Research on Image Segmentation Algorithm Based on Information Theory

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Abstract. With the development of science and technology, the requirements of images processing have been greatly improved. When processing complicated images, segmentation can be firstly applied, which makes the characteristics of the image more distinct, and divides the image into multiple regions with homogeneous properties for subsequent processing. And the image segmentation algorithm based on information theory is one significant technology, which can identify, compress, and encode images, with relatively high accuracy. This paper introduces the principles of information theory and image segmentation, and research on image segmentation algorithm based on information theory, hoping to promote the better development of image processing.

1. Introduction

Information theory is mainly based on the communication system. It regards the information circulation system as an abstract whole which includes communication systems such as television, telephone, fax, and radar, as well as genetic systems, visual systems, and human social management system. Information theory is an experience that has been summed up in long-term practice and belongs to a cross-disciplinary discipline. Entropy is an uncertainty density of the probability that occurs an event, which can be used to understand the amount of information contained.

Image segmentation has a high application value, and its application range is also extensive, such as industrial production, product inspection and monitoring systems, military field and image processing. Image segmentation is the extraction of useful pixels in an image and an image-based image segmentation algorithm is also applied. Digital image segmentation is a key step in digital image processing, and the accuracy of segmentation directly affects the effectiveness of subsequent tasks. The entropy-based image segmentation method minimizes the loss of image information [1].

2. Segmentation Algorithm Based on Information Theory

This paper mainly introduces several popular methods of image segmentation using information theory principles, such as the method of image segmentation based on the maximum entropy grey threshold selection, the so-called threshold method, is essentially the pixel of different attributes [2]. The essential purpose is to divide the pixels of a given image into two categories: one belongs to the target and the other belongs to the background.
The grey level of the pixel $(X, Y)$ is denoted as $g(X, Y)$, and the grey level range of the image is denoted as $G = 0, 1, 2, \ldots, L - 1$ and $k$ is set as the segmentation threshold, the segmentation result can be expressed as a binary image function $G_k(X, Y)$, where $g_0, g_1$ are different grey levels [1]:

$$G_k(X, Y) = \begin{cases} 
g_0, & g(X, Y) < k 
g_1, & g(X, Y) \geq k 
\end{cases}$$

It can be seen that the calculation result of the threshold segmentation method depends on the selection of the threshold [3].

2.1. Maximum between-cluster variance method
This method was proposed by Japan’s Nobuyuki Otsuka in 1979. It is derived based on the principle of least squares. The basic idea is to divide the histogram into two groups of thresholds, when the variance of two groups is maximum, the extreme value determination threshold at this time is calculated. The basic idea of the maximum inter-class variance method is to divide the pixels and determine the threshold of the appropriate filaments by maximizing the distance between the various types of divisions [4].

Let the number of pixels of the grey value $i$ in the image be $n_i$, and the total number of pixels $N = \sum_{i=0}^{L-1} n_i$, the probability of occurrence of each greyscale is $p_i = \frac{n_i}{N}$. Set the grayscale $k$ threshold to divide the image into two regions. The pixels with greyscale of $[0, k]$ and the pixels with greyscale of $[k + 1, L - 1]$ belong to regions $A$ and $B$ respectively, then the probabilities of regions $A$ and $B$ respectively are

$$\omega_A = \sum_{i=0}^{k} p_i, \quad \omega_B = \sum_{i=k+1}^{L-1} p_i$$

The average grey level of the areas $A$ and $B$ is

$$\mu_A = \frac{1}{\omega_A} \sum_{i=0}^{k} i p_i, \quad \mu_B = \frac{1}{\omega_B} \sum_{i=k+1}^{L-1} i p_i$$

The average grey level of the whole figure is

$$\mu = \sum_{i=0}^{L-1} i p_i = \omega_A \mu_A + \omega_B \mu_B$$

The overall variance of the two regions is

$$\sigma^2 = \omega_A (\mu_A - \mu)^2 + \omega_B (\mu_B - \mu)^2$$

The maximum entropy segmentation algorithm has a better image segmentation effect on the boundary between the target and the background, but the algorithm has poor processing ability on the image edge. The OTSU segmentation algorithm has strong recognition ability for image edges, but the algorithm is not good for image segmentation with the blurred boundary between target and background. Aiming at the above problems, a maximum entropy image segmentation algorithm based on the maximum inter-class variance is proposed [5]. This algorithm can not only segment the image with blurred boundary between target and background, but also effectively identify the edge of the image. The experimental results show that the algorithm proposed in this paper is better than the traditional maximum inter-class variance algorithm and maximum entropy algorithm for segmentation of images with blurred boundaries between targets and backgrounds and has better validity and robustness [6].

2.2. Consistency criterion method
The consistency criterion method is based on the consistency of grey scales in the target area and the consistency of grey levels in the background area for image segmentation. The consistency criterion measures whether the segmentation method can effectively extract the target from the background, so it is a general guideline for measuring the quality of image segmentation [3].

Let $R_i$ be the segmentation area, $N_i$ is the number of pixels in the region, and $g(X, Y)$ is the grey level of the pixel. For the segmentation threshold $t$, the consistency criterion function is, where $C$ is the normalization factor.
\[ U(t) = 1 - \frac{\sigma_i^2 + \sigma_j^2}{C} \]

\[ \sigma_i^2 = \sum_{(x,y) \in R} (g(x,y) - \mu_i)^2, \quad \mu_i = \frac{\sum_{(x,y) \in R_i} g(x,y)}{N_i} (i = 1, 2) \]

It can be seen that \( 0 \leq U(t) \leq 1 \). When \( U(t) \) is closer to 1, the segmentation has better result [7]. The maximum grey level \( k \) of \( U(t) \) is the optimal threshold.

2.3. Maximum entropy threshold selection

The information entropy of the image reflects the overall overview of the model [8]. If the image contains a target, the amount of information is greatest at the intersection of the target and the background.

Image segmentation is performed using binarization. The ultimate goal is to segment the target from the background, dividing the image into two regions: the target region and the background region, thus obtaining a binary copy [9]. In a multi-grey image containing a target, there must be a grayscale \( k \), which is used as a threshold to make the image optimally binarized.

Let \( k \) divide the multi-gradation image into two regions: the grey value of one region is \([0, k]\), and the probability distribution is \( P(t) = \sum_{i=0}^{k} p(X,Y) \), the entropy of this region is \(-P(t)\ln P(t)\); the grey value of the other region is \([k + 1, L - 1]\), the probability distribution is \( 1 - P(t) \), and the entropy of the region is \(- (1 - P(t)) \ln (1 - P(t)) \) [2]. Then the total entropy is

\[ H(P(t)) = -P(t)\ln P(t) - (1 - P(t))\ln (1 - P(t)) \]

According to information theory, entropy should be maximized when the target is best segmented from the background. Therefore, the maximum \( t \) of \( H(P(t)) \) is the optimal threshold.

The following describes how to use the principle of maximum entropy to select the grey threshold to segment the image based on the maximum inter-class variance method and the consistency criterion method [10].

Let the segmentation threshold obtained by the maximum inter-class variance method be \( t_1 \), then \( H(P(t_1)) = -P(t_1)\ln P(t_1) - (1 - P(t_1))\ln (1 - P(t_1)) \). The segmentation threshold obtained by the consistency criterion method is \( t_2 \), then \( H(P(t_2)) = -P(t_2)\ln P(t_2) - (1 - P(t_2))\ln (1 - P(t_2)) \). In order to make the divided binary image have the largest inter-class variance and maximum consistency at the same time, the selected threshold \( t \) should satisfy \( \min(t_1, t_2) \leq t \leq \max(t_1, t_2) \). Since \( P \) is an increasing function of \( t \), \( \min(P(t_1), P(t_2)) \leq P(t) \leq \max(P(t_1), P(t_2)) \) can be obtained[11]. According to the maximum entropy method, the optimal threshold \( t^* \) should be \( t^* = \arg \max H(P(t)) \).

![Figure 1. The optimal threshold \( H(P(t)) \) varying with \( P(t) \) changing](image)

Since \( 0 \leq P(t_1) < P(t_2) \leq 0.5 \), \( t^* = t_2 \). That is, when \( P(t_1), P(t_2) \in [0,0.5] \), \( t^* = \max(t_1, t_2) \). Similarly, when \( P(t_1), P(t_2) \in [0.5,1.0] \), \( t^* = \min(t_1, t_2) \). It is important to note that when \( P(t_1) \in [0,0.5] \), \( P(t_2) \in [0.5,1.0] \) or \( P(t_2) \in [0,0.5] \), \( P(t_1) \in [0.5,1.0] \). When \( t^* \) is taken, it should satisfy \( P(t^*) = 0.5 \) [12].
3. Conclusion
After entering the information age, computer technology is more and more developed, and the application of information entropy is more comprehensive, which is no longer limited to the field of communication. Information entropy plays a significant role in image segmentation, providing development for image processing. To ensure the better use of information entropy, image segmentation algorithm based on information theory can be applied to image processing. This paper introduces the concept and development of information theory and the definition and application of image segmentation. The image segmentation algorithm based on information theory is studied to ensure the accuracy of image segmentation. Information entropy has specified complexity in the process of calculation; therefore, when researching information-based image segmentation algorithms, it is necessary to continually improve the theory and technology.

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