A simplified model for analysis of Side-impact Velocity Based on Energy Method

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Abstract. With the increasing number of automobile, the frequency of road traffic accident increases in the same way. Therefore, accident reconstruction is required as effective method to make confirmation of liabilities. This paper set up an energy method based car-car side impact velocity computing model. In this model, firstly, Pc-crash was used as a simulation platform, using the message leaved by car-car side collision accident such as stop position, stop angle and collision trajectory. Next the relationship between total deformation energy and pre-impact velocity was also analyzed in the model. Then pre-impact velocity was constructed in the computing model based on the principle of energy conservation and momentum conservation. Finally, a numerical simulation was provided to test the validity of the results. It is shown that the car-car side impact velocity calculation model was effective for the accident analysis and could be used in practical cases.

1. Introduction
The handle of traffic accidents always includes field investigation of an accident, accident analysis and confirmation on the liability of traffic accident. And the accident process analysis is mainly basis of confirmation on liability. Obviously it is a great significance to have the accident reconstructed for the analysis on the cause of the accident and ruling the responsibility fast and fairly.

Table.1 data statistics data statistics for different types of collision accidents [1] (2012)

|                  | number of accident (%) | number of injury (%) | direct property loss (%) |
|------------------|------------------------|---------------------|--------------------------|
| front impact     | 5.2                    | 6.91                | 6.31                     |
| side-impact      | 39.38                  | 46.41               | 34.72                    |
| rear-impact      | 14.93                  | 7.03                | 23.21                    |
| opposite direction collision | 0.87              | 1.24                | 0.43                     |
| Same direction collision | 3.01              | 2.92                | 1.58                     |
| total            | 63.39                  | 64.51               | 66.25                    |
In fact, the number of both the traffic accident and the injury caused by collision is much larger than the others. Among the collision accident, side-impact takes over a great majority. Table 1 shows a statistic data indicating a big proportion by side-impact of Chinese car accidents in 2012. In the process of determination of traffic accident liabilities, it was very difficult to determine the pre-impact speed [2]. Mc Lundie et al. apply finite element method to establish the model of vehicle and bicycle and explore the possibility of finite element method for the simulation of two wheeled vehicle accident[3]. Lin Qingfeng et al. make a quantitative analysis on the accident of the car and the two rounds according to a large number of bicycle accident data and crash test, which mainly research the rider throwing model, debris throwing model and bicycle throwing model etc[4]. Yang Bo et al. apply the image measurement and video method on the accident reconstruction, and combine them and the finite element method to simulate the model of accident[5]. However, in case of initial data is missing or parameter is uncertainty, simplified model still can be used.

In this paper, simplified model for calculating vehicle collision speed was established finally according to energy conservation methods combine with the conservation of momentum and software simulation. This model presents the relationship between energy loss and velocity through software simulation, calculates with high accuracy by several simple parameters.

Collision motion

2. Theoretical analysis and Computation of Collision

No matter which part of collision process: pre-crash, crash or post-crash, the analysis of energy and force involved in the collision stages is complex, for facilitating the analysis and assuming that:

(1) During the collision the location in earth’s axis keeps stable, the change of displacement in the instant of collision is ignored;

(2) The force like Gravity, ground friction force, driving force, air drag and the tiny change of energy loss is neglected;

(3) The energy before and after the collision complies with energy conservation principle;

(4) Parameters would not change in the collision such as quality, centroid position, moment of inertia, wheel track.

2.1 Kinematic Equation

According to above assumptions, in this paper, the principle of energy conservation was utilized, neglecting the loss of the total deformation energy. Formula can be given as:

\[
\frac{1}{2} (m_1 v_{10}^2 + m_2 v_{20}^2 + I_1 \omega_{10}^2) = \frac{1}{2} (m_1 v_1^2 + I_1 \omega_1^2 + m_2 v_2^2 + I_2 \omega_2^2) + \Delta E. \tag{1}
\]

Where \(v_{10}, v_{20}, v_1, v_2, \omega_{10}, \omega_{20}, \omega_1, \omega_2\), represents the translation velocity and rotate velocity of vehicle 1 and vehicle 2 before and after impact respectively. \(\Delta E\) is the loss of energy.

Considering that the work done by four wheel frictional forces is along the wheel-track \(L\), then the value of the work is equals to variation of kinetic energy, according to:

\[
m_{fg}fL = \Delta E_c = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2. \tag{2}
\]

Where \(f\) is the friction coefficient between locked wheels and the ground, assuming that all the rotate and translation speed have a linear behavior and both motion finish at the same time (this can be verified easily when all wheel are locked), parameter set as:

\[
\omega = k t + \omega_0. \tag{a}
\]

\[
v = g t + v_0. \tag{b}
\]
\[
\frac{a}{x} = \frac{1}{2} (\omega_t + \omega_0) t \\
= \frac{1}{2} (v_t + v_0) t = \frac{\omega_t + \omega_0}{v_t + v_0}.
\] (c)

According to equations above, \( \omega_t + v_t = 0 \) therefore:

\[
\frac{a}{x} \approx \frac{\omega}{v}.
\] (3)

Substitution equation (3) into (2) we can get:

\[
\Delta E_c = \frac{1}{2} v^2 (m + I \frac{a^2}{x^2}).
\] (4)

Combining equation (2), (3) and (4), it is possible to obtain the post-impact velocity as:

\[
v = \sqrt{\frac{2m g f L}{m + I (a^2/x^2)}}.
\] (5)

\[
\omega = v \frac{a}{x}.
\] (6)

According to the principle of energy conservation and momentum conservation, formula can be written as:

\[
\frac{1}{2} (m_{10}v_{10}^2 + I_{10} \omega_{10}^2 + m_{20}v_{20}^2 + I_{20} \omega_{20}^2) = f_1 m_1 g L_1 + f_2 m_2 g L_2 + \Delta E_c.
\] (7)

\[
m_1 v_{10n} + m_2 v_{20n} = m_1 v_{1n} + m_2 v_{2n},
\] (8)

As Figure 1, the longitudinal direction of vehicle 1 before impact was taken as \( t \)-axis and the lateral as \( n \)-axis.

\[
v_{10n} = v_{10}, \quad v_{10t} = 0.
\] (9)

When the angle between \( t \)-axis and longitudinal of post-impact of vehicle 1 is \( \alpha \), where the post-impact speed is \( v_1 \), we can obtain:

\[
v_{1t} = v_1 \cos \alpha.
\]

\[
v_{1n} = v_1 \sin \alpha.
\] (10)
In the similar way, the angle of vehicle 2 is $\beta$ and velocity is $v_2$, we can get:

\[
\begin{align*}
    v_{2t} &= v_2 \cos \beta, \\
    v_{2n} &= v_2 \sin \beta.
\end{align*}
\]  

$v_1$ and $v_2$ can be obtained via Eqs (5). Now pre-velocity $v_{10}$, $v_{20}$ can be calculated based on Eqs (7), (8), (9), (10), (11). The resolving still exists two unknown parameters, which is the distance covered by the tyre in the process of collision ($L$), and the energy loss ($\Delta E$).

### 2.2 The distance $L$

Considered the collision of wheel lock, the adhesion coefficient of the four wheels in each direction is constant. Although in reality all wheel locking is not common, this assumption can cover most of the accidents for a suitable resistance factor. In the case of all-wheel lock, adhesion coefficient of the four wheels is constant in all directions. After collision, the vehicle is rotated and translated around its centroid. One wheel trajectory is a circle and its radius is $r$. In order to simplify the calculation, the $r$ was select the medium value. The centroid trajectory distance is $X (\vec{X} = \alpha d)$, where the angle of rotation after the collision is $\alpha$ and the radius of the center of mass of the vehicle is $d$.

In road accidents, $d$ is usually greater than $r$, which is believed that translation is more important than rotation. Because $d$ approaches infinity, the motion is close to the real translation and the wheel travel distance $L$ will approach the centroid distance $X$. Conversely, if the vehicle is primarily rotating, the trajectory of each wheel is significantly longer relative to the center of mass. The ratio of the locus of the wheel to the center-of-mass locus is close to $r/d$. Approximate formula for $L$ can be deduced as:

\[
    L = \vec{X}(1 + 0.14 \frac{r}{d}).
\]

The formula was verified with high precision and calculated faster than the integral[6].

### 2.3 Total deformation energy $E$

According to the hypothesis, deformation energy is the dominated loss. In order to analyze the relationship between total deformation energy and collision speed, We used the popular accident reconstruction software pc-crash to do simulation experiment.

In simulation, vehicle-1 was taken as the one crashed car under the range of 40km/h-80km/h, and at the same time vehicle-2 was the collision-car under 20-40km/h. Vehicle-2 hit vehicle-1 in different collision angle $\theta (\theta > 45^\circ$, as shown in Figure 1). Collision occurred in $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ of side of vehicle-1.

In order to keep other parameters constants in simulation, we observed that the total deformation energy is proportional to the impact velocity of vehicle-2, which presents the tendency of a linear function or the quadratic function. The curve fitting could be made on the maximum and minimum of curve series, as shown in Figure 3.
It is can be seen from Figure 3 that the change area of total deformation is relatively concentration. When incident angle is greater than 45°, the maximum of total deformation mainly focus on the curve of the 1/4 collision point with 45° incident angle and the speed of vehicle-2 is 30km/h. The minimum of total deformation is mainly concentrated on the curve of the 1/2 collision point with 110° incident angle and the speed of vehicle-2 is 30km/h.

The trend line is used to fit the total deformation energy with the increase of collision speed, and the two trend lines are obtained. The upper and lower boundary curve equations are as follows:

$$E_{K_{max}} = 1847.7v_{20} + 47590$$

$$E_{K_{min}} = 1465.425v_{20} + 19036$$

(13)  (14)

The total deformation energy in Eq. (15) is related only to the velocity before collision of the two vehicles. Substituting this equation into Eq. (1), the velocity before collision can be solved. The parameters required for this equation can be obtained from a measurement or from an existing model, which is verified below.

3. Case study

3.1 Case description and calculation

Two same type car stop after right-angle side-collision. Through investigation in accident scene, the position of two cars shows in Figure 4. The arrow means the direction of velocity. All information of this case can be found in table 2:

$$E_{K} = 1656.56v_{20} + 33313$$

(15)
Table 2 The parameters of the accident measured in scene

| parameters                        | vehicle-1 | vehicle-2 |
|-----------------------------------|-----------|-----------|
| mass (kg)                         | 1400      | 1400      |
| length (m)                        | 4.68      | 4.68      |
| width (m)                         | 1.7       | 1.7       |
| Angle with X direction before collision (deg) | 0         | 90        |
| Angle with X direction after collision (deg) | 61        | 70        |
| the distance covered by the central of mass of the vehicle after impact (m) | 9.8       | 6.1       |
| Friction coefficient between tyre and ground | 0.6       | 0.6       |
| wheel track (front/rear) (m)      | 1.414/1.422 | 1.414/1.422 |

According to formula (Eq.12), L can be calculated as:

$L_i = X_i + 0.14 \alpha_i r_i \quad (i = 1, 2)$

Where $X_1 = 9.8$, $\alpha_1 = 61$, $r_1 = 1.5$, $X_2 = 6.1$, $\alpha_2 = 20$, $r_2 = 1.5$, $L_1 = 10.02$, $L_2 = 6.17$, $I_i = 0.1478 m.t.I_i \pm 4.8\% \quad (i = 1, 2)$

Now moment of inertia $I$ can be obtained as above: $I_1 = 1373$, $I_2 = 1373$ and post-impact velocity can be calculated via formula (5) as:

$$v_1 = \sqrt{\frac{2m_1gt_1L_1}{m_1 + I_1(\alpha_1^2 / x_1^2)}} = 10.79 m / s$$

$$v_2 = \sqrt{\frac{2m_2gt_2L_2}{m_2 + I_2(\alpha_2^2 / x_2^2)}} = 8.13 m / s$$

The momentum conservation in x-axis as shown in Figure 3:

$$m_1v_{10}\cos0 + m_2v_{20}\cos90 = m_1v_1 \cos61 + m_2v_2 \cos70$$

$$v_{10} = 8.138 m / s = 29.29 km / h \text{ can be obtained.}$$

$$\frac{1}{2}(m_1v_{10}^2 + m_2v_{20}^2) = f_1m_1gL_1 + f_2m_2gL_2 + E_k$$

Where $E_k = 1656.56 \times 3.6v_{10} + 33313$, combine with above formula, $v_{20} = 18.05 m / s = 64.95 km / h$.

3.2 Model validation

Generally deformation energy can be calculated by some methods like energy grid and stiffness coefficient method, where energy grid has a widely application. It divided the front of vehicle into several parts and calculated the energy absorbed by each part, which can be used in uncertain damage mode or complex relationship between the force and residual deformation energy. However, those methods obtained by collision experiment need certain financial and power [7], and therefore it can’t
be widely applicable. According to the area covered by the energy grid and the deformation curve in this case, the total energy loss of the collision deformation absorption is 134704J. The total deformation energy obtained by the above model is 140956J.

3.2.1 Stiffness coefficient method
According to a lot of positive walls collision test, Campbell [8] provided the linear relation between the residual deformations and collision velocity \(v\), and consequently deformation energy can be calculated by following formula.

\[
E = \int_0^L \left[ \frac{1}{2} B \times C_R^2 + A \times C_R + A^2 / (2B) \right] d\omega.
\]

(17)

\(A\) = Force per unit width with zero residual deformation (N/m);
\(B\) = Stiffness coefficient per unit width (N/m²);
\(C_R\) = Residual deformation (m);
\(\omega\) = Width of vehicle damaged (m).

Fig. 5 Deformable contours
The measurement data of deformation on car body in this case can be found as follows: For car-1, \(C_1=0.01\) m, \(C_2=0.2\) m, \(C_3=0.32\) m, \(C_4=0.3\) m, \(C_5=0.15\) m, \(C_6=0.01\) m, \(\omega = 3.20\) m. For car-2, \(C_1=0.37\) m, \(C_2=0.4\) m, \(C_3=0.35\) m, \(C_4=0.42\) m, \(C_5=0.4\) m, \(C_6=0.35\), \(\omega = 1.70\) m. The National Highway Traffic Safety Administration (NHTSA) divided vehicle into seven categories on the basis of the wheelbase length [8], and reference value of parameters like A, B, G was given. According to this case, parameter value selected as follows: A = 303N/cm and B = 39.3N/cm² for stiffness reference coefficient of car-1; A = 555.2N/cm B = 38.61N/cm² for car-2, now the total plastic deformation energy loss in the case can be obtained based on Eqs (17), which is 138350.8J.

3.2.2 Simulation on Pc-crash

Fig. 6 Parameter input interface
Fig. 7 Result of simulation
Table 3 Vehicle parameters before the collision on simulation

| No. | Simulation velocity | Angle with X-axis |
|-----|---------------------|-------------------|
| Vehicle-1 | 28km/h | 86.5° |
| Vehicle-2 | 69km/h | 3° |

Table 4 The comparison of stop location between simulation and real accident

| No. | Vehicle-1 | Vehicle-2 |
|-----|-----------|-----------|
| X-axis | Accident scene measurement | 2.09 | 3.95 |
|       | Simulation on pc-crash | 2.27 | 3.94 |
| Y-axis | Accident scene measurement | 5.73 | 8.96 |
|       | Simulation on pc-crash | 5.67 | 8.01 |
| Rotation angel | Accident scene measurement | -20° | 61° |
|       | Simulation on pc-crash | -20.3° | 60.3° |

Figure 6 shows that the test result in the pc-crash simulation is approximately equal to the actual measured value, which means simulation results is effective. The vehicle impact speed in Figure 4 is close to the velocity gotten by model above, and finally the value of the deformation energy by the simulation is 132598.09 J.

Table 5 The value of deformation energy calculated by four methods

|                  | Simulation (E1) | Energy grid (E2) | Stiffness coefficient (E3) | Model (Ek) |
|------------------|-----------------|-----------------|---------------------------|------------|
| Total deformation energy | 132598.09J | 134704.00J | 138350.80J | 140956.00J |
| Error rate
\[ (E_k - E_i) / E_i \] | 6.30% | 1.80% | 4.60% | —— |

3.2.3 Velocity result comparison of two methods

Formula for car-car side collision velocity based on the ‘speed technical evaluation for vehicles involved in representative road accidents’ (in this case the longitudinal adhesion correction coefficient \(k_1, k_2\) take 1.0) as:

\[
v_1 = \left( \frac{m_2}{m_1} \right) \sqrt{2\phi_2 g k_2 s_2} \sin \beta + \sqrt{2\phi_1 g k_1 s_1} \cos \alpha \times 3.6 = 29.43 km / h
\]

\[
v_2 = \left( \frac{m_1}{m_2} \right) \sqrt{2\phi_2 g k_2 s_2} \sin \alpha + \sqrt{2\phi_2 g k_2 s_2} \cos \beta \times 3.6 = 64.84 km / h
\]

Table 6 the contrast of pre-collision velocity calculated in three methods

|                  | analogue simulation | Standard formulas. | Model |
|------------------|---------------------|-------------------|-------|
| Pre-impact velocity | \(v_{10a}\) | \(v_{20a}\) | \(v_{10b}\) | \(v_{20b}\) | \(v_{10}\) | \(v_{20b}\) |
|                  | 28km/h | 69km/h | 29.43km/h | 64.84km/h | 29.29km/h | 64.95km/h |
| Error rate
\[ (v_j - v_{j0}) / v_j \] | 4.60% | 5.90% | 0.40% | 0.20% | —— | —— |
Through the comparison result in table 5, it’s easy to found that the model presented in this study gives modest differences in comparison with the other methods, since errors in the pre-velocity are less than 6%, and those differences are considered acceptable, which can be applied to the real case and calculating the deformation effectively. Although rigidity coefficient and deformation energy grid figure methods are common methods to most kinds of collision accidents, they all need different coefficient to particular car which is obviously unrealistic. The simple model we studied here can calculate pre-velocity and the total deformation with high precision, and bring convenience for the analysis of the accident.

4. Conclusion
In this research, the calculation model of side-impact speed based on methods of energy is established. According to the fitted curve from the designed experiments on Pe-crash simulation software on the relationship between plastic deformation with the pre-velocity speed, and combining with the principle of conservation of momentum and conservation of energy, this model can be applied both in the case when wheel partially locked or all locked when it have an appropriate drag factor. Simplified model was validated through a case through comparing the result with Pe-crash software simulation, stiffness coefficient method, the energy grid method and the standard speed calculation formula, the errors are been thought considerable. The model has a great advantage that, the only necessary initial data for the model are the angle and distance changed, the distance between the contact area of wheels the center of mass ,and the friction coefficient between the road and tyre with locked wheels, those parameters can be found in the field measurement. However this model is not applicable to special circumstances, special weather, but still has a wide scope of application.

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