Thermal properties of the exotic $X(3872)$ state via QCD sum rule

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In this work we investigate the $X(3872)$ meson with quantum numbers $J^{PC} = 1^{++}$ in the framework of the thermal QCD sum rules method. We use a diquark-antidiquark current with the corresponding quantum numbers and calculate the two-point correlation function including contributions of non-perturbative condensates up to six dimensions. Analysis of the obtained thermal sum rule allows us to study contributions of a medium to the mass and coupling constant of the $X(3872)$ resonance. Our numerical calculations demonstrate that the mass and the meson-current coupling constant are insensitive to the variation of temperature up to $T = 110$ MeV, however after this point; they start to fall by increasing the temperature. At deconfinement temperature, the meson-current coupling constants attain roughly to 34% of the vacuum value.

I. INTRODUCTION

Understanding the non-perturbative properties of QCD is one of the most difficult and intriguing research topics of the in strong interactions. The investigation of the hadron spectrum can play an important role in achieving this goal. According to QCD, not only traditional mesons and baryons, but also exotic particles such as glueball, hybrid and tetraquark states can be observed. In the last decade, observation of charmonium-like $P_c^+$, bottomonium-like $A_0, A_1$ and the pentaquark states $(3872)$ in experiments and detailed examination of these states have revealed significant information on exotic particles. The observations of these unconventional states with some properties beyond the standard quark model have motivated different theoretical interpretations such as molecule and tetraquark models $[5,23]$. The examination of exotic particles is very important to understand the investigation of heavy ion collisions. But for understanding the investigation of heavy ion collisions, we need to know the thermal properties of these particles. For this reason, it is important to examine the exotic particles in the medium.

In 2003 the Belle collaboration announced the discovery of the $X(3872)$ particle $[1]$. This particle was confirmed shortly thereafter by the CDF $[2]$, D0 $[3]$, BaBar $[26,27]$ and LHCb $[28]$ collaborations by analyzing the $B^-\rightarrow \pi^-[X\rightarrow J/\psi\pi^+\pi^-]$, $B^-\rightarrow K^-[X\rightarrow J/\psi\pi^+\pi^-]$, $B^+\rightarrow K^+[X\rightarrow \psi(2S)\gamma]$ and $B^+\rightarrow K^+[X\rightarrow \psi\gamma]$ decays.

In the last decades, QCD sum rules have successfully been used to investigate different properties of the conventional mesons and baryons as reviewed in Refs. $[29-32]$. The thermal version of QCD sum rules has been successfully used to study the thermal properties of mesons $[33,40]$ as a reliable and well-established approach. The QCD sum-rules have been also extended to investigate exotic hadrons $[47,52]$. In this article we use the Thermal QCD sum rules method for exploration of the $X(3872)$ resonance with quantum numbers $J^{PC} = 1^{++}$. We consider it as a diquark-antidiquark bound state. By using relevant interpolating current we calculate the two-point correlation function including contributions of nonperturbative condensates up to six dimensions. Equating the expression of the correlation function obtained using the operator product expansion (OPE) and its hadronic representation we derive thermal QCD sum rules for parameters of the $X(3872)$ state.

This work is organized in the following manner. In Sec. II we derive the thermal sum rules to calculate mass and coupling constant of the resonance $X(3872)$. Section III is devoted to numerical analysis, where we write down values of parameters used in computations, and also present our results for the mass and coupling constant of the $X(3872)$ state. The appendix contains the explicit expression of the two-point thermal spectral density $\rho^{QCD}(s,T)$.

II. MASS AND COUPLING CONSTANT OF THE $X(3872)$ STATE AT FINITE TEMPERATURE

To calculate the mass and coupling constant of the $X(3872)$ state in the framework of the thermal QCD sum rules we start from the correlation function

$$\Pi_{\mu\nu}(q,T) = i \int d^4 x e^{iqx} \langle T \{J_\mu(x)J_\nu^\dagger(0)\} \rangle, \quad (1)$$

where $J_\mu(x)$ is the interpolating current of the $X(3872)$ state, $T$ is the temperature and $T$ indicates the time ordering operator. The thermal average of any operator $A$ in thermal equilibrium can be expressed as:

$$\langle A \rangle = Tr e^{-\beta H} A / Tr e^{-\beta H}, \quad (2)$$

where $H$ is the QCD Hamiltonian, and $\beta = 1/T$ is the inverse of the temperature $T$.

We consider the $X(3872)$ state as the resonance with quantum numbers $J^{PC} = 1^{++}$. Then in the diquark-antidiquark model the current $J_\mu(x)$ is expressed by the
following expression \[61\]
\[
J_\mu(x) = \frac{i \epsilon \bar{\epsilon}}{\sqrt{2}} \left\{ \left[ \bar{q}_a(x) C \gamma_5 c_b(x) \right] \left[ \gamma^\mu \gamma_\nu C T^\nu(x) \right] + \left[ \bar{q}_a(x) \gamma_\mu C T^\mu(x) \right] \right\},
\]
where \(q\) is one of the light \(u\) or \(d\) quarks. Here we have introduced the short-hand notations \(\epsilon = \epsilon_{abc}\) and \(\bar{\epsilon} = \epsilon_{dec}\). In Eq. \(6\) \(a, b, c, d, e\) are color indexes and \(C\) is the charge conjugation matrix.

In order to derive QCD sum rule expression we first calculate the correlation function in terms of the physical degrees of freedom. Performing integral over \(x\) in Eq. \(1\), we get
\[
\Pi^{\text{Phys}}_{\mu\nu}(q, T) = \frac{(0|J_\mu|X(q)\rangle_T\langle X(q)|J^\dagger_\nu|0\rangle_T}{m_X^2(T) - q^2} + \ldots,
\]
where \(m_X(T)\) is the temperature-dependent mass of \(X(3872)\), and dots stand for contributions of the higher resonances and continuum states. We define the temperature-dependent meson-current coupling constant \(f_X(T)\) through the matrix element
\[
\langle 0|J_\mu|X(q)\rangle_T = f_X(T)m_X(T)\epsilon_\mu, \quad (4)
\]
with \(\epsilon_\mu\) being the polarization vector of the \(X(3872)\) state. Then in terms of \(m_X(T)\) and \(f_X(T)\), the correlation function can be written in the form
\[
\Pi^{\text{Phys}}_{\mu\nu}(q, T) = \frac{m_X^2(T)f_X^2(T)}{m_X^2(T) - q^2} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_X^2(T)}\right) + \ldots.
\]

The required sum rules can be obtained by using coefficient, \(\Pi^{\text{phys}}_{\mu\nu}(q, T)\), corresponding to the structure \(-g_{\mu\nu}\). After performing the Borel transformation, the physical sides is found as
\[
\mathcal{B}_q \Pi^{\text{Phys}}_{\mu\nu}(q, T) = m_X^2(T)f_X^2(T)e^{-m_X^2(T)/M^2}. \quad (6)
\]

The correlation function in the QCD side, \(\Pi^{\text{QCD}}_{\mu\nu}(q, T)\), has to be determined employing the quark-gluon degrees of freedom. To this end, we contract the heavy and light quark fields and find for the correlation function \(\Pi^{\text{QCD}}_{\mu\nu}(q, T)\) in the diquark-antidiquark picture the following expression:

\[
\Pi^{\text{QCD}}_{\mu\nu}(q, T) = -\frac{i}{2} \int \frac{d^4 x}{(2\pi)^4} e^{-i q \cdot x} \left\{ \left[ \bar{c}_a(x) \gamma_\mu S^{ab}_c(x) \right] \bar{c}_b(x) \gamma_\nu c_a(x) - \left[ \bar{c}_a(x) \gamma_\nu S^{ab}_c(x) \right] \bar{c}_b(x) \gamma_\mu c_a(x) \right\},
\]

In Eq. \((7)\) we use the notation
\[
\tilde{S}^{ij}_{c}(q)(x) = C S^{ijT}_{c}(q)(x) C,
\]
with \(S^{ij}_{c}(q)(x)\) and \(\tilde{S}^{ij}_{c}(q)(x)\) being the light and heavy quark propagators at the finite temperature, respectively. The quark propagator in vacuum was investigated, in detailed, in many studies in external spinor and gauge fields. In the result of these investigations, the quark propagator can be written in terms of quark and gluon condensates \[30, 61\]. At finite temperature breakdown of the Lorentz invariance by the choice of the reference frame and appearance of the residual \(O(3)\) symmetry the new operators arise in operator product expansions and, therefore, the thermal propagator includes new terms compared with the vacuum quark propagators \[62\].
the mass and meson-current coupling constant are derived after fixing the same structures in both $\Pi^\text{phys}_{\mu\nu}(q, T)$ and $\Pi^\text{QCD}_{\mu\nu}(q, T)$. As in the physical side of the sum rule, in its QCD side the structure $-g_{\mu\nu}$ has been taken into account. In the case $q = 0$, i.e. in the rest frame of the resonance particle, we can write $\Pi^\text{QCD}_{\mu\nu}(q^2_0, T)$ as the dispersion integral,

$$\Pi^\text{QCD}_{\mu\nu}(q^2_0, T) = \int_{2m^2_0}^{s_0(T)} \frac{\rho^\text{QCD}(s, T)}{s - s_0} ds + ...,$$

(11)

where $\rho^\text{QCD}(s, T)$ is the corresponding spectral density. The main question of this section is the calculation of $\rho^\text{QCD}(s, T)$. In the present work we include into our sum rules the quark, gluon and mixed condensates up to six dimensions. For computation of the components of the spectral densities we use the technical methods presented in Ref. [63]. Results of our calculations are collected in the appendix. Let us note that we have used the following relation to express the gluon condensate in terms of the gluonic part of the energy-momentum tensor $\Theta^\text{QCD}_{\mu\nu}$ (see for details Ref. [62]):

$$\langle T^\text{QCD}G_{\mu\nu}G_{\mu\nu}\rangle = \frac{1}{24}[g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}](g^\text{QCD}G_{\lambda\sigma})$$

$$+ \frac{1}{6}[g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu} - 2(u_\alpha u_\mu g_{\beta\nu} - u_\alpha u_\nu g_{\beta\mu} - u_\beta u_\mu g_{\alpha\nu} + u_\beta u_\nu g_{\alpha\mu})](u^\lambda \Theta^\text{QCD}_{\alpha\beta} u^\sigma).$$

(12)

Applying the Borel transformation on the variable $q^2_0$ in the invariant amplitude $\Pi^\text{QCD}(q^2, T)$, equating the obtained expression with the relevant part of $\sum B^\text{phys}_{\mu\nu}(q, T)$, and subtracting the continuum contribution, we finally obtain the required sum rule. Thus, the mass of the $X(3872)$ state can be evaluated from the sum rule

$$m^2_X(T) = \frac{\int_{2m^2_0}^{s_0(T)} ds s \rho^\text{QCD}(s, T)e^{-s/M^2}}{\int_{2m^2_0}^{s_0(T)} ds \rho^\text{QCD}(s, T)e^{-s/M^2}},$$

(13)

whereas to extract the numerical value of the meson-current coupling constant $f_X(T)$ we employ the formula

$$f^2_X(T)e^{-m^2_X(T)/M^2} = \frac{1}{m^2_X(T)} \int_{2m^2_0}^{s_0(T)} ds \rho^\text{QCD}(s, T)e^{-s/M^2}.$$  

(14)

### III. NUMERICAL ANALYSIS

The QCD sum rules for the mass and coupling constant of the $X(3872)$ state at finite temperature contain as parameters various quark, gluon and mixed vacuum condensates. Their values are collected in Table I.

| Parameters | Values |
|------------|--------|
| $m_c$      | 1.28 ± 0.03 GeV [64] |
| $\langle \bar{q}q \rangle$ | $(-0.24 ± 0.01) \times 10^3$ GeV$^3$ [29, 30] |
| $\langle \bar{q}Gq \rangle$ | $(0.012 ± 0.004) \times 10^4$ GeV$^4$ [29, 30] |
| $m^2_0$    | $(0.8 ± 0.1) \times 10^2$ GeV$^2$ [29, 30] |

TABLE I: Input parameters.

For the temperature-dependent continuum threshold for the $X(3872)$ state we use the parametrization obtained from the lattice QCD graphics presented in Ref. [68]:

$$\langle \Theta^0 \rangle = \frac{\langle 0|\bar{q}q|0 \rangle}{1 + e^{(113.867[1.010(1.010)T^2 + 4.99216(1.010)T^2 + 1.10042(1.010)T^2 + 0.10141[1.010]T^2])}},$$

and is valid up to a critical temperature $T_c = 190$ MeV and $\langle 0|\bar{q}q|0 \rangle$ is the vacuum condensate of the light quarks.

For the gluonic and fermionic parts of the energy density we use the parametrization obtained in Ref. [38] from the lattice QCD graphics presented in Ref. [68]:

$$\langle G^2 \rangle = \langle 0|G^2|0 \rangle \left[1 - 1.65 \left(\frac{T}{T_c}\right)^{8.735} + 0.04967 \left(\frac{T}{T_c}\right)^{0.7211}\right].$$

(16)

where $\langle 0|G^2|0 \rangle$ is the gluon condensate in the vacuum.

The temperature-dependent continuum threshold for the $X(3872)$ state is one of the auxiliary parameters that should also be determined. We used the continuum threshold in terms of temperature [42, 70]:

$$s_0(T) = s_0 \left[1 - \left(\frac{T}{T_c}\right)^8\right] + 4m^2_0 \left(\frac{T}{T_c}\right)^8,$$

(17)

where $s_0$ is the continuum threshold at $T = 0$. This parameter is not arbitrary and depends on the energy of the first excited state with the same quantum numbers as the chosen interpolating currents for the $X(3872)$ state. For $s_0$, we take the interval

$$16.5 \text{ GeV}^2 \leq s_0 \leq 17.0 \text{ GeV}^2,$$

in which the physical quantities show relatively weak dependence on it. According to the philosophy of the used method, the physical quantities should be practically independent the
auxiliary parameters $M^2$. Also, in order to fix the working window for the Borel parameter $M^2$, we require the convergence of the OPE, as well as the suppression of the contributions arising from the higher resonances and continuum, in other words exceeding of the pole contribution over the ones coming from the higher dimensional condensates. As a result, for the mass and coupling constant calculations we find the range of $M^2$

$$3.5 \text{ GeV}^2 \leq M^2 \leq 5.5 \text{ GeV}^2, \quad (19)$$
as reliable for our purposes. It is worth noting that in this interval the dependence of the mass and meson-current coupling constant on $M^2$ is stable, and we may expect that the sum rules give the correct results. In order to demonstrate independence of physical quantities from $M^2$ and $s_0$, we plot the mass and meson-current coupling constant versus $M^2$ at different fixed values of the continuum threshold $s_0$ at $T = 0$ and vice versa in Figs. 1 and 2. From these figures, we see that these quantities depend on both $M^2$ and $s_0$ very weakly in their working intervals.

![FIG. 1](image1.png)

**FIG. 1:** (a) The mass of the $X(3872)$ state as a function of $M^2$ for different fixed values of $s_0$ at $T = 0$. (b) The same as (a) but for the coupling parameter $f_X$.

![FIG. 2](image2.png)

**FIG. 2:** (a) The mass of the $X(3872)$ state as a function of $s_0$ for different fixed values of $M^2$ at $T = 0$. (b) The same as (a) but for the coupling parameter $f_X$.

The final task is to investigate the variations of the mass and coupling parameter of the $X(3872)$ state with
respect to temperature. For this purpose, we plot these quantities as a function of temperature in Fig. 3. This figure indicates that the mass and coupling constant of the \(X(3872)\) state remain approximately unchanged up to \(T \approx 0.11\) GeV, however, after this point, they start to diminish rapidly by increasing the temperature. Near the critical or deconfinement temperature, the coupling constant reaches approximately 34\% of its value in vacuum, while the mass decreased by 26\%. From this figure we deduce the result on the meson-current coupling constant and mass in vacuum as presented in Table II. In the case of the mass, our result within the uncertainties is in good agreement with those of [52, 64]. On the other hand, the result of the meson-current coupling is smaller than the result in the literature [52]. It can be checked in the future experiments.

![Graphs](image)

**FIG. 3:** (a) The mass of the \(X(3872)\) state as a function of temperature for different values of \(s_0\) and at fixed value of \(M^2 = 4.5\) GeV^2. (b) The same as (a) but for the coupling parameter \(f_{X}\).

### IV. CONCLUSIONS

In the present work we have investigated the diquark-antidiquark \(X(3872)\) meson by calculating its spectroscopic parameters in the framework of the thermal QCD sum rules method. The analysis of the obtained thermal sum rule allows us to study contributions of a medium to the mass and coupling constant of the \(X(3872)\) resonance. Our numerical calculations demonstrate that the mass and meson-current coupling constant are insensitive to the variation of the temperature up to \(T = 110\) MeV, however after this point; they start to fall by increasing of the temperature. At deconfinement temperature, the meson-current coupling constants attain roughly to 34\% of its vacuum value. But decreasing of the mass and current coupling with the temperature does not mean a stability of the particle under consideration. To make a conclusion about the stability of the particle one has to calculate its decay width. Indeed, apart from the mass and coupling constant, the decay width of the particle depends also on other temperature-dependent parameters. In Ref. [41] by explicit calculations of the decay width, it was shown that, despite decreasing of the mass and coupling constant of pseudoscalar particles their decay widths increase with the temperature revealing their unstable nature. Therefore, decreasing the mass and coupling of the particle with temperature does not automatically lead to decreasing its width. In the future our aim is to investigate the temperature dependent decay width of the \(X(3872)\) state.

The considerable decrease in the values of mass and coupling parameter can be considered as a sign of the quark-gluon plasma phase transition. Also, the obtained behavior in terms of temperature can be used in the analysis of the heavy-ion collision experiments. Our predictions for the spectroscopic properties of the \(X(3872)\) state can be checked in future experiments.

### TABLE II: Values of mass and coupling constant of the \(X(3872)\) state in the vacuum.

|          | \(m_{X(3872)}\) (MeV) | \(f_{X(3872)} \times 10^2\) (GeV^4) |
|----------|------------------------|-------------------------------------|
| Present Work | 3885 ± 85              | 0.31 ± 0.12                         |
| Experiment [64] | 3871.69 ± 0.17       | -                                   |
| [52]    | 3873 ± 127             | 0.56 ± 0.19                         |
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Appendix: The two-point thermal spectral density $\rho^{QCD}(s, T)$

In this appendix we have collected the results of our calculations of the spectral density

$$\rho^{QCD}(s, T) = \rho^{pert.}(s) + \sum_{k=3}^{6} \rho_k(s, T), \quad (A.1)$$

necessary for the evaluation of the mass and coupling constant of the temperature-dependent $X(3872)$ state from the QCD sum rules. $\rho_k(s, T)$ denote the nonperturbative contributions to $\rho^{QCD}(s, T)$ and $g_s = 4\pi\alpha_s$. The explicit expressions for $\rho^{pert.}(s)$ and $\rho_k(s, T)$ are presented below as the integrals over the Feynman parameters $z$ and $w$:

$$\rho^{pert.}(s) = \frac{1}{3072\pi^6} \int_0^{1-t^2} dz \int_0^{1-z} dw \frac{wz}{t^8}$$

$$\times \left[ (swzh - m^2_z((w + z))^2 \right.$$

$$\times \left[ 35h^2w^2z^2s^2 - 26hwzw(w + z)sm_z 
+ 3t^2(w + z)^2m^2_z\theta[L], \right]$$



$$\rho^3(s, T) = \frac{(uw)m_c}{64\pi^4} \int_0^{1-t^2} dz \int_0^{1-z} dw \frac{\left( (w + z)m^2_z - hwz \right)}{t^2}$$

$$\times (w + z) \left[ 7swzh - 3m^2_zt(w + z) \right] \theta[L]$$

$$\rho^4(s, T) = \frac{1}{36864\pi^4} \frac{G^2}{\pi} \int_0^{1-t^2} dz \int_0^{1-z} dw \frac{wz}{h^2t^6}$$

$$\times \left[ 480h^4s^2w^4 - hwz(60w^2(w - 1)^3 \right.$$

$$+ w(w - 1)z(120 + w(353w - 345)) + z^2(879w \right.$$

$$+ w^2(907w - 15989) - 60) + z^3(1577w \right.$$

$$- 594) + 4z^4(321w - 220) + 346z^5 \right.$$$$+ m^4_z \right.$$

$$\times \left[ 60w^3(w - 1)^3 + 3w^2(w - 1)z(60 - 155w \right.$$$$+ 111w^2) + 4w^2(240w + w^2(179w - 362) - 45) \right.$$$$+ z^2(705w + w(868w - 1465) - 60) + z^3(637w^2 \right.$$$$- 827w + 210) + 2z^4(127w - 96) + 42z^5 \right.$$$$+ m^4_z \right.$$

$$\times \left[ \alpha_s(u\Theta^u) \right.$$$$+ \frac{1}{9216\pi^5} \int_0^{1-t^2} dz \int_0^{1-z} dw \frac{wz}{h^2t^6} \left[ 240h^4s^2w^2z^4 \right.$$$$- 3m^4_z \right.$$

$$\times (20w^3(w - 1) + 15w^2z(w - 1)^2(7w - 4) \right.$$$$+ 4w^2w^2(w - 1)(64w^2 - 74w + 15) + z^3(w - 1) \right.$$$$\times (20 - 257w + 384w^2) + z^4(86 - 411w + 361w^2) \right.$$$$+ 2z^5(97w - 56) + 46z^6 \right.$$$$+ m^4_zhwzw \left( -4sz \right.$$$$\times z(-15w^2(w - 1)^2 + wz(w - 1)^2(109w - 1299) \right.$$$$+ z^2(w - 1)(379w - 144) + 4z^3(103w^2 - 72) \right.$$$$+ 144z^4 \right.$$$$+ 15z(4w^2(w - 1)^3 + wz(w - 1)^2 \right.$$$$\right.$$}

$$\times (5shwz - 3m^2_zt(w + z))\theta[L], \quad (A.2)$$

$$\rho^5(s, T) = \frac{1}{108\pi^2} \int_0^{1-t^2} dz \int_0^{1-z} dw \left[ 9m^2_z(u\Theta^u) \right.$$$$- 3m_c(u\Theta^u)(w\Theta^u) + 20z(u\Theta^u)^2 \right] \theta[L]. \quad (A.6)$$

In the expressions above we have used the notations:

$$L = \frac{m^2_z(w^3 + w^2(2z - 1) + (z^2 + 2wz)(z - 1)) - swzh}{t^2}$$

$$\Phi = \frac{m^2_z[3w^3 + z(z - 1)(2w + z) + w^2(2z - 1)]}{wzh}$$

$$t = w^2 + (z - 1)(w + z), \quad h = w + z - 1. \quad (A.7)$$
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