Finite element modeling of the pull-apart formation: Implication for tectonics of Beng Co pull-apart basin, Southern Tibet

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The tectonic deformation and state of stress are significant parameters to understanding the active structure, seismic phenomenon and overall ongoing geodynamic condition of any region. In this paper, we have examined the state of stress and crustal deformation during the formation of the Beng Co pull-apart basins produced by an en-échelon strike-slip fault systems using 2D Finite Element Modeling (FEM) under plane stress condition. The numerical modeling technique used for the experiments is based on FEM which enables us to analyze the static behavior of a real and continuous structures. We have used three sets of models to explore how the geometry of model (overlap fault and pre-existing weak shear zone) and applied boundary conditions (pure strike-slip, transpressional and transtensional conditions) influence the development of state of stress and deformation during the formation of pull-apart basins. Modeling results presented here are based on five parameters: (i) distribution, orientation and magnitude of maximum (σH max) and minimum (σH min) horizontal compressive stress; (ii) magnitude and orientation of displacement vectors; (iii) distribution and concentration of strain; (iv) distribution of fault type and; (v) distribution and concentration of maximum shear stress (τmax) contours. The modeling results demonstrate that the deformation pattern of the en-échelon strike-slip pull-apart formation is mainly dependent on the applied boundary conditions and amount of overlap between two master strike-slip faults. When the amount of overlap of the two master strike-slip faults increases, the surface deformation gets wider and longer, but when the overlapping between two master strike-slip faults is zero, block rotation observed is significant and only narrow and small surface deformation is obtained. These results imply that overlapping between two master strike-slip faults is a significant factor in controlling the shape, size and morphology of the pull-apart basin formation. Results of numerical modeling further show that the pattern of the distribution of maximum shear stress (τmax) contours prominently depends on the amount of overlapping between two master strike-slip faults and applied boundary conditions. In case of more overlapping between two master strike-slip faults, τmax was mainly concentrated at the two corners of the master faults which later reduces and finally reaches zero at the centre of the pull-apart basin, whereas in case of no overlapping, τmax was largely concentrated at the two corners and tips of the master strike-slip faults. These results imply that the distribution and concentration of the maximum shear stress (τmax) is mainly governed by the amount of overlapping between the master strike-slip faults in the en-échelon pull-apart formation. Numerical results further highlight that the distribution patterns of the displacement vectors are mostly dependent on the amount of overlapping and applied boundary conditions in the en-échelon pull-apart formation.

Key words: State of stress, deformation regime, pull-apart formation, numerical modeling, Southern Tibet.

INTRODUCTION

Pull-apart basins are the prominent feature of topographic depression structures formed as a result of crustal extension associated with either right-lateral right-stepping or left-lateral left stepping en-échelon strike-slip fault systems (Katzman et al., 1995; Petrunin and Sobolev, 2008). They usually show a rhombic to spindle shape and occur at different ranges of size from small sag ponds of few millimeters up to several kilometers.
such as the Dead Sea basins (Burchfiel and Steward, 1966; Sylvester, 1988). The ratio between the length and width of the pull-apart basins mainly varies between 3 and 4 (Ayden and Nur, 1982), but recorded pull-apart basins from different parts of the world show significance differences in their geometry and structural characteristics (Ayden and Nur, 1982; Gamond, 1983; Bahat, 1983). Several mechanisms have been proposed for the formation of the pull-apart basins but the common types of mechanism are: (i) Local extension between two en-échelon basement strike-slip fault segments; (ii) A distributed simple strike-slip shear mechanism and; (iii) The Riedel shear mechanisms (Figure 1). The relative motion of the crust blocks involved in a pull-apart system can either be parallel or oblique, and can be divided into pure strike-slip, transtensional or transpressional conditions (Figure 2).

Pull-apart basins are the preferred sites of concentrated fracturing (Connolly and Cosgrove, 1999), elevated heat flow (Hu et al., 2001) and intense seismicity (Armijo et al., 1986, 1989, 2002). Moreover, they have significant economical importance and can confine hydrocarbon (Harding, 1990), significant mineralization (Recherds et al., 2001) and geothermal fields (Monastero et al., 2005). In recent years, many pull-apart basins have been studied extensively in several parts of the world (Burchfiel and Stewart, 1966; Segall and Pollard, 1980; Armijo et al., 1986, 1989; Basile and Brun, 1999; Harding, 1990; Recherds et al., 2001; Monastero et al., 2005; Petrunin and Sobolev, 2008). Several continental pull-apart basins have also been documented in the Tibetan Plateau (Armijo et al., 1986, 1989) but, so far, very few studies that focus on the pull-apart basin have been done. Present study is the first attempt to model numerically Beng Co pull-apart basin in the Southern Tibet.

Numerical modeling is a powerful tool, which provides useful insights that are beyond direct observations e.g. stress state, characteristics structures, sequential evolution of the basin, deformation pattern during evolution of the basin, possible temperature regime and rheology during and after the pull-apart formation. Therefore, numerical models have been extensively applied for studying the pull-apart basins (Segall and Pollard, 1980; Bahat, 1983; Du and Aydin, 1993; Gölke et al., 1994; Katzman et al., 1995; Petrunin and Sobolev, 2006 and 2008). Segall and Pollard (1980) used the analytical models based on the infinitesimal strain theory. They maximized the displacement near the middle of the faults with the application of remote external stress. These models provide significant clues to the orientations of different faults which can develop inside the overstep area. Gölke et al. (1994) analyzed the vertical displacement and topographic variations in the releasing overstep along the master strike-slip faults by using finite element model. Katzman et al. (1995) applied the 3D boundary element models of pull-apart basin and compared the modeling results to the Dead Sea Basin. Their results show that the basin deformation mainly depends on the width of the shear zone and on the amount of the overlapping between the basin-bounding faults. Petrunin and Sobolev (2006 and 2008) presented the 3D thermo-mechanical models of the pull-apart basin developed at an overstepping of an active continental transform faults, and found that the thickness of the brittle layer beneath the basin has significant role in controlling the dimension and deformation pattern of the basin. From their modeling they further conclude that the deep narrow pull-apart basins are relatively well developed in cold lithosphere, as in the Dead Sea Basin and requires very low friction at major faults (Petrunin and Sobolev, 2008). Although numerical modeling studies have been applied extensively for simulating deformation in the pull-apart basins, but much less is known generally about the kinematics or geodynamics within the shallow pull-part structure, as it is filled by unconsolidated sediments, high structurally disrupted or crystallizing materials (veins/plutons).

The purpose of this paper is to understand the relationship among fault geometry, applied boundary conditions (pure strike-slip, transtensional and transpressional conditions) and imposed displacements with state of stress and tectonic deformation pattern within a releasing overstep along the two en échelon strike-slip pull-apart formation, applying different sets of models (Figure 3). We have used a series of 2D finite element calculations incorporating elastic rheology under plane stress condition. Mohr-Coulomb failure criterion is assumed to predict the position, orientation and type of faults in the pull-apart basin formation.

GEOLOGY AND TECTONIC SETTING

The tectonic evolution and uplift of the Tibet Plateau are a result of tectonic events which occurred due to Indian and Asian plate convergence (Molnar and Tapponier, 1975). The continuing northward movement of the Indian plate for the past 10 years has lead to the Tibet Plateau experiencing widespread extension as indicated by the large scale normal faults and strike-slip zones that made several extensional features such as graben, rift-systems and pull-apart basins in Late Quaternary time (Armijo et al., 1986; Molnar and Tapponier, 1977; Mercier et al., 1987). The tectonic evolution and contemporary states of stress on the Tibetan plateau are mainly governed by E-W extension and N-S compression. The present day average state of stress of the Tibetan Plateau is subject to an extensional ($\sigma_3$) axis trending 112° ± 6° and the minimum horizontal stress ($\sigma_{H\ min}$) trajectory trends WNW-ESE. The compressional ($\sigma_1$) axis trends 022° ± 6° and the maximum horizontal stress ($\sigma_{H\ max}$) trends N-S to
NNE - SSW direction, roughly parallel to the Indian-Eurasian convergence in the central part of the India-Asia collision zone (Mercier et al., 1987).

The Beng Co is an en échelon strike-slip pull-apart basin named after the 25 km long and 7 km wide Beng Co Lake. It developed within the Late Ceneozoic time (Armijo, 1986), and is located at 31°10’ N and 91°10’E (Figure 4). Beng Co pull-apart basin is about 40 km long with an average strike of north 122°E originating from the long side of Beng Co and extending toward the NW and SE strike-slip fault zone. Geological field observations along the Beng Co can identify two major fault strands and they are composed of series of en échelon pull-apart basins. An en échelon arrangement of the mole tracks in the field implies possible evidence of the right-lateral strike-slip nature of the Beng Co pull-apart (Armijo et al., 1989). The Beng Co Fault Zones (BCFZ) cut obliquely across folded Jurassic black shale and calcschists, whereas the southern branch of the fault zone runs mostly in the granites and the Jurassic shales (Allegere et al., 1984). The northern exposure of the BCFZ cuts highly folded, early-to-middle Cretaceous red sandstone, which lies non-conformably upon the Jurassic shales (Armijo et al., 1989). Further northwest, it passes through the area where ophiolites have been thrust southward on the Jurassic shales and truncates towards the gently folded conglomerates. The southern branch of the BCFZ lies along the southern edge of a NW-SE granite range.

**SEISMICITY OF THE REGION**

The Tibet Plateau is one of the highest and most active region of the world, which evolved as a consequence of the collision between India and Eurasia landmasses about 50 millions ago (Molnar and Tapponier, 1975). The continuous northward penetration of Indian crust within Eurasia resulted in significant amount of stress accumulation, causing intense seismicity and active tectonic nature of the plateau. In the Tibetan Region, seismicity is observed mostly from shallow to intermediate depths (Torre et al., 2007). Generally, the seismic pattern shows
diffusion in nature and does not follow any known particular tectonic trends. The focal mechanisms solutions here are predominantly of normal and strike-slip type, which are further attributed to the large scale E-W extension of the region (Torre et al., 2007).

The field observations provide several evidences of Quarternary displacements, ruptures and large offsets on either side of the Beng Co pull-apart basin. Several prominent, continuous and fresh surface breaks with large numbers of paleoseismic events along the zone imply that the Beng Co pull-apart region is seismotectonically active in contemporary time. Evidence includes several major earthquakes including November 17 and 18, 1951 (Ms = 8); 17 August, 1952; 28 December,
Figure 3. Plan view of the formation of pull-apart basin geometry (a) before and (b) during and (c) after tectonic deformation. Source: Joshi and Hayashi (2009).

1951 and 12 July, 1972, which show a magnitude of (M) > 6, and are located near the southern extremity of the Beng Co pull-apart (Figure 4).

MODELING TECHNIQUE

The numerical modeling technique used for the experiments is based on a Finite Element Modeling (FEM) which enables us to analyze the static and behavior of a real and continuous structure. FEM has successfully proved to be a powerful method for simulating pull-apart basin geometries and deformation mechanisms (Segall and Pollard, 1980; Bahat, 1983; Du and Aydin, 1993; Gölke et al., 1994; Katzman et al., 1995; Petrunin and Sobolev, 2006 and 2008). In this study, we applied a 2D-finite element software package developed by Hayashi (2008), which has been used widely by Joshi and Hayashi, (2008a, 2008b), Chamlagain and Hayashi (2007), Dwivedi and Hayashi (2009), Koirala and Hayashi (2009). Similar to most mesh-based numerical methods, bodies of rocks in this program are represented by triangular elements and each element is assigned appropriate material properties, such as density, Young’s modulus, cohesion and angle of internal friction. The mesh deforms and moves with respect to material and is able to compute appropriate deformation in the program. The details of mathematical formulations about the software package have already been described by Hayashi (2008).

Model set up

The dimension of the models are 42 km in length and 7.5 km in width, which mimics the natural dimension of the Beng Co pull apart basin adopted after Armijo et al. (1989) (Figure 4). We simplified the model and divided its area into triangular mesh and several domains. The initial mesh of the model consists of 546 nodal points, 984 triangular elements and two master right-lateral strike-slip faults. In the model, we assumed that the upper crust is a brittle layer and is treated as elastic material. In order to simulate the brittle deformation mechanism of the model, we adopt elastic rheology under plane stress conditions. In our model, the crust up to 20 km is considered to behave as an elastic material because of its brittle nature and presence of earthquake and faults. Rocks forming
the brittle crust of the earth contain heterogeneity, which may result in differences compared with our homogeneous and uniform model. In spite of these limitations, our models are still able to yield valuable information related to the pull-apart formation.

**Boundary conditions**

For the modeling purpose, a two dimension Cartesian rectangular simplified model which shows original geometry of the Beng Co pull-apart basin has been adopted after Armijo et al. (1989) (Figure 4). Far-field plate velocity boundary conditions are enforced at the either side of the Beng Co Fault Zones (BCFZ). The brittle crust is divided into three simple domains, which may exhibit dissimilar rock layer properties. Domain 1 and 2 represent the southern and northern flank of the pre-existing BCFZ, and domain 3 represents surrounding regions. We consider typical two types of models (i) a model with a pre-existing pull-apart basin and (ii) a model without pre-existing pull-apart basin. The model without a pre-existing pull-apart basin is further tested into different overlap/separation ratios (Model B and Model C). We imposed three types of reasonable boundary conditions to mimic the possible natural strike-slip environment of the pull-apart formation. These displacement boundary conditions are (i) pure strike-slip (ii) transtensional and (iii) transpressional conditions (Figure 5). The empirical 100 to 500 m displacements were imposed from northern-left and southern-right corners in different boundary environments, and only 10% of imposed displacement is considered for transtensional and transpressional conditions for modeling (Figure 5).

**BC1: Pure strike-slip model**

The pure en échelon strike-slip boundary conditions were obtained by moving the upper left-hand and lower-right hand corners using displacement in the left (-X) and right (+X) directions while the lower and upper edges are fixed (Figure 5a). This boundary condition explores the effect of pure-strain-strike slips movements on the overall stress field and faulting regime on the pull-apart formation.

**BC2: Transtensional model**

The transtensional boundary conditions were simulated by moving the upper left - hand and lower - right hand corners using displacement in the left (-X) and right (+X)
directions, and adding an outward displacement in left (-Y) and right (+Y) directions to the lower and upper edges of the model respectively (Figure 5b). This boundary condition provides the opportunity to understand the distribution and orientation of the stress field and deformation style of the transtensional environment of the pull-apart formation.

**BC3: transpressional model**

In order to investigate the state of stress and overall deformation of the strike-slip pull-apart basin we applied transpressional boundary condition. The transpressional boundary condition was obtained by moving the upper left-hand and lower-right hand corners, using displacement in the left (-X) and right (+X) directions, and adding an inward displacements in left (-Y) and right (+Y) directions to the lower and upper edges, respectively (Figure 5c).

**Mechanical parameters and rock domain property**

The mechanical properties such as density ($\rho$), Young's modulus ($E$), Poisson's ratio ($\nu$), angle of internal friction ($\phi$) and cohesive strength ($c$) are important rock parameters in the FEM analysis (Table 1). The density ($\rho$) was obtained from the interval velocity of the individual rock domain, using the relation proposed by Barton (1986) and compared them with published velocity model (Zhao et al., 1993) from southern Tibet. Seismic P-wave ($V_p$) and S-wave ($V_s$) velocities are chosen from the published literature of the study area (Cogan, et al., 1998). Two independent elastic constants were used: Young’s modulus and Poisson’s ratio to solve the following elastic equations in the brittle part of the lithosphere (Timoshenko and Goodier, 1970; Hayashi, 2008):

$$ E = \rho V_p^2 \frac{(1 + \nu)(1 - 2\nu)}{(1 - \nu)} $$

$$ \nu = \frac{1}{2} \left[ 1 - \frac{1}{(V_p - V_s)^2 - 1} \right] $$

Where $E$ - Young’s modulus, $\nu$ - Poisson’s ratio, $\rho$ - density of rock, $V_p$ - seismic P-wave velocity and $V_s$ - seismic S-wave velocity.

In performing FEM calculation, the whole model is divided into 3 domains and each domain has been allocated distinct rock layer properties on the basis of predominant rock types (Table 1). In case of Model A, we assume that BCFZ is pre-existing weak shear zones.
which allowed us to adopt the value of Young’s modulus less compared to other rock domain (Pauselli and Federico, 2003). And C was obtained from the Handbook of Physical Constants (Clark, 1966).

**MODELING RESULTS**

To understand the various factors that control the induced state stress and deformation pattern of pull-apart basin formation, we have carried out a number of modeling experiments for two characteristic models (i) with a pre-existing pull-apart basin and (ii) without a pre-existing pull-apart basin. In case of without pre-existing pull-apart basin we further calculated by two separate models (Models B and C), where Model B represents no overlap ratio between the master faults and Model C corresponds to considerable overlap between two master faults. Each of these models were run for the three most common types of boundary conditions of pull-apart formation (i) BC1: pure strike-slip (ii) BC2: transtensional, and (iii) BC3: transpressional. The modeling results are represented based on (i) the maximum (\(\sigma_{H_{\text{max}}}\)) and minimum (\(\sigma_{H_{\text{min}}}\)) horizontal principle stress (ii) magnitude and orientation of the displacement vectors (iii) distribution and magnitude of the strain type and (iv) concentration and distribution of the maximum shear stress (\(\tau_{\text{max}}\)) contours. The direction and magnitude of the maximum compressive stress axis and minimum compressive stress axis are represented by \(\sigma_1\) and \(\sigma_3\). Furthermore, we calculated faulting regime on the Beng Co pull-apart basin based on the relation and position of the \(\sigma_1\), \(\sigma_2\) and \(\sigma_3\), using Mohr-Coulomb failure criterion.

**Model A: Pre-existing pull-apart basin**

Figures 6, 7 and 8 illustrate the orientation of the maximum (\(\sigma_{H_{\text{max}}}\)) and minimum (\(\sigma_{H_{\text{min}}}\) ), horizontal principle stress trajectories, strain distribution, displacement vectors contour lines of maximum shear stress (\(\tau_{\text{max}}\)) and development of a faulting regime for Model A in the pure strike-slip boundary condition. The calculated \(\sigma_{H_{\text{max}}}\) trajectories show almost N-S directional orientation with uniform distribution in the model and minor variation in the upper left and lower right corners, which corresponds to the direction of maximum shortening of the Tibetan Plateau (Figures 6a, 7a and 8a). Similarly, \(\sigma_{H_{\text{min}}}\) trajectories show more or less E-W orientation, which is consistent with the direction of maximum extension in the Tibetan Plateau (Figures 6b, 7b and 8b). However, some discrepancy was observed in the corners of the models which might be due to boundary effect. The orientations of the displacement vectors show prominent difference among three boundary conditions. The major discrepancy was obtained at the upper-right and lower-left corners of the pull-apart basin (Figures 6c, 7c and 8c). Figures 6d, 7d and 8d illustrate the predicted strain partitioning for Model A, where high extensional strain is mainly concentrated along pre-existing weak pull-apart zone. This is due to weak rheology, and is consistent with the applied model geometry. The predicted faulting pattern shows almost similar predominantly strike-slip type of faults for all boundary conditions (Figures 6e, 7e and 8e). Figures 6f, 7f and 8f show concentration and distribution patterns of modeled maximum shear stress (\(\tau_{\text{max}}\)) contours for all three boundary conditions, where \(\tau_{\text{max}}\) is largely confined at the central part of the pull-apart basin.

**Without pre-existing pull-apart basin**

In this case, two models (Models B and C) were used to calculate state of stress and deformation regime of the Beng Co pull-apart basin for understanding the effect of different overlap on the stress distribution. Figures 9-14 illustrate the calculated maximum (\(\sigma_{H_{\text{max}}}\)) and minimum (\(\sigma_{H_{\text{min}}}\) ) horizontal principle stress trajectories, strain distribution, displacement vectors, contour lines of maximum shear stress (\(\tau_{\text{max}}\)) and development of faulting regime of the Models B and C. In both models, orientations of the \(\sigma_{H_{\text{min}}}\) trajectories show more or less E-W directed orientation for all boundary conditions, which is consistent with E-W extension environment of the Tibetan Plateau. A comparison of the Models B and C shows that although the general stress (\(\sigma_{H_{\text{min}}}\) ) patterns remain similar, there are significance differences in the distribution and concentration of \(\tau_{\text{max}}\) (Figures 9f - 14f).

### Table 1. Mechanical parameters applied for the different domains in finite element modeling.

| Rock domain | \(\rho\) (kg/m\(^3\)) | \(E\) (GPa) | \(c\) (MPa) | \(\phi\) (deg.) |
|-------------|------------------|-------------|-------------|----------------|
| Domain 1    | 2900             | 60.0        | 24.0        | 50.0           |
| Domain 2    | 2000             | 01.0        | 10.0        | 31.0           |
| Domain 3    | 2000             | 01.0        | 10.0        | 31.0           |

**Abbreviations:** \(\rho\) = density; \(E\) = Young’s modulus; \(c\) = cohesion; \(\phi\) = Angle of internal friction.
Figure 6. Results of model A for pure strike-slip boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) trajectories (c) Strain distribution (d) Displacement vectors (e) Faulting regime and (f) Distribution of maximum shear stress ($\tau_{\text{max}}$) contours under 100 m boundary displacement condition at 10 km depth. Source: Joshi and Hayashi (2009).

Figure 7. Results of Model A for pure transtensional boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) Trajectories (c) Strain distribution (d) Displacement vectors (e) Faulting regime and (f) Distribution of maximum shear stress ($\tau_{\text{max}}$) contours under 100 m boundary displacement condition at 10 km depth. Source: Joshi and Hayashi (2009).
Figure 8. Results of model A for transpressional boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) trajectories (c) strain distribution (d) displacement vectors (e) faulting regime and (f) distribution of maximum shear stress ($\tau_{\text{max}}$) contours at 100 m boundary displacement condition under 100 m boundary displacement condition at 10 km depth. Source: Joshi and Hayasiy (2009).

Figure 9. Results of model B for pure strike-slip boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) trajectories (c) strain distribution (d) displacement vectors (e) faulting regime and (f) distribution of maximum shear stress ($\tau_{\text{max}}$) contours under 100 m boundary displacement condition at 10 km depth. Source: Joshi and Hayasiy (2009).
significant variations between Models B and C (Figures 9c - 14c). There are no considerable differences observed in the predicted strain partitioning among both models, where strain is mainly concentrated along the fault zone which is due to weak rheology. The predicted faulting pattern of the model exhibits almost similar predominantly strike-slip types of faults that have developed for all boundary conditions.

**Model B: without overlap on the pull-apart basin**

Model B illustrates the results of numerical simulation in the case of no pre-existing pull-apart basin and zero overlap of the two master strike slip faults in the model. Figures 9, 10 and 11 illustrate the orientation of $\sigma_{H,\text{max}}$ and $\sigma_{H,\text{min}}$ trajectories, displacement vectors, strain concentration, distribution of $\tau_{\text{max}}$ contours and faulting regimes. In this model orientation of $\sigma_{H,\text{max}}$ trajectories, strain concentration and faulting regimes show similar results for all boundary conditions at the same displacement, compared to models A and C. However, the results of displacement vectors and distribution of $\tau_{\text{max}}$ show considerable differences. Figures 9f, 10f and 11f show how differently $\tau_{\text{max}}$ is distributed for the different boundary conditions in Model B. Similarly, figures 8c, 9c and 10c illustrate the principal variations of predicted displacement vectors among three boundary conditions (that is BC1, BC2 and BC3) for Model B.

**Model C: with fault overlap on the pull-apart basin**

Model C predicted the results of numerical simulation taking into account pre-existing overlap of the two master strike slip faults in the Beng Co pull-apart basin. Figures 12, 13 and 14 illustrate the orientation of $\sigma_{H,\text{max}}$ and $\sigma_{H,\text{min}}$ trajectories, displacement vectors, strain concentration, distribution and accumulation of $\tau_{\text{max}}$ and overall faulting regimes for Model C. Results show that there are no considerable variations of the distribution and orientation of the predicted $\sigma_{H,\text{max}}$ and $\sigma_{H,\text{min}}$ trajectories, strain partitioning and faulting regime. Nevertheless, high discrepancies do exist in case of displacement vectors (Figures 12c, 13c and 14c) and distribution and concentration of $\tau_{\text{max}}$ contours (Figures 12f, 13f and 14f).

**DISCUSSIONS**

**Effect of pre-existing weak shear zone of pull-apart basin**

We first explore the effect of a pre-existing weak shear zone of pull-apart basin on the stress field and deformation pattern during formation of the pull-apart basin. Figures 7, 8 and 9 illustrate the modeling results of a pre-existing weak shear zone of Beng Co strike-slip pull-apart basin. In order to quantify the relative importance of a pre-existing strike-slip weak shear zone on the pull-apart basin the modeling results are compared between Models A and B. A close examination of results of these two models demonstrates that major disparities exist in the horizontal displacement vectors and distribution and concentration of $\tau_{\text{max}}$ contour lines, whereas minor differences also exist with regards to the orientation and magnitude of the horizontal principal stresses and deformation pattern.

**Effect of change in boundary conditions in pull-apart formation**

Boundary conditions are important factors for controlling the stress state and deformation patterns of the model. Therefore, we explore the effect of a change in boundary conditions on the stress field and deformation style in the formation of the pull-apart basin. In order to investigate the effect of boundary conditions in stress field and deformation patterns, we have tested three types (i) pure strike-slip (ii) transtensional and (iii) transpressional of boundary conditions. Figures 6 - 14 illustrate predicted modeling results of the $\sigma_{H,\text{max}}, \sigma_{H,\text{min}}, \tau_{\text{max}}$ displacement vector, strain partitioned and faulting regime for all three models. Modeling results clearly demonstrated that the distribution and concentration of $\tau_{\text{max}}$ and displacement vectors in each boundary condition varies significantly, while orientations of the $\sigma_{H,\text{max}}$ and $\sigma_{H,\text{min}}$ are moderately influenced and faulting regime is not effected by changing applied boundary conditions.

**Effect of change in fault overlap in pull-apart development**

The models that are considered here include two en-échelon faults having (i) with fault overlap (ii) zero fault overlap between them. Figures 9c - 14c illustrate the different results of the orientation of horizontal displacement vectors, and distribution and concentration of the $\tau_{\text{max}}$ contours, where the effect of a change in fault overlap can be seen. The large rotation of the horizontal displacement vector appears in the central part of the pull-apart basin with zero overlap model (Model B), while no significant rotation of displacement vector was observed in the overlap model (Model C). The model B produced a tentative rectangular pull-apart basin, whilst model C produced a narrow and small pull-apart basin. These results of numerical modeling imply that fault overlap geometry has a considerable influence on the change in shape, size and morphology of the pull-apart formation; this is consistent with other studies of Gölke et al. (1994). Our modeling results show that the change in fault overlap is significantly related to the size of the pull-apart formation such that if fault overlap increases, the
Figure 10. Results of Model B for pure transtensional boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) Trajectories (c) Strain distribution (d) Displacement vectors (e) Faulting regime and (f) distribution of maximum shear stress ($\tau_{\text{max}}$) contours under 100 m boundary displacement condition at 10 km depth. Source: Joshi and Hayasiy (2009).

Figure 11. Results of model B for transpressional boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) Trajectories (c) Strain distribution (d) displacement vectors (e) Faulting regime and (f) distribution of maximum shear stress ($\tau_{\text{max}}$) contours under 100 m boundary displacement condition at 20 km depth. Source: Joshi and Hayasiy (2009).
size of the pull-apart basin also increases considerably. Similarly, fault overlap geometry has significant effect on $\sigma_{H\min}$ orientation and distribution and concentration of the $\tau_{max}$ contours but there is no effect on the faulting regime (Figures 9-14).

Figure 12. Results of Model C for pure strike-slip boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) trajectories (c) Strain distribution (d) Displacement vectors (e) Faulting regime and (f) Distribution of maximum shear stress ($\tau_{max}$) contours under 100 m boundary displacement condition at 10 km depth. Source: Joshi and Hayashi (2009).

Figure 13. Results of model C for transpressional boundary conditions. (a) Maximum compressional stress ($\sigma_1$) trajectories (b) Maximum extensional stress ($\sigma_3$) Trajectories (c) Strain distribution (d) Displacement vectors (e) Faulting regime and (f) Distribution of maximum shear stress ($\tau_{max}$) contours under 100 m boundary displacement condition at 10 km depth. Source: Joshi and Hayashi (2009).

Effect of change in displacement in pull-apart formation

The applied displacement is another factor that strongly influences the magnitude and orientation of the stress...
CONCLUSION

A two-dimension finite element numerical model was used to simulate the strike-slip pull-apart basin formation. We examine the state of stress and deformation associated with the right-lateral en-échelon Beng Co pull-apart basin in the southern Tibetan Plateau. In this paper, we have considered three models each incorporating three different boundary conditions (pure strike-slip, transtensional and transpressional) with different amounts of fault overlap of the master strike-slip fault systems. Our modeling results demonstrate that the deformation pattern of the en-échelon strike-slip pull-apart formation is mainly dependent on the applied boundary conditions and the amount of overlap between two master strike-slip faults. When the amount of overlap of the shear zone increases, the surface deformation gets wider and longer between master faults, but if zero overlap exists between the two strike-slip fault systems then the narrow and block rotation is observed within the pull-apart basin. We therefore conclude that overlap between two en-échelon strike-slip faults is a significant factor in controlling the overall shape, size and morphology of the pull-apart basin formation.

The pattern of the rotation of displacement vectors and maximum shear stress ($\tau_{\text{max}}$) distribution contours are also highly dependent on the applied boundary conditions and amount of overlap. In the case of a larger overlap, $\tau_{\text{max}}$ is mainly concentrated at the two corners of the master strike-slip faults and reduces toward the centre of the pull-apart basin, whereas for zero overlap conditions, $\tau_{\text{max}}$ is largely concentrated at the two corners and tips of the master strike-slip faults. These results imply that the concentration and distribution of the maximum shear stress ($\tau_{\text{max}}$) is principally governed by amount of overlap between the master strike-slip faults in the en-échelon
pull-apart formation.
Finally, our modeling results clearly demonstrate that the adopted geometry and applied boundary conditions have a remarkable role in controlling the shape, size, stress of state and deformation pattern during the pull-apart development.

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Appendix A:
Appendix A is quoted from “Theoretical basis of FE simulation software package” page 84 to 89 written by Hayashi (2008).

2D elastic problem

The principle of virtual work is described that the external works done by virtual displacement equals the internal work done by virtual strain.

Let us consider a certain element within a domain concerned as shown in Figure A1. When small displacement \( \mathbf{u} \), which is called virtual displacement, is applied to deform the element without disturb the balance of system, the external work is written as:

\[
W = \mathbf{u}^T \mathbf{f}^e,
\]

Where \( \mathbf{u}^e = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \) and \( \mathbf{f}^e = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \)

Writing in vector form:

\[
U = \int_s (\mathbf{e})^T \mathbf{s} \, ds
\]

According to the principle of virtual work, both must be equated.

\[
\mathbf{W} = \mathbf{U} (\mathbf{e})^T \mathbf{f}^e = \int_s (\mathbf{e})^T \mathbf{s} \, ds
\]

(1)

Then, to obtain the practical form of (1), we assume the displacement within element as a function of coordinates. Since the simplest relation is linear, we take linear relation as follows:

\[
\mathbf{u} = a_0 + a_1 x_1 + a_2 x_2
\]

Substituting the values of coordinate and displacement at nodes into this equation, we have:

\[
\mathbf{u}_N = \begin{bmatrix} 1 & x_{N1} \\ 1 & x_{N2} \end{bmatrix} \mathbf{a}
\]

Writing in vector form:

\[
\mathbf{u}^e = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ 1 & x_{31} & x_{32} \end{bmatrix} \mathbf{a} = \mathbf{C} \mathbf{a}
\]

The coefficient vector \( \mathbf{a} \) is derived from the equation,

\[
\mathbf{a} = \mathbf{C}^{-1} \mathbf{u}^e
\]

\[
\mathbf{C}^{-1} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{21} & \Delta_{31} \\ \Delta_{12} & \Delta_{22} & \Delta_{32} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} \end{bmatrix}
\]

Where \( \Delta = \text{det} \mathbf{C} \) and \( \Delta_{ij} = \text{cofactor of } \mathbf{C} \)

Therefore, the inner displacement is represented in terms of nodal displacements.

\[
\mathbf{u} = \mathbf{C}^{-1} \mathbf{u}^e = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{bmatrix} \mathbf{u}^e
\]
Replacing as 
\[ u = \phi N \ u_N, \]
we have 
\[ u_1 = \phi_1 N \ u_{N1} \quad \text{and} \quad u_2 = \phi_2 N \ u_{N2}. \]

Since we will consider 2D situation, displacement has 2 components as \( u_1 \) and \( u_2 \).

Writing them in vector form,
\[ \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 & 0 \end{pmatrix} \begin{pmatrix} u_{N1} \\ u_{N2} \end{pmatrix} \]

Then, exchanging the order of nodal displacements,
\[ u_1 u_2 u_{N1} u_{N2} u_{N3} u_{N4} \Rightarrow u_1 u_{N2} u_2 u_{N1} u_{N3} u_{N4} \]

\[ \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \phi_1 & 0 & \phi_2 & 0 & \phi_3 & 0 \\ 0 & 0 & \phi_1 & 0 & \phi_2 & \phi_3 \end{pmatrix} \begin{pmatrix} u_{N1} \\ u_{N2} \\ u_{N3} \end{pmatrix} \]

\[ = \Phi \mathbf{u}^t \]

Then, we can represent strain by nodal displacements as:
\[ \mathbf{e} = \begin{pmatrix} e_{11} \\ e_{12} \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & 0 & \epsilon_{12} & 0 & \epsilon_{13} & 0 \\ 0 & \epsilon_{12} & 0 & \epsilon_{22} & 0 & \epsilon_{23} \end{pmatrix} \mathbf{u}^t \]

\[ = \mathbf{B} \mathbf{u}^t \]

\[ \mathbf{B} = \frac{1}{\Delta} \begin{pmatrix} \Delta_{12} & 0 & \Delta_{13} & 0 & \Delta_{14} & 0 \\ 0 & \Delta_{12} & 0 & \Delta_{23} & 0 & \Delta_{24} \\ \Delta_{13} & 0 & \Delta_{12} & 0 & \Delta_{23} & \Delta_{34} \end{pmatrix} \]

\[ \text{Where} \]

As for stress vector, according to the constitutive law of elasticity,
\[ \mathbf{s} = \begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{13} \end{pmatrix} = \mathbf{D} \mathbf{e} \]

For example, in case of plane strain
\[ \mathbf{D} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{pmatrix} \]

Then, according to the principle of virtual work,
\[ (\mathbf{M}^t)^T \mathbf{f}^t = \int \mathbf{b}^T \mathbf{M}^t \mathbf{s} \, dS \]

\[ = (\mathbf{e}^t)^T \int \mathbf{B}^T \mathbf{D} \mathbf{B} \mathbf{e} \, dS \]

\[ \mathbf{f}^t = \mathbf{K}^t \mathbf{u}^t \]

This is called the stiffness equation of element.

Superposing every stiffness equations of element, we obtain the stiffness equation of whole domain.

\[ \mathbf{f} = \mathbf{K} \mathbf{u} \]

If body force \( \mathbf{f}_b \) is considered, the principle of virtual work need be modified as:
\[ (\mathbf{M}^t)^T \left( \mathbf{f}^t - \int \mathbf{f}_b \, dS \right) = \int \mathbf{b}^T \mathbf{M}^t \mathbf{s} \, dS \]

Fault analysis

We introduce how the Mohr-Coulomb criterion is combined into the FE software package (Table 1).

When we consider the analysis in plane strain condition, it is possible to calculate the value of third principal stress \( \sigma^* \), which is perpendicular to \( \sigma_1 - \sigma_2 \) plane using the equation:

\[ \sigma^* = \nu (\sigma_1 + \sigma_2) \quad (2) \]

Where \( \nu \) is the Poisson ratio (Timoshenko and Goodier, 1970). After comparing the values of \( \sigma_1, \sigma_2 \) and \( \sigma^* \), we can recognize the newly defined \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) as the maximum, intermediate and minimum principal stresses respectively. As shown in Figure A2, the Mohr-Coulomb criterion is written as a linear relationship between shear and normal stresses,
\[ \tau = c + \sigma \tan \phi \]  
\[ \text{(3)} \]

Where \( c \) and \( \phi \) are the cohesive strength and the angle of internal friction, respectively. Failure will occur when the Mohr’s circle first touches the failure envelope (3). It will happen when the radius of the Mohr’s circle, \( \frac{\sigma}{2} \), is equal to the perpendicular distance from the center of the circle at \( \frac{\sigma - \sigma_t}{2} \) to the failure envelope,

\[ \left( \frac{\sigma - \sigma_t}{2} \right)_{\text{failure}} = c \cos \phi + \left( \frac{\sigma + \sigma_t}{2} \right) \sin \phi \]  
\[ \text{(4)} \]

According to Melosh and Williams (1989), the proximity to failure \( (P_f) \) is the ratio between the calculated stress and the failure stress, which is given by:

\[ P_f = \left[ \left( \frac{\sigma - \sigma_t}{2} \right)_{\text{failure}} \right] \]  
\[ \text{(5)} \]

When the ratio reaches one \( (P_f = 1) \), failure occurs, but when \( P_f < 1 \) stress is within the failure envelope, rock does not fail. The proximity to failure \( P_f \) reveals which parts of the model are close to failure or already failed by generating faults. The type of faulting has been determined by the Anderson’s theory (1951). According to his theory three classes of faults (normal, strike slip and thrust) result from the three principal classes of inequality that may exist between the principal stresses. The judgment in the program `failure.state.func` in FE package was realized.

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Appendix

Figure A1. Force vector and virtual displacement vector work at each nodal point in a certain finite element (Hayashi, 2008).

Figure A2. Failure envelope and Mohr's circle in $\sigma - \tau$ space. $c$ is cohesion and $\phi$ is angle of internal friction (Hayashi, 2008).