Cosmological constraint on the light gravitino mass from CMB lensing and cosmic shear

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Abstract. Light gravitinos of mass \( \lesssim O(10) \) eV are of particular interest in cosmology, offering various baryogenesis scenarios without suffering from the cosmological gravitino problem. The gravitino may contribute considerably to the total matter content of the Universe and affect structure formation from early to present epochs. After the gravitinos decouple from other particles in the early Universe, they free-stream and consequently suppress density fluctuations of (sub-)galactic length scales. Observations of structure at the relevant length-scales can be used to infer or constrain the mass and the abundance of light gravitinos. We derive constraints on the light gravitino mass using the data of cosmic microwave background (CMB) lensing from Planck and of cosmic shear from the Canada France Hawaii Lensing Survey survey, combined with analyses of the primary CMB anisotropies and the signature of baryon acoustic oscillations in galaxy distributions. The obtained constraint on the gravitino mass is \( m_{3/2} < 4.7 \) eV (95% C.L.), which is substantially tighter than the previous constraint from clustering analysis of Ly-\( \alpha \) forests.

Keywords: weak gravitational lensing, dark matter theory

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1 Introduction

The existence of gravitino is predicted in particle physics models with local supersymmetry (SUSY), or in supergravity models. The gravitino mass $m_{3/2}$ is related to the energy scale of SUSY breaking, which is one of the most important quantities in low-scale SUSY models that are invoked to solve the gauge hierarchy problem (see, e.g., [1], for a review). However, it is well-known that gravitino can cause serious problems in the cosmological context (the so-called cosmological gravitino problems [2]). If gravitino is stable (or very long-lived), its relic abundance may contradict with the observed energy density of dark matter unless $m_{3/2}$ is very small. Gravitino with $m_{3/2} \lesssim \mathcal{O}(10)\text{ eV}$ has attracted particular attention, because such light gravitinos can evade the cosmological gravitino problem [3] and provide baryogenesis mechanisms (e.g., thermal leptogenesis [4]) that work only at very high temperature.

Not only probed by collider experiments (e.g., LHC) with direct and indirect signatures, the existence or the abundance of light gravitinos has also been constrained from cosmological observations [5]. Light gravitinos produced from thermal plasma in the early Universe behave effectively as warm dark matter. Gravitinos with sizable velocity dispersions free-stream over a cosmological distance until they become non-relativistic. Gravitinos suppress the growth of matter density fluctuations and imprint characteristic signatures in the matter power spectrum at and below the free-streaming length. One can therefore constrain, in principle, the mass of light gravitinos from cosmological observations of large-scale matter distribution. To present, observations of Ly-α forests [6] give a stringent constraint on the gravitino mass as $m_{3/2} < 16\text{ eV}$ (95% C.L.). It is important to notice that cosmological constraints provide upper limits to $m_{3/2} < 16\text{ eV}$, whereas particle collider experiments generally provide lower limits.

The objective of the present paper is to improve the cosmological constraint on $m_{3/2}$ using independent observations of CMB and the large-scale structure. Weak gravitational lensing is a powerful probe of matter distribution. The coherent distortion of images of distant galaxies can be used to infer the foreground matter distribution. There are two background sources for weak gravitational lensing: one is lensing of cosmic microwave background
radiation, and the other is lensing of distant galaxies. The difference in the source redshifts enables us to probe a wide spatial range of large-scale structure. Previous studies [7, 8] suggest that the two observables can be indeed utilized to constrain $m_{3/2}$.

There is another motivation to consider light gravitinos as a possible matter content of the Universe. It has been claimed that $\sigma_8$, the amplitude of matter density fluctuations normalized at $8\, h^{-1}\text{Mpc}$, derived from observations of CMB anisotropies is higher than the values derived from observations of large-scale structure at low redshifts, such as weak gravitational lensing and the abundance of galaxy clusters [9, 10]. The apparent tension may infer some mechanism that suppress matter density fluctuations at late epochs. For example, massive active or sterile neutrinos [9–13], decaying dark matter [14], and various baryonic physics [13, 15] have been proposed. The existence of light gravitinos may offer another intriguing resolution for this tension, in quite a similar way to massive neutrinos. Clearly, it is important to perform a fully consistent cosmological parameter estimate including $m_{3/2}$ as one of the primary parameters. This problem will be addressed in detail in appendix A. However, it is difficult to resolve this tension only with light gravitinos.

The rest of the paper is organized as follows. In section 2, we briefly summarize particle physics aspects of the gravitino. We discuss how light gravitinos affect large-scale structure in the Universe in section 3. In section 4, we describe the basics of two observational probes that are used to derive constraints on gravitino mass $m_{3/2}$. We explain in detail the observational data and methods to extract posterior distributions of cosmological parameters including $m_{3/2}$ in section 5. In section 6, we present constraints on the mass of light gravitinos from these two observational probes combined with CMB and baryon acoustic oscillation (BAO) observations. We give concluding remarks in section 7.

## 2 Light gravitino

Light gravitinos are realized typically in gauge-mediated SUSY breaking (GMSB) scenarios [16–21]. Let us review briefly basics of GMSB models and the current constraints from LHC. In GMSB models, a characteristic relation is supposed to hold among masses of the gravitino and SUSY particles (sparticles) in the Standard Model (SM) sector. In fact, the discovery potential of GMSB models by LHC experiments largely relies on these sparticle masses, and thus the current constraints on the gravitino mass are often model-dependent.

The masses of the gravitino and sparticles originate from spontaneous SUSY breaking. As a consequence, the masses are proportional to the SUSY breaking scale $\langle F \rangle$. The gravitino mass is given by

$$m_{3/2} = \frac{\langle F \rangle}{\sqrt{3}M_{\text{pl}}},$$

(2.1)

with $M_{\text{pl}} \simeq 2.43 \times 10^{18}\,\text{GeV}$ being the reduced Planck mass. Sparticles in the SM sector acquire masses from the SUSY breaking through messenger fields, whose mass scale we denote as $M_{\text{mess}}$. With the gauge couplings denoted by $g_1 = \sqrt{5/3}g'$, $g_2 = g$, and $g_3$, where $g'$ and $g$ are the conventional electro-weak gauge couplings, gaugino mass and sfermion mass squared are approximately given by

$$M_a \sim \frac{g_a^2}{16\pi^2} \frac{\langle F \rangle}{M_{\text{mess}}}$$

and

$$m_{\tilde{f}_i}^2 \sim \sum_a C_a^{(i)} \left( \frac{g_a^2}{16\pi^2} \frac{\langle F \rangle}{M_{\text{mess}}} \right)^2,$$

(2.2)

respectively, where the indices $a$ and $i$ respectively indicate a SM gauge group and a flavor of the sfermion, and $C_a^{(i)}$ is the quadratic Casimir invariant. Lower bounds on their masses
are placed by collider experiments directly and indirectly as discussed below. With some model-dependence, the mass constraints can be translated into lower bounds on the SUSY breaking scale $|⟨F⟩|$, or the gravitino mass $m_{3/2}$.

Stringent constraints on GMSB models come from the Higgs mass $m_h = 125$ GeV measured by LHC [22, 23]. In the minimal supersymmetric Standard Model (MSSM), the Higgs mass is bounded from above at tree level, $m_{h,\text{tree}} < m_Z = 91$ GeV. This requires a large contribution of radiative corrections from top-stop loops, $m_{h,\text{loop}}^2 \sim m_t^4/(16\pi^2v^2) \ln (m_{t}/m_{\tilde{t}})$, where $m_t$ ($m_{\tilde{t}}$) and $v$ are the mass of top quark (stop) and the vacuum expectation value of Higgs, respectively. To achieve the observed large Higgs mass, the stop mass is required to be as large as $m_{\tilde{t}} = O(10–100)$ TeV, which correspondingly places a lower bound on the gravitino mass. For example, in a class of GMSB models with $N_5$ copies of messenger fields in the $\mathbf{5} + \overline{\mathbf{5}}$ representation of SU(5),\footnote{In order to maintain perturbative unification of the SM gauge couplings, $N_5$ needs to be not too large (typically $N_5 \leq 5$). For details of the model, especially the explicit formulas of sparticle masses, we refer to, e.g., [1, 8].} one obtains a bound $m_{3/2} > 300$ eV with $N_5 = 1$ (60 eV with $N_5 = 5$) [24], provided that the coupling between the messengers and a SUSY breaking field $\lambda$ is perturbative (i.e., $|\lambda| < 1$). As will be discussed in the next section, such light gravitinos are ruled out or only marginally allowed in order for their thermal relic density not to exceed the observed density of dark matter. However, the bound is model-dependent, and $m_{3/2} = O(1–10)$ eV may be possible if the coupling is non-perturbative $|\lambda| > 1$ or a singlet Higgs (i.e., next to MSSM) is introduced [25]. Interestingly, the former may offer a hidden baryon as a main component of dark matter [26].

Less stringent lower bounds on the gravitino mass are obtained also from direct SUSY searches in LHC that seek for production of sparticles (mostly squarks and gluinos) decaying into energetic SM particles and gravitinos. For example, in the same GMSB model as mentioned above, with $M_{\text{mess}} = 250$ TeV and $N_5 = 3$ (10 + $\overline{\mathbf{10}}$ of SU(5)) being fixed, a bound $|\lambda(F)|/M_{\text{mess}} > 63$ TeV is obtained from analysis of events with at least one tau lepton and zero or one light lepton in 20 fb$^{-1}$ of the LHC 8 TeV run [27]. Assuming a perturbative coupling $|\lambda| < 1$, this bound leads to $m_{3/2} > 3.7$ eV. We also note that in the future International Linear Collider may allow us to measure the light gravitino mass directly from decay of a “long-lived” next-to-lightest supersymmetric particle [28, 29].

3 Effects on large-scale structure of the Universe

In what follows, we assume that the reheating temperature is so high that light gravitinos are once in equilibrium with a thermal bath in the very early universe. As the background temperature decreases, the gravitino decouples from other particles at some point, and its relic abundance in the Universe is fixed. The relic abundance is approximately estimated from the relativistic degrees of freedom (hereafter denoted as $g_{*3/2}$) in a thermal plasma at the gravitino decoupling. In [7, 30], the Boltzmann equation is solved for the gravitino number density in GMSB models. For a messenger mass scale $M_{\text{mess}} \sim 100$ TeV, $g_{*3/2} \sim 90$ with only mild dependence on $m_{3/2}$ in 1–100 eV [7]. Throughout the present paper, we set $g_{*3/2} = 90$ as a canonical value.

At late times, thermal relic gravitinos act as warm dark matter particles that can be characterized by the temperature (velocity dispersion) and mass. The phase-space distribution of the gravitinos is given by the Fermi-Dirac distribution with two degrees of freedom because only the (spin $\pm 1/2$) goldstino components virtually interact with other particles.
If there is no significant entropy production after the gravitino decoupling, the gravitino temperature is given in terms of the standard neutrino temperature $T_\nu = 1.95$ K as

\[ T_{3/2} = \left( \frac{g_{*\nu}}{g_{*3/2}} \right)^{1/3} T_\nu = 0.96 \text{ K} \left( \frac{g_{*3/2}}{90} \right)^{-1/3}, \tag{3.1} \]

where $g_{*\nu} = 10.75$ is the relativistic degree of freedom at the neutrino decoupling. Then the effective number of neutrino species accounting for the gravitino is given by

\[ N_{3/2} = \left( \frac{T_{3/2}}{T_\nu} \right)^4 = \left( \frac{g_{*\nu}}{g_{*3/2}} \right)^{4/3} = 0.059 \left( \frac{g_{*3/2}}{90} \right)^{-4/3}. \tag{3.2} \]

At late epochs, light gravitinos are non-relativistic, so that the energy density can be estimated as

\[ \Omega_{3/2} h^2 = 0.13 \left( \frac{m_{3/2}}{100 \text{ eV}} \right) \left( \frac{g_{*3/2}}{90} \right)^{-1}. \tag{3.3} \]

We note that dark matter cannot consists solely of the gravitino in the cosmological model with thermally produced gravitinos considered here. This is because $m_{3/2}$ needs to be as large as 86 eV in order to account for the observed dark matter density $\Omega_{3/2} h^2 \approx 0.11$ [31], which clearly contradicts the existing constraint from Ly-$\alpha$ forest, $m_{3/2} < 16$ eV [6]. In what follows, we assume that dark matter consists of the light gravitino and some additional CDM constituents, so that

\[ \Omega_{\text{dm}} = \Omega_{\text{cdm}} + \Omega_{3/2}. \tag{3.4} \]

For example, within the framework of GMSB, where the gravitino is usually the lightest SUSY particle, a messenger baryon or QCD axion can be the extra CDM.

The primary effects of light gravitinos on structure formation are of twofold. The epoch of matter-radiation equality is slightly delayed (corresponding to a larger $a_{\text{eq}}$), and the matter fluctuations at small length scales are suppressed. Regarding the first effect, light gravitinos are relativistic in the early universe and can contribute to the radiation energy. However, the contribution of the light gravitinos is too small ($N_{3/2} \approx 0.059$) to change the epoch of the equality appreciably when compared to current observational sensitivities.\(^2\)

In order to constrain the mass of light gravitino, we consider the latter effect. Light gravitinos free-stream with a sizable velocity, in a similar manner to massive neutrinos. Within the free-streaming scale, gravitinos do not cluster, and thus act effectively as “drag” of the growth of matter fluctuations. Thus, typically, matter fluctuations at late times are smaller than in the conventional CDM model. The characteristic length scale can be estimated as [8]

\[ k_J = a \sqrt{\frac{4\pi G \rho_m}{\langle v^2 \rangle}} \bigg|_{a=a_{\text{eq}}} \simeq 0.86 \text{ Mpc}^{-1} \left( \frac{m_{3/2}}{100 \text{ eV}} \right)^{1/2} \left( \frac{g_{*3/2}}{90} \right)^{5/6}. \tag{3.5} \]

Massive neutrinos affect the growth of structure, but the temperature and the energy density of massive neutrinos are different from those of the light gravitinos. For massive neutrinos the resulting suppression scale eq. (3.5) differs.

\(^2\)For reference, the current constraint on the effective number of massless neutrino species is $N_{\text{eff}} = 3.04 \pm 0.2$ from CMB and baryon acoustic oscillation (BAO) in galaxy distributions [31], which is not sensitive enough to measure $\Delta N_{\text{eff}} = 0.059(= N_{3/2})$. Furthermore, if the neutrino species has a total mass of $\mathcal{O}(1)$ eV, the constraint on $\Delta N_{\text{eff}}$ should be even less stringent since such species are already more or less non-relativistic at the equality.
Figure 1. Linear matter power spectra at \( z = 0 \) for three different values of \( m_{3/2} \) computed with the \textsc{CLASS} code. For comparison, we also show the spectrum with massive neutrinos with the total mass \( M_\nu = 0.3 \text{ eV} \). The total dark matter density \( \Omega_{\text{dm}} = \Omega_{\text{cdm}} + \Omega_{3/2} \) or \( \Omega_{\text{dm}} = \Omega_{\text{cdm}} + \Omega_\nu \) is fixed to be \( \Omega_{\text{cdm}} \) of the result of Planck 2015 TT,TE,EE+lowP dataset [31]. Upper panel shows the absolute values and lower panel shows the fractional differences with respect to the standard \( \Lambda \text{CDM} \).

To demonstrate the effect of the free-streaming of light gravitinos, in figure 1 we plot linear matter power spectra at \( z = 0 \) in the presence of the light gravitino with different masses. The power spectra are computed using the Boltzmann code \textsc{CLASS} [32, 33] with the base cosmological parameters of the Planck 2015 TT,TE,EE+lowP dataset [31]. At large scales, the linear matter power spectra are rather insensitive to \( m_{3/2} \). The characteristic suppression wave number is larger for larger \( m_{3/2} \), because the light gravitinos become non-relativistic earlier and the free-streaming length becomes smaller. On the other hand, the fraction of gravitinos in the matter content increases with increasing \( m_{3/2} \), and then the power is more effectively suppressed. The figure suggests that, in order to probe the gravitino mass of our interest (i.e., \( m_{3/2} = \mathcal{O}(1-10) \text{ eV} \)), we need a cosmological probe that is sensitive to matter power spectrum around \( k = \mathcal{O}(0.01-0.1) \text{ Mpc}^{-1} \).

### 4 Observational probes

Weak gravitational lensing effect is one of the most powerful probes of matter power spectrum at scales \( k = \mathcal{O}(0.01-0.1) \text{ Mpc}^{-1} \). To constrain the gravitino mass, we use recent data from CMB lensing and cosmic shear observations. In the following, we briefly review these observations in order. Following the previous section, we in this section adopt the cosmological parameters from the Planck 2015 TT,TE,EE+lowP dataset [31] with fixed \( \Omega_{\text{dm}} = \Omega_{\text{cdm}} + \Omega_{3/2} \).

**CMB lensing.** CMB photons travel through the gravitational potential between the last scattering surface and the observer point. The collective gravitational lensing effect is
called CMB lensing and, in principle, contains information of the matter distribution in the Universe at $z \simeq 2 - 3$ (e.g., see ref. [34] for a review). The CMB lensing is described by the lensing potential $\phi(\hat{n})$,

$$\phi(\hat{n}) = -2 \int_0^{\chi_*} d\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*) f_K(\chi)} \Psi(\chi \hat{n}, \eta(\chi)), \quad (4.1)$$

where $\chi$ is a radial comoving distance from us, $\chi_*$ is a comoving distance to the last scattering surface, $\eta(\chi)$ is a conformal time at which the photon is at $\chi \hat{n}$, $\Psi$ is a gravitational potential. $f_K(\chi)$ is a comoving angular diameter distance,

$$f_K(\chi) = \begin{cases} K^{-1/2} \sin(K^{1/2} \chi) & (K > 0) \\ \chi & (K = 0) \\ (-K)^{-1/2} \sin[(-K)^{1/2} \chi] & (K < 0), \end{cases} \quad (4.2)$$

where $K$ is a curvature of the Universe. Throughout this paper, we assume a flat universe with $K = 0$.

The power spectra of the lensing potential contains rich information of the matter distribution at intermediate redshifts. The lensing potential can be expanded with spherical harmonics as

$$\phi(\hat{n}) = \sum_{\ell,m} \phi_{\ell m} Y_{\ell m}(\hat{n}). \quad (4.3)$$

The angular power spectrum of $\phi$ is then defined by

$$\langle \phi_{\ell m} \phi_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{mm'} C_{\ell}^{\phi \phi}. \quad (4.4)$$

We can calculate $C_{\ell}^{\phi \phi}$ by using eq. (4.1) and Limber approximation [35]$^3$ as follows:

$$C_{\ell}^{\phi \phi} = \int_0^{\chi_*} d\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*) f_K(\chi)} \left[ -2 \frac{f_K(\chi_* - \chi)}{f_K(\chi_*) f_K(\chi)} \right]^2 P_\Psi(\ell/f_K(\chi), \eta(\chi)), \quad (4.5)$$

where $P_\Psi(k, \eta(\chi))$ is the power spectrum of the gravitational potential, related with the matter density through the Poisson equation.

In eq. (4.5), it is sufficient to consider fluctuations to linear order, because we mainly focus on the range of $40 < \ell < 400$ where non-linear gravitational growth is unimportant [36]. With the linear approximation, $P_\Psi$ is expressed as

$$P_\Psi(k, \eta(\chi)) = |T_\Psi(k, \eta(\chi))|^2 P_K(k), \quad (4.6)$$

where $T_\Psi$ is the transfer function and $P_K$ represents the power spectrum of the primordial curvature fluctuations.

**Cosmic shear.** Images of distant galaxies are distorted by weak gravitational lensing effect of foreground matter distributions. One can characterize the distortion by the following 2D matrix:

$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} \equiv \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}, \quad (4.7)$$

$^3$The exact expression of $C_{\ell}^{\phi \phi}$ is found in [36], while eq. (4.5) is an accurate approximation for $\ell \gtrsim 10$ where we are interested in.
where $\kappa$ is convergence, $\gamma$ is shear, and $\theta$ and $\beta$ represent the observed position and the true position of a source, respectively. In the weak lensing limit (i.e., $|\kappa|, |\gamma| \ll 1$), each component of $A_{ij}$ can be related to the second derivative of the gravitational potential $\Psi$ [37, 38].

By using the Poisson equation, one can relate the convergence field to the matter overdensity field $\delta$ as [37, 38]

$$\kappa(\hat{n}) = \int_0^{\chi_H} d\chi G(\chi) \delta[\chi, \eta(\chi)],$$  \hspace{1cm} (4.8)

where $G(\chi)$ is the lensing efficiency. Given a source galaxy distribution $p(\chi)$ normalized to $\int d\chi p(\chi) = 1$, $G(\chi)$ is given by

$$G(\chi) = \frac{3}{2} \frac{\Omega_m}{a(\chi)} \left( \frac{H_0}{c} \right)^2 f_K(\chi) \int_\chi^{\chi_H} d\chi' p(\chi') \frac{f_K(\chi' - \chi)}{f_K(\chi')},$$  \hspace{1cm} (4.9)

where $\chi_H$ is a comoving distance to the horizon. The convergence power spectrum can then be computed as

$$C_{\kappa \kappa}^\ell = \int_0^{\chi_H} \frac{d\chi}{f_K^2(\chi)} G^2(\chi) P_\delta(\ell / f_K(\chi), \eta(\chi)), \hspace{1cm} (4.10)$$

where $P_\delta$ is the three dimensional matter power spectrum. The direct observables are the two-point correlation functions (2PCFs) of cosmic shear, $\xi_{\pm}(\vartheta)$ (see ref. [39] for details). Via Hankel transformation, $\xi_{\pm}(\vartheta)$ can be related with $C_{\kappa \kappa}^\ell$ as

$$\xi_{+, -}(\vartheta) = \int_0^{\ell} \frac{\ell d\ell}{2\pi} J_0(\ell \vartheta) C_{\kappa \kappa}^\ell,$$  \hspace{1cm} (4.11)

where the plus (minus) sign corresponds to the Bessel function of the first kind $J_0$ ($J_4$).

We calculate convergence power spectra and CMB lensing potential spectra using CLASS. Non-linear correction is computed by the HALOFIT fitting formula [40] modified to incorporate massive neutrinos [41]. At late epochs, both massive neutrinos and light gravitinos are non-relativistic and behave similarly. We can easily modify the code based on the one with massive neutrinos to incorporate the effect of light gravitinos. We simplify call the modified one HALOFIT in the following.

In figure 2, we show the power spectra of CMB lensing potential $C_{\ell}^{\phi \phi}$ computed with different masses of the light gravitinos. We also plot the measurement of the Planck mission [42]. Figure 3 compares the non-linear matter power spectra calculated using CLASS with HALOFIT and the $N$-body simulation results of [8] at $z = 0.3, 0.6$ in order to check the validity of the HALOFIT treatment. These two results are consistent within 10% level. In figure 4, the comparison of 2PCFs of cosmic shear calculated from CLASS with measured 2PCFs by the CFHTLenS survey [43] are shown. We find suppression of the power spectrum and of the correlation function for models with light gravitinos.

5 Data set and parameter estimation

To estimate cosmological parameters including the mass of light gravitinos, we perform the Markov chain Monte Carlo (MCMC) analysis. For this purpose, we make use of the publicly available code MontePython [45] with the Metropolis-Hastings sampling. The data used in this study are summarized below.
Figure 2. CMB lensing auto-power spectrum in the presence of light gravitinos. The gravitino mass is set to be 0 eV (cyan line) and 4 eV (magenta line) as in figure 1. The red points show the Planck 2015 measurement [42]. We adopt cosmological parameters from the Planck 2015 TT,TE,EE+lowP dataset [31] for this plot. The offset of the data points and the model curves, which adopt Planck 2015 TT,TE,EE+lowP parameters, can be interpreted as the often-claimed tension in $\sigma_8$ between observations of CMB anisotropies of temperature and polarizations and observations of large-scale structure.

Figure 3. We compare the non-linear matter power spectra calculated by CLASS (solid lines) with HALOFIT and the simulation results of [8] (points with error bars) at $z = 0.3, 0.6$. Only in this plot, we adopt cosmological parameters from Planck 2013 temperature-only results [44] because N-body simulations in [8] adopted Planck 2013 parameters.
Figure 4. We compare the 2PCFs of cosmic shear computed with CLASS (solid lines) and the measurements of the CFHTLenS [43] (red points). The left (right) panel shows \( \xi_+ (\theta) \) (\( \xi_- (\theta) \)). The cyan (magenta) line shows the result with the gravitino mass 0 eV (4 eV). In the model calculations, the redshift distributions of the source galaxies are included as the weight function. We adopt the cosmological parameters from Planck 2015 TT,TE,EE+lowP dataset [31]. Notice the apparent tension in \( \sigma_8 \) as in figure 2.

**Planck TT,TE,EE+lowP.** The Planck satellite provides CMB anisotropy maps along with a subset of the polarization data. The measurement is done with the angular resolution of \( \sim 10 \) arcmin at the frequencies of 25–1000 GHz, allowing the estimation of CMB power spectra for \( \ell \lesssim 2000 \) without significant contaminants from astrophysical sources. We use measurements of the angular power spectra of CMB temperature and polarizations anisotropies made by Planck. The dataset consists of the three auto power spectra of the temperature (\( C_{TT}^{\ell} \)), the E-mode (\( C_{EE}^{\ell} \)) and the B-mode polarizations (\( C_{BB}^{\ell} \)), and the cross power spectrum of the temperature and the E-mode polarization (\( C_{TE}^{\ell} \)). The ranges of multipole are \( \ell = 2–2508 \) for \( C_{TT}^{\ell} \), \( \ell = 2–1996 \) for \( C_{EE}^{\ell} \) and \( C_{BB}^{\ell} \), and \( \ell = 2–29 \) for \( C_{BB}^{\ell} \). We use the publicly available Planck likelihood code [46].

**Planck lensing.** The CMB maps provided by the Planck satellite enable us to estimate the lensing potential over approximately 70% of the sky [42]. The reconstruction of the lensing potential from the observed CMB maps has been performed by the quadratic estimator [47]. Details of the reconstruction are described in [42]. The reconstructed lensing potential has been detected at a significance of \( \sim 40 \sigma \) from the combined analysis with the CMB temperature and polarizations data. We use the angular auto power spectrum of lensing potential (\( C_{\phi \phi}^{\ell} \)) in the range of multipole of \( \ell = 40–400 \).

**CFHTLenS 2PCFs.** The Canada France Hawaii Lensing Survey (CFHTLenS) is an imaging survey in five optical bands with the sky coverage of 154 square degrees. The CFHTLenS achieved an effective weighted number density of \( \sim 11 \) galaxies per square arcminutes with shape and photometric redshift estimates [48], allowing robust and accurate weak lensing analysis [49]. We use the 2PCFs of galaxy’s ellipticities in the CFHTLenS measured by [43]. The measured 2PCFs are known as the good estimator of \( \xi_{\pm} \) in the absence of intrinsic alignment (IA) of galaxies [39]. In [43], 2PCFs have
been measured for the source galaxies with the redshift of 0.2 < z < 1.3. It is expected that the broad redshift distribution of source galaxies makes the IA contribution sub-dominant (see, e.g., [50]). We check that the effect of IA is negligible by removing the data of small angular separation (see section 6). The correlation functions are binned with 21 bins in the range 0.9 arcmin ≤ θ ≤ 300 arcmin. We consider a Gaussian likelihood function of the 2PCFs with the covariance matrix as in [43].

**BOSS.** The Baryon Oscillation Spectroscopic Survey (BOSS) is designed to obtain spectra and redshifts for 1.35 million galaxies covering 10,000 square degrees in the Sloan Digital Sky Survey (SDSS) [51]. BOSS includes the galaxy catalog from SDSS data releases 10 [52] and 11 [53], allowing us to select red galaxies in the spectroscopic redshift range of 0.2 < z < 0.7. The BAO feature has been detected in the clustering of galaxies from the BOSS at a significance of \( \sim 7\sigma \) [54], and it has been fully utilized to measure the cosmic distance scale with a 1% precision. We use the measurement of the cosmic distance scale in [54] to improve the cosmological constraints derived from the CFHTLenS. We consider the ratio of the cosmic distance measure \( D_V \) and the sound horizon at the drag epoch \( r_s \). The measure of \( D_V/r_s \) is performed for two galaxy samples named LOWZ (\( z_{\text{eff}} = 0.32 \)) and CMASS (\( z_{\text{eff}} = 0.57 \)). The result is given by [54]

\[
D_V/r_s(\text{LOWZ}) = 8.47 \pm 0.17, \tag{5.1}
\]

\[
D_V/r_s(\text{CMASS}) = 13.77 \pm 0.13. \tag{5.2}
\]

We explore two cosmological models with and without light gravitinos. As a base model without gravitinos, we assume the conventional flat power-law \( \Lambda \)CDM model with six parameters (\( \Omega_{\text{cdm}} h^2 \), \( \Omega_b h^2 \), 100\( \theta_s \), \( \ln(10^{10} A_s) \), \( n_s \), \( \tau_{\text{reio}} \)). As its extension, \( \Lambda \)CDM+gravitino model has an additional parameter \( m_{3/2} \). We assume that all neutrinos are massless with the effective degree of freedom \( N_{\text{eff}} = 3.046 \). Because massive neutrinos and light gravitinos affect the matter power spectrum in a similar manner [8], and their effects are roughly additive, the upper bounds on gravitino mass we derive in the following are conservative. Theoretical nonlinear matter power spectrum is computed by CLASS with the HALOFIT nonlinear corrections.

We note that there is one caveat associated with the applicability of the HALOFIT formula to our model. Originally, it is calibrated by simulations with the total neutrino mass \( M_\nu \lesssim 0.6 \text{eV} \) [41]. It is not clear if the same formula applies to models with light gravitinos with mass \( \gtrsim \mathcal{O}(1) \text{eV} \). However, the results of HALOFIT and N-body simulations with light gravitino of \( m_{3/2} = 4 \text{eV} \) show reasonable agreement (figure 3) at observable angular scales of \( \ell \lesssim \mathcal{O}(100) \). In addition, as we shall see in the next section, light gravitinos with a significantly larger mass, \( m_{3/2} \gtrsim 10 \text{eV} \) is already disfavored observationally in light of the Planck (TT,TE,EE+lowP)+lensing. Thus we expect that using the HALOFIT model gives reasonably accurate result in the parameter ranges we consider here.

6 Constraints on gravitino mass

Figure 5 shows the posterior distributions of \( m_{3/2} \) marginalized over all the other cosmological parameters, and figure 6 shows the two-dimensional posterior distributions of the total matter

\[^4\text{Angular scales of CMB lensing measurements are much larger and nonlinear corrections are even less relevant.}\]
Figure 5. Posterior distributions of the gravitino mass from three different data sets. In all figures we abbreviate “Planck TT,TE,EE+lowP” to “Planck”.

Figure 6. Two-dimensional posterior distributions of the total matter density $\Omega_m$ and the gravitino mass $m_{3/2}$ from three different data sets.

| Data set                                           | $m_{3/2}$ [eV]                        |
|----------------------------------------------------|---------------------------------------|
| Planck (TT,TE,EE+lowP)+lensing                     | < 4.9 (95% C.L.)                      |
| Planck (TT,TE,EE+lowP)+lensing + CFHTLenS + BOSS   | $2.1 \pm 1.2$ (68% C.L.), < 4.7 (95% C.L.) |

Table 1. Constraints on the mass of light gravitinos.
density and the gravitino mass. Table 1 summarizes the constraints of $m_{3/2}$ with two different data sets. Our upper bounds $m_{3/2} \lesssim 4.7 \text{eV}$ (95% C.L.) are about three times more stringent than the previous one derived from Ly-$\alpha$ forest observations, $m_{3/2} < 16 \text{eV}$ [6]. When combined with the cosmic shear data, the width of the posterior distribution becomes twice smaller, although the upper bounds are virtually unchanged. Clearly adding cosmic shear observations is effective to constrain $m_{3/2}$, although cosmic shear by itself alone cannot constrain $m_{3/2}$ very tightly (see appendix A and ref. [8]). We have examined the robustness of our results by removing the cosmic shear 2PCFs at small angular scales $\theta < 10$ arcmin, where the discrepancy between the simulation and HALOFIT results is larger, while within 10%. We have confirmed that removing the small scale data leads to only small changes and for the mass constraint, the difference is within 1σ. The slight (only around 68% C.L.) preference for nonzero $m_{3/2}$ in the “all” data set arises from the tension in estimations of $\sigma_8$, which we shall discuss in appendix A.

7 Conclusions

We have derived cosmological constraints on the gravitino mass $m_{3/2} = O(1) \text{eV}$. In the late-time universe, such light gravitinos can contribute to the total matter density, and act as a warm component, causing suppression of the matter density fluctuations at $k = O(0.01–0.1) \text{Mpc}^{-1}$. Interestingly, cosmological observations and collider experiments can probe a SUSY breaking scale in a complemental way.

We have used two cosmological observations: the CMB lensing power spectrum from Planck, and the two-point correlation function of cosmic shear from the CFHTLenS. The combination enables us to measure the amplitude of the matter power spectrum at a broad range of scales, which is essential to probe the light gravitino mass. Combining the two data with primary CMB power spectra and galaxy clustering, we have obtained a stringent upper bound on the light gravitino mass $m_{3/2} < 4.7 \text{eV}$ (95% C.L.). Our constraint is considerably tighter than the previous constraint from Ly-$\alpha$ forest [6].

Measurements of cosmic shear will be improved significantly in the near future. High-quality data from, e.g., DES [55]5 and Hyper Suprime-Cam [56]6 will not only improve the present bound, but also will allow us to explore a broader range of cosmological scenarios such as those with nonthermally produced gravitinos with a low reheating temperature or ones diluted by a late entropy production. We expect the future observations will provide us with knowledge of physics at high energy scales and in the early universe.

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5http://www.darkenergysurvey.org/.
6http://www.naoj.org/Projects/HSC/index.html.
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7}
\caption{Two-dimensional posterior distributions of the total matter density $\Omega_m$ and the amplitude of the fluctuation $\sigma_8$. The top and bottom panels show the cases without and with light gravitinos, respectively. For the purpose of comparison, in the bottom panel, the dashed lines show the contours of distributions at 68\% C.L. results without light gravitinos.}
\end{figure}

A Implications for $\sigma_8$ tension

In this appendix, we address the apparent tension in the estimate of $\sigma_8$. We perform the MCMC analysis using only CHFTLenS and BOSS data. Without the CMB, $m_{3/2}$ is almost unconstrained and can be as large as $O(10)$ eV. Because applying \textsc{Halofit} to such models can significantly compromise the parameter estimate, we impose a top-hat prior on $m_{3/2}$ in $[0, 10]$ eV. Since we use only the low-redshift large-scale structure probes, i.e., CFHTLenS and BOSS, the baryon density and the optical depth are not constrained effectively. We fix these parameters as the Planck 2015 TT,TE,EE+lowP values: $\Omega_b h^2 = 0.0225$, $\tau_{\text{reio}} = 0.0079$ [31].

Figure 7 shows the contours of the two-dimensional posterior distribution of the total matter density $\Omega_m = \Omega_b + \Omega_{\text{cdm}} + \Omega_{3/2}$ and the amplitude of the matter fluctuation $\sigma_8$ in models with and without light gravitinos. The one-dimensional posterior distributions
Figure 8. Posterior distributions of $\sigma_8$ from different datasets. Solid and dashed lines correspond to the cases with and without light gravitinos, respectively.

of $\sigma_8$ is also shown in figure 8. Light gravitinos suppress the matter power spectrum and effectively lower the fluctuation amplitude as we have shown in sections 3 and 4. As a result, the distributions of $\sigma_8$ shift downward (figures 7 and 8). We have checked that removing small scale data ($\theta < 10$ arcmin) of 2PCFs results in only minor changes of the posterior distributions, and thus the main results remain robust if we use all the 2PCF data. Although the tension between the two data sets with gravitinos, CHFTLenS+BOSS (the blue solid line) and Planck+lensing (the yellow solid line), is slightly mitigated, it is unlikely that invoking the light gravitino helps reconcile the data at a sufficient level.

References

[1] S.P. Martin, A supersymmetry primer, Adv. Ser. Direct. High Energy Phys. 18 (1998) 1 [hep-ph/9709356] [inSPIRE].

[2] S. Weinberg, Cosmological constraints on the scale of supersymmetry breaking, Phys. Rev. Lett. 48 (1982) 1303 [inSPIRE].

[3] T. Moroi, H. Murayama and M. Yamaguchi, Cosmological constraints on the light stable gravitino, Phys. Lett. B 303 (1993) 289 [inSPIRE].

[4] M. Fukugita and T. Yanagida, Baryogenesis without grand unification, Phys. Lett. B 174 (1986) 45 [inSPIRE].

[5] E. Pierpaoli, S. Borgani, A. Masiero and M. Yamaguchi, Formation of cosmic structures in a light gravitino dominated universe, Phys. Rev. D 57 (1998) 2089 [astro-ph/9709047] [inSPIRE].

[6] M. Viel, J. Lesgourgues, M.G. Haehnelt, S. Matarrese and A. Riotto, Constraining warm dark matter candidates including sterile neutrinos and light gravitinos with WMAP and the Lyman-\alpha forest, Phys. Rev. D 71 (2005) 063534 [astro-ph/0501562] [inSPIRE].

[7] K. Ichikawa, M. Kawasaki, K. Nakayama, T. Sekiguchi and T. Takahashi, Constraining Light Gravitino Mass from Cosmic Microwave Background, JCAP 08 (2009) 013 [arXiv:0905.2237] [inSPIRE].

[8] A. Kamada, M. Shirasaki and N. Yoshida, Weighing the Light Gravitino Mass with Weak Lensing Surveys, JHEP 06 (2014) 162 [arXiv:1311.4323] [inSPIRE].
[9] J. Hamann and J. Hasenkamp, *A new life for sterile neutrinos: resolving inconsistencies using hot dark matter*, JCAP **10** (2013) 044 [arXiv:1308.3255] [SPIRE].

[10] R.A. Battye and A. Moss, *Evidence for Massive Neutrinos from Cosmic Microwave Background and Lensing Observations*, Phys. Rev. Lett. **112** (2014) 051303 [arXiv:1308.5870] [SPIRE].

[11] BOSS collaboration, F. Beutler et al., *The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: Signs of neutrino mass in current cosmological data sets*, Mon. Not. Roy. Astron. Soc. **444** (2014) 3501 [arXiv:1403.4599] [SPIRE].

[12] R.A. Battye, T. Charnock and A. Moss, *Tension between the power spectrum of density perturbations measured on large and small scales*, Phys. Rev. D **91** (2015) 103508 [arXiv:1409.2769] [SPIRE].

[13] N. MacCrann, J. Zuntz, S. Bridle, B. Jain and M.R. Becker, *Cosmic discordance: are Planck CMB and CFHTLenS weak lensing measurements out of tune?*, Mon. Not. Roy. Astron. Soc. **451** (2015) 2877 [arXiv:1408.4742] [SPIRE].

[14] K. Enqvist, S. Nadathur, T. Sekiguchi and T. Takahashi, *Decaying dark matter and the tension in σ₈*, JCAP **09** (2015) 067 [arXiv:1505.05511] [SPIRE].

[15] K. Osato, M. Shirasaki and N. Yoshida, *Impact of Baryonic Processes on Weak-lensing Cosmology: Power Spectrum, Nonlocal Statistics and Parameter Bias*, Astrophys. J. **806** (2015) 186 [arXiv:1501.02055] [SPIRE].

[16] M. Dine and W. Fischler, *A Phenomenological Model of Particle Physics Based on Supersymmetry*, Phys. Lett. B **110** (1982) 227 [SPIRE].

[17] C.R. Nappi and B.A. Ovrut, *Supersymmetric Extension of the SU(3) × SU(2) × U(1) Model*, Phys. Lett. B **113** (1982) 175 [SPIRE].

[18] L. Álvarez-Gaumé, M. Claudson and M.B. Wise, *Low-Energy Supersymmetry*, Nucl. Phys. B **207** (1982) 96 [SPIRE].

[19] M. Dine and A.E. Nelson, *Dynamical supersymmetry breaking at low energies*, Phys. Rev. D **48** (1993) 1277 [hep-ph/9303230] [SPIRE].

[20] M. Dine, A.E. Nelson and Y. Shirman, *Low energy dynamical supersymmetry breaking simplified*, Phys. Rev. D **51** (1995) 1362 [hep-ph/9408384] [SPIRE].

[21] M. Dine, A.E. Nelson, Y. Nir and Y. Shirman, *New tools for low energy dynamical supersymmetry breaking*, Phys. Rev. D **53** (1996) 2658 [hep-ph/9507378] [SPIRE].

[22] ATLAS and CMS collaborations, *Combined Measurement of the Higgs Boson Mass in pp Collisions at √s = 7 and 8 TeV with the ATLAS and CMS Experiments*, Phys. Rev. Lett. **114** (2015) 191803 [arXiv:1503.07589] [SPIRE].

[23] CMS collaboration, *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. B **716** (2012) 30 [arXiv:1207.7235] [SPIRE].

[24] M.A. Ajaib, I. Gogoladze, F. Nasir and Q. Shafi, *Revisiting mGMSB in Light of a 125 GeV Higgs*, Phys. Lett. B **713** (2012) 462 [arXiv:1204.2856] [SPIRE].

[25] T.T. Yanagida, N. Yokozaki and K. Yonekura, *Higgs Boson Mass in Low Scale Gauge Mediation Models*, JHEP **10** (2012) 017 [arXiv:1206.6589] [SPIRE].

[26] S. Dimopoulos, G.F. Giudice and A. Pomarol, *Dark matter in theories of gauge-mediated supersymmetry breaking*, Phys. Lett. B **389** (1996) 37 [hep-ph/9607225] [SPIRE].

[27] ATLAS collaboration, *Search for supersymmetry in events with large missing transverse momentum, jets, and at least one tau lepton in 20 fb⁻¹ of √s = 8 TeV proton-proton collision data with the ATLAS detector*, JHEP **09** (2014) 103 [arXiv:1407.0603] [SPIRE].

[28] S. Matsumoto and T. Moroi, *Studying very light Gravitino at the ILC*, Phys. Lett. B **701** (2011) 422 [arXiv:1104.3624] [SPIRE].
[29] R. Katayama et al., *Full simulation study of very light gravitino at the ILC*, LC-REP-2013-010 (2013).

[30] E. Pierpaoli, S. Borgani, A. Masiero and M. Yamaguchi, *The formation of cosmic structures in a light gravitino dominated universe*, Phys. Rev. D 57 (1998) 2089 [astro-ph/9709047] [inSPIRE].

[31] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, arXiv:1502.01589 [inSPIRE].

[32] J. Lesgourgues, *The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview*, arXiv:1104.2932 [inSPIRE].

[33] D. Blas, J. Lesgourgues and T. Tram, *The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes*, JCAP 07 (2011) 034 [arXiv:1104.2933] [inSPIRE].

[34] A. Lewis and A. Challinor, *Weak gravitational lensing of the CMB*, Phys. Rept. 429 (2006) 1 [astro-ph/0601594] [inSPIRE].

[35] N. Kaiser, *Weak gravitational lensing of distant galaxies*, Astrophys. J. 388 (1992) 272 [inSPIRE].

[36] W. Hu, *Weak lensing of the CMB: A harmonic approach*, Phys. Rev. D 62 (2000) 043007 [astro-ph/0001303] [inSPIRE].

[37] M. Bartelmann and P. Schneider, *Weak gravitational lensing*, Phys. Rept. 340 (2001) 291 [astro-ph/9912508] [inSPIRE].

[38] D. Munshi, P. Valageas, L. Van Waerbeke and A. Heavens, *Cosmology with Weak Lensing Surveys*, Phys. Rept. 462 (2008) 67 [astro-ph/0612667] [inSPIRE].

[39] P. Schneider, L. van Waerbeke, M. Kilbinger and Y. Mellier, *Analysis of two-point statistics of cosmic shear: I. estimators and covariances*, Astron. Astrophys. 396 (2002) 1 [astro-ph/0206182] [inSPIRE].

[40] R. Takahashi, M. Sato, T. Nishimichi, A. Taruya and M. Oguri, *Revising the Halofit Model for the Nonlinear Matter Power Spectrum*, Astrophys. J. 761 (2012) 152 [arXiv:1208.2701] [inSPIRE].

[41] S. Bird, M. Viel and M.G. Haehnelt, *Massive Neutrinos and the Non-linear Matter Power Spectrum*, Mon. Not. Roy. Astron. Soc. 420 (2012) 2551 [arXiv:1109.4416] [inSPIRE].

[42] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XV. Gravitational lensing*, arXiv:1502.01591 [inSPIRE].

[43] M. Kilbinger et al., *CFHTLenS: combined probe cosmological model comparison using 2D weak gravitational lensing*, Mon. Not. Roy. Astron. Soc. 430 (2013) 2200 [arXiv:1212.3338] [inSPIRE].

[44] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XVI. Cosmological parameters*, Astron. Astrophys. 571 (2014) A16 [arXiv:1303.5076] [inSPIRE].

[45] B. Audren, J. Lesgourgues, K. Benabed and S. Prunet, *Conservative Constraints on Early Cosmology: an illustration of the Monte Python*, JCAP 02 (2013) 001 [arXiv:1210.7183] [inSPIRE].

[46] PLANCK collaboration, N. Aghanim et al., *Planck 2015 results. XI. CMB power spectra, likelihoods and robustness of parameters*, Submitted to: Astron. Astrophys. (2015) [arXiv:1507.02704] [inSPIRE].

[47] T. Okamoto and W. Hu, *Cosmic Microwave Background lensing reconstruction on the full sky*, Phys. Rev. D 67 (2003) 083002 [astro-ph/0301031] [inSPIRE].
[48] T. Erben et al., *CFHTLenS: The Canada-France-Hawaii Telescope Lensing Survey — imaging data and catalogue products*, Mon. Not. Roy. Astron. Soc. 433 (2013) 2545 [arXiv:1210.8156] [inSPIRE].

[49] C. Heymans et al., *CFHTLenS: The Canada-France-Hawaii Telescope Lensing Survey*, Mon. Not. Roy. Astron. Soc. 427 (2012) 146 [arXiv:1210.0032] [inSPIRE].

[50] D. Kirk, A. Rassat, O. Host and S. Bridle, *The cosmological impact of intrinsic alignment model choice for cosmic shear*, Mon. Not. Roy. Astron. Soc. 424 (2012) 1647 [arXiv:1112.4752] [inSPIRE].

[51] SDSS collaboration, D.G. York et al., *The Sloan Digital Sky Survey: Technical Summary*, Astron. J. 120 (2000) 1579 [astro-ph/0006396] [inSPIRE].

[52] SDSS collaboration, C.P. Ahn et al., *The Tenth Data Release of the Sloan Digital Sky Survey: First Spectroscopic Data from the SDSS-III Apache Point Observatory Galactic Evolution Experiment*, Astrophys. J. Suppl. 211 (2014) 17 [arXiv:1307.7735] [inSPIRE].

[53] SDSS-III collaboration, S. Alam et al., *The Eleventh and Twelfth Data Releases of the Sloan Digital Sky Survey: Final Data from SDSS-III*, Astrophys. J. Suppl. 219 (2015) 12 [arXiv:1501.00963] [inSPIRE].

[54] BOSS collaboration, L. Anderson et al., *The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: baryon acoustic oscillations in the Data Releases 10 and 11 Galaxy samples*, Mon. Not. Roy. Astron. Soc. 441 (2014) 24 [arXiv:1312.4877] [inSPIRE].

[55] Dark Energy Survey collaboration, T. Abbott et al., *The dark energy survey*, astro-ph/0510346 [inSPIRE].

[56] S. Miyazaki et al., *Hyper Suprime-Cam*, Proc. SPIE 8446 (2012) 84460Z.