Gravitational Wave Test of the Strong Equivalence Principle

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The Strong Equivalence Principle (SEP) holds the full essence and meaning of the General Theory of Relativity as the nonlinear relativistic theory of gravitation. It asserts the universal coupling of gravity to all matter and its interactions including the gravitational interaction and the gravitational self energy. We point out that confirming the gravitational coupling to gravitons, and hence to the gravitational waves, is the direct test of the SEP. We show that the near simultaneous detection of gravitational waves and gamma rays from the merger of binary neutron stars provides a unique and the most precise test of the SEP, better than a part in $10^9$, which is also the only test of the SEP in the radiation sector.

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I. INTRODUCTION

The Einstein Equivalence Principle (EEP) that asserts the local equivalence of all physical phenomena in a uniform gravitational field $g$ and an accelerated frame with $a = -g$ was postulated by Einstein on the basis of the weak equivalence principle (WEP), or the universality of the ratio of the gravitational to inertial mass. The WEP in turn rests empirically on the observed Universality of Free Fall (UFF), which is the fact that the gravitational acceleration does not depend in the mass or other material properties of the body in free fall. Due to the equivalence of the mass and energy, UFF implies that the gravitational ac-
acceleration does not depend on the binding energies of the standard model interactions in the falling body. The formulation of the General Theory of Relativity as a geometrical theory presupposes that the EEP includes the gravitational phenomena as well. Then, one has to insist on the UFF of the gravitational binding energy of the falling bodies, leading to the Strong Equivalence Principle (SEP). Generalizing from the equivalence of the gravitational mass and the inertial mass, The SEP asserts the universal coupling of gravity to all matter and their interaction energy, including that of the gravitational interaction, highlighting the nonlinear nature of gravity. Hence, at the fundamental microscopic level, the SEP affirms the universal gravitational coupling to gravitons themselves. This completes the Einstein Equivalence Principle as the grand generalization that asserts the local equivalence of all physical phenomena, including gravitational phenomena, in a uniform gravitational field and in a uniformly accelerated frame. The SEP is the final frontier of explicit tests of the principle of equivalence.

A significant test of the SEP requires that the precision in the measurement of the differential acceleration $\delta a/g$ in the gravitational field $g$ is better than the fractional amount of the gravitational self energy relative to the total energy $\varepsilon_g/E$. Since $E$ is essentially the rest mass energy of a massive test body, $\varepsilon_g/Mc^2 \approx GM^2/R(Mc^2) = GM/Rc^2$. Hence, a significant test requires the observational access to the relative trajectories of a pair of planetary to stellar scale test objects, in the gravitational field of a third body.

The SEP has been tested and verified directly in the ‘free fall’ of the Earth and the Moon towards the Sun employing the Nordtvedt effect, by monitoring the orbital distance of the Moon from the Earth in Lunar Laser Ranging (LLR). This feat was possible because of the impressive long term precision achieved, of several millimeters, in the determination of the orbital distance of $3.7 \times 10^8$ m. This translated to a test of the UFF and the WEP with a precision of 2 parts in $10^{13}$. With the gravitational self energy of $E_g/Mc^2 \approx 5 \times 10^{-10}$ for the Earth, and relatively negligible self energy for the Moon, one obtains the significant observational constraint on any violation of the SEP of $< 0.04\%$, which is remarkable considering the smallness of the contribution of the gravitational self energy of these bodies to their total mass-energy.

The only better observational constraint on the SEP is obtained by comparison of the orbits of a triple stellar system PSR J0337+1715, consisting of a binary system of a millisecond radio pulsar (366 Hz) and a white dwarf, with a 1.6-day orbital period, that itself is in a much longer period orbit (327 days) with another white dwarf. The orbits can be monitored with the radio pulses from the neutron star (pulsar). The gravitational self energy of a neutron start is about 10\%, and it is comparatively negligible for the while dwarf. With the limit of 2 parts in $10^6$ on the universality of free fall in this triple stellar system, the SEP is established with a precision of $10^{-5}$, which is more than an order of magnitude tighter than the LLR result. However, the analysis involved in obtaining this constraint is necessarily more elaborate and complicated, owing the fact that the entire information on the orbits is extracted from the observation of the pulses from the pulsar in the system.

There is evidently a paucity of accessible tests of SEP, owing to the extreme weakness of the gravitational interaction. There is only one Earth-Moon-Sun system in this universe for which we have easy access to, for the study of such a deep and characteristic foundational issue of gravity. Pulsar-white dwarf systems provide test systems to study situations in which there is a large contribution of the gravitational self energy to the total mass energy, but the precision is limited because the galactic gravitational field in which they free fall is very small. An observationally convenient and accessible triple stellar system is a rare chance...
that is very hard to come by. In this context, we have found an entirely new way for the precision tests of the SEP and GTR, leveraging the realization that the gravitational waves are the purest and manifest form of gravitational energy. Gravitational waves conceptually correspond to real gravitons, whereas the notional gravitational self energy in bulk matter corresponds to the virtual carriers of the interaction. In fact, the binding energy or the self energy, both in case of electromagnetism and gravitation, is the absence of real energy, rather than its presence; that much positive energy is released in making the bound system. The manifest form of the gravitational energy is the gravitational wave, just as the manifest form of the electromagnetic energy is the electromagnetic wave. Therefore, testing for the gravitational coupling of the gravitons themselves is the ultimate test of the SEP. The photons can provide the ideal reference for the comparison. Clearly, a test of the universal coupling of gravitational waves to the gravity of bulk matter is the most direct test of SEP.

A new window for this direct and reliable precision test of SEP opened with the detection of near simultaneous gravitational waves (GW) and gamma rays from the merger of binary neutron stars [9, 10]. This unique test has the clear possibility of scores of future detections, allowing crucial statistical reliability. The central idea of the test stems from the realizations that gravitational waves are propagating form of pure gravitational energy, released directly from the gravitational binding energy of the binary system. Then, the gravitational effects of massive structures, like galaxy clusters, on the propagation of pristine gravitational energy, relative to the same effects on photons, is a transparent direct test of SEP. The gamma rays serve as the reference matter-energy, without any significant gravitational energy, providing a complete test. The Shapiro delay is a first order test, proportional to the integrated gravitational potential in the intervening space [11], and the gravitational bending is a second order test, proportional to the gradient of the potential. Hence, the Shapiro delay provides a much better test of the SEP than the gravitational bending. But the latter is the one that resembles the ‘free fall’ tests. We examine both effects for the completeness of the discussion.

II. SENSITIVITY OF THE TESTS OF THE SEP

The classic test of the SEP along the lines of the tests of the universality of free fall involves comparing the relative trajectory of two bodies in the gravitational fall towards a third massive body. What is measured is the difference in acceleration \( \delta a \) in the gravitational field \( g \). The ratio \( \delta a/g \) is identical to the Eötvös WEP parameter \( \eta \). When the relative contribution of the gravitational self energy is larger than the sensitivity \( \eta \) if the test, one gets a useful test of the SEP. There are three key quantities to consider while estimating the precision of the test of the SEP, denoted by the symbol \( \Delta \). One is the gravitational acceleration at the location of the falling bodies, \( g \). The second parameter is the difference in the ratio of the gravitational energy in the falling bodies to their total energy. Since the total energy is essentially the rest mass energy,

\[
\delta \varepsilon_g = \left| \frac{\varepsilon_{g1}}{m_1c^2} - \frac{\varepsilon_{g2}}{m_2c^2} \right|
\]

(1)

The third parameter is of course the precision achieved in the test of the universality of free fall from factual observation (including both the statistical and systematic errors). This is the precision to which the differential acceleration of free fall is constrained relative to the
local gravitational field, $\delta a/g$. Clearly, for a given precision in the measurement of the differential acceleration, a better test is obtained when the actual gravitational acceleration $g$ is larger. The precision of the test of WEP is determined by

$$\eta \equiv \left( \frac{m_1}{m_g} \right)_1 - \left( \frac{m_2}{m_g} \right)_2 = \delta a/g$$

Then the sensitivity for the test of SEP is given by $\Delta = \eta/\delta \varepsilon_g$.

For the solar system tests, $g \simeq 10^{-2} - 10^{-3}$ m/s$^2$ and the precision (2$\sigma$) achieved in observing $\delta a/g$ (for LLR) is about $10^{-13}$ m/s$^2$. But the quantity $\delta \varepsilon_g$ for the Earth-Moon system is only $4.6 \times 10^{-10}$. Therefore, the constraint on the violation of the SEP is limited to

$$\Delta = \frac{\eta}{\delta \varepsilon_g} \leq \frac{10^{-13}}{4.6 \times 10^{-10}} \simeq 2 \times 10^{-4}$$

The test employing the free fall of a neutron star-white dwarf system towards the galaxy can take advantage of the much higher gravitational energy of the neutron star ($\delta \varepsilon_g > 10^{-1}$), but much of that advantage is offset by the tiny galactic gravitational acceleration of only $g \simeq 2 \times 10^{-10}$ m/s$^2$. Yet, impressive constraints on $\eta$ have been obtained of the order of $\eta \leq 10^{-3}$, which translates to the constraint on SEP of $\Delta = \eta/\delta \varepsilon_g \leq 10^{-2}$ \cite{12}.

In contrast, a triple stellar system like the PSR J0337+1715 in which a neutron star-white dwarf system orbits another neutron star, allows the better constraint through painstaking analysis of the orbital data and careful modeling \cite{7, 8}. The gravitational acceleration (with the 327 day orbit) is comparable to the LLR case, $g \simeq 5 \times 10^{-3}$ and the difference in the gravitational self energy is $\delta \varepsilon_g \simeq 10^{-1}$. The precision achieved in estimating the differential acceleration is $\eta \leq 2 \times 10^{-6}$. This provides the strongest constraint on SEP to date of $\Delta \leq 2 \times 10^{-5}$.

The GW test of the SEP is a class apart from the tests involving massive bodies in two respects. A gravitational wave is pristine gravitational energy, propagating over cosmological distances. Therefore, the ratio of the gravitational energy to the total energy is unity for GW, $\varepsilon_g/E = 1$. Then we need another entity that can co-propagate with GW with negligible gravitational energy, for comparison, and that is light, for which $\varepsilon_g/E \simeq 0$. Thus $\delta \varepsilon_g = 1$. The differential comparison is possible if both the GW signal and the electromagnetic signal from the same source is observed. Fortunately, we have one confirmed astrophysical event with these criteria satisfied, in the gravitational wave detection in the LIGO-Virgo detectors and the Gamma ray detection by the Fermi satellite, GW170817+GRB \cite{9, 10}.

### III. GW TESTS OF THE SEP

The triad of interferometric gravitational wave (GW) detectors, LIGO_Hanford, LIGO_Livingston, and Virgo, sensed the arrival of gravitational waves from the inspiral and the merger of a binary neutron star (BNS) system, on the 17th August 2017 \cite{9}. Named GW170817+GRB, it became the one-of-a-kind event (so far) because of the near simultaneous detection of gamma rays, within about 1.7 s, by the Fermi gamma ray satellite \cite{10}. This detection with the three GW detectors allowed the delineation of a relatively precise localization area in the sky, which led to the identification of the source galaxy as NGC 4993 at a distance of 40 megaparsecs ($10^{24}$ m). What is relevant for our test of the SEP is that the gravitational waves and the gamma rays have to traverse this vast distance in the
FIG. 1: The relative positioning and parameters of the source galaxy, the solar system and the Virgo galaxy cluster for the calculation of the propagation delay and gravitational bending.

gravitational presence of the mighty Virgo cluster of galaxies, before they pass through the gravitational field of our Milky Way galaxy, and arrive on the Earth.

The geometrical configuration of the GW-GRB event relative to the Earth and the Virgo cluster is indicated in the figure 1. The universal gravitational coupling of the Virgo cluster mass to the gravitational energy of the GW and the electromagnetic energy of the gamma rays would manifest in two physical effects. One is the Shapiro time delay which of first order in \( \alpha = \phi/c^2 \simeq GM/Rc^2 \), and the second is the time delay due to the gravitational bending, which is of second order in \( \alpha \) (the gravitational bending itself, which is first order in the potential, cannot be observed for the gravitational waves).

The Shapiro delay is the excess propagation delay of the waves due to the gravitational potential, \( \int ds(2\phi/c^3) \). Any violation of the SEP due to a nonuniversal coupling to the gravitational energy of GW will alter this delay, relative to the Shapiro delay of the photons. One already know that the electromagnetic energy in bulk matter obeys the UFF to the very high precision of \( \eta_{em} \leq 10^{-9} \), because the ratio of the total electromagnetic energy in the atom to its rest mass energy is \( E_{em}/mc^2 > 10^{-5} \), and the best tests of the WEP have reached \( \eta \leq 1.5 \times 10^{-14} \). Therefore, we the gravitational coupling of the gamma rays would obey the WEP to better than a few parts in \( 10^9 \). With the galactic potential \( \phi_{mw}/c^2 \simeq 10^{-6} \), the Shapiro delay during the propagation over 30 Kpc (\( \sim 10^{21} \) m) in the Milky Way galaxy itself amounts to about 100 days, as has been analyzed in a comparison of the velocity of light and GW in the BNS merger event GW170817+GRB \[10\]. However, we should consider the Virgo cluster, because the main contribution of the gravitational potential in our cosmological neighbourhood is from this galaxy cluster. The Shapiro delay is scalar cumulative effect, steadily increasing with the same sign. There are no other theoretical complication like fixing a metric because we know with certainty that the propagation is in the unique \( k \simeq 0 \) FLRW metric of our factual universe. Therefore, the calculation with the mass of the Virgo cluster gives a reliable lower limit to the gravitational effect between NGC 4993 and the Earth. Because the propagation duration \( t \) (over the distance \( >10 \) Mpc) in the average potential \( \phi_V/c^2 \simeq 1.5 \times 10^{-6} \) of the Virgo cluster is more than \( 10^{15} \) s, a very conservative value for the gravitational Shapiro delay is readily estimated from \( \delta t = \int ds(2\phi/c^3) \) as

\[
\delta t > \frac{2\phi_V}{c^2} t \simeq 3 \times 10^9 \text{ s}
\]
which is to be compared with the observed 1.7 s between the arrival times of the gravitational waves and the gamma rays. This implies the gravitational coupling of propagating gravitational energy to the source masses is the same as the gravitational coupling of electromagnetic energy within $\Delta \simeq 6 \times 10^{-10}$. Note that any kind of electromagnetic factor along the path can only introduce further delays in the propagation of the photons; hence the factual constraint is always better than our conservative constraint. The similar propagation of the GW and the gamma rays from GW170817+GRB has yielded a very high precision test of the SEP at better than part in a billion!

We can also get a constraint on the SEP from the similar bending of the GW and light in the field of the Virgo cluster, albeit with lower precision. This is similar to the free fall test of two bodies in the gravitational field of a third body, with the additional aspect of relativistic propagation of the tested entities. If there are many massive structures, distributed around the path of propagation, the precise calculation of the resultant gravitational bending is complicated, and requires the detailed catalogue of matter distribution. Unlike the scalar Shapiro delay, the gravitational bending by a large structure like the Virgo cluster can be partially nulled, if there are several smaller structures that are closer to the path of propagation. If the gravitational waves and the gravitational energy in them did not experience the same ‘free fall’ as the photons in the gravitational field of the Virgo cluster, there would have been significant discrepancy in the time of arrival of the gravitational waves and the gamma rays.

The angle of bending due to the gravity of a compact source is well known as $\alpha = 4GM/c^2R$. Since the bending is small, the difference in the distance of propagation between the deflected path and the unperturbed path is (figure 1),

$$\delta L = \frac{L}{\cos \alpha} - L \approx \frac{L}{1 - \alpha^2/2} - L \simeq L\alpha^2/2$$

The excess delay due the gravitational bending is then $\delta t = \delta L/c = L\alpha^2/2c$. Hence, this effect is of second order in $\alpha$. A more accurate expression from the lensing equation is $\delta t \simeq (1+z)\alpha^2D_LD_S/2cD_{LS}$, which corrects for the redshift distance of the source. The time delay due to the bending of waves propagating at the velocity $c$ in the gravitational field of a mass distribution of size smaller than the distance between the source and the observer can now be estimated referring to the figure 1.

Conservatively taking $3 \times 10^{14} M_\odot$ as the total mass of the Virgo cluster of galaxies and its dark matter halo within a radius of about 3 Mpc (and the ‘impact parameter’ of 10 Mpc), the bending angle $\alpha$ is approximately $6 \times 10^{-6}$ rad [16]. This is consistent with the gravitational field of the Virgo cluster at the local group, estimated from the infall velocity of about 200 km/s of the local group towards Virgo cluster. The deflection of the path translates to the gravitational bending delay $L\alpha^2/2c > 3 \times 10^4$ s, where we have taken $L$ as half the distance to NGC 4993 (this is consistent with the combination of distances appearing in the lensing equation). This has to be compared to the 1.7 s delay between the GW from the BNS merger and the GRB. Since the gravitational waves are pure gravitational energy in propagation, this observed universality of ‘free fall’ under the gravitational action of the mighty cluster of matter on both electromagnetic waves and gravitational waves readily proves the Strong Equivalence Principle, with the tightness of the constraint on any violation smaller than $\Delta < 6 \times 10^{-5}$. As expected, the Shapiro delay constraint on the SEP that we obtained in this work is by far the most stringent, $\Delta \leq 6 \times 10^{-10}$. 
IV. CONCLUDING REMARKS

A unique test of the strong equivalence principle, which asserts the universal gravitational coupling to the gravitational energy itself, is devised by recognizing that the propagating gravitational waves are entirely gravitational energy. A comparison of the propagation of gravitational energy with gamma photons, from the merger event of the binary neutron stars detected by the LIGO-Virgo detectors, yielded the most stringent test of the SEP, with the constraint on any violation $\Delta \leq 6 \times 10^{-10}$. Besides, this is the only test of the SEP in the radiation sector. The design sensitivity of the upgraded advanced GW detectors is 170-300 Mpc for binary neutron star mergers, which is 3 times the sensitivity they had when the event GW170817 happened. This means that one can expect an order of magnitude higher event rate at the full sensitivity, expected by 2025, and about 3 to 10 BNS events/year, with source identification. With many such events, the statistical precision and confidence in our unique test of the SEP will steadily improve. In such a scenario, there is no doubt that this gravitational wave test will remain the most precise confirmation of the strong equivalence principle.

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