Conserved Currents
in
Supersymmetric
Quantum Cosmology? *†

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Abstract

In this paper we investigate whether conserved currents can be sensibly defined in supersymmetric minisuperspaces. Our analysis deals with $k = +1$ FRW and Bianchi class–A models. Supermatter in the form of scalar supermultiplets is included in the former. Moreover, we restrict ourselves to the first-order differential equations derived from the Lorentz and supersymmetry constraints. The “square-root” structure of N=1 supergravity was our motivation to contemplate this interesting research. We show that conserved currents cannot be adequately established except for some very simple scenarios. Otherwise, equations of the type $\nabla_a J^a = 0$ may only be obtained from Wheeler-DeWitt–like equations, which are derived from the supersymmetric algebra of constraints. Two appendices are included. In appendix A we describe some interesting features of quantum FRW cosmologies with complex scalar fields when supersymmetry is present. In particular, we explain how the Hartle-Hawking state can now be satisfactorily identified. In appendix B we initiate a discussion about the retrieval of classical properties from supersymmetric quantum cosmologies.

1 Introduction

N=1 supergravity [1]-[3] constitutes a “square-root” [4]-[6] of gravity: in finding a physical state $\Psi$, it is sufficient to solve the Lorentz and supersymmetry constraints of the theory. The algebra of constraints then implies that $\Psi$ will consequently obey the Hamiltonian constraints $^1$. This property suggests that supersymmetry may induce interesting and advantageous features within a quantum cosmological scenario. In fact, the supersymmetry and Lorentz constraints lead in many cases to simple first-order differential equations in the bosonic variables (cf. ref. [1]).

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1For a review on the canonical quantization of supersymmetric minisuperspaces see, e.g., ref. [7, 8].
This contrasts with the situation in non-supersymmetric quantum cosmology: a second-order Wheeler-DeWitt equation has to be solved, employing specific boundary conditions. Therefore, it is quite tempting to address some problems of usual quantum cosmology from a supersymmetric point of view. In particular, the issue of probability densities for a quantum state $\Psi$ and conservation equations of the type $\nabla_a J^a = 0$.

As established by C. Misner, we can derive the conserved current $J^a \sim \Psi^* \nabla^a \Psi - \Psi \nabla^a \Psi^*$ from the Wheeler-DeWitt equation of superspace. It satisfies $\nabla_a J^a = 0$, where $\nabla_a$ constitutes the corresponding covariant derivative. We may associate with the current $J^a$ a flux across a surface $\Sigma$. In particular, $\Sigma$ may be defined as the hypersurface of constant value of the corresponding timelike coordinate in a minisuperspace. Moreover, a conserved probability can then be defined from $J$ on the set of classical trajectories. However, this conserved current can be afflicted from difficulties with negative probabilities.

This situation bears obvious similarities with the case of a scalar field $\Phi$ satisfying a Klein-Gordon equation. In this case, the surface $\Sigma$ is usually of constant physical time. But the fact that $J^0[\Phi]$ may be negative led to the discovery of the Dirac equation. From Dirac’s equation a new conserved current was derived, with the advantage of inducing positive definite probabilities. Subsequently, the concepts of anti-particles and second quantization were introduced. The important point to emphasize here is that the Dirac equation constitutes a “square-root” of the Klein-Gordon equation. But how far can we stretch this tempting analogy between, on the one hand, the Klein-Gordon and Dirac equations and, on the other hand, the Wheeler-DeWitt and the equations obtained from the supersymmetry and Lorentz constraints?

Within a standard quantum cosmological formulation, the possibility that $J^0$ can be positive or negative merely corresponds to having both expanding and collapsing classical universes. The flow will intersect a generic $\Sigma$ at different times. In other words, $J^0$ being negative is due to a bad choice of $\Sigma$ and does not lead necessarily to a third quantization. However, the choice of $\Sigma$ as a surface of constant $S$ within a semiclassical minisuperspace approximation is quite satisfactory: the flow associated with $\nabla S$ intersects them once and only once.

The objective of this paper (see also ref. [39]) is precisely to investigate if positive definite conserved currents can be defined in supersymmetric quantum cosmology. Namely, in the sense of those retrieved from the Dirac equation in standard quantum field theory. If supersymmetric conserved currents could be obtained, then the $J^0$ negative values in non-supersymmetric quantum cosmology would be seen from a whole new perspective. In particular, maybe the presence of supersymmetry could induce conserved currents which would correspond in standard quantum cosmology to “select” appropriate trajectories and hypersurfaces $\Sigma$ in minisuperspace.

The approach that we employ here is based on a differential operator representation for the fermionic variables. This constitutes the correct procedure. In fact, it is totally consistent with the existence of second-class constraints and subsequent Dirac brackets in supergravity theories. These then imply that fermionic variables and their Hermitian conjugates are intertwined within a canonical coordinate–momentum relation (see ref. [39], [40, 41] for further details).

It should be pointed out that other authors have pursued objectives similar to ours [13, 40, 41]. However, their approaches involved “square-root” formalisms of gravity distinct from the one we employ here. In particular, rigid supersymmetry was used in ref. [13]. The approach present in [41] is not supersymmetric. Furthermore, a wave function arranged as a vector was used in ref. [31, 32, 33].

The function $S$ that we mention in the text represents an approximate solution of the Lorentzian Hamilton-Jacobi equation. In a semiclassical case the wave function is of the WBK form $\Psi \sim Ce^{-W}$, where $W$ and $C$ are both complex, $W = I_R - iS$ and $|\nabla S| \gg |\nabla I_R|$. Subsequently, $\nabla S$ satisfies $\nabla \cdot J = 0$ with $J \sim e^{-I_R}|C|^2|\nabla S|$.
with a bewildering interpretation of universe-anti-universe states \cite{15}. In ref. \cite{10}-\cite{14} the crucial role of the Lorentz constraints does not seem to have been properly dealt with or is even absent.

This paper is then organized as follows. In section 2 we analyse a $k = +1$ FRW model in the framework of N=1 supergravity with a scalar supermultiplet \cite{13}-\cite{18}, \cite{22}-\cite{25}, \cite{28}-\cite{30}. Some improvements concerning the results present in \cite{22}-\cite{27}, \cite{28,29} are included. We also point to specific differences between our FRW model and the ones analysed in ref. \cite{40,48}. The analysis of these differences subsequently assist us in establishing if (Dirac-like \cite{38}) conserved currents are allowed in supersymmetric quantum cosmology. In section 3 we consider Bianchi class–A models obtained within pure N=1 supergravity \cite{9}-\cite{13}, \cite{46,47}. Some results previously presented in the literature are rectified. In addition, we describe how the presence of anisotropy prevent us from establishing generic conservation equations. Anisotropy and matter lead in general to a mixing of fermion (Grassman-valued) sectors in $\Psi$. This is also a direct consequence of the Lorentz invariance of the theory, which is absent in ref. \cite{42}-\cite{45}. As a result, the first-order differential equations derived from the supersymmetry constraints become coupled. Only the use of subsequent Wheeler-DeWitt–type equations can provide consistent physical solutions. But in doing so we are effectively placing any discussion of conserved currents and probability densities back in the context of usual quantum cosmology \cite{26}. Our conclusions and discussions are presented in section 4. In appendix A we present some features of $k + 1$ FRW cosmologies with complex scalar fields that may be relevant in supersymmetric quantum cosmology. Finally, we initiate in appendix B a discussion on the topic of retrieving classical properties from supersymmetric quantum cosmological models.

2 Supersymmetric FRW $k = +1$ models

Let us consider the action of the more general theory of N=1 supergravity in the presence of gauged supermatter (see eq. (25.12) in ref. \cite{3}). Our physical variables include the tetrad $e^{A A'} \mu$ (in 2-component spinorial form) and the gravitinos which are represented by $\psi^A \mu$, $\bar{\psi}^{A'} \mu$. The “overline” denotes Hermitian conjugation. The tetrad for a $k = +1$ FRW model can be be written as

$$
e^{a \mu} = \begin{pmatrix} N(\tau) & 0 \\ 0 & a E^a_i \end{pmatrix}, \quad e^a_\mu = \begin{pmatrix} N(\tau)^{-1} & 0 \\ 0 & a(\tau)^{-1} E^a_i \end{pmatrix},$$

where $\hat{a}$ and $i$ run from 1 to 3 and $E_{\hat{a}i}$ is a basis of left-invariant 1-forms on the unit $S^3$ with volume $\sigma^2 = 2\pi^2$. This ansatz reduces the number of degrees of freedom provided by $e^{A A' \mu}$. Hence, a consistent ansatz for $\psi_A \mu$ and $\bar{\psi}^{A'} \mu$ is required. We take $\psi_A^0$ and $\bar{\psi}^{A'}_0$ to be functions of time only and

$$\psi_A^0 = e^{A A'} e_i^A \bar{\psi}_A^0, \quad \bar{\psi}^{A'}_i = e^{A A'} i^A \psi_A^0.$$  \hspace{1cm} (2)

We have introduced the new spinors $\psi_A$ and their Hermitian conjugate, $\bar{\psi}^{A'}$, which are also functions of time only \cite{3}, \cite{7, 14-18}, \cite{30}. The scalar supermultiplet present in the action will consist of spatially homogeneous complex scalar fields $\phi, \bar{\phi}$ and their spin-$1/2$ partners $\chi_A(t), \bar{\chi}_A(t)$. Any vector field and supersymmetric partners are taken henceforth to be zero. Moreover, we choose a two-dimensional spherically symmetric Kähler geometry\footnote{For a flat two-dimensional Kähler geometry we will basically get the same physical information \cite{13}-\cite{18}, \cite{22}-\cite{24}, \cite{28,29}.}.

Using the Ansätze previously described, the action of the full theory is reduced to one with a finite number of degrees of freedom. The analysis of the reduced theory becomes simpler if we...
redefine the fermionic fields, $\chi_A, \psi_A$ as follows. First, we take

$$\hat{\chi}_A = \frac{\sigma a^2}{2\pi(1 + \phi \bar{\phi})} \chi_A, \quad \hat{\psi}_A = \frac{\sigma a^2}{2\pi(1 + \phi \bar{\phi})} \bar{\psi}_A \quad \text{(3)}$$

and

$$\hat{\psi}_A = \frac{\sqrt{3}}{2\pi} \sigma a^2 \psi_A, \quad \hat{\psi}_A' = \frac{\sqrt{3}}{2\pi} \sigma a^2 \bar{\psi}_A'. \quad \text{(4)}$$

In addition, we use unprimed spinors, and, to this end, we define

$$\bar{\psi}_A = 2n_B^A \bar{\psi}_B, \quad \bar{\chi}_A = 2n_B^A \bar{\chi}_B. \quad \text{(5)}$$

These redefinitions allow for simple Dirac brackets to be obtained \cite{7,22,23,24}, namely

$$[\chi_A, \bar{\chi}_B]_D = -i \epsilon_{AB}, \quad [\psi_A, \bar{\psi}_B]_D = i \epsilon_{AB}. \quad \text{(6)}$$

Furthermore,

$$[a, \pi_a]_D = 1, \quad [\phi, \pi_\phi]_D = 1, \quad [\bar{\phi}, \pi_{\bar{\phi}}]_D = 1, \quad \text{(7)}$$

and the rest of the brackets are zero. At this point we choose $(\chi_A, \psi_A, a, \phi, \bar{\phi})$ to be the coordinates of the configuration space and $(\bar{\chi}_A, \bar{\psi}_A, \pi_a, \pi_\phi, \pi_{\bar{\phi}})$ to be the momentum operators in this representation. Hence, quantum mechanically we may take (with $\hbar = 1$)

$$\bar{\chi}_A \rightarrow -\frac{\partial}{\partial \chi_A}, \quad \bar{\psi}_A \rightarrow \frac{\partial}{\partial \psi_A}, \quad \pi_a \rightarrow \frac{\partial}{\partial a}, \quad \pi_\phi \rightarrow -i \frac{\partial}{\partial \phi}, \quad \pi_{\bar{\phi}} \rightarrow -i \frac{\partial}{\partial \bar{\phi}}. \quad \text{(8)}$$

Implementing all these redefinitions, the supersymmetry constraints have the differential operator form

$$S_A = -\frac{i}{\sqrt{2}}(1 + \phi \bar{\phi})\chi_A \frac{\partial}{\partial \phi} - \frac{1}{2\sqrt{6}} a \bar{\psi}_A \frac{\partial}{\partial a} - \sqrt{\frac{3}{2}} \sigma a^2 \psi_A$$

$$- \frac{5i}{4\sqrt{2}} \bar{\phi} \chi_A \psi_B \frac{\partial}{\partial \chi_B} - \frac{1}{8\sqrt{6}} \psi_B \bar{\psi}_B \frac{\partial}{\partial \psi_B} - \frac{i}{4\sqrt{2}} \bar{\phi} \chi_A \psi_B \frac{\partial}{\partial \psi_B}$$

$$- \frac{5}{4\sqrt{6}} \chi_A \psi_B \frac{\partial}{\partial \chi_B} + \frac{\sqrt{3}}{4\sqrt{2}} \chi_B \psi_B \frac{\partial}{\partial \chi_B} + \frac{1}{2\sqrt{6}} \psi_A \chi_B \frac{\partial}{\partial \chi_B}, \quad \text{(9)}$$

together with its Hermitian conjugate, obviously using eq. (3). The Lorentz constraints take the form

$$J_{AB} = \psi_A \bar{\psi}_B - \chi_A \bar{\chi}_B = 0, \quad \text{(10)}$$

which implies that the most general form for the wave function of the universe is

$$\Psi = A + B \psi_C \psi_C + C \psi_C \chi_C + D \chi_C \chi_C + E \psi_C \psi_C \chi_D, \quad \text{(11)}$$

where $A, B, C, D, E$ are functions of $a, \phi, \bar{\phi}$, only.

Let us now see which physical states can be derived from the Lorentz and supersymmetry constraints. From the constraint (3), its Hermitian conjugate and eq. (11) we will get four equations from $S_A \Psi = 0$ and another four equations from $\bar{S}_A' \Psi = 0$:  

4
\[
(1 + \phi \bar{\phi}) \frac{\partial A}{\partial \phi} = 0, \quad (1 + \phi \bar{\phi}) \frac{\partial E}{\partial \phi} = 0,
\]
(12)
\[
\frac{a}{2\sqrt{6}} \frac{\partial A}{\partial a} + \sqrt{\frac{3}{2}} \sigma^2 a^2 A = 0, \quad \frac{a}{\sqrt{6}} \frac{\partial E}{\partial a} - \sqrt{6} \sigma^2 a^2 E = 0,
\]
(13)
\[
(1 + \phi \bar{\phi}) \frac{\partial B}{\partial \phi} + \frac{1}{2} \bar{\phi} B + \frac{a}{4\sqrt{3}} \frac{\partial C}{\partial \phi} - \frac{7}{4\sqrt{3}} C + \frac{\sqrt{3}}{2} \sigma^2 a^2 C = 0,
\]
(14)
\[
\frac{a}{\sqrt{3}} \frac{\partial B}{\partial a} - 2\sqrt{3} \sigma^2 a^2 B - \sqrt{3} B + (1 + \phi \bar{\phi}) \frac{\partial C}{\partial \phi} + \frac{3}{2} \phi C = 0,
\]
(15)
\[
\frac{a}{\sqrt{3}} \frac{\partial D}{\partial a} + 2\sqrt{3} \sigma^2 a^2 D - \sqrt{3} D - (1 + \phi \bar{\phi}) \frac{\partial C}{\partial \phi} - \frac{3}{2} \phi C = 0,
\]
(16)
\[
(1 + \phi \bar{\phi}) \frac{\partial D}{\partial \phi} + \frac{1}{2} \phi D - \frac{a}{4\sqrt{3}} \frac{\partial C}{\partial \phi} + \frac{7}{4\sqrt{3}} C + \frac{\sqrt{3}}{2} \sigma^2 a^2 C = 0.
\]
(17)

We can see that (12), (13) constitute decoupled equations for \(A\) and \(E\). Eq. (14) and (15) constitute coupled equations between \(B\) and \(C\), while eq. (16), (17) are coupled equations between \(C\) and \(D\). These equations can be decoupled employing \(B = \tilde{B}(1 + \phi \bar{\phi})^{-\frac{1}{2}}, C = \frac{\bar{\phi}}{\sqrt{3}}(1 + \phi \bar{\phi})^{-\frac{1}{2}}, \quad D = \tilde{D}(1 + \phi \bar{\phi})^{-\frac{1}{2}}\). We can then eliminate \(\tilde{B}\) and \(\tilde{D}\) to get two partial differential equations which imply that \(C = 0\) (cf. ref. [22, 23]).

Let us multiply the first eq. in (13) by \(E\), then multiply the second by \(A\). Their addition results in
\[
\frac{\partial (A \cdot E)}{\partial a} = 0.
\]
(18)

Eq. (18) seems to suggest a relation vaguely similar to \(\nabla \cdot J = 0\). For the case of pure N=1 supergravity, eq. (12)-(17) are reduced to just eq. (13). Hence, eq. (18) would constitute a (very simple) conservation–type equation obtained directly from the supersymmetry and Lorentz constraints.

It is interesting to notice that eq. (12)-(13) imply
\[
A = f(\bar{\phi}) e^{-3\sigma^2 a^2}, \quad E = g(\phi) e^{3\sigma^2 a^2},
\]
(19)

where \(f, g\) are anti-holomorphic and holomorphic functions of \(\phi, \bar{\phi}\), respectively. It seems unsatisfactory that we cannot obtain from the Lorentz and supersymmetry constraints the explicit dependence of \(\Psi\) on \(\phi, \bar{\phi}\) (see ref. [13-14, 22-23, 28, 29]). However, this apparent drawback can be circumvented as follows. Basically, we introduce the variables \(r^2 = \phi \bar{\phi}\) with \(\phi = re^{i\theta}\). These new variables effectively decouple the two degrees of freedom associated with \(\phi\) and \(\bar{\phi}\). But more importantly, they will assist us in establishing if generic conserved currents can be defined in a supersymmetric \(k = +1\) FRW minisuperspace with complex scalar fields.

In fact, equations (12) can then be written as
\[
\frac{\partial A}{\partial r} - i \frac{1}{r} \frac{\partial A}{\partial \theta} = 0 ,
\]
(20)
\[
\frac{\partial E}{\partial r} + i \frac{1}{r} \frac{\partial E}{\partial \theta} = 0 .
\]
(21)
Multiply eq. (20) by $\phi$ and eq. (21) by $A$. It follows from their subtraction that

$$\frac{\partial (A \cdot E)}{\partial \theta} - ir \left( \frac{\partial E}{\partial r} A - \frac{\partial A}{\partial r} E \right) = 0. \quad (22)$$

Let us now take $C = 0$. Multiply eq. (15) by $D$ and eq. (16) by $B$. Adding them, we get a relation similar to (18)

$$D_a (B \cdot D) = 0, \quad (23)$$

with the generalized derivative $D_a \equiv \partial_a - \frac{\phi}{r}$. Directly from eq. (14), (13), (16), (17) we obtain

$$B = a^3 h(\bar{\phi})(1 + \bar{\phi}\phi)\frac{1}{2} e^{3\sigma^2 a^2}, \quad D = a^3 k(\phi)(1 + \bar{\phi}\phi)\frac{1}{2} e^{-3\sigma^2 a^2}, \quad (24)$$

where $h, k$ are anti-holomorphic and holomorphic functions of $\phi, \bar{\phi}$, respectively. Using again $r^2 = \bar{\phi}\phi$ and $\phi = re^{i\theta}$, we obtain from eq. (14)–(17) that

$$(1 + r^2) \frac{\partial B}{\partial r} - i \frac{1 + r^2}{r} \frac{\partial B}{\partial \theta} + rB = 0, \quad (25)$$

$$(1 + r^2) \frac{\partial D}{\partial r} + i \frac{1 + r^2}{r} \frac{\partial D}{\partial \theta} + rD = 0. \quad (26)$$

Multiply eq. (25) by $D$ and eq. (26) by $B$. Then divide by $1 + r^2$. Their subtraction eventually leads to

$$\frac{\partial (B \cdot D)}{\partial \theta} - ir \left( \frac{\partial B}{\partial r} D - \frac{\partial D}{\partial r} B \right) = 0. \quad (27)$$

The general quantum state corresponding to a $k = 1$ FRW supersymmetric model with a scalar supermultiplet is now given by

$$\Psi = c_1 r^{\lambda_1} e^{-i\lambda_1 \theta} e^{-3\sigma^2 a^2} + c_2 a^2 r^{\lambda_3} e^{-i\lambda_3 \theta} (1 + r^2)^{\frac{1}{2}} e^{3\sigma^2 a^2} \psi_C \psi_C$$

$$+ c_3 a^2 r^{\lambda_4} e^{i\lambda_4 \theta} (1 + r^2)\frac{1}{2} e^{-3\sigma^2 a^2} \chi_C \chi_C + c_4 a^2 e^{i\lambda_2 \theta} e^{3\sigma^2 a^2} \psi_C \psi_C \chi_D \chi_D, \quad (28)$$

where $\lambda_1...\lambda_4$ and $c_1...c_4$ are constants. Notice the explicit form of $A, B, D, E$ in (28) in contrast with eq. (19), (24) and ref. [15]-[17], [22]-[25], [28, 29]. If we had use $\phi = \phi_1 + i\phi_2$ then the corresponding first-order differential equations would lead to $A = d_1 e^{-3a^2 a^2} e^{k_1 (\phi_1 - i\phi_2)}$, $B = d_3 e^{-3a^2 a^2} (1 + \phi_1^2 + \phi_2^2)e^{k_3 (\phi_1 - i\phi_2)}$, $B = d_4 e^{-3a^2 a^2} (1 + \phi_1^2 + \phi_2^2)e^{k_4 (\phi_1 + i\phi_2)}$, $E = d_2 e^{3a^2 a^2} e^{k_2 (\phi_1 + i\phi_2)}$.

As far as a generalization of relation (18) is concerned, the bosonic coefficients present in eq. (28) satisfy attractive relations in a 3-dimensional minisuperspace:

$$\frac{\partial (A \cdot E)}{\partial a} + \frac{\partial (A \cdot E)}{\partial \theta} - ir \left( \frac{\partial E}{\partial r} A - \frac{\partial A}{\partial r} E \right) = 0, \quad (29)$$

$$D_a (B \cdot D) + \frac{\partial (B \cdot D)}{\partial \theta} - ir \left( \frac{\partial B}{\partial r} D - \frac{\partial D}{\partial r} B \right) = 0. \quad (30)$$

However, the presence of the terms $ir \left( \frac{\partial E}{\partial r} A - \frac{\partial A}{\partial r} E \right)$ and $ir \left( \frac{\partial B}{\partial r} D - \frac{\partial D}{\partial r} B \right)$ in eq. (29) and (30), respectively, clearly prevent us from obtaining conservation equations of the type $\nabla \cdot J = 0$. The reason can be identified with the variable $\theta$ no longer being a cyclical coordinate when supersymmetry is present (see eq. (32) below). To understand this argument, let us consider a

\[\text{In fact, neither } r \text{ or } \phi_1, \phi_2 \text{ (defined from } \phi = \phi_1 + i\phi_2) \text{ but in the non-supersymmetric description only } r \text{ is non-cyclical.}\]
be a constant. And this will imply the absence of satisfactory conserved currents. However, in the corresponding supersymmetric scenario there are terms in the action that do not allow \( \theta \) to be a cyclical coordinate. So, \( \pi_\theta \) would not be a constant. And this will imply the absence of satisfactory conserved currents.

In fact, the canonical momenta conjugate to \( r \) and \( \theta \) take the following form:

\[
\pi_r = \frac{2}{(1 + r^2)^2} \sigma a^3 \frac{\partial}{\partial t} + \frac{\sigma a^3 e^{-i\theta}}{\sqrt{2(1 + r^2)^2}} 3n_{AA}\chi^A\bar{\psi}^A + \frac{\sigma a^3 e^{i\theta}}{\sqrt{2(1 + r^2)^2}} 3n_{AA}\chi^A\psi^A
\]

\[
\pi_\theta = \frac{2\sigma a^3}{(1 + r^2)^2} \frac{\partial}{\partial t} + \frac{5\sigma a^3 e^{-i\theta}}{\sqrt{2(1 + r^2)^2}} \chi^A\bar{\psi}^A - \frac{3\sigma a^3 e^{i\theta}}{\sqrt{2(1 + r^2)^2}} \chi^A\psi^A
\]

The relevant point for our argument is that equations (31) and (32) directly prevent us to obtain relations like \( \nabla J = 0 \). To see this, notice the last four terms in equations (31) and (32). The origin of these specific terms can be traced back to the action of N=1 supergravity which implies that \( \theta \) is no longer a cyclical variable.

In addition, equations (31) and (32) and the terms just mentioned are basically translated into the last two terms present in equations (29) and (30), which are obtained from the supersymmetry constraints \( S_A\Psi = 0 \) and \( \bar{S}_A\Psi = 0 \). This follows directly from the usual Hamiltonian \( H \sim \rho \bar{q} - L \), which involves a term \( \sigma a^3 \left[ \left( \frac{\partial}{\partial t} \right)^2 + ir^2 \left( \frac{\partial}{\partial \theta} \right)^2 \right] \). But at this point we may emphasize as well the following.

It is precisely the last two terms in each of the equations (31) and (32) that will allow us to obtain explicitly the contributions of the kinetic terms of \( r \) and \( \theta \) in the supersymmetry constraints. These are then read from the coefficients of the Lagrange multipliers \( \psi_0^A, \tilde{\psi}_0^A \) in the Hamiltonian \( H \). But the last four terms in both eq. (21) and (22) (which also include the ones with \( \psi_0^A, \tilde{\psi}_0^A \)) are also a direct consequence of the terms in the action \( \mathcal{S} \) that imply \( \theta \) not being a cyclical coordinate. Thus, the fact that the coordinate \( \theta \) is no longer being a cyclical coordinate can be interpreted as inherited from local supersymmetry, which is now a feature of the reduced model. This can be summarized as follows: a relation as \( \nabla \cdot J = 0 \) cannot be sensibly defined due to the absence of cyclical coordinates, which are ultimately due to the presence of supersymmetry. A similar situation would occur in usual quantum cosmology with a matter Lagrangian taken from the Wess-Zumino model, due to the non-trivial interaction with fermion fields.

### 3 Bianchi class-A models

Bianchi class-A models obtained within pure N=1 supergravity are analysed in this section, closely following ref. [12] (see also ref. [16, 17, 19, 20]). A left-invariant basis [19] is employed, where

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5The author is grateful to S. Kamenshchik for having pointed out this to him.
the spatial metric $h_{ij}$ can be expanded as $h_{ij} = h_{pq}(t)E^p_i E^q_j$. The left-invariant invariant basis $E^p_i$ further satisfy $\partial_i E^p_j = C^p_{qr} E^q_i E^r_j$, $C^p_{qr} = m^p \varepsilon_{eqr}$ are the structure constants of the Bianchi group which allow to identify each Bianchi model. In this basis the tetrad $e^{AA'}_p(t)$ is time-dependent only. Notice that $h_{pq} = -e_{AA'}_p e^{AA'}_q$. In addition, we have $\psi^A_0 = \psi^A_0(t), \psi^A_1 = \psi^A_1(t) E^p_i$ with similar restrictions imposed on $\bar{\psi}^{A'}_p$. An important feature of Bianchi class-A models is that anisotropic gravitational degrees of freedom are now present. Hence, more gravitino modes are allowed to be included. In such a way a more realistic insight on the full theory of N=1 supergravity can be obtained.

Let us focus on the question if conserved currents can be constructed in these supersymmetric models. As we mentioned before, our approach bears significant differences as far as ref. [12-14] are concerned. The Lorentz constraint seems either absent in ref. [42]-[45] or truncated to some extent ref. [41]. Such absence may enforce additional restrictions, either in the spectrum of solutions or even in the general validity of the results. Our approach involves instead a fermionic differential operator representation and the complete Lorentz constraints. This is of some relevance since the wave functional ought to be a Grassman-algebra-valued expression and not just a wave function arranged as a vector [11-15].

The Lorentz invariance implies that $\Psi$ can only contain fermionic terms with an even number of $\psi^A_p$. Thus, we can decompose (and not restrict!) $\Psi$ into fermionic parts of zeroth (bosonic), quadratic, quartic up to sixth (fermionic filled) order, formally denoted by $\psi^0, \psi^2, \psi^4, \psi^6$. Moreover, we employ the general decomposition of $\psi^A_i$ as $\psi^A_{BB'} = e_{BB'}^i \psi^A_i$, and $\psi_{ABB'} = -2n^C_{BB'} \gamma_{ABC} + \frac{2}{3} (\beta_{A} B_{BB'} + \beta_{AB} B_{BB'}) - 2 \varepsilon_{ABn} \gamma_{C, BB},$ where the $\gamma_{ABC} = \gamma_{(ABC)}$, and $\beta^A$ fields denote the spin-$\frac{3}{2}$ and $\frac{1}{2}$ modes of the gravitino, respectively. The wave function of the universe can then be symbolically written as

$$\Psi = A(h_{pq}) + \psi^2 + \psi^4 + F(h_{pq}) \beta^A \beta_B \gamma^{BCD} \gamma_{BCD}^2, \tag{33}$$

where $A$ and $F$ are the coefficients of the bosonic ($\psi^0$) and fermionic filled ($\psi^6$) sectors; the middle sectors $\psi^2$ and $\psi^4$ require further elements and will be discussed later in this section.

The supersymmetry constraints for a generic Bianchi class-A minisuperspace have the form [12]

$$S_A = \sigma m^p e_{AA'} p \bar{\psi}^A_q - \frac{1}{2} i \kappa^2 \left[ (1-s) p^p_{AA'} \bar{\psi}^A_q + s \bar{\psi}^A_q p^p_{AA'} \right], \tag{34}$$

together with its Hermitian conjugate, where we take $\kappa^2 = 8 \pi$. The parameter $s$ represents the ambiguity of operator ordering, which comes from noncommutativity of $\psi^A_p, \bar{\psi}^A_q, p^p_{AA'}$. $p^p_{AA'}$ constitutes the conjugate momenta to $e^A_q$. Quantum mechanically, the $S_A$ supersymmetry constraint takes the form

$$\zeta m^p e_{AA'} p \bar{\psi}^A_q + \psi^A_q \frac{\partial}{\partial e_{AA'}^p} + s \bar{\psi}^A_q e_{AA'}^p = 0, \tag{35}$$

where $\zeta = \frac{2\sigma}{m^p}$ and $S_A$ is just the Hermitian conjugate. We have chosen the following representation [12]

$$\bar{\psi}^A_q = -i \hbar D^A_q \frac{\partial}{\partial \psi^A_q}, \tag{36}$$

$$p^p_{AA'} = -i \hbar \frac{\partial}{\partial e_{AA'}_p} + \frac{1}{2} i \hbar \sigma \varepsilon \psi^A_q D^B_{tq} \frac{\partial}{\partial \psi^B_t}, \tag{37}$$

8
where
\[ D^{AA'}_{pq} = \frac{i}{\sigma \sqrt{\det h}} h_{pq} n^{AA'}_{p} + \frac{\varepsilon_{pqr}}{\sigma} e^{AA'r} \]  
and $\sigma$ is the volume of the hypersurface of quantization.

It can be checked from equations (33), its Hermitian conjugate and the wave function (33) that the bosonic coefficients $A$ and $F$ in (33) satisfy, respectively, the equations
\[
\begin{align*}
\left[ 2 \frac{\partial}{\partial h_{pq}} - \zeta m^{pq} - s h_{pq} \right] A &= 0, \\
\left[ 2 \frac{\partial}{\partial h_{pq}} + \zeta m^{pq} + s h_{pq} \right] F &= 0.
\end{align*}
\]  
The corresponding solutions are
\[
\begin{align*}
A &= c_1 (\det h_{pq})^{\frac{1}{2}} e^{\frac{1}{2} \zeta m_{pq} h_{pq}}, \\
F &= c_2 (\det h_{pq})^{-\frac{1}{2}} e^{-\frac{1}{2} \zeta m_{pq} h_{pq}},
\end{align*}
\]  
where $c_1, c_2$ are constants. Let us now multiply eq. (39) by $F$ and eq. (40) by $A$. We find after adding them that
\[
\frac{\partial (A \cdot F)}{\partial h_{pq}} = 0.
\]  
As expected, eq. (44) constitutes the generalization of eq. (18). Moreover, eq. (44) also represents the decomposition of the result present in ref. [45] concerning a (positive-definite) probability amplitude conservation. However, it is viewed in this section within a fermionic differential operator representation [8, 7].

Let us now consider the middle fermionic sectors of $\Psi$. As far as these sectors are concerned, consistency can only be achieved (see ref. [46, 47]) from the use of the Hamiltonian constraint derived from the Dirac bracket $[S_A, \tilde{S}_{A'}]_D$. However, this back to basics [26] procedure clearly move us from the purpose of using solely the first-order differential equations generated by the supersymmetry constraints. Any conserved currents (and positive-definite probability densities) must then be addressed within the Wheeler-DeWitt-type equations obtained in [46, 47].

The situation concerning the full theory of N=1 supergravity is even more helpless. As we mention in the introduction, a conservation equation of the form
\[
\nabla^a J_a = 0
\]  
can be derived in general relativity with the assistance of the Wheeler-DeWitt equation [35]. But this is achieved without making any assumptions about the space-time geometry. With respect to N=1 supergravity the $\tilde{S}_A \Psi = 0$ constraint reads
\[
\left( \varepsilon^{ijk} e_{AA'i}^{3s} D_j \psi^A_k \right) \Psi - \frac{1}{2} \kappa^2 \psi^A_i \frac{\delta \Psi}{\delta e_{AA'i}} = 0,
\]  
where $e_{AA'i}^{3s} = -\sigma \varepsilon_{pqr} e_{AA'p}$, $D(e_{AA'i}) = \det \left(-\frac{1}{2} C_{AA'i}^{pq} \right)$. This gives us the wave function in the representation $\tilde{\Psi}(e_{AA'}, \psi^A)$ and leads to a factor of $h^{-1}$ (via $D^{-1}(e_{AA'})$). In particular, it intertwines $F$ in (33) with $\tilde{\Psi}$, whose equation is substantially easier to derive. But the inverse transformation does not involve a factor of $h$; see ref. [3] for the reasons of this asymmetry.
while the $S_A \Psi = 0$ constraint in the $\tilde{\Psi}(e_{AA'i}, \tilde{\psi}^{A'i})$ representation is given by

$$\left(\varepsilon^{ijk}e_{AA'i}, 3sD_j\psi^{A'i}\right) \tilde{\Psi} + \frac{1}{2}\kappa^2 \tilde{\psi}^{A'i} \frac{\delta \tilde{\Psi}}{\delta e_{AA'i}} = 0. \quad (46)$$

In the homogenous case, the $\psi^{A'i}$ and $\tilde{\psi}^{A'i}$ can be arbitrarily chosen and hence cancelled out in eq. (45) and (46) [10, 12, 20]. Furthermore, we can use either of the representations $\Psi$ or $\tilde{\Psi}$: their bosonic coefficients are related by a Fourier transformation generalizing (41). But the inhomogenous case is clearly different. The gravitinos cannot be cancelled out. Moreover, $\Psi$ can only have states with infinite fermion number in full N=1 supergravity [52].

### 4 Our message

The purpose of this paper was to investigate if conservation equations [38] of the type $\nabla \cdot J = 0$ could be sensibly defined in supersymmetric minisuperspaces. In section 2 we considered $k = +1$ FRW models with and without supermatter in the form of scalar supermultiplets $(\phi, \tilde{\phi}; \chi_A, \tilde{\chi}_A)$. Bianchi class-A models in pure N=1 supergravity as well as the full theory were discussed in section 3. We restricted ourselves to the first-order differential equations derived from the Lorentz and supersymmetry constraints. Moreover, we employed here a differential operator representation for the fermionic variables. This constitutes the correct approach due to the existence of second–class constraints and subsequent Dirac brackets. These then imply that the fermionic variables and their Hermitian conjugates are intertwined within a canonical coordinate-momentum relation. The “square-root” structure present in the algebra of constraints of N=1 supergravity was the main motivation for our study. Namely, the fact that the above mentioned first-order differential equations act relatively to the Wheeler-DeWitt equation in a way similar to the standard procedure in quantum field theory relating the Klein-Gordon and Dirac equations [38].

In the supermatter case our results were twofold. First, we showed how the explicit dependence of the wave functional $\Psi$ on $\phi, \tilde{\phi}$ could be brought about. This was done by introducing the the transformation $\phi = re^{i\theta} = \phi_1 + i\phi_2$ directly in the supersymmetry constraints. In particular, the no-boundary wave function has now been adequately identified, in contrast with previous comments in ref. [22]-[25], [28, 29].

Our second result also followed from the use of this transformation. In fact, the variable $\theta$ is no longer a cyclical coordinate if supersymmetry is present. This should be contrasted with the situation in plain quantum cosmology. Basically, $\theta$ being a non-cyclical coordinate is caused by the presence of specific terms in the momenta $\pi_r$ and $\pi_\theta$, which lead to the expressions $\frac{\delta (A \cdot E)}{\delta \phi} + \frac{\delta (A \cdot E)}{\delta \theta} - ir \left( \frac{\delta E}{\delta r} A - \frac{\delta A}{\delta r} E \right) = 0$ and $D_a (B \cdot D) + \frac{\delta (B \cdot D)}{\delta \theta} - ir \left( \frac{\delta B}{\delta \tau} D - \frac{\delta D}{\delta \tau} B \right) = 0$ (see eq. (29) and (30)). And these expressions clearly prevent us from obtaining a relation like $\nabla \cdot J = 0$ directly from the supersymmetry constraints. But among those specific terms in $\pi_r$ and $\pi_\theta$ (that prevent $\nabla J = 0$ to be obtained), we can also identify the ones which are necessary to retrieve the supersymmetry constraints in a usual canonical formalism. Hence, the absence of satisfactory conserved currents may be ultimately related to the presence of supersymmetry in our supermatter model. Furthermore, our results provide a dissimilar perspective with regard to ref. [40]. There, a particular square-root of a non-supersymmetric FRW model with complex scalar fields was used. Our model is sustained instead by a “square-root” quantization inherited from N=1 supergravity. Concerning homogeneous models in pure N=1 supergravity, we obtain a rather simple conservation equation for the FRW case. With respect to Bianchi models, our results are summarized in eq. (44). This expression represents essentially the conservation of the probability amplitude
mentioned in ref. [45] but viewed here within our canonical approach. Namely, by employing a differential operator representation for the fermionic variables. This should be compared with ref. [41]-[45], where the Lorentz constraint is either absent or truncated. Moreover, the supersymmetric wave function in [41]-[45] is arranged in a vector, leading to universe—anti-universe states.

Overall, our message in this paper is that generic conserved currents do not seem feasible to obtain directly from the supersymmetry constraints equations. Only for very simple scenarios does this becomes possible. Otherwise, conserved currents (and consistent probability densities) may only be obtained upon the use of subsequent Wheeler-DeWitt–like equations. These are derived through the associated supersymmetric algebra of constraints.

In our view, the fundamental reason for our conclusions is related with the following. A physical supersymmetric wave functional $\Psi$ takes values in a Grassman algebra. Such algebra is formed by complex linear combinations of products of anti-commuting elements such as the gravitino $\psi_i^A$. Hence, $\Psi \left[ \psi_{\mu}^{AA'}, \psi_{\mu}^{A}, \psi^{A'}, \phi, \bar{\phi}, \chi^A, \bar{\chi}^{A'} \right]$ embodies more than a wave function arranged as a vector and satisfying a Dirac-like equation (see ref. [41]-[45]). Furthermore, the first-order differential equations derived from the supersymmetry, Lorentzian and Grassmanian-valued $\Psi$ constitute more than a simple set of conditions. They rather represent the action of the supersymmetry constraints on different fermionic representations of $\Psi$, related by a (coordinate–momentum) fermionic Fourier transformation [6, 14]–[18].

Finally, we included two appendices. One contains additional comments on quantum FRW closed cosmologies with complex scalar fields in the presence of supersymmetry. The other appendix initiates a discussion on the retrieval of classical properties from a supersymmetric quantum cosmological scenario.

A Supersymmetric quantum FRW models with complex scalar fields

In this appendix we will compare some of the features of supersymmetric quantum cosmological FRW models in the presence of complex scalar fields with the corresponding situation in non-supersymmetric quantum cosmology. To begin with, let us mention that FRW quantum cosmological models with complex scalar fields can be found in, e.g., ref. [48]. A characteristic method employed in [48] was the use of the following transformation for the complex scalar field: $\phi \rightarrow r e^{i\theta}$. This allowed to identify $\theta$ as cyclical coordinate and hence a constant conjugate momentum: $\pi_{\theta} \sim r^2 a^3 \partial \theta / \partial t$.

In this paper we also investigated closed FRW models with complex scalar fields but in the context of supersymmetric quantum cosmology. The definition $\phi = r e^{i\theta}$ was also employed but directly in the supersymmetry constraints. Notice that in ref. [16, 17] this was only employed in subsequent Wheeler-DeWitt–type equations. One of the advantages of in substituting $\phi = r e^{i\theta}$ in the supersymmetry constraints is to obtain the explicit dependence of $\Psi$ on $\phi, \bar{\phi}$. This was missing in ref. [15]-[17], [22]-[25], [28, 29].

In addition, this approach also allowed our results to be compared with the ones present in [48]. In fact, the bosonic coefficients in expression (28) correspond to particular solutions that can be obtained in the framework of ref. [48]. Namely, if a specific factor ordering for $\pi_{a}, \pi_{r}, \pi_{\theta}$ is used in the Wheeler-DeWitt equation together with a judicious choice of integration
constants. The point is that the constraint \([\bar{H}],\) its Hermitian conjugate, the Lorentz constraint and expression \([11]\) imply \(\frac{\partial A}{\partial \phi} = 0\) and \(\frac{\partial E}{\partial \phi} = 0\) (i.e., eqs. \([20], [21]\)) and hence the solutions you found here for \(A\) and \(E\). Let us now see how such expressions can be obtained from the Hamiltonian constraint. It is then important to notice that the Hamiltonian constraint includes a bosonic term \(\pi_i \pi_i \sim (\pi_r - i \pi_\theta)(\pi_r + i \pi_\theta)\). Quantum mechanically, we can write it as \(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}\) This was in fact the choice made in ref. \([48]\). But notice that we could also have \((\pi_r - i \pi_\theta)(\pi_r + i \pi_\theta)\) as \(\left(\frac{\partial}{\partial r} - i \frac{1}{r} \frac{\partial}{\partial \theta}\right) \left(\frac{\partial}{\partial r} + i \frac{1}{r} \frac{\partial}{\partial \theta}\right)\) which is different from \(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}\). Hence, the presence of supersymmetry induces a particular set of solutions for \(\Psi\), which can only be obtained from the Wheeler-DeWitt equation if a particular factor ordering for the canonical momenta is selected in the Hamiltonian constraint. As a consequence, specific and explicit exact solutions (say, \(e^{-3\sigma^2a^2} l^\lambda e^{i\lambda \theta}\)) can be found in the gravitational and matter sectors.

A comparison between quantum FRW models with and without supersymmetry can be further enhanced by mentioning the following. Firstly, it is interesting to remark that when supersymmetry is present the situation changes considerably. The variables \(\{a, r\}\) become all intertwined and the analysis is less simple. Furthermore, the constants of separation \(\lambda_1, ..., \lambda_4\) seem to correspond to \(i \equiv \sqrt{-1}\) times the eigenvalues of \(\pi_\theta\) permitted in \([48]\).

Finally, let us mention that the no-boundary wave function corresponds to the bosonic coefficient \(A\). We stress that only by employing \(\phi = re^{i\theta} = \phi_1 + i \phi_2\) can such claim become fully justified. The exponential factors \(e^{\pm 3\sigma^2a^2}\) are to be viewed as \(e^{\pm 1}\), where \(I\) is Euclidean action for a classical solution \(\text{without}\) matter outside or inside a three-sphere with radius \(a\) (see ref. \([14, 15]\)). In the absence of matter the Hartle-Hawking state \([35]\) is therefore given by \(\Psi_{HH} = \psi_A \psi^A e^{3\sigma^2a^2}\). The solution \(\Psi = e^{-3\sigma^2a^2}\) bears quantum wormhole properties \([16, 17, 28, 29, 34]\). Furthermore, we get the expressions \(A = d_1 e^{-3\sigma^2a^2} c_1 (\phi_1 - i \phi_2)\) and \(E = d_2 e^{3\sigma^2a^2} c_2 (\phi_1 + i \phi_2)\) from the decomposition \(\phi = \phi_1 + i \phi_2\). For each of them the constant \(k\) (as defined from the equations \((9)-(23)\) of ref. \([34]\)) has to be \(k = d_1^2 - d_2^2 = 0\). This means that the scalar flux associated with \(\text{two independent massless scalar fields}\) \(\phi_1\) and \(\phi_2\) is now absent. Consequently, there is also no lower bound for \(a\). Such result is quite curious. In fact, it is not apparent how the solutions for \(A\) and \(E\) could represent a wormhole connecting two asymptotic regions. For a related discussion concerning the retrieval of wormhole states see ref. \([29]\).

B Can classical properties be retrieved from supersymmetric quantum cosmologies?

In this appendix we present some comments concerning the retrieval of classical features in supersymmetric quantum cosmology. The main reason is that the topic of conserved currents (and positive-definite probability amplitudes) is particularly relevant in quantum cosmology and closely related with the emergence of classical properties from quantum models \([31]\). Hence the question we raise here: would the presence of supersymmetry introduce any significative changes concerning the retrieval of classical properties in the framework of quantum cosmology?

An answer to this question is currently being sought after \([54]\). In order to understand the relevance of retrieving classical properties from supersymmetric quantum cosmologies, let us mention
the following.

In plain quantum cosmology, conserved probability currents can be obtained by requiring the wave function of the universe to be of the form $\Psi_{WKB} \sim e^{iS}$. As consequence, classical properties can emerge from $\Psi_{WKB}$. But what can be obtained from $e^I$? When $\Psi$ is an exponential $e^I$ rather than an oscillatory function, $I$ corresponds to the action of an Euclidean rather than a Lorentz geometry. This situation occurs when no matter is present, and the dominant saddle-point contribution to the path-integral solution is a real Euclidean solution of the field equations [53]. This conclusion holds for a variety of homogeneous models [53]. But the important fact to notice is that a wave functional $e^I$ is not peaked around a set of Euclidean solutions. It fails to predict classical correlations between bosonic coordinate and momenta. In contrast, a wave function $e^{iS}$ is peaked around a set of classical Lorentzian trajectories [31, 32].

In supersymmetric quantum cosmology most of the solutions that have been found include only the exponential of the Euclidean action $e^{\pm I}$ [7, 9]-[30], [46, 47]. This means that these solutions present in [10, 17] do not induce any classical Lorentzian geometry. Thus the current framework for supersymmetric Bianchi models may require additional elements in order to get oscillating $e^{iS}$ solutions. Maybe that would provide the means to establish a relation similar to $\nabla \cdot J = 0$, $J \sim e^{-\frac{1}{8} |C^2| \nabla S}$, which is only valid in a minisuperspace approximation. However, solutions with oscillating properties were obtained in ref. [20, 21] for a very simple FRW model with $\Lambda \neq 0$ and within pure N=1 supergravity.

Finally, the relation between supersymmetry and first-order differential equations may also provide another interesting perspective. Previous analysis based on the second-order Wheeler-DeWitt equation has shown a peculiar behaviour when constructing wave packets [53] in quantum cosmology. Perhaps the general superposition present in the wave functional (11) may point out to alternative insights.

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