DMC-PID Cascade Control Algorithm for the Constant Rate Temperature Rising Process in Differential Scanning Calorimeter

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ABSTRACT: In view of the poor tracking performance of conventional control method in differential scanning calorimeter during constant speed heating progress, a DMC-PID cascade control algorithm for differential scanning calorimetry is proposed. Firstly, based on the predictability and optimality of the DMC algorithm, the DMC controller is used in the main circuit to control the generalized object composed of the PID controller and the reheating furnace. Secondly, on the basis of the characteristics that PID controller can improve the dynamic of the controlled object, the PID controller is used in the secondary loop of the control system to improve the linearity of the controlled object. Finally, the proposed control method is applied to differential scanning calorimetry constant temperature control process. Experimental results show that the proposed method can effectively reduce the static error and improve the tracking performance of the control process.

1. Introduction

Differential scanning calorimeter (DSC) is a thermal analysis instrument for measuring the relationship between the heat flux difference of samples and the reference material and temperature or time under the programmed temperature.[1] DSC is widely used in the research and development of materials and food, performance testing and quality control.[2] Using the instrument to conduct thermal analysis experiments, only when the temperature of the differential scanning calorimeter is controlled linearly, the thermodynamic parameters are measured accurate and effective.[3] Therefore, the heating process is required to be stable and not overshoot and the control precision of temperature and temperature change rate of DSC is very important for the application of thermal analyzer.[4]

At present, the most commonly used algorithm for temperature control of reheating furnace is PID control. Du [5] proposed a PID control algorithm with constant set values in small zones for constant
temperature control of the furnace and analyzed the parameter selection principle under the different control conditions. The parameter tuning of the algorithm is complex and the dynamic response performance cannot meet the control requirements of high precision thermal analysis instrument. Zhou [6] proposed a fuzzy PID control algorithm to achieve the constant speed temperature control. The algorithm has high control precision and small overshoot. However, the expert experience is difficult to obtain, and the subjectivity of fuzzy modeling has great influence on control effect. Wang [7] had designed a Smith predictor PI controller for large time-delay controlled objects. The method has fast dynamic response, but the control accuracy and robustness cannot meet the requirements of the instrument. Zhang [8] proposed an acceleration control method and the method was used for the constant temperature control of the high temperature furnace in the laboratory. The control precision of this method depends on the operation experience.

For constant-speed temperature control, the above control algorithms have the problem of poor temperature tracking, that is, there is always a certain static difference between the measured value and the set value. The static error could cause the worse accuracy of DSC in the initial and final stages of heating. That is, in the initial stage, the linearity of the heating curve is poor, and in the final stage of the experiment, there was a problem that the maximum temperature could not be reached.

In this paper, a DMC-PID cascade control algorithm for DSC is proposed, which is based on the characteristics of predictive control and optimal control. The predictive controller is used in the main loop of the control system to reduce the static error between the measured value and the set value and improve the tracking characteristics of the control system. PID controller is used in the secondary loop of the control system. The closed-loop feedback of the PID controller accelerates the dynamic response of the controlled object, improves the linearity of the controlled object and the sensitivity to parameter changes. The control algorithm proposed in this paper is applied to the constant speed temperature rise control of DSC. The experimental results show that the control algorithm can effectively reduce the static error and improve the tracking characteristics of the control process.

2. Dynamic Matrix Control Algorithm
Predictive control is a model-based control method that predicts the future output of an object based on its historical information and future input. State equation, transfer function and step response, can all be used as predictive models. The basic characteristics of the model predictive control algorithm are summarized as follows: model prediction, rolling optimization and feedback correction. The dynamic matrix algorithm, adopting the linear step response as the mathematical model, is the most widely used MPC algorithm.

2.1 Predictive model
We assume the predictive horizon length is $P$, the length of the control horizon is $M$, the model length is $N$ and the step response $a_{ij}(t)$ is the output $y_i$'s response of each input $u_j$. The model vector $a_{ij}(t)=[a_{ij}(1), a_{ij}(2), \ldots, a_{ij}(N)]$, $i=1, 2, \ldots, p$ and $j=1, 2, \ldots, m$ is composed of the sampling points value of $a_{ij}(t)$. First of all, the predictive model of controlled output $y_i$ by the action of controlled input $u_j$ is concerned. When $u_j$ has only one increment $\Delta u_i$, the predicted output value of $y_i$ in future time $N$ is

$$\hat{y}_{i, N}(k) = \hat{y}_{i, 00}(k) + a_{ij} \Delta u_i(k) \quad (1)$$

where

$$\hat{y}_{i, 00}(k) = \begin{bmatrix} \hat{y}_{i, 0}(k+1|k) \\ \vdots \\ \hat{y}_{i, N}(k+N|k) \end{bmatrix}$$

and

$$\hat{y}_{i, 00}(k) = \begin{bmatrix} \hat{y}_{i, 0}(k+1|k) \\ \vdots \\ \hat{y}_{i, P}(k+P|k) \end{bmatrix}$$

From time $k$, if $u_j$ has $M$ continuous incremental changes as $\Delta u_j(k)$, $\Delta u_j(k+1)$ $\cdots \Delta u_j(k+M-1)$, the predictive value of $y_i$ in the future time $P$ is:

$$\hat{y}_{i, P0}(k) = \hat{y}_{i, 00}(k) + A_{ij} \Delta u_i(k) \quad (2)$$
where \( \hat{\hat{y}}_{i,u}(k) = \begin{bmatrix} \hat{y}_{i,u}(k+1|k) \\ \vdots \\ \hat{y}_{i,u}(k+P|k) \end{bmatrix} \) and \( \hat{\hat{y}}_{i,p}(k) = \begin{bmatrix} \hat{y}_{i,p}(k+1|k) \\ \vdots \\ \hat{y}_{i,p}(k+P|k) \end{bmatrix} \).

When \( y_i \) is under the coaction of \( M \) control inputs, it is superimposed by linear theory. If every \( u_j \) has instant change \( \Delta u_j(k) \), similar equation of (1) can be expressed as equation (3):

\[
\hat{y}_{i,u}(k) = \bar{y}_{i,u}(k) + \sum_{j=1}^{m} a_{ij} \Delta u_j(k) \tag{3}
\]

When \( u_j \) changes \( M \) times from time \( k \), \( \Delta u_j(k) \), \( \cdots \), \( \Delta u_j(k+M-1) \) would be generated. Similar equation of (2) can be expressed as (4):

\[
\hat{y}_{i,p}(k) = \bar{y}_{i,p}(k) + \sum_{j=1}^{m} A_{ij} \Delta u_{i,M}(k) \tag{4}
\]

When all predictive values are combined into one vector, the predictive model of multi-variable system is given by equation (5):

\[
\hat{y}_{p}(k) = \hat{y}_{p}(k) + A \Delta u_{i}(k) \tag{5}
\]

2.2 Rolling optimization

Optimal control law can be obtained by minimizing the performance indicator in each control cycle. The performance indicator can be expressed as equation (6):

\[
J(k) = \left[ w(k) - \hat{y}_{p}(k) \right]^T R w(k) \tag{6}
\]

where \( w(k) \) is the matrix of the controlled output set values, and \( Q=\text{block-diag}(Q_p,\cdots,Q_p) \) is the error weighted matrix of the output variable. \( R=\text{block-diag}(R_p,\cdots,R_p) \) is the weighted matrix of the input variable.

In the case of no constraints, using equation (5) and equation (6), we can get the optimal control increment which can be expressed as equation (7):

\[
\Delta u_{i}(k) = (A^T Q A + R)^{-1} A^T Q (w(k) - \hat{y}_{p}(k)) \tag{7}
\]

2.3 Feedback correction

In time \( k \), predictive values of controlled objects in future time can be obtained by equation (5), where \( \hat{y}_{i}(k+1|k) \) is the predictive value of each output variables in time \( k+1 \). The error vector which is expressed by equation (8) is obtained by the comparison between the actual output predictive value in time \( y_i(k+1) \) and time \( k+1 \).

\[
e(k+1) = \begin{bmatrix} e_i(k+1) \\ \vdots \\ e_p(k+1) \end{bmatrix} = \begin{bmatrix} y_i(k+1) - \hat{y}_{i1}(k+1|k) \\ \vdots \\ y_p(k+1) - \hat{y}_{ip}(k+1|k) \end{bmatrix} \tag{8}
\]

The predictive value is corrected by \( e(k+1) \), which can be given by:

\[
\hat{y}_{cor}(k+1) = \hat{y}_{i1}(k) + H e(k+1) \tag{9}
\]

where \( H = \begin{bmatrix} k_1 \cdots k_1 \\ \vdots \\ k_p \cdots k_p \end{bmatrix} s.t. i=1,\cdots,p \).

Due to the change of the time baseline, the original predictive value in time \( k+1 \) given by equation (10) can be inferred by the shift of the corrected predictive vector \( \hat{y}_{cor}(k+1) \).

\[
\hat{y}_{0}(k+1) = S \hat{y}_{cor}(k+1) \tag{10}
\]

where \( S = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} \) is the shift matrix.
3. Difficulties of DSC temperature control
The heating furnace of DSC has the characteristics of non-linearity and thermal inertia. (1) Nonlinearity: Due to the influence of specific heat capacity and temperature phase control, DSC heating furnace has non-linear characteristics, which shows that the response of heating process with different control variables shows different dynamic and steady-state characteristics. (2) Thermal inertia: Because of the inertia of the heat conduction of the controlled object, there is a time lag in the control of temperature rise, that is, the current time control effect, which can only be shown after a period of delay. The delay of this response will lead to a longer time of the output transition process, thus making the dynamic quality of the control system worse, or even worse. There is divergent oscillation.

DSC is a precise thermal analysis instrument. Its temperature control system needs to meet higher requirements. According to the National Metrological Verification Regulations of the People's Republic of China-Differential Scanning Calorimeter (JJG 936-2012), the DSC should meet the requirements as follows:

(1) The deviation of heating rate is \( \leq \pm 2\% \).
(2) The static error of heating process is \( \leq \pm 1^\circ C \).
(3) The control process should have short adjustment time and small overshoot.

4. DMC-PID Cascade Control Algorithms for DSC Temperature Control
The conventional PID controller is lacking of the prediction, and the constant-speed heating process of DSC is the variable-set-value control that the set-value is changing according to the predetermined heating rate. Therefore, the conventional PID algorithm cannot track the change of the set value during the constant-speed heating process of DSC

Model predictive control (MPC) is an optimal control algorithm [9]. It decomposes the optimal control problem with a long time into several shorter time optimal control problems, and solves the optimal solution with the minimum difference between the set value and the predicted value. [10] Therefore, the control amount of predictive control algorithm at the current time is calculated by optimizing the minimum difference between the predicted value at the current time and the future time and the reference trajectory, so as to realize the optimal tracking of the reference trajectory. The predictive control algorithm can minimize the static error between the set value and the actual value, and it can improve the tracking characteristics of the control system.

However, there are two problems in the application of predictive control algorithm to constant speed temperature rise control of DSC. (1) DSC heating furnace has non-linear characteristics, and the predictive model in predictive control algorithm is a linear model, which must satisfy the principle of linear superposition. (2) In the hardware system of DSC, single chip microprocessor is used as the microprocessor, and the operation space is limited. Nonlinear model predictive control is too computational to be realized in single chip computer. The heating process of DSC is a slow response process. In order to ensure high control accuracy, the control cycle is generally small. With the increase of model length, the calculation amount of predictive control algorithm increases exponentially. Therefore, the dynamic response time of controlled object should be reduced as much as possible.

According to the negative feedback theory [11], the PID negative feedback can accelerate the dynamic response of the controlled object. By adding the PID controller, the generalized object composed of the PID controller and the DSC heating furnace is taken as the controlled object of the predictive controller. Thus, the non-linear object is transformed into the object with better linearity. As the dynamic response of the system is accelerated, the length of the predictive model is greatly reduced in the same sampling period, which reduces the operation load of the DMC controller and saves the operation space.

Based on the above analysis, we introduce the cascade control algorithm [12] which is consist of the DMC controller and PID controller into the constant temperature rise of DSC. This method can improve the tracking performance while guarantee the control accuracy. The structure of the control system is shown in Fig. 1. The main controller of the main loop is the DMC controller shown by the red dotted line in the figure2, and the sub-controller of the secondary loop is the PID controller, which improves
the characteristics of the controlled object. The generalized object is composed of the PID controller and the DSC heating furnace, which is the controlled object of the DMC controller. In the figure, is the set value of temperature, is the real-time temperature value, is the control output of DMC controller, is the control output of PID controller.

Figure 1 Control system structure

4.1 Main Controller

Model predictive control algorithm is optimal control algorithm which minimizes a performance index in each control cycle to obtain the optimal control law [13]. The control amount obtained by rolling optimization is the optimal solution at the minimum difference between the set value and predicted value. Thus, the main controller is designed as the MPC controller to improve the tracking characteristics of the system.

(1) Prediction model

The controlled object of the main controller is the generalized object in the dashed line in Figure 2. The step response model is the step test model of the generalized object which is obtained as the follow steps. Firstly, the parameters of PID control are adjusted. Secondly, the step input is given to the generalized object which is composed of the PID controller and the DSC heating furnace. Finally, the step response is sampled as , where the sampling period is . The prediction model is a set of parameters composed of sampling values , where is the time domain of modeling. The sampling period must conform the Shannon’s theorem, which requires that the parameters of the prediction model describe the dynamic information of the controlled object as completely as possible.

(2) Rolling optimization

Formula (2) is used to optimize the optimal control amount. In order to determine the open-loop predictive value in the future, the selection of predictive time domain will affect the stability and rapidity of the control. If the predictive model is accurate enough, the controlled object can output the expected value closely at each sampling point. The control time domain represents the change number of future control variables in the optimization performance index [14].

The real-time control increment of the optimal control amount is taken as the input setting value of the secondary loop PID controller, and the actual control amount is applied to the generalized object.

(3) Feedback correction

The robustness of DMC algorithm is mainly guaranteed by feedback correction. The error between the predicted output value and the actual output value corrects the predicted output value to the actual output value as far as possible, thus guaranteeing the robustness of the system. [15].
4.2 Sub controller

The secondary control loop adopts the PID controller. \( y(k) \) is the real-time temperature measurement value in the heating furnace in Figure 2. \( u_f(k) \) is the real-time output of the PID controller. The transfer function of the PID control algorithm [16] is as follows:

\[
G_c(s) = k_p \left[ 1 + \frac{1}{k_i s} + k_d s \right]
\]  

(11)

where \( k_p \) is the scale factor of the PID controller; \( k_i \) is the integration factor of the PID controller; \( k_d \) is the differential factor of the PID controller.

The critical proportion method [17], response curve method [18] and so on are usually used to tune the parameters of the PID controller.

4.3 Algorithm Implementation steps

Step1: Set \( k = 1 \), initialize controller parameters, including predictive time domain, control time domain, error weight matrix, control matrix, proportional coefficient, integration time and differential time.

Step2: Predictive model vectors are obtained by step test of generalized objects.

Step3: Measure the actual temperature output value and get the prediction error.

Step4: The output predicted value is corrected by the prediction error.

Step5: The control increment is calculated by formula (3). The real-time control increment is taken as the output setting value of the PID controller.

Step6: The control amount is calculated by the PID controller and applied to the DSC heating furnace.

Step7: The output predicted value is calculated by formula (1).

Step8: Let \( k = k + 1 \) and return to step 3.

5. Experimental Result

5.1 Experimental results of DSC temperature control

The control strategy proposed in this paper is experimented on DSC. The temperature rate is set at 10 ℃/min. The experimental results are shown in Fig. 2 and Fig. 3 by using the compound multi-PID control algorithm and the cascade control algorithm proposed in this paper. The ordinate is the temperature (℃) and the abscissa is time (min).

5.2 Control effect evaluation

The temperature control effect of DSC is evaluated by the linearity of temperature curve and the accuracy of temperature control. The Integral Square Error (ISE) is used to evaluate the linearity the heating curve which is calculated by formula (12).

\[
J_1 = ISE = \int_0^\infty e^2(t) dt
\]  

(12)

The Temperature Accuracy Error (TAE) is used to evaluate the accuracy of temperature control which is calculated by formula (13).

\[
J_2 = TAE = \frac{1}{n} \sum_{i=1}^{n} e_i
\]  

(13)

The ISE and TAE of the compound multi-PID control algorithm and the cascade control algorithm proposed in this paper are shown in Table 1.

| Control Algorithm                  | ISE     | TAE   |
|------------------------------------|---------|-------|
| The compound multi-PID control algorithm | 80419   | 9.34  |
| The cascade control algorithm      | 651.5   | 0.81  |

Table 1. The ISE and TAE of DSC temperature control
According to Fig. 2 and Fig. 3, it can be seen that there is a significant static difference between the set value and the output value of the compound multi-PID control algorithm, and there is almost no static difference between the set value and the output value of the cascade control algorithm mentioned in this paper. According to the control effect evaluation index in Table 1, the deviation square integral value of the control algorithm proposed in this paper is 651.5, which is reduced by two orders of magnitude compared with 80419 of the compound multi-PID control algorithm. The temperature accuracy deviation of the cascade control algorithm proposed in this paper is 0.81 degrees. It is less than the technical specifications of the verification regulation. The temperature accuracy error of the composite multi-PID algorithm is 9.34 degrees, which does not meet the technical requirements of the verification regulation. The experimental results show that the control algorithm proposed in this paper can effectively reduce the static error between the measured value and the set value under the requirement of ensuring the accuracy of the heating rate, and has good tracking characteristics.

6. Conclusion

In order to solve the problem of static difference between output value and set value in constant-speed heating process of DSC, a cascade control algorithm of DMC-PID for DSC is proposed based on the
analysis of constant-speed heating characteristic of DSC. The proposed control algorithm is applied to
the constant-speed heating process of DSC. Experimental results show that the proposed control
algorithm can effectively reduce the static error between the output value and the set value, and it has
better tracking characteristics and effectively improves the performance of the control system.

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