Possible Formation Scenario of the Quark Star of Maximum Mass around $0.7 \, M_\odot$  

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ABSTRACT  

If there exists the quark star of maximum mass $\sim 0.7 M_\odot$ as suggested by recent Chandra observations, we show that the general relativistic collapse of a neutron star of mass greater than the maximum mass of the neutron star with angular momentum parameter $q \equiv cJ/GM^2 > 1$ may lead to such a strange star. Here $J$ and $M$ are the angular momentum and the gravitational mass of the neutron star, respectively. Under the cosmic censorship hypothesis, such a star cannot be a black hole directly. The jet formed in the soft core might explode the outer envelope and leave the quark star of mass $\sim 0.7 M_\odot$. The remnant quark star has $\lesssim 10^{53}$ erg rotational energy so that the formation of the quark star may be related to the central engine of GRBs. The detailed numerical simulations are urgent to confirm or refute this scenario. (version 3 May 2002)  

Subject headings: stars: neutron  

1. Introduction  

Recently Drake et al. (2002) have made Deep Chandra LETG+HRC-S observations of the isolated neutron star candidate RXJ185635-3754 and found that the X-ray spectrum is well-represented by a $\sim 60$ eV black body. Their data contain no evidence for pulsation. The derived column density favors the distance to RXJ185635-3754 to be $D \sim 140$ pc instead of $D \sim 60$ pc (Walter 2001). Although Walter & Lattimer (2002) claim that the distance is $D \sim 120$ pc and the heavy-element atmosphere of Pons et al. (2002) should be taken, we here follow the arguments by Drake et al. (2002).  

From the observed flux, the distance and the observed temperature of RXJ185635-3754, only the radiation radius defined by $R_\infty \equiv R/\sqrt{1 - R_g/R}$ is determined since the observed luminosity and the observed temperature suffer from the gravitational redshift. For the given $R_\infty$, $M$ is given as a function of $R$ as  

$$R_g = \frac{2GM}{c^2} = R(1 - \left(\frac{R}{R_\infty}\right)^2).$$  

(1)  

$M$ is positive only for $0 < R < R_\infty$ and has the maximum value $R_\infty c^2/G3\sqrt{3} = 0.7 M_\odot (R_\infty/5.6 \text{ km})$ at $R = R_\infty/\sqrt{3}$. If $R_\infty$ is $\sim 5.6 \text{ km} D_{140}$ as suggested by Drake et al. (2002), the maximum mass is $\sim 0.7 M_\odot D_{140}$ where $D_{140}$ is $D$ in the unit of 140 pc. Therefore $R_\infty$ can not determine the radius of the star but gives us the upper limit of the mass as well as the upper limit of the radius of RXJ185635-3754.  

X-ray luminosity of RXJ185635-3754 is $\sim 6 \times 10^{31}$ erg/s $D_{140}^2$. If this is due to the accretion from a uniform medium (Bondi & Hoyle 1944) we have the relation as  

$$\pi \left(\frac{2GM}{V^2}\right)^2 \rho V \epsilon = 6 \times 10^{31} \text{ ergs}^{-1} D_{140}^2,$$  

(2)  

where $V$ is the velocity of RXJ185635-3754 and $\epsilon$ is the conversion efficiency of the accreting matter to the radiation and is $\sim 0.3 c^2$ for the quark star with the maximum mass $\sim 0.7 M_\odot$ (Witten 1984). From the above equation we have  

$$\left(\frac{M}{M_\odot}\right)^2 \left(\frac{V}{200 \text{ kmps}^{-1}}\right)^{-3} \left(\frac{n}{10^4 \text{ cm}^{-3}}\right) \left(\frac{\epsilon}{0.3 c^2}\right) = 0.5 D_{140}^2$$  

(3)  

Since the proper motion of RXJ185635-3754 is 0.3 arc-second/yr $\sim 200 D_{140} \text{ kmps}^{-1}$ (Walter 2001)
1 the required number density of the interstellar matter is rather high even for $M \sim M_\odot$. While the column density to RXJ185635-3754 is estimated as $\sim 10^{20}$ cm$^{-2}$ (Drake et al. 2002) so that the high density region should be local with the maximum size of $\sim 100$ AU. Such a local high density region of the size $< 100$ AU is observed in the interstellar medium using 21cm HI line by Frail et al. (1994) so that RXJ185635-3754 may be passing through the high density region now.

If $M$ is $\sim 0.1 M_\odot$, the required density of the interstellar matter is so high that the column density may be in conflict with the observed one or we need very small ($\sim$AU) high density region in the interstellar matter. In the latter case we expect the change of the luminosity in a time scale of 10 days or so, while only $\sim 2\%$ change of the luminosity has been observed in the 19 months (Drake et al. 2002) so that the mass is much greater than $\sim 0.1 M_\odot$. In this Letter, assuming the mass and the radius of RXJ185635-3754 are $\sim 0.7 M_\odot$ and $\sim 3.2$ km, we propose a possible formation scenario of such an object.

2. Spherically Symmetric Model

First such a small radius object can not be the neutron star (Lattimer & Prakash 2000). Even for the softest equation of state of the high density nuclear matter, $R_\infty$ is larger than 10 km (Lattimer & Prakash 2000). Except for the free neutron case (Oppenheimer & Volkoff 1939), the maximum mass of the neutron star is greater than $0.7 M_\odot$ (Lattimer & Prakash 2000), which is in conflict with the maximum mass of RXJ185635-3754 from the observed $R_\infty$. The strange quark star is one of the candidate for such an object like RXJ185635-3754. In the quark star the baryon number is distributed among three quarks $u$, $d$ and $s$ so that the degenerate energy decreases by a factor of 0.89 compared with the proton and neutron nuclear matter (Witten 1984). In this sense the strange quark matter is stable although it contains unstable strange quarks. This reason is essentially the same as that the unstable neutron exists stably in the stable nucleus. In MIT bag model with the large bag constant $B = (245$ MeV)$^4$ suggested from the finite-temperature QCD lattice data (Peshier, Kampfer & Soff 2001), the maximum mass of the quark star $M_{\text{MAX}}$ is given by

$$M_{\text{MAX}} = 0.7 M_\odot \frac{B}{(245 \text{MeV})^4}^{-1/2}, \quad (4)$$

$$R = 3.9 \text{km} \frac{B}{(245 \text{MeV})^4}^{-1/2}, \quad (5)$$

$$\rho_c = 1.5 \times 10^{16} \text{g cm}^{-3} \frac{B}{(245 \text{MeV})^4}, \quad (6)$$

where $R$ and $\rho_c$ are the radius and the central density of the maximum mass quark star (Witten 1984). Note here that the central density of the quark star with $M_{\text{MAX}} \sim 0.7 M_\odot$ is much higher than the central density of the neutron star $\sim 10^{15}$ g cm$^{-3}$. Therefore to make a quark star from the neutron star we must consider how to increase the central density of the neutron star. If the bag constant is smaller, for example $B = (145$ MeV)$^4$ as usual, the maximum mass and the radius are $2 M_\odot$ and 11 km, respectively. However, in this case with $R_\infty$ = 5.6 km, $M$ becomes small $\sim 0.15 M_\odot$ which is in conflict with the X-ray luminosity of RXJ185635-3754 as stated in §2 so that we prefer the larger bag constant.

A naive way to increase the central density of the neutron star is to add the mass to the maximum mass neutron star by accretion from the companion star or the fall back gas in the supernova event (MacFadyen, Woosley, & Heger 2001). However the maximum mass of the neutron star is $1.4 M_\odot \sim 2 M_\odot$ (Lattimer & Prakash 2000), which is at least two times larger than the putative maximum mass of the quark star relevant to RXJ185635-3754. The most plausible outcome of such a collapse is not the formation of the quark star but the black hole. In a sense the soft equation of state of the strange quark matter ($P = (\rho - 4B)/3$) is favorable to the formation of the strong shock in the supernova explosion as shown by Takahara & Sato (1988). If the equation of state is soft, the large density gradient is accumulated to produce the strong shock while for the hard equation of state, the small density gradient is enough to produce the pressure gradient to support the falling gas in the collapse. However the shock is the strongest just before the formation of the black hole, that is, when the mass of the col-

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1Here we assume that the optical counterpart is the same object as RXJ185635-3754 although the luminosity in the optical band can not be explained by the Rayleigh-Jeans tail of $\sim 60$ eV black body spectrum of RXJ185635-3754 (Pons et al. 2002; Drake et al. 2002; Walter & Lattimer 2002).
lapsing star is just a bit smaller than the maximum mass of the neutron star in the spherically symmetric model (Van Riper & Arnett 1978). Since in our case of the formation of the quark star, the mass of the collapsing matter is much larger than the maximum mass of the quark star so that we expect the formation of the black hole. The detailed numerical simulations might be needed to confirm that the shock can not explode the outer envelope and the black hole is formed.

3. Rotating Model

I propose another promising scenario. First let us review the simulations of general relativistic collapse of rotating stars. Nakamura (1981) first performed the numerical simulations of general relativistic collapse of axially symmetric stars of mass 10$M_\odot$ leading to the formation of rotating black holes. In these simulations the equation of state is hard with $\gamma = 2$ in the limit of $\rho \gg \rho_{nuc}$ and the rotation is differential with various values of initial angular momentum (Nakamura 1981). Nakamura & Sato (1981a) performed similar simulations with soft equation of state of $\gamma = 4/3$ for two kinds of rotation laws with various values of angular momentum. The results of these simulations crucially depend on one parameter $q$ defined by

$$q = \frac{cJ}{GM^2}, \quad (7)$$

where $J$ and $M$ are the total angular momentum and the gravitational mass of the initial rotating star, respectively. When $q \lesssim 1$ the black hole is formed irrespective of initial rotation laws, the equation of state and the initial density distribution (Nakamura & Sato 1981b). If $q \gtrsim 1$, the black hole is not formed but the expanding ring, the expanding disk or the jets appear depending on initial rotation laws, equation of state and initial density distribution. This conclusion was confirmed by Stark & Piran (1985) and Shibata (2000).

Physical reason for the above result is as follows: Let us consider the collapsing star of mass $M$ and the angular momentum $J$. The radius $R_b$ when the centrifugal force balances the gravitational force is given by

$$\frac{GM}{R_b^3} \sim \frac{J^2}{M^2R_b^3}. \quad (8)$$

Using $q$, we can express $R_b$ as

$$R_b \sim \frac{GM}{c^2q^2}. \quad (9)$$

Equation (9) means that if $q \lesssim 1$, $R_b$ is smaller than the black hole, which suggests that when the rotation becomes important, the star itself is already inside the black hole. Therefore it is natural that the black hole is formed irrespective of initial rotation laws, the equation of state and the initial density distribution.

In the sense of general relativity, the results of the simulations support the cosmic censorship hypothesis (Penrose 1973) which says that the singularity does not exist outside the event horizon in nature. Under this hypothesis, the Kerr metric with $a/m = q < 1$ is known to be unique as the ultimate space-time of the gravitational collapse (Robinson 1975). Note here that $q$ defined in Eq. (7) and $a/m$ in Kerr metric are the same quantity. If the whole of the collapsing star with $q \gtrsim 1$ becomes a black hole, that is the counter example of the cosmic censorship hypothesis. However numerical simulations have not yet found such a case.

In relation to the formation of the quark star, Nakamura & Sato (1981a,b) showed an interesting case of the jet formation. In their model of A146 which is the general relativistic collapse of the rotating star with $\gamma = 4/3$ equation of state and the initial $q = 1.46$, the jet appeared in the final phase of the collapse. This is in accord with the result of Van Riper & Arnett (1978); Takahara & Sato (1988). In the rotating collapse, when the size of the star becomes $\sim R_b$ in Eq. (9), the collapse perpendicular to the rotation axis halts while the collapse along the rotation axis proceeds. When the equation of state is soft, the large density gradient accumulates to produce the jet. While if the equation of state is hard, this is not so. In reality Nakamura, Oohara & Kojima (1987) performed the simulations of the accreting neutron star with rotation. They used the equation of state with the maximum mass 1.4$M_\odot$ and simulated the collapse of the rotating neutron star of mass 1.9$M_\odot$. They found no jets. The final result was the differentially rotating neutron star.

Recently Baumgarte, Shapiro & Shibata (2000) computed the equilibria of differentially rotating neutron stars. They stated that some of the mod-
els exceed the Kerr limit, that is, \( q > 1 \). If we add certain amount of matter to exceed the maximum mass neutron star keeping \( q > 1 \), the collapse will start. At \( R_b \), the density is estimated as

\[
\rho_b \sim \frac{M}{f R_b^3} = 6.7 \times 10^{16} \text{gcm}^{-3} f^{-1} q^{-6} \left( \frac{M}{3M_\odot} \right)^{-2},
\]

where \( f \sim 1 \) is the form factor of the rotating star. Since \( \rho_b \) can be in the quark phase for appropriate parameters, the equation of state in the central core can be soft at the centrifugal bounce. Then the matter will collapse along the rotational axis to form the jet like A146 in Nakamura & Sato (1981a,b). If the jet is strong enough to explode the outer envelope and leave the central core of mass \( \sim 0.7 M_\odot \), the strange quark star might be formed. The important point here is that if the cosmic censorship hypothesis is correct as suggested by numerical simulations, the whole of the rotating neutron star of mass greater than the maximum mass of the neutron star with \( q > 1 \) can not be a black hole directly while if \( q = 0 \) it is possible to be a black hole directly.

The detailed numerical simulations are urgent to confirm or refute the present scenario of formation of the strange quark star of maximum mass around 0.7 \( M_\odot \).

4. Discussion

Olinto (1987) showed that a seed of strange matter in a neutron star will convert the star into a strange star. As a seed, the quark nugget formed at the QCD phase transition in the early universe (Witten 1984) can be considered. Then the capture cross section of the quark nugget by the neutron star is given by

\[
\sigma \sim \pi R_{NS} \frac{GM_{NS}}{V_{rel}^2} \sim 10^{18} \text{cm}^2 \left( \frac{V_{rel}}{100 \text{kmms}^{-1}} \right)^{-2},
\]

where \( R_{NS}, \ M_{NS} \) and \( V_{rel} \) are the radius of the neutrons star, the mass of the neutron star and the relative velocity, respectively. In order that at least a single quark nugget should be captured in the age of the universe \( (t_U) \) for a certain neutron star, the number density of the quark nugget \( (n_{nugget}) \) should be

\[
n_{nugget} > \frac{1}{\sigma V_{rel} t_U} = 10^{-42} \text{cm}^{-3} \left( \frac{V_{rel}}{100 \text{kmms}^{-1}} \right) \left( \frac{t_U}{10^{10} \text{y}} \right)^{-1}.
\]

Since the mass density of the quark nugget is at most the same as the dark matter density, each mass of the quark nugget \( m_{nugget} \) should be smaller than \( \sim 10^{12} \text{g} \). However such a small mass quark nugget may not be formed due to the evaporation \( (m_{nugget} > 10^{20} \text{g} \text{ for survival}) \) (Bhattacharjee et al. 1993)). Therefore to explain the possible strange quark star of RXJ185635-3754, the quark nugget should have been captured by chance by the progenitor neutron star.

The putative mass of \( \sim 0.7 M_\odot \) is just the mass of the typical white dwarf (Bragaglia Renzi & Bergeron 1995). Therefore if we can compress the white dwarf of density \( \sim 10^6 \text{gcm}^{-3} \) to \( \sim 10^{16} \text{gcm}^{-3} \), the strange quark star of mass \( 0.7 M_\odot \) might be formed. A possible process is the tidal pinching proposed by Luminet & Pichon (1989). If the moderate massive black hole of mass \( \sim 2000 M_\odot \) exists, a white dwarf passing inside the tidal radius of the black hole may be pinched and the density increases by more than a factor 50 (Luminet & Pichon 1989). Although the existence of such an intermediate mass black hole has been suggested (Matsumoto et al. 2002), the number density of the white dwarfs is small \( \sim 0.01 \text{pc}^{-3} \) so that the pinching number in the age of the universe for a single intermediate mass black hole is very small as

\[
N_{pinch} \sim n_{WD} \frac{6GM_{BH}}{c^2} \left( \frac{M_{BH}}{2000 M_\odot} \right)^2 \left( \frac{V_{rel}}{100 \text{kmms}^{-1}} \right)^{-1} \left( \frac{t_U}{10^{10} \text{y}} \right).
\]

Therefore to explain the possible strange quark star of RXJ185635-3754, the white dwarf should have been tidally pinched by the intermediate mass black hole by chance.

In any scenario, if the quark star of mass \( M_{qs} \sim 0.7 M_\odot \) is formed, the huge binding energy \( \sim 4 \times 10^{53} \text{erg} \) is liberated. The major part of this energy may be used to explode the outer envelope of the progenitor neutron star. If, however, only \( \sim 10\% \) of this energy is deposited as the kinetic energy, it is enough to explain the luminous supernova such as SN1998bw. In the rotating model, the
remnant quark star should have an initial value of $q_{qs} \lesssim 1$. Then the rotational energy is estimated as $E_{\text{rot}} \sim (GM_{qs}/Rc^2)^2 q_{qs}^2 M_{qs} c^2 \sim 10^{53}$ erg. This energy is larger than or comparable to the energy needed for GRB so that the formation of the quark star may be related to the central engine of GRB.

I would like to thank Chiba, Inutsuka, Tatsumi and Tsuru for useful comments. This work was supported in part by Grant-in-Aid of Scientific Research of the Ministry of Education, Culture, and Sports, No.14047212 and No.14204024.

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