Designing physically realizable state observer for estimating the kinematic parameters of the road train

V G Volkov$^{1,2}$ and D N Demyanov$^1$

$^1$Naberezhnye Chelny Institute, Kazan Federal University, Syuyumbike 10A, Naberezhnye Chelny, Russian Federation

$^2$Scientific and Technical Center of PJSC "KAMAZ", Transportnyi proezd 70, Naberezhnye Chelny, Russian Federation
demyanovdn@mail.ru

Abstract. In this paper the observability of state variables of the linear dynamic system, describing truck and semitrailer lateral motion on a high constant longitudinal velocity and minor rotation angles of steered wheels, is analyzed. The synthesis of a physically implementable reduced state observer for estimation of immeasurable state variables on the lateral displacement magnitude of the vehicle’s frontal part is realized.

1. Introduction

Currently, one of the most effective approaches to transportation of bulky and heavy loads is the use of road trains of various types, for example, truck tractors with semi-trailers and trucks with trailers on a rigid coupling. Due to the large kinematic turning radius, considerably big weight and dimensions of the above-described vehicles particularly the development of effective active driver assistance systems that enhance driving stability and controllability is getting an acute problem [1].

As a rule, for its operation, such systems require the fullest possible information about the current parameters of the kinematic motion (state variables), such as the lateral component of the velocity vector, the angular velocity and the folding train angle. Obtaining this information through direct measurements is significantly difficult due to the complexity and high cost of measurement equipment suitable for the use under the road speed conditions of high vibration load. The use of monitoring devices to retrieve information about the unobservable (invisible) part of the state vector is more appropriate.

Thus, the aim of this study is to assess the observability of the linearized model of road trains lateral movement, to develop physically realizable reduced observing device for it and to study its efficiency.

2. A description of the model used

We consider the linear model of road train lateral movement on absolutely flat surface, resulting in the study [2]. This model written in a matrix form is a system of equations:
\[
\begin{bmatrix}
\dot{v}_x \\
\omega_{1x} \\
\omega_{2x}
\end{bmatrix} = M^{-1} \begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \end{bmatrix} \begin{bmatrix}
\dot{v}_x \\
\omega_{1x} \\
\omega_{2x}
\end{bmatrix} + M^{-1}
\begin{bmatrix}
(k_i - \delta_i N_i)
\end{bmatrix} + \theta_i
\]
\[
M = \begin{bmatrix}
m_1 + m_2 & m_1 d_1 & -m_2 d_2 & 0 & 0 & 0 \\
m_1 d_1 & J_z + m_1 d_1^2 & 0 & 0 & 0 & 0 \\
-m_2 d_2 & 0 & J_z + m_2 d_2^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
\[
A_2 = \begin{bmatrix}
-v_x \left( \sigma (m_1 + m_2)v_x - \frac{k_1 l_1 - k_2 l_2}{v_x} \right) - \frac{k_1 l_1^2 + k_2 l_2^2}{v_x} - m_2 d_1 v_x \\
\frac{m_1 d_1}{v_x} \\
1 \\
0
\end{bmatrix}
A_3 = \begin{bmatrix}
\frac{k_3 l_3 + k_4 l_4 + k_5 l_5}{v_x} \\
0 \\
\frac{k_3 l_3^2 + k_4 l_4^2 + k_5 l_5^2}{v_x} \\
0
\end{bmatrix}
A_4 = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
A_5 = \begin{bmatrix}
\delta_i N_3 + \delta_i N_4 + \delta_i N_5 - (k_i + k_4 + k_5) \\
k_i l_3 + k_i l_4 + k_i l_5 \\
v_x \\
0 \\
0
\end{bmatrix}
A_6 = \begin{bmatrix}
k_i + k_4 + k_5 - (\delta_i N_3 + \delta_i N_4 + \delta_i N_5) \\
0 \\
0 \\
- (k_i l_3 + k_4 l_4 + k_5 l_5) \\
0 \\
0
\end{bmatrix}
\]

Here \(v_x\) – the lateral speed of the tractor truck in the moving coordinate system associated with the body of the tractor; \(\omega_{1x}, \omega_{2x}\) – angular speed of the tractor truck and semi-trailer; \(\phi_1, \phi_2\) – rotation angles of tractor truck and semi-trailer bodies respectively; \(Y\) – the value of lateral displacement of the fifth wheel (saddle-coupling unit) in the absolute coordinate system associated with the road, \(\theta\) – the turning angle of the steering wheel.

Mass coefficients and moments of inertia – \(m_1, m_2, J_1, J_2, d_1, d_2\), as well as of the distance from fifth wheel to the centers of mass of the tractor truck and semi-trailer; \(l_i, k_i, \delta_i, N_i\) – the distance from the rotation axis to the center of mass of the corresponding level, the automobile drag coefficient, the rolling resistance coefficient, and the normal reaction of the road for the \(i\)-th train truck axle; \(v_x\) – longitudinal component of velocity in the moving coordinate system; \(\sigma\) – factor taking into account the rotating masses.
3. Investigation of the system observability

Since the system under the study consists of six state variables, each of which can be measured or observed, totally there are 64 variants of output vector, two of which, corresponding to the cases when the system is fully measurable and not measurable, may be excluded from the consideration.

To reduce the order of the observer and, as a consequence, reduce the requirements for computing resources of on-board information and control system, we estimate the fundamental possibility of constructing a reduced observer, restoring not the entire state vector, but only its unmeasured part [3]. For this purpose we consider the sequence of each of the possible combinations of state variables that determine the measurable output signal. After that, by a non-degenerate transformation of the state vector we present the system (1) in block form:

\[
\begin{bmatrix}
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A_1 & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
y \\
z
\end{bmatrix}
+ \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix}
\theta,
\]
\[
y = \begin{bmatrix} I & 0 \end{bmatrix}
\begin{bmatrix}
y \\
z
\end{bmatrix}.
\]

Here \(y\) and \(z\) is measurable and immeasurable part of the state vector, respectively, in new coordinates.

Observability of the system (2) for each case we define analyzing the pair of matrices \((A_{22}, A_{12})\) on the observability per Kalman [3]. The analysis results show that the reduced observer can be constructed for any set of measurable state variables, provided that it contains the value of lateral displacement of the fifth wheel coupling.

4. Observer Synthesis

Considering the previous results, it can be assumed that the most simple and least expensive way to determine the state vector of the model (1) is a measurement of \(Y\) and all other variables using a reduced observer of the 5 order. However, the direct measurement of \(Y\) is associated with considerable difficulties in the physical realization, so it seems appropriate to consider alternatives.

The analysis of existing and future hardware configurations of automotive control systems [4] allowed proposing a sufficiently effective and simple solution to the problem. We will use the front part of the lateral displacement of the tractor as measured variable value, measurement of which can be carried out, for example, with the help of the subsystem used to track road lanes.

The reduced observer resulting equation looks like:

\[
\dot{z} = (A_{22} - LA_{21}) \dot{z} + (A_{21} - LA_{11} + (A_{22} - LA_{22}) L) y + (B_2 - LB_1) u,
\]

where \(L\) is the matrix of observation.

The value of \(L\) can be determined using known methods [3], by specifying requirements for the quality indicators of evaluation process:

\[
L = \begin{bmatrix}
-20.3967 & 39.4035 & -144.4212 & 12.8957 & -8.3916
\end{bmatrix}^T.
\]

Figure 1 presents the results of numerical modeling in the environment of MATLAB / Simulink process of the observer work (3) during the typical road train truck maneuver.

![Figure 1 - Evaluation of the rotation angle of the semi-trailer body](Image)
It can be seen that even with non-zero initial conditions the estimation error over time asymptotically approaches zero.

5. Conclusions
The research of observability of a dynamic system, describing the lateral movement of the train truck, has been carried out. The magnitude of lateral displacement of the tractor truck front part, determined by using the image processing algorithms, has been used as a measurable variable for constructing the reduced observer. Results of the study can be used in practice in the development of active driver assistance systems.

6. Acknowledgment
This work was supported by RFBR (grant № 16-38-00042).

References
[1] Smirnov G A 1990 Wheeled Vehicles Motion Theory (Moscow: Mechanical Engineering) 352 p
[2] Dem'yanov D N 2014 Development of the linearized model of the motion train truck Proceedings of the final scientific conference pp 375–80
[3] Kuzovkov N T 1976 Modal control and watching devices (Moscow: Mechanical Engineering) 184 p
[4] Belousov B N, Naumov S V and Ksenevich T I 2015. Trends in the development of automotive control systems Automotive Industry No 1 pp 33–36