Critical conductance of the chiral 2d random flux model

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Abstract

The two-terminal conductance of a random flux model defined on a square lattice is investigated numerically at the band center using a transfer matrix method. Due to the chiral symmetry, there exists a critical point where the ensemble averaged mean conductance is scale independent. We also study the conductance distribution function which depends on the boundary conditions and on the number of lattice sites being even or odd. We derive a critical exponent $\nu = 0.42 \pm 0.05$ for square samples of even width using one-parameter scaling of the conductance. This result could not be obtained previously from the divergence of the localization length in quasi-one-dimensional systems due to pronounced finite-size effects.

Key words: critical conductance, random flux, chiral symmetry, critical exponent, localization length

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1. Introduction

The electronic transport properties of disordered materials depend essentially on the physical symmetry of the system under examination. The well known standard symmetry classes—orthogonal, unitary, and symplectic—are appropriate for describing situations that occur in the presence or absence of time reversal symmetry, and broken spin rotational symmetry, respectively. Recently, an additional chiral symmetry has been attracted considerable interest for disordered two-dimensional systems [1,2,3,4]. This special symmetry may appear if the underlying lattice is bipartite [5], at which eigenfunctions can occupy only one of the two sub-lattices. In two-dimensional lattice models with diagonal disorder, the chiral symmetry is not observed so that for orthogonal and unitary symmetry all electronic states are localized. However, for non-diagonal disorder situations, like the motion of quantum particles subject to random magnetic fluxes as investigated in the present contribution, the eigenvalues occur in pairs $\pm \epsilon_i$, a characteristic feature of the chiral symmetry [5]. Appertaining to the chiral symmetry is a critical point in the center of the tight-binding band with a diverging localization length at energy $E = 0$ [6,7] and a scale independent critical conductance.

2. Model and Method

We consider a two-dimensional tight-binding lattice model where the perpendicular random magnetic field is introduced via complex hopping terms. These are chosen such that the magnetic flux per plaquette is given by the sum of the random Peierls

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phases along the two bonds in the \( z \)-direction

\[ 2\pi \phi_m = \alpha_{m,m+a_z} - \alpha_{m+a_z,m+a_z} \]

The corresponding Hamiltonian with nearest neighbor hopping is:

\[
H = -\sum_m \left( c_{m+a_x}^\dagger c_m + c_{m-a_x}^\dagger c_m + e^{i\alpha_{m,m+a_x}} c_{m+a_x}^\dagger c_m + e^{-i\alpha_{m,m-a_x}} c_{m-a_x}^\dagger c_m \right)
\]

where the width \( L \) (\( x \)-direction) and the length \( L_z \) (\( z \)-direction) of the sample is given in units of the lattice constant \( a \). The \( c_m^\dagger \) and \( c_m \) create or annihilate a Fermi particle at the site \( m \), respectively. The random fluxes are distributed uniformly, \(-f/2 \leq \phi_m \leq f/2\), with probability density \( p(\phi_m) = 1/f \), and the randomness is maximal for \( f = 1 \) used in the following. The average magnetic flux through the system is zero.

We calculate the dimensionless two-terminal conductance \([8]\) for systems of length \( L_z \) via

\[
g = \text{Tr}\{t^\dagger t\} = \frac{1}{N} \sum_i \frac{1}{\cosh^2(x_i/2)}. \quad (2)
\]

Here, \( t \) is the transmission matrix with the \( x_i \) parameterizing its eigenvalues. \( N \) is the number of open channels. Dirichlet boundary conditions are imposed in the transversal direction.

### 3. Conductance

Fig. 1 confirms the theoretical prediction \([3]\) that for quasi one-dimensional systems (Q1D), \( \langle \ln g \rangle \) decreases much slower for \( L \) being odd than for the same system with \( L \) even. This is easy to understand from Eq. (2) and from the mean values of the parameters \( x_i \). In systems with chiral symmetry, the \( x_i \) assume both positive and negative values. When ordered in absolute values, \(|x_1| \leq |x_2| \leq \ldots \), it follows from symmetry reasons that \( x_1 = 0 \) for \( L \) odd, but \( x_1 = -(x_2) \neq 0 \) when \( L \) is even. This is confirmed in Fig. 2 which shows the probability distribution \( p(|x_1|) \) both for \( L \) odd and even. For \( L \) even, \( p(|x_1|) \) converges to the Gaussian distribution with non-zero mean when the length \( L_z \) of the system increases, but \( p(|x_1|) \) remains half-Gaussian for \( L \) odd.

The different form of the distribution of \( x_i \) influences the transmission through very long Q1D systems and thus the conductance. Fig. 3 shows the distribution of the logarithm of the conductance \( p(\ln g) \). In very long Q1D systems, the conductance is determined by the

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**Fig. 1.** The dependence of \( \ln g \) on the square root of the aspect ratio \( L_z/L \) (length to width).

**Fig. 2.** The probability distribution \( p(|x_1|) \) for odd (\( L = 65 \)) and even (\( L = 66 \)) sample widths.

**Fig. 3.** The distribution of \( \ln g \). The solid lines are given by the function (4). Note the divergence of \( p(\ln g) \) for \( \ln g \to 0 \) when \( L \) is odd.
first channel, \( g = \cosh^{-2}(x_1) \), so that
\[
p(ln g) = \int dx_1 p(x_1) \delta(ln g - 2 \ln \cosh x_1).
\]
We find
\[
p(g = \ln g) = \frac{2}{\sqrt{2\pi} \sigma} \coth(\tilde{x}/2) \exp(-\tilde{x}^2/2\sigma),
\]
where \( \tilde{x} = 2 \ln[e^{-y/2} + \sqrt{e^{-y} - 1}] \), and \( \sigma = \langle x_1^2 \rangle \). For \( L \) even, \( p(x_1) \) is Gaussian with a non-zero mean value. Consequently, \( p(ln g) \) is Gaussian, too. The singularity of \( \coth(\tilde{x}) \) in Eq. (4) is eliminated by the exponential decrease of \( p(x_1) \). However, for \( L \) odd, \( p(x_1) \) possesses a maximum at \( x_1 = 0 \) so that \( p(ln g) \) is singular for \( \ln g \to 0 \). Since this singularity survives for any length of the system, there is a non-zero probability to obtain a sample with conductance \( g \) of order of unity. This explains the absence of exponential localization in these systems, we find \( \langle g \rangle \sim L_+^{1/2} \) for \( L \) odd instead as shown in Fig. 1.

4. Scaling of the conductance

Single parameter scaling theory requires that the conductance is a function of only one parameter in the vicinity of the critical point,
\[
\langle g(E, L) \rangle = F[L/\xi(E)].
\]
Here \( \xi(E) \) is a correlation length which diverges as \( \xi(E) \propto |E|^{-\nu} \). In Ref. [9] we have analyzed the scaling of the smallest Lyapunov exponent of Q1D systems with \textit{odd} width and found the critical exponent \( \nu \approx 0.35 \pm 0.03 \). As also shown in Ref. [9], no scaling analysis of the Lyapunov exponent is possible for \( L \) even because of strange finite size effects. Therefore, we verify in this paper the one parameter scaling for 2D square samples \( L \times L \) with even \( L \) using the mean conductance (Fig. 4). First, we subtract from the conductance the finite size correction, given by the irrelevant scaling term \( c/L \),
\[
\langle g(E = 0, L) \rangle = g_c - c/L.
\]
where \( c = 2.06 \) and \( g_c = 1.489 \pm 0.001 \). Then, the data scale to the universal curve, \( \langle g \rangle - c/L = F[EL^{1/\nu}] \) with a critical exponent \( \nu = 0.42 \pm 0.05 \), consistent with our recent result for the divergence of the localization length in samples having odd width [9].

5. Conclusion

The two-terminal conductance of a random flux model has been investigated numerically. It shows critical behavior at the band center due to the chiral unitary symmetry of the system. The critical exponent obtained from a finite size scaling analysis agrees with the one that governs the divergence of the localization length. These results support the notion of universality at the chiral critical point.

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