Semi-supervised Anomaly Detection on Attributed Graphs

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Abstract
We propose a simple yet effective method for detecting anomalous instances on an attribute graph with label information of a small number of instances. Although with standard anomaly detection methods it is usually assumed that instances are independent and identically distributed, in many real-world applications, instances are often explicitly connected with each other, resulting in so-called attributed graphs. The proposed method embeds nodes (instances) on the attributed graph in the latent space by taking into account their attributes as well as the graph structure based on graph convolutional networks (GCNs). To learn node embeddings specialized for anomaly detection, in which there is a class imbalance due to the rarity of anomalies, the parameters of a GCN are trained to minimize the volume of a hypersphere that encloses the node embeddings of normal instances while embedding anomalous ones outside the hypersphere. This enables us to detect anomalies by simply calculating the distances between the node embeddings and hypersphere center. The proposed method can effectively propagate label information on a small amount of nodes to unlabeled ones by taking into account the node’s attributes, graph structure, and class imbalance. In experiments with five real-world attributed graph datasets, we demonstrate that the proposed method achieves better performance than various existing anomaly detection methods.

1. Introduction
Anomaly detection is an important task in machine learning, which is a task of identifying anomalous instances, called anomalies, in a dataset (Chandola et al., 2009; Chalapathy & Chawla, 2019). Anomaly detection methods have been used in a wide variety of applications such as intrusion detection (Dokas et al., 2002), fraud detection (Kou et al., 2004), and medical care (Keller et al., 2012).

Although many anomaly detection methods have been proposed such as one-class support vector machines (OSVM) (Schölkopf et al., 2001), autoencoder (AE) (Sakurada & Yairi, 2014), and isolation forest (Liu et al., 2008), these methods typically assume that instances are independent and identically distributed (i.i.d.). However, in many real-world applications, instances are often explicitly connected with each other, i.e., they have graph structures. For example, in botnet detection on the Internet, each host is connected by its communication (Bilge et al., 2012). In anomalous user detection on social networking services, users are connected by their social relationships (Egele et al., 2015). Such graphs, i.e., each node in a graph has instance (attributes) information, are called attributed graphs or attributed networks.

To detect anomalies on attributed graphs, many methods have been proposed (Li et al., 2017; Peng et al., 2018; Ding et al., 2019; Li et al., 2019; Akoglu et al., 2015). By considering graph structure as well as instance information, these methods often perform better than anomaly detection methods for i.i.d. data. Most existing methods aim to find anomalies on attributed graphs in an unsupervised fashion, i.e., not considering the label (normal and anomalous) information of nodes. However, label information, which is valuable for anomaly detection, may be usable in practice. Semi-supervised learning methods for an attributed graph can use this label information to classify unlabeled instances (Kipf & Welling, 2017; Yang et al., 2016; Hamilton et al., 2017; Wu et al., 2019; Rong et al., 2020; Xu et al., 2020; Sen et al., 2008). Although these methods are effective for standard classification tasks, they do not perform well when the number of anomalous training instances is small, which is common in anomaly detection tasks due to their rarity.

In this paper, we propose a novel semi-supervised anomaly detection method for an attributed graph in which there is a class imbalance. We focus on detecting anomalies on the attributed graph by using the graph structure as well as labeled and unlabeled instance information 1. In the pro-

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posed approach, we embed all nodes on the attribute graph in the latent space to better discriminate between anomalous and normal instances. Specifically, the proposed method is based on graph convolutional networks (GCNs) (Kipf & Welling, 2017), which can output node embeddings given an attributed graph while considering both graph structure and instance information effectively by stacking graph convolutional layers. With the proposed method, to obtain node embeddings specialized for anomaly detection, the parameters of the GCN are trained to minimize the volume of a hypersphere that encloses the node embeddings of normal instances while embedding anomalous ones outside the hypersphere. In anomaly detection tasks, it is important to model a data description of the normal class because the number of anomalies is too small for their description to be modeled (Chandola et al., 2009; Chalapathy & Chawla, 2019). By minimizing the volume of the hypersphere enclosing normal node embeddings, we can effectively learn a brief data description of the normal class, which is effective in one-class classification tasks (Tax & Duin, 2004; Ruff et al., 2018). In addition, to embed anomalous instances outside the hypersphere, the proposed method uses a differential area under the curve (AUC) loss as the regularizer, which can effectively extract anomalous information even though the number of anomalous training instances is small (Iwata & Yamanaka, 2019; Kumagai et al., 2019). Even if a small amount of nodes have label information on the attributed graph, this information can be effectively propagated to other nodes by using both the graph structure and the attributes of all nodes with the GCNs. As a result, the proposed method can accurately detect anomalies on the attributed graph. Figure 1 illustrates the proposed method.

Our main contributions are summarized as follows:

- We proposed a simple yet effective semi-supervised anomaly detection method on attributed graphs. Our method learns node embeddings specialized for anomaly detection in such a way that normal node embeddings are placed in a hypersphere while anomalous ones lie outside the hypersphere based on GCNs.

- Through the experiments using five real-world attributed graph datasets, we demonstrated that the proposed method performs better than various existing anomaly detection methods.

2. Related Work

Anomaly detection, which is also called outlier detection or novelty detection, has been widely studied (Chandola et al., 2009; Chalapathy & Chawla, 2019). Many unsupervised anomaly detection methods have been proposed such as OSVM based methods (Schölkopf et al., 2001; Chalapathy et al., 2018), AE based methods (Sakurada & Yairi, 2014), and density based methods (Zong et al., 2018; Mahadevan et al., 2010). Recently, deep one-class classification (Ruff et al., 2018) which is closely related to the proposed method, has showed promising results on i.i.d. data. This method learns compact representations of instances by minimizing a data-enclosing hypersphere in output space, which is an extension of the classical support vector data description (Tax & Duin, 2004) to the deep learning. However, this method is not applicable to the attributed graph. The proposed method utilizes this data-enclosing hypersphere approach to learn node embeddings for the normal instances on the attributed graph. Some studies focus on supervised anomaly detection methods that use both anomalous and normal information to obtain anomaly detectors (Iwata & Yamanaka, 2019; Chawla et al., 2002; Yamanaka et al., 2019; Ruff et al., 2019). All these methods assume that instances are i.i.d. and cannot use any graph structure information.

Unsupervised anomaly detection methods on attributed graphs have been proposed (Akoglu et al., 2015). Early methods use graph structure information only such as node degree, common neighbors, and egonet features to detect anomalies on the graph (Xu et al., 2007; Akoglu et al., 2010; Tong & Lin, 2011). Recent methods such as residual analysis based methods (Li et al., 2017; Peng et al., 2018) and graph AE based methods (Ding et al., 2019; Li et al., 2019) use node instance information as well as graph structure information to improve performance. These methods do not use label (anomalous and normal) information. In contrast,
the proposed method uses the label information to improve anomaly detection performance.

Semi-supervised learning for graph structured data has been proposed (Zhu et al., 2003; Zhou et al., 2004; Sen et al., 2008). These methods propagate label information of a small amount of nodes to unlabeled nodes by using both node instances and a graph structure. By taking advantage of the progress of graph neural networks (GGNs) including GCNs, these methods have achieved state-of-the-art results on various semi-supervised node classification tasks (Kipf & Welling, 2017; Yang et al., 2016; Hamilton et al., 2017; Wu et al., 2019; Rong et al., 2020; Xu et al., 2020). However, these methods do not assume the class imbalance and thus are not appropriate for anomaly detection tasks.

Few methods aim to detect anomalies on attributed graphs considering both label information and class imbalance like the proposed method. ImVerde (Wu et al., 2018) is a semi-supervised learning method based on random walks considering the class imbalance. One method for rare category characterization (Zhou et al., 2018) uses curriculum learning to learn node representations. In addition to learn classifier networks, both methods use the random-walks for learning node embeddings, which have many hyperparameters such as walk lengths, context sizes, and sampling numbers (Perozzi et al., 2014; Grover & Leskovec, 2016; Tang et al., 2015). The proposed method is based on simple GCNs for classifiers and thus will be convenient in practice. In addition, these methods are difficult to apply to one-class classification tasks, which contain only normal training instances. In contrast, the proposed method can be used since it explicitly models representations of the normal class.

3. Proposed Method

3.1. Task

Let $\mathcal{G} = (\mathcal{V}, A, X)$ be an undirected attributed graph, where $\mathcal{V}$ represents a set of nodes $\{v_1, \ldots, v_N\}$, $A \in \mathbb{R}^{N \times N}$ represents a symmetric adjacency matrix where its $(n, m)$-th element $a_{nm} > 0$ denotes that there is an edge between node $v_n$ and $v_m$ and $a_{nm} = 0$ denotes a no-edge, and $X = [x_1, \ldots, x_N]^T \in \mathbb{R}^{N \times D}$ represents the set of instances, where $x_n$ is $D$-dimensional attribute vector of $n$-th node $v_n$. The index sets for anomalous and normal training nodes are represented as $\mathcal{A} = \{n|v_n \text{ is anomalous}\}$ and $\mathcal{N} = \{n|v_n \text{ is normal}\}$, respectively. We assume that the label information is given for a small amount of nodes on the attribute graph, i.e., $|\mathcal{A} \cup \mathcal{N}| \ll N$. In addition, we assume the class imbalance, i.e., $|\mathcal{A}| \ll |\mathcal{N}|$, since anomalies rarely occur. We note that the proposed method is applicable even when there are no anomalous nodes, $|\mathcal{A}| = 0$.

Our task is to estimate anomaly scores of unlabeled nodes on the graph, $\mathcal{V} \setminus (\mathcal{A} \cup \mathcal{N})$, so that the anomaly score becomes high (low) when the instance is anomalous (normal), given the attributed graph $\mathcal{G}$ and its label information $\mathcal{A} \cup \mathcal{N}$.

3.2. Anomaly Scores

We define the anomaly score for each node as follows:

$$a(v_n) := \|h_n - c\|^2,$$

where $h_n \in \mathbb{R}^K$ is a $K$-dimensional learned node embedding of the $n$-th node, $c \in \mathbb{R}^K$ is a pre-determined center vector, and $\| \cdot \|$ is the Euclidian norm. This anomaly score takes a small (large) value when node embedding $h_n$ is close to (far from) center vector $c$. Therefore, to detect anomalies accurately, we want to learn node embeddings in such a way that node embeddings for normal instances are placed close to center vector $c$ while those of anomalous instances are far away from $c$. We will explain how to learn these node embeddings in the next subsection.

3.3. Model

The proposed method learns node embeddings specialized for anomaly detection on the attributed graph on the basis of GCNs, which are proposed by Kipf and Welling (2017). The GCNs learn $K$-dimensional node embedding $h_n \in \mathbb{R}^K$ for the $n$-th node by applying multiple layer transformations to attribute vector $x_n$. Specifically, the $(\ell + 1)$-th layer $H^{(\ell+1)} = [h_n^{(\ell+1)}, \ldots, h_N^{(\ell+1)}]^T$ is calculated from the previous $\ell$-th layer $H^{(\ell)}$ with the following propagation rule:

$$H^{(\ell+1)} = \sigma(\tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} H^{(\ell)} W^{(\ell)}),$$

where $\tilde{A} = A + I$ is the adjacency matrix of the graph $\mathcal{G}$ with added self-connections, $\tilde{D} \in \mathbb{R}^{N \times N}$ is the degree matrix of $\tilde{A}$, which is a diagonal matrix where its $(n, n)$-th element is $\sum_{m=1}^{N} a_{nm} + 1$, $W^{(\ell)}$ is a layer-specific trainable weight matrix, and $\sigma(\cdot)$ is an activation function such as the ReLU. The initial node embeddings are set to the original attribute vectors, $H^{(0)} := X$. Note that the proposed method is applicable even when the graph does not have attributes $X$ by regarding the identity matrix as $X = I$ as described in previous studies (Mehta et al., 2019). By decomposing equation (2) for each node, we can see that $h_\mathcal{N}^{(\ell+1)}$ is represented as follows:

$$h_n^{(\ell+1)} = \sigma \left( \frac{1}{d_n + 1} W^{(\ell)} \top h_n^{(\ell)} + \sum_{m=1}^{N} \frac{a_{nm}}{\sqrt{(d_n + 1)(d_m + 1)}} W^{(\ell)} \top h_m^{(\ell)} \right),$$

where $d_n$ is the degree of the $n$-th node, $d_n := \sum_{m=1}^{N} a_{nm}$. This equation means that node embeddings of the next layer are calculated using node embeddings of its connected nodes.
We construct the objective function of the proposed method where \( \lambda \) is the hyperparameter that controls the influence of the differentiable AUC loss. By omitting the AUC regularizer or \( \lambda = 0 \), this objective function can be used even though there is no anomalous label information. By adding the AUC regularizer, the proposed method can learn more sophisticated node embeddings that can detect anomalies accurately. Note that, when there are no anomalous instances or \( \lambda = 0 \), we should use the GCNs without bias terms, use unbounded activation functions such as the ReLU, and avoid using an all-zero vector as center vector \( c \) to prevent a hypersphere collapse, in which any node embedding converge to the center vector \( c \) (Ruff et al., 2018).

The parameters of the GCN can be optimized by minimizing (6) with any gradient-based optimization methods. Even though few nodes have label information, the proposed method can effectively and efficiently propagate this information to other nodes on the basis of the GCN.

### 3.4. Estimation

After learning the parameters of the GCN by minimizing (6), the anomaly score of unlabeled instance on the graph \( a(v_a) \) is obtained as \( a(v_a) = \|\mathbf{h}_a - c\|^2 \). Although the proposed method described in this paper is transductive, which means we can only estimate anomaly scores of instances that are already observed in the graph at training time, we can easily extend it to inductive, which means we can estimate anomaly scores for unobserved instances at training time, by applying inductive variants of GNNs such as Planetoid (Yang et al., 2016) and GraphSage (Hamilton et al., 2017).

### 4. Experiments

We demonstrated the effectiveness of the proposed method using five real-world attributed graph datasets. To measure anomaly detection performance on the attributed graphs, we used AUC, which is a well used measure for anomaly detection tasks. All experiments were conducted on a Linux server with an Intel Xeon CPU and a NVIDIA GeForce GTX 1080 GPU.

### 4.1. Data

We used five real-world attributed graph datasets: Cora, Citeseer (Cite), Pubmed (Pub), Amazon-Photo (Photo), and Amazon-Computers (Comp). Cora, Cite, and Pub are public datasets widely used in previous studies (Kipf & Welling, 2017; Yang et al., 2016; Wu et al., 2018; 2019)\(^2\). All of them are citation networks, each node corresponds to one scientific publication, and the edge represents the citation relationship between two publications. Each publication is represented by a bag-of-words attribute vector. Photo and Comp are also well used public datasets.

\(^2\)https://github.com/kimiyoung/planetoid
datasets (Shchur et al., 2018). These datasets are segments of the Amazon co-purchase graph (McAuley et al., 2015), where nodes represent goods and the edge indicates that two goods are frequently bought together. Each product review is represented by a bag-of-words attribute vector. For all datasets, the edges are unweighted, i.e., $a_{nm} = 1$ if there is a link between $v_n$ and $v_m$. Each attribute was linearly rescaled to $[0, 1]$. The statistics of the datasets are summarized in Table 1. Although these datasets have several classes, we created a binary class problem for each dataset by regarding the smallest class as anomalous and the remaining classes as normal following the previous studies (Wu et al., 2018; Zhou et al., 2018). The average anomaly rates of Cora, Cite, Pub, Photo, and Comp are 0.06, 0.07, 0.21, 0.04, and 0.02, respectively. For each dataset, we evaluated anomaly detection performance by changing the ratio of labeled instances and all instances within $\{2.5\%, 5\%, 10\%\}$. For each case, we used 10% of all instances for validation and the remaining for testing instances. We randomly generated ten training/validation/testing datasets for each case and evaluated the average test AUC over ten sets.

### 4.2. Comparison Methods

We evaluated two variants of the proposed method: Ours-AN and Ours-N. Ours-AN is the method explained in Section 3, which uses both anomalous and normal label information. Ours-N uses only normal label information. The proposed method was implemented by using PyTorch (Paszke et al., 2017) and PyTorch Geometric (Fey & Lenssen, 2019).

We compared the proposed method with the following nine methods.

**OSVM** is the one-class support vector machine (Schölkopf et al., 2001). The OSVM finds the maximal margin hyperplane which separates the given normal data from the origin in a RKHS. We used the RBF kernel.

**DOC-N** is the deep one-class classification (One-class Deep SVDD) (Ruff et al., 2018), which is a recently proposed unsupervised anomaly detection method for i.i.d. data. This method uses the feed-forward neural network to output embeddings and aims to minimize the volume of the hypersphere that encloses the embeddings of normal instances.

Although this objective is also used in the proposed method, DOC-N cannot use any graph structure information.

**DOC-AN** is a supervised extension of DOC-N. We added the differentiable AUC regularizer to the objective function of the OCD-N. We included this method in the comparison methods to evaluate the effectiveness of considering the attributed graph structure in the proposed method.

**DSAD** is the deep semi-supervised anomaly detection (Ruff et al., 2019), which is an extension of DOC-N for semi-supervised anomaly detection. This method uses anomalous and normal labeled instances and unlabeled instances to learn the anomaly detector for i.i.d. data.

**NN** is the feed-forward neural network classifier for i.i.d. data. The parameters of NN are trained by minimizing the cross entropy loss.

**SLGCN** is the semi-supervised learning method based on the GCNs (Kipf & Welling, 2017). The parameters of the GCNs are trained by minimizing the cross entropy loss of labeled nodes.

**DW** is the DeepWalk (Perozzi et al., 2014), which is a famous unsupervised node embedding method for graph structured data. This method learns node embeddings on the basis of skip gram models (Mikolov et al., 2013), which are applied to the sequences of random-walks. We used negative sampling instead of hierarchical softmax to improve performance the same as (Grover & Leskovec, 2016). After learning node embeddings, logistic regression was used as classifiers the same as (Wu et al., 2018; Zhou et al., 2018).

**DOM** is the Dominant (Ding et al., 2019), which is a recently proposed unsupervised anomaly detection method on attributed graphs. This method uses an autoencoder framework to reconstruct the original attributed graph (graph structure and node attributes). The anomaly scores are defined as a weighted sum of the graph structure and node attribute reconstruction errors.

**ImVerde** is a recently proposed semi-supervised anomaly detection method for class imbalanced attributed graphs (Wu et al., 2018). This method learns node embeddings on the basis of a vertex-diminished random-walk model that reduces the transition probability to one node each that it has visited to deal with the class imbalance. We used the authors’ implementation.

OSVM and DOC-N are anomaly detection methods for i.i.d. data, which learn from normal training instances. Note that DOC-N uses unlabeled instances as well as normal instances for training assuming that there are fewer anomalies than normal instances in the original paper. However, training with only normal instances showed better results in

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1. https://pytorch-geometric.readthedocs.io/en/latest/

4. https://github.com/jwu4sml/ImVerde
our experiments, so we report the results of DOC-N learned with normal instances in this paper. NN and DOC-AN are supervised learning methods for i.i.d. data, which learn from both anomalous and normal training instances. DSAD learns from anomalous and normal training instances and unlabeled instances. We used DSAD in the transductive setting, i.e., unlabeled training instances are equivalent to testing instances, for a fair comparison. These five methods do not use any graph structure information. DOM uses both graph structure and instance information but not label information. DW uses both the graph structure and anomalous and normal label information. SLGCN and ImVerde use the graph structure, instances, and anomalous and normal label information, which is the same as Ours-AN. For the proposed method, DOC-N, DOC-AN, DSAD, NN, and SLGCN, the three-layer feed-forward neural networks with 32 hidden nodes and ReLU activation were used. For DOM, the three-(two-)layer feed-forward neural network with 32 hidden nodes and ReLU activation was used for the encoder (the decoder). For ImVerde, we used the three-layer feed-forward neural network for the classifier the same as the authors’ implementation.

4.3. Hyperparameters

For Ours-AN, DOC-AN, DSAD, NN, SLGCN, DW, and ImVerde, we selected hyper-parameters by using validation AUC. For Ours-N, OSVM, DOC-N, and DOM, hyper-parameters were selected on the basis of the average anomaly score on validation normal instances since these methods do not use any anomalous label information for training. For OSVM, the kernel parameter was selected from \( \{10^{-3}, 10^{-2}, \ldots, 10^3\} \). For DW, we used the following typical parameters: the length of a walk was 80, the window size of neighbor in a random walk sequence was 10, and the number of negative samples was 10. For logistic regression of DW, regularization parameter \( C \) was chosen from \( \{10^{-2}, 10^{-1}, \ldots, 10^2\} \). For DOM, the balancing parameter of structure and attribute reconstruction \( \alpha \) was set to 0.5, which is the recommended value in the original paper. For ImVerde, we followed the parameters used in the author’s implementation. For the proposed method, DOC-N, DOC-AN, DSAD, DW, and DOM, and ImVerde, the dimension for node embeddings \( K \) was set to 32. For Ours-AN and DOC-AN, regularization parameter for the AUC regularizer \( \lambda \) was chosen from \( \{1, 10, \ldots, 10^4\} \). For DSAD, the weighting parameter for labeled instances \( \eta \) was selected from \( \{10^{-2}, 10^{-1}, \ldots, 10^4\} \). For DOC-N and DSAD, we set weight regularization parameter as \( 10^{-6} \) the same as in the original papers. In addition, we used the weights from the encoder part of trained AE for initialization and set hypersphere center \( c \) to the mean of the node embeddings for normal instances after performing an initial forward pass, which is a recommended procedure (Ruff et al., 2018; 2019). Following this, for Ours-N, we used the same procedure except for changing the AE as the graph AE (Kipf & Welling, 2016). For Ours-AN and DOC-AN, we did not use pre-training weights since both methods worked well without pre-training weights due to the AUC regularizer in our preliminary experiments. We set hypersphere center \( c \) to the mean of the node embeddings for normal instances after performing an initial forward pass. For all methods, we used the Adam optimizer (Kingma & Ba, 2014) with a learning rate of 0.001, and the maximum number of epochs was 500, 500, 1000, and 1000 for Cora, Cite, Pub, Photo, and Comp, respectively. We used early-stopping based on the validation data to avoid over-fitting.

4.4. Results

First, we quantitatively evaluated the anomaly detection performance of the proposed method. Tables 2 – 4 show the average and standard deviation of test AUCs for each dataset with the different ratio of labeled and all instances. In Tables 2 – 4, boldface denotes the best and comparable methods according to the paired t-test at a significance level of 5%. Ours-AN showed the best/comparable test AUCs in almost all cases (13 of 15). Overall, methods that use anomalous label information (i.e., Ours-AN, DOC-AN, DSAD, NN, SLGCN, and ImVerde) performed better than other methods, which indicates the usefulness of anomalous label information for anomaly detection tasks. Although DW uses both anomalous and normal label information, it performed poorly. This was most likely because this method takes a two-step approach, i.e., learning classifiers after learning node embeddings, and thus discriminative information disappeared in the process of learning node embeddings. As for methods that do not use anomalous label information (i.e., Ours-N, OSVM, DOC-N, and DOM), Ours-N performed the best in almost all cases (13 of 15). Although DOC-N, DOC-AN, and DSAD aim to minimize the volume of the instances-enclosing hyperspheres like the proposed method, the proposed method outperformed them because it takes the graph structure information into account. As a result, these results indicate the effectiveness of the proposed method in settings in which both anomalous and normal labels or only normal labels are available for training.

Next, we visualized the learned node embeddings to quantitatively evaluate the proposed method. Figure 2 shows the node embeddings on Cora learned by Ours-AN, Ours-N, DOC-N, DOC-AN, DW, and SLGCN when 10% of all instances were labeled. For SLGCN, we used the hidden layer as the node embeddings. We used t-distributed stochastic neighbor embeddings (t-SNE) (Maaten & Hinton, 2008) to reduce the dimensions of the node embeddings from 32 to 2. As for methods that use both anomalous and normal label information for learning node embeddings (i.e., Ours-AN, DOC-AN, and SLGCN), Ours-AN was able to learn better
Third, we investigated the dependency of the regularizer weight for the AUC regularizer $\lambda$ for Ours-AN and DOC-AN, which use the regularizer. Figure 3 shows the average test AUCs by changing $\lambda$ when the rate of labeled and all instances was 2.5%. For all datasets, Ours-AN outperformed DOC-AN in almost all $\lambda$, which indicates the robustness of the proposed method against $\lambda$. The best $\lambda$ of Ours-AN differed across datasets. With Photo, large $\lambda$, which corresponds to minimize the AUC loss only in (6), performed the best although the small value ($\lambda = 1$) performed better with Cora, Cite, Pub, and Comp.

Fourth, we investigated the dependency of the dimension of embeddings $K$ for the proposed method. Figure 4 shows the average test AUCs by changing $K$ when the rate of labeled and all instances was 2.5%. We compared Ours-AN and Ours-N with the embedding based methods: DOC-AN, DOC-N, DSAD, DW, DOM, and ImVerde. Ours-AN consistently performed well in all $K$ for all datasets. As for methods that do not use anomalies for training, Ours-N performed better than DOC-N with almost all $K$. Overall, these results suggest the proposed method is robust against the value of $K$.

Lastly, we investigated the training times of 500 epochs for Ours-AN, Ours-N, and SLGCN on Cora when 10% of all instances were labeled and $K = 32$. The training times of Ours-AN, Ours-N, and SLGCN were 4.92, 3.56, and 2.46 seconds, respectively. Since Ours-AN uses the AUC regularizer, it took more training time than Ours-N. However, the proposed method was able to learn fast enough.

5. Conclusions

In this paper, we proposed a novel semi-supervised anomaly detection method on attribute graphs. The proposed method...
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utilizes graph GCNs to extract node embeddings considering both label information of small nodes and attribute information of all nodes with the graph structure. To learn useful node embeddings for anomaly detection, the proposed method minimizes the volume of the hypersphere that encloses normal node embeddings while embedding anomalous ones outside the hypersphere. In experiments using five real-world attributed graph datasets, the proposed method outperformed various existing anomaly detection methods in settings in which both anomalous and normal labels or only normal labels are available for training.

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Figure 2. Visualization of the learned node embeddings on Cora. Red and blue points represent anomalous and normal training instances, respectively. Orange and sky blue points represent anomalous and normal testing instances, respectively.

Figure 3. Average test AUCs of each dataset when $\lambda$ was changed.
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Figure 4. Average test AUCs of each dataset when $K$ was changed.

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