Lorentz invariance violation and electromagnetic field in an intrinsically anisotropic spacetime

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Abstract

Recently, Kostelecky [V. A. Kostelecky, Phys. Lett. B 701, 137 (2011)] proposed that the spontaneous Lorentz invariance violation (sLIV) is related to Finsler geometry. Finsler spacetime is intrinsically anisotropic and induces naturally Lorentz invariance violation (LIV). In this paper, the electromagnetic field is investigated in locally Minkowski spacetime. The Lagrangian is presented explicitly for the electromagnetic field. It is compatible with the one in the standard model extension (SME). We show the Lorentz–violating Maxwell equations as well as the electromagnetic wave equation. The formal plane wave solution is obtained for the electromagnetic wave. The speed of light may depend on the direction of light and the lightcone may be enlarged or narrowed. The LIV effects could be viewed as influence from an anisotropic media on the electromagnetic wave. In addition, the birefringence of light will not emerge at the leading order in this model. A constraint on the spacetime anisotropy is obtained from observations on gamma–ray bursts (GRBs).

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1. INTRODUCTION

At experimentally attainable energy scales, Einstein’s special relativity (SR) is compatible with the present observations. However, the SR is believed to be modified at higher energy scales, such as the Planck scale which involves the effects of quantum gravity [1–14]. The study on the string theory reveals that the Lorentz symmetry could be broken spontaneously in the perturbative framework [12–14]. The spontaneous Lorentz invariance violation (sLIV) involves nonzero vacuum expectation values of certain tensor fields. It characterizes the anisotropy of spacetime since nonzero vacuum expectations of tensor fields are related to certain preferred directions.

To demonstrate spirit of the sLIV, we review shortly the spontaneous symmetry breaking in the electroweak theory. The electroweak theory involves a Higgs field acquiring a nonzero vacuum expectation value, which leads to the mass terms of other particles. Similarly, certain tensor fields acquire nonzero vacuum expectation values in the sLIV framework. However, these expectation values take along the spacetime indices, which are different from the scalar one in the standard model (SM). Therefore, the velocities of particles and fields may influence propagations and interactions, respectively.

Actually, the sLIV terms are added into the Lagrangian of fields by considering the gauge invariance, renormalizability, etc. The vacuum expectation values of tensor fields become the coupling constants in the sLIV terms. This approach to introduce the sLIV effects is called the standard model extension (SME) [13, 14], which is an effective field theory irrelative to the ultimately underlying theory. Obviously, the spacetime background is still Minkowskian in the SME. However, Minkowski spacetime should be amended together with Lagrangian of particles and fields if the Lorentz symmetry is violated (no matter spontaneously or not).

Recently, Kostelecky [15] proposed that the SME is closely related to Finsler spacetime which is intrinsically anisotropic. The coupling constants in the sLIV terms could be related to certain fixed preferred directions in the Finsler structure. The most fundamental reason is that Finsler geometry [16, 18] gets rid of the quadratic restriction on the spacetime structure such that the Finsler metric depends on directions of the spacetime. In addition, the isometric transformations reveal that non–Riemannian Finsler spacetime possesses fewer symmetries than Riemann spacetime [19–22]. These characters imply that Finsler spacetime is intrinsically anisotropic.
Einstein’s special relativity resides in a flat Riemann spacetime, namely Minkowski spacetime. Similarly, the special relativity with LIV effects may reside in a flat Finsler spacetime. In fact, it is found that the SME-related Finsler spacetime is indeed flat in the sense of Finsler geometry [15]. For instance, the simplest SME model, with only one nonvanishing coupling constant $a_{\mu}$ in the sLIV terms, leads to a flat spacetime of Randers–Finsler geometry [23]. Actually, the flat Finsler spacetime is called locally Minkowski spacetime [17], which could be viewed as a generalization of Minkowski spacetime. In addition, doubly special relativity (DSR) [1–5] was found to be incorporated into Finsler spacetime [24], as well as very special relativity (VSR) [10, 25].

As the LIV corresponds to new spacetime, it is valuable to investigate physics compatible with the LIV effects. In this paper, we try to set up equations of motion for the electromagnetic field in locally Minkowski spacetime. A Lagrangian is proposed for the electromagnetic field in such a spacetime. The LIV effects are induced into the Lagrangian in a natural way. The amended Maxwell equations are obtained via the variation of action. A formal plane wave solution is obtained for the electromagnetic wave. The dispersion relation is modified for the electromagnetic wave. We also study the electromagnetic field at the first order of LIV effects. We compare these perturbative results with those in the SME framework. Relations and differences are discussed between Finsler spacetime and the SME. An interpretation is proposed for the LIV effects as influence of an anisotropic media. In addition, a constraint on the spacetime anisotropy could be obtained from astrophysical observations on gamma-ray bursts (GRBs).

The rest of the paper is arranged as follows. In section 2, we briefly discuss the spacetime in Finsler spacetime, especially the locally Minkowski spacetime. We propose an electromagnetic field model in locally Minkowski spacetime in section 3. In section 4, we study this model at the first order of LIV effects and compare it with the SME. The anisotropic media is invoked to interpret the LIV effects on the electromagnetic field. In section 5, a constraint on the LIV effects is obtained from the Fermi-observations of GRBs in a specific locally Minkowski metric. Conclusions and remarks are listed in section 6.
2. SPACETIME ANISOTROPY

Finsler spacetime is defined on the tangent bundle $TM := \bigcup_{x \in M} T_x M$ instead of the manifold $M$. Each element of $TM$ is denoted by $(x, y)$, where $x \in M$ and $y \in T_x M$. Finsler geometry originates from the integral of the form \[ \int_a^b F(x, y) \, d\tau, \] where $x$ denotes a position and $y := dx/d\tau$ denotes a so-called 4–velocity. The integrand $F(x, y)$ is called a Finsler structure, which is a smooth, positive and positively 1–homogeneous function defined on the slit tangent bundle $TM \backslash \{0\}$. The positive 1–homogeneity denotes the character $F(x, \lambda y) = \lambda F(x, y)$ for all $\lambda > 0$. The Finsler metric is defined as

$$g_{\mu\nu}(x, y) := \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} \left( \frac{1}{2} F^2 \right).$$

Together with its inverse tensor, it is used for raising and lowering indices of tensors. Note that the Finsler metric becomes Riemannian if it does not depend on $y$.

A Finsler spacetime $(M, F)$ is called locally Minkowski spacetime if there is no dependence on $x$ for Finsler structure $F$, namely $F = F(y)$. Therefore, the Finsler metric $g_{\mu\nu}$ only depends on $y$ according to (2). In such a spacetime, connections and curvatures vanish. Therefore, it is flat and maximally symmetric. The vanishment of the connections implies that a free particle follows a straight line. It also implies that locally Minkowski spacetime belongs to Berwald spacetime. All tangent spaces of Berwald spacetime are linearly isomorphic to one common Minkowski–normed linear space. Physically, this character implies that the laws of physics are common at each position in such a spacetime.

In Finsler spacetime, 4–velocity of a free particle is given by the Finsler geodesic equation

\[ \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}(x, \frac{dx}{d\tau}, \frac{dx}{d\tau}, \frac{dx}{d\tau}) \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0, \]

where $\Gamma$ denotes the connection. The Finsler geodesic originates from variation of an integral of the Finsler line element of the form (1). In locally Minkowski spacetime, the connections vanish, particularly. The Finsler geodesic equation (3) becomes

$$\frac{d^2 x^\mu}{d\tau^2} = 0.$$
Its solution gives a constant vector to \( y \), which means that \( y \) is independent on \( x \). In this paper, \( y \) denotes 4–velocity of a free photon along the Finsler geodesic. For a charged particle, such as electron, it would interact with the electromagnetic field. The Finsler geodesic equation should be modified. An extra term related to electromagnetic force \( F^\mu(x) \) should be added to the right hand side (r.h.s.) of the Finsler geodesic equation. The velocity of the charged particle is given by the solution of the modified geodesic equation. Thus, it depends on \( x \).

3. ELECTROMAGNETIC FIELD IN LOCALLY MINKOWSKI SPACETIME

An advantage of studying the LIV in Finsler spacetime is that the principle of relativity is preserved automatically. As in Minkowski spacetime, we define the 4–potential 1–form of electromagnetic field in locally Minkowski spacetime

\[
A := A_\mu(x)dx^\mu .
\]  

(5)

It preserves the internal U(1) gauge symmetry. The electromagnetic 4–potential is chosen as such a form that its 2–form excludes the terms \( dx^\mu \wedge \delta y^\nu \) and \( \delta y^\mu \wedge \delta y^\nu \) whose physical meaning is unclear. Therefore, the field strength 2–form is given by

\[
F := dA = \frac{1}{2!} F_{\mu\nu}(x)dx^\mu \wedge dx^\nu ,
\]  

(6)

where

\[
F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}
\]  

(7)

is invariant under the U(1) gauge group.

One of the Maxwell equations is given by the Bianchi identity \( dF = 0 \),

\[
\frac{\partial F_{\mu\nu}}{\partial x^\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x^\mu} + \frac{\partial F_{\lambda\mu}}{\partial x^\nu} = 0 .
\]  

(8)

It is similar to the one in Minkowski spacetime. In addition, the contravariant field strength \( F^{\mu\nu} \) is given via raising the indices of the covariant 2–form (7) by the Finsler metric \( g^{\mu\nu}(y) \) of locally Minkowski spacetime, namely \( F^{\mu\nu} = g^{\mu\sigma}g^{\nu\lambda}F_{\sigma\lambda} \). In this way, the covariant character is preserved in locally Mikowski spacetime.

We follow the form of the Lagrangian for the electromagnetic field but replace the space-time metric \( \eta_{\mu\nu} \) by the Finsler metric \( g_{\mu\nu}(y) \). In this way, the Lagrangian could
reduce back to the one in Minkowski spacetime when locally Minkowski spacetime reduces into Minkowski spacetime. The Lagrangian takes the form

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \]

(9)

The LIV effects are introduced via contracting the spacetime indices by the Finsler metric. The Lagrangian is invariant under coordinate transformations.

In locally Minkowski spacetime, an orthogonal base is given by \( \{ \frac{\partial}{\partial x^\mu} \} \) and its dual base is \( \{ dx^\mu \} \). The action of the electromagnetic field could be given by

\[ I = \int \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) d\Omega, \]

(10)

where \( d\Omega = \sqrt{-\det g_{\mu\nu}(y)} d^4 x \) denotes the invariant volume element at each position \( x \). The variation of action (10) with respect to \( A_\mu \) is given by

\[ \int \left( \frac{\partial L}{\partial A^\mu} - \frac{\partial}{\partial x^\sigma} \frac{\partial L}{\partial \left( \frac{\partial A^\mu}{\partial x^\sigma} \right)} \right) \delta A^\mu d\Omega = 0. \]

(11)

It gives the familiar Euler–Lagrangian equation

\[ \frac{\partial L}{\partial A^\mu} - \frac{\partial}{\partial x^\sigma} \frac{\partial L}{\partial \left( \frac{\partial A^\mu}{\partial x^\sigma} \right)} = 0. \]

(12)

The Euler–Lagrange equation supplements the Maxwell equations

\[ g^{\mu\nu} \frac{\partial F_{\mu\sigma}}{\partial x^\nu} = 0. \]

(13)

The equations (8) and (13) form a complete set of equations of motion for the electromagnetic field in locally Minkowski spacetime.

The Maxwell’s equation (13) could be rewritten in terms of \( A_\sigma \) as

\[ g^{\mu\nu} \frac{\partial^2 A_\sigma}{\partial x^\mu \partial x^\nu} = 0, \]

(14)

under the Lorentz gauge

\[ g^{\mu\nu} \frac{\partial A_\mu}{\partial x^\nu} = 0. \]

(15)

The above equation is the so-called electromagnetic wave equation. It has a formal plane wave solution

\[ A_\sigma \propto \epsilon_\sigma e^{-ik_\mu x^\mu} = \epsilon_\sigma e^{-ig_{\mu\nu}k^\mu x^\nu}, \]

(16)
where \( \varepsilon^\sigma \) denotes a polarization and \( k^\mu \) denotes a wavevector of the electromagnetic plane wave. By substituting (16) into (14), we obtain a dispersion relation for the electromagnetic plane wave

\[
k_\mu k^\mu = g_{\mu\nu} k^\mu k^\nu = 0.
\]

(17)

Its form is as similar as the one in the Lorentz invariant electrodynamics. However, it is modified by the Finsler metric \( g_{\mu\nu} \) since the contraction of spacetime indices is implicated via this metric.

### 4. LORENTZ INVARIANCE VIOLATION

Observations do not show signals of the LIV effects at the present attainable energy scales [30]. This fact implies that the LIV effects should be very tiny. We could extract the LIV effects by expanding the Finsler metric into

\[
g^{\mu\nu}(y) = \eta^{\mu\nu} + h^{\mu\nu}(y).
\]

(18)

In this way, the first–order LIV effects are extracted and characterized completely by \( h_{\mu\nu} \).

At the leading order, the Lagrangian (9) of the electromagnetic field could be expanded into

\[
L = -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{2} \eta^{\mu\rho} h^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma},
\]

(19)

\[
:= L_{LI} + L_{LIV},
\]

(20)

where \( L_{LI} = -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \) denotes the Lorentz invariant term while the LIV term is given as

\[
L_{LIV} = -\frac{1}{8} (\eta^{\mu\rho} h^{\nu\sigma} - \eta^{\nu\rho} h^{\mu\sigma} - \eta^{\mu\sigma} h^{\nu\rho} + \eta^{\nu\sigma} h^{\mu\rho}) F_{\mu\nu} F_{\rho\sigma}.
\]

(21)

In the above equation, we have anti–symmetrized the indices \( \mu\nu \) and \( \rho\sigma \). In the SME framework, meanwhile, the CPT–even sLIV term in the Lagrangian of the electromagnetic field is given by [14]

\[
L_{SME} = -\frac{1}{4} k^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma},
\]

(22)

where \( k^{\mu\nu\rho\sigma} \) denotes a dimensionless parameter which characterizes the level of the sLIV effects. In the SME, the parameter \( k^{\mu\nu\rho\sigma} \) is given by hand.
In locally Minkowski spacetime, however, we could relate this parameter to deformation parameter \( h^{\mu \nu} \) of the spacetime from Minkowski to locally Minkowski. By comparing (21) and (22), we obtain a relation
\[
k^{\mu \nu \rho \sigma} = \frac{1}{2} \left( \eta^{\mu \rho} h^{\nu \sigma} - \eta^{\nu \rho} h^{\mu \sigma} - \eta^{\mu \sigma} h^{\nu \rho} + \eta^{\nu \sigma} h^{\mu \rho} \right).
\]
In the SME, the parameter \( k \) is a constant since the energy and momentum are conserved [14]. In locally Minkowski spacetime, the geodesic equation (11) of photon gives \( y \) a constant vector along the geodesic. Thus, \( h \) is a constant and the r.h.s. of equation (23) is also a constant. These inferences mean that the LIV electromagnetic field model obtained in locally Minkowski spacetime is compatible with the perturbative results in the SME. In addition, there are ten independent components for \( h^{\mu \nu} \) while nineteen for \( k^{\mu \nu \rho \sigma} \) [14]. Only components of the form \( k^{\mu \nu \mu \sigma} \) are possibly nonvanishing in locally Minkowski spacetime. Furthermore, the birefringence of light will not emerge at the leading order in this Finsler model of electromagnetic field. The reason is that all Weyl components of \( k^{\mu \nu \rho \sigma} \) vanish at the leading order [31]. These predictions distinguish the electromagnetic field model in locally Minkowski spacetime from the SME–based one.

As \( L_{SME} \) does in the SME, the LIV term \( L_{LIV} \) also denotes the Lorentz–violating interactions at first order for the electromagnetic field in locally Minkowski spacetime. Traditionally, the observations on the electromagnetic field give rise to the most stringent tests of the Lorentz symmetry. An incomplete list includes: the LIV could lead to the anisotropy of the speed of light which is tested by the Michelson–Morley experiment [32–36]; there is an atomic clock experiment named as the Hughes–Drever experiment [37, 38] which is used to test the variation of the SME coefficients with the movement of the Earth; the observations from distant galaxies give severe limits on the birefringence of light [39, 40], and etc. For a more detailed summarization on the observations of the (s)LIV effects, see for example citations [30, 41] and references therein.

The Maxwell equations (8) and (13) could be rewritten as
\[
\frac{\partial F_{\mu \nu}}{\partial x^\lambda} + \frac{\partial F_{\nu \lambda}}{\partial x^\mu} + \frac{\partial F_{\lambda \mu}}{\partial x^\nu} = 0,
\]
\[
\eta^{\mu \nu} \frac{\partial F_{\mu \sigma}}{\partial x^\nu} + h^{\mu \nu} \frac{\partial F_{\mu \sigma}}{\partial x^\nu} = 0.
\]
The second equation includes the LIV effects while the first one is not related to dynamics.
The electromagnetic wave equation (14) could be expanded as
\[
\eta^{\mu\nu} \frac{\partial^2 A_\sigma}{\partial x^\mu \partial x^\nu} + h^{\mu\nu} \frac{\partial^2 A_\sigma}{\partial x^\mu \partial x^\nu} = 0 ,
\] 
where the first term denotes the electromagnetic wave equation in Minkowski spacetime and the second term denotes the terms related to the LIV effects.

There is a solution for this wave equation (26) at first order
\[
A_\mu = A_{0\mu} + A_{1\mu} ,
\]
where \(A_{0\mu}\) and \(A_{1\mu}\) denote the zero–order solution and the first–order solution, respectively. The zero–order solution \(A_{0\mu}\) satisfies the electromagnetic wave equation in Minkowski spacetime, namely
\[
\eta^{\mu\nu} \frac{\partial^2 A_{0\sigma}}{\partial x^\mu \partial x^\nu} = 0 .
\]
It has a plane wave solution \(A_{0\sigma} \propto \epsilon_\sigma e^{-\eta_{\mu\nu} k^\mu x^\nu}\). Therefore, we could obtain an equation for the first–order solution \(A_{1\mu}\) as
\[
\eta^{\mu\nu} \frac{\partial^2 A_{1\sigma}}{\partial x^\mu \partial x^\nu} = -h^{\mu\nu} \frac{\partial^2 A_{0\sigma}}{\partial x^\mu \partial x^\nu}
= h^{\mu\nu} \eta_{\mu\rho} \eta_{\nu\kappa} k^\rho k^\kappa A_{0\sigma}
= h_{\mu\nu} k^\mu k^\nu A_{0\sigma} ,
\]
where we have contracted indices with \(\eta\) in the third equal. The r.h.s. of the above equation behaves like a source of the electromagnetic field, which could be viewed as influence from a slightly anisotropic media on the electromagnetic wave. Furthermore, the dispersion relation (17) could be expanded into
\[
\eta_{\mu\nu} k^\mu k^\nu = -h_{\mu\nu} k^\mu k^\nu .
\]
It is also called the lightcone. The lightcone is enlarged if the r.h.s. of the above equation is negative, while narrowed if the r.h.s. of the above equation is positive. It depends on concrete characters of the LIV effects \(h_{\mu\nu}\). The spatial speed of light could be superluminal if the lightcone is enlarged while it is subluminal if the lightcone is narrowed, and vice versa [42–44]. In addition, the speed of light could depend on its direction since there could be of direction–dependence for \(h_{\mu\nu}\). These could be tested by observations, such as the Michelson–Morley experiment.
5. CONSTRAINT FROM GAMMA-RAY BURSTS

In this section, a specific locally Minkowskian metric is postulated and investigated. The speed of light is obtained to be subluminal and the lightcone is found to be squeezed. In addition, a constraint on the level of the LIV effects is gained from the Fermi-observations of GRBs.

To discuss detailed predictions on the LIV effects, we postulate the locally Minkowski spacetime as

\[ g^{\mu\nu} = \text{diag}(1 + ay^0, -1, -1, -1) \, , \]  

where \(|ay^0| \ll 1\) is assumed and \(a\) is positive. \(F\) has been normalized \(F(\tilde{y}) = 0\), and \(y^\mu = \tilde{y}^\mu/F(\tilde{y})\). The 4-velocity of a particle is related to 4-momentum of this particle. Thus, \(y\) could be characterized by \(k\). In the simplest case, \(y\) is a linear function of \(k\), namely \(y \propto k\) as similar as that in quantum mechanics. In this way, the metric could be written as

\[ g^{\mu\nu} = \text{diag} \left( 1 + \frac{k^0}{M}, -1, -1, -1 \right) \, , \]  

where the constant \(M\) is a high-energy scale into which \(a\) has been absorbed. The perturbative metric deviation is given by

\[ h^{00} = -h_{00} = \frac{k^0}{M} \, , \]  

and other components vanish. The energy scale \(M\) implies a scale for possible occurrence of the LIV effects. Meanwhile, it reveals that the LIV effects are suppressed severely by this scale. Therefore, the LIV effects are expected to be most possibly observed in the ultra-high energy physics, such as the Planck scale.

With the spatially isotropic metric (32), the electromagnetic wave equation (14) (or (26)) becomes

\[ \left[ \left( 1 + \frac{k^0}{M} \right) \frac{\partial^2}{\partial t^2} - \nabla^2 \right] A_\sigma = 0 \, , \]  

where \(\nabla\) denotes the 3D divergence. Comparing this equation with that in the Lorentz-invariant electrodynamics, we obtain the speed of light as

\[ c = \left( 1 + \frac{k^0}{M} \right)^{-\frac{1}{2}} \approx 1 - \frac{k^0}{2M} \, . \]  

It implies that a photon with energy \(k^0 > 0\) would propagate subluminally in such a spacetime. Meanwhile, higher the photon energy is, slower it propagates. On the other hand, the
dispersion relation (17) (or (30)) becomes
\[ \eta_{\mu \nu} k^\mu k^\nu = \frac{k^0 (k^0)^2}{M} > 0 . \] (36)

It implies that the lightcone is squeezed. Higher the photon energy is, more severely its lightcone is squeezed. These are consistent with the prediction that the speed of light (35) is “subluminal”.

The above predictions could be tested by the astrophysical observations on GRBs. The reason is that the above LIV effects could be accumulated after photons traveling a cosmological distance. The Fermi satellite has observed several GRBs with photon energy larger than 100 MeV in recent years. It has been shown that GeV photons arrive several seconds later than MeV photons [45–48]. The observed time lag for two photons with energy \( k^0_{\text{high}} \) and \( k^0_{\text{low}} \) consists of two parts [49]

\[ \Delta t_{\text{obs}} = \Delta t_{\text{LIV}} + \Delta t_{\text{int}} , \] (37)

where \( \Delta t_{\text{int}} \) denotes the intrinsic emission time delay, and \( \Delta t_{\text{LIV}} \) represents the flying time difference induced by the LIV effects. According to the magnetic jet model [50], \( \Delta t_{\text{int}} \) could be evaluated. In such a model, photons with energy less than 10 MeV can escape when the jet radius is beyond the Thomson photosphere radius, i.e., the optical depth is \( \tau_T \sim 1 \). Nevertheless, GeV photons will be converted into electron–positron pairs at this radius, but can escape later when the pair–production optical depth \( \tau_{\gamma \gamma}(k^0) \) drops below unity. One can calculate the time delay \( \Delta t(k^0) \) for the emissions of GeV and 100 MeV photons relative to MeV photons (see detailed discussions in Ref.[49]). The intrinsic time delay is

\[ \Delta t_{\text{int}} = \Delta t(k^0_{\text{high}}) - \Delta t(k^0_{\text{low}}) . \]

Therefore, we could obtain the LIV–induced time delay \( \Delta t_{\text{LIV}} \) according to (37), see Table I.

Consider two photons emitted at the same spacetime point, the arrival time delay between them could be written as [49, 51, 52]

\[ \Delta t_{\text{LIV}} = \frac{\Delta k^0}{2M} D(z) , \] (38)

where we have used the equation (35). The cosmological distance \( D \) is defined as [49, 51, 52]

\[ D(z) := H_0^{-1} \int_0^z \frac{(1 + z')dz'}{\sqrt{\Omega_M(1 + z)^3 + \Omega_\Lambda}} , \] (39)

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TABLE I: Table taken from the Ref. [49]. The LIV–induced time delay $\Delta t_{LIV}$ and the LIV energy scale $2M$ derived from the Fermi–observations of four GRBs: GRB 080916c, GRB 090510, GRB 090902b and GRB 090926. The observed time lags $\Delta t_{obs}$ were collected from Ref. [45–48]. The LIV energy scale was shown to be $2M \sim 10^{20}$ GeV, which is consistent with the Planck energy.

| GRB     | $k_{0\text{low}}$ | $k_{0\text{high}}$ | $\Delta t_{obs}$ | $\Delta t_{LIV}$ | $K(z)$ | $2M$   |
|---------|-------------------|--------------------|------------------|------------------|--------|--------|
| 080916c | 100               | 13.22              | 12.94            | 0.24             | 4.50×10^{18} | 10.02×10^{19} |
| 090510  | 100               | 31                 | 0.20             | 0.14             | 7.02×10^{18} | 9.73×10^{19} |
| 090902b | 100               | 11.16              | 9.5              | 0.10             | 3.38×10^{18} | 9.94×10^{19} |
| 090926  | 100               | 19.6               | 21.5             | 0.20             | 6.20×10^{18} | 9.59×10^{19} |

where $H_0 \approx 72 \text{km} \cdot \text{sec}^{-1} \cdot \text{Mpc}^{-1}$ denotes the Hubble constant, $\Omega_M \approx 0.3$ and $\Omega_\Lambda \approx 0.7$ are densities of matter and cosmological constant, respectively. In this way, the LIV energy scale is given by

$$2M = \frac{\Delta k^0}{\Delta t_{LIV}} D(z) \quad (40)$$

To reveal the LIV effects, one depicts the $\Delta t_{LIV}/(1+z)$ vs. $K(z)$ plot, where $K(z)$ is defined as

$$K(z) := \frac{\Delta k^0}{1+z} D(z) \quad (41)$$

The slope of this plot denotes the inverse of the level of the LIV effects, i.e., $(2M)^{-1}$.

In Ref. [49], Chang et al. took advantage of the Fermi–observations of four GRBs to estimate the level of the LIV effects. The four GRBs are GRB 080916c [45], GRB 090902b [46], GRB 090510 [47] and GRB 090926 [48], respectively. Their LIV–induced time lags $\Delta t_{LIV}$ and $K(z)$ were calculated and listed in Table I. Their $\Delta t_{LIV}/(1+z)$ vs. $K(z)$ plot was given by Fig. II. The slope of the fit line was obtained $(2M)^{-1} \sim 10^{-20}$ GeV$^{-1}$. Correspondingly, the LIV energy scale was shown to be $2M \sim 10^{20}$ GeV, which is consistent with the Planck energy scale. Therefore, we would expect to observe the spacetime anisotropy near the planck scale in future astrophysical and cosmological observations.
FIG. 1: Figure taken from Ref. 49. The $\Delta t_{LIV}/(1+z)$ vs. $K(z)$ plot for the four GRBs observed by the Fermi satellite. The slope of the fit line was shown to be $(2M)^{-1} \sim 10^{-20} \text{GeV}^{-1}$.

6. CONCLUSIONS AND REMARKS

Conclusions and remark are listed as follows. Finsler geometry gets rid of the quadratic restriction on the form of the spacetime structure. It is intrinsically anisotropic. The SME–related sLIV effects on the classical point–like particles have been related to this kind of intrinsically anisotropic spacetime. In principle, the laws of physics should be studied in the intrinsically anisotropic spacetime if the Lorentz symmetry is violated. In this paper, we proposed that locally Minkowski spacetime could be a suitable platform to characterize the possible LIV effects. The reason is that locally Minkowski spacetime is the flat and maximally symmetric non–Riemannian Finsler spacetime.

We studied the electromagnetic field in the locally Minkowski spacetime. The Lagrangian with LIV effects was constructed for the electromagnetic field via replacing the spacetime metric with the Finsler metric. It was found that the obtained Lagrangian is invariant under the coordinate transformations, which preserves validation of the principle of relativity. We obtained the Maxwell equations via the Bianchi identity and the variation of action. We presented a formal plane wave solution. The dispersion relation is modified for the electromagnetic wave. The lightcone might be enlarged or narrowed, depending on concrete characters of the LIV effects. The approach proposed in the paper could be generalized straightforward to study the non–Abelian gauge fields with the LIV effects in locally Minkowski spacetime.
To demonstrate the LIV effects clearly in locally Minkowski spacetime, we expanded the Lagrangian of the electromagnetic field around the Minkowski background. The explicit LIV term was especially extracted from the Lagrangian. It is noteworthy that this LIV term could be reduced back to the sLIV one in the SME formally at first order. It reveals that our results are compatible with the previous works in the framework of SME. However, the LIV effects originate in departure from Minkowski spacetime to locally Minkowski spacetime. There are fewer independent parameters for the LIV effects in locally Minkowski spacetime. The birefringence of light would not appear in our model, which is consistent with the astronomical observations. In addition, the LIV influence on the electromagnetic wave was found to behave like a source of the electromagnetic field. It could be interpreted as influence from a slightly anisotropic media on the electromagnetic field.

To discuss phenomenological predictions on the LIV effects, we investigate a specific locally Minkowski metric. The electromagnetic wave equation was studied and the light was found to propagate subluminally. On the other hand, we obtained a squeezed lightcone. Both characters are consistent with each other. Another important feature of this metric was that the lightcone becomes more severely squeezed as increase of the photon energy. These features were tested by the Fermi–observations on the GRBs. The LIV effects accumulate when the light propagates from distant GRBs. The GeV photons were found to arrive at the Earth later than the MeV photons. This observation gave a severe constraint on the LIV energy scale, i.e., $10^{20}$ GeV. We would expect to observe the spacetime anisotropy near this energy scale in future astrophysical and cosmological observations.

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