Multiple-model switching control for vibration suppression of planar membrane structures

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Abstract
Membrane space structures have received widespread attention because of their small packaging volume and low mass. However, because membranes are very flexible and lightly damped, vibration suppression in membrane structures is very difficult. The objective of this study was to solve this problem. The first part of this article describes the influence of wrinkling in a membrane structure on the structure’s vibration characteristics. On this basis, the vibration deformations of a wrinkled square membrane structure were derived from the dynamic equations, and the correctness of this vibration model was verified by numerical simulation and experiment. A multi-model system is proposed to simulate the dynamic response of a membrane structure under different boundary conditions. In combination with the drive system, a multi-model switching control method based on adaptive and proportional–integral–derivative control is proposed. Under the initial disturbance, when the vibration amplitude dropped below 0.01 mm, the vibration duration was reduced to 2.96 s, compared with the duration of 12.37 s without control. The duration was shortened by approximately 39.7%, compared with the duration of 4.91 s achieved by the traditional proportional–integral–derivative control method, and by approximately 15.9% compared with the 3.52 s achieved by the out-plane control method. When there were multiple disturbances and the proposed method was used, the boundary displacement loadings did not increase when a certain value was exceeded. This prevented the breaking of the membrane by overstretching and provides a theoretical foundation for setting the initial pre-stress values.

Keywords
Membrane, dynamic analysis, vibration, control, multiple-model switching

Introduction
Membrane enables the development of ultra-lightweight and extremely large space structures, such as solar sails, large reflectors, solar arrays, and membrane mirrors, which are used in innovative space missions. Surface accuracy and structural stability are two significant factors that affect the performance of membrane structures. However, the inherent flexibility of membrane structures is detrimental to the in-orbit stability and maneuverability, and the vibration duration after a disturbance in space is longer because of the air-free environment. Therefore, an effective control...
method is needed for the effective and rapid suppression of vibration.

To develop a vibration model, He et al.\(^4\) considered a flexible robotic manipulator as a concise spring–mass system. Liu et al.\(^5,6\) studied the nonlinear dynamic response of orthotropic membranes under an impact load by analytical and numerical approaches. Li et al.\(^7\) studied the dynamic response of a pre-stressed orthotropic circular membrane under an impact load based on the principle of virtual displacement and solved by the Krylov–Bogoliubov–Mitropolsky perturbation method. However, for a membrane structure, wrinkling is a ubiquitous phenomenon that affects the vibration characteristics.\(^8\) Therefore, in dynamic modeling, the wrinkling information must be considered. Fang et al.\(^9\) used the distributed transfer function (DTF) method to predict the structural natural frequencies and mode shapes of an 8 m membrane reflecting antenna. The in-plane wrinkling shape and natural vibration characteristics were analyzed by Kukhashan and Pellegrino\(^10\) using the iterative membrane properties (IMP) method. The dynamic analysis of a partially wrinkled annular and three-sided membrane has been presented by Hossain et al.\(^11,12\) and the penalty parameter modified material (PPMM) model, which has been previously introduced by Liu et al.\(^13\) was used in ABAQUS to investigate how wrinkling affects the transverse vibration behavior. Then the stress distribution of the wrinkled membrane structure obtained by the nonlinear buckling finite element method (FEM) was introduced into the modal analysis to predict the natural vibration frequencies and modes.\(^14\) This study used modal analysis to establish the vibration model and estimate the vibration deformations of a partially wrinkled planar membrane structure.

General control methods can be broadly characterized as passive, active, semi-active, and hybrid.\(^15\) For active vibration suppression, some modern techniques are studied. Gasteratos\(^16\) designed a fuzzy-gray controller, David et al.\(^17\) designed a fractional-order proportional–integral–derivative (PID) controller, and Zhao et al.\(^18\) presented a novel adaptive neural network based on sliding mode control strategy for image stabilization systems. In the field of the dynamic control of membranous structures, Renno et al.\(^19\) investigated nonlinear control techniques for a one-dimensional membrane strip with an attached piezoelectric ceramic transducer (PZT) bimorph. Subsequently, Ruggiero and Inman\(^20\) designed a linear quadratic (LQ) regulator control system using distributed bimorph actuators to eliminate any detrimental vibration of the membrane mirror. Ferhat and Sultan\(^21\) investigated the LQ regulator using variational-based solutions for the second-order form of a membrane structure. Liu et al.\(^22\) designed a \(H\)\(_\infty\) robust controller to suppress the nonlinear vibration of the structure based on the linearized model; however, the wrinkles in the membrane were ignored. Another study investigated a square membrane with all four edges clamped and with four PZT bimorph actuators attached to it.\(^23\) A passive method for vibration suppression using catenary cables along the membrane edges has been proposed.\(^24\) In addition, Sakamoto\(^25\) proposed a web-like cable membrane structure. An LQ regulator was designed based on the web-cable substructure, and small actuators were attached to the interfaces between the web cables and the membrane.\(^26,27\) However, attaching the actuators directly to large membranes is very difficult, mainly due to the difficulty of wiring the devices. In addition, a web-cable structure with many actuators or passive dampers increases the total mass of the system.

In our previous study,\(^28\) static analysis of wrinkled membrane structures was made, and a wrinkle-wave model was proposed to more accurately describe the wrinkling details. In the dynamic analysis presented in this article, the wrinkle shapes were considered as the initial state, which is the vibration equilibrium position. Moreover, considering the influence of wrinkling on the structural natural frequencies and mode shapes, a multi-model vibration system for a wrinkled square membrane structure was established to approximate the dynamic response under different boundary conditions. The proposed multi-model switching control method based on adaptive and PID control can effectively and rapidly suppress the vibration by adjusting the boundary displacement loadings without attaching extra actuators to the membrane. The correctness and effectiveness of the method were verified by nonlinear finite element simulations and experimental measurements. Moreover, the results obtained by this study provided a theoretical foundation for setting the initial pre-stress values.

Dynamic analysis

Solar sails are typical planar membrane structures, wherein a square membrane is stressed by two pairs of equal and opposite displacement loadings \(V_X\) and \(V_Y\), with side length \(L\).\(^29\) It is assumed that the membrane is isotropic with Young’s modulus \(E\) and Poisson’s ratio \(\nu\), and that the constitutive material is linearly elastic.\(^29\)

Vibration model for wrinkled membrane structure

Without considering the external disturbance and external damping, the bending deformation potential energy of a membrane structure with local wrinkles can be expressed as follows\(^30\)

\[\sum_{i=1}^{n} \int_{0}^{L} \frac{1}{2} \sigma_{ii} \left( \frac{d u_i}{d X} \right)^2 \, dx + \sum_{i=1}^{n} \int_{0}^{L} \frac{1}{2} \sigma_{jj} \left( \frac{d v_j}{d Y} \right)^2 \, dx \]
\[ E_{p1} = \frac{1}{2} \frac{Eh_m^2}{12(1 - \nu^2)} \int \left\{ \left( \nabla^2 w_v \right)^2 - 2(1 - \nu) \right\} dxdy \]

where \( w_v \) is the vibration configuration function and \( h_m \) is the thickness of the membrane.

The strain potential energy of the membrane structure is expressed as follows

\[ E_{p2} = \frac{Eh_m}{2(1 - \nu^2)} \int \left( \varepsilon_v^2 + \varepsilon_y^2 + 2\nu \varepsilon_x \varepsilon_y + \frac{1}{2} \gamma_{xy}^2 \right) dxdy \]

\[ \varepsilon_v = \frac{1}{2} \left( \frac{\partial w_v}{\partial x} \right)^2 - \left( \frac{\partial w_z}{\partial x} \right)^2 \]

\[ \varepsilon_y = \frac{1}{2} \left( \frac{\partial w_v}{\partial y} \right)^2 - \left( \frac{\partial w_z}{\partial y} \right)^2 \]

\[ \gamma_{xy} = \frac{\partial w_v}{\partial x} \frac{\partial w_v}{\partial y} - \frac{\partial w_z}{\partial x} \frac{\partial w_z}{\partial y} \]

where \( w_z \) is the out-of-plane deformation caused by the wrinkles.

The kinetic energy of free vibration is

\[ E_v = \frac{1}{2} \rho h_m \int \left( \frac{\partial w_v}{\partial t} \right)^2 dxdy \]

where \( \rho \) is the density of the membrane.

The vibration deformation \( w_v \) and wrinkling deformation \( w_z \) can be expressed in the form of a series, as follows

\[ \begin{align*}
    w_v(x, y, t) &= \sum w_v(x, y) q_v(t) \\
    w_z(x, y) &= \sum w_z(x, y)
\end{align*} \]

By substituting equation (4) into equations (1) and (2), we can obtain the total potential energy, as follows

\[ E_p = E_{p1} + E_{p2} = \frac{1}{2} \left( k_{ijkl} q_{ijkl} + k_{ij} q_j + k'_{ijkl} \right) \]

By substituting equation (4) into equation (3), we obtain the following relationship

\[ E_v = \frac{1}{2} m_{ij} q_i q_j \]

where \( k_{ijkl}, k'_{ijkl}, k_{ij} \), and \( m_{ij} \) can be expressed as follows

\[ k_{ijkl} = \frac{Eh_m}{4(1 - \nu^2)} \int \left( \frac{\partial w_v}{\partial x} \frac{\partial w_v}{\partial x} \frac{\partial w_z}{\partial x} \frac{\partial w_z}{\partial x} + \frac{\partial w_v}{\partial y} \frac{\partial w_v}{\partial y} \frac{\partial w_z}{\partial y} \frac{\partial w_z}{\partial y} + 2 \frac{\partial w_v}{\partial x} \frac{\partial w_v}{\partial y} \frac{\partial w_z}{\partial x} \frac{\partial w_z}{\partial y} \right) dxdy \]
The solutions of equation (9) can be written as

\[ q = \phi e^{int} \]

and introduced into equation (9) as follows

\[ (K - \omega^2 M)\phi e^{int} = 0 \]  

(11)

where \( \omega \) is a group of natural frequencies \( \omega_1, \omega_2, \ldots, \omega_i, \ldots, \omega_n \); \( \omega_i \) is the natural frequency of the \( i \)th mode, and the corresponding \( \phi_i \) is the vibration mode of the \( i \)th mode. The mode shape for the first mode is shown in Figure 1. The wrinkling shape can be observed near the corners, while the maximum vibration deformation is located at the central point. When the vibration at the central point is effectively suppressed, the vibration amplitudes at other points on the membrane will be reduced.

**Control-oriented dynamic modeling**

The degrees of freedom are denoted by \( n \), the number of inputs is denoted by \( r \), and the number of outputs is denoted by \( s \). By considering the external disturbance and external damping, the dynamic equation in nodal coordinates can be expressed as follows

\[
\begin{align*}
M\ddot{q} + D\dot{q} + Kq &= B_0u + B_1u_d \\
y &= Cq
\end{align*}
\]

(12)

where \( q \) is the nodal displacement; \( u = [V_X \ V_Y]^T \) represents the displacement loading inputs; \( u_d \) is the disturbance input; \( B_0 \) and \( B_1 \) are the input matrices; \( C \) is the output displacement matrix; \( M, D, \) and \( K \) are the mass, damping, and stiffness matrices, respectively. However, note that \( K = K_s + K_L + K_{NL} \).

By transforming the system to modal coordinates, a modal displacement matrix \( q_m \) is introduced and satisfies the condition \( q = \Phi q_m \), where \( \Phi \) is the mode shapes matrix expressed as follows

\[
\Phi = [\phi_1 \ \phi_2 \ \cdots \ \phi_n] = \begin{bmatrix} \phi_{11} & \phi_{21} & \cdots & \phi_{n1} \\ \phi_{12} & \phi_{22} & \cdots & \phi_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1n} & \phi_{2n} & \cdots & \phi_{nn} \end{bmatrix}
\]

(13)

where \( \phi_{ij} \) is the \( j \)th displacement of the \( i \)th mode.

Then the dynamic equation in modal coordinates can be obtained by multiplying the left side by \( \Phi^T \)

\[
\begin{align*}
M_m\ddot{q}_m + D_m\dot{q}_m + K_mq_m &= \Phi^TB_0u + \Phi^TB_1u_d \\
y &= C\Phi q_m
\end{align*}
\]

(14)

where the modal mass matrix \( M_m \), modal damping matrix \( D_m \), and modal stiffness matrix \( K_m \) satisfy the following conditions

\[
\begin{align*}
M_m &= \Phi^T M \Phi, \quad D_m = \Phi^T D \Phi, \\
K_m &= \Phi^T K \Phi, \quad D_m = \alpha_1 K_m + \alpha_2 M_m
\end{align*}
\]

(15)

where \( \alpha_1 \) and \( \alpha_2 \) are the Rayleigh damping scale coefficients.31

The state-space expression of the vibration model can be expressed as follows

\[
\begin{align*}
\dot{x}_p &= A_p x_p + B_p u + B_{d1}u_d \\
y &= C_p x_p
\end{align*}
\]

(16)

where

\[
\begin{align*}
A_p &= \begin{bmatrix} 0 & I \\ -M_m^{-1}K_m & -M_m^{-1}D_m \end{bmatrix} \\
B_p &= M_m^{-1}\Phi^TB_0, \quad B_{d1} = M_m^{-1}\Phi^TB_1 \\
C_p &= [C_m \ 0], \quad x_p = [q_m \ u], \quad u = [V_X \ V_Y]
\end{align*}
\]

Generally, the natural frequencies and mode shapes can be obtained using modal analysis. Then, the dynamic model in modal coordinates can be constructed. However, for a flexible membrane structure, wrinkling certainly affects the vibration characteristics. In the first case, the wrinkling details should be introduced to identify the predominate modes for dynamic analysis. The mode energies are the sum of the kinetic energies and potential energies, as follows

\[
E_i = \frac{1}{2} M_m \phi_i^2 + \frac{1}{2} K_m \phi_i^2
\]

(17)
where the modal mass matrix $M_{mi}$, modal stiffness matrix $K_{mi}$, and mode shapes $\varphi_i$ are calculated by introducing the wrinkling results.

The mode energy contribution ratio $\eta_i$ of the first 15 modes can be obtained as follows

$$\eta_i = \frac{E_i}{\sum_{i=1}^{15} E_i}$$

(18)

As shown in Figure 2, the sum of the kinetic energy for the first six modes accounts for approximately 90% of the total energy. Then, a dimensionally reduced model can be obtained by retaining the first six modes.

**Transient analysis**

The disturbance point is shown in Figure 3(a). To simulate the dynamic response, an impulse displacement of amplitude of 4 mm was applied for 0.02 s.

The parameters of the membrane structure are listed in Table 1. Twenty-one points distributed on the diagonal line $\overline{AB}$ (Figure 3(a)) were selected. In the case of $V_X = V_Y = 0.3$ mm, the numerical results were obtained by the finite element software ANSYS. First, the wrinkling details were obtained by nonlinear buckling analysis; on this basis, the transient analysis was carried out, and the vibration curves on $\overline{AB}$ were

![Figure 2. Kinetic energy ratios of the first 15 modes.](image1)

![Figure 3. Vibration curve: (a) square membrane model, (b) vibration curves on the diagonal line $\overline{AB}$, and (c) vibration curve at the central point.](image2)
obtained through post-processing of the numerical solution, as shown in Figure 3(b). As can be seen, the maximum vibration amplitude appeared at the central point. The vibration curves at the central point are shown in Figure 3(c) for both the dimension reduction model (DRM) and the FEM. The agreement between the analytical results with the DRM and the numerical results obtained with the FEM proves that the dimensionally reduced vibration model can express the inherent characteristics of the full model.

By maintaining $V_Y = 0.25$ mm, $V_X$ gradually increases to 1.0 mm. Then, large-scale wrinkles appear in the central region, as shown in Figure 4(a), and act as a vibration equilibrium state. The out-of-plane deformation, at point $P$, which is the vibration equilibrium position, is 0.82 mm in the numerical results and 0.80 mm in the analytical results obtained by the wrinkle-wave model. Figure 4(b) shows the vibration curves at point $P$. The analytical values are also close to the numerical results, and the vibration amplitude error is less than 0.02 mm, which further validates the analytical model.

**Multiple-model switching controller design**

The natural frequencies and mode shapes change with the boundary loadings applied to the membrane structure, which leads to changes in the vibration model. Thus, more than one model must be established to approximate the dynamic performance of the system under different displacement loadings. It is assumed that the tension is in the range of 0.1–1.8 mm, such that $0.1 \leq V_X, V_Y \leq 1.8$ mm. Then, this range is divided into various sub-ranges, and different vibration sub-models are established to estimate the vibration characteristics at different intervals. As the number of sub-ranges increases, the models become more accurate. In this study, 13 vibration sub-models were used.

**Drive system**

A voice coil motor (VCM) was used as the actuator because of its small size and high-response speed. The equivalent circuit diagram of the coil circuit is shown in Figure 5. Here, $u_a$ is the input voltage, $i_a$ is the armature current, $L_a$ is the armature inductance, and $R_a$ is the motor resistance. The back-electromotive force produced by the operation of the motor is expressed as follows

$$e_a = K_v i_a$$  \hspace{1cm} (19)
where \( K_s \) denotes the motor force constant and \( v_a \) denotes the speed of the armature when cutting the magnetic field lines.

The voltage balance equation is derived as follows

\[ u_a = K_s v_a + i_a R_a + L_a \frac{di_a}{dt} \quad (20) \]

The inertial force of the electromagnetic force that overcomes the mover is expressed as follows

\[ F_e = F_m + kV \quad (21) \]

where \( m \) is the total mass of the mover.

Let \( k \) be the dynamic friction coefficient and \( V \) be the output displacement of the motor. Then, the dynamic force balance equation can be expressed as follows

\[ F_e = F_m + kV \quad (22) \]

The transfer function between the input voltage and the output displacement can be obtained as follows

\[ G(s) = \frac{V(s)}{u_a(s)} = \frac{K_s}{L_a ms^3 + (L_a k + R_a m)s^2 + (R_a k + K_s^2)s} \quad (23) \]

By combining the vibration sub-models (equation (16)) and the drive model (equation (23)), we can obtain the multiple-model dynamic system with the state-space matrices \((A_g, B_g, C_g)\).

**Model reference adaptive control**

The control objective is to make the system output \( y \) track the desired trajectory \( y_r \). Let the output error be \( e_y = y - y_r \), and let \( e_{\dot{y}_y} = \dot{e}_y = y - y_r \). Then, the multiple-model dynamic system \((A_g, B_g, C_g)\) can be expanded as follows

\[
\begin{align*}
\dot{x} &= Ax + Bu_a + B_d u_d + B_y y_r \\
y &= Cx
\end{align*}
\quad (24)
\]

where

\[
\begin{align*}
x &= \begin{bmatrix} e_{\dot{y}_y} \\ x_g \end{bmatrix}, & u_a &= \begin{bmatrix} u_{ax} \\ u_{ay} \end{bmatrix}, & A &= \begin{bmatrix} 0 & C_g \\ 0 & A_g \end{bmatrix} \\
B &= \begin{bmatrix} 0 \\ B_g \end{bmatrix}, & B_i &= \begin{bmatrix} -I \\ 0 \end{bmatrix} \\
B_{dd} &= \begin{bmatrix} 0 \\ B_d \end{bmatrix}, & C &= \begin{bmatrix} 0 & C_g \end{bmatrix}
\end{align*}
\]

In the above extension model, the system output \( y \) is introduced as a state variable. Subsequently, a reference model system for equation (24) is developed, and the reference state matrix satisfies the matching equation, as follows

\[ A_m = A + BK_x^T \quad (25) \]

The LQ regulator is used to obtain the optimal gain matrix \( K_x \), and the quadratic performance index is expressed as follows

\[ J = \int_{t_0}^{t_f} (x^T Q x + u^T R u) d\tau + x^T(t_f)Q_f x(t_f) \quad (26) \]

where \( t_0 \) and \( t_f \) are the initial and terminal time, respectively; \( Q, Q_f \) denote the positive semi-definite symmetric matrix; and \( R \) is a positive definite symmetric control input weighting matrix. The optimal gain matrix \( K_x \) is obtained as follows

\[ K_x^T = -R^{-1}B^T P_s \quad (27) \]

where \( P_s \) satisfies the following equations

\[ -P_s = P_s A + A^T P_s + Q - P_s BR^{-1}B^T P_s \quad (28) \]

By substituting equation (27) into equation (25), we can obtain the reference model system, as follows

\[ \dot{x}_m = A_m x_m + B_y y_r \quad (29) \]

Then, we can rewrite the system dynamic equation (23), as follows

\[ \dot{x} = A_m x + B(u_a - K_x^T x) + B_d u_d + B_y y_r \quad (30) \]

The tracking error of the states is defined as follows

\[ e = x - x_m \quad (31) \]

To achieve the tracking objective, that is, \( e = 0 \), the adaptive state feedback controller is constructed as follows

\[ u_a = K_x^T x \quad (32) \]

where \( K_x \) is the estimated gain matrix \( K_x \).

The standard adaptive law based on the gradient algorithm is expressed as follows

\[ \dot{K}_x = \Gamma x^T P B, \quad \Gamma = \Gamma^T > 0 \quad (33) \]

where \( P \) is the unique positive definite symmetric solution for \( A_m^T P + PA_m = -Q \), and \( Q = Q^T > 0 \).

**Proof.** Consider the following Lyapunov function

\[ V(e, DK_x) = e^T Pe + tr(DK_x^T G^{-1} DK_x) \quad (34) \]
In combination with equation (25), we can obtain the following expression

\[ V = C_0 e^T Q e - 2e^T PB \Delta K^T x + 2tr(\Delta K^T (\Gamma^{-1} \hat{\Gamma} - x e^T PB)) \]

where \( K_p, K_i, K_d \) are the proportion coefficient, integral coefficient, and differential coefficient, respectively.

Therefore, the condition for Lyapunov stability is satisfied.

**PID-model reference adaptive control system**

The dynamic system consists of 13 vibration sub-models and different reference models were designed for the different sub-models. Before outputting the control signal \( u_a \), we must determine the sub-range of the displacement loadings \( V_x, V_y \) from the previous step, and the corresponding adaptive law changes based on the different sub-models. The model reference adaptive controller acts as a compensator, the output is \( u_{a1} \), and an external PID controller acts as a coarse tracking controller, the output is \( u_{a2} \), which can be expressed as follows

\[ u_{a1} = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{d}{dt} e(t) \]  

where \( K_p, K_i, K_d \) are the proportion coefficient, integral coefficient, and differential coefficient, respectively.

These two controllers constitute a multi-model switching PID-model reference adaptive control (MRAC) system. Figure 6 shows the flowchart of the system, and Figure 7 shows the block diagram of the PID-MRAC system. The third part of equation (8) is ignored when designing the controller; however, the nonlinear term was added in the dynamic model.

The control signal \( u_a \) consists of two components, as follows

\[ u_a = \lambda_1 u_{a1} + \lambda_2 u_{a2} \]

where \( \lambda_1 \) and \( \lambda_2 \) are the control input weighting coefficients. When the vibration displacement \( |y| \geq 0.5 \text{ mm} \), \( \lambda_1 = 0.8, \lambda_2 = 0.2 \), the PID controller plays an important role in rapidly increasing the boundary displacement loadings, and increasing the stiffness of the membrane structure. When \( |y|<0.5 \text{ mm} \), \( \lambda_1 = 0, \lambda_2 = 1 \), the PID controller does not work and the displacement loadings are adjusted by the MRAC controller. A limiting filter is used, the maximum allowable deviation of two sampling times is 0.1 mm, which ensures that the boundary displacement loadings will not increase when a certain value is exceeded, and prevents the membrane from breaking by overstretching.

**Experiment**

**Experimental design**

Figure 8(a) shows a square membrane made of Kapton. As can be seen in the enlarged local drawing, the vertex was connected to a force sensor with a cable, which was used to obtain initial pre-stress values and monitor the
force. A carbon fiber material was chosen for the cable. Because this material has Young’s modulus of 193 GPa, the elastic deformation can be ignored. The tensions were adjusted by two servo VCM actuators. When the VCM in the X-direction moved by 1 mm, which is similar to a pair of equal and opposite displacement loadings, $V_X = 0.5$ mm was applied to the membrane vertices in the X-direction, and the same was applied to the Y-direction. Then, the out-of-plane vibration displacement was measured as the output of the closed-loop control system using a position-sensitive detector (PSD). An impact hammer installed behind the membrane structure was used to simulate the disturbance. The block diagram of the experimental system is shown in Figure 8(b).

**Results under initial disturbance**

The vibration curves at the central point under the same disturbance with $V_X = V_Y = 0.5$ mm are shown in Figure 9. Because of the measurement errors and the deformation of the membrane itself, the vibration amplitudes obtained by experiment were larger than the analytical results. However, the maximum error
between the analytical results and numerical results was 0.025 mm, and the local enlarged drawing shows that the vibration trends obtained analytically, experimentally, and using the FEM are consistent. This provides additional proof that the analytical model can describe the dynamic characteristics of the vibration system.

Both the traditional PID control method and the PID-MRAC method were applied to the dynamic vibration system. Figure 10 shows the vibration curves, and it can be seen that the vibration was adequately suppressed. When the vibration amplitude dropped below 0.01 mm, the vibration duration without control was approximately 12.37 s; this was reduced to approximately 2.96 s with the PID-MRAC method, while it required approximately 4.91 s with the traditional PID control method alone. Combined with the enlarged local drawing shown in Figure 10, we concluded that the proposed PID-MRAC method is more effective in suppressing vibration compared with the traditional PID control method.

Figure 11(a) shows the relative position of the VCM. The displacements $v_x, v_y$ in the X- and Y-directions were the same as with the traditional PID control method, whereas with the PID-MRAC method, $v_x$ was not equal to $v_y$. The absolute displacement loadings $V_X, V_Y$...
applied to the membrane structure were half the corresponding total displacements for the VCM, as shown in Figure 11(b). The corresponding forces \( T_x, T_y \) applied to the membrane are shown in Figure 11(c).

The proposed method is an in-plane control method. The method proposed by Ferhat and Sultan\(^2\) attached the actuators to the membrane is thus an out-of-plane control method. As shown in Figure 12(a), two actuators were attached to the corners of the membrane, and an adaptive controller was designed for the multi-model vibration system. Figure 12(b) shows the vibration curves, and these indicate that the vibration displacement was smaller with the in-plane control method. In addition, the partially enlarged drawing shows that the vibration duration was approximately 3.52 s with the out-of-plane control method, which amounts to a lengthening of approximately 15.9% compared with the duration obtained by the proposed method. In space, the vibration duration after the disturbance will be longer, because of the air-free environment, the effectiveness of the proposed method will therefore be greater. Moreover, attaching the actuators directly to the membrane can change the local vibration characteristics, and this cannot be ignored in engineering applications.

**Results under multiple disturbances**

As can be seen in Figure 11(b), the final steady-state displacement loading was about 0.86 mm. New disturbances were then applied to the structure after the initial disturbance. The absolute displacement loadings \( V_x, V_y \) applied to the membrane structure are shown in Figure 13. Four more disturbances were applied to the membrane, and small adjustments were made after the third disturbance. When the boundary displacement loadings reached a certain value, the final steady-state displacement loading was about 0.99 mm, and the corresponding natural frequency was approximately 36.47 Hz. Therefore, applying an appropriate pre-stress to make the initial natural frequency around 36.47 Hz gave the membrane structures high anti-interference capabilities. The vibration curves shown in Figures 14 indicate that the vibration was effectively suppressed with the proposed control method, and the vibration amplitudes dropped below 0.01 mm within 3 s.

**Conclusion**

A dimension reduction multi-model vibration system that accounted for the influence of wrinkles was developed to estimate the dynamic characteristics of a membrane structure. A multi-model switching approach based on adaptive control and PID control is also proposed to effectively and rapidly suppress the vibration. A comparison of the analytical, numerical, and experimental results verified the validity of the proposed vibration model. The preceding discussion outlines the
effectiveness of the vibration suppression achieved by adjusting the boundary displacement loadings without attaching actuators directly to the membrane surface.

The main conclusions drawn from this study can be summarized as follows:

1. Wrinkling information was introduced during dynamic modeling, and a control-oriented multi-model vibration system was established. This model reflected the vibration characteristics and propagation regularity of a planar membrane structure with local wrinkles.

2. An in-plane control method was proposed, and a PID-MRAC system was designed. The vibration of the membrane was effectively suppressed without attaching extra actuators to the surface of the membrane, and the maximum vibration deformation was reduced by adjusting the boundary displacement loadings.

3. An experimental system was designed to verify the effectiveness of the proposed method. Under the initial disturbance, the vibration duration with the PID-MRAC method was reduced to 2.96 s, which amounts to a shortening of approximately 39.7% compared with the traditional PID control method, and approximately 15.9% compared with the out-of-plane control method. With multiple disturbances, the vibration was effectively suppressed within 3 s, and the boundary displacement loadings did not increase when a certain value was exceeded. This prevented the membrane from breaking by overstretching and provides a theoretical foundation for setting the initial pre-stress values.

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References
1. Bonin AS and Seffen KA. De-wrinkling of pre-tensioned membranes. Int J Solids Struct 2014; 51: 3303–3313.
2. Ruggiero EJ. Modeling and control of SPIDER satellite components. PhD Dissertation, Department of Mechanical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, 2005.
3. Luo YJ, Xing J, Niu Y, et al. Wrinkle-free design of thin membrane structures using stress-based topology optimization. *J Mech Phys Solids* 2017; 102: 277–293.

4. He W, Ouyang YC and Hong J. Vibration control of a flexible robotic manipulator in the presence of input deadzone. *IEEE T Ind Inform* 2017; 13: 48–59.

5. Liu CJ, Todd MD, Zheng ZL, et al. A nondestructive method for the pretension detection in membrane structures based on nonlinear vibration response to impact. *Struct Health Moni* 2018; 17: 67–79.

6. Liu CJ, Deng XW, Liu J, et al. Impact-induced nonlinear damped vibration of fabric membrane structure: theory, analysis, experiment and parametric study. *Compos Part B-Eng* 2019; 159: 389–404.

7. Li D, Zheng ZL, He C, et al. Dynamic response of prestressed orthotropic circular membrane under impact load. *J Vib Control* 2018; 24: 4010–4022.

8. Ciprian CD. On the nonlinear membrane approximation and edge-wrinkling. *Int J Solids Struct* 2016; 82: 85–94.

9. Fang H, Yang B, Ding H, et al. Dynamic analysis of large in-space deployable membrane antennas. In: The 13th international congress on sound and vibration, Vienna, Austria, 2 July 2006. Pasadena, CA: Jet Propulsion Laboratory, National Aeronautics and Space Administration.

10. Kukathasan S and Pellegrino S. Nonlinear vibration of wrinkled membranes. In: 44th *AIAA/ASME/ASCE/AHS structures, structural dynamics, and materials conference*, Norfolk, VA, 7–10 April 2003, paper no. AIAA 2003-1747. Reston, VA: American Institute of Aeronautics and Astronautics.

11. Hossain NMA, Jenkins CH, Woo K, et al. Transverse vibration analysis for partly wrinkled membranes. *J Spacecraft Rockets* 2006; 43: 626–637.

12. Hossain NMA, Jenkins CH, Woo K, et al. Wrinkles and gravity effects on transverse vibration of membranes. In: 46th *AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics & materials conference*, Austin, TX, 18–21 April 2005, paper no. AIAA 2005-1978. Reston, VA: American Institute of Aeronautics and Astronautics.

13. Liu X, Jenkins CH and Schur WW. Large deflection analysis of pneumatic envelopes using a penalty parameter modified material model. *Finite Elem Anal Des* 2001; 37: 233–251.

14. Wang C, Li Y, Du X, et al. Simulation analysis of vibration characteristics of wrinkled membrane space structure. *Int J Space Struct* 2007; 22: 239–246.

15. Kandasamy R, Cui F, Townsend N, et al. A review of vibration control methods for marine offshore structures. *Ocean Eng* 2016; 127: 279–297.

16. Gasteratos A. Active camera stabilization with a fuzzy-grey controller. *Eur J Mech Environ Eng* 2009; 2: 18–20.

17. David SA, de Sousa RV, Valentim CA Jr., et al. Fractional PID controller in an active image stabilization system for mitigating vibration effects in agricultural tractors. *Comput Electron Agr* 2016; 131: 1–9.

18. Zhao F, Dong M, Qin Y, et al. Adaptive neural-sliding mode control of active suspension system for camera stabilization. *Shock Vib* 2015; 2015: 542364.

19. Renno JM, Inman DJ and Chevva KR. Nonlinear control of a membrane mirror strip actuated axially and in bending. *AIAA J* 2009; 47: 484–493.

20. Ruggiero EJ and Inman DJ. Modeling and vibration control of an active membrane mirror. *Smart Mater Struct* 2009; 18: 095027.

21. Ferhat I and Sultan C. System analysis and control design for a membrane with bimorph actuators. *AIAA J* 2015; 53: 2110–2120.

22. Liu X, Cai G, Peng F, et al. Nonlinear vibration control of a membrane antenna structure. *Proc IMechE Part G: J Aerospace Engineering* 2019; 233: 3273–3285.

23. Ferhat I and Sultan C. LQR using second order vector form for a membrane with bimorph actuators. In: 23rd *AIAA/AHS adaptive structures conference*, Kissimmee, FL, 5–9 January 2015, paper no. AIAA 2015-1512. Reston, VA: American Institute of Aeronautics and Astronautics.

24. Mikulas MM and Adler AL. Rapid structural assessment approach for square solar sails including edge support cords. In: 44th *AIAA/ASME/ASCE/AHS structures, structural dynamics, and materials conference*, Norfolk, VA, 7–10 April 2003, paper no. AIAA 2003-1447. Reston, VA: American Institute of Aeronautics and Astronautics.

25. Sakamoto H. Dynamic wrinkle reduction strategies for membrane structures. PhD Dissertation, Department of Aerospace Engineering Sciences, University of Colorado at Boulder, Boulder, CO, 2004.

26. Sakamoto H, Park KC and Miyazaki Y. Dynamic wrinkle reduction strategies for cable-suspended membrane structures. *J Spacecraft Rockets* 2005; 42: 850–858.

27. Dinh TD, Rezaei A, Panurari W, et al. A shape optimization approach to integrated design and nonlinear analysis of tensioned fabric membrane structures with boundary cables. *Int J Solids Struct* 2016; 83: 114–125.

28. Liu MJ, Huang J and Wang YL. Analysis of wrinkled membrane structures based on a wrinkle-wave model. *AIP Adv* 2017; 7: 015301.

29. Johnson L, Alexander L, Baggett RM, et al. NASA’s In-space propulsion technology program: overview and update. In: 40th *AIAA/ASME/SAE/ASEE joint propulsion conference and exhibit*, Fort Lauderdale, FL, July 2004, paper no. AIAA 2004-3841, https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/2005000112.pdf

30. Li YL, Wang CG and Tan HF. Research on free vibration of wrinkled membranes. In: *The 5th International conference on nonlinear mechanics*, Shanghai, China, 11–14 June 2007, pp.649–654. China: Shanghai University Press.

31. Zhang J, Huang J, Qiu LL, et al. Analysis of reflector vibration-induced pointing errors for large antennas subject to wind disturbance: evaluating the pointing error caused by reflector deformation. *IEEE Antenn Propag M* 2015; 57: 46–61.

32. Xie Y, Zhao T and Cai GP. Model reduction and active control for a flexible plate. *Acta Mech Solida Sin* 2011; 24: 467–476.

33. Zhang ZJ, Zhou HB and Duan J. Design and analysis of a high acceleration rotary-linear voice coil motor. *IEEE T Magn* 2017; 53: 1–9.