Moment Invariants Technique for Image Analysis and Its Applications: A Review

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Abstract. The Moment invariant is a feature extraction technique used to extract the global features for shape recognition and identification analysis. There are many types of Moment Invariants technique since it was introduced. To date, many applications still use the Moment Invariant technique as feature extraction technique to extract the features of any images. The reason why the Moment Invariants still valid till today because its capabilities to analyze the image due to its invariant features of an image based on rotation, translation and scaling factors. Therefore, this review paper focuses to elaborate the history of Moment invariants and its applications in related fields. The summary about the advantages and disadvantages of Moment Invariants techniques will be described at the end of this review paper.

Keyword: Moment Invariants, Feature Extraction, Image Analysis, Image Processing, Neural Network, Classification.

1. Introduction

The percentage of correct classification for the image has a high impact on a classifier system. Therefore, the higher percentage of image classification yields a better classifier. Image classification depends on feature extraction techniques and thus these techniques gain potential efforts by researchers and developers to optimize the recognition and classification. Many feature extraction techniques have been introduced by researchers, however, there are still more rooms for further enhancing both the feature extraction techniques and the classifier system. The core of any good performance classification system is measured by its feature extractor engine, i.e. the Moment Invariants techniques.

In this paper, the fundamental of the Moment Invariant will be described. Generally, the aim of this paper is to study and discuss the features of the Moment Invariants technique since its introduced and its applications, which are related to the extraction and classification process. The Moment Invariant is one of the popular techniques that can produce the feature vector based on the Rotation, Translation, and Scale (RTS) invariants. This research involves the analysis of shape images that can be utilized for recognition and classification process. The purpose of shape recognition is to ease the computer to learn and process the images without any human intervention. There are several approaches found in the literature review to solve the problem of feature extraction related to the shape analysis.
The shape analysis based on Moment Invariants produces invariant properties errors when the shape is examined under various invariant properties factors, such as the Rotation, Translation and Scaling (RTS) based on the numerous studies in the literature review. Among the three invariant properties factors (RTS), the Scaling factor was found to produce more errors compared to the others. Isnanto [17] had shown that the Moment Invariants are sensitive to the scaling factor. In their work, the Hu Moment Invariant technique is utilized to classify the herb leaves images by using the Rotation, Translation and Scaling (RTS) invariant factors in order to examine the recognition performance of the system. The results have shown that the images observed via the rotation and translation factors produced the lowest error compared to that of the images inspected under the scaling factors.

In the year 2018, two improvements of the Moment Invariants technique to reduce the invariant properties errors have been proposed by Zhi [59] and Batioua [4]. The first improvement method suggested by Zhi [59] is based on the new derivation model of the scaling invariants directly from the Krawtchouk polynomials. The binary characters were used to verify the performance of the proposed method. The results show that the percentage of the image correct recognition based on the proposed method had improved of about 1% to 3% compared to that of the conventional Krawtchouk Moment invariant technique. Having said that the new approach method is solely based on the optimization of the scale invariants of Krawtchouk moments, however, the method employed is not optimal and there are still opportunities for further enhancements.

The second improvement emphasizes on the combination of two Moment Invariants techniques proposed by Batioua [4]. Three Moment Invariants techniques have been analyzed alongside with the Racah moment, namely the Tchebichef moment, Krawtchouk moment and the Dual-Hahn moment. The butterfly data set influenced by the RTS invariants and the mixed transformation between the RTS invariants was used in order to examine the proposed techniques. The results have shown that the Racah-Dual-Hahn moment gives better performance compared to the Racah-Krawtchouk with a percentage difference of about 0.31%. However, the results obtained for each moment without the combination with Racah moment (i.e. Krawtchouk and Dual-Hahn) show that the Krawtchouk moment produces better classification performance as compared to the Dual-Hahn moment by up to 3% difference. The results were also found to be influenced by the RTS invariants properties errors, particularly the scaling factor error. The best feature vector with less error of invariant properties error will give the better classification performance.

This paper is organized as follows. The history of the Moment Invariants is first introduced in Section 2. The types of the different Moment Invariants in the previous works are stated in Section 3. The application related to the Moment Invariant techniques are briefly discussed in Section 4. For summary will be summarized in Section 5.

2. History of Moment Invariants
Moment Invariants is a technique used to extract the global features for shape recognition and identification analysis. It has been widely used in many applications, especially recognition due to its ability to produce feature vectors to represent the image. This Moment Invariant technique generally used to extract the shape properties of the binary images. This is also known as Silhouette moments where it refers to the moments that have been calculated from the binary images [35].

The general computation of any moment type of order \( p \) and \( q \) and of an image intensity function \( f(x,y) \) of \( N \times M \) pixels size is characterized in (1).

\[
m_{pq} = NF \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \Pi_{pq}(x,y)f(x,y)
\]  

(1)

where \( NF \) refers to the normalization factor, the \( \Pi_{pq} \) relates to the moments kernel comprising of the product of the specific polynomial of order \( p \) and \( q \) [39], [40], which form the orthogonal basis. The
name of the new moment family is denoted based on the type of Kernel’s polynomials by resulting in a wide range of moment types.

The first 2-D moments was introduced by Hu [16]. He proposed 2-D geometric moments of a distribution function for an image as a structured element of what he called the ‘Moment Invariants’. In his work, Hu derived a set of Moment Invariants based on the theory of algebraic function in order to define the orthogonal invariants to linear transformation which are the translation, scaling and rotation factors. This function is also known as Geometric Moment Invariant (GMI). The general description of GMI function \( m_{pq} \) of the order \((p+q)\) for two-dimensional continuous function \( f(x,y) \) for \( p, q = 0, 1, 2, 3 \ldots \) is defined in (2). In the digital image analysis, the Equation (2) can be presented as (3) where \( f(x,y) \) is a pixel value for an image of size \( M \times N \). The central moments in (3) can be defined based on Equation (4) where \( \bar{x} = m_{10}/m_{00} \) and \( \bar{y} = m_{01}/m_{00} \). Then, the invariant properties with scaling factor can be produced using (5) where it is used to normalize the central moment. The six functions presented by Hu [16] are defined as (6), which are invariant to rotation, translation and scale.

\[
m_{pq} = \int \int x^p y^q f(x,y) dx dy \tag{2}
\]

\[
m_{pq} = \sum_{x=1}^{N} \sum_{y=0}^{M} x^p y^q f(x,y) \tag{3}
\]

\[
\mu_{pq} = \sum_{x=1}^{N} \sum_{y=1}^{M} (x-\bar{x})^p (y-\bar{y})^q f(x,y) \tag{4}
\]

\[
\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{p+q+2}} \tag{5}
\]

\[
\phi_1 = (\eta_{20} + \eta_{02})
\]

\[
\phi_2 = (\eta_{20} - \eta_{02})^2 + 4 \eta_{11}^2
\]

\[
\phi_3 = (\eta_{30} - 3 \eta_{12})^2 + (3 \eta_{21} - \eta_{03})^2
\]

\[
\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2
\]

\[
\phi_5 = (\eta_{30} - 3 \eta_{12})(\eta_{30} + \eta_{12})[3(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3 \eta_{21} + \eta_{03})
\]

\[
(\eta_{21} + \eta_{03})(3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]
\]

\[
\phi_6 = (\eta_{20} - \eta_{02})(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4 \eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \tag{6}
\]

3. Types of Moment Invariants

Besides the Geometric Moment Invariants introduced by Hu [16], there has been numerous studies performed to improve and develop new type of moment invariants. Teague [45] suggested the use of the Legendre Invariants (LMI) based on the continuous orthogonal Legendre polynomials. The LMI is proposed in order to give a solution to the inherent weakness of the Geometric Moment Invariants, since the GMI gives the information with high redundancy. The GMI is the projection of the intensity function of an image onto specific monomials, which do not construct an orthogonal basis. This drawback of the conventional moment can be solved by using the orthogonal moments since their kernels are orthogonal polynomials. The corresponding moments are given the feature of minimum information redundancy through its property of orthogonality, in other words, the different part of the image is represented by different moment orders. Generally, researchers had utilized the LMI in image-based application. For instance, Dehghan [6] adopted the LMI technique to develop a Farsi
handwritten identification system. Haddadnia [14], on the other hand, used the LMI technique to extract the facial features in the face recognition system.

Teague [45] also introduced the Zernike Moment Invariant (ZMI) which is derived based on the continuous orthogonal Zernike polynomials. This method was proposed to improve the conventional Geometric Invariants especially for the rotation purposes. This technique has been proven to be more robust in the noise situation. Mukundan and Ramakrishnan [35] identified that the ZMI is able to reduce the invariant properties error even though the image is under the influence of the independent characteristics. The ZMI usually used in the recognition task which requires rotation invariance since it is generally defined using a polar coordinate.

Flusser & Suk [10] has proposed the Moment Invariant technique called the Affine Moment Invariant (AMI) by extending the GMI technique to the general affine transformations. The AMI can be used in many applications such as the character recognition and shape classification since they have the ability to identify the affine-deformation which usually occurs in the real shape recognition problems. The next Moment Invariant is the Tchebichef Moment Invariant (TMI) technique which is developed by Mukundan [36]. The TMI technique was generated based on the discrete Tchebichef polynomials equation. In their work, a new set of orthogonal moment function has been presented which can be successfully adapted as the pattern features in the 2-D image analysis. The TMI technique was proven to be better than the conventional orthogonal moments, such as the LMI and ZMI techniques, and hence this technique have been adopted for numerous image recognition and classification systems.

Yinan [55] had developed the formula of Moment Invariants in order to reduce the error of Rotation, Translation and Scaling factors. The GMI computation technique has been improved in order to make it stronger against the rotation, translation and scaling. A set of equations based on the three conditions related to the GMI techniques have also been introduced. The three conditions were used to remove the influence of the zero-order central moment and the scaling factor in discrete image. The new formula of improvement for the GMI technique is known as United Moment Invariant (UMI). The UMI is an extension of GMI that tends to reduce the effects of scaling factors in the Moment Invariant function. The calculations of the UMI technique are quite similar to that of the GMI technique.

Yap [54] had proposed the Krawtchouk Moment Invariant (KMI) by using the concept of Krawtchouk polynomial function with the implementations of linear combinations of the Geometric Moment. The weighted Krawtchouk polynomials have been developed to ensure the numerical stability by scaling the Krawtchouk polynomials. The Krawtchouk moment is then was derived based on the weighted Krawtchouk polynomials. The significance of the Krawtchouk Moment Invariants is the capability to ensure no involvement of any numerical approximation since the weighted Krawtchouk polynomials are discrete. Therefore, the KMI technique is observed to be well suited as the shape features in the analysis of 2-D images and have shown that the Krawtchouk Moment Invariants performed considerably better than the GMI technique.

Zhou [60] recommended the Hahn Moment Invariant (HMI) based on the discrete orthogonal Hahn polynomials. The related functions of the Hahn polynomials have been investigated in order to define the Hahn moment in order to produce the feature images with minimum information redundancy. In later years, the new set of the Moment Invariant was introduced by Zhu [63], namely the Dual Hahn Moment Invariant (DHMI). The new set of the DHMI technique was derived based on the discrete orthogonal functions, the Dual Hahn polynomials. The weighted Dual Hahn polynomials have been developed to ensure the numerical stability by scaling the Dual Hahn polynomials. The Dual Hahn Moment Invariant is developed based on a linear combination of the Geometric Moments. The Dual Hahn moments is capable to perform better in the image reconstruction aspect in both the noise-free and noisy conditions.

In addition to the Moment Invariants, a set of discrete orthogonal moments, namely the Racah Moment Invariant (RMI) has been suggested by Zhu [62]. The RMI technique has been proposed based on the discrete Racah polynomials. The methodology of the RMI technique is similar to that of the KMI and DHMI techniques because the Racah polynomials has been normalized using the
weighted Racah polynomials in order to ensure numerical stability. Moreover, another set of orthogonal moments based on the Pollaczek polynomials has been proposed by Raj [42], which is also known as Pollaczek Moment Invariant (PMI). The weighted polynomials are also obtained to ensure the numerical stability by normalizing the Pollaczek polynomials.

The Bessel-Fourier moments (BFM) based on the Bessel function has been proposed by Xiao [51]. The Bessel-Fourier moments are generally more suitable for rotation invariant in image analysis. Besides that, the Gaussian-Hermite Moments has also been recommended by Yang and Dai [53] based on a set of orthonormal weighted Hermite polynomials. It has been discussed that the Geometric Moment Invariant has been utilized in order to allow the Gaussian-Hermite moment to be more invariant against the rotation and translation factors.

All the literatures on the moment invariants obtained from the past 50 years had shown to be relevant until today and has attracted researchers towards developing new moment families for a better image analysis. The new type of Moment Invariants was also proposed, which is called the separable moments. The separable moments are developed by utilizing combination of various polynomials for each dimension. The first separable moment was introduced by Zhu [61]. He presented some of the basic properties of the proposed separable Moment, such as Chebyshev–Legendre moments (CLM), Chebyshev–Gegenbauer moments (CGM) and Gegenbauer–Legendre moments (GLM), and additionally for the separable Moment such as Krawtchouk–Hahn moments (KHM), Tchebichef–Krawtchouk moments (TKM) and Tchebichef–Hahn moments (THM). The new Moment Invariant has been introduced by Karakasis [22], namely the Generalized Dual Hahn Moment Invariant (GDHMI). The GDHMI is proposed by using the concept of Generalized Duan Hahn polynomial function with the implementations of linear combinations of the Geometric Moment.

The following Moment Invariant is the Miexner Moment Invariant (MMI), which is developed based on the Miexner polynomials [43]. The MMI technique has been proposed as a solution to reduce the computational cost since the use of orthogonal moment is limited to the high computational cost. The MMI technique is developed by deriving the algebraic function based on the Geometric Moment Invariant. Hmimid [15], in the same year, had developed a set of moment invariant based on the orthogonal Chalier polynomials, namely the Charlier Moment Invariant (CMI). The Geometric Moment Invariant has also been utilized to allow the CMI technique to be more invariant against the rotation, translation and scale factors, as well as to improve the computational cost. Subsequently, a set of Moment Invariants technique has also been suggested by Pandey [37], namely the Bateman Moment Invariant (BMI) which is developed based on the Bateman polynomials. The scale and translation invariant of Bateman moment is determined based on the linear combination of Geometric Moment Invariant. The comparison between the BMI and other Moment Invariant techniques have demonstrated that the BMI technique has a great potential in the field of shape analysis.

More recently, Karmouni [23] developed a separable moment to reconstruct the image, which is called the Krawtchouk-Tchebichef Moment Invariants (KTMI). The KTMI has been proposed based on the Tchebichef and Krawtchouk polynomials. In the experimental analysis, the KTMI technique is examined by utilizing the binary and gray-scale images. The results obtained by Karmouni [23] have shown that the KTMI technique is effective in regards to the global approach of the image reconstruction. Meanwhile, Batioua [4] had proposed another separable moment in order to improve the recognition performance and invariability. The objective of this proposed separable moment is to compare the classical moment invariants and separable moment invariant based on the Racah polynomials. The Racah moments have been combined with other three moment invariant techniques, namely the Tchebichef, Krawtchouk and Dual-Hann moments. The butterfly data set influenced by the RTS invariants and mixed transformation are used in order to examine the performance of the proposed techniques. On the other hand, Zhi [59] had proposed a new method to improve the translation and scale invariants of the Krawtchouk moments directly from the Krawtchouk polynomials since the derivation of these function is not based on any polynomials. The translation and scale invariants of Krawtchouk moment are achieved either by normalizing the image or by using a combination of the corresponding invariants of the geometric moments. The proposed method is
verified using the binary character images. The result shows that the proposed method is invariant
under image translation and scale and can be used in the image analysis field.

Table 1 summarizes the evolution of the Moment Invariants that have been introduced and
proposed by the other researchers.

| Year | Reference | Method |
|------|-----------|--------|
| 1962 | Hu [16]   | Developed the first Moment Invariants called as Geometric Moment Invariants (GMI) |
| 1980 | Teague [45]| Proposed the Legendre Moment Invariant (LMI) by using Legendre Polynomials |
| 1980 | Teague [45]| Proposed the Zernike Moment Invariant (ZMI) based on the Orthogonal Zernike Polynomials |
| 1993 | Flusser & Suk [10]| Proposed the Affine Moment Invariants (AMI) by extending the GMI based on general affine transformations. |
| 2001 | Mukundan [36]| Introduction of Tchebichef Moment Invariant (TMI) based on discrete Tchebichef polynomials |
| 2003 | Yinan [55]| Development of new formula called the United Moment Invariant (UMI) in order to improve the GMI computation technique |
| 2003 | Yap [54]| Development of Krawtchouk Moment Invariant (KMI) by using Krawtchouk Polynomials based on implementation of linear combinations of GMI |
| 2005 | Zhou [60]| Proposed the Hahn Moment invariants (HMI) by using discrete orthogonal Hahn polynomials |
| 2007 | Zhu [63]| Proposed the Dual Hahn Moment Invariant (DHMI) based on Dual Hahn Polynomials |
| 2007 | Zhu [62]| Proposed the Racah Moment Invariant (RMI) based on Racah polynomials |
| 2008 | Raj [42]| Proposed the Pollaczek moment Invariant (PMI) by using Pollaczek Polynomials |
| 2010 | Xiao [51]| Proposed the Bessel-Fourier moments (BFMs) based on Bessel function |
| 2011 | Yang & Dai [53]| Proposed the Gaussian-hermite Moment invariant (GHMI) by using set of weighted Hermite polynomials |
| 2012 | Zhu [61]| Introduction of Separable Moment Invariants by using combination of difference polynomials for each dimension |
| 2013 | Karakasis [22]| Proposed the Generalized Dual Hahn Moment (GDHMI) invariants based on Generalize Dual Hahn Polynomials |
| 2014 | Sayyouri & Hmimid [43]| Proposed the Meixner Moment invariant (MMI) based on Meixner Polynomials |
| 2014 | Hmimid [15]| Developed the Charlier Moment invariant (CMI) by using Chalier Polynomials |
| 2016 | Pandey [37]| Proposed the Bateman Moment Invariant (BMI) based on Bateman Polynomials |
| 2017 | Karmouni [23]| Proposed the new separable moment namely Krawtchouk-Tchebichef Moment Invariant (KTMI) in order to improve the method in image reconstruction |
| 2018 | Batioua [4]| Developed another new separable moment based on Racah Polynomials by using three (3) moment invariant techniques which are the Tchebichef moment, Krawtchouk moment and Dual-hann moment |
| 2018 | Zhi [59]| Introduction of new technique based on the Krawtchouk moment in order to improve the translation and scale invariants. |
Based on the Table I, it can be seen that the Moment Invariant techniques have been used widely by many researchers to date, particularly, in order to improve the invariant properties error occurred in the Moment Invariant technique. Most of the Moment Invariant methods are developed based on specific polynomials namely Zernike, Krawtchouk, Tchebichef and so on.

4. Application of Moment Invariants
The previous section presents the fundamental and theoretical parts of the Moment Invariants, which serves as the main part of image processing since it was introduced in 1962. The applications of the moments and their invariants increases year by year and it have been widely utilized for image recognition and classification in many applications in regard to image analysis. This method is successfully adopted along with the other techniques in order to produce an efficient system. Thus, the next section will elaborate some of the related works that have been carried out previously in many applications by using Moment Invariant as the feature extraction technique.

4.1. Character Recognition
The Moment Invariant techniques are commonly used to extract the shape of the character image in various forms. The application of the Moment Invariants techniques provides many interesting features due to their capability in preserving the invariant properties against rotation, translation and scaling of the shape image such as the classification of character for many languages. For instance, Kef [28] utilized the Moment Invariant to extract the features of the Arabic words. In order to understand the use of the Geometric Moment, different moment techniques have been used to distinguish the comparison between the techniques, where the types of Moment Invariants employed are the Hu, Zernike, Tchebichef and Legendre Moment Invariant techniques. To do so, the IFN/ENIT database had been used while the neural network classifier has been adopted to recognize the Arabic words. As a result, the Zernike Moment Invariant has been found to generate the best recognition performance compared to other techniques.

Wu [50] developed a system to recognize the Chinese character. The Racah Moment Invariant was used to extract the global features of the Chinese character. For the classification method, the Euclidean distance is used to measure the distance between a pair of Chinese characters. The results demonstrated that the Racah Moment Invariant can efficiently recognize the Chinese character since the Chinese characters have very similar shapes.

Next, Fitriana [9] developed a Handwriting Digit Recognition (HDR) system using United Moment Invariant (UMI). This system is developed to overcome the problem due to the distortion of handwriting. For this purpose, the UMI is used to extract the feature image from MNIST dataset which contains 10000 images with digits from 0 to 9. The Self Organizing Maps (SOM) is then used to recognize the actual digit. As the result, the system has shown to produce a very good performance of classification. Similarly, Kaur [26] developed the verification system for the handwritten signatures using the Zernike Moment Invariant in order to extract the global shape descriptors, since the Zernike moments are orthogonal and able to produce minimum redundancy between object shape. The performance of the system was evaluated based on Artificial Neural Network to classify the handwritten signature images.

Figure 1. Chinese character used by Zhan [57]

Zhan [57] presents a method to retrieve the Chinese character calligraphy images. The Hu Moment Invariant was utilized to extract the image feature in order to set up the image database. For the
validation purposes, the image quality is measured based on the percentage of precision and recall by using the Shape Context (SC) algorithm. This has been done by detecting the similarity of shape feature based on region and edge information. The retrieve processes is divided into two stages, including (a) retrieving the image based on Hu Moment Invariant and (b) perform the second retrieval by using Shape Context (SC) algorithm.

Likewise, Gharde [13] applied the Moment Invariant to recognize the handwritten Devanagari numerical and Vowel languages. The Devanagari is an alphabetic script that consists of 13 vowels, 34 consonants and 10 numerals which is used by various Indian languages, for example, Marathi, Hindi, Konkani and Nepali. For the handwritten samples extraction task, the two type Moment Invariants were hybridized, which includes the Geometric Moment Invariant and Affine Moment Invariant. For the classification task, the Support Vector Machine (SVM) is used to classify the handwritten numerals while the Fuzzy Gaussian Membership (FGM) function is used to identify handwritten vowels.

Meanwhile, Wafi [48] utilized the Moment Invariant technique to classify the character images. The Geometric Moment Invariant was used to extract the feature vectors from each image while, the Artificial Neural Network which are Multilayer Perceptron and Simplified Fuzzy ARTMAP were used for the classification purpose. The feature vectors produced from the Geometric Moment Invariants technique are used as the input features for Neural Network classification process. The results show that both Neural Networks techniques have produced good classification performance with overall accuracy above 90% by using the Geometric Moment Invariant technique.

More recently, other researchers had further extended the utilization of Moment Invariant techniques in character recognition concept to develop a system to recognize the language gesture. For instance, Mathur [33] proposed a system to recognize the sign language based on five modules consists of gesture segmentation, real time detection, key frame extraction for removing redundant frames, the feature extraction phase using Zernike Moment Invariant and lastly gesture recognition. They compared the Geometric Moments and Zernike Moments in terms of the number of signs correctly classified and, the proposed system gives a satisfactory recognition performance when the Zernike moments is used as the shape descriptors.

4.2. Face and Facial Recognition
The feature extraction technique also had been implemented in the field of face and facial recognition system. For instance, Karaali [21] utilized the Zernike Moment Invariant to extract the features of face in arbitrary images. The multipose face detection system is developed based on the segmentation of images into skin-coloured blobs. The moment technique is used to compute the images under scale, translation and rotation invariant in order to learn a statistical model of face and non-face regions. The CVL database is used to test the system and it shows that this system provides better results as compared to the well-known Viola-Jones face detector.

Meanwhile, Papakostas [38] had used the Geometrical Moments technique to compute the center mass of the binary segment. A methodology to recognize the human face based on thermal infrared images has been proposed. Besides that, a method for face localization in order to remove irrelevant image content has also been suggested. For the feature extraction task, the Zernike and Tchebichef Moments were used to extract the feature of cropped face image. Then, three type of Minimum Distance Classifier (MDC) were used as classifier technique such as the Euclidean Distance (ED), Chi Square Distance and Manhattan Distance. The proposed technique has been tested with 20 persons consist of 70 images each and the results have demonstrated that the proposed technique has the greatest accuracy in recognizing the human faces.

Tuba [46] also proposed a system to detect the face based on skin by using the Geometric Moment Invariants as region descriptors. The GMI was used to extract the features of invariants moment before the classification task. Then, the Multilayer Perceptron neural network was used to classify the feature vector produced by GMI. The results show that the proposed system gives good accuracy for human faces detection regardless of size, orientation and viewpoint.
Next, Gautam [12] further utilized the Moment Invariant technique to recognize the facial expression. The Krawtchouk Moment Invariants was employed to extract the Region of Interest (ROI) based on spatial pyramid levels. Then, the Neighbourhood Component Analysis (NCA) was used to reduce the dimension space between the feature vector in order to select the relevant feature with high expression information only. The Support Vector Machine (SVM) technique has been used as the classifier. The system had been tested with the CK+ and JAFFE database and have shown to produce almost 96% of correct recognition.

Meanwhile, Kaur [27] had compared the performance of the continuous and discrete orthogonal moment in the system of face recognition. The continuous moments are represented by the Legendre and Zernike Moments while the discrete moments are represented by the Krawtchouk and Tchebichef Moments. Some important properties in this comparative analysis need to be considered such as invariance to rotation, scale translation, orientation. The results show that all the moments produce a good recognition performance but the discrete moments is found to be the best as compared to the continuous moments for face recognition task.

Zaeri [56] proposed a method which is useful for thermal face recognition system with low resolution. The Geometric Moment Invariant is used to extract the features that represent several emotions such as neural, anger and smiling of face expression. The proposed system had been tested with 20 different subjects and consists of a total of 1500 frontal images. The system can achieve almost 94% of recognition accuracy using the number of thermal face images as the test subject. More recently, Fachrurrozi [8] used the Geometric Moment Invariants as feature extraction method in order to extract the human facial expression. The facial expressions will be classified based on the multiple face images. The Viola-Jones method is used for face detection process and the Multi Class SVM method is used for classification process. The results show the sad expression give the greatest accuracy with 77.26%.

4.3. Finger and Palm Print
The feature extraction technique also had been applied in the fingerprint and palm print systems in order to create a personal identification. For instance, Jucheng [19] developed a fingerprint recognition system based on the Moment Invariant for cloud computing communications. The Geometric and Zernike moments were assembled in order to secure the cloud computing communications. This system only allows the authorized users to access the data. The proposed system is based on an effective pre-processing such as the extraction using Moment Invariant and the classification using SVM classifier. Thus, this system is able to handle the various input conditions that occurs in the communication of cloud computing.

Figure 2. Example of fingerprint images used by Kahyaei [20]

Kahyaei [20] proposed the fingerprint recognition system by using the Moment Invariant technique. There are three stages of the proposed system which the first stage involves the pre-processing, background removal and contrast enhancement while the second stage focuses on extracting the feature of fingerprint by using the Zernike moments since the data of the fingerprint is
influenced by the size, translation and rotations factors. The last stage is the recognition phase where the Euclidean distance is utilized to match the fingerprint image between the input samples and the stored templates. In order to evaluate the proposed system, two data sets comprising of FVC 2004 and FVC 2006 are used. The result shows that the system is fast and accurate.

Li [29] proposed a system to recognize the finger-vien based on the Moment Invariant techniques. The Hu and Zernike Moment Invariants are used since the finger-vien image is affected by rotation, translation and noise factors. The system is able to identify the characteristics of the finger-vien as well as improving the percentage of correct recognition. The errors produced from the influence of the image rotation, translation and noise in the finger-vien recognition process are also reduced.

Figure 3. Example of finger-vien image used by Li [29]
(a) The image before threshold division (b) The image after binarization (c) The image after reverse

4.4. Medical Image Analysis
The Moment Invariant also had been adopted widely in the field of medical image analysis. For example, Vaidya [47] utilized the Moment Invariants techniques to extract the features of blood vessels of retinal fundus images using four moment invariant techniques including the Geometric moment invariants (GMI), Legendre moment invariants (LMI) and Zernike moment invariants (ZMI). The four moment invariants techniques are compared to find the best technique of Moment Invariant based on the higher accuracy of vessel identification and segmentation by using Neural Networks technique. The results show that the LMI produced higher accuracy compared to other Moment Invariants.

Meanwhile, Zhang [58] developed the algorithm based on Moment Invariant technique that can be used in medical robot to detect alcohol use disorder from structural magnetic resonance imaging of the brain. The algorithm was tested with 30 alcoholics and 30 non-alcoholic participants. In the algorithm stage, the Geometric Moment Invariants are used to extract the global features while the single hidden layer Neural Network is used as the classifier. The proposed algorithm is able to detect alcoholics with best performance and this algorithm can be adopted to be used in the medical robots appliances.

Figure 4. Slide comparison images used by Zhang [58]
(a) non-alcoholic (b) alcoholic
4.5. Others

The feature extraction technique also had been applied in other fields in which involves the image analysis. For instance, Soon [44] developed a Vehicle Logo Recognition (VLR) system using the Tchevichef and Legendre Moment Invariant techniques as the feature extraction technique. They analysis has been carried out using 240 dataset images of six different types of vehicle logos from a public dataset. The Minimum-Mean Distance (MMD) has been utilized as a classifier technique to classify the vehicle logos. As the result, the propose system shows that the Tchebichef Moment Invariants produce better result compared to the Legendre Moment Invariant.

On the other part, Wafi [49] used the Moment Invariant techniques in order to analyze the performance of extraction the shape properties of the image. The three Moment Invariant techniques which are Geometric Moment, United Moment and Zernike Moment were used for the comparison purposes. The categories of shape properties are divided to scaling and rotation factors. For the analysis part, the intraclass analysis technique is used to measure the similarity of feature vector for produced from these Moment invariant techniques. As a result, it shows the Geometric and United Moment Invariants produce the small value of percentage error compared to Zernike Moment Invariants technique.

Kaur [25] investigated the Krawtchouk Moment Invariant (KMI) performance as the feature extraction technique for object recognition. The hand images were used to analyse the performance of KMI based on recognition accuracy, rotational invariance, scale invariance, computational time and feature vector size. The dataset consists of 21 objects under varying illumination conditions. The Zernike Moment Invariant (ZMI) is also used to compare its performance to that of the KMI technique and the result shows that the KMI gives higher accuracy for object recognition compared to the ZMI technique.

Clemente [5] proposed an automatic system to recognize the military vehicles based on Moment Invariant technique. The Krawtchouk Moment invariants was used to develop an algorithm that can provide robust performance for target recognition, identification and characterization. The proposed algorithm was used to provide a more reliable solution to the system from the SAR image with higher capabilities in discriminating between different subclasses of targets and in noisy environments. The MSTAR dataset is used to demonstrate the effectiveness of the proposed system. The experimental results obtain that the Krawtchouk-based algorithm shows improved performance particularly, on the characterization target. Meanwhile, Alp and Keles [2] proposed a method for Hidden Markov Model (HMM) based on Hu Moment Invariant technique in order to recognize the action from video streams. A modified Motion History Images (MHI) is used to compute the proposed method. The experimental result shows that the proposed method is able to produce good percentage of correct classification using the Weizmann dataset.

Moment Invariant also had been utilized in many applications for the industrial purposes. For instance, Jie [18] utilized the Hu Moment Invariants technique to extract the shape features of axis.
The studies were performed on an axis orbit to diagnose the fault of the rotor system. The main idea of the research is to propose a method of combining the fractal dimension with Moment Invariants as the feature vectors. Then, the back propagation neural network is used as a classifier. As the results, the method shows high recognition speed and high recognition accuracy as well as efficient in diagnosing the fault of the rotor system by suing the axis orbit.

Figure 6. Example of ship images used by Du [7]
(a) Original video image (b) Result of target detection

Du [7] utilized the Moment Invariant technique to extract the ships features in order to classify the ship targets from the intelligent video surveillance system. The Hu Moment Invariant was used to extract the regional shape eigenvalues of the ship targets. Only seven regional shape eigenvalues of highest distinction will be selected from the 43 regional shape eigenvalues extracted to create the ship shape feature vector. The shape feature library is established based on the different types of ships such as bulk carrier, container ship and tank. The KNN is used as a classifier technique to classify the ship classification purpose. As a result, the system shows good accuracy for ship classification purposes.

Figure 7. Example of leafs from Flavia dataset used by Lukic [32]

Meanwhile, Lukic [32] used the Hu Moment Invariant concept in developing an algorithm to recognize the leaf images. The different colors, shapes and textures of leaf are used as features for the classification purpose. The Artificial Neural Networks including the KNN and SVM were used as the classifiers techniques. The Flavia data set is utilized to test the proposed algorithm and the result attained is better in accuracy compared to the other approaches.

Lu [31] proposed a detection method of high voltage bushing by using the Hu invariant moments. This method is proposed according to the special characters of the complex environment of transformer substation and the characters of pictures obtained by the inspection robot. Then, the
features of the potential positions are extracted by using the Hu Moment technique. For the classification task, the Support Vector Machine (SVM) is used to classify and recognize the position of high voltage bushing. As the results, the proposed detection method is able to remove the complex background of the substation.

Recently, Li [30] presents a method to classify the Peanut images by using the Hu moment Invariant. The proposed method is classifying the peanut into three categories which are one peanut, two peanuts and three peanuts since the different peanuts have different prices. Then, The SVM is used to classify the peanut based on the peanut number.

![Figure 8. Example of binary image of peanut used by Li [30]](image)

Funatsu [11] proposed the method to detect the license play by using the Moment Invariant technique. They used Hu Moment Invariant in order to measure the inter-vehicle distance since the Hu Moment is robust to scaling and rotation of the object. As the experimental result, the proposed method was able to increase the percentage of correct detection of the license plate with robust against shadow and weather change. The proposed method also can improve the processing time.

Moreover, Monge-Alvarez [34] has proposed an automatic cough detection system based on smartphone. They used Hu Moment Invariant to extract the feature of the audio signals. The k-Nearest Neighbours (KNN) is used as classifier to classify the extracted features. The system has been evaluated with diversity of noisy backgrounds which is the real cough audio data is contaminated with a variety of noises such as noise from indoor and outdoor environments and non-cough events *i.e.* sneeze, laugh and speech. This evaluation was complemented by using the real patient data from an outpatient clinic. The experimental result obtained indicated that the system is able to detect cough event with high sensitivity and can be specificities in a variety of noisy environments.

Adam [1] utilized the Zernike Moment Invariants as a feature extraction method to extract the feature of fruit plant leaf. The Backpropagation algorithm used for classification purpose. The result shows the success rate obtained 78% of accuracy by using 100 test data.

Meanwhile, Raffaitin [41] used the Hu Moment concept as a detection algorithm which is implemented on a FPGA system by using hardware description language. This algorithm is able to recognize a target shape over a test image. The two finite state machines are used in order to implement the computation of Hu Moment and the NLM filter. As a result, it shows that the hardware implementation is much faster that software in the computation of the image moments.

More recently, Asmara [3] had proposed an identification system to identify the type of toga plants using leaf images. The Geometric Moment Invariants is used to extract the features of the leaf images. While, the Canny edge is used to recognize the leaf textures. For the classification process, the K-Nearest is used as classification technique and the result shows, the system yields 80% classification accuracy.
5. Summary
This study summarizes and discusses the fundamental concepts and related works regarding the Moment Invariant techniques and its applications in various fields, which have been avidly pursued by scientists and engineers in the past. The Moment Invariant is one of the promising techniques of shape analysis, which has caught the attention of many researchers in the recent years, resulting in the rapid research in this field. The survey has also showed the continuous research on optimizing or enhancing the percentage of correct classification of an image based on the feature extractor. The error produced from the invariant properties especially from the scaling factor effects the feature vector and thus reduces the image classification performance.

Table 2 shows few relevant reviews that emphasized on the error due to scaling factor. Based on the Table 2, only the Zhi-technique is able to reduce the scaling factor error using the Krawtchouk Moment Invariant (KMI). The other Moment Invariants techniques listed in Table II, however, shows that the scaling factor produces higher error within the context of different studies conducted by past researchers.

| Authors   | Notes                                                      | Results                  |
|-----------|------------------------------------------------------------|--------------------------|
| Zhi [59]  | Improved the Krawtchouk Moment Invariant                   | Able to reduce the scaling factor error |
| Batioua [4] | Proposed Separable Moment Invariant called as Racah Krawtchouk Moment Invariants | The scaling factor produces higher error compared to the other factors |
| Isnanto [17] | Classification of leaves based on the RTS factors |                          |
| Yaakob [52] | Comparison between six Moment Invariant techniques based on rotation and scaling factors |                          |

Meanwhile, Table 3 shows the reviews based on the performance of correct classification. In general, Table 3 shows that the percentage of correct classification can be improved by using different techniques based on the generated feature vectors.

| Authors   | Notes                                                      | Results                  |
|-----------|------------------------------------------------------------|--------------------------|
| Batioua [4] | Investigated the Comparison between the traditional Moment Invariant and Separable Moment Invariants based on the percentage of correct classification | The RKMI produces percentage of correct classification of about 50%, relatively larger than the other techniques |
| Gautam [12] | Recognition of facial expression by using Krawtchouk Moment and Support Vector Machine | The features extracted from KMI produces efficiency about 95% |
| Soon [44]  | Studied comparison between Tchebichef MI and Legendre MI based on percentage of correct classification | The TMI improves the percentage of correct classification by 3% compared to LMI |
| Kaur [25]  | Investigated the comparison between the Krawtchouk and Zernike Moment based on percentage of correct classification by using rotated and scaling images | The KMI gives higher percentage of correct classification by 9% compared to ZMI |
Here are some extracted points to achieve the objectives of this study. The first point is, to date, the moment invariants still being used by many researchers for some applications. Secondly, the feature vectors generated from the Moment Invariants techniques will affect the result of the percentage of correct classification. If the feature vectors produced the lower invariant properties errors, it will contribute a higher percentage of correct classification compared to the moment invariants produced the feature vectors with higher invariant properties errors. Next, there is few published information regarding the error produced by the scaling factor and the method to reduce or eliminate it.

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