Theory of non-Fermi liquid near a diagonal electronic nematic state on a square lattice

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We study effects of Fermi surface fluctuations on a single-particle lifetime near the diagonal electronic nematic phase on a two-dimensional square lattice. It has been shown that there exists a quantum critical point (QCP) between the diagonal nematic and isotropic phases. We study the longitudinal fluctuations of the order parameter near the critical point, where the singular forward scattering leads to a non-Fermi liquid behavior over the whole Fermi surface except along the $k_x$- and $k_y$-directions. We will also discuss the temperature and chemical potential dependence of the single-particle decay rate.

Introduction — In strongly correlated electronic materials, electrons can organize themselves into a pattern with periodicity differing from that of the underlying lattice along a particular direction. The phase with such a non-trivial charge ordering breaks a discrete translational symmetry of underlying lattice along one direction, and has been named as stripe. The stripe physics has been of great interest to both theoretical and experimental condensed matter physicists, especially because its quantum fluctuation may play an important role in understanding the mechanism of high temperature superconductivity. The clear evidence of such a novel paired stripe order was reported by Mori et al in electron diffraction experiment on (La,Ca)MnO$_3$.

It has been proposed that phases with fluctuating stripes can be generic ground states of a doped Mott insulator introduced by the hole doping. These intermediate forms of matter have been dubbed as electronic smectic and nematic phases. While the smectic breaks a translational symmetry, the electronic nematic phase breaks a discrete rotational symmetry of underlying lattice, which can be viewed as fluctuating stripes. The electronic nematic phase has been referred as Pomeranchuk instability of Fermi liquid, and has been studied in theoretical models such as t-J, (extended) Hubbard, and Fermi liquid models. The electronic nematic phase has been suggested as a phase responsible for novel phenomena observed in strongly correlated systems. These phenomena include, for example, the anisotropic resistivities in high Landau levels of quantum Hall systems, the anisotropic scattering patterns observed in one of high temperature superconductors, and the metamagnetic transition and anisotropic transports in Sr$_3$Ru$_2$O$_7$.

In addition to the anisotropic nature of electronic nematic phase, it was shown that the gapless Goldstone mode of the nematic phase in continuum model leads to the non-Fermi liquid behavior in single-particle scattering rate on Fermi surface except along the symmetric directions. However, using the same model of quadrupole density interaction on the square lattice, it was shown that the putative continuous transition is preempted by a first order transition due to the van Hove singularity. The regime of nematic phase diminishes exponentially as one approaches the continuum limit of small density with approximately circular Fermi surface. Therefore, there is no critical Fermi surface fluctuations for the non-Fermi liquid behavior in realistic systems with underlying lattice; both transverse (phase) and longitudinal (amplitude) modes are gapped. On the other hand, it was later found that additional uniform interaction in the forward scattering channel suppresses the first order transition. This restores the non-Fermi liquid behavior at the Fermi surface, except along the zone diagonal direction, via the coupling to the fluctuation of order parameter at the quantum critical point.

In this paper, we present theory of non-Fermi liquid near the diagonal nematic phase. It is worthwhile to note that there are two distinct nematic states in the square lattice. Previous studies provide understanding of phenomena related to Fermi surface distortion along the parallel direction of $k_x$- or $k_y$-axis of the Brillouin zone. The diagonal nematic phase can be characterized by a Fermi surface distortion along the diagonal direction of the Brillouin zone. It is shown that the transition between the diagonal nematic and isotropic phases is second order, because it does not involve the splitting of the van Hove singularity in the density of states. We find the non-Fermi liquid behavior in the single-particle life time at the Fermi surface , through the coupling to the longitudinal mode of diagonal nematic order, except along the zone parallel directions ($x$- and $y$-axis of Brillouin zone). Our numerical results of self energy show the expected exponent of $2/3$ in frequency similar to the parallel nematic phase and the continuum model. However, the physical origin of the non-Fermi liquid behavior is different for these two cases.

Model — The model Hamiltonian for the parallel and the diagonal nematic phases on a square lattice is written as follows,

\[ H = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} - \sum_{k k' q \sigma' \sigma} [F_2(q) \zeta_1(k) \zeta_1(k') + G_2(q) \zeta_2(k) \zeta_2(k'')] \times c_{k+\frac{q}{2} \sigma}^\dagger c_{k'-\frac{q}{2} \sigma'}^\dagger c_{k+\frac{q}{2} \sigma'} c_{k'-\frac{q}{2} \sigma}, \]

where $\zeta_1(k) = \cos k_x - \cos k_y$, and $\zeta_2(k) = 2 \sin k_x \sin k_y$ are the form factors originating from the quadrupole density-diffusion interaction, and $\varepsilon_k$ is the tight binding dispersion. We assume short range quadrupolar interactions of $F_2(q)$ and $G_2(q)$ with the form...
The competition between the diagonal and the parallel nematics leads to the suppression of both phases, while they coexist in a finite window of chemical potential. The order parameters associated with the parallel ($\Delta_P$) and diagonal ($\Delta_D$) nematic phases are given by

$$\Delta_P = F_2 \sum_k (\cos k_x - \cos k_y) \langle c^\dagger_k c_k \rangle,$$

$$\Delta_D = G_2 \sum_k (2 \sin k_x \sin k_y) \langle c^\dagger_k c_k \rangle.$$  

Solving the above mean-field equation, Eq. (2), one can obtain the behavior of nematic order parameters as one varies the ratio of $F_2/G_2$. The transition to the diagonal nematic ordered state from isotropic liquid phase occurs above a critical value of interaction $G_2$, and it is second order as a function of chemical potential. The combination of the diagonal and the parallel nematics leads to the suppression of both phases, while they coexist in a finite window of chemical potential. The longer range hopping, such as $t'$, is ignored in this study, since qualitative features are not affected by a finite $t'$.  

To study the behavior of non-Fermi liquid near the QCP, we set our parameter as follows: $F_2 N_0 = 0.1$ and $G_2 N_0 = 0.196$, where $N_0 = 1/(2\pi t^2)$ is the density of states at the Fermi surface. With these parameters, the QCP occurs near $\mu_c/(2t) = -0.2$. While the parallel nematic also occurs at smaller value of $\mu$ within the above parameter space, below we will present the results near the $\mu_c$, since the non-Fermi liquid behavior is realized only near the QCP associated with the diagonal nematic phase. We first consider the dynamical effective interaction between the quasi-particles.  

**Susceptibility in Random Phase Approximation (RPA)** — The effective interaction mediated by the collective modes at RPA level is given by

$$\chi_N(q, \nu) = \frac{G_2(q)}{1 - G_2(q) \Pi_N^0(q, \nu)},$$

and the bare dynamical polarizability of the diagonal nematics is given by

$$\Pi_N^0(q, \nu) = -\int \frac{d^2 p}{(2\pi)^2} \frac{f(\epsilon_p + q/2) - f(\epsilon_p - q/2)}{\epsilon + i\eta - (\epsilon_p + q/2 - \epsilon_p - q/2)} \zeta_2(p)^2,$$

where $f(\epsilon)$ is the Fermi function. Near the nematic transition, the denominator in Eq. (3) becomes very small for $q$ slightly above the QCP. The diagonal nematic fluctuations have a propagating mode along the zone parallel directions and the mode becomes diffusive along the zone diagonals. This can be contrasted with the nematic Goldstone modes in the continuum case.  

In the inset of Fig. 1 we plot the dispersion of these modes. For the highly damped mode along the diagonal direction, the dispersion is determined by the peak position of the imaginary part of the effective interaction in the forward scattering limit, and eventually non-Fermi liquid behavior. We carried out numerical integration of the above equation. In Fig. 1, we plot the imaginary part of the effective interaction for the diagonal nematic is shown for small values of $q$ for $\mu/2t = -0.22$ slightly above the QCP. The diagonal nematic fluctuations has a propagating mode along the zone parallel directions and the mode becomes diffusive along the zone diagonals. The imaginary part of the effective interaction becomes very small for $q \to 0$ and $\nu \to 0$, if $\nu$ vanishes faster than $q$. This results in a singular behavior of the effective interaction in the forward scattering limit, and eventually non-Fermi liquid behavior.  

In the inset of Fig. 1 we plot the dispersion of these modes. For the highly damped mode along the diagonal direction, the dispersion is determined by the peak position of the energy of the diagonal nematic fluctuations for small $q$. For $\mu/2t = -0.22$ slightly above the QCP, the diagonal nematic fluctuations has a propagating mode along the zone parallel directions and the mode becomes diffusive along the zone diagonals. This can be contrasted with the nematic Goldstone modes in the continuum case.  

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**Quasiparticle Self-energy** — To capture this singular behavior of the effective interaction, we expand the $\Pi_N^0$ in the small $q$ and small $\omega/q$ limit as follows.

$$\Pi_N^0 \approx a + bq^2 + ic\frac{\omega}{q} + O\left(\frac{\omega^2}{q^2}\right)$$

where coefficients $a, b$ and $c$ can be determined numerically, and depend on the underlying band structure and temperature. The closeness to the QCP can thus be quantified.
by quantity \( \delta = 1 - G_2(0)a \). To understand how the decay rate for single-particle excitations is modified by the collective fluctuations, we calculate the imaginary part of the self-energy. At one-loop level, the imaginary part of the self-energy in the real frequency is given by

\[
\text{Im}\Sigma(k, \omega) = -\int \frac{d\nu}{2\pi} \int \frac{d^2q}{(2\pi)^2} \zeta_2(k) \zeta_2(k + \frac{q}{2}) \times
\]

\[
\left[ b(\nu) + f(\nu + \omega) \right] \text{Im} \chi_N(q, \nu) \text{Im} G(k + q, \omega + \nu)
\]

In the lowest order approximation, we use the non-interacting Green’s function and \( \text{Im} G_0(k, \omega) = -\pi\delta(\omega - \epsilon_k) \) in Eq. (6). Figure 2 shows the imaginary parts of the self energies near the QCP both in the symmetric phase (\( \mu/2t = -0.3, -0.22 \)) near the QCP (\( \mu/2t = -0.2 \)) and inside the diagonal nematic phase (\( \mu/2t = -0.18 \)) at the Fermi surface along the \( (\pi, \pi) \) direction in the low \( T \) limit. At small \( \omega \), the self-energy shows clear deviation from the Fermi liquid behavior with the asymptotic behavior \( \text{Im}\Sigma \sim |\omega|^\alpha, \alpha \leq 1 \). The scattering rate is the largest close to the QCP at \( \mu/2t = -0.2 \), and self-energy shows a strong non-Fermi liquid behavior with \( \alpha \sim 2/3, 2/7, 2/11 \). This indicates that the quasiparticle excitations are not well defined approaching the Fermi surface, which rules out a Fermi-liquid description. This non-Fermi liquid behavior exists at all parts of the Fermi surface except at four points along the \( x \) and \( y \) axes, where the quasiparticle is well defined. In Fig. 2, we also compute the contribution from the regular part of \( \Pi_N^0 \) to the self-energy, and we find the amplitude is always smaller than that of the singular part. This is due to the smaller density of states at the Fermi surface along the nodal directions. This should be contrasted to the parallel nematic order, where the density of the states at the Fermi surface is large due to the closeness of the Van Hove singularity near \( (\pi, 0) \). The Fermi liquid behavior is restored far from the QCP (\( \mu = -0.3 \)), where the singular and the regular parts overlap at low \( \omega \).

In Fig. 3 we show the imaginary part of the self-energy at the Fermi surface in the nodal direction with \( \mu/2t = -0.2 \) at different temperatures. This corresponds to the quantum critical regime above the QCP. Finite temperature effects enter through the finite temperature boson and fermi functions, as well as the temperature dependence of the expansion coefficients in \( \Pi_N^0 \). In addition to the singular contribution of \( \Pi_N^0 \), the regular part is also included in the calculation. In the quantum critical regime at \( T > 0 \), the self-energy consists both contributions from the classical and the quantum fluctuations. The quantum part of the self-energy obeys \( (\omega/T) \) scaling in the quantum critical regime, while the classical part dominates at \( \omega = 0 \), and scales as \( \text{Im}\Sigma(k_F, \omega) \propto T^{-\xi}(T) \propto \sqrt{T/\ln T} \). We observe this scaling of the classical fluctuations in the \( \text{Im}\Sigma(k_F, \omega) \); however, our numerical precision cannot separate these two contributions.

Conclusion—A consequence of the nematic order is the distortion of the underlying Fermi surface due to the development of the nematic phase. It has been proposed that the Fermi surface distortion and critical Fermi surface fluctuations play an important role in strongly correlated electron systems. Recent experiments in \( Sr_1-xRu_2O_7 \) near the metamagnetic QCP suggest that the formation of a spin-dependent nematic phase can explain the experimental observation of the strong anisotropy in the magnetoresistivities. It has been also suggested that the in-plane anisotropy of spin dynamics in detwinned \( YBa_2Cu_3O_{7-\delta} \) have indicated a possible existence of two dimensional anisotropic liquid crystalline phase in high temperature cuprates. Close to the Pomeranchuk instability of the electronic system, the dynamics of the lattice distortion can be dramatically amplified. It has been argued that electron-phonon interaction indeed plays an important role in the high-\( T_c \) superconductors. The anisotropic form factors of the \( B_{1g} \) buckling phonons has the form of \( (\cos k_x - \cos k_y)^{1/2} \). Therefore, it is interesting to note that the effective interaction between electrons after integrating out the phonon mode has the form of \( \chi_1(k) \) in our Eq. (1) for...
the small $q$ limit. While the existence of a nematic phase still need further investigation, it is tempting to speculate that the non-Fermi liquid behaviors in these systems can be partially attributed to the closeness to the nematic-isotropic transition, where the Fermi surface becomes soft.

In summary, we have studied the fluctuation effects of the diagonal electronic nematic order in the square lattice, using a model Hamiltonian with a quadrupole density-density interaction. We found that the collective mode associated with the diagonal nematic fluctuations has a strong anisotropy with diffusive peaks around the Fermi surface except along the $(0, \pm \pi)$ and $(\pm \pi, 0)$ directions. Close to the QCP, this mode becomes critical and strongly influences the decay rate of the quasi-particle; the imaginary part of the self-energy shows a non-Fermi liquid behavior. We found the decay rate at the Fermi surface is highly anisotropic and reaches a maximum at the QCP. It will be interesting to explore the effects of the collective mode on charge and heat transports, Raman spectroscopy, and electron-phonon interaction near the transition.

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