A NEW DYNAMIC MODEL TO OPTIMIZE THE RELIABILITY OF THE SERIES-PARALLEL SYSTEMS UNDER WARM STANDBY COMPONENTS

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Abstract. Redundancy allocation problem (RAP) is a common technique for increasing the reliability of systems. In this paper, a new model for the RAP is introduced that takes into account the warm standby and mixed strategy, the model dynamics, and the type of the strategy in redundancy allocation problems. A recursive formula is first obtained for the reliability function in the dynamic warm standby and mixed redundancy strategies that leverages the success mode analysis and works for any arbitrary failure-time distribution. Failure rates for warm standby units change before and after their replacement with a damaged unit, and, therefore, the reliability function in warm standby varies with time (i.e., the model is dynamic). Although dynamic models are commonplace in practice, they are more challenging to assess than static models, which have been mainly considered in the literature. An optimization problem is then formulated to select the best redundancy strategy and redundancy levels. Genetic algorithm and particle swarm optimization are leveraged to solve the problem. Finally, the efficiency of the presented method is verified through a numerical example. The experimental results verify that the proposed model for RAP significantly improves the system reliability, which can be of vital importance for system designers.

1. Introduction. System reliability is leveraged as a design factor in all industries and its optimization assures a high performance during systems’ life cycle. The overall system reliability can be increased by allocating redundancies to its subsystems. Therefore, there has been a surge of interest recently in RAP [22, 5, 8, 24, 43] to maximize reliability. Reliability maximization results in customer satisfaction maximization and gaining a large market share [52, 26]. Using RAP to improve the reliability of systems and products will in turn help organizations remain more competitive in the market [20, 24]. RAP has been used for vast variety of system’s structures, including series [32], parallel series [57, 14, 47, 17, 55, 2, 56, 36, 58, 50, 35], and k-out-of-n systems [13, 33, 20, 10], to name a few. There are generally two

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classes of RAP: the first class is limited to component selection from a discrete set of subsystems with a priori given characteristics, such as reliability, weight, cost, and volume. The second class also determines the type of components that must be used in each subsystem as well as their redundancy levels. i.e., the component’s reliability is not assumed to be known a priori and is a design parameter to be selected. Moreover, in this class, the characteristics of the component (i.e., cost, volume, and weight) are specified as functions of the component’s reliability [9, 45].

Active and standby strategies are two typical strategies leveraged in the literature of RAP to determine how redundant components must be utilized. In the active redundancy strategy, all non-damaged components are operating throughout the mission, even though only a portion of them can successfully achieve the mission [15]. In the standby redundancy strategy, one or multiple components remain operational while some other components are kept in a standby mode as redundant components [3]. Upon a failure in the operating components, a standby unit is activated and its mode is switched to operating to take over the responsibility of the failed component [38, 34]. Standby strategies are very helpful in complex industries [12]. The levels of standby redundancy determine the tradeoff between the energy consumption and the recovery time required for switching standby component to the operation mode [16, 44].

A warm-standby model considers both hot and cold standby components [48]. A hot standby component operates concurrently with the online unit. However, a hot standby component is typically subject to a reduced failure rate during the warm standby compared to its operational mode [49]. On the other hand, a cold standby component is not powered before being activated. Therefore, it is fully shielded from the operating stress which makes its hazard function to be zero [46, 23]. The warm standby model integrates both hot and cold standby. The warm standby model consumes less power compared to the hot standby case and requires shorter initialization and less recovery time compared to the cold standby case [21]. An example of warm standby systems is keeping extra generating units for power systems in the standby mode. As another example, wireless sensor networks also utilize the warm standby redundancy strategy to make a tradeoff between their energy consumption and the recovery time required for switching the backup non-operational sensors to their operational mode [1]. In this paper, the focus is on the warm-standby systems because of their variety of applications, and, because both hot and cold strategies are of their special cases. As explained before, for warm-standby systems, since failure rates change after using standby components to replace the faulty units, the standby components have time-dependent failure behaviors which in turn make their reliability function time-dependent. This makes optimizing their reliability more challenging. Optimizing reliability with the dynamic model about warm standby components has been considered in [44, 45, 1]. However, these studies are limited to only two or three standby components. More specifically, reliability for systems with two warm standby components has been studied in the past [44, 1]. However, studies for three components are scarce [45]. Analysis for more than three standby units is very complex and difficult even in cold standby samples when the probability distribution function (pdf) of the life time on the online unit is optional. This becomes more difficult when the warm standby is due to the failure rate of its components depending on time. However, none of the above mentioned papers provided a formulation for calculating reliability for systems with warm standby components, and their results are limited to the case
where each subsystem only uses warm components. Gregory et al. [21] suggested two optimization problems, called component allocation and sequencing problem, as well as component distribution and sequencing problem, based on a reliability evaluation algorithm. Components are limited to be warm standby [21].

Due to the complexity of calculations and the time dependence of warm components, very few studies have been conducted on RAP with a variety of redundancy strategies that each subsystem can use. Existing approaches are limited to static models in which the reliability functions are not time-dependent, which is not realistic. More specifically, Suprasad et al. [46] proposed a static model to calculate system reliability of k-out-of-n systems in which each subsystem can be active or warm standby. Gong et al. [20] analyzed the reliability of static systems with warm and cold standby for each subsystem to solve the RAP. They used an algorithm based on the Markov process. Sharifi et al. [42] used three strategies in the RAP, i.e., active, warm standby, or no redundancy, in series-parallel systems. Hadipour et al. [23] used a mixed strategy under which any one of the active, warm, or mixed strategies may be selected for each subsystem. However, besides being static, in sharp contrast to our approach, objective functions are maximizing the minimum of mean time and minimizing the total system costs. In this work, a novel model is introduced that utilizes a redundancy strategy that combines active and warm-standby strategies and is called a mixed redundancy strategy. A recursive formula is then provided by analyzing the success scenarios about both warm standby and mixed strategy. To our knowledge, no comprehensive formula for these strategies has been presented in the literature, despite the fact that in reality, the system design must simultaneously consider both redundancy strategies. To perform reliability analysis, the analytical integral method is used. In the integration method, since the component failure time can follow any desired distribution, any distribution can be used for the probability density function of the reliability. This is in contrast to Markov-based methods that are applicable only when the component failure behavior follows the exponential distribution.

Since in RAP reliable optimization problems are integer [4] and it has been proven that these problems are NP-hard [30] it is necessary to use an appropriate optimization technique to solve them. Several traditional techniques have been used to solve the RAP. But, when the complexity of the problem increases, these techniques fail to obtain optimal solutions. Also, these techniques need high computational sources. Therefore researchers have applied relevant optimization technique from the group of meta-heuristics algorithms to find a solution for them [51]. To solve the problem of redundancy allocation, meta-heuristic algorithms have been widely used [6, 37, 31]. In this paper, GA [19, 30, 59, 11, 41, 50] and PSO algorithms [27, 54, 7, 39], as well-known successful meta-heuristics algorithms in solving high-dimensional non-convex optimization problems, are used in solving the proposed optimization problem.

The main contributions of this paper are as follows: First, a dynamic mathematical model for optimizing the reliability of a system with \( n \) warm standby components in series-parallel systems with optional failure distributions is introduced. Second, a mixed strategy in which each subsystem can simultaneously use different components (active and warm standby) is used in the proposed model. Third, different redundancy strategies are used in this model in each subsystem to maximize reliability (i.e., selecting components from available options, selecting redundancy type from allowed choices, and determining redundancy levels in different subsystems).
The rest of the paper is organized as follows. The problem statement is presented in Section 2. Mathematical problem modeling is provided in Section 3. Section 4 presents Research method. Section 5 provides a benchmark example to verify the effectiveness of the proposed redundancy strategy. Finally, the conclusion is provided in Section 6.

2. Problem statement. Reliability optimization problems must encode various constraints, including weight, cost, and volume of the system components. This paper introduces a nonlinear integer programming problem for reliability optimization of a series-parallel system while including these mentioned constraints. Series-parallel systems are of the most widely used layouts in various industries for which the main set is placed in series, and redundancy components are added to each of the series components to improve reliability. The system remains intact as long as at least one of the entire series sets operators. In this section, notifications are first provided and then the mathematical model of a series-parallel system with \( S \) subsystems with linear constraints on weight and cost is presented. More specifically, a complex integer programming model is formed that takes into account a new redundancy strategy. Moreover, there are different reliability, weight, cost, and other specifications for each selected type. In addition, there are system-level constraints on weight and cost. Finally, the decisions variables in maximizing reliability are component choices, redundancy strategy type, and redundancy levels. Table 1 shows notations in proposed model.

Model assumption

A) All system components and their failure are independent of each other.
B) Components have two operating states: they either operate or do not operate.
C) There are different components to choose from for each subsystem.
D) The type of component must be the same in each subsystem.
E) There are no maintenance or repair operations for the components.
F) The series-parallel system has an incomplete switching system.

3. Mathematical model. The main aim of redundancy allocation is to maximize reliability with constraints given by weight, volume, cost, and the number of redundant components. To model a reliability problem, therefore, these constraints must be taken into account according to the underlying assumptions of the problem [32]. Specifically, in this paper, RAP is examined to maximize reliability with weight and cost constraints. The general model of the nonlinear programming of complex integer RAP is usually formulated as follows.

\[
\text{max} \quad R(t; z, n_A, n_S) \tag{1}
\]

\[
\text{S.t} \quad \sum_{i=1}^{S} w_i z_i n_i \leq W, \quad z_i \in \{1, 2, \cdots, m_i\}, \quad \forall i = 1, 2, \cdots, S. \tag{2}
\]

\[
\sum_{i=1}^{S} C_i z_i n_i \leq C, \quad z_i \in \{1, 2, \cdots, m_i\}, \quad \forall i = 1, 2, \cdots, S. \tag{3}
\]

\[
n_i = n_{Ai} + n_{Si}, \quad n_i \in \{1, 2, \cdots, n_{\text{max}, i}\}, \quad \forall i = 1, 2, \cdots, S. \tag{4}
\]

The objective function in Eq. (1) represents the system reliability, while the decision variables are the type of components, the redundancy strategy, and level, as well as the number of active and standby components of the subsystem. Eqs. (2) and
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| Table 1. Notations in proposed model |
|-------------------------------------|
| **Sets**                            |
| $A$ The set of subsystems that use active strategy. |
| $S$ The set of subsystems that use standby strategy. |
| $N$ The set of subsystems that do not use any redundancy. |
| $M$ The set of subsystems that use mixed redundancy. |
| $Z$ The set of chosen component. |
| **Decision variables**               |
| $z_i$ Index of chosen component for the subsystem $i$. |
| $n_i$ The total number of components that are used in subsystem $i$. |
| $n_{S,i}$ The number of warm standby components of the subsystem $i$. |
| $n_{A,i}$ Number of active redundant components in the subsystem $i$. |
| **Parameters**                      |
| $n_{max,i}$ Upper bound for $n_i$. |
| $m_i$ The number of components to be chosen from for subsystem $i$. |
| $C_{iz_i}$ Cost and weight for subsystem $i$ for the $z_i$th available component. |
| $R_{iz_i}(t)$ Reliability at time $t$ for subsystem $i$ for the $z_i$th available component. |
| $\lambda_{iz_i}$ The failure rate for subsystems $i$ for the $z_i$th component. |
| $\lambda_{iz_i}$ The reduced failure rate for subsystems $i$ for the $j$th component. |
| $W$ Upper bound for weight. |
| $C$ Upper bound for the cost. |
| $t$ Mission time. |
| $T_{iz_i}^{nS}$ The lifetime of the $z_i$th component of $n$th standby for the subsystem $i$ in standby mode. |
| $T_{iz_i}^{nS}$ The lifetime of the $z_i$th standby component of $n$th standby for the subsystems in operation mode. |
| $T_{iz_i}^{nA}$ The lifetime of the $z_i$th all active component of for subsystem $i$. |
| $T_{iz_i}^{nA,iz_i}$ The lifetime of the $z_i$th all active component of for subsystem $i$. |
| $f_{iz_i}^{nS}(t)$ Pfd for the $n$th warm standby failure arrival of the $z_i$th component for the subsystem $i$. |
| $f_{iz_i}^{nA}(t)$ Pfd for the active failure arrival of the $z_i$th component for the subsystem $i$. |
| $f_{iz_i}^{nA,iz_i}(t)$ Pfd for the maximum failure times of $nA$, the number of the $z_i$th component for the subsystem $i$. |
| $R_{iz_i}^{nS}$ Reliability for $n^t$th warm standby component of the $z_i$th component for the subsystem $i$. |
| $R_{iz_i}^{nS}$ Reliability for $n^t$th warm standby component of the $z_i$th component for the subsystem $i$. |
| $R_{iz_i}^{mix,nS}(t)$ Switching reliability of the $z_i$th component for subsystem $i$ at a time $t$. |
| $R(t, z, n_A, n_S)$ Pfd for the maximum failure times of $nA$, number of the $z_i$th component for the subsystem $i$. |

(3) encode the cost and weight constraints, respectively. Eq. (4) demonstrates the maximum number of components allowed to be used in each subsystem.

3.1. **Objective function.** The objective function in the model is the product of four factors, each one representing a redundancy allocation strategy. i.e., it is assumed here that there are four types of strategy choices for each subsystem of which it can choose from to maximize its reliability. Therefore, the objective function (i.e., the reliability of the whole system), which is obtained by a general form for calculating reliability for all failure time distributions, is given by:

$$R(t, z, n_A, n_S) = \prod_{i \in A} (1 - (1 - R_{iz_i}(t))^{n_A_i}) \times \prod_{i \in S} R_{iz_i}^{nS}(t)$$

$$\times \prod_{i \in N} R_{iz_i}(t) \times \prod_{i \in M} R_{iz_i}^{mix,nS}(t). \quad (5)$$

The first factor in this function is the reliability of subsystems with active redundancy strategy, its second factor is the reliability of subsystems with warm standby strategy, its third factor is the reliability of subsystems with no strategy, and, finally, the fourth factor describes the reliability of subsystems with a mixed strategy. The second and fourth factors of the objective function are defined as follows, in which Eq. (6) shows component reliability formula with warm standby strategy and Eq.
shows component reliability formula with mixed standby strategy.

\[
R_{iz_i}^{S}(t) = R_{iz_i}^{(n-1)S} + \int_{0}^{t} f_{iz_i}^{\text{active}}(t_1) R_{iz_i}^{1S}(t_1) \int_{0}^{t-t_1} f_{iz_i}^{1S}(t_2) R_{iz_i}^{2S}(t_2) \times \int_{0}^{t-t_1-t_2} f_{iz_i}^{1S}(t_3) \rho(t_1 + t_2 + t_3) \\
\times R_{iz_i}^{3S}(t_3) R_{iz_i}^{3S}(t - t_1 - t_2 - t_3) \\
\times \ldots \times \int_{0}^{t-t_1-t_2-\ldots-t_{n-2}} f_{iz_i}^{(n-2)S}(t_{n-1}) \rho(t_1 + \ldots + t_{n-1}) \\
\times R_{iz_i}^{(n-1)S}(t_{n-1}) R_{iz_i}^{(n-1)S}(t - t_1 - \ldots - t_{n-1}) \\
\times d(t - t_1 - \ldots - t_{n-1}) \times \ldots \times d(t_1).
\]

(6)

\[
R_{iz_i}^{\text{mix},nS}(t) = R_{iz_i}^{\text{mix},(n-1)S}(t) + \int_{0}^{t} f_{iz_i}^{\text{max},nA}(t_1) R_{iz_i}^{0}(t_1) \\
\times \int_{0}^{t-t_1} f_{iz_i}^{1S}(t_2) R_{iz_i}^{2S}(t_2) \int_{0}^{t-t_1-t_2} f_{iz_i}^{2S}(t_3) \\
\times R_{iz_i}^{\text{Switch}}(t_1 + t_2 + t_3) R_{iz_i}^{3S}(t_3) \\
\times R_{iz_i}^{3S}(t - t_1 - t_2 - t_3) \times \ldots \\
\times \int_{0}^{t-t_1-t_2-\ldots-t_{n-2}} f_{iz_i}^{(n-2)S}(t_{n-1}) \\
\times R_{iz_i}^{\text{Switch}}(t_1 + \ldots + t_{n-1}) R_{iz_i}^{(n-1)S}(t_{n-1}) \\
\times R_{iz_i}^{(n-1)S}(t - t_1 - \ldots - t_{n-1}) \\
\times d(t - t_1 - \ldots - t_{n-1}) \times \ldots \times d(t_1).
\]

(7)

The following is a complete description of each of these strategies, and that how the objective function formula is obtained in each of the strategies. Four redundancy scenarios as described in Table 2.

| Scenarios | nA; | nS; | Redundancy strategy |
|-----------|-----|-----|---------------------|
| S1        | 1   | 0   | No redundancy       |
| S2        | > 1 | 0   | Active redundancy strategy |
| S3        | = 1 | ≥ 1 | warm-standby redundancy strategy |
| S4        | > 1 | ≥ 1 | Mixed redundancy strategy |

3.1.1. Redundancy strategies. Each subsystem has the option to choose among one of four possible redundancy strategies. Subsystem reliability can then be evaluated by Eq. (8).

\[
R(t; z, n_A, n_S) = \prod_{i \in A} R_i(t; z, n_A; \text{active}) \times \prod_{i \in S} R_i(t; z, n_S; \text{standby}) \\
\times \prod_{i \in N} R_i(t; z; N R) \times \prod_{i \in M} R_i(t; z, n_A, n_S; \text{mixed}).
\]

(8)

The following 4 cases may be considered for every subsystem strategy:
None Redundancy Strategy. The subsystems with non-redundancy have one active component and Eq. (9) shows the reliability of subsystems with no redundant components.

\[
\prod_{i \in N} R_i(t; z; NR) = \prod_{i \in N} R_{iz_i}(t). \tag{9}
\]

The Active Redundancy Strategies. A subsystem with an active redundancy strategy is analogous to a parallel configuration for which all its components operate at the same time. Eq. (10) shows the reliability for the subsystem \(i\) that used the active redundancy strategy.

\[
\prod_{i \in A} R_i(t; z, n_A; \text{active}) = \prod_{i \in A} (1 - (1 - R_{iz_i}(t))^{n_A}). \tag{10}
\]

The Warm Standby Strategy. In this work, each of the warm-standby units has two different time-to-failure distributions, i.e., \(T'\) and \(T\). The random variable \(T\) captures the time-to-failure of a component when it operates online. However, after replacing a faulty unit, the failure parameters and distributions of a standby unit would change. Therefore, the time-to-failure distribution \(T'\) is used after replacing the primary faulty unit. The failure behavior for standby elements in warm-standby systems is time-dependent. i.e., their failure rates differ before and after they are used to replace an online faulty unit. For this reason, both cases must be considered.

A warm standby subsystem initially has an online component that operates with a full failure rate \(\lambda_{iz_i}\) while other standby components operate with a reduced failure rate \(\lambda_{iz_i}^-\). The primary working component fails at any time; the first spare in-line component replaces it. A subsystem and consequently the entire system fail when all units failed. In this paper, \(f_{iz_i}(t)\) and \(f_{\text{active}}(t)\) are considered to have exponential distributions. For an exponential distribution with parameter \(\lambda_{iz_i}\), the probability density and is obtained by

\[
f_{iz_i}(t) = \lambda_{iz_i} e^{-\lambda_{iz_i} t} \tag{11}
\]

To gain reliability, we must obtain the incompatible states and put them together. In the following analysis, the standby components of the system are designated as \(1, 2, \cdots, n\).

Reliability for 2 units (one active and one standby): \(n = 1\) (Standby)

State 1: The active component is functioning at the time \(t\). Eq. (12) represents the probability that the active component operates until time \(t\).

\[
P_r(\text{State 1}) = P_r(T_{iz_i}^{\text{active}} > t) = R_{iz_i}^{\text{active}}(t). \tag{12}
\]

State 2: The active component is failed at the time \(t_1 < t\), and the switch is functioning at the time \(t_1\) and standby component has not failed until the time \(t_1\) (with a lower failure rate \(\lambda_{iz_i}^-\)) and the standby component has not failed from \(t_1\) to \(t\) (with failure rate \(\lambda_{iz_i}\)) and is functioning at operating mode. Eq. (13) represents the mathematical formula of state 2.

\[
P_r(\text{State 2}) = P_r(T_{iz_i}^{\text{active}} \leq t \cap T_{iz_i}^{1S} > t_1 \cap T_{iz_i}^{1S} > t - t_1)
\]
\[
= \int_0^t f_{iz_i}^{\text{active}}(t_1) R_{iz_i}^{\text{Switch}}(t_1) \times R_{iz_i}^{'}(t_1) R_{iz_i}^{1S}(t - t_1) dt_1, \tag{13}
\]

\[
R_{iz_i}^{1S}(t) = P_r(\text{State 1}) + P_r(\text{State 2}), \tag{14}
\]
\[ R_{iz_i}^{1S}(t) = R_{iz_i}^{active}(t) + \int_0^t f_{iz_i}^{active}(t_1) R_{iz_i}^{Switch}(t_1) \times R_{iz_i}^{1S}(t_1) R_{iz_i}^{1S}(t - t_1) \, dt_1. \] (15)

Since states 1 and state 2 are mutually exclusive events, Eq. (14) is written as the sum of states 1 and 2. By substituting Eqs. (12) and (13) on Eq. (14), one obtains Eq. (15).

**Reliability for 3 units (one active and two standbys):** \( n = 2 \) (Standby)

**State 1:** The active component is functioning at the time \( t \)

**State 2:** The active component is failed at the time \( t_1 < t \), and the switch is functioning at the time \( t_1 \) and the standby component has not failed until the time \( t_1 \) (with a lower failure rate \( \lambda_{iz_i}^- \)) and the standby component has not failed from \( t_1 \) to \( t \) (with failure rate \( \lambda_{iz_i} \)) and is functioning at operating mode.

**State 3:** The active component is failed at the time \( t_1 < t \), and the switch is functioning at the time \( t_1 \) and the first standby component has not failed until the time \( t_1 \) (with a lower failure rate \( \lambda_{iz_i}^- \)) and the first standby component failed at the time \( t_2 \), and the switch is functioning at the time \( t_2 \), the second standby component has not failed until the time \( t_2 < t \) (with a lower failure rate \( \lambda_{iz_i}^- \)) and the standby component has not failed from \( t_2 \) to \( t \) (with failure rate \( \lambda_{iz_i} \)) and is functioning at operating mode. Eq. (21) represents reliability for two standby components.

\[ R_{iz_i}^{2S} = p_r(\text{State } 1) + p_r(\text{State } 2) + p_r(\text{State } 3), \] (16)
\[ R_{iz_i}^{2S}(t) = R_{iz_i}^{1S}(t) + P_r(\text{State } 3), \] (17)
\[ P_r(\text{State } 3) = P_r(T_{iz_i}^{1S} \leq t) \bigcap T_{iz_i}^{2S} \geq t_1 \bigcap T_{iz_i}^{1S} \geq t - t_1 - t_2 \] (18)
\[ R_{iz_i}^{2S}(t) = R_{iz_i}^{1S}(t) + \int_0^t f_{iz_i}^{active}(t_1) R_{iz_i}^{1S}(t_1) \int_0^{t-t_1} f_{iz_i}^{2S}(t_2) \times R_{iz_i}^{Switch}(t_1 + t_2) R_{iz_i}^{2S}(t_2) R_{iz_i}^{2S}(t - t_1 - t_2) \, dt_2 \, dt_1. \] (19)

By comparing Eq. (14) with Eq. (16), one obtains Eq. (17). Eq. (18) represents the mathematical formula of state 3. By substituting Eqs. (15) and (18) on Eq. (16), one obtains Eq. (19). As it can be seen, the reliability for \( n \) standby units depends on the success of all \( n \) states, which can be calculated recursively. In fact, in success state analysis, only one success state is added to calculate the reliability of \( n \) standby units compared to \( n-1 \) standby, and the previous standby success states are repeated. Therefore, a recursive formula can be obtained in calculating reliability as follows:

\[ R_{iz_i}^{1S}(t) = R_{iz_i}^{active}(t) + \int_0^t f_{iz_i}^{active}(t_1) R_{iz_i}^{Switch}(t_1) R_{iz_i}^{1S}(t_1) R_{iz_i}^{1S}(t - t_1) \, dt_1, \] (20)
\[ R_{iz_i}^{2S}(t) = R_{iz_i}^{1S}(t) + \int_0^t f_{iz_i}^{active}(t_1) R_{iz_i}^{1S}(t_1) \int_0^{t-t_1} f_{iz_i}^{2S}(t_2) \times R_{iz_i}^{Switch}(t_1 + t_2) R_{iz_i}^{2S}(t_2) R_{iz_i}^{2S}(t - t_1 - t_2) \, dt_2 \, dt_1, \] (21)
\[ R_{iz_i}^{3S}(t) = R_{iz_i}^{2S}(t) + \int_0^t f_{iz_i}^{active}(t_1) R_{iz_i}^{1S}(t_1) \int_0^{t-t_1} f_{iz_i}^{1S}(t_2) R_{iz_i}^{Switch}(t_1 + t_2 + t_3) \times R_{iz_i}^{2S}(t_2) \int_0^{t-t_1-t_2} f_{iz_i}^{2S}(t_3) R_{iz_i}^{3S}(t_3). \]


Using the same procedure, the formula can be obtained for three standby units as shown by Eq. (22). By comparing Eq. (20), Eqs. (21), and (22), the final reliability formula, which is given in Eq. (24) can be calculated for \( n \) warm standby components which is a recursive formula for reliability. Eqs. (23) and (24) also shows reliability for \( n \) warm standby components.

\[
\prod_{i \in S} R_i(t; z, n_i; \text{standby}) = R^{nS}_{iz_i}(t) \tag{23}
\]

\[
P^{nS}_{iz_i}(t) = R^{(n-1)S}_{iz_i} + \int_0^t f^{\text{active}}_{iz_i}(t_1) R^{1S}_{iz_i}(t_1) \int_0^{t-t_1} f^{1S}_{iz_i}(t_2) R^{2S}_{iz_i}(t_2) \times \dots \times \int_0^{t-t_1-t_2} f^{1S}_{iz_i}(t_3) R^{\text{Switch}}_{iz_i}(t_1 + t_2 + t_3) R^{3S}_{iz_i}(t_3) R^{3S}_{iz_i}(t - t_1 - t_2 - t_3) \times \dots \times \int_0^{t-t_1-t_2-\cdots-t_{n-2}} f^{(n-2)S}_{iz_i}(t_{n-1}) R^{\text{Switch}}_{iz_i}(t_1 + \cdots + t_{n-1}) R^{(n-1)S}_{iz_i}(t_{n-1}) \times R^{(n-1)S}_{iz_i}(t - t_1 - \cdots - t_{n-1}) d(t - t_1 - \cdots - t_{n-1}) \times \cdots \times d(t_1). \tag{24}
\]

Reliability for subsystems with the mixed strategy: The mixed strategy integrates the traditional active strategy with the warm standby strategy. In this mixed strategy, levels of active and warm-standby redundancies are showed by \( nA_i \) and \( nS_i \), respectively, which can have different values for each subsystem \( i \). The goal is then to pick these values for each subsystem to maximize the overall system's reliability. The redundancy level in each subsystem \( i \), which is \( n_i \), is also a decision variable calculated as \( n_i = nA_i + nS_i \).

The structure of a series-parallel system is shown in Figure 1, which utilizes the presented redundancy strategy for redundant components. As can be seen, levels of active and warm-standby redundancies differ in each subsystem and \( nA_i \) primary components are initially online for the subsystem \( i \). In each subsystem, only one component needs to operate at any time and all active components are online and work at the beginning of the process. If the current online active component fails, an available standby unit replaces it in a predefined order. The number of active units and warm-standby units must satisfy \( nA_i \geq 1 \) and \( nS_i \geq 1 \), respectively. As can be seen, mixed strategy is more flexible than standby and active strategies as it considered them simultaneously.

Reliability formula for mixed redundancy strategy:
Using the same strategy as performed to obtain the recursive formula for the warm standby strategy, the formula for the mixed strategy can be obtained.

Reliability for one standby component in mixed redundancy:

\( n = 1 \) (Standby)

State 1: At least one of the active components operated until time \( t \).

State 2: All the active redundant components are failed at the time \( t_1 < t \), and the switch is functioning at the time \( t_1 \) and standby component has not failed until the time \( t_1 \) (with a lower failure rate \( \lambda_{iz_i} \)), the first standby component has not failed from \( t_1 \) to \( t \) (with failure rate \( \lambda_{iz_i} \)) and is functioning at operating mode.

\[
P_r(\text{State 1}) = P_r(T^{\text{active}}_{\text{max}, iz_i} > t) = (1 - (1 - R^{\text{active}}_{iz_i}(t))^{nA_i})
\]

\[
P_r(\text{State 2}) = P_r(T^{\text{active}}_{\text{max}, iz_i} \leq t \cap T^{1S}_{iz_i} > t_1 \cap T^{1S}_{iz_i} > t - t_1)
\]
Figure 1. Series-parallel system

\[
R^{\text{mix,1S}}_{iz_i}(t) = P_r(\text{State 1}) + P_r(\text{State 2}),
\]

\[
R^{\text{mix,2S}}_{iz_i}(t) = P_r(\text{State 1}) + P_r(\text{State 2}) + P_r(\text{State 3}),
\]

Eq. (25) represents the probability that all the active components fail at a time between zero and the mission time \( t \), and the switching system activates the first standby component to operate from time \( t_1 \) to \( t \). Since state 1 and state 2 are mutually exclusive events, Eq. (27) is written as their summation. By substituting Eqs. (25) and (26) on Eq. (27), one obtains Eq. (28).

**Reliability for two standby components in mixed redundancy**

\( n = 2 \) (Standby)

**State 1:** At least one of the active components operated until times \( t \).

**State 2:** All the active redundant components are failed at the time \( t_1 < t \), and the switch is functioning at time \( t_1 \), standby component has not failed until the time \( t_1 \) (with a lower failure rate \( \lambda_{iz_i}^{-} \)) and the standby component has not failed from time \( t_1 \) to \( t \) (with failure rate \( \lambda_{iz_i}^{iz} \)) and is functioning at the operating mode.

**State 3:** All the active redundant components are failed at the time \( t_1 < t \), and the switch is functioning at the time \( t_1 \), the first standby component has not failed until the time \( t_1 \) (with a lower failure rate \( \lambda_{iz_i}^{-} \)), the first standby component failed at the time \( t_2 \), and the switch is functioning at the time \( t_2 \) and the second standby component has not failed until the time \( t_2 \) (with a lower failure rate \( \lambda_{iz_i}^{iz} \)), and the second standby component has not failed from time \( t_2 \) to \( t \) (with failure rate \( \lambda_{iz_i}^{iz} \)) and functioning at operating mode.
\begin{align*}
& \leq t_2 \bigcap \mathcal{T}^{2S}_i > t_2 \bigcap \mathcal{T}^{2S}_i > t - t_1 - t_2, \\
& R^{\text{mix,2S}}_{iz_i}(t) = R^{\text{mix,1S}}_{iz_i}(t) + \int_{0}^{t} f_{iz_i}^{\max,nA_i}(t_1) R^{1S}_{iz_i}(t_1) \int_{0}^{t-t_1} f_{iz_i}^{1S}(t_2) \\
& \times R_{iz_i}^{\text{Switch}}(t_1 + t_2) R^{2S}_{iz_i}(t_2) R^{2S}_{iz_i}(t - t_1 - t_2) d(t_2)d(t_1). \tag{31}
\end{align*}

By substituting Eq.(29) and (30) on Eq. (29), one obtains Eq. (31). Eq. (30), which shows the mathematical formula of state 3, where \( f_{iz_i}^{\max,nA_i}(t) \) is calculated as follows.

\[ f_{iz_i}^{\max,nA_i}(t) = nA_i[F_{iz_i}(t)]^{nA_i-1}f_{iz_i}(t). \tag{32} \]

Where \( f_{iz_i}(t) \) in Eq. (32), is the density function of the time-to-failure for the \( z^i \)th component of subsystem \( i \) and \( F_{iz_i}(t) \) is its cumulative distribution function. Using the same procedure as the mixed reliability for \( n \) standby components, the final formula for the mixed warm standby in Eqs. (33) and (34) can be calculated.

\[
\prod_{i \in M} R_i(t; z, n_A, n_s, \text{mixed}) = R^{\text{mix,nS}}_{iz_i}(t) \tag{33}
\]

\[
R^{\text{mix,nS}}_{iz_i}(t) = R^{\text{mix,(n-1)S}}_{iz_i}(t) + \int_{0}^{t} f_{iz_i}^{\max,nA_i}(t_1) R^{1S}_{iz_i}(t_1) \\
\times \int_{0}^{t-t_1} f_{iz_i}^{1S}(t_2) R^{2S}_{iz_i}(t_2) \\
\times \int_{0}^{t-t_1-t_2} f_{iz_i}^{2S}(t_3) R_{iz_i}^{\text{Switch}}(t_1 + t_2 + t_3) \\
\times R^{2S}_{iz_i}(t - t_1 - t_2 - t_3) \times \cdots \\
\times \int_{0}^{t-t_1-t_2-\cdots-t_{n-2}} f_{iz_i}^{(n-2)S}(t_{n-1}) \\
\times R_{iz_i}^{\text{Switch}}(t_1 + \cdots + t_{n-1}) R^{(n-1)S}_{iz_i}(t_{n-1}) \\
\times R^{(n-1)S}_{iz_i}(t - t_1 - \cdots - t_{n-1}) \times \cdots \times d(t_1). \tag{34}
\]

3.2. Constraints. The mathematical model presented in this paper has three constraints, including cost, weight, and the number of system components. Eqs. (35) and (36) show that the number of components in each subsystem can be increased to an extent that the maximum cost and weight, respectively, do not exceed their permitted values. Moreover, Eq. (37) shows the number of components used in each subsystem cannot exceed the maximum number of components assigned to it.

\[
\sum_{i=1}^{S} c_{iz_i} n_i \leq C, \quad z_i \in \{1, 2, \cdots, m_i\}, \quad \forall i = 1, 2, \cdots, S. \tag{35}
\]

\[
\sum_{i=1}^{S} w_{iz_i} n_i \leq W, \quad z_i \in \{1, 2, \cdots, m_i\}, \quad \forall i = 1, 2, \cdots, S. \tag{36}
\]

\[
n_i = n_{A,i} + n_{S,i}, \quad z_i \in \{1, 2, \cdots, m_i\}, \quad \forall i = 1, 2, \cdots, S. \tag{37}
\]

Also, the number of components that can be used in the subsystem cannot be more than the maximum number of components assigned to each subsystem. The number of components in each subsystem is equal to the number of active and standby components in which the subsystem is used.
4. Research methodology. The problem under study is the selection of components, redundancy levels and redundancy strategies to maximize the system reliability on weight and cost constraints as well as the number of system components. The formulated problem is a complex integer programming problem. To solve the proposed optimization problem, since the complex form of the objective function used in the developed model is a nonlinear, non-differentiable, non-convex, and integer function, standard optimization methods and existing techniques for convex optimization are not appropriate. The non-convexity of the cost function can lead these algorithms to find a local optimum with poor performance. Given that meta-heuristic algorithms do not require the derivative and other ancillary information, and that they can escape local optimum solutions [7], they are very suitable to solve the formulated problem. Therefore, meta-heuristics algorithms are used to solve the presented mathematical model of the problem. In meta-heuristics algorithms, the solution representation techniques have profound effects on the effectiveness of results [51]. Every meta-heuristic algorithm has inherent limitations and thus several variants of the original meta-heuristic algorithms are presented to enhance their performances. One way to modify these algorithms is to change the searching mechanism by adding some operators to the nature of the problem [29]. A max-min mutation operator is added to the standard GA in [58] to improve its performance. The efficiency of the min-max mutation-based GA has been proven in the reliability optimization problem [19, 3, 2, 5, 4]. PSO algorithm, as one of the well-known successful meta-heuristics algorithms in solving high-dimensional non-convex optimization problems, is used in solving the proposed optimization problem. The GA and PSO algorithms are both coded with MATLAB R2019b.

4.1. GA and PSO algorithms: Fine-tuning and comparison. In this part of the article, since there is no benchmark to evaluate the proposed model and compared the results obtained with, the proposed optimization problem is solved using two of the most famous meta-heuristic algorithms, called GA and PSO, and their performance is compared. Since the values obtained from meta-heuristic algorithms are very sensitive to their parameters, the Taguchi experimental design method [40] is used to adjust them.

4.1.1. Tuning the GA and PSO parameters. To run the GA and PSO, it is necessary to effectively tune their parameter values, as the efficiency of these algorithms is influenced by modifying their parameters. To tune the parameters of the GA (i.e., the population size (Popsize)), the crossover rate ($p_c$), the percent of max-min mutation ($p^m_1$), the percent of general mutation ($p^m_2$) and parameters of PSO: the initial inertia weight ($w_{min}$), the final inertia weight ($w_{max}$), the acceleration coefficients ($c_1, c_2$) and the Taguchi experiment design method [40] is leveraged. To this end, three different levels are selected for each of the parameters. So, with four parameters in both algorithms, a design is employed with 9 experiments. The Taguchi orthogonal design L9 is selected to run the experiments Table 2 shows different combinations of parameter levels in the design of experiments. Since in this paper more reliability if of utmost desire, the value of $S/N$ the ratio is calculated from Eq. (38) where $y_i$ denote the response in the $i^{th}$ experiment and $n$ denotes the number of orthogonal arrays, on which the performance of the experiments is based. The highest $S/N$ ratio determines the optimal level for each factor. Tables 4 and 5 show Taguchi’s experimental results on test problems for the GA and PSO.
\[ S/N = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \right) \]  

(38)

Figures 2 and 3 show the mean of \( S/N \) ratios. The optimal levels obtained are shown in Table 3.

Table 3. Controllable factors and their levels

| Parameters | Notations | Levels | Optimal levels |
|------------|-----------|--------|----------------|
| Popsizes   | A         | 150    | 100            |
|           | B         | 0.8    | 0.5            | 0.4  |
|           | C         | 0.3    | 0.2            | 0.1  |
|           | D         | 0.3    | 0.2            | 0.1  |

Table 4. Taguchi experimental results on test problem for the GA

| Exp NO. | Popsizes | \( p_c \) | \( p_m^1 \) | \( p_m^2 \) | \( S/N \) |
|---------|----------|----------|-------------|-------------|---------|
| 1       | 1        | 1        | 1           | 1           | -0.47   |
| 2       | 1        | 2        | 2           | 1           | -0.43   |
| 3       | 1        | 3        | 3           | 1           | -0.32   |
| 4       | 2        | 1        | 2           | 2           | -0.24   |
| 5       | 2        | 2        | 3           | 2           | -0.23   |
| 6       | 2        | 3        | 1           | 2           | -0.44   |
| 7       | 3        | 1        | 3           | 3           | -0.41   |
| 8       | 3        | 2        | 1           | 3           | -0.46   |
| 9       | 3        | 3        | 2           | 3           | -0.36   |

Table 5. Taguchi experimental results on test problem for the PSO

| Exp NO. | \( c_1 \) | \( c_2 \) | \( \bar{w}_{\text{max}} \) | \( \bar{w}_{\text{min}} \) | \( S/N \) |
|---------|----------|----------|----------------|----------------|---------|
| 1       | 1        | 1        | 1              | 1              | -0.47   |
| 2       | 1        | 2        | 2              | 1              | -0.24   |
| 3       | 1        | 3        | 3              | 1              | -0.41   |
| 4       | 2        | 1        | 2              | 2              | -0.34   |
| 5       | 2        | 2        | 3              | 2              | -0.23   |
| 6       | 2        | 3        | 1              | 2              | -0.24   |
| 7       | 3        | 1        | 3              | 3              | -0.27   |
| 8       | 3        | 2        | 1              | 3              | -0.41   |
| 9       | 3        | 3        | 2              | 3              | -0.23   |
4.2. Genetic algorithm (GA). Since RAP is an NP-hard problem, we used GA to solve the proposed model. GA is a meta-heuristic optimization algorithm that has been widely and effectively utilized for solving combinatorial optimization problems [11]. GA method is one of the most famous meta-heuristic methods that were first introduced by Holland [25]. We present the details of the implementation of a modified GA to solve the RAP. This includes chromosome structure that encodes the solution, fitness function, selection of parents to generate a new population, crossover, mutation, and stopping criteria.

4.2.1. Chromosome definition. In the presented GA solution, each candidate solution is a chromosome. In the proposed model, the type of component, the number of active components, and the number of warm components used in each subsystem
are the decision variables. The chromosome structure of GA represents these decision variables introduced in Section 2. More specifically, the chromosome used in the GA is a $3 \times 14$ matrix; each row represents one of the decision variables. Rows 1, 2, and 3 of this matrix indicate the type of component used in each subsystem, the number of active components, and the number of warm components, respectively. A chromosomes structure is shown in Figure 4 considered for the case where $S = 14$. Or instance, in this example, the first subsystem ($S = 1$), uses three of the first type of components in parallel with all three active redundancy strategies, and the Eighth subsystem uses one of the second types of the component in parallel with all one redundancy strategies. Figure 5 shows the structure of obtained of model solution, which depicts the chromosomes structure.

![Figure 4. Chromosome representation (solution encoding)](image)

![Figure 5. Representation of a solution](image)

4.2.2. Fitness function. The fitness function is considered to be a weighted sum of the penalty of constraints violation and the objective function (reliability). i.e., the constraints are encoded in the fitness function such that if a solution violated the constraint, the objective function is summed with a large amount of penalty. This penalty guarantees the feasibility of the solutions, including the final solution. Therefore, the objective function and the constraints presented in Section 3 were defined in the Fitness Function. The objective function in the model is the product of four factors each one representing a redundancy allocation strategy. Warm strategy and mixed strategy Due to its time-dependent are very complex. To implement two factors in MATLAB, we used the exponential distribution formula in the model instead of the probability density function before entering the objective function.
into the MATLAB environment. Thus, considering that it is easy to integrate from
this formula, we calculated the integral wherever necessary and finally, we obtained
the formula for these, e.g., we explain the formula for the warm strategy here which
is as follows.

\[
R_{iz,iz}^n(t) = R_{iz,iz}^{(n-1)S} + \int_0^t f_{iz}^{\text{active}}(t_1) R_{iz,iz}'^{1S}(t_1) \int_0^{t-t_1} f_{iz}^{1S}(t_2) R_{iz,iz}'^{2S}(t_2)
\]

\[
\times \int_0^{t-t_1-t_2} f_{iz}^{1S}(t_3) R_{iz,iz}^{\text{Switch}}(t_1 + t_2 + t_3)
\]

\[
\times R_{iz,iz}^{3S}(t_3) R_{iz,iz}^{3S}(t - t_1 - t_2 - t_3)
\]

\[
\times \cdots \times \int_0^{t-t_1-t_2-\cdots-t_{n-2}} f_{iz}^{(n-2)S}(t_{n-1}) R_{iz,iz}^{\text{Switch}}(t_1 + \cdots + t_{n-1})
\]

\[
\times R_{iz,iz}^{(n-1)S}(t_{n-1}) R_{iz,iz}^{(n-1)S}(t - t_1 - \cdots - t_{n-1})
\]

\[
\times d(t - t_1 - \cdots - t_{n-1}) \times \cdots \times d(t_1),
\]

where

\[
f_{iz}(t) = \lambda_{iz} e^{-\lambda_{iz} t}, \quad (40)
\]

\[
R_{iz,iz}(t) = \int f_{iz,iz}(t) = e^{-\lambda_{iz} t}, \quad (41)
\]

\[
R_{iz,iz}^n(t) = R_{iz,iz}^{(n-1)S} + \int_0^t f_{iz}^{\text{active}}(t_1) R_{iz,iz}'^{1S}(t_1) \int_0^{t-t_1} f_{iz}^{1S}(t_2) R_{iz,iz}'^{2S}(t_2)
\]

\[
\times \int_0^{t-t_1-t_2} f_{iz}^{1S}(t_3) R_{iz,iz}^{\text{Switch}}(t_1 + t_2 + t_3) R_{iz,iz}^{3S}(t_3) R_{iz,iz}^{3S}(t - t_1 - t_2 - t_3)
\]

\[
\times \cdots \times \int_0^{t-t_1-t_2-\cdots-t_{n-2}} f_{iz}^{(n-2)S}(t_{n-1})
\]

\[
\times R_{iz,iz}^{(n-1)S}(t_{n-1}) R_{iz,iz}^{(n-1)S}(t - t_1 - \cdots - t_{n-1})
\]

\[
= (\lambda_{iz})^n/n! \times e^{-\lambda_{iz} t} \]

\[
\times \left( \sum_{k=0}^{n-2} 1 - ((\lambda_{iz} + \lambda_{iz}^{\text{Switch}})^k/k!) e^{-(\lambda_{iz} + \lambda_{iz}^{\text{Switch}})(t)} \right) \quad (42)
\]

By Substituting Eqs. (40) and (41) in the Eq. (39) one obtains Eq. (42).

4.2.3. Initial population. Pop chromosomes are produced randomly within their
feasible range for the generated initial population. In this paper, the population
size (Pop) is selected to be 100 after some preliminary experiments.

4.2.4. Selection. The tournament selection is utilized in this paper for the selection
of chromosomes to perform crossover and mutation operators. In the tournament
selection method, for all chromosomes in the population (called Pop), the objective
function must be calculated. It then selects four chromosomes randomly to pick the
one with the largest fitness function among them as the parent for producing a new
population. To select Pop parents, this method will be repeated Pop times.

4.2.5. Crossover. The max-min crossover operator [46] is performed with a rate of
0.8 in this paper. The crossover operator will generate four offspring from each
of two selected parents. Two premier chromosomes will be selected for the next
population out of parents and offspring with the best fitness values. Therefore,
we will have Pop chromosome at the end of the crossover operation. The double-point crossover, as well as the max-min crossover, is used to generate off-springs from parents, as shown in Figure 6. Max–min crossover determines the subsystems with the highest reliability and all relative genes for each parent are exchanged with the same genes in other parents. The function for the crossover in MATLAB is presented as bellow:

```matlab
function [y1 y2]=MaxMinCrossover(x1,x2)

y1 = x1;
 [~, SortedReli1]=sort(x1.SubReli);

y1.Solution(:,SortedReli1(1))=x2.Solution(:,SortedReli1(1));
y1.Solution(:,SortedReli1(end))=x2.Solution(:,SortedReli1(end));

y2 = x2;
 [~,SortedReli2]=sort(x2.SubReli);

y2.Solution(:,SortedReli2(1))=x1.Solution(:,SortedReli2(1));
y2.Solution(:,SortedReli2(end))=x1.Solution(:,SortedReli2(end));

end.
```

![Figure 6. Max-min crossover operator](image)

4.2.6. **Mutation.** In this paper, a max-min mutation operator [57] with a rate of 0.3 is utilized. The mutation operator increases the diversity of the population of solutions and thus avoids trapping into a local optimum. In this work, the max-min mutation operator introduced by [31] is used. In the max-min mutation operator, for each candidate solution in the population, the subsystems with the lowest and highest reliability are randomly mutated. The values of genes for these subsystems are randomly changed at a mutation rate. To implement mutation, 30 percent of the population is randomly selected and their genes values are randomly modified by a probability of 0.2. Then, the fitness function for the new mutated chromosome is evaluated and is compared with the fitness of the old pre-mutation chromosome. If the new fitness is better, it replaces the old chromosome. The max-min mutation is shown in Figure 7 for a specific example, and, as can be seen, the mutation operator transfers one generation to another.

4.2.7. **Stopping criteria.** The GA optimization process for the presented model is stopped after a determined number of iteration (which was 100 iterations in this paper).
generate local responses where velocity, \( \text{Gbest} \). As the particle moves over time, the position of the best point of the objective function in optimization problems. The position of a particle at the current stage is a component of its previous position, its own best position (Pbest), and knowledge of global or their neighborhood’s (Gbest). As the particle moves over time, the position of the \( i^{th} \) particle changes [54].

More specifically, let \( X_i(t) \) denote the position of the particle at time step \( t \) in the search space. By adding a velocity, \( V_i(t) \) to the position of each particle, a new position for the particle can be considered. Let \( p_i(t) \) and \( p_g(t) \) be the best position of the \( i^{th} \) particle during its motion and the best position of the population in the iteration \( t \), respectively. Eq. (44) shows how the position of the particle is updated.

\[
\begin{align*}
x_i(t + 1) &= x_i(t) + v_i(t + 1) \\
v_i(t + 1) &= wv_i(t) + c_1r_1(p_i(t) - x_i(t)) + c_2r_2(p_g(t) - x_i(t))
\end{align*}
\]

(43)

(44)

where \( w \) represents the weight of inertia and the coefficients \( c_1 \) and \( c_2 \) are known as the acceleration coefficients. Also, \( r_1 \) and \( r_2 \) are random numbers in the range of zero and one with uniform distribution. The inertia weight parameter regulates the ability of the particle population to explore optimal local areas. In a way, large amounts of inertia increase random search and generate new answers, while lower values of inertia increase the PSO’s ability to generate local responses [40].

Therefore, in this study, to create better answers and comprehensive exploration in the search space, the inertia weight is initialized with a high value to explore in the beginning. In this paper, inertia weight is reduced from 0.9 to 0.3 and the coefficients \( c_1 \) and \( c_2 \) are selected constant values 1.5 according to the experiments. The structure of particles in stocktickerPSO is the same as the structure of chromosomes in GA based on Figure 4. The fitness function defined in the stocktickerPSO (42) is similar to the GA.
5. **A numerical example.** To verify the effectiveness of the presented mixed redundancy strategy, it is implemented in an example. To this end, an example is adapted from Fyffe et al [16]. For each subsystem, four component choices are considered and the \( k \)-Erlang distribution, weight, cost parameters are predetermined. Since in warm-standby strategy the component failure rate is different from the standby and operating modes and does not follow the Erlang distribution, exponential distributions are considered for all components and a reduced failure rate is added, which are shown in Table 6. The overall goal is to improve the reliability of the system at 100 time units while taking into account the cost constraint as \( C = 130 \) and the system weight constraint as \( W = 170 \). Each subsystem can select from active, warm-standby, or mixed redundancy strategies or none of them.

**Table 6.** Data for the illustrative example

|     | Choice 1 \((j = 1)\) | Choice 2 \((j = 2)\) | Choice 3 \((j = 3)\) | Choice 4 \((j = 4)\) |
|-----|-----------------|-----------------|-----------------|-----------------|
| 1   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 2   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 3   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 4   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 5   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 6   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 7   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 8   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 9   | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 10  | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 11  | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 12  | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 13  | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |
| 14  | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) | \(\lambda_{ij}\) |

The proposed GA and PSO are both coded with MATLAB R2019b. The results of GA and PSO are shown in Tables 7 and 8. The CPU time to get a solution for GA and PSO are 18700, 13690 seconds, respectively.
**Table 7.** Numerical results of model by GA

|   | i  | z_i | n_A | n_S | Redundancy |
|---|----|-----|-----|-----|------------|
|1  | 1  | 1   | 3   | 0   | Active     |
|2  | 2  | 2   | 2   | 1   | Standby    |
|3  | 3  | 4   | 3   | 0   | Active     |
|4  | 4  | 3   | 1   | 1   | Standby    |
|5  | 5  | 2   | 2   | 4   | Mixed      |
|6  | 6  | 2   | 2   | 0   | Active     |
|7  | 7  | 2   | 1   | 1   | Standby    |
|8  | 8  | 2   | 2   | 2   | Mixed      |
|9  | 9  | 3   | 2   | 2   | Mixed      |
|10 | 10 | 3   | 1   | 1   | Standby    |
|11 | 11 | 3   | 3   | 1   | Mixed      |
|12 | 12 | 1   | 1   | 2   | Standby    |
|13 | 13 | 2   | 1   | 1   | Standby    |
|14 | 14 | 3   | 2   | 0   | Active     |

System Reliability: 0.9823
System weight: 170
System cost: 116

**Table 8.** Numerical results of model by PSO

|   | i  | z_i | n_A | n_S | Redundancy |
|---|----|-----|-----|-----|------------|
|1  | 1  | 3   | 3   | 0   | active     |
|2  | 2  | 2   | 1   | 1   | Standby    |
|3  | 3  | 4   | 2   | 0   | active     |
|4  | 4  | 3   | 2   | 4   | Mixed      |
|5  | 5  | 2   | 4   | 0   | Active     |
|6  | 6  | 4   | 1   | 0   | Active     |
|7  | 7  | 2   | 2   | 0   | Active     |
|8  | 8  | 2   | 1   | 1   | Standby    |
|9  | 9  | 1   | 2   | 2   | Mixed      |
|10 | 10 | 3   | 1   | 1   | Standby    |
|11 | 11 | 3   | 2   | 0   | Active     |
|12 | 12 | 1   | 1   | 2   | Mixed      |
|13 | 13 | 1   | 2   |     | Mixed      |
|14 | 14 | 3   | 2   | 0   | Active     |

System Reliability: 0.9432
System weight: 170
System cost: 118
Table 9. Comparison between the computational of GA and PSO

| Algorithm     | PSO  | GA  | System reliability | Resource consumed cost | Resource consumed Weight |
|---------------|------|-----|--------------------|------------------------|--------------------------|
|               | 0.9432 | 0.9823 | System reliability |                       |                          |
|               | 170   | 170 | Resource consumed cost |                       |                          |
|               | 118   | 116 | Resource consumed Weight |                    |                          |

Figure 8. Convergence diagram of the best implementation of the GA

Figure 9. Convergence diagram of the best implementation of the PSO

As the results of Table 7 shows, subsystems 1, 3, 6, 14 use an active strategy, subsystems 2, 4, 7, 10, 12, 13 use a standby strategy, and subsystems 5, 8, 9, 11 use a mixed strategy redundancy. The reliability of the system is found to be 0.9823.
As the results of Table 8, subsystems 1, 3, 5, 6, 7, 11, 14 use an active strategy, subsystems 2, 8, 10 use a standby strategy, and subsystems 4, 9, 12, 13 use a mixed strategy redundancy. The reliability of the system is found to be 0.9432. The results clarify that both GA and PSO algorithms achieved a high level of reliability, which certifies the efficiency of the model, regardless of the optimization algorithm used for solving it. Although, the results of Table 9 show the superiority of GA in comparison to PSO. Figures 8 and 9 show the convergence diagram of the best implementation of the GA and PSO implemented on the designed model, respectively, in which GA converges after about 45 iterations and PSO converges after about 50 iterations.

Table 10 demonstrates the comparison results for the new dynamic model with four strategies as explained before and the paper that used three strategies in the RAP, i.e. active, warm standby, or no redundancy, in series-parallel systems [23]. The reliability for warm components in a system with 14 subsystems has only been investigated in [23]. In this paper, GA and HGA algorithms are used to solve the model. The results in a Table 9 clearly show the superiority of our approach compared to [42]. The results clarify that both GA and PSO algorithms achieved a high level of reliability, which certifies the efficiency of the model, regardless of the optimization algorithm used for solving it.

Table 10. Comparison results among proposed mixed strategy and other redundancy strategies

| Strategy; i | Warm standby (GA) | Warm standby (HGA) | Proposed mixed (GA) | Proposed mixed (PSO) |
|-------------|-------------------|--------------------|---------------------|----------------------|
|             | \( Z_i \) | \( n_i \) | \( Z_i \) | \( n_i \) | \( n_{A_X} \) | \( n_{s_X} \) | \( Z_i \) | \( n_{active} \) | \( n_s \) |
| 1           | 3     | 2     | 3     | 2     | 1     | 3     | 0     | 3     | 3     |
| 2           | 1     | 1     | 1     | 2     | 2     | 2     | 1     | 2     | 1     |
| 3           | 3     | 3     | 3     | 3     | 4     | 3     | 0     | 4     | 2     |
| 4           | 3     | 1     | 1     | 3     | 3     | 1     | 1     | 3     | 2     |
| 5           | 1     | 1     | 1     | 2     | 2     | 2     | 4     | 2     | 4     |
| 6           | 2     | 2     | 2     | 2     | 2     | 2     | 0     | 4     | 1     |
| 7           | 3     | 1     | 1     | 3     | 2     | 1     | 1     | 2     | 0     |
| 8           | 1     | 1     | 1     | 3     | 2     | 2     | 2     | 2     | 1     |
| 9           | 3     | 3     | 3     | 3     | 3     | 2     | 2     | 1     | 2     |
| 10          | 2     | 4     | 2     | 4     | 3     | 1     | 1     | 3     | 1     |
| 11          | 1     | 4     | 1     | 4     | 3     | 3     | 1     | 3     | 2     |
| 12          | 1     | 2     | 1     | 2     | 1     | 1     | 2     | 1     | 2     |
| 13          | 2     | 2     | 2     | 2     | 2     | 1     | 1     | 1     | 2     |
| 14          | 3     | 3     | 3     | 4     | 3     | 2     | 0     | 3     | 2     |

| System reliability | 0.4269 | 0.4403 | 0.9823 | 0.9432 |
| System weight      | 118    | 118    | 116    | 118    |
| System cost        | 170    | 170    | 170    | 170    |

The practical implication of this result is that the optimal solution includes a mixture of four redundancies. A mixed redundancy strategy leads to a noticeable improvement in the reliability of complex systems. However, mixed redundancy alone may not always provide higher reliability as compared to warm or active redundancies and a combination of strategies should be considered. To find optimal designs to improve system reliability, it is necessary to investigate factors such as system resource constraint, component choices, switch failures, redundancy strategies, and warm standby failures. Therefore, considering that the proposed model uses a variety of redundancy strategies, it can be very effective in improving reliability. It can be observed that the system reliability is significantly increased by...
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using a mixed strategy in some subsystems. Therefore, the results can help system
designers in improving reliability when standbys are warm in the system such as
nuclear power plants, radioactive components, large-scales complex equipment.

Remark 1. Please note that this paper provides a solution to the redundancy al-
location problem which involves the selection of components and redundancy levels
to maximize system reliability given various system-level constraints. This is a com-
mon and extensively studied problem involving system design, reliability engineer-
ing, and operations research. It is becoming increasingly important for managers
and system designers to develop efficient solutions to this reliability optimization
problem because many manufacturers are becoming more complex.

6. Conclusion. A new dynamic model was introduced for RAP with warm standby
and active components in series-parallel systems. Due to the time dependence of the
warm standby components, the objective function was obtained using the success
mode analysis. The problem was formulated under a nonlinear integer program-
ing model due to several cost and weight constraints. In general, solving RAPs in
real cases is not easy, especially for large-scale situations. Therefore, it is suggested
to use meta-heuristic methods to solve such a difficult and complex problem. For
this purpose, GA and PSO have been developed as useful algorithms for RAP. A
well-known example in this field was used to evaluate the effectiveness of the pro-
posed model. The high reliability of both algorithms indicates the Performance of
the proposed model.

The new strategy provides more flexibility for reliability analysts and leads to
a significant increase in the reliability of complex systems. In the proposed math-
ematical model, only one type of component is used in each subsystem, while in
subsequent studies, different types of components can be used in each subsystem.
Also, as the number of parallel components of the subsystem increases, the reli-
bility of the system increases, but on the other hand, the design cost increases
accordingly. Therefore, it is better in the designed model, in addition to the objec-
tive function of maximizing reliability, to add another objective function called cost
minimization to the model so that the balance between reliability and cost can be
better established. One of the limitations of the present study is that in the formula
obtained in this paper, given that the number of integrals increases by increasing
the number of warm standby components, for more complex distributions such as
the normal distribution, computing reliability becomes very difficult and complex.
The future work is to consider more complex distributions to analyze reliability. In
addition, applications of stochastic constraints such as chance constraints can be
considered to obtain more useful information about the model dynamics.

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