Design of an Intelligent Reflecting Surface aided mmWave Massive MIMO using X-Precoding

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**ABSTRACT** In this paper, we consider an intelligent reflecting surface (IRS) aided millimeter wave (mmWave) massive multiple-input multiple-output (MIMO) system using hybrid beamforming/combining. To enhance error performance, we adopt X-code (or X-precoder), a low-complexity precoding technique for traditional MIMO channels, to encode information symbols. We first derive an upper bound on word error rate (WER), based on which we design jointly IRS phase shifts and X-code (or X-precoder) to minimize WER. Specifically, we propose two algorithms for IRS design exploiting alternating optimization and gradient ascent optimization methods. Then we devise X-code and X-precoder, respectively, by minimizing average WER over all channel realizations and WER for each channel realization. We also provide their diversity analysis. Further, we present the procedure of decoupling fully digital beamformer/combiner at transceiver into the optimal hybrid one. Finally, simulation results show that both IRS optimization algorithms have similar WERs whereas gradient ascent approach has a lower computational complexity. Simulations demonstrate that the designed X-code (or X-precoder) provides a significant performance gain.

**INDEX TERMS** Diversity, intelligent reflecting surface (IRS), massive multiple-input multiple output (MIMO), millimeter wave (mmWave), precoding, word error rate, X-code, X-precoder

**I. INTRODUCTION**

Driven by the ever-increasing demand of transmission capacity and spectral efficiency, the emerging technologies such as millimeter wave (mmWave) communications and massive multiple-input multiple-output (MIMO), have been enabled in the fifth-generation (5G) network [1]–[4]. It is known that mmWave bands can offer order of magnitude greater bandwidth, but mmWave also incurs high path loss, leading to unfavorable communication environment [5], [6]. As a means to compensate severe propagation loss, large antenna arrays in massive MIMO are utilized to achieve high beamforming gains and coverage extension. Short wavelengths associated with high frequency facilitate the use of massive MIMO thanks to the reduced sizes of antenna arrays [4]. Moreover, mmWave signals are sensitive to blockage, even a small obstacle, such as human body, can result in 20dB decrease in signal strength [7]. Hence, it is necessary to develop a new technology to cope with this issue.

**A. INTELLIGENT REFLECTING SURFACE**

Recently, a promising technology, intelligent reflecting surface (IRS) has drawn an increasing attention to combat blockage in mmWave and assist massive MIMO to achieve better performance [8], [9]. IRS is a two-dimensional meta-surface which consists of a large array of passive scattering and low-cost elements. Each element is controlled by an embedded controller to change electromagnetic properties (e.g. amplitudes and phase shifts) of incident radio frequency (RF) signals [10]. By smartly adjusting passive elements at IRS, RF signals can be added coherently to achieve advantageous propagation conditions, such as improving energy efficiency [11]–[14], increasing capacity [15]–[25], decreasing error rate [26], mitigating user interference [27]–[29], and so on.

**B. IRS-ASSISTED MASSIVE MIMO**

A lot of research has been conducted for the optimization design and performance analysis of IRS-aided mmWave massive MIMO systems (see [11]–[30] and references therein).
It is worth mentioning that the authors in [18] developed an alternating optimization algorithm to maximize the capacity. This algorithm iteratively optimizes one reflecting phase of IRS at a time while others are fixed, which however has large computational complexity. Another research focus is on the joint design of hybrid beamforming and IRS reflection elements (see [14], [22]–[26] and references therein). For example, the authors in [23] designed jointly active and passive beamforming to maximize spectral efficiency by proposing a manifold optimization algorithm. This approach has the advantage of creating favorable channel matrix with small condition number. On the other hand, the error performance of IRS-aided mmWave massive MIMO has been rarely investigated. In [26], the authors focused on broadband transmission and proposed a geometric mean decomposition-based algorithm, leading to a satisfactory bit error rate (BER) performance.

**C. PRECODING USING X-CODE (OR X-PRECODER)**

To further enhance performance of IRS-aided mmWave massive MIMO without introducing extra significant complexity, we consider to use X-code (or X-precoder), a low-complexity precoding technique proposed in [31], [32] for traditional MIMO. In [31], [32], a MIMO channel was first decomposed into a number of parallel subchannels via a singular value decomposition (SVD) operation. Then X-code (or X-precoder) was used to pair the subchannels with good and bad diversities to improve overall diversity, and an optimal pairing strategy was proposed. Essentially, X-code and X-precoder share the same precoding matrix structure, in which the parameters are designed differently. The parameters for X-precoder was designed to optimize the error performance for each channel realization, while those for X-code were designed to optimize average error performance over all channel realizations. Finally, a low complexity maximum likelihood (ML) detection was conducted to each pair rather than the entire MIMO channel.

**D. OUR CONTRIBUTIONS**

In this paper, we consider an IRS-assisted mmWave massive MIMO, where both transmitter (Tx) and receiver (Rx) are deployed with large numbers of antennas, followed by RF chains. The information symbols are precoded by X-code (or X-precoder) [31], and processed by digital/analog active beamforming. The data transmission arrives at the Rx via an IRS, followed by analog/digital combining. Finally, ML detection is used to recover the information symbols. Under such a setting, we perform a joint design of IRS, X-code (or X-precoder), as well as transceiver beamforming to improve overall error performance. Detailed contributions are summarized below.

- We conduct performance analysis and derive an upper bound on word error rate (WER). As a result, we propose a system design criterion, where we design jointly IRS phase shifts and X-code (or X-precoder) to minimize the upper bound on WER.

- Based on the design criterion, we first design the IRS phase shifts. To find the optimized IRS phase shifts, we explore two optimization methods: alternating optimization and gradient ascent optimization. We also compare their computational complexity and convergence. By simulations, we verify that both methods lead to similar performance, whereas gradient ascent approach has lower computational complexity than alternating approach.

- We then design the X-code (or X-precoder) to improve error performance. It is known in [31] that both X-code and X-precoder adopt the same precoding matrix structure (see (4)), but their parameters in (4) should be designed differently. Hence, for X-code, we design the parameters in (4) to minimize average WER over all channel realizations, while for X-precoder, we design them to minimize WER for each channel realization. For both schemes, we provide diversity analysis. Simulation results show a significant performance gain.

- We also design the digital (or hybrid) active beamforming to improve the overall system performance. Simulations demonstrate the effectiveness of our design.

The rest of the paper is organized as follows. Section II introduces system model. Section III presents problem formulation. Section IV presents IRS phase shifts design. Section V provides X-precoding design, followed by design of hybrid precoding/combinning in Section VI. Simulation results and conclusions are given in Sections VII and VIII.

**Notations:** Vectors are boldface letters and matrices are boldface capital letters. Scalars are lower case letters. \( A_{i,j} \) represents \( i \)-th row and \( j \)-th column entry in matrix \( A \). The \( ||A||_F \) denotes Frobenius norm of \( A \). Superscripts \( T \) represents transpose and \( H \) represents conjugate transpose. \( \mathbb{R}^{a \times b} \) and \( \mathbb{C}^{a \times b} \) represent space of matrix with size \( a \times b \) in real value and complex value, respectively. The notation tr(\( \cdot \)) denotes the trace operation, and \( |A| \) denotes the cardinality of a set \( A \). The operator diag\( \{a_1, \ldots, a_n\} \) creates a \( n \times n \) matrix with the vector values along the diagonal. The operators \( \Re(\{\mathbf{u}\}) \) and \( \Im(\{\mathbf{u}\}) \) are used to obtain the real and imaginary parts of a complex vector \( \mathbf{u} \). The notation \( E[\cdot] \) denotes the expectation operation. The \( _pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q, x) \) stands for a hypergeometric function [33] and \( Q(\cdot) \) stands for the Q-function. Also, \( f'(\cdot) \) and \( f''(\cdot) \) denote the first and second order derivatives of the function \( f(\cdot) \). And \( \mathcal{CN}(0, \sigma^2) \) is complex Gaussian distribution with zero mean and variance \( \sigma^2 \).

**II. SYSTEM MODEL**

We consider a point-to-point mmWave massive MIMO IRS-aided wireless communication system, transmitting \( N_s \) data streams (see Fig. 1). At Tx, there are \( N_t \) transmit antennas with \( N_{RF}^t \) RF chains, while at Rx, there are \( N_r \) receive antennas with \( N_{RF}^r \) chains, where \( N_s \leq \min(N_{RF}^t, N_{RF}^r) \ll \min(N_t, N_r) \). An IRS equipped with \( M \) passive elements is deployed to enhance communication between Tx and Rx.
A. TRANSMITTER SITE

At the Tx, the information symbol vector \( u = [u_1, \ldots, u_{N_s}]^T \in \mathbb{C}^{N_s} \) is assumed to be drawn from the normalized \( M \)-quadrature amplitude modulation (QAM) square constellation, i.e.,

\[
u_i = \xi \cdot (u_R + j u_I), \quad i = 1, 2, \ldots, N_s,
\]

where

\[
\xi = \left( \frac{2M - 2}{3} \right)^{-\frac{1}{2}}
\]

is used such that the average symbol energy \( E_s = 1 \), and \( u_R \) and \( u_I \) are real and imaginary parts of the non-normalized QAM symbol associated with \( u_i \). Then the information vector \( u \) is first precoded by the X-code (or X-precoder)\(^1\), proposed in \cite{31} (see details in Section II-A1), yielding

\[
s = [s_1, \ldots, s_{N_s}]^T = \Phi u \in \mathbb{C}^{N_s},
\]

where \( \Phi \in \mathbb{C}^{N_s \times N_s} \) is the X-code matrix \cite{31} (see (6) in Section II-A1). The symbol vector \( s \) is further processed by a digital baseband precoder \( F_{BB} \in \mathbb{C}^{N_s \times N_s} \) and an analog RF precoder \( F_{RF} \in \mathbb{C}^{N_s \times N_s} \), leading to the transmitted signal vector

\[
x = F_{RF}F_{BB}\Phi u = F_{RF}F_{BB}s,
\]

where \( s \) is assumed to satisfy \( \mathbb{E}[ss^H] = I_{N_s} \). Also, the transmit power constraint is enforced by normalizing \( F_{BB} \) such that \( \|F_{RF}F_{BB}\|^2_{F} = N_s \).

1) Precoding using X-code

As introduced above, we consider the information symbol vector is precoded by X-code to improve the error performance and gain better channel diversity.

We first recall X-code, proposed in \cite{31} for MIMO system that achieves the best diversity order, and then we show how we adapt the X-code in our system. Without loss of generality, we consider an even \( N_s \). In \cite{31}, a MIMO channel was decomposed (via a SVD operation) into a number of parallel subchannels, and the X-code was used to pair the subchannels with \textit{good and bad diversities}, yielding a total of \( \frac{N_s}{2} \) subchannel pairs. Further, an optimal pairing strategy was proposed in \cite{31}, where each pair has the pairing list of

\[
(k, N_s - k + 1)
\]

for \( k = 1, \ldots, \frac{N_s}{2} \), which is further encoded by the following \( 2 \times 2 \) precoding submatrix

\[
A_k = \begin{bmatrix}
\cos(\phi_k) & \sin(\phi_k) \\
-\sin(\phi_k) & \cos(\phi_k)
\end{bmatrix} \in \mathbb{R}^{2 \times 2},
\]

where \( \phi_k \) denotes the phase rotation in \( \Phi \). Collecting all \( A_k \), \( \forall k \), we form the entire X-code precoding matrix \( \Phi \) as

\[
\begin{bmatrix}
\cos(\phi_1) & \sin(\phi_1) \\
\vdots & \vdots \\
-\sin(\phi_{\frac{N_s}{2}}) & \cos(\phi_{\frac{N_s}{2}})
\end{bmatrix}
\begin{bmatrix}
\cos(\phi_{\frac{N_s}{2}}) & \sin(\phi_{\frac{N_s}{2}}) \\
-\sin(\phi_1) & \cos(\phi_1)
\end{bmatrix}
\]  

In addition, in \cite{31}, phases \( \phi_k \) in (6) and the pairing strategy are designed to improve the overall MIMO diversity.

In this paper, we adopt the same pairing strategy as (4) and the same precoding matrix structure as (6) with submatrices (5). Further, to avoid increasing transmit power, we impose orthogonality constraint on each \( A_k \), which consists of a single phase \( \phi_k \), for \( k = 1, \ldots, \frac{N_s}{2} \).

Later in Section V, we will design the optimal \( \phi_k \), \( \forall k \), to achieve the improved error performance and channel diversity.

B. IRS

As shown in Fig. 1, an IRS is used to assist communications between the Tx and Rx. The IRS has \( M \) passive elements,
each of which can be adjusted to change phase shift by an embedded controller. Let \( \Theta = [\theta_1, \theta_2, \ldots, \theta_M] \) and \( \Theta = \text{diag} \{ \beta e^{j\phi_1}, \beta e^{j\phi_2}, \ldots, \beta e^{j\phi_M} \} \in \mathbb{C}^{M \times M} \) be diagonal phase shift matrix for IRS, where \( \theta_m \in [0, 2\pi) \) and \( \beta \in [0, 1] \) are the phase and amplitude introduced by each IRS element. For simplicity, we set \( \beta = 1 \) to maximize signal reflection in the sequel of this paper.

C. CHANNEL MODEL

From Fig. 1, we let \( G \in \mathbb{C}^{M \times N_t} \) and \( R \in \mathbb{C}^{N_r \times M} \) denote the Tx-IRS and IRS-Rx links, respectively. Then \( H = R \Theta G \in \mathbb{C}^{N_r \times N_t} \) stands for the cascade channel from the Tx to Rx through IRS. Following the Saleh-Valenzuela (SV) channel model [34] for mmWave, assuming the uniform planar arrays (UPAs) at Tx and Rx, the Tx-IRS and IRS-Rx channels are given by

\[
G = \sqrt{\frac{N_t M}{N_t N_R}} \sum_{i,t} \alpha_{t,i} a_{r,i}^{\text{irs}}(\eta_{i,t}^r, \vartheta_{i,t}^r) a_{t,i}^{\text{tx}}(\eta_{i,t}^t, \vartheta_{i,t}^t),
\]

\[
R = \sqrt{\frac{N_r M}{N_r N_T}} \sum_{i,t} \beta_{t,i} a_{t,i} a_{r,i}^{\text{rs}}(\eta_{i,t}^r, \vartheta_{i,t}^r) a_{r,i}^{\text{tx}}(\eta_{i,t}^t, \vartheta_{i,t}^t),
\]

where \( N_t \) and \( N_r \) are the number of transmitting and receiving antennas, respectively.

Using (10), we can decouple analog \( F_{\text{RF}} \) and digital \( F_{\text{BB}} \) beamformers according to strategies in [34], [38]–[40], indicating the hybrid approach performs similarly to fully digital one with a small number of RF chains.

A. DIGITAL PRECODING/COMBINING BEAMFORMING

Let \( F = F_{\text{RF}} F_{\text{BB}} \in \mathbb{C}^{N_r \times N_t} \) and \( W = W_{\text{RF}} W_{\text{BB}} \in \mathbb{C}^{N_t \times N_r} \) denote the digital precoder and combiner [34]. To obtain the optimal \( F \) and \( W \), we apply a SVD on \( H \in \mathbb{C}^{N_r \times N_t} \), yielding

\[
H = U \Sigma V^H = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^H \\ V_2^H \end{bmatrix},
\]

where \( U \in \mathbb{C}^{N_r \times N_r} \) and \( V \in \mathbb{C}^{N_t \times N_t} \) are unitary matrices, and \( \Sigma \) is diagonal matrix of the diagonal entries of singular values in decreasing order. We have submatrices \( U_1 \in \mathbb{C}^{N_r \times N_t} \), \( \Sigma_1 \in \mathbb{C}^{N_r \times N_t} \), and \( V_1 \in \mathbb{C}^{N_t \times N_r} \). The submatrices \( U_2, \Sigma_2 \), and \( V_2 \) are not considered here for computation. Assuming equal power allocation, a fixed X-code matrix \( \Phi \) and an IRS phase shift matrix \( \Theta \), the optimal solution is given by [34]

\[
F^* = V_1, \quad W^* = U_1.
\]

Using (10), we can decouple analog \( F_{\text{RF}} \) and digital \( F_{\text{BB}} \) beamformers according to strategies in [34], [38]–[40], indicating the hybrid approach performs similarly to fully digital one with a small number of RF chains.

B. PERFORMANCE ANALYSIS AND SYSTEM DESIGN CRITERION

Substituting (10) into (8) yields

\[
y = \Sigma_1 \Phi u + \tilde{n} = Bu + \tilde{n},
\]

where \( B = \Sigma_1 \Phi \) is the effective channel matrix, and \( \Phi \) is given in (6). Here, \( \tilde{n} = U_1^H \tilde{n} \) is the noise vector having the same statistic as \( n \).

Let \( \sigma_1 > \sigma_2 > \cdots > \sigma_{N_r} \) denote the diagonal elements of \( \Sigma_1 \) (i.e., the singular values determined by the cascade channel matrix \( H \)). Following X-code pairing strategy in (4), each pair has an equivalent subchannel matrix, \( B_k \in \mathbb{R}^{2 \times 2} \), for \( k = 1, 2, \ldots, \frac{N_t}{2} \), given by

\[
B_k = \text{diag} \{ \sigma_k, \sigma_{N_r-k+1} \} A_k
= \begin{bmatrix} \sigma_k \cos(\phi_k) & \sigma_k \sin(\phi_k) \\ -\sigma_{N_r-k+1} \sin(\phi_k) & \sigma_{N_r-k+1} \cos(\phi_k) \end{bmatrix}.
\]
where $A_k$ is in (5). Let the $k$-th pair be $u_k = [u_k, u_{N_s-k+1}]^T$. As discussed before, a ML detector is adopted at the Rx to detect real and imaginary components of $u_k$ as

$$
\Re(\hat{u}_k) = \arg\min u_k \in \mathcal{S} \|\Re(y_k) - B_k \Re(u_k)\|^2,
\Im(\hat{u}_k) = \arg\min u_k \in \mathcal{S} \|\Im(y_k) - B_k \Im(u_k)\|^2,
$$

(13)

where $\hat{u}_k$ is ML detector output for the $k$-th pair.

Let $P$ denote WER for real component for a given channel realization $B_k$,

$$
P = \frac{2}{N_s} \sum_{k=1}^{N_s/2} \hat{P}_k,
$$

(14)

where $\hat{P}_k$ represents WER for real component of $k$-th pair. Since WERs for real and imaginary parts are same, the overall WER for a given channel $B_k$ is

$$
P = 1 - (1 - P)^2.
$$

Consider the real component of $k$-th pair. For a given channel $B_k$, let $P(\Re(\hat{u}_k) \rightarrow \Re(\hat{v}_k))$ denote the pairwise error probability (PEP), i.e., the probability when $\Re(\hat{u}_k)$ is transmitted and $\Re(\hat{v}_k)$ is detected, and

$$
d_k = \Re(\hat{u}_k) - \Re(\hat{v}_k) = [d_k, d_{N_s-k+1}]
$$

(16)

denotes pairwise difference vector of the real component of the $k$-th pair. We obtain the following upper bound.

**Proposition 1.** For a fixed channel realization $B_k$, the WER for real component of $k$-th pair is upper bounded by

$$
\hat{P}_k \leq \frac{1}{|S_k|} \sum_{\Re(\hat{u}_k) \in S_k} \sum_{d_k \neq 0} P(\Re(\hat{u}_k) \rightarrow \Re(\hat{v}_k))
$$

$$
= \frac{1}{|S_k|} \sum_{\Re(\hat{u}_k) \in S_k} \sum_{d_k \neq 0} Q\left(\frac{\|B_k d_k\|^2}{2N_0}\right),
$$

(17)

where $|S_k|$ denotes the cardinality of $S_k$ (a collection of signal sets for the $k$-th pair of real components) and $|S_k| = M$ for $M$-QAM signalling.

**Proof.** The first inequality follows the standard union bound on PEP for a given channel realization [41]. Second step follows direct computation of PEP with Gaussian noise. □

**Example 1.** We let

$$
a = \sigma^2_k, \quad b = \sigma^2_{N_s-k+1},
$$

(18)

and obtain the upper bound on WER for 4-QAM and a given $B_k$, as

$$
\hat{P}_k \leq Q\left(\frac{a \cos^2(\phi_k) + b \sin^2(\phi_k)}{N_0}\right) + \frac{1}{2} Q\left(\frac{a(1 + \sin(2\phi_k)) + b(1 + \sin(2\phi_k))}{N_0}\right) + \frac{1}{2} Q\left(\frac{a(1 - \sin(2\phi_k)) + b(1 + \sin(2\phi_k))}{N_0}\right).
$$

**Remark 1. (System design criterion)** To minimize overall WER $P$, we need to minimize $\hat{P}_k, \forall k$, and the upper bound in (17) by jointly designing X-code matrix $\Phi$ and the IRS phase matrix $\Theta$ (or equivalently, IRS phase shift vector $\theta$).

**IV. IRS PHASE SHIFT MATRIX DESIGN**

Based on the design criterion, we design IRS phase shift matrix $\Theta$ with fixed X-precoding matrix $\Phi$ and obtain the following Proposition.

**Proposition 2.** For a fixed $B_k$ and $M$-QAM signalling, the WER for the real component is upper bounded by

$$
P \leq -\kappa \text{tr}(\text{HH}^H) + \frac{(M - 1)}{2},
$$

(19)

where $\kappa$ is a very small positive real number.

**Proof.** See Appendix A. □

Hence, our objective problem becomes

$$
\max_{\Theta} \text{tr}(\text{HH}^H)
$$

s.t. $\Theta = \text{diag}\{e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_M}\}.
$$

(20)

Below we provide two different approaches to solve the problem in (20).

**A. ALTERNATING OPTIMIZATION**

Let $r_i \in \mathbb{C}^{N_t}$ be $i$-th column of $R$ and $g_i \in \mathbb{C}^{1 \times N_t}$ be $i$-th row of $G$, we express $H$ as

$$
H = \sum_{i=1}^{M} e^{j\theta_i} r_i g_i.
$$

(21)

Similar to [18], we can obtain

$$
\text{tr}(\text{HH}^H) = \text{tr}(r_m g_m^H r_m^H) + \left\| \sum_{i=1, i \neq m}^{M} e^{j\theta_i} r_i g_i \right\|^2_F
$$

$$
+ 2 \text{Re}\left( e^{-j\theta_m} r_m^H \left( \sum_{i=1, i \neq m}^{M} e^{j\theta_i} r_i g_i \right) g_m^H \right).
$$

(22)

**Proposition 3.** With any given $\{\theta_i\}_{i=1, i \neq m}$, the optimal value of $\theta_m$ is given by

$$
\theta_m^* = \text{Phase}\left( \sum_{i=1, i \neq m}^{M} e^{j\theta_i} r_i g_i \right) \text{g}_m^H.
$$

(23)
Proof. The result can be easily obtained by observing (22).

Within this context, we can first generate a set of IRS phase shifts $\theta$ and then iteratively optimize $\theta_m$, for $m = 1, 2, \cdots, M$, one by one with the other angles fixed. The algorithm above is summarized in Algorithm 1.

**Algorithm 1 Alternating Optimization Approach for Solving (20)**

1: Initialize $\theta = [\theta_1, \theta_2, \cdots, \theta_M].$
2: for $m = 1 \rightarrow M$ do
3: Obtain optimal value of $\theta_m$ of IRS according to (23).
4: end for
5: Check convergence. If yes, stop; if not, go to step 2.
6: Obtain optimal IRS phase shift vector $\theta \leftarrow \theta_m^{\star}$.

Note in Algorithm 1, it can be seen that the value of objective function (20) is non-decreasing during each iteration, which guarantees the convergence of Algorithm 1.

**B. GRADIENT ASCENT OPTIMIZATION**

Let $\hat{G} = GG^H$ and we obtain

$$
\text{tr}(HH^H) = \text{tr}(R\hat{G}\hat{G}^H R^H)
= Q + 2 \sum_{p=1}^{M-1} \sum_{q=p+1}^{M} \left| \hat{G}_{p,q} \sum_{i=1}^{N_r} R_{i,p} R_{i,q}^H \right| 
\times \cos \left( \theta_p - \theta_q + \arg \left\{ \hat{G}_{p,q} \sum_{i=1}^{N_r} R_{i,p} R_{i,q}^H \right\} \right),
$$

where $Q$ is a real number which is given by

$$
Q = \sum_{i=1}^{N_r} \sum_{j=1}^{M} |R_{i,j}|^2 \times \hat{G}_{i,j}.
$$

(24)

It can be seen that (24) is a multi-variable function $F(\theta)$, which increases fastest when the initial data goes into the direction of positive gradient. The gradient of function $F(\theta)$ for each $\theta_m$, $m = 1, 2, \cdots, M$, is given by

$$
\nabla F(\theta_m) = -2 \sum_{q=m+1}^{M} \left| \hat{G}_{m,q} \sum_{i=1}^{N_r} R_{i,m} R_{i,q}^H \right| 
\times \sin \left( \theta_m - \theta_q + \arg \left\{ \hat{G}_{m,q} \sum_{i=1}^{N_r} R_{i,m} R_{i,q}^H \right\} \right)
+ 2 \sum_{p=1}^{m-1} \left| \hat{G}_{p,m} \sum_{i=1}^{N_r} R_{i,p} R_{i,m}^H \right| 
\times \sin \left( \theta_p - \theta_m + \arg \left\{ \hat{G}_{p,m} \sum_{i=1}^{N_r} R_{i,p} R_{i,m}^H \right\} \right).
$$

(25)

After we obtain gradient, we can update the set of IRS phase shifts $\theta$ with a step size $\delta$,

$$
\theta^{(n+1)} = \theta^{(n)} + \delta \nabla F(\theta),
$$

where $\delta \in \mathbb{R}$ is a small number. Accordingly, we summarize the above algorithm in Algorithm 2.

**Algorithm 2 Gradient Ascent Approach for Solving (20)**

1: Initialize $\theta^{(1)} = [\theta_1, \theta_2, \cdots, \theta_M].$
2: repeat
3: Compute the gradient $\nabla F(\theta)$ according to (26);
4: Choose step size $\delta$;
5: Update $\theta^{(n+1)} = \theta^{(n)} + \delta \nabla F(\theta)$;
6: until The gap between values of objective function in (20) between two iterations is below threshold $\varepsilon$.
7: Obtain optimal IRS phase shift vector $\theta$.

In Algorithm 2, the step size $\delta$ is chosen to make sure $\text{tr}(HH^H)^{(n+1)} \geq \text{tr}(HH^H)^{(n)}$ during each iteration, which means the objective function in (20) converges into at least a local maximum optimal value.

**C. COMPUTATIONAL COMPLEXITY COMPARISON**

With the same initial set of $\theta$, the dominant computational complexity comes from IRS phase shift optimization during each iteration. The complexity of alternating optimization approach can be described as $O(N_r N_t M I_1)$, where $I_1$ represents the total number of iterations until convergence. By contrast, the complexity of gradient ascent approach is given by $O(M I_2)$, where $I_2$ represents total number of iterations required to reach convergence.

With $N_r = 16$ and $N_t = 36$ antennas employed at the Tx and Rx respectively, the convergence behavior of two IRS optimization algorithms is shown in Fig 2. The terms "Alter" and "Grad" respectively represent alternating and gradient ascent optimization approaches. It can be seen that alternating algorithm converges quicker than gradient ascent approach, where the previous method attains the maximum value of $\text{tr}(HH^H)$ within 5 iterations and the last one attains the maximum value within 25 iterations. Nevertheless, the gradient ascent approach has much lower computational complexity during each iteration and thus lower overall complexity, as shown in Fig 3. With the same number of passive elements at IRS, the value of $N_r N_t I_1$ is much higher than $I_2$, even if $I_2$ is larger than $I_1$. Thus, alternating optimization algorithm has much higher complexity, especially for large-array Tx and Rx.

**V. PRECODING DESIGN: X-CODE AND X-PRECODER**

As discussed before, both X-code and X-precoder employ the same pairing strategy in (4), and the same precoding matrix structure in (6) (or equivalently, the submatrix structure in (5)). Note that the submatrix in (5) is parameterized by a single phase $\phi_k$, which needs to be designed in both cases to improve the error performance. The only difference between
function (pdf)
\[ f(\sigma_i) = \frac{1}{\Gamma(\lambda_i)\varphi_i^{\lambda_i}}\sigma_i^{\lambda_i - 1}e^{-\frac{\sigma_i}{\varphi_i}}, \quad i = 1, \ldots, N_s, \] (28)
where \( \sigma_i \sim \Gamma(\lambda_i, \varphi_i) \) follows gamma distribution, and \( \lambda_i, \varphi_i \) represent the shape and scale parameters defining this gamma distribution.

Recall (16), and we can obtain the following Proposition.

Proposition 4. For M-QAM signalling, the theoretical upper bound of average WER for real component of the \( k \)-th pair with X-code, where \( k = 1, \ldots, 17 \), can be expressed as

\[
\bar{P}_k \leq \frac{1}{|\mathcal{M}|} \sum_{R(u_k) \in S_k} \sum_{d_k \neq 0} \mathbb{E}_{B_k} \left[ Q \left( \sqrt{\frac{|B_k d_k|^2}{2N_0}} \right) \right]
\]

\[
\leq \frac{1}{|\mathcal{M}|} \sum_{R(u_k) \in S_k} \sum_{d_k \neq 0} \left[ \frac{1}{12} F_0 \left( \frac{\lambda_k + 1}{2}, \frac{\lambda_k}{2} + 1; \frac{A^2 \varphi_i^2}{N_0} \right) \right. 
\]

\[
\times 2 F_0 \left( \frac{\lambda_{N_s-k+1} + 1}{2}, \frac{\lambda_{N_s-k+1}}{2}; \frac{B^2 \varphi_i^2}{3N_0} \right) + 
\]

\[
\times 2 F_0 \left( \frac{\lambda_{N_s-k+1}}{2}, \frac{\lambda_{N_s-k+1}}{2}; \frac{4A^2 \varphi_i^2}{3N_0} \right) \Bigg] .
\] (29)

where \( F_0(\cdot) \) denotes hypergeometric function, the solid horizontal line indicates an empty parameter sequence, \( \lambda_k \) \( (\lambda_{N_s-k+1}) \) and \( \varphi_i \) \( (\varphi_{N_s-k+1}) \) represent shaping and scale parameters of pdfs \( f(\sigma_k) \) \( f(\sigma_{N_s-k+1}) \) in (28), and

\[
A = \cos(\phi_k)d_k + \sin(\phi_k)d_{N_s-k+1}
\]
\[
B = \cos(\phi_k)d_{N_s-k+1} + \sin(\phi_k)d_k.
\] (30)
Proof. See Appendix B. \hfill \blacksquare

We let $h(\phi_k)$ denote the upper bound in (29), a function of $\phi_k$. To find $\phi_k^*$ that minimizes $h(\phi_k)^2$, we need to solve
\begin{equation}
  h'(\phi_k^*) = 0,
\end{equation}
which unfortunately does not have an explicit analytical solution. However, since X-precoding matrix is fixed over all channel realizations, the optimal $\phi_k^*$ can be performed numerically off-line. Alternatively, we can adopt $\frac{\pi}{4}$, $\forall k$, instead of the exact $\phi_k^*$, for simplicity. It approaches the error performance of exact optimal $\phi_k^*$ as demonstrated in Section VII.

2) Diversity analysis for X-code

Here, we provide diversity analysis for the X-coded system. We have the following Proposition.

**Proposition 5.** Consider each pair adopts an optimal $\phi_k^*$, $k = 1, \ldots, \frac{N}{2}$, with 4-QAM signaling. At high SNRs, the diversity gain of each pair is lower bounded by $\max\left(\frac{\lambda_k}{2}, \frac{\lambda_{N_k-k-1}}{2}\right)$. Further, the lower bound on the diversity of the system is determined by the $k$-th pair’s one, i.e., $\max\left(\frac{\lambda_k}{2}, \frac{\lambda_{N_k-k+1}}{2}\right)$, where $k'$ is defined as
\begin{equation}
  k' = \min_{k \in \mathcal{K}} \left(\max\left(\frac{\lambda_k}{2}, \frac{\lambda_{N_k-k+1}}{2}\right)\right),
\end{equation}
where $\mathcal{K} = \{1, \ldots, \frac{N}{2}\}$.

**Proof.** See Appendix C. \hfill \blacksquare

Later in Section VII, we will evaluate the proposed diversity analysis via simulations.

**B. OPTIMAL DESIGN OF X-PRECORDER**

As mentioned before, in contrast to X-code, the phase $\phi_k$ of X-precoder submatrix in (6) changes during every channel realization. Thus we need to design $\phi_k$, $\forall k$, that minimizes the upper bound on WER in (17). To simplify notation, we define $f(\phi_k)$, a function of $\phi_k$, as
\begin{equation}
  f(\phi_k) = \frac{1}{|S_k|} \sum_{\mathbf{d_k} \neq 0} \sum_{\mathbf{u_k} \in S_k} Q\left(\sqrt{\frac{|B_k d_k|^2}{2N_0}}\right),
\end{equation}
which is the upper bound on WER of real components in the $k$-th pair, as in (17). We also only consider $\phi_k \in (0, \frac{\pi}{4}]$. The first derivative of $f(\phi_k)$ is given by
\begin{equation}
  f'(\phi_k) = \frac{1}{|S_k|} \sum_{\mathbf{d_k} \neq 0} \sum_{\mathbf{u_k} \in S_k} -\frac{1}{4\sqrt{\pi N_0}} \times \exp\left(-\frac{X \cos^2(\phi_k) + Y \sin^2(\phi_k) + Z \sin(2\phi_k)}{4N_0}\right) \times \frac{2Z \cos(2\phi_k) + (Y - X) \sin(2\phi_k)}{\sqrt{X \cos^2(\phi_k) + Y \sin^2(\phi_k) + Z \sin(2\phi_k)}},
\end{equation}
\begin{equation}
  \text{with}
\begin{align*}
  X &= ad_k^2 + bd_{N_k-k+1}^2, \\
  Y &= bd_k^2 + ad_{N_k-k-1}^2, \\
  Z &= (a - b)d_{N_k-k-1},
\end{align*}
\end{equation}
where $a, b$ are given in (18) and $d_k, d_{N_k-k-1}$ are given in (16). We then show the selection of optimal $\phi_k^*$ in following Proposition.

**Proposition 6.** The optimal $\phi_k^*$ of X-precoding of $k$-th pair is given by
\begin{equation}
  \phi_k^* = \begin{cases} 
  \frac{\pi}{4} & \frac{\sigma_k}{\sigma_{N_k-k+1}} \leq \tau, \\
  \frac{\pi}{4} & \frac{\sigma_k}{\sigma_{N_k-k-1}} > \tau, 
\end{cases}
\end{equation}
where $\phi_0$ is root of function $f'(\phi_0)$ in (34) and $\phi_0 \notin \{0, \frac{\pi}{4}\}$. $\tau$ is threshold which is root of second derivative of function $f'(\phi = \frac{\pi}{4})$, which is given by
\begin{equation}
  f''(\phi = \frac{\pi}{4}) = \frac{1}{|S_k|} \sum_{\mathbf{d_k} \neq 0} \sum_{\mathbf{u_k} \in S_k} \frac{\exp\left(-\frac{X + Y + 2Z}{2N_0}\right)}{8\sqrt{2\pi N_0}^3 (X + Y + 2Z)^2} \left[4N_0 ((X - Y)^2 + 4XZ + 4YZ + 8Z^2)\right].
\end{equation}
Proof. It can be seen that $f'(\phi_0) = f'(\phi_k = \frac{\pi}{4}) = 0$. In the case of $\frac{\sigma_k}{\sigma_{N_k-k+1}} \leq \tau$, $f'(\phi_k) < 0$ for all $\phi_k \in (0, \frac{\pi}{4}]$ and $f''(\phi_k = \frac{\pi}{4}) > 0$, which means $f(\phi_k)$ is monotonically decreasing and has minimum value when $\phi_k = \frac{\pi}{4}$. In the case of $\frac{\sigma_k}{\sigma_{N_k-k+1}} > \tau$, $f'(\phi_k) \leq 0$ when $\phi \in (0, \phi_0)$ and $f''(\phi_k) > 0$ when $\phi = (\phi_0, \frac{\pi}{4})$. Also, we have $f''(\phi_k = \frac{\pi}{4}) < 0$ under this case. Thus, we force (34) to zero to determine the value of $\phi_0$ and force (36) to zero at $\phi_k = \frac{\pi}{4}$ to find the value of threshold $\tau$, where both problems can be solved in MATLAB. \hfill \blacksquare

The problem of solving $\phi_k^*$ in X-precoder is complex. Nonetheless, we still are able to derive an analytic solution for lower order modulation, discussed below.

**Proposition 7.** The optimal $\phi_k^*$ of X-precoding under 4-QAM can be expressed as
\begin{equation}
  \phi_k^* = \begin{cases} 
  \frac{\pi}{4} & \sin^{-1}\left[\sqrt{\frac{\frac{a}{a-b}}{2} + \frac{a}{a-b} - \frac{a}{a-b} + \frac{b}{a-b}^2}\right], \frac{a}{b} \leq 3 \\
  \frac{\pi}{4} & \sin^{-1}\left[\sqrt{\frac{\frac{a}{a-b}}{2} - \frac{a}{a-b} + \frac{b}{a-b}^2}\right], \frac{a}{b} > 3
\end{cases}
\end{equation}

**Proof.** See Appendix D. \hfill \blacksquare

**Remark.** (Diversity discussions) It can be easily shown that, the lower bounds on the diversity gain of X-code in Proposition 5 (for each pair and the entire system) can be also used for the X-precoder case, since the derivation uses the phase $\frac{\pi}{4}$ for all pairs which may be the suboptimal solution to some pairs in the X-precoder case (see (35), (37)).
VI. HYBRID PRECODING/COMBINING DESIGN

After designing the IRS phase shift matrix $\Theta_{\text{opt}}$, we have the optimal $H_{\text{opt}} = R\Theta_{\text{opt}}G = U_{\text{opt}}\Sigma_{\text{opt}}V_{\text{opt}}^H$. Then the fully digital precoding/combining matrices can be obtained according to (10). In the following, we need to decouple fully digital precoder/combiner into hybrid analog and digital precoder/combiner. Such a problem can be formulated as

$$\min_{F_{\text{RF}}, F_{\text{BB}}} \| F_{\text{RF}} - F_{\text{RF}}F_{\text{BB}} \|_F$$

s.t. $| (F_{\text{RF}})_{i,j} | = 1,$

$$| F_{\text{RF}}F_{\text{BB}} \|_F = N_s.$$  (38)

This problem can also be applied to decoding side. Specifically, we can apply manifold optimization based algorithm in [39] to solve this decoupling problem (see details in [39]).

In summary, the overall algorithm of jointly optimization of transceiver and IRS is summarized in Algorithm 3.

Algorithm 3 Proposed Algorithm for Jointly Design of Transceiver and IRS

Input $R, G, N_s, N_{RF}^t, N_{RF}^r, N_0$

1: Obtain optimal passive beamforming matrix $\Theta_{\text{opt}}$ at IRS based on Algorithm 1/2 in Section III.
2: Obtain optimal X-code or X-precoder $\phi_{X}^*$ in Section IV.
3: Calculate optimal fully digital precoding/combining matrices $F_{\text{RF}}, F_{\text{BB}}$, and hybrid combining matrices $W_{\text{RF}}, W_{\text{BB}}$, based on manifold optimization approach.
4: Obtain hybrid analog and digital precoding matrices $F_{\text{RF}}, F_{\text{BB}}$, and hybrid combining matrices $W_{\text{RF}}, W_{\text{BB}}$.

Output $\Theta_{\text{opt}}, \phi_{X}^*, F_{\text{RF}}, F_{\text{BB}}, W_{\text{RF}}, W_{\text{BB}}$

VII. SIMULATION RESULTS

In this section, we evaluate the WER performance for IRS-aided mmWave massive MIMO system with (or without) X-code (or X-precoder). We model propagation environment as $N_{\text{cl}} = 8$ clusters and $N_{\text{ray}} = 10$ rays per cluster. The azimuth and elevation angles of arrival and departure follow Laplacian distributed [34], [42]. The complex channel gains have distribution $\alpha_{c,i,j}, \beta_{c,i,j} \sim C\mathcal{N}(0, 1)$. The inner-element spacing is half-wavelength.

In all simulations, we set $N_s = 4$ and transmit power at Tx as $P_t = N_s$. There are $N_t = 36$ and $N_r = 16$ antennas employed at the Tx and the Rx, respectively. The number of passive element at IRS is set to $M = 10 \times 10$. RF chains are set to $N_{\text{RF}}^t = N_{\text{RF}}^r = 6$. Totally $10^6$ Monte-Carlo MATLAB simulation runs are adopted for average.

In Fig 5, we demonstrate average WERs versus transmit SNRs of IRS-aided scheme under the case of 4-QAM, where “digital” and “hybrid” respectively represent fully digital precoding/combining and hybrid analog and digital precoding/combing. The terms “withXP” and “withoutXP” respectively represent the system scheme with and without X-precoder, where X-precoding matrices change based on (37). We compare two IRS design methods, alternating (“alter”) and gradient ascent (“grad”) optimization algorithms. We observe that utilization of X-precoder can achieve significantly better WER performance compared to IRS-aided scheme without X-precoder. For example, at WER of $10^{-3}$, the IRS scheme with X-precoder outperforms its counterpart without X-precoder by approximately 6.2dB. It also can be seen that the hybrid precoding/combining performs closely to fully digital precoding/combining, indicating that hybrid precoding/combining with small number of RF chains can asymptotically approach fully digital precoding/combining. In addition, the error performance of two IRS optimization approaches are similar. However, our proposed gradient ascent algorithm has lower overall computational complexity as shown in Fig. 3.

In Fig 6, we illustrate WERs of the system using X-code and X-precoder, respectively. Using distribution fitting tool in MATLAB, the optimized singular values have following distributions: $\sigma_1 \sim \Gamma(14.81, 45.86)$, $\sigma_2 \sim \Gamma(7.28, 26.18)$, $\sigma_3 \sim \Gamma(10.71, 9.28)$ and $\sigma_4 \sim \Gamma(12.44, 5.02)$. Then the optimal X-code are obtained as in (31). We firstly compare WERs of the optimal X-code and the X-code with $\frac{\pi}{4}$ for all pairs. It shows that both X-code (the optimal one and the one with $\frac{\pi}{4}$) have similar performance, demonstrating the simple choice of $\frac{\pi}{4}$ is a practically good solution. Secondly, we compare WERs of these two X-code and X-precoder and find they perform similarly at practical SNRs (or upto WER of $10^{-5}$), and then X-code get slightly worse than X-precoder at high SNRs (or beyond WER of $10^{-5}$). This is owing to the fact that X-code’s phase design is done for average WER and X-precoder’s phase design is performed for the WER of each channel realization. In addition, we evaluate the simulated diversity versus the diversity lower bound demonstrated in Proposition 5 for the optimal X-coded system, where we have $k' = 2$. We find that the system diversity is lower bounded by $\max (\frac{\lambda_1}{\lambda_2}, \frac{\lambda_2}{\lambda_1}) = 5.36$, while the simulated diversity is about 5.37, demonstrating a close match between the diversity

![Figure 5: WER versus SNR under the case of 4-QAM scheme, where $N_t = 36$, $N_r = 16$ and $M = 100$.](image-url)
lower bound and simulated diversity.

Similar observations can be obtained for Fig. 7, where 4-QAM modulation and different system setups are considered, i.e., $N_t = 49$, $N_r = 9$ and $M = 144$. Similar to the above distribution fitting, the optimized singular values’ distributions are obtained as: $\sigma_1 \sim \Gamma(16.06, 53.43)$, $\sigma_2 \sim \Gamma(6.39, 32.70)$, $\sigma_3 \sim \Gamma(9.02, 10.43)$ and $\sigma_4 \sim \Gamma(10.96, 5.70)$. Then the diversity lower bound presented in Proposition 5 is $\max \left( \frac{\lambda_1}{2}, \frac{\lambda_2}{2} \right) = 4.51$, which is close to the simulated $4.53$.

In Fig. 7, we observe two IRS optimization algorithms approach similar results, and hybrid precoding/combining performs closely to fully digital precoding/combining. These observations can be also found in Fig. 8 with 16-QAM signalling and X-precoder.

Finally, in Fig. 9, we demonstrate error performance of IRS-aided communication system with different value of IRS elements. Assuming UPA is employed at IRS, we observe that WERs decrease as $M$ increases.

VIII. CONCLUSIONS

In this paper, we considered an IRS-aided mmWave massive MIMO with hybrid beamforming/combining, where the information symbols are precoded by X-code (or X-precoder). We first designed transceiver digital (or hybrid) active beamforming. Then we derived an upper bound on WER, based on which we jointly optimize IRS phase shifts and X-code (or X-precoder) to enhance WER performance. Specifically, we devised two IRS optimization algorithms based upon alternating and gradient ascent optimization approaches and compared their computational complexity and convergence. Further, we designed X-code and X-precoder to minimize the average WER and WER, respectively. We also provided their diversity analysis. Finally, by simulations, we observe that gradient ascent optimization has same performance as alternating optimization, but with a lower computational complexity. Also, we observe that adopting X-code and X-precoder techniques can significantly improve error performance.

APPENDIX A

PROOF OF PROPOSITION 2

For $M$-QAM signalling, using the improved exponential bounds in [43]

$$Q(x) \leq \frac{1}{2} \exp \left( -\frac{x^2}{2} \right), \quad x > 0,$$

then (17) becomes

$$\hat{P}_k \leq \frac{1}{2M} \sum_{\mathcal{R}(\mathbf{w}_k) \in \mathcal{S}_k} \sum_{d_k \neq 0} \exp \left( -\frac{||\mathbf{B}_kd_k||^2}{4N_0} \right)$$

To simplify notation, we let $y = \frac{||\mathbf{B}_kd_k||^2}{4N_0}$. We consider $0 < y < \frac{1}{\kappa} - \epsilon$, where $\kappa, \epsilon$ are some positive real numbers, such that the following inequality is satisfied

$$\exp(-y) \leq -\kappa y + 1.$$

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Then \( (40) \) becomes
\[
\hat{P}_k \leq \frac{1}{2M} \sum_{\mathcal{S}(u_k) \in S_k} \sum_{d_k \neq 0} (-\kappa y + 1) = \frac{(M - 1)}{2} \left( \frac{M}{M - 1} \right) \sum_{\mathcal{S}(u_k) \in S_k} \sum_{d_k \neq 0} (-\kappa y + 1) = \frac{(M - 1)}{2} \left( -\kappa E(y) + 1 \right).
\]
(42)
where the operator \( E(\cdot) = \sum_{\mathcal{S}(u_k) \in S_k} \sum_{d_k \neq 0} (-\kappa y + 1) \) is to obtain the average squared distance of each pairwise component of \( d_k \) and
\[
E(y) = \frac{a + b \mathcal{M} E_s}{4N_0 \mathcal{M} - 1}.
\]
(43)
Substituting (43) into (42) yields
\[
\hat{P}_k \leq \frac{(M - 1)}{2} \left( -\kappa (a + b) \mathcal{M} E_s + 1 \right).
\]
(44)
For \( \mathcal{M} \)-QAM signalling, we have set \( E_s = 1 \). Using (14), we have
\[
\hat{P} \leq \frac{-\kappa M 4^s}{4N_0 N_s} \left( \frac{\sigma_i^2}{4N_0 N_s} \right)^2 + \frac{(M - 1)}{2} \left( -\kappa (a + b) \frac{\mathcal{M} \mathcal{M} E_s}{4N_0 \mathcal{M} - 1} \right) \leq \frac{-\kappa M tr(\mathbf{H} \mathbf{H}^H)}{4N_0 N_s} + \frac{(M - 1)}{2}.
\]
(45)
where \( (I) \) becomes equality when \( N_s = \text{rank}(\mathbf{H}) \) and the result in (19) can be easily observed.

**APPENDIX B**

**PROOF OF PROPOSITION 4**

The average PEP of the \( k \)-th pair is given by
\[
\text{PEP}_k = \mathbb{E}_{\mathbf{B}_k} \left[ Q \left( \frac{\left| \mathbf{B}_k \mathbf{d}_k \right|^2}{2N_0} \right) \right].
\]
(46)
To facilitate expression manipulations, we approximate \( Q \)-function as [43]
\[
Q(x) \approx \frac{1}{12} e^{-\frac{x^2}{2}} + \frac{1}{4} e^{-\frac{x^2}{2}}.
\]
(47)
We model singular values \( \sigma_k \) and \( \sigma_{N_s-k+1} \) as random variables \( \omega_1 \) and \( \omega_2 \). Then, \( \text{PEP}_k \) can be expressed as
\[
\text{PEP}_k = \frac{1}{\Gamma(\lambda_k) \Gamma(\lambda_{N_s-k+1}) \varphi_k \varphi_{N_s-k+1}} \int_0^\infty \int_0^\infty \omega_1^{\lambda_k-1} \omega_2^{\lambda_{N_s-k+1}-1} \exp \left( -\frac{\omega_1}{\varphi_k} - \frac{\omega_2}{\varphi_{N_s-k+1}} \right) \left[ \frac{1}{12} \exp \left( -\frac{\omega_1^2 A^2 + \omega_2^2 B^2}{4N_0} \right) + \frac{1}{4} \exp \left( -\frac{\omega_1^2 A^2 + \omega_2^2 B^2}{3N_0} \right) \right] d\omega_1 d\omega_2.
\]
(48)
From [44], we have function in the form
\[
\Phi(p, q, m) = \int_0^\infty x^p e^{-q x^2 - m x} \ dx = \frac{m \Gamma(p + 1)}{2^{p+2} q^{-3/2}} \left( \frac{p + 2}{2} \right) u \left( 2, 2, \frac{m^2}{4q} \right),
\]
(49)
where \( U(\cdot) \) denotes confluent hypergeometric function of the second kind. From [45, 13.6.21], we can replace \( U(\cdot) \) function as
\[
U(a, b, z) = z^{-a} F_0(a, 1 + a - b; -z^{-1}) \]
(50)
Then the result in (29) can be obtained by substituting (49) and (50) into (48).

**APPENDIX C**

**PROOF OF PROPOSITION 5**

For the \( k \)-th pair, we set the near-optimal phase \( \phi_k = \frac{\pi}{4} \), 4-QAM signalling, and (19) becomes
\[
\hat{P}_k \leq \mathbb{E}_{\sigma_k, \sigma_{N_s-k+1}} \left[ 2Q \left( \frac{\sigma_k^2 + \sigma_{N_s-k+1}^2}{2N_0} \right) \right] + \frac{1}{2} Q \left( \frac{2\sigma_k^2}{N_0} \right) + \frac{1}{2} Q \left( \frac{2\sigma_{N_s-k+1}^2}{N_0} \right).
\]
(51)
At high SNRs, we can discard the first part in the summation due to its smaller value compared to the sum of the other two parts. We also have
\[
\frac{N_0}{4\sigma_k^2} \rightarrow 0, \quad \frac{N_0}{4\sigma_{N_s-k+1}^2} \rightarrow 0.
\]
(52)
From [45, 13.2.18], we can obtain that
\[
U(a, b, z) = \frac{\Gamma(b-1)}{\Gamma(a)} z^{-a} + \frac{\Gamma(1-b)}{\Gamma(a+b)} z^{-a+b} + O(z^{-a-b}),
\]
\[
z \rightarrow 0, 1 \leq b < 2, b \neq 1.
\]
(53)
Solving (51) using the rule in (53) yields
\[ \bar{P}_k < \frac{1}{48} F_0 \left( \frac{\lambda_k + 1}{2} \frac{\lambda_k}{2} - \frac{4\varphi_k^2}{N_0} \right) + \frac{1}{16} F_0 \left( \frac{\lambda_k + 1}{2} \frac{\lambda_k}{2} - \frac{16\varphi_k^2}{3N_0} \right) \]
\[ + \frac{1}{48} F_0 \left( \frac{(\lambda_k - k - 1)(\lambda_k - k + 1)}{2} \frac{\lambda_k}{2} - \frac{4\varphi_{k-1}^2}{N_0} \right) + \frac{1}{16} F_0 \left( \frac{(\lambda_k - k - 1)(\lambda_k - k + 1)}{2} \frac{\lambda_k}{2} - \frac{16\varphi_{k-1}^2}{3N_0} \right) \]
\[ \propto N_0^{\max} \left( \frac{\lambda_k}{2}, \frac{\lambda_k - k + 1}{2} \right). \]

(54)

Since we select the phase \( \frac{\pi}{4} \), then \( \max \left( \frac{\lambda_k}{2}, \frac{\lambda_k - k + 1}{2} \right) \) is the lower bound of the diversity gain of the \( k \)-th pair. Applying (14) to the average WER, it can be easily verify that the lower bound on the diversity of the system is determined by the pair with the minimum value of \( \max \left( \frac{\lambda_k}{2}, \frac{\lambda_k - k + 1}{2} \right) \) for \( \forall k \in K \).

APPENDIX D
PROOF OF PROPOSITION 7

According to (19), it can be found that WER is dominated by the minimum value of summation parts. Let
\[ d_1 = a \cos^2(\phi_k) + b \sin^2(\phi_k), \]
\[ d_2 = a \sin^2(\phi_k) + b \cos^2(\phi_k), \]
\[ d_3 = a(1 + \sin(2\phi_k)) + b(1 - \sin(2\phi_k)), \]
\[ d_4 = a(1 - \sin(2\phi_k)) + b(1 + \sin(2\phi_k)). \]

(55)

Then the optimization problem can be written as
\[ \phi_k^* = \arg \max_{\phi_k \in [0, \frac{\pi}{4}]} \min \{d_1, d_2, d_3, d_4\}. \]

(56)

Since \( \sigma_1 > \sigma_2 > \cdots > \sigma_{N_0} \), we can figure out that \( d_1 > d_2 \) and \( d_3 > d_4 \). Also, \( d_2 > d_4 \) only if \( b > 1 - \frac{1}{\sin^2(\phi_k) + \sin(2\phi_k)} = g(\phi_k). \)

(57)

It can be observed that \( g(\phi_k) \) is a monotonically increasing function and reaches maximum at \( g(\phi_k = \frac{\pi}{4}) = \frac{1}{3} \). Thus, we have two cases to consider.

If \( \frac{\pi}{4} \leq d_2 \), then \( d_2 \) will be mimin over \( \{d_1, d_2, d_3, d_4\} \). Since \( d_2 \) is monotonically increasing, the optimal X-precoding should be set to \( \phi_k^* = \frac{\pi}{4} \) to maximum \( d_4 \), which leads to minimum WER.

If \( \frac{\pi}{4} > d_2 \), suppose \( d_2 = d_4 \) at \( \phi_k^* \). Let \( d_4 \) be mimin over the range \( \phi_k \). \( d_4 \) is minim. \( d_4 \) is monotonically decreasing, therefore we can conclude that optimal design is \( \phi_k^* = \phi_k \). By solving function
\[ g(\phi_k) = 1 - \frac{1}{\sin^2(\phi_k) + \sin(2\phi_k)} = b, \]

(58)

we can obtain the solution in (37).

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