Calculation Method of Take-off Weight for Single-seat Ultralight General Aircraft

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Abstract. With the reform of low-altitude airspace, there will be a lot of room for development of ultralight general aircraft. However, it is difficult to accurately calculate weight of single-seat ultralight general aircraft in the past. This paper deduces a method for quickly calculating the takeoff weight in the general aircraft concept design stage, and combines the design data of some typical single-seat ultralight general aircraft to modify the coefficient of the formula. Thus the aircraft takeoff weight calculation formula can more accurately estimate the maximum takeoff weight of a single-seat ultralight aircraft. Taking a single-seat aircraft as an example, the effectiveness of the method is tested, and the error of the take-off weight calculation before and after the parameter correction are compared and analyzed. The calculation method and results have certain reference value.

1. Introduction
According to past statistics, there are approximately 300,000 general aircraft worldwide. In the United States, large areas of low-altitude airspace are available for free flight, and the number of ultralight general aircraft accounts for more than 70% of the total aircraft. In China, since the promulgation of the "Opinions on Further Deepening China's Low-altitude Airspace Management Reform" in 2010, the number of general-purpose aircraft has grown significantly. In the future, with the further reform of low-altitude airspace, China's general aircraft market will show great potential for development.

The existing takeoff weight calculation method has a large estimation error for a single-seat ultralight general aircraft. In aircraft design, the takeoff weight is the basis for determining the thrust and wing area used for takeoff. After the take-off weight, the thrust used for take-off, and the area of the wing are determined, the two most important parameter wing load and thrust-to-weight ratios that affect the performance of the aircraft can be initially determined. This paper deduces a general method for aircraft takeoff weight calculation, which divides the data used in aircraft takeoff weight calculation into two categories [1]. One type is directly obtained data, which can be directly inquired according to aircraft design requirements and related standards, specifications, design. Another type of data is indirectly obtained. This type of data is obtained through long-term aircraft design experience and data statistics, and then converted into a certain proportion of the maximum takeoff weight by the formula. Only the two unknown data we are concerned with are the maximum takeoff weight and the empty weight. These two weight data are statistically calculated and have a certain relationship. Based on this, the first estimate of the take-off weight of a single-seat ultralight general aircraft is obtained.
2. Calculation method

The total takeoff weight of the design is the total weight at the beginning of the mission, which is not necessarily the same as the "maximum takeoff weight" [2]. Later in the conceptual design phase, a higher value than the maximum takeoff weight \( M_{TO} \) will be chosen to estimate the quality of the structural and system components, in order to prevent subsequent quality increases in the design process [3]. At the beginning of the conceptual design phase, except for special instructions, the total takeoff weight is assumed to be the design weight [2].

The formula for the takeoff weight of the aircraft is:

\[
W_{TO} = W_{OE} + W_{PL} + W_{mfuel}
\]  

(1)

In the formula, \( W_{OE} \) is using empty weight; \( W_{PL} \) is the commercial load weight; \( W_{mfuel} \) is the mission oil weight.

The aircraft’s using empty weight definition formula:

\[
W_{OE} = W_{E} + W_{tfo} + W_{crew}
\]  

(2)

In the formula, \( W_{E} \) is empty weight; \( W_{tfo} \) is the weight of lubricating oil and unusable fuel; \( W_{crew} \) is the weight of the aircraft occupant.

The formula for calculating the weight of lubricating oil and unusable fuel is:

\[
W_{tfo} = M_{tfo} \cdot W_{TO}
\]  

(3)

In the formula, \( M_{tfo} \) is the coefficient of lubricating oil and unusable fuel, which is usually small. For ultralight general aircraft, the difference in coefficient of lubricating oil and unusable fuel is relatively large. It can be estimated by the following approximate formula:

\[
M_{tfo} \approx 0.017447 M_f^{2/3} W_{TO}^{-1/3}
\]  

(4)

In the formula, \( M_f \) is the fuel coefficient.

The formula for defining the fuel coefficient is:

\[
M_f = \frac{W_{mfuel}}{W_{TO}}
\]  

(5)

The task oil redefinition formula is:

\[
W_{mfuel} = W_{fuel} + W_{rfuel}
\]  

(6)

In the formula \( W_{fuel} \) is the fuel consumed during the mission. \( W_{rfuel} \) is the fuel that must be used to ensure the completion of the mission. There are two ways to determine the backup fuel:
- A percentage of the oil used to perform the task.
- The voyage oil to the alternate airport.

For ease of calculation, choose the first method to determine the reserve fuel weight:

\[
W_{rfuel} = M_{res} \cdot W_{tfo}
\]  

(7)

In the formula, \( M_{res} \) is the reserve fuel coefficient, and often different values are selected according to different flight missions.

The task oil weight calculation formula is:
\[
W_{\text{fuel}} = (1 - M_{\text{ff}}) \cdot W_{\text{TO}}
\] (8)

The \(M_{\text{ff}}\) in the formula is the fuel weight ratio factor. The fuel consumption of each segment is determined by an empirical formula.

The mission profile of a common single-seat ultralight general aircraft is often divided into the engine warm-up phase, the pre-flight taxiing phase, the aircraft take-off phase, the aircraft climb phase, the aircraft cruise phase, the aircraft descent phase, and the aircraft taxi landing phase. As shown in Figure 1. The fuel ratio factor for each stage is the ratio of the mass at the end of each stage to the beginning of the stage [4].

![Figure 1. Airplane mission profile.](image)

The phase 1: The engine warm-up phase. At the beginning of this phase, the weight is \(W_{\text{TO}}\), the weight at the end is \(W_1\), and the fuel oil reference data for this segment is 0.99-0.998.

The phase 2: the stage of the aircraft slip. At the beginning of this phase, the weight is \(W_1\), and at the end, the weight is \(W_2\). The fuel oil reference data for this segment is 0.99-0.998.

The phase 3: the aircraft take-off phase. At the beginning of this phase, the weight is \(W_2\), and at the end, the weight is \(W_3\). The fuel oil reference data for this segment is 0.99-0.998.

The phase 4: the aircraft climbing stage. At the beginning of this phase, the weight is \(W_3\), and at the end, the weight is \(W_4\), and the reference data of the fuel coefficient of this section is 0.98-0.995.

The phase 5: the aircraft cruise phase. At the beginning of this phase, the weight is \(W_4\), and at the end, the weight is \(W_5\). The reference data of the fuel coefficient of this section is 0.863-0.99.

The phase 6: the stage of aircraft decline. At the beginning of this phase, the weight is \(W_5\), and at the end, the weight is \(W_6\). The reference data of the fuel coefficient of this section is 0.985-0.995.

The phase 7: Landing taxiing stage. The weight at the beginning of this phase is \(W_6\), and the weight at the end is \(W_7\). The reference data for the fuel coefficient of this segment is 0.99-0.998.

The fuel scale factor is:

\[
M_{\text{ff}} = (W_1/W_{\text{TO}}) \prod_{i=1}^{7} (W_{i+1}/W_i)
\] (9)

For propeller aircraft, the fuel weight scale factor for the cruise phase can be calculated using Breguet formula [5]:

\[
R = f_R \left( \frac{\eta_p}{C_P} \right) \frac{L}{D} \ln \frac{W_4}{W_5}
\] (10)

In the formula, \(R\) is the range, the unit is km, \(\eta_p\) is the propeller efficiency, \(L/D\) is the lift-to-drag ratio when cruising, \(C_P\) is the engine fuel consumption rate, the unit is kg/kw/h, \(f_R\) is the coefficient, when the fuel consumption unit is kg/kw/h, it is equal to 367 [6].
Table 1. Typical values of cruise phase.

| Aircraft type       | L/D  | $C_p$  | $\eta_r$ |
|---------------------|------|--------|----------|
| Single-engine aircraft | 8~10 | 0.304~0.426 | 0.8      |

Reorganize the above formula to get:

$$M_{TO} = W_E + W_{f_{to}} + W_{crew} + W_{PL} + W_{mfuel}$$  \hspace{1cm} (11)

Substituting the above formula into the formula (11) to get:

$$W_{TO} = \frac{W_E + W_{PL} + W_{crew}}{1 - M_{f_{to}} - (1 + M_{res})(1 - M_{f})}$$  \hspace{1cm} (12)

Define the coefficient:

$$C = 1 - M_{f_{to}} - (1 + M_{res})(1 - M_{f})$$  \hspace{1cm} (13)

When the number of aircraft occupants is determined, the commercial load weight and crew weight in the aircraft takeoff weight estimate are determined.

$$W_{PLC} = W_{PL} + W_{crew}$$  \hspace{1cm} (14)

Substituting the formulas (13) and (14) into the formula (12):

$$W_E = C \cdot W_{TO} - W_{PLC}$$  \hspace{1cm} (15)

In Equation (15), except that the $W_{TO}$ and $W_E$ are unknown, all other parameters can be calculated by the mission requirements or the formula given before.

According to the aircraft design manual, the following relationship can be obtained for the statistics of the takeoff weight and the empty weight of a large number of light general-purpose aircraft:

$$\log W_E = (\log W_{TO} - A) / B$$  \hspace{1cm} (16)

The values of A and B can be found in Table 3, and the unit is kg.

Table 2. Values for A, B [7].

| A  | B   |
|----|-----|
| -0.1040 | 1.1162 |

Equations 1.15 and 1.16 can be used to obtain values for maximum takeoff weight and empty weight.

3. Calculation method correction

The takeoff weight estimation method described above is extensive, but many of the newly designed ultralight general aircraft after 2000 have used composite materials, and the quality of the ultralight general aircraft has been further reduced, which makes the estimation results have certain errors. In order to make the estimation result more accurate, we modify the coefficient of this method.
3.1. Correction of coefficients $A$ and $B$

The coefficients $A$ and $B$ in Table 2 are obtained by counting a large number of propeller aircraft weight data, and it is estimated that there may be some error in the single-seat ultralight general aircraft. In order to reduce the error, data on the maximum take-off weight and the empty weight of some single-seat ultralight aircraft were collected.

Table 3. Typical ultra-light general-purpose aircraft weight data [8].

| No. | Aircraft                        | $W_{TO}$/kg | $W_{E}$/kg |
|-----|--------------------------------|-------------|------------|
| 1   | Kolb Firestar (USA)             | 226         | 115        |
| 2   | Golden Cicle T-Bird (USA)       | 272         | 122        |
| 3   | Seahawk Condor (USA)            | 215         | 106        |
| 4   | Gull 2000 (USA)                 | 250         | 113        |
| 5   | Rans S-17tinger (USA)           | 239         | 114        |
| 6   | Interplane Griffon103 (Czech)   | 235         | 115        |
| 7   | Aerolite 103                    | 204         | 114        |
| 8   | Aviasud Sirocco (France)        | 250         | 115        |
| 9   | CMF Shadow C&D (USA)            | 386         | 158        |
| 10  | Advanced Aviation Carrera 150   | 249         | 118        |
| 11  | Quicksilver MX Sprint (USA)     | 236         | 112        |
| 12  | Maxair Drifter DR532Rocket (USA)| 227         | 109        |
| 13  | Weedhopper (USA)                | 250         | 113        |
| 14  | Fling K sky raider (USA)        | 249         | 113        |
| 15  | Aerolites Bearcat (USA)         | 317         | 130        |
| 16  | Flightstar Spyder (USA)         | 292         | 126        |
| 17  | MB O2 souricette (France)       | 200         | 95         |
| 18  | Fisher Avenger (USA)            | 270         | 113        |
| 19  | Lazair (Canada)                 | 240         | 95         |

Using the least squares method to fit the relationship function between the maximum takeoff weight and the empty weight, the values of the regression coefficients $A$ and $B$ for the single-seat ultralight general aircraft are obtained, $A=-0.8090$, $B=1.5527$.

3.2. Correction of coefficient $C$

The value of the coefficient $C$ in Equation (15) is affected by the value of the fuel coefficient $M_{ff}$. The mission profile can be divided into seven typical mission segments. In the seven mission segments, fuel consumption during the engine warm-up phase and the taxi start phase does not affect the take-off weight. Since the fuel for these two mission segments is already consumed before the aircraft takes off, the ratio of the end weight of the two mission segments to the weight at the beginning of the segment should be 1 [10]. Adding the fuel consumption of these two task segments to the calculation may also bring some error to the takeoff weight estimation.

3. Examples and analysis

Taking a single-seat ultralight general aircraft as an example, the use of the above estimation method is explained, and the estimation result of the method before and after the coefficient correction is compared with the actual aircraft weight value. Analyze the calculation results and Test the impact of two types of coefficient corrections on the estimation results. Aircraft missions are shown in Table 5.
Table 4. Flight mission requirements.

| Mission       | Mission details |
|---------------|-----------------|
| Crew          | 85KG            |
| Takeoff run distance | 46m            |
| Landing deceleration distance | 46m            |
| Cruising speed  | 100KM/h        |

The fuel proportionality factor for each stage of the aircraft can be obtained by performing similar tasks with reference to the same type of aircraft, as shown in Table 6.

Table 5. Weight fractions for various mission segments.

| Task segment | Take description | Wi+1/Wi |
|--------------|------------------|---------|
| 1            | Warm up          | 0.995   |
| 2            | Sliding start    | 0.997   |
| 3            | Take off         | 0.998   |
| 4            | Climb            | 0.992   |
| 5            | Cruise           | 0.93    |
| 6            | Decline          | 0.993   |
| 7            | Landing          | 0.993   |

Therefore, the fuel proportional coefficient $M_{ff}$ is 0.900, the $M_{ff}$ is 0.018, and the $M_{res}$ design is given as 0.20. From the formula 1.15, $C = 0.9$ can be calculated. A, B check single-propeller aircraft regression coefficient table can get $A = -0.1040$, $B = 1.1162$. Draw an image of the 1.15 and 1.16 equations.

Figure 2. Corrected the relationship between the takeoff weight and the empty machine weight.

The intersection is the solution of the equation, $W_{to} = 365kg$, $W_{E} = 245kg$.

According to the second chapter, the least squares regression calculation is used to obtain the corrected parameters $A$ and $B$ and the parameter $C$ obtained by ignoring the fuel consumption during the warm-up phase and the running phase. Substituting $A$, $B$, and $C$ into the formulas (15) and (16), respectively. Then draw the images of Equations (15) and (16) to get. Then draw the images of Equations (15) and (16) to get.
After correcting the parameters, a new set of solutions can be obtained: $W_{\text{to}}=215\text{kg}$, $W_{\text{e}}=105\text{kg}$. Compare the actual takeoff weight and empty weight of the ultralight general-purpose aircraft in the example, and take off weight and empty weight before and after the correction of parameters A and B, as shown in the following table.

**Table 6.** Comparison and relative error of take-off weight before and after correction of an aircraft.

| Actual value of the aircraft | Estimated value before parameter A and B correction | Estimated value after parameter A and B correction |
|-----------------------------|-----------------------------------------------------|--------------------------------------------------|
| Takeoff weight              | Empty weight                                        | Takeoff weight Empty weight                       |
| 250kg                       | 115kg                                               | 365kg 245kg                                       |

The results in the analysis table are as follows: Before the coefficient correction, the above method has a large error in the takeoff weight estimation of a single-seat ultralight general aircraft, and cannot be directly used in engineering. By comparing the weight of a single-seat general aircraft and correcting the regression coefficients A and B using the least squares method, the estimation results are relatively accurate. The relative error between the estimated weight and the true weight is approximately 10% and can be used as a reference for the takeoff weight in the project. The coefficient C is again compared and analyzed separately for the takeoff weight estimate. As shown below.

**Figure 3.** Corrected take-off weight and empty machine weight curve.

**Figure 4.** Curve between the takeoff weight and the empty machine weight before and after the parameter C correction.
From the image, it is difficult to see how the coefficient C affects the solution of the equation before and after the correction. The image of the 1.15 equation before and after the coefficient C correction is almost coincident. Therefore, it is concluded that in the engineering calculation, the correction of the fuel weight ratio coefficient of each flight stage has little effect on the take-off weight of the aircraft, which is negligible in the actual aircraft design.

4. Conclusion

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The method in this paper can calculate the take-off weight of a single-seat ultralight general-purpose aircraft more accurately. This method has certain reference value for the future takeoff weight estimation of ultralight general-purpose aircraft.

The correction of the coefficients A and B is effective for improving the accuracy of the take-off weight estimation of the single-seat ultralight general-purpose aircraft. The correction of the coefficient C, that is, the correction of the fuel weight ratio factor, has little effect on the take-off weight of a single-seat ultralight general-purpose aircraft, which can be neglected in engineering applications.

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