Mathematical modeling to determine a radius for a watering-place used by flocks of sheep in distant arid grazing lands

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Abstract. The paper introduces a computer-based methodology, the aim of which is to determine a watering-place radius for flocks of sheep grazing in distant pasture lands within the arid ecosystem. All calculations leading to the quantitative values are performed through mathematical modeling which is the base of the methodology we propose. The mathematical modeling is performed through some analytical method including the sequential parameters calculation with mathematical formulas. The object was theoretically described, an algorithm was compiled in the form of a block diagram, and a computer program SUARU in the QBASIC language was developed. A series of computational experiments was carried out, aimed at analyzing, interpreting and comparing the analytical modeling outcomes. At the same time, graphical dependencies are presented to show clearly the dependence between the calculation results and the source data. The calculation methodology allows modeling the object of study depending on various values of significant parameters and applying it in practice by drawing up appropriate recommendations for farms which deal with sheep breeding when grazing lands of the arid ecosystems are used.

1. Introduction
Different animal species choose different survival strategies in the locality with the arid climate [1]. In contrast to wild animal, agricultural animals are subordinated to the life conditions organized for them by a human. It is a human, who determines the size of livestock (a number of animals) according to available resources of fodder and water, as well as the methods of how grazing lands will be used. In a dry climate, sheep are the most common farm animals. To feed animals, depending on the grazing land yield, a different grazing area is required [2]. The size of a grazing land affects the distance to the watering point (from 200 to 2000 m) and the diversity of vegetation [3].

The intensity of grazing land use significantly affects the diversity of vegetation, grazing land yield and the ecological situation in it [4-6]. Livestock can negatively affect both the flora and fauna of the
area [7]. At the same time, reducing the number of flocks is useful for the environmental protection of grazing lands [8]. The issue of modeling grazing lands for specific climatic conditions enables to reach the efficient animal breeding [9]. There are some models to determine the economic efficiency of using grazing lands and livestock in different climatic conditions [10, 11].

One of the main problems of keeping the livestock is availability of drinking water [12]. The most important and main parameter that determines the water supply for the pasture territory is the radius of the watering-place or watering point. This is due to the fact that the livestock productivity, a size of grazing areas, a number of watering points, the degree of their concentration and the amount of capital investment in the construction of watering points depend on the radius of a watering-place. The threshold distance for animals as recommended in [13] is: for sheep - 3 km, for cattle - 6 km. So, to determine the optimal distance is becoming an issue of great economic importance. And this is due to the fact that any unreasonable decrease in the radius of a watering-place for farm animals causes additional financial costs, when the pasture areas are flooded.

In turn, an unreasonable increase in the radius of the watering-place entails an increase in energy needed for animals when they are moving more that can be the reason for them to be fatigue. And it entails a decrease in animals’ productivity.

Therefore the purpose of this work is to develop a mathematical model assisting to determine the radius of a watering-place for sheep under the pasture conditions of arid ecosystems and to do the analysis of various factors that influence it.

To verify to what extent the goal is reachable, it is necessary to analyze the graphical results showing the changes in the radius values defined through the parameters stated in the study. The choice to use the mathematical modeling method is due to the fact that this method allows processing certain data and performing a computational task, the purpose of which is to determine the radius of the watering-place for a flock of sheep of a certain size.

2. Materials and methods

2.1 Study object
The object of the study is the radius of the watering-place for flocks of sheep grazing in the pasture conditions within the arid ecosystem. The calculation of the quantitative values regarding the radius of the watering-place for flocks of sheep through mathematical modeling was performed for the conditions typical to an arid distant pasture. This takes into account the average speed of the grazing flocks, the frequency of watering and grazing per day. Besides, the average speed of a flock was calculated depending on the front of the flock, the number of sheep in it, their need in grazing lands, yield and coefficient of permissible grazing of the pasture territory.

2.2 Research methods
The research methods were based on the well-known principles of the mathematical modeling with the help of some analytical calculus method. The idea of this method is in the sequential calculation of parameters by the mathematical formulas, involving the relation between the studied object and the essential characteristics of a grazing land of the arid ecosystems. In addition, the methods of theoretical analysis and synthesis, some methods of comparison and generalization were used in the study.

2.3 Research tools
The modern computer based on the 7th generation processor and the Windows 10 platform was used as the main modeling tool. Auxiliary calculations and computational experiments were carried out with the computer program SUARU in the QBASIC language, developed by the authors. The dependency graphs were carried out with a graphical editor which Microsoft Office Excel offers.
3. Research results

There are different points of view on the methodology to determine the radius of a watering-place. There are known some methods: simple, but approximate; labor-intensive based on technical and economic indicators; tabular ones based on averaged regulatory data; and the analytical ones used to calculate the radius of a watering-place. The disadvantages of the methods used in practice are introduced in [14].

In addition, it is known that with a decrease in the radius of the watering-place, the productivity of animals grazing in a pasture increases. But with a decrease in the radius of the watering-place, the capital investments in the construction of watering points increase. Therefore, the necessity to determine the optimal radius of the watering-place based on the parameter showing the actual movement of the flocks across the pasture is undoubtedly more justified because it is simpler and more accurate than other approaches (methods). The subject of this study is to define the factors affecting the radius of the watering for flocks of sheep grazing in the pasture conditions of the arid ecosystem.

In general, the functional dependence of the radius of the watering-place on many factors can be represented as follows:

$$ R = \varphi (N_a; Y; W_f; \upsilon_g; \rho_g; f_g; L_g; T_g; F; \alpha_c), $$

where $N_a$ – the number of animals in a flock (animals); $Y$ – average pasture productivity per grazing days, t/ha; $W_f$ – watering frequency; $\upsilon_g$ – the average flock speed when grazing, m/s; $\rho_g$ – flock density during grazing, animal/ha; $f_g$ – front of sheep grazing, m; $L_g$ – the actual distance gone by a flock during grazing, m; $T_g$ – flock grazing frequency per day, s; $F$ – the average daily need of an animal for pasturing feed, kg/animal; $\alpha_c$ – coefficient of permissible pasturing of the pasture territory.

Theorectical basis and the approach proposed by G.S. Gumarov to determine the environmentally friendly value of the radius of a watering-place for farm animals were described well in [14, 15].

The research methods were based on the well-known principles of the mathematical modeling theory with the help of some analytical calculus method. The idea of this method is in the sequential calculation of parameters by the mathematical formulas, involving the relation between the studied object and the essential characteristics of the pasture arid ecosystems.

The average value of the front of the flock grazing is determined by the formula [14]:

$$ f_g = 2 \sqrt{\frac{10000N_a}{\pi \rho_g}}, $$

where 10000 - the conversion factor of the values denoting the pasture area from hectares to square meters; $N_a$ - number of sheep in a flock (animals); $\rho_g$ – flock density when grazing, animal / ha.

The average speed of the flocks when grazing is calculated as [15]:

$$ \upsilon_g = \frac{N_a F}{0,1 \alpha_c YT_g}, $$

where $F$ – the average daily need of sheep in pasturing feed, kg/animal; 0.1 – conversion factor t/ha in kg/m²; $\alpha_c$ – coefficient of permissible grazing of the pasture territory; $Y$ – average pasture productivity per grazing days, t/ha; $T_g$ – flock grazing frequency per day, sec.

The radius of the watering-place is calculated with the formula [14, 15]:

$$ R = \frac{\upsilon_g T_g}{2(W_f - 1)}, $$

where $W_f$ - is a watering frequency.

To study the object by the method of mathematical modeling, an algorithm was compiled in the form of a block diagram, and the computer program SUARU in the QBASIC language was developed.

The calculation of the quantitative values regarding the radius of the watering-place for flocks of
sheep through the mathematical modeling was performed for the conditions typical to an arid distant pasture. This takes into account the average speed of the flocks of sheep while grazing, the frequency of watering and grazing per day. Besides, the average speed of flock moving when grazing was calculated depending on the front of the flock grazing, the number of sheep in it, their need for pasturing feed, yield and coefficient of permissible grazing of the pasture territory.

At the same time, in relation to the conditions of Western Kazakhstan on the basis of reference and normative literature the following initial data were used: \(N_a = 200; 400; 600; 800; 1000\) animal; \(Y = 0.05; 0.20; 0.35; 0.50; 0.65; 0.80\) t/ha; \(\rho_g = 2000\) animal/ha; \(W_f = 2; F = 3\) kg/animal; \(\alpha_c = 0.65; T_g = 10\) hours.

A series of computational experiments based a computer was carried out, aimed at analyzing, interpreting and comparing the analytical modeling outcomes. At the same time, graphical dependencies are presented to clearly show the connection of the calculation results on the source data.

As a result of the data processing by the SUARU program, dependences of the flock watering radius on the grazing land yield were performed at \(F = 3\) kg/animal, \(\rho_g = 2000\) animal/ha, \(W_f = 2\) and \(T_g = 10\) hours.

Figure 1. Dependence of the radius of the watering-place on grazing land yield

An analysis of the dependencies shows that with an increase in the number of sheep in a flock from 200 to 1000 animals, i.e. 5 times, the radius of the watering-place increases 2.23 times. At the same time, this trend is observed for any grazing land yield. With an increase in this parameter from 0.2 t/ha to 0.35 t/ha, i.e. by 175%, the radius of the watering-place decreases by 1.75 times. Thus, the analysis of the calculated dependences \(R = \varphi(Y)\) indicates that the radius of the watering place increases with a decrease in the yield of the pasture territory and with an increase in a number of animals in a flock, and this increase is power-law.

In order to make clear the real possibilities in organizing mechanized water supply across the pasture territories, taking into account environmental requirements and increasing the efficiency of pasture for sheep, the results of our work can gain the practical importance. To prove it, we will apply these results in practice by drawing up appropriate recommendations for farms who deal with sheep breeding in pasture conditions of the arid ecosystems. The case of Western Kazakhstan we use as an example.

Recommended value of the ecologically safe radius of the watering-place for grazing lands is: Naryn Sands – 2.5 ... 2.9 km; Taysogan, Mangyshlak, Karynzharyk, Ustyurt – 2.3 ... 2.7 km; Caspian strip – 2.2 ... 2.5 km; Trans-Ural steppes – 3.5 ... 4 km; Aral Karakums, Baiganinsky – 2 ... 2.3 km;
Irgiz – 1.5 ... 1.8 km; Big and Small Badgers – 1.2 ... 1.4 km; Karabutak – 1.7 ... 2 km.

At the same time, when calculating the radius of the watering-place, for the above-mentioned largest grazing lands of Western Kazakhstan according to the proposed methodology, the following data were used: \( N_a = 2000 \) animals of sheep (the limit of livestock served by a watering point); flock density when grazing \( \rho_g = 1500 \) and \( 2000 \) animal/ha; watering frequency \( W_f = 2 \); the average daily need of animals for pasturing feed \( F = 6 \) kg/animal; the coefficient of permissible grazing of the pasture territory \( \alpha_c = 0.65 \); \( T_g = 10 \) hours; the average yield of the grazing land was taken according to the statistical data for the area under consideration.

4. Discussion
The aim of the study was to identify the influence of various grazing land conditions of the arid ecosystems on the radius of the watering-place for sheep flocks through mathematical modeling. The actions guided by the goal made it possible to model the object of the study depending on various significant parameters. Establishing and visualizing the nature of the various factors and their impact on the radius of sheep-flocks watering in the arid pasture conditions is of scientific value of the work performed. The results of the work carried out give reasons for practicing it by drawing up appropriate recommendations for farms within involved in the production of sheep products in pasture conditions of the arid ecosystems.

The scientific novelty of this work lies in the application of the method of mathematical modeling when studying the environmentally safe value of the radius of the watering-place for sheep flocks grazing in the conditions of a distant arid pasture.

The theoretical significance of the work lies in the fact that the results of the study can be used to analytically determine the ecologically safe value of the radius of the watering hole for flocks of sheep contained in the conditions of arid pasture.

The practical value of the work lies in the fact that the results and recommendations can be used by research institutions, sheep farmers, as well as educational institutions in training and retraining of senior and middle-class managers. The results of the work in economic practice can bring some economic benefits due to the rational operation of pastures, will help prevent degradation and improve the environmental situation on transhumant pastures of the arid ecosystems.

It is known that in sheep breeding the herd structure includes the following age-and-gender groups: stud rams, teaser rams, sheep for breeding, ewes, young ewes, one-year ewes and wedders. The flock is shaped depending on many factors: the purpose of certain breeds, breeding characteristics, gender and age of sheep (f/m) and etc. Moreover, large farms have the opportunity to allocate animals according to their wool color, size, pedigree and etc. Determination of the watering-place radius according to the method described above for each specific flock containing a particular gender and age group of animals is a task that requires its specific solution.

5. Conclusion
As a result of the analytical modeling, quantitative values of the radius of the watering-place are introduced in a graphical format. The calculations performed and the methodology obtained on their basis allow us to conclude that the radius of the watering-place depends on the average speed of animals (flocks of sheep) when grazing, the frequency of watering and grazing frequency per day. The main influence on the distance is exerted by the grazing land yield and the number of sheep in a flock.

The calculation methodology allows modeling the object of the study depending on various values of significant parameters and applying it in practice by drawing up appropriate recommendations for farms which deal with sheep products in pasture conditions of the arid ecosystems.

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