Update of the electromagnetic effective coupling constant $\alpha(M_Z^2)$

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Abstract

We recalculate the hadronic contribution to the effective electromagnetic coupling constant $\alpha(M_Z^2)$ using quasi-analytical approximations for the cross-section of the $e^+e^-\rightarrow\text{hadrons}$ proposed earlier by one of the authors (N.V.K., Mod.Phys.Lett.A9(1994)2825). We find that $\alpha^{-1}(M_Z^2) = 128.98 \pm 0.06$. 
For LEP1 observables at the Z-pole the largest effect from pure QED corrections is the change in the effective electromagnetic constant when going from $q^2 = 0$, where the fine structure constant $\alpha^{-1} = 137.036$ is measured to $q^2 = M_Z^2$ \[\Box\]. The change from $\alpha \equiv \alpha(q^2 = 0)$ to $\alpha(M_Z^2)$ is related to the photon vacuum polarisation function $\Pi(q^2)$ via the relation

$$\alpha(q^2) = \frac{1}{1 - \Pi(q^2)},$$

where in the leading log approximation

$$\Pi(q^2) = \Pi_{\text{lept}}(q^2) + \Pi_{\text{had}}(q^2),$$

and the hadronic contribution $\Pi_{\text{had}}(q^2)$ can be directly determined by the total cross-section of $e^+e^-$-annihilation into hadrons \[\Box\]

$$\sigma_h(s) = \sigma_{\text{tot}}(e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}) = \frac{4\pi\alpha^2}{3s}R(s),$$

So the main problem in the calculation of $\alpha(M_Z^2)$ is the evaluation of the integral (4). There are several papers \[2\] - \[9\] devoted to the calculation of the integral (4).

In this paper we recalculate the hadronic contribution to $\alpha(M_Z^2)$ using quasianalytical approximations for the cross-section of $e^+e^- \rightarrow \text{hadrons}$ proposed earlier by one of the authors \[4\]. The motivation for such recalculation is the appearance of new data on $e^+e^-$-annihilation into hadrons in low energy region and new value of the effective strong coupling constant $\alpha_s(M_Z^2)$. We find that

$$\alpha^{-1}(M_Z^2) = 128.98 \pm 0.06 \quad \text{(6)}$$

In ref.[4] the integral (4) has been calculated using quasianalytical approximation for $R(s)$. Namely, low energy contribution for $R(s)$ has been taken from experimental data and the theoretical ansatz has been used for high energy contribution.
for \( R(s) \). Two slightly different methods have been used. The first method numerically accounts for all resonances \((\rho, \omega, \phi, J/\psi, \Upsilon, \ldots)\) and low energy continuum region \( \sqrt{s} \leq 2.3 \) GeV. The continuum contribution to \( R(s) \) is determined by step-like approximation. In the approximation when strong coupling constant \( \alpha_s = 0 \) our ansatz has the form [4]

\[
R_c(s) = 2\theta(s - s_1) + \frac{4}{3}\theta(s - s_2) + \frac{1}{3}\theta(s - s_3),
\]

where we took \( s_1 = (2.3\text{GeV})^2 \), \( s_2 = (4.4\text{GeV})^2 \) and \( s_3 = (12\text{GeV})^2 \). The parameters \( s_2 \) and \( s_3 \) correspond to the charm and beauty thresholds correspondingly. So in the first method we use the relation

\[
\Pi_{had}(s) = \Pi_{2m_\pi-2.3\text{GeV}}(s) + \Pi_{r,J/\psi,\Upsilon}(s) + \Pi_c(s)
\]

Here \( \Pi_{2m_\pi-2.3\text{GeV}}(s) \) is the contribution of the low energy region \( \sqrt{s} \leq 2.3\text{GeV} \), \( \Pi_{r,J/\psi,\Upsilon} \) is the contribution of \( J/\psi \), \( \Upsilon \)-resonances and their radial excitations and \( \Pi_c(s) \) is the contribution of the continuum which is described by the formula (7) in zero approximation. In our paper we use the value [5]

\[
\Pi_{2m_\pi-2.3\text{GeV}}(M_Z^2) = (6.06 \pm 0.25) \times 10^{-3}
\]

for the contribution of the low energy region. For the \( J/\psi \), \( \Upsilon \)-resonances and their radial excitations we use the formula [3]

\[
\Pi_{res}(M_Z^2) = \sum_i \frac{3\Gamma_{ee,i}}{M_i} \frac{\alpha}{(\alpha(M_i^2))^2},
\]

where \( M_i \) and \( \Gamma_{ee,i} \) are the mass and leptonic width of the i-th resonance, respectively, and the effective QED coupling constant at the resonance scale is used. An account of QCD and quark mass corrections lead to the appearance of the factors in formula (7)

\[
(1 + \frac{2m_q^2}{s}) \sqrt{(1 - \frac{4m_q^2}{s})}[1 + \frac{\alpha_s}{\pi}f_1(\frac{m_q^2}{s}) + (\frac{\alpha_s}{\pi})^2f_2(\frac{m_q^2}{s}) + (\frac{\alpha_s}{\pi})^3f_3(\frac{m_q^2}{s}) + \ldots]
\]

[4]

[5]
The coefficients \( f_2(0) \) and \( f_3(0) \) have been calculated in refs. [10] and the function \( f_1(\frac{m^2_q}{s}) \) is approximately determined by the expression

\[
f_1(x) = \frac{4\pi}{3} \left[ \frac{\pi}{2} v(x) - \frac{3 + v(x)}{4} \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) \right],
\]

\[
v(x) = \sqrt{(1 - 4x)}
\]

(12)

(13)

The second method of the calculations consists of taking into account low energy region \( \sqrt{s} \leq 2.3 \) GeV in the calculation of the integral (4) and an account of the high energy contribution to \( \Pi_{\text{had}}(M^2_Z) \) is performed by taking into account nonzero c- and b-quark masses. So in the second method our ansatz has the form

\[
\Pi_{\text{had}}(s) = \Pi_{2m_c-2.3\text{GeV}}(s) + \Pi_c(s),
\]

(14)

where in the approximation when strong coupling constant \( \alpha_s = 0 \) our ansatz for \( R_{\text{cont}}(s) \) is

\[
R_{\text{cont}}(s) = 2\theta(s-s_1) + \frac{4}{3}\theta(s-4m^2_c)(1+2\frac{m^2_c}{s})\sqrt{(1-\frac{4m^2_c}{s})} + \frac{1}{3}\theta(s-4m^2_b)(1+2\frac{m^2_b}{s})\sqrt{(1-\frac{4m^2_b}{s})}
\]

(15)

Nonzero c- and b-quark masses in formula (15) effectively take into account the contribution of \( J/\psi \)- and \( \Upsilon \)-resonances and their radial excitations. Such approach works rather well in the method of QCD sum rules.

In our numerical calculations we used the value of the effective strong coupling constant equal to [11]

\[
\alpha_s(M^2_Z) = 0.119 \pm 0.006
\]

(16)

and the values of the c- and the b- pole quark masses equal to [12]

\[
m^\text{pole}_c = (1 + \frac{4}{3} \frac{\alpha_s(m_c(m_c))}{\pi}) + ...m_c(m_c) = 1.4 \cdot (1 \pm 0.1)\text{GeV},
\]

(17)

\[
m^\text{pole}_b = (1 + \frac{4}{3} \frac{\alpha_s(m_b(m_b))}{\pi}) + ...m_b(m_b) = 4.5 \cdot (1 \pm 0.1)\text{GeV}
\]

(18)
In the method 1 the inverse effective electromagnetic coupling constant \( \alpha^{-1}(M_Z^2) \) is represented in the form

\[
\alpha^{-1}(M_Z^2) = \alpha^{-1} - \Delta \alpha^{-1}(l) - \Delta \alpha^{-1}(< \sqrt{s} < 2.3 GeV) - \Delta \alpha^{-1}(J/\psi, \Upsilon) - \Delta \alpha^{-1}(c) - \Delta \alpha^{-1}(t)
\]  

(19)

Here \( \Delta \alpha^{-1}(l) \) is the leptons contribution, \( \Delta \alpha^{-1}(< \sqrt{s} < 2.3 GeV) \) is the low energy contribution, \( \Delta \alpha^{-1}(J/\psi, \Upsilon) \) is the contribution of \( J/\psi, \Upsilon \)-resonances and their radial excitations, \( \Delta \alpha^{-1}(c) \) is the continuum contribution and \( \Delta \alpha^{-1}(t) \) is the top-quark contribution. In the estimation of \( \Delta \alpha^{-1}(l) \) we have used two loop approximation.

To estimate the uncertainties we assumed the uncertainties related with the choice of \( s_2 \) and \( s_3 \) to be equal 20 pecent. In our calculations we used the values \( f_2\left(\frac{m_c^2}{s}\right) \) and \( f_3\left(\frac{m_b^2}{s}\right) \) at \( m_q^2 = 0 \). However the dependence of our results in method 1 on the value of \( c \)- and \( b \)-quark masses is rather small and also three and four loop contributions are also small numerically. As it has been mentioned before in the estimation of low energy hadron contribution into \( \alpha^{-1}(M_Z^2) \) we used the results of ref. [5]. Numerically we have found [1]

\[
\Delta \alpha^{-1}(l) = 4.313,
\]

(20)

\[
\Delta \alpha^{-1}(< \sqrt{s} < 2.3 GeV) = 0.830 \pm 0.034,
\]

(21)

\[
\Delta \alpha^{-1}(J/\psi, \Upsilon) = 0.160 \pm 0.008(0.016),
\]

(22)

\[
\Delta \alpha^{-1}(c) = 2.757 \pm 0.019(\alpha_s) \pm 0.024(h.c.) \pm 0.003(m_c, m_b) \pm 0.032(s_2) \pm 0.008(s_3),
\]

(23)

\[
\Delta \alpha^{-1}(m_t = 174 GeV) = -\frac{4}{3\pi} \cdot \frac{1}{15} \frac{M_Z^2}{m_t^2} \approx -0.008
\]

(24)

In formula (23) the uncertainties \( 0.019(\alpha_s), 0.003(m_c, m_b), 0.032(s_2), 0.008(s_3) \) are determined by the uncertainties in the determination of \( \alpha_s(M_Z^2) \), \( c \)- and \( b \)-quark
masses, parameters of the spectrum $s_2$ and $s_3$. The uncertainty $0.024(h.c.)$ is our estimate of the higher order contributions. We assume that such uncertainty is equal to one half of the difference between calculated value which takes into account QCD corrections up to 3 loops and one loop contribution or numerically it coincides with 3 loop contribution. Note that we assumed 20 percent uncertainty in the determination of $s_2$ and $s_3$ that is rather conservative estimate. In formula (22) in the estimation of the overall error we assumed the errors in the determination of the contribution of different resonances are statistically independent and obtained the error equal to 0.008. The number in brackets corresponds to the case when we simply sum up the errors from different resonances. Assuming that all errors are statistically independent we find

$$\alpha^{-1}(M_Z^2) = 128.98 \pm 0.06(0.13)$$

(25)

In formula (25) the number in brackets corresponds to the case when we simply sum up the errors.

In method 2 the inverse effective electromagnetic coupling constant $\alpha^{-1}(M_Z^2)$ can be represented in the form

$$\alpha^{-1}(M_Z^2) = \alpha^{-1} - \Delta \alpha^{-1}(l) - \Delta \alpha^{-1}(\sqrt{s} < 2.3 GeV) - \Delta \alpha^{-1}(c) - \Delta \alpha^{-1}(t)$$

(26)

Here $\Delta \alpha^{-1}(c)$ is the contribution of heavy quark continuum and heavy quark resonances. Numerically we have found

$$\Delta \alpha^{-1}(c) = 2.950 \pm 0.033(\alpha_s) \pm 0.038(h.c.) \pm 0.030(m_c) \pm 0.009(m_b)$$

(27)

Here uncertainties $0.033(\alpha_s), 0.038(h.c.), 0.0030(m_c), 0.008(m_b)$ are the uncertainties determined by the uncertainties of $\alpha_s(M_Z^2)$, higher order corrections, c-quark mass an b-quark mass. Assuming that all errors are statistically independent we find that

$$\alpha^{-1}(M_Z^2) = 129.95 \pm 0.07(0.14)$$

(28)
in the method 2. The number in brackets in the formula (28) corresponds to the simple summation of the errors. So we have found that both methods 1 and 2 lead to similar values for $\alpha^{-1}(M_Z^2)$ with the similar errors. However we believe that method 1 is more reliable since the errors related with the $\alpha_s(M_Z^2)$ uncertainty and the uncertainty of higher order corrections are smaller in method 1. Besides, in the estimation of the errors in method 1 we assumed very conservative estimates in the uncertainties related with the choice of $s_2$ and $s_3$ (20 percent). For instance, if we assume 10 percent uncertainty in the choice of $s_2$ and $s_3$ our error in formula (28) will be 0.05(0.10). Therefore we quote in the abstract our estimate of $\alpha^{-1}(M_Z^2)$ obtained using the method 1. The value obtained in our paper is very similar to the value $\alpha^{-1}(M_Z^2) = 128.97\pm0.06(exp.)\pm0.07(theor.)$ obtained in ref.[4]. The decrease of the errors is related mainly with the better calculation of low energy contribution $\Delta\alpha^{-1}(M_Z^2)(\sqrt{s} < 2.3GeV)$ and the better determination of the $\alpha_s(M_Z^2)$. As it has been mentioned before there are several recent calculations of $\alpha^{-1}(M_Z^2)$. Table 1 shows a comparison of some recent estimates of $\alpha^{-1}(M_Z^2)$.

To conclude, in this paper we have recalculated the value of $\alpha^{-1}(M_Z^2)$ using two methods of ref.[4] and new experimental data. We have found that both methods give similar results, however as it has been mentioned before we believe that the first method is more reliable.

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Table 1. Some recent values of $\alpha^{-1}(M_Z^2)$

| $\alpha^{-1}(M_Z^2)$          | ref.     |
|--------------------------------|----------|
| 128.87 ± 0.12                  | [3]      |
| 128.97 ± 0.06(exp.) ± 0.07(thor.) | [4]      |
| 0 128.99 ± 0.06                 | [5]      |
| 128.89 ± 0.06                  | [6]      |
| 128.96 ± 0.06                  | [7]      |
| 128.896 ± 0.090                | [8]      |
| 128.98 ± 0.06                  | this paper |
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