Calculation of the power requirement for soil cutting by rotary tool blade

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Abstract. The tilling specific energy requirement can be reduced by optimizing design and operating parameters of the rotary tillage tools of rotary tillage machines. The objective of this study was to develop a technique for calculation the power requirement for soil cutting by a blade of the previously proposed rotary tillage tool as function of the rotation angle and the parameters of the rotary tool. The power requirement dependence on this angle and on two dimensionless parameters, the relative depth and the kinematic parameter, was obtained and analyzed. It was shown that the instantaneous power requirement changes its magnitude periodically with the change of the rotation angle. The computational experiment has shown, that instantaneous and maximum dimensionless powers requiring for soil cutting by the blade monotonically increase with the increase of relative depth. In the range of variation of the kinematic parameter from 3 to 5, when the relative depth is doubled, the maximum instantaneous power requirement increases by 1.5-1.6 times, and when it is tripled, the maximum instantaneous power requirement increases by 1.9-2.3 times. The obtained formula can be used for optimizing the constructive and the functional parameters of the rotary tool. Due to the generality of the basic assumptions, the technique can be mainly transferred to other rotary tillage tools as well.

1. Introduction

Rotary tillers and rotary plows allow to set the required degree of soil breaking. Any type of soil, including heavy and soddy soils, can be tilled with high quality in one pass of a tillage machine, by choosing an appropriate ratio (kinematic parameter) of the peripheral speed to the forward speed of the rotary tools.

However, despite of the important advantages, rotary tillers and rotary plows in agriculture have not yet become widespread due to the high expenditure of energy for soil cultivation and lower performance than performance of tillage implements [1]. Therefore reducing the expenditure of energy for tillage by optimizing the structural and operating parameters of the tillage tools of rotary tillers and rotary plows is an actual problem.

There are three main approaches to the problem: experimental, theoretical, and semi-empirical.

Some researchers [2–3] have studied the effect of various operating regimes such as depth, forward speed and angular speed on the power requirement or the torque characteristics of different types of rotary tiller blades in a laboratory soil bin or in a field. Others researchers have studied in a laboratory soil bin or in a field the effect of blade shapes on the power requirement [4–6], the torque characteristics and the specific tilling energies [7–8].
The advantages of the experimental approach include the reliability of the results obtained, and the disadvantages of the experimental approach include excessive cost of the researches and complexity of applying the obtained results to similar conditions.

Many early theoretical studies were well reviewed [9]. A theoretical calculator to obtain the power requirement of a rotary tiller was developed recently [10]. The modern researchers have used the soil-cutting simulation method [11] and Finite Element Method [12–14] for effective optimization of the rotary blades.

The advantages of the theoretical approach include the lack of the need for complex equipment, and the disadvantages include the usage of unsubstantiated assumptions that reduce the reliability of the obtained results.

Semi-empirical relations are constructed as a result of a combination of qualitative considerations (in particular, dimensional considerations) and the treatment of experimental results or other statistics or are derived from other relations of the same nature [15]. Every semi-empirical model is built on the basis of physical laws, but the empirical coefficients of this model are determined by statistical processing of the results of experimental studies (both laboratory and full-scale).

A mathematical model was proposed for the rotary tiller power requirement based on the work done by a blade during the cutting and throwing the soil slice [16]. In the model the influence of different kinds of blades and different soils is considered using two adjustable constants based on the experiments.

The advantages of the semi-empirical approach include the fact that semi-empirical models are easier to be verified compared to theoretical ones, and they are more adequate compared to empirical (statistical) models. The disadvantages include the lack of universality of the obtained results.

The objective of this article is to develop a technique for calculating the instantaneous power requirement for soil cutting with a blade of rotary tillage tool (RTT) as a function of the RTT rotation angle and the RTT constructive and functional parameters.

2. Materials and methods

The RTT consists of the flange, to which four identical flat vanes are attached. These vanes are quarters of the plate, bounded by an ellipse. The blades of the vane minor semi-axes are located in a vertical plane, and the vanes constitute some angle with it. The projection of all the vanes on the plane is a circle of radius $r$, equal to the minor semi-axis of the ellipse. A model sample of the RRT and the RRT row on a shaft of rotary plough are shown in figure 1 a, b. [17]

![Figure 1](image1.png)

**Figure 1.** A model sample of the RRT (a) and the RRT row on a shaft of rotary plough (b).

This rotary tool can be used also in other rotary machines, which perform various agricultural operations. It can effectively work as a furrow opener in seeders [18], as a ridger in rotary cultivators [19] and as an active coulter (power-take-off driven) in ploughing units [20].

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For calculating the RTT strength and optimizing the RTT operation process a mathematical model of the RTT interaction with soil was developed [17]. This model allows to determine the resultant force of soil resistance to cutting by the rotary tool blade.

Several basic assumptions were made for the model development. They are: 1) the RTT moves with a constant forward velocity $v_0$, 2) the rotary tool rotates around the axis of rotation with a constant angular velocity $\omega$, and 3) during operation the blades of rotary tool are immersed to a constant maximal depth $h$ into almost homogeneous soil.

For characterizing the RTT operating regime the two constant dimensionless parameters were used: the ratio of the rotor peripheral velocity to forward speed of the rotary tillage machine (the kinematic parameter) $\lambda = \omega r / v_0$ and the maximal relative depth $\xi = h/r$.

The minor semi-axis blade of a RTT cuts the soil, performing a plane-parallel motion in a vertical plane. The direction of elementary soil reaction force $d\mathbf{R}$ acting on an infinitesimal elementary segment of the blade cutting part are shown in the figure 2.

\[ R_x = \frac{Qr}{\lambda} \int_{\alpha_0}^{\lambda} \frac{(u \cos \alpha + 1) du}{\sqrt{u^2 + 2u \cos \alpha + 1}}, \]  
\[ R_z = \frac{Qr}{\lambda} \int_{\alpha_0}^{\lambda} \frac{u \sin \alpha du}{\sqrt{u^2 + 2u \cos \alpha + 1}}, \]  
where $\alpha_0 = 1 - l(\alpha, \xi)/r$, $l(\alpha, \xi)$ is the length of cutting part of the blade $BD$ corresponding to the rotation angle value $\alpha$ and the maximal depth of the blade equal to $h$ [m], and $Q$ is the average specific force of cutting per unit length of the blade [N/m] (empirical coefficient).

If we choose as the positive direction of the moments of forces the direction coinciding with the RTT rotation direction, then the resultant moment of soil resistance to cutting will be given by the formula

\[ M_o = \frac{Qr^2}{\lambda^2} \int_{\alpha_0}^{\lambda} \frac{u(u + \cos \alpha) du}{\sqrt{u^2 + 2u \cos \alpha + 1}}. \]
The integrals in formulas (1) – (3) are integrals depending on the variable $\alpha$ and on the parameters $\lambda$, $\varphi_0$. They are easily calculated by the well known method of undetermined coefficients. But the obtained explicit expressions are cumbersome, so sequential designations is used.

For this purpose, we introduce the three functions of two variables:

$$F_1(\alpha, u) = (u^2 + 2u\cos\alpha + 1)^{1/2},$$
$$F_2(\alpha, u) = \ln(u + \cos\alpha + F_1(u, \alpha)),$$
$$F_3(\alpha, u) = 0.5(u - \cos\alpha - F_1(u, \alpha)). \quad (4)$$

Then the components of resultant of soil reactions and the resultant moment of soil resistance to cutting by the blade take the following forms [17]:

$$R_c = -Qr(G_1(\alpha, \lambda, \varphi_0)\cos\alpha + G_2(\alpha, \lambda, \varphi_0)\sin^2\alpha)/\lambda,$$
$$R_z = -Qr(G_1(\alpha, \lambda, \varphi_0) - G_2(\alpha, \lambda, \varphi_0)\cos\alpha\sin\alpha)/\lambda,$$
$$M_o = -Qr^2(G_3(\alpha, \lambda, \varphi_0) - 0.5G_2(\alpha, \lambda, \varphi_0)\sin^2\alpha)/\lambda^2, \quad (5)$$

where the functions $G_i(\alpha, \lambda, \varphi_0) = F_i(\alpha, \lambda) - F_i(\alpha, \lambda, \varphi_0)$, $i = 1, 2, 3$.

The sequential expressions (4) – (5) have a very simple structure, and are programmed without any difficulty. They allow to calculate the values of $R_c$, $R_z$, and $M_o$ at any given values of the parameters $\alpha$, $\lambda$, and $\varphi_0$ with the help of a computer or even a programmable calculator.

Through simplifying the system of forces of elementary soil reactions acting on the RTT blade to the center $O$, we get the main vector of this forces $R$ applied at the center $O$, and the pair with principal moment of the elementary reactions $M_o$. The projections of the main vector on the coordinate axes and the magnitude of the main moment are determined by the formulas (5). Therefore, the power that is developed by the system of forces of elementary soil reactions is equal to the power that the specified vector $R$ and the specified pair of forces with the developed moment $M_o$. This power is known to be determined by using the equality

$$W = R_cv_0 + M_o\omega. \quad (6)$$

We note that the force $R$ develops positive power, since its direction coincides with the direction of forward velocity. The pair of forces with a moment $M_o$ develops a negative power, since its moment is directed opposite to the direction of rotation.

To reduce the number of significant parameters, we introduce dimensionless projection, moment, and power by the formulas

$$R'_c = \frac{R_c}{Q\cdot r}; \quad M'_o = \frac{M_o}{Q\cdot r^2}; \quad W' = \frac{W}{Q\cdot r\cdot v_0}. \quad (7)$$

Then the dimensionless instantaneous power of the system of elementary soil reactions is determined by using the relation

$$W' = R'_c + \lambda\cdot M'_o \quad (8)$$

To use the formulas (4) – (8) it is firstly need to determine the length $l(\alpha, \xi)$ of the cutting part of the blade. This determination was considered in an article of the authors [21]. Figure 2, a shows the projection onto the $Oxz$ plane of the blade part interacting with the soil at the initial (first) stage of cutting process, when the blade is in contact only with the uncut soil surface and is not in contact with the soil surface formed by the previous blade. It follows from this figure that the determined length is given by the formula $l(\alpha, \xi) = r + (r - h)/\cos\alpha$. Hence, for the first stage the next formula is valid

$$\varphi_0 = (\xi - 1)/\cos\alpha. \quad (9)$$
Since from the equation \( l = 0 \) the next equality implies \( \phi_0 = 1 \), then the last condition determines the angle of the beginning of soil cutting with a blade. As follows from (9), the soil cutting begins at the angle value \( \alpha_0 = \arccos(\xi - 1) \).

Formula (9) is valid only until the moment of contact of the blade edge with the upper surface of the cut soil slice in the point \( D_0 \) (figure 2, b). At the final (second) stage of soil cutting the edge of the considered RTT blade crosses the upper surface of the slice in some point \( D \) of the trochoid formed by the previous blade. In mentioned above article of the authors it was shown that at this stage the equation (9) is replaced by the next equation

\[
\phi_0 = \cos\alpha' / \cos\alpha.
\]

where the angle \( \alpha' \) is determined from transcendental equation

\[
[2\pi/z_b + (\alpha - \alpha')] \lambda' \cos\alpha + \sin(\alpha - \alpha') = 0,
\]

where \( z_b \) is the number of rotary blades.

The angle \( \alpha' \) is a function of the angle \( \alpha \), which is defined implicitly by the transcendental equation (11). The angle \( \alpha' \) can be found by means of solving the equation (11) using one of the known numerical methods. The substitution of this function into the equality (10) allows us to express \( \phi_0 \) through the angle \( \alpha \). Thus, equalities (10), (11) and (4)–(8) at the final stage of cutting process determine the dimensionless power requirement for cutting as the composite functions of the angle \( \alpha \).

The point \( D_0 \) is located on the soil surface and upper surface of slice. Therefore, if we substitute \( \alpha' = \arccos(\xi - 1) \) into equation (10), we obtain the transcendental equation for determining the angle \( \alpha = \alpha_1 \) of the completion of cutting process of the uncut soil surface (initial stage) and the beginning of cutting process of the upper surface of soil slice (final stage). This equation is solved numerically for different values of \( \xi \).

The value of angle \( \alpha = \alpha_2 \), corresponding to the completion of soil cutting process by a blade of the RTT in the point \( K \) of intersection of cycloids at the bottom of a furrow (figure 2, b), is also determined by using the equation (11). At this point \( \phi_0 = 1 \), \( \alpha' = 2\pi - \alpha_2 \), so we have the other transcendental equation \( \sin\alpha_2 = (\pi - \alpha_2 - \pi/z_b) / \lambda \). The equation is also solved numerically. If we pass from the angle \( \alpha_2 \) to the angle \( \alpha_2' \) by \( \alpha_2' = \alpha_2 - \pi \), then equation (11) transforms into the well-known equation, determining the location of crests at the bottom of a furrow during the operation of a rotary tiller [22].

3. Results and discussion

The technique for calculating the instantaneous power requirement for soil cutting with a blade of RTT as a function of the RTT rotation angle and the RTT constructive and functional parameters was developed, and the objective of this article has been achieved.

To carry out the calculations, the program was written in the internal programming language of the computer mathematics system Maple 17.

For \( z_b = 4 \) the calculation results are presented in the graphical form in the figure 3. In the figure 3, \( \alpha \), there are three graphs of dimensionless required power \( (-W') \) for soil cutting with a RTT blade depending on the angle of rotation of the blade \( \alpha \) for the kinematic parameter \( \lambda = 3 \) and three relative depths \( \xi = 0.2, 0.4, \) and \( 0.6 \).
As follows from this figure, the angle at which the blade edge begins to cut the soil decreases with the increase of $\xi$, and the cutting of the soil ends at the same angle $\alpha$ regardless of the relative depth.

Initially, at the first stage of cutting, the instantaneous power ($-W^*$) increases with an increase in the angle of rotation $\alpha$ with a high rate, then it reaches its maximum value and begins to decrease. The rate of its decrease sharply increases at the final cutting stage, the beginning of which corresponds to the break points on the function graphs ($-W^*$). After the period of $360^\circ/z_b = 90^\circ$, the blade of the next RTT begins to interact with the soil, therefore, the indicated change in instantaneous power ($-W^*$) is repeated again. Figure 3 shows that instantaneous and maximum dimensionless power requirement for soil cutting with a RTT blade are monotonically increasing functions of the parameter $\xi$.

According to calculations, in the range of variation of $\lambda$ from 3 to 5, with an increase in relative depth by a factor of two, the maximum instantaneous power increases by $1.5\text{–}1.6$ times, and when it increases by a factor of three, the maximum instantaneous power increases by $1.9\text{–}2.3$ times.

Figure 3, b presents three graphs of the dimensionless required power ($-W^*$) for cutting the soil with a RTT blade depending on the angle of blade rotation $\alpha$ for the value of the relative depth $\xi = 0.5$ and three values of the kinematic parameter $\lambda = 3, 3.5, 4$. As it follows from this figure, for a constant relative depth and different values of the parameter $\lambda$, soil cutting begins at the same angle $\alpha$, and ends at a different values.

4. Conclusion
On the basis of the developed mathematical model of soil cutting with a blade of RTT, a technique for calculating the instantaneous power requirement for soil cutting depending on the angle of blade rotation and the two dimensionless parameters, the relative depth and the kinematic parameter, has been worked out.

Using the program written on the basis of the constructed technique, the computer experiment was conducted on a personal computer, which showed that:

- instantaneous and maximum dimensionless power requirement for soil cutting with a RTT blade are monotonically increasing functions of the relative depth;
- in the range of variation of $\lambda$ from 3 to 5, when the relative depth is doubled, the maximum instantaneous power increases by $1.5\text{–}1.6$ times, and when it is tripled, the maximum instantaneous power increases by $1.9\text{–}2.3$ times.
The proposed technique allows to select rationally the parameters of the RTT by using the criterion of the minimum specific energy costs for soil cutting and thereby to reduce the energy expenditure of soil cultivation.

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