Study on damage-crack criterion of bedding limestone based on ultimate tensile strain

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Abstract. The ultimate tensile strain can be used as the criterion to assess material damage. However, typical sedimentary rocks contain structural planes arranged in various directions, resulting in anisotropy of the tensile properties. Determining the ultimate tensile strain of rock with bedding of different dip angles is the fundamental problem. According to the limit analysis method, the endpoint of the elastic stage was taken as the critical state to judge splitting, and the ultimate tensile strain and directions of rock with different bedding angles were determined. Based on the Weibull distribution of the rock element strength and the Lemaitre principle of strain equivalence, the evolution equation of the damage variable was derived with increasing tensile strain. The tensile criterion and the calculation method of damage stress were established based on the ultimate tensile strain, and the numerical simulation program with the tensile damage was constructed in Fast Lagrangian Analysis of Continua (FLAC³D). Finally, this model was applied to stability analysis for excavated surrounding rock in an ideal circular tunnel. The damage evolution has a negative exponential relationship with the first principal strain. The plastic zone, displacement field, stress field, and damaged zone are all in accordance with the general law. The results can provide a basis for examining the tensile characteristics of a layered rock mass.

1. Introduction
A large number of laboratory tests, in-situ tests, and engineering practice show that the lateral deformation of a rock mass under the action of a vertical force is closely related to the accumulation and expansion of tensile cracks [1-2]. The judgment of rock fracture is based mainly on two criteria. The first criterion is about the tensile stress criterion. Griffith [3] suggested that there were many micro-cracks in a material, and the stress concentration can be generated around and at the end of these micro-cracks under the action of external forces. The destruction of the material usually started from the seam end and expanded gradually, eventually led to the destruction of the material. Lu et al. [4] discussed the fracture types of non-metallic materials, the fracture mechanism, and the discriminants of three types of cracking based on the application of Griffith strength theory of non-metallic materials. Peng et al. [5] obtained the critical stress formula of rockburst in the excavation of underground caverns in terms of the tensile strength of the rock using the Griffith strength theory. The compressive strength to tensile strength ratio can be used to describe the intensity of rockburst. Liu et al. [6] analyzed the meso-crack initiation mechanism of rock caused by the local maximum tensile stress at the defect end of rock in a compressive stress field. They proposed an empirical criterion for crack initiation prediction. Zhang et al. [7] supplemented and modified the tensile stress crack growth criterion for brittle materials, which can determine the step size of crack growth.
The second criterion is the ultimate tensile strain. Zhao et al. [8] suggested that the magnitude of the ultimate tensile strain of a rock material can be determined by the ratio of uniaxial tensile strength $R_t$ and elastic modulus $E$. Moreover, the surrounding rock was considered to enter the loose zone when the tensile strain of the surrounding rock exceeded its ultimate tensile strain. Zheng et al. [9-11] used a numerical limit analysis method to determine the ultimate strain of geotechnical materials under unidirectional stress. Currently, most failure criteria refer to the stress criterion when it comes to tensile fracture. The stress is renewed during the elastic-plastic calculation, usually through the plastic flow rule. This is not the actual stress but accumulated strain. Therefore, using the ultimate tensile strain criterion to judge the extent of damage has an obvious advantage. In addition, the process from loading to bearing the capacity loss of rock material is a step-by-step process. Hence, most yield criteria cannot reflect the process from local failure to the overall failure of the material. Accordingly, the “ultimate tensile strain” is proposed as the starting point to judge the initial damage of a rock mass, and the material mechanical parameters deteriorate gradually, which reflects an evolutionary process from local damage to global failure.

2. Determination of ultimate tensile strain of limestone samples with different bedding angles

2.1. Geometric model in numerical Experiment
The specimen was a Brazilian split disc, 50 mm × 25 mm in diameter and height, respectively. Figure 1 presents the loading mode; the axial loading rate was $1.0 \times 10^{-7}$ m/s.

![Figure 1. Numerical model and monitoring](image)

(a) Loading mode  (b) Numerical model  (c) Layout of the monitoring points

2.2. Constitutive model and parameters
As an extension of the Mohr-Coulomb model, the ubiquitous-joint model can establish the independent yield criteria for rock blocks and discontinuous planes, respectively, and consider the influence of the dominant structures on the mechanical mechanism and failure of rock masses. Limestone is generally distributed in layers (figure 1(a)), and the ubiquitous-joint model is the best choice to establish a Brazilian splitting model for bedding limestone with an implied mesh (figure 1(b)). Table 1 lists the required parameters of the model.

| Material          | Bedding angles $\theta$ /° | Elastic Modulus $E$ / GPa | Poisson's ratio $\mu$ | Unit weight $\gamma$ /kN·m³ | Friction angle $\phi$ /° | Cohesive strength $c$ /MPa | Tensile strength $R_t$ /MPa |
|-------------------|---------------------------|---------------------------|-----------------------|-----------------------------|--------------------------|---------------------------|-----------------------------|
| Limestone block   | 0                         | 4.33                      | 0.208                 | 26.0                        | 42.0                     | 15.0                      | 8.22                        |
|                   | 15                        | 3.93                      | 0.214                 |                             |                          |                           |                             |
|                   | 30                        | 3.08                      | 0.233                 |                             |                          |                           |                             |
|                   | 45                        | 2.99                      | 0.245                 |                             |                          |                           |                             |
|                   | 60                        | 2.56                      | 0.251                 |                             |                          |                           |                             |
|                   | 75                        | 2.54                      | 0.256                 |                             |                          |                           |                             |
|                   | 90                        | 2.50                      | 0.259                 |                             |                          |                           |                             |
| Bedding plane     | ——                        | ——                        | ——                    | ——                          | 30.0                     | 5.0                       | 3.99                        |
2.3. Results and analysis

2.3.1. Setting of the monitoring points. According to Zheng’s point of view [10], the displacement convergence criterion of the critical points was adopted to monitor the ultimate strain of stratified limestone at various angles in the ultimate state. In this simulation, there were 17 monitoring points, as shown in Figure 1(c).

2.3.2. P-ε1 curves. Brazilian splitting simulation tests were carried out for seven groups of rock samples with different bedding angles (0°, 15°, 30°, 45°, 60°, 75°, and 90°). The first principal strain ε1 was calculated by calling the self-defined FISH function, and the load P and ε1 were recorded in each step. The P-ε1 curves at the 17 monitoring points of each group were plotted, as shown in Figure 2. Each group of curves was quite different, but the three stages of elasticity, plasticity, and failure were generally observed. During the initial loading, the deformation curves showed an approximately linear upward trend, exhibiting elastic characteristics with increasing P. The samples were then partially damaged to enter the plastic state with a section of the curve with a higher slope. Finally, failure occurred when the samples were in the ultimate state. The endpoint of the elastic stage was defined as the proportional limit, which is also the starting point of damage. The first principal strain corresponding to this point is called the ultimate tensile strain. The ultimate tensile strain at different monitoring points of each group of samples was relatively constant, but the corresponding load was different when damage occurred. Each sample with different bedding angles had different initial damage locations and different ultimate tensile strains, suggesting that the bedding angles played a controlling role in the mechanical properties of the layered rock mass.

Taking the sample with β=0° as an example, when the loading of stratified limestone started from 60% of the failure load, damage plastic deformation occurred at monitoring points 1, 5, 2, and 8, successively. With the continuous increase in load, the subsequent monitoring points gradually suffered damage and failure. The corresponding ultimate tensile strain at these points was very close, approximately 0.145%, when damage occurred. The disk was destroyed when the first principal strain at each point in the disk exceeded 0.145%, reflecting the process of rock samples gradually developing from local damage to overall failure under the action of an external load.
2.3.3. Distribution of $\varepsilon_1$. Figure 3 shows the distribution of the first principal strain of rock samples with different inclination angles under the critical failure state. The maximum first principal strain at this time is called the failure tensile strain. When the bedding angle was 30° and 45°, the failure tensile strain was 0.03%, which was distributed mainly in the upper and lower parts of the center of the disc. When $\beta=90^\circ$, the failure tensile strain was 0.35%, which was distributed mainly in the center of the disc. With increasing bedding angle, the failure tensile strain gradually moved from the top and bottom to the center of the disk.

![Figure 3. Distribution of $\varepsilon_1$ in the critical failure state](image)

**Table 2.** Ultimate and failure tensile strains of the samples with different bedding angles

| Bedding angle | 0°  | 15° | 30° | 45° | 60° | 75° | 90° |
|---------------|-----|-----|-----|-----|-----|-----|-----|
| Ultimate tensile strain / % | 0.145 | 0.115 | 0.120 | 0.082 | 0.050 | 0.143 | 0.182 |
| Failure tensile strain / % | 0.213 | 0.165 | 0.162 | 0.125 | 0.078 | 0.172 | 0.240 |
| Position | 13 | 2, 10 | 3, 6, 11 | 2, 7, 10 | 3, 10 | 3, 10 | 6, 7, 13 |
| Direction$^a$ | 0° | -15° | -30° | -45° | -60° | -75° | 0° |

$^a$ Direction refers to the angle from the first principal strain to the horizontal plane: positive in the clockwise direction and negative in the counterclockwise direction.
Table 2 lists the magnitude, occurrence point, and direction of the ultimate tensile strain and failure tensile strain of rock samples with different bedding angles. The first principal strain decreased initially and then increased with increasing angle. The first principal strain was anisotropic in the process of disc splitting.

3. Crack criterion based on ultimate tensile strain

3.1. Expression of the crack criterion

The tensile fracture of rock is caused by the actual strain of rock exceeding the ultimate tensile strain [12]. The expression of crack criterion is

\[ f' = \varepsilon_1 - [\varepsilon] \] (1)

where, \( \varepsilon_1 \) is the first principal tensile strain, and \([\varepsilon]\) is the ultimate tensile strain.

When \( f' > 0 \), the element is in the state of damage cracking. In this case, the damage variable, \( D \), should be calculated; otherwise, \( D = 0 \).

3.2. Crack damage evolution

On the micro-level, the damage manifests as the formation and expansion of the micro-cracks in the rock mass. On the macro level, the damage manifests as the gradual deterioration and complete loss of the mechanical parameters of the rock mass. The fracture of layered rock mass begins with the expansion of internal micro-cracks, and the existence of micro-cracks greatly weakens the tensile properties of the layered rock mass.

The probability density of element failure of the rock damage variable is expressed as follows [13]:

\[ \frac{dD}{d\varepsilon} = \phi(\varepsilon) \] (2)

The integration of Equation (2) results in

\[ D = \int_0^\varepsilon \phi(\varepsilon) d\varepsilon = 1 - \exp\left(-\frac{1}{m} B \left(\frac{\varepsilon}{\varepsilon_c}\right)^m\right) \] (3)

where \( B \) is the parameter related to the size of the rock sample and material deformation characteristics; \( E \) is the tensile modulus of the rock; \( m \) can be calculated from the uniaxial tensile test results as follows [14]:

\[ m = \frac{1}{\ln(E\varepsilon_c / \sigma_c)} \] (4)

Table 3. Fitting parameters of the rock samples

| \( \beta \) | \( E_c(\%) \) | \( B \) | \( m \) |
|-----------|-------------|-------|-------|
| 0°        | 0.213       | 80.61 | 5.27  |
| 15°       | 0.165       | 69.32 | 5.87  |
| 30°       | 0.162       | 58.52 | 7.79  |
| 45°       | 0.125       | 50.23 | 7.25  |
| 60°       | 0.078       | 37.52 | 7.32  |
| 75°       | 0.172       | 28.53 | 8.48  |
The change rule of the damage variable, $D$, with increasing cumulative strain, $\varepsilon$, can be obtained, as shown in Figure 4. The damage variables of each group of specimens showed an increasing trend with increasing cumulative strain. The growth rate begins to decrease and continues to accelerate. The rate then tends to be stable after increasing to a certain value and finally slows down until the damage value $D\approx 1.0$.

![Figure 4. Curves between the damage variable-strain](image)

### 3.3. Elastoplastic damage constitutive equation

The relationship between the damage stress status and the stress status calculated using the elastoplastic model may be expressed as follows [15]:

$$\sigma_{ij}^d = (1-D)\sigma_{ij} + \frac{D}{3}\sigma_{kk}\delta_{ij}$$

(5)

where $\sigma_{ij}$ is the stress component suitable for the elastic-plastic constitutive model. The value of $\delta_{ij}$ depends on the following expression:

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

(6)

### 4. Model validation

The ideal circular tunnel calculation model in the literature was adopted [16]. The excavation radius was 1.0 m. The initial in-situ stress was 30MPa in all three directions, and the physical and mechanical parameters of rock mass were $K=3.9$GPa, $G=2.8$GPa, $C=3.45$MPa, and $\phi=30^\circ$. Figure 5 shows the calculation model.

![Figure 5. 1/4 model and boundary condition](image)

Figures 6 and 7 show the circumferential stress, radial stress, and radial displacement of the rock around the hole at different positions along the radius direction, which are similar to the conventional calculation results.
Figure 6. Distribution of stress

Figure 7. Distribution of displacement

Figure 8 shows the distribution of the rock damage coefficient around the ideal round hole after excavation according to the definition of the damage coefficient. The maximum and minimum damage coefficients were 0.0418 and 0.00134, respectively, which decreased gradually from the excavation face to the deep surrounding rock, and has a certain corresponding relationship with the distribution of the surrounding rock failure zone, as shown in Figure 9. In the tensile fracture zone, the damage was most serious. In contrast, in the elastic non-failure zone, the damage value was zero, and the intermediate transition zone showed a gradient distribution. After excavation, the stress distribution was more uniform, and the damage was not severe because the research object was an ideal round tunnel and the initial in-situ stress was axisymmetric. Therefore, the results considering the stress and displacement after damage are similar to those not considering the damage characteristics.

Figure 9. Failure zones

5. Conclusion

(1) During the elastic-plastic simulation calculation, the stress is renewed usually through the plastic flow rule, not the actual stress, but strain accumulates. Therefore, the ultimate tensile strain criterion to judge rock damage has a more obvious advantage than the tensile stress criterion.

(2) The Brazilian splitting simulation tests were carried out for seven groups of rock samples with different bedding angles. The ultimate tensile strain at different monitoring points of each group of samples is a relatively constant value. Each sample with different bedding angles had different initial damage locations and different ultimate tensile strains, suggesting that the bedding angles play a controlling role in the mechanical properties of a layered rock mass.

(3) When the first principal strain exceeds the ultimate tensile strain, the rock mass is subjected to tensile crack damage. The damage evolution has a negative exponential relationship with the first principal strain. The new stress should be calculated according to the damage elasto-plastic model. The model is applied to calculate the stability of the surrounding rock of an ideal circular tunnel, and the stress distribution, displacement distribution, damage zone, and plastic zone obtained are all in accordance with the general law.

References

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Acknowledgments
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