Permeability and flow through a packed bed of beads in a rectangular cross-section affected by overlapping wall effects

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The beads in a packed bed are usually distributed randomly. Therefore, on a macroscopic scale, a fluid flows uniformly through such a porous medium. For thin porous media, deviations are known near the regions of confinement. This so-called wall effect is extensively investigated in literature for circular confinements. In this work, the wall effect and the flow through porous media in a rectangular domain is studied. This has great importance during the initial filling process of lithium-ion batteries with the liquid electrolyte. Such batteries consist of collector foils, on which the porous electrodes are attached. Their thickness is only 5 - 10 times a particle diameter \( d_p \) and may be modeled as a monodisperse bed of spheres. The filling flow is extremely slow and may be in the pre-Darcy flow region, wherefore the validity of Darcy’s law is experimentally checked. Then, the local porosity function depending on the distance to the wall is experimentally determined. For very thin porous media, the wall effect of each wall may overlap each other, which is taken into account by using an overlapping model from the literature. The widely used model of Carman-Kozeny [1] relates the permeability with the porosity. Its validity in the near-wall region is checked by comparing experimentally-obtained mean pore diameters to those related to the model of Carman-Kozeny. The flow through a porous media is computed and compared to experimental results from literature for circular confinement and preliminary optically-matched PIV measurements.

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1 Basics

The wall-adjacent beads can at most have point contact to the wall. Therefore, the beads cannot be distributed randomly but exhibit a characteristically, highly-ordered arrangement. This order propagates into the porous media and fades out with increasing distance from the wall. It affects the local porosity \( \varepsilon(z) \), which is defined as the free area to the total area in a confinement-parallel plane at a distance of \( z \). The local porosity \( \varepsilon(z) \) can be described as a damped oscillation, which reaches the core porosity \( \varepsilon_c \) after a certain distance \( z \). In circular confinement, the influence of a wall on the local porosity \( \varepsilon(z) \) is negligible for \( z > 10d_p \) and the influence on the volumetric flow rate may be neglected for a diameter of the circular confinement \( D_c > 50d_p \) [2]. To distinguish between different flow types, the Reynolds number \( Re_{d_b} = \frac{g d_b v}{\eta} \) is used, with the fluid density \( \rho \), the dynamic viscosity \( \eta \), and the mean velocity in the pores \( v \). The characteristic length scale should be the mean pore diameter, which can be related to the hydraulic diameter \( d_h \) and the particle diameter \( d_p \) [3]:

\[
d_h = \left(\frac{2}{3}\right) \cdot \left(1 - \varepsilon\right)^{-1/3} d_p.
\]

(C1)

Creeping flow is observed for low Reynolds numbers \( Re_{d_b} < 2.3 \), where the pressure drop is linearly related to the mean velocity \( \nabla p = \eta \bar{v}/k \). The permeability \( k \) can be related to the hydraulic diameter \( d_h \) [1]:

\[
k = \frac{d_h^2 v}{80}.
\]

(C2)

Several authors even claim a separate flow regime for extremely low Reynolds numbers, mostly citing [4] with \( Re_{d_b} = \frac{g d_b v}{\eta} < 10^{-5} \).

2 Results

Pre-Darcy flow regime: In the first experimental setup, the onset of the pre-Darcy flow regime is investigated. In order to decrease \( Re_{d_b} \), while still having a measurable pressure drop, glycerine is used as the working liquid. It flows through a sufficiently-thick packing \( (D > 200d_p) \) to neglect the wall effect. Controlling temperature and moisture, the pressure drop perfectly follows equation (2) down to the lowest measured \( Re_{d_b} = 1 \cdot 10^{-8} \) with a deviation below 3 %. Thereby, no pre-Darcy regime is observed.

Permeability depending on \( d_b \): In the second experiment, the validity of equation (2) is checked in the wall-effect region, where the beads are not arranged randomly. A box with beads is filled with a resin. After curing, it is cut into slices. The bead-free area and the wettable length of the solid in the cut plane are measured. This gives data points for the hydraulic diameter \( d_h \) at certain local porosities \( \varepsilon(z) \) in the near-wall region. This is used to check the aforementioned fundamental dependency of the law of Carman-Kozeny in equation (1). The experimental data give for low porosities 30 % lower values.

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A different relation basing on fractal mathematics [5] is developed, which represents the trend of the data and (deviations <10 %). The velocity profile is computed using the Brinkman-equation [6] and compared to published data for the velocity profile and porosity in a thin circular domain [7]. Compared to equation (2), our model gives velocity profiles visibly closer to the experimental data, especially at low porosities.

**Local porosity:** The local porosity $\varepsilon(z)$ is measured by engaging the method of displacement [8]. A bed of $d_p = 6$ mm beads in a transparent rectangular container is filled with a certain volume of water. Measuring the height of the water and the volume leads to the total porosity in the filled region. Usually, this method is not that precise, as capillary effects corrugate the free interface. A dodecane layer is placed as a second liquid on top of the water mixed with sodium oleate, which totally flattens the free interface. The mean values of several experiments are fitted to a function used for circular confinements [2]. Compared to cylindrical confinement, the wavelength of the oscillation is comparable, but the porosity and damping rate are remarkably smaller. If the wall effects of two opposing walls overlap, mainly the wavelength is altered and modeled by [2].

Their model is adjusted to our experimental results and used with our permeability model to compute the flow through a porous medium for different core porosities $\varepsilon_c$. For rectangular confinements, the wall effect is only negligible if the distance of opposing walls is $D_e = 70d_p$, which is a direct result of the higher-ordered beads in rectangular confinement. If $D_e = 10d_p$, the wall effect increases the effective permeability by 10 % (for $\varepsilon_c = 0.45$) up to 25% (for $\varepsilon_c = 0.3$).

**PIV measurements:** The vertical velocity component $v(x, y, z)$ of a vertical flow through a packed bed of beads in rectangular confinement is planned to be measured (cf. fig. 1A). The beads are made of Duran with a refractive index of $n = 1.47$. The working fluid used is rapeseed oil with the same refractive index $n_{oil} = 1.47$, cf. fig 1B. The temperature dependence of $n_{oil}$ is used for optical fine adjustment. The modeled porosity function $\varepsilon(z)$ from the previous section is used to compute the velocity profile $v(z)$, averaged in the $x$ and $y$-directions. The first test run is located at a distance of $d_p/2$ from the camera-sided confinement. The vertical velocity component $v(y, z)$ is averaged vertically (crosses in fig. 1C). To obtain smooth and sinusoidal experimental data, the velocities for different packed beds must be ensemble-averaged at different distances $x$ to the camera-sided confinement. This is also the reason for deviations to both models, with which ensemble-averaged velocities can be determined. In fig. 1C an oscillating run of all curves can be observed. In both models and the preliminary experimental results, the second maximum is greater than the first one. Both models overpredict the velocity in the first maximum, since the experimental data are obtained not near one wall, but in the corner of two walls, where a greater influence of friction is evident. Comparing the minima and maxima, the fractal model (solid line) seems to better predict the velocity profile than the law of Carman-Kozeny (dashed line).

**Outlook:** Future effort will be spent in measuring the local porosity in rectangular confinement with two overlapping wall effects. As the method of displacement is not applicable, a switch to e.g. a $\mu$CT is necessary. The flow rate and pressure drop are intended to be measured along the velocity field for different wall distances.

![Fig. 1](image-url) **Fig. 1:** A: scheme of experimental setup B: measurement section with Duran beads (not fine adjusted) C: by mean flow velocity $\bar{v}$ normalized velocity $v$ against the distance from the confinement

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