Model Identification and Equivalence Method for Grid Optimal Operation

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Abstract. In this paper, on the basis of simplification idea of external transfer consistence of internal power flow in power grid, the network identification method was used to find the local power grid with equivalent value, and WARD equivalent value was used to equalize the local power grid to the boundary node, because the power transfer relationship between internal nodes and boundary nodes of equivalent power grid is basically consistent, equivalent unit could reflect the overall output characteristics of equivalent power grid, there was no need to concern the power distribution of internal node of equivalent power grid, furthermore, the schedulable capacity and cost curve of schedulable resources of equivalent power grid could be calculated, on this basis, the dispatching optimization of power generation of large power grid were conducted. The example analysis showed that the equivalent method in this paper could maintain higher model equivalent accuracy, and the overall optimal solution of power generation dispatching of large-scale power grid was realized.

Keywords: Network identification, equivalence, economic dispatch, security constraint.

1. Introduction

With the expansion of power grid scale and deepening of interconnection degree, considerable variables and constraints have brought great challenges to the economic dispatch problem, the economic dispatch of security constraint of interconnected large power grid has become a mathematical program problem with large-scale multi-objective complex constraint [1-2]. The large-scale power grid directly solves dispatching program of the whole network with many constraints, even if the constraints such as power grid security are all converted into linear constraints, the power generation cost function is linearized, the nonlinear problem is transformed into the linear program problem, if the problem scale to be solved is huge, the computation will still be too large to reduce the solution efficiency.

The solution scale can be greatly reduced through power grid equivalence, the key issue is how to transfer the equivalent network information to the equivalent point in the equivalence process, the achievement of lossless compression of the local power grid is the key to solve the overall optimization of dispatching program of large power grid [3-5]. Therefore, in allusion to the problem of excessive computation of economic dispatch of large-scale power grid, in addition to linearization of the nonlinear model, the power grid model should be reasonably divided under the premise of ensuring optimality,
variable dimension and overall computation scale can be reduced, a large number of scholars have conducted related explorations in order to complete the goal of optimal computation within a reasonable time frame and find the best feasible solution [6-8].

In this paper, the power grid model identification and equivalence were used as techniques, WARD equivalent principle was used to moderately simplify the local power grid, and connected to the main grid to form the equivalent model, moreover, the schedulable resources of the local grid were taken as equivalent unit model, on this basis, the computation of the optimal dispatching of the whole network was carried out, and finally the computation efficiency and feasibility of the method in this paper were verified through the example analysis.

2. Network Optimization Methods

At present, the large-scale scheduling optimization problems are handled from the angle of decomposition coordination and equivalence simplification to obtain the final feasible solution. The local power grid is simplified, and as long as the equivalent power grid obtained by simplification can reflect the original network characteristics, the large power grid can be transformed into the smaller scale power grid. Network transformation, simplification, and equivalence are often needed in power grid computations, network transformation can transform the original network into the form that is easy for computation; network simplification can replace parts of the network that do not require detailed analysis with simplified network, keep the parts that need detailed analysis; network equivalence can greatly reduce the network scale studied.

The federated computation of multiple areas can realize the trans-provincial optimum distribution of resources, but simple combination cannot adapt to the business characteristics of power grid dispatching program, and cannot avoid the surge problem of computation scale. The method based on equivalence or decomposition can improve computational efficiency by reducing the optimization scale, the key is how to maintain the accuracy of model compression. If there are some important branches or sections in the equivalent network present different power flow transfer characteristics to the outside, the power distribution of some internal units will also affect the power flow distribution of the whole network and cause unknown interference to the security and stability of the power grid.

Some local networks in the power grid have special properties: the power distribution of their internal nodes has little impact on the power flow of the boundary nodes, and there are a large number of radial networks or relatively independent networks in the power grid, for example, regional power grids and some provincial grid connected by a small number of tie lines, the power grids with this special nature can be used as a whole for dispatch, optimization and computation of power generation. In order to determine this type of network, it is necessary to find it out through identification method, and then simplify and connect it.

3. Network Identification and Simplification

The networks with different voltage classes are connected through the main transformer, if their internal nodes have the similar transfer relationship to outside, their internal network structure must be the connected network, and the network identification method is identified step by step in accordance with the voltage order.

The selection of boundary nodes: power grids with different voltage classes are connected through transformers, boundary nodes are selected as main transformer nodes of different voltage classes, and cluster analysis and judgment are conducted for the power transfer factor from the network node to the upper main transformer.

Identification order: in order to be convenient for the development of computations, the network identification is carried out successively in accordance with orders of voltage classes, and the low-to-high way of voltage class is adopted to carry out network identification step by step and reduce the network scale.
Step one: without considering line maintenance and load transfer, starting from the low voltage class, the lower voltage side of the higher-class main transformer as the boundary node, the boundary node transfer matrix $A_{\text{tran},x}$ of other nodes to the xkV main transformer is established:

$$
A_{\text{tran},x} = \begin{bmatrix} a_{1,1} & a_{1,n} \\ M & O \\ a_{m,1} & a_{m,n} \end{bmatrix}
$$

(1)

In the formula, the element $a_{m,n}$ is the transfer coefficient, which represents the transfer coefficient of the network internal node n to the main transformer node m.

Step two: for the internal node i, its corresponding relationship with the external main transformer node can be expressed as $[a_{ij}, a_{ij}, L, a_{ij}]$, it represents the position in $m \times n$ dimensional space. By judging the mutual distance of node i in $m \times n$ dimensional space, clustering forms node group, Euclidean distance and maximum single point distance are used as the judgment basis[9-11], and their distance expressions can be expressed separately as:

$$
d_{i,j} = \sqrt{\sum_{k=1}^{m} (a_{k,i} - a_{k,j})^2}
$$

(2)

$$\max \{|a_{1,i} - a_{1,j}|, |a_{2,i} - a_{2,j}|, L, |a_{m,i} - a_{m,j}|\}
$$

(3)

The threshold is set for two farthest distances to conduct aggregated analysis, prevent nodes with large transfer coefficients from being clustered together.

Step three: the breadth-first search is conducted for the identified node group (including related main transformer nodes), and determine the connected network of the node group.

Step four: checking the nodes involved in the maintenance line: when the branch circuit is broken, the change of transfer coefficient of each generator node i and boundary node $\max \{\Delta a_{1,i}, \Delta a_{2,i}, L, \Delta a_{m,j}\}$ is measured and calculated, when it is less than the setting discrimination threshold, it shows that the condition of the equivalent network does not affect the power flow external influence to the outside power node; when this condition is not met, it is necessary to repeat cluster analysis for the existing node set under the new network structure.

Step five: setting the minimum threshold for the number of nodes included in the simplified network, namely virtual nodes, to ensure the efficiency of network simplification.

Step six: after grid of the voltage class is identified, the network that meets the simplification conditions is simplified by WARD, and re-connected to other network parts to form the expanded networks, and then virtual node identification is conducted for the network with higher voltage class, and then move on to step one;

Step seven: when all voltage classes of the whole network have been identified, the schedulable capacity and cost curve of equivalent network are generated;

Step eight: the dispatch optimization and computation of power generation are conducted for the extended network model with equivalent units and other units;

Step nine: the power generation program of the equivalent units is decomposed to form the power generation program of the internal units of the original equivalent network.

In step six, the networks that meet the simplification conditions are firstly simplified to form equivalent network, and then connected to the rest of power grids to form extended network models of the whole network. It is divided into two steps:

1) The internal nodes and corresponding branches in the network to be equivalent are eliminated, number the equivalent networks, ensure that the same number is connected by the boundary nodes of the equivalent networks, and write grid admittance matrix $Y_{re}$ of all boundary nodes in order;
2) The new admittance matrix of the boundary nodes of the equivalent network is connected with the grid admittance matrix $Y_{re}$ [12-13], form the simplified grid admittance matrix, if there are $n$ equivalent networks, the admittance matrix is $Y_{BB}^n$, then

$$Y_{re}^n = Y_{re} + \text{diag}(Y_{BB}^n, Y_{BB}^n, \ldots, Y_{BB}^n, 0, 0)$$  \hspace{1cm} (4)

In the formula, $\text{diag}(\bullet)$ is the block diagonal matrix formed by the new admittance matrix of boundary nodes.

The model connection is completed to form a global network expansion model.

![Fig.1 Flow chart of network identification and simplification](image)

### 4. Schedulable Capacity and Cost Curve of Equivalent Power Grid

The schedulable capacity of the equivalent network includes the output adjustment range and rate of the equivalent unit, for the equivalent network without adjustable power supply, it does not have the scheduling ability, and its output adjustment range and rate are zero. For the equivalent network with adjustable power supply, assuming that the equivalent network has $M$ branches connected to the outside, the following adjustment capacity solving model of equivalent network is established:

$$\min F = \sum_{i=1}^{N} \sum_{j=1}^{T} u_{ij} c_{ij} (P_{ij}^{lim}) + \sum_{i=1}^{T} \sum_{k=1}^{M} c_i (P_{grid})$$ \hspace{1cm} (5)

In the formula: $T$ is the number of optimized time; $N$ is the number of bidding units included in the virtual node; $P_{ij}^{lim}$ is the feasible limit of the output of unit $i$ during $t$ period; $u_{ij}$ is the start-stop status of unit $i$ during $t$ period, 0 is shutdown, 1 is startup; $c_{ij}$ is the bidding curve of unit $i$ during $t$ period; $P_{grid}$ is the exchange power of the external network to the virtual node; $c_i$ is the price guide curve of
the external network, when its value is lower than the minimum value of all unit bidding, the virtual node output reaches the lower limit, when its value is greater than the maximum value of all unit bidding, the virtual node output reaches the upper limit.

Power balance constraints:
\[
\sum_{i=1}^{N} u_{i,t} P_{i,t}^{\text{lim}} + \sum_{k=1}^{M} P_{k,t}^{\text{grid}} = D_{t}^{\text{node}}
\]  
(6)

In the formula, \(D_{t}^{\text{node}}\) is the virtual node load.

Unit operation needs to meet:
\[
u_{i,t} P_{i,t}^{\text{min}} \leq u_{i,t} P_{i,t}^{\text{lim}} \leq u_{i,t} P_{i,t}^{\text{max}}
\]  
(7)

In the formula, \(P_{i,t}^{\text{max}}\) and \(P_{i,t}^{\text{min}}\) are the upper and lower limits of unit output.

Branch security constraints:
\[
\sum_{i \in I_{n}} s_{i,j,t}(P_{i,t}^{\text{lim}} + P_{i,t}^{\text{grid}} - l_{i,j,t}) \leq S_{j,t}^{\text{max}}, \ j \in S
\]  
(8)

In the formula, \(I_{n}\) is the set of network internal nodes, \(l_{i,j,t}\) is the node load, \(S\) is the branch set of the virtual nodes, \(S_{j,t}^{\text{max}}\) is the transmission limit of branch \(j\), \(s_{i,j,t}\) is the sensitivity of the power injection of node \(i\) to section \(S_{j}\).

The output upper and lower limit all units in the equivalent network are directly summed, the network transmission limit cannot be summed, the above model can use maximum and minimum price guide curve of the input external network obtain the limit power output of the internal node unit under the limit condition considering the network constraints. The solution steps:

Firstly, the formulas of (5)-(8) are used to determine the output value of the units when the output of the equivalent network reaches the upper and lower limit;

Finally, the schedulable adjustment range of the equivalent network is calculated by the output value of the units:
\[
p_{i,t}^{\text{node, max}} = \sum_{i=1}^{N} P_{i,t}^{\text{lim}}, \ c_{k} > \forall c_{i,t}
\]  
(9)

\[
p_{i,t}^{\text{node, min}} = \sum_{i=1}^{N} P_{i,t}^{\text{lim}}, \ c_{k} < \forall c_{i,t}
\]  
(10)

In the formula: \(p_{i,t}^{\text{node, max}}\) and \(p_{i,t}^{\text{node, min}}\) are the output upper and lower limits of the virtual node during \(t\) period, respectively.

The maximum adjustment rate of the equivalent network is not greater than the adjustment rate sum of the adjustable units in the network internal nodes, namely:
\[
\Delta p_{i}^{\text{node}} \leq \sum_{i \in L} \Delta p_{i}
\]  
(11)

In the formula: \(\Delta p_{i}^{\text{node}}\) represents the adjustment rate of the virtual node, \(\Delta p_{i}\) represents the climbing limit among unit times.

The load distribution of the network internal nodes does not affect the external power flow, so the total load of the internal nodes can be equivalent to the load of the equivalent network, namely:
\[
D_{t}^{\text{node}} = \sum_{i \in L} l_{i,j,t}
\]  
(12)

The forming of bidding curve of equivalent network:

The units in the equivalent network present power flow transfer consistency to the outside, so the different schemes of unit output distribution in the equivalent network will not affect the external power injection relationship. The load distribution and internal topology of the equivalent network are different
in different periods, and the price power curves of the equivalent network are different in different periods. The training model of total power and price is established, including objective function:

$$\min F = \sum_{i=1}^{N} h_{i,j}c_{i,j}(P_{i,j})$$

(13)

In the formula: $N$ is the number of bidding units included in the virtual nodes; $P_{i,j}$ is the output of unit $i$ during $t$ period; $c_{i,j}$ is the bidding curve of unit $i$ during $t$ period.

In addition to unit operation constraints and branch security constraints, the constraints also include load balancing constraint:

$$\sum_{i \in L} P_{i,j} = D_j, D_j \in \Omega_j$$

(14)

$D_j$ is different total output values selected for training, $\Omega_j$ is the training value set, and they can be selected in total capacity of zero-sum unit based on experience and historical data.

5. Example Analysis

The total installed capacity of a power grid is 41203MW, including 32722MW capacity of directly dispatched unit of the main network, 88 power plants, 8481MW installed capacity of local area networks and below, 6025 power plants, 1450MW adjustable unit capacity connected to voltage class below 220kV, there are 180 substations of 220kV and 344 transformers in the whole network, including 24 substations including power generations, 37 transformers, 153 main transformers of 110kV with connection, 125 main transformers of 35kV, the method in this paper is used to identify the network equivalent value, and calculate the power generation dispatch program before 96-point.

Table 1 is the statistical results of power plants in the whole network, 35kV buses and branches and above, the number of power plants in the whole network exceeds 6,000, nearly 5000 buses and 10000 branches above 35kV. Table 2 is the network computation scale after the grid model equivalence is conducted through network identification; there are 47 equivalent units of 110kV, 71 equivalent units of 35kV, and 43 equivalent units below 35kV.

| Table 1 | Statistics of power grid scale in the whole network |
|---------|-----------------------------------------------|
| name    | main network | local network | whole network model |
| power plant | 88           | 6 025         | 6 113          |
| unit    | 207          | 13 992        | 14 199         |
| bus     | 1 082        | 3 779         | 4 861          |
| branch  | 2 148        | 7 418         | 9 566          |

| Table 2 | Power grid scale after equivalence |
|---------|-----------------------------------|
| Name    | 110kV network | 35kV network | below 35kV | simplified regional grid | main network | equivalent model |
| Unit    | 47            | 71           | 43         | 161                      | 207          | 368             |
| Bus     | 129           | 212          | 73         | 404                      | 1 082        | 1 486           |
| branch  | 301           | 391          | 101        | 793                      | 2 148        | 2 941           |
Through network identification and equivalence, the number of units that need to be directly optimized was greatly reduced to 368 units, including 207 directly dispatched units of the main network and 161 equivalent units. The mixed integer programming algorithm is used, on the basis of the power grid model equivalence and the integration of the whole network (35kV and network above), power generation program optimization is conducted, respectively, the comparison of optimal results are shown in Table.3. On the basis of equivalent mode of power grid model, the computation time is only 209s, which reduces 89% lower than the 1933s of the whole network integration method, and the difference of optimization target is only 0.18%.

### Table.3 Optimal computation results

| computation mode         | computation time (s) | optimization target ( per unit) |
|--------------------------|----------------------|---------------------------------|
| model extension          | 209                  | 1.0018                          |
| whole network integration| 1933                 | 1.0000                          |

In order to further verify the feasibility of this method, 30 typical network states of the power grid throughout the year are selected, and the power generation dispatching optimization is conducted based on the power grid model equivalent method, the comparison of optimization results are shown in Table.4. The results show that the average computation time is 211s, which reduces 90% lower than the 2027s of the whole network integration mode, the average deviation of the optimization target is 0.12%, and the maximum deviation is 0.19%, which meets the actual computation needs.

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**Fig.2 110kV virtual node in a local network**
Table.4 Statistical results of 30 optimization computations of a power grid

| computation mode       | average computation time (s) | average optimization target (per unit) | maximum deviation of optimization target (per unit) |
|------------------------|-----------------------------|--------------------------------------|-----------------------------------------------|
| model equivalence      | 211                         | 1.0012                               | 0.0019                                        |
| whole network integration | 2 027                   | 1                                    | 1                                             |

6. Conclusion
In this paper, WARD equivalence was used to conduct equivalence simplification of local power grids that have external power flow transfer consistency, the equivalent whole network equivalent model was formed, on this basis, the dispatching optimization of power generation in the whole network could be conducted, the example proved that the equivalent method in this paper was more efficient, and its advantages are:

1) By simplifying the local power grids into equivalent units, the equivalent units can retain the main power flow characteristics of the original local power grids, thereby simplifying the overall power grid structure and providing conditions for optimal computation of large-scale power grid.

2) Through the computation of schedulable capacity, as equivalent units, the local power grids participated in the optimization of the whole grid power generation program, the coordination of economic and safety of the main power grid and the local power grid were achieved, and gave play to the benefits of global resource allocation.

3) The high accuracy of simplified network was retained, and the computation scale was controlled within a feasible range, it could avoid the multi-round iteration problem of the decomposition and coordination computation method, and it was of great significance to improve the scheduling control of the complex network.

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