Uncertainty estimation for extrapolation of the ITS-90 down to the boiling point of nitrogen from the triple point of argon

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Abstract. We estimated uncertainty components associated with the extrapolation of temperature scales down to the boiling point of nitrogen (77.352 K) from a temperature scale of the standard resistance thermometers calibrated at the triple points of argon, mercury and water based on the international temperature scale of 1990. The extrapolation below the triple point of argon is done by using the deviation function of the ITS-90 obtained by the calibration down to the triple point of argon. The standard uncertainty at the boiling point of nitrogen due to the extrapolation is estimated to be 0.17 mK on the assumption that the differences between the two temperature scales calibrated down to the triple points of argon and oxygen are described by a symmetric, rectangular a priori probability distribution.

1. Introduction

National Metrology Institute of Japan (NMIJ) supplies calibration services for long-stem type standard platinum resistance thermometers (SPRTs) down to the triple point of argon (83.8058 K), which is one of the defining fixed points of the International Temperature Scale of 1990 (ITS-90) [1, 2]. Some calibration laboratories in Japan are using liquid nitrogen comparison baths for calibration of their customers' thermometers by comparing them with long-stem type SPRTs calibrated down to the triple point of argon, although the temperature of the liquid nitrogen comparison baths (the boiling point of nitrogen) is 6.5 K below the temperature of the triple point of argon.

In this paper, we estimate uncertainty components associated with the extrapolation of temperature scales down to the boiling point of nitrogen (77.352 K) from the temperature range obtained by calibration at the triple points of argon, mercury (234.3156 K) and water (273.16 K). The extrapolation below the triple point of argon is done by using the deviation function of the ITS-90 obtained by calibration down to the triple point of argon. We estimate the differences between the extrapolated temperature scale and a temperature scale obtained by calibration down to the triple point of oxygen (54.3584 K) for capsule type SPRTs [3]. From our results, the standard uncertainty at the boiling point of nitrogen due to the extrapolation is estimated to be 0.17 mK on the assumption that the differences between the above two temperature scales are described by a symmetric, rectangular a priori probability distribution. We also estimate propagation of uncertainties of measurements at the triple points of argon, mercury and water down to around the boiling point of nitrogen using usual uncertainties for calibration of long stem type SPRTs at NMIJ [3]. The uncertainty component due to the propagation at the boiling point of nitrogen is five times larger than that estimated from the difference of the above two temperature scales. The detailed results of our estimation are described in the following sections.

2. Differences between temperature scales obtained by calibration down to the triple point of argon and down to the triple point of oxygen

Fig. 1 shows differences between TSO2 and TSAR for five capsule-type SPRTs calibrated at NMIJ. Hereafter, we refer to the temperature scale obtained by calibration down to the triple point of argon and its extrapolation as "TSAR". And the temperature scale obtained by calibration down to the triple point of oxygen is called "TSO2".
Figure 1. Differences between the temperature scale obtained by calibration down to the triple point of oxygen (54.3584 K), and the temperature scale obtained by calibration down to the triple point of argon (83.8058 K) and extrapolated below, for five capsule-type SPRTs measured at NMIJ [3]. The vertical axis $\delta T_{54K-84K}$ is the deviation of the temperature of the former temperature scale from that of the latter one.

Fig. 2 includes a part of Fig.1 in the temperature range from 77 K to 84 K. In addition, to have a larger data set to explore and confirm the generalness of the data, the differences between TSAR and TSO2 are estimated also by using the results of the key comparison CCT-K2 reported by Steele et al. [5] and shown in Fig. 2 from 77 K to 84 K. As shown in Figs. 1 and 2, the differences are within 0.6 mK down to the boiling point of nitrogen.

Figure 2. Differences, $\delta T_{54K-84K}$, between the temperature scale obtained by calibration down to the triple point of oxygen and the extrapolation of the temperature scale obtained by calibration down to the triple point of argon for capsule type SPRTs. The red lines are from the same data as those shown in Fig. 1. In addition, the blue lines are estimated by using the results of the key comparison reported by Steele et al. [5].

NMIJ has recommended calibration labs in Japan to add the above uncertainty component due to the extrapolation into their uncertainty budget for comparative calibration of their customers' thermometers by using liquid nitrogen baths and the TSAR as a reference temperature scale for the comparative calibration. Combined standard uncertainties of measurements by using the liquid nitrogen comparison baths for their calibration services are ten times or more as large as the uncertainty component estimated from the differences of the temperature scales obtained by calibration down to the triple points of oxygen and argon. Thus, it is confirmed that the extrapolation
scarcely affects the uncertainty of measurements for their calibration services using the liquid nitrogen comparative baths.

To allow TSAR as a temperature scale down to the boiling point of nitrogen, uncertainty component due to the differences between the temperature scales TSAR and TSO2 is to be estimated. On the assumption that the differences between the temperature scales are described by a symmetric, rectangular a priori probability distribution, the standard uncertainty, $\sigma/\sqrt{3}$, is calculated, where $\sigma$ is half of the estimated width of the distribution. Thus, the standard uncertainty component due to the extrapolation is estimated to be 0.17 mK at the boiling point of nitrogen, which is consistent with the additional component in the NPL budget for comparison calibration in liquid nitrogen reported by Rusby [4] and Steur [6].

3. Uncertainty propagated from measurements at the triple points of argon mercury and water.

Fig. 3 shows uncertainties propagated from measurements at the triple points of argon mercury and water for long stem type SPRTs [3]. The uncertainties of measurements at the triple points of argon, mercury and water are assumed to be 1.1 mK, 0.4 mK and 0.19 mK, respectively. These uncertainties are usual ones for calibration services for long stem type SPRTs at NMIJ.

![Figure 3](image-url)

**Figure 3.** Uncertainties propagated from measurements at the triple points of argon, mercury and water for long stem type SPRTs [3].

The total uncertainty propagated from these fixed-point measurements increases with decreasing temperature below the temperature of the triple point of argon, and it is 20 % larger at the boiling point of nitrogen than that at the triple point of argon. And the total uncertainty due to the propagation at the boiling point of nitrogen is five times larger than that estimated from the difference of the temperature scales, TSAR and TSO2, described in the above section.

NMIJ has recommended calibration labs in Japan to estimate the above propagation of the uncertainties in addition to their usual uncertainty components for comparative measurements by using liquid nitrogen baths and the TSAR.

Fig. 4 shows temperature differences, $\Delta T$, between TSAR of long stem type SPRTs (9 thermometers) and TSO2 for a capsule type SPRT, which were obtained by using a liquid nitrogen comparison bath of NMIJ. The error bars in Fig. 4 are the combined uncertainties for the extrapolated temperature scales for the long-stem type SPRTs, which are dominated by the uncertainties propagated from measurements at the fixed points as shown in Fig. 3. The error bars do not include the uncertainty component of the comparison measurement of thermometers using the liquid nitrogen bath and it is 1 mK. As shown in Fig. 4, TSAR for the long stem type thermometers agrees with the temperature scale TSO2 for the capsule type SPRTs within the combined uncertainty for the extrapolation of the temperature scale, which is dominated by the propagation of uncertainties as shown in Fig. 3 [3]. This is consistent with reports of Hermier et al. and EURAMET Technical Guide No. 1 [7, 8].
4. Summary
Uncertainty components associated with the extrapolation of temperature scales down to the boiling point of nitrogen from the temperature scale obtained by calibration at the triple points of argon, mercury and water, TSAR, are estimated from differences between TSAR and the temperature scale obtained by calibration down to the triple point of oxygen, TSO2. It is confirmed the differences are within 0.6 mK down to the boiling point of nitrogen. The standard uncertainty at the boiling point of nitrogen due to the difference of the temperature scales TSAR and TSO2 is estimated to be 0.17 mK on the assumption that the differences are described by a symmetric, rectangular a priori probability distribution.

A propagation of uncertainties of measurements at the triple points of argon, mercury and water down to the boiling point of nitrogen is also estimated by using the usual uncertainty for calibration of long stem type SPRTs at NMIJ. The total uncertainty due to the propagation at the boiling point of nitrogen is five times larger than that estimated from the differences of the temperature scales and dominates the combined uncertainty to use the temperature scale, TSAR, down to the boiling point of nitrogen. For usual calibration services, the dominant uncertainty component of which is ten times or more as large as the uncertainty due to the differences of the above two temperature scales, it will be no problem to use the extrapolated temperature scale TSAR for comparative calibration of thermometers using liquid nitrogen baths.

References
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