Dynamics of the superconducting condensate in the presence of a magnetic field.  
Channelling of vortices in superconducting strips at high currents

D. Vodolazov, B.J. Baelus, and F.M. Peeters

Department Natuurkunde,
Universiteit Antwerpen (Campus Drie Eiken),
B-2610 Antwerpen, Belgium

On the basis of the time-dependent Ginzburg-Landau equation we studied the dynamics of the superconducting condensate in a wide two-dimensional sample in the presence of a perpendicular magnetic field and applied current. We could identify two critical currents: the current at which the pure superconducting state becomes unstable ($J_{c1}$) and the current at which the system transits from the resistive state to the superconducting state ($J_{c2} < J_{c1}$). The current $J_{c2}$ decreases monotonically with external magnetic field, while $J_{c1}$ exhibits a maximum at $H^*$. For sufficient large magnetic fields the hysteresis disappears and $J_{c1} = J_{c2} = J_c$. In this high magnetic field region and for currents close to $J$, the voltage appears as a result of the motion of separate vortices. With increasing current the moving vortices form 'channels' with suppressed order parameter along which the vortices can move very fast. This leads to a sharp increase of the voltage. These 'channels' resemble in some respect the phase slip lines which occur at zero magnetic field.

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It is well-known that the resistive state in superconducting wires or stripes with diameter/width less or comparable to the coherence length $\xi$ is realized through the appearance of phase slip centers (PSC). The phenomenological theory of phase slip centers (PSC) was first proposed in Ref. 2. According to this theory a PSC is a region with size of order $\xi$ where the order parameter is strongly suppressed. The normal current density produced by the oscillation of the order parameter in the phase slip center decays on a larger distance scale $\xi_0 \gg \xi$.

Current-voltage characteristics of such a system is usually irreversible. It is possible to distinguish two critical current densities: $J_{c2}$ - current density at which the superconducting state becomes unstable and $j_{c1} < J_{c2}$ - current density below which the phase slip solution does not exist in the system. In recent work it was claimed that the current $j_{c1}$ is defined by the competition between two relaxation times: the relaxation time of the absolute value of the order parameter $\tau_{||}$ and the relaxation time of the phase of the order parameter $\tau_{\phi}$. The phase slip solution does exist when roughly $\tau_{||} / \tau_{\phi}$.

Because an applied magnetic field suppresses the order parameter and leads to the appearance of screening currents in the sample it is naturally to expect that it affects the phase slip process in the superconductor. For example in Ref. 3 it was shown that a parallel (to the direction of the injected current) magnetic field modifies the critical currents $j_{c1}$, $J_{c2}$ and the stair structure of the current-voltage characteristics.

If we apply a perpendicular magnetic field the situation becomes more complicated. In this case the screening currents induced by the magnetic field decreases the current density $j$ on one side of the superconductor and increases $j$ on the other side of the sample (see Fig. 1) and the current density in some part of the stripe becomes smaller than $j_{c1}$ (if total current is equal to $J = j_{c1} W$). In accordance with Ref. 2, in that part of the sample the phase slip process cannot be realized. We should increase the applied current in order to satisfy the condition $j > j_{c1}$ in any point of the line connecting the two opposite sides of the strip. Further increasing $H$ the slope of $j(|x|)$ increases (at current $J = J_{c1} (H)$) and at the moment when the current density on the left side of the sample reaches $j_{c2}$ (see Fig. 1) both critical currents becomes equal to each other $J_{c2} = J_{c1}$. At field $H = H^* \sim (j_{c2} - j_{c1}) / W$.

To check the above predictions we studied the current-voltage characteristics of quasi-two-dimensional superconductors using the generalized time-dependent Ginzburg-Landau (TDGL) equation. The latter was first

FIG. 1: Schematic current density distribution in a superconducting stripe in a perpendicular magnetic field at the critical currents $J_{c1} (H)$ (solid lines) and $J_{c2} (H)$ (dotted lines). In the insert a schematic view of the considered set up is shown.

$J = j_{c1} W$.
written down in the work of Ref.\textsuperscript{2}
\[
\frac{u}{\sqrt{1 + \gamma^2|\psi|^2}} \left( \frac{\partial}{\partial t} + i\varphi + \frac{\gamma^2}{2} \frac{\partial|\psi|^2}{\partial t} \right) \psi = (\nabla - iA)^2\psi + (1 - |\psi|^2)\psi. \tag{1}
\]
and should be supplemented with the equation for the electrostatic potential
\[
\Delta \varphi = \text{div}(\text{Im}(\psi^*(\nabla - iA)\psi)), \tag{2}
\]
which is nothing else than the condition for the conservation of the total current in the wire, i.e. div\(J = 0\). In Eqs. (1,2) all the physical quantities (order parameter \(\psi = |\psi|e^{i\varphi}\), vector potential \(A\) and electrostatic potential \(\varphi\)) are measured in dimensionless units: the vector potential \(A\) and momentum of superconducting condensate \(p = \nabla\varphi - A\) is scaled by \(\Phi_0/(2\pi\xi)\) (where \(\Phi_0\) is the quantum of magnetic flux), the order parameter is in units of \(\Delta_0\) and the coordinates are in units of the coherence length \(\xi(T)\). In these units the magnetic field is scaled by \(H_{c2}\) and the current density by \(j_0 = c\Phi_0/8\pi^2\Lambda^2\xi\). Time is in units of the Ginzburg-Landau relaxation time \(\tau_{GL} = 4\pi\sigma_n\lambda^2/c^2 = 2T\hbar/\pi\Delta_0^2\), and the electrostatic potential \(\varphi\) is in units of \(\varphi_0 = c\Phi_0/8\pi^2\xi\lambda\sigma_n = \hbar/2e\tau_{GL}\) \((\sigma_n\) is the normal-state conductivity). The parameter \(u\) is about 5.79 according to Ref.\textsuperscript{2}. We also put \(A = (0, Hx, 0)\) in Eq. (1,2) because we considered the case when the effect of the current-induced magnetic field is negligible.

In our calculations we mainly used vacuum-superconductor boundary conditions \((\nabla - iA)|\psi| = 0\) and \(\nabla\varphi = 0\) except for the regions where current was injected (see insert in Fig. 1). In those points we used the normal metal-superconductor boundary conditions \(\psi = 0\) and \(\nabla\varphi = -j\).

We found two jumps in the current-voltage characteristics (see Fig. 2). The first one is connected with the transition of the sample from the superconducting to the resistive state (but with \(|\psi| \neq 0\)). The second transition is the one from the resistive to the normal state. The hysteresis connected to the second transition survives till very high magnetic field. The latter is connected with the stability of the normal state carrying current at temperatures lower than the critical temperature (see for example Ref.\textsuperscript{2}). For such a system it was shown that the normal state may exist (at \(T < T_c\)) till very low current densities if there is no finite superconducting nucleus. In our case we inject current through part of the cross-section of the sample (see inset of Fig. 1) and in this way we provide a more optimal condition for the nucleation of superconductivity in the corners of the superconducting sample where the current density is minimal even in normal state. We want to stress that the second hysteresis crucially depends on the geometrical parameters of the sample and it is not the subject of the present paper to study that effect. We only study here the first hysteresis which is almost sample-independent.

In Fig. 3 we present the dependencies of \(J_{c1}(H)\) and \(J_{c2}(H)\) for different widths of the sample. We found that in accordance with our predictions the current \(J_{c1}\) increases at low magnetic fields and at some \(H = H^*\) it becomes equal to \(J_{c2}\). Beyond \(H^*\) that type of hysteresis no longer exists in the system and the critical current \(J_c = J_{c1} = J_{c2}\) decreases with increasing magnetic field. Only for relatively narrow samples with \(W \sim \xi\) there is a second peak in the \(J_c(H)\) dependence. The reason for that peak is probably connected with strong nonlinear effects. Indeed, the current density is related to the momentum of the superconducting condensate \((p = \nabla\varphi - A)\) as \(j = p(1 - p^2)\) in accordance to the Ginzburg-Landau relation. The second peak occurs approximately at a field \(H\), when the first vortices penetrate the sample in absence of injected current. At \(H = H_s\) the value of \(p\) on the edges is close to unity for samples with \(W \sim \xi\). It means that the term \((1 - p^2)\) may be very important in that range of fields. In wider stripes the above effect is negligible and there is no second peak in the \(J_c(H)\)
curve.

For magnetic fields larger than $H_s$, vortices will penetrate the superconductor and occupy the central part of the sample\cite{9,10}. If one injects current into the sample the vortex-filled region starts to move to one side of the sample and when the vortex dome touches the sample boundary the vortices will leave the sample\cite{9,10}. This value of the current is the critical one. It is essential that the distribution of the current density in the vortex dome is practically constant in the resistive state (in narrow samples with $W < \lambda = \lambda^2/d$) even in the absence of bulk pinning\cite{11}. At fields $H > H_s$ the vortex dome occupies almost the whole sample and it means that at $J \approx J_{c1}(0) = j_{c1}W$ the conditions for the occurrence of phase slip lines (or vortex channels) will be fulfilled and it will lead to the appearance of a stair structure in the $I-V$ characteristics and to a sharp increase of the voltage at $J > J_{c1}(0)$\cite{12}. It also follows that at relatively large magnetic field the current should be field-independent. The lower boundary of the field-independent region is defined by the condition that the current density distribution should be almost uniform ($H_{lower} \approx H_s$). The upper boundary $H_{upper}$ depends on how large is magnetic field and how strong the order parameter is suppressed in vortex filled region because according to Refs.\cite{6,7} both these factors affect the value of $j_{c1}$ (it leads to a decrease of it).

We found that the voltage sharply increases at $J > J_c$ when vortex slip lines or vortex channels appear in the sample (compare Fig. 2 and Fig. 4 at $H=0.3$ and $H=0.6$). In contradiction with the above prediction the current at which the vortex channeling occurs decreases with increasing $H$ (see Fig. 2). The most probable reason is that the current density strongly varies over the width of the sample for $H > H_s$ because even close to $H_{c2}$ there will be only three rows of vortices (see Fig. 4).

Our results coincide qualitatively with recent experimental results on the $I-V$ characteristics of carbon nanotube (compare Fig. 13 in Ref.\cite{8} with Fig. 3 in our paper) measured at different magnetic fields. In that work a non-monotonous dependence of $J_{c1}$ on $H$ was observed at low magnetic fields. In Ref.\cite{8} a stair structure in $I-V$ characteristics of Mo/Si multi-layer stripes was found which appears practically at the same value of the injected current, as predicted in this paper, in wide field region.

In conclusion, we found that an external magnetic field strongly affects the resistive state of mesoscopic wires and stripes. It leads to a shrinking of the hysteresis in the current-voltage characteristic at relatively high values of magnetic field. If the magnetic field is perpendicular to the current direction the dependence $J_{c1}(H)$ is a non-monotonous function of $H$ at low magnetic field. At high magnetic fields $J_{c1}$ and $J_{c2}$ monotonically decreases with increasing $H$ for both perpendicular and parallel orientation of the applied magnetic field.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{Contour plot of the order parameter in the resistive state of a superconducting strip at different values of the magnetic field and applied current. The width of the sample is $9\xi$, the length is $40\xi$, and we used the parameter $\gamma = 10$, $\langle j \rangle = J/(j_0W)$.}
\end{figure}

\begin{thebibliography}{12}
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\item[$^\dagger$] Electronic address: peeters@nia.ua.ac.be
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