Constraining compact star properties with nuclear saturation parameters

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A set of hadronic equations of state (EoSs) derived from relativistic density functional theory and constrained by terrestrial experiments, astrophysical observations, in particular by the GW170817 event, and chiral effective field theory (χEFT) of neutron matter is used to explore the sensitivity of the EoS parameterization on the few nuclear matter characteristics defined at the saturation density. We find that the gross properties of compact stars are most sensitive to the isoscalar skewness coefficient \( Q_{\text{sat}} \) and the isovector slope coefficient \( \lambda_{\text{sym}} \) around saturation density, since the higher order coefficients, such as \( k_{\text{sym}} \), are fixed by our model. More specifically, (i) among these \( Q_{\text{sat}} \) is the dominant parameter controlling both the maximum mass and the radii of compact stars while \( \lambda_{\text{sym}} \) is constrained somewhat by χEFT of neutron matter; (ii) massive enough \((M \sim 2 \, M_\odot)\) compact stars featuring both hyperons and \( \Delta \) resonances can be obtained if the value of \( Q_{\text{sat}} \) is large enough; (iii) the emergence of \( \Delta \)'s reduces the radius of a canonical mass \((M \sim 1.4 \, M_\odot)\) compact star thus easing the tension between the predictions of the relativistic density functionals and the inferences from the X-ray observation of nearby isolated neutron stars.

I. INTRODUCTION

Compact stars are unique laboratories for studies of dense matter. The hadronic core of a compact star extends from half up to a few times the nuclear saturation density \( \rho_{\text{sat}} \). Currently, the most rigorous constraint on the high-density behavior of the equation of state (EoS) comes from the observations of a few massive pulsars with masses \( \sim 2 \, M_\odot \) [1–3]. These observations set a lower bound on the maximum mass predicted by any EoS of dense matter. The recent detection of gravitational waves from the binary neutron star inspiral event GW170817 [4] allowed to place constraints on the tidal deformability of compact stars and thus to put additional constraints on the EoS of dense matter [5–12]. The GW170817 event is complementary to the mass measurements indicated above as it allows one to put constraints on the properties (specifically, radius and deformability) of a canonical-mass \((M \sim 1.4 \, M_\odot)\) neutron star.

The details of the composition of compact stars at high densities are not fully understood yet and the possibilities include hyperonization [13–25], the appearance of \( \Delta \) resonances [12, 13, 26–33] and transition from hadronic to quark matter [34–41]; for recent reviews see Refs. [42, 43, 45]. The onset of hyperons entails a considerable softening of the EoS and thus reduces the maximum mass of corresponding sequences of compact stars compared to those based on purely nucleonic EoS [17–24, 50]. The existence of new degrees of freedom in the core of a neutron star cannot be confirmed or ruled out so far on the basis of astrophysical observations alone.

The physics of nuclear systems at and somewhat below the saturation density and zero temperature is well constrained by the studies of finite nuclei. The EoS of isospin symmetric nuclear matter around saturation density is well constrained because physical observables that are dominated by the isoscalar sector have been measured with very high precision. On the other hand, the isovector sector remains poorly determined as the measurements of the observables that are sensitive to the isovector channel lack the necessary precision. Pure neutron matter sets the limiting behavior of isovector properties of nuclear matter. In particular, the neutron matter EoS, obtained by solving the many-body Hamiltonian derived from chiral effective field theory (χEFT), is expected to be reliable up to densities \( \sim 1.3 \rho_{\text{sat}} \) [51]. This allows one to gauge the phenomenological theories of isospin asymmetrical matter by requiring that in the limit of pure neutron matter the ab-initio results for the EoS are reproduced.

Clearly, any viable EoS must simultaneously satisfy the constraints from experimental and theoretical studies of nuclear systems near the saturation density and the observational constraints deduced from studies of compact stars. In this work, we present a density functional based parametrization of the dense matter EoS for the hadronic matter that (i) reproduces the saturation properties of isospin-symmetric nuclear

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matter; (ii) in the limit of pure neutron matter matches the $\chi$EFT-based ab initio results for the EoS of neutron matter, (iii) allows for strangeness in the form of hyperons as well as for $\Lambda$ resonances; (iv) produces compact star sequences with $M_{\text{max}} \gtrsim 2 M_\odot$ and $R_{\text{max}} \lesssim 13.8$ km, where $R_{\text{max}}$ is the radius of $1.4 M_\odot$ mass star. The key new feature of our study is the mapping of the density-functional based EoS onto a generic one that is parametrized in terms of a few observables of nuclear systems at saturation, which we call characteristic parameters or characteristics, see Eq. (9) below. Note that the low-order characteristic parameters are known from the data on nuclei and are often referred to as “nuclear empirical parameters”. We use the former term below to refer to the full parameter set entering this equation, among which some are not constrained experimentally.

Similar explorations were previously carried out using Skyrme density functionals in order to constrain the symmetry energy by evaluating the neutron skin [52, 53], giant monopole resonances [54], and the electric dipole polarizability [55]. Correlations between the critical density of $\Delta$-formation and the maximum mass of compact stars within the nonlinear (NL) density functional theory has also been studied in Refs. [32, 56]. Furthermore, the tidal polarizability of a neutron star has been applied to constrain the symmetry energy within the NL-density functional theory [57]. Also, nucleonic EoS based on the Taylor expansions around the saturation density has been applied to assess the effect of the high-order characteristics [58–60] as well as to put potential constraints among them [60, 61].

The paper is organized as follows. In Sec. II we outline the framework necessary to compute the stellar structure and the general properties of asymmetric nuclear matter. In Sec. III we show how the uncertainties in the values of nuclear matter characteristics at the saturation influence the parameters of the compact stars. This is combined with the constraints on the available parameter space set by the current theoretical and observational information. Finally, Sec. IV summarizes our concluding remarks.

II. THEORETICAL FRAMEWORK

A. EoS of hadronic matter

We use here the standard form of the Hartree density functional in which Dirac baryons are coupled via meson fields [62, 63]. The theory is Lorentz invariant and, therefore, preserves causality when applied to high-density matter. The baryons interact via exchanges of $\sigma$, $\omega$ and $\rho$ mesons, which comprise the minimal set necessary for a quantitative description of nuclear phenomena [64]. In addition, we consider two hidden-strangeness mesons ($\sigma^*$, $\phi$) which describe interactions between hyperons [15, 23, 25, 50].

The Lagrangian is given by the sum of the free baryonic and mesonic Lagrangians [31, 50], which we do not write down, and the interaction Lagrangian which reads

$$\mathcal{L}_\text{int} = \sum_B \bar{\psi}_B \left( -g_{\sigma B} \sigma - g_{\omega B} \omega - g_{\rho B} \rho \cdot \tau \right) \psi_B + \sum_{D} \left( \psi_B \rightarrow \psi_D \right),$$

(1)

where $\psi$ stands for the Dirac spinor and $\psi^r$ for the Rarita-Schwinger spinor. Index $B$ labels the spin-1/2 baryonic octet, which comprises nucleons $N \in \{n, p\}$, and hyperons $Y \in \{\Lambda, \Sigma^{0, -}, \Xi^{0, -}\}$, while index $D$ refers to the spin-3/2 resonance quartet of $\Delta$’s ($\Delta \in \{\Delta^{++}, \Delta^{+}, \Delta^0\}$) which are treated as Rarita-Schwinger particles [65]. The mesons couple to the baryon octet and $\Lambda$’s with the strengths determined by the coupling constants $g_{nB}$ and $g_{dB}$, which are functionals of the vector density. The Lagrangian (1) is minimal, as it does not contain (a) isovector-scalar $\delta$ meson [66] and (b) the $\pi$ meson and the tensor couplings of vector meson to baryons (both of which arise in the Hartree-Fock theories [50, 67–70]), which are beyond the scope of this paper.

In the nucleonic sector, the meson-nucleon ($mN$) couplings are given by [71, 72]

$$g_{mN}(\rho_t) = g_{mN}(\rho_{\text{sat}}) f_{mN}(r),$$

(2)

where $r = \rho_t/\rho_{\text{sat}}$ and $\rho_t$ is the baryon vector density. For the isoscalar channel, one has

$$f_{mN}(r) = a_1 + \frac{m_b}{m_b(r + d_m)^2} + m_s(r + d_m)^2, \quad m = \sigma, \omega, \rho,$$

(3)

with $f_{mN}(1) = 1$, $f''_{mN}(0) = 0$ and $f''_{mN}(1) = f''_{\omega N}(1)$. The density dependence for the isovector channel is taken in an exponential form

$$f_{mN}(r) = e^{-a_3 (r - 1)}, \quad m = \rho.$$

(4)

It is seen that if we fix in the Lagrangian (1) the baryon and meson masses to be (or close to) the ones in the vacuum then properties of infinite nuclear matter can be computed uniquely in terms of seven adjustable parameters. These are the three coupling constants at saturation density ($g_{\sigma N}, g_{\omega N}, g_{\rho N}$), and four parameters ($a_1, b_1, a_3, a_4$) that control their density dependence.

The vector meson-hyperon ($mY$) couplings are given by the SU(6) spin-flavor symmetric quark model [18, 21, 23, 50] whereas the scalar meson-hyperon couplings are determined by fitting to certain preselected properties of hypernuclear systems. We determine the coupling constants, $g_{\sigma Y}$, using the following hyperon potentials in the symmetric nuclear matter at saturation density $\rho_{\text{sat}}$ [73, 74]:

$$U_\Lambda^{(N)} = -U_\Sigma^{(N)} = -30$ MeV, \quad U_\Xi^{(N)} = -14$ MeV. (5)

Physically, the $\Lambda\Lambda$ bond energy provides a rough estimate of the $U_\Lambda^{(N)}$ potential at the average $\Lambda$ density ($\approx \rho_{\text{sat}}/5$) inside a hypernucleus [17, 75, 76]. We adopt the value

$$U_\Lambda^{(N)}(\rho_{\text{sat}}/5) = -0.67$ MeV, \quad (6)
which reproduces the most accurate experimental value to date [77]. This information we use to fix the value of the coupling $g_{\sigma\Lambda}$. It has been shown in Refs. [75, 76] that the bond energy can be approximated by the $U^{\Lambda}(\Lambda)$ potential if the rearrangement term in the medium field between double-$\Lambda$ and single-$\Lambda$ hypernuclei is negligible. The coupling of remaining hyperons $\Xi$ and $\Sigma$ to the $\sigma^*$ is constrained by the relation $g_{\sigma\Xi} = g_{\sigma\Lambda} = g_{\sigma\Lambda}$. Detailed discussions of hyperon potentials can be found, e.g., in Refs. [24, 50, 78].

The isoscalar meson-$\Lambda$ ($m\Lambda$) couplings are uncertain, as no consensus has been reached yet on the magnitude of the $\Delta$ potential in nuclear matter. The studies of the scattering of electrons and pions off nuclei and photoabsorption which are based on a phenomenological models [79, 80] indicate that the $\Delta$ isoscalar potential $V_\Delta$ should be in the range [29]

$$-30 \text{ MeV} + V_N(\rho_{\text{sat}}) \leq V_\Delta(\rho_{\text{sat}}) \leq V_N(\rho_{\text{sat}}),$$

(7)

where $V_N$ is the nucleon isoscalar potential. The studies of $\Delta$ production in heavy-ion collisions [81, 82] suggest a less attractive potential [83].

$$V_N(\rho_{\text{sat}}) \leq V_\Delta(\rho_{\text{sat}}) \leq 2/3V_N(\rho_{\text{sat}}).$$

(8)

At the same time, the isovector meson-$\Delta$ couplings are largely unknown. Below, we limit ourselves to the case where $R_{\Delta \omega} = g_{\Delta \omega}/g_{N\omega} = 1.1$, $g_{\Delta \rho}/g_{N\rho} = 1.0$ and $g_{\Delta \Lambda}$ is determined by fitting to the $\Delta$-potential at saturation density $\rho_{\text{sat}}$. The value $R_{\Delta \omega} = 1.1$ allows one to obtain a physical solution for very attractive $\Delta$-potentials [31]. Note that we assume that the hyperon and $\Delta$ potentials scale with density as the nucleonic potential, therefore their high-density behavior is inferred from that of the nucleons. Such an assumption has its justification in the quark substructure of these constituents (where $\Delta$'s involve three-body bound states of light quarks only, as nucleons, and strangeness-1 hyperons involve bound states of two light and one heavy quark). However, first-principle computations which may confirm our assumption are still lacking.

Once the coupling constants are determined, one could compute the EoS of the stellar matter by implementing the additional conditions of weak equilibrium and change neutrality that prevail in neutron stars. We further match smoothly our EoS for the core to that of the crust EoS given in Refs. [84, 85] at the crust-core transition density $\rho_{\text{sat}}/2$. The integral parameters of a compact star, in particular, the mass and the radius, are then computed from the Tolman-Oppenheimer-Volkoff (TOV) equations [86, 87].

### B. Characteristic parameters of nuclear matter

As is well known, the EoS of isospin asymmetric nuclear matter can be expanded close to the saturation and the isospin symmetrical limit in power series

$$E(\chi, \delta) \approx E_{\text{sat}} + \frac{1}{21}K_{\text{sat}}\chi^2 + \frac{1}{31}Q_{\text{sat}}\chi^3 + E_{\text{sym}}\delta^2 + L_{\text{sym}}\delta^3 \chi + O(\chi^4, \chi^2 \delta^2),$$

(9)

where $\chi = (\rho - \rho_{\text{sat}})/3\rho_{\text{sat}}$ and $\delta = (\rho_n - \rho_p)/\rho$. The coefficients of the density-expansion given by the first line of Eq. (9) are known as the empirical parameters of nuclear matter in the isoscalar channel, specifically, the saturation energy $E_{\text{sat}}$, the incompressibility $K_{\text{sat}}$, and the skewness $Q_{\text{sat}}$. The isovector characteristics associated with the expansion away from the symmetrical limit [the second line in Eq. (9)] are the symmetry energy parameter $E_{\text{sym}}$ and its slope parameter $L_{\text{sym}}$. The higher-order terms in the expansion (9), which are not shown here, have been studied in Refs. [58, 60].

It is then seen that, per definition, the various characteristics of the bulk nuclear matter are the coefficients of the expansion of the energy density close to the saturation density and isospin-symmetrical limits (note that $\delta$ appears in even powers only). In order to fully determine the parameters of our relativistic density functional, we specify [in addition to the parameters appearing in Eq. (9)] the value of the Dirac mass $M_P^{\text{sym}}$ at the saturation, which is important for a quantitative description of finite nuclei, e.g., spin-orbit splitting.

Thus, given the five macroscopic characteristics in Eq. (9) together with the preassigned values of $\rho_{\text{sat}}$ and $M_P^{\text{sym}}$, we are in a position to determine uniquely the seven adjustable parameters of the density functional defined above. Having this in mind, our strategy would be to vary individually these macroscopic characteristics within their acceptable ranges and to examine the influence of these variations on the EoS of dense matter and properties of compact stars. In this manner, we explore the correlation(s) between specific properties of nuclear matter and/or compact stars and each parameter entering Eq. (9). Of particular interest are the quantities which arise at a higher order of the expansion, specifically, $Q_{\text{sat}}$ and $L_{\text{sym}}$. Their values are weakly constrained by the conventional fitting protocol used in constructing the density functionals, i.e., the procedure which involves usually fits to nuclear masses, charge radii and neutron skins, see for instance Refs. [59, 89–91]. It is worthwhile to mention that there is a strong correlation between $L_{\text{sym}}$ and the neutron skins [92–95]. Unfortunately, the uncertainties in the determination of neutron skins are large and as a consequence, the experimental constraints on the theory are weak [96, 97].

For our analysis below we adopt as a reference the DD-ME2 parametrization [88]. It has been tested on the entire nuclear chart with great success and agrees with experimentally known bounds on the empirical parameters of nuclear matter.
In Table I we list the characteristic parameters of symmetric nuclear matter at saturation density according to the DD-ME2 parametrization.

The coefficients of the terms in the expansion (9) that are higher than the second order in $\chi$ and $\delta$ are highly model dependent [58, 89–91]. For example, nonrelativistic Skyrme/Gogny models predict negative $Q_{\text{sat}}$ value [58, 89, 90], whereas relativistic mean-field models predict positive $Q_{\text{sat}}$ value [58, 91]. Note that once the free parameters of our density functional are fixed using the low-order characteristics, these higher-order characteristics are predicted by the density functional, i.e., these are not free parameters in the present setup.

III. RESULTS AND DISCUSSIONS

A. Low-density neutron matter

As mentioned above, we consider as characteristics the five expansion coefficients in Eq. (9) plus the values of $\rho_{\text{sat}}$ and $M_{\text{D}}$ at saturation density. We then further restrict the set of EoS by choosing only those which reproduce the result for neutron matter at densities below and around saturation derived from the ab initio calculations based on $\chi$EFT for densities up to $\sim 1.3\rho_{\text{sat}}$ [51].

While we fix the characteristics at saturation density, the nuclei are most sensitive to the physics at densities that are below the saturation density. Indeed, it has been recognized by several authors [53, 104–108] that a variety of nuclear models which fit the properties of nuclear systems predict almost identical values of symmetry energy for the density $\rho_c = 0.11 \text{ fm}^{-3}$. Motivated by this, we hold the value of the symmetry energy $E_{\text{sym}}(\rho_c)$ [instead of $E_{\text{sym}}(\rho_{\text{sat}})$] constant when $L_{\text{sym}}$ is being varied.

In Fig. 1 we show the EoS which are compatible with the neutron matter EoS and which lie within the allowed band region obtained from studies based on $\chi$EFT. These EoS are obtained by changing $L_{\text{sym}}$ (upper panel) or $Q_{\text{sat}}$ (lower panel), while keeping all other characteristics at their default values of DD-ME2 parametrization. It is seen that the uncertainties in the values of these parameters allowed by the $\chi$EFT have a minor influence on the behavior of the EoS at saturation density. However, they significantly affect the behavior of the EoS at higher densities (above $\sim 2\rho_{\text{sat}}$). The energy of neutron matter below $\rho_c (\rho_{\text{sat}})$ becomes larger for the model with smaller $L_{\text{sym}}$ ($Q_{\text{sat}}$) [58, 60].

We also illustrate in Fig. 1 some typical cases for the neutron matter EoS that are outside the $\chi$EFT band. For $L_{\text{sym}} = 80 \text{ MeV}$ considerable deviation from $\chi$EFT result in the very low-density regime is observed, although this value is still consistent with the bounds $L_{\text{sym}} = 58.7 \pm 28.1 \text{ MeV}$ obtained from the combined analysis of astrophysical constraints and terrestrial experiments [109]. As seen in Fig. 1, the energy is slightly overestimated compared to the $\chi$EFT calculations for $Q_{\text{sat}} = -1000 \text{ MeV}$ in the very low-density regime. This shows that the influence of $Q_{\text{sat}}$ on the behavior of the EoS at subsaturation density is vanishing for $Q_{\text{sat}} \lesssim -500 \text{ MeV}$. In the following, we shall restrict our attention to those EoS models which satisfy the constraints on low-density neutron matter from $\chi$EFT calculations [51].

The discussion above (see Fig. 1) is based on the DD-ME2 parameterization. Since there is compensation between the isoscalar and isovector channels, a change in the parameterization for isoscalar channel will change the constraints for the isovector channel, and vice versa. However, the change of the Lagrangian or even the form of the functional (for example to the NL form [57]) does not change the general features deduced above. In addition, our setup does not allow us to vary freely the higher order parameters, such as $K_{\text{sym}}$ or $Z_{\text{sat}}$ in Table I, because once the low-order characteristic is fixed, the higher order ones are the predictions of our density functionals. This is in contrast to the models based entirely on Taylor expansions [58–61], where higher-order characteristics were varied at will.
FIG. 2. Gross properties of compact stars for nucleon–hyperon (NY) and nucleon–hyperon–Delta-resonance (NYΔ) compositions. The maximum mass $M_{\text{max}}$ (upper panels), the corresponding radii $R_{\text{max}}$ (middle panels), and the radii $R_{M_{\text{sat}}}$ for the canonical mass stars (lower panels) are varied by tuning individually the energy $E_{\text{sat}}$ (a), the density $\rho_{\text{sat}}$ (b), and the Dirac mass $M'_{\text{ff}}$ (c), with the remaining parameters being fixed. The vertical shading in each figure indicates the effect of varying the values of parameters around their mean value considering 1σ deviation. The yellow shadings show the mass of PSR J0348+0432 [2]. The light-blue shadings indicate the spreads of the upper limit on the radius for a canonical $1.4M_\odot$ mass neutron star set by recent analysis of the tidal deformability determined from the GW170817 event [5–10].

B. Uncertainties in characteristics and compact stars

We now study the correlations between the gross properties of compact stars and each nuclear characteristics at saturation density. We base our exploration on the DD-ME2 interaction by varying individually the seven characteristics within their empirical uncertainty ranges. (Recall that we vary one characteristic at a time, i.e., all others are held fixed at their default values defined by the DD-ME2 parametrization.) It is worthwhile to point out that although the five macroscopic characteristics listed in Eq. (9) together with $\rho_{\text{sat}}$ and $M'_{\text{ff}}$ simultaneously affect the EoS of dense matter, they are treated as independent of each other in the present analysis.

Figures 2 and 3 show the gross properties of compact stars with hyperon (NY) and Delta resonance (NYΔ) compositions. We vary individually the isoscalar characteristics $E_{\text{sat}}$, $\rho_{\text{sat}}$, $M'_{\text{ff}}$, $K_{\text{sat}}$ and $Q_{\text{sat}}$, and the isovector characteristics $E_{\text{sym}}$ and $L_{\text{sym}}$. For illustrative purposes we fix the meson-Δ couplings by assuming the Δ potential satisfies the condition $V_\Delta(\rho_{\text{sat}}) = V_{\Sigma}(\rho_{\text{sat}})$. It should be mentioned that one has to modify all the 5 parameters in isoscalar sector in order to vary the $\rho_{\text{sat}}$, $E_{\text{sat}}$, and $M'_{\text{ff}}$, while one needs to modify only the 3 density-dependent parameters instead to vary the characteristics $K_{\text{sat}}$ and $Q_{\text{sat}}$. In this context, variations of $K_{\text{sat}}$ and $Q_{\text{sat}}$ do not impact the meson-hyperon and meson-Δ couplings at nuclear saturation density.

If hyperons and no Δ’s are admixed in the stellar matter, the first hyperon to appear is the Σ, which is followed by the Ξ− hyperon. The Σ hyperons are disfavored due to their repulsive potential at nuclear saturation density. This sequence of hyperon thresholds is consistent with the recent relativistic hypernuclear computations of Refs. [19, 50, 78]. As a result, the hyperons appear in compact stars with masses with $M_{\text{max}} \geq 1.5M_\odot$, i.e., masses larger than the canonical pulsar mass. When Δ resonances are taken into account by taking $V_\Delta(\rho_{\text{sat}}) = V_{\Sigma}(\rho_{\text{sat}})$, Δ$^-$ is the first isobar to be populated around $2\rho_{\text{sat}}$; its number density grows and reaches the number density of protons at $\sim 3\rho_{\text{sat}}$. At even higher densities it is gradually replaced by the Ξ$^−$ hyperons around $4\rho_{\text{sat}}$. It has been shown that Δ resonances soften the EoS at low densities but stiffen it at high densities [31]. (The corresponding particle content of matter will be discussed below in Fig. 5.) It is thus seen that the overall trends are rather similar when varying individually the characteristics for NY and NYΔ matter. The difference between the two compositions is clearly reflected in the radius of a canonical neutron star. Note that in the entire parameter space considered, the purely nucleonic EoS models always predict a maximum mass of neutron star which is larger than $2M_\odot$.

It is seen from Fig. 2 that the maximum mass of a star $M_{\text{max}}$ (the corresponding radius $R_{M_{\text{sat}}}$) decreases with $E_{\text{sat}}$ and $\rho_{\text{sat}}$, while it increases with $M'_{\text{ff}}$. The radius for a canonical star $R_{M_{\text{sat}}}$ exhibits similar correlation. These features indicate that the gross properties of compact stars are sensitive to the values of $\rho_{\text{sat}}$ and $M'_{\text{ff}}$. Since the parametrizations presented in Fig. 2 all satisfy the $\chi$EFT constraint for neutron matter, we show instead the 1σ deviations that are evaluated from the available density-dependent relativistic mean-field (DDRMF) parametrizations (DD-ME [72, 88], DD [110–112], PKDD [113] and TW99 [71]). It is clearly seen that this model is well constrained with respect to the characteristics $\rho_{\text{sat}}$, $E_{\text{sat}}$, and $M'_{\text{ff}}$ within ~ 2%. Therefore, the effect of varying the value of $\rho_{\text{sat}}$ (or $M'_{\text{ff}}$) around the mean value at the level of 1σ deviation is not significant. It is worthwhile to mention that even the lowest order characteristics, for instance, the $E_{\text{sat}}$ and $\rho_{\text{sat}}$ could be different among different type of models, and the differences could be larger than the standard deviations. Indeed, while nonrelativistic models predict $\rho_{\text{sat}} \approx 0.160 \pm 0.004$ fm$^{-3}$ [89, 90], the relativistic models have a significantly smaller value $\rho_{\text{sat}} \approx 0.150 \pm 0.003$ fm$^{-3}$ [50, 91]. In this context, the difference in the saturation density from relativistic and nonrelativistic models yields already considerable effects on gross properties of compact stars; see Fig. 2(b).

We now turn our attention to the assessment of the effects of higher order characteristics which are shown in Fig. 3. The vertical shading indicates the constraints from $\chi$EFT calculations. Besides this, we have checked that all the constrained parameter sets can reasonably reproduce the binding energies and charge radii of a number of (semi-)closed-shell nuclei with ~ 2% relative deviation. As seen from Fig. 3 (a) and (c), the maximum mass $M_{\text{max}}$ is independent of the symmetry energy $E_{\text{sym}}$, while it shows a weak negative correlation with the slope parameter $L_{\text{sym}}$. The corresponding radius of the
maximum-mass star $R_{\text{M, max}}$ is essentially independent on the value of $L_{\text{sym}}$ (and $E_{\text{sym}}$), while the radius of a canonical neutron star $R_{\text{M, sat}}$ is strongly and almost linearly dependent on the value of $L_{\text{sym}}$. It is interesting to note that the key two astrophysical constraints available presently, i.e., $M_{\text{max}} \gtrsim 2.0M_{\odot}$ and $R_{\text{M, sat}} \leq 13.8$ km, favor small $L_{\text{sym}}$. As pointed out in Refs. [92, 114, 115], there is a correlation between the radius of 1.4$M_{\odot}$ stars and the magnitude of $L_{\text{sym}}$. However, once the constraints placed by $\chi$EFT calculations are implemented, the isovector characteristics $E_{\text{sym}}$ and $L_{\text{sym}}$ have a very small influence on the gross properties of compact stars. For example, the variations in the maximum mass turn out to be of the order of 0.05 $M_{\odot}$. The variations in the radius of a canonical neutron star are of the order of 0.4 km.

We further find that $M_{\text{max}}$, $R_{\text{M, max}}$ and $R_{\text{M, sat}}$ display positive correlation with the isoscalar characteristics $K_{\text{sat}}$ and $Q_{\text{sat}}$, as shown in Fig. 3 (b) and (d), respectively. The correlations are almost linear for $K_{\text{sat}}$ but more complex for $Q_{\text{sat}}$. While isoscalar skewness $Q_{\text{sat}}$ induces variation in both the maximum mass and radius, it largely controls the maximum mass of compact stars because it is most effective in modifying the EoS at supra-saturation densities. The seemingly stronger correlation between $M_{\text{max}}$ and $Q_{\text{sat}}$ compared to the correlations of $M_{\text{max}}$ with the other six variables is because of the relatively larger uncertainty in $Q_{\text{sat}}$. In fact, the quality of the resultant model depends not only on the form of the functional but, in addition, on the data used for its calibration. Indeed, even within the same functional form, the spread in values of $Q_{\text{sat}}$ is very large, covering the range $\sim -500 < Q_{\text{sat}} < 500$ MeV [71, 88]. The constraint, $M_{\text{max}} \gtrsim 2.0M_{\odot}$, favors larger values for $Q_{\text{sat}}$ (or $K_{\text{sat}}$), but the constraint, $R_{\text{M, sat}} \leq 13.8$ km, favors smaller values for $Q_{\text{sat}}$ (or $K_{\text{sat}}$). Notice also the significant reduction of $R_{\text{M, sat}}$ for negative values of $Q_{\text{sat}}$ as seen in Fig. 3 (d). We present a set of alternative parametrizations that preserve these values of $p_{\text{sat}}$, $E_{\text{sat}}$, and $M_{\text{D}}^*$ as DD-ME2, but produce different values of $K_{\text{sat}}$ and/or $Q_{\text{sat}}$ in Table III of the Appendix.

It is interesting to notice that the maximum mass gradually saturates at the value $2.1M_{\odot}$ with an increase of the $Q_{\text{sat}}$. We conclude that our study indicates an upper limit $\sim 2.1M_{\odot}$ on the maximum mass of compact stars with hyperon mixing. This prediction will be further confirmed below, specifically in Fig. 6. Interestingly, the value we find is compatible with the most recent inferences on the maximum mass of neutron stars, $M_{\text{max}} \sim 2.17M_{\odot}$ [116–118].

We now compare purely nucleonic compact stars with those which contain hyperonic matter. To support a purely nucleonic star with a mass of about $2.0M_{\odot}$, $Q_{\text{sat}}$ needs to be just $Q_{\text{sat}} \gtrsim -650$ MeV, leading to a value of $R_{\text{M, sat}}$ can be small as $\sim 11.8$ km. Once one allows for hyperonization, $Q_{\text{sat}}$ has to be, at least, as large as $\sim 300$ MeV. Thus, once the value of the maximum mass of a compact star is pinned down, it will put a stringent upper limit on the $Q_{\text{sat}}$ parameter. However, such a limit will largely depend on the composition of matter.

In Fig. 4 we show the EoS models of stellar matter and the mass-radius (hereafter MR) relations for purely nucleonic, hyperon admixed and hyperon-$\Lambda$ admixed matter for the allowed ranges of $Q_{\text{sat}}$ (left panels) and $L_{\text{sym}}$ (right panels). It is clearly seen that the same microscopic and astrophysical constraints lead to different EoS and MR relation depending on the assumed particle content. The appearance of hyperons and/or $\Delta$ resonances softens the EoS from baryon density $\rho \sim 2.5\rho_{\text{sat}}$, which corresponds to the threshold of the $\Lambda$ and/or $\Delta$ production. Furthermore, the displayed MR relations
show that $L_{\text{sym}}$ has an appreciable effect on the radii of less massive stars ($M \leq 1.4M_\odot$), whereas the $Q_{\text{sat}}$ has more significant effect on the radii of heavier stars ($M \geq 1.4M_\odot$). The canonical neutron stars are just at the intersection where the effects of $Q_{\text{sat}}$ and $L_{\text{sym}}$ on the radii are comparable, which implies that some combinations of $Q_{\text{sat}}$ and $L_{\text{sym}}$ can lead to the same $R_{M_{\odot}}$. Therefore, $Q_{\text{sat}}$ or $L_{\text{sym}}$ values alone are insufficient to characterize the low-density (up to around $2\rho_{\text{sat}}$) behavior of EoS within relativistic density functional theory. Our conclusion is consistent with that in recent metamodeling for the nucleonic EoS [59]. Notice however that in our models the hyperons/resonances could appear already in canonical neutron stars.

Finally, we examine the effect of varying the value of $Q_{\text{sat}}$ (upper panel) and $L_{\text{sym}}$ (lower panel) on particle fraction which is shown in Fig. 5. By changing the value of $Q_{\text{sat}}$ in the interval $[300, 800]$ MeV which corresponds to $\sim 50\%$ variations around its default value $\sim 480$ MeV from DD-ME2, we observe that the effect of changing $Q_{\text{sat}}$ on the onset density of $\Lambda^-$ and $\Lambda$ appears to be rather small, while its effect on the onset density of $\Xi^-0$ is more visible. This is because $Q_{\text{sat}}$ characterizes the medium- and high-density behavior of the isoscalar component of EoS. As a result, the particle fractions shown in Fig. 5 (a) and (b) differ to some for $\rho \geq 3.5\rho_{\text{sat}}$. Varying the value of $L_{\text{sym}}$ in the interval $[40, 60]$ MeV which corresponds to $\sim 20\%$ variations around its default DD-ME2 value $\sim 50$ MeV, we find that a larger value of $L_{\text{sym}}$ pushes up the threshold density of $\Lambda^-$, while the onsets of $\Lambda$ and $\Xi$ are shifted down. Since within our model the isospin asymmetry of stellar matter generally decreases with the increase of density, and the coupling constant of vector meson decreases exponentially with density [see Eq. (4)], the isovector field is largely suppressed at a higher density. As a result, the particle fractions shown in Fig. 5 (c) and (d) become identical for density $\gtrsim 4\rho_{\text{sat}}$.

In closing this section let us note that the value of $M_{\text{max}}$ for a compact star is basically determined by the isoscalar characteristics of the EoS, i.e., $\rho_{\text{sat}}$, $K_{\text{sat}}$, and $Q_{\text{sat}}$. The so-called “hyperon puzzle” [43] is therefore mainly related to the isoscalar skewness coefficient $Q_{\text{sat}}$ that characterizes the medium- and high-density behavior of EoS. On the other hand, the radius of the star is determined by both the isoscalar and isovector characteristics of the EoS. The constraints on neutron matter EoS coming from $\chi$EFT do not allow for a wide variation of $L_{\text{sym}}$, therefore one is left with the potential variations of $Q_{\text{sat}}$ for the determination of the radius of a $1.4M_\odot$ star. Thus, we conclude that the observations of massive compact stars and advanced determinations of stellar radii will potentially constrain the value of $Q_{\text{sat}}$ within our model setup. It should be stressed again that the theoretically inferred value of $Q_{\text{sat}}$ depends on the composition of matter, and to a certain extent, the detailed form of the density functional.
M (MeV)

FIG. 6. Contour plots for the gross properties of compact stars in the parameter space spanned by \( L_{\text{sym}} \) and \( Q_{\text{sat}} \) (both in MeV) with two fixed values of \( K_{\text{sat}} = 220 \) and 280 MeV. Shown are the maximum mass and the radius of a canonical 1.4\( M_\odot \) mass star with \( NY \) (a, b) and \( NY\Lambda \) (c, d) compositions. The solid lines indicate the configurations that have a maximum mass equal to 1.97\( M_\odot \). The dashed lines show the constraints from \( \chi^{\text{EFT}} \) calculations.

C. Observational constraints in the \( Q_{\text{sat}}-L_{\text{sym}} \) plane

Having established some general trends by varying only one of the parameters (i.e. only one of the dimensions in the parameter space) we would like to explore next the parameter space when two dimensions are varied. Our focus will be on the characteristics \( Q_{\text{sat}} \) and \( L_{\text{sym}} \) which are less constrained so far. We use alternative parametrizations that produce \( K_{\text{sat}} = 220 \) and 280 MeV, respectively, but preserve these values of \( \rho_{\text{sat}} \), \( E_{\text{sat}} \), and \( M_P^2 \) as DD-ME2. Figure 6 shows the value of the maximum-mass star, and the radius of a canonical 1.4\( M_\odot \) star with \( NY \) and \( NY\Lambda \) compositions, computed by simultaneously varying both \( Q_{\text{sat}} \) and \( L_{\text{sym}} \).

Comparing Fig. 6 (a) and (b), we observed that, (i) to satisfy the constraints imposed by \( \chi^{\text{EFT}} \), the value of \( L_{\text{sym}} \) has to be smaller for the EoS model which has small \( K_{\text{sat}} \), in order to enhance the contribution from the symmetry energy at very low density, see Fig. 1 (a); (ii) to support compact stars with the maximum mass \( M_{\text{max}} \geq 1.97 M_\odot \), the value of \( Q_{\text{sat}} \) has to be larger for the EoS model which has small \( K_{\text{sat}} \), to compensate the smaller contribution from the \( K_{\text{sat}} \); (iii) the maximum masses \( M_{\text{max}} \) (radii of 1.4\( M_\odot \) stars) predicted by EoS models with \( K_{\text{sat}} = 220 \) MeV are typically \( \sim 0.1M_\odot \) (\( \sim 0.3 \) km) smaller than those by EoS models with \( K_{\text{sat}} = 280 \) MeV; as already observed in Fig. 3 (b). As a consequence, the uncertainties in the isovector characteristics will impact the uncertainty intervals of the isoscalar characteristics, and vice versa; the uncertainty in a lower-order characteristic will impact somewhat the uncertainty interval of a higher-order characteristics. Such an interplay between the characteristic parameters have been discussed previously in the metamodeling approach to nuclear EoS [119].

Consider now the radius of a canonical 1.4\( M_\odot \) star with \( NY \) composition. For those EoS models that satisfy the \( \chi^{\text{EFT}} \) and \( M_{\text{max}} \geq 2 M_\odot \) constraints, the predicted radius of a 1.4\( M_\odot \) star spans from 12.8 km (defined by the EoS with \( K_{\text{sat}} = 220 \), \( L_{\text{sym}} \approx 35 \), and \( Q_{\text{sat}} \approx 450 \) MeV) to 13.6 km (defined by the EoS with \( K_{\text{sat}} = 280 \), \( L_{\text{sym}} \approx 60 \), and \( Q_{\text{sat}} \approx 800 \) MeV). Notice that the values \( K_{\text{sat}} = 220 \) MeV and \( L_{\text{sym}} \approx 35 \) MeV adopted here are already very close to the current lower bound of constraints on them placed by the combined analysis of terrestrial experiments [101, 109]. Therefore, the parameter space left for further reduction of the radius of a 1.4\( M_\odot \) star appears to be rather small.

When the \( \Delta \) resonances are taken into account and one sets \( V_\Delta(\rho_{\text{sat}}) = V_N(\rho_{\text{sat}}) \) the magnitude of \( Q_{\text{sat}} \) may be taken to be slightly smaller (\( \sim 100 \) MeV), than in the case of \( NY \) mat-
FIG. 8. Radii of canonical mass stars \( R_{\text{M_{\odot}}} \) versus (a) the slope of symmetry energy \( L_{\text{sym}} \) and (b) the isoscalar skewness coefficient \( Q_{\text{sat}} \) at saturation density. The open symbols mark the cases where the maximum-mass is below \( 1.97M_{\odot} \). The vertical shadings indicate the constraints from \( \gamma \)EFT calculations [51]. The light-blue shadings show the spreads of the upper limit for canonical 1.4\( M_{\odot} \) neutron stars set by recent analysis of the tidal deformability determined by GW170817 event [5–10]. The dashed lines mark the most likely value of 12.4 km set in Ref. [7].

FIG. 9. EoS models and the corresponding MR relations for \( NY\Delta \) stellar matter in the parameter space \( (Q_{\text{sat}}, L_{\text{sym}}, V_{\Delta}) \) (in MeV) that support a 1.4\( M_{\odot} \) neutron star with radius about 12.5 km. The yellow shading represents the probable (2\( \sigma \)) radii of neutron stars estimated in Ref. [7].

...ter in order to obtain a 2\( M_{\odot} \) star. Indeed the inclusion of \( \Delta \) resonances results in a larger maximum mass [31].

In Fig. 7 we summarize the EoS models (upper panels) and the corresponding MR relations (lower panels) for \( NY \) and \( NY\Delta \) compositions with \( K_{\text{sat}} = 220 \) MeV (left panels) and \( K_{\text{sat}} = 280 \) MeV (right panels) respectively, restricting the \( (Q_{\text{sat}}, L_{\text{sym}}) \) space to that shown in Fig. 6. We also show the constraints on the EoS obtained at the 90% posterior credible level (90%CI) from the binary neutron star merger event GW170817 [9], the posterior for the mass and radius of each binary component using EoS-insensitive relations in Ref. [9], as well as the probable (2\( \sigma \) region) radii of neutron stars estimated from a very large range of hadronic EoS by imposing constraints on the maximum mass and the tidal deformability [7]. Since the exotic degrees of freedom were not considered in Ref. [9], the band corresponding to 90%CI constraints on the high-density regime are not shown here. As can be seen from Fig. 7 (a) and (c), for the low-density region \( 0.5 \leq \rho/\rho_{\text{sat}} \leq 3 \), our collection of EoS are fully consistent with the inference of Ref. [9]. The radii predicted by those models for a star with the canonical mass 1.4\( M_{\odot} \) lie close to the upper range of the radii inferred from the analysis of tidal deformability from the binary neutron star inspiral event GW170817 [7, 9]. The MR relations generated by EoS models which have \( K_{\text{sat}} = 220 \) MeV appear to be in better agreement with the 2\( \sigma \) domain inferred in Ref. [7]. However, it should be noted that, below the onset density of hyperons/resonances (~ 2.5\( \rho_{\text{sat}} \)), our EoS models span a small region of the inferred band, namely our EoS models do not represent all possible models compatible with GW170817, but only a subset of them.

Finally, it is worthwhile to note that if the vector meson-hyperon couplings are drawn from within the SU(3) flavor symmetric model [19, 50], rather than SU(6) spin-flavor sym-

D. Canonical mass stars with small radii

As mentioned in the previous section, the constraints on the tidal deformability have allowed for the determination of the statistically most probable radius of a 1.4\( M_{\odot} \) neutron stars. For instance, by imposing constraints on the maximum mass and on the dimensionless tidal deformability, it has been shown that a purely hadronic neutron star has \( 12.0 \leq R_{\text{M_{1.4}}} \leq 13.5 \) km with a 2\( \sigma \) confidence level, with a most likely value of 12.4 km [7]. In Ref. [8] the binary neutron star mergers with different prior choices of masses have been analyzed. Using Bayesian parameter estimation the authors concluded that the radius range is \( 8.9 < R_{\text{M_{1.4}}} < 13.2 \) km, with an average value of \( R_{\text{M_{1.4}}} = 10.8 \) km [8].

The possibility that hyperonic stars have small radii in the range above is as exciting as it is challenging for nuclear theory. Notice that small radii demand a sufficiently soft EoS below 2-3\( \rho_{\text{sat}} \), while the observed large masses require that the same EoS must be able to evolve into a stiff EoS at high densities. In Ref. [31], it was argued that \( \Delta \) resonances soften the EoS at low densities but stiffen it at high densities, resulting in significantly reduced radii and larger maximum masses of compact stars. The \( \Delta \) resonances are therefore an interesting degree of freedom for the modeling of small-radius stars. The effects of \( \Delta \) resonances have been illustrated for the case of \( V_{\Delta} = V_{\chi} \) in previous sections. We next explore further their effects by varying the \( \Delta \)-potential.

Figure 8 shows the radius for a 1.4\( M_{\odot} \) star as a function of \( L_{\text{sym}} (Q_{\text{sat}}) \) and \( V_{\Delta} \), while the remaining characteristic parameters are set as default values in Table I. As expected, the
changes in the \( \Delta \)-potential \( V_\Delta \) have a stronger effect on the radius. The appearance of \( \Delta \) resonances reduces the radius of a canonical star by up to 1 km for a reasonably attractive \( \Delta \)-potential \( V_\Delta = 5/3V_N \), thus producing a radius which is closer to the inferred most likely value 12.4 km obtained by Ref. [7]. It is worth noticing that the reduction is not sensitive to the values of \( Q_{\text{sat}} \) and/or \( L_{\text{sym}} \).

We present in Fig. 9 several EoS models and the corresponding MR relations for \( N\Delta \) matter in the parameter space \( (Q_{\text{sat}}, L_{\text{sym}}, V_\Delta) \) that reproduce a 1.4\(M_\odot \) neutron star with a radius about 12.5 km. The particle fractions for two EoS models are shown in Fig. 10 for illustration. As can be seen in such configuration the \( \Delta^- \) appears already at \( \sim 1.5\rho_{\text{sat}} \), and \( \Delta^0 \) appears at intermediate densities. At the very central part of a canonical neutron star, the concentration of \( \Delta^- \) resonance is close to that of protons.

Finally, in Fig. 11 we show the limits on the radius of a 1.4\(M_\odot \) canonical neutron star from the work which used the data on GW170817 event, along with the radii obtained from our EoS models assuming purely nucleonic (\( N \)), hyperon admixed (\( NY \)), and hyperon-\( \Delta \) admixed (\( N\Delta \)) particle composition. We recall that our limits are set on the \( (K_{\text{sat}}, Q_{\text{sat}}, L_{\text{sym}}) \) parameter space, by restricting \( K_{\text{sat}} \in [220, 280] \) MeV. The value of the \( \Delta \)-potential \( V_\Delta \) is varied from 2/3\( V_N \) to 5/3\( V_N \). Further reduction of the radius up to 2 km can be obtained if one further decrease the \( V_\Delta \), see Ref. [31] for a detailed discussion. It is clearly seen from Fig. 11 that our estimate of the upper limit of \( R_{M_{1.4}} \) is consistent with other analyses [7–10]. The upper limit is in fact set by the purely nucleonic EoS, whereas the lower limit essentially depends on the assumed particle composition. Our hyperon-\( \Delta \) admixed EoS models \( (V_\Delta >> 5/3V_N) \) place a lower limit of about 11.6 km. Interestingly, this limit is rather close to the one set by the purely nucleonic EoS models. This underlines our argument that while the EoS uniquely determines the MR relation, it does not allow one to extract information on the composition of dense matter. A canonical-mass star with small radius could be interpreted not only as a purely nucleonic object, but also as hypernuclear star admixed with \( \Delta \) resonances [12, 31] or, alternatively, a hybrid star containing a quark matter core [11, 120]. Furthermore, it appears that \( R_{M_{1.4}} \leq 11 \) km is marginally compatible with our present knowledge of the nuclear, hypernuclear and \( \Delta \) resonance physics data. Canonical mass stars with small radii (less than 11 km) may therefore indicate the possibility of hadron-quark phase transition at density around \( 2\rho_{\text{sat}} \) [5, 7, 10, 11, 39].

IV. SUMMARY

Using the EoS for hadronic matter satisfying the latest constraints from both terrestrial nuclear experiments and astrophysical observations at saturation, as well as \( \chi \)EFT of low-density neutron matter, we found that the gross properties of compact stars are very sensitive to the higher-order empirical parameters of nuclear matter around the saturation density, specifically the isoscalar skewness \( Q_{\text{sat}} \) and isovector slope \( L_{\text{sym}} \). These are not well constrained from the experimental side, while \( L_{\text{sym}} \) is constrained somewhat by \( \chi \)EFT.

We observe that the \( Q_{\text{sat}} \) is the dominant parameter controlling both the maximum mass and the radius of a compact star. This is due to the fact that, on the one hand, the isovector characteristics \( E_{\text{sym}} \) and \( L_{\text{sym}} \) in Eq. (9) weakly influence the maximum mass, on the other hand, the strong restriction on the allowed values of \( L_{\text{sym}} \) coming from \( \chi \)EFT does not allow for noticeable variations in the radius.

Another important point is that the upper limit on \( Q_{\text{sat}} \) is essentially dependent on the assumed particle composition of stellar matter. Our exploration of the parameter space shows that hyperonic stars more massive than 2\( M_\odot \) would require
TABLE II. Meson masses and meson-nucleon coupling constants in the DD-ME2 parametrization [88], whereby \( g_{\sigma N} \) refer to the values at the saturation density.

| \( m_\sigma \) | \( m_\omega \) | \( m_\rho \) | \( g_{\sigma N} \) | \( g_{\omega N} \) | \( g_{\rho N} \) | \( a_\sigma \) |
|---|---|---|---|---|---|---|
| 550.1238 | 783.0000 | 763.0000 | 10.5396 | 13.0189 | 3.6836 | 0.5647 |

\( a_\sigma \) | \( b_\sigma \) | \( c_\sigma \) | \( d_\sigma \) | \( a_\omega \) | \( b_\omega \) | \( c_\omega \) | \( d_\omega \)
|---|---|---|---|---|---|---|---|
| 1.3881 | 1.0943 | 1.7057 | 0.4421 | 1.3892 | 0.9240 | 1.4620 | 0.5647 |

TABLE III. Alternative parametrization of the density dependence of the couplings in the isoscalar channels for the indicated values of \( K_{\text{sat}} \) (MeV) and/or \( Q_{\text{sat}} \) (MeV). The values of \( g_{\sigma N} \) and \( g_{\omega N} \) are the same as in the DD-ME2 parametrization, see Table II.

| \( K_{\text{sat}} \) | \( Q_{\text{sat}} \) | \( a_\sigma \) | \( b_\sigma \) | \( c_\sigma \) | \( d_\sigma \) | \( a_\omega \) | \( b_\omega \) | \( c_\omega \) | \( d_\omega \)
|---|---|---|---|---|---|---|---|---|---|
| 200 | 480 | 1.4851 | 1.1012 | 1.8753 | 0.4216 | 1.4843 | 0.8786 | 1.5293 | 0.4669 |
| 220 | 480 | 1.4469 | 1.1074 | 1.8214 | 0.4278 | 1.4469 | 0.9013 | 1.5110 | 0.4745 |
| 240 | 480 | 1.4088 | 1.1015 | 1.7498 | 0.4365 | 1.4096 | 0.9165 | 1.4803 | 0.4822 |
| 260 | 480 | 1.3707 | 1.0802 | 1.6573 | 0.4485 | 1.3722 | 0.9212 | 1.4334 | 0.4938 |
| 280 | 480 | 1.3328 | 1.0401 | 1.5413 | 0.4650 | 1.3348 | 0.9117 | 1.3670 | 0.4938 |
| 300 | 480 | 1.2953 | 0.9756 | 1.3970 | 0.4885 | 1.2976 | 0.8815 | 1.2740 | 0.5115 |
| 250 | -600 | 1.3501 | 0.1798 | 0.3299 | 1.0052 | 1.3788 | 0.1467 | 0.2905 | 1.0711 |
| 250 | -300 | 1.3406 | 0.3380 | 0.5619 | 0.7702 | 1.3611 | 0.2813 | 0.4915 | 0.8235 |
| 250 | 0 | 1.3477 | 0.5546 | 0.8807 | 0.6152 | 1.3612 | 0.4655 | 0.7647 | 0.6602 |
| 250 | 300 | 1.4077 | 0.8555 | 1.3353 | 0.4996 | 1.3752 | 0.7205 | 1.1493 | 0.5385 |
| 250 | 600 | 1.4077 | 1.2814 | 2.0136 | 0.4069 | 1.4049 | 1.0809 | 1.7136 | 0.4410 |
| 250 | 900 | 1.4730 | 1.9201 | 3.0965 | 0.3281 | 1.4571 | 1.6107 | 2.5947 | 0.3584 |
| 220 | 0 | 1.3993 | 0.6123 | 1.0183 | 0.5721 | 1.4140 | 0.4990 | 0.8630 | 0.6215 |
| 220 | 300 | 1.4244 | 0.8934 | 1.4672 | 0.4766 | 1.4306 | 0.7284 | 1.2282 | 0.5210 |
| 220 | 600 | 1.4657 | 1.2752 | 2.1075 | 0.3977 | 1.4610 | 1.0361 | 1.7367 | 0.4381 |
| 250 | 900 | 1.5312 | 1.8144 | 3.0791 | 0.3290 | 1.5106 | 1.4654 | 2.4873 | 0.3661 |
| 280 | 0 | 1.2987 | 0.4670 | 0.7119 | 0.6843 | 1.3107 | 0.4051 | 0.6354 | 0.7243 |
| 280 | 300 | 1.3147 | 0.7809 | 1.1600 | 0.5361 | 1.3206 | 0.6825 | 1.0316 | 0.5684 |
| 280 | 600 | 1.3494 | 1.2550 | 1.8662 | 0.4226 | 1.3485 | 1.1020 | 1.6520 | 0.4492 |
| 280 | 900 | 1.4143 | 2.0165 | 3.0866 | 0.3286 | 1.4038 | 1.7774 | 2.7166 | 0.3503 |

\( Q_{\text{sat}} \gtrsim 200 \text{ MeV} \), leading to a radius \( R_{M_{1A}} \gtrsim 12.8 \text{ km} \). Including in the composition, in addition, the \( \Delta \) resonances reduce the radius of a canonical mass star by about 1 km for a reasonably attractive \( \Delta \)-potential, in agreement with previous findings [12, 29, 31].

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Appendix: Meson-nucleon coupling constants

The parameters of the DD-ME2 effective interaction are shown in Table II. As we already discussed in the text, this model is well constrained with respect to the characteristics \( \rho_{\text{sat}}, E_{\text{sat}}, \) and \( M_{D}^* \). In Table III we further present a set of alternative parametrizations that preserve these values of \( \rho_{\text{sat}}, E_{\text{sat}}, \) and \( M_{D}^* \), but produce different values of \( K_{\text{sat}} \) and/or \( Q_{\text{sat}} \). Notice that to this end one needs to modify only the parameters in functions \( f_{\sigma N} \) and \( f_{\omega N} \) [see Eq. (3)] that control the density dependence of the couplings in the isoscalar sector. As the density dependence of the couplings in the isovector sector is parametrized by an exponential form given by Eq. (4), the modification for isovector sector is rather simple: one first determines \( g_{\rho N} \) by the preassigned value of \( E_{\text{sym}} \) and then fixes \( a_{\rho} \) by the desired value of \( L_{\text{sym}} \).
