The no-reflection regime of radar detection of cosmic ray air showers

M I Bakunov\(^1\), A V Maslov\(^1\), A L Novokovskaya\(^1\) and A Kryemadhi\(^2\)

\(^1\) University of Nizhny Novgorod, Nizhny Novgorod, Russia
\(^2\) Department of Math, Physics, and Statistics, Messiah College, Mechanicsburg, PA 17055, USA
E-mail: bakunov@rf.unn.ru

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Abstract

The ionization front of a cosmic ray air shower propagates in the atmosphere with almost the speed of light in vacuum, i.e., faster than a radio wave in the air. There can be no reflection of a radar signal from such a front. Instead, an additional transmitted wave, which travels behind the front in the backward direction, is generated. We study the frequencies, propagation directions, and amplitudes for the waves excited at the front and discuss their use for radar detection of air showers.

1. Introduction

The problem of radar detection of extensive air showers (EASs)—cascades of particles created in the Earth’s atmosphere by ultrahigh energy cosmic rays—has been challenging radio astronomers for more than 70 years, since the 1941 paper by Blackett and Lovell [1], where the concept of radar detection was proposed. The original concept was based on the view that an EAS produces a long narrow cylinder of ionization in the atmosphere, which can give detectable reflections of the sounding radio wave from a powerful ground-based radio transmitter. However, several years of unsuccessful experimental efforts of Lovell to detect the radar reflection from EASs using a specially built radio telescope in Jodrell Bank [2] had led to the loss of interest in this concept for decades. After sporadic publications [3–5], a new surge of interest to the topic was ignited in 2001 by Gorham [6], who revisited the concept of radar detection and estimated that EASs caused by primary particles with energies above \(10^{18}\) eV can be detected with radars in the VHF frequency range (30–100 MHz). New experimental efforts to observe radar echoes from EASs have been made by several groups including LAAS group [7], Brookhaven National Laboratory [8], Jicamarca Radio Observatory [9, 10], MU radar group [11], MARIACHI experiment [12], and TARA project [13, 14]. However, no reliable experimental results on EAS detection using radars have been obtained up to now. Nevertheless, due to potential merits of covering a large detection area and operating with 100% duty cycle the radar concept of EAS detection continues to attract the attention of researchers [15, 16].

In our recent papers [17, 18], we drew attention to the relativistic effects and effects of plasma nonstationarity as crucial factors that can determine the frequency, propagation direction, and amplitude of the radar signal reflected from EAS. Indeed, the free electron lifetime is as short as \(\sim 10\) ns at sea level to \(\sim 100\) ns at 10 km altitude [19–21]. This lifetime determines the length of the ionization trail behind the ionization front of EAS. Assuming that the electrons (and positrons) in the trail are slow whereas the front moves with almost the speed of light [22–25], the length of the trail can be estimated as \(\sim 3\) to \(\sim 30\) m at sea level and 10 km altitude, respectively. Thus, instead of expected reflection from a long plasma column with the lifetime of \(\sim (1\ \mu s–1\ s)\), as in the original concept of radar detection [1], the wave is scattered from a relativistically moving plasma cloud where the particles are created at the front of the cloud and disappear at its rear end due to attachment to oxygen (see also [26]). As shown in [17, 18, 26], the return signal in this case is characterized by a large Doppler shift and high directionality.

In [17, 18], the analysis was made in the assumption that the ionization front of EAS moves with a moderate relativistic factor \(\gamma\) (for example, \(\gamma < 70\) for the refractive index of air \(n = 1.0001\) at a 10 km altitude), i.e., slower than a radio wave in the air. A radar signal incident on such a (subluminal) front produces a reflected wave, which can be used for detecting the EAS. In the more commonly used model of EAS [22–25], the ionization...
front of EAS propagates in the atmosphere with almost the speed of light in vacuum, i.e., faster than a radio wave in the air. There can be no reflection of a radar signal from such a (superluminal) front. Instead, an additional transmitted wave, which travels in the plasma behind the front in the backward direction, will be generated [27]. We analyze this superluminal case in the present paper. The frequencies, propagation directions, and amplitudes for the waves excited at the front are calculated and possible use of these waves for radar detection of EAS is discussed.

The paper is organized as follows. Section 2 formulates a general problem of the interaction of a plane wave with an ionization front moving in the air with a superluminal velocity and gives its solution. In particular, the frequencies and propagation directions of the secondary waves as well as their amplitudes are obtained. Section 3 analyzes the properties of the waves obtained in section 2 in two particular cases: with and without collisions in the plasma created behind the front. Section 4 describes the properties of the secondary waves for the case of a highly collisional plasma, such as created by an EAS, and discusses some issues related to the radar technique of EAS detection. Section 5 gives conclusions.

2. Problem and solution

2.1. Formulation of the problem

We consider the following scheme of EAS radar detection, see figure 1(a). The ionization front of EAS moves with a velocity $V \approx c$ vertically in the Earth’s atmosphere toward the surface and creates charged particles (electrons and positrons). The particles, which have relativistic initial velocities, deaccelerate rapidly within a thin region (the so-called pancake [28]) and form immobile plasma (or ionization trail) behind the front. A ground-based radio-transmitter illuminates the EAS with a radio wave in the medium frequency range. The Doppler-shifted transmitted waves in the plasma that propagate toward the surface can be detected by the receiving antenna.

To calculate the return signal, we adopt the following simplified theoretical model that is expected to capture the basic aspects of the radio wave transformation on the ionization front of an EAS, which moves faster than the radio waves in the air. We consider a plasma front that moves with a relativistic velocity $V$ vertically in the air that has refractive index $n$, see figure 1(b). We neglect the finite thickness of the pancake and assume that the charged particles with density $N$ are created at the front with zero velocities. We also neglect the plasma inhomogeneity in the transverse direction. The sounding wave at frequency $\omega_0$ is TE polarized (its electric field is along $z$) and is incident under an angle $\theta_0$. The electric field of the incident wave is

$$E_i(x, y, t) = \hat{z}E_0 \exp\left(i\omega_0 t - ig_0 x - ih_0 y\right),$$

where $g_0 = n(\omega_0/c)\cos\theta_0$ and $h_0 = n(\omega_0/c)\sin\theta_0$. Our aim is to find the secondary waves that are produced. In particular, we are interested in the regime when the velocity of the front $V$ exceeds the velocity of the waves in the air. In this case, $\beta = V/c > 1/n$ and, therefore, no reflected wave can exist ahead of the moving front. All secondary waves exist in the created plasma only. The described model neglects the finite thickness of the pancake region where the initially created plasma particles deaccelerate rapidly. The motion of the plasma does not affect the dispersion equation for the waves inside it. The amplitudes of the excited waves can on the other hand be affected both by the motion of the plasma
and the profile of the plasma density. The relativistic motion of electrons within the pancake is expected to increase the amplitudes of the secondary waves at the expense of the kinetic energy of the electrons. The assumption of immobile plasma behind the front allows us to estimate the lower limit on the amplitudes of the waves, thus ensuring detectability of the wave. The assumption of the rapid plasma creation should be valid if the region of the plasma growth is smaller than the spatial scale (such as wavelength) of the excited waves. Accounting adequately for plasma non-uniformity and deacceleration effects would require a much more complicated model that had to be approached numerically. The purpose of the present model is to understand some basic properties of the waves excited at the superluminal front.

Although our consideration refers specifically to the case of a vertical EAS, which is a common situation for the EASs created by ultra-high energy cosmic ray protons, the reflection of the wave from an inclined EAS can be readily obtained by specifying the corresponding angle $\theta_0$ between the direction of the EAS motion and the incident wave.

2.2. Kinematic properties of the transmitted waves

In order to find the secondary waves created by the incident wave (1), we take the transmitted field behind the front in the general form

$$E_t(x, y, t) = \hat{z}E_t \exp\left(\imath\omega_t t - \imath\mathbf{g}_t \cdot \mathbf{x} - \imath h_t y\right),$$

(2)

where $\omega_t$ is the frequency of the transmitted wave, $g_t$ and $h_t$ are the projections of the propagation wavevector. There are several waves that can satisfy equation (2). Their frequencies and wavevectors can be found from the conservation of the phases at the front and the dispersion relation for the waves in the plasma. The phase conservation gives $h_t = h_0$ and

$$\omega_t + g_0 V = \omega_t + g_t V.$$  \hspace{1cm} (3)

The dispersion equation for the plasma is

$$\omega^2 + \beta^2 = \frac{\omega_p^2}{c^2} \epsilon(\omega), \quad \epsilon(\omega) = n^2 \left(1 - \frac{\omega_p^2}{(\omega - \imath \nu \omega)}\right),$$  \hspace{1cm} (4)

where $\omega_p = \sqrt{4\pi e^2 N/(mn^2)}$ is the plasma frequency and $\nu$ is the collision rate.

It is convenient to normalize all frequencies to the incident frequency $\omega_0$ and introduce $f_t = \omega_t/\omega_0$, $f_p = \omega_p/\omega_0$, and $f_\nu = \nu/\omega_0$. By using equations (3) and (4) we obtain an equation for $f_t$:

$$f_t^3 + Af_t^2 + Bf_t + C = 0.$$  \hspace{1cm} (5)

The coefficients in equation (5) are

$$A = -1 - f_\nu - i f_p,$$  \hspace{1cm} (6)

$$B = f_r + n^2 \beta^2 f_p^2 \left(1 - n^2 \beta^2\right) + i f_p \left(f_r + 1\right),$$  \hspace{1cm} (7)

$$C = -i f_p f_r,$$  \hspace{1cm} (8)

where

$$f_r = 1 + \frac{2n\beta \left(n\beta + \cos \theta_0\right)}{1 - n^2 \beta^2}.$$  \hspace{1cm} (9)

Expression (9) coincides formally with the frequency of the reflected wave when it exists at $n\beta < 1$ (see [17, 18]). However, unlike [17, 18], in the superluminal case under investigation ($n\beta > 1$) the parameter $f_r$ is negative.

Solving equation (5) gives three complex frequencies which we denote as $f_t^{(1,2,3)}$.

The propagation of energy for the transmitted waves can be calculated by finding the Poynting vector. This gives the following expression for the projection of the group velocity on the $x$ axis:

$$v_{gs} = \frac{2c \text{ Re}(g_t \omega_t)}{n^2 \left|\omega_t\right|^2 + \omega_p^2 + c^2 \left|\mathbf{h}_t\right|^2 + \left|\mathbf{g}_t\right|^2}.$$  \hspace{1cm} (10)

The group velocity for each transmitted wave can be found by substituting the corresponding $\omega_t$, $h_t$, and $g_t$ into equation (10).
2.3. Amplitudes of the transmitted waves
To find the amplitudes of the transmitted waves, we use the continuity of $B_y$, $E_z$, and $\partial E_z/\partial t$ at the moving front [29, 30]. This gives

$$\frac{E_i^{(1)}}{E_0} = f_i^{(1)} \left( f_i^{(2)} - 1 \right) \left( f_i^{(3)} - 1 \right)$$

and similar expressions can be obtained for $E_i^{(2,3)}$. Expression (11) is valid for the cases with and without collisions in the plasma. The effect of collision is incorporated into the frequencies $f_i^{(1,2,3)}$. It is interesting to note that expression (11) is similar to the expression for the reflection coefficient obtained in [17]. The only difference is the minus sign that comes from the fact that the reflected and transmitted fields are on opposite sides of the front.

3. General analysis

3.1. Without collisions in the plasma
Let us first analyze the case of a collisionless plasma for which $\nu = 0$ in equation (4). In this case, the solutions of equation (5) can be written in a simple analytical form

$$f_i^{(1,2)} = 1 + \frac{f_p - 1}{2} \left( 1 \pm \sqrt{1 - \eta} \right), \quad f_i^{(3)} = 0,$$  

where $\eta = (1 - n^2 \beta^2)f_p^2/(n\beta + \cos \theta_0)$. When $n\beta > 1$, all three solutions correspond to the waves that satisfy the causality condition and therefore, all three waves, which we denote as T1, T2, and T3, can be excited.

Under condition

$$|1 - n^2 \beta^2| f_p^2 \ll 1$$  

we obtain approximate frequencies $f_i^{(1,2)}$:

$$f_i^{(1)} \approx f_i - \frac{n\beta}{n\beta + \cos \theta_0} \frac{f_p^2}{2},$$  

$$f_i^{(2)} \approx 1 + \frac{n\beta}{n\beta + \cos \theta_0} \frac{f_p^2}{2}.$$  

Approximation (13) is particularly useful in two interesting cases. The first case is when the front creates a low density plasma behind it but its velocity can lie in a wide range. The second one, which is focused on in this paper, is when the front has a highly relativistic velocity ($|1 - n\beta| \ll 1$) but the plasma density can lie in a wide range.

The T1 wave has an extremely high frequency upshift which is determined mostly by the closeness of $n\beta$ to 1, see equation (14):

$$f_i^{(1)} \approx f_i \approx \frac{1 + \cos \theta_0}{1 - n\beta}, \quad \left| f_i^{(1)} \right| \gg 1.$$  

The upshift depends very weakly on the plasma density $f_p$. Note that $f_i^{(1)} < 0$ but $g_i^{(1)} > 0$. The wave propagates in the $-x$ direction with the group velocity that is approximately equal to $c/n$, i.e., slightly smaller than that of the front $V$. Note that when $n\beta < 1$, this wave would propagate in the plasma faster than the front and, therefore, would not exist. Instead, a reflected wave would appear. The reflected frequency would be given by equation (14) with $f_p = 0$ which also coincides with equation (9).

The frequency for the T2 wave depends rather strongly on the plasma density but there is no highly relativistic upshift, unlike for the T1 wave. Moreover, the wave propagates in the $+x$ direction for rather low plasma densities when $f_p < f_p^*$ with $f_p^* = \sqrt{(2n\beta + (1 + n^2 \beta^2) \cos \theta_0)} \cos \theta_0$ and in the $-x$ direction when $f_p > f_p^*$. The group velocity and its projection on $x$ as functions of $f_p$ are shown in figure 2(a). As $f_p$ increases, the group velocity decreases, reaches a minimum and then starts to increase. The increase of $f_p$ also leads to the reversal of the power flow at some value of $f_p$. For normal incidence, $\theta_0 = 0$, the minimum of the group velocity and the turning point coincide and take place at $f_p^* = 1 + n\beta$. We note that the change of polarization of the incident wave from TE to TM at very small angles $\theta_0$ would lead to a resonant excitation of Langmuir waves behind the front [29]. Since $n\beta > 1$, the wave skinning regime cannot be realized unlike in the case of $n\beta < 1$ [29]. The angle of propagation for the T2 wave is shown in figure 2(b). For very low $f_p$, the transmitted wave
propagates at the same angle as the incident wave. The increase of $f_p$ leads to the rotation of the wave. At large densities, the wave follows the front, i.e., $\theta \to 180^\circ$.

The T3 wave describes the so-called free-streaming mode which consists of a self-consistent distribution of the magnetic field and currents in the plasma [30]. Since $f^{(3)} = 0$, neither the velocity of the front nor the plasma density affects the frequency of this mode.

The amplitudes of the transmitted waves can be obtained from equations (11) and (12):

$$\frac{E^{(1)}_1}{E_0} = 1 - \sqrt{1 - \eta} \approx - \frac{1 - n^2 \beta^2}{(n \beta + \cos \theta_0)^2} \frac{f_p^2}{4},$$

$$\frac{E^{(2)}_1}{E_0} = 1 + \sqrt{1 - \eta} \approx 1 + \frac{1 - n^2 \beta^2}{(n \beta + \cos \theta_0)^2} \frac{f_p^2}{4},$$

$$\frac{E^{(3)}_1}{E_0} = 0,$$

where the approximations are obtained under condition (13) using equations (14) and (15). According to equation (17), the highly frequency upshifted wave T1 has a rather small amplitude in the regime described by equation (13). The T2 has an amplitude that is practically equal to the amplitude of the incident wave, see equation (18). Thus, if the T2 wave follows the front, $f_p > f_p$, then one can expect significant energy carried by that wave in the $-x$ direction. The T3 wave has zero electric field associated with it, see equation (19). However, its magnetic field is non-zero and is given by

$$\frac{B^{(3)}_x}{E_0} = n \sin \theta_0 \cdot \frac{n^2 \beta^2 f_p^2}{1 + n^2 \beta^2 + 2n \beta \cos \theta_0 + n^2 \beta^2 f_p^2},$$

$$\frac{B^{(3)}_y}{E_0} = \frac{1 + n \beta \cos \theta_0}{\beta} \cdot \frac{n^2 \beta^2 f_p^2}{1 + n^2 \beta^2 + 2n \beta \cos \theta_0 + n^2 \beta^2 f_p^2}.$$

The dc current associated with the T3 mode can be found directly from its magnetic field.

3.2. With collisions in the plasma

We now include collisions in the plasma, i.e., $\nu \neq 0$ in equation (4). We discuss here in detail the case when the collision rates are not very high $\nu \omega p \lesssim 10$. This allows one to track how the properties of the secondary waves change gradually in the presence of collisions and compare with the case of a collisionless plasma described in section 3.1. The case of very high collision rates is given in section 4.

In the presence of collisions, one has to solve cubic equation (5). In principle, any cubic equation has an analytical solution but due to its complicated form, especially for a cubic equation with complex coefficients such as equation (5), it is not particularly useful for the analysis here. We, therefore, give only numerical results without general analytical formulas.
The frequency of the T1 wave for $\beta \approx n_1$ can be approximated by

$$f_{t}^{(1)} \approx \frac{1 + \cos \theta_0}{1 - n\beta} \frac{1}{f_p} \frac{1 - n\beta}{2 \left(1 + \cos \theta_0\right)^2}.$$  

Due to a large real part of the frequency, the role of collisions is rather small. The amplitude for the T1 wave is

$$E_{f}^{(1)} \approx \frac{1 - n\beta}{2 \left(1 + \cos \theta_0\right)^2} f_p^2.$$  

Similar to the case without collisions, the amplitude of the T1 wave is small.

The complex frequencies for the T2 and T3 waves are shown in figure 3. When plotting, we fixed the ratio $\nu/\omega_p$ so that $f_p = \omega_p/\omega_0$, for a fixed density and collision rate of the plasma. When the collision rate is rather small, $\nu/\omega_p = 0.5$ in figure 3, the real part of $f_{t}^{(2)}$ increases monotonically starting from 1 as in the absence of collisions, see equation (15). Its imaginary part also increases but remains smaller than the real part. The frequency $f_{t}^{(3)}$ of the free streaming mode acquires both real and imaginary parts as compared to $f_{t}^{(2)} = 0$ in the absence of collisions. The imaginary part of $f_{t}^{(3)}$ is larger than its real part.

When the collision rate becomes larger, $\nu/\omega_p = 1$ in figure 3, the T2 and T3 modes become strongly mixed as evidenced by the peculiar behavior of $f_{t}^{(2,3)}$ around $f_p = 2$. The real parts of $f_{t}^{(2)}$ and $f_{t}^{(3)}$ become attracted to each other and eventually cross each other with further increase of $\nu/\omega_p$. The values of $f_{t}^{(2)}$ and $f_{t}^{(3)}$ when the real parts of $f_{t}^{(2)}$ and $f_{t}^{(3)}$ intersect depend slightly on the angle of incidence, see frames (a) and (c) in figure 3.

At large collision rates, $\nu/\omega_p = 4$ in figure 3, the T2 and T3 waves have properties that are drastically different from the case of weak collisions. The real part of $f_{t}^{(2)}$ becomes close to 1 and shows a weak tendency to decrease with increasing $f_p$. The imaginary part of $f_{t}^{(2)}$ increases with $f_p$. The T2 wave is, therefore, affected only weakly by the highly collisional plasma. The frequency of T3, has a very large imaginary part which makes the mode confined to a very narrow region just behind the front.

The properties of the T2 and T3 waves are illustrated further by the dependence of the x projections of their group velocities on $f_p$, see figure 4. For low collision rate, $\nu/\omega_p = 0.5$, the T2 wave reverses its propagation direction at some value of $f_p$. This reversal is accompanied by a monotonic increase of its frequency, see figure 3. The T3 wave acquires some small group velocity in the presence of collisions and can also reverse its propagation direction, although in the opposite manner as compared to that of the T2 wave. At large collision rates,
\( \nu/\omega_p = 4 \), however, the behavior of the modes is drastically different. The T2 wave propagates always in the +\( x \) direction and the magnitude of its group velocity decreases monotonically to zero with increasing \( f_p \). The frequency of this mode is very close to 1, see figure 3. The T3 wave propagates in the −\( x \) direction and the magnitude of its group velocity increases from 0 to that of the front with increasing \( f_p \). This behavior is consistent with the large value of the imaginary part of \( f_t^{(3)} \) and the localization of the mode behind the front.

The amplitudes of the T2 and T3 waves are shown in figure 5. At low collision rates, \( \nu/\omega_p = 0.5 \), the amplitude of the T2 wave is almost equal to that of the incident wave. The presence of collisions also gives rise to a finite electric field of the free-streaming mode T3. At moderate collision rates, \( \nu/\omega_p = 1 \), the amplitudes of the T2 and T3 waves show very high peaks. However, the electric fields of the modes are directed in the opposite directions and therefore, the total electric field does not exhibit resonant behavior. At high collision rates, \( \nu/\omega_p = 4 \), the normalized amplitude of the T2 wave is very close to 1. The normalized amplitude of the T3 wave increases with \( f_p \) but remains less than 1 in the investigated range. We can, therefore, summarize the properties of the T2 and T3 waves depending on the collision rate \( \nu/\omega_p \).

At low collision rates, the T2 wave increases its normalized frequency monotonically from 1 at \( f_p \rightarrow 0 \) and simultaneously reverses its propagation angle from \( \theta_0 \) to 180°. Its amplitude is very close to that of the incident wave. The free-streaming mode T3 acquires some very small group velocity. At large collision rates, the T2 wave has approximately the same frequency as the incident wave. It propagates always away from the front and its group velocity along \( x \) goes to zero at large \( f_p \). The T3 wave becomes strongly localized near the front.

4. Application to EAS detection

A specific feature of an EAS is a very high value of the collision rate in the created plasma. Although its specific value is not known accurately, it is expected to be in the \( 10^{10}–10^{11} \) s\(^{-1} \) range. Therefore, the ratio \( \nu/\omega_p \) is somewhere around \( 10^2–10^4 \).

The frequencies of the transmitted waves calculated by solving cubic equation (5) for \( \nu/\omega_p = 10^3 \) are shown in figure 6 and the corresponding amplitudes are plotted in figure 7. The large values of the collision rate and \( nP \approx 1 \) also allow one to derive the following approximate formulas for the complex frequencies of the transmitted waves:
\begin{align}
\frac{f_1^{(1)}}{f_r} & \approx f_r - \frac{f_p f_r^2 (1 - \eta \beta)}{2 \left[ (1 + \cos \theta_0)^2 + (1 - \eta \beta)^2 f_r^2 \right]}, \\
\frac{f_1^{(2)}}{f_r} & \approx 1 + i \frac{f_p^2}{2 \left( 1 + \cos \theta_0 \right)} \frac{1}{f_r}, \tag{24}
\end{align}

Figure 6. Real (a), (c), (e) and imaginary (b), (d), (f) parts of the frequencies for the T1, T2, and T3 waves as functions of $f_p$ for $n = 1.0002$, $\beta = 1$, $\chi \eta \omega_p = 10^4$, and two incident angles: $\theta_0 = 0, 45^\circ$. In frame (a), the frequencies $\text{Re} f_1^{(1)}$ are shifted by $f_1^{(1)}(\theta_0 = 0) = -10001.00$ and $f_1^{(1)}(\theta_0 = 45^\circ) = -8536.39$. In frame (f), the two lines are indistinguishable.

Figure 7. Amplitudes for the T1, T2, and T3 waves as functions of $f_p$ for the same parameters as in figure 6.

$$\text{Amplitudes for the T1, T2, and T3 waves as functions of $f_p$ for the same parameters as in figure 6.}$$
\[ f_t^{(3)} \approx \frac{f_p^2 (1 + \cos \theta_0)}{2 \left( 1 + \cos \theta_0 \right)^2 + (1 - n\beta)^2 f_p^2} + if_p. \] (26)

The T1 wave has a very high frequency which is determined by the real part of \( f_t^{(1)} \), see figure 6(a). The frequency upshift depends rather strongly on the angle incident just like in the case of moderate collision rates, see equation (22). For example, at \( f_p \to 0 \) and \( n\beta = 1.0002 \), we obtain \( f_t^{(1)}(\theta_0 = 0) = -10001.00 \) and \( f_t^{(1)}(\theta_0 = 45^\circ) = -8536.39 \). As \( f_p \) increases, the real part of \( f_t^{(1)} \) becomes slightly more negative, i.e., its absolute value increases. Note that the density dependent part of \( f_t^{(1)} \) is basically similar to that for the T3 wave, compare frames (a) and (e) in figure 6. Although the imaginary part of \( fi^{(1)} \) is rather large, see figure 6(b), it is still much smaller than the real part. This means that the transmitted wave packet has a rather significant length behind the front.

The real part of the normalized frequency for the T2 wave is very close to 1 and the imaginary part is rather small, see figures 6(c), (d). However, the T2 wave propagates in the +x direction and, therefore, cannot be detected. The imaginary part of the normalized frequency for the T3 wave is very large, see figure 6(f). This means that the T3 wave is concentrated in a very small region behind the front and, therefore, its energy is very small.

Based on the properties of the transmitted waves, it appears that the T1 wave has the highest chances of being detected using the radar technique. Since it propagates in the -x direction it will eventually escape the plasma column (for example, after the ionizing particles lose the energy required for further ionization) and then reach the receiver. The wave should carry a detectable power. For example, for typical values of \( \theta_0 = 45^\circ \), \( n\beta = 1.0002 \), \( \nu \rho_p = 10^5 \), \( \omega_0/(2\pi) = 50 \) MHz, and \( \omega_0/(2\pi) = 1 \) MHz (\( f_p = 50 \)) we obtain \( |E_1|^2/|E_0| \approx 0.015 \), see figure 7(b). This amplitude is comparable to the amplitude of the wave reflected from a subluminal (with \( n\beta < 1 \)) ionization front creating plasma with the same values of \( \omega_0 \) and \( \nu \). The main difficulty in detecting the T1 wave, just as the wave reflected from a subluminal front, is the strong dependence of its frequency on the angle of incidence. Indeed, since \( \omega = f_t^{(1)} \approx f_r \approx (1 + \cos \theta_0)/(1 - n\beta) \) the grazing incidence gives the frequency upshift a factor of two smaller than the normal incidence. For example, for \( n\beta = 1.0002 \) the frequency changes in the interval \( |\text{Re} \omega^{(1)}|/(2\pi) \approx 5-10 \) GHz when \( \theta_0 \) varies from 90° to 0°. In general, the properties of the transmitted T1 wave are very similar to that of the reflected wave that would exist for subluminal, \( n\beta < 1 \), ionization front.

5. Conclusion

To conclude, we studied the basic aspects of the radio wave transformation on the ionization front of an EAS, which moves faster than the radio waves in the air. In this regime, the reflected wave cannot exist and there are three transmitted waves in the EAS plasma. We investigated the frequencies, propagation directions, and amplitudes of the transmitted waves in cases with and without collisions in the plasma. Simple analytical formulas for the frequencies and amplitudes of the transmitted waves in the highly relativistic case were derived.

For weakly collisional plasma, there can exist two transmitted waves that propagate toward the surface. One wave, that moves toward the surface in a rather dense plasma only, has the frequency and amplitude that are close to that of the incident wave. The other wave is highly frequency upshifted but its amplitude is much smaller than that of the incident wave. In the highly collisional plasma, such as created by an EAS, only the highly frequency upshifted wave can propagate toward the surface behind the ionization front. Despite its small amplitude, this wave can potentially be detected by a receiver on the Earth’s surface. We also showed that the properties of this wave (frequency, propagation direction, and amplitude) are similar to the reflected wave that would exist if the ionization front moved with the velocity slightly below the speed of light.

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