Phenomenology of transverse single spin asymmetries in inclusive processes

M. Anselmino¹, M. Boglione² and F. Murgia³

¹Dipartimento di Fisica Teorica, Universit`a di Torino and INFN, Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy

²Dept. of Physics and Astronomy, Vrije Universiteit Amsterdam, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

³Dipartimento di Fisica, Universit`a di Cagliari and INFN, Sezione di Cagliari, CP 170, I-09042 Monserrato (CA), Italy

Abstract

A phenomenological description of single transverse spin asymmetries for the inclusive production of hadrons in \( p^{-}p \) and \( \ell^{-}p \) processes is discussed within pQCD and a straightforward generalization of the factorization theorem with the inclusion of parton intrinsic transverse motion. Fits to existing data, predictions for new processes and interpretation of recent results are presented.

QCD formalism for \( A^{\uparrow}B \rightarrow CX \) at leading twist, \( k_{\perp} = 0 \) : \( A_{N} = 0 \)

It is well known that perturbative QCD and the factorization theorem at leading twist [1, 2] can be used to describe the large \( p_{T} \) production of a hadron \( C \) resulting from the interaction of two polarized hadrons \( A \) and \( B \):

\[
\frac{E_{C} d^{3}\sigma^{A,S_{A}+B,S_{B} \rightarrow C+X} d^{3}p_{C}}{d^{3}p_{C}} = \sum_{a,b,c,d;\{\lambda\}} \rho^{a/A,S_{A}}_{\lambda_{a},\lambda'_{a}} f_{a/A}(x_{a}) \otimes \rho^{b/B,S_{B}}_{\lambda_{b},\lambda'_{b}} f_{b/B}(x_{b}) \otimes M^{*}_{\lambda_{c},\lambda_{d};\lambda_{a},\lambda_{b}} \otimes D^{\lambda_{c},\lambda'_{c}}_{\lambda_{c},\lambda'_{c}}(z),
\]

where \( \otimes \) denotes the usual convolutions [see, for example, Ref. [3] for details]. \( \rho^{a/A,S_{A}}_{\lambda_{a},\lambda'_{a}}(x_{a}) \) is the helicity density matrix of parton \( a \) inside the polarized hadron.

*Talk delivered by M. Anselmino at the Fifth Workshop on Quantum Chromodynamics, January 3-7, 2000, Villefranche, France.
$A$; similarly for $\rho^{b/B,SB}_a(x_b)$. The $\hat{M}_{\lambda_a,\lambda_b;\lambda_v,\lambda_h}$'s are the helicity amplitudes for the elementary process $ab \rightarrow cd$; if one wishes to consider higher order (in $\alpha_s$) contributions also elementary processes involving more partons should be included. $D^{\lambda_c,\lambda_c'}_{\lambda_c',\lambda_C}(z)$ is the product of fragmentation amplitudes for the $c \rightarrow C + X$ process

$$D^{\lambda_c,\lambda_c'}_{\lambda_c',\lambda_C} = \oint_{X,\lambda_X} D_{\lambda_X,\lambda_C;\lambda_c} \hat{D}^*_{\lambda_X,\lambda_C;\lambda_c'} \cdot$$

where the $\oint_{X,\lambda_X}$ stands for a spin sum and phase space integration of the undetected particles, considered as a system $X$. The usual unpolarized fragmentation function $D_{C/c}(z)$, i.e. the density number of hadrons $C$ resulting from the fragmentation of an unpolarized parton $c$ and carrying a fraction $z$ of its momentum, is given by

$$D_{C/c}(z) = \frac{1}{2} \sum_{\lambda_c,\lambda_C} D^{\lambda_c,\lambda_c}_{\lambda_c',\lambda_C}(z).$$

For simplicity of notations we have not shown in Eq. (1) the $Q^2$ scale dependences in the unpolarized distribution and fragmentation functions, $f$ and $D$.

In the case in which only one hadron, say $A$, is polarized (orthogonally to the scattering plane) Eq. (1) reads

$$E \rho^{a/A,\uparrow}_a(x_a) \rho^{b/B,\downarrow}_b(x_b) \hat{M}_{\lambda_a,\lambda_b;\lambda_v,\lambda_h} \hat{M}^*_{\lambda_a,\lambda_b;\lambda_v,\lambda_h} \cdot D^{\lambda_c,\lambda_c'}_{\lambda_c',\lambda_C}(z).$$

Eqs. (1) and (4) hold at leading twist and large $p_T$ values of the produced hadron; the intrinsic $k_\perp$ of the partons have been integrated over and collinear configurations dominate both the distribution functions and the fragmentation processes; one can then see that, in this case, there cannot be any single spin asymmetry. In fact, total angular momentum conservation in the (forward) fragmentation process [see Eq. (2)] implies $\lambda_c = \lambda_c'$; this, in turns, together with helicity conservation in the elementary process, implies $\lambda_a = \lambda_a'$. If we further notice that, by parity invariance, $\sum_{\lambda_C} D^{\lambda_c,\lambda_c}_{\lambda_c',\lambda_C}$ does not depend on $\lambda_c$ and that $\sum_{\lambda_a,\lambda_c,\lambda_d} |\hat{M}_{\lambda_c,\lambda_d;\lambda_v,\lambda_h}|^2$ is independent of $\lambda_a$, we remain with $\sum_{\lambda_a} \rho^{a/A,\uparrow}_a(x_a) = 1$. Moreover, in the absence of intrinsic $k_\perp$ and initial state interactions, the parton density numbers $f_{a/A}(x_a)$ cannot depend on the spin of $A$ and any spin dependence disappears from Eq. (1), so that:

$$d\sigma^{A\uparrow B \rightarrow CX} - d\sigma^{A\downarrow B \rightarrow CX} = 0. \tag{5}$$

Therefore one does not expect any sizeable single spin (or left-right) asymmetry:

$$A_N \equiv \frac{d\sigma^{\uparrow}(p_T) - d\sigma^{\uparrow}(-p_T)}{d\sigma^{\uparrow}(p_T) + d\sigma^{\uparrow}(-p_T)} = \frac{d\sigma^\uparrow(p_T) - d\sigma^\uparrow(-p_T)}{2d\sigma^{\text{unp}}}. \tag{6}$$
This is contradicted by data showing large values of $A_N$ in $p^+p \rightarrow \pi X$ and $\bar{p}^+p \rightarrow \pi X$ processes.

**Intrinsic $k_\perp$ effects in fragmentation and/or distribution functions: $A_N \neq 0$**

The above conclusion may be avoided by considering the transverse motion of the quarks relatively to the parent hadron or of the observed hadron relatively to the fragmenting quark. The original suggestion that the intrinsic $k_\perp$ of the quarks in the distribution functions might give origin to single spin asymmetries was first made by Sivers; such an effect is not forbidden by QCD time reversal invariance provided one takes into account soft initial state interactions among the colliding hadrons.

A similar suggestion for a possible origin of single spin asymmetries was later made by Collins, concerning transverse momentum effects in the fragmentation of a polarized quark. A consistent phenomenological application of these ideas can be found in a series of papers. More recently, a further possible source of single spin effects, related to distribution functions, was discussed by Boer.

When allowing for parton intrinsic motion spin effects can remain in new – spin and $k_\perp$ dependent – distribution or fragmentation functions. We list these functions in the sequel.

$\Delta N f_{q/p^\uparrow}(x, k_\perp)$ is the difference between the density numbers $\hat{f}_{q/p^\uparrow}(x, k_\perp)$ and $\hat{f}_{q/p^\downarrow}(x, k_\perp)$ of quarks $q$, with all possible polarization, longitudinal momentum fraction $x$ and intrinsic transverse momentum $k_\perp$, inside a transversely polarized proton with spin $\uparrow$ or $\downarrow$:

$$\Delta N f_{q/p^\uparrow}(x, k_\perp) = \hat{f}_{q/p^\uparrow}(x, k_\perp) - \hat{f}_{q/p^\downarrow}(x, k_\perp)$$

where the second line follows from the first one by rotational invariance. Notice that $\Delta N f_{q/p^\uparrow}(x, k_\perp)$ vanishes when $k_\perp \to 0$; parity invariance also requires $\Delta N f$ to vanish when the proton transverse spin has no component perpendicular to $k_\perp$, so that

$$\Delta N f_{q/p^\uparrow}(x, k_\perp) \sim k_\perp \sin \alpha$$

where $\alpha$ is the angle between $k_\perp$ and the $\uparrow$ direction.

$\Delta N f$ by itself is a leading twist distribution function, but its $k_\perp$ dependence, when convoluted with the elementary partonic cross-section, results in twist-3 contributions to single spin asymmetries. This same function (up to some factors) has also been introduced in Ref. – where it is denoted by $f_{1T}^\perp$ – as a leading twist $T$-odd distribution function. The exact relation between $\Delta N f$ and $f_{1T}^\perp$ is discussed in Ref.

A function analogous to $\Delta N f_{q/p^\uparrow}(x, k_\perp)$ can be defined for the fragmentation process of a transversely polarized parton, giving the difference between the density numbers $\hat{D}_{h/q^\uparrow}(z, k_\perp)$ and $\hat{D}_{h/q^\downarrow}(z, k_\perp)$ of hadrons $h$, with longitudinal momentum
fraction $z$ and transverse momentum $\mathbf{k}_\perp$ inside a jet originated by the fragmentation of a transversely polarized quark with spin $\uparrow$ or $\downarrow$:

$$\Delta^N D_{h/q}(z, \mathbf{k}_\perp) \equiv \hat{D}_{h/q}(z, \mathbf{k}_\perp) - \hat{D}_{h/q}(z, -\mathbf{k}_\perp)$$

(9)

A closely related function is denoted by $H_1^+$ in Refs. [9, 11] and its correspondence with $\Delta^N D$ is discussed in Ref. [10]. Again we expect

$$\Delta^N D_{h/q}(z, \mathbf{k}_\perp) \sim k_\perp \sin \beta$$

(10)

where $\beta$ is the angle between $\mathbf{k}_\perp$ and the $\uparrow$ direction.

Similarly one can introduce the difference $\Delta^N f_{q^+/p}(x, \mathbf{k}_\perp)$ between the density numbers $\hat{f}_{q^+/p}(x, \mathbf{k}_\perp)$ and $\hat{f}_{q^+}(x, \mathbf{k}_\perp)$ of quarks $q$, with spin $\uparrow$ or $\downarrow$, longitudinal momentum fraction $x$ and intrinsic transverse momentum $\mathbf{k}_\perp$, inside an unpolarized proton:

$$\Delta^N f_{q^+/p}(x, \mathbf{k}_\perp) \equiv \hat{f}_{q^+/p}(x, \mathbf{k}_\perp) - \hat{f}_{q^+}(x, \mathbf{k}_\perp)$$

(11)

$$= \hat{f}_{q^+}(x, \mathbf{k}_\perp) - \hat{f}_{q^+}(x, -\mathbf{k}_\perp).$$

A closely related function, denoted by $h_1^+$, was discussed in Ref. [8].

Finally, although not relevant for the single spin asymmetries we consider here, one could introduce the difference between the density numbers $\hat{D}_{h^+/q}(z, \mathbf{k}_\perp)$ and $\hat{D}_{h^+/q}(z, \mathbf{k}_\perp)$ of hadrons $h$, with longitudinal momentum fraction $z$ and transverse momentum $\mathbf{k}_\perp$ inside a jet originated by the fragmentation of an unpolarized quark $q$:

$$\Delta^N D_{h^+/q}(z, \mathbf{k}_\perp) \equiv \hat{D}_{h^+/q}(z, \mathbf{k}_\perp) - \hat{D}_{h^+}(z, \mathbf{k}_\perp)$$

(12)

$$= \hat{D}_{h^+}(z, \mathbf{k}_\perp) - \hat{D}_{h^+}(z, -\mathbf{k}_\perp).$$

Such a function might prove useful in tackling the longstanding problem of hyperon polarization in inclusive $p - N$ processes [12]. A closely related function is denoted by $D_{1T}^\perp$ in Refs. [8, 11].

Assuming that the factorization theorem, Eq. (4), holds when parton intrinsic motion is taken into account, and using the new functions (7), (9) and (11), one has, at leading order in $k_\perp$, for the $p^+p \rightarrow \pi X$ process:

$$\frac{E_\pi d^3\sigma^\uparrow}{d^3p_\pi} - \frac{E_\pi d^3\sigma^\downarrow}{d^3p_\pi} = \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} \times \bigg\{ \int d^2k_\perp \Delta^N f_{a/p}(x_a, \mathbf{k}_\perp) f_{b/p}(x_b) \frac{d\hat{\sigma}}{dt}(x_a, x_b, \mathbf{k}_\perp) D_{\pi/c}(z)$$

$$+ \int d^2k_\perp h_{1a/p}(x_a) f_{b/p}(x_b) \Delta_{NN}^{\pi \sigma}(x_a, x_b, \mathbf{k}_\perp) \Delta^N D_{\pi/c}(z, \mathbf{k}_\perp)$$

$$+ \int d^2k_\perp h_{1a/p}(x_a) \Delta^N f_{b^+/p}(x_b, \mathbf{k}_\perp') \Delta_{NN}^{\pi \sigma}(x_a, x_b, \mathbf{k}_\perp') D_{\pi/c}(z) \bigg\}.$$
where the second line corresponds to the so-called Sivers effect \[5\], the third to Collins effect \[1\] and the fourth one to the mechanism recently proposed by Boer \[8\]. Apart from the new functions $\Delta^N f$ and $\Delta^N D$, the other quantities appearing in Eq. (13) are the unpolarized quark distribution and fragmentation functions, $f$ and $D$; the unpolarized cross-section for the elementary process $ab \rightarrow cd$, $d\sigma/dt$; the transverse spin content of the proton:

$$h_1^{q/p} = f_{q^+/p^+}(x) - f_{q^+/p^+}(x)$$

(14)

and the elementary double spin asymmetries, computable in pQCD:

$$\Delta_{NN} = \frac{d\hat{\sigma}(a^+ b\rightarrow c^+ d)}{dt} - \frac{d\hat{\sigma}(a^+ b\rightarrow c^+ d)}{dt},$$

(15)

$$\Delta'_{NN} = \frac{d\hat{\sigma}(a^+ b\rightarrow c^+ d)}{dt} - \frac{d\hat{\sigma}(a^+ b\rightarrow c^+ d)}{dt} .$$

(16)

An equation with the same structure and a similar physical meaning as Eq. (13) can be found in Ref. \[13\], where the new functions $\Delta^N f$ and $\Delta^N D$ are replaced by higher twist parton correlation functions.

Eq. (13), or some of its terms, can be used for a phenomenological description of single spin asymmetries. We next summarize what has been done.

**Fits to existing data, predictions**

**Sivers effect only**

In Refs. \[3\] and \[4\] the scheme of Eq. (13) was adopted, taking into account only Sivers effect; to simplify things it was assumed – in this very first application of the idea of intrinsic quark motion and spin dependence – that the integral over $k_\perp$ is dominated by configurations in which $k_\perp$ lies in the scattering plane [that is, $\sin \alpha = 1$ in Eq. (8)] and its magnitude equals some average value \[3\]:

$$\frac{1}{M} k_\perp^0(x_a) = 0.47 x_a^{0.68} (1 - x_a)^{0.48},$$

(17)

where $M = 1$ GeV/c$^2$.

The residual $x_a$ dependence in $\Delta^N f_{a/p^+}$ not coming from $k_\perp^0$ was taken to be of the simple form

$$N_a x_a^{\alpha_a} (1 - x_a)^{\beta_a},$$

(18)

where $N_a$, $\alpha_a$ and $\beta_a$ are free parameters. Only $u$ and $d$ quark contributions to $\Delta^N f_{a/p^+}$ were considered.

One then ends up with the simple expression:

$$\int d^2k_\perp \Delta^N f_{a/p^+}(x_a, k_\perp) \left[ \frac{d\hat{\sigma}}{dt}(k_\perp) - \frac{d\hat{\sigma}}{dt}(-k_\perp) \right]$$

$$\simeq \frac{k_\perp^0(x_a)}{M} N_a x_a^{\alpha_a} (1 - x_a)^{\beta_a} \left[ \frac{d\hat{\sigma}}{dt}(k_\perp^0) - \frac{d\hat{\sigma}}{dt}(-k_\perp^0) \right],$$

(19)
where \( k_0^0(x_0) \) is given by Eq. (17) and, choosing \( xz \) as the scattering plane and \( z \) as the direction of the incoming polarized proton, \( k_\perp^0 = (k_{\perp}^0, 0, 0) \).

Inserting this result into the first two lines of Eq. (13), one can compute the single spin asymmetry (6) in terms of the parameters \( \alpha_a, \beta_a, N_a \); only the leading valence quark contributions to \( \Delta^N f \) were considered in the numerator of \( A_N \), while all leading order pQCD elementary processes were included in the denominator. The parameters were fixed by performing a best fit to the experimental data, with the results shown in Fig. 1, corresponding to the \( \Delta^N f \) functions:

\[
\Delta^N f_{u/p^\uparrow}(x, k_\perp^0) = 6.92 \, x^{2.02}(1-x)^{1.06} \quad \Delta^N f_{d/p^\uparrow}(x, k_\perp^0) = -2.33 \, x^{1.44}(1-x)^{4.62} \ \ (20)
\]

It is clear from Fig. 1 how Sivers effect alone allows a good fit of the experimental data. Let us add a few more comments:

- The resulting expressions of \( \Delta^N f_{q/p^\uparrow} \), Eq. (20), are reasonable and quite acceptable; in particular they satisfy the positivity condition \( |\Delta^N f_{q/p^\uparrow}| \leq 2 \, f_{q/p} \).
- The opposite sign of \( \Delta^N f_{u/p^\uparrow} \) and \( \Delta^N f_{d/p^\uparrow} \) is expected from transverse momentum conservation.
- It is easy to explain, within this scheme, why almost opposite values of \( A_N^{\pi^+} \) and \( A_N^{\pi^-} \) do not imply, as one might naively expect, \( A_N^{\pi^0} \simeq 0 \) [3].
- This same scheme, with the same \( \Delta^N f \) functions (20), was used – without any free parameter – to compute \( A_N \) in \( \bar{p}^\uparrow p \to \pi X \); the agreement with data [4] is good [7].

**Collins effect only**

A similar analysis was performed in Ref. [3] taking into account only Collins effect. Again, it was assumed that the main contribution comes from an average \( k_\perp^0 \) value, with a simple parametrization of \( \Delta^N D_{\pi/c}(z, k_\perp^0) \) for all leading valence quarks:

\[
\Delta^N D_{\pi/c}(z, k_\perp^0) = \frac{k_0^0(z)}{M} N_c \, z^{\alpha_c}(1-z)^{\beta_c}, \quad (21)
\]

where \( N_c, \alpha_c \) and \( \beta_c \) are free parameters and

\[
\frac{k_0^0(z)}{M} = 0.61 \, z^{0.27} (1-z)^{0.20}, \quad (22)
\]

with \( M = 1 \) GeV/c\(^2\).

Another free parameter is contained in the expression of \( h_{1q/p}^q(x) = P_{q/p}^\uparrow \ f_{q/p}(x) \) where \( P_{q/p}^\uparrow \) is the transverse polarization of quark \( q \) inside the transversely polarized proton.
The experimental data \cite{4} were best fitted as in Fig. 2, with the resulting $\Delta^N D$ function and $P_{u/p}^\uparrow$ values \cite{3}:

\begin{align}
\Delta^N D_{\pi/q}(z,k_1^0) &= -0.13 z^{2.60} (1 - z)^{0.44} \quad z \leq 0.977 \\
\Delta^N D_{\pi/q}(z,k_1^0) &= -2 D_{\pi/q}(z) = -2.20 z^{-1} (1 - z)^{1.2} \quad z > 0.977 \\
P_{u/p}^\uparrow &= \frac{2}{3} \quad P_{d/p}^\uparrow = -0.88.
\end{align}

The quality of the fit is comparable to that obtained using only Sivers effect, shown in Fig. 1. However, some comments are now necessary:

- In order to fit the data, $\Delta^N D_{\pi/q}$ has to saturate at large $z$ the positivity constraint $|\Delta^N D_{\pi/q}| \leq 2 D_{\pi/q}$. Otherwise, the values of $A_N$ at large $x_F$ would be much too small.

- The resulting value of $h_1^{d/q}$ is

\begin{equation}
\Delta^N D_{\pi/q}(x) = -0.88 f_{d/p}(x),
\end{equation}

which violates the Soffer’s bound \cite{14}

\begin{equation}
|h_1^{d/p}| \leq \frac{1}{2}(f_{d/p} + \Delta d).
\end{equation}

$\Delta d$ is the $d$ quark helicity distribution, which, being in most parametrization of data negative, makes bound \cite{25} very strict.

- A parametrization of $\Delta^N D$ which satisfies Soffer’s bound was used in Ref. \cite{15}, with a good resulting fit, provided one allows $\Delta d$ to become positive at large $x$ values; however, also in this case the positivity constraint has to be saturated at large $z$.

We can at this point conclude that the phenomenological approach to single spin asymmetries based on the generalization of the factorization theorem with the inclusion of quark intrinsic motion and on the introduction of the new spin and $k_\perp$ dependent functions \cite{7}, \cite{9} and \cite{11} is indeed promising and worth being pursued. The relative contributions of the several terms in Eq. \cite{13} is still unknown; a first guess would indicate that Sivers effect is indeed necessary, while Collins effect alone leads to unreasonably large values of $|\Delta^N D|$, the Collins function. The third effect, originating from the quarks in the unpolarized hadron has not been studied yet, although it should not be relevant for large and positive $x_F$ values. We turn now to a study of Collins effect in DIS.
Fragmentation of a polarized quark in semi-inclusive DIS

The inclusive production of hadrons in DIS with transversely polarized nucleons, $\ell N^\uparrow \rightarrow \ell hX$, is the ideal process to study Collins effect; in such a case, in fact, Sivers effect, which requires initial state interactions [3, 16], is negligible and any single spin asymmetry must originate from spin dependences in the fragmentation of a polarized quark.

If one looks at the $\gamma^* N^\uparrow \rightarrow hX$ process in the $\gamma^* N$ c.m. system, the elementary interaction is simply a $\gamma^*$ hitting a transversely polarized quark, which bounces back and fragments into a jet containing the detected hadron. The hadron $p_T$ in this case coincides with its $k_{\perp}$ inside the jet; the fragmenting quark polarization can be computed from the initial quark one.

The spin and $k_{\perp}$ dependent fragmentation function for a quark with momentum $p_q$ and a transverse polarization vector $P_q$ ($p_q \cdot P_q = 0$) which fragments into a hadron with momentum $p_h = z p_q + p_T$ ($p_q \cdot p_T = 0$) can be written as:

$$D_{h/q}(p_q, p_T) = \hat{D}_{h/q}(z, p_T) + \frac{1}{2} \Delta^N D_{h/q}(z, p_T) \frac{P_q \cdot (p_q \times p_T)}{|p_q \times p_T|}$$

where $\hat{D}_{h/q}(z, p_T)$ is the unpolarized fragmentation function and $\Delta^N D$ is the same function as introduced in Eq. (9). Notice that – as required by parity invariance – the only component of the polarization vector which contributes to the spin dependent part of $D$ is that perpendicular to the $q-h$ plane; in general one has:

$$P_q \cdot \frac{p_q \times p_T}{|p_q \times p_T|} = P_q \sin \Phi_C,$$

where $P_q = |P_q|$ and we have defined the Collins angle $\Phi_C$.

Eq. (26) leads to a possible single spin asymmetry in semi-inclusive DIS off nucleons with polarization $\pm P$,

$$A_N^h(x, y, z, \Phi_C, p_T) = \frac{2(1-y)}{1 + (1-y)^2} P_T \sin \Phi_C,$$

which is given by [17]

$$A_N^h(x, y, z, \Phi_C, p_T) = \frac{\sum_p e_p^2 h_1^p(x) \Delta^N D_{h/q}(z, p_T)}{2 \sum_p e_p^2 f_{q/p}(x) D_{h/q}(z, p_T)} \frac{2(1-y)}{1 + (1-y)^2} P_T \sin \Phi_C,$$

where $P_T$ is the nucleon polarization vector component transverse with respect to the $\gamma^*$ direction; $x$ and $y$ are the usual DIS variables. If one collects data at different kinematical values one should integrate over the relevant $x$ and $y$ regions so that Eq. (29) reads:

$$A_N^h(z, \Phi_C, p_T) = \frac{\sum_p \int dx dy e_p^2 h_1^p(x) 2(1-y)/(xy)^2 \Delta^N D_{h/q}(z, p_T) P_T \sin \Phi_C}{2 \sum_p \int dx dy e_p^2 f_{q/p}(x) (1 + (1-y)^2)/(xy)^2 D_{h/q}(z, p_T)}.$$
Some preliminary data on $A_N^π$ have recently appeared \cite{18, 19} and have been analysed in Ref. \cite{17} using Eq. (29). By saturating the unknown values of $h_{1u/d/p}$ with the Soffer’s bound \cite{25} lower bounds for the Collins function have been obtained for the fragmentation of a $u$ quark into a $π^+$. From SMC data \cite{19} one has

$$\frac{|\Delta^N D_{π/q}(⟨z⟩, ⟨p_T⟩)|}{2 D_{π/q}(⟨z⟩, ⟨p_T⟩)} \gtrsim (0.24 \pm 0.15) \quad ⟨z⟩ \simeq 0.45, \quad ⟨p_T⟩ \simeq 0.65 \text{ GeV}/c,$$  \hspace{1cm} (31)

and from HERMES data \cite{18}

$$\frac{|\Delta^N D_{π/q}(z, p_T)|}{2 D_{π/q}(z, p_T)} \gtrsim 0.20 \pm 0.04(\text{stat.}) \pm 0.04(\text{syst.}) \quad z \geq 0.2.$$ \hspace{1cm} (32)

If confirmed, HERMES and SMC data indicate a large value of the Collins function, which might then play a significant role in other processes. In particular, it would be of great interest to compare the single spin asymmetry measured in the inclusive process $ℓp^+ → πX$ with that measured by E704 Collaboration \cite{4} in $p^+p → πX$ processes: if the origin of the asymmetry is mainly in Collins mechanisms similar results should be found in both cases. More data on single transverse spin asymmetries will be available in the future from operating or progressing facilities like JLAB, RHIC and COMPASS.

**Acknowledgements**

One of us (M.A.) would like to thank the organizers of the Workshop for the kind invitation.
References

[1] J.C. Collins, \textit{Nucl. Phys.} \textbf{B396} (1993) 161;

[2] J.C. Collins, \textit{Nucl. Phys.} \textbf{B394} (1993) 169; J.C. Collins, S.H. Heppelmann and G.A. Ladinsky, \textit{Nucl. Phys.} \textbf{B420} (1994) 565

[3] M. Anselmino, M. Boglione and F. Murgia, \textit{Phys. Rev.} \textbf{D60} (1999) 054027

[4] D.L. Adams \textit{et al.}, \textit{Phys. Lett.} \textbf{B264} (1991) 462; A. Bravar \textit{et al.}, \textit{Phys. Rev. Lett.} \textbf{77} (1996) 2626

[5] D. Sivers, \textit{Phys. Rev.} \textbf{D41} (1990) 83; \textit{Phys. Rev.} \textbf{D43} (1991) 261

[6] M. Anselmino, M. Boglione and F. Murgia, \textit{Phys. Lett.} \textbf{B362} (1995) 164

[7] M. Anselmino and F. Murgia, \textit{Phys. Lett.} \textbf{B442} (1998) 470

[8] D. Boer, \textit{Phys. Rev.} \textbf{D60} (1999) 014012

[9] D. Boer and P. Mulders, \textit{Phys. Rev.} \textbf{D57} (1998) 5780

[10] M. Boglione and P. Mulders, \textit{Phys. Rev.} \textbf{D60} (1999) 054007

[11] R. Jacob and P.J. Mulders, Proceedings of SPIN 96, World Scientific 1997, e-Print Archive: \texttt{hep-ph/9610297}

[12] M. Anselmino, D. Boer, U. D’Alesio, F. Murgia, in preparation

[13] J.W. Qiu and G. Sterman, \textit{Phys. Rev.} \textbf{D59} (1999) 014004

[14] J. Soffer, \textit{Phys. Rev. Lett.} \textbf{74} (1995) 1292

[15] M. Boglione and E. Leader, \textit{Phys. Rev.} \textbf{D61} (2000) 114001

[16] M. Anselmino, M. Boglione, J. Hansson and F. Murgia, \textit{Eur. Phys. J.} \textbf{C13} (2000) 519

[17] M. Anselmino and F. Murgia, e-Print Archive: \texttt{hep-ph/0002120}, to appear in \textit{Phys. Lett. B}

[18] HERMES Collaboration (A. Airapetian \textit{et al.}), DESY-99-149, Oct. 1999, e-Print Archive: \texttt{hep-ex/9910062}; H. Avakian (on behalf of the HERMES collaboration), \textit{Nucl. Phys.} \textbf{B79} (Proc. Suppl.) (1999) 523

[19] A. Bravar (on behalf of the SMC collaboration), \textit{Nucl. Phys.} \textbf{B79} (Proc. Suppl.) (1999) 520
Figure 1: Fit of the data on $A_N$ for the process $p^+p \rightarrow \pi X$ \cite{4}, as obtained in Ref. \cite{7}, assuming that only Sivers effect is active; the upper, middle, and lower sets of data and curves refer respectively to $\pi^+$, $\pi^0$, and $\pi^-$.

Figure 2: Fit of the data on $A_N$ for the process $p^+p \rightarrow \pi X$ \cite{4}, as obtained in Ref. \cite{3}, assuming that only Collins effect is active; the upper, middle, and lower sets of data and curves refer respectively to $\pi^+$, $\pi^0$, and $\pi^-$. 

11