Power spectrum of halo intrinsic alignments in simulations

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ABSTRACT
We use a suite of N-body simulations to study intrinsic alignments (IA) of halo shapes with the surrounding large-scale structure in the \(\Lambda\)CDM model. For this purpose, we develop a novel method to measure multipole moments of the three-dimensional power spectrum of the \(E\)-mode field of halo shapes with the matter/halo distribution, \(P(\ell)\delta_E(k)\) (or \(P(\ell)hE(k)\)), and those of the auto-power spectrum of the \(E\) mode, \(P_{EE}(k)\), based on the \(E/B\)-mode decomposition. The IA power spectra have non-vanishing amplitudes over the linear to nonlinear scales, and the large-scale amplitudes at \(k \lesssim 0.1 \ h\ Mpc^{-1}\) are related to the matter power spectrum via a constant coefficient (\(A_{\text{IA}}\)), similar to the linear bias parameter. We find that the cross- and auto-power spectra \(P_{\delta E}\) and \(P_{EE}\) at nonlinear scales, \(k \gtrsim 0.1 \ h\ Mpc^{-1}\), show different \(k\)-dependences relative to the matter power spectrum, suggesting a violation of the nonlinear alignment model commonly used to model contaminations of cosmic shear signals. The IA power spectra exhibit baryon acoustic oscillations, and vary with halo samples of different masses, redshifts and cosmological parameters (\(\Omega_m, S_8\)). The cumulative signal-to-noise ratio for the IA power spectra is about 60\% of that for the halo density power spectrum, where the super-sample covariance is found to give a significant contribution to the total covariance. Our results demonstrate that the IA power spectra of galaxy shapes, measured from imaging and spectroscopic surveys for an overlapping area of the sky, can be powerful tools to probe the underlying matter power spectrum, the primordial curvature perturbations, and cosmological parameters, in addition to the standard galaxy density power spectrum.

Key words: cosmology: theory – large-scale structure of Universe – gravitational lensing: weak – methods: numerical

1 INTRODUCTION
There are many ongoing and planned imaging and spectroscopic surveys covering a wide area of the sky (e.g., Takada et al. 2014). These surveys aim to address the fundamental questions in cosmology: properties of the primordial perturbations that are seeds of the present-day cosmic structures, and the physical nature of dark matter and dark energy that are introduced to explain the dominant source of gravity and the cosmic accelerating expansion in the late-time universe (e.g., see Weinberg et al. 2013, for a review).

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The cold dark matter (CDM) dominated structure formation model predicts that shapes of galaxies interact with the surrounding gravitational (tidal) field in large-scale structure, and it induces intrinsic (not lensing-induced) correlations between galaxy shapes in the common large-scale structure, so-called intrinsic alignments (IA) (Croft & Metzler 2000; Catelan et al. 2001; Crittenden et al. 2002; Jing 2002). Usually the IA effect is considered as one of the most important physical systematic effects in the cosmic shear analysis (Hirata & Seljak 2004; Heymans et al. 2006) (see also Joachimi et al. 2015; Kiessling et al. 2015; Kirk et al. 2015; Troxel & Ishak 2015, for a review). Only very recently several theoretical works have started considering the IA effects as cosmological signals (Schmidt & Jeong 2012; Chisari & Dvorkin 2013; Schmidt et al. 2015; Kagai et al. 2018; Okumura et al. 2019; Okumura & Taruya 2020; Taruya & Okumura 2020). The IA correlations have been indeed measured from observational data, especially for early-type red galaxies (Mandelbaum et al. 2006; Okumura et al. 2009; Singh et al. 2015; Johnston et al. 2019; Samuroff et al. 2019; Yao et al. 2020).

Based on the above background, there have been analytical and numerical attempts to develop an accurate model of the IA effects. For an analytical approach it is usually assumed that galaxy shapes are tracers of the underlying gravitational tidal field that is sourced by the total matter (mainly dark matter) distribution in large-scale structure, and this model is called the linear (tidal) alignment model (Hirata & Seljak 2004). Then, the linear theory or perturbation theory of structure formation can be used to express the IA correlations in terms of the power spectrum or higher-order moments of matter and tidal fields (also see Blazek et al. 2015; Blazek et al. 2019). For the cosmic shear analysis, an empirical model, the so-called nonlinear alignment model (Bridle & King 2007), is often used to model the IA contamination to the cosmic shear signals at nonlinear scales, where the linear matter power spectrum appearing in the IA correlation is replaced with the nonlinear matter power spectrum. There are also simulation-based studies using cosmological N-body simulations (Jing 2002; Xia et al. 2017; Piras et al. 2018; Osato et al. 2018; Okumura et al. 2017, 2019; Sunayama et al. 2020) and cosmological hydrodynamical simulations (Tenneti et al. 2015; Codis et al. 2015; Velliscig et al. 2015a; Chisari et al. 2015b; Chisari et al. 2017a,b; Tugendhat & Schäfer 2018). Moreover, the halo model approach has been recently developed to model the IA effects of galaxies at nonlinear scales, more specifically inside the host halos (Schneider & Bridle 2010; Fortuna et al. 2020).

However, most of the previous studies are on the real- or configuration-space IA correlations, except for the perturbation theory based studies (e.g., Blazek et al. 2019). Hence the purpose of this paper is to develop a novel method to measure the three-dimensional power spectrum of the IA effects, using the E/B-mode decomposition method developed in the cosmic microwave polarization and the cosmic shear. We then apply the method to shapes of halos measured from a suite of N-body simulations, generated in Nishimichi et al. (2019), and estimate the auto-power spectra of the halo shape E/B modes and the cross-power spectrum of the E mode with the surrounding matter or halo distribution. Since the halo shapes are a spin-2 field defined in the two-dimensional plane perpendicular to the line-of-sight direction, the IA power spectra break the statistical isotropy and display anisotropic modulations depending on the angle between wavevector and the line-of-sight direction, just like the redshift-space power spectrum of galaxies. We use the measured IA power spectra to study a validity of the linear and nonlinear alignment models, the baryon acoustic oscillations, the information content (the cumulative signal-to-noise ratio) and the redshift-space distortion effect, compared to the standard power spectrum of halo density field. We also examine how the IA power spectra vary with halo samples of different masses, redshifts and cosmological parameters. In doing these, we pay special attention to the fact that keeping the three-dimensional Fourier modes in the IA power spectrum measurements enables one to extract the full information of two-point statistics, compared to the angular or projected correlation functions that are often studied in analogy to the cosmic shear correlations. The method developed in this paper can be applied to imaging and spectroscopic galaxy surveys observing the same area of the sky, where galaxy shapes are measured from the imaging data and the three-dimensional positions of galaxies are obtained from the spectroscopic data. This is the case, e.g., for the BOSS survey combined with the Subaru HSC survey (Ai-hara et al. 2018), the Subaru HSC/PFS surveys (Takada et al. 2014), the ESA Euclid1 and the NASA WFIRST2.

This paper is structured as follows. In Section 2, we review the intrinsic alignment model, mainly the tidal/linear alignment model, and define notations and quantities used in this paper. In Section 3, we give details of our simulations and describe the methods to measure the ellipticities of dark matter halos and the IA power spectra from the ellipticity/shear field. In Section 4, we present our results. We give conclusion and discussion in Section 5.

2 INTRINSIC ALIGNMENT MODEL

2.1 Preliminaries

Here we briefly review the IA model in large-scale structure. The IA model is based on the assumption that the shear tensor, defined by shapes of galaxies or halos at a redshift \(z\), \(g_{ij}(x; z)\), originates from the gravitational tidal tensor at a redshift \(z_{\Lambda}\) higher than \(z\) around the epoch of the formation of the galaxy of interest, i.e.,

\[ g_{ij}(x; z) \propto K_{ij}(x; z_{\Lambda}), \]

where

\[ K_{ij}(x; z) = \left( \nabla_i \nabla_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) \Phi(x; z), \]

and \(\Phi(x, z)\) is the gravitational potential field, or the metric perturbation in the General Relativity framework. As stressed in Hirata & Seljak (2004), the relationship of Eq. (1) is expected to hold only on large scales in the linear regime, in analogy with the linear bias model that relates the spatial distributions of galaxies and matter on large scales via a proportionality factor, i.e., a linear bias coefficient. The

1 https://www.cosmos.esa.int/web/euclid
2 https://wfirst.gsfc.nasa.gov

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gravitational potential field is related to the mass density fluctuation field via the Poisson equation as

$$\nabla^2 \Phi(x, z) = 4\pi G \rho_m(z) a^2 \delta(x, z),$$

where $a$ is the scale factor ($a = 1/(1 + z)$), $\rho_m(a)$ is the mean mass density at redshift $z$, and $\delta(x; z)$ is the mass density fluctuation field.

We can observe the “shape” of individual galaxies projected onto the sky, which is on a two-dimensional plane perpendicular to the line-of-sight direction, under the flat-sky approximation (this would be a good approximation as a galaxy size is very small compared to the curvature scale of the celestial sphere). In other words we cannot observe a three-dimensional shape of the galaxy. Hence we define an "observed" shear of a galaxy or halo as

$$\gamma_{ij}(x; z) \equiv \left( P_{i\ell k}(\hat{n}) P_{j\ell l}(\hat{n}) - \frac{1}{2} P_{i\ell l}(\hat{n}) P_{j\ell k}(\hat{n}) \right) \delta_{kl}(x; z),$$

where $\hat{n}$ is the unit vector of the line-of-sight direction, and $P_{i\ell k}(\hat{n}) \equiv \delta_{ij} - \hat{n}_i \hat{n}_j$ is the projection tensor onto the plane perpendicular to the line-of-sight direction. Throughout this paper, we refer to the coordinate components as $x = (x^1, x^2, x^3)$ and do not use $x^3 = z$ to avoid confusion with redshift “$z$”. If we set the $x^3$-direction to the line-of-sight direction, i.e., $\hat{n} \parallel x^3$, $\gamma_{ij}$ is expressed as

$$\gamma_{ij}(x; z) = \begin{pmatrix} \gamma_+ & \gamma_\times & 0 \\ \gamma_\times & -\gamma_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Since the shear tensor is traceless and symmetric, $\gamma_{ij}$ has two degrees of freedom for which we introduce the two components, $\gamma_+, \gamma_\times$, in analogy to the weak lensing shear (Bartelmann & Schneider 2001; Dodelson 2017). The IA model relates the shear tensor to the tidal field as

$$\gamma_+(x; z) \equiv -\frac{C_1}{4\pi G} \nabla_1^2 \nabla_2^2 \Phi(x; z) \delta_{\Lambda},$$

$$\gamma_\times(x; z) \equiv -\frac{2C_1}{4\pi G} \nabla_1^2 \nabla_2 \Phi(x; z) \delta_{\Lambda}.$$

Throughout this paper we adopt the plane-parallel or distant-observer approximation. Following the convention in the literature (Hirata & Seljak 2004), we introduced a prefactor $C_1/4\pi G$, and $C_1$ is a constant factor that has a dimension of $\rho^{-1}$. $C_1$ is a proportionality factor that depends on properties of sample galaxies or halos, e.g., luminosity (for galaxies), mass, redshift, cosmological parameters and so on. The minus sign is conventionally taken so that, if shapes of galaxies and halos are elongated along the direction of the mass accretion from the surrounding structures, $C_1$ turns to be positive.

Using the Poisson equation, Eq. (6) can be expressed in Fourier space as

$$\gamma_{+(k \cdot \hat{n})}(k; z) = -C_1 \Omega_m \rho_c(0)(1 + \Omega_{\Lambda}) \frac{f}{f_{+(k \cdot \hat{n})}}(k; z) \delta(k; z),$$

where $\rho_c(0)$ is the critical density today and we have defined the function $f(x, z)$, following Blazek et al. (2015), as

$$f_{+(k \cdot \hat{n})}(k) \equiv (1 - \mu_2) \cos(2\phi, \mu \sin(2\phi,$$

where $\mu = \hat{n} \cdot \hat{k} = \cos$ and $\phi = \tan^{-1}(k_3/k_2)$. The factor $(1 - \mu_2^2)$ in the kernel, $f_{+(k \cdot \hat{n})}$, reflects the fact that the IA shear arises from Fourier modes in two-dimensional plane perpendicular to the line-of-sight direction, $k_{\perp}$. For example, Fourier modes along the line-of-sight direction, which have $\mu = \pm 1$, do not cause the observed IA shear. This is opposite to the redshift-space distortion due to peculiar velocities of galaxies, which arise from Fourier modes along the line-of-sight direction. If we take $\Omega_{\Lambda}$ to be in the matter dominated era for an epoch of the IA generation, the amplitude of tidal field on linear scales is constant in time, and therefore the IA reflects the primordial tidal field. This model is called the primordial alignment model, and in this case we have

$$\gamma_{+(x, y)}(k, z) = -\Omega_{\Lambda} C_1 \rho_c(0) \frac{\Omega_m}{D(z)} \frac{f_{+(x, y)}}{f_{+(k \cdot \hat{n})}}(k) \delta(k; z),$$

where $D(z)$ is the linear growth factor. We set $C_1 \rho_c(0) = 0.0134$ as we employ the normalization $D(z = 0) = 1$ in this work, following the convention in Joachimi et al. (2011). We use the dimensionless parameter $\Omega_{\Lambda}$ to characterize the amplitude of the IA signal.

For an practical measurement of the IA effect, we further take into account the effect of redshift-space distortion (RSD) caused by peculiar velocities of galaxies or halos. We will discuss the RSD effect in a separate section later.

### 2.2 $E/B$ decomposition of the IA power spectrum

As we described, the galaxy shape induced by the IA effect is a spin-2 field by definition. This is a useful property, and we can use the $E/B$-mode decomposition of the observed galaxy shape field that gives a unique decomposition of the two degrees of freedom in the spin-2 field. The $E$ mode is a physical mode caused by the scalar gravitational potential, and the $B$ mode is a non-physical mode that cannot be generated by the scalar mode in the linear regime, so served as an indicator of systematic errors in actual measurements. However, note that the $E$ and $B$ modes are mixed in the nonlinear regime or if the field is a nonlinear field of the underlying scalar fields, which indeed occurs in the IA power spectrum as we will show later. In analogy with CMB polarization (Zaldarriaga & Seljak 1997; Kamionkowski et al. 1997) and weak lensing (Crittenden et al. 2002), the $E/B$-mode decomposition is non-local in real space, while it is “local” in Fourier space. From Eq. (7), we can define the $E/B$ modes of galaxy shapes, denoted as $\gamma_{E/B}$:

$$\gamma_E(k) = \gamma_+(k) \cos 2\phi + \gamma_\times(k) \sin 2\phi,$$

$$\gamma_B(k) = -\gamma_+(k) \sin 2\phi + \gamma_\times(k) \cos 2\phi.$$

From these equations, in this paper we consider the following 3D power spectra to study the IA effect:

$$\langle \gamma_{E}^{\text{BE}}(k) \delta(k') \rangle \equiv P_{E/E}(k)(2\pi)^3 \delta_3^D(k + k'),$$

$$\langle \gamma_{E}^{\text{BE}}(k) \gamma_{E}^{\text{BE}}(k') \rangle \equiv P_{E/E}(k)(2\pi)^3 \delta_3^D(k + k').$$

where $\delta_3^D(k)$ is the 3D Dirac delta function, $P_{E/E}(k)$ is the cross-power spectrum between the mass density field and the $E$ mode of galaxy shape, and $P_{E/E}(k)$ is the auto-power spectrum of the $E$-mode field. We should emphasize that, although the $E/B$ modes are defined with respect to the Fourier modes $k_{\perp}$ in the “two-”dimensional plane perpendicular to the line-of-sight direction, the power spectra are given as a function of the three-dimensional wavevector, $k$. In addition, the power spectra are not only a function of the scalar $k(=|k|)$, but also depends on the direction of $k$. These
3D power spectra contains the full information on the IA effect at the level of two-point statistics. Usually an actual measurement of the IA effect is done in the projected correlation function, i.e., integrating the above 3D power spectrum information along the line-of-sight direction, in analogy with the weak lensing measurement. As we will show, this projection leads to a loss of the underlying information. For the B-mode power spectra, \( \langle Y_B Y_B \rangle = 0 \) for the IA caused by the scalar tidal field in the linear regime, and \( \langle Y_B \delta \rangle = \langle Y_B \gamma \rangle = 0 \) due to the statistical parity invariance. These give a useful sanity check of residual systematic errors in actual measurements.

For convenience of our discussion, we define the multipole moments of the IA power spectrum as

\[
P_{XY}^{(f)}(k) = \frac{2\ell + 1}{2} \int d\mu L_\ell(\mu)P_{XY}(k, \mu),
\]

where the subscripts \( X \) and \( Y \) are labels for \( \delta \) (density), \( h \) (halos), or \( E \) (shape), and \( L_\ell(\mu) \) is the Legendre polynomial of order \( \ell \). Due to the geometrical nature of \( E/B \) modes of the intrinsic galaxy shapes, the following relations between the multipole moments are expected to hold:

\[
\frac{p_{EE}^{(2)}}{p_{EE}^{(0)}} = -1, \quad \frac{p_{EE}^{(4)}}{p_{EE}^{(0)}} = \frac{10}{7}, \quad \frac{p_{EE}^{(4)}}{p_{EE}^{(0)}} = \frac{3}{7}.
\]

For the cross-power spectrum, \( P_{E\delta} \) or \( P_{\delta E} \), the above relation always holds because it comes from the geometrical factor \( (1 - \mu^2) \) in the definition of the projected shapes, and thus does not rely on the specific IA model. On the other hand, for the auto-power spectrum, \( P_{EE} \), the above relation holds in the linear regime to a good approximation, but is not exact even in the linear regime (small \( k \)) due to the nonlinear shot-noise contribution (see below). Note that the projection effects do not cause the higher-order moments beyond the 2nd- or 4th-order moments for \( P_{E\delta}(P_{E\delta}) \) and \( P_{EE} \) in real space, respectively.

Plugging Eq. (7) into Eq. (12), we find that the linear IA model predicts the power spectra to be given as

\[
P_{\delta E}(k, \mu; z) = -A_{IA}C_1P_{\delta}(k; z),
\]

\[
P_{EE}(k, \mu; z) = \left[ A_{IA}C_1P_{\delta}(k; z) \right]^2
\]

where \( p_{\delta E}^{(0)}(k; z) \) is the linear matter power spectrum. This is called the linear alignment model. If we replace \( P_{\delta E}^{(0)}(k) \) with the nonlinear matter power spectrum, denoted as \( P_{\delta N}(k) \), it gives the nonlinear alignment model (Brüggen & King 2007), which has been often used in the weak lensing cosmology analysis (e.g., Hikage et al. 2019). These alignment models predict the specific relations between \( P_{\delta E} \) and \( P_{EE} \) via the same coefficient with respect to the matter power spectrum. The above equations are found to satisfy Eq. (14).

Note that the shear field estimated by using shapes of galaxies or halos \( \gamma_i \) is a density-weighted field because we can sample the shape field only at the positions of halos/galaxies and the halos/galaxies are biased tracers of the underlying matter density field, i.e., \( \gamma_{ij} = (1 + \delta_{gh})Y_{ij} \) (see Appendix A for details) (also see Seljak & McDonald 2011, for a similar discussion on the redshift-space distribution field of galaxies). At the leading order, its Fourier transform is written as

\[
\tilde{\gamma}_{ij}(k) = \left[ 1 + b_{g/h}\delta_{\text{lin}} \right] \tilde{\gamma}_{ij}(k)
= \int \frac{d^3k^*}{(2\pi)^3}\gamma_{ij}(k - k^*) \left[ 2\pi \delta_D(k^*) + b_{g/h}\delta_{\text{lin}}(k^*) \right],
\]

where \( b_{g/h} \) is a linear galaxy/halo bias. Therefore \( P_{EE} \) and \( P_{BB} \) have \( O(\sigma_{\text{lin}}^4) \) correction terms in addition to Eq. (16). These nonlinear terms of the fluctuation fields lead to a leakage of \( E \) mode into \( B \) mode, as we will discuss below.

In order to predict the IA effect beyond linear theory, one might want to use the perturbation theory of structure formation (Bernardeau et al. 2002) or the effective field theory of large-scale structure (McDonald & Roy 2009; Baumann et al. 2012). For this kind of approach, one can write down a general expansion of the IA field in terms of series of the underlying matter fields and possibly additional counter terms, with coefficients for each term (see Blazek et al. 2015; Schmidt et al. 2015; Blazek et al. 2019; Schmitz et al. 2018; Vlah et al. 2020, for recent works).

### 3 MEASUREMENT METHOD OF IA POWER SPECTRA FROM N-BODY SIMULATIONS

In this section, we describe details of \( N \)-body simulations and the halo catalogs, the way to quantify shapes of halos, and the way to measure the IA power spectra from the simulations.

#### 3.1 \( N \)-body simulations and halo catalogs

In this paper, we use a subset of the \( N \)-body simulation data in Dark Quest (Nishimichi et al. 2019), more exactly the high-resolution (HR) suite constructed in the paper, and the associated halo catalogs. All the \( N \)-body simulations are performed with 2048\(^3\) particles in a comoving cubic box with \( 1 \) h\(^{-1}\) Gpc on a side. We simulate the time evolution of particle distribution using the parallel Tree-Particle Mesh code Gadget2 (Springel 2005). We employ the second-order Lagrangian perturbation theory (Scoccimarro 1998; Crocce et al. 2006; Crocce & Scoccimarro 2006; Nishimichi et al. 2009; Valageas & Nishimichi 2011a,b) to set up the initial displacement vector and the initial velocity of each \( N \)-body particle, where we use the publicly available linear Boltzmann code CAMB (Lewis et al. 2000) to compute the linear matter power spectrum at the initial redshift for the fiducial cosmology. As for the fiducial cosmological model, we employ the best-fit flat-geometry \( \Lambda \)CDM model that is supported by the Planck CMB data (Planck Collaboration et al. 2016): \( \omega_b, \omega_c, \Omega_\Lambda, \ln(10^{10}A_s), n_s = (0.2225, 0.1198, 0.6844, 3.094, 0.9645) \). We set \( \sigma_8 = 0.0064 \) for the density parameter of massive neutrinos to set up the initial condition, but we use a single \( N \)-body particle species to represent the “total matter” distribution and follow the subsequent time evolution of their distribution (i.e., we do not consider distinct particles for the massive neutrinos). The Planck model has, as derived parameters, \( \Omega_\Lambda = 0.3156 \) (the present-day matter density parameter), \( \sigma_8 = 0.831 \) (the present-day rms linear mass density fluctuations within a
top-hat sphere of radius $8 \ h^{-1} \ Mpc$) and $h = 0.672$ for the Hubble parameter. The particle mass is $1.02 \times 10^{10} \ h^{-1} \ M_\odot$ for the fiducial Planck cosmology.

For each simulation realization, we identify halos in the post-processing computation, using a phase space finder, Rockstar (Behroozi et al. 2013) (also see Nishimichi et al. 2019, for details). We use the Rockstar outputs to infer the mass of each halo; throughout this paper, we use the virial mass definition, $M_{\text{vir}}$. The center of each halo is estimated from the center-of-mass location of a subset of member particles in the inner part of halo, which is considered as a proxy of the mass density maximum. Throughout this paper we use halos with masses greater than $10^{13} \ h^{-1} M_\odot$, which contains more than 100 $N$-body particles. We use the outputs of $N$-body realizations at 21 redshifts in the range of $0 \leq z \leq 1.48$, evenly stepped by the linear growth factor for the fiducial Planck cosmology (see Nishimichi et al. 2019, for details).

### 3.2 Measurements of halo shapes

We now need to quantify the “shape” of individual halos. Since dark matter halos are not relaxed nor in dynamical equilibrium and do not have any clear boundary, there is no unique definition of halo shape. What we can observe from data is only the “shape” of a galaxy, or that of star distribution, and those stars would form in the center around the mass density maximum in each host halo due to baryonic dissipative processes forming stars. Hence, in order to estimate a “central-galaxy-like” shape of each halo, we use the following inertia tensor of $N$-body particle distribution in each halo (Osato et al. 2018) (also see Bett 2012; Tenen & Bet. et al. 2015, for the similar definition):

$$I_{ij} = \sum_p w_p \Delta x_p^i \Delta x_p^j,$$  

(18)

where $\Delta x_p = x_p - x_h$, $x_h$ is the position of the halo center for each halo, $x_p$ is the position of the $p$-th member particle in the halo, $w_p(r_p)$ is the radial weight function and $r_p$ is the radius in the triaxial coordinate system defined by using the iterative scheme (see Appendix C for the details). From the above consideration, we employ $w_p = 1/r_p^2$; we upweight contributions from inner particles around the mass density maximum, assuming that those particles are more gravitationally bound and are proxies of stars if a galaxy forms in the halo (see Masaki et al. 2013, for the similar discussion).

Taking $x$-direction as the line-of-sight direction, we define two components to characterize the ellipticity of each halo, from the inertia tensor, as

$$\epsilon_x \equiv \frac{I_{11} - I_{22}}{I_{11} + I_{22}}, \quad \epsilon_z \equiv \frac{2I_{12}}{I_{11} + I_{22}}.$$  

(19)

In an actual observation, we can see only the “projected” distribution of stars in each galaxy, and therefore the above definition would be appropriate for the definition of the halo ellipticity or closer to what we can estimate from the light distribution of each galaxy. In Appendix C, we study how the IA power spectra are changed if a different definition of halo shapes is used. A brief summary of the results is as follows. The ellipticities of individual halos are sensitive to how to define the halo shapes, and the large-scale IA amplitudes ($A_{I\alpha}$), measured from the simulations, also vary with the halo shape definitions. However, the shapes of IA power spectrum and the signal-to-noise ratio of the IA power spectrum remain almost unchanged. Hence the results we show in the following are valid.

Fig. 1 shows the distribution of halo ellipticities measured from one simulation realization, for different halos samples, defined according to the halo mass ranges. Note that the distribution satisfies the normalization condition: $\int d\epsilon P(\epsilon) = 1$. The figure shows $\epsilon \sim 0.5$ as typical halo ellipticities, with a wide distribution. However, as we will show later, the IA effect arises from a correlated part between shapes of different halos, which corresponds to a few percent in the ellipticity amplitude, much smaller than the random intrinsic shapes. Thus the random intrinsic shapes give a dominant source of statistical errors in a measurement of the IA effect.

As in the weak lensing convention (Bernstein & Jarvis 2002), we define the halo shape “shear” from the second-order moments (Eq. 19). We convert the halo shapes into the spin-2 shear, via the following relation, in order to compare with the IA theory given in terms of the gravitational tidal field:

$$\gamma(\epsilon, x) = \frac{1}{2R} \epsilon_{x}\epsilon_{z}$$

(20)

where $R \equiv 1 - (\epsilon_z^2)$ is the responsivity (Bernstein & Jarvis 2002) and $\epsilon_{x}^2, \epsilon_{z}^2$ is the intrinsic variance of halo shapes per component, defined as $\epsilon_{x}^2 \equiv \sum_h \epsilon_x^2 h, \epsilon_{z}^2$ (z is equivalent). Typically $R \approx 0.9$ for our halo samples as indicated in Fig. 1.

The halo shape, given by Eq. (18), is considered as a representative tracer of the underlying ellipticity/shear field or theoretically the tidal field in the IA model, which has an analogy to the peculiar velocity field of galaxies (Kaiser

\[\text{Figure 1. Distributions of ellipticity amplitudes of halo shapes, defined as } \epsilon = \sqrt{\epsilon_x^2 + \epsilon_z^2}, \text{ for halo samples in the mass ranges denoted by the legend. Here we show the results at } z = 0.484. \text{ For the two-component ellipticities, the probability distribution is given as } P(\epsilon_x, \epsilon_z) = \epsilon_x P(\epsilon_x, \epsilon_z). \text{ Hence the top panel shows the probability distribution of ellipticity amplitude, } \epsilon^2 P(\epsilon), \text{ which satisfies the normalization } \int d\epsilon P(\epsilon) = 1. \text{ The lower panel shows the non-normalized distribution, i.e., the number of halos, at each bin of } \epsilon. \text{ For illustrative purpose, we adopt the logarithmic scale of } y\text{-axis.}\]
1987) or the weak lensing field (van Waerbeke 2000)3 for the definition. In this work, we consider a number-density weighted ellipticity field:

\[
\gamma_{\ell}(\mathbf{x}) = \frac{1}{n_h} \sum_{\mathbf{x}_h} \gamma^h_{\ell}(\mathbf{x} - \mathbf{x}_h)
\]

where \(n_h\) is the mean number density of halos, and \(x_h\) denotes the position of halos. It is useful to get access to this field on regular grids to make use of the Fast Fourier Transform. To do so, we use the cloud-in-cell (CIC) assignment (Hockney & Eastwood 1981) to interpolate the ellipticities sampled at the positions of halos, to the entire simulation box (see Appendix A for details). Throughout this paper we employ 10243 grids to define the halo shear fields. Finally we perform a Fourier-transformation of the fields to compute the \(E/B\)-mode fields from Eqs. (10) and (11), \(E(k)\) and \(B(k)\). After the decomposition, we measure the power spectrum from each realization; in the next section we consider the following power spectra:

\[
\left\{ P_{\delta}(k), P_{\delta}(k), P_{\delta E}(k), P_{\delta E}(k), P_{\delta E}(k) \right\}.
\]

where “\(\delta\)” and “\(h\)” denote the density fields of matter and halos, respectively. We also give a discussion on the \(B\)-mode power spectrum in Appendix B.

4 RESULTS

4.1 Power spectra of matter, halos and shapes

In Fig. 2 we show the cross-power spectrum between the \(E\)-mode field of halo shapes and the matter density field, \(P_{\delta E}\), measured for halos with masses in the range \([10^{12}, 10^{12.5}] h^{-1} M_\odot\) in simulation outputs at \(z = 0.48\). The symbols are the average among the 20 realizations, and the errorbars indicate the statistical error for a volume of \((1 h^{-1} \text{ Gpc})^3\), computed from the realization-to réalization scatters. First, the cross-power spectrum displays \(\langle \delta \rangle\) as the physical correlation of halo shapes with the large-scale matter distribution arises on scales beyond the halo shape correlation scales.

In Fig. 3, we show the dimensionless power spectra, defined by \(\Delta_{\delta E}^2 = k^3 P_{\delta E}/2\pi^2\), to study the typical amplitude of the halo shape \(E\)-mode field. Recalling that the dimensionless power spectrum at a particular \(k\) corresponds to the real-space variance per unit logarithmic wavenumber interval at the corresponding length scale, e.g., \(\Delta_{\delta E}^2(k) \sim \langle |\mathbf{\delta_{E}}|^2 \rangle_{R=1/k}\), one can find \(\delta \sim O(1)\) at \(k\) a few \(O(0.1) h \text{ Mpc}^{-1}\) from the gray points showing \(\Delta_{\delta E}^2\). Then comparing the amplitudes of \(\Delta_{\delta E}^2\) bias coefficient, e.g., as seen from the ratio of the halo-matter cross power spectrum to the matter power spectrum, \(P_{\delta E}/P_{\delta}\) at \(k \to 0\) with a constant coefficient \(b_1\). The scale-independent (constant) ratio is a confirmation of the linear alignment model. This large-scale correlation is as expected in the standard \(\Lambda\) CDM model with an adiabatic Gaussian initial condition that is employed in our simulations, as follows. The formation and evolution of individual halos are governed by local physics or physical quantities within a few Mpc scales around each halo. Hence, as long as the physical correlation of halo shapes with the large-scale matter distribution arises on scales beyond the halo scales, it should originate from the gravitational interaction and the primordial perturbations. Since there is only a single degree of freedom in the perturbations at large scales in the adiabatic initial conditions, the power spectra of the IA (halo shape) fields at linear scales should be related to the matter power spectrum via a constant factor (also see Desjacques et al. 2018, for the similar discussion on halo bias).

The lower panel of Fig. 2 shows the ratio of the monopole or quadrupole moment to the matter power spectrum.
and $\Delta_{E}^{2}$ tells $E$ a few $O(0.01)$, i.e., a few per cent for the IA shear amplitude at $k \approx 0.1$ h Mpc$^{-1}$. This means that, if the halo $E$-mode field is smoothed within a volume of scales $R_{\text{min}} \sim 1/k \sim$ a few 10 Mpc, the $E$-mode amplitude is of the order of 0.01. This $E$-mode amplitude can be compared to the intrinsic random shape, $\gamma_{N} \sim 0.2$ (Fig. 1 and Eq. 20 taking into account the relation $\gamma = \epsilon/(2\kappa)$ with responsivity $\kappa \sim 0.9$). Thus, the large-scale IA shear is measurable only in a statistical sense, e.g., via the correlation function or power spectrum for the two-point statistics. At the nonlinear scales $k \gtrsim 0.1$ h Mpc$^{-1}$, the shear IA amplitude appears greater as shown by the lower panel, but the boosted amplitudes are likely due to the higher-order contribution of density perturbation as explained around Eq. (17).

In Fig. 4 we show the auto power spectra of the $E$-mode shape fields. Here we first substracted the shot noise term from the measured power spectrum, and then computed the multipole moments of power spectrum. In Appendix B we in detail describe how to estimate the shot noise term due to the discrete nature of the intrinsic shapes of halos in each simulation. Note that the shape noise contributes only to the monopole moment. The monopole and quadrupole moments display different $k$-dependences at $k \gtrsim 0.1$ h Mpc$^{-1}$. This means that a simple geometrical relation between the monopole and quadrupole moments, given by Eq. (14), does not hold for the auto spectrum especially at $k \gtrsim 0.1$ h Mpc$^{-1}$, unlike that of the cross-power spectra (the relation for the hexadecapole moment is not clear due to the larger errors). This implies that the higher-order contributions to the auto-power spectra cause non-trivial angular modulations, which are also found from a perturbation theory calculation in Blazek et al. (2015). In Appendix B, we show that the $B$-mode auto power spectrum displays a deviation from the simple shot noise, with a weak-scale dependence (see Fig. B1). We believe that this is ascribed to the “renormalized” shot noise arising from the small-scale nonlinear terms as discussed in Blazek et al. (2019), which has an analogy to the renormalization of bias parameters (McDonald 2006; McDonald & Roy 2009). This term should equally contribute to the $E$-mode power spectrum.

The IA effect is one of the most important systematic effects in cosmic shear cosmology (Hildebrandt et al. 2017; Troxel et al. 2018; Hikage et al. 2019; Hamana et al. 2020). In this context, there are two contributions, called “II” and “GI”, to the cosmic shear power spectrum, which correspond to $P_{EE}$ and $P_{AI}$, respectively. In cosmic shear analyses, the following relation is often assumed based on the linear alignment model (Eq. 9):

$$P_{EE}(k) = A_{IA}c(z)(1 - \mu^{2})P_{\delta}(k),$$

$$P_{AI}(k) = A_{IA}c(z)(1 - \mu^{2})P_{\delta}(k),$$

where $c(z)$ is a constant number at a particular redshift, defined from Eq. (9) as $c(z) \equiv C_{\nu}\rho_{0}H_{0}/D(z)$. The cosmic shear is a projected field of the underlying matter density field along the line-of-sight direction, so the power spectrum corresponds to the one evaluated at $\mu = 0$ in the above equation because $k_{\perp} = k(1 - \mu^{2})^{1/2}$ (see Section 4.5 for a similar discussion). If we use the nonlinear power spectrum for $P_{\delta}(k)$, it corresponds to the nonlinear alignment (NLA) model (Bridle & King 2007).

Here we address the validity of the linear and nonlinear alignment models by comparing the expressions (Eq. 23).
with the IA power spectra measured in simulations. Fig. 5 shows the comparison for the two halo samples with different mass ranges, $M_{\text{vir}} = [10^{12}, 10^{12.5}]$ or $[10^{13}, 10^{13.5}] h^{-1} M_{\odot}$, respectively, at different redshift outputs. For comparison purpose, we arbitrarily scaled the ratio of $P_{\text{EE}}/P_{\delta}$ so that it becomes unity at small-$k$ limit; equivalently we set $A_{\text{IA}}(\zeta) = 1$ in Eq. (23) at each redshift output. Then we apply the same normalization to the ratio of the auto-power spectrum at the respective redshift. Hence, if the linear and nonlinear alignment model (Eq. (23)) holds, all the curves go to unity at small $k$, and the ratios of the cross- and auto-power spectra at respective redshift should have the same $k$ dependence (see text for details). Note that the results of Figs. 2 and 4 correspond to one snapshot, and we subtracted the shape noise or shot noise contribution from the measured $P_{\text{EE}}$.

Fig. 5 also shows that, at lower redshifts or as nonlinear structure formation more evolves, the ratio of the cross-power spectrum, $P_{\text{SE}}$, has smaller amplitudes compared to that at small $k$ bins in the linear regime. This implies that $P_{\text{SE}}$ has a weaker response to the nonlinear clustering. This would be partly ascribed to the spin-2 field nature of halo shape (shear) field; the halo shear field can be both negative or positive in the nonlinear regime depending on the orien-
tation of the halo ellipticities even for the fixed ellipticity model, while the mass density fluctuations are always positive in such nonlinear regions. Or equivalently the IA amplitude, while the mass density fluctuations are always constant in time, whenever the IA correlation is measured (even if the number of the halos significantly changes across different redshifts). This is because the halos of same mass form from the primordial density peaks of the same Lagrangian volume and the large-scale relation/correlation between the halo shapes and the primordial tidal field has no time dependence in the Lagrangian picture.

Now we consider the samples for a fixed number density. A spectroscopic survey of galaxies is sometimes designed to keep a constant number density over a range of redshifts for the cosmological analysis purpose (e.g., Dawson et al. 2013; Takada et al. 2014). The ongoing and upcoming spectroscopic surveys are in the range of \( \bar{n} = 10^{-5}, 10^{-3} (\text{Mpc}^{-3}) \). The redshift evolution of \( A_{IA} \) depends on the number density of a sample; \( A_{IA} \) decreases with the increase of redshift for a low density sample such as \( \bar{n} = 10^{-5} (\text{Mpc}^{-3}) \). \( A_{IA} \) appears to be almost constant with respect to redshifts for \( 10^{-4} (\text{Mpc}^{-3}) \) and \( A_{IA} \) increases with redshift for \( 10^{-3} (\text{Mpc}^{-3}) \). Thus the \( A_{IA} \) amplitude depends on the selection of halos or the nature of the halo sample. Finally, we comment on a connection of the results in Fig. 6 to the IA effects of galaxies. We can consider the \( A_{IA} \) amplitude shown in Fig. 6 is the maximum case, since we consider the halo shapes. Since the physics and evolution of galaxies are more complicated, and galaxy shapes would have a misalignment with the halo shapes to some degrees (Okumura et al. 2009), the \( A_{IA} \) coefficients for galaxies would be smaller even if the galaxies of interest are central galaxies and reside in halos in the mass range we have considered so far. We also note that the \( A_{IA} \) amplitude varies with the definition of halo shapes even for the same sample of halos, as shown in Appendix C.

Fig. 6 indicates \( A_{IA} \sim 20 \) for halos with \( 10^{13} h^{-1} M_{\odot} \) at \( z \sim 0.5 \), which roughly corresponds to the host halos of the SDSS luminous red galaxies, and this is larger than \( A_{IA} \sim 8 \) implied from the actual SDSS data (Okumura et al. 2009; Singh et al. 2015). As discussed in Appendix C, if we employ a crude definition of the halo shapes in simulations, it leads to about halved value of \( A_{IA} \) even for the same sample of halos. In addition, actual galaxies might have a misalignment with the orientations of the host halos, and this also leads to a smaller \( A_{IA} \) value inferred from galaxy shapes, compared to the halo shapes (Okumura et al. 2009). A random misalignment of about 30 degree between the major axes of halo and galaxy orientations leads to about factor of 2 smaller value of \( A_{IA} \). Thus an actual value of \( A_{IA} \) is sensitive to the definition of halo shapes and the properties of galaxies relative to host halos, so the results of Fig. 6 can be considered as an example of \( A_{IA} \) values that dark matter halos could have. Or the parameter \( A_{IA} \) should be considered as a “nuisance” parameter, because the genuine value of \( A_{IA} \) is difficult to predict from the first principles.

4 If the linear alignment model (Eq. 6) holds, the linear IA coefficient \( (A_{IA})_{\perp} \) for halos of the same mass would be the same and constant in time, whenever the IA correlation is measured (even if the number of the halos significantly changes across different redshifts). This is because the halos of same mass form from the primordial density peaks of the same Lagrangian volume and the large-scale relation/correlation between the halo shapes and the primordial tidal field has no time dependence in the Lagrangian picture.

5 Nevertheless, note that, even for this case, the signal-to-noise ratios of IA power spectrum is not largely changed as in the main results we show below.
**Figure 6.** The halo mass dependence and the redshift evolution of the large-scale IA coefficient, $A_{IA}$, which is estimated according to Eq. (24) and theoretically corresponds to the coefficient of linear alignment model in Eq. (9). **Left panel:** The results for the samples of halos in a given mass range, as denoted by the legend. **Right:** Similar to the left panel, but the results for samples of halos with a fixed number density, where we define each sample by selecting halos from the ranked list of masses starting from the most massive halo until the number density of selected halos matches the target number density.

**Figure 7.** The ratio of the cross-power spectrum of matter and halo $E$-field to the linear matter power spectrum without BAO wiggles (no wiggle), for halos samples in different mass bins at $z = 0.484$. Here we use the fitting formula in Eisenstein & Hu (1998) to compute the no-wiggle matter spectrum for the Planck cosmology, the same model used in the simulations. For easier comparison we arbitrarily normalize the ratio in such that all the curves have the similar amplitudes up to $k = 0.05 \ h\ Mpc^{-1}$ (we employed the normalization on individual realization basis). We here consider the two samples of halo masses, where the sample of $M_h = [10^{12}, 10^{13.5}] h^{-1} M_{\odot}$ roughly corresponds to a typical mass of halos hosting the BOSS CMASS galaxies. We also show the ratios for the matter-halo cross spectrum, $P_{sh}$, and for the linear matter power spectrum with BAO wiggles, which are similarly normalized (arbitrarily scaled in the $y$-direction).

Planck cosmology. We arbitrarily normalize all the cross-power spectra so that the ratio, $P_{SE}(k)/P_{\delta}(k)$, is close to unity at $k$ bins up to $k = 0.05 \ h\ Mpc^{-1}$ in each realization.

The IA power spectrum displays clear BAO features as in the power spectrum of the halo density field. Thus the IA power spectrum can be used to measure the BAO scales (Okumura et al. 2019). Perhaps more interestingly, while the power spectrum of the halo density field has a boost in the amplitude at $k \gtrsim 0.1 \ h\ Mpc^{-1}$ in the nonlinear regime, the IA power spectrum displays a weaker boost in the amplitude at such nonlinear scales; for less massive halos with $M = 10^{12-12.5} h^{-1} M_{\odot}$ the amplitude stays almost unchanged as that of the linear power spectrum. This could be interpreted as follows. Consider an overdensity region in the initial linear density field, at a sufficiently high redshift, i.e., in the linear regime. The Lagrangian volume of such a region shrinks due to the gravitational instability, and the density contrast accordingly grows due to the mass conservation. A larger number of halos form in such an overdensity region. Thus the mass density or number density of halos have a boost in the amplitude, reaching $\delta \gg 1$, in the overdensity region. On the other hand, there is no conservation law for...
the halo shapes or tidal fields. Even in the highly nonlinear regime, ellipticities of halo shapes still stay in the range of \(|\gamma| \leq 1\) or never goes beyond unity, unlike the density contrast. Hence the IA power spectrum should have a weaker response to the nonlinear clustering, at least in the power spectrum amplitudes. Nevertheless, the observed IA field is a galaxy density-weighted field (see below), and the observed halo shapes is expressed as \((1 + \delta)\). The prefactor \((1 + \delta)\) can lead to a boost in the IA power spectrum, which partly explains a boost in the IA power spectrum for the halo sample with \(M = 10^{13.5-13.5} \, h^{-1} M_{\odot}\). These are interesting results.

4.4 Signal-to-noise ratio

How much information does the IA power spectrum carry, compared to the standard halo power spectrum? To address this question, we study the cumulative signal-to-noise ratio \((S/N)\) over a range of \(k_{\text{min}} \leq k \leq k_{\text{max}}\), defined by

\[
\left(\frac{S}{N}\right)^2 = \sum_{k=k_{\text{min}}}^{k_{\text{max}}} \beta^2(k) |C_{\ell \ell}(k)|^{-1} \beta^2(k),
\]

where \(C_{\ell \ell}(k)\) is the covariance matrix between the \(\ell\)– and \(\ell'\)-th multipole moments of power spectra and \(|C_{\ell \ell}(k)|^{-1}\) is the inverse of the covariance matrix. Given an estimator of the power spectrum, the covariance matrix is defined as

\[
C_{\ell \ell}(k) = \left\{ \hat{P}^E(k) \hat{P}^{E E}(k) \right\} - \langle \hat{P}^E(k) \rangle \langle \hat{P}^{E E}(k) \rangle,
\]

and \(\beta(k)\) is defined as \(\langle \hat{P}(k) \rangle = \langle \hat{P}^{E E}(k) \rangle\). Throughout this paper, we adopt \(k_{\text{min}} = 0.04 \, h \, \text{Mpc}^{-1}\) for the minimum wavenumber and \(\Delta n_k = 0.215\) for the width of the \(k\)-bin in the \(S/N\) calculation. The covariance can be generally broken down into three contributions (Takada & Hu 2013): the Gaussian (G) covariance, the connected non-Gaussian (cNG) covariance, and the super-sample covariance (SSC), respectively. The covariance contribution to the IA power spectrum has not been studied. For the Gaussian field, the covariance has only the Gaussian contribution. The non-Gaussian covariances (cNG plus SSC) arise from the nonlinear mode coupling, more specifically the four-point correlation function (trispectrum) of the fields.

To accurately estimate the covariance matrices of the halo and IA power spectra, we use a set of the simulation realizations following the method in Li et al. (2014). We use a suite of 20 simulations in Nishimichi et al. (2019) each of which has a \(1 \, h^{-1} \, \text{Gpc}\) box size. We subdivide each box into 64 subvolumes of size \(250 \, h^{-1} \, \text{Mpc}\) each. Thus we have \(N_{\text{sub}} = 20 \times 64 = 1280\) subboxes in total. We measure the power spectrum, \(P_{XY}\), from each of the subboxes, and then take the standard estimator to obtain the covariance of the sub-volume power spectra:

\[
C_{\ell \ell}(k) \equiv \frac{1}{N_r - 1} \sum_{a=1}^{N_r} \left( \hat{P}_{XY,a}(k) - \langle \hat{P}_{XY,a}(k) \rangle \right) \times \left( \hat{P}_{XY,a}(k) - \langle \hat{P}_{XY,a}(k) \rangle \right),
\]

where \(N_r\) is the number of subvolume realizations, i.e., \(N_r = 1280\) in our case. Note that we do not include the correction factor in Hartlap et al. (2007), as it is only 2% effect in the covariance given a sufficient number of the realizations. The covariance estimated in this way includes the contribution of the SSC covariance, and therefore serves as an estimator of the total covariance given in Eq. (26).

In the following, we scale the covariance by a factor of \((250 \, h^{-1} \, \text{Mpc})^2/1000 \, h^{-1} \, \text{Mpc}^3\) to approximately obtain the covariance for the volume of \(1 \, h^{-1} \, \text{Gpc}^3\), a typical volume of ongoing galaxy surveys such as the SDSS BOSS survey. Due to violation of the periodic boundary conditions in the subvolume, the estimated power spectrum is biased by the window function at low \(k\) bins. We corrected for this bias by multiplying the estimated power spectrum by a factor of \(P(k_i)/P_{\text{sub}}(k_i)\) at each \(k\) bin, where \(P(k_i)\) is the power spectrum estimated from the original simulations with periodic boundary conditions (see around Eq. 53 in Li et al. 2014, for the details).

Fig. 8 shows the cumulative \(S/N\) for the halo power spectrum \(\langle P_{\text{hh}}(k) \rangle\), the monopole and quadrupole moments of cross-power spectrum of halo and E mode \(\langle P_{EE}(k)\rangle\), and the monopole of E-mode auto spectrum \(\langle P_{EE}(k)\rangle\) as a function of the maximum wavenumber \(k_{\text{max}}\). We show the results at \(z = 0.484\) and 0 in the left and right panels, respectively, and here we consider the halo sample with \(M_{\text{vir}} = 10^{12-12.5} \, h^{-1} M_{\odot}\). First, the \(S/N\) values for all the spectra are saturated at \(k \gtrsim 0.4 \, h \, \text{Mpc}^{-1}\) because the shot noise or shape noise is dominated in the covariance. Second, the \(S/N\) value for the monopole moment of \(P_{EE}\) can be greater than 200 at \(k_{\text{max}} \gtrsim 0.3 \, h \, \text{Mpc}^{-1}\) for a survey volume of \(1 \, h^{-1} \, \text{Gpc}^3\), and is about 60% of that for the density power spectrum for the same halo sample, \(P_{\text{hh}}\). This is not so bad, and this results imply that we can measure \(P_{\text{hh}}\) from the same galaxy survey in addition to \(P_{\text{hh}}\). If the galaxy shapes have a misalignment with the halo shape, the \(S/N\) for the galaxy IA spectrum would be smaller than shown in this plot. Comparing the left and right panels manifest that the \(S/N\) values are higher for higher redshifts, for a halo sample with a fixed mass threshold.

How important are the connected non-Gaussian covariance and the super-sample covariance important for the results in Fig. 8? In the following we address this question. First, we can analytically estimate the Gaussian covariance \(C^G\) and then estimate the cumulative \(S/N\) for the Gaussian case, which gives a maximum information content of the \(S/N\) value we could extract from the observed cosmological field. Once the power spectra of “X” and “Y” fields \((X,Y = \delta, h, E)\) are given, the Gaussian covariance matrix is given, as shown in Guzik et al. (2010) (also see Kobayashi et al. 2020), by

\[
C^G_{\ell \ell}(k) = \frac{\delta_{ij}}{N_{\text{mode}}(k)} (2\ell + 1)(2\ell' + 1) \int_{-1}^{1} dp L_{\ell}(p) L_{\ell'}(p) \times \left[ \hat{P}_{XY}^2(k, \mu) + \hat{P}_{XX}(k, \mu) \hat{P}_{YY}(k, \mu) \right],
\]

where \(N_{\text{mode}}(k)\) is the number of Fourier modes that are used for the power spectrum estimation at the \(\ell\)-th \(k\) bin.

\[6\] Exactly speaking, the SSC covariance does not scale with a survey volume as \(1/V\), and more rapidly decreases than the scaling. However, the relative decrease compared to \(1/V\) is not a strong function of \(V\) (a very slowly-varying function of \(V\)) as shown in Fig. 1 of Takada & Hu (2013). The \(S/N\) value for the case including the SSC contribution might be changed by 5–10%, but the discussion here is qualitatively valid.
with width $\Delta k$. For a mode satisfying $k_i \gg 2\pi/L$, $N_{\text{mode}} (k_i) \simeq 4\pi k_i^2 \Delta k / (2\pi / L)^3$, where $L$ is the size of survey volume (the side length of simulation box in our case). The Gaussian covariance matrix is diagonal, meaning no correlation between different $k$ bins. Also note that the auto power spectra of $P_{XX}$ and $P_{YY}$ include the shot noise or the shape noise contribution.

Furthermore, to study the impact of the connected non-Gaussian covariance ($C_{cNG}$), we use a different set of simulations; we run a set of 1000 small-box simulations of $250 \ h^{-1} \ Mpc$ size, where we employ $512^3$ particles to keep the same particle/force resolution as in the fiducial simulations, but employ the periodic boundary conditions. Then, we measure the power spectrum from each small-box realization, and then estimate the covariance matrix similarly to Eq. (27). The covariance matrix estimated from the small-box simulations does not include the SSC contribution, but does includes the contributions of $C_G$ and $C_{cNG}$ in Eq. (26).

Fig. 9 shows the $S/N$ values of $P_{hE}$ obtained by using the full covariance matrix, the Gaussian covariance matrix ($C_G$) alone, and the covariance matrix without the super-sample covariance contribution ($C_G + C_{cNG}$), in the calculation of Eq. (25). First, all the results fairly well agree with each other up to $k_{\text{max}} \approx 0.2 \ h \ Mpc^{-1}$, meaning that the Gaussian covariance is a good approximation up to this wavenumber. Second, comparing the gray and red points tells us the connected non-Gaussian covariance is significant and reduces the $S/N$ value by about 10, 20 and 30% at $k_{\text{max}} \approx 0.3$.
The cumulative S/N for the cross-power spectrum for the projected fields of halos and $E$ mode, $P_{E|E}^{2D}$, compared to that for the monopole moment of the 3D power spectrum $P_{E|E}^{3D}$ in Fig. 8. To define the projected fields, we consider the redshift slice centered at $z = 0.484$ and with radial width of 250 $h^{-1}$ Mpc. To have a fair comparison between the 2D and 3D case, we assume a survey volume of $1$ ($h^{-1}$ Gpc)$^3$ for both cases, where the geometry of 2D case corresponds to (2000)$^2$ x (250) ($h^{-1}$ Mpc)$^3$. We use the 1280 subboxes to compute the covariance matrix for the 2D spectrum, and the covariance includes the full contributions including the SSC covariance (see text for details).

0.5 and 0.8 $h$ Mpc$^{-1}$, respectively. Third, comparing the red and blue points, we can find that the SSC further reduces the cumulative S/N value by up to 20% at $k_{max} \geq 0.2$ $h$ Mpc$^{-1}$, meaning that the SSC gives a significant contribution to the total covariance at the nonlinear scales. The 20% loss corresponds to about 40% smaller survey volume as S/N scales roughly with the volume as $S/N \propto V^{1/2}$. The relative importance of SSC to other covariance terms looks similar to the case of weak lensing covariance (Sato et al. 2009; Takada & Jain 2009; Takada & Hu 2013). In other words, the SSC term needs to be taken into account if one properly uses the IA power spectrum for cosmology. To further study the SSC effect, the separation simulation technique using anisotropic expansion in the local background would be useful (St"{u}cker et al. 2020; Masaki et al. 2020).

### 4.5 2D vs 3D IA power spectrum

We have so far assumed that both three-dimensional positions and shapes of halos are available. This is the case that imaging and spectroscopic galaxy surveys for the same patch of the sky are available for the IA power spectrum measurements. With the advent of deep wide-area multi-band imaging surveys such as the Subaru HSC survey (Aihara et al. 2018), the Kilo-Degree survey (KiDS; Kuijken et al. 2015), the Dark Energy Survey (DES; Abbott et al. 2018; Becker et al. 2016), the Rubin Observatory’s Legacy Survey of Space and Time (LSST; LSST Science Collaboration et al. 2009), Euclid (Laureijs et al. 2011) and WFIRST (Spergel et al. 2015), it is natural to ask whether photometric surveys can be used for the IA power spectrum measurements, where the precise radial position (or distance) of individual halos (galaxies) is not available. To address this question, in this section we investigate how uncertainties in the galaxy redshifts affect our results. Here we define the projected shear field as

$$\gamma_{ij}^{2D}(x_\perp) \equiv \int dx_3 \ p(x_3) \gamma_{ij}(x_\perp, x_3),$$

where $p(x_3)$ is the radial selection function satisfying the normalization condition, $\int_0^{\chi_{max}} dx_3 \ p(x_3) = 1$. We employ a simple radial selection function given by $p(x_3) = 1/\Delta \chi$ for $\chi - \Delta \chi/2 \leq x_3 \leq \chi + \Delta \chi/2$, and otherwise $p(x_3) = 0$, where $\chi$ is the mean comoving distance to the survey slice (survey volume) and $\Delta \chi$ is the width of the redshift slice. We define $E/B$ modes similarly to Eqs. (10) and (11) because the shear field is defined in the two-dimensional plane perpendicular to the line-of-sight direction. The power spectrum of the projected field, e.g., the cross-power spectrum of the projected halo and $E$-mode fields is given by

$$\left\langle \gamma_{ij}^{2D}(k_\perp) \delta^{2D}(k_\perp') \right\rangle \equiv P_{E|E|k_\perp}(2\pi)^2 \delta^2_{2D}(k_\perp + k_\perp'),$$

where $\delta^2_{2D}(k)$ is the two-dimensional Dirac function. As can be found in Takahashi et al. (2019) (see Eq. 29 in their paper), the 2D power spectrum is related to the monopole moment of the 3D power spectrum as

$$P_{E|E|k_\perp}(k_\perp) \approx \frac{1}{\Delta \chi} P_{E|E}(k = k_\perp; z = \bar{z}).$$

Here we used the notation “$\approx$” because the above equation is exact if we can ignore time evolutions of the fields within the redshift slice we consider (under the distant observer approximation). The prefactor, $1/\Delta \chi$, in the above equation accounts for the fact that the fluctuation fields are diluted by the radial projection. Here we consider the projected wavenumber $k_\perp$ for comparison purpose with the 3D power spectrum, and the 2D power spectrum is related to the angular power spectrum if the projected field is defined on the celestial sphere, via $C_{E|E}(\ell) = (1/\bar{\chi}^2) P_{E|E}(k_\perp = \ell/\bar{\chi})$. Hence the following results for the 2D power spectrum are equivalent to what we have for the angular power spectrum.

To have a quantitative comparison of the information contents in the 3D and 2D IA power spectra, we consider the following specifications for a hypothetical imaging survey. We consider the mean redshift for $z = 0.484$, corresponding to $\bar{\chi} = 1278$ $h^{-1}$ Mpc for the Planck cosmology, and a redshift slice with radial width $\Delta \chi = 250$ $h^{-1}$ Mpc around $\bar{\chi}$. Recalling the relation $\Delta \chi \approx \Delta z/H(\bar{\chi})$, the radial width corresponds to the redshift width $\Delta z/(1+z) \approx 0.074$. Although we here consider a top-hat selection around $\bar{\chi}$ for simplicity, the radial selection roughly corresponds to a photo-$z$ accuracy of $\sigma_{\alpha} \sim 0.04$ on individual galaxies, if we assume that the radial selection corresponds to the $2\sigma$ width of photometric errors. This is comparable to or slightly better than the typical photo-$z$ accuracy for red galaxies as found in the ongoing imaging surveys such as the Subaru HSC survey (Tanaka et al. 2018). As we did for Fig. 8, we divide each simulation of $1$ ($h^{-1}$ Gpc)$^3$ into 64 subboxes each of which has a size of 250 $h^{-1}$ Mpc on a side. Then we first project the halo and shear fields along the $x^2$-axis to define the projected fields, and compute the 2D power spectrum from each subbox. We then compute the covariance from the 1280 subboxes. To have a fair comparison, we scale the covariance to

![Figure 10](image-url)
that for a volume of $1 \left( h^{-1} \text{Gpc}^3 \right)$, corresponding to a geometry of $1 \left( h^{-1} \text{Gpc}^3 \right) = (2000)^3 \times (250) \left( h^{-1} \text{Mpc}^3 \right)$, where $250 h^{-1} \text{Mpc}$ is the radial width. The covariance matrix estimated in this way includes all the contributions including the SSC covariance (see also Takahashi et al. 2019).

In Fig. 10 we compare the cumulative $S/N$ values for the 2D and 3D cross power spectra of the halo density field and $E$ mode. The 2D power spectrum has about only a halved information of the 3D spectrum due to the number of available Fourier modes at a certain $k$-bin in the 2D Fourier space compared to the 3D case. Thus a spectroscopic survey is advantageous to explore the IA signals. In order to explore the full IA information at the level of two-point statistics, we need both imaging and spectroscopic surveys for the same region of the sky. As we describe above, the $S/N$ value for the angular IA power spectrum is the same as that of 2D spectrum in Fig. 10.

### 4.6 Dependences of the IA power spectra on cosmological parameters

How does the IA power spectrum varies with cosmological parameters? To address this question, we study how the IA power spectrum depends on the two cosmological parameters, $S_8$ and $\Omega_m$. Here $S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$ is a parameter to characterize the clumpiness of the universe today, and is the primary parameter to which weak lensing or cosmic shear cosmology is the most sensitive (Hikage et al. 2019). Since the IA effect is one of the most important, physical systematic effects in cosmic shear cosmology, we study how the IA power spectra depend on these parameters. To do this, we run a set of $N$-body simulations where either of $S_8$ or $\Omega_m$ is shifted from their fiducial value of Planck cosmology by $\pm 5\%$, but other parameters are kept to their fiducial values. Note that, when we vary $S_8$ with $\Omega_m$ being fixed to its Planck value, we vary $\sigma_8$ alone by an amount corresponding to $\pm 5\%$ change in $S_8$. We also use the same initial seeds for one particular realization of the Planck cosmology simulations in order to reduce scatters due to the sample variance. Then we compute the fractional variations in the IA power spectra, computed as

$$
\frac{\partial \ln P_{XY}}{\partial \ln p_{\text{cosmo}}} = \frac{P_{XY}[1 + \epsilon]p_{\text{cosmo}} - P_{XY}[1 - \epsilon]p_{\text{cosmo}}}{2\epsilon P_{XY}[p_{\text{cosmo}}]},
$$

(32)

where $p_{\text{cosmo}} = S_8$ or $\Omega_m$, $\epsilon = 0.05$, and $X,Y$ are either of halo ($h$) and/or the IA $E$-mode ($E$), respectively. The fractional differences quantify scaling relations of the IA power spectrum with the cosmological parameters in the vicinity of the Planck cosmology in two-dimensional parameter space of ($S_8, \Omega_m$), given by

$$
P_{EE, P_{EE}} \propto \sigma_h^2 \Omega_{m}^2.
$$

(33)

Fig. 11 shows the results. Although the fractional changes look noisy at small $k$ bins, the IA power spectra display characteristic scale-dependent responses to these parameters. The changes get flattened at larger $k$ bins, meaning that changes in these parameters cause an almost scale-independent change in the IA power spectra, just like an overall factor. The value of each curve in $y$-axis roughly gives the scaling indices $p$ or $q$ in Eq. (33) at each scale of $k$ bins. For the impact of IA effect on the cosmic shear power spectrum for cosmological models around the Planck cosmology, one needs to further take into account the dependence of the prefactor in Eq. (9), $\Omega_m/D(z)$, on $\Omega_m$.

### 4.7 IA power spectra in redshift space

We have so far considered the real- or configuration-space fields. However, actual observables for a spectroscopic survey are not real-space fields, but rather defined in redshift space. Redshift-space distortions (RSD) due to peculiar velocities of galaxies (halos in our case) (Kaiser 1987) cause the observed positions of halos to be modulated compared to those in real space.

Compared to the standard RSD effect on halos’ positions, halo shapes are not affected by the RSD effect (Singh et al. 2015; Okumura & Taruya 2020). That is, the shear field in redshift space is invariant under a mapping between real and redshift space:

$$
\gamma^R(x) = \gamma^E(x),
$$

(34)

where quantities with superscripts “$S$” and “$R$” denote the quantities in redshift and real space, respectively, the real- and redshift-space mapping is given by $x_1 = x_1, x_2 = x_2, x_3 = x_3 + v_3/aH$, and $v_3$ is the line-of-sight component of peculiar velocity. As we discussed around Eq. (17), however, the shear field estimated from a survey is sampled only at halo’s positions, and is affected by the RSD effect on the density field of halos as

$$
\tilde{\gamma}^R(x) = \left[ 1 + \delta_3^h(x) \right] \gamma^E(x).
$$

(35)

On large scales in the linear regime, the redshift-space density fluctuation field of halos is expressed as

$$
\delta_3^h(k) = (1 + \beta\mu^2)\delta_3^h(k),
$$

(36)

where $\beta$ is the RSD distortion parameter, defined as $\beta = (1/3)\ln D/\ln a$. The multiplicative factor $1 + \beta\mu^2 > 0$ leads to a boost in the amplitude of redshift-space density fluctuation field compared to the real-space density field on large scales (small $k$). Eq. (34) tells that the RSD effect on the shear field arises from the nonlinear term of fluctuation fields, $\delta^3$. Hence the observed shear field on large scales in the linear regime, where $|\delta h| < 1$, is equivalent to the real-space shear field, i.e., free of the RSD effect. However, on smaller scales the observed shear field is affected by the RSD effect, and receives additional $\mu$-modulations, giving characteristic anisotropic patterns in the observed IA shear field (see Singh et al. 2015; Okumura & Taruya 2020, for the study on the IA correlation functions in configuration space).

In Fig. 12, we study the multipole moments of IA power spectra in redshift space, compared to the real-space IA spectra. To compute the RSD effect on the halo distribution in simulations, we adopt the bulk motion of each halo that is estimated from the average of velocities of $N$-body particles in a core region of each halo (see Kobayashi et al. 2020, for details). As we described, the monopole moment of the redshift-space auto-power spectrum of $E$ mode, $P_{EE}^{\text{red}}$, is the same as that of the real-space power spectrum on large scales (small $k$) as expected. On the other hand, the monopole moment of the redshift-space cross spectrum of halo and $E$ fields, $P_{hE}^{\text{red}}$, receives a boost in the amplitude due to the RSD effect, similarly to the effect on the halo...
The IA shear amplitude is about a few per cent at $z = 0.484$, and the monopole moments of $P_{hE}$ and $P_{EE}$. For comparison, we also show the dependences for $P_{hh}$.

5 DISCUSSION AND CONCLUSIONS

In this work we have developed a novel method to measure the three-dimensional IA power spectra from shapes of halos (as a proxy of galaxy shapes) using a suite of high-resolution $N$-body simulations for the Planck cosmology. Our findings are summarized as follows:

- The Fourier-space analysis of halo shapes allows for a straightforward decomposition of the halo shapes into the $E$ and $B$ modes, as in the CMB polarization field and the cosmic shear field.

- The IA power spectra (the cross spectra of the halo density field and the IA $E$-mode and the auto spectrum of the $E$-mode) display non-vanishing amplitudes on all scales from the linear to nonlinear regimes. This means that the primordial fluctuations and gravity in large-scale structure induce a correlation between halo shapes and the matter distribution and between the shapes of different halos on scales much greater than a size of halos (scales of physics inherent in halo formation, a few Mpc at most). The IA power spectra on large scales are related to the matter power spectrum, with a constant coefficient, as in the linear bias relation of the power spectrum of halo intrinsic alignments in simulations (Fig. 10). Hence the IA power spectrum can be used to probe the underlying matter power spectrum, very much like what is done using the galaxy power spectrum.

- The negative sign of the cross power spectrum of halo density and $E$ mode means that the major axis of halo shapes tend to be statistically aligned with the minor axis of the tidal field, i.e., the direction of mass accretion onto the halos, which is consistent with the previous simulation results.

- The IA power spectrum for more massive halos have the greater amplitudes (Fig. 6). If we consider the halo sample in a fixed mass bin, the large-scale IA coefficient ($A_{IA}$) asymptotically approaches to a constant value at higher redshift. This is as expected for the primordial tidal alignment model (Hirata & Seljak 2004), implying that the halos shapes of a fixed mass scale at higher redshift retain the information on the primordial tidal field. At lower redshifts, the $A_{IA}$ amplitude decreases, probably reflecting the fact that the halo shapes lose the initial memory to some extent due to the mergers or mass accretion in the nonlinear regime.

- The IA power spectra display BAO features as in the density power spectrum, confirming the similar finding for the real-space IA correlation function (Okumura et al. 2019). In addition, the cross power spectrum of halo density and the IA $E$-mode shows a weaker boost in the amplitude at nonlinear scales compared to the halo density power spectrum, due to the spin-2 nature of the IA field.

- The cumulative signal-to-noise ratio ($S/N$) for a measurement of the cross power spectrum of halo density and the IA $E$-mode is about 60% of that of the halo density power spectrum (Fig. 8). The super-sample covariance arising from the long-wavelength fluctuations comparable or greater than a size of survey volume gives a significant contribution to the total covariance as in the covariance of cosmic shear power spectrum (Fig. 9). The two-dimensional power spectra of the projected IA field, measured from an imaging survey, suffers from about factor of 2 loss in the information content of the 3D IA power spectrum (Fig. 10).

- The IA power spectra in redshift space, the direct ob-
Figure 12. Comparison between the monopole (left-column panels), quadrupole (middle-column) and hexadecapole (right-column) moments in real and redshift space, for $P_{hh}$, $P_{hE}$ and $P_{EE}$. We show the power spectra for the halo sample with $M_{\text{vir}} = 10^{12.5} \, h^{-1} M_\odot$ at $z = 0.484$.

As we have shown, the IA power spectra can be powerful tools to extract the information on the matter power spectrum, properties of the primordial matter (tidal) perturbations and the cosmological parameters. Thus it would be interesting to explore how the IA power spectrum improves the power to constrain cosmological parameters, when combined with the standard density power spectrum. This offers additional opportunities that can be attained for imaging and spectroscopic surveys if the two surveys observe the same patch of the sky, where the imaging survey is needed to measure shapes of galaxies and the spectroscopic survey is needed to know the three-dimensional spatial position of the galaxies. As we showed, having spectroscopic redshifts leads to a significant boost in the $S/N$ compared to an imaging survey alone.

In particular, the cross-power spectrum of the galaxy density field and galaxy shapes looks very promising. As we showed, the IA shear has the similar amplitudes (a few percent in ellipticities) to the cosmic shear, i.e., weak lensing shear due to large-scale structure in the foreground.
This would not be surprising because both the effects arise from the gravitational field. Even if both imaging and spectroscopic surveys are available, the auto-power spectra of galaxy shapes would suffer from the cosmic shear contamination due to foreground large-scale structures; we cannot distinguish the IA effect and the cosmic shear from the measured power spectra. On the other hand, this is not the case for the cross spectrum as long as spectroscopic surveys are available, because the IA cross spectrum we are interested in are on scales up to a few $100 h^{-1}$ Mpc at most, arising from pairs of galaxies separated by such scales (one is for shapes and the other is for the positions) in the common large-scale structure, and the cosmic shear on galaxy shapes by other galaxy would be negligible (recall that cosmic shear builds up by large-scale structures over Gpc scales along the line-of-sight direction). Since galaxy shapes at higher redshifts might retain more information on the primordial tidal fields (higher $A_{IA}$ coefficients), imaging and spectroscopic surveys for higher redshifts might be more powerful tools of cosmology from joint measurements of the galaxy density and IA power spectra in redshift space. Such high-redshift galaxy surveys are, for example, the Subaru HSC and PFS surveys (Takada et al. 2014). These are all interesting directions, and are our future work.

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APPENDIX A: DENSITY-WEIGHTED FIELD

In this section we describe how to make grid assignments of the halo density and shape fields measured in $N$-body simulation realizations. Throughout this section, we omit the subscripts $\{+,-,h\}$ and write $\epsilon_n(x)\eta_n(x)\delta_R(x)$ as $\epsilon(x)\eta(x)\delta(x)$ for notational simplicity unless specifically mentioned.

First, the halo number density field $\hat{n}(x)$ can be formally written as

$$\hat{n}(x) = \sum_{i=1}^{N} \delta_D^3(x - x_i).$$  \hfill (A1)

where $N$ is the total number of halos and $x_i$ is the position of the $i$-th halo. Here the mean halo number density is

$$\bar{\hat{n}} = \frac{\int_V d^3x \, \hat{n}(x)}{\int_V d^3x} = \frac{N}{V}.$$  \hfill (A2)

By using an arbitrary weighting function $W(x)$, we can discretize this field, i.e., evaluate it at the grid point,

$$x_{\text{grid}} = m L_{\text{grid}} \ (m \in \mathbb{Z}^3),$$  \hfill (A3)

as

$$\hat{n}(x_{\text{grid}}) = \int_V d^3x \, W(x_{\text{grid}} - x) \hat{n}(x).$$  \hfill (A4)

where $W(x)$ satisfies the normalization condition

$$\int_V d^3xW(x) = 1.$$  

For example, the NGP assignment is given as

$$W_{\text{NGP}}(x) = \begin{cases} 1/L_{\text{grid}}^3 & \text{if } |x_1|, |x_2|, |x_3| < L_{\text{grid}}/2, \\ 0 & \text{otherwise} \end{cases}$$

and then Eq. (A4) becomes

$$\hat{n}(x_{\text{grid}}) = \frac{1}{V_{\text{grid}}} \sum_{i=1}^{N_{h_{\text{grid}}}} n_h L_{\text{grid}}^3.$$  \hfill (A5)

where $N_{h_{\text{grid}}}$ is the number of halos in a grid. Therefore the halo number density contrast is calculated by

$$\hat{\delta}(x_{\text{grid}}) = \frac{\hat{n}(x_{\text{grid}}) - \bar{\hat{n}}}{\bar{\hat{n}}}.$$  \hfill (A7)

Next we consider the ellipticity field. We have a set of ellipticities of dark matter halos $\{\epsilon_i|i = 1, \cdots, N\}$ from a simulation realization and we assume that the ellipticity field is sampled at their position, i.e., $\epsilon_i = \epsilon(x_i)$. Here we define the discretized ellipticity field in analogy with the density field (Eq. A6) as

$$\hat{\epsilon}(x_{\text{grid}}) = \frac{1}{V_{\text{grid}}} \sum_{i=1}^{N_{h_{\text{grid}}}} \epsilon_i = \frac{1}{V_{\text{grid}}} \sum_{i=1}^{N_{h_{\text{grid}}}} \epsilon(x_i).$$  \hfill (A8)

$$= \int_V d^3x \, W_{\text{NGP}}(x_{\text{grid}} - x) \epsilon(x) \sum_{i=1}^{N} \delta_D^3(x - x_i)$$

$$= \int_V d^3x \, W_{\text{NGP}}(x_{\text{grid}} - x) \epsilon(x) \hat{\delta}(x).$$  \hfill (A9)

Therefore $\hat{\epsilon}(x) = \epsilon(x)\hat{\delta}(x)$. Finally, by redefining $\hat{\epsilon}(x) \rightarrow \hat{\epsilon}(x)/\bar{\hat{\delta}}$, we obtain $\hat{\epsilon}(x) = (1 + \hat{\delta}(x))\epsilon(x)$. Note that we use the cloud-in-cells (CIC) assignment kernel, a higher order
scheme than NGP, in the analyses presented in the main text. This can be achieved simply by replacing $W_{\text{NGP}}$ with $W_{\text{CIC}}$ in the above expressions.

APPENDIX B: SHAPE NOISE

Here we discuss the shape noise. The measured auto-power spectra of halo shape $E, B$ fields, $P_{XX}$ ($X = (E, B)$), have the shape noise contribution that arises due to a finite number sampling of the shape fields at the halo positions. Unlike the cosmic shear field, there are two contributions. One is the standard Poisson shot noise term that arises when shapes of different halos are completely uncorrelated, and corresponds to the shape noise term in the cosmic shear power spectrum (Hikage et al. 2019). The other is from the non-linear evolution of IA (Blazek et al. 2019). The IA power spectrum itself arises from physical correlation of halo shapes and halo distribution in the same large-scale structure, and this non-Poisson shot noise term contributes the total shot noise term. Taking advantage of the spin-2 field of halo shape field, we can disentangle the two contributions. This is also the case for actual observations, and is not the case for the density power spectrum. One way to estimate the Poisson shot noise is as follows; first, rotate orientation of individual halo ellipticity with random angle, measure the power spectrum in the same way to actual measurements, repeat the random-orientation measurements many times, and then estimate the variance from the many realizations. This erases correlated IA effects between different halos keeping the distribution of halos (keeping the clustering of halos). In an actual observation, this method can automatically take into account the effects of masks and boundary of survey footprints (Shirasaki et al. 2019). We perform the random-orientation measurements 10000 times for each of 20 each simulation realization, this method can automatically take into account the responsivity $R_{\text{in}}$ from the distribution of halo ellipticities in Fig. 1 taking into account the responsivity $R$. For comparison, we also show $P_{BB}$ without subtracting the Poisson shot noise term. Both the power spectra agree with the Poisson shot noise term at sufficiently large $k$ as expected.

Interestingly the $B$-mode power spectrum shows a clear deviation from the Poisson shot noise. The extra contribution is considered as the “renormalized” term arising from the $k \to 0$ limit of higher-order terms in the $B$-mode power spectrum (McDonald & Roy 2009) (also see Blazek et al. 2019). In particular, it converges to a certain $k$-independent constant in $k \to 0$ limit. The difference between the constant values at the limits of $k \to \infty$ and $k \to 0$ could be recognized as the difference between the (bare) number density $\bar{n}_h$ and the effective number density $\bar{n}_{\text{eff}}$ which is defined by

$$\bar{n}_{\text{eff}}^{-1} = \frac{1}{\bar{n}_h^2} \lim_{k \to 0} P^{(0)}_{EE/BB}(k).$$

(B1)

In this work we estimate $\bar{n}_{\text{eff}}$ for our halo samples from simulation by minimizing the $\chi^2$ statistics:

$$\chi^2 \equiv \sum_{k, \ell < 0.05 h\text{ Mpc}^{-1}} \frac{[P_{BB}^{(0)}(k) - \sigma_{BB}^2/\bar{n}_{\text{eff}}]^2}{\sigma_{BB}^2},$$

(B2)

where $\sigma_{BB}^2$ is the variance of $P_{BB}^{(0)}(k)$ of 20 simulation realizations. We can safely estimate the constant offset by using $k$ modes in the sufficiently linear regime. Once again, we should note that the discrepancy from the Poisson shot noise can be estimated from actual data, by comparing the Poisson shot noise, estimated by the above method, and the measured $B$-mode power spectrum.

In Fig. B2 we show the relative difference of the number density, $\Delta n \bar{n}_h \equiv \bar{n}_{\text{eff}}/\bar{n}_h - 1$. The non-Poisson shot noise compared to the Poisson shot noise is roughly 5–10% for all the halo samples we consider.

APPENDIX C: A DEPENDENCE ON THE DEFINITIONS OF INERTIA TENSOR

In this section, we discuss how our results are sensitive to how to define the inertia tensor of individual halo shapes. To characterize the halo shape, we adopt the reduced inertia tensor using the radial weighting function $w_p(r_p) = 1/r_p^2$ (see Osato et al. 2018) in this paper, i.e.,

$$I_{ij} = \sum_p \frac{\Delta x_i^p \Delta x_j^p}{r_p^2}. \quad (C1)$$

The triaxial radius of each particle, $r_p$, is defined by using the following iterative scheme. First, we use member particles within the virial radius $r_{\text{vir}}$ in each halo and compute the above inertia tensor $I_{ij}$, where $r_p^0 = \mid x_p - x_{\text{vir}} \mid$, i.e., spherical weighting. Second, we diagonalize $I_{ij}$ and obtain the eigenvectors $e_a, e_b, e_c$, corresponding to the three principal axes $a, b, c$ ($a > b > c$). Then we can define the triaxial radius for each particle as

$$r_p^{(1)} = \sqrt{(x_p \cdot e_a)^2 + (x_p \cdot q e_b/s)^2 + (x_p \cdot q e_c)^2}, \quad (C2)$$

where $q = c/a, s = b/a$ are the axis ratios. Third, by using member particles which satisfy $r_p^{(1)} < r_{\text{vir}}$, we redefine the inertia tensor replacing $r_p^0$ with $r_p^{(1)}$:

$$I_{ij}^{(1)} = \sum_p \frac{\Delta x_i^p \Delta x_j^p}{r_p^{(1)}}. \quad (C3)$$

We perform the second and third step calculations iteratively until $q$ and $s$ converge to within 1% precision and we finally use the converged inertia tensor $I_{ij}$ to define the ellipticities and measure the IA power spectra.

In Fig. C1 we compare the “iterative” ellipticity of individual halos using the converged inertia tensor from the iteration, $I_{ij}$, with the “non-iterative” ellipticity using the first-step inertia tensor, $I_{ij}^{(0)}$. The non-iterative ellipticity tends to show more round shape due to the spherical weighting; the rms ellipticities per component are $(\epsilon_{E/BB}^{\text{iter}}, \epsilon_{E/BB}^{\text{non-iter}}) = (0.33, 0.16)$ for the halo sample with $M_{\text{vir}} = 10^{12.5} h^{-1} M_\odot$ at $z = 0.484$. That is, the rms ellipticities are a factor of
2 different depending on the halo shape definition. We find the similar results for halo samples of other mass scales. We show the overall IA signal and the large-scale amplitude, $A_{\perp}h$, in Fig. C2 for both cases. The figure shows that the overall amplitudes of the IA power spectra are changed by the different definitions of halo shapes, but the shapes of the power spectra remain unchanged. That is, the change in the halo shape definition leads to a change in the overall amplitudes of the IA power spectra. The dependences of $A_{\perp}h$ on masses and redshifts are also changed. Nevertheless, the qualitative behavior remains unchanged.

In Figure C3, we compare the ratios of the cross IA spectrum, $P^{(c)}_{BB}$, to the statistical error at each $k$ bin, where we estimate the error from the 20 realizations of $1 \ (h^{-1} \text{Gpc})^3$ volume. This gives an estimate of the signal-to-noise ratio of the band power measurement at each $k$ bin. It is clear that the different definition of halo shapes do not change the signal-to-noise ratio. That is, even if the intrinsic rms ellipticities of halo shapes are changed by a factor of 2, the signal-to-noise ratio for the IA power spectra is almost unchanged. Hence the main results shown in the main text are not changed, irrespective of the halo shape definition, although we should keep in mind that the $A_{\perp}h$ values are sensitive to the halo shape definition. Hence the $A_{\perp}h$ amplitude for a given halo sample should be considered as a nuisance parameter.
Figure C2. The dark-color data points are the same as those in Fig. 2 (left panel) and Fig. 6 (right), respectively. The lighter-color points are the results for the different definition of halo shapes, the non-iterative case in Fig. C2. The lower panel in the right panel shows the ratio of $A_{\text{IA}}$ of the non-iterative case to that of the iterative case.

Figure C3. Comparison of the signal-to-noise ratio for the band power measurement of the IA cross power spectrum at each $k$-bin, for the different halo shape definitions.