Modeling and Dynamic Behavior Analysis of the ICPT System with Multiple Receivers

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Abstract. This paper investigates modeling and dynamic behavior of inductively coupled power transfer system with multiple receivers. Firstly, by using the idea of decomposition to set up model of the inductively coupled power transfer system with multiple receivers. Then, by simulation, analyzing dynamic behavior of the inductively coupled power transfer system with two receivers. The proposed modeling method is help to clearly illustrate the dynamic behavior of each subsystem. And results indicate variation of coupling parameters has a significant influence on the dynamic behavior of the system.

Introduction

Wireless power transfer technology has attracted more and more people attention in the past several decades due to its wide range of applications such as mobile phones [1], biomedical implants [2] and electric vehicles [3]. Therefore, inductively coupled power transfer (ICPT) system has been investigated deeply by relevant scholars at home and abroad. However, at present, the ICPT system is becoming more complex so as to meet various needs of people, which results in many problems, for example the modeling, the complicated dynamic behavior analysis and so on.

First of all, for the modeling method of the ICPT system, the circuit theory method [4,5] and generalized state space averaging (GSSA) modeling method [6,7] are adopted mainly in past research. Unfortunately, the former is not reveal the dynamic behavior of the ICPT system. For the latter, this modeling method can only be used for analyzing the simple ICPT system. But for the complex ICPT system such as the ICPT system with multiple receivers, this approach will be confronted with the intricate modeling problem. Besides, the dynamic behavior of the ICPT system will be also more complex with the increase of subsystems.

Motivated by the above discussion, in this paper, a novel modeling approach for the ICPT system with multiple receivers is proposed by utilizing the idea of decomposition. This method can not only be used for analyzing the dynamic behavior of the single subsystem, but also for studying the whole system, which can help us to put forward some targeted strategies for locally problematic subsystems to improve the dynamic performances of these subsystems and reduce cost.

The paper is organized as follows. Section 2 describes model of the ICPT system with multiple receivers. In section 3, the dynamic behavior of the ICPT system with multiple receivers is analyzed by simulation when the receive systems are moved randomly. Conclusions are drawn in section 4.

Model Description

A detailed description of the ICPT system can be found in [8], here we mainly discuss modeling.

The ICPT system with multiple receivers is depicted in figure 1. The whole ICPT system consists of a transmit system and \( n \) receive systems. Generally, for the ICPT system with multiple receivers, the position of the transmit system is fixed while the positions of the receive systems can be moved randomly. The interconnected relationship is established by the magnetic coupling (i.e., the coupling...
parameter $M_0$). By using the circuit theory, differential equations of the ICPT system with multiple receivers are expressed as follows.

$$\begin{align*}
\left\{ \begin{array}{l}
s(t)E_{DC} = u_0(t) + i_0(t)R_0 + L_0 \frac{di_0}{dt} + M_{01} \frac{di_1}{dt} + \cdots + M_{0n} \frac{di_n}{dt} \\
0 = L_1 \frac{di_1}{dt} + M_{01} \frac{di_0}{dt} + i_1 R_1 + i_1 R_{L1} + u_0 + M_{12} \frac{di_2}{dt} + \cdots + M_{1n} \frac{di_n}{dt} \\
\vdots \\
0 = L_n \frac{di_n}{dt} + M_{0n} \frac{di_0}{dt} + i_n R_n + i_n R_{L_n} + u_0 + M_{1n} \frac{di_1}{dt} + \cdots + M_{(n-1)n} \frac{di_{(n-1)}}{dt} \\
C_0 \frac{du_0}{dt} = i_0 \\
C_1 \frac{du_1}{dt} = i_1 \\
\vdots \\
C_n \frac{du_n}{dt} = i_n
\end{array} \right.
\end{align*}$$

where $s(t) = \begin{cases} 
1 & kT \leq t < \frac{T}{2} (2k + 1) \\
-1 & \frac{T}{2} (2k + 1) \leq t < (k + 1)T
\end{cases}$, $k$ is zero or positive integer, $T$ denotes a work circle of the inverter.

From equations (1), we can see that the model is very complex and highly difficult to analyze directly. Therefore, it is essential to further improve the model.

For $\sum_{j=1,j\neq i}^{n} M_{ji} \frac{di_j}{dt}$, since $u_0 = L_j \frac{di_j}{dt}$, it follows that $\sum_{j=1,j\neq i}^{n} M_{ji} \frac{di_j}{dt} = \sum_{j=1,j\neq i}^{n} \frac{M_{ji}}{L_j} u_0(t)$, $i=0, 1, 2, \ldots, n$.

Besides, since the ICPT system is in resonance, according to circuit theory, we know that $u_0(t) = -u_0(t)$. It is noted that due to reciprocity of the mutual inductance, $M_{ji} = M_{ij}$. Then by using the idea of decomposition, we can obtain state space model of each subsystem as follows.
\[
\begin{align*}
\frac{di_0}{dt} &= \frac{1}{L_0} s(t) E_{DC} - \frac{1}{L_0} u_0(t) - \frac{1}{L_0} i_0(t) R_0 + \sum_{j=1}^{n} \frac{M_{0j}}{L_0 L_j} u_j(t) \\
\frac{du_0}{dt} &= \frac{1}{C_0} i_0 \\
\frac{di_1}{dt} &= -\frac{1}{L_1} i_1 R_1 - \frac{1}{L_1} i_1 R_{L1} - \frac{1}{L_1} u_1 + \sum_{j=0, j\neq 1}^{n} \frac{M_{1j}}{L_1 L_j} u_j(t) \\
\frac{du_1}{dt} &= \frac{1}{C_1} i_1 \\
&\vdots \\
\frac{di_n}{dt} &= -\frac{1}{L_n} i_n R_n - \frac{1}{L_n} i_n R_{Ln} - \frac{1}{L_n} u_n + \sum_{j=0}^{n-1} \frac{M_{nj}}{L_n L_j} u_j(t) \\
\frac{du_n}{dt} &= \frac{1}{C_n} i_n
\end{align*}
\]

Define the state variable \( x_i = (u_i, i) \), the state space model of the \( i \)-th subsystem is described as follows.

\[
\dot{x}_i = A_i x_i + B_i v + f_i
\]

where

\[
A_i = \begin{bmatrix}
0 & 1/C_i \\
-1/L_i & -1/(R_i + R_{Lj})/L_i
\end{bmatrix},
B_i = \begin{bmatrix}
0 \\
1/L_0
\end{bmatrix},
B_{i(i \neq 0)} = \begin{bmatrix}
0 \\
0
\end{bmatrix},
\nu \text{ denotes input control, and } v(t) = s(t) \cdot E_{DC},
\]

\[
f_i = \sum_{j=0, j\neq i}^{n} \frac{M_{ij}}{L_j L_i} u_j, \quad i=0, 1, 2, \ldots, n.
\]

**Dynamic Behavior Analysis of the ICPT with Multiple Receivers**

In this section, the dynamic behavior of the ICPT system with two receivers as shown in figure 2 will be analyzed. First of all, the relevant parameters of the system are adopted as follows.

\( E_{DC} = 100V \), \( \omega_0 = 44367.8rad/s \), \( L_0 = L_1 = L_2 = 127\mu F \), \( R_0 = 6\Omega \), \( R_1 = R_2 = 3\Omega \), \( M_{01} = 0.8\mu F \), \( C_0 = C_1 = C_2 = 4\mu F \), \( M_{01} = M_{02} = 12.7\mu F \), \( R_{L1} = 100\Omega \), \( R_{L2} = 150\Omega \), \( M'_{12} = [0.8 + 0.8\sin(\omega_0 t)]\mu H \), \( M'_{01} = M'_{02} = [63.5 + 63.5\sin(\omega_0 t)]\mu H \).

![Figure 2. ICPT system with two receivers.](image-url)
Figure 3. Phase plots of ICPT system with two receivers when subsystems are not moved randomly.

Figure 4. Phase plots of ICPT system with two receivers when subsystems are moved randomly.
Phase plots of the ICPT system with two receivers are presented in figure 3 and figure 4 respectively when the positions of the subsystems are not moved randomly and moved randomly. Subplots (a) and (b) show the dynamic behavior of the whole ICPT system with two receivers, and subplots (c) and (d) show the dynamic behavior of the subsystems 0 and 1 respectively. From these figures, we can see that the ICPT system with two receivers does not occur chaos phenomenon and is stable. Then, when the positions of the subsystems are moved randomly, from subplots (e) and (f), it is easy to see that the dynamic behavior of the whole ICPT system becomes very complicated. Besides, from subplots (g) and (h), we can also clearly see the variation of the dynamic behavior of each subsystem, which can help us to more accurately research the dynamic behavior characteristics of the complex ICPT system.

**Summary**

In this work, by adopting the idea of decomposition to establish the model of the ICPT system with multiple receivers. Based on the model, the dynamic behavior of the ICPT system with multiple receivers was researched. The simulation results show that the dynamic behavior of the ICPT system becomes more complex with the change of the coupling parameters. The proposed model method has an important guiding significance for the modeling, analysis and control of the complex ICPT system.

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