1. INTRODUCTION

Neutrinos from cosmic ray interactions in the atmosphere and from solar fusion processes have already opened a new era of neutrino physics as we study the masses and mixing parameters of neutrinos [1]. Much higher energy neutrinos hold the promise to reveal aspects of particle physics, but also to help us understand mechanisms for cosmic acceleration and the environment in astrophysical sources where high energy cosmic rays are produced [2].

Neutrinos are unique in their properties of being undeflected by magnetic fields, and for traversing astronomical distances unabsorbed. Large under-water and under-ice experiments using Cherenkov radiation are dedicated to detecting neutrinos from astrophysical sources, and other experiments such as air shower arrays and air fluorescence detectors have the possibility to detect neutrinos via horizontal air showers. Combined, these experiments probe sources of neutrinos over a phenomenal range of energies, potentially up to the highest energies seen in cosmic ray experiments.

An essential ingredient for the interpretation of neutrino results is the neutrino-nucleon cross section. We review here the status of the neutrino-nucleon cross section up to $E_{\nu} = 10^{12}$ GeV. Experimental results from $e p$ scattering at HERA make the standard model neutrino-nucleon cross section very well understood up to approximately $10^7$ GeV. At higher energies, extrapolations of the parton distribution functions (PDFs) make for some uncertainties in the theoretical predictions. We review the different approaches to the high energy extrapolations.

At the highest energies, one considers the possibility that non-standard model physics may play a role in neutrino interactions. The contribution of mini-black holes to the neutrino-nucleon cross section is briefly discussed as an example of how non-standard model physics comes into play at high energies. In addition, there have been proposals that standard model electroweak instantons may contribute significantly to the ultrahigh energy neutrino cross section. These additions to the standard model (perturbative) cross section are discussed in the fourth section.

2. STANDARD MODEL FORMALISM

The differential cross section for neutrino scattering with an isoscalar nucleon $N$

$$\nu_{\mu}(k) N(p) \rightarrow \mu(k') X ,$$

written in terms of $x = Q^2/(2p \cdot q)$, $Q^2 = -q^2$

$q = k - k'$, $y = p \cdot q/(p \cdot k)$, nucleon mass $M$ is

$$\frac{d^2 \sigma}{dx \ dy} = \frac{2G_F^2 M E_{\nu}}{\pi(1 + Q^2/M_W^2)^2} \times \{ q(x, Q^2) + (1 - y)^2 \bar{q}(x, Q^2) \} .$$  

The quantities $q(x, Q^2)$ and $\bar{q}(x, Q^2)$ are the parton distribution functions (PDFs) describing the quark and antiquark content of the nucleon.
In Eq. 2, the high energy behavior of the cross section can be qualitatively understood. At low energy, the conventional growth of the cross section linearly with neutrino energy comes from the fact that at low $Q^2$, the $W$-boson propagator and the PDFs are nearly $Q$ independent. Measurements in this linear regime are made up to energies of approximately 450 GeV [5]. At higher energies, the $Q^2$ dependence becomes important in two ways. There is a power suppression from the boson propagator, and there is a logarithmic growth of the PDFs with increasing $Q^2$. This last feature was first pointed out in the context of rising neutrino cross sections by Andreev, Berezinsky and Smirnov in Ref. [6]. Qualitatively, the propagator limits the value of $Q^2$ to approximately $Q^2 \sim M_W^2 \sim 10^6$ GeV$^2$, so for an incident neutrino energy $E_\nu$, one is probing a range of $x$ values of approximately

$$x \sim \frac{10^4}{(E_\nu/\text{GeV})}.$$

(3)

At the highest energies discussed here, $E_\nu = 10^{12}$ GeV, this translates to $x \sim 10^{-8}$, well below the measured region for parton distribution functions at $Q^2 \sim M_W^2$ [3,4]. In the next section, we discuss the parton distribution functions and their extrapolations to small values of parton $x$. For an extensive and readable discussion of deep inelastic scattering and the parton distribution functions, the reader is referred to Ref. [7].

### 3. PARTON DISTRIBUTION FUNCTIONS AND EXTRAPOLATIONS

#### 3.1. DGLAP Evolution

The cross section for neutrino-nucleon scattering has been evaluated over the past twenty years with a number of different input parton distribution functions and extrapolations to small $x$ values. In the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) formalism [5], valid for large $Q^2$ and moderate $x$, parton distribution functions of quarks, antiquarks and gluons are extracted from global analyses and evolved from a reference $Q_0^2$ to larger values of $Q^2$. Typically, modern analyses are performed in the range of $10^{-6} - 10^{-5} \leq x \leq 1$ with $Q_0^2 = 1.25 - 1.69$ GeV$^2$ [9,10]. An alternate approach of Gluck, Reya and Vogt [11] is to dynamically generate the sea quark and gluon distributions from a scale $Q_0^2 = 0.8$ GeV$^2$ using DGLAP evolution and requiring consistency with the experimental data. The GRV PDFs are parametrized between $10^{-9} \leq x \leq 1$. At $Q \sim M_W$, all of these approaches yield comparable quark and antiquark PDFs.

Gluon splitting to quark-antiquark pairs,

$$g \rightarrow q\bar{q}$$

(4)

is responsible for the generation of small $x$ sea distributions, so by studying $xg(x, Q)$ at small $x$, one gets insight into the sea distributions as well. At leading order, it has been shown that for gluon distributions parametrized at small $x$ by

$$xg(x, Q_0^2) \sim A(Q_0^2) x^{-\lambda} \quad x \ll 1,$$

(5)

with moderate values of $\lambda$ ($\lambda \simeq 0.3 - 0.4$), the gluon distribution approximately evolves with the same power law

$$xg(x, Q^2) \sim A(Q^2) x^{-\lambda}.$$

(6)

Taking this as a guide, one is led to extrapolate sea quark PDFs below $x_{\text{min}} = 10^{-6} - 10^{-5}$ by matching, e.g.,

$$x\bar{q}(x, Q^2) = \left(\frac{x_{\text{min}}}{x}\right)^\lambda x\bar{q}(x_{\text{min}}, Q^2)$$

(7)

where $\lambda$ is determined for each flavor from the PDFs at $Q = M_W$. This is in contrast to the double-logarithmic-approximation extrapolations [12] appropriate for

$$xg(x, Q_0^2) \sim \text{constant} \quad x \ll 1$$

(8)

which was the standard expectation before HERA results showed otherwise.

The results of this extrapolation using the NLO CTEQ6 parton distribution functions [9] in the DIS scheme are shown in Fig. 1 as a function of incident energy for neutrino and antineutrino charged current scattering. We use the leading order matrix element squared to evaluate the cross section. At low energy, one sees the increase in the cross section proportional to the neutrino energy, gradually moderated by the $W$-boson propagator. Above $\sim 10^9$ GeV, the neutrino and antineutrino cross sections are nearly equal, indicating that the sea quarks dominate the scattering.
with neutrinos above that energy. Other evaluations of the cross sections using a variety of PDFs, with DGLAP evolution, yield similar results [14,15,16].

3.2. BFKL/DGLAP

At ultrahigh neutrino energies, the values of $x$ probed are such that $\alpha_s \ln(1/x)$ can be large. The DGLAP formalism sums powers of $\ln(Q^2/Q_0^2)$ but does not sum powers of $\ln(1/x)$. Theoretical efforts have been directed to include summations of $\ln(1/x)$ corrections, the Balitsky, Fadin, Kuraev and Lipatov (BFKL) formalism [17]. Because of the $Q^2$ evolution of the PDFs are so important, only a unified BFKL/DGLAP approach is useful for UHE neutrinos. A combined BFKL/DGLAP evaluation of the neutrino-nucleon cross section has been made by Kwiecinski, Martin and Stasto (KMS) [18]. The approach calls for generalized parton distribution functions, which, when integrated over parton transverse momentum, yield the usual PDFs. They take as their canonical PDFs the GRV parametrization, and their results for the charged current cross section are shown by the dashed line in Fig. 2. Also shown in Fig. 2 is the CTEQ6 result with a power law extrapolation for comparison. The close correspondence of the two curves suggests that the additional $\ln(1/x)$ corrections are small compared to the DGLAP evolution. This is consistent with other theoretical work on unifying the BFKL and DGLAP approaches [19].

3.3. Saturation

The growth of the cross section with energy must eventually saturate to preserve unitarity [13,20]. In parton model language, saturation comes from

$$gg \rightarrow g, \quad (9)$$

gluon recombination. In principle, gluon recombination would come into the evolution equations.
via a non-linear term. A first estimate of the saturation effect is to consider
\[
\frac{\alpha_s}{Q^2} x g(x, Q^2) \sim \pi R^2
\]
(10)
where \(\pi R^2\) is the transverse size of the proton disk. For \(Q^2 \sim M_W^2\), this lead to an estimate of \(x \sim 10^{-17}\) for the relevant scale of \(x\) \[21\].

Kutak and Kwiecinski (KK) have instead evaluated a unified BFKL/DGLAP equation with a nonlinear term accounting for gluon recombination. Their results \[22\] together with the Kwiecinski, Martin and Stasto \[18\] curve without recombination are shown in Fig. 3. Also shown are Kutak and Kwiecinski’s results using the Golec-Biernat and Wusthoff (GBW) color dipole model \[23\] as an alternative to the unified BFKL/DGLAP plus gluon recombination approach.

The cross sections are remarkably consistent at the highest energies. The lower curve in Fig. 3 is only about a factor of two lower than the uppermost curve. This is a reasonable estimate of the range of predictions. Additional work on saturation appears in Refs. \[24,25\], with results that lie in the same range. Other authors have suggested that in fact the ultrahigh energy neutrino cross section is significantly enhanced by QCD effects \[26\], however, one awaits a quantitative demonstration of the effect in this context.

4. NON-PERTURBATIVE AND NON-STANDARD MODEL CONTRIBUTIONS

4.1. Non-perturbative electroweak instantons

Electroweak instanton contributions to the neutrino-nucleon cross section have been discussed in several references \[27,28\]. The idea is that standard model electroweak instanton processes can result in neutrino-nucleon conversion to multiparticle final states. They are exponentially suppressed because it is tunneling phenomenon with an energy barrier, the sphaleron energy, equivalent to approximately 8 TeV. Because it is a tunneling process between two vacua, the transition rate is exponentially suppressed at low energies, but can become quite large at high energies.

The actual calculation of the electroweak instanton contribution to the neutrino-nucleon cross section is subject to large uncertainties, both in the energy at which the transition is unsuppressed, and the normalization of that large cross section. Two different approaches, one using a perturbative approach with an instanton background field \[29\], and the other using a semi-classical approach \[30\], give widely different thresholds for the large cross section: the first at \(\sim 30\) TeV neutrino-parton center of mass energy, and the other at an energy scale roughly 10 times larger. The estimates of the magnitude of the cross section above the scale at which these interactions turn on is also widely varying, on the order of three orders of magnitude for the work of \[27\] and \[28\]. The ability of neutrino telescopes to constrain instanton induced events has been explored \[27,28\], with the conclusion that it may...
be difficult despite the very large cross sections.

4.2. Large extra dimensions

Non-standard model contributions to the neutrino-nucleon cross section are not well constrained at ultrahigh energies. Indeed, interactions at the millibarn level may help to explain the highest energy cosmic ray events by interpreting them as being produced by neutrinos with anomalously strong interactions. One model that has an anomalously large neutrino cross section is the model of \( n \) large compact extra dimensions [31]. Theoretical analyses have shown that one consequence of large extra dimensions is the possibility to produce mini-black holes in neutrino-nucleon interactions [32,33,34,35,36]. These mini-black holes decay into hadrons and leptons [38], and so mimic showers produced by cosmic ray interactions in the atmosphere.

In these mini-black hole calculations, one takes the neutrino-parton cross section to be

\[
\hat{\sigma}(\nu j \rightarrow BH) = \pi R_S^2 \left| M_{BH}^{} \right| \Theta(\sqrt{\hat{s}} - M_{BH}^\text{min})
\]

where the Schwarzschild radius \( R_S^{} \) is given by

\[
R_S^{} = \frac{1}{M_D^{} \left[ M_{BH}^{} \left( \frac{2^n \pi^{\frac{n-1}{2}} \Gamma(\frac{3+n}{2})}{2+n} \right) \right]^\frac{1}{n+1}}.
\]

The geometrical cross section is evaluated for neutrino-parton center of mass energy \( \sqrt{\hat{s}} \) larger than some minimum black hole mass \( M_{BH}^\text{min} \), which should be larger than the scale of the extra dimensions \( M_D^{} \) so that the semiclassical approach is a reasonable one. The neutrino-nucleon cross section then involves the integral over parton \( x \) of the neutrino-parton cross section in Eq. (11) multiplied by the parton distribution function.

An illustration of the enhanced neutrino-nucleon cross section from mini-black hole production is shown in Fig. 4. For this figure, the scale of the extra dimensions is set at \( M_D^{} = 2 \) TeV, and the cross section is shown for a range of minimum black hole masses, for \( n = 4 \) (solid line) and 6 (dashed line) extra dimensions. The standard model cross section is shown by the dot-dashed line for reference.

Figure 4. The black hole production cross section for neutrino-nucleon scattering with \( n = 4 \) (solid) and \( n = 6 \) (dashed) extra dimensions for \( M_{BH}^\text{min} = 2, 10 \) and 20 TeV, given \( M_D^{} = 2 \) TeV. The standard model cross section is also shown (dot-dashed).

While there are uncertainties in evaluating contributions from non-standard model physics, ultrahigh energy neutrinos may offer an unparalleled opportunity to explore particle physics in new energy regimes [37], whether it is large extra dimensions, supersymmetry or some other extension of the standard model.

5. FINAL REMARKS

Kusenko and Weiler have made the point that the cross section plays different roles in neutrino air shower events and upward-going events [40]. As an example, they point to the case of tau neutrinos. Tau neutrinos can produce horizontal air showers, but they can also produce upward air showers. The upward air showers come from the two step process of first \( \nu_\tau \) charged current interaction in the Earth producing a tau,
which emerges from the Earth to decay in the atmosphere. Event rates for neutrino interactions with air nuclei producing horizontal air showers increase linearly with the cross section. On the other hand, the upward air shower rate from tau decays is suppressed by large cross sections. Indeed, at ultrahigh energies, the upward air showers will be nearly horizontal, because neutrino fluxes incident at more than a “skimming” angle will be extinguished due to the short interaction length of the neutrino \[41\]. If ultrahigh energy neutrino fluxes are large enough, one might get an effective measurement of the neutrino cross section by comparing these two processes.

Neutrino messengers from astrophysical sources show the promise of revealing the nature of the sources themselves. Neutrino detection will give us more information about particle physics in energy regimes far beyond the reach of terrestrial accelerators, testing the standard model and extensions to new physics at ultrahigh energies.

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