Indirect measurement of triple-Higgs coupling at an electron-positron collider with polarized beams

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Abstract

We examine the possibility of using single-Higgs production at an $e^+e^-$ collider with polarized beams to measure, or constrain, indirectly a possible anomalous triple-Higgs coupling, which can contribute to the process via one-loop diagrams. In the dominant process $e^+e^-\rightarrow ZH$, longitudinally polarized beams can lead to an improvement in the cross section by 50\% for $e^-$ and $e^+$ polarizations of $-0.8$ and $+0.3$, respectively. This corresponds to an improvement in the sensitivity to the triple-Higgs coupling of about 18\% for a centre-of-mass energy of 250 GeV and an integrated luminosity of 2 ab$^{-1}$, making a strong case of beam polarization. This also implies that with polarized beams, the luminosity needed to get a particular sensitivity is less by about 33\% as compared to that needed with unpolarized beams. Even when only the $e^-$ beam is polarized $-0.8$, the improvement in the sensitivity is about 8\%. We also study the effect of longitudinal beam polarization on the sensitivity to the triple-Higgs coupling of Higgs production through the subdominant process $e^+e^-\rightarrow H\bar{\nu}\nu$ occurring through $WW$ fusion.

1 Introduction

Experiments at the Large Hadron Collider (LHC) since the discovery of the Higgs boson have been steadily enhancing the accuracy in the measurement of its couplings to various standard model (SM) particles and the results show good conformity with theoretical predictions within the SM. An exception to this trend of high accuracy measurements is the scalar self-couplings,
which as yet have not been determined with very good precision. The scalar self-couplings correspond to \( \lambda_3 \) and \( \lambda_4 \) in the following terms in the scalar Lagrangian

\[
\mathcal{L}_{3H} = -\lambda_3 H^3, \\
\mathcal{L}_{4H} = -\lambda_4 H^4.
\]

In the SM, these couplings are related to the physical Higgs mass \( m_H \) and the scalar vacuum expectation value \( v \) by

\[
\lambda_3 = \lambda_4 v; \quad \lambda_4 = m_H^2 / (2v^2).
\]

It will be the task of future experiments at the LHC as well as at the proposed lepton colliders to determine \( \lambda_3 \) and \( \lambda_4 \) with greater precision and check the SM relations of eq. (3). It is of course possible that the underlying theory is not the SM, but an extension of the SM. In that case, the above couplings would be the couplings in an effective theory, and they may not obey the relations (3). Deviations of these couplings from their SM values have been discussed in the context of the standard model effective field theory (EFT), where effective interactions induced by new physics are written in terms of higher-dimensional operators, suppressed by a high-energy scale, the effective theory presumed to be valid at energies much lower than this scale. Thus, for example, \( \mathcal{L}_{3H} \) would get a contribution from a dimension-six operator, \(-\lambda(H^1 H)^3\), see for example, [1].

A determination of the triple-Higgs coupling \( \lambda_3 \) can be carried out through a process where two or more Higgs bosons are produced. First of all, such a process needs a high centre-of-mass (c.m.) energy for the interacting particles, partons in the case of hadron colliders, or leptons in the case of lepton colliders. Moreover, it has been found that in the SM [2], there is destructive interference between the one-loop diagrams contributing to the process \( gg \rightarrow HH \), making the total cross section extremely small. Thus, the accuracy of the determination of \( \lambda_3 \) is low.

A suggestion was made by McCullough [3] that the triple-Higgs coupling could be measured through its contribution in one-loop diagrams in single-Higgs production. The process considered in [3] was \( e^+ e^- \rightarrow ZH \), in which, making an assumption that only \( \lambda_3 \) deviates from its SM value,

\[
\lambda_3 = \lambda^\text{SM} v (1 + \kappa),
\]

it would be possible to put a limit on the fractional deviation \( \kappa \) which is of the order of 28% at \( e^+ e^- \) c.m. energy of 240 GeV, with an integrated
luminosity of 10 \text{ab}^{-1}$, expected to be available at TLEP (currently known as FCC-ee) \cite{4}. This estimate is based on the assumption, made for the sake of concreteness, that there are no other contributions to an effective $ZZH$ vertex.

The proposal to construct a linear $e^+e^-$ collider, the so-called International Linear Collider (ILC), in Japan may reach a decision in the near future. We concentrate mainly on the ILC in this work, though our discussion is also important in the context of other $e^+e^-$ colliders. Since in the initial run the ILC would collect 2 \text{ab}^{-1}, rather than 10 \text{ab}^{-1} being anticipated for the FCC-ee, it seems advisable to look for some means of enhancing the sensitivity of the ILC. We therefore turn to beam polarization for the purpose. We have examined to what extent the sensitivity of the single-Higgs production processes to the triple-Higgs coupling can be improved by the use of longitudinal beam polarization. It is well-known that suitable longitudinal polarization could lead to a larger cross section for $ZH$ production, and may also help in suppressing background. We find that beam polarization unfortunately does not improve the relative contribution of the the triple-Higgs coupling. However, it does improve the sensitivity because of the increase in statistics. A similar improvement is also found for single Higgs production via $WW$ fusion, though the dependence on beam polarization is somewhat different. Thus, to achieve the same sensitivity for $\kappa$, a collider with polarized beams would need much less luminosity as compared to one without beam polarization. This makes a good case for beam polarization at future $e^+e^-$ colliders.

We have systematically investigated this issue quantitatively, making comparisons of the results that can be achieved without polarization and using electron and/or positron beam polarization.

Here we give a quantitative summary of our results. We assume that for a c.m. energy of 250 GeV, an integrated luminosity of 2 \text{ab}^{-1} would be available with polarized beams. The degree of polarization assumed is $-0.8$ for electrons and $+0.3$ for positrons. We have assumed that, as estimated in \cite{5,6}, a precision of about 0.9\% could be possible in the measurement of the cross section for $e^+e^\to ZH$. We then find that the accuracy of determination of the parameter $\kappa$ that could be obtained from the measurement of the cross section with polarized beams is about 57\% as compared to 70\% in the absence of polarization, an enhancement in sensitivity of about 18\%.

To put it differently, at a collider like the FCC-ee, if operating at 240 GeV with polarized beams, the same sensitivity as anticipated in ref. \cite{3} could be achieved with an integrated luminosity of about 6.66 \text{ab}^{-1}. We also found
that the sign-inverted combination of polarizations, viz., +0.8 and −0.3 respectively for electrons and positrons, gives worse sensitivity as compared to the unpolarized case.

The plan of the remaining paper is as follows. In the next section, we summarize the different approaches to the measurement of the Higgs self-coupling at different colliders. In section 3, we present the formalism, including analytical expressions, for calculating the effect of the tree-level and the loop contribution to the single-Higgs production processes. The effect of polarization is discussed in Section 4. After presenting the numerical results in Section 5, the last section (6) contains our conclusions and a discussion of the results.

2 Various methods for measurement of Higgs self-coupling

We discuss here in brief various methods suggested for the measurement of the Higgs self-coupling.

Indirect constraints can be obtained from electroweak precision measurements. A bound of −15.0 < κ < 16.4 was obtained [7,8] by this method.

As mentioned earlier, the dominant approach to the measurement of the Higgs self-coupling is through Higgs pair production. The first possibility is Higgs pair production at the LHC, and then at the future high luminosity run of the LHC (HL-LHC). It is expected that at HL-LHC, with a c.m. energy of 14 TeV and an integrated luminosity of 3 ab\(^{-1}\), with sophisticated jet substructure techniques, a limit of \(\lambda_3 < 1.2\lambda_{3}^{SM}\) may be set [9]. Adding more Higgs decay channels, this accuracy might be improved, see, for example, [10].

A future hadron collider at 100 TeV may probe the coupling, using the \(b\bar{b}\gamma\gamma\) channel for HH decay, to an accuracy of 30% [11]. For a discussion at a 100 TeV FCC-hh collider in the \(t\bar{t}hh\) channel see Ref. [12].

A global fit which includes several inputs including that from double-Higgs production was shown to enable limits on \(\lambda_3/\lambda_{3}^{SM}\) in the interval [0.1,2.3] [13]. A recent work [14] proposes restricting to certain kinematic regions to improve the accuracy of measurement at the LHC.

There have been various suggestions for the construction of \(e^+e^-\) colliders with c.m. energy ranging from a few hundred GeV to a few TeV. After the discovery of the Higgs boson with mass of about 125 GeV, the dominant sug-
gestion is to construct a linear collider, the ILC, which would first operate at a c.m. energy of 250 GeV, enabling precise measurement of Higgs properties, through an abundant production of a $ZH$ final state. There have also been other proposals, as for example, the Compact Linear Collider (CLIC), the Future Circular Collider (FCC-ee) and Circular Electron Positron Collider (CEPC) where electron and positron beams would be collided, providing a clean environment to study couplings of SM particles, and possibly look for new physics, if any.

Double Higgs production can also be used to measure $\lambda_3$ at an $e^+e^-$ collider. For example, $\lambda_3$ would be determined with an accuracy of 83\% at the proposed International Linear Collider (ILC) at c.m. energy of 500 GeV, improving to 21\% and 13\% with luminosity and energy upgrades [5,15,16].

Some other recent studies on Higgs coupling measurements at electron-positron colliders, though not necessarily limited to triple-Higgs couplings, are [17]-[37]. We will discuss the most relevant ones towards the end.

As mentioned earlier, single Higgs production in association with $Z$ at an $e^+e^-$ collider was suggested by McCullough [3] as an indirect measurement of $\lambda_3$ through loop contributions. Following McCullough’s suggestion, there have been proposals to measure $\kappa$ in various single-Higgs production processes at the LHC [1,38–40] and at $e^+e^-$ colliders [17–19]. For example, the Run I single-Higgs data at the LHC leads to $−13.6 \leq \lambda_3/\lambda_3^{SM} \leq 16.9$ [39].

Single Higgs production has also been proposed at the future LHeC collider as a possibility for the measurement of $\lambda_3$ [41]. The best limits anticipated in this paper on $\kappa$ are [-0.10, 4.07] for an integrated luminosity of 3 ab$^{-1}$ proton and electron beam energies of 7 TeV and 60 GeV, respectively.

HL-LHC would eventually be able to determine $\kappa$ to a good degree of precision, of the order of 50\% at 68\% confidence level [20,42]. However, it is expected that results from an $e^+e^-$ collider would be able to improve on this accuracy, especially when combined with the HL-LHC results.

We now review the current limits on the Higgs self-coupling. Current constraints from the analysis of the $\sqrt{s} = 8$ TeV (Run I) and the $\sqrt{s} = 13$ TeV (Run II) data from the LHC are weak. Direct searches constrain $\lambda_3$ to $−14.5 \leq \lambda_3/\lambda_3^{SM} \leq 19.1$ (Refs. [38] and [43]) from Run I data. From the Run II data, using $\gamma\gamma b\bar{b}$ final state, ATLAS has obtained a limit of $−8.2 < \lambda_3/\lambda_3^{SM} < 13.2$ [44], whereas the corresponding CMS limit is $−11 < \lambda_3/\lambda_3^{SM} < 17$ [45]. ATLAS has also studied $b\bar{b}b\bar{b}$ final state, and reports a limit of $\lambda_3/\lambda_3^{SM} < 17$ [46]. In Ref. [47], a recent study of Higgs boson pair decaying into various channels has constrained the ratio $−5 < \lambda_3/\lambda_3^{SM} < 12$
with 95% C.L.

In this work we pursue further the possibility of measuring $\kappa$ at future $e^+e^-$ colliders. In the context of $e^+e^-$ colliders, particularly the ILC, the possibility of utilizing beam polarization and its advantages have received much attention. Suitable longitudinal beam polarization could help in improving the sensitivity for many different processes and suppressing unwanted background [6, 48]. The use of polarization in the context of Higgs properties is also discussed in [49, 50]. It is expected that at the ILC, polarizations of 80% and 30% would be possible respectively for electron and positron beams for c.m. energy of 250 GeV [6]. We therefore proceed in the next section with a calculation of the loop contribution to the cross section for $e^+e^- \rightarrow ZH$, and assume this combination of polarizations to estimate the sensitivity of determination of $\lambda_3$ in the process.

Figure 1: Tree diagram for the process $e^+e^- \rightarrow ZH$.

Figure 2: One-loop diagrams for the process $e^+e^- \rightarrow ZH$ which include the triple-Higgs coupling.
3 Formalism

Here we briefly describe the formalism for the determination of $\lambda_3$ from single-Higgs production at non-leading order. The calculation of the effect of the triple-Higgs coupling at one-loop level follows the work in refs. [3,39].

The Feynman diagrams for the process $e^+e^- \rightarrow Z\ H$ are shown in Figs. 1 and 2. The former shows the tree-level diagram, whereas the latter shows the one-loop diagrams which have contribution from the triple-Higgs coupling.

The Feynman diagrams for the $WW$-fusion process $e^+e^- \rightarrow H\nu\overline{\nu}$ are shown in Figs. 3 and 4. The former shows the tree-level diagram, whereas the latter shows the one-loop diagrams which have contribution from the triple-Higgs coupling.

Figure 3: Tree diagram for the process $e^+e^- \rightarrow H\nu\overline{\nu}$ occurring through $WW$ fusion.

Figure 4: One-loop diagrams for the process $e^+e^- \rightarrow H\nu\overline{\nu}$ occurring through $WW$ fusion which include the triple-Higgs coupling.
The process $e^+e^- \rightarrow ZH$ as well as the $WW$ fusion contribution to $e^+e^- \rightarrow \nu \bar{\nu} H$ involve a $VV^*H$ vertex, where $V = Z$ in the former case, and $V = W$ in the latter. The $VV^*H$ vertex can be written as

$$\Gamma_{\mu
u}^{VVH} = g_V m_V [(1 + \mathcal{F}_1)g_{\mu\nu} + \mathcal{F}_2 k_{1\mu} k_{2\nu}],$$

where $k_1, k_2$ are the momenta (assumed directed inwards) of the gauge bosons carrying the respective polarization indices $\mu, \nu$. This form assumes that the gauge bosons are either on-shell, satisfying $k_i\mu e^{\mu}(k_i) = 0 (i=1,2)$, or couple to a conserved current, so that the terms with $k_{1\mu}$ or $k_{2\nu}$ can be dropped. Here $m_V$ is the mass of the gauge boson, $g_W$ is the weak coupling and $g_Z = g_W / \cos \theta_W$, $\theta_W$ being the weak mixing angle. The quantities $\mathcal{F}_{1,2}$ are functions of bilinear invariants constructed from the momenta.

Isolating the contribution of the triple-Higgs coupling $\lambda_3$, the form factors $\mathcal{F}_{1,2}$ for the two processes of $Z^* \rightarrow ZH$ and $W^+W^-\rightarrow H$ can be written at one-loop order in terms of the Passarino-Veltman (PV) functions [51] as follows.

$$\mathcal{F}_1(k_1^2, k_2^2) = \frac{\lambda_{SM}^\alpha(1 + \kappa)}{(4\pi)^2} \left(-3B_0 - 12(m^2_H C_0 - C_{00} - \frac{9m^2_H (\kappa + 1)}{2})B_1'\right),$$

$$\mathcal{F}_2(k_1^2, k_2^2) = \frac{\lambda_{SM}^\alpha(1 + \kappa)}{(4\pi)^2} 12(C_1 + C_{11} + C_{12}).$$

For the process $W^+W^-\rightarrow H$, the arguments of the PV functions are

$$B_0 \equiv B_0(m^2_H, m^2_H, m^2_H), \quad C_0 \equiv C_0(m^2_H, k_1^2, k_2^2, m^2_H, m^2_H, m^2_W),$$

and analogously for the functions $B_1'$ and the tensor coefficients $C_1, C_{11}$ and $C_{12}$. For the process $Z^* \rightarrow ZH$, the arguments of the PV functions are

$$B_0 \equiv B_0(m^2_H, m^2_H, m^2_H), \quad C_0 \equiv C_0(m^2_H, s, m^2_Z, m^2_H, m^2_H, m^2_Z),$$

and analogously for the functions $B_1'$ and the tensor coefficients $C_1, C_{11}$ and $C_{12}$. The above expressions are evaluated to first order in the parameter $\kappa$ for consistency, as there would be higher-loop contributions at order $\kappa^2$ which are not being included. We use the package LoopTools [52] to evaluate the PV integrals.

In a realistic situation, there would be contribution at tree level from an anomalous $ZZH$ vertex. This beyond standard model (BSM) contribution
would be model dependent, depending on the anomalous couplings. In a
specific model, as for example, a two Higgs doublet model, the tree-level \( ZZH \)
vertex of the SM would be modified by a known constant, determined by
certain mixing angles in the theory. In an effective field theory approach, the
anomalous contribution would be unknown, though the higher-dimensional
operator would be suppressed by powers of the cut-off scale. Thus, a way
should be found of extracting the loop-level contribution, separating the tree
level BSM contribution. The extraction of \( \kappa \) is possible either from the cross
section using the different energy dependences of the two contributions [3],
or from the differential cross section, using details of \( Z \) decay distributions.
It has also been pointed out that anomalous coupling of the Higgs to a top-
quark pair can affect the determination of the triple-Higgs coupling, though
this pollution is small at low energies [53].

In the next section we evaluate the cross section in the presence of beam
polarization, including the contribution of \( \kappa \) at the one-loop level, which was
just discussed.

4 Polarized cross section

The differential cross section for the process \( e^+e^- \rightarrow ZH \) with longitudinally
polarized beams may be written as [54]

\[
\frac{d\sigma_L}{d\Omega} = (1 - P_L\bar{P}_L)[A_L + B_L \sin^2 \theta],
\]

where \( \theta \) is the angle between \( Z \) and \( e^- \) directions, \( P_L, \bar{P}_L \) are the degrees
of longitudinal polarizations of \( e^- \), \( e^+ \) beams, and \( A_L \) and \( B_L \) can be split
into their respective tree-level SM contributions \( A_{L,SM} \) and \( B_{L,SM} \) and the ex-
tra contributions \( \Delta A_L \) and \( \Delta B_L \) coming from the one-loop trilinear Higgs
couplings. The expressions for these are as follows.

\[
A_L = A_{L,SM} + \Delta A_L, \tag{11}
\]

\[
B_L = B_{L,SM} + \Delta B_L, \tag{12}
\]

where

\[
A_{L,SM} = B_{L,SM} \frac{2m_Z^2}{|q|^2} = (g_V^2 + g_A^2 - 2g_Vg_A P_L^{\text{eff}})K_{SM}, \tag{13}
\]

\[
K_{SM} = \frac{\alpha^2|q|}{2s\sin^4 2\theta_W} \frac{m_Z^2}{(s - m_Z^2)^2}, \tag{14}
\]
\( q \) being the momentum of the \( Z \), \( g_{V,A} \) the SM couplings of the \( Z \) to \( e^+e^- \),

\[
g_V = -1 + 4 \sin^2 \theta_W, \quad g_A = -1, \tag{15}\]

and

\[
P_L^{\text{eff}} = \frac{P_L - \bar{P}_L}{1 - P_L \bar{P}_L}. \tag{16}\]

The expressions for the contributions from anomalous triple-Higgs couplings are

\[
\Delta A_L = 2 \mathcal{F}_1 (g_V^2 + g_A^2 - 2g_V g_A P_L^{\text{eff}}) K^{\text{SM}}, \tag{17}\]

\[
\Delta B_L = 2 \left( \mathcal{F}_1 + \mathcal{F}_2 \sqrt{s} q^0 \right) \frac{|g|^2}{2m_Z^2} (g_V^2 + g_A^2 - 2g_V g_A P_L^{\text{eff}}) K^{\text{SM}}. \tag{18}\]

The partial cross section, with a cut-off \( \theta_0 \) on forward and backward directions is obtained by integrating \( \theta \) between limits \( \theta_0 \) and \( \pi - \theta_0 \). The corresponding expression is

\[
\sigma_L(\theta_0) = (1 - P_L \bar{P}_L) 4\pi \cos \theta_0 [A_L + \left( 1 - \frac{1}{3} \cos^2 \theta_0 \right) B_L]. \tag{19}\]

We see from the above equations that the beam polarization dependence of the SM contribution as well of the contribution of the triple-Higgs coupling to the cross section is through the same factor of \( (1 - P_L \bar{P}_L)(g_V^2 + g_A^2 - 2g_V g_A P_L^{\text{eff}}) K^{\text{SM}} \). This is also true for the differential cross section.

The differential cross section for the process \( e^+e^- \rightarrow H\nu\bar{\nu} \) has two contributions, one from the process \( e^+e^- \rightarrow ZH \), with \( Z \) decaying into \( \nu\bar{\nu} \), and the other from the \( WW \) fusion process \( e^+e^- \rightarrow W^+\nu W^-\bar{\nu} \rightarrow H\nu\bar{\nu} \). There is also a small interference term between the amplitudes for these two mechanisms, which we neglect. The contribution from \( ZH \) production followed by \( Z \rightarrow \nu\bar{\nu} \) is the same as that obtained for \( ZH \) production earlier here, multiplied by the branching ratio for \( Z\nu\bar{\nu} \), which is about 20%. The matrix element squared for the \( WW \) fusion process summed over final polarizations may be written as

\[
\Sigma |M|^2 = \frac{g_W^6 m_W^2 (1 - P_L)(1 + \bar{P}_L)}{(q_1^2 - m_W^2)^2 (q_2^2 - m_W^2)^2} \left[ (1 + \mathcal{F}_1)^2 p_1 \cdot p_2 p_2 \cdot p_3 \
+ (1 + \mathcal{F}_1) \mathcal{F}_2 \left\{ s_0 (p_1 \cdot p_4 p_2 \cdot p_3 - \frac{1}{2} p_1 \cdot p_4 p_3 \cdot p_4 - \frac{1}{2} p_2 \cdot p_3 p_3 \cdot p_4) \
+ p_1 \cdot p_4 p_2 \cdot p_3 (2 p_3 \cdot p_4 - p_2 \cdot p_3 - p_1 \cdot p_4) \
+ p_1 \cdot p_3 p_2 \cdot p_4 (p_1 \cdot p_4 + p_2 \cdot p_3) \right\} \right]. \tag{20}\]
5 Numerical Results

For our numerical calculations, we use $m_W = 80.385$ GeV, $m_Z = 91.1876$ GeV, $g_W = \sqrt{8m_W^2(G_F/\sqrt{2})}$, $\sin^2 \theta_W = 0.22$ and $m_H = 125$ GeV. We assume electron longitudinal polarization of $P_L = -0.8$, and positron longitudinal polarization $\bar{P}_L = +0.3$. For our analysis of the sensitivity, we assume a modest integrated luminosity of $2 \text{ ab}^{-1}$, which for simplicity is taken to be the same for unpolarized beams as well as all polarization combinations. The efficiency of measurement of the final state is taken from earlier works \cite{5,6,16} to be 0.9% at $\sqrt{s} = 250$ GeV, and appropriately scaled for other energies. It may be mentioned that the efficiencies expected at other colliders are 0.4% at FCC-ee for $\sqrt{s} = 240$ GeV and luminosity 10 ab$^{-1}$ \cite{4}, 3.8% at CLIC for $\sqrt{s} = 350$ GeV and luminosity 500 fb$^{-1}$ \cite{55}, and 0.5% at CEPC for $\sqrt{s} = 240$ or 250 GeV and luminosity 5.6 ab$^{-1}$ \cite{34}.

Taking up the case of longitudinal polarization for the process $e^+e^- \rightarrow ZH$ first, we present in Table 1 our results for the polarization dependence of the cross section $\sigma_L$, as well as the 1 $\sigma$ limit that can be obtained on $\kappa$ from the polarized cross section for various values of c.m. energy. We also present the fractional change $\delta \sigma_L$ in the cross section to the parameter $\kappa$ in the form of $\delta \sigma_L/\sigma_L$ for unit change in $\kappa$, following ref. \cite{3}. As mentioned earlier, this quantity does not change with polarization for a given $\sqrt{s}$. This observation has also been made in \cite{19}, and follows from the fact mentioned earlier that the polarization dependence of the cross section at tree level and at one-loop in the triple-Higgs couplings is identical, both arising from a single $Z$ at the $e^+e^-$ vertex \cite{54}. In this case, we assume the cut-off on the production angle of the $Z$ to be zero, since it was found that the sensitivity worsens with increasing cut-off angle. A negative sign in the table signifies that the cross section decreases with increasing $\kappa$.

The points worth noting are that there is an improvement of sensitivity in the measurement of $\kappa$ when both electron and positron beams are polarized. The sensitivity improves even when only the electron beam is polarized. The increase in sensitivity is related to increase in the cross section by the factor $(1 - P_L \bar{P}_L)(1 - 2g_V g_A P_L^{\text{eff}}/(g_V^2 + g_A^2))$, which for the best case is approximately 0.5. It can be checked that for the case of electron and positron polarizations being respectively +0.8 and −0.3, this factor is less than 1, and corresponds to degradation of the sensitivity.

For the process $e^+e^- \rightarrow H\nu\bar{\nu}$ which includes the Higgsstrahlung process followed by $H$ decay into 3 species of neutrino pairs, as well as the $WW$
Table 1: The cross section $\sigma_L$ for $ZH$ production, the fractional change in the cross section $\sigma_L$ for unit value of $\kappa$ and the sensitivity to $\kappa$ for various c.m. energies $\sqrt{s}$ and combinations of $e^-$ and $e^+$ longitudinal polarizations $P_L$ and $\bar{P}_L$, respectively. An integrated luminosity of 2 ab$^{-1}$ is assumed.

| $\sqrt{s}$ | $P_L$ | $\bar{P}_L$ | $\sigma_L$ (fb) | $\delta\sigma/\kappa$ | $\kappa_{\text{lim}}$ (%) |
|------------|------|-------------|-----------------|----------------------|----------------------|
| 250        | 0    | 0           | 242             | 1.278                | 70.0                 |
|            | -0.8 | 0           | 288             | 1.278                | 64.2                 |
|            | -0.8 | +0.3        | 364             | 1.278                | 57.2                 |
| 350        | 0    | 0           | 129             | 0.284                | 315                  |
|            | -0.8 | 0           | 153             | 0.284                | 289                  |
|            | -0.8 | +0.3        | 193             | 0.284                | 257                  |
| 500        | 0    | 0           | 56.9            | -0.203               | -440                 |
|            | -0.8 | 0           | 67.6            | -0.203               | -403                 |
|            | -0.8 | +0.3        | 85.3            | -0.203               | -359                 |
| 1000       | 0    | 0           | 12.7            | -0.433               | -206                 |
|            | -0.8 | 0           | 15.1            | -0.433               | -189                 |
|            | -0.8 | +0.3        | 19.1            | -0.433               | -169                 |

fusion process, the two contributions have to be added. There is also a small interference term, which is neglected for present purposes. Moreover, we make a simplifying assumption that the accuracy of measurement is the same as in the case of $e^+e^- \rightarrow ZH$ process. Our results are presented in Table 2. Since for values of $\sqrt{s}$ less than or reaching 500 GeV the $HZ$ production process dominates, the results are similar to those shown in Table 1. For higher energies, however, even though the cross section for $WW$ fusion dominates, the sensitivity of this process to $\kappa$ is somewhat less, since there is a partial cancellation between the two mechanisms, the dependence on $\kappa$ of the $HZ$ production cross section being negative. Thus, while longitudinal polarization does help, the $HZ$ production channel with $Z$ decay into visible channels is still better for the measurement of $\kappa$, as compared to the $\nu\bar{\nu}$ channel. However, it is possible to combine data from these two processes at the ILC to get a sensitivity which is better than individual sensitivities of the two processes.
The cross section $\sigma_L$ for $H\nu\bar{\nu}$ production, the fractional change in the cross section $\sigma_L$ for unit value of $\kappa$ and the sensitivity to $\kappa$ for various c.m. energies $\sqrt{s}$ and combinations of $e^-$ and $e^+$ longitudinal polarizations $P_L$ and $\bar{P}_L$, respectively. An integrated luminosity of $2\text{ ab}^{-1}$ is assumed.

### Table 2

| $\sqrt{s}$ (GeV) | $P_L$ | $\bar{P}_L$ | $\sigma_L$ (fb) | $\frac{\delta \sigma}{\sigma}/\kappa$ | $\kappa_{\text{lim}}$ (%) |
|------------------|-------|-------------|-----------------|--------------------------------------|--------------------------|
| 250              | 0     | 0           | 56.4            | 1.148                                | 77.9                     |
|                  | −0.8  | 0           | 71.9            | 1.094                                | 72.4                     |
|                  | −0.8  | +0.3        | 91.3            | 1.090                                | 64.5                     |
| 350              | 0     | 0           | 56.6            | 0.313                                | 286                      |
|                  | −0.8  | 0           | 86.0            | 0.318                                | 228                      |
|                  | −0.8  | +0.3        | 111             | 0.318                                | 201                      |
| 500              | 0     | 0           | 86.6            | 0.254                                | 352                      |
|                  | −0.8  | 0           | 149             | 0.275                                | 248                      |
|                  | −0.8  | +0.3        | 193             | 0.277                                | 216                      |
| 1000             | 0     | 0           | 214             | 0.296                                | 302                      |
|                  | −0.8  | 0           | 384             | 0.299                                | 224                      |
|                  | −0.8  | +0.3        | 499             | 0.299                                | 196                      |

### 6 Conclusions and Discussion

The determination of the Higgs self-coupling is an important issue for the confirmation of the SM or establishing the veracity of a possible extension of the SM. Since the cross section for Higgs pair production, which could measure the triple Higgs coupling directly, is small, an indirect measurement through the process of associated single Higgs production may have an upper hand. Following an earlier suggestion [3] for such a measurement in the process $e^+e^- \rightarrow ZH$, where $\lambda_3$ could be measured through its one-loop contribution, we have investigated the effect of beam polarization on such a measurement. Again, it is assumed that the only source of additional contribution is the $\kappa$ term, and we work in the lowest order in $\kappa$. Our calculation shows that the fractional change in the cross section arising from one-loop contribution of the triple-Higgs coupling itself does not depend on polarization. Nevertheless, since the cross section does improve with polarization, we find that reasonable longitudinal beam polarizations of −0.8 and +0.3 as foreseen for the ILC leads to an improvement of about 19% in the sensitivity to $\kappa$. Even if only the electron beam is polarized, there is still an improvement in the sensitivity by about 8%.
This also implies that if polarized beams are used, the same sensitivity as obtained with unpolarized beams can be achieved with a 33% lower luminosity of 1.34 ab\(^{-1}\). We have also calculated the effect of beam polarization for a final state \(H\nu\bar{\nu}\) which gets contribution from \(ZH\) production with \(Z\) decaying into \(\nu\bar{\nu}\) as well as from \(WW\) fusion. Here we do find that the fractional change in the cross section with \(\kappa\) depends on the degree of beam polarization. However, for somewhat higher energies, where the total cross section is higher, this change has opposite signs for the \(ZH\) and \(WW\) fusion contributions, and tends to reduce the sensitivity. We find an improvement of 16% in the sensitivity as compared to the case of unpolarized beams for a c.m. energy of 250 GeV. The data from \(H\nu\bar{\nu}\) and \(H\mu\bar{\mu}\) final states may be combined to get a much better sensitivity. Again, the sign-reversed polarization combination \((+0.8,-0.3)\) gives a worse sensitivity.

We now discuss our results in the context of other recent relevant studies. As mentioned earlier, it is expected that HL-LHC, at the end of its run will be able to determine \(\kappa\) with an accuracy of about 50%. There have been various studies suggesting how these results could be combined with results from the planned \(e^+e^-\) experiments, see, for example, [20]. However, to be able to compare our results, we restrict ourselves to results which could be obtained separately from \(e^+e^-\) experiments.

The idea of using single-Higgs production \(e^+e^- \rightarrow HZ\) was suggested by [3]. This work did not consider the effect of beam polarization, which we have included. We have also included in the study the process \(e^+e^- \rightarrow H\nu\bar{\nu}\), not included in [3], and which gets additional contribution from \(WW\) fusion.

In the context of ILC, in a recent report it was estimated that Higgs pair production at \(\sqrt{s} = 500\) GeV and an integrated luminosity of 4 ab\(^{-1}\), \(\kappa\) could be determined to an accuracy of 27% using \(e^+e^- \rightarrow ZH\) [35] with a run involving certain mix of polarization. Using \(e^+e^- \rightarrow ZH\), the accuracy expected is 40% for the full run of ILC in the energy range 250 GeV to 500 GeV [35], with most of the contribution from the Higgs loop stated to be coming below 350 GeV. The report does not mention the process \(e^+e^- \rightarrow H\nu\bar{\nu}\), however. The results in [35] are obtained using detailed simulation, which we have not attempted. Again, they report on results using a certain scheduling which includes runs with all beam polarization combinations, with view to optimizing sensitivity to several EFT couplings. If we do a simple-minded extrapolation of our results from Tables 1 and 2 to an integrated luminosity of 4 ab\(^{-1}\) for a run at \(\sqrt{s} = 250\) GeV a combination of the \(ZH\) and \(H\nu\bar{\nu}\) data would lead us to an accuracy of about 37% for unpolarized beams,
and about 30% for both beams polarized in the combination (−0.8, +0.3). It thus seems that from the point of view of triple-Higgs couplings, the accuracy mentioned in [35] can be achieved, or perhaps even superceded with the use of the same integrated luminosity without an energy upgrade.

At the CEPC, with a precision of 0.5% on the measurement of the $ZH$ cross section at 240 GeV and an integrated luminosity of 5.6 ab$^{-1}$, $\kappa$ can be constrained to 35% [34]. This is an update on the original proposal [4] alluded to by [3], which we mentioned earlier. As CEPC is not designed to have polarized beams, our suggestion for the use of polarization does not have a direct impact on this result. However, a similar capability would be possible at ILC with polarized beams with a lower luminosity of about 5 ab$^{-1}$. Similarly, at FCC-ee, the capability of measurement of $\kappa$ is expected to be similar to that at CEPC [29].

Proposals at CLIC, on the other hand, have considered the possibility of only the electron beam having 80% polarization. In a recent study in Ref. [30] it is shown that with an integrated luminosity of 5 ab$^{-1}$ and $\sqrt{s} = 3$ TeV, CLIC will be able to measure trilinear Higgs coupling with an uncertainty of −7% and 11% with C.L. 68%. Single Higgs production at this high energy would not be significant. On the other hand, the integrated luminosity proposed for CLIC to operate at lower energies of 350 or 380 GeV is low. However, our study allows us to anticipate that if CLIC is operated at lower energy with longitudinal polarization for a sufficient length of time, $ZH$ production would allow a significant limit to be put on $\kappa$.

Unlike our present work which discusses only the total cross sections, better sensitivity might perhaps be obtained with use of differential decay distributions of charged leptons, and needs to be investigated in detail. A significant work points out the possibility of measuring $\kappa$ directly from single Higgs production making use of a certain T-odd kinematic asymmetry of decay leptons, which is not affected by anomalous tree-level couplings [36]. This work estimates a limit on $\kappa$ of order 1 for an integrated luminosity of 30 fb$^{-1}$ making use of beam polarization as well as $\tau$ polarization in $Z$ decay.

The discussion here has included only statistical uncertainties. It is appropriate to mention possible systematic uncertainties which may be anticipated. These would be mainly related to $b$ tagging when using the decay $H \rightarrow b\bar{b}$ for Higgs detection, luminosity measurement, and polarization measurement. The systematic errors in luminosity and polarization measurements are each estimated to be 0.1% [16,35]. The systematic errors due to
$b$-tagging efficiency are estimated to be $0.3\% \sqrt{0.250/L}$, where $L$ is the integrated luminosity in $ab^{-1}$ [35]. Thus the systematic uncertainty of the ILC option we consider would be within about 0.1%. It is thus clear that the systematic uncertainties are much lower than the statistical uncertainties in the measurement of the Higgs coupling, and may be ignored.

It is hoped that we have made a case for exploring seriously the possibility of implementing longitudinal polarization at future $e^+e^-$ colliders. While polarization of both electron and positron beams would be extremely useful, even a high electron beam polarization in the absence of positron polarization would serve a useful purpose.

We have assumed for simplicity that a run with polarized beams would collect the full integrated luminosity of $2\ ab^{-1}$. Also, we have made a simplified assumption regarding the detection efficiencies. A more realistic simulation, including proper isolation cuts and detector efficiencies, would be needed to determine the actual improvement in the sensitivity in a practical situation. However, our results do provide a reasonable first estimate of the advantage of beam polarization.

As mentioned earlier, there may be a BSM tree-level contribution to the process through an anomalous $ZZH$ coupling, and this will have to be subtracted before extracting the one-loop effect of $\kappa$ discussed here. One way, is to use results from two different energies, as was discussed in ref. [3]. Another possibility of separating the two contributions would be the use of different kinematic correlations in the final state arising from $Z$ decay. As mentioned earlier, the work of ref. [36] makes uses of such a kinematic asymmetry which does not get contribution from tree-level $ZZH$ couplings. Our treatment assumes that the production process proceeds through a virtual $Z$. There would also be a contribution from a virtual $\gamma$ state at tree level through an anomalous $\gammaZH$ coupling, or at one-loop level, though not from the triple-Higgs coupling. Though we have not taken into account this possibility, it is possible, as shown in [54], to use either more than one beam polarization combination or more than one polar-angle cut-off to determine separately the $Z^*$ contribution. Ref. [54] also shows how transverse beam polarization, if available, may be used to achieve the same purpose.

**Acknowledgement** SDR acknowledges support from the Department of Science and Technology, India, under the J.C. Bose National Fellowship programme, Grant No. SR/SB/JCB-42/2009.
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