Hypersonic Bose–Einstein condensates in accelerator rings

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Some of the most sensitive and precise measurements—for example, of inertia, gravity and rotation—are based on matter-wave interferometry with free-falling atomic clouds. To achieve very high sensitivities, the interrogation time has to be very long, and consequently the experimental apparatus needs to be very tall (in some cases reaching ten or even one hundred metres) or the experiments must be performed in microgravity in space.

Cancelling gravitational acceleration (for example, in atomtronic circuits and matter-wave guides) is expected to result in compact devices with extended interrogation times and therefore increased sensitivity. Here we demonstrate smooth and controllable matter-wave guides by transporting Bose–Einstein condensates (BECs) over macroscopic distances. We use a neutral-atom accelerator ring to bring BECs to very high speeds (16 times their sound velocity) and transport them in a magnetic matter-wave guide for 15 centimetres while fully preserving their internal coherence. The resulting high angular momentum of more than 40,000ħ per atom (where ħ is the reduced Planck constant) gives access to the higher Landau levels and the hypersonic velocities achieved, combined with our ability to control potentials with picokelvin precision, will facilitate the study of superfluidity and give rise to tunnelling and a large range of transport regimes of ultracold atoms.

Coherent matter-wave guides are expected to enable interaction times of several seconds in highly compact devices and to lead to portable guided-atom interferometers for applications such as inertial navigation and gravity mapping.

Ring-shaped atom circuits are excellent candidates for guided matter-wave Sagnac interferometry, Josephson oscillations of angular momentum, and atomtronic applications such as quantized conductance through a constriction. In the past decades, several ring traps, mainly based on magnetic and optical dipole trapping, have been implemented. In small traps the deleterious effects of corrugations can be avoided by operating at velocities well below the critical velocity of superfluid BECs. This has allowed ring-shaped traps to be used in fundamental studies, for example, on the correspondence between superfluidity and Bose–Einstein condensation and on the hysteresis of flux in a ring-shaped atomtronic circuit. Atom-chip-based magnetic waveguides have been a very successful platform for many cold-atom experiments requiring non-trivial geometries and have been used to study the propagation of BECs in matter-wave guides.

One of the main problems of chip-based waveguides and complex optical potentials is that small modulations in the confining potential are almost unavoidable. Methods have been developed to reduce the effect of this roughness, for example, by modulating the currents in microchip traps or by using optimal control theory. Most previous experiments were performed in a regime in which the effect of the roughness of the guiding potential is suppressed by the superfluidity of the BECs. Atom interferometers, however, need to operate at high speeds and low atom densities, where the energy shifts associated with the superfluid properties are vanishingly small. Therefore, the presence of any corrugation or roughness in the guiding potential of atom interferometers leads to a coupling of the forward momentum to transversely excited states, thus scrambling the phase of the interferometer and severely limiting the distances over which the atoms can be guided coherently.

Here we report the first experimental realization of matter-wave guiding of BECs over large distances while completely preserving their internal coherence. In our ring-shaped waveguides, BECs travel at hypersonic speeds of 28 mm s⁻¹ for as long as 148 mm without any appreciable additional heating or reduction in lifetime as compared to the static case. The peak-to-peak roughness of our waveguide is smaller than our measurement limit of 189 pK, which corresponds to a maximum difference in gravitational potential of less than 2 nm over the whole ring (radius R = 443 μm). The traps and waveguides presented here are based on magnetic time-averaged adiabatic potentials (TAAPs), where the shape of the waveguide is defined by a simple d.c. quadrupole field and by the polarization and amplitudes of homogeneous fields oscillating at audio and radio frequencies. The field-generating coils are large and distant when compared to the atomic ensemble. This limits the highest spatial frequency of the trapping potential in the azimuthal direction to 4πR, that is, a maximum of two minima per turn. Any harmful corrugation or roughness due to imperfections in the wires falls off exponentially with the distance from the coils (about 50 mm) divided by the relevant length scale (≪1 mm), resulting in perfectly smooth TAAP waveguides. The unavoidable imperfections of the field-generating coils cause a modulation of the magnetic field close to its surface. For a given spatial frequency k, the amplitude of this modulation decreases by a factor of e⁻²k ≈ 0.006 over a distance z (ref. 5).

The minimum spatial frequency of interest is related to the inverse of the diameter of the ring-shaped waveguide as k = 2π/1 (mm), which—together with the distance between the coils and the atoms being a minimum of 50 mm—implies a reduction in coil-induced defects by a factor of more than 10⁻¹³. This has to be compared to atom-chip-based traps, which use the shape of close-by wires to define the trapping potential and optical dipole potentials using the spatial distribution of the light. Both can create much more complex structures and thus have small but unavoidable corrugations of the wave-guiding potential. Figure 1 shows absorption images of atoms in a ring-shaped TAAP waveguide, with Fig. 1a showing static atoms and Fig. 1b–d showing atoms guided at high speeds. The images in the lower panels show the error of fits of a smooth potential to the images directly above them. We note that there is no trace of the ring visible in the error, which demonstrates the smoothness of the atom distribution and thus of the waveguide itself.

Our TAAP traps and waveguides are generated by combining a.c. and d.c. magnetic fields at three different time-scales: d.c., audio frequency and radio frequency. The radio-frequency fields (Brf(t)) dress the states of the atoms in the d.c. magnetic quadrupole field (Bq) yielding adiabatic states, which are time-averaged by a spatially homogeneous magnetic field (Bω(t)) oscillating at audio frequencies (ωm).

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The frequency of the time-averaging field is chosen such that the magnetic spin of the atoms can follow adiabatically but the centre of mass of the atoms remains virtually unchanged \( \omega_{\text{rf}}/(2\pi) = 5 \text{ kHz} \). Starting from a d.c. magnetic quadrupole trap and a vertically polarized radio-frequency field, we can produce a ring-shaped waveguide by applying a vertical audio-frequency field. If we tilt the audio-frequency field from its vertical axis, then the ring tilts as well, and gravity creates an azimuthal trapping potential along the ring. The full time-dependent magnetic field is

\[
B = \alpha \begin{pmatrix} x \\ y \\ z \end{pmatrix} + B_m \sin \omega_{\text{rf}} t \begin{pmatrix} \delta \cos \phi_0 / 1 \\ \delta \sin \phi_0 / 1 \\ 0 / 1 \end{pmatrix} + B_{\text{rf}} \sin \omega_{\text{rf}} t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1}
\]

where the first term of the sum stands for a d.c. quadrupole \((B_{\text{dc}})\) field of gradient \( \alpha \). The second term represents an audio-frequency modulation field \((B_{\text{rf}}(t))\), which is mainly in the vertical \((z)\) direction but can be tilted by a small angle \( \delta \) in the direction \( \phi_0 \). The final term of the sum is a radio-frequency field \((B_{\text{rf}}(t))\) with a linear polarization in the vertical direction. Near the resonance \( h\omega_{\text{rf}} = |\mu_B| |g| B \), the radio-frequency field \( B_{\text{rf}}(t) \) dresses the atoms in the magnetic field. This turns the quadrupole trap into a shell-like trap\(^{29}\). If the modulation frequency \((\omega_{\text{rf}})\) of the homogeneous field \( B_m \) is small compared to the Larmor frequency \((\Omega_L = |\mu_B| |g| B)\) but high compared to the eventual radial trapping frequency \((\omega_{\text{r}})\), then the atoms are trapped in the time-adiabatic potential, which results in a ring-shaped matter-waveguide of radius \( R \approx h\omega_{\text{rf}} / |\mu_B| |g| B_{\text{dc}} \) from the zero-field point\(^{22}\). A detailed description of the TAAP potentials is provided elsewhere\(^{18,27}\). Near the core of the waveguide, the TAAP ring potential can be described in cylindrical coordinates \((r, \phi, z)\) as \(^{18,27}\)

\[
V(r, \phi, z) = \hbar \Omega_{\text{rf}} t + \frac{1}{2} m \omega_r^2 (r - R)^2 + \frac{1}{2} m \omega_z^2 z^2 - \frac{\delta m g R}{\cos(\phi - \phi_0)} \tag{2}
\]

where \( \Omega_{\text{rf}} \) is the Rabi frequency associated with the radio-frequency field and \( \omega_r \) and \( \omega_z \) are the radial and axial trapping frequencies, respectively, \( g \) is Earth's gravitational acceleration and \( m \) is the mass of an atom. For \( \delta = 0 \), equation (2) represents a circular waveguide. Figure 1 shows ultracold atomic clouds in such ring-shaped potentials. The radial and axial confinement frequencies are \( \omega_r = \omega_0 (1 + \beta_0^2)^{-1/4} \) and \( \omega_z = 2 \omega_0 [1 - (1 + \beta_0^2)^{-1/2}]^{1/2} \), respectively, where \( \beta_0 = |m| g |\mu_B| B_{\text{dc}}(h\omega_{\text{rf}}) \) is the index of modulation of the time-averaging field and \( \omega_0 = |m| g |\mu_B| B_{\text{dc}}(h\omega_{\text{rf}}) \) is the radial trapping frequency of an adiabatic shell potential in the absence of modulation\(^{27,29}\).

One of the most interesting aspects of a circular waveguide is its ability to guide atoms with extreme precision at very high angular momentum. Figure 1d, for example, shows atoms travelling at an angular momentum of \( L = m R^2 \phi = 17,000 \text{ h} \) per atom. This is an ideal starting point, for example, for the excitation of quantum Hall states and well defined higher-lying Landau levels. In order to accelerate the atoms one needs to create an azimuthal potential along the ring. For this we apply a small horizontal modulation field, which tilts \( B_{\text{rf}}(t) \), and thus the ring, in the direction of the horizontal modulation field \((\phi_0)\) by an angle \( \delta/2 \) with respect to the horizontal direction (see equation (1) with \( \delta = 0 \)). The gravitational potential of the atoms then creates a trap in the azimuthal direction at \( \phi_0 \). The azimuthal trapping frequency is then simply that of a tilted pendulum \( \omega = \sqrt{g R / 2 \delta} \). By adjusting the amplitudes of the modulation fields in the two horizontal Helmholtz coil pairs we can freely move the position \( \phi_0 \) of the minimum around the ring. The basic idea of our accelerator ring is to load a BEC into a static, tilted ring and then modulate the amplitudes of the modulation field in the \( x \) and \( y \) directions so that the minimum of the trap accelerates along the ring and then transports the BEC at a constant angular velocity over large distances.

A sudden acceleration, however, would excite centre-of-mass oscillations of the ultracold cloud. Adiabatic acceleration, on the other hand, would take prohibitively long. An elegant solution is provided by optimal control theory and its so-called 'bang-bang' scheme\(^{23}\), which compensates the force due to a constant acceleration by an opposite force due to an offset in the position of the atomic cloud relative to the centre of the moving harmonic trap. We do this by instantaneously shifting the position of the trap forward by \( \Delta \phi = \phi / \phi_0^2 \) exactly at the moment when we start the acceleration \( \phi \). In the accelerated frame, the atoms then stay exactly at the bottom of the effective trapping potential. Once the target velocity is reached, we abruptly stop the acceleration \( \phi = 0 \) and change the phase back by \( \omega_0 \), thus placing the atomic cloud at the bottom of the trap moving at a constant velocity. The uniform acceleration causes a uniform force on the atoms, which in turn corresponds to a shift of the parabolic azimuthal trapping potential. We correct this by a displacement of the trapping potential in the opposite direction. However, in practice the trapping frequency is not entirely independent of the angular velocity, which necessitates fine-tuning of the size of the phase jump. The same method can be used to decelerate the atoms. Figure 2 shows the measured positions of BECs (dots) and the corresponding theoretical prediction (lines) during acceleration in a TAAP ring, which range from \( 2\pi \times 25 \text{ rad s}^{-2} \) to \( 2\pi \times 400 \text{ rad s}^{-2} \), reaching angular velocities of up to \( 2\pi \times 20 \text{ rad s}^{-1} \), which corresponds to an angular momentum of \( 44,600h \). The deviations from the theoretical curves result from small changes in the trapping frequency during the acceleration. The centrifugal force associated with the rapid rotation in the ring forces the atoms outwards from a radius of \( 436(2) \text{ \mu m} \) in the static trap to \( 443.4(4) \text{ \mu m} \) at an angular speed of \( 2\pi \times 10 \text{ rad s}^{-1} \) (all uncertainties are 1 s.d.). Care has to be taken to ensure that the change in trapping frequency does not cause parametric heating of the sample. Here, for an acceleration of \( 2\pi \times 50 \text{ rad s}^{-2} \) to a final angular velocity of \( \phi = 2\pi \times 10 \text{ Hz} \), the azimuthal trapping frequency decreases within 0.2 s from \( \omega_0 = 2\pi \times 9.17(3) \text{ Hz} \) to its final value of \( 2\pi \times 7.76(1) \text{ Hz} \). Therefore, \( \omega_0 / \omega_0 = 0.02 \approx 1 \), which means that this change in trapping frequency is fully adiabatic even in the azimuthal direction\(^{27}\). Nevertheless, the phase jumps have to be optimized to take this change in trapping frequency into account. By making small adjustments of the phase jumps both at the beginning and the end of the acceleration, we can suppress any oscillation of the condensate in the final trap.
In the experiments described here, we start with BECs of $3 \times 10^5$ atoms at 32 nK in a static trap in the ring, accelerate them at $2\pi \times 50 \text{ rad s}^{-2}$ for 200 ms and continue the transport at a constant angular speed of $\dot{\phi}_0 = 2\pi \times 10 \text{ rad s}^{-1}$ for up to 14.3 s or 39.8 cm (see Fig. 3). The phase jumps are $\Delta \phi = \pm 0.5 \text{ mrad}$. The inset of Fig. 3a shows a bi-modal fit to the atomic density after 41 round-trips corresponding to a total transport distance of 11.4 cm. Figure 3b shows the deviation of the position of the atoms from the programmed trajectory at a constant speed of $2\pi \times 10 \text{ rad s}^{-1}$. The angular position of the atoms was fitted to the sum of three sine waves: two at a fixed rotation frequency of $2\pi \times 10 \text{ rad s}^{-1}$ and its first harmonic $2\pi \times 20 \text{ rad s}^{-1}$, plus one decaying sine wave of arbitrary frequency. The fitted amplitudes were 140(10) mrad, 40(10) mrad and 70(10) mrad, respectively. The third sine wave corresponds to an azimuthal centre-of-mass motion of the atomic cloud in the moving trap at 7.76(1) Hz, decaying with a 1/e time constant of 5.3 s. This very precise measurement of the centre-of-mass oscillation (see Fig. 3b) can be used to eliminate any final oscillation in the moving trap by fine-tuning the phase jumps at the beginning and end of the acceleration. The extremely fine control of the velocity of the atoms in the TAAP accelerator ring for neutral atoms could be used for highly controlled atomic collision experiments. The 10 Hz and 20 Hz modulations originate from the non-perfect flatness of the ring. A fit to a static thermal cloud in Fig. 1a reveals a slight modulation of the azimuthal potential at angular frequencies of $2\pi$ (till of the ring) and $4\pi$ (polarization-induced warping). This reappears during the transport as a micro-motion\(^0\) in the moving trap at exactly the final angular velocity and its harmonic.

If BECs move more slowly than their speed of sound, then superfluidity allows them to flow around obstacles without dissipation or heating. This is the regime where most guided condensates have been operated so far\(13,15,16,20,21,31\). At larger velocities though, any corrugation of the guiding potential couples the forward motion of the atoms to their transverse degrees of freedom, leading to a distortion of the BEC, heating or atom loss, and eventually to the destruction of the BEC\(17,21\). Therefore, one can probe the roughness of a guiding potential by propagating BECs at high velocities. For our TAAP waveguides, we do not observe any change in the lifetimes of the thermal clouds (3.3 s) and BECs (5.3 s) when comparing the atomic clouds in the moving and static traps. (The thermal lifetime corresponds to the point where the thermal atom number has dropped to 1/e of its original value, whereas the BEC lifetime is the time required for the BEC to vanish completely.) The measured heating rate of $3 \pm 1 \text{ nK s}^{-1}$ is the same in the moving and static cases, giving an upper limit on the additional heating rate induced by the guiding of 1 nK s\(^{-1}\) and 32 pK mm\(^{-1}\). The BEC lifetime of 5.3 s is particularly impressive because the atoms travel in the waveguide at a hypersonic speed of 16 times the Landau critical velocity in the BEC. At such high velocities, any roughness or corrugation of the guide would couple the longitudinal velocity to transverse excitations and thus rapidly destroy the coherence of the condensate. Therefore, the absence of any measurable heating is only possible because the TAAP matter-wave guides are perfectly smooth.

To the best of our knowledge, this is the first demonstration of lossless hypersonic transport of condensates in a matter-wave guide. Such smooth guides form an ideal base for guided matter-wave interferometry. The circular waveguide lends itself naturally to Sagnac interferometry, where the atoms take one or multiple round-trips in opposite directions. Atoms propagating through two opposite halves of the ring form a Michelson interferometer, which—owing to the potentially very long interaction time—can be made superbly sensitive to gravitational acceleration and even gravitational waves. The ability to create BECs at an extremely high and well controllable angular momentum per atom is the ideal starting position for highly correlated angular momentum states, such as Laughlin or quantum Hall states\(^2\). The transfer to the necessary harmonic trap can be achieved by simply ramping down the amplitude of the vertical time-averaging field and thus adiabatically transferring the atoms from the ring trap to the harmonic bottom of a shell-type trap.

To explore this further, we study the free propagation of a BEC in the waveguide eventually filling the full ring. We start by characterizing the flatness of the ring potential ($\delta = 0$) using a static cloud. We load $10^6$ thermal atoms at 430 nK in a static ring trap of radius 470 $\mu$m and allow them to thermalize for 6.5 s. We fit a Maxwell–Boltzmann distribution...
to the atomic column density (see Methods) to determine the potential energy landscape. For sufficiently long expansion times, that is, $t_{\text{exp}} \gg \omega_1^{-1}$, the radial atomic distribution yields the temperature. For sufficiently short expansion times, that is, $t_{\text{exp}} \ll \omega_1^{-1}$, the azimuthal distribution of the atoms provides the azimuthal shape of the trapping potential. For $\omega_1^{-1} \ll t_{\text{exp}} \ll \omega_1^{-1}$ both can be achieved in a single absorption image, such as those in Fig. 1. The azimuthal dependence of the theoretical description of TAAP rings for arbitrary modulation and radio-frequency polarization\(^\text{18}\) can be rewritten using only the first two cylindrical harmonics. The azimuthal potential can therefore be parameterized as

$$V_\phi = V_\phi \left[ 1 + h_1 \cos(\phi + \phi_1) + h_1 \cos(2\phi + \phi_2) \right]$$  \hspace{1cm} (3)$$

where $V_\phi$ is the energy scale, $h_1$ and $h_2$ are arbitrary amplitudes, and $\phi_1$ and $\phi_2$ are angular positions\(^\text{18}\). We fit our experimental images to a Maxwell–Boltzmann distribution for $V_r + V_\phi$ (see Methods). The fact that the fit residual in Fig. 1e contains no trace of the original ring in Fig. 1a shows that the highest spatio-angular frequency present in the azimuthal potential is indeed $2\phi$, further supporting the extreme smoothness of the TAAP potentials. The fit yields $2 \times 10^5$ atoms at 502 nK and modulation amplitudes of $h_1 = 0.11$ and $h_2 = 0.20$ at $\phi_1 = -118^\circ$ and $\phi_2 = +115^\circ$, respectively. This corresponds to a maximum potential difference of 250 nK, which corresponds to a gravitational shift of only $\Delta z = 2.4 \mu m$ across the ring.

We also studied the propagation of ultracold atoms in a ring without azimuthal confinement. To investigate this, we accelerated nearly pure BECs of $10^5$ atoms to an angular velocity of $2\pi \times 10^{10}$ rad s\(^{-1}\). We then slowly removed the azimuthal trapping potential by ramping $\delta$ to zero and allowed the atomic cloud to fill the entire ring. The atomic density distribution after a hold time of 2 s and a time-of-flight expansion of 6.3 ms can be seen in Fig. 1d. Again, we do not observe any additional heating or atom loss compared to the static trap, nor do we observe any decay of the angular momentum of the atoms over a timescale of 10 s. We use a fit based on equation (3) to analyse the atom distribution in Fig. 1d and determine the effective azimuthal modulation of the waveguide potential, and find $h_1 = 0.003$, $h_2 = 0.002$ and a temperature of 28 nK. In a static trap this would correspond to a total variation of the waveguide potential by 189 pK or to a difference in height of only 1.8 nm over a distance of 1 mm—that is, a gravitational potential due to an angular misalignment of 2 μrad. The reason for the difference between the moving and static atoms can be understood by studying the moving atoms in their co-moving frame. The moving atoms experience a periodic modulation of the trapping potential at the rotation frequency (10 Hz). This perturbation is not resonant with the trapping frequency of $\omega = 2\pi \times 7.8$ Hz, and thus leads only to micro-motion in the moving frame and a small density modulation in the rest frame.

In conclusion, the TAAP waveguides demonstrated here fulfil a long standing goal of atomtronics: the coherent transport of atoms and BECs over long distances and in non-trivial geometries. We have demonstrated virtually excitationless acceleration of ultracold thermal clouds and BECs to hyperionic velocities (Mach 16) and extremely high angular momentum (17,000h per atom). The atoms were transported in an ultrasmooth atomtronic waveguide over macroscopic distances, with BECs reaching distances of 15 cm and ultracold thermal clouds distances of up to 40 cm without any sign of additional heating compared to the stationary case. The matter-wave guides show no sign of roughness, with an effective flatness smaller than our measurement sensitivity of 189 pK or 1.8 nm equivalent in gravitational height. Such extremely flat effective potentials open very interesting possibilities in the study of one-dimensional physics or ultralow energy interactions. These neutral-atom ring accelerators open the way to many scientific and applied measurements, such as the precise excitation of Landau levels in quantum Hall states, hyperionic transport phenomena and highly controlled collision experiments.\(^\text{3,2}\). Our newly found ability to control barriers with picokelvin precision will enable new regimes of conduction and tunnelling of ultracold atoms through mesoscopic barriers and channels.\(^\text{1,12}\). This demonstration of ultrasmooth TAAP waveguides is an important step towards guided matter-wave interferometry, which will lead to increased sensitivity for fundamental physics and for applications such as inertial navigation and gravitational sensing using highly compact atomtronic devices.
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METHODS
Preparation of the initial atom cloud. Following ref. 33, we first load $^{87}\text{Rb}$ atoms in the $|F=1, m_F=-1\rangle$ state (F and $m_F$ are the hyperfine and magnetic quantum numbers) from a magneto-optical trap into a magnetic quadrupole trap. After performing radio-frequency evaporation, we transfer the atom cloud into a hybrid trap consisting of a weak quadrupole field and a crossed-beam optical dipole trap. We evaporatively cool the atoms to 50 nK to obtain BECs of $3 \times 10^5$ atoms, which we then adiabatically transfer into a tilted ring trap by ramping down the power of the dipole beams. We do not observe any substantial heating, atom number loss, or shape or centre-of-mass fluctuation due to the transfer sequence. The parameters of the final ring trap are $\alpha = 70 \text{ G cm}^{-1}$, $B_0 = 1.4 \text{ G}$, $\delta = 0.37$, $\omega_{\phi}/(2\pi) = 5.02 \text{ kHz}$ and $\omega_{\theta}/(2\pi) = 2.55 \text{ MHz}$. The measured radial, axial and azimuthal trapping frequencies of the tilted ring trap are 85.3(4) Hz and 46.2(3) Hz and 9.17(3) Hz, respectively. From the radial trapping frequency we calculate the radio-frequency coupling strength $h \Delta \tau = \sqrt{|g_{1}\mu_{B}\hbar}/(2n \times 357 \text{ kHz})$.

Absorption imaging and fitting. The images of Fig. 1 were taken using absorption imaging, where one sends resonant light through the atom cloud. The transmitted light is then imaged onto a charge-coupled device camera, from which one can then deduce the column density of the atoms via a modified version of the Beer-Lambert law34. In Fig. 1a the atoms are nearly at rest. For Fig. 1b–d, we accelerated the atoms to an angular speed of $2\pi \times 10^{-6} \text{ rad/s}$ in the clockwise direction and guided them for 0.2 s, 0.452 s and 2 s, respectively. From the radial trapping frequency we calculate the radio-frequency coupling strength $h \Delta \tau = \sqrt{|g_{1}\mu_{B}\hbar}/(2n \times 357 \text{ kHz})}$.

Oscillations in the moving trap. The angular position of the atomic cloud in Fig. 3b was fitted for the entire dataset with

$$\phi(t) = \phi_0 + (2\pi \times 10) + a_1 \sin \left(2\pi \times 10 t + \phi_1\right) + a_2 \sin \left(2\pi \times 20 t + \phi_2\right) + a_3 e^{-t/\tau} \sin \left(2\pi \times \omega_{\phi} t + \phi_3\right),$$

where $a_1$ and $a_2$ are the quadratic and the linear oscillation amplitudes, respectively. The fitting yields $\phi_0 = 2.2 \text{ rad}$, $\phi_1 = 3.3 \text{ rad}$, $\phi_2 = 3.5 \text{ rad}$ and $\phi_3 = 0.5 \text{ rad}$. No drift in phase or amplitude within our estimated resolution was detected for the 10- and 20-Hz oscillations over the full 14.3 s. The final term, $a_3 e^{-t/\tau} \sin \left(2\pi \times \omega_{\phi} t + \phi_3\right)$, is a centre-of-mass damped oscillation of the atomic cloud in the moving trap. The decay time constant for the azimuthal trap oscillation at 7.76(1) Hz in the moving trap is $\tau = 5.3 \text{ s}$.

Internal coherence of the BEC. A zero-temperature BEC has a flat phase distribution at rest and a simple gradient when at motion. At higher temperatures, however, thermal excitations can occur, which then result in random phase gradients35. Phase gradients correspond to a flow of the condensate. Inside the trap, the superfluidity strongly suppresses density fluctuations from being generated by these phase fluctuations. In time-of-flight free expansion, however, the density of the condensate drops very quickly and with it the critical superfluid velocity. Once the critical velocity drops below the velocity associated with the phase fluctuations, density patterns (usually stripes for elongated BECs) form. Such density patterns have been observed experimentally36. The absence of such density modulations in long-time-of-flight images thus proves phase coherence across the condensates. We do not observe any such fringes up to our maximum expansion time of 24 ms.


c = \frac{nU_c}{m}

(4)

where $U_c = 4\pi \hbar^2 a_{\text{b}}/m$ is the bosonic interaction parameter, and $a_{\text{b}}$ and $m$ are the scattering length and mass of $^{87}\text{Rb}$, respectively. The density of atoms in the BEC is $n$. In our case, the density of the gas takes the parabolic shape of the trap. The maximum speed of sound occurs at the centre of the trap, where the peak density $n = n_{\text{max}}$, is

$$n_{\text{max}} = \frac{1}{8\pi} \left[15N \frac{m w_{\text{int}}}{h^2 a_{\text{b}}^2 g_{1}^2}\right]^{2/5}

(5)\]
where $\omega_{3\omega} = \left(\omega_\omega\omega_\omega\right)^{\frac{1}{3}} = 2\pi \times (46\ \text{Hz} \times 85\ \text{Hz} \times 7.8\ \text{Hz})^{\frac{1}{3}} = 31.2\ \text{Hz}$. For $N = 3 \times 10^5$ atoms this results in a maximum speed of sound in the BEC of $c_{\text{max}} = 1.75\ \text{mm s}^{-1}$.

The condensate travels along the ring at a speed of $v = \omega \times R = 2\pi \times 10\ \text{Hz} \times 443\ \mu\text{m} = 27.8\ \text{mm s}^{-1}$, which is 16 times its peak speed of sound (Mach number $M_{\text{max}} = v/c_{\text{max}} = 16$), well above the velocity at which its superfluidity allows it to flow frictionless around defects in the waveguide. The absence of any heating associated with its motion thus experimentally proves the extreme smoothness of the TAAP waveguides.

**Angular momentum of guided atoms.** In the case of moving atoms in a flat ring waveguide (Fig. 1d), we measure the angular momentum using the time-of-flight method. The ring potential is suddenly switched off and we let the cloud fall under gravity for different times of flight. Owing to the in-trap angular momentum, the cloud radius increases with the time of flight. We fit the cloud radius with $R(t) = R(0) \sqrt{1 + \left(\Omega t\right)^2}$, where $R(0)$ is the in-trap radius at an angular speed of $\Omega$. After 1 s of hold time in the flat ring waveguide, we measure an angular speed of $2\pi \times 10.01(0.06)\ \text{rad s}^{-1}$, while the programmed $\Omega$ is $2\pi \times 10\ \text{rad s}^{-1}$.

**Radius of an annular atom cloud under rotation.** The centrifugal force on the atoms, together with their harmonic radial confinement, causes the radius of the waveguide to increase. The radius of the ring increases as a function of the angular speed according to $R(\Omega) = R_0\left(1 - \frac{\Omega^2}{\omega_r^2}\right)^{-1}$, where $\omega_r$ is the radial trapping frequency and $R_0$ is the ring radius at $\Omega = 0$. In line with this prediction, the ring radius increases from $436(2)\ \mu\text{m}$ in a static ring to $443.4(4)\ \mu\text{m}$ at an angular speed of $2\pi \times 10\ \text{rad s}^{-1}$.

**Data availability**
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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