A Note on Nonperturbative Instability in String Theory

Oleg Andreev∗†

Humboldt–Universität zu Berlin, Institut für Physik
Invalidenstraße 110, D-10115 Berlin, Germany

Abstract

We investigate the possibility that stringy nonperturbative instabilities are described by worldsheet methods. We focus on the case of open bosonic string theory, where the D-instanton plays a role of the bounce, i.e. it describes barrier penetration. In the process, we compute the exponential factor in a decay probability.

PACS: 11.25.Sq

Keywords: nonperturbative instability, D-instanton

It is well-known since the seventies that the perturbative string S-matrix defined as expectation values of worldsheet vertex operators reproduces field theory scattering amplitudes [1]. If one thinks of string theory as field theory with infinite number of fields living in higher dimensions then this statement is equivalent to saying that scattering amplitudes of string theory near its trivial perturbative vacuum are reproduced by worldsheet methods. It is a big problem to understand the vacuum structure of string theory. It is clear that some vacua may be unstable in perturbative or even in nonperturbative sense. In the last case, it seems natural to ask whether one can compute decay probabilities as for example it was done by instanton methods in ordinary field theory [2]. It is clear that this is in principle a solvable problem. However, in practice it turns out to be very hard to deal even with several fields not saying an infinite number of them. A hope maybe that worldsheet methods are appropriate again. A good motivation for this comes from the fact that there are representations of string theory effective actions $S$ via worldsheet objects [3, 4]. These representations are based on partition functions of strings propagating in background fields. Moreover, the actions evaluated at solutions of the corresponding equations of motion coincide with the partition functions namely,

$$S(\lambda^i) = cZ(\lambda^i) \quad ,$$

where $\lambda^i$ are string fields and $Z$ is the partition function. Here, we also include a normalization constant $c$. The extrema of the actions have the known meaning within the worldsheet theory: they are conformal backgrounds. So, the right hand side of Eq.(1) represents a partition function of two-dimensional theory at its fixed point. This simplifies explicit computations of partition functions since there are no UV divergencies anymore. Let us now assume that equations of motion associated with

∗e-mail: andreev@physik.hu-berlin.de
†Permanent address: Landau Institute, Moscow, Russia
the Euclidean action $S_E$ obtained from $S$ by analytic continuation admit a solution $\lambda_i^0$ which from the field theory point of view can be recognized as the bounce, i.e. it describes a decay of some false vacuum $\lambda_i^0$. Then, assuming that the relation (1) is also valid in the Euclidean case, we can immediately get the following representation for the exponential factor in a decay probability per unit time per unit volume

$$w \sim \exp \left[ -cZ_E(\lambda_i^0) \right].$$

(2)

Notice that we do not include $S_E(\lambda_i^0)$ into the exponent.

The purpose of this note is to give an example of explicit computations. To do so, we consider open bosonic string theory where a big progress has been recently achieved in understanding of D-brane decay as open string tachyon condensation [5]. In particular, this was achieved by using a toy field theory model [6] and a background independent open string field theory [4]. Indeed, they turned out to be useful tools to gain intuition on the physics and carry out some explicit calculations. It turns out that they are useful for our purpose as well.

To gain some intuition, we begin with a scalar field theory in Euclidean $p+1$ dimensional space with action

$$S_{ft} = \tau_p \int d^{p+1}x e^{-T \left( 1 + T + \frac{1}{2} \alpha' \partial_i T \partial_i T \right)}.$$

(3)

This theory was used as a toy model for tachyon condensation on unstable branes in [3]. In this context, it describes the open string tachyon living on an unstable $p$-brane whose tension is $\tau_p$. Note that the tension includes a factor of the dilaton (string coupling constant $g$).

One remarkable observation is that the theory belongs to a set of field theory models whose equations of motions admit exact solutions [9]. In particular, a set of exact spherically symmetric solutions associated with the action (3) is given by

$$T_n(x) = \frac{1}{2\alpha'} \left( x_0^2 + \cdots + x_{n-1}^2 \right) - n,$$

(4)

where $n$ ranges from 1 to $p+1$. In the context of tachyon condensation, these solutions are interpreted as the lower dimensional branes. Indeed, they almost reproduce the famous descent relations for the D-brane tensions [6].

It is easy to find the bounce among the set of the solutions. It corresponds to $n = p+1$. Indeed, only in this case the Euclidean action (3) evaluated at the solution is finite. Moreover, one can immediately check that $T_{p+1}$ obeys the boundary conditions for the bounce in the sense of Coleman:

$$\lim_{x_0 \to \pm \infty} T_{p+1}(x) = +\infty, \quad \partial_0 T_{p+1}\big|_{x_0=0} = 0,$$

where $\partial_0 = \partial / \partial x_0$ and $x_0$ is treated as Euclidean time.

At this point we should mention that in the context of tachyon condensation $T_{p+1}(x)$ is called as the D-instanton. Thus, what we have learned from this toy field theory model is a hint on the
physical meaning of the D-instanton: it might describe a decay of an unstable vacuum through barrier penetration. In our case, the unstable vacuum corresponds to $T = +\infty$ (see Fig. 1).

To complete our discussion of the field theory model, let us compute the exponential factor in a decay probability of the unstable vacuum. Evaluating the action at $T_{p+1}(x)$, we obtain

$$S_{ft}(T_{p+1}) = \tau_p e^{p+1}(2\pi\alpha')^{\frac{p+1}{2}}$$

that results in the following expression for the exponential factor

$$w \sim \exp \left[ -\tau_p e^{p+1}(2\pi\alpha')^{\frac{p+1}{2}} \right].$$

Since $S_{ft}$ evaluated at $T = +\infty$ vanishes, there is no the corresponding contribution in (7).

So far we have just noticed that the D-instanton might be interpreted as a bounce in the sense that it describes a decay of some unstable vacuum. However, our discussion given within the field theory model has two disadvantages: it can not in principle provide us with the desired representation of the decay factor (2). The relation of the solution $T_{p+1}(x)$ with the D-instanton might seem not sufficiently convincing. Fortunately, both of these disadvantages disappear in the background independent open string field theory [4]. To the leading order in derivatives, its Euclidean action is simply [7, 8]

$$S_E = \tau_p \int d^{p+1} x e^{-T} \left( 1 + T + \alpha' \partial_i T \partial_i T + \ldots \right),$$

where the dots stand for an infinite number of higher derivative terms. These terms can in principle be considered as a result of integration over the other open string modes that modifies the action (3). It turns out that this modification of the action does not have a strong influence on the existence of a set of exact spherically symmetric solutions like (4).

We now have

$$T_n(x) = \frac{t}{2\alpha'} \left( x_0^2 + \cdots + x_{n-1}^2 \right) + a,$$
where \( a \) and \( t \) are some parameters which will be determined later.

We will again specialize to \( T_{p+1} \) because it results in a finite action and obeys the boundary conditions (3). Due to these reasons, we will call it the bounce. It turns out that the action evaluated at the bounce can be rewritten as a function of the parameters in the following form [4]

\[
S_E(a, t) = \tau p \left[ 1 + \beta_a \frac{\partial}{\partial a} + \beta_t \frac{\partial}{\partial t} \right] Z_E(a, t),
\]

(10)

where \( \beta_a = -a - (p + 1)t, \beta_t = -t \) and \( Z_E(a, t) = (2\pi \alpha'/t)^{p+1} \exp\left[ -a \right] \frac{\Gamma(1 + t)}{\Gamma(p + 1)}. \) \( \gamma \) denotes the Euler’s constant.

The parameters in (10) are determined by demanding that \( S_E(a, T) \) is stationary under their variations. It is a simple task to do so for \( a \) [5]. Indeed, in this case a simple algebra leads to

\[
a(t) = (p + 1) \left[ -t + t \frac{\partial}{\partial t} \left( \gamma t + \ln \Gamma(1 + t) \right) \right].
\]

As a consequence, the action for the bounce reduces to

\[
S_E(a(t), t) = \tau p Z_E(a(t), t).
\]

(11)

Before going on, it is time to remind the meaning of the entries on the right hand side of Eq.(10) as objects of the underlying worldsheet theory [4]. Consider open bosonic string in Euclidean target space in the presence of the tachyon background whose profile is similar to \( T_{p+1} \). This is a simple choice of the background for which the partition function can be computed exactly. In a special scheme, it is given by \( Z_E(a, t) \) [4]. As to \( \beta_a \) and \( \beta_t \), they are the renormalization group (RG) beta functions. A simple RG analysis shows that the parameters flow from zero in the UV to infinity in the IR. The last means that all \( X^i \) in the path integral are subject to the Neumann boundary conditions at the UV and the Dirichlet boundary conditions at the IR.

Having reminded the worldsheet theory, we have all at our disposal to achieve the purpose. First, let us note that the formula (11) is the desired representation for the effective action [3]. At this point, we have only to check that it results in a finite action for the bounce. \( t \) is easily found from the correspondence between the fixed points of RG on the worldsheet and extrema of the effective action. It is unique and given by its value in the IR fixed point. Next, plugging \( t = \infty \) into the partition function, we get \( Z_E(a(\infty), \infty) \equiv Z_D = (4\pi^2 \alpha')^{p+1/2} \). From the worldsheet point of view, it is of course the expected result as the string path integral is finite for the Dirichlet boundary conditions. Second, the desired relation between the bounce and the D-instanton follows from a canonical construction of the D-instanton within the worldsheet theory (see, e.g. [11]) as at \( t = \infty \) all \( X^i \) are subject to the Dirichlet boundary conditions. Finally, we finds for the exponential factor in the decay probability

\[
w \sim \exp \left[ -\tau p Z_D \right].
\]

(12)

Let us conclude by several short remarks:

(i) So far, there is not known any partition function representation for string theory effective action that includes all string modes. It is the reason why we dealt with the background independent open string field theory.

---

2Note that at the beginning this assumes the use of parameters (bare couplings) which differ from the parameters in \( T_{p+1} \). The parameters \( a \) and \( t \) are the renormalized couplings.

3This relation is obvious for the UV as it directly follows from Eq.(10) but it is far from obvious for the IR.
(ii) Our analysis of the actions for the bounce is in fact similar to the computation of the descent relations between brane tensions in [1, 8].

(iii) The idea that the D-instantons lead to nonperturbative effects like \( \exp(-O(1/g)) \) is an old one (see, e.g., [11, 12]). In particular, from the point of view [12], the right hand side of Eq. (12) with \( p = 25 \) is interpreted as the open string partition function with the Dirichlet boundary conditions for all \( X^i \). This means that what we found can be called as the partition function representation for the exponential factor in a decay probability.

(iv) We have made no attempt to study quantum corrections. It is clear that it would include higher genera of two-dimensional surfaces and, as a consequence, appearance of closed string modes. This makes the problem more involved than even including gravity within the field theory analysis [13].

(v) A perturbative instability of the standard bosonic open string vacuum (\( T = 0 \) of Fig.1) has been discussed in [14], where its decay rate has been evaluated via one-loop computations.

Acknowledgements. We have been benefited from discussions with L. Alvarez-Gaume, I. Bars, and J. Polchinski. We also would like to thank H. Dorn and A.A. Tseytlin for comments and reading the manuscript. This work is supported in part by DFG under grant No. DO 447/3-1 and the European Commission RTN Programme HPRN-CT-2000-00131.

References

[1] J. Scherk, Nucl.Phys. B31 (1971) 222;
    A. Neveu and J. Scherk, Nucl.Phys. B36 (1972) 155;
    J. Scherk and J.H. Schwarz, Nucl.Phys. B81 (1974) 118;
    T. Yoneya, Prog.Theor.Phys. 51 (1974) 1907.

[2] M.B. Voloshin, I.Yu. Kobzarev, and L.B. Okun', Yad. Fiz. 20 (1974) 1229;
    S. Coleman, Phys.Rev. D15 (1977) 2929;
    C.G. Callan and S. Coleman, Phys.Rev. D16 (1977) 1762.

[3] E.S. Fradkin and A.A. Tseytlin, Nucl.Phys. B261 (1985) 1.

[4] E. Witten, Phys.Rev. D46 (1992) 5467; Phys.Rev. D47 (1993) 3405;
    S.L. Shatashvili, Phys.Lett. B311 (1993) 83.

[5] A. Sen, JHEP 9912 (1999) 027; Int.J.Mod.Phys. A14 (1999) 4061.

[6] J.A. Minahan and B. Zwiebach, JHEP 0009 (2000) 029; JHEP 0103 (2001) 038.

[7] A.A. Gerasimov and S. Shatashvili, JHEP 0010 (2000) 034.

[8] D. Kutasov, M. Marino, and G. Moore, JHEP 0010 (2000) 045.

[9] J. Goldstone and R.L. Jaffe, as cited in [3].

[10] J. Polchinski, “TASI lectures on D-branes”, Report No. NSF-ITP-96-145, hep-th/9611050

[11] S. Shenker, in “Random Surfaces and Quantum Gravity”, edited by O. Alvarez, E. Marinari, and P. Windey, Plenum, New York 1991.

[12] J. Polchinski, Phys.Rev. D50 (1994) R6041.

[13] S. Coleman and F. De Luccia, Phys.Rev. D21 (1980) 3305;
    E. Witten, Nucl.Phys. B195 (1982) 481.

[14] K. Bardakci and A. Konechny, “Tachyon condensation in boundary string field theory at one loop”, hep-th/0105093.
    B. Craps, P. Kraus, and F. Larsen, JHEP 0106 (2001) 062;
    O. Andreev and T. Ott, “On One-Loop Approximation to Tachyon Potentials”, hep-th/0109187.