Violation of Time-Reversal Invariance and CPLEAR Measurements

Luis Alvarez-Gaumé, Costas Kounnas†, Smaragda Lola
CERN Theory Division, CH-1211 Geneva, Switzerland

Panagiotis Pavlopoulos
Institut für Physik, University of Basle CH-4056,
and CPLEAR Collaboration, CH-1211 Geneva Switzerland

ABSTRACT

Motivated by the recent CPLEAR measurement on the time-reversal non-invariance, we review the situation concerning the experimental measurements of charge conjugation, parity violation and time reversibility, in systems with non-Hermitean Hamiltonians. This includes in particular neutral meson systems, like $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$. We discuss the formalism that describes particle-antiparticle mixing and time evolution of states, paying particular emphasis to the orthogonality conditions of incoming and outgoing states. As a result, we confirm that the CPLEAR experiment makes a direct measurement of violation of time-reversal without any assumption of unitarity and $CPT$-violation. The asymmetry which signifies $T$-violation, is found to be independent of time and decay processes.

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† On leave from Ecole Normale Supérieure, 24 rue Lhommond, F-75231, Paris Cedex 05, France.
E-mail addresses: alvarez@nxth04.cern.ch, kounnas@nxth04.cern.ch, magda@mail.cern.ch, Noulis.Pavlopoulos@cern.ch
1 Introduction

Recently, the CPLEAR experiment at CERN, reported the first direct observation of time-reversal violation in the neutral kaon system [1]. This observation is made by comparing the probabilities of a $\bar{K}^0$ state transforming into a $K^0$ and vice-versa. CPLEAR produces initial neutral kaons with defined strangeness from proton-antiproton annihilations at rest, via the reactions

$$p\bar{p} \longrightarrow \begin{cases} K^-\pi^+K^0 \\ K^+\pi^-\bar{K}^0 \end{cases},$$

and tags the neutral kaon strangeness at the production time by the charge of the accompanying charged kaon. Since weak interactions do not conserve strangeness, the $K^0$ and $\bar{K}^0$ may subsequently transform into each-other via oscillations with $\Delta S = 2$. The final strangeness of the neutral kaon is then tagged through the semi-leptonic decays of the type

$$K^0(\bar{K}^0) \longrightarrow e^\pm \pi^\mp \nu(\bar{\nu}),$$

where, a positive (negative) lepton charge is associated with a $K^0$ ($\bar{K}^0$).

In this way, among other quantities, CPLEAR also measured the asymmetry

$$A_T^{\exp} = \frac{R(\bar{K}^0 (t = 0) \longrightarrow e^+\pi^-\nu (t = \tau)) - R(K^0 (t = 0) \longrightarrow e^-\pi^+\bar{\nu} (t = \tau))}{R(K^0 (t = 0) \longrightarrow e^+\pi^-\nu (t = \tau)) + R(\bar{K}^0 (t = 0) \longrightarrow e^-\pi^+\bar{\nu} (t = \tau))}, \quad (1)$$

which parametrizes the difference of the probability that an initial $\bar{K}^0(t_i)$ oscillates to a final $K^0(t_f)$, from the probability that an initial $K^0(t_i)$ oscillates to a final $\bar{K}^0(t_f)$. The average value of $A_T^{\exp}$ was found over the time interval from $1\tau_S$ to $20\tau_S$ (where $\tau_S$ is the lifetime of the short-lived kaon), to be different than zero by $4\sigma$ and this has been interpreted by CPLEAR as the first direct measurement of time-reversal non-invariance.

However, doubts have been expressed whether the experiment does provide such a direct evidence for $T$-violation. The basic argument is that decay processes enter in the observables, making $CP$-violation manifest. The observed effect is then attributed to these irreversible processes, rather than $T$-violation. It is also argued that this is only a direct effect of the decaying states being non-orthogonal.

The aim of this work is to clarify these points. In order to do so, we are going to re-discuss the formalism that describes the particle-antiparticle mixing and time evolution of states in the kaon system. Since the Hamiltonian $H$ of the system is non-Hermitean, the various masses, widths and eigenstates have to be found by using bi-unitary transformations\(^1\). This is

\(^1\) Indeed, there exist unitary matrices $V_L$ and $V_R$ such that $V_L^\dagger H V_R = H_{\text{diagonal}}$. The form of the two unitary matrices is found by diagonalizing the Hermitean combinations $HH^\dagger$ and $H^\dagger H$, while the physical states are defined by “rotations” of the initial ones, via the same matrices $V_L$ and $V_R$. 

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equivalent to identifying the form of the matrices and the eigenstates, by looking consistently at the correct orthogonality conditions for the incoming and outgoing states. The analysis is done in section 2, where we describe the states in the vector space of the system, its dual, as well as the dual complex space. In section 3, we are going to show that the theoretical asymmetry which arises directly from the definition of $T$-violation, is independent of time and decay processes. In section 4, we point out that this is also true for the experimental asymmetry that CPLEAR uses, which differs from the theoretical one due to the appearance of the semileptonic decays in the process. In the same section, we show that since the experiment uses a specific search-channel, rather than summing over all possible modes, no unitarity or CPT-invariance arguments enter in the analysis. Finally, in section 5 we present a summary of the basic points and conclude that the CPLEAR experiment indeed makes a direct measurement of $T$-violation.

2 Definition of states in the incoming $\mathcal{H}_{in}$ and outgoing $\mathcal{H}_{out}$ dual spaces

We denote by $\mathcal{H}_{in}$ and $\mathcal{H}_{out}$ the Hilbert space of incoming and outgoing (dual) states, respectively.

$$\mathcal{H}_{in} \equiv \{ |\Psi_{I}^{in} \rangle , I = 1, 2, \ldots, n \}, \quad \mathcal{H}_{out} \equiv \{ < \Psi_{I}^{out} | , I = 1, 2, \ldots, n \} , \quad (2)$$

$n$ is the dimension of the space and $|\Psi_{I}^{in} \rangle$ and $< \Psi_{I}^{out} |$ are the right- and left- eigenstates of the effective Hamiltonian $H$:

$$H |\Psi_{I}^{in} \rangle = \lambda_{I} |\Psi_{I}^{in} \rangle , \quad < \Psi_{I}^{out} | H = < \Psi_{I}^{out} | \lambda_{I} . \quad (3)$$

In this basis, the effective Hamiltonian is diagonal and can be expressed in the following form in terms of the incoming and outgoing states:

$$H = \sum |\Psi_{I}^{in} \rangle \lambda_{I} < \Psi_{I}^{out} | , \quad \text{with} \quad < \Psi_{I}^{out} | \Psi_{J}^{in} > = \delta_{IJ} , \quad (4)$$

where the unity operator $1$ takes the usual form:

$$1 = \sum |\Psi_{J}^{out} \rangle < \Psi_{J}^{out} | . \quad (5)$$

---

2 Technically, we assume that the Hamiltonian $H$ is an $n \times n$ matrix with $n$ well-defined left- and right-eigenvectors, to avoid some pathological cases that are irrelevant in the $K^{0} - \bar{K}^{0}$ system.
Up to this point, we do not assume that $H$ is Hermitean; $H \neq H^\dagger$. This implies that the conjugate states $|\Psi^\text{out}_I\rangle$ and $|\Psi^\text{in}_I\rangle$ are not isomorphic to their duals:

$$|\Psi^\text{out}_I\rangle \equiv <\Psi^\text{out}_I| \neq |\Psi^\text{in}_I\rangle,$$

$$<\Psi^\text{in}_I| \equiv |\Psi^\text{in}_I\rangle^\dagger \neq |\Psi^\text{out}_I\rangle.$$

The vectors, $|\Psi^\text{out}_I\rangle$ and $<\Psi^\text{in}_I|$ are eigenstates of the $H^\dagger$ operator but they are not eigenstates of $H$:

$$H^\dagger |\Psi^\text{out}_I\rangle = \lambda^*_I |\Psi^\text{out}_I\rangle,$$

$$<\Psi^\text{in}_I| H^\dagger = <\Psi^\text{in}_I| \lambda^*_I.\tag{7}$$

Only if the effective Hamiltonian is Hermitean, (i.e. $H = H^\dagger$), the conjugate outgoing states become isomorphic to the incoming ones, $|\Psi^\text{out}_I\rangle = |\Psi^\text{in}_I\rangle$; in this case the eigenvalues $\lambda_I = \lambda^*_I$ are real.

When $H \neq H^\dagger$, the time evolution of the incoming and outgoing states $|\Psi^\text{in}_I(t_i)\rangle$ and $|\Psi^\text{out}_I(t_f)\rangle$ are obtained from $|\Psi^\text{in}_I\rangle$ and $|\Psi^\text{out}_I\rangle$, using the evolution operators $e^{-iHt_i}$ and $e^{-iH^\dagger t_f}$ respectively:

$$|\Psi^\text{in}_I(t_i)\rangle = e^{-iHt_i} |\Psi^\text{in}_I\rangle,$$

$$|\Psi^\text{out}_I(t_f)\rangle = e^{-iH^\dagger t_f} |\Psi^\text{out}_I\rangle.\tag{8}$$

From the above equations, follows the evolution of the conjugate states:

$$<\Psi^\text{in}_I(t_i)| = <\Psi^\text{in}_I| e^{iHt_i},$$

$$<\Psi^\text{out}_I(t_f)| = <\Psi^\text{out}_I| e^{iH^\dagger t_f}.\tag{9}$$

In view of our later discussion, it is important to stress here that the inner products among incoming and outgoing states do not obey the usual orthogonality conditions. Indeed,

$$<\Psi^\text{out}_I|\Psi^\text{out}_J\rangle \neq \delta_{IJ} \quad \text{and} \quad <\Psi^\text{in}_I|\Psi^\text{in}_J\rangle \neq \delta_{IJ}.\tag{10}$$

On the other hand, the physical incoming and outgoing eigenstates obey at all times the orthogonality conditions

$$<\Psi^\text{out}_I(t_f)|\Psi^\text{in}_J(t_i)> = <\Psi^\text{out}_I|e^{-iH^\dagger \Delta t}\Psi^\text{in}_J> = e^{-i\lambda_J \Delta t} \delta_{IJ}.\tag{11}$$

We now proceed to discuss particle-antiparticle mixing in the neutral kaon system.
3 Particle-antiparticle mixing in the neutral kaon system

The $K^0, \bar{K}^0$ states are produced under strong interactions and are strangeness eigenstates. Moreover, they obey the relations:

$$CP \ |K^\text{in}_0\rangle = |\bar{K}^\text{in}_0\rangle,$$
$$T \ |K^\text{in}_0\rangle = <K^\text{out}_0|,$$
$$CPT \ |K^\text{in}_0\rangle = <\bar{K}^\text{out}_0|.$$  \hspace{1cm} (12)

These states are admixtures of the physical incoming ($|K^\text{in}_S\rangle$ and $|K^\text{in}_L\rangle$) and outgoing ($<K^\text{out}_S|$ and $<K^\text{out}_L|$) states of the full Hamiltonian and obey the following orthogonality conditions:

$$<K^\text{out}_L|K^\text{in}_S\rangle = 0,$$  \hspace{1cm} (13)

$$<K^\text{out}_S|K^\text{in}_L\rangle = 1.$$  \hspace{1cm} (13)

The physical states, are the left and right eigenvectors of the effective Hamiltonian of the system, $H \equiv M - i\Gamma/2$:

$$H \ |K^\text{in}_L\rangle = \lambda_L \ |K^\text{in}_L\rangle,$$
$$H \ |K^\text{in}_S\rangle = \lambda_S \ |K^\text{in}_S\rangle,$$
$$<K^\text{out}_L|H = <K^\text{out}_L|\lambda_L,$$
$$<K^\text{out}_S|H = <K^\text{out}_S|\lambda_S.$$  \hspace{1cm} (14)

Since $H$ is not Hermitean, this implies in general that the incoming and outgoing eigenvectors in the $K^0, \bar{K}^0$ base are not related simply by complex conjugation.

Without loss of generality, we can express the physical incoming states in terms of $|K^\text{in}_0\rangle$ and $|\bar{K}^\text{in}_0\rangle$ as:

$$|K^\text{in}_S\rangle = \frac{1}{N_S} \left( (1 + \alpha)|K^\text{in}_0\rangle + (1 - \alpha)|\bar{K}^\text{in}_0\rangle \right),$$
$$|K^\text{in}_L\rangle = \frac{1}{N_L} \left( (1 + \beta)|K^\text{in}_0\rangle - (1 - \beta)|\bar{K}^\text{in}_0\rangle \right).$$  \hspace{1cm} (15)

where $\alpha$ and $\beta$ are complex variables associated with $CP, T$ and $CPT$-violation, and $N_L, N_S$ are normalization factors to be discussed below. Similar relations exist for the dual outgoing states:

$$<K^\text{out}_S| = \frac{1}{N_S} \left( (1 + \bar{\alpha})<K^\text{out}_0| + (1 - \bar{\alpha})<\bar{K}^\text{out}_0| \right),$$
$$<K^\text{out}_L| = \frac{1}{N_L} \left( (1 + \bar{\beta})<K^\text{out}_0| - (1 - \bar{\beta})<\bar{K}^\text{out}_0| \right).$$  \hspace{1cm} (16)
The parameters \((\alpha, \beta)\) and \((\tilde{\alpha}, \tilde{\beta})\) that are associated with the incoming and outgoing states respectively, are not independent but are related through the orthogonality conditions (eqs.13) valid for the physical states:

\[
< K^{\text{out}}_L | K^{\text{in}}_S > = 0 \Rightarrow \tilde{\beta} = -\alpha , \\
< K^{\text{out}}_S | K^{\text{in}}_L > = 0 \Rightarrow \tilde{\alpha} = -\beta , \\
< K^{\text{out}}_S | K^{\text{in}}_S > = 1 \Rightarrow N_S \tilde{N}_S = 2(1 - \alpha \beta) , \\
< K^{\text{out}}_L | K^{\text{in}}_L > = 1 \Rightarrow N_L \tilde{N}_L = 2(1 - \alpha \beta) .
\] (17)

The above relations indicate that, while the normalizations \(\tilde{N}_{S,L}\) can be expressed in terms of \(N_{S,L}\), the latter remain unspecified. This ambiguity however will not affect any measurable quantity. Thus we can always choose

\[
N \equiv N_S = \tilde{N}_S = N_L = \tilde{N}_L = \sqrt{2(1 - \alpha \beta)} .
\] (18)

Let us write down for completeness the inverse transformations that express the \(K^0, \bar{K}^0\) states in terms of \(K_L, K_S\):

\[
|K^0_in> = \frac{1}{N} \left( (1 - \beta) |K^0_in>_S + (1 - \alpha) |K^0_in>_L \right) ,
\]

\[
|\bar{K}^0_in> = \frac{1}{N} \left( (1 + \beta) |K^0_in>_S - (1 + \alpha) |K^0_in>_L \right) ,
\] (19)

and

\[
< K^0_out| = \frac{1}{N} \left( (1 + \alpha) < K^0_out|_S + (1 + \beta) < K^0_out|_L \right) ,
\]

\[
< \bar{K}^0_out| = \frac{1}{N} \left( (1 - \alpha) < K^0_out|_S - (1 - \beta) < K^0_out|_L \right) .
\] (20)

In the basis of the states \(K_L, K_S, H\) can be expressed in terms of a diagonal \(2 \times 2\) matrix

\[
H = |K^0_in>_S > \lambda_S < K^0_out| + |K^0_in>_L < K^0_out| ,
\] (21)

while in the basis of \(K^0, \bar{K}^0, H\) takes the following form:

\[
H_{ij} = \frac{1}{2} \begin{pmatrix}
(\lambda_L + \lambda_S) - \Delta \lambda \frac{(\alpha - \beta)}{1 - \alpha \beta} & \Delta \lambda \frac{(1 + \alpha \beta)}{1 - \alpha \beta} + \Delta \lambda \frac{\alpha + \beta}{1 - \alpha \beta} \\
\Delta \lambda \frac{(1 + \alpha \beta)}{1 - \alpha \beta} - \Delta \lambda \frac{\alpha + \beta}{1 - \alpha \beta} & (\lambda_L + \lambda_S) + \Delta \lambda \frac{\alpha - \beta}{1 - \alpha \beta}
\end{pmatrix} .
\] (22)

Here,

\[
\Delta \lambda = \lambda_L - \lambda_S , \quad \lambda_L = m_L - \frac{i}{2} \Gamma_L , \quad \lambda_S = m_S - \frac{i}{2} \Gamma_S ,
\]
where $m_S, m_L$ are the $K_S, K_L$ masses and $\Gamma_S, \Gamma_L$, the $K_S, K_L$ widths. From eq.(22), we can identify the $T$, $CP$- and $CPT$- violating parameters. Indeed:

- Under $T$–transformations,

$$< K_0^\text{out} | H | K_0^\text{in} > \leftrightarrow < \bar{K}_0^\text{out} | H | K_0^\text{in} >,$$

thus, the off-diagonal elements of $H$ are interchanged. This indicates that the parameter $\epsilon \equiv (\alpha + \beta)/2$, which is related to the difference of the off-diagonal elements of $H$, measures the magnitude of the $T$-violation:

$$\frac{2}{N^2} \epsilon = \frac{< K_0^\text{out} | H | K_0^\text{in} > - < \bar{K}_0^\text{out} | H | K_0^\text{in} >}{2 \Delta \lambda}. \quad (23)$$

- Under $CPT$–transformations,

$$< K_0^\text{out} | H | K_0^\text{in} > \leftrightarrow < \bar{K}_0^\text{out} | H | \bar{K}_0^\text{in} >,$$

and therefore, the parameter $\delta \equiv (\alpha - \beta)/2$, related to the difference of the diagonal elements of $H$, measures the magnitude of $CPT$-violation.

$$\frac{2}{N^2} \delta = \frac{< \bar{K}_0^\text{out} | H | \bar{K}_0^\text{in} > - < K_0^\text{out} | H | K_0^\text{in} >}{2 \Delta \lambda}. \quad (24)$$

- Under $CP$–transformation,

$$< K_0^\text{out} | H | K_0^\text{in} > \leftrightarrow < \bar{K}_0^\text{out} | H | \bar{K}_0^\text{in} >,$$

and simultaneously

$$< K_0^\text{out} | H | \bar{K}_0^\text{in} > \leftrightarrow < \bar{K}_0^\text{out} | H | K_0^\text{in} >,$$

thus, both the diagonal and the off-diagonal elements of $H$ are interchanged. Then, the parameters $\alpha = \epsilon + \delta$ and $\beta = \epsilon - \delta$, usually denoted as $\epsilon_S$ and $\epsilon_L$, are the ones which measure the magnitude of $CP$-violation in the decays of $K_S$ and $K_L$ respectively.

### 4 Direct measurement testing time-reversibility

The meaning of classical time-reversal invariance is unambiguous. A system at a final classical configuration retraces its way back to some initial configuration by reversing the velocities. As a result of time-reversal invariance, initial and final quantum mechanical states are interchanged with identical positions and opposite velocities:

$$T \left[ < \Psi^\text{out}(t_f) | \Phi^\text{in}(t_i) > \right] = < \Phi^\text{out}(t_f) | \Psi^\text{in}(t_i) >. \quad (25)$$

\[3\] $2/N^2 \approx 1$, in the linear approximation.
In order to test time reversibility, one has to compare the magnitude of the probability $|\Psi^{\text{out}}(t_f)\rangle|\Phi^{\text{in}}(t_i)\rangle|^2$ with that of the time-reversed process $|\Phi^{\text{out}}(t_f)\rangle|\Psi^{\text{in}}(t_i)\rangle|^2$. Any possible difference in the two probabilities will signal deviations of time-reversibility. In that case, the process is not equivalent to its time reversed one, resulting in time-reversal violation. In the neutral kaon system, at a given time $t_i$ one has an initial strangeness eigenstate, such that $|K_0^{\text{in}}(t_i)\rangle = |\Psi^{\text{in}}(t_i)\rangle$. At some later time $t_f$, one finds a final strangeness eigenstate $\bar{K}_0^{\text{out}}(t_f)\rangle = <\Phi^{\text{out}}(t_f)|$. According to time-reversibility, we may conclude that the above process should have the same probability with the reversed one, namely, an initial $|\bar{K}_0^{\text{in}}(t_i)\rangle$ to be transformed into a final $<K_0^{\text{out}}(t_f)|$. Then, for the kaon system we can write for the case of time-reversal invariance:

$$
|<\bar{K}_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle|^2 = |<K_0^{\text{out}}(t_f)|\bar{K}_0^{\text{in}}(t_i)\rangle|^2.
$$

(26)

Any deviation from the above equality will definitely signal time-reversal violation. The comparison of the probabilities of a $\bar{K}_0$ transforming into $K_0$, and $K_0$ transforming into $\bar{K}_0$ can demonstrate a departure from time-reversal invariance. More explicitly, such a departure is manifest in the asymmetry

$$
A_T = \frac{P_{KK}(|\Delta t|) - P_{\bar{K}\bar{K}}(|\Delta t|)}{P_{KK}(|\Delta t|) + P_{\bar{K}\bar{K}}(|\Delta t|)},
$$

$$
= \frac{|<\bar{K}_0^{\text{out}}(t_f)|\bar{K}_0^{\text{in}}(t_i)\rangle|^2 - |<K_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle|^2}{|<K_0^{\text{out}}(t_f)|\bar{K}_0^{\text{in}}(t_i)\rangle|^2 + |<\bar{K}_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle|^2},
$$

(27)

known in the literature as the Kabir asymmetry [3].

The time evolution from $t_i$ to $t_f$ is induced by the effective Hamiltonian $H$:

$$
A_{K_0\to\bar{K}_0} = <\bar{K}_0^{\text{out}}(t_f)|K_0^{\text{in}}(t_i)\rangle = <\bar{K}_0^{\text{out}}|e^{-iH\Delta t}|K_0^{\text{in}}>,
$$

$$
A_{\bar{K}_0\to K_0} = <K_0^{\text{out}}(t_f)|\bar{K}_0^{\text{in}}(t_i)\rangle = <K_0^{\text{out}}|e^{-iH\Delta t}|\bar{K}_0^{\text{in}}>. \quad (28)
$$

Inserting the unity operator

$$
1 = |K_L^{\text{in}}><K_L^{\text{out}}| + |K_S^{\text{in}}><K_S^{\text{out}}|,
$$

(29)

to the right of the evolution operator $e^{-iH\Delta t}$ and using the fact that $K_{L,S}$ are Hamiltonian eigenstates, we obtain:

$$
A_{K_0\to\bar{K}_0} = <\bar{K}_0^{\text{out}}|K_L^{\text{in}}><K_L^{\text{out}}|K_0^{\text{in}}|e^{-i\lambda_L\Delta t} + <\bar{K}_0^{\text{out}}|K_S^{\text{in}}><K_S^{\text{out}}|K_0^{\text{in}}|e^{-i\lambda_S\Delta t}
$$

$$
= \frac{1}{N^2} (1 - \alpha)(1 - \beta) \left(e^{-i\lambda_S\Delta t} - e^{-i\lambda_L\Delta t}\right), \quad (30)
$$

where $N$ is the normalization constant, $\lambda_L$ and $\lambda_S$ are the corresponding eigenvalues.
and

\[ A_{K_0 \to K_0} = < K_0^{\text{out}} | K_L^{\text{in}} > < K_L^{\text{out}} | K_0^{\text{in}} > e^{-i\lambda_L \Delta t} + < K_0^{\text{out}} | K_S^{\text{in}} > < K_S^{\text{out}} | K_0^{\text{in}} > e^{-i\lambda_S \Delta t} = \frac{1}{N^2} (1 + \alpha)(1 + \beta) (e^{-i\lambda_S \Delta t} - e^{-i\lambda_L \Delta t}). \quad (31) \]

We see that the time-dependent factor \( g(\Delta t) \equiv (e^{-i\lambda_S \Delta t} - e^{-i\lambda_L \Delta t}) \), whose absolute value square is given by

\[ |g(\Delta t)|^2 = e^{-\Gamma_S \Delta t} + e^{-\Gamma_L \Delta t} - 2\cos(m_L - m_S) \Delta t e^{-\frac{\Gamma_S + \Gamma_L}{2} \Delta t}, \quad (32) \]

is common in both amplitudes and therefore will cancel in the asymmetry \( A_T \), which becomes time-independent\(^2\). Thus

\[ A_T = \frac{|(1 + \alpha)(1 + \beta)|^2 - |(1 - \alpha)(1 - \beta)|^2}{|(1 + \alpha)(1 + \beta)|^2 + |(1 - \alpha)(1 - \beta)|^2}. \quad (33) \]

Making the substitutions \( \alpha = \epsilon + \delta \) and \( \beta = \epsilon - \delta \), and keeping only linear terms, one finds that

\[ A_T \approx 4Re \ [\epsilon]. \quad (34) \]

We note therefore that a non-zero value for \( A_T \) signals a direct measurement of \( T \)-violation without any assumption about \( CPT \) invariance.

To make clear the misunderstandings in the literature\(^3\)–\(^11\) (with the exception of ref.\(^12\)) we need to introduce the adjoint outgoing states:

\[ < K_S^{\text{in}} | = \frac{1}{N^*} ((1 + \alpha^*) < K_0^{\text{in}} | + (1 - \alpha^*) < \bar{K}_0^{\text{in}} |), \]
\[ < K_L^{\text{in}} | = \frac{1}{N^*} ((1 + \beta^*) < K_0^{\text{in}} | - (1 - \beta^*) < \bar{K}_0^{\text{in}} |). \quad (35) \]

Notice that the adjoint states \( < K_S^{\text{in}} | \) and \( < K_L^{\text{in}} | \), are not orthogonal to \( |K_S^{\text{in}} > \) and \( |K_L^{\text{in}} > \):

\[ < K_S^{\text{in}} | K_S^{\text{in}} > = \frac{1 + |\alpha|^2}{|1 - \alpha \beta|}, \quad < K_L^{\text{in}} | K_L^{\text{in}} > = \frac{1 + |\beta|^2}{|1 - \alpha \beta|}, \]
\[ < K_S^{\text{in}} | K_L^{\text{in}} > = \frac{\alpha^* + \beta}{|1 - \alpha \beta|}, \quad < K_L^{\text{in}} | K_S^{\text{in}} > = \frac{\alpha + \beta^*}{|1 - \alpha \beta|}, \]
\[ \rightarrow < K_S^{\text{in}} | K_L^{\text{in}} > + < K_L^{\text{in}} | K_S^{\text{in}} > = \frac{2Re [(\alpha + \beta)]}{|1 - \alpha \beta|} = \frac{4Re [\epsilon]}{|1 - \alpha \beta|}. \quad (36) \]
In linear order in $\epsilon$ and $\delta$, the approximate equality

$$A_T \approx < K^i_S | K^i_L > + < K^i_L | K^i_S > \approx 4 \text{Re} \left[ \epsilon \right],$$

(37)

holds. This relation resulted in some misleading conclusion in the literature, namely that $A_T \neq 0$ is not associated with $T$-violation, but rather with the non-orthogonality of the physical incoming states $K^i_L$ and $K^i_S$ states, and with the violation of $CPT$. However, as we already stressed, (i) the relevant physical states $< K^i_L |$ and $| K^i_S >$ are always orthogonal (see eq. (13)) and (ii) $A_T$ is by definition the magnitude of $T$-violation, without any assumption about the validity of $CPT$ or even unitarity.

To better illustrate the misunderstanding, let us imagine that the $CP$-violating part $\beta$ of $K_L$ is zero. In this case $\epsilon = -\delta$, so that $T$ is violated together with $CPT$, with $CP$ invariance in the $K_L$ decays. Besides, if $CPT$ is assumed, then $\delta = 0$ and $\epsilon = \alpha = \beta$. In that case, clearly, $T$-violation is identical to $CP$-violation.

5 The CPLEAR measurement

Up to now, we described the behaviour of the theoretical asymmetry that stems directly from the definition of $T$-reversal. However, as we mentioned in the introduction, CPLEAR uses semi-leptonic decays in order to tag the strangeness of the final states and therefore the experimental asymmetry of eq. (1) is:

$$A_T^{exp} = \frac{\overline{R}_+ (\Delta t) - R_- (\Delta t)}{\overline{R}_+ (\Delta t) + R_- (\Delta t)},$$

(38)

where

$$\overline{R}_+ (\Delta t) = \left| < e^+ \pi^- \nu(t_f) | K^0_0(t_f) > < K^0_0(t_i) | \bar{K}^0_0(t_i) > \right|^2,$$

$$R_- (\Delta t) = \left| < e^- \pi^+ \bar{\nu}(t_f) | \bar{K}^0_0(t_f) > < K^0_0(t_i) | K^0_0(t_i) > \right|^2.$$  

(39)

The basic idea here is the following: There are in principle four semi-leptonic decays for neutral kaons:

$$K^0 \rightarrow e^+ \pi^- \nu, \quad \bar{K}^0 \rightarrow e^- \pi^+ \bar{\nu},$$

$$K^0 \rightarrow e^- \pi^+ \bar{\nu}, \quad \bar{K}^0 \rightarrow e^+ \pi^- \nu.$$  

(40)

Among them, the first two are characterized by $\Delta S = \Delta Q$ and are allowed, while the others are characterized by $\Delta S = -\Delta Q$ and would be forbidden if no oscillations between $K^0$ and $\bar{K}^0$ were occurring. By looking therefore at the “wrong-sign” leptons, one studies $K^0 - \bar{K}^0$ conversions.
As we see from the above expressions, the squared matrix elements
\[
|< e^+ \pi^- \nu(t_f) | K_0^{\text{in}}(t_f) > |^2 \equiv |a|^2 |1 - y|^2 ,
\]
\[
|< e^- \pi^+ \bar{\nu}(t_f) | \bar{K}_0^{\text{in}}(t_f) > |^2 \equiv |a|^2 |1 + y|^2 ,
\]
(41)
enter in the calculation and are parametrized by the quantity \(y\) \([8, 11]\), which describes \(CPT\)-violation in semileptonic decays, when the \(\Delta S = \Delta Q\) rule holds. Moreover, although the \(\Delta S = \Delta Q\) rule is expected from the Standard Model to be valid up to order \(10^{-14}\) \([8]\), the experimental limit before CPLEAR was much larger \([13]\). For this reason, two quantities (denoted by \(x\) and \(\bar{x}\) \([8, 11]\)), which are related to violation of the \(\Delta S = \Delta Q\) in the decays, have been retained in the analysis \([1]\). These parameters were found to be very small, and will not concern us further.

Even if \(y\) is included in the calculation the time-independence of the asymmetry still holds. However, \(y\) does enter in the asymmetry calculation:
\[
A_T = \frac{|(1 + \alpha)(1 + \beta)|^2 |1 - y|^2 - |(1 - \alpha)(1 - \beta)|^2 |1 + y|^2}{|(1 + \alpha)(1 + \beta)|^2 |1 - y|^2 + |(1 - \alpha)(1 - \beta)|^2 |1 + y|^2}.
\]
(42)
In particular for the linear approximation one finds that
\[
A_T^{\text{exp}} \approx 4 \text{Re } [\epsilon] - 2 \text{Re } [y].
\]
(43)
Since \(y\) has also been measured by the experiment and is found to be close to zero \([14]\), we conclude that the non-zero value of \(A_T^{\text{exp}}\) is due to \(T\)-violation.

One basic point to emphasize here, is that CPLEAR uses only one out of the possible decaying channels, and therefore its measurements are independent of any unitarity assumption and the possible existence of invisible decay modes. An interesting question to ask, however, is what information one could obtain from previous measurements plus unitarity \([15, 8, 14]\). Unitarity implies the relations
\[
< K_L^{\text{in}} | K_S^{\text{in}} > = \Sigma_f < K_L^{\text{in}} | f^{\text{in}} > < f^{\text{out}} | K_S^{\text{in}} > ,
\]
\[
< K_S^{\text{in}} | K_L^{\text{in}} > = \Sigma_f < K_S^{\text{in}} | f^{\text{in}} > < f^{\text{out}} | K_L^{\text{in}} > ,
\]
(44)
where \(f\) stands for all possible decay channels. Making the additional assumption that the final decay modes satisfy the relation \(| f^{\text{in}} > = | f^{\text{out}} > \equiv | f^{\text{out}} > |\) (which is equivalent to making use of \(CPT\)-invariance of the final state interactions), it is possible to calculate the sum \(< K_L^{\text{in}} | K_S^{\text{in}} > + < K_S^{\text{in}} | K_L^{\text{in}} >\), by measuring only the branching ratios of kaon decays. This is what is done in \(K_L, K_S\) experiments, where only the incoming kaon states are used. In the linear approximation, this sum is equal to \(4 \text{ Re } [\epsilon]\) (see eq. (37)). However, this is an indirect determination of \(T\)-violation, and would not have been possible if invisible decays or \(CPT\)-violation in the final states interactions were present \([17]-[20]\). This is to be contrasted with the results of CPLEAR, which do not rely at all on unitarity and thus on the knowledge of other decay channels than the one used in the analysis.
6 Concluding comments

Motivated by the recent CPLEAR report on the first direct observation of time-reversal non-invariance, we attempted to clarify the situation on measurements of charge conjugation, parity violation and time reversibility, in systems with non-Hermitean Hamiltonians. To do so, we re-discussed the formalism of the neutral kaon system, paying particular attention in the definition of states in the vector space of the system, but also in its dual and in the dual complex spaces. This allows a consistent implementation of the orthogonality conditions for the incoming and outgoing states, used to describe particle-antiparticle mixing and the time evolution of the system.

As a result, we confirm that the asymmetry measured by CPLEAR, is directly related to the definition of $T$-violation. In addition, it does not get affected by time and decay processes. Finally, the experiment uses only one out of the possible decaying channels, therefore its results are independent of any $CPT$ or unitarity assumption, and the possible existence of invisible decay modes. We conclude therefore that, CPLEAR indeed made the first direct measurement of $T$-violation.

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