Examination of an isospin dependent single-nucleon momentum distribution and the high-density behavior of nuclear symmetry energy

Gao-Feng Wei,1,2,3,* Qi-Jun Zhi,1,2 Xin-Wei Cao,3 and Zheng-Wen Long4

1School of Physics and Electronic Science, Guizhou Normal University, Guiyang 550025, China
2Guizhou Provincial Key Laboratory of Radio Astronomy and Data Processing, Guizhou Normal University, Guiyang 550025, China
3School of Mechanical and Material Engineering, Xi’an University of Arts and Sciences, Xi’an 710065, China
4College of Physics, Guizhou University, Guiyang 550025, China

Within an improved transport model using as an input nucleon momentum profiles from a parameterized isospin dependent single-nucleon momentum distribution with a high momentum tail induced by short-range correlations (SRCs), we employ the 197Au + 197Au collisions at 400 MeV/nucleon to examine on one hand effects of these short-range correlated nucleons in the colliding nuclei on the pion and flow observables in probing the nuclear symmetry energy, and on the other hand how reliable are this isospin dependent single-nucleon momentum distribution as well as the corresponding parameter settings. Besides the significant effects of these correlated nucleons in the colliding nuclei on the pion and flow observables are observed, we also found that the theoretical simulation of the 197Au + 197Au collisions with this momentum distribution using two sets of parameters extracted from the experimental analysis (HMT-exp. parameter) and the self-consistent Green’s function (SCGF) predictions (HMT-SCGF parameter), respectively, can reproduce the neutron elliptic flows of the FOPI-LAND experiment and the \( \pi^-/\pi^+ \) ratios of the FOPI experiment under the symmetry energy setting in a soft range with parameter \( x \) no less than 1. Therefore, we can roughly conclude that this parameterized isospin dependent single-nucleon momentum distribution is reliable for isospin-asymmetric nucleonic matter, and both the HMT-SCGF parameter and the HMT-exp. parameter can not be ruled out according to the available experimental information at present. Moreover, compatible with the previous constraints on the slope value \( L \) of the symmetry energy, our results indicate that the upper limit of the \( L \) is not more than 59.872 MeV.

I. INTRODUCTION

The determination of equation of state (EoS) of densely isospin-asymmetric nucleonic matter has always been a fascinating problem in nuclear physics and nuclear astrophysics due to its vital importance in studying the structure of radioactive nuclei and evolution of the compact stars, and thus attracted much attention in the past few decades, see, e.g., Refs. [1–9] for comprehensive reviews. Nevertheless, the predictions on the EoS of densely isospin-asymmetric nucleonic matter, especially its density dependent symmetry energy term at supersaturation density are still discrepant, even controversial at high baryonic density [10–12], although many isospin signals have been proposed aiming to detect the EoS of densely isospin-asymmetric nucleonic matter, see, e.g., Refs. [13–25]. This is mainly because the isovector part of nuclear interactions is much weaker than its isoscalar part, and thus these isospin signals can usually be interfered with by other factors in theoretical simulations and experimental measurement. Therefore, some recent efforts on the strategic studies [26] and covariance analysis [27] of these isospin observables as well as some comparative examinations [28, 29] between different theoretical communities have been carried out to better understand the origins of these discrepancies. Actually, besides these efforts, some studies on the deficiency of mechanism itself in theoretical simulations are also the correct direction to the solutions of these discrepancies, for instance, the pion potential [30–35] and the \( \Delta \) isovector potential [36, 37] have been confirmed that they can all interfere with the sensitivities of the \( \pi^-/\pi^+ \) ratio in probing the symmetry energy using the heavy-ion collisions (HICs).

On the other hand, the momentum distribution of nucleons in a nucleus and/or nucleonic matter, as the direct reflection of nuclear interactions at short distances, has always been a long-standing interest in nuclear physics [38–50]. In particular, the discovery of correlated nucleons pairs in a \( ^{12}\text{C} \) nucleus in high energy electron scattering experiments at the Jefferson Laboratory (J-Lab) [51] arouses the higher enthusiasm in studying the momentum distribution of nucleons and their SRCs in the past decades [52–79]. Qualitatively, people have already gained some general knowledge on nucleon momentum distribution, i.e., the tensor components of the nuclear interactions usually push a few nucleons from low momentum to high momentum, and thus leading to a HMT above the nucleon Fermi surface and a corresponding low momentum depletion (LMD) below the nucleon Fermi surface in the nucleon momentum distribution [40, 41]. Moreover, a qualitative consensus that the \( np \) dominance of the short-range correlated pairs in the HMT has been confirmed by various theoretical and ex-
experimental investigations, see, e.g., Refs. [60–63]. Quantitatively, the experimental results in the J-Lab suggest that about 20% nucleons are in the HMT in a nucleus from light $^{12}$C [51] even to heavy $^{208}$Pb [63, 79]. Also, the systematic analyses of these results in experiments at the J-Lab indicate that the fraction of nucleons in the HMT is about 25% in symmetric nuclear matter (SNM) at the saturation density $\rho_0$ [63, 67], it is however, the theoretical calculations using various many-body theories become to deviate significantly from this suggested fraction for the HMT in SNM at $\rho_0$. For example, the self-consistent Green’s function approach employing the Av18 interactions predicts only 11–13% nucleons in the HMT for SNM at $\rho_0$ [52], while the latest Brueckner-Hartree-Fock calculations go so far as to suggest a wide ranges for nucleons in the HMT in SNM at $\rho_0$ from about 10% using the N3LO450 interaction to about 20% using the Av18, Paris, or Nij93 interactions [72]. More generally, guided by these studies, a parameterized isospin dependent single-nucleon momentum distribution with a HMT induced by SRCs is proposed for isospin-asymmetric nucleonic matter in Refs. [69, 78], and the corresponding two sets of parameters from experimental analysis [63, 67] and the SCGF predictions [52] are also extracted, i.e., the HMT-exp. parameter and the HMT-SCGF parameter. With the HMT-exp. parameter, the nucleons in the HMT can reach a fraction 28% (1.5%) for SNM (PNM) at $\rho_0$, while the HMT-SCGF parameter only leads to a fraction 12% (4%) for the HMT in SNM (PNM) at $\rho_0$. To this situation, a natural question is how reliable are these extracted fractions for the HMT in the nucleon momentum distribution. To answer this question, at least qualitatively, we attempt to use the available data to check the reliability of the extracted fraction based on this isospin dependent single-nucleon momentum distribution as well as the corresponding parameter settings.

In nucleus-nucleus collisions, as a direct input the momentum distribution of nucleons in a colliding nucleus can be reflected by the final reaction products. Naturally, the colliding nuclei with different fraction of correlated nucleons in the HMT is likely to be detected from the final reaction products. With this consideration in mind, in this paper we conduct a typical reaction of $^{197}$Au + $^{197}$Au collisions at 400 MeV/nucleon available in several experiments, and thus expect to extract some information about the HMT through comparing the final reaction products with the available data. As expected, with the HMT-exp. parameter, the theoretical simulation of $^{197}$Au + $^{197}$Au collisions at a impact parameter $b=7.2$ fm can reproduce the neutron elliptic flows of the FOPILAND experiment under the symmetry energy setting in a soft range with parameter $x$ no less than about 0.9; while with the HMT-SCGF parameter, the theoretical simulations of central Au+Au collisions can well fit the $\pi^-/\pi^+$ ratios of the FOPI experiment with the symmetry energy setting in a soft range with parameter $x$ no less than 1. Consequently, taking these results an overall, we can conclude that the upper limit of the slope value $L$ of the symmetry energy is not more than 59.872 MeV. This constraint is compatible with that on the $L \leq 60.4$ MeV in the ASY-EOS experiment [80, 81] and also within the range from other analyses including that on the $L \leq 66$ MeV from the electric dipole polarizability [82] as well as that on the $L \leq 70$ MeV from the charge radii of mirror nuclei [83].

**II. THE IMPROVED IBUU MODEL**

**A. Initialization of the model**

The present study is carried out in the framework of an isospin- and momentum-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model [84, 85]. As the first step, the startup of the IBUU model is the initialization of colliding nuclei in the coordinate and momentum spaces. Specifically, we initialize the spatial distributions of nucleons using the density profiles generated from the Skyrme-Hartree-Fock calculations and then sample them via monte carlo methods [86] into the corresponding spherical shells, i.e.,

$$ x = r \sin \theta \cos \phi; \quad y = r \sin \theta \sin \phi; \quad z = r \cos \phi, $$

$$ r = R(x_1); \quad \cos \theta = 1 - 2x_2; \quad \phi = 2\pi x_3, $$

where $x_1$, $x_2$ and $x_3$ are three independent random numbers, and $R$ is the radius of the initial colliding nuclei. To consistent with the initialization of nucleons in the coordinate spaces, we use the local Thomas-Fermi approximation in each spherical shell of radius $r$ to calculate the corresponding Fermi momentum for these nucleons, i.e.,

$$ k_F(r) = \left[3\pi^2 \hbar^3 \rho(r) \right]^{1/3}, $$

and then initialize them in the momentum space with the method similar to that used in the coordinate space. However, as we have mentioned that there exists a HMT above the nucleon Fermi surface and a corresponding LMD below the nucleon Fermi surface in the nucleon momentum distribution, we should also consider these high momentum nucleons during the initialization of colliding nuclei in momentum space. Therefore, we also use an isospin dependent single-nucleon momentum distribution [69, 78] in the isospin-asymmetric nucleonic matter with the parameterized expression of

$$ n^j_k(\rho, \delta) = \left\{ \begin{array}{ll} \Delta_j + \beta_j I(|k|/k^j_F), & 0 < |k| \leq k^j_F, \\ C_j (k^j_F/|k|)^4, & k^j_F < |k| \leq \phi_j k^j_F, \end{array} \right. $$

to initialize the nucleons of the colliding nuclei in the momentum space. Here, the parameters $\Delta_j$, $C_j$, $\beta_j$ and $\phi_j$ depend linearly on the isospin asymmetry $\delta$ in the general form of $Y_j = Y_0 (1 + Y_1 \tau_3^1 \delta)$, where the value...
three different momentum distributions of the

\[ \frac{2}{(2\pi)^3} \int_0^\infty n_k^{l}(\rho, \delta) d\mathbf{k} = \frac{1}{3\pi^2} \]

is used.

The parameter of \( \tau_3 \) in the isospin-asymmetric matter into the distribution space, we should transform above nucleon distribution of the nucleon momentum distribution with and without range correlated pairs in the HMT. To show the difference in the initial momentum distribution with and without the consideration of the nucleon momentum distribution with and without the consideration of the HMT induced by SRCs, we plot in Fig. 1 three different momentum distributions of the Au nucleus, i.e., without a HMT (w/o HMT), with a HMT calculated by the HMT-SCGF parameter (with HMT-SCGF) and with a HMT but calculated by HMT-exp. parameter (with HMT-exp.). Obviously, except for the observed LMD below nucleon Fermi surface and the corresponding HMT above the nucleon Fermi surface, the nucleon momentum profiles generated from this isospin dependent momentum distribution are very similar to those without a HMT, and thus preliminarily indicating the feasibility of initialization using this isospin dependent momentum distribution in the momentum space. Certainly, the reliability of specific parameter settings based on this distribution still needs to be checked as shown in the following parts. Moreover, because the nucleons in the HMT calculated with the HMT-exp. parameter are more than those with the HMT-SCGF parameter, more apparent LMD below the Fermi surface and the corresponding HMT above the nucleon Fermi surface can be seen in calculations with the HMT-exp. parameter compared to that with the HMT-SCGF parameter. Therefore, some sensitive observables of the momentum distribution using \( ^{197}\text{Au}+^{197}\text{Au} \) collisions. Moreover, we have also checked effects of the value \( \beta_1 \) in the allowed range on the nucleon momentum distribution in \( ^{197}\text{Au} \) nucleus and found that the nucleon momentum distribution of the \( ^{197}\text{Au} \) nucleus is less influenced by the value of \( \beta_1 \) parameter, it is thus we randomly take the value for the \( \beta_1 \) in the allowed range in this study.

B. Interaction used in the model

As far as the nuclear interaction used in the present model, we take similar to our recent work \( ^{89, 90} \) the expression in the form of

\[
U(\rho, \delta, \bar{p}, \tau) = \frac{A_u(x)}{A_0} P^x - A_t(x) P^x + \frac{B}{2}(\frac{2\rho_x}{\rho_0})^\sigma (1 - x) + \frac{2B}{\sigma + 1} \left( \frac{\rho}{\rho_0} \right)^\sigma (1 + x) \frac{P^x}{\rho} \left[ 1 + (\sigma - 1) \frac{\rho_x}{\rho} \right] + 2C_i \int d^3p' f_{-}(p') \frac{f_{+}(p')}{1 + (p - p')^2/\Lambda^2} + 2C_u \int d^3p' f_{-}(p') \frac{f_{+}(p')}{}{1 + (p - p')^2/\Lambda^2}.
\]

This nuclear mean-field interaction has been improved for more reasonable consideration that assuming the interaction between neutrons (protons) depends on the neutron (proton) density instead of total density, and thus can effectively distinguish the density dependences of the in-medium \( nn \), \( pp \) and \( np \) interactions in the effective many-body force term \( ^{91, 92} \). Correspondingly, the parameters \( A_t(x) \) and \( A_u(x) \) are expressed in forms of

\[
A_t(x) = A_{t0} - \frac{2B}{\sigma + 1} \left[ (1 - x) \frac{\sigma}{\sigma + 1} - \frac{1 + x}{2} \right],
\]

\[
A_u(x) = A_{u0} + \frac{2B}{\sigma + 1} \left[ (1 - x) \frac{\sigma}{\sigma + 1} - \frac{1 + x}{2} \right].
\]

The parameter \( x \) embedded in above expressions is to mimic the slope value \( \beta \equiv 3\rho(dE_{sym}/d\rho) \) of the nuclear symmetry energy at the saturation density \( \rho_0 \) predicted by various many-body theories without changing the value of symmetry energy \( E_{sym}(\rho) \) at \( \rho_0 \) and
TABLE I: The parameters used in the present IBUU model

| parameters               | w/o HMT | with HMT |
|--------------------------|---------|----------|
| $A_{0}$ (MeV)            | −96.963 | −96.963  |
| $A_{u0}$ (MeV)           | −36.963 | −36.963  |
| $C_{l}$ (MeV)            | −40.820 | −24.719  |
| $C_{u}$ (MeV)            | −119.368| −135.469 |
| $B$ (MeV)                | 141.963 | 141.963  |
| $\sigma$ (MeV)          | 1.2652  | 1.2652   |
| $\Lambda/p_f$           | 2.424   | 2.424    |
| $L(x=−1)$ (MeV)          | 149.309 | 181.183  |
| $L(x=0)$ (MeV)           | 88.654  | 120.528  |
| $L(x=1)$ (MeV)           | 27.999  | 59.872   |
| $L(x=2)$ (MeV)           | −32.657 | −0.783   |

any properties of symmetric nuclear matter. Moreover, this mean-field interaction can also better fit the high-momentum behaviors of the nucleon optical potential extracted from nucleon-nucleus scattering experiments [92, 93]. Using empirical constraints on properties of nuclear matter at the saturation density $\rho_0 = 0.16$ fm$^{-3}$, i.e., the isoscalar constraints on symmetric nuclear matter including the banding energy $E_0(\rho_0)$, the incompressibility $K_0$, the isoscalar effective mass $m^*_s$, isoscalar potential at infinitely large nucleon momentum $U_0^\infty(\rho_0)$, as well as isovector constraints on the isospin-asymmetric nuclear matter including the symmetry energy $E_{sym}(\rho_0)$ and the symmetry potential at infinitely large nucleon momentum $U_{sym}^\infty(\rho_0)$, we can fix the values of these parameters, i.e., $A_{0}$, $A_{u0}$, $B$, $C_{l}$, $C_{u}$, $\sigma$, and $\Lambda$. Generally, without the consideration of correlations in a nuclear system, the kinetic part of the symmetry energy is calculated from the free Fermi gas model as $E^{\text{kin}}_{sym}(\rho) = 8\pi\rho^2/3m^2\rho^2\rho\rho^{2/3}$, it is however, under the consideration of the SRCs, this expression should be modified because the kinetic part of nuclear symmetry energy is reduced significantly due to the SRCs induced by tensor force according to some solid evidences from microscopic many-body theories [53–57] as well as experimental analysis findings [63, 64, 67]. On the other hand, to our considered problems, we should incorporate the effects of tensor force or the tensor component of nucleon-nucleon potential into nuclear effective interactions and thus consider the effects of SRCs induced by tensor force on reaction products during reactions. To this end, we have readjusted these parameters marked as with HMT as shown in Table I. In the actual readjustment, considering that a large proportion of nucleons are uncorrelated and only minority of nucleons are correlated, therefore, it is suitable to assume that the kinetic symmetry energy also holds for the 2/3 regularity with respect to density. Moreover, according to a microscopic Brueckner-Hartree-Fock calculation using the Av18 interactions plus the Urbana IX three-body force [54–56], the symmetry energy at $\rho_0$ is almost completely contributed from its potential part. It is therefore we use an expression $E^{\text{sym}}_{sym}(\rho) = 12.5[(\rho/\rho_0)^{2/3} − 1]$ for the kinetic part of the symmetry energy to meet these demands under the inclusion of SRCs induced by tensor force similar to previous studies [70]. Certainly, it needs to be emphasized that the two sets of parameters are obtained through fitting the same constraints on the nuclear matter at $\rho_0$, i.e., $E_0(\rho_0) = −16$ MeV, $K_0 = 230$ MeV, $m^*_s = 0.7m$, $U_0^\infty(\rho_0) = 75$ MeV, $E_{sym}(\rho_0) = 32.5$ MeV and $U_{sym}^\infty(\rho_0) = −30$ MeV. Shown in Fig. 2 are the density- and momentum-dependent isoscalar potential $U_0(\rho, p)$ and symmetric potential $U_{sym}(\rho, p)$ calculated by the two sets of parameters, respectively. As expected, the isoscalar potentials $U_0(\rho, p)$ under the consideration of SRCs are completely identical with those without the consideration of SRCs regardless of low densities or high densities due to only the symmetry energy is used as the criterion of the correlation-driven effects. Actually, the reduction of kinetic symmetry energy is the
only effect that we are able to identify as correlation-driven as indicated in Ref. [57]. Nevertheless, with the consideration of SRCS, the symmetric potentials become to deviate significantly from those without the consideration of SRCS regardless of low densities or high densities. Naturally, after decomposing the single-nucleon potential in Eq. (5) according to the well-known Lane approximation [94], i.e., $U_{p,n}(\rho, \delta, p) \approx U_0(\rho, p) + U_{\text{sym}}(\rho, p)(\tau \delta)$ with $\tau = 1$ for neutrons and $-1$ for protons, one can found that the nuclear interaction in Eq. (5) employing the with HMT parameter is essentially different from that using the w/o HMT parameter although an identical expression of nuclear interaction in form is used in these two different scenarios. Therefore, the tensor force effects will be naturally generated in the scenario with the SRCS through the evolution of nucleons momenta during reactions, i.e., $d\vec{p}/dt = -\nabla U$ with $U \equiv U_{p,n}(\rho, \delta, p)$ for simplicity. Here, we emphasize again that the tensor force effects generated during reactions are just because of the corresponding readjustment of the parameters embedded in Eq. (5) that is resulting from the modification of kinetic part of the symmetry energy. While the initialization effects of momentum distribution under the inclusion of the SRCS are also naturally considered through the evolution of equations of motions, i.e., $d\vec{r}/dt = \vec{p}/\sqrt{p^2 + m^2}$. Moreover, for the HMT-SCGF parameter and HMT-exp. parameter, the shapes of the nucleon momentum distributions are similar to each other, their differences are only the initial maximum momentum and the specific fraction of high momentum nucleons, it is therefore the utilization of identical nuclear interaction in form for these two cases, i.e., the single-nucleon potential (5) employing the with HMT parameter, is suitable. Certainly, the differences of initial maximum momentum and the specific fraction of high momentum nucleons will get the nuclear interaction (5) for these two cases to be essentially different through its momentum dependent parts, i.e., the momentum dependent $C_l$ and $C_u$ terms. On the other hand, as a response to the change of symmetry potential, one may expect that the symmetry energy will be different at low and high densities between with and without the SRCS. This expectations are confirmed in Fig. 3. Consistent with the changes of symmetric potentials, the symmetry energy becomes softer (stiffer) at low (high) densities with the SRCS compared to that without the SRCS. The specific slop values $L$ at $\rho_0$ in these two cases are also shown in the bottom of Table I. Generally speaking, the initialization with SRCS will get the neutrons and protons to form pairs, reducing the isospin asymmetry of the reaction system and thus reducing the symmetry energy effects of the observables, while the enhanced (reduced) symmetry potential at low (high) densities will compensate (reduce) the symmetry energy effects of the observables at low (high) densities. Naturally, we can in advance evaluate that at high densities the protons will be more bound and neutrons will be less bound during reactions. Certainly, the final reaction products are affected by both the strength of symmetric potential and the initial momentum distribution of nucleons in the colliding nuclei.

\begin{align*}
e_\text{E}(\vec{r}, t) &= \frac{e^2}{4\pi \varepsilon_0} \sum_n Z_n \frac{c^2 - v^2}{(c R_n - \vec{R}_n \cdot \vec{v}_n)^3} (c \vec{R}_n - R_n \vec{v}_n), \\
\bar{e}(\vec{r}, t) &= \frac{e^2}{4\pi \varepsilon_0} \sum_n Z_n \frac{c^2 - v^2}{(c R_n - \vec{R}_n \cdot \vec{v}_n)^3} \vec{v}_n \times \vec{R}_n,
\end{align*}

due to they can also appreciably affect some isospin observables such as the $\pi^-/\pi^+$ ratio and the neutron-proton differential transverse flow in HICs at intermediate energies especially around 400 MeV/nucleon, see., e.g., Refs. [89, 90] for more details. Moreover, to improve the accuracies of theoretical simulations of HICs, we have also considered the pion potential and the $\Delta$ isovector potential in the present study. Specifically, when the pionic momentum is higher than 140 MeV/c, we adopt the pion potential based on the $\Delta$-hole model of the form used in Ref. [98], and when the pionic momentum is lower than 80 MeV/c, we use the pion potential of the form used in Refs. [99, 100], while for the pionic momentum falls into the range from 80 to 140 MeV/c, an interpolative pion potential constructed by O. Buss in Ref. [98] is used. As far as the effects of this pion potential on the isospin observables such as the $\pi^-/\pi^+$ ratio in HICs, we refer readers to see Ref. [33] for more details. For the $\Delta$ potential, guided by the earlier studies [36, 37] and according to the decay mechanism of $\Delta$ resonance, we use an isospin dependent $\Delta$ potential in the present study.

![FIG. 3: (Color online) The density dependence of nuclear symmetry energy calculated with (Open symbols) and without (Solid symbols) the consideration of SRCS induced by tensor force.](image-url)
i.e.,
\[ U(\Delta^{++}) = f_\Delta U(p), \]  
\[ U(\Delta^+) = f_\Delta \left[ \frac{1}{3} U(n) + \frac{2}{3} U(p) \right], \]  
\[ U(\Delta^0) = f_\Delta \left[ \frac{2}{3} U(n) + \frac{1}{3} U(p) \right], \]  
\[ U(\Delta^-) = f_\Delta U(n). \]  

Certainly, the quantitative results of the \( \Delta \) potential is still inconclusive at present, it is however, considering the fact that the depth of nucleon potential is approximately \(-50\) MeV, while that of the \( \Delta \) potential is empirically constrained around \(-30\) MeV \cite{101, 102}, it is therefore we additionally introduce an identical factor \( f_\Delta = 2/3 \) for them to meet this demand in spite of this factor maybe different for the \( \Delta \) resonances with different charged states.

III. RESULTS AND DISCUSSIONS

Now, let’s check the influences of the SRCs on the final reaction products in \( {}^{197}\text{Au} + {}^{197}\text{Au} \) collisions. First, we examine effects of the SRCs on the pion observable. To this end, the dynamic ratio \( (\pi^-/\pi^+)_{\text{like}} \) defined as
\[ (\pi^-/\pi^+)_{\text{like}} = \frac{2}{3} \Delta - \frac{1}{3} \Delta_0 + \frac{1}{3} \Delta^+, \]  
can be used to check the effects of the SRCs on the dynamic production of pions. Certainly, the dynamic ratio \( (\pi^-/\pi^+)_{\text{like}} \) will naturally become the free \( \pi^-/\pi^+ \) ratio at the end of reactions due to all the \( \Delta \) resonances will decay into nucleons and pions. Shown in Fig. 4 is the time evolution of dynamic ratio \( (\pi^-/\pi^+)_{\text{like}} \) in central \( {}^{197}\text{Au} + {}^{197}\text{Au} \) collisions at 400 MeV/nucleon with and without the consideration of the SRCs. First, it can be seen that, without the consideration of the SRCs in the colliding nuclei, the dynamic ratio \( (\pi^-/\pi^+)_{\text{like}} \) during reactions and thus the \( \pi^-/\pi^+ \) ratio at the end of reactions are larger on the whole than those with the consideration of the SRCs, regardless of the HMT-SCGF parameter or HMT-exp. parameter is used in calculations. Second, except for the observation consistent with the established systematics for pion production \cite{103}, i.e., the \( \pi^-/\pi^+ \) ratio is sensitive to the density dependent nuclear symmetry energy \( E_{\text{sym}}(\rho) \), and the softer symmetry energy usually leads to a higher \( \pi^-/\pi^+ \) ratio, reflecting a more neutron-rich participant region formed in the reaction, we can also observe a decreased sensitivity of the \( \pi^-/\pi^+ \) ratio to the nuclear symmetry energy with the consideration of the SRCs in the colliding nuclei, and this phenomenon is more apparent for the case with the HMT-exp. parameter. Actually, the second observation can be easily understood within the reached consensus about the short-range correlated nucleon-nucleon pairs in the HMT. That is, due to the \( np \) dominance of the short-range correlated pairs in the HMT \cite{60-63}, the higher fraction of nucleons in the HMT naturally leads to more \( np \) pairs in the reaction system, and thus causes a more reduction of isospin asymmetry of the reaction system; naturally, the dynamic ratio \( (\pi^-/\pi^+)_{\text{like}} \) during reactions and thus the final \( \pi^-/\pi^+ \) ratio show a decreased sensitivity to the nuclear symmetry energy. Moreover, the \( \pi^-/\pi^+ \) ratio as a sensitive observable of the symmetry energy at high densities mainly reflects the properties of the symmetry potential at high densities, it is therefore the weakened symmetry potential at the high densities with SRCs naturally gets the \( \pi^-/\pi^+ \) ratio showing a reduced sensitivity to the nuclear symmetry energy. Nevertheless, to understand the first observation, we need to look at the dynamic multiplicities of \( \pi^- \) and \( \pi^+ \) in a quantitative manner in the following.

![Figure 4: (Color online) Time evolution of the \( (\pi^-/\pi^+)_{\text{like}} \) ratio in central \( {}^{197}\text{Au} + {}^{197}\text{Au} \) collisions at 400 MeV/nucleon with the symmetry energy ranging from the superhard of \( x = -1 \) to the supersoft of \( x = 2 \). The panels (a) and (b) are the results without and with the consideration of the SRCs in the colliding nuclei, respectively.](image1)

![Figure 5: (Color online) Multiplicities evolution of dynamic \( \pi^+ \) (a) and \( \pi^- \) (b) in central \( {}^{197}\text{Au} + {}^{197}\text{Au} \) collisions at 400 MeV/nucleon with and without the consideration of the SRCs in the colliding nuclei. The symmetry energy parameter \( x = 1 \) is used in panels (a) and (b).](image2)
namic $\pi^-$ and $\pi^+$ with a certain symmetry energy parameter $x = 1$ in the same reaction, which are determined by $\pi^- + \Delta^- + \frac{2}{3} \Delta^0$ and $\pi^+ + \Delta^+ + \frac{2}{3} \Delta^0$ according to the decay mechanism of $\Delta$ resonances. It is obvious to see that more $\pi^-$ and $\pi^+$ are produced with than without the consideration of SRCs in the colliding nuclei. Consequently, the dynamic ratio $(\pi^-/\pi^+)_\text{like}$ during reactions and thus the final $\pi^-/\pi^+$ ratio may alter to different extent according to the specific fraction of the HMT used in the nucleon momentum distribution. More specifically, according to the following formula

$$R \equiv \frac{|(\pi^+)_{\text{with}} - (\pi^+)_{\text{w/o}}|}{|(\pi^+)_{\text{with}} + (\pi^+)_{\text{w/o}}|/2} \times 100\%,$$

we can calculate the relative effects of the SRCs on the final multiplicities of $\pi^-$ and $\pi^+$, the corresponding results employing the HMT-exp. parameter and HMT-SCGF parameter are shown in Fig. 6, respectively. It is seen that, regardless of HMT-SCGF parameter or HMT-exp. parameter is used in calculations, the increment of $\pi^+$ is more than that of $\pi^-$, and thus leading to a corresponding reduction of the $\pi^-/\pi^+$ ratio. This is the reason why we see in Fig. 4 the $\pi^-/\pi^+$ ratio becomes small on the whole with the consideration of the SRCs in the colliding nuclei. One may suspect why the increment of $\pi^+$ is more than that of $\pi^-$ in reactions for a certain parameter settings. Actually, just as we discussed in above, the weaken symmetry potential at high densities also gets protons more bound and neutrons less bound, and thus generates more $\pi^+$ than $\pi^-$. Moreover, compared with the HMT-SCGF parameter, the HMT-exp. parameter can lead to more high momentum nucleons in the colliding nuclei and thus more increment of $\pi^+$ than $\pi^-$ as well as more $np$ pairs, it is therefore the corresponding $\pi^-/\pi^+$ ratio naturally has more smaller values and weaker sensitivity to the nuclear symmetry energy.

Before checking the effects of the SRCs on the collective flows of nucleons, let’s first compare the final multiplicities of $\pi^-$ and $\pi^+$ as well as their $\pi^-/\pi^+$ ratios with the available data to examine the reliability of this isospin dependent single-nucleon momentum distribution as well as these two sets of parameters for isospin asymmetric matter. On the other hand, the predictions on the high density behavior of nuclear symmetry energy using the pion signals are mainly through comparing with the experimental data in FOPI collaboration [103]. Therefore, to constrain roughly the high density behavior of nuclear symmetry energy simultaneously, we show in panels (b) and (c) of Fig. 7 the final multiplicities of $\pi^-$ and $\pi^+$ generated in central $^{197}\text{Au} + ^{197}\text{Au}$ collisions under the consideration of the SRCs to compare with the FOPI data. As a comparison, we also show the corresponding results of central $^{197}\text{Au} + ^{197}\text{Au}$ collisions without the consideration of the SRCs in the colliding nuclei. Obviously, without the SRCs in the colliding nuclei, the multiplicities of both $\pi^-$ and $\pi^+$ are much underestimated on the whole in calculations using all these $x$ parameters; on the contrary, the multiplicities of both $\pi^-$ and $\pi^+$ are much overestimated in calculations using the HMT-exp. parameter. While for the case with the HMT-SCGF parameter, the theoretical simulations of $^{197}\text{Au} + ^{197}\text{Au}$ collisions can reproduce the multiplicities of both $\pi^-$ and $\pi^+$ of the FOPI experiment with the symmetry energy parameter $x$ setting approximately in the range from 0 to 2. Certainly, to reduce the systematic errors in probing the symmetry energy using the multiplicities of charged pions, their ratios $\pi^-/\pi^+$ are usually used as more effective observable of the symmetry energy in HICs at intermediate energies. To this end, we compare in panel (a) of Fig. 7 the FOPI experimental data with the corresponding ratios $\pi^-/\pi^+$ of charged pions in the same reactions with and without the consideration of the SRCs. It is obvious to see that with the HMT-SCGF parameter the theoretical simulations of $^{197}\text{Au} + ^{197}\text{Au}$ collisions can indeed reproduce the $\pi^-/\pi^+$ ratio of the FOPI data quite well under the symmetry energy setting in a soft range with parameter $x$ no less than 1. This result implies that the upper limit of the slope value $L$ of the symmetry energy is not more than 59.872 MeV. Certainly, one can also observe that without the consideration of the SRCs the ratio $\pi^-/\pi^+$ of charged pions can also reproduce the FOPI experimental data even with all the $x$ parameter used here. However, it needs to be emphasized that this consistence maybe just a mathematical coincidence because the multiplicities of both $\pi^-$ and $\pi^+$ deviate too much from the FOPI experimental data.

![Fig. 6](image_url)

**FIG. 6:** (Color online) Relative effects of the SRCs on the final multiplicities of $\pi^-$ (a) and $\pi^+$ (b) in central $^{197}\text{Au} + ^{197}\text{Au}$ collisions at 400 MeV/nucleon.

We now investigate effects of the SRCs on the collective flows of nucleons. As a typical collective flow, the elliptic flows of nucleons defined as

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{p_t^2},$$

are widely used as the probe of the isovector part of nuclear interactions in HICs at intermediate energies as well as the properties of the hot and dense matter formed in the early stage of HICs at relativistic energies, see,
are the elliptic flows of nucleons in with those from the corresponding experiment during ex-
our theoretical simulations of the elliptic flows of nucleons
sity behavior of nuclear symmetry energy, we compare
ucleon momentum distribution as well as the correspond-
model consistently favor a moderately soft prediction for
semitcentral
ativistic quantum molecular dynamics (UrQMD) \[\text{TüQMD}\] model and ultrarel-
iments of the SRCs in the colliding nuclei. Thus, leading to an appreciable reduction of the
crement of correlated nucleons in the HMT are found to cause more
Au+Au collisions at 400 MeV/nucleon without and with the consideration of the SRCs in the colliding nuclei.

FIG. 7: (Color online) Final multiplicities of $\pi^-$ and $\pi^+$ and the corresponding $\pi^-/\pi^+$ ratios compared with those from the FOPI experiment in central $^{197}$Au + $^{197}$Au collisions at 400 MeV/nucleon without and with the consideration of the SRCs in the colliding nuclei.

FIG. 8: (Color online) Elliptic flows of neutrons (a) and protons (b) compared with those from the FOPI-LAND experiment in semicentral $^{197}$Au + $^{197}$Au collisions with a impact parameter $b = 7.2 \text{ fm}$ at 400 MeV/nucleon with and without the consideration of the SRCs in the colliding nuclei.

e.g., Refs. [104–108]. Presently, predictions on nuclear symmetry energy using the elliptic flow of nucleons are mainly through comparing with the available data in FOPI-LAND and/or ASY-EOS experiments [80, 109]. For example, based on these data, the Tübingen quantum molecular dynamics (TüQMD) [110] model and ultrarelativistic quantum molecular dynamics (UrQMD) [111] model consistently favor a moderately soft prediction for nuclear symmetry energy at high densities. Therefore, to further check the reliability of this isospin dependent nucleon momentum distribution as well as the corresponding parameter settings and also constrain the high density behavior of nuclear symmetry energy, we compare our theoretical simulations of the elliptic flows of nucleons with those from the corresponding experiment during ex-
aming the effects of the SRCs on the elliptic flows of nucleons. Shown in Fig. 8 are the elliptic flows of nucleons in semicentral $^{197}$Au + $^{197}$Au collisions with a impact parameter $b = 7.2 \text{ fm}$ and beam energy of 400 MeV/nucleon. Obviously, with the consideration of the SRCs in the colliding nuclei, the elliptic flows of both neutrons and protons are decreased due to the correlations enhancing the difficulties of squeezing out these nucleon-nucleon pairs, and this phenomena are special apparent for the case with the HMT-exp. parameter because of more corre-
lected nucleon-nucleon pairs formed. Again, due to the dominance of the $np$ pairs gets the isospin asymmetry of the reaction system to be reduced, we can also observe a reduced sensitivity of elliptic flows to the nuclear symmetry energy. Moreover, it can be seen that the elliptic flows of protons in FOPI-LAND experiment are failed to reproduced in our theoretical simulations regardless of using the HMT-SCGF parameter or HMT-exp. parameter. Interestingly, it is however, we found that with the symmetry energy setting in a soft range with parameter $x$ no less than about 0.9, the elliptic flows of neutrons in FOPI-LAND experiment can be well reproduced by theoretical simulations of $^{197}$Au + $^{197}$Au collisions employing the HMT-exp. parameter, this constraint on the symmetry energy coincides qualitatively with the previous predictions using the TüQMD [110] and UrQMD [111] models. So far, according to what we have learned from elliptic flow signals and taking the above obtained from pion signals an overall, we can roughly conclude two points. First, the parameterized isospin dependent nucleon momentum distribution given in Eq. (4) is reliable for isospin-asymmetric nucleonic matter, and both the HMT-SCGF parameter and the HMT-exp. parameter can not be ruled out according to the available experimental information at present. Second, compatible with the results of the ASY-EOS experiment [80, 81] on the upper limit of the slope value $L \leq 60.4 \text{ MeV}$ of the symmetry energy, our results indicate that the upper limit of the $L$ is not more than 59.872 MeV. It is noted that this constraint is also within the range from other analyses including that on the $L \leq 66 \text{ MeV}$ from the electric dipole polarizability [82] as well as that on the $L \leq 70 \text{ MeV}$ from the charge radii of mirror nuclei [83].

IV. SUMMARY

Within an improved IBUU transport model using as an input nucleon momentum profiles from an isospin dependent single-nucleon momentum distribution with a HMT induced by SRCs, in this paper we have investigated effects of the SRCs on the pion and flow observables in Au+Au collisions at 400 MeV/nucleon. Compared to the case without the SRCs in the colliding nuclei, these correlated nucleons in the HMT are found to cause more increment of $\pi^+$ than $\pi^-$ in $^{197}$Au + $^{197}$Au collisions, and thus leading to an appreciable reduction of the $\pi^-/\pi^+$ ratio; while for the flow observables, these correlated nucle-
ons are found to decrease the squeezed out elliptic flows of both neutrons and protons due to their correlations enhancing the difficulties of anisotropic emission of these nucleons. Moreover, due to the dominance of the $np$ pairs in the HMT and thus a corresponding reduction of the isospin asymmetry of the reaction system as well as the weakened symmetry potential at high densities, both pion and flow observables are found to show a reduced sensitivity to the nuclear symmetry energy. On the other hand, through comparing the pion observable as well as the flow observable with the available data, we have also checked the reliability of the used isospin dependent single-nucleon momentum distribution as well as the corresponding two sets of parameters. It is found that the theoretical simulation of $^{197}$Au + $^{197}$Au collisions with this momentum distribution using the HMT-exp. parameter and the HMT-SCGF parameter, respectively, can reproduce the neutron elliptic flows of the FOPI-LAND experiment and the $\pi^{-}/\pi^{+}$ ratios of the FOPI experiment under the symmetry energy setting in a soft range with parameter $x$ no less than 1. Therefore, we can roughly conclude that this parameterized isospin dependent single-nucleon momentum distribution is reliable for isospin-asymmetric nucleonic matter, and both the HMT-SCGF parameter and the HMT-exp. parameter can not be ruled out according to the available experimental information at present. Moreover, compatible with the results of the ASY-EOS experiment on the upper limit of the slope value $L \leq 60.4$ MeV of the symmetry energy, our results indicate that the upper limit of the $L$ is not more than 59.872 MeV. This constraint is also within the range from other analyses including that on the $L \leq 66$ MeV from the electric dipole polarizability as well as that on the $L \leq 70$ MeV from the charge radii of mirror nuclei.

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