Spin-orbit scattering effect on critical current in SFIFS tunnel structures

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Abstract

The spin-orbit scattering effect on critical current through superconductor/ferromagnet (SF) bilayers separated by an insulator (SFIFS tunnel junction) have been investigated for the case of absence of the superconducting order parameter oscillations (thin $F$ layers). The analysis is based on a microscopic theory for proximity coupled $SF$ bilayer for different bilayer parameters (boundary transparency, proximity effect strength, relative orientation and value of the $F$ layers exchange field). We find that the spin-orbit scattering considerably modifies dc Josephson current in SFIFS tunnel junction. In contrast to a simple physical picture, the reduction of the exchange field effects is nonlinear in character getting its maximum in the field’s region where the critical current enhancement or the transition to the $\pi$-state take place. Hence, for understanding the various experimental results on tunnel structures with thin $F$ layers, the coupled effects of the exchange interaction and spin-orbit scattering must be considered.

I. INTRODUCTION

Nowadays progress in nanotechnology made it possible to produce nanostructures with new physical phenomena. This has led to renewed attention to hybrid systems consisting of superconductor ($S$) and ferromagnetic ($F$) metals, and displaying rich and elegant physics and having potential applications. Transport properties of $SF$ structures with artificial geometry have turned out to be quite unusual. These have been treated by several authors [1-9], and the obtained results show that in ferromagnet the Cooper pair potential not only exponentially decays but also has an oscillatory character; i.e., in a ferromagnet the density of Cooper pairs is spatially inhomogeneous and the superconducting order parameter contains nodes where the phase changes by $\pi$. This causes the exchange field dependence of the Josephson coupling energy, and if the exchange energy $H_{\text{exc}}$ in magnetic layer exceeds a certain value, a crossover from 0-phase to $\pi$-phase superconductivity takes place. The phenomenon has been theoretically described for $SFS$ weak links with thick $F$ layer [1-5]. The crossover to the $\pi$-state even in the absence of the order parameter oscillations in thin $F$ layer was also predicted for SFIFS tunnel junctions (where $I$ is an insulator) with parallel alignment of layer’s magnetization [6]. For an antiparallel orientation, the possibility of the critical current enhancement by the exchange interaction in SFIFS junctions with thin $F$ layers was discussed both for small [7] and bulk [8] $S$ layer thicknesses. Experimentally, the $\pi$-phase superconductivity has been observed for $SFS$ weak links [9] and SIFS [10] and SNFNS [11] tunnel junctions with thick $F$ layers, while the enhancement of the dc Josephson current has not been detected until now.

Some features should be taken into account for theory to be adjustable for experimental results on $SF$ structures, and the spin-flip processes are among the important ones. These processes can be induced, e.g., by the spin-orbit scattering centers presented in the film; other important source of the spin-flip processes for nanoscale hybrid structures is a strong electric field arising near metal-metal boundaries [12]. The basic physics behind spatial
oscillations of induced superconductivity in $SF$ sandwiches with spin-orbit scattering has been recently discussed in [3,13]. As is known, in the presence of spin-orbit scattering the electron spin is no longer a good quantum number and the electron will change its spin state during a characteristic time $\tau_{SO}$, while the Cooper pair will mix with its spin-exchanged counterpartner. This causes a pair to ”feel” an exchange field which changes sign at a rate proportional to $1/\tau_{SO}$, decreasing the average ferromagnetic field experienced by the pair. That means, that the spin-flip processes not only modify the oscillation length but also lead to an extra decay of the Cooper pair potential and, at a critical strength, these scattering processes can completely suppress the $\pi$-phase superconductivity.

The scenario of the $0 - \pi$-phase transition, as well as the mechanisms of critical current enhancement, in the limit of thin $F$ layers differ from those for thick $F$ layers. However, the basic physics behind spin-orbit scattering effect for the $SF$ sandwiches where there is no induced order parameter oscillations, has not been discussed till now. The purpose of this paper is to clarify the mechanisms of influence of spin-orbit scattering on critical current in $SFIFS$ tunnel structures with thin $F$ layers. Our analysis is based on the extension of calculations [6,8,14], so as to include the effects of the spin-orbit scattering. Namely, we study the tunnel junction with leads formed by the proximity coupled $SF$ bilayers of a massive $s$-wave superconductor and a thin $F$ metals, when the spin-orbit scattering processes take place in ferromagnetic layers. The microscopic model of the proximity effect for $SF$ bilayer is employed to discuss the case of arbitrary value of $SF$ boundary transparency, ferromagnetic exchange field, and proximity effect strength (Sec.II). Critical current of symmetrical $SFIFS$ junction is discussed in Sec.III where analytical solutions have been obtained for some particular cases. As we shall see, spin-flip scattering plays a major role in transport properties of superconductor-ferromagnet structures with thin $F$ layers and should be considered for understanding the various experimental results. We close with a Conclusion.

II. SUPERCONDUCTIVITY OF SF BILAYER WITH SPIN-ORBIT SCATTERING

We consider the case when both metals are in the dirty limit condition $\xi_{S,F} \geq l_{S,F}$, and the $S$ layer is thick, $d_S \gg \xi_S$, while the $F$ layer is thin, $d_F \ll \min(\xi_F, \sqrt{D_F/2\pi T_C})$. Here $\xi_S = (D_S/2\pi T_C)^{1/2}$ and $\xi_F = (D_F/2H_{exc})^{1/2}$ are the coherence lengths of the $S$ and $F$ metals; $l_S$ and $l_F$ are the electron mean free paths, $d_{S,F}$ are the thicknesses, and $D_{S,F}$ are the diffusion coefficients of the $S$ and $F$ metals, respectively. These conditions make it possible to neglect the reduction of the critical temperature of the $SF$ bilayer compared to that of the bulk $S$ metal, and to imply constant superconducting properties through $F$ layer thickness. Throughout this work the $F$ layer will be treated as a single domain film with the spin-orbit scattering centers, while there are no such centers in the $S$ layer. Then, the superconducting properties of the $SF$ bilayer are described by the Usadel equations [15]. The latter can be written as (the domain $x \geq 0$ is occupied by the $S$ metal and $x < 0$ by the $F$ metal):

$$\Phi_{S\sigma} = \Delta_S + \xi_S^2 \frac{\pi T_C}{2G_{S\sigma}} [G_{S\sigma}^2 \Phi_{S\sigma}']' ,$$

(1)
\[ \Phi_{F\pm} = \xi^2 \frac{\pi T_C}{\omega_\pm G_{F\pm}^2}[G_{F\pm}^2 \Phi_{F\pm}']' + \alpha_{SO} G_{F\pm}(\frac{\Phi_{F\pm}}{\omega_\pm} - \frac{\Phi_{F\pm}}{\omega_\pm}) \]  

(2)

Here \( \tilde{\omega}_\sigma = \omega + i \sigma H_{exc} \), \( \sigma = \pm \), and \( \omega \equiv \omega_n = \pi T(2n + 1), \) \( n = \pm 1, \pm 2, ... \) is Matsubara frequency; \( \alpha_{SO} = 2/3 \tau_{SO} \) and \( \tau_{SO} \) is the spin-orbit scattering time; the pair potential \( \Delta_S \) is determined by the usual self-consistency relations (see, e.g., [16]). We define the \( x \) axis as perpendicular to the film surfaces and the prime denotes differentiation with respect to a coordinate \( x \). The modified Usadel functions [17] \( \Phi_{S\sigma} = \omega F_{S\sigma}/G_{S\sigma}, \Phi_{F\sigma} = \tilde{\omega}_\sigma F_{F\sigma}/G_{F\sigma} \), where \( G_{F,S} \) and \( F_{F,S} \) are normal and anomalous Green’s functions for the \( F \) and \( S \) material, respectively, are introduced to take into account the normalized confinement of the Green’s functions \( G^2 + F F = 1 \); here \( \tilde{F}(\omega, H_{exc}) = F^*(\omega, -H_{exc}) \), and \( \Phi_{S\sigma} = \omega F_{S\sigma}/G_{S\sigma} \) (see below). We also assume that for a nonsuperconducting \( F \) metal the bare value of the order parameter \( \Delta_F = 0 \), however, the Cooper pair correlation function \( F_F \neq 0 \) due to the proximity effect. Eqs. (2) for the \( F \) metal are the generalization of the Usadel equations for the case when the spin-orbit scattering is present (for more details see Ref. [18]). At temperatures close to \( T_C \), when \( G_{S(F)\sigma} \approx 1 \) and \( \Phi_{S(F)\sigma}/\tilde{\omega}_\sigma \approx F_{S(F)\sigma} \), Eqs. (2) have been simplified getting the form of Eqs. (26) of Ref. [3]. If the spin-orbit processes are absent \( (\tau_{SO} \to \infty) \), the spin ”up” and ”down” subbands do not mix with each other and the equation (2) obtains the usual form (see, e.g., [6,14]).

Eqs. (1), (2) should be supplemented with the boundary conditions. In the bulk of the \( S \) metal the pair potential is equal to the BCS order parameter \( \Delta_0(T) \) at the temperature \( T : \Phi_S(\infty) = \Delta_S(\infty) = \Delta_0(T) \), while at the free (dielectric) interface of the \( F \) metal: \( \Phi_{F\pm}^\prime (d_F) = 0 \). Assuming that there are no spin-flip processes at the \( SF \) interface, the boundary conditions at this interface (see Ref. [14] for details) can easily be generalized for the case of two fermionic subband in the form:

\[ \gamma \xi G_{F\pm}^2 \Phi_{F\pm}^\prime /\tilde{\omega}_\pm = \xi_S G_{S\pm}^2 \Phi_{S\pm}^\prime /\omega, \]  

(3)

\[ \gamma_{BF} \xi G_{F\pm} \Phi_{F\pm} /\tilde{\omega}_\pm = \tilde{\omega}_\pm G_{S\pm}(\Phi_{S\pm} /\omega - \Phi_{F\pm} /\tilde{\omega}_\pm) \]  

(4)

The parameters \( \gamma \) and \( \gamma_{BF} \) involved in these relations are given by \( \gamma = \rho_S \xi_S / \rho_F \xi \), \( \gamma_{BF} = R_B / \rho_F \xi \), where \( \rho_S,F \) are the normal state resistances of the \( S \) and \( F \) metals, \( R_B \) is the product of \( SF \) boundary resistance and its area. In Eqs. (1) and (3), and below we have used the effective coherence length \( \xi = (D_F/2\pi T_C)^{1/2} \) thus providing the regular crossover to both limits \( T_C \gg H_{exc} \to 0 \) and \( H_{exc} \gg T_C \). The relation (3) provides the continuity of the supercurrent flowing through the \( SF \) interface at any value of the interface transparency for spin ”up” and ”down” fermionic subband separately, while the condition (4) accounts for the quality of the electric contact. An additional physical approximation for the relations (3) and (4) to be valid is the assumption that spin discrimination by the interface is unimportant, i.e., the interface parameters involved are the same for both spin subband. Generalization of Eqs. (3) and (4) to the \( SF \) interface with different transmission probabilities for ”up” and ”down” spin quasiparticles is straightforward, however the case we consider contains all the new physics we are interested in and is simpler. In such ferromagnetic metals as \( Ni, Gd \), etc. the polarization of the electrons at low temperatures is not more than 10%, and one can expect the model under consideration reflects the transport properties of these ferromagnets hybrid structures.
Due to a small thickness of the $F$ metal the proximity effect problem can be reduced to the boundary problem for the $S$ layer and a relation for determining $\Phi_{F\pm}$ at $x = 0$. There are three parameters which enter the model: $\gamma_M = \gamma d_F/\xi$, $\gamma_B = \gamma_B d_F/\xi$ and the energy of the exchange field $H_{exc}$. Using the system of equations (3) and (4), one can obtain the equations determining the unknown value of the functions $\Phi_{F\pm}(0)$ and boundary conditions for $\Phi_{S\pm}$. Due to these boundary conditions all the equations for the functions $\Phi_{S\pm}$ and $\Phi_{F\pm}$ are coupled. In the general case, the problem needs self-consistent numerical calculations (similar to those as, e.g., in Ref. [19]). Here, however, we will not discuss the quantitative solution, but pay attention to the qualitative one to consider new physics we are interested in.

We will hereinafter assume that $\alpha_{SO} \ll 1$ and $d_F/\xi \ll 1$ and solve differential equations by an iteration procedure finding the corrections to $\Phi_{F\pm}(x)$ and $\Phi_{S\pm}(x)$ in small parameters $d_F/\xi$ and $\alpha_{SO}$. We also restrict ourselves to the quite realistic experimental case of a bilayer with a weak proximity effect and low transparency of the $SF$ boundary, i.e., $\gamma_M \ll 1$ and $\gamma_B \gtrsim 1$. Then the problem is simplified (see Ref. [18] for details) and reduced to the Usadel Eqs. (1) for the $S$ layer with boundary conditions (3), while Eq. (4) reduces to

$$\xi_S G_{S\pm}(0) \approx \bar{\gamma}_M \bar{\omega}_\pm \frac{\Phi_{S\pm}}{\pi T C_{\pm}} \left\{ 1 \mp 2 \alpha_{SO} \frac{i H_{exc}}{\bar{\omega}_+ \bar{\omega}_-} \left( \frac{\omega}{\gamma_{B} \bar{\omega}_\pm} \frac{G_S}{\omega} \bar{A}_\pm^2 \right) \right\} |_{x=0},$$

where we abbreviated $A_\pm = [1 + 2\bar{\gamma}_B G_S \bar{\omega}_\pm + \bar{\gamma}_B^2 \bar{\omega}_\pm^2]^{1/2}$, $G_S = \omega/(\omega^2 + \Delta_0^2)^{1/2}$ and $\bar{\gamma}_B(M) \equiv \gamma_{B(M)}/\pi T C$ (functions that are multiplied by $\alpha_{SO}$ have been used in the limit $\alpha_{SO} \to 0$).

Now, the equation for $F$ layer function $\Phi_{F\pm}(0)$ has the form:

$$\Phi_{F\pm}(0) \approx G_{S\pm}(0) \approx G_S \Phi_{S\pm}(0) \left\{ 1 \mp 2 \alpha_{SO} \frac{i H_{exc}}{\bar{\omega}_+ \bar{\omega}_-} \left( \frac{G_S}{\omega} \bar{A}_\pm^2 \right) \right\} |_{x=0},$$

In zeroth approximation in $\gamma_M$ the $\Phi_{S\pm}'(0) = 0$. So, we can neglect the suppression of superconductivity in the $S$ layer assuming that $\Phi_{S\pm}(x)$ is spatially homogeneous: $\Phi_{S\pm}(x) = \Delta_S(x) = \Delta_0(T)$. In the next order in $\gamma_M$, by linearizing the Usadel equations for the $\Phi_{S\pm}(x)$ and making use of the relation (3), the general solution of the linearized equation (1) is given by

$$\Phi_{S\pm}(x) = \Delta_0 \left\{ 1 - C_\pm \exp(-\beta x/\xi_S) \right\},$$

where $\beta^2 = (\omega^2 + \Delta_0^2)^{1/2}/\pi T C$. Substituting this solutions into the boundary relations (5), we get for $C_\pm$:

$$C_\pm \approx \frac{\bar{\gamma}_M \beta \bar{\omega}_\pm}{\gamma_{B} \beta \bar{\omega}_\pm + \omega A_\pm} \left\{ 1 \mp 2 \alpha_{SO} \frac{i H_{exc}}{\bar{\omega}_+ \bar{\omega}_-} \left( \frac{G_S}{\gamma_{B} \bar{\omega}_\pm} \frac{G_S^2 \Delta_0^2}{\omega^2 A_\pm^2} \right) \right\}$$

Using Eqs. (7) for $x = 0$ and relations (6), one can find expressions for $\Phi_{F\pm}(\omega, 0)$. However we will not show here these expressions because of their cumbersome structure. For $H_{exc} \to 0$ the quantity $\bar{\omega}_\pm \to \omega$, and solution (7), (8) reproduces the results obtained in Ref. [16] for an $SN$ bilayer. If $\tau_{SO} \to \infty$, the expressions restore earlier results for $SF$ bilayer (see Refs. [6,14]).

Let us give some comments on the results obtained. For the structure under consideration, one can expect a kind of induced magnetic properties for $S$ layer, that are the result of
the effect similar to the superconducting proximity effect [20]. In accordance with Eqs. (7), (8), we see that due to proximity induced magnetic correlations even in the $S$ layer fermionic symmetry of subbands has been lost and Cooper pair mixes with its spin-exchanged counterpartner.

III. CRITICAL CURRENT OF SFIFS TUNNEL JUNCTION

We assume that both banks of the Josephson SFIFS tunnel junction are formed by equivalent $SF$ bilayers, and the transparency of the insulating layer is small enough to neglect the effect of the tunnel current on the superconducting state of the electrons. The plane $SF$ boundary can have arbitrary finite transparency, but it is large compared to the transparency of the junction barrier. The transverse dimensions of the junction are supposed to be much less then the Josephson penetration depth, so all quantities depend only on a single co-ordinate $x$ normal to the interface surface of the materials. Using the above obtained results we investigate the influence of the spin-flip scattering on critical Josephson current in SFIFS tunnel junction. The critical current of the ($SF)_L I (FS)_R$ tunnel contact can be written in the form (see, e.g. Ref. [6]):

$$j_C = \left(\frac{eR_N}{2\pi T_C}\right)I_C = \frac{T}{T_C} \text{Re} \sum_{\omega > 0, \sigma = \pm} \frac{G_{F\sigma} \Phi_{F\sigma}}{\omega_{\sigma}} |L G_{F\sigma} \Phi_{F\sigma} |_R$$

where $R_N$ is the resistance of the contact in the normal state; the subscript $L$ ($R$) labels quantities referring to the left (right) bank and the sign of the exchange field depends on mutual orientation of the bank magnetizations.

A. Parallel orientation of the layer’s magnetizations

For parallel alignment of the layer’s magnetizations, i.e., with $\bar{\omega}_L = \bar{\omega}_R$, the expression for critical current reads:

$$j_C^{FM} = \frac{T}{T_C} \text{Re} \sum_{\omega > 0, \sigma = \pm} \frac{G_{S\sigma}^2 \Phi_{S\sigma}^2}{\omega^2} \{1 + 4\sigma \alpha_{SO} \gamma_B \frac{iH_{exc}}{\omega + \bar{\omega}_-}\} \times$$

$$\{1 + 2\tilde{\gamma}_BG_{S\sigma} \bar{\omega}_\sigma + \bar{\gamma}_B^2 \bar{\omega}_\sigma^2 + \sigma 4\alpha_{SO} \bar{\gamma}_B \frac{iH_{exc}}{\omega + \bar{\omega}_-} G_{S\sigma}^2 \Phi_{S\sigma} \bar{\Phi}_{S\sigma} \omega^2 \}^{-1}$$

We begin with an analytical consideration for the case of a vanishing effective pair breaking parameter near the $SF$ boundary, $\gamma_M = 0$; i.e., the influence of the $F$ layer on the superconducting properties of the $S$ metal can be neglected and the order parameter in the $S$ region is spatially homogeneous: $\Phi_{S\pm}(x) = \Delta_0(T)$ (see, Eqs. (7) and (8)). For the amplitude of the Josephson current we then obtained

$$j_C^{FM} \approx 2 \frac{T}{T_C} \sum_{\omega > 0} \frac{\Delta_0^2}{\Delta_0^2 + \omega^2} \{1 + 2\tilde{\gamma}_B \omega G_{S\sigma} + \bar{\gamma}_B^2 (\omega^2 - H_{exc}^2)\} \times$$

$$\{1 + 2\tilde{\gamma}_B \omega G_{S\sigma} + \bar{\gamma}_B^2 (\omega^2 - H_{exc}^2)\}^2 + 4H_{exc}^2 \bar{\gamma}_B (G_{S\sigma} + \bar{\gamma}_B \omega) \}^{-1}$$
If $H_{\text{exc}} \to 0$ the expression (11) restores the result for $\text{SNINS}$ junction (see, e.g., Eq.(28a) of Ref.[17] for parameters value under consideration). As is seen from this expression, for large enough $H_{\text{exc}}$, the supercurrent changes its sign, i.e., with increasing magnetic energy the junction crosses over from 0-phase type to $\pi$-phase type superconductivity. However, the spin-flip processes exert influence upon junction’s tendency to set in the $\pi$-phase state.

On Fig. 1 we plot a family of the Josephson current amplitude (11) as function of $H_{\text{exc}}$ when $\gamma_B = 2$, $\gamma_M = 0$ for various values of the spin-orbit scattering intensity $\alpha_{SO}$. Here we also show the function $\delta j_{\text{SF}}^F = j_{\text{SF}}^F(\alpha_{SO}) - j_{\text{SF}}^F(\alpha_{SO} = 0)$. The main feature here is that the increase of the exchange energy pulls the $\text{SFIFS}$ junction to $\pi$-state. The new result of this figure is that, as intensity of spin-orbit scattering processes increases, the critical current amplitude, $j_{\text{SF}}^F(\alpha_{SO})$, for the $\pi$-state decreases. The spin-orbit scattering reduces the effect of exchange field, and this reduction has nonlinear character with its maximum in the region where transition to the $\pi$-state take place.

A more realistic case is shown on Figs. 2, where we plot the results of numerical calculations of the critical current, Eq. (10), for the tunnel junction when suppression of the $S$ layer superconductivity occurs due to proximity effect ($\gamma_M \neq 0$). The function $\delta j_{\text{SF}}^F$ has also been shown. Again, the main result of these calculations is that the spin-flip processes can sizably reduce the $\text{SFIFS}$ junction tendency to $\pi$-phase state. In contrast to a simple physical picture, the suppression has nonlinear behavior getting its maximum near $H_{\text{exc}}$ region of cross over from 0-type to $\pi$-type superconductivity. While our results have been obtained for the $SF$ bilayers with a weak proximity effect and low boundary transparency, qualitatively the conclusions are valid for arbitrary $\gamma_B$ and $\gamma_M$ values.

**B. Antiparallel orientation of the layer’s magnetizations**

To be definite, we took $\tilde{\omega}_L = \omega + iH_{\text{exc}}, \tilde{\omega}_R = \omega - iH_{\text{exc}}$. Now the critical current of the $(SF)_L I(\text{FS})_R$ tunnel contact can be given in the form

$$j_{\text{SF}}^F = 2\frac{T}{T_C} \text{Re} \sum_{\omega > 0} \frac{G_S + \Phi_S + G_S - \Phi_S}{\omega^2} \left\{ \left( G_{S+} + \bar{\gamma}_B \tilde{\omega} \right)^2 + \frac{G_{S+}^2 + \Phi_S^2}{\omega^2} \left[ 1 + 4\alpha_{SO} \frac{iH_{\text{exc}}}{\tilde{\omega} \tilde{\omega}} \right] \right\}^{-1} \times \left\{ \left( G_{S-} + \bar{\gamma}_B \tilde{\omega} \right)^2 + \frac{G_{S-}^2 + \Phi_S^2}{\omega^2} \left[ 1 - 4\alpha_{SO} \frac{iH_{\text{exc}}}{\tilde{\omega} \tilde{\omega}} \right] \right\}^{-1}$$

(12)

Fist we assume that one can neglect the suppression of the superconductivity in the $S$ layer ($\gamma_M = 0$). Then, after simple but cumbersome algebra we have for the critical current

$$j_{\text{SF}}^F \approx 2\frac{T}{T_C} \sum_{\omega > 0} \frac{\Delta_0^2}{\Delta_0^2 + \omega_0^2} \left[ 1 - \frac{4\alpha_{SO}}{\Delta_0^2} \frac{\Delta_0^2}{\Delta_0^2 + \omega^2} \left( \frac{\omega H_{\text{exc}}^2}{\tilde{\omega} \tilde{\omega}} \right) \right] \left[ \left( 1 + 2\bar{\gamma}_B \omega G_S + \bar{\gamma}_B^2 \left( \omega^2 - H_{\text{exc}}^2 \right) \right)^2 + 4H_{\text{exc}}^2 \bar{\gamma}_B^2 G_S^2 \left( \omega^2 + H_{\text{exc}}^2 \right) \right]^{-1/2}$$

(13)

For $H_{\text{exc}} \to 0$ expression (13) restores the result for $\text{SNINS}$ junction (Eq. (28a) [17]). One can see, that for $\omega < H_{\text{exc}}$ the expression in figured brackets is low than one. As a result, the 0-phase state across the contact has been preserved, and in some interval of the exchange field energy the enhancement of dc Josephson current occurs. The spin-orbit processes exert upon this behavior, reducing the current enhancement.
On Fig. 3 we show a family of the Josephson current amplitude (13) as function of $H_{\text{exc}}$ when $\gamma_B = 2$, $\gamma_M = 0$ for various values of the spin-orbit scattering intensity $\alpha_{SO}$. Here we also plot the function $\delta j^\text{AF} = j^\text{AF}(\alpha_{SO}) - j^\text{AF}(\alpha_{SO} = 0)$. The main and unusual feature here is that the increase of the exchange energy enhances the SFIFS junction critical current. The new result of this figure is that, as the intensity of spin-orbit scattering processes increases, the tendency of the $j^\text{AF}(\alpha_{SO})$ to enhance decreases. The spin-orbit scattering reduces the effect of exchange field, and this reduction is nonlinear in character with its maximum in the most interesting for experimentalists region. More general cases are shown on Fig. 4, where we illustrate the results of numerical calculations of the critical current, Eq. (12), taking into account a suppression of the $S$ layer superconductivity by the proximity effect ($\gamma_M \neq 0$). The function $\delta j^\text{AF}$ has also been shown. One can see again, that the spin-orbit suppression has nonlinear behavior getting its maximum in the most interesting for experimental investigation region of $H_{\text{exc}}$. Qualitatively, these results are valid for arbitrary values of $\gamma_B$ and $\gamma_M$.

IV. CONCLUSION

We do not consider here the experimental situation, which is now unclear an controversial even for the SF hybrid structures with thick ferromagnetic layers. Let us only note, that spin-orbit scattering is relevant for ferromagnetic conductors containing large $Z$ number. Magnetic inhomogeneity of the materials, such as the $F$ layer multidomain structure, domain walls, inhomogeneous "cryptoferromagnetic" state imposed by superconductor also give nonzero probability amplitude for the spin-flip scattering. For nanoscale hybrid structures strong electric field arising near metal-metal boundaries is also an important source of the spin-flip processes. In the absence of any precise information about the magnetic structure of the samples used in experiments on SF sandwiches, we restrict ourselves to making only qualitative calculation.

In conclusion, we investigate the spin-orbit scattering effect on critical current of SFIFS tunnel junction for the case of thin $F$ layers, when the superconducting order parameter oscillation is absent. Instead, the parameter phase jumps at the SF interfaces. The analysis is based on a microscopic theory for proximity coupled $S$ and $F$ layers. The main result of our calculations is that the spin-flip processes can sizably modify the behavior of the $dc$ Josephson current versus the exchange field for SFIFS tunnel junction. We find that the reduction of the exchange field effects is nonlinear in character getting its maximum in the ferromagnetic field’s region where the critical current enhancement or the transition to the $\pi$-state occur. Hence, for understanding the various experimental results on tunnel structures with thin $F$ layers, the coupled effects of the exchange interaction and spin-orbit scattering must be considered.

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Figure Captures

Fig. 1. Critical current of SFIFS tunnel junction with parallel orientation of the F layers magnetization $j_C^{FM}$ versus exchange energy for various $\alpha_{SO}/\Delta_0 = 0, 0.05, 0.1$ and 0.15; the SF interface transparency is low $\gamma_B = 2$ and proximity effect is weak $\gamma_M = 0$. The additive part of critical current due to spin-orbit scattering, $\delta j_C^{FM}$, is also shown. $T = 0.1T_C$.

Fig. 2. Critical current of SFIFS tunnel junction with parallel orientation of the F layers magnetization $j_C^{FM}$ versus exchange energy for various $\gamma_M = 0, 0.05, 0.1, 0.15; \gamma_B = 2$ and $\alpha_{SO}/\Delta_0 = 0.1$. The additive part of critical current due to spin-orbit scattering, $\delta j_C^{FM}$, is also shown. $T = 0.1T_C$.
Fig. 3. Same as on Fig.1 but for antiparallel orientation of the $F$ layers magnetization. The additive part of critical current due to spin-orbit scattering, $\delta j_C^{AF}$, is also shown.

Fig. 4. Same as on Fig.2 but for parallel orientation of the $F$ layers magnetization. The additive part of critical current due to spin-orbit scattering, $\delta j_C^{FM}$, is also shown.
\[ \gamma_B = 2 \]
\[ \gamma_M = 0 \]

\[ \frac{\alpha_{SO}}{\Delta_0} = 0.15 \]

\[ \begin{align*}
\alpha_{SO}/\Delta_0 & = 0.15 \\
& = 0.05 \\
& = 0.0 \\
\end{align*} \]
Fig. 2
\[ \gamma_B = 2 \]
\[ \gamma_M = 0 \]
\[ \alpha_{SO}/\Delta_0 = 0.0 \]
\[ 0.05 \]
\[ 0.1 \]
\[ 0.15 \]
\[ j_{C}^{AF} \]

\[ \gamma_{B} = 2 \]
\[ \alpha_{SO}/\Delta_0 = 0.1 \]

\[ \gamma_{M} = 0.0 \]
\[ 0.05 \]
\[ 0.1 \]
\[ 0.15 \]

\[ \delta j_{C}^{AF} \]

\[ H_{\text{exc}}/\Delta_0 \]

Fig. 4