Inner synchronization of Boolean networks with time delays

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Abstract. The model of Boolean networks with time delays is proposed, whose delays are multiple state delays for each node. Inner synchronization problem of the Boolean networks is investigated, and necessary and sufficient condition of synchronization is established based on the semi-tensor product of matrices. Moreover, illustrative examples show the efficiency of the proposed results.

1. Introduction

Boolean network (BN) is a simplified logical dynamic system to model the gene regulatory network, which was first proposed by Kauffman in 1969 [1]. Boolean networks (BNs) have been successfully used in describing cellular networks, neural networks, biological evolution models, and physical models etc [2-5]. Boolean networks are simple models of system, but they can provide a general method to simulate some real systems. Therefore, it is very important to study Boolean network.

Synchronization of dynamic systems is an important dynamic behavior. In last few years, synchronizations of complex networks have been deeply studied in many fields, such as physics, biology, neuroscience, computer science and mathematics. With the development of the semi-tensor product of matrices (STP)[6], STP is an effective tool to study control problem of Boolean networks. Now, the synchronization of complex networks has been extended to Boolean networks. And, many results have been obtained on the research of synchronization of Boolean networks[7-9]. As we know, time delay phenomenon is common, and there are time delays in many real systems, such as economic systems, biological systems and gene networks. The synchronization of two Boolean networks with time delays was studied, and necessary and sufficient conditions of synchronization are established with the semi-tensor product of matrices[10]. Recently, Li et al.[11] designed state feedback controller to make synchronization of two coupled Boolean networks with time delays, derived a necessary condition for the existence of a state feedback controller, and provided the feedback control design procedure for the achieving synchronization of two coupled Boolean networks.

The above research work mainly studied synchronization problem for two coupled Boolean networks. However, different from the synchronization of multiple Boolean networks, synchronization of all nodes in the same Boolean network is less studied, and it can also be referred to as the inner synchronization. Zhang et al.[12] gave the concept of inner synchronization for the first time, and derived necessary and sufficient conditions of synchronization in Boolean network without any time delays. This paper establishes inner synchronization model of Boolean network with time delays, and discusses the influence of time delay on the inner synchronization. With the semi-tensor product of matrices, give algebraic representation of Boolean network, and derives necessary and sufficient conditions of inner synchronization.
The rest of this paper is organized as follows. Some concepts and results of semi-tensor product and Boolean network are presented in Section 2. Inner synchronization theorem for Boolean network is proved with a rigorous mathematical method in Section 3. Numerical examples are given in Section 4, and conclusions are drawn in the final section.

2. Preliminaries

In this section, we first introduce the semi-tensor product, some concepts, notations and lemmas, and give algebraic representation of Boolean network with time delays.

2.1. Semi-tensor product of matrices

Daizhan Cheng and his colleagues first introduced the semi-tensor product (STP), and apply it to algebraic representation and dynamic behavior analysis of Boolean network [6]. The definition of semi-tensor product can be described as follows:

Definition 1 [6]. For $M \in \mathbb{M}_{m \times n}$, $N \in \mathbb{M}_{p \times q}$, their STP $C = M \hat{\otimes} N$, is defined as follows:

$$C = M \hat{\otimes} N = (M \otimes I_p)(N \otimes I_p),$$

where $l = \text{lcm}[n, p]$, is the least common multiple of $n$ and $p$, and $\otimes$ is the Kronecker product (tensor product), $\mathbb{M}_{m \times n}$ is a set of $m \times n$ real matrices. $\hat{\otimes}$ is left semi-tensor product, called semi-tensor product for short. Obviously, semi-tensor product of matrices is a generalization of conventional matrix product. In the following, we omit the symbols of left semi-tensor product.

Some notations with respect to STP are given as follows:

$\mathbb{I}_n$ is the $n$th column of $n$ dimensional identity matrix; $\mathbb{D}_n \triangleq \{1, 2, \ldots, n\}$, $\mathbb{D}_2 \triangleq \{0, 1\}$, $\mathbb{D} \triangleq \mathbb{D}_n \times \mathbb{D}_n$ is called a logical matrix, denoted by $L_{m \times n}$, and by identifying True = 1 $\delta^e$, False = 0 $\delta^o$.

Now, some related results and lemmas are presented in the following.

(1) A swap matrix $W \in \mathbb{M}_{m \times n}$, which is an $mn \times mn$ matrix, defined as

$$W = \delta \mathbb{I}_m \mathbb{I}_n \mathbb{I}_m \mathbb{I}_n = \delta \mathbb{I} \mathbb{I} \mathbb{I} \mathbb{I},$$

where $\mathbb{I}$ is an identity matrix.

(2) The dummy matrix is defined as $E_{xy} \triangleq \delta \mathbb{I}_y \mathbb{I}_x \mathbb{I}_y \mathbb{I}_x$. for any two logical variables $x, y \in \mathbb{D}_2$, then $E_{xy} = y$.

(3) Power reducing matrix $M_r(x) = \delta \mathbb{I}_1 \mathbb{I}_x \mathbb{I}_1 \mathbb{I}_x \mathbb{I}_x \mathbb{I}_x \mathbb{I}_x \mathbb{I}_x$, is a $2^m \times 2^m$ logical matrix. And if $x \in \mathbb{D}_2^m$, then $xx = M_r(x)$.

Lemmas 1[6]: If $A \in \mathbb{M}_{m \times n}$, and $Z \in \mathbb{R}^n$ is a column vector, then $ZA = (I_r \otimes A)Z$.

Lemmas 2[6]: let $X = (x_1, x_2, \ldots, x_m)^T$ and $Y = (y_1, y_2, \ldots, y_n)^T$ be two column vectors, then

(1) $XY = W_{[mn]}XY$

(2) $XY = W_{[mn]}XY$.

2.2. Algebraic representation of Boolean networks

In the paper, the Boolean network with $n$ nodes can be described as

$$x_i(t+1) = \tilde{f}_i(x_i(t), x_1(t), \ldots, x_n(t))$$

where $x_i(t) (i=1, 2, \ldots, n)$ is the $i$th node state of Boolean network at time $t$ ($t=0,1,2,\ldots$), $\tilde{f}_i$ is the Boolean function defined as $D^n \rightarrow D$ of the $i$th node.

The following lemma is fundamental for the matrix expression of the logical function.
Lemmas 3[6]: \( f(x_1, x_2, \ldots, x_n): D^* \to D \) is a logical function, then there exists a unique matrix \( M_f \in L_{2^n \times 2^n} \), called the structure matrix of \( f \) such that
\[
f(x_1, x_2, \ldots, x_n) = M_f x_1 x_2 \cdots x_n \in \Delta, \quad f_i \in \Delta, \quad i = 1, 2, \ldots, n, \quad (\Delta - D).
\]
From Lemmas 3, Eq.(1) can be written as
\[
x(t + 1) = F(t)x(t)
\]
where \( F \in L_{2^n \times 2^n} \) is called the transition matrix of Boolean network (1). Eq(3) is algebraic representation of Eq.(1).

The inner synchronization problem of Boolean network (1) has been investigated in Refer.[6]. Next, we will extend the inner synchronization of Boolean network to that of Boolean network with time delays.

3. Main results

3.1. the inner synchronization model of the Boolean networks

Consider a Boolean network consisting of \( n \) identical nodes with time delays, which is described by
\[
x_i(t + 1) = f_i(x_1(t), x_2(t), \ldots, x_i(t), x_{i+1}(t), \ldots, x_n(t), (t - \tau), \ldots, x_n(t - \tau))
\]
\[
x_2(t + 1) = f_2(x_1(t), x_2(t), \ldots, x_i(t), x_{i+1}(t), \ldots, x_n(t), (t - \tau), \ldots, x_n(t - \tau))
\]
\[...
\]
\[
x_n(t + 1) = f_n(x_1(t), x_2(t), \ldots, x_i(t), x_{i+1}(t), \ldots, x_n(t), (t - \tau), \ldots, x_n(t - \tau))
\]
where \( x_i \in D, \quad f_i : D^{n(\tau + 1)} \to D \) is a Boolean function, \( i = 1, 2, \ldots, n, \tau \) is a non-negative integer that denote the inherent state delay of BN.

For above BN(4) with time delays, we have following definition of inner synchronization.
Definition 1: For all nodes \( x_i(0), x_i(1), \ldots, x_i(t), \in \{0, 1\}^n (i = 1, 2, \ldots, n) \), in Boolean network (4), if there is a positive integer \( k \) such that \( t \geq k \) and \( x_i(t) = x_i(t+1) = \cdots = x_n(t) \), then the Boolean network (4) achieves inner synchronization.

3.2. the inner synchronization criterion of the Boolean networks

Assume that \( X(t) = x_1(t)x_2(t)\ldots x_n(t) \), then Eq.(4) can be expressed as:
\[
x_i(t + 1) = M_ix(t)x(t - 1)\ldots x(t - \tau)
\]
\[
x_2(t + 1) = M_ix(t)x(t - 1)\ldots x(t - \tau)
\]
\[...
\]
\[
x_n(t + 1) = M_ix(t)x(t - 1)\ldots x(t - \tau)
\]
where \( M_i \in L_{2^n \times 2^n} \) is a structure matrix of \( f_i, \quad i = 1, 2, \ldots, n \). let \( Y(t) = X(t)x(t - 1)\ldots x(t - \tau) \), then Eq.(5) can be converted into the following
\[
x_i(t + 1) = M_iY(t)
\]
\[
x_2(t + 1) = M_iY(t)
\]
\[...
\]
\[
x_n(t + 1) = M_iY(t)
\]
Form the formula above, we have
\[
X(t+1) = x_1(t+1)x_2(t+1)\ldots x_n(t+1)
\]
\[ Y(t+1) = X(t+1)X(t)...X(t+1-\tau) \]
\[ = F_i Y(t)X(t)...X(t+1-\tau) \]
\[ = F_i X(t)X(t-\tau)...X(t+1-\tau)X(t-\tau)...X(t+1-\tau)X(t-\tau) \]
\[ = F_i (I_{w_{\tau \tau}}W_{[w_{\tau \tau}]}^n)X(t)X(t-\tau)...X(t+1-\tau)X(t-\tau) \]
\[ = F_i (I_{w_{\tau \tau}}W_{[w_{\tau \tau}]}^n)M_s(2^{\tau}Y(t) \]
\[ = Y(t+1) = F_i (I_{w_{\tau \tau}}W_{[w_{\tau \tau}]}^n)M_s(2^{\tau})Y(t) \]
Thus
\[ Y(t+1) = Y(t), \quad t = 1, 2, 3, ... \]

Obviously, \( Y(t+1) = F_iY(t), \quad t = 1, 2, 3, ... \)

Through the derivation above it can be found that Eq. (7) is algebraic representation of Eq. (4), and two formulas are equivalent. Now we will discuss the inner synchronization conditions of Eq. (4) by using Eq. (7).

**Theorem 1:** The inner synchronization of BN (4) can be achieved if and only if there is a positive integer \( k \) such that

\[ Col(F^k) \subseteq \{ \delta_{\alpha, \beta}^{(1)}, \delta_{\alpha, \beta}^{(2...k)} \} \]

where \( F = F_i (I_{w_{\tau \tau}}W_{[w_{\tau \tau}]}^n)M_s(2^{\tau}) \).

**Proof (Sufficiency):** Suppose that there is a positive integer \( k \) such that \( Col(F^k) \subseteq \{ \delta_{\alpha, \beta}^{(1)}, \delta_{\alpha, \beta}^{(2...k)} \} \), apparently, we get

\[ Col(F^{k+i}) \subseteq ... \subseteq Col(F^{k+i}) \subseteq Col(F^k) \subseteq \{ \delta_{\alpha, \beta}^{(1)}, \delta_{\alpha, \beta}^{(2...k+i)} \}, \quad i \geq 1 \]

Because
\[ Y(t+1) = F_iY(t), \quad for \quad t > k \quad and \quad any \quad Y(l) \in \delta_{\alpha, \beta}^{(i)}, \quad we \quad can \quad get \]
\[ F_iY(l) = \delta_{\alpha, \beta}^{(i)}, \quad or \quad F_iY(l) = \delta_{\alpha, \beta}^{(i)} \].

However,
\[ Y(t) = X(t)X(t-\tau)...X(t-k\tau) \]
\[ = x_i(t)...x_i(t) x_i(t-l)...x_i(t-l)...x_i(t-\tau)...x_i(t-\tau) = \delta_{\alpha, \beta}^{(i)}, \quad or \]
\[ Y(t) = x_i(t)...x_i(t) x_i(t-l)...x_i(t-l)...x_i(t-\tau)...x_i(t-\tau) = \delta_{\alpha, \beta}^{(i)} \].

This is to say,
\[ x_i(t) = x_i(t)... = x_i(t-l) = x_i(t-l)... = x_i(t-\tau) = x_i(t-\tau) = \delta_{\alpha, \beta}^{(i)} \]
\[ = x_i(t-\tau) = x_i(t-\tau) = \delta_{\alpha, \beta}^{(i)}, \quad or \]
\[ x_i(t) = x_i(t)... = x_i(t) = x_i(t-l) = x_i(t-l)... = x_i(t-\tau) = x_i(t-\tau) = \delta_{\alpha, \beta}^{(i)} \].
\[ x_n(t-\tau) = \delta^2_2. \]

Hence, when time is \( t-\tau \), the inner synchronization of BN (4) can be achieved.

(Necessity): Assume that the BN (4) gets inner synchronization from time \( k-\tau \), then we have

\[ x_i(k) = x_2(k) = x_3(k) = x_i(k-1) = x_2(k-1) = x_3(k-1) = x_i(k-\tau) \]

or

\[ x_i(k) = x_2(k) = x_3(k) = x_i(k-1) = x_2(k-1) = x_3(k-1) = x_i(k-\tau) \]

Consequently,

\[ Y(k) = x_i(k)x_2(k)x_3(k)x_i(k-1)x_2(k-1)x_3(k-1)x_i(k-\tau)x_2(k-\tau)x_3(k-\tau) = \delta^3_{2n(i)} \]

or

\[ Y(k) = x_i(k)x_2(k)x_3(k)x_i(k-1)x_2(k-1)x_3(k-1)x_i(k-\tau)x_2(k-\tau)x_3(k-\tau) = \delta^3_{2n(i)} \]

So,

\[ Y(k+1) = Y(k) = \delta^3_{2n(i)}, \text{ or } Y(k+1) = Y(k) = \delta^3_{2n(i)}. \]

In addition,

\[ Y(k+1) = F^3Y(l) \]

And,

\[ F^3x_1(1)x_2(1)x_3(1)x_1(0)x_2(0)x_3(0)x_1(1-\tau)x_2(1-\tau)x_3(1-\tau) = \delta^3_{2n(i)} \]

or

\[ F^3x_1(1)x_2(1)x_3(1)x_1(0)x_2(0)x_3(0)x_1(1-\tau)x_2(1-\tau)x_3(1-\tau) = \delta^3_{2n(i)} \]

Since the initial values is arbitrary, we will get

\[ \text{Col}(F^3) = \delta^3_{2n(i)}, \text{ or } \text{Col}(F^3) = \delta^3_{2n(i)}. \]

This is, \( \text{Col}(F^4) \leq \{ \delta^1_{2n(i)}, \delta^3_{2n(i)} \} \). Therefore, Eq.(8) holds.

It is worth mentioning that a Boolean network can achieve inner synchronization, but the Boolean network by simply introducing delays do not always synchronize. This conclusion will be confirmed in the following simulation examples.

### 4. Examples

The Whether a Boolean network can achieve inner synchronization depends on dynamic evolution rules and state delays for nodes. In order to verify the effectiveness in Theorem 1, we present two examples in the following.

**Example 1.** A logical Boolean network with time delays is described as

\[
\begin{align*}
    x_i(t+1) &= x_i(t) \land x_i(t-1) \\
    x_2(t+1) &= x_2(t-1) \\
    x_3(t+1) &= x_3(t-1) \\
    x_1(t+1) &= x_1(t) \land x_1(t) \leftrightarrow x_1(t-1)
\end{align*}
\]

\[ (9) \]

It can be written as the following equivalent equations:

\[
\begin{align*}
    x_i(t+1) &= M_1X(t)X(t-1) \\
    x_2(t+1) &= M_2X(t)X(t-1) \\
    x_3(t+1) &= M_3X(t)X(t-1) \\
    x_1(t+1) &= M_1Y(t) \\
    x_2(t+1) &= M_2Y(t) \\
    x_3(t+1) &= M_3Y(t) \\
    Y(t+1) &= FY(t)
\end{align*}
\]

\[ (10) \]

where \( X(t) = x_1(t)x_2(t)x_3(t), \ Y(t) = X(t)X(t-1). \)

By using STP, we have

\[ x_i(t+1) = M_iE_iW_{an}[2,2](I_2 \otimes E_2)(I_8 \otimes E_3)(I_8 \otimes E_4)(I_8 \otimes E_4)(I_8 \otimes W_{an}[4,2])Y(t), \]

\[ (11) \]

\[ (12) \]
\[ x_i(t+1) = E_x(I_2 \otimes E_x)(I_2 \otimes E_x)W_{\text{mm}}[8,2](I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2])Y(t), \]

\[ x_i(t+1) = M_1(I_2 \otimes M_1)(I_2 \otimes E_x)(I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2])Y(t), \]

So, \( M_1 = E_x(I_3 \otimes E_x)(I_3 \otimes E_x)W_{\text{mm}}[8,2](I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2]), \)

\[ M_2 = E_x(I_3 \otimes E_x)(I_3 \otimes E_x)W_{\text{mm}}[8,2](I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2]), \]

\[ M_3 = M_1(I_2 \otimes M_1)(I_2 \otimes E_x)(I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2]), \]

\[ F = M_1(I_2 \otimes M_1)M_1(2^{2n})(I_2 \otimes M_1)M_1(2^{2n})(I_2 \otimes W[2^n,2^n])M_1(2^n), \]

\[ M_1 = \delta[1,2,2,1], \]

Next, we verify that \( F \) Eq.(9) satisfies the following condition

\[ \text{Col}(F^*) \subseteq \{\sigma^a, \sigma^b, \sigma^c, \sigma^d\} \quad (13) \]

With the help of MATLAB programming calculation, we obtain

\[ F^* = F^* = ... = \delta[1,64,1,64,1,64,1,64,...,64]. \]

Hence, when \( k = 6 \) and \( \tau = 1 \), BN (4) meets sufficient condition for achieving inner synchronization (in Theorem 1). In the following, we observe evolution trend of node state in Eq.(9) for any initial values.

Theoretically, \( \tau = 1 \), there are the 64 possible initial combinations in total. However, the equation has only one delay variable \( 3(t-1) \), so we only need to verify 16 different initial values.

For example, we select initial values to be \( (x_1(0), x_2(0), x_3(0)) = (1, 1, 0) \) and \( (x_1(1), x_2(1), x_3(1)) = (0, 0, 0) \), then \( (x_1(0), x_2(0), x_3(0)) = (0, 0, 0) \rightarrow (x_1(2), x_2(2), x_3(2)) = (0, 0, 0) \rightarrow ... \)

So, when \( \tau = 1 \), three node states is \{0\}. Where asterisk ‘*’ can be 0 or 1.

In the same way, it is not difficult to derive results of the rest of the other 15 cases. In addition, only when \( (x_1(0), x_2(0), x_3(0)) = (1, 1, 1) \) and \( (x_1(1), x_2(1), x_3(1)) = (1, 1, 1) \), does node states reach \{1\} (True). In the rest of the cases, node states reach \{0\} (False).

Example 2. Consider a following Boolean network

\[ Y(t+1) = X(t+1) \wedge x_i(t) \wedge x_j(t), \]

\[ x_i(t) = E_x(I_2 \otimes E_x)W_{\text{mm}}[4,2](I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2])Y(t), \]

\[ x_i(t) = M_1(I_2 \otimes M_1)(I_2 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2])(I_3 \otimes E_x)(I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2])Y(t), \]

\[ F = M_1(I_2 \otimes M_1)M_1(2^{2n})(I_2 \otimes M_1)M_1(2^{2n})(I_2 \otimes W[2^n,2^n])M_1(2^n), \]

\[ M_1 = M_1(W[2^n,2^n])M_1(I_2 \otimes E_x)W_{\text{mm}}[4,2](I_3 \otimes E_x)(I_3 \otimes E_x)(I_3 \otimes E_x)(I_3 \otimes E_x)(I_3 \otimes W_{\text{mm}}[4,2])Y(t), \]

By calculation, we obtain

\[ F^2 = \delta[1,1,64,1,64,1,64,1,64,...,64]. \]

\[ F^2 = \delta[1,1,64,1,64,1,64,1,64,...,64]. \]
From the above calculation results, it is found that when $k (>3)$ is even or odd, we get two different matrices, respectively. Obviously, $F$ in Eq.(15) cannot satisfy necessary condition in Theorem 1, so it means Eq.(15) cannot synchronize.

See the example below.

Let initial values be \((x_1(0), x_2(0), x_3(0)) = (\ast, 0, \ast)\) and \((x_1(1), x_2(1), x_3(1)) = (1, 1, 0)\), then

\[
(x_1(1), x_2(1), x_3(1)) = (1, 1, 0) \rightarrow (x_2(2), x_2(2), x_2(2)) = (0, 0, 1) \rightarrow (x_1(3), x_1(3), x_1(3)) = (1, 1, 0)...
\]

i.e. \((1,1,0) \leftrightarrow (0,0,1)\).

Clearly node states of Eq.(15) form a cycle with length 2, and Eq.(15) cannot achieve inner synchronization.

Two example above verify sufficient and necessary conditions for the theorem 1 separately.

5. Conclusion

We propose the model of Boolean network time delays, and present algebraic representation of Boolean network. With the semi-tensor product of matrices, Inner synchronization problem of BNs with time delays is investigated, and necessary and sufficient condition of synchronization is given. Also, two examples verify the sufficient and necessary conditions for achieving inner synchronization.

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