Fermi bubbles: the collimated outburst needed to explain forward-shock edges

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ABSTRACT
The bipolar, nonthermal, high-latitude lobes known as the Fermi bubbles (FBs) are thought to originate from a massive energy release near the Galactic centre (GC). We constrain the FB engine and the circumgalactic medium (CGM) by analytically and numerically modeling the FB edges as strong forward shocks, as inferred from recent observations. A non-directed energy release produces shocks too spherical to account for observations even for a maximally massive Galactic disc, critical CGM rotation, or injection effectively offset from the GC. In contrast, collimated injection nearly perpendicular to the disc can account for observations in both ballistic (free expansion) and slowdown regimes, as we show using a simple stratified evolution model verified by hydrodynamic simulations. FBs still in their ballistic regime require injection (at \(z \approx 100\) pc heights in our model) with a half-opening angle \(\theta \approx 4^\circ\), a normalized velocity \(\beta^{-2} = v/(0.01c) \gtrsim 0.4\), and an energy \(E \gtrsim 2\beta^{-2} \times 10^{55}\) erg, launched \(T \approx 3.3\beta^{-1}\) Myr ago, showing a distinctive low-pressure region behind the bubble head. Slowing-down (mass accumulated) FBs require a faster injection, a thinner jet, a smaller \(E/(\beta^{-2}\theta^2)\), and a comparable \(T\), and follow a ballistic stage that must reach a height \(z_s \gtrsim 5\) kpc.

Key words: ISM: jets and outflows – Galaxy: centre – Galaxy: halo

1 INTRODUCTION
The Fermi bubbles (FBs) are bipolar, non-thermal lobes, extending to latitudes \(|b| \approx 50^\circ–55^\circ\) above and below the Galactic centre (GC). Their high-latitude signature was first revealed in \(\gamma\)-rays, by the Fermi Gamma-ray Space Telescope (Dobler et al. 2010; Su et al. 2010, henceforth S10), but subsequently also in microwaves (Dobler & Finkbeiner 2008; Dobler 2012; Planck Collaboration 2013) and in X-rays (Keshet & Gurwich 2018, hereafter KG18). The edges of the FBs are distinct and easily traced (Keshet & Gurwich 2017, hereafter KG17), linking them to large (Sofue 2000), intermediate (Bland-Hawthorn & Cohen 2003) and small (Baganoff et al. 2003) scale nonthermal features, and thus identifying the FBs as arising from a major, \(> 10^{55}\) erg event (Veilleux et al. 2005, and references therein) near the GC.

The origin of the FBs is still uncertain, present models including a starburst (Carretti et al. 2013; Lacki 2014; Sarkar et al. 2015b, hereafter S15b), an SMBH jet (Cheng et al. 2011; Guo & Mathews 2012; Zubovas & Nayakshin 2012), SMBH accretion wind (Mou et al. 2014, 2015), or steady star formation (Crocker 2012). Due to their dynamical, nonthermal nature, and the vast energy involved, an accurate interpretation of the FBs is important for understanding the energy budget, structure, and history of our Galaxy.

Correctly interpreting the FBs is important for understanding additional, possibly related phenomena, such as ‘chimneys’ which may play a role in the transport of energy to the base of the FBs (Ponti et al. 2019), symmetric lobes on a few 100 pc scales (Heywood et al. 2019), and magellanic echoes (Bland-Hawthorn et al. 2019), where two bipolar ionization radiation cones are associated with the GC took place \(~3.5\) Myr ago. Recent UV absorption survey of quasar sightlines passing through the FBs have been used to map the kinematics of associated cold clouds. The velocity of the clouds are symmetric in both hemispheres, decreases from \(\sim 900\) km s\(^{-1}\) at low latitude to \(\sim 350\) km s\(^{-1}\) at high latitudes (Fox et al. 2015; Bordoloi et al. 2017; Karim et al. 2018; Ashley et al. 2020).

In spite of their dramatic appearance in the \(\gamma\)-ray sky,
even the nature of the FB edges is not widely agreed upon. Even models have interpreted the FB edge as either an outgoing shock (Fujita et al. 2013), or a termination shock of a wind (Lacki 2014), or a discontinuity (Crocker 2012, S15b). More recent studies provided evidence that the edges are forward shocks. This evidence is based on the little variation of the \(\gamma\)-ray spectrum with position along the edge, indicating a strong, Mach \(M > 5\) shock (KG17), on a faint X-ray shell inside the FB edges found by stacking ROSAT data and consistent with \(M > 4\) forward shocks (KG18), and on a combined analysis of the microwave and \(\gamma\)-ray spectrum (Keshet & Gurwich, in preparation). These claims are, however, in tension with some other results (Kataoka et al. 2013; Carretti et al. 2013; Kataoka et al. 2021).

The identification of the FB edges as strong forward shocks simplifies, in a sense, their analysis, because their supersonic motion renders them less sensitive to details of the energy release and CGM properties than the alternative, discontinuity or reverse shock models. We thus use hydrodynamic simulations to study which properties of the FB engine and of the CGM are consistent with the observed morphology of the FBs. In particular, we use the edges identified by applying gradient filters to the \(\text{Fermi}\) data (KG17), and the projection of the toy model for the FBs in 3D (KG18).

The paper is organised as follows. In \(\S\)2, we describe the methodology and the numerical setup. The analysis of putative FBs generated by an isotropic, non-directed injection of energy is presented in \(\S\)3, and shown to yield bubbles too wide to be consistent with the FBs. In \(\S\)4, we study FBs generated by the collimated injection of both momentum and energy, presenting a stratified evolution model in \(\S\)4.1 and the corresponding simulations in \(\S\)4.2. Our results are summarised and discussed in \(\S\)5. We present the Galactic model implemented in the simulations in \(\S\)A, and representative convergence tests in \(\S\)B.

2 METHOD

We simulate a simple numerical model of the Galaxy, inject energy and in some cases also momentum rapidly near the Galactic centre, and examine the resulting, evolved FBs with an emphasis on the robustly-measured geometry of the expanding, forward shock. The parameters of the Galactic model and of the injecting engine are varied, to quantify the dependence of the observed quantities upon the assumptions, and to identify the physical condition which could account for the observed FB edges.

The analysis is based on hydrodynamic simulations using the publicly available Eulerian grid code PLUTO-v4.0 (Mignone et al. 2007). The simulations are axisymmetric and non-relativistic, modeling a viscous fluid with no heat conduction, with an ideal gas of polytropic index \(\gamma = 5/3\), without cosmic-rays, and with a mean particle mass \(\mu_m m_p\), where \(m_p\) is the proton mass and \(\mu_m \approx 0.6\). These assumptions are reasonable for the present problem, as we focus mainly on the geometry of the shock, at \(\gtrsim\) kpc scales. Spherical coordinates \((r, \theta, \phi)\) centered on the GC are used, with polar coordinate \(\theta = \pi/2\) along the Galactic disc and a frozen azimuthal coordinate \(\phi\). The computation box extends from \(r_0\) to 15 kpc in the radial direction and from \(\theta = 0\) to \(\pi/2\) in the polar direction. The inner boundary in the radial direction is set to have the values corresponding to the hydrostatic equilibrium, except where we inject jet energy. The outer boundary in the radial direction is set to have zero gradient. In the polar direction, \(\theta = 0\) and \(\pi/2\) boundaries are set to be axisymmetric and reflective, respectively. For convenience, we introduce also a cylindrical coordinate system \((R, \phi, z)\), centered on the GC with axial coordinate \(z\) along \(\theta = 0\).

The Galactic model and its numerical setup follow the work of Sarkar et al. (2015a, hereafter S15a), as detailed in \(\S\)A. The model includes two gas components of the interstellar medium (ISM): a hot, \(4 \times 10^8\) K, disc-like component and a warm, \(2 \times 10^6\) K, halo component. The model assumes that the gas components are in hydrostatic equilibrium with the gravity of dark matter, the disc, and the bulge, and each component is initially isothermal. The warm component associated with the disc rotates approximately with the stars; a rotation parameter \(f_d \leq 1\) is introduced to quantify the ratio between halo rotation and stellar rotation (see \(\S\)A). The halo has a global baryon fraction \(f_h = 0.16\), consistent with the cosmic value of \(\Omega_m/\Omega_m\). The fixed parameters of our nominal Galactic model are provided in Table 1. Table 2 for jetted bubble simulations, and in Table 3 for non-directed bubble simulations.

We consider two different modes of FB engines, operating over a short, \(\ll\) Myr timescale near the GC, as illustrated in Figure 1: energy only (henceforth: non-directed) or both energy and momentum (henceforth: jetted) injection. The non-directed mode can arise, for example, from starburst activity at some distance from the GC, or from a dissipated jet at some height above the GC, so we consider such injection both close to the GC (around \(r_0\); henceforth GC injection) and at intermediate latitudes (around \(r \sim\) kpc; IL injection). Jetted emission can arise from a relativistic jet launched from the central black hole (CBH) in the GC, and is represented for simplicity as simultaneous energy and momentum in the \(\theta = 0\) direction, injected at \(r_0\). As we show below, jetted injection can lead to two dif-

| Parameter | Definition | Value |
|-----------|------------|-------|
| \(M_{\text{vir}} [M_\odot]\) | Virial mass | \(1.2 \times 10^{12}\) |
| \(r_{\text{vir}} [\text{kpc}]\) | Virial radius | 250 |
| \(d [\text{kpc}]\) | DM core radius | 6.0 |
| \(r_s [\text{kpc}]\) | Scale radius | 20.8 |
| \(c_{\text{vir}} = r_{\text{vir}}/r_s\) | Concentration parameter | 12 |
| \(a_d [\text{kpc}]\) | Disc scale length | 3.0 |
| \(b_d [\text{kpc}]\) | Disc scale height | 0.4 |
| \(a_b [\text{kpc}]\) | Bulge scale radius | 2.0 |
| \(\rho_{d0} [\text{m}_p \text{ cm}^{-3}]\) | Disc mass density | 1.0 |
| \(\rho_{h0} [\text{m}_p \text{ cm}^{-3}]\) | Halo mass density | 0.019 |
| \(T_d [\text{K}]\) | Disc temperature | \(4 \times 10^4\) |
| \(T_h [\text{K}]\) | Halo temperature | \(2 \times 10^6\) |
| \(T_{\text{cmz}} [\text{K}]\) | CMZ temperature | \(10^3\) |
| \(f_d\) | Disc gas rotation | 0.975 |
| \(f_{\text{cmz}}\) | CMZ rotation | 0.0 |
| \(r_{\text{max}} [\text{kpc}]\) | Outer boundary of the box | 15 |
| \(r_{\text{gd}} [\text{kpc}]\) | Sun Galactocentric radius | 8.5 |
different types of FBs: ballistic bubbles for high energies, and slowing-down, non-ballistic for low energies.

We detail the two (GC and IL) non-directed modes in § 2.1, and the two (ballistic and slowdown) jetted modes in § 2.2. Shear in the flow, especially during the injection of slowdown jetted bubbles, can give rise to substantial Kelvin-Helmholtz instabilities (KHI), which are dealt with in § 2.3. The simulation results are finally projected and compared to observations, in a method outlined in § 2.4. Convergence tests are provided in § B.

2.1 Non-directed injection

We begin with the simple, non-directed injection of energy with zero momentum, either from the vicinity of the CBH (GC), or from intermediate latitudes (IL). We assume that the injected energy is thermalised within a conical region (with half opening angle $\theta_i$) around the rotation axis of the Galaxy. The injected energy is thus added to the internal energy above the injection point, at some radial range $r_1 < r < r_2$, as illustrated by the orange shaded region in Figure 1. The internal energy is added at a constant rate

$$\dot{E}_j = \frac{E_j}{\Delta t_j}$$

(1)

during a short time interval $t_j$, after which injection is turned off. Here, both the total injected energy $E_j$ and the injection volume

$$V_j = \frac{4\pi}{3}(r_2^3 - r_1^3)(1 - \cos \theta_i)$$

(2)

are defined for the full sky, summing both hemispheres.

With the above general setup, we explore the simulated appearance of the FBs for different central engines and Galactic structures, by varying the properties of energy injection and of the Galactic model, as detailed in Table 2. Such variations are essential because, as shown in § 3, extreme — and even implausible — choices of the Galactic model parameters are needed in order to produce simulated FBs of any resemblance to the observations.

2.2 Jetted injection

Next, consider jetted injection, introducing both energy and momentum in the $\theta \approx 0$ direction, approximately perpendicular to the Galactic plane. Injection into the computational domain is implemented by modifying the $r_0$, inner boundary within the half opening angle $\theta_i$ of the jet, introducing a luminosity $L_j = E_j/t_j$ over a short time interval $t_j$, in the form of kinetic energy due to momentum in the $\hat{r}$ direction. This injection scheme is illustrated by the hatched cyan region in Figure 1. The quantities $E_j$ and $L_j$ take all injection parameters above and below, pertain to the full sky, i.e., to both hemispheres combined.

The energy injected is predominantly kinetic, so the injected mass is taken as

$$\dot{M}_j = \frac{2L_j}{v_j^2}$$

(3)

where $v_j$ is the non-relativistic velocity of the injected gas; relativistic corrections are omitted. The area of the injection boundary is given by

$$A_j = 4\pi(1 - \cos \theta_i)r_0^2$$

(4)

so the mass density of the jet material injected at $r_0$ is given by

$$\rho_j = \frac{\dot{M}_j}{A_jv_j}$$

(5)

Kinetic energy is assumed to dominate at the base of the jet, so the thermal energy flux $(\gamma - 1)^{-\gamma}Pv_j$ is taken to be negligible with respect to the kinetic energy flux $(1/2)\rho_jv_j^3$ throughout the injection boundary. Here, $P$ is the gas pressure.

The bubbles formed by jetted injection are not very sensitive to the Galactic model in the ballistic case, and depend mostly on the halo distribution for the slowdown case. The minimum grid necessary for convergence is a $1024 \times 512$ uniform grid in the $r$ and $\theta$ directions for the ballistic case, and a $1536 \times 768$ uniform grid for the slowdown case.

2.3 Overcoming KHI

The emergence of strong shear, in particular for jetted injection, gives rise to KHI that corrugate the discontinuities.

In particular, we vary the mass of the Galactic disc and of the bulge, which can modify the resulting bubble morphology, while keeping the virial mass of the galaxy fixed. We also vary the rotation frequency of the Galactic components, which affect their structure and therefore also the appearance of the shock. The geometrical parameters of the injection volume have little effect near the GC, but they do have some impact at high latitudes, where we consider in particular a narrow, $\theta_i \approx 5^\circ$ cone representing a putative dissipated jet.

We find converged results with a minimal resolution of $1024 \times 256$ with a stretched grid along the $r$ and $\theta$ directions, with no need to introduce viscosity. All of our production runs for the non-directed set-ups have this resolution.

\footnote{See http://plutocode.ph.unito.it/files/userguide.pdf for details.}
thus leading to expensive simulations with a short time step and early dissipation of the kinetic energy. We avoid these instabilities, especially at the early stages of jet formation, because details of the early evolution are in any case poorly understood and not well-constrained by observations.

One modification we thus incorporate is smoothing of the injected velocity gradient, by switching from a step function at $\theta_j$ to a continuous, linear velocity profile

$$v(r = r_0, \theta < \theta_j; t < t_j) = \left(1 - \frac{\theta}{\theta_j}\right) v_j,$$

in which case the tabulated parameter $v_j$ refers to the maximal, axial velocity. We keep $\rho_j$ uniform, so Equation 5 is replaced by

$$\rho_j = \frac{L_j}{2\pi \left(1 - 6\frac{\theta}{\theta_j} - \sin(\theta_j)\right)^{3/2} r_j^2 v_j^3}.$$

More importantly, we incorporate viscosity, modeled for simplicity as the isotropic, dynamic viscosity of a nonmagnetized plasma (Braginskii 1958; Spitzer 1962)

$$\mu \simeq 6 \times 10^3 \frac{\ln \Lambda}{37} \left(\frac{T}{10^4 \text{K}}\right)^{5/2} \frac{\text{gm cm}^{-1} \text{s}^{-1}}{v_j^2},$$

where $T$ is the temperature in Kelvin and $\ln \Lambda$ is the Coulomb logarithm. To prevent exceedingly small time steps, we cap the viscosity at $\mu_{\text{max}} = 10 \text{ g cm}^{-1} \text{s}^{-1}$, which is sufficient for present purposes. Indeed, KHI of wavelengths $\lambda$ are suppressed by viscosity (e.g., Lamb 1932; Chandrasekhar 1961; Roediger et al. 2013)

$$\mu \gtrsim \frac{\rho_0 \Delta v}{64 q_j v_j^2},$$

where $\Delta v$ is the shear velocity and $q_j$ is the density contrast across the discontinuity. If we replace $\lambda$ with the scale of the bubble at a given time ($z$), we will have the value necessary to insure KHI suppression at all scales. At small scales, this upper limit keeps increasing as the bubble grows, but beyond $z \gtrsim 0.35 \text{kpc}$, the density drops rapidly as the bubble grows. At $z \sim 0.35 \text{kpc}$ our simulation shows that $\rho_0 = 0.3 \text{ cm}^{-3}$ and $\rho_0 = 2 \times 10^{-3} \text{ cm}^{-3}$ whereas, $\Delta v = 0.1 c$ is chosen for our nominal simulation, which gives, $\mu \sim 13 \text{ g cm}^{-1} \text{s}^{-1}$ as the necessary threshold to suppress all KHI modes.

### 2.4 Projection and comparison with observations

In order to compare the simulated bubble to FB observations, we project the simulation box for an observer at the solar position, taken as $(r, \theta, \phi) = (8.5 \text{ kpc}, \pi/2, \pi)$. We use the Projection Analysis Software for Simulations (PASS; S15a), which computes the surface brightness by integrating the emissivity along the line of sight (LOS),

$$F_X(l, b) = \frac{1}{4\pi} \int_{\text{LOS}} n^2 \varepsilon(T, Z) ds,$$

where $n$ is the particle number density and $T$ is the temperature. The emissivity $\varepsilon(T, Z)$ is computed assuming a metallicity $Z = 0.2 Z_\odot$ by interpolating the MEKAL from xspec (Mewe et al. 1985, 1986; Liedahl et al. 1995; Arnaud & Rothenflug 1985) data; we use the $2.0 - 10.0 $ keV band, sufficient for our purposes. The LOS is extended beyond the simulation box, out to $r = 100 \text{kpc}$, assuming steady-state equilibrium with the background gravitational field, extending the temperature and density profiles of our Galactic model.

Given the projected image of the simulation at a given time $t$, we identify the bubble edges by applying a gradient filter, of $2^\circ$ width. The maximal latitude $b$ of the projected bubble, and its maximal longitudinal half width $\Delta l$, are then measured as a function of $t$. The age $T$ of the bubble is estimated by requiring $b(T) = 52^\circ$, as inferred from FB observations. The measured bubble edge at this time is compared to the edges inferred by applying a gradient filter to the Fermi map (6$^\circ$ width for southeast bubble; KG17) and to the projection of a FB toy model (KG18).

### 3 NON-DIRECTED INJECTION

We simulate various non-directed injection models, as described in §2.1, and compare the bubbles as they reach $b \simeq 52^\circ$ with the observed FBs. The parameters of select non-directed injection simulations, which are displayed in the following figures, are detailed in Table 2. The table also provides the central measured properties of each simulated bubble: its age $T$ and maximal half width $\Delta l$.

Figure 2 shows the Fermi-like bubbles obtained for our nominal GC (S1; top panels) and IL (S2; bottom panels) injection simulations. The figure shows the $2 - 10 \text{ keV}$ surface brightness maps (left panels) and the spatial distributions of density and temperature (in cylindrical coordinates; right panels). For comparison with observations, FB edge contours traced by a gradient filter (KG17, red dotted curve) and obtained from projecting a toy model (KG18; solid blue) are superimposed on the surface brightness maps. For this purpose, we adopt the south eastern FB edge, which is better measured than its northern and western counterparts due to the abundance of dust in the northern hemisphere and the elongation of the bubbles to the west.

As the upper panel of Figure 2 shows, our nominal GC injection model produces bubbles with strong forward shocks that are approximately spherical, and quite inconsistent with the more elongated, thinner morphology of the observed FBs. Such quasi-spherical bubbles are expected as the shock propagates far from the GC, into an initial COM density that itself becomes quasi-spherical at $z \gtrsim 1 \text{kpc}$ distances from the Galactic plane. For injection near the GC, the initial blastwave is close to spherical and accumulates mass rapidly, so the transition from an initial quasi-ballistic propagation into an approximately self-similar propagation occurs relatively quickly. Indeed, after $\sim 0.5 \text{ Myr}$, we find $R \propto t^{0.6}$ and $z \propto t^{0.6}$ evolution, close to the spherical self-similar $r \propto t^{4/7}$ solution expected for the $\rho_0 \propto r^{-3/2}$ radial profile implemented by our Galactic model at large radii. The inner structure of the bubble presents a reverse shock, separated from the forward shock by a deformed contact discontinuity.

As the lower panel of Figure 2 shows, our nominal IL injection model produces forward shocks similar to those of the GC injection, and again far too spherical to account for the FB observations. Such injection, representing for example the dissipation of a jet at some intermediate latitude, avoids the initial propagation of the blastwave through the high
of the bubbles is noticeably different for IL injection, presenting reverse shocks reflected from the Galactic plane at low latitude and a closed contact discontinuity structure at mid-latitudes.

After its ballistic stage, the evolution of the bubbles becomes sensitive to the properties of the CGM distribution. It is therefore natural to ask if there are any plausible modifications to the Galactic model that could alter the simulated bubbles and produce an acceptable match to the observed FBs. As the FBs are substantially thinner than the bubbles in Figure 2, a better fit to the observations requires enhancing the collimation of the simulated bubbles. Such collimation can be established by considering flatter, more disc-like mass distributions near the origin of the bubbles. This, in turn, can be facilitated by a faster CGM rotation, a more massive disc, or a more massive bulge. These three modifications are considered, separately and combined, in Figure 3 for GC injection, and in Figure 4 for IL injection.

In the first three rows of these two figures, we consider a radical modification of a single parameter: (i) a nearly maximal halo rotation parameter, $f_b = 0.9$; (ii) doubling the disc mass, $M_d = 1.2 \times 10^{11} \, M_\odot$; and (iii) doubling the bulge mass, $M_b = 4 \times 10^{10} \, M_\odot$. The fourth row simultaneously combines all three modifications, to examine their cumulative effect. The masses of the Galactic stellar components are varied while keeping the dark matter mass of the Galaxy constant; this changes the local gravity near the disc, but does not alter the total mass of the Galaxy (including dark matter) by more than $\sim 10\%$. The effect of changing the gravity translates to a slightly different gas density profile near the disc. A similar effect could also be obtained by varying the temperature profile of each component, but our present construction of the CGM profile does not accommodate a variable temperature.

As the two figures show, these modifications to the CGM model, although extreme, are still insufficient to provide the necessary collimation of the bubbles. This conclusion holds even when all modifications are combined, for both GC and IL injection, and our results are converged and not sensitive to the addition of viscosity. In both injection methods, the modifications do somewhat enhance the collimation, in particular when all modifications are combined with IL injection. However, the resulting bubbles are still too wide and too spherical to account for the observations. The collimation is associated with taller and younger bubbles, reaching $z \approx 10$ kpc and $T \sim 1.5$ Myr for the thinnest of the bubbles in the figures. Extrapolation suggests that
a good match to the FB morphology would require unrealistically massive central Galactic components and young bubbles.

We conclude that the rapid injection of energy near the GC, or even at intermediate latitudes above it, cannot explain the morphology of the FBs as forward shocks, for any Galactic model consistent with present observations. Our results are consistent with previous studies Zhang & Guo (2020, hereafter ZG20), which focused on a specific Galactic model and so could not make the above general claim.

## 4 Jetted Injection

As concluded in §3, we are unable to reproduce the observed FB geometry using non-directed injection, for any plausible Galactic model. We are therefore reduced to considering jetted injection scenarios, where a new dimensional scale
is introduced into the problem by means of the conserved momentum \( p_\perp = p_j \) deposited in the axial, \( z \) direction, or interchangeably by the initial radial velocity \( v_j \) introduced in the small, \( 0 < \theta < \theta_j \) opening cone.

With such jetted injection models, we find that the observed FB geometry can be nicely reproduced in two distinct ways. A sufficiently energetic, slow initial jet can reproduce acceptable edges for simulated FBs that are presently still in their ballistic stage, whereas the converse — an initially fast, low-energy jet — can reproduce acceptable edges of non-ballistic FBs already in their slowdown phase.

At early times, the injected flow evolves ballistically in both cases, with the head of the bubble (subscript \( H \)) moving at a near constant velocity \( v_H(t) \equiv v_H \equiv v_j \). After the flow accumulates a mass \( M \) larger than the initial deposited mass \( M_j \), the flow inevitably starts slowing down, with \( v_H(t) \sim t^{-\alpha} \) typically following a power-law decay with index \( \tau_\gamma > 0 \). It is not yet known if the FBs observed today are in the former, ballistic stage or in the latter, slowdown stage, so we examine both scenarios. One can further split the ballistic forward evolution of a jetted bubble into two stages according to the sideways expansion being either ballistic or non-ballistic (Irwin et al. 2019), but we avoid this distinction here.

The mass \( M \) swept up by the bubbles is strongly dominated, for plausible Galactic models, by the halo, CGM component, exceeding the swept-up disc mass by about two orders of magnitude. For a \( \rho_h \propto r^{-\alpha} \) CGM distribution, the present-day FBs are thus in the ballistic stage if the injected mass \( M_j \) satisfies

\[
\frac{M_j}{2} \equiv \frac{E_j}{v_j^2} \gtrsim \frac{M}{2} \equiv \pi \theta_j^3 \int_0^r r^2 \rho_h(r) \, dr \sim \frac{\pi \theta_j^3 \rho_h(z_H) z_H^2}{3 - \alpha},
\]

where \( z_H \approx 10 \) kpc is the present-day height of the bubbles, and we approximated \( \theta_j \) as small; recall that \( E \) and \( M \) pertain to the sum of both hemispheres. Conversely, the FBs have already transitioned into the slowdown phase if \( M_j \lesssim M \), which may be written for \( \alpha = 3/2 \) as

\[
\frac{E_j}{3 \times 10^{55} \text{erg}} \lesssim \left( \frac{\theta_j}{0.01} \right)^2 z_{10} \rho_4 \approx (10^3 \beta_{\perp})^3 z_{10} \rho_4,
\]

where we defined \( z_{10} \equiv z_H/10 \) kpc, \( \theta_j \equiv \theta/5^\circ \), \( \rho_4 \equiv \rho_h(z_H)/10^4 m_p \text{ cm}^{-3} \), and the sideways initial velocity \( \beta_{\perp} \equiv \beta_j/\theta_j \). One can equivalently replace \( v_j \) by the conserved \( 2E_j/p_j \).

After developing in §4.1 a simple analytic model for the jetted bubbles in both regimes, we simulate the two scenarios in §4.2. We examine different variants of such bubbles, as described for jetted injection in §2.2, and compare the bubbles as they reach \( b \approx 52^\circ \) with the model and with the observed FBs.

4.1 Stratified model for jetted bubbles

Before discussing the simulated bubbles, it is useful to derive some relevant scaling relations using a simple toy model. Sufficiently far above the Galactic disc, the mass accumulated by the bubble is dominated by halo material, which has an approximately one-dimensional distribution around a narrow jet. The ambient medium can therefore be approximated as an isothermal, \( \rho \propto z^{-\alpha} \), planar atmosphere, where in our simulations \( \alpha \approx 1.5 \). The fairly narrow FBs, when interpreted as arising from thin initial jets, are thus amenable to a simplified model that approximates the flow behind the head of the bubble as quasi two-dimensional, propagating perpendicular to the \( z \) axis.

Such a piecewise planar, or stratified, model is sufficient for capturing the main properties of the simulated flow, as we show for the ballistic bubbles in §4.1.1, and for bubbles in their slowdown phase in §4.1.2. More sophisticated modeling, for example using the Kompaneets approximation (Irwin et al. 2019, and references therein, for a simplified ambient model and \( \gamma = 4/3 \)) is less useful here, because we find that (i) the ballistic phase plays an important role in the FBs; (ii) even in the slowdown phase, the injected momentum plays a key role; and (iii) post-shock pressure variations across the bubble surface are not small for slowing-down FBs.

We find that the structure and velocity of the jet are modified in the initial, injection stage, especially in the high \( v_j \) regime used to produce bubbles reaching the slowdown phase. We therefore use subscript \( k \) instead on \( j \) to refer to the actual jet parameters immediately after its \( 0 < t < t_j \) launching.

4.1.1 Ballistic regime

Consider our simple jetted setup deep in the ballistic regime. Here, the bubble has a height

\[
z_H(t) \equiv v_j t,
\]

so an observed FB with \( z_H \approx 10 \) kpc would be of age \( \tau \approx z_H/v_k \approx 3.3 z_{10} \beta_{\perp}^{-1} \) Myr.

\[
\beta_{\perp} \equiv \beta_j/0.01. \text{ At its top, the bubble head has a width}
\]

\[
R \approx \theta_h z_H \approx 0.9 z_{10} \theta_j \text{ kpc},
\]

where the forward shock is strong, of Mach number

\[
\Upsilon \approx \frac{v_j}{c_s} \approx 14 \beta_{\perp} \left( \frac{T_h}{2 \times 10^6 \text{ K}} \right)^{-1/2}.
\]

Here, we defined \( \theta_j \equiv \theta_j/5^\circ \). At lower, \( z < z_H \) heights, the bubble initially has a half-width

\[
R(z,t=\tau_{\theta_h}) \approx \theta_h z,
\]

but later necessarily expands due to the pressure of the shocked CGM.

Let us parameterise the geometry of the bubble as \( R_b(z,t) \approx \Theta(z,t) \), in the small angle, \( \Theta \ll 1 \) limit. A crude way to estimate the shape \( R_b(z,t=T) \) of the evolved ballistic bubble is to consider each fixed-\( z \) slice as a separate layer, independently expanding sideways, i.e., in the \( R \) direction. The pressure transferred to this slice as it is first shocked by the head of the bubble is, according to the shock jump conditions, \( P(z) \approx \Theta(z,t)^{-3/2} \rho_h(z) v_H(z)^2 \). For simplicity, we approximate the subsequent slice evolution as sideways expansion with a characteristic velocity \( v_H(z,t) = f_R R_b \), where \( f_R \) is a dimensionless factor of order unity, chosen as \( f_R \approx 1/2 \) in light of the nearly linear velocity profile anticipated behind a cylindrical strong shock. We roughly equate the associated kinetic energy per unit
height with the thermal energy initially deposited by the head, $4\rho_b(z)\pi R_b^2(z, t)v_h^2(z, t)/2 \simeq \pi(z\theta_t)^2(3/2)P(z)$.

Solving the resulting ODE,

$$2z\dot{\Theta} = 3v_H(z)\theta_k,$$

with the ballistic behavior $v_H(z) \simeq v_k$ and the initial condition (17), we then obtain

$$\Theta(z, t)^2 \simeq 3\left(\frac{v_k t}{z} - 1\right)\theta_k + \theta_k^2,$$

in good agreement with the FB shape inferred from observations. For small $\theta_t$, and for $z$ not too close to $z_H$, we may approximate the bubble shape by

$$R_b(z, t)^2 \simeq 3(z_H(t) - z)\theta_{0}.$$

Note that for a fixed $z \gg R$, at late times $R(z) \sim t^{1/2}$ agrees with the expected self-similar cylindrical expansion.

For small $\theta_t$, the maximal half-width

$$R_{\text{max}}(t) \simeq \frac{v_k t}{2}\sqrt{3\theta_k},$$

is obtained at the bubble half-height, $z \simeq z_H/2$. Hence, for the evolved bubble

$$R_{\text{max}}(t = T) \simeq \frac{z_H}{\sqrt{3\theta_k}} \simeq 2.6\theta_{0}/z_{10}\ kpc.$$

Equation (21) also implies that the latitudinal and longitudinal widths of the bubble at any given time are

$$\Delta \beta \simeq \frac{z_H}{R_{\text{co}}}, \simeq 20.1^{\beta_{-2}}\ t_{\text{Myr}};$$

$$\Delta \eta \simeq \frac{R_{\text{max}}(t)}{R_{\text{co}}} \simeq 5.3^{\beta_{-2}}\ t_{\text{Myr}}\ \theta_{0}^{\beta_{-2}/2}.$$

Non-ballistic corrections, becoming gradually larger as the swept up mass $M$ approaches the injected mass $M_j$ near the slowdown phase, can render the bubbles somewhat older and wider than estimated here. The geometry of the bubble edge for $z_H = 10\ kpc$ is demonstrated in Figure 5 (left panel). This one-zone approximation for the sideways expansion leaves out substructure associated with the contact discontinuity and the reverse shock.

The shock is fastest at the top of the bubble, where the downstream temperature is

$$T_s \simeq \frac{3\mu_m m_p}{16k_B} v_k^2 \simeq 1.2 \times 10^8\beta_{-2}^2\ K.$$

Note that the electron temperature may be considerably lower than the ion temperature close to the shock, as the ion-electron equilibration time-scale can be shorter than the dynamical time-scale ($z_H/T$) of the shock (e.g., KG18). The sideways velocity $v_R = \dot{r}_R/\dot{\Theta}$ of the shock at a given $z$ and $t$ is given by

$$v_R \simeq \frac{3v_k\theta_k}{4\Theta(z)} \simeq \frac{3}{4}\frac{1}{\dot{\Theta}}\left[1 + \frac{3\theta_k}{\theta_{0}}\left(z_H - 1\right)\right],$$

indicating that the temperature behind a strong sideways shock,

$$T_R \simeq \frac{3}{16}\frac{\mu_m m_p}{k_B} \left(\frac{3v_k\theta_k}{4\Theta}\right)^2,$$

is constant at a given $\Theta$, but gradually declines in time at a fixed $z$. The evolution of the shape and temperature is shown in the right panel of Fig 5. Note that, the above temperature is only derived from the sideways expansion and is expected to fail near $\Theta \sim \theta_t$ since the shock velocity in this region will be dominated by vertical motion of the jet.

Figure 5 (middle panel) illustrates the vertical profiles $n(z) \simeq 4(\mu_m m_p)^{-1} \rho_b(z)\theta_t^2/\Theta(z)^2$ of particle number density and $P(z) \simeq (3/4)\mu_m m_p(z)\theta_t^2/\Theta(z)^2$ of pressure, for the one-zone model, assuming a homogeneous isothermal expansion near the axial.

The above arguments suggest that the FBs may indicate a ballistic flow arising from an abrupt injection of energy and momentum, provided that the ratio $E_j/v^2 \times P_j/E_j$ is

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sufficiently high to satisfy (11), that the opening angle of the jet is of order 4°, and that the FB age is given by (14). This possibility is studied numerically in §4.2.1.

4.1.2 Slowdown regime

Next, consider the same jetted setup, but now deep in the slowdown regime. Here, the nearly planar shock of area \( \sim \pi h^2 \) (the exact value is immaterial for the present argument) at the head of the jet propagates at an increasingly slower velocity \( v_H(t) \propto t^{-\beta} \) into the approximately \( \rho_0 \propto z^{-\alpha} \) CGM.

We approximate the momentum of the shocked mass, \( p_s \approx f_0 \rho_0(z_H) H^2 v_M/(3-\alpha) \), as some fraction \( f_0 \) of the product of the swept-up mass \( M \) and the velocity \( v_M \approx (3/4)v_H \) inferred from the shock jump conditions just behind the shock, where all velocities are measured in the Galactic frame. Fresh mass accreted through the shock raises the momentum of the bubble at a rate \( p_{\text{s}}^\prime = \pi (\theta_h z_H)^2 \rho_0(z_H) v_H v_M \), so momentum conservation implies that \( dp_s/dt + p_{\text{s}}^\prime = 0 \).

The resulting ODE,
\[
\frac{\partial}{\partial \ln \rho_0(z_H)} \frac{z_H^2 v_H}{\partial \ln t} = -f_q^{-1},
\]
where \( f_q \equiv f_q/(3-\alpha) \), leads to \( v_H \propto z_H^{-(3-1)/2-\alpha} \), and hence to the late-time power-law indices
\[
0 < \tau_s \equiv \frac{d \ln z_H}{d \ln t} \approx \frac{f_q}{1 + (4-\alpha)f_q},
\]
and
\[
0 < \tau_v \equiv \frac{d \ln v_H}{d \ln t} = 1 - \tau_s = \frac{1 + (3-\alpha)f_q}{1 + (4-\alpha)f_q}.
\]

For \( \alpha = 3/2 \), typical estimates \( 1/5 \lesssim f_q \lesssim 1/3 \) yield a small \( 2/15 \lesssim \tau_s \lesssim 2/11 \) and a near-unity \( 9/11 \lesssim \tau_v \lesssim 13/15 \), as indicated by the simulations shown below.

More precisely, twice integrating Eq. (27) yields
\[
z_H(t > t_c) \approx \left[ 1 + \left( \frac{t}{t_s} - 1 \right)^{-\tau_s} \right] z_s,
\]
where \( z_s \) and \( t_s \) are the height and time of the transition from ballistic to slowdown phases, derived toward the end of this subsection. The implied age of the bubble,
\[
\tau \approx \tau_v + \left( \frac{z_H}{z_s} \right)^{1/\tau_s} t_s,
\]
is the sum of a linear, \( t_s \propto z_s \) ballistic period and a non-linear, \( \propto (z_H/z_s)^{1/\tau_s} \) slowdown period. As \( \tau_s \) is a small number, a low transition height \( z_s \) would imply that \( \tau \) is dominated by the slowdown phase. A given bubble age thus imposes a lower limit on \( z_s \),
\[
\frac{z_s}{z_H} > \left( \frac{z_H}{z_s} \right)^{1/\tau_s} \left( \frac{z_H}{z_s} \right)^{1/\tau_s} = \left( \frac{z_H}{z_s} \right)^{1/\tau_s} t_s/t_s,
\]
and slowdown at \( z_s < z_H \) would require \( \beta h \tau \gtrsim z_H/c \), as illustrated in Figure 6. For example, for a \( z_H = 10 \) kpc, \( \tau = 3 \) Myr bubble and \( f_q = 1/4 \), the FBs can be in the slowdown phase only if \( z_s \gtrsim 3.1 \beta h^{-1/2} \) kpc \( \gtrsim 3.1 \) kpc (blue arrow in the figure); requiring slowdown implies that the average normalized velocity till \( t_s \) is \( \beta h > z_H/c \tau \approx 0.01 \).

To estimate the shape of the bubble in the slowdown regime, assume for simplicity that the sideways expansion remains ballistic for some duration after \( t_s \). We solve the same sideways expansion ODE (18) as in the ballistic case, but now incorporating the slowing down vertical velocity
\[
v_H(z > z_s) \approx (z/z_s)^{-(3-1)/2-\alpha} v_k,
\]
which corresponds to a Mach number
\[
\Upsilon \approx \frac{v_k}{c_s} \approx 14 \left( z/z_s \right)^{-(3-1)/2-\alpha} \beta_h \left( \frac{T_h}{2 \times 10^6 \text{k}\text{K}} \right)^{-1/2}
\]
for the top of the bubble. The initial sideways-expansion condition (17) of the ballistic regime now pertains only to \( t < t_s \), and is supplemented by the slowdown-phase condition
\[
R_0[z = z(t > t_s), t > t_s] \approx \theta_h z.
\]
For \( z < z_s \), the bubble geometry is still formally given by (19), whereas above the transition height the bubble narrows sharply with increasing \( z \),
\[
\Theta(z > z_s, t)^2 \approx 3 \left( \frac{z_H(t)}{z} \right)^{1/\tau_s} - 1 \right) \tau_s \theta_h + \theta_h^2.
\]

The combined solution (19) and (36) is illustrated in Figure 5 for \( f_q = 1/4 \). For small \( \theta_h \) and for \( z \) not too close to \( z_H \), we may approximate
\[
R_0[z > z_s, t]^2 \approx 3 \theta_h \tau_s z_H^{1/\tau_s} z^{-\tau_v/\tau_s}.
\]
Anticipating the widest part of the bubble at \( z \approx z_s \), Eq. (19) indicates that
\[
R_{\text{max}} = (3 \theta_h z_s)^{1/2} \approx 2.0 \left( \beta / \theta_h z_s \right)^{1/2} \text{kpc}
\]
in the \( t \approx t_s \) Myr \( \gg t_s \) regime. As the bubble necessarily narrows from \( z_s \) to \( z_H \), the observed shape of the FBs indicates that \( z_s \gtrsim z_H/2 \).

In addition to the stratified flow approximation, the above solution to the shape of the bubble in the slowdown phase is also based on the simplifying assumptions of a sudden transition from ballistic to slowdown behaviors, and a fixed head opening-angle \( \Theta[z_H(t), t] = \theta_h \) at all times. In
practice, the transition is smooth, and the tip of the head may gradually expand. Consequently, while the non-ballistic profile in Figure 5 provides a reasonable approximation to the simulated slowing-down bubbles, the sharp break at \( z_s \) is smoothed out in our simulations. Therefore, instead of estimating the maximal width of the evolved bubble at \( z = z_s \),

\[
R_{\text{max}}(t = \tau) \sim (3\pi\theta_0)^{1/2} \left( \frac{2H}{z_s} \right)^{1/2} z_s ,
\]

we may for simplicity use \( z \approx z_H/2 \), such that

\[
R_{\text{max}}(t = \tau) \approx \left( \frac{3}{2} \theta_0 v_h z_H \right)^{1/2} .
\]

As \( v_h \tau > z_H \) in the slowdown regime, nominal parameters yield \( \theta \lesssim 2^\circ \).

A jet generating FBs in the slowdown regime is faster and lower in energy and in mass than its ballistic counterpart, and thus more sensitive to the numerical setup. Assuming that the setup loads the jet with an additional, spurious mass \( M_0 \) and slows it down to a velocity \( v_0 < v_j \), we equate the total injected mass with an uncertainty factor \( M \), assuming that the setup loads the jet with an additional, spurious mass \( M_0 \) and thus more sensitive to the numerical setup. As- simulated slowing-down bubbles, the sharp break at \( z \) may gradually expand. Consequently, while the non-ballistic simulations described below in §4.2.2, respectively. In the case of FBs in the slowdown phase, the numerical launching of the faster, less energetic jet is more delicate, and the outcome is more sensitive to details. For simplicity, above we neglected the finite, \( t_j \approx 0.4 \) Myr injection stage of the jet, during which interactions with ambient gas over a substantial, \( z_j \lesssim t_j v_j \approx 1.2 \) (\( t_j / 0.04 \) Myr)(\( \beta_j / 0.1 \)) kpc height can modify the jet parameters. In particular, for a sufficiently high \( v_j \), the mass \( M_0 \) accumulated during this stage can exceed the injected mass \( M_1 \), the jet can be slowed down by a factor \( v_h / v_j \) of a few, and it can become broadened or pinched by a noticeable factor \( \theta_0 / \theta_j \).

For a fast injected jet, the strong initial shear may disrupt the jet, reshape it, focus it, or broaden it via partial thermalisation of the jet-base through viscosity or KHI. Consider in particular the viscous radial acceleration of gas around the jet, boosting the initial \( v_0 = 0 \) to \( v_R(t) \approx (t v_1 \mu) / (\rho \Delta R \Delta z) \), where \( \Delta R \approx (t \mu / \rho)^{1/2} \) is the width of the shear layer and \( \Delta z \) is the typical scale for flow variations in the \( z \) direction. Then the effective opening angle becomes

\[
\theta_{\text{eff}} \approx v_R(t) / v_j \approx (t \mu / \rho)^{1/2} / \Delta z \propto \mu^{1/2} z \lesssim t_j ,
\]

with some dependence on \( t_j \) arising from \( t \) and possibly also from \( \rho \) and \( \Delta z \), depending on details.

We conclude that the FBs are either presently in the ballistic stage, or were at the ballistic stage as recently as when the bubbles were at a height \( z_s \gtrsim 5 \) kpc, based on four different observations: (i) the bubbles are not very old, so Eq. (32) constrains \( z_s \); (ii) the Mach number is high, so Eq. (34) similarly constrains \( z_s \); (iii) the bubbles are fairly cylindrical, as discussed below Eq. (38); and (iv) the initial opening angle cannot be too small, so \( z_s \) is constrained by Eq. (39).

\[
\frac{z_s}{z_H} \approx \left( \frac{R_{\text{max}} / z_H}{\sqrt{30} \theta_0} \right)^{2/3} \approx 0.9 \left( \frac{4 R_{\text{max}} / z_H}{\theta_0^2} \right)^{-4/9} ,
\]

where the last estimate pertains to \( f_q = 1/4 \). Due to this weak dependence upon \( \theta_0 \), a small, say \( z_s < 1 \) kpc would require an early jet opening angle smaller than an arcsecond.

Adopting \( z_s \gtrsim 5 \) kpc, Eq. (41) implies that

\[
\frac{E_{\text{BB}}}{(\beta \theta_0)^2} \gtrsim 0.4 (4 f_q) C_4 .
\]

In both ballistic and slowdown scenarios, a high Mach number \( \Upsilon \) at the top of the bubble requires \( \beta \lesssim 0.4 \Upsilon_5 \), where \( \Upsilon_5 = \Upsilon / 5 \), and the thickness of the bubbles requires \( \theta_0 \gtrsim 4^\circ \). These results become approximate equalities in the ballistic case, whereas an extended slow-down phase requires a much faster and narrower jetted injection. The FBs are in the ballistic stage if they are energetic or slow, \( E_{\text{BB}} \gtrsim 3(\beta \theta_0)^2 \), in which case \( E_{\text{BB}} \gtrsim 2 \beta^2 \gtrsim 0.2 \beta^2 \) \( \Upsilon_s \) and \( \Upsilon \approx 3.3 \beta^{-2} \) Myr \( \lesssim \Upsilon_5 \) Myr. The FBs are in the slowdown phase if \( E_{\text{BB}} \gtrsim 3(\beta \theta_0)^2 \), in which case \( 0.05 \Upsilon_s \theta_0^2 \lesssim 2 \beta \gtrsim 2 \beta^2 \) \( \Upsilon_s \approx 1.4 (\beta \theta_0)^{-1} \) Myr \( \lesssim 4 (\Upsilon s \theta_0)^{-1} \) Myr. Such slowing-down FBs can still be energetic, provided that \( \beta \) is large, but their age would then be of order a Myr only if \( \theta_0 \) is small.

4.2 Jetted simulations

We carry out a suite of simulations that generate Fermi-like bubbles in both ballistic and slowdown phases. The parameters and resulting bubble properties for a sample of such simulations are listed in Table 3. Figure 7 shows our nominal ballistic (setup J1; top row) and slowdown (J2; bottom panel) simulated bubbles. Unlike non-directed injection, jetted injection gives rise to forward shocks that do agree well with the observed FB edges, as inferred from the stratified models of §4.1 and confirmed by our simulations. As the figure demonstrates, this agreement holds in both ballistic and slowdown regimes, albeit with different parameters, as seen in the table.

The figure indicates that the inner structure of the bubble in the ballistic and slowing-down bubbles differ substantially, the former showing an inner cylindrical shock surrounded at its top by an irregular contact discontinuity surface, and the latter showing a bubble-like contact discontinuity trailing the shock. The two types of bubbles differ qualitatively also in their evolution and parametric dependence, as outlined in §4.1 and explored numerically below.
In our fiducial ballistic simulation, denoted J1, an energy younger as its ballistic stage was very short due to the much faster ballistic (top; setup J1) and slowdown (bottom; setup J2) phases. Simulated FBs generated by jetted injection, in their $z_{\text{FB}}$ values.

4.2.1 Ballistic bubbles

In our fiducial ballistic simulation, denoted J1, an energy $3 \times 10^{56}$ erg is injected with $\beta_j = 0.01$ inside an opening angle $\theta_j = 5^\circ$ for a duration $t_j = 0.04$ Myr, resulting in a Fermi-like bubble reaching the designated latitude at $T \simeq 3.3$ Myr, as depicted in Figure 7 (top row). The contact discontinuity (seen towards the top of the bubble) and inner shock are also evident in the pressure and entropy distributions presented in Figure 8 (left panel). These distributions indicate a non-homogeneous inner structure, with a very low pressure region behind the bubble head. However, the pressure just behind the shock and throughout the bubble is fairly uniform, varying by only a factor of a few throughout the surface of the bubble.

The low pressure region can be identified as the unperturbed jet material that has been pinched by the high pressure region behind the ballistically expanding shock. In a simple picture, the region should be conical in shape, with a head half-width $R_{\text{hp}} \simeq z_H \theta_j$ and a length $\Delta z_{\text{hp}} \simeq v_j R_{\text{hp}}/c_s$ given by the $R_{\text{hp}}$ sound crossing time, where $c_s \simeq (3/16)^{1/2} v_j$ is the sound speed behind the shock. For $v_j = 0.01$ c and $\theta_j = 5^\circ$, $\Delta z_{\text{hp}} \simeq 1.5$ kpc is consistent with the simulated region in Figure 7.

The figure also shows the temporal evolution of the bubble, in space (middle panel) and in projection (right). After the initial $\approx 0.6$ Myr, the height and maximal half-width of the bubble are well fit (green curves) by $z_H \sim 3.33 t_f^{0.92} \pm 0.084$ kpc and $R_{\text{max}} \sim 0.79 t_f^{0.92} \pm 0.02$ kpc, where $t_f \equiv t/\text{Myr}$. This evolution is consistent with the expected linear behavior (13) and (21), with $v_k \simeq v_j$ and $\theta_k \simeq \theta_j$ almost unaffected by the initial transients, and with only a $\sim 8\%$ slowdown during $0 < t < T$. The projected coordinates are well fit at this stage by $b \simeq 22^\circ.2 \ t_f^{0.77} \pm 0.02$ and $\Delta l \sim 5^\circ \ t_f^{0.99} \pm 0.02$, in agreement with the model (see Eq 23).

The ballistic bubbles and their internal structures are not much affected by viscosity, as the associated jet velocities ($\beta_j \sim 0.01$) are relatively low. We have confirmed this behavior with several viscous simulations, of Spitzer viscosity capped at $\mu_{\text{max}} = 10$ g cm$^{-1}$ s$^{-1}$ (as discussed in sec 2) and even with an excessive, fixed $\mu = 40$ g cm$^{-1}$ s$^{-1}$ throughout the simulation box.

We carry out a suite of simulations that generate Fermi-like bubbles in their ballistic stage. A representative sample of such simulations, including our fiducial setup J1 and its variants J1a through J1d, are summarised in the middle columns of Table 3, and are presented in Figure 9 at the times (displayed in the upper left corner of each row) when each bubble reaches the designated latitude. The table provides the central measured properties of each simulated bubble: its age $T$ and maximal half-width $\Delta l$.

By separately varying each of the setup parameters, one at a time, we derive the functional dependencies of the bubble age

$$T \simeq 3.2 \left( \frac{E_j}{3 \times 10^{56} \text{ erg}} \right)^{-0.030 \pm 0.004} \beta_j^{-0.96 \pm 0.01} \theta_j^{-0.003 \pm 0.031} \times \left( \frac{t_f}{0.04} \right)^{0.007 \pm 0.004} \text{ Myr},$$

its maximal half-width

$$R_{\text{max}} \simeq 2.2 \left( \frac{E_j}{3 \times 10^{56} \text{ erg}} \right)^{-0.032 \pm 0.005} \beta_j^{-0.11 \pm 0.02} \theta_j^{0.40 \pm 0.03} \times t_f^{0.017 \pm 0.002} \text{ kpc},$$

and maximal longitude

$$\Delta l \simeq 10^\circ.3 \left( \frac{E_j}{3 \times 10^{56} \text{ erg}} \right)^{-0.024 \pm 0.004} \beta_j^{-0.07 \pm 0.02} \theta_j^{0.41 \pm 0.03} \times t_f^{0.014 \pm 0.005},$$

valid in the close vicinity of the fiducial J1 setup. Such individual parametric scans are illustrated in Figure 10. Here, we defined $t_{f,0.04} \equiv t_f/0.04$ Myr. The fits (45) and (46) are in good agreement with the model Eqs. (14) and (22), showing that the anticipated age $T \propto \beta_j^{-1}$ and half-width $R_{\text{max}} \propto \theta_j^{1/2}$ of the ballistic bubble are nearly independent of other parameters.

As expected, these scaling relations break down when the setup parameters are modified substantially away from the J1 setup. In particular, if (only) the energy is lowered considerably, non-ballistic corrections emerge, and we obtain

$$T \sim 3.1 + \left( \frac{E_j}{2.1 \times 10^{56} \text{ erg}} \right)^{-0.80 \pm 0.03} \text{ Myr},$$

and

$$\Delta l \simeq 13^\circ.8 + 1^\circ.0 \left( \frac{E_j}{3.0 \times 10^{56} \text{ erg}} \right)^{-0.39 \pm 0.01},$$

as shown in Figure 10. These results indicate that $\lesssim 7\%$ non-ballistic corrections are present in the J1 bubble.

The properties of the simulated ballistic bubbles are approximately independent of the injection parameters $t_j$ and $r_0$ (in the nominal ranges $0.25 < t_{f,0.04} < 2$ and $r_0 \sim 1$ kpc).
0.05 < \rho_0 < 0.25). The results are also fairly independent of the Galactic model parameters. In particular, even extreme changes to the disc mass and halo rotation parameters have a rather negligible effect on the shape and age of the bubble.

### 4.2.2 Slowing-down bubbles

In our fiducial slowing-down FB simulation, denoted J2, an energy of $2.5 \times 10^{55}$ erg is injected with velocity $\beta_j = 0.1$ inside an opening angle $\theta_j = 4^\circ$ for a duration of $t_j = 0.04$ Myr. This results in a Fermi-like bubble reaching the designated latitude at $\tau \simeq 2.4$ Myr, as depicted in Figure 7 (bottom row). For such a lower energy, higher velocity jet, the amount of thermalisation near the jet-base is not numerically resolved, due to unresolved KHI. To deal with such artefacts, we introduce viscosity adopting Spitzer viscosity with a $\mu_{\text{max}} = 10 \text{ g s}^{-1} \text{cm}^{-1}$ upper limit, as discussed in section 2.

The contact discontinuity trailing all parts of the slowing-down shock in the figure, as well as in the corresponding pressure and entropy distributions presented in Figure 11 (left panel). Unlike in the ballistic bubble, this figure does not show any evidence of an inner shock or a low-pressure region trailing the bubble head, due to an earlier thermalisation of the low momentum $(2E_j/c \beta_j)$ jet for a given energy, as explained in section 4. The pressure is fairly uniform in the inner parts of the bubble, but pressure variations behind the shock span more than an order of magnitude across the surface of the bubble.

We find that in the fast injection domain, the initial jet emerging by the injection time $t_j$ is substantially reshaped by the strong interactions with the ambient gas. Unlike the slow jets studied for ballistic bubbles, which are initially sustained as cones of opening angle $\theta_k \simeq \theta_j$, the present fast jets substantially narrow at their top, presenting as top-pinched bubbles. The upper structure of these initial jets is subsequently shaped by the interaction with the ambient gas, rather than by the initial jet parameters. Indeed, we find that the simulation results in the high velocity regime are not sensitive to the initial setup parameters $3^\circ \lesssim \theta_j \lesssim 5^\circ$ and $0.05 < \rho_0/\text{kpc} < 0.3$. Instead, a dependence on viscosity and injection time emerges, as anticipated in §4.1.3.

Figure 11 also shows (middle and right panels) the temporal evolution of the bubble. At early, $t \lesssim 0.5$ Myr times, the bubble grows approximately linearly both in height and in width, as expected in the ballistic phase. For the fast jets in this regime, the injection stage has a much stronger effect in slowing and reshaping the jet. From Eqs. (13) and (21), we estimate that $v_k \simeq 0.06c$, and $\theta_k \simeq 0.3^\circ$ (independent of $\theta_j$). At late times ($t \gtrsim 0.5$ Myr), the bubble height grows noticeably slower, while its widening is only slightly diminished, as expected in the slowdown phase. The ballistic-to-slowdown transition is gradual, with the bubble radius following a power-law fit. The results are also fairly independent of the Galactic model parameters. In particular, even extreme changes to the disc mass and halo rotation parameters have a rather negligible effect on the shape and age of the bubble.

### Table 3

| Category       | Values |
|----------------|--------|
| J1             | J1a    | J1b    | J1c    | J1d    |
| J2             | J2a    | J2b    | J2c    | J2d    |
| $M_\text{d}$ [10$^{10} M_\odot$] | 6.0    | 6.0    | 6.0    | 6.0    |
| $M_\text{b}$ [10$^6 M_\odot$]   | 2.0    | 2.0    | 2.0    | 2.0    |
| $f_\text{b}$       | 0      | 0      | 0      | 0      |
| $E_j$ [10$^{56}$ erg] | 3.0    | 3.0    | 3.0    | 3.0    |
| $\beta_j = v_j/c$ | 0.01   | 0.01   | 0.01   | 0.01   |
| $t_j$ [Myr]       | 0.04   | 0.04   | 0.04   | 0.04   |
| $\theta_j$        | 5$^\circ$ | 5$^\circ$ | 5$^\circ$ | 5$^\circ$ |
| $r_0$ [kpc]       | 10     | 10     | 10     | 10     |
| $\mu_{\text{max}}$ [g s$^{-1}$ cm$^{-1}$] | 16$^\circ$ | 16$^\circ$ | 16$^\circ$ | 16$^\circ$ |
| $\Delta t$ [Myr]  | 3.3    | 3.3    | 2.2    | 3.2    | 2.4    | 1.5    | 2.5    | 2.0    | 1.0    |

**Figure 8.** Thermal structure (left panel) and spatial (middle) and projected (right) evolution of our fiducial ballistic bubble (setup J1). Left panel: logarithmic pressure (in erg cm$^{-3}$; left half of left panel) and entropy (shown as the adiabat $Pn^{-\gamma}$ in erg cm$^2$ units; right half) distributions highlight the inner shock and contact discontinuity. Middle and right panels: the temporal evolution of the height (red discs with left axis) and half-width (black squares; right axis) of the bubble is shown, along with power-law fits (solid green for $t > 0.6$ Myr; see text) and the cutoff time (dot-dashed blue) at the designated latitude.
transition can therefore be identified at \( t_\ast \approx 0.5 \) Myr and \( z_\ast \approx 8 \) kpc.

In the slowdown phase, the slow growth in height is well-fit by a power law, \( z_H(t > 0.6 \) Myr) \( \approx 8.5t_\ast^{1.14} \) kpc, consistent with Eq. (28) provided that \( f_\ast \approx 1/5 \). The widening of the bubble is not precisely a power law in this stage, slowing from \( R_{\max}(t > 0.6 \) Myr) \( \approx 1.4t_\ast^{0.6} \) kpc to \( R_{\max}(t > 1.8 \) Myr) \( \approx 1.5t_\ast^{0.5} \). The latter nicely matches (both in normalization and in power-law) the expected late-time behavior (38) for the above parameters. The projected dimensions of the bubble are best fit for \( t > 0.6 \) Myr by \( b \approx 46^\circ.36t_\ast^{1.13} \) and \( \Delta l \approx 8^\circ.66t_\ast^{0.7} \).

We carry out a suite of simulations that generate Fermi-like bubbles in their slowing-down phase. A representative sample of such simulations, including our fiducial setup J2 and its variants J2a through J2d, are summarised in the right columns of Table 3, and are depicted in Figure 12. The observed height and width of the FBs strongly constrain the initial conditions of the simulations, such that large deviations from the J1 or J2 parameters generally yield simulated bubbles inconsistent with observations.

By separately varying each of the J2a setup parameters, one at a time, we derive the functional dependencies of the bubble age in the close vicinity of the J2a setup,

\[
\Upsilon \approx \frac{E_j}{3 \times 10^{55} \text{ erg}} \left( \frac{E_j}{3 \times 10^{55} \text{ erg}} \right)^{-1.51 \pm 0.06} \beta_{-1}^{-1.92 \pm 0.10} \mu_{10}^{1.58 \pm 0.06} \times 0.55 \pm 0.05 \text{ Myr},
\]

(maximal half-width,

\[
R_{\max} \approx 1.95 \left( \frac{E_j}{3 \times 10^{55} \text{ erg}} \right)^{-0.77 \pm 0.05} \beta_{-1}^{1.13 \pm 0.07} \mu_{10}^{1.2 \pm 0.1} \times 0.37 \pm 0.03 \text{ kpc},
\]

(maximal longitude,

\[
\Delta l \approx 13^\circ.1 \left( \frac{E_j}{3 \times 10^{55} \text{ erg}} \right)^{-0.79 \pm 0.05} \beta_{-1}^{-1.26 \pm 0.08} \mu_{10}^{1.13 \pm 0.07} \times 0.37 \pm 0.04 \text{ kpc},
\]

and Mach number of the head,

\[
\Upsilon \approx 3.73 \left( \frac{E_j}{3 \times 10^{55} \text{ erg}} \right)^{1.27 \pm 0.13} \beta_{-1}^{-1.14 \pm 0.14} \mu_{10}^{-1.0 \pm 0.1} \times 0.40 \pm 0.05 \text{ kpc}.
\]

These individual parametric scans are illustrated in Figure 13. Notice that the energy-dependencies in the top row are not well-fit by a power law.
These results are generally consistent with our model, considering the limited dynamical range explored. For $z_s \approx 8$ kpc and $\tau_s = 0.14$, the age of the bubble in Eq. (31) scales approximately as $T \propto v_s \tau_s^{1.2}$. Combining this with Eqs. (41) and (42) yields $T \propto E_s^{-1.5} \beta_j^{1.0} R_{\max}^{1.0}$, in qualitative agreement with the fit (50). Equation (40) then gives $R_{\max} \propto E_s^{-0.5} \beta_j^{1.5} R_{\max}^{1.0}$, in fairly good agreement with the fit (51). Taking the time derivative of Eq. (30) yields, for late-times when $z_H(t)$ is a power-law, $T \propto (\tau_s z_H(t))/(\tau_c z_s) \propto T^{-1}$, in qualitative agreement with the fit (53). Taking into account corrections to $z_H(t)$ gives $T \propto (v_t/c_s) (z_H/z_s)^{-1.2} \propto v_t T^{-1.2} \propto E_s^{1.8} \beta_j^{1.3} R_{\max}^{1.8}$, partly consistent with (53). Some positive dependence of $\theta_{\text{eff}}$ upon the injection time is expected in (42); the $t_j$ dependence of the fits (50)–(53) would match the model if $\theta_{\text{eff}} \propto t_j^{0.2}$.

The slowing-down bubbles are more sensitive to the Galactic model than their ballistic counterparts. Increasing the disc mass significantly lowers both the age and the width of the slowing-down bubble, as it leads to a steeper, $\alpha > 2$ CGM density profile. This effect, along with the slower sideways expansion inside the disc, lead to younger bubbles more pinched at their base, as seen in the bottom panel of Figure 12. Other parameters have a more modest effect. Introducing extreme CGM rotation, for example, renders the bubbles slightly thinner, but does not substantially modify their age, as shown in the bottom panel of the figure.

5 SUMMARY AND DISCUSSION

We model the FBs under the assumption that their edges are strong forward shocks, as based on recent evidence ([1]). We simulate a bubble in one Galactic hemisphere as either a jet emanating from CBH or as a collimated injection of both energy and momentum as a jet emanating from CBH on $\sim 100$ pc scales ([4]). The evolved FBs, computed for various injection and CGM parameters, are compared to observations in order to constrain both the FB engine and the Galactic model. The study combines analytic modeling with converged ([8]) axisymmetric hydrodynamic PLUTO-v4.0 simulations ([2]), which implement various hydrostatic models of the Galaxy ([8]) including halo, disc, and bulge components.

A putative non-directed FB engine, injecting energy but not momentum, leads to a fairly spherical bubble due to the fast sideways expansion after the bubble emerges from the
and bulge masses, fast and even maximal CGM halo rotation, and combining the above effects; see Table 2. We conclude that a non-directed engine fails to generate FBs with forward shock edges consistent with observations for any plausible Galactic model.

In contrast, an outflow collimated into a thin jet, launched near the GC approximately perpendicular to the disc, can reproduce the observed morphology of the FB edge as a strong forward shock. The evolution can be crudely understood using a stratified model (§4.1), in which the gas behind the head of the jet is approximated as expanding sideways at a fixed z, as verified by numerical simulations (§4.2). The bubble initially evolves ballistically, $z_H \propto t$, until it accumulates sufficient mass and starts slowing down, with momentum conservation leading to a very slow, $z_H \propto t^{0.1-0.2}$ subsequent growth (Equation 28). As the transition height $z_t \gtrsim 5$ kpc (Figure 6) can be above or below the present-day $z_H \gtrsim 10$ kpc tip of the bubble, the FBs can be either in the ballistic phase (large $E/\beta E_0$; Equation 11) or in the slowdown phase (small $E/\beta E_0$; Equation 12). In both regimes, we identify plausible injection parameters (Table 3) that yield bubbles (modeled: left panel of Figure 5; simulated: Figure 7) consistent with the observed edges (KG17) and X-ray shell (KG18) of the FBs.

If the injection is sufficiently energetic or the jet is slow, $E_{55} \gtrsim 3(\beta \psi \theta_0)^2$, then the FBs are still ballistic. A high Mach number $\Upsilon$ at the top of the bubble then requires $\beta \psi \gtrsim 0.35 \Upsilon_5$, and the thickness of the bubbles requires $\theta_0 \gtrsim 4^\circ$. The injected energy (both hemispheres) is then $E_{55} \gtrsim 2\beta^2 \theta_0^2 \gtrsim 0.2 \Upsilon_5 t$ and the age of the bubbles is $T \simeq 3.3 \beta^{-1}_2$ Myr $\lesssim 8 \Upsilon_5^{-1}$ Myr. Our simulations show bubbles broadly consistent with the model in terms of structure (Figure 5 and the top panel of Figure 7), evolution (Figure 8), and parameter dependence (Figure 9 and Figure 10). We identify a pronounced low-pressure region behind the head of the simulated ballistic bubble, consisting of unperturbed jet material. The ballistic nature of the bubble renders it nearly independent of the Galactic model.

In contrast, if the injection is sufficiently fast or of low energy, $E/\beta E_0$ is small (Equation 12) and the observed FBs are already in their slowdown phase. The injected energy (both hemispheres) is then $E_{55} \lesssim 3(\beta \psi \theta_0)^2$, in which case $0.05 \Upsilon_5 \theta_0^2 \lesssim E_{55} \lesssim 2\beta^2 \theta_0^2$ and $T \approx 1.4(\beta \psi \theta_0)^{-1}$ Myr $\lesssim 4(\Upsilon \theta_0)^{-1}$ Myr. Such slowing-down FBs can still be energetic, provided that $\beta \psi$ is large, but their age would then be of order a Myr only if $\theta_0$ is small. Our simulations again show bubbles broadly consistent with the model in terms of structure (bottom panel of Figure 7), evolution (Figure 11), and parameter dependence (Equation 11 and Figure 13). These bubbles have a more regular inner structure, without the aforementioned low-pressure region. The edges of the bubbles in the slowdown phase are somewhat dependent on the disc mass.

While the FB edges and X-ray shell can be reproduced equally well if observed in the ballistic or slowdown phases, the evolution of the inner structure of the bubble can distinguish between the phases. The ballistic FB shows an inner cylindrical shock surrounded at its top by an irregular contact discontinuity surface, whereas the slowing-down FB shows a bubble-like contact discontinuity trailing the shock. The high, $T \gtrsim 5$ Mach number of the forward shock, inferred from observations (KG17; KG18), is consistent with...
with respect to the nistic of the FB energetics, being directly related to the emerge on we model as $\theta$
edges as forward shocks imply a collimated injection, which $\theta$ too wide, incorporating the disc component exacerbates the discrepancy.

Figure 14 shows that while the ZG20 setup already leads to FBs avoiding the propagation of the jet through the disc. Fig-
t $t$ with $j$
number imposes strong constraints on the injection setup. $z$
≃$shocks are too weak (with $\Upsilon$
ulated FBs are too wide (reaching $R$
and declining shock Mach number. However, the resulting $z$
parable to our directed injection with slowing-down FBs, here. The preferred (case A) ZG20 scenario is roughly com-
considering a much narrower parameter range than studied
ulated FBs with forward shock edges from jet injection,
ure 14 shows that while the ZG20 setup already leads to FBs too wide, incorporating the disc component exacerbates the discrepancy.

We have shown that FB observations identifying the edges as forward shocks imply a collimated injection, which we model as $\theta \lesssim 4^\circ$ jets along the Galactic $z$ axis as they emerge on $\sim 100$ pc scales. Sufficiently small inclinations with respect to the $z$ axis, of order $\theta \lesssim 30^\circ$, say, may still provide a good match to the FBs, but we could not explain forward-shock FB edges with jets of large inclinations. It is generally believed that highly collimated jets are launched by a rapidly spinning CBH along the spin axis (e.g., McK-inney et al. 2013). While a jet perpendicular to the Galactic plane does not seem consistent with the presently inferred spin orientation (e.g., Gravity Collaboration et al. 2018), rapid changes in spin direction may be natural for such a CBH (e.g., Dotti et al. 2012). Therefore, our results impose a challenge on modeling the launching of the jet and its collimation on small scales.

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APPENDIX A: GALACTIC MODEL

The galactic model implemented in the simulations contains gas components which are in a steady state equilibrium with a static gravitational potential. The potential consists of a cylindrically symmetric stellar disc and spherically symmetric bulge potential. The potential consists of a gas component which are in a steady state equilibrium with the total gravitational potential $\Phi(R,z)$.

For the gaseous components, we consider a circumgalactic medium (halo), an ISM disc, and a central molecular zone (CMZ) component. Each of these components follows an independent steady-state equilibrium with the total gravitational potential $\Phi(R,z)$. For example, the density distribution of the ISM disc component, in equilibrium, can be written as (for a derivation, see Sarkar et al. 2015a)

$$\rho_d(R,z) = \exp \left\{ - \frac{\Phi(R,z) - \Phi_0 - \frac{1}{2} \left[ \Phi(R,0) - \Phi_0 \right] f_d^2}{c_d^2} \right\},$$

where the potential $\Phi_0 \equiv \Phi(0,0)$ and ISM disc mass-density $\rho_d(0,0)$ are evaluated at the Galactic centre, $c_d \equiv c_d(T_d) = (k_B T_d/m_p)^{1/2}$ is the isothermal sound speed (including turbulence) of the disc gas, and $f_d$ is the rotation velocity ratio between the disc gas $v_{\phi,d}(R)$ and stellar component $v_{\phi,c}(R) = (-R \, \Phi(R) / \rho(R) h^2)^{1/2}$ at any given cylindrical radius $R$. We assume this ratio and the sound speed to be independent of $R$ and $z$, in order to obtain an analytical expression for the density distribution.

We obtain the density distributions of the other components in a similar way. For the halo and the CMZ, the density distributions are, respectively,

$$\rho_h(R,z) = \exp \left\{ - \frac{\Phi(R,z) - \Phi_0 - \frac{1}{2} \left[ \Phi(R,0) - \Phi_0 \right] f_h^2}{c_h^2} \right\},$$

and

$$\rho_{\text{CMZ}}(R,z) = \exp \left\{ - \frac{\Phi(R,z) - \Phi_0 - \frac{1}{2} \left[ \Phi(R,0) - \Phi_0 \right] f_{\text{CMZ}}^2}{c_{\text{CMZ}}^2} \right\},$$

where, $\rho_{\text{CMZ}} \equiv \rho_{\text{CMZ}}(0,0)$, $c_{\text{CMZ}} \equiv c_{\text{CMZ}}(T_{\text{CMZ}})$, and rotation factors $f$ are defined analogously to $f_d$ in Eq. (A4).

The total density at any computational cell is therefore given as $\rho_{\text{tot}}(R,z) = \rho_h(R,z) + \rho_d(R,z) + \rho_{\text{CMZ}}(R,z)$. Since each of these components may have a different rotation speed, the effective rotation speed $v_{\phi,\text{eff}}$ at any computational cell is given by

$$v_{\phi,\text{eff}}(R,z) = \sqrt{f_h^2 \rho_h(R,z) + f_d^2 \rho_d(R,z) + f_{\text{CMZ}}^2 \rho_{\text{CMZ}}(R,z) / \rho_{\text{tot}}(R,z)}.$$  

Notice that, although we include the CMZ component, it is set to zero in practice in the simulations shown.

APPENDIX B: CONVERGENCE TESTS

Figure B1 shows the contemporary projected FB images obtained for non-directed, GC injection (setup S1) for stretched $\{r, \theta\}$ grids of dimensions $\{1024 \times 256\}$ (left) and $\{2048 \times 512\}$ (right). The two images are sufficiently similar to warrant adopting $\{1024 \times 256\}$ as the base resolution for non-directed simulations.

For directed injection, we find that a higher resolution is
**Figure B1.** Same as top panel of Figure 2 (setup S1), but for simulation grids $1024 \times 256$ (left) and $2048 \times 512$ (right) in the $\{r, \theta\}$ directions.

**Figure B2.** Same as top panel of Figure 7 (setup J1), but for $\theta_j=4^\circ$, with simulation grids $\{1024 \times 512\}$ (left) and $\{2048 \times 1024\}$ (right).

**Figure B3.** Same as top panel of Figure 12 (setup J2a) but for simulation grids $\{1536 \times 768\}$ (left) and $\{3072 \times 1536\}$ (right).

needed to guarantee convergence. In the ballistic case, shown in Figure B2, we find that $\{1024 \times 512\}$ suffices as our base resolution. In the slowdown regime, shown in Figure B3, a base resolution $\{1536 \times 768\}$ is sufficient.

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