Probing the field-induced variation of the chemical potential in $Bi_2Sr_2CaCu_2O_y$ via the magneto-thermopower measurements

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Approximating the shape of the measured in textured $Bi_2Sr_2CaCu_2O_y$ magneto-thermopower (TEP) $\Delta S(T,H)$ by asymmetric linear triangle of the form $\Delta S(T,H) \simeq S_p(H) \pm B^\pm(H)(T_c - T)$ with positive $B^-(H)$ and $B^+(H)$ defined below and above $T_c$, we observe that $B^+(H) \approx 2B^-(H)$. In order to account for this asymmetry, we explicitly introduce the field-dependent chemical potential of holes $\mu(H)$ in the Ginzburg-Landau theory and calculate both an average $\Delta S_{av}(T,H)$ and fluctuation $\Delta S_{fl}(T,H)$ contributions to the total magneto-TEP $\Delta S(T,H)$. As a result, we find a rather simple relationship between the field-induced variation of the chemical potential in this material and the above-mentioned magneto-TEP data around $T_c$, viz. $\Delta \mu(H) \propto S_p(H)$.

As is well-known, the variation of the chemical potential $\mu$ of free carriers in an applied magnetic field $H$ provides a direct information about the magnetization structure inside a superconducting sample. Namely, the field-induced change of the chemical potential in superconducting state reads $\Delta \mu(H) \equiv (\mu(H) - \mu(0)) = -M(H)H/n$, where $M(H)$ is the field-induced magnetization, and $n$ the carrier number density. At the same time, due to the existence of the so-called compensation effect, it is rather difficult to observe field-induced modulations of $\mu$ in bulk samples since in equilibrium any field-induced variations of $\mu$ will be completely canceled by similar variations caused by the magnetostriuctive changes of the volume. However, this compensation does not occur in thin films and oriented powders. And thus we can expect to see some tangible changes of $\mu(H)$ in layered (anisotropic) structures as well. On the other hand, in view of its carrier sensitive nature, thermopower (TEP) measurements seem to be the most adequate tool for probing the field-induced changes of the chemical potentials. Indeed, TEP results have already proved to be useful for providing reasonable estimates for such important physical parameters as the Fermi energy, Debye temperature, interlayer spacing etc. Studying the observable magneto-TEP $\Delta S(T,H) = S(T,H) - S(T,0)$ also provides important insights into different aspects of the material in the mixed state (when $H_{c1} < H < H_{c2}$). When experimental results are presented in the form of the above-defined $\Delta S(T,H)$ one observes that its temperature dependence has a Λ-like shape asymmetric around $T_c$ where it reaches its magnetic field-dependent peak value $S_p(H) \equiv \Delta S(T_c,H)$. Then, for small fields, approximating the shape of $\Delta S(T,H)$ by the asymmetric linear triangle of the form $\Delta S(T,H) \simeq S_p(H) \pm B^\pm(H)(T_c - T)$, (1)

with positive slopes $B^-(H)$ and $B^+(H)$ defined for $T < T_c$ and $T > T_c$, respectively, one finds (see Fig.1) that $B^+(H) \approx 2B^-(H)$ in the vicinity of $T_c$.

In the present paper, using the Ginzburg-Landau theory and utilizing some typical magneto-TEP data on textured $Bi_2Sr_2CaCu_2O_y$, we discuss the mixed-state behavior of the magneto-TEP (and in particular the origin of the asymmetry given by Eq.(1)) via the corresponding behavior of the chemical potential in applied magnetic field.

It is well-known that for external fields $H$ such that $H_{c1} < H < H_{c2}$ and for the Ginzburg-Landau parameter $\kappa \gg 1$, the magneto-TEP $\Delta S(T,H)$ is proportional to the strength of the external field. To describe the observed behavior of the magneto-TEP both below and above $T_c$, we can roughly present it in a two-term contribution form

$\Delta S(T,H) = \Delta S_{av}(T,H) + \Delta S_{fl}(T,H)$, (2)

where the average term $\Delta S_{av}(T,H)$ is assumed to be non-zero only below $T_c$ (since in the normal state the TEP of high-$T_c$ superconductors (HTS) is found to be very small), while the fluctuation term $\Delta S_{fl}(T,H)$ should contribute to the observable $\Delta S(T,H)$ for $T \simeq T_c$. In what follows, we shall discuss these two contributions separately within a mean-field theory approximation.

a. Mean value of the magneto-TEP: $\Delta S_{av}(T,H)$. Assuming that the net result of the magnetic field is to modify the chemical potential (Fermi energy) $\mu$ of quasiparticles, we can write the generalized GL free energy functional $\mathcal{G}$ of a superconducting sample in the mixed state as
\[ G[\psi] = a(T)|\psi|^2 + \frac{\beta}{2}|\psi|^4 - \mu|\psi|^2. \] (3)

Here \( \psi = |\psi|e^{i\phi} \) is the superconducting order parameter, \( \mu(H) \) stands for the field-dependent in-plane chemical potential of quasiparticles; \( a(T, H) = a(H)(T - T_c) \) and the GL parameters \( a(H) \) and \( \beta(H) \) are related to the critical temperature \( T_c \), zero-temperature BCS gap \( \Delta_0 = 1.76k_B T_c \), the out-of-plane chemical potential of quasiparticles (Fermi energy) \( \mu_c(H) \), and the total particle number density \( n \) as \( a(H) = \beta(H)n/T_c = 2\Delta_0k_B/\mu_c(H) \). In fact, in layered superconductors, \( \mu = \mu_c/\gamma^2 \approx m_{ab}^*(J_d/2b)^2 \), where \( d \) and \( J_c \) are the interlayer distance and coupling energy within the Lawrence-Doniach model, and \( \gamma = \sqrt{m_c^2/m_{ab}^*} \) is the mass anisotropy ratio. The magnetic field is applied normally to the \( ab \)-plane where the strongest magneto-TEP effects are expected. In what follows, we ignore the field dependence of the critical temperature since for all fields under discussion \( T_c(H) = T_c(0)(1 - H/H_{c2}) \approx T_c(0) \equiv T_c \).

As usual, the equilibrium state of such a system is determined from the minimum energy condition \( \partial G/\partial |\psi| = 0 \) which yields for \( T < T_c \)

\[ |\psi|^2 = \frac{\alpha(H)(T_c - T) + \mu(H)}{\beta(H)} \] (4)

Substituting \( |\psi|^2 \) into Eq. (3) we obtain for the average free energy density

\[ \Omega(T, H) \equiv G[\psi] = -\frac{[\alpha(H)(T_c - T) + \mu(H)]^2}{2\beta(H)} \] (5)

In turn, the magneto-TEP \( \Delta S(T, H) \) can be related to the corresponding difference of transport entropies

\[ \Delta S(T, H) = \Delta S(T, H)/en, \] where \( e \) is the charge of the quasiparticles. Finally the mean value of the mixed-state magneto-TEP reads (below \( T_c \))

\[ \Delta S_{av}(T, H) = S_{p,av}(H) - B_{av}(H)(T_c - T), \] (6)

with

\[ S_{p,av}(H) = \frac{\Delta\mu(H)}{eT_c}, \] (7)

and

\[ B_{av}(H) = \frac{8\Delta_0k_B\Delta\mu(H)}{eT_c^2\mu^2(0)}. \] (8)

Before we proceed to compare the above theoretical findings with the available experimental data, we first have to estimate the corresponding fluctuation contributions to the observable magneto-TEP, both above and below \( T_c \).

b. Mean-field Gaussian fluctuations of the magneto-TEP: \( \Delta S_{fl}(T, H) \). The influence of superconducting fluctuations on transport properties of HTS (including TEP and electrical conductivity) has been extensively studied for the past few years (see, e.g., and further references therein). In particular, it was found that the fluctuation-induced behavior may extend to temperatures more than 10K higher than the respective \( T_c \).

Let us consider now the region near \( T_c \) and discuss the Gaussian fluctuations of the mixed-state magneto-TEP \( \Delta S_{fl}(T, H) \). Recall that according to the theory of Gaussian fluctuations, the fluctuations of any observable, which is conjugated to the order parameter \( \psi \) (such as heat capacity, susceptibility, etc) can be presented in terms of the statistical average of the square of the fluctuation amplitude \( \langle (\delta\psi)^2 \rangle \) with \( \delta\psi = \psi - \psi_0 \). Then the TEP above \((+)\) and below \((-)\) \( T_c \) have the form of

\[ S^\pm_{fl}(T, H) = A < (\delta\psi)^2 > = \frac{A}{Z} \int d|\psi|(\delta\psi)^2 e^{-\Sigma[\psi]}, \] (9)

where \( Z = \int d|\psi|e^{-\Sigma[\psi]} \) is the partition function with \( \Sigma[\psi] = G[\psi] = G[\psi_0] \). \( A \) is a coefficient to be defined below. Expanding the free energy density functional \( G[\psi] \)

\[ G[\psi] \approx G[\psi_0] + \frac{1}{2} \left[ \frac{\partial^2 G}{\partial \psi^2} \right]_{\psi = \psi_0} (\delta\psi)^2, \] (10)

around the mean value of the order parameter \( \psi_0 \), which is defined as a stable solution of equation \( \partial G/\partial |\psi| = 0 \) we can explicitly calculate the Gaussian integrals. Due to the fact that \( |\psi_0|^2 \) is given by Eq.(4) below \( T_c \) and vanishes at \( T \geq T_c \), we obtain finally

\[ S^\pm_{fl}(T, H) = \frac{Ak_BT_c}{4\alpha(H)(T_c - T) + 4\mu(H)}, \] (11)

\[ S^\mp_{fl}(T, H) = \frac{2Ak_BT_c}{(T_c - T) - 2\mu(H)}, \] (12)

As we shall see below, for the experimental range of parameters under discussion, \( \mu(H)/\alpha(H) \gg |T_c - T| \). Hence, with a good accuracy we can linearize Eqs.(11) and (12) and obtain for the fluctuation contribution to the magneto-TEP

\[ \Delta S_{fl}(T, H) \approx S^\pm_{p,fl}(H) + B^\pm_{fl}(H)(T_c - T), \] (13)

where

\[ S^\pm_{p,fl}(H) = -\frac{Ak_BT_c\Delta\mu(H)}{4\mu^2(0)}, \quad S^\mp_{p,fl}(H) = -2S^\pm_{p,fl}(H), \] (14)

and

\[ B^\pm_{fl}(H) = -\frac{3Ak^2_BT_c\Delta\mu(H)}{\gamma^2\mu^4(0)}, \quad B^\mp_{fl}(H) = -2B^\pm_{fl}(H). \] (15)
Furthermore, it is reasonable to assume that $S_p^+ = S_p^-$, where $S_p^- = S_{p,av} + S_{p,fl}$ and $S_p^+ = S_{p,fl}$. Then the above equations bring about the following explicit expression for the constant parameter $A$, namely $A = 4\mu^3(0)/3keBT_c^2$. This in turn leads to the following expressions for the fluctuation contribution to peaks and slopes through their average counterparts (see Eqs.(7) and (8)): $S_{p,fl}(H) = (2/3)S_{p,av}(H)$, $S_{p,fl}(H)^{-} = -(1/3)S_{p,av}(H)$, $B_{fl}(H) = -(1/2)B_{av}(H)$, and $B_{fl}(H) = B_{av}(H)$. Finally, the total contribution to the observable magneto-TEP reads (Cf. Eq.(1))

$$\Delta S(T, H) = S_p(H) \pm B^\pm(H)(T_c - T), \quad (16)$$

where

$$S_p(H) = \frac{2\Delta \mu(H)}{3eT_c}, \quad B^+(H) = B_{fl}^+(H) = 2B^-(H), \quad (17)$$

and

$$B^{-}(H) = B_{av}(H) + B_{fl}(H) = \frac{4\Delta \mu(H)}{eT_c\gamma^2\mu^2(0)}. \quad (18)$$

Let us compare now the obtained theoretical expressions with the typical experimental data on textured $Bi_2Sr_2CaCu_2O_y$ for the slopes $B^\pm(H)$ and the peak $S_p(H)$ values for $H = 0.12T$ (see Fig.1): $S_p = 0.16 \pm 0.01 \mu V/K$, $B^- = 0.012 \pm 0.001 \mu V/K^2$, and $B^+ = 0.027 \pm 0.003 \mu V/K^2$. First we notice that the calculated slopes $B^+(H)$ above $T_c$ are twice their counterparts below $T_c$, i.e., $B^+(H) = 2B^-(H)$ in a good agreement with the observations. Using $\gamma \approx 55$ and $d = 1.2nm$ for the anisotropy ratio and interlayer distance in this material, we obtain reasonable estimates of the field-induced changes of the in-plane chemical potential (Fermi energy) $\Delta \mu(H)$ (along with its zero-field value $\mu(0)$) and the interlayer coupling energy $J_c$. Namely, $\mu(0) \approx 1.6meV$, $\Delta \mu(H) \approx 0.02meV$, and $J_c \approx 4meV$. Furthermore, relating the field-induced variation of the in-plane chemical potential to the change of the corresponding magnetization $M(H)$, viz.

$$\Delta \mu(H) = -\frac{M(H)H}{n_h}, \quad (19)$$

where $M(H)$ for $H_{c1} \ll H \ll H_{c2}$ has a form
denoted as (recall that the lower critical field for this material is $H_{c1} = (\phi_0/4\pi\lambda_{ab}^2)\ln \kappa \approx 40G$ with $\lambda_{ab} \approx 250nm$, $\xi_{ab} \approx 1nm$, and $\kappa \approx 250$)

$$\mu_0 M(H) = \frac{2\phi_0}{\sqrt{3}\lambda_{ab}^2} \left(\ln \left[\frac{3\phi_0}{4\pi\lambda_{ab}^2(H - H_{c1})}\right]\right)^{-2} - H, \quad (20)$$

we obtain $n_h \approx 2.5 \times 10^{27}m^{-3}$ for the hole number density in this material, in reasonable agreement with the other estimates of this parameter. Fig.2 shows $\Delta \mu(H)$ calculated according to Eq.(19) with the experimental data points deduced (via Eq.(17)) from the magneto-TEP measurements on the same sample. As is seen, the data are in a good agreement with the model predictions. And finally, using the above parameters (along with the critical temperature), we find that $\mu(H)/\alpha(H) \approx 100K$ which justifies the use of the linearized Eq.(13) since, as is seen in Fig.1, the observed magneto-TEP practically vanishes for $|T_c - T| \geq 15K$.

In conclusion, to probe the variation of chemical potential $\Delta \mu(H)$ of quasiparticles in anisotropic materials under an applied magnetic field, we calculated the mixed-state magneto-thermopower $\Delta S(T, H)$ in the presence of field-modulated charge effects near $T_c$. Using the available magneto-TEP experimental data on textured $Bi_2Sr_2CaCu_2O_y$, field-induced behavior of in-plane $\Delta \mu(H)$ was obtained along with reasonable estimates for its zero-field value (Fermi energy) $\mu(0)$, interlayer coupling energy $J_c$, and the hole number density $n_h$ in this material.

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FIG. 1. A typical pattern of the observed magneto-TEP $\Delta S(T, H)$ of textured $Bi_2Sr_2CaCu_2O_y$ at $H = 0.12T$. The best fit to the data points according to Eq.(1) yields $S_p(H) = 0.16 \pm 0.01\mu V/K$, $B^-(H) = 0.012 \pm 0.001\mu V/K^2$, and $B^+(H) = 0.027 \pm 0.003\mu V/K^2$ for the peak and slopes.

FIG. 2. The change of the chemical potential $\Delta \mu(H)$ in applied magnetic field calculated according to Eq.(19). The experimental points are deduced from the magneto-TEP data on $Bi_2Sr_2CaCu_2O_y$ and related to $\Delta \mu(H)$ via Eq.(17).