POSSIBLE LARGE DIRECT CP VIOLATIONS
IN CHARMLESS B-DECAYS

– Summary Report on the PQCD method –

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Abstract

We discuss the perturbative QCD approach for the exclusive two body B-meson decays to light mesons. We briefly review its ingredients and some important theoretical issues on factorization approach. We show numerical results which are compatible with present experimental data for the charmless B-meson decays. Speciailly we predict the possibility of large direct CP violation effects in $B^0 \to \pi^+ \pi^- \ (23 \pm 7\%)$ and $B^0 \to K^+ \pi^- \ (17 \pm 5\%)$. In the last section we investigate two methods to determine the weak phases $\phi_2$ and $\phi_3$ from $B \to \pi\pi; K\pi$ processes. We obtain bounds on $\phi_2$ and $\phi_3$ from present experimental measurements.

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Possible Large Direct CP Violations in Charmless B-meson Decays

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Abstract. We discuss the perturbative QCD approach for the exclusive two body B-meson decays to light mesons. We briefly review its ingredients and some important theoretical issues on factorization approach. We show numerical results which are compatible with present experimental data for the charmless B-meson decays. Specially we predict the possibility of large direct CP violation effects in $B^0 \rightarrow \pi^+\pi^-$ (23 7%) and $B^0 \rightarrow K^+\pi^-$ (17 5%). In the last section we investigate two methods to determine the weak phases $\phi_2$ and $\phi_3$ from $B \rightarrow \pi\pi; K\pi$ processes. We obtain bounds on $\phi_2$ and $\phi_3$ from present experimental measurements.

INTRODUCTION

The aim of the study on weak decay in B-meson is two folds: (1) To determine precisely the elements of Cabibbo-Kobayashi-Maskawa (CKM) matrix[1, 2] and to explore the origin of CP-violation at low energy scale, (2) To understand strong interaction physics related to the confinement of quarks and gluons within hadrons.

Both tasks complement each other. An understanding of the connection between quarks and hadron properties is a necessary prerequisite for a precise determination of CKM matrix elements and CP-violating Kobayashi-Maskawa(KM) phase[2].

The theoretical description of hadronic weak decays is difficult since nonperturbative QCD interaction is involved. This makes it difficult to seek the origin of CP violation at asymmetric B-factories. In the case of B-meson decays into two light mesons, the factorization approximation [3, 4, 5, 6] offer some understanding of branching ratios. In the factorization approximation, it is argued that, because the final-state mesons are moving so fast that it is difficult to exchange gluons. So, soft final-state interactions can be neglected(color-transparancy argument[7, 8]), and we can express the amplitude in terms of product of decay constants and transition form factors. These amplitudes are real. It predicts vanishing CP asymmetries. In this approach, we can not calculate non-factorizable contributions and annihilation contributions.

Recently two different QCD approaches beyond naive and general factorization assumption were proposed: (1) QCD-factorization in heavy quark limit [9, 10] in which non-factorizable terms and $a_t$ are calculable in some cases. (2) PQCD approach [11, 12, 13] including the resummation effects of the transverse momentum carried by
partons inside meson. In this talk, we discuss some important theoretical issues in the PQCD factorization and numerical results for charmless B-decays.

**INGREDIENTS OF PQCD**

**Factorization in PQCD:** The idea of perturbative QCD is as follows: When heavy B-meson decays into two light mesons, it can be shown that the hard process is dominant. A hard gluon exchange is needed to boost the spectator quark (which is almost at rest) to large momentum so that it can pair up with the fast moving quark to form a meson. Also, it can be shown that the final-state interaction, if any, is calculable, i.e. soft gluon exchanges between final state hadrons are negligible.

So the process is dominated by one hard gluon exchanged between spectator quark and quarks involved in the weak decay. It can be shown that all possible diagrams, contributing to the decay amplitude, can be cast into a convolution of this hard amplitude and meson wave functions.

Let’s start with the lowest-order diagram for $B \rightarrow K \pi$. There are diagrams which have infrared divergences. It can be shown that divergent parts can be absorbed into the light-cone wave functions. Their finite pieces are absorbed into the hard part. Then in a natural way we can factorize the amplitude into two pieces: $G \ H(Q; \mu) \ \Phi(n; \mu)$ where $H$ contains the hard part of the dynamics and is calculable using perturbation theory. $\Phi$ represents a product of wave functions which contains all non-perturbative dynamics.

Based on the perturbative QCD formalism developed by Brodsky and Lepage [15], and Botts and Sterman [16], three scale factorization theorem can be proven[14] inclusion of the transverse momentum components which was carried by partons inside meson.

We have three different scales: electroweak scale: $M_W$, hard interaction scale: $t \ \mathcal{O}(\bar{\Lambda} m_b)$, and the factorization scale: $1/b$ where $b$ is the conjugate variable of parton transverse momenta. The dynamics below $1/b$ is completely non-perturbative and can be parameterized into meson wave functions which is universal and process independent. In our analysis we use the results of light-cone distribution amplitudes (LCDAs) by Ball [17, 18] with light-cone sum rule.

The amplitude in PQCD is expressed as

$$A(C \ \psi) \ H(\psi) \ \Phi(\nu) \ \exp \ s(P; b) \frac{d \mu}{\mu} \frac{\gamma_q(\alpha_s \ \mu)}{1+b} (1)$$

where $C(\psi)$ are Wilson coefficients, $\Phi(\nu)$ are meson LCDAs and variable $t$ is the factorized scale in hard part.

**Sudakov Suppression Effects:** There are set of diagrams which contain powers of double logarithms $\ln^2(P; b)$. They come from the overlap of collinear and soft divergence in radiative corrections to meson wave functions, where $P$ is the dominant light-cone component of a meson momentum.

Fortunately they can be summed. The summation of these double logarithms leads to a Sudakov form factor $\exp [ s(P; b)]$ in Eq.(1), which suppresses the long distance contributions in the large $b$ region, and vanishes as $b > 1=\Lambda_{QCD}$. 

The Sudakov factor can be understood as follows: Even a single gluon emission does not allow the formation of exclusive final state. So, the exclusive two body decays are proportional to the probability that no gluon is emitted during the hard process. The Sudakov factor leads to this probability. When two quarks are far apart (i.e. large b, thus small $k_\perp$), their colors are no longer shielded. So, when quarks undergo hard scattering, they can not help but emit soft gluons. Since Sudakov factor suppresses small $k_\perp$ region, $k_\perp^2$ flowing into the hard amplitudes becomes large:

$$k_\perp^2 \sim O(\bar{\Lambda} M_B)$$  

and the singularities are removed.

In earlier analysis, $k_\perp$ and the Sudakov factor have been neglected and it was found that the amplitude is infrared singular. It is clear that such naive analysis is in error.

Thanks to the Sudakov effect, all contributions to the $B \to \pi$ form factor come from the region with $\alpha_s = \pi < 0$ [12] as shown in Figure 2. It indicates that our PQCD results are well within the perturbative region.

**Threshold Resummation:** The other double logarithm is $\alpha_s \ln^2 (1/x)$ from the end point region of the momentum fraction $x$ [19]. This double logarithm is generated by the
corrections of the hard part in Figure 2. This double logarithm can be factored out of the hard amplitude systematically, and its resummation introduces a Sudakov factor $S_t(t) = 1.78 \ln (t / \mu^2)$ with $\mu = 0.3$ into PQCD factorization formula. The Sudakov factor from threshold resummation is universal, independent of flavors of internal quarks, twists and topologies of hard amplitudes, and decay modes.

Threshold resummation[19] and $k_\gamma$ resummation [16, 20, 21] arise from different subprocesses in PQCD factorization and suppresses the end-point contributions, making PQCD evaluation of exclusive $B$ meson decays reliable. We point out that these resummation effects are crucial. Without these resummation effects, the PQCD predictions for the $B \to K$ form factors are infrared divergent. The $k_\gamma$ resummation renders the amplitudes finite, and suppresses two-parton twist-3 contributions to reasonable values.

**Power Counting Rule in PQCD:** The power behaviors of various topologies of diagrams for two-body nonleptonic $B$ meson decays with the Sudakov effects taken into account has been discussed in details in [22]. The relative importance is summarized below:

\[
\text{emission : annihilation : nonfactorizable} = 1 : \frac{2m_0}{M_B} : \frac{\tilde{\Lambda}}{M_B};
\]

with $m_0$ being the chiral symmetry breaking scale. The scale $m_0$ appears because the annihilation contributions are dominated by those from the $(V-A)(V+A)$ penguin operators, which survive under helicity suppression. In the heavy quark limit the annihilation and nonfactorizable amplitudes are indeed power-suppressed compared to the factorizable emission ones. Therefore, the PQCD formalism for two-body charmless nonleptonic $B$ meson decays coincides with the factorization approach as $M_B \to \infty$.

### TABLE 1. Amplitudes for the $B^0 \to \pi^+ \pi^-$ decay

| Amplitudes | twist-2 contribution | Twist-3 contribution | Total |
|------------|----------------------|----------------------|-------|
| $Re (f_F^T)$ | 3.44 $\times$ 10^2 | 5.00 $\times$ 10^2 | 8.44 $\times$ 10^2 |
| $Im (f_F^T)$ | -1.26 $\times$ 10^3 | -4.76 $\times$ 10^3 | -6.02 $\times$ 10^3 |
| $Re (f_F^P)$ | 2.52 $\times$ 10^6 | -3.30 $\times$ 10^4 | -3.33 $\times$ 10^4 |
| $Im (f_F^P)$ | 8.72 $\times$ 10^7 | 3.81 $\times$ 10^3 | 3.81 $\times$ 10^3 |
| $Re (M^T)$ | 7.26 $\times$ 10^4 | -1.39 $\times$ 10^6 | -7.25 $\times$ 10^4 |
| $Im (M^T)$ | -1.62 $\times$ 10^3 | -2.91 $\times$ 10^4 | 1.53 $\times$ 10^3 |
| $Re (M^P)$ | -1.67 $\times$ 10^5 | -1.67 $\times$ 10^7 | 1.66 $\times$ 10^5 |
| $Im (M^P)$ | -3.52 $\times$ 10^5 | 6.56 $\times$ 10^6 | -2.87 $\times$ 10^5 |
| $Re (M^\mu)$ | -7.37 $\times$ 10^5 | 2.50 $\times$ 10^6 | -7.42 $\times$ 10^5 |
| $Im (M^\mu)$ | -3.43 $\times$ 10^5 | -2.04 $\times$ 10^5 | -5.47 $\times$ 10^5 |

we adopted $\phi_3 = 80^0$, $R_b = \sqrt{\rho^2 + \eta^2} = 0.38$, $m_0^B = 1.4 GeV$ and $\omega_B = 0.40 GeV$. 

$\bar{\Lambda}$ denotes the chiral symmetry breaking scale.
However, for the physical value $M_B \approx 5$ GeV, the annihilation contributions are essential. In Table 1 and 2 we can easily check the relative size of the different topology in Eq.(3) by the penguin contribution for W-emission ($f_\pi F^P$), annihilation($f_B F_P$) and non-factorizable($M_P$) contributions, which is shown in Figure 4. Specially we show the relative size of the different twisted light-cone-distribution-amplitudes (LCDAs) for each topology. Actually twist-3 contributions in larger than twist-2 contributions.

Note that all the above topologies are of the same order in $\alpha_s$ in PQCD. The nonfactorizable amplitudes are down by a power of $1=m_b$, because of the cancellation between a pair of nonfactorizable diagrams, though each of them is of the same power as the factorizable one. I emphasize that it is more appropriate to include the nonfactorizable contributions in a complete formalism. The factorizable internal-$W$ emission contributions are strongly suppressed by the vanishing Wilson coefficient $a_2$ in the $B \to \psi K$ decay.
decays [23], so that nonfactorizable contributions become dominant [24]. In the $B \rightarrow D\pi$ decays, there is no soft cancellation between a pair of nonfactorizable diagrams, and nonfactorizable contributions are significant [23, 25].

In QCDF the factorizable and nonfactorizable amplitudes are of the same power in $1=m_b$, but the latter is of next-to-leading order in $\alpha_s$ compared to the former. Hence, QCDF approaches FA in the heavy quark limit in the sense of $\alpha_s \gg 0$. Briefly speaking, QCDF and PQCD have different counting rules both in $\alpha_s$ and in $1=m_b$. The former approaches FA logarithmically ($\alpha_s \approx 1=\ln m_b \gg 0$), while the latter does linearly ($1=m_b \gg 0$).

THE COMPARISON OF PQCD AND QCDF

End Point Singularity and Form Factors: If calculating the $B \rightarrow \pi^+ \pi^- \pi^0$ decay parameter $F_B^{\pi \pi}$ at large recoil using the Brodsky-Lepage formalism [15, 26], a difficulty immediately occurs. The lowest-order diagram for the hard amplitude is proportional to $1=(k_1 x_1^2)$, $x_1$ being the momentum fraction associated with the spectator quark on the $B$ meson side. If the pion distribution amplitude vanishes like $x_3$ as $x_3 \gg 0$ (in the leading-twist, i.e., twist-2 case), $F_B^{\pi \pi}$ is logarithmically divergent. If the pion distribution amplitude is a constant as $x_3 \gg 0$ (in the next-to-leading-twist, i.e., twist-3 case), $F_B^{\pi \pi}$ even becomes linearly divergent. These end-point singularities have also appeared in the evaluation of the nonfactorizable and annihilation amplitudes in QCDF mentioned above.

When we include small parton transverse momenta $k_?$, we have

$$\frac{1}{x_1 x_2^2 M_B^4 (x_3 M_B^2 + k_3^2)} \left( \frac{1}{[x_1 x_3 M_B^2 + k_1^2] [x_1 x_3 M_B^2 + k_3^2]} \right)$$

and the end-point singularity is smeared out.
In PQCD, we can calculate analytically space-like form factors for $B \rightarrow PV$ transition and also time-like form factors for the annihilation process [22, 27].

Strong Phases: While strong phases in FA and QCDF come from the Bander-Silverman-Soni (BSS) mechanism[28] and from the final state interaction (FSI), the dominant strong phase in PQCD come from the factorizable annihilation diagram[11, 12, 13] (See Figure 5). In fact, the two sources of strong phases in the FA and QCDF approaches are strongly suppressed by the charm mass threshold and by the end-point behavior of meson wave functions. So the strong phase in QCDF is almost zero without soft-annihilation contributions.

**FIGURE 5.** Different sources of strong phase: (a) Factorizable annihilation, (b) BSS mechanism and (c) Final State Interaction
**Dynamical Penguin Enhancement vs Chiral Enhancement:** As explained before, the hard scale is about 1.5 GeV. Since the RG evolution of the Wilson coefficients $C_{4,6}(\mu)$ increase drastically as $t < M_B=2$, while that of $C_{1,2}(\mu)$ remain almost constant, we can get a large enhancement effects from both wilson coefficients and matrix elements in PQCD.

In general the amplitude can be expressed as

$$Amp = \langle \bar{\psi}_{1,2} a_4 m_0^{PV}(\mu) a_6 \rangle < K\pi \vec{O} \vec{B} >$$

with the chiral factors $m_0^{PV}(\mu) = m_0^{P} = \sqrt{m_1(\mu) + m_2(\mu)}$ for pseudoscalar meson and $m_0^{V} = m_0^{V}$ for vector meson. To accommodate the $B \to K\pi$ data in the factorization and QCD-factorization approaches, one relies on the chiral enhancement by increasing the mass $m_0$ to as large values about 3 GeV at $\mu = m_b$ scale. So two methods accomodate large branching ratios of $B \to K\pi$ and it is difficult for us to distinguish two different methods in $B \to PP$ decays. However we can do it in $B \to PV$ because there is no chiral factor in LCDAs of the vector meson.

We can test whether dynamical enhancement or chiral enhancement is responsible for the large $B \to K\pi$ branching ratios by measuring the $B \to \phi K$ modes. In these modes penguin contributions dominate, such that their branching ratios are insensitive to the variation of the unitarity angle $\phi_3$. According to recent works by Cheng et al. [29], the branching ratio of $B \to \phi K$ is $(2.7) \times 10^{-6}$ including 30% annihilation contributions in QCD-factorization approach (QCDF). However PQCD predicts $(1.2) \times 10^{-6}$ [22, 34]. For $B \to \phi K$ decays, QCDF gets about $(9.1) \times 10^{-6}$ [30], but PQCD have $(15.1) \times 10^{-6}$ [35]. Because of these small branching ratios for $B \to PV$ and $VV$ decays in QCD-factorization approach, they can not globally fit the experimental data for $B \to PP, VP$ and $VV$ modes simultaneously with same sets of free parameters ($\phi_H, \phi_{H'}$) and ($\phi_{A}, \phi_{A'}$) [31].

**Fat Imaginary Penguin in Annihilation:** There is a folklore that annihilation contribution is negligible compared to W-emission one. For this non-reason annihilation contribution was not included in the general factorization approach and the first paper on QCD-factorization by Beneke et al. [9]. In fact there is a suppression effect for the operators with structure $(V A)(V A)$ because of a mechanism similar to the helicity suppression for $\pi \to \mu \nu$. However annihilation from the operators $O_{5,6,7,8}$ with the structure $(S P)(S' + P')$ via Fiertz transformation possess no such helicity suppression, and in addition, they lead to large imaginary value. The real part of factorized annihilation contribution becomes small because there is a cancellation between left-hand-side gluon exchanged one and right-hand-side gluon exchanged one as shown in Table 1. This mostly pure imaginary annihilation amplitude is a main source of large CP asymmetry in $B \to \pi^+\pi^-$ and $K^+\pi^-$. In Table 7 we summarize the CP asymmetry in $B \to K(\pi)\pi$ decays.

**NUMERICAL RESULTS**

**Branching ratios and Ratios of CP-averaged rates:** The PQCD approach allows us to
calculate the amplitudes for charmless B-meson decays in terms of ligh-cone distribution amplitudes up to twist-3. We focus on decays whose branching ratios have already been measured. We take allowed ranges of shape parameter for the B-meson wave function as $\omega_B = 0.36 \pm 0.04$ which accommodate to reasonable form factors, $F^{BK}(0) = 0.27 \pm 0.33$ and $F^{BK}(0) = 0.31 \pm 0.40$. We use values of chiral factor with $m_0^2 = 1.3$ GeV and $m_0^2 = 1.7$ GeV. It can be seen that the branching ratios for $B \to K(\pi\pi)[11, 12, 13, 32], \rho(\omega)\pi[33], K\Phi[22, 34] K(\phi[35]$ and $K(\pi[36]$ are in reasonable agreement with present experimental data (see Table 3, 4, 5 and 6).

**TABLE 3.** Branching ratios of $B \to \pi\pi K\pi$ and $K\bar{K}$ decays with $\phi_3 = 80^0$, $R_b = \rho^2 + \eta^2 = 0.38$. Here we adopted $m_0^2 = 1.5$ GeV and $m_0^2 = 1.7$ GeV. Unit is $10^{-6}$. (07/2002 data)

| Decay Channel | CLEO  | BELLE  | BABAR |
|---------------|-------|--------|-------|
| $\pi^+ \pi$   | $4.3^{+1.5}_{-2.2}$ | $0.5$  | $5.4$  | $1.2$  | $0.5$  | $4.7$  | $0.6$  | $0.2$  | $7.0^{+2.0}_{-3.3}$ |
| $\pi^+ \pi^0$ | $4.5^{+4.2}_{-2.6}$  | $1.5$  | $7.4$  | $2.3$  | $0.9$  | $5.5^{+1.0}_{-0.9}$  | $0.6$  | $0.3$  | $3.5^{+1.0}_{-0.9}$  |
| $\pi^0 \pi^0$ | $< 5.2$  | $< 6.4$  | $< 3.4$  | $0.5$  | $0.4$  | $0.5$  | $0.4$  |

| Decay Channel | CLEO  | BELLE  | BABAR |
|---------------|-------|--------|-------|
| $K^+ \pi$     | $17.2^{+2.5}_{-2.4}$ | $1.2$  | $22.5$ | $1.9$  | $18$  | $17.9^{+0.9}_{-0.9}$  | $0.7$  | $15.5^{+3.4}_{-2.4}$ |
| $K^0 \pi$     | $18.2^{+4.9}_{-4.8}$ | $1.6$  | $19.4$ | $3.4$  | $1.6$  | $17.5^{+1.8}_{-1.2}$  | $1.3$  | $16.4^{+3.3}_{-2.3}$ |
| $K^+ \pi^0$   | $11.6^{+1.0}_{-1.0}$ | $1.3$  | $13.0$ | $2.5$  | $1.3$  | $12.8^{+1.3}_{-1.2}$  | $1.0$  | $9.5^{+1.9}_{-1.5}$  |
| $K^0 \pi^0$   | $14.6^{+5.9}_{-5.2}$ | $3.3$  | $8.0$  | $3.2$  | $1.6$  | $8.2^{+3.3}_{-2.7}$  | $1.2$  | $8.6^{+0.8}_{-0.8}$  |

| Decay Channel | CLEO  | BELLE  | BABAR |
|---------------|-------|--------|-------|
| $K^+ K$       | $< 1.9$  | $< 0.9$  | $< 0.6$  | $0.6$  |
| $K^0 K$       | $< 5.4$  | $< 2.0$  | $< 1.3$  | $1.4$  |
| $K^+ K^0$     | $< 1.3$  | $< 4.1$  | $< 7.8$  | $1.4$  |

| Decay Channel | CLEO  | BELLE  | BABAR |
|---------------|-------|--------|-------|
| $\rho^0 \pi$  | $27.6^{+8.4}_{-7.5}$ | $4.2$  | $20.9^{+6.0}_{-5.9}$ | $28.9$ | $5.4$  | $4.3$  | $27.0$  |
| $\rho^0 \pi^0$| $10.4^{+3.3}_{-3.2}$ | $2.4$  | $8.9^{+2.5}_{-2.8}$  | $24$  | $8$  | $3$  | $5.4$  | $0.02$  |

| Decay Channel | CLEO  | BELLE  | BABAR |
|---------------|-------|--------|-------|
| $\omega \pi$  | $11.3^{+2.9}_{-2.9}$ | $1.4$  | $4.2^{+2.0}_{-1.8}$  | $0.5$  | $6.1^{+2.4}_{-1.8}$  | $0.7$  | $5.5$  | $0.01$  |
| $\omega \pi^0$| $11.3^{+2.9}_{-2.9}$ | $1.4$  | $4.2^{+2.0}_{-1.8}$  | $0.5$  | $6.1^{+2.4}_{-1.8}$  | $0.7$  | $5.5$  | $0.01$  |

**TABLE 4.** Branching ratios of $B \to \rho \pi$ and $\omega \pi$ decays with $\phi_2 = 75^0$, $R_b = \rho^2 + \eta^2 = 0.38$. Here we adopted $m_0^2 = 1.5$ GeV and $m_0^2 = 1.7$ GeV. Unit is $10^{-6}$. (07/2002 data)

| Decay Channel | CLEO  | BELLE  | BABAR |
|---------------|-------|--------|-------|
| $\rho \pi$    | $27.6^{+8.4}_{-7.5}$ | $4.2$  | $20.9^{+6.0}_{-5.9}$ | $28.9$ | $5.4$  | $4.3$  | $27.0$  |
| $\rho^0 \pi$  | $10.4^{+3.3}_{-3.2}$ | $2.4$  | $8.9^{+2.5}_{-2.8}$  | $24$  | $8$  | $3$  | $5.4$  | $0.02$  |

**TABLE 5.** Branching ratios of $B \to \phi K$ decays with $\phi_3 = 80^0$, $R_b = \rho^2 + \eta^2 = 0.38$. Here we adopted $m_0^2 = 1.5$ GeV and $m_0^2 = 1.7$ GeV. Unit is $10^{-6}$. (07/2002 data)

| Decay Channel | CLEO  | BELLE  | BABAR |
|---------------|-------|--------|-------|
| $\phi K$      | $5.5^{+2.4}_{-1.8}$ | $0.6$  | $11.2^{+2.2}_{-2.4}$ | $0.4$  | $7.7^{+1.6}_{-1.3}$  | $0.8$  | $10.2^{+3.9}_{-2.9}$ |
| $\phi K^0$    | $< 12.3$ | $8.9^{+2.4}_{-2.9}$  | $1.0$  | $8.4^{+1.3}_{-1.3}$  | $0.8$  | $9.5^{+2.0}_{-2.0}$  |
| $\phi K^0$    | $10.6^{+6.4}_{-4.8}$ | $1.8$  | $< 36$  | $9.7^{+4.2}_{-3.8}$  | $1.7$  | $16.0^{+5.2}_{-4.8}$ |
| $\phi K^0$    | $11.5^{+3.2}_{-2.7}$ | $1.7$  | $15.8$  | $6$  | $3.8$  | $6.4^{+2.4}_{-2.4}$  | $1.1$  | $14.9^{+3.8}_{-3.8}$ |
have a totally opposite sign to those of QCD factorization. Different sources of strong phases. Our predictions for CP asymmetry on (both magnitude and sign) is a crucial way to test factorization models which have 25% in factorized annihilation diagrams in PQCD approach, we predict large CP asymmetry (ππ decay). Because we have a large imaginary contribution from decay constant of B-meson and from light-cone distribution amplitudes, we consider rates of CP-averaged branching ratios, which is presented in Table 7.

In order to reduce theoretical uncertainties from decay constant of B-meson and from light-cone distribution amplitudes, we consider rates of CP-averaged branching ratios, which is presented in Table 7.

**Table 6.** Branching ratios of $B \rightarrow K \pi$ decays with $\phi_3 = 0^0$, $R_b = 0.38$. Here we adopted $m_b^0 = 1.2$ GeV and $m_b^0 = 0.36$ GeV. Unit is 10^-6 (07/2002 data).

| Decay Channel | CLEO | BELLE | BABAR | PQCD |
|---------------|------|-------|-------|------|
| $K^0 \pi$     | $7.6^{+3.5}_{-3.0} 1.6$ | $16.2^{+4.4}_{-3.8} 2.4$ | $15.5 3.4 1.8$ | $10.0^{+5.3}_{-4.9}$ |
| $K \pi$       | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$   |
| $K \pi^0$     | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$   |
| $K_0\pi^0$    | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$    | $16.6^{+5.2}_{-5.0}$   |

**Table 7.** Ratios of CP-averaged rates in $B \rightarrow K\pi\pi$ decays with $\phi_3 = 0^0$, $R_b = 0.38$. Here we adopted $m_b^0 = 1.5$ GeV and $m_b^0 = 1.7$ GeV.

| Quantity | Experiment | PQCD | QCDF[37] |
|----------|------------|------|----------|
| $Br(\pi^-\pi^0K^-)$/$Br(\pi^-K^-)$ | $0.25 0.04$ | $0.30 0.09$ | $0.5 1.9$ |
| $Br(\pi^-K^-)$/$Br(\pi^-K^-)$ | $1.05 0.27$ | $0.78 1.05$ | $0.9 1.4$ |
| $2Br(\omega^0K^-)$/$Br(\pi^-K^-)$ | $1.25 0.22$ | $0.77 1.60$ | $0.9 1.3$ |
| $\tau(\pi^-)Br(\pi^-K^-)$/$\tau(\pi^-)Br(\pi^-K^-)$ | $1.07 0.14$ | $0.70 1.45$ | $0.6 1.0$ |

**Table 8.** CP-asymmetry in $B \rightarrow K\pi\pi$ decays with $\phi_3 = 40^0$, $R_b = 0.38$. Here we adopted $m_b^0 = 1.3$ GeV and $m_b^0 = 1.7$ GeV.

| Direct $A_{CP}$ (%) | BELLE (07/02) | BABAR (07/02) | PQCD | QCDF[38] |
|---------------------|---------------|---------------|------|----------|
| $\pi^+K$            | $6 9^{+6.2}_{-5.0}$ | $10.2 5.0 1.6$ | $12.9 21.9$ | $5 9$ |
| $\pi^0K$            | $2 19^{+2.2}_{-2.0}$ | $9.0 9.0 1.0$ | $10.0 17.9$ | $7 9$ |
| $\pi K^0$           | $46 15^{+7.2}_{-6.0}$ | $4.7 13.9$ | $0.6 1.5$ | $1 1$ |
| $\pi^+\pi$          | $94^{+25}_{-31} 9.0$ | $30 25 4$ | $16.0 30.0$ | $6 12$ |
| $\pi^+\pi^0$        | $30 30^{+6.2}_{-5.0}$ | $3 18 2$ | $0.0$ | $0.0$ |

**CP Asymmetry of $B \rightarrow \pi\pi;K\pi$:** Because we have a large imaginary contribution from factorized annihilation diagrams in PQCD approach, we predict large CP asymmetry (25%) in $B^0 \rightarrow \pi^+\pi$ decays and about 15% CP violation effects in $B^0 \rightarrow K^+\pi$. The detail prediction is given in Table 8. The precise measurement of direct CP asymmetry (both magnitude and sign) is a crucial way to test factorization models which have different sources of strong phases. Our predictions for CP-asymmetry on $B \rightarrow K\pi\pi$ have a totally opposite sign to those of QCD factorization.
DETERMINATION $\phi_2$ AND $\phi_3$ IN $B \to \pi \pi; K \pi$

One of the most exciting aspects of present high energy physics is the exploration of CP violation in B-meson decays, allowing us to overconstrain both sides and three weak phases $\phi_1 (= \beta)$, $\phi_2 (= \alpha)$ and $\phi_3 (= \gamma)$ of the unitarity triangle of the CKM matrix and to check the possibility of New Physics.

The “gold-plated” mode $B_d \to J=\psi K_s[39]$ which allow us to determine $\phi_1$ without any hadron uncertainty, recently measured by BaBar and Belle collaborations[40]: $\phi_2 = (25 \pm 4 \Omega)^0$. There are many other interesting channels with which we may achieve this goal by determining $\phi_2$ and $\phi_3[41]$.

In this paper, we focus on the $B \to \pi^+ \pi$ and $K \pi$ processes, providing promising strategies to determine the weak phases of $\phi_2$ and $\phi_3$, by using the perturbative QCD method.

A: Extraction of $\phi_2$ from $B \to \pi^+ \pi$

Even though isospin analysis of $B \to \pi \pi$ can provide a clean way to determine $\phi_2$, it might be difficult in practice because of the small branching ratio of $B^0 \to \pi^0 \pi^0$. In reality in order to determine $\phi_2$, we can use the time-dependent rate of $B^0 \to \psi \pi$ including sizable penguin contributions. The amplitude can be written by using the c-convention notation:

$$A (B^0 \to \pi^+ \pi) = V_{ub} V_{ud}^* A_u + V_{cb} V_{cd}^* A_c + V_{tb} V_{td}^* A_t;$$

$$= V_{ub} V_{ud} (A_u - A_t) + V_{cb} V_{cd} (A_c - A_t);$$

$$= (\mathcal{F}_c e^{i\delta_c} e^{i\theta_3} + \mathcal{P}_c e^{i\delta_p})$$

(6)

Penguin term carries a different weak phase than the dominant tree amplitude, which leads to generalized form of the time-dependent asymmetry:

$$A (\psi) = \frac{\Gamma (B^0 \to \psi \pi^+ \pi)}{\Gamma (B^0 \to \pi^+ \pi)} = \frac{S_{\pi\pi}}{C_{\pi\pi}} \sin (\Delta t) + C_{\pi\pi} \cos (\Delta t)$$

(7)

where

$$C_{\pi\pi} = \frac{1}{1 + \mathcal{P}_c e^{i\delta_p}}; \quad S_{\pi\pi} = \frac{2 \text{Im} \phi_{\pi\pi}}{1 + \mathcal{P}_c e^{i\delta_p}}$$

(8)

satisfies the relation of $C_{\pi\pi}^2 + S_{\pi\pi}^2 = 1$. Here

$$\lambda_{\pi\pi} = \frac{\mathcal{F}_c}{\mathcal{P}_c + \delta_{\pi\pi}} = e^{2i\phi_2} \frac{1 + R e^{i\delta} e^{i\theta_3}}{1 + R e^{i\delta} e^{i\phi_2}}$$

(9)

with $R_c = \mathcal{P}_c = \mathcal{P}_c$ and the strong phase difference between penguin and tree amplitudes $\delta = \delta_{\pi\pi} - \delta_T$. The time-dependent asymmetry measurement provides two equations for $C_{\pi\pi}$ and $S_{\pi\pi}$ in terms of three unknown variables $R_c$, $\delta$ and $\phi_2$. 


When we define $R_{\pi\pi} = \overline{BR} (B^0 \rightarrow \pi^+ \pi^- ) = \overline{BR} (B^0 \rightarrow \pi^+ \pi^- ) j_{rec}$, where $\overline{BR}$ stands for a branching ratio averaged over $B^0$ and $\bar{B}^0$, the explicit expression for $S_{\pi\pi}$ and $C_{\pi\pi}$ are given by:

$$R_{\pi\pi} = 1 + 2R_c \cos \phi \cos \phi + R_c^2$$

$$R_{\pi\pi} S_{\pi\pi} = \sin 2\phi \phi + 2R_c \cos \phi \cos \phi \cos \delta + R_c^2 \sin 2\phi$$

$$R_{\pi\pi} C_{\pi\pi} = 2R_c \phi \phi \sin \phi$$

If we know $R_c$ and $\delta$, then $\phi_2$ can be determined by the experimental data on $C_{\pi\pi}$ versus $S_{\pi\pi}$.

Since PQCD provides $R_c = 0.23^{+0.07}_{-0.05}$ and $41^\circ < \delta < 32^\circ$, the allowed range of $\phi_2$ at present stage is determined by $55^\circ < \phi_2 < 100^\circ$ as shown in Figure 6.

According to the power counting rule in the PQCD approach [22], the factorizable annihilation contribution with large imaginary part becomes subdominant and give a negative strong phase from $i\pi \delta (\phi^2 \times M_B^2)$. Therefore we have a relatively large strong phase in contrast to QCD-factorization ($\delta = 0^\circ$) and predict large direct CP violation effect in $B^0 \rightarrow \pi^+ \pi^-$ with $A_{\text{cp}} (B^0 \rightarrow \pi^+ \pi^-) = (23 \pm 7)\%$, which will be tested by more precise experimental measurement within two years. Since the data by Belle collaboration[42] is located outside allowed physical regions, we considered only the recent BaBar measurement[43] with 90% C.L. interval taking into account the systematic errors:

$$S_{\pi\pi} = 0.02 \ 0.34 \ 0.05 \ [-0.54, +0.58]$$

**FIGURE 6.** Plot of $C_{\pi\pi}$ versus $S_{\pi\pi}$ for various values of $\phi_2$ with $\phi_1 = 25.5^\circ$, $0 < R_c < 0.30$ and $41^\circ < \delta < 32^\circ$ in the PQCD method. Here we consider the allowed experimental ranges of BaBar measurement within 90% C.L. Dark areas is allowed regions in the PQCD method for different $\phi_2$ values.
\[ C_{\pi\pi} = 0.30 \, 0.25 \, 0.04 \, [-0.72, +0.12]. \]

The central point of BaBar data corresponds to \( \phi_2 = 78^o \) in the PQCD method.

Denoting \( \Delta \phi_2 \) by the deviation of \( \phi_2 \) due to the penguin contribution, derived from Eq.9, it can be determined with known values of \( R_c \) and \( \delta \) by using the relation \( \phi_3 = 180^o - \phi_1 - \phi_2 \). In Figure 7 we show PQCD prediction on the relation \( \Delta \phi_2 \) versus \( \phi_2 \). For allowed regions of \( \phi_2 = (55, 100)^o \), we have \( \Delta \phi_2 = (8, 16)^o \). Main uncertainty comes from the uncertainty of \( J_{ub} \). The non-zero value of \( \Delta \phi_2 \) demonstrates sizable penguin contributions in \( B_0 \to \pi^+ \pi^- \) decay.

**B. Extraction of \( \phi_3 (= \gamma) \) from \( B_0 \to K^+ \pi^- \) and \( B^+ \to K^0 \pi^+ \)**

By using tree-penguin interference in \( B_0 \to K^+ \pi^- \) (\( T^0 + P^0 \)) versus \( B^+ \to K^0 \pi^+ \) (\( P^0 \)), CP-averaged \( B \to K \pi \) branching fraction may lead to non-trivial constraints on the \( \phi_3 \) angle[44]. In order to determine \( \phi_3 \), we need one more useful information on CP-violating rate differences[45]. Let’s introduce the following observables:

\[
R_K = \frac{\overline{B} R (B_0 \to K^+ \pi^-) \tau_+}{\overline{B} R (B^+ \to K^0 \pi^+) \tau_0} = 1 - 2 r_K \cos \delta \cos \phi_3 + r_K^2 \sin^2 \phi_3
\]

(13)
\[
A_0 = \frac{\Gamma(B^0 \to K^+ \pi^-) \Gamma(B^0 \to K^- \pi^+)}{\Gamma(B^0 \to K^0 \pi^+) + \Gamma(B^0 \to K^0 \pi^+)} = A_{cp}(B^0 \to K^+ \pi^-) R_K = 2 r_K sin \phi_3 sin \delta:
\] (14)

where \( r_K = \frac{\Gamma(B^0 \to P^0 \pi^0)}{\Gamma(B^0 \to P^0 \pi^0)} \) is the ratio of tree to penguin amplitudes in \( B^0 \to K^0 \pi^0 \) decays and \( \delta = \delta_T - \delta_P \) is the strong phase difference between tree and penguin amplitudes. After eliminate \( sin \delta \) in Eq.(8)-(9), we have

\[
R_K = 1 + \frac{r_K^2}{2} \left( 4 r_K^2 cos^2 \phi_3 - A_0^2 cot^2 \phi_3 \right):
\] (15)

Here we obtain \( r_K = 0.201 \) to 0.201 0.037 from the PQCD analysis[12] and \( A_0 = 0.110 \) 0.065 by combining recent BaBar measurement on CP asymmetry of \( B^0 \to K^0 \pi^0 \): 

\[
A_{cp}(B^0 \to K^+ \pi^-) = 0.105 \pm 0.65 \%
\] [43] with present world averaged value of \( R_K = 1.10 \pm 0.15 \) [46].

As shown in Figure 3, we can constrain the allowed range of \( \phi_3 \) with 1\( \sigma \) range of World Averaged \( R_K \) as follows:

For \( cos \delta > 0 \), \( \phi_3 = 0.164 \): we can exclude \( 0^\circ \) \( \phi_3 \) \( 6^\circ \) and \( 24^\circ \) \( \phi_3 \) \( 75^\circ \).

For \( cos \delta > 0 \), \( \phi_3 = 0.201 \): we can exclude \( 0^\circ \) \( \phi_3 \) \( 6^\circ \) and \( 27^\circ \) \( \phi_3 \) \( 75^\circ \).

For \( cos \delta > 0 \), \( \phi_3 = 0.238 \): we can exclude \( 0^\circ \) \( \phi_3 \) \( 6^\circ \) and \( 34^\circ \) \( \phi_3 \) \( 75^\circ \).

For \( cos \delta < 0 \), \( \phi_3 = 0.164 \): we can exclude \( 0^\circ \) \( \phi_3 \) \( 6^\circ \).

For \( cos \delta < 0 \), \( \phi_3 = 0.201 \): we can exclude \( 0^\circ \) \( \phi_3 \) \( 6^\circ \) and \( 35^\circ \) \( \phi_3 \) \( 51^\circ \).

For \( cos \delta < 0 \), \( \phi_3 = 0.238 \): we can exclude \( 0^\circ \) \( \phi_3 \) \( 6^\circ \) and \( 24^\circ \) \( \phi_3 \) \( 62^\circ \).

From the table 2, we obtain \( \delta_P = 157^\circ \), \( \delta_T = 140^\circ \) and the negative value of \( cos \delta \): \( cos \delta = 0.91 \). Therefore the maximum value of the excluded region for the \( \phi_3 \) strongly
depends on the uncertainty of $\mathcal{F}_{ab}$. When we take the central value of $r_K = 0.201$, $\phi_3$ is allowed within the ranges of $51^\circ < \phi_3 < 129^\circ$, because of the symmetric property between $R_K$ and $\cos \delta$, which is consistent with the result by the model-independent CKM-fit in the $(\rho, \eta)$ plane.

**SUMMARY AND OUTLOOK**

In this paper we have discussed ingredients of PQCD approach and some important theoretical issues with numerical results by comparing experimental data. The PQCD factorization approach provides a useful theoretical framework for a systematic analysis on non-leptonic two-body $B$-meson decays. This method explain sucessfully present experimental data upto now. Specially PQCD predicted large direct CP asymmetries in $B^0 \to \pi^+ \pi^-$; $K^+ \pi^-$ decays, which will be a crucial for distinguishing our approach from others in future precise measurement.

We discussed two methods to determine weak phases $\phi_2$ and $\phi_3$ within the PQCD approach through 1) Time-dependent asymmetries in $B^0 \to \pi^+ \pi^-$; $K^+ \pi^-$ processes via penguin-tree interference. We can get interesting bounds on $\phi_2$ and $\phi_3$ from present experimental measurements. More detail works on other methods in $B \to \pi\pi; K\pi$ [47] and $D \to \pi$ processes will appeare in elsewhere [48].

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**REFERENCES**

1. N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
2. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
3. M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C 29**, 637 (1985).
4. M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C 34**, 103 (1987).
5. A. Ali, G. Kramer and C.-D. Lu, *Phys. Rev.* **D 58**, 094009 (1998).
6. Y.-H. Chen, H.-Y. Cheng, B. Tseng and K.-C. Yang, *Phys. Rev.* **D 60**, 094014 (1999).
7. G.P. Lepage and S.J. Brodsky, Phys. Rev. **D 22**, 2157 (1980); S.J. Brodsky, hep-ph/0208158.
8. J.D. Bjorken, Nucl. Phys. (Proc. Suppl.) **B 11**, 325 (1989).
9. M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *Phys. Rev. Lett.* **83**, 1914 (1999).
10. M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, *Nucl. Phys. B* **591**, 313 (2000).
11. Y.-Y. Keum, H.-N. Li, and A.I. Sanda, *Phys. Lett. B* **504**, 6 (2001).
12. Y.-Y. Keum, H.-N. Li, and A.I. Sanda, *Phys. Rev. D* **63**, 074006 (2001).
13. Y.-Y. Keum, and H.-N. Li, *Phys. Rev. D* **63**, 054008 (2001).
14. C.H. Chang and H.-N. Li, *Phys. Rev. D* **55**, 5577 (1997).
15. see first one in ref.[5].
16. J. Botts, and G. Sterman, *Nucl. Phys.* B 225, 62 (1989).
17. P. Ball, *JHEP* 9809, 005 (1998).
18. P. Ball, *JHEP* 9901, 010 (1999).
19. H.-N. Li, hep-ph/0102013.
20. J.C. Collins, and D.E. Soper, *Nucl. Phys.* B 193, 381 (1981).
21. H.-N. Li and G. Sterman, *Nucl. Phys.* B 381, 129 (1992).
22. C.-H. Chen, Y.-Y. Keum and H.-N. Li, *Phys. Rev.* D 64, 112002 (2001).
23. T.-W. Yeh and H.-N. Li, *Phys. Rev.* D 56, 1615 (1997).
24. Y.-Y. Keum, in preparation.
25. Y.-Y. Keum, T. Kurimoto, H.-N. Li, C.-D. Lu, and A.I. Sanda, in preparation.
26. A. Szczepaniak, E.M. Henley and S. Brodsky, *Phys. Lett.* B 243, 287 (1990).
27. T. Kurimoto, H.-N. Li and A.I. Sanda, *Phys. Rev.* D 65, 014007 (2002).
28. M. Bander, D. Silverman and A. Soni, *Phys. Rev. Lett.* 43, 242 (1979).
29. H.-Y. Cheng and K.-C. Yang, *Phys. Rev.* D 64, 074004 (2001). H.-Y. Cheng Y.-Y. Keum and K.-C. Yang, *Phys. Rev.* D 65, 094023 (2002).
30. H.-Y. Cheng and K.-C. Yang, *Phys. Lett.* B 511, 40 (2001).
31. G. Zhu, talk at the 3rd workshop on Higher Luminosity B factory, Aug.6-7,2002, Kanegawa, Japan.
32. C.-D. Lu, K. Ukai and M.-Z. Yang, *Phys. Rev.* D 63, 074009 (2001).
33. C.-D. Lu and M.-Z. Yang, *Eur. Phys. J.* C 23, 275 (2002).
34. S. Mishima, *Phys. Lett.* B 521, 252 (2001).
35. C.-H. Chen, Y.-Y. Keum and H.-N. Li, hep-ph/0204166 (To be published in Phy. Rev. D).
36. Y.-Y. Keum and H.-N. Li, in preparation; Y.-Y. Keum, in preparation.
37. M. Neubert, hep-ph/0011064.
38. M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, *Nucl. Phys.* B 606, 245 (2001).
39. A.B. Carter and A.I. Sanda, *Phys. Rev. Lett.* 45 (1980) 952; *Phys. Rev.* D23, 1567 (1981); I.I. Bigi and A.I. Sanda, Nucl. Phys. B193, 85 (1981); I.I. Bigi and A.I. Sanda, CP Violation, Cambridge University Press, Cambridge, 2000.
40. BaBar Collaboration (B. Aubert et al.), *Phys. Rev. Lett.* 87 (2001) 091801. Belle Collaboration (K. Abe et al.), *Phys. Rev. Lett.* 87 (2001) 091802.
41. Recent works: R. Fleischer and J. Matias, hep-ph/0204101; M. Gronau and J.L. Rosner, Phys.Rev.D65 (2002) 013004, Erratum-ibid.D65 (2002) 079901; Phys. Rev. D65 (2002) 093012; hepph/0205323; C.D. Lu and Z. Xiao, hepph/0205134.
42. Belle Collaboration (K. Abe et al.), Belle-preprint 2002-8 [hep-ex/0204002].
43. BaBar Collaboration (B. Aubert et al.), BaBar-Pub-02-09 [hep-ex/0207055].
44. R. Fleischer and T. Mannel, *Phys. Rev.* D57, (1998) 2752; M. Neubert and J.L. Rosner, *Phys. Lett.* B441 (1998) 403; *Phys. Rev. Lett.* 81, (1998) 5076.
45. M. Gronau and J.L. Rosner, Phys.Rev.D65 (2002) 013004, Erratum-ibid.D65 (2002) 079901.
46. R. Bartoldus, talk on Review of rare two-body B decays at FPCP workshop, May 17, 2002.
47. Y.-Y. Keum, hep-ph/0209002; hep-ph/0209208 (Accepted in Phys. Rev. Lett.); Y.Y. Keum et al., in preparation.
48. Y.Y. Keum et al., in preparation.