Mode formation of free surface rotating flow between concentric vertical cylinders

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Abstract. Mode exchanges of flows between concentric and rotating cylinders with vertical axes have been studied by numerical and experimental approaches. The lengths of the cylinders are finite, and the inner cylinder rotates and the outer cylinder is stationary. The bottom end wall of the annulus is a fixed solid wall and the top is a free surface between a working liquid and the air, and the wall condition is axially asymmetric. This gives one of the modifications of Taylor-Couette system. In this system, the normal mode flow has an inward flow on the lower end wall and an outward flow near the free surface. In experiment, visualized flows are observed from the top and the side of the cylinders, and the position of the free surface is measured. Numerical methods are based on the unsteady axisymmetric equations, and the gravitational acceleration and the surface tension are considered to formulate the dynamics of the free surface. The experimental result and numerical result show the primary mode, the secondary normal mode and the anomalous mode, which are similar to the modes found in the symmetric system. The regions where the primary modes and the normal secondary modes appear are determined in the space spanned by the Reynolds number and the aspect ratio.

1. Introduction
Among studies on various modes found in Taylor-Couette flows, some have been conducted on modulated and/or extended systems, as well as the original flows between two concentric cylinders. In this paper, a free surface is introduced as a new degree of freedom which determines the flow system. The axes of the cylinders are parallel to the direction of the gravitational acceleration and the air-liquid interface appears at the top of the vertical annulus, while the bottom end wall of the annulus is fixed and stationary.

One free surface flow around a rotating cylinder is found in a viscous fingering. In the studies of this flow [1, 2], the interest is on the thin shear layer and the Hele-Shaw flow, and the centrifugal effect of the rotation is not considered. Campero et al. [3], Baier and Graham [4] and Dluska et al. [5], for example, examined the behavior of the immiscible liquids in Taylor-Couette system and investigated the behavior of the liquid-liquid interface. The main parameters of the flows they
concerned were the difference of the density and the rotation speed of the cylinder. They showed the effect of these parameters on axially stratified flows and radially banded flows.

Some of the present authors performed detailed experiments on Taylor-Couette system which had a vertical axis and a free surface at the top, and determined the bifurcation diagram of the flow [6, 7]. Linek and Ahlers [8] studied the selection of the wave numbers formed in free-surface Taylor-Couette flows with varying aspect ratios and specified the stability of the vortices. Experimental results by Djeridi et al [9, 10] and Atkhen et al [11] presented the motion of air bubbles captured by the ventilation near the free surface at the upper end. Ammar et al [12] showed the experimental result of the effect of the Reynolds number and the aspect ratio on the stability of the flow. The consequences of these studies are very informative to investigate the formation of the flow modes.

We direct our attention to these free surface flows as shown in figure 1 and consider the mode exchanges of the asymmetric Taylor-Couette system. No ventilation at the free surface is expected and the flow is different from that measured by Djeridi et al and Atkhen et al. In the present paper, we especially discuss the flow patterns found at small aspect ratios less than 5.7 and present the result obtained by the numerical and experimental approaches.

2. Model

2.1. Formulation

The outer cylinder of the system is stationary and the inner cylinder is rotating, and the radii of these cylinders are \( r_{out} \) and \( r_{in} \), respectively. The axes of the cylinders are vertical and they are parallel to the direction of the gravitational acceleration. The bottom end wall of the cylinders is a solid wall, and the top is the free surface between the working fluid and the air. The height of the quiescent working fluid is \( L \). The reference length \( D \) is the radial gap \( r_{out} - r_{in} \), the reference velocity \( U \) is the azimuthal velocity component of the inner cylinder and the reference time is given by the fraction of the reference length to the reference velocity. The kinematic viscosity, the density and the surface tension of the working fluid are \( \nu \), \( \rho \) and \( \sigma \), respectively. The dimensionless parameters determining the flow are the Reynolds number \( Re \), the Weber number \( We \) and the Froude number \( Fr \):

\[
Re = \frac{UD}{\nu}, \quad We = \frac{\rho DU^2}{\sigma}, \quad Fr = \frac{U}{\sqrt{gD}}, \quad (1)
\]

where \( g \) is the acceleration of gravity. The geometrical parameters are the radius ratio \( \eta = \frac{r_{in}}{r_{out}} \) and the aspect ratio \( \Gamma = \frac{L}{D} \).

The governing equations are represented in the cylindrical coordinate system \((r, \theta, z)\) and they are the equation of continuity, the momentum equations and the conservation equations of \( F \) which is the fraction of the working fluid in confined flow regions (VOF function):

\[
\nabla \cdot \mathbf{u} = 0, \quad (1)
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \frac{g}{Fr^2}, \quad (2)
\]
\[
\frac{\partial F}{\partial t} + \nabla \cdot (uF) = 0. \tag{3}
\]

In these equations, \( u = (u, v, w)^T \) is the velocity vector and \( g \) is the gravitational vector. The dimensionless time is \( t \) and the time in the viscous scale is defined as \( t_{vis} = t / Re \).

The effect of the surface tension is formulated by evaluating the radial curvature \( \kappa \) from the displacement of the surface \( z_s(r) \),

\[
\kappa = \frac{\partial^2 z_s / \partial r^2}{(1 + (\partial z_s / \partial r)^2)^{3/2}}, \tag{4}
\]

and the increase of the pressure. While the circumferential curvature may also be considered [13], we have confirmed that it is small and negligible.

The convection terms of the equations are modeled by the third-order upstream difference method and other terms are expressed by the central difference method. The time integration is the fractional method. The velocity boundary condition is the non-slip condition, and the Neumann type condition is applied to the pressure equation. Initially, the fluid is at rest in the entire flow field, and the angular velocity of the inner cylinder suddenly or gradually increases to attain a prescribed Reynolds number.

2.2. Experiment

Two sets of experimental apparatuses were used. The small one has cylinders with radii of 20 mm and 30 mm and the large one comprises cylinders with 40 mm and 60 mm, and the radius ratio \( \eta = 0.667 \). In the system of the infinite cylinders, the critical Reynolds number for the onset of Taylor vortices is 76.3 [14]. A small amount of aluminum flakes was used to visualize the flow. The top views and the side views of the flows were recorded by a still camera and a video camera.

2.3. Conditions

The fluid property may depend on the date and time of the run of the experiment. Average values of the kinematic viscosity \( \nu \), the density \( \rho \) and the surface tension \( \sigma \) are introduced for the comparison between the experimental results and the numerical results. These values for the smaller apparatus are 8.9 mm²/s, 1124 kg/m³ and 7.0 \( \times \) \( 10^{-2} \) N/m, respectively, and those for the larger apparatus are 9.5 mm²/s, 935 kg/m³ and 2.1 \( \times \) \( 10^{-2} \) N/m, respectively. The acceleration of the gravity is 9.8 m/s². The radius ratio is fixed to be 0.667.

3. Results

3.1. Experimental observation of free surface flows

The experimental observations of the flow from the top are shown in figure 2. These are the photos

**Figure 2.** Experimental observations of the free surface flow in the top view. The aspect ratio is 5.7. The white circular plane in the center is the inner cylinder rotating in the counterclockwise direction and the surrounding thin rim is the acrylic outer cylinder.
taken at the lager apparatus and the aspect ratio $\Gamma$ is 5.7. The inner cylinder rotates in the counterclockwise direction. At the Reynolds numbers 1259, 1424, 1697 and 1877, the Weber numbers are 318, 407, 579 and 707, and the Froude numbers are 1.35, 1.53, 1.82 and 2.01, respectively. The flow at $Re = 1259$ is axisymmetric. Though some circumferential variations of the flow pattern appear in the case at $Re = 1424$, they are weak and the symmetry is almost kept. This implies that the present formulation based on the axisymmetric equations is valid at the Reynolds number below one thousand and few hundreds. When the Reynolds number is higher, traveling waves emerge and the flow pattern becomes irregular. The dynamics of the wave and the vorticity on the free surface induces interesting and important phenomena, and many researches and reviews have been conducted [15, 16, 17, 18]. This problem is outside of the scope of the present paper and it will be discussed elsewhere.

3.2. Primary modes and secondary modes
When the system is symmetric and the aspect ratio is very small, Cliffe [19] and Furukawa et al. [20], for example, presented their numerical results of the critical loci of the anomalous one-cell mode (single cell flow) and the normal two-cell mode in the $(Re, \Gamma)$ plane. Figure 3 shows the diagram at the aspect ratio about unity, which is obtained by our three-dimensional calculation. In the figure, Nn

![Figure 3](image)

**Figure 3.** Critical loci for the one-cell mode (A1) and the normal two-cell mode (N2) under the condition that the two end wall are fixed (numerical result).

![Figure 4](image)

(a) Critical locus for N1 and N3.  
(b) Critical locus for N3 and N5.

**Figure 4.** Critical loci for the normal one-cell mode (N1), the normal three-cell mode (N3) and the normal five-cell mode (N5) under the condition that the bottom end wall is fixed and the top is the free surface (numerical result).
represents the normal $n$-cell mode and $A_n$ denotes the anomalous $n$-cell mode. When the Reynolds number is higher, the secondary mode appears, which is a normal mode but has a different number of cells from the one the primary mode has. Cliffe [21] determined the exchanges between the primary mode and the secondary normal mode in the symmetric system. In the asymmetric system, the normal mode flow has an odd number of cells. Figure 4 shows the diagrams where the primary mode $N_1$ and the secondary normal mode $N_3$ appear and the primary mode $N_3$ and the secondary normal mode $N_5$ appear in the asymmetric system. The Weber number $We$ and the Froude number $Fr$ at $Re = 1000$ are 201 and 1.07, respectively. As the aspect ratio becomes higher, the critical Reynolds number at the appearance of the secondary mode decreases. At the Reynolds number between the ranges where the secondary modes appear, the normal three-cell mode was found.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Experimental observations of the anomalous two-cell mode in the asymmetric system. The rotating inner cylinder and the stationary outer cylinder are on the left and the right sides, respectively. The upper row shows the whole flow regions and the lower row shows the flows near the free surface at the top.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Numerical results of the normal mode flows and the anomalous mode flows in the asymmetric system.}
\end{figure}
3.3. Anomalous mode

In the asymmetric system, the flow in the anomalous mode has an outward flow on the bottom end wall and/or an inward flow near the free surface. Figure 5 shows the experimental observations of the anomalous two-cell mode. The smaller apparatus is used and the Weber number and the Froude number are 45.8 and 1.71 at $Re = 600$, respectively. The rotating inner cylinder is on the left side and the stationary outer cylinder is on the right side. The flow on the bottom end wall is outward and the lower cell is an anomalous cell rotating in the counterclockwise direction, while the upper cell is a normal cell with an outward flow at the free surface.

Figure 6 includes the numerically predicted anomalous modes obtained by sudden or gradual accelerations of the inner cylinder. Each figure represents the contour of the Stokes’ stream function in the $(r, z)$ plane. Cells rotating in the counterclockwise direction are represented by the warmer color. When the aspect ratio is 2.0, for example, the upper cell in the two-cell mode flow is anomalous at $Re = 200$ and the lower cell is anomalous at $Re = 800$, and the flow is the normal three-cell mode at $Re = 1000$. The lower anomalous cell accompanies extra cells at the corners between the bottom end wall and the cylinder walls. When the aspect ratio is 2.4 and 2.6, the anomalous mode has four cells and the anomalous cell is found near the bottom end.

4. Conclusions

Exchanges of the flow modes were studied in the axially asymmetric Taylor-Couette system. The axes of the cylinders are vertical and the free surface appears at the top. The Reynolds number at which the flow keeps its axisymmetry was estimated by the experimental observation. The numerical method was used to determine the mode exchange diagram between the primary modes (the one-cell mode and the three-cell mode) and the secondary normal modes (the three-cell mode and the five-cell mode). The experimental result showed the anomalous mode with an outward flow on the bottom end wall. The numerical result predicted the anomalous mode with an inward flow near the top, as well as the anomalous mode found in the experiment. As a future work, the study on the relation between the free surface curvature and the aspect ratio and the effect of the free surface on the vortex structure are considered.

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References

[1] Michalland S, Rabaud M and Couder Y 1993 Europhysics Letters 22-1 17
[2] Benilov E S and O’Brien S B G 2005 Physics of Fluids 17-5 052106-1
[3] Campero R J and Vigil R D 1999 Industrial & Engineering Chemistry Research 38 1094
[4] Baier G and Graham M D 2000 Physics of Fluids 12-2 294
[5] Dluska E Wronski S and Ryszczuk T 2004 Experimental Thermal and Fluid Science 28 467
[6] Toya Y Nakamura I Yamashita S and Ueki Y 1994 Acta Mechanica 102 134
[7] Nakamura I and Toya Y 1996 Acta Mechanica 111 33
[8] Linek M and Ahlers G 1998 Physical Review 58-3 3168
[9] Djéridi H J-F FAVÉ J-Y Billard and D H Fruman 1999 Experiments in Fluids 26 233
[10] Djeridi H Gabillet C G and Billard Y 2004 Physics of Fluids 16-1 128
[11] Atkhen K Fontaine J and Wesfreid J E 2000 Journal of Fluid Mechanics 422 55
[12] Ammar M AHCÈNE B and Eddine S S 2005 Canadian Journal of Chemical Engineering 83 652
[13] Batchelor G K 1967 An Introduction to Fluid Dynamics (Cambridge: Cambridge University Press) chapter 1
[14] Koschmeider E L 1993 Bénard Cells and Taylor Vortices (Cambridge: Cambridge University Press) chapter 11
[15] Wu J and Wu M 1996 Vorticity dynamics on boundaries Advances in Applied Mechanics 32 eds
Hutchinson J W and Wu T Y (Boston: Academic Press) pp 119-275

[16] Sarpkeya T 1996 *Annual Review of Fluid Mechanics* **28** 83

[17] Lundgren T and Koumoutsakos P 1999 *Journal of Fluid Mechanics* **382** 351

[18] Dopazo C, Lozano A and Barreras F 2000 *Physics of Fluids* **12-8** 1928

[19] Cliffe K A 1983 *Journal of Fluid Mechanics* **135** 219

[20] Furukawa H Watanabe T Toya Y and Nakamura I 2002 *Physical Review E* **65** 036306-1

[21] Cliffe K A 1988 *Journal of Fluid Mechanics* **197** 57