Anomalous prompt photon production in hadronic collisions at low-$x_T$

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Abstract

We investigate the discrepancy that exists at low-$x_T = 2p_T/\sqrt{s}$ between the next-to-leading order QCD calculations of prompt photon production and the measured cross section. The central values of the measured cross section are of order 100% larger than QCD predictions in this region. It has been suggested that the bremsstrahlung contribution may account for this discrepancy. The quark fragmentation function $D_{γ/q}(z)$ has not been measured and an exactly known asymptotic form is normally used in calculations. We examine the effect of much larger fragmentation functions on the QCD predictions. After illustrating the effect of the large fragmentation functions in some detail for recent CDF data at $\sqrt{s}=1.8$ TeV, we perform a $\chi^2$ fit to 8 prompt photon data sets ranging in CMS energy from 24 GeV to 1.8 TeV. While a large fragmentation function normalization may prove to play an important role in resolving the discrepancy, the present theoretical and experimental uncertainties prevent any definite normalization value from being determined.

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1 Introduction

Prompt photon production in hadronic collisions is an important testing ground for QCD. Recently, measurements of inclusive prompt photon production at CMS energy $\sqrt{s} = 1.8$ TeV have been published by the CDF collaboration, and a comparison with QCD calculations at next-to-leading order has been carried out [1]. While the QCD predictions give good qualitative agreement, the central values of the measured cross section in the low-$p_T$ region are of order 100% larger than standard QCD calculations. The same discrepancy exists between theory and experiment for data taken by UA1 and UA2 at the CMS energy $\sqrt{s} = 630$ GeV.

The NLO QCD calculation consists of three parts.

$$\sigma = \sigma^{LL} + \sigma^{NLL} + \sigma^{ANOM}$$

where the three terms indicated are the leading logarithm (improved Born approximation) contribution, the next-to-leading logarithm piece, and the so-called “anomalous” part, in which a photon is emitted collinearly from an outgoing parton. This is just the bremsstrahlung process. While the first two parts of the QCD calculation are reliably calculated (given the fact that the quark and gluon distribution functions are reasonably well constrained) the third, the bremsstrahlung process, is not. The calculation of the bremsstrahlung process involves an unmeasured set of non-perturbative functions, the parton $\to$ photon fragmentation functions. While there is a standard parametrization [2] for these functions used by most theorists in QCD calculations, this parametrization is only valid at asymptotically large energy scales where the effects of boundary conditions are vanishing. In this paper we examine the effect of large fragmentation functions on the QCD predictions of inclusive prompt photon production and compare with experimental data.

This paper is organized as follows. In Section 2 we discuss the NLO QCD calculation of the bremsstrahlung component of prompt photon production and we elaborate on certain aspects of the fragmentation functions involved in this calculation. In Section 3 we first illustrate the existing discrepancies between theory and experiment for the UA1 and UA2 data at $\sqrt{s} = 630$ GeV and CDF data at $\sqrt{s} = 1.8$ TeV. Concentrating on the CDF data, we examine the effects...
of the theoretical uncertainties due to the choice of scales and distribution functions on the discrepancy. We then go on to show the results of the QCD prompt photon calculation including larger fragmentation functions. We compare our results with the standard calculations, and with the CDF data. We again examine the effect of changing the scales and parton densities on the theoretical predictions. In Section 4 we proceed to compare the results of the calculation with and without a large set of fragmentation functions to data at center of mass energies $\sqrt{s}=1800, 630, 546, 63, \text{ and } 24 \text{ GeV}$. A $\chi^2$ analysis is performed and the results are discussed. Section 5 is reserved for the conclusions.

2 Bremsstrahlung calculation; Fragmentation functions

The anomalous part of the QCD calculation of the inclusive prompt photon production differential cross section $\sigma(AB \to \gamma + X)$ is given at the leading logarithm level by

$$\sigma^{ANOM} = \sum_{abcd} \int F_{a/A}(x_a, \mu_R^2) F_{b/B}(x_b, \mu_F^2) D_{\gamma/c}(z, M_f^2) \frac{d\hat{\sigma}}{dv} dx_a dx_b dz dv. \quad (1)$$

In this equation $v = 1 + \hat{t}/\hat{s}$, where $\hat{t}$ and $\hat{s}$ are the Mandlestam variables of the subprocess, and $x_a$, $x_b$, and $z$ are the fractional momenta of the initial state partons and the photon, respectively. The subprocess cross section $d\hat{\sigma}/dv$, which describes the parton level scattering process $ab \to cd$, is convoluted with the parton distribution functions $F_{a/A}$ and $F_{b/B}$ and the parton $\to$ photon fragmentation functions $D_{\gamma/c}$. The three scales in the calculation are the renormalization scale $\mu_R$, associated with the strong coupling constant, the factorization scale $\mu_F$ where the parton distribution functions are evaluated, and the fragmentation scale, $M_f$, associated with the parton $\to$ photon fragmentation process. At sufficiently high order in perturbation theory, the total calculated cross section should be insensitive to the chosen values of these scales.

The fragmentation and distribution functions are nonperturbative objects and are treated as measured inputs within the framework of perturbative QCD.\footnote{Given a set of functions at some scale the evolution of them to other energy scales is perturbatively calculable.}

While many experiments such as deep inelastic scattering have given rise to a
fairly well constrained set of quark and gluon distribution functions, the parton → photon fragmentation functions have not been measured. However, owing to the pointlike nature of the photon these fragmentation functions differ essentially from the parton distribution functions in that at asymptotically large scales the fragmentation functions are perturbatively calculable. The asymptotic form for the quark and gluon fragmentation functions, which includes the summation to all orders in perturbation theory of soft and collinear gluon emission at leading logarithm level, is parametrized as follows [2]

\[ zD_{\gamma/q}^{LL}(z, Q^2) = F \left[ \frac{e^2(2.21 - 1.28z + 1.29z^2)z^{0.049}}{1 - 1.63\ln(1 - z)} + 0.0020(1 - z)^{2.0}z^{-1.54} \right] \]

\[ zD_{\gamma/g}^{LL}(z, Q^2) = F \frac{0.194}{8}(1 - z)^{1.03}z^{-0.97}, \]

where \( F = (\alpha/2\pi) \ln\left( Q^2/\Lambda_{QCD}^2 \right) \). Theorists generally use this asymptotic form when doing QCD calculations of prompt photon production and comparing to the experimental data [2, 3, 4]. However, the boundary conditions on the evolution equations may be important at scales where experimental data has been taken. The boundary conditions on the evolution equations are simply the initial fragmentation functions at some scale \( Q_0 \), where \( Q_0 \) is defined to be the scale above which the perturbative evolution equations are assumed to be valid. We take this scale to be 2 GeV. We consider the possibility that the physics at scales below \( Q_0 \) builds up a set of functions at \( Q_0 \) with a larger normalization than that of the asymptotic form evaluated at the starting scale.

In order to make the analysis tractable we consider starting quark and gluon fragmentation functions which have the same shape as the asymptotic forms but which have normalizations \( N_0 \) times that of the asymptotic forms evaluated at the scale \( Q_0 \). Of course, as the scale increases these large fragmentation functions evolve and approach the asymptotic form normalization. In figure 1a we show the up-quark fragmentation function both for the asymptotic form and for a large fragmentation function with \( N_0 = 20 \). The asymptotic form grows logarithmically with the scale and this factor has been divided out in fig.1a. We show in this plot the large fragmentation function evaluated at the starting scale 2 GeV, the scale 10 GeV and the scale 100 GeV. In figure 1b we show the normalization of the large fragmentation function vs. the scale \( Q \), for values of
$N_0$ equal to 10, 20, and 30. Because of isolation cuts, many experiments are only sensitive to large values of fragmentation function fractional momenta. For this reason, the relative normalization $N(Q^2)$ shown in fig.1b is given by

$$N_0(Q^2) = \frac{\int_{0.85}^{1} D_{NL}^{LL}(z, Q^2) \, dz}{\int_{0.85}^{1} D_{LL}^{asym}(z, Q^2) \, dz}$$

(4)

where the denominator is the asymptotic form fragmentation function of eq.2. Note that the relative normalization decreases rapidly in the first 10 or 20 GeV beyond the starting scale $Q_0$. This feature is well suited for resolving the discrepancy between the QCD prompt photon predictions and experimental data at low transverse momenta. If the starting fragmentation function normalization is much larger than the asymptotic form normalization, the smaller $p_T$ values will probe this large normalization, while at larger $p_T$ values most of the initial large normalization will have evolved away. Thus, in the low-$p_T$ region the predicted cross section will markedly increase relative to the standard calculations, while at higher photon transverse momenta we can expect results similar to the standard QCD predictions.

It is of interest to estimate the largest initial fragmentation function normalization that may be physically relevant. An order of magnitude estimate for an upper bound comes from demanding that at the starting scale $Q_0$ the total number of photons fragmented from some parent quark with fractional momenta above some cut-off $z_0$ be less than some small number of $\mathcal{O}(\alpha/\alpha_s)$, say $\frac{1}{40}$, times the total number of hadrons that are fragmented by the quark. We show in figure 2 the maximum normalization $N_{max}$ as a function of $z_0$ coming from this constraint. We assume a hadron multiplicity of $\sqrt{2E}$ for a parent quark of CMS energy $E$, in GeV units, and have fixed the parent quark energy at 3 GeV. Taking $z_0$ to be 0.3, we see from fig. 2 that we can tolerate an initial normalization which is $\sim 20$ times larger than that of the asymptotic form.

Another more quantitative bound can be estimated by examining the parton $\to$ photon process in the crossed channel, namely the photon $\to$ quark fragmentation function. In this case the photon structure function in the non-perturbative regime can estimated by appealing to vector meson dominance (VMD) arguments. In ref.[6] the $\gamma \to$ vector meson fragmentation function is hypothesized to be equal to the $\gamma \to \pi^0$ fragmentation function, which is extracted from fits to $e^+e^- \to \pi^0X$ data. We find from the analysis of ref.[6] that
using the VMD hypothesis to specify the fragmentation function boundary conditions limits \( N_0 \) to be less than 2 at leading order. However, due to the many theoretical uncertainties involved in the VMD procedure, we will not respect this limit. Rather, we will go ahead and vary \( N_0 \) freely in order to determine whether the present experimental prompt photon data can constrain it directly.

We will find that in order to account for a substantial increase in the prompt photon production cross section at low transverse momenta we will need to consider initial fragmentation function normalizations which are in some cases 30 or more times larger than the normalization of the asymptotic form (which is probably unreasonably large). At first sight this may be surprising since at large center of mass energies and low-\( p_T \) it is well known that the bremsstrahlung contribution can dominate the other photon production processes. However, in order to reduce the background photons from \( \pi^0 \) and \( \eta \) decay the data samples are taken with the imposition of an isolation cut. These isolation cuts effectively remove most of the bremsstrahlung events from the data sample. Typically only photon fractional momenta values of \( z > 0.85 \) are allowed. Additionally, at the lowest \( p_T \) values measured in the experiments the fragmentation function normalizations will have evolved considerably down from any large starting normalization.

\section{Results}

We use the algorithm of Baer \textit{et al.} \cite{4} to calculate the next-to-leading order QCD cross section for prompt photon production in hadronic collisions. See their paper and references therein for the relevant formulae. The calculation involves a Monte Carlo integration wherein various isolation criteria are easily implemented. To simplify our analysis we choose to set the renormalization scale equal to the factorization scale, \( \mu^2 = \mu_R^2 = \mu_F^2 \).

In figure 3 we compare our results for prompt photon production with CDF data \cite{1} taken at \( \sqrt{s} = 1.8 \, \text{TeV} \), and with UA1 and UA2 data \cite{6, 7} taken at 630 GeV. Here we are using the standard asymptotic form parametrization of the quark and gluon fragmentation functions given in eqs.(2) and (3) (i.e. \( N_0 = 1 \)). We show the results for the scales \( \mu^2 = M^2_f = p_T^2 / 4 \) and the distribution functions Morfin & Tung Set 1 (M-T1) \cite{8}. Both the CDF data and the corresponding calculation include an isolation cut, such that the total hadronic energy inside a
cone of size $\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2} = 0.7$ centered on the photon be less than 2 GeV ($\eta$ and $\phi$ are the pseudo-rapidity and azimuthal angle, respectively). The UA1 data and the corresponding calculation include an isolation cut such that the total hadronic energy inside a cone around the photon of size $\Delta R = 0.4$ be less than 10% of the photonic energy. The isolation criteria for the UA2 data shown in fig. 3 is not as simple, but in an earlier UA2 publication of prompt photon data it was demonstrated that the data sample was consistent with having assumed an isolation cut such that the hadronic energy in a cone around the photon of size $\Delta R = 0.78$ be less than 3.5 GeV. We use this isolation criteria in our calculation. The UA1 and UA2 isolation criteria give very similar results and we show the curve corresponding to the UA1 isolation criteria in fig. 3. The data samples have the overall scale normalization uncertainties shown in the figures. The error bars in the figures include statistical and systematic errors added in quadrature. The discrepancy in the low-$p_T$ region is apparent for all three data sets.

The discrepancy is more easily seen on a linear scale. In figure 4a we show the relative difference between data and theory, $(\text{data-theory})/\text{theory}$, versus $p_T$ for the CDF data. This figure illustrates the theoretical uncertainty due to changing the scales and distribution functions. The reference theory calculation on this plot includes M-T1 distribution functions and scales equal to $p_T^2$. The results using the DFLM260 [10] distribution functions are shown and give about a 20% larger cross section at low-$p_T$, decreasing to less than 10% of the M-T1 result at large $p_T$. Also in this figure we see that changing the scales to $\mu^2 = M_f^2 = p_T^2/4$ increases the results of the predictions by 10-15% (for a given set of distribution functions). The results are relatively insensitive to changes in the factorization scale $M_f^2$. For standard initial normalizations ($N_0 = 1$) the results change by $\lesssim 5\%$ when considering $M_f^2 = p_T^2$ and $M_f^2 = p_T^2/4$ or $M_f^2 = 4 \times p_T^2$. At larger initial fragmentation function normalization values the fragmentation scale dependence is larger, but generally it is $\lesssim 10\%$ for starting normalization values less than 40.

The discrepancy is readily observed in fig. 4a for the case of $p_T^2$ scales and M-T1 distribution functions. For these choices the central values of the three lowest $p_T$ bins are 60, 51 and 117% larger than the theoretical calculation, and have a $\chi^2$/d.o.f. of 5.3. For comparison, the central values of the next
lowest three \( p_T \) bins average less than 9\% larger than the theoretical curve. The choices corresponding to the largest theoretical curve are \( p_T^2/4 \) scales and DFLM distribution functions. In this case there is certainly no glaring discrepancy in the low-\( p_T \) region, although the central value of the lowest \( p_T \) bin is still 80\% larger than the prediction.

In figure 4b we illustrate the discrepancy between data and theory for the UA1 and UA2 data at \( \sqrt{s} = 630 \) GeV. As in fig.4a the largest low-\( p_T \) region discrepancy corresponds to the theoretical calculation which includes M-T1 parton densities and \( p_T^2 \) scales. For these choices the lowest four \( p_T \) bins average 97\% larger than the expectation. The discrepancy is least, as for the CDF data, in the case of DFLM parton densities and scales equal to \( p_T^2/4 \). In this case the \( \chi^2/d.o.f. \) of the lowest four \( p_T \) bins is 3.2.

We show in figure 5a the qualitative improvement in the comparison of CDF data and theory if we include a large initial normalization for the fragmentation functions. We show the curves for \( N_0 = 30 \) and \( 40 \) for the case of M-T1 distribution functions and \( p_T^2 \) scales. The large \( N_0 \) curves qualitatively account for the rise in the cross section in the low-\( p_T \) region while retaining the good description at large \( p_T \). The best fit (defined by the smallest \( \chi^2 \); see Sec.4) for the initial normalization \( N_0 \) for this choice of scales and distribution functions is \( N_0 = 34 \). Of course, the best fit value for the starting normalization \( N_0 \) will change with the distribution function and scale choices. If the scale \( \mu^2 \) is chosen to be smaller (\( e.g. \mu^2 = p_T^2/4 \)) the QCD prediction will increase overall, thus a smaller normalization will be favored. As an illustration, we show in figure 5b the standard (\( N_0 = 1 \)) curve and the best fit (\( N_0 = 17 \)) curve for the scale choice \( \mu^2 = M_f^2 = p_T^2/4 \). Similarly, as the DFLM distribution functions give larger cross sections, they will also give smaller values of the best fit starting normalization. Figure 5c shows the DFLM curve for the standard calculation (\( N_0 = 1 \)) and the best fit initial normalization in this case, \( N_0 = 17 \), for the choice of scales \( \mu^2 = M_f^2 = p_T^2 \).

If we move from the choice of distribution function and scale of fig. 5a to the one of DFLM distribution functions and the scale choice \( \mu^2 = p_T^2/4 \), we can expect an even smaller best fit initial fragmentation function normalization value, as both of these choices increase the theoretical values of the cross section relative to the choices of fig. 5a. We find the best fit value in this case is \( N_0 = 4 \).
However, it is perhaps worth noting that this case gives the worst fit to the data. The central values of all the data points (except for the three lowest $p_T$ bins) fall below the theoretical calculation (even in the case $N_0 = 1$) and correspondingly the $\chi^2$ value for this choice of scales and distribution functions is the largest $\chi^2$ of the four choices considered.

We have seen that while varying the choice of scales and distribution functions in the theoretical calculation at $\sqrt{s}=1.8$ TeV the best fit value for the initial normalization $N_0$ varies from 4 to 34. The uncertainty in the QCD calculation is too great to yield a well constrained value for the normalization. However, even if the QCD calculation were under better control, we will see in the next section that the experimental errors are too large to lend a best fit normalization value any statistical significance.

4 $\chi^2$ analysis

In order to understand the statistical significance of the qualitative improvement achieved by stipulating a large initial fragmentation function normalization, and to see if such a large fragmentation function is consistent with other lower energy experimental data, we vary the initial fragmentation function normalization and perform a $\chi^2$ analysis. In most of the data sets there is an overall scale normalization uncertainty. In such cases we evaluate the $\chi^2$/d.o.f. by varying the overall normalization of all the data points and finding the normalization $N$ which renders the $\chi^2$ formula

$$\chi^2/{\text{d.o.f.}} = \frac{1}{L} \sum_{i=1}^{L} \frac{(\sigma^{\text{exp}}(p_T_i) - \sigma^{\text{theory}}(p_T_i))^2}{(\Delta \sigma^{\text{exp}}(p_T_i))^2} + \frac{(N - 1)^2}{\Delta N^2}$$

a minimum. $\Delta N$ is the experimental normalization error. For $\Delta \sigma^{\text{exp}}$ we add the statistical and systematic errors in quadrature. If part of the systematic error is correlated the true $\chi^2$ values for that data sample will be larger than the ones evaluated using eq.(5). In figures 6 we show the $\chi^2$ distributions for three representative data sets; the CDF data at $\sqrt{s} =1.8$ TeV, the combined $\chi^2$ for the UA1 and UA2 data sets at $\sqrt{s} =630$ GeV, and the NA24 data taken at $\sqrt{s} =23.76$ GeV. In fig.6a the scales are set to $\mu^2 = M_f^2 = p_T^2$ and the DFLM distribution functions are used. These three data sets show a favored
initial fragmentation function normalization value much larger than the standard value of 1 (the values are \( N_0 = 17, 24, \) and 19 respectively). However, for both the 1.8 TeV and the 630 GeV data, the difference in the \( \chi^2 \) value at the minimum and that for an initial normalization of 1 (or zero) is less than 0.5. Thus, while the experimental data favors a large initial fragmentation normalization, the experimental errors are such that a standard initial normalization (or even zero initial normalization) gives almost as good a fit.

For the low energy data set shown, the situation is quite different. The reason the \( \chi^2 \) distribution corresponding to the NA24 data set shows a strong dependence on the initial fragmentation function normalization is that this data set has no isolation criteria applied. The \( \chi^2 \) plot in fig.6a shows the value \( N_0 = 1 \) is actually ruled out at the 68% confidence level. However, by changing the scales and/or parton distribution functions in the calculation the value \( N_0 = 1 \) can be accommodated. This is shown in fig.6b, in which the scales in the calculation have been set to \( p_T^2/4 \). All three distributions in fig.6b show minima at smaller values of \( N_0 \) than in fig.6a, and in particular the NA24 data in this case shows a minimum at \( N_0 = 5 \). The \( \chi^2/d.o.f. \) value corresponding to \( N_0 = 1 \) differs by less than 1 from the \( \chi^2/d.o.f. \) value at the minimum.

We now discuss the combined \( \chi^2 \) plots for all of the data samples considered. The data samples included are the CDF data at \( \sqrt{s} = 1.8 \) TeV and \( \eta = 0 \) \[1\]; the UA1 data at \( \sqrt{s} = 630 \) GeV and \( \sqrt{s} = 546 \) GeV, both for \( \eta = 0 \) and 1.1 \[3\]; the UA2 data at \( \sqrt{s} = 630 \) GeV, \( \eta = 0 \) \[4\]; R806 data at \( \sqrt{s} = 63 \) GeV and \( \eta = 0 \) \[11\]; and NA24 data at \( \sqrt{s} = 23.76 \) GeV with \(-0.65 < \eta < 0.52\) \[12\]. Eight QCD calculations were performed for each of the data sets corresponding to the distribution function choices M-T1 and DFLM, and the scale choices \( \mu^2 = n * p_T^2; \ M_f^2 = m * p_T^2 \) for \((n, m) = (1,1), (1,4), (\frac{1}{4}, \frac{1}{4}), \) and \((\frac{1}{4}, 1)\). We show in figure 7a the \( \chi^2 \) distribution as a function of the initial fragmentation function normalization \( N_0 \) for both sets of structure functions considered and for the scale choice \( \mu^2 = M_f^2 = p_T^2 \). In this figure the initial normalization values corresponding to the lowest \( \chi^2 \) values are \( N_0 = 24 \) and \( N_0 = 19 \) for the M-T1 and DFLM distribution function choices, respectively. In figure 7b we show the results for the scales equal to \( p_T^2/4 \). Here the minimum \( \chi^2 \) values correspond to the normalizations \( N_0 = 8 \) and \( N_0 = 4 \) for the M-T1 and DFLM distribution function choices, respectively. The \( \chi^2 \) distributions corresponding to the same
$\mu^2$ scale choices as in figs. 7 but different $M_f^2$ scale choices give results similar to those shown in figs. 7.

The $\chi^2$ plots all show very mild dependence on the initial normalization $N_0$. A mild dependence for the higher energy data sets can be explained as follows. In the case of the CDF data the theoretical predictions and the experimental data agree within the (large) experimental errors for 11 out of 12 of the highest $p_T$ data points. The data set, even with a discrepancy for the two or three lowest $p_T$ data points, has a low $\chi^2$/d.o.f. The $\chi^2$/d.o.f. value changes very little if the theory is modified in such a way that the three lowest $p_T$ data points are also brought into agreement.

In none of the eight cases considered is the difference between the $\chi^2$ values corresponding to the standard normalization $N_0 = 1$ and the normalization at the minimum of the $\chi^2$ plot greater than 0.4. Thus, because the experimental errors are large, the quantitative improvement in the comparison of theory and data obtained by including a larger fragmentation function normalization is small. However, we do find it somewhat suggestive that in the individual $\chi^2$ distributions for all of the data sets considered and for all choices of scales and parton densities considered (64 plots) the minima were more often than not at initial fragmentation normalization values greater than 16.

5 Conclusions

We find that including large boundary conditions in the evolution equations of the quark $\to$ photon and gluon $\to$ photon fragmentation functions may play an important role in resolving the discrepancy between next–to–leading order QCD calculations and experimental data of the inclusive prompt photon production cross section in the low-$x_T$ region. The fragmentation function evolutions were performed at leading order here. Perhaps including large initial fragmentation function normalizations in a full next–to–leading analysis would lead to a better constrained set of best fit initial normalization values. The inclusive prompt photon data sets considered here favor a larger initial normalization, but the theoretical uncertainties arising from scale variations and parton density uncertainties are too large to give a definite value. A $\chi^2$ analysis reveals that, additionally, the experimental errors are too large to give any statistical sig-
nificance to what appears to be a qualitative improvement in the comparison of theory and data by including a larger fragmentation function normalization. In order to satisfactorily resolve the discrepancy, both theoreticians and experimentalists will have to further refine their respective results. On the theoretical side, more tightly constrained proton structure functions and higher order calculations would certainly stabilize the predictions. On the experimental side we note that if $N_0$ is large the photon production cross section is particularly sensitive to the isolation criteria (either the isolation cone size or the amount of hadronic energy allowed in the cone). A constraint on the initial fragmentation normalization could come from an experimental determination of the dependence of the photon production cross section on the isolation criteria.

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Figure captions

Figure 1
(a) The up quark fragmentation function versus $z$. The curves are shown for $N_0=20$ at scales $Q_0 = 2$ GeV, and $Q=10$ and 100 GeV. The asymptotic form ($N_0=1$) is also shown.
(b) The normalization of the up quark fragmentation function versus the scale $Q$ for initial normalization values $N_0 = 10, 20$ and $30$. The normalization is $N_0$ times the asymptotic form normalization at the scale $Q_0 = 2$ GeV. The dotted lines show the asymptotes at $Q_0=2$ GeV and $N=1$.

Figure 2
The maximum initial normalization for the up quark fragmentation function $N_{max}$ versus the photon fractional momenta cut-off $z_0$, assuming the number of photons fragmented from a 3 GeV quark with fractional momenta $z > z_0$ is $\frac{1}{30}$ times the number of hadrons fragmented by the quark.

Figure 3
The inclusive prompt photon differential cross section $E d^3\sigma/dp^3$ versus the photon transverse momenta. The data is from CDF at $\sqrt{s}=1.8$ TeV and UA1 and UA2 at $\sqrt{s}=630$ GeV. The curves are the QCD predictions using Morfin-Tung Set 1 distribution functions and scales equal to $p_T^2/4$. Note the experimental normalization uncertainties.

Figure 4
The relative difference between data and theory, (data-theory)/theory, is shown vs. transverse momenta. The reference theory calculation includes $p_T^2$ scales with M-T1 parton densities. Also shown are the QCD results for scales $p_T^2/4$ and DFLM distribution functions.
(a) CDF data at $\sqrt{s}=1.8$ TeV.
(b) UA1 and UA2 data, $\sqrt{s}=630$ GeV.
Figure 5
The relative difference between data and theory, (data-theory)/theory, vs. transverse momenta for the CDF data at $\sqrt{s}=1.8$ TeV. The reference theory calculations include the standard asymptotic form fragmentation functions.

(a) M-T1 distribution functions and scales $\mu^2 = M_f^2 = p_T^2$ are used. Initial fragmentation function normalizations $N_0=30$ and $N_0=40$ are shown. The best fit initial normalization value in this case is $N_0=34$.

(b) The results for M-T1 distribution functions with scales $\mu^2 = M_f^2 = p_T^2/4$. The best fit initial fragmentation function normalization value $N_0=17$ is shown.

(c) DFLM parton densities are used, and the scales are set equal to $p_T^2$. The best fit normalization value $N_0=17$ is shown.

Figure 6
The $\chi^2$ distributions vs. the initial fragmentation function normalization $N_0$ for three data sets; the CDF data at $\sqrt{s} = 1.8$ TeV, the combined $\chi^2$ for the UA1 and UA2 data at $\sqrt{s} = 630$ GeV, and NA24 data at $\sqrt{s} = 23.76$ GeV. All the data sets are at zero rapidity. The DFLM distribution functions were used.

(a) The scales are set to $\mu^2 = M_f^2 = p_T^2$.

(b) The scales are set to $\mu^2 = M_f^2 = p_T^2/4$.

Figure 7
The combined $\chi^2$ distributions vs. the initial fragmentation function normalization $N_0$ for eight data sets; see text.

(a) The scales are set to $\mu^2 = M_f^2 = p_T^2$.

(b) The scales are set to $\mu^2 = M_f^2 = p_T^2/4$. 