Full Quantum Theory of $C_{60}$ Double-slit Diffraction

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In this paper, we apply the full new method of quantum theory to study the double-slit diffraction of $C_{60}$ molecules. We calculate the double-slit wave functions of $C_{60}$ molecules by Schrödinger equation, and calculate the diffraction wave function behind the slits with the Feynman path integral quantum theory, and then give the relation between the diffraction intensity of double-slit and diffraction pattern position. We compare the calculation results with two different double-slit diffraction experiments. When the decoherence effects are considered, the calculation results are in good agreement with the two experimental data.

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1. Introduction

Matter waves diffraction is a well established field in physics which has become a large field of interest throughout the past years [1, 2]. Diffractions with de-broglie waves have been demonstrated for electrons and neutron, and extensively used for fundamental tests of quantum-mechanical prediction [3-5]. Recently, there are classical and quantum methods to study interference and diffraction [6, 7]. As a matter of fact, matter-wave interference and diffraction are quantum phenomena and its full description needs quantum mechanical approach [8, 9]. At present, Phenomena of diffraction have been studied in many experiments [10-12]. Such as the electron diffraction experiment in the crystal by Davison and Germer in 1927. The electron single and double slit diffraction experiment by J nsson in 1961. The neutron single and double slit diffraction experiment by Anton Zeilinger, Roland G hler and C.G.Shull in 1988, and these experiments have been explained by some theoretical workers. In view of quantum mechanics, the $C_{60}$ has the nature of wave, and the wave is described by wave function $\psi(r, t)$, which can be calculated with the Schrödinger wave equation [13, 14]. The wave function $\psi(r, t)$ has statistical meaning, $|\psi(r, t)|^2$ can be explained as particle’s probability density at the definite position [15, 16]. In this paper, we apply the full new method of quantum theory to study the double-slit diffraction of $C_{60}$ molecules, and the $C_{60}$ wave functions can be divided into three parts. The first is the incoming area, the $C_{60}$ wave function is a plane wave. The second is the slit area, where the $C_{60}$ wave function can be calculated by the Schrödinger wave equation. The third is the diffraction area, where the $C_{60}$ wave function can be obtained by Feynman path integral quantum theory. Otherwise, we give the relation between the diffraction intensity of double-slit and diffraction pattern position. When we consider the decoherence effects in calculation, we find that the theory results are in good agreement with the experimental data.

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2. Quantum approach of $C_{60}$ diffraction

In an infinite plane, we consider a double-slit, its width $a$, length $b$ and the slit-to-slit distance $d$ are shown in FIG. 1. The $x$ axis is along the slit length $b$ and the $y$ axis is along the slit width $a$. We calculate the $C_{60}$ wave function in the left slit with the Schrödinger equation, and the $C_{60}$ wave function of the right slit can be obtained easily. At time $t$, we suppose that the incoming plane wave travels along the $z$ axis. It is

$$\psi_0(z, t) = \hat{A} \exp \frac{i}{\hbar} (pz - Et),$$

(1)

where $A$ is a constant. The time-dependent Schrödinger equation is

$$\left[ -\frac{\hbar^2}{2M} \nabla^2 + V(r) \right] \psi(\vec{r}, t) = E \psi(\vec{r}, t),$$

(2)

where $M(E)$ is the mass(energy) of the $C_{60}$. The potential in the left slit is

$$V(x, y, z) = \begin{cases} 0 & 0 \leq x \leq b, \quad -\frac{d}{2} - a \leq y \leq -\frac{d}{2}, \quad 0 \leq z \leq c, \\ \infty & \text{otherwise,} \end{cases}$$

(3)

where $c$ is the thickness of slit.

When $V = 0$, the time-independent Schrödinger equation in left slit is

$$-\frac{\hbar^2}{2M} \nabla^2 \psi_1(\vec{r}) = E \psi_1(\vec{r}),$$

(4)

the partial differential Eq. (4) can be solved by the method of separation of variable.

$$\frac{\partial^2 \psi_1(\vec{r})}{\partial x^2} + \frac{\partial^2 \psi_1(\vec{r})}{\partial y^2} + \frac{\partial^2 \psi_1(\vec{r})}{\partial z^2} + \frac{2ME}{\hbar^2} \psi_1(\vec{r}) = 0,$$

(5)

the wave function $\psi_1(x, y, z)$ satisfies the boundary conditions

$$\psi_1(0, y, z) = \psi_1(b, y, z) = 0,$$

$$\psi_1(x, -a - \frac{d}{2}, z) = \psi_1(x, -\frac{d}{2}, z) = 0.$$

(6)

By the method of separation of variable $\psi_1(x, y, z) = X(x)Y(y)Z(z)$, the general solution of Eq. (4) is

$$\psi_{1mn}(x, y, z) = \sum_{m,n} \left( D_{mn} \sin \frac{n\pi x}{b} \cos \frac{m\pi y}{a} + D'_{mn} \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \right) \exp \left[ i \sqrt{\frac{2ME}{\hbar^2} - \frac{n^2\pi^2}{b^2} - \frac{m^2\pi^2}{a^2}} z \right],$$

(7)
the general solution of time-dependent Schrödinger equation is

\[ \psi_{1mn}(x, y, z, t) = \sum_{m,n} \left( D_{mn} \sin \frac{n\pi x}{b} \cos \frac{m\pi y}{a} + D'_{mn} \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \right) \exp \left[ i \sqrt{\frac{2ME}{\hbar^2} - \frac{n^2\pi^2}{b^2} - \frac{m^2\pi^2}{a^2} z} \right] e^{-iE_it}, \]  

(8)

since the wave functions are continuous at \( z = 0 \), we have

\[ \vec{\psi}_0(x, y, z, t) \mid_{z=0} = \vec{\psi}_{1mn}(x, y, z, t) \mid_{z=0}. \]  

(9)

Then

\[ A_1 = \sum_{m,n} \left( D_{mn} \sin \frac{n\pi x}{b} \cos \frac{m\pi y}{a} + D'_{mn} \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \right), \]  

(10)

we can obtain the Fourier coefficient \( D_{mn} \) and \( D'_{mn} \) by Fourier transform

\[ D_{mn} = \frac{4}{ab} \int_{-\frac{d}{2}}^{\frac{d}{2}} \int_{-\frac{d}{2}}^{\frac{d}{2}} A \sin \frac{n\pi x}{b} \cos \frac{m\pi y}{a} \, dx \, dy \]
\[ = -\frac{16A}{(2m+1)(2n+1)\pi^2} \sin \frac{(2m+1)\pi}{2a} d \quad m, n = 0, 1, 2, 3, \]  

(11)

\[ D'_{mn} = \frac{4}{ab} \int_{-\frac{d}{2}}^{\frac{d}{2}} \int_{-\frac{d}{2}}^{\frac{d}{2}} A \sin \frac{n\pi x}{b} \sin \frac{m\pi y}{a} \, dx \, dy \]
\[ = -\frac{16A}{(2m+1)(2n+1)\pi^2} \cos \frac{(2m+1)\pi}{2a} d \quad m, n = 0, 1, 2, 3, \]  

(12)

substituting (11) and (12) into (8), we can obtain the \( C_{60} \) wave function in the left slit,

\[ \psi_1(x, y, z, t) = -\sum_{m,n} \frac{16A_1}{(2m+1)(2n+1)\pi^2} \exp \left[ i \sqrt{\frac{2ME}{\hbar^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2} z} \right] \exp \left[ -\frac{i}{\hbar} E_it \right] \]
\[ \cdot \left[ \sin \frac{(2m+1)\pi}{2a} d \sin \frac{(2n+1)\pi x}{b} \cos \frac{(2m+1)\pi y}{a} \right. \]
\[ + \cos \frac{(2m+1)\pi}{2a} d \sin \frac{(2n+1)\pi x}{b} \sin \frac{(2m+1)\pi y}{a} \right]. \]  

(13)

Similarly, we can obtain the \( C_{60} \) wave function in the right slit

\[ \psi_2(x, y, z, t) = -\sum_{m,n} \frac{16A_2}{(2m+1)(2n+1)\pi^2} \exp \left[ i \sqrt{\frac{2ME}{\hbar^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2} z} \right] \exp \left[ -\frac{i}{\hbar} E_it \right] \]
\[ \cdot \left[ \sin \frac{(2m+1)\pi}{2a} d \sin \frac{(2n+1)\pi x}{b} \cos \frac{(2m+1)\pi y}{a} \right. \]
\[ - \cos \frac{(2m+1)\pi}{2a} d \sin \frac{(2n+1)\pi x}{b} \sin \frac{(2m+1)\pi y}{a} \right]. \]  

(14)
3. The wave function of $C_{60}$ diffraction

With Path Integral approach, we can calculate the $C_{60}$ wave function in the diffraction area. $C_{60}$ diffraction area diagram is shown in FIG. 2. $p_0$ is the position of a point on the surface ($z=c$), its position vector is $\vec{r}_0$, $p$ is an arbitrary point in the diffraction area and its position vector is $\vec{r}$. At time $t_0$, the particles at position $p_0$. By Eq. (13), we have

$$\psi_1(\vec{r},t) = -\sum_{m,n} \frac{16A_1}{(2m+1)(2n+1)\pi^2} \exp\{i\sqrt{\frac{2ME}{\hbar^2} - \frac{(2n+1)^2\pi^2}{b^2} - \frac{(2m+1)^2\pi^2}{a^2}}c\} \exp\{-\frac{i}{\hbar}Et\} \times \sin \frac{(2m+1)\pi}{2a}d\sin \frac{(2n+1)\pi x_0}{b} \cos \frac{(2m+1)\pi y_0}{a} + \cos \frac{(2m+1)\pi}{2a}d\sin \frac{(2n+1)\pi x_0}{b} \sin \frac{(2m+1)\pi y_0}{a}.$$

The diffraction wave function can be calculated by Path Integral formula[11], it is

$$\psi_1(\vec{r},t) = \int k(\vec{r};\vec{r}_0,t_0)\psi_1(\vec{r}_0,t_0)dr_0,$$

the propagator $k(\vec{r};\vec{r}_0,t_0)$ is

$$k(\vec{r};\vec{r}_0,t_0) = \left[\frac{M}{2\pi i\hbar(t-t_0)}\right]^\frac{3}{2} \cdot \exp\{\frac{iMR^2_1}{2\hbar(t-t_0)}\},$$

where $R$ is the distance between $p_0$ and $p$, and $(t-t_0)$ is the time of $C_{60}$ propagating from $p_0$ to $p$.

Substituting Eq. (17) into Eq. (16), we have

$$\psi_1(\vec{r},t) = \int \left[\frac{M}{2\pi i\hbar(t-t_0)}\right]^\frac{3}{2} \cdot \exp\{\frac{iMR^2_1}{2\hbar(t-t_0)}\}\psi_0(\vec{r}_0,t_0)dr_0,$$

where $dr_0 = dx_0dy_0$.

From Fig. 2, there is

$$R^2_1 = |\vec{r} - \vec{r}_0|^2 = (x-x_0)^2 + (y-y_0)^2 + (z-c)^2 \approx x^2 + y^2 + z^2 - 2xx_0 - 2yy_0 - 2zc + x_0^2 + y_0^2 + c^2$$

In Eq. (19), $x_0^2$, $y_0^2$ and $c^2$ are second-order infinitesimal, they can be ignored, then we have

$$R^2_1 = r^2 - 2r \sin \alpha \cdot x_0 - 2r \sin \beta_1 \cdot y_0 - 2r \cos \theta \cdot c,$$
where \( \alpha \) is the angle between \( r \) and \( yz \) plane, \( \beta_1 \) is the angle between \( R_1 \) and \( xz \) plane, \( \theta \) is the angle between \( r \) and \( z \) axis.

Substituting Eq. (15) and (20) into Eq. (18), the diffraction wave function of the left slit is

\[
\psi_{\text{out1}}(\vec{r}, t) = \frac{M}{2\pi i \hbar(t - t_0)} \left[ \exp \left( -\frac{i}{\hbar} E_{t_0} \right) \exp \left[ -\frac{i}{\hbar} \sum m = 0 \right. \left. \frac{16A_1}{(2m + 1)(2n + 1)^2} \exp \left[ i \sqrt{2ME/h^2 - \left( \frac{(2n + 1)^2 + (2m + 1)\pi}{a} \right)^2} \cdot c \right] \right] \right]
\]

i.e.,

\[
k = \frac{2\pi}{\lambda} = \frac{MV}{\hbar},
\]

so

\[
\left[ \frac{M}{2\pi i \hbar(t - t_0)} \right]^2 = \left[ \frac{MR_1}{2\pi i \hbar(t - t_0)} \right]^2 = \left( \frac{k}{2\pi i} \right)^2 = \left( -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \right) \left( \frac{k}{2\pi i} \right)^2,
\]

and

\[
\exp \left[ \frac{iMr^2}{2\hbar(t - t_0)} \right] = \exp \left( \frac{ikr}{2} \right).
\]

Substituting Eqs. (23)-(25) into Eq. (21), the diffraction wave function of the left slit is

\[
\psi_{\text{out1}}(\vec{r}, t) = \left( -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \right) \left( \frac{k}{2\pi i} \right)^2 \exp \left( \frac{ikr}{2} \right) \exp \left( -\frac{i}{\hbar} E_{t_0} \right) \exp \left[ -\frac{i}{\hbar} \sum m = 0 \right. \left. \frac{16A_1}{(2m + 1)(2n + 1)^2} \exp \left[ i \sqrt{2ME/h^2 - \left( \frac{(2n + 1)^2 + (2m + 1)\pi}{a} \right)^2} \cdot c \right] \right] \]

\[
\int_0^b \exp \left[ -\frac{iM r}{\hbar(t - t_0)} \sin \alpha x_0 \right] \sin \frac{(2m + 1)\pi x_0}{2a} \int_{-\frac{\pi}{4} - a}^{\frac{\pi}{4}} \exp \left[ -\frac{iM r}{\hbar(t - t_0)} \cdot y_0 \right] \sin \frac{(2m + 1)\pi y_0}{a} dy_0
\]

\[
\cos \frac{(2m + 1)\pi}{a} y_0 dy_0 + \cos \frac{(2m + 1)\pi d}{2a} \int_{-\frac{\pi}{4} - a}^{\frac{\pi}{4}} \exp \left[ -\frac{iM r}{\hbar(t - t_0)} \cdot y_0 \right] \sin \frac{(2m + 1)\pi y_0}{a} dy_0.
\]
Similarly, the diffraction wave function of the right slit is

$$
\psi_{\text{out}2}(\vec{r}, t) = (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i)\left(\frac{k}{2\pi r}\right)^2 \exp\left(\frac{ikr}{2}\right) \exp\left(-\frac{i}{\hbar} E_0 t\right) \exp[-ik\cos\theta \cdot c] \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{-16 A_2}{(2m+1)(2n+1)\pi^2} \exp[i\sqrt{2ME/b} - \left((2n+1)\pi^2 - \left((2m+1)\pi^2\right)^2\right) c] \int_{0}^{b} \exp[-ik\sin\alpha \cdot x_0] \sin\left(\frac{2n+1}{b}\pi x_0 dx_0\right) \sin\left(\frac{2m+1}{2a}\pi d \cdot \int_{\frac{a}{2}}^{\frac{d+a}{2}} \exp[-ik\sin\beta_2 \cdot y_0] \right) \left[\cos\left(\frac{2m+1}{a}\pi y_0 dy_0\right) - \cos\left(\frac{2m+1}{2a}\pi d \cdot \int_{\frac{a}{2}}^{\frac{d+a}{2}} \exp[-ik\sin\beta_2 \cdot y_0\sin\left(\frac{2m+1}{a}\pi y_0 d y_0\right)]\right] \right] (27)
$$

Where $\beta_2$ is the angle between $R_2$ and $xz$ plane, from FIG. 2, since $a + d \ll L$, there is $\beta_1 \simeq \beta_2 \approx \beta$ and $\theta$, $\alpha$ and $\beta$ are satisfied the relation: $\cos^2\theta + \sin^2\alpha + \sin^2\beta = 1$.

The total diffraction wave function for the double-slit is

$$
\psi(x, y, z, t) = c_1 \psi_{\text{out}1}(x, y, z, t) + c_2 \psi_{\text{out}2}(x, y, z, t),
$$

where $c_1, c_2$ satisfy the equation

$$
c_1^2 + c_2^2 = 1.
$$

For the double-slit diffraction, we can obtain the relative diffraction intensity $I$ on the display screen

$$
I \propto |\psi(x, y, z, t)|^2 = c_1^2|\psi_{\text{out}1}(x, y, z, t)|^2 + c_2^2|\psi_{\text{out}2}(x, y, z, t)|^2 + 2c_1c_2Re[\psi_{\text{out}1}^*(x, y, z, t)\psi_{\text{out}2}(x, y, z, t)].
$$

(30)

4. The relative diffraction intensity $I$ on the display screen

Decoherence is introduced here using a simple phenomenological theoretical model that assumes an exponential damping of the interferences [17], i.e., the decoherence is the dynamic suppression of the interference terms owing to the interaction between system and environment. Eq. (28) describes the coherence state coherence superposition, without considering the interaction of system with external environment. When we consider the effect of external environment, the total wave function of system and environment for the double-slit factorizes as [17]

$$
\psi_{\text{out}}(x, y, z, t) = c_1 \psi_{\text{out}1}(x, y, z, t) \otimes |E_1 >_t + c_2 \psi_{\text{out}2}(x, y, z, t) \otimes |E_2 >_t,
$$

(31)

where $\otimes|E_1 >_t$ and $\otimes|E_2 >_t$ describe the state of the environment. Now, the diffraction intensity on the screen is given by [3]

$$
I = (1 + |\alpha_t|^2)[c_1^2|\psi_{\text{out}1}(\vec{r}, t)|^2 + c_2^2|\psi_{\text{out}2}(\vec{r}, t)|^2 + 2c_1c_2|\psi_{\text{out}1}(\vec{r}, t)\psi_{\text{out}2}(\vec{r}, t)|] + 2c_1c_2|\psi_{\text{out}1}(\vec{r}, t)\psi_{\text{out}2}(\vec{r}, t)|],
$$

(32)

where $\alpha_t = |E_2|/|E_1| >_t$, and $\Lambda_t = 2|\alpha_t|^2$. Thus, $\Lambda_t$ is defined as the quantum coherence degree. The fringe visibility of $n$ is defined as [17]

$$
v = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}},
$$

(33)

where $I_{\text{max}}$ and $I_{\text{min}}$ are the intensities corresponding to the central maximum and the first minimum next to it, respectively. The value for the fringe visibility of $\nu = 0.625$ is obtained in the experiment [18], and the quantum coherence degree $v \simeq \Lambda_t$ [17]. Eq. (32) is the diffraction intensity of $C_{60}$ double-slit diffraction including decoherence effects, and Eq. (30) is the diffraction intensity of $C_{60}$ double-slit diffraction considering coherence superposition.
5. Numerical result

Next, we shall give out the calculation results of $C_{60}$ double-slit diffraction intensity $I$, and compare the calculation results with experiment data, which were carried out by two diffraction experiment device with different experiment parameters and experiment data in Ref. [18] and Ref. [19]. In Ref. [18] and Ref. [19], the authors have given the relation between diffraction intensity and pattern position. In Eqs. (26) and (27), we should convert diffraction angle $\beta$ to positions from FIG. 2, we can find the relation is $\sin \beta = \frac{R}{R}$, where $R = |\vec{r}|$ and $L$ is the distance between the slit and screen, which are shown in FIG. 2. In calculation, we take the same experiment parameters in Ref. [18] and Ref. [19]. Firstly, we study the experiment [18], its slits width $a = 47.5 \text{nm}$, the wave length of the $C_{60}$ $\lambda = 2.4 \times 10^{-12} \text{m}$, the two slits distance $d = 52.5 \text{nm}$, the distance between the slit and screen $L = 1.25 \text{m}$. The theory parameters are taken: $A_1 = 1.6 \times 10^{12}$, $A_2 = 1.7 \times 10^{12}$, $c_1 = 0.915$, $c_2 = 0.40345(|c_1|^2 + |c_2|^2 = 1)$, and the quantum coherence degree $\nu = 0.53$, which is calculated by Eq. (33) with the experiment data [18]. From Eq. (33), we can obtain the calculation result, which is shown in FIG. 3. In FIG. 3, the solid line is theoretical calculation curve and the circle points are experiment data. we find that the theoretical result is in accordance with the experiment data in Ref. [18]. Finally, we study the experiment [19], its slits width $a = 42 \text{nm}$, the wave length of the $C_{60}$ $\lambda = 4.8 \times 10^{-12} \text{m}$, the two slits distance $d = 86 \text{nm}$, the distance between the slit and screen $L = 1.25 \text{m}$. The theory parameters are taken: $A_1 = 5.35 \times 10^{13}$, $A_2 = 2.1 \times 10^{13}$, $c_1 = 0.9075$, $c_2 = 0.42(|c_1|^2 + |c_2|^2 = 1)$, and the quantum coherence degree $\nu = 0.88$, which is calculated by Eq. (33) with the experiment data [19]. From Eq. (33), we can obtain the calculation result, which is shown in FIG. 4. In FIG. 4, the solid line is theoretical calculation curve and the circle points are experiment data. we find that the theoretical result is also in accordance with the experiment data in Ref. [19].

6. Conclusion

In conclusion, we apply the new full quantum theory to study the double-slit diffraction of $C_{60}$ molecules, and compare the theoretical prediction with experimental data taken from Refs. [18] and [19]. When the decoherence effects are considered, we can find the theoretical results are in accordance with two experimental data. This approach has universal applicability, such as, it can also study the diffraction of electron, neutron and atom, which include their multi-slit and grating diffractions.
FIG. 4: Comparing the calculation result with the experiment data [19].

1 Anton Zeilinger, Roland Gahler, C. G. Shull and Walter Mampe, Rev. Mod. Phys. 60, 1067 (1988)
2 Brian J Smith and M GRaymer, New J.phys.9,414(2007)
3 Tschernitz M,GahlerR,ZeilingerA,eta.lPrecision measurement of single slit diffraction with very cold neutrons [J].
Phys Lett A.26,365-379.(1992)
4 W. Schöllkopf and P. J. Toennies, Science 266,1345(1994)
5 O. Carnal and J. Mlynek, Phys. Rev. Lett. 66, 2689 (1991)
6 O.Nairz, M. Arudt and A. Zeilinger, J. Mod. Opt. 47, 2811 (2000)
7 D.Writh,”Beam widths of a diffracted laser using four proposed methods,”Opt.Quantum Electron.24,S1129-
S1135(1992)
8 S. Kunze, K. Dieckmann and G. Rempe, Phys. Rev. Lett. 78, 2038 (1997)
9 Torres-Ruiz F A, Lima G, Delgado A 2010 Phys. Rev. A.81,042104(2010)
10 A. Viale, M. Vicari and N. Zanghi, Phys. Rev. A 68, 063610 (2003).
11 R. Tumulka, A. Viale and N. Zanghi, Phys. Rev. A 75, 055602 (2007)
12 J.D. Jackson Classical Electrodynamics (Chichester: John Wiley-Sons) chap 10 p 579. (1999)
13 Angelo M D, Chekhova M V, Shih Y, Two-Photon Diffraction and Quantum Lithography[J]. Phys. Rev. Lett. 87,
013602-4,(2001).
14 D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, Observation of single-Photon Ghost Interference
and Diffraction[J]. Phys. Rev. Lett. 74,3600-3603,(1995)
15 Brown R H, Twiss R Q. A test of a new type of stellar interferometer on sirius [J]. Nature,178,1046-1048.(1956)
16 Francis A J, White H E Fundamentals of Optics (McGraw-Hill, Inc. Press, New York, 1976) p. 315-326,(1976)
17 A. Viale, M. Vicari, N. Zanghi, Phys. Rev. A 68, 063610 (2003)
18 M. Arudt, O. Nairz, J. Voss-Andreae, C. Kweller, G.Vander Zouw and A. Zeilinger, Nature 401, 680 (1999)
19 Olaf Nairz Markus Arndt, and Anton Zeilingerb.Quantum interference experiments with large molecules [J]. American
Association of Physics Teachers.A-1090(2002)