The fractal analysis of the gravitational field and topography of the Mars

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Abstract. The aim of this paper is to represent the results of a structured fractal analysis of gravitational and topographical parameters of Mars on the basis of data obtained from the space missions. To analyze Martian fractal structures the observations from the data received from boards of the space missions including «Mars Global Surveyor» has been used. The models of relief and Mars’ gravitational field have been constructed on the basis of harmonic analysis of the expansion in spherical functions of the satellite observations data. As a result, fractal dimensions of Martian relief’s anomalies and Mars’ gravitational potential by longitude and latitude have been determined. Mean values of the fractal dimensions \( D \) have been obtained as well: mean fractal dimensions of Martian topographic model by latitude \( D = 0.86 \), by longitude \( D = 0.88 \); mean fractal dimensions of Martian gravitational potential anomalies by latitude \( D = 1.06 \), by longitude \( D = 1.092 \).

1. Introduction
For evolution of complex celestial systems investigation the methods of multiparametric statistical physics [1, 2] are used. Particularly, during the analysis of non-linear processes and structures of natural objects, including determination of its properties, fractal geometry plays the key role, this also applies both to gravimetric and topographical parameters of Mars [3, 4]. Dilated symmetry and spatio-temporal heterogeneity are common properties of various physical system. These properties in, in case of a scale change, lead to basic geometrical parameters invariance [5, 6]. Fractal dimension is a quantitative measure characterizing structure of an object. The connection between geometrical parameters of an object and processes of its formation may be investigated by an analysis of fractal dimensions. This is why development of methods for fractal structure of celestial systems determination is a relevant issue. During the space missions to the bodies of Solar system [7, 8] a significant amount of data on geophysics, morphology, gravitational fields and other planet’s atmosphere has been accumulated. As a result of these investigations comparative planetology has been significantly developed. This field focuses on the problems related not just to cosmogony and the theory of Solar system’s bodies evolution, but to the analysis of processes of physical nature as well. The purpose of this work is to conduct the fractal structured analysis of gravimetric and topological parameters of Mars on the basis of the space missions’ data.

2. The method of Martial structures fractal analysis
The models of gravitational field and the physical surface of Mars variations may be considered as fractal structures. To investigate them the corresponding approaches of statistical physics and multi-parameter analysis are required. The formulation diversity of «fractal dimension», ways to determine that by the results of experiments allow to consider fractal analysis as a separate field of the dynamical systems theory. The fact, that fractal dimension of an object’s structure is different from topological one, indicates there is some significant complexity in structural organization of an object being investigated (distribution in space). As a model of a fractal object the scale-invariant fractal curve – the fractal Weierstrass-Mandelbrot function is used. From the one hand the model of fractal is a recursive one, i.e. each part of it repeats the whole model and reproduces that at different scales without any change. From the other hand, real natural systems can not be described as one fractal and
present interdependent multifractals where each fractal included in a set has its dimension. Multifractal analysis estimates a system in a form of fractal dimensions spectrum. Thus, such an approach provides sufficient accuracy of global fractal structures description on the basis of certain local areas investigation. In this work as a mathematical algorithm the Minkovsky method on determination and analysis of dimension has been used which is a simplified option of Hausdorff–Besicovitch dimension but at the same time provides sufficient accuracy and reliability.

Let $M$ be a limited set in the plane and we define $N(\sigma)$ as a minimal number of an area being investigated covers whose combination covers $M$ [9]. In the general case the set $M$ has a dimension of $D=\dim M$, $0 \leq D \leq 2$, if at $\sigma \rightarrow 0$ the number of covers $N(\sigma)$ increases as $R/\sigma^D$, where $R$ is a positive constant called $D$-measure of the set $M$. For $R$ in the general case we may write:

$$R = \lim_{\sigma \rightarrow 0} \sigma^D N(\sigma).$$

Expression (1) is not convenient when calculating since it contains the value of $D$-measure. The fractal dimension $D$ may be determined according to the following expression [9]:

$$D = \dim M = \lim_{\sigma \rightarrow 0} \frac{\ln N(\sigma)}{\ln \frac{1}{\sigma}},$$

as

$$\lim_{\sigma \rightarrow 0} \frac{\ln N(\sigma)}{\ln \frac{1}{\sigma}} = \lim_{\sigma \rightarrow 0} \frac{\ln R}{\ln \frac{1}{\sigma}} = \lim_{\sigma \rightarrow 0} D \frac{\ln \frac{1}{\sigma} + \ln R}{\ln \frac{1}{\sigma}} = D.$$  

In order to analyze Martian structures the data received from the boards of the spaceships, particularly “Mars Global Surveyor” [10–12], has been used. The model of topographical anomalies is based on height measurements from the board of the spaceship, while the model of gravitational potentials includes investigations of Martian artificial satellites’ orbits and gravitational observations made by other methods. The constructed geoid of Mars (as it is well known, names of gravimetric and topographical characteristics for the Solar system bodies coincide with terrestrial ones and only different for the Moon) defines both gravity potential and difference between normal and real gravity potentials. With this model one can investigate the gravitational field near the surface of Mars and mass distribution in bowels of Mars. Determination of the basic level surface on Mars is set by a point on its surface that coincides with the geoid’s surface or a certain value of the potential. Thus, depending the gravimetric potential the geoid may be located above the physical surface of Mars as well as under that which is similar to Earth’s geoid [3]. The obtained models of variations of the gravitational field and the topographical surface of Mars have been studied on the basis of the Minkovsky fractal dimension’s values (see equation (2)). The analysis of the obtained dimension has allowed to conduct a comparative analysis of various geophysical Martian parameters.

3. Harmonic models of topography and gravitational field of Mars

Using harmonic analysis of expansion in spherical functions of the topographical data received from boards of «Mars Global Surveyor» a topographical model of Mars has been constructed [13]:

$$h(\varphi, \lambda) = \sum_{n=0}^{N} \sum_{m=0}^{n} \left( \tilde{C}_{nm} \cos m\lambda + \tilde{S}_{nm} \sin m\lambda \right) \cdot \tilde{P}_{nm}(\cos \varphi) + \varepsilon,$$

where $h(\varphi, \lambda)$ – function of altitude; 
$\varphi, \lambda$ – latitude, longitude (known parameters); 
$\tilde{C}_{nm}, \tilde{S}_{nm}$ – normalized harmonic amplitudes; 
$\tilde{P}_{nm}$ – normalized associated Legendre functions; 
$\varepsilon$ – random regression error.
This equation has also been used for an analysis of positional observations made in Engelhardt Astronomical Observatory [14].

On the basis of gravimetric measurements a harmonic model of Mars’ gravitational potential has been constructed [15]:

\[
U(\varphi, \lambda, r) = \frac{GM}{r} \left[ 1 + \sum_{n=0}^{\infty} \left( \frac{R_0}{r} \right)^n \sum_{m=0}^{n} (c_{nm} \cos m\lambda + s_{nm} \sin m\lambda) \cdot \tilde{P}_m(\sin \varphi) \right] + \varepsilon, \tag{5}
\]

where \( G \) – gravitational constant;
\( GM \) – gravitational field of Mars (42,828.31 km\(^3\)/sec\(^2\));
\( \varphi, \lambda, r \) – spherical coordinate of reference point;
\( R_0 \) – equatorial radius of planet.

4. Study of Martian fractal structures and the results of analysis

The topographical model of Mars has been cut by planes in order to pick out macroprofiles of the planet’s surface on parallels with latitudes of the following values: -80, -60, -40, -20, 0, 20, 40, 60, 80 degrees. As an example for the latitude of 20 degrees this may be seen on the figure 1. The units systems on both x and y axis correspond to the measured data and are not critical, since the profiles are considered as self-similar structures.

![Figure 1. Macroprofile of Mars topography on the latitude of 20 degrees. X-axis corresponds to longitude (\( \lambda \)) in degrees, Y-axis corresponds to altitude in meters (\( h \)).](https://example.com/image1.png)

Similarly to this the models of gravity’s anomalies on the parallels with the following latitudes: -80, -60, -40, -20, 0, 20, 40, 60, 80 degrees. As an example for the latitude of 0 degrees this may be seen on the figure 2.

For each cut fractal dimensions according to equation (2) have been calculated, where \( N \) – number of covers in the investigation area. The division into covers starts with value of 24 for number of covers to be integer. The obtained values for the topographical model and the model of gravity field’s anomalies are represented in figure 3.

Analysis of figure 3 shows that for the topographical data the variations of fractal dimensions vary within 0.8–1.0. Considering the fact that the more significantly topological dimensions of a structure
differs from fractal one, the more complex formation of a system is, so the similarity of topographical formations on Mars is sufficiently defined. The variations of gravity’s anomalies fractal dimensions change within 0.6–1.2. Thus, the gravity field of Mars may be described as a topological dissimilar system with a complex structure in latitude measurements.

**Figure 2.** Macroprofile of Mars gravity anomalies on the latitude of 0 degrees. X–axis corresponds to latitude in degrees (λ), Y–axis corresponds to unit of acceleration mGal (0.001 cm/sec²) (U).

**Figure 3.** Fractal dependence of topographical model and gravity anomalies by Mars latitudes. X–axis corresponds to latitudes in degrees (φ), Y–axis corresponds to fractal dimension values D. Line 1 corresponds to fractal dimensions of Mars topographical model; line 2 corresponds to fractal dimensions of gravity anomalies on Mars.
The next stage is determination of fractal dimensions by longitude. To conduct this, cuts of Mars models by Martian meridians in certain Martian longitudes within 0–320 degrees with a step of 25 degrees have been made. As a result, the curve on figure 4 has been plotted.

![Figure 4](image_url)

**Figure 4.** Dependence of fractal dimension of topographical model and gravity anomalies on longitude of Mars. X-axis corresponds to longitude in degrees (λ), Y-axis corresponds to values of fractal dimension D. Line 1 corresponds to fractal dimensions of gravity anomalies on Mars; line 2 corresponds to fractal dimensions of Mars topographical model.

Analysis of figure 4 well is in well agreement with the conclusions drawn for the analysis of figure 3. It only should be said that topographical fractal dimensions behave more evenly while gravitational fractal dimensions show more anomalies than in case of figure 3.

5. **Summary and conclusions**

In this work the fractal dependences for Martian topographic model and anomalies of gravitational potential by longitude as well as by latitude have been defined; the values of mean fractal dimensions have been obtained: mean fractal dimensions of Martian topographic model by latitude \( \bar{D} = 0.86 \), by longitude \( \bar{D} = 0.88 \); mean fractal dimensions of Martian gravitational potential anomalies by latitude \( \bar{D} = 1.06 \), by longitude \( \bar{D} = 1.092 \).

It is also found that use of fractal method for topological and gravimetric structures variations allows to estimate them by topological inhomogeneity and to draw conclusion about their structural similarity.

Thus, investigation of fractal structures is a very important and interesting direction in the studies of physical objects as well as natural systems [16]. A lot of materials are heterogeneous. Structures of disordered materials or connections between their structures and formation processes or properties are still not enough studied. The reason for it is that the methods of study of such structures have been developed recently. The qualitative feature of fractal objects is their inherent invariance when rescaling. Fractal geometry allows to estimate inhomogeneity of complex physical systems quantitatively.
We should also note that use of fractal analysis in this work allows to introduce independent estimates of Martian macrostructure which leads to new approaches to interpret physical processes taking place on Mars. The further use of fractal comparative analysis at reduce of space measurements data will surely bring some interesting results which will allow to solve certain problems of space astrometry.

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