Vibration analysis for framed structures using the finite-difference time-domain method based on the Bernoulli-Euler beam theory

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Abstract: A vibration analysis method for structure-borne sound transmission in framed structures using an implicit finite-difference time-domain (FDTD) method is proposed in this paper. The prediction of structure-borne sound is difficult owing to the complexity of the vibration mechanism in building structures. As a powerful means of analyzing structure-borne sound, wave-based numerical techniques have the potential to solve the problem accurately by virtue of their flexibility from the viewpoint of modeling the object. For this reason, we model the target structure as a composition of beam elements and calculate the characteristics of the wave propagation using the FDTD method. Using the beam model, we can decrease the dimension of the simulated field to one dimension, compared with the situation that the field is discretized by three-dimensional solid elements. This results in a reduction of the computational cost. In this paper, the basic theory of the calculation method for a model with beam elements is described and the results of a case study of a multilayered frame structure are discussed.

Keywords: Structure-borne sound, Finite-difference time-domain method, Bernoulli-Euler beam theory, Frame model

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1. INTRODUCTION

Structure-borne sound transmission in building structures affects the quality of sound environments in rooms, and adequate consideration is necessary in the planning phase of architecture. However, it is difficult to predict structure-borne sound accurately because of the complex mechanism of vibration propagation. As prediction methods, the followings have been investigated: methods using an empirical formula [1], an analytical method [2], an energy-based method such as statistical energy analysis (SEA) [3] and a wave-based numerical method such as the finite-element method (FEM) [4].

The authors have been investigating an efficient prediction method for structure-borne sound employing the finite-difference time-domain (FDTD) method. In the method, the computational cost, which can be a critical bottleneck in wave-based numerical analysis, is reduced by modeling the building as a composition of plate elements and/or beam elements. Using this model, the dimension of the simulated vibration field is reduced to one dimension (beam elements) or two dimensions (plate elements), in contrast to the situation in which the architecture is modeled by three-dimensional solid elements, and as a result, it can contribute to memory saving and faster simulation. In our previous paper [5], the validity of the calculation method for the plate model was confirmed. As the next step, in this paper, the validity of the method for a beam model and its applicability to a multilayered frame structure are investigated.

Regarding the prediction method using the beam model, Tanaka and Kuga [6] investigated the propagation characteristics of vibration on a beam structure using an analytical method. On the other hand, Nawaoka et al. [7]...
investigated the calculation of structure-borne sound for framed structures using a numerical method. Their method employs wave-based theory; however, it is not based on a discrete method. In contrast, spatially discrete methods such as the boundary element method (BEM), FEM and FDTD have high flexibility in the modeling accuracy of the structure. Among them, numerical schemes in the time domain such as FDTD and the constrained interpolation profile (CIP) method are widely used. Using such a method, the time development characteristics of vibration propagation can be directly obtained, greatly contributing to understanding the mechanism of wave propagation. In the investigation of the vibration characteristics of structure-borne sound transmission in building structures, such a time-domain scheme can be an effective prediction method.

In this paper, the basic theory of the analysis for a model with beam elements is described and the results of a case study for a multilayered frame model are discussed.

2. THEOREY OF THE FDTD ANALYSIS

2.1. Governing Equations

The Governing equations for a bending wave and a quasi-longitudinal wave in a beam model are described as follows. Equations (1)–(3) describe the bending wave, quasi-longitudinal wave and distortional wave on a beam existing along the x direction, respectively:

\[
\begin{align*}
EI \frac{\partial^4 w}{\partial x^4} + \xi EI \frac{\partial}{\partial t} \frac{\partial^3 w}{\partial x^4} + \mu m \frac{\partial^2 w}{\partial t^2} + m \frac{\partial^2 w}{\partial t^2} &= q, \\
E \frac{\partial^2 u}{\partial x^2} - \rho \frac{\partial^2 w}{\partial t^2} &= 0, \\
GJ \frac{\partial^2 \theta_d}{\partial x^2} - \rho I \frac{\partial^2 \theta_d}{\partial t^2} &= 0.
\end{align*}
\]

Here, \( w \) is the displacement of the out-of-plane bending vibration, \( u \) is that of the in-plane vibration, \( \theta_d \) is the distortional angle, \( \xi \) and \( \mu \) are coefficients used to model the damping characteristics of the material, \( q \) is an external pressure, \( I \) is the moment of inertia \((I = bh^3/12)\) and \( E, m, b, h \) and \( GJ \) are the Young’s modulus, mass per unit length, width of the beam, thickness of the beam and St. Venant’s torsional rigidity, respectively. The damping characteristics are considered for only the bending wave in this study, because the out-of-plane deformation of the material significantly contributes to the sound radiation. The damping characteristics are modeled by setting appropriate values for the coefficients \( \xi \) and \( \mu \), which are determined in Sect. 3.2.

2.2. Discretization

The basic equations of Eqs. (1)–(3) have fourth-/second-order differential systems, which are approximated by the following finite-difference equations:

\[
\begin{align*}
\frac{\partial^4 w_i}{\partial x^4} &= \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + O(\Delta x^2), \\
\frac{\partial^2 w_i}{\partial x^2} &= \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} + O(\Delta x^2),
\end{align*}
\]

where \( i \) is the discrete grid number in space. Equations (4) indicates that the parameter \( w_i \) is calculated using the neighboring five parameters including itself, as shown in Fig. 1. In Eq. (5), \( w_i \) is calculated using the neighboring three parameters. When we calculate the parameter \( w_3 \) in Fig. 1(a) or \( w_2 \) in Fig. 1(b) defined at the boundary, two or one virtual cell must be considered for each situation, respectively. The boundary condition of the beam can be modeled by setting appropriate values for the virtual parameters.

In the case that multiple beam elements are rigidly connected with each other, as shown in Fig. 2, the vibration transmission through each element can be simulated by considering the relationship between the virtual parameters defined at the boundary of each beam. Then, the relationship between the parameters is defined on the basis of the continuity conditions concerning the bending, quasi-longitudinal and distortional wave motion of each beam. The theory concerning these continuity conditions is described in the next section.

The procedure used to approximate the basic equations is described herein. Spatial and time differential terms in the basic equations are approximated by finite differences, and the time transient response is calculated by an implicit
method. The space derivative of the parameter is approximated by Eqs. (4) and (5). For the approximation of the time derivative, the following one-sided difference approximation is used:

$$\frac{\partial^2 w^n}{\partial t^2} = \frac{2w^n - 5w^{n-1} + 4w^{n-2} - w^{n-3}}{\Delta t^2} + O(\Delta t^2),$$  \hspace{1cm} (6)

where $n$ is the time step. As a result, the following discretized equations for the bending wave, quasi-longitudinal wave and distortional wave are obtained:

$$
(2 + \mu \Delta t)u_{n+1}^i + \left(1 + \frac{\xi}{\Delta t}\right) \frac{EI}{m} \Delta t^2 w_{n+1}^{i+2} - 4w_{n+1}^{i+1} + 6w_{n+1}^{i+1} - 4w_{n+1}^{i+1} + w_{n+1}^{i+2} - 2u_{n+1}^i + \frac{\xi EI}{m} \Delta t \frac{w_{n+1}^{i+2} - 4w_{n+1}^{i+1} + 6w_{n+1}^{i+1} - 4w_{n+1}^{i+1} + w_{n+1}^{i+2}}{\Delta x^4}
$$

$$
-2u_{n+1}^i + \frac{E\Delta t^2 u_{n+1}^{i+2} - 2u_{n+1}^{i+1} + u_{n+1}^{i+2}}{\Delta x^4}
$$

$$
-2\theta_{n+1}^{d,1} + \frac{GJ\Delta t^2 \theta_{n+1}^{d,1} - 2\theta_{n+1}^{d,1} + \theta_{n+1}^{d,1}}{\Delta x^2}
$$

(7)

(8)

(9)

where $i$ is the discrete grid number in the $x$ direction. In these equations, the parameters at time step $n+1$ on the left side are unknown, and those at time steps $n$, $n-1$ and $n-2$ on the right side are already known. Finally, the obtained discrete equations and the continuity conditions for the junctions between elements are solved as simultaneous equations.

2.3. Composite Model with Multiple Beam Elements

A method for the vibration analysis of a frame model composed of three beam elements is next described. In this model, the three beam elements are connected at one junction with a right-angled joint. The discrete mesh form of this model is shown in Fig. 3. In the figure, the parameters concerning the bending wave, quasi-longitudinal wave and distortional wave are shown. For the bending motion, vibration in two directions is considered as shown in the figure. The continuity conditions for the joint of Beam 1 are considered as follows. First, the physical parameters of the displacement and the rotation angle at the boundary of Beam 1 are made equal to those of Beams 2 and 3. Second, the bending moment, which acts at the boundary of Beam 1, is balanced with those of Beams 2 and 3. Third, the shear force acting at the boundary of Beams 1 and 2, and the in-plane force acting at the boundary of Beam 3 are balanced. Lastly, the distortional angle acting at the boundary of Beam 1 equals the rotation angle at the edge of Beams 2 and 3. In total, five continuity conditions are considered in this simulation. Here, the rotational angle $\theta_h$, bending moment $M_t$, shear force $F_t$ and in-plane force $T_t$ are described as follows:

$$\theta_h = \frac{\partial w}{\partial x},$$  \hspace{1cm} (10)

![Fig. 3 Discrete form of the beam model composed of three beam elements.](image-url)
\[ M_x = -EI \frac{\partial^2 w}{\partial x^2}, \]  
\[ F_x = -EI \frac{\partial^3 w}{\partial x^3}, \]  
\[ T_x = EA \frac{\partial u}{\partial x}. \]

These equations are transformed to discrete forms and are rewritten by considering the continuity conditions.

1. Displacement:
\[
\frac{3}{2} w_x^{nB1,1} - \frac{1}{2} w_y^{nB1,1} = \frac{u_x^{nB3} + u_y^{nB3}}{2},
\]  
\[
\frac{3}{2} w_x^{nB2,1} - \frac{1}{2} w_y^{nB2,1} = \frac{u_x^{nB3} + u_y^{nB3}}{2},
\]  
\[
\frac{3}{2} w_x^{nB3,1} - \frac{1}{2} w_y^{nB3,1} = \frac{u_x^{nB1} + u_y^{nB1}}{2},
\]  
\[
\frac{3}{2} w_x^{nB2,2} - \frac{1}{2} w_y^{nB2,2} = \frac{u_x^{nB1} + u_y^{nB1}}{2},
\]  
\[
\frac{3}{2} w_x^{nB1,2} - \frac{1}{2} w_y^{nB1,2} = \frac{u_x^{nB2} + u_y^{nB2}}{2},
\]  
\[
\frac{3}{2} w_x^{nB3,2} - \frac{1}{2} w_y^{nB3,2} = \frac{u_x^{nB2} + u_y^{nB2}}{2}.
\]

2. Rotation angle:
\[
\frac{w_x^{nB1,1} - w_y^{nB1,1}}{\Delta x_{B1,1}} = \frac{w_x^{nB3,1} - w_y^{nB3,1}}{\Delta x_{B1,1}},
\]  
\[
\frac{w_x^{nB2,1} - w_y^{nB2,1}}{\Delta x_{B2,1}} = \frac{w_x^{nB3,2} - w_y^{nB3,2}}{\Delta x_{B2,1}},
\]  
\[
\frac{w_x^{nB1,2} - w_y^{nB1,2}}{\Delta x_{B1,2}} = \frac{w_x^{nB2,2} - w_y^{nB2,2}}{\Delta x_{B2,2}}.
\]

3. Bending moment:
\[
\frac{u_x^{nB1,1} - 2u_x^{nB1,1} + u_y^{nB1,1}}{\Delta x_{B1,1}^2} + \frac{u_x^{nB3,1} - 2u_x^{nB3,1} + u_y^{nB3,1}}{\Delta x_{B1,1}^2} = 0,
\]  
\[
\frac{u_x^{nB2,1} - 2u_x^{nB2,1} + u_y^{nB2,1}}{\Delta x_{B2,1}^2} + \frac{u_x^{nB3,2} - 2u_x^{nB3,2} + u_y^{nB3,2}}{\Delta x_{B2,1}^2} = 0,
\]  
\[
\frac{u_x^{nB1,2} - 2u_x^{nB1,2} + u_y^{nB1,2}}{\Delta x_{B1,2}^2} + \frac{u_x^{nB2,2} - 2u_x^{nB2,2} + u_y^{nB2,2}}{\Delta x_{B2,2}^2} = 0.
\]

4. Shear force and in-plane force:
\[ - EI \frac{u_x^{nB1,1}}{\Delta x_{B1,1}^2} - 3u_x^{nB1,1} + 3u_x^{nB1,1} = \frac{u_y^{nB1,1}}{\Delta x_{B1,1}^2},
\]  
\[ - EI \frac{u_x^{nB2,1}}{\Delta x_{B2,1}^2} - 3u_x^{nB2,1} + 3u_x^{nB2,1} = \frac{u_y^{nB2,1}}{\Delta x_{B2,1}^2},
\]  
\[ + EA \frac{u_x^{nB1,1} - u_y^{nB1,1}}{\Delta x_{B1,1}^2} = 0,
\]  
\[ - EI \frac{u_x^{nB1,2}}{\Delta x_{B1,2}^2} - 3u_x^{nB1,2} + 3u_x^{nB1,2} = \frac{u_y^{nB1,2}}{\Delta x_{B1,2}^2},
\]  
\[ - EI \frac{u_x^{nB2,2}}{\Delta x_{B2,2}^2} - 3u_x^{nB2,2} + 3u_x^{nB2,2} = \frac{u_y^{nB2,2}}{\Delta x_{B2,2}^2},
\]  
\[ + EA \frac{u_x^{nB1,2} - u_y^{nB1,2}}{\Delta x_{B1,2}^2} = 0.
\]

(5) Distortional angle and rotation angle:
\[ \theta_{d,1}^{nB2} = \frac{\left( u_x^{nB1,1} - w_y^{nB1,1} \right)}{\Delta x_{B1,1}}, \]
\[ \theta_{d,1}^{nB1} = \frac{u_x^{nB2,1} - w_y^{nB2,1}}{\Delta x_{B1,1}}, \]
\[ \theta_{d,1}^{nB3} = \frac{u_x^{nB1,2} - w_y^{nB1,2}}{\Delta x_{B1,2}}, \]

where \( u, w \) and \( \theta \) are the in-plane and out-of-plane displacements and the distortional angle shown in Fig. 3, respectively. \( B_{m,n} \) in each superscript means the direction \( n \) of beam \( m \). The 18 unknown parameters at the junction shown in Fig. 3 are solved by considering the 18 equations above. Then, these discrete equations and the basic equations of Eqs. (7)–(9), are solved together as simultaneous equations. To solve the equations, PARDISO [8] was used.

2.4. Numerical Settings

In this section, the procedure used to determine the numerical settings concerning the spatial and time intervals for simulation of the bending wave and quasi-longitudinal wave on an acrylic beam is described.

2.4.1. Determination of the spatial interval

First, the spatial interval was set as follows.

Bending wave It has been shown that FDTD requires a grid spacing of 10 cells per wavelength [9]. The wavelength of the bending wave is described as follows:
\[ \lambda = \sqrt{\frac{EI}{m}} \sqrt{\frac{2\pi}{f}}, \]  
(32)

where \( f \) is the frequency. This equation means that the wavelength is inversely proportional to the square root of the frequency. In this study, the target upper frequency in the simulation is 2 kHz, and the value of \( \lambda / 10 \) at this frequency is calculated to be 0.014 m using Eq. (32). Here, as the physical properties of the acrylic beam, a Young’s modulus of 5.6 \( \times 10^6 \) N/m\(^2\) and a density of 1,150 kg/m\(^3\) were given for a beam with a cross section of 0.02 m \( \times \) 0.02 m. Therefore, the spatial interval was set to be 0.01 m which is less than the obtained number of 0.014 m.

Quasi-longitudinal wave The wavelength of the quasi-longitudinal wave is described as follows:

\[ \lambda = \frac{1}{f} \sqrt{\frac{E}{\rho}}, \]  
(33)

where \( \rho \) is the density. This equation means that the wavelength decreases as the frequency increases. The value of \( \lambda / 10 \) at the upper frequency, 2 kHz, is calculated to be 1.1 m. Therefore, the spatial interval for the quasi-longitudinal wave was set to be 0.01 m, which is the same as that of the bending wave. This method of applying the same interval for both waves simplifies the calculation.

2.4.2. Determination of the time interval

Second, the time interval was set as follows. It has been shown that an implicit FDTD method provides unconditional stability [10]; however, a large time step results in numerical error [11]. To illustrate the numerical error due to the time step, a simple calculation for a beam was performed.

Bending wave A calculation for a beam with 500 mm length was performed. The size of the discrete grid was 10 mm and the discrete time interval was set to be 10.416 or 83.333 \( \mu \)s (sampling rates of 96 and 12 kHz, respectively). The damping coefficients \( \xi \) and \( \mu \) were set to be 0, and the boundary condition of the beam was a fixed edge with no absorption of the vibration. For the vibration source, an impulsive source including frequency components up to 2 kHz was set following the detailed procedure described in Sect. 4.1. In addition, the other physical parameters were the same as those described in Sect. 4.1. The energy decay curves of the velocity obtained at the center of the beam, which was excited at a point near the edge, are shown in Fig. 4(a). The results were obtained in RMS form with a time constant of 10 ms. For the sampling rate of 12 kHz, the level of the velocity is attenuated by approximately 12 dB at a time of 1.0 s. For the sampling rate of 96 kHz, the attenuation level is reduced to approximately 5 dB at a time of 1.0 s. The attenuation characteristics at a sampling rate of 192 kHz are almost the same as those at 96 kHz, and no improvement in accuracy caused by the finer time interval is seen.

From the obtained results described above, a common interval of 10.416\( \mu \)s for the discrete grid of 10 mm is applied for both waves.

3. IDENTIFICATION OF LOSS FACTOR

The loss factor of the acrylic beam was measured...
through an excitation test, and the damping parameters were determined on the basis of measurement results.

3.1. Measurement of the Loss Factor

The loss factor of the acrylic beam was estimated by the impulse response decay method [12]. The experimental setup is shown in Fig. 5. The measurement was performed on an acrylic beam of 1,800 mm length. The beam was suspended in the air, as shown in the figure, and hit by an impulse hammer (PCB, 086C02) at S1, and the transient responses of the vibration velocities at five receiving points, R1 to R5 were, measured by an acceleration pickup (RION PV-90B). From the results, the reverberation time $T$ in which the vibration energy of the beam is attenuated by 60 dB compared with the maximum energy level in the response was calculated by the integrated impulse response method. The loss factor in each 1/3 Oct. band was calculated from the calculated reverberation time $T$ as follows [13]:

$$\eta = \frac{2.2}{f \cdot T}.$$  \hspace{1cm} (34)

where $f$ is the center frequency of the 1/3 Oct. band. Then, an FIR filter was used for filtering. The loss factor was calculated for five conditions. The loss factor under 100 Hz could not be calculated because the FIR filter was longer than the actual reverberation decay time. The averaged values of the loss factor and their standard deviations are shown in Fig. 6. While the values have deviations of up to approximately 0.02 in some frequency bands, the loss factors are distributed in the range from 0.02 to 0.04 in the frequency bands from 100 Hz to 2 kHz.

3.2. Numerical Model Used in FDTD Calculation

The characteristics of the loss factor can be modeled by setting appropriate coefficients for $\xi$ and $\mu$ in Eq. (1). The loss factor $\eta$ is described as follows [13]:

$$\eta = 2\pi f \xi + \frac{\mu}{2\pi f}.$$  \hspace{1cm} (35)

An appropriate pair of coefficients, $\xi$ and $\mu$, which can describe the measured characteristics was searched for by the least-squares method. As a result, $\xi = 3.40 \times 10^{-6}$ and $\mu = 32.64$ were obtained, and the loss factor (a) shown in Fig. 6 by the outlined circle was obtained. Note that the simulated frequency dependence of the loss factor has a curved shape (not flat), and for this reason the difference in the loss factor between the simulation and the actual phenomena is larger at higher/lower frequencies than at medium frequencies, especially above 1.25 kHz as shown in Fig. 6. For frequencies under 100 Hz, different values of the coefficients were set. However, the loss factor in this frequency range was not obtained by the measurement. For this reason, another pair of coefficients was searched for so that the loss factor was distributed around 0.04 in the range from 10 Hz to 100 Hz. As a result, $\xi = 7.80 \times 10^{-5}$ and $\mu = 3.20$ were selected. Note that loss factors (a) and (b) have the same value in the 100 Hz band as shown in Fig. 6.

4. BASIC STUDY

To carry out a basic validation of the proposed simulation method, a basic study of an excitation test for a beam model was performed, and the results are compared with the FDTD calculation results.
4.1. Investigation Models

The experimental setup of the beam is shown in Fig. 7. In the measurement, the specimen was excited by an impulse hammer, and the velocity at each part of the specimen shown in the figure was measured. The driving point S1 was hit, and the velocity at R1 and R2 on the other side was measured.

In the simulation, the suspended beam in the air was modeled by setting a free edge condition. The boundary condition is described as follows:

\[
- EI \frac{\partial^2 w}{\partial x^2} = 0, \quad (36) \\
- EI \frac{\partial^3 w}{\partial x^3} = 0. \quad (37)
\]

The discrete forms of these continuity equations and the basic equations of Eqs. (7)–(9) are solved together as simultaneous equations. The size of the discrete grid is 10 mm and the discrete time interval is 10.416 ms. For the physical properties of the acrylic beam, the same parameters as those in Sect. 2.4.1 were given. The loss factor is considered by setting the coefficients described in Sect. 3.2. As an impulsive force, the time transient characteristics of an impact force with a Gaussian profile in the time domain are used as shown in Fig. 8(a). The time transient characteristics in FDTD are set to ensure a similar wave shape to that in the measurement. In the figure, examples of the measured excitation force are also shown. Then, the peak values at a time of 2 ms are set to 400 N by amplifying them. The small peak existing around 2.8 ms is caused by the reaction of the beam, and it is considered to be unavoidable in the case of manual excitation for such a deformable material with a slender shape. The frequency characteristics of these signals are shown in Fig. 8(b).

In this study, the frequency components up to 2 kHz are used for investigation. However, only in the case of the measurement for the single-beam model, an impulse hammer, the head of which is made of rubber instead of plastic as for the other conditions, was used in order to obtain stable excitation characteristics of force. For only this condition, the impulsive force has frequency components up to 1 kHz, and the results are also shown up to this frequency.

In order to compare the values obtained by calculation and measurement, the velocities \(v(f)\) in the frequency domain are transformed to the following impedance level \(Z(f)\):

\[
Z(f) = 10 \log_{10} \left( \frac{|F(f)|}{|v(f)|} \right)^2, \quad (38)
\]

where \(Z_0 (= 1 \text{ Pa})\) is the criterion value for \(Z(f)\). Then, \(v(f)\) is obtained through an FFT of the time-domain results.

In this study, two kinds of loss factor are used, as described in Sect. 3, and as a result, we obtain two kinds of vibration characteristics for over 100 Hz and for under 100 Hz by using the loss factors (a) and (b) in Fig. 6, respectively. Hereafter, we illustrate these two results together in one figure for simplicity. As an example, the calculated impedance level for the beam model obtained at...
receiver R1 is shown in Fig. 9. In this figure, the characteristics of the impedance level under 100 Hz and those over 100 Hz are smoothly connected with each other at 100 Hz. This is because the loss factors (a) and (b) shown in Fig. 6 have the same value in the 100 Hz band.

4.2. Results

The calculation and measurement results for the beam model are shown in Fig. 10. Only for this model, the upper-limit frequency of the impact force is lower than the other models as described in the previous section, and the results for up to 1 kHz are shown. The calculation results show similar characteristics to the measurement results at both R1 and R2. To evaluate the similarity between the calculation and measurement results quantitatively, the impedance levels at both R1 and R2 are converted to 1/3 Oct. band values, as shown in Fig. 11, and these values are comparatively plotted in Fig. 12. In the figure, the regression equations and correlation coefficients are shown. The gradient of the regression curve $\alpha$ and the correlation coefficient $R$ are also shown in Fig. 10. The results obtained at both R1 and R2 have correlation coefficients of over 0.9 and gradients of approximately 1.0, and the results agree well.

5. VALIDATION OF THE CONNECTION METHOD

To validate the connection method between the beam elements, the vibration characteristics of frame models are calculated, and the results are compared with measurement results.

5.1. Investigated Models

An excitation test for frame models was performed. Detailed shapes of the models are shown in Fig. 13. In this study, Type-A and Type-B models are investigated. Type-A is a simple frame-type structure composed of 12 beam elements with eight junctions. Type-B is a multilayered frame model with a more complex structure composed of 44 beam elements with 24 junctions. Through investigation of the Type-A model, basic validation of the proposed connection method is carried out. Through the Type-B
model, the applicability of the method to more complex structures with an increased number of junctions is validated.

The models are composed of acrylic beams with a cross section of $20\text{mm} \times 20\text{mm}$. These acrylic elements are jointed with each other by welding them. In the measurement, an excitation force was applied to each model by hitting the surface with an impulse hammer. Then, the model was suspended in the air. In each model, the driving point S1 was hit, and the velocities at R1, R2 and R3 were measured.

In the simulation, the acrylic frame is modeled as a structure composed of multiple pieces of beams. For the frame-type model or the multilayered frame model, 12 or 44 beam elements are used, respectively.

Vibration propagation between each beam is modeled following the procedure described in Sect. 2. The numerical parameters of the discrete size, the time interval and the physical properties of the material are set to be the same as those described in Sect. 2.4.1.

5.2. Results

The results of the measurement and calculation for the Type-A model are shown in Fig. 14. The correlation between the calculation and measurement results was also calculated following the same procedure as that in Fig. 10, and is shown in each figure. While the calculation results for R1, R2 and R3 generally describe the typical characteristics of the measurement, some discrepancy in the frequency of each peak/dip and its value is seen for R2 and R3. The correlation coefficients $R$ for R2 and R3 are under 0.8 and are relatively low compared to those for the other conditions. One of the reasons for this discrepancy is considered to be that the assumption of a rigid connection between each beam element in the simulation does not agree well with the rigidity of the connection between acrylic beams in the measurement. Nevertheless, the basic validation of the proposed method was confirmed through comparison between the calculation and measurement results.

Next, the results of the measurement and calculation for the Type-B model are shown in Fig. 15. The correlation coefficients are also shown. For R1, the calculation result has similar frequency characteristics to the measured result. For R2 and R3, the frequency characteristics of the measured results have more complicated characteristics.
than that of R1 with slightly different features in the detailed peak/dip characteristics, especially under 100 Hz. However, the calculated results generally describe the tendency of the measured ones. As a quantitative evaluation, these results have correlation coefficients $R$ of approximately 0.7 for R2 and R3, and the trend is reasonably similar to that for the Type-A model. The reason for the relatively low correlation for R1 is the disagreement of the level difference under 50 Hz. While the number of junctions between beam elements is greater in the Type-B model than in the Type-A model, a significant decrease in the simulation accuracy is not seen for the Type-B model.

To summarize the results of investigating the Type-A and Type-B models, the calculation results generally describe the peak/dip characteristics of the measurement. However, partial discrepancies between the calculation and measurement results were also seen in both models, which may be caused by the disagreement of the rigidity of the junctions between the calculation and measurement.

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**Fig. 14** Results of the transfer impedance level for Type-A model.

**Fig. 15** Results of the transfer impedance level for Type-B model.
In this section, the calculation costs are evaluated. These calculations were performed by a personal computer with an Intel Core 2 Quad Q9550 processor (2.83 GHz). While this PC has four core processors, only a single processor was used for simplicity. The employed FDTD is a time-dependent method, and the calculation time depends on the duration of the response. In this paper, all the responses are calculated for a sufficient duration for the power of the vibration velocity/sound pressure response to decay to $-60 \text{dB}$ relative to the maximum value.

For the discretization of the Type A model investigated in Sect. 5, a mesh with 2,544 cells was used. The required memory for this calculation is approximately 2.1 MB. The required time for calculating a time-transient response of 1.17 s was 38 s. For the Type B model, a mesh with 8,400 cells was used, and the required memory for this calculation is approximately 3.7 MB. A calculation time of 43 s was required for a time-transient response of 1.40 s. From these results, the proposed method has a relatively low computational cost, and the practical investigation of case studies involving multiple models is possible in a relatively short time.

7. CONCLUSION

A vibration analysis scheme for a beam structure using an implicit FDTD method was proposed, which was used to simulate the vibration characteristics of a single-beam model, a frame-type model and a multilayered frame model composed of multiple beam elements.

In this study, the bending deformation of a beam structure was simulated using the Bernoulli-Euler beam theory, which is formulated in a simple way and models the deformation of a beam in a one-dimensional vibration field. As a result of applying the FDTD scheme to this simple theory, faster simulation based on a wave phenomenon can be performed with lower storage capacity required.

To validate the proposed method, the calculation and measurement results were compared and discussed. In the study of a single-beam model, the calculation and measurement results agreed well, and validity of the finite-difference discretization in time and space for the Bernoulli-Euler beam equation was confirmed. Next, a case study of a frame-type model was performed and the validity of the proposed connection method for the bending and quasi-longitudinal waves between beam elements was confirmed. Lastly, a more practical case study of a multilayered frame model was performed. In this case, the calculation results generally described the frequency characteristics of the measurement results, and it was confirmed that the accuracy of the simulation is not significantly reduced compared with the frame-type model in spite of the increased number of junctions. As a result, the applicability of the proposed method to a model with a more complex structure was confirmed. On the other hand, partial discrepancies of the peak/dip frequency between the calculation and measurement results were also seen. One of the reasons for this is considered to be that the assumption of a rigid connection between each beam element in the simulation does not agree well with the rigidity of the connection between acrylic beams in the measurement. More detailed investigation of this issue is required.

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