Evolution of supermassive black hole spins in the ΛCDM cosmology

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Abstract. Over the last years, several observations suggest that SMBHs likely reside at the centres of all spheroid galaxies. Even more interestingly, their properties seem to correlate with the bulge luminosity – or stellar velocity dispersion – and the bulge mass, suggesting a single mechanism for assembling BHs and forming spheroids in galaxies. We have been investigating the cosmological co-evolution of galaxies and their central SMBH in hierarchical models of galaxy formation, using the semi-analytic model developed in Durham University. We focus on the spin of the SMBHs and study how does it evolve during accretion of gas and mergers with other SMBHs. We conclude that the global SMBH spin distribution in the present universe may be bimodal, provided that the gas is fed into the hole through a self-gravity limited accretion disk.

1. Introduction

Astrophysical supermassive blacks holes (SMBHs) are expected to possess angular momentum $J_{\text{bh}} = a G M_{\text{bh}}^2 / c$, where $M_{\text{bh}}$ is the mass of the black hole (BH) and $a$ is the spin parameter ($0 \leq a \leq 1$). The spin is believed to have a significant impact in the close vicinity of the BH. For example, it determines the efficiency of converting matter into radiation in an accretion disk and it is believed to influence the formation and direction of the radio jets in active galactic nuclei (AGN) [11], [27], [8]. In addition, it is of special interest in the modelling of BH binaries in gravitational wave astronomy [33], [1], [2], [15], [14].

The evolution of spins with time is closely related to the channels that contribute to the growth of SMBHs. It is known that the spin changes due to accretion of gas and mergers with other BHs [3], [41], [42], [21]. Each mechanism of BH growth has varying effects on the spin evolution. For example, accretion of gas that co-rotates with the BH should spin up the hole [3], whereas, capturing of small BHs with randomly oriented spins should spin down the hole [21]. Thus, the spin could be used to study and determine which mechanism most influences BH growth.

Motivated by the above discussion we present a model of BH spin evolution in the context of the ΛCDM cosmology, by performing a series of simulations using the GALFORM semi-analytic model [16]. The paper is organised as following. In Section 2 we review the different mechanisms of BH growth in the ΛCDM cosmology. In Sections 3 and 4 we detail how accretion and mergers influence the spin of supermassive back holes. Finally, in Section 5 we present our simulations and main results.
2. The growth of SMBHs in hierarchical cosmologies

The evolution of BH mass fits naturally in the ΛCDM model of galaxy formation [44], [6], where structures grow hierarchically: the small structures form first, then evolve through mergers into the largest ones observed. In the event of a galaxy merger, the dark matter halo and baryonic matter of the less massive galaxy (satellite) sink in the gravitational potential of the massive central galaxy as a result of dynamical friction [7]. If the mass of the satellite galaxy is comparable to the central galaxy, the galaxies after the merger completely disrupt and form an elliptical galaxy (major merger). The remnant galaxy is characterised by intense star formation as the available cold gas from both progenitors is transformed rapidly into stars. In addition, part of the cold gas reservoir feeds the central SMBH. It has been argued for a long time that AGN must be powered by accretion onto these SMBHs through a disk of cold gas [26]. For highly efficient accretion activity the disk/BH system dominates the energetics of the nucleus and the galaxy becomes visible as a quasar. SMBHs at the centres of galaxies observed in the local universe are regarded as the remnants of those quasar phases at early epochs.

SMBHs accumulate part of their mass via binary mergers with other SMBHs. The formation of a BH-BH binary and the subsequent coalescence of it is a natural evolutionary stage for a SMBH if the host galaxy has experienced multiple mergers. As the galaxies in the dark-matter haloes merge the SMBHs sink to the centre via dynamical friction from distant stars. The transition to a bound binary state after the galaxy merger is an open issue in theoretical astrophysics, however, it is well known that once the separation of the two BHs becomes small enough, gravitational radiation carries away the remaining angular momentum of the binary. The removal of energy from a SMBH binary due to gravitational wave emission leads to a gradual shrinkage of the relative separation of the two members and circularises the orbit. There is a point where the eccentricity reaches zero and the orbit circularises. At that time the two BHs are very close to each other, and strong gravitational waves are emitted copiously. The radiated energy is so large, that two SMBHs which are a few AU apart will lose all their potential energy within a couple of minutes and inevitably coalesce. After the coalescence is completed the binary enters the *ringdown* phase, where the merged members settle into a quiescent remnant hole.

Diffuse gas in haloes undergoing quasi-hydrostatic cooling is also believed to contribute to the growth of SMBHs. As the baryonic content of the satellite halo collapses during galaxy mergers it gets shocked and heated up to its virial temperature with a subsequent phase characterised by cooling and the formation of cooling gas flows towards the core of the central halo. In massive halos at late redshifts the shocking of the gas occurs at a radius comparable to the radius of the dark matter halo. These haloes have cooling times longer than the free-fall time of the gas and, thus, the gas is added to a quasi-static atmosphere surrounding the galaxy rather than simply fall towards the centre. This atmosphere – the *hot halo* regime – is in pressure supported hydrostatic equilibrium and extends up to the virial radius of the dark matter halo. The galaxy is then supplied by the hot halo with fresh gas via cooling flows, which also feed the central SMBH.

Bower et al. [13] adopt a BH growth model where during a galaxy merger the BH accretes a fixed fraction of the available cold gas in the galaxy. The amount of gas deposited into the BH is set by an efficiency factor, which determines what fraction of the available gas reservoir for star formation accretes into the hole. The value of that parameter is chosen to fit the local $M_{\text{BH}} - M_{\text{bulge}}$ relation [28], [32], [29], [18]. An additional process of cold gas accretion that contributes to the BH mass in the Bower et al. model is the *disk instabilities* triggered by dynamical instabilities (see [16] and references therein). When the mass of the galactic disk becomes sufficiently gravitating, gas collapses towards the centre when the disk equilibrium is distorted, e.g., by an encounter with a satellite galaxy. A fraction of that gas is directly accreted into the BH, while the rest undergoes star formation. We hereafter refer to the accretion of cold gas triggered by disk instabilities and galaxy mergers as *quasar mode* and to the accretion from
quasi-hydrostatic haloes as radio mode, following the terminology adopted by Bower et al. [13].

3. Evolution of spin due to accretion

Once the SMBH seed forms at the centre of a galaxy, accretion usually initiates the growth era. We assume that the accretion disk is formed in the equatorial plane of the hole. As proposed by Lynden-Bell [26], the gas particles in the disk gradually lose angular momentum due to viscous torques exerted by magnetic fields and radially drift inwards until they reach the inner edge of the disk. The inner edge of an accretion disk is usually taken as the location of the last stable orbit (LSO) around the BH. The LSO is a function of the hole’s angular momentum and can be written as [4]:

\[ R_{\text{LSO}} = R_{\text{Sch}} \left\{ 3 + Z_2 \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2} \right\} \]

with \( R_{\text{Sch}} = 2GM_{\text{bh}}/c^2 \) and

\[ Z_1 = 1 + (1 - a^2)^{1/3} \left[ (1 + a)^{1/3} + (1 - a)^{1/3} \right] \]

\[ Z_2 = (3a^2 + Z_1^2)^{1/2} \]

(2)

The change in the hole’s spin induced by the accretion of \( dM_0 \) is governed by the differential equation

\[ \frac{da}{d \ln M_{\text{bh}}} = \frac{1}{M_{\text{bh}}} \frac{c^3 \tilde{\ell}_{\text{LSO}}}{\tilde{e}_{\text{LSO}}} - 2a. \]

(4)

This was integrated by Bardeen [3] using the explicit expressions for \( \tilde{e} \) and \( \tilde{\ell} \), resulting in the following solution,

\[ \begin{align*}
\text{for} \quad 1 & \leq M_{\text{bh}}^f / M_{\text{bh}} \leq R_{\text{LSO}}^{1/2} : \quad a^f = \frac{1}{3} R_{\text{LSO}}^{1/2} M_{\text{bh}}^{-1} \left[ 1 - \left( \frac{3R_{\text{LSO}}/M_{\text{bh}} - 2}{3R_{\text{LSO}}/M_{\text{bh}} - 2} \right)^{1/2} \right] \\
\text{for} \quad M_{\text{bh}}^f / M_{\text{bh}} \geq R_{\text{LSO}}^{1/2} : \quad a^f = 1,
\end{align*} \]

(5)

where, \( M_{\text{bh}}^f \) and \( a^f \) are the final mass and spin of the BH. The expressions in (5) govern the evolution of \( a \) during accretion from an initial state of a co-rotating or counter-rotating thin disk (we define a disk to be thin if its semi-thickness is much smaller than its radius, i.e., \( H << R \)). According to Bardeen’s law, a non-rotating BH (\( R_{\text{LSO}} = 6R_{\text{Sch}} \)) will be spun up to a maximum rotation \( (a = 1, R_{\text{LSO}} = R_{\text{Sch}}) \) after increasing its mass by \( (\sqrt{6} - 1)M_{\text{bh}} \). For a maximally rotating BH in a counter-rotating accretion disk \( (a = -1, R_{\text{LSO}} = 9R_{\text{Sch}}) \), an increase in mass of \( 2M_{\text{bh}} \) is required in order for the BH to be spun down to \( a = 0 \) and subsequently spun up to maximum rotation. As implied by Eq. (5), further accretion into the BH keeps \( a \) equal to one.

Bardeen’s calculations are limited to the process of angular momentum transport between the accreted matter and BH without taking into account other processes that may influence the spin evolution. In fact, Thorne [40] argued that the accretion disk radiates and part of this radiation will be accreted by the hole. Capturing of photons with angular momentum opposite to that of the BH will then produce a counteracting torque that prevents spin up beyond the limiting value of \( a = 0.998 \).
3.1. The case of misaligned accretion disks

In the general case where the accretion disk does not lie on the equatorial plane of the BH but has a random orientation relative to the angular momentum of the BH, the evolution of the spin turns out to be a complicated process. Following the discussion in King et al. [22], we assume that a thin disk is inclined at some random angle $\theta$ relative to the orientation of the hole’s angular momentum vector $J_{bh}$. We denote the angular momentum of the disk as $J_d$ (see [22] and [42] for a discussion on the nature of $J_d$), and define the total angular momentum vector $J_{tot}$ as

$$J_{tot} = J_{bh} + J_d,$$

(6)

with $\hat{J}_{bh} \cdot \hat{J}_d = \cos \theta$. The angle $\theta$ is defined such that $0 \leq \theta \leq \pi$; the values $\theta = 0$ and $\theta = \pi$ correspond to full alignment and anti-alignment. It is obvious that, $J_{tot}$ is a constant vector and its orientation in space is fixed. Its algebraic expression is given by

$$J^2_{tot} = J^2_{bh} + J^2_d + 2J_{bh}J_d \cos \theta.$$  

(7)

When the vectors $J_{bh}$ and $J_d$ are misaligned, the tilted orbits of the particles in the accretion disk experience a torque due to the Lense-Thirring effect [25], which causes the plane of the accretion disk to precess about the rotational axis of the BH [43], [5], [38]. If the torque is strong enough compared to the internal viscosity, the inner parts of the disk will be forced to rotate in the equatorial plane of the hole resulting in a warped disk (see figure 1). The end effect of the Lense-Thirring precession is a BH aligned or anti-aligned with the accretion disk. The expression for the magnitude of $J_{tot}$ allows us to examine the final configuration of the system. If $J^2_{bh} > J^2_{tot}$ anti-alignment occurs, which requires

$$\cos \theta < -\frac{J_d}{2J_{bh}}.$$  

(8)

Hence, a BH with $\theta > \pi/2$ and $2J_{bh} > J_d$, eventually anti-aligns with the accretion disk.

Further investigation of the warped disks requires the assumption of a thin disk model. We use the solution from the Shakura-Sunyaev disk [39] for our analysis. In the standard Shakura-Sunyaev disk model the analytic expression for the warp radius depends on the values of $J_{bh}$, $M_{bh}$, and the accretion rate $\dot{M}$ of the BH. In terms of the Schwarzschild radius, $R_{warp}$ is written as [42]

$$\frac{R_{warp}}{R_{Sch}} = 3.6 \times 10^3 a^{5/8} \left( \frac{M_{bh}}{10^8 M_\odot} \right)^{1/8} f_{Edd}^{-1/4} \left( \frac{\nu_2}{\nu_1} \right)^{-5/8} \alpha^{-1/2}.$$  

(9)
Here $\alpha$ is the Shakura-Sunyaev viscosity parameter, $\nu_1$ and $\nu_2$ are the accretion and warp propagation viscosity respectively and $f_{\text{Edd}} = L/L_{\text{Edd}}$ is the Eddington ratio. The accretion luminosity $L$ and Eddington luminosity $L_{\text{Edd}}$ are defined as

$$L = \epsilon \dot{M} c^2,$$  \hspace{1cm} (10)

with $\epsilon$ denoting the accretion efficiency, and

$$L_{\text{Edd}} = \frac{4\pi GM_{\text{bh}} c}{\kappa} = 1.4 \times 10^{46} \left( \frac{M_{\text{bh}}}{10^8 M_\odot} \right) \text{ erg s}^{-1},$$  \hspace{1cm} (11)

where $\kappa$ is the electron scattering opacity. The mass of the disk inside the radius $R_{\text{warp}}$ is then

$$M_d(R_{\text{warp}}) = \dot{M} t_{\nu_1} \nu_1 (R_{\text{warp}}),$$  \hspace{1cm} (12)

where the accretion timescale is given by

$$t_{\nu_1} = \frac{R_{\text{warp}}^2}{\nu_1} = 3 \times 10^6 \ a^{7/8} \left( \frac{M_{\text{bh}}}{10^8 M_\odot} \right)^{11/8} f_{\text{Edd}}^{-3/4} \left( \frac{\nu_2}{\nu_1} \right)^{-7/8} \alpha^{-3/2} \text{ yr}.$$  \hspace{1cm} (13)

We can define the total angular momentum $J_d$ passing through $R_{\text{warp}}$ as,

$$J_d(R_{\text{warp}}) \lesssim M_d(R_{\text{warp}}) (GM_{\text{bh}} R_{\text{warp}})^{1/2}.$$  \hspace{1cm} (14)

In terms of the anti-alignment criterion this gives,

$$\frac{J_d}{2J_{\text{bh}}} = \frac{M_d}{a M_{\text{bh}}} \left( \frac{R_{\text{warp}}}{R_{\text{Sch}}} \right)^{1/2} = 10^{-9} f_{\text{Edd}} \left( \frac{t_{\nu_1}}{1 \text{ yr}} \right) \left( \frac{R_{\text{warp}}}{R_{\text{Sch}}} \right)^{1/2} \ a^{-1}. \hspace{1cm} (15)$$

Derivation of the above quantity shows whether the anti-alignment condition in a misaligned disk is satisfied.

### 3.2. Self-gravity limited accretion disks

The BH growth process in AGN environments involves vast amounts of accreted gas, often comparable to the initial mass of the accreting hole. This amount of mass settles to the accretion disk around the hole, which often extends in radius to several thousands of gravitational radii, $R_{\text{Sch}}$. It is usually assumed that the available mass fuel, $M_{\text{acc}}$, is consumed in a single accretion episode, thus, providing a supply of constant angular momentum. In this case, the amount of mass consumed is usually enough to spin up the hole up to a Kerr parameter close to 0.99, even if the BH is maximally spinning in a counter-rotating disk ($a = -1$). Inevitably, subsequent accretion episodes during major galaxy mergers act to spin up the BH to maximum rotation.

Recently, however, King et al. [23] argued that the end effect of the accretion growth channel might be completely different if we take into account the fact that an accretion disk becomes self gravitating at some radius $R_{\text{sg}}$ where its mass exceeds $M_{\text{sg}} \sim (H/R)M_{\text{bh}}$. As a result the mass of the disk is limited by its self gravity to $\Delta M_{\text{episode}} \ll M_{\text{acc}}$, which gives rise to a series of $N \sim M_{\text{acc}}/\Delta M_{\text{episode}}$ well separated accretion episodes. These accretion episodes are assumed to be randomly oriented around the BH since observations suggest that there is no apparent relation between the accretion disk (or radio jet) orientation and the host galaxy disks [24]. Outside $R_{\text{sg}}$ the gas is believed to undergo intense star formation which could provide a qualitative explanation for the existence of the ring of stars seen in the near vicinity of the central BH in the Milky Way [17].
We note here that in a large population of accreting BHs with $J_d < 2J_{bh}$, we should expect, in principle, counteralignment to occur in a fraction \[ f = \frac{1}{2} \left( 1 - \frac{J_d}{2J_{bh}} \right), \] of the accretion episodes. Thus, a succession of episodes that comprise a small fraction of the total mass accreted must have $J_d \ll J_{bh}$, which implies that $f \approx 1/2$. Therefore, counter- and co-alignment are equally possible outcomes of the Lense-Thirring effect. Such a succession of chaotic accretion episodes should then systematically spin down the BH, resulting in a global spin distribution oscillating around zero.

The physics of self-gravitating accretion disks is described in Pringle [35]. Briefly, the criterion that the self-gravity of a disk is negligible is the requirement that the gravitational force along the $\hat{z}$ direction be only due to the central BH. This can be expressed as the surface density of the disk, $\Sigma$, being negligible compared to the quantity $M_{bh}H/R^3$, or in terms of the disk mass,

\[ M_d(< R) \ll \frac{H}{R} M_{acc}. \] (17)

The self-gravity becomes marginally important when $M_d \simeq H/RM_{acc}$, which condition gives the mass of a self-gravitating disk [23],

\[ M_{sg} = \left( \frac{H}{R} M_{acc} \right)_{R=R_{sg}} = 2.13 \times 10^5 \, \epsilon^{-5/27} \left( \frac{M_{bh}}{10^8 M_\odot} \right)^{23/27} f_{\text{Edd}}^{5/27} a^{-2/17}. \] (18)

As we have described in Section 3.1 the disk once formed with a non-zero misalignment angle about the spin axis of the hole, it will be subject to the Lense-Thirring precession. If the radius, $R_{sg}$, is greater than $R_{warp}$ then,

\[ J_d = J_d(R_{warp}). \] (19)

If, however, $R_{sg} < R_{warp}$, then the entire disk will be subject to Lense-Thirring precession, thus, aligning itself on the equatorial plane of the BH.

4. Spin evolution under binary coalescences

We now examine the evolution of spin during BH-binary coalescences. Following the discussion in Section 2, at the moment of the final merger the smaller BH adds its angular momentum at the LSO to the spin of the larger one,

\[ \tilde{l}_{\text{iso}} = \pm \frac{2}{3\sqrt{3}} \left[ 1 + 2 \left( 3 \frac{R_{\text{iso}}}{M_{bh}} - 2 \right)^{1/2} \right]. \] (20)

Note that, $\pm$ corresponds to co-rotating and counter-rotating equatorial orbits respectively. Even if the progenitor holes do not possess any angular momentum, the final remnant will preserve the residual orbital momentum of the binary (i.e., the angular momentum that has not been radiated away), thus, the remnant BH will belong to the Kerr family. We note here, that in the field of numerical simulations, recent remarkable breakthroughs in numerical relativity have provided robust simulations of BH mergers by solving directly the Einstein equations in a fully relativistic frame [15], [20], [31]. These simulations have been exploring the parameter space for different configurations of initial masses and spins, allowing accurate measurements of the final spin. For example, a merger of two equal mass BHs with $a = 0$ results in a Kerr BH with $a^f \approx 0.69$. Analytic fits extend the predictions for $a^f$ to the entire space of parameters and
reproduce all the available numerical data. In the case where the masses are unequal, the final spin can be estimated by the analytic expression

\[ a_f \approx 2\sqrt{3}\frac{q}{(1+q)^2} - 2.029\frac{q^2}{(1+q)^4}, \]  

where \( q \) expresses the mass ratio \( M_2/M_1 \) [9].

A more generic semi-analytic fitting formula for predicting the final spin of any given binary configuration in the parameter space has been provided by Rezzolla et al. [36], [37]. In their analysis, they assume that the final spin vector can be expressed as

\[ a_f = \frac{1}{(1+q)^2} [ \left( a_1 + a_2 \right) q^2 + \ell q ] . \]  

Here \( a_{1,2} = cJ_{1,2}/(GM_{1,2}^2) \) and \( \ell = \tilde{\ell}/(M_1M_2) \), with \( \ell \) defining the difference between the orbital angular momentum when the binary is widely separated, and the angular momentum radiated away in gravitational waves before the merger,

\[ \tilde{\ell} = L - J_{\text{rad}}. \]  

The vector \( \ell \) is taken to be parallel to the orbital momentum vector throughout the evolution of the binary, an assumption not strictly valid since the system could radiate away angular momentum in a non-symmetric way. However, the error imposed by this assumption is relatively small for the binary configurations studied in their simulations. A further assumption is that the mass radiated away in gravitational waves is neglected in the calculations, as it accounts for only a minor fraction (5 – 7%) of the total mass-energy of the binary configurations analysed. Under these assumptions, Rezzolla et al. proposed an analytic expression for the magnitude of the final spin, given by

\[ |a_{\text{fin}}| = \frac{1}{(1+q)^2} \left[ |a_1|^2 + |a_2|^2 q^4 + 2|a_2||a_1|q^2 \cos \alpha + 2 \left( |a_1| \cos \beta + |a_2|q^2 \cos \gamma \right) |\ell|q + |\ell|^2 q^2 \right]^{1/2}, \]  

with the cosine angles \( \alpha, \beta \) and \( \gamma \) defined as

\[ \cos \alpha \equiv \hat{a}_1 \cdot \hat{a}_2, \quad \cos \beta \equiv \hat{a}_1 \cdot \hat{\ell}, \quad \cos \gamma \equiv \hat{a}_2 \cdot \hat{\ell}. \]  

The final spin as given by expression (24) is in a good agreement when compared with numerical data, with residuals being restricted to less than 3%.

5. Evolution of spin in hierarchical cosmologies: numerical simulations

5.1. Method

We perform a series of numerical simulations in which we explore the effect of gas accretion and mergers on the spin of SMBHs. The basic tool for our predictions is the GALFORM semi-analytic model [16], which implements the Bower et al. [13] model for galaxy evolution in the ΛCDM cosmology. BHs evolve their mass in accordance with the model developed by Malbon et al. [30]. We study the cosmological spin evolution of SMBH seeds for a total number of \( \sim 2.1 \times 10^7 \) galaxies in a co-moving volume of \( 1.35 \times 10^8 \) (Mpc/h)^3 from redshift 127 to redshift zero. Here \( h \) is defined in \( H_0 = h \times 100 \) km s^{-1}Mpc^{-1}, where \( H_0 \) is the Hubble constant at redshift zero.

During the evolution of the BHs we take into account spin changes due to accretion of gas and mergers, as described in Sections 3 and 4. The initial population of seeds is assumed to be non-rotating and “pre-galactic” – it could represent the end product of massive Population III stars.
or collapsed clumps of gas in the early universe. These seeds grow via accretion of gas during galaxy mergers and disk instabilities, the radio mode accretion, and BH-BH mergers. Figure 2 shows the contribution from the different growth channels to the final BH mass at different redshifts. As it is illustrated, at high redshifts SMBHs with masses up to \( \sim 10^8 \, M_\odot \) built their mass almost exclusively through the quasar mode. On the contrary, accretion of gas from the hot halo in massive galaxies (radio-mode accretion) and BH mergers contribute significantly only to BHs with masses \( \gtrsim 10^8 \, M_\odot \) at low redshifts.

To model the physics of the accreted gas, we assume that prior to its collapse it forms an accretion disk whose physics is adequately described by the standard Shakura-Sunyaev disk model. We assume that the available gas to be fed into the BH after a galaxy merger or disk instability accretes over a timescale equal to the dynamical timescale of the galactic bulge that hosts the BH.

The disk is chosen to be randomly oriented relative to the spin axis of the hole. The inclination angle \( \theta \) is chosen from a flat distribution of cosine values between \(-1\) and \(1\). If \( \theta \neq 0^\circ, 180^\circ \), we assume that the disk precesses around the BH as described in Section 3.1. In brief, for a given BH mass \( M_{\text{bh}} \) we determine the numerical values of the warp parameters, \( R_{\text{warp}} \), \( M_d(R_{\text{warp}}) \), and the angular momenta \( J_d(R_{\text{warp}}) \) and \( J_{\text{bh}} \). The ratio \( J_d/2J_{\text{bh}} \) allows us through the criterion in relation (8) to check if anti-alignment or alignment occurs. We then allow the BH to accrete an amount of gas equal to \( M_d(R_{\text{warp}}) \) and evolve the spin using relation (5). Finally, we update the masses of the BH and we estimate the hole’s new spin and the disk precession by calculating the new angle \( \theta \). This process is repeated until: (a) the disk aligns or counteraligns (\( \theta = 0^\circ \) or \( 180^\circ \)) with the hole’s spin, in which case we just consume the rest of the available gas in a single step and evolve the spin according to (5), (b) the accretion disk is entirely consumed, without being able to align or counteralign itself with the spin. In almost all of our cases we find that the accretion disks ultimately align (or anti-align) themselves on the equatorial plane of the BH.

In addition to the standard accretion model, we decide to test the model proposed by King...
Figure 3. The distribution of spins at different redshifts (a) and in different mass ranges (b) for aligned binary mergers.

et al. [23] where the mass of the accretion disk is limited by its self-gravity. We repeat the same steps as previously taking, however, $M_d = M_{sg}$. Once $M_{sg}$ is consumed we update the mass of the hole, determine $M'_{sg}$ computed from the updated hole mass, and settle a new disk in a random orientation around the BH with $M_d = M'_{sg}$.

Finally, we consider the spin changes due to binary coalescences. Following the merger of two galaxies in GALFORM, each harbouring a SMBH, we expect the one hosted by the satellite to sink toward the SMBH of the central galaxy and eventually form a binary. In our mode BH mergers occur in gas-rich environments, where we assume that torques from the accreting gas suffice to align the spin of the BHs with the orbital axes prior the collapse [12]. We, therefore, take $\alpha = \beta = \gamma = 1$ in relation (24). Furthermore, we constrain our results to BH masses greater than $\sim 10^6 M_\odot$ in accordance with the current $M_{bh} - M_{bulge}$ observations. In addition, we assume that all binaries will ultimately coalesce on a very short timescale, which effectively allows us to coalesce the binary right after the host galaxies merge.

During the merger, the smaller BH is taken to plunge into the larger one carrying along its angular momentum at the LSO. At this point we neglect any mass loss due to gravitational emission, as it corresponds to a small fraction of the total mass-energy of the system. The spin of the final remnant is estimated using the semi-analytic fitting formulae form Rezzolla et al. [36], [37], as described in Section 4.

5.2. Results

In order to establish clearly the spin evolution under binary coalescences and elucidate the contribution of mergers to the global spin distributions, we first allow the BHs to evolve in our model ignoring the effects of accretion. The histograms in figure 3 (a) show the distributions of spins at different redshifts. As demonstrated in these plots, at high redshifts there are almost no mergers occurring in the universe. As redshift decreases, BHs gradually form binaries and merge with each other and eventually at redshift zero nearly 19% of the BHs have experienced at least one merger. What is apparent in the distribution of spins at redshift zero is that it has a strong peak centred around $a^f \sim 0.69$. We recall that $a^f \sim 0.69$ is the maximum spin
Figure 4. (a) The spin evolution with redshift due to BH-BH mergers and accretion. Different colours represent different accretion scenarios. (b) The distribution of BH spins at $z = 0$ for different mass bins.

that a remnant BH can have when its progenitors are non-spinning and corresponds to the case where the members of the binary have equal masses ($q = 1$). Thus, the region of the histogram with $a_f \leq 0.69$ is dominated by BHs with non-spinning progenitors. BHs with final spins higher than 0.69 are the end product of aligned binaries with spinning members since mergers of holes with parallel spins have the tendency to spin up the remnant (see [10]). Finally, a strong peak appears at the low-spin end of the distribution since most of the BH seeds never experience any merger.

A clear prediction of figure 3 (a) is that BH mergers in the ΛCDM universe are not a very common phenomenon. For example at redshift zero about 6% of the BHs have experienced more than one merger and only $\sim 8\%$ have had a recent major merger ($q > 0.1$). The rare nature of mergers is a consequence of the dominance of accretion as a growth channel (see figure 2).

When we consider the post-merger spins for different BH masses we find that the distribution of spins depends strongly on the mass range chosen. This is demonstrated for $z = 0$ in figure 3 (b), where we show the distribution of spins for different back hole mass ranges. As can be seen, high spin values are common only for massive BHs ($M_{bh} \gtrsim 10^8 M_\odot$). These BHs are hosted by massive elliptical galaxies and have experienced at least one major merger in their past history. Hence, they acquire high spins since aligned mergers with comparable BH masses act to spin BHs up quite efficiently and, thus, account for the high-spin end of the distribution at redshift zero. On the contrary, small BHs built their mass mainly through accretion (see figure 2) and repeated minor mergers. Therefore, BHs with $M_{bh} \lesssim 10^7 M_\odot$ do not acquire very high spins. BHs in the range $10^7 M_\odot \lesssim M_{bh} \lesssim 10^8 M_\odot$ attribute a small fraction of their mass to mergers which allow them to spin up to $a_f \sim 0.7 - 0.8$. The mild bimodality of spins seen here is reflected in the global spin distribution at redshift zero as BHs in this mass range dominate the mass function of BHs in the present universe.

We now simulate the effect of accretion on the evolution of BH spins. The accretion is modelled as in Volonteri [42] where we assume that the available gas is accreted during a single accretion event, and alternatively as in King et al. [23] where we constrain the amount...
of accretion to the self-gravity limited mass (see Section 3.2). We henceforth refer to these two different scenarios as *prolonged* and *chaotic* accretion models respectively.

Figure 4 (a) reports how the spin evolves under the chaotic and prolonged accretion models. In both cases it is evident that accretion dominates the spin evolution, resulting in spin distributions different from those obtained under mergers only. If we assume chaotic feeding of gas to the BH we find a bimodal distribution of spins. The bimodality is an expression of the nature of accretion in our model. In order to illustrate this better we plot in figure 4 (b) the spin distributions at $z = 0$ in different mass ranges. When BHs acquire their mass through chaotic accretion episodes, the spin evolution depends strongly on the hole’s mass. Low mass BHs have moderate spins, very close to $a^f = 0.1$, and account for the low-spin peak in the bottom panel of figure 4. More massive BHs have much higher spins and populate the high-spin end of the global spin distribution at redshift zero.

In our model, low mass BHs ($M_{bh} \lesssim 10^8 M_\odot$) built up their mass mainly through the quasar mode of accretion. For example, for a BH with mass $10^8 M_\odot$ a typical accretion event contains $10^8 M_\odot$ of gas. However, if we assume that the size of the disk is limited by its self gravity, the mass of the gas that settles itself on the disk cannot be more than $10^6 M_\odot$. Thus, a $10^8 M_\odot$ accretion event gives rise to $\sim 100$ accretion episodes with $\Delta M_{\text{episode}} \ll M_{bh}$, all randomly oriented around the BH. As a consequence, these repeated episodes produce a net spin-down to modest values centred around $a^f \sim 0.1$.

For more massive BHs the mass of a self-gravity limited disk can be much higher (remember $M_{\text{self}} \propto M_{bh}^{1.85}$ from Eq. (18)). For example a BH of $\sim 10^8 M_\odot$ can sustain a disk of up to $10^9 M_\odot$. Such massive BHs accumulate their mass primarily via accretion during the radio mode. However, the radio mode is characterised by low mass accretion episodes; typical amounts of gas consumed during the radio mode for a $10^8 M_\odot$ BH are of the order of $10^5 M_\odot$. This amount of gas can be accreted in a single episode, resulting in high final spins. This is a common feature for BHs with high masses as above that mass limit the radio mode is the dominant growth channel. Thus, all BHs with $\gtrsim 10^8 M_\odot$ are found to rotate rapidly.

When we consider the prolonged accretion model we find that accretion is a very efficient mechanism for spinning up BHs even at very high redshifts. A great fraction of the back holes have accreted enough gas to be spun-up up to $a^f \approx 1$, even if those BHs originally accrete via retrograde accretion. This results in a significant population of rapidly rotating remnants in the present universe for any given BH mass. If however, there is not enough gas available to the hole and the disk is counter-rotating the BH are on average spun down. As a result, a long tail extending down to $a^f \approx 0$ is also present.

6. Discussion - Conclusions

We have presented a model for studying the evolution of SMBH spins in the ΛCDM cosmology. We found that the different astrophysical processes that influence the evolution of SMBHs may have a significant effect on the global spin distribution in the present universe depending on the accretion scenario we assume. For example, in the prolonged model, accretion of gas dominates the evolution of spins through cosmic time, resulting in rapidly rotating BHs. However, if we assume that gas is fed into the BH through a self-gravity limited disk we obtain a bimodal spin distribution. In this case, high spin values is a property of BHs with masses $\gtrsim 10^8 M_\odot$. These BHs are hosted in our cosmological model by massive elliptical galaxies. On the contrary, spiral galaxies usually host small BHs ($M_{bh} \lesssim 10^8 M_\odot$) which according to the chaotic accretion-scenario should possess moderate spins.

This is an important result of our model as it could provide insight into the radio loudness of AGNs. If the spin is the BH parameter that determines the strength of the jet in an AGN environment [11], then according to our model since elliptical galaxies have rapidly rotating SMBHs they should host AGNs with powerful radio jets. On the contrary, AGNs in spiral
galaxies must have quiescent jets for the reason that their central SMBHs have on average very low spins. This is indeed what observation suggest (see [34] and references therein), since radio jets in AGNs hosted by elliptical galaxies can be about $10^3$ times more radio powerful than those in AGNs hosted by spiral galaxies. The relation between the SMBH spin and the radio properties of AGNs will be studied in a forthcoming paper.

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