Ground-state cooling of an magnomechanical resonator induced by magnetic damping

Ming-Song Ding¹, Li Zheng², and Chong Li¹

¹School of Physics, Dalian University of Technology, Dalian 116024, China
²Science and Engineering College, Dalian Polytechnic University, Dalian 116034, China

(Dated: February 10, 2022)

Abstract: Quantum manipulation of mechanical resonators has been widely applied in fundamental physics and quantum information processing. Among them, cooling the mechanical system to its quantum ground state is regarded as a key step. In this work, we propose a scheme which one can realize ground-state cooling of resonator in a cavity magnomechanical system. The system consists of a microwave cavity and a small ferromagnetic sphere, in which phonon-magnon coupling and cavity photon-magnon coupling can be achieved via magnetostrictive interaction and magnetic dipole interaction, respectively. After adiabatically eliminating the cavity mode, an effective Hamiltonian for exploring the radiation-pressure-mediated interaction, respectively. After adiabatically eliminating the cavity mode, an effective Hamiltonian for exploring the radiation-pressure-mediated interaction is obtained. Within experimentally feasible parameters, we demonstrate that the ground-state cooling of the magnomechanical resonator can be achieved by extra magnetic damping. Unlike optomechanical cooling, magnomechanical interaction is utilized to realize the cooling of resonators. We further illustrate the ground-state cooling can be effectively controlled by the external magnetic field.

I. INTRODUCTION

In recent years, the ferrimagnetic systems has attracted considerable interest, which can realize the strong light-matter interactions. Among them, the cavity-magnon system has shown its extensive application prospects, in including the bistability of cavity magnon polaritons [1], the light transmission [2], the electromechanical elements. In addition, the cavity magnomechanical system provides a promising platform for the study of macroscopic quantum phenomena.

Here, we explore theoretically the ground-state cooling of the magnomechanical resonator with experimentally reachable parameters. The system consists of a YIG sphere placed in a low-Q microwave cavity, and an uniform external bias magnetic field. Base on the highly dissipative cavity mode, the system is effectively transformed into a two-mode system, which is composed of a magnon mode and a mechanical mode. Then we give the expression of the final phonon number in the full quantum theory; it illustrates that the magnon mode can be utilized to cool the resonator to its ground-state. In order to achieve better cooling effect, a drive magnetic field is introduced to enhance the magnon-phonon coupling. In addition, the ground state cooling can be well controlled by adjusting the external magnetic field without changing other parameters, which provides an additional degree of freedom.

The structure of the paper is as follows. In Sec. II, we frist establish a physical model and give the linearized Hamiltonian. Then, by assuming the cavity mode is highly dissipative, the effective Hamiltonian by adiabatically eliminating optical mode is given. We also show the equations of motion. In Sec. III, we give the ana-

Quantum manipulation of mechanical resonators has been widely applied in fundamental physics and quantum information processing. Among them, cooling the mechanical system to its quantum ground state is regarded as a key step. In this work, we propose a scheme which one can realize ground-state cooling of resonator in a cavity magnomechanical system. The system consists of a microwave cavity and a small ferromagnetic sphere, in which phonon-magnon coupling and cavity photon-magnon coupling can be achieved via magnetostrictive interaction and magnetic dipole interaction, respectively. After adiabatically eliminating the cavity mode, an effective Hamiltonian for exploring the radiation-pressure-mediated interaction is obtained. Within experimentally feasible parameters, we demonstrate that the ground-state cooling of the magnomechanical resonator can be achieved by extra magnetic damping. Unlike optomechanical cooling, magnomechanical interaction is utilized to realize the cooling of resonators. We further illustrate the ground-state cooling can be effectively controlled by the external magnetic field.
lytic expression of extra magnetic damping for the magnomechanical resonator, then the final phonon number is studied. We also find the ground-state cooling of an magnomechanical resonator can be achieved with the experimentally feasible parameters. The effects of external magnetic field on the cooling are also studied. Finally, we make a conclusion based on the results obtained in sec. IV.

II. MODEL AND EFFECTIVE HAMILTONIAN

We utilize a hybrid system as shown in Fig. 1(a), which consists a microwave cavity and a small sphere (a highly polished single-crystal YIG sphere of diameter 1mm is used in [20]). The YIG sphere is placed near the maximum microwave magnetic field of the cavity mode, and we add an adjustable external magnetic field $H$ in $z$-axis, which establishes the magnon-photon coupling [18, 20], and the rate of coupling can be tuned by the position of YIG sphere in the cavity. The adjusting range of bias magnetic field $H$ is between 0 and $IT$ [20].

Because of the magnetostrictive effect, YIG sphere can be used as an excellent mechanical resonator. Therefore, the term of coupling between magnons and phonons can be introduced into Hamiltonian of our system as mentioned in [37]. Based on it, we have the vibrational mode (phonons) of the sphere.

There are three modes in this system: cavity photon mode, magnon mode and phonon mode. The equivalent coupled-harmonic-resonator model is given in Fig. 1(b), and we assume that the cavity mode is highly dissipative. Here, a microwave source is used to directly drive the magnon mode, therefore, the magnomechanical coupling can be enhanced [20, 36]. Moreover, the size of the sphere we considered is much smaller than the wavelength of the microwave field. Accordingly, the interaction between cavity microwave photons and phonons due to the effect of radiation pressure is neglected. After making a frame rotating at the drive frequency $\omega_d$ and using the rotating-wave approximation, the total Hamiltonian of hybrid system can be written as ($\hbar = 1$)

$$H_{\text{total}} = -\Delta_a a^\dagger a - \Delta_m m^\dagger m + \omega b^\dagger b + g_{ma}(a^\dagger m + m^\dagger a) + g_{mb}(m^\dagger b + b^\dagger m) + i(\varepsilon_d m^\dagger - \varepsilon_g^* m),$$

where $\Delta_a = \omega_d - \omega_a$ and $\Delta_m = \omega_d - \omega_m$ are the detunings, $\omega_d$ denotes the resonance frequency of the mechanical mode. A uniform magnon mode in the YIG sphere at frequency $\omega_m = \gamma_a H$, where $\gamma_a/2\pi = 28GHz/T$ is gyromagnetic ratio, and we set $\omega_m$ at the frequency of Kittel mode [14] (uniformly precessing mode), which can strongly couple to the microwave cavity photons leading to cavity polaritons. $a(a^\dagger), m(m^\dagger)$ and $b^\dagger(b)$ are the annihilation(creation) operators of the cavity mode, magnon mode and mechanical mode, respectively. In addition, $g_{ma}$ and $g_{mb}$ are the coupling rates of the magnon-cavity interaction and magnon-phonon interaction. $i(\varepsilon_d m^\dagger - \varepsilon_g^* m)$ is the Hamiltonian which describes the external driving of the magnon mode.

As we know, $g_{mb}$ is much weak in the experiment [37]. The magnetostrictive coupling strength is determined by the mode overlap between the uniform magnon mode and the phonon mode. J. Q. You et al. designed an experimental setup, where the YIG sphere can be directly driven by a superconducting microwave line which is connected to the external port of the cavity [18, 20]. Rabi frequency $\varepsilon_d = \frac{2\Delta}{\gamma_g}\sqrt{MB_0}$ (under the assumption of the low-lying excitations) stands for the coupling strength of the drive magnetic field [36]. The amplitude and frequency are $B_0$ and $\omega_d$ respectively, the total number of spins $M = \rho V$, where $V$ is the volume of the sphere. Furthermore, $\rho = 4.22 \times 10^{27} m^{-3}$ is the spin density of the YIG sphere.

The dynamics of the system can be linearized, through a series of calculations, we have the linearized Hamiltonian (see Appendix A)

$$H_{\text{lin}} = -\Delta_a a^\dagger a - \Delta_m m^\dagger m + \omega b^\dagger b + (Gm^\dagger + G^* m)(b + b^\dagger) + (g_{ma} m^\dagger a + g_{ma}^* m a^\dagger),$$

where $\Delta_m = \Delta_m - g_{mb}(\beta + \beta^*)$ is the modified detuning of the magnon mode. For the parameters we considered here, $g_{mb}(\beta + \beta^*) \ll \Delta_m$, so we can approximately have $\Delta_m \approx \Delta_m$. $G = \eta g_{mb}$ can be regarded as the coherent-driving-enhanced magnomechanical coupling strength with $\eta$ the average magnetic field of magnon mode. We have $\beta = -ig_{mb}|\eta|^2/(i\omega_b + \gamma_b)$ by solving steady-state Langevin equations, and $\eta$ is given by

$$\eta = \frac{\varepsilon_d(-i\Delta_a + \kappa_a)}{g_{ma}^2 + (-i\Delta_m + \kappa_m)(-i\Delta_a + \kappa_a)},$$

where $\kappa_a, \kappa_m$ and $\gamma_a$ are the losses of microwave cavity mode, magnon mode and mechanical mode, respectively. Because $\eta$ is affected by the driving field, We can enhance $G$ by tuning the external driving field $\varepsilon_d$. Note that the nonlinearity in $\Delta_m$ comes from $|\eta|^2$ intrinsically.

The quantum Langevin equations of the linearized Hamiltonian $H_{\text{lin}}$ in Eq.(2) are given by

$$\dot{a} = (i\Delta_a - \kappa_a) a - ig_{ma} m - \sqrt{2\kappa_a} a_{in},$$

$$\dot{m} = (i\Delta_m - \kappa_m) m - ig_{ma} a - iG(b + b^\dagger) - \sqrt{2\kappa_m} m_{in},$$

$$\dot{b} = (-i\omega_b - \gamma_b) b - i(G^* m + Gm^\dagger) - \sqrt{2\gamma_b} b_{in},$$

where $a_{in}, m_{in}$ and $b_{in}$ are the corresponding noise operators, the correlation functions can be found in the appendix A. To make the following result within experimental realizations, the parameters are in accord with recent cavity magnomechanical work [36–38], i.e.,
highly dissipative. In the limit that \( \kappa_m \gg g_m a \) the effective Hamiltonian by assuming the mode resonator and make our calculations more convenient, we show a effective Hamiltonian by assuming the mode resonator and make our calculations more convenient, we...
E, and F are related to the energy swapping, counter-rotating-wave interaction between the magnon mode and the phonon mode, and the dissipation of magnon mode.

In order to better prove the accuracy of $H_{eff}$ obtained, we show the fidelity between the exact state $\rho$ and the effective state $\rho_{eff}$ as a function of $t(\omega_b^{-1})$. The parameters we chosen are the same as those in Fig. 3.

III. MAGNOMECHANICAL COOLING

We analyze the ground-state cooling of the whole system in the cases of weak and strong magnomechanical coupling. In the weak magnomechanical coupling regime, similar to dealing with such problems in optomechanical systems, we use quantum noise approach to get the extra magnetic damping $\Gamma_{md}$ for the magnomechanical resonator (see Appendix B)

$$\Gamma_{md} = -2i\text{Im}\Sigma(\omega_b)$$

$$= 2|G|^2 \text{Re}\left[\frac{1}{-i(\omega_b + \Delta_{eff}) + \kappa_{eff}} - \frac{1}{-i(\omega_b - \Delta_{eff}) + \kappa_{eff}}\right].$$

By solving the rate equation in the steady state, the final phonon number can be analytically described by

$$n_f = \frac{A_+ + \gamma_b n_{th}}{\Gamma_{md} + \gamma_b},$$

where $A_+$ is the heating rate, its detailed expressions can be found in Appendix B. $n_{th} = (e^{\omega_b/k_B T} - 1)$ is thermal phonon number of the mechanical resonator, $T$ is the environmental temperature and $k_B$ is the Boltzmann constant. Here, $n_f$ can also be regarded as the cooling limit. And it can be divided into two parts, one part is the classical cooling limit $\gamma_b n_{th}/(\Gamma_{md} + \gamma_b)$, the other part is the quantum cooling part $A_+/\Gamma_{md}$. Eqn. (11) and Eqn. (12), $n_f$ decreases with the increase of $\Gamma_{md}$. Fig. 3(b) shows the final phonon number $n_f$ under different $\Delta_{eff}/\omega_b$ with two coupling strengths $G/\omega_b = 0.15$ and $G/\omega_b = 0.075$, respectively, it corresponds to the heating rate $A_+$ originates from the quantum backaction.

Fig. 3(a) shows the extra magnetic damping $\Gamma_{md}$ under different $\Delta_{eff}/\omega_b$ with two coupling strengths $G/\omega_b = 0.15$ and $G/\omega_b = 0.075$, respectively. It can be seen that the maximum magnetic damping (gain) is located at the point $\Delta_{eff} = -\omega_b$. From Eqn. (11) and Eqn. (12), $n_f$ decreases with the increase of $\Gamma_{md}$. Fig. 3(b) shows the final phonon number $n_f$ under different $\Delta_{eff}/\omega_b$ with two coupling strengths, respectively. It shows the minimum $n_f$ is located at the detuning point $\Delta_{eff} = -\omega_b$. When $G/\omega_b = 0.15$ and $G/\omega_b = 0.075$, the minimums of $n_f$ are about $10^{-2}$ and $10^{-1}$ respectively, ground-state cooling of the magnomechanical resonator can be achieved. Moreover, compared to the two different $G$ in Fig. 3(a) and Fig. 3(b), it can be known that the stronger $G$, the better effect of ground-state cooling in weak coupling regime ($G < \kappa_{eff}$).
In order to study the influence of magnon mode on the cooling of the oscillator, in Fig. 4, we plot the final phonon number $n_f$ under different external magnetic field $H$ with two coupling strengths $G/\omega_b = 0.15$ (red solid curve) and 0.1 (blue dotted curve). The other parameters are the same as those in Fig. 3.

![Graph](image)

**FIG. 4.** The final phonon number $n_f$ versus the external magnetic field $H$ with two coupling strengths $G/\omega_b = 0.15$ (red solid curve) and 0.1 (blue dotted curve). The other parameters are the same as those in Fig. 3.

The recent work shows the strong coupling between magnon mode $m$ and mechanical mode $b$ can be achieved. From Eq.(10), the covariance approach is used to calculate the mean phonon number $N_{bs}$ in steady state (see Appendix C), and this approach is applicable to both cases of strong coupling and weak coupling.

$$N_{bs} \simeq \frac{4 |G|^2 + \kappa_{eff}^2}{4 |G|^2 (\gamma_b + \kappa_{eff})} \gamma_b n_{th}$$

$$+ \frac{4\omega_b^2 (\kappa_{eff}^2 + 8 |G|^2) + \kappa_{eff}^2 (\kappa_{eff}^2 - 8 |G|^2)}}{16\omega_b^2 (4\omega_b^2 + \kappa_{eff}^2 - 16 |G|^2)}.$$  \hspace{1cm} (13)

For $G/\omega_b = 0.4$ ($G > \kappa_{eff}$), the other parameters are the same as those in Fig. 3, we have $N_{bs} \simeq 0.257$ by Eq.(13). Under the same parameters, the numerical solution of the mean phonon number obtained from the quantum master equation is shown in Fig. 5. Here, we only need to numerically calculate the mean values of all the second-order moments instead of the matrix elements of the density operator $\rho$ (see Appendix C). The cut-off of the density matrix is not necessary and the solutions are exact. It can be seen that the numerical solution $N_{bs}$ tends to be stable at 0.28 after a period of oscillation. This agrees roughly with the analytical result. Note that the analytical and numerical solution are both obtained under condition $\Delta_{eff} = -\omega_b$. The results show that ground-state cooling can be achieved in strong coupling regime.

The dynamical stability condition of our system can be given by the Routh-Hurwitz criterion [54]. Y. C. Liu, et al discussed a similar two-mode coupled system [53]. Referring to their results, the steady-state condition here is $|G|^2 < \omega_b^2/4 + \kappa_{eff}^2/16$ with the detuning $\Delta_{eff} = -\omega_b$. And the parameters used here are all satisfied with the stability condition.

Our discussion is on the premise of low-lying excitation, which is $\langle m^4 \rangle \ll 2N s = 5N$ ($N$ is the total number of spins). For a 1-mm-diam YIG sphere [18, 20], $N = N V = 2.2 \times 10^{18}$, where $\Lambda = 4.22 \times 10^{27}$ is the spin density of the YIG and $V$ is the volume of the sphere. For $G/2\pi = 4 MHz$, we have $|\eta| = 4 \times 10^7$. Finally $\langle m^4 \rangle = 1.6 \times 10^{15} \ll 5N \approx 1.1 \times 10^{19}$, the condition of low-lying excitation is satisfied.

IV. CONCLUSIONS

In summary, we have studied ground-state cooling in a cavity magnomechanical system, which has three modes: cavity photon mode, magnon mode and phonon mode. The magnon and mechanical modes are coupled to each other through the magnetostrictive interaction. By assuming the cavity mode is highly dissipative, we adiabatically eliminate cavity mode, and the effective Hamiltonian is given. Which is consist of two mode: magnon mode and phonon mode. Then we study the final phonon number numerically and analytically. And we find the ground-state cooling of magnomechanical resonator can be achieved by using experimentally feasible parameters. Different from the existing optomechanical cooling system, the extra magnetic damping is the reason of cavity magnomechanical cooling intrinsically. In other words, we can utilize magnon mode to achieve the cooling of mechanical mode. Furthermore, the ground-state cooling can be well controlled by adjusting the magnetic field without changing other parameters, which provides an additional degree of freedom.

Because the cavity magnomechanical system has intrinsic great tunability, low loss, and promising integration with electromechanical elements, we believe that the proposed scheme provides a promising platform to the further investigation of cooling of mechanical resonator.
And we hope it opens up new way to the foundations of quantum physics and applications. In addition, cooling the mechanical system to its quantum ground state is also an important guarantee for realizing quantum operations in quantum information processing.

ACKNOWLEDGEMENTS

We thank Y. X Zeng for his fruitful discussion. This work was supported by National Natural Science Foundation of China (NSFC): Grants Nos. 11574041 and 11375036.

Appendix A: linearization of Hamiltonian

From Eq.(1), the quantum Langevin equations (QLEs) of the system are given by

\[
\dot{a} = (i\Delta_a - \kappa_a)a - ig_{ma}m - \sqrt{2\kappa_a}a_{in},
\]
\[
\dot{m} = (i\Delta_m - \kappa_m)m - ig_{ma}a - ig_{mb}(b + b^\dagger) + \varepsilon_d - \sqrt{2\kappa_m}m_{in},
\]
\[
\dot{b} = (-i\omega_b - \gamma_b)b - ig_{mb}m_{\dagger}m - \sqrt{2\gamma_b}b_{in},
\]

where \(a_{in}, m_{in}\) and \(b_{in}\) are the corresponding noise operators with zero mean values, and the correlation functions for these noise operators can be written as

\[
\langle a_{in}(t)a_{in}^\dagger(t') \rangle = \delta(t - t'),
\]
\[
\langle a_{in}^\dagger(t)a_{in}(t') \rangle = 0,
\]
\[
\langle m_{in}(t)m_{in}^\dagger(t') \rangle = \delta(t - t'),
\]
\[
\langle m_{in}^\dagger(t)m_{in}(t') \rangle = 0,
\]
\[
\langle b_{in}(t)b_{in}^\dagger(t') \rangle = (n_{th} + 1)\delta(t - t'),
\]
\[
\langle b_{in}^\dagger(t)b_{in}(t') \rangle = n_{th}\delta(t - t'),
\]

where \(n_{th}\) is thermal phonon number of the mechanical resonator, and it can be regarded as \(n_{th} = (e^{\hbar\omega_b/k_B T} - 1)\), \(T\) is the environmental temperature and \(k_B\) is the Boltzmann constant.

Then we rewrite each Heisenberg operator as a sum of its steady-state mean value and the quantum fluctuations, i.e., \(a = a + a', m = m + m'\) and \(b = b + b'\). By separating the classical and quantum components, the quantum Langevin equations (QLEs) can be rewritten as

\[
\dot{a} = (i\Delta_a - \kappa_a)a - ig_{ma}m - \sqrt{2\kappa_a}a_{in},
\]
\[
\dot{m} = (i\Delta_m - \kappa_m)m - ig_{mb}\eta(b + b^\dagger) + G_m^1 + G^m_m(b + b^\dagger),
\]
\[
\dot{b} = (-i\omega_b - \gamma_b)b - ig_{mb}(\eta^*m + \eta m^\dagger) - ig_{mb}m_{\dagger}m + \sqrt{2\gamma_b}b_{in},
\]

\[
H_{\text{lin}} = -\Delta_a a^\dagger a - \Delta_m m_{\dagger}m + \omega_bb^\dagger b + \eta g_{mb}(b + b^\dagger) + \eta g_{mb}(\eta^*m + \eta m^\dagger) - \eta g_{mb}m_{\dagger}m + \sqrt{2\gamma_b}b_{in},
\]

where \(\Delta_m = \Delta_m - g_{mb}(\beta + \beta^*)\). Here, under the strong driving condition, the nonlinear terms \(ig_{mb}(b + b^\dagger)\) and \(ig_{mb}m_{\dagger}m\) can be neglected, then we have linearized quantum Langevin equations, and the Hamiltonian in Eq.(1) is rewritten as an linearized Hamiltonian

\[
H_{\text{lin}} = -\Delta_a a^\dagger a - \Delta_m m_{\dagger}m + \omega_bb^\dagger b + \eta g_{mb}(b + b^\dagger) + \eta g_{mb}(\eta^*m + \eta m^\dagger) - \eta g_{mb}m_{\dagger}m + \sqrt{2\gamma_b}b_{in},
\]

Appendix B: weak coupling

Similar to the method used in cavity optomechanical systems, we study the cooling rate of the magnomechanical resonator by using the quantum noise spectrum of the magnetic force.

In weak coupling regime \((G < \kappa_{eff})\), the quantum noise approach is feasible. From Eq.(10), \(F_m(t) = \eta g_{mb}(b + b^\dagger) + \eta g_{mb}(\eta^*m + \eta m^\dagger) - \eta g_{mb}m_{\dagger}m + \sqrt{2\gamma_b}b_{in}\).
\[ \hat{m}\ddot{x} + \hat{m}\omega_0^2 x \text{ and } \dot{\hat{p}} + \hat{m}\omega_0^2 x = -\frac{1}{x_{ZPF}} [G^*m(t) + Gm^1(t)], \]

the magnetic force operator can be obtained as

\[ F_m(t) = -\frac{1}{x_{ZPF}} [G^*m(t) + Gm^1(t)], \quad (B5a) \]

where \( \hat{m} \) is the effective mass of the mechanical resonator, \( \hat{p} \) is the momentum operator, \( x = x_{ZPF}(b + b^\dagger) \) is the position operator and \( x_{ZPF} = \sqrt{1/(2\hat{m}\omega_0)}(x_{ZPF} \text{ is the zero-point fluctuation amplitude of the mechanical oscillator}) \). Using the Fourier transform of the autocorrelation functions, the quantum noise spectrum is given by

\[ S_{FF}(\omega) = \int \langle F_m(t)F_m(0) \rangle e^{i\omega t} \, dt. \quad (B6a) \]

In the absence of the magnomechanical coupling, the term of \( m \) in the Langevin equations of the effective Hamiltonian \( H_{eff} \) can be expressed as

\[ -i\omega m(\omega) = (i\Delta_{eff} - \kappa_{eff})m(\omega) - \sqrt{2\kappa_{eff}m_{eff,in}}, \quad (B7a) \]

Here, we transform \( m(t) \) to the frequency domain. Then using Eq.\( (B5a) \) and Eq.\( (B6a) \), the spectral density of the magnetic force is given by

\[ S_{FF}(\omega) = \frac{2\kappa_{eff} |G(\omega)|^2}{x_{ZPF}^2}, \]

where the response function is

\[ \chi(\omega) = \frac{1}{-i(\omega + \Delta_{eff}) + \kappa_{eff}}. \quad (B9a) \]

The cooling and heating rates can be given by \( A_- = S_{FF}(\omega)x_{ZPF}^2 \) and \( A_+ = S_{FF}(\omega)x_{ZPF}^2 \), respectively. And they correspond to the rates for absorbing and emitting a phonon, respectively.

By considering the magnomechanical coupling, the Langevin equations of the effective Hamiltonian \( H_{eff} \) are given by

\[ -i\omega m(\omega) = (i\Delta_{eff} - \kappa_{eff})m(\omega) - iG[b(\omega) + b^\dagger(\omega)] - \sqrt{2\kappa_{eff}m_{eff,in}(\omega)}, \quad (B10a) \]

\[ -i\omega b(\omega) = (-i\omega_b - \gamma_b)b(\omega) - i[G^*m(\omega) + G(\omega)m^1(\omega)] - \sqrt{2\gamma_b b_n(\omega)}, \quad (B10b) \]

where noise operators \( m_{eff,in}(\omega) \) and \( b_n(\omega) \) are the corresponding noise operators in the frequency domain. from which we obtain

\[ b(\omega) \simeq \frac{\sqrt{2}\gamma_b b_n(\omega) - i\sqrt{2}\kappa_{eff}f(\omega)}{i\omega - i[\omega_b + \Sigma(\omega)] - \gamma_b}, \]

where \( f(\omega) = G^*\chi(\omega)m_{eff,in}(\omega) + G\chi^*(-\omega)m_{eff,in}(\omega) \). The reason for the approximation is we consider \( \omega \simeq \omega_b \), in this way, the terms containing \( b^\dagger(\omega) \) can be neglected. \( \Sigma(\omega) = -i|G|^2[\chi(\omega) - \chi^*(-\omega)] \) is the magnomechanical self energy. Then the frequency shift \( \delta\omega_b \) and the extra magnetic damping \( \Gamma_{md} \) are given as

\[ \delta\omega_b = Re\Sigma(\omega), \quad \Gamma_{md} = -2Im\Sigma(\omega) = A_- - A_. \quad (B12a) \]

Using similar methods in cavity optomechanical systems, from the rate equation for the probability \( P_n(t) \) and the average phonon number \( \bar{n} = \sum_{n=0}^{\infty} nP_n [27] \), and after making \( \bar{n} = 0 \), we have the final phonon number \( n_f \)

\[ n_f = \frac{A_+ + \gamma_b n_{th}}{\Gamma_{md} + \gamma_b}. \quad (B13a) \]

### Appendix C: Strong coupling

Because of the introduction of external drive magnetic field, the whole system can enable strong coupling \( (G > \kappa_{eff}) \) between magnon mode \( m \) and mechanical mode \( b \) by adjusting the intensity of driving field.

For the linear regime under strong driving, the mean phonon number can be computed exactly by the quantum master equation. From the effective Hamiltonian in Eq.\( (10) \), the quantum master equation can be described by

\[ \dot{\rho} = i[\rho, H_{eff}] + \kappa_{eff}\mathcal{L}(m)\rho + \gamma_b\rho, \quad (C14a) \]

where the Lindblad superoperators are given by

\[ \mathcal{L}(\rho) = \rho \omega b^\dagger b - b^\dagger b \rho - \rho b^\dagger b \omega, \quad (C15a) \]

Since the Hamiltonian is linear, we can only calculate the mean values of all the second-order moments, Such as \( \langle m^1m \rangle, \langle b^\dagger b \rangle, \langle m^1b \rangle, \langle mb \rangle, \langle m^2 \rangle, \langle b^2 \rangle \) and the conjugation of the last four terms. In the stable regime, under the conditions \( \Delta_{eff} = -\omega_b \) and \( 4|G|^2/(\gamma_b\kappa_{eff}) \gg 1 \), \( N_{bs} \) is calculated as

\[ N_{bs} \simeq \frac{4|G|^2 + \kappa_{eff}^2}{4|G|^2(\gamma_b + \kappa_{eff})}\gamma_b n_{th} \]

\[ \frac{4\omega_b^2(\kappa_{eff}^2 + 8|G|^2) + \kappa_{eff}^2(\kappa_{eff}^2 - 8|G|^2)}{16\omega_b^2(4\omega_b^2 + \kappa_{eff}^2 - 16|G|^2)}. \quad (C16a) \]

Note that no cut-off of the density matrix is required in this solution, and it holds for both weak and strong coupling regimes.
[46] S. Grölacher, J. B. Hertzberg, M. R. Vanner, S. Gigan, K. C. Schwab, and M. Aspelmeyer, Demonstration of an ultracold microoptomechanical oscillator in a cryogenic cavity, Nat. Phys. 5, 485 (2009).
[47] Y. S. Park and H. Wang, Resolved-sideband and cryogenic cooling of an optomechanical resonator, Nat. Phys. 5, 489 (2009).
[48] Meenehan S M, Cohen J D, MacCabe G S, et al. Pulsed excitation dynamics of an optomechanical crystal resonator near its quantum ground state of motion[J]. Physical Review X, 2015, 5(4): 041002.
[49] Zhou B, Li G. Ground-state cooling of a nanomechanical resonator via single-polariton optomechanics in a coupled quantum-dot–cavity system[J]. Physical Review A, 2016, 94(3): 033809.
[50] Fogarty T, Landa H, Cormick C, et al. Optomechanical many-body cooling to the ground state using frustration[J]. Physical Review A, 2016, 94(2): 023844.
[51] Z. Q. Yin, W. L. Yang, L. Y. Sun, and L. M. Duan, Phys. Rev. A 91, 012333 (2015).
[52] Y. C. Liu, Y. F. Xiao, X. S. Luan, and C. W. Wong, Phys. Rev. Lett. 110, 153606 (2013).
[53] Y. C. Liu, W. H. Yu, C. W. Wong, and Y. F. Xiao, Chin. Phys. B 22, 114213 (2013).
[54] E. X. DeJesus and C. Kaufman, Routh-Hurwitz criterion in the examination of eigenvalues of a system of nonlinear ordinary differential equations, Phys. Rev. A 35, 5288 (1987).
[55] Y. X. Zeng, J. Shen, T. Gebremariam, and C. Li, Quantum. Inf. Process 18, 205 (2019).
[56] Y. X. Zeng, T. Gebremariam, M. S. Ding, and C. Li, J. Opt. Soc. Am. B 35, 2334 (2018).