An Analytical Framework for Device-to-Device Communication in Cellular Networks

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Abstract

This paper presents an analytical framework that enables characterizing analytically the spectral efficiency achievable by D2D (device-to-device) communication links integrated within a cellular network. This framework is based on a stochastic geometry formulation with a novel approach to the modeling and spatial averaging of interference, which facilitates obtaining compact expressions, and with the added possibility of incorporating exclusion regions to protect cellular users from excessive interference from active D2D transmitters. To illustrate the potential of the framework, a number of examples are provided. These examples confirm the hefty potential of D2D communication in situations of strong traffic locality as well as the effectiveness of properly sized exclusion regions.

Index Terms

D2D communication, overlay, underlay, spectral efficiency, stochastic geometry, Poisson point process.

I. INTRODUCTION

Device-to-device (D2D) communication, currently being touted as a potential ingredient of 5th-generation wireless networks [2]–[5], allows users in close proximity to establish direct communication links, replacing two long hops via the base station (BS) with a single shorter hop. Provided there is sufficient spatial locality in the wireless traffic, this can bring about several...
benefits: reduced power consumption, lower end-to-end latency, reduced backhaul loads, higher bit rates, and especially a much higher system spectral efficiency thanks to the denser spectral reuse. While the ad-hoc nature of D2D communication results in more complex and irregular topologies, unlike in traditional ad-hoc networks a D2D integrated cellular system can benefit from infrastructure assistance to perform efficient user discovery and smart scheduling.

The 3rd Generation Partnership Project (3GPP) is in the process of studying and standardizing D2D communication for cellular networks [6]–[8]. Several problems associated with D2D have been identified and are being explored by academia [9], [10]. While specific problems such as mode selection [11], [12], resource optimization [13]–[15], power allocation [16], [17], and advanced interference management techniques [18], [19] are being tackled, it is necessary to understand the fundamental performance limits of a D2D integrated cellular network.

Most previous works on D2D relied on simulations to explore system-level aspects [11]–[20]. Recognizing that stochastic geometry tools allow for models that are more amenable to analytical treatment, and arguably more representative of the heterogeneous structure of emerging wireless networks [21], some recent works [22]–[24] modeled the user locations via PPP (Poisson point process) distributions and analytically tackled the system-level performance of D2D communication.

In this paper, we continue down the path of [22], but with a different approach to model interference and with a controllable degree of spatial averaging, altogether completing a powerful and flexible framework that allows characterizing in simpler form—sometimes even in closed form—the spectral efficiencies achievable with D2D communication. The framework accommodates both underlay or overlay options, where respectively the D2D communication reuses the existing uplink or utilizes dedicated spectrum. We present several examples of how this framework can be leveraged to gauge the benefits of D2D, hoping that it can serve other researchers as they further explore the possibilities offered by D2D. Specifically, the examples demonstrate how the tools developed can be put to use, to answer the following questions:

- How often is direct D2D better than two hops (uplink-downlink) via the BS?
- How many D2D links can be packed per channel, without compromising their link spectral efficiency (bits/s/Hz)?
- How much better is the system spectral efficiency (bits/s/Hz/cell) given the denser spectral reuse?
II. System Model

We consider an interference-limited cellular network where the BSs are regularly placed on a hexagonal grid. At each BS, cellular transmissions are orthogonalized while multiple D2D links share each time-frequency signaling resource. Transmitters and receivers have a single antenna and each receiver knows the fading of only its own link, be it cellular or D2D. Our focus is on a given time-frequency resource, where one cellular uplink and/or (underlay/overlay) multiple D2D links are active in each cell.

To facilitate the readability of the equations, we utilize distinct fonts for the cellular and D2D variables.

A. User Locations

The locations of the transmitters, both cellular and D2D, are modeled relative to the location of a given receiver under consideration. For the cellular uplink, the receiver under consideration is a BS whereas, for the D2D link, it is a D2D receiver. In either case, and without loss of generality, we place such receiver at the origin and index the intended transmitter with zero. All other transmitters (interferers) are indexed in order of increasing distance within each class (cellular and D2D). For the receiving BS, the intended transmitter is always the closest cellular transmitter while, for the D2D receiver, the intended transmitter need not be the closest D2D transmitter.

1) Cellular Uplink: To study this link, we place a receiving BS at the origin and locate an intended cellular transmitter uniformly within the cell associated with that BS (cf. Fig. 1), which is circular with radius $R$ and denoted by $B(0, R)$. There is thus one and only one cellular transmitter within $B(0, R)$, and its distance to the BS at the origin is denoted by $r_0$.

The cellular interferers from other cells are all outside $B(0, R)$, modeled via a PPP $\Phi$ with density $\lambda = \frac{1}{\pi R^2}$. With this density made to coincide with the number of BSs per unit area, this has been shown to be a fine model for a network with one cellular transmitter per cell [22], [25].

The D2D interferer locations form another independent PPP $\Phi$ with density $\lambda = K\lambda$, reflecting the fact that there are, on average, $K$ active D2D links per cell.

2) D2D Link: To study this link, we place a D2D receiver at the origin and locate its intended D2D transmitter at a distance $r_0$. 

Fig. 1. Cellular uplink with D2D. Located at the origin is a receiving BS and shown with a square marker in the surrounding circle is its intended cellular transmitter; shown with square markers outside the circle are the cellular interferers; shown with diamond markers are the D2D interferers.

The cellular and D2D interferers conform to $\Phi$ and $\Phi$, respectively. Note that the cellular interferers can be arbitrarily close to the D2D receiver, a point whose implications are discussed later.

B. Received Signal

We denote by $P$ and $P$ the (fixed) powers transmitted by cellular and D2D users, respectively, with ratio $\mu = P/P$. And, in order to present the results in a more general fashion, we define a binary parameter $\alpha \in \{0, 1\}$ that distinguishes between underlay ($\alpha = 1$) and overlay ($\alpha = 0$).

1) Cellular Uplink: The BS at the origin observes

$$y = \sqrt{P r_0^{-\eta} H_0 b_0} + z$$  \hspace{1cm} (1)

where the first term is the signal from the intended cellular user while the second term represents the interference

$$z = \sum_{k=1}^{\infty} \sqrt{P r_k^{-\eta} H_k b_k} + \alpha \sum_{j=1}^{\infty} \sqrt{P r_j^{-\eta} H_j b_j}$$  \hspace{1cm} (2)

whose first summation spans the other-cell cellular users in $\Phi \setminus B(0, R)$ and whose second summation spans all the D2D transmitters in $\Phi$. In turn, $\eta > 2$ is the pathloss exponent for
cellular links, \( r_k \) represents the distance between the \( k \)th cellular transmitter and the BS at the origin, \( H_k \) denotes the corresponding fading, and \( b_k \) is the symbol transmitted by the \( k \)th cellular transmitter. Similarly, \( r_j \) represents the distance between the \( j \)th D2D transmitter and the BS at the origin, \( H_j \) denotes the corresponding fading and \( b_j \) is the symbol transmitted by the \( j \)th D2D transmitter. The fading coefficients \( H_k \) and \( H_j \) are independent identically distributed (IID) complex Gaussian random variables with zero mean and unit variance, i.e., drawn from \( \mathcal{CN}(0, 1) \). Likewise, \( b_k \sim \mathcal{CN}(0, 1) \) and \( b_j \sim \mathcal{CN}(0, 1) \).

2) D2D Link: To analyze this link, we shift the origin to the D2D receiver of interest, which observes

\[
y = \sqrt{P r_0^{-\eta}} H_0 b_0 + z
\]

where the first term is the signal from the intended D2D transmitter while the second term is the interference

\[
z = \alpha \sum_{k=1}^{\infty} \sqrt{P r_k^{-\eta}} H_k b_k + \sum_{j=1}^{\infty} \sqrt{P r_j^{-\eta}} H_j b_j
\]

received from the D2D transmitters in \( \Phi \) and the cellular transmitters in \( \Phi \). For these user-to-user signals, we consider a different pathloss exponent \( \eta > 2 \).

III. INTERFERENCE MODELING

A key differentiating feature of our framework is the interference modeling, expounded in this section and validated later in the paper. We depart from the approach in [22] where \( z \) and \( z \) are distributed as per (2) and (4), with all the products of signal and fading Gaussian variates therein, and also from the approach in [26] where a Gamma distribution with matched moments is fitted.

Rather, recognizing that both \( z \) and \( z \) consist of a large number of independent terms whose fading is unknown by the receiver of interest, we model their short-term distributions as zero-mean complex Gaussian with matched conditional covariances \( \sigma^2 = \mathbb{E}[|z|^2|\{r_k, r_j\}] \) and \( \sigma^2 = \mathbb{E}[|z|^2|\{r_k, r_j\}] \), respectively, where the expectations are over the data and fading distributions. The conditional covariance \( \sigma^2 \), which represents the power of \( z \) for given interferer locations, is easily found to equal

\[
\sigma^2 = \sum_{k=1}^{\infty} P r_k^{-\eta} + \alpha \sum_{j=1}^{\infty} P r_j^{-\eta}
\]
while its D2D counterpart $\sigma^2$ equals

$$\sigma^2 = \alpha \sum_{k=1}^{\infty} \Pr r_k^{-\eta} + \sum_{j=1}^{\infty} \Pr r_j^{-\eta}. \quad (6)$$

Besides the central limit theorem, there are information-theoretic arguments in favor of modeling the aggregate interference as complex Gaussian with a power dictated by the locations of the interferers: if the exact distribution of the interference is either unknown or ignored by the receiver, with a decoder designed to handle Gaussian noise, then the achievable spectral efficiency is precisely as if the interference were indeed Gaussian \[27\].

Returning to our modeling approach, both (5) and (6) contain an infinite number of terms, of which a handful largely dominate the total interference power. In recognition of this, we condition on the interferer locations within a circle surrounding the receiver of interest and replace the aggregate interference emanating from outside that circle with its expected (over the interference locations) value. As we shall see, this expected value can be representative of most instances of the interference outside the circle—by virtue of the law of large numbers—and, thanks to the potency of stochastic geometry, this expected value can be computed explicitly.

The introduction of the averaging circle allows reducing the number of variables retained in the formulation without the significant loss of information brought about by a complete averaging (a frequent recourse in stochastic geometry analyses). This allows establishing the performance for specific locations of the users within the circle, which are the dominant ones, and not only the average performance over all such locations. The radius of the averaging circle then becomes a modeling parameter that should be chosen to balance simplicity (the smaller the circle, the few interferers that are explicitly retained) and accuracy (the smaller the circle, the less fidelity in representing interference instances with their average). As a rather natural choice, we make the size of the circle coincide with that of a cell, $\mathcal{B}(0, R)$, and later validate this choice.

With that choice for the averaging circle, the interference power in (5) can be rewritten as

$$\begin{align*}
\sigma^2 = \alpha \sum_{j=1}^{K'} \Pr r_j^{-\eta} + \alpha \sum_{j=K'+1}^{\infty} \Pr r_j^{-\eta} + \sum_{k=1}^{\infty} \Pr r_k^{-\eta} \\
&= \underbrace{\sigma^2_{\text{in}}}_{\substack{\text{in} \ K' \ \text{D2D transmitters in } \Phi \cap \mathcal{B}(0, R) \text{ for the given geometry realization}}} + \underbrace{\sigma^2_{\text{out}}}_{\substack{\text{out} \ \text{transmitters in } \Phi \setminus \mathcal{B}(0, R) \text{ and } \Phi \setminus \mathcal{B}(0, R)}}
\end{align*} \quad (7)$$

where $\sigma^2_{\text{in}}$ corresponds to the $K'$ D2D transmitters in $\Phi \cap \mathcal{B}(0, R)$ for the given geometry realization whereas $\sigma^2_{\text{out}}$ corresponds to the transmitters in $\Phi \setminus \mathcal{B}(0, R)$ and $\Phi \setminus \mathcal{B}(0, R)$. Recalling
that \( \mathbb{E}[K'] = K \), the expectation of \( \sigma_{\text{out}}^2 \) over the PPPs equals
\[
\sigma_{\text{out}}^2 = \alpha \mathbb{E} \left[ \sum_{j=K'+1}^{\infty} P r_j^{-\eta} \right] + \mathbb{E} \left[ \sum_{k=1}^{\infty} P r_k^{-\eta} \right] \tag{8}
\]
\[
= \alpha \int_{r=R}^{\infty} 2\pi K \lambda P r^{1-\eta} dr + \int_{r=R}^{\infty} 2\pi \lambda P r^{1-\eta} dr \tag{9}
\]
\[
= \frac{2(\alpha K P + P)}{(\eta - 2) R^\eta} \tag{10}
\]
where (9) follows from Campbell’s theorem [28] and (10) is obtained by evaluating the integrals and substituting \( \lambda = \frac{1}{\pi R^2} \).

Similarly, for the D2D link, considering \( B(0, R) \) around the D2D receiver at the origin, the interference power in (6) can be rewritten as
\[
\sigma^2 = \sum_{j=1}^{K'} P r_j^{-\eta} + \alpha \sum_{k=1}^{K''} P r_k^{-\eta} + \sum_{j=K'+1}^{\infty} P r_j^{-\eta} + \alpha \sum_{k=K'+1}^{\infty} P r_k^{-\eta} \tag{11}
\]
where \( \sigma_{\text{in}}^2 \) corresponds to the \( K' \) D2D transmitters in \( \Phi \cap B(0, R) \) and the \( K'' \) uplink cellular transmitters in \( \Phi \cap B(0, R) \), whereas \( \sigma_{\text{out}}^2 \) corresponds to the transmitters in \( \Phi \setminus B(0, R) \) and \( \Phi \setminus B(0, R) \). Noting that \( \mathbb{E}[K'] = K \) and \( \mathbb{E}[K''] = 1 \), the expectation of \( \sigma_{\text{out}}^2 \) over the PPPs gives
\[
\sigma_{\text{out}}^2 = \frac{2(\alpha K P + P)}{(\eta - 2) R^\eta}. \tag{12}
\]

A. SIR of the Cellular Uplink

Under the foregoing model for the interference, with power \( \sigma_{\text{in}}^2 + \sigma_{\text{out}}^2 \), and recalling the intended signal term from (1), the instantaneous SIR of the uplink is
\[
\text{SIR} = \frac{P r_0^{-\eta} \mathbb{E} \left[ |H_0 b_0|^2 \right]}{\sigma_{\text{in}}^2 + \sigma_{\text{out}}^2} \tag{13}
\]
\[
= \rho |H_0|^2 \tag{14}
\]
where the expectation in (13) is over \( b_0 \), conditioned on the fading \( H_0 \), while
\[
\rho = \frac{r_0^{-\eta}}{\alpha \mu \sum_{j=1}^{K'} r_j^{-\eta} + \frac{2(\alpha \mu K+1)}{(\eta - 2) R^\eta}} \tag{15}
\]
is the local-average SIR at the BS. Further normalizing all the terms by \( R \),
\[
\rho = \frac{a_0^{-\eta}}{\alpha \mu \sum_{j=1}^{K'} a_j^{-\eta} + \frac{2(\alpha \mu K+1)}{\eta - 2}} \tag{16}
\]
where \( a_0 = \frac{r_0}{R} \), \( a_0 = \frac{r_0}{R} \), \( a_k = \frac{r_k}{R} \) and \( a_j = \frac{r_j}{R} \) are normalized distances. Indeed, our formulation is interference limited and therefore invariant to the absolute scale of the network.
B. SIR of the D2D Link

Similarly, for the D2D link, the instantaneous SIR is

\[
\text{SIR} = \frac{P_r - \eta}{\sigma_{in}^2 + \sigma_{out}^2}
\]

where the expectation is over \( b_0 \), conditioned on the fading, and

\[
\rho = \frac{r_0^{-\eta}}{\sum_{j=1}^{K'} r_j^{-\eta} + \frac{a}{\mu} \sum_{k=1}^{K''} r_k^{-\eta} + \frac{2(K + \alpha/\mu)}{(\eta-2)R^\eta}}
\]

is the local-average SIR at the D2D receiver of interest.

IV. SIR DISTRIBUTION

For a specific network geometry, i.e., given the normalized distances \( a_0 \) and \( \{a_j\}_{j=1}^{K'} \), the value of \( \rho \) becomes determined. Since \( |H_0|^2 \) is exponentially distributed with unit mean, it follows from (14) that the SIR exhibits an exponential distribution with local-average \( \rho \) and hence its conditional CDF (cumulative distribution function) is

\[
F_{\text{SIR}|\rho}(\gamma) = 1 - e^{-\gamma/\rho}.
\]

Similarly, given the normalized distances \( a_0 \), \( \{a_j\}_{j=1}^{K'} \) and \( \{a_k\}_{k=1}^{K''} \), the value of \( \rho \) becomes determined and it follows from (18) that

\[
F_{\text{SIR}|\rho}(\gamma) = 1 - e^{-\gamma/\rho}.
\]

Let us now validate the foregoing distributions, and with that the interference modeling approach that underpins them.

Example 1. Consider an overlay system (\( \alpha = 0 \)) with an average of \( K = 10 \) D2D links per cell and with typical values for the power ratio \( \mu \) and for the pathloss exponents (cf. Table I). To render the system as typical as possible, \( K' \) is set to its expected value and for \( j = 1, \ldots, K' \), \( a_j \) is set to the expected value of the normalized distance to the \( j \)th nearest neighboring point in a PPP with density \( \lambda \) [29]. The resulting \( a_1, \ldots, a_{K'} \) are given in Table I where \( \Gamma(\cdot) \) is the
TABLE I

SYSTEM PARAMETERS

| Parameter | Value                  |
|-----------|------------------------|
| \( K = \lambda \) | 10                     |
| \( \mu = \frac{p}{p_0} \) | 0.1                    |
| \( \eta, \eta \) | 3.5, 4.5               |
| \( K' \) | \( K \)                |
| \( a_j \) | \( \frac{\Gamma(0.5+j)}{\sqrt{K} \Gamma(j)} \) |

Fig. 2. CDF of instantaneous SIR for cellular and D2D links in an overlay system with the parameters of Table I and with \( a_0 = 0.5, 0.95 \) and \( a_0 = 0.05, 0.1 \).

Gamma function. Shown in Fig. 2 are the distributions in (21)-(22) alongside the corresponding numerically computed distributions with \( z \) as in (2) and \( z \) as in (4). A very satisfactory agreement is observed for cellular links corresponding to both average (\( a_0 = 0.5 \)) and cell-edge (\( a_0 = 0.95 \)) user locations, and for D2D links with \( a_0 = 0.05 \) and \( a_0 = 0.1 \).

Similarly good agreements are observed for other overlay settings, and also for underlay settings, supporting our interference modeling approach.
V. Link Spectral Efficiency

We now turn our attention to the ergodic spectral efficiency, arguably the most operationally relevant quantity in contemporary systems where codewords span many fading realizations across frequency (because of the wide bandwidths) and time (because of hybrid-ARQ) [30].

A. Specific Network Geometry

For a specific network geometry, i.e., for given $\rho$ and $\varrho$, the spectral efficiency of the cellular uplink is

$$C(\rho) = \mathbb{E}[\log_2(1 + \text{SIR}|\rho)]$$

$$= \int_0^{\infty} \log_2(1 + \gamma) \ dF_{\text{SIR}|\rho}(\gamma)$$

$$= e^{1/\rho} E_1 \left( \frac{1}{\rho} \right) \log_2 e$$

where $E_1(\zeta) = \int_1^{\infty} t^{-1} e^{-\zeta t} dt$ is an exponential integral.

Similarly, the spectral efficiency of the D2D link is obtained as

$$C(\varrho) = e^{1/\varrho} E_1 \left( \frac{1}{\varrho} \right) \log_2 e.$$  \hspace{1cm} (26)

Example 2. Shown in Fig. 3 is a comparison of the spectral efficiency in (25) against its numerically computed counterpart with $z$ as in (2). For the system parameters in Table I, the match is excellent, further validating our interference modeling approach.

Arguably in fact, as mentioned earlier, the spectral efficiency obtained with our analytical approach might be more operationally relevant than the one obtained with the interference $z$ as in (2) because it is unlikely that the receiver can learn such distribution, and even if it could a standard decoder for Gaussian noise might be featured.

B. Average Network Geometry

As an alternative to the characterization for specific network geometries presented in the previous section, we can choose to characterize the average spectral efficiency over all possible such geometries. Although, as mentioned earlier, the quantities thus obtained—which are favorite outcomes in stochastic geometry—are less informative, they do allow calibrating system-level benefits.
Fig. 3. Cellular uplink spectral efficiency in the overlay system as function of the normalized link distance.

To effect the spatial averaging, slightly different approaches are computationally more convenient in our stochastic geometry framework depending on the type of link (cellular/D2D) and on the case (underlay/overlay). Hence, each one is separately presented next.

1) Cellular Uplink: For this link, we retain an averaging circle equal to the cell size for the cellular interferers. Since all cellular interferers are by definition outside that circle, the spectral efficiency is then a functional of the random locations of the intended cellular user (uniformly distributed within the cell) and the D2D interferers. An expectation over all such locations yields the average spectral efficiency.

The underlay and overlay cases are presented separately beginning with the former.

**Proposition 1.** With underlay, the cellular uplink average spectral efficiency over all network geometries is

\[
\bar{C} = 2 \log_2 e \int_0^\infty \frac{1}{\gamma + 1} \int_0^1 a e^{-\gamma^2 a^2} - (\gamma \mu)^2 a^2 K \Gamma(1 - \frac{2}{\eta}) da d\gamma
\]

(27)

which, for \( \eta = 4 \), simplifies to

\[
\bar{C} = \frac{\sqrt{\pi} e^{-\pi \mu K^2/4}}{2 \log_2 2} \int_0^\infty \frac{\text{erf} \left( \sqrt{\gamma} + \frac{\sqrt{\pi} \mu K}{2} \right) - \text{erf} \left( \frac{\sqrt{\pi} \mu K}{2} \right)}{\sqrt{\gamma} (1 + \gamma)} d\gamma
\]

(28)

where \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \) is the error function.
Proof. See Appendix A.

With overlay ($\alpha = 0$), as the cellular link is not subject to D2D interference, the expectation is only over $a_0$ and a compact result involving only the Meijer-G function

$$G^{m,n}_{p,q}\left(z \begin{bmatrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \vrule & b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{bmatrix}\right)$$

and the cellular pathloss exponent $\eta$ is obtained.

**Proposition 2.** With overlay, the cellular uplink average spectral efficiency over all network geometries is

$$\bar{C} = \frac{2 \log_2 e}{\eta} G^{2,2}_{2,3}\left(\frac{2}{\eta - 2} \begin{bmatrix} 0, \frac{\eta - 2}{\eta} \vrule & 0, \frac{-2}{\eta} \end{bmatrix}\right).$$

Proof. See Appendix B.

The versatility of our interference modeling approach is in full display here, facilitating a closed-form expression for a quantity that in previous analyses had only been obtained in integral form.

2) **D2D link:** For the D2D link, the average performance is characterized in two steps. First, we establish the average over all possible interferer locations but for a fixed D2D link distance. To do so, we directly average the link spectral efficiency over the spatial locations of all cellular and D2D interferers in the network, without any prior replacement of interference by its expected value, i.e., without invoking any averaging circle.

**Proposition 3.** For a given $a_0$, the D2D link spectral efficiency averaged over all network geometries equals

$$\bar{C}(a_0) = \log_2(e) \int_0^\infty \frac{1}{\gamma + 1} e^{-\gamma^{2/\eta} a_0^2 \Gamma\left(1 - \frac{2}{\eta}\right)} \left(K + \frac{\alpha}{\sqrt{\pi}}\right) \Gamma\left(\frac{K}{2}\right) \sin\left(K a_0^2\right) \cos\left(K a_0^2\right) \mathrm{d}\gamma$$

which, for $\eta = 4$, reduces to

$$\bar{C}(a_0) = 2 \log_2(e) \left[\sin\left(K a_0^2\right) \sin\left(K a_0^2\right) - \cos\left(K a_0^2\right) \cos\left(K a_0^2\right)\right]$$

where $K = \sqrt{\pi}(K + \frac{\alpha}{\sqrt{\pi}})$ while the trigonometric integrals $\sin(\cdot)$ and $\cos(\cdot)$ are respectively given by $\sin(x) = \int_x^\infty \frac{\sin(t)}{t} \mathrm{d}t$ and $\cos(x) = -\int_x^\infty \frac{\cos(t)}{t} \mathrm{d}t$. 
Proof. See Appendix C. ■

The above result, of interest in itself, can be then further averaged over the distribution of $a_0$. In the absence of empirical data on actual user behavior, [22] and [23] propound a Rayleigh distribution, which we next apply to our formulation.

The normalized interferer distances $\{a_j\}_{j=1}^{\infty}$ to the points in the PPP $\Phi$ satisfy $\mathbb{E}[a_j] \propto \frac{1}{\sqrt{K}}$ [29]. Recognizing this, we have $a_0$ conform to a Rayleigh distribution with mean $\frac{\sqrt{\pi}}{2\sqrt{K}}$ where $\delta > 0$. Then, as $K$ increases, the average length of both the intended D2D link and the interfering D2D links shrink with $\frac{1}{\sqrt{K}}$. This is a desirable behavior, attempting to model a natural reduction in transmission range as the user density intensifies.

The underlay and overlay cases are presented separately beginning with the former.

**Proposition 4.** Let $a_0$ be Rayleigh-distributed with mean $\frac{\sqrt{\pi}}{2\sqrt{K}}$. With underlay, the D2D link spectral efficiency averaged over all network geometries equals

$$
\bar{C} = \int_0^{\infty} \frac{\log_2 e}{(\gamma + 1) \left[ 1 + \frac{\gamma^2/\eta}{\Gamma \left( 1 - \frac{2}{\eta} \right)} \right]} \frac{\log_2 e}{1 + \frac{\gamma^2/\eta}{\Gamma \left( 1 - \frac{2}{\eta} \right)}} d\gamma
$$

(33)

which, for $\eta = 4$, reduces to

$$
\bar{C} = \frac{\delta^2 \pi^{3/2}}{1 + \frac{1}{K\mu^{2/\eta}}} \log_2 e - \log_2 \left( \delta^4 \pi \left( 1 + \frac{1}{K\mu^{2/\eta}} \right)^2 \right).
$$

(34)

Proof. See Appendix D. ■

**Proposition 5.** Let $a_0$ be Rayleigh-distributed with mean $\frac{\sqrt{\pi}}{2\sqrt{K}}$. With overlay, the D2D link spectral efficiency averaged over all network geometries is

$$
\bar{C} = \int_0^{\infty} \frac{\log_2 e}{(\gamma + 1) \left[ 1 + \frac{\gamma^2/\eta}{\Gamma \left( 1 - \frac{2}{\eta} \right)} \right]} \frac{\log_2 e}{1 + \frac{\gamma^2/\eta}{\Gamma \left( 1 - \frac{2}{\eta} \right)}} d\gamma
$$

(35)

which, for $\eta = 4$, reduces to

$$
\bar{C} = \frac{\delta^2 \pi^{3/2} \log_2 e - \log_2 (\delta^4 \pi)}{1 + \delta^4 \pi}.
$$

(36)

Proof. See Appendix D. ■

Note that the above result is independent of $K$, as a consequence of the aforediscussed choice for the distribution of $a_0$. 

Example 3. Shown in Fig. 4 are the average spectral efficiencies in (30) and (35) alongside the respective numerically computed values with \( z \) as in (2) and \( z \) as in (4). The match is excellent, once again evincing the goodness of our interference modeling approach. For the D2D link, the distance is Rayleigh-distributed with \( \delta = 0.3 \).

VI. Benefits of D2D

In order to gauge the benefits of D2D integrated cellular network and to demonstrate the usefulness of the framework developed in earlier sections, we provide some examples comparing the uplink and D2D spectral efficiencies. Unless otherwise specified, the system parameters in Table I are applied. Comparisons are done separately for overlay and underlay.

A. Overlay D2D

Example 4. For an overlay system, equating the link spectral efficiencies \( C(\rho) \) and \( C(\varrho) \) for the parameters in Table I we obtain the relationship

\[
a_0 = 0.512 a_0^{4.5} \frac{4.5}{38}
\]  

(37)
Fig. 5. Contour plot for the relationship in (37). Within the unshaded region, $C(\varrho) > C(\rho)$.

A contour plot of which is presented in Fig. 5. Within the unshaded region, the D2D link has a better spectral efficiency than a corresponding uplink transmission from the same user would have, and thus D2D becomes advantageous. The share of geometries for which $C(\varrho) > C(\rho)$ for a given value of $a_0$ is easily obtained as $\mathbb{P}[a_0 > x] = 1 - x^2$ where $x$ is the corresponding $x$-axis value of the contour. Some such shares are displayed, e.g., for $a_0 = 0.15$ D2D is preferable in $80\%$ of situations.

The above example shows how, for a rather typical network geometry, from a link vantage D2D is very often a better alternative than cellular communication via the BS even if only the uplink is considered, with the downlink taken for granted. With the resource costs of both uplink and downlink considered, the appeal of D2D would increase even further.

**Example 5.** Considering overlay and a fixed link distance $a_0 = 0.08$ for all D2D pairs, CDFs of $C(\varrho)$ and $C(\rho)$ over the values of $a_0$ and $\{a_j\}_{j=1}^{K'}$ are plotted in Fig. 6. Even for very high density $K$, with the system brimming with D2D interference, thanks to their short range many D2D links enjoy higher spectral efficiencies than the corresponding cellular uplink.

The foregoing example indicates that the number of D2D links that can coexist on a given signaling resource is large, and to better appreciate the benefits of such dense spectral reuse
Fig. 6. CDFs of cellular and D2D link spectral efficiencies in an overlay system with $a_0 = 0.08$.

Fig. 7. Average system spectral efficiency in an overlay system with a fixed D2D link distance $a_0$.

we next turn our attention to the system spectral efficiency (bits/s/Hz/cell), which reflects the benefits of this reuse.

**Example 6.** Since there are $K$ active D2D links per cell on average, the average system spectral efficiency of the D2D traffic is $\bar{C}(a_0)$ whereas, for the cellular uplink, the average system
spectral efficiency is $\bar{C}$ as there is only one active cellular user per cell. Shown in Fig. 7 is the comparison of these quantities as function of $K$, for various $a_0$.

Example 6 prompts the following observations:

- For each value of $a_0$, there is an optimum “load” $K$.
- For a very wide range of $K$, and especially around its optimum value, the D2D system spectral efficiency is much higher than its cellular counterpart.
- The D2D advantage grows with a shrinking $a_0$, approaching two orders of magnitude for a reasonable value of $a_0 = 0.09$.

B. Underlay D2D

Next, we turn our attention to underlay systems.

Example 7. Considering underlay and a fixed link distance $a_0 = 0.08$ for all D2D pairs, CDFs of $C(\rho)$ and $C(\varrho)$ over the values of $a_0$, $\{a_j\}_{j=1}^{K'}$ and $\{a_k\}_{k=1}^{K''}$ are plotted in Fig. 8.

![CDFs of cellular and D2D link spectral efficiencies in an underlay system with $a_0 = 0.08$.](image)

Example 7 makes it evident that, in an underlay system, the cellular uplink spectral efficiency is severely affected by the additional interference caused by cochannel D2D transmissions. In
the next example, we look at the average system spectral efficiency achieved by underlaid D2D communication.

Fig. 9. Average system spectral efficiency of the D2D links with fixed D2D link distance $a_0 = 0.12$ and underlay, for different values of $K$, $\mu$ and $\beta$.

**Example 8.** In Fig. 9, the average system spectral efficiency (bits/s/Hz/cell) achieved by underlaid D2D links with $a_0 = 0.12$ is plotted until its peak value by varying $K$, for different values of $\mu$. Suppose that we want the average uplink spectral efficiency to satisfy $\overline{C} \geq \beta \overline{C}|_{K=0}$ where $\overline{C}|_{K=0}$ denotes the average uplink spectral efficiency without D2D and $\beta > 0$ parametrizes its degradation. For instance, $\beta = 0.8$ corresponds to less than 20% degradation. The maximum average system spectral efficiency of D2D for different values of $\beta$ are indicated in the figure.

Again, we see that the interference caused by D2D transmissions onto cochannel cellular users is a limiting factor. This encourages us to look into ways by which the cellular uplink can be protected from the D2D interferers, which is the focus of the next section.

**VII. UNDERLAI D2D WITH EXCLUSION REGIONS**

One way to reduce the interference seen in the uplink of an underlay system is to have exclusion regions around the BSs wherein the D2D transmitters are not scheduled on the underlay resource. Specifically, let us consider circular exclusion regions of radius $d_{\text{ex}}$ (cf. Fig. 10) and
normalized radius \( a_{\text{ex}} = \frac{d_{\text{ex}}}{R} \). With the introduction of such exclusion regions, the D2D interferer locations no longer conform to a homogeneous PPP, which makes the analysis difficult. To proceed, we model the D2D interferer locations outside the averaging circle \( B(0, R) \) as belonging to a different homogeneous PPP \( \Phi \) with a scaled-down density \( \tilde{\lambda} = p\lambda \), where \( p = 1 - a_{\text{ex}}^2 \) such that \( \tilde{\lambda} \) coincides with the average number of active D2D transmitters per unit area. The goodness of this model is validated in a later example.

Under the foregoing model, the interference power emanating from outside the averaging circle \( B(0, R) \) is averaged over the random locations. In the case of the cellular uplink, this gives

\[
\overline{\sigma^2_{\text{out}}} = \frac{2 (pKP + P)}{(\eta - 2) R^\eta}
\]  \hspace{1cm} (38)

whereas, for the D2D link,

\[
\overline{\sigma^2_{\text{out}}} = \frac{2 (pKP + P)}{(\tilde{\eta} - 2) R^{\tilde{\eta}}}. \hspace{1cm} (39)
\]

For the cellular uplink, the interference power from the transmitters inside the averaging circle

![Fig. 10. A Cellular uplink with underlaid D2D, where the D2D transmitters are not present in circular exclusion regions of radius \( d_{\text{ex}} \) around the BSs. Located at the origin is a receiving BS and shown with a square marker within the circle of radius \( R \) is its intended cellular transmitter; shown with square markers outside the circle are the cellular interferers; shown with diamond markers are the D2D interferers.](image)
\( B(0, R) \) is given by
\[
\sigma_m^2 = \sum_{j=1}^{K'} P r_j^{-\eta}
\]
(40)

where the \( K' \) D2D transmitters are located within an annulus with inner radius \( d_{ex} \) and outer radius \( R \) denoted by \( \mathcal{A}(d_{ex}, R) \). The locations of the D2D transmitters within \( \mathcal{A}(d_{ex}, R) \) conform to the points of the PPP \( \Phi \) with density \( \lambda \).

As of the D2D link, recall that to study it we shift the origin to the D2D receiver under consideration. The interference power from the transmitters inside the averaging circle \( B(0, R) \) is given by
\[
\sigma_m^2 = \sum_{j=1}^{K''} P r_j^{-\eta} + \sum_{k=1}^{K''} P r_k^{-\eta}
\]
(41)

where the \( K'' \) cellular interferer locations conform to the points of the PPP \( \Phi \) in \( B(0, R) \) while the \( K'' \) D2D interferer locations are difficult to model in general due to the asymmetry of the voids in the averaging circle \( B(0, R) \) that result from the exclusion regions. We shall turn to this issue later in the section.

A. SIR Distributions

The local-average SIRs of the cellular uplink and the D2D links are given by
\[
\rho = \frac{a_0^{-\eta}}{\mu \sum_{j=1}^{K'} a_j^{-\eta} + \frac{2(\mu K+1)}{\eta-2}}
\]
(42)
\[
\varrho = \frac{a_0^{-\eta}}{\sum_{j=1}^{K''} a_j^{-\eta} + \frac{1}{\mu} \sum_{k=1}^{K''} a_k^{-\eta} + \frac{2(\mu K+1/\mu)}{\eta-2}}
\]
(43)

Given the values of \( \rho \) and \( \varrho \), i.e., conditioning on the link distances inside \( B(0, R) \), the instantaneous SIRs become exponentially distributed as in Section [IV]. In the following example, we validate the interference modeling with exclusion regions.

Example 9. Consider a cellular uplink in an underlay system where D2D transmitters cannot occupy the circular exclusion regions. Let the normalized uplink distance be \( a_0 = 0.6 \) while \( K = 10, \mu = 0.1 \) and \( \eta = 3.5 \). The D2D interferers within the annulus \( \mathcal{A}(d_{ex}, R) \) are placed at the normalized distances \( a_j = \frac{\Gamma(0.5+j)}{\sqrt{K(j)}} \), if \( a_j > a_{ex} \), for \( j = 1, \ldots, 10 \). Shown in Fig. 11 is the comparison of \( F_{\text{SIR}|\rho}(\gamma) \) against the corresponding numerically computed CDF of instantaneous
SIR, for different values of $a_{\text{ex}}$. The numerical simulations are done with regularly spaced circular exclusion regions within which the D2D interferers are not present as depicted in Fig. 10. Satisfactory agreement can be seen for the cases of no exclusion region ($a_{\text{ex}} = 0$) and exclusion regions with $a_{\text{ex}} = 0.35$ and $a_{\text{ex}} = 0.67$.

![Simulation vs Analytical CDF for SIR](image)

**Fig. 11.** CDF of instantaneous SIR of the cellular uplink in an underlay system with the parameters of Table I, for $a_{\text{ex}} = 0, 0.35, 0.67$.

### B. Link Spectral Efficiency

For specific network geometries, i.e., given the values of the $\rho$ and $\varrho$, the link spectral efficiencies $C(\rho)$ and $C(\varrho)$ are defined as in Section V and not be repeated here for the sake of brevity. Then, those expressions can be further expected over the locations of the interferers inside the averaging circle, leading to the results that follow.

**Proposition 6.** In an underlay system with normalized exclusion regions of radius $a_{\text{ex}}$, the uplink spectral efficiency averaged over all geometries is

$$
\bar{C} = \frac{2 \log_2 e}{\nu \rho K} \int_0^\infty \frac{1}{\gamma + 1} \int_0^1 a e^{-\gamma a^2} \frac{2(\nu + K + 1)}{\eta - 2} - \frac{2K}{\gamma} \left[ a_{\text{ex}}^\nu E_{2+\nu} \left( \frac{\eta}{\nu} \mu a_{\text{ex}}^\nu \right) - E_{2+\nu} \left( \frac{\eta}{\nu} \mu a_{\text{ex}}^\nu \right) \right] da d\gamma.
$$

where $E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt$ is the generalized exponential integral and $p = 1 - a_{\text{ex}}^2$.

**Proof.** See Appendix E.
Next, we turn our attention to the D2D links. Since the asymmetry of the voids present inside the circle \( B(0, R) \) makes it difficult to model the D2D interferer locations corresponding to \( \sigma_{in}^2 \), we upper-bound the interference power in order to obtain a lower bound on the average spectral efficiency. Specifically, we fill the voids inside \( B(0, R) \) and regard the D2D interferers as conforming to a PPP with density \( \lambda \) within \( B(0, R) \), which can only increase the amount of interference.

**Proposition 7.** In an underlay system with normalized exclusion regions of radius \( a_{ex} \) and with a given \( a_0 \),

\[
\bar{C}(a_0) \geq \frac{\log_2 e}{e^{K+1}} \int_0^\infty \frac{1}{\gamma + 1} e^{-\gamma a_0^2(\frac{\lambda}{\eta-2} + \frac{2}{\pi})} [K E_{2+\eta}(\gamma a_0^2) + E_{2+\eta}(\frac{\pi}{\eta} a_0^2)] d\gamma. \tag{45}
\]

**Proof.** See Appendix F. \[\blacksquare\]

**Example 10.** With exclusion regions around the BS, the average system spectral efficiency (bits/s/Hz/cell) of the underlaid D2D links with fixed D2D pair distance \( a_0 \) becomes \( pK\bar{C}(a_0) \). In Fig. 12 the analytical lower-bound on such average system spectral efficiency is contrasted against the exact results obtained numerically for \( a_0 = 0.12 \), \( \mu = 0.1 \) and \( \eta = 4.5 \).

![Fig. 12. Average system spectral efficiency per cell of the underlaying D2D links. The analytical lower-bound and simulation results are contrasted for different values of \( K \).](image)

Our final example aims at illustrating the effectiveness of the exclusion regions.
**Example 11.** With a constraint of less than 20% degradation for the average uplink spectral efficiency, parametrized by $\beta = 0.8$ in the relationship $C \geq \beta C|_{K=0}$, the achievable average system spectral efficiency (bits/s/Hz/cell) of the underlaid D2D links $pK\bar{C}(a_0)$ computed utilizing the analytical lower-bound in (45) is plotted against the number of underlaid D2D links per cell $pK$, for different values of $a_{ex}$ and $\mu$. Similar to Example 8, $a_0 = 0.12$, $\eta = 3.5$ and $\eta = 4.5$ are chosen. It can be seen that exclusion regions allow to pack more D2D links per cell, for a given degradation of the average uplink performance, thereby achieving a higher system spectral efficiency.

![Graph](image)

**Fig. 13.** With $\beta = 0.8$, achievable average system spectral efficiency of the underlaid D2D links with fixed D2D link distance $a_0 = 0.12$, parameterized by $\mu$ and $a_{ex}$.

**VIII. Summary**

By means of a novel model for the interference, the framework introduced in this paper enables analytical characterizations of the spectral efficiency of both underlay and overlay D2D communication. Advantageously, this framework allows characterizing the performance of specific network geometries in addition to the average thereof, and it yields compact expressions that, in some cases, amount even to closed forms.
The framework has been developed for single-antenna transmitters and receivers, and hence a generalization to multiantenna setups would be a logical follow-up. With multiple antennas per transceiver, even denser spectrum reuse and/or higher-rate links would be possible.

Altogether, D2D communication seems to offers a prime opportunity for network densification in the face of local traffic. The increase in system efficiency that it could bring about is very high, even if no attempt is made to optimize the scheduling of D2D transmissions. Strong interference does arise as a problem for a share of the users, and smart scheduling could alleviate this issue. We have seen a glimpse of that by simply introducing fixed-size exclusions regions around the BSs, something that was attempted by means of simulations in a single-cell model [31] and that we have incorporated into our multicell analytical framework. More sophisticated schemes where the exclusion regions are dynamically modified could be even better and apply also to D2D links, where without smart scheduling arbitrary proximity is possible. In that sense, the extension of the analytical framework to include schemes such as FlashLinQ [32] or ITLinQ [33], [34] would be another natural extension.

APPENDIX A

PROOF OF PROPOSITION [1]

The uplink spectral efficiency averaged over all geometries is computed as

$$\bar{C} = \mathbb{E}[C(\rho)]$$  (46)

where the expectation is over $\rho$. Expanding the above equation,

$$\bar{C} = \mathbb{E} \left[ \mathbb{E} \left[ \log_2(1 + \text{SIR}|\rho) \right] \right]$$  (47)

$$= \mathbb{E} \left[ \int_{0}^{\infty} \frac{\log_2 e}{\gamma + 1} \left( 1 - F_{\text{SIR}|\rho}(\gamma) \right) d\gamma \right]$$  (48)

$$= \int_{0}^{\infty} \frac{\log_2 e}{\gamma + 1} \left( 1 - \mathbb{E} \left[ F_{\text{SIR}|\rho}(\gamma) \right] \right) d\gamma$$  (49)

$$= \int_{0}^{\infty} \frac{\log_2 e}{\gamma + 1} \left( 1 - F_{\text{SIR}}(\gamma) \right) d\gamma$$  (50)

where the outer and inner expectations in (47) are over $\rho$ and over the fading, respectively. We next compute $F_{\text{SIR}}(\gamma)$ and then use it to evaluate (50).

Recall, from (15), that the local-average SIR in the presence of an averaging circle is

$$\rho = \frac{P r_0^{-\eta}}{\sigma_m^2 + \sigma_{\text{out}}^2}$$  (51)
where $\sigma_{out}^2$ is the spatial average of $\sigma_{out}^2$. For this computation though, it is more convenient to retain the averaging circle equal to a cell size only for the cellular interferers, while not applying it to D2D interferers (or, equivalently, taking its size to infinity). This relaxation can only make the model, whose goodness was already validated, even tighter. With it, the local average SIR becomes

$$\rho = \frac{r_0^{-\eta}}{\alpha \mu \sum_{j=1}^{\infty} r_j^{-\eta} + \frac{2}{(\eta-2)R^\eta}}.$$  \hspace{1cm} (52)

and the conditional CDF of the SIR becomes

$$F_{\text{SIR} | \rho}(\gamma) = 1 - e^{-\gamma \frac{r_0^\eta}{\pi R^2} \sum_{j=1}^{\infty} e^{-\gamma \alpha \mu r_j^\eta r_j^{-\eta}}}.$$  \hspace{1cm} (53)

Conditioning on $r_0$ and averaging $F_{\text{SIR} | \rho}(\gamma)$ over the interference

$$F_{\text{SIR} | r_0}(\gamma) = 1 - e^{-\gamma \frac{r_0^\eta}{\pi R^2} \sum_{j=1}^{\infty} e^{-\gamma r_j^\eta r_j^{-\eta}}}.$$  \hspace{1cm} (54)

$$= 1 - e^{-\gamma \frac{r_0^\eta}{\pi R^2} \sum_{j=1}^{\infty} e^{-\gamma r_j^\eta r_j^{-\eta}}}.$$  \hspace{1cm} (55)

$$= 1 - e^{-\gamma \frac{r_0^\eta}{\pi R^2} \sum_{j=1}^{\infty} e^{-\gamma r_j^\eta r_j^{-\eta}}}.$$  \hspace{1cm} (56)

where the expectation in (54) is over the PPP $\Phi$, (55) follows from the definition of the probability generating functional (PGFL) of the PPP [28], and (56) follows from the variable change $\gamma r_0^\eta x^{-\eta} = u$ and the relation $\pi \lambda = K / R^2$. Employing integration by parts in (56) and invoking $a_0 = \frac{r_0}{R}$, we obtain

$$F_{\text{SIR} | a_0}(\gamma) = 1 - e^{-\gamma a_0^\eta \frac{a_0^\eta}{\pi R^2} - \alpha (\gamma \mu)^{\frac{\eta}{2}} a_0^\eta K \Gamma(1-\frac{\eta}{2}).}.$$  \hspace{1cm} (57)

The density of $a_0$ for a user uniformly located in the cell is

$$f_{a_0}(a) = \begin{cases} 2a & 0 \leq a \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$  \hspace{1cm} (58)

We further average $F_{\text{SIR} | a_0}(\gamma)$ over the above distribution of $a_0$ to get $F_{\text{SIR}}(\gamma)$, which plugged into (50) yields the average spectral efficiency in (27).
APPENDIX B
PROOF OF PROPOSITION 2

With overlay ($\alpha = 0$), the local-average SIR of the cellular uplink is

$$\rho = \frac{\eta - 2}{2a_0^\eta}. \quad (59)$$

Averaging $C(\rho)$ over $a_0$ via the density in (58) yields

$$\bar{C} = \log_2(e) \int_0^1 e^{\frac{2a_0^\eta}{\eta - 2}} E_1\left(\frac{2a_0^\eta}{\eta - 2}\right) 2a \, da. \quad (60)$$

When $x$ is positive and real, $E_1(x) = -E_i(-x)$ where $E_i(x) = \int_{-x}^{\infty} \frac{-e^{-t}}{t} \, dt$. Utilizing this relation in (60) and then evaluating the integral by virtue of the identity given in [35], we obtain the final expression in (30).

APPENDIX C
PROOF OF PROPOSITION 3

Following the same approach as in Appendix A, we replace the average interference $\bar{\sigma}^2_{\text{out}}$ with its exact value $\sigma^2_{\text{out}}$ in the definition of $\varrho$, i.e., we do not apply the averaging circle to the D2D interference. Then,

$$\varrho = \frac{r_0^{-\eta}}{\sum_{j=1}^{\infty} r_j^{-\eta} + \frac{\alpha}{\mu} \sum_{k=1}^{\infty} r_k^{-\eta}}. \quad (61)$$

The conditional CDF of SIR, given $r_0$, $\{r_j\}_{j=1}^{\infty}$ and $\{r_k\}_{k=1}^{\infty}$, is

$$F_{\text{SIR}|r_0}(\gamma) = 1 - e^{-\gamma r_0^{-\eta}} \left(\sum_{j=1}^{\infty} r_j^{-\eta} + \frac{\alpha}{\mu} \sum_{k=1}^{\infty} r_k^{-\eta}\right) \quad (62)$$

$$= 1 - \prod_{j=1}^{\infty} e^{-\gamma r_0^{-\eta} r_j^{-\eta}} \prod_{k=1}^{\infty} e^{-\gamma \frac{\alpha}{\mu} r_0^{-\eta} r_k^{-\eta}}. \quad (63)$$

Maintaining the conditioning on the desired link distance $r_0$, we average (63) over the PPPs $\Phi$ and $\Phi_0$ to get

$$F_{\text{SIR}|r_0}(\gamma) = 1 - \mathbb{E}_\Phi \left[ \prod_{j=1}^{\infty} e^{-\gamma r_0^{-\eta} r_j^{-\eta}} \right] \mathbb{E}_\Phi \left[ \prod_{k=1}^{\infty} e^{-\gamma \frac{\alpha}{\mu} r_0^{-\eta} r_k^{-\eta}} \right] \quad (64)$$

$$= 1 - e^{-\gamma r_0^{-\eta} \alpha \mu \Gamma\left(1 - \frac{1}{\eta}\right)} \left(K + \alpha \mu \Gamma\left(1 - \frac{2}{\eta}\right)^2 \right). \quad (65)$$
where the expectations in (65) are computed as in Appendix A with the substitution $a_0 = \frac{r_0}{R}$.

Thus, the average spectral efficiency of the D2D links conditioned on $a_0$ becomes

$$
\bar{C}(a_0) = \int_0^\infty \frac{\log_2(e)}{\gamma + 1} (1 - F_{\text{SIR}|a_0}(\gamma)) \, d\gamma
$$

(66)

$$
= \log_2(e) \int_0^\infty \frac{e^{-\gamma \frac{2}{\eta} a_0^2 (1-\frac{2}{\eta})}}{\gamma + 1} \, d\gamma
$$

(67)

which is unwieldy for general $\eta$. However, for $\eta = 4$, (67) reduces to

$$
\bar{C}(a_0) = 2 \log_2(e) \int_0^\infty \frac{x e^{-x Ka_0^2}}{x^2 + 1} \, dx
$$

(68)

which follows from the variable change $\sqrt{\gamma} = x$ in (66) with $K = \sqrt{\pi(K + \frac{\alpha}{\sqrt{\mu}})}$. By virtue of [36, 3.354.2], (68) turns into the claimed expression in (32).

**APPENDIX D**

**PROOF OF PROPOSITIONS 4 AND 5**

The distribution of $a_0$ is given by

$$
f_{a_0}(a) = \frac{2Ka}{\delta^2} e^{-Ka^2/\delta^2} \quad a > 0.
$$

(69)

Averaging out the conditional CDF in (65) over $a_0$, we obtain the CDF of unconditional SIR

$$
F_{\text{SIR}}(\gamma) = 1 - \int_0^\infty e^{-\gamma \frac{2}{\eta} a_0^2 (1-\frac{2}{\eta}) (K+\alpha \mu \delta^2)} \frac{2Ka}{\delta^2} e^{-Ka^2/\delta^2} \, da
$$

(70)

$$
= 1 - \frac{1}{1 + \gamma^{2/\eta} \delta^2 \Gamma \left(1 - \frac{2}{\eta} \right) \left(1 + \frac{\alpha}{K\mu^{2/\eta}} \right)}.
$$

(71)

The D2D link spectral efficiency, averaged over all possible geometries is computed by plugging (71) into

$$
\bar{C} = \int_0^\infty \frac{\log_2(e)}{\gamma + 1} (1 - F_{\text{SIR}}(\gamma)) \, d\gamma.
$$

(72)

Setting $\alpha = 1$ and $\alpha = 0$, we obtain Propositions 4 and 5 respectively.
APPENDIX E

PROOF OF PROPOSITION 6

The conditional CDF of SIR is

\[ F_{\text{SIR}|\rho} (\gamma) = 1 - e^{-\gamma r_0^\eta \sigma_0^2 \text{out} \sum_{j=1}^{K'} e^{-\gamma \mu r_0^\eta r_j^{-\eta}}}. \]

(73)

Conditioning on \( r_0 \) and averaging \( F_{\text{SIR}|\rho} (\gamma) \) over the distances \( \{r_j\}_{j=1}^{K'} \) to the D2D interferers in the annulus \( A(d_{\text{ex}}, R) \), we obtain

\[ F_{\text{SIR}|a_0} (\gamma) = 1 - e^{-\gamma r_0^\eta \sigma_0^2 \text{out} \sum_{j=1}^{K'} \exp \left( -2\pi \lambda \int_{d_{\text{ex}}}^R \left( 1 - e^{-\gamma \mu r_0^\eta x^{-\eta}} \right) x \, dx \right) \]

(74)

\[ = 1 - e^{-pK - \gamma a_0^\eta \frac{2(\mu K + 1)}{\eta - 2} - \frac{2K}{\eta} \left( a_0^\eta E_{2+p} \left( \frac{\gamma \mu a_0^\eta}{a_0^\eta} \right) - E_{2+p} \left( \mu a_0^\eta \right) \right)} \]

(75)

where (75) follows from the definition of the PGFL of PPP, the integral is solved following the approach in Appendix A, and we substitute \( r_0 R = a_0 \) to obtain (76). The expression in (76) is further averaged over \( a_0 \) via (58) to get \( F_{\text{SIR}} (\gamma) \), which plugged into (50) yields the claimed result in (44).

APPENDIX F

PROOF OF PROPOSITION 7

The conditional CDF of SIR is

\[ F_{\text{SIR}|e} (\gamma) = 1 - e^{-\gamma r_0^\eta \sigma_0^2 \text{out} \prod_{j=1}^{K''} e^{-\gamma r_0^\eta r_j^{-\eta}} \prod_{k=1}^{K'''} e^{-\frac{2}{\mu} r_0^\eta r_k^{-\eta}}}. \]

(77)

Under the assumption that the D2D interferers within the circle \( B(0, R) \) belong to a PPP of density \( \lambda \), and conditioning on \( r_0 \), we average out \( F_{\text{SIR}|e} (\gamma) \) over the distances \( \{r_j\}_{j=1}^{K''} \) to get \( F_{\text{SIR}|a_0} (\gamma) \)

\[ F_{\text{SIR}|a_0} (\gamma) = 1 - e^{-\gamma r_0^\eta \sigma_0^2 \text{out} \prod_{j=1}^{K''} e^{-\gamma r_0^\eta r_j^{-\eta}} \sum_{k=1}^{K'''} e^{-\frac{2}{\mu} r_0^\eta r_k^{-\eta}}} \]

(78)

\[ = 1 - e^{-\gamma r_0^\eta \sigma_0^2 \text{out} \exp \left( -2\pi \lambda \int_0^R \left( 1 - e^{-\gamma r_0^\eta x^{-\eta}} \right) x \, dx \right) \} \cdot \exp \left( -2\pi \lambda \int_0^R \left( 1 - e^{-\frac{2}{\mu} r_0^\eta x^{-\eta}} \right) x \, dx \right) \]

(79)

\[ = 1 - e^{-\gamma a_0^\eta \frac{2}{\eta - 2} (pK + 1) \frac{2}{\mu} \left( \frac{KE_{2+p} \left( \frac{\gamma a_0^\eta}{a_0^\eta} \right) + E_{2+p} \left( \frac{2 a_0^\eta}{\mu a_0^\eta} \right)}{\lambda} \right)} \]

(80)
where (79) follows from the definition of the PGFL of PPP, the integrals are solved following the approach in Appendix A and we substitute \( r = a_0 \) to get (80). Then, (45) is obtained by substituting (80) into (66).

REFERENCES

[1] G. George, R. K. Mungara, and A. Lozano, “Overlaid device-to-device communication in cellular networks,” IEEE Global Telecommun. Conf., accepted, 2014.

[2] F. Boccardi, R. W. Heath Jr., A. Lozano, T. Marzetta, and P. Popovski, “Five disruptive technology directions for 5G,” IEEE Commun. Mag., vol. 52, no. 2, pp. 74–80, Feb. 2014.

[3] J. G. Andrews, S. Buzzi, W. Choi, S. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, “What will 5G be?,” IEEE Journal on Sel. Areas in Communications, vol. 32, no. 7, July 2014.

[4] B. Bangerter, S. Talwar, R. Arefi, and K. Stewart, “Networks and devices for the 5G era,” IEEE Commun. Mag., vol. 52, no. 2, pp. 90–96, Feb. 2014.

[5] S. Andreev, A. Pyattaev, K. Johnsson, O. Galinina, and Y. Koucheryavy, “Cellular traffic offloading onto network-assisted device-to-device connections,” IEEE Commun. Mag., vol. 52, no. 4, pp. 20–31, Apr. 2014.

[6] 3GPP TR 22.803 V1.0.0, “Feasibility study for proximity services.” Tech. Rep., 3rd Generation Partnership Project 3GPP, www.3gpp.org, 2012.

[7] L. Lei, Z. Zhong, C. Lin, and X. Shen, “Operator controlled device-to-device communications in LTE-advanced networks,” IEEE Wireless Commun. Mag., vol. 19, no. 3, pp. 96–104, June 2012.

[8] X. Lin, J.G. Andrews, A. Ghosh, and R. Ratasuk, “An overview of 3GPP device-to-device proximity services,” IEEE Commun. Mag., vol. 52, no. 4, pp. 40–48, Apr. 2014.

[9] G. Fodor, E. Dahlman, G. Mildh, S. Parkvall, N. Reider, G. Miklós, and Z. Turányi, “Design aspects of network assisted device-to-device communications,” IEEE Trans. Commun., vol. 50, no. 3, pp. 170–177, Mar. 2012.

[10] A. Asadi, Q. Wang, and V. Mancuso, “A survey on device-to-device communication in cellular networks,” IEEE Commun. Surveys Tuts., to appear, 2014.

[11] H. Min, W. Seo, J. Lee, S. Park, and D. Hong, “Reliability improvement using receive mode selection in the device-to-device uplink period underlaying cellular networks,” IEEE Trans. Wireless Commun., vol. 10, no. 2, pp. 413–418, Feb. 2011.

[12] K. Doppler, C. Yu, C. B. Ribeiro, and P. Janis, “Mode selection for device-to-device communication underlaying an LTE-advanced network,” in Proc. IEEE Wireless Commun. and Networking Conf., Apr. 2010.

[13] Y. Pei and Y-C Liang, “Resource allocation for device-to-device communications overlaying two-way cellular networks,” IEEE Trans. Wireless Commun., vol. 12, no. 7, pp. 3611–3621, July 2013.

[14] C. Yu, K. Doppler, C. B. Ribeiro, and O. Tirkkonen, “Resource sharing optimization for device-to-device communication underlaying cellular networks,” IEEE Trans. Wireless Commun., vol. 10, no. 8, pp. 2752–2763, Aug. 2011.

[15] D. Feng, L. Lu, Y. Yuan-Wu, G.Y. Li, G. Feng, and S. Li, “Device-to-device communications underlaying cellular networks,” IEEE Trans. Commun., vol. 61, no. 8, pp. 3541–3551, Aug. 2013.

[16] G. Fodor and N. Reider, “A distributed power control scheme for cellular network assisted D2D communications,” in Proc. IEEE Global Telecommun. Conf., Dec. 2011, pp. 1–6.
[17] S. Shalmashi, G. Miao, and S. B. Slimane, “Interference management for multiple device-to-device communications underlaying cellular networks,” in Proc. IEEE Int. Symp. Pers., Indoor, Mobile Radio Commun., Sept. 2013, pp. 223–227.

[18] H. Tang, C. Zhu, and Z. Ding, “Cooperative MIMO precoding for D2D underlay in cellular networks,” in Proc. IEEE Int. Conf. Commun., June 2013, pp. 5517–5521.

[19] W. Xu, L. Liang, H. Zhang, S. Jin, J.C.F. Li, and M. Lei, “Performance enhanced transmission in device-to-device communications: beamforming or interference cancellation?,” in Proc. IEEE Global Telecommun. Conf., Dec 2012, pp. 4296–4301.

[20] B. Kaufman, J. Lilleberg, and B. Aazhang, “Spectrum sharing scheme between cellular users and ad-hoc device-to-device users,” IEEE Trans. Wireless Commun., vol. 12, no. 3, pp. 1038–1049, Mar. 2013.

[21] J. G. Andrews, F. Baccelli, and R. K. Ganti, “A tractable approach to coverage and rate in cellular networks,” IEEE Trans. Commun., vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[22] X. Lin, J. G. Andrews, and A. Ghosh, “Spectrum sharing for device-to-device communication in cellular networks,” Available online: http://arxiv.org/abs/1305.4219/, 2013.

[23] Q. Ye, M. Al-Shalash, C. Caramanis, and J. G. Andrews, “Resource optimization in device-to-device cellular systems using time-frequency hopping,” Available online: http://arxiv.org/abs/1309.4062/, 2013.

[24] N. Lee, X. Lin, J. G. Andrews, and R. W. Heath Jr., “Power control for D2D underlaid cellular networks: Modeling, algorithms and analysis,” Available online: http://arxiv.org/abs/1305.6161, 2013.

[25] T. D. Novlan, H. S. Dhillon, and J. G. Andrews, “Analytical modeling of uplink cellular networks,” IEEE Trans. Commun., vol. 12, no. 6, pp. 2669–2679, June 2013.

[26] R. W. Heath Jr., M. Kountouris, and T. Bai, “Modeling heterogeneous network interference using Poisson point processes,” IEEE Trans. Signal Processing, vol. 61, no. 16, pp. 4114–4126, Aug. 2013.

[27] A. Lapidoth and S. Shamai, “Fading channels: how perfect need ”perfect side information” be?,” IEEE Trans. on Inform. Theory, vol. 48, no. 5, pp. 1118–1134, 2002.

[28] D. Stoyan, W. Kendall, and J. Mecke, Stochastic Geometry and Its Applications, John Wiley and Sons, 2nd edition, 1996.

[29] M. Haenggi, “On distances in uniformly random networks,” IEEE Trans. Inform. Theory, vol. 51, no. 10, pp. 3584–3586, Oct. 2005.

[30] A. Lozano and N. Jindal, “Are yesterday’s information-theoretic fading models and performance metrics adequate for the analysis of today’s wireless systems?,” IEEE Commun. Mag., vol. 50, no. 11, pp. 210–217, Nov. 2012.

[31] M. Ni, L. Zheng, F. Tong, J. Pan, and L. Cai, “A geometrical-based throughput bound analysis for device-to-device communications in cellular networks,” Available online: http://arxiv.org/abs/1404.2366v1, 2014.

[32] X. Wu, S. Tavildar, S. Shakkottai, T. Richardson, J. Li, R. Laroia, and A. Jovicic, “FlashLinQ: A synchronous distributed scheduler for peer-to-peer ad hoc networks,” IEEE/ACM Trans. Networking, vol. 21, no. 4, pp. 1215–1228, Aug. 2013.

[33] N. Naderializadeh and A.S. Avestimehr, “ITLinQ: A new approach for spectrum sharing in device-to-device communication systems,” IEEE Journal on Sel. Areas in Communications, to appear, 2014.

[34] R. K. Mungara, X. Zhang, A. Lozano, and R. W. Heath Jr., “On the spatial spectral efficiency of ITLinQ,” in Proc. Annual Asilomar Conf. Signals, Syst., Comp., Nov. 2014.

[35] Wolfram Research, “The Wolfram functions website,” Available online: http://functions.wolfram.com/06.35.21.0015.01, 2001.

[36] I. S. Gradsteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, San Diego, 7th edition, 2007.