From Nondynamic Errors to Dynamic Errors – A Structural Deployment

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Abstract. Errors represent a special kind of signals. There are many types of errors. The most prominent ones are systematic and/or random errors, as well as constant and/or time dependent errors. Different types may overlap. The modelling procedure of error quantities is an important issue. There are processes of interest, error sources, error processes, error superpositions, and error compensations, a veritable error chain. The review at hand develops and interprets some structures and properties of this chain.

Introduction
Errors have been discussed continuously, with more or less power of persuasion and success. At present, intentions to neglect errors are fashionable. However, there seems to be a revival of the error to come, since concepts and structures are still debatable.

Error analysis, error description, and error correction are no easy tasks. In some fields, in production processes for example, an error is just an ad hoc information about a particular property value: Some report is due and decisions can be taken. However, there are processes, for example continuous, real-time, on-line measurement processes, where time dependent acquisition results are error prone. One has to reconstruct the original values or trajectories of quantity of interest in real time. Further on, one may want to separate the errors from the erroneous quantities by whatever means. The latter procedure is notably malicious due to time dependent errors and dynamic processes.

The following sections provide an introduction to a deeper and more systematic understanding of this tricky area. In principle, all fields of Science and Technology are concerned, but not all provide the necessary background for an appropriate handling. In this respect, Signal and System Theory (SST), a field, which is famous for consistent logical statements and for mathematically rigorous relations, delivers valuable tools to analyse, describe, and solve the diverse challenges.

Up to now, errors in Metrology are seldom addressed by Signal and System Theory (SST). The main reasons are their demanding theoretic capabilities, which seem not to account for specific needs in daily practice. This is a misjudgement. Quite the contrary, the definition of relations between quantities as the description of processes of interest always utilise the very same concepts and tools. This is important, since the measurement and observation chain normally consists of many interacting subprocesses, which are quite diverse indeed. Therefore, the following treatment of error quantities and error processes has to take a broad view.

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1. Quantities intended to be acquired and processes of interest

Important: No process P of interest is measured or observed, only its defined and specified quantities are! At an early stage of any metrological procedure, it is important to properly define the quantities intended to be measured and possibly observed. The model of a particular quantity (signal) with its properties (structure plus parameters) is theoretically and / or empirically identified (calibrated) either from scratch or more likely from measurement, based on a hypothesis of the model structure.

For an acquisition of quantities of a process P, we need a measurement process M, which includes a sensor process S and a reconstruction process RS.

Defined abstract error signals come into existence within systems and are visible within system blocks of Signal Relation Graphs (SRG). This may be surprising, since we know that signals are often disturbed by their environment and that vice versa signals may disturb their environment. But we have to be aware that mathematical models do not map the real world as such. They just describe signals and relations between signals. Therefore, a disturbance of a signal is considered a relation between signals and thus a model of a disturbed system. In other words, all procedures concerning signals occur in systems.

2. Ideal measurement procedures

The idealistic hypothesis assumes that defined output quantities \( y(t) \) of process P with actual properties are acquired by an ideal (nominal) measurement process, denoted by \( M_N \). This assumed ideal measurement process \( M_N \) serves as a reference and as a benchmark for further definitions, discussions, and decisions, which always take place on the model level and not on the real-world level. An ideal (nominal) measurement process \( M_N \) is described by the constant transfer identity matrix \( G_{MN} = I \), which means that \( y(t) = G_{MN} y(t) - I y(t) \). For this fictitious, ideal situation, this relation is remodelled, in order to include the total measurement error \( e_y(t) \), which fictitiously is zero: \( y(t) - y(t) = e_y(t) = 0 \).

3. Almost ideal measurement procedures

Since no ideal measurement process \( M \) exists, the question comes up, what a almost ideal measurement process \( M \) is, whose measurement errors \( e_y(t) \) will almost equal zero. Such a definition does not just depend on the properties of the measurement process \( M \), but on the quantities \( y(t) \) of process \( P \), intended to be measured and in addition on the requirements of the measurement task. Therefore, any measurement procedure by means of an almost ideal measurement process \( M \) is a compromise.

4. Errors in nonideal, dynamic systems

As a first step, an extension of the so-called ideal system is made, in order to enable nonideal structures as well as descriptions of the different error signal types. Therefore, a general nonideal system \( P \) is assumed to have two inputs, the input signal vector \( u(t) \) and the disturbance signal vector \( v(t) \), and, in addition, two outputs, the output signal vector \( y(t) \) and the loading signal vector \( z(t) \), which influences precedent systems. Internal subsystems, for example nonlinear and / or dynamic subsystems, are embedded, in order to enable the description of error signal vectors \( e(t) \). This concept is unique and valid for all further systems.
For this extension, the conventional *state description* (SD) of a dynamic system includes one additional column for the additional, disturbing input signal vector $v(t)$, and one additional row for the additional, loading output signal vector $z(t)$. A vector-matrix differential equation describes the possible internal dynamic relations. This structure serves almost all applicational needs. Nonlinear effects are includable.

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Ev(t) \\
y(t) &= Cx(t) + Du(t) + Fv(t) \\
z(t) &= Gx(t) + Hu(t) + Jv(t)
\end{align*}
$$

or summing up

$$
\begin{bmatrix}
x(t) \\
y(t) \\
z(t)
\end{bmatrix} =
\begin{bmatrix}
A & B & E \\
C & D & F \\
G & H & J
\end{bmatrix}
\begin{bmatrix}
x(t) \\
u(t) \\
v(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
$$

An exemplifying and rather symmetric Signal Relation Graph (SRG) visualises this concept, including the dynamic core, which is able to consider and summarise all dynamic contributions within the nonideal system.

The next step uses this concept by focusing on a nonideal measurement system $M$ as a combination of a desired, defined and specified ideal subsystem $MN$ and an annoying error subsystem $E$. This procedure is called error separation (error pull out, system decomposition), and can be seen here for example in a parallel connection. It is assumed that the resulting error signal vector $e_y(t)$ from the error system $E$ interferes with the nominal output signal vector $y_{nom}(t)$ of the ideal subsystem $MN$, by linear and / or nonlinear interaction.

The parallel connection is not the only possibility to describe the structure of influences of an error signal vector $e(t)$. Signal and System Theory (SST) provide three and only three possibilities of
connecting two systems according to the basic connection rules: series, parallel and feedback connection.

The example of the parallel connection presents the situation of a linear, time invariant (LTI), nondynamic nominal measurement system MN, characterised by a throughput matrix $D_{MN}$. The parallel measurement error system E is a dynamic system. It is sensitive to an external disturbance signal vector $v(t)$ and it loads preceding systems by the loading signal vector $z(t)$. The error signal vector $e_x(t)$ interferes linearly with the nominal output signal vector $y_{nom}(t)$, resulting in an erroneous output signal vector $\hat{y}(t) = y_{M}(t)$.

\[
\begin{bmatrix}
x(t) \\
y(t) \\
z(t)
\end{bmatrix} =
\begin{bmatrix}
A & B & E \\
C & (D_{MN} + D) & F \\
G & H & J
\end{bmatrix}
\begin{bmatrix}
x(t) \\
u(t) \\
v(t)
\end{bmatrix}
\]

We conclude that a measurement result signal vector $\hat{y}(t) = y_{M}(t)$ of a nonideal measurement system M is error prone due to three and only three possible error sources: internal system transfer errors, errors due to external disturbances, and errors due to loading effects on preceding systems of interest.

Note that the state signal vector $x(t)$ is the generating vector of the dynamic measurement error components $e_x(t)$ and $e_z(t)$. The nondynamic error components $e_y(t)$ and $e_u(t)$ also belong to the internal transfer errors.

Each error signal component $e_x(t)$ is a member of the well-known error classification, like linearity error, sampling error, hysteresis error, model error, dynamic error, and so on. They are transfer responses of the error subprocess E, described by the nine parameter matrices A to J.

We also see that the influences of nondynamic and dynamic errors are clearly distinguished. However, a strategy concerning random signals is not directly visible. If there are random components in the input signals $u(t)$ to be measured (measurands), they proceed on two parallel paths to the measurement result signals $\hat{y}(t)$. One path leads there directly, as specified through the nominal system MN, causing no measurement error signals at all. The second path leads there indirectly via the error system EM, causing
nondynamic and dynamic random error signals. This means that only one random partition of the resulting output signals is error prone, the other random partition belongs to the signal of interest. Note that both random signals are correlated. For an error correction, a separation of these two components has to be performed, a tricky endeavour. If there are random components in the disturbing input signals \( v(t) \), they will affect the measurement result signals \( \hat{y}(t) \) only as random errors.

Remark: The derived structure of the error system represents the maximal thinkable version. It is obvious that for an analytical and / or empirical analysis all elements of the parameter matrices must be known. This is an immense enterprise indeed, in spite of the fact that most elements may equal zero in practice, which makes the matrices sparse. Nevertheless, an overview on such a basic concept helps to design less comprehensive models by reducing systematically and gradually the overall model.

The nominal system needn’t be a nondynamic system. If the process of interest were a signal filter system, the nominal system would be a dynamic system, specified and described by an amplitude and phase transfer function in the frequency domain. Deviations from these transfer functions would lead to a spectral error transfer function vector \( e(f) \).

So far and further on, all derived structures have assumed that the diverse error components are uncorrected. Resulting output error signal vectors \( e_y(t) \) and \( e_z(t) \) may in principle be corrected by inverting means within a subsequent reconstruction system to be added. It has to be mentioned that many reconstruction tasks fail due to the strict conditions of invertibility and observability. The procedure is known under different terms: Deconvolution, inversion, inference, filtering, equalisation, restitution, and so on.

Example: A dynamic sensor process \( S \) with a defined nominal behaviour is assumed to be nonideal, since it is of first order according to a priori knowledge. Thus, a simple model of the nonideal sensor system \( S \) can be established. In this case, the appropriate reconstruction process \( R \) is simple too. Both together constitute the (hopefully) ideal measurement system \( M \), since the dynamic measurement error should have been corrected by the reconstruction process \( R \), which has the inverse transfer function of the sensor process \( S \): \( g_M = g_R g_S = 1 \).

### 5. Errors in nonideal dynamic systems in static state

If signals \( y(t) \) intended to be measured, and disturbance signals \( v(t) \) intended to be prevented, are (temporarily) constant, the resulting, formerly time dependent output signal vector \( \hat{y}(t) \) of a dynamic measurement system \( M \) becomes constant too after a certain transition time. Then the dynamic system will remain in a static (steady, equilibrium) state: The normally time dependent state signals \( x(t) \) are constant and the integrators are inactive. A dynamic system in a static state does not reveal the inherently existing dynamic error components. Only the now time independent, nondynamic error components are visible.

For the static state of a dynamic error system \( E \), all derivatives \( \dot{x}(t) \) of the linear differential equation, which cause the dynamic error components, equal zero. For the nonideal dynamic measurement system in static state the State Description (SD) delivers:

\[
\begin{bmatrix}
0 \\
\dot{y}(t) \\
\dot{z}(t)
\end{bmatrix} =
\begin{bmatrix}
A & B & E \\
C & (D_{MN} + D) & F \\
G & H & J
\end{bmatrix}
\begin{bmatrix}
x(t) \\
u(t) \\
v(t)
\end{bmatrix}
\]

Obviously, the description degrades to a set of algebraic equations. We subsume them by inserting the static state signal vector \( x \) into the output equations and get the output signal vectors \( y \) and \( z \) for the general and error behaviour.
What about stationarity within a nonideal dynamic system? If the time dependent, deterministic and / or random input signal vectors $y(t)$ and $v(t)$ are, or become stationary, the resulting output signal vector $\hat{y}(t)$ of a dynamic measurement system are, or become stationary too after a certain transition time. The dynamic system is then in a stationary state: The state signal vector $x(t)$ is time dependent, but stationary. Dynamic and nondynamic errors are effective.

Stationarity of a signal means that all deterministic and random characteristic values and functions of a signal, like mean values, variance values, autocorrelation functions, spectral power density functions, and others, are time independent, which means, are constant. However, these signals, including the error signals, change with time $t$ within these constraints.

6. Errors in nonideal, nondynamic systems

No differential equations are involved to describe this case. Thus all subsystems are assumed nondynamic. No dynamic errors appear, but all other errors of the error classification are possible. As a consequence, the measurement of time dependent signals $y(t)$ of a system $P$ of interest by a nonideal, but nondynamic measurement system $M$ is not a dynamic measurement procedure at all, though the measurement result vector $y_M(t)$ will of course be time dependent! The same is true for the time dependent disturbing signals $v(t)$. The shape of the different signals may change, due to internal nonlinearities for example.

The measurement of time dependent random signal vectors $y(t)$ of a system $P$ by a nonideal, but nondynamic measurement process $M$ does by no means lead to dynamic measurement errors! Resulting output error signal vectors $e_y(t)$ and $e_z(t)$ of the measurement process $M$ are combined in the total measurement error vector $e_M(t)$.

$$e_M(t) = \begin{bmatrix} e_y(t) \\ e_z(t) \end{bmatrix} = \begin{bmatrix} e_{yu}(t) + e_{yy}(t) \\ e_{zu}(t) + e_{yz}(t) \end{bmatrix} = \begin{bmatrix} D_{yu} & F_{yy} \\ H_{zu} & J_{yz} \end{bmatrix} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = P_{ME} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix}$$
7. Ideal systems

An ideal system reacts exactly according to defined and specified rules. Concerning a measurement procedure, there will be no measurement errors at all, even if the signals to be measured are random.

The simple vector-matrix equation describes the ideal nondynamic system.

\[ y(t) = D u(t) \]

with

\( D \) throughput matrix

If we are interested in probabilistic properties of the output vector \( y(t) \) of process \( P \), we use this description too. As an example, we want to know the arithmetic mean values \( \mu_y \) of the output vector \( y(t) \):

\[ \mu_y = D \mu_u \]

with

\( D \) throughput matrix

Conclusion

The rather unknown field of dynamic errors has been introduced and visualised by the State Description (SD) and corresponding Signal Relation Graphs (SRG). The pivots of the endeavour are the diverse structures of error sources.

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