Analytical solution of the problem of acceleration of cargo by a bridge crane with constant acceleration at elimination of swings of a cargo rope

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Abstract. Limitation of the swing of the bridge crane cargo rope is a matter of urgency, as it can significantly improve the efficiency and safety of the work performed. In order to completely dampen the pendulum swing after the break-up of a bridge or a bridge-crane freight cart to maximum speed, it is necessary, in the normal repulsion control of the electric motor, to split the process of dispersion into a minimum of three gaps. For a dynamic system of swinging of a bridge crane on a flexible cable hanger in a separate vertical plane, an analytical solution was obtained to determine the temporal dependence of the cargo rope angle relative to the gravitational vertical. The resulting analytical dependence of the cargo rope angle and its first derivative can break the process of dispersing the cargo suspension point into three stages of dispersal and braking with various accelerations and enter maximum speed of movement of the cargo suspension point. In doing so, the condition of eliminating the swings of the cargo rope relative to the gravitational vertical is fulfilled. Provides examples of the maximum speed output constraints-to-time when removing the rope swing.

1. Introduction
The movement of the cargo, suspended by a cargo rope, is carried out by means of a bridge crane (BC), working in warehouses and workshops. The movement of the cargo is possible in the complete elimination of the unmanaged components of the pendulum swings. That is, with the vertical position of the cargo rope in the steady-state driving mode. This will not only reduce the time of the BC cycle, but also increase the efficiency and safety of the work performed [1, 2, 3]. Known methods for synthesizing the path of a suspension point [3, 5, 6, 7, 8, 9, 10, 11, 12] have a general disadvantage in the form of a relatively large margin of error in implementing both the vertical and the linear coordinates of the moving cargo. In general, the angle of deflection of the BC freight cable from the vertical is not monitored or controlled. The time of movement of the cargo is increasing.
In order to completely dampen the pendulum swings after the dispersion of the bridge or the BC freight cart to the maximum speed, it is necessary to move the point of the cargo suspension (the bridge or the freight cart of the BC) with a variable acceleration. This requires the use of a frequency-
controlled electric drive. Or, if you are using conventional relay management, you need to break the acceleration process by a minimum of three gaps.

2. Formulation of the problem
Accept as an assumption that the dispersion of the bridge (or the BC freight cart) to a certain maximum speed when the appropriate electric motor is turned on, is performed with a constant acceleration of $a_1$. Break the process of reaching the bridge maximum speed by three gaps. At the end of the third gap, the bridge should be dispersed to the maximum speed $v_{\text{max}}$, and the suspension point acceleration changes to zero. The angle of deflection of the freight cable $q$ and its first derivative $\dot{q}$ at that time is zero.

In the first and third periods, the racing acceleration of $a_1$ will be carried out. It is created by the bridge drive. The second period between the first and the third can be described as moving the bridge in inertia with slow braking. Braking is done with a constant acceleration of negative sign $a_2$, under the influence of friction forces in the drive. There is no positive acceleration in the dispersion of the bridge drive. A relatively small deceleration acceleration of $a_2$ is not created by the bridge braking mechanisms. It is created by friction forces arising in the transmissions of the kinematic steam of the drive and the transmission engine.

3. Theoretical provisions
For a mathematical description of the swing of the BC cargo on the cable hanger, a known mathematical model of the flat pendulum with fading vibrations is used. For the small corners of the cargo rope, it is described by the linear differential equation (DE) of the second order [13, 14, 15, 16]:

$$\ddot{q} + \frac{x}{L} + b\dot{q} + g \cdot q / L = 0,$$

where $L$ is the length of the BC’s cargo rope from the movable point of the freight trolley (chain centre of the roller block) to the centre of the mass of the cargo, $m$; $b = 2b_{\text{fr}}m$; $b_0$ – the coefficient of viscous friction, given to the angular coordinate, gives the measure of the energy dissipation, N∙m/Rad; $m$ – mass of cargo, kg; $q$, $\dot{q}$, $\ddot{q}$ – the angle of deflection of the BC’s cargo rope from the gravitational vertical and its first two derivatives in time, respectively, Rad, Rad/s, Rad/s$^2$; $g = 9.81$ – acceleration of free fall, m/s$^2$; $\dot{x}$ – linear acceleration of the cargo suspension point in the horizontal direction of the freight trolley, m/s$^2$.

Assumptions were made about: - the minor angle of deflection of the cargo rope from the gravitational vertical (not more than 1-3°); - the constancy length of the cargo rope $L$ in the process of moving the cargo; - the constancy racing acceleration $a_1$ and constancy acceleration the coast down under the forces of inertia and friction $a_2$; - the negligible of the small influence of the mass of the transportable cargo and the moving links of the BC to the controlled acceleration parameter of the suspension point that takes values from the discrete series $\ddot{x} = [a_1; a_2]$.

Analytic decision DE (1) has been obtained for constant acceleration:

$$q(t) = C_1 \cdot e^{-\left(\frac{L b + \sqrt{L^2 \cdot b^2 - 4 \cdot L \cdot g}}{2 L^2 \cdot b^2 - 4 \cdot L \cdot g}\right)} \cdot e^{\left(\frac{L b - \sqrt{L^2 \cdot b^2 - 4 \cdot L \cdot g}}{2 L^2 \cdot b^2 - 4 \cdot L \cdot g}\right)}$$

The expression (2) resulted in an expression of the speed of change in the inclination of the BC's cargo cable also in analytical form. The expression $\dot{q}(t)$ is not given due to its cumbersomeness.
To define permanent integrations $C_1$ and $C_2$, a system of two equations of function angle $q(t)$ and its first derivative $\dot{q}(t)$ at time $t = 0$ (initial conditions) was drawn:

\[
\begin{align*}
q_0 &= C_1 + C_2 - \frac{a_1(Lb + L^2 b^2 - 4Lg)}{2gL b^2 - 4Lg} - \frac{a_1(Lb - L(4g - Lb^2))}{2gL b^2 - 4Lg}; \\
\dot{q}_0 &= \frac{a_1(Lb + L^2 b^2 - 4Lg)}{4Lg} - \frac{2L}{4Lg} - \frac{a_1(Lb - L(4g - Lb^2))}{4Lg} + \frac{a_1(Lb + L^2 b^2 - 4Lg)}{4Lg}; \\
&\quad + \frac{2a_1(Lb - L^2 b^2 - 4Lg)}{4Lg} - \frac{ba_1(Lb - L^2 b^2 - 4Lg)}{4Lg} - \frac{ba_1(Lb + L^2 b^2 - 4Lg)}{4Lg}.
\end{align*}
\]

Where $q_0$ is the value of the angle of the cargo rope at time $t = 0$; $\dot{q}_0$ – the value of the first derivative of the angle of the cargo rope at the time $t = 0$.

The solution of the system (3) relatively unknown $C_1$ and $C_2$ allowed their analytical expressions to be obtained:

\[
\begin{align*}
C_1 &= \frac{a_1\sqrt{-L(4g - Lb^2)} + 2Lq_0 + q_0g\sqrt{-L(4g - Lb^2)} + Lb\dot{q}_0}{2g\sqrt{-L(4g - Lb^2)}}; \\
C_2 &= \frac{2Lg\dot{q}_0 - a_1\sqrt{-L(4g - Lb^2)} - q_0g\sqrt{-L(4g - Lb^2)} + Lb\dot{q}_0}{2g\sqrt{-L(4g - Lb^2)}}.
\end{align*}
\]

The use of the resulting analytical constraints (2), (4) opens the possibility of optimizing the process of dispersing the BC bridge without using simulation modelling and numerical solution DE (1). The acceleration is carried out in three periods, with the condition of the total elimination of the swings at the end of the third period of dispersal (on the output to a specified constant speed).

4. Experimental results

As an example of the practical use of the analytic expressions received, the values of the three time intervals (racing acceleration – moving the bridge in inertia with slow braking – racing acceleration) $T_1$, $T_2$ and $T_3$, respectively, are optimized to the extent that the target function is minimized

\[
f = |q_{end}| + |\dot{q}_{end}|,
\]

where $q_{end}$ is the value of the deflection angle of the cargo rope at the end of the time of the dispersion $t = T_1 + T_2 + T_3$; $\dot{q}_{end}$ – the value of the first derivative of the angle of deflection of the cargo rope at the final time of the dispersion of $t = T_1 + T_2 + T_3$. 

3
The results of a computational experiment on bridge racing in the mode of eliminating cargo swings upon reaching the maximum speed: the dependence of the target function (a) on the time $T_2$ at $v_{\text{max}}=1.5$ m/s; the optimum process of intermittent racing (b) at $v_{\text{max}}=1.5$ m/s; the dependency of the target function (c) from time $T_2$ at $v_{\text{max}}=1$ m/s; the optimum process of intermittent racing (d) at $v_{\text{max}}=1$ m/s.

The acceleration of the cargo suspension point during the racing and slow braking of the bridge assumed values $a_1=0.25$ m/s; $a_2=-0.05$ m/s. In order to reduce the size of the task and the number of independent factors in the computational experiment, an additional condition was adopted for the equality of the first and third periods (racing). In this case, the values of the maximum velocity $v_{\text{max}}$, accelerations and time intervals are related to each other by the dependence

$$2a_1T_1 + a_2T_2 = v_{\text{max}}.$$ 

When taking the value of the time of slow braking $T_2$ between two equal racing periods as the only independent experiment factor, the value of the latter will be calculated from the dependence

$$T_1 = T_3 = (v_{\text{max}} - a_2T_2)/(2a_1).$$

The values of the angle of the cargo rope $q$, as well as its first derivative, were calculated by (2) for a finite time of each of the three periods and served as initial conditions in (4) for the following period. Some results of the computational experiment are shown in Figure 1 for the values of the maximum velocity $v_{\text{max}}=1.5$ m/s ($a$, $b$) and $v_{\text{max}}=1$ m/s ($c$, $d$). The optimal process of intermittent racing at $v_{\text{max}}=1.5$ m/s is achieved at $T_2=0.142$ s, $T_1=T_3=3.014$ s. At $v_{\text{max}}=1$ m/s, the analogous optimal values of the time periods were $T_2=0.959$ s, $T_1=T_3=2.096$ s. The total discontinuous acceleration time was 6.17 and 5.151 s for velocities $v_{\text{max}}=1.5$ m/s and $v_{\text{max}}=1$ m/s, respectively.

5. Conclusions

Based on the known DE of swings of a plane pendulum with a movable suspension point and attenuation, the analytical dependences of the slope angle of the cargo rope and its first derivative on time are obtained with constant acceleration of the suspension point. They allow for the separation of the process of acceleration of the bridge or the BC cargo trolley at least three stages (racing with constant acceleration – forward motion with slow braking – further racing with constant acceleration)
to enter the mode of the specified maximum speed of the cargo suspension point when the condition of complete elimination of the swings of the cargo rope around the gravitational vertical.

To advantages of the developed method it is necessary to attribute shortness and limited character of deviations of corners of a cargo rope from a vertical. The use of the method for optimizing the acceleration process of a bridge or a BC freight trolley does not require the implementation of simulation and numerical integration of DE. After the acceleration, the cargo rope and the cargo are in an upright position. At the same time, there are no deviations and no cargo rope swings. The method can be used in automatic control systems of BC with relay control of moving links for the acceleration of cargo in the mode of eliminating the residual swings of the cargo rope.

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