The quantum speed up as advanced knowledge of the solution

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Abstract

With reference to a search in a database of size $N$, Grover states: "What is the reason that one would expect that a quantum mechanical scheme could accomplish the search in $O(\sqrt{N})$ steps? It would be insightful to have a simple two line argument for this without having to describe the details of the search algorithm". The answer provided in this work is: "because any quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of the information that specifies the solution of the problem". In database search, knowing in advance 50% of the $n$ bits that specify the database location, brings the search from $O(2^n = N)$ to $O(2^{n/2} = \sqrt{N})$. This empirical rule, unnoticed so far, holds for both quadratic and exponential speed ups and is theoretically justified in three steps: (i) once the physical representation is extended to the production of the problem on the part of the oracle and to the final measurement of the computer register, quantum computation is reduction on the solution of the problem under a relation representing problem-solution interdependence, (ii) the speed up is explained by a simple consideration of time symmetry, it is the gain of information about the solution due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution; this advanced knowledge of the solution reduces the size of the solution space to be explored by the algorithm, (iii) if $\Im$ is the information acquired by measuring the content of the computer register at the end of the algorithm, the quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of $\Im$, which brings us to the initial statement.

The fact that a problem solving and computation process can be represented as a single interaction, sheds light on our capability of perceiving (processing) many things together at the same time in the so called "present".
1 Premise

We represent an entire problem solving and computation process as a single interaction: the reduction of the forward evolution (ending with the state before measurement) on the backward evolution (the same unitary transformation but ending with the state after measurement) under a relation representing problem-solution interdependence
1. The relation applies to an infinite set of variables representing the amplitudes of the computational basis vectors throughout the quantum process; reduction changes these variables from the values assumed in the forward evolution to the values assumed in the backward evolution. This representation emphasizes the interdependence between all the parts of the information processing and turns out to be useful in two areas: the speed up of the quantum algorithms (their higher efficiency with respect to the corresponding classical algorithms) and the unity of perception.

For what concerns the quantum algorithms, we extend their physical representation to the production of the problem on the part of the oracle and to the measurement of the computer register at the end of the algorithm. The quantum process becomes reduction on the solution of the problem under the problem-solution interdependence relation. This highlights an empirical fact unnoticed so far: a quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of the information acquired in the final measurement of the computer register. The reader interested in this rule can go to section 4. For its theoretical explanation, he should also read section 3.

The fact that an entire problem solving and computation process can be represented as a single interaction, sheds light on the unity of perception. The reader interested in this part can go directly to section 5, which also provides the links to the notions developed in the previous sections. Here below we provide an overview of the work.

2 Overview

Solving a problem requires two steps. A problem solving step (deriving, from the formulation of the problem, the solution algorithm) and a computation step (running the algorithm). The latter step is generally oblivious of the former. We unify the two steps into a single physical interaction, represented as a hypothetical many body interaction in the classical framework (a useful conceptual reference), as a measurement interaction in the quantum framework, as follows.

For the conventional representation of computation, we adopt the so called ”circuit model”. At the logical level, a set of Boolean variables (representing the logical state of the computer register) undergoes a sequence of elementary reversible/deterministic input output transformations, changing input values into

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1 We do not require that reduction is objective in character. What is considered to be objective here is the whole quantum process – preparation, unitary evolution, and measurement – in fact represented as a single interaction. We use the term ”reduction” in its mathematical meaning of projection on the subspace of an eigenvalue of the measured observable.
output values. Through this sequence, an overall input is transformed into an overall output representing the solution of the problem. The sequence can be represented as a linear Boolean network, a series of reversible Boolean gates with no feedback loops and no preassigned values on the outputs, where the input logically propagates to the output in a deterministic way. At the physical level, a computer register undergoes a sequence of deterministic transformations (corresponding to the logical ones) that change input states of the register into output states until the overall output is reached. Reading the register content in the overall output state yields the solution of the problem. This representation of computation is oblivious of the problem (in the quantum algorithms, of the oracle that produces the problem), an omission that makes the speed up conceptually unexplained.

Computation is unified with problem solving as follows. We consider the non-linear Boolean network representing the original problem, to be distinguished from the above said linear network, which represents the algorithm that solves the problem. The nonlinear network is generally exponentially smaller, has logically irreversible gates or partial gates, feedback loops (outputs feeding back into inputs), and outputs with preassigned values. Solving this network means finding a Boolean assignment that satisfies all network elements (gates, wires, preassigned Boolean values). The non linear network is not directly solvable by the deterministic propagation of an input into an output; it is usually transformed into an exponentially larger linear network (representing the algorithm that solves the problem) solvable by this kind of propagation. Here we proceed in a different way, identifying a physical interaction that directly produces the solutions of the nonlinear network. We replace the Boolean variables \( x_i \) of the nonlinear network by ratios between real non-negative variables \( X_i/Q \), the network constraints by equations on these ratios. For \( Q > 0 \), the solutions of this system of equations correspond to the solutions of the nonlinear network. The relation between variables (the ratios) established by the system of equations represents problem-solution interdependence. The solutions are produced by a quantum measurement interaction, first visualized as a hypothetical classical many body interaction\(^2\). This latter is inspired to a well known paradox of classical mechanics: statically, the application of external forces to a perfectly rigid body is balanced by infinitely many distributions of stress inside the body, against one distribution if the body is flexible. This paradox is ported to a perfectly rigid body made of moving parts, whose coordinates \( X_i \) and \( Q \) are submitted to mechanical constraints representing the system of equations. In the initial configuration all coordinates are zero. By applying a force to the "input" part of coordinate \( Q \), the many distributions of stress inside the body find a combination of movements of the machine parts that satisfies all the constraints at the same time. In more detail, pushing the input part from \( Q = 0 \) to \( Q > 0 \), brings in the many body problem, the non-determination of the dynamics (because of perfect coincidence of interaction times between many bodies).

\(^2\)This visualization allows to bring to quantum computation the mechanical engineering notion that the absence of dissipation of a process does not imply its mechanical invertibility.
By applying the principle of sufficient reason, we postulate a many body interaction that produces a solution of the problem with probability proportional to the mass of the machine parts moved to produce that solution. In reality, the kinematics and the statistics of this hypothetical classical interaction represent a quantum measurement interaction. Configuration space becomes phase space. The ratios $X_i/Q$ represent the populations of (the reduced density operators of) the qubits of a quantum register in the surrounding of measurement. The many body interaction is replaced by the measurement interaction, which changes these population variables from the values immediately before to the values immediately after measurement. The problem-solution interdependence relation linearly extends to an infinite set of variables representing the amplitudes of the computational basis vectors throughout the quantum process (these latter variables are function of the above population variables). Under the extended relation, the measurement interaction changes the forward evolution, ending with the state before measurement, into the backward evolution, ending with the state after measurement that encodes the solution of the problem. In this "relational" representation, quantum computation is reduction on the solution under the relation representing problem-solution interdependence.

Applied to the quantum algorithms, this representation of problem solving and computation (for short, computation from now on) explains the quantum speed up, the fact that some quantum algorithms are much faster than the corresponding classical algorithms: the physical representation of computation must be extended to the production of the problem on the part of the oracle and to the final measurement of the register’s content. Then any known quantum algorithm becomes reduction on the solution under the relation representing problem-solution interdependence. The quantum speed up is explained by a simple consideration of time-symmetry. It is the gain of information about the solution of the problem due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution. This advanced knowledge of the solution reduces the size of the solution space to be explored by the algorithm: let $\mathcal{I}$ be the information acquired by measuring the content of the computer register at the end of the algorithm, the quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of $\mathcal{I}$. This is verified for the algorithms of Deutsch, Grover, and Simon, and also holds in the case of the generalized Simon’s algorithm, thus for the hidden subgroup algorithms.

The notion that an entire problem solving and computation process can be condensed into a single interaction (an idealized classical many body interaction or a quantum measurement interaction) can also explain the unity of perception, the fact that we perceive (assumedly, process) many things together at the same time in the so called "present". Tackling into account many constraints at the same time is exactly what a classical many body interaction, or a quantum measurement interaction, does. The physical representation of the notion of

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3We have chosen the term “advanced” because it is reminiscent of the “advanced wave”, going backward in time, of Cramer’s transactional interpretation of quantum mechanics.
"present" is the instant of the interaction in the classical case, the time interval spanned by backdated reduction in the quantum case.

3 The notion of relational computation

We develop the representation of computation as reduction of the forward evolution on the backward evolution under a relation representing problem-solution interdependence.

3.1 The notion of fundamental relation in classical and quantum physics

The idea that many things are processed all together at the same time, standing at the basis of relational computation, is formalized by resorting to a notion of the Gestalt theory (e.g. Mulligan and Smith, 1988). The wholeness/unity of a physical situation implies that there is a relation – a "simultaneous dependence" in the language of the theory – between all the quantitative variables describing it.

An example of fundamental relation in classical physics is "force equal mass times acceleration" in the case of a point mass. In view of what will follow, it should be noted that this relation is implicitly assumed to be objectively perfect. If we see it as a mechanism, whose degrees of freedom are the variables related by the law, this mechanism should be perfectly accurate, rigid, and reversible – it is not the case that Newton’s second law gets deformed and jams because of mechanical flexibility or irregularity, or dissipates because of friction.

Another important feature of the fundamental relations that we find in Nature, is that they can be nonfunctional, which is also the case of Newton’s law. The change of any one variable is correlated with an identical change of the product or ratio of the other two variables but does not determine their individual changes. Correspondingly, Newton’s law can host nondeterminism in the form of the many body problem.

An example of functional simultaneous dependence in quantum physics is the correlation between the polarizations of two photons in an entangled polarization state. We should note an important difference with respect to the classical notion of simultaneity. Now, if the two measurements are successive in time, there is simultaneous dependence between two results displaced in time (this is of course the notion of correlation between measurement outcomes in an entangled state). Simultaneous dependence is time symmetric: it is not the case that the former result determines the latter, causality between the two results is mutual.

An example of nonfunctional simultaneous dependence is the partial OR relation between the three orthogonal components of a spin 1 particle.

The requirement that the relation representing problem-solution interdependence is perfect, is essential in the case of the classical many body interaction, it is absorbed into the quantum principle and the notion of qubit (Finkelstein,
Coral Gables Conference, 1969 and Phys. Rev., 1969) in the case of the measurement interaction. That infinite classical precision can be dispensed for because of quantization has already been noted by Finkelstein (Finkelstein, 2008).

3.2 Relational computation as a hypothetical classical many-body interaction

We postulate a many body interaction inspired to a well known paradox of classical mechanics: statically, the application of external forces to a perfectly rigid body is balanced by infinitely many distributions of stress inside the body, against one distribution if the body is flexible. This paradox is ported to a perfectly rigid body made of moving parts, whose coordinates are submitted to mechanical constraints representing the problem. Applying a force to an "input part" brings in the many body problem. It is reasonable to postulate that the many distributions of stress inside the body find a combination of movements of the body’s parts that satisfies all the constraints at the same time.

It is interesting to note that giving up the limitation to two-body interaction marks the departure from classical computation. The fundamental physical model of classical computation is the bouncing ball model of reversible computation (Fredkin and Toffoli, 1982). Here the variables at stake are ball positions and momenta. Outside collisions, there is no simultaneous dependence between the variables of different balls, which are independent of each other. During collision, there is simultaneous dependence between the variables of the colliding balls, but this is confined to ball pairs (there can be several collisions at the same time, but involving independent ball pairs, with no simultaneous dependence between the variables of different pairs). The simultaneous collision between many balls is avoided to avoid the many body problem, the non-determination of the dynamics.

Instead, by assuming a perfect simultaneous dependence between all computational variables, one can devise an idealized classical machine that, thanks to a many body interaction, nondeterministically produces the solution of a (either linear or non-linear) system of Boolean equations under the simultaneous influence of all equations.

Let us start with the simple problem of finding the solutions \((x = 0, y = 1\) and \(x = 1, y = 0\)) of the single Boolean equation \(y = \overline{x}\). The mechanical principle that produces these solutions can easily be extended to any system of Boolean equations. Let \(Q, X,\) and \(Y\) be real non-negative variables. The Boolean problem is transformed into the problem of finding the solutions, for \(Q > 0\), of the system of equations

\[
Q = X + Y, \tag{1}
\]

\[
Q^2 = X^2 + Y^2. \tag{2}
\]

\(Q = 0\) implies \(X = Y = 0\), while \(\frac{X}{Q}\) and \(\frac{Y}{Q}\) are undetermined. When \(Q > 0\), \(\frac{X}{Q}\) coincides with the Boolean variable \(x\) and \(\frac{Y}{Q}\) with \(y = \overline{x}\). Equations \(\Pi\)
and (2), representing the problem constraint \( y = x \), establish a nonfunctional relation, a simultaneous dependence, between the variables \( Q, X, \) and \( Y \). To introduce the notion of relational computation, it is useful to think that the solutions are produced under this relation through a many body interaction as follows. We put a differential gear between coordinate \( Q \) (the input of the gear) and coordinates \( X \) and \( Y \) (the two outputs of the gear), which implements equation (1). We put another differential gear between the squares of these coordinates, namely between the auxiliary coordinates \( Q', X', \) and \( Y' \) connected through parabolic cams to \( Q, X, \) and \( Y \), so that \( Q' = Q^2, \; X' = X^2, \) and \( Y' = Y^2 \), which implements equation (2).

The initial machine configuration is \( Q = X = Y = 0 \); it can be argued that any motion of the part of coordinate \( Q \) from \( Q = 0 \) to \( Q > 0 \) produces a solution in a nondeterministic way, as follows. The many body problem is the problem of the non-determination of the dynamics in the case of perfect coincidence between interaction times of many bodies – which is the case if we try and push part \( Q \) out of \( Q = 0 \). Here we postulate a solution to the many body problem by applying the principle of sufficient reason. The motion of part \( Q \) could be obtained by applying a force to it. In fact, there is no reason for either \( X \) or \( Y \), in a mutually exclusive way, not to move with \( Q \), since either movement offers zero static resistance to the force – there is only the inertia of the machine parts.

By playing with inertia, we can also tune the probabilities that \( X \) and \( Y \) move. For example, we can ask that, on average (over an ensemble of repetitions of the interaction), there is equipartition of energy among the machine degrees of freedom. This implies that the probability that the parts (either \( Q \) and \( X \) or \( Q \) and \( Y \) ) move, is proportional to the mass of the parts. Under this assumption, and by assuming for example even masses for \( X \) and \( Y \), the values of the coordinate ratios change from \( \frac{X}{Q} = \frac{Y}{Q} = \frac{1}{2} \) before interaction to \( \frac{X}{Q} = 1 \) and \( \frac{Y}{Q} = 0 \) or (in a mutually exclusive way) \( \frac{X}{Q} = 0 \) and \( \frac{Y}{Q} = 1 \) after interaction. This hypothetical behavior is an arbitrary prolongation of classical computation to the many body case. In reality, it describes quantum measurement, as shown in the next section.

We should note that, unlike deterministic reversible processes, the present process is not invertible. For example, we can think of connecting the input part to an ideal spring charged when \( Q = 0 \). On the one side, there would be oscillations without dissipation. On the other, at each oscillation, the movement of the input part from \( Q = 0 \) to \( Q > 0 \) would randomly drag either \( X \) or \( Y \) in a mutually exclusive way.

The extension of this computation mechanism to the problem of solving a generic system of Boolean equations is provided in the Appendix.

We should note that this form of computation is essentially different from the causal propagation of an input into an output. For example, it can produce two inputs such that their product is a preassigned output (the nonlinear Boolean network for this problem is the network for the multiplication of two integer numbers, with a preassigned value on the output). If this were an input-
output propagation (which is not), one should say that inputs are produced with advanced knowledge of the output.

3.3 Relational computation as a quantum process

The kinematics and the statistics of the classical many body interaction can describe a quantum measurement interaction. Considering the previous example $y = x$, the motion from $Q = 0$ to $Q > 0$ is analogous to measuring two qubits $X$ and $Y$ in the entangled state $|0\rangle_X |1\rangle_Y + e^{i\delta} |1\rangle_X |0\rangle_Y$, where $\delta$ is an arbitrary (even random) phase. The configuration space of the classical machine becomes the phase space of the two qubits, the normalized coordinates of the machine parts (the ratios $\frac{X}{Q}$, $\frac{Y}{Q}$) represent the populations of (the reduced density operators of) the qubits at time $t_r$, at the end of the unitary evolution stage of the quantum process – we are thus dealing with "population variables". The measurement interaction changes these population variables from the values assumed in the forward evolution to the values assumed in the backward evolution. The correspondence between normalized coordinates and population variables is:

$$\frac{X}{Q} = x_{00}(t_r) = 1 - x_{11}(t_r), \quad \frac{Y}{Q} = y_{00}(t_r) = 1 - y_{11}(t_r),$$  

(3)

where $x_{00}(t_r)$ is the variable representing the population of qubit $X$ in $|0\rangle_X$ ($0$) during the interaction, etc. The problem-solution interdependence relation is the same as before:

$$Q = X + Y,$$  

(4)

$$Q^2 = X^2 + Y^2.$$  

(5)

In the transition from $Q = 0$ to $Q > 0$, the population variables change from the values before reduction $x_{00}(t_r) = x_{11}(t_r) = y_{00}(t_r) = y_{11}(t_r) = \frac{1}{2}$ to the values after reduction, one of the two mutually exclusive sets of values $x_{00}(t_r) = 0$, $x_{11}(t_r) = 1$, $y_{00}(t_r) = 1$, $y_{11}(t_r) = 0$ and $x_{00}(t_r) = 1$, $x_{11}(t_r) = 0$, $y_{00}(t_r) = 0$, $y_{11}(t_r) = 1$.

It should be noted that simultaneous dependence (functionally) extends to the amplitudes of the basis vectors throughout the unitary evolution stage of the quantum process – for example the process of entangling $X$ and $Y$ starting from a sharp preparation of the two qubits. Amplitudes should be represented by variables, function of the above population variables. To this end, we define the state of the quantum system at time $t_r$ as a function of the population variables:

$$|\psi, t_r\rangle = \sum_{i=1}^{n} \alpha_i(t_r) |i\rangle,$$  

(6)

where $|i\rangle$ is the $i$-th computational basis vector and

$$\alpha_i(t_r) = f_i[x_{00}(t_r), y_{00}(t_r)].$$  

(7)
The transition from \( Q = 0 \) to \( Q > 0 \) changes together the population variables \( x_{00}(t_r), y_{00}(t_r) \) and the amplitudes \( \alpha_i(t_r) \) from the values assumed in the forward evolution to the values assumed in the backward evolution. The entire unitary evolution can be represented as a unitary transformation of \( |\psi, t_r\rangle \):

\[
\forall t : |\psi, t\rangle = U(t, t_r)^\dagger |\psi, t_r\rangle, \tag{8}
\]

where \( U(t, t_r) \) is the unitary transformation undergone by the quantum system from \( t \) to \( t_r \). Under the relation established by equations \( \ref{eq:3} \), \( \ref{eq:4} \), \( \ref{eq:5} \), \( \ref{eq:6} \), \( \ref{eq:7} \), and \( \ref{eq:8} \), reduction changes the forward evolution into the backward evolution. This way of representing together the unitary evolution and the measurement stage of the quantum process – as the reduction of the forward evolution onto the backward evolution under a relation representing problem-solution interdependence, is essential for the explanation of the speed up.

It should be noted that, under the above relation, any two amplitudes \( \alpha_i(t_1) \) and \( \alpha_j(t_2) \), with \( t_1 \leq t_2 \), also depend from one another in a time symmetric way. In other words, it is not the case that the change (from the forward to the backward value) of the amplitude at time \( t_1 \) causes the change of the amplitude at the later time \( t_2 \); causality is mutual, like in the measurement of two entangled polarizations. In this sense, we can say that there is simultaneous dependence between all the amplitudes of the computational vectors throughout the entire quantum process.

4 Relational computation and the quantum algorithms

We apply relational computation to the quantum algorithms. It suffices to represent together (as a single physical interaction) the production of the problem on the part of the oracle, the unitary evolution stage of the quantum process, and the final measurement of the content of the computer register. This shows that quantum computation is reduction on the solution of the problem under the problem-solution interdependence relation, and explains the speed up.

We need to consider only the input and the output of the unitary transformation performed by the quantum algorithm, therefore no previous knowledge of the subject is required to the reader. For an introductory description of the quantum algorithms in the same notation used here, see (Kaye et al., 2007).

4.1 Grover’s algorithm

The problem addressed by Grover’s algorithm (Grover 1996) is database search. It can be seen as a game between two players with a chest of \( N \) drawers; the first player (the oracle) hides a ball in drawer number \( k \). The second player must find where the ball is. Opening drawer \( x \) to check whether the ball is in it amounts to computing the Kronecker function \( \delta(k, x) \), which is 1 if \( k = x \) and
0 otherwise. Here $k \in \{0, 1\}^n$, where $n = \log_2 N$ (for simplicity, we can assume that $N$ is a power of 2).

The value of $k$ chosen by the first player is hardwired inside a black box that, for each input $x$, computes $\delta(k, x)$. This black box is given to the second player, who has to find the value of $k$ by computing $\delta(k, x)$ for different values of $x$ (i.e., by opening different drawers to check whether the ball is in it). In the classical case, to find the value of $k$, $\delta(k,x)$ must be computed the order of $N$ times, in the quantum case the order of $\sqrt{N}$ times – there is a quadratic speed up.

We review the original Grover’s algorithm. The first player chooses a black box with a given value of $k$, and gives it to the second player. Instead of trying, one by one, single values of $x$, the second player prepares an $n$-qubit register $X$ in an even weighted superposition of all the possible values of $x$, and computes $\delta(k, x)$ in quantum parallelism. For example, with $N = 4$ and $k = 01$, the algorithm unitarily changes the input state

$$\frac{1}{2\sqrt{2}} (|00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X) (|0\rangle_V - |1\rangle_V),$$

into the output state

$$\frac{1}{\sqrt{2}} |01\rangle_X (|0\rangle_V - |1\rangle_V)$$

(10)

(the quantum network implementing this transformation, as well as the function of register $V$, can be disregarded here). Measuring the content of register $X$ in the output state (10) yields the solution. In the present case ($N = 4$), this is obtained by computing $\delta(k, x)$ only once.

Thus, in the original algorithm, the solution is obtained in a deterministic way (for simplicity, we put ourselves in those values of $N$ where the probability of error of Grover’s algorithm is zero); relational computation, namely reduction on the solution of the problem under the problem-solution interdependence relation, is completely hidden. This is because the random generation of $k$ is not represented physically.

We extend the physical representation by adding an ancillary $n$-qubit register $K$ prepared in an even weighted superposition of all the possible values of $k$. The black box that, given an input $x$, computed $\delta(k, x)$ for a specific value of $k$, is now replaced by a black box that, given the inputs $k$ and $x$, computes $\delta(k, x)$. The extended algorithm computes $\delta(k, x)$ as before, but now for a superposition of combinations of values of $k$ and $x$. The input state is now:

$$\frac{1}{4\sqrt{2}} (|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K) (|00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X) (|0\rangle_V - |1\rangle_V),$$

where the superposition hosted in register $K$ can indifferently be coherent or incoherent (in which case each element of the superposition should be multiplied...
by a random phase factor). The extended algorithm unitarily transforms the
input state (11) into the output state:

\[
\frac{1}{2\sqrt{2}} (|00\rangle_K |00\rangle_X + |01\rangle_K |01\rangle_X + |10\rangle_K |10\rangle_X + |11\rangle_K |11\rangle_X) \left( |0\rangle_V - |1\rangle_V \right),
\]

where each value of \(k\) is entangled with the corresponding solution found by
the second player (the same value of \(k\) but in register \(X\)). The final measurement
of the contents of registers \(K\) and \(X\) in state (12) determines the moves of both
players. The reduction induced by measuring the content of register \(K\), back-
dated to before running the algorithm, yields the original Grover’s algorithm.
Thus, by representing together the production of the problem on the part of
the oracle, the unitary evolution, and the measurement of the register content,
we find reduction on the solution of the problem under the problem-solution
interdependence relation. The present relation is a straightforward generaliza-
tion (from two one-qubit registers to two two-qubit registers) of the relation
established by equations (3), (4), (5), (6), (7), and (8) – section 3.3.

The nondeterministic production of the contents of the two registers by
quantum measurement, can be seen as mutual determination between such con-
tents, like in the measurement of two entangled polarizations. This justifies
the square root speed up, as follows. We cannot say that reading the content of \(K\)
at the end of the algorithm determines the content of \(X\), namely that choosing
the drawer to hide the ball in determines the drawer the ball is found in – this
would be the classical game with no mutual determination. For the same rea-
son we cannot say that reading the content of \(X\) determines the content of
\(K\), that looking inside a drawer at the end of the algorithm creates the ball in it.
Mutual determination is symmetrical, it should be represented by saying that
the contents of the two registers are determined by reading the first (second)
bit of register \(K\) and the second (first) bit of register \(X\).

Then Grover’s algorithm is equivalent to the following game. We arrange
the \(N\) drawers in a matrix of \(\sqrt{N}\) columns and \(\sqrt{N}\) rows. At the end of
the algorithm, the first player determines (say) the row by reading the first bit
of register \(K\). The second player determines the column by reading the second
bit of register \(X\), say that this reading is 1. The reduction induced by
the second player, backdated to before running the algorithm, changes the ini-
tial preparation of register \(K\): \(\frac{1}{2} (|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K)\) (eq. 11), into
\(\frac{1}{\sqrt{2}} (|01\rangle_K + |11\rangle_K)\), thus determining the column before running the algorithm.

In this picture, Grover’s algorithm searches just the row randomly chosen by
the first player, which justifies the \(\sqrt{N}\) computations of \(\delta (k, x)\) – the quadratic
speed up. Grover’s algorithm is equivalent to a classical search in a database
of size \(\sqrt{N}\) (we should symmetrize for the exchange of columns and rows). See
also (Castagnoli, 1997), (Castagnoli et al., 1999), (Castagnoli and Finkelstein,
2001).

The same justification holds in the case that the value of \(k\) is already deter-
mined before running the algorithm, like in virtual database search. This case is
indistinguishable from the random generation of \(k\) at the end of the algorithm,
where backdating reduction makes $k$ predetermined.

If we think that $k$ is predetermined, the initial superposition in register $K - \frac{1}{2}(|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K)$ represents the initial ignorance of the value of $k$ on the part of the second player. Its reduction to $\frac{1}{\sqrt{2}}(|01\rangle_K + |11\rangle_K)$, due to the second player reading 1 in the second bit of register $X$ at the end of the algorithm, gives – before running the algorithm – an information gain of one bit. This is 50% of the information acquired by reading the solution, namely the two bits of register either $K$ or $X$ (the information content of one register is redundant with respect to the content of other register), or one bit of $K$ and the other bit of $X$.

For a database of size $N$, the reduction of ignorance about the solution due to backdating, to before running the algorithm, 50% of the information acquired by reading the solution, is:

$$\Delta_\Omega = \frac{1}{2} \log_2 N.$$  \hspace{1cm} (13)

Summing up, in the relational representation:

1. Quantum computation is reduction on the solution of the problem under the relation representing problem-solution interdependence.

2. The speed up is the reduction of the initial ignorance of the solution due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution, namely 50% of the information acquired by reading the solution. This advanced knowledge of the solution reduces the size of the solution space to be explored by the algorithm.

3. Let $\Omega$ be the information acquired by measuring the content of the computer register at the end of the algorithm; the quantum algorithm takes the time taken by a classical algorithm (the reference classical algorithm) that knows in advance 50% of $\Omega$.

In the present case of Grover’s algorithm, knowing in advance 50% of the $n$ bits that specify the database location, reduces the size of the solution space (of the database) from $2^n = N$ to $2^{n/2} = \sqrt{N}$. The quantum algorithm working on a database of size $N$ takes the time taken by a classical algorithm working on a database of size $\sqrt{N}$. See also (Castagnoli, 2007, 2008, 2008).

We should note that the information acquired by measuring the content of the computer register at the end of the algorithm, specifies both the problem (the oracle’s random choice) and the solution. Therefore, ”reduction on the solution of the problem” should be understood as reduction on the solution and the problem. However, in Grover’s algorithm, the information acquired by reading the solution and the problem coincides with the information acquired by reading either one, since either one is a function of the other.

4If we assume that the initial superposition hosted in register $K$ is incoherent, this information gain is the decrease of the von Neumann entropy of the (reduced density operator of the) content of register $K$ during measurement.
We should also note that point 3, besides being theoretically justified within the relational representation of computation, is an empirical fact holding for all quantum algorithms – once the physical representation is extended to the production of the problem on the part of the oracle and to the final measurement of the register’s content.

4.2 Deutsch’s algorithm

We consider the revisitation of Deutsch algorithm (Deutsch, 1985) due to Cleve et al. (1996). Now the oracle chooses at random one of the four binary functions $f_k : \{0, 1\} \rightarrow \{0, 1\}$ (see table [14] – $k \equiv k_1k_2$ is a two-bit string belonging to $\{0, 1\}^2$):

$$
\begin{array}{|c|c|c|c|}
\hline
x & f_{00}(x) & f_{01}(x) & f_{10}(x) & f_{11}(x) \\
\hline
0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 \\
\hline
\end{array}
$$

Note that $k_1 = f_k(0)$ and $k_2 = f_k(1)$. Then the oracle gives to the second player the black box hardwired for the computation of that function. The second player, by trying function evaluation for different values of $x$, must find out whether the function is balanced (i.e. $f_{01}$ or $f_{10}$, with an even number of zeroes and ones) or constant (i.e. $f_{00}$ or $f_{11}$). This requires two function evaluations in the classical case, just one in the quantum case (this has been the first speed up ever found).

In the conventional quantum algorithm, the second player prepares two one-qubit registers $X$ and $V$ in the input state

$$\frac{1}{2} (|0\rangle_X + |1\rangle_X) (|0\rangle_V - |1\rangle_V) \quad (15)$$

(the function of register $V$ can be disregarded here). With just one function evaluation, the algorithm unitarily produces the output state

$$\frac{1}{\sqrt{2}} |0\rangle_X (|0\rangle_V - |1\rangle_V) \quad (16)$$

if the function is constant and

$$\frac{1}{\sqrt{2}} |1\rangle_X (|0\rangle_V - |1\rangle_V) \quad (17)$$

if the function is balanced. Thus, at the end of the algorithm, register $X$ contains the solution of the problem ($0 \equiv$ constant, $1 \equiv$ balanced). The speed up is obtained in a deterministic way, but also in this case the random generation of $k$ is not represented physically. We extend the physical representation by adding an ancillary two-qubit register $K$ prepared in a superposition (indifferently coherent or incoherent) of all the possible values of $k$. The input state is now:

$$\frac{1}{4} (|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K) (|0\rangle_X + |1\rangle_X) (|0\rangle_V - |1\rangle_V). \quad (18)$$
The extended algorithm, given the inputs $k$ and $x$, computes $f(k, x) \equiv f_k(x)$, yielding the output:

$$\frac{1}{2\sqrt{2}} [(|00⟩_K - |11⟩_K) |0⟩_X + (|01⟩_K - |10⟩_K) |1⟩_X] (|0⟩_V - |1⟩_V).$$  \hspace{1cm} (19)$$

The measurement of the content of registers $K$ and $X$ in the output state determines the moves of both players (the random choice of the function on the part of the first player and the answer provided by the second player). The information acquired is 2 bits – the two bits of register $K$ (the content of register $X$ is a function of the content of $K$, therefore the information contained in $X$ is redundant). The quantum algorithm takes the time taken by a classical algorithm working on a solution space reduced in size because one bit of information about the content of register $K$, either $k_1 = f_k(0)$ or $k_2 = f_k(1)$, is known in advance. This algorithm must acquire the other bit of information by computing either $f_k(1)$ or $f_k(0)$. Thus the reference classical algorithm has to perform just one function evaluation like the quantum algorithm. This verifies point 3.

4.3 Simon's and the hidden subgroup algorithms

In Simon's algorithm, a first player (the oracle) chooses at random a function among the set of the "periodic" functions $f_k : \{0, 1\}^n \rightarrow S$, where $S \subseteq \{0, 1\}^n$. The "periodic" function $f_k$, where $k = k_1...k_n$ is a string of $n$ Boolean values (excluding the all zeroes string), is such that $f_k(x) = f_k(y)$ if and only if $x = y$ or $x = y \oplus k$. Here $x$ and $y$ are variables belonging to $\{0, 1\}^n$ (also represented as $n$ bit strings) and $\oplus$ denotes bitwise addition modulo 2 (see the following example). Then he gives to the second player a black box that, given an input $x$, computes $f_k(x)$. The second player should find the hidden string $k$ (among $2^n - 1$ possible strings) through function evaluation. To find the value of $k$ with probability, say, $\frac{2}{3}$, $f_k(x)$ must be computed the order of $2^\frac{n}{2}$ times in the classical case, the order of $3n$ times in the quantum case. There is an exponential speed up.

Let us exemplify. With $n = 2$ and $S = \{0, 1\}$, there are three "periodic" functions (up to permutation of function values leaving the hidden string unaltered):

| $x$ | $f_{01} (x)$ | $f_{10} (x)$ | $f_{11} (x)$ |
|-----|-------------|-------------|-------------|
| 00  | 0           | 0           | 0           |
| 01  | 0           | 1           | 1           |
| 10  | 1           | 0           | 1           |
| 11  | 1           | 1           | 0           |

The original Simon's algorithm is as follows. The second player prepares an $n$-qubit register $X$ in an even weighted superposition of all the possible values of $x$, and an $m$-qubit register $F$ (devoted to contain the result of function evaluation) in a sharp state (standing for a blank register). In the present example, the input state of the algorithm is thus:
\[
\frac{1}{2} (|00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X) |0\rangle_F .
\] (21)

Then he performs function evaluation on the superposition of all the possible values of \(x\), obtaining the intermediate output (say that \(k = 10\), see \(f_{10}(x)\) in table 20):

\[
\frac{1}{2} \left[ (|00\rangle_X + |10\rangle_X) |0\rangle_F + (|01\rangle_X + |11\rangle_X) |1\rangle_F \right].
\] (22)

Now he applies the Hadamard transform (still a unitary transformation) to the state of register \(X\), obtaining the overall output:

\[
\frac{1}{2} \left[ (|00\rangle_X + |01\rangle_X) |0\rangle_F + (|00\rangle_X - |01\rangle_X) |1\rangle_F \right].
\] (23)

In the overall output state (23), for each value of the function, register \(X\) hosts an even weighted superposition of the \(2^n - 1\) strings \(h_j = h_{j1}h_{j2}...h_{jn}\) orthogonal to \(k\) – such that, for all \(j\), \((\sum_{i=1}^{n} h_{ji}k_{ji})\) modulo 2 = 0; in the example, \(h_1 \equiv 00\) and \(h_2 \equiv 01\) are the two strings orthogonal to \(k \equiv 10\). Note that, in (23), only the phase of the even weighted amplitudes depend on the value of \(f_k(x)\).

Therefore, by measuring the content of \(X\) in (23), one obtains at random one of the \(h_j\). The entire process (initial preparation of registers \(X\) and \(F\), unitary transformation, and measurement of the content of \(X\)) is iterated until obtaining \(n - 1\) different \(h_j\), which allows to find \(k\) by solving a system of \(n - 1\) modulo 2 linear equations.

This formulation of Simon’s algorithm leaves the number of iterations of the algorithm unbounded. Alternatively, we can iterate the algorithm a fixed number of times, which leaves a certain probability of failing to find the solution. If the algorithm is iterated, say, \(6n\) times, the probability of obtaining \(n - 1\) different \(h_j\), thus of finding the solution, is about \(\frac{5}{9}\) (e. g. Kaye et al., 2007).

The computation time taken by the \(6n\) iterations Simon’s algorithm, in terms of number of elementary logical operations, is the order of \(n^3\), against the order of \(2^n\) of the classical algorithm.

Now we extend Simon’s algorithm to represent the random choice of the hidden string on the part of the oracle. We add an auxiliary \(n\)-qubit register \(K\), prepared in an even weighted superposition of the \(2^n - 1\) possible values of \(k\). The black box that (given input \(x\)) computed \(f_k(x)\) for a specific \(k\), is now replaced by a black box that, given the inputs \(k\) and \(x\), computes \(f(k, x) \equiv f_k(x)\). The input state is now:

\[
\frac{1}{2\sqrt{3}} \left( |01\rangle_K + |10\rangle_K + |11\rangle_K \right) \left( |00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X \right) |0\rangle_F ,
\] (24)

\(^5\)Throughout this article, the term “solution” should be understood in the broader sense of “information acquired at the end of the quantum algorithm”, here leading to the solution of the problem with a classical post processing.
where the superposition hosted in register $K$ is indifferently coherent or incoherent. The overall output state \((23)\) becomes now:

\[
\frac{1}{2} |0\rangle_K [(|00\rangle_X + |10\rangle_X) |0\rangle_F + (|00\rangle_X - |10\rangle_X) |1\rangle_F] \\
+ \frac{1}{2} |1\rangle_K [(|00\rangle_X + |01\rangle_X) |0\rangle_F + (|00\rangle_X - |01\rangle_X) |1\rangle_F] \\
+ \frac{1}{2} |11\rangle_K [(|00\rangle_X + |11\rangle_X) |0\rangle_F + (|00\rangle_X - |11\rangle_X) |1\rangle_F].
\]

The random choice of a value of $k$ on the part of the first player is obtained by measuring the content of register $K$ in \((25)\). This puts register $K$ in a sharp state and registers $X$ and $F$ in a state of the form \((23)\). The second player measures the content of register $X$ (thus collecting the first of the $h_j$) then, leaving register $K$ in its sharp state and working only on registers $X$ and $F$, iterates up to a total of $6n$ times the original Simon's algorithm (thus collecting the other $6n - 1$ values of $h_j$).

In the case that the algorithm finds the solution (when there are at least $n - 1$ different $h_j$, the probability of this being $\frac{8}{9}$), there is mutual determination between the content of register $K$ and the content of register $X$ in each of its $6n$ measurements. In more detail: measuring the content of $K$ projects the state of register $K$ on a single value of $k$; which value of $k$ is the result of mutual causality between measuring the content of $K$ and the successive measurements of the content of $X$. This is completely similar to measuring the polarizations of two photons in an entangled polarization state; the polarization measured first is the result of mutual causality between this measurement and the successive measurement of the second polarization; it is not the case that the first result determines the second, nor that the second determines the first, causality is mutual even across an interval of time.

Now, backdating 50% of the information acquired by reading the content of register $X$ (50% of $6n$ readings), reduces the size of the problem not with certainty (like in the case of Grover's and Deutsch's algorithms) but with probability $\frac{2}{3}$ (with good approximation, the probability of finding the hidden string with $3n$ readings is in fact $\frac{2}{3} = 1 - \sqrt{1 - \frac{8}{9}}$). Moreover, with this probability, the size of the problem is reduced from $2^n - 1$ to 1.

The fact that the reference classical algorithm disposes of $n - 1$ different $h_j$, implies that the quantum algorithm (whose $h_j$ comprise those of the reference algorithm) also disposes of them: $\frac{2}{3}$ is thus the probability that both algorithms have enough information to identify the solution. Point 3, the fact that the quantum algorithm takes the time taken by a reference classical algorithm that knows in advance 50% of the information about the solution, applies in a probabilistic way. With probability $\frac{2}{3}$, we put ourselves in the case that both algorithms dispose of $n - 1$ different $h_j$ (the $\frac{1}{3}$ probability that this is not the case, goes down exponentially fast with the number of iterations of the quantum algorithm, it is $\approx (\frac{1}{3})^9$ with $6gn$ iterations). In this case, the reference
algorithm has simply to sort out \( n - 1 \) different \( h_j \) and solve the related system of linear equations. Thus, the quantum algorithm takes a time the order of \( n^3 \), the reference algorithm the order of \( n^2 \), not so different in comparison with the order of \( 2^n \) taken by classical computation.\(^\text{[a]}\)

Perhaps a more significant way of comparing classical computation and the reference algorithm is to say that:

4. classical computation has to find the hidden string by solving a system of nonlinear equations in \( n \) variables (we can think of the formulation of the problem in terms of real nonnegative variables), the reference classical algorithm – thanks to the advanced knowledge of 50% of the contents of register \( X \) (measured in the successive iterations of the algorithm) – a system of linear equations in the same order of variables.

The fact that points 1, 2, 3, and 4 hold for Simon’s algorithm implies that they hold for a larger class of algorithms, because of the tight similarity between Simon’s algorithm, the generalized Simon’s algorithm, and the hidden subgroup formulation of the quantum algorithms (Mosca and Ekert, 1999) – like finding orders, finding the period of a function (the quantum part of Shor’s factorization algorithm), finding discrete logarithms, etc. (Kaye, 2007). In all cases, utilizing 50% of the information acquired in a batch of iterations of the quantum algorithm reduces the probability of finding the solution of an amount that goes exponentially to zero with the number of iterations in the batch, and reduces the problem of solving a nonlinear system in \( n \) variables to the problem of solving a linear system in a number of variables of the same order. Point 4 holds replacing the expression ”hidden string” by ”hidden subgroup”.

5 Relational computation and the unity of perception

For wholeness of perception, as it appears in introspective analysis, I mean the following. For example, in this moment, I see the room in which I am working, an armchair, the window, the garden, and the Mediterranean Sea on the background. In my visual perception, besides some aspects that are addressed by artificial intelligence, like the recognition of patterns, there is another thing that should be addressed by a physical information theory, the both obvious and striking fact that I see so many things together at the same time. What I see is close to a digital picture whose specification would require a significant amount of information. And, apparently, we can perceive a significant amount

\(^\text{[a]}\)The two errors, the lower probability of finding the solution and the lower computational cost of the reference algorithm (with respect to the quantum algorithm), tend to compensate with one another. The compensation is full in the following perspective. We iterate the quantum algorithm other 6\( n \) times, just to feed the reference algorithm with other 3\( n \) values of \( h_j \). In this way the probability of finding the solution is the same for the original 6\( n \) iterations quantum algorithm and this enriched reference algorithm. If we ascribe the cost of the enrichment entirely to the reference algorithm, also computational costs are the same.
of information simultaneously all together, in the so called "present". Another example is our capability of grasping the solution of a problem. Reasonably, when we grasp the solution, we should take into account at the same time the statement of the problem, the solution, and the logical connection in between.

In the assumption that perception is information processing, perceiving many things at the same time poses the question: what form of computation can process many things together at the same time in the so called "present", and what is the physical counterpart of the introspective notion of "present", such that an entire computation process can occur in it? The mechanism of relational computation can provide an answer.

In the idealized classical case, an entire computation process is condensed into an instantaneous many body interaction (sections 3.1, 3.2). The physical counterpart of the introspective notion of "present" is here the instant of the interaction.

In the quantum case, a state can hold any amount of information, which is processed together "at the same time" by the sequence: preparation, unitary transformation, and measurement. "At the same time" since there is simultaneous dependence between all the amplitudes of the computational basis vectors at any pair of times along the process (the change of one amplitude, from the forward to the backward value, changes the other and vice-versa, in a time symmetric way). Correspondingly the measurement interaction changes the entire forward evolution into the backward evolution (sections 3.1, 3.2, and 3.3). The physical counterpart of the notion of "present" is here the time interval spanned by backdated reduction.

The observation that, in visual perception, we take into account many things at the same time acquires a literal meaning. Taking into account many things at the same time is exactly what many body interaction, or reduction of the forward evolution on the backward evolution, does.

By the way, if the physical basis of consciousness is a fundamental nondeterministic problem solving mechanism, it is difficult to think that consciousness is the passive witness of a deterministic classical process. Moreover, the deterministic, two-body character of classical computation prevents taking into account many (so to speak, more than two) things together at the same time, or (reasonably) hosting consciousness either.

The present model might provide some theoretical ground to the empirical notion of premonition. Point 3 of sections 4.1 through 4.3, put in anthropomorphic language, says that the quantum algorithm is quicker than the classical algorithm because it disposes of an anticipated partial knowledge of the solution it will find in the future – knowing in advance 50% of it. For the possibility that consciousness interacts with systems displaced in time up to 500 milliseconds in the past and from milliseconds to months into the future, see (Sheehan, 2008).

The present identification between the notions of simultaneous dependence, physical law, and perception has a precedent in Plato's notion of Form (the Greek word Eidos translates into Form, Idea, or Vision). In Phaedo: "Ideas are objective perfections that exist in themselves and for themselves, at the same time they are the cause of natural phenomena, they keep phenomena
bound together and constitute their unity” (Abbagnano, 1958). The Ideas of our mind are clearly identified with physical laws; as well known, Platonic Ideas are also perfect mathematical objects. The usual Platonist interpretation of this ambivalence is that the mind can access an autonomous and objective world of perfect mathematical ideas. A more physical interpretation is the other way around, the ideas in our head – our perceptions – are instances of physical laws, namely of objectively perfect, nonfunctional simultaneous dependences.

The present idea that "grasping the solution of a problem" implies reduction under a simultaneous dependence representing problem-solution interdependence, is parallel to another statement of the theory of Forms: "To know the Form of X is to understand the nature of X; so the philosopher who, for example, grasps the Form of justice, knows not merely what acts are just, but also why they are just” (Flew, 1984).

Also Plato's notion that the mind can access the objective perfection of the world of ideas, while material objects are imperfect (a formulation of the mind-body problem) is reflected in the present model, where perception is the instantiation, in the quantum level of the brain, of an objectively perfect physical law.

The present fundamental problem-solving mechanism matches in my judgement with: (i) the Orchestrated Objective Reduction theory of consciousness of Hameroff and Penrose (Hameroff and Penrose, 1996): it is a way of seeing problem-solving, specifically of a form relying on the non linearity of reduction, in Objective Reduction, (ii) the idea, also present in that theory, that the very existence of consciousness depends on our capability of accessing the Platonic world of objectively perfect mathematical Ideas (Penrose, 1994), and (iii) Stapp’s argument (Stapp, 1993) that consciousness is incompatible with classical locality (here seen as two body interaction, unable to process many things together at the same time) and compatible with quantum non-locality and reduction (here seen as many body interaction, or reduction under problem-solution interdependence, capable of processing many things together at the same time).

6 Conclusions

In the admission of their same authors, the quantum speed ups are still little understood. In his 2001 paper (Grover, 2001), Grover states: "What is the reason that one would expect that a quantum mechanical scheme could accomplish the search in $O\left(\sqrt{N}\right)$ steps? It would be insightful to have a simple two line argument for this without having to describe the details of the search algorithm”.

Casting the present argument in two lines, the answer is:

Because any quantum algorithm takes the time taken by a classical algorithm that knows beforehand 50% of the information specifying the solution of the problem.

The theoretical justification of this empirical fact can be segmented as follows:
1. once the physical representation is extended to the production of the problem on the part of the oracle, quantum computation is reduction on the solution of the problem under the relation representing problem-solution interdependence;

2. the speed up is the reduction of the initial ignorance of the solution due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution; this advanced knowledge of the solution reduces the size of the solution space to be explored by the algorithm;

3. thus, if $\mathcal{I}$ is the information acquired by measuring the register content at the end of the algorithm, the quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of $\mathcal{I}$.

The expounded mechanism could be used, by reverse engineering, for the search of new quantum algorithms. For example, Grover’s algorithm can be seen as the symmetrization (for exchange of all the possible ways of getting in advance the 50% of $\mathcal{I}$) of a classical algorithm that does database search in a solution space of quadratically reduced size. The 50% rule could be used to investigate which problems are liable of being solved with a quadratic or exponential speed up, the question of why the speed up is quadratic for unstructured problems, exponential for a very limited class of structured problems, not others, etc.

The relevance of the rule to this question can be seen as follows. Let us think of a crossword puzzle, the 50% rule means knowing in advance 50% of the characters of the word; if the word is a random sequence of characters, this advanced knowledge yields a quadratic speed up, if the word is structured, it can yield much more.

From another standpoint, relational computation provides a comparevi-sion of disparate forms of computation. One goes from deterministic classical computation to nondeterministic quantum computation by giving up the limitation to two body interaction.

It is interesting to compare reversible computation (Finkelstein, Coral Gables Conference, 1969, Bennett, 1973 and 1982, Benioff, 1982, Fredkin and Toffoli, 1982) with relational computation. In the present context, the former form of computation can be seen as a special case of the latter:

- in reversible computation, the physical process is limited to the unitary evolution and is thus deterministic in character; the two notions of absence of dissipation and process invertibility overlap;

- in relational computation, the physical process is extended to the sequence unitary evolution and measurement and is thus nondeterministic in character; the overlapping between the two notions is released, the absence of dissipation can go together with the non-invertibility of the process (section 3.2).

Summing up, nondeterminism (the non-invertibility of the process) here becomes a fundamental feature of quantum computation. It is not loosing track of
a time-invertible propagation spreading through too many degrees of freedom; it is capability of making choices satisfying many (like in many body) constraints at the same time.

Furthermore, being the condensation of an entire computation process into a single physical interaction (a classical many body interaction or a quantum reduction), the present nondeterministic form of computation can represent the information processing standing at the basis of perception. It explains an essential feature of conscious perception, the fact that (e. g. in visual perception) we see many things together at the same time. Taking into account many constraints at the same time is exactly what classical many body interaction, or quantum reduction of the forward evolution on the backward evolution, does; "at the same time" is the instant of the interaction in the classical case, the time interval spanned by backdated reduction in the quantum case.

Appendix

The computation mechanism expounded in section 3.2 is easily extended to solve any system of Boolean equations, namely (without loss of generality) a network of \( n \) partial OR gates \( \text{POR}(x_{i,1}, x_{i,2}, x_{i,3}) = 1 \) (\( i = 1, \ldots, n \)) and \( m \) wires \( x_{i,j} = x_{h,k} \) (for \( m \) assignments of \( i, j, h, k \)). The truth table of the partial OR gate \( i \) is given in the right side of table 26.

\[
\begin{array}{c|ccc}
\text{part} & x_{i,1} & x_{i,2} & x_{i,3} \\
\hline
X_{i,1} & 0 & 1 & 1 \\
X_{i,2} & 1 & 0 & 1 \\
X_{i,3} & 1 & 1 & 0 \\
\end{array}
\]

The problem solving machine for this network is defined as follows. For all \( i \), each row \( j \) (\( j = 1, 2, 3 \)) of the truth table is associated with a mechanical part of coordinate \( X_{i,j} \) (table 26 – we also say that part \( X_{i,j} \) is labelled by the values of the Boolean variables \( x_{i,1}, x_{i,2}, x_{i,3} \) appearing in the corresponding row. \( Q \) and \( Q^2 \) (with a parabolic cam in between) are now the inputs of \( n \) pairs of differential gears, one linear and the other nonlinear, as before. This time, each linear gear \( i \) has three outputs of coordinates \( X_{i,j} \) (\( j = 1, 2, 3 \)), each nonlinear gears their squares (with parabolic cams in between) so that, for all \( i \): \( Q = \sum_{j=1}^{3} X_{i,j} \) and \( Q^2 = \sum_{j=1}^{3} X_{i,j}^2 \). Therefore, the motions of the parts of each triplet \( (X_{i,1}, X_{i,2}, X_{i,3}) \) are mutually exclusive with one another. If part \( X_{i,j} \) moves with \( Q \), we understand that \( x_{i,1}, x_{i,2}, x_{i,3} \) assume the values appearing in the corresponding row.

This is justified by the following implementation of the wires. For example, let us assume that \( x_{i,1} = x_{h,2} \), which means either \( x_{i,1} = x_{h,2} = 0 \) or \( x_{i,1} = x_{h,2} = 1 \). Looking at table 26, one can see that this wire must be represented by the equations \( X_{i,1} = X_{h,2} \) and \( X_{i,2} + X_{i,3} = X_{h,1} + X_{h,3} \). In fact, if the part that moves in the first triplet is \( X_{i,1} \), this implies \( x_{i,1} = 0 \) (see the intersection between first row and first column of table 26). Then, to satisfy the wire, the part that moves in the second triplet must be \( X_{h,2} \), so that also \( x_{h,2} = 0 \) (intersection between second row and second column of table 26 – having replaced the subfix \( i \) by \( h \)). This justifies the first equation. If instead the
part that moves in the first triplet is either \( X_{i,2} \) or \( X_{i,3} \), this implies \( x_{i,1} = 1 \) (intersection between second or third row and first column). Then, to satisfy the wire, the part that moves in the second triplet must be either \( X_{h,1} \) or \( X_{h,3} \), so that also \( x_{h,2} = 1 \) (intersection between first or third row and second column). This justifies the second equation. In general, we require that the sums \( \sum_j X_{i,j} \), with \( j \) running over the labels with the same value of the same Boolean variable, are conserved across different triplets (e.g. we understand that \( x_{i,1} = x_{h,2} \) and \( x_{h,2} \) are the same Boolean variable in triplets \( i \) and \( h \)). These linear equations (representing the wires) are implemented by systems of gears between the parts involved.

At this point the thought machine is completed. By applying a force to the input part \( Q \), the machine’s motion from \( Q = 0 \) to \( Q > 0 \) produces a solution (provided that there is one, otherwise the machine is jammed) under the simultaneous influence of all the problem constraints: in each triplet, there is only one part that moves, the labels of all the parts that move make up a Boolean assignment that solves the network.

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