ON THE FEKETE-SZEGÖ PROBLEM ASSOCIATED WITH LIBERA TYPE CLOSE-TO-CONVEX FUNCTIONS

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ABSTRACT. The main purpose of this paper is to introduce a new comprehensive subclass of analytic close-to-convex functions and derive Fekete-Szegö inequalities for functions belonging to this new class by using a different way. Various known special cases of our results are also pointed out.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

(1.1)

which are analytic in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$. Also let $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of univalent functions in $U$.

For $f \in \mathcal{S}$ given by (1.1), Fekete and Szegö [12] proved a noticeable result that

$$|a_3 - \mu a_2^2| \leq \begin{cases} 
3 - 4\mu & , \mu \leq 0 \\
1 + 2 \exp \left( \frac{-2\mu}{1-\mu} \right) & , 0 \leq \mu \leq 1 \\
4\mu - 3 & , \mu \geq 1 
\end{cases}$$

(1.2)

holds. The result is sharp in the sense that for each $\mu$ there is a function in the class under consideration for which equality holds.

The coefficient functional

$$\phi_\mu (f) = a_3 - \mu a_2^2 = \frac{1}{6} \left( f'''(0) - \frac{3\mu}{2} (f''(0))^2 \right)$$

on $f \in \mathcal{A}$ represents various geometric quantities as well as in the sense that this behaves well with respect to the rotation, namely

$$\phi_\mu (e^{-i\theta} f (e^{i\theta} z)) = e^{2i\theta} \phi_\mu (f) \quad (\theta \in \mathbb{R}).$$

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In fact, other than the simplest case when 
\[ \phi_0 (f) = a_3, \]
we have several important ones. For example,
\[ \phi_1 (f) = a_3 - a_2^2 \]
represents \( S_f (0) / 6 \), where \( S_f \) denotes the Schwarzian derivative
\[ S_f (z) = \left( \frac{f''(z)}{f'(z)} \right)' - \frac{1}{2} \left( \frac{f''(z)}{f'(z)} \right)^2. \]

Thus it is quite natural to ask about inequalities for \( \phi_\mu \) corresponding to subclasses of \( S \). This is called Fekete-Szegő problem. Actually, many authors have considered this problem for typical classes of univalent functions (see, for instance [1, 4–10, 14–17]).

A function \( f \in A \) is said to be starlike of order \( \beta \) (\( 0 \leq \beta < 1 \)) if it satisfies the inequality
\[ \Re \left( \frac{zf'(z)}{f(z)} \right) > \beta \quad (z \in \mathbb{U}). \]

We denote the class which consists of all functions \( f \in A \) that are starlike of order \( \beta \) by \( S^*(\beta) \). It is well-known that \( S^*(\beta) \subset S^*(0) = S^* \subset S \).

Let \( 0 \leq \alpha, \beta < 1 \). A function \( f \in A \) is said to be close-to-convex of order \( \alpha \) and type \( \beta \) if there exists a function \( g \in S^*(\beta) \) such that the inequality
\[ \Re \left( \frac{zf'(z)}{g(z)} \right) > \alpha \quad (z \in \mathbb{U}) \]
holds. We denote the class which consists of all functions \( f \in A \) that are close-to-convex of order \( \alpha \) and type \( \beta \) by \( C(\alpha, \beta) \). This class is introduced by Libera [16].

In particular, when \( \beta = 0 \) we have \( C(\alpha, 0) = C(\alpha) \) of close-to-convex functions of order \( \alpha \), and also we get \( C(0, 0) = C \) of close-to-convex functions introduced by Kaplan [13]. It is well-known that \( S^* \subset C \subset S \).

Keogh and Merkes [14] stated the Fekete-Szegő inequalities for functions in the classes \( S^*(\beta) \) and \( C \), respectively, as follows:

**Theorem 1.** For \( 0 \leq \beta < 1 \), let \( f(z) \) given by \((1.1)\) belongs to the function class \( S^*(\beta) \). Then for any real number \( \mu \),
\[ |a_3 - \mu a_2^2| \leq (1 - \beta) \max \{1, |3 - 2\beta - 4\mu (1 - \beta)|\}. \]
Theorem 2. If \( f \in C \) and if \( \mu \) is real, then
\[
|a_3 - \mu a_2^2| \leq \begin{cases}
3 - 4\mu, & \mu \leq \frac{1}{3} \\
\frac{1}{3} - \frac{4}{9\mu}, & \frac{1}{3} \leq \mu \leq \frac{2}{3} \\
1, & \frac{2}{3} < \mu \leq 1 \\
4\mu - 3, & \mu \geq 1
\end{cases}.
\]
For each \( \mu \), there is a function in \( C \) such that equality holds.

Recently, Darus and Thomas [9] generalized the results of Theorem 2 for functions \( f \in C(\alpha, \beta) \) as the following.

Theorem 3. Let \( f \in C(\alpha, \beta) \) and be given by (1.1). Then for \( 0 \leq \alpha, \beta < 1 \),
\[
3|a_3 - \mu a_2^2| \leq \begin{cases}
(3 - 2\beta)(3 - 2\alpha - \beta) - 3\mu(2 - \alpha - \beta)^2, & \mu \leq \frac{2(1-\beta)}{3(2-\alpha-\beta)} \\
1 - 2\alpha + \beta(3 - 2\beta) + \frac{4}{3\mu}(1 - \beta)^2, & \frac{2(1-\beta)}{3(2-\alpha-\beta)} \leq \mu \leq \frac{2}{3} \\
3 - 2\alpha - \beta, & \frac{2}{3} \leq \mu \leq \frac{2(2-\beta)(3-2\alpha-\beta)}{3(2-\alpha-\beta)^2} \\
(2\beta - 3)(3 - 2\alpha - \beta) + 3\mu(2 - \alpha - \beta)^2, & \mu \geq \frac{2(2-\beta)(3-2\alpha-\beta)}{3(2-\alpha-\beta)^2}
\end{cases}.
\]
For each \( \mu \), there is a function in \( C(\alpha, \beta) \) such that equality holds.

Al-Abbadi and Darus [3] introduced a general subclass \( U_{\alpha, \beta}^\delta \) of close-to-convex functions as follows:

**Definition 1.** For \( 0 \leq \lambda < 1, 0 \leq \alpha < 1 \) and \( 0 \leq \beta < 1 \), let the function \( f \in \mathcal{A} \) be given by (1.1). Then the function \( f \in U_{\alpha, \beta}^\delta \) if and only if there exist \( g \in S^*(\beta) \) such that
\[
\Re \left( zf'(z) + \lambda z^2 f''(z)\right) > \alpha \quad (z \in \mathbb{U}).
\]

Now we define a more comprehensive class of close-to-convex functions of order \( \alpha \) and type \( \beta \):

**Definition 2.** Let \( 0 \leq \alpha < 1 \) and \( 0 \leq \delta \leq \lambda \leq 1 \). We denote by \( K_{\lambda, \delta}(\alpha, \beta) \) the class of functions \( f \in \mathcal{A} \) satisfying
\[
\Re \left( z f'(z) + (\lambda - \delta + 2\lambda\delta) z^2 f''(z) + \lambda\delta z^3 f'''(z)\right) > \alpha \quad (z \in \mathbb{U}),
\]
where $g \in S^*(\beta); 0 \leq \beta < 1$.

**Remark 1.** (i) For $\delta = 0$, the class $K_{\lambda, \delta}(\alpha, \beta)$ reduces to the class $U_{\lambda, \alpha}^\beta$ defined in Definition 1.

(ii) For $\delta = 0$ and $\alpha = 0$, the class $K_{\lambda, \delta}(\alpha, \beta)$ reduces to the class $U_{\lambda}^\beta$ which consists of functions $f \in A$ satisfying

$$
\Re \left( \frac{zf'(z) + \lambda z^2 f''(z)}{g(z)} \right) > 0 \quad (z \in U),
$$

where $g \in S^*(\beta); 0 \leq \beta < 1; 0 \leq \lambda \leq 1$. This class introduced and studied by Al-Abbadi and Darus [2].

(iii) For $\delta = 0$ and $\lambda = 0$, the class $K_{\lambda, \delta}(\alpha, \beta)$ reduces to the class $C(\alpha, \beta)$.

2. Preliminary Results

We denote by $P$ a class of the analytic functions in $U$ with $p(0) = 1$ and $\Re (p(z)) > 0$.

We shall require the following lemmas.

**Lemma 1.** [11] Let $p \in P$ with $p(z) = 1 + c_1 z + c_2 z^2 + \cdots$. Then

$$
|c_n| \leq 2 \quad (n \geq 1).
$$

**Lemma 2.** [17] Let $p \in P$ with $p(z) = 1 + c_1 z + c_2 z^2 + \cdots$. Then for any complex number $\nu$

$$
|c_2 - \nu c_1^2| \leq 2 \max \{1, |2\nu - 1|\},
$$

and the result is sharp for the functions given by

$$
p(z) = \frac{1 + z^2}{1 - z^2} \quad \text{and} \quad p(z) = \frac{1 + z}{1 - z}.
$$

**Lemma 3.** [18] Let the function $g$ defined by

$$
g(z) = z + \sum_{k=2}^{\infty} b_k z^k \quad (z \in U) \tag{2.1}
$$

belongs to the function class $S^*(\beta) \ (0 \leq \beta < 1)$. Then we have

$$
|b_2| \leq 2 (1 - \beta)
$$

and

$$
|b_3| \leq (1 - \beta)(3 - 2\beta).
$$
Lemma 4. Let the function \( f \) given by (1.1) belongs to the function class \( K_{\lambda, \delta} (\alpha, \beta) \). Then

\[
(1 + \lambda - \delta + 2\lambda \delta) |a_2| \leq 2 - \alpha - \beta
\]

and

\[
3 (1 + 2\lambda - 2\delta + 6\lambda \delta) |a_3| \leq (3 - 2\alpha - \beta) (3 - 2\beta).
\]

Proof. Let the function \( f \in K_{\lambda, \delta} (\alpha, \beta) \) be of the form (1.1). Therefore, there exists a function \( g \in S^{*} (\beta) \), defined in (2.1), so that

\[
\Re \left( \frac{zf''(z)}{g(z)} + (\lambda - \delta + 2\lambda \delta) z^2 f''(z) + \lambda \delta z^3 f'''(z) \right) > \alpha \quad (z \in U).
\]

It follows from the above inequality that

\[
\frac{zf''(z)}{g(z)} + (\lambda - \delta + 2\lambda \delta) z^2 f''(z) + \lambda \delta z^3 f'''(z) = \alpha + (1 - \alpha) p(z),
\]

with

\[
p(z) = 1 + c_1 z + c_2 z^2 + \cdots \in \mathcal{P}.
\]

The equality (2.5) implies the equality

\[
z + \sum_{k=2}^{\infty} k [(1 - \lambda + \delta) + k (\lambda - \delta) + k (k - 1) \lambda \delta] a_k z^k
\]

\[
z + \sum_{k=2}^{\infty} b_k z^k = 1 + (1 - \alpha) \sum_{k=1}^{\infty} c_k z^k.
\]

Equating coefficients of both sides, we have

\[
2 (1 + \lambda - \delta + 2\lambda \delta) a_2 = b_2 + (1 - \alpha) c_1
\]

and

\[
3 (1 + 2\lambda - 2\delta + 6\lambda \delta) a_3 = b_3 + (1 - \alpha) b_2 c_1 + (1 - \alpha) c_2.
\]

Using Lemma 1 and Lemma 3 in (2.6) and (2.7), we easily get (2.2) and (2.3), respectively. \( \Box \)

Remark 2. In Lemma 4, letting \( \delta = 0; \delta = 0, \alpha = 0; \) or \( \delta = \lambda = 0 \), we have [3, Lemma 3], [2, Lemma 2.3] and [2, Lemma 3], respectively.
3. Main Results

In this section, we begin by solving the Fekete-Szegö problem for functions belonging to the class $K_{\lambda,\delta}(\alpha, \beta)$ when $\mu \in \mathbb{C}$.

**Theorem 4.** Let $f(z)$ given by (1.1) belongs to the function class $K_{\lambda,\delta}(\alpha, \beta)$. Then, for any complex number $\mu$,

$$3 \left( 1 + 2\lambda - 2\delta + 6\lambda \delta \right) |a_3 - \mu a_2^2| \leq (1 - \beta) \max \{ 1, |3 - 2\beta - \mu \Psi_{\lambda,\delta}(\beta)| \}$$

$$+ 2 (1 - \alpha) \max \left\{ 1, \left| 1 - \mu \frac{\Psi_{\lambda,\delta}(\alpha)}{2} \right| \right\}$$

$$+ 4 (1 - \alpha) (1 - \beta) \left| 1 - \mu \frac{\Psi_{\lambda,\delta}(0)}{2} \right|,$$

where

$$\Psi_{\lambda,\delta}(s) = \frac{3 \left( 1 + 2\lambda - 2\delta + 6\lambda \delta \right)}{(1 + \lambda - \delta + 2\lambda \delta)^2} (1 - s).$$

**Proof.** Let the function $f \in K_{\lambda,\delta}(\alpha, \beta)$ be of the form (1.1). For the simplicity, we set

$$\tau = 1 + \lambda - \delta + 2\lambda \delta, \quad \sigma = 1 + 2\lambda - 2\delta + 6\lambda \delta.$$

From (2.6) and (2.7), we obtain

$$3\sigma (a_3 - \mu a_2^2) = \left( b_3 - \mu \frac{3\sigma}{4\tau^2} b_2^2 \right) + (1 - \alpha) \left( c_2 - \mu \frac{3\sigma (1 - \alpha)}{4\tau^2} c_1^2 \right)$$

$$+ (1 - \alpha) \left( 1 - \mu \frac{3\sigma}{2\tau^2} \right) b_2 c_1.$$  \hfill (3.2)

So we have

$$3\sigma |a_3 - \mu a_2^2| \leq \left| b_3 - \mu \frac{3\sigma}{4\tau^2} b_2^2 \right| + (1 - \alpha) \left| c_2 - \mu \frac{3\sigma (1 - \alpha)}{4\tau^2} c_1^2 \right|$$

$$+ (1 - \alpha) \left| 1 - \mu \frac{3\sigma}{2\tau^2} \right| |b_2| |c_1|.$$

Hence, by means of Theorem 4 and Lemmas 1-3, we get desired result. \hfill $\square$

Letting $\delta = 0$ in Theorem 4 we get following consequence.

**Corollary 1.** Let $f(z)$ given by (1.1) belongs to the function class $\mathcal{U}^3_{\lambda,\alpha}$. Then, for any complex number $\mu$,

$$3 \left( 1 + 2\lambda \right) |a_3 - \mu a_2^2| \leq (1 - \beta) \max \{ 1, |3 - 2\beta - \mu \Psi_{\lambda}(\beta)| \}$$
\[ +2 (1 - \alpha) \max \left\{ 1, \left| 1 - \mu \frac{\Psi_\lambda(\alpha)}{2} \right| \right\} \]
\[ +4 (1 - \alpha) (1 - \beta) \left| 1 - \mu \frac{\Psi_\lambda(0)}{2} \right| , \]

where

\[ \Psi_\lambda(s) = \frac{3(1 + 2\lambda)}{(1 + \lambda)^2} (1 - s) . \]  \hspace{1cm} (3.3)

Letting \( \delta = 0 \) and \( \alpha = 0 \) in Theorem 4, we get following consequence.

**Corollary 2.** Let \( f(z) \) given by (1.1) belongs to the function class \( \mathcal{U}_\lambda^{\beta} \). Then, for any complex number \( \mu \),

\[ 3 (1 + 2\lambda) \left| a_3 - \mu a_2^2 \right| \leq (1 - \beta) \max \left\{ 1, \left| (3 - 2\beta) - \mu \Psi_\lambda(\beta) \right| \right\} + 2 \max \left\{ 1, \left| 1 - \mu \frac{\Psi_\lambda(\alpha)}{2} \right| \right\} \]
\[ +4 (1 - \beta) \left| 1 - \mu \frac{\Psi_\lambda(0)}{2} \right| , \]

where \( \Psi_\lambda \) is defined by (3.3).

Letting \( \delta = 0 \) and \( \lambda = 0 \) in Theorem 4 we get following consequence.

**Corollary 3.** Let \( f(z) \) given by (1.1) belongs to the function class \( \mathcal{C}(\alpha, \beta) \). Then, for any complex number \( \mu \),

\[ 3 \left| a_3 - \mu a_2^2 \right| \leq (1 - \beta) \max \left\{ 1, \left| 3 - 2\beta - 3\mu (1 - \beta) \right| \right\} \]
\[ + 2 (1 - \alpha) \max \left\{ 1, \left| 1 - \mu \frac{3(1 - \alpha)}{2} \right| \right\} \]
\[ + 2 (1 - \alpha) (1 - \beta) \left| 2 - 3\mu \right| , \]

Now we prove our main result when \( \mu \) is real.
Theorem 5. Let \( f(z) \) given by \((1.1)\) belongs to the function class \( \mathcal{K}_{\lambda, \delta}(\alpha, \beta) \). Then

\[
3(1 + 2\lambda - 2\delta + 6\lambda\delta) |a_3 - \mu a_2^2| \leq \begin{cases} 
(3 - 2\beta)(3 - 2\alpha - \beta) - \mu \frac{3(2 - \alpha - \beta)^2(1 + 2\lambda - 2\delta + 6\lambda\delta)}{(1 + \lambda - \delta + 2\lambda\delta)^2}, \\
\text{if } \mu \leq \frac{2(1 - \beta)(1 + \lambda - \delta + 2\lambda\delta)}{3(2 - \alpha - \beta)(1 + 2\lambda - 2\delta + 6\lambda\delta)}, \\
1 - 2\alpha + \beta(3 - 2\beta) + \frac{4(1 - \beta)^2(1 + \lambda - \delta + 2\lambda\delta)^2}{3(1 + 2\lambda - 2\delta + 6\lambda\delta)\mu}, \\
\text{if } \frac{2(1 - \beta)(1 + \lambda - \delta + 2\lambda\delta)^2}{3(2 - \alpha - \beta)(1 + 2\lambda - 2\delta + 6\lambda\delta)} \leq \mu \leq \frac{2(1 + \lambda - \delta + 2\lambda\delta)^2}{3(1 + 2\lambda - 2\delta + 6\lambda\delta)}, \\
(2\beta - 3)(3 - 2\alpha - \beta) + \mu \frac{3(2 - \alpha - \beta)^2(1 + 2\lambda - 2\delta + 6\lambda\delta)}{(1 + \lambda - \delta + 2\lambda\delta)^2}, \\
\text{if } \mu \geq \frac{2(2 - \beta)(3 - 2\alpha - \beta)(1 + \lambda - \delta + 2\lambda\delta)^2}{3(2 - \alpha - \beta)^2(1 + 2\lambda - 2\delta + 6\lambda\delta)}.
\end{cases}
\]

(3.4)

For each \( \mu \), there is a function in \( \mathcal{K}_{\lambda, \delta}(\alpha, \beta) \) such that equality holds.

Proof. Let \( f \in \mathcal{K}_{\lambda, \delta}(\alpha, \beta) \) be given by \((1.1)\) and let us define the function

\[
F_{\lambda, \delta}(z) = (1 - \lambda + \delta)f(z) + (\lambda - \delta)zf'(z) + \lambda\delta z^2 f''(z).
\]

(3.5)

Then it is worthy to note that the condition \((1.3)\) is equal to

\[
\Re \left( \frac{zF_{\lambda, \delta}(z)}{g(z)} \right) > \alpha \quad (z \in \mathbb{U}).
\]

Since \( g \in \mathcal{S}^*(\beta) \), by the definition of close-to-convex function of order \( \alpha \) and type \( \beta \), we deduce that \( F_{\lambda, \delta} \in \mathcal{C}(\alpha, \beta) \). By the definition of close-to-convex function class \( \mathcal{C}(\alpha, \beta) \), there exists two functions \( p, q \in \mathcal{P} \) with

\[
p(z) = 1 + c_1 z + c_2 z^2 + \cdots
\]

and

\[
q(z) = 1 + q_1 z + q_2 z^2 + \cdots
\]
such that

\[ \frac{zF'_{\lambda,\delta}(z)}{g(z)} = \alpha + (1 - \alpha) p(z) \quad \text{with} \quad \frac{zg'(z)}{g(z)} = \beta + (1 - \beta) q(z). \]

We assume that the function \( F_{\lambda,\delta} \) is of the form

\[ F_{\lambda,\delta}(z) = z + \sum_{k=2}^{\infty} A_k z^k \quad (z \in \mathbb{U}). \quad (3.6) \]

By Theorem 3, \( F_{\lambda,\delta} \in \mathcal{C}(\alpha, \beta) \) implies that

\[
\begin{align*}
3 \left| A_3 - \rho A_2^2 \right| & \leq \begin{cases} 
(3 - 2\beta) (3 - 2\alpha - \beta) - 3\rho (2 - \alpha - \beta)^2, & \rho \leq \frac{2(1-\beta)}{3(2-\alpha-\beta)} \\
1 - 2\alpha + \beta (3 - 2\beta) + \frac{4}{3\rho} (1 - \beta)^2, & \frac{2(1-\beta)}{3(2-\alpha-\beta)} \leq \rho \leq \frac{2}{3} \\
3 - 2\alpha - \beta, & \frac{2}{3} \leq \rho \leq \frac{2(2-\beta)(3-2\alpha-\beta)}{3(2-\alpha-\beta)^2} \\
(2\beta - 3) (3 - 2\alpha - \beta) + 3\rho (2 - \alpha - \beta)^2, & \rho \geq \frac{2(2-\beta)(3-2\alpha-\beta)}{3(2-\alpha-\beta)^2},
\end{cases}
\end{align*}
\]

(3.7)

Now equating the coefficients of (3.5) and (3.6), we obtain

\[ A_2 = \tau a_2, \quad A_3 = \sigma a_3, \]

where \( \tau \) and \( \sigma \) defined by (3.1). Hence we get from the above equalities that

\[ |A_3 - \rho A_2^2| = |\sigma a_3 - \rho \tau^2 a_2^2| = \sigma \left| a_3 - \rho \frac{\tau^2}{\sigma} a_2^2 \right|. \]

Taking \( \rho = \mu \sigma / \tau^2 \) in (3.7), we get desired estimate (3.4).

Finally, sharpness of the results in (3.4) is getting by

(i) in Case 1: upon choosing

\[ c_2 = 2, \quad q_1 = 3, \quad b_2 = 2 (1 - \beta), \quad b_3 = (3 - 2\beta) (1 - \beta), \]

(ii) in Case 2: upon choosing

\[ c_1 = \frac{2 (1 - \beta) (2\tau^2 - 3\sigma \mu)}{3 (1 - \alpha) \sigma \mu}, \quad c_2 = 2, \]

\[ q_1 = q_2 = 2, \]
\[ b_2 = 2(1 - \beta), \quad b_3 = (3 - 2\beta)(1 - \beta). \]

(iii) in Case 3: upon choosing
\[
\begin{align*}
c_1 &= 0, \quad c_2 = 2 \\
q_1 &= 0, \quad q_2 = 2 \\
b_2 &= 0, \quad b_3 = 1 - \beta,
\end{align*}
\]

(iv) in Case 4: upon choosing
\[
\begin{align*}
c_1 &= 2i, \quad c_2 = -2 \\
q_1 &= 2i, \quad q_2 = -2 \\
b_2 &= 2(1 - \beta)i, \quad b_3 = (3 - 2\beta)(\beta - 1).
\end{align*}
\]

Remark 3. Note that our method proves easily the theorems given by [3, Theorem 1] and [2, Theorem 3.1] with a different way.

Corollary 4. In Theorem 5, letting \( \delta = 0; \delta = 0, \alpha = 0; \delta = \lambda = 0, \alpha = \beta = 0, \)
we have [3, Theorem 1], [2, Theorem 3.1], Theorem 3 and Theorem 2, respectively.

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