An Efficient Wave Approach for Simulating the Propagation of Impulsive Signals in Hydroacoustic Waveguides on the Sea Shelf

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Abstract. The paper proposes an efficient wave method for simulating the propagation of impulsive signals in hydroacoustic waveguides of the sea shelf. The method of normal modes calculates the acoustic field in a wide frequency band. Then the inverse Fourier transform of the acoustic field is performed and the impulse response of the waveguide is restored. The signal replica is then calculated as a convolution of the impulse response and the signal. The advantages of this approach are as follows. Convolution is cyclical – there are no restrictions on the duration of the signal. Not only calculated, but also experimentally determined impulse response can be used. At the discretion of the researcher, the fields of individual modes can be excluded, add noise in the frequency or time domain, simulate the movement of the source, the impact of wind waves. Restriction - conditions for uniformity of the waveguide along the distance. A number of examples are considered, in which the possibility of determining the acoustic properties of the bottom is studied.

1. Introduction

The sea shelf is characterized by a wide range of depths, from units to hundreds of meters [1]. The sea shelf is a hydroacoustic waveguide bounded from above by a free, fully reflecting surface and from below by a partially reflecting bottom. Calculation of sound fields generated by sources located inside the water layer is necessary when solving problems of inversion of bottom properties [2–8], geophysics [9], ecology [10,11], military technologies for searching for underwater objects and sound underwater communication [12].

The conditions of sound propagation in the shelf waveguide are determined by two competing factors: the sound speed profile (SSP) in the water layer and the acoustic properties of the layered bottom [1,13]. On a shelf with depths of tens or hundreds of meters, in the range of medium frequencies (hundreds of Hertz), the SSP can have a predominant effect if there is a pronounced minimum on it. In the case of depths of a unit or tens of meters, i.e., when the thickness of the water layer is comparable to the length of the sound wave, and the SSP is practically constant, the bottom will have the main influence. The above is especially true for the northwestern part of the Black Sea shelf, where the minimum on the SSP is located at shallow depths of 50 – 60 m [1].
Calculations of tonal sound fields are now well developed. Depending on the waveguide environment, the following methods are used: normal modes, parabolic equation, ray method [13]. Calculating impulse fields is much more difficult. In shallow water conditions, only wave calculations are used, most often the method of normal modes.

Usually, a impulsive field is simulated according to a standard computational scheme: direct Fourier transform of a signal; calculation of the acoustic field in the source frequency band; multiplication of the signal spectrum with the acoustic field in the frequency domain; inverse Fourier transform [13].

The disadvantage of this approach is as follows. The arrays to be multiplied must have the same dimension; therefore, a limited set of signals of limited duration is algorithmically embedded in the computational program, which deprives the researcher of the arbitrariness he needs to select signals for solving various problems of shelf acoustics.

The purpose of the article is to present and test a different approach to simulating an impulsive signal, the meaning of which is as follows. An acoustic field in a wide frequency band is calculated by any wave method. Then the inverse Fourier transform of the acoustic field is carried out and the wave impulse response (WIR) of the waveguide is restored. The waveguide replica to a signal with any type of modulation is calculated further as a convolution of a signal with an impulse response.

Since the signal and the waveguide are separate objects, the convolution method does not have the disadvantages listed above. The convolution is cyclical – there are no restrictions on the signal duration. Can be used not only calculated, but also experimental WIR [14]. At the discretion of the researcher, the fields of individual modes can be excluded, noise can be added in the frequency or time domain, and the motion of a source can be simulated and the effect of wind waves. The only limitation is the condition of the waveguide homogeneity along the range.

2. Simulation of waveguide impulse response and signal replica

Let us consider in Figure 1 a model of a horizontally stratified hydroacoustic waveguide with a water layer depth \( h \), an arbitrary SSP \( c_1(z) \), the water density \( \rho_1 \) is constant, the source is at a depth \( z_s \), the receiver is at a depth \( z \), the distance between the source and the receiver is \( r \). The acoustic properties of the transition bottom layer depend on depth and frequency [15 – 17]. The speed of sound in the layer is \( c_2(z,f) \), and the loss tangent \( \beta_2(z,f) \), and the density \( \rho_2 \) is constant. Half-space "H" with constant acoustic properties: \( c_H, \rho_H, \beta_H \). Cyclic frequency \( \omega = 2\pi f, f, \text{Hz.} \)

The sound field in the waveguide is represented as the sum of normal modes [1,13]:

\[
p(r,z,\omega) = \sum_{l=1}^{\infty} Q_l(z_s,\omega) \varphi(z,\omega) H_0^{(1)}(\omega, r),
\]

2 Figure 1. Shallow water waveguide model
where $Q_l$ is the coefficient of excitation of the normal mode with number $l$, $k_1$ is the Hankel function, $\phi_l(z,\omega)$ are the depth dependence of the normal modes profiles determined from the equation

$$
\frac{d^2 \phi_l(z,\omega)}{dz^2} + (k_1^2 - \xi_l^2)\phi_l(z,\omega) = 0,
$$

where $\xi_l$ is the horizontal wave number, $k_1 = \omega/c_1(z)$ is the acoustic wave number.

The boundary conditions for (1) require zero pressure on the surface, equality of pressures and normal components of the particle velocity at the boundaries of liquid layers, and equality of impedances at the water layer–bottom boundary.

To solve the differential spectral problem (3), it is transformed into a finite-difference problem by introducing a grid in depth, an algebraic spectral problem is constructed, and the eigenvalues and eigenvectors of the algebraic problem are found [13]. The shortly described method was implemented in the "KRAKENC" program, which was used to calculate the horizontal wavenumbers and mode profiles. In the case of the simplest Pekeris waveguide, the sound field can be calculated using well-known formulas [1,18].

The phase and group velocities of the normal modes are determined by the formulas: $v_l = \omega/\text{Re}(\xi_l)$, $u_l = \Delta\omega/\Delta\text{Re}(\xi_l)$, respectively.

To simulate the WIR, the fields of individual modes at fixed frequencies are calculated so that the sampling theorems are satisfied. At frequencies below the critical value, the normal mode field is set equal to zero. In practice, the frequency step was chosen $\Delta f = 1$ Hz, the number of samples $N = 2^{14}$, which gives the upper frequency $f_u = N\cdot \Delta f$, the sampling frequency $f_s = 2f_u$. Accordingly, in the time domain, the interval between samples is $\Delta t = 1/f_s$, the total observation time is $t_0 = N\cdot \Delta t$.

Carrying out the inverse Fourier transform (IFFT) of the fields of all modes separately, we obtain temporal realizations of WIR individual normal modes: $h_l(r,z,t) = \text{IFFT}(p_l(r,z,\omega)\exp(i\omega(t_0-\tau)))$, where $t_0 = r/c_1$ is the time of arrival of the water wave, $r$ is the time of advancing the start of observation. Then, summing the fields of normal modes in the time domain, we obtain a multimode WIR waveguide: $h = \sum h_l(r,z,t)$.

Representing the waveguide as a multipole with $N$ inputs and outputs (in accordance with the depth sampling step), one should determine the partial impulse responses $h_{ij}$, $i = 1, 2 \ldots M$, $j = 1, 2 \ldots M$, each of which displays the signal on the $j$-th output when giving the $i$-th input of the delta function. The collection of functions $h_{ij}$ forms the matrix $h(t) = (h_{ij}(t))$. Then the vector of the output signal $R = (R_j)$ can be calculated as $R = s \otimes h$, where $s = (s_i)$ is the row vector of the input signal, $\otimes$ is the symbol of the convolution.

3. Simulation results

3.1. Two types of impulse response of the shelf waveguide

In shallow water conditions, dispersion distortions of signals arise due to the difference in the group velocities of the modes and the dependence of the group velocity of each mode on frequency. The acoustic properties of the bottom and the SSP have a complex and opposite effect on the dispersion law of normal modes. In the case of a constant SSP, the modes with lower numbers, the group velocities of which are higher, will be the first to arrive at the receiving point – Figure 3 (a). One can see the arrival of the leading ground (head) wave propagating along the boundary between the water layer and the bottom, the arrival of the 1st and then other modes, the maximum amplitude is observed at the beginning of the wave process. At the end of the realization, the Airy wave arrives [18].

If the SSP has a minimum, a sound channel is formed in the water layer, the sound fields of lower modes propagate inside the channel with a lower group velocity – see Figure 2 (b). In this case, the modes with high numbers and lower amplitudes are the first to arrive at the receiving point. The wave process ends with the arrival of the 1st mode – see Figure 2 (b).
3.2. Study of the possibility of determining the acoustic properties of the bottom

One of the most important problems in shallow water acoustics is to determine the acoustic and physical properties of marine sediments, which make up a transition layer several meters thick. These are mainly unconsolidated sediments composed of a mixture of sand with silt, clay and other clastic materials [2–9].

Let us take the acoustic properties of the bottom actually measured in the marine experiments SAX99 (point No.1) and TREX13 (point No.2, VLA1) [19,20]. Let us reconstruct the frequency dependences of the speed of sound and attenuation in the framework of the GSEC theory [15–17]. Consider a waveguide with $c(z) = \text{const}$, depth $h = 20$ m (location SAX99). At point 1 – clean medium sand, at point 2 – a layer of mud covers finer sand. This bottom is typical for ports and harbors.

Let us check the possibility of distinguishing between these two types of bottom by performing a mathematical experiment.

First, by emitting a tone pulse signal and registering a replica, we will increase the distance between the source and the receiver until the responses of a waveguide with two types of bottom become distinguishable.

For points 1 and 2, the critical frequencies of the 1st and 2nd normal modes are respectively \{51, 155\} and \{77, 232\} Hz. A tone pulse with a frequency of $f = 200$ Hz, a duration of 10 periods and replicas to this pulse are shown in Figure 3.

The signal replica is passed through a low-pass filter, the amplitude is normalized to the replica of the waveguide with the bottom number 1. For the pulses to separate and take the position shown in Figure 4 along the time axis, a distance $r = 30c_1 \approx 45$ km is required! Propagating over such a distance, the fields of higher normal modes already decay, only the 1st mode remains. As you can see, the different replica lag makes it possible to determine the group velocity of the 1st mode at a frequency of 200 Hz: bottom 1 – $u_1(200 \text{ Hz}) = 30c_1/(t_0 + 0.1903) = 1518.4$ (m/s), bottom 2 – $u_1(200 \text{ Hz}) = 30c_1/(t_0 + 0.2378) = 1516$ (m/s), where $t_0 = 30$ s. Note that for the bottom number 2 (mud on top of the sand), the spectrogram shows traces of the 2nd mode, excited by the transient process when the pulse is instantaneously turned on. However, since the signal frequency is less than the critical frequency of

![Figure 2. Wave impulse response and spectrograms: a) SSP is constant; b) waveguide with a minimum on the sound speed profile.](image-url)
the 2nd mode (232 Hz), the end of the wave process is clean, without beats. On the contrary, for bottom no. 1, the signal frequency is higher than the critical 2nd mode (155 Hz), and in the end part, beats are seen between the signal replicas transferred by the 1st and 2nd modes.

Figure 3. Source pulse and waveguide replica with bottom no. 1 and no. 2. Left – tonal pulse of the source, right – chirp pulse.

The next signal is linear frequency modulated (chirp), so as to match it with the dispersion law of the 1st mode. Signal $R = A \sin(2\pi(50 + (300 - 50)/0.4)t)$ with Gaussian envelope $A(t)$. The frequency changes from 50 Hz to 300 Hz in a time of 0.4 s. A waveguide replica to this signal is shown in Figure 3. As you can see, such a signal turned out to be in the best agreement with the dispersion law of the 1st mode in the waveguide with bottom no. 2. The replica was compressed 8 times. The spectrogram shows short-term “flashes” caused by transient processes of signal switching on and off.

Pulse compression is provided only at a certain distance between the source and the receiver. Changing the distance violates the conditions for the best matching, and the law of changing the phase of the signal to match it with the dispersion law of the waveguide must be selected again.

To accumulate dispersion distortions at short distances, it is possible to carry out multiple recursion of impulses between two receivers-transceivers.

4. Conclusion
An original numerical-analytical method for reconstructing waveguide responses to signals with unlimited duration and with arbitrary types of modulation, based on the principle of convolution of a signal with an impulse response, is presented. The originality of the method lies in the fact that the impulse response of the waveguide is formed in the time domain by adding the impulse responses of individual modes. The impulse characteristics of individual normal modes are reconstructed using the inverse discrete Fourier transform of the acoustic field of a separate normal mode.

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Acknowledgments
The investigation is carried out within the framework of the SevSU internal grant "Development of theoretical models for physical methods of research of the Black Sea shelf", project No. 41/06-31.