ISAR Imaging Based on Block Sparse Smoothed L0 Norm Recovery Algorithm

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Abstract. In order to obtain high resolution inverse synthetic aperture radar (ISAR) sparse images, a block sparse signal recovery ISAR imaging algorithm is proposed by considering the cluster characteristics of target in this paper. Firstly, the ISAR sparse imaging model is established, the imaging is converted to L0 norm optimization problem. Secondly, one negative exponential function sequence is used as smoothed function to approach the block L0 norm. Finally, the revised step is added to ensure solving the optimization problem along the steepest descent gradient direction and the cost function is updated for the next loop. Simulation results verify the proposed algorithm is effective.

1. Introduction
Inverse synthetic aperture radar (ISAR) has the ability of achieving high-resolution image of target in all weather and day/night conditions, which is used widely in many military and civilian applications [1-2]. To obtain high resolution ISAR image, the coherent processing interval (CPI) must be long enough. In order to improve the cross-range resolution in a short CPI, super-resolution method can be used.

By utilizing the sparse distribution of a signal, sparse signal recovery algorithm has the property of super-resolution and has been used in ISAR/SAR imaging [3-4]. Because ISAR target is relatively small compared with the imaging region and the strong scatterers can occupy many resolution cells, the target scatters have block or group structure. If the structure of the sparse signal is exploited, the better recovery performance can be obtained.

In order to improve ISAR imaging performance, a block sparse signal recovery ISAR imaging algorithm is proposed. The proposed block sparse signal recovery ISAR imaging algorithm doesn’t need the information of the number of blocks. Real target imaging results show that this approximation is valid.

2. ISAR imaging model
Assume the signal transmitted is a linear chirp signal:
where $\hat{t}$ is the fast time, $f_c$ denotes the carrier frequency, $\gamma$ is the chirp rate. $T_a$ is pulse duration, rect($\cdot$) is rectangle pulse function. Assume the distance unit includes $K$ strong scatterers, the signal is

$$y(t) = \sum_{k=1}^{K} x_k \cdot \text{rect}\left(\frac{t}{T} \right) \cdot \exp(-j2\pi f_c t) + n$$

where $x_k$ and $f_k$ are the $k$th scattering centers’ amplitude and Doppler frequency. $n$ is the additive noise. The time sequence is $t = [1:N]^T \cdot \Delta t$, $\Delta t = 1/f_r$ is the time interval, $f_r$ is the pulse repetition frequency.

$N = T_a / \Delta t$ is the number of pulses. $\Delta f_d$ is the Doppler frequency resolution, the sparse Doppler sequence is $f_d = [1:M] \cdot \Delta f_d$, $M = f_c / \Delta f_d$, $M$ is the number of Doppler units corresponding to $\Delta f_d$. The the basis matrix is $\Psi = \{\varphi_1, \varphi_2, \ldots, \varphi_m, \ldots, \varphi_M\}$, $\varphi_m(t) = \exp(-j2\pi f_d(m)t)$, $0 < m \leq M$.

The received discrete signal can be rewritten as

$$y = A x + n$$

where $x$ is the reflection coefficient vector.

To estimate $x$, the following optimization strategy can be used:

$$\hat{x} = \arg \min \|x\|_2 \text{ subject to } \|y - A \hat{x}\|_2 < \eta$$

where $\eta$ is a small positive number relating the norm of $n$.

Mohimani presented a smoothed L0 norm method [5]. The Gauss function $G_\sigma(x) = \sum_i \exp\left(-\frac{x_i^2}{\sigma}\right)$ was used to approach the L0 norm when the parameter $\sigma$ approaches zero.

A two-layer method was used to solve the sparse signal recovery problem. In this paper, a negative exponential function $F_\sigma(x) = \sum_i \exp\left(-\frac{x_i}{\sigma}\right)$ is proposed to obtain an approximate L0 norm solution.

When the parameter $\sigma \rightarrow \infty$, $F_\sigma(x)$ approaches $l_1$ norm. Thus the negative exponential function can search for the sparse solution at the very beginning of iteration. By utilizing the inherent block structure of target, a block sparse signal recovery algorithm for ISAR imaging is proposed.

3. Block sparse recovery ISAR imaging algorithm

A block sparse signal can be expressed as follows

$$x = [x_1, \ldots, x_d, x_{d+1}, \ldots, x_{2d}, \ldots, x_{N-d+1}, \ldots, x_N]^T$$

where $x[i]$ is the $i$th block with size $d$. $T$ is the transpose.

Define $I(s)$ as follow:

$$I(s) = \begin{cases} 1 & s \neq 0 \\ 0 & s = 0 \end{cases}$$

By denoting the mixed $l_2/l_0$ norm

$$\|x\|_{2,0} = \sum_{i=1}^{Q} I(\|x[i]\|_2)$$
A vector $x$ is block $K$ sparse if $\|x\|_{2,0} \leq K$. Therefore, the optimization problem will be:

$$\hat{x} = \arg\min x \|x\|_{2,0} \quad \text{s.t.} \quad \|y - \Psi x\|_2 < \varepsilon \quad (8)$$

where $\varepsilon$ bounds the $l_1$ norm of the noise.

$F_\sigma(x)$ for block sparse signal $x$ can be expressed as $F_\sigma(x) = \sum_{i=1}^{Q} f_\sigma(x_i)$, where $f_\sigma(x_i) = e^{-\|x_i\|_\sigma}$. Therefore, we can write:

$$\|x\|_{2,0} \approx Q - \sum_{i=1}^{Q} e^{-\|x_i\|_\sigma} = \Delta - F_\sigma(x) \quad (9)$$

So the final optimization problem is:

$$\hat{x} = \lim_{\sigma \to 0} \arg\min_{\sigma} F_\sigma(x) \quad \text{s.t.} \quad \|y - \Psi x\|_2 < \varepsilon \quad (10)$$

A two-layer method and gradient projection are used to solve the sparse signal recovery problem based on smoothed L0 norm [5]. In the outer layer, a decreasing sequence $\sigma_1, \sigma_2, \cdots, \sigma_j$ approaches to zero; in the inner layer, the steepest descent algorithm is used to obtain the minimum solution.

The steepest descent algorithm should decrease along descent direction. However, it can not ensure that the cost function decrease along the descent direction. Therefore, the step is added that check whether the cost function decrease in each iteration process. If the direction isn't descent, the midpoint of the current point and the before point is used. The new algorithm is called block sparse smoothed L0 algorithm (simplified as BSSL0) in this paper. The total BSSL0 optimization algorithm can be summarized as follows:

- **Initialization**
  1. Let $\hat{x}_0$ be equal to the minimum $l_2$ norm solution of $y = \Psi x$, obtained by
     $$\hat{x}_0 = \Psi^H (\Psi \Psi^H)^{-1} y$$
  2. Choose a suitable decreasing sequence for $\sigma: [\sigma_1, \cdots, \sigma_j]$.

- For $j = 1, \cdots, J$:
  1. Let $\sigma = \sigma_j, x = \hat{x}_{j-1}, \beta = \frac{J - j + 1}{J}$
  2. Minimize the function $F_\sigma(x)$ on the feasible set $x = \{x: \|\Psi x - y\|_2 < \varepsilon\}$

- For $l = 1, \cdots, L$:
  a) Let $\delta$ be gradient of $F_\sigma(x), \gamma = \frac{L - l + 1}{L}$
  b) For $x$, let $x_1 \leftarrow x - \mu \sigma \delta$ (where $\mu$ is a small positive constant)
  c) If $\|\Psi x_1 - y\|_2 > \varepsilon$, project $x$ back into the feasible set:
     $$x_1 \leftarrow x_1 + \Psi^H (\Psi \Psi^H)^{-1} (y - \Psi x_1)$$
  d) compare step
     $$Fun2 = F_\sigma(x_1)$$
     While $Fun2 > Fun1$
     $$x_2 = (x_1 + x)/2$$
     $$Fun2 = F_\sigma(x_2)$$
     If $Fun2 < Fun1$
\[ x = x_2 \]

3. Set \( \hat{x}_j = x \).

- Final solution is \( x = \hat{x}_j \).

The choice of step size factor \( \mu \) is important. For a large step size, it may not converge, but for a very small step size, the computation efficiency is low. The step size should decrease when the searching point approaches the minimum solution. We select a larger step size at the beginning of the search, when the searching point approaches the minimum solution, the step size should decrease. So in the proposed algorithm, the step size \( \mu = \beta \gamma \left( \max(|x|) / L_0 \sigma / L_1 \right) \) is used. \( L_0 \) and \( L_1 \) are two large numbers to control the step size.

4. Simulation results

The real data of the Yak-42 plane is used to illustrate the performance. The carrier frequency is 10 GHz with signal bandwidth of 400 MHz, the pulse repetition frequency is 100 Hz, i.e., 256 pulses are used in this experiment. Because the volume of the target is not large, we choose four discrete units as a block. The imaging results of BP algorithm [6], SBL algorithm[7], PC-SBL algorithm [8] and BSSL0 algorithm are presented in Fig.1, Fig.2 and Fig.3. Several weak points are around in the images obtained by BP algorithm when the number of pulses is small. The images obtained by BSSL0 algorithm are similar to PC-SBL algorithm, which are more intensive than BP algorithm and Bayesian algorithm.

![Fig.1 Reconstructed images of different algorithms with 16 Pulses (a) BP (b) SBL (c) PC-SBL (d) BSSL0](image-url)
Fig. 2 Reconstructed images of different algorithms with 32 pulses (a) BP (b) SBL (c) PC-SBL (d) BSSL0

Fig. 3 Reconstructed images of different algorithms with 64 pulses (a) BP (b) SBL (c) PC-SBL (d) BSSL0

5. Conclusions
By exploiting the block sparse structure of target, a block sparse signal recovery ISAR imaging algorithm is proposed to improve target image. A negative exponential function sequence is used as smoothed function to approach the block L0 norm. The revised step is added to ensure the searching direction is decrease compared traditional smoothed L0 norm recovery algorithm. The experiment results verify that the proposed algorithm can improve ISAR imaging quality.

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