Drag and diffusion coefficients of heavy quarks in hard thermal loop approximations

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The drag and diffusion coefficients of heavy quarks propagating through quark gluon plasma (QGP) have been evaluated using Hard Thermal Loop (HTL) approximations. The HTL corrections to the relevant propagators and vertices have been considered. It is observed that the magnitudes of both the transport coefficients are changed significantly from values obtained by earlier approaches where either (i) the t channel divergence in $T = 0$ pQCD matrix element is shielded simply by Debye mass. or (ii) only HTL resummed propagator is used ignoring the HTL corrections at the interaction vertices. The implications of these changes in the transport coefficients on the heavy ion phenomenology have been discussed.

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I. INTRODUCTION

The study of the transport coefficients of strongly coupled system is a field of high contemporary interest both theoretically and experimentally. In one hand, the calculation of the lower bound on the shear viscosity ($\eta$) to entropy density ($s$) ratio ($\eta/s$) within the framework of AdS/CFT model [1] has ignited enormous interests among the theorists. On the other hand, the experimental study of the $\eta/s$ for cold atomic systems and QGP and their similarities have generated huge interest across various branches of physics (see [2] for a review). In general, the interaction of probes with a medium brings out useful information about the nature of the medium. Since the magnitude of the transport coefficients are sensitive to the coupling strength, hence these quantities can be adapted as useful quantities to characterize a medium. In the context of probing QGP, expected to be produced in ultra-relativistic heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies, we choose the heavy quarks (HQs), charm and beauty, as probes. That is, we would like to extract the drag and diffusion coefficients of the QGP by studying the propagation of HQs through QGP. Selection of HQs as probes has several advantages, such as (i) they are produced very early in the collisions and remain extant throughout the evolution of the QGP. As a result, the HQs witness the entire evolution of the system. It is expected that the HQ thermalization time is larger than the light quarks by a factor $m/T$ where $m$ is the mass of the HQ and $T$ is the temperature. Therefore, the HQs cannot remain out of equilibrium in QGP. (ii) The chances of HQs getting thermalized in the system is weaker and hence do not dictate the bulk properties of QGP. Moreover, the observed transverse momentum suppression ($R_{AA}$) of leptons originating from the decays of heavy flavours produced in nuclear collisions as compared to those produced in proton-proton (pp) collisions at the same colliding energy [3–5] offer us an opportunity to estimate the drag and diffusion coefficients of QGP.

In the present circumstances a description of the motion of the non-equilibrated HQs in the background of equilibrated system of QGP is required. An appropriate foundation is provided by the Fokker-Planck (FP) equation [6, 7], which reads as follows:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p) f + \frac{\partial}{\partial p_i} [B_{ij} f] \right]$$

(1)

where $f$ stands for the momentum-space distribution of the particle (HQs here) undergoing Brownian motion in the thermal bath of QGP. The question of HQ thermalization can be addressed by comparing the solution of FP equation with the HQ’s thermal distribution at any given time. $A_i$ and $B_{ij}$ of Eq. (1) are related to the drag and diffusion coefficients respectively. Hence, the interactions of the HQs with the QGP are incorporated in $A_i$ and $B_{ij}$. That means, $A_i$ and $B_{ij}$ can supply information about the nature of the QGP [8–19]. The issue of HQ thermalization in QGP can also be addressed experimentally by measuring the elliptic flow ($v_2$) of leptons from the decays of HQs. Therefore, the evaluation of the drag and diffusion coefficients of QGP become extremely important. We will see below that drag (diffusion) coefficients are, essentially, momentum (square of the momentum) transfer weighted over the squared interaction matrix element ($|M|^2$). This indicates that an accurate evaluation of $|M|^2$ is of vital importance. In the present work we attempt to estimate both these coefficients by using the techniques of thermal field theory in hard...
thermal loop (HTL) approximations using resummed gluon propagator and one loop corrections to relevant vertices. Some of the earlier attempts \[20, 21\] lack the vertex corrections which is necessary for maintaining gauge-invariance.

The two main elastic processes which contribute to the transport coefficients are: \( Q + q \rightarrow Q + q \) and \( Q + g \rightarrow Q + g \). Here \( Q \) stands for heavy (light) quarks and \( q \) denotes gluon. The \( |M|^2 \) for these processes contain \( t \)-channel divergence which are normally regulated by introducing thermal mass \( (m_D) \) for the exchanged gluons \[12, 17\] i.e. by replacing \( t \) by \( t - m_D^2 \) in the denominator of the matrix elements. In the present work, instead of shielding the divergences simply by (static) Debye mass we will use the HTL approximated gluon propagator in the \( t \)-channel diagrams with vertex correction in a self-consistent way.

The paper is organized as follows. In the next section the general expressions for the drag and diffusion coefficients are outlined. In section III we briefly discuss the effective gluon propagators in HTL approximations. The significance of the effective three gluon \((ggg)\) and quark-gluon \((qgg)\) vertices correction is discussed in the context of gauge invariance. Section IV is devoted for presenting results on the drag and diffusion coefficients. The summary and conclusions of the present work is presented in section V. The appendix contains the detailed derivation of the matrix elements required for the evaluation of drag and diffusion coefficients.

II. THE DRAG AND DIFFUSION COEFFICIENTS

In terms of the transition rates the collision integral of the Boltzmann transport equation can be written as \[3\]:

\[
[\frac{df}{dt}]_{\text{collisions}} = \int d^3k [w(p + k, k)f(p + k) - w(p, k)f(p)].
\]

where \( w(p, k) \) is the collision rate, say for the processes, \( Q(p) + g(q) \rightarrow Q(p - k) + g(q + k) \), where the quantities within the bracket denotes the corresponding momenta of the particle. Using Landau approximation i.e. by expanding \( w(p + k, k) \) in powers of \( k \) and keeping up to quadratic term, the Boltzmann transport equation can be written as \[12, 17\]

\[
\frac{df}{dt} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_j} [B_{ij}(p)f] \right],
\]

where the kernels are defined as

\[
A_i = \int d^3kw(p, k)k_i,
\]

and

\[
B_{ij} = \frac{1}{2} \int d^3kw(p, k)k_ik_j.
\]

For \( |p| \rightarrow 0, A_i \rightarrow \gamma p_i \) and \( B_{ij} \rightarrow D\delta_{ij} \) where \( \gamma \) and \( D \) stand for drag and diffusion coefficients respectively. The drag and diffusion coefficients have recently been evaluated within the ambit of AdS/CFT \[22\] and pQCD \[23\] and their importance for jet quenching have been discussed. Eq. \[3\] is a nonlinear integro-differential equation known as the Landau kinetic equation. The appearance of parton distribution in the expression for \( \omega \) makes Eq. \[3\] a non-linear one. For the problem under consideration one of the colliding partners (light quarks or gluons) is in equilibrium. In such a situation the distribution function which appears in \( w \) can be replaced by thermal distribution. As a consequence Eq\[3\] becomes a linear partial differential equation, known as Fokker-Planck (FP) equation. The \( T \) dependence of the transport coefficients enter through the thermal distribution appearing in \( \omega \).

As mentioned above the drag and diffusion coefficients are related to the quantities \( A_i \) and \( B_{ij} \). Both these coefficients can be calculated from the following expression \[12\] with appropriate choice of the function \( F(p') \),

\[
\langle F(p') \rangle = \frac{1}{512\pi^4} E_p \int_0^{\infty} q dq d(cos\chi) \frac{s - m^2}{s} f(q) \int_{-1}^{1} d(\phi_{c.m.}) \int_0^{2\pi} d(cos\chi) F(p')
\]

where \( g_Q \) is the HQ degeneracy, \( F(p' = p - k) \) is a function of \( p, q \) and CM frame scattering angles and \( cos\chi \) can be obtained from,

\[
s = p^2 + q^2 + 2(E_pE_q - |\vec{p}||\vec{q}|cos\chi)
\]
$\theta_{c.m.}$ and $\phi_{c.m.}$ are polar and azimuthal angles of q respectively. Drag ($\gamma$) can be obtained by the following replacement in Eq. [4]

$$F(p') = 1 - \frac{p.p'}{p'^2} \quad (8)$$

For determining diffusion ($D$) we substitute,

$$F(p') = \frac{1}{4} \left[ p'^2 - \frac{(p.p')^2}{p^2} \right] \quad (9)$$

in Eq. [9]

### III. RESUMMED GLUON PROPAGATOR, EFFECTIVE THREE GLUON AND QUARK-QUARK-GLUON VERTICES IN HTL APPROXIMATION

As discussed before the calculation of drag and diffusion coefficients involve the evaluation of amplitudes for processes like $Q + q \rightarrow Q + q$ and $Q + g \rightarrow Q + g$ [24]. The amplitudes from bare perturbation theory contains t-channel divergence due to low four-momentum, $P = (\omega, \vec{p})$ gluon exchange. This divergence can be regulated by introducing thermal mass of gluon. Here, we study the HTL approximations [25] and resummation of gluon propagator which will enable us to regulate the t-channel divergence in a self-consistent way and hence will lead to comparatively more reliable values of the transport coefficients.

Our aim is to find out HTL approximated self-energy of gluon which goes as an input to the resummed gluon propagator to be used as effective thermal propagator regularizing the t channel divergence. The gluon self-energy in HTL approximation is discussed in detail in Ref. [26, 27]. In this section we give only an outline of the scheme. There are four diagrams which contribute to gluon self-energy (Fig. 1). The loop integrations can be written down easily if we keep in mind that the loop-momentum, $K = (k_0, \vec{k})$ is ‘hard’ compared to external gluon momentum, i.e. $P >> Q$ which enables us to use simplified $ggg$ vertex [26]. Our goal will be to find out $T^2$ contributions of self-energy because the momentum integration is cut-off at the momentum scale\~ $T$ due to the presence of thermal distribution function. That is we can take-up the momentum integration $\int_0^{\infty} kdk$ which blows up as $k \rightarrow \infty$, but, insertion of thermal distribution function makes it finite even at $k \rightarrow \infty$, 

![Feynman diagrams contributing to gluon self-energy upto one loop. (a)ghost-gluon loop. (b)four-gluon vertex. (c) quark-antiquark pair creation. (d)three-gluon vertex.](image-url)
The HTL corrections to $ggg$ of the strong interaction. 

The leading contribution in Eq. (10) is given by $k \sim T$. If we are interested in high-temperature limit we can assume $|\vec{k}| >> |\vec{p}|$ and approximate related quantities accordingly. 

The effective gluon propagators evaluated in one loop order in HTL approximation enter in the transport coefficients evaluated for the processes displayed in Figs. 8 and 11. For low momentum transfer (i.e. $\sim gT$ where $g$ is the colour charge, $g = \sqrt{4\pi \alpha_s(T)}$ and $\alpha_s(T)$ is the strong coupling), one has to use the resummed propagator [20]. The resummed gluon propagator, which is given by:

$$\Delta^{\mu\nu} = \frac{P^{\mu\nu} - P_T^{\mu\nu} + (\alpha - 1) P^\mu P^\nu}{P^2 + \Pi_L}$$ (11)

will need HTL approximated $\Pi_L$ and $\Pi_T$. The transverse and longitudinal self-energies, $\Pi_T$ and $\Pi_L$ are given by

$$\Pi_L(P) = (1 - x^2)\pi_L(x), \quad \Pi_T(P) = \pi_T(x)$$ (12)

where $x = \omega / q$ and scaled self-energies $\pi_T$ and $\pi_L$ are given by [26],

$$\pi_T(x) = m_D^2 \left[ \frac{x^2}{2} + \frac{x^4}{4} (1 - x^2) ln \left( \frac{1 + x}{1 - x} \right) - i \frac{\pi}{4} x (1 - x^2) \right]$$ (13)

$$\pi_L(x) = m_D^2 \left[ 1 - \frac{x}{2} ln \left( \frac{1 + x}{1 - x} \right) - i \frac{\pi}{4} x \right]$$ (14)

where $m_D$ in Eq. (13) is the thermal mass of gluon; and is given by $m_D^2 = g^2 T^2 (C_A + N_f / 2) / 6$, we use 2-loop perturbative temperature-dependent coupling for our calculation [20] i.e. in the present calculations the strong coupling runs with temperature. In addition to the HTL corrections to the propagator we introduce the HTL corrections to the $ggg$ and $qqg$ vertices in t-channel diagram of $Qg \rightarrow Qg$ process and $Qg \rightarrow Qg$ processes to maintain gauge-invariance [31]. The involvement of heavy quarks in the dynamical processes helps us to approximate heavy quark (HQ) propagators and HQ - HQ - $g$ vertex by their $T = 0$ counterparts. But the matrix elements under consideration have their origin from cutting through the heavy quark self-energy diagram using effective gluon propagator. Hence inclusion of effective $qqg$ and $ggg$ vertex becomes inevitable. Some recent works [32] refrain from using effective vertices because the gluons and light quarks are ‘hard’ as they are in thermal bath; and following the argument of [26], the uncorrected $ggg$ vertex ($\mathcal{O}(T)$), then, dominates over corresponding HTL vertex correction ($\mathcal{O}(g^3T)$). So, the vertex-corrections can approximately be neglected though one should maintain gauge-invariance by as gauge-symmetry is a sacred symmetry of the strong interaction.

From the previous discussion we have seen that the leading behaviour in temperature of gauge particle self-energies is proportional to $T^2$. This result can be generalized to $N$-point functions, computed at the one-loop approximation. The HTL corrections to $ggg$ and $qqg$ vertices can be obtained from [20], (see Appendix for detailed discussion)

**IV. RESULTS**

The drag and diffusion coefficients of HQ, propagating through QGP and suffering elastic collisions, evaluated by using the HTL approximated effective gluon propagators and effective $ggg$ and $qqg$ vertices are denoted by $\gamma_{HTL}$ and $D_{HTL}$ respectively. To present the results of our calculations we use the following notations. The drag and diffusion coefficients evaluated with bare vertices and propagators will be denoted by $\gamma$ and $D$ respectively (the Debye mass is introduced in the t-channel propagator to shield the infra-red divergences). The $\gamma_{HTL}$ and $D_{HTL}$ are calculated with the HTL approximated propagator and vertices. All these quantities, i.e. $\gamma$, $D$, $\gamma_{HTL}$ and $D_{HTL}$ are evaluated with the same kinematic approximations [26] to make the comparison of the bare and HTL approximated quantities meaningful.

The variations of drag with temperature for HQs at momentum, $p = 1$ GeV are displayed in Fig. 2. The results clearly indicate an enhancement and a more rapid variation of $\gamma_{HTL}$ compared to $\gamma$. The increase is more prominent
for charm than beauty. We have explicitly checked that in the static limit \((\omega/q = x \rightarrow 0)\) the \(\gamma_{HTL}\) approaches \(\gamma\). Results displayed in Fig. 2 indicate that at \(T = 400\) MeV the \(\gamma_{HTL}\) is about two times \(\gamma\) for charm quark. Whereas, \(\gamma_{HTL}\) for bottom is about 40\% more than \(\gamma\). We also observe that this difference increases with the increase in temperature.

The variation of \(\gamma_{HTL}\) and \(\gamma\) with momentum is depicted in Fig. 3 for \(T = 300\) MeV. The \(\gamma_{HTL}\) is greater than \(\gamma\) for the entire momentum range considered here. Again, drag being a measure of the HQs energy loss \(28\), increase in drag results in more suppression of heavy flavours, which will have crucial consequences in understanding the heavy flavour suppression measured at RHIC and LHC energies. The momentum dependence of drag is distinctly affected if drag results in more suppression of heavy flavours, which will have crucial consequences in understanding the heavy flavour suppression measured at RHIC and LHC energies. The momentum dependence of drag is distinctly affected if we consider the HTL resummation technique. For a 5 GeV charm the

\[
\gamma_{HTL}\text{ for bottom is about } 40\% \text{ more than } \gamma.
\]

In Fig. 4 the interaction rate (see \(33\) for details) of the HQs with the QGP is shown as function of temperature. The rate is increased when the HTL corrections in both the propagator and the vertices are taken into account. The inclusion of the HTL approximated gluon propagator and vertices increases the likelihood of charm or bottom being equilibrated with the medium. The measured non-zero elliptic flow of heavy flavours at RHIC (through the single electron spectra originated from the semileptonic decays of the heavy flavoured mesons) and LHC energies indicate that the HQs in the QGP phase follow the collective motion of the background QGP, indicating thermalization of HQs. Therefore, the increase of the interaction rate (hence decrease in the interaction time scale) will have important implications in understanding the data on elliptic flow of heavy flavours.

In Figs. 5 and 6 the diffusion coefficients \(D_{HTL}\) and \(D\) are plotted with \(T\) (for \(p = 1\) GeV) and \(p\) (for \(T = 300\) MeV) respectively. At \(T = 300\) MeV, the magnitude of \(D_{HTL}\) is almost 2.5 times \(D\) in case of charm. For beauty quark, the ratio of \(D_{HTL}\) to \(D\) is \(\sim 1.4\). The momentum dependence of diffusion is significantly modified too. A 5 GeV charm diffuses 3.3 times more in momentum space when effective vertices and propagators are used. In case of bottom, the effect of effective propagators and vertices are less than that of charm. These changes in drag and diffusion coefficients originate from the spectral modification of the \(t\)-channel gluons due to its interaction with the thermal bath. In the static limit \(\pi_T \rightarrow 0\) and \(\pi_L \rightarrow m_T^2\). The appearance of non-zero \(\pi_T\) makes \(\gamma_{HTL}\) larger than \(\gamma\). The inclusion of the vertex correction terms also introduces thermal fluctuations in the present formalism which, together with the resummed propagator, ultimately increases the magnitude of the drag and diffusion coefficients of HQ.

Now some comments on the magnitude of the values of \(D_{HTL}\) and \(D\) are in order here. The value of the diffusion coefficients in spatial co-ordinate, \(D_{HTL}^{\mu}\) can be estimated from the value of drag by using the relation \(D_{HTL}^{\mu} = T/(\omega_{HTL})^{-1}\). In Fig. 7 we plot \(D_{HTL}^{\mu}\) multiplied by the inverse of the thermal de Broglie length, \(\Lambda = 1/(2\pi T)\). The results clearly indicate that the \(D_x\) remains well above the quantum bound.

V. SUMMARY AND CONCLUSION:

In summary, we have taken into account the HTL modifications of the gluon spectral function and the \(qgq\) and \(ggg\) vertices in evaluating the drag and diffusion coefficients of HQs propagating through the QGP. The deviations between \(\gamma_{HTL}\) and \(\gamma\) and \(D_{HTL}\) and \(D\) is found to be substantial. The enhanced drag will result in higher suppression of the HQ momentum spectrum. The increase in drag will also enhance the chances of HQ getting equilibrated with the bulk of the system. These results will have crucial consequences on the observables like nuclear suppressions, \(R_{AA}(p_T)\) and elliptic flow, \(v_2\) of heavy flavours measured at RHIC and LHC experiments.

VI. APPENDIX: CALCULATING MATRIX ELEMENTS FROM HARD THERMAL LOOP(HTL) PERTURBATION THEORY

A. Symbols and Expressions we use:

In this appendix we evaluate the following matrix elements for the processes \(Qg \rightarrow Qg\) (Fig. 8) and \(Qq \rightarrow Qg\) (Fig. 11) in a thermal medium applying HTL approximations. First, we define the following useful quantities \(26\) required to write down the gluon propagator in thermal medium. Let \(u_\mu\) be the fluid four-velocity, with normalization condition \(u_\mu u^\mu = 1\). Then any four-vector \(P^\mu\) can be decomposed into components parallel and perpendicular to the fluid velocity:

\[
\omega = P.u, \quad \tilde{P}_\mu = P_\mu - u_\mu (P.u)
\]

\[(15)\]
where

$$P^2 = \omega^2 - p^2$$

$$\tilde{P}^2 = -p^2$$

Eqs. 15 and 16 are valid in the local rest frame of fluid, i.e. in a frame where $u = (1, \vec{0})$. Similarly a tensor orthogonal to $u_\mu$ can be defined as,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

The longitudinal and transverse projection tensors $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ respectively are defined as \[20\]

$$P_L^{\mu\nu} = -\frac{1}{P^2 p^2} (\omega P^\mu - P^2 u^\mu)(\omega P^\nu - P^2 u^\nu)$$

$$P_T^{\mu\nu} = \tilde{g}_{\mu\nu} + \frac{\tilde{P}_\mu \tilde{P}_\nu}{P^2}$$
which are orthogonal to $P^\mu$ as well as to each other, i.e.

$$P_\mu P^\mu_L = P_\mu P^\mu_T = P_{L\nu} P^\nu_T = 0 \tag{20}$$

But,

$$P_i^{\mu\rho} P_{i\nu}^{\rho} = P_{i\nu}^{\mu}, \quad i = L, T \tag{21}$$

The transverse and longitudinal self-energies of gluon at non-zero temperature are given by:

$$\Pi_T(P) = (1 - x^2)\pi_L(x), \quad \Pi_L(P) = \pi_T(x) \tag{22}$$

respectively, where $x = \omega/p$ and scaled self-energies $\pi_T$ and $\pi_L$ are given by \cite{26},

$$\pi_T(x) = m_D^2 \left[ \frac{x^2}{2} + \frac{x}{4} (1 - x^2) \ln \left( \frac{1 + x}{1 - x} \right) - i \frac{\pi}{4} x (1 - x^2) \right]$$

\tag{23}
FIG. 6: (Color online) Variation of diffusion of heavy quarks with momentum in a QGP bath of temperature 300 MeV.

FIG. 7: Comparison of coordinate space diffusion of charm quark with thermal de Broglie wavelength.

and

\[
\pi_L(x) = m_D^2 \left[ 1 - \frac{x}{2} \ln \left( \frac{1}{1-x} \right) + i \frac{\pi}{2} x \right]
\]

respectively. Non-zero real and imaginary parts of the self-energies corresponds to the shift of the pole of the propagator and to the different physical processes take place in the medium. With the help of the quantities defined above we can now write down the gluon propagator with momentum \( P \) using Dyson-Schwinger equation:

\[
\Delta^{\mu\nu} = \frac{\Pi_T^{\mu\nu}}{-P^2 + \Pi_T} + \frac{\Pi_L^{\mu\nu}}{-P^2 + \Pi_L} + (\alpha - 1) \frac{P^{\mu} P^{\nu}}{P^2}
\]

where \( \alpha \) is a gauge-fixing parameter taken to be unity in this literature.
B. Calculating \( Qg \rightarrow Qg \) Matrix Element:

This process contains three Feynman diagrams corresponding to the channels s, t and u. Since heavy quarks are not thermalized, we use bare HQ propagators as well as bare HQ-gluon vertex for s channel and u channel diagrams. Consequently, we use naive perturbation theory results \([12, 24]\) for \(|M_s|^2\), \(|M_t|^2\) and cross-term \(M_sM_u^*\). On the other hand, we have to use effective propagator as well as effective three-gluon vertex for \(t\) channel diagram. Hence, \(|M_t|^2\) and cross-terms \(|M_sM_u^*|\) as well as \(|M_sM_t^*|\) are drastically different from their \(T = 0\) counterparts. We write down \(M_s\), \(M_t\) and \(M_u\) for the process under discussion.

\[
\begin{align*}
-iM_s &= \mp(P_3)(-ig\gamma^\nu t^b_{jk})i\frac{P_t^j + P_t^k}{s - m^2}(-ig\gamma^\mu t^a_{ki})u(P_1)\epsilon_\mu\epsilon_\nu^* \\
-iM_u &= \mp(P_3)(-ig\gamma^\mu t^a_{ki})i\frac{P_t^j - P_t^k}{u - m^2}(-ig\gamma^\nu t^b_{kj})u(P_1)\epsilon_\mu\epsilon_\nu^* \\
-iM_t &= \mp(P_3)(-ig\gamma^\nu t^b_{jk})i\epsilon_\mu\epsilon_\nu^* u(P_1)(-i\Delta_{\alpha\delta})gf_{abc}\Gamma_\mu^\alpha\Gamma_\nu^\delta u^* 
\end{align*}
\]  

(26)

\(\) \(\) \(\) \(\) \(\) 

Here, \(s = (P_1 + P_2)^2\), \(t = (P_4 - P_2)^2\) and \(u = (P_1 - P_4)^2\) are Mandelstam variables. Now, according to the requirement of the gauge invariance we have used both three-gluon \((ggg)\) effective HTL vertex \((\Gamma_\mu^\alpha)\) and HTL resummed gluon propagator \((\Delta_{\alpha\delta})\). We note that \(ggg\) effective vertex \(\Gamma_\mu^\alpha\nu^\beta\) is given by two parts: (a) the \(T = 0\) \(ggg\) vertex \((C_\mu^\alpha\nu^\beta)\) and (b) the one-loop HTL correction to (a), \(\delta\Gamma_\mu^\alpha\nu^\beta\). So we can write:

\[
\Gamma_\mu^\alpha\nu^\beta = C_\mu^\alpha\nu^\beta + \delta\Gamma_\mu^\alpha\nu^\beta
\]  

(29)

where \(C_\mu^\alpha\nu^\beta = [(2P_1 - P_2)^\mu g^{\beta\nu} + (2P_2 - P_4)^\mu g^{\alpha\nu} + (P_1 - P_4)^\mu g^{\alpha\nu}]\) is three-gluon vertex at zero temperature.

The diagrams which contribute to HTL correction to \(ggg\) vertex are given in Fig. The expression for HTL \(ggg\) vertex correction is:

\[
\delta\Gamma_\mu^\alpha\nu^\beta = 2m_D^2 \int \frac{d\Omega}{4\pi} \hat{K}_\mu\hat{K}_\nu\times \left[ \frac{i\omega_r}{(P.K)(R.K)} - \frac{i\omega_q}{(P.K)(Q.K)} \right]
\]  

(30)

The momenta \(P\), \(Q\), \(R\) are defined in Fig. with \(P + Q + R = 0\). For simplicity we assume three momentum transfer, \(\hat{p}\), to be zero. In this approximation we can simplify Eq. into:

\[
\delta\Gamma_\mu^\alpha\nu^\beta = -2m_D^2 \int \frac{d\Omega}{4\pi} \hat{K}_\mu\hat{K}_\nu\frac{|\hat{q}|cos\theta}{R.K.Q.K}
\]  

(31)

where \(\cos\theta\) is the angle \(\hat{q}\) (considered to be along z-axis) makes with \(\hat{K}\).

The complex-conjugate of Eq. can be written as:

\[
\delta\Gamma_\mu^\alpha\nu^\beta^* = -2m_D^2 \int \frac{d\Omega}{4\pi} \hat{K}_\mu\hat{K}_\nu\frac{|\hat{q}|cos\theta}{R.K.Q.K}
\]

(32)
Calculating t-channel diagram

The t-channel diagram for the process $Qg \rightarrow Qg$ has the following amplitude square:

$$
\frac{8}{g^4} \frac{M_t M_f}{\hat{s}} = [4(m^2 - P_1 \cdot P_3)g^{\alpha\alpha'} + 4P_1^\alpha P_3^\alpha' + 4P_3^\alpha P_1^\alpha']
\times \Delta_{ab} \Delta_{a'b'}^* \Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'} g_{\mu\nu}^\ast g_{\mu\nu'}^\ast
$$

$$
= [4(m^2 - P_1 \cdot P_3)g^{\alpha\alpha'} + 4P_1^\alpha P_3^\alpha' \Delta_{ab} \Delta_{a'b'}^* + 4P_3^\alpha P_1^\alpha' \Delta_{ab} \Delta_{a'b'}^*]
\times (\Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'}) ,
$$

(33)

where $m$ is the mass of Heavy Quark (HQ). The term $\Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'}$ is actually given by the following terms:

$$
\Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'} = \Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'} \sim O(m_D^0) + O(m_D^2) + O(m_D^4),
$$

(34)

We are left with the contributions from (a) product of uncorrected vertices ($O(m_D^0)$), (b) product of corrected and uncorrected vertices ($O(m_D^2)$) and (c) that of HTL corrections ($O(m_D^4)$). Calculation of part (a) can be performed by taking explicitly the form of the HTL resummed gluon propagator. We are not writing down the full expression for part (a) simply because it is too long. However, one can evaluate part (b) as well as (c) with the assumption that the three momentum transfer, $\hat{p}$ is negligibly small. Here, we will illustrate the calculation of the terms of Eq. (33), having contributions of ($O(m_D^2)$) and ($O(m_D^4)$).

It is evident from Eq. (33) that we need to evaluate the quantity $\Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'}$ as discussed below:

$$
\Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'} = C_{\mu\nu}^{t\delta_{\mu\nu'}} + 2C_{\mu\nu}^{t\delta_{\mu\nu'}} \Gamma_{t}^{\mu\nu} \Gamma_{t}^{\mu\nu'} + 2 \delta_{\mu\nu} \delta_{\mu\nu'} \Gamma_{t}^{\mu\nu}
$$

$$
= C_{\mu\nu}^{t\delta_{\mu\nu'}} + 2 \left[ -2m_D^2 \int \frac{d\Omega}{4\pi} \hat{K}_1 \cdot \hat{K}_1 \hat{K}_2 \cdot \hat{K}_2 \cos \theta \right]
$$

$$
+ 4m_D^2 \int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} (\hat{K}_1 \cdot \hat{K}_2)^2 \hat{K}_1 \cdot \hat{K}_2 \cos \theta
$$

$$
\times \frac{|\vec{q}|^2 \cos \theta_1 \cos \theta_2}{Q \cdot \hat{K}_1 \cdot \hat{K}_2 \cdot Q \cdot \hat{K}_2 \cdot \hat{K}_2} ,
$$

(35)

where $\Omega_1 (\Omega_2)$ is the solid angle $\hat{K}_1 (\hat{K}_2)$ makes with $Q$ chosen to be along z-axis (Fig 10). From three-momentum conservation at the three-gluon vertex we get, $\vec{p} + \vec{q} + \vec{r} = 0 \quad p = 0 \rightarrow q = -r$; $|\vec{q}| = |\vec{r}|$. Hence $\vec{r}$ is aligned along
the negative z axis. Now, choosing \( \hat{K}_r = (-i, \hat{k}_r), [r = 1, 2] \), a light-like unit vector, we can write three-unit vectors \( \hat{k}_r \) as below:

\[
\hat{k}_r = \sin \theta_1 \cos \phi_1 \hat{i} + \sin \theta_1 \sin \phi_1 \hat{j} + \cos \theta_1 \hat{k}
\]

(36)
in the spherical co-ordinate.

\[
(\hat{K}_1 \cdot \hat{K}_2)^2 = (-1 + \hat{k}_1 \cdot \hat{k}_2)^2
\]
\[
= \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 (\phi_1 - \phi_2) + \cos^2 \theta_1 \cos^2 \theta_2
\]
\[
+ \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2) \cos \theta_1 \cos \theta_2
\]
\[
- 2(\sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2) + \cos \theta_1 \cos \theta_2) + 1
\]

\[Q. \hat{K}_r = -i Q_4 + |\vec{q}| \cos \theta_r \quad \text{(Euclidean)}\]

\[R. \hat{K}_r = -i R_4 - |\vec{r}| \cos \theta_r\]

(37)

So the integrand in Eq. 35 is, now, entirely in terms of \( \theta_r \) and \( \phi_r \). Having done so, we will find out an analytic expression for right hand side of Eq. 33.

**Calculation of \( |\vec{M}|^2 \):**

Equation 33 can be written in the following way,

\[
\frac{8}{g^4} |\vec{M}|^2 = [A + B + C] \times [\#1 + \#2 + \#3],
\]

(38)

where A, B, C and \#1, \#2 and \#3 are already defined. Now let us evaluate the products one by one.

**Calculation of \((A + B + C) \times \#1\):**

This is the part of \( |\vec{M}|^2 \) which involves propagator correction only and no vertex correction. Since heavy quarks may not thermalize, we use \( T = 0 \) results for \( |\vec{M}|^2_{s}, |\vec{M}|^2_{u}, \vec{M}_{s} \vec{M}_{u}^{*} \) in our formalism. Results for \( |\vec{M}|^2_{t}, \vec{M}_{s} \vec{M}_{t}^{*}, \vec{M}_{u} \vec{M}_{t}^{*} \) has a quite long expression which we have not written here.

**Calculation of \( A \times \#2\):**

This calculation can be done following the procedure as delineated below:
\[ A \times \varphi^2 = 4(m^2 - P_1 \cdot P_3)g^{\alpha\alpha'} \Delta_{\alpha\delta} \Delta_{\alpha'\delta'}^* \times 2 \left[ -2m_D^2 \int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} (\hat{K}_1 \cdot \hat{K}_2)^2 \hat{K}_1^\delta \hat{K}_2^\alpha \times \frac{|q|^2 \cos \theta \cos \phi_2}{Q \cdot \hat{K}_1 \cdot \hat{K}_2} \right] \]

Writing the entire expression for transverse and longitudinal projection operators and \( C_{\mu\nu}^\delta \), we find that the terms we need to evaluate in Euclidean space contains the following: (a) \( g_{\delta\delta'} \hat{K}_1^\delta \hat{K}_2^\alpha \), (b) \( u_\delta u_\delta' \hat{K}_1^\delta \hat{K}_2^\alpha \), (c) \( (P \cdot \hat{K} - \omega u_\delta \hat{K}) \) and (d) \( (\omega P \cdot \hat{K} - P^2 u_\delta \hat{K}) \), save the coefficients. (a) is zero because \( \hat{K} \) is light-like. (b) is -1. (c) and (d) both give zero by virtue of the assumption of vanishing three-momentum transfer. Finally, we get

\[ A \times \varphi^2 = -\frac{16tm_D^2}{|t - \Pi T|^2} \left[ 2 - a_1 \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right], \tag{40} \]

where \( a_1 = \frac{Q_0}{|q|} \), after analytic continuation to Minkowski space.

**Calculation of \((B + C) \times \varphi^2\):**

Can be shown to be zero if we average over the directions of heavy quark momenta.

**Calculation of \(A \times \varphi^3\):**

The calculation is depicted below:

\[ A \times \varphi^3 = 4(m^2 - P_1 \cdot P_3)g^{\alpha\alpha'} \Delta_{\alpha\delta} \Delta_{\alpha'\delta'}^* \times 4m_D^4 \int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} (\hat{K}_1 \cdot \hat{K}_2)^3 \hat{K}_1^\delta \hat{K}_2^\alpha \times \frac{|q|^2 \cos \theta_1 \cos \theta_2}{Q \cdot \hat{K}_1 \cdot \hat{K}_2} \tag{41} \]

Eq. 41 involves the following,

\[ g_{\delta\delta'} \left( \delta \Gamma_{\mu\nu} \cdot \Gamma_{\alpha'\mu\nu}^\delta \right) = 4m_D^4 \int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} (\hat{K}_1 \cdot \hat{K}_2)^3 \frac{|q|^2 \cos \theta_1 \cos \theta_2}{Q \cdot \hat{K}_1 \cdot \hat{K}_2} \tag{42} \]

After expanding \((\hat{K}_1 \cdot \hat{K}_2)^3\), we retain only those terms which will yield non-zero contribution in Eq. 41. The final result for part (a) gives:

\[ g_{\delta\delta'} \left( \delta \Gamma_{\mu\nu} \cdot \Gamma_{\alpha'\mu\nu}^\delta \right) = \frac{3m_D^4}{2|q|^2} \left[ 2 - a_1 \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2 \]

\[ + \frac{m_D^4}{2|q|^2} \left[ \frac{2}{3} + 2a_1^2 - a_1 \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2 \]

\[ + \frac{3m_D^4}{2|q|^2} \left[ \frac{4}{3} - 2a_1^2 - (a_1 - a_1^3) \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2 \tag{43} \]

where we have used

\[ \int \frac{d\Omega}{4\pi} \frac{\cos (2n + 1) \theta}{(-ia + \cos \theta)(-ia - \cos \theta)} = 0 \quad [n \in \mathbb{I}^+] \]

\[ \int_0^{2\pi} \cos (\phi_1 - \phi_2) d\phi_1 d\phi_2 = 0 \tag{44} \]
and have utilized known results for

$$\int \frac{d\Omega}{4\pi} \frac{\cos^{2n}\theta}{(-ia + \cos \theta)(-ia - \cos \theta)} \ [n \in \mathbb{I}^+]$$  \hspace{1cm} (45)

and

$$u\delta u' \left( \delta \Gamma^{\mu \nu} \cdot \Gamma^{\nu \mu}_\rho \right) = -\frac{2m_D^4}{|q|^2} \left[ 2 - a_1 \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2$$  \hspace{1cm} (46)

The rest are zero as:

$$P \hat{K} - \omega u \hat{K} = -i\omega + i\omega = 0 \hspace{1cm} (47)$$

and

$$\omega P \hat{K} - P^2 u \hat{K} = \omega P \hat{K} - \omega^2 u \hat{K} = \omega(-i\omega + i\omega) = 0 \hspace{1cm} [\text{because } P^2 = \omega^2, \text{ in the approximation } \vec{p} = 0] \hspace{1cm} (48)$$

**Calculation of B × t**:

This term has vanishing contribution if average over the directions of $P_1$ or $P_3$ are taken. Same is true for the term $C \times t$.

- Hence the expression for $|M_t|^2$ due to both the vertex and propagator corrections becomes:

$$\frac{|t - \Pi_T|^2}{2t} \frac{8}{g^4} \frac{8}{g^4} \frac{m_D^4}{|q|^2} \left[ 2 - a_1 \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2 + \frac{m_D^4}{|q|^2} \left[ \frac{2}{3} + 2a_1^2 - a_1^3 \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2$$

$$+ \frac{3m_D^4}{2|q|^2} \left[ \frac{4}{3} - a_1^2 - (a_1 - a_1^3) \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2 - 8m_D^2 \left( 2 - 2a_1 \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right)$$  \hspace{1cm} (49)

**Contribution of M_M^*_t**:

$M_M^*_t$ can be calculated applying all the assumptions and techniques already discussed. So we can directly write down the results (with vertex and propagator corrections).

$$|t - \Pi_T|^2 \frac{16}{g^4} (s - m^2) \text{Re} M_M^*_t = -8tm_D^2 \left[ \left( 1 - \frac{a_1}{2} \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right) (t - \text{Re} \Pi_T) \right]$$  \hspace{1cm} (50)

Similar procedure may be followed to obtain the corresponding expressions for $M_u M_t^*$.

$$|t - \Pi_T|^2 \frac{16}{g^4} (u - m^2) \text{Re} M_u M_t^* = 8tm_D^2 \left[ \left( 1 - \frac{a_1}{2} \log \left( \frac{a_1 + 1}{a_1 - 1} \right) \right) (t - \text{Re} \Pi_T) \right]$$  \hspace{1cm} (51)
C. $Qq \rightarrow Qq$ Matrix Element from HTL approximation:

$$-iM_t = \Sigma(P_{1})(-ig\gamma^\mu t^a_{ji})u(P_4)[-i\Delta_{\mu\nu}]$$

$$\Sigma(P_{1})(-ig\Gamma^\nu t^a_{lk})u(P_2)$$

(52)

$i, j, k, l$ ($i \neq j, k \neq l$) are quark colours and ‘a’ is the colour of intermediary gluon with polarizations $\mu, \nu$. The term $\Gamma^\nu$ denotes the HTL vertex (Fig. 12) correction term upto one loop and is given by the following expression.

$$\Gamma^\nu = \gamma^\nu + m^2_f \int \frac{d\Omega}{4\pi} \frac{(\hat{k}^\nu \hat{k})}{(Q, \hat{k})(R, \hat{k})},$$

(53)

where $m_f = gT/\sqrt{6}$ is the thermal mass of fermions. After squaring and averaging over spin and colour and using Eq. 25 we get

$$\sum |M_{Qq}|^2 = g^4 C_{Qq} \left[ 4(m^2 - P_1 P_3)g^{\mu\nu} + 4P^\mu_1 P^\nu_3 + 4P^\mu_3 P^\nu_1 \right] \Delta_{\mu\nu} \Delta^{\mu\nu}_{\mu'\nu'} Tr \left[ P_k \Gamma^\nu P_k \Gamma^\nu' \right]$$

(54)

First we calculate the trace involving the vertex correction term.

$$Tr \left[ P_k \Gamma^\nu P_k \Gamma^\nu' \right] = Tr \left[ P_k \gamma^\nu P_k \gamma^\nu' \right] - m^2_f \int Tr \left[ P_k \gamma^\nu P_k \gamma^\nu' \right] \frac{\hat{k}^\nu}{(P_1, \hat{k})(P_2, \hat{k})} d\Omega$$

$$-m^2_f \int Tr \left[ P_k \gamma^\nu P_k \gamma^\nu' \right] \frac{\hat{k}^\nu}{(P_1, \hat{k})(P_2, \hat{k})} d\Omega$$

$$+ m^2_f \int Tr \left[ P_k \gamma^\nu P_k \gamma^\nu' \right] \frac{\hat{k}_1^\nu}{(P_1, \hat{k}_1)(P_2, \hat{k}_1)} \frac{\hat{k}_2^\nu}{(P_1, \hat{k}_2)(P_2, \hat{k}_2)} d\Omega_d d\Omega_2$$

(55)

After having calculated the trace and performed the relevant integration we arrive at the final result which constitutes of two parts: the first part comes solely from the correction due to HTL propagator and the rest is attributed to corrections due to HTL propagator as well as HTL vertex upto one loop approximation. The first part due to
propagator correction only is given by:

\[
\sum \frac{|M_{Q_0}|^2}{4C_{Q_0}g^4} = 2\frac{P_1.P_T.P_2.P_T.P_1}{(t-\Pi_T)^2} + 2\frac{P_2.P_L.P_3.P_L.P_1}{(t-\Pi_L)^2} + 2\frac{P_3.P_T.P_2.P_T.P_3}{(t-\Pi_T)^2} + 2\frac{P_3.P_L.P_2.P_L.P_3}{(t-\Pi_L)^2} \\
+ 2A\frac{P_1.P_L.P_2.P_T.P_1 + P_2.P_T.P_3.P_T.P_1 + P_3.P_T.P_3.P_T.P_3}{(t-\Pi_T)^2} + 2\frac{P_1.P_L.P_2.P_L.P_1 + P_2.P_L.P_3.P_L.P_1 + P_3.P_T.P_3.P_L.P_3}{(t-\Pi_L)^2} \\
- 2P_4.P_2 \left[ \frac{P_3.P_T.P_1}{(t-\Pi_T)^2} + \frac{P_3.P_L.P_1}{(t-\Pi_L)^2} \right] - 2P_3.P_4 \left[ \frac{P_4.P_T.P_2}{(t-\Pi_T)^2} + \frac{P_4.P_L.P_2}{(t-\Pi_L)^2} \right] \\
+ P_3.P_4.P_2 \left[ \frac{2}{(t-\Pi_T)^2} + \frac{1}{(t-\Pi_L)^2} \right] + m^2 \left[ \frac{2}{(t-\Pi_T)^2} + \frac{2}{(t-\Pi_L)^2} \right]
\]

where \( C_{Q_0} = \frac{3}{2} \) is the color factor, \( t = (P_1 - P_3)^2 \), \( A = t^2 - t(Re\Pi_T + Re\Pi_L) + Re\Pi_T\Pi_L \) and we have used the following relations.

\[
\Delta^{\mu\nu} \Delta^{*\rho} = \frac{\mathcal{P}^{\mu\nu}_T}{(t-\Pi_T)^2} + \frac{\mathcal{P}^{\mu\nu}_L}{(t-\Pi_L)^2} \\
|\Delta|^2 = \Delta^{\mu\nu} \Delta^{*\nu} = \frac{2}{(t-\Pi_T)^2} + \frac{1}{(t-\Pi_L)^2}
\]

Using eqs. 15 [43] we can show that

\[
P_1.P_L.P_2 = P_3.P_L.P_4 = P_4.P_L.P_3 = P_2.P_L.P_3
\]

where all the calculations are done in the rest frame of fluid element. Therefore, the terms due to both HTL approximated vertices and propagator are:

\[
\sum \frac{|M_{Q_0}|^2}{g^4C_{Q_0}} = -32t \frac{m_f^2}{|t-\Pi_T|^2} + \frac{4t^2m_f^4}{|t-\Pi_T|^2|\vec{q}|^2 a_1} \ln \left( \frac{a_1 + 1}{a_1 - 1} \right) + 16ta_1 \frac{m_f^2}{|t-\Pi_T|^2} \ln \left( \frac{a_1 + 1}{a_1 - 1} \right)
\]

\[
+ \frac{4tm_f^2}{|t-\Pi_T|^2|\vec{q}|^2} \left[ 2 - a_1 \ln \left( \frac{a_1 + 1}{a_1 - 1} \right) \right] + \frac{2t^2m_f^4}{|t-\Pi_T|^2|\vec{q}|^4} \left[ \frac{a_1}{2} \ln \left( \frac{a_1 + 1}{a_1 - 1} \right) - \frac{1}{2a_1} \ln \left( \frac{a_1 + 1}{a_1 - 1} \right) \right]^2
\]

where \( a_1 = Q_0/|\vec{q}| \) in Minkowski space.

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