Flow patterns and heat transfer in electro-thermo-convection

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Abstract. In this article we study the electro-thermo-convective phenomena in a dielectric liquid layer placed between two electrodes and subjected to the simultaneous action of an electric field and a thermal gradient. The full set of coupled equations: Navier-Stokes, Electro-Hydro-Dynamic (EHD) and Heat Transfer equations are directly solved in a 2D cavity using a finite volume method. In order to characterize the influence of the electric field on heat transfer we first heat the liquid till the thermal steady state is obtained and then apply the electric potential and charge are injected from the lower electrode. Two cases of heating are considered: from the lower electrode and from a lateral wall (left or right). We show that both flow pattern and Nusselt number strongly depend on the following non-dimensional characteristic parameters: electrical parameter, Rayleigh number, Prandtl number and mobility parameter. The development of the convective motion passing from a purely thermal convection to a purely electrical convection is investigated as well as the number of electro-thermo-convective rolls. As a consequence of the analysis of the combined effect of electric and thermal fields on the flow structure and on Nusselt number, we have also evaluated the heat transfer enhancement due to electro-convection.

1. Introduction
The combined effects of an electric field and a thermal gradient simultaneously applied to a horizontal dielectric liquid layer leads to very complex physical interactions in the flow. It has been experimentally shown [1][2] that the heat transfer across an insulating liquid could be increased by one order of magnitude. This augmentation of heat transfer is due to the development of secondary motions that result from two principal body forces: Coulomb forces acting on any free charges present in the liquid and dielectric forces. In this article we work on dielectric liquids of very low intrinsic conductivity with free space charges coming from ion injection phenomena at only one electrode. The charge induced by the injection process is dominant, often by more than one order of magnitude compared to that induced by the volume electrical conduction mechanisms. In EHD the convection induced by charge injection in a dielectric liquid is a problem as fundamental as the one of Rayleigh-
Benard in non isothermal fluid mechanics. The action of the electric field on the space charge density arising from a unipolar injection has the same destabilizing role than the thermal field when the fluid is heated from below in Rayleigh-Benard problems [3]. When the space charge only results from ion injection, the coupling between the conservation equations of momentum, electric charge and energy is ensured via the Coulomb and the buoyancy forces. This coupling results from the direct interaction between the velocity, temperature and charge perturbations and from the non direct interaction between the velocity and the charge. So far most of the authors who have been working on electro or electro-thermo convective problems in horizontal planar layers of dielectric liquids have only used a stability analysis [4]-[7]. There have been just a few numerical simulations on pure EHD convection problems [8]-[10]. For the first time the electro-hydro-dynamic problem coupled with energy and Navier-Stokes equations in a 2D cavity was solved in [11]. A direct numerical simulation (we start the computation with the fluid at rest and do not impose the fluid velocity field at any time) based on a finite volume method was developed. In [12] several numerical simulations are carried out to quantitatively measure the combined effect of both electric and thermal fields on heat transfer. We are particularly interested in the configurations that give the maximum of heat transfer and for which EHD heat exchanger applications could be considered. That is the reason why we have studied the different configurations of injection and heating: heating from below and charge injection from lower and upper wall. In this paper, we use the same method we developed in [11]. We focus on the convective mechanisms responsible for fluid motion and heat transfer in the case of a strong injection of electric charges (C=10) from the lower electrode coupled with a heating from the lower electrode or from a lateral wall (left or right). The variation of Nusselt number with electrical parameter $T$ for different values of the non-dimensional parameters (Rayleigh number $Ra$, Prandtl number $Pr$ and mobility parameter $M$) is analyzed. The flow patterns for a confined cavity of length 2 are given.

2. Statement of the problem

2.1. Basic governing equations

We consider a dielectric liquid layer of thickness $H$ enclosed between two electrodes of length $L_x$ and subjected to a thermal gradient $\Delta \theta = \theta_0 - \theta_1$. Between the two electrodes a potential difference $\Delta V = V_0 - V_1$ is applied. The electric charge of density $q_0$ is injected from lower electrode (Electrode 0). The heating can come from either from the lower electrode or from the lateral wall (left or right). Figure 1 displays the case where the heating is from below.

![Figure 1 The dielectric liquid layer](image)

The general set of equations for an incompressible fluid flow (see for example [13], [14]) is given below. It includes the Navier-Stokes equations (conservation of mass and momentum with electrical and buoyancy effects), the energy equation (with Boussinesq assumption), the equation of conservation of electric charge, Poisson equation (Gauss theorem) and the relation giving the electric field from the electric potential $V$.)
\[ \nabla \vec{u} = 0 \]  
\[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\nabla p + \Delta \vec{u} + RT \vec{C} q \vec{E} + \frac{Ra}{Pr} \nabla \theta \cdot \vec{e}_z \]  
\[ \frac{\partial \theta}{\partial t} + \vec{u} \cdot \nabla \theta = \frac{1}{Pr} \Delta \theta \]  
\[ \frac{\partial q}{\partial t} + \nabla \left((q \vec{u} + R \vec{E})\right) = 0 \]  
\[ \Delta V = -q \vec{E} = -\nabla V \]  
\[ \vec{E} = -\nabla V \]  

where \( \vec{u} \) is the velocity, \( p \) the static pressure, \( q \) the electric charge density in the liquid, \( \vec{E} \) the electric field, \( \varepsilon \) the permittivity of the fluid, \( \theta \) the absolute temperature, and \( V \) the electric potential, \( q\vec{E} \) is the Coulomb force. We introduce the following non-dimensional scales: \( H \) for length, the applied voltage \( \Delta V = V_0 - V_1 \) for electric potential, \( \Delta V/H \) for electric field, \( \varepsilon_0 \rho V/H^2 \) for current density, \( \nu/H \) for velocity (\( \nu \) is the kinematic viscosity), \( \rho_0 \varepsilon^2/H^2 \) for pressure, \( \Delta \theta = \theta_0 - \theta_1 \) for temperature, and \( H^2/\nu \) for time. The following dimensionless quantities appear: \( Ra = g \beta \Delta \theta H \nu \kappa \), \( T = \varepsilon \Delta V/\rho v K \), \( C = \rho \varepsilon H^2/\varepsilon \Delta V \), \( M = \sqrt{\varepsilon/\rho K^2} \), \( Pr = \nu/\kappa \), \( R = T/M^2 \), where \( Ra \) is the Rayleigh number (\( g \) the gravity and \( \kappa \) the thermal diffusivity), \( T \) is the electric instability parameter, \( C \) is a measure of the injection level (\( q_0 \) the injected charge), \( M \) is the mobility parameter (\( K \) ion mobility), \( Pr \) is the Prandtl number, \( R \) is electrical Reynolds number. Here we consider the case of a strong injection \( C=10 \), the liquid properties being therefore independent of temperature and the dielectric force \(-1/2 \varepsilon \nabla \vec{E}^2 \) much lower than the Coulomb force so that we neglected it [1][15].

2.2. Initial and boundary conditions

Our set of equations is time dependent and we start from the fluid initially at rest. We also start the heating of the liquid from the lower electrode (or from the left lateral wall) at time \( t=0 \). When the thermal steady state is obtained, we apply the electric potential difference between the two electrodes so that the electric charges are injected at that time in the bulk.

On horizontal and lateral walls we apply no-slip boundary conditions. The boundary conditions in the two different cases of heating: from below or from the left lateral wall are presented in figure 2a and figure 2b, respectively. The charge injection in the both cases is always from lower electrode.

a) case 1 (heating from lower electrode)  
b) case 2 (heating from left lateral wall.)

![Figure 2 Boundary conditions](image-url)
2.3. Numerical method
The set of coupled partial differential equations (1)-(5) are integrated using the finite volume method [16]. The numerical procedure based on the Augmented Lagrangian method [17] is explained in [11]. The calculation is fully transient. Special care is provided on the transport equation (4) for charge density $q$ because of its hyperbolic nature. The SMART algorithm has been utilized in that way.

3. Results and discussions
The combined effect of the thermal gradient and electric potential (with charge injection) on the flow patterns and on Nusselt number is analyzed. We first heat (at $t=0$), from the lower electrode and apply the electric potential when the thermal steady state is obtained. The thermal and electrical fields then act simultaneously. The whole simulation duration is set to $t=50$ (dimensionless time).

We consider a rectangular cavity of height $H=1$ and length $L_x=2$ and two cases:
- **Case 1**: Heating and strong injection of electric charges from the lower electrode.
- **Case 2**: Heating from the lateral wall (left or right one) and strong injection of electric charges from the lower electrode.

3.1. Flow patterns
The global effect of convective heat transfer is usually analysed from Nusselt number $Nu$, which is the ratio of the mean heat flux by convection to the flux by conduction for the same temperature difference. It is the reason why we compute and study the mean Nusselt number $\overline{Nu} = \frac{1}{2} \int_0^1 \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \, dx$ at the hot wall of the cavity. In the non-stationary state the Nusselt number fluctuates more or less around a mean value and stabilises when a well developed convective state (following a transition one) is reached. The thermo-convective flow patterns depend on two instability criteria: the Rayleigh number $Ra$ and the electrical parameter $T$ as well as on the non-dimensional parameters $Pr$ and $M$ numbers. Figure 3 shows the stream function evolution at different times in a pure thermo-convection case (without electric field and without any injection of electric charges) in the two following cases of heating: from below (upper row of figure 3), from the left lateral wall (lower row of figure 3). Typical two roll patterns are obtained when heating from below whereas two rolls appear just at the beginning of the simulation when heating from the left. In this latter case the right roll moves very slowly and the left one grows until the right one totally disappears. The system finally gets its stationary state regime with just one roll.

Figure 4 shows the flow structure of a four roll stationary pattern obtained with a pure electro-convection simulation with a charge injection from the lower electrode. For a confined cavity the wave length of the more instable mode is about 0.5 [11] for $Ra=0$ and 1.0 for $T=0$.

| t=2 | t=5 | t=50 |
|-----|-----|------|
| ![Stream function for t=2](image1) | ![Stream function for t=5](image2) | ![Stream function for t=50](image3) |

Figure 3 Stream function for a pure thermo-convection ($T=0$), $Ra=100$ heating from below (upper row) and the left lateral wall (lower row)

Figure 4 Flow patterns for a pure electro-convection ($Ra=0$), $T=300$, $Pr=40$, $M=10$. 
It is reason why the 2 rolls are obtained for the pure thermo convective case and four for the pure electro convective case, in this cavity of length \(L_x=2\).

In figure 5 the streamlines are given at the stationary state for different values of \(T\) in cases 1 and 2 (injecting from below and heating either from below (upper row) or from the left wall (lower row)).

In case 1 (heating from below) for lower values of \(T\) \((T=150)\) two thermo-convective cells appear. When \(T\) is greater than the typical threshold of electro convection \((T=200 \text{ and } 300)\) the number of rolls changes to four instead of two rolls. Time evolution of the flow is presented in details in [15]. In case 2 (heating from a lateral wall) just one thermo-convective cell appears for \(T=150\). When increasing \(T\) up to \(T=200\), secondary structures composed of four rolls of different shapes appear in the bulk, while at \(T=300\) only four uniform rolls are obtained. We can see that electro-convection imposes its proper structure (four rolls). For higher values of \(T\) we always obtain a four roll pattern. Therefore, it exists a certain specific value of \(T\) \(T_{ch}\) which corresponds to the change from a one to four roll pattern. \(T_{ch}\) depends on the values of all the other parameters \((Ra, Pr \text{ and } M)\). When the electric effects are dominant (pure electro-convection) the flow structure is a four roll pattern whereas when the buoyancy effect is dominant (pure thermo-convection) it gives rise to only one roll for lateral heating or two rolls for heating below.
3.2. *Influence of the characteristic parameters (M and Pr) on the heat transfer*

In order to quantitatively measure the influence of electro-convection on heat transfer we have to present the variation of Nusselt number with electrical parameter \( T \) for different values of the non-dimensional parameters \( Ra, Pr \) and \( M \).

In Figure 6 we plot the Nusselt number as a function of \( T \) for two different values of number \( M \) (\( M=10 \) and \( M=60 \)) in the case of heating from below and heating from the left lateral wall. We see that \( Nu \) increases with \( T \) for small values of \( M \) and it does not change with \( T \) for large \( M \). Mobility parameter characterizes the electro-hydrodynamic properties of the liquid: the smaller value of \( M \) the higher mobility \( K \) of the electric charges and therefore the better induced convection and heat transfer. The jump of the curve for \( M=10 \) corresponds to the value of \( T \) for which four rolls appear.

In Figure 7 the time evolution of \( Nu \) for different \( Pr \), for two values of \( T \) (0 and 200) and for fixed values of parameters, in the case of heating from below is presented. For \( T=0 \) \( Nu \) is almost independent of \( Pr \). The 3 other curves (\( T=200 \) and \( Pr \) varying) coincide from \( t=10 \) to \( t=25 \) (the injection starts at \( t=25 \)). From this point, we observe a brusque increase of the Nusselt number which ends with a constant value. The heat transfer has therefore increased significantly in comparison with the case of pure thermo-convection due to injected charge. This increase is significant for large \( Pr \) numbers (\( Pr>>1 \)) for which convection is more effective than conduction.

In Figures 8 and 9 we have plotted the mean Nusselt number as a function of \( T \) for two Rayleigh number \( (Ra=2000, 10000) \) and different values of \( Pr \). Case 1 (heating from below) is on Figure 8 and case 2 (heating from left lateral wall) on Figure 9. The charge injection is still from the lower electrode. In both cases, we can clearly see that \( Nu \) increases significantly with \( T \) for large \( Pr \). This increase of \( Nu \) with the electric potential (contained in \( T \)) is related to the dynamic mixing due to the turbulent activity in the flow. For large values of \( T \) (\( T>250 \)) \( Nu \) does not depend on \( Ra \), which coincides with the experimental results [1][2]. This point is discussed in details in [11].

![Figure 8: Nusselt number against T in case 1 (heating from below and injecting from lower electrode) for different values of Pr and Ra.](image-url)
The jumps and twists of the curves in Figure 8 and Figure 9 correspond to the change from two to four roll patterns (see also Figure 5). There are some differences in the behaviour of the curves in the two cases of heating: the growth is smooth in case 1 while in the other case there is a sudden increase of $Nu$ for large $Pr$. However, the most notable feature is that $Nu$ does not depend on Rayleigh number for high enough values of $T$. It means that the electrical forces dominate and increase the heat transfer.
In figure 10 the two curves (cases 1 and 2) of $Nu(T)$ are compared. $Nu$ has larger values in case 1 (heating from below rather than case 2 (heating from lateral wall) whatever $T$. The behaviour of the curves is different but for large values of $T$, which correspond to higher electric potential, both tend to the same value of $Nu$. In both cases the increase of Nusselt number is due to the charge injection. It is therefore obvious that the heat transfer is enhanced by the electrical activity in the flow and this increase is significant for higher $Pr$ numbers.

4. Conclusion
A direct numerical simulation of the effect of the electric forces on the heat transfer in a dielectric liquid layer has been presented. Two different cases of heating (from the lower electrode and from a lateral wall, left or right) with an injection from the lower electrode are considered. It has been demonstrated that $Nu$ increases with $Pr$ rapidly. The most important feature is that for high enough values of $T$, $Nu$ becomes almost independent of Rayleigh. It means that the electrical forces increase and dominate the heat transfer. We have also shown that the heat transfer augmentation could be obtained for small values of the mobility parameter ($M=10$). For high electric fields and large $Pr$ numbers the comparison between the values of Nusselt number when heating from below with the ones when heating from the left wall gives almost the same values.

Finally, the flow patterns have been obtained for different $T$, $Ra$, $Pr$ and $M$. When heating from below two and four rolls have been obtained for Rayleigh and $T$ number greater than the typical thresholds of the thermo or electro-convective phenomena. When heating from a lateral wall (left or right) one or four convective flow patterns are obtained depending on whether $T$ number is lower or greater than a certain specific value $T_{ch}$. No threshold for Rayleigh number has been observed in the latter case.

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