f = \frac{1}{2} \int_{-\infty}^{\infty} d\lambda - \frac{\lambda}{2} J_1 \left( \frac{\lambda}{2} (A + B) \right) \delta A \left[ A_+(\lambda) e^{i\frac{\lambda}{2}(A-B)} + A_-\left(\lambda\right) e^{-i\frac{\lambda}{2}(A-B)} \right]$$

$$+ A_\left(\lambda\right) e^{-i\frac{\lambda}{2}(A-B)} + J_0 \left( \frac{\lambda}{2} (A + B) \right) \left[ \delta A_+(\lambda) e^{i\frac{\lambda}{2}(A-B)} + \delta A_-\left(\lambda\right) e^{-i\frac{\lambda}{2}(A-B)} \right]$$

$$- A_\left(\lambda\right) i\frac{\lambda}{2} \delta A e^{-i\frac{\lambda}{2}(A-B)} . \quad (153)$$

If we now substitute the asymptotic expansions of $J_0$ and $J_1$ we can factorize $(A + B)^{-\frac{1}{2}}$ and the remaining terms can be collected to give

$$\delta f \sim \frac{1}{2} \int_{-\infty}^{\infty} d\lambda \left[ \pi \lambda (A + B) \right]^{-\frac{1}{2}} \delta \left[ A_+(\lambda) e^{i\lambda A} e^{-i\frac{\lambda}{2}} + A_-\left(\lambda\right) e^{-i\lambda A} e^{i\frac{\lambda}{2}} \right]$$

$$= (A + B)^{-\frac{1}{2}} \delta F_+. \quad (154)$$

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We need also to calculate
\[
\delta ((A + B) \partial_+ f) = 
\frac{1}{2} \int_{-\infty}^{\infty} d\lambda \frac{\lambda}{2} \left\{ \delta \left( \partial_+ AA_+ (\lambda) e^{i \frac{\lambda}{2} (A - B)} \right) (A + B) \left[ i J_0 \left( \frac{\lambda}{2} (A + B) \right) \right] 
- i J_1 \left( \frac{\lambda}{2} (A + B) \right) \partial_+ AA_+ (\lambda) e^{i \frac{\lambda}{2} (A - B)} \left[ i J_0 \left( \frac{\lambda}{2} (A + B) \right) \partial A 
- i \frac{\lambda}{2} (A + B) J_1 \left( \frac{\lambda}{2} (A + B) \right) \right] 
- \delta \left( \partial_+ AA_- (\lambda) e^{-i \frac{\lambda}{2} (A - B)} \right) (A + B) \left[ i J_0 \left( \frac{\lambda}{2} (A + B) \right) + J_1 \left( \frac{\lambda}{2} (A + B) \right) \right] 
- \partial_+ AA_- (\lambda) e^{-i \frac{\lambda}{2} (A - B)} \left[ i J_0 \left( \frac{\lambda}{2} (A + B) \right) \partial A 
- i \frac{\lambda}{2} (A + B) J_1 \left( \frac{\lambda}{2} (A + B) \right) \partial A \right] \right\} ,
\]
where we have used the identity
\[
J'_1 (x) = J_0 (x) - \frac{J_1 (x)}{x} .
\]

Asymptotically
\[
\delta ((A + B) \partial_+ f) \sim 
\frac{i}{2} \int_{-\infty}^{\infty} d\lambda \left( \frac{\lambda (A + B)}{\pi} \right) \frac{i}{2} \delta \left( \partial_+ A \left( A_+ (\lambda) e^{i \frac{\lambda}{2} A_+ A_- + i \frac{\lambda}{2} A_- A_+} \right) \right) 
\sim (A + B)^{\frac{1}{2}} \delta \left( \partial_+ F \right) .
\]

In a similar way it can be checked that when \( x^+ \to \infty \)
\[
\delta f \sim (A + B)^{-\frac{1}{2}} \delta F_- ,
\]
\[
\delta ((A + B) \partial_- f) \sim (A + B)^{\frac{1}{2}} \delta \partial_- F .
\]

With these results it is clear now that the symplectic form \( \omega \) can be written in the following way
\[
\omega = \int_{x^+ = -\infty} dx^+ - 4 \delta A \delta \partial_+ a + \delta F_+ \delta \partial_+ F 
+ \int_{x^+ = \infty} dx^+ - 4 \delta B \delta \partial_- b + \delta F_- \delta \partial_- F .
\]
In conclusion, defining

\[ X^+ = A, \quad \Pi_+ = -4\partial_+ a + 2\frac{\partial_+^2 A}{\partial_+ A}, \]  
\[ X^- = B, \quad \Pi_- = -4\partial_- b + 2\frac{\partial_-^2 B}{\partial_- B}, \]  

the constraints can be cast into the form of a free field theory

\[ C_\pm = \pm \left( \partial_\pm X^\pm \Pi^\pm + (\partial_\pm F)^2 \right), \]  

and the expression for \( \omega \)

\[
\begin{align*}
\omega &= \int_{x^-=-\infty} dx^+ \delta X^+ \delta \Pi_+ + \delta F_+ \delta \partial_+ F \\
&\quad + \int_{x^+=\infty} dx^- \delta X^- \delta \Pi_- + \delta F_- \delta \partial_- F,
\end{align*}
\]

shows that the transformation from the initial variables to the new ones \( X^\pm, \Pi^\pm, F, \pi_F = \tilde{F} \) is canonical.

We must note that the above free field representation has important consequences in the quantization of the theory. In contrast with pure dilaton gravity, for which the contributions to the Virasoro central charge of the two free fields cancel in the Schrödinger quantization, we have now a completely different situation and there is no way to avoid a non-vanishing center with \( c = 1 \) coming from the contribution of the field \( F \)

\[
[C_\pm(x), C_\pm(\tilde{x})] = i (C_\pm(x) + C_\pm(\tilde{x})) \, \delta'(x - \tilde{x}) \mp \frac{i}{24\pi} \delta'''(x - \tilde{x}),
\]

\[
[C_+(x), C_-(\tilde{x})] = 0.
\]

As a byproduct this explains why it has not been possible to find any solution to the quantum constraints and the theory has been mainly studied in the framework of the reduced phase-space \([17]\) which sweeps the anomaly under the carpet. Nevertheless, it is possible to modify the quantum constraints in such a way that the anomaly cancels. The addition of a term depending on the fields \( X^\pm \)

\[
C_\pm(x) \pm \frac{1}{48\pi} \left[ \frac{X^{\pm m}}{X^{\pm}} - \left( \frac{X^{\pm n}}{X^{\pm}} \right)^2 \right],
\]

27
produces a cancellation of the anomaly and allows to find states that are annihilated by all the quantum constraints. The above quantum modification of the constraints was introduced in Ref. [6] in the context of the CGHS theory and it was motivated by a mechanism based on an embedding-dependent factor ordering of the constraints [26]. This type of quantum corrections were also introduced in Ref. [27] to achieve a Virasoro algebra of central charge \( c = 26 \) also in the context of the (1-loop corrected) CGHS theory. The modified constraints also appeared in a more geometrical context. The gravity part of these quantum constraints

\[ -iX^{\pm'} \frac{\delta}{\delta X^\pm} \pm \frac{1}{48\pi} \left( \frac{X^{\pm''}}{X^{\pm'}} - \left( \frac{X^{\pm''}}{X^{\pm'}} \right)^2 \right), \tag{168} \]

are just the right-invariant vector fields of the Virasoro group with \( c = -1 \) [28], where the fields \( X^\pm, X^\pm \neq 0 \) are the diffeomorphism group parameters.

The modified quantum constraints can be solved in terms of "gravitationally dressed" oscillators defined by the relations

\[ \hat{f}(X) = \frac{1}{2\sqrt{\pi}} \int \frac{dk}{|k|} e^{ikX} \hat{\alpha}(k) + c.c. , \tag{169} \]

and it suggests to consider the gauge \( X^\pm = \pm x^\pm \) which corresponds to the light-cone gauge of critical string theory, as a natural one since the quantum modification of (167) vanishes. This might imply that the reduced phase-space treatment of the theory [17], which is based on the gauge-fixing \( X^\pm = \pm x^\pm \), could be equivalent to the proper Dirac quantization approach. In the CGHS theory, the gauge \( X^\pm = \pm x^\pm \) corresponds to the Kruskal gauge and the 1-loop reduced phase-space quantization [29] in this gauge turns out to be compatible with the covariant approach.

### 5.2 Models with non-vanishing potential

Our purpose now is to study the theory obtained by adding a potential term \( \lambda^2 V(\phi) \) to the model (117). In the hamiltonian formalism the constraints take the form

\[
H = -\frac{1}{2} \pi_\rho \pi_\phi + 2(\phi'' - \phi' \rho') - \rho'' \lambda^2 V(\phi) + \frac{1}{2} \left( \frac{1}{\phi} \pi_f^2 + \phi(f')^2 \right), \tag{170}
\]

\[
P = \rho' \pi_\rho - \pi_\rho' + \phi' \pi_\phi + \pi_f f'. \tag{171}
\]
We have seen that, for the theory with a vanishing potential, the central charge in the Schrödinger representation is $c = 1$, but we must stress that this result can be maintained for an arbitrary potential. This follows immediately from the fact that the pure dilaton-gravity sector contributes with $c = 0$ and the remaining sector contributes with $c = 1$, irrespective of the particular form of the potential. So this suggests that the free field realization of the Einstein-Rosen midisuperspace could be extended to models with a non-vanishing potential. Using the transformation which converts a generic pure dilaton gravity model into a free field theory, the constraints become

$$\partial_\pm \tilde{X}_\pm \tilde{\Pi}_\pm + \frac{1}{2} \phi (\partial_\pm f)^2 = 0,$$

although now the fields $\tilde{X}_\pm, \tilde{\Pi}_\pm$ are not chiral and $\phi$ in an involved function of $\tilde{X}_\pm, \tilde{\Pi}_\pm$. Because of $\partial_\mp G_\pm = 0$, we can write the constraints into the form

$$\partial_\pm X_\pm \Pi_\pm + \frac{1}{2} (\partial_\pm F)^2 = 0,$$

if we define

$$X_\pm = \tilde{X}_\pm |_{x^\mp = x_0^\mp}, \quad \Pi_\pm = \tilde{\Pi}_\pm |_{x^\mp = x_0^\mp},$$

and

$$(\partial_\pm F)^2 = \phi (\partial_\pm f)^2 |_{x^\mp = x_0^\mp},$$

where $x_0^\pm$ are arbitrary. By analogy with the previous analysis of the Einstein-Rosen midisuperspace the natural choice for $x_0^\pm$ are the surfaces at null infinity $x_0^\pm = \pm \infty$. Assuming that there is not incoming (outgoing) matter at null infinity $x^+ = \infty (x^- = -\infty)$ the fields $X_\pm, \Pi_\pm$ coincide with the canonical free field variables of the pure dilaton gravity theory, as in the model with a vanishing potential. Furthermore, the field $F$ is similar to the corresponding free field of the Einstein-Rosen model (147) and therefore inherits the reflecting property (148). It is important to stress now that a similar reflecting boundary condition was introduced in the one-loop corrected CGHS theory [30] due to the appearance of a time-like singularity curve. The interaction of the matter field with the boundary line was a crucial ingredient in the approach of Ref. [27] and now we have seen that the existence of a free field $F$ with a reflecting boundary condition naturally emerges in the canonical analysis of the dilaton-coupled theory.
Up to now we have transformed the constraints into those of a free field theory where the fields $X^\pm, \Pi^\pm$ are chiral and the field $F$ satisfies the free field equation. We want to prove now that this imply that the transformation going from the initial variables to the new ones $X^\pm, \Pi^\pm, F, \pi_F = \dot{F}$ is canonical. To this end, let us consider the variation of the lagrangian density of the theory under an infinitesimal diffeomorphism $\delta$

$$\delta \chi \left( \sqrt{-g} \mathcal{L} \right) = \sqrt{-g} \nabla_{\mu} (\chi^\mu \mathcal{L}) . \quad (176)$$

If the lagrangian of the theory is zero over solutions of the hamiltonian equations of motion the above equation can be rewritten as

$$\delta \chi \left( \sqrt{-g} \mathcal{L} \right) = -2G_{++} \partial_- \chi^+ - 2G_{--} \partial_+ \chi^- + \partial_\mu j^\mu \left( \delta \chi X^\pm, \delta \chi \Pi^\pm, \delta \chi F \right) = 0 . \quad (177)$$

This property can be easily checked when the potential is a power of the dilaton (as is the relevant case of spherically symmetric Einstein gravity $V(\phi) \propto \phi^{-2}$). In this case the lagrangian

$$\mathcal{L} = R\phi + \lambda^2 \phi^{a-1} - \frac{1}{2} \phi \left( \nabla f \right)^2 , \quad (178)$$

turns out to be a total derivative

$$\mathcal{L} = (1 - a) \Box \phi , \quad (179)$$

when restricted to the solutions of the hamiltonian equations

$$R + \lambda^2 a \phi^{a-1} - \frac{1}{2} \left( \nabla f \right)^2 = 0 , \quad (180)$$

$$\Box \phi = \lambda^2 \phi^a . \quad (181)$$

Therefore the equivalent lagrangian

$$\mathcal{L}' = \mathcal{L} - (1 - a) \Box \phi , \quad (182)$$

satisfies the desired property and leads to the same canonical structure, up to boundary terms. If we choose the field $\chi$ in equation (177) to be $\chi^+ = x^-, \chi^- = 0$, then we have

$$2G_{++} = \partial_\mu j^\mu = \partial_+ j^+ + \partial_- j^- . \quad (183)$$
Now we assume that the constraints have the quadratic form (173). So we arrive at

$$-\Pi_+\partial_+ X^+ - (\partial_+ F)^2 = \partial_+ j^+ + \partial_- j^-, \quad \text{(184)}$$

Observing that

$$\delta\chi X^+ = \chi^\mu \partial_\mu X^+ = x^- \partial_+ X^+, \quad \delta\chi X^- = 0, \quad \text{(185)}$$

$$\delta\chi \Pi^+ = x^- \partial_+ \Pi_+, \quad \delta\chi \Pi^- = 0, \quad \text{(186)}$$

it is not difficult to see that the only possible expression for $j^-$ compatible with (183) and which is a scalar density is

$$j^- = -\Pi^+ \delta X^+ - \partial_+ F \delta F_+. \quad \text{(187)}$$

An analogous argument choosing $\chi^+ = 0, \chi^- = x^+$ shows that

$$j^+ = -\Pi^- \delta X^- - \partial_- F \delta F_. \quad \text{(188)}$$

Taking into account now that the Noether current $j^\mu$ can be interpreted as the symplectic current potential for the 2-form $\omega$, where the variation symbol $\delta$ plays the role of the exterior differential on phase-space [31], we arrive at the conclusion that the symplectic form

$$\omega = \int_{x^- = -\infty} dx^+ \delta j^- + \int_{x^+ = \infty} dx^- \delta j^+, \quad \text{(189)}$$

becomes

$$\omega = \int_{x^- = -\infty} dx^+ \delta X^+ \delta \Pi_+ + \delta F_+ \delta \partial_+ F$$

$$+ \int_{x^+ = \infty} dx^- \delta X^- \delta \Pi_- + \delta F_- \delta \partial_- F, \quad \text{(190)}$$

and this shows that the free field variables $X^\pm, \Pi_\pm, F_\pm, \partial_\pm F$ are, in fact, canonical. Although the result only applies when the potential is a power of the dilaton we believe that a more detailed analysis could eliminate this restriction.
6 Conclusions and Final Comments

Stimulated by the paper [7] a lot of studies of 2D dilaton gravity and black holes have been developed from different viewpoints. In the absence of matter fields an exact Dirac quantization for all the dilaton gravity theories has been given in Ref. [8]. These theories admit a unified description in terms of Poisson-Sigma models [32], which generalizes the gauge theoretical formulation of the CGHS model [33] and the well-known extra symmetry of the later model can also be generalized for a generic model [34]. This provides additional reasons for the exact solvability of these theories giving rise to a finite dimensional phase-space. Moreover, the path-integral results are in accordance with this since the exact effective action has been shown to exactly coincide with the classical one [35]. However, in the presence of matter fields, needed to have the Hawking radiation, the theories are no longer topological and one must quantize an infinite number of degrees of freedom. For the CGHS model, and when the matter field is minimally coupled to gravity, it is still possible to have control on the matter-coupled theory because the dilaton-gravity sector of the theory can be canonically mapped into a set of two free fields with signature (-1,1) [6, 9]. Therefore, due to the fact that the matter sector itself is a free field the full theory is similar to a bosonic string theory with a three dimensional Minkowski target space. In this paper we have shown that this crucial property of the CGHS model remains valid for a generic model of 2D dilaton gravity minimally coupled to a scalar field. This way the well-known canonical equivalence of a Liouville field and a free field can be understood in a more general setting. Furthermore the intriguing analogy of the Liouville field as a longitudinal target space coordinate arising in no-critical string theory [36] can also be generalized for a generic theory of dilaton gravity. The longitudinal target space coordinates $X^\pm$ are related to the logarithm of the conformal factor and the dilaton.

The main drawback of considering theories with a minimal coupling to matter is that they do not mimic properly the propagation of a scalar field on a four-dimensionally geometry. In the general situation the matter field has a 2D dilaton coupling and, therefore, a natural question arises. Is still possible to map the matter-coupled theory into a free field theory? This question has been analyzed in this paper and the answer is in the affirmative. For theories with a potential of the form $V(\phi) = \phi^a$, which includes spherically
symmetric gravity, we have shown that a canonical transformation converts
the theory, up to a boundary term, into a free field theory with a Minkowskian
target space. This canonical equivalence emerges when one combine the
previous result on pure dilaton gravity and the explicit form of the free field
representation of the dilaton-coupled theory with a vanishing potential. The
existence of a free field $F$ which automatically incorporates the reflecting
condition (148) seem to indicate that could be an adequate variable to study
the problem of back reaction in the more realistic setting of dilaton-coupled
models. We shall explore this and related questions in future publications.

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