A simple assessment on the hierarchy problem

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We consider a generalization of the Dirac field equation that is very simple but such that if the Higgs boson were a top-quark condensate then the hierarchy problem may be lightened.

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I. INTRODUCTION

The problem of the incompatibility between the cosmological constant and the Higgs field is very well known, and it comes from the fact that whenever the Higgs boson acquires the vacuum expectation value which minimizes its potential, the minimum of the potential gives rise to an effective cosmological constant that is negative and one-hundred and twenty orders of magnitude off the empirically measured value. To deal with this problem it is customary to hope that eventually we could find another mechanism producing an effective cosmological constant but with a positive value and the same order of magnitude, and also a fine-tuning of the two contributions up to the one-hundred and twentieth decimal place, in order to have a positive and small effective cosmological constant in the end; if no such fine-tuning or additional mechanism producing an effective cosmological constant is ever found, then the only possibility we have to avoid the effective cosmological constant coming from the spontaneous symmetry breaking mechanism is renouncing to the spontaneous symmetry breaking.

Torsion symmetry breaking can occur without being spontaneous is by being dynamical, and one possibility is to have the Higgs field as a fermion bound-state subject to Nambu–Jona-Lasinio types of potentials. But as one might then wonder, where does this NJL potential come from?

The torsional completion of gravity is the theory that we obtain when we do not constrain the most general metric-compatible connection to be symmetric in the two lower-indices, yielding a geometry that is endowed with torsion as well as curvature; when applied to the case of Dirac matter fields, torsion couples to the spin density in the same way in which curvature couples to the energy density of the Dirac matter field, and the resulting theory is therefore called Sciama-Kibble-Einstein-Dirac theory, or SKED theory for short. In it the Dirac matter fields, torsion couples in terms of least-order derivative field equations.

II. DIRAC FIELDS WITH NON-STANDARD MASS-LIKE TERMS

In this paper, we take (1+3)-dimensional spacetimes filled with $\frac{1}{2}$-spin spinor fields, and all the fields will be coupled in terms of least-order derivative field equations.

All definitions we will employ hereafter are given in references [6–10] where the SKED theory giving rise to the mentioned NJL potential is given: nevertheless in the following we are not going to employ the torsional completion of gravity for Dirac matter fields because we are not interested in the mechanism of condensation for the fermions themselves, but only on the possibility to compute the mass of the condensate in terms of the mass of the fermions in such a way that the condensate scalar is allowed to be less massive than the condensing fermions.

We introduce $\gamma_a$ known as Clifford matrices because they are such that $2\mathcal{N} h_{ij} = \{\gamma_i,\gamma_j\}$ known as Clifford algebra in terms of which $\frac{1}{4}\{\gamma_i,\gamma_j\} = \sigma_{ij}$ are the generators of the spinorial transformation with which spinors transform and also $\{\gamma_i,\sigma_{jk}\} = i\varepsilon_{ijkq}\pi\gamma^q$ are the relations implicitly giving the parity-odd matrix used to obtain the left-handed and right-handed chiral projections of the spinor field itself; the spinorial covariant derivative is denoted with the notation $\nabla_{\mu}$ as it is usually done.

By neglecting torsion and gravity, the Dirac field is described by the following Lagrangian density

$$L = i\bar{\psi}\gamma^\mu \nabla_\mu \psi - i\bar{b}\pi \psi - m\bar{\psi}\psi$$  \hspace{1cm} (1)

in which two mass-like terms have been included [11–12].

The mass-like term given with constant $m$ is the parity-even mass term as usual while the mass-like term given with constant $b$ is a new parity-odd mass term: since the standard model of particle physics is maximally parity-violating, the presence of such terms is not to be regarded as surprising, and on the other hand its presence may affect the hierarchy problem quenching the incompatibility...
between the standard model of particle physics and cosmology as we are going to show in the following [13].

By varying (1) with respect to the spinor field we get

\[ i\gamma^\mu \nabla_\mu \psi - m\psi - ib\pi \psi = 0 \]  \hspace{1cm} (2)

which can be worked out in several different ways. First of all by applying to this equation another Dirac derivative operator we may work out the expression

\[ \nabla^2 \psi + (m^2 + b^2) \psi = 0 \]  \hspace{1cm} (3)

as a spinorial field equation of Klein-Gordon type.

Further by multiplying (2) on the left with gamma matrices and the adjoint spinor field and splitting real and imaginary parts we obtain the following expressions

\[
\nabla^\alpha (\overline{\psi} \psi) + 2 \left( \overline{\psi} \sigma^{\alpha\mu} \nabla_\mu \psi - \nabla_\mu \overline{\psi} \sigma^{\alpha\mu} \psi \right) - 2b \overline{\psi} \sigma^\alpha \pi \psi = 0 \tag{4}
\]

\[
\nabla_\mu \left( \overline{\psi} \sigma^{\alpha\mu} \psi \right) = 0 \tag{5}
\]

\[
\nabla_\mu \left( i\overline{\psi} \sigma^{\alpha\mu} \psi \right) + \frac{i}{2} \left( \overline{\psi} \nabla^\alpha \psi - \nabla^\alpha \overline{\psi} \psi \right) - m\overline{\psi} \sigma^\alpha \psi = 0 \tag{6}
\]

\[
\nabla_\mu \left( \overline{\psi} \sigma^{\alpha\mu} \pi \psi \right) + \frac{i}{2} \left( \overline{\psi} \nabla^\alpha \pi \psi - \nabla^\alpha \overline{\psi} \pi \psi \right) + \frac{b_i}{2} \overline{\psi} \sigma^\alpha \pi \psi = 0 \tag{7}
\]

\[
\nabla^\alpha \left( i\overline{\psi} \pi \psi \right) + 2i \left( \overline{\psi} \sigma^{\alpha\mu} \pi \nabla_\mu \psi - \nabla_\mu \overline{\psi} \sigma^{\alpha\mu} \pi \psi \right) + \frac{b_i}{2} \overline{\psi} \sigma^\alpha \pi \psi = 0 \tag{8}
\]

which are known in literature as Gordon decompositions.

It is possible to work out field equations (3) or to take the divergence of (4) and (5) and re-compose them together with (4) and (5) themselves in order to get

\[
\nabla^2 \left( \overline{\psi} \psi \right) + 4 i \nabla_\alpha \overline{\psi} \sigma^{\alpha\mu} \nabla_\mu \psi - 4b m\overline{\psi} \pi \psi + 4b \overline{\psi} \pi \psi = 0 \tag{10}
\]

\[
\nabla^2 \left( i\overline{\psi} \pi \psi \right) + 4 i \nabla_\alpha \overline{\psi} \sigma^{\alpha\mu} \pi \nabla_\mu \psi + 4m^2 \overline{\psi} \pi \psi - 4mb \overline{\psi} \psi = 0 \tag{11}
\]

as scalar/pseudo-scalar equations of Klein-Gordon type.

In a quantum theory of fields, the derivatives of the fields are the momenta of the particle according to the prescription \(i \nabla_\alpha \psi = P_\alpha \psi\) so that one term is shown to be negligible for plane-waves and therefore we may simplify the field equations down to the following expressions

\[
\nabla^2 \left( \overline{\psi} \psi \right) - 4bm\overline{\psi} \pi \psi + 4b \overline{\psi} \pi \psi = 0 \tag{12}
\]

\[
\nabla^2 \left( i\overline{\psi} \pi \psi \right) + 4m^2 \overline{\psi} \pi \psi - 4mb \overline{\psi} \psi = 0 \tag{13}
\]

as the most general field equations of Klein-Gordon type.

Both (3) and (12-13) have positive mass term, so that they give rise to real masses, according to

\[
M_{sp}^2 = m^2 + b^2 \tag{14}
\]

and

\[
M_p = 2|b| \tag{15}
\]

\[
M_p = 2|m| \tag{16}
\]

in terms of the parameters of the mass-like terms.

By combining (14-15-16) we see that the parameters can be removed obtaining the final expression

\[
4M_{sp}^2 = M_p^2 + M_p^2 \tag{17}
\]

as a relationship linking only the masses of the fields.

By taking the spinor as the heaviest quark, if the scalar field is the Higgs boson then we have that the pseudo-scalar boson must have \(M_p \approx 322 \text{ GeV}\) as mass value.

If this pseudo-scalar boson exists and with this mass then the scalar boson can have a mass smaller than the fermion mass, and the hierarchy problem alleviated.

III. CONCLUSION

In this paper, we noticed that in a parity-violating standard model the Dirac field may have a non-standard parity-odd mass-like term beside the standard parity-even mass-like term, and we have proceeded to obtain the ensuing field equations for the fermion condensing into a scalar and a pseudo-scalar, showing that if the pseudo-scalar is more massive than the scalar can be less massive than the fermion, lightening the hierarchy problem in what may be the simplest way possible.

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