The Electromagnetic Effects In Isospin Symmetry Breakings Of $q\bar{q}$ Systems

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May 29, 2017

Abstract

The isospin symmetry breakings of $q\bar{q}$ are investigated in the QCD sum rule method. The electromagnetic effects are evaluated following the procedure requiring that the electromagnetic effects for charged mesons be gauge invariant. We find that the electromagnetic effects are also dominant in the isospin violations of the $\rho$ mesons, which have been shown to be the case in the mass splittings of pions. The numerical results for the differences of pion decay constants and the masses of $\rho$ mesons are presented, which are consistent with the data.

PACS number(s): 13.40.Dk, 11.30.Hv, 12.38.Lg
The origin of mass differences in isospin multiplets has long been of great interest in nuclear and particle physics as a source of information about symmetry violations. Hadronic isospin violations are particularly important in that they arise from quark mass differences in Quantum Chromodynamics (QCD) (see Ref. [1] for a review of the early work in this area), and of course electromagnetic effects. Among the first applications of the method of QCD sum rules was the study of isospin violations in the $\rho - \omega$ system[2], in which it was recognized that the isospin splitting of the light-quark condensates can produce effects as large as the current-quark mass splittings and electromagnetic effects. Recently the QCD sum rule method has been used to study the neutron-proton mass difference[3, 4], the octet baryon mass splittings[5] and the mass differences in the charmed meson systems (the $D$ and $D^*$ scalar and vector mesons)[6].

The isospin symmetry breakings of pions have been discussed extensively in the chiral perturbation theory, in which the electromagnetic effects are shown to be dominant in the mass splittings of pions. Thus, a consistent treatment of the electromagnetic effect is particularly important for the isospin symmetry breakings in the $q\bar{q}$, $q = u$ or $d$, systems. In comparison with hadronic quark models, the QCD sum rule method for calculating isospin splittings has the advantage that one can directly use QED field theory rather than rely on models to estimate Coulomb corrections. In Ref. [8], a systematic approach for the electromagnetic effects in the operator product expansion for the $q\bar{q}$ systems has been developed, in which the problem of the gauge invariance for the charged $q\bar{q}$ states was resolved. This enables us to investigate the violations of the isospin symmetry due to all three effects, the nonperturbative effects, the quark mass differences and the electromagnetic effects, consistently in the QCD sum rule method. The nonperturbative effect is due to the difference of the quark condensate, which arises from the up and down quark mass difference. The results
of our studies\cite{11} in the heavy-light quark systems are in good agreement with the available data. The focus of this paper is to extend our early studies to the light-light quark systems, such as the $\pi$ and $\rho$ systems.

The basic approach of the QCD sum rule is to study the two-point correlation function in the Wilson operator product expansion (OPE), defined by

$$\bar{\Pi}_{\mu\nu}(q^2) = i \int d^4x e^{iqx} < T(J_\mu(x)J_\nu(0)) > = \Pi(q^2)(q_\mu q_\nu - q^2 g_{\mu\nu}) + \Pi_1(q^2)q_\mu q_\nu,$$

(1)

where

$$J_\mu(x) = \bar{q}_1(x)\gamma_\mu q_2(x).$$

(2)

The $\Pi(q^2)$ in Eq. (1) can be written as

$$\Pi(q^2) = \Pi_0(q^2) + \Pi_{\text{em}}(q^2),$$

(3)

where $\Pi_0(q^2)$ is the leading term, and $\Pi_{\text{em}}$ represents the contributions from the electromagnetic effects. The quantity $\Pi_0(q^2)$ has been evaluated in the literature. Thus we extend the results of $\Pi_0(q^2)$ in Refs. \cite{2, 7} by including the flavor dependences of each term. In order to study the violations of the isospin symmetry, the electromagnetic effects $\Pi_{\text{em}}(q^2)$ should also be written in the framework of the operator product expansion. The leading electromagnetic effects in this approach are the two-point functions from a two-loop perturbative contribution, whose Feynman diagrams are shown in Fig. 1. For the charge neutral current $\Pi_{\text{em}}^0(q^2)$, one could simply obtain the two-point functions by changing the gluons in QCD to the photons in QED in Fig. 1, since the two-point functions has been calculated in QCD\cite{2, 7}.

$$\Pi_{\text{em}}^0(q^2) = -\frac{1}{4\pi^2 e_q^2 \alpha_e/\pi} \ln \frac{-q^2}{\mu^2},$$

(4)

where $e_q$ is the quark charge and $\mu$ is the infrared cutoff. Obviously, this result is gauge invariant.
Following the procedure in Ref. [8], there are additional Feynman diagrams shown in Fig. 2 for the charged current so that the $\Pi_{\text{em}}^{\pm}(q^2)$ for the charged $q\bar{q}$ systems is gauge invariant. Thus, the two point function $\Pi_{\text{em}}^{\pm}(q^2)$ for the charged $q\bar{q}$ systems is

$$\Pi_{\text{em}}^{\pm}(q^2) = -\frac{1}{4\pi^2} e_u e_{\bar{d}} \frac{\alpha}{\pi} \ln \frac{-q^2}{\mu^2}. \quad (5)$$

The leading nonperturbative terms due to the electromagnetic effects are the four quark condensates, of which the Feynman diagrams are shown in Fig. 3 and Fig. 4. We present the expressions for $\Pi(q^2)$

$$\Pi_{\rho}^{\pm} = -\frac{1}{4\pi^2} (1 + \frac{\alpha_s}{\pi}) \ln \frac{-q^2}{\mu^2} - \frac{1}{4\pi^2} e_u e_{\bar{d}} \frac{\alpha}{\pi} \ln \frac{-q^2}{\mu^2} + \frac{1}{q^4} (m_u \langle \bar{d}d \rangle + m_d \langle \bar{u}u \rangle) + \frac{1}{12q^4} \frac{\alpha_s}{\pi} G^2$$

$$+ \frac{8}{81q^6} g_s^2 \langle \bar{u}u \rangle \langle \bar{d}d \rangle - \frac{8}{81q^6} g_s^2 (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2)$$

$$+ \frac{7}{27q^6} e_u e_{\bar{d}} (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2) + \frac{1}{4q^6} e^2 e_{\bar{d}} \langle \bar{u}u \rangle \langle \bar{d}d \rangle, \quad (6)$$

and

$$\Pi_{\rho}^{0} = -\frac{1}{4\pi^2} (1 + \frac{\alpha_s}{\pi}) \ln \frac{-q^2}{\mu^2} - \frac{1}{4\pi^2} e_u e_d \frac{\alpha}{2\pi} \ln \frac{-q^2}{\mu^2} + \frac{1}{q^4} (m_d \langle \bar{d}d \rangle + m_u \langle \bar{u}u \rangle) + \frac{1}{12q^4} \frac{\alpha_s}{\pi} G^2$$

$$+ \frac{28}{81q^6} g_s^2 (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2) + \frac{7}{27q^6} e^2 (e_u^2 \langle \bar{u}u \rangle^2 + e_d^2 \langle \bar{d}d \rangle^2), \quad (7)$$

for the $\rho$ systems, and

$$\Pi_{\pi}^{\pm} = -\frac{1}{4\pi^2} (1 + \frac{\alpha_s}{\pi}) \ln \frac{-q^2}{\mu^2} - \frac{1}{4\pi^2} e_u e_d \frac{\alpha}{\pi} \ln \frac{-q^2}{\mu^2} + \frac{1}{q^4} (m_u \langle \bar{d}d \rangle + m_d \langle \bar{u}u \rangle) + \frac{1}{12q^4} \frac{\alpha_s}{\pi} G^2$$

$$- \frac{88}{81q^6} g_s^2 \langle \bar{u}u \rangle \langle \bar{d}d \rangle - \frac{11}{27q^6} e^2 e_u e_d (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2), \quad (8)$$

and

$$\Pi_{\pi}^{0} = -\frac{1}{4\pi^2} (1 + \frac{\alpha_s}{\pi}) \ln \frac{-q^2}{\mu^2} - \frac{1}{4\pi^2} e_u e_d \frac{\alpha}{\pi} \ln \frac{-q^2}{\mu^2}$$
for the \(\pi\) systems, in which we have assumed the vacuum dominance and factorization hypothesis for the four quark condensates following Ref. [2]. In the Appendix we present expressions in which the vacuum dominance approximation has not been implemented. The sensitivity to the possible deviation from vacuum dominance approximation will be discussed later.

The isospin symmetry breakings of these systems are determined by the difference between \(\Pi^\pm(q^2) - \Pi^0(q^2)\), and we have

\[
\Pi_\rho - \Pi_\rho^\pm = -\frac{1}{8\pi^2}(e_u - e_d)^2 \frac{\alpha}{\pi} \ln \frac{q^2}{\mu^2} + \frac{1}{27q^6}\pi \alpha (e_u - e_d)^2 \langle \bar{u}u\rangle^2
\]

\[
+ \frac{4}{9q^4}g^2_s(\langle \bar{u}u\rangle - \langle \bar{d}d\rangle)^2 + \frac{1}{q^4}(m_u - m_d)(\langle \bar{u}u\rangle - \langle \bar{d}d\rangle)
\] (10)

for \(\rho\) states, and

\[
\Pi_\pi^0 - \Pi_\pi^\pm = -\frac{1}{8\pi^2}(e_u - e_d)^2 \frac{\alpha}{\pi} \ln \frac{q^2}{\mu^2} - \frac{44}{27q^6}\pi \alpha (e_u - e_d)^2 \langle \bar{u}u\rangle^2
\]

\[
- \frac{44}{81q^6}g^2_s(\langle \bar{u}u\rangle - \langle \bar{d}d\rangle)^2 + \frac{1}{q^4}(m_u - m_d)(\langle \bar{u}u\rangle - \langle \bar{d}d\rangle)
\] (11)

for pions. The first two terms in Eqs. (10) and (11) represent the electromagnetic effects, and the last two terms correspond to the isospin violations in the quark masses and condensates. Qualitatively, the contributions from the electromagnetic effects and the differences of the quark masses and condensates to the isospin symmetry breakings of \(\pi\) and \(\rho\) states should have the same order of magnitudes. Thus, Eqs. (10) and (11) show explicitly that the contributions from the differences of quark masses and condensates are of higher order relative to the electromagnetic effects. This has been shown in Ref. [1] for pions sometime ago, while Eq. (10) suggests that the electromagnetic effects is also dominant in the isospin symmetry breakings of \(\rho\) mesons.
We adopt the standard one resonance plus a continuum model for the spectral density \( \text{Im}\Pi_{\rho}(s) \) at the hadronic level:

\[
\text{Im}\Pi_{\rho}(s) = \pi f_{\rho\rho}^2 \delta(s - m_{\rho}^2) + \frac{1}{4\pi} \left( 1 + \frac{\alpha_s}{\pi} + \frac{e_u^2 + e_d^2}{2} \frac{\alpha}{\pi} \right) \theta(s - s_{\rho\rho})
\]

where \( f_{\rho\rho} \) is related to the electronic width of the \( \rho \) meson and \( s_{\rho\rho} \approx 1.5 \text{GeV}^2 \) is the \( \rho \) meson continuum threshold. We use \( s_{\pi} \approx 0.75 \text{GeV}^2 \) in the pseudo vector channel so that the sum rule is dominated by the pion and the \( a_1(1260) \) meson contributes only to the continuum. Although it is impossible to extract the pion masses in the QCD sum rule framework, it is still possible to calculate their decay constants with the physical masses as the inputs \[2, 7\]. After making the Borel transformation and transferring the continuum contribution to the right hand side, the final sum rules are:

\[
f^2_{\rho\rho} e^{-\frac{m_{\rho^\pm}^2}{M_B^2}} = \frac{1}{4\pi^2} \left( (1 + \frac{\alpha_s(M_B^2)}{\pi} + e_u e_d \frac{\alpha}{\pi}) M_B^2 (1 - e^{-\frac{s_{\rho^\pm}}{M_B^2}}) - m_u a_d + m_d a_u + \frac{\langle \alpha_s G^2 \rangle}{12 M_B^2} - \frac{4 \bar{\alpha}_s a_u a_d}{9 \pi M_B^2} + \frac{4 \bar{\alpha}_s a_u^2 + a_d^2}{81 \pi M_B^2} - \frac{7}{54} e_u e_d \frac{\alpha a_u^2 + a_d^2}{\pi M_B^2} - \frac{1}{8} e^2 \bar{\alpha} a_u a_d \right) \]

and

\[
f^2_{\rho\rho} e^{-\frac{m_{\rho^-}^2}{M_B^2}} = \frac{1}{4\pi^2} \left( (1 + \frac{\alpha_s(M_B^2)}{\pi} + e_u e_d \frac{\alpha}{\pi}) M_B^2 (1 - e^{-\frac{s_{\rho^-}}{M_B^2}}) - m_u a_d + m_d a_u + \frac{\langle \alpha_s G^2 \rangle}{12 M_B^2} - \frac{14 \bar{\alpha}_s a_u^2 + a_d^2}{81 \pi M_B^4} - \frac{7 \alpha e_u e_d a_u^2 + e_d^2 a_d^2}{54 \pi M_B^4} \right) \]

for \( \rho \) mesons, and

\[
f^2_{\pi \pi^\pm} e^{-\frac{m_{\pi^\pm}^2}{M_B^2}} = \frac{1}{4\pi^2} \left( (1 + \frac{\alpha_s(M_B^2)}{\pi} + e_u e_d \frac{\alpha}{\pi}) M_B^2 (1 - e^{-\frac{s_{\pi^\pm}}{M_B^2}}) - m_u a_d + m_d a_u + \frac{\langle \alpha_s G^2 \rangle}{12 M_B^2} - \frac{44 \bar{\alpha}_s a_u a_d}{81 \pi M_B^4} + \frac{11}{54} e_u e_d a_u^2 + a_d^2 \right)
\]
and

$$f^2 \pi^2 e^{-\frac{m^2}{M^2}} = \frac{1}{4\pi^2} \left( (1 + \frac{\alpha_s(M^2_B)}{\pi}) + \frac{e_u^2 + e_d^2}{2} \alpha \right) M^2_B (1 - e^{-\frac{s}{M^2_B}})$$

$$- \frac{m_u a_u + m_d a_d}{M^2_B} + \frac{\langle G^2 \rangle}{12 M^2_B}$$

$$+ \frac{22 \tilde{\alpha}_s a_u^2 + a_d^2}{81 \pi} + \frac{11 \alpha e_u^2 a_u^2 + e_d^2 a_d^2}{54 \pi} \right\} \right)$$

(16)

where $e_T = e_u - e_d$, $e_u = \frac{2}{3}$, $e_d = -\frac{1}{3}$, $m_u = 5.1$ MeV, $m_d = 8.9$ MeV, $a_q = -(2\pi)^2 \langle \bar{q} q \rangle$, $q = u, d$, $\alpha = \frac{1}{137}$, $\alpha_s(M^2_B) = \frac{4\pi}{\ln(100M^2_B)}$, $\tilde{\alpha}_s = 0.7$, and the values of the various condensates $a_q$ are standard[2] $\bar{a} = \frac{a_u + a_d}{2} = 0.55$ GeV$^3$, and $\langle g_s^2 G^2 \rangle = 0.474$ GeV$^4$.

The mass sum rules for $\rho$ mesons are obtained by taking the ratio of Eq. (13) and the resulting equation of the first derivative with respect to $\frac{1}{M^2_B}$. The $\Delta m_\rho$ is shown in Fig. 5. The threshold parameters for the stable sum rules are $s_\rho^\pm = 1.502$ GeV$^2$ and $s_\rho^0 = 1.500$ GeV$^2$. The mass of $\rho^\pm$ is found to be 770 MeV, which reproduces the results in Refs. [2, 7] and is in good agreement with data. Our numerical analysis shows that contributions from the differences of quark masses and condensates are indeed negligible. We have the resulting $\Delta m_\rho = -0.25$ MeV, and this is not inconsistent with the experimental value from the Particle Data Group, of which $\Delta m_\rho = 0.3 \pm 2.2$ MeV[13].

With the physical pion masses $m_{\pi^\pm} = 139.57$ MeV and $m_{\pi^0} = 134.98$ MeV as inputs, one could extract the decay constants of pions. The decay constant $f_{\pi^\pm}$ from these sum rules is 131 MeV, and we present the sum rule for $\Delta f_{\pi} = f_{\pi^\pm} - f_{\pi^0}$ in Fig. 6. The corresponding threshold parameters are $s_{\pi^\pm} = 0.78$ GeV$^2$ and $s_{\pi^0} = 0.70$ GeV$^2$. Our result is $\Delta f_{\pi} = 4.5$ MeV, while the data in the Particle Data Group for $f_\pi$ suggest $\Delta f_{\pi} = 11.9 \pm 4.0$ MeV[13].

Our numerical analysis shows that the quantity $\Delta f_{\pi}$ is dominated by the second term in Eq. [11], which comes from the four quark condensates. Thus, it is very sensitive to the vacuum dominance approximation adopted in our approach. The
possible violation of this approximation was discussed in Ref. [14]. Following Ref. [14], we introduce a factor $1 + \kappa$ in front of the QED induced four quark condensates based on the factorisation assumption in (13), (14), (15) and (16). A nonvanishing $\kappa$ signals the breakdown of the factorisation hypothesis. We find that $\Delta m_\rho$ is insensitive to $\kappa$ since it is dominated by the electromagnetic radiative correction. However, the $\Delta f_\pi$ is increased to 8.3MeV with $\kappa = 1$, which is in good agreement with the PDG data. We estimate $\kappa = 0.5 \sim 1.5$. More accurate data on $\Delta f_\pi$ will determine how much the vacuum dominance approximation is being violated.

In summary, we have presented a field theory calculation of the electromagnetic effects in the isospin symmetry breakings of the $\pi$ and $\rho$ mesons. We have shown explicitly that the isospin symmetry breaking induced by the electromagnetic effects are dominant in the $\pi$ and $\rho$ systems. Our results show that the electromagnetic effects in isospin symmetry breakings of pions are much larger than that of $\rho$ mesons. This is consistent with the available data, although we have not included the effects due to the $\rho - \omega$ mixings in the mass splittings of $\rho$ mesons. Certainly, more accurate data are needed to test our theoretical approach, and to determine the possible deviation from the factorization of the QED-induced four quark condensates.

This work is supported in part by the Natural Science Foundation and the Doctoral Program of State Education Commission of China. The financial support by Peking University are also gratefully acknowledged.
Appendix

We present the expressions for the $\pi^0$ and $\rho^0$ systems in which the vacuum dominance approximation has not been implemented.

\[
\Pi_{\rho^0} = -\frac{1}{4\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right) \ln \frac{-q^2}{\mu^2} - \frac{1}{4\pi^2} \frac{e_u^2 + e_d^2}{2} \frac{\alpha}{\pi} \ln \frac{-q^2}{\mu^2} \\
+ \frac{1}{q^4} (m_d \langle \bar{d}d \rangle + m_u \langle \bar{u}u \rangle) + \frac{1}{12 q^4} \left(\frac{\alpha_s}{\pi} G^2\right) \\
+ \frac{1}{q^6} g_s^2 \left[ (\bar{u} \gamma_\mu \frac{\lambda^a}{2} u)(\bar{u} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} u) + (\bar{d} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} d)(\bar{d} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} d) \right] \\
+ \frac{2}{q^6} e^2 \left[ (\bar{u} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} u)(\bar{u} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} u) + (\bar{d} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} d)(\bar{d} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} d) \right] \\
+ \frac{1}{q^6} e^2 \left[ \sum_{q=u,d,s} e_\mu \bar{q} \gamma^\mu q \right] + \left( e_\mu \bar{d} \gamma^\mu d \right) \left( \sum_{q=u,d,s} e_\mu \bar{q} \gamma^\mu q \right),
\]

for the $\rho^0$ system and

\[
\Pi_{\pi^0} = -\frac{1}{4\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right) \ln \frac{-q^2}{\mu^2} - \frac{1}{4\pi^2} \frac{e_u^2 + e_d^2}{2} \frac{\alpha}{\pi} \ln \frac{-q^2}{\mu^2} \\
+ \frac{1}{q^4} (m_d \langle \bar{d}d \rangle + m_u \langle \bar{u}u \rangle) + \frac{1}{12 q^4} \left(\frac{\alpha_s}{\pi} G^2\right) \\
+ \frac{1}{q^6} g_s^2 \left[ (\bar{u} \gamma_\mu \frac{\lambda^a}{2} u)(\bar{u} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} u) + (\bar{d} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} d)(\bar{d} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} d) \right] \\
+ \frac{2}{q^6} e^2 \left[ (\bar{u} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} u)(\bar{u} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} u) + (\bar{d} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} d)(\bar{d} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} d) \right] \\
+ \frac{1}{q^6} e^2 \left[ \sum_{q=u,d,s} e_\mu \bar{q} \gamma^\mu q \right] + \left( e_\mu \bar{d} \gamma^\mu d \right) \left( \sum_{q=u,d,s} e_\mu \bar{q} \gamma^\mu q \right),
\]

for the $\pi^0$ system. Similarly we may derive expressions for $\Pi_{\rho^\pm}$ and $\Pi_{\pi^\pm}$. 
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Figure Captions

1. The two loop perturbative corrections for charge neutral currents

2. Additional diagrams generated by $J^e_\mu(x)$ for charged quark systems.

3. The electromagnetic interaction induced four quark condensates for charge neutral currents.

4. The additional four quark condensates for charged currents.

5. The Borel mass dependence of $\Delta m_\rho$.

6. The Borel mass dependence of $\Delta f_\pi$. 
