CONTROL OF NONLINEAR DYNAMICS OF ELECTROMECHANICAL SYSTEMS

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This article proposes to analyze the processes in the most widely used at present frequency-controlled AC drives, as in automatic control systems with dynamic non-linearities, and structural correction methods, improving their dynamics. For the first time, dynamic formulas of transfer functions of a torque driver in an asynchronous motor with frequency control, taking into account the slip and frequency of the stator voltage, are proposed. Methods for constructing families of frequency characteristics of such electromechanical systems with “frozen” but different values of the frequency of the stator voltage and slip is described. In the Simulink application of the MATLAB software environment, families of frequency characteristics were constructed corresponding to nonlinear transfer functions. The nonlinear transfer functions obtained in this work made it possible to substantiate the structural solutions of linearizing frequency-controlled electric drives and, significantly, increasing their efficiency. Such a solution turned out to be a positive feedback on the current value of the stator current with a dynamic link. This dynamic link ensures the effective action of a positive connection without disturbing the stability of electromechanical systems. The experiments fully confirmed the correctness of the mathematical expressions obtained for nonlinear links of systems and their correction. This paper is an example of how the initial complicated (but more accurate!) Interpretation of nonlinearity allowed us to find a new best solution to the problem of controlling a complex dynamic object.

Keywords: AC drive, mathematical analysis, dynamic nonlinearity, frequency response, dynamic correction, positive feedback.

Introduction
The widespread use of frequency converters for controlling asynchronous motors in recent years has created the impression that there are no problems in the field of automated electric drive (AED). However, attempts to study in depth the dynamic characteristics of such electric drives make it necessary to return to the study of the problems of control of nonlinear systems. Frequency controlled AEDs are a highly non-linear system. The “main” parameter determining the non-linearity of these systems is the variable frequency of the supply voltage. Unlike the stationary non-linearities of the regulatory systems considered in the 80s and 90s of the 20th century, the variable frequency in the AED changes its frequency response.

Variable frequency, strictly speaking, does not allow the use of a mathematical apparatus designed for AC drives, based on vector analysis, since the vector representation of variables over time implies the constancy of the frequency of the supply voltage, or the frequency of rotation of these vectors. However, due to the absence of another, vector methods are used in most research or educational works on AC drives, despite the fact that the authors quite often recognize the illegality of such an approach.

Problem statement
The most effective engineering method for assessing the dynamics of electric drives is the method of frequency analysis. Direct application of this method to asynchronous electric drives is hampered by the presence of significant non-linearities in them. The construction of the frequency characteristics of
such systems involves a number of inevitable assumptions. After considering the different versions of these assumptions, the calculation of the dynamic mechanical characteristic set forth in the Usoltsev’s monograph [1] turned out to be the most acceptable. The calculation a repelled by equation 1.36 on p. 23 [1].

It establishes a connection between the current moment \( m \) and slip \( \beta \) at the nominal frequency \( \omega_{1\text{nom}} \):

\[
m = \frac{2M_k}{(1+T_2'p)[\frac{2M_k}{\beta}(1+T_2'p) + \frac{P}{S_k}]},
\]

(1)

where \( T_2' = \frac{L_s}{R_s} \) — the transient time constant of the rotor, \( \beta = \frac{\omega_2}{\omega_1} \) — the relative slip, \( M_k \) — the critical moment, \( S_k \) — the critical slip at the nominal frequency \( \omega_{1\text{nom}} \).

At the beginning of the working characteristic (for \( M \approx 0, \beta \geq 0 \)), the transfer function is simplified and reduces to a dynamic link of the 1st order:

\[
m = \frac{2M_k}{(1+T_2'p)[\frac{2M_k}{\beta} + \frac{P}{S_k}]} = \frac{2M_k(\omega_1 - \omega_1)}{(1+T_2'p)S_k}\omega_1
\]

(2)

At the same time, the transfer function linking the absolute slip and the torque developed by the motor will look as follows:

\[
W_D(p) = \frac{m}{\Delta \omega} = \frac{2M_k}{(1+T_2'p)S_k}\omega_1.
\]

However, the results of experiments given in [2–6, 12, 14] showed that it is incorrect to extend this formula to all operating modes.

**Solution**

Equation (1) allowed us to propose another variant of linearization, in which the initial equation takes the form:

\[
m \left( T_2'p^2 + 2T_2'p + 1 + \left( \frac{\beta}{S_k} \right)^2 \right) = \frac{2M_k}{S_k} \beta (1 + T_2'p).
\]

(3)

Then, the equation connecting moment \( m \), relative slip \( \beta \) and engine parameters \((T_2' – \text{transition time constant}; M_k, S_k – \text{critical moment and critical slip, depending on the frequency } \omega_1)\) takes the form:

\[
m = \frac{2M_k(T_2'p+1)S_k\beta}{(1+T_2'p)^2S_k^2+\beta^2}.
\]

(4)

and the transfer function linking the absolute slip and moment will take the form:

\[
W(p) = \frac{2M_k(T_2'p+1)S_k}{\omega_1[(1+T_2'p)^2S_k^2+\beta^2]}.
\]

(5)

where \( \omega_1 \) — the frequency of the stator voltage.

The block diagram of the drive in the working area will take the form shown in Fig. 1.

![Fig. 1. Structural diagram of an asynchronous motor in the working area](image)

The transfer function of the torque driver changes as the stator voltage and slip frequency varies, i.e. is essentially nonlinear.

It should be noted that at \( \beta = 0 \), the transfer function, as well as the structural diagram, exactly coincide with the linear transfer function and structural diagram for the asynchronous drive, given in the Usoltsev’s monograph [1]. In the proposed non-linear interpretation, the formula and block diagram
explain some of the problems of an asynchronous electric drive. To this end, it is proposed to consider the transfer functions and the corresponding frequency characteristics at “frozen”, but different values of the stator voltage frequency and slip. Moreover, instead of the traditional characteristics of the control object, it will be necessary to consider “families”, grouped by varying stator voltage (its frequency) or slip [7].

Below, the frequency characteristics of an asynchronous electric drive with frequency control based on low-power squirrel cage induction motor are shown in Figs. 2 and 3. They are built in the Simulink application of the MATLAB software [8–11].

Amplitude and phase frequency characteristics of the motor at a stator voltage frequency of 10 Hz and slip corresponding to low (0.2 $M_n$) and nominal loads as shown in Fig. 2. Fig. 3 shows similar characteristics for a stator voltage frequency of 50 Hz.

![Frequency responses of an electric motor at a stator voltage frequency of 10 Hz and slip, corresponding to low ($W_1$) and nominal ($W_2$) loads](image)

Fig. 2. Frequency responses of an electric motor at a stator voltage frequency of 10 Hz and slip, corresponding to low ($W_1$) and nominal ($W_2$) loads

The given frequency responses well explain some problems of AC drive. When operating at low frequencies of the stator voltage, the phase shifts significantly change with changing load (and slip), which leads to instability and inefficient operation at low speeds (Fig. 2). Comparison of frequency responses at frequencies of stator voltage of 10 and 50 Hz shows that in the range from 10 to 100 rad/s the phase shifts of frequency responses have significantly different values – from 25 to –45 electric degree. This means that during acceleration and deceleration, the phase shifts change in such a way that a system with a stability margin at a frequency of 50 Hz can become unstable. The frequency of the stator voltage will be 10 Hz. This may be the reason for the different oscillation of the drive at different frequencies of the stator voltage, which were noted in [7]. Thus, the nonlineairties of the transfer functions of the link of the torque generator (Fig. 1) require linearization to increase the efficiency of the electric drive and the same behavior at different frequencies.
One of the widely used methods of linearization are various types of so-called “Transvector” control. With this control, dynamic links reverse to the dynamic links of the motor are formed in the control device, which are adapted to different modes of operation. Since ideal adaptation is impossible in a real drive, the transfer functions incorporated in the software of frequency converters and real asynchronous motors can vary for a number of reasons: a number of parameters are difficult to measure; the structure of a real electric motor is much more complicated than a model; some parameters may change during operation. Dynamic links are quite complicated, because the equivalent transfer functions of the frequency converter – asynchronous motor can contain resonant links in some modes. These links lead to control failures, to high-frequency harmonics, and to differences in dynamics at different speeds, which were observed during the experiments [1].

Other options for linearizing a torque driver are of interest [7, 14, 15].

Consider the option of applying local feedback on the electromagnetic moment in this structure. The structural diagram is shown in Fig. 4.

![Fig. 3. Frequency responses of an electric motor at a stator voltage frequency of 50 Hz and slip, corresponding to low ($W_1$) and nominal ($W_2$) loads](image)

$$\omega_{ref} \quad \frac{2M_k(T_2'p + 1)S_k}{\omega_1[(1 + T_2'p)^2S_k^2 + \beta^2]} \quad M \quad W_{DPF}$$

**Fig. 4. Block diagram of AED with local feedback on electromagnetic moment**
In this case, the transfer function of the torque driver will take the form:

\[ W_{eqv} = \frac{2M_k(T_2p + 1)}{\omega_1[1 + (T_2p)^2S_k^2 + 2B_1]} = \frac{2M_kS_k(T_2p + 1)}{\omega_1[1 + (1 + T_2p)^2S_k^2 + 2M_kS_k(T_2p + 1)]W_{DPF}}. \]

Under the following condition:

\[ \omega_1B_1^2 = -2M_kS_k(T_2p + 1)W_{DPF}, \]

that is, if the corrective element will have the following transfer function:

\[ W_{DPF} = -\frac{\omega_1B_1^2}{2M_kS_k(T_2p + 1)^2}, \]

then the transfer function of the torque driver takes the form:

\[ W_{eqv} = \frac{2M_kS_k(T_2p + 1)}{\omega_1[1 + (1 + T_2p)^2S_k^2]} = \frac{2M_k}{\omega_1[1 + (1 + T_2p)^2S_k^2]} \]

that is, it becomes a linear link, independent of slip (load), and completely coincides with the transfer function (2), given in the Usolshev’s monograph [1] for small loads. Pay attention to the formula (8). The dynamic link is a first-order inertia with a coefficient that ultimately depends on the frequency of the stator voltage and on the absolute slip. The sign (−) in front of the formula means that the feedback must be positive. Let's call this connection – dynamic positive feedback (DPF+). It should be noted that the correction of the coefficient of frequency is very easy to implement in frequency converters. Thus, the proposed positive feedback, selected by condition (6), makes it possible to compensate for the external load and the nonlinearity of the asynchronous electric drive, spreading the transfer function of the motor as a 1st order link for any β values. In addition, the block diagram (Fig. 1) and the transfer function of the moment drive link (5) connecting the moment and slip allow us to offer an estimate of the efficiency of the moment drive algorithm: the algorithm that generates the necessary moment with the smallest absolute slip will be more effective [12–15].

Next, we consider the correction of the asynchronous electric drive with the parameters corresponding to the frequencies of the supply voltage (FSV) of 10 and 50 Hz. The initial frequency responses are shown in Fig. 2 and 3. The transfer functions of the initial AED with such parameters and the transfer functions of the corrective links are presented in Table 1. The frequency characteristics of the initial and adjusted AED are shown in Figs. 2, 5 and 6 for the supply voltage frequency of 10 and 50 Hz, respectively.

### Table 1

| FSV  | * | \(W_{AED}(p)\) | \(W_{PFFB}\) |
|------|---|----------------|-------------|
| 10 Hz| \(\beta_1\) | \(0.036p + 0.226\) | \(0.002p + 0.229p + 1.38\) | \(0.0038p + 0.226\) |
|      | \(\beta_2\) | \(0.038p + 0.226\) | \(0.002p + 0.229p + 4.52\) | \(0.0038p + 0.226\) |
| 50 Hz| \(\beta_1\) | \(0.027p + 1.548\) | \(0.006p^2 + 0.628p + 20.56\) | \(0.27p + 1.548\) |
|      | \(\beta_2\) | \(0.027p + 1.548\) | \(0.006p^2 + 0.628p + 21.19\) | \(0.27p + 1.548\) |

*\(\beta_1\) – corresponds to slip at low load, \(\beta_2\) – corresponds to slip at rated load.

As expected, the frequency responses of the AED with the structural correction proposed in the work are close to the frequency responses of the 1st order linear link. In widely used AEDs, it is very difficult to realize the link by mechanical moment. Given that the moment is equal to \(I_1\Psi_2\) and in almost all calculations it is assumed that the rotor flux linkage is constant, you can replace the original signal in this local connection with the effective value of the stator current, or its active component, which is calculated in all frequency converters.
Fig. 5. Frequency responses of the torque driver link: initial ($W_1$, $W_2$) and corrected ($W_3$, $W_4$) for the supply voltage frequency 10 Hz

Fig. 6. Frequency responses of the torque driver link: initial ($W_1$, $W_2$) and corrected ($W_3$, $W_4$) for the supply voltage frequency 50 Hz

For stator current feedback, the linearization condition will vary slightly:

$$\omega_1 \beta^2 = -2M_k S_k (T_2 p + 1) \frac{W_{PFB}}{\Psi_2}.$$  

This expression shows that when controlling the flux linkage, the linearization conditions can be refined, thereby ensuring high quality regulation.
On the other hand, it is easy to show that with some inaccuracy in the fulfillment of the linearization condition, i.e.

\[
\frac{\beta}{s_k} - \frac{2M_2(T_2p+1)W_1}{W_2} \neq 0 = \Delta.
\]

The transfer function and frequency response of the torque driver will differ slightly from the transfer function and frequency response of the first-order linear link.

Consider the case of deviation of the parameters of the corrective element by 5% for the frequencies of the supply voltage of 10 and 50 Hz. The frequency characteristics of the link of the torque driver with accurate correction \((W_1, W_2)\) and the deviation of the transmission coefficient of the correction link \(\pm 5\% (W_3, W_4)\) are presented in Figs. 7 and 8.

\[
W_{PFB1} = \frac{3.84}{0.0038p + 0.226}
\]
\[
W_{PFB2} = \frac{4}{0.0038p + 0.226}
\]

Fig. 7. Frequency response of the torque driver and transfer functions of the correction link for the stator voltage frequency of 10 Hz, accurate \((W_i)\) and when the transfer ratio of the correction link deviates by 5% \((W_i)\)

\[
W_{PFB1} = \frac{3.84}{0.0042p + 0.226}
\]
\[
W_{PFB2} = \frac{3.84}{0.0042p + 0.226}
\]

Fig. 8. Frequency response of the torque driver and transfer functions of the correction link for the stator voltage frequency of 50 Hz, accurate \((W_i)\) and when the transfer ratio of the correction link deviates by 5% \((W_i)\)
Conclusion

Thus, the proposed method for analyzing processes in an asynchronous drive with frequency control according to changing transfer functions and frequency characteristics (“families” of functions and characteristics with frozen frequency and slip parameters) made it possible to propose an effective correction, in terms of controllability of a nonlinear dynamic structure, allowing increase its effectiveness.

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УПРАВЛЕНИЕ НЕЛИНЕЙНОЙ ДИНАМИКОЙ ЭЛЕКТРОМЕХАНИЧЕСКИХ СИСТЕМ

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Предлагается анализировать процессы в наиболее широко применяемых в настоящее время частотно-регулируемых электроприводах переменного тока, как в системах автоматического регулирования с динамическими нелинейностями, и структурные методы коррекции, совершенствующие их динамику. Впервые предложены динамические формулы передаточных функций формирователя вращающегося момента в асинхронном электродвигателе с частотным управлением, учитывающие скольжение и частоту статорного напряжения. Изложены методы построения семейств частотных характеристик таких электромеханических систем при "замороженных", но различных значениях частоты статорного напряжения и скольжения. В приложении Simulink программной среды MATLAB построены семейства частотных характеристик, соответствующие нелинейным передаточным функциям. Полученные в работе нелинейные передаточные функции позволили обосновать структурные решения, линеаризующие частотно-регулируемые электроприводы и существенно повышающие их эффективность.

Таким решением оказалась положительная обратная связь по действующему значению тока статора с динамическим звеном. Именно это динамическое звено обеспечивает эффективное действие положительной связи без нарушения устойчивости электромеханических систем. Эксперименты полностью подтвердили правильность полученных математических выражений для нелинейных звеньев систем и их коррекции. Данная работа является примером того, как исходная усложненная (но более точная!) интерпретация нелинейности позволила найти новое лучшее решение задачи управления сложным динамическим объектом.

Ключевые слова: электропривод переменного тока, математический анализ, динамическая нелинейность, частотная характеристика, динамическая коррекция, положительная обратная связь.

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