The Adjoint SU(5) constrained by a Z₄ flavour symmetry

C. Simões
CFTP and Departamento de Física, Instituto Superior Técnico,
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
E-mail: csimoes@cftp.ist.utl.pt

Abstract. We study the neutrino mass spectrum in the context of the adjoint SU(5) model constrained by a Z₄ flavour symmetry that imposes the nearest-neighbour interaction form on both up- and down-quark mass matrices. We show that at least three adjoint fermionic fields are required in order to reproduce the light neutrino mass spectrum.

1. Introduction
In the Standard Model neutrinos are massless particles, however neutrino oscillation experiments reveal that they have mass. This problem can be solved in the context of Grand Unified Theories (GUT) of which the SU(5) model [1] is an example. In that model light neutrino masses can be generated through the seesaw mechanism due to the presence of right-handed neutrinos.

Ref. [2] studied the SU(5) model in the case where three right-handed neutrinos were considered and a Z₄ flavour symmetry imposed, in order to give the Nearest-Neighbour-Interaction (NNI) form [3]

\[
\text{NNI} = \begin{pmatrix}
0 & * \\
* & 0 \\
0 & *
\end{pmatrix},
\]

(1)
to the both up- and down-quark mass matrices. Despite the good agreement with the neutrino data, the study does not solve the problem with relation to the \(M_e = M^T_d\) at GUT scale being incompatible with the down-type quark and charged lepton mass hierarchy observed at low energy scales. Such problems can be solved in the context of adjoint SU(5) [4, 5].

The purpose of this work was to study the light neutrino mass spectrum obtained in the case of the adjoint SU(5) model where the Z₄ flavour symmetry is used to impose the NNI form on both up- and down-quark mass matrices [6]. In this model, the adjoint fermionic field, 24, is responsible for the generation of the neutrino masses through type-I [7, 8, 9, 10], type-III [11, 12] and radiative [13, 14, 15] seesaw mechanisms.

This work is organized as follows: in section 2 we present the SU(5) × Z₄ model; in section 3 we discuss the obtained neutrino mass matrices and their compatibility with experimental data, then we conclude.
2. The model

The fermionic particle content of this model, based on the adjoint-SU(5) model [4, 5], is composed of three generations of 10 and 5* multiplets plus \( n \) copies of the adjoint fermionic representations, 24. The adjoint fermionic field is responsible for the generation of neutrino masses through type-I and type-III seesaw mechanism. There is also the possibility to generate the neutrino masses through radiative seesaw which is explored in Refs. [16, 6].

The neutrino oscillation data is consistent with the presence of just one 24 fermionic field [4, 5], however due to the implementation of the \( \mathbb{Z}_4 \) flavour symmetry we will see that in this model at least three adjoint fermionic fields are needed. The fermionic fields decompose as

\[
5^* = d^c \oplus L, \\
10 = Q \oplus u^c \oplus e^c, \\
24 = \rho_8 \oplus \rho_3 \oplus \rho_{(3,2)} \oplus \rho_{(3,2)} \oplus \rho_0,
\]

while the Higgs sector is composed by one adjoint multiplet, \( 24_H \), one quintet, \( 5_H \), and one 45 dimensional representation, \( 45_H \), decomposed as

\[
5_H = T_1 \oplus H_1, \\
24_H = \Sigma_5 \oplus \Sigma_3 \oplus \Sigma_{(3,2)} \oplus \Sigma_{(3,2)} \oplus \Sigma_0, \\
45_H = S_8 \oplus S_{(6,1)} \oplus S_{(3,3)} \oplus S_{(3,2)} \oplus S_{(3,1)} \oplus T_2 \oplus H_2,
\]

where \( H_1 \) and \( H_2 \) are the low energy doublets and \( T_1 \) and \( T_2 \) are the coloured SU(3) triplets.

The \( 24_H \) is responsible for breaking the SU(5) gauge group down to the SM one, \( SU(3)_c \times SU(2)_L \times U(1)_Y \), through the vacuum expectation value (VEV),

\[
\langle 24_H \rangle = \frac{v_{24}}{\sqrt{60}} \text{diag}(2, 2, 2, -3, -3).
\]

At the GUT scale, \( \Lambda \), the 45 Higgs field acquires a VEV as

\[
\langle 45_H^{\beta 5} \rangle = v_{45} \left( \delta^\beta_5 - 4 \delta^0_4 \delta^1_3 \right),
\]

where \( \alpha, \beta = 1, \ldots, 4 \); the Higgs quintet gets the VEV, \( \langle 5_H^{\alpha} \rangle = \delta^\alpha_3 v_5 \) where \( \alpha = 1, \ldots, 5 \) breaking the SM gauge group down to \( SU(3)_c \times U(1)_{\text{em}} \). The two VEVs are related by \( v^2 = v_5^2 + 24v_{24}^2 = (246.2 \text{ GeV})^2 \). The Higgs doublets contained in the \( 5_H \) and \( 45_H \) representations generate, at the electroweak scale, the fermion masses through the Yukawa interactions. In fact at low energy we end up with a two Higgs doublet model.

The main goal of this work was to study the neutrino sector in the presence of a \( \mathbb{Z}_4 \) flavour symmetry as proposed in Ref. [2]. This symmetry imposes the NNI form [3] for both up- and down-quark mass matrices. We have shown in Ref. [2] that to have the quark mass matrices in the NNI form, only two independent charges were needed, \( Q(5_H) = \phi \) and \( Q(10_3) = q_3 \); furthermore, we have chosen the 45 Higgs field to couple to the bilinear term \( 10_3 10_3 \) and hence \( Q(45_H) = -2q_3 \). The charges of \( 5^*_1 \) and \( 10_1 \) fields are written in terms of \( \phi \) and \( q_3 \) as,

\[
Q(5^*_1) = (q_3 + 2\phi, -3q_3, -q_3 + \phi), \quad Q(10_1) = (3q_3 + \phi, -q_3 - \phi, q_3).
\]

The adjoint fermionic fields are free under \( \mathbb{Z}_4 \) while the charge of \( 24_H \) is zero to guarantee the existence of the flavour symmetry below the GUT scale. For further details see Ref. [2].
The most general Yukawa interactions are given by
\[
-L_Y = (\Gamma^1_u)_{ij} 10_i 10_j 5_H + (\Gamma^2_u)_{ij} 10_i 10_j 45_H \\
+ (\Gamma^1_d)_{ij} 10_i 5^*_i 5_H + (\Gamma^2_d)_{ij} 10_i 5^*_i 45_H \\
+ (\Gamma^1_D)_{ik} 5^*_i 24_k 5_H + (\Gamma^2_D)_{ik} 5^*_i 24_k 45_H \\
+ M_{kl} \text{tr} (24_k 24_l) + \lambda_{kl} \text{tr} (24_k 24_l 24_H) + \text{H.c.},
\] (7)
where \(\Gamma^1,\Gamma^2\) and \(M\) are symmetric complex matrices and \(\Gamma^1,\Gamma^2\) are just complex matrices; the SU(5) indices were omitted and \(i, j\) are generation indices and \(k, l = 1, \ldots, n\).

Thus, the up- and down-quark masses as well as the charged lepton masses are given by
\[
M_u = 4 \left( \Gamma^1_u + \Gamma^1_u^T \right) v_5 - 8 \left( \Gamma^2_u - \Gamma^2_u^T \right) v_{45},
\]
\[
M_d = \Gamma^1_d v_5^* + 2\Gamma^2_d v_5^*,
\]
\[
M_e = \Gamma^1_e v_5^* - 6\Gamma^2_e v_{45}^*,
\] (8)
the form of the matrices \(\Gamma^1,\Gamma^2\) can be found in Ref. [2]. From the last two lines in Eq. (8) one sees that the mismatch between the down-type and charged lepton matrices is given by
\[
M_d - M_e^T = 8\Gamma^2_d v_{45}^*.
\] (9)

The mass matrices of the fermionic fields responsible for the neutrino masses, through type-I, type-III and radiative seesaw, \(\rho_0, \rho_3\) and \(\rho_8\), are given respectively by
\[
M_{\rho_0} = \frac{1}{4} \left( M - \frac{v_{24}}{\sqrt{30}} \lambda \right),
\]
\[
M_{\rho_3} = \frac{1}{4} \left( M - \frac{3v_{24}}{\sqrt{30}} \lambda \right),
\]
\[
M_{\rho_8} = \frac{1}{4} \left( M + \frac{2v_{24}}{\sqrt{30}} \lambda \right).
\] (10)

The form of the matrices \(M\) and \(\lambda\) is not yet known, however from Eq. (7) and since \(24_H\) is trivial under \(Z_4\) one can say that the two matrices have the same form; this form depends only on the \(Z_4\) charges of the adjoint fermionic fields.

The effective neutrino mass matrix, \(m_\nu\), obtained from type-I and type-III seesaw, is given by
\[
m_\nu = -m_D^{\rho_0} M_{\rho_0}^{-1} m_D^{\rho_0 T} - m_D^{\rho_3} M_{\rho_3}^{-1} m_D^{\rho_3 T},
\] (11)
where the matrices \(m_D^{\rho_0}\) and \(m_D^{\rho_3}\) are given by
\[
m_D^{\rho_0} = \frac{\sqrt{15}}{2} \left( \frac{v_5}{5} \Gamma_D + v_{45} \Gamma_D^2 \right),
\]
\[
m_D^{\rho_3} = -v_5 \Gamma_D + 3 v_{45} \Gamma_D^2.
\] (12)

In this report, for simplicity we will not consider the contribution from radiative seesaw; details on the effects of such mechanism can be found on Ref. [6]. However, it is important to emphasize that this does not change the conclusions, since their contribution to the \(m_\nu\) matrix is similar to the one from type-I and type-III seesaw mechanism.

The details concerning the unification of gauge couplings, proton decay and leptogenesis in this model can be seen in Ref. [6].
3. Effective Neutrino Textures

In the previous section, we have anticipated that reproducing the neutrino masses requires at least the introduction of three 24 fermionic fields. In order to clarify this statement, we have scanned the $\mathbb{Z}_4$ charges of the fields for the cases where just one, two or three adjoint fermionic fields are present. Notice that, as already mentioned, the $\mathbb{Z}_4$ charges of the adjoint fermionic fields are free. The resulting effective mass matrices, $m_\nu$, are sketched in Table 1; we have called by the same name matrices with the same form and defined $X_g = P_g X P_g^T$ where $\{P_g\}$ are the $3 \times 3$ permutation matrices.

Looking at the obtained mass matrices one can conclude that the cases with one and two 24 fermionic fields can not reproduce the neutrino oscillation data [17]. Hence the only possibility to have a neutrino mass spectrum compatible with data is by introducing three adjoint fermionic representations. It is interesting to note that the effective neutrino mass matrices found here for the case of three 24 fermionic fields are the same as the ones found in Ref. [2], where a similar $\mathbb{Z}_4$ flavour symmetry was imposed in the context of three right-handed neutrinos. This is a consequence of the flavour symmetry behind the model.

At this point some comments are in order. The effective neutrino mass matrices given in Eq. (11) require that both $M_{\nu}$ and $M_{\rho}$ are non singular. However, as the $\mathbb{Z}_4$ charges of the adjoint fermionic fields are free one can have $\mathbb{Z}_4$ charges that lead to $|M| = 0$. In such case the usual seesaw mechanism is substituted by the singular seesaw mechanism [18, 19, 20, 21]. Just for illustration, we give in the Appendix an example for the singular seesaw mechanism.

It is not possible, in any of the cases where the singular seesaw applies, to obtain the neutrino mass spectrum compatible with experimental data. In the case of two adjoint fermionic fields and when the Majorana matrix has rank 1, the neutrino mass spectrum is composed by one very heavy neutrino, two electroweak neutrinos, one light seesaw neutrino and one massless neutrino. For the case where three 24 fermionic representations are added and the Majorana matrix has rank 1, the neutrino spectrum consists of one heavy neutrino, two electroweak neutrino, two 24 fermionic fields.

Table 1. The effective neutrino mass matrices for one, two and three 24 fermionic representations, where $X_g = P_g X P_g^T$ with $\{P_g\}$ the $3 \times 3$ permutation matrices.

| number of 24 fermionic fields | $m_\nu$ |
|-------------------------------|---------|
| 1                             | $A = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $A_{12} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & 0 & 0 \end{pmatrix}$, $B_{12} = \begin{pmatrix} * & 0 & * \\ 0 & 0 & 0 \\ * & 0 & * \end{pmatrix}$ |
| 2                             | $A = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & 0 \end{pmatrix}$, $C = \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & * & * \\ 0 & 0 & 0 \\ * & 0 & * \end{pmatrix}$ |
| 3                             | $C = \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & 0 \end{pmatrix}$, $\text{NNI}_{13} = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ * & 0 & 0 \end{pmatrix}$, $E = \begin{pmatrix} 0 & * & 0 \\ 0 & 0 & * \\ * & 0 & 0 \end{pmatrix}$, $C_{12} = \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$, $\text{NNI}_{132} = \begin{pmatrix} * & * & 0 \\ 0 & * & 0 \\ * & 0 & 0 \end{pmatrix}$, $E_{12} = \begin{pmatrix} * & * & 0 \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$ |
one seesaw neutrino and two massless neutrinos; on the other hand, in the case where the rank of \( M \) is two one has two heavy neutrinos, two electroweak neutrinos, one intermediate neutrino and one massless neutrino. None of these cases can reproduce the experimental data hence we end up with only three 24 fermionic representations where neutrino masses are generated through type-I and type-III seesaw mechanism.

In Ref. [2], a study was performed of the compatibility of the six textures in the last line of Table 1 and only the textures \( E \) and \( E_{12} \) were found to be phenomenologically compatible with the actual experimental data. In Table 2 are sketched all possible \( Z_4 \) charges for the fermionic and Higgs fields of the model; it is also shown the matrices \( m_{\nu_0}^p \) and \( M_{\nu_0} \) for \( \alpha = 0, 3 \). Notice that the matrix \( M_{\nu_0} \) has the same zero pattern as \( M_{\rho_0} \), likewise the matrix \( m_{\nu_0}^p \) has the same zero pattern as \( m_{\rho_0}^p \).

Table 2. The \( Z_4 \) charges for the fermionic and bosonic fields of the model in the case where the effective neutrino textures are phenomenologically viable. \( Q(45_H) = -2 Q(10_3) \) and \( \alpha = 0, 3 \).

| \( Q(5^*_1) \) | \( Q(10_1) \) | \( Q(5_H) \) | \( Q(24_i) \) | \( M_{\nu_0} \) | \( m_{\nu_0}^p \) | \( m_{\nu} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| (3,1,0)   | (0,2,1)  | 1                | (1,2,3) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & * & 0 \\ 0 & * & 0 \\ 0 & * & * \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & * \end{pmatrix} \) |
| (1,3,0)   | (0,2,3)  | 3                |                     | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) |
| (1,3,2)   | (2,0,3)  | 1                | (0,1,3) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) |
| (3,1,2)   | (2,0,1)  | 3                |                     | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) |
| (2,0,1)   | (1,3,0)  | 1                | (1,2,3) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) |
| (2,0,3)   | (3,1,0)  | 3                |                     | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) |
| (0,2,3)   | (3,1,2)  | 1                | (0,1,3) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) |
| (0,2,1)   | (1,3,2)  | 3                |                     | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & * & 0 \end{pmatrix} \) |

In what follows we will explore the phenomenological consequences of the two viable textures, \( E \) and \( E_{12} \), confronting the predictions found for the charged-lepton mass matrices and effective neutrino mass matrices with the observable neutrino data at \( M_Z \) energy scale. This is possible since the \( Z_4 \) flavour symmetry is preserved till the electroweak symmetry breaking and therefore the form of the matrices \( \Gamma_d^{1,2}, \Gamma_D^{1,2} \) and \( M_{\rho_0,\rho_3} \) remains unchanged.

The charged lepton mass matrix, \( M_e \), and the effective neutrino mass matrices, \( m_\nu \), can be written without loss of generality as

\[
M_e = K_e \begin{pmatrix} 0 & A_e & 0 \\ A'_e & 0 & B_e \end{pmatrix}, \quad m_\nu = P_g \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu e^{i\phi} \end{pmatrix} P_g^T. \tag{13}
\]
Table 3. The neutrino oscillation parameters at 3σ range from Ref. [17] for both normal and inverted hierarchies.

| parameters | NH        | IH        |
|------------|-----------|-----------|
| $\Delta m_{21}^2$ $[10^{-5}\text{eV}^2]$ | 7.12 – 8.20 |           |
| $|\Delta m_{31}^2| [10^{-3}\text{eV}^2]$ | 2.31 – 2.74 | 2.21 – 2.64 |
| $\sin^2 \theta_{12}$ | 0.27 – 0.37 |           |
| $\sin^2 \theta_{13}$ | 0.017 – 0.033 |           |
| $\sin^2 \theta_{23}$ | 0.26 – 0.68 | 0.37 – 0.67 |

Note that $m_\nu$ has two possibilities that correspond to $E$ and $E_{12}$, in such case the permutation matrix, $P_g$, that enters in Eq. (13) is respectively the $3 \times 3$ identity matrix, $P_e$, or the matrix $P_{12}$ given by,

$$P_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{14}$$

The constants $(A, A', B, B', C)_e$ and $(A, B, C, D)_\nu$ can be taken real and positive and the diagonal phase matrix $K_e$ can be parameterised as $K_e = \text{diag}(e^{ix_1}, e^{ix_2}, 1)$.

In order to confront the predictions from the charged lepton mass matrices and the effective neutrino mass matrices with the neutrino oscillation data one needs to diagonalize both $M_e$ and $m_\nu$ and hence compute the leptonic mixing matrix. The Pontecorvo-Maki-Nakagawa-Sakata (PMNS)[22, 23, 24] matrix, $U$, is given by

$$U = U_e^T P_g U_\nu, \tag{15}$$

where $U_e$ and $U_\nu$ are the charged leptons and neutrino diagonalizing matrices, respectively.

In the numerical analysis we have used the neutrino oscillation data [17] sketched in Table 3 and the charged-lepton masses at $M_Z$ scale, from Ref. [2],

$$m_e(M_Z) = 0.486661305 \pm 0.000000056 \text{ MeV}, \quad m_\mu(M_Z) = 102.728989 \pm 0.000013 \text{ MeV},$$

$$m_\tau(M_Z) = 1746.28 \pm 0.16 \text{ MeV}, \tag{16}$$

and varied all the input parameters within their allowed range taking into account the appropriate values for the free parameter of Eq (13); for more details see Refs. [2, 6].

We have found that the mass matrix $E$ is compatible with a normal hierarchy spectrum while the mass matrix $E_{12}$ is compatible with inverted hierarchy. For the normal hierarchy case, where the effective neutrino mass matrix has the form $E$, we found the lightest neutrino mass in the range $m_1 = [0.353; 20.884] \times 10^{-3} \text{ eV}$ while for the inverted hierarchy we found $m_3 = [2.575; 15.335] \times 10^{-3} \text{ eV}$. For more details see Ref. [6].

4. Conclusions
In this work we have studied the phenomenological consequences of the adjoint $\text{SU}(5) \times \mathbb{Z}_4$ in the presence of adjoint fermionic fields. The $\mathbb{Z}_4$ flavour symmetry imposed leads to the NNI form for both up- and down-quark mass matrices. Due to the $\text{SU}(5)$ symmetry the charged lepton mass matrix has also NNI form.
In this model, neutrinos get light masses through type-I, type-III and radiative seesaw mechanisms mediated, respectively, by the $\rho_0$, $\rho_3$ and $\rho_8$ components of the 24 fermionic fields. It is important to note that the contribution from radiative seesaw does not change the form of the effective neutrino mass matrix; details on radiative seesaw can be found in Ref. [6].

Since the adjoint fermionic fields are not constrained by any symmetry gauge, their $Z_4$ charges are free leading to several different patterns for the effective neutrino mass matrices. We have studied all these patterns and showed that at least three 24 fermionic fields were required in order to reproduce a light neutrino mass spectrum compatible with the actual oscillation data. Furthermore, we found that only the mass matrices $E$ and $E_{12}$ in Table 1 are phenomenologically viable and compatible with a normal and inverted neutrino mass spectrum, respectively.

Acknowledgments
I would like to thank the organizers of DISCRETE2012 for the opportunity to present my work. I would like to thank Juan Antonio Aguilar-Saavedra and Filipe Joaquim for a visit to Universidad de Granada, where this talk was prepared, and the Granada group for their great hospitality. This work was supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) under the contract SFRH/BD/61623/2009 and the project PEst-OE/FIS/UI0777/2011.

Appendix
The singular seesaw [18, 19, 20, 21] applies when the determinant of the Majorana matrix, $M_R$, that enters in the usual seesaw formula $m_\nu = -m_D M_R^{-1} m_D^T$ is zero; in that case $M_R^{-1}$ is not defined.

The idea of the singular seesaw is to “promote” the heavy neutrino fields with zero mass into light neutrinos and then apply the usual seesaw formula to the new matrices thus obtained.

Let us consider, just as example, the case of two 24 fermionic representations and the Dirac, $m_D$, and the Majorana, $M_R$, matrices given by,

$$M_D = \begin{pmatrix} 0 & A \\ B & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad M_R = \begin{pmatrix} 0 & 0 \\ 0 & C \end{pmatrix},$$

The effective neutrino matrix is of the following form,

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & A \\ 0 & 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & B & 0 & 0 & 0 \\ A & 0 & 0 & 0 & C \end{pmatrix},$$

the light neutrino mass matrix is

$$m_\nu = m_L - m_D M_R^{-1} m_D^T,$$

where the matrices $m_L$, $m_D$ and $M_R$ are given by

$$m_L = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B \\ 0 & 0 & 0 & 0 \\ 0 & B & 0 & 0 \end{pmatrix}, \quad m_D = \begin{pmatrix} A \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad M_R = C.$$

It is worth mentioning that the singular seesaw can be used in the case where there is at least one heavy state with non-zero mass, otherwise $M_R^{-1}$ in the seesaw formula is not defined.
References

[1] Georgi H and Glashow S 1974 Phys.Rev.Lett. 32 438–441
[2] Emmanuel-Costa D and Simões C 2012 Phys.Rev.C 85 016003 (Preprint 1102.3729)
[3] Branco G C, Lavoura L and Mota F 1989 Phys. Rev. D 39 3443
[4] Fileviez Pérez P 2007 Phys.Rev. D 76 071701 (Preprint 0705.3589)
[5] Fileviez Pérez P 2007 Phys. Lett. B 654 189–193 (Preprint hep-ph/0702287)
[6] Emmanuel-Costa D, Simões C and Tórtola M (Preprint 1303.5699)
[7] Minkowski P 1977 Phys.Lett. B 67 421
[8] Yanagida T 1979 Conf.Proc. C7902131 95 in Proc. of the Workshop on Unified Theory and Baryon Number in the Universe, KEK, March 1979
[9] Gell-Mann M, Ramond P and Slansky R 1979 Conf.Proc. C790927 315–321 to be published in Supergravity, P. van Nieuwenhuizen & D.Z. Freedman (eds.), North Holland Publ. Co., 1979
[10] Mohapatra R N and Senjanovic G 1980 Phys.Rev.Lett. 44 912
[11] Foot R, Lew H, He X and Joshi G C 1989 Z.Phys.C 44 441
[12] Ma E 1998 Phys.Rev.Lett. 81 1171–1174 (Preprint hep-ph/9805219)
[13] Zee A 1980 Phys.Lett. B 93 389
[14] Wolfenstein L 1980 Nucl.Phys.B 175 93
[15] Fileviez Perez P and Wise M B 2009 Phys. Rev. D 80 053006 (Preprint 0906.2950)
[16] Kannike K and Zhuridov D V 2011 JHEP 07 102 (Preprint 1105.4546)
[17] Forero D, Tórtola M and Valle J 2012 Phys.Rev. D 86 073012 (Preprint 1205.4018)
[18] Johnson R, Ranfone S and Schechter J 1986 Phys.Lett. B 179 355
[19] Glashow S L 1991 Phys.Lett. B 256 255–257
[20] Fukugita M and Yanagida T 1991 Phys.Rev.Lett. 66 2705–2707
[21] Allen T, Johnson R, Ranfone S, Schechter J and Valle J 1991 Mod.Phys.Lett. A6 1967–1976
[22] Pontecorvo B 1957 Sov. Phys. JETP 6 429
[23] Pontecorvo B 1958 Sov. Phys. JETP 7 172–173
[24] Maki Z, Nakagawa M and Sakata S 1962 Prog. Theor. Phys. 28 870–880