Observational Probes of Holography with Quantum Coherence on Causal Horizons

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There is much recent development towards interferometric measurements of holographic quantum uncertainties in an emergent background space-time. Despite increasing promise for the target detection regime of Planckian strain spectral density, the latest motivating theoretical ideas have not been connected to a phenomenological model of observables measured in a realistic experiment. This manuscript proposes a candidate model, based on the central hypothesis that all horizons are universal boundaries of coherent quantum information — where the decoherence of space-time happens for the observer. The prediction is motivated by ’t Hooft’s algebra for black hole information that gives coherent states on horizons, whose spatial correlations were shown by Verlinde and Zurek to also appear on holographic fluctuations of causal boundaries in flat space-time (conformal Killing horizons). Time-domain correlations are modeled from Planckian jitters whose coherence scales match causal diamonds, motivated by Banks’ framework for the emergence of space-time and locality. The universality of this coherence on causal horizons compels a multimodal research program probing concordant signatures: An analysis of cosmological data to probe primordial correlations, motivated by Hogan’s interpretation of well-known CMB anomalies as coherent fluctuations on the inflationary horizon, and upcoming 3D interferometers to probe causal diamonds in flat space-time. Candidate interferometer geometries are presented, with a modeled frequency spectrum for each design.

Introduction

The Fermilab Holometer [1–4], proposed in 2009 and in operation through 2019, was the first instrument commissioned to directly search for quantum uncertainties in space-time, posited to be measurable in the regime of Planckian strain spectral density if the background space-time is emergent from a quantum system with holographic total degrees of freedom [5–8]. This line of research, initially based on heuristic arguments about the large correlations necessary for such dimensional reduction of information in the Planckian degrees of freedom of background space-time, was met with much controversy. In recent years, however, the motivating theoretical ideas have been reformulated with much more rigorous treatment by Banks, Verlinde, and Zurek [9–11]. This new understanding has yet to result in a phenomenological model of observables measured in a realistic experiment, which is the aim of this manuscript. Presently, two major experiments are being advanced to succeed the Holometer with significant new technologies: the GQuEST experiment being planned by a Caltech-based team [12], and an experiment already being commissioned at Cardiff University [13, 14] as part of the Quantum-enhanced Interferometry for New Physics (QI) research program under the UK’s Quantum Technologies for Fundamental Physics (QTFP) initiative. The QI experiment at Cardiff was specifically designed with the future capability to construct the most promising configurations proposed here.

A standard perspective, based on the framework of local effective field theory, is that quantum effects in space-time manifest at the Planck scale \( \ell_P \equiv c t_P \), where \( t_P \equiv \sqrt{\hbar G/c^5} = 5.4 \times 10^{-44} \) sec. However, there are strong indications that space-time is emergent from the thermodynamics of covariant causal structures [15, 16], and that the holographic entropy of a black hole \( S_{BH} = k_B A/4 \ell_P^2 \) (where \( A \) is the area of the horizon) can be generalized to a covariant entropy bound that might apply universally [17–19]. If one takes this seriously in flat space-time, the density of information in a causal diamond — the number of microscopic degrees of freedom per volume — decreases as the inverse of the system scale. This picture necessitates that the space-time degrees of freedom have large correlations in the infrared, holistically dependent on the extent of the system, nonlocally encoded via the bounding causal structures.

An estimate of holographic space-time uncertainties in this picture [5–11] says that for an observational system of scale \( L = c \tau \), a causal diamond delimiting its measurement should embody fluctuations of variance \( \langle \delta L/L \rangle^2 \sim t_P/\tau \). As a frequency-domain signal, the strain \( h = \delta L/L \) has a power spectral density (PSD) of \( \sim t_P \) (in units of inverse frequency) over a bandwidth \( \sim 1/\tau \) (in units of frequency). This “Planckian random walk” scaling is analogous to the standard quantum positional uncertainty \( \langle \Delta x^2 \rangle \sim \hbar/\tau m \) of a mass \( m \) over time \( \tau \), and corresponds to the scale of quadrupolar distortions a black hole horizon needs in order to radiate at the standard Hawking flux, one graviton of wavelength \( c \tau \) per time \( \tau \). Heuristically, if each causal diamond smaller than the system size adds a Planck-scale jitter with a coherence scale matching its boundary, they accumulate like a random walk over \( \sim \tau/t_P \) nested layers, creating spacelike fluctuations much larger than expected in standard local effective field theory by the same factor.

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This paper presents a candidate model of quantum space-time phenomenology in this regime of Planckian strain spectral density. A few important theoretical developments are reviewed, moving beyond the heuristic arguments above and rigorously establishing that low-energy, macroscopic quantum gravity could have significant departures from classicality of this magnitude due to holography. A set of general principles is proposed to encapsulate the characteristic physics implied by these developments. Two geometric configurations are identified in which real-world observables measured in an interferometer are expected to couple to the posited effect, and their design parameters optimized such that the magnitude of the quantum space-time uncertainty and the instrumental response are set at levels attainable in realistic state-of-the-art instruments in the near future. Experimental signatures are modeled, including time-domain and frequency-domain spectra for the geometries proposed.

The reader is strongly encouraged to give careful consideration to the key literature cited in the following section. These seminal works represent important, fundamental departures from the mainstream paradigms of the quantum gravity community.

**Background and Motivation**

It has long been understood that standard local effective field theory results in infrared catastrophes at surprisingly small scales. The Cohen-Kaplan-Nelson bound [20], derived by simply counting all the possible modes of the QFT in a given system volume and imposing the black hole entropy bound while insisting that there be no states with Schwarzschild radius larger than the system size, shows that for an ultraviolet cutoff at the QCD scale $\Lambda \sim 200$ MeV $\approx 1.7 \times 10^{-20} m_P$, the system is subject to an infrared cutoff at the $L \sim \Lambda^{-2} \approx 4 \times 10^{-39} \ell_P \approx 60$ km scale (in Planck units) to avoid such failures. This UV-IR scaling coincides neatly with the Planckian strain spectral density scaling discussed here: if the system size $L$ serves as an IR bound, the corresponding Planckian random walk uncertainty for the underlying space-time $\langle \delta L^2 \rangle \propto \sqrt{L} \sim \Lambda^{-1}$ matches the localization scale for field modes at the UV cutoff [21].

This problem has not attracted much serious effort towards a solution, despite being one of the most significant failures framing the fundamental issues in quantum gravity, partly because of blind spots in our standard frameworks, and partly because addressing these shortcomings requires difficult sacrifices of key notions we have presupposed as essential building blocks in our physical theories. The lesson of the CKN bound is that a definite background space-time manifold that is smooth all the way down to Planckian microstates simply contains too much information (relative to no background), when the true degrees of freedom of the quantum space-time system are far more limited, necessitating large correlations in the background space-time itself. Banks has argued for years that the AdS/CFT duality is failing to account for these background degrees of freedom in its holographic projection, and that the cosmological constant is a boundary condition for these omitted degrees of freedom [22, 23]. In AdS space, the built-in boundary with its negative curvature structure hides this infrared issue for most quantum fields by construction. In contrast, consider the problems encountered trying to describe quantum space-time modes using gravitational effective field theory in a finite volume, say, a causal diamond [24]: we want to use unitarity, which means we have to use a non-Lorentz-invariant gauge, and furthermore, the only gauge-invariant things we calculate in gravity are scattering amplitudes at infinity — if we insist that the time slices stay within the causal boundary, as one ought for the problem discussed here, the solution would require a non-translation-invariant gauge. There is no known well-defined finite space-time setup for gravitational effective field theory that meets these conditions.

The same flaw was alternatively explained from the general relativist’s perspective in an illuminating essay by Hollands and Wald [25]. The key issue is that standard quantum field theory does not fully capture the nonlocality of quantum mechanics even in the low-energy effective degrees of freedom. In quantum mechanics, nothing happens on definite paths in space-time (world lines); everything exists in a superposition state. The active gravity from the mass-energy of such a state is necessarily nonlocal, and so is its effect on space-time, including the distortion of causal structures. This concern is usually dismissed as an insignificant correction, and only considered in the context of thought experiments where extremely large coherent quantum states of matter can actually create indeterminate superpositions of different causal orderings [26]. However, the Hollands-Wald manuscript demonstrates that these nonlocal correlations in the background metric in fact entirely invalidate the basic underlying construction of QFT in the context of the space-time vacuum. The renormalization of a field theory requires subtracting infinities that are interpreted as the physical modes of the vacuum, but this process is necessarily global in character: “An individual mode will have no way of knowing whether its own subtraction is correct unless it ‘knows’ how the subtractions are being done for all the other modes” [25]. To enforce locality and covariance, the standard QFT prescription for decomposing a quantum field into modes resorts to acausally, unphysically constraining the system at infinity.
independently in all directions. This is true even in a low-energy, flat space-time example, where the renormalization prescription subtracts the vacuum energy of the modes that hypothetically would have been present in a globally Minkowskian universe, instead of the ones that are actually present in a particular QFT system. In the real universe, distortions in causal structure caused by modes with wavenumber \( k \to 0 \), including events at distance \( R \to \pm \infty \), reshape the QFT mode decomposition and their mapping onto events in space-time on smaller scales. The typically assumed classical background with definite causal structure, on which the fields are defined, is subtly altered at all scales in a way that makes it impossible to achieve local and covariant renormalization without considering the holistic aspects of the QFT system. According to Hollands and Wald, this failure to correctly subtract from the low energy modes of the QFT is the source of the cosmological constant problem. A meticulous, informative manuscript by Stamp [27] has also laid out similar fundamental issues, with a proposed solution of rewriting the path integral formalism of QFT with world lines that are reformulated to include these nonlocal correlations in space-time.

A promising framework for constructing locality from a quantum system is found in the Holographic Space-Time (HST) theory proposed by Banks and Fischler [22, 28–30]. In this theory, which is formulated without any background space-time, every causal diamond has its own Hilbert space, whose dimension scales holographically with the size of the causal boundary. Nested causal diamonds are subspaces via tensor factorizations, and overlapping causal diamonds are similarly handled via tensor products. Information is delocalized within the boundaries of causal diamonds, which determine the entanglement of geometrical states. Entangled states of different world lines undergo “fast scrambling” outside of intervals set by causal diamonds that scale with their separation. The viewpoint is that “The Holographic Principle tells us that both the causal structure and conformal factor of the space-time geometry are encoded as properties of the quantum operator algebra. The space-time geometry is not a fluctuating quantum variable, but, in general, only a thermodynamic / hydrodynamic property of the quantum theory.” This theory is general enough to accommodate field theory in the appropriate limits, although a formulation recovering the Standard Model has not yet been enunciated. Importantly, it provides a precisely controlled framework for the amount of correlation between the quantum geometric information contents of different causal volumes demarcated by their respective causal diamonds. Even though a well-defined connection to real space-time observables remains unclear, the framework provides basic principles about the scaling and localization of quantum geometric information that will guide our estimates of time-domain correlations for experimental measurements in the next section. We will rely extensively on the notion that fast scrambling of information happens on the boundaries of causal diamonds (conformal Killing horizons), and that we may measure coherent quantum effects of geometry in our time-domain correlations when our observables are appropriately coupled to space-time within those causal boundaries.

For the nonlocal spatial correlations, it is instructive to take a brief detour to consider a concrete example that cannot be captured by a field theoretic approach [31]. Consider a case where massive particles are decaying to EPR-like states of oppositely propagating photons, and the direction of decay is indeterminate — as in, the photons are created in S-wave states, as isotropic superpositions of all directions. The gravitational shock waves from such decays result in coherent quantum gravitational fluctuations of causal diamond boundaries on all scales. Statistically, the angular correlations are approximately quadrupolar and independent of scale. For a measurement system in which the number of decaying particles is limited by the gravitational binding energy corresponding to its size, with an ultraviolet cutoff at the Planck mass, the amount of distortion in causal structure scales linearly with the duration of the measurement, just as in the holographic Planckian random walk scaling described above. This example is not intended to be an appropriate model for our proposed interferometric experiment, where the instrument measures fluctuations in empty space-time and there is no active source at the center of the causal diamond. In a realistic model, the fabric of space-time is constructed without a background as in the HST theory, via intersecting causal diamonds whose vacua are filled with virtual states that pop in and out of existence (zero mean, nonzero mean square power). The coherent quantum gravitational fluctuations on these causal diamond boundaries originate from the relationships among different observers and the incoming / outgoing information among their respective causal diamonds.

A pathbreaking model of quantum information on black hole horizons derived by ‘t Hooft [32–36] helps us develop intuition for what kind of coherent quantum gravitational fluctuations are possible on causal horizons when quantum modes are traveling in and out of the boundary. This model separates the transverse operators from the radial ones, and combines a quantum treatment of the radial components of particle states together with the radial components of gravitational frame dragging. For a particle state entering the horizon of an eternal Schwarzschild black hole, by

† These works comprise judicious arguments well grounded in classical quantum mechanics and general relativity, and should be regarded separately from the more controversial works on interpretations of quantum mechanics by the same author.
correctly accounting for the gravitational back reaction across the surface of the horizon, the model finds it entangled with another particle state at its antipode leaving the horizon. The antipodal entanglement is in both space and time—between antipodes in 3-space, with a sign inversion in time—such that the position operator of an outgoing particle is associated with the momentum operator of an incoming one at the antipode, and vice versa. The resulting topology appears quite strange, but this solution is mathematically shown to uniquely restore unitarity to black hole information when the effects of gravitational back reaction are properly included. All of the information is nonlocally encoded on the boundary, as coherent states on the horizon. Reminiscent of the Banks-Fischler theory where causal diamond boundaries are fast scramblers, the causal horizon in this black hole model constitutes a kind of quantum-classical boundary— a boundary of coherent quantum information. In this perspective, to an observer outside the boundary, the delocalized interior may as well not exist. The entire black hole is a single quantum object, akin to a hydrogen atom, and its quantum states can be expanded in terms of spherical harmonics. Unlike the previous example with approximately quadrupolar correlations, the expansion here only accepts odd harmonics due to the antipodal antisymmetry between incoming and outgoing modes. The operator algebra, written in null coordinates $u^\pm$, is:

\[
\begin{align*}
    u^\pm(\Omega) &= \sum_{\ell,m} u^{\pm}_{\ell m} Y_{\ell m}(\Omega) \\
p^\pm(\Omega) &= \sum_{\ell,m} p^{\pm}_{\ell m} Y_{\ell m}(\Omega) \\
    [u^{\pm}_{\ell m}, p^{\pm}_{\ell' m'}] &= i\delta_{\ell\ell'}\delta_{m m'} \\
    u^{-}_{\ell m} &= \frac{8\pi G}{\ell^2 + \ell + 1} p^{+}_{\ell m} \\
    \left[ u^{+}_{\ell m} , u^{-}_{\ell' m'} \right] &= \frac{8\pi G}{\ell^2 + \ell + 1} \delta_{\ell\ell'} \delta_{m m'} 
\end{align*}
\]  

A further breakthrough was made by Verlinde and Zurek [9] when a similar angular correlation function was calculated for quantum gravitational fluctuations on causal diamonds (conformal Killing horizons) in holographic flat space-time. A key conjecture is established, that there is a universal effect of metric fluctuations near horizons, because standard thermodynamics near a black hole horizon equally applies near null surfaces associated with boundaries of finite regions in flat space-time, e.g. a Rindler type horizon. Using a topological black hole coordinate transformation on a causal diamond in flat space-time, the paper gives the following correlation function on the bounding 2-sphere:

\[
\left\langle \delta u^-(r_1) \delta u^+(r_2) \right\rangle = \frac{1}{\sqrt{2\pi}} \ell_p L \cdot G(r_1, r_2) \quad \text{where} \quad G(r_1, r_2) = \sum_{\ell,m} \frac{Y_{\ell,m}(r_1) Y^*_{\ell,m}(r_2)}{\ell^2 + \ell + 1}
\]  

Importantly, this correlation scales linearly with the system size $L$ like a Planckian random walk, differing from the one generated by Eq. (1) by a factor $L/(\sqrt{8\pi \ell_p})$. The difference originates from a shortcoming of the ’t Hooft prescription in handling holography: it only quantizes and calculates gravitational effects for the radial parts of the states, and in the transverse direction, simply applies a mode cutoff (or an angular filter) at the Planck mass where the transverse gravitational effects become significant. While this provisionally gives the correct number of degrees of freedom, a proper quantum treatment of the transverse parts is clearly needed. The Verlinde-Zurek model also only calculates fluctuations for the longitudinal components, but the effect is derived based on a 3D holographic system, borrowing from Marolf’s estimation of the quantum width of a black hole horizon from its thermodynamic fluctuations [37].

Verlinde and Zurek have reproduced this result in the context of AdS/CFT [10], using thermodynamic arguments that carefully avoid the flaws of field theory pointed out in the Banks-Fischler and Hollands-Wald papers and take into account some of the holistic aspects of the QFT system. This was followed up by a stupendous, elegant paper by Banks and Zurek [11], which generalizes a key result from the AdS/CFT calculation: thermodynamic fluctuations in the modular Hamiltonian of a causal diamond are equal to the entanglement entropy. The new conjecture, deriving from near-horizon thermodynamics via a conformal field theory of the vacuum states, applies broadly to flat space, the cosmological horizon of dS, and AdS Ryu-Takayanagi diamonds, but not to large diamonds or black holes in the bulk of AdS. It should be noted, however, that all of these descriptions based on field theory and metric fluctuations are conceptually different from the ’t Hooft models. They do not retain the insight from ’t Hooft that the space-time correlations are described by coherent quantum states that macroscopically extend across the entire horizon, which cannot be described by local metric fluctuations. Indeed, while the Verlinde-Zurek calculation gives the same spherical harmonic modes that respect the spherical symmetry of the system, it does not give an $S$-matrix in the same way that ’t Hooft’s “hydrogen atom” model of the quantum black hole handles information propagating in and out of the system. As such, the Verlinde-Zurek model also does not feature any antipodal antisymmetry between incoming and outgoing modes like the ’t Hooft model does, and does not select for the odd harmonics.
A most compelling reason to study coherent quantum fluctuations on horizons comes from signatures of primordial fluctuations — the only known observations of active gravitational effects from quantum states. It is generally understood that the large scale structure we see today originates from distortions in space-time caused by quantum fluctuations in the early universe, which became permanently “frozen in” as they crossed the inflationary horizon. The cosmic microwave background on large angular scales also preserves a mostly intact map of the gravitational potential on our cosmic horizon during this process. In the absence of a theory of quantum gravity, existing models of inflation resort to patching QFT modes onto a classical background space-time. This works very well at smaller angular scales $\ell \gtrsim 30$, where most of the analysis is done to avoid cosmic variance, or the difference between possible realizations of the universe from a quantum process. However, at the largest angular scales $\ell \lesssim 30$, numerous well-known anomalies persist with $p$-values in the $0.1 – 0.5\%$ range [38], which Hogan, Meyer, and colleagues have shown only become more compelling with more careful analysis [39–41]. For example, the temperature anisotropy 2-point correlation function is conspicuously close to exact zero at $90^\circ$ angle (within $3\mu\text{K}^2$), along with a strange dominance of odd multipoles over even ones in the angular power spectrum ($p < 0.003$ at $\ell = 28$) [38, 39]. The standard interpretation is to assume that these are just individual statistical anomalies, but interpreted in the context of possible symmetries on the horizon that might cause a conspiracy of interrelated anomalies, our universe looks exceedingly rare among thousands of simulated realizations based on the standard model [40, 41]. Given that this QFT based model was never founded on consistent, well-motivated principles — with perhaps even some unphysical approximations used, such as plane wave modes — it would behoove the community to seriously consider alternatives. There may be signatures of quantum gravity that are easily reachable simply by updating models of inflation with coherent delocalized quantum states on inflationary horizons instead of local effective fields on classical backgrounds [41–44].

Design Principles and Interferometer Geometry

The ’t Hooft and Verlinde-Zurek models provide much insight about what kind of quantum fluctuations — and correlation modes thereof — are to be expected on causal horizons. However, translating these theorized signatures to observables measured in a real experiment is a highly nontrivial process. In a full theory of quantum gravity, merely identifying the operators corresponding to the observables would be sufficient. The ’t Hooft algebra, Eq. (1), comes closest to identifying the microstates, but it was derived on a Schwarzschild metric, and it is unclear how to map it onto a background independent Hilbert space for flat space-time. It also does not quantize the transverse modes at all, and does not give the correct magnitude of thermodynamic fluctuations on the horizon. The Verlinde-Zurek model, Eq. (2), on the other hand, simply gives you correlated metric fluctuations from the entanglement entropy of a CFT without identifying the quantum states, but is calculated for a causal diamond in flat space-time. One might naively expect this to be directly measurable in a Michelson interferometer that inscribes the diamond (Fig. 1), but the expression contains a logarithmic divergence as $|\mathbf{r}_1 - \mathbf{r}_2| \to 0$. This “autocorrelation” limit can be absorbed into the definition of $L$ for a macroscopic device where only the low $\ell$ modes correlated over larger transverse scales are detectable, but interpreted straightforwardly as metric fluctuations on causal diamonds, regulated only by a Planckian transverse mode cutoff, the steep difference in correlation power at fine angular scales would lead to a blurring of astrophysical images over cosmological distances, which is already ruled out by existing data [45].

In our design, we will take seriously the ’t Hooft picture of coherent quantum states extended across the entire causal horizon. There is no definite effect until a specific measurement has been made, where the observable is subject to the specific scales and characteristics of the device (such as a telescope). The quantum indeterminacies are not mapped onto local metric fluctuations. Because the eventual full theory must be independent of a classical definite background, we will insist that space-time cannot be measured without specifying an answer to the “relative to what?” question, just as a gravitational potential cannot be defined locally without a reference to another observer. This leads us to the first guiding principle of our experimental design:

1. All causal horizons are universal boundaries of coherent quantum information, where the decoherence of space-time happens for the observer.

This is posited as an alternative to the Verlinde-Zurek postulate about the universality of metric fluctuations on horizons. Similar to Verlinde-Zurek, we take the view that the coherent states on the ’t Hooft black hole horizon must analogously also

FIG. 1. Null trajectories inside a simple Michelson interferometer exactly trace the surface of a matching causal diamond.
apply to causal diamonds in flat space-time (conformal Killing horizons). The “fast scrambling” on the boundaries of causal diamonds in the Banks-Fischler HST framework may be akin to transitions between coherent states and decohered ones in the ’t Hooft model. Decoherence and scrambling are of course different processes, but their effects are interrelated, and the connection between them is an interesting topic for further research [46]. In general, there is robust, increasing awareness that in solving the black hole information problem, the community needs to elucidate the nonlocality of quantum mechanical states on causal horizon scales, and that a quantum theory of space-time should start with a formulation based on Hilbert spaces instead of field theory [47–51].

There are also hints from studies of quantum foundations that the quantum-classical boundary for information on the ’t Hooft black hole horizon should also be relevant to Unruh horizons formed by accelerated frames in flat space-time. For example, we have seen renewed interest in the Wigner’s friend gedankenexperiment in recent years, as analyses of a modified version involving Bell-type states showed that the quantum theory could not consistently describe the use of itself, resulting in a no-go theorem for observer-independent facts [52, 53]. However, a common loophole in all of the proofs is the assumption of a classical formulation of locality based on a definite background space-time. An intriguing manuscript by Durham, where the same problem was studied in the context of an Unruh horizon, reveals the role of the horizon as a kind of quantum-classical boundary [54]. If the background fabric of space-time is stitched out of Hilbert spaces that are bounded by causal horizons which determine the localization and coherence of quantum geometric information, we might have a path towards resolving this paradox.

As a corollary to the first principle, we propose a second postulate:

2. The states of space-time are quantized on null surfaces — black hole horizons, light cones, et cetera — without a background. These surfaces set the symmetries of quantum geometry. The states have coherence scales matching the size of the causal boundary along its surface. However, along the world line — e.g. between layers of parallel light cones or nested causal diamonds — the states are independent and uncorrelated at the Planck scale [11].

This is our modern version of Wheeler’s vision for a theory of space-time that captures the nonlocality of quantum mechanics [55]. It also reflects some of the lessons we learned from the first-generation Holometer. That instrument was designed on the naive premise that we might be able to measure some noncommutativity of space-time in a simple Michelson setup similar to Fig. 1, based on a toy model of holographic dimensional reduction. However, since the device actually measured translational degrees of freedom along two spatial dimensions, the proposal faced much (justified) criticism that the targeted effect was not Lorentz invariant. As the trajectory of light inside the interferometer traces the boundary of a causal diamond, the null result from this experiment should be thought of as a verification of an exact symmetry, and tells us that we should probe effects that preserve this structure going forward. Our model will be designed such that all fluctuations are on these causal surfaces, preserving the shape of the light cones.

If each causal diamond layer has Planckian “thickness” along the world line, and has an independent Planck scale fluctuation that is coherent along its surface, how do we design a geometric configuration that maximizes the accumulation of these fluctuations? In Banks-Zurek [11], the authors adopt a model of nested causal diamonds, starting from the origin and growing along the world line by a Planck time for each successive layer until it reaches the system size of the interferometer. But we do not actually measure all of those different sized layers. A single-shot measurement of the largest diamond delimiting the measurement would presumably contain all of the accumulated uncertainty, but what is the diamond fluctuating relative to? In a theory without a background, a single-shot measurement simply defines what the space-time is. In any case, we cannot make such a single-shot measurement of sufficient sensitivity.

Of course, an interferometer also only measures differential fluctuations over time, not absolute lengths one can compare against an expected value. Any given measurement just establishes a nominal value of measured phase, and each successive measurement gives a relative phase compared to the earlier measurement. Each photon entering the interferometer traces the surface of an equally sized causal diamond delimiting the measurement, at different times. If the time offset $\tau$ is smaller than the total duration $2L/c$, the causal diamonds partially overlap. The overlapping volume of space-time must be consistent between the two measurements, and the non-overlapping interval can potentially contribute to differential fluctuations if coupled to the right observable (boundaries offset by $n$ Planckian layers have differential uncertainty that scales as $\sqrt{n}$). If $\tau$ is larger than $2L/c$, the two measurements are causally disconnected and independent. This means that for maximum quantum indeterminacy, or noise power, we want each measurement to pass through as many Planckian layers of causal surfaces as possible, and for maximum statistical power, we want the total integration time to contain as many independent samples of duration $2L/c$ as possible.

All of these are general statements about the quantum geometric information. Is the space-time fluctuation actually measurable in a device, however? Does the uncertainty actually couple to an experimental observable?
As stated above, a straightforward interpretation of Verlinde-Zurek [9] as local metric fluctuations runs into significant phenomenological issues. However, while Verlinde-Zurek demonstrated mathematically that the correlations on the ‘t Hooft black hole can also be modeled in flat space-time, they diverged conceptually from the way ‘t Hooft intended his model to be understood [32–36]. Here, we will try to recover some of ‘t Hooft’s original interpretation of the quantum states, and combine it with some of the lessons from our phenomenological work to infer the observables and measurements which may give us irreducible quantum uncertainties.

To do so, we will borrow from Dirac’s old approach to writing down quantum commutation relations in a Lorentz covariant way. Dirac sets up the following modified \( \delta \)-function to define localization on light cones [56]:

\[
\Delta(x) = 2\delta(x_\mu x^\mu|x_0|) \quad \Delta(-x) = -\Delta(x)
\]  

(3)

This is a \( \delta \)-function that vanishes everywhere except on the light cone, but unlike the typical \( \delta(x_\mu x^\mu) \), has a sign flip between future and past light cones. It is even at spacelike separations but odd in timelike directions, which results in a 4D point-parity antisymmetry that combines time and space. Dirac uses this function to establish covariant commutation relations for field operators: \([A_\mu(x), A_\nu(x')] = g_{\mu\nu}\delta(x - x')\).

A common way of understanding space-time fluctuations at the Planck scale is to think of the virtual states popping in and out of the vacuum as small Planck sized black holes. Fig. 2 is a schematic representation of a Dirac light cone, where we may imagine at the origin a Planck sized virtual black hole as modeled by ‘t Hooft. If we insist that quantum space-time has to be quantized on this covariant boundary, the states constrained to this structure follow the same 4D antisymmetry as the ‘t Hooft black hole states in Eq. (1). This means that if we match the null cone foliation in the rest frame of the Planck black hole to the null cone foliation in distant asymptotically flat space-time, we could establish a form of equivalence principle, where the holographic correlations have the same effect on states of light tangent to a light cone in flat space-time as they do on tangential components of field states near a black hole horizon. Of course, this is what one would have expected from considerations of Unruh horizon entropy and Verlinde-Zurek’s success using topological black hole transformations, but the difference here is that we may be able to preserve ‘t Hooft’s nonlocal coherent quantum states with antipodal antisymmetry. As ‘t Hooft asserts [34], “If it is true that vacuum fluctuations include virtual black holes, then the structure of space-time is radically different from what is usually thought.” But perhaps our modern paradigms just needed to return to the ideas that Wheeler and Dirac prefigured [55, 56].

We now make statements about the specific quantum states and the observables that may lead to a detectable effect. One part is straightforwardly adopted from the works cited above; one part will rely on some intuitive reasoning:

3. The coherent quantum geometric states on causal diamonds have a spherical harmonic expansion. Due to frame dragging, the entire null horizon is a single quantum object with nonlocally extended states. Its relation to incoming / outgoing information resembles a hydrogen atom, as described by the ‘t Hooft gravitational \( S \)-matrix.

4. As expected for a system with spherical symmetry described by spherical harmonic states, the quantum space-time system accepts rotational solutions. To measure an irreducible quantum uncertainty, we want two incompatible observables — e.g. two transverse fluctuations with respect to orthogonal rotational axes.

Why do we propose measuring rotational fluctuations? We arrive at this design by way of first considering the radial fluctuations. As noted before, a straightforward interpretation of Eq. (2) as local metric fluctuations would lead to a blurring of astrophysical images over large distances, which is ruled out by data [45]. Local metric fluctuations are likely not the correct interpretation — predictions of scalar gravitational potential fluctuations are quite risky anyway, since they are the same in every frame. The angular correlations are likely from coherent nonlocal quantum states, rather than being described by a Green’s function in a local framework as in Verlinde-Zurek. In this case, subtleties about the specific measurements must be considered. The naive approach would be to make longitudinal measurements of null radial fluctuations, say, using the directional arms of a Michelson interferometer, with some variation in the geometry to measure angular correlation functions — e.g. an interferometer where the angle between
the two arms can be changed, or two cross-correlated interferometers where one is offset by a varying angle relative to the other. The first-gen Holometer, with a simple Michelson geometry, was designed on the premise of non-commuting directions. But in the ’t Hooft spherical harmonic states, it is unclear that there is anything non-commuting between different directions. Radial measurements in two different directions might simply result in the system “collapsing” to the same state, since the two radii can be correlated on the same spherical harmonic state (unlike the case of local metric fluctuations). Typically, making a continuous measurement of the same quantum system with the same observable results in the Zeno effect, where the system never decays away from the measured state. It is of course possible that in a system of quantum space-time, the decay time for each Planckian null foliation layer is simply a Planck time, which would result in continuous stochastic generation of additional quantum geometric noise, similar to the toy model in Ref. [31] but with the angular correlations in Verlinde-Zurek [9]. Such an effect should still not be measurable in a simple Michelson interferometer, because it is inconsistent with existing astrophysical data, just like the interpretation of Eq. (2) as local metric fluctuations. Here, the “relative to what?” question may be relevant. It may be that in the absence of a background, a local measurement determined by the causal diamond of a single observer might simply define the space-time for that observer — not just for a single-shot measurement as previously considered, but for all successive measurements as well. We will revisit this hypothesis later.

How about radial fluctuations between two observers? After all, the gravitational potential cannot be defined locally without a reference elsewhere. Furthermore, unlike in the toy model of Ref. [31], in empty space-time, there is no source generating the fluctuations; there are only virtual states that pop in and out of existence. Such effects have zero mean and nonzero mean square power — the energy and information are being exchanged relationally. All observers and physical systems create their own space-time without a background, comprising these relations, woven together in a way that respects consistency conditions for overlapping causal volumes, perhaps as in the Banks-Fischler HST framework. This picture lends itself naturally to the ’t Hooft gravitational S-matrix approach. One observer’s causal diamond consists of a outgoing light cone of information as well as an incoming light cone of external information from other observers, from the larger Hilbert space outside one’s own Hilbert space. How do we detect irreducible quantum uncertainties in this model? In the ’t Hooft model, the nonzero commutator is between incoming and outgoing modes at opposite directions. It is very difficult to design an interferometer geometry that measures the two concurrently.

If we may relax the requirement of what constitutes a “concurrent” measurement of two incompatible observables, we could consider using two interferometers to test the ’t Hooft commutator. So far, we have been considering a traditional interferometer setup, where each photon is split by a beamsplitter and exists in a delocalized superposition of two arms until the interference is measured, in which case we can reasonably assume that the geometries of both arms would couple to a single photon state. If, instead, we allow using two different devices to measure two different observables, we could for example consider two rhombic Mach-Zehnder interferometers connected back-to-back or colocated in opposite directions, sharing vertices in the middle where there are both incoming and outgoing light beams, in which case the irreducible uncertainty could manifest between the two measurements (as opposed to in their cross-correlations). However, unlike the case of metric fluctuations, it is unclear that quantum states of space-time would show irreducible uncertainties when measured by two different observers with their own causal diamonds. A study of such couplings between quantum space-time and the states of photons and detectors is left to future work.

Measuring rotational fluctuations could offer a way to avoid these tricky issues. We expect a spherically symmetric system expanded into spherical harmonic modes to naturally accept rotational solutions. In the context of the preceding discussion on the emergence of space-time without a background, we expect there to be an indeterminacy of direction, as the observer’s inertial reference frame must also be emergent. The ’t Hooft “hydrogen atom” model of a black hole — which we know can be mapped onto causal diamonds in flat space-time — gives us much motivation to explore this possibility. Consider a picture where we have the orbital model of an atom, but instead of the wavefunction representing probability densities of electrons, it represents (heuristically) point fluctuations in the local gravitational potential or metric. A scalar curvature fluctuation (again, loosely speaking) at a particular point on a causal horizon layer should cause phase shift effects in all directions, including both radial and transverse directions (of course adjusted for the mapping from a black hole horizon to causal diamond layers in flat space-time). This means that for each spherical harmonic mode, there should be angularly correlated transverse shifts as well as radial shifts. Just like the spherical harmonics of an atom correspond to rotational eigenstates of the electron cloud, the spherical harmonics of a causal horizon should have similar rotational solutions. In the ’t Hooft black hole model, because only the radial components were quantized, angular filters are used at scales where the transverse gravitational effects become significant in order for the system to respect holographic total entropy. Quantum geometric uncertainties in the transverse direction, angularly correlated by coherent states extended across the horizon, provide a natural mechanism for how this Planckian transverse mode cutoff is physically implemented in the system.
Notably, the transverse fluctuations do not run into the logarithmic divergence issue at small angles that we saw for the radial fluctuations. It does not result in a detectable blurring of astrophysical images, because while a wavefront distorted by \( \langle \delta R^2 \rangle \sim \sqrt{\ell P R} \sim 10 \mu \text{m} \) from a source \( R \sim 1 \text{ Mpc} \) away is detectable in a telescope (say, differentially between the whole aperture versus a subarea with 1/10 of the radius), a transverse fluctuation of \( \langle \delta R^2 \rangle \sim 10 \mu \text{m} \) is far smaller than the transverse width of the wavefront from the optical diffraction of X-rays or \( \gamma \)-rays, not to mention the aperture that is focusing the light. Also, for an interferometric apparatus, the measured transverse fluctuation scales with the duration of time that the photon trajectory spends coupled to the transverse direction (say, in a bent portion of the arm), so the logarithm is controlled by the added linear factor of the transverse extent of the system.

It is quite probable that Eqs. (1) and (2) are not exactly correct, because they both only calculated radial modes, and that when the transverse directions are properly handled, including the quantum gravitational effects, there are no sharp peaks in correlation power at small angular scales. For example, the toy model in Ref. [31], which uses classical general relativity to calculate the gravitational shock waves of particles being emitted in indeterminate directions from isotropic decay events, shows very little power at higher order modes. If this model is closer to the correct solution, and if systems of quantum space-time have new noise being constantly generated in a stochastic manner (say, with a Planck decay time between null foliations after each measured eigenstate), it may be that we can simply use straight Michelson interferometers with varying angles, without the issue with local versus relational measurements that we posited for a background independent emergence of space-time. Such possibilities may be considered in future work.

Here, we will aim to identify designs where rotational degrees of freedom are expected to show irreducible quantum geometric uncertainties. A natural way to do so is to draw an analogy with the angular momentum algebra for the atomic orbital, and couple the detector to two incompatible observables associated with non-commuting operators: transverse fluctuations with respect to two orthogonal rotational axes in three dimensions. In hindsight, this explains why the second-gen Holometer (Michelson with one bent arm) did not see a signal: in going from two orthogonal translational observables (first-gen) to a rotation in two dimensions, we ended up with only one physical degree of freedom being measured. If the design proposed here detects an effect, we expect the magnitude to be of similar order as the radial effect suggested by Verlinde-Zurek, since both should be derived from the same physical phenomena and mechanism in flat space-time. Perhaps there are nuances of this physical system that can affect the specific shape of the projected time-domain correlation, but as we will see later, the frequency spectrum of the expected signal is fairly insensitive to subtle differences at this level. Since we are seeking the equivalent of the Bohr hydrogen atom model for quantum gravity, we should be able to reach projections that are serviceable for experimental design.

Towards this aim, we present candidate Michelson interferometer geometries (Fig. 3) with both arms bent, where each arm measures one rotational degree of freedom respectively and the two rotational axes are orthogonal to each other. Photons inside an interferometer, once they enter through the beamsplitter, are delocalized across both arms.

**FIG. 3.** Interferometer geometries designed to optimize detector coupling to irreducible quantum geometric noise. (a) All angles are 60\(^\circ\) and all segments equal. The horizontal plane of one arm and the vertical plane of the other intersect at 90\(^\circ\). (b) The two bent segments intersect at 90\(^\circ\) at their midpoints, and the center of the bent segments is exactly orthogonal to the radial vector from the beamsplitter. The ratio between the bent segments and the radial segments is 2:3 (not depicted to scale).
as superposition states until they are measured at the output port set up at nearly perfect destructive interference. This means that each photon is coupled to the geometries of both arms “at the same time,” forcing a true concurrent measurement of two incompatible observables. To optimize the detector coupling to correlated fluctuations, we want the space-time being measured in the two arms to be separated by an angle (with respect to the beamsplitter) that maximizes the spherical harmonic correlations. For example, if we imagine (heuristically) two tracer photons traveling concurrently through the two arms of configuration (a) in Fig. 3, the angle between the tracer photons is in the 45 – 60° sweet spot of Fig. 4, and for configuration (b) in Fig. 3, the analogous angle is in the 0 – 30° sweet spot of Fig. 4. In calculating the angular correlation functions of Fig. 4, we have taken only the odd harmonic modes of the correlation function \( G(r_1, r_2) \) in Eq. (2), without the \( \ell = 1 \) mode. In general, we expect the \( \ell = 0 \) and 1 modes to be unobservable in a local measurement, because the former corresponds to the average scalar curvature of the system, and the latter simply defines the inertial reference frame of the emergent space-time background.

**Numerical Model and Projected Spectra**

The design principles can be used to construct a numerical model for estimated signals. We will use configuration (a) in Fig. 3 as an example since it is the more nontrivial setup, and the same calculation will apply to configuration (b).

An interferometer is sensitive to the phase difference between light that traveled through its two arms. Due to the conventions of gravitational wave interferometry (where the light travel time is much shorter than the sampling time of the output readout), the measurements are typically characterized by the differential arm length, \( \text{DARM}(t) = [S_2(t) - S_1(t)]/2 \), where \( S_{\alpha,1}(t) \) are the optical path lengths of the two arms measured by a photon that reached the detector at time \( t \). The time-domain correlation function for signals measured an offset interval \( \tau \) apart is:

\[
\text{DARM Corr} (\tau) \equiv \langle \text{DARM} (t) \text{DARM} (t + \tau) \rangle \equiv \frac{1}{4} \sum_{\alpha,\beta}^{1,2} (-1)^{\alpha + \beta} \langle S_{\alpha}(t) S_{\beta}(t + \tau) \rangle
\]

In a realistic experiment, we will likely use the cross-correlation between two identical interferometers that are nearly colocated and coaligned. This provides two independent measurements of quantum fluctuations in the same macroscopic space-time. If the instruments are carefully isolated such that there is minimal correlated systematic uncertainty, the remaining noise — e.g., incoherent photon Poisson noise — can be averaged down over large sets of data. It is important to note that both interferometers must measure the same pair of incompatible observables in order to detect an irreducible quantum geometric uncertainty that is correlated between the two devices. If, say, two differently shaped interferometers were each measuring one out of a pair of incompatible observables, this would perhaps create an irreducible uncertainty, but the quantum fluctuations would not show up in the cross-correlation between...
FIG. 5. Schematic diagram of interferometer correlations. Left: The two arms are bent in orthogonal directions, with arm 1 (red) measuring rotations with respect to the $x$-axis and arm 2 (green) measuring rotations around the $z$-axis. The $y$-axis is defined along the radial segment of arm 1, with the bent segment going into the first quadrant of the $y$-$z$ plane. All of arm 2 sits in the first quadrant of the $x$-$y$ plane. Each pair of radial vectors denoted A (blue) or B (yellow) represents a pair of “coincident” points along the two arms that share the same causal diamond. Right: Both arms (red and green) follow the same trajectory in a diagram of radius versus time. The whole measurement (through both arms) is delimited by a large black causal diamond. Another measurement in the same instrument, offset in time by $\tau$, is represented by a thick violet trajectory and a corresponding large black causal diamond. The blue and yellow causal diamonds share an overlapping causal volume, shaded grey, where the space-time should be consistent. The small shaded light blue and light yellow causal subvolumes contribute to differential quantum geometric fluctuations between their two parent causal diamonds.

their signals. This would be akin to having two orthogonally oriented detectors measuring the spin of an electron at the same time; obviously the two measurements are not correlated at all. It may be possible in the future, as some experiments are exploring, to achieve sufficiently low noise floors in a single interferometer by utilizing new technologies, such as single photon detectors (to eliminate shot noise) and cryogenics (to reduce thermal noise).

We wish to calculate the correlated effects on $S_{2,1}(t)$ as a function of the offset $\tau$. To estimate these time-domain correlations, we will adopt a simplified heuristic version of the Banks-Fischler HST framework. In Fig. 5, we see two causal diamonds (blue and yellow) corresponding to two positions along the space-time trajectory of the light inside the interferometer. The overlapping causal volume, shaded in grey, represents a region where the space-time should be consistent. If the two causal diamonds were entirely overlapping, there would be no relative difference between their space-times. In a relational view of constructing local space-time, measuring the same causal diamond does not give you any differential fluctuation. The small shaded causal subvolumes (light blue and light yellow) represent the added information that is distinct to the respective causal diamonds (blue or yellow). Both diamonds are holographic in information, and the total quantum uncertainty, or variance, scales linearly with the size of each diamond like a random walk, but the relative fluctuation power scales with the offset between the two diamonds. The causal horizons are made out of null sequences of 2-spheres, each of which have two geometric degrees of freedom on its boundary.

We can translate this picture into a semiclassical calculational framework. Imagine the detector’s space-time foliated with past and future null cones at approximately Planckian intervals, since that is the layer spacing where we expect uncorrelated and independent quantum geometric uncertainties. In other words, we have a new blue or yellow causal diamond roughly every Planck time (with the prefactor in Eq. (2) providing the nominal normalization). This means that the light blue or light yellow region is a Planck sized causal volume, which can be thought of as the virtual ’t Hooft black hole in Fig. 2, with the Dirac light cones in future and past directions extending out from it. We will parameterize the observer’s space-time in terms of these Dirac null cones, denoted by the coordinates $\mathcal{S}^{\pm} = t \pm |r|/c$.

As the light travels along its space-time trajectory in Fig. 5, it encounters a new Planckian “bit” of quantum geometric uncertainty for each null cone foliation it passes through (it does not encounter any new information or uncertainty while traveling along one null cone, as this is a single coherent state), and its phase couples to this geometric uncertainty if the path is configured the right way. To obtain a heuristic model with the correct scaling
behavior, we posit that for each Planck “thickness” Dirac light cone, the two transverse degrees of freedom on a given 2-sphere at radius $|\mathbf{r}|$ both contribute Planckian quantum geometric fluctuations, with the following covariance structure between the transverse rotational fluctuations of two nearly parallel position vectors $\mathbf{r}$ and $\mathbf{r}'$ measured at times $t$ and $t'$ respectively (written in terms of the null cone coordinates $\mathcal{F}^\pm$ and $\mathcal{F}^\mp$):

\[
\text{cov} \left( \delta L^\pm_z(\mathcal{F}^+, \mathcal{F}^-), \delta L^\pm_z(\mathcal{F}^+, \mathcal{F}^-) \right) = \begin{cases} \frac{1}{4\sqrt{2\pi}} \hat{G}(\mathbf{r}, \mathbf{r}') \mathbf{G}(\mathbf{r}, \mathbf{r}') \cos \theta, & |\mathcal{F}^\pm - \mathcal{F}^\mp| < \frac{1}{2}t_P \quad \text{or} \quad |\mathcal{F}^\pm - \mathcal{F}^\mp| < \frac{1}{2}t_P \\ 0, & \text{otherwise} \end{cases}
\]

\[
\text{cov} \left( \delta L^\pm_z(\mathcal{F}^+, \mathcal{F}^-), \delta L^\pm_z(\mathcal{F}^+, \mathcal{F}^-) \right) = 0
\]

Here, we again take the modified version of $\mathbf{G}(\mathbf{r}, \mathbf{r}')$ in Fig. 4 where $\ell = 3, 5, 7, \ldots$, and the subscripts $z$ and $x$ denote rotational fluctuations around those respective axes. Rotational measurements with respect to two different orthogonal axes are not correlated, although necessary to observe irreducible quantum uncertainties, as we will see shortly. Note that the Planckian covariance interval $t_P$ applies to any combination of $\mathcal{F}^\pm$ and $\mathcal{F}^\mp$. This implements the ‘t Hooft correlations between incoming and outgoing information in a virtual Planck black hole, which asymptotically maps the past and future parts of the Dirac light cone in Fig. 2 (there are two parity changes that cancel out, one from the antipodal identification involving space-time and one from the odd harmonic modes). A normalization factor of $1/\sqrt{2\pi}$ was used based on the prefactor in Eq. (2). We could have chosen a different normalization for the Planckian variance, and instead modified the null foliation spacing to get the same overall prefactor for the macroscopic uncertainty, but this does not affect any results going forward. An additional factor of $1/4$ accounts for two factors of $1/2$ dividing the total variance: there are two transverse degrees of freedom on the 2-sphere boundary, and the total uncertainty arises from the combined effect of incoming and outgoing information. For the latter, as ‘t Hooft carefully explains, the antipodal identification involving $u^\pm$ and $p^\pm$ states does not result in a double cover of the geometric degrees of freedom; in fact, it fixes the one-to-two mapping of the standard interpretation. This subtlety is not fully implemented here due to the difficulty of correctly identifying the relevant microstates including the transverse degrees of freedom, but we expect the effect to be similar, because the characteristic rate of position shift in each Planckian fluctuation is approximately one Planck length per Planck time — as in, in Planck units, for each step in the Planckian random walk, the position change and the rate of change are both unity. As noted before, specifics about the physical characteristics of the effect at this level generally do not significantly change the final frequency spectrum projected, although they can modify the intermediate level features of the time-domain correlation.

What about for position vectors that are not almost in the same direction? Fig. 5 shows that we need to calculate a correlation involving four different points in the light path. Since an interferometer output is sensitive to the optical path difference between the two arms, and we are correlating two outputs measured at different times, we consider the differential effect on the signal from two orthogonal rotations in arms 1 and 2, correlated between two pairs of points $A$ and $B$ along the light path of each arm (where the light heuristically “arrives at different times,” even though the photons are actually delocalized). To avoid cluttering up our equations too much, we write down the simplified case of a covariance on a single Planckian null boundary layer, such that the condition of the point pairs $A$ and $B$ being on the same Dirac null cone foliation in Eq. (5) is already satisfied (accounting for the different spatial positions $\mathbf{r}$ and laboratory times $t$ where the light path intersects the Planckian null boundary layer):

\[
\text{cov} \left( \delta L^A_x(\mathbf{r}_2), \delta L^B_x(\mathbf{r}_2) - \delta L^B_x(\mathbf{r}_1) \right) = \cos \theta \left( \delta L^A_x(\mathbf{r}_2), \delta L^B_x(\mathbf{r}_2) \right) + \cos \left( \delta L^A_x(\mathbf{r}_1), \delta L^B_x(\mathbf{r}_1) \right)
\]

We see that there is no covariance between rotational fluctuations in orthogonal directions, so the cross-terms drop out. We can also see, however, that measuring two incompatible observables along orthogonal rotational axes results in an irreducible quantum indeterminacy, by writing down the further simplified case where $A = B$:

\[
\text{cov} \left( \delta L^A_x(\mathbf{r}_2), \delta L^B_x(\mathbf{r}_2) \right) + \cos \left( \delta L^A_x(\mathbf{r}_1), \delta L^B_x(\mathbf{r}_1) \right) = \delta L^A_x(\mathbf{r}_2) \delta L^B_x(\mathbf{r}_2) \cos \theta \geq 2 \left| \delta L^A_x(\mathbf{r}_2) \right| \left| \delta L^B_x(\mathbf{r}_1) \right|
\]

\[
\approx \frac{1}{2\sqrt{2\pi}} \hat{G}(\mathbf{r}_2, \mathbf{r}_1) \sum_{\ell = 3}^{\text{odd}} \frac{1}{4\pi} \frac{2\ell + 1}{\ell^2 + \ell + 1} P_\ell(\mathbf{r}_2 \cdot \mathbf{r}_1)
\]

Here, $\mathbf{r}_1$ and $\mathbf{r}_2$ denote unit vectors in the directions of $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. We may find the angular correlator in Eq. (9) odd since it pertains to two arms measuring orthogonal rotational degrees of freedom, which we claimed are not correlated. However, the idea here is that the amount of incompatibility between these observables depends on
the amount of correlation there is for each of those degrees of freedom respectively. For example, the z rotation at \( \mathbf{r}_1 \), \( \delta L^z_\perp(\mathbf{r}_1) \), is correlated to the z rotation at \( \mathbf{r}_2 \) by a factor \( P_L(\mathbf{r}_1 \cdot \mathbf{r}_2) \), and therefore when we measure the x rotation at \( \mathbf{r}_1 \), which has an irreducible uncertainty with \( \delta L^z_\perp(\mathbf{r}_1) \), the amount of quantum indeterminacy we encounter is proportional to this angular correlator. The angular correlations become considerably more complicated when there are four points, as in \( A \neq B \). A simple way to visualize this would be to consider one more angular correlator for each \( \ell \) mode, \( P_L(\mathbf{r}_1 \cdot \mathbf{r}_2) = P_L(\mathbf{r}_1 \cdot \mathbf{r}_2^\ell) \), although obviously this angular correlation is not independent from the one in Eq. (9). In any case, we are not too concerned about conceptual subtleties in the effects of the angular correlators. Again, as will soon become clear in our results, the final shape and magnitude of the projected frequency spectrum are quite stable to changes in the details of implementing the angular correlators.

We are finally ready to evaluate the time-domain differential arm length signals. First, we write a semiclassical expression coupling \( S_{2,1}(t) \) to the Planckian transverse fluctuations along the light path:

\[
S_{2,1}(t) = \int_{t-\mathcal{T}}^{t} \left[ \hat{\mathbf{r}}_{2,1}(t) + \sum_{\pm} \hat{L}^\pm_{z,x}(t, \mathbf{r}_{2,1}(t)) \hat{\theta}_{z,x} \right] \cdot \frac{\hat{\mathbf{r}}_{2,1}(t)}{c} \, dt \tag{10}
\]

Here, \( \mathcal{T} = 2L/c \) is the light travel time inside the interferometer, and \( \hat{\theta}_{z,x} \) denotes a unit vector in the transverse direction corresponding to a rotation around the z or x axis. The pair of indices \((2,1)\) and \((z,x)\) should be matched, as each arm is only sensitive to its respective rotational axis. The first term in the square brackets is simply the light propagating along its path, and the second holographic space-time fluctuation term \( \hat{L}^\pm_{z,x}(t, \mathbf{r}_{2,1}(t)) \) is defined as:

\[
\hat{L}^\pm_{z,x}(t, \mathbf{r}_{2,1}(t)) = \frac{\delta L^\pm_{z,x}(t, \mathbf{r}_{2,1}(t))}{t_p} \frac{d\mathcal{F}^\pm(t, \mathbf{r}_{2,1}(t))}{dt} \tag{11}
\]

This should be interpreted as an expression of “fluctuation rate” in the context of a Planckian random walk, where the step size \( \delta \mathcal{F}^\pm = t_p \) is used as the first denominator. Note that we have two terms of \( \hat{L}^\pm_{z,x}(t, \mathbf{r}_{2,1}(t)) \) because for each point \((t, \mathbf{r}_{2,1}(t))\) we have both past and future light cones passing through it, denoted by the \pm symbols. The first fraction here is the familiar transverse rotational fluctuation from Eqs. (5–9), just divided by \( t_p \) to express the fact that this Planckian jitter happens within a null cone foliation layer of Planck time thickness. The second fraction represents the fact that the relationship between \( \mathcal{F}^\pm \) and \( t \) is nonlinear; since the light has nonzero inward or outward radial velocity throughout its trajectory, one Planck time in laboratory time does not equal one Planck time in the null cone coordinates. The light path sometimes intersects many layers of null cone foliations quickly, and sometimes it runs parallel to them, not passing through any at all. We now rewrite \( S_{2,1}(t) \) in terms of null cone coordinates:

\[
S_{2,1}(t) = c\mathcal{T} + \int_{t-\mathcal{T}}^{t} \frac{\delta L^\pm_{z,x}(\mathcal{F}^+, \mathbf{r}_{2,1}(\mathcal{F}^+))}{t_p} \hat{\theta}_{z,x} \cdot \frac{\hat{\mathbf{r}}_{2,1}(\mathcal{F}^+)}{c} \, d\mathcal{F}^+ + \int_{t-\mathcal{T}}^{t} \frac{\delta L^\pm_{z,x}(\mathcal{F}^-, \mathbf{r}_{2,1}(\mathcal{F}^-))}{t_p} \hat{\theta}_{z,x} \cdot \frac{\hat{\mathbf{r}}_{2,1}(\mathcal{F}^-)}{c} \, d\mathcal{F}^- \tag{12}
\]

This can be combined with Eq. (4) to form the full expression for the time-domain DARM correlation function, where we will reparameterize \( \mathbf{r}_{2,1}(t) \) to set \( \mathcal{T} = T \) without loss of generality (as in, the light was injected at \( t = 0 \)):

\[
\text{DARM Corr} \left( \tau \right) = \frac{1}{4} \sum_{\zeta, \eta} \sum_{\alpha, \beta} \left( \begin{array}{c} 2x \\ 1x \end{array} \right) (-1)^{\alpha+\beta} \left( \int_0^\tau \frac{\delta L^\alpha_{\zeta}(\mathcal{F}^\zeta, \mathbf{r}_\alpha(\mathcal{F}^\zeta))}{t_p} \hat{\theta}_\alpha \cdot \frac{\hat{\mathbf{r}}_\alpha(\mathcal{F}^\zeta)}{c} \, d\mathcal{F}^\zeta \right. \\
\left. \times \int_0^{\tau+\tau} \frac{\delta L^\beta_{\eta}(\mathcal{F}^\eta, \mathbf{r}_\beta(\mathcal{F}^\eta))}{t_p} \hat{\theta}_\beta \cdot \frac{\hat{\mathbf{r}}_\beta(\mathcal{F}^\eta)}{c} \, d\mathcal{F}^\eta \right) \tag{13}
\]

\[
= \frac{1}{4t_p^2} \sum_{\zeta, \eta} \sum_{\alpha} \int_0^\tau \hat{\theta}_\alpha \cdot \frac{\hat{\mathbf{r}}_\alpha(\mathcal{F}^\zeta)}{c} \, d\mathcal{F}^\zeta \int_0^{\tau+\tau} \hat{\theta}_\alpha \cdot \frac{\hat{\mathbf{r}}_\alpha(\mathcal{F}^\eta)}{c} \, d\mathcal{F}^\eta \left( \delta L^\alpha_{\zeta}(\mathcal{F}^\zeta, \mathbf{r}_\alpha(\mathcal{F}^\zeta)) \delta L^\beta_{\eta}(\mathcal{F}^\eta, \mathbf{r}_\beta(\mathcal{F}^\eta)) \right) \tag{14}
\]

Here, we have used Eq. (7) to eliminate the cross-terms evaluating correlations between the two arms, as they measure orthogonal rotational degrees of freedom. In fact, the quantity in the angle brackets in Eq. (14), summed over the two arms \( \alpha = (2, z) \) or \((1, x)\), is precisely the quantity on the right hand side of Eq. (7). Since this covariance is only nonzero within a Planckian foliation layer, we can use the term to eliminate one of the integrals. Its contribution will be a value of \( \sim t_p^2 \) over an microscopic interval \( t_p \), for an integral of \( \sim t_p^2 t_p \). The remaining integral will run over the system scale \( \sim T \), so combined with the prefactor \( \sim 1/t_p^2 \), the entire expression should evaluate to a value \( \sim c\mathcal{T} \), just as we expected. Unfortunately, due to the complexity, most of the calculation will be done numerically.
FIG. 6. The time-domain DARM correlation and frequency-domain power spectral density (PSD) for each of the proposed geometric configurations (a) and (b), estimated for interferometers of size $L = 7.5$ m. The characteristic scales of the holographic space-time fluctuations for this system size are $\text{DARM Corr} (\tau) \sim \ell P L \approx 1.2 \times 10^{-34} \text{m}^2$ and $\tau \sim L/c = 25$ ns in the time domain, and $\text{PSD} (f) \sim \ell P L^2 \approx 3 \times 10^{-42} \text{m}^2/\text{Hz}$ and $f \sim c/L \approx 40 \text{MHz}$ in the frequency domain. We see that all estimated spectra are 2–3 orders of magnitude smaller than the characteristic scales, due to the instrument’s partial coupling to the effect, the angular correlation functions, and the division of fluctuation power among many different modes.

A few notes about the calculation: The function $r_{2,1}(\mathcal{I}^\pm)$ is a parameterization of the light trajectory inside the interferometer in terms of the null cone coordinates $\mathcal{I}^\pm$. The inner product $\hat{\theta}_{z,x} \cdot \hat{r}_{2,1}(\mathcal{I})/c$, representing the projection of the transverse rotational fluctuations onto a direction tangent to the light path, can take positive or negative values depending on whether the light is in the outgoing or incoming portion of the trajectory. This angular factor is applied on top of the four Legendre polynomial angular correlators discussed above, between two points within each arm and between the two arms (the mode summation in Eq. (9) is only applied once, of course). The $(\zeta, \eta) = (\pm, \pm)$ integrals, representing correlations on null cone foliations in the same direction, and the $(\zeta, \eta) = (\pm, \mp)$ integrals, representing correlations between past and future null cones, obviously do not all run the entire range, as the covariance is only nonzero when there is overlap in the null cone times. In practice, the integral in Eq. (14) is evaluated in 16 segments for an interferometer that has bent and radial portions whose lengths are identical between the two arms, accounting for the fact that we only need to evaluate it for $\tau > 0$ since the DARM correlation is symmetric.

Finally, we evaluate the power spectral density in the frequency domain. Using the engineering convention, where the fluctuation power in the negative frequencies is folded into the positive frequencies, the PSD is:

$$\text{PSD} (f) \equiv 4 \int_0^\infty \text{DARM Corr} (\tau) \cos (2\pi f \tau) \, d\tau \quad (15)$$

The time-domain and frequency-domain results are plotted in Fig. 6 for both configurations (a) and (b) presented in Fig. 3. We see that the overall time-domain structures are similar for the two interferometer geometries, and that the peak frequencies and the distribution of power at those frequencies only differ by about 10–20% despite the very different designs. Using configuration (a) as an example, the general time-domain structure owes to the three scales at play here: the round-trip distance of the whole arm, and the one-way and round-trip distances of the bent
segment. The former decides the overall extent of the correlation, and the latter two scales determine the locations of the sharp discontinuities in the slope, as the correlation shifts between outgoing and incoming light, and between null cone foliations in the same direction versus crossings of past / future null cones. The same structure mostly holds for configuration (b), although the correlation involves more scales because the bent and radial segments are different in length. One additional feature of note in this configuration is that some of the corners are due to the points midway through the bent segments where the two arms intersect orthogonally. As explained before, the logarithmic divergence at such a point is well-controlled in our model, but this point is still a maximum in the angular correlation.

These spectra are within reach of the interferometer planned at Cardiff University [13, 14], despite being 2 – 3 orders of magnitude smaller than the characteristic scales corresponding to a simple Planckian random walk.

Conclusion

A phenomenological model was presented that describes how the most recent mathematical models of space-time fluctuations on holographic causal boundaries [9–11] might connect to observables measured in a specific instrument, along with a proposed design geometry for potential detection. We relied on a conceptual picture of quantum space-time where the background independence of general relativity is intricately linked to the rejection of local realism in quantum mechanics. In this picture, locality and the fabric of space-time are both built relationally. Causal horizons are where space-time states decohere for an observer, forming a quantum-classical boundary where an indeterminate quantum state becomes a concrete measured reality. Based on this conjecture, we built an interferometer design optimized for a robust signature, and calculated an approximate projected spectrum. Because we take seriously the distortion of causal structure from quantized gravitational states on all scales [25, 27, 31–36], the interferometer configurations and space-time fluctuation modes proposed here are distinct from those expected from models of metric fluctuations based on conformal field theory and entanglement entropy [9–11].

In recent years, there has been increasing awareness that there are aspects of quantum mechanics and quantum information that simply cannot be captured within a framework based on local field theory, and that a solution for quantum gravity should start with delocalized quantum states and Hilbert spaces rather than fields and metrics [22, 25, 28–30, 49–51]. This is vividly exemplified through black hole models or various views of cosmic evolution where quantum nonlocality, entanglement entropy, or the holographic infrared problem has to be considered at the scale of the entire horizon [20, 23, 32–37, 41–44, 48]. In particular, several colleagues have pointed out that this kind of physics does not just apply to black hole causal horizons or null boundaries; the same holistic aspects of the theory should also apply to flat space-time, especially with care of treating finite causal volumes and null surfaces [5–11, 22, 23, 25, 28–31, 49]. Such arguments resulted in predictions of macroscopic quantum correlations for holographic causal diamonds in flat space-time [5–11]. The earlier proposals were founded on heuristic arguments and met with some skepticism from the community, but the later ideas have been much more rigorously formulated and solidly grounded in mainstream techniques such as entanglement entropy and bulk-boundary relationships in conformal field theories. As an aside, some of the same characteristic physics was also found in a model that used localized graviton fields (not holographic in their degrees of freedom) but with other exotic ideas from quantum information research such as squeezed vacuum states [57]. It is promising that several different approaches — heuristic arguments counting holographic degrees of freedom [5–8], gravitational frame dragging of coherent quantum states [31], topological mappings between black holes and flat space-time [9], conformal field theory and entanglement entropy [10, 11], and squeezed graviton states [57] — have all led to phenomenological regimes similarly reachable by state-of-the-art interferometers. This provides perfect motivation for developing more advanced experimental techniques [12–14, 58, 59]. The recognition of a promising target phenomenological regime has inspired much research, including the funding of a major theory collaboration [60] and two new experiments being advanced [12–14]. There is also the possibility that solving the nature of quantum space-time and quantum information on causal horizons is the key to resolving the intractable paradoxes we encounter in quantum foundations [54]. Perhaps most importantly, in the rare observational data where we expect see active gravitational effects from quantum states, the cosmic microwave background, we already see compelling “anomalies” — or signatures — that match the features found in our model: exact symmetry of zero correlation at 90º and negative correlations at angular separations approaching the antipodes, just as one would expect from gravitational back reaction effects if quantum states were nonlocally extended across the horizon.

The viability of this proposed experimental design makes it a key prong of a strongly motivated multimodal research program, combining foundational theory and phenomenology, reframed statistical analyses of cosmological observations, and a laboratory experiment. We may finally be approaching empirical studies of quantum space-time.
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