Rate Dependent Behavior of Reinforced Concrete Using Irregular Lattice Model

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Abstract. In this study, a numerical approach using irregular lattice model is developed to simulate failure behavior of reinforcement concrete (RC) at high strain rates of loading. In the irregular lattice model, the material domain is discretized by the Delaunay/Voronoi dual tessellation with their lattice connectivity of spring sets. Generally, cementitious materials have rate dependency on their mechanical properties with different rates of loading. During the numerical analysis, in order to achieve this material characteristic, a rheological unit with a combination of springs and dashpots is introduced into the rigid-body-spring elements of the lattice model. The material response in every time step is obtained based on the time integration scheme. To model the reinforcement within the matrix, a semi-discrete typed approach is adopted into the lattice model, in which the stiffness of a reinforcement is transformed into a corresponding spring set of concrete element.

1. Introduction
As the destruction of structures due to earthquake and explosion load is becoming more frequent, concerns about mechanism of structure destruction loom large in the computational analysis area. In general, the destruction of a structure induced from dynamic loads such as earthquake and collision is greatly influenced by rates of loading as well as a magnitude and location of the applied load. John and Shah conducted a beam-column joint experiment showing that the fracture behavior depends on the load rates [1]. Therefore, rate dependency should be considered when performing the dynamic analysis of the structure.

In this paper, the rate dependency of the material is expressed by implementing a visco-plastic components to the spring set of rigid-body-spring-network (RBSN) model, which is a kind of irregular lattice model. In addition, for better representation of reinforcement behavior, three-dimensional (3D) dynamic model was developed to supplement the deficiency of two-dimensional (2D) dynamic model [2-3]. Through comparison of fracture configuration and the behavior between the experiment and the numerical analysis, the model was validated.

2. Methodology

2.1. Modeling of concrete
RBSN model consists of random lattices which is developed based on the spring model firstly proposed by Kawai [4], its arbitrary shape is known to be efficient for fracture analysis of heterogeneous material [5-6]. The mesh is constructed by following sequences: (1) random point generation; (2) Delaunay tessellation of the point set; and (3) construction of Voronoi tessellation (figure 1). The elements of
generated mesh are connected by the stiffness constraint condition of nodes i and j of two adjacent Voronoi cells. In the generated mesh, node i and j of arbitrary two adjacent Voronoi cells are connected to elements. The element consists of a spring set (figure 2). The element matrix is comprised of a sizeless spring set that has six degree of freedom which exists in the centroid of facet between the two Voronoi cells. The stiffness of the spring set to the translational motion at the three individual local axis can be expressed as follows:

\[
k_n = \frac{E(1-\nu)}{(1+\nu)(1-\nu)} \frac{A_i}{h_{ij}}, k_s = \frac{E}{h_{ij}}, k_r = \frac{E}{(1+\nu) h_{ij}}
\]

The rotational stiffness according to each local axis is as follows:

\[
k_{\theta} = \frac{E J_p}{h_{ij}}, k_{\varphi} = \frac{EI_{22}}{h_{ij}}, k_{\psi} = \frac{EI_{11}}{h_{ij}}
\]

where \(E\) = concrete elastic modulus; \(\nu\) = Poisson ratio; \(A_{ij}\) = Voronoi facet area of element ij; \(h_{ij}\) = length of element ij; \(J_p\) = polar moment of inertia; and \(I_{11}, I_{22}\) = principal moment of inertia. The concrete fracture analysis can be effectively performed with the help of randomly divided element which can express the non-uniformity of the material. Concrete fracture is determined by crack band theory which is suggested by Bažant and Oh [7]. The RBSN model assumes the cell is a rigid body, so the destruction occurs along with the spring set of the Voronoi facet. If the resultant stress \(\sigma_r\) applied to the Voronoi facet is larger than the reference failure stress, the spring set is gradually damaged [8].

2.2. Modeling of reinforcement

In the material domain, the rebar is automatically discretized into a series of structural elements similar to those representing the matrix. The segmented reinforcement elements have axial stiffness \(k_r\) at the point that passing through the matrix Voronoi facet:
where \( E_r \) = elastic modulus of the reinforcement; \( A_r \) = cross sectional area; and \( L \) = discretized reinforcement length. After reaching the yield stress \( f_y \), the rebar stiffness was multiplied by the damage factor of the rebar \( \alpha \) to express stiffness reduction effect. The stiffness matrix of a rebar is substituted into stiffness matrix of concrete by transformations such as coordinate transformation. By using this methodology called semi-discrete method, further addition of degree of freedom to the system of equation was disregarded for faster computation and adequate analysis [10].

2.3. Rate dependence
The dynamic change factor of concrete varies according to the strain rate; from this characteristics, concrete is called rate-dependent material. In order to implement the dynamic behavior of a material depending on the rate of load, the physical mechanism according to the strain rate should be realized. Concrete is more sensitive to strain rate changes under tensile load than under compression load [11-12]. The reason why the strength of the material increases in tensile behavior is due to the voids in the concrete and the viscosity of free water act at the stage of pre-cracking: this we call Stefan effect [13]. In this analysis, rate dependency was expressed by connecting a rheological visco-plastic model(\( \varepsilon^vp \)) to the spring set(\( \varepsilon^p \)) of the matrix facet as shown in figure 4 [14-16].

![Figure 3](image_url)  
**Figure 3.** Various experimental data dynamic increase factor for tensile strength [12]

![Figure 4](image_url)  
**Figure 4.** Rheological model of visco-plastic [14]
3. Analysis program verification

3.1. Four-point bending test

Four-point bending test was simulated at 380mm/s loading rate. As shown in figure 5a, the geometry of the beam used for the analysis is from the experimental work done by Kulkarni and Shah [2]; the length and depth of the concrete beam is 1826mm and 152mm. The concrete beam is singly reinforced at the effective depth from the top with three steel bars providing a reinforcement ratio of 1.38% with a diameter of 9.5mm. The concrete mesh generated through Delaunay-Voronoi dual tessellation consists of 5980 nodes and 35876 elements; the reinforcement has 402 elements. In order to improve the speed of the computation and to accurately analyze the fracture pattern, the mesh was densely generated at the bottom of the beam (figure 5b.). The material properties used for the analysis are set identical to the values from the actual experiment and are as follows: elastic modulus of concrete $E_c = 32.9 \text{GPa}$; static tensile strength of concrete $f_t = 4.3 \text{MPa}$; elastic modulus of steel $E_s = 200 \text{GPa}$; and yield stress of steel $f_y = 518 \text{MPa}$. The results were compared with the data from the experiment, 2D model [3], and 3D model.

![Figure 5. Four-point bending test of reinforced concrete beam: (a) test specimen configuration; (b) RBSN model](image)

3.2. Results

As a numerical result of four-point bending test using 3D dynamic RBSN, among the shear cracks and flexural cracks, flexural cracks were dominantly generated around the midspan (figure 6.c). The crack pattern through this analysis was very similar to the crack pattern from the experiment (figure 6.a). However, no cone failure appeared in the area where the load is applied because no failure was considered in the analysis at the compression part. The same phenomenon appears in the analysis results of 2D dynamic RBSN (figure 6.b).

In the fracture pattern of the 2D analysis, the cracks occurred on both sides of the beam which did not occur in the actual experiment. Also, minor cracks were formed around reinforcements because three reinforcements are recognized as one in 2D analysis (figure 6.b). Such problem can be solved with 3D analysis which represents more similar rate dependent failure mode when compared to the actual experiment. For more detailed failure mode analysis, the load-deflection curves were compared using data from experiments, 2D analysis, and 3D analysis (figure 7). In the case of the load-deflection curve obtained from 2D and 3D analysis, the load-deflection curve was drawn after smoothing process due to heavy fluctuation caused by the transfer of load waves in the conductor. Since the initial slope of the load-deflection curve from the experiment is gradually increased, flexural failure is dominant in this beam. However, the load carrying capacity of the rebar was not sufficiently expressed in the model at
high loading rates due to no consideration of rate dependency of rebar. This caused a difference in the initial slope value. Despite the fact that there exist a slight discrepancy at the initial part of the graph, the overall results from 3D analysis is suitable to describe the fracture mechanisms at high loading rates; the crack pattern and the curve achieved from 3D analysis represents more similar results than from 2D analysis.

![Figure 6](image.png)

**Figure 6.** Fracture configuration under load rate of 380mm/s: (a) actual experiment; (b) 2D dynamic RBSN; and (c) 3D dynamic RBSN (this study)

![Figure 7](image.png)

**Figure 7.** Load-deflection curve of four-point bending test

4. Conclusion
This study conducted 3D dynamic RC analysis using semi-discrete modeling approach. The model is developed to complement the shortcomings of pre-developed 2D dynamic RC analysis; when multiple rebars are placed in the same depth, in the case of 2D numerical analysis program, the rebars are recognized as a single rebar. Along with the increased freedom from the 3D analysis, the more precise results were achieved. However, few assumptions were reflected in the analysis: (1) the bond between the matrix and inface were considered to be perfectly bonded; and (2) the initial load carrying capacity was not sufficiently considered at high loading rate for not considering the rate dependency of the rebar. Further consideration should be included so that the model demonstrate better at higher rate loads such as impact load.
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