Random systems and replica field theory

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I. INTRODUCTION

There are two aspects to the statistical physics of systems with quenched disorder. One is
the static aspect, namely to understand the properties of the Boltzmann Gibbs distribution,
the other is the dynamical problem. In these lectures I would like to concentrate on some
recent developments about the statics. The interested reader can find some brief description
of the recent works on dynamics in [1]. A system with quenched disorder contains two
types of variables, $q$ and $t$, which equilibrate on very different time scales. The thermalized
variables $t$ are assumed to reach thermal equilibrium (this is an assumption of the static
problem, it is in general not true for real materials, and then the dynamical approach is
needed at low temperatures). The quenched variables $q$ are given a priori and do not
thermalize. In the problems we shall study there are many quenched variables, which means
a number which diverges in the thermodynamic limit where the number of quenched variables
diverges. One sample of the disordered system corresponds to one set of values of $q$. We
would need an infinite amount of information to just describe one sample. So in practice
what we know is the probability distribution, $P[q]$, of $q$.

In practice there are many systems in this category which have been studied in recent
years. A canonical example is the spin glass problem [2,3]. The techniques and ideas from
the spin glass mean field theory have been applied to many other systems, like directed
polymers (see for instance [4,7]). One could also mention the Random Field Ising Model
(RFIM) [8,9], the protein folding problem [10,11], and several problems outside of the usual
physics ones, i.e. where the ‘energy’ function is not a real physical energy, like neural
networks (see for instance [12]), or optimization problems (see for instance [3]).

Thermal equilibrium means that the probability of a given configuration of thermalized
variables is given in terms of the energy $H[q,t]$ by the Boltzmann weight:

$$P_q[t] = \frac{1}{Z[q]} \sum_{[t]} e^{-\beta H[q,t]}$$

(1)
where \( Z[q] \) is the partition function. So there is one such probability distribution for each sample. We face a difficult problem which is to characterize the set of the \( P_q[t] \). Fortunately it turns out that some quantities are self averaging. This means that they become sample independent (i.e. independent of \( q \)) in the thermodynamic limit, a typical central limit theorem. This is the case for thermodynamic quantities, and it is easy to derive this result for finite dimensional systems with finite range and bounded interactions. The properties which are not self averaging may also be interesting (for instance the nature of the ground state of the travelling salesman problem, or the folding of a protein), but this is an algorithmic problem. Here we shall keep to analytic studies of self averaging quantities and properties.

One standard approach of statistical physics uses mean field methods to work out the phase diagram of the system. This is efficient away from the second order phase transitions. Around such transitions the renormalisation group can be used. It has been gradually understood that in many disordered systems this very general and powerful approach fails. The fundamental reason for this failure is that the free energy functional possesses many secondary minima (metastable states) which can be quite different from the minimum, and are not taken into account in the standard approach. A striking example is provided by the RFIM. It has been shown that, order by order in perturbation theory the critical exponents of a \( d \) dimensional Ising system with a quenched random field are equal to those of the pure system in dimension \( d - 2 \) \[13\]. This is one of the few results in field theory which is known to hold to all orders in perturbation, and a beautiful derivation can be obtained through supersymmetry \[14\]. However this result differs from the simple domain wall argument of Imry and Ma \[15\], and it would predict a lower critical dimension equal to 1, while there are by now exact results which show that this lower critical dimension is equal to two \[16\] \[17\]. It has been realized long ago that the failure of the perturbative approach is related to the existence of many metastable states in this problem \[18\].

What one needs is a systematic and non perturbative method which is able to handle the field theory of disordered systems with many metastable states. Such a method can be obtained with the use of replicas, to handle randomness, together with a gaussian variational method. It turns out that the possibility of breaking replica symmetry allows to handle the metastable states. Such an approach was used on the problem of heteropolymers or proteins \[10,11\], but this is a complicated problem which requires additional assumptions \[19\]. It has been developed as a systematic field theoretic method to handle manifolds in random media in \[24,25\], and used since on several problems. Before turning to this case, let us just comment on the general meaning of replica symmetry breaking (rsb). The replica method allows to compute extensive thermodynamic quantities like the free energy. Using the property of self averageness, we need to compute the average of the logarithm of the partition function, which is written as:
\begin{equation}
\ln(Z) \equiv \sum_{[q]} \mathcal{P}(\Pi) \ln(Z(\Pi)) = \lim_{\rightarrow' \infty} \frac{Z(\Pi)}{n} = \lim_{\rightarrow' \infty} \frac{Z(\Pi)}{n}
\end{equation}

The \( n \)'th power of the partition function can in turn be written as the partition function for \( n \) replicas of the original sample (with the same disorder). Disorder averaging leads to a pure problem (no disorder), with an effective attraction between the replicas:

\begin{align}
\overline{Z(q)^n} = \sum_{[q]} \mathcal{P}(\Pi) Z(\Pi) = \\
= \sum_{[q]} \mathcal{P}(\Pi) \sum_{\cup_\infty,\ldots,\cup_{\downarrow}} \exp -\beta \left( \mathcal{H}[\Pi, \cup_\infty] + \ldots + \mathcal{H}[\Pi, \cup_{\downarrow}] \right) \equiv \sum_{\cup_\infty,\ldots,\cup_{\downarrow}} \exp -\beta \left( \mathcal{H}_{t_{\{\{\cup_\infty, \ldots, \cup_{\downarrow}\}\}}{}} \right)
\end{align}

The replica Hamiltonian \( H_{\text{eff}}[t_1, \ldots, t_n] \) is obviously invariant under permutations of the \( n \) replicas. The problem is to know whether this symmetry is preserved or spontaneously broken in the thermodynamic limit. As first found in spin glass theory (for a review see [3]), such a spontaneous symmetry breaking can occur, and is related to the appearance of a spin glass phase. Roughly speaking, if the system can have generically many metastable states, each replica can condense in one or the other such state, leading to this \( \text{rsb} \) phenomenon. The problem is in the term ‘generically’: In order for \( \overline{Z(q)^n} \) to be typical of the generic behaviour, we need that \( n \) go to zero, and therefore this \( \text{rsb} \) requires understanding the properties of the permutation group when \( n \rightarrow 0 \). This was achieved by Parisi in 1979 and is reviewed in [3]. A more rigorous definition of \( \text{rsb} \) can be found using two real replicas (i.e. \( n = 2 \)), coupled through an extensive energy term of strength \( \epsilon \) [20]. We consider two replicas with the same disorder governed by the Hamiltonian:

\begin{equation}
H_\epsilon[q,t_1,t_2] = H[q,t_1] + H[q,t_2] - \epsilon \delta[t_1 - t_2]
\end{equation}

(Here the last term is written in a symbolic way. It will in fact depend on the problem at hand. What I mean is an extensive term which is the integral of a local interaction between the two replicas, and the interaction is attractive if \( \epsilon > 0 \) and repulsive otherwise). Calling \( F(\epsilon) \) the quenched average of the free energy for this two replica system, the onset of \( \text{rsb} \) is signaled by a non analyticity of \( F(\epsilon) \) at \( \epsilon = 0 \). The idea is the following: the difficulty in random systems is that we do not know what is the conjugate field which selects one given state (such a knowledge depends on the sample and requires an infinite amount of information). In the above procedure each replica plays the role of the conjugate field for the other one. Practical implementations of this idea can be found in [20] for spin glasses and in [7] for directed polymers.
II. MANIFOLDS IN RANDOM MEDIA

Consider a $D$ dimensional manifold described in the solid on solid approximation by a $N$ component vector field $\omega(x)$ where $x$ is the $D$ dimensional vector of internal coordinates. The Hamiltonian describing the system is the sum of a rigidity term and an external potential:

$$h[\omega] = \frac{1}{2} \int dx \sum_{\mu=1}^{D} \left( \frac{\partial \omega}{\partial x_{\mu}} \right)^2 + \int dx \ V(x, \omega(x)).$$

(5)

The external potential is random and gaussian. Its correlations are given by [23]:

$$V(x, \omega) V(x', \omega') = -\delta^{(D)}(x-x') \ N \ f \left( \frac{[\omega - \omega']^2}{N} \right).$$

(6)

In most of the cases the relevant part of the correlation function of the potential is its asymptotic behaviour for large transverse distances, which we suppose to be described by a power law:

$$f \left( \frac{\omega^2}{N} \right) \sim \omega^2 \gg 1 \ \frac{g}{2(1-\gamma)} \left( \frac{\omega^2}{N} \right)^{(1-\gamma)}.$$  

(7)

One rather general class of problems (for a review see for instance [21,24]) consists in understanding the transverse fluctuations of the manifold, governed by the exponent $\zeta$ defined by:

$$\langle [\omega(x+l) - \omega(x)]^2 \rangle \sim_{l \gg a} l^{2\zeta}.$$  

(8)

The general model is mainly described by three parameters: $D$ is the internal dimension of the manifold, $N = d - D$ is its codimension, and $\gamma$ characterizes the large distance correlations of the potential. For $N = 1$ the manifold is an interface, for $D = 1$ it is a directed polymer, for $N = D$ it describes the elastic deformations of a $D$ dimensional crystal with impurities, and for $D = 3, N = 2$ it describes the elastic deformations of a vortex lattice in certain pinning regimes of type two superconductors [26].

III. THERMAL FLUCTUATIONS WITHOUT DISORDER

If $V = 0$, the fluctuations are purely thermal. The propagator is $1/k^2$, leading to $\zeta = 0$ if $D > 2$ (the manifold is flat), and $\zeta = (2-D)/2$ if $D < 2$ (the manifold is rough), and all these results are independent from $N$. 

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IV. RANDOM FORCES

Another simple case is that of random forces, where the pinning term of the Hamiltonian is $-\int dx \ f(x) \omega(x)$, with gaussian random forces of strength $F$. This may be relevant for the deformation of a cristal lattice on small enough length scales so that the displacement of an atom is much smaller than the correlation length of the potential. The problem is simple because there are no metastable states. One gets immediately

$$<\omega(k)> = \frac{f(k)}{k^2}$$  \hspace{1cm} (9)

so that:

$$<\omega^\alpha(k)><\omega^\beta(k')> = \delta^{\alpha\beta} F^2 \delta(k+k') \frac{1}{k^4}.$$  \hspace{1cm} (10)

Clearly the $1/k^4$ term leads to a $\zeta$ exponent which is zero above $D = 4$, and equal to $(4-D)/2$ in $2 < D < 4$ (In $D < 2$ the lateral fluctuations between two points at distance $x$ depend on the total size of the manifold). The thermal fluctuations are much weaker since the connected correlation function is:

$$<\omega^\alpha(k)-<\omega^\alpha(k)>)^2> = \frac{T}{k^2}.$$  \hspace{1cm} (11)

The lesson from this computation is that the disorder fluctuations are dominant, and they lead to a typical "dimensional reduction" result, namely the fact that the value of $\zeta$ for the disordered system in $D$ dimension is equal to the value of $\zeta$ for the thermal fluctuations of the pure system in dimension $D - 2$. This is exact for random forces.

V. RANDOM POTENTIAL: VARIATIONAL APPROACH

This is a complicated problem with many metastable states. A simple argument is that of Imry and Ma [15]. Let me just do it in the case of an interface ($N = 1$) in the RFIM, which corresponds to $\gamma = 1/2$. A bump of size $\omega \simeq w$ on a length scale $L$ costs an elastic energy $L^{D-2}w^2$, while the typical gain from the pinning energy is of order $\sqrt{L^D}w$. Optimizing $w$ leads to $w \simeq L^{(4-D)/3}$, so that $\zeta = (4-D)/3$. Generalizing this argument to arbitrary correlations of the potential, we find:

$$\zeta = \frac{4-D}{2(1+\gamma)}.$$  \hspace{1cm} (12)

This is a reasonable guess for the exponent $\zeta$, but it is difficult to improve it and it has no reason to be exact (a well known counter example is the case $N = D = 1, \gamma = 3/2$, where it is known [33,32] that $\zeta = 2/3$, while the above formula leads to $\zeta = 3/5$).
What one would like is a general field theoretic approach similar to that of the previous sections to handle this problem. A first step is to use perturbation theory. Although this is not strictly necessary, it is convenient to introduce replicas. The effective replica Hamiltonian after disorder averaging is:

\[
H_n = \frac{1}{2} \int dx \sum_{a=1}^{n} \sum_{\mu=1}^{D} \left( \frac{\partial \omega_a}{\partial x_\mu} \right)^2 + \mu \frac{1}{2} \int dx \sum_{a=1}^{n} (\omega_a(x))^2 
\]

\[
+ \frac{\beta}{2} \int dx \sum_{a,b} N f \left( \left( \omega_a(x) - \omega_b(x) \right)^2 \right) .
\]

The usual perturbative treatment of the Hamiltonian \(H_n\) consists in expanding \(f\) in powers of \(\omega\); the quadratic part gives the free propagator \(F_{ab}\):

\[
F_{ab}(k) = \frac{\delta_{ab}}{k^2 + n2\beta f'(0)} + \frac{2\beta f'(0)}{k^2(k^2 + n2\beta f'(0))} .
\]

For \(n \to 0\), the presence of the \(1/k^4\) term immediately leads to \(\zeta = (4 - D)/2\), as in the random force case. This result holds at higher orders in perturbation. We face a typical situation where perturbation theory, which is unable to handle the metastable states, gives qualitatively wrong results.

The method developed in [24] uses a variational method which evaluates the best quadratic Hamiltonian to compute the quenched free energy with the replica method. I shall describe only the main ideas here, leaving all the (sometimes complicated) technical steps aside. The reader is referred to [24] for a full presentation. The case \(D = 0\), studied in [29–31] is a somewhat simpler exercise which is also of interest. The most general quadratic Hamiltonian is:

\[
H_v = \frac{1}{2} \int dk \; G_{a,b}^{-1}(k) \; \omega_a(x)\omega_b(x)
\]

Optimizing the variational free energy leads to a simple gap equation for the self energy \(\sigma_{ab}\):

\[
G_{a,b}^{-1}(k) = k^2 \delta_{ab} - \sigma_{ab}
\]

\[
\sigma_{ab} = 2\beta f' \left( \frac{1}{\beta} \int dk \left[ G_{aa}(k) + G_{bb}(k) - 2G_{ab}(k) \right] \right) , \; a \neq b
\]

\[
\sigma_{aa} = - \sum_{b(\neq a)} \sigma_{ab}.
\]

It is easy to seek a replica symmetric solution to these equations, i.e. to assume that \(\sigma_{ab} = \sigma\). One immediately finds again the famous \(1/k^4\) term in the propagator, leading
to $\zeta = (4 - D)/2$. This result is wrong mathematically because this saddle point is not a local minimum of the variational free energy. It is also wrong physically. The reason is the following: The most general replica symmetric quadratic Hamiltonian describes a situation in which:

$$Z^n \simeq \int d\omega_a \exp \left( -\frac{\beta}{2} \int dk \left( k^2 \sum_a \omega_a(k)\omega_a(-k) - \sigma \sum_{a,b} \omega_a(k)\omega_b(-k) \right) \right).$$

(20)

Disentangling the square, one gets:

$$Z^n \simeq \int d[f] \exp \left( -\frac{1}{2\beta\sigma} \int dk \lvert f(k) \rvert^2 \right) \left[ \int d\omega \exp \left( -\frac{\beta}{2} \int dk \left( k^2 \lvert \omega(k) \rvert^2 - \omega(k)f(-k) \right) \right) \right]^n.$$

(21)

In this form it is clear that the situation we describe is nothing but a random force problem, with a quenched random force with a distribution $\exp \left( -\frac{1}{2\beta\sigma} \int dk \lvert f(k) \rvert^2 \right)$. The replica symmetric quadratic Hamiltonian is bound to describe the random force problem, a problem without metastable states. To get better results one needs to break replica symmetry.

The rsb solution has been worked out using the Ansatz developed by Parisi for the spin glass problem (see the review in [3]). This Ansatz describes hierarchical $n \times n$ matrices in the $n \to 0$ limit by continuous functions on the interval $u \in [0, 1]$. Therefore the self energy becomes a function of this internal parameter $u$. We get a spectrum of masses, which results in a non trivial behaviour of the correlation function. The wandering exponent turns out to be identical to the one derived above through the Imry Ma argument. Therefore it provides a microscopic ”derivation” of this result. I shall not reproduce here these technical computations, but rather sketch in the next section the physical interpretation of the result. Let me point out a few properties of the gaussian rsb variational method. It is a very versatile method which can be used on many problems. It is basically a Hartree approximation which becomes exact in the large $N$ limit. A systematic expansion around $N = \infty$ is thus in principle possible, although technically difficult. Such an expansion was carried to first order on the RFIM, because the leading Hartree approximation had no rsb (see [3]). In the case of the random manifold, some attempts have been made [29,34], but the full expansion has not yet been worked out. Only recently has the replica technical apparatus needed for this expansion been developed [35].

VI. PHYSICAL INTERPRETATION OF THE SOLUTION

It is very instructive to work out the physical content of the rsb solution, in the same spirit as we did for the rs solution before. The idea is to consider seriously the quadratic
Hamiltonian with rsb, and deduce what kind of physical situation it describes. This is not an easy task. Again I will mention only the results, referring the reader to [24] for the derivations. Let us concentrate on one degree of freedom \( \omega = \omega(x) - \omega(0) \).

We have already seen before that in the rs case \( \omega \) feels a random force. It means that for a given sample there is one value of the force \( f \), and the probability distribution of \( \omega \) is \( c^f \exp(-a(\omega - f)^2) \).

The next stage of approximation uses a single breaking step in Parisi’s hierarchical construction. Its physical interpretation is as follows: For each sample one generates a set of favoured values of \( \omega \), called \( \omega_\alpha \), together with a set of weights \( W_\alpha \). These variables \( \omega_\alpha \) and \( W_\alpha \) are quenched random variables. The favoured values \( \omega_\alpha \) are independent variables, with a distribution

\[
P(\omega_\alpha) = \frac{1}{2\pi q_1^{N/2}} \exp \left( -\frac{\omega_\alpha^2}{2q_1} \right) \quad (22)
\]

As for the weights \( W_\alpha \), they are derived from some "free energy" variables \( f_\alpha \) through:

\[
W_\alpha = e^{-\beta f_\alpha} \sum_\nu e^{-\beta f_\nu} . \quad (23)
\]

The \( f_\alpha \) are independent random variables with an exponential distribution such that the average number of states with free energy less than \( f \) (i.e. weight \( W_\alpha \) greater than \( e^{-\beta f} \)) is:

\[
\mathcal{N}(f) \sim e^{\rho f} , \quad (24)
\]

For a given sample, that is given the variables \( \omega_\alpha \) and \( W_\alpha \), the Boltzmann probability for \( \omega \) is given by:

\[
P(\omega) = c^f \sum_\alpha W_\alpha \exp \left( -\frac{[\omega - \omega_\alpha]^2}{2q_0} \right) \quad (25)
\]

This distribution of \( \omega \) is characterized by three numbers \( q_0, q_1, \rho \), which are the variational parameters which are determined by the variational method at this one step rsb stage. The above formula (25) is somewhat surprising. It parametrizes the Boltzmann distribution of the degree of freedom \( \omega \) by a weighted sum of gaussians. Such a representation clearly allows for the existence of several metastable states contributing simultaneously to \( P(\omega) \), which is nice. The surprising point is that such a situation is possible using a gaussian approximation. The power of the method comes from the combined used of the gaussian variational method together with the breaking of replica symmetry. An elementary example would be to look at a particle in a symmetric double well potential, and approximate its Boltzmann weight by a gaussian. At low enough temperature the best gaussian is shifted from the origin and breaks the inversion symmetry in the problem. To restore it one approximates the
Boltzmann distribution of the particle by the sum of the two symmetric shifted gaussians, and one finds a pretty good approximation to the exact distribution at low temperatures. The situation is exactly analogous here, but the symmetry at hand is the replica permutation symmetry.

The full construction consists in iterating the above procedure. For two steps of rsb one must divide each gaussian in \( (25) \) into a weighted sum of subgaussians, introducing two new parameters (one ‘q’ parameter for the width of the subgaussians, one ‘\( \rho \)’ parameter for the distribution of their relative weights). The construction is then iterated an infinite number of times, adding each time two new variational parameters. Clearly in this way we can generate very complicated probability distributions for \( \omega \).

To summarize, the formalism of broken replica symmetry provides a large space for the probability distribution of \( P(\omega) \) (when one changes sample). The probability distributions so generated have the following properties:

a) They depend on many parameters so that we have a large variety of choices.

b) If we want, we can construct these probabilities in such a way that the system is scaling invariant at large distances, characterized for instance by a very non trivial scaling exponent \( \zeta \).

c) The expectation values can be computed explicitly so that this solution can be taken as the starting point of a perturbative expansion.

d) Last, but not least, the gaussian rsb Ansatz becomes exact when the dimension of the space goes to infinity.

An interesting example of application of this very general scheme to the vortices in superconductors can be found in \[26–28,36\].

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