Constituent Quark Masses and the Electroweak Standard Model

M. D. Scadron
Physics Department, University of Arizona, Tucson, AZ 85721, USA
E-mail: scadron@physics.arizona.edu

R. Delbourgo
School of Mathematics and Physics, University of Tasmania
GPO Box 252-21, Hobart 7001, Australia
E-mail: Bob.Delbourgo@utas.edu.au

G. Rupp
Centro de Física das Interacções Fundamentais, Instituto Superior Técnico,
P-1049-001 Lisboa, Portugal
E-mail: george@ist.utl.pt

Abstract.
Constituent quark masses can be determined quite well from experimental data in several ways and one can obtain fairly accurate values for all six $m_q$. The strong quark-meson coupling $g = 2\pi/\sqrt{3}$ arises from the quark-level linear $\sigma$ model, whereas $e$ and $\sin\theta_w$ arise from weak interactions when the heavy $M_W$ and $M_Z$ are regarded as resonances in analogy with the strong KSFR relation. The Higgs boson mass, tied to null expectation value of charged Higgs components, is found to be around 317 GeV. Finally, the experimental CPV phase angle $\delta$ and the three CKM angles $\Theta_1$, $\Theta_2$, $\Theta_3$ are successfully deduced from the 6 constituent quark masses following Fritzsch’s approach.

PACS numbers: 12.15.-y, 12.40.Vv, 14.65.-q, 13.40.Em
1. Introduction

One of the principal complaints about the electroweak standard model (EWSM) is that it contains too many parameters, in fact no fewer than 19 of them, even if one disregards massive neutrinos. Aside from this plethora, there is the matter of the quark masses being “current” masses, and somewhat far removed from effective “constituent” masses due to dynamical QCD contributions, which may in principle change with quark flavour, because the current masses are nonzero. The values of current quark masses are often the subject of debate, but much less controversy surrounds the constituent quarks. The purpose of this paper is to demonstrate the usefulness of naive constituents to shed light on and reduce some of the parameters of the standard model.

In Secs. 2–5 we show that the $u, d, s, c, b$ constituent masses may be reliably found by various experiments and are perfectly consistent with chiral Goldberger-Treiman relations (GTRs). In Sec. 6 we indicate the analogies between strong interactions and the EWSM, by regarding the $Z$ and $W$ bosons as resonances, when invoking strong-interaction VMD and KSRF-type relations. In our treatment the $W$ couples to sources in $(V - A)$ form (as originally found for low-mass hadrons and leptons by Sudarshan & Marshak and Feynman & Gell-Mann), while $Z$ coupling to leptons is largely axial.

As a by-product we predict $m_t \approx 176.7$ GeV, via a GTR construction, and arrive at a plausible Higgs mass $M_H \approx 316.7$ GeV. Finally, in Sec. 7 we attempt to “predict” the mixing angles, by following Fritzsch’s approach, but using constituent quarks instead of current quarks, and show that the agreement with experiment is quite reasonable.

2. Light-quark mass difference

The simplest way to estimate the $u$-$d$ quark mass difference is via the neutral- and charged-kaon mass difference (neglecting the small error):

$$m_d - m_u \approx m_{\bar{s}d} - m_{\bar{s}u} = m_{K^0} - m_{K^+} = (497.648 - 493.677) \text{ MeV} \approx 3.97 \text{ MeV}. \quad (1)$$

The kaon mass itself follows from knowledge of the chiral-breaking current masses $m_{n_{\text{cur}}} (n = u, d)$ and $m_{s_{\text{cur}}}$, but for the kaon mass difference we need predict only the constituent-quark mass difference $m_d - m_u$. Equation (1) is compatible with the charged-$\Sigma$ baryon mass difference

$$2(m_d - m_u) \approx m_{dds} - m_{uus} = m_{\Sigma^-} - m_{\Sigma^+} = (1197.45 - 1189.37) \text{ MeV} \approx 8.08 \text{ MeV}, \quad (2)$$

or

$$m_d - m_u \approx 4.04 \text{ MeV}. \quad (3)$$

If we include the $\Sigma^0$ in the latter estimate, we get the mass splittings $m_{\Sigma^-}(dds) - m_{\Sigma^0}(uds) = 4.81$ MeV and $m_{\Sigma^0}(uds) - m_{\Sigma^+}(uus) = 3.27$ MeV, with average constituent-quark mass difference $m_d - m_u = 4.04$ MeV, which is equal to (3) and close to (1).
Besides the 4 MeV mass scale from (1) and (3), an approximate \( m_d - m_u \) mass difference follows from higher resonances, namely
\[
\begin{align*}
m_{d} - m_{u} & \approx m_{sd} - m_{su} = m_{K^{*0}} - m_{K^{*+}} = (896.10 - 891.66) \text{ MeV} \approx 4.44 \text{ MeV}, \\
m_{d} - m_{u} & \approx m_{dc} - m_{uc} = m_{D^{+}} - m_{D^{0}} = (1869.3 - 1864.5) \text{ MeV} \approx 4.8 \text{ MeV},
\end{align*}
\]
though it is noticeable that these estimates deteriorate as the masses rise. [Parenthetically, it is reassuring that (1) is in agreement with the nucleon mass difference; this can be estimated from the Coleman-Glashow \( \lambda^3 \) tadpole for \( n-p \) nucleons \[1\]
\[
\left( H_{\text{tad}}^3 \right)_{n-p} \approx 2.5 \text{ MeV},
\]
less the proton current-current Hamiltonian density
\[
(H_{JJ})_{p} = \frac{3\alpha}{2\pi} m_{p} \left[ \ln(\frac{\Lambda_{p}}{m_{p}}) + \frac{1}{4} \right] \approx 1.2 \text{ MeV},
\]
for UV cutoff \( \Lambda \approx 1.05 \text{ GeV} \). Then
\[
m_{n} - m_{p} \approx 2.5 \text{ MeV} - 1.2 \text{ MeV} = 1.3 \text{ MeV},
\]
very near data \[2\] \( m_{n} - m_{p} \approx 1.293 \text{ MeV} \). Since (6) requires a cutoff, we note that \[1\] predicts a similar UV cutoff \( \Lambda = 1.02 \text{ GeV} \) for (squared) pion masses.

Or we can justify the usual \( \Delta I = 1 \) group-theory predictions in nuclear physics to support Ref. \[1\], via a constituent-quark loop, for \( m_{d} - m_{u} = 4 \text{ MeV} \) \[4\].

3. Light-quark mass sum

One way to get at the average nonstrange quark mass \( \hat{m} \equiv (m_{u} + m_{d})/2 \) is to employ the nonrelativistic quark model (NRQM) \[5\] connecting nucleon magnetic moments with those of the underlying quarks:
\[
\begin{align*}
\mu_{p} &= \frac{4}{3} \mu_{u} - \frac{1}{3} \mu_{d} , \\
\mu_{n} &= \frac{4}{3} \mu_{d} - \frac{1}{3} \mu_{u} ,
\end{align*}
\]
where \( \mu_{i} = e_{i}/2m_{i} \) for constituent quark masses, with quark charges \( e_{u} = 2e/3 \), \( e_{d} = -e/3 \), and nucleon Bohr magneton scaled to \( e/2M_{N} \). Equations (8) tell us that
\[
\begin{align*}
\mu_{p} &= \frac{e}{18} \left[ \frac{8}{m_{u}} + \frac{1}{m_{d}} \right] , \\
\mu_{n} &= -\frac{e}{18} \left[ \frac{4}{m_{d}} + \frac{2}{m_{u}} \right].
\end{align*}
\]
Since \( m_{d} - m_{u} \approx 4 \text{ MeV} \) (see Sec. 2), from \( \mu_{p} \) in (9) and the experimental \[2\] value \( \mu_{p} = 2.792847 e/2m_{p} \), we obtain (in MeV) a quadratic equation for \( \hat{m} \):
\[
\hat{m}^{2} - \frac{m_{p}}{2.792847} (\hat{m} + \frac{14}{9}) = 0 ,
\]
whose only positive solution reads
\[
\hat{m} \approx 337.5 \text{ MeV} .
\]
Thus the light quark masses are
\[
\begin{align*}
m_{d} &\approx 339.5 \text{ MeV} , \\
m_{u} &\approx 335.5 \text{ MeV} .
\end{align*}
\]
Backtracking to (9), the ratio
\[- \frac{\mu_n}{\mu_p} = \frac{4m_u + 2m_d}{m_u + 8m_d} \approx 0.6623\] (13)
is close to data \[2\]
\[- \frac{\mu_n}{\mu_p} = \frac{1.913043}{2.792847} \approx 0.6850 \, .\] (14)

A second way of obtaining \( \hat{m} \) is through the chiral-symmetric GTR
\[\hat{m} \approx f_\pi g \approx (93 \text{ MeV}) \frac{2\pi}{\sqrt{3}} \approx 337.4 \text{ MeV},\] (15)
which is satisfyingly close to (11). Here, \( f_\pi \approx 93 \text{ MeV} \), and the meson-quark coupling is \( g = 2\pi/\sqrt{3} \), determined from infrared QCD \[6\], the \( Z = 0 \) compositeness condition \[7\], or the quark-level linear \( \sigma \) model (QLL\( \sigma \)M) \[8, 9\].

### 4. Strange quark mass

Because the nonstrange GTR (15) matches the magnetic-moment prediction (11) so nicely, it invites us to extend the GTR to kaons, i.e.,
\[\frac{1}{2} (m_s + \hat{m}) = f_K g,\] (16)
where \( f_K/f_\pi \approx 1.22 \) from data [2]. First dividing (16) by (15), the meson-quark coupling \( g \) divides out:
\[\frac{m_s + \hat{m}}{2\hat{m}} = \frac{f_K}{f_\pi} \approx 1.22 \implies \frac{m_s}{\hat{m}} \approx 1.44,\] (17)
whereupon we deduce that
\[m_s \approx 1.44 \hat{m} = 486.0 \text{ MeV}.\] (18)
Stated alternatively,
\[\frac{m_s + \hat{m}}{2} \approx f_K \frac{2\pi}{\sqrt{3}} \approx 411.6 \text{ MeV} \implies m_s \approx 485.7 \text{ MeV},\] (19)
where we have made explicit use of the value \( g = 2\pi/\sqrt{3} \).

It is worth recalling that the almost pure \( \bar{s}s \) vector \( \phi(1020) \) mass is about twice \( m_s \), viz.
\[m_s \approx \frac{1}{2} m_{\phi(1020)} = 510 \text{ MeV},\] (20)
as originally stated in Ref. [5]. (This works less well for lighter quarks, since \( m_\rho/2 \approx 388 \text{ MeV} \) is about 10% larger than our earlier estimate of \( \hat{m} \).)

To conclude this section, we tabulate here the predicted and measured [2] magnetic moments of several ground-state baryons, as obtained in the simple constituent quark model, with the above-derived quark masses \( m_u = 335.5 \text{ MeV} \), \( m_d = 339.5 \text{ MeV} \), and \( m_s = 486.0 \text{ MeV} \). From Table 1 we see that this naive picture works surprisingly well. Of course, we are aware of more sophisticated approaches to compute baryon magnetic moments (see e.g. [10] and references therein). However, the reasonable overall agreement in the table indicates that our derived constituent quark masses are close to the ideal ones.
SU straightforwardly to think that one can simply generalize the GTRs that work so nicely for light quarks, mesons, no such constants have been measured so far. Besides, it would be rather naive to turn to the medium-heavy, depending on which of the few experiments one picks. Moreover, in the case of the D mesons, no such constants have been measured so far. Besides, it would be rather naive to think that one can simply generalize the GTRs that work so nicely for light quarks straightforwardly to SU(4) and SU(5). A fancy way to include mass and energy-scale corrections was suggested very recently [11]. However, for the purpose of the present paper, we rather prefer to proceed as in [20]. Thus, we estimate the loosely-bound charm and bottom masses as one-half the vector $c c J / Ψ(1S)$ and $b b Υ(1S)$ masses, respectively, or

$$m_c \sim \frac{3096.92}{2} \text{MeV} \approx 1550 \text{MeV},$$

$$m_b \sim \frac{9460.30}{2} \text{MeV} \approx 4730 \text{MeV}.$$  

(21) (22)

As general confirmation that we are in the right ballpark (to within about 10%), we turn to the medium-heavy, $D$ and $B$ pseudoscalar-meson mass differences; we have the collection of values

$$D^+(dc; 1869.4) - D^0(uc; 1864.6) \approx 4.8 \text{MeV} \sim m_d - m_u,$$

$$D^+_s(bs; 1968.3) - D^+(dc; 1869.4) \approx 98.9 \text{MeV} \sim m_s - m_d,$$

$$B^0_s(bs; 5369.6) - B^0(ud; 5279.4) \approx 90.2 \text{MeV} \sim m_s - m_d,$$

$$D^+_c(bc; 6400) - B^+(bu; 5279.0) \approx 1121 \text{MeV} \sim m_c - m_u,$$

$$B^0(ud; 5279.4) - D^-(cd; 1869.4) \approx 3410 \text{MeV} \sim m_b - m_c.$$  

(23) (24) (25) (26) (27)

The medium-heavy baryons tell much the same story, since the data [2] give

$$\Xi_c^0(dsc; 2471.8) - \Xi_c^+(usc; 2466.3) = 5.5 \pm 1.8 \text{MeV} \sim m_d - m_u.$$  

(28)
\[ \Xi_c^+(usc; 2466.3) - \Lambda_c^+(udc; 2284.9) \approx 181.4 \text{ MeV} \sim m_s - m_d \quad (29) \]
\[ \Omega_c^0(ssc; 2697.5) - \Sigma_c^0(ddc; 2452.2) \approx 245.3 \text{ MeV} \sim 2(m_s - m_d) \quad (30) \]
\[ \Xi_{c0}^+(ccd; 3518.7) - \Sigma_c^0(ddc; 2452.2) \approx 1066.5 \text{ MeV} \sim m_c - m_d \quad (31) \]
\[ \Lambda_b^0(udb; 5624) - \Lambda_b^+(udc; 2284.9) \approx 3339.1 \text{ MeV} \sim m_b - m_c \quad . \quad (32) \]

We see that the constituent \( c \) and \( b \) quark masses deduced from Eqs. (26, 27, 31, 32) are indeed roughly compatible (\( \pm 5-10\% \)) with our 1\(^{-} \)-\( \chi \)-charmonium and bottomonium estimates in Eqs. (21) and (22), respectively. Unfortunately, we cannot test the idea on charm-bottom or double-bottom baryons, as they have not yet been observed. The results for \( m_s - m_d \) are less nice, as the pseudoscalar-meson estimates (24, 25) yield \( \sim 95 \) MeV, while the baryons in Eqs. (29, 30) give on average \( \approx 142 \) MeV. Nonetheless, the latter value is close to \( (486.0 - 339.5) \text{ MeV} = 146.5 \text{ MeV} \) from Eqs. (12, 18). Incidentally, not only do the \( \bar{q}q \) pseudoscalar/vector meson and \( qqq \) baryon resonances follow this same \( u, d, s, c, b \) quark-mass pattern, but so do the \( \bar{q}q \) scalars (see [9], second paper).

6. Very heavy top quark and the standard model

Since the observed \( m_t = 174.3 \pm 5.1 \) GeV is so much heavier than \( m_u, m_d, m_s, m_c, m_b \), present approaches prefer linking the heavy \( m_t \) with the heavy \( W \) and \( Z \) bosons, in the context of the electroweak standard model (EWSM [13]). The basic formulae are

\[ \frac{g_w^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}, \quad (33) \]
\[ f_w = \left[ \sqrt{2} G_F \right]^{-\frac{1}{2}} \approx 246.2 \text{ GeV}, \quad (34) \]
\[ \sin^2 \Theta_w = \left( \frac{\bar{e}}{g_w} \right)^2 \approx 0.2284 \sim \left( 1 - \frac{M_W^2}{M_Z^2} \right), \quad (35) \]

implying \( g_w = 0.65326, \) for \( G_F = 11.6637(1) \times 10^{-6} \) GeV\(^{-2} \) and observed \( M_W = 80.425(38) \) GeV, with \( \bar{e}^2/4\pi \approx 1/128.91 \) at the scale \( M_Z = 91.1876(21) \) GeV.

This is indeed a beautiful theory for the very heavy \( W \) and \( Z \) bosons, wherein the \( W \) and the \( Z \) are treated as elementary particles; the left-handed fermion fields transform as \( SU(2) \) doublets, but the right-handed fields as \( U(1) \) singlets. What makes the theory doubly impressive is the fact that one can compute radiative corrections reliably, because it is renormalizable. Although the EWSM predictions are all very accurate, they are saddled with 19 arbitrary parameters (see [2], page 160 and section VII). Furthermore, the quark masses are “current” masses and do not incorporate dynamical contributions.

Instead, we attempt to reduce these 19 parameters by treating the heavy \( W \) and \( Z \) bosons as resonances, like one does in strong interactions (SI), using e.g. vector-meson dominance (VMD) [14] concepts and KSRF [15] identities, not usually considered in the EWSM. For example, one knows that the \( \rho \) meson approximately obeys the KSRF relation

\[ m_\rho = \sqrt{2} (g_{\rho \pi \pi} g_\rho) \frac{1}{2} f_\pi , \quad (36) \]
where \( g_{\rho \pi \pi} \simeq 5.95 \) comes from the \( \rho \) decay width,

\[
\Gamma_{\rho \pi \pi} = \frac{g_{\rho \pi \pi}^2}{6\pi m_\rho^2} q^3 \approx (150.3 \pm 1.6) \text{ MeV},
\]

and \( g_\rho \simeq 4.96 \) is determined by the much smaller decay width,

\[
\Gamma_{\rho e^+e^-} = \frac{e^4 m_\rho}{12\pi g_\rho^2} \approx (7.02 \pm 0.11) \text{ keV} .
\]

(In passing, note that the chiral QLLσM predicts \([17]\) \( g_{\rho \pi \pi}/g_\rho = 6/5 \), in excellent agreement with the above data ratio.)

The weak-interaction analogue of the KSRF relation \((36)\) is obtained by the substitutions

\[
m_\rho \rightarrow M_W , \quad \sqrt{g_{\rho \pi \pi} g_\rho} \rightarrow \frac{g_w}{2} , \quad \sqrt{2} f_\pi \rightarrow f_w ,
\]

where the weak coupling simulates \( g_\rho \tau^+/2 \), and the charged \( W \) requires a \( \sqrt{2} \) in the weak (VEV) decay constant. Indeed the weak KSRF relation

\[
M_W = \frac{1}{2} g_w f_w ,
\]

corresponds precisely to the famous EWSM relation \([13]\). Other physicists have also searched for the relation between the EWSM and high-energy resonances \([19]\).

Extending this VMD scheme to the heavy \( Z \) boson, the analogue of the \( \rho_0 \rightarrow e^+e^- \) rate in \((38)\) above determines the coupling constant \( g_Z \) of the \( Z \) boson to electrons:

\[
\Gamma_{Ze^+e^-} = \frac{e^2 \bar{e}^2 M_Z}{12\pi g_Z^2} = (83.913 \pm 0.126) \text{ MeV} ,
\]

Inserting \( e^2/4\pi = 1/137.036 \) and \( \bar{e}^2/4\pi = 1/128.91 \) at the \( Z \) mass, we arrive at

\[
|g_Z| \approx 0.50761 ,
\]

Now the tree-level vector and axial-vector couplings of \( Z \) to the leptons get modified, from \( g_V^e = -1/2 + 2\sin^2\theta_w, g_A^e = -1/2 \) to

\[
g_V^e = -0.50123(26) , \quad g_A^e = -0.03783(41) ,
\]

by radiative corrections. Nevertheless, \( Z \) remains largely axial (since \([2]\) \( \sin^2\theta_w = 0.23120(15) \)), and the difference \( V-A \) coupling is

\[
g_{V-A}^e = 0.4633(34) .
\]

This is reasonably close to the EWSM value \( 2\sin^2\Theta_w = 0.462 \) and is also supported by the ratio of \((40)\) to the conventional EWSM rate for \( Z \), namely

\[
\Gamma_{Ze^+e^-} = \left( \frac{g_w}{4} \right)^2 \frac{M_Z^3}{12\pi M_W^2} = 82.936 \text{ MeV},
\]

which yields the alternative expression (compatible with the data above)

\[
\sin^2\Theta_w = 1 - (g_w g_Z/4e\bar{e})^2 = 0.23118 .
\]
More interestingly, we may estimate the very heavy top quark mass \( m_t \) via a GTR as we did for the lighter quarks. Here we have to be careful to take account of an EWSM factor of \( 2\sqrt{2} \) and the (V-A) VMD coupling \( g_Z/2 \). In this way we get

\[
m_t = 2\sqrt{2} f_w \frac{|g_Z|}{2} = \sqrt{2} (246.2 \text{ GeV}) (0.50761) \approx 176.7 \text{ GeV} ,
\]

compatible with data \([2]\) at \( 174.3 \pm 5.1 \text{ GeV} \).

Lastly, we examine the scalar Higgs-boson mass in the spirit of B. W. Lee’s null tadpole sum for the \( SU(2) \) LoM \([21]\), characterizing the true vacuum (as obtained in \([8, 9]\))

\[
m_{\sigma}^2 = 4 \hat{m}^2 + m_{\sigma}^2 .
\]

For the EW model the analogue of this relation is the vanishing expectation value of the charged Higgs components; this constraint produces

\[
m_H^2 = 4m_t^2 - 2M_W^2 - M_Z^2 \approx (316.7 \text{ GeV})^2 ,
\]

as originally found in \([22]\). A somewhat smaller Higgs mass, about 216 GeV, results from a recent renomalization-group resummation of all leading-logarithm contributions \([23]\). Of course, we are well aware that most physicists favour a much smaller Higgs mass, of the order of 100 GeV. For instance, the 2004 PDG review “Electroweak model and constraints on new physics” claims \([2\), page 122\] “The data indicate a preference for a small Higgs mass”, arriving at a central global-fit value of \( M_H = 113^{+56}_{-40} \) GeV. However, the detailed analyses in \([24]\) showed that such “predictions” should be taken with a great deal of caution, leading to the conclusion that even Higgs masses in the range 500–1000 GeV cannot be excluded on the basis of LEP data.

7. Mixing angles and conclusion

The 19 parameters of the EWSM (not extended to massive neutrinos) are:

(a) 9 fermion masses, i.e., the six \( u, d, s, c, b, t \) quark and the three \( e, \mu, \tau \) lepton masses.

(b) 3 gauge couplings, namely \( \alpha_s = g^2/4\pi \approx \pi/3 \), for \( g = 2\pi/\sqrt{3} \) \([6, 8]\), and \( g_w, g_w' \), with the derived electromagnetic coupling \( e = g_w g_w' / \sqrt{g_w^2 + g_w'^2} \).

(c) 3 vacuum or mass scales, i.e., \( v = \langle 0 | \phi_H | 0 \rangle = f_w, m_H \) (our (49)), and \( \Theta_{QCD} = 0 \) (see the \( U_A(1) \) problem \([25]\)).

(d) 3 quark CKM mixing angles and 1 CPV phase angle \( \delta \). The data (see \([2\), page 130\) give:

\[
V_{ud} \sim V_{es} \sim \cos \Theta_c \sim 0.974 \quad \Rightarrow \quad \Theta_c \sim 13.1^\circ ,
\]

\[
V_{us} \sim V_{cd} \sim \sin \Theta_c \sim 0.224 \quad \Rightarrow \quad \Theta_c \sim 12.9^\circ ,
\]

\[
V_{cb} \sim V_{ts} \sim \sin \Theta_2 \sim 0.041 \quad \Rightarrow \quad \Theta_2 \sim 2.35^\circ ,
\]

\[
V_{tb} \sim \cos \Theta_2 \sim 0.9991 \quad \Rightarrow \quad \Theta_2 \sim 2.43^\circ ,
\]

\[
V_{td} \sim V_{ub} \sim \sin \Theta_3 \sim 0.0066 \quad \Rightarrow \quad \Theta_3 \sim 0.38^\circ .
\]
Constituent Quark Masses and the Electroweak Standard Model

so we may conclude that $\Theta_c \sim 13^\circ$, $\Theta_2 \sim 2.39^\circ$, $\Theta_3 \sim 0.38^\circ$. Lastly, the angle $\delta$, has been measured as [2] $\delta = (3.27 \pm 0.12) \times 10^{-3}$. Note, too, that $\Theta_c/\Theta_2 : \Theta_2/\Theta_3 \sim 6$.

Given the consistent pattern of the 6 quark masses in Secs. 3–6, we try to make use of them in the manner of Fritzsch [26], but adopting instead constituent quark masses. Using our values for $m_q$, this approach predicts [27] $\phi_{sd} = 22.9^\circ$, and approximately

$$\sin 2\phi_{cu} \approx \sqrt{\frac{m_s - m_d}{m_c - m_u}} \approx 0.347 \quad \text{or} \quad \phi_{cu} \approx 10.2^\circ, \quad (55)$$

$$\Theta_c = \phi_{sd} - \phi_{cu} \approx 22.9^\circ - 10.2^\circ = 12.7^\circ, \quad (56)$$

$$\sin 2\Theta_2 \approx \sqrt{\frac{m_c - m_u}{m_t - m_c}} \approx 0.083 \quad \text{or} \quad \Theta_2 \approx 2.4^\circ, \quad (57)$$

$$\Theta_3 \sim \Theta_2/6 \sim 0.4^\circ, \quad (58)$$

Thus we see that the predicted CKM angles $\Theta_c$ and $\Theta_2$ are reasonably near the observed ones in (50–53). Furthermore, the CPV phase angle $\delta$ can be estimated as [28]

$$\delta = \frac{\alpha}{\pi} \ln \left(1 + \frac{\Lambda^2}{m_t^2}\right) = 3.34 \times 10^{-3}, \quad (59)$$

taking for $m_t$ our predicted value of about 177 GeV (see [17]), and for the UV cutoff $\Lambda$ the Higgs mass $m_H \approx 317$ GeV, as derived in [19]. This value for $\delta$ is compatible with data [2] at $(3.27 \pm 0.12) \times 10^{-3}$.

In conclusion, in the present paper we have employed constituent quark masses, instead of the usual current masses, to reduce part of the arbitrariness of the 19 parameters in the SM. Our nonstrange constituent quark mass follows from the QLLSM $gf_\pi = \hat{m}$. A further justification for using constituent quark masses is S. Weinberg’s mended-chiral-symmetry paper [29], which predicted the sigma width as 9/2 times the rho width, resulting in the value $4.5 \times (150.3 \pm 1.6)$ MeV = $(676.4 \pm 7.2)$ MeV, astonishingly near the QLLSM prediction for the sigma mass in terms of constituent quark masses: $m_\sigma = 2 \times 337.5$ MeV = 675 MeV. This (near) equality of the sigma mass and width is crucial to obtain the correct amplitude magnitude for the $\Delta I = 1/2$ weak $K_S \rightarrow 2\pi$ decay, via a sigma-pole graph (see e.g. Ref. [30]). Then, in his immediately following paper [31], Weinberg stated in the abstract: “An explanation is offered why quarks in the constituent quark model should be treated as particles with axial coupling $g_A = 1$ and no anomalous magnetic moment.” Well, note that our constituent-quark GTRs in Eqs. [15,16] do indeed have $g_A = 1$ at this quark level. Summarizing, constituent quark masses always make chiral contact with data, while current quark masses are more problematic.

Wrapping up, the analysis of this paper allows us to fix 13 of the 19 parameters from the experimental data [2] and also provides a link with the present EWSM. Moreover, employing once again $(V - A)$ currents, we can derive [32,33] two additional relations among $G_F$, $m_\mu$, and $m_e$, thus further reducing the arbitrariness of the remaining 6 parameters of the Standard Model.
Constituent Quark Masses and the Electroweak Standard Model

Acknowledgments

One of us (MDS) is indebted to V. Elias for prior discussions and collaboration. We also thank F. Kleefeld for very useful comments. This work was partly supported by the Fundação para a Ciência e a Tecnologia of the Ministério da Ciência, Tecnologia e Ensino Superior of Portugal, under contracts POCI/FP/63437/2005 and POCI/FP/63907/2005.

References

[1] Delbourgo R, Liu D and Scadron M D 1999 Phys. Rev. D 59 113006 (Preprint hep-ph/9808253)
[2] Eidelman S et al. (Particle Data Group) 2004 Phys. Lett. B 592 1
   Also see EWSM review: Erler J and Langacker P Sect. 10 of 2004 Phys. Lett. B 592 1 (Preprint hep-ph/0407097)
[3] Coleman S R and Glashow S L 1964 Phys. Rev. 134 B671
[4] The Coleman-Glashow scale in Eq. (5) has nucleon matrix element
   \[ \langle N | H_{\text{tad}}^3 | N \rangle = (-2.5 \text{ MeV}) 2m_N \approx -4700 \text{ MeV}^2, \] (60)
   near the group-theory predictions
   \[ \langle H_{\text{tad}}^3 \rangle_{\Delta K} = \Delta m_K^2 - \Delta m_{\pi}^2 \approx -5220 \text{ MeV}^2, \] (61)
   and the quark-loop \( I = 1 \) tadpole predictions (for \( m_d - m_u \approx 4 \text{ MeV}, m \approx 337 \text{ MeV}, m_{\omega} \approx 984.7 \text{ MeV}, g_\rho \approx 5.03 \))
   \[ \langle n^0 | H_{\text{tad}}^3 | \eta_{NS} \rangle = - (m_d - m_u) \left[ 2m + \frac{16m^3}{m_{\pi}^2} \right] \approx -5230 \text{ MeV}^2, \] (63)
   \[ \langle \omega | H_{\text{tad}}^3 | \rho \rangle = - (m_d - m_u) \left[ \frac{g_\rho^2 \langle \eta \rangle m}{4\pi^2} + \frac{16m^3}{m_{\omega}^2} \right] \approx -5127 \text{ MeV}^2, \] (64)
   where \( NS \) stands for nonstrange. Note that Eqs. (60–64) have roughly the same scale.
   Besides Ref. [4], see Coon S A and Scadron M D 2000 Rev. Mex. Fis. 46S1 23
[5] Beg M A B, Lee B W and Pais A 1964 Phys. Rev. Lett. 13 514
[6] Elias V and Scadron M D 1984 Phys. Rev. Lett. 53 1129
[7] Babukhadia L R, Elias V and Scadron M D 1997 J. Phys. G 23 1065 (Preprint hep-ph/9708431)
[8] Salam A 1962 Nuovo Cim. 25 224
[9] Weinberg S 1963 Phys. Rev. 130 776
[10] Scadron M D 1998 Phys. Rev. D 57 5307 (Preprint hep-ph/9712425)
[11] Delbourgo R and Scadron M D 1995 Mod. Phys. Lett. A 10 251 (Preprint hep-ph/9910242)
[12] Scadron M D, Kleefeld F, Rupp G and van Beveren E 2003 Nucl. Phys. A 724 391 (Preprint hep-ph/0211275)
[13] Scadron M D, Rupp G, Kleefeld F and van Beveren E 2004 Phys. Rev. D 69 014010 (Erratum 2004 Phys. Rev. D 69 059901) (Preprint hep-ph/0309109)
[14] Isgur N and Karl G 1980 Phys. Rev. D 21 3175
[15] Kleefeld F 2005 Czech. J. Phys. 55 1123 (Preprint hep-th/0506140)
[16] Mattson M et al. (SELEX Collaboration) 2002 Phys. Rev. Lett. 89 112001 (Preprint hep-ex/0208014)
[17] Ocherashvili A et al. (SELEX Collaboration) Preprint hep-ex/0406033
[18] Weinberg S 1967 Phys. Rev. Lett. 19 1264
[19] Salam A 1968 Elementary Particle Theory ed N Svartholm (Stockholm: Almquist and Wiksell) p 367
[20] Glashow S L, Iliopoulos J and Maiani L 1970 Phys. Rev. D 2 1285
Constituent Quark Masses and the Electroweak Standard Model

[14] Sakurai J J 1960 Ann. Phys., NY 11 1 (1960) Sakurai J J 1969 Currents and Mesons (Chicago: University of Chicago Press) Chap 3
[15] Kawarabayashi K and Suzuki M 1966 Phys. Rev. Lett. 16 255
Riazuddin and Fayyazuddin 1966 Phys. Rev. 147 1071
[16] Sakurai J J 1966 Phys. Rev. Lett. 17 552
[17] Bramon A, Riazuddin and Scadron M D 1998 J. Phys. G 24 1 (Preprint hep-ph/9709274)
[18] Chan L H 1977 Phys. Rev. Lett. 39 1124
Novozhilov V Y 1989 Phys. Lett. B 228 240
[19] Hung P Q and Sakurai J J 1978 Nucl. Phys. B 143 81 (Erratum 1979 Nucl. Phys. B 148 538)
Bjorken J D 1979 Phys. Rev. D 19 335
[20] Bramon A and Scadron M D 1992 Europhys. Lett. 19 663
[21] Lee B W 1972 Chiral Dynamics (New York: Gordon and Breach) p 12
[22] Veltman M J G 1981 Acta Phys. Polon. B 12 437
Scadron M D 1993 Phys. Atom. Nucl. 56 1595 (1993 Yad. Fiz. 56N11 245)
Nambu Y 1989 Proc. Conf. on Dynamical Symmetry Breaking (Nagoya) pp 1–10
Lopez Castro G and Pestieau J 1995 Mod. Phys. Lett. A 10 115 (Preprint hep-ph/9504350)
Fang Z Y, Lopez Castro G, Lucio J L and Pestieau J 1997 Mod. Phys. Lett. A 12 1531 (Preprint hep-ph/9612430)
[23] Elias V, Mann R B, McKeon D G C and Steele T G 2003 Phys. Rev. Lett. 91 251601 (Preprint hep-ph/0304153)
[24] Consoli M and Hioki Z 1995 Mod. Phys. Lett. A 10 2245 (Preprint hep-ph/9505249)
Consoli M and Hioki Z 1995 Mod. Phys. Lett. A 10 845 (Preprint hep-ph/9503288)
Consoli M and Ferroni F 1995 Phys. Lett. B 349 375 (Preprint hep-ph/9501371)
[25] Kekez K, Klabucar D and Scadron M D 2000 J. Phys. G 26 1335 (Preprint hep-ph/0003234)
Also see Kekez K, Klabucar D and Scadron M D 2005 Circumventing the axial anomalies and the strong CP problem Preprint hep-ph/0503141. This paper finds Θ_{QCD} = 0, which solves the strong CP problem.
[26] Fritzsch H 1977 Phys. Lett. B 70 436
Fritzsch H 1978 Phys. Lett. B 73 317
[27] Paver N and Scadron M D 1985 Nuovo Cimento A 88 447
Paver N and Scadron M D 1984 Nuovo Cimento A 79 57
In the spirit of these papers, we can relate the mixing angle \( \phi_{sd} \), via a \( \kappa^0 \) scalar-meson (\( \bar{d}s \)) tadpole transition, to the \( \kappa \) decay constant \( f_\kappa \) as
\[
\sin 2\phi_{sd} = \frac{\sqrt{2} |f_\kappa| g}{m_s - m_d}.
\]
Note that the GTR for the scalar \( \kappa \) meson reads \( |f_\kappa| g = (m_s - \hat{m})/2 \), predicting \( |f_\kappa| = 20.468 \) MeV for \( g = 2\pi/\sqrt{3} \). Or in a more model-independent way,
\[
|f_\kappa| = \frac{m_s - \hat{m}}{m_s + \hat{m}} f_K = f_K - f_\pi = 0.22 f_\pi = 20.46 \text{ MeV},
\]
where we have used (17). Substitution into (15) then yields, with \( g = 2\pi/\sqrt{3} \) and \( m_s - m_d = 146.5 \) MeV (from (12,18)),
\[
\sin 2\phi_{sd} = 0.7165 \Rightarrow \phi_{sd} = 22.9^\circ .
\]
[28] Choudhury S R and Scadron M D 1996 Phys. Rev. D 53 2421
Scadron M D, Rupp G and van Beveren E 2004 Mod. Phys. Lett. A 19 2267 (Preprint hep-ph/0310059)
[29] Weinberg S 1990 Phys. Rev. Lett. 65 1177
[30] Van Beveren E, Kleefeld F, Rupp G and Scadron M D 2002 Mod. Phys. Lett. A 17 1673 (Preprint hep-ph/0204139)
[31] Weinberg S 1990 Phys. Rev. Lett. 65 1181
[32] Sudarshan E C G and Marshak R E 1958 Phys. Rev. 109 1860
predict, via $V - A$ currents,
\[ \Gamma^{V-A}(\mu \to e\nu\bar{\nu}) \approx \frac{G_F^2m_\mu^5}{192\pi^3}, \] (68)
less a 0.44% radiative-rate correction.

Also see Nagels M M et al. 1976 *Nucl. Phys. B* 109 1

Present data says \[ \Gamma^{\text{exp}}(\mu \to e\nu\bar{\nu}) = \frac{\hbar}{\tau_\mu} = 2.99592(6) \times 10^{-16} \text{ MeV}, \] (69)

for $\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$. This implies \[ G_F = 11.6637(1) \times 10^{-6} \text{ GeV}^{-2}. \]

Note that, ignoring the very small electron mass, the much larger $W$ mass predicts an EWSM $e\bar{\nu}$ weak decay rate \[ \Gamma(W \to e\bar{\nu}) = \frac{G_F^2}{6\pi\sqrt{2}} M_W^3 = 227.6 \text{ MeV}, \] (70)

for $G_F = 11.6637(1) \times 10^{-6} \text{ GeV}^{-2}$ and $M_W = 80.425 \text{ GeV}$, which is very close to data \[ \text{ at } (227.7 \pm 5.6) \text{ MeV}. \]

Another typical $V - A$ process is $\pi^\pm \to \ell^\pm \nu$, which predicts
\[ \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)} = \frac{m_\mu^2(m_\ell^2 - m_\mu^2)^2}{m_\ell^2(m_\ell^2 - m_\mu^2)^2} = 1.23 \times 10^{-4}, \] (71)

agreeing remarkably well with experiment \[ \text{viz. } (1.230 \pm 0.004) \times 10^{-4}. \] Note that this result is totally at variance with naive phase-space arguments, thus showing the predictive power of $V - A$ at low energies.

See e.g. Scadron M D 1991 *Advanced Quantum Theory* 2nd edition (Berlin Heidelberg New York: Springer-Verlag) ISBN 3-540-53681-7 p 270

[33] Feynman R P and Gell-Mann M 1958 *Phys. Rev.* 109 193