Best values of parameters for interacting HDE with GO IR-cutoff
in Brans-Dicke cosmology

A. Khodam-Mohammadi\textsuperscript{1*}, E. Karimkhani\textsuperscript{1†} and A. Sheykhi\textsuperscript{2 ‡}

\textsuperscript{1} Department of Physics, Faculty of Science, 
Bu-Ali Sina University, Hamedan 65178, Iran

\textsuperscript{2} Physics Department and Biruni Observatory, 
College of Sciences, Shiraz University, Shiraz 71454, Iran

Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), 
P.O. Box 55134-441, Maragha, Iran

Abstract

We investigate the interacting holographic dark energy (HDE) with Granda-Oliveros (GO) IR-cutoff in the framework of Brans-Dicke (BD) cosmology. We obtain the equation of state (EoS) parameter of HDE, \(w_D\), the effective EoS parameter \(w_{\text{eff}}\), the deceleration parameter \(q\) and the squared of sound speed \(v_s^2\) in a flat FRW universe. We show that at late time the cosmic coincidence problem can be alleviated. Also we show that for non-interacting case, HDE can give a unified dark matter-dark energy profile in BD cosmology, except that it cannot solve the coincidence problem in the future. By studying the equation of state parameter, we see that the phantom divide may be crossed. Using the latest observational data, we calculate the best values of the parameters for interacting HDE in BD framework. Computing the deceleration parameter implies that the transition from deceleration to the acceleration phase occurred for redshift \(z \geq 0.5\). Finally, we investigate the sound stability of the model, and find that HDE with GO cutoff in the framework of BD cosmology can lead to a stable DE-dominated universe favored by observations, provided we take \(\beta = 0.44\) and \(b^2 < 0.35\). This is in contrast to HDE model in Einstein gravity which does not lead to a stable DE dominated universe.

\textsuperscript{*} khodam@basu.ac.ir
\textsuperscript{†} E.karimkhani91@basu.ac.ir
\textsuperscript{‡} asheykhi@shirazu.ac.ir
I. INTRODUCTION

Nowadays, countless and precise cosmological observations, type Ia supernova (SNIa) \[1\], the 9th year data of the WMAP mission \[2\], the cosmic microwave background radiation \[3\] and the large scale structure (LSS) \[4\], confirm that our Universe is currently undergoing a phase of accelerated expansion. The provenance of acceleration may be caused due to an un-known energy component with negative pressure, called *dark energy* (DE). According to cautious analysis of cosmological observations, about %73 of the total energy content of the universe has been occupied by DE, around %23 pressureless dark matter (DM), and around %4 of total energy content is denoted to the normal baryonic matter. However, at the present, the radiation part can be ignored in comparison to other components. Despite of mysterious nature of DE, during the past decades, many candidates have been nominated in order to describe DE. See \[5\] and references therein. The first and simplest candidate of dark energy is the cosmological constant $\Lambda$, with a constant equation of state (EoS) parameter $w_\Lambda = -1$. Although this model is consistent very well with all observations, it suffers the cosmic coincidence problem. The solution of the cosmic coincidence problem requires that our universe behaves in such a way that the ratio of DM to DE densities must be a constant of order unity or varies more slowly than the scale factor and finally reaches to a constant of order unity \[6–8\]. In order to solve this problem, the dynamical DE models have been proposed. Some analysis on the ‘SNIa’ observational data reveals that the time varying DE model gives a better fit compare with a cosmological constant \[9\].

Among of many dynamical DE models, the so called HDE model, based on holographic principle proposed by ’t Hooft \[10\] and Susskind \[11\], has attracted a lot of attention. According to the holographic principle, the number of degrees of freedom of a physical system scales with its area instead of its volume. The development of holographic principle for our purpose was put forwarded by Cohen et al. \[12\] specify an infrared cutoff length scale and then by Hsu \[13\] and Li \[14\], who applied HDE model for solving the DE puzzle. In this model the energy density is written by $\rho_D = 3\eta^2 M_{\text{pl}}^2 / L^2$, where $L^2$ is proportional to the area which provides an IR-cutoff, $M_{\text{pl}}$ is the Planck mass and the numerical constant $3\eta^2$ is introduced for convenience and utility \[13–15\]. At following, we would work in the framework of *natural unit*, where ($c = h = 1$). The IR-cutoff ’$L$’ plays an essential role in HDE model. If $L$ is chosen as particle horizon, the HDE cannot produce an acceleration expansion \[16\],
while for future event horizon, Hubble scale $L = H^{-1}$, and apparent horizon (AH), as an IR-cutoff, the HDE can simultaneously drive accelerated expansion and solve the coincidence problem [8, 17, 18]. Thereupon, Gao et al. [19] recommended that the HDE density may inversely be proportional to the Ricci scalar curvature. Succeeding this, Feng [20] studied this model in the framework of BD cosmology. Afterward on Granda-Oliveros [21, 22] proposition, a new cutoff based on wholly dimensional basis, which adds a term including the first derivation of the Hubble parameter, was introduced. This cutoff looked alike the Ricci scalar of the FRW metric but with two free parameters, $\rho_D = 3M_{\text{pl}}^2(\gamma H^2 + \beta H)$, where $\gamma$ and $\beta$ are constant parameters of order unity. This model depends on local quantities, avoiding in this way the causality problem that is exist in the holographic dark energy based on the event horizon [23]. Despite of some success in corresponding with other DE models like scalar field [24], Chaplygin gas [25] and study of cosmological evolution [26, 27] and obtaining the cosmological constrain on this model [28], this form of cutoff unable to fit the most recent growth data on structure formation in the ordinary Einstein general relativity frame work [29, 30].

For the reason that the HDE density belongs to a dynamical cosmological constant, we need a dynamical framework to accommodate it instead of Einstein gravity. The best choice for this, is BD theory which is the scalar tensor theory and was invented first by Jordan [31] and then ripened by brans and Dicke [32]. This theory is based on Mach’s principle, which is a fundamental principle to explain the origin of inertia. In attempting to incorporate Mach’s principle, the BD theory introduces a time dependent inertial scalar field $\varphi$, which plays the role of the gravitational constant $G$, so that $\varphi(t) \propto 1/G$ and is determined by the matter field distributions. So the gravitationnal fields are described by the metric $g_{\mu\nu}$ and the BD scalar field $\varphi$, which has the dimension $[\varphi] = [M]^2$. In BD theory, the scalar field $\varphi$ couples to gravity via a coupling parameter $\omega$ and it has been generalized for various scalar tensor theories. Therefore, the investigation on the holographic models of DE in the framework of BD theory, has of great interest and have been accomplished in [33].

The combination of BD field and HDE can accommodate $w_D = -1$ crossing for the EoS parameter of non-interacting HDE [34]. It was shown that in BD cosmology when an interaction between DE and DM is taken into account, the transition from normal state $w_D > -1$ to the phantom regime $w_D < -1$ can be more easily accounted compared to the Einstein gravity. Also, in [35], it has been demonstrated that the accelerated expansion will
not be achieved in BD theory without interactions, when the Hubble horizon is taken as the IR cut-off. Howbeit, when the event horizon takes the role of IR cut-off, an accelerating universe is obtained. Furthermore, the phantom crossing is more easily achieved when the matter and the HDE undergo an exotic interaction [36].

In this paper we investigate the HDE in BD cosmology using a GO as IR-cutoff. We assume the BD field as a power law of the scale factor, \( \varphi \propto a^n \), and as it has mentioned in [36, 37] that there is no compelling reason for this choice. However it has been shown that for small \( |n| \) it leads to consistent result. Usually in papers which investigate the DE in the BD cosmology with different IR-cutoff [8, 20, 36, 37], authors assume that matter evolves as \( \rho_m \propto \rho_{m_0}a^{-3} \) or some other assumptions like this for \( \rho_m \) and by use of this, they calculated the EoS parameter of DE. This assumption had been also considered in interacting case, which has been given more complicated or sometimes wrong relations for cosmological quantities. In fact by considering interaction between DE-DM, energy conservation is not valid separately for each component. Here we are attempting to provide a more generic way, without any restricted supposition, to calculate the EoS parameter and some other cosmological parameters of our interest.

It must be noted that, although, there is not any evidence for existence any direct interaction between DM and DE, no known symmetries prevent such interaction. On the other hands, such a choice for interacting term don’t make any discordance with observation as mentioned in Ref. [38]. In addition, the latest cosmological constraints on the coupled DE models, in which the quintessence scalar field non-minimally couples to the cold DM, from the recent Planck measurements has been done [39].

The last main task that must be investigated is the stability analysis of model by calculating the squared of sound speed (\( v_s^2 = dp/d\rho \)) [40]. The analysis of squared of sound speed could say us about the perturbation growth and the structure formation at present. However this quantity does not enough insight to say the model is surely stable, but at least can show sounds of instability of the model. The sign of \( v_s^2 \) plays a crucial role in determining the instability of the background evolution. If \( v_s^2 < 0 \), it means that we have the classical instability of a given perturbation. In contrast \( v_s^2 > 0 \), leaves chance for greeting a stable universe against perturbations.

This paper is outlined as follows. In section II a brief review of the HDE in BD cosmology is given. In section III we describe the physical contest which we are working in and we
derive the EoS parameter of DE, \( w_D \), effective EoS parameter \( (w_{\text{eff}}) \) and the deceleration parameter \( q \) in a flat FRW universe, for various choice of interaction. The quantity of interest for analyzing the coincidence problem is the ratio \( u = \rho_m/\rho_D \), which we make a spacious discussion on this quantity in this section. In section \[ IV \] the stability analysis through the dynamical DE model is studied. In section \[ V \] we give a detailed discussion on all calculations. We summarize our results in section \[ VI \].

II. GENERAL FORMALISM

We start with a brief review on HDE in the framework of BD cosmology. The BD field equation can be written as \[ 32, 41 \]

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{\omega}{\varphi^2}(\nabla_{\mu}\varphi\nabla_{\nu}\varphi - \frac{1}{2}g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}\varphi) - \frac{1}{\varphi}(\nabla_{\mu}\nabla_{\nu}\varphi - g_{\mu\nu}\nabla_{\alpha}\nabla^{\alpha}\varphi) = \frac{1}{\varphi}T_{\mu\nu},
\]

where \( \varphi \) is the BD scalar field which is allowed to vary with space and time and \( \omega \) is the generic dimensionless parameter of the BD theory. In this theory, the total energy momentum tensor \( T_{\mu\nu} = \text{diag}(-\rho, p, p, p) \), minimally couples to the gravity and there is no interaction between the scalar field \( \varphi \) and the matter field. General relativity is a particular case of the BD theory, corresponding to \( \omega \rightarrow \infty \) \[ 41 \]. In a FRW universe, with line element metric

\[
 ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right],
\]

the BD field equations take the form \[ 42 \]

\[
 3 \left( \frac{a^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{2} \frac{\dot{\varphi}^2}{\varphi^2} + 3 \frac{\ddot{a}}{a} \varphi = \frac{1}{\varphi} (\rho_m + \rho_D)
\]

\[
 2 \frac{\ddot{a}}{a} + \frac{a}{a^2} + \frac{k}{a^2} + \frac{1}{2} \frac{\dot{\varphi}^2}{\varphi^2} + 2 \frac{\dot{a}}{a} \frac{\dot{\varphi}}{\varphi} = -\frac{1}{\varphi} p_D,
\]

where \( a(t) \) is the dimensionless scale factor of the universe, \( \rho = \rho_m + \rho_D \) is the total energy density and the curvature parameter \( k = -1, 0, 1 \), represent spatially open, flat and closed universe, respectively. Also for simplicity we assuming \( \varphi = \varphi(t) \). The equation of motion for BD scalar field is given by \( (2\omega + 3)\nabla_{\mu}\nabla^{\mu}\varphi = T \), where \( T \) is trace of energy momentum tensor \( T_{\mu\nu} \). Since the DM is pressureless, thus the total pressure equals the DE pressure, \( p = p_D \). We further assume that both components do not conserve separately but interact
with each other in such a manner that the continuity equations take the form

\[
\dot{\rho}_D + 3H(1 + w_D)\rho_D = -Q, \tag{5}
\]

\[
\dot{\rho}_m + 3H\rho_m = Q, \tag{6}
\]

where \(w_D = p_D/\rho_D\), denotes the EoS of DE, and \(Q\) stands for the interaction term. It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction \(Q\). It is worth noting that the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor \(H\)) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i) \(Q \propto H\rho_D\), (ii) \(Q \propto H\rho_m\), or (iii) \(Q \propto H(\rho_m + \rho_D)\). Thus hereafter we consider only the first case, namely \(Q = b^2H\rho_D = \Gamma\rho_D\), where \(b^2\) is a coupling constant.

III. HDE WITH GO CUTOFF IN FLAT BD THEORY

We assume the energy density of HDE with GO cutoff in BD theory in the form

\[
\rho_D = 3\varphi \left(\gamma H^2 + \beta \dot{H}\right), \tag{7}
\]

where \(M_{pl}^2\) has been replaced by \(\varphi\) in BD cosmology as mentioned previously. Assume the BD field is proportional to the scale factor as, \(\varphi \propto a^n\), it then follows that

\[
\frac{\dot{\varphi}}{\varphi} = nH, \quad \frac{\ddot{\varphi}}{\varphi} = n^2H^2 + n\dot{H}, \quad \dddot{\varphi} = (n + \frac{\dot{H}}{H^2})H. \tag{8}
\]

Using relations (8), Eqs. (3) and (4) for flat FRW universe reduce to

\[
\rho_D = \frac{\varphi H^2}{(1 + u)} \left[ 3(1 + n) - \frac{\omega n^2}{2} \right], \tag{9}
\]

\[
\rho_D = -\frac{H^2\varphi}{w_D} \left[ 3 + \frac{\dot{H}}{H^2}(2 + n) + n^2 + \frac{\omega n^2}{2} + 2n \right], \tag{10}
\]

where \(u = \rho_m/\rho_D\) is the ratio of energy densities. Differentiating Eq. (11) with respect to the cosmic time \(t\), gives

\[
\dot{\rho}_D = 3\dot{\varphi} \left(\gamma H^2 + \beta \dot{H}\right) + 3\varphi \left(2\gamma \dot{H}H + \beta \ddot{H}\right). \tag{11}
\]
Equating Eqs. (9) and (10), leads to
\[
\frac{\dot{H}}{H^2} = -\frac{3w_D[(1 + n) - \frac{\omega n^2}{6}]}{(2 + n)(1 + u)} - \frac{3 + n^2 + 2n + \frac{\omega n^2}{2}}{(2 + n)}.
\] (12)

On the other hand, by inserting Eq. (7) in (9), we arrive at
\[
\frac{\dot{H}}{H^2} = \frac{(1 + n) - \frac{\omega n^2}{6}}{(1 + u)\beta} - \frac{\gamma}{\beta}.
\] (13)

It’s useful here to examine the signature of the deceleration parameter,
\[
q = -1 - \frac{\ddot{a}}{aH^2}.
\]

Since \(H = \dot{a}/a\), we have \(\ddot{a}/a = \dot{H} + H^2\) and from (13), it follows that
\[
q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{\gamma}{\beta} - \frac{(1 + n) - \frac{\omega n^2}{6}}{(1 + u)\beta}.
\] (14)

Despite of the obtained equation for deceleration parameter in [37], here we see that the BD field, \(\varphi\), doesn’t appear in Eq. (14) and therefore it caused to make the best estimate for \(q\) and \(u\) parameters. Recent constrain on interacting DE models in BD cosmology, gives the best value of \(n \approx 0.005\) and \(\omega \approx 1000\) [43, 44]. Because of very small value of \(n\), only terms of order \(\omega n\) and \(\omega n^2\) gets important and we can neglect again terms of order \(n\) and \(n^2\) in calculations. At present time, \(u \approx 0.4\), the deceleration parameter \(q\) can obviously be negative if
\[
\omega n^2 < 6 - 8.4\beta \left(\frac{\gamma}{\beta} - 1\right),
\] (15)

where it can give a bound on \(\omega n^2\) at present by obtaining \(\beta\) and \(\gamma\). By equating Eqs. (12) and (13), the EoS parameter \(w_D\), of the HDE in BD theory is given by
\[
w_D = \frac{1}{3} \left[A(1 + u) - \frac{2 + n}{\beta}\right]
\] (16)

where
\[
A = -\frac{1}{1 + n - \frac{\omega n^2}{6}} \left[3 + n^2 + 2n + \frac{\omega n^2}{2} - \frac{(2 + n)\gamma}{\beta}\right].
\] (17)

Neglecting terms of order \(n \approx 0.005\) while keeping \(\omega n^2 \approx 0.025\), we find
\[
w_D = -\frac{2}{3\beta} - (1 + u) \left(1 - \frac{4\gamma}{\beta} - \frac{2\omega n^2}{6}\right).
\] (18)

Also by considering Eq. (12) and solving it for \(w_D\) in term of \(q\) \((q = -1 - \dot{H}/H^2)\), we obtain
\[
w_D = -\frac{(1 + u)}{3} \left(1 + n + \frac{n^2 + \frac{\omega n^2}{2} - q(2 + n)}{1 + n - \frac{\omega n^2}{6}}\right)
\]
\[
\approx -(1 + u) \left(\frac{2 + \omega n^2 - 4q}{6 - \omega n^2}\right),
\] (19)
where in the last step, we have neglected again terms of order $n$ and $n^2$ and keep only terms of order $\omega n \approx 5.0$ and $\omega n^2 \approx 0.025$. Taking $u \approx 0.4$ and $q = -0.6$ for the present time, which has been parameterized recently by Pav ón et al. [45], and using Eq (19), we could estimate the EoS parameter, $w_{D0} \approx -1.04$, which is consistent with observational data of [46, 47] and WMAP9+SNLS+HST data [48].

The effective EoS parameter comes out to be

$$w_{\text{eff}} = \frac{p}{\rho} = \frac{w_D}{1 + u} = -\left(\frac{2 + \omega n^2 - 4q}{6 - \omega n^2}\right). \quad (20)$$

It is worth while to mention that the acceleration ($q \leq 0$) in BD cosmology started at $w_{\text{eff}} \leq -(2 + \omega n^2)/(6 + \omega n^2) \approx -0.34$ which has a very small difference from $-1/3$. This is only due to $\omega n^2$ term. The quantity $w_{\text{eff}}$ can also determine whether Universe evolves in phantom phase or not. Besides, by considering super acceleration phase, where $\dot{H} > 0$ (i.e. $q < -1$), the effective EoS parameter (20) reduced to $w_{\text{eff}} < -1$, which coincides perfectly with phantom phase.

By taking the time Derivative of $u$, and using Eqs. (5) and (6), we can obtain

$$\dot{u} = 3Hu \left[w_D + \frac{b^2}{3} \left(\frac{1 + u}{u}\right)\right]. \quad (21)$$

If we now define the e-folding $x$ with definition $x = \ln a = -\ln(1 + z)$, where $z = a^{-1} - 1$, is the redshift parameter and using the fact that $d/d(x) = \frac{1}{H} d/d(t)$, then Eq. (21) yields

$$u' = u \left[A(1 + u) - \frac{2 + n}{3\beta}\right] + b^2 (1 + u) \quad (22)$$

where prime denotes derivative with respect to $x$ and $\Gamma/H = b^2$ is the interaction parameter. By solving the differential equation (22), the ratio of energy densities, $u(x)$, would be gained as

$$u(x) = \frac{1}{2\beta A} \left\{C \tan \left[\frac{Cx}{2\beta} + \arctan \left(\frac{9\beta A - 5n + 5\beta b^2 - 10}{5C}\right)\right] - \beta A + 2 + n - \beta b^2\right\} \quad (23)$$

where the parameter $C$ is given by

$$C = \sqrt{4A\beta(n + 2) - (\beta b^2 - 2 - A\beta - n)^2}. \quad (24)$$

In limiting case of ordinary Einstein general relativity, (where $n \to 0$, $\omega n^2 \to 0$, $\omega \to \infty$), we find

$$w_D = -\left[1 - \frac{\gamma}{3\beta}(1 + u) + \frac{2}{3\beta}\right]. \quad (25)$$
For a DE dominated universe, where $u = 0$, it reduces to the EoS parameter of Granda-Oliveros [21]

$$w_D = -1 - \frac{2}{3\beta}(1 - \gamma).$$  \hspace{1cm} (26)

However, a comparison between Eqs. (25) and (18) shows that crossing the phantom divide line for the HDE in BD gravity can be more easily achieved for than when resort to the Einstein gravity. The effective EoS parameter with above limiting case becomes

$$w_{\text{eff}} = \frac{p}{\rho} = \frac{w_D}{1 + u} = -\left(\frac{1 - 2q}{3}\right).$$  \hspace{1cm} (27)

By taking $q = -0.6$ [45], we have $w_{\text{eff}} = -0.733$, which shows the quintessence phase of the universe for present time. In section V, we will discuss on all obtained quantities.

Using the continuity equation (6), the energy density of dark matter in the interacting case yields

$$\frac{\rho_m}{\rho_{m_0}} = \exp[-3x + 3b^2(F(x) - F(x_0))]$$  \hspace{1cm} (28)

where $\rho_{m_0}$ is current value of matter energy density and

$$F(x) = \int u(x)dx = \frac{1}{2A} \times$$

$$\left\{x(-A - b^2 + \frac{1}{\beta}) + \ln \left(1 + \tan \left[Cx \frac{2\beta}{1 + \arctan \left(\frac{9\beta A - 5n + 5\beta b^2 - 10}{5C}\right)}\right]^2\right)\right\}$$  \hspace{1cm} (29)

and $F(x_0)$ is the current value of $F(x)$. In comparison to the standard matter density law $\rho_m \propto \exp(-3x) = a^{-3}$, it shows that $\rho_m/\rho_{m_0} = \exp[3b^2(F(x) - F(x_0))]$, which is related to interaction parameter, BD parameter and $n$. This departure of standard law is small. More discussion will be left by Sec. VI It is worth while to mention that for non-interacting case, $b^2 = 0$, the matter energy density can be given by the standard law. The dark energy density is also given by $\rho_D = u(x)\rho_m$.

The evolution of gravitational constant $G \propto 1/\varphi$ by considering $\varphi \propto a^n$, is obtained as $|\dot{G}/G| = \dot{\varphi}/\varphi = nH$. The upper bound of this quantity with given value $n = 0.005$, has a good agreement with latest constraint on the evolution of $G$ at present which is $|\dot{G}/G| < 10^{-11} yr^{-1}$ [43, 49]. Also if we consider the another form of evolution of $\Delta G/G_0 \sim -n\ln(a)$ or $\dot{G}/G_0 \sim -nH$, which is suggested in some theories(e.g. in fact theoretical QFT models [29, 50, 51]), ($G_0$ is current value of $G$), very slow evolution of $G$ is also expected by our given value of $n = 0.005$. 
IV. SOUND STABILITY OF THE MODEL

From observations we know that our universe is in a DE dominated phase. Thus any viable DE model should result a stable DE dominated universe. One simple way to check such a stability for any new DE model is to discuss the behavior of the square sound speed \( v_s^2 = dp/d\rho \) in a DE dominated universe. The sign of \( v_s^2 \) plays a crucial role in determining the stability of the background evolution. If \( v_s^2 < 0 \), it means that we have the classical instability of a given perturbation. In contrast \( v_s^2 > 0 \), leaves chance for greeting a stable universe against perturbations. However, this does not enough insight to say the model is surely stable but at least can show sounds of instability in the model. This approach has been used for exploring some DE models. For example in the authors investigated the behavior of the square sound speed for HDE as well as the agegraphic DE models and found both of these models are instable against background perturbations. Also it was shown that chaplygin gas and tachyon DE have positive squared speeds of sound with, \( v_s^2 = -w \), and thus they are supposed to be stable against small perturbations. The stability of the GDE models was studied in, and it was shown that the GDE models are not capable to result a stable DE dominated universe. Also, a same procedure was considered in to show the stability of the GDE in the chameleon BD theory.

In our model, \( \rho = \rho_D(1 + u) \) and for pressureless CDM, \( p = p_D \), the quantity \( v_s^2 \) for a flat FRW universe is obtained as

\[
v_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{\dot{\rho}_D w_D + \rho_D \dot{w}_D}{\rho_D(1 + u) + \rho_D \ddot{u}}
\]  

(30)

Taking the time derivative of Eq. (16) and using Eqs. (5) and (21), we can obtain the following equation for sound speed with respect of \( x \)

\[
v_s^2 = -\frac{w_D \left( A\dot{u}(x) - 3w_D - b^2 - 3 \right) + \frac{b^2}{3} A \left( 1 + u(x) \right)}{3(1 + u(x) + w_D)}.
\]  

(31)

Detailed discussion will be given in the next section.

V. DISCUSSION

In this work for simplicity and in order to reduce the unconstrained parameters, we choose \( \gamma = 2\beta \). It is a reasonable assumption since in Refs. the numerical values for these
parameters has been obtained by use of observational data in Einstein gravity, which confirm that $\gamma \approx 2\beta$. Also this assumption is very accurate in Ricci DE model [24].

We start our analysis by computing $u(x)$ with respect to $x$ and then the behavior of all cosmological quantities such as the EoS of DE, $w_D$, effective equation of state, $w_{\text{eff}}$, deceleration parameter, $q$, and $v^2_s$ are discussed. At present time where $x = 0 (z = 0)$, the following values of cosmological parameters “$u = 0.4, q = -0.6, \omega n^2 = 0.025, |n| \approx 0$” are assumed, which have been chosen as the best values of recent observational data [43, 45].

First, we discuss on the deceleration parameter $q$ in the spatially flat FRW case. From the mentioned cosmological data and Eq. (14), the parameter $\beta$ is calculated as $\beta \approx 0.44$. As it is shown in Fig. 1, in order to have the present acceleration expansion ($q < 0$), an upper limit $\beta < 0.71$ is found and from Eq. (13), for supper acceleration expansion ($\dot{H} > 0$), this limit reduced to $\beta < 0.36$. Therefore, it shows that Ricci DE model which is a famous model of HDE with Ricci scaler as IR cutoff, namely $\rho_D = 3C^2 \varphi (2H^2 + \dot{H})$, the acceleration expansion can be achieved in BD cosmology provided $C^2 < 0.71$.

In Fig. 2 the evolution of $u(x)$ is plotted versus $x$ for various interacting parameters. It shows that $u(x)$ is bounded which depend on interaction parameter. By increasing the interacting parameter, $u(x)$ evolves slower and will reach to a finite value according to interacting parameter at infinity and at past the upper limit of $u(x)$ is of order unity and it reached to a smaller saturated value, for larger values of $b^2$. For example $u(x)$ is bounded between (3.00-0.13) for $b^2 = 0.4$ and (3.27-0.06) for $b^2 = 0.2$. The same of this behavior
is seen in Refs. [59, 60]. For non-interacting case $u(x)$ vanishes at late time. Therefore it can alleviate the coincidence problem only for interacting model. Behavior of matter energy density versus $x$ for various $b^2$, is illustrated in Fig. 3 and Fig. 4 shows the ratio of matter energy density with standard one in versus $x$. These figures show that the departure of matter energy density with respect to standard law (or non-interacting case) become smaller at future (see figure 3) and it become larger by increasing the interaction term (see figure 4). The evolution of dark energy density is plotted in Fig. 5. As it is shown, $\rho_D$ reduce to a finite non zero value for interacting DE models.
Inserting $u(x)$ from (23) into (14), the deceleration parameter, $q$ is calculated with respect to $x$ for the best value of $\beta \approx 0.44$. Figure 4 shows that for all values of $b^2$, the deceleration parameter transits from deceleration ($q > 0$) to acceleration ($q < 0$) at $x < -0.4$ which corresponds to $z > 0.5$. However by increasing $b^2$ the transition point moves to older Universe. Further it shows that the interaction will affect on late time acceleration. Increasing $b^2$ corresponds with decreasing $q$ at any time.

In figure 7 the EoS parameter of HDE (18) with respect to $x$ is plotted for $\beta \approx 0.44$. It shows that for all values of $b^2$ the EoS parameter behaves similar to $q$. Likewise, from Eq.
we find that the acceleration phase is started from $w_D \leq -0.47$ and at present it gives $w_D = -1.04$ which shows a phantom DE behavior. The effective EoS parameter, which can determine the phases of acceleration of the universe, plotted in Fig. 8 for $\beta \approx 0.44$. It shows that for all values of $b^2$ the universe transits from quintessence ($-0.34 > w_{\text{eff}} > -1$) to phantom phase ($w_{\text{eff}} < -1$). From this figure we see that at early time, roughly $x < -2$ which can be translated to $z > 6.4$, and hence $w_{\text{eff}} \to 0$, which indicates a pressureless DM dominated universe, for $b^2 = 0$. However, for $b^2 > 0$, the effective EoS parameter tends to finite negative value. Therefore, in non-interacting case, HDE can give a unified DM-DE
profile in BD cosmology. It must be noted that the parameter $\beta$ plays a crucial role in $w_{\text{eff}}$. This fact shows in Fig. 9. At present, by decreasing $\beta < 0.71$, the universe tends to phantom phase in such away that at $\beta = 0.35$, phantom wall is crossed and bellow this, the phantom phase is achieved. At late time it approaches to a finite value, according to values of $\beta$. At last we study on the squared of sound speed. As it is shown in Fig. 10, from now to past, $x \leq 0$, $v_s^2$ is negative for $b^2 \geq 0.35$. Also increasing $\beta$, results a reduction of $v_s^2$. It is worth noting that for $b^2 < 0.35$, and $\beta = 0.44$, we find $v_s^2 > 0$ and hence our model can lead to a stable DE dominated universe favored by observations at the present time.
On the other hand, choosing values $\beta = 0.44$ and $b^2 > 0.35$ leads to instable DE dominated universe. This implies that with increasing $b^2$ we have more instability in the universe.

VI. CONCLUDING REMARKS

In this paper, we studied interacting HDE model in the framework of BD cosmology. As system's IR cutoff we chose GO cutoff inspired by Ricci scalar curvature proposed by [21]. We investigated cosmological implications of this model in ample details. First, deriving the energy density ratio, $u(x)$ versus $x = -\ln(1 + z)$, showed that this ratio is bounded between finite values of order unity according to interaction parameter so that by increasing $b^2$, $u(x)$ evolves slower and will reach to a larger finite values at infinity. It was also demonstrated that the cosmic coincidence problem can only be alleviated in the interacting model at late time. After that, we calculated the deceleration parameter and the EoS parameter of HDE, effective EoS parameter, and the squared of sound speed ($v_s^2$) in the case of $\gamma = 2\beta$ (Ricci DE case). From the analysis of $q$, we found that for the spatially flat FRW, the best value of $\beta$ is $\beta \approx 0.44$. Present acceleration expansion, put an upper limit on $\beta < 0.71$ and for supper acceleration expansion, this limit reduced to $\beta < 0.36$. We found that the interaction affects on the transition point of deceleration to acceleration. More interaction moves the transition point to older universe.

The analysis of $w_D$ showed that the acceleration is started from $w_D \leq -0.47$ and at the
present time the phantom DE phase with $w_D = -1.04$ can be achieved. We found that for both interacting and non-interacting cases, the phantom divide of HDE can be crossed with suitable choice of the parameter, $\beta = 0.44$. Also the analysis of $w_{\text{eff}}$ demonstrated that $\beta$ plays a crucial role in phase of the evolution of the universe. At present, by decreasing $\beta < 0.71$, the universe tends to phantom phase in such a way that at $\beta = 0.36$, phantom wall is crossed and bellow this the phantom phase is achieved. Considering interaction revealed that for all values of $b^2$ the universe transits from quintessence ($-0.34 > w_{\text{eff}} > -1$) to phantom phase ($w_{\text{eff}} < -1$). Also, we showed that in non-interacting case, HDE can give a unified DM-DE model in BD cosmology except that it would not solve the coincidence problem at far future.

We also showed that in limiting case where $\omega \to \infty$, our model reduces to the HDE model with GO cutoff in standard cosmology [21]. We found that in the framework of BD cosmology, crossing the phantom divide line for the EoS parameter of HDE with GO cutoff can be more easily achieved for than when resort to the Einstein gravity.

Finally, we investigate sound instability $v_s^2 = \frac{dP}{d\rho}$ of the model. If $v_s^2$ is positive the HDE would be stable against perturbations. When $v_s^2$ is negative we encounter the instability in the background spacetime. Analyzing the speed of sound $v_s^2$ indicates that HDE with GO cutoff in the framework of BD cosmology can lead to a stable DE dominated universe favored by observations, provided we take $\beta = 0.44$ and $b^2 < 0.35$. This is in contrast to HDE model in Einstein gravity which does not lead to a stable DE dominated universe [52].

**Acknowledgment**

The work of A. Sheykhi has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Iran.

[1] A. G. Riess et al., Astron. J. 116, 1009 (1998)
[2] G. Hinshaw et al. Astrophys. J. Suppl. 208 19,(2013)
[3] S. Hanany et al., Astrophys. J. Lett. 545, L5 (2000); C.B. Netterfield et al., Astrophys. J. 571, 604 (2002); D.N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).
[4] M. Colless et al., Mon. Not. R. Astron. Soc. 328, 1039 (2001); M. Tegmark et al., Phys. Rev.
D 69, 103501 (2004); S. Cole et al., Mon. Not. R. Astron. Soc. 362, 505 (2005); V. Springel, C.S. Frenk, and S.M.D. White, Nature (London) 440, 1137 (2006).

[5] T. Padmanabhan, Phys. Rep. 380, 235 (2003); E. J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006).

[6] Y. Bisabr, Phys. Rev. D 82, 124041 (2010).

[7] A. Khodam-Mohammadi and M. Malekjani, Gen Relativ Gravit 44, 1163 (2012)

[8] W. Zimdahl and D. Pavón, Class. Quant. Grav. 24, 5461 (2007).

[9] J. Sola and H. Stefancic, Phys. Lett. B 624, 147 (2005); I.L. Shapiro, J. Sola, Phys. Lett. B 682, 105 (2009).

[10] G. ’t Hooft, [arXiv:gr-qc/9310026].

[11] L. Susskind, J. Math. Phys. (N.Y.) 36, 6377 (1995).

[12] A. G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Rev. Lett. 82, 4971 (1999).

[13] S. D. H. Hsu, Phys. Lett. B 594, 13 (2004) [arXiv:hep-th/0403052].

[14] M. Li, Phys. Lett. B 603, 1 (2004) [arXiv:hep-th/0403127].

[15] Y. S. Myung, Phys. Lett. B 649, 247 (2007).

[16] S. D. H. Hsu, Phys. Lett. B 669, 275 (2008).

[17] D. Pavón, and W. Zimdahl, Phys. Lett. B 628, 206 (2005).

[18] A. Sheykhi, Class. Quant. Grav. 27, 025007 (2010).

[19] C. Gao, X. Chen and Y. G. Shen, arXiv:0712.1394 [astro-ph].

[20] C. Feng, arXiv:0806.0673 [hep-th].

[21] L.N. Granda, and A. Oliveros, Phys. Lett. B 669, 275 (2008).

[22] D. A. Easson, P. H. Frampton, G. F. Smoot, Phys. Lett. B 696, 273 (2011).

[23] L.N. Granda, and A. Oliveros, Phys. Lett. B 671, 199 (2009), [arXiv:0810.3663] [gr-qc].

[24] A. Khodam-Mohammadi, Mod. Phys. Lett. A 26, 2487 (2011).

[25] M. Malekjani, A. Khodam-Mohammadi, Int. J. Mod. Phys. D 20, 281 (2011).

[26] M. Sharif, Abdul Jawad, Eur. Phys. J. C 72, 2097 (2012).

[27] M. Malekjani, A. Khodam-Mohammadi, N. Nazari-Pooya, Astrophys. Space Sci. 332, 515 (2011).

[28] Miao Li, Xiao-Dong Li, Jun Meng, Zhenhui Zhang, Phys. Rev. D 88, 023503 (2013).

[29] S. Basilakos, D. Polarski, J. Sola, Phys. Rev. D 86, 043010 (2012) [arXiv:1204.4806] [gr-qc].

[30] S. Basilakos, J. Sola, Phys. Rev. D 90, 023008 (2014) [arXiv:1402.6594] [astro-ph].
[31] P. Jordan, , Nature 164, 637 (1955), Schwerkraft und Weltall (Friedrich Vieweg und Sohn, Braunschwig)
[32] C. Brans, and R.H. Dicke, Phys. Rev. 124, 925 (1961).
[33] Y. Gong, Phys. Rev. D 70 (2004) 064029; H. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B 628,11 (2005); A. Sheykhi, Phys. Lett. B 681,205 (2009); B. Bertotti, L. Iess, P. Tortora, Nature 425, 374 (2003); V. Acquaviva, L. Verde, JCAP 12, 001 (2007); L. Xu, J. Lu, W. Li, arXiv:0905.4174 [astro-ph.CQ]; M.R. Setare, M. Jamil, Phys. Lett. B 690,1 (2010).
[34] A. Sheykhi, Phys. Lett. B 681, 205 (2009).
[35] L.X. Xu, W.B. Li, and J.B. Lu, Eur. Phys. J. C 60 135 (2009).
[36] M. Jamil et al. Int. J. Theor. Phys, 51 ,604 (2012).
[37] N. Banerjee, and D. Pavon, Phys. Lett. B 647, 447 (2007).
[38] R-G. Cai, and Q. Su, Phys. Rev. D 81, 103514 (2010).
[39] Jun-Qing Xia, JCAP 11, 022 (2013).
[40] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75 559 (2003).
[41] S. Weinberg, Gravitation and Cosmology, John Wiley & Sons, Inc. (1972).
[42] N. Banerjee, and D., Pavon, Class. Quant. Grav. 18, 593 (2001).
[43] J. Lu, W. Wang, L. Xu, Y. Wu, Eur. Phys. J. Plus 126, 92 (2011), arXiv:1105.1868v3 [astro-ph].
[44] J. Lu, L. Ma, M. Liu, Y. Wu,International Journal of Modern Physics D 21, 1250005 (2012) arXiv:1203.4906v1 [astro-ph].
[45] D. Pavon et al., Phys. Rev. D 86, 083509 (2012).
[46] Planck Collaboration, arXiv:1303.5076.
[47] J. Q. Xia, H. Li, X. Zhang, Phys. Rev. D 88, 063501 (2013)
[48] N. Suzuki et al., Astrophys. J. (ApJ), 746, 85 (2012).
[49] J-O. Xia, H. Li, and X. Zhang, Phys. Rev. D 88, 063501 (2013).
[50] S. Ray and U. Mukhopadhyay, Int. J. Mod. Phys. D 16, 1791 (2007).
[51] J. Grande, J. Sola et al, arXiv:1001.0259v2 [astro-ph.CO].
[52] Y. S. Myung, Phys. Lett. B 652 223 (2007).
[53] K. Y. Kim, H. W. Lee and Y. S. Myung, Phys. Lett. B 660 (2008) 118.
[54] V. Gorini, A. Kamenshchik, U. Moschella, V. Pasquier and A. Starobinsky, Phys. Rev. D 72
(2005) 103518.

[55] H. Sandvik, M. Tegmark, M. Zaldarriaga and I. Waga, Phys. Rev. D 69 (2004) 123524.

[56] E. Ebrahimi and A. Sheykhi, Int. J. Mod. Phys. D 20 (2011) 2369;
    E. Ebrahimi and A. Sheykhi, Int. J. Theor. Phys. 52 (2013) 2966.

[57] Kh. Saaidi, arXiv: 1202.4097.

[58] Y. Wang, and L. Xu, Phys. Rev. D 81, 083523 (2010).

[59] J. Grande, J. Sola, H. Stefancic, JCAP 0608, 011 (2006).

[60] J. Grande, A. Pelinson, J. Sola, Phys Rev D 79 043006 (2009).