Smooth expansion rate data to reconstruct cosmological matter perturbations

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Abstract. The existing degeneracy between different dark energy and modified gravity cosmologies at the background level may be broken by analyzing quantities at the perturbative level. In this work, we apply a non-parametric smoothing (NPS) method to reconstruct the expansion history of the Universe ($H(z)$) from model-independent cosmic chronometers and high-$z$ quasar data. Assuming a homogeneous and isotropic flat universe and general relativity (GR) as the gravity theory, we calculate the non-relativistic matter perturbations in the linear regime using the $H(z)$ reconstruction and realistic values of $\Omega_m$ and $\sigma_8$ from Planck and WMAP-9 collaborations. We find a good agreement between the measurements of the growth rate and $f\sigma_8(z)$ from current large-scale structure observations and the estimates obtained from the reconstruction of the cosmic expansion history. Considering a recently proposed null test for GR using matter perturbations, we also apply the NPS method to reconstruct $f\sigma_8(z)$. For this case, we find a $\sim 3\sigma$ tension (good agreement) with the standard relativistic cosmology when the Planck (WMAP-9) priors are used.

Keywords: cosmological parameters from LSS, dark energy theory, galaxy clustering

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1 Introduction

The discovery of cosmic acceleration poses a very fundamental question to theoretical cosmology: does acceleration reflect the existence of new fields in high energy physics or the need for modifications of the standard gravity theory on cosmological scales? This question becomes even more complicated as it is possible to construct modified gravity (MG) scenarios that produce the same cosmic expansion of general relativistic dark energy (DE) models [1]. Such degeneracy lies in the background level, where the description of the Universe is made from a perfectly symmetric and simplified model. Therefore, the use of cosmological observables, such as the Hubble parameter or the luminosity or angular diameter distances, does not seem to be enough to determine if the cosmic acceleration is a geometrical or a dynamical effect. When a more realistic scenario is considered, in which the universe has geometrical and energy fluctuations, it is possible to describe the growth and evolution of overdensity and underdensity regions. Since different classes of models produce, in general, characteristic predictions of the growth of the cosmic structures, perturbative quantities, such as the growth rate \( f \) and index \( \gamma \), are believed to be important tools to distinguish MG from DE models (see, e.g., [1–3]).

From the theoretical side, some null tests have been proposed to probe the validity of the standard cosmology [4–6]. In these tests, the relation between different observables must be set at specific values, otherwise, there would be a violation of one or more assumptions used to derive the test. In refs. [7, 8] it was presented a new null test involving both measurements of cosmic expansion and matter perturbations. For the arguments mentioned earlier, it is expected that such null tests involving perturbative observables will be more efficient than others using only background quantities.

On the other hand, to obtain information about the functional behavior of dynamical and kinematic variables, parametric and non-parametric methods have been used. In both cases, the final reconstructed quantity is constrained by observational data. In parametric approaches (see, e.g., [9–13]), a prior functional form is used to describe the observations, whereas in the non-parametric approaches (see, e.g., [14–26]), it is commonly assumed a correlation between each data point.
In this work, we apply a non-parametric smoothing (NPS) method \cite{23–28} to reconstruct the evolution of matter perturbations from background data, such as the measurements of the cosmic expansion rate $H(z)$. We apply the method to a sample of $H(z)$ data from cosmic chronometers \cite{29–34}, lying in the redshift $0.070 \leq z \leq 1.2$, and high-$z$ quasar data at $z \approx 2.3$ \cite{35}. The cosmic chronometer data have been obtained from the differential age method for passively evolving galaxies of ref. \cite{36}, which is aimed to be cosmological and stellar population synthesis model-independent \cite{37}. On the other hand, current measurements of the expansion rate from quasar data have been obtained using the three-dimensional correlation function of the transmitted flux fraction in the Ly$\alpha$-forest of high-$z$ quasars, as reported in ref. \cite{35}. In particular, the application of this latter technique to a sample of 48,640 quasars provided a measurement of $H(z)$ within $\sim 3\%$ accuracy at $z = 2.3$, which imposes tight bounds on cosmological parameters when combined with current $H_0$ measurements and other cosmological data sets (see, e.g., \cite{38} for a recent analysis). We follow the approach presented in ref. \cite{39} (see also \cite{26}) to reconstruct perturbative quantities from background observables and compare them with current measurements of growth rate and index. Finally, we also use the NPS method to reconstruct the $f\sigma_8(z)$ observable and evaluate the null test proposed in refs. \cite{7, 8}.

This paper is organized as follows: in section 2.1 we summarize the treatment of linear matter perturbations of ref. \cite{39} and the perturbative null test proposed in refs. \cite{7, 8}. We also introduce the basic equations of the matter perturbation theory and a brief explanation on how to construct the null test. In section 3 we present the observational data and the non-parametric method used to reconstruct the evolution of $H(z)$ and the perturbative quantities. We present the results of our reconstructed cosmic expansion, the matter perturbation analysis and the calculation of the null test in section 4. We end this paper with the main conclusions in section 5.

2 Matter perturbation equations

2.1 Matter perturbation description

In this paper, we follow closely the discussion presented in ref. \cite{26}. We consider a homogeneous and isotropic universe composed by matter and unclustering DE fluid separately conserved and assume that the correct theory of gravity is GR. Under this assumptions, the evolution of the matter density contrast, $\delta(\vec{x}, t) \equiv \rho(\vec{x}, t) - \rho(t) / \rho(t)$, inside the Hubble sphere, is governed by the second order differential equation \cite{40}:

\begin{equation}
\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0 ,
\end{equation}

where the dot denotes derivative with respect to cosmic time.

Eq. (2.1) can be solved in terms of the follow set of integral equations \cite{39, 41}

\begin{align}
\delta(D) &= 1 + \delta'_0 \int_0^D [1 + z(D_1)] dD_1 + \frac{3}{2} \Omega_m \int_0^D [1 + z(D_1)] \left( \int_0^{D_1} \delta(D_2) dD_2 \right) dD_1 , \quad (2.2a) \\
\delta'(D) &= \delta'_0 [1 + z(D)] + \frac{3}{2} \Omega_m [1 + z(D)] \int_0^D \delta(D_1) dD_1 . \quad (2.2b)
\end{align}

where $D$ represents the adimensional physical distance, defined by

\begin{equation}
D = H_0 \int_t^{t_0} \frac{dt}{a(t)} = H_0 \int_0^z \frac{dz_1}{H(z_1)} .
\end{equation}

\begin{align*}
\delta(D) &= 1 + \delta'_0 \int_0^D [1 + z(D_1)] dD_1 + \frac{3}{2} \Omega_m \int_0^D [1 + z(D_1)] \left( \int_0^{D_1} \delta(D_2) dD_2 \right) dD_1 , \quad (2.2a) \\
\delta'(D) &= \delta'_0 [1 + z(D)] + \frac{3}{2} \Omega_m [1 + z(D)] \int_0^D \delta(D_1) dD_1 . \quad (2.2b)
\end{align*}

where $D$ represents the adimensional physical distance, defined by

\begin{equation}
D = H_0 \int_t^{t_0} \frac{dt}{a(t)} = H_0 \int_0^z \frac{dz_1}{H(z_1)} .
\end{equation}
The prime denotes the derivative with respect to $D$ and $\Omega_{m0}$ is the matter density parameter at $z = 0$. Note that the integral solution of the matter density contrast is not coupled with its first derivative. Conversely, to solve $\delta'$ we need to know the density contrast.

In this work, we use realistic values of the density parameter provided by current CMB experiments. To obtain a unique solution for the second order differential equation (2.1), we need to fix the two integration constants. In the solution (2.2a) these constants are the values of the matter density contrast and its first derivative at the present time. $\delta(z = 0)$ is fixed by the normalization of (2.2a) at $z = 0$ whereas the second constant, $\delta'(z = 0) = \delta'_0$, can be fixed analyzing the behavior of the solution at very high redshift, where it is expected that $\delta \propto a$ (see [26] for more details). The integral solution (2.2a) involves only information about the matter density parameter at the current epoch and the normalized expansion rate ($H(z)/H_0$). Prior knowledge of these two quantities determines univocally the matter density contrast [1, 42]. As we can see from the previous equations, in the context of a given cosmology, there is not an explicit dependence between the matter perturbations and the value of the Hubble constant ($H_0$).

We define the growth rate of the linear density contrast as

$$f(z) \equiv \frac{d \ln \delta}{d \ln a} = -\frac{(1 + z)H_0 \delta'}{H(z) \delta},$$

and the rate of change of the amplitude of clustering,

$$f\sigma_8(z) \equiv f(z)\sigma(z) = \frac{d \sigma_8(a(z))}{d \ln a},$$

where

$$\sigma_8(z) = \sigma_8\delta(z)/\delta(0)$$

is the rms amplitude of mass fluctuations in terms of $\delta(z)$ [8] and $\sigma_8$ is the present linear mass dispersion on a sphere of radius $8h^{-1}$Mpc. These two quantities can be inferred from the observations of the large-scale structure by analyzing the matter power spectrum or from the gravitational lensing data [43, 44].

The measurements of $f(z)$ and $f\sigma_8(z)$ from current large-scale structure observations and the estimates obtained from eqs. (2.1), (2.4) and (2.5) along with the reconstruction of the Hubble parameter are independent. A tension between these independent estimates would be a clear evidence of non-standard cosmology or a violation of the assumptions used to derive eq. (2.1), i.e.:

- The Universe is not flat, homogeneous and isotropic in large scales. For instance, in a LTB cosmology the perturbation equations have to be modified, as shown in refs. [45, 46].

- The DE and matter components are not separately conserved and the matter density does not decrease proportional to $a^{-3}$. Examples of this kind are models with decaying of dark energy into dark matter or vice versa [47–50].

- The correct theory of gravity is not the GR or the DE fluid is clustering. In both cases, $G$ is no longer a constant and can be written as an effective gravitational function which depends on the scale and time $G_{\text{eff}} = G_{\text{eff}}(k, t)$ [3, 51–53].

We refer the reader to ref. [39] for a more detailed discussion of this topic.
Finally, we define the growth index, which we write as a function of \( z \) as [54–56],

\[
\gamma(z) \equiv \frac{\ln f(z)}{\ln \Omega_m(z)},
\]

where

\[
\Omega_m(z) = \frac{\Omega_{m0}(1+z)^3H_0^2}{H^2(z)}.
\]

The main importance of the growth index is that it constitutes a powerful tool to characterize gravity theories. For instance, the ΛCDM model is characterized by \( \gamma = 6/11 \) and, in general, slow varying DE models are predict \( \gamma \approx 3(w-1)6w-5 \) [56, 57]. In other scenarios, like DGP model, \( \gamma = 11/16 \) [57] whereas for the \( f(R) \) gravity model discussed in ref. [58] the growth index is constrained to the interval \( 0.40 < \gamma < 0.43 \) [3].

2.2 Null test for GR

It is possible to construct a null test for GR from matter density perturbations using the equations that govern their evolution. For this reason, the hypotheses used so that eq. (2.1) correctly describes the evolution of \( \delta(z) \) must be satisfied. The null test is constructed as follows [7, 8]. First, let us assume that eq. (2.1) results from the Euler-Lagrange equations with \( \delta \) as generalized coordinate, i.e.,

\[
\frac{d}{da} \left( \frac{\partial L(a, \delta, \dot{\delta})}{\partial \dot{\delta}} \right) - \frac{\partial L(a, \delta, \dot{\delta})}{\partial \delta} = 0.
\]

In this case, we can associate the follows Lagrangian \( L \) and Hamiltonian \( H \) to the system:

\[
L = \frac{1}{2}a^3\delta'(a)^2H(a)/H_0 + \frac{3\Omega_{m0}}{4a^2H(a)/H_0}\delta(a)^2,
\]

\[
H = \frac{1}{2}a^3\dot{\delta}'(a)^2H(a)/H_0 - \frac{3\Omega_{m0}}{4a^2H(a)/H_0}\delta(a)^2.
\]

In this system, the Hamiltonian depends explicitly on the ‘time’ parameter \( a \) which implies that \( H \) is not a constant of ‘motion’. Finding a symmetry direction of the system, we construct the first integral of motion,

\[
\Sigma = a^3H(a)/H_0\delta'(a)e^{-\int_{a_0}^{a} \frac{3\Omega_{m0}\delta(x)}{x^2H(x)^2/R_0^2\delta'(x)} dx}.
\]

This normalized quantity constitutes a null test of GR involving a perturbative variable. It is still possible to rewrite eq. (2.11) in terms of the observables \( H(z) \) and \( f\sigma_8(z) \), i.e. (considering \( a_0 = 1 \) and \( G_{\text{eff}} = G \))

\[
O(a) = \frac{a^2H(a)f\sigma_8(a)}{H_0f\sigma_8(1)} \times \exp \left( -\frac{3}{2} \Omega_{m0} \int_1^a \frac{\sigma_8(a = 1)}{x^4H(x)^2/H_0^2f\sigma_8(x)} dx \right).
\]

In addition to the assumptions used to obtain eq. (2.1) and its solution (2.2a), this test requires that there will not be tension between the cosmic expansion and matter density growth data. If these hypotheses are satisfied, the value of \( O(z) \) must be unity.
3 Data and Hubble parameter reconstruction

3.1 Data

Measurements of the expansion rate are important observational probes of the late-time cosmic acceleration. The cosmic chronometer approach is developed using the differential ages of massive and passively evolving old elliptical galaxies [36]. In contrast to other observables that depend on integrated quantities along the line of sight, cosmic chronometers estimates are independent of the spatial geometry of the Universe. The cosmic expansion corresponds to the estimate of the age change of the Universe for a given variation of the redshift. This information can be obtained by the analysis of the galaxy ages with their respective $z$. Considering passively evolving galaxies at approximately the same redshift, it is possible to obtain an estimate of the cosmic expansion using the expression $H(z) = -1/(1+z)\Delta z/\Delta t$. Here, $\Delta z$ is the difference between the redshift of two galaxies and $\Delta t$ the difference between their ages.

One of the main feature of cosmic chronometer approach lies in the fact that their estimates are cosmological model-independent, although there can be dependence on stellar population synthesis models at high redshift. As emphasized in ref. [37], this latter dependence does not appear until $z \approx 1.2$. For this reason, we use only 16 measurements of the $H(z)$ from cosmic chronometers, up to $z = 1.04$. As discussed in ref. [37] we increase slightly (20%) the error bar of the highest-$z$ data point to account for the uncertainties of the stellar population synthesis models. We also add to our sample two measurements of $H(z)$ from quasar data at very high-$z$, i.e., $z = 2.34$ [59] and $z = 2.36$ [60], which were obtained by determining the BAO scale from the correlation function of the Lyα-forest systems. To perform our analysis, we complement the dataset used in ref. [26] with the most recent expansion rate measurement, $H(z = 0.4293) = 91.8 \pm 5.3$ [64].

3.2 Non-parametric smoothing

In order to perform a model-independent reconstruction of the expansion rate of the Universe from the cosmic chronometer and high-$z$ quasar data, we applied the non-parametric method
Figure 2. The evolution of the matter density parameter calculated using the reconstruction of $H(z)$ shown in figure 1(a) and current estimates of $\Omega_{m0}$ from the Planck collaboration (a) and from the WMAP-9 collaboration (b). The shaded regions correspond to the 1σ and 2σ confidence intervals.

proposed in refs. [23–25, 28]. This method has been very useful in the reconstruction of the luminosity and the physical distances and the Hubble parameter. In its general form, and taking into account the data errors, the smoothing quantity is obtained as

$$H^s(z, \Delta) = H^g(z) + N(z) \sum_i \frac{[H(z_i) - H^g(z_i)]}{\sigma^2_H(z_i)} \times K(z, z_i),$$  \hspace{1cm} (3.1)$$

where $H^s(z, \Delta)$ is the smoothed quantity, $H^g(z_i)$ is the initial guess model, $H(z_i)$ is the observational data, $\sigma_H(z_i)$ is the data error, $\Delta$ is the smoothing scale and $N(z)$ is the normalization factor given by:

$$N(z)^{-1} = \sum_i \frac{K(z, z_i)}{\sigma^2_H(z_i)}. \hspace{1cm} (3.2)$$

Following the procedure presented in ref. [25], in this work we adopt a Gaussian kernel ($K(z, z_i) = \exp(-(z - z_i)^2/2\Delta^2)$) to perform the reconstruction. Due to the iterative application of the smoothing function, the dependence on the initial guess model becomes insignificant and the reconstruction is effectively model-independent (see, e.g. [23–25, 27, 28]).

In order to calculate the 1σ confidence level we use the approach developed in ref. [61] (see also [25]). In this case the error is given by:

$$\sigma_{H^s(z)} = \left( \sum_i v_i^2 \hat{\sigma}^2 \right)^{1/2}, \hspace{1cm} (3.3)$$

$\sigma_{H^s(z)}$ being the 1σ confidence level of $H(z)$, $v_i$ the smoothing factor ($v_i = N(z)K(z, z_i)/\sigma^2_H(z_i)$) and $\hat{\sigma}^2$ is the estimate of the error variance given by [25, 61]:

$$\hat{\sigma}^2 = \frac{\sum_i (H(z_i) - H^s(z_i))^2}{\sum_j (1 - v_j(z_j))}. \hspace{1cm} (3.4)$$

As done in ref. [25] we increase the value of the 1σ confidence level ($\sigma_{H^s(z)}$) by 30%.
Figure 3. Reconstruction of the growth rate of the matter perturbations. (a) The growth rate obtained assuming the Planck $\Omega_m^0$ value. (b) The same as in the previous panel assuming the WMAP-9 $\Omega_m^0$ value. The dashed line corresponds to the growth rate calculated using the $H(z)$ reconstruction via GP of the ref. [26]. The data points were taken from table II of ref. [67] and the shaded regions represent the $1\sigma$ and $2\sigma$ confidence levels.

The final reconstruction depends on the smoothing scale. For instance, for very small values of $\Delta$, the reconstructed curve tends to follow more closely the data points with several bumps. On the other hand, if $\Delta$ is too large, the curve is too smooth, departing significantly from the data. For this reason it is convenient to estimate an optimal value of $\Delta$. We select the $\Delta$ value that minimizes the cross-validation function given by

$$CV(\Delta) = \frac{1}{n} \sum_i (H(z_i) - H^s_i(z_i|\Delta))^2,$$

where $H^s_i(z_i|\Delta)$ denotes the reconstructed expansion at $z = z_i$ without taking into account the data point $(z_i, H(z_i))$ for a given $\Delta$. For the cosmic chronometer and high-$z$ quasar data (see section 3.1) the $\Delta$ value that asymptotically minimizes eq. (3.5) is $\sim 1.4$.

4 Results

In order to perform our non-parametric reconstruction of the Hubble parameter, we apply the method described in section 3.2 to the cosmic chronometer and high-$z$ quasar data presented in section 3.1. The final reconstruction is shown in figure 1(a) along with the time variation of the scale factor, i.e., $\dot{a} = H(z)/(1 + z)$ (figure 1(b)). It is important to explore this latter quantity because in the following analysis of matter perturbations we assume the existence of a matter-dominated epoch and, therefore, the existence of a decelerated phase in the cosmic expansion. In figure 1(b), the transition redshift (deceleration/acceleration) corresponds to the minimum of the function at $z_t \simeq 0.48$ which is in agreement with recent estimates of this quantity [62–64]. It is worth mentioning that the values of the Hubble constant and the transition redshift derived from the NPS method are, respectively, $\sim 4\%$ higher and $\sim 29\%$ smaller than the values obtained from the Gaussian Process approach [26].

In figures 2(a) and 2(b) we show the matter density parameter (eq. (2.8)) obtained from the smoothed $H(z)$ function and two priors for $\Omega_m^0$ given by Planck collaboration [65], $\Omega_m^0 = 0.308 \pm 0.012$, and by WMAP-9 collaboration, $\Omega_m^0 = 0.279 \pm 0.025$ [66]. Using the
Figure 4. The rate of change of the clustering amplitude, $f\sigma_8(z)$. The solid line corresponds to the reconstruction while the shaded regions represent the 1σ and 2σ confidence levels. For comparison, we also show the ΛCDM prediction (dashed line). (a) The rate of change of the clustering amplitude obtained assuming the Planck $\Omega_{m0}$ and $\sigma_8$ values. (b) The same as in the previous panel assuming the WMAP-9 $\Omega_{m0}$ and $\sigma_8$ values. The data points were taken from table I of ref. [7].

Figure 5. The growth index $\gamma(z)$ of matter perturbations. The solid line corresponds to the reconstruction from the NPS method whereas the shaded regions represent the 1σ and 2σ confidence levels. (a) The growth index obtained assuming the Planck $\Omega_{m0}$ value. (b) The same as in the previous panel assuming the WMAP-9 $\Omega_{m0}$ value.

$H(z)$ and the $\Omega_{m0}$ priors, as mentioned in section 2.1, we also calculate the matter density contrast by solving the integral (2.2a). As discussed above, the evolution of $\delta(z)$ is totally determined by the normalized cosmic expansion and by the matter density content at the present day when we fix the integration constant $\delta'_0$. From the perturbation theory we know exactly how is the behavior of the density contrast in the matter dominated epoch. We know that in GR the matter density growth is proportional to $a = 1/(1 + z)$ when $\Omega_m(z) \approx 1$, which is expected at high-$z$. This behavior is more easily identifiable if we analyze the growth factor ($g(z) = (1 + z)\delta(z)$) at high redshift. In this regime the function $g(z)$ must be constant. We explore different values for the constant $\delta'_0$ and we choose the one when we reach the expected evolution of the growth factor close to the highest redshift of our sample, $z = 2.34$. The final growth factor function is very sensitive to the $\delta'_0$ value, therefore we can obtain
an accurate estimate of this integration constant. The resulting $\delta(z)$ depends on the $\Omega_{m0}$ and hence the correct $\delta'_0$ value also depends on the matter content. For the matter density contrast calculated with Planck and WMAP-9 $\Omega_{m0}$ values we infer $\delta'_0 = 0.518 \pm 0.003$ and $\delta'_0 = 0.488 \pm 0.003$, respectively.

In figure 3 we show the resulting reconstruction of the growth rate. In this plot we also display the $f(z)$ measurements compiled in ref. [67]. The previous estimate of the $\delta'_0$ constant is very important in the determination of the $f(z)$. From the alternative expression given by eq. (2.4), in terms of $\delta$ and $H(z)$, we can show that the growth rate at the present epoch and $\delta'_0$ are related through $f(0) = -\delta'_0$.

In order to compare our results with the ones presented in ref. [26], we also plot the growth rate calculated using the reconstruction of the cosmic rate via GP (See figure 1(a)). This result clearly shows the sensitivity of the perturbative quantities to the expansion rate. In this case, the main difference between the Hubble function reconstructions (via NPS and GP) is its value at $z = 0$. Note that, when we consider a cosmological model with cosmic expansion given by $H(z) = H_0 E(z)$, the matter perturbations are independent of the Hubble constant because they are determined only by $H(z)/H_0$, but in the case of a non-parametric reconstruction, the value of the normalization ($H_0$) will affect the function $H(z)/H_0$ in the entire redshift range.

The other observable, $f \sigma_8(z)$, is shown in figure 4. In this case we also need information about the $\sigma_8$ parameter to determine the function (eq. (2.5)) from the reconstructed cosmic expansion. We use the Planck and WMAP-9 collaboration values, $\sigma_8 = 0.815 \pm 0.009$ and $\sigma_8 = 0.821 \pm 0.023$, respectively. In figure 4 we also show the $f \sigma_8(z)$ measurements compiled in ref. [7]. These data, $f(z)$ and $f \sigma_8(z)$, are usually obtained from data of large-scale galaxy distribution and are derived assuming a fiducial cosmology. The compatibility between the reconstructed quantities from the $H(z)$ function and the observational data shows a good agreement with the standard cosmological model, that is, with the hypothesis of a homogeneous and isotropic universe filled with matter and DE fluid covariantly conserved in the framework of GR (see section 2.1). An agreement between this reconstructed approach and the data was also pointed out in ref. [26] using GP to obtain the $H(z)$ function. There-
fore, we notice that this agreement does not depend on the non-parametric method used to reconstruct the cosmic expansion.

The last perturbative quantity defined in section 2.1, $\gamma(z)$, is calculated with the reconstructed growth rate (figure 3) and with the matter density parameter (figure 2) for both values of $\Omega_m$ from CMB experiments. At $z = 0$, we found $\gamma_0 = 0.56 \pm 0.11$ (2$\sigma$) and $\gamma_0 = 0.56 \pm 0.13$ (2$\sigma$) for the Planck and WMAP-9 values of $\Omega_m$, respectively. These results are in good agreement with the results presented in ref. [26]. The reconstruction of the growth index is shown in figure 5.

Finally, we also apply the NPS method to the $f\sigma_8(z)$ data displayed in figure 4 to obtain the evolution of this quantity in a non-parametric form. With the $H(z)$ and $f\sigma_8(z)$ observables reconstructed and the $\sigma_8$ and $\Omega_m$ estimates from CMB experiments, we calculate the perturbative null test $O(z)$, discussed in section 2.2. The results are shown in figure 6. We note that the non-parametric reconstruction of the quantity $O(z)$ shows a tension between the standard model prediction and the data at $\approx 3\sigma$ level for the results obtained with Planck collaboration priors. As it was noticed in ref. [7] the value of the $O(z)$ test is very sensitive to the processing of the data. In our case it correspond to the smoothing scale to perform a reconstruction. We explore different $\Delta$ values to reconstruct the $f\sigma_8(z)$, however, the tension at low redshift remains. This tension between the growth rate data and the Planck results has been pointed out in refs. [68, 69]. On the other hand, the same reconstruction performed with WMAP-9 priors is consistent with the standard cosmology.

5 Conclusions

In this work we have performed a reconstruction of perturbative quantities using a non-parametric smoothing method applied to current data of the cosmic expansion. The fact that both the method of reconstruction and the data set are model-independent reduces the possibility of biased results.

Analyzing the growth rate, $f(z)$, and the rate of change of the clustering amplitude, $f\sigma_8(z)$, calculated by solving the perturbation eqs. (2.2a), (2.4) and (2.5) and comparing with their observational estimates, we have not found any deviation from the predictions of the standard cosmology. This is consistent with the results reported in ref. [26], where a similar analysis was performed using a different reconstruction method.

The previous result is confirmed by calculating the $O(z)$ test with the reconstructed $H(z)$ and $f\sigma_8(z)$ evolutions via NPS using the values of $\sigma_8$ and $\Omega_m$ from the WMAP-9 collaboration. However, the result is not the same when we calculate the perturbative null test using the corresponding values from the Planck collaboration. In this case, we have found a violation of the null test at $\approx 3\sigma$ level, which is not evident in figures 3(a) and 4(a), showing the high sensitivity of the $O(z)$ test when compared with the usual perturbative quantities. Such tension deserves further investigation and seems to be in line with the results discussed in ref. [69], where a $3\sigma$ discrepancy was reported using the same data sets and a different approach. We have calculated the growth index and found that it is more effectively constrained at $z = 0$, $\gamma_0 = 0.56 \pm 0.11$ (2$\sigma$). Such result is compatible with the $\Lambda$CDM expected value ($\gamma = 6/11$) and its first derivative $\gamma'_0 \approx -0.015$ [3].

Finally, it is worth emphasizing that GP and NPS are independent reconstruction methods which may produce different results for the same data sets, as shown in ref. [25]. The fact that the growth index estimate obtained in this work is in good agreement with the results presented in ref. [26] using the GP method shows the robustness of both analyses.
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