Spin Waves in 2D ferromagnetic square lattice stripe

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Abstract

In this work, the area and edges spin wave calculations were carried out using the Heisenberg Hamiltonian and the tridiagonal method for the 2D ferromagnetic square lattice stripe, where the SW modes are characterized by a 1D in-plane wave vector \( q_x \). The results show a general and an unexpected feature that the area and edge spin waves only exist as optic modes. This behavior is also seen in 2D Heisenberg antiferromagnetic square lattice. This absence of the acoustic modes in the 2D square lattice is explained in [1] by the fact that the geometry constrains for NN exchange inside the square lattice allow only optical modes. We suggest that this unexpected behavior of spin waves in the 2D square lattice may be useful in realizing an explanation for HTS.

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I. INTRODUCTION

Spin waves in 2D magnetic systems are very interesting both experimentally [2-6] and theoretically [7-13]. For example, these systems are relevant to our understanding of high temperature superconductors [14-20], and are the basis of many technological applications of ultrathin ferromagnetic films (e.g., magnetic memory and storage devices, switches, giant magnetoresistance, etc), as well as in the new promising field of spintronics (see [21-26]).

Many theoretical techniques has been used to study spin waves (SWs) in 2D and 3D Heisenberg magnets [10]. Some examples are the Holstein-Primakoff (HP) method [1], the “boson mean-field theory” [7, 8] where Schwinger bosons are used to represent the spin operators, and the “modified spin-wave theory” [11] where the Dyson-Maleev transformation is used to represent the spin operators. Additionally, the semi-classical approaches [27, 28] are widely employed for SWs at long wavelengths (or small wavevectors). In recent years, ultra-thin magnetic nanostructures have been fabricated and studied extensively [22, 29-32], for their SW dynamics. It is often useful to distinguish between the propagating SW modes and the localized SW modes. In a film geometry these are usually referred to as ”volume” (or ”bulk”) modes and ”surface” modes respectively. Thus, new theoretical studies are needed where the surfaces (and/or interfaces) are important and where the localized SW modes are considered in detail.

In this work our aim is to study SW modes at low temperatures (compared to the Curie temperature $T_c$). We do this for ultra-thin ferromagnets with typically one atom thickness, specifically a 2D stripe with a finite number of atomic rows (a nanoribbon). We employ the Heisenberg model assume, for simplicity, a square lattice. We study both the “area” SWs (that propagate across the stripe width) and localized “edge” SWs. This work is very interesting for its expected novel fundamental physics and promising application in magnetic devices.

An operator equation-of-motion technique known as the tridiagonal matrix method [33-36] will be conveniently employed here to calculate the SW spectra and, in particular, to distinguish between the edge modes and area modes of the ferromagnetic square lattice nanoribbons.

II. THEORETICAL MODEL

The system initially under study is a 2D Heisenberg ferromagnetic stripe (or nanoribbon) in the xy-plane. We assume a square lattice with lattice constant $a$ and we take the average spin
alignment of the magnetic sites to be in the \( z \) direction, which is also the direction of the applied magnetic field. The nanoribbon is of finite width in the \( y \) direction with \( N \) atomic rows (labeled as \( n = 1, \cdots , N \)) and it is infinite in the \( x \) direction (\( -\infty \leftrightarrow \infty \)). The position vector for each site is given by \( \mathbf{r} = a(m,n,0) \), where \( m \) is an integer from \( -\infty \) to \( \infty \), and \( n \) is the row number with \( n = 1, 2, \cdots , N \) (see Figure 1).

![Diagram of a 2D Heisenberg ferromagnetic square lattice nanoribbon](image)

**FIG. 1.** Geometry of a 2D Heisenberg ferromagnetic square lattice nanoribbon. The spins are in the \( xy \)-plane and the average spin alignment is in \( z \) direction. The nanoribbon is finite in \( y \) direction with \( N \) atomic rows (\( n = 1, \cdots , N \)).

The total Hamiltonian of the system is given by

\[
\hat{H}_{\text{Total}} = -\frac{1}{2} \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - g\mu_B H_0 \sum_i S_i^Z - \sum_i D_i (S_i^Z)^2, \tag{1}
\]

the first term is the Heisenberg NN exchange term, the second term is the Zeeman energy term due to an applied field \( H_0 \), and the third term represents the uniaxial anisotropy. The summations over \( i \) and \( j \) run over all the sites, where the NN exchange \( J_{ij} \) has the “bulk” value \( J \) when either \( i \) and \( j \) are in the interior of the nanoribbon, and a modified value \( J_e \) when \( i \) and \( j \) are both at the edge of the nanoribbon (i.e., in row \( n = 1 \) or \( n = N \)). Similarly, for the uniaxial anisotropy term, we assume a value \( D \) when the site \( i \) is inside the nanoribbon, and a modified value \( D_e \) for sites at the edge of the nanoribbon.

To calculate the SWs for this system at low temperatures \( T \ll T_c \) where the spins are well aligned such that the thermal average of \( S^z \approx S \) for each spin, we use the Holstein-Primakoff (HP) transformation and follow similar procedures to [1] to express the total Hamiltonian in terms of
boson operators. We arrive to the expression

$$\hat{H}_{\text{Total}} = E_0 + \hat{H}_s$$  \hspace{1cm} (2)

where the constant term $E_0$ is the energy of the ground state for the ferromagnetic system given by

$$E_0 = S^2 \left( -\frac{1}{2} \sum_{i,j} J_{i,j} - \sum_i D_i \right) - g\mu_B H_0 \sum_i S,$$

and the operator term $\hat{H}_s$ has the following quadratic form

$$\hat{H}_s = -\frac{1}{2} S \sum_{i,j} J_{i,j} \left( b_i b_j^\dagger + b_j b_i^\dagger - b_j^\dagger b_j - b_i b_i^\dagger \right) + \sum_i \left[ g\mu_B H_0 + (2S - 1)D_i \right] b_i^\dagger b_i$$  \hspace{1cm} (4)

where $b_i^\dagger$ and $b_i$ are the creation and the annihilation boson operators.

In order to diagonalize $\hat{H}_s$ and obtain the SW frequencies, we may consider the time evolution of the creation and the annihilation operators $b_i^\dagger$ and $b_i$, as calculated in the Heisenberg picture in quantum mechanics. In this case, the equation of motion \[37–41\] (using units with $\hbar = 1$) for the annihilation operator $b_i$ is

$$\frac{db_i(t)}{dt} = i \left[ H, b_i(t) \right] = (g\mu_B H_0 + (2S - 1)D_i) b_i(t) - \frac{1}{2} S \sum_{i,j} J_{i,j} \left( b_i(t) - b_j(t) \right)$$  \hspace{1cm} (5)

where the commutation relation between $b_i^\dagger$ and $b_i$ in

$$[b_i, b_j^\dagger] = \delta_{ij}, \quad [b_j^\dagger, b_i] = -\delta_{ij}, \quad [b_i, b_i] = [b_j^\dagger, b_j^\dagger] = 0.$$  \hspace{1cm} (6)

was used, as well as the operator identity $[AB, C] = A[B, C] + [A, C]B$.

The equation of motion for the creation operator $b_i^\dagger$ is easily obtained by taking the Hermitian conjugation of Equation (5), giving

$$\frac{db_i^\dagger(t)}{dt} = -(g\mu_B H_0 + (2S - 1)D_i) b_i^\dagger(t) + \frac{1}{2} S \sum_{i,j} J_{i,j} \left( b_i^\dagger(t) - b_j^\dagger(t) \right).$$  \hspace{1cm} (7)

The dispersion relations of the SWs (i.e., the energy or frequency versus wavevector) can now be obtained by solving the above operator equations of motion. The SW energy is related to the SW frequency using $E = \hbar \omega$, and a Fourier transform for the operators from the time representation to the frequency representation is made:

$$b_j(x, t) = \int_{-\infty}^{\infty} b_j(x, \omega)e^{-i\omega t}d\omega,$$

$$b_j^\dagger(x, t) = \int_{-\infty}^{\infty} b_j^\dagger(x, \omega)e^{-i\omega t}d\omega.$$  \hspace{1cm} (8)
On substituting Equation (8) in Equations 5 and 7, we get

\[
\left( \omega - (g\mu_B H_0 + (2S - 1)D_j) - \frac{1}{2} S \sum_i J_{i,j} \right) b_j(\omega) + \frac{1}{2} S \sum_i J_{i,j} b_i(\omega) = 0,
\]

\[
\left( \omega + (g\mu_B H_0 + (2S - 1)D_j) + \frac{1}{2} S \sum_i J_{i,j} \right) b_j^\dagger(\omega) - \frac{1}{2} S \sum_i J_{i,j} b_i^\dagger(\omega) = 0.
\]  (9)

Since the nanoribbon extends to \( \pm \infty \) in the \( x \) direction, we may introduce a 1D Fourier transform to wavevector \( q_x \) along the \( x \) direction for the boson operators \( b_j^\dagger \) and \( b_j \) as follows:

\[
b_j(x, \omega) = \frac{1}{\sqrt{N_0}} \sum_{q_x} b_n(q_x, \omega) e^{iq_x m a},
\]

\[
b_j^\dagger(x, \omega) = \frac{1}{\sqrt{N_0}} \sum_{q_x} b_n^\dagger(q_x, \omega) e^{iq_x m' a},
\]  (10)

where \( N_0 \) is the (macroscopically large) number of spin sites in any row. The transformed operators obey the following commutation relations:

\[
\left[ b_n(q_x, \omega), b_n^\dagger(q_x', \omega) \right] = \delta_{q_x, q_x'}.
\]  (11)

By substituting Equation (10) into Equation (9) and rewriting the summations, we get the following set of coupled equations:

\[
\left( \omega - (g\mu_B H_0 + (2S - 1)D_j) - \frac{1}{2} S (2J_\epsilon + J) + \frac{1}{2} S J_\gamma(q_x) \right) b_N(q_x, \omega)
+ \frac{1}{2} S J b_{N-1}(q_x, \omega) = 0 \quad \text{for } n = N
\]

\[
\left( \omega - (g\mu_B H_0 + (2S - 1)D_j) - \frac{1}{2} S (4J) + \frac{1}{2} S J_\gamma(q_x) \right) b_n(q_x, \omega)
+ \frac{1}{2} S J (b_{n+1}(q_x, \omega) + b_{n-1}(q_x, \omega)) = 0 \quad \text{for } N > n > 1
\]

\[
\left( \omega - (g\mu_B H_0 + (2S - 1)D_{j'}) - \frac{1}{2} S (2J_\epsilon + J) + \frac{1}{2} S J_\gamma(q_x) \right) b_1(q_x, \omega)
+ \frac{1}{2} S J b_2(q_x, \omega) = 0 \quad \text{for } n = 1.
\]  (12)

The first and the third equations refer to the edges \( n = N \) and \( n = 1 \) for the nanoribbon system, and we have defined \( J_\gamma(q_x) = 2 \cos(q_x a) \). Similar results can be found for the equations involving the creation operator \( b_j^\dagger \).

The above coupled equations can conveniently be written in matrix form as

\[
(-\Omega I + A)b = 0,
\]

\[
(\Omega I + A)b^\dagger = 0,
\]  (13)
where $b$ and $b^\dagger$ are $N \times 1$ column matrices whose elements are the boson operators $b_n(q_x, \omega)$ and $b_n^\dagger(q_x, \omega)$. Also $\Omega = \omega/SJ$ is a dimensionless frequency. The second equation is redundant in that it does not give rise to any new physical modes, so we will therefore ignore it. Here $I$ is the $N \times N$ identity matrix and $A$ is the following tridiagonal $N \times N$ matrix:

\[
A = \begin{pmatrix}
  a_s & -1 & 0 & 0 & \cdots \\
  -1 & a & -1 & 0 & \cdots \\
  0 & -1 & a & -1 & \cdots \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  \cdots & a & -1 & 0 & \cdots \\
  \cdots & -1 & a & -1 & \cdots \\
  \cdots & 0 & -1 & a_s & \\
\end{pmatrix}.
\]

The following dimensionless quantities have been defined:

\[
a_s = \frac{2(g \mu_B H_0 + (2S - 1)D_x) - S(2J_e + J) + SJ\gamma(q_x)}{SJ},
\]

\[
a = \frac{2(g \mu_B H_0 + (2S - 1)D) - S(4J) + SJ\gamma(q_x)}{SJ}.
\]

It is convenient to denote a new matrix by $A' = -\Omega I + A$, so

\[
A' = \begin{pmatrix}
  a'_s & -1 & 0 & 0 & \cdots \\
  -1 & a' & -1 & 0 & \cdots \\
  0 & -1 & a' & -1 & \cdots \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  \cdots & a' & -1 & 0 & \cdots \\
  \cdots & -1 & a' & -1 & \cdots \\
  \cdots & 0 & -1 & a'_s & \\
\end{pmatrix},
\]

where

\[
a'_s = \frac{-\omega + 2(g \mu_B H_0 + (2S - 1)D_x) - S(2J_e + J) + SJ\gamma(q_x)}{SJ},
\]

\[
a' = \frac{-\omega + 2(g \mu_B H_0 + (2S - 1)D) - S(4J) + SJ\gamma(q_x)}{SJ}.
\]

The new tridiagonal matrix $A'$ may be separated into two terms, following the approach in \[35, 36, 42\], as

\[
A' = A_0 + \Delta,
\]
where
\[
A_0 = \begin{pmatrix}
a' & -1 & 0 & 0 & \cdots \\
-1 & a' & -1 & 0 & \cdots \\
0 & -1 & a' & -1 & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\cdots & a' & -1 & 0 & \cdots \\
\cdots & -1 & a' & -1 & \cdots \\
\cdots & 0 & -1 & a' & \cdots 
\end{pmatrix},
\]
(19)
\[
\Delta = \begin{pmatrix}
\Delta & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\cdots & 0 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & \Delta & \cdots 
\end{pmatrix},
\]
(20)
and the element \( \Delta = \alpha' - a' \). In this way all the edge properties have been separated into the matrix \( \Delta \). The inverse of a finite-dimensional tridiagonal matrix with constant diagonal elements such as \( A_0 \) is well known \([34, 36]\) and can be expressed as
\[
(A_0^{-1})_{ij} = \frac{x^{i+j} - x^{j-i} + x^{2N+2-(i+j)} - x^{2N+2-(j-i)}}{(1 - x^{2N+2})(x - x^{-1})}. \tag{21}
\]
Here \( x \) is a complex variable defined such that \( |x| \leq 1 \) and \( x + x^{-1} = a' \).

On noting that \( A' = (A_0 + \Delta) = A_0(1 + A_0^{-1}\Delta) \), the dispersion relations are obtained by the condition \([28, 42]\) that \( \det A' = 0 \), which implies
\[
\det(I + A_0^{-1}\Delta) = 0. \tag{22}
\]

Using the previous equations, the matrix \( M = (I + A_0^{-1}\Delta) \) can next be written in a partitioned form \([43]\):
\[
M = \begin{pmatrix}
M_{1,1} & 0 & M_{1,N} \\
M_{2,1} & M_{2,N} \\
\vdots & \ddots & \vdots \\
M_{N-1,1} & M_{N-1,N} \\
M_{N,1} & 0 & M_{N,N}
\end{pmatrix}, \tag{23}
\]
where the nonzero elements of $M$ can be written as

$$M_{i,j} = \delta_{i,j} + \delta_{1,j}(A_0^{-1})_{i,j} \Delta + \delta_{N,j}(A_0^{-1})_{i,N} \Delta,$$  \hspace{1cm} (24)

and the determinant of $M$, which is required for Equation (22), can be calculated to give

$$\det(M) = (M_{1,1})^2 - (M_{1,N})^2 = \left(1 + \frac{x^2 + x^{2N} - x^{2N+2} - 1}{(1 - x^{2N+2})(x - x^{-1}) \Delta}\right)^2 - \left(\frac{2x^{N+1} - x^{N-1} - x^{N+3}}{(1 - x^{2N+2})(x - x^{-1}) \Delta}\right)^2.$$  \hspace{1cm} (25)

After some more algebraic steps the condition for $\det(M) = 0$ can be written as

$$\left[\left(1 - x^{2N+2}\right)(x - x^{-1}) + \left(x^2 + x^{2N} - x^{2N+2} - 1\right)\Delta\right] + \eta \left(2x^{N+1} - x^{N-1} - x^{N+3}\right) \Delta = 0,$$  \hspace{1cm} (26)

where $\eta = \pm 1$. This result is formally similar to an expression obtained in the study of finite thickness ferromagnetic slabs [34]. Also, it follows by analogy with previous work [34, 35] that the solutions with $|x| = 1$ correspond to the area modes (those propagating across the width of the stripe) while those with $|x| < 1$ correspond to the localized edge modes.

Before presenting a general analysis of Equation (26) for the SW frequencies in Section III, we next examine some special cases.

A. Special case of $N \to \infty$

It is of interest to study the behavior of the model in the special case when the ribbon (or stripe) becomes very wide. This is the limit of $N \to \infty$ in Equation (26). Since $|x| < 1$ for edge modes, the terms of order $x^N \to 0$ as $N \to \infty$, giving for these modes

$$(1 + x \Delta) = 0 \quad \Rightarrow \quad x = -\frac{1}{\Delta}.$$  \hspace{1cm} (27)

This is the same formal expression as obtained in the case of a semi-infinite Heisenberg ferromagnet [34, 35] when $N \to \infty$.

For the edge mode localization condition $|x| < 1$ to be satisfied in this special case, Equation (27) implies $|\Delta| > 1$, which gives two possibilities. The first one is $\Delta > 1$ and the second one is $\Delta < -1$.

From the definitions of $a'_s$ and $a'$ we have

$$\Delta = \frac{2(2S - 1)(D_e - D) - S(2J_e - 3J) + S(J_e - J)\gamma(q_e)}{SJ},$$
which depends on the physical parameters $D_e$, $D$, $S$, $J_e$ and $J$ of the ferromagnetic stripe and on the wavevector component $q_x$. Since $\gamma(q_x) = 2 \cos(q_x a)$ has its maximum when $\cos(q_x a) = 1$ and its minimum when $\cos(q_x a) = -1$, it follows that the minimum value $\Delta_{\text{min}}$ and the maximum value $\Delta_{\text{max}}$ for $\Delta$ correspond to

$$\Delta_{\text{max/min}} = \frac{2(2S - 1)(D_e - D) - S(2J_e - 3J) \pm 2S(J_e - J)}{SJ}$$

In simple cases we might have $D_e = D$ i.e., the edge perturbation to the anisotropy is negligible. Then, denoting the ratio between the edge exchange and area exchange by $r = J_e / J > 0$, we have

$$\Delta_{\text{max}} = 1 \quad \Delta_{\text{min}} = -4r + 5.$$

The two cases $\Delta > 1$ and $\Delta < -1$ give the following ranges for $r$:

$$r < \frac{5}{4} \quad r > \frac{6}{4}.$$  \hspace{1cm} (28)

These are the ranges of the ratio exchange $r = J_e / J$ for which the edge modes exist at some $q_x$ value in this special case ($N \to \infty$) for the square lattice.

The conditions are modified if edge perturbations in the anisotropy are included.

**B. Case of large finite $N$**

Another interesting case is when $N$ becomes sufficiently large that the two solution $x^+$ and $x^-$ for $\eta = \pm 1$ of Equations (26) can be obtained by an iterative approach used in reference [34]. Since $|x| < 1$ for edge modes, all terms of order $x^N$ in Equations (26) are small for sufficiently large $N$ and the two solution $x^+$ and $x^-$ become closer to the solution $x_0 = -\Delta^{-1}$ for the special case of $N \to \infty$.

To use a first order iteration for Equations (26) we must rewrite them in the forms $x^\pm = F^\pm(x_0)$. We get the following two first order iteration approximate solutions

$$x^\pm = -\frac{1}{x_{ap/amp}}$$ \hspace{1cm} (29)

where $x_{ap}$ is equal to

$$x_0^{2N+3} + \Delta x_0^{2N+2} - x_0^{2N+1} - \Delta x_0^{2N} + \Delta x_0^{N+3} - 2\Delta x_0^{N+1} + \Delta x_0^{N-1} + \Delta$$

and $x_{am}$ is equal to

$$x_0^{2N+3} + \Delta x_0^{2N+2} - x_0^{2N+1} - \Delta x_0^{2N} - \Delta x_0^{N+3} + 2\Delta x_0^{N+1} - \Delta x_0^{N-1} + \Delta.$$
FIG. 2. Calculated values of $x^+$ and $x^-$ for several values of $\Delta < -1$ and for $N$ from 10 to 100.

This approximation is valid provided $|\Delta| > 1$. Figure 2 shows calculated values of $x^+$ and $x^-$ using illustrative values of $\Delta$ chosen as $-1.05$, $-1.1$ and $-1.3$ when $N$ has values from 10 to 100. The iterative approach is found to work well for $N > 10$ in this case.

III. NUMERICAL CALCULATIONS

More generally, the dispersion relations can be obtained by solving Equation (26) using a numerical calculation for any finite $N$. The number of rows $N$ and the value of $\Delta$ are first substituted in the Equations (26), and then the polynomial equations are solved for $x$ which can be used to obtain the dispersion relations. Since the solutions for $x$ may have complex roots, one of the ways to solve such equations is to use Laguerre’s method for finding the roots of polynomials.
By rearranging Equations (26) we have:

First polynomial
\[
x^{2N+4} + \Delta x^{2N+3} - x^{2N+2} - \Delta x^{2N+1} - \Delta x^{N+4} + 2\Delta x^{N+2} - \Delta x^N
\]
\[-\Delta x^3 - x^2 + \Delta x + 1 = 0
\]

Second polynomial
\[
x^{2N+4} + \Delta x^{2N+3} - x^{2N+2} - \Delta x^{2N+1} + \Delta x^{N+4} - 2\Delta x^{N+2} + \Delta x^N
\]
\[-\Delta x^3 - x^2 + \Delta x + 1 = 0
\]

Both polynomials are of degree \(2N + 4\), and they are applicable for all \(N\) equal to 3 and above. We note that there is a special case when \(N = 3\), since the two power indices \(2N + 1\) and \(N + 4\) become equal to 7. The obtained values for \(x\) must satisfy the conditions for physical SW modes mentioned earlier. The edge SW modes are localized on the edge and their amplitudes decay exponentially inside the nanoribbon. This requires that \(x\) must be real and less than 1 for edge modes. The area modes are oscillating waves inside the nanoribbon, and so

\[
x \in \mathbb{R} \text{ and } |x| < 1 \text{ for edge modes}
\]
\[
x = e^{iq_m} \text{ and } |x| \leq 1 \text{ for area modes}
\]

In the previous section the ranges for \(r\) for edge modes to exist were obtained algebraically for the special cases \(N \to \infty\) and large \(N\). We can use numerical calculations to obtain the ranges of \(\Delta\) for smaller \(N\) that satisfies the conditions for which both edge modes and area modes exist.

For that purpose, a Fortran program was written to solve the two polynomials using Laguerre’s method where two subroutines \texttt{zroots} and \texttt{laguer} are adapted from [44].

The values for minimum positive (P) and maximum negative (N) of \(\Delta\) for even (E) and odd (O) rows number \(N\) that satisfy (32) are computed from the first polynomial (F) 30 and from the second polynomial (S) 31 and displayed in figure 3 for edge modes and in figure 4 for area modes.

Figure 3 shows the behavior of minimum positive for \(\Delta\) that satisfies the conditions (32) for the existence of edge modes. It is clear from the figure, that in the range of rows number \(N \leq 20\), the minimum positive of both odd rows number of first polynomial 30 and even rows number for second polynomial 31 are approximately the same and are exponentially decaying to a nearly constant value of 1.02. In the same rows number range, the minimum positive of even rows for first polynomial 30 and of odd rows for second polynomial 31 is nearly constant and equal to 0.95.
FIG. 3. The values for minimum positive (P) and maximum negative (N) of $\Delta$ for even (E) and odd (O) rows number $N$ that satisfies edge modes (32), are computed from the first polynomial (F) and from the second polynomial (S).

FIG. 4. The values for minimum positive (P) and maximum negative (N) of $\Delta$ for even (E) and odd (O) rows number $N$ that satisfies area modes (32) are computed from the first polynomial (F) and from the second polynomial (S).
After $N$ equal to 20, the minimum positive of both polynomials is independent of stripe (ribbon) width, (i.e. rows number), and it is also independent of rows number parity, whether even or odd. As the rows number increases, the minimum positive is convergent to an approximate constant value of 1.02.

The same Figure 3 displays the maximum negative of $\Delta$ for first polynomial $30$ and second polynomial $31$. Here, the maximum negative shows different behavior from that of the above minimum positive as it is independent of rows number parity for both polynomial. In the range of rows number $N \leq 20$, as in case of minimum positive, the maximum negative of first polynomial $30$ is nearly constant and equal to -0.96, while the maximum negative of second polynomial $31$ exponentially increases to a nearly constant value of 1.02. As in the case of the minimum positive, after $N$ equal to 20, the maximum negative of both polynomials is independent of stripe (ribbon) width (i.e. rows number), and rows number parity, whether even or odd. As $N$ increases, the maximum negative value is convergent to approximately constant value equal to -1.02.

The conclusion from Figure 3 is that edge modes, in small rows number $N \leq 20$, are dependent on both the stripe width and the rows number parity. This is an indication for the interaction between the two edges in the small range of rows number. As $N$ increases above 20, the edge modes become independent on both the stripe width and the rows number parity. In this case both minimum positive and maximum negative of both polynomials are stripe width and rows number parity independent. This is an indication for disappearing of the interaction between the two edges after $N = 20$. That behavior agrees with result for the special case of $N$ become large, as discussed above. Also, it is noted in the range of $N$ larger than 20, that the difference between minimum positive and maximum negative is nearly constant and independent of the stripe width and rows number parity.

Figure 4 shows the behavior of minimum positive and maximum negative for $\Delta$ that satisfies the conditions (32) for area modes. It is clear that both minimum positive and maximum negative are independent of rows number parity. While they depend on the stripe width, in the range of rows number from 5 to 10 the two values are constant. After $N = 10$, the value of minimum positive is increasing linearly with the stripe width, while the value of maximum negative is decreasing linearly with the stripe width. The difference between minimum positive and maximum negative is increasing as the stripe width increases and it is mostly independent on the rows number parity.

To obtain the dispersion relations for the above system, a Fortran program was written to solving the first polynomial $30$ and the second polynomial $31$ by Laguerre’s method using two subrou-
tines zroots and laguer adopted from [44], which are used before. The values of physical parameters for calculating these dispersion relations are chosen as follow: \( S = 1, \ J = 1, \ D = D_e = 0 \) and \( g\mu_BH_0 = 0.3J \). The chosen value for the ratio between the edge exchange and area exchange is equal to \( r = J_e/J = 0.04 \), which is satisfies the existence condition [28] for edge mode. The chosen values of \( q_xa \) are run from 0 to \( \pi \) corresponding to the first Brillouin zone center and boundary respectively.

IV. SW DISPERSION RELATIONS

The numerical results for calculating the dispersion relations of 2D square lattice using the above algorithm and physical parameters are displayed in Figures 5-8. These figures show plots of SW frequency in terms of the dimensionless quantity \( \omega/SJ \) as a function of the dimensionless wavevector \( q_xa \) for various numbers of rows. The two polynomials [30] and [31] have even power in \( x \), and therefore the area and edge modes are symmetric about \( \omega/SJ = 0 \). As a result, we have chosen to show only the positive frequency branches.

All the figures display similar general features for ferromagnetic 2D stripes. Since every polynomial gives \( N \) area modes, the number of area modes are equal to twice the number of rows \( N \). These area modes are upper bounded by the frequency obtained when the value of \( x \) is equal to \( = 1 \), and lower bounded by the frequency obtained when the value of \( x \) is equal to \( = -1 \), as shown in the Figures. In the upper of area modes, some spin wave frequencies cross each other. As \( N \) increases, the number of areas modes inside their boundary increases twofold, which then merge into an areas modes continuum.

The figures show that in all cases that there are two optic edge modes appearing above the area modes region, these two edge modes look like extension for their counterpart area modes. As \( N \) increases, the difference between the two edge modes is decreasing which is seen too for their counterpart area modes.

V. DISCUSSION

The dispersions relations of area and edges spin waves, and the effect of the stripe width on them for 2D ferromagnetic square lattice stripes, have been studied using the tridiagonal method. The result shows the same unexpected feature: the area and edge spin waves only exist in optic
FIG. 5. Area and edge spin waves modes (in units of $S J$) plotted against the wavevector $q_x a$ for stripe with width $N = 3$, where $x = 1$ and $x = -1$ are the upper and lower boundary for area modes.

FIG. 6. Area and edge spin waves modes (in units of $S J$) plotted against the wavevector $q_x a$ for stripe with width $N = 4$, where $x = 1$ and $x = -1$ are the upper and lower boundary for area modes.
FIG. 7. Area and edge spin waves modes (in units of $S J$) plotted against the wavevector $q_x a$ for stripe with width $N = 7$, where $x = 1$ and $x = -1$ are the upper and lower boundary for area modes.

FIG. 8. Area and edge spin waves modes (in units of $S J$) plotted against the wavevector $q_x a$ for stripe with width $N = 8$, where $x = 1$ and $x = -1$ are the upper and lower boundary for area modes.
modes. This behavior is also seen in 2D Heisenberg antiferromagnetic square lattice experimental and theoretically [46-48], the absence of the acoustic modes could be explained that the square lattice support only optics modes, which need more studies to be completely understood.

Our conclusion, that the unexpected behavior of spin waves in the 2D square lattice of existence in only optic modes if included in the HTS theories may lead to an explanation for HTS. Since it is known that HTS is linked to 2D square antiferromagnetic lattice, and we expect that the Optic spin wave could mediate the electrons using their spin degree of freedom and Pauli exclusion principle for the formation of cooper pair with much less energy than cooper pair created by phonon mediated electrons using their electric charge [15, 49-53].

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