Can the Higgs be seen in rapidity gap events at the Tevatron or the LHC?

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Abstract

Double diffractive Higgs production at $pp$ (or $\bar{p}p$) colliders continues to attract attention as a potential signal in the search for the boson. We present improved perturbative QCD estimates of the event rates for both the exclusive and inclusive double diffractive Higgs processes, paying particular attention to the survival probability of the rapidity gaps. We find that the major uncertainty is in the prediction for the survival probability associated with soft rescattering. We show that an analogous process, the double diffractive production of a pair of jets with large values of $E_T$, has an event rate which makes it accessible at the Tevatron. Observation of this process can therefore be used as a luminosity monitor for two-gluon exchange processes, such as the production of a Higgs boson with rapidity gaps on either side.

1 Introduction

A central problem in particle physics is to find a good signal with which to identify the Higgs boson at the present and forthcoming hadron colliders, the Tevatron and the LHC. This has become more important since it appears likely that the Higgs boson will be beyond the reach of LEP2. One possibility which, at first sight, looks attractive is to select events with a large rapidity gap on either side, where the conventional background is relatively low \cite{1}–\cite{5}. From an experimental point of view the exclusive signal looks particularly promising

\begin{equation}
    p + p \to p + \text{gap} + H + \text{gap} + p,
\end{equation}
and similarly for $p\bar{p}$ events. For an exclusive process we have the possibility of good experimental resolution on the Higgs boson mass, $M_H$, whereas in an inelastic collision the event rate is higher but the large multiplicity of secondary particles poses an additional problem in identifying the Higgs boson. The main question is whether the production rate of Higgs events with rapidity gaps is large enough.

The cross section for double diffractive rapidity gap Higgs production can be estimated using perturbative QCD but unfortunately it is found\cite{6} that it is strongly suppressed by rescattering and QCD radiative effects. Despite this, very optimistic predictions of the exclusive event rate persist, and are frequently cited in experimental proposals. The purpose of this paper is to improve the reliability of the perturbative QCD predictions by going beyond double log accuracy so as to give believable estimates of the event rate and to settle the present ambiguities. We will see that this improvement will lead to some enhancement of the event rate as compared to our previous estimates\cite{6, 7}. Moreover, since the basic QCD mechanism for Higgs production is the same as that for the double diffractive central production of a pair of large $E_T$ jets, we also estimate the event rate for the latter process (which has a larger cross section) so that it may be used as a pomeron-pomeron luminosity monitor for rapidity gap Higgs production. Indeed such dijet data are already accessible at the Tevatron, see, for example,\cite{8} and references therein. Thus we have a valuable check on the QCD predictions for process (1).

As mentioned above, the cross section for Higgs production via the exclusive process (1) is suppressed by the small survival probability of the rapidity gaps. The survival probability $w$ is given by the product of two factors\footnote{We do not discuss the multiple (or “pile-up”) interactions of high luminosities.}

$$w = S_{\text{spec}}^2 T^2. \quad (2)$$

First, the gaps may be filled by soft parton rescattering and, second, by QCD bremsstrahlung from the two fast coloured gluons which annihilate into the Higgs boson, see Fig. 1. The probability $S_{\text{spec}}^2$ not to have any extra soft rescattering was estimated in\cite{2, 3, 9, 10} to be about $S_{\text{spec}}^2 \approx 0.1$, up to a factor of 2. This suppression agrees with the simple phenomenological estimate\cite{11}

$$S_{\text{spec}}^2 = \langle e^{-\Omega(\rho_T)} \rangle \simeq \left(1 - \frac{2\sigma^D}{\sigma_{\text{tot}}} \right)^2, \quad (3)$$

where $\Omega$ is the opacity (or optical density) of the proton and $\langle \ldots \rangle$ indicates the appropriate average over the impact parameter $\rho_T$; $\sigma^D$ is the sum of the elastic and diffractive dissociation cross sections and $\sigma_{\text{tot}}$ is the $pp$ (or $p\bar{p}$) total cross section. We will return to discuss the determination of $S_{\text{spec}}^2$ in detail in Section 5.

The factor $T^2$ in (2) is the probability not to radiate gluons in the hard subprocess $gg \rightarrow H$, and is incorporated in the perturbative QCD calculation of the exclusive amplitude. To suppress the QCD radiation we have to screen the colour of the annihilating gluons by an additional $t$-channel gluon, as in Fig. 1. The most optimistic scenario is to assume that this gluon, which
screens the colour, does not couple to the Higgs and that it has small virtuality $Q_T^2$ to enhance the probability of screening via a large value of $\alpha_S$. However there is a probability of relatively hard gluon emission coming from distance scales $\lambda \gtrsim 1/M_H$, but shorter than the characteristic transverse size ($\sim 1/Q_T$) at which the colour flow is screened. Such gluons fill up the rapidity gaps. It is found \cite{6} that the typical values of $Q_T$ of the screening ‘soft’ gluon are indeed much smaller than $M_H$, but yet are sufficiently large for perturbative QCD to be applicable.

Since there has been much recent interest in the double diffractive Higgs signal, both for experiments at the Tevatron and the LHC (see, for example, \cite{12}), it is timely to reassess the estimate of the event rate.

\section{Double diffractive exclusive Higgs production}

In Ref. \cite{6} the production amplitude for the exclusive process \cite{11}, derived from perturbative QCD, was given by

$$
M(pp \to p + H + p) = A \pi^3 \int \frac{dQ_T^2}{Q_T^4} e^{-S(Q_T^2, M_H^2)} f(x_1, Q_T^2) f(x_2, Q_T^2),
$$

(4)

where $f(x, Q_T^2)$ is the unintegrated gluon density of the proton and $A$ is a factor associated with the $gg \to H$ vertex$^2$

$$
A = (\sqrt{2} G_F)^{\frac{1}{2}} \alpha_S(M_H^2)/3\pi.
$$

(5)

The exponential is the conventional double log Sudakov form factor which is the probability not to emit bremsstrahlung gluons (one of which is shown in Fig. 1a by $p_T$) in the interval $Q_T \lesssim p_T \lesssim M_H/2$. The upper bound of $p_T$ is clear, and the lower bound occurs because there is destructive interference of the amplitude in which the bremsstrahlung gluon is emitted from a “hard” gluon $k_i$ with that in which it is emitted from the screening gluon. That is there is no emission when $\lambda \simeq 1/p_T$ is larger than the separation $d \sim 1/Q_T$ of the two $t$-channel gluons in the transverse plane, since then they act effectively as a colour-singlet system. So the Sudakov form factor (that is the probability not to have bremsstrahlung) is given by the Poisson distribution $\exp(-S)$, where the mean multiplicity of bremsstrahlung is

$$
S(Q_T^2, M_H^2) = \int_{Q_T^2}^{M_H^2/4} \frac{C_A \alpha_S(p_T^2)}{\pi} \frac{dE}{E} \frac{dp_T^2}{p_T^2}.
$$

(6)

Here $E$ and $p_T$ are the energy and transverse momentum of an emitted gluon in the Higgs rest frame.

Note that the amplitude \cite{11} for the exclusive process is written for forward outgoing protons, that is for a Higgs boson produced with small transverse momentum $q_T$. Indeed, the presence of the proton form factors suppresses large $q_T$ production, and the exclusive cross section $d\sigma/dy_H|_0$

$^2$We assume that $M_H$ is below $m_t$ and hence is far from the $t\bar{t}$ threshold.
is calculated assuming form factors $\exp(bt_i/2)$ at the proton vertices, with $b = 5.5$ GeV$^{-2}$. Here the notation means that the cross section is to be evaluated for Higgs rapidity $y_H = 0$.

Eq. (4) is the perturbative QCD estimate of the exclusive amplitude to double log accuracy. We are now able to improve the prediction by (i) including the effect of using skewed or off-diagonal gluon distributions and (ii) using a more precise definition of the unintegrated gluon distribution. With these improvements the amplitude (4) may be rewritten, to single log accuracy

$$\mathcal{M}(pp \to p + H + p) = A \pi^3 \int \frac{dQ_T^2}{Q_T^4} f_g(x_1, x'_1, Q_T^2, M_H^2/4) f_g(x_2, x'_2, Q_T^2, M_H^2/4)$$

(7)

where $f_g(x, x', Q_T^2, M_H^2/4)$ denotes the skewed or off-diagonal unintegrated gluon density in the initial proton. The diagonal density is defined such that the probability to find a gluon (with transverse momentum $Q_T$ and momentum fraction $x$ in the interval $dQ_T^2dx$) is $f_g(dQ_T^2/Q_T^2)(dx/x)$. These unintegrated distributions are the quantities which enter when we apply the $Q_T$-factorization theorem [13] to the evaluation of the Feynman diagram of Fig. 1a. The procedure of how to calculate $f_g(x, x, Q_T^2, \mu^2)$ from the conventional integrated gluon $g(x, Q_T^2)$ is described in Ref. [14]. Here we will use the form proposed by DDT [15].

$$f_g(x, x, Q_T^2, \mu^2) = \frac{\partial}{\partial \ln Q_T^2} \left[ T(Q_T, \mu) xg(x, Q_T^2) \right],$$

(8)

where $T(Q_T, \mu)$ is the survival probability that the gluon with $x, x' = x$ and transverse momentum $Q_T$ remains untouched in the evolution up to the hard scale $\mu (= M_H/2)$. $T$ is the result of resumming the virtual ($\propto \delta(1-z)$) contributions in the DGLAP evolution equation and is given by [14]

$$T(Q_T, \mu) = \exp \left( - \int_{Q_T^2}^{\mu^2} \frac{\alpha_s(k_i^2)}{2\pi} \frac{dk_i^2}{k_i^2} \int \left[ zP_{gg}(z) + \sum_q P_{qg} \right] dz \right).$$

(9)

The derivative $\partial T/\partial \ln Q_T^2$ in (8) cancels the virtual DGLAP term in $\partial (xg)/\partial \ln Q_T^2$. To be precise the equation for $f_g$ is a little more complicated than (8) (see eq. (3) of [14]). However in the relevant small $x$ and $Q_T \ll M_H$ region, (8) is sufficiently accurate for our purposes. Note that after integrating (8) up to scale $\mu$ we do indeed get back the integrated gluon distribution

$$\int_0^{\mu^2} f_g(x, x, Q_T^2, \mu^2) \frac{dQ_T^2}{Q_T^2} = T(\mu, \mu) xg(x, \mu^2) = xg(x, \mu^2).$$

(10)

To double log accuracy we see from (4) that $T = \exp(-S)$ with $S$ given by (3). In fact we will work to single log accuracy and hence it follows from (4) that

$$T(Q_T, \mu) = \frac{\alpha_s(Q_T^2)}{\alpha_s(\mu^2)} e^{-S}. $$

(11)

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The single log accuracy of (4) may be established using $Q_T$-factorization [13]. The crucial point is that in a physical gauge (for example, the planar gauge $A^{\mu}_n n_\mu = 0$ with the gauge vector $n_\mu$ chosen as the longitudinal component of the Higgs 4-momentum) any additional gluon which embraces the Higgs boson (that is which connects the upper and lower parts of the diagram in Fig. 1a) gives neither a DGLAP or BFKL collinear logarithm.
We comment on this result in Section 7.

Now we must consider the skewed effect coming from the fact that the screening gluon carries a much smaller momentum fraction, that is \(x'_i \ll x_i\). As a consequence we have

\[
f_g(x,x',Q^2_T,M^2_H/4) = R_g \frac{\partial}{\partial \ln Q^2_T} \left[ \sqrt{T}(Q_T,M_H/2) x_g(x,Q^2_T) \right]
\]

where the \(\sqrt{T}\) arises because the survival probability is only relevant to the hard gluon. The multiplicative factor \(R_g\) is the ratio of the off-diagonal \(x'_i \ll x_i\) integrated distribution to the conventional diagonal one \(x_g(x,Q^2_T)\). In terms of the Operator Product Expansion both the diagonal and off-diagonal (or skewed) distributions are given by the expectation values of the same conformal operators \([17, 18, 19]\). It was shown \([20]\) that for \(x \ll 1\) the expectation values for the diagonal and skewed distributions are the same. Hence the skewed distribution is completely determined in terms of the conventional diagonal gluon. Indeed for \(x'_i \ll x_i \ll 1\) we have \([20]\)

\[
R_g = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma \left( \lambda + \frac{5}{2} \right)}{\Gamma(\lambda + 4)}
\]

where \(\lambda\) governs the small \(x\) behaviour of the diagonal gluon \(x_g(x,Q^2_T) \propto x^{-\lambda}\).

Note that with the \(\sqrt{T}\) in \((12)\) the amplitude \((4)\) contains the same Sudakov suppression factor, \(\exp(-S)\), as in \((4)\). However amplitude \((7)\) includes two improvements in comparison to the previous perturbative QCD form \((4)\). We now include the \(\partial T/\partial \ln Q^2_T\) contribution, see \((8)\), and also the single log part of the skewed effect, that is the factor \(R_g\) in \((12)\). Both improvements enhance the exclusive \(pp \to p + H + p\) event rate. This is particularly true at FNAL energies where the \(\partial T/\partial \ln Q^2_T\) dominates the \(\partial g/\partial \ln Q^2_T\) contribution in \((8)\). Moreover the enhancement due to \(R_g\) is non-negligible. In the relevant kinematic domain we have \(R_g \simeq 1.2(1.4)\) leading to an enhancement factor \(R^4_g \simeq 2(4)\) at LHC (Tevatron) energies.

The values of the double diffractive exclusive Higgs cross section at Tevatron and LHC energies that are obtained from \((7)\) are presented in Section 4. We emphasize that this perturbative QCD calculation is based on the unintegrated gluon distribution \(f_g\) obtained from \((12)\). It has been checked to be realistic in the sense that it gives reasonable cross sections for diffractive vector meson (\(\rho, J/\psi, \Upsilon\)) production \([21]\) and for large \(q_t\) prompt photon hadroproduction at the Tevatron energy \([14]\). We will see the importance of this comment in Section 7.

### 3 DOUBLE DIFFRACTIVE INCLUSIVE HIGGS PRODUCTION

The cross section for the inclusive process

\[
p + p \to X + \text{gap} + H + \text{gap} + Y
\]

\(^4\)It was shown explicitly in \([16]\) that when \(x' \ll x\) only the self-energy of the hard \(x\) gluon contributes to the survival probability to leading log accuracy. Note that the gluon with \(x' \simeq 0\) is almost “at rest” (that is \(Q_L \ll Q_T\)) and there is no possibility of QCD radiation.

\(^5\)Strictly speaking \([13]\) was only proved for integrated gluons \([23]\). However it is expected to hold equally well for the unintegrated distribution.
is much larger than for the exclusive process \( \Pi \), see Ref. [6]. Here the initial protons may be broken up and so the transverse momentum of the Higgs is no longer limited by the proton form factor, and hence the Sudakov suppression is weaker. Unfortunately we can no longer achieve single log accuracy. The momenta transferred, \( t_i = (Q - k_i)^2 \), are large and hence we cannot express the “blobs” in Fig. 1 in terms of the gluon density of the proton. At present, the corresponding skewed (large \( t_i \)) unintegrated gluon distribution is not known. So we must use the (BFKL-type) non-forward amplitude. On the other hand for inclusive production we allow emission in a larger part of the phase space and so the QCD suppression is weaker, and the more approximate (essentially the double log) expression should give a satisfactory estimate of the event rate.

Let us recall the main QCD formulae needed to estimate the inclusive cross section. The partonic quasi-elastic subprocess is \( ab \rightarrow a' + \text{gap} + H + \text{gap} + b' \). For example, for the subprocess \( gg \rightarrow g + H + g \) \( [6] \), the cross section is given by

\[
\frac{d\sigma}{dy_H} = A^2 \alpha_S^4 \frac{81}{29\pi} \int \frac{dQ^2}{Q^2} \frac{dQ'^2}{Q'^2} \frac{dt_1}{t_1} \frac{dt_2}{t_2} e^{-(n_1 + n_2 + n_3 + n_4 + S_1 + S_2 + S_3 + S_4)/2},
\]  

(15)

where the primes indicate quantities occurring in the complex conjugate amplitude to that shown in Fig. 1b. Now the suppression due to QCD radiative effects comes from the double log resummations \( \exp(-n_i/2) \) in the BFKL non-forward amplitudes, as well as from the Sudakov form factors \( \exp(-S(k_T^2, M_H^2)) \) which arise from the requirement that there is no gluon emission in the interval \( k_T < p_T < M_H/2 \). The leading logarithmic contribution again comes from the asymmetric configuration of the \( t \)-channel gluons, \( Q_T \ll k_{iT} \). The amplitude for no gluon emission with \( Q_T < p_T < k_{iT} \) in the gap \( \Delta \eta_i \) is \( \exp(-n_i/2) \) where

\[
n_i = \frac{3\alpha_S}{\pi} \Delta \eta_i \ln \left( \frac{k_{iT}^2}{Q_T^2} \right).
\]  

(16)

The inclusive cross section is obtained by convoluting the parton-parton cross sections with the parton densities. The results are presented in the next section.

### 4 Cross sections for exclusive and inclusive Higgs production

The predictions for the double diffractive Higgs production cross sections are presented in Table 1. The values \( \sigma_{\text{excl}} \) given for exclusive production are obtained from the most complete

\[6\] There is a second \( \ln(k_T^2/Q_T^2) \) arising from the BFKL evolution, which when resummed gives the BFKL non-forward amplitude \( \Phi(\Delta \eta) \exp(-n_i/2) \). Here \( \Delta \eta \) plays the role of \( \ln(1/x) \) in the BFKL evolution. For the energies and rapidity gaps of interest this BFKL enhancement is small, that is \( \Phi(\Delta \eta) \approx 1 \). Of course, a more precise calculation to single \( \ln Q_T \) accuracy may give a larger amplitude due to less QCD radiative suppression. On the other hand the NLO \( \ln(1/x) \) corrections decrease the forward \( (t_i = 0) \) BFKL amplitude. Thus we will still take \( \Phi(\Delta \eta) \approx 1 \).
perturbative QCD calculation that can be made at the present stage. They are based on (7) using the unintegrated gluon distribution of (12). That is, the distributions are calculated to single log accuracy (as in (8) and (9)) and include the skewed effect \( R_g \) of (13). We also include the \( \alpha_s \) correction to the \( gg \to H \) vertex factor. That is \( A^2 \) given by (5) is multiplied by the regularized virtual correction \( [22] \), or so-called \( K \)-factor,

\[
A^2 \to A^2 \left( 1 + \frac{\alpha_s(M_H^2)}{\pi} \left[ \pi^2 + \frac{11}{2} \right] \right) \simeq 1.5 A^2. \tag{17}
\]

The predictions of the inclusive cross section \( \sigma_{\text{incl}} \) in Table 1 are obtained using the BFKL non-forward amplitude and are valid to double log accuracy. As mentioned above, the unintegrated gluon distributions are unknown at large momentum transfer \( t_i \), and so, at present, we cannot improve our estimates as we have done for the exclusive case. However the values of \( \sigma_{\text{incl}} \) do include the factor of (17).

Finally all the cross sections in Table 1 include the survival probability \( S_{\text{spec}}^2 = 0.1 \) arising from soft rescattering effects. As we shall see in the next section, the uncertainty in the value of \( S_{\text{spec}}^2 \) gives by far the largest uncertainty in the predicted cross sections.

Table 1: The cross sections \( \sigma = d\sigma/dy|_{0} \) (in fb) for the central production of a Higgs boson in \( p\bar{p} \) (or \( pp \)) collisions at \( \sqrt{s} = 2 \) and 14 TeV via the exclusive or inclusive process of Fig. 1 and via \( WW \) fusion. The inclusive cross sections are shown for (parton level) rapidity gaps \( \Delta \eta = 2 \), and also for \( \Delta \eta = 3 \). The tabulated cross sections are obtained using \( S_{\text{spec}}^2 = 0.1 \), but see the discussion in Section 5.

| \( M_H \) (GeV) | \( \sigma_{\text{excl}} \) | \( \sigma_{\text{incl}} \) | \( \sigma_{\text{incl}}(WW) \) |
|----------------|----------------|----------------|----------------|
|                | \( \Delta \eta = 2 \) | \( \Delta \eta = 2 \) | \( \Delta \eta = 2 \) |
| Tevatron \((\sqrt{s} = 2 \text{ TeV})\) | \( 100 \) | 0.071 | 1.1 (0.09) | 0.49 (0.031) |
|                  | \( 120 \) | 0.030 | 0.62 (0.05) | 0.41 (0.026) |
|                  | \( 140 \) | 0.018 | 0.38 (0.03) | 0.35 (0.022) |
|                  | \( 160 \) | 0.008 | 0.25 (0.02) | 0.30 (0.019) |
| LHC \((\sqrt{s} = 14 \text{ TeV})\)  | \( 100 \) | 2.4 | 49 (5.5) | 21.6 (5.6) |
|                  | \( 120 \) | 1.4 | 36 (3.9) | 20.6 (5.4) |
|                  | \( 140 \) | 0.86 | 28 (2.9) | 19.7 (5.2) |
|                  | \( 160 \) | 0.55 | 21 (2.3) | 18.8 (5.0) |

We note that the values of \( \sigma_{\text{excl}} \) are enhanced in comparison to our previous predictions \( [3] \), which were based on the low \( x \) limit for the unintegrated gluon density. That is we took

\[
f = \frac{\partial(xg(x,Q_T^2))}{\partial \ln Q_T^2}, \tag{18}
\]
assuming that \( \ln(1/x) \gg \ln(M_H/2Q_T) \). The predictions in Table 1, however, are based on the improved expression (12) for the skewed unintegrated gluon density. For example at LHC energies both logarithms \( r_x \equiv \ln(1/x) \simeq 4.5 \) and \( r_T \equiv \ln(M_H/2Q_T) \simeq 3.5 \) are of comparable size and the derivative of \( \sqrt{T} \) in (12) implies a correction of about \( 1 + r_T/r_x \simeq 1.8 \) to (18), and an enhancement of the cross section \( d\sigma_{\text{excl}} \propto f_g^4 \) by a factor \( (1.8)^4 \simeq 11 \). Next we have an enhancement due to the skewed effect, \( R_g \) in (12). At first sight we might expect that the off-diagonal gluon distribution, \( f_g(x,x') \simeq \sqrt{xg(x)x'g(x')} \), would be much larger than the diagonal density, \( f_g(x,x) \), since \( x' \ll x \) and \( x'g(x') \) grows rapidly as \( x' \to 0 \). However it was shown [23] that in the leading \( \ln(1/x) \) limit

\[
f_g(x,x') = f_g(x,x),
\]

Nevertheless beyond leading \( \ln(1/x) \) the ratio is found to be \( R_g \simeq 1.2(1.4) \) at LHC (Tevatron) energies, leading to an enhancement \( R_g^4 \simeq 2(4) \), which is included in \( \sigma_{\text{excl}} \) in Table 1. The third improvement is the inclusion of the single logarithmic contribution in (9) for the survival probability \( T \). These contributions allow for the kinematic constraints on gluon emission and enlarge the value of \( T \). (It is well known that the double log expression of the type of that was used in [3], overestimates the suppression.)

Table 2 shows how the QCD prediction for the exclusive cross section \( \sigma_{\text{excl}} \) would be reduced if we omit the various improvements one-by-one. \( \sigma_{\text{excl}} \) becomes \( \sigma_1 \) if we switch off the single log contribution to \( T \) in (8) and return to the double log formula of [3]. If we then omit the \( \partial T/\partial \ln Q_T^2 \) term in (8, 12) for \( f_g \) the cross section reduces to \( \sigma_2 \), and finally if we omit the skewed effect factor \( R_g \) we obtain \( \sigma_3 \).

Table 2: The reduction in the cross section \( \sigma_{\text{excl}} \to \sigma_1 \to \sigma_2 \to \sigma_3 \) due to the omission of the various QCD improvements one-by-one, as detailed in the text. \( \sigma \equiv d\sigma/dy|_0 \) in fb.

| \( M_H \) (GeV) | Tevatron (\( \sqrt{s} = 2 \) TeV) | LHC (\( \sqrt{s} = 14 \) TeV) |
|-----------------|----------------|----------------|
|                 | 120  | 160 | 120  | 160 |
| \( \sigma_{\text{excl}} \) | 0.030 | 0.008 | 1.4 | 0.55 |
| \( \sigma_1 \) | 0.012 | 0.003 | 0.56 | 0.21 |
| \( \sigma_2 \) | \( 0.8 \times 10^{-4} \) | \( 0.8 \times 10^{-5} \) | 0.03 | 0.007 |
| \( \sigma_3 \) | \( 0.2 \times 10^{-4} \) | \( 0.2 \times 10^{-5} \) | 0.012 | 0.003 |

We should discuss the \( Q_T^2 \) integration which is necessary to compute the exclusive amplitude (7). We take the lower limit to be \( Q^2_0 = 1.25 \) GeV\(^2 \), since this is the starting value of the MRS [24] partons that we use. The saddle points of the \( d\ln Q_T^2 \) integration are at about \( Q_{SP}^2 = 3.2 \) GeV\(^2 \) and 1.5 GeV\(^2 \) for the LHC and Tevatron energies respectively.
The prediction for the rapidity distribution for double diffractive exclusive Higgs production is shown in Fig. 2. After integration over rapidity we obtain\[ \sigma_{\text{excl}} = 5.7 \text{ fb} \] for \( M_H = 120 \text{ GeV} \) at \( \sqrt{s} = 14 \text{ TeV} \).

We see from Table 1 that the values of the cross section for the inclusive process (14) are much larger than the exclusive cross section. Recall that the reasons are that (i) the QCD suppression is not so strong and (ii) the Higgs boson may be produced with a larger transverse momentum. However we see that for a large rapidity gap, \( \Delta \eta = 3 \), the inclusive rate falls to a value comparable to that for the exclusive process.

Up to now we have dealt with a purely perturbative QCD expression. All the next-to-leading kinematically enhanced effects are under control and so we may hope that the higher order \( \alpha_S \) effects will only give a 20–40\% correction. Much more uncertainty comes from the soft non-perturbative effects. First there is the contribution coming from \( Q_T^2 < Q_0^2 = 1.25 \text{ GeV}^2 \). We can estimate it using GRV partons \[ 29 \] (where we may take \( Q_0 \simeq 0.6 \text{ GeV} \)) or by freezing the anomalous dimension of the MRS gluon for \( Q_T < Q_0 \), but still allowing \( \alpha_S \) to run in (9). In both cases the contributions are comparable and not too large at the LHC energy; \( \sigma_{\text{excl}} \) increases by \( \lesssim 20 \% \) for \( M_H = 120 - 160 \text{ GeV} \). Note that it is the inclusion of the Sudakov suppression effects which makes the perturbative QCD estimates infrared stable and hence the predictions reliable. On the other hand at the Tevatron energy the position of the saddle point is rather low (\( Q_{\text{SP}}^2 \simeq 1.5 \text{ GeV}^2 \)) and the contribution from \( Q_T < Q_0 \) enlarges the cross section by about a factor of 2.

The main uncertainty, however, does not come from any of the above effects, but arises from the survival probability of the rapidity gaps with respect to the soft rescattering effects. As we shall see below it appears that we have been too optimistic to use in Table 1 the canonical value \( S_{\text{spec}}^2 = 0.1 \) at LHC energies.

5 Suppression due to soft rescattering

Here we will show that the main uncertainty in the calculation of the double diffractive cross sections arises from the estimate of how much soft rescattering fills up the rapidity gaps; that is, in the estimate of the probability, \( S_{\text{spec}}^2 \), for soft rescattering not to occur.

To begin, we consider the effect of a single elastic rescattering, which is shown in Fig. 3a where the blob represents the whole \( pp \) (or \( p\bar{p} \)) elastic amplitude including the absorptive corrections. It is straightforward to show that this simple rescattering amplitude gives a survival probability

\[
S_{\text{spec}} = \left( 1 - \frac{\sigma_{\text{tot}}}{4\pi(B_{\text{el}} + 2b)} \right)
\]

\[ (21) \]

\[ ^7 \text{This value may be compared to the original estimate implied by Bialas and Landshoff [3] of } \sigma_{\text{excl}} \simeq 100 \text{ fb}, \text{ which should be multiplied by } 8S_{\text{spec}}^2 \simeq 1. \text{ The factor of 8 is necessary to allow for the identity of the gluons.} \]
where $B_{el}$ is the slope of the elastic differential cross section ($d\sigma/dt \sim \exp(B_{el}t)$), $b$ determines the $t$ dependence of exclusive Higgs production (via the proton form factors $\sim \exp(bt/2)$ in the amplitude) and $\sigma_{tot}$ is the $pp$ (or $p\bar{p}$) total cross section. For example at LHC energies, where we expect $\sigma_{tot} \approx 100 \text{ mb}$ and $B_{el} \approx 20 \text{ GeV}^{-2}$, (21) gives

$$S_{\text{spec}} \approx 0.65,$$

if we take $b = 5.5 \text{ GeV}^{-2}$ as before.

To allow for dissociation in the rescattering process (shown by the heavier intermediate states in Fig. 3b) we multiply $\sigma_{tot}$ by a factor $C^2 > 1$, where

$$C = 1 + \frac{\sigma(\text{target dissociation})}{\sigma_{el}}.$$  \hspace{1cm} (23)

Here $\sigma_{el}$ is the elastic $pp$ cross section. Note that, for the above example, if $C$ becomes greater than 1.25, then we would have already obtained a negative value for $S_{\text{spec}}$. This is a warning that we need to be careful in the precise values that we assume for $\sigma_{tot}$, $B_{el}$ and $C$ at LHC energies. Alternatively, we can sum up the effect of multiple rescattering using a model which embodies unitarity and therefore has $S_{\text{spec}} > 0$ built in.

Let us consider the eikonal model sketched in Fig. 3c. In this model we have

$$\sigma_{tot} = \frac{2}{C^2} \int d^2 \rho_T \left(1 - e^{-\Omega(\rho_T)/2}\right),$$  \hspace{1cm} (24)

$$\sigma_{el} = \frac{1}{C^2} \int d^2 \rho_T \left(1 - e^{-\Omega(\rho_T)/2}\right)^2$$  \hspace{1cm} (25)

where the impact parameter $\rho_T$ is the transverse coordinate of the incoming proton with respect to the target proton, and $\Omega(\rho_T)$ is the optical density (or opacity) of the interaction. Here $\exp(-\Omega)$ reflects the absorption of the incoming beam, and $\exp(-\Omega/2)$ describes the reduction of the amplitude at impact parameter $\rho_T$. Thus

$$S_{\text{spec}} = \langle e^{-\Omega(\rho_T)/2} \rangle$$  \hspace{1cm} (26)

where the average is taken over the $\rho_T$ dependence, $\exp(-\rho_T^2/4b)$, of the amplitude for exclusive Higgs production

$$\mathcal{M} \propto e^{b(t_1+t_2)/2}.$$  \hspace{1cm} (27)

That is we have

$$S_{\text{spec}} = \int d^2 \rho_T e^{-\Omega(\rho_T)/2} e^{-\rho_T^2/4b} \frac{\int d^2 \rho_T e^{-\rho_T^2/4b}}{\int d^2 \rho_T e^{-\rho_T^2/4b}}.$$  \hspace{1cm} (28)

For the opacity we take the Gaussian form

$$\Omega(\rho_T) = \frac{C^2\sigma(s)}{2\pi B} e^{-\rho_T^2/2B},$$  \hspace{1cm} (29)
where \( \sigma(s) = \sigma_0(s/s_0)^\Delta \) corresponds to the Pomeron exchange amplitude shown by the double lines in Fig. 3c. We take the slope of the Pomeron amplitude to be

\[
\frac{B}{2} = B_0 + 2\alpha_P' \ln(s/s_0). \tag{30}
\]

We tune the parameters \((\sigma_0, B_0, \Delta, \alpha_P')\) of the eikonal model to describe the behaviour of \(\sigma_{\text{tot}}\) and the pp elastic scattering data throughout the ISR to Tevatron energy range \((30 < \sqrt{s} < 1800 \text{ GeV})\). Finally we study the predictions for \(S_{\text{spec}}\) for two relevant values of the enhancement parameter \(C\) of \((23)\), namely \(C = 1.15\) and \(C = 1.3\). The smaller value of \(C\) is obtained if we include only the nucleon resonance excitations \((27)\); in terms of partons it means that we account mainly for valence quark rescattering. On the other hand at the larger (LHC) energies we have to include rescattering of partons with small \(x\) (‘wee’ partons), and in this case \(C \approx 1.3\) is more appropriate. Both choices allow a satisfactory fit of \(\sigma_{\text{tot}}\) and the elastic data\(^8\), and the values of \(S_{\text{spec}}^2\) obtained from \((28)\) are shown in Table 3.

Table 3: The probability \(S_{\text{spec}}^2\), that the rapidity gaps survive rescattering, calculated using the eikonal model \((28)\) for two values of the enhancement factor \(C\) of \((23)\), namely \(C = 1.15\) and \(C = 1.3\) (expected to be appropriate for Tevatron and LHC energies respectively). The slope \(b\) for exclusive Higgs production, \((27)\), is expected to be 5.5 \(\text{GeV}^{-2}\), but smaller values are not excluded (see text).

| \(b\) \(\text{GeV}^{-2}\) | Tevatron \((\sqrt{s} = 2 \text{ TeV})\) | LHC \((\sqrt{s} = 14 \text{ TeV})\) |
|---|---|---|
| \(C = 1.15\) | \(C = 1.3\) | \(C = 1.15\) | \(C = 1.3\) |
| 5.5 | 0.11 | 0.034 | 0.054 | 0.011 |
| 4 | 0.07 | 0.013 | 0.029 | 0.003 |
| 2.5 | 0.04 | 0.003 | 0.012 | 0.0003 |

From the results shown in Table 3 we see that the survival probability \(S_{\text{spec}}^2\) depends sensitively on the value of the slope \(b\) (of \((27)\)), that is on the spatial \(\langle \rho_T \rangle\) distributions of partons inside the proton, see \((28)\). Unfortunately we cannot be completely sure that the value we have adopted, \(b = 5.5 \text{ GeV}^{-2}\), is correct. This value is obtained by assuming that the distribution of colour dipoles (gluons) mimicks the electric charge distribution of the proton. However in diffractive \(J/\psi\) electroproduction the slope is observed to be \(b_\psi \approx 4 \text{ GeV}^{-2}\) \((28)\). Since this process is also mediated by two-gluon exchange, another choice for \(b\) could be 4 \(\text{GeV}^{-2}\). Moreover the \(J/\psi\) slope is given by

\[
b_\psi = b_0 + 2\alpha_P' \ln(1/x_\psi), \tag{31}\]

where \(1/x_\psi \approx W^2/M_\psi^2\). Thus, since the \(J/\psi\) HERA data \((28)\) sample \(x \sim 10^{-3}\), whereas Higgs production at the LHC samples \(x \sim 10^{-2}\), it suggests that \(b\) is about \(2\alpha_P' \ln 10\) less than \(b_\psi\).

\(^8\)The parameter \(\Delta\), which specifies the Pomeron intercept, is found to be \(\Delta = 0.10\) and \(\Delta = 0.13\) in order to fit the “soft” data, taking \(C = 1.15\) and \(C = 1.3\) respectively.
For this reason we also show results in Table 3 for $b = 2.5$ GeV$^{-2}$, which if it were true would give much lower values of $S^2_{\text{spec}}$. However the evidence from HERA is conflicting. Large $Q^2$ open $q\bar{q}$ or $\rho$ meson electroproduction [28] are observed to have slope $b_\rho \approx 7$ GeV$^{-2}$, in the same region of $x$ as $J/\psi$. From this point of view the lower $J/\psi$-motivated values of $b$ look anomalous. So, on balance, the value $b = 5.5$ GeV$^{-2}$ looks to be the most realistic.

Taking $b = 5.5$ GeV$^{-2}$, we see from Table 3, that at the Tevatron we have $S^2_{\text{spec}} \approx 0.1$, which is the canonical value that we have used. However at the LHC the corresponding value is $S^2_{\text{spec}} \approx 0.05$. Even worse, for the more relevant choice of enhancement factor, $C = 1.3$ at LHC energies, we predict $S^2_{\text{spec}} \approx 0.01$.

We have used other models to estimate the survival probability $S^2_{\text{spec}}$ and, given the values of $C$ and $b$, we have found essentially the same results as in Table 3. The reason is interesting. At high energies the centre of the proton becomes black, that is $\Omega \gg 1$ and $\exp(-\Omega/2) \approx 0$ in (28). Hence the main contribution to $S_{\text{spec}}$ comes from the peripheral region $\rho_T \gtrsim \rho_0$, where $\rho_0$ is where the proton starts to become transparent, i.e. $\Omega(\rho_T \gtrsim \rho_0) \lesssim 1$. Thus, as long as the model describes $\sigma_{\text{tot}}$ and elastic $pp$ data, the prediction for $S_{\text{spec}}$ does not depend crucially on the details, but is controlled essentially by the proton radius (or $B_{\text{el}} \approx \sigma_{\text{tot}}^2/16\pi \sigma_{\text{el}}$) and the slope $b$.

Let us finally comment on $S^2_{\text{spec}}$ for a Higgs boson produced by $WW$ or $\gamma\gamma$ fusion. It is not excluded that the radius of the quark distributions in the proton is larger than that of the gluons. If this were the case, then $S^2_{\text{spec}}$ for $WW$ fusion would be larger than those shown in Table 3. The most exciting example is $\gamma\gamma \to H$ production. This process takes place at very large impact parameter $\rho_T$, and here $S_{\text{spec}} \simeq 1$. In Ref. [29] the $\gamma\gamma \to H$ cross section at the LHC was predicted to be 0.3 fb for $M_H \approx 120$ GeV. This would be close to our prediction of 0.6 fb for production by two-gluon exchange if we were to take a survival factor of $S^2_{\text{spec}} = 0.01$, instead of $S^2_{\text{spec}} = 0.1$. If indeed this is the case and, moreover, if we were to assume that $b < 5.5$ GeV$^{-2}$ is correct, then we would have the following hierarchy

$$1 \approx S^2_{\text{spec}}(\gamma\gamma \to H) \gg S^2_{\text{spec}}(WW \to H) \gg S^2_{\text{spec}}(IP/IP \to H),$$

where $IP/IP$ denotes the two-gluon exchange mechanism of Fig. 1. Of course for the default choice $b = 5.5$ GeV$^{-2}$ (i.e. assuming the spatial distributions of quarks and gluons to be the same) we have

$$S^2_{\text{spec}}(WW \to H) = S^2_{\text{spec}}(IP/IP \to H).$$

6 The dijet monitor

The previous section demonstrates that the “Achilles heel” of the calculation of the Higgs production cross sections is the uncertainty in the soft survival factor $S^2_{\text{spec}}$. Fortunately there
is a way to experimentally measure $S_{\text{spec}}^2$ by observing the double diffractive production of a pair of high $E_T(\sim M_H/2)$ jets with rapidity gaps on either side of the pair. The process is described by the same Feynman diagrams, both in the case of the exclusive process (11) and also for inclusive production (14). Essentially we need simply to replace the $gg \rightarrow H$ subprocess by that for $gg \rightarrow \text{dijet}$. The dijet event rate is much larger than that for the Higgs signal and so the collider experiments should be able to directly test the QCD estimates and measure $S_{\text{spec}}^2$.

QCD estimates of the rapidity gap dijet rate were given in [7]. In Table 4 we present improved numerical results in a kinematic range comparable to that for Higgs production. We use the same prescription that was employed to calculate the Higgs production cross sections presented in Table 1. That is for exclusive dijet production we integrate from $Q_T = Q_0$ to $Q_T = E_T$ over skewed unintegrated gluons (12), calculating the QCD radiative survival factor $T$ to single log accuracy. However the NLO $K$-factor for the subprocess $gg \rightarrow \text{dijet}$ is omitted. This correction depends on the “jet finding” algorithm. Usually the size of the jet cone is chosen in such a way that the effective NLO $K$-factor is close to 1. For comparison with Table 1 we continue to use the canonical soft rescattering factor $S_{\text{spec}}^2 = 0.1$, although we note from Section 5 the true factor may be smaller.

In practice it is impossible to study a purely exclusive dijet production process, analogous to (11). We cannot distinguish a bremsstrahlung gluon emitted in the dijet rapidity interval from a gluon which belongs to one of the high-$E_T$ jets. We have therefore chosen rapidity gaps such that bremsstrahlung is only forbidden for $|\eta_g| > 2$ in the dijet centre of mass frame. Of course the QCD radiative suppression will be much smaller in this case. For example, for semi-exclusive production of $E_T = 50$ GeV jets at LHC energies, we have a typical survival factor $T(Q_{SP}, M_H/2) \approx 0.5$ at the saddle point $Q_{SP}^2 \approx 2 \text{ GeV}^2$ of the $d \ln Q_T^2$ integration.

The double diffractive dijet cross sections are much larger than those for Higgs production. For example if we take a dijet bin of size $\delta E_T = 10$ GeV for each jet and $\delta(\eta_1 - \eta_2) = 1$ we estimate, for $E_T = 50$ GeV jets at LHC energies,

$$d\sigma_{\text{excl}}/d\eta|_0 \approx 38 \text{ pb}, \quad d\sigma_{\text{incl}}/d\eta|_0 \approx 240 \text{ pb} \quad (34)$$

where $\eta \equiv (\eta_1 + \eta_2)/2$, and the rapidity gaps are taken to be $\Delta\eta(\text{veto}) = (\eta_{\text{min}}, \eta_{\text{max}}) = (2, 4.1)$ for the inclusive case (see [7] for the definition of the dijet kinematics). For 30 GeV jets at the Tevatron the corresponding cross sections are

$$d\sigma_{\text{excl}}/d\eta|_0 \approx 17 \text{ pb}, \quad d\sigma_{\text{incl}}/d\eta|_0 \approx 150 \text{ pb}. \quad (35)$$

The numbers in square brackets in Table 4 correspond to double diffractive $b\bar{b}$ dijet production. For $M_H$ in the range that we consider, this process is the main background to the double diffractive Higgs signal. However the event rate of $b\bar{b}$ jets is more than two orders of magnitude lower than the gluon dijet rate. Even after integration over a $\delta E_T = 10$ GeV interval the rate is comparable to the Higgs cross section. We conclude the $b\bar{b}$ background should not be a problem.
We emphasize that the rapidity intervals chosen in our calculations refer to partonic rapidities. Table 4 shows that the cross sections depend sensitively on the size of the rapidity gaps, $\Delta \eta_{\text{veto}}$. Practical estimates require Monte Carlo simulations appropriate to the specific experimental cuts and which include treatment of possible initial state radiation and the hadronization of the large $E_T$ jets.

Table 4: The cross sections $\sigma = \frac{d\sigma}{dp_T^2d\eta d\Delta \eta}|_{\eta=0}$ (in fb/GeV$^2$) for the exclusive and inclusive double diffractive dijet production, for three values of the rapidity difference of the two jets $\Delta \eta \equiv \eta_1 - \eta_2$. The results are shown for different collider energies ($\sqrt{s}$) and different transverse energy ($E_T$) of the jets. The two jets are taken to have the same $E_T$. The rapidity gaps are defined by the intervals $\pm \Delta \eta_{\text{veto}} \equiv \pm (\eta_{\text{min}}, \eta_{\text{max}})$ [7]. The numbers in the square brackets are the $b\bar{b}$ component of the dijet signal.

| $\sqrt{s}$ | $E_T$ | $\Delta \eta$ | $\sigma_{\text{excl}}(jj)$ | $\sigma_{\text{incl}}(jj)$ | $\sigma_{\text{incl}}(b\bar{b})$ | $\Delta \eta_{\text{veto}} = (2, 4.1)$ | $\Delta \eta_{\text{veto}} = (1.5, 4.6)$ |
|----------|-------|-------------|----------------|----------------|----------------|----------------|----------------|
| 2 TeV    | 50 GeV| 0           | 0.97 [0.008]   | 5.2 [0.04]     | 0.28 [0.002]   |                 |                 |
|          |       | 1           | 0.76 [0.005]   | 4.4 [0.03]     | 0.23 [0.0015]  |                 |                 |
|          |       | 2           | 0.31 [0.001]   | 2.4 [0.01]     | 0.11 [0.0004]  |                 |                 |
| 2 TeV    | 30 GeV| 0           | 29 [0.23]      | 240 [1.9]      | 15 [0.12]      |                 |                 |
|          |       | 1           | 22 [0.13]      | 220 [1.5]      | 13 [0.09]      |                 |                 |
|          |       | 2           | 11 [0.03]      | 140 [0.5]      | 8 [0.3]        |                 |                 |
| 14 TeV   | 50 GeV| 0           | 38 [0.30]      | 240 [1.9]      | 24 [0.19]      |                 |                 |
|          |       | 1           | 31 [0.22]      | 240 [1.6]      | 23 [0.16]      |                 |                 |
|          |       | 2           | 19 [0.08]      | 200 [0.7]      | 18 [0.07]      |                 |                 |

7 Comparison with other QCD-based predictions

We have argued that perturbative QCD gives reliable estimates for double diffractive Higgs and dijet production, up to the uncertainty in the soft rescattering effects. It is therefore important to understand the origin of the difference with recent more optimistic estimates of the event rates.

Double diffractive high $E_T$ dijet production has been recently estimated by Berera [30]. His non-factorized N(L)DPE model is similar to our perturbative QCD approach. However there are some differences in application. First, in [30] a fixed coupling $\alpha_S(E_T^2)$ was used in the double log form of the Sudakov suppression factor, whereas here (and in Ref. [7]) we use the more appropriate running coupling $\alpha_S(p_T^2)$ inside the integration $Q_T^2 < p_T^2 < E_T^2$. Second, dijet production cannot be a pure exclusive process since it is impossible to forbid extra emission in the central rapidity interval occupied by the dijets. Thus we have, at best, a semi-exclusive
reaction in which the suppression\footnote{So, the Sudakov suppression appears to be much weaker for dijet production than for the analogous Higgs process. We can easily gain insight into the origin of this difference by recalling a similar phenomenon in the radiative effects accompanying the production of narrow and wide heavy resonances (see, for example, the discussion of well known QED effects in Ref. [31]). The energetic bremsstrahlung pushes the initial state off the resonance energy for the non-radiative process. Thus, the narrow resonance could be produced only if we damp radiation with energy exceeding the resonance width. The wider the resonance, the larger is the phase space available for emission and, therefore, the less pronounced is the Sudakov suppression.} is only relevant in some rapidity interval $\delta \eta_{\text{veto}}$, see Ref. [4]. When this is taken into account the predictions [30] are reduced by an amount that roughly compensates for the enhancement due to the use of $\alpha_S(E_T^2)$. The most important numerical difference between [30] and our predictions is due to treatment of the gluon exchanges. In Ref. [30] (and also in [25], as we discuss below) a non-perturbative two-gluon model is used in which the gluon propagator is modified so as to reproduce the total cross section. On the contrary, we have used a realistic unintegrated gluon density, determined from conventional gluons of global parton analyses, which has been found to give a consistent description of other processes described by perturbative QCD.

It was emphasized in [30] (see also [7]) that the above non-perturbative normalisation based on the value of the elastic (or total) cross section fixes the diagonal gluon density at $\hat{x} \sim \ell_T/\sqrt{s}$ where the transverse momentum $\ell_T$ is small, namely $\ell_T < 1 \text{ GeV} < Q_0$. Thus the value of $\hat{x}$ is even smaller than

$$x' \approx Q_T/\sqrt{s} \ll x \approx M_H/\sqrt{s}.$$  \hspace{1cm} (36)

However, the gluon density grows as $x \to 0$ and so it is clear that such a non-perturbative gluon normalisation will overestimate the double diffractive cross section.

We now turn to the very optimistic (“brave”) estimate for double diffractive exclusive Higgs production, process (1), that has recently been presented in [25]. For $M_H = 100 \text{ GeV}$ the prediction is $d\sigma/dy \approx 20 \text{ fb}$, even for Tevatron energies, see Fig. 2. We are unable to justify this estimate. First the radiative suppression $T^2 (\equiv S^2_{\text{par}}$ in [25]) $\simeq 0.1$ which is much larger than our determination. The phenomenological estimate of $T^2$ in [25] is based on the known hadron multiplicity $N_{\text{had}}$ measured in the rapidity interval $\Delta y = \ln(M_H^2/s_0)$ in a soft hadron-hadron collision. This multiplicity increases as $\ln(M_H^2/s_0)$ and has nothing to do with the double logarithmic bremsstrahlung, where the mean number of emitted gluons $S \propto \ln \left(\frac{M_H^2}{4\Lambda^2_{\text{QCD}}}\right)$. Next, motivated by the BLM prescription [32], the coupling $\alpha_S$ in the $gg \to H$ matrix element (that is in the analogue of (11)) is evaluated in [25] at a low scale $Q_0$ rather than $M_H$. However to apply the BLM procedure consistently we must ascertain which part of the gluon self-energy insertions are already included in the survival factor $T$ (or $S^2_{\text{par}}$), and which part should be attributed to $\alpha_S$. Indeed in calculating $T$ to single log accuracy we obtained the pre-exponential factor

$$\left[ \frac{\ln(Q_T^2/\Lambda^2_{\text{QCD}})}{\ln(M_H^2/4\Lambda^2_{\text{QCD}})} \right]^{-1} = \frac{\alpha_S(Q_T^2)}{\alpha_S(M_H^2/4)},$$  \hspace{1cm} (37)

see (11). This factor reflects the fact that, as usual, the double log approximation overestimates the kinematically available phase space for emission. That is the probability not to
bremsstrahlung a gluon, $T$, is larger than $\exp(-S)$. Thus, in conclusion, as far as the single log corrections are already included in the $T$ factor, we must use $\alpha_S(M_H^2)$ in (5), together with the $K$-factor of (17) evaluated at scale $\mu = M_H$.

From Table 1 and Fig. 2 we see that the perturbative QCD predictions for $d\sigma/dy_H$ show a strong increase with increasing energy, which arises because of the growth of the gluon densities $xg(x, Q^2_T)$ with increasing $1/x \simeq s/M_H^2$. On the contrary the non-perturbative two-gluon-exchange-type phenomenological models have no $x$ dependence. The predictions of these models depend only weakly on energy through the energy dependence of the “soft” cross section which is used to normalise the two-gluon exchange contribution. The same arguments apply to the production of a pair of high $E_T$ jets. Therefore an experimental study of the dijet production rate as a function of the collider energy will clearly be able to discriminate between the perturbative QCD determinations and the non-perturbative model approaches.

8 Summary

We have calculated the cross sections for exclusive and inclusive double diffractive Higgs boson, and also dijet, production in the central region, at both LHC and Tevatron energies. That is

$$pp \rightarrow p + (H \text{ or } jj) + p$$

(38)

$$pp \rightarrow X + (H \text{ or } jj) + Y$$

where + denotes a rapidity gap. These processes are driven by ‘asymmetric’ two gluon exchange, with the colour screening gluon being comparatively soft, but still in the perturbative QCD domain.

All the important perturbative QCD corrections were included in the calculation. A prescription for the unintegrated gluon distribution, up to single log accuracy, was used. The major uncertainty comes from the non-perturbative sector, namely from the value of the survival factor $S_{\text{spec}}^2$ — the small probability to have no secondaries from soft rescattering populating the rapidity gaps. We found that this probability $S_{\text{spec}}^2$ depends sensitively on the spatial distribution of gluons inside the proton. In the tables in which we presented the Higgs and dijet cross section predictions, we took $S_{\text{spec}}^2 = 0.1$, but at LHC energies our estimates of $S_{\text{spec}}^2$ indicate that the value is most likely to be an order of magnitude smaller.

To overcome the normalisation uncertainty due to the lack of knowledge of $S_{\text{spec}}^2$, we proposed that measurements of the double diffractive production of dijets would act as a luminosity monitor for the two-gluon exchange processes of (38). The estimates of the dijet cross section are such that the process should be readily observable at the Tevatron and at the LHC. In particular measurements of jets with $E_T \sim M_H/2$ would enable the cross section for the double diffractive production of the Higgs boson to be reliably predicted, since the two processes are
driven by the same $S_{\text{spec}}$ factor in the same kinematic region. Unfortunately even our most optimistic predictions for the Higgs process are considerably smaller than previous estimates, and would make the process hard to observe at the Tevatron and the LHC.

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Figure Captions

Fig. 1 Diagrams for (a) exclusive, and (b) inclusive, double diffractive Higgs production of transverse momentum $q_T$ in high energy $pp$ (or $\bar{p}p$) collisions. The QCD radiative corrections (such as the emission of the gluon of transverse momentum $p_T$) suppress the number of rapidity gap events via Sudakov form factors, $\exp(-S)$. For inclusive production $q_T$ can be much larger and the Sudakov suppression is weaker; however there are additional QCD radiative effects from the double log resummations $\exp(-n_i/2)$ in the BFKL non-forward amplitudes.

Fig. 2 The perturbative QCD predictions of the rapidity distribution for double diffractive exclusive Higgs production at LHC and Tevatron energies. We also show the recent prediction by Levin [25], which is discussed in Section 7.

Fig. 3 Diagram (a) illustrates the absorptive correction to exclusive Higgs production, assuming that only elastic $pp$ rescattering occurs, with an amplitude $\text{Im} A = s \sigma_{\text{tot}}$. Diagram (b) includes both elastic and inelastic intermediate states. Diagram (c) is an eikonal representation of (b), where the double line denotes Pomeron exchange.
(a) Exclusive production

(b) Inclusive production

Fig. 1
$\frac{d\sigma_H}{dy}$ (fb)
Fig. 3