Large Scale Magnetic Fields: Galaxy Two-Point correlation function

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Abstract

We study the effect of large scale tangled magnetic fields on the galaxy two-point correlation function in the redshift space. We show that (a) the magnetic field effects can be comparable to the gravity-induced clustering for present magnetic field strength $B_0 \approx 5 \times 10^{-8}$ G, (b) the absence of this signal from the present data gives an upper bound $B_0 < \sim 3 \times 10^{-8}$ G, (c) the future data can probe the magnetic fields of $\approx 10^{-8}$ G. A comparison with other constraints on the present magnetic field shows that they are marginally compatible. However if the magnetic fields corresponding to $B_0 \approx 10^{-8}$ G existed at the last scattering surface they will cause unacceptably large CMBR anisotropies.

Key Words: Cosmology:theory—Large-scale structure of the universe—Magnetic fields—MHD

1 Introduction

Spatially coherent magnetic fields are ubiquitous in galaxies and galaxies clusters (for a recent review see Widrow 2002). It is not well understood whether these fields are flux-frozen primordial fields or they originated by the dynamo amplification of small seed fields. The existence of magnetic fields at larger scales (\sim 1 Mpc) cannot yet be inferred from direct observations (for a summary of results Kronberg 1994, Widrow 2002). If these large scale primordial fields exist they can serve, depending on their strength, either of these two scenarios. Therefore their existence might be of great importance to understand the way galaxies and clusters formed. Also they can be dynamically important in shaping the large scale structure in the universe (Wasserman 1978).

Wasserman (1978) showed that the large scale magnetic fields can generate density and velocity perturbations which could be responsible for the formation of structures in the universe. Kim, Olinto & Rosner (1996) studied this hypothesis in more detail. In this paper, we attempt to understand the effect of primordial magnetic fields on the galaxy two-point correlation function.

One of the most important diagnostic of structures formation in the universe is the two-point correlation function of the galaxy distribution (Peebles 1980). Recently the two-point correlation function in the redshift space was accurately determined at large scales (\sim 10 Mpc) (Peacock et al. 2001). This gives the best statistical evidence of the large scale velocity flows in the universe. Analysis shows this velocity flow to be consistent with the assumption that the structures in the universe formed from gravitational instability (Hawkins et al. 2002, Peacock et al. 2001). The present and up-coming galaxy surveys like 2dF (Colless et al. 2001) and SDSS (York et al. 2000) are large enough to do precision cosmology. Therefore it is of interest to ask whether galaxy distribution at the present epoch could also be affected by causes other than gravitational instability.

In this paper we estimate the two-point correlation function in redshift space from primordial tangled magnetic field. The velocity flow in the presence of magnetic fields will not be pure gradient as is the case of gravity. Such velocity flows might leave unique geometrical signals in the correlation function. On the other hand lack of such signatures in the data can put meaningful upper bounds on the large scale magnetic fields.

The most direct method to infer the presence of intergalactic magnetic fields for $z \sim 3$ is to study the Faraday rotation of polarized emission from extra-galactic sources (Rees & Reinhardt 1972, Kronberg & Simard-Normandin 1976, Vallée 1990, Kolatt 1998, Blasi, Burles & Olinto 1999). The existence of these fields can also be constrained at the last scattering surface from CMBR anisotropy and spectral distortion measurements (Barrow, Ferreira & Silk 1997, Subramanian & Barrow 1998, Jedamzik, Katalinic & Olinto 2000) and Faraday rotation of the polarized component of the CMBR anisotropies (Kosowsky & Loeb 1996). If magnetic fields existed at even higher redshifts, they can also affect the primordial nucleosynthesis (see e.g. Widrow 2002 for detailed discussion).

In the next section we briefly discuss the equations of motion of the cosmological fluid in the presence of magnetic fields and other preliminaries for this study. In §3 we calculate the two-point correlation function in the redshift space including the effect of magnetic fields. In §4 we present our results and compare them with other constraints on the present magnetic fields. In §5 we give our conclusions. Throughout
this paper we use the currently-favoured background cosmological model: spatially flat with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ (Perlmutter et al. 1999, Riess et al. 1998). For numerical work we use $\Omega_b h^2 = 0.02$ (Tytler et al. 2000) and $h = 0.7$ (Freedman et al. 2001).

2 MAGNETO-HYDRODYNAMICS EQUATIONS

In linearized Newtonian theory the equations of magneto-hydrodynamics for one fluid model \(^1\) in the co-moving coordinates are (Wasserman 1978):

\[
\frac{d(a \mathbf{v}_b)}{dt} = -\nabla \phi + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi \rho_b} \tag{1}
\]

\[
\nabla \cdot \mathbf{v}_b = -a \delta_b \tag{2}
\]

\[
\nabla^2 \phi = 4\pi G a^2 (\rho_{DM} \delta_{DM} + \rho_b \delta_b) \tag{3}
\]

\[
\frac{\partial(a^2 \mathbf{B})}{\partial t} = \frac{\nabla \times (\mathbf{v}_b \times a^2 \mathbf{B})}{a} \tag{4}
\]

\[
\nabla \times \mathbf{v} = 0 \tag{5}
\]

In Eq. \((1)\) we have neglected the pressure gradient term on the right hand side as it is important at Jeans’ length scales ($\ll 1$ Mpc before re-ionization and $\approx 1$ Mpc after re-ionization). Our interest here is to study scales at which the perturbations are linear at the present epoch, $\gg 10 h^{-1}$ Mpc. Eq. \((1)\) and Eq. \((2)\) can be combined to give:

\[
\frac{\partial^2 \delta_b}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta_b}{\partial t} - 4\pi G (\rho_{DM} \delta_{DM} + \rho_b \delta_b) = -\frac{\nabla \cdot (\mathbf{v} \times \mathbf{B})}{4\pi \alpha^2 \rho_b} \tag{6}
\]

Here the subscript ’b’ refers to the baryonic component and the subscript ’DM’ refers to the dark matter. We do not give here the evolution of the dark matter perturbations which is well known (Peebles 1980) and can be solved using the equations above by dropping the magnetic field terms. It was shown by Wasserman (1978) that Eq. \((6)\) admits a growing solution, i.e. tangled magnetic fields can lead to growth of the density perturbation. These solutions are discussed in Appendix A. In writing Eq. \((4)\) we have assumed the medium to have infinite conductivity. It can be simplified further by dropping the right hand side of the equation as it is of higher order, this gives:

\[
\mathbf{B}(x, t) a^2 = \text{constant}. \tag{7}
\]

We assume the tangled magnetic field to be a statistically homogeneous and isotropic vector random process. In this case the two-point correlation function of the field in Fourier space can be expressed as (Landau & Lifshitz 1987):

\[
\langle B_i(q) B_j^*(k) \rangle = \delta_0^3(q - k) \left( \delta_{ij} - q_i q_j / q^2 \right) B^2(q) \tag{8}
\]

3 TWO-POINT CORRELATION FUNCTION IN REDSHIFT SPACE

The density field in the universe is a statistically homogeneous and isotropic random process in the real space. However in redshift space it becomes both statistically inhomogeneous and anisotropic (for a detailed discussion on this point see Hamilton 1998 and reference therein). The measured position of an object, \(s\), is related to the real position, \(r\) at the present redshift as:

\[
s = r + H_0^{-1} (\dot{r} \cdot \mathbf{v}_b) \hat{r} \tag{9}
\]

In linearized theory the density inhomogeneity in redshift space is related to the density inhomogeneity in the redshift space as:

\[
\delta^s(r) = \delta(r) - \hat{r} \nabla + \alpha(r/r) \hat{r} \cdot \mathbf{v}_b \tag{10}
\]

Here $\alpha(r) = \partial \ln r^2 n(r)/\partial \ln r$ is the logarithmic derivative of the survey selection function $n(r)$. Galaxy correlation function is expected to be biased with respect to the underlying density field correlations (Bardeen et al. 1986). Recent results show that optical galaxies are unbiased tracers of the underlying density field at linear scales ($\gg 10$ Mpc) (Lahav et al. 2002, Verde et al. 2002). We take the value of linear bias to be one throughout this paper.

We use the plane parallel approximation in this paper (Kaiser 1987). In this approximation the term containing $\alpha(r)$ in Eq. \((10)\) can be dropped and lines of sight to two different sources on the sky can be taken to be parallel. This is valid either if the sources are far away and/or the angle between the two lines of sight $\theta \ll 1$ (see Hamilton 1998 for details). The two-point correlation function in redshift space can then be written as:

\[
\langle \delta^s(r') \delta^s(r + r) \rangle = \langle \left( \delta(r') - \hat{z} \nabla \cdot \mathbf{v}_0(r) \right) \left( \delta(r + r') - \hat{z} \nabla \cdot \mathbf{v}_0(r + r') \right) \rangle \tag{11}
\]

\(^1\) in this model all the three components of the matter, electrons, protons and the neutral hydrogen particles share the force applied on the small ionized component in the post-recombination era and the small ionized component prevents the magnetic fields from decaying.
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Here $\hat{z}$ is the unit vector in the $z$ direction, which is also chosen to be the line of sight. The density contrast and velocity perturbation in Eq. (11) get contribution from both the magnetic field induced perturbations and gravity-induced perturbations (the solution of the homogeneous part of Eq. (8)). We assume the ‘initial condition’ perturbations to be uncorrelated with the magnetic field induced perturbations. This is justified as the only coupling between these components comes from the potential fluctuation term (Eq. (4)). As $\rho_b \ll \rho_{DM}$ (Peacock et al. 2001, Tytler et al. 2000), this correlation can be neglected or in other words we assume the baryons to contribute negligibly to the potential fluctuations. These simplifications allow us to separate the magnetic field induced density and velocity perturbations. For the rest of this paper we only deal with magnetic field induced perturbations and $\delta_b$ and $v_b$ refer to these perturbations.

The time dependent part of Eqs (11) and (13) can be solved to give (Appendix A):

$$\delta_b(r, t) = g(t) \nabla \cdot (\mathbf{v} \times \mathbf{B})$$  \hspace{1cm} (12)

$$v_b(r, t) = q(t) (\nabla \times \mathbf{B}) \times \mathbf{B}$$  \hspace{1cm} (13)

Eq. (11) has three terms: the density-density correlation, the density-velocity correlations and the velocity-velocity correlation. For fluctuations seeded by magnetic fields the three terms in Eq. (11) can be expanded, using Eq. (8), as:

$$\langle \delta_b(r') \delta_b(r + r') \rangle = g^2(t) \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \exp[(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r}] B^2(k_1) B^2(k_2) \times$$

$$\left[ k_1^2 + 3k_1^2k_2^2 + (\mathbf{k}_1 \cdot \mathbf{k}_2)(k_2^2 - k_1^2) + (\mathbf{k}_1 \cdot \mathbf{k}_2)^2 \left( \frac{k_1^2}{k_2^2} - 3 \right) + \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^3}{k_1^2k_2^2}(k_2^2 - k_1^2) \right]$$

$$\langle \delta_b(r') \mathbf{\hat{z}} \cdot \nabla \mathbf{v}_b(r + r') \rangle = g(t)q(t) \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \exp[(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r}] B^2(k_1) B^2(k_2) \times$$

$$\left[ k_1^4 + 3k_1^2k_2^2 + (k_1^2 + k_2^2)k_1k_2k_2 + ( \mathbf{k}_1 \cdot \mathbf{k}_2 ) \left( 5k_1^2k_2^2 + \frac{k_1^2k_2^2}{k_1^2} + 3k_1k_2k_2 \right) \right] + (\mathbf{k}_1 \cdot \mathbf{k}_2)^2 \left( \frac{k_1^2}{k_2^2} + \frac{k_2^2}{k_1^2} - \frac{k_1k_2}{k_1^2} \right)$$

$$\langle \mathbf{\hat{z}} \cdot \nabla \mathbf{v}_b(r + r') \rangle = g(t)q(t) \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \exp[(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r}] B^2(k_1) B^2(k_2) \times$$

$$\left[ k_1^4 + 3k_1^2k_2^2 + (k_1^2 + k_2^2)k_1k_2k_2 + ( \mathbf{k}_1 \cdot \mathbf{k}_2 ) \left( \frac{2k_1^2k_2^2}{k_1^2} + \frac{2k_1^2k_2^2}{k_2^2} + k_1k_2k_2 \right) \right] + (\mathbf{k}_1 \cdot \mathbf{k}_2)^2 \left( k_1^2 + 9k_1^2k_2^2 + 6k_1^2k_2^2 + 2k_1^2k_2^2 + 6k_1^2k_2^2 + 2k_1^2k_2^2 \right)$$

(14)

Here $k_z = \mathbf{k} \cdot \mathbf{\hat{z}}$ is the angle between the k-mode and the line of sight.

$$\langle \mathbf{\hat{z}} \cdot \nabla \mathbf{v}_b(r) \mathbf{\hat{z}} \cdot \nabla \mathbf{v}_b(r + r') \rangle = g^2(t) \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \exp[(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r}] B^2(k_1) B^2(k_2) \times$$

$$\left[ k_1^4 + 3k_1^2k_2^2 + (k_1^2 + k_2^2)k_1k_2k_2 + ( \mathbf{k}_1 \cdot \mathbf{k}_2 ) \left( \frac{2k_1^2k_2^2}{k_1^2} + \frac{2k_1^2k_2^2}{k_2^2} + k_1k_2k_2 \right) \right] + (\mathbf{k}_1 \cdot \mathbf{k}_2)^2 \left( k_1^2 + 9k_1^2k_2^2 + 6k_1^2k_2^2 + 2k_1^2k_2^2 + 6k_1^2k_2^2 + 2k_1^2k_2^2 \right)$$

(15)

For studying fluctuations in redshift space, quantities of interest are the angular moments of the redshift space two-point correlation function along the line of sight (Hamilton 1992):

$$\xi_c(r) = \int d\mu_r P_c(\mu_r) \langle \delta^*(r') \delta^*(r + r') \rangle$$

(16)

Here $\mu_r = \mathbf{r} \cdot \mathbf{\hat{z}}$ is the angle between the separation between the two points and the line of sight and $P_c(\mu_r)$ are Legendre polynomials. Integrating Eqs (14), (15), and (16) over angles and writing in terms of moments about the line of sight, we get:

$$\langle \delta_b(r') \delta_b(r + r') \rangle = g^2(t) \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} B^2(k_1) B^2(k_2) P_0(\mu_r) \times$$

$$\left[ j_0(k_1r) j_0(k_2r) \left( \frac{2}{3} k_1^4 + 2k_1^2k_2^2 \right) + j_2(k_1r) j_2(k_2r) \left( \frac{2}{3} k_1^4 - 2k_1^2k_2^2 \right) \right]$$

(17)

$$\langle \delta_b(r') \mathbf{\hat{z}} \cdot \nabla \mathbf{v}_b(r + r') \rangle = g(t)q(t) \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} B^2(k_1) B^2(k_2) \times$$

$$\left[ P_0(\mu_r) \left[ j_0(k_1r) j_0(k_2r) \left( \frac{1}{9} k_1^4 + \frac{4}{9} k_1^2k_2^2 \right) + j_1(k_1r)j_1(k_2r) \left( \frac{31}{15} k_1^3k_2 \right) + \right. \right.$$

$$\left. j_2(k_1r) j_2(k_2r) \left( \frac{1}{9} k_1^4 + \frac{7}{9} k_1^2k_2^2 \right) + \frac{1}{5} k_1^2k_2 \left( j_1(k_1r)j_1(k_2r) + j_1(k_1r)j_3(k_2r) + j_3(k_1r)j_1(k_2r) + \right. \right.$$

$$\left. \right. + \frac{2}{15} j_3k_2 j_3(k_2r) \right] \right] + P_2(\mu_r) \left[ j_2(k_1r) j_0(k_2r) \left( -\frac{2}{3} k_1^3k_2 - 2k_1^2k_1 - \frac{4}{3} k_1^2k_2^2 \right) \right]$$

(18)
In normalizing the power spectrum we use a sharp-k filter with

\[ -\frac{229}{75} k_1^4 k_2 j_1(k_1 r) j_1(k_2 r) + j_2(k_1 r) j_2(k_2 r) \left( \frac{50}{63} k_1^2 k_2^2 + \frac{8}{63} k_1^4 \right) \\
- j_4(k_1 r) j_2(k_2 r) \left( \frac{8}{35} k_1^2 k_2^2 + \frac{8}{35} k_1^4 \right) \]  

(19)

\[ \langle \mathbf{\nabla} \cdot \mathbf{v}_n(r) \mathbf{\nabla} \cdot \mathbf{v}_n(r + r') \rangle = q^2(t) \int \frac{k_1^2 dk_1}{2\pi^2} \frac{k_2^2 dk_2}{2\pi^2} B^2(k_1) B^2(k_2) \times \]

\[ \left\{ P_0(\mu) \left[ j_0(k_1 r) j_0(k_2 r) \left( \frac{4}{15} k_1^4 - \frac{46}{225} k_1^2 k_2^2 \right) + j_1(k_1 r) j_1(k_2 r) \left( \frac{28}{15} k_1^2 k_2^2 \right) + \right. \right. \]

\[ + j_2(k_1 r) j_2(k_2 r) \left( \frac{1082}{7875} k_1^4 + \frac{441}{1575} k_1^2 k_2^2 \right) + j_3(k_1 r) j_1(k_2 r) \left( \frac{466}{525} k_1^2 k_2^2 - \frac{13}{225} k_1 k_2^2 \right) \]

\[ + \left. j_4(k_1 r) j_2(k_2 r) \left( \frac{107}{175} k_1^4 - \frac{58}{225} k_1^2 k_2^2 \right) + j_5(k_1 r) j_5(k_2 r) \left( \frac{328}{1225} k_1^2 k_2^2 - \frac{704}{11025} k_1 k_2^2 \right) \right. \]

\[ + \frac{48}{175} k_1^2 k_2 j_4(k_1 r) j_5(k_2 r) \right] \left. + P_2(\mu) \left[ j_2(k_1 r) j_0(k_2 r) \left( \frac{16}{21} k_1^4 - \frac{7}{15} k_1^2 k_2^2 \right) \right. \right. \]

\[ + j_0(k_1 r) j_2(k_2 r) \left( \frac{8}{105} k_1^4 + \frac{148}{105} k_1^2 k_2^2 \right) - j_1(k_1 r) j_1(k_2 r) \left( \frac{282}{75} k_1^2 k_2^2 \right) \]

\[ + j_2(k_1 r) j_2(k_2 r) \left( \frac{1136}{11025} k_1^4 + \frac{470}{1029} k_1^2 k_2^2 \right) + j_3(k_1 r) j_3(k_2 r) \left( \frac{312}{245} k_1^2 k_2^2 - \frac{208}{1225} k_1 k_2^2 \right) \]

\[ + j_4(k_1 r) j_2(k_2 r) \left( \frac{288}{1225} k_1^4 - \frac{632}{1575} k_1^2 k_2^2 \right) - \frac{792}{1225} k_1^2 k_2 j_4(k_1 r) j_5(k_2 r) \right] \left. + P_4(\mu) \left[ j_4(k_1 r) j_0(k_2 r) \left( \frac{32}{105} k_1^4 + \frac{96}{175} k_1^2 k_2^2 \right) \right. \right. \]

\[ + j_2(k_1 r) j_2(k_2 r) \left( \frac{832}{18375} k_1^4 + \frac{1536}{1715} k_1^2 k_2^2 \right) + j_3(k_1 r) j_3(k_2 r) \left( \frac{176}{1225} k_1^4 - \frac{488}{1575} k_1^2 k_2^2 \right) \]

\[ + \frac{48}{529} k_1^2 k_2 j_4(k_1 r) j_5(k_2 r) - j_5(k_1 r) j_5(k_2 r) \left( \frac{512}{8085} k_1^4 + \frac{352}{1020} k_1^2 k_2^2 \right) \]

\[ - \frac{48}{315} k_1^2 k_2 j_5(k_1 r) j_5(k_2 r) + \frac{1296}{8575} k_1^2 k_2 j_4(k_1 r) j_5(k_2 r) \right] \left. + \frac{128}{735} k_1^2 k_2 j_5(k_1 r) j_5(k_2 r) + \right. \]

\[ \frac{70128}{473473} k_1^4 j_6(k_1 r) j_2(k_2 r) \right\} \]  

(20)

We normalize the RMS of magnetic field which can be written as:

\[ B_0^2 \equiv \langle B_i(x) B_i(x) \rangle = \frac{1}{\pi^2} \int dk k^2 B^2(k) \]  

(21)

The magnetic field power spectrum is taken to be power law:

\[ B^2(k) = A k^n \]  

(22)

In normalizing the power spectrum we use a sharp-k filter with \( k_c = 1 \text{ Mpc}^{-1} \) (Subramanian & Barrow 2002). This gives:

\[ A = \frac{\pi^3 (3 + n)}{k_c^{(3 + n)}} B_0^2 \]  

(23)

### 4 RESULTS

Our main results are given in Eqs (18), (19) and (20). The quantities of interest, as they are directly measured by observations, are the line-of-sight angular moments of redshift-space correlations (Eq. (17)). As in the case of purely gravitational clustering, the three non-vanishing moments are the zeroth, second, and the forth; these are the coefficient of \( P_0(\mu) \), \( P_2(\mu) \), and \( P_4(\mu) \), respectively. However the ‘geometry’ of magnetic field induced perturbations is more complicated. We collect appropriate terms from Eqs (18), (19) and (20) to calculate the three moments (Eq. (17)).

In Figure 1 we plot the three moments of correlation function for \( B_0 = 5 \times 10^{-8} \text{ G} \) and magnetic spectral index \( n = 1 \) at the present epoch. For comparison the gravity induced moments are also plotted for the currently favoured value of \( \beta = 0.43 \) (Peacock et al. 2001). Note that only scales above 10 h^{-1} \text{ Mpc} are plotted, even though magnetic fields effects are larger at smaller scales. It is because non-linear clustering effects become important at smaller scales. In redshift space the effects of non-linear clustering become dominant at larger scales as compared to the real space (for discussion and references see Hamilton 1998). At scales smaller than \( \approx 10 \text{ h^{-1} Mpc} \), the
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random velocities wash out the information contained in linear coherent flows. In gravitational clustering scenario the second moment of
the correlation function is driven to zero and changes sign below these scales (the finger-of-god effect; see Peacock et al. 2001 for recent
observational evidence of cross-over from coherent flow to random flow). As we anticipate the overall velocity flows to be dominated by
gravitational effects, in keeping with the good observational evidence in its support (Peacock et al. 2001, Hawkins et al. 2002), it is difficult
to get information about magnetic field induced coherent flows at scales smaller than \( \simeq 10 h^{-1} \) Mpc from data.

Can the present data show the evidence of large scales magnetic fields? Currently the largest redshift survey is the 2dF galaxy survey
(Colless et al. 2001). Its most recent analyses contain nearly 2,20000 galaxies (Hawkins et al. 2002). Two kind of statistics commonly used
to infer large scale coherent flows are the angular-averaged two-point function (the zeroth moment) and the ratio of the second and the zeroth
moment (Hawkins et al. 2002). If magnetic field is present with a power law power spectrum then its presence will show itself as oscillations
in the two-point functions. The present data does not show such oscillations, so it can be used to put upper bound on the strength of the
magnetic fields. It should however be borne in mind that interpretation of such statistics is difficult as the incoherent velocity flow continue
to be important, though sub-dominant, at larger scales also (see e.g. Hatton & Cole 1998; Peacock et al 2001). To distinguish the effect of
magnetic field induced flows from uncertainty owing to incoherent flows a typical correlation function resolution of \( \simeq 0.1 \) would be required
for scales \( \gtrsim 10 h^{-1} \) Mpc (Peacock et al. 2001). This already rules out the case shown in Figure 1. The current data constrains the magnetic
field strength to be \( B_0 \lesssim 3 \times 10^{-8} \) G. A better way to detect magnetic field effects or put upper limit on the magnetic field strength might be
to extract the forth moment of the correlation function. As seen in Figure 1 magnetic field effect dominate over the pure gravity contribution
for even a smaller value of magnetic field strength. However it is noisier to extract the forth moment (Hamilton 1998) and it has not so far
been computed from the galaxy data.

From future data like SDSS galaxy survey (York et al. 2000) it might be possible to detect the effects of even smaller magnetic fields on
two-point correlation function. Notwithstanding the effects of incoherent flows and other systematics, the theoretical limit on the error of two
point correlation function can be calculated. Landy and Szalay (1993) showed that by a judicious choice of two-point correlation function
estimator, error on it is given by the Poisson noise, i.e. \( \Delta \xi = 1/n_p(r)^{1/2} \); \( n_p(r) \) is the number of galaxy pairs for separation between \( r \)
and \( r + dr \). It can be written as \( n_p(r) = f(r)n(n-1)/2 \), where \( n \) is the total number of objects in the sample and \( f(r) \) is the fraction of
pairs at a separation \( r \). Taking \( n = 10^6 \) from SDSS galaxy survey and \( f(r) = 10^{-4} \) for some scale of interest, \( \Delta \xi \simeq 10^{-4} \). To compare it
with the magnetic field contribution, we show the expected signal in Figure 2 for a normalization \( B_0 = 1.5 \times 10^{-8} \) G for two values of
magnetic spectral indices. For theoretical errors on correlation function, the SDSS galaxy survey might be able to extract the magnetic field
contribution for \( r \lesssim 30 h^{-1} \) Mpc.

As seen in Figure 2, changing the value of magnetic field spectral index \( n \) doesn’t change our conclusions much. For smaller value of \( n \)
(theoretically \( n > -3 \)) there is more power at larger scales. However, qualitatively the results don’t change.

4.1 Comparison with other constraints

Several methods have been applied to determine the strength intergalactic magnetic fields at \( z \lesssim 3 \). The most direct method is to study
the Faraday rotation of the polarized radiation of the extra-galactic sources. For tangled magnetic fields the average rotation measure (RM)
along any line of sight will be zero but it has non-zero RMS that can be used to constrain the intergalactic magnetic field (for a derivation
see Appendix B). The current upper limit on on RM fluctuations is: \( \lesssim 2 \) rad m\(^{-2} \) for \( z \simeq 3 \) (Vallée 1990). This gives an upper bound:
\( B_0 \lesssim 2-3 \times 10^{-8} \) G (Appendix B). The magnetic field power spectrum can be constrained by correlating the rotation measure of extra-
galactic radio sources (Kollat 1998). Using this approach it was shown that a few hundred radio sources can be used to constrain the
magnetic field strengths \( \lesssim 10^{-9} \) G at scales 10–50 Mpc (Kollat 1998). To compare it with the magnetic field strengths used in this paper
we show in Figure 4 the Magnetic field RMS smoothed over different scales. For \( B_0 = 1.5 \times 10^{-8} \) this method can be used to constrain
magnetic fields for scale \( \gtrsim 10 \) Mpc.

Blasi, Burles and Olinto (1999) obtain bounds on magnetic field strengths at scales from Jeans’ to Horizon scale assuming Lyman-\( \alpha \) clouds to trace the matter inhomogeneities and that the magnetic fields are frozen in the plasma. They showed that magnetic field RMS
smoothed at different scales \( \lesssim 10^{-8} \) G at these scales.

All these constraints rule out \( B_0 \gtrsim 3 \times 10^{-8} \) G which is also ruled out by the current galaxy data. Our requirement that \( B_0 \gtrsim 10^{-8} \) G
for the magnetic field to leave detectable signal at linear scales is marginally acceptable by these bounds.

Subramanian and Barrow (1998,2002) calculated the level of CMBR anisotropies if the tangled magnetic fields existed at the last
scattering surface. They concluded that a magnetic field strength \( B_0 \approx 3 \times 10^{-9} \) G can cause temperature fluctuations at the level \( \approx 10 \) \( \mu \)K
for \( \ell \approx 1000–3000 \). This is comparable with the detected level of anisotropies at these scales (Mason et al. 2002). Therefore the magnetic
field required to make appreciable effect on the two-point correlation function could not have existed at the last scattering surface.

5 CONCLUSIONS

In this paper we investigated the role of primordial tangled magnetic fields in shaping the large scale structure in the universe at present. In
particular we calculated the two-point correlation function in the redshift space in the presence of these fields for linear scales \( \gtrsim 10 h^{-1} \) Mpc
at the present epoch. Our results can be summarized as:
1. Magnetic field contribution to the clustering in redshift space is comparable to the gravity-driven clustering for magnetic field strength $B_0 \simeq 5 \times 10^{-8}$.
2. Present data might have shown the presence of tangled magnetic fields in the two-point correlation function for $B_0 \simeq 3 \times 10^{-8}$.
3. Comparing this with the other bounds on the magnetic field strength at the present epoch such large magnetic field are mostly likely already ruled out. Therefore present data is consistent with this requirement.
4. On-going galaxy surveys like SDSS can probe magnetic field $B_0 \simeq 10^{-8}$.
5. $B_0 \simeq 10^{-8}$ is still too large to be compatible with the CMBR anisotropy constraints on the primordial magnetic fields. Therefore if the magnetic field signature is ever detected in the two-point correlation function, it would imply that these magnetic fields originated in the post-recombination era.

ACKNOWLEDGMENTS

I would like to thank John Barrow, Ofer Lahav, Martin Rees, Tarun Saini, and Kandaswamy Subramanian for useful discussions and suggestions. I would also like to thank the hospitality of Institute of Astronomy, Cambridge, UK, where a part of this work was done.

APPENDIX A

We start with Eq. (6). The Green function for the homogeneous part can be written as:

$$G^+ (t, t') = c(t')u_1 (t) + d(t')u_2 (t)$$  (24)

with

$$c(t) = \frac{1}{a^2(t)} \frac{u_2(t)}{(u_1(t)u_2(t) - u_1(t)u_2(t))}$$  (25)

$$d(t) = \frac{1}{a^2(t)} \frac{u_1(t)}{(u_1(t)u_2(t) - u_1(t)u_2(t))}$$  (26)

Here $u_1(t)$ and $u_2(t)$ are the growing and decaying solutions of the left hand side of Eq. (6). For a spatially flat universe with non-zero cosmological constant:

$$u_1(t) = H \int_0^\infty \frac{da}{H^2 a^3}$$  (27)

$$u_2(t) = H$$  (28)

Here $H$ the expansion rate is:

$$H^2 = H_0^2 \left( \Omega_m (1+z)^3 + \Omega_\Lambda \right)$$  (29)

The solution of Eq. (6) can be written as:

$$\delta(x, t) = -\int dt' G^+ (t, t') \frac{\nabla \cdot (\nabla \times B(x, t')) \times B(x, t')}{4\pi \rho_0 (t')}$$  (30)

Using Eq. (6) the time dependent part of $\delta(x, t)$ can be written as:

$$g(t) = \int dt' G^+ (t, t') \frac{1}{4\pi a^0 (t') \rho_0 (t')}$$  (31)

The time dependent part of the velocity field can be got by directly integrating Eq. (1) and using Eq. (6). For the magnetic field as the source of large scale velocities:

$$q(t) = \frac{1}{a(t)} \int dt' \frac{1}{4\pi a^3 (t') \rho_0 (t')}$$  (32)

The first term on the right hand side of Eq. (11) gives the usual gravitational instability growth.

APPENDIX B

The rotation measure of a source at a redshift $z$ is:

$$RM = 8.1 \text{rad m}^{-2} \left( \frac{n_e (t_0)}{10^{-5} \text{cm}^{-3}} \right) \left( \frac{H_0^{-1}}{1 \text{Mpc}} \right) \int_0^z dz' \frac{(1+z')^2 \mathbf{J}_B (r)}{(\Omega_m (1+z')^3 + \Omega_\Lambda)^{3/2}}$$  (33)
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Here $n_e(t_0)$ is the present (ionized) electron density, $\hat{z}.B(r)$ is the component of magnetic field along the line of sight at the present epoch. As the magnetic field is tangled, the line of sight component has zero mean which implies, $\langle RM \rangle = 0$. The average RMS of the rotation measure is:

$$\langle (RM)^2 \rangle^{1/2} = 8.1 \text{rad} m^{-2} \left( \frac{n_e(t_0)}{10^{-5} \text{ cm}^{-3}} \right) \left( \frac{H_0}{1 \text{ Mpc}} \right) \times \left( \int_0^z dz'' \int_0^{z'} (1+z')^2 (1+z'')^2 \frac{1}{(\Omega_m(1+z')^3 + \Omega_\Lambda)^{1/2}(\Omega_m(1+z'')^3 + \Omega_\Lambda)^{1/2}} \right)^{1/2} \int \frac{d^3 k}{(2\pi)^3} (1 - \mu^2) \exp[i\mu(kr'' - kr')] B^2(k)$$

Using Eq. (33) this expression can be simplified to:

$$\langle (RM)^2 \rangle^{1/2} = 8.1 \text{rad} m^{-2} \left( \frac{n_e(t_0)}{10^{-5} \text{ cm}^{-3}} \right) \left( \frac{H_0}{1 \text{ Mpc}} \right) \times \left( \int_0^z dz'' \int_0^{z'} (1+z')^2 (1+z''')^2 \frac{1}{(\Omega_m(1+z')^3 + \Omega_\Lambda)^{1/2}(\Omega_m(1+z''')^3 + \Omega_\Lambda)^{1/2}} \times \int \frac{d^3 k}{(2\pi)^3} (1 - \mu^2) \exp[i\mu(k(r'' - r''')] B^2(k) \right)^{1/2} \tag{34}$$

Here $\mu = \hat{z}.k$. Carrying out angular integral in the $k$-space one obtains:

$$\langle (RM)^2 \rangle^{1/2} = 8.1 \text{rad} m^{-2} \left( \frac{n_e(t_0)}{10^{-5} \text{ cm}^{-3}} \right) \left( \frac{H_0}{1 \text{ Mpc}} \right) \times \left( \int_0^z dz'' \int_0^{z'} (1+z')^2 (1+z'')^2 \frac{1}{(\Omega_m(1+z')^3 + \Omega_\Lambda)^{1/2}(\Omega_m(1+z'')^3 + \Omega_\Lambda)^{1/2}} \times \int \frac{d^3 k}{(2\pi)^3} j_z(k(r'' - r'')) B^2(k) \right)^{1/2} \tag{35}$$

This expression can be further simplified by using the fact that it falls rapidly as $r \equiv r' - r''$ increases, so most of the contribution comes from small $r$. This allows one to write $r \simeq H^{-1}(z)(z' - z'')$. Making a further change in variable one gets:

$$\langle (RM)^2 \rangle^{1/2} \simeq 8.1 \text{rad} m^{-2} \left( \frac{n_e(t_0)}{10^{-5} \text{ cm}^{-3}} \right) \left( \frac{H_0}{1 \text{ Mpc}} \right) \times \left( \int_0^z dz' (\Omega_m(1+z')^3 + \Omega_\Lambda)^{1/2} H_0 \int_0^{r_{max}} dr \int \frac{d^3 k}{(2\pi)^3} \frac{j_z(kr)(kr')}{kr} B^2(k) \right)^{1/2} \tag{36}$$

For $\Omega_0 h^2 = 0.02$ one gets $n_e(t_0) = 2.3 \times 10^{-7} \text{ cm}^{-3}$ for a fully ionized universe. For magnetic spectral index $n = 1$ and $z = 3$, Eq. (36) can be numerically solved to give:

$$\langle (RM)^2 \rangle^{1/2} = 0.7 \text{rad} m^{-2} h^{-1/2} \left( \frac{B_0}{10^{-8} \text{ G}} \right) \tag{37}$$

For magnetic spectral index $n = -2$, the normalization changes from 0.7 to 1.2.

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Figure 1. The moments of two-point correlation function in redshift space are plotted for $B_0 = 5 \times 10^{-8} \, G$ (with superscript 'B') to indicate magnetic field). Also shown are the three moments for linear gravitational clustering (superscript 'G') for $\beta = 0.43$ (Peacock et al. 2001).

Figure 2. The absolute value of the moments of the two-point correlation function is plotted for two values of magnetic spectral index for $B_0 = 1.5 \times 10^{-8} \, G$. The line styles for different moments is the same as Figure 1. Left Panel: magnetic spectral index $n = 1$, Right Panel: magnetic spectral index $n = -2$. 

\[ r \, (h^{-1} \, \text{Mpc}) \]

\[ \xi(r) \]

\[ B_0 = 5 \times 10^{-8} \, G \]
Figure 3. The RMS of the smoothed magnetic field fluctuations is plotted as a function of the smoothing scale for two values of magnetic spectral index for $B_0 = 1.5 \times 10^{-8} \text{ G}$.