FORMATION OF ISOTHERMAL DISKS AROUND PROTOPLANETS. I. INTRODUCTORY THREE-DIMENSIONAL GLOBAL SIMULATIONS FOR SUB-NEPTUNE-MASS PROTOPLANETS

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\begin{abstract}
The regular satellites found around Neptune ($\approx 17 M_\oplus$) and Uranus ($\approx 14.5 M_\oplus$) suggest that past gaseous circumplanetary disks may have co-existed with solids around sub-Neptune-mass protoplanets (~< 17 $M_\oplus$). These disks have been shown to be cool, optically thin, and quiescent, with low surface densities and low viscosities. Numerical studies of the formation are difficult and technically challenging. As an introductory attempt, three-dimensional global simulations are performed to explore the formation of circumplanetary disks around sub-Neptune-mass protoplanets embedded within an isothermal protoplanetary disk at the inviscid limit of the fluid in the absence of self-gravity. Under such conditions, a sub-Neptune-mass protoplanet can reasonably have a rotationally supported circumplanetary disk. The size of the circumplanetary disk is found to be roughly one-tenth of the corresponding Hill radius, which is consistent with the orbital radii of irregular satellites found for Uranus. The protoplanetary gas accretes onto the circumplanetary disk vertically from high altitude and returns to the protoplanetary disk again near the midplane. This implies an open system in which the circumplanetary disk constantly exchanges angular momentum and material with its surrounding prenatal protoplanetary gas.
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\textbf{Key words:} hydrodynamics – methods: numerical – planets and satellites: formation

\textbf{Online-only material:} color figures

1. INTRODUCTION

Regular satellites of giant planets in the solar system are commonly postulated to form in circumplanetary disks (CPDs, hereafter) surrounding their parent planets. The process of forming a CPD accumulates gas and rock-ice solids, which are thought to be raw material of satellites, from the parent protoplanetary disk. Properties of regular satellites, such as their chemical compositions, differentiation, total mass, as observed for the outer planets (Jupiter, Saturn, Uranus, and Neptune) in the solar system, place tight constraints on their precursors. Icy satellites would not have formed if the CPDs formed were too hot for the volatiles to condense. Very dense CPDs would not have enough time to allow satellites to accrete mass and to survive through fast orbital decays. Understanding the CPDs is a key step toward understanding the formation of satellites.

Several models for CPDs have been proposed to account for the formation of satellite systems around giant planets. The solid enhanced minimum mass (SEMM) disk model (Mosqueira & Estrada 2003a, 2003b) suggests that CPDs that formed around gas giants (Jupiter and Saturn) may be optically thick inside and optically thin outside, while those that formed around ice giants (Uranus and Neptune) may be isothermal to the background nebula throughout the disk due to their low surface density. While the inner Saturnian satellites can only acquire volatiles after the disk cools down, Uranian/Neptunian satellites may accrete (icy) mass immediately after the formation of Uranus/Neptune. The CPD that formed around Uranus/Neptune is supposed to be cool, quiescent, and laminar, with low viscosity. The low surface density also suggests that the self-gravity of gas in those isothermal CPDs can be neglected as a first approximation (see Mosqueira et al. (2010) for the discussions on the Jovian and Saturnian systems). Canup & Ward (2002, 2006) proposed an alternative “gas-starved” CPD produced by a slow inflow of gas and solids. Similar to SEMM disks, such CPDs are expected to be optically thin and laminar, and with low viscosity, as expected for the formation of Galilean satellites. The major difference between the two competing models is how a CPD accretes gas and solids. More recently, a gas-free tidal-spread model is advocated (Crida & Charnoz 2012) to explain the puzzling mass–distance trend as observed in the outer three giant planets. However, this picture is hard to reconcile with the formation of Galilean satellites, which may still need to be assembled in a gaseous environment.

The formation of CPDs is complex and is best studied with numerical simulations (Lubow et al. 1999; D’Angelo et al. 2002, 2003; Tanigawa & Watanabe 2002; Bate et al. 2003; Machida et al. 2008, 2010; Machida 2009; Ayliffe & Bate 2009b, 2012; Ward & Canup 2010; Martin & Lubow 2011; Tanigawa et al. 2012; Uribe et al. 2013). It is especially technically challenging to simulate the formation of CPDs surrounding low-mass protoplanets like Uranus and Neptune. For example, for a 17 $M_\oplus$ protoplanet embedded within a protoplanetary disk with an aspect ratio of 0.05 at a heliocentric distance of 5.2 AU, the corresponding Hill radius is only half of the scale height of the protoplanetary disk. The CPD formed around this protoplanet is deeply embedded inside the scale height of the disk. The gas motions are three-dimensional in the surroundings of low-mass protoplanets. These CPDs are not isolated objects attached only to the protoplanets. They are constantly exchanging material and are disturbed by the bigger protoplanetary disks. For motions in and out of the CPDs in such an environment, the inclusion of geometric curvature is also necessary. Three-dimensional global simulations are required to cover the system. Using high spatial resolution without introducing softening lengths is the key to correctly following the gas dynamics of CPDs without distorting.
gas behavior around protoplanets for a CPD whose size is small compared to the corresponding Hill sphere (Quillen & Trilling 1998; Martin & Lubow 2011).

Due to the technical challenges, only a handful of numerical investigations have been attempted for the CPDs around low-mass protoplanets. Bate et al. (2003) performed three-dimensional global simulations to explore the formation of CPDs around protoplanets of one Earth mass to one Jupiter mass embedded in locally isothermal protoplanetary disks. Owing to insufficient spatial resolution (≈0.015 AU), CPDs are resolved only for protoplanetary masses greater than 32 $M_\oplus$, making the formation of CPDs around protoplanets of mass less than 32 $M_\oplus$ inconclusive. Ayliffe & Bate (2009a) performed sophisticated three-dimensional local smooth particle hydrodynamic simulations including self-gravity of gas and interstellar grain opacities to study the accretion of gas onto prescribed solid cores. Ayliffe & Bate (2009b) found that no CPDs can form around sub-Neptune-mass protoplanets even with a locally isothermal equation of state. The conclusion obtained from those numerical experiments, however, poses a challenge to our current understanding of satellite formation. Crida & Charnoz (2012) proposed that regular satellites of Uranus and Neptune formed from ancient massive rings rather than in a CPD, as partly motivated by Ayliffe & Bate (2009b).

Improvements that address these challenges are needed in state-of-the-art numerical codes to attack the physical problems. Several new features have been implemented into our Antares code (Yuan & Yen 2005). The original two-dimensional Cartesian code is extended to solve the three-dimensional hydrodynamic equations in cylindrical coordinates so that the effects of the third dimension and geometric curvature are included in this setup. Global simulations are performed to avoid the use of an artificial boundary condition in the azimuthal direction. Nested mesh refinement (NMR) is also implemented to resolve both the protoplanetary disk (≈10 AU) and CPDs (≈0.01 AU) that are of different dynamical ranges, and enables us to concentrate computational power on the surroundings of protoplanets. The use of softening length is avoided in this work so as not to distort the potential of protoplanets. This allows us to properly follow the gas behavior of CPDs. The global disk is relaxed for eight orbital times measured at the location of protoplanets for a carefully prepared initial condition. The perturbation in the protoplanetary gas caused by low-mass protoplanets is relatively small and can easily be overwhelmed without being started in an initial state that is as close to equilibrium as possible.

We address the formation of CPDs around sub-Neptune-mass protoplanets under the context of satellite formation using three-dimensional global hydrodynamical simulations. This work is parallel to the locally isothermal work of Ayliffe & Bate (2009a) but is performed with a different numerical scheme. This paper is structured as follows. The physical models and a brief introduction of the numerical method are described in Section 2. Analytic expectations based on a simple assumption and results obtained from numerical simulations are presented and analyzed in Section 3. A brief summary, discussions, and implications of our findings are arranged in Section 4.

2. MODELS

2.1. Governing Equations

We explore the formation of CPDs formed around sub-Neptune-mass protoplanets with masses of 4, 8, and 16 $M_\oplus$. Since the Hill radii of these models are smaller than the scale height of the protoplanetary disk, three-dimensional models are important for correctly following the flow patterns around protoplanets. The governing equations are described using cylindrical coordinates ($r, \phi, z$) co-rotating with protoplanets and with the origin centered at a host star of one solar mass. Here, we ignore the small shift in position between the central star and the center of mass. The globally isothermal gaseous disk then evolves based on the following governing equations without considering the self-gravity and the viscosity of gas (D’Angelo et al. 2002; Skinner & Ostriker 2010):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \phi} (\rho v_\phi) + \frac{\partial}{\partial z} (\rho v_z) = 0,$$  

$$\frac{\partial}{\partial t} (\rho v_r) + \frac{1}{r} \frac{\partial}{\partial r} [r (\rho v_r^2 + p)] + \frac{1}{r} \frac{\partial}{\partial \phi} (\rho v_r v_\phi) + \frac{\partial}{\partial z} (\rho v_r v_z) = \left(\rho v_\phi^2 + p\right)/r + \rho \left(2\Omega_B v_\phi + \Omega_p r - \frac{\partial \Phi}{\partial r}\right),$$  

$$\frac{\partial}{\partial t} (\rho v_\phi) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r v_\phi) + \frac{1}{r} \frac{\partial}{\partial \phi} \left(\rho v_\phi^2 + p\right) + \frac{\partial}{\partial z} (\rho v_\phi v_z) = -\left(\rho v_r v_\phi + \frac{1}{r} \frac{\partial \Phi}{\partial \phi}\right),$$  

$$\frac{\partial}{\partial t} (\rho v_z) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r v_z) + \frac{1}{r} \frac{\partial}{\partial \phi} (\rho v_r v_\phi) + \frac{\partial}{\partial z} \left(\rho v_z^2 + p\right) = -\rho \frac{\partial \Phi}{\partial z},$$

with $\rho$ being the volume density; ($v_r$, $v_\phi$, $v_z$) the velocity components in the radial, azimuthal, and vertical directions observed in the rotating frame, respectively; $p = c_s^2 \rho$ the isothermal gas pressure with $c_s$ the sound speed; $\Omega_B$ the orbital angular speed of the protoplanet; and $\Phi$ the total gravitational potential contributed from the central star and the protoplanet.

Equation (1) is the continuity equation, and Equations (2)–(4) are the momentum equations in the radial, azimuthal, and vertical directions, respectively. Note that the left-hand sides of Equations (1)–(4) are arranged in a conservation form, which allows us to make use of the original exact Riemann solver used for the Cartesian Antares with minimum modifications (Yuan & Yen 2005). The first terms on the right-hand sides of Equations (2) and (3) are geometric source terms. The Coriolis force, the fictitious centrifugal force due to the non-inertial frame, and the gravitational forces contributed from the central star and the protoplanet are organized together in the rightmost of Equations (2)–(4). The gravitational potential $\Phi$ has two parts given by

$$\Phi = \frac{-GM_p}{\sqrt{r^2 + z^2}} - \frac{GM_\odot}{\sqrt{R^2 + z^2}},$$

where $M_\odot$ is one solar mass, $M_p$ denotes the mass of the protoplanet, $G$ is the gravitational constant, and $r$ and $R$ are the radial distances measured from the central star and from the protoplanet, respectively. We note that the softening length is not introduced explicitly in our models. $R$ is related to the coordinates ($r, \phi, z$) through the following relation:

$$R^2 = r^2 + r_p^2 - 2rr_p \cos(\phi).$$
where $r_p$ is the orbital radius of the protoplanet, which is fixed at 5.2 AU for all models in this work. The isothermal sound speed, $c_s$, is chosen through the following relation:

$$c_s = \frac{\Omega_p^2}{r_p} = 0.05,$$

with $H$ being the disk thickness at $r_p$ and $v_{\text{Kep}} = \sqrt{GM_\odot/r_p} = \Omega_p r_p$ the corresponding Keplerian orbital velocity. The corresponding temperature at this particular orbital radius, $r_p$, can be estimated with a standard model of solar minimal nebula (Hayashi 1981; Hayashi et al. 1985) through the relation:

$$T = 280.0 \left( \frac{L}{L_\odot} \right)^{1/4} \left( \frac{r_p}{1\ \text{AU}} \right)^{-1/2} \text{K},$$

where $L$ and $L_\odot$ are the protostellar and solar luminosities, respectively. The gas temperature at the orbital radius is then evaluated to be $T = 123 \text{ K}$ if $L = L_\odot$ is adopted. Since we are interested in the local phenomena in the neighborhood of protoplanets, also given that the possibility that CPDs that formed around sub-Neptune-mass planets are optically thin (Mosqueira & Estrada 2003a), we further assume the temperature is uniform in the whole computational domain. It should be noted here that, in general, a global simulation using constant temperature is not an appropriate assumption. However, since the CPDs that formed around low-mass protoplanets only accrete surrounding gas of nearly the same temperature and the orbits of protoplanets are fixed at a constant radius, we expect that the global isothermal assumption adopted in this particular work would not dramatically affect our general conclusions for the properties of CPDs. The purpose of performing global simulations is to provide an appropriate background flow and to avoid using artificially imposed boundary conditions in the azimuthal direction.

### 2.2. Computational Setup

Three-dimensional global isothermal simulations are performed to study the formation of CPDs around sub-Neptune-mass protoplanets. Since the simulations evolved in a frame co-moving with protoplanets, we fix the location of protoplanets at $(r_p, \phi_p, z_p) = (5.2, 0, 0) \text{ AU}$. The choice of placing protoplanets at 5.2 AU is mainly for the comparison between our conclusions and that of Ayliffe & Bate (2009b), though it is also supported by the model proposed by Thommes et al. (2002). The protoplanetary disk is modeled in the region $r \in [2, 8] \text{AU}$, $\phi \in [0, 2\pi]$ radians, and $z \in [0, 1] \text{ AU}$. The root grid (coarsest grid, zeroth level) is covered by $200 \times 816 \times 25$ cells uniformly distributed in the radial, azimuthal, and vertical directions, respectively. The cell numbers are chosen such that the shape of cells close to a protoplanet is nearly a cube. The protoplanet is placed in the corner of cells to avoid singularity of gravitational potential. We note that no protoplanet surface is modeled and therefore any effects related to the boundary layer around protoplanets are not studied in this work. A possible impact of the presence of a boundary layer is the release of gravitational energy into the surroundings as a source of heat when the cores still accrete planetesimals. Since the CPDs in which we are interested are at the late stage of planet formation and are inviscid, we assume that the accretion onto core surfaces has almost subsided, although satellitesimals may still fall in due to gas drag or tidal torques (Canup & Ward 2006). Furthermore, if we simply adopt the mean density estimated for the ice-rock core of Uranus (Podolak et al. 1995) and apply it to evaluating the core radius of our models (see Table 1), we will find that the numerical resolution near the protoplanets is about four to seven times these core sizes. The physics near the “surface” of solid cores is not resolved in this work and therefore is neglected. The outer gaseous envelope, which is not considered in the models of ice-rock core sizes, contributes roughly an additional 20% to 30% radial extension in the sizes based on the models constructed for the internal structures of Uranus and Neptune (Podolak et al. 1995). We note that during the epoch when protoplanets initially formed, the entropy in the gaseous envelopes may have been high, so the envelopes could have been more extended than those that are presently observed.

Since the dynamical range involved in the formation of a CPD spans three orders of magnitude from 10 AU to $10^{-2}$ AU, we adopt the numerical technique of NMR for the evolution of both the protoplanetary and CPDs. With this technique, spatial resolution uniformly increases in a nested fashion, i.e., a uniform mesh of a higher spatial resolution of level $l$ is embedded within a uniform coarse mesh of level $l - 1$. In this way, computational power can be concentrated around the areas of scientific interests, i.e., the CPDs in our work. The volume covered by higher-level grids follows the rules:

$$|r - r_p| = \frac{3}{2^l},$$

$$|\phi - \phi_p| = \frac{\pi}{2 \cdot 2^l},$$

$$|z - z_p| = \frac{1}{2^l},$$

where $l$ is an integer running from 1 to 6, representing the level of refinements. The spatial resolution of level $l$ doubles

Table 1: Properties of CPDs

| Planet Mass $M_p$ (M$_\oplus$) | $R_H$ (AU) | Disk Size $a$ (AU) | Radius of $h/R = 0.5$ $(R_H)$ (estimated) | Radius of $h/R = 0.5$ $(R_H)$ (simulation) | $R_{\text{core}}$ $b$ (AU) | $N_{\text{K}}$ $c$ $d$ |
|-------------------------------|-----------|-------------------|-----------------------------------------------|-----------------------------------------------|----------------------|------------------|
| 4                             | 0.0837    | 0.008             | 0.055                                         | 0.049                                         | 9.0 x 10^{-3}       | 134              |
| 8                             | 0.1054    | 0.01              | 0.085                                         | 0.082                                         | 1.13 x 10^{-4}      | 168              |
| 16                            | 0.1328    | 0.016             | 0.137                                         | 0.164                                         | 1.43 x 10^{-4}      | 212              |

Notes:

$^a$ The disk size is determined by the turning point of specific angular momentum shown in Figure 4.

$^b$ The numerical resolution near the protoplanets is $6.25 \times 10^{-4}$ AU.

$^c$ The mean density of the ice-rock core adopted here is $2.35 \text{ g cm}^{-3}$, which is estimated based on the model for the core of Uranus (Podolak et al. 1995).

$^d$ The effective number of cells used to resolve a Hill radius in terms of the finest spatial resolution.
that of the level \( l - 1 \). With this rule, the finest spatial resolution is \( 6.25 \times 10^{-4} \) AU (compared with Jupiter's radius, \( 4.77 \times 10^{-4} \) AU). In terms of the finest spatial resolution, the corresponding Hill radii for protoplanets with 4, 8, and 16 \( M_\oplus \) located at 5.2 AU will be resolved with cell numbers \( N_{\text{grid}} = 134, 168, \) and 212, respectively. Since the frame is co-rotating with protoplanets, the grid structure is static in time. Because the equations we solve for are symmetric with respect to the midplane, only the north hemisphere is considered. The initial disk density profile was chosen to be axisymmetric and to follow:

\[
\rho(r, \phi, z) \propto \frac{1}{r^2} \exp \left( -\frac{GM_\odot z^2}{2r^2c_s^2} \right), \tag{12}
\]

where the volume density at the disk midplane scaling as \( r^{-2} \) is an arbitrary choice. Using different power laws is not expected to affect the results significantly because of the small size of a CPD compared with the characteristic length of the protoplanetary disk. The volume density is scale free because the self-gravity of gas is neglected in this work. The initial velocity \((v_r, v_\phi, v_z)\) in the co-rotating frame reads:

\[
(v_r, v_\phi, v_z) = (0, v_{Kep} - \Omega_pr, 0), \tag{13}
\]

where \( v_{Kep} \) denotes the circular Keplerian velocity orbiting the central star for a given radius. The physical boundary conditions are all fixed using the initial condition described above except that the boundary at the midplane is reflective. Note that no boundary in the azimuthal direction needs to be specified in global simulations.

2.3. Numerical Method

Three-dimensional simulations are performed with a high-order Godunov code known as Antares, in which hydrodynamic fluxes on cell interfaces are obtained from the exact/approximate Riemann solution (Yuan & Yen 2005). We employ the finite volume method to solve the hydrodynamic equations outlined in Section 2.1. The nested grid used in this work is based on the adaptive mesh refinement engine implemented for resolving a huge dynamical range. For the current work, computational domains are refined according to Equations (9)–(11) only at the beginning of a simulation. The grid structure is then fixed without change.

The grid arrangement of levels looks the same as shown in Figure 1 of D’Angelo et al. (2002). Unlike the staggered grid used in their work, we put all the variables at cell centers. The second-order accuracy in space is achieved by utilizing slope limiters, while the second-order accuracy in time is implemented with the Runge–Kutta method of second order (RK2). A global time step, \( \Delta t \), is determined by fulfilling the Courant–Friedrichs–Lewy (CFL) condition for all grids of different levels. The boundary of each grid is surrounded by two layers of ghost cells. The variables of ghost cells that are adjacent to the physical boundaries are described by Equations (12) and (13). For the ghost cells between levels \( l \) and \( l + 1 \), variables of ghost cells on level \( l + 1 \) are evaluated by conservative interpolations between the associated cells of level \( l \) (Li & Li 2004). The flux correction and the variable restriction are forced between levels to ensure the conservation of conservative variables between levels.

The exact Riemann solver is used for the calculation of hydrodynamic fluxes. We found that the exact Riemann solver is more robust than the approximate Riemann solver of HLL-type when the softening length is not explicitly introduced. In the Antares code, the Riemann problem is first solved in an iterative way. If it fails, the bisection method is used to find the solution.

2.4. Preparation for the Initial Condition

A good initial condition is especially important for the growth of CPDs around sub-Neptune-mass planets. The energy involved in the relaxation process purely from the numerical discretization may be comparable to the potential energy of low-mass protoplanets. This may potentially bias our conclusions, making the evolution of the first few orbits untrustworthy. To get around this problem, we first relax the initial condition as

![Figure 1. Initial condition prepared for three-dimensional simulations. Before the planetary potential is turned on, the imposed basic state is relaxed for eight orbital times. Images (a) and (b) show the volume density (on logarithmic scale) cutting through the plane \( \phi = 0 \) at \( t = 0 \) and \( t = 8 \) orbits, respectively. (c) The vertically integrated surface density along the line \( \phi = 0 \) after the relaxation (dashed line) is almost identical to that of the imposed initial condition (red solid). The analytic profile (black solid) is obtained by direct integrating Equation (12) for a given midplane density from \( z = 0 \) to \( \infty \). The good match inside 6 AU indicates that the vertical structure of the disk is well-resolved with our coarsest numerical resolution, while the deviation in the outer disk reflects that the computational domain is not large enough to include all disk gas. Since the self-gravity of gas is not taken into account, both the volume density and the surface density are scale free. (A color version of this figure is available in the online journal.)](image-url)
described in Section 2.2 for eight orbital times (measured at the position of protoplanets, i.e., 5.2 AU) until the protoplanetary disk accommodates itself to the nested grid structures. After the relaxation, we expect the new configuration should not deviate much from the imposed initial condition.

Figure 1 illustrates the result after a relaxation of eight orbital times. Figures 1(a) and (b) show the volume density cutting through the plane defined by $\phi = 0$ (on a logarithmic scale) at $t = 0$ and $t = 8$ orbits, respectively. Figure 1(c) shows the vertically integrated surface density along $\phi = 0$. The surface density after relaxation (dashed line) is almost identical to the imposed initial condition (red solid line). We adopt the relaxed configuration as our real initial condition throughout this work. The potentials of protoplanets are gradually turned on and reach their full strength after 0.4 orbital time. The analytic radial profile (black solid) of the surface density integrated directly by Equation (12) from $z = 0$ to $\infty$ is also shown in Figure 1(c) for comparison. The good match between the analytic profile and the numerical setup inside 6 AU indicates that the vertical structure of the protoplanetary disk is well resolved with the coarsest numerical resolution, whereas the deviation from the analytical profile in the outer disk reflects that the computational domain is not large enough to include the high altitude disk gas. Since the CPDs around low-mass protoplanets only accrete material from the nearby surroundings and the Hill radii of our models are less than the scale height of the protoplanetary disk, we expect that missing high altitude gas in the outer disk will not alter our general conclusions.

3. RESULTS

The scientific goal of this work is to show that the expected CPDs, which are associated with the formation of satellites, can reasonably form around sub-Neptune-mass protoplanets in an isothermal, inviscid and non-self-gravitating protoplanetary disk. The upper panel of Figure 2 shows the edge-on view of volume density cutting through the plane defined by $\phi = 0$. The bottom panel shows the corresponding surface density integrated vertically for the finest grid, i.e., from $z = 0$ to 0.0156 AU as shown in the figures. It is clear that the disk structure is increasingly evident with increasing protoplanetary mass due to the density contrast between CPDs and high altitude protoplanetary gas. That is, for protoplanets with lower masses the boundaries between CPDs and protoplanetary gas are blurred since the pressure support becomes increasingly important compared with the gravity of protoplanets. As a result, the structure of CPD formed around a 4 $M_\oplus$ protoplanet looks more like an oblate spheroid.

The CPD that forms around a protoplanet of 16 $M_\oplus$ reaches a steady state in a few orbital times, while the CPDs surrounding protoplanets of 4 and 8 $M_\oplus$ develop non-steady spiral shocks and remain constantly disturbed by protoplanetary gas. This phenomenon can be understood by looking at the flow lines of protoplanetary gas (Figure 3(a)) together with the density contrast between CPDs and the surrounding protoplanetary gas (Figure 2). Figure 3(a) shows the bird’s-eye view of six streamlines that follow the tracks of protoplanetary gas moving through the CPD that formed around a protoplanet of 16 $M_\oplus$. These streamlines are integrated both forward (positive) and backward (negative) from the zero-time points shown as the green dots. The time elapse from the zero-time points is coded with different colors along the streamlines. That is, the bluer color is farther back in time while the redder is more ahead in time. The green dots are uniformly placed on a circle centered at $(r, \phi, z) = (5.2, 0, 0.001)$ AU with its normal along the $z$-axis. The radius of the circle is chosen to be 0.016 AU defined as the size of the CPD using the turning point seen in the plot of specific angular momentum (discuss below). The streamlines show that high altitude protoplanetary gas accretes onto the CPD vertically, circling the central protoplanet for several times before it returns again as the protoplanetary gas. This figure implies that this CPD is constantly exchanging angular momentum and material with protoplanetary gas and explains how the CPD gets disturbed. Nevertheless, the high density contrast as seen from the edge-on view for the 16 $M_\oplus$ protoplanet (Figure 2(c)) makes this CPD less subject to disturbance from the protoplanetary gas. On the contrary, those CPDs that formed around 4 or 8 $M_\oplus$ protoplanets develop non-steady shocks due to the impinging of high altitude protoplanetary gas.
Another important conclusion we can draw from Figure 3(a) is that for sub-Neptune-mass protoplanets a fraction of gas situated inside but near $R_H/10$ is not bound to the central protoplanets. This naturally leads to the asymmetry of CPD as seen in Figure 2(c), since the protoplanetary gas in the vicinity of the protoplanet is also not entirely symmetric with respect to the protoplanet. The lopsidedness of the CPD is sustained by the protoplanetary gas and may have an impact on the long-term planet migration due to its proximity.

The scale height of an isothermal CPD can be estimated analytically by assuming that the CPD is vertically hydrostatic, i.e.,

$$-rac{c_s^2}{\rho} \frac{\partial \rho}{\partial z} = -\Phi_R.$$  \hspace{1cm} (14)

As a result, $\rho(R,z) = \rho_0(R) \exp[-(\Phi(R,z) - \Phi(R,z = 0))/c_s^2]$, where the total potential $\Phi(R,z)$ is defined by Equation (5) and $\rho_0(R)$ is the volume density in the midplane.

For a given planetocentric radius, the scale height, $h$, is then estimated as the standard deviation of a Gaussian function fitted for the vertical volume density.

The estimated aspect ratios of CPDs as functions of planetocentric distance for our models are shown as the solid curves in Figure 3(b). The radial distances are normalized with the corresponding Hill radius. Evidently, isothermal CPDs are flaring with increasing distance. The degree of disk flaring increases with decreasing protoplanetary mass. This plot indicates that disk-like objects are expected to form around sub-Neptune-mass protoplanets and should be resolved with sufficient spatial resolution. Based on this result, if one defines a radius that corresponds to the aspect ratio $h/R = 0.5$ as the size of a disk, with our finest spatial resolution, the inner part of CPDs with aspect ratios less than 0.5 will be covered with 7, 14, and 29 cells for protoplanets with masses 4, 8, and 16 $M_\oplus$, respectively. The radii that correspond to the aspect ratio $h/R = 0.5$ can be either read directly from Figure 3(b) or found in Table 1 (column 4). The choice of $h/R = 0.5$ is somewhat arbitrary. However, it helps us estimate the spatial resolution needed to resolve a CPD. We note that our spatial resolution placed around protoplanets is fairly sufficient to resolve a CPD formed around a 16 $M_\oplus$ protoplanet, while it only marginally resolves the CPD formed around a 4 $M_\oplus$ protoplanet.

The dashed curves shown in Figure 3(b) are the aspect ratios of CPDs extracted from our numerical simulations using the Gaussian fit. These curves are obtained by azimuthally averaging the structure of CPDs over the last eight orbital periods of simulations. If again taking a radius that corresponds to $h/R = 0.5$ as the disk size (see Table 1, column 5), the results obtained for the models of 8 and 16 $M_\oplus$ closely follow the corresponding solid lines until 0.1$R_H$, which is consistent with the disk size defined by the turning point of the specific angular momentum. The deviation seen in the model of 4 $M_\oplus$ is expected from the edge-on view shown in Figure 2(a), since the vertical extent of the CPD is blurred with the surrounding protoplanetary gas. Although the gas situated beyond 0.1$R_H$ should not be considered as part of the CPDs, the aspect ratios obtained for all simulation models are generally thinner than the corresponding solid lines. This is because, within the Hill radius, protoplanetary gas is vertically perturbed by the gravity of the protoplanet. As a result, ram pressure exerted from the top may significantly reduce the expected scale heights.

Figure 4 shows the specific angular momentum measured in the rotating frame as a function of planetocentric radius. The red dashed lines represent the ideal Keplerian motions, while the vertical black lines mark the locations of one-third of the Hill radii. The inner CPD surrounding the 16 $M_\oplus$ protoplanet is nearly Keplerian, while the one surrounding the 4 $M_\oplus$ is sub-Keplerian. This result is consistent with the edge-on view shown in Figure 2, since the latter CPD is embedded in a partially pressure-supported envelope. If we take the turning point seen in

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**Figure 3.** (a) Bird’s-eye view of streamlines (solid lines) traced in a CPD that formed around a protoplanet of 16 $M_\oplus$. The dots represent the zero-time points for tracing the streamlines. They are arranged on a circle with its normal along the $z$-axis and centered at $(r, \phi, z) = (5.2$ AU, 0, 0.001 AU). The radius of the circle is chosen to be 0.016 AU defined as the size of the CPD (see Table 1). These streamlines are traced both forward (redder, positive number) and backward (bluer, negative number) using colors to denote the time elapse from the zero-time points. The unit of time is (AU km) $^{-1}$. (b) The solid lines denote the estimated scale heights of CPDs using semi-analytic models for planets with 4, 8, and 16 $M_\oplus$, while the dashed lines are the corresponding scale heights obtained by azimuthally averaging the structure of CPDs over the last eight orbital times of the simulations. (A color version of this figure is available in the online journal.)
the specific angular momentum as the size of the CPDs (Ayliffe & Bate 2009b; Bu et al. 2013), as listed in Table 1, the disk sizes will be roughly $R_H/10$, much less than $R_H/3$ suggested by the kinematic argument (Quillen & Trilling 1998).

4. DISCUSSION AND SUMMARY

4.1. Discussion

In the isothermal limit, the gas behavior around sub-Neptune-mass protoplanets in this work is different from that of Ayliffe & Bate (2009a). While they found no disk-like structure around protoplanets of masses less than $33 M_⊕$, our results indicate a CPD of expected vertical profile can reasonably form around a protoplanet with a mass at least down to $8 M_⊕$. The exact reason that leads to the discrepancy is not clear. It seems that the difference in spatial resolution should not be the main cause, since those adopted in Ayliffe & Bate (2009a) ($\approx 3 \times 10^{-4} R_H$) are much better than what is done in this work ($\approx 5 \times 10^{-3} R_H$ for the $16 M_⊕$ protoplanet). Beyond this specific aspect, some differences between Ayliffe & Bate (2009a) and our work still exist. Gas viscosity, which presumably originates from magnetic turbulence, is not explicitly included in this work. On the contrary, artificial viscosity is a standard procedure when using a particle-based hydrodynamic code. The impact of viscosity on the formation and on the structure of CPDs is not well understood. Two-dimensional simulations in shearing boxes by Bu et al. (2013) suggested that viscosity may be responsible for transferring angular momentum out of disks and facilitates mass accretion onto CPDs. On the other hand, as shown in this work, the gas motion around low-mass protoplanets is fully three-dimensional. It is not clear how viscosity would work as a CPD accretes gas vertically. It has been tested and reported that a grid-based code is especially suitable for problems in which the physics of interest is in the region of rapidly changing density (Tasker et al. 2008). As shown in Figure 2(c), the density contrasts can be more than three orders of magnitude between CPDs and the surrounding gas. Whether or not differences in the numerical schemes can generate the main differences requires further investigation.

In this work, the sizes of CPDs determined by the turning points of specific angular momentum are substantially smaller than $R_H/3$, which is obtained from a purely kinematic argument (Quillen & Trilling 1998). A size of $R_H/3$ is usually expected for the CPDs around more massive protoplanets (Ayliffe & Bate 2009b). The turning point of the specific angular momentum marks a radius beyond which gas does not orbit the protoplanet and is often taken as the outer edge of a CPD (Ayliffe & Bate 2009b; Bu et al. 2013). On the other hand, the kinematic argument neglects the effect of thermal pressure and is applied to protoplanets massive enough that their gravitational forces dominate over the thermal pressure inside the Hill radii. In this case, the sizes of CPDs are limited by the conservation of angular momenta of the inflow gas. For the low-mass regime explored in this work, thermal pressure is important compared with the gravitational forces of the protoplanets inside the Hill radii. Keplerian disks can only be expected for the regions close enough to the protoplanets (due to the inverse square law of gravity), resulting in smaller CPDs around low-mass protoplanets. The second and the third columns of Table 1 indicate that more massive protoplanets tend to have larger CPDs in terms of Hill radii. We conclude that the size of CPDs increases with the mass of protoplanets and $R_H/3$ should be taken as an upper limit of disk size.

The sizes of CPDs may impact calculations of torques exerted on protoplanets. The CPD and its protoplanet are often assumed to be a gravitationally bound system so that their internal interaction would not have a long-term effect on the protoplanetary migration. Based on this assumption, either a planetocentric radius inside which the interaction between the CPD and the protoplanet is excluded or a softening length comparable to the size of CPDs is often introduced in the study of planet migration. In fact, the region that can be excluded in the calculation of migration causing torques still lacks consensus, especially for low-mass protoplanets, primarily due to the poor spatial resolution in the vicinity of protoplanets. Efforts have been made to quantify the region to exclude the migration causing torques for massive protoplanets. Two-dimensional simulations suggest that a significant fraction of the total torque exerted on a massive protoplanet is from the region $R_H/2 < R < R_H$ (Crida et al. 2009). Three-dimensional simulations by Ayliffe & Bate (2009b) showed that a CPD surrounding a Jovian mass protoplanet may extend to $\approx R_H/3$ inside which the gas exerts no migration causing torque to the protoplanet. Ayliffe & Bate (2010) explored thermal effects on the Type I migration timescale for low-mass protoplanets ($10–33 M_⊕$) using more realistic protoplanetary surfaces located at 0.03 $R_H$, where the gravitational forces of protoplanets diminish to zero. The treatment of the protoplanetary surface allows the envelope close to the core to develop self-consistently. Since, in this mass regime, a discrepancy exists between their results and
ours, a straightforward interpretation cannot be drawn. More recently, Tanigawa et al. (2012) studied the gas accretion flow onto a CPD that formed around a Jovian mass protoplanet using three-dimensional local nested-grid hydrodynamic simulations with a spatial resolution that was one-fourth of that of the present Jupiter radius. They found that the outward radial velocity increases significantly at a planetocentric distance of about $R_{H}/5$, pushing the planetocentric boundary, within which one may exclude gaseous torque, further inside $R_{H}/5$. On the other hand, our results indicate that the torques exerted on low-mass protoplanets require a careful calculation for the gas situated inside one-tenth of the Hill radius. Owing to the proximity of CPDs to the protoplanets, the lopsidedness of the CPD sustained by protoplanetary gas moving in and out of the CPD may affect planet migration in the long run. The softening length should be used with caution when a low-mass planet is free to migrate.

Since protoplanetary gas moves in and out of the CPDs, the systems around low-mass protoplanets may be easily perturbed. As shown in Figures 2(a) and (b), nonsteady spiral shocks may appear in CPDs surrounding protoplanets with masses less than $16 M_{\oplus}$. These nonsteady structures are not likely due to numerical instability since they are only prominent in the models with lower masses, i.e., 4 and $8 M_{\oplus}$, and disappear in the model with $16 M_{\oplus}$. This trend is physically expected since lower density contrast between the CPDs and gas infalling at high altitudes as seen in Figures 2(a) and (b) makes these CPDs more vulnerable to disturbances from protoplanetary gas. We also performed two-dimensional local and global simulations (unpublished) for the same mass regime using the same numerical code, Antares. For the two-dimensional global simulations, CPDs are disturbed with epicyclic frequency, though the disturbances are from the midplane rather than from high altitudes. By contrast, only steady CPDs are observed for two-dimensional local simulations, where the outgoing/injecting boundary conditions are applied in the azimuthal direction (Tanigawa & Watanabe 2002; Bu et al. 2013). This simple tests suggest that the disturbance is feedback from the perturbed protoplanetary gas, which can only be properly modeled in global simulations. The nonsteady shocks are more likely due to a physical origin rather than a numerical artifact since we would otherwise expect to see them in two-dimensional local simulations.

The isothermal three-dimensional global simulations in this work show interesting implications for planet formation without the inclusion of viscosity, grains, self-gravity, and planet migration. Current understanding of the core accretion model was mainly based on results obtained in one-dimensional calculations (Pollack et al. 1996; Lissauer et al. 2009; Movshovitz et al. 2010; Mordasini et al. 2012), which adopted the assumed spherical symmetry of core-nucleated gas envelopes surrounding solid cores of 1–15 earth masses. Lissauer et al. (2009) and Movshovitz et al. (2010) suggest that, due to grain growth, the grain opacities in the envelopes of protoplanets can be three to four orders of magnitude less than the interstellar level, resulting in more rapid heat loss. Our results indicate that in the limit of vanishing opacity, instead of direct contraction, the centrifugal barrier in the conservation of angular momentum leads to the formation of CPD. As a result, low grain opacities might imply that one-dimensional models of the protoplanetary envelopes might be geometrically oversimplified. However, when thermal support is important, the protoplanets may remain approximately spherical up to $100 M_{\oplus}$ then undergo gravitational collapses that lead to the formation of CPDs (Ayliffe & Bate 2012; Lissauer et al. 2009; Mordasini et al. 2012).

4.2. Summary

We address the formation of CPDs around sub-Neptune-mass protoplanets using three-dimensional global hydrodynamic simulations. The CPDs, inside which satellites grow, are believed to form after the protoplanets have accreted most of their final masses. While inferences from observations expect the CPDs to be cool and optically thin, the processes of forming CPDs and accreting material are not well understood. By implementing NMR in our Antares code, protoplanetary disks and CPDs can be properly resolved and evolved together. Cylindrical coordinates are adopted for the effects of curvature. An initial condition in equilibrium was carefully prepared to study the interaction between the protoplanetary and CPDs. Our findings are summarized as follows.

1. The size of isothermal CPDs formed around sub-Neptune-mass protoplanets is estimated to be $R_{H}/10$ using the turning point of specific angular momentum.
2. The vertical structures of CPDs formed around $8$ and $16 M_{\oplus}$ protoplanets fit well with the estimated scale heights based on the assumption of vertical hydrostatic equilibrium, while the CPD around $4 M_{\oplus}$ resembles an oblate spheroid embedded in protoplanetary gas.
3. The streamlines around the protoplanets enter the CPDs almost vertically from high altitudes and return to the protoplanetary disk in heliocentric orbits near the midplane. A CPD is an open system, constantly exchanging angular momentum and material with the outer protoplanetary disk. The lopsidedness of CPDs with respect to the protoplanet is maintained by flows in and out of the system for tens of orbits, whose symmetry is broken by geometric curvature. This indicates that torque contributed by gas inside $R_{H}/10$ might have a long-term influence on planet migration.
4. The CPDs formed around $4$ and $8 M_{\oplus}$ protoplanets may develop non-steady shocks due to disturbances in the protoplanetary gas, while those CPDs around more massive protoplanets are relatively steady because of the high density contrast between CPDs and the ambient gas.
5. In the limit of vanishing opacity, the presence of a CPD implies that a centrifugal barrier needs to be overcome before a low-mass protoplanet can grow to a Jupiter size.

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