Heavy resonance production in high energy nuclear collisions

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Abstract

We estimate freezeout conditions for $s$, $c$, and $b$ quarks in high energy nuclear collisions. Freezeout is due either to loss of thermal contact, or to particles “wandering” out of the region of hot matter. We then develop a thermal recombination model in which both single-particle (quark and antiquark) and two-particle (quark-antiquark) densities are conserved. Conservation of two-particle densities is necessary because quarks and antiquarks are always produced in coincidence, so that the local two-particle density can be much larger than the product of the single-particle densities. We use the freezeout conditions and recombination model to discuss heavy resonance production at zero baryon density in high energy nuclear collisions.
I. INTRODUCTION

One of the striking features of high energy proton-proton, proton-nucleus and nucleus-nucleus collisions is the production of heavy resonances. Since these resonances appear as a consequence of raising the collision energy, it is natural to ask, “Can heavy resonances be used experimentally to signal a deconfined phase of matter?” Enhanced strangeness production was in fact proposed as one of the first potential signals of the Quark Gluon Plasma (QGP). \[J/\psi\] production \[2\] can be looked upon as changing the focus to the heavier charm (c) quark from the much lighter strange (s) quark. Here we aim at a unified treatment of heavy quark composites, extending the scope to include also the bottom (b) quark.

In view of the very different masses of the s, c, and b quarks, and of the mesons that contain these quarks, it is reasonable to assume that the various flavors will stop interacting with the expanding collision system at different times and thus carry “frozen out” information about different stages of the time evolution, reflecting their last interactions. This calls for a detailed examination of the freezeout conditions for each flavor. The concept of different freezeout times for different species is well known in studies of the Early Universe, and has also been discussed in the context of nuclear collisions at lower energies. \[3\]

Our physical picture assumes thermal equilibrium, and chemical equilibrium between resonances with the same heavy quark content (but not between heavy quarks and antiquarks) at high temperature. We allow resonances to depart from chemical equilibrium after freezeout. In determining the freezeout chemical potentials we pay particular attention to the local quark-antiquark (Q\bar{Q}) pair density. Since quarks and antiquarks are always created in pairs, the local pair density may be significantly higher than the product of the single-particle densities. We take this important effect into account in terms of a pair chemical potential (see Section \[\text{II}\]). This pair chemical potential is the main difference between our recombination model and others that have been used recently to study strange \[4,5\] and heavier exotic \[6\] resonances.

In Section \[\text{II}\] we discuss two conditions which may signal that heavy resonances cease to interact with the hot matter, and estimate freezeout temperatures and proper times for s, c, and b quark resonances in selected AA collisions at CERN’s Super Proton Synchrotron (SPS, for which $\sqrt{s} = 20$ GeV/nucleon) and future Large Hadron Collider (LHC, $\sqrt{s} = 7$ TeV/nucleon), and at Brookhaven’s future Relativistic Heavy Ion Collider (RHIC, $\sqrt{s} = 200$ GeV/nucleon). We obtain a rich variety of freezeout scenarios, depending on the properties of the transition from the hadronic phase to the high temperature (deconfined) phase. We also calculate the mean distance a heavy quark moves before freezeout and the mean number of interactions it has; the mean number of interactions is high enough that thermal recombination models can be expected to work for heavy ion collisions at or above SPS energy, but not necessarily for collisions of lighter ions. In Section \[\text{II}\] we describe our thermal recombination model. We then give an illustrative application to J/ψ suppression, and calculate freezeout densities and the fractions of c and b quarks that end up in c\bar{c} and b\bar{b} resonances. Finally, in Section \[\text{IV}\] we summarize our work.
II. FREEZEOUT CONDITIONS

In this section, we estimate the freezeout conditions for the different heavy quark flavors. We consider two mechanisms for freezeout. In subsection II A we discuss freezeout due to loss of thermal contact, which we refer to as “contact” freezeout. In subsection II B we discuss freezeout by emission, which we estimate by treating the quarks as random walkers, and hence refer to as “wandering” freezeout. We then summarize the results of the two freezeout scenarios in subsection II C.

The s quark mass, \( m_s \), is approximately equal to the quantum chromodynamics (QCD) transition temperature, \( T_c \approx 150 – 200 \) MeV, while the c and b quark masses, respectively \( m_c \) and \( m_b \), are much greater. [We use \( m_s = 0.2 \) GeV, \( m_c = 1.5 \) GeV, and \( m_b = 5 \) GeV.] Thus, freezeout conditions for c and b quarks are slightly different than those for s quarks. Because of this, we estimate freezeout conditions under each scenario for c and b quarks together, and then for s quarks.

A. Loss of thermal contact

One possibility is that freezeout occurs when the thermal contact between the heavy quark and the hot matter ends. In this case, the temperature at freezeout, \( T_f \), is determined by setting the mean free path equal to the mean distance to leave the hot matter. If the hot matter forms an infinite cylinder of radius \( R_f \) at freezeout, the initial location is equally likely to be anywhere within the cylinder, and any initial direction of motion is equally likely, the mean distance to leave the hot matter is \( 4R_f/3 \). The thermal heavy quark mean free path is

\[
\lambda_Q(T) = \frac{v_Q(T)}{\Gamma_Q(T)},
\]

where \( v_Q \) is the velocity and \( \Gamma_Q \) is the collision rate. For heavy quarks in deconfined matter, we take

\[
v_Q(T) = \left( \frac{3T}{m_Q} \right)^{1/2},
\]

\[
\Gamma_Q(T) = \frac{4}{3\pi} T,
\]

\[
\lambda_Q(T) = \frac{3\pi}{4} \sqrt{3/m_Q T},
\]

where we use \( \Gamma_Q \) from Ref. [7] with \( N = 3 \) colors, Casimir factor \( C_f = 4/3, N_f = 3 \) light flavors (u, d, s) and strong coupling constant \( g = 2 \). We have dropped terms of order \( v_Q \ln(1/v_Q) \) in these equations, as these are small and do not affect our results much.

In the coexistence region, the mean free path is

\[
\lambda_Q(T_c) = \frac{\lambda_Q^{(d)}(T_c) \lambda_Q^{(h)}(T_c)}{(1-x)\lambda_Q^{(d)}(T_c) + x\lambda_Q^{(h)}(T_c)},
\]
where $x$ is the fraction of matter in the deconfined phase. We estimate mean free paths in hadronic matter at $T_c$ by assuming that the density of scatterers is proportional to the entropy density in the two phases, and that cross sections remain constant:

$$\lambda^{(h)}(T_c) = \frac{\nu}{2} \lambda^{(d)}(T_c), \quad \text{(6)}$$

where $\nu = s_d(T_c)/s_h(T_c)$ is the ratio of entropy densities in the two phases at $T_c$. This is equivalent to assuming that the number of scatterers is proportional to the entropy, and that the phase transition dilutes the scatterers without affecting their cross sections; the extra factor of $1/2$ occurs because quarks must travel in mesons (which we treat as two quarks) in the hadronic phase. We thus obtain

$$\lambda^{(c)} = \frac{\nu}{2 + (\nu - 2)x} \lambda^{(d)}(T_c). \quad \text{(7)}$$

For hadronic gas, we expect the mean free path to be

$$\lambda^{(h)}_Q(T) = \frac{\nu}{\rho \sigma_Q} v_Q(T), \quad \text{(8)}$$

where $\rho$ is the density of light particles (with $m \ll T$) and $\sigma_Q$ is the mean cross section. The thermal velocity is approximately the same for heavy quarks and mesons containing heavy quarks, so at $T_c$ we find

$$\sigma_Q = \frac{72}{\pi^3} \nu^{-1} T_c^{-2} \approx \frac{2.3}{\nu} T_c^{-2}, \quad \text{(9)}$$

assuming continuity of $\lambda_Q$, and that $\rho$ is given by the density of a massless pion gas,

$$\rho = \frac{\pi^2}{27} T_c^3. \quad \text{(10)}$$

For $T_c = 150 - 200$ MeV and $\nu = 5 - 10$, $\sigma_Q = 2 - 8$ mb, in reasonable agreement with currently used values for heavy meson cross-sections. However, different heavy quark mesons may have very different mean free paths, so it is not clear that all meson species containing any given heavy quark flavor will freeze out at the same time. This issue is beyond the scope of this paper, so we leave it for future work.

We extrapolate to lower temperatures by assuming that $\sigma_Q$ is constant and $\rho \propto T^3$, so

$$\lambda^{(h)}_Q(T) = \frac{\nu}{2} \lambda^{(d)}_Q(T_c) \left(\frac{T_c}{T}\right)^{5/2}. \quad \text{(11)}$$

This estimate of $\lambda^{(h)}_Q$ is reasonably accurate near $T_c$. However, for $T \ll m_\pi$ the mean free path will grow exponentially with $T^{-1}$ as the density drops, so freezeout will occur before $T$ drops much below $m_\pi$.

For freezeout occurring from deconfined matter, we obtain

$$4 \frac{R_f}{3} = \frac{3\pi}{4} \sqrt{3/m_Q T_f} \rightarrow T_f = \frac{243\pi^2}{256} m_Q^{-1} R_f^{-2} \approx 9 m_Q^{-1} R_f^{-2}. \quad \text{(12)}$$
Freezeout occurs for $T \geq T_c$ if
\[ R_f \leq \frac{9\pi\sqrt{3}}{16} (m_\eta T_c)^{-1/2} \simeq 3 (m_\eta T_c)^{-1/2}. \]  
(13)

“Contact” freezeout occurs from the coexistence region as long as
\[ R_f \leq \frac{9\pi\sqrt{3}}{32} \nu (m_\eta T_c)^{-1/2} \simeq 1.5 \nu (m_\eta T_c)^{-1/2}, \]  
(14)
and occurs from normal hadronic matter for larger values of $R_f$. Note that in all equations above $R_f$ can depend on $\tau_f$ or $T_f$, so that the equations are valid even with transverse expansion of the hot matter as long as $R_f$ is treated properly.

The $s$ quark mass is small, so its velocity can be taken to be unity, and the mean free path is
\[ \lambda_s^{(d)}(T) = \frac{1}{\Gamma_s^{(d)}(T)} = 2 T^{-1}, \]  
(15)
where we obtain $\Gamma_s^{(d)}$ for light quarks from Ref. [7]. We then find that “contact” freezeout of $s$ quarks from deconfined matter occurs when
\[ T_f = \frac{3}{2} R_f^{-1}. \]  
(16)
Freezeout thus occurs at or above $T_c$ if
\[ R_f \leq \frac{3}{2} T_c^{-1}. \]  
(17)

We estimate the mean free path for an $s$ quark in the coexistence region just as we did for the $c$ and $b$ quarks. The thermal velocity in hadronic matter is still approximately unity near $T_c$, as the mass of the lightest meson containing an $s$ quark is not too much greater than the transition temperature. If we again assume that the number of scatterers is proportional to the entropy, and cross sections are constant, we obtain Eq. (7) as before. This is equivalent to assuming a cross section
\[ \sigma_s = \frac{27/\pi^2}{\nu} T_c^{-2} \simeq 3 - 10 \text{ mb}, \]  
(18)
which again is not an unreasonable value. We find that “contact” freezeout of $s$ quarks in the coexistence region occurs if
\[ R_f \leq \frac{3}{4} \nu T_c^{-1}. \]  
(19)
For larger values of $R_f$, the $s$ quarks will freeze out from normal hadronic matter.
B. Emission ("wandering")

A second possibility is that freezeout occurs when the quarks leave the hot matter. This happens when the path length becomes equal to the mean distance to leave the cylinder. We calculate the path length by treating the quarks as random walkers, with thermal velocities and mean free paths. The rms distance moved by a quark (or antiquark) of flavor $Q$, $r_Q$, from creation at proper time $\tau_0 \simeq 0$ to freezeout at proper time $\tau_f$ is given by

$$r_Q^2 = \int_0^{\tau_f} d\tau v_Q[T(\tau)] \lambda_Q[T(\tau)], \quad (20)$$

where $v_Q$ and $\lambda_Q$ are respectively the thermal velocity and mean free path for flavor $Q$, evaluated at $T(\tau)$. We relate $T$ and $\tau$ by treating the deconfined matter as a non-interacting gas with 16 boson and 24 fermion degrees of freedom, assuming that entropy is conserved, and that the final state can be treated as a massless Bose gas, so that

$$\pi R_f^2 \tau \times \frac{74\pi^2}{45} T^3 = 3.6 \frac{dN}{dy}, \quad (21)$$

where $dN/dy$ is the total rapidity density (charged plus neutral). As we neglect transverse expansion, we set $R_f$ equal to the nuclear radius,

$$R_f = 1.2 A^{1/3} \text{ fm.} \quad (22)$$

The assumption that $\tau_0 = 0$ is good for $c$ and $b$ quarks, which are only created early in the collision, but not necessarily for $s$ quarks which are created as long as there is any deconfined matter. However, we make this approximation for simplicity.

"Wandering" freezeout of heavy quarks occurs in deconfined matter when

$$\left(\frac{4}{3} R_f\right)^2 \leq \frac{9\pi}{4m_Q} \tau_c \rightarrow dN/dy \geq \frac{37 \times 64\pi^2}{3^8} m_Q T_c^3 R_f^4 \simeq 3.6 m_Q T_c^3 R_f^4, \quad (23)$$

where $\tau_c$ is the proper time at which the deconfined matter reaches $T_c$. Including transverse expansion increases $R_f$ and decreases $T$ for fixed $\tau$, making it less likely that $c$ or $b$ quarks freeze out before the hot matter reaches $T_c$. The condition for "wandering" freezeout in the coexistence region is

$$dN/dy \geq \frac{2368\pi^2}{6561 \{1 + (\nu - 1)[(\nu - 1) - (\nu - 2) \ln 2]\}} m_Q T_c^3 R_f^4, \quad (24)$$

if there is no transverse expansion. For smaller values of $dN/dy$, "wandering" freezeout occurs during the hadronic phase.

For "wandering" of $s$ quarks, we again take Eq. (20), but with $v_Q = 1$. Using $\lambda_s(T)$ from Eq. (15), we find that $s$ quarks freeze out from deconfined matter if

$$dN/dy \geq \frac{1184\pi^3}{2187} T_c^3 R_f^4 \simeq 16 T_c^4 R_f^4. \quad (25)$$

"Wandering" freezeout from the coexistence region will occur if
\[ dN/dy \geq \frac{1184\pi^3}{729 \left\{ 3 + 4(\nu - 1) \left[ (\nu - 1) - (\nu - 2) \ln 2 \right] \right\}} T_c^4 R_f^4. \]  

(26)

For smaller values of \( dN/dy \), “wandering” freezeout takes place from hadronic matter. Again, including transverse expansion would shorten the duration of the hot matter at any entropy density, so the above inequalities give solid lower limits on the values of \( dN/dy \) needed for “wandering” freezeout.

C. Freezeout summary

We have evaluated freezeout conditions for \( s \), \( c \), and \( b \) quarks in proton collisions at SPS, RHIC, and LHC energies. We find that the mean number of collisions before freezeout,

\[ n_f = \int_0^{\tau_f} d\tau \Gamma \left[ T(\tau) \right], \]  

(27)

is much less than unity for proton collisions at any feasible energy. Thus, the behavior of heavy quarks in proton collisions probably cannot be described by the quasi-equilibrium models used here, and so predictions for proton collisions are beyond the scope of this paper.

In Table I, we show freezeout conditions for the various heavy quark flavors in selected AA collisions at SPS, RHIC, and LHC. We use four scenarios, with \( \nu = 5 \) and 10, and \( T_c = 150 \) and 200 MeV. This gives a reasonable range of values for \( T_c \), while \( \nu = 10 \) is near the upper limit for a strong first-order QCD phase transition, and \( \nu = 5 \) gives a moderately strong phase transition. These values are compatible with lattice gauge theory results.

The number of collisions per quark is only weakly dependent on \( \nu \), \( T_c \), and flavor. For \( S+S \) collisions at SPS, \( n_f \simeq 1 \), so quasi-equilibrium models are not ridiculous but are also not very well justified. For \( Au+Au \) collisions, \( n_f \) is significantly larger than unity, and thus quasi-equilibrium models are at least self-consistent even at SPS. This is in general agreement with Parton Cascade Model (PCM) results for \( Au+Au \) collisions at RHIC and LHC, [10] which show that all quarks follow approximately hydrodynamic behavior, with mean momenta following the temperature of the surrounding matter.

None of the quark flavors freeze out above \( T_c \). The \( s \) quarks freeze out before the end of coexistence in any collision if \( \nu = 10 \). For \( \nu = 5 \), they may freeze out in the hadronic phase, especially if \( T_c \) is relatively high. In general, freezeout of \( s \) quarks typically occurs at \( T_f \simeq T_c \).

The \( c \) quarks also freeze out in the coexistence region if \( \nu = 10 \), except in heavy ion collisions at SPS and RHIC if \( T_c \) is large. If \( \nu = 5 \), they freeze out below \( T_c \). Since these are upper limits, it is likely that \( c \) quarks also freeze out in the coexistence region if there is a strong QCD phase transition, but they freeze out from hadronic matter if the transition is weak.

The \( b \) quarks may freeze out in the coexistence region in \( Au+Au \) collisions at LHC, but only if the transition is strong and \( T_c \) is relatively low. In all other cases considered, they freeze out far below \( T_c \). Thus, accurate predictions of \( b \) quark physics in \( Au+Au \) collisions below LHC energy (and possibly at LHC, if \( T_c \simeq 200 \) MeV) probably requires accurate cross sections for the various \( b \) mesons in hadronic matter, and possibly the use of a microscopic hadronic cascade code.
The energy required for freezeout in the coexistence region will be lower in pA and AB collisions than in AA collisions. However, including transverse expansion will raise the energy thresholds, as might more realistic estimates of heavy resonance cross sections in hadronic matter, so the values given here are probably lower bounds for freezeout before the end of coexistence.

III. THE THERMAL RECOMBINATION MODEL

We use a thermal recombination model that is similar to the models of Refs. [4, 5]. The major difference is that we introduce a pair chemical potential to account for the fact that the quarks and antiquarks are always produced in pairs, so the local density of pairs may be much larger than the product of the quark and antiquark densities:

\[ \rho^{(2)}_{Q\overline{Q}}(x, x) \gg \rho_{Q}(x) \rho_{\overline{Q}}(x). \]  

(28)

We assume that the heavy quark flavors (s, c, and b) are far from chemical equilibrium before they stop interacting thermally, [9] so that the densities of heavy quarks and antiquarks at thermal freezeout are separately conserved. The chemical potential for hadron species \( n \) is determined by the quark, antiquark, and pair chemical potentials, respectively \( \mu_{Q}, \mu_{\overline{Q}}, \) and \( \mu_{Q\overline{Q}} \):

\[ \mu_{n} = \sum_{Q} \left( k_{n}^{(Q)} \mu_{Q} + k_{n}^{(Q)} \mu_{\overline{Q}} + k_{n}^{(Q\overline{Q})} \mu_{Q\overline{Q}} \right). \]  

(29)

Here \( k_{n}^{(Q)} \) (\( k_{n}^{(Q)} \)) is the number of quarks (antiquarks) of flavor \( Q \), and \( k_{n}^{(Q\overline{Q})} \) is the number of pairs (the smaller of \( k_{n}^{(Q)} \) and \( k_{n}^{(Q)} \)). We assume that the number of hadrons of species \( n \) produced is proportional to its density at temperature \( T_{f} \) and chemical potential \( \mu_{n} \).

The chemical potentials are determined by conservation of quark and antiquark densities, respectively \( \rho_{Q} \) and \( \rho_{\overline{Q}} \), and the local two-particle quark-antiquark density, \( \rho^{(2)}_{Q\overline{Q}} \). [We use the more compact notation \( \rho^{(2)}_{Q\overline{Q}} = \rho^{(2)}_{Q\overline{Q}}(x, x) \) for the remainder of this paper.] Since there is no chemical equilibrium between quarks and antiquarks, the constraint \( \mu_{Q\overline{Q}} = -\mu_{Q} \) does not apply. The equations are

\[ \rho_{Q} = \sum_{n_{b}} \frac{g_{n_{b}} k_{n_{b}}^{(Q)}}{2 \pi^{2}} \int_{0}^{E_{max}(p_{T})} \frac{p^{2} \, dp}{e^{[E_{Q}(p_{T})-\mu_{n_{b}}]/T_{f}} - 1} \]  

(30)

\[ \rho_{\overline{Q}} = \sum_{n_{b}} \frac{g_{n_{b}} k_{n_{b}}^{(Q)}}{2 \pi^{2}} \int_{0}^{E_{max}(p_{T})} \frac{p^{2} \, dp}{e^{[E_{\overline{Q}}(p_{T})-\mu_{n_{b}}]/T_{f}} - 1} \]  

(31)

\[ \rho^{(2)}_{Q\overline{Q}} = \sum_{n_{b}} \frac{g_{n_{b}} k_{n_{b}}^{(Q\overline{Q})}}{2 \pi^{2}} \int_{0}^{E_{max}(p_{T})} \frac{p^{2} \, dp}{e^{[E_{Q\overline{Q}}(p_{T})-\mu_{n_{b}}]/T_{f}} - 1} \]  

(32)

\[ \mu^{'}_{n} = \mu_{n} + \mu_{Q\overline{Q}}. \]  

(33)
\[ E_n(p) = \left( p^2 + m_n^2 \right)^{1/2}. \] (34)

Here the sums are over bosons \((n_b)\) and fermions \((n_f)\), while \(m_n\) and \(g_n\) are respectively the mass and degeneracy for hadron \(n\).

We estimate \(\rho_{Q\overline{Q}}^{(2)}\) by again treating the quarks as random walkers. We assume that the \(Q\) and \(\overline{Q}\) are distributed randomly over a sphere centered on the point at which the pair was formed, such that the rms distances from the center are \(r_Q\). We thus obtain

\[ \rho_{Q\overline{Q}}^{(2)} \approx \rho_Q \left[ \rho_{\overline{Q}} + \frac{3}{4\pi} \left( \frac{3}{5 r_Q^2} \right)^{3/2} \right], \] (35)

where \(\rho_Q\), \(\rho_{\overline{Q}}\) and \(r_Q\) are determined at freezeout. We obtain the rhs of Eq. (32) by averaging the \(Q\overline{Q}\) pairs in hadrons over the same sphere to get the mean value of \(\rho_{Q\overline{Q}}^{(2)}\) for the hadrons.

For copiously produced quarks, \(\rho_{Q\overline{Q}}^{(2)} \approx \rho_Q \rho_{\overline{Q}}\), so the third constraint equation (32) just gives \(\mu_{Q\overline{Q}} \approx 0\).

### A. J/\(\psi\) suppression

One possible application of this model is to understand the phenomenon of \(J/\psi\) suppression. The prediction \cite{2} is that \(\sigma\) pairs will tend to form \(J/\psi\) mesons less often in collisions in which QGP is formed, as a result of interactions of the pair with the deconfined matter. Thus, if QGP is formed in nuclear collisions, production of \(J/\psi\) mesons will be suppressed relative to production of \(\sigma\) pairs.

In this section, we show how suppression of heavy \(Q\overline{Q}\) mesons arises in our model. For illustrative purposes, we consider a case for which there are only three hadron species. Species 1 hadrons contain a single \(Q\) and have degeneracy \(g_1\) and mass \(m_1\). Species 2 hadrons are the antiparticles of species 1, so they contain a single \(\overline{Q}\) and have degeneracy \(g_1\) and mass \(m_1\). Species 3 hadrons contain a \(Q\overline{Q}\) pair, and have degeneracy \(g_2\) and mass \(m_2 \simeq 2m_1\). This is similar to the situation for \(c\) quarks, where the \(D\) and \(\overline{D}\) could be regarded as species 1 and 2 and the \(J/\psi\) as species 3.

Because heavy quarks freeze out at \(T_f \ll m_Q\), we use Maxwell-Boltzmann statistics for the heavy hadrons. We take \(\rho_Q = \rho_{\overline{Q}}\), because heavy quarks are always produced in pairs so the net density is always zero. We then solve the constraint equations, obtaining

\[ \frac{\rho_3}{\rho_1} = \frac{1}{2} \left\{ -1 + \left[ 1 + \frac{4g_2}{g_1^2} \left( \frac{2\pi m_2}{m_1^2 T_f} \right)^{3/2} \right] e^{(2m_1 - m_2)/T_f} \frac{\rho_{Q\overline{Q}}^{(2)}}{\rho_Q} \right\}^{1/2}. \] (36)

It is clear from eq. (36) that the relative densities of the resonances depends only on \(T_f\) and the local density, \(\rho_{Q\overline{Q}}^{(2)}/\rho_Q\). This remains true for more complicated resonance spectra as long as all resonances have at most one quark and one antiquark, if the flavor is heavy enough to justify the use of Maxwell-Boltzmann statistics. These conditions are met by the flavors \(c\) and \(b\), so studies of the relative production of \(c\) and \(b\) resonances (except for exotic
multi-heavy hadrons \cite{1}) will be insensitive to \( \rho_c \) and \( \rho_b \). Thus, the only way to determine \( \rho_c \) and \( \rho_b \) (in the absence of exotic multi-heavy hadrons) is through comparisons of heavy and light resonance production. Since there are many multi-strange baryons, \( \rho_s \) can be determined solely by studying strange resonances, as long as these multi-strange baryons are detected.

\textbf{B. Freezeout density estimates}

We estimate freezeout densities from rapidity densities, staying as close as possible to predictions supported by data. In the absence of transverse expansion,

\[ \rho_Q = \rho_{\bar{Q}} = \frac{dN_Q/dy}{\pi R_f^2 \tau_f}, \]

and we obtain \( \rho_{Q(\bar{Q})}^{(2)} \) from eq. (35). The predicted freezeout densities are shown in Table II; for the interested reader, we explain our estimates of \( dN_Q/dy \) below. The main point of interest is that \( \rho_{Q(\bar{Q})}^{(2)} \gg \rho_Q \rho_{\bar{Q}} \) for \( b \) quarks in all collisions considered, and for \( c \) quarks except in Au+Au collisions at LHC. For \( s \) quarks, the difference is less than 10% in most collisions, which is probably within experimental error and thus not of great interest.

The PCM reproduces strange particle production in \( \bar{p}p \) collisions at \( \sqrt{s} = 546 \) GeV fairly well, \cite{11} so we assume that it gives reasonable predictions for \( dN_s/dy \) in \( AA \) collisions at RHIC and LHC. \cite{12} The factor of 5 increase in \( s \) production from RHIC to LHC is roughly consistent with most \( s \) quarks being produced thermally, which gives \( dN_s/dy \propto (dN/dy)^2/A^{2/3} \) for \( AA \) collisions with the same initial temperature. \cite{9} For \( S+S \) collisions at SPS, we use data \cite{13} to estimate \( dN_s/dy \). For \( Au+Au \) collisions at SPS, we extrapolate down from RHIC energy by assuming that \( dN_s/dy \propto dN/dy \).

PCM results for \( c \) and \( b \) production in nuclear collisions \cite{12} may be overestimates, especially for lower energies, because of the “flavor excitation” processes that are included. PCM results for charm production in \( pp \) collisions match perturbative results, \cite{14} if only flavor creation is included but not flavor excitation. \cite{15} However, flavor excitation is included in PCM simulations of nuclear collisions, so they may overestimate the number of heavy quarks that are produced. This overestimate is relatively small at LHC, where the two production mechanisms have approximately equal weight, but is large below LHC energy, where flavor excitation dominates.

Using PCM results without flavor excitation from Fig. 4 of Ref. \cite{12}, \( dN_c/dy\)\( |_{y=0} \approx 2 \) for central Au+Au collisions at RHIC and 60 at LHC. \cite{16} Simple superposition of perturbative \( pp \) collision predictions, \cite{14} assuming that \( dN_c/dy \) scales as \( A^{1/3} \) and that the \( c \) quarks are spread over 4 units of rapidity at RHIC and 5 units of rapidity at LHC, gives respectively 3 and 10. These rapidity intervals are compatible with PCM results, but smaller than those predicted for \( pp \) collisions. \cite{17}

Even without flavor excitation, the PCM predicts a large increase in charm production (compared to simple superposition of \( pp \) collisions) in Au+Au collisions at LHC but not at RHIC. The estimated thermal charm production \cite{3} is \( dN_c/dy\)\( |_{y=0} \approx 2 \) at RHIC and 10 at LHC, which is too small to account for all of the excess charm at LHC. We make a
conservative estimate, taking perturbative predictions \[17\] and scaling \(dN_c/dy\) as \(A^{4/3}\), to generate the numbers in Table 2. For \(b\) quarks, we obtain all rapidity densities by taking perturbative QCD predictions for \(p\bar{p}\) collisions \[17\] and assuming that \(dN_b/dy\) scales as \(A^{4/3}\) (simple superposition). \[18\]

C. Predictions for \(Q\bar{Q}\) meson production

Thermal recombination models have been used to analyze strange particle production data from ultrarelativistic nuclear collisions, \[4,5\] so we only briefly discuss mesons containing \(s\) quarks, and instead concentrate on \(c\) and \(b\) quark mesons. The only significant difference between our model and others in current use \[4–6\] is the inclusion of the pair chemical potential, which adds an extra degree of freedom to the models. However, we expect that \(\rho_s \simeq \rho_s\rho_T\) in nearly all collisions, so the addition of this extra degree of freedom will give little change to the predictions of the model, while reducing its predictive power. Thus, while the use of \(\mu_s\) may be justified theoretically, it is probably not worthwhile including it when analyzing collision data. Other models also often include a baryon chemical potential, which could be added to our model in the obvious manner.

We show predictions for \(f_Q\), the fraction of quarks of flavor \(Q\) that form \(Q\bar{Q}\) mesons, in Table III. We include all confirmed mesons and baryons, as well as those unconfirmed mesons that have masses less than the highest confirmed meson mass for each flavor. \[19\] For simplicity, we take \(\mu_s = 0\). The uncertainties in \(f_c\) and \(f_b\) are very large, as they depend very strongly on \(\nu\) and \(T_c\).

For fixed \(\nu\) and \(T_c\), both \(f_c\) and \(f_b\) drop from \(S+S\) to \(Au+Au\) collisions at SPS. The magnitude of the drop in \(f_b\) is largest for large \(T_c\) and large \(\nu\), and could provide a constraint on the two quantities as decrease varies from a factor of 1.2 to 4.5. The decrease in \(f_c\) could provide a second, tighter, constraint, as \(f_c\) is between 3 and 30 times smaller in \(Au+Au\) collisions than in \(S+S\) collisions at SPS.

The energy dependences of \(f_b\) and \(f_c\) are also interesting. For all cases considered here, \(f_b\) decreases monotonically with increasing energy. The behavior of \(f_c\) is non-monotonic, so changes are smaller and more theory-dependent. Consequently, \(f_c\) will probably provide useful constraints on \(\nu\) and \(T_c\) only if \(\nu \simeq 10\) and \(T_c \simeq 150\) MeV, in which case \(f_c\) increases from SPS to LHC.

IV. CONCLUSIONS

In nuclear collisions at SPS, RHIC, and LHC, \(s\) and \(c\) quarks are most likely to freeze out during the period of two-phase coexistence (at the QCD transition temperature, \(T_c\)) if \(\nu\) is large corresponding to a strong QCD phase transition. For moderate values of \(\nu\) all heavy quark species are most likely to freeze out from hadronic matter. The \(b\) quarks freeze out at the lowest temperatures. Since these estimates depend mainly on the temperature dependence of the entropy density, but are independent of the exact shape, we expect similar conclusions to hold even if the transition is not first order as long as there is a reasonably sharp change in the entropy density. The mean number of collisions per quark is essentially flavor-independent, and large enough to justify the use of quasi-equilibrium models in heavy
ion collisions at and above SPS energy. Quasi-equilibrium models are only marginally self-consistent for S+S collisions at SPS, and are not expected to work for pp collisions at any attainable energy.

In the absence of exotic multi-heavy baryons, density ratios for the various $c$ and $b$ resonances will constrain the local freezeout densities, $\rho_{QQ}^{(2)}/\rho_Q$, but not the global densities, $\rho_Q$. Thus, determination of $\rho_c$ and $\rho_b$ may require measurement of heavy to light resonance ratios. The local and global densities are predicted to be almost equal for $s$ quarks in all collisions considered, and for $c$ quarks in collisions at LHC. The local $b$ quark density is much higher than the global density in all collisions considered here.

Measurements of the fractions of $c$ and $b$ quarks forming $c$ and $b$ mesons, respectively $f_c$ and $f_b$, may help to constrain $\nu$ and $T_c$. This appears particularly promising in light of the strong dependence of all other quantities on the entropy density ratio and on the critical temperature.

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TABLE I. Freezeout conditions for $s$, $c$, and $b$ quarks. A “c” after $T_f$ indicates that freezeout was due to loss of thermal contact, while “w” denotes freezeout due to emission.

| $Q$ | $A$ | $dN/dy$ | $\nu$: 5 | 10 | 5 | 10 | $\tau_f$ (fm/$c$): 150 | 200 | 150 | 200 |
|-----|-----|---------|-----------|----|----|----|-----------------|-----|-----|-----|
| $s$ | 32  | 85      | 150c      | 150c| 198c| 200c| 2.6            | 5.8 | 2.1 | 2.0 |
|     | 197 | 1000    | 145w      | 150w| 162c| 200c| 18             | 36  | 18  | 16  |
|     | 197 | 2000    | 150w      | 150w| 176w| 200w| 23             | 46  | 23  | 21  |
|     | 197 | 3500    | 150w      | 150w| 197w| 200w| 28             | 55  | 27  | 25  |
| $c$ | 32  | 85      | 140c      | 150c| 176c| 200c| 5.8            | 12  | 5.8 | 2.9 |
|     | 197 | 1000    | 116w      | 150w| 138c| 182c| 36             | 30  | 21  | 18  |
|     | 197 | 2000    | 134w      | 150w| 149w| 198w| 46             | 39  | 33  | 29  |
|     | 197 | 3500    | 150w      | 150w| 168w| 200w| 55             | 49  | 41  | 35  |
| $b$ | 32  | 85      | 110c      | 145c| 138c| 182c| 12             | 10  | 6.0 | 5.2 |
|     | 197 | 1000    | 91w       | 120w| 108c| 143c| 75             | 65  | 44  | 38  |
|     | 197 | 2000    | 104w      | 138w| 117w| 154w| 98             | 84  | 70  | 60  |
|     | 197 | 3500    | 117w      | 150w| 131w| 173w| 121            | 103 | 87  | 75  |

| $r_Q$ (fm) | $n_f$ | $s$ | 32  | 85 | 2.9 | 2.5 | 2.6 | 1.9 | 1  | 1  | 1  | 1  |
|            |       | 197 | 1000| 9.3| 9.3 | 8.2 | 8.5 | 5  | 4  | 4  | 3  |
|            |       | 197 | 2000| 9.3| 9.3 | 9.3 | 9.3 | 8  | 7  | 6  | 5  |
|            |       | 197 | 3500| 9.3| 9.3 | 9.3 | 9.3 | 11 | 11 | 9  | 8  |
| $c$ | 32  | 85 | 3.2 | 2.5 | 2.4 | 2.0 | 1  | 1  | 1  | 1  |
|     | 197 | 1000| 9.3| 9.3 | 8.0 | 8.1 | 6  | 5  | 4  | 3  |
|     | 197 | 2000| 9.3| 9.3 | 9.3 | 9.3 | 9  | 8  | 7  | 6  |
|     | 197 | 3500| 9.3| 9.3 | 9.3 | 9.3 | 13 | 12 | 10 | 8  |
| $b$ | 32  | 85 | 3.1 | 3.2 | 2.3 | 2.3 | 2  | 1  | 1  | 1  |
|     | 197 | 1000| 9.3| 9.3 | 7.9 | 7.9 | 7  | 4  | 4  | 4  |
|     | 197 | 2000| 9.3| 9.3 | 9.3 | 9.3 | 13 | 11 | 8  | 7  |
|     | 197 | 3500| 9.3| 9.3 | 9.3 | 9.3 | 19 | 16 | 13 | 11 |
TABLE II. Freezeout densities.

| $T_c$ (MeV) | $\rho_Q$ ($10^{-5}$ fm$^{-3}$) | $\rho_Q^{(2)}/\rho_Q^{(1)}$ |
|-------------|-------------------------------|-----------------------------|
| $A$ | $dN/dy$ | $dn_Q/dy$ | $\nu$: 5 | 10 | 5 | 10 | $\rho_Q^{(2)}/\rho_Q^{(1)}$ | 150 | 200 | 150 | 200 |
| $s$ | 32  | 85  | 0.06 | 11  | 18  | 22  | 33  | 30  | 41  | 37  | 45  |
| 197 | 1000  | 5  | 5.5  | 6.4  | 9.2  | 11  | 3.5  | 3.1  | 3.4  | 3.0  |
| 197 | 2000  | 50  | 1400  | 1500  | 1600  | 1800  | 1.0  | 1.0  | 1.0  | 1.0  |
| 197 | 3500  | 250  | 5800  | 6000  | 6400  | 7200  | 1.0  | 1.0  | 1.0  | 1.0  |
| $c$ | 32  | 85  | 0.0003 | 0.06 | 0.06 | 0.11 | 0.13 | 6700 | 5600 | 8100 | 6800 |
| 197 | 1000  | 0.3 | 0.03  | 0.03  | 0.05  | 0.05  | 530  | 450  | 500  | 430  |
| 197 | 2000  | 1  | 0.13  | 0.16  | 0.19  | 0.22  | 100  | 89  | 75  | 65  |
| 197 | 3500  | 5  | 0.02  | 0.14  | 0.17  | 0.20  | 0.23  | 1.0  | 1.0  | 1.0  | 1.0  |
| $b$ | 32  | 85  | 0.0003 | 0.06 | 0.06 | 0.11 | 0.13 | 6700 | 5600 | 8100 | 6800 |
| 197 | 1000  | 0.003 | 0.03  | 0.03  | 0.05  | 0.05  | 530  | 450  | 500  | 430  |
| 197 | 2000  | 0.02 | 0.13  | 0.16  | 0.19  | 0.22  | 100  | 89  | 75  | 65  |
| 197 | 3500  | 0.4 | 0.02  | 0.14  | 0.17  | 0.20  | 0.23  | 1.0  | 1.0  | 1.0  | 1.0  |

TABLE III. Predicted $c\bar{c}$ and $b\bar{b}$ meson fractions.

| $T_c$ (MeV): | $f_c$ | $f_b$ |
|-------------|-------|-------|
| $A$ | $dN/dy$ | $\nu$: 5 | 10 | 5 | 10 | $\nu$: 5 | 10 | 5 | 10 |
| 32  | 85  | 0.066  | 0.075  | 0.029  | 0.021  | 0.62  | 0.16  | 0.36  | 0.072  |
| 197 | 1000  | 0.021  | 0.002  | 0.008  | 0.001  | 0.50  | 0.053  | 0.20  | 0.016  |
| 197 | 2000  | 0.009  | 0.004  | 0.004  | 0.001  | 0.20  | 0.013  | 0.070  | 0.005  |
| 197 | 3500  | 0.009  | 0.010  | 0.005  | 0.002  | 0.075  | 0.008  | 0.027  | 0.002  |