Finite Size of Electron in $e^+e^- \rightarrow \gamma\gamma$ Annihilation in differential cross section of centre-of-mass energies 55 - 207 GeV

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Abstract

The experimental data of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction from the VENUS, TOPAS, OPAL, DELPHI, ALEPH and L3 collaborations, collected from 1989 to 2003, are used to test the QED in performing a comprehensive $\chi^2$ test to search for an excited electron mass $m_{e^*}$ and a finite annihilation length in a direct contact term approach. The experimental data of the differential cross section compared to the QED cross section in a $\chi^2$ test allows to set an approximately $5 \times \sigma$ significance on the mass of an excited electron to $m_{e^*} = 308 \pm 14$ GeV. A similar $5 \times \sigma$ significance effect was found for a charge distribution radius of the electron $r_e = (1.57 \pm 0.07) \times 10^{-17}$ cm.

Keywords: QED; Contact interaction; Beyond standard model

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1 Introduction of the experimental and theoretical evidence of the electron

The discoveries of Coulomb’s law 1785 and the magnetism 1820, open 1869 the possibility to investigate charged particle beams with Cathode Ray Tubes.

The law from Charles-Augustin de Coulomb states that the magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product
of the magnitudes of charges and inversely proportional to the square of the distance between them. [1].

Three discoveries in 1820 established the foundation of magnetism. Hans Christian Ørsted demonstrated that a current-carrying wire is surrounded by a circular magnetic field [2]. André-Marie Ampère showed that parallel wires with currents attract one another if the currents are in the same direction and repel if they are in opposite directions [3]. Jean-Baptiste Biot and Félix Savart announced empirical results about the forces that a current-carrying long, straight wire exerted on a small magnet, determining the forces were inversely proportional to the perpendicular distance from the wire to the magnet [4].

Cathode rays (electron beam or e-beam) are streams of electrons observed in discharge tubes. If an evacuated glass tube is equipped with two electrodes and a voltage is applied, glass behind the positive electrode is observed to glow, due to electrons emitted from the cathode (the electrode connected to the negative terminal of the voltage supply). They were first observed in 1869 by German physicist Julius Plücker and Johann Wilhelm Hittorf [5] and were named in 1876 by Eugen Goldstein Kathodenstrahlen, or cathode rays. [6] In 1897, British physicist J. J. Thomson showed that cathode rays were composed of a previously unknown negatively charged particle, which was later named the electron. Cathode-ray tubes (CRTs) use a focused beam of electrons deflected by electric or magnetic fields to render an image on a screen. [7].

Sir Joseph John Thomson credited 1897 with the discovery of the electron, the first subatomic particle to be discovered. He showed that cathode rays were composed of previously unknown negatively charged particles, which he calculated must have bodies much smaller than atoms and a very large charge-to-mass ratio. [8]. Finally Millikan and Fletcher measured 1909 the mass or charge separately in the oil drop experiment [9].

Soon after the experimental discovery of the electron as an object with charge and mass, the first models of the electron proposed by Abraham 1903 [10] and Lorentz 1903 [11, 12] are developed. The electron was visualised as an extended spherical electrical charged object with a total field energy. The models were based on the assumption of a homogenous distribution of charge density, what opens the possibility to calculate the mass of the electron of electromagnetic origin, but introduces the problem of preventing the electron from flying apart under the single force of Coulomb repulsion. Abraham argued that non-electromagnetic forces (the Poincare stress) were necessary to prevent Lorentz’s contractile electrons from exploding. This testifies the impossibility to construct an electron model within electrodynamics at this time. Later Dirac [13] introduces a point-like model of electron and noted the most attractive idea of the Lorentz model [11] concerning the electromagnetic origin of the electron mass. To extend this idea to all known particles at this time was in conflict with the neutron. Dirac also points out: “That the electron treated as a point charge the difficulties of the infinite Coulomb energy
are avoided in the equations by a procedure of direct omission or subtracting of unwanted terms, but in their physical interpretation the finite size of the electron reappears in a new sense, the interior of the electron being a region of space through which signals can be transmitted faster than light”.

The first to introduce the idea of electron spin was Arthur Compton in 1921. In a paper on investigations of ferromagnetic substances with X-rays. [14] he wrote: “Perhaps the most natural, and certainly the most generally accepted view of the nature of the elementary magnet, is that the revolution of electrons in orbits within the atom give to the atom as a whole the properties of a tiny permanent magnet ”. The electron magnetic moment $\mu_s$ is intrinsically connected to electron spin $S$ via the equation $\mu_s = -g_s\mu_BS/\hbar$. The spin g-factor is approximately $g_s \sim 2$. Finally the existence of quantised electron spin angular momentum is inferred by the Stern - Gerlach experiment. After its conception by Otto Stern in 1921, the experiment was first successfully conducted by Walther Gerlach in early 1922 [15]. In the experiment, silver atoms were sent through a spatially varying magnetic field, which deflected them before they struck a glass slide detector screen. Particles with non-zero magnetic moment are deflected, due to the magnetic field gradient, from a straight path. The existence of the electron spin can also be inferred theoretically from the spin - statistics theorem and from the Pauli exclusion principle and vice versa, given the particular spin of the electron, one may derive the Pauli exclusion principle. [16].

The existence of quantised particle spin introduces the possibility to scatter polarised particle beams on different targets to investigate spin depending interaction. In nuclear physics scattering experiments used polarized beam [17] and electrostatic accelerators like Tandem accelerators in an energy range from centre of mass energies like 1.2 MeV [18] to 20 MeV [19]. In principle three type of polarized beam get developed, the atomic beam source using the technique of the Stern - Gerlach experiment [20], after the discovery of the Lamb shift 1947 [21], the Lamb-shift source [22] and the crossed - beam source. [23].

From 1926 until today more classical models of point-like spinning particles are developed. All these models are plagued from the fact, that it is complicated to construct a stable point particle including a single repulsive coulomb force raging from a distance zero to infinite.

One point-like models goes back to the Schrödinger suggestion [24] relating the electron spin to its Zittebewegung motion - trembling motion due to the rapid oscillation of a spinning particle about its classical worldline. The approach based on the concept of Zitterbewegung was motivated by attempts to understand the intrinsic nature of the electron spin and it involved studying the fundamental questions of quantum mechanics [25].

Other types of classical models of point-like spinning particles encounter the problem of divergent self-energy for a point charge and approach this problem in the frame of various
generalisations of the classical Lagrangian terms with higher derivatives or extra variables and then restricting undesirable effects by applying geometrical or symmetry constraints.

A recent development of model for searches on the electron structure has been opened in 1965 by the discovery of the Kerr - Newman solution to the Einstein - Maxwell equations. This novel model is used by I. Dymnikova to introduce a “Image of the Electron Suggested by Nonlinear Electrodynamics Coupled to Gravity”. All so far discussed electron models are plagued with the coulomb repulsion, what causes severe problems to calculate a stable lifetime of the electron. The coupling to gravity opens the possibility to introduce the De Sitter (1872- 1934) spaces geometry, what includes in the model an attractive / repulsive force depending about the distance from the origin. The De Sitter space, for example discussed in distinguish between Schwarzschild and De Sitter black wholes, describes a force what has boundaries in inner distance with attractive force and an other distance with repulsive force. Under these circumstances is the model very attractive to avoid singularities and infinite unwanted field integrals.

The model uses the electron input parameters: charge, mass, spin, magnetic moment and Compton wave length, in addition to the Planck constant, speed of light and the gravitational constant G. In the output conclusion I. Dymnikova points out : “ We see that the behaviour of solutions for electromagnetic fields determines typical generic features of regular nonlinear electrodynamics minimally coupled to gravity spinning objects: Interior de Sitter vacuum disk ( equ. 8 page 132 ) has the properties of a perfect conductor and ideal diamagnetic. The superconducting current flowing over the edge of the disk replaces the ring singularity of the Kerr - Newman geometry, powers the electromagnetic fields of an object, and provides the origin of its intrinsic magnetic momentum. For the electromagnetic soliton with the parameters of the electron $j = 79.277 \, \text{A}$.”

Not directly addressed in is, that the superconducting current $j$ implements an infinite lifetime of the electron. An electric current through a loop of superconducting wire can persist indefinitely with no power source. The experimental limit of $\tau_e$ is so far , stable $\tau_e > 6 \times 10^{28} \, \text{yr}$.The superconducting electric ring current $j$ is in the model a homogen not clustered electric current. Under this condition an electric dipole moment is very unlikely because a moving charge cluster is missing. Within the Standard Model of elementary particle physics, such a dipole is predicted to be non-zero but very small, at most $d_e < 10^{-38} \, \text{e-cm}$.

The model of is actual the optimal description of the electron and opens a window between particle physics and gravitation. Missing is the experimental confirmation that the De Sitter vacuum disk is a geometrical extended object.

In the Standard Theory of Quantum Mechanics ( STQ ) in particular of Quantum Electro
Dynamic (QED), is a finite size of the electron not addressed. The electron behaves like a point-particle and a wave. These "duality paradox" is excepted in the "Copenhagen Interpretation of Quantum Mechanics"[35]. The problem of infinities, for example that the charge and mass of a naked electron would be infinite, could be solved via renormalisation[36] but is still in discussion today[37].

The question is, is this point-like behaviour correct from zero up to the sizes of the Planck-Scale $1.616255 \times 10^{-35}$ m or is it possible to detect a deviation from the point-like structor of the electron after the "Sitter vacuum disk"[32] at much lower scales? The QED is for this investigation the ideal theory, because every deviation from QED $e^+ e^- \rightarrow \gamma \gamma$ scattering in differential or total cross section, could be interpreted as a deviation from the point-like structure of the electron or new physics.

A experiment to test this point like structure of the electron requests, a well defined signal in a detector with low background, high statistic and high test energy. The ideal experimental setup for this goal would be an $e^+ e^-$ storage ring accelerator. The available data from the $e^+ e^-$ accelerators favour two experiments to test the finite size of an electron the $e^+ e^- \rightarrow \gamma \gamma$ and $e^+ e^- \rightarrow e^+ e^-$ reactions. Both reactions are shown in Figure 1.

The two $\gamma$'s in the final state of the reaction $e^+ e^- \rightarrow \gamma \gamma$ are undistinguishable. The reaction performs for this reason via the t- and u-channel. The s-channel is forbidden, by the law of angular momentum conservation. As consequent this reaction is more sensitive to the long range QED interaction. The two $\gamma$'s in the final state are left-handed and right-handed polarised. They couple to total spin zero. Under these circumstances the s-channel with spin one for $\gamma$ and $Z^0$ is forbidden. The reaction is a pure annihilation reaction. The $e^+$ and $e^-$ in the initial state completely annihilate to two $\gamma$'s in the final state. The signal in a detector contains ONLY two gammas, it is easy to subtract background. The reaction $e^+ e^- \rightarrow \gamma \gamma$ is a very well defined test of QED.

The Bhabha reaction $e^+ e^- \rightarrow e^+ e^-$ is a mixed reaction, via the scattering in the s-channel and t-channel. In particular at energies around the $Z^0$ pole, the $Z^0$ is dominant. The annihilation channel is superimposed by elastic scattering. The $e^+$ and $e^-$ in the initial state and final state are identical. The reaction $e^+ e^- \rightarrow e^+ e^-$ is a test of a superimposition of short range Weak Interaction and long range QED interaction.

In the actual experimental investigation the differential cross section of the $e^+ e^- \rightarrow \gamma \gamma$ reaction was used to study the possibility of a deviation from QED using all experimental data generated from 1989 by different electron - positron colliders up to today. The reaction was investigated by the VENUS[38] collaboration 1989 from energies $\sqrt{s} = 55$ GeV - 57 GeV, OPAL[39] collaboration 1991 at the $Z^0$ pole at $\sqrt{s} = 91$ GeV, TOPAS[40] collaboration 1992 at $\sqrt{s} = 57.6$ GeV, ALEPH[41] collaboration 1992 at the $Z^0$ pole $\sqrt{s} = 91.0$ GeV, DELPHI[42]
Figure 1: Lowest order $e^+e^- \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow e^+e^-$ reactions. The reaction $e^+e^- \rightarrow \gamma\gamma$ proceeds via the t- and u-channel. The Bhabha $e^+e^- \rightarrow e^+e^-$ channel proceeds via the s- and t-channel.
collaboration from 1994 to 2000 at energies $\sqrt{s} = 91.0$ GeV to 202 GeV, L3 collaboration from 1991 to 1993 at the $Z^0$ pole range from $\sqrt{s} = 88.5$ GeV - 93.7 GeV, L3 collaboration 2002 from $\sqrt{s} = 183$ GeV - 207 GeV and OPAL collaboration 2003 from $\sqrt{s} = 183$ GeV - 207 GeV. In summary cross sections of the $e^+e^- \rightarrow \gamma\gamma$ reaction have been measured at centre-of-mass energies in total from $\sqrt{s} = 55$ GeV to 207 GeV by these six collaborations. Possible deviations from QED were studied in terms of contact interaction and excited electron exchange shown in Figure 2.

Figure 2: Lowest-order Feynman diagrams of $e^+e^- \rightarrow \gamma\gamma$ reaction. Figure (a) represents QED, (b) contact interaction and (c) excited electron exchange.

Figure 2a represents the lowest-order Feynman diagrams, Figure 2b the QED direct contact term with scale parameters $\Lambda_+$ and $\Lambda_-$ sensitive to an extended annihilation radius of the $e^+e^- \rightarrow \gamma\gamma$ reaction and Figure 2c: the Feynman diagrams sensitive to the mass of an excited electron $m_{e^*}$. All the collaborations used the data from differential cross section to derive limits on compositeness scales $\Lambda_+$ and $\Lambda_-$ in the direct contact term interaction of the diagram shown in Figure 2b and search for the scale of the excited electron $\Lambda_{e^*} = f(m_{e^*})$ (33) in the t, u-channel of the diagram shown in Figure 2c.
The VENUS collaboration [38] set 1989 the most stringent limits on $\Lambda_+ > 81$ GeV, $\Lambda_- > 82$ GeV and published on Table 11 page 186, an overview of all other collaborations worked on the subject at 1989. In common for all analyses was, that the significance of the fit values to the data $1/\Lambda^4$ and $1/\Lambda^4_{e^\ast}$ was for all parameters below a significance $1\times\sigma$ and negative. For example, the L3 collaboration set 2002 [44] limits on $\Lambda_+ > 400$ GeV, $\Lambda_- > 300$ GeV and $m_{e^\ast} > 310$ GeV, including negative fit parameters with a significance below $1\times\sigma$. The latest publication 2013 from “The LEP Electroweak Working Group” [45] analysed the data from the differential cross section of all LEP detectors of the LEP II energy range $\sqrt{s} = 133$ GeV to 207 GeV. The group set limits on $\Lambda_+ > 431$ GeV, $\Lambda_- > 339$ GeV and $m_{e^\ast} > 366$ GeV, including negative fit parameters with a significance of close to $2\times\sigma$.

The analyses of VENUS, TOPAS, OPAL DELPHI, L3, ALEPH and “The LEP Electroweak Working Group” are limited to a certain energy range and luminosity. It is interesting to notice, that the values of the different limits between the L3 paper [44] and the “The LEP Electroweak Working Group” [45] are very similar, but the significance suddenly is not below $1\times\sigma$ any more. The higher statistics of the input data to the fit, do not change very much the limits, but increase the significance.

In this paper the statistic was increased to perform a global FIT, using all data from these six research projects to investigate $\Lambda_+$, $\Lambda_-$ and $m_{e^\ast}$ for energies from $\sqrt{s} = 55$ GeV to 207 GeV including the associated luminosities. After the first result of the global FIT [46] the significance of the fit parameter increase to approximately $5\times\sigma$. For security reasons, the whole procedure was repeated with different program structure, and again got a significance of $5\times\sigma$ from the $\chi^2$ test. This is the first time, an analysis of the differential cross of the $e^+e^- \rightarrow \gamma\gamma$ reaction shows a significant deviation from the QED.

The importance of the $5\times\sigma$ effect demands, to describe the procedure of the whole $\chi^2$ analysis in full technical detail. First, the theoretical framework to calculate the differential and total cross section of the QED $e^+e^- \rightarrow \gamma\gamma$ reaction, in particular which type of radiative corrections and how many are used in the calculation of the cross sections. Second, the measurement of the absolute and differential cross section, the global $\chi^2$ test of the differential cross section, for a heavy electron $m_{e^\ast}$ and a finite size of electron $\Lambda_6$. Third, visibility of the significance of the differential cross section $\chi^2$ test in the total cross section, the systematic errors of $\chi^2$ test and total cross section approximation. Finally, the discussion of the history and latest analysis results of $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction and the conclusion.
2 Theoretical framework

The physical interactions are dictated by symmetry principles of local gauge invariance, in connection with conserved physical quantities of a local region of space. The Lagrangian formalism highlights the connection between symmetries and conservation laws.

The Euler Lagrange equation, in the minimum of the action path integral, defines the Lagrangian density of the Dirac equation for a free particle with spin 1/2 in (1).

\[ \mathcal{L}_{\text{Dirac}} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \] (1)

With \( \Psi \) the fermion field, \( \bar{\Psi} \equiv \Psi + \gamma^0 \) its adjoint spinor, \( \gamma^\mu \) the gamma matrices, \( \partial_\mu = \partial/\partial x_\mu \) the covariant derivative and \( m \) the mass of the particle. The request of local gauge invariance yields the QED Lagrangian function in (2).

\[ \mathcal{L}_{\text{QED}} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi + e\bar{\Psi}\gamma^\mu A_\mu \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \] (2)

With \( A_\mu \) the gauge field, the mass \( m_A = m_e = 0 \), \( e \) charge of the electron, \( e\bar{\Psi}\gamma^\mu A_\mu \Psi \) the interaction term and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \).

2.1 The lowest order cross section of \( e^+e^- \rightarrow \gamma\gamma \)

The interaction term of the QED Lagrangian defines the Born cross section of the \( e^+e^- \rightarrow \gamma\gamma \) reaction. The mathematical formalism uses the \( M \)-matrix of (3)

\[ M_{fi} = -e \int \bar{\Psi}\gamma^\mu A_\mu d^4x \] (3)

Where, ( \( f \) ) stands for final and ( \( i \) ) for initial state. The differential cross section for the \( e^+e^- \rightarrow \gamma\gamma \) reaction is displayed in (4). The derivation of the cross section uses the square of the \( M \)-matrix, including the t - and u - channels of the Feynman graphs of Figure 2a and neglecting the electron mass \( m_e \approx 0 \) for high energies.

\[ \frac{d\sigma_0}{d\Omega} = \frac{S}{64\pi^2 s p_i} |M|^2 = \frac{\alpha^2}{s} \frac{1 + \cos^2(\theta)}{k - \cos^2(\theta)} \] (4)

In (4) is \( S = 1/2 \) statistical factor, \( \sqrt{s} \) the centre-of-mass energy, the momentum \( p_f = p_i \), \( k = E_{e^+} / |\vec{p}_{e^+}| \approx 1 \) for high energies \( E_{e^+} \) and \( \alpha = e^2/4\pi \). The angle \( \theta \) is the photon scattering angle with respect to the \( e^+e^- \) - beam axis. The Born total cross section is calculated in (5).
\[\sigma^0 = \frac{1}{2!} \frac{\alpha^2}{s} \int_0^{2\pi} d\phi \int_{-1}^{+1} \frac{1 + \cos^2\theta}{k - \cos^2\theta} d(\cos\theta)\]

\[= \frac{2\pi\alpha^2}{s} (\ln(s/m_e^2) - 1)\]  

(5)

With increasing statistics, of the \(e^+e^- \rightarrow \gamma\gamma(\gamma)\) reaction, it is absolutely necessary to include radiative corrections.

### 2.2 Radiative corrections of the QED \(e^+e^- \rightarrow \gamma\gamma(\gamma)\) cross section.

The radiative corrections of the QED \(e^+e^- \rightarrow \gamma\gamma(\gamma)\) cross section include in total of 22 corrections for virtual, soft and hard photons.

The first set of eight virtual photon corrections is shown in the Feynman graphs of Figure 3.

![Figure 3: Third order of eight virtual photon corrections Feynman graphs of the \(e^+e^- \rightarrow \gamma\gamma(\gamma)\) reaction.](image)

The second set, of four soft real photon initial state corrections, including the six hard photon corrections are shown in the Feynman graphs of Figure 4.
Figure 4: Third order Feynman graphs of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction, four soft initial photon corrections and six hard photon corrections.
Since the exact analytic expression is not known, the corrections of the Feynman diagrams from Figure 3 and Figure 4 are calculated by numerical simulations. This is performed using an event generator [49] for the reaction $e^+ e^- \rightarrow \gamma \gamma (\gamma )$, where the lowest order total cross section $\sigma_{\text{Born}}$ and differential cross section $\left( \frac{d\sigma}{d\Omega} \right)_{\text{Born}}$ is improved numerically by adding the higher order correction to $O(\alpha^3)$ in (6).

$$\left( \frac{d\sigma}{d\Omega} \right)_{\alpha^3} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Born}} (1 + \delta_{\text{virtual}} + \delta_{\text{soft}} + \delta_{\text{hard}}) \quad (6)$$

where, $\delta_{\text{virtual}}$ is the virtual correction and $\delta_{\text{soft}}$ and $\delta_{\text{hard}}$ are the soft- and hard-Bremsstrahlung corrections. To use the event generator the radiative corrections are calculated in two steps. First, when experimentally, the energies of the photons from initial state radiation (soft Bremsstrahlung) are too small to be detected, the reaction can be treated as 2-photon final state. Second, when the photon energies from initial state radiation are above $k_3/|p_+| = k_0 << 1$ (hard Bremsstrahlung with $p_+$ momentum of $e^+$), the process is treated as 3 - photon final state.

### 2.2.1 Virtual and soft radiative corrections of the $e^+ e^- \rightarrow \gamma \gamma (\gamma )$ cross section.

If the energies of the photons from initial state radiation (soft Bremsstrahlung) are too small for detection $k_3/|p_+| = k_0 << 1$, the reaction can be treated as 2-photon final state in (7).

$$e^+(p_+) + e^-(p_-) \rightarrow \gamma(k_1) + \gamma(k_2) \quad (7)$$

The equations for $\delta_{\text{virtual}} + \delta_{\text{soft}}$ are in (8) to (12).

$$\delta_{\text{soft}} + \delta_{\text{virtual}} = -\frac{2}{\pi} \left\{ 2(1 - 2v)(lnk_0 + v) + \frac{3}{2} \right\}$$

$$- \frac{1}{3} \frac{\pi^2}{2(1 + \cos^2 \theta)} \times \left[ -4v^2(3 - \cos^2 \theta) - 8v \cos^2 \theta \right.$$

$$+ 4uv(5 + 2\cos \theta + \cos^2 \theta)$$

$$+ 4wv(5 - 2\cos \theta) + \cos^2 \theta$$

$$- u(5 - 6\cos \theta + \cos^2 \theta)$$

$$- w(5 + 6\cos \theta + \cos^2 \theta)$$

$$- 2u^2(5 + 2\cos \theta + \cos^2 \theta)$$

$$- 2w^2(5 - 2\cos \theta + \cos^2 \theta) \right\} \quad (8)$$

12
\[ v = \frac{1}{2} \ln \left( \frac{s}{m_e^2} \right) \] (9)

\[ u = \frac{1}{2} \ln \left( \frac{2(e + \cos \theta)}{m^2} \right) \] (10)

\[ w = \frac{1}{2} \ln \left( \frac{2(e - \cos \theta)}{m^2} \right) \] (11)

\[ m = \frac{m_e}{|p_+|} \] (12)

The mass of the electron \( m_e \) is still included in this equation. The total cross section of the two \( \gamma \) final state is (13)

\[
\sigma^{2\gamma} = \sigma_0 + \frac{2\alpha^3}{s} [2(2v - 1)^2 \ln k_0 + \frac{4}{3} v^3 + 3v^2
\]

\[ + \left( \frac{2}{3}\pi^2 - 6 \right) v - \frac{1}{12} \pi^2] \] (13)

2.2.2 Hard radiative corrections of the \( e^+e^- \to \gamma\gamma(\gamma) \) cross section.

The soft-Bremsstrahlung photon energy is limited by a value \( k_3/|p_1| = k_0 \ll 1 \). If the energies of the photons from initial state radiations are above \( k_0 \), the process is treated as 3 - photon final state (14)

\[ e^+ (p_+) + e^- (p_-) \to \gamma(k_1) + \gamma(k_2) + \gamma(k_3) \] (14)

For the differential cross section of \( e^+e^- \to \gamma\gamma(\gamma) \) it is necessary to introduce two additional parameters in the phase space. The calculation in (15) is performed in the extreme relativistic limit 50.

\[
\frac{d\sigma}{d\Gamma_{ijk}} = \frac{d\sigma}{d\Omega_id\Omega_kdx_k} = \frac{\alpha^3}{8\pi^2s} w_{ijk} F(i,j,k) \] (15)

\[
w_{ijk} = \frac{x_ix_k}{y(z_j)}, x_i = \frac{k_{i0}}{|p_+|}, y(z_j) = 2e - x_k + x_kz_j \] (16)

\[ z_j = \cos(\alpha_{ik}) \] (17)

\[ x_l = \frac{E_{l}}{|p_+|} \] (18)

13
\[ F(i, j, k) = \sum_p \left[ -2m^2 \frac{k_j'}{k_k^2 k_i} - 2m^2 \frac{k_j}{k_k^2 k_i} + \frac{2}{k_k k_k'} \left( \frac{k_j^2 + k_{j'}^2}{k_k k_k'} \right) \right] \]
\[ = \sum_p M(i, j, k) \]

(20)

\( \alpha_{ik} \) is the angle between \( k_i \) and \( k_j \). \( P \) binds all permutations of \( \{1, 2, 3\} \). The quantities \( k_i \) and \( k_i' \) are

\[ k_i = x_i(e - \cos(\theta_i)) \]
\[ k_i' = x_i(e + \cos(\theta_i)) \]

(21)

(22)

where \( \theta_i \) is the angle between the momentum of the \( i \)-th photon and \( |\vec{p}_+| \). The total \( 3 \times \gamma \) cross section is (23).

\[ \sigma^{3\gamma} = \frac{1}{3!} \int d\Gamma_{ijk}, i, j, k \in \{1, 2, 3\} \]

(23)

The integral runs over all phase spaces: \( k_0 < x_i < 1 \).

For practical calculation (23) can be approximated by an analytical approach. The photons get sorted after energy, \( E_{\gamma_1} \geq E_{\gamma_2} \geq E_{\gamma_3} \) where \( \gamma_1 \) and \( \gamma_2 \) are treated as annihilation photons, and \( \gamma_3 \) as hard Bremsstrahlung photon. The total cross section after integrations is (24) [49].

\[ \sigma^{3\gamma} = \frac{2\alpha^3}{s} \left[ 3 - \left( \ln \frac{4}{m^2} - 1 \right)^2(2lnk_0 + 1) \right] \]

(24)

2.3 The \( e^+e^- \rightarrow \gamma\gamma(\gamma) \) total cross section.

The differential cross section for the hard-Bremsstrahlung process is not analytically known, but the total integrated cross section, adding the \( e^+e^- \rightarrow \gamma\gamma \) plus \( e^+e^- \rightarrow \gamma\gamma(\gamma) \) reactions can be calculated (25) and (26).

\[ \sigma_{tot} = \sigma(2\gamma) + \sigma(3\gamma) \]
\[ = \sigma^0 + \frac{2\alpha^3}{s} \left[ \frac{4}{3}v^3 - v^2 + \left( \frac{2}{3} \pi^2 - 2 \right)v + 2 - \frac{1}{12} \pi^2 \right] \]

(25)

(26)
2.4 The numerical calculation of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ differential cross section.

The third order differential cross section is obtained by a numerical Monte Carlo generator [49]. The program generates three $\gamma$'s events sorted after the energies $E_{\gamma 1} \geq E_{\gamma 2} \geq E_{\gamma 3}$, with the correct mixture for soft $k_3/|p_1| = k_0 \ll 1$ and hard QED corrections shown in Figure 3 and Figure 4. The angle $\alpha$ between the $E_{\gamma 1}$ and $E_{\gamma 2}$ event $\alpha_{\min} < \alpha < \alpha_{\max}$ is connected to the scattering angle $\theta$ by $|\cos \theta|$.

The differential cross section $(d\sigma/d\Omega)_i$ at an angle $\theta$, an energy $E_{tot}$ and for an angle bin width $\Delta(|\cos \theta|)$ is (27).

\[
(d\sigma/d\Omega)_i = \frac{1}{2\pi \Delta(|\cos \theta|)} \sigma_{tot} N_i \tag{27}
\]

The scattering angle is $|\cos \theta| = (|\cos \theta_1| + |\cos \theta_2|)/2$, where $\theta_1$ is the scattering angle of $E_{\gamma 1}$ and $\theta_2$ is the scattering angle of $E_{\gamma 2}$, $N_i$ is the number of events in an angle bin width $\Delta(|\cos \theta|)$ and $N$ the total amount of events used in the generator.

The Monte Carlo generator together with (27) is used to calculate distributions of differential cross sections as function of $|\cos \theta|$ including 5 parameters. As example for 189 GeV the following parameters are used: $E_{tot} = 189$ GeV, $14 < \alpha < 166$ equal $|\cos \theta| < 0.97$, 50 $N_i$ bins $\Delta(|\cos \theta|)$, the limit soft/hard $k_0 = 0.01$ and $N$ one million events. To produce an analytical expression for further calculations, the angular distribution of the differential cross section (27) is fitted by a $\chi^2$ fit using a polynomial with 6 parameters $p_1$ to $p_6$ shown in (28).

\[
(d\sigma/d\Omega)_{QED} = (d\sigma/d\Omega)_{Born} \times \\
\left(1 + p_1 + p_2 e^{x^{1.2}} + p_4 x + p_5 x^2 + p_6 x^3\right)|_{x=|\cos \theta|} \tag{28}
\]

For the example of $E_{tot} = 189$ GeV, the result of the 6 parameter fit is shown in (29).

\[
p_1 = 0.2869 \tag{29}
\]
\[
p_2 = -0.51851
\]
\[
p_3 = 0.19946
\]
\[
p_4 = -0.39652
\]
\[
p_5 = -0.41213
\]
\[
p_6 = 0.70428
\]

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2.5 Deviations from QED

If the QED is a fundamental theory, it should be possible to describe the experimental parameters of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction up to the Grand Unification scale. So far it was possible to test this fundamental character of the QED up to $E < 100$ GeV. New non-QED phenomenon could become visible at higher energy scales. An energy scale characterised by a cutoff parameter $\Lambda$ can be used as a threshold point for QED breakdown, and for underlying new physics. Different models are discussed in [51]. Deviations from QED with the reaction $e^+e^- \rightarrow \gamma\gamma(\gamma)$ [52] are investigated in a program, initiated between 1991 and 2002 by the Swiss Federal Institute of Technology in Zurich (ETHZ) and USTC Hefei (University of Science and Technology of China). In this paper, the focus is on a model of an excited state of electron and finite size of electron.

2.5.1 Heavy electron mass

This model is based on the assumption that the electron has an excited state, and the reaction $e^+e^- \rightarrow \gamma\gamma$ proceeds via exchange of a virtual excited electron $e^*$ in the t- and u-channel, shown in the Feynman graph of Figure 2c. The new interaction is described by a new coupling between the excited electron and the electron, and between the excited electron and the photon field. A magnetic interaction term was introduced by A. Litke [55] in (30).

\[ L_{e^*} = \frac{e\lambda}{2m_{e^*}} \bar{\Psi}_e \sigma_{\mu\nu} \Psi_e F^{\mu\nu} \] (30)

With $\lambda$ the relative magnetic coupling strength to the QED magnetic coupling, $m_{e^*}$ the mass of the excited electron and $\sigma^{\mu\nu} = \frac{1}{2} [\gamma^\mu, \gamma^\nu]$. A. Litke changed the magnetic interaction term the QED differential cross section too (31).

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{QED} \left[ 1 + \frac{s^2}{2m_{e^*}^2} \left( 1 - \cos^2\theta \right) F(\cos\theta) \right] \] (31)

With $\sqrt{s}$ centre-of-mass energy and $F(\cos\theta)$ in (32).

\[ F(\cos\theta) = \left( 1 + \frac{s}{2m_{e^*}^2} \frac{1 - \cos^2\theta}{1 + \cos^2\theta} \right) \times \left[ \left( 1 + \frac{s}{2m_{e^*}^2} \right)^2 - \left( \frac{s}{2m_{e^*}^2} \right)^2 \cos^2\theta \right]^{-1} \] (32)

The condition $s/m_{e^*}^2 \ll 1$ would simplify the function $F(\cos\theta) \simeq 1$. Under these circumstances (31) and (32) reduces to (33).
\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{QED} (1 \pm \delta_{\text{new}}) \\
= \left( \frac{d\sigma}{d\Omega} \right)_{QED} \left[ 1 \pm \frac{s^2}{2\Lambda^2} \left( 1 - \cos^2 \theta \right) \right]
\]  

(33)

The scale \( \Lambda_+ = \Lambda_{e^*} \) (Figure 2c) is related to the mass of the excited electron \( m_{e^*} \) by \( \Lambda_+^2 = m_{e^*}^2/\lambda \). The negative contribution \( \Lambda_- \) is added for symmetry reasons [55].

2.5.2 Minimal interaction length includes finite size of electron.

The QED differential and total cross section of the \( e^+ e^- \rightarrow \gamma\gamma(\gamma) \) reaction is calculated under the condition that the electron is point-like, without limited interaction length and coupled to the vacuum as shown in Figure 3 and Figure 4. So far no experimental evidence exist, the unlimited interaction length of the electron exist from zero energies up to the Planck scale \( M_P \). If at an energy scale \( \Lambda \) between \( 0 < \Lambda < M_P \) in the \( e^+ e^- \rightarrow \gamma\gamma(\gamma) \) reaction, a finite interaction length appears, this \( \Lambda \) defines a size of an object the annihilation occurs. It is possible to calculate the size of the object via the generalised uncertainty principle [56–58] or via the electromagnetic energy \( E \) and wave length \( \lambda \) [59] of the light the object submits.

The wavelength \( \lambda \) of the gammas must be smaller or equal to the size of the interaction area. If the energy \( \Lambda \) of the size of the interaction area is known, the frequency \( \nu \) of the gammas is known via \( \Lambda = E = \hbar \times \nu \). This frequency \( \nu \) is connected to the size of the object via the wave length \( \lambda \) to the equation \( \nu \times \lambda = c \). The energy scale \( \Lambda \) define under these circumstances the size of the interaction area and in consequence the size of the electron involved in the annihilation area.

It is possible to construct several effective Lagrangians containing nonstandard \( \gamma e^+ e^- \) or \( \gamma\gamma e^+ e^- \) couplings which are \( U(1)_{em} \) gauge invariant and only differ in their dimensions [53]. In the lowest order effective Lagrangian, this reaction contains operators of dimension 6, 7 and 8 [54]. In the further discussion, we concentrate on the simplest operator of dimension 6, with the effective Lagrangian of (34).

\[
\mathcal{L}_6 = i\bar{\Psi} \gamma_\mu (\bar{D}_\nu \Psi) (g_6 F^{\mu\nu} + \tilde{g}_6 \tilde{F}^{\mu\nu})
\]  

(34)

The coupling constant \( g_n = \sqrt{4\pi}/\Lambda^{(n-4)} \) \((n = 6)\), is related to the mass scale \( \Lambda \), \( D_\mu = \partial_\mu - ieA_\mu \) is the QED covariant derivative, and \( \tilde{F}^{\mu\nu} \) is the dual of the electromagnetic tensor \( \tilde{F}^{\alpha\beta} = \frac{1}{2} \varepsilon^{\alpha\beta\mu\nu} F_{\mu\nu} \). The differential cross section to \( \mathcal{L}_6 \) is (35).
\[
\left( \frac{d\sigma}{d\Omega} \right)_T = \left( \frac{d\sigma}{d\Omega} \right)_{QED} \left[ 1 + \delta_{\text{new}} \right] \\
= \left( \frac{d\sigma}{d\Omega} \right)_{QED} \left[ 1 + \frac{s^2}{2\alpha} \left( \frac{1}{\Lambda^4} + \frac{1}{\tilde{\Lambda}^4} \right) (1 - \cos^2\theta) \right]
\] (35)

We use \( \Lambda = \tilde{\Lambda} = \Lambda_6 \), higher order terms like \( \Lambda_7 \) or \( \Lambda_8 \) of \( \delta_{\text{new}} \) are omitted.

To search for a deviation of QED it is common to use a \( \chi^2 \) tests. This test compares the QED cross section to the experimental measured cross section. In the test is the QED cross section modified by a non QED direct contact term threshold energy scale \( \Lambda \) after equation (35). If the \( \chi^2 \) test indicates a minimum for a finite threshold energy scale \( \Lambda \) this energy of the cutoff parameter \( \Lambda \) defines via the two equations \( \Lambda = E = \hbar \times \nu \) and \( \nu \times \lambda = c \) a finite size of the area the \( e^+ e^- \) annihilation must occur. For \( \lambda = r_e \) is this a measure for size of the electron shown in (36) including \( \hbar \) the Planck constant and \( c \) the speed if light.

\[
r_e = \frac{\hbar c}{\Lambda}
\] (36)

Equation (36) is generic, the calculation using the generalised uncertainty principle generates the same equation. It is interesting to notice that in equation (36) for \( \Lambda \to \infty \) the size of the object will be \( r_e \to 0 \). In consequence the point-like QED would be correct to infinite energies.

3 The measurement of the absolute and differential cross section

The \( e^+ e^- \to \gamma\gamma(\gamma) \) reaction initiates in a storage \( e^+ e^- \) ring accelerators a background free signal in the detector. For example, Figure 5 shows an event from LEP in the L3 detector. Shown is the position and energy storage of a \( e^+ e^- \to \gamma\gamma(\gamma) \) event perpendicular to the \( e^+ e^- \) beam axis in the electromagnetic BGO calorimeter. The charge sensitive detectors of the inner trackers, the outer hadron calorimeter, and the muon chambers are free from any background signal.

The total cross section is a function of the number of events \( N \) in an angular range of the scattering angle \( 0 < |\cos\theta| < |\cos\theta|_{\text{max}} \), the luminosity \( L \) and the efficiency \( \varepsilon \) of the detector for \( e^+ e^- \to \gamma\gamma \) events (37).

\[
\sigma_{\text{tot}} = \frac{N}{\varepsilon L}
\] (37)
Figure 5: $e^+e^- \rightarrow \gamma\gamma(\gamma)$ event in the L3 detector at CERN [61]
The differential cross section at an angular $\theta_i$ is (38).

$$\left(\frac{d\sigma}{d\Omega}\right)_i = \frac{1}{2\pi \Delta(|\cos\theta|)_i} \cdot \frac{N_i}{L \cdot \epsilon_i} \quad (38)$$

$N_i$ is the number of events in the bin (i), $\epsilon_i$ is the selection efficiency in the bin region. The angular is calculated after (39).

$$|\cos\theta|_i = \frac{\int_{|\cos\theta|\in i} |\cos\theta| \cdot \left(\frac{d\sigma}{d\Omega}(|\cos\theta|)\right)^{\text{Born}}_{e^+e^-\rightarrow\gamma\gamma} \cdot d|\cos\theta|}{\int_{|\cos\theta|\in i} \left(\frac{d\sigma}{d\Omega}(|\cos\theta|)\right)^{\text{Born}}_{e^+e^-\rightarrow\gamma\gamma} \cdot d|\cos\theta|} \quad (39)$$

The angle in the event axis $|\cos\theta| = 0.5 \ast (|\cos\theta_i| + |\cos\theta_{i+1}|)$ is weighted with the ratio of the Born differential cross section $\left(\frac{d\sigma}{d\Omega}(|\cos\theta|)\right)^{\text{Born}}_{e^+e^-\rightarrow\gamma\gamma}$ to the QED reaction $e^+e^-\rightarrow\gamma\gamma$. The weighting increases $|\cos\theta|_1$ at ($|\cos\theta| \sim 1$) approximately 0.5%. The bin width is $\Delta(|\cos\theta|_i) = |\cos\theta_{i+1}| - |\cos\theta_i|$ of bin (i) and (i + 1).

### 3.1 The differential cross section from VENUS, TOPAS, ALEPH, L3, and OPAL.

If the signal of the deviation of the QED $e^+e^-\rightarrow\gamma\gamma$ reaction is hidden in the angular distribution of the differential cross section, it is interesting to study the shape of the differential cross sections. The expected deviation between experiment and QED differential cross section is one the percent level. It would be for this reason very helpfully to develop a plot to see with bare eye a deviation between experiment and QED cross section. For this reason a plot is introduced to scale to an energy $E_{\text{scale}}$, weighted to the number $N_i$ at this energy $E_i$ in (40) and respect the possible different binning of the differential cross section $\left(\frac{d\sigma}{d\Omega}\right)^i_{|\cos\theta|_j}$ to the scaled differential cross section the bin (j) counts from $i = 1$ to $i = r$ and the angle $|\cos\theta|_j$ is calculated in (39).

$$\left(\frac{d\sigma}{d\Omega}\right)_{|\cos\theta|_j}^{\text{scale}} = \frac{\sum_r N_i \left(\frac{d\sigma}{d\Omega}\right)^i_{|\cos\theta|_j} \cdot \left(\frac{E_i}{E_{\text{scale}}}\right)^2}{\sum_i N_i} \quad (40)$$

In a first step of the analysis, a series of plots is introduced to show the differential cross section from the different collaborations scaled to an energy of 91.2 GeV and compared it to the QED differential cross section at this energy.

The VENUS collaboration published in [38], in Table 2 the luminosity, in Table 3 the $e^+e^-\rightarrow\gamma\gamma$ candidates, in Table 4 the angular distribution of the $e^+e^-\rightarrow\gamma\gamma$ candidates and Table 8 the differential cross section at $\sqrt{s} = 55.0$ GeV, 56.0 GeV, 56.5 GeV and 57.6 GeV.
together with the used bins $\cos\theta$. The TOPAS collaboration published in [40] Table 1 the luminosity at $\sqrt{s} = 57.0$ GeV and in Table 2 the differential cross section together with the bin width. Figure 6 displays the data from VENUS and TOPAS scaled with (40) to $\sqrt{s} = 91.2$ GeV. The black line is the $QED - \alpha^3$ differential cross section at $\sqrt{s} = 91.2$ GeV.

![Figure 6: Differential cross section of the $e^+e^- \rightarrow \gamma\gamma$ reaction from VENUS and TOPAS. The black line is the QED-$\alpha^3$ cross section](image)

The ALEPH collaboration published in [41], in Table 8.2 the differential cross section at $\sqrt{s} = 91.2$ GeV together with bin width and on page 321 the luminosity. Figure 7 displays the data from ALEPH scaled with (40) to $\sqrt{s} = 91.2$ GeV. The black line is the $QED - \alpha^3$ differential cross section at $\sqrt{s} = 91.2$ GeV.

The DELPHI collaboration published in [42] data from 1994, 1998 and 2000. In 1994 in Table 1 the luminosity at $\sqrt{s} = 91.25$ GeV and in Table 2 the differential $e^+e^- \rightarrow \gamma\gamma$ cross section, together with the bin width and number of events per bin. In 1998 in Table 2 the luminosities of $e^+e^- \rightarrow \gamma\gamma$ reaction, the number of events at $\sqrt{s} = 91.25$ GeV, 130.4 GeV, 136.3 GeV, 161.5 GeV 172.4 GeV and 182.7 GeV. In Table 3 the differential cross section at $\sqrt{s} = 133$ GeV, 161 GeV, 172 GeV and 183 GeV together with the bin width and number of events per bin. In 2000 on page 71, the luminosity of $\sqrt{s} = 188.63$ GeV, 191.6 GeV, 195.5 GeV, 199.5 GeV and 201.6 GeV. In Table 3 is shown, for the same energy, the differential cross section,
Figure 7: Differential cross section of the $e^+e^- \rightarrow \gamma\gamma$ reaction from ALEPH. The black line is the QED-$\alpha^3$ cross section.
the number of events and the bin width. Figure 8 displays the data from DELPHI scaled with $\sqrt{s} = 91.2$ GeV. The black line is the $QED - \alpha^3$ differential cross section at $\sqrt{s} = 91.2$ GeV.

Figure 8: Differential cross section of the $e^+e^- \rightarrow \gamma\gamma$ reaction from DELPHI. The black line is the QED-$\alpha^3$ cross section.

The L3 collaboration published data in 1995 [43], 2000 [48] and 2002 [44]. The 1995 data are focused on the energy $\sqrt{s} = 91.2$ GeV, display the luminosity on page 141 and the differential cross section on Table 1 together with bin size and number of events in the bin. The 2000 data include the luminosity for $\sqrt{s} = 183$ GeV and $\sqrt{s} = 189$ GeV on page 201. The 2002 data include the luminosity for $\sqrt{s} = 192$ GeV, 196 GeV, 200 GeV, 202 GeV, 205 GeV and 207 GeV in Table 1. In Table 4, for all energies of the 2000 and 2002 data, the data events per efficiency and the bin size are shown. Figure 9 shows all the L3 differential cross section data from $\sqrt{s} = 91$ GeV to 207 GeV, scaled with $\sqrt{s} = 91.2$ GeV. The black line is the $QED - \alpha^3$ differential cross section at $\sqrt{s} = 91.2$ GeV.

The OPAL collaboration published data 1991 [39] for $\sqrt{s} = 91.0$ GeV. At page 533 and Table 1 the luminosity and in Table 3 the differential cross section the number of events including the bin size. In the year 2003 [47] data for $\sqrt{s} = 183$ GeV, 189 GeV, 192 GeV, 196 GeV, 200 GeV, 202 GeV, 205 GeV and 207 GeV. In Table 6 number of events per bin, bin size, and efficiency.
Figure 9: Differential cross section of the $e^+e^- \rightarrow \gamma\gamma$ reaction from L3. The black line is the QED- $\alpha^3$ cross section.
Figure 10 shows all the OPAL differential cross section data from $\sqrt{s} = 91$ GeV to 207 GeV, scaled with (40) to 91.2 GeV. The black line is the QED $\alpha^3$ differential cross section at $\sqrt{s} = 91.2$ GeV.

![Figure 10: Differential cross section of the $e^+e^- \rightarrow \gamma\gamma$ reaction from OPAL. The black line is the QED $\alpha^3$ cross section.](image)

The differential cross section measured from VENUS, TOPAS, ALEPH, L3 and OPAL at energies from $\sqrt{s} = 55$ GeV to 207 GeV is summarised in Figure 11. All data are scaled with (40) to 91.2 GeV. The black line is the QED $\alpha^3$ differential cross section at $\sqrt{s} = 91.2$ GeV.

It is absolutely visible from Figure 6 to Figure 11 that all measured differential cross sections agree in the error bars, approximately with the QED cross section. It is not possible to see with pure eye an extensive deviation from the experimental to the QED cross section. This supports the assumption that the deviation between experiment and QED is on the percent level.

4 Global $\chi^2$ test of the differential cross section, for a heavy electron $m_{e^*}$ and a finite size of electron $\Lambda_6$

A more sensitive method to search for a deviation from QED in the differential cross sections measurements from VENUS, TOPAS, OPAL, DELPHI, ALEPH and L3 collaborations with
Figure 11: Differential cross section of the $e^+e^- \rightarrow \gamma\gamma$ reaction from VENUS, TOPAS, ALEPH, L3 and OPAL. The black line is the QED $\alpha^3$ cross section.

Table 1: The luminosity used from the VENUS, TOPAS, ALEPH, DELPHI, L3 and OPAL experiment.

| GeV | VENUS   | TOPAS   | ALEPH   | DELPHI  | L3      | OPAL   |
|-----|---------|---------|---------|---------|---------|--------|
| 55  | 2.34 pb$^{-1}$ [38] |         |         |         |         |        |
| 56  | 5.18 pb$^{-1}$ [38] |         |         |         |         |        |
| 56.5| 0.86 pb$^{-1}$ [38] |         |         |         |         |        |
| 57  | 3.70 pb$^{-1}$ [38] |         |         |         |         |        |
| 57.6| 52.26 pb$^{-1}$ [40] |         |         |         |         |        |
| 91  |         | 8.5 pb$^{-1}$ [41] | 36.9 pb$^{-1}$ [42] | 64.6 pb$^{-1}$ [43] | 7.2 pb$^{-1}$ [39] |         |
| 133 |         | 5.92 pb$^{-1}$ [42] |         |         |         |        |
| 162 |         | 9.58 pb$^{-1}$ [42] |         |         |         |        |
| 172 |         | 9.80 pb$^{-1}$ [42] |         |         |         |        |
| 183 |         | 52.9 pb$^{-1}$ [42] | 54.8 pb$^{-1}$ [44] | 55.6 pb$^{-1}$ [47] |         |        |
| 189 |         | 151.9 pb$^{-1}$ [42] | 175.3 pb$^{-1}$ [44] | 181.1 pb$^{-1}$ [47] |         |        |
| 192 |         | 25.1 pb$^{-1}$ [42] | 28.8 pb$^{-1}$ [44] | 29.0 pb$^{-1}$ [47] |         |        |
| 196 |         | 76.1 pb$^{-1}$ [42] | 82.4 pb$^{-1}$ [44] | 75.9 pb$^{-1}$ [47] |         |        |
| 200 |         | 82.6 pb$^{-1}$ [42] | 67.5 pb$^{-1}$ [44] | 78.2 pb$^{-1}$ [47] |         |        |
| 202 |         | 40.1 pb$^{-1}$ [42] | 35.9 pb$^{-1}$ [44] | 36.8 pb$^{-1}$ [47] |         |        |
| 205 |         |         |         |         | 74.3 pb$^{-1}$ [44] | 79.2 pb$^{-1}$ [47] |
| 207 |         |         |         |         | 138.1 pb$^{-1}$ [44] | 136.5 pb$^{-1}$ [47] |
the according luminosities, is to use (41) to perform a $\chi^2$ test with the CERNLIB program MINUIT [63].

$$\chi^2 = \sum_{i,j} \left\{ \frac{\frac{d\sigma}{d\Omega}^{\text{meas}}(|\cos\theta|_i, E_j) - \frac{d\sigma}{d\Omega}^{\text{new}}(|\cos\theta|_i, E_j, \Lambda)}{\Delta \frac{d\sigma}{d\Omega}^{\text{meas}}(|\cos\theta|_i, E_j)} \right\}^2$$

The term $\frac{d\sigma}{d\Omega}^{\text{meas}}(|\cos\theta|_i, E_j)$ is the differential cross section measured at an angular bin $(i)$ and an energy $(j)$. $\frac{d\sigma}{d\Omega}^{\text{new}}(|\cos\theta|_i, E_j, \Lambda)$ is the $QED - \alpha^3$ differential cross section at an angular bin $(i)$, an energy $(j)$ and a test parameter $\Lambda$. The term $\Delta \frac{d\sigma}{d\Omega}^{\text{meas}}(|\cos\theta|_i, E_j)$ is the standard error of the mean value of the measurements. The standard error is the quadratic sum of the statistical error and systematic error, it will be discussed in chapter 6. The $\chi^2$ test needs nearby the details of the differential cross section, the luminosity at the different energies the data are taken. In Table 1 is given a summary of all luminosities from VENUS, TOPAS, ALEPH, DELPHI, L3 and OPAL used for the $\chi^2$ test.

4.1 Global $\chi^2$ test for heavy electron $m_{e^*}$.

For the $\chi^2$ fit in (41) it is necessary to use the differential QED cross section (28) for every energy $E_{\text{tot}}$ under investigation, the differential cross section for the heavy electron (31) and (32). Inspecting (41) it is possible to perform the $\chi^2$ fit for every collaboration and measured $\sqrt{s}$ energy separately. Some collaboration publishes directly the experimental differential cross section needed for the fit. If this is not the case, the luminosity, the number of events per bin, the width and the efficiency is published. This data allows with (38) and (39) to calculate the differential cross section. The theoretical $QED - \alpha^3$ differential cross section is calculated by using the numerical calculation of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction discussed in chapter 2.4 for every energy under test. In the cross section (33) for the test, the parameter $\left(1/\Lambda^4_+ + \frac{1}{\text{GeV}^4}\right)$ is used. The single $\chi^2$ fit result is displayed in Table 2 sorted by $\sqrt{s}$ energy, collaboration, value of the the fit parameter $\left(1/\Lambda^4_+[1/\text{GeV}^4]\right)$ and the the quality parameter of the fit $\chi^2/dof$.

In total, 34 fit values $\left(1/\Lambda^4_+[1/\text{GeV}^4]\right)$ are calculated. Out of these 34 values are six positive, more as 4/5 of the samples have negative sign. The fits have a negative tendency in $1/\Lambda^4_+[1/\text{GeV}^4]$. The $\chi^2$ fit allows to calculate the fit value $\left(1/\Lambda^4_+[1/\text{GeV}^4]\right)$ for every collaboration, sorted after the energies $\sqrt{s}$ the collaboration measured as displayed in Figure 12.

The detailed overview of the $\chi^2$ test is shown in Figure 12 from ALEPH, Delphi, L3, OPAL, TRISTAN (TOPAS and VENUS), LEP 1, LEP 2 and all groups separately support the negative trend of $\left(1/\Lambda^4_+[1/\text{GeV}^4]\right)$ with increasing amount of data. This trend is also visible in Table 3 where the parameters $\left(1/\Lambda^4_+[1/\text{GeV}^4]\right)$ and $\left(\chi^2/dof\right)$ are sorted after TRISTAN, LEP 1, LEP 2 and ALL Data.
Table 2: The fit parameter (1/λ\textsuperscript{4}[1/GeV\textsuperscript{4}]) for all collaborations and the \(\sqrt{s}\) energies, including the quality parameter of the fit \(\chi^2/\text{dof}\).

| GeV | VENUS (\(\chi^2/\text{dof}\)) | TOPAS | ALEPH | DELPHI | L3 | OPAL |
|-----|-----------------------------|-------|-------|--------|----|------|
| 56  | \(-4.26 \pm 2.52\) \times 10^{-8}\n\chi^2/\text{dof} = 12.90/8 |       |       |        |    |      |
| 56  | \((3.24 \pm 1.88) \times 10^{-8}\n\chi^2/\text{dof} = 9.48/8 |       |       |        |    |      |
| 66  | \(-2.11 \pm 3.96\) \times 10^{-7}\n\chi^2/\text{dof} = 4.93/8 |       |       |        |    |      |
| 77  | \((1.19 \pm 2.02) \times 10^{-8}\n\chi^2/\text{dof} = 8.82/8 |       |       |        |    |      |
| 81  | \((0.20 \pm 1.04) \times 10^{-7}\n\chi^2/\text{dof} = 5.27/4 |       |       |        |    |      |
| 89  | \((0.11 \pm 1.01) \times 10^{-7}\n\chi^2/\text{dof} = 2.67/4 |       |       |        |    |      |
| 102 | \((3.95 \pm 2.04) \times 10^{-7}\n\chi^2/\text{dof} = 1.03/4 |       |       |        |    |      |
| 106 | \((0.37 \pm 1.10) \times 10^{-7}\n\chi^2/\text{dof} = 7.84/0 |       |       |        |    |      |
| 120 | \((0.88 \pm 1.42) \times 10^{-7}\n\chi^2/\text{dof} = 8.07/4 |       |       |        |    |      |
| 122 | \((1.11 \pm 1.51) \times 10^{-7}\n\chi^2/\text{dof} = 2.94/4 |       |       |        |    |      |
| 207 | \((0.94 \pm 5.99) \times 10^{-7}\n\chi^2/\text{dof} = 23.6/9 |       |       |        |    |      |

Table 3: The parameter 1/λ\textsuperscript{4}[GeV\textsuperscript{4}] for the combined \(\chi^2\) test

| TRISTAN | \(2.49 \pm 5.05\) \times 10^{-9}\n\chi^2/\text{dof} = 50.0/41 |
|---------|---------------------------------------------------------------|
| LEP 1   | \(-9.20 \pm 6.90\) \times 10^{-10}\n\chi^2/\text{dof} = 32.3/41 |
| LEP 2   | \(-1.10 \pm 0.20\) \times 10^{-10}\n\chi^2/\text{dof} = 267/203 |
| All Data| \(-1.11 \pm 0.20\) \times 10^{-10}\n\chi^2/\text{dof} = 351/287 |
Figure 12: The $\chi^2$ fit parameter ($1/\Lambda_+^4[1/GeV^4]$) including the fit error, for all collaborations and measured energies.
Table 4: Summary of heavy electron $\chi^2$ tests $(1/\Lambda^4)_{top}[GeV^{-4}]$

| Heavy electron mass $m_{e^*}$ | $-(1.11 \pm 0.20) \times 10^{-10}$ | $\Lambda_+^2 = m_{e^*}/\lambda$ | $m_{e^*} = 308 \pm 14 \text{ GeV}$ |

The numerical values of $1/\Lambda^4[GeV^{-4}]$ in Table 3 from TRISTAN at energies from $\sqrt{s} = 55 \text{ GeV}$ to 57.6 GeV are positive with a big error bar. The LEP 1 data at $\sqrt{s} = 91 \text{ GeV}$ measured at the $Z^0$-pole, are already negative with approximately a statistical significance of ONE standard deviation $\sigma$. These data are taken with much less luminosity as the LEP 2 data, from energy 133 GeV to 207 GeV. This is visible in the size of the error bar in Table 3. The LEP 2 data dominate the global fit result with a negative fitted parameter about FIVE $\sigma$ at energies from $\sqrt{s} = 133 \text{ GeV}$ to 207 GeV.

The essential point of the global $\chi^2$ fit for the mass of a heavy electron, can be extracted from the minimum of $\chi^2$ of all data from 55 GeV to 207 GeV as function of $1/\Lambda^4 \times 10^{-9}$ in Figure 13.

The best fit result from Table 3, Figure 12 and Figure 13 after the MINUIT output is shown in Table 4.

After the output of MINIUT in Table 4 is the minimum of the global fit parameter of the $\chi^2$ test $(1/\Lambda^4)_{top} = - (1.11 \pm 0.20) \times 10^{-10}[GeV^{-4}]$. The statistical significance of one standard deviation is $\sigma = 0.20 \times 10^{-10}[GeV^{-4}]$. The total statistical significance of the minimum of the $\chi^2$ test is approximately $5 \times \sigma$. Including $\Lambda_+^2 = m_{e^*}/\lambda$ for the mass of the heavy electron $m_{e^*}$ and setting relative magnetic coupling strength to the QED magnetic coupling $\lambda = 1.0$, the mass of the heavy electron $m_{e^*}$ and the estimated error from $\chi^2$ to this mass is displayed in...
Table 5: $\chi^2$ tests $(1/\Lambda)^4_{\text{top}}[\text{GeV}^{-4}]$ finite size and heavy electron.

| Summary $r_{\text{electron}}$ and $m_{e^*}$ | Test $r_{\text{electron}}$ $r = (h \times c)/\Lambda$ $r = (1.57 \pm 0.07) \times 10^{-17}$ cm |
|---------------------------------------------|-----------------------------------------------|
| $-(4.05 \pm 0.73) \times 10^{-13}$ | $r = (1.11 \pm 0.20) \times 10^{-10}$ $\Lambda^2_+ = m^2_{e^*}/\lambda$ $m_{e^*} = 308 \pm 14$ GeV |

Table 4.

4.2 Global $\chi^2$ test for finite size of an electron.

For the electron finite size test of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction, the same data set from VENUS, TOPAS, OPAL, DELPHI, ALEPH and L3 collaboration, of the differential cross section and luminosity as for the test of the heavy electron (33), are used. The equations to test a deviation from QED with the assumption of a heavy electron (33) and to test via a direct contact term, a finite size of an electron (35) are up to the constant $\alpha$ the same. The angular term $(1 - \cos^2\theta)$ in both equations is the same, also the centre-of-mass energy $s$ term. Inserted is (35) including $\Lambda = \tilde{\Lambda} = \Lambda_6$ without higher order terms, in the part $\frac{d\sigma}{d\Omega}$ of the $\chi^2$ test (11).

The minimum from the program MINUIT for the $\chi^2$ test on the $(1/\Lambda)^4$ axis is $(1/\Lambda)^4_{\text{top}} = -(4.05 \pm 0.73) \times 10^{-13}[\text{GeV}^{-4}]$, with $\Lambda_{\text{top}} = 1253.53 \pm 226$ GeV. The statistical significance of one standard deviation is $\sigma = (0.73 \times 10^{-13})[\text{GeV}^{-4}]$. The total statistical significance of the minimum of the $\chi^2$ test is approximately $5 \times \sigma$.

It is possible to calculate the significance $\sigma$ with the p-value $\sigma = f(p(\chi^2, \text{deg.free}))$. In the $\chi^2$ test of the finite size of the electron is the $\chi^2 = 400$ on the $\chi^2$ axis and the degree of freedom $\text{deg.free}=254$ after Table 2. This calculation results in the same approximately $5 \times \sigma$. The mathematical details to calculate the p-value and the error of $r$ are in 62 (Appendix 4.a and 4.b).

The difference of the constant factor $\alpha$ in the $\chi^2$ test for the heavy electron and finite interaction length of the direct contact term does not change the significance of the analysis. For this reason, the focus is on the $\chi^2$ test for the finite size of an electron, on the final test result without the details of the heavy electron test. Both test results are displayed in Table 5.
5 Visibility of the significance of the differential cross section $\chi^2$ test in the total cross section.

The sensitivity of the differential cross section to the deviation from the measured experimental data to the theory of a heavy electron $m_e^*$ or an extended electron $r_{\text{electron}}$ is visible in the $\chi^2$ test of the $1/\Lambda^4[1/\text{GeV}^4]$ value in Table 2, Table 3 and Figure 12. The data measured between $55.0 \text{ GeV} < E_{\text{cm}} < 91 \text{ GeV}$ from TRISTAN and LEP I are not significant. The significance of $1/\Lambda_{\text{top}}^4[1/\text{GeV}^4]$ gets important from $91.0 \text{ GeV} < E_{\text{cm}} < 207 \text{ GeV}$. The global $\chi^2$ test uses the big data set of the differential cross section, without the data of the total cross section. The significance of the $\chi^2$ test should be visible also in the total cross section. The plots of Figure 6 to Figure 11 display no direct deviation visible with pure eye from the QED differential cross section. It is interesting to investigate whether is this correct also for the total cross section of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction.

5.1 Calculation of the total cross section of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction.

It is not possible to study directly the sensitivity of the $\chi^2$ test to the total experimental cross section $\sigma(tot)$ of the $e^+e^- \rightarrow \gamma\gamma$ reaction between $55 \text{ GeV}$ to $207 \text{ GeV}$. All the collaborations measured the total cross section $\sigma(tot)$ at one energy, at a different range of the angle $\theta$ and at different selection efficiencies $\varepsilon$. In common, all the detectors compare the experimental total cross section $\sigma(tot)$ to the QED total cross section $\sigma(QED)$ for the detector and use for the calculation of this cross section a very similar numerical Monte Carlo generator [49].

This fact can be used, to introduce an approach for an imaginary L3 detector, as a leading detector for the whole energy range. According to Table 1 $\sigma(tot)$ is measured in total at $17 \sqrt{s}$ energies, $9 \sqrt{s}$ energies are measured by more than one detector and $8 \sqrt{s}$ energies by only one detector. If in the measurement of $\sigma(tot)$, more as one detector is involved, the approach allows to add together the data of different detectors together, this decreases the statistical error of $\Delta\sigma(\text{stat})$. A detailed investigation, Hint for a minimal interaction length in $e^+e^- \rightarrow \gamma\gamma$ annihilation in total cross section of centre - of - mass energies $55 - 207 \text{ GeV}$ [62] allow to calculate from a $\sqrt{s}$ energy of $55.0 \text{ GeV}$ to $207.0 \text{ GeV}$ the total measured experimental cross section for $\sigma(tot) = \sigma(tot, \text{meas.})$, the statistical error of $\Delta\sigma(\text{stat})$, the Ratio $R(\text{exp}) = \sigma(tot, \text{meas.}) / \sigma(tot, \text{QED})$ of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction and the statistical error $\Delta R(\text{stat})$. The numerical values are shown in Table 6.

The $\sigma(tot)$ value of all detectors and $\Delta\sigma(\text{stat})$ of Table 6 compared to the total QED cross section $\sigma(QED)_{\text{tot}}$ is displayed in Figure 14.

Visible from eye, no substantial disagreement between $\sigma(tot)$ of the measured $e^+e^- \rightarrow \gamma\gamma(\gamma)$
Figure 14: The $\sigma(\text{tot})$ of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction of all detectors as function of centre-of-mass energy $\sqrt{s}$. The data (points) are compared to QED prediction (solid line).
Table 6: Summary of $\sigma(tot)$, $\Delta\sigma(stat)$ and ratio $R(exp)$ and $\Delta R(stat)$

| $\sqrt{s}$ GeV | $\sigma(tot)$ | $\Delta\sigma(stat)$ pb | R(exp) | $\Delta R(stat)$ |
|-----------------|---------------|--------------------------|--------|-----------------|
| 55              | 124.746±13.1736 | 0.92001 ± 0.09716       |        |                 |
| 56              | 150.623 ± 9.7176    | 1.15000 ± 0.07419       |        |                 |
| 56.5            | 141.633 ± 22.9310   | 1.10000 ± 0.17810       |        |                 |
| 57              | 135.456 ± 10.7933   | 1.07000 ± 0.08526       |        |                 |
| 57.6            | 125.311 ± 1.9970    | 1.01000 ± 0.01610       |        |                 |
| 91              | 50.3103 ± 0.86517   | 0.98764 ± 0.01698       |        |                 |
| 133             | 26.5472 ± 5.80853   | 1.09604 ± 0.23981       |        |                 |
| 162             | 16.0640 ± 2.42633   | 0.98462 ± 0.14872       |        |                 |
| 172             | 15.6375 ± 2.64851   | 1.08187 ± 0.18324       |        |                 |
| 183             | 12.6404 ± 0.34388   | 0.99219 ± 0.02699       |        |                 |
| 189             | 11.7626 ± 0.18843   | 0.98582 ± 0.01579       |        |                 |
| 192             | 11.0253 ± 0.46129   | 0.95427 ± 0.03993       |        |                 |
| 196             | 11.2978 ± 0.27689   | 1.02004 ± 0.02500       |        |                 |
| 200             | 10.1373 ± 0.26604   | 0.95400 ± 0.02504       |        |                 |
| 202             | 10.1199 ± 0.37855   | 0.97204 ± 0.03636       |        |                 |
| 205             | 9.98539 ± 0.32275   | 0.98865 ± 0.03196       |        |                 |
| 207             | 9.66178 ± 0.23860   | 0.97594 ± 0.02410       |        |                 |

reaction from $55.0 \text{ GeV} \leq \sqrt{s} \leq 207 \text{ GeV}$ and the $\sigma(QED)_{tot}$ prediction can be seen at this scale. Visible is the decrease of the statistical error originated from the increase of the statistic of more as one detector is involved in the analysis. After the global $\chi^2$ fit in Figure 12 a deviation from the QED cross sections $\sigma(QED)_{tot}$ is predicted at LEP2 energies. To increase the visibility of the effect of the 5 $\sigma$ deviation from QED in the $\chi^2$ test of the differential cross section to the total cross section, it is useful to display all the data in the Ratio $R(exp) = \sigma(tot, meas.) / \sigma(tot, QED)$. For this reason, we display in Figure 15 and Figure 16 the ratio $R(exp)$ from Table 6 and the statistical error $\Delta R(stat)$ for two scales.

In Figure 15 and Figure 16 deviate the experimental data from the QED prediction above approximately $\sqrt{s}=180 \text{ GeV}$. It indicates a negative interference with $\sigma(tot, QED) > \sigma(tot, measured)$.

The green line in both Figures, is the ratio $R(\Lambda_6) = \sigma(QED)_{tot}^{L3}/\sigma(QED+\Lambda_{top})_{tot}^{L3}$ calculated from the Monte Carlo generator [49]. In the calculation is used, the pure QED differential cross section and QED differential cross section including $\Lambda_6$ at 7 energies between 91.2 GeV and 200.0 GeV from the L3 paper Table 3 [44] and $\Lambda_{top} = 1253.53 \text{ GeV}$. These numerical values, are approached with an analytic toy function (42).

$$R(\Lambda_6) = C04 + C01 \cdot (-Tanh[C03 \cdot \sqrt{s} + C02])$$ (42)
Figure 15: Ratio $\sigma_{(\text{tot,meas.})} / \sigma_{(\text{tot,QED})}$ of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction of all detectors as function of centre-of-mass energy $\sqrt{s}$, at low scale. The experimental data (points), QED prediction (solid blue line) and ratio prediction from $R(\Lambda_6)$ calculation (green line).

Figure 16: Ratio $\sigma_{(\text{tot,measured.})} / \sigma_{(\text{tot,QED})}$ of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction as Figure 15 on expanded scale. The green line is the ratio prediction from the $R(\Lambda_6)$ calculation.
The constant factors $C_01 = 0.0732964$, $C_02 = -3.06655$, $C_03 = 0.0127994$ and $C_04 = 0.928311$ are the best fit to the data. At $\sqrt{s} = 91.2$ GeV is $R(\Lambda_6) = 0.99$ at $\sqrt{s} = 200.0$ GeV is $R(\Lambda_6) = 0.96$ approximately 4.0 % lower as the QED value. It is important to notice that the total cross section $\sigma(tot)$ is not involved in the total $\chi^2$ fit of the differential cross section. As visible in the green line in Figure 15 and Figure 16 is the deviation of $\sigma(tot, measured)$ from $\sigma(tot, QED + \Lambda_6)$ in the range of the statistical error of $\sigma(tot, measured)$.

In summary a deviation between total cross section of the measured data to QED is visible with pure eye shown in Figure 15 to Figure 16, in contrast to differential measured cross section to QED as shown in Figure 6 to Figure 11.

6 Systematic errors of $\chi^2$ test and total cross section approximation

6.1 Systematic error of the $m_{e^{+}}$ and direct contact term $\chi^2$ test.

The default error $\Delta \left[ \frac{d\sigma}{d\Omega}^{meas}(|cos\theta|, E_j) \right]$ in (41) is usually the quadratic sum of the statistical error and systematic error. The data from the different groups show in the differential cross section the statistical error. The systematic error was not published for every group in detail. For this reason the statistical error as default error, is used in the analysis.

Systematic errors arise from the luminosity evaluation, the selection efficiency, the background evaluation, the choice of the QED-$\alpha^3$ theoretical cross section as the reference cross section, the choice of the fit procedure, the type of the fit parameter, and of the scattering angle in $|cos\theta|$ for comparison between data and theoretical calculations. The maximum estimated error for the value of the fit, from the luminosity, selection efficiency, and background evaluations is approximately $\delta \Lambda/\Lambda = 0.01$. The choice of the theoretical QED cross section was studied with 1882 $[e^+e^- \rightarrow \gamma\gamma(\gamma)]$ events from the L3 detector [64] and [49]. In the worst case, of scattering angles close to 90°, the $|cos(\theta)|_{experiment} \sim 0.05$ would result in $(\delta \Lambda/\Lambda)_{\delta |cos\theta|} = 0.01$. The total systematic error is $\delta \Lambda/\Lambda \approx 0.015$. For a small sample of $[e^+e^- \rightarrow \gamma\gamma(\gamma)]$ events, the fit values were compared between $\chi^2$, Maximum-Likelihood, Smirnov-Cramer von Mises, and Kolmogorov test, with and without binning [65]. An approximate $\delta \Lambda/\Lambda = 0.005$ effect, was estimated for the overall fit with the fit parameter $P = (1/\Lambda)^4$. In summary the systematic errors are negligible compared to the statistical error of the experimental data.

6.2 The systematic error of the total cross section approximation.

In Table 6 and Figure 16 appears above $\sqrt{s} > 91.2$ GeV a small deviation in $R(exp)$ from the $\sigma(QED)_{tot}$ cross section. The systematic error of the measured total cross section of detector,
\( \sigma(\text{exp})_{\text{tot}}^{\text{det}} \) and total QED cross section of \( \sigma(\text{QED})_{\text{tot}}^{\text{det}} \) above \( \sqrt{s} > 91.2 \) GeV, is for L3 \(^{44}\) (Table 3) \( 0.10 \) pb \(<\Delta\sigma(\text{meas.})_{\text{sys}} < 0.13 \) pb and \( \Delta\sigma(\text{QED})_{\text{sys}} = 0.1 \) pb, for DELPHI \(^{42}\) (Table 4, year 2000) \( 0.09 \) pb \(<\Delta\sigma(\text{meas.})_{\text{sys}} < 0.14 \) pb and for OPAL \(^{47}\) (Table 7) \( 0.05 \) pb \(<\Delta\sigma(\text{meas.})_{\text{sys}} < 0.08 \) pb. According to these tables the \( \Delta\sigma_{\text{sys}} \) values behave statistically. No change as the function of the energy of the systematic errors above \( \sqrt{s} > 91.2 \) GeV could be observed. This excludes the possibility, that the deviation of \( R(\text{exp}) \) from \( R(\text{QED}) \) could be originated from an energy \( \sqrt{s} \) behaviour of the systematic error. More information in \(^{62}\).

7 Discussion of the history and latest analysis results of \( e^+e^- \to \gamma\gamma(\gamma) \) reaction.

Since the VENUS collaboration 1989 published the data of the \( e^+e^- \to \gamma\gamma(\gamma) \) reaction, numerous other collaborations studied this reaction and extended the search to the Bhabha reaction \( e^+e^- \to e^+e^- (\gamma) \), for example \(^{71}\).

7.1 Search for a heavy excited electron \( m_{e^*} \).

The study of the single fit parameter originated from one detector (\( 1/\Lambda^4[1/\text{GeV}^4] \)) displayed in Table 2 allow to set upper limits on \( m_{e^*} \). This is in agreement with the experimental results of VENUS, TOPAS, ALEPH, DELPHI, L3, and OPAL. These collaborations detected in their fits similar low significance, which allowed to set upper limits on \( m_{e^*} \). The most stringent limit was set by L3 \(^{44}\) to \( m_{e^*} > 310 \) GeV on 95% CL. The effect of a larger amount of input data on the \( \chi^2 \) test gets visible in Table 2 if all the 34 parameters (\( 1/\Lambda^4[1/\text{GeV}^4] \)) are compared. From total 34 single fits in Table 2 are 28 with a negative sign and 6 with a positive sign, which exhibits a trend of a negative interference between QED and the assumption of an excited electron exchange shown in Figure 2. This trend is more significant, if the fits like in Table 3 and Figure 12 are grouped after data from TRISTAN (VENUS plus TOPAS), LEP I and LEP II and “All Data” or grouped after the detector ALEPH, Delphi, L3, OPAL, TRISTAN, LEP1 (At the \( Z^0 \) - pole ), LEP2 (133 GeV to 207 GeV) and “ALL Data”. In particular, the Delphi, L3 and OPAL data at higher energies are negative in \( 1/\Lambda^4[1/\text{GeV}^4] \). The leading parameter in the significance of \( 1/\Lambda^4[1/\text{GeV}^4] \) is the \( \sqrt{s} \)-energy and the number of \( e^+e^- \to \gamma\gamma(\gamma) \) events, displayed in Figure 12 and Figure 13.

As discussed in the introduction, is the approach of an excited electron \( e^* \) shown in Figure 2 is only possible via the Mandelstam variables in the \( t \) - and \( u \) - channel. The \( s \) - channel is highly suppressed by angular conservation, because the intermediated particle would be a \( \gamma \) or \( Z^0 \) with spin ONE. The two \( \gamma \)'s in the final state are able to couple to spin ZERO or spin TWO but not to spin ONE. The two \( \gamma \)'s in the final state couple for this reason to spin ZERO,
it means they are left-right hand polarised. This configuration leads to a mass of the excited electron to $m_{e^*} = 308 \pm 56$ GeV.

It is important to notice, that similar values with lower significance are published from L3 [44] (Page 35) and OPAL [47]. Considering a coupling of the electron to a photon via chiral magnetic interaction L3 [44] (Page 36, equ. (4)) the global fit value $(1/\Lambda)^4_{top} = -(1.11 \pm 0.02) \times 10^{-10}[GeV^{-4}]$ is very similar to the L3 value with lower significance [44] (Page 36) $1/\Lambda^4_{e^*} = -(0.9^{+2.0}_{-1.7}) \times 10^{-10}[GeV^{-4}]$.

In the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction at the $Z^0$ pole at $\sqrt{s} = 91.2$ GeV, leads the suppression of the s-channel to a perfect agreement of $R(\Lambda^6_{\nu}) = \sigma(tot,meas)/\sigma(tot,QED) = 0.999$. If the s-channel is involved in a Bhabha like reaction $e^+e^- \rightarrow e^+e^-(\gamma)$ a different QED test is used to search for $m_{e^*}$. In this case, the search for heavy neutral $L^0$ or charged leptons $L^\pm$ is performed to use pair-production in the s-channel via $\gamma$ and $Z$ boson exchange, similar to Bhabha scattering shown in Figure 1. The expectation is, that the numerical values for the mass of an excited electron or for limits on this mass depend, on the new test reaction and on the theoretical background (review in [66] and [72]) to interpret the $\Lambda$ values, for example in $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction from the Lagrange function (30) or equation (4) in [44]. Lower limits were set by the L3-collaboration [66] on 95 % CL for pair-production of neutral heavy leptons, depending on model (Dirac or Majorana), from $m_{L^*} > 102.7$ GeV to $m_{L^*} > 80.5$ GeV. In addition the L3-collaboration [66] set lower limits at 95 % CL for pair-produced charged heavy leptons from $m_{L^*} > 102.6$ GeV to $m_{L^*} > 100.8$ GeV. Similar limits on 95 % CL are set by the OPAL-collaboration on long-lived charged heavy leptons and charginos by $(m_{L^*},m_{\text{chargino}}) > 102.0$ GeV and lower limits on neutral $L^0$ and charged $L^\pm$ heavy leptons [68].

The HERA H1 collaboration performed a search for heavy leptons [69]. Best fit limits for an $e^*$ production in the HERA mass range was obtained in the $\gamma$ final state. A compositeness scale parameter $\Lambda$ excluded values below approximately 300 GeV.

CMS collaboration is looking for long-lived charged particles in pp collision [72]. A signal sample of a modified Drell-Yan production of long-lived leptons was studied. In the Drell-Yan process, a quark from one hadron and an antiquark from the second hadron annihilate to create a pair of leptons, through the exchange of a virtual photon or $Z^0$ in the s-channel. Drell-Yan signals with $|Q|=1e$ are excluded below masses of 574 GeV/$c^2$.

### 7.2 Search for a finite size of an electron.

The mathematical approach to test the finite size of an electron via a direct contact term in Figure 2 is very similar to the approach of an excited electron (33) and (35). The similarity in the significance of $5 \times \sigma$ in the $\chi^2$ fit is not surprising, for the value of $\Lambda_{top} = 1253.53 \pm 226$ GeV. But the theoretical input is very different. The direct contact term tests the distance of
annihilation between $e^+$ and $e^-$, but the $m_{e^*}$ test uses an intermediated electron $e^*$ to search for a heavy electron $m_{e^*}$.

If the electron has a finite extension, the search with the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ - reaction at high energies is, a competition between the Lorentz Contraction of the object and the size of the object in rest. Including the Lorentz Contraction at $\sqrt{s} = 207$ GeV the electron radius would be $3.2 \times 10^{-14}$ m bigger than the charge radius of the proton $0.87 \times 10^{-15}$ m. Under these circumstances, it seems possible to assume that a charge distribution inside this electron volume exists. The effective Lagrangian of (34) is electromagnetic. The annihilation of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ - reaction would test the long-range direct contact to the charge distribution.

7.3 Visibility of $r_{electron}$ from the $\chi^2$ fit, to the total cross section in the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ - reaction.

The global $\chi^2$ test under discussion uses the differential cross section measured from different detectors. To investigate the deviation from the total measured $e^+e^- \rightarrow \gamma\gamma(\gamma)$ cross section from QED total cross section, it is necessary to introduce an approach for a common total cross section in the energy range from $55$ GeV $< \sqrt{s} < 207$ GeV shown in Figure 14. In the first view, is the agreement between the total measured cross section $\sigma_{\text{tot}}$ and the QED cross section $\sigma_{\text{tot,QED}}$ excellent. To test in more details, the agreement between both cross sections, it is necessary to study the ratio $\sigma_{\text{tot, meas.}} / \sigma_{\text{tot, QED}}$ in Figure 15 and Figure 16. A deviation from $\sigma_{\text{tot, meas.}} / \sigma_{\text{tot, QED}} < 1.0$ appears above $\sqrt{s} = 180.0$ GeV, indicating, as in the $\chi^2$ test, a negative interference. These findings agree with bigger statistical error bars of the measurements from L3 [44] ( Fig. 2 ) and DELPHI [42] (2000 , Fig. 2 ).

The green line in Figure 15 and Figure 16, is the ratio $R(\Lambda_6) = \sigma(QED)_{\text{tot}}^{L3}/\sigma(QED+\Lambda_{\text{top}})_{\text{tot}}^{L3}$ calculated from the Monte Carlo generator, using the pure QED differential cross section and QED differential cross section including $\Lambda_6$. The deviation from the total QED cross section appears at the same $\sqrt{s}$ energy as the measured cross section.

8 Conclusion

The differential cross section of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction measured from the VENUS, TOPAS, OPAL, DELPHI, ALEPH and L3 collaborations, was used to test the QED. In common all these collaborations analyse a negative deviation from the QED, but with low significance. In particular the test of the total cross section of LEP2 and our comparison of the QED with the measurement in Figure 16 support these negative trends. The high rate of input data, allowed us to perform a comprehensive $\chi^2$ test to search for an excited electron mass $m_{e^*}$ and
a finite annihilation length in direct contact term approach. These extensive data rates from the collaborations measuring the differential cross section, allow for the first time to set an approximately $5 \times \sigma$ significance on the mass of an excited electron to $m_{e^*} = 308 \pm 56$ GeV. A similar significance $5 \times \sigma$ effect was detected for a charge distribution radius of the electron $r = (1.57 \pm 0.07) \times 10^{-17}$ cm.

Extensive measurements and analyses are performed to search for quark and lepton composites in contact interaction [70] in the Bhabha channel Figure 1. A hint for axial-vector contact interaction, in the data on $e^+e^- \rightarrow e^+e^-(\gamma)$ scattering from ALEPH, DELPHI, L3 and OPAL at centre-of-mass energies 192 - 208 GeV was detected at $\Lambda = 10.3^{+2.8}_{-1.6}$ TeV [71].

Depending on the experimental test, the electron exhibits two extensions. In the case of the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction, only the QED long range interaction is tested. The weak interaction via $Z^0$, is suppressed by angular momentum conservation. This results in a $\Lambda_{top} = 1253.53 \pm 226$ GeV. In the case of the Bhabha reaction $e^+e^- \rightarrow e^+e^-(\gamma)$, the short range weak and QED interaction is involved. The Bhabha channel is dominating because the differential cross section in Bhabha channel is much bigger than in the pure QED channel. The contribution of the $Z^0$ in the reaction, leads to a $\Lambda = 10.3^{+2.8}_{-1.6}$ TeV, about 8 times bigger as in the $e^+e^- \rightarrow \gamma\gamma(\gamma)$ reaction. The conclusion of these findings is, that in the electron an outer and inner core exists. Two attributes are combined in one particle.

It is possible to use these attributes to introduce “Extended Particle Models” [73]. In a publication “Image of the Electron Suggested by Nonlinear Electrodynamics Coupled to Gravity” [30] an electromagnetic spinning soliton with the electron parameters introduces an electron including a de Sitter vacuum disk. This de Sitter vacuum disk generates an electric and magnetic field. These fields open the possibility to construct a wave function of the electric field. If this get connected with a “Scheme of geometrical extended fundamental particles and anti-particles” [73] and the Lorenz Contraction of the radius from the experiment, the radius from the experiment and the radius calculated via the theoretical scheme agrees at approximately $r_e \sim 1.57 \times 10^{-17}$ [cm] [73,74]. The numerical coincidence between [30], [73,74] and the experiment support that the electron is NOT a point.

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