Detonation propagation from a cylindrical tube into a diverging cone

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Abstract
The characteristics of the propagation of a detonation from a cylindrical tube of constant cross section into a diverging cone were experimentally investigated using the smoked-foil technique for three explosive gas mixtures: C$_2$H$_2$+2.5O$_2$, 2H$_2$+O$_2$+4.5Ar, and C$_2$H$_4$+3O$_2$+0.44N$_2$. The initial pressure and the cone enlargement angle were varied as governing parameters. The results were summarized in terms of the ratio between the inner diameter of the cylindrical tube through which a detonation initially propagated and the detonation cell width $d/\lambda$ and of the cone enlargement half angle $\theta$. Four patterns of detonation propagation were observed in the diverging cone: continuous propagation, re-initiation on the cone wall, re-initiation apart from the cone wall, and failure. The obtained results were qualitatively consistent with past experimental results reported by other researchers. However, quantitatively, the obtained results were dependent on the explosive gas mixtures, particularly on the so-called critical tube diameter. Actually, when the critical values of $d/\lambda$ against detonation failure were normalized by that corresponding to $\theta = 90^\circ$, a single curve unifying all data to some degree was obtained. In addition, some characteristics of the detonation behavior in the diverging cone were explained by simple models.

Keywords: Detonation, Diverging cone, Failure, Smoked foil, Critical tube diameter

1. Introduction
The most successful application of gaseous detonations thus far can likely be considered as the detonation thermal spray (Poorman et al., 1955) (Tucker Jr., 1994). In thermal spraying, melted or highly heated high-performance materials are sprayed onto a surface with high speed to produce a thick surface coating (Tucker Jr., 1994). The technology in which gaseous detonations are repetitively generated in a combustor is sometimes called pulse-detonation technology (PDT), and a thermal-spray gun utilizing PDT was first developed in the middle of the 20th century (Poorman et al., 1955). Tucker Jr. (1994) remarked “The detonation gun coatings have some of the highest bond strengths and lowest porosities of the thermal spray coatings. They have been the benchmark against which the other coatings have been measured for years.”

Table 1 summarizes the specifications of developed detonation guns for thermal spray, including a detonation gun developed by the authors of the present study (Hiroshima University, Japan). The distinguishing characteristics of our detonation gun for thermal spray are its small size and its operability at high frequencies (Endo et al., 2016). It is expected that the small combustor results in a thin coating per cycle, thereby resulting in a smooth uniform cross-sectional structure of the coating, with the high-frequency operation compensating for the thin coats due to the small combustor. However, the small inner diameter of our detonation gun results in a short potential core of the hot-gas jet, thereby resulting in a short stand-off distance of the thermal spray, which is the distance between the exit of the thermal-spray gun and the surface to be coated. In fact, the typical stand-off distance for our thermal-spray detonation gun is only 50 mm for ceramic coatings. Sometimes, the object to be sprayed has a complicated shape, and therefore the stand-off distance of the thermal spray cannot be arbitrarily short. Accordingly, this issue of short
The stand-off distance of thermal spray is governed by the length of the potential core of the hot-gas jet. As the length of the potential core of a gas jet is scaled by the initial diameter of the gas jet (Kleinstein, 1964) (Witze, 1974), a larger inner diameter of the exit of a spraying gun is likely to be effective for longer thermal spray stand-off distances. There are two choices for this purpose: to enlarge the inner diameter of the combustor entirely, or to enlarge the inner diameter of the downstream portion of the combustor near the exit only. To keep the pulse-detonation combustor (PDC) small and operable at high frequencies, the run-up distance and time to deflagration-to-detonation transition (DDT) from the ignition in the PDC should be short. Because of this constraint, it is desirable that the inner diameter of the upstream portion of the PDC near the spark plug, where the DDT occurs, is small (Jost, 1946) (Baumann et al., 1961) (Laderman and Oppenheim, 1962) (Nettleton, 1987) (Li et al., 2006). The shape of the PDC for thermal spray thus should have a small inner diameter at the upstream portion near the spark plug, and a large inner diameter at the downstream portion near the exit. In addition, the discontinuous enlargement of the inner diameter at the middle part of a PDC forms a recirculation zone in its downstream region, causing challenges in the high-frequency operation of the PDC as the purging of the hot burned gas between the pulse-detonation cycles becomes difficult. Accordingly, the shape of the PDC for thermal spray should have a smaller-inner-diameter upstream tube and a larger-inner-diameter downstream tube connected in the middle by a diverging cone. Therefore, to design a PDC for thermal spray, the enlargement angle of the connecting diverging cone must be determined so that a detonation can pass through the diverging cone successfully.

As a result of the literature survey, it was found that the published experimental results on a detonation that initially propagates in a channel with a constant cross-sectional area and subsequently enters a diverging channel are limited, including three-dimensional cases using a diverging cone (Kogarko, 1956) (Gubin et al., 1982) (Borisov et al., 1989) (Khasainov et al., 2005) and two-dimensional cases using a pair of non-parallel plates (Strehlow and Salm, 1976) (Thomas et al., 1986). In particular, the dependence of the detonation behavior on the explosive gas species is mostly unknown quantitatively. In this study, the results of fundamental experiments are presented in which a detonation

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Table 1  Developed detonation guns for thermal spray.

| Name or developer of detonation gun | Length [m] | Inner diameter [mm] | Operating Frequency [Hz] | Reference |
|------------------------------------|------------|---------------------|--------------------------|-----------|
| D-Gun                              | 1          | 25                  | 4-8                      | Gill (1990), Tucker Jr. (1994), Knotek (2001) |
| Institute of Materials Science Problems, Academy of Science of the Ukrainian S.S.R. | 2          | 22                  | 1-6                      | Kharlamov (1987) |
| ADK-1M                             | 1.85       | 16-25               | 1-5                      | Kharlamov (1987) |
| KORUND                             | 1.2        | 26                  | 2-4                      | Saravanan et al. (2000) |
| ob                                 | 1.25       | 25                  | 4-6                      | Du et al. (2005) |
| Shock Wave Laboratory, RWTH Aachen University, Germany | 2          | 25                  | 5                        | Henkes and Olivier (2014) |
| Perun P                            | 0.66 or 1.1| 21                  | 3.3 or 6.6               | Niemi et al. (1994) |
| HFPD                               | 0.25-0.9   | 15-20               | 45-75                    | Higuera et al. (2002), Hidalgo et al. (2006), Parco et al. (2008), Mayoral et al. (2008), Mubarok et al. (2013) |
| Hiroshima University, Japan        | 0.35-0.65  | 10                  | 150                      | Endo et al. (2016) |
initially propagates in a cylindrical tube with a constant cross-sectional area and subsequently enters a diverging cone. Either C$_2$H$_2$+2.5O$_2$, 2H$_2$+O$_2$+4.5Ar, or C$_2$H$_4$+3O$_2$+0.44N$_2$ was used as the explosive gas mixture. The behavior of the detonation in the diverging cone was investigated using a smoked plate similar to Khasainov et al. (2005). C$_2$H$_2$+2.5O$_2$ was used to confirm the validity of our experimental method by comparing our results with those in Khasainov et al. (2005). The other explosive gas mixtures were chosen for their use in detonation applications and the non-dimensional activation energy $E_a/(RT_{\text{CN}})$, where $E_a$ is the effective activation energy of the detonative chemical reaction of the explosive gas mixture, $R$ is the specific gas constant for the unburned explosive gas mixture, and $T_{\text{CN}}$ is the temperature of the unburned explosive gas mixture just behind the leading shock wave of the Chapman–Jouguet (CJ) detonation. The non-dimensional activation energy $E_a/(RT_{\text{CN}})$ gives the relative sensitivity of the chemical-reaction rate to the relative perturbation of the unburned-gas temperature just behind the leading shock wave of a perturbed detonation. Therefore, the behavior of a perturbed detonation is largely influenced by $E_a/(RT_{\text{CN}})$. The values of $E_a/(RT_{\text{CN}})$ are 4.8–6.2 for C$_2$H$_2$+2.5O$_2$ (Ng et al., 2005) (Manzhalei, 1977), 4.6–5.0 for 2H$_2$+O$_2$+4.5Ar (Schultz and Shepherd, 2000), and 8.0–8.6 for C$_2$H$_4$+3O$_2$+0.44N$_2$ (Schultz and Shepherd, 2000). 2H$_2$+O$_2$+4.5Ar was chosen as it is a gas species with a relatively small $E_a/(RT_{\text{CN}})$, and C$_2$H$_4$+3O$_2$+0.44N$_2$ was chosen for its relatively large $E_a/(RT_{\text{CN}})$.

In the following, we first describe the measurement of the detonation cell width. Next, the experiments on the behavior of a detonation in a diverging cone are described, where the initial pressure and the cone enlargement angle are varied as governing parameters. The results are summarized in terms of the ratio between the inner diameter of the detonation tube with an outer diameter of 12 mm and a thickness of 1 mm, the upstream end of which was at 15 mm from the side wall of the most downstream region of the detonation tube. For each explosive gas mixture, we measured the obtained critical values of $d/\lambda$ against the detonation failure is presented for convenience. Finally, the results are summarized.

2. Measurement of the detonation cell width

The detonation cell width $\lambda$ (Lee, 1984) of a detonation propagating steadily in a cylindrical tube was measured using the experimental setup shown in Fig. 1. The explosive gas mixture, namely C$_2$H$_2$+2.5O$_2$, 2H$_2$+O$_2$+4.5Ar, or C$_2$H$_4$+3O$_2$+0.44N$_2$, was premixed using a circulation pump. The experimental apparatus was composed of a small ignition sub-chamber, detonation tube, and dump tank. The length and inner diameter of the detonation tube were 2 m and 102.3 mm, respectively. The small ignition sub-chamber with an inner diameter of 102.3 mm and a length of 16.7 mm was separated from the detonation tube by a Mylar film with a thickness of 150 mm. Alternatively, when the detonation tube and the dump tank were filled with C$_2$H$_2$+2.5O$_2$, the ignition sub-chamber was filled with 2H$_2$+O$_2$ at 150 kPa. Alternatively, when the detonation tube and the dump tank were filled with C$_2$H$_4$+3O$_2$+0.44N$_2$, the ignition sub-chamber was filled with C$_2$H$_4$+3O$_2$ at 150 kPa. To promote the DDT in the detonation tube, we installed a Shchelkin spiral with a total length of 600 mm, a pitch of 50 mm, and a blockage ratio of 0.41, comprising a copper tube with an outer diameter of 12 mm and a thickness of 1 mm, the upstream end of which was at 15 mm from the Mylar film, where the distance between the downstream end of the Shchelkin spiral and the first ion probe was therefore 885 mm. To measure the detonation cell width, we installed a smoked stainless-steel foil with a width along the circumferential direction of 180 mm, a length along the central axis of 400 mm, and a thickness of 100 mm, on the side wall of the most downstream region of the detonation tube. For each explosive gas mixture, we measured the detonation cell width $\lambda$ at various initial pressure $p_0$. On the detonation propagation speed, the root-mean-square of $D/D_C - 1$, where $D$ is the detonation propagation speed measured by using the ion probes shown in Fig. 1 and $D_C$ is the CJ detonation speed (Gordon and McBride, 1996), was 4.2% for C$_2$H$_2$+2.5O$_2$, 1.5% for 2H$_2$+O$_2$+4.5Ar, and 5.3% for C$_2$H$_4$+3O$_2$+0.44N$_2$.

![Fig. 1. Experimental setup for the cell-width measurement.](image-url)
Figure 2 shows the detonation cell width $\lambda$ measured in the arrangement shown in Fig. 1 as a function of the initial gas pressure $p_0$. As the detonation cell width is too small to measure at relatively high pressures, an empirical formula for the measured $\lambda$ in mm as a power-law function of $p_0$ in kPa was derived from the linear fitting of $\ln \lambda$ against $\ln p_0$. The obtained empirical formulas are summarized in Table 2 and shown by lines in Fig. 2. For nondimensionalization of the inner diameter of the detonation tube $d$ by the detonation cell width $\lambda$ in the following sections, the value of $\lambda$ corresponding to the initial gas pressure $p_0$ calculated by the empirical formulas summarized in Table 2 is used.

### 3. Experiments on the detonation propagation from a cylindrical tube into a diverging cone

#### 3.1 Experimental arrangement

Figure 3(a) shows the setup for the experiments on the detonation propagation from a cylindrical tube into a diverging cone. The experimental apparatus illustrated in Fig. 3(a) was installed at the connecting portion between the detonation tube and the dump tank shown in Fig. 1. A cylindrical tube with an inner diameter of 40 mm was inserted into the detonation tube with an inner diameter of 102.3 mm (see Fig. 1), and a diverging cone was attached to the 40 mm cylindrical tube, where the diverging cone was inside the dump tank shown in Fig. 1. The enlargement half angle of the diverging cone $\theta$ was either 5°, 10°, 20°, 30°, 40°, 60°, or 90°. To investigate the characteristics of the detonation propagation inside the diverging cone, a 2-mm-thick smoked plate made of SUS304 was installed along the central axis of the 40-mm-diameter cylindrical tube and the diverging cone as shown in Fig. 3(a), where the edge of the smoked plate against which the incident detonation impacted was shaped as a knife edge for minimizing the perturbation on the detonation propagating from the cylindrical tube into the diverging cone.

The length of the diverging cone along the central axis was determined by estimating the distance $h$ along the central axis between the entrance of the diverging cone and the position at which the head of the rarefaction wave propagating from the entrance of the diverging cone reaches the central axis. The shock-rarefaction interaction for this

![Figure 2: Dependence of the detonation cell width on the initial gas pressure.](image)

![Figure 3(a): Experimental setup for the detonation propagation from a cylindrical tube into a diverging cone.](image)

![Figure 3(b): Schematic of the shock-rarefaction interaction.](image)

Table 2  Empirical formulas for $\lambda$ (in mm) as a function of the initial gas pressure $p_0$ (in kPa).

| Mixture           | Empirical formula |
|------------------|-------------------|
| $\text{C}_2\text{H}_4+2.5\text{O}_2$ | $\lambda = 13.5p_0^{0.972}$ |
| $2\text{H}_2+\text{O}_2+4.5\text{Ar}$ | $\lambda = 453p_0^{1.24}$ |
| $\text{C}_2\text{H}_4+3\text{O}_2+0.44\text{N}_2$ | $\lambda = 38.3p_0^{0.999}$ |
estimation method is shown in Fig. 3(b). The head of the rarefaction wave propagating from the entrance of the diverging cone moves in the unburned gas just behind the leading shock wave of the detonation propagating at the CJ detonation speed $D_{\text{CJ}}$ at the local sound speed $a_{\infty}$. As the flow speed of the unburned gas just behind the leading shock wave along the central axis is $u_{\infty}$, the speed $V$ at which the head of the rarefaction wave moves along the leading shock wave toward the central axis can be geometrically calculated from $\left(D_{\text{CJ}} - u_{\infty}\right)^2 + V^2 = a_{\infty}^2$. Here, $u_{\infty}$ and $a_{\infty}$ can be calculated from the formulas for an inert shock wave as $u_{\infty} = \frac{2a_0}{\gamma_0 + 1}\left(M_{\text{CJ}} - \frac{1}{M_{\text{CJ}}} \right)$ and $a_{\infty} = \frac{\sqrt{2\gamma_0 M_{\text{CJ}}^2 - (\gamma_0 - 1)} \left(\gamma_0 - 1\right) M_{\text{CJ}}^2 + 2}{(\gamma_0 + 1) M_{\text{CJ}}}$, respectively, in which $\gamma_0$ and $a_0$ are the specific-heat ratio and sound speed of the initial unburned gas, respectively, and $M_{\text{CJ}} = D_{\text{CJ}}/a_0$ is the propagation Mach number of the CJ detonation. Alternatively, the angle $\chi$ shown in Fig. 3(b), at which the line intersection between the leading shock wave and the head of the rarefaction wave approaches the central axis, can be given by $\tan \chi = (d/2)/h = V/D_{\text{CJ}}$.

From this relation, the distance $h$ along the central axis between the entrance of the diverging cone and the position at which the head of the rarefaction wave propagating from the entrance of the diverging cone reaches the central axis can be estimated by

$$h = \frac{D_{\text{CJ}}}{\sqrt{a_{\infty}^2 - (D_{\text{CJ}} - u_{\infty})^2}} \frac{d}{2}.$$  (1)

We calculated the values of $h$ for initial conditions of 25 $^\circ$C and 100 kPa for the three mixtures using parameters summarized in Table 3, and the results are $h = 1.27d$ for C$_2$H$_2$+2.5O$_2$, $h = 1.04d$ for H$_2$+O$_2$+4.5Ar, and $h = 1.25d$ for C$_2$H$_4$+3O$_2$+0.44N$_2$. From these calculations, the length of the diverging cone along the central axis was designed to be 50 mm, corresponding to 1.25$d$ as shown in Fig. 3(a). For $\theta = 90^\circ$, the length and width of the smoked plate in the diverging cone were 270 mm and 230 mm, respectively.

Figure 4 shows the detonation cell width measured by the smoked plate inserted into the 40 mm cylindrical tube shown in Fig. 3(a) just upstream of the entrance of the diverging cone. The empirical formulas summarized in Table 2 are also shown by lines in Fig. 4. As shown in Fig. 4, some agreement exists between the measured cell widths just upstream of the entrance of the diverging cone and the empirical formulas summarized in Table 2, suggesting that the disturbance of the detonation induced by the collision against the edge of the smoked plate was attenuated to some degree before entering the diverging cone. In other words, the degree of agreement between the data and the empirical formulas in Fig. 4 shows the preciseness of the present experiments.

Figure 5 compares the present results and the published results (Khasainov et al., 2005) for C$_2$H$_2$+2.5O$_2$ for the detonation propagation in the diverging cone by “GO” and “NO GO” expressions. The case of $\theta = 90^\circ$ corresponds to the critical-tube-diameter problem (Lee, 1984). For reference, the typical result for the critical-tube-diameter problem for acetylene-oxygen mixtures (Mitrofanov and Soloukhin, 1965), $d/\lambda = 13$, is shown by the broken horizontal line in Fig. 5. As shown in Fig. 5, the results obtained in the present study agree well with the published results (Khasainov et al., 2005) not only qualitatively but also quantitatively, and are consistent with the result of the critical-tube-diameter problem ($d/\lambda = 13$). Therefore, it is judged that the experimental method in the present study is adequately valid.

### 3.2 Results

The smoked-plate records obtained in the present study can be classified into four patterns. Figure 6 shows the

| Mixture       | $a_0$ [m/s] | $\gamma_0$ | $D_{\text{CJ}}$ [m/s] | $M_{\text{CJ}}$ | $a_{\infty}$ [m/s] | $u_{\infty}$ [m/s] | $E_c/(RT_{\text{CJ}})$ |
|---------------|-------------|-------------|------------------------|-----------------|---------------------|---------------------|---------------------|
| C$_2$H$_2$+2.5O$_2$ | 329.8       | 1.3294      | 2424.9                 | 7.3523          | 1026.2              | 2043.4              | 4.8–6.2             |
| 2H$_2$+O$_2$+4.5Ar | 362.8       | 1.5274      | 1808.4                 | 4.9852          | 971.76              | 1373.6              | 4.6–5.0             |
| C$_2$H$_4$+3O$_2$+0.44N$_2$ | 329.5       | 1.3453      | 2314.4                 | 7.0239          | 1004.1              | 1933.6              | 8.0–8.6             |

For these calculations, NASA CEA (Gordon and McBride, 1996) was used.
typical results of the four patterns: (1) “continuous propagation (specified by a circle hereafter),” where the cellular structure of the detonation is never lost and the detonation continues to propagate in the diverging cone (Fig. 6(a)); (2) “re-initiation on the cone wall (specified by a square hereafter),” where the detonation almost disappears after entering the diverging cone and is re-initiated later on the cone wall (Fig. 6(b)); (3) “re-initiation apart from the cone wall (specified by a triangle hereafter),” where the detonation almost disappears after entering the diverging cone and is re-initiated later apart from the cone wall (Fig. 6(c)), which is similar to the re-initiation in the critical-tube-diameter problem (Pintgen and Shepherd, 2009); and (4) “failure (specified by a cross hereafter),” where the detonation disappears after entering the diverging cone and is not re-initiated within 50 mm from the entrance of the diverging cone although the length of the smoked plate was longer than 50 mm (Fig. 6(d)).

In Fig. 5, the data represented by “GO” include the data for “continuous propagation,” “re-initiation on the cone wall,” and “re-initiation apart from the cone wall,” and the data represented by “NO GO” correspond to the data for “failure.” In addition, Fig. 7 shows the smoked-plate records for the cases of $2\text{H}_2\text{O}_2+4.5\text{Ar}$ and $\theta = 20^\circ$ at various initial pressures. The distance between the edge of the entrance of the diverging cone and the location of the detonation re-initiation on the cone wall became shorter as the detonation cell width became shorter as a tendency. Finally, when the detonation cell width is so short that the distance between the edge of the entrance of the diverging cone and the location of the detonation re-initiation on the cone wall is in the same order of magnitude as the detonation cell width, “re-initiation on the cone wall” cannot be distinguished from “continuous propagation.”

Figure 8 summarizes the experimental results obtained in the present study. The experimental conditions for “failure” can be described as $d/\lambda < (d/\lambda)_c$, where $(d/\lambda)_c$ is the critical value of $d/\lambda$ between the “GO” and “NO GO” conditions and is a function of the enlargement half angle of the diverging cone $\theta$. Furthermore, $(d/\lambda)_c$ depends on the gas species. When $\theta$ increased from $0^\circ$ to $90^\circ$, $(d/\lambda)_c$ increased initially then became almost constant when $\theta$ was approximately greater than $40^\circ$. When $\theta$ was very small, “continuous propagation” was observed. When $\theta$ increased slightly, “re-initiation on the cone wall” was observed for $d/\lambda$ slightly larger than $(d/\lambda)_c$ for $2\text{H}_2\text{O}_2+4.5\text{Ar}$ and $2\text{H}_2\text{O}_2+2.5\text{O}_2$; however, this was not the case for $\text{C}_2\text{H}_4+3\text{O}_2+0.44\text{N}_2$. When $\theta$ further increased, “re-initiation apart from the cone wall” was observed for $d/\lambda$ larger than $(d/\lambda)_c$. Finally, for $\theta$ between approximately $40^\circ$ and $90^\circ$, $(d/\lambda)_c$ was almost constant. Additionally, although the result for the case of $\text{C}_2\text{H}_4+3\text{O}_2+0.44\text{N}_2$, $\theta = 40^\circ$, and $d/\lambda = 26.0$ was judged as “re-initiation on the cone wall” as shown in Fig. 8(b), this judgement was actually very difficult because “re-initiation on the cone wall” was observed on one side of the smoked flat plate shown in Fig. 3(a) but “re-initiation apart from the cone wall” was observed on the other side of the same smoked plate. This seems to show some capricious nonlinear nature of detonations in mixtures with large $E_s/(RT_N)$ originated from uncontrolled subtle disturbances in the initial and/or boundary conditions of experiments.

### 3.3 Discussion with simple models

Figure 9(a) shows the situation when the leading shock wave of a steady detonation propagating at the CJ
detonation speed \( D_{\text{CJ}} \) is passing through the entrance of a diverging cone whose enlargement half angle is \( \theta \). At this moment, the unburned gas just behind the leading shock wave, which is specified by the subscript \( vN \), moves along the central axis at the flow speed \( u_{\text{N}} \). Meanwhile, as viewed from the unburned gas just behind the leading shock wave, the cone wall moves apart from the gas at a speed of \( u_{\text{N}} \tan \theta \).

When \( \theta \) is small, the dynamics of the unburned gas just behind the leading shock wave shown in Fig. 9(a) is treated by the piston model depicted in Fig. 9(b). In this model, a rarefaction wave is created in the unburned gas just...
behind the leading shock wave propagating toward the central axis along the leading shock wave, and the gas near the cone wall is accelerated outward to \( u_N \tan \theta \) through the rarefaction wave. As only the moment when the leading shock wave of a steady detonation is passing through the entrance of a diverging cone is considered, this rarefaction wave is treated as a planar self-similar rarefaction wave. When the pressure of the gas accelerated through the rarefaction wave outward to the speed of \( u_N \tan \theta \) is \( p_a \) and a shock wave whose pressure is \( p_a \) propagates along the cone wall. Specifically, the propagation Mach number of the shock wave \( M_s \) propagating on the cone wall at the moment when the leading shock wave of a steady detonation is passing through the entrance of a diverging cone is calculated as follows. The specific-heat ratio of the unburned gas \( \gamma_0 \) can be easily calculated from the chemical composition of the unburned gas, where the subscript 0 denotes the initial state. The propagation Mach number of the CJ detonation \( M_{CJ} \) can be obtained through chemical equilibrium software (Gordon and McBride, 1996). First, from \( \gamma_0 \) and \( M_{CJ} \), considering an inert shock wave, we can write as 

\[
\frac{u_N}{a_N} = (M_{CJ}^2 - 1) \sqrt{\left(\frac{\gamma_0 M_{CJ}^2 - \frac{\gamma_0 - 1}{2}}{\frac{\gamma_0 - 1}{2} M_{CJ}^2 + 1}\right)} \quad \text{and} \quad \frac{p_{\infty}}{p_0} = \frac{2 \gamma_0 M_{CJ}^2 - (\gamma_0 - 1)}{\gamma_0 + 1}.
\]

Next, considering a planar self-similar rarefaction wave, we can write as 

\[
\frac{P_a}{p_0} = \left(1 - \frac{\gamma_0 - 1}{2} \frac{u_N}{a_N} \tan \theta \right)^{\frac{2 \gamma_0}{1 - \gamma_0}} \frac{P_{\infty}}{p_0}.
\]

Finally, considering an inert shock wave, we can write as 

\[
M_s = \sqrt{\frac{\gamma_0 + 1}{2 \gamma_0} \left(\frac{\gamma_0}{\gamma_0 + 1} \frac{P_{\infty}}{p_0} + \frac{\gamma_0 - 1}{2} \frac{u_N}{a_N} \tan \theta \right)^2}.
\]

When \( \theta \) is large, for example \( \theta = 90^\circ \), the dynamics of the unburned gas just behind the leading shock wave shown in Fig. 9(a) is treated by the shock-tube model depicted in Fig. 9(c). In this model, the unburned gas just behind the leading shock wave is treated as the high-pressure gas in the shock tube, and the unburned gas in the initial conditions is treated as the low-pressure gas in the shock tube. Further, the shock wave created in the low-pressure gas
in this shock tube is considered as the shock wave propagating on the cone wall at the moment when the leading shock wave of a steady detonation is passing through the entrance of a diverging cone. Specifically, the propagation Mach number of the shock wave \( M_s = D_w/a_w \) propagating on the cone wall when the leading shock wave of a steady detonation is passing through the entrance of a diverging cone is calculated as follows. First, \( \gamma_0 \), \( M_{C1} \), and \( \frac{\rho_{C1}}{\rho_0} \) are obtained similarly to the piston model. Next, from \( \gamma_0 \) and \( M_{C1} \), considering an inert shock wave, we can write as
\[
a_0 = a_{\infty} \frac{(\gamma_0 + 1)M_{C1}}{\sqrt{2\gamma_0 M_{C1}^{-2} - (\gamma_0 - 1)}}. \]
Finally, for the obtained values of \( \frac{\rho_{C1}}{\rho_0} \) and \( a_0/a_{\infty} \), the value of \( M_s \) is solved iteratively to satisfy the shock tube equation
\[
\frac{\rho_{C1}}{\rho_0} = \frac{2\gamma_0 M_s^{-2} - (\gamma_0 - 1)}{\gamma_0 + 1} \left[ 1 - \frac{a_0}{a_{\infty}} \left( \frac{M_s - 1}{M_s} \right) \right]^{\frac{\gamma_0 - 2}{\gamma_0 - 1}}.
\]

Using the parameters shown in Table 3, the calculations for the above models were carried out, and their results are summarized in Fig. 10. In Fig. 10, the calculation results for the diffracted shock waves by the CCW theory (Whitham, 1974) are also plotted just for reference although the CCW theory cannot be applied, in principle, to such a case that the side wall of a channel changes its direction discontinuously. The Mach number of the diffracted shock wave \( M_s \) by the CCW theory was obtained as the solution of the equation \(\lambda(M_1) + \delta = \lambda(M_{C1})\), where \(\lambda(M) = \int_0^M \frac{\lambda(M)}{M' - 1} dM\), \(\lambda(M) = \left(1 + \frac{2}{\gamma + 1} \mu^2 \right) \left(1 + 2\mu + \frac{1}{M^2} \right)\), and \(\mu^2 = \frac{(\gamma - 1)M^2 + 2}{2\gamma M^2 - (\gamma - 1)}\). The propagation Mach number \(M_s = 0.8M_{C1}\) is approximately the lower limit of marginal detonations with enlarged cells (Strehlow and Crooker, 1974). Therefore, the results shown in Fig. 10 suggest that \(\theta < 10^\circ\) is the condition under which “continuous propagation” can be observed. This nearly agrees with the experimental results shown in Fig. 8 for \(2H_2+O_2+4.5Ar\) and \(C_2H_2+2.5O_2\). The propagation Mach number \(M_s = 0.5M_{C1}\) is approximately the shock Mach number for which autoignition occurs usually in the vicinity of the side wall and the onset of detonation may be triggered (Lee, 2008). Therefore, the results shown in Fig. 10 suggest that “re-initiation on the cone wall” is observable approximately for \(10^\circ < \theta < 30^\circ\) for \(2H_2+O_2+4.5Ar\), and \(10^\circ < \theta < 25^\circ\) for \(C_2H_2+3O_2+0.44N_2\) and \(C_2H_2+2.5O_2\). This is consistent with the experimental results for \(2H_2+O_2+4.5Ar\) and \(C_2H_2+2.5O_2\). However, for \(C_2H_2+3O_2+0.44N_2\), “re-initiation on the cone wall” was rarely observed, and even for \(10^\circ < \theta < 25^\circ\), “continuous propagation” was observed when the detonation was able to propagate in the diverging cone. This issue will be discussed later. If \(\theta\) is sufficiently large that \(M_s\) calculated by the shock-tube model is significantly larger than \(M_s\) calculated by the piston model, the influence of the cone wall on the propagation characteristics of the detonation in the diverging cone is negligible and the diverging-cone problem treated in the present study is equivalent to the critical-tube-diameter problem. This is consistent with the experimental results that \((d/d\lambda)_{\infty}\) is nearly constant for \(\theta\) between approximately \(40^\circ\) and \(90^\circ\).

As shown in Fig. 8, the experimental results for \(C_2H_2+3O_2+0.44N_2\) are qualitatively different from those for \(2H_2+O_2+4.5Ar\) and \(C_2H_2+2.5O_2\) for \(10^\circ < \theta < 20^\circ\). This is likely because the non-dimensionalization activation energy \(E_a(RT_{\infty})\) for \(C_2H_2+3O_2+0.44N_2\) is larger than for \(2H_2+O_2+4.5Ar\) and \(C_2H_2+2.5O_2\). That is, for \(C_2H_2+3O_2+0.44N_2\), the chemical reaction is sensitively activated by the insignificant temperature increase due to the collision of the transverse waves against the cone wall, and thereby the cellular structure of the detonation is persistently maintained due to the micro-explosions induced by the sensitively activated chemical reaction. The transverse waves in the cellular structure of a detonation propagate in the unburned gas just behind the leading shock wave at speeds slightly higher than the sound speed (Strehlow, 1970). Because the transverse wave propagates almost at the sound speed in the coordinate system moving with the gas and the gas behind the leading shock wave is accelerated through the rarefaction wave toward the diverging cone wall, the transverse wave inevitably collides with the cone wall irrespective of the value of \(\theta\). However, the gas in the vicinity of the cone wall is isentropically cooled through the rarefaction wave, and therefore the temperature after the collision of the transverse wave against the cone wall is lower when \(\theta\) is larger. Considering the Riemann invariant, the sound speed of the gas in the vicinity of the cone wall \(a_w\) is written as
\[
a_w = a_{\infty} \left(1 - \frac{\gamma_0 - 1 u_{\infty} \tan \theta}{2a_{\infty}}\right).
\]
Therefore, the condition \(\frac{\gamma_0 - 1}{2} \leq \frac{a_{\infty}}{u_{\infty} \tan \theta}\) is roughly required for the temperature increase and thereby micro-explosion due to the collision of the transverse wave against the cone wall. Because \(\gamma_0 \approx 1.4\) in the present experiments, the above condition can be written as \(0.2 \leq \frac{a_{\infty}}{u_{\infty} \tan \theta}\). We found that
\[
\frac{a_{\infty}}{u_{\infty} \tan \theta} = \sqrt{\left( \frac{\gamma_0 M_{e_i} - \frac{\gamma_0 - 1}{2} M_{e_i}^2 + 1}{(M_{e_i}^2 - 1) \tan \theta} \right)}
\]
and its dependence on \( \theta \) is shown in Fig. 11. For \( 10^\circ \leq \theta \leq 20^\circ \), the condition \( 0.2 \ll \frac{a_{\infty}}{u_{\infty} \tan \theta} \) is satisfied; therefore, the micro-explosion by the collision of the transverse wave against the cone wall and the maintenance of the detonation are expected if the non-dimensional activation energy \( E_a/(RT_{e_i}) \) is large enough, in addition \( E_a/(RT_{e_i}) \) of \( C_2H_4+3O_2+0.44N_2 \) is larger than the other two gases. This is a crude estimation; however, the result is consistent with the experimental results for \( 10^\circ \leq \theta \leq 20^\circ \).

3.4 A curve unifying the data

Although \( (d/\lambda)_\theta \) as a function of \( \theta \) depends on the gas species, it is convenient to express its general behavior by a simple function if possible. For this purpose, we plotted the ratio of \( (d/\lambda)_\theta \) to that corresponding to \( \theta = 90^\circ \) for each gas species in Fig. 12. For these plots, we used \( (d/\lambda)_{\theta=90^\circ} = 19.4 \pm 0.8 \) for \( 2H_2+O_2+4.5Ar \), \( (d/\lambda)_{\theta=90^\circ} = 22.9 \pm 2.1 \) for \( C_2H_4+3O_2+0.44N_2 \), and \( (d/\lambda)_{\theta=90^\circ} = 14.8 \pm 2.9 \) for \( C_2H_2+2.5O_2 \). On the error and representative value of \( (d/\lambda)_{\theta=90^\circ} \), the maximum value of \( d/\lambda \) at which the detonation propagation was failed and the minimum value of \( d/\lambda \) at which the detonation propagation was not failed are the two ends of the error, and the representative value is their average. An error bar in Fig. 12 was evaluated from both of the errors of \( (d/\lambda)_{\theta=90^\circ} \) and \( (d/\lambda)_{\theta=90^\circ} \). The curve depicted in Fig. 12 shows a fitting function based on the sigmoid function:

\[
\frac{(d/\lambda)_{\theta=90^\circ}}{(d/\lambda)_{\theta=90^\circ}} = \frac{1}{1 + \exp \left( \frac{(\theta_{\text{hit}} - \theta)/\alpha}{\lambda} \right)}
\]
where \( \theta_{\text{hit}} \) and \( \alpha \) are the parameters to fit this function to the experimental results of \( (d/\lambda)_{\theta} \). It should be noted that this functional form was chosen not by any physical consideration but by its shape only. As shown in Fig. 12, using the values of \( \theta_{\text{hit}} = 22.13^\circ \) and \( \alpha = 7.6361^\circ \), the fitting function unifies the data to some degree. Therefore, this function can be used to predict \( (d/\lambda)_{\theta=90^\circ} \) from \( \theta \) and \( (d/\lambda)_{\theta=90^\circ} \), which is the so-called critical tube diameter.

4. Conclusion

The characteristics of a detonation propagating from a cylindrical tube with a constant cross section into a diverging cone were experimentally studied using three explosive gas mixtures: \( C_2H_2+2.5O_2 \), \( 2H_2+O_2+4.5Ar \), and \( C_2H_4+3O_2+0.44N_2 \). We observed four detonation propagation patterns in the diverging cone: continuous propagation, re-initiation on the cone wall, re-initiation apart from the cone wall, and failure. We proposed simple models to explain the observed propagation behavior. Furthermore, we found that the behavior of the detonations inside the diverging cone essentially depended on the gas species, where the non-dimensional activation energy \( E_a/(RT_{e_i}) \) was likely the key
parameter. In addition, we presented a simple function unifying the experimental results on the critical values of $d/\lambda$ against detonation failure in terms of the so-called critical tube diameter ($d/\lambda_{cr}$). Regarding the design of the combustor of a pulse-detonation thermal-spraying gun, we can use this function because any “GO” case can be allowed for it.

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