Rough Set Approach for Analyzing the Effect of Viscoelastic and Micropolar Parameters on Hiemenz Flow in Hydromagnetics

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ABSTRACT

This research describes the hydromagnetic problem of two dimensional Hiemenz flow for a micropolar, viscoelastic, incompressible, viscous, electrically conducting fluid, impinging perpendicularly onto a plane in the presence of a transverse magnetic field. An approach based on the rough set theory is introduced where the mathematical model which describes the problem is first transformed into a dimensionless form. Then it is solved by using the Runge–Kutta numerical integration procedure in conjunction with shooting technique. Finally a set of maximally generalized decision rules (classification rules) are generated by using rough sets methodology.

Keywords: Viscoelastic fluids; Micropolar fluids; Hiemenz Flow; Rough set theory; feature selection; Hydromagnetics.

1) INTRODUCTION

In recent past the attention of many scientists was attracted to viscoelastic fluids due to application of this kind of fluids in industrial engineering, chemical industries, biomedical engineering, paints, polymers and technological applications since this type of fluids retains old distortions and its new behavior depends on previous distortions due to its “elastic” nature. Beard and Walters [1] developed the first model which describe and simulate the viscous fluids then many scientists and engineers studied and analyzed the flow and heat transfer characteristics of viscoelastic fluids as a type of non-Newtonian fluids [2-10].

Hiemenz was the first one who studied the two dimensional flow of a fluid near a stagnation point and show that the governing equations which describe the flow can be reduced to an ordinary differential equation with the aid of similarity transformation [11] then Several studies were elaborated by researchers to study Hiemenz flow in different ways to include various physical effects in hydromagnetics [12-14]. Also, Micropolar fluids are introduced by Eringen [15] and he characterized the structure of these fluids and define it physically as it consist of rigid, randomly oriented (or spherical) particles suspended in a viscous medium and theses particles can rotate with their own spins and microrotations, since the deformation of fluid particles is ignored. Then Eringen [16] extended his investigation of micropolar elasticity and many researchers and engineers focus their efforts in studying micropolar fluids as it has a great role in industrial applications Examples include exotic lubricants, food industry, biological and bio-medical sciences. For excellent review see [17-19].

The problem of reducing has been investigated for many numerous applications in different fields, since the irrelevant and redundant features in the dataset lead to low accuracy. There are two main approaches to reduce the input dimensionality, namely feature extraction and feature selection. Rough set theory was used as a tool to reduce the dimensionality as well as dealing with uncertainty in datasets. Many heuristic algorithms are proposed based on rough set theory, also numerous approached based on rough set theory and other theory are investigated to extract decision rules and reduce the dimensionality of dataset [20-30].

NOMENCLATURE

\( \overline{u} \) and \( \overline{v} \) Velocity components along \( x \) and \( y \) axes
\( X \) and \( Y \) Dimensionless velocity component in the \( x \)-and \( y \)-direction
\( N \) angular velocity
\( \nu \) Kinematic viscosity
\( \sigma \) electrical conductivity of fluid
\( K' \) weissenberg number
\( B \) induced magnetic field
\( a \) constant
\( \mu \) Dynamic viscosity
\( K \) vortex viscosity
\( \gamma \) spin gradient viscosity
\( j \) microinertia per unit mass
\( k_0 \) viscoelastic parameter
\( M \) Hartman Number
In this paper, we consider the effect of a transverse magnetic field on the Hiemenz flow (the two-dimensional flow near a stagnation point) of micropolar viscoelastic fluids. The governing Equations are solved by using the Runge–Kutta numerical integration procedure in conjunction with shooting technique. We present numerical results for a range of values of the Hartman number, of the viscoelastic parameter, and of the material properties of the fluid. Besides, the outcomes are elaborated graphically for involved variables.

2) MATHEMATICAL FLOW MODEL

Let us consider two-dimensional flow of a viscous, incompressible, electrically conducting, micropolar, viscoelastic fluid impinging perpendicularly onto a plane directed along the \( x \)-axis, as shown in Fig. 1. The flow is embedded in a uniform magnetic field of constant strength \( H_0 \).

![Flow model and coordinate system](image)

The governing equations which describe the mathematical model for this problem take the form [33]:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0, \\
\frac{\partial \bar{u}}{\partial x} + \nu \frac{\partial \bar{u}}{\partial y} = a^2 x + \left( \nu + \frac{k}{\rho} \right) \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{k}{\rho} \frac{\partial N}{\partial y} + \frac{\sigma B^2}{\rho} (a \bar{u} - \bar{u}) - k \left( \bar{u} \frac{\partial^3 \bar{u}}{\partial x \partial y^2} + \bar{v} \frac{\partial^3 \bar{u}}{\partial x^3} + \bar{u} \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial \bar{u}}{\partial y} \frac{\partial^2 \bar{u}}{\partial x \partial y} \right), \\
\frac{\partial N}{\partial x} + \nu \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j} \frac{\partial^3 N}{\partial y^2} - k \frac{2N + \frac{\partial \bar{u}}{\partial y}}{\rho j},
\]

According to [33] Boundary conditions for the stagnation-point flow are as follows:

\[
y = 0: \quad \bar{u} = 0, \quad \bar{v} = 0, \quad N = -m \frac{\partial \bar{u}}{\partial y} \\
y \to \infty: \quad \bar{u} = ax, \quad N = 0
\]
The partial differential conservation equations (1)-(4) are thereby converted into dimensionless system by defining the stream function \( \psi \) as:

\[
\vec{u} = \frac{\partial \psi}{\partial y}, \quad \vec{v} = -\frac{\partial \psi}{\partial x}
\]  

(5)

And the following non-dimensional variables are introduced:

\[
\eta = \sqrt{\frac{a}{u}} \psi, \quad \psi = \sqrt{av} \, xf \, (\eta), \quad N = \sqrt{av} \, x g \, (\eta)
\]  

(6)

Also to facilitate numerical solutions for low values of the viscoelastic parameter \( k_0 \), the following formalization expressions are used:

\[
f = f_0 + k_0 f_1 + k_0^2 f_2 + \ldots
\]

\[
g = g_0 + k_0 g_1 + k_0^2 g_2 + \ldots
\]

(7)

The final mathematical model was obtained as:

\[
(1 + \Delta) f_0'' + \Delta g_0' + f_0 f_0' + 1 - f_0' + M^2 (1 - f_0') = 0
\]  

(8)

\[
\lambda_0 g_0' - \Delta B_1 (2g_0 + f_0') - f_0' g_0 + g_0 f_0 = 0
\]  

(9)

\[
(1 + \Delta) f_1'' + \Delta g_1' + f_0 f_0' - 2f_0 f_1' - M^2 f_1' - (2f_0 f_0' - f_0 f_0' - f_0'') = 0
\]  

(10)

\[
\lambda_1 g_1' - \Delta B_1 (2g_1 + f_1') - f_0' g_1 - f_1' g_0 + g_0 f_1 + g f_0 = 0
\]  

(11)

\[
(1 + \Delta) f_2'' + \Delta g_2' + f_0 f_0' + f_0 f_1' - 2f_0 f_2' - f_1' + M^2 f_2' - (2f_0 f_0' + f_0 f_0' - f_0 f_0' - f_0') = 0
\]  

(12)

\[
\lambda_2 g_2' - \Delta B_1 (2g_2 + f_2') - f_0' g_2 - f_1' g_1 - f_2' g_0 + g_0 f_2 + g f_1 + g f_0 = 0
\]  

(13)

Subject to the boundary conditions:

\[
f_0(0) = f_1(0) = f_2(0) = f_0'(0) = f_1'(0) = f_2'(0) = 0
\]

\[
f_0'(\infty) = 1, \quad f_1'(\infty) = f_2'(\infty) = 0
\]  

(14)

\[
g_0(0) = -mf_0'(0), \quad g_1(0) = -mf_1'(0), \quad g_2(0) = -mf_2'(0)
\]

\[
g_0(\infty) = g_1(\infty) = g_2(\infty) = 0
\]

3) ANALYSIS

The system of equations (8)-(13) subject to the boundary condition in (4) was solved by the aid of Runge–Kutta numerical-integration procedure in conjunction with a shooting technique. The results of these calculations are divided into two parts. The first part represents the case of viscoelastic fluids where the surface values \( f'(0) \) of the velocity gradient at \( \Delta = 0 \) are shown in table 1. While the
second part represents the case of micropolar viscoelastic fluid, where the surface values $f''(0)$ of the velocity gradient and the surface values $g'(0)$ of the microrotation gradients for various nonzero values of $\Delta$ are shown in table 2.

| U  | M   | K₀   | $f''(0)$ |
|----|-----|------|----------|
| X₁ | 0.0 | 0    | 1.23259  |
| X₂ | 0.0 | 0.025| 1.26972  |
| X₃ | 0.0 | 0.05 | 1.29336  |
| X₄ | 0.2 | 0    | 1.24857  |
| X₅ | 0.2 | 0.025| 1.27933  |
| X₆ | 0.2 | 0.05 | 1.31009  |
| X₇ | 0.4 | 0    | 1.29537  |
| X₈ | 0.4 | 0.025| 1.32725  |
| X₉ | 0.4 | 0.05 | 1.35913  |
| X₁₀| 0.6 | 0    | 1.36988  |
| X₁¹| 0.6 | 0.025| 1.40360  |
| X₁²| 0.6 | 0.05 | 1.43732  |
| X₁₃| 0.8 | 0    | 1.46798  |
| X₁₄| 0.8 | 0.025| 1.50423  |
| X₁₅| 0.8 | 0.05 | 1.54048  |
| X₁₆| 1.0 | 0    | 1.58533  |
| X₁₇| 1.0 | 0.025| 1.62475  |
| X₁₈| 1.0 | 0.05 | 1.66418  |
| X₁₉| 1.2 | 0    | 1.71804  |
| X₂₀| 1.2 | 0.025| 1.76124  |
| X₂₁| 1.2 | 0.05 | 1.80444  |
| X₂₂| 1.4 | 0    | 1.86285  |
| X₂₃| 1.4 | 0.025| 1.91038  |
| X₂₄| 1.4 | 0.05 | 1.95790  |
| X₂₅| 1.6 | 0    | 2.01715  |
| X₂₆| 1.6 | 0.025| 2.06952  |
| X₂₇| 1.6 | 0.05 | 2.12189  |
| X₂₈| 2.0 | 0    | 2.34666  |
| X₂₉| 2.0 | 0.025| 2.41013  |
| X₃₀| 2.0 | 0.05 | 2.47361  |
| X₃₁| 3.0 | 0    | 3.24095  |
| X₃₂| 3.0 | 0.025| 3.33951  |
| X₃₃| 3.0 | 0.05 | 3.43806  |
| X₃₄| 5.0 | 0    | 5.14796  |
| X₃₅| 5.0 | 0.025| 5.34429  |
| X₃₆| 5.0 | 0.05 | 5.54062  |
| X₃₇| 10.0| 0    | 10.07474 |
| X₃₈| 10.0| 0.025| 10.63314 |
| X₃₉| 10.0| 0.05 | 11.25077 |
Table 2. Values of \( f^{\cdot}(0) \) and \( g^{\cdot}(0) \) for various values of \( M \), \( k_0 \) and \( \Delta \), for a second-order viscoelastic fluid.

| U   | M  | \( k_0 \) | \( \Delta \) | \( f^{\cdot}(0) \) | \( g^{\cdot}(0) \) |
|-----|----|----------|----------|----------------|----------------|
| X1  | 0.0| 0        | 0.5      | 1.00365        | 0.04813        |
| X2  | 0.0| 0.025    | 0.5      | 1.01735        | 0.04844        |
| X3  | 0.0| 0.05     | 0.5      | 1.03106        | 0.044876       |
| X4  | 0.2| 0        | 0.5      | 1.01669        | 0.04876        |
| X5  | 0.2| 0.025    | 0.5      | 1.03056        | 0.04834        |
| X6  | 0.2| 0.05     | 0.5      | 1.04444        | 0.04865        |
| X7  | 0.4| 0        | 0.5      | 1.05489        | 0.04897        |
| X8  | 0.4| 0.025    | 0.5      | 1.06927        | 0.04895        |
| X9  | 0.4| 0.05     | 0.5      | 1.08365        | 0.04927        |
| X10 | 0.6| 0        | 0.5      | 1.11573        | 0.04959        |
| X11 | 0.6| 0.025    | 0.5      | 1.13094        | 0.04989        |
| X12 | 0.6| 0.05     | 0.5      | 1.14615        | 0.05021        |
| X13 | 0.8| 0        | 0.5      | 1.19581        | 0.05054        |
| X14 | 0.8| 0.025    | 0.5      | 1.21216        | 0.05107        |
| X15 | 0.8| 0.05     | 0.5      | 1.22851        | 0.05140        |
| X16 | 1.0| 0        | 0.5      | 1.29164        | 0.05173        |
| X17 | 1.0| 0.025    | 0.5      | 1.30942        | 0.05241        |
| X18 | 1.0| 0.05     | 0.5      | 1.32720        | 0.05308        |
| X19 | 1.2| 0        | 0.5      | 1.40001        | 0.05383        |
| X20 | 1.2| 0.025    | 0.5      | 1.41949        | 0.05418        |
| X21 | 1.2| 0.05     | 0.5      | 1.43897        | 0.05452        |
| X22 | 1.4| 0        | 0.5      | 1.51827        | 0.05529        |
| X23 | 1.4| 0.025    | 0.5      | 1.53969        | 0.05564        |
| X24 | 1.4| 0.05     | 0.5      | 1.56112        | 0.05599        |
| X25 | 1.6| 0        | 0.5      | 1.64429        | 0.0673         |
| X26 | 1.6| 0.025    | 0.5      | 1.66789        | 0.05710        |
| X27 | 1.6| 0.05     | 0.5      | 1.69148        | 0.05746        |
| X28 | 2.0| 0        | 0.5      | 1.91341        | 0.05950        |
| X29 | 2.0| 0.025    | 0.5      | 1.94199        | 0.05989        |
| X30 | 2.0| 0.05     | 0.5      | 1.97057        | 0.06028        |
| X31 | 3.0| 0        | 0.5      | 2.64380        | 0.06544        |
| X32 | 3.0| 0.025    | 0.5      | 2.68810        | 0.06588        |
| X33 | 3.0| 0.05     | 0.5      | 2.73241        | 0.06633        |
| X34 | 5.0| 0        | 0.5      | 4.20127        | 0.07352        |
| X35 | 5.0| 0.025    | 0.5      | 4.28933        | 0.07406        |
| X36 | 5.0| 0.05     | 0.5      | 4.37739        | 0.07459        |
| X37 | 10.0| 0     | 0.5     | 8.22464        | 0.08223        |
| X38 | 10.0| 0.025  | 0.5     | 8.48811        | 0.08299        |
| X39 | 10.0| 0.05   | 0.5     | 8.75158        | 0.08375        |
| U   | M   | K₀   | Δ   | \( f' \left( 0 \right) \) | \( -g' \left( 0 \right) \) |
|-----|-----|------|-----|----------------|----------------|
| X40 | 0.0 | 0    | 1.5 | 0.76688        | 0.12087        |
| X41 | 0.0 | 0.025| 1.5 | 0.77215        | 0.12129        |
| X42 | 0.0 | 0.05 | 1.5 | 0.77741        | 0.12172        |
| X43 | 0.2 | 0    | 1.5 | 0.77691        | 0.12151        |
| X44 | 0.2 | 0.025| 1.5 | 0.78224        | 0.12194        |
| X45 | 0.2 | 0.05 | 1.5 | 0.78757        | 0.12236        |
| X46 | 0.4 | 0    | 1.5 | 0.80632        | 0.12336        |
| X47 | 0.4 | 0.025| 1.5 | 0.81185        | 0.12379        |
| X48 | 0.4 | 0.05 | 1.5 | 0.81737        | 0.12422        |
| X49 | 0.6 | 0    | 1.5 | 0.85321        | 0.12622        |
| X50 | 0.6 | 0.025| 1.5 | 0.85906        | 0.12666        |
| X51 | 0.6 | 0.05 | 1.5 | 0.86490        | 0.12710        |
| X52 | 0.8 | 0    | 1.5 | 0.91502        | 0.12984        |
| X53 | 0.8 | 0.025| 1.5 | 0.92130        | 0.13029        |
| X54 | 0.8 | 0.05 | 1.5 | 0.92758        | 0.13074        |
| X55 | 1.0 | 0    | 1.5 | 0.98905        | 0.13397        |
| X56 | 1.0 | 0.025| 1.5 | 0.99587        | 0.13444        |
| X57 | 1.0 | 0.05 | 1.5 | 1.00269        | 0.13490        |
| X58 | 1.2 | 0    | 1.5 | 1.07286        | 0.13838        |
| X59 | 1.2 | 0.025| 1.5 | 1.08032        | 0.13886        |
| X60 | 1.2 | 0.05 | 1.5 | 1.08778        | 0.13934        |
| X61 | 1.4 | 0    | 1.5 | 1.16438        | 0.14291        |
| X62 | 1.4 | 0.025| 1.5 | 1.17258        | 0.14341        |
| X63 | 1.4 | 0.05 | 1.5 | 1.18077        | 0.14391        |
| X64 | 1.6 | 0    | 1.5 | 1.26195        | 0.14745        |
| X65 | 1.6 | 0.025| 1.5 | 1.27096        | 0.14797        |
| X66 | 1.6 | 0.05 | 1.5 | 1.27995        | 0.14848        |
| X67 | 2.0 | 0    | 1.5 | 1.47042        | 0.15624        |
| X68 | 2.0 | 0.025| 1.5 | 1.48129        | 0.15679        |
| X69 | 2.0 | 0.05 | 1.5 | 1.49217        | 0.15735        |
| X70 | 3.0 | 0    | 1.5 | 2.03655        | 0.17598        |
| X71 | 3.0 | 0.025| 1.5 | 2.05329        | 0.17604        |
| X72 | 3.0 | 0.05 | 1.5 | 2.07002        | 0.17669        |
| X73 | 5.0 | 0    | 1.5 | 3.24417        | 0.20237        |
| X74 | 5.0 | 0.025| 1.5 | 3.27708        | 0.20318        |
| X75 | 5.0 | 0.05 | 1.5 | 3.30999        | 0.20400        |
| X76 | 10.0| 0    | 1.5 | 6.36472        | 0.23245        |
| X77 | 10.0| 0.025| 1.5 | 6.46101        | 0.23370        |
| X78 | 10.0| 0.05 | 1.5 | 6.55830        | 0.23494        |
| U | M   | K₀  | Δ   | f' (0) | -g' (0) |
|---|-----|-----|-----|-------|--------|
|  X79 | 0.0 | 0   | 5   | 0.47168 | 0.25896 |
|  X80 | 0.0 | 0.025 | 5    | 0.47284 | 0.25933 |
|  X81 | 0.0 | 0.05  | 5   | 0.47400 | 0.25970 |
|  X82 | 0.2 | 0    | 5   | 0.47765 | 0.26071 |
|  X83 | 0.2 | 0.025 | 5   | 0.47882 | 0.26108 |
|  X84 | 0.2 | 0.05  | 5   | 0.48000 | 0.26146 |
|  X85 | 0.4 | 0    | 5   | 0.49521 | 0.26581 |
|  X86 | 0.4 | 0.025 | 5   | 0.49642 | 0.26719 |
|  X87 | 0.4 | 0.05  | 5   | 0.49764 | 0.26657 |
|  X88 | 0.6 | 0    | 5   | 0.52346 | 0.27384 |
|  X89 | 0.6 | 0.025 | 5   | 0.52474 | 0.27423 |
|  X90 | 0.6 | 0.05  | 5   | 0.52602 | 0.27462 |
|  X91 | 0.8 | 0    | 5   | 0.56108 | 0.28425 |
|  X92 | 0.8 | 0.025 | 5   | 0.56244 | 0.28465 |
|  X93 | 0.8 | 0.05  | 5   | 0.56381 | 0.28506 |
|  X94 | 1.0 | 0    | 5   | 0.60662 | 0.29642 |
|  X95 | 1.0 | 0.025 | 5   | 0.60809 | 0.29684 |
|  X96 | 1.0 | 0.05  | 5   | 0.60957 | 0.29726 |
|  X97 | 1.2 | 0    | 5   | 0.65865 | 0.30977 |
|  X98 | 1.2 | 0.025 | 5   | 0.66025 | 0.31021 |
|  X99 | 1.2 | 0.05  | 5   | 0.66186 | 0.31065 |
| X100 | 1.4 | 0    | 5   | 0.71593 | 0.32383 |
| X101 | 1.4 | 0.025 | 5   | 0.71768 | 0.32429 |
| X102 | 1.4 | 0.05  | 5   | 0.71943 | 0.32475 |
| X103 | 1.6 | 0    | 5   | 0.77738 | 0.33821 |
| X104 | 1.6 | 0.025 | 5   | 0.77929 | 0.33869 |
| X105 | 1.6 | 0.05  | 5   | 0.78121 | 0.33917 |
| X106 | 2.0 | 0    | 5   | 0.90960 | 0.36692 |
| X107 | 2.0 | 0.025 | 5   | 0.91188 | 0.36745 |
| X108 | 2.0 | 0.05  | 5   | 0.91417 | 0.36797 |
| X109 | 3.0 | 0    | 5   | 1.27182 | 0.43286 |
| X110 | 3.0 | 0.025 | 5   | 1.27525 | 0.43350 |
| X111 | 3.0 | 0.05  | 5   | 1.27868 | 0.43414 |
| X112 | 5.0 | 0    | 5   | 2.05015 | 0.53342 |
| X113 | 5.0 | 0.025 | 5   | 2.05667 | 0.53428 |
| X114 | 5.0 | 0.05  | 5   | 2.06318 | 0.53514 |
| X115 | 10.0 | 0  | 5 | 4.06866 | 0.67589 |
| X116 | 10.0 | 0.025 | 5   | 4.08709 | 0.67718 |
| X117 | 10.0 | 0.05  | 5   | 4.10552 | 0.67848 |
Then applying the proposed approach based on rough set theory which summaries as:

Step 1: discretize the decision table by using the Boolean reasoning algorithm.

Step 2: compute the reduct of the discretized decision table with the aid of genetic algorithm.

Step 3: generate a set of maximally generalized decision rules (classification rules).

The following flowchart represents the complete steps to extract the set of classification rules.

![Flowchart](image)

**Fig. 2: Complete Steps to Extract Decision Rules**

It worth noting that in this stage we will use software called ROSETTA which is an RST analysis toolkit. Table 3 shows part of the rule set extracted by using rough set methodology which explained in fig. 2.
It is noted that in the case of viscoelastic fluids, \( f'(0) \) is proportional to the friction factor. And in the case of micropolar viscoelastic fluids, \( f'(0) \) is proportional to the friction factor and \( g'(0) \) is proportional to the wall couple stress.

**CONCLUSION**

This paper suggests the use of rough set theory to process and extract rules for analyzing the effect of viscoelastic and micropolar parameters on Hiemenz flow in hydromagnetics. The results of this study indicate that as the micropolar parameter \( \Delta \) increases, the friction factor decreases. A similar behavior is noted when the Hartman number and the viscoelastic parameter increase. The absolute value of the microrotation gradient is found to increase with increasing Hartman number, micropolar parameter, and viscoelastic parameter. Also, the obtained results are in good agreement with previous studies. The technique has been simplified logic-based rules, reduces the time and resources required to build knowledge.

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