Redshifts and paradigm shifts: in defence of Hubble’s Law

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Abstract

We consider the recent proposal by the International Astronomical Union and others to rename Hubble’s law as the Hubble-Lemaître law in order to recognise the scientific contributions of Georges Lemaître. We find the proposal problematic from a historical and a philosophical perspective; in particular, we find that the proposal conflates Hubble’s observation of an empirical relation between redshift and distance for the spiral nebulae with Lemaître’s derivation of a universal law of cosmic expansion from the general theory of relativity. We note that the first of these phenomena is merely one manifestation of the second, an important distinction that has been somewhat overlooked in recent years. We suggest that maintaining the distinction may be helpful in the context of contemporary puzzles concerning the current rate of cosmic expansion.
1. Introduction

The recognition that we inhabit an expanding universe is one of the most startling discoveries of 20th century science. There is little doubt that, at least in the popular science literature, this discovery is primarily attributed to the work of one individual, the famous American astronomer Edwin Hubble (see for example Carey 1995 p 426; Greene 2004 p 229; Aughton 2008 pp 240-242). Even within the physics community, Hubble is closely associated with the discovery of cosmic expansion, as evidenced by the naming of key cosmic parameters such as Hubble’s law, the Hubble constant, the Hubble flow and the Hubble time.

In recent years, many scholars have pointed out that the acclaim afforded to Hubble is somewhat misleading, as the discovery that we live in an expanding universe was the result of the painstaking work of a number of different physicists (see for example Kragh and Smith 2003; Nussbaumer and Bieri 2011; van den Bergh 2011a; Way 2013). In particular, it has been noted that the moniker Hubble’s law – often loosely understood as a law of cosmic expansion – overlooks the seminal contribution of Georges Lemaître, the first to describe the redshifts of the spiral nebulae in the context of an expanding universe (Kragh 1996 p 30; Nussbaumer and Bieri 2009 p 127; van den Bergh 2011b). Such scholarship recently culminated in a formal proposal by the General Assembly of the International Astronomical Union (IAU) to rename the Hubble law as the “Hubble-Lemaître law”. The IAU proposal (see Appendix) was advanced at the 30th meeting of the union in August 2018 and had four stated aims:

(i) to pay tribute to both Georges Lemaître and Edwin Hubble for their fundamental contributions to the development of modern cosmology
(ii) to honour the intellectual integrity of Georges Lemaître that made him value more the progress of science rather than his own visibility
(iii) to highlight the role of the IAU General Assemblies in fostering exchanges of views and international discussions
(iv) to inform the future scientific discourses with historical facts

Members of the IAU worldwide voted electronically on the resolution on October 26th 2018 and it was passed with a majority of 78% . As stated in the accompanying press release:¹

An electronic vote has been conducted among all members of the International Astronomical Union, and the resolution to recommend renaming the Hubble law as the Hubble-Lemaître law has been accepted. The Hubble-Lemaître law describes the effect by which objects in an expanding Universe move away from each other with a velocity proportional to their distance. This resolution was proposed in order to pay tribute to both Lemaître and Hubble for their fundamental contributions to the development of modern cosmology.

¹ https://www.iau.org/news/pressreleases/detail/iau1812/
We note that particular reference was made in the IAU press release to the importance of historical considerations in their deliberations:

One of the IAU’s roles is to foster exchanges of views and international discussions – and it strives to contribute to scientific discourses with historical facts. To honour the intellectual integrity and the supremely significant discovery by Georges Lemaître, the IAU is pleased to recommend that the expansion of the Universe be referred to as the Hubble-Lemaître law.

To this effect, background information to inform the vote was provided by the Resolutions Committee to IAU members, as listed in the press release. Unfortunately, we find some of that material questionable from a historical perspective, as detailed below. More generally, we find the IAU proposal problematic from both a historical and a philosophical perspective, as it appears to conflate two distinct scientific advances, Hubble’s discovery of an approximate empirical relation between redshift and distance for certain astronomical bodies and Lemaître’s derivation of an exact law of cosmic expansion from the general theory of relativity.

In this article, we first provide a brief history of the discovery of Hubble’s law, from Vesto Melvin Slipher’s early observations of the spectra of the spiral nebulae to Hubble’s observations of an approximately linear relation between the redshifts and distances of some spirals in 1929. In section 3, we recall the emergence of the first non-static theoretical models of the universe in the 1920s, and in section 4, we recall the gradual adoption of the community of the paradigm of an expanding universe in the wake of these advances in both theory and observation. In section 5, we review the history and justification of the naming of Hubble’s law, and in section 6, we argue that the proposal to rename the law is not good history, good philosophy or good physics. In a short coda, we suggest that maintaining a clear distinction between an empirical redshift/distance relation obeyed by certain celestial bodies and a general law of cosmic expansion may be helpful in the context of today’s debate concerning the current rate of cosmic expansion.

2. A brief history of observation

In 1909, Vesto Melvin Slipher, a young astronomer working at the Lowell observatory in Flagstaff, Arizona, was assigned the task of studying the spectrum of light from the spiral nebulae. For this work, Slipher had at his disposal a 24-inch refracting telescope by Alvan Clark and a spectrograph made by Brashear. Experimenting carefully over many months, Slipher discovered that satisfactory spectra of some spirals could be obtained using a
spectrograph fitted with a camera lens of very short focus, a prism of high angular dispersion and a wide collimator slit. His key discovery was that the procurement of clear spectra depended critically on the speed of the spectrograph, rather than the size of the aperture of the telescope (Hoyt 1980).

By 1917, Slipher had measured spectra for 25 spiral nebulae (Slipher 1917). For all but the four closest, the characteristic spectral lines of the nebulae were shifted to longer wavelengths. Assuming these redshifts represented the Doppler effect, it appeared that the nebulae were receding from the observer at radial velocities $v$ given by

$$v = zc$$

where $z$ represented the fractional change in wavelength $\Delta \lambda/\lambda$ and $c$ was the speed of light. Of particular interest were the large recession speeds implied by Slipher’s observations, ranging from 150 to 1100 km/s (figure 1). Such large velocities were a great anomaly and suggested to some that the spirals could not be gravitationally bound by the Milky Way. Thus, Slipher’s redshift observations became well-known as an argument for the ‘island-universe’ hypothesis, the theory that the spiral nebulae constituted distinct galaxies far beyond the Milky Way (Smith 1982 p 22; Nussbaumer and Bieri 2009 p 57). However, the debate could not be settled until the distances to the spirals had been measured. By 1922, Slipher had amassed radial velocities for 41 spirals, almost all of which were redshifted. Although he never formally published the full collection, the data became known to theorists when they were published in a seminal textbook on general relativity by Arthur Stanley Eddington (Eddington 1923 p 162) and in papers by astronomers such as Gustav Strömberg, as discussed below.

An intriguing feature of Slipher’s data was that the faintest nebulae appeared to exhibit the largest redshifts; thus if the spectral shifts truly represented outward radial velocities, it seemed the most distant spirals were receding at the highest velocities. This phenomenon attracted the attention of many astronomers and theorists during the early 1920s; some were motivated by a prediction that light from distant objects would be redshifted in the de Sitter universe (see below), others by the more traditional problem of determining the solar motion relative to the spirals (Smith 1979; Trimble 2012). Thus, a number of astronomers such as Carl Wirtz, Ludwig Silberstein, Knut Lundmark and Gustav Strömberg attempted to ascertain a definitive redshift/distance relation for the nebulae and for other distant astronomical objects.
(Wirtz 1922, 24; Silberstein 1924; Lundmark 1924; Strömberg 1925). However these efforts were unsuccessful due to great uncertainties in the distances of the nebulae.\(^2\)

An important advance in determining astronomical distances was achieved by the noted astronomer Edwin Hubble in the 1920s. Early estimates of nebular distances were carried out using apparent magnitude as a measure of distance (Hubble 1926; Smith 1982 pp 110-111). However, working at the world's largest telescope, the 100-inch Hooker reflector at the Mt Wilson observatory, Hubble was able to resolve stars known as Cepheid variables in several of the nebulae. This was an important breakthrough as he was then able to employ Henrietta Leavitt’s period-luminosity relation to determine their distances (Smith 1982 pp 111-126; Nussbaumer and Bieri 2009 pp 60-62). By the mid-1920s, Hubble had convincingly demonstrated that several spiral nebulae lay far beyond the limits of the Milky Way, settling the ‘island universe’ debate at last (Hubble 1925, 1926).

The next step was to investigate the relation between the redshifts of the spiral nebulae and their distances, using the new data.\(^3\) By 1929, Hubble had amassed reliable estimates of the distances of 24 spirals using a variety of methods including the use Cepheid variables as standard candles, the brightness of ‘involved stars’ and the mean luminosity of nebulae; combining these with Slipher’s redshift data, and a few redshift measurements acquired at Mt Wilson by Milton Humason, Hubble obtained the graph shown in figure 2. Assuming the redshifts corresponded to velocities of recession, a linear relation between radial velocity and distance could be discerned, despite considerable scatter. As Hubble stated in the paper: “the results establish an approximately linear relation between the velocities and distances among nebulae for which velocities have been previously published” (Hubble 1929). We note that the distances of the closest seven nebulae were estimated by observing Cepheid stars within the nebulae; the next thirteen distances were estimated by observing the most luminous stars in nebulae and assuming an upper limit of absolute magnitude \(M = -6.3\); the remaining four objects had distances assigned on the basis of the mean luminosities of the nebulae in a cluster. Finally, the single cross represents a mean velocity/distance ratio for 22 nebulae whose distances were estimated using the method of apparent magnitude of nebulae (Hubble 1929).

Many commentators have noted that the quality of the data only marginally support the conclusion of a linear relation between velocity and distance for the nebulae (Kragh 1996 p 18; 2000).

\(^2\) In addition, many of the bodies studied were not sufficiently distant to manifest such a relation, as discussed in (Smith 1979; Duerbeck and Seitter 2000).

\(^3\) According to Hubble’s assistant Milton Humason, this study was inspired by discussions of the redshifts of the nebulae at the 1928 IAU meeting in Leiden (Humason 1965; Christianson 1995 pp 187-188).
Longair 2006 p 110; Ostriker and Mitton 2013 p 73; Peacock 2013). As pointed out by all these authors, it is likely that Hubble’s conclusion was influenced by an important data point cited in the paper but not shown in the graph, i.e., Humason’s measurement of an apparent velocity of 3779 km/s for a nebula at an estimated distance of 7.8 megaparsecs (Hubble 1929; Humason 1929). We note also that Hubble did not acknowledge his use of Slipher’s redshift data in the paper and this is perhaps one reason the result later became known as Hubble’s law (see below).

By the time the graph of figure 2 was published, Hubble had already embarked on a program to extend the study to even more distant nebulae. Using a state-of-the-art spectrograph with a specially designed Rayton camera lens in conjunction with the great telescope at Mt Wilson, he and Humason successfully measured redshifts and distances for forty more spirals, demonstrating a linear relation between velocity and distance out to a distance eighteen times that of figure 2 (Hubble and Humason 1931; Nussbaumer and Bieri 2009 pp 134-136).

It should not be concluded from this section that “Hubble discovered the expanding universe”, as is often stated in the popular literature. Such a statement confuses observation with theory and astronomy with cosmology, as discussed below. Indeed, Hubble himself declined to interpret his data in such a manner, as discussed in section 4. It is much more accurate to say that Hubble’s 1929 graph provided the first observational data that could be interpreted as evidence in support of the hypothesis of the expanding universe. But what was this hypothesis?

3. A brief history of theory

In 1917, Einstein attempted the first relativistic model of the universe (Einstein 1917a), an important test for his newly-minted general theory of relativity. As he remarked to the Dutch astronomer Willem de Sitter: “For me... it was a burning question whether the relativity concept can be followed through to the finish, or whether it leads to contradictions” (Einstein 1917b). However, assuming a cosmos of closed spatial curvature with a static, uniform distribution of matter, Einstein soon found that the covariant field equations of relativity gave a null solution. His answer was to modify the field equations by adding a new term known the cosmological constant term. Einstein then showed that, applied to the universe as a whole, the modified field equations had the simple solution

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4 No evidence for a non-static universe was known to Einstein at the time. The assumption of closed spatial curvature arose from Einstein’s desire to render his model compatible with his understanding of Mach’s Principle. See (Smeek 2014; O’Raifeartaigh et al. 2017) for further details.
\[ \lambda = \frac{\kappa \rho}{2} = \frac{1}{R^2} \]  

where \( \lambda \) was the cosmological constant and \( \kappa, \rho \) and \( R \) represented the Einstein constant, the mean density of matter and the radius of the cosmos respectively (Einstein 1917a). Thus, Einstein’s 1917 model of the cosmos gave an apparently satisfactory relation between the size of the universe and the amount of matter it contained. Indeed, Einstein even attempted a rough estimate of the size of the universe in his correspondence around this time, although he soon realised that such calculations were unreliable (O’Raifeartaigh et al. 2017).

In July 1917, de Sitter noted that Einstein’s modified field equations allowed an alternate cosmic solution, namely the case of a universe with no matter content (de Sitter 1917).\(^5\) Einstein was greatly perturbed by de Sitter’s empty model as it was in direct conflict with his understanding of Mach’s Principle in these years (Smeenk 2014; O’Raifeartaigh et al. 2017; Realdi 2019). Einstein made this criticism public in a paper of 1918 and also suggested that de Sitter’s model contained a spacetime singularity (Einstein 1918a). After an intervention by Felix Klein, Einstein privately accepted that the latter criticism was unjustified (Einstein 1918b); however, he never formally retracted his criticism and it is clear from his writings around this time that that he did not consider the de Sitter solution a realistic model of the universe (O’Raifeartaigh et al. 2017; Realdi 2019). By contrast, many astronomers took a keen interest in de Sitter’s cosmology, because of a prediction that the light from distant objects in the de Sitter universe would be redshifted. Indeed, this prediction acted as a spur for the investigations of Wirtz, Silberstein, Lundmark and Strömberg mentioned in section 3.\(^6\)

In 1922, the Russian physicist Alexander Friedman suggested that non-static solutions of the Einstein field equations should be considered in relativistic models of the cosmos (Friedman 1922). Starting from the modified field equations and assuming a positive spatial curvature for the cosmos, he derived the two differential equations

\[ \frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho \]  

\[ \frac{2R''}{R} + \frac{R'^2}{R^2} + \frac{c^2}{R^2} - \lambda = 0 \]  

In this model, Einstein’s matter-filled three-dimensional universe of closed spatial geometry was replaced by an empty four-dimensional universe of closed \textit{spacetime} geometry.

\(^5\)See (Realdi 2019) for a review.
linking the time evolution of the cosmic radius $R$ with the mean density of matter $\rho$ and the cosmological constant $\lambda$. However, few physicists paid attention to Friedman’s time-varying cosmology, possibly because the work was quite technical and made no connection to astronomy. Worse, Einstein publicly criticized the paper on the basis that it contained a mathematical error (Einstein 1922). When it transpired that the error lay in Einstein’s analysis, he retracted his criticism (Einstein 1923a). However, an unpublished draft of Einstein’s retraction demonstrates that he did not consider Friedman’s cosmology to be realistic: “to this a physical significance can hardly be ascribed” (Einstein 1923b).\(^7\)

A few years later, the Belgian physicist Georges Lemaître independently derived similar differential equations for the radius of the cosmos from Einstein’s field equations (Lemaître 1927). The motivation for this study was quite different to the case of Friedman; aware of Slipher’s redshift observations and of Hubble’s emerging measurements of the vast distances of the spirals, Lemaître suggested that the redshifts of the nebulae were a manifestation of a general expansion of space from a pre-existing static cosmos of radius $R_0 = 1/\sqrt{\lambda}$. Considering the effect of a stretching of space on light emitted at a position $\sigma_1$ and received at a position $\sigma_2$, Lemaître established a landmark connection between the new cosmology and astronomy by deriving the simple expression

$$\frac{\Delta \lambda}{\lambda} = \frac{R_2}{R_1} - 1$$

(5)

for the fractional change in wavelength due to cosmic expansion at cosmic radii $R_1$ and $R_2$. He suggested that the effect would be observed as an apparent Doppler effect

$$\frac{v}{c} = \frac{R_2}{R_1} - 1 = \frac{R'}{R}r$$

(6)

where $R'$ was the time derivative of cosmic radius and $v$ and $r$ were the recession velocities and distances $r$ of the nebulae respectively.\(^8\)

Noting carefully that the equivalence was only approximately valid at certain velocities and distances, Lemaître then estimated the rate of cosmic expansion by inserting observational

\(^7\)A detailed account of this episode can be found in (Nussbaumer and Bieri 2009 pp 91-92).

\(^8\)Lemaître’s 1927 article can be found in English translation in (Luminet 2013).
data into equation (6). Taking redshifts and distances for 42 spiral nebulae from Strömberg and Hubble respectively (Strömberg 1925; Hubble 1926), Lemaître divided the mean velocity of the nebulae by the mean distance: “Using the 42 nebulae appearing in the lists of Hubble and Strömberg...and taking account of the proper velocity of the Sun...one finds a mean distance of 0.95 megaparsecs and a radial velocity of 600 Km/sec, i.e., 625 Km/sec at 10^6 parsecs”. Inserting the last two figures into equation (6),\footnote{This the first estimate of what is now known as the Hubble constant. We note that almost all of the redshift data in Strömberg’s paper was provided by Slipher.} he obtained

\[ \frac{R'}{R} = \frac{v}{cr} = \frac{625 \times 10^5}{10^6 \times 3.08 \times 10^{18} \times 3 \times 10^{10}} = 0.68 \times 10^{-27} \text{ cm}^{-1} \]  

(7)

Two aspects of Lemaître’s analysis are worth emphasizing. In the first instance, the nebular distances are taken from Hubble’s publication of 1926 and thus almost all of them were estimated using the method of apparent magnitude. In this paper, Hubble himself was mindful of the many uncertainties associated with the method. Throughout the paper, Hubble described the method of measuring nebular distance by apparent magnitude as a ‘working hypothesis’ and stressed that “reliable values of distances, and hence of absolute magnitudes, are restricted to a very few of the brightest nebulae.....the number of known distance is too small to serve as a basis for estimates in the range in absolute magnitude among nebulae in general” (Hubble 1926). In the second instance, Lemaître does not employ a linear relation between redshift and distance that is already established; instead he predicts the existence of such a relation from theory. This is seen most clearly in an important footnote to the section, where Lemaître notes that recent attempts to establish a relation between \( v \) and \( r \) indicate only a very weak correlation due to the uncertainties in nebular distance and suggests that a systematic error may be avoided by considering the ratio of mean velocity divided by mean distance (Luminet 2013; O’Raifeartaigh 2019). Unfortunately, this footnote was omitted from the 1931 translation of the paper, as discussed below.

Lemaître’s paper also received very little attention at first. One reason is undoubtedly the fact that it was published in French in a lesser-known Belgian journal. However, it is known that both Eddington and de Sitter received copies of the paper when it was first published and neither paid attention until 1930. Having recently considered the matter in some detail (O’Raifeartaigh 2019), we have suggested that another reason Lemaître’s hypothesis fell on stony ground may be the preliminary nature of the observational data used in the paper. As the
nebular distances cited were established using a method assumed to be prone to large errors, it is likely that many readers were not yet convinced of the reality of a linear redshift/distance relation for the nebulae. In any event, Lemaître himself did little to promote his model in the next few years, perhaps due to the lack of interest from Eddington and de Sitter and a negative reaction from Einstein. As is well known, when Lemaître and Einstein met in 1927, the world’s most famous physicist declared expanding models of the cosmos ‘tout à fait abominable’ and added that a similar hypothesis had already been suggested by the Russian physicist Alexander Friedman! (Lemaître 1930, 1958; Nussbaumer and Bieri 2009 pp 111-113).

4. The paradigm shift

The publication of Hubble’s graph of 1929 was an important milestone for astronomers and theorists alike. For example, at the January meeting of the Royal Astronomical Society in 1930, Willem de Sitter pointed out that a linear relation between distance and radial velocity for the nebulae could not be explained in the context of his own cosmology or that of Einstein. In the ensuing discussion, de Sitter and Eddington speculated that a new model of the cosmos was needed. This discussion was published in The Observatory (de Sitter 1930a) and came to the attention of Lemaître, who wrote to Eddington to remind him of his 1927 paper (Lemaître 1930). Eddington immediately grasped the significance of Lemaître’s work and quickly made others aware of it (Eddington 1930; de Sitter 1930b). Eddington also arranged for the paper to be translated and republished in the widely-read Monthly Notices of the Royal Astronomical Society. The article duly appeared, although the passage where a co-efficient of cosmic expansion is estimated from observational data (see section 3 above) was reduced to a single line: “From a discussion of available data, we adopt \( R'/R = 0.68 \times 10^{-27} \) cm⁻¹” (Lemaître 1931a). It has recently been confirmed that the translation and revision of the paper was carried out by Lemaître himself (Livio 2011), as discussed below. Meanwhile, a number of articles on Friedman-Lemaître cosmologies with varying cosmic parameters were published (Eddington 1930, 1931; de Sitter 1930c, 1931; Tolman 1930, 1931, 1932; Heckmann 1931, 1932; Robertson 1932, 1933). Einstein himself overcame his earlier distrust of time-varying models of the cosmos and proposed two dynamic models during this period, the Friedman-Einstein model of 1931 and the Einstein-de Sitter model of 1932 (Einstein 1931; Einstein and de Sitter 1932). Thus by the early 1930s, it seemed to many that an astonishing new phenomenon, the

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10 We note that redshift/distance data from both Hubble and de Sitter were discussed.
expanding universe, had been discovered that could be explained in a natural way in the context of the general theory of relativity.

However, such changes in scientific worldview are rarely instant or unanimous, as pointed out by philosophers of science such as Thomas Kuhn and Imre Lakatos (Kuhn 1977; Lakatos 1981). Certainly, Hubble’s redshift/distance data were soon accepted, despite some initial objections from Harlow Shapley (Smith 1979; Kragh 1996 p 19). One reason was undoubtedly the use of Cepheid variables to measure the distances of some nebulae. Hubble’s status as a leading astronomer working at the world’s largest telescope may also have played a role (Trimble 1996, 2012). A third factor was the publication of similar data for nebulae at much greater distance in the years that followed (Hubble and Humason 1931). Thus, by the mid-1930s, few doubts remained concerning the validity of the reshift/distance relation.

By contrast, the interpretation of Hubble’s data in terms of cosmic expansion was far from settled in this period. One obvious problem was that most expanding models seemed to predict an age for the universe that was problematic. Many theoreticians noted that for the simplest models, Hubble’s estimated rate of expansion of 500 (km/s)/Mpc implied a universe that had been expanding for about two billion years (Kragh 1996 pp 73-76). This was a curious figure if it represented the age of the cosmos, since experiments from radioactivity suggested that the earth was at least four billion years old! Thus, several alternative explanations for the recession of the nebulae were offered in these years. The best known of these was a hypothesis from the Swiss physicist Fritz Zwicky that light from distant stars might be redshifted due to a loss of energy as it travelled over vast distances in interstellar space (Zwicky 1929). Indeed, quite a number of physicists made similar suggestions, a class of theories that became known as ‘tired-light’ theories (see Kragh 2017 for a review). Another hypothesis was that the redshifts of the nebulae represented a Doppler effect due to the movement of galaxies into neighbouring space, a suggestion that was advanced in the context of the so-called kinematic cosmology of Edward Milne (Milne 1934). Thus, in the 1930s many astronomers and theorists kept an open mind regarding the meaning of the redshifts.11 As Hubble and Tolman remarked in a joint publication (Hubble and Tolman 1935) on observational and theoretical investigations of the nebulae:

> Until further evidence is available, both the present writers wish to express an open mind with respect to the ultimately most satisfactory explanation of the nebular red-shift and, in the presentation of purely observational findings, to continue to use the phrase “apparent”

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11 See (Plaskett 1933) for a contemporaneous discussion of objections to the expanding universe.
velocity of recession. They both incline to the opinion, however, that if the red-shift is not due to recessional motion, its explanation will probably involve some quite new physical principles.

As the years progressed, non-relativistic explanations for the redshift/distance relation of the nebulae were eventually ruled out (Kragh 2017). However, some astronomers, including Hubble himself remained agnostic on the subject throughout their careers. Hubble’s attitude to his redshift/distance observations is perhaps most clearly seen in his last public address, the 1953 Darwin Lecture of the Royal Astronomical Society (Hubble 1953):

I propose to discuss the law of red-shifts—the correlation between distances of nebulae and displacements in their spectra. It is one of the two known characteristics of the sample of the universe that can be explored and it seems to concern the behaviour of the universe as a whole. For this reason it is important that the law be formulated as an empirical relation between observed data out to the limits of the greatest telescope. Then, as precision increases, the array of possible interpretations permitted by uncertainties in the observations will be correspondingly reduced. Ultimately, when a definite formulation has been achieved, free from systematic errors and with reasonably small probable errors, the number of competing interpretations will be reduced to a minimum.

Hubble’s failure to embrace the thesis of cosmic expansion has become the subject of much comment in recent years, and it is certainly somewhat ironic in the context of modern nomenclature such as the Hubble expansion and the Hubble flow (see below). However, it should be borne in mind that his careful demarcation between observation and explanation was not uncommon amongst astronomers at the time, particularly in cases where the explanation involved abstruse theories such as the general theory of relativity (North 1965 p 237; North 1990; Kragh 1996 pp 69-70). Indeed, it could be stated that astronomers of this period were engaged in ‘cosmology by accident rather than design’, as noted by the historian Robert Smith (Smith 2019). We also note that it was not until the mid-1950s that the time-scale difficulty associated with expanding cosmologies was resolved, due to important revisions in the cosmological distance scale (Trimble 1996; Longair 2019).

5. On the naming of Hubble’s law

In time, the velocity/distance graph of figure 2 became known as ‘Hubble’s law’. It is not entirely clear when this nomenclature became the norm. Certainly, there are copious references to ‘Hubble’s observations’ and ‘Hubble’s relation’ in the cosmological papers of the early 1930s cited in section 4. By 1933, at least two specific references to ‘Hubble’s law’ had
appeared in the literature, in papers by Edward Arthur Milne and by Arthur Geoffrey Walker (Milne 1933; Walker 1933). Both theoreticians were well-known and their nomenclature may have been influential. However, the use of the moniker ‘Hubble’s law’ only seems to have become widespread in the 1950s, possibly through its use in popular books such as ‘The Creation of the Universe’ by George Gamow (Gamow 1952 p 37) and ‘The Expansion of the Universe’ by Pierre Couderc (Couderc 1952 pp 108-110). Indeed, it is interesting to note that Lemaître himself employed the nomenclature “la loi de Hubble” in his review of the French edition of the latter book, as discussed below.

In our view, this nomenclature is quite reasonable, given Hubble’s groundbreaking measurements of the distances to the nebulae, his combination of the distances with Slipher’s observations to obtain the first evidence for a linear velocity/distance relation in 1929, and his extension of the relation to much larger distances with the assistance of Humason in the years to follow. We have argued elsewhere (O’Raifeartaigh 2013) that the graph of 1929 could have become known as the ‘Hubble-Slipher graph’; however, Hubble’s failure to acknowledge Slipher’s data in the 1929 paper rendered this a remote possibility. In any case, it is not unusual for scientific laws to be named after the last observer to put the capstone in place (see below). Thus we do not agree with authors who cite Hubble’s law as an example of Stigler’s law of eponymy, i.e., as an example of the phenomenon that “no scientific discovery is named after its original discoverer” (Stigler 1980).

6. On the proposed renaming of Hubble’s law

As mentioned in section 1, many scholars have recently argued that Hubble’s law should be renamed, a proposal that has culminated in a recent vote by the International Astronomical Union to rename the law as the ‘Hubble-Lemaître law’. We now consider this proposal from the perspective of the brief history laid out in sections 2-5 above.

We recall first that Hubble’s law was understood for many years as an empirical relation between the redshifts and distances of many spiral nebulae, generally interpreted as velocities of recession. Lemaître did not provide any measurements of redshift or distance of the nebulae, nor did he establish the linearity of the redshift/distance relation. Instead, he predicted a linear relation between velocity and distance as a manifestation of a general expansion of space derived from relativistic cosmology; assuming such a relation existed, he estimated a co-

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12 See (Kragh 2018).
13 See for example (Kragh and Smith 2003; Block 2012; Belinkiy 2015; Shaviv 2011).

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efficient of cosmic expansion using mean values for velocity and distance taken from the observational data of Slipher and Hubble. That this calculation was something of a provisional ‘guesstimate’ can be seen from the fact that it appears only as a single line in the English version of the paper (Lemaître 1931a). Indeed, Lemaître’s own attitude towards the astronomical data he used in 1927 is made clear in the covering letter that accompanied his 1931 translation: “I do not think it is advisable to reprint the provisional discussion of radial velocities which is clearly of no actual interest” (Lemaître 1931b). The same attitude can be seen in a comment made by Lemaître many years later in a review of Pierre Couderc’s book: “Naturellement, avant la découverte et l’étude des amas de nébuleuses, il ne pouvait être question d’établir la loi de Hubble” or “Naturally, before the discovery and study of the clusters of nebulae, there could be no question of establishing Hubble’s law” (Lemaître 1950). Similarly, in a review article written in 1952, Lemaître wrote: “Hubble and Humason established from observation the linear relation between velocity and distance which was expected for theoretical reasons and which is known as the Hubble velocity-distance relation” (Lemaître 1952).

Historical concerns similar to the above regarding the renaming of Hubble’s law have also been raised by the astronomer Virginia Trimble and the historian Helge Kragh (Trimble 2012; Kragh 2018). In addition, we find the proposal problematic from the point of view of the philosophy of physics. In our view, one should not conflate an empirical relation between two observables with a general law of spatial expansion derived from cosmological theory. As pointed out by the noted cosmologist and astronomer Edward Harrison (Harrison 2000 p 275):

The redshift-distance law \( zc = HL \)…is the observers’ linear law first established by Slipher’s redshift measurements and Hubble’s distance determinations. Its proper name is the Hubble law. From the time of its discovery most cosmologists have realised that in its linear form it is only approximately true. On the other hand, the velocity-distance law \( V=HL \)…is the theorists’ linear law that follows automatically from the assumption that expanding space is uniform (isotropic and homogeneous). This law, often improperly referred to as the Hubble law, is of central importance in modern cosmology and is rigorously true in all uniform universes.

One obvious distinction between the two laws is that a simple relation between redshift and distance is not observed in the case of nebulae (or other astronomical bodies) at relatively close distance, as the effects of cosmic expansion are overwhelmed by local gravitational effects. Indeed, this phenomenon hampered the early investigations of the redshift/distance relation of

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14 Here the word ‘actual’ is clearly used in the sense of ‘contemporary’ (O’Raifeartaigh 2019).
15 This sentence is mistranslated in (Block 2012) as discussed below.
astronomical bodies discussed in section 2. Another distinction is that the equivalence expressed in equation (6) of Lemaître’s analysis is only approximately valid, as he was careful to point out; it is easily seen that the equation requires relativistic correction for distances corresponding to superluminal velocities, for example. Moreover, a clear distinction should be drawn in principle between a cosmological redshift, caused by the stretching of space, and a Doppler shift of wavelength due to motion; indeed, some celestial bodies can exhibit both effects simultaneously.16

More generally, philosophers of science distinguish between laws of science that are empirical relations between observables, limited in range, and laws of universal application derived from theory. As the philosopher Peter Caws (Caws 1965 p 85) put it:

The distinction between hypotheses and empirical generalizations suggests a distinction between two different kinds of scientific law, one corresponding to empirical generalizations which are accepted as true and the other to hypotheses which are accepted as true. In the latter category would fall, for instance, the law of conservation of energy; energy is not observed, but rather the penetration of bullets or the compression of strings, so that any statement about it must be hypothetical.

Thus Ohm’s law, an empirical relation between current and voltage observed to hold in some materials, is merely one manifestation of Maxwell’s more general laws of electromagnetism. Similarly, Boyle’s law, an empirical relation between pressure and volume obeyed by most gases at constant temperature, is merely one example of a more general ideal gas law, nowadays posited in terms of the kinetic theory of gases. There are many other examples of scientific laws that are really empirical manifestations of much more general laws, from Snell’s laws of reflection and refraction to Einstein’s law of the photoelectric effect.

In this context, Hubble’s redshift/distance relation is seen as a particular manifestation of a much more universal law, the Friedman-Lemaître law of cosmic expansion. The two are not equivalent, not least because the former is of limited validity, as pointed out above. In addition, it should be borne in mind that Hubble’s law is merely one manifestation of cosmic expansion. Other manifestations exist, notably the frequency range of the cosmic microwave background (CMB). Indeed studies of the CMB provide a key alternative measurement of cosmic expansion, as discussed below. We note in passing that the discovery of the cosmic microwave background is routinely attributed to the empirical observations of the radio astronomers Arno Penzias and Robert Wilson (Penzias and Wilson 1965), although a

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16 See (Harrison 1993) for a full technical discussion of this point.
theoretical explanation for their observations was provided by Robert Dicke’s group (Dicke et al. 1965) and the phenomenon was earlier predicted by Ralph Alpher and Robert Herman (Apher and Herman 1948).\(^\text{17}\)

**On the motivation for the IAU proposal**

Given all of the above, one might wonder about the rationale for a renaming of Hubble’s law. Reading through the literature linked to the IAU proposal, we find that the proposal reflects a widespread recognition that Lemaître has not received due recognition for his pioneering cosmological contributions, while Hubble has received rather more than his due. One reason for this is that the meaning of the moniker ‘Hubble’s law’ appears to have mutated over the years. It is now commonplace to find Hubble’s law cited as a general law of cosmic expansion, from articles in popular science magazines to papers in technical journals (Kragh and Smith 2003). Indeed, the expression ‘Hubble expansion’ is now a commonplace even in textbooks, along with ‘Hubble flow’ and ‘Hubble time’ (see for example Coles and Lucchin 2002 pp 13-15). It is quite difficult to pinpoint when this nomenclature became the norm, although it appears to have taken root after the discovery of the cosmic microwave background and become more pronounced thereafter (Kragh 2018). For example, a simple citation analysis suggests 168 citations for Hubble and 60 for Lemaître in the period 1965-69; this trend becomes even more marked in later years, with 193 citations for Hubble versus 6 for Lemaître in the period 1980-84 (Trimble 2012). Thus, there is little doubt that an intermingling of the two laws has already occurred, with the phenomenon of cosmic expansion now routinely associated with Hubble alone. In this context, does it not make sense to redress the balance by acknowledging Lemaître?

We take the view that it is not good practice to address a common historical error by introducing another; in our view, to refer to Hubble’s law as the Hubble-Lemaître law simply institutionalizes the confusion by formally crediting Hubble with the discovery of cosmic expansion. Rather than compounding the error, it would surely be better to honour Lemaître by preserving the distinction between a general law of cosmic expansion derived from cosmological theory and an empirical relation between redshift and distance observed for certain astronomical bodies. As pointed out above, this is not just a question of semantics; instead it is good history, good philosophy and good physics.

\(^\text{17}\) See (Peebles 1993 pp 139–148) for a review.
In addition, we feel bound to point out a number of questionable statements in the background material provided in conjunction with the IAU proposal (see Appendix). Considering point 3 first, it is stated that “at the time of publication, the limited popularity of the Journal in which Lemaître’s paper appeared and the language used made his remarkable discovery largely unperceived by the astronomical community”. This is undoubtedly true to some extent, but several scholars have recently pointed out that the journal in question was reasonably well-known to European physicists (Luminet 2013; Lambert 2015 pp 132-133). It should also be noted that a linear relation between redshift and distance for the nebulae had not yet been established empirically in 1927 and it is likely that the community was not yet ready to embrace the concept of an expanding universe at this point (Peebles 2015 pp 11-12; Lambert 2015 pp 132-133; O’Raifeartaigh 2019). Turning to point 4 of the proposal, it is stated that “both George Lemaître...and the American astronomer Edwin Hubble...attended the 3rd IAU Assembly in Leiden in July 1928 and exchanged views [4] about the relevance of the redshift vs distance observational data of the extragalactic nebulae to the emerging evolutionary model of the universe”. In fact, there is no evidence that Hubble and Lemaître met at this meeting, as pointed out by Helge Kragh, nor does reference [4] (Humason 1965) suggest such a thing (Kragh 2018). While it is very likely that Hubble engaged in many discussions about the nebulae with astronomers at this meeting due to his position as the Chair of the Commission on Nebulae and Star Clusters, there is no evidence that he had discussions with anyone about the relevance of such observations to theoretical cosmology; instead it is reported that he returned to the US intrigued by the simple observation that the most distant nebulae appeared to exhibit the largest redshifts (Humason 1965; Christianson 1995 pp 187-188). Finally, it is stated in item 5 that “Soon after the publication of his papers, the cosmic expansion became universally known as the Hubble law”. This is not the case, as has been pointed out in detail in section 4 (see also (Kragh 2018)).

We note finally that the bibliography accompanying the IAU proposal draws particular attention to a paper by David Block (Block 2012). We are puzzled by the highlighting of this article as an aid to historical deliberations. In the first instance, the paper is founded on a curious speculation (that Lemaître’s 1931 paper was censored by third parties), a conjecture that was disproven before publication (Livio 2011). In the second instance, the paper features a quotation from Lemaître that is an unfortunate mistranslation: “Of course, before the discovery and study of clusters of nebulae, there was no point to establish the Hubble law, but only to calculate its coefficient” (Block 2012) As pointed out in section 5, Lemaître in fact uses the phrase “il ne pouvait être question d’établir la loi de Hubble” (Lemaître 1950) which should
be translated as “there could be no question of establishing Hubble’s law”, a very different statement.

Conclusions

We do not support the proposal by the IAU and others to rename Hubble’s law as the Hubble-Lemaître law. In our view, such a change does not represent good historical practice as it conflates Hubble’s observation of a linear relation between redshift and distance for the nebulae with Lemaître’s derivation of a general law of cosmic expansion from relativity. It is also not good philosophy as it conflates an empirical relation between two observables, limited in validity, with a universal law of general application derived from cosmological theory. In addition, we note that the background material provided by the IAU to its members contains several historical inaccuracies.

Coda

At the end of the 20th century, measurements of the redshift/distance relation of type Ia supernovae led to the startling discovery that the rate of cosmic expansion is increasing (Riess et al. 1998; Perlmutter et al. 1999). The cause of this phenomenon, dubbed dark energy, is not known, but can be represented in theoretical models of the cosmos with the use of a cosmological constant term in Einstein’s field equations (see section 2). Evidence of cosmic acceleration has also been found in satellite studies of the cosmic microwave background (Ade et al. 2016), leading to the so-called concordance or Λ-CDM model of the universe. However, a perplexing conflict has emerged; while the most recent estimates of the current rate of cosmic expansion $H_0$ from measurements of the redshifts and distances of type Ia supernovae suggest a value of $H_0 = 73.24 +/- 1.74$ (km/s)/Mpc (Riess et al. 2016), estimates from measurements of the cosmic background radiation suggest a value of $H_0 = 67.4 +/- 0.5$ (km/s)/Mpc (Aghanim et al. 2018). The supernova studies correspond directly to Hubble’s redshift/distance investigations of the 1920s and 30s, and thus some scholars suspect that our calibration of cosmological distance may once again be subject to systematic errors (see section 4). On the other hand, other scholars point out that measurements of $H_0$ from studies of the cosmic microwave background are not made directly, but inferred from a number of cosmic parameters, many of which are model dependent; thus the conflict may indicate that something is awry with our standard model of the cosmos (Freedman 2017). At the time of writing, it is
not known where the source of the discrepancy lies, although the advent of new astronomical techniques such as ‘multi-messenger’ astronomy (i.e., the detection of astronomical events by both gravitational and electromagnetic waves) may soon cast some light on the puzzle (Abbott et al. 2017; Chen et al. 2018). In the meantime, the impasse serves as a useful reminder that an empirical relation between redshift and distance for certain celestial bodies is merely one manifestation of our expanding universe.
Figure 1. Radial velocities in km/s of 25 spiral nebulae measured by VM Slipher (reproduced from Slipher 1917). Negative terms indicate velocities of approach while positive velocities are receding.

| Nebula     | Vel.   | Nebula     | Vel.   |
|------------|--------|------------|--------|
| N.G.C. 221 | -300  km |
| 224        | -300  | 598        | +160   |
| 1023       | +300   | 1068       | +1100  |
| 2683       | +400   | 3031       | -30    |
| 3115       | +600   | 3379       | +780   |
| 3521       | +730   | 3623       | +800   |
| 3627       | +650   | 4258       | +500   |
| N.G.C. 4526| +580 km |
| 4565       | +1100  |
| 4594       | +1100  |
| 4649       | +1090  |
| 4735       | +290   |
| 4826       | +150   |
| 5005       | +900   |
| 5955       | +450   |
| 5194       | +270   |
| 5236       | +500   |
| 5866       | +650   |
| 7331       | +500   |

Figure 2. Graph of apparent velocity vs distance for the spiral nebulae (reproduced from Hubble 1929). Filled circles represent data where solar motion was corrected for individual nebulae; open circles represent data where solar motion was corrected for nebulae in groups.
Appendix

THIRTIETH GENERAL ASSEMBLY

RESOLUTIONS PRESENTED TO THE XXXth GENERAL ASSEMBLY

RESOLUTION B4

on a suggested renaming of the Hubble Law

Proposed by the IAU Executive Committee

The XXX General Assembly of the International Astronomical Union,

considering

1. that the discovery of the apparent recession of the galaxies, which is usually referred to as the “Hubble law”, is one of the major milestones in the development of the science of Astronomy during the last 100 years and can be considered one of the founding pillars of modern Cosmology;

2. that the Belgian astronomer Georges Lemaître, in 1927 published (in French) the paper entitled “Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques” [1]. In this he first rediscovered Friedman’s dynamic solution to Einstein’s general relativity equations that describes an expanding universe. He also derives that the expansion of the universe implies the spectra of distant galaxies are redshifted by an amount proportional to their distance. Finally he uses published data on the velocities and photometric distances of galaxies to derive the rate of expansion of the universe (assuming the linear relation he had found on theoretical grounds);

3. that, at the time of publication, the limited popularity of the Journal in which Lemaître’s paper appeared and the language used made his remarkable discovery largely unperceived by the astronomical community;

4. that both Georges Lemaître (an IAU member since 1925 [2]) and the American astronomer Edwin Hubble (an IAU member since 1922 [3]) attended the 3rd IAU General Assembly in Leiden in July 1928 and exchanged views [4] about the relevance of the redshift vs distance observational data of the extragalactic nebulae to the emerging evolutionary model of the universe;

5. that Edwin Hubble, in 1929 published the paper entitled “A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae” [5] in which he proposed and derived
the linear distance-velocity relation for galaxies, ultimately including new velocity data
in his 1931 paper with Humason [6]. Soon after the publication of his papers, the cosmic
expansion became universally known as the “Hubble law”;

6. that, in 1931, on invitation by the Journal Monthly Notices of the Royal Astronomical
Society, G. Lemaître translated in English his original 1927 paper [7], deliberately
omitting the section in which he derived the rate of expansion because he “did not find
advisable to reprint the [his] provisional discussion of radial velocities which is clearly
of no actual interest, and also the geometrical note, which could be replaced by a small
bibliography of ancient and new papers on the subject” [8];

desiring

7. to pay tribute to both Georges Lemaître and Edwin Hubble for their fundamental
contributions to the development of modern cosmology;

8. to honour the intellectual integrity of Georges Lemaître that made him value more the
progress of science rather than his own visibility;

9. to highlight the role of the IAU General Assemblies in fostering exchanges of views
and international discussions;

10. to inform the future scientific discourses with historical facts;

resolves

11. to recommend that from now on the expansion of the universe be referred to as the
“Hubble-Lemaître law”.

[1] Annales de la Société Scientifique de Bruxelles, A47, p. 49-59 (1927)
[2] Lemaître, G. 1950, Ann d’ Ap., 13, 344, as translated by David L Block, 2012, in Georges
Lemaître: Life, Science and Legacy, eds. R.D. Holder and S. Mitton, Astrophysics and Space
Science Library, Springer-Verlag: Berlin, Vol. 395, p. 89
[3] IAU Transactions Vol. 1, 1922
[4] Humason (https://www.aip.org/history-programs/niels-bohr-library/oral-histories/4686), as
reported by Sidney van den Bergh, 2011, JRASC, Vol. 105, p. 197
[5] Proceedings of the National Academy of Science, USA, 15, 168 (1929)
[6] "The velocity-distance relation among extra-galactic nebulae", Astrophysical Journal, Vol
74, p. 43-80 (1931)
[7] Monthly Notices of the Royal Astronomical Society, Vol. 91, p.483-490 (1931)
[8] Georges Lemaître, quoted by Mario Livio in Nature, Volume 479, Issue 7372, pp. 171-173
(2011)
References

Abbott, B.P. et al. 2017. A gravitational-wave standard siren measurement of the Hubble constant. *Nature* **551**: 85–88

Ade, P. et al. 2016. Planck 2015 results. XIII. Cosmological parameters. *Astron. & Astrophys.* **594** (A13):1-63

Aghanim, N. et al. 2018. Planck 2018 results. VI. Cosmological parameters. Physics ArXiv preprint https://arxiv.org/abs/1807.06209

Alpher, R. A. and R.C. Herman 1948. On the relative abundance of the elements. *Phys. Rev.* **74**(12): 1737–1742

Aughton, P. 2008. *The Story of Astronomy: from Babylonian Stargazers to the Search for the Big Bang*. Quercus, London.

Belenkiy, A. 2013. The waters I am entering no one yet has crossed: Alexander Friedmann and the origins of modern cosmology. In *Proceedings of the Conference ‘Origins of the Expanding Universe’* (Eds M. Way and D. Hunter) ASP Conf. Ser. **471**: 71-96.

Belenkiy, A. 2015. Discovery of Hubble’s law as a series of type III errors. *The Physics Teacher* **53**: 20-24.

Block, D. 2012. Georges Lemaître and Stigler’s law of eponymy. In *Georges Lemaître; Life Science and Legacy* (eds R.D. Holder and S. Mitton). Astrophysics and Space Science Library **395** Springer, Berlin 89-96.

Carey, J. *The Faber Book of Science*. Faber and Faber. London.

Caws, P. 1965. *The Philosophy of Science: A Systematic Account*. Van Nostrand, New York.

Christianson, G. 1995. *Edwin Hubble: Mariner of the Nebulae*. Chicago University Press, Chicago.

Chen, H., Fishbach, M. and D. E. Holz. 2018. A two per cent Hubble constant measurement from standard sirens within five years. *Nature* **562**: 545–547.

Coles, P. and F. Lucchin 2002. *Cosmology: The Origin and Evolution of Cosmic Structure*. Wiley & Sons, Chichester.

Couderc, P. 1952. *The Expansion of the Universe*. Faber and Faber, London.

de Sitter, W. 1917. On Einstein’s theory of gravitation and its astronomical consequences. Third paper. *MNRAS* **78**: 3-28.

de Sitter, W. 1930a. Proceedings of the RAS. *The Observatory* **53**: 37-39.

de Sitter, W. 1930b. On the magnitudes, diameters and distances of the extragalactic nebulae, and their apparent radial velocities. *Bull. Ast. Inst. Neth.* **5**: 157- 171.
de Sitter, W. 1930c. The expanding universe. Discussion of Lemaître’s solution of the equations of the inertial field. Bull. Astron. Inst. Neth. 5: 211-218.
de Sitter, W. 1931. The expanding universe. Scientia 49:1-10.
Dicke, R. H. et al. 1965. Cosmic black-body radiation. ApJ 142: 414–419.
Duerbeck, H. W. and W.C. Seitter. 2000. In Hubble's shadow: Early research on the expansion of the universe. Act. Hist. Ast. 10: 120-147.
Eddington, A.S. 1923. The Mathematical Theory of Relativity. Cambridge University Press, Cambridge.
Eddington, A.S.1930. On the instability of Einstein’s spherical world. Month. Not. Roy. Astron. Soc. 90: 668-678.
Eddington A.S.1931. The recession of the extra-galactic nebulae. Proc. Roy. Soc. A 133, 3-10
Einstein, A. 1917a. Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie. Sitz. König. Preuss. Akad. 142-152. Or ‘Cosmological considerations in the general theory of relativity’ CPAE 6 (Doc. 43).
Einstein, A. 1917b. Letter to Willem de Sitter, March 12th. CPAE 8 (Doc. 311).
Einstein, A. 1918a. Kritisches zu einer von Hrn. De Sitter gegebenen Lösung der Gravitationsgleichungen. Sitz. König. Preuss. Akad. 270-272. Or “Critical comment on a solution of the gravitational field equations given by Mr. de Sitter” CPAE 7 (Doc. 5).
Einstein A. 1918b, Letter to Felix Klein, June 20th. CPAE 8 (Doc. 567).
Einstein, A. 1922. Bemerkung zu der Arbeit von A. Friedmann “Über die Krümmung des Raumes” Zeit. Phys. 11: 326. Or ‘Comment on A. Friedmann’s paper “On The Curvature of Space” ’ CPAE 13 (Doc. 340).
Einstein, A.1923a. Notiz zu der Arbeit von A. Friedmann “Über die Krümmung des Raumes” Zeit. Phys. 16: 228. Or ‘Note to the paper by A. Friedmann “On the Curvature of Space” ’ CPAE 14 (Doc. 51).
Einstein, A. 1923b. Notiz zu der Arbeit von A. Friedmann “Über die Krümmung des Raumes” . The Albert Einstein Archives. Doc. 1-26.
Einstein, A. 1931. Zum kosmologischen Problem der allgemeinen Relativitätstheorie. Sitz. König. Preuss. Akad. 235-237.
Einstein, A. and W. de Sitter. 1932. On the relation between the expansion and the mean density of the universe. Proc. Nat. Acad. Sci. 18 (3): 213-214
Einstein, A. 1946. The structure of space according the the general theory of relativity. Appendix 4 to Relativity: The Special and the General Theory. Princeton University Press, Princeton (4th Ed).
Freedman, W.L. 2017. Cosmology at a crossroads. *Nature Astronomy* **1**: 0121

Friedman, A. 1922. Über die Krümmung des Raumes. *Zeit. Physik*. **10**: 377-386. Available in English translation as ‘On the curvature of space’ *Gen. Rel. Grav.* **31**(12): 1991-2000 (1999).

Gamow, G. 1952. *The Creation of the Universe*. Viking Press, New York.

Greene, B. 2004. *The Fabric of the Cosmos*. Penguin, London.

Harrison, E. 2000. *Cosmology: The Science of the Universe*. Cambridge University Press, Cambridge. 2nd ed.

Harrison, E. 1993. The redshift-distance and velocity-distance laws. *ApJ* **403**: 28-31.

Heckmann, O. 1931. Über die Metrik des sich ausdehnenden Universums. *Nach. Gesell. Wiss. Göttingen, Math.-Phys. Klasse* **2**: 126-131.

Heckmann, O. 1932. Die Ausdehnung der Welt in ihrer Abhängigkeit von der Zeit. *Nach. Gesell. Wiss. Göttingen, Math.-Phys. Klasse* **2**: 181-190.

Hoyt, W.G. 1980. V.M. Slipher (1875-1969). *Nat. Acad. Sci.* **52**: 441-449.

Hubble, E. 1925. Cepheids in spiral nebulae. *The Observatory* **48**: 139-142

Hubble E. 1926. Extragalactic nebulae, *ApJ* **64**: 321-369.

Hubble, E. 1929. A relation between distance and radial velocity among extra-galactic nebulae. *Proc. Nat. Acad. Sci.* **15**: 168-173.

Hubble, E. 1953. The law of red-shifts (George Darwin Lecture). *Mon. Not. Roy. Ast. Soc.* **113**: 658-666.

Hubble, E. and M.L. Humason 1931. The velocity-distance relation among extra-galactic nebulae. *ApJ* **74**: 43-80.

Humason, M. L. 1929. The large radial velocity of N. G. C. 7619. *Proc. Nat. Acad. Sci.* **15**: 167-168.

Humason, M.L. 1965. Interview by Bert Shapiro, Niels Bohr Library & Archives, American Institute of Physics https://www.aip.org/history-programs/niels-bohr-library/oral-histories/4686

Kragh, H. 1996. *Cosmology and Controversy*. Princeton University Press, Princeton.

Kragh, H. 2017. Is the universe expanding? Fritz Zwicky and early tired-light hypotheses. *J. Ast. Hist. Herit.* **20**(1): 2-12.

Kragh, H. 2018. Hubble Law or Hubble-Lemaître Law? The IAU Resolution. Physics ArXiv 1809.02557

Kragh, H. and R. Smith. 2003. Who discovered the expanding universe? *Hist. Sci.* **41** :141-163.
Kuhn, T. 1977. *The Essential Tension: Selected Studies in Scientific Tradition and Change*. University of Chicago Press, Chicago 165-177.

Lakatos, I. 1981. History of science and its rational reconstructions. In *Scientific Revolutions*. Oxford University Press, Oxford (Ed. I. Hacking).

Lambert, D. 2015. *The Atom of the Universe: The Life and Work of Georges Lemaître*. Copernicus Center Press, Krakow.

Lemaître, G. 1927. Un univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Ann. Soc. Sci. Brux.* A47: 49-59. Republished in English translation in (Luminet 2013).

Lemaître, G. 1930. Letter to A.S. Eddington, February. Archives Lemaître, Université Catholique de Louvain.

Lemaître, G. 1931a. A homogeneous universe of constant mass and increasing radius, accounting for the radial velocity of the extra-galactic nebulae. *Mon. Not. Roy. Ast. Soc.* 91: 483-490.

Lemaître, G. 1931b. Letter to W. Smart, March 9th. Correspondence of the Royal Astronomical Society.

Lemaître, G. 1950. Compte rendu de P. Couderc; L’expansion de l’universe. *Ann d’Astro.* 13: 344-345.

Lemaître, G. 1952. Clusters of nebulae in an expanding universe. *Mon. Not. Ast. Soc. SA* 11: 110-117.

Lemaître, G. 1958. Rencontres avec Einstein. *Rev. Quest. Sci.* 129: 129-132.

Livio, M. 2011. Lost in translation: mystery of the missing text solved. *Nature* 479: 171-173.

Longair, M. 2006. *The Cosmic Century: A History of Astrophysics and Cosmology*. Cambridge University Press, Cambridge.

Longair, M. 2019. Observational and astrophysical cosmology 1940-1980. In *The Oxford Handbook of the History of Modern Cosmology* (eds H. Kragh and M. Longair). Oxford University Press, Oxford. 206-244.

Lundmark, K. 1924. The determination of the curvature of space-time in de Sitter’s world, *Mon. Not. Roy. Ast. Soc.* 84: 747-757.

Luminet, J-P. 2013. Editorial note to ‘A homogeneous universe of constant mass and increasing radius, accounting for the radial velocity of the extra-galactic nebulae’. *Gen. Rel. Grav.* 45(8): 1619-1633.
Milne, E. A. 1933. World-relations and the "cosmical constant". *Mon. Not. Roy. Ast. Soc.* **94**: 3-9.

North, J.D. 1965. *The Measure of the Universe; A History of Modern Cosmology*. Dover publications, New York.

North, J.D. 1990. The early years. In *Modern Cosmology in Retrospect*. (Eds B. Bertotti et al.) Cambridge University Press, Cambridge pp 11-30.

Nussbaumer, H. and L. Bieri. 2009. *Discovering the Expanding Universe*. Cambridge University Press, Cambridge.

Nussbaumer H and L. Bieri. 2011. Who discovered the expansion of the universe? *The Observatory* **131**: 394-398.

O’Raifeartaigh, C. 2013. The contribution of V.M. Slipher to the discovery of the expanding universe. In *Origins of the Expanding Universe: 1912-1932* (Eds M.Way and D. Hunter) Ast. Soc. Pacific. Conf. Series **471**: 49-61.

O’Raifeartaigh, C. 2019. Eddington, Lemaître and the discovery of the expanding universe. Physics Arxiv [https://arxiv.org/abs/1907.12297](https://arxiv.org/abs/1907.12297)

O’Raifeartaigh, C., O’Keeffe, M., Nahm, W. and S. Mitton. 2017. Einstein’s 1917 static model of the cosmos: a centennial review. *Eur. Phys. J (H)* **42**(3): 431-474.

Ostriker, J.P. and S. Mitton. 2013. *Heart of Darkness: Unravelling the Mysteries of the Invisible Universe*. Princeton University Press, Princeton.

Peacock. J. A. 2013. Slipher, galaxies, and cosmological velocity fields. In *Origins of the Expanding Universe: 1912-1932* (Eds M.Way and D. Hunter) Ast. Soc. Pacific. Conf. Series **471**: 3-25.

Peebles, P. J. E. 1993. *Principles of Physical Cosmology*. Princeton University Press, Princeton.

Peebles, P.J.E. 2015. Preface to ‘*The Atom of the Universe*’ (Lambert 2015) pp 9-14.

Penzias, A. and R. W. Wilson. 1965. A measurement of excess antenna temperature at 4080 Mc/s. *ApJ* **142**(1): 419–421.

Perlmutter S. et al. 1999. Measurements of Ω and Λ from 42 high redshift supernovae. *ApJ* **517**: 565-586.

Realdi, M. 2019. Relativistic models and the expanding universe. In *The Oxford Handbook of the History of Modern Cosmology* (eds H.Kragh and M. Longair). Oxford University Press, Oxford. 76-119.

Riess, A. G. et al. 1998. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* **116**: 1009-1038
Riess, A. G. et al. 2016. A 2.4% determination of the local value of the Hubble constant. *ApJ* 826: 56.

Robertson, H.P. 1932. The expanding universe. *Science* 76: 221-226

Robertson, H.P. 1933. Relativistic cosmology. *Rev. Mod. Phys.* 5(1): 62-90

Shaviv, G. 2011. Did Edwin Hubble plagiarize? Physics ArXiv:1107.0442

Silberstein L. 1924. Radial velocities and the curvature of space-time. *Nature* 114(2862): 347-348.

Slipher, V. M. 1917. Nebulae. *Proc. Am. Phil. Soc.* 56: 403-409.

Smeenk, C. 2014. Einstein’s role in the creation of relativistic cosmology. In *The Cambridge Companion to Einstein* (eds Michel. Janssen and Christoph Lehner) Cambridge University Press, Cambridge 228-269.

Smith, R.W. 1979. The origins of the velocity-distance relation. *J. Hist. Ast.* 10: 133-164.

Smith R.W. 1982. *The Expanding Universe: Astronomy’s ‘Great Debate’ 1900-1931*. Cambridge University Press, Cambridge.

Smith, R.W. 2019. Observations and the universe. In *The Oxford Handbook of the History of Modern Cosmology* (Eds H. Kragh and M. Longair). Oxford University Press, Oxford. 39-76.

Stigler, S.M. 1980. Stigler’s law of eponymy. *Trans. NY Acad. Sci.* 39: 147-157

Strömgren, G. 1925. Analysis of radial velocities of globular clusters and non-galactic nebulae. *ApJ* 61: 353-363.

Tolman, R.C. 1930. More complete discussion of the time-dependence of the non-static line element for the universe. *Proc. Nat. Acad. Sci.* 16: 409-420

Tolman, R.C. 1931. On the theoretical requirements for a periodic behaviour of the universe. *Phys. Rev.* 38: 1758-1771

Tolman, R.C. 1932. On the behaviour of non-static models of the universe when the cosmological term is omitted. *Phys. Rev.* 39: 835-843

Trimble, V. 1996. H₀: the incredible shrinking constant 1925-1975. *Pub. Ast. Soc. Pac.* 108: 1073-1082

Trimble, V. 2012. Eponyms, Hubble’s Law, and the three princes of parallax. *The Observatory* 132: 33-35.

van den Bergh, S. 2011a. Discovery of the expansion of the universe. *J. Roy. Ast. Soc. Can.* 105: 197.

van den Bergh, S. 2011b. The curious case of Lemaitre’s equation no. 24. *J. Roy. Ast. Soc. Can.* 105: 151–152.
Walker, A. G. 1933. Distance in an expanding universe. *Mon. Not. Roy. Ast. Soc.* 94: 159 – 184.

Way, M. 2013. Dismantling Hubble's legacy? In *Origins of the Expanding Universe: 1912-1932* (Eds M. Way and D. Hunter) Ast. Soc. Pacific. Conf. Series 471: 97-133

Wirtz, C. 1922. Einiges zur Statistik der Radialbewegungen von Spiralnebeln und Kugelsternhaufen. *Ast. Nach.* 215: 349-359

Wirtz, C. 1924. De Sitters Kosmologie und die Radialbewegungen der Spiralnebel *Ast. Nach.* 222: 21-31.