The $N_t = 4$ finite temperature phase transition for lattice QCD with a weak chiral 4-fermion interaction.

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Abstract

We study the finite temperature phase transition of lattice QCD with an irrelevant chiral 4-fermion interaction and two massless quark flavours, on $8^3 \times 4$ and $12^2 \times 24 \times 4$ lattices. The strength of the 4-fermion interaction was reduced to half the minimum value used in previous simulations, to study how the nature of this phase transition depends on this additional interaction. We find that the transition remains first order as the 4-fermion coupling is reduced. Extending our earlier studies indicates that for sufficiently large 4-fermion coupling, the transition is probably second order.
I. INTRODUCTION

In a recent series of papers \[1, 2\], we have used a new staggered fermion action modified to include an irrelevant 4-fermion interaction which preserves the flavour symmetries of the standard staggered action, to study the chiral symmetry restoring finite temperature phase transition of QCD. (For earlier work by others proposing similar actions see \[3, 4\].) In particular, we study the transition with two massless quark flavours. Such simulations are impossible with the standard action, since the Dirac operator becomes singular at zero quark mass. On lattices with temporal extent \(N_t = 4\), the transition appeared to be first order \(1\), while at \(N_t = 6\) it appeared to be second order as expected \(5\), but with the critical indices of a tricritical point \(2\), rather than the \(O(4)/O(2)\) indices expected from universality arguments. We interpreted these results to indicate that at these large lattice spacings \(a\), the additional interactions due to discretization, which vanish as \(a^2\), are large enough to affect the nature of the transition. For this reason, we are currently performing simulations at \(N_t = 8\). One such interaction is our additional 4-fermion interaction, so we need to vary its coefficient in an attempt to determine how important it is in determining the nature of the phase transition. At \(N_t = 6\) we ran at 2 different (small) values of this coupling and concluded that although the \(\beta_c = 6/g_c^2\) of the transition changed as this coupling was varied, its nature did not. In this paper we present the results of studies of the dependence of the \(N_t = 4\) transition on the 4-fermion coupling.

The action we use is the staggered lattice version of the continuum Euclidean Lagrangian density,

\[
\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathcal{D} + m)\psi - \frac{\lambda^2}{6N_f}[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau_3\psi)^2]. \tag{1}
\]

For details of the staggered lattice transcription of this Lagrangian, using auxiliary fields to render it quadratic in the fermion fields, thus allowing simulations, we refer the reader to our earlier papers. The largest value of \(\gamma = 3/\lambda^2\) used in our previous simulations at \(N_t = 4\), and the only value for which we determined the nature of the transition, was \(\gamma = 10\). We present the results of new simulations at \(\gamma = 20\) on \(8^3 \times 4\) and \(12^2 \times 24 \times 4\) lattices, which indicate that the transition remains first order. We have also extended the studies of the \(\gamma = 5\) transition reported in our earlier work. These new studies show no sign of the meta-stability observed at \(\gamma = 10, 20\), suggesting that the \(\gamma = 5\) transition is second order.
In section 2 we present the results of these simulations. Section 3 gives our conclusions.

II. SIMULATIONS AT $N_t = 4$, $N_f = 2$ AND $\gamma = 20, 5$.

We performed simulations using the hybrid molecular dynamics method with noisy fermions allowing us to tune to 2 flavours. We ran on $8^3 \times 4$ and $12^2 \times 24 \times 4$ lattices to observe finite size effects. On both lattices our molecular dynamics time increment was chosen to be $dt = 0.05$, which appeared adequate. Our quark mass was set to zero and $\gamma = 20$.

For the $8^3 \times 4$ simulations we ran for 20,000 molecular dynamics time units for $\beta = 5.27$ and 7 $\beta$ values in the range $5.285 \leq \beta \leq 5.3$, and 10,000 time units for 3 $\beta$s outside this range. We observed evidence for metastability with 2-state signals both in the chiral condensate and in the Wilson line (Polyakov loop) for $5.285 \lesssim \beta \lesssim 5.3$, with the clearest signals at $\beta = 5.29$ and $\beta = 5.295$. A histogram of the chiral condensate at $\beta = 5.29$ is shown in figure 1 showing two clearly separated peaks. From this we would conclude that there is a first order transition. We estimate the transition to occur at $\beta = \beta_c = 5.2925(50)$.

For our $12^2 \times 24 \times 4$ simulations, we ran for 50,000 time units at $\beta = 5.2875$, $\beta = 5.289$ and $\beta = 5.29$, close to the transition, and for shorter ‘times’ at 8 other $\beta$ values in the range $5.265 \leq \beta \leq 5.35$. We observed clear signals for metastability with a well defined 2-state signal at $\beta = 5.289$ and $\beta = 5.29$, but not outside this region, clear evidence that there is a first order transition at $\beta_c = 5.289(1)$. The time evolution of the chiral condensate at $\beta = 5.289$ is shown in figure 2. A histogram showing the two-state signal in the chiral condensate at $\beta = 5.289$ is shown in figure 3. The separation of the peaks in this histogram is $\approx 0.56$, while that on the $\beta = 5.29$ histogram on the $8^3 \times 4$ lattice is $\approx 0.61$. The decrease in going to the larger lattice is small, leading us to the conclusion that the first order transition is real, and not a small lattice artifact. For the Wilson line, the peak separation is $\approx 0.22$ for the larger lattice and $\approx 0.32$ for the smaller lattice. Most of this change comes from a rise in the value of the Wilson line for the low temperature phase, and might well be a fermion screening effect on the larger lattice.

Finally, figure 4 shows the two order parameters for the $12^2 \times 24 \times 4$ lattice simulations.
FIG. 1: Histogram of $\langle \sqrt{\langle \bar{\psi}\psi \rangle^2 - \langle \bar{\psi}\gamma_5\xi\psi \rangle^2} \rangle$ measurements spaced at 2 time-unit intervals on an $8^3 \times 4$ lattice at $\beta = 5.29$.

as functions of $\beta$, showing how abrupt the transition is.

Since our earlier studies at stronger 4-fermion couplings were inadequate to determine the nature of the phase transition, we have extended the $\gamma = 5$ simulations to $12^2 \times 24 \times 4$ lattices and added some $\beta$ values in the crossover region to our $8^3 \times 4$ simulations. For the larger lattice we ran for 50,000 time units for each $\beta$ in the range $5.415 \leq \beta \leq 5.440$. Figure 5 shows the chiral condensate and Wilson line for the $12^2 \times 24 \times 4$ lattice. The transition appears smooth, and the time dependence of these order parameters shows no
FIG. 2: Time evolution of $\langle \sqrt{\langle \bar{\psi} \psi \rangle^2 - \langle \bar{\psi} \gamma_5 \xi_5 \psi \rangle^2} \rangle$ on a $12^2 \times 24 \times 4$ lattice at $\beta = 5.289$.

sign of metastability for any of these $\beta$ values. This suggests that the $\gamma = 5$ chiral transition is second order and occurs at $\beta = 5.425(5)$

III. CONCLUSIONS

We have simulated lattice QCD with 2 flavours of massless staggered quarks and an additional chiral 4-fermion term on $N_t = 4$ lattices. The chiral finite temperature transition at weaker 4-fermion coupling ($\gamma = 20$) appears to be first order, as it was at stronger 4-fermion coupling ($\gamma = 10$). The chiral condensate in the confined phase at the transition
FIG. 3: Histogram of $\langle \sqrt{\langle \bar{\psi}\psi \rangle^2 - \langle \bar{\psi}\gamma_5\xi\psi \rangle^2} \rangle$ on a $12^2 \times 24 \times 4$ lattice at $\beta = 5.289$.

was 0.60(1) at $\gamma = 20$, compared with 0.76(1) for $\gamma = 10$ (in the deconfined phase this condensate would vanish). The change in the Wilson line across the transition is 0.23(1) for $\gamma = 20$, compared with 0.33(1) at $\gamma = 10$. A priori, this softening of the transition as the 4-fermion coupling weakens might suggest that it would become second order at vanishing 4-fermion coupling. However, we see that at stronger coupling, $\gamma = 5$, the transition is even softer and probably second order, so the situation is not so simple.

We suggest that the order of the transition is determined primarily by the ‘standard’ ($\gamma$ independent) part of the action, and hence by $\beta$. On $N_t = 4$ lattices, $\beta_c = 5.289(1)$ at
FIG. 4: Chiral condensate and Wilson line on a $12^2 \times 24 \times 4$ lattice as functions of $\beta$ at $\gamma = 20$.

$\gamma = 20$ compared with $\beta_c = 5.327(2)$ at $\gamma = 10$ and $5.425(5)$ at $\gamma = 5$. At $\gamma = 10, 20$ the lower values of $\beta_c$ mean larger lattice artifacts, which we suggest make the transition first order. The larger $\beta_c$ at $\gamma = 5$, has smaller lattice artifacts, which we suggest leave it second order. It is interesting to note that $\beta_c$ at $\gamma = 5$ and $N_t = 4$ is very close to $\beta_c$ at $\gamma = 20$ for $N_t = 6$, which we have determined to be second order (tricritical), which is consistent with this scenario. The reason the discontinuity in the $\gamma = 10$ chiral condensate is greater than that at $\gamma = 20$ is that the stronger 4-fermion coupling enhances condensate formation. The larger $\beta$ of the $\gamma = 10$ transition increases the Wilson line just above the transition over the corresponding $\gamma = 20$ value. Such a scenario suggests that at zero 4-fermion coupling $\gamma = \infty$, the transition would remain first order. As was noted in reference [6], the presence of
FIG. 5: Chiral condensate and Wilson line on a $12^2 \times 24 \times 4$ lattice as functions of $\beta$ at $\gamma = 5$. This first order transition could explain the failure of universal critical scaling at the $N_t = 4$ finite temperature transition for lattice QCD with 2 staggered fermion flavours as $m \to 0$.

Finally, we should comment that, because this 4-fermion term also breaks the same flavour symmetries as the standard staggered action, it has the potential to significantly increase this breaking and make improving this action more difficult. We are planning hadron spectroscopy calculations to determine how bad this breaking might be. However, this additional $\mathcal{O}(a^2)$ symmetry breaking could be reduced to $\mathcal{O}(a^4)$ by addition of the terms required to make this 4-fermion interaction that of the staggered lattice implementation of the $SU(4) \times SU(4)$ or $(SU(2) \times SU(2)) \times (SU(2) \times SU(2))$ Gross-Neveu model. The
only problem is that such an action has a complex fermion determinant, essential to the physics of interest. However, changing some of the couplings from imaginary to real removes this difficulty, and it might be possible to analytically continue results from real values to imaginary values of these couplings, as has been suggested for similar terms in the highly improved staggered actions proposed by the HPQCD collaboration [11]. We note that the 4-fermion term in our action, and at least some of the additional 4-fermion terms which we are suggesting, are present in the HPQCD action after a Fierz transformation.

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References

[1] J. B. Kogut, J.-F. Lagaé and D. K. Sinclair, Phys. Rev. D58, 034504 (1998).
[2] J. B. Kogut and D. K. Sinclair, Phys. Lett. B492, 228 (2000); J. B. Kogut and D. K. Sinclair, Phys. Rev. D64, 034508 (2001).
[3] K. I. Kondo, H. Mino and K. Yamawaki, Phys. Rev. D39, 2430 (1989).
[4] R. C. Brower, Y. Shen and C.-I. Tan, Boston University preprint BUHEP-94-3 (1994);
R. C. Brower, K. Orginos and C.-I. Tan, Nucl. Phys. B(Proc. Suppl.) 42, 42 (1995).
[5] R. Pisarski and F. Wilczek, Phys. Rev. D29, 338 (1984).
[6] C. Bernard et al. (MILC collaboration) Nucl. Phys. B(Proc. Suppl.) 63A-C, 400 (1998); Phys. Rev. D61, 054503 (2000).
[7] G. Boyd with F. Karsch, E. Laermann and M. Oevers, Proceedings of 10th International Conference Problems of Quantum Field Theory, Alushta, Crimea, Ukraine (1996).
[8] S. Aoki, et al. (JLQCD Collaboration), Phys. Rev. D57, 3910 (1998).
[9] E. Laermann, Nucl. Phys. B(Proc. Suppl.) 63A-C, 114 (1998).
[10] D. J. Gross and A. Neveu, Phys. Rev. D10, 3235 (1974).
[11] Q. Mason, et al, [hep-lat/0209152] (2002).