The strength of crystalline color superconductors

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Outline of the talk

- Motivations
- Color Superconductors
- Stressing the Superconductor
- Crystalline phases
- Conclusion and Outlook
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Reviews: hep-ph/0011333, hep-ph/0102047, hep-ph/0509068, hep-ph/0202037 (HDET)
In studying extremely dense matter one has to rely on (QCD-inspired) phenomenological approaches and it is difficult to make numerical predictions.

**Aiming at stars:** finding signatures of the existence of deconfined quark matter in compact stars. If matter is deconfined it will also be color superconducting.
Motivations

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Isolated neutron stars lose angular momentum mainly through dipole radiation (and particles wind). Glitches are sudden speed-up in the rotational frequency.

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1. They occur without warning
2. Large variety of time scales
3. Statistically any pulsar glitches
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**One needs:**
- Superfluid vortices
- A rigid structure where vortices are pinned
Nambu-Jona-Lasinio model

Free Lagrangian

\[ \mathcal{L}_0 = \bar{\psi} (i \not{\! \! \! \partial} + \mu \gamma_0) \psi \]

Where

\[ \mu_{ij}^{\alpha \beta} = (\mu_b \delta_{ij} - \mu_Q Q_{ij}) \delta^{\alpha \beta} + \delta_{ij} \left( \mu_3 T_3^{\alpha \beta} + \frac{2}{\sqrt{3}} \mu_8 T_8^{\alpha \beta} \right) \]

is the chemical potential matrix \( i,j=1,2,3 \) flavor, \( \alpha, \beta=1,2,3 \) color indices.

Color and electrical chemical potentials are introduced by hand. In QCD the gauge fields drive the system to a neutral state.
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Local Fermi interaction

\[ \mathcal{L}_I = G \bar{\psi}(x) \Gamma \bar{\psi}(x) \psi(x) \Gamma \psi(x) \]
SUPERCONDUCTOR

- **Cooper theorem**: At high density and sufficiently low temperature any arbitrarily weak interaction leads to Cooper pairs.
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**BCS-like superconductor**

- Breaking of gauge symmetries and magnetic field is expelled: *Meissner effect*

- Fermionic excitation spectrum is gapped.
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**Color superconductor**

- The interaction in the **color 3 channel** is attractive
- Formation of a $<qq>$ condensate with total momentum 0 and total spin 0
- No need to resort to phonons: Color Superconductivity is driven by the color interaction and is a robust phenomenon. Large gap $\Delta \sim 10 - 100$ MeV
Stressing the superconductor

\[ \mu_u = \mu - \delta \mu \quad \mu_d = \mu + \delta \mu \]
Stressing the superconductor

Clogstone limit: for $\delta \mu > \delta \mu_c$ the BCS-like superconductors is not energetically favoured.

The cost $\sim \delta \mu$ of promoting fermions to the average energy level $\mu$ is bigger than the energy gained in the pairing $\sim \Delta$. 

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$$p^F_d = p^F_u + \mu_e \quad p^F_u = \mu - \frac{2}{3}\mu_e \quad p^F_s = p^F_u - \mu_e$$
Larkin-Ovchinnikov and Fulde-Ferrel (LOFF) for $\delta \mu_c < \delta \mu < \delta \mu_2$ ($\delta \mu_c \approx \Delta_0/\sqrt{2}$, $\delta \mu_2 \approx 0.754\Delta_0$) **anisotropic phase** is favored with Cooper pairs of total momentum $2q$.

Therefore the LOFF phase corresponds to a **non-homogeneous superconductor**, with a spatially modulated condensate in the spin 0 channel.
We can generalize the LOFF condensate as a sum of plane waves

\[ \langle \psi_{\alpha_i}(x) C\gamma_5 \psi_{\beta_j}(x) \rangle \propto \sum_{I=1}^{3} \Delta_I \sum_{q^a_I \in \{q^a_I\}} e^{2i q^a_I \cdot r} \epsilon_{I\alpha\beta} \epsilon_{Iij} \]

For each value of I the set of vectors \( \{q^a_I\} \) with \( a = 1, n \), describes a crystalline structure. Therefore we have 3 interacting crystalline structures.
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**Simplifying assumptions**: Near the transition point we assume that the Fermi momenta in the broken phase are equal to the Fermi momenta in the normal phase.

This means that we can take $\Delta_2 = \Delta_3 = \Delta$ and $\Delta_1 = 0$. This reduces the problem to 2 interacting crystals.
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Two plane waves

- Condensate determined by 2 vectors $q_2$ and $q_3$.
- $|q_2| = |q_3| = q$ fixed,
- Free parameters are $\Delta$ and $\cos \varphi = \hat{q}_2 \cdot \hat{q}_3$
- This case can be solved without employing the Ginzburg-Landau expansion.

$$\psi_i(x) \rightarrow e^{i\mathbf{k_i} \cdot \mathbf{x}} \psi_i(x)$$
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The interaction Lagrangian becomes

$$\mathcal{L}_\Delta = \Delta e^{2i(q_2+k_1+k_3) \cdot x} \psi_1 \psi_3 + \Delta e^{2i(q_3+k_1+k_2) \cdot x} \psi_1 \psi_2$$

taking

- $2q_2 = k_1 + k_3$
- $2q_3 = k_1 + k_2$

it becomes space independent
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For $\varphi = 0$

Taking $2q_2 = k_1 + k_3$
$2q_3 = k_1 + k_2$

it becomes space independent

$u(-v)$ $d(v)$ $s(v)$ $k_1 = 0$ $k_2 = 2q$ $k_3 = 2q$
$k_1 = 2q$ $k_2 = 0$ $k_3 = 0$
In the HDET approximation the free-energy is

\[ \Omega = -\frac{\mu^2}{4\pi^2} \sum_{a=1,18} \int_{-\delta}^{+\delta} dl \int \frac{dv}{4\pi} \left| E_a(v, l) \right| + \frac{2\Delta^2}{G} - \frac{\mu_e^4}{12\pi^2} \]

Gap and free-energy in the ground state
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\[
\Omega = -\frac{\mu^2}{4\pi^2} \sum_{a=1,18} \int_{-\delta}^{+\delta} dl_{\parallel} \frac{dv}{4\pi} |E_a(v, l_{\parallel})| + \frac{2\Delta^2}{G} - \frac{\mu_e^4}{12\pi^2}
\]

Gap and free-energy in the ground state

The **favored structure** corresponds to **small angles between q_1 and q_2**. This is an important result because it gives the recipe for more complicated structures: **avoid overlapping between pairing regions**.
Crystalline structures

Using this recipe more complicated crystal structures can be built: K. Rajagopal and R. Sharma hep-ph/0605316

CX-structure

2cube45z-structure

♦ These crystalline structures have been determined by minimizing the free energy of the system in a Ginzburg-Landau approximation.
♦ Can they provide the rigid structure we need for the glitches?
What is the strength of these crystal structures? How they behave under stress?

The shear modulus $\nu$ describes the response of a crystal to an external stress.

$$\nu_{ij}^I = \frac{\sigma_{ij}^I}{2s_{ij}^I} \quad \text{for } i \neq j$$

$\sigma_{ij}^I$ are the components of the stress tensor acting on the crystal $I$

$s_{ij}^I$ are the components of the strain matrix of the crystal $I$
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To evaluate $\nu$ one studies the fluctuations of the crystal structures. **Phonons** fields $u_I(x)$ are position and time dependent fluctuation of the condensate

$$\Delta_I(r) \rightarrow \Delta_I(r)e^{2iq^I \cdot u_I(x)}$$
Vertices

\[ \Delta_I \]

\[ \epsilon_{I\alpha\beta} \]

\[ \phi_I \frac{f_I}{f} \]

\[ 2q_I \]

\[ \alpha, i \]

\[ \Delta_I \]

\[ \epsilon_{I\alpha\beta} \]

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\[ 2q_I \]

\[ \alpha, i \]

\[ \beta, j \]
Vertices

\[ \Delta_I \]

\[ \epsilon_{I \alpha \beta} \epsilon_{I \alpha} \]

\[ \phi_I \]

\[ \frac{1}{f_I} \]

\[ \alpha, i \rightarrow \beta, j \]

\[ 2q_I \]

Self-energy

\[ \Delta_I \]

\[ \frac{1}{p + q_I + \mu_j + \mu_k} \]

\[ \epsilon^{I \alpha \beta} \epsilon^{I \alpha \beta} \]

\[ 2q_I \]

\[ \phi_I \]

\[ \frac{1}{f_I} \]

\[ \alpha, i \rightarrow \beta, j \]

\[ 2q_I \]

\[ \Delta_I \]

\[ \frac{1}{p - q_I + \mu_j + \mu_k} \]

\[ \phi_I \]

\[ \frac{1}{f_I} \]

\[ \alpha, i \rightarrow \beta, j \]

\[ 2q_I \]

\[ \Delta_I \]

\[ \frac{1}{p - q_I - \mu_k} \]

\[ \phi_I \]

\[ \frac{1}{f_I} \]

\[ \alpha, i \rightarrow \beta, j \]

\[ 2q_I \]

\[ \Delta_I \]

\[ \frac{1}{p + q_I + \mu_j + \mu_k} \]

\[ \phi_I \]

\[ \frac{1}{f_I} \]

\[ \alpha, i \rightarrow \beta, j \]
Here $2q^a_I \cdot u_I = \phi^a_I$. Integrating out the fermionic fields we get the effective action for the phonon fields at the order $u^2$

$$S[u] \sim \int d^4x \sum_I \mu^2 |\Delta_I|^2 \sum_{a} \left[ \partial_0(\hat{q}^a_I \cdot u_I)\partial_0(\hat{q}^a_I \cdot u_I) - (\hat{q}^a_I \cdot \vec{d})(\hat{q}^a_I \cdot u_I)(\hat{q}^a_I \cdot \vec{d})(\hat{q}^a_I \cdot u_I) \right]$$
We find $\nu \equiv \text{stress/strain} \sim \mu^2 \Delta^2 \gg \nu_{\text{Iron}}$ and the crystalline LOFF phase behaves as a very rigid solid. Since a global U(1) is spontaneously broken it is a solid and a superfluid!!
Summary

* A large mismatch between Fermi momenta leads to the breakdown of the BCS phase

* Gapless homogeneous phases (at least in weak coupling) are ruled out

* The LOFF phase is a candidate in this regime

* This phase shows a remarkable property: it has a **rigid structure and is a the same time superfluid**

* Connection with glitches? May (some of) them originate in the core of neutron star?
Pairing regions

Graphical representation of the region where pairing is stronger

hep-ph/0603076

In the HDET approximation the gap equation can be written as

\[ \Delta \propto \int_{-\delta}^{+\delta} dl_{\parallel} \int d\nu f(\nu, l_{\parallel}) \]

where

\[ f(\nu, l_{\parallel}) = \sum_{a=1}^{n} \frac{\partial E_a}{\partial \Delta} \text{Sign}(E_a) \]

contour plot of this function:

\[ k_1=0, \ k_2=k_3=2q \]

\[ k_1=2q, \ k_2=k_3=0 \]
The ideal environment: Neutron stars

- **atmosphere**
- **outer crust**: lattice of ions
- **inner crust**: neutron rich nuclei, electrons and superfluid neutrons
- **outer core**: superconducting protons, electrons and superfluid neutrons
- **inner core**: ?

10-15 Km

$M \sim 1.4 \, M_{\text{sun}}$
Neutron stars are observed as rotating objects known as pulsars. The range of frequency of rotation is milliseconds to hundreds of seconds. Firstly observed in 1967, by Jocelyn Bell and Anthony Hewish.
Neutron star as lighthouse

Pulsar Geometry

Relativistic Outflow of Charged Particles on Open Field Lines

Magnetic Field Line

Corotating Plasma on Closed Field Lines

Rotating, Magnetized Neutron Star

Pencil Beam Radiation Pattern (fan beam also possible)

Pulsed X-Rays from Neutron Star in Crab Nebula (30 pulses per second)

Light Cylinder

Magnetic Axis

Rotation Axis

Off Pulse

On Pulse

EPN arxiv

Crab

33ms

Vela

89ms

B0329+54

0.7145s
Glitch mechanism I

Sudden angular momentum change due to variation of the inertial momentum
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Starquake model (Ruderman ‘69)
Glitch mechanism I

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As the star slows down the gravitational stress abruptly decreases the momentum of inertia breaking the crust. Because of the “ballerina” effect the star speeds-up
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Quite appealing, because simple and general. However, cannot account for the large and frequent glitches of the Vela-like stars and for the slow spin-up of the Crab.

It is not yet completely ruled out if core-quakes are allowed.
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One needs: A solid-like structure
Glitch mechanism II

Sudden angular momentum transfer to the crust from the neutron superfluid of the inner crust
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Sudden angular momentum transfer to the crust from the neutron superfluid of the inner crust

When a superfluid is put in rotation it is thread by superfluid vortices

A superfluid can reduce its angular momentum only by diluting the vortex density

If vortices are pinned to some external structure they cannot dilute and the velocity of the superfluid remains unchanged
Glitch mechanism II

Sudden angular momentum transfer to the crust from the neutron superfluid of the inner crust

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A superfluid can reduce its angular momentum only by diluting the vortex density

If vortices are pinned to some external structure they cannot dilute and the velocity of the superfluid remains unchanged

Superfluid neutron vortices are pinned to the lattice of ions: the angular momentum of the superfluid cannot change!

In a glitch the superfluid transfers the angular momentum to the crust

One needs:
- Superfluid vortices
- A rigid structure where vortices are pinned
**Motivations**

**Phases of matter**
- **H**: Hadronic phase
- **QGP**: Quark-Gluon Plasma
- **CSC**: Color Superconductor

**Diagram**
- **T** (temperature) axis
- **ρ** (density) axis
- Neutron stars
- QGP
- CSC
At high densities one has to rely on phenomenological approaches and it is difficult to make numerical predictions.

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