Robust H-infinity Control of Discrete Time-delay Generalized System with Saturating Actuator

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ABSTRACT: This thesis studies robust H-infinity control problem of discrete time-delay generalized system with saturating actuator, aiming to design robust H-infinity controller of system. Based on sufficient condition allowable in discrete time-delay generalized system, this thesis used linear matrix inequality to give the sufficient condition of H-infinity performance \( r \). It also applied feasible solution of linear matrix inequality to give expression of robust H-infinity controller. In the first part, this thesis will provide the sufficient condition of time-delay system with saturating actuator having robust H-infinity performance and the design method of robust controller.

Keywords: discrete generalized system; saturating actuator; time-delay; robust H-infinity control; linear matrix inequality

1 INTRODUCTION

In actual control problems, time delay and actuator saturation are two common non-linear phenomena. They both cannot only deteriorate system performance; but can also make system unstable. Many scholars have made separate studies of time-delay system and system with saturating actuator. Compared with linear system, time-delay system control is more complicated in system analysis and synthesis due to the introduction of status and control variables. Therefore, the studies made in recent years have mainly focused on developing means to stabilize time-delay system \(^{[1-4]}\). These means mainly include Lyapunov method, optional quadratic form method, finite spectrum assignment method, and various inequations. All these methods have been used to obtain the equanimity condition. Although several methods \(^{[5-8]}\) have been developed to deal with saturating problem and succeeded in application, our understanding about the influence that saturation has on system response is far away from being enough to solve the problem of controlling system with actuator saturation. For univariate actuator saturating system \(^{[7]}\), this thesis used Popov principle to analyze system stability. For multivariate situation, this thesis mainly used inequations and LQ optimization method \(^{[8-9]}\).

Naturally, what needs to be further studied is the robust H-infinity control of system with time delay and actuator saturation. The methods used to separately deal with time delay and actuator saturation problems as mentioned above cannot be taken to directly solve the robust H-infinity control problem of time-delay system with actuator saturation. Based on this observation, Reference \(^{[10]}\) gives the existence condition and design method of system robust H-infinity controller according to time-delay generalized system with actuator saturation. Based on the above, this thesis considers discrete time and is the first to give the sufficient condition for time-delay system with saturating actuator to have robust H-infinity performance. It also tells the design method of robust controller.

2 PROBLEM DESCRIPTIONS AND PREREQUISITE KNOWLEDGE

Consider the uncertain discrete time-delay generalized system with actuator saturation as shown below:
The saturating function can be described as:

\[
\text{sat}(u(k)) = (\text{sat}(u_1(k)), \text{sat}(u_2(k)), \ldots, \text{sat}(u_m(k)))^T,
\]

where

\[
\begin{cases}
  u^*_1, & u_1(k) > u^*_1 \\
  u_1^*, & u_1(k) \leq u_1^* \\
  -u_1^*, & u_1(k) < -u_1^* \\
\end{cases}
\]

Break this saturating function into the following form:

\[
\text{sat}(u(k)) = \frac{1}{2} u(k) + dv(u(k))
\]

The aim of this thesis is System (1). Construct status feedback controller \( u(k) = k_x x(k) \), and make the closed-loop system constructed by them allowable and can contain H-infinity performance \( r \). In order to verify, this thesis gives the following definitions and lemmas.

Definition 1[11]: System (1) is called system with H-infinity performance \( r \).

\[
\begin{align*}
  &\|Ex(k + 1) - Ax(k) + A_x x(k - \tau) + Bu(k) + B_i \omega(k)\|_2 \\
  &\|z(k) - C x(k) + C_i x(k - \tau) + D \omega(k)\|_2 \\
  &\|x(k) - \varphi(k), k \in [-\tau, 0]\|
\end{align*}
\]

(5)

If it is in zero input condition which means \( u(k) = 0 \), the two conditions as shown below can be met:

1. The system is asymptotically stable.
2. H-infinity norm of the transfer function \( G_{\omega}(k) \) for the external disturbance \( \omega(k) \) to be outputted to \( z(k) \) is not more than constant \( r \) (\( r > 0 \)), meaning \( \|z(k)\|_2 < r \|\omega(k)\|_2 \) and \( \forall \alpha \in L_2[0, \infty) \) can be met in the initial condition \( x(k) = 0 \) (\( k \in [-\tau, 0] \)).

Lemma 1[12]: For any vector \( x \) of \( n \)-dimension, \( y \) can satisfy \( 2x^T y \leq x^T x + y^T y \).

Lemma 2[13]: For any given symmetric matrixes \( \Xi \) and \( \Psi \), inequation \( \Xi + \Psi F(k) \Phi \Phi^T F^T(k) \Psi^T < 0 \) exists in \( F(k) \) if \( F^T(k) F(k) \leq I \). When there exists and only have one scalar \( \varepsilon > 0 \), then \( \Xi + \varepsilon \Psi \Psi^T + \varepsilon^{-1} \Phi \Phi^T \Phi < 0 \).

Lemma 3[14]: (Schur Complement Lemma) For any given symmetric matrix \( S \), the following three equations are of equal value:

\[
\begin{align*}
  &\Xi = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix} < 0; \\
  &S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0; \\
  &S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.
\end{align*}
\]

Lemma 4[15]: The sufficient condition allowable in time-delay discrete generalized system

\[
\begin{align*}
  &Ex(k + 1) = Ax(k) + A_x x(k - \tau) \\
  &z(k) = C x(k) + C_i x(k - \tau) \\
  &x(k) = \varphi(k), k \in [-\tau, 0]
\end{align*}
\]

is: there are reversible symmetric matrix \( P \) and positive definite matrix \( Q \), then the following two inequalities can be met:
\[ E^T PE \geq 0, \quad \begin{pmatrix} A^T PA + Q - E^T PE & A^T PA_d \\ A_d^T PA_d & A_d^T PA_d - Q \end{pmatrix} < 0 \]

3 KEY RESULTS

3.1 H-infinity performance analysis of closed-loop system

For uncertain discrete time-delay generalized system with saturating (1), closed-loop system can be obtained with the function of Status Feedback Controller (6):

\[
1 \begin{pmatrix} 1 & (1 ) ( ) ( )2 \\
1 & ( ) () () () \\
1 & () () \end{pmatrix} dv( ( )) \quad (7)
\]

\[ x(k) = \phi(k) , \quad k \in [-r, 0] \]

Among which,

\[
A_e = A + \Delta A \cdot B_e = B + \Delta B \cdot A_{e1} = A_d + \Delta A_d , \\
B_{e1} = B_1 + \Delta B_1 , \\
C_e = C + \Delta C \cdot C_{e1} = C_1 + \Delta C_1 , \quad D_e = D + \Delta D
\]

It can firstly conclude the sufficient condition for closed-loop system (7) to allow and contain H-infinity performance \( r \).

Theorem 1 for closed-loop system (7) and a given constant \( r > 0 \), if there exists positive definite matrices \( P \) and \( Q \), the following matrix inequations can be met:

\[
B_{e1}^T PB_{e1} - r^2 I < 0 \quad (8)
\]

\[
\Phi = \begin{pmatrix} S_1 & A_{e1}^T PA_{e1} & A_{e1}^T PB_{e1} \\ * & 2A_{e1}^T PA_{e1} - Q & A_{e1}^T PB_{e1} \\ * & * & 2B_{e1}^T PB_{e1} - r^2 I \end{pmatrix} < 0 \quad (9)
\]

Among which, \( * \) refers to transposed matrices of corresponding matrices: \( A_b = (A + \frac{1}{2} B_1 K) \),

\[ S_1 = 2A_{e1}^T PA_{e1} + Q + K^T B_{e1}^T PB_{e1} K - E^T PE \]

then closed-loop (7) is allowable and contains H-infinity performance \( r \).

Demonstration: Firstly, demonstrate that closed-loop (7) is allowable.

Assume there are positive definite matrices \( P \) and \( Q \) which can satisfy the conditions of assumption, formula (9) can be transformed as follows according to Schur Complement Lemma:

\[
\begin{pmatrix} A_{e1}^T PA_{e1} + Q - E^T PE & A_{e1}^T PA_{e1} \\ * & A_{e1}^T PA_{e1} - Q \end{pmatrix} \quad (10)
\]

\[ + \begin{pmatrix} A_{e1}^T PB_{e1} \\ * \end{pmatrix} \left( \begin{pmatrix} 0 \\ \frac{0}{0} \end{pmatrix} P(0, 0) + \begin{pmatrix} 0 \\ \frac{0}{0} \end{pmatrix} P(0, 0) \right)
\]

The following inequation can be obtained by the condition that \( P \) is a positive definite matrix and \( \Phi < 0 \):

\[
\begin{pmatrix} A_{e1}^T PA_{e1} + Q - E^T PE & A_{e1}^T PA_{e1} \\ * & A_{e1}^T PA_{e1} - Q \end{pmatrix} \quad (11)
\]

Based on the Inequation (8) in the assumption, the inequation given above can be concluded according to Schur Complement Lemma. The following inequation can be obtained from the above one:

\[
\begin{pmatrix} A_{e1}^T PA_{e1} + Q - E^T PE & A_{e1}^T PA_{e1} \\ * & A_{e1}^T PA_{e1} - Q \end{pmatrix} \quad (12)
\]

As \( P \) is a positive definite matrix, it is obvious that \( E^T PE \geq 0 \) can be met. According to Lemma 4, we can know that closed-loop (7) is allowable.

The second step is to demonstrate closed-loop (7) contains H-infinity performance \( r \).

Take the generalized Lyapunov function \( V(x(k)) \) of system as follows:

\[
V(x(k)) = x^T(k)E^T PEx(k) + \sum_{i=1}^{m} x^T(k - i)Qx(k - i)
\]

As \( P \) and \( Q \) are positive definite matrices, it is obvious that \( V(x(k)) > 0 \) can be met and its forward difference is:
$\Delta V(x(k)) = V(x(k+1)) - V(x(k))$
\[ = x^T(k)A_0^T P A_0 x(k) + x^T(k)A_0^T P A_1 x(k) + x^T(k)Q x(k) - x^T(k)E^T P E x(k) + x^T(k)A_{ri}^T P B_{ri} \omega(k) + x^T(k - \tau)A_{ri}^T P A_r x(k) + x^T(k - \tau)A_{ri}^T P A_{ri} x(k - \tau) + x^T(k - \tau)A_{ri}^T P B_{ri} \omega(k) - x^T(k - \tau)Q x(k - \tau) + \omega^T(k)B_{ri}^T P A_r x(k) + \omega^T(k)B_{ri}^T P A_{ri} x(k - \tau) + \omega^T(k)B_{ri}^T P B_{ri} \omega(k) + 2x^T(k)A_{ri}^T P B_{ri} \delta v(u(k)) + 2x^T(k - \tau)A_{ri}^T P B_{ri} \delta v(u(k)) + 2\omega^T(k)B_{ri}^T P B_{ri} \delta v(u(k)) + (\delta v(u(k)))^T B_{ri}^T P B_{ri} \delta v(u(k)) \]

As $P$ is a positive definite matrix, there is matrix $U$ to make $T^T P U = U P$. The following inequation can be concluded according to Lemma 1 and Inequation (5):

$2x^T(k)A_{ri}^T P B_{ri} \delta v(u(k)) \leq x^T(k)A_0^T P A_0 x(k)$  
\[ + \frac{1}{4} x^T(k)K^T B_{ce}^T P B_{ce} K x(k) \]
\[ + \omega^T(k)B_{ri}^T P B_{ri} \omega(k) \]

The following inequation can be obtained based on the four inequations shown above:

$\Delta V(x(k)) \leq \xi^T W \xi$

In which,

$\xi = (x(k)^T \ x^T(k - \tau) \ \omega^T(k))^T$

$W = \begin{pmatrix} S_1 & A_{ri}^T P A_1 & A_{ri}^T P B_{ri} \\ * & 2A_{ri}^T P A_1 - Q & A_{ri}^T P B_{ri} \\ * & * & 2B_{ri}^T P B_{ri} \end{pmatrix}$

Take

$J_\infty = \sum_{k=0}^{\infty} {\Delta V(k)} + z^T(k)z(k) - r^2 \omega^T(k)\omega(k) \)

We can obtain:

$J_\infty = \sum_{k=0}^{\infty} {\Delta V(k)}$  

Then, it can be seen from the assumption of $\Phi < 0$ that:

$J_\infty < 0$

Based on the initial condition and $V'(x(k)) > 0$, it can be concluded that $z^T(k)z(k) - r^2 \omega^T(k)\omega(k) < 0$.

That is,

$\|z(k)\|_2 < r \|\omega(k)\|_2$

Thus, closed-loop (7) contains H-infinity performance $r$.

In order to turn the Inequation (8) and (9) in Theorem 1 into LMI matrix inequations, this thesis reached deductions as follows:

Deduction 1: For closed-loop (7) and the given constant $r > 0$, if there are positive definite matrices $P$ and $Q$, the following matrix inequations can be met:

\[ \begin{bmatrix} -r^2 I & B_{ri}^T \\ * & -P^{-1} \end{bmatrix} < 0 \]

\[ \begin{bmatrix} \Theta_1 & \Theta_2 \\ \Theta_2^T & \Theta_3 \end{bmatrix} < 0 \]

Among which, $*$ refers to transposed matrix of corresponding matrix:

\[ A_0 = (A_r + \frac{1}{2} B_{ri} K) \cdot \Theta_1 = \begin{pmatrix} Q - E^T P E & 0 & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -r^2 I \end{pmatrix} \]

\[ \Theta_2 = \begin{pmatrix} K^T B_{ce}^T A_0^T & 0 & 0 & A_0^T C_{ce}^T \\ 0 & 0 & A_{ri}^T C_{ri}^T \\ 0 & 0 & B_{ri}^T B_{ri}^T C_{ri}^T \end{pmatrix} \]

$\Theta_3 = \text{diag}(-P^{-1}, -P^{-1}, -P^{-1}, -P^{-1}, -P^{-1}, -I)$

Thus, closed-loop (7) is allowable and it contains H-infinity performance $r$.

Demonstration: Obviously, it can be concluded that Inequation (10) and Inequation (8) are of equal value according to Schur Complement Lemma.

In the next step, we will demonstrate Inequation (11) and Inequation (9) are of equal value.

The following can be proved by Theorem 1:
\[
\Phi = \begin{pmatrix}
Q - E^\top P E & 0 & 0 \\
0 & -Q & 0 \\
0 & 0 & -r^2 I
\end{pmatrix} + \begin{pmatrix}
A_0^\top \\
B_{c_1}^\top \\
C_{c_1}^\top
\end{pmatrix} P(A_0, 0, 0) + \begin{pmatrix}
0 \\
A_{c_1}^\top \\
B_{c_1}^\top
\end{pmatrix} P(0, A_{c_1}, 0) + \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix} P(0, 0, B_{c_1}) + \begin{pmatrix}
A_0^\top \\
A_{c_1}^\top \\
B_{c_1}^\top
\end{pmatrix} P(A_0, A_{c_1}, B_{c_1}) + \begin{pmatrix}
C_{c_1}^\top \\
D_{c_1}^\top
\end{pmatrix} (C_{c_1}, C_{c_1}, D_{c_1}) + \begin{pmatrix}
K_r^\top B_{c_1}^\top \\
0 \\
0
\end{pmatrix} P(B_{c_1} K_r, 0, 0)
\]

It can be known from Schur Complement Lemma that:

\[
\begin{pmatrix}
\Theta_1 & \Theta_2 \\
\Theta_2^\top & \Theta_3
\end{pmatrix} < 0 \text{ shares the same value with } \Phi < 0,
\]

that is Inequation (11) and Inequation (9) are of equal value. Thus, the deduction exists.

3.2 Design of status feedback controller

Theorem 2 for closed-loop (7) and a given constant \( r > 0 \), if there are positive definite matrixes \( X \) and \( Z \), matrix \( Y \) and scalars \( \varepsilon_i \) (\( \varepsilon_1, \varepsilon_2 > 0 \)), the following matrix inequation can be met:

\[
EX =XE
\]

\[
\begin{pmatrix}
-r^2 I & B_{c_1}^\top & N_4^\top \\
* & \varepsilon_i M_i M_i^\top & -X \\
* & * & -\varepsilon_i I
\end{pmatrix} < 0 \quad (13)
\]

\[
\Psi = \begin{pmatrix}
\Psi_{11} & \Psi_{12} \\
\Psi_{21} & \Psi_{22}
\end{pmatrix} < 0
\quad (14)
\]

Among which,

\[
\Psi_{11} = \begin{pmatrix}
\Psi_{111} & \Psi_{112} \\
\Psi_{121} & \Psi_{122}
\end{pmatrix},
\]

\[
\Psi_{111} = \begin{pmatrix}
-E^\top X^\top E & 0 & 0 \\
0 & -Z^\top & 0 \\
0 & 0 & -r^2 I
\end{pmatrix}
\]

\[
\Psi_{112} = \begin{pmatrix}
Y_1^\top B_{c_1}^\top & X_1^\top A_{c_1} + \frac{1}{2} Y_1^\top B_{c_1}^\top & X_1^\top C_{c_1} \\
0 & 0 & Z_1^\top A_{c_1}^\top \\
0 & 0 & Z_1^\top C_{c_1}^\top \\
0 & 0 & B_1^\top \\
0 & 0 & B_1^\top \\
0 & 0 & D_1
\end{pmatrix}
\]

\[
\Psi_{21} = \begin{pmatrix}
\Sigma_1 & \Sigma_2 \\
\Sigma_2^\top & \Sigma_3
\end{pmatrix}
\]

\[
\Sigma_1 = \begin{pmatrix}
-X + \varepsilon_i M_i M_i^\top & \frac{1}{2} \varepsilon_i M_i M_i^\top & 0 & 0 \\
* & G & 0 & 0 \\
* & * & \varepsilon_i M_i M_i^\top - X & 0 \\
* & * & * & \varepsilon_i M_i M_i^\top - X
\end{pmatrix}
\]

Among which,

\[
G = \varepsilon_i M_i M_i^\top + \frac{1}{4} \varepsilon_i M_i M_i^\top - X
\]

\[
\Sigma_2 = \begin{pmatrix}
\frac{1}{2} \varepsilon_i M_i M_i^\top & 0 \\
\varepsilon_i M_i M_i^\top + \frac{1}{4} \varepsilon_i M_i M_i^\top & 0 \\
\varepsilon_i M_i M_i^\top & 0 \\
\varepsilon_i M_i M_i^\top & 0
\end{pmatrix}
\]

\[
\Sigma_3 = \begin{pmatrix}
V & 0 \\
0 & \varepsilon_i \sum_{k=5} M_i M_i^\top - I
\end{pmatrix}
\]

Among which,

\[
V = \varepsilon_i (M_i M_i^\top + M_i M_i^\top + M_i M_i^\top + M_i M_i^\top) - X
\]

\[
\Psi_2 = \begin{pmatrix}
X_1^\top N_1^\top & Y_1^\top N_3^\top & X_1^\top N_5^\top & 0 & 0 & X_1^\top \\
0 & Z_1^\top N_1^\top & 0 & 0 & 0 & Z_1^\top N_3^\top \\
0 & 0 & N_3^\top & 0 & 0 & N_5^\top \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
\Psi_3 = \text{diag}(-\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I, -\varepsilon_i I, -Z)
\]

Thus, there exists status feedback controller \( u(k) = YX^{-1} x(k) \) to make closed-loop (7) allowable and contains H-infinity performance \( \rho \).

Demonstration: Firstly, demonstrate that Inequation (13) and Inequation (10) in Deduction 1 are of equal value.

Introduce \( B_{c_1} = B_1 + \Delta B_1 \) into the Inequation (1) in
Deduction 1, then it can be concluded that:

\[
\begin{pmatrix}
-2^2 & B_i^T \\
* & -P^{-1}
\end{pmatrix} = \begin{pmatrix}
-2^2 & B_i^T \\
* & -P^{-1}
\end{pmatrix} + \Pi_1\Pi_3^T (\Pi_1\Pi_3^T)^T < 0
\]

Among which,

\[
\Pi_1 = \left( \begin{array}{c} 0 \\ M_4 \end{array} \right), \Pi_3 = \left( \begin{array}{c} N_4 \\ 0 \end{array} \right), \Pi_2 = F_4
\]

From Lemma 2, it can be known that the above equation is applicable in all conditions of

\[
F_4^T F_4 \leq I_{g_4}
\]

When there exists and there’s only one scalar \( \epsilon > 0 \) to make

\[
\begin{pmatrix}
-2^2 & B_i^T \\
* & -P^{-1}
\end{pmatrix} + \epsilon \Pi_1\Pi_3^T + \frac{1}{\epsilon} \Pi_3^T \Pi_3 < 0
\]

It can be known from Schur Complement Lemma that the above inequality shares the same value with

\[
\begin{pmatrix}
-2^2 & \Pi_1^T \\
* & -\epsilon I
\end{pmatrix} < 0
\]

Take \( P^{-1} = X \) and introduce it into the above inequality, then we can have the following inequality by operation:

\[
\begin{pmatrix}
-2^2 & B_i^T \\
* & -P^{-1}
\end{pmatrix} + \epsilon_i M_4 M_4^T - X \begin{pmatrix} N_i^T \\ 0 \end{pmatrix} < 0
\]

That is Inequation (13) and Inequation (1) in Deduction 1 are of equal value.

Next, we can demonstrate Inequation (14) and Inequation (11) in Deduction 1 are of equal value.

Introduce Inequation (3) into Inequation (11) of Deduction 1, we can obtain

\[
\Lambda_3 = \begin{pmatrix}
\kappa^2 B_i^T (A + \frac{1}{2} Rk)^T & 0 & 0 & (A + \frac{1}{2} Rk)^T C^T \\
0 & A_i^T & 0 & A_i^T C_i^T \\
0 & 0 & B_i^T & B_i^T D_i^T
\end{pmatrix}
\]

\[
R = \begin{pmatrix}
\kappa^2 \Delta B_i^T (\Delta A + \frac{1}{2} \Delta Rk)^T & 0 & 0 & (\Delta A + \Delta Rk)^T \Delta C_i^T \\
0 & \Delta C_i^T & 0 & \Delta C_i^T C_i^T \\
0 & 0 & \Delta R_i^T & \Delta R_i^T \Delta D_i^T
\end{pmatrix}
\]

where \( 0_{r \times r_1} \) and \( 0_{r_2 \times r_2} \) refer to null matrices of corresponding dimensions.

To make further transformation of Inequation (1), we can obtain:

\[
\Lambda + \begin{pmatrix}
0_{g \times g} \\
R^T \\
0_{g \times g_2}
\end{pmatrix} = \Lambda + \Gamma F H + (\Gamma F H)^T < 0
\]

Among which,

\[
F = \text{diag}(F_1, F_2, F_3, F_4, F_5, F_6, F_7)
\]

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

It can be known from Lemma 2 that the above inequality can satisfy all uncertain conditions of Inequation (3). When there exists and there’s only one scalar \( \epsilon > 0 \) to make

\[
\Lambda_3 = \begin{pmatrix}
\kappa^2 B_i^T (A + \frac{1}{2} Rk)^T & 0 & 0 & (A + \frac{1}{2} Rk)^T C^T \\
0 & A_i^T & 0 & A_i^T C_i^T \\
0 & 0 & B_i^T & B_i^T D_i^T
\end{pmatrix}
\]

\[
R = \begin{pmatrix}
\kappa^2 \Delta B_i^T (\Delta A + \frac{1}{2} \Delta Rk)^T & 0 & 0 & (\Delta A + \Delta Rk)^T \Delta C_i^T \\
0 & \Delta C_i^T & 0 & \Delta C_i^T C_i^T \\
0 & 0 & \Delta R_i^T & \Delta R_i^T \Delta D_i^T
\end{pmatrix}
\]
\[ \Lambda + \varepsilon_2 \Gamma T^T + \frac{1}{\varepsilon_2} H^T H < 0. \]

It can be known from Schur Complement Lemma that the above inequation equals to
\[
\begin{pmatrix} \Lambda + \varepsilon_2 \Gamma T^T & H^T \\ H & -\varepsilon_2 I \end{pmatrix} < 0
\]

We can obtain the following inequation by operation:
\[
\begin{pmatrix} \Lambda + \varepsilon_2 \Gamma T^T & H^T \\ H & -\varepsilon_2 I \end{pmatrix} = \begin{pmatrix} \Omega_1 & \Omega_2 \\ \Omega_2^T & \Omega_3 \end{pmatrix} < 0
\]

In which,
\[
\Omega_1 = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^T & \Omega_{22} \end{pmatrix}, \quad \Omega_2 = \begin{pmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^T & \Lambda_3 \end{pmatrix}
\]

\[
\Omega_{11} = \begin{pmatrix} Q - E^T P E & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -r^2 I \end{pmatrix}, \quad \Omega_{12} = -P^T + \varepsilon_2 M_6^T \frac{1}{2} \varepsilon_2 M_6^T
\]

\[
\Lambda_1 = \begin{pmatrix} \frac{1}{2} \varepsilon_2 M_3^T M_3^T & 0 \\ * & G & 0 \\ * & * & \varepsilon_2 M_2^T M_2^T - P^T & 0 \\ * & * & * & \varepsilon_2 M_4^T M_4^T - P^T \end{pmatrix}
\]

\[
\Lambda_2 = \begin{pmatrix} \varepsilon_2 M_1^T M_1^T + \frac{1}{4} \varepsilon_2 M_3^T M_3^T & 0 \\ \varepsilon_2 M_2^T M_2^T & 0 \\ \varepsilon_2 M_4^T M_4^T & 0 \end{pmatrix}
\]

\[
\Lambda_3 = \begin{pmatrix} \varepsilon_2 \left( M_1^T M_1^T + M_4^T M_4^T \right) + G_i & 0 \\ 0 & \varepsilon_2 \sum_{i=5}^{\infty} M_i^T M_i^T - I \end{pmatrix}
\]

\[
\begin{align*}
\Omega_2 &= \begin{pmatrix} N_1^T & 0 & K^T N_3^T & 0 & N_5^T & 0 & 0 \\ 0 & N_2^T & 0 & 0 & 0 & N_5^T & 0 \\ 0 & 0 & 0 & N_4^T & 0 & 0 & N_5^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\
\Omega_3 &= \text{diag}(\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I)
\end{align*}
\]

\[
G_i = \varepsilon_2 M_1^T M_1^T + \frac{1}{4} \varepsilon_2 M_3^T M_3^T - P^{-1}
\]

It can be know from Schur Complement Lemma that the above inequation equals to
\[
\hat{\Omega} = \begin{pmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} \\ \hat{\Omega}_{12}^T & \hat{\Omega}_{22} \end{pmatrix} < 0
\]

In which,
\[
\hat{\Omega}_{11} = \begin{pmatrix} -E^T P E & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -r^2 I \end{pmatrix}, \quad \hat{\Omega}_{12} = \frac{1}{2} \varepsilon_2 M_6^T \frac{1}{2} \varepsilon_2 M_6^T
\]

\[
\begin{pmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} \\ \hat{\Omega}_{12}^T & \hat{\Omega}_{22} \end{pmatrix} \hat{\Omega}_3 = \begin{pmatrix} -E^T P E & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -r^2 I \end{pmatrix}
\]

\[
\hat{\Omega}_3 = \text{diag}(\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I, -\varepsilon_2 I)
\]

Multiply the left part of
\[
\hat{\Omega}_4 = \begin{pmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} \\ \hat{\Omega}_{12}^T & \hat{\Omega}_{22} \end{pmatrix} < 0
\]

by
\[
J = \text{diag}(P^{-1}, (Q^{-1})^T, 1, 1, 1, 1, 1, 1, 1, 1)
\]

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And multiply the left part by $J^T$. Take $X = P^{-1}$, $Y = KP^{-1}$ and $Q^{-1} = Z$. Based on $EX = XE$, we can obtain the following equation by operation:

$$\Psi = \begin{pmatrix} \Psi_1 & \Psi_2 \\ \Psi_2^T & \Psi_3 \end{pmatrix} < 0$$

That is Inequation (14) and Inequation (11) of Deduction 1 are of equal value.

Thus, there is status feedback controller $u(k) = YX^{-1}x(k)$ to make Closed-Loop (7) allowable and can contain H-infinity performance $r$. Thus, the theorem can be proved.

4 CONCLUSIONS

This thesis firstly conducted analysis of the stability of uncertain discrete time-delay generalized closed-loop system with saturating actuator. It gave a sufficient condition allowable in closed-loop system, and obtained the sufficient condition for system to contain H-infinity performance. Then, this thesis reached the conclusion about the sufficient condition and corresponding control law existing in system status feedback robust H-infinity controller.

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