Future Small $x$ Physics with $ep$ and $eA$ Colliders

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The interaction of spatially small dipoles with nucleons, nuclei is calculated in the DGLAP approximation at the top of HERA energies and found to be close to the $S$-channel unitarity limit in the case of the color octet dipoles. The DGLAP analyses of the current diffractive data appear to support this conclusion as they indicate a $\sim 30 - 40\%$ probability of the gluon induced diffraction for $Q^2 \sim 4\text{GeV}^2$. The need for the high-precision measurements of the $t$-dependence of inclusive and exclusive diffraction for pinpointing higher twist effects in the gluon sector is emphasized. The $eA$ collisions at HERA would provide a strong amplification of the gluon densities allowing to reach deep into the regime of nonlinear QCD evolution. Connection between the leading twist nuclear shadowing and leading twist diffraction in $ep$ scattering is explained. The presented model independent results for the nuclear shadowing for light nuclei indicate much larger shadowing for the gluon sector than for the sea quark sector. It is argued that HERA in $eA$ mode would be able to discover a number of new phenomena including large gluon shadowing, large nonlinearities in parton evolution, small $x$ color transparency in the vector meson production followed by color opacity at $x \leq .005$, large probability of inclusive diffraction. Implications for the nucleus-nucleus collisions at LHC are discussed as well.

1. INTRODUCTION

Fascination with small $x$ - physics can be traced back to the old questions - why is the cross section of nucleon-nucleon interactions at high energies large and slowly growing with $s$? Can the interaction of a heavy onium state with hadrons remain weak even at very high energies? Is it possible to exploit the increase of the parton distributions at small $x$ to produce superdense hadron matter in the laboratory?

A unique advantage of DIS for studying dynamics of strong interactions is the possibility to tune the resolution scale. Nuclei add another dimension - possibility to vary the thickness of the target. Hence it is possible to investigate the transition from soft nonperturbative dynamics to perturbative dynamics: up to what virtualities (or down to which distances) is nonperturbative QCD important? In addition, how does the transition depend on the incident energy?

The fundamental questions one can address in DIS at small $x$ are: How large is the cross section of interaction of small “hadrons” (oniums) with nucleons/nuclei at high energies? Is a kind of the Froissart bound for DIS close to the HERA kinematics for the gluon induced processes?, Would nuclei be transparent to a high-energy $J/\psi$, $\Upsilon$? How far down in $x$ it is necessary to go for the observation of a break down of the linear DGLAP evolution equations? What are implications of nonlinear QCD evolution for the physics of $AA$ collisions?

In this talk we first focus on $ep$ scattering. We discuss the interaction of spatially small color singlet clusters with hadrons at high energies. The $S$-channel unitarity constraint on this interaction implies that, in the case of color octet dipoles interaction may already be close to the unitarity limit already at higher HERA energies, implying a large probability of diffraction when the hard process is induced by the interaction with a gluon. The use of the QCD factorization theorem | 4 for inclusive diffraction allows the extraction
2. ELECTRON-PROTON PHYSICS

2.1. The interaction of small color singlets with hadrons

At small $x$ the average longitudinal distances (Ioffe distances) in the correlator of the e.m. currents which determine $F_{2p}(x,Q^2)$ are $l_{coh} \sim \frac{1}{2 m_N x}$. At HERA $l_{coh}$ can reach $10^3$ fm. Hence in the target rest frame the small $x$ processes occur in three steps. Initially the $\gamma^*$ transforms into $q\bar{q}$ pair. Next, the 33 color dipole of transverse size $b$ interacts with the target. This cross section rapidly grows with increase of energy $|\mathcal{F}_{p\rightarrow \text{had}}|$:

$$\sigma^{\text{incl}}_{q\bar{q},N}(E_{\text{inc}}) = \frac{\pi^2}{3} b^2 \alpha_s(Q^2)xG_N(x,Q^2 \simeq \frac{\lambda}{b^2}) \text{,} \quad (1)$$

where $\lambda(x \simeq 10^{-3}) \approx 9, x = \frac{Q^2}{2 m_N E_{\text{inc}}}$. For color octet dipole the cross section is enhanced by the ratio of Casimir operators of color group $SU(3)_{c} \cdot C_F(8)/C_F(3) = 9/4$ $|\mathcal{F}_{p\rightarrow \text{had}}|$:

$$\sigma^{\text{incl}}_{g g,N}(E_{\text{inc}}) = \frac{3\pi^2}{4} b^2 \alpha_s(Q^2)xG_N(x,Q^2 \simeq \frac{\lambda}{b^2}) \text{,} \quad (2)$$

These equations are another form of the QCD evolution equations. For the fit to the energy dependence $|\mathcal{F}_{p\rightarrow \text{had}}|$ $\lambda(s,Q^2) \propto s^{\alpha(Q^2)-1}$, one finds that due to increase of the steepness of the gluon density with virtuality, $\alpha(Q^2)$ gradually increases with $Q^2$:

$$\alpha(4 GeV^2) \approx 0.2, \alpha(40 GeV^2) \approx 0.4. \quad (3)$$

2.2. Pomeron: one, two, ... too many?

Satan: “You were selling spoiled sturgeon fish.”
Merchant: “No - sturgeon was of regular quality”
Satan: “Sturgeon could only be the prime quality there is no other one.”

“Master and Margaret” M.Bulgakov

Eqs.3 & 2 clearly demonstrate a qualitative difference of the strong interaction dynamics in DIS at small $x$ from the soft physics of the vacuum exchange. This object was derived by V.Gribov within the high-energy Regge theory as the rightmost singularity in the complex angular momentum plane. It was named Pomeron by M.Gell-Mann after I.Pomeranchuk who was the first to predict the total cross sections of particle(antiparticle)- hadron scattering to be equal at asymptotic energies. The key feature of the Regge trajectories is their universality - independence of the trajectories on the process.

Such a fit is useful in the practical applications although QCD predicts a behavior $\propto a + b \ln \frac{1}{x} + c \ln^2 \frac{1}{x}$ for the HERA energy range since radiation of $\propto 1-2$ hard gluons is possible in the multiRegge kinematics at HERA.
Clearly the behavior of the cross sections, in the kinematics where DGLAP describes the $Q^2$ evolution of the parton densities, violates such universality in a gross way.

It is sometimes suggested that one can save the idea of the Pomeron in DIS processes by introducing two Pomeron trajectories with different intercepts. Clearly a smooth change of the effective power with $Q^2$ indicates that this hypothesis is not equivalent to eq.\[1\].

Another manifestation of the breakdown of the universality is the change of the pattern of the $t$-dependence of the quasi two-body processes expected in pQCD \[7\]. Let us consider for example the process $\gamma^* + p \rightarrow V + p$ where $V$ is a vector meson.

In the soft regime the $t$ dependence of the differential cross section changes with $s$ as

$$\frac{d\sigma}{dt} = f(s,0)\phi(t) \left( \frac{s}{s_0} \right)^{2\alpha_F - 2} \simeq f(s,0)\phi(t) \exp(2\alpha' t \ln(s/s_0)). \quad (4)$$

In the impact parameter space this amounts to an increase of the radius of the projectile hadron with increase of the incident energy. This can be understood as due to multiperipheral structure of the soft process - fast partons of the projectile decay into slower partons with a random change of the position in the impact parameter space of $b \sim \frac{1}{k_t}$ where $k_t$ is the soft scale (transverse momentum of the emitted parton). The resulting random walk in the impact parameter space leads to a growth of $\langle b^2 \rangle$ proportional to the number of steps, that is to the length of the rapidity interval. Hence the size grows $\propto \Delta y$, leading to the shrinkage of the cone given by eq.\[3\].

In the hard regime the parton (gluon) emissions are governed by the hard scale (Fig.\[1\]). Hence the random walk produces much smaller increase of $\langle b^2 \rangle$: $\alpha_{\text{hard}} \ll \alpha_{\text{soft}}$.

Therefore with an increase of $Q^2$ one expects three phenomena to happen \[7\]: (i) a decrease of the contribution of the upper end of the ladder to the slope, $B \left( f(t) = \exp(Bt) \right)$ which should tend to the limiting value determined by the two-gluon form factor of the nucleon - experimentally $B_{2g} \approx 4 \text{ GeV}^{-2}$. (ii) Gradual disappearance of the energy dependence of the slope. (iii) A fast increase of the cross section with energy $\sim |xG(x,Q^2)|^2 \propto \frac{Q^2}{x^{\alpha'}}$. However this pattern can be sustained only until the transverse momenta in the perturbative ladder degrade to the level corresponding to the soft physics. At this point the further steps in the ladder will be soft and will lead to increase of the slope (see the sketch Fig.\[2\]) and lead to the values of $\alpha'$ which are likely to approach, at very high energies, values similar to those for the soft physics.

Obviously the discussed pattern is qualitatively...
different from the soft Pomeron type physics. Hence, one should be extremely careful with the use of the word “Pomeron” in the DIS regime.

Current HERA data confirm the prediction of the convergence of the slope of $\rho$-meson production at large $Q^2$ to that for $J/\psi$ (section 2.5). However the HERA data are not sufficiently accurate so far to determine $\alpha'$ for DIS processes in the HERA energy range and the change of the slope between the HERA range and the fixed target experiment range is still a matter of debates [4 11]. Nevertheless one can obtain clear indications of nonuniversality of $\alpha'$ by combining HERA and SLAC linac data. Indeed, assuming for a moment that $\alpha'$ is universal we would find that $B \approx 4 \text{ GeV}^{-2}$ for $W = 200 \text{ GeV}$ as measured at HERA would lead to $B \approx 1 \text{ GeV}^{-2}$ at $W \sim 6 \text{ GeV}$, while the SLAC linac data at that energy correspond to $B \approx 3 \text{ GeV}^{-2}$.

Hence we conclude that an accurate measurement of the energy dependence of the $t$-dependence would provide a unique way to determine the transition from soft to hard physics and provide a direct evidence for non-Regge type behavior of the interaction in the vacuum channel in DIS.

2.3. Can cross sections grow rapidly forever?

A fast increase of the inelastic cross section of the small dipole -nucleon interaction cannot continue indefinitely. The black body limit: $\sigma_{\text{diff}} \leq \sigma_{\text{tot}}/2$ indicates that this is impossible.

Nucleon case

The analysis is much simpler than in the case of soft interactions since the radius of the system practically does not grow with energy (see discussion in the previous section and Fig.6).

It follows from the optic theorem that [4 11]:

$$\sigma_{\text{el}} = (1 + \eta^2) \frac{\sigma_{\text{tot}}^2}{16\pi B_{2g}}$$

(5)

where $\eta$ is the ratio of real and imaginary parts of the amplitude. Using the Gribov-Migdal relation for $\eta$:

$$\eta = \frac{\pi}{2} \frac{d \ln \sigma_{\text{tot}}}{d \ln s}$$

(6)

and $\eta \geq 0.4$ for $Q^2 \sim 10 \text{ GeV}^2$ from eqs.4 11 we find a kind of the Froissart bound:

$$\sigma_{\text{tot}} \leq \frac{8\pi B_{2g}}{1 + \eta^2} \approx 35 \text{mb},$$

(7)

for $Q^2 = 10 \text{ GeV}^2$ and therefore $\sigma_{\text{inel}} \leq 18 \text{mb}$. The limit becomes tighter with increase of $Q^2$ due to increase of $\eta$ with $Q^2$.

To illustrate numerically to what extent this limit is relevant for dynamics at HERA and beyond let us consider $Q^2 \sim 10 \text{ GeV}^2$ and $x \sim 10^{-4}$ and take $xG_N \sim 20$ and $\alpha_s = .25$. Clearly such an estimate carries substantial uncertainties, in particular because eqs.4 12 were derived in the leading $\alpha_s \log Q^2$ approximation. The cross section of interaction of the color triplet dipoles with nucleon is still much smaller than the discussed limit: $\sigma_{\text{inel}} \sim 7 \text{ mb}$. Using eq.5 we also get $\frac{\sigma_{\text{el}}}{\sigma_{\text{inel}}} \approx 0.1$. At the same time for the color octet dipole eq.6 gives $\sigma_{\text{inel}}(x = 10^{-4}, Q^2 = 10\text{GeV}^2) \approx 14 \text{mb}$. This is pretty close to the limit and corresponds to: $\sigma_{el}/\sigma_{\text{tot}} \sim 1/3$. This indicates that decomposition of cross section over twists in this situation becomes unreliable, also one should expect a large probability of diffraction in the gluon induced processes. Applying the AGK rules [4 12] for the diagrams corresponding to the higher twist terms (like attachment of four gluons to the dipole) we find that they lead to final states with double, triple, ... multiplicity. This provides an evidence for the large fluctuations of the parton densities in the regime close to the unitarity limit [4 11].

It is worth emphasizing that a large value of $xG_N(x, Q^2)$, which results in a breakdown of the linear evolution equations, was generated within DGLAP predominantly due to $\log Q^2$ evolution rather than solely due to $\ln 1/x$ effects which would be the BFKL approximation. Qualitatively the large gluon density is due to the presence of large number of gluons already on a low resolution scale (gluon carry about the same mo-

\footnote{It is worth noting that there is no direct relation between the higher twist effects in diffraction and in the total cross of DIS. In the diffraction case we could use the optic theorem to single out the leading term in the scattering of small clusters. In the case of the total cross section several competing terms enter with opposite signs, cf. [4 11].}
momentum as quarks at this scale) and they generate a lot of gluons in the evolution. (At HERA kinematics \( xG(x, Q^2) \approx 20 \) but only 1-3 gluons are due to multiRegge kinematics). When energies approach the unitarity limit the increase of \( \sigma(b, x) \) should slow down to a rate which may be rather similar to the case of soft physics. One may expect that in this limit a soft regime will be reached with a genuine universal Pomeron.

Nucleus case. In the case of a sufficiently heavy nucleus the inelastic cross section cannot exceed the geometric size of the nucleus:

\[
\sigma^{\text{inel}}_{q\bar{q}-\text{A}}(b, s) = \frac{\pi^2}{3} b^2 \alpha_s xG_A(x, Q^2) = \lambda/b^2 < \pi R_A^2
\]

Hence \( xG_A(x, Q^2 = 10)/A \leq 120 A^{-1/3} \). For central impact parameters the inequality is a factor of 1.5 times stronger:

\[
xG_A^{\text{cent. imp. par.}}(x, Q^2 = 10)/A \leq 80 A^{-1/3}. \tag{9}
\]

For the interactions induced by the color octet dipole the constraint is a factor of 9/4 stronger:

\[
xG_A^{\text{centr. imp. par.}}(x, Q^2 = 10)/A \leq 35 A^{-1/3}, \tag{10}
\]

which for \( A = 200 \) is substantially smaller than \( xG_N(10^{-4}, 10) \sim 20 \).

Thus we conclude that at the HERA kinematics quark induced interactions do not reach the unitarity limit but perhaps the gluon induced interactions are close to it (estimates at lower Q are more uncertain but the trend is that the approach to the unitarity limit should occur earlier for smaller Q). If the gluon shadowing were small, the quark induced interaction at small impact parameters for heavy nuclei would be close to the unitarity limit for \( Q^2 \sim 10 \text{ GeV}^2 \) leading to large nonlinear effects in the QCD evolution for quark sector.

In the case of heavy nuclei the unitarity constraint definitely points to a modification of the dynamics for the gluon induced interaction even if the gluon shadowing is large.

Extension of the x-range by two orders of magnitude at TESLA-HERA collider would correspond to an increase of the gluon densities by a factor of 3 for \( Q^2 = 10 \text{ GeV}^2 \). It will definitely bring quark interactions at this scale into the region where DGLAP will break down. For the gluon-induced interactions it would allow the exploration of a non-DGLAP hard dynamics over two orders of magnitude in \( x \) in the kinematics where \( \alpha_s \) is small while the fluctuations of parton densities are large.

2.4. Inclusive diffraction

We discussed above that a large probability of diffraction in DIS would be a clear signal for the onset of a new nonlinear dynamics. Since this characteristic of DIS can be directly measured, it provides a more direct probe than analysis of inclusive structure functions which depend on the nonperturbative boundary conditions which are not constrained by the theory strongly enough.

The current situation with diffraction at HERA is already very intriguing. One expects, based on the QCD factorization theorem for diffractive processes [1], that the diffractive structure functions \( f^D_1(\beta, Q^2, x_F, t) \) satisfy DGLAP evolution equations. Here \( \beta = x/x_F \). The data seem to be consistent with an early scaling for \( Q^2 \geq 4 \text{ GeV}^2 \) for most of \( \beta \) range and a large probability of diffraction - \( P_q \approx 10\% \) for \( \gamma^* p \) scattering poses rather strong constrains on the dynamics.

This experimental observation is highly non trivial. Indeed, if the DGLAP evolution for the fragmentation would be valid for \( Q \geq Q_0 = 0.7 \text{ GeV} \) where quark and gluon parton densities tend to zero at \( x \rightarrow 0 \) (the GRV scenario) the rapidity gap probability in DIS would be \( \ll P_q \approx 0.1 \) observed at HERA. (Naively, if one assumes factorization for the inclusive cross sections, the assumption of an early factorization for the leading baryon production should be reasonable as well. Indeed, the overlapping integral for the leading partons (present at the scale \( Q_0 \) ) with the emitted partons to form a low \( p_t \) leading nucleon should be very small since the partons which are generated in the evolution have \( p_t \geq Q_0 \) and rather small light-cone fractions).

Another interesting feature of the current data (shared by the diffractive data from the proton colliders) is a very important role of the gluons in the diffractive events. In the parlor of diffractive community - the “Pomeron” is predominantly built of gluons. To quantify this statement it is
convenient to define the probability of diffraction for the action of the hard probe which couples to a parton $j$:

$$P_j(x, Q^2) = \frac{\int f_j^D (x, x_F, Q^2, x_F, t) dt dx_F}{f_j(x, Q^2)}.$$  \hfill (11)

If the interaction in the gluon sector at small $x$ reaches strengths close to the unitarity limit we should expect that $P_g$ is rather close to 1/2 and much larger than $P_q$. Using eq.2 for $\sigma_{\text{color octet dipole}}^{\text{incl}}$ we calculate $P_g(x = 10^{-4}, Q^2 = 10 \text{ GeV}^2) \approx 1/3$. Indeed, using the global fit of Alvero, Collins and Whitmore \cite{14} we find (Fig.3): $P_g \gg P_q$, and $P_g(x \leq 10^{-3}) \approx 0.4(0.3)$ for $Q^2 = 4(10) \text{ GeV}^2$!

It is also interesting that diffraction when parameterized in terms of the effective Pomeron trajectory corresponds to $\alpha_P(t = 0) = 1.15 - 1.2$ which is somewhat larger than what is usually observed in the soft domain.

A word of caution: the direct measurements of $f_g^D$ are available for a limited range of $x_F$ and for rather large $Q^2$. “Backward” evolution neglects higher twist effects which may slow down this evolution and reduce $P_g$ at $Q \sim Q_0$. However backward evolution should be a small effect for $Q^2 \sim 15 \text{ GeV}^2$ where $P_g \sim 0.3$. So we conclude that the data confirm the expectation that interactions in the gluon sector are strong. They are not corresponding to the black limit since $P_g$ is substantially smaller than 0.5. Also, the measured slope of diffraction suggests that configurations involved in the total cross section of diffraction have a rather large size while the slope of the cross section of diffraction in the gluon induced processes was not measured so far.

Due to the large distances involved in the diffraction we can think of diffraction as “elastic” scattering of $q\bar{q}$ or $gg$ configurations of the target (if we define the process at a given $Q$ scale, changing the scale leads to a mixing of these configuration, and the addition of new intermediate states like $q\bar{q}g$). Hence it is instructive to treat diffraction in a complementary $S$-channel picture -the eigen states of the scattering matrix \cite{15}. Use of the optic theorem leads to

$$\sigma_{\text{eff}}(x, Q^2) = \frac{16\pi d\sigma_{\text{diff}}/dt}{\sigma_{\text{tot}}} \bigg|_{t=0}$$  \hfill (12)

and allows to extract the average cross section for the configurations which contribute to quark and gluon induced diffraction: $\sigma_{\text{eff}}^{(q)}(x, Q)$. We find $\sigma_{\text{eff}}^{(q)}(x = 10^{-4}, Q = 4) \sim 30 \text{mb}$ and even larger value of $\sigma_{\text{eff}}^{(g)}(x = 10^{-4}, Q = 2) \sim 50 \text{mb}$ which indicates presence of superstrong interactions in the gluon sector (Fig.4). These strong interactions should be due to some hard dynamics of strong gluon fields since the $\gamma + p \rightarrow J/\psi + p$ data suggest that pQCD for these $Q$ occupies most of the rapidity interval.

This argument for superstrong gluon interactions at small $x$ is complementary to the unitarity argument given above.

Several avenues of studies are possible in $ep$ scattering to investigate relative role of soft and hard physics in the gluon induced diffraction: detailed investigation of charm diffraction, diffractive dijet production, energy dependence of the $t$-slope. The study of scattering off nuclei - nuclear shadowing, diffraction off nuclei would be able to provide direct answers to these questions.

Figure 3. Probabilities of rapidity gap events for quark and gluon induced diffraction calculated using the best fit of \cite{14}.
tering amplitude of the hadron state, \(\sigma\) wave function of the photon \(\Psi\) can be written as a convolution of the light-cone leading twist processes. In the small processes may be calculated as rigorously as the tons \([16]\) has demonstrated that for large \(Q\) these tons production by longitudinally polarized photons the QCD factorization theorem for exclusive meson production in DIS were performed. A proof of and theoretical studies of the exclusive vector mesons 

2.5. Vector meson production - a quest for a low Q gluonometer

Over the last few years a lot of experimental and theoretical studies of the exclusive vector meson production in DIS were performed. A proof of the QCD factorization theorem for exclusive meson production by longitudinally polarized photons \([16]\) has demonstrated that for large \(Q\) these processes may be calculated as rigorously as the leading twist processes. In the small \(x\) limit the amplitude of the process - \(A(\gamma_L^+ + p \rightarrow V + p)\) can be written as a convolution of the light-cone wave function of the photon \(\Psi_{\gamma^+\rightarrow(q\bar{q})}\), the scattering amplitude of the hadron state, \(\sigma(q\bar{q}T)\), and the wave function of the vector meson, \(\psi_V\). \(\sigma(b, s)\) is expressed through the skewed gluon density of the target \([8]\). In impact parameter space (suppressing the \(z\)-dependences)

\[
A = \int d^2 b \psi_{\gamma^+, L}(b) \sigma(b, s) \psi_V(b).
\]

The parameter free leading twist answer for the process is

\[
\frac{d\sigma^L_{KN \rightarrow V_n}}{dt}|_t=0 = 12\pi^3 \Gamma_{V \rightarrow e^+e^-} M_V \cdot
\]

\[
\alpha_s^2(Q) \eta_V^2 \left(1 + i \frac{\alpha_s}{2\pi} \frac{d\sigma_{\text{skewed}}(x, Q^2)}{d\eta_V^2} \right) \alpha_{EM} Q^2 N_c^2 (2)  \]

Here, \(\Gamma_{V \rightarrow e^+e^-}\) is the decay width of \(V \rightarrow e^+e^-\); \(\eta_V \equiv \frac{1}{2} \int d^2 k \phi_V(z, q^2) \rightarrow 3\) for \(Q^2 \rightarrow \infty\).

In the leading twist \(b=0\) in \(\psi_V(b)\). Finite \(b\) effects in the meson wave function in the overlap integral \([13]\) appear to be one of the major sources of higher twist effects at small \(x\). They strongly reduce the cross section at intermediate \(Q^2\). However the transverse sizes essential in the amplitude may remain small, down to rather small \(Q^2\) (Fig.4). Hence it is likely that the study of \(\rho,\phi\)-meson production at intermediate \(Q^2 \sim 5 - 10\) \(GeV^2\) will allow the study of the gluon densities at \(Q^2\) down to \(2-3 GeV^2\). The best way to check the quality of \(\rho,\phi\)-mesons as low-Q gluonometers would be to study color transparency in the production of these mesons at intermediate \(Q^2\), see discussion in section 3.7.1.

Experiments at HERA have confirmed a number of the QCD predictions \([8]\) for the vector meson production: (i) The rate of the increase with energy for \(\rho\) production for \(Q^2 \geq 20 GeV^2\) and for \(J/\psi\) production is \(xG_N(x, Q^2_{eff})^2 \propto W^{0.8}\) for \(Q^2_{eff} \sim 4 GeV^2\). (Note that the soft physics expectation is \(xW^{0.32}\)). The models which take into account higher twist effects based on eq.\([13]\) \([11, 18]\) are able to to describe the HERA data at lower \(Q^2\). The inclusion of the quark exchange allows to reproduce the magnitude of the fixed target data as well. \([18]\). (ii) absolute cross section of \(\gamma\) production is reasonably reproduced. \([13]\). (iii) The extensive data of H1 \([24]\) on the energy and \(Q^2\) dependence of \(J/\psi\) production agree with predictions of \([13]\) and confirm need for large higher...
twist effects. The model of Ryskin [21] which neglects these effects leads to a factor $\sim 4 - 5$ faster decrease of the cross section with $Q^2$. (iv) the dominance of $\sigma_L$ over $\sigma_T$ at $Q^2 \gg m_V^2$ is confirmed (iv) The $\phi/\rho$ ratio has increased a factor $\sim 4$ as compared to the value at $Q \sim 0$ and reached the value close to the QCD prediction of $1.2* (2/9)$ (where $2/9$ is the SU(3) result), similar trend is observed for the $J/\psi/\rho$ ratio. (v) Convergence of the $t$-slopes of $\rho$ meson production at large $Q^2$ and the $J/\psi$ slope (which weakly depends on $Q^2$) has been observed at HERA. The $Q^2$ dependence of $B$ can be described semiquantitatively as a result of the finite $b$ in the vector meson wave function, see Fig.6. (vi) The data are consistent with a slower shrinkage of diffractive cone for these processes than for soft phenomena, see discussion in section 2.2.

2.6. BFKL dynamics - small dipole - small dipole scattering

The cross section of scattering of two small dipoles was calculated in the leading log $1/x$ approximation neglecting $\ln Q^2$ evolution by BFKL [2]. Calculation of next to leading log $1/x$ effects were reported a year ago which demonstrated a large reduction of the rate of increase with energy, as compared to the leading log $1/x$ approximation. To a large extent this reflected the smallness of the phase space - emission of one gluon in this approximation requires at least about two units of rapidity (at HERA this corresponds to emission of $\leq 3$ gluons). A number of attempts to perform resummation of NLO BFKL were reported recently. The current scenario is that $\sigma \propto s^{0.25 \pm 0.05}$ and that diffusion in the impact parameter space may be rather weak and not bring an onset of nonperturbative regime over a significant energy range [23].

This scenario leads to a slower energy dependence of the cross section than eq.4 for the scattering of a small dipole off a large hadron. Such a pattern seems unrealistic and reflects that in the HERA range the $\ln Q^2$ effects which are neglected
in the BFKL model are very important. Hence the quantitative comparison of the model with the data is hardly possible at present. It would be more promising to study the \( p_t \) ordering of the produced hadrons which is qualitatively different in BFKL and DGLAP approximations. Therefore detection of particles over large rapidity intervals is necessary - to emit three gluons between dipoles in BFKL kinematics \( \Delta y \geq 2 \) between the jets one needs a detector with acceptance of \( \geq 8 \) units in rapidity. Promising direction of the study of the BFKL regime is \( \gamma^* - \gamma^* \) scattering at large \( Q_1^2 \sim Q_2^2 \). In this case contribution of the scattering of two small dipoles is enhanced by a power of \( Q^2 \) as compared to the scattering of a small dipole on a large dipole [24].

2.7. Main focus of the future studies

Since the most dramatic effects are expected to be happening in the processes directly coupled to gluons the major focus of the studies should be a measurement of the quantities more directly sensitive to the gluon dynamics and to the onset of hard (pQCD) regime. This includes measurement of \( \sigma_L (x, Q^2) \), the absolute value, \( W, Q^2, t \) dependence of diffraction for the exclusive production both for \( \sigma_L \) and \( \sigma_T \) (vector mesons, \( \pi \pi \) ....), for charm, dijets, etc.

The key issue is to establish nonuniversality of the vacuum exchange - the \( Q^2 \) and flavor dependence of the rate of the change of the \( t \)-slopes with energy, and of the energy dependence of the diffractive processes, and especially the difference between the hard diffraction induced by hard \( \gamma^* \)-gluon and by \( \gamma^* \)-quark scattering.

Another direction of the research would be looking for manifestations of BKFL-type dynamics in the hard processes spanning maximal possible range in rapidity.

To achieve these objectives one would need to perform high precision DIS measurements both at lower energies down to \( W \sim 10 \text{GeV} \) and beyond the energies of HERA - future \( e p \) colliders. Accurate measurements of the \( t \) dependence of diffractive processes requires improving the forward acceptance of the collider detectors. The extension of the detector rapidity acceptance range much further into the proton fragmentation region would be necessary to reach the rapidity ranges of \( \Delta y \sim 6 - 8 \) which would allow studies of a new pattern of \( p_t \) ordering important in the kinematics where \( \log \frac{1}{x} \) effects become essential.

3. SMALL \( x \) PHYSICS IN \( eA \) SCATTERING

3.1. High-Energy \( eA \) scattering - History

Thirty years ago nuclei were suggested as a direct way to observe hadron properties of light, see review and references in [23].

The striking effect was that in spite of the small value of \( \sigma_{\gamma N} \), the screening was observed for \( \gamma^* \)-nucleus cross sections: \( \frac{\sigma_{\gamma A}}{\sigma_{\gamma N}} < 1 \). The reason is that a virtual photon transforms to a hadronic (quark-gluon) configuration \(|h|\) which propagates distances \( l_{coh} = \frac{2\mu}{Q^2 + M^2_h} \approx \frac{1}{2 m_N x} \) in the DIS limit, which by far exceed the nucleus size of \( 2R_A \) (large Ioffe distances). This key feature of space-time development of the high-energy DIS is a backbone of many analyses of small \( x \) DIS at HERA which we discussed in section 2.

Hence there are two distinctive kinematics for DIS electron - nucleus scattering. At \( x \geq 0.2 \) nuclear effects in the total cross section originate from the short-range nuclear structure, while at small \( x \ll 0.1 \), \( l_{coh} \gg 2R_A \) cross section is practically not sensitive to details of the nuclear wave function.

3.2. Structure functions of nuclei in the black disk limit

In the limit \( l_{coh}(x) \gg 2R_A \ (x \ll 10^{-2} \) for \( A=200) \) there is a deep connection between the \( \gamma^* A \) scattering and \( R_h^e e^- (M^2) \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+ \mu^-) \) [10]

\[
\sigma_{tot}(\gamma^*A) = \frac{\alpha}{3\pi} \int \frac{\sigma_{tot}(|h'' - A|)R_h e^- (M^2)M^2}{(Q^2 + M^2_h)^2} dM^2, \quad (15)
\]

If one makes an assumption natural for the soft strong interaction dynamics that ALL hadronic configurations \(|h|\) in \( \gamma^* \) at large enough \( \nu \equiv E_{\gamma^*} \) would interact with a large nucleus with \( \sigma_{tot}(|h'' - A|) \sim 2\pi R_A^2 \) a gross violation of the Bjorken scaling should take place for \( Q^2 = \ldots \)
\[ \frac{1}{Q^2} F_{2A}(x, Q^2) = \frac{\pi R_A^2}{12\pi^2} x e^{-\ln x + 1} \]

where \( x_0 = \frac{1}{2m_N R_A} \).

Since the Bjorken scaling was considered a natural behavior of the cross sections at large \( Q^2 \) this prediction of the gross violation of the Bjorken scaling was referred by Bjorken as the Gribov paradox. Bjorken suggested the parton model solution of the paradox: the Aligned Jet Model - only a small fraction \( \sim 1/Q^2 \) of configurations in \( \gamma^* \) which have small \( k_t \) (large transverse size) could interact with hadron-like cross sections [26]. The rest of configurations would not interact (parton model) or interact with in a color transparent way (pQCD of intermediate energies).

In QCD, due to a fast increase with energy of the cross section of interaction of a “small color dipole” with a nucleon, the Gribov scenario becomes ones again a viable possibility in the dipole with a nucleon, the Gribov scenario became a viable possibility in the dipole with a nucleon, the Gribov scenario became a viable possibility in the dipole with a nucleon, the Gribov scenario became

\[ x G_A(x, Q^2)/Q^2 = \frac{\pi R_A^2}{8\pi^2} x_0 \ln x. \]  

Hence \( x G_A(10^{-4}, 2^2 = 10)/A \sim 10^{-80} \) which is about 6 for \( A = 200 \) and should be compared to the current fits which give \( x G_N(10^{-4}, Q^2 = 10) \sim 20 \). Thus eq. (16) implies that at HERA a strong reduction of the gluon densities in heavy nuclei should be observed. The interaction in the gluon sector in the case of heavy nuclei would be close to the black body limit corresponding to a drastic violation of the DGLAP evolution.

3.3. Summary of the HERA 95-96 \( eA \) study

The study performed within the framework of the workshop “Future of HERA” [28] has identified several fundamental questions which can be addressed within the electron-nucleus program: (i) amplification of the nonlinear effects expected in QCD at small \( x \), (ii) the dynamics of high-energy interactions of small color singlet systems with nuclei, (iii) investigation of propagation of quarks and gluons through nuclear matter - energy losses, \( p_t \) broadening, extra hadron production, (iv) connection to the heavy ion physics.

HERA would allow to use three complementary tools: inclusive parton densities - quark and gluon nuclear shadowing, exclusive and inclusive hard diffraction, leading and central hadron production in inelastic \( eA \) collisions. Measurements with a set of nuclei with a luminosity \( 10 \text{pb}^{-1}/A \) per nucleus would allow high precision studies of the dependence of various characteristics of DIS as a function of \( A \) at 100 times smaller \( x \) than in the fixed target experiments (which could study a small fractions of the key observables available for a collider).

Frequent switching between different ions would lead to a strong reduction of the detector related systematic errors for various nuclear ratios. Systematic errors due to the radiative corrections are also small.

In particular, it would be possible to measure the difference the slopes \( d(F_2^A/F_2^D)/d\ln Q^2 \) and \( dF_2^p/d\ln Q^2 \) with accuracy \( \sim 5\% (1\text{pb}^{-1}/A \text{ per nucleus}) \) and measure directly \( G_A/G_N \) with accuracy of better than \( 10\% (10-20\text{pb}^{-1}/A \text{ per nucleus}) \). This would allow to observe \( 10\% \) deviations from the DGLAP predictions.

To achieve the desired luminosity it would be necessary to have luminosity decreasing approximately \( \propto \frac{1}{A} \). With a reasonable capital investment this condition can be almost satisfied for \( A \sim 16 \), while to reach this goal for heavy nuclei it appears necessary to perform cooling of heavy nuclei. In principle, the cooling may be easier for heavy ions than for protons. Its feasibility at HERA is currently is under investigation [28].

The development of the capabilities to accelerate nuclei would make it possible also to perform studies of the neutron parton densities using the deuteron beams and the proton tagging and hence to measure the nonsinglet parton densities. If the scheme for the accelerator of the polarized deuteron could be developed [6] this would allow to measure simultaneously the singlet and nonsinglet \( g_{1N} \) which would be very beneficial for the spin program.

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5 M.S. is indebted to F. Willeke and J. Maidment for discussions of the accelerator related issues.

6 V. Skrinsky, private communication
3.4. Infinite momentum frame approaches to nuclear shadowing

There exist two complementary approaches to descriptions of the small x physics: one is based on the use of the parton frame (fast nucleus) another on the use of the nucleus rest frame.

The fast frame descriptions are convenient for the pQCD analyses, modeling of nonlinear effects. They usually assume that there is no shadowing in the leading twist at $Q_0 \approx 200$ [29, 30, 31, 22, 33].

The rest frame descriptions allows to visualize connection to the soft physics and to the ep diffraction [34, 35, 36, 37, 38, 39, 40, 41, 42, 43].

So far the effects of the higher twist effects were not treated explicitly in these approaches.

In the fast frame nucleon looks as a "MATRESHKA": the fast partons in nucleons are contracted to a pancake of radius $R_A$ and a small longitudinal size $z = 2r_N \frac{m_N}{p_N}$ while the small $x$ partons form pancakes of times larger size ($\langle x_v \rangle \sim 0.2$ is average $x$ of the valence quarks). A nucleus looks as a collection of MATRESHKA's with valence quarks not overlapping and clouds of partons of different nucleons with $x \ll \langle x_v \rangle \frac{Q}{R_A}$ completely overlapping.

One can consider small $x$ parton density per unit transverse area: $\frac{1}{2} \rho_0 R_A x G_N(x,Q^2)$, where $\rho_0 \sim 0.16 fm^{-3}$ is the average nuclear density. At central impact parameters parton density/unit area is 1.5 times larger: $2 \rho_0 R_A x G_N(x,Q^2)$. Hence one finds a large enhancement of the parton densities/area in nuclei as compared to nucleons. For the central impact parameters $G_A(x,Q^2) \approx A^{1/2} \frac{\delta G_A}{\delta x} \approx 6_{A=200}$, if $\frac{Q}{R_A} \approx 1$.

One can study interaction of overlapping tubes of partons in the perturbative strongly interacting phase In the simplest model of Mueller and Qiu [29] nonlinear effects are described by the fan diagrams. The additional nonlinear term contribution $\delta G_A(x,Q^2)$ in the evolution equation for $G_A(x,Q^2)$:

$$Q^2 \frac{\partial}{\partial Q^2} \frac{\delta x G_A(x,Q^2)}{A} =$$

$$-\frac{81}{16} \frac{A^{1/3}}{Q^2 r_0^2} \alpha_s^2(Q^2) \int_x^1 \frac{du}{u} [u G_N(u,Q^2)]^2.$$

For $A \sim 200$ this correction is three times larger than for the proton and would strongly reduce $\frac{\delta G_A}{\delta x}$ at $Q^2 = 4 GeV^2, x \sim 10^{-3} - 10^{-4}, A = 200$ as compared to the DGLAP predictions (Fig.7).

Note however that if this correction is large it certainly cannot be trusted since other effects should become important. Current models of nonlinear effects try to account for multinucleon effects, etc. They involve many simplifying assumptions. The overall trend is an expectation of large gluon shadowing even if starting from the assumption that at a low normalization point the leading twist shadowing is small.

Though this approach is seemingly qualitatively different from the rest frame approach which we started with and which we will discuss in the next subsection these descriptions are in fact dual.

When viewed in the rest frame deep inelastic scattering corresponds to the scattering off a small dipole off the nucleus. The dipole resolves strong gluon fields at the scale $Q \sim \frac{1}{R_A}$ and propagates much longer distances $2R_A$ versus $2r_N$ through the gluon fields leading to amplification of nonlinear effects. Therefore large nonlinear effects are expected based on the space-time picture in both reference frames both in the struc-
ture of the final states and in a very large gluon shadowing which should evolve with $Q^2$ in a way substantially different from the DGLAP expectations.

3.5. The leading twist shadowing for parton densities and hard diffraction in electron-proton scattering

There exists a deep connection between high-energy diffraction and phenomenon of nuclear shadowing [8]. Qualitatively it is due to a possibility of small momentum transfer $-t_{\text{min}} = x_p^2 m_N^2$ where $x_p = x_b (1 + M_{\text{dif}}^2/Q^2)$. If $\sqrt{-t} \leq \text{“average nucleon momentum in D(A)”}$ the amplitudes of diffractive scattering off proton and off neutron interfere. The corresponding double scattering diagram for the $\gamma^* D$ scattering leads to a diffractive final state. The Abramovskii, Gribov, Kancheli theorem [12] then implies that this interference for a deuteron or a light nucleus: (i) increases the diffractive total cross section by $\Delta \sigma_{\text{dif}} = \sigma_2$, (ii) decreases total cross section by $\Delta \sigma_{\text{tot}} = -\sigma_2$, (iii) decreases cross section of inelastic interactions with a single nucleon by $\Delta \sigma_{\text{single}} = -4\sigma_2$, (iv) results in simultaneous inelastic interactions with two nucleons with $\sigma_{\text{double}} = 2\sigma_2$. The sum of partial contributions gives the contribution of the diffraction to the total cross section: $A \sigma_{\text{tot}}(eN) - \sigma_{\text{tot}}(eA) \equiv \Delta \sigma_{\text{tot}} = \Delta \sigma_{\text{dif}} + \Delta \sigma_{\text{single}} + \sigma_{\text{double}}$.

Thus there exists a direct connection between $\sigma_{\text{dif}}(eN)$ at $t \sim 0$ and $\Delta \sigma_{\text{tot}}(eA)$ for light nuclei. Neglecting small corrections due to the real part of the diffractive amplitude one finds:

$$\Delta \sigma_{\text{tot}}^d = \frac{d \sigma_{\text{dif}}(ep)}{8\pi R_D^2} \bigg|_{t=0},$$

where $R_D$ is the deuteron radius.

As we discussed in section 2 the QCD factorization theorem is valid for DIS diffraction: the $Q^2$ evolution of diffractive structure functions at fixed $x_p, p_t$ is described by DGLAP. By comparing the QCD diagrams for hard diffraction and for nuclear shadowing due to scattering off two nucleons (Fig.9) one can prove [27] that in the low thickness limit the leading twist nuclear shadowing is unambiguously expressed through the diffractive parton densities $f^D_j(x_p, Q^2, x, t)$ of ep scattering:

$$f_{j/A}(x, Q^2)/A = f_{j/N}(x, Q^2) - \frac{1}{2} \int d^2 b \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{\infty} dx_{\mathbf{p}} \cdot f_{j/N}(\beta, Q^2, x_{\mathbf{p}}, t)|_{t^2=0} \rho_A(b, z_1)\rho_A(b, z_2) \cdot \cos(x_{\mathbf{p}} m_N(z_1 - z_2)), \quad (20)$$

where $f_{j/A}(x, Q^2), f_{j/N}(x, Q^2)$, are inclusive parton densities; $\rho_A(r)$ is the nucleon density in the nucleus. At not too small $x$ one should add a term related to the longitudinal distances comparable to internucleon distances in nuclei. This additional term can be evaluated using information on enhancement of the gluon and valence quark parton densities at $x \sim 0.1$ in the normalization point as an input. It slightly diminishes nuclear shadowing at higher $Q^2$ via the $Q^2$ evolution. Thus for light nuclei at small enough $x$ the shadowing effect for $1 - f_{j/A}(x, Q^2)/A f_{j/N}(x, Q^2) \propto \sigma_{\text{eff}}^g(x, Q^2) \equiv 16\pi d \sigma_{\text{dif}}/dt|_{t=0}/\sigma_{\text{tot}}$. As we discussed before, the analyses of the HERA hard diffractive data indicate that $\sigma_{\text{eff}}^g(x \leq 10^{-3}, Q = 2) \sim 50 mb$, $\sigma_{\text{eff}}^g(x, Q = 2) \sim 3.5$.

If one follows indications from HERA that in the $Q^2 \geq 4 \text{ GeV}^2$ range the leading twist already dominates in diffraction (this is well established for quarks, but may not be a good approximation for the gluons (see discussion in section 2.2) one predicts: (i) a significant shadowing in the quark channel with small theoretical uncertainty, (ii) a much larger (a factor of 3 for light nuclei) shadowing in the gluon channel subject to the question of the higher twist effects (Fig.8).

Note that in many models of nuclear shadowing which did not incorporate information about the gluon induced diffraction it was expected that the shadowing in the gluon channel would be smaller than in the quark channel.

The Gribov theory agrees well with the high precision NMC data if one uses the HERA diffractive data (this involves a certain extrapolation of the HERA data to smaller $W$) [43]. A study of inclusive parton densities at the HERA $x, Q^2$ range would (i) establish shadowing in the quark channel, and check connection between diffraction and
Figure 8. Diagrams for hard diffraction in $ep$ scattering and for the leading twist nuclear shadowing

Figure 9. The dependence of $G_A/AG_N$ and $F_{2A}/AF_{2N}$ on $x$ for $Q=2$ (dashed), 5 (dotted), 10 GeV (solid) curves calculated in the quasieikonal model using diffractive parton densities of [14].
shadowing starting from nonperturbative domain of $Q^2 \sim 0$ where shadowing strongly depends on $Q^2$, (ii) find out whether interactions in the gluon channel are superstrong ($\sim 50$ mb) at HERA for intermediate $Q \sim 2 - 3$ GeV, (iii) study coherent interactions of $\gamma^*$ with three, four, ... nucleons in the scattering off heavy nuclei. Such interactions are sensitive to the fluctuations of the interaction strength in the quark and gluon channel and check the role of weakly interacting components - this information cannot be obtained from $ep$ scattering. For $A=200$ fluctuations may reduce the gluon shadowing at $x = 10^{-3} - 10^{-4}$ by a factor up to $\sim 1.5 \ [27]$. Hence the study of shadowing for $A \geq 40$ would provide a complementary information to the case of light nuclei; (iv) verify connection of diffraction and shadowing via investigation of the $Q^2$ dependence of shadowing between $Q^2 = 0$ and 4 GeV$^2$ which should be strong due to a strong $Q^2$ dependence of the diffraction contribution to $\sigma_{\gamma \to N}$.

3.6. Implications for the heavy ion collisions at LHC

The production of minijets is often considered as an effective mechanism of producing high densities in the head on heavy ion collisions. For the central rapidities minijets are produced due to collisions of partons with $x_{jet} = \frac{2p_t}{\sqrt{s_{NN}}}$ For heavy ion collisions at LHC $\sqrt{s_{NN}} \sim 4$ TeV and the gluon-gluon collisions are responsible for production of most of the minijets. Therefore the gluon nuclear shadowing would lead to a reduction of the rate of the jet production due to the leading twist mechanism by a large factor up to $p_t \sim 10 GeV/c$, see Fig.11 where we give results of calculation in the quasieikonal and fluctuation models of shadowing. The nuclear gluon shadowing leads to a similar very strong reduction of the heavy onium production in $pA$ and $AA$ collisions at LHC energies for $y_{c.m.} \sim 0$ and small $p_t$. However nuclear shadowing leads to other contributions to the jet production not included in the factorization models - fragmentation of the spectator partons belonging to the dipoles, decay of the ladders involved in the multiple interactions with the dipoles. Hence the overall effect could be a smaller suppression and additional emission of soft particles over a large rapidity range. Experimental studies of $eA$ collisions would be of great help in quantifying these effects.

3.7. Study of diffraction with nuclei
3.7.1. Exclusive channels: \( M = \rho, J/\psi, \pi\pi, ... \)

The QCD factorization theorem for exclusive processes \([10]\) leads to [1]

\[
\frac{d\sigma}{dt}(\gamma^* A \to MA)|_{t=0} = \frac{d\sigma}{dt}(\gamma^* N \to MN)|_{t=0}
\]

\[
= \left[ \frac{F_2^A(x,Q)}{F_2^N(x,Q)} \right]^2 = G_A^2(x,Q) G_N^2(x,Q)
\]

(21)

Hence in the region \( x \geq 0.01 \) the color transparency regime of weak absorption is expected. Establishing down to what \( Q^2 \) this regime would hold would allow to determine down to what virtualities production of vector mesons could be used as a low \( Q^2 \) gluonometor.

At smaller \( x \) the gluon shadowing would lead to a dramatic decrease of the yield of vector meson production at small \( x \) (yields are still large to measure the cross section accurately) - the color opacity phenomenon - strong absorption of small color dipoles by nuclei at ultra-high energies. This phenomenon is a qualitative departure from the experience of QED where cross section of interaction of small electric dipoles practically does not depend on the incident energy. The predicted \( A \)-dependence for coherent \( J/\psi \) production is about the same as for photoproduction of \( \rho \)-mesons!?

3.7.2. Inclusive and semiinclusive channels:

Prime objectives of the study of the inclusive diffraction is to obtain a direct answer to the question how “black” are diffractive interactions. We expect an increase of the rapidity gap probability with \( A \). For the heavy nuclei the fraction of events where a nucleus remains intact would reach \( \sim 30\% \) for the quark induced hard diffraction and nearly \( 50\% \) for the gluon induced diffraction \([11, 27]\). Also, the rapidity gap probability would depend on \( p_t \) of the jet is a rather peculiar way - contrary to naive expectations it would increase between the aligned jet kinematics and the \( p_t \) range where gluons give dominant contribution.

Overall the study of inelastic diffraction would: provide a direct test of the strengths of interaction in the quark and gluon sector - identify the channels for which interaction is close to black for heavy nuclei. This information will be complementary to the study of the \( A \) dependence of \( G_A/G_N \) and allow to clarify the origin of gluon component of the “Pomeron”.

3.8. Nondiffractive hadron production

Numerous data on hadron-nucleus scattering at fixed target energies indicate that the multiplicities of the leading hadrons \( N_A(z) \equiv \frac{1}{\sigma_{tot}[aA]} \int d\sigma(x) z A \rightarrow h + X \) decrease with increase of \( A \). Here \( z \) is the light-cone fraction of the projectile "a" momentum carried by the hadron "h".

On the contrary, the QCD factorization theorem for the inclusive hadron production in DIS implies that in the case of electron-nucleus scattering no such dependence should be present in the DIS case. This indicates that there should be an interesting transition from the soft physics dominating in the interactions of real photons with nuclei to the hard physics in the inclusive hadron production in the DIS kinematics. It would be manifested in the disappearance of the \( A \)-dependence of the leading spectra at large \( z \):

\[ N_A(z,Q^2) = N_N(z,Q^2), \text{for } z \geq 0.2, Q^2 \geq \text{ few GeV}^2. \]

At the same time the transverse spectra are expected to be broadened for nuclei \( (\Delta p_t^2 \propto A^{1/3}) \) due to the QCD analog the Landau-Migdal-Pomeranchuk effect, see discussion and references in \([44]\).

At small \( x \) a new interesting phenomenon should emerge for smaller \( z \) due to the presence of diffraction and nuclear shadowing. Indeed, the diffraction originates from the presence in the wave function of \( \gamma^* \) of partons with relatively small virtualities which screen the color of the leading parton(partons) with large virtuality and can rescatter elastically from a target (several target nucleons in the case of nuclear target). Inelastic interactions of these soft partons with several nucleons should lead to a plenty of new revealing phenomena in small \( x \) DIS eA scattering, which resemble hadron-nucleus scattering but with a shift in rapidity from \( y_{max}(current) \) related to the average rapidities of these soft partons. This shift can be expressed through the average masses of the hadron states produced in
the diffraction:

\[ y_{\text{soft partons}} \sim y_{\text{max}} - \ln\left(\frac{\langle M_{\text{dif}}^2 \rangle}{\mu^2}\right), \quad (22) \]

where \( \mu \sim 1\text{GeV} \) is the soft scale. Partons with these rapidities will interact in multiple collisions and lose their energy leading to a dip in the ratio \( \eta_A(y) \equiv N_A(y)/N_p(y) \) (Fig.12). At the same time these multiple interactions should generate a larger multiplicities at smaller rapidities. Application of the AGK rules indicates that for \( y \leq y_{\text{soft partons}} - \Delta \), where \( \Delta = 2 - 3 \) the hadron multiplicity in the case of nuclei will be enhanced by the factor: \( \eta_A(y) = AF_{2p}(x, Q^2)/F_{2A}(x, Q^2) \). At the rapidities close to the nuclear rapidities a further increase of \( \eta_A(y) \) is possible due to formation of hadrons inside the nucleus.

One also expects a number of phenomena due to long range correlations in rapidity. This includes: (a) local fluctuations of multiplicity in the central rapidity region, e.g. the observation of a broader distribution of the number of particles per unit rapidity, due to fluctuations of the number of wounded nucleons [1]. These fluctuations should be larger for the hard processes induced by gluons, for example the direct photon production of two high \( p_t \) dijets. (b) Correlation of the central multiplicity with the multiplicity of neutrons in the forward neutron detector, etc.

To summarize, the small \( x \) eA physics is one of the last unexplored frontiers of QCD. A plenty of new nonlinear QCD phenomena are to be discovered in these studies: the gluon nuclear shadowing, the high-energy color transparency, the perturbative color opacity, ... **These are just expected discoveries - what about unexpected ones?** Studies of eA collisions would provide decisive tests of the interface of pQCD and soft physics, allow to reach understanding of the space-time picture of the high-energy strong interactions. Important applications of these studies include the measurement of the nonsinglet parton densities in eD scattering, as well as numerous connections to the RHIC and LHC heavy ion physics.

HERA has produced exciting glimpses of new small \( x \) strong interaction gluon dynamics. The aim now is to produce a definitive proof and observe high density hard QCD.

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