Unique ergodicity for foliations in $\mathbb{P}^2$ with an invariant curve

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Abstract

Consider a foliation in the projective plane admitting a projective line as the unique invariant algebraic curve. Assume that the foliation is generic in the sense that its singular points are hyperbolic. We show that there is a unique positive $dd^c$-closed $(1,1)$-current of mass 1 which is directed by the foliation and this is the current of integration on the invariant line.

A unique ergodicity theorem for the distribution of leaves follows: for any leaf $L$, appropriate averages of $L$ converge to the current of integration on the invariant line. This property is surprising because for most of foliations as above, the leaves (except the line at infinity) are dense in $\mathbb{P}^2$ and one could expect that they spend a significant amount of hyperbolic time in every open set.

To obtain the results, we use an extension of our theory of densities for currents. The talk is based in a recent joint work with Nessim Sibony.