RETINOTOPIC MECHANICS DERIVED USING CLASSICAL PHYSICS

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SUMMARY

The concept of a cell’s receptive field is a bedrock in systems neuroscience, and the classical static description of the receptive field has had enormous success in explaining the fundamental mechanisms underlying visual processing [1]. Borne out by the spatio-temporal dynamics of visual sensitivity to probe stimuli in primates, I build on top of this static account with the introduction of a new computational field of research — “retinotopic mechanics”. At its core, “retinotopic mechanics” assumes that during active sensing receptive fields are not static but can shift beyond their classical extent [2-3]. Specifically, the canonical computations and the neural architecture that supports these computations are inherently mediated by a neurobiologically inspired force field (e.g., $r \propto \frac{-1}{\Delta M}$) [4-5]. For example, when the retina is displaced because of a saccadic eye movement from one point in space to another, cells across retinotopic brain areas are tasked with discounting the retinal disruptions such active surveillance inherently introduces. This neural phenomenon is known as spatial constancy. Using “retinotopic mechanics”, I propose that to achieve spatial constancy or any active visually mediated task, retinotopic cells, namely their receptive fields, are constrained by eccentricity dependent elastic fields – pEØs. I propose that pEØs are self-generated by the visual system and allow receptive fields the ability to predictively shift beyond their classical extent to future post-saccadic location such that neural sensitivity which would otherwise support intermediate eccentric locations likely to contain retinal disruptions is transiently blunted.
INTRODUCTION

Retinotopic mechanics has two principle aims. One, provide a general mathematical abstraction grounded in Newtonian physics that can reveal the mechanical laws retinotopic cells use to shift their spatial extent beyond their classical surround during active vision. Two, to predict the innate architecture used by the visual system to ensure that cells appropriately shift their spatial extent such that retinotopic organization is always maintained.

ELASTIC FIELDS

The underlying architecture of this framework includes a retinotopic field \( \phi_r \). \( \phi_r \) consists of a two-dimensional hexagonal grid of overlapping retinotopic receptive fields – RFs – that tile visual space. A RF is characterized by three parameters: (a) \( p_i \), the location of the RF centre \( (x_i, y_i) \) at \( t_0 \) (i.e., at a state of equilibrium); \( p_i \) determines the eccentricity \( e \) of a RF; (b) \( s_i \), the radius of a RF, which is proportional to \( e \). Empirically speaking, during active vision, RFs are not passive sensors but can exhibit transient spatial shifts which predict the location in space that is relevant for the impending task. [6]. However, there are limits to the extent these transient retinotopic shifts occur so that retinotopic organization is always maintained. Consequently, this framework extends the concept of a cell’s receptive field by introducing its elastic fields – pEØs. pEØs behave like spring loaded sensors and are self-generated by the visual system. The annulus \( (a) \) that defines a pEØ span the region immediately beyond the classical extent of a RF. Depending on the biological system in question, \( a \) may or may not be proportional to the eccentricity of a RF, while in both cases pEØs are omnidirectional with respect to a RF. This eccentricity and non-eccentricity dependency can be mathematically expressed as follows:

\[
\begin{align*}
a &= 1 * ecc + \eta_1 \\
a &= 0 + \eta_2
\end{align*}
\]

FORCE FIELD

Another layer within this framework is a force field \( \phi_f \). \( \phi_f \) exerts its influence on a RF, thus causing it to change its \( p_i \) at \( t_{on} \). In theory, \( \phi_f \) can include an infinite number of forces. However, the capacity of most, if not all sensory systems, are limited in the number of stimuli they can attend to at a single point in time. Further, an organism must prioritize relevant stimuli and ignore the irrelevant ones. With this in mind, a force field at the very least should include three external forces – the centripetal force \( (\vec{F}_C) \), a convergent force \( (\vec{F}_P) \), and a translational force \( (\vec{F}_T) \) – and an internal force – the equilibrium force \( (\vec{F}_E) \). The external
forces are exerted by corresponding time-varying masses: $M_c$ – the mass subtended at the central region in a sensory space to which the organism is currently attending, $M_p$ – the mass subtended at a peripheral site within a sensory space to which the organism wants to attend to in the future, and $M_s$ – a virtual mass at infinity in the direction of and parallel to the impending self-generated movement, in this case, an eye movement. These three principal time ($t$) varying masses can be mathematically expressed as follows:

\[
m_c(t) = \begin{cases} 
0, & t < t_{x1} \\
c_1 t^n, & t_{x1} < t < t_{x2} \\
\text{const}, & t_{x2} < t < t_{x3} \\
c_1 t^{-n}, & t_{x3} < t < t_{x4}
\end{cases}
\]

\[
m_p(t) = \begin{cases} 
0, & t < t_{y1} \\
c_2 t^m, & t_{y1} < t < t_{y2} \\
\text{const}, & t_{y2} < t < t_{y3} \\
c_2 t^{-m}, & t_{y3} < t < t_{y4}
\end{cases}
\]

\[
m_s(t) = \begin{cases} 
0, & t < t_{z1} \\
c_3 t^p, & t_{z1} < t < t_{z2} \\
\text{const}, & t_{z2} < t < t_{z3} \\
c_3 t^{-p}, & t_{z3} < t < t_{z4}
\end{cases}
\]

The following are the possible forces a RF$_i$ at $t_{0+n}$ can experience:

\[
\vec{F}_C = \vec{U}_{M_c} \cdot M_c \cdot D_1^t \\
\vec{F}_P = \vec{U}_{M_p} \cdot M_p \cdot D_2^t \\
\vec{F}_T = \vec{U}_{M_s} \cdot M_s \cdot |s|
\]

where $\vec{U}_{M_c}$, $\vec{U}_{M_p}$, and $\vec{U}_{M_s}$ are the unit vectors in the direction of the three masses. $D_1^t$ and $D_2^t$ is the spatial difference between a RF$_i$ and $M_c$ or $M_p$ raised to a scalar $\varepsilon$ – a distance exponent, which is the principal parameter in the model used to determine whether retinotopic remapping obeys an inverse distance rule or a proportional distance rule. $|s|$ is the magnitude of the impending eye movement. A resultant force $\vec{F}_R$, depending on the condition, could then include a single (e.g., $\vec{F}_T$), two (e.g., $\vec{F}_C$ and $\vec{F}_p$), or, in the most dynamic case, all three external forces and the internal force:

\[
\vec{F}_{R} = k_c \cdot \vec{F}_C + k_p \cdot \vec{F}_p + k_s \cdot \vec{F}_T + \vec{F}_e
\]
where \( k_c \) is obtained by calculating the average of \( D1_{t_0}^{\infty} \) across RFs and then taking its reciprocal. The same expression is used to compute \( k_p \), but, in this case, the average of \( D2_{t_0}^{\infty} \) across RFs is used. It is worth noting that for a given \( \varepsilon \), \( k_c \) and \( k_p \) is recalculated. Note once this is obtained, it does not change at \( t_{0+n} \). Furthermore, with \( k_c \) and \( k_p \), a range of magnitude of forces for \( F_C \) is obtained. Next, a look-up table is constructed, where \( k_s \) is selected such that \( F_T \) and \( F_E \) were in the same order of magnitude as \( F_C \) and \( F_P \). Note that the offset of \( F_C \) constitutes the onset of eye movement. Further, these external forces exert their influence on RFs in discrete time increments of 1ms, where \( F_E \) is proportional to the movement of RFs (\( \Delta d \)).

**NEURAL RESPONSE ESTIMATION**

The time varying forces perturbs each RF. Perturbed RFs capture spatiotemporal retinotopic displacements. The concerted movements of the constellations of RFs manifest in time-varying modulation of density at a given location of visual space. Each RF is modelled as a bivariate gaussian kernel function \( G \). To obtain a probability density estimate, which is equivalent to changes in population neural sensitivity, the following equation is used:

\[
G(u) = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2} u^t u}
\]

\[
\hat{f}(x, B) = \frac{1}{n} \sum_{i=1}^{n} G_B(x - (x_{i1}, x_{i2}))
\]

where \( x_{i1}, x_{i2} \) denote a sample from the bivariate distribution. \( B \) denotes the bandwidth used, with \( G_B \) as a non-negative and symmetric function (\( \int G_B(u)du = 1 \)), defined in bivariate terms as \( B^{-\frac{1}{2}} G(B^{-\frac{1}{2}}u) \).

**CONCLUSION**

To conclude, retinotopic mechanics has revealed in the context of neural remapping that attentional and oculomotor signals available around the time of an eye movement give rise to an inverse force field (\( r \propto \frac{1}{\Delta M} \)). The inverse force field acts on receptive fields across different retinotopic brain areas, while being restricted within their pEØs [4-5].
FUTURE DIRECTION

Note that biological systems that possess smaller heads or bodies do not initially surveil their environment using their eyes as it is observed in primates. For example, rodents move their heads quite frequently because it is less energetically expensive. Because of this, these systems initially surveil their environment by orienting their head (i.e., head saccades) towards \( \mathbf{M}_p \), to which their eyes later follow [7]. Conversely, in drosophila, which are much smaller in their overall size when compared to rodents, these organisms initially surveil their environment with their bodies (i.e., body saccades)[8]. With this in mind, retinotopic mechanics need not focus on retinotopic brain areas per se but can be extended to biological systems such as those mentioned above that do not possess a sensory organ that has a high spatial resolution (i.e., a fovea). For these systems, the framework retinotopic mechanics provides can be used to understand how craniontopic (head-centered) or body-centered cells in concert with retinotopic cells predictively shift their spatial extent before the onset of impending movement towards a location in space that is task-relevant.

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