Polarization state of a biphoton:
quantum ternary logic

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Abstract

Polarization state of biphoton light generated via collinear frequency-degenerate spontaneous parametric down-conversion is considered. A biphoton is described by a three-component polarization vector, its arbitrary transformations relating to the SU(3) group. A subset of such transformations, available with retardation plates, is realized experimentally. In particular, two independent orthogonally polarized beams of type-I biphotons are transformed into a beam of type-II biphotons. Polarized biphotons are suggested as ternary analogs of two-state quantum systems (qubits).
Polarization state of a single photon is described by a two-dimensional normalized polarization vector. As any quantum system with two basic states \([1]\), an arbitrarily polarized photon can represent a qubit - a quantum bit of information \([2]\) used in quantum computation. Several quantum logical operations with photon qubits have been proposed, which make use, in addition to photons, of atoms or ions \([3]\). Recently, quantum gates were suggested based only on photons, some of them serving as polarization qubits and some as location qubits \([4]\).

In this paper, we consider a quantum system formed by two correlated photons - a biphoton emitted via frequency-degenerate collinear spontaneous parametric down-conversion (SPDC). Its polarization state is assumed to be arbitrary. In this general case, the biphoton can be described by the state vector \([5]\)

\[
\Psi = c_1|2, 0\rangle + c_2|1, 1\rangle + c_3|0, 2\rangle, \quad (1)
\]

where \(c_i = d_i e^{i\phi_i}\) are complex amplitudes and the notation \(|N_x, N_y\rangle\) means a state with \(N_x\) photons in the horizontal \((x)\) polarization mode and \(N_y\) photons in the vertical \((y)\) polarization mode, with \(N_x + N_y = 2\). The normalization condition is \(\sum_i |c_i|^2 = 1\). In most cases, the total phase of the state \((1)\) is not essential, so one can assume \(\phi_2 = 0\), and the three-component state of a biphoton is given by four real parameters. One can introduce the ‘polarization vector’ of a biphoton,

\[
e = (c_1, c_2, c_3). \quad (2)
\]

In the most general form, the state \((1)\) can be prepared via SPDC generated in three nonlinear crystals with common coherent pumping. The states \(|2, 0\rangle\) and \(|0, 2\rangle\) are generated via type-I SPDC, and the state \(|1, 1\rangle\) via type-II SPDC \([6]\). According to Eq. \((1)\), a biphoton is a three-state system, similarly to a particle with spin 1. Arbitrary transformations of polarization vectors \(e\) are given by unitary \(3 \times 3\) matrices \(G\), \(G^+G = I, \det G = 1\), which form a three-dimensional representation of the SU(3) group, see \([7]\). This type of symmetry, which is well-known in nuclear physics but seems to be new for optics, could be used for
developing ‘ternary logic’ in quantum computation. To each of the three basic states $|2, 0\rangle$, $|1, 1\rangle$, and $|0, 2\rangle$, one can assign one of the digits 0, 1, and 2. The advantage of ‘ternary’ quantum logic over binary logic (qubits) is the larger number of states that can be covered by an $n$-element quantum register: $3^n$ instead of $2^n$. The first question arising here is how one can ‘switch’ between these three basic states or their combinations.

According to the properties of the SU(3) group, an arbitrary transformation $G$ of the vector $\mathbf{e}$ is given by eight real parameters. Linear lossless elements (retardation plates and polarization rotators) introduced into the biphoton beam transform the vector $\mathbf{e}$ but cannot give all possible matrices $G$. A transformation of this kind can be characterized by three independent parameters and therefore, it only realises a three-dimensional representation of the SU(2) group, which leaves invariant the polarization degree $P$. At the same time, two of the basic states in superposition (1) have $P = 1$ and one has $P = 0$.

However, by passing from the basis $|2, 0\rangle, |1, 1\rangle, |0, 2\rangle$ to the basis

$$
\Psi_+ = \frac{|2, 0\rangle + |0, 2\rangle}{\sqrt{2}} \equiv |+, -\rangle \\
\Psi_- = \frac{|2, 0\rangle - |0, 2\rangle}{\sqrt{2}} \equiv |+ 45^\circ, -45^\circ\rangle, \\
\Psi_0 = |1, 1\rangle \equiv |x, y\rangle,
$$

one obtains three states that can be transformed one into another by means of only retardation plates. Indeed, all three vectors of the new basis have $P = 0$. They all correspond to pairs of correlated photons with orthogonal polarizations: right and left circular, linear at $\pm 45^\circ$ to x, and along x and y. In this work, we experimentally realize transformations between these states.

The states $\Psi_+, \Psi_-, \Psi_0$ have much in common with quantum ternary logic states (‘trits’) suggested in [10]. Indeed, the Bell states $\frac{1}{\sqrt{2}}(|H\rangle|H\rangle \pm |V\rangle|V\rangle)$ of [10] correspond to $\Psi_\pm$, the Bell state $\frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)$ corresponds to $\Psi_0$, and the Bell state $\frac{1}{\sqrt{2}}(|H\rangle|V\rangle - |V\rangle|H\rangle)$ has no sense in the case of indistinguishable photons. It is essential that unlike the states introduced in [10], all states considered here relate to a single spatial mode. This is an important practical advantage, since it removes the requirement of equalizing optical paths,
which was necessary in [10].

In experiment (Fig. 1), we use a type-I lithium iodate crystal pumped by cw He-Cd laser with wavelength 325 nm and vertical polarization. The pump is split into two collinear beams, so that horizontally polarized SPDC with $e = (1, 0, 0)$ is generated in two spatially separated domains. The pump radiation is suppressed by the cutoff filter F. After the crystal, the SPDC radiation from one of the domains is passed through a $\lambda/2$ plate oriented at 45 degrees to the initial polarization. The corresponding biphoton polarization vector becomes $(0, 1, 0)$. Both SPDC beams are then joined together by means of a polarizing beamsplitter PBS1. In fact, this part of the setup is a Mach–Zehnder interferometer with the nonpolarizing beamsplitter for the pump at the input and a polarizing beamsplitter for biphoton radiation at the output. After the interferometer, the state is

$$\Psi = \frac{1}{\sqrt{2}}(|2, 0\rangle + e^{i\phi}|0, 2\rangle), \tag{4}$$

where the phase $\phi$ is varied by means of a piezoelectric element (PE) shifting the mirror at the input of the beamsplitter. Preparation of the biphoton state is accomplished by introducing a retardation plate RP (either a half wave plate or a quarter wave plate) after the beamsplitter.

A half wave plate with the optic axis oriented at the angle $\chi$ to the horizontal direction transforms the state into the state of the form with

$$|c_1|^2 = |c_3|^2 = \frac{1 - \sin^2 4\chi \sin^2 \frac{\phi}{2}}{2}, |c_2|^2 = \sin^2 4\chi \sin^2 \frac{\phi}{2}. \tag{5}$$

At $\phi = \pi$ and $\chi = \frac{\pi}{8}, \frac{3\pi}{8}, \ldots$, $|c_1| = |c_3| = 0$, i.e., the state is completely transformed into the state $|1, 1\rangle$. In our notation, this is the transition $\Psi_- \rightarrow \Psi_0$. Note that if $\phi = 0$, the state is invariant to the action of a half wave plate, $\Psi_+ \rightarrow \Psi_+$. Similarly, for a quarter wave plate oriented at the angle $\chi$,

$$|c_2|^2 = \sin^2 2\chi (\cos^2 \frac{\phi}{2} + \cos 2\chi \sin \frac{\phi}{2})^2, \tag{6}$$

and the transformation from the state to the state $|1, 1\rangle$ is achieved at $\phi = 0$, $\chi = \frac{\pi}{4}$. 

This describes the transition $\Psi_+ \rightarrow \Psi_0$. At the same time, a quarter wave plate with $\chi = \frac{\pi}{4}$ leaves $\Psi_-$ invariant.

Transitions from the states $\Psi_-, \Psi_+$ to the state $\Psi_0$ can be demonstrated experimentally by measuring the second-order correlation function of the final state,

$$G_{xy}^{(2)} \equiv \langle \Psi | E^{(-)}_x E^{(-)}_y E^{(+)}_x E^{(+)}_y | \Psi \rangle,$$

(7)

where $E^{(\pm)}_{x,y}$ are field operators for the modes $x$ and $y$. Indeed, for a state of the form $|\Pi\rangle$, we have $G_{xy} \sim |c_2|^2$. The correlation function $G_{xy}$ is measured by means of a polarizing beamsplitter PBS2, two photodetectors, D1 and D2, and a coincidence circuit CC (Fig. 1). The pinhole PH with diameter 1 mm and the interference filter IF with FWHM $\Delta \lambda = 10$nm and central wavelength $\lambda = 650$nm are used for the spatial and frequency selection of the SPDC collinear frequency-degenerate radiation. The coincidence counting rate $R_c$, which is proportional to $G_{xy}$, is measured either as a function of the optical path length variation (phase $\phi$ variation) introduced by the piezoelectric element or as a function of the retardation plate orientation (angle $\chi$ variation).

The experimental dependencies obtained with the half wave plate are shown in Figures 2 and 3. First, we fix the orientation of the plate, $\chi = \frac{\pi}{8}$, and measure $R_c$ as a function of $\phi$, which is determined by the voltage applied to the piezoelectric element (Fig. 2). In the minima, the state at the output of the interferometer is $\Psi_+$, which stays the same after the half wave plate. At the maxima, the interferometer creates the state $\Psi_-$, which is then transformed into $\Psi_0$ by the half wave plate. Fixing the phase $\phi$ at a maximum ($\phi = \pi$), we measure the dependence of $R_c$ on the half wave plate angle $\chi$ (Fig. 3a). High coincidence counting rate at the maxima of this dependence (in comparison with accidental coincidence counting rate, which is less than $0.1 \text{ sec}^{-1}$) indicates that the state $|1, 1\rangle$ is formed. However, to check that $\Psi_-$ is fully transformed into $\Psi_0$, we need to measure the correlation functions $G_{xx}$ and $G_{yy}$, which are proportional to $|c_1|^2$ and $|c_3|^2$, respectively. Such measurements are performed by introducing an additional block before the polarizing beamsplitter PBS2. This block (framed by a dashed line in Fig. 1) includes a polarizer
selecting \( x \) or \( y \) polarization and a half wave plate rotating the polarization by \( \frac{\pi}{4} \). With this block introduced into the setup, \( R_c \) is proportional to \( G_{xx} \sim |c_1|^2 \) or \( G_{yy} \sim |c_3|^2 \), depending on the polarizer orientation. For instance, Fig. 3b shows the dependence of \( |c_1|^2 \) on \( \chi \) for the phase \( \phi \) being the same as for Fig. 3a. One can see that at the angles \( \chi \) where maxima of \( |c_2|^2 \) are observed (Fig. 3a), the amplitude \( |c_1| \) (and similarly, \( |c_3| \)) is almost completely suppressed (Fig. 3b). The background coincidence counting rate in Fig. 3b (the visibility of the interference pattern is 90%) can be explained by nonequal losses for the states \(|2,0\rangle\) and \(|0,2\rangle\).

Similarly, to perform the transformation \( \Psi_+ \rightarrow \Psi_0 \), one should use a quarter wave plate as RP in Fig. 1. The phase \( \phi \) in this case should be equal to 0. In Fig. 4, the dependence of \( G_{xy} \sim |c_2|^2 \) on \( \chi \) at \( \phi = 0 \) is shown. In accordance with Eq. (6), the period of this dependence is twice larger than in the case of the half wave plate.

All dependencies shown in Figs. 2-4 demonstrate nonclassical interference with high visibility. If both biphoton states \(|2,0\rangle\) and \(|0,2\rangle\) generated in separate spatial domains are projected onto a single polarization direction \( [11] \), one can observe interference in coincidences regardless of the delay introduced between the SPDC beams. In principle, the crystal inside the interferometer can be replaced by two separate crystals, placed at different distances from the beamsplitter. The only condition for the interference is that the arms of the interferometer should not differ by more than the pump coherence length. This property is due to the collinear degenerate phase matching used in our experiment. In a similar interference experiment with noncollinear SPDC performed previously \( [12] \), equality of the optical path lengths for two crystals was required. \( [13] \)

Another paradoxical feature of this experiment should be pointed out. The state \(|1,1\rangle\), which is what one calls ‘a type-II biphoton’, is produced by two independent ‘type-I biphoton states’ \(|2,0\rangle\) and \(|0,2\rangle\). At the same time, the biphoton flux is so low (about hundreds of \( s^{-1} \)) that biphotons, if considered as ‘wavepackets’ with coherence length \( l_{coh} = \frac{\lambda^2}{\Delta \lambda} \sim 40 \mu \), almost never overlap. This shows that unlike single photons, biphotons should not be viewed as independent wavepackets \( [3,14] \).
Thus, we have demonstrated switching between the three states $\Psi_-, \Psi_+, \text{and } \Psi_0$: the transitions $\Psi_- \rightarrow \Psi_0, \Psi_+ \rightarrow \Psi_0$ are performed by half wave and quarter wave retardation plates, respectively. Note that the transition $\Psi_- \rightarrow \Psi_+$ can be performed by introducing a $\pi$ phase shift between $x$- and $y$- polarized light, i.e., by inserting a half wave plate with the axes parallel to $x, y$ directions. It is worth noting that all these transformations are reversible.

A remarkable property of retardation plates is that they leave invariant the number of biphotons, i.e., do not split photon pairs. This could be used for developing ‘biphoton’ communication systems where biphotons propagate along a single direction, for instance, in an optical fiber, and are transformed by retardation plates.

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Figure Captions

Fig. 1. The experimental setup. CW radiation of He-Cd laser at 325 nm is fed into a Mach-Zehnder interferometer, so that two coherent pump beams excite collinear frequency-degenerate SPDC in different spatial domains of a LiIO$_3$ crystal. The cutoff filter F suppresses the pump radiation, and the $\lambda/2$ plate rotates polarization of the SPDC light in one of the arms by $\pi/2$. The piezoelectric element PE is used for path length (phase $\phi$) variation. The polarizing beamsplitter PBS1 joins two SPDC beams together. The retardation plate RP, either $\lambda/2$ or $\lambda/4$, can be rotated by angle $\chi$. The registration part of the setup includes the interference filter IF and the pinhole PH selecting SPDC radiation, the polarising beamsplitter PBS2, two detectors D1, D2, lenses L1, L2, focusing the radiation on the detectors, and the coincidence circuit CC. The framed block including a polarizer P and a $\lambda/2$ plate is introduced for measuring $G_{xx}, G_{yy}$; without this block, $G_{xy}$ is measured.

Fig. 2. Coincidence counting rate $R_c \sim G_{xy} \sim |c_2|^2$ as a function of the optical path length variation (phase $\phi$ in Fig. 1). The $\lambda/2$ plate after the Mach-Zehnder interferometer is oriented at $\pi/8$. Maxima of the dependence correspond to the $\Psi_-$ state formed at the output of the interferometer; the half wave plate transforms it into $|1, 1\rangle$. In the minima, the state at the output of the interferometer is $\Psi_+$, and it is invariant to the action of the half wave plate.

Fig. 3. Coincidence counting rate corresponding to $G_{xy}$ (a) and to $G_{xx}$ (b) as a function of the angle $\chi$ of $\lambda/2$ plate. In the lower case, the framed block in Fig. 1 is inserted. For both dependencies, the phase $\phi$ introduced by the piezoelectric element is $\pi$, i.e., the state at the output of the interferometer is $\Psi_-$. 

Fig. 4. Coincidence counting rate corresponding to $G_{xy}$ as a function of the angle $\chi$ of $\lambda/4$ plate. The phase $\phi$ introduced by the piezoelectric element is 0, i.e., the state at the output of the interferometer is $\Psi_+$. In the maxima, the plate transforms it into the state $\Psi_0$; in the minima, it leaves it invariant.
Coincidences per 70 sec

Angle of $\lambda/4$ plate (deg)