Integrated Sensing and Over-the-Air Computation: Dual-Functional MIMO Beamforming Design

Xiaoyang Li, Fan Liu, Ziqin Zhou, Guangxu Zhu, Shuai Wang, Kaibin Huang, and Yi Gong

Abstract

To support the unprecedented growth of the Internet of Things (IoT) applications and the access of tremendous IoT devices, two new technologies emerge recently to overcome the shortage of spectrum resources. The first one, known as integrated sensing and communication (ISAC), aims to share the spectrum bandwidth for both radar sensing and data communication. The second one, called over-the-air computation (AirComp), enables simultaneous transmission and computation of data from multiple IoT devices in the same frequency. The promising performance of ISAC and AirComp motivates the current work on developing a framework that combines the merits of both called integrated sensing and AirComp (ISAA). Two schemes are designed to support multiple-input-multiple-output (MIMO) ISAA simultaneously, namely the shared and separated schemes. The performance metrics of radar sensing and AirComp are evaluated by the mean square errors of the estimated target response matrix and the received computation results, respectively. The design challenge of MIMO ISAA lies in the joint optimization of radar sensing beamformers and data transmission beamformers at the IoT devices, and data aggregation beamformer at the server, which results in complex non-convex problem. To solve this problem, an algorithmic solution based on the technique of semidefinite relaxation is proposed. The results reveal that the beamformer at each sensor needs to account for supporting dual-functional signals in the shared scheme, while dedicated beamformers for sensing and AirComp are needed to mitigate the mutual interference between the two functionalities in the separated scheme. The use case of target location estimation based on ISAA is demonstrated in simulation to show the performance superiority.

Index Terms

Integrated sensing and communication (ISAC), over-the-air computation (AirComp), multiple-input multiple-output (MIMO), beamforming, multi-user interference.

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I. INTRODUCTION

In the near future, tens of billions of Internet-of-things (IoT) devices are expected to be connected in our societies, which requires tremendous spectrum resources [1]. To overcome the limitation of available spectrum resources in practice, two promising technologies are proposed. The first is integrated sensing and communication (ISAC), which utilizes the same spectrum bandwidth for both radar sensing and data communication [2]. Prompted by ISAC, the spectrum resources for radar sensing can be exploited for data communication [3]. The second is over-the-air computation (AirComp), which realizes fast wireless data aggregation by simultaneous transmissions and exploiting analog-wave addition in a multi-access channel [4]. AirComp is regarded as a promising approach for saving the spectrum resources by enabling data transmission of multiple IoT devices in the same frequency band.

To facilitate efficient implementation for IoT, it is natural to tackle the shortage of spectrum resources by pursuing the fusion of the two corresponding technologies: ISAC and AirComp. The resultant design challenge lies in the joint optimization of their key operations at beamformers, namely radar sensing beamformer and data transmission beamformer at the IoT devices, and data aggregation beamformer at the server. This motivates the current work on developing a framework called integrated sensing and AirComp (ISAA).

In ISAA framework, an IoT system is considered for supporting radar sensing and AirComp simultaneously. To be specific, multiple multi-antenna IoT sensors transmit radar signals to detect target and data symbols to a multi-antenna server for data fusion via AirComp. The dual-functional radar sensing and AirComp can be achieved by two schemes, namely shared scheme and separated scheme. In the shared scheme, the whole antennas at each sensor are exploited for transceiving a signal vector both serving as a radar pulse and a data carrier. In the separated scheme, the antennas at each sensor are divided into two groups for radar sensing and data transmission. As the key performance indicators for radar sensing and AirComp are the mean square errors (MSE) of target estimation and function computation respectively, there exists a natural tradeoff between the performance of sensing and AirComp, which is reflected in the beamforming design. This introduces coupling between sensing and AirComp, and hence necessitates the joint design of radar signal beamforming, data transmission beamforming, and data aggregation beamforming. The beamforming designs together with the performance analysis for both the shared and separated schemes are investigated in this paper.

The contributions of this work are summarized as follows.
• **Transmission and Aggregation Beamforming Design in Shared Scheme:** As the signal transmitted by each sensor in the shared scheme serves as both the radar pulse and data carrier, only one beamformer need to be designed at each sensor, which is known as the transmission beamformer. The radar sensing target can be extracted from the statistic information of the reflected signal by exploiting the law of large-number and *maximum likelihood estimation* (MLE). As for AirComp, a beamformer at the server is deployed for equalizing the received signals, which is known as the data aggregation beamformer. The joint transmission and data aggregation beamforming design is formulated as a semidefinite programming problem for minimizing the computation error in AirComp under the constraints of radar sensing requirement and power budget for each sensor. The solving approach based on semidefinite relaxation is applied to obtain the desired design.

• **Radar and Communication Beamforming Design in Separated Scheme:** As there are two signals transmitted by each sensor with one for radar sensing and another for data transmission, two corresponding beamformers are needed to be designed at each sensor. Moreover, the existence of radar signal will exacerbate the interference at the server, which results in a more complex performance metrics of AirComp. The coupling relationship between radar sensing beamformer, data transmission beamformer, and data aggregation beamformer makes the optimization problem even harder to be solved. To deal with such problem, an orthonormal constraint for radar sensing beamforming is introduced to obtain the desired beamforming design.

• **Comparison between Shared and Separated Schemes:** The simulations are conducted to compare the performances of the shared and separated schemes. Particularly, the impedance of AirComp in the shared scheme lies in the limited dimension of beamformers for supporting dual-functional signals, while the radar signals in the separated scheme will exacerbate the interference on AirComp. The radar sensing MSE in the separated scheme solely depends on the maximum MSE tolerance for reducing the AirComp error, while the sensing MSE in the shared scheme is relevant to multiple parameters such as the number of antennas.

• **Target Location Estimation based on ISAA:** To illustrate the performance of ISAA design, the use case of target location estimation is conducted. Specifically, the target location is estimated locally by each sensor based on the information extracted from its received radar signals and transmitted to the server via AirComp. The averaged estimated target location received by the server is compared with the ground truth.
Organization: The remainder of the paper is organized as follows. Section II reviews the state-of-the-art techniques of AirComp and ISAC. Section III introduces the ISAA system model. Section IV and V present the shared and separated beamforming designs for achieving the best ISAA performance, respectively. Section VI further illustrates the performance of target location estimation based on ISAA. Simulation results are provided in Section VII, followed by concluding remarks in Section VIII.

II. STATE-OF-THE-ART OF AIRCOMP AND ISAC

A. Development of AirComp

The idea of AirComp can be traced back to the pioneering work studying function computation in sensor networks [5], where the distributed sensing values are analog modulated and transmitted over a multi-access channel for reliable function computation at a server. The importance of the work lies in the counter-intuitive finding that interference caused by simultaneous transmission can be exploited to facilitate computation. The transmission synchronization over sensors was further investigated in [6].

Driven by the need of fast data aggregation in IoT, AirComp has been applied for supporting function computation via data transmission from multiple sensors to the server. The functions that can be calculated by AirComp has the general format $h(\cdot)$ as shown below:

$$y = h(x_1, x_2, \cdots, x_K) = f\left(\sum_{k=1}^{K} g_k(x_k)\right), \quad (1)$$

where $\{x_k\}$ represents the distributed data samples, $f(\cdot)$ and $g_k(\cdot)$ represent post-processing at the server and pre-processing at a device, respectively. The summation in (1) is achieved by simultaneous analog transmission to exploit the wave-addition of the multi-access channel. Consequently, the function computation is performed “over-the-air” and the result is directly received by the server. The class of functions having the above form is known as nomographic functions such as averaging and geometric mean. Typical functions in this class are summarized in Table I. Simultaneous transmission in AirComp achieves low latency independent of the number of devices and saves the spectrum resources.

To accelerate the computation of multiple functions, the features of multi-modal sensing as well as the prevalence of antenna arrays at both servers and devices were exploited to enable the multiple input multiple output (MIMO) AirComp [7]. In MIMO AirComp, the spatial degrees-of-freedom is leveraged to spatially multiplex multi-function computation simultaneously and
Table I: Examples of nomographic functions in AirComp.

| Name               | Expression                          |
|--------------------|-------------------------------------|
| Arithmetic Mean    | $y = \frac{1}{K} \sum_{k=1}^{K} x_k$ |
| Weighted Sum       | $y = \sum_{k=1}^{K} \omega_k x_k$  |
| Geometric Mean     | $y = \left( \prod_{k=1}^{K} x_k \right)^{1/K}$ |
| Polynomial         | $y = \sum_{k=1}^{K} \omega_k x_k^\beta_k$ |
| Euclidean Norm     | $y = \sqrt{\sum_{k=1}^{K} x_k^2}$  |

reduce computation errors by noise suppression [8]. Along this vein, the MIMO AirComp was integrated with the wireless power transfer technique to achieve self-sustainable AirComp for low-power devices [9]. The reduced-dimension design of AirComp was investigated in [10] for clustered IoT networks. To overcome the reliance on channel station information (CSI), a blind MIMO AirComp technique without requiring CSI access was proposed in [11] for low-complexity and low-latency IoT networks. Facing the practical scenario with fading channels, the optimal power control was designed in [12] to deal with the channel distortion, while the tradeoff between the computation effectiveness and the energy efficiency was analyzed in [13]. The hybrid beamforming for massive MIMO AirComp was studied in [14].

Due to its promising performance in fast function computation, AirComp has been deployed in a series of IoT applications, including federated edge learning (FEEL) [15]–[23], reconfigurable intelligent surface (RIS) assisted communication [24], unmanned aerial vehicle (UAV) communication [25], autonomous driving [26], wireless control system [27], and MapReduce over the edge cloud network [28]. As for FEEL, the wave-addition of AirComp is in perfect match with the aggregation of local training results at the server for global model updating, which has attracted great research efforts. The existing literatures have investigated the implementation of AirComp in FEEL from different perspectives, including communication-learning tradeoff [15], devices scheduling [16], update compression [17], beamforming design [18], hyper-parameters control [19], learning rate control [20], power control [21], digital modulation [22], and data privacy [23].

Despite the wide applications of AirComp in learning and communication systems, most of the existing literatures focus on the spectrum reuse for multiple communication links. In contrast, the spectrum reuse for both radar sensing and data communication remains as an uncharted area in AirComp, which deserves to be investigated in this paper.
B. Development of ISAC

The origin of ISAC can be traced back to the early work in radar communication, where information was embedded into a group of radar pulses [29]. In practice, the S-band (2-4 GHz) and C-band (4-8 GHz) occupied by radar applications might be shared with communication systems [30]. Consequently, a series of literatures focus on investigating the co-existence of radar and communications systems. Particularly, an opportunistic spectrum sharing scheme was proposed in [31], where the communication signals are sent when the spectrum is not occupied by radar. Despite its easy implementation, the radar and communication functions cannot work simultaneously. To overcome such drawback, a null-space projection method was carried out to support the co-existence of MIMO radar sensing and communication [32], where the radar signals are projected onto the null-space of the interference channel for the communication link. Nevertheless, such projection might harm the optimality of radar signal beamforming and thus result in performance loss for the radar sensing.

For improving the performance of radar sensing and communication, a bunch of researches have been conducted. In [33], the designs of radar beamformer and communication covariance matrix were jointly optimized to maximize the radar sensing signal to interference plus noise ratio (SINR) subject to specific capacity and power constraints. As for co-existence of MIMO radar and multi-user MIMO (MU-MIMO) communications, a robust beamforming design with imperfect CSI was proposed in [34], where the radar sensing accuracy is maximized under the SINR requirements by communication and the power budget. Moreover, the multi-user interference was exploited as a source of transmission power in [35], based on which a novel beamforming design was proposed. As for signal receiving, a communication receiver was designed in [36] to demodulate the communication data while cancelling the radar interference. It should be noted that the side-information including CSI, radar probing waveforms, and communication modulation formats need to be exchanged between the radar and communication devices to support the coexistence. Though such cooperation might be achieved by deploying a control center connecting both systems via a wireless link or a backhaul channel, the implementation will impose extra complexity on the system [37].

To avoid the side-information exchange, an advanced co-existence scheme was proposed in [38], where a dual-functional system supporting both radar and communications was designed. From the perspective of information theory, the performance of radar and communications were unified based on the rate distortion theory [39]. The implementation in practice was conducted by
the dual-functional waveform design, which supports target detection as well as data transmission simultaneously [40]. Along this vein, the integrated radar and communication waveform was designed in single antenna systems [41]. The work of [42] brought the integrated waveform design into the MIMO systems, where the information bits are embed in the sidelobe of the radar transmitting beampattern. Accounting for the multi-user communication system, a series of transmitting beamforming designs were carried out in [43] w.r.t. both shared and separated schemes. Aiming at reducing signal distortion, the constant modulus waveforms was further utilized in the dual-functional beamforming design [44].

The benefit of spectrum sharing makes ISAC a popular technology that has been applied in a series of systems, such as millimeter-wave radar and communication networks [45], RIS systems [46], smart home [47], edge learning systems [48], vehicular networks [49], and UAV systems [50]. Particularly, an IEEE 802.11ad-based radar was deployed in millimeter-wave band for supporting an automotive radar and communication network [45]. To mitigate the multi-user interference in ISAC, the joint waveform and discrete phase shift design was carried out by employing RIS [46]. As for smart home, the traditional sensing devices were empowered with communication capability, while the sensing capability of WiFi signals were enhanced [47]. ISAC was further applied to accelerate the edge learning process by designing wireless signals for the dual purpose of dataset generation and uploading. In vehicular networks, the wireless sensing functionality was exploited to acquire vehicles’ states and facilitate the communication [49]. In UAV-enabled ISAC system, the maneuver and beamforming designs were jointly optimized to communicate with multiple users and sense potential targets simultaneously [50].

Among the rich literatures on ISAC, the mitigation of interference is regarded as a headache problem due to the coexistence of radar and communication signals in the same frequency band. Fortunately, the interference can be harnessed by AirComp for fast function computation. Therefore, the combination of ISAC and AirComp will be a promising approach for improving the spectrum efficiency.

### III. System Model

The considered MIMO ISAA system comprises one common target, one access point (AP) equipped with $N_a$ antennas, and $M = |\mathcal{M}|$ radar sensors equipped with $N_s$ antennas. Each radar sensor can simultaneously transmit probing signals to detect the target and data symbols to the AP for AirComp. The ISAA phase is divided into $T$ time slots. The operations of different
sensors are synchronized using a reference clock broadcast by the server (see e.g., [6]). The CSI between the AP and each sensor is estimated individually at each sensor from broadcasted pilot signals and then passed to the AP subsequently. For simplicity, channels are assumed to vary following the block-fading model. In other words, each channel remains fixed within a phase and varies over different phases. The maximum transmit power of each sensor is $P$. Two ISAA schemes are considered in this paper, namely the shared scheme and the separated scheme. The notations are summarized in Table II.

![Figure 1: Integrated sensing and AirComp system.](image)

### Table II: Notation.

| Symbol | Definition | Symbol | Definition |
|--------|------------|--------|------------|
| $M$    | Number of sensors | $G_{im}$ | Target response matrix between sensor $i$ and $m$ |
| $K$    | Number of AirComp functions | $H_m$ | Data transmission channel from sensor $m$ to AP |
| $N_a$  | Number of antennas at the AP | $Q_{im}$ | Direct radar channel between sensor $i$ and $m$ |
| $N_s$  | Number of antennas at each sensor | $R_m$ | Radar signal channel from sensor $m$ to AP |
| $n_c$  | Noise of data transmission channel | $\eta_m$ | Sensing MSE threshold of sensor $m$ |
| $T$    | Number of time slots in ISAA phase | $C_{im}$ | Data reflection channel between sensor $i$ and $m$ |
| $P$    | Power budget of each sensor | $O_{im}$ | Direct data channel between sensor $i$ and $m$ |
| $s_m[t]$ | Radar signals of sensor $m$ at time $t$ | $\Omega_m[t]$ | Interference signal for sensor $m$ at time $t$ |
| $d_m[t]$ | Data symbols of sensor $m$ at time $t$ | $\hat{Y}_m$ | Sufficient statistic matrix of sensor $m$ |
| $x_m[t]$ | Signals transmitted by sensor $m$ at time $t$ | $y_m[t]$ | Target reflection signal of sensor $m$ at time $t$ |
| $n_r$  | Noise of radar signal channel | $N_c$ | Number of antennas at sensor for data transmission |
| $W_m$  | Data transmission beamformer of sensor $m$ | $N_r$ | Number of antennas at sensor for radar sensing |
| $F_m$  | Radar signal beamformer of sensor $m$ | $N_{tx}$ | Number of radar transmitting antennas at sensor |
| $A$    | Data aggregation beamformer at AP | $N_{rx}$ | Number of radar receiving antennas at sensor |
A. Shared Scheme

As shown in Fig. 1 (a), all the \( N_s = N_{tx} + N_{rx} \) antennas at each sensor are shared for both radar sensing and data transmission, where \( N_{tx} \) antennas are for signal transmitting and \( N_{rx} \) for receiving. The signals transmitted by the sensors have dual functions for both sensing the target and carrying the data symbols to the AP simultaneously. The data symbols to be transmitted by the \( m \)-th sensor at the \( t \)-th slot can be expressed as a vector denoted by \( s_m[t] \in \mathbb{C}^{K\times1} \), where \( K \) represents the number of functions to be calculated via AirComp. The data symbols are assumed to be i.i.d. among different sensors and functions with zero mean and unit variance, i.e., \( \mathbb{E}[s_m[t]s_m^H[t]] = I \) and \( \mathbb{E}[s_m[t]s_i^H[t]] = 0, \forall i \neq m \). The diagram of shared scheme is shown in Fig. 2 and described as follows.

\[
\text{Data symbol } s_1[t] \quad \text{Transmission beamformer } W_1 \quad \text{AirComp channel } H_1 \quad \text{AWGN } A \quad \text{Target response matrix } \hat{G}_{11} \quad \text{Estimated parameter } \hat{G}_{11} \quad \text{Interference } \Omega_1 \quad \text{Data aggregation beamformer } W_M \quad \text{AirComp output} \quad \text{Data symbol } s_M[t] \quad \text{Transmission beamformer } W_M \quad \text{AirComp channel } H_M \quad \text{AWGN } A \quad \text{Target response matrix } \hat{G}_{M} \quad \text{Estimated parameter } \hat{G}_{MM} \quad \text{Interference } \Omega_M \quad \text{Data symbol } s_{MM}[t] \quad \text{Transmission beamformer } W_{MM} \quad \text{AirComp channel } H_{MM} \quad \text{AWGN } A \quad \text{Target response matrix } \hat{G}_{MM} \quad \text{Estimated parameter } \hat{G}_{MM} \quad \text{Interference } \Omega_M
\]

Figure 2: Diagram of shared scheme.

After transmitting beamforming \( W_m \in \mathbb{C}^{N_{tx}\times K} \), the signal \( x_m[t] \in \mathbb{C}^{N_{tx}\times1} \) transmitted by the \( m \)-th sensor can be expressed as

\[
x_m[t] = W_m s_m[t]. \tag{2}
\]

Due to the limited transmit power of each sensor, the beamformer design should satisfy the power constraint:

\[
\text{tr}(W_m W_m^H) \leq P, \forall m. \tag{3}
\]
The target reflection signal \( y_m[t] \in \mathbb{C}^{N^r \times 1} \) received at the \( m \)-th sensor can be expressed as

\[
y_m[t] = G_{mm} W_m s_m[t] + \Omega_m[t] + n_r[t],
\]

where \( \Omega_m[t] = \sum_{i \in M \setminus \{m\}} G_{im} W_i s_i[t] + \sum_{i \in M \setminus \{m\}} Q_{im} W_i s_i[t] \) is the interference signal, \( G_{im} \in \mathbb{C}^{N^r \times N^r} \) and \( Q_{im} \in \mathbb{C}^{N^r \times N^t} \) denotes the target response matrix (TRM) and direct radar channel (DRC) between the \( i \)-th and \( m \)-th sensors, respectively. \( n_r[t] \in \mathbb{C}^{N^r \times 1} \) is an additive white Gaussian noise (AWGN) vector with distribution \( \mathcal{N}_{N^r}(0, \sigma_n^2 I) \).

According to [51], the sufficient statistic matrix \( \hat{Y}_m \in \mathbb{C}^{N^r \times K} \) can be derived as

\[
\hat{Y}_m = \frac{1}{T} \sum_{t=1}^{T} y_m[t] s_m^H[t] = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{M} G_{im} W_i s_i[t] s_m^H[t] + \frac{1}{T} \sum_{t=1}^{T} \sum_{i \in M \setminus \{m\}} Q_{im} W_i s_i[t] s_m^H[t] + \frac{1}{T} \sum_{t=1}^{T} n_r[t] s_m^H[t].
\]

According to the law of large-number, when the number of slots \( T \) is large enough, one can get

\[
\frac{1}{T} \sum_{t=1}^{T} s_i[t] s_m^H[t] \approx \mathbb{E}_t [s_i[t] s_m^H[t]] = 0,
\]

\[
\frac{1}{T} \sum_{t=1}^{T} s_m[t] s_m^H[t] \approx \mathbb{E}_t [s_m[t] s_m^H[t]] = I.
\]

Therefore, the sufficient statistic matrix can be expressed as

\[
\hat{Y}_m = G_{mm} W_m + N_m,
\]

where \( N_m = \frac{1}{T} \sum_{t=1}^{T} n_r[t] s_m^H[t] \in \mathbb{C}^{N^r \times K} \). The distribution of \( N_m \) is given in the lemma below.

**Lemma 1** (Distribution of \( N_m \)). \( N_m \sim \mathcal{MN}_{N^r \times K}(0, \sigma_r^2 I_{N^r \times N^r}, \sigma_n^2 I_{K \times K}) \)

**Proof:** See Appendix A. \( \square \)

The corresponding probability density function (PDF) of \( \hat{Y}_m \) is

\[
p(\hat{Y}_m; G_{mm}) = \frac{1}{(2\pi\sigma_r^2/T)^{N^r \times K/2}} e^{-\frac{T}{2\sigma_r^2} \| [\hat{Y}_m - G_{mm} W_m]^H (\hat{Y}_m - G_{mm} W_m) \|_F^2}.
\]

Therefore, the maximum likelihood estimation (MLE) of \( G_{mm} \) can be found by minimizing the log-likelihood function

\[
L(\hat{G}_{mm}) = \text{tr}[((\hat{Y}_m - G_{mm} W_m)^H (\hat{Y}_m - G_{mm} W_m))].
\]

The derivatives of \( L(\hat{G}_{mm}) \) w.r.t. \( \hat{G}_{mm} \) is

\[
\frac{\partial L(\hat{G}_{mm})}{\partial G_{mm}} = 2\hat{G}_{mm} W_m W_m^H - 2\hat{Y}_m W_m^H.
\]
By setting the derivatives of \( L(\hat{G}_{mm}) \) w.r.t. \( \hat{G}_{mm} \) as zero, the MLE of \( G_{mm} \) can be obtained as

\[
\hat{G}_{mm} = \hat{Y}_m W^H_m (W_m W^H_m)^{-1}, \forall m.
\] (12)

Accordingly, the mean square error (MSE) of estimating \( G_{mm} \) can be computed as

\[
\text{MSE}(G_{mm}) = \mathbb{E}\left\{ \| G_{mm} - \hat{G}_{mm} \|^2 \right\} = \frac{N_{rx}\sigma^2_r}{T} \text{tr}\left\{ (W_m W^H_m)^{-1} \right\}.
\] (13)

Given the sensing MSE threshold \( \eta_m \), the sensing quality requirement of the \( m \)-th sensor is

\[
\frac{N_{rx}\sigma^2_r}{T} \text{tr}\left\{ (W_m W^H_m)^{-1} \right\} \leq \eta_m, \forall m.
\] (14)

Due to the long distance between the data source and the AP, the target reflection signal vanishes at the AP. Therefore, the corresponding received signal \( \hat{z}[t] \in \mathbb{C}^{K \times 1} \) at the AP can be expressed as

\[
\hat{z}[t] = A^H \sum_{m=1}^{M} \mathbf{H}_m W_m s_m[t] + A^H n_c[t],
\] (15)

where \( \mathbf{H}_m \in \mathbb{C}^{N_a \times N_{tx}} \) is the channel between the AP and the \( m \)-th sensor, \( A \in \mathbb{C}^{N_a \times K} \) is the data aggregation beamformer at the AP. \( n_c \in \mathbb{C}^{N_a \times 1} \) is a AWGN vector with distribution \( \mathcal{N}_{N_a \times 1}(0, \sigma^2_c \mathbf{I}) \), which is statically independent of \( s_m[t] \).

Due to the nature of analog transmission, the accuracy of AirComp is prone to the distortion by channel fading and noise. As the goal of AirComp is to accurately compute certain functions, the computation error becomes a natural performance metric. Following the existing literatures (see, e.g., [8], [9]), the error is measured via the MSE between the estimated function value and the ground truth one, i.e.,

\[
\mathbb{E}_t \left[ \text{tr} \left( \left( \sum_{m=1}^{M} (A^H \mathbf{H}_m W_m - \mathbf{I}) s_m[t] + A^H n_c[t] \right) \left( \sum_{m=1}^{M} (A^H \mathbf{H}_m W_m - \mathbf{I}) s_m[t] + A^H n_c[t] \right)^H \right) \right] = \sum_{m=1}^{M} \text{tr} \left( (A^H \mathbf{H}_m W_m - \mathbf{I})(A^H \mathbf{H}_m W_m - \mathbf{I})^H \right) + \sigma^2_c \text{tr}(AA^H).
\] (16)

B. Separated Scheme

As shown in Fig. 1 (b), the antennas at each sensor are divided into two groups with \( N_a = N_r + N_c \), where \( N_r \) antennas are for radar sensing and \( N_c \) for data transmission. The \( N_r \) antennas are further divided into two groups with \( N_r = N_{tx} + N_{rx} \), where \( N_{tx} \) antennas are for radar signal transmitting and the remaining \( N_{rx} \) for receiving. The data symbols transmitted by the \( m \)-th sensor at the \( t \)-th slot can be expressed as a vector denoted by \( d_m[t] \in \mathbb{C}^{K \times 1} \), where
$K$ represents the number of functions to be calculated via AirComp. The data symbols are assumed to be i.i.d. among different sensors and functions with zero mean and unit variance, i.e., $E_t[d_m[t]d_m^H[t]] = I$ and $E_t[d_m[t]d_i^H[t]] = 0, \forall i \neq m$. The radar signals transmitted by the $m$-th sensor at the $t$-th slot can be expressed as a vector denoted by $s_m[t] \in \mathbb{C}^{K \times 1}$, which satisfies $E_t[s_m[t]s_m^H[t]] = I$ and $E_t[s_m[t]s_i^H[t]] = 0, \forall i \neq m$. The data stream signals are statically independent of the radar signals, i.e., $E_t[s_i[t]d_m^H[t]] = 0, \forall i, m$. The diagram of separated scheme is shown in Fig. 3 and described as follows.

The signal transmitted by the $m$-th sensor can be expressed as

$$x_m[t] = \begin{bmatrix} W_m d_m[t] \\ F_m s_m[t] \end{bmatrix}, \quad (17)$$
where $W_m \in \mathbb{C}^{N_r \times K}$ is the data transmission beamformer, $F_m \in \mathbb{C}^{N_t \times K}$ is the radar sensing beamformer. Due to the limited transmit power of each sensor, the beamformer design should satisfy the power constraint:

$$\text{tr}(W_m W_m^H) + \text{tr}(F_m F_m^H) \leq P, \forall m.$$  \hfill (18)

The target reflection signal $y_m[t] \in \mathbb{C}^{N_r \times 1}$ received at the $m$-th sensor can be expressed as

$$y_m[t] = G_{mm}F_ms_m[t] + \Omega_m[t] + n_r[t].$$  \hfill (19)

where $\Omega_m[t] = \sum_{i=1}^M C_{im}W_id_i[t] + \sum_{i \in M \setminus \{m\}}(G_{im}F_is_i[t] + Q_{im}F_is_i[t] + O_{im}W_id_i[t])$. For the $i$-th and $m$-th sensors, $G_{im} \in \mathbb{C}^{N_r \times N_t}$ is the TRM, $Q_{im} \in \mathbb{C}^{N_r \times N_t}$ is the DRC, $C_{im} \in \mathbb{C}^{N_r \times N_c}$ is the data reflection channel, and $O_{im} \in \mathbb{C}^{N_r \times N_c}$ is the direct data channel. $n_r[t] \in \mathbb{C}^{N_r \times 1}$ is an AWGN vector with distribution $\mathcal{N}_{N_r}(0, \sigma_r^2 I)$.

According to [51], the sufficient statistic matrix $\hat{Y}_m \in \mathbb{C}^{N_r \times K}$ can be derived as

$$\hat{Y}_m = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^M (G_{im}F_is_i[t] + C_{im}W_id_i[t])s_m^H[t]$$

$$+ \frac{1}{T} \sum_{t=1}^T \sum_{i \in M \setminus \{m\}} (Q_{im}F_is_i[t] + O_{im}W_id_i[t])s_m^H[t] + \frac{1}{T} \sum_{t=1}^T n_r[t]s_m^H[t].$$  \hfill (20)

According to the law of large-number, when the number of slots $T$ is large enough, one can get

$$\frac{1}{T} \sum_{t=1}^T s_i[t]s_m^H[t] \approx \mathbb{E}_t[s_i[t]s_m^H[t]] = 0,$$  \hfill (21)

$$\frac{1}{T} \sum_{t=1}^T s_m[t]s_m^H[t] \approx \mathbb{E}_t[s_m[t]s_m^H[t]] = I,$$  \hfill (22)

$$\frac{1}{T} \sum_{t=1}^T d_i[t]s_m^H[t] \approx \mathbb{E}_t[d_i[t]s_m^H[t]] = 0.$$  \hfill (23)

Therefore, the sufficient statistic matrix can be expressed as

$$\hat{Y}_m = G_{mm}F_m + N_m,$$  \hfill (24)

where $N_m = \frac{1}{T} \sum_{t=1}^T n_s[t]s_m^H[t] \in \mathbb{C}^{N_r \times K}$. Following the similar analysis in the shared scheme, it can be derived that $N_m \sim \mathcal{MN}_{N_r \times K}((0, \frac{\sigma_r}{\sqrt{T}} I_{N_r \times N_r}, \frac{\sigma_r}{\sqrt{T}} I_{K \times K})$. The corresponding PDF of $\hat{Y}_m$ is

$$p(\hat{Y}_m; G_{mm}) = \frac{1}{(2\pi\sigma_r^2/T)^{N_rK/2}} e^{-\frac{T}{2\sigma_r^2} \text{tr}(\hat{Y}_m - G_{mm}F_m)^H(\hat{Y}_m - G_{mm}F_m)}$$  \hfill (25)
By minimizing the log-likelihood function, the MLE of $G_{mm}$ can be obtained as

$$
\hat{G}_{mm} = \hat{Y}_m F_m^H (F_m F_m^H)^{-1}, \forall m
$$

(26)

Accordingly, the MSE of estimating $G_{mm}$ can be computed as

$$
\text{MSE}(G_{mm}) = \mathbb{E} \left\{ \| G_{mm} - \hat{G}_{mm} \|^2 \right\} = \frac{N_{rx} \sigma^2}{T} \text{tr} \left\{ (F_m F_m^H)^{-1} \right\}
$$

(27)

Given the sensing MSE threshold $\eta_m$, the sensing quality requirement of the $m$-th sensor is

$$
\frac{N_{rx} \sigma^2}{T} \text{tr} \left\{ (F_m F_m^H)^{-1} \right\} \leq \eta_m, \forall m.
$$

(28)

Due to the long distance between the data source and the AP, the target reflection signal vanishes at the AP. Therefore, the corresponding received signal $\hat{z}[t] \in \mathbb{C}^{K \times 1}$ at the AP can be expressed as

$$
\hat{z}[t] = A^H \sum_{m=1}^{M} (H_m W_m d_m[t] + R_m F_m s_m[t]) + A^H n_c[t],
$$

(29)

where $H_m \in \mathbb{C}^{N_a \times N_c}$ and $R_m \in \mathbb{C}^{N_a \times N_{rx}}$ are the channels between the AP and the $m$-th sensor for data symbols and radar signals, respectively. $A \in \mathbb{C}^{N_a \times K}$ is the data aggregation beamformer at the AP. $n_c \in \mathbb{C}^{N_a \times 1}$ is a AWGN vector with distribution $\mathcal{N}_{N_a \times 1}(0, \sigma^2_c I)$, which is statically independent of $s_m[t]$ and $d_m[t]$. The corresponding MSE between the estimated function value and the ground truth one can be expressed as

$$
\mathbb{E}_t \left[ \left\| A^H \sum_{m=1}^{M} (H_m W_m d_m[t] + R_m F_m s_m[t]) + A^H n_c[t] - \sum_{m=1}^{M} d_m[t] \right\|^2 \right] = \sum_{m=1}^{M} \text{tr} \left( (A^H H_m W_m - I)(A^H H_m W_m - I)^H \right) + \sum_{m=1}^{M} \text{tr} \left( A^H F_m W_m F_m^H R_m A \right) + \sigma^2_c \text{tr}(A A^H).
$$

IV. Dual-functional Shared Beamforming Design

The shared scheme for radar sensing and AirComp in the ISAA system can be formulated as a joint optimization problem over transmission beamformer $W_m$ at each sensor and aggregation beamformer $A$ at the AP. Specifically, given the MSE in (16) together with the power constraint in (3) and the sensing quality constraint in (14), the problem can be formulated as

$$
\min_{A, \{W_m\}} \sum_{m=1}^{M} \text{tr} \left( (A^H H_m W_m - I)(A^H H_m W_m - I)^H \right) + \sigma^2_c \text{tr}(A A^H)
$$

(PI)

s.t. $\text{tr} \left( (W_m W_m^H)^{-1} \right) \leq \frac{T \eta_m}{N_{rx} \sigma^2}, \forall m$,

$$
\text{tr}(W_m W_m^H) \leq P, \forall m.
$$
Problem P1 is difficult to solve due to its non-convexity. The lack of convexity arises from the coupling between the transmitting and aggregation beamformers. To deal with such problem, the optimal transmission beamforming design is given in the following proposition.

**Proposition 1** (Optimal Transmission Beamformer at Sensor). For the shared beamforming design, given the data aggregation beamformer $A$, the computation error is minimized by adopting the following zero-forcing transmission beamformer at the sensors:

$$ W_m = (H_m^H A A^H H_m)^{-1} H_m^H A, \forall m. \quad (31) $$

*Proof*: See Appendix B. \qed

By adopting the zero-forcing transmitting beamformer, the corresponding problem can be formulated as

$$ \begin{align*}
\min_{A} & \quad \sigma_c^2 \text{tr}(AA^H) \\
\text{s.t.} & \quad \text{tr} \left( H_m^H A A^H H_m \right) \leq \frac{T \eta_m}{N_x \sigma_r^2}, \forall m, \\
& \quad \text{tr} \left( (H_m^H A A^H H_m)^{-1} \right) \leq P, \forall m.
\end{align*} \quad (P2) $$

Since $\text{tr} \left( (H_m^H A A^H H_m)^{-1} \right)$ is neither convex nor concave over $A \in \mathbb{C}^{N_a \times K}$, the problem (P2) is non-convex. By introducing new variable $\hat{A} = A A^H$, the problem can be formulated as

$$ \begin{align*}
\min_{\hat{A}} & \quad \sigma_c^2 \text{tr}(\hat{A}) \\
\text{s.t.} & \quad \text{tr} \left( H_m^H \hat{A} H_m \right) \leq \frac{T \eta_m}{N_x \sigma_r^2}, \forall m, \\
& \quad \text{tr} \left( (H_m^H \hat{A} H_m)^{-1} \right) \leq P, \forall m, \\
& \quad \text{rank}(\hat{A}) = K, \\
& \quad \hat{A} \succeq 0.
\end{align*} \quad (P3) $$

By applying the semidefinite relaxation (SDR), the problem can be formulated as

$$ \begin{align*}
\min_{\hat{A}} & \quad \sigma_c^2 \text{tr}(\hat{A}) \\
\text{s.t.} & \quad \text{tr} \left( H_m^H \hat{A} H_m \right) \leq \frac{T \eta_m}{N_x \sigma_r^2}, \forall m, \\
& \quad \text{tr} \left( (H_m^H \hat{A} H_m)^{-1} \right) \leq P, \forall m, \\
& \quad \hat{A} \succeq 0.
\end{align*} \quad (P4) $$

Then, the convexity of problem (P4) is established in the following lemma.
Lemma 2 (Convexity of Problem (P4)). Problem P4 is a convex problem.

Proof: See Appendix C.

Upon solving the problem (P4) via a convex problem solver (e.g., the cvx toolbox in MATLAB) and attaining the globally optimal solution $\hat{\mathbf{A}}^*$, the next task is to retrieve from it a feasible solution to the original problem denoted by $\hat{\mathbf{A}}$. Since the rank of $\hat{\mathbf{A}}^*$ might be larger than $K$, the Gaussian randomization algorithm proposed in [52] can be applied to extract $\hat{\mathbf{A}}$ from $\hat{\mathbf{A}}^*$. The main procedure is summarized in Algorithm 1.

**Algorithm 1 Gaussian Randomization Algorithm for ISAA**

- **Initialization**: Given an SDR solution $\hat{\mathbf{A}}^*$, and the number of random samples $N$.
- **Gaussian Random Sampling**:
  1. Perform eigen decomposition $[\hat{\mathbf{V}}_{\hat{\mathbf{A}}}, \Sigma_{\hat{\mathbf{A}}}]=\text{eig}(\hat{\mathbf{A}}^*)$.
  2. Generate $N'$ random matrices $\mathbf{Z}_n \sim \mathcal{CN}(0, \mathbf{I})$ with $\mathbf{Z}_n \in \mathbb{C}^{N_a \times K}$, $\mathbf{0} \in \mathbb{C}^{N_a \times K}$ and $\mathbf{I} \in \mathbb{C}^{N_a \times N_a}$.
  3. Retrieve $N$ feasible solutions $\{\mathbf{A}_n = \hat{\mathbf{V}}_{\hat{\mathbf{A}}} \Sigma_{\hat{\mathbf{A}}}^{-1/2} \mathbf{Z}_n^H\}$, such that the constraints $\text{rank}(\mathbf{A}_n \mathbf{A}_n^H) = K$, $\text{tr}((\mathbf{H}_m^H \mathbf{A}_n \mathbf{A}_n^H \mathbf{H}_m)^{-1}) \leq P$, and $\text{tr}(\mathbf{H}_m^H \mathbf{A}_n \mathbf{A}_n^H \mathbf{H}_m) \leq \frac{T_{\text{m}}}{{N_r \sigma_r^2}}$, $\forall m$ can be enforced.
  4. Select the best $\mathbf{A}_n$ that leads to the minimum objective, namely $\mathbf{A}_n^* = \arg \min_n \sigma_c^2 \text{tr}(\mathbf{A}_n^H \mathbf{A}_n)$.
  5. Output $\hat{\mathbf{A}} = \mathbf{A}_n^*$ as the approximated optimal data aggregation beamformer.

Remark 1 (Coupling Relationship of Radar Sensing and AirComp in the Shared Scheme). As shown in problem (P1), one transmission beamformer needs to be designed at each sensor for supporting both radar sensing and AirComp functionalities, which is further correlated to the data aggregation beamformer design at the server via zero-forcing. Therefore, the shared scheme need to guarantee the sensing MSE requirements at the price of sacrificing the AirComp accuracy.

V. Dual-Functional Separated Beamforming Design

In contrast to the shared scheme, the separated scheme should take the joint optimization of data transmission beamformer $\mathbf{W}_m$, radar sensing beamformer $\mathbf{F}_m$, and data aggregation beamformer $\mathbf{A}$ into consideration. Specifically, given the MSE in (30) together with the power constraint in (18) and the sensing quality constraint in (28), the problem can be formulated as
\[
\begin{align*}
\min_{A, \{W_m\}, \{F_m\}} \quad & \sum_{m=1}^{M} \text{tr} \left( (A^H H_m W_m - I)(A^H H_m W_m - I)^H \right) \\
& + \sum_{m=1}^{M} \text{tr} \left( A^H R_m F_m F_m^H R_m^H A \right) + \sigma_c^2 \text{tr}(AA^H) \\
\text{(P5)} \quad \text{s.t.} \quad & \text{tr} \left( (F_m F_m^H)^{-1} \right) \leq \frac{T \eta_m}{N_{rx} \sigma_r^2}, \forall m, \\
& \text{tr}(W_m W_m^H) + \text{tr}(F_m F_m^H) \leq P, \forall m.
\end{align*}
\]

Following the similar approach of solving problem (P1), the zero-forcing data transmission beamformer is adopted to minimize the AirComp MSE, i.e.,
\[
W_m = (H_m^H A A^H H_m)^{-1} H_m^H A, \forall m.
\] (32)

The corresponding problem can be formulated as
\[
\begin{align*}
\min_{A, \{F_m\}} \quad & \sum_{m=1}^{M} \text{tr} \left( F_m F_m^H R_m A A^H R_m \right) + \sigma_c^2 \text{tr}(AA^H) \\
\text{(P6)} \quad \text{s.t.} \quad & \text{tr} \left( (F_m F_m^H)^{-1} \right) \leq \frac{T \eta_m}{N_{rx} \sigma_r^2}, \forall m, \\
& \text{tr}(H_m^H A A^H H_m)^{-1} + \text{tr}(F_m F_m^H) \leq P, \forall m.
\end{align*}
\]

The problem (P6) is non-convex due to the coupling variables \( F_m \) and \( A \) in the objective function.

Following a common approach in the MIMO beamforming literatures (see e.g., [53]–[55]), the radar sensing beamformer \( F_m \) is constrained to be an orthonormal matrix. Mathematically, one can write \( F_m = \sqrt{\alpha_m} D_m \) with \( D_m \) being a tall unitary matrix and thus \( D_m D_m^H = I \), while \( \alpha_m \) is a positive scaling factor. The corresponding problem can be formulated as
\[
\begin{align*}
\min_{A, \{\alpha_m\}} \quad & \sum_{m=1}^{M} \alpha_m \text{tr} \left( R_m^H A A^H R_m \right) + \sigma_c^2 \text{tr}(AA^H) \\
\text{(P7)} \quad \text{s.t.} \quad & \frac{N_{tx}}{\alpha_m} \leq \frac{T \eta_m}{N_{rx} \sigma_r^2}, \forall m, \\
& \text{tr}((H_m^H A A^H H_m)^{-1}) + \alpha_m \leq P, \forall m.
\end{align*}
\]

It can be observed that the increasing of \( \alpha_m \) will result in larger MSE. Therefore, the minimum of MSE over \( \alpha_m \) is achieved when the minimum \( \alpha_m^* \) is adopted for all \( m \), i.e.,
\[
\alpha_m^* = \frac{N_{tx} N_{rx} \sigma_r^2}{T \eta_m}, \forall m.
\] (33)
By introducing $\hat{A} = AA^H$ and applying the SDR, the problem can be formulated as

$$
\min_{\hat{A}} \sum_{m=1}^{M} \frac{N_{tx}N_{rx}\sigma^2_r}{T\eta_m} \text{tr}\left( R^H_m \hat{A} R_m \right) + \sigma^2_c \text{tr}(\hat{A})
$$

(P8) \quad \text{s.t.} \quad \text{tr}\left( (H^H_m \hat{A} H_m)^{-1} \right) + \frac{N_{tx}N_{rx}\sigma^2_r}{T\eta_m} \leq P, \forall m,

$$
\hat{A} \succeq 0.
$$

It can be easily proved that problem (P8) is convex due to the linear objective and convex constraints. Upon attaining the globally optimal solution of problem (P8) by the cvx toolbox in MATLAB, denoted by $\hat{A}^*$, the remaining task is to convert it into a feasible solution of the original problem, denoted by $\hat{A}$, of rank $K$. To this end, the Gaussian randomization based Algorithm 1 is applied.

**Remark 2** (Coupling Relationship of Radar Sensing and AirComp in the Separated Scheme). As shown in problem (P8), the existence of radar signals results in extra AirComp error denoted by $\sum_{m=1}^{M} \frac{N_{tx}N_{rx}\sigma^2_r}{T\eta_m} \text{tr}\left( R^H_m \hat{A} R_m \right)$. To mitigate the interference on AirComp caused by radar signals, the radar sensing beamformers are designed to achieve the maximum tolerance of sensing MSE, i.e., $\frac{N_{tx}\sigma^2_r}{T} \text{tr}\left( (F_m^H F_m)^{-1} \right) = \eta_m$.

![Figure 4: Location Estimation System based on ISAA.](image)

**VI. TARGET LOCATION ESTIMATION BASED ON ISAA**

In this section, the ISAA scheme was applied for the use case of target location estimation. Particularly, the location of the target is estimated by all $M$ sensors based on their own locations.
as well as the information of angle and distance extracted from the reflected radar signals. The estimated location of the target by each sensor is then transmitted to the server via AirComp, and thus the server will obtain the averaged estimated target location. As shown in Fig. 4, the TRM $G_{mm}$ is composed of a phase delay matrix $\Phi(\theta_m)$ and a complex amplitude $\beta_m$ of the received signal, i.e., $G_{mm} = \beta_m \Phi(\theta_m)$.

Let $\varphi_{pq}(\theta_m)$ denote the element of $\Phi(\theta_m)$ in $p$-th row and $q$-th column, then $\varphi_{pq}(\theta_m) = \exp\{-j\omega[\tau_p(\theta_m) + \tau_q(\theta_m)]\}, \quad (34)$

where $\omega$ represents the angular velocity, $\tau_p(\theta_m)$ represents the transmitting time delay between the 1-st and $p$-th antennas, $\tau_q(\theta_m)$ represents the receiving time delay between the 1-st and $q$-th antennas. According to [51], the phase delay between the $p$-th and $q$-th antennas at the $m$-th sensor can be expressed as

$$\varphi_{pq}(\theta_m) = \exp\{-\frac{2\pi j}{\lambda}(y_p^{(m)} + y_q^{(m)}) \sin \theta_m\}, \quad (35)$$

where $y_p^{(m)}$ and $y_q^{(m)}$ denote the location of the $p$-th and $q$-th antennas at the $m$-th sensor, respectively. Following the derivation of $G_{mm}$, the MLE of $\beta_m$ and $\theta_m$ can be found by minimizing the log-likelihood function:

$$L(\beta_m, \theta_m) = \text{tr}[\hat{Y}_m - \beta_m \Phi(\theta_m)W_m]^H (\hat{Y}_m - \beta_m \Phi(\theta_m)W_m). \quad (36)$$

The derivatives of $L(\beta_m, \theta_m)$ w.r.t. $\beta_m$ is

$$\frac{\partial L(\beta_m, \theta_m)}{\partial \beta_m} = 2\beta_m \text{tr}(W_m^H \Phi^H(\theta_m)\Phi(\theta_m)W_m) - 2\text{tr}(W_m^H \Phi^H(\theta_m)\hat{Y}_m). \quad (37)$$

Setting the derivatives as zero, one can get

$$\hat{\beta}_m = \frac{\text{tr}(W_m^H \Phi^H(\theta_m)\hat{Y}_m)}{\text{tr}(W_m^H \Phi^H(\theta_m)\Phi(\theta_m)W_m)}, \forall m. \quad (38)$$

By replacing $\beta_m$ with $\hat{\beta}_m$ in $L(\beta_m, \theta_m)$, one can get

$$L(\theta_m) = \text{tr}(\hat{Y}_m^H\hat{Y}_m) - \frac{\text{tr}^2(W_m^H \Phi^H(\theta_m)\hat{Y}_m)}{\text{tr}(W_m^H \Phi^H(\theta_m)\Phi(\theta_m)W_m)} \quad (39)$$

Therefore, one can get

$$\hat{\theta}_m = \arg \max_{\theta_m} \frac{\text{tr}^2(W_m^H \Phi^H(\theta_m)\hat{Y}_m)}{\text{tr}(W_m^H \Phi^H(\theta_m)\Phi(\theta_m)W_m)} \quad (40)$$

It should be noted that $\hat{\theta}_m$ cannot be expressed in closed form. Therefore, the grid search or golden section search can be applied to find the numerical results. On the other hand, the distance $d_m$ between the target and the $m$-th sensor can be estimated following the free space
propagation law [56]. Based on the estimated parameters (distance $\hat{d}_m$ and angle $\hat{\theta}_m$) and its own location $(0, y_m)$, the $m$-th sensor can obtain its local estimation of the target location denoted by $\hat{z}_m = [\hat{x}_m, \hat{y}_m]^T$ via

$$\hat{x}_m = \hat{d}_m \sin \hat{\theta}_m,$$

$$\hat{y}_m = y_1^{(m)} + \hat{d}_m \cos \hat{\theta}_m.$$  

The target location estimated by the $m$-th sensor is then modulated into data symbols represented by $s_m = [x_m, y_m]^T$, where

$$x_m = \frac{\hat{x}_m - \bar{x}}{\sigma_x},$$

$$y_m = \frac{\hat{y}_m - \bar{y}}{\sigma_y}.$$  

with $\bar{x}$ and $\bar{y}$ denoting the statistic values of the target location at x-axis and y-axis, while $\sigma_x$ and $\sigma_y$ denote the statistic standard deviation. After transmission beamforming $W_m$, the data symbols are transmitted to the AP. In the shared scheme, the signals received at the AP can be expressed as

$$\hat{s} = A^H \sum_{m=1}^{M} H_m W_m s_m + A^H n_c,$$  

where $\hat{s} = [x', y']^T$. The averaged estimated target location can be derived as $z_m = [x' + \bar{x}, y' + \bar{y}]^T$. The performance of radar sensing and AirComp will be evaluated based on the simulation results in section VII.

VII. SIMULATION

In this section, the performance of our proposed ISAA framework is evaluated by simulation on MATLAB, where the radar sensing and AirComp channel models in shared and separated schemes are simulated based on (4), (15), (19), and (29). The performance metric is the normalized AirComp MSE, defined by $MSE/M$ with the AirComp MSE given in (16) and (30) for shared and separated schemes, respectively. The simulation parameters are set as follows unless specified otherwise. The number of time slots is $T = 1000$. The number of computed functions is set to be $K = 10$. There are $M = 10$ sensors each equipped with $N_s = 12$ antennas and one AP with $N_a = 15$ antennas. In the shared scheme, $N_{tx} = 6$ antennas are for signal transmitting and $N_{rx} = 6$ antennas are for signal receiving. In the separated scheme, $N_c = 4$ antennas are for data transmission and $N_r = 8$ antennas are for radar sensing, where $N_{tx} = 4$ antennas are for
First, the normalized AirComp MSE versus the number of antennas at the AP is evaluated in Fig. 5 for both the shared and separated schemes. It can be observed that the normalized AirComp MSE decreases with the increasing number of antennas at the AP. This is because more antennas at the AP will enlarge the dimension of data aggregation beamformer, and thus exploit the diversity gain for achieving lower AirComp MSE. Moreover, the separated scheme has better performance than the shared one under the current system settings. The reason is that the dual-functional signals in the shared scheme make it hard to design one common beamformer for supporting both radar sensing and AirComp, while the interference caused by radar signals in the separated scheme can be effectively mitigated by the dedicated beamformer design for AirComp signals.
Fig. 6 demonstrates the normalized AirComp MSE versus the number of antennas at each sensor in both the shared and separated schemes. One can observe that the normalized AirComp MSE monotonically increases with the increasing number of antennas at each sensor, since more antennas at sensors will result in larger dimension of TRM to be estimated and thus more stringent sensing constraints. Therefore, the beamformers need to guarantee the requirements of radar sensing at the price of scarifying the performance of AirComp. Moreover, the performance of shared scheme becomes better than that of the separated one with the increasing number of antennas at each sensor. Such phenomenon is caused by double-folded effects of deploying more antennas at each sensor. On one hand, more antennas at each sensor will result in larger dimension of beamforming matrix for supporting the dual-functionality of signals in the shared scheme. On the other hand, more antennas for radar sensing at each sensor will exacerbate the interference on AirComp in the separated scheme.

Fig. 7 illustrates the curves of the normalized AirComp MSE versus the number of sensors for both the shared and separated schemes. It is shown that the increasing number of sensors will result in higher normalized AirComp MSE, as more connected sensors make it harder to design one common data aggregation beamformer to equalize the channels among different sensors. Moreover, the increasing trend of normalized AirComp MSE in the separated scheme is more drastic compared with the shared scheme, since larger number of sensors will exacerbate the interference of radar signals on AirComp.
Fig. 7: Normalized AirComp MSE versus the number of sensors.

Fig. 8: Normalized AirComp MSE versus the number of functions to be computed.

Fig. 8 further shows the curves of the normalized AirComp MSE versus the number of functions to be computed in both the shared and separated schemes. One can observe that the normalized AirComp MSE increases with the number of functions, which indicates that higher computation throughput is at a cost of declining accuracy. Moreover, the separated scheme always performs better than the shared one no matter how many functions need to be computed, which implies that the former is more robust against the varying number of functions.
B. Radar Sensing Performance of ISAA

As for radar sensing, the averaged sensing MSE among all sensors versus the number of antennas at the AP is evaluated in Fig. 9 for both the shared and separated schemes. One can observe that the averaged sensing MSE decreases with the increasing number of antennas at the AP in the shared scheme, which indicates that the enlarged dimension of aggregation beamformer will result in higher degree of freedom for achieving lower sensing MSE. In contrast, the averaged sensing MSE won’t change with the number of antennas at the AP in the separated scheme, since the radar sensing constraint is tighten for mitigating the interference of radar signals on AirComp. Therefore, the sensing MSE only depends on the sensing quality requirement and is irrelevant with other parameters.

Fig. 10 compares the averaged sensing MSE in shared and separated schemes versus the number of antennas at each sensor. On one hand, it can be observed that the averaged sensing MSE increases with the increasing number of antennas at each sensor in the shared scheme, since more antennas at each sensor will result in larger dimension of TRM to be estimated and thus larger sensing MSE. On the other hand, the averaged sensing MSE doesn’t change with the number of antennas at the sensors in the separated scheme as it only depends on the sensing quality requirement.

Fig. 11 further illustrates the curves of the averaged sensing MSE versus the number of sensors in the shared and separated schemes. It is shown that the averaged sensing MSE increases with
the number of sensors in the shared scheme, since more connected sensors will result in larger multi-user interference. Nevertheless, there is no significant change of averaged sensing MSE in the separated scheme as it solely depends on the average of sensing quality requirements.

C. Target Location Estimation based on ISAA

The use case of target location estimation based on ISAA is demonstrated in Fig. 12. The ground truth location of the target is set as (5, 30) m. 10 sensor are located at the range [0, 20]
m on the Y-axis with 2 m distance between each other. The number of antennas at each sensor is set as $N_{tx} = N_{rx} = 2$ with 0.1 m space between each other. The information to be estimated and transmitted is a vector which contains the two-dimensional location of the target. It can be observed that the estimated target location by each sensor based on radar sensing is a little deviating from the ground truth, while the application of AirComp can alleviate such deviation by averaging the measured values of sensors over transmission.

![Figure 12: Target location estimation based on ISAA.](image)

**VIII. CONCLUDING REMARKS**

In this paper, an ISAA framework has been proposed for enabling the simultaneous radar sensing and Aircomp to improve the spectrum efficiency in IoT systems. To this end, two designs known as the shared and separated schemes have been investigated. In the shared scheme, all the antennas at each sensor are exploited for transceiving dual-functional signals. In the separated scheme, the whole antennas at each sensor are divided into two groups for supporting radar sensing and Aircomp respectively. The design challenge lies in the joint optimization of the beamformers for radar sensing, data transmission and aggregation, which results in non-convex quadratic problems. The semidefinite relaxation together with the Gaussian randomization algorithm are applied for deriving the tractable solutions. This work points to the promising new research area of ISAA where many interesting research issues warrant further investigation, such as sensor scheduling, vehicular tracking, and target surface estimation.
APPENDIX

A. Proof of Lemma 1

By letting \( S_m = [s_m[1], s_m[2], ..., s_m[T]] \in \mathbb{C}^{K \times T} \) and \( N_r = [n_r[1], n_r[2], ..., n_r[T]] \in \mathbb{C}^{N_{rx} \times T} \), one can get \( N_m = \frac{1}{T} N_r S_m^H \in \mathbb{C}^{N_{rx} \times K} \). The vectorization of \( N_r \) is a Gaussian random vector denoted by vec\((N_r) \sim \mathcal{N}_{N_{rx}T \times 1}(0, \sigma_r^2 I_{N_{rx}T \times N_{rx}T})\). Correspondingly, the vectorization of \( N_m \) can be expressed as

\[
\text{vec}(N_m) = \text{vec}\left( \frac{1}{T} I_{N_{rx} \times N_{rx}} N_r S_m^H \right) = \frac{1}{T} (S_m \otimes I_{N_{rx} \times N_{rx}}) \text{vec}(N_r),
\]

which is a linear transformation of vec\((N_r)\). Therefore, \( n_m \sim \mathcal{N}_{N_{rx}K \times 1}(0, \Sigma) \), where \( \Sigma = \mathbb{E}\left[ \sigma_r^2 \left( S_m \otimes I_{N_{rx} \times N_{rx}} \right) \left( S_m \otimes I_{N_{rx} \times N_{rx}} \right)^H \right] = \mathbb{E}\left[ \frac{\sigma_r^2}{T} (S_m S_m^H) \otimes I_{N_{rx} \times N_{rx}} \right] = \frac{\sigma_r^2}{T} I_{N_{rx} \times N_{rx} \times K} \). Under such condition the PDF of \( n_m \) is

\[
p(n_m|0, \Sigma) = \frac{1}{(2\pi)^{N_{rx}K/2}|\Sigma|^{1/2}} e^{-\frac{1}{2} n_m^H \Sigma^{-1} n_m} \\
= \frac{1}{(2\pi)^{N_{rx}K/2}|\Sigma|^{1/2}} e^{-\frac{T}{2\sigma_r^2} n_m^H n_m} \\
= \frac{1}{(2\pi)^{N_{rx}K/2}} \left( \frac{\sigma_r}{\sqrt{T}} I_{N_{rx} \times N_{rx}} \right)^{K/2} \left( \frac{\sigma_r}{\sqrt{T}} I_{K \times K} \right)^{N_{rx}/2} e^{-\frac{T}{2\sigma_r^2} \text{vec}(n_m)^H \Sigma \text{vec}(n_m)} \\
= (\mathbb{E}[\Sigma])^{N_{rx}K/2} \left( \frac{\sigma_r}{\sqrt{T}} I_{N_{rx} \times N_{rx}} \right)^{K/2} \left( \frac{\sigma_r}{\sqrt{T}} I_{K \times K} \right)^{N_{rx}/2} e^{-\frac{T}{2\sigma_r^2} \text{vec}(n_m)^H \Sigma \text{vec}(n_m)} \\
= p(N_m|0, \frac{\sigma_r}{\sqrt{T}} I_{N_{rx} \times N_{rx}}, \frac{\sigma_r}{\sqrt{T}} I_{K \times K}).
\]

B. Proof of Proposition 1

Given the AirComp MSE minimization objective provided in (16), it can be observed that both \( \sum_{m=1}^M \text{tr} \left( (A^H H_m W_m - I)(A^H H_m W_m - I)^H \right) \) and \( \sigma_c^2 \text{tr}(AA^H) \) are positive. Therefore, given any data aggregation beamformer \( A \), the inequality

\[
\sum_{m=1}^M \text{tr} \left( (A^H H_m W_m - I)(A^H H_m W_m - I)^H \right) + \sigma_c^2 \text{tr}(AA^H) \geq \sigma_c^2 \text{tr}(AA^H)
\]

always holds. It is easy to verify that setting \( W_m \) to have the zero-forcing structure in (31) enforces

\[
\sum_{m=1}^M \text{tr} \left( (A^H H_m W_m - I)(A^H H_m W_m - I)^H \right) = 0,
\]

and thus achieves the equality in (48).
C. Proof of Lemma 2

Since the item $(\mathbf{H}_m^H \hat{\mathbf{A}}_m)^{-1}$ is convex over $\hat{\mathbf{A}}$ and $\text{tr}(\mathbf{X})$ is linear over $\mathbf{X}$, the function $\text{tr}((\mathbf{H}_m^H \hat{\mathbf{A}}_m)^{-1})$ is convex over $\hat{\mathbf{A}}$ according to the composition rule [58]. Since other constraints as well as the objective function are linear functions over $\hat{\mathbf{A}}$, problem (P4) is convex.

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