Abstract
Motivated by recent advances in neuroscience, in this work, we explore the emergent behaviour of quantum systems with a dynamical biologically-inspired qubits interaction. We use a minimal model of two interacting qubits with an activity-dependent dynamic interplay as in classical dynamic synapses that induces the so-called synaptic depression, that is, synapses that present synaptic fatigue after heavy presynaptic stimulation. Our study shows that in absence of synaptic depression the 2-qubits quantum system shows typical Rabi oscillations whose frequency decreases when synaptic depression is introduced, so one can maintain active a given qubit for large period of time. This creates a population imbalance between the qubits even though the Hamiltonian is Hermitian. This imbalance can be sustained in time by introducing a small energy shift between the qubits. In addition we report that long time entanglement between the two qubits raises naturally in the presence of synaptic depression. Moreover, we propose and analyse a plausible experimental setup of our 2-qubits system which demonstrates that these results are robust and can be experimentally obtained in a laboratory.

1 Introduction
Learning and information processing are key topics of science that have recently been pushed to the quantum domain. In the last years, thanks to the development of quantum computers, there is an increasing interest in the design of autonomous devices to perform certain tasks with a quantum improvement developing therefore the fields of Quantum Machine Learning and Quantum Artificial Intelligence [1, 2]. In this direction there have been proposal of autonomous machines that can estimate a state or a quantum unitary [3, 4], as well as algorithms of quantum reinforcement learning [5, 6], and quantum neuronal networks (QNN) [7].

In the field of QNN there have been theoretical proposals of models of single quantum neurons [8, 9, 10], as well as networks such as the perceptron [11, 12].
and Hopfield's [13][14]. The main interest here has been to see if such quantum versions of neural networks are able to improve the properties of classification and pattern recognition compared with classical ones. All these approaches use versions of binary neurons that are substituted by qubits, with very simple interactions between the units. However, classical neural networks and biological inspired neural population models include other important element that has been shown to have a prominent role on neural computation, i.e., the synapses [15], since the transmission of the information encoded in firing patterns of the neurons is transmitted by the synapses to postsynaptic neurons through highly non-linear processes. These include, among others, the biophysical processes that control the trafficking and recycling of neurotransmitter molecules at the synapses and which are the responsible for the transmission of the electrical signals among interconnected neurons. In the last decades, neuroscientists have extensively studied the role of such synaptic processes can have on the processing of information in the brain. In particular, it has been reported in different neural media that due to the incoming presynaptic activity, synapses can reduce (synaptic depression) or increase (synaptic facilitation) its capability to transmit the incoming electrical signals [16], that clearly shows that actual synapses are dynamic or activity-dependent entities. Moreover, such dynamic synapses have strong computational implications [17] in the behaviour of classical neural networks, including among others a strong effect on storage capacity [18][19], the appearance of dynamical memories [20][21] and the emergence of stochastic multiresonances during processing of weak stimuli [22], to name a few.

With this motivation, in this work, we explore the emergent behaviour of quantum systems with a dynamical biologically-inspired qubits interaction. We use a minimal model of two interacting qubits with an activity-dependent dynamic interplay as in classical dynamic synapses. Although our study can be easily generalised for dynamic synapses which include both synaptic depression and synaptic facilitation, we here only report results concerning the case of depressing synapses, that is, synapses that present synaptic fatigue after heavy presynaptic stimulation. We observe that in absence of synaptic depression our 2-qubits quantum system shows typical Rabi oscillations. However, when synaptic depression is introduced such Rabi oscillations decrease their frequencies so one can maintain active a given qubit for large period of time. This creates an asymmetry between the qubits even though the Hamiltonian is Hermitian. This asymmetry can be sustained in time by introducing a small energy shift between the qubits. We also study the effect of dynamic interaction depression in the creation of entanglement between the two qubits, probing that long time entanglement raises naturally. Moreover, by analysing the behaviour of a plausible experimental setup of our 2-qubits system, we demonstrated that these results are robust and can be experimentally demonstrated in a laboratory.

1.1 Model: Two interacting qubits Hamiltonian with short-term depression

Our model is based on two qubits with an $XY$ interaction in the form
\[ H(t) = \epsilon_1 \sigma_z^1 + \epsilon_2 \sigma_z^2 + \frac{\Omega}{2} r(t) \left( \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+ \right), \tag{1} \]

where \( \sigma_z^i \) are Pauli matrices for the \( i \)th qubit, \( \epsilon_i \) are the one-site energies, \( \sigma_i^\pm \) are the creation/destruction spin operators acting on site \( i \), \( \Omega \) is a parameter characterising the qubits interaction strength, and \( r(t) \) is the time-dependent parameter we will use to model the synaptic depression. By tuning the parameter \( r(t) \) the interaction between the spins can be switched on and off. This parameter will be used to introduce the synaptic dynamics in our system by making it time-dependent. This kind of XY Hamiltonians have been broadly studied in different fields as quantum transport \[23, 24, 25\] and quantum biology \[25, 26\].

To tune the variable \( r(t) \), we use a biologically-inspired dynamics. From the neuroscience perspective, it is now well known that the postsynaptic response of chemical synapses can vary in scales from milliseconds to minutes, in addition to more familiar long-term plastic effects due to the incoming presynaptic activity \[28, 29, 30, 31\]. Thus, synaptic efficacy can decrease due to depletion of neurotransmitters inside the synaptic button after heavy presynaptic activity inducing the so called short-term depression (STD). Additionally, the postsynaptic response can be enhanced due to the growth of residual intracellular calcium concentration after the opening of the voltage gated calcium channels due to successive arrival of presynaptic action potentials to the synaptic button \[32, 33\]. This is well known to increase the neurotransmitter release probability and therefore the postsynaptic response, inducing the so called short-term synaptic facilitation (STF). Both synaptic processes, i.e. STD and STF, can coexist and compete in actual synapses inducing complex emergent behaviour \[21, 19, 22, 34\] and resulting in strong computational implications \[17\]. As a first step, we are going to consider here synapses including only STD, which can be described by monitoring the time dependence of the fraction \( r(t) \) of neurotransmitters which are ready to be released after the arrival of an action potential (in the present quantum model this fraction will modulate the interaction between both qubits, that is why it is named as the time-dependent variable of the Hamiltonian), and which follows the dynamics

\[ \frac{dr(t)}{dt} = \frac{1 - r(t)}{\tau} - U r(t) \delta(t - t_{sp}), \tag{2} \]

where the parameter \( U \) is the release probability, \( \tau \) is the neurotransmitter recovering time, and \( t_{sp} \) is the time at which the presynaptic spike arrives at the synapse (note that this is defined for classical systems). The presence of the Dirac delta function \( \delta(t) \) in the second right-hand term of \( \tag{2} \) indicates that this term is only present at \( t = t_{sp} \). The dynamics \( \tag{2} \) indicates that each time a presynaptic spike occurs, a constant portion \( U r(t_{sp}) \) of the resources is released into the synaptic cleft, and then the remaining fraction \( (1 - r(t)) \) becomes available again at rate \( 1/\tau \).

It is straightforward to see that when \( \tau \) is small the level of synaptic depression is also low since the variable \( r(t) \) quickly recover, from the lower values
originated by the release of neurotransmitter due to the arrival of a presynaptic spike, to its maximum value \( r_{\text{max}} = 1 \). When \( \tau \) is enlarged such recovering becomes slow and \( r(t) \) will take a long time to recover so it remains small for a while. Then, the synaptic response which is proportional to \( r(t) \) will be more depressed the larger the \( \tau \). Therefore we can use \( \tau \) as a parameter to control the level of STD. In classical neural networks including STD, the variable \( r(t) \) in fact modulates the synaptic strength, namely \( \omega_{ij} \), between the presynaptic \( j \) neuron and the postsynaptic \( i \) neuron. Then, the postsynaptic neurons receives an input \( h_i(t) = \omega_{ij} r(t) s_j \), where \( s_j = 1, 0 \) is the neuron state variable, and which can be seen as an energy term per neuron [15].

In our quantum case we cannot define the time of spike arrival \( t_{sp} \) so we rely in an approximation. In a steady state condition, the term including the Dirac delta function in Eq. (2) can be approximated by

\[
Ur_{\text{stat}} \langle \sigma^+ \sigma^- \rangle,
\]

where \( \langle \sigma^+ \sigma^- \rangle \) is the population of the qubit, that is the quantum analog to the classical \( \langle s_i \rangle \). Then one has

\[
r_{\text{stat}} = \frac{1}{1 + \tau U \langle \sigma^+ \sigma^- \rangle}.
\]

Then, for each qubit, one can hypothesise a quantum version of (2) as follow

\[
\frac{dr_i(t)}{dt} = \frac{1 - r_i(t)}{\tau} - Ur_i(t) \langle \sigma^+_i \sigma^-_i \rangle \quad i = 1, 2.
\]

(3)

In a general system there are as many values of \( r_i \) as neurons. In our two qubit case we will work with only one depression parameter. We focus our study to see how the population of qubit 1 depresses the interaction Hamiltonian. and then substitute in (1) \( r = r_1(t) \) so the final dynamics of our system is given by a set of Eqs.

\[
\frac{d\rho(t)}{dt} = -i[H(t), \rho(t)],
\]

\[
\frac{dr(t)}{dt} = \frac{1 - r(t)}{\tau} - Ur(t) \langle \sigma^+_1 \sigma^-_1 \rangle(t).
\]

(4)

One difficulty in the design of quantum systems with STD is that the mean value of the excitation of a qubit is not measurable and it is difficult to engineer non-linear dynamics as the one given in Eqs. (4). Because of that we have also explored the possibility of implementing STD by a measurement-based protocol that could be easily implemented in realistic systems as cold atoms [35, 36] and trapped ions [37, 38] (see Section 5 for the model details).

2 Results

We first analysed how the level of STD in the qubits interaction term affects the behaviour of our 2 qubits system. The results are summarised in Fig. 1.
Figure 1: Effect of dynamic synapses in an ideal two interacting qubits system. From top to bottom the level of synaptic depression in the system is decreased using, respectively, \( \tau = 500, 100, 10, 0.001 \). Other parameter values are \( U = 0.5, \Omega = 0.05 \). In the left panel it is shown the time dependence of the population (green line) of the first qubit as well as the Hamiltonian parameter \( r(t) \) for a symmetric Hamiltonian \( \epsilon_1 = 0, \epsilon_2 = 0 \). In the right panel the same parameters are displayed for \( \epsilon_1 = 0 \) and \( \epsilon_2 = 0.1 \). The complex interplay among these two variable modulated the shape and frequency of the emergent Rabi oscillations in a nontrivial way. Time given in natural units. Be aware of the different time scales of the plots.

where the occupation probability of qubit one, \( p_{1}^{\uparrow}(t) = \langle \sigma_{1}^{+}\sigma_{1}^{-} \rangle(t) \) (green line) is displayed as a function of time as well as \( r(t) \) (purple line). We have considered a relatively low interaction strength (\( \Omega = 0.05 \)) and an initial state in the form \( \rho_{I} = |01\rangle\langle01| \). In the left panel we observe the case of a symmetric system with \( \epsilon_1 = \epsilon_2 = 0 \). Due to the interchange form of the interaction Hamiltonian both qubits will oscillate between their ground and excited states. In the case of no depression, \( \tau \ll 1 \), these oscillations are equivalent between both qubits but when the depression time increases a difference between the qubits arises. Qubit one starts being exited for a longer time than qubit two because when it is excited the depression parameter \( r(t) \) is reduced. This difference increases with \( \tau \) and can use to create a population imbalance between the qubits. Furthermore, if the energy of the qubits is not equal this population imbalance becomes more important, as it is shown in the right panel of Fig. 1. Independently of the energy difference the qubit responsible of the STD increases its population until reaching a value close to 1. Interestingly, this behaviour happens for any energy difference, even very small ones. The population imbalance grows faster for medium values of \( \tau \).

The complex interplay between the qubits dynamics and the depression can be used to create long time entanglement in the system (cf. Fig. 2). In the
Figure 2: Negativity of the system as a function of time. From top to bottom the level of synaptic depression in the system is decreased using, respectively, $\tau = 500, 100, 10, 0.001$. Other parameter values are $U = 0.5$, $\Omega = 0.05$. In the left panel it is shown the time dependence of the population (green line) of the first qubit as well as the Hamiltonian parameter $r(t)$ for a symmetric Hamiltonian $\epsilon_1 = 0$, $\epsilon_2 = 0$. In the right panel the same parameters are displayed for $\epsilon_1 = 0$ and $\epsilon_2 = 0.1$. Time given in natural units. Be aware of the different time scales of the plots.

In the case with no depression the entanglement between the qubits, measured by the negativity \cite{39,40}, oscillates between 0 and the maximum value 0.5 within a time window of $\Delta t \approx 40$. For the energy symmetric case, when the value of the parameter $\tau$ increases the negativity presents a similar behaviour that the first qubit population, increasing the time the system is entangled. This effect, for both the symmetric and asymmetric cases, is displayed in Fig. 2. In the left panel it is shown for the symmetric case and we can observe that the system still presents oscillations but it is entangled for a longer time than in the non-depression limit (see the plotted time scale in all panels). In the right panel, we observe the case with energy imbalance. In this case, there is a fast increase of negativity from a separable state to a maximum-entangled one followed by a slow decay of entanglement. The entanglement life is proportional to the STD variable $\tau$, being very long for $\tau \gg 1$. This method to create entanglement using the present biological inspired synaptic mechanism, is deterministic and autonomous, in the sense that it does not require the application of any specific measurement or rotation in the system.

Finally, we have also studied the dynamics of the system under the measurement-based scenario described in Sec 3. In this case, as we cannot know the value of the average population, we perform a measurement of qubit 1 in the $\{0, 1\}$ basis. This measurement makes the system collapse, and depending on its output, we associate to a qubit collapsed state, namely $s_c$, the value 0 or 1, which
then affect the dynamics of the depression variable $r_i(t)$ (see Methods section). The behaviour of the system is shown in Fig. 3. In the left panel the population of the first qubit is displayed for three different trajectories for different values of the depression parameter ($\tau = 10, 1, 0.01$), while in the right panel shows the average behaviour. For small values of the depression parameter $\tau$ both qubits perform small oscillations and the measurement process makes stochastic random jumps. In this case the average population of both qubits is 0.5 as it is expectable. When $\tau$ increases the oscillations when the first qubit is excited are depressed. This makes this configuration more expectable and there are more jumps up than down. The result is a population imbalance that can be appreciated in the average behaviour (left panels). Interestingly, in this measurement-based approach the effect is more relevant for smaller values of $\tau$ than in the previous scenario. This happens because of the binary value of the collapse state variable $s_c$ in this case (See section 3).

3 Methods

3.1 Measurement-based method

In this section, we explain the measurement based framework we used to undermine the difficulty of working with a non-linear von Neumann equation. The main difficulty of the original model is to design a system with a dynamics that
depends on the average population $\langle \sigma^+ \sigma^- \rangle$ as the one given by Eq. 6. An easier way to proceed would be to leave the system evolving according to its Hamiltonian and perform a measurement of the first qubit population. The variable $r(t)$ of the Hamiltonian now follows the dynamics

$$\frac{dr(t)}{dt} = \frac{1 - r(t)}{\tau} - U r(t) s_c, \quad (5)$$

where $s_c$ is a binary variable which can be 1 or 0 depending on the outcome of the measurement. The algorithm that gives the evolution is now the following:

1. At the beginning we select $s_c = 1$. For a time $t_m$ the system evolves following the equations

$$\frac{d\rho(t)}{dt} = -i[H(t), \rho(t)],$$

$$\frac{dr(t)}{dt} = \frac{1 - r(t)}{\tau} - U r(t) s_c. \quad (6)$$

2. At $t = t_m$ the measurement is done, the system collapsed and the variable $s_c$ is adjusted to the outcome. From a simulation perspective this is done in the following way: We choose an uniform random number $x_{\text{rand}}$ and then make the choice

$$s_c = \begin{cases} 
1 & \text{if } x_{\text{rand}} < \langle \sigma^+ \sigma^- \rangle \\
0 & \text{if } x_{\text{rand}} > \langle \sigma^+ \sigma^- \rangle
\end{cases}$$

3. Go to 1.

4 Conclusions

In this paper we have proposed a novel model of quantum neurons with synaptic depression. This model is inspired by the synaptic plasticity that happens in biological neuronal networks and it is based in a non-linear interaction of the qubits. This complex interactions allows the emergence of interesting behaviours such that population imbalance between the qubits and long-lasting entanglement generation. These behaviours are more appealing if an energy shift between the qubits is introduced, even if it is small. Furthermore, we have proposed a measurement-based protocol that can be implemented in real experimental devices.

This work opens the door to the study of complex quantum neuronal networks with dynamical synapses. Several questions are still open as what is the effect of synapse facilitation, and if quantum dynamics can improve the retrieval capacity of neuronal networks. The possibility of multipartite entanglement generation by this kind of model is also interesting to study.
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