Semileptonic charm decay as a test for the spectator model in the $B_c$-meson

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Abstract

The $b$ and the $c$ quark compete with each other in decays of the $B_c$-meson. We emphasize the need of obtaining reliable signals of $c \to s$ in order to assess the merits of the spectator approximation in calculating the relative strengths of the two types of decays. This, we argue, can be done by considering the decay $B_c \to B_s(B_s^*)l^-\bar{\nu}_l$, followed by semileptonic decays of the $B_s$. We suggest looking for like-sign dileptons together with a $D_s$ at the BTeV or LHC-B experiments, and show that such signals can be made background-free by suitable event selection.

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The recent discovery of the $B_c$-meson, consisting of a $b$ and a $c$ quark, opens up rather interesting possibilities whose test may confront one with some ill-understood aspects of heavy flavour physics [1]. So far, the Collider Detector at Fermilab (CDF) have confirmed observations of the $B_c$ [2]. In addition, the ALEPH, OPAL and DELPHI collaborations at the Large Electron Positron (LEP) collider at CERN have reported candidates for a similar designation [3–5]. The decay channel utilised in all these searches is one where the $b$-quark decays first into a $c$, producing a $J/\psi$, either semi-leptonically or with one or more pions. This is because the $J/\psi$ is easily identified through the decay into two leptons, and the additional leptonic and pionic tracks passing close to the decay vertex provide a viable reconstruction of the $B_c$ [6]. The best fit for the $B_c$ mass obtained from this channel also shows a fair agreement with that predicted by potential model calculations [7].

However, even a naive estimate of the lifetimes of the $b$ and the $c$ quark shows that, in the $B_c$ meson, the charm decay may take precedence over the bottom decay. This is due to the fact that, unlike $b \to c$, the decay $c \to s$ is not suppressed by any off-diagonal element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Such a suppression tends to offset the effect of the mass of the $b$-quark. The annihilation channels of $B_c$ are also important [8]. Both these types of decays have already been studied in different theoretical approaches. To be more specific, the semileptonic channels $B_c \to J/\psi l \bar{\nu}_l$ and $B_c \to B^*_s l \bar{\nu}_l$ have been studied using the spectator approximation and different types of form-factors, e.g., BSW [9], ISGW [10], ACCMM [11] etc. No definite conclusion can yet be drawn, however, regarding the preferability of a non-relativistic potential [12] to a relativistic one, when a $c$ decays into an $s$, while both of them interact with the $b$-quark all along [13]. It has also been pointed out that the massive spectator for the latter mode significantly affects the phase space that would have been otherwise available with a light spectator [14]. Summing everything up, one can say that the decay widths for the two semileptonic modes might as well be just comparable.

The above remarks show that there is still considerable scope for improving our understanding of the $B_c$-system where two heavy quarks compete with each other in the decay processes and at the same time influence the transverse motions of each other. In order to clearly establish the validity of a given theoretical picture, it is important at this stage to experimentally identify signatures of decays induced by $c \to s$. In this note, we suggest ways of extracting the corresponding final states in the semileptonic decay

$$B_c \to B^*_s l \bar{\nu}_l. \tag{1}$$

One of the reasons for concentrating on semileptonic decays is to ensure that our basic question does not get swamped by additional theoretical uncertainties concerning, for example, the legitimacy of factorisation. Thus, our numbers will depend only on the parametrization of the semileptonic decays using spectator approximation, and any observed deviation from these numbers will mean significant non-spectator physics contribution.

As will be clear from the discussion below, it is profitable for us to consider final states which are again produced through semileptonic decays of the $B_s$. A realistic testing ground for such decays will be provided by experiments dedicated to B-studies at hadronic colliders, typical examples being the BTeV and LHC-B experiments [15]. Depending on the integrated luminosity achieved ultimately, these machines can be expected to produce $10^{11} - 10^{12}$ $b\bar{b}$-pairs per year. Thus, with a fragmentation ratio of $3.8 \times 10^{-4}$ ($5.4 \times 10^{-4}$) for the $B_c$ ($B^*_c$) meson [16], approximately $10^{8} - 10^{9} B_c s$ can be available according to a rather conservative estimate. One

\footnote{$B^*_c$ decays almost entirely to $B_c$, so these two numbers just add up to give the total fragmentation ratio.}
may expect a higher yield if one includes the possibility of having a $B_c$ on one side and $B$-meson of a different type on the other \[17\].

The decay $B_c \to B_s^{(*)} l\bar{\nu}$ is followed by the $B_s$ decaying into one of its allowed channels. (The $B_s^{(*)}$ decays almost entirely into a $B_s$.) These include final states consisting of $D_s \pi$, $D_s 3\pi$ as well as the semileptonic channel $D_s^{(*)} l\nu$. In this letter, we propose to utilise the semileptonic channel alone. Despite suffering from a small branching fraction, this has an unique advantage. Because of the high rate of $B_s \to B_s^{(*)}$ oscillation, both set of final states $D_s^{(*)} l^- \pi^+$ and $D_s^{(*)} l^+ \pi^-$ are equally probable. Thus in approximately half the cases one expects like-sign dileptons together with a $K^0$ can give rise to leptons of either sign. We shall show that they can be rather easily removed by kinematic cuts.

Calculation of the rates for the exclusive semileptonic decays is straightforward in a form-factor approach. The hadronic matrix elements relevant for the decay of a generic pseudoscalar meson $P_0$ may be parametrized as

$$
\langle P(k) | q_1 \gamma_\mu (1 - \gamma_5) q_2 | P_0(p) \rangle = f_+ (p + k)_\mu + f_- q_\mu
$$

$$
\langle V(k, \epsilon) | q_1 \gamma_\mu (1 - \gamma_5) q_2 | P_0(p) \rangle = \frac{2V_{P_0V}}{m_0 + m_V} \epsilon^{\mu \nu \alpha \beta} \epsilon^\nu p^\alpha k^\beta
+ i \left[ (m_0 + m_V) A_{P_0V}^{(1)} \epsilon^*_\mu
- \epsilon^*_\cdot p \left\{ \frac{A_{P_0V}^{(2)}}{m_0 + m_V} (k + p)_\mu + \frac{2}{q^2} \left[ A_{P_0V}^{(3)} - A_{P_0V}^{(0)} \right] q_\mu \right\} \right]
$$

In eq.(2), $q \equiv p - k$ and $P$ and $V$ correspond to generic pseudoscalar and vector mesons respectively. The value of the form-factors $f_\pm$, $V$ and $A^{(i)}$ and their $q^2$-dependence can be calculated within a specific model. For the rest of our analysis, we shall use the results obtained within the Bauer-Stech-Wirbel (BSW) formalism \[18\] which we compile in Table 1. We have checked that our results are insensitive to, say, a 20% variation of the form-factors, including that arising from the uncertainty in the parameter $\omega$ in the BSW framework.

In the remainder of our analysis we shall assume that the pions and/or photons resulting from the decay of an excited state(s) to the corresponding ground state will remain untagged. Since the signal consists of a $D_s$ together with a pair of like-sign dileptons, the relevant backgrounds emanate from processes that include these particles in the final state apart from other undetected ones. The most important processes\footnote{There also exist additional channels with intermediate $D_s^{(*)}$ and/or $D^{(*)}$ states cascading into the respective ground states. Since the phase space distributions are quite akin to those above, these channels cannot be distinguished unless the accompanying photon(s) are detectable. We shall assume that this is not the case and add on such contributions to those of eqs. (3a) and (3b).} in this context are

$$
B_d^0 \to D_s D^- \to D_s K^0 l^- \nu \ , \quad \text{(3a)}
$$

and

$$
B_d^0 \to D_s D^- \to D_s K^0 l^- \nu \to D_s K^0 \pi l^- \nu \ . \quad \text{(3b)}
$$
Table 1: Form-factors relevant to the decays of eq.(1) as calculated within the BSW formalism [18]. The values are shown for $q^2 = 0$ only. The relevant $c\bar{s}$ poles are: 2.60 GeV ($f^+$), 2.11 GeV ($V_{P0}$), 2.53 GeV ($A_{P0V}^{(1,2)}$). The other form-factors do not contribute in the limit of vanishing lepton masses. All masses and $\omega$ are in GeVs. All other form-factors are obtained from Ref. [18].

The $K^0$ has a 50% probability each of being in the $K_L$ or the $K_S$ state. With the $K_S$ having a negligibly small semileptonic branching ratio, it is the $K_L$ which can decay ($BR \simeq 65\%$) into $\pi l \nu_l$ ($l = e, \mu$), $l^+$ and $l^-$ being produced with equal probability.

As $m(D_s) - m(K^{*0})$ is considerably smaller than $m(D_s) - m(K^0)$, lepton from the $D_s$ decay will have significantly different energy spectra in the two cases. Similarly, the leptons in the signal events will have an entirely different profile. We aim next to quantify this difference in a frame-independent manner.

The first kinematic distribution on which we focus our attention is $\Gamma /\Gamma /d\ell \ell$ where $m_{\ell \ell}$ is the invariant mass of the dilepton system. Note that this is a Lorentz invariant distribution and is unaffected by the extent to which the parent $B_{c,s}$ are boosted. For the signal, this distribution shows a peak in the neighbourhood of 1 GeV (see Fig. 1), and has little dependence on the form-factors. The figure clearly shows that the background peaks at a considerably lower invariant mass compared to the signals. This is because each of the two leptons in the background process is produced at a later stage of the cascade, thereby suffering a degradation that lowers the invariant mass.

It must be borne in mind though, that Fig. 1 exhibits only the normalized distributions. However, at a hadronic machine, the production rate for $B_d^0$ is nearly 3 orders of magnitude higher than that for $B_c$. Again, while the branching fractions for the backgrounds of eqs. (3a) and (3b) are $1.1 \times 10^{-3}$ and $4.7 \times 10^{-4}$ respectively [18], that for the signal chain is approximately $0.08$ $B$ where

$$B = Br(B_c \longrightarrow B_s \ell \nu) + Br(B_c \longrightarrow B_s^* \ell \nu) .$$

Using the theoretical prediction of $B \sim 0.2$ [1], it is easy to see that the background can, a priori, be much larger than the signal. However, as Fig. 2 shows, one could still separate the signal from the background by imposing a cut on the invariant mass for the dilepton pair:

$$m_{\ell \ell} > m_{\text{min}} .$$

Note that this includes the factor of half due to the fact that we consider only the like-sign dilepton case.

The invariant mass for the signal falls far short of the $J/\psi$ and hence such backgrounds can easily be eliminated without losing any of the signal.
Clearly, with a strong cut on \( m_{\ell\ell} \) most of the background can be eliminated while losing a relatively smaller fraction of the signal. For example, with a \( m_{\text{min}} \) of 0.8, 1.2 and 1.6 GeV, one retains 64.5%, 33.8% and 11.4% of the signal respectively. For the same set of cuts, the background (3a) gets reduced to 16.7%, 0.86% and 0.001% respectively, while background (3b) falls to 3.4%, 0.01% and 0% of its unrestricted value.

Although a relatively large value of \( m_{\text{min}} \) would eliminate all of the background, one needs to ensure that the events retained are sufficiently numerous for them to be considered a signal. To make a judicious choice, one thus has to consider the event rates expected. The latter is obviously given by the product \( (0.08 B N \epsilon) \), where \( N \) is the number of \( B_c \)s produced, and \( \epsilon \) is the experimental efficiency of detecting the final state. Now, \( \epsilon \) is the same for both the signal and the background, while the number of \( B_d \)s produced can be approximated\(^5\) to be 500\( N \).

At the BTeV experiment, \( N \simeq 10^8 \) even with a conservative estimate. The rate may be slightly higher at the LHC-B. The identification efficiency for each lepton is about 90%. The \( D_s \) is mostly to be recognised through the \( \phi\pi \) and \( K^*K \) channels, in which the combined branching ratio is 6%. Each of these will give rise to three tracks. Including the trigger efficiency and all kinematic cuts necessary for proper identification, the average efficiency for these three tracks being faithfully reconstructed into a \( D_s \) is approximately 2.5%. Thus, with the requirement of proper identification of the other lepton (coming from \( B_c \)-decay), the net efficiency of detection of the final states of our concern turns out to be \( \epsilon \sim 0.001 \). It should be noted that the parent \( B_c \) is sufficiently boosted, so that the lepton coming from its decay at the first step stays more or less in the same rapidity interval as the \( D_s \) decay tracks and the lepton emanating from \( B_s \)-decay.

In Table 2, we present, for a given \( N\epsilon \), the number of signal \( (S) \) and background \( (B) \) events as a function of \( m_{\text{min}} \). The significance \( S/\sqrt{B} \) (which is proportional to \( \sqrt{N\epsilon} \)) is then easy to

\(^5\)Approximately only 0.1% of the \( b \) quarks produced end up in \( B_c \)s while most of them hadronize into \( B_d \) and \( B^\pm \) with nearly equal probability.
Figure 2: The fraction of the signal and background retained on imposition of a lower bound on the invariant mass of the lepton pair. The solid line corresponds to the signal, while the short- and long-dashed lines refer to backgrounds from the decay chains (3a) and (3b) respectively.

calculate. Since the branching ratios for all other decays in both the signal and the background
cascades are rather well-known, it should be possible from the above estimates to measure $B$, the sum of the branching ratios for $B_c \rightarrow B_s^{(*)}\ell\nu$ and match it with the different theoretical predictions. It is interesting to note that, even without imposing any kinematical cuts, it would be possible to have a $5\sigma$ signal as long as $B > 0.17$.

Further distinction of the signal from the backgrounds can be achieved by considering different angular distributions. These distributions in the laboratory frame require the $B_c$ to be boosted appropriately, and one can perform a detailed simulation only by knowing the machine parameters. On the other hand, some interesting features can be observed for the distributions

| $m_{\text{min}}$ (GeV) | Signal | Background |
|------------------|--------|------------|
| $B_{\text{d}}$  | $B_{\text{c}}$ | $B_{\text{d}}$ |
| 0.0 | 8000 $B_{\text{d}}$ | 55000 $B_{\text{c}}$ | 23500 $B_{\text{d}}$ |
| 0.2 | 7820 $B_{\text{d}}$ | 49980 $B_{\text{c}}$ | 19330 $B_{\text{d}}$ |
| 0.4 | 7240 $B_{\text{d}}$ | 36610 $B_{\text{c}}$ | 10410 $B_{\text{d}}$ |
| 0.6 | 6320 $B_{\text{d}}$ | 21100 $B_{\text{c}}$ | 3640 $B_{\text{d}}$ |
| 0.8 | 5160 $B_{\text{d}}$ | 9180 $B_{\text{c}}$ | 813 $B_{\text{d}}$ |
| 1.0 | 3900 $B_{\text{d}}$ | 2780 $B_{\text{c}}$ | 98 $B_{\text{d}}$ |
| 1.2 | 2700 $B_{\text{d}}$ | 474 $B_{\text{c}}$ | 3.7 $B_{\text{d}}$ |
| 1.4 | 1770 $B_{\text{d}}$ | 28 $B_{\text{c}}$ | 0 $B_{\text{d}}$ |
| 1.6 | 910 $B_{\text{d}}$ | 0 $B_{\text{c}}$ | 0 $B_{\text{d}}$ |

Table 2: The number of signal and background events retained as a function of $m_{\text{min}}$ for $N\epsilon = 10^5$. We assume that 500 as many $B_{\text{d}s}$ are produced as $B_{\text{c}s}$. 
in the rest frame of the $B_c$.

Let us, for example, consider, for the signal process $B_c \to B_s(B^*_s)$, the distribution in the opening angle between the reconstructed $D_s$ and the slower of the two leptons (which, in most of the cases, is the one coming from the primary decay). As shown in figure 3, the distribution is a slowly falling one, as opposed to the corresponding case for the backgrounds, where a peaking for large angle is evident. This can be explained by noting that the background leptons originate from $B_d^0 \to D_s(D^*_s)D^-$ followed by cascade decay of the $D^-$. Since the momentum of the lepton in the rest frame of the $D^-$ is smaller than the momentum of the $D^-$ itself, the leptons have trajectories close to that of the $D^-$. The latter, on the other hand, moves back-to-back with respect to the $D_s$ in the $B_c$ rest frame, making it imperative for the leptons, too, to have a large opening angle against the $D_s$.

With increasing $m_{\min}$, events with more energetic leptons, or with large opening angles between them, are filtered out. For the signal events, this results in a slightly enhanced likelihood of small opening angle between the slow lepton and the $D_s$. For the background, on the other hand, an increase is $m_{\min}$ does not change this particular distribution to a perceptible extent. This can be traced to the event topology discussed in the last paragraph.
Though the distinctions suggested above are most noticeable in the rest frame of the $B_c$, they can be utilised if the latter can be reconstructed even at a statistical level. If such reconstruction is possible, the study of the angular distributions will enable one to filter out the signals even with a somewhat relaxed invariant mass cut and thus to be left with a larger number of events.

In conclusion, the study of $D_s$ plus like-sign dilepton signals can serve as extremely useful pointers to the the decay $c \rightarrow s$ taking precedence over $b \rightarrow c$ in a $B_c$ meson. Using invariant mass cuts on the dileptons, one can reduce the main backgrounds to a large extent while still preserving a sufficient number of signal events. A careful analysis of such events at the hadronic B-factories can therefore reveal useful information enabling one to ascertain the correctness of different theoretical claims regarding the decay of the charm quark with a b-spectator.

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