Bose enhancement and the ridge

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We point out that Bose enhancement in a hadronic wave function generically leads to correlations between produced particles. We show explicitly, by calculating the projectile density matrix in the Color Glass Condensate approach to high-energy hadronic collisions, that the Bose enhancement of gluons in the projectile leads to azimuthal collimation of long range rapidity correlations of the produced particles, the so-called ridge correlations.

The ridge structure observed in high multiplicity p-p and p-Pb collisions at the Large Hadron Collider (LHC) triggered an intense activity aimed at understanding the possible physical origin of correlations between emitted particles. Two basic ideas have been put forward in this context (see others in [3]). According to one idea, the origin of the correlations is the same as in similar ridge correlations observed earlier in heavy ion collisions at the Relativistic Heavy Ion Collider [4] and the LHC [5]. Namely, the angular collimation is due to flow effects in the final state [6]. The qualitative features of the high multiplicity p-p and p-Pb data, including the dependence on masses of produced particles, are well described by the hydrodynamic-based models. It is nevertheless challenging to explain how the spatially small system produced in the final state in p-p collisions can sustain the collective behavior necessary for local equilibration.

The second suggestion is that the final state correlations carry the imprint of the partonic correlations that exist in the initial state. Three different variants of such initial state effects have been discussed in the literature: local anisotropy of target fields [7], spatial variation of partonic density [8] and finally the “glasma graph” contributions to particle production [9] within the Color Glass Condensate (CGC) approach to high-energy hadronic scattering. While the physical origin of the first two effects is quite clear, the physics behind glasma graphs has not been elucidated in the literature. On the other hand, numerical calculations based on the glasma graph approach have been very successful in reproducing the systematics of ridge correlations [10]. It is therefore important to understand the physics that underlies these numerical results.

The purpose of this note is to point out that there exists a general quantum mechanical mechanism that leads to positive correlations of emitted particles with similar quantum numbers. It is operative when the wave function of an incoming hadron is dominated by bosons (gluons), and is due to Bose enhancement in this wave function. After recalling the basic derivation, we will show that this is precisely the physical mechanism that underlies the glasma graph calculation of hadron production in p-p and p-A collisions. The mechanism itself is however more general, has been widely used for identical mesons in heavy-ion collisions, see e.g. [11], and should lead to similar correlations in other observables, for example in di-photon production [12]. Analogously, for fermions in the initial state one expects the opposite effect, namely Pauli blocking. We briefly discuss which final state observables could be sensitive to the initial state Pauli blocking.

The prototypical textbook calculation of Bose enhancement proceeds as follows [13]. Consider a state with fixed occupation numbers of N species of bosons at different momenta, \(|\{n^i(p)\}\rangle = \prod_{i,p} \frac{1}{\sqrt{n^i(p)!}} |a^i(p)\rangle |0\rangle, \quad i = 1, \ldots, N,\) with a finite volume V and periodic boundary conditions so that momenta are discrete. The state is translationally invariant with mean particle density

\[ n \equiv \langle \{n^i(p)\} |a^{i\dagger}(x)a^{i}(x)|\{n^i(p)\}\rangle = \sum_{i,p} n^i(p). \] (1)

Hereafter we take \(\sum_i n^i \approx \int d^3p/(2\pi)^3\). The 2-particle correlator in coordinate space is

\[ D(x, y) \equiv \langle \{n(p)\} |a^{i\dagger}(x)a^{j\dagger}(y)a^{j}(x)a^{i}(y)|\{n(p)\}\rangle. \] (2)

This is calculated by going to momentum space, where the operator averages are simple:

\[ \langle \{n(p)\} |a^{i\dagger}(p)a^{j\dagger}(q)a^{j}(l)a^{i}(m)|\{n(p)\}\rangle \]
\[ = \langle \{n(p)\} |\delta(p-l)\delta(q-m)a^{i\dagger}(p)a^{j\dagger}(q)a^{j}(q)a^{i}(l)|\{n(p)\}\rangle \]
\[ = \delta(p-l)\delta(q-m)\sum_{i,j} n^i(p) n^j(q) + \delta(p-m)\delta(q-l)\sum_{i,j} n^i(p) n^j(q), \] (3)

where we have neglected the terms where all momenta are equal, which are suppressed by a phase space factor.
Using this, the result for $D(x, y)$ reads

$$D(x, y) = n^2 + \sum_{i} \left| \int \frac{d^3p}{(2\pi)^3} \ e^{ip(x-y)} n^i(p) \right|^2. \quad (4)$$

The last term expresses the Bose enhancement. It vanishes when the points are very far away, and gives $O(1/N)$ enhancement when the points coincide. The $O(1/N)$ suppression of the second term relative to the first one is due to the fact that the second term contains a single sum over the species index. The physics is that only bosons of the same species are correlated with each other. Technically the origin of this additional contribution is the “wrong contraction” term in eq. (3).

The Bose enhancement is a generic phenomenon, and is not tied specifically to the state with fixed number of particles. An overwhelming majority of pure states or quantum density matrices exhibit Bose enhancement at some degree. There is however one type of states that do not exhibit such behavior, notably classical-like coherent states. Consider a coherent state

$$|b(x)\rangle \equiv \exp\{i \int d^3x \ b^i(x)(a^i(x) + a^{ii}(x))\} |0\rangle. \quad (5)$$

A trivial calculation in this state gives

$$\langle b(x)|a^{ii}(x)a^i(x)|b(x)\rangle = b^i(x)b^i(x), \quad (6)$$

$$\langle b(x)|a^{ij}(x)a^j(x)a^i(y)|b(x)\rangle = b^i(x)b^j(y)b^j(y),$$

so $D(x, y) = n(x)n(y)$. Thus, in order to exhibit Bose enhancement, a state has to be nonclassical.

As stated above, we want to demonstrate that the angular collimation arising from the glasma graph calculation owes its existence to the Bose enhancement in the projectile wave function. Following [11, 14], we consider the calculation of inclusive two particle production. The glasma graphs that contribute to this observable come in two varieties, see fig. [1]. Type A graphs give the contribution whereby two gluons from the incoming projectile wave function scatter independently on the target. The incoming gluons have transverse momenta $k_1$ and $k_2$ respectively. While propagating through the target the first particle picks up transverse momentum $p - k_1$ and the second particle picks up transverse momentum $q - k_2$, so that the outgoing particles have momenta $p$ and $q$. Type B graphs from the projectile point of view are “interference graphs”, in the sense that the final state gluon with momentum $p$ comes from the projectile gluons with different momenta in the amplitude and complex conjugate amplitude. Assuming similar factorization for projectile and target fields (charge densities), Type B graphs can be reinterpreted as Type A but with gluons originating from the target. In the following we will only discuss the Type A contribution, keeping in mind the complementary interpretation of the Type B contribution.

The Type A contribution to the double inclusive gluon production can be written as

$$C \int_{k_1, k_2} \langle in|a^{ii}_a(k_1)a^{ij}_b(k_2)a^{ij}_a(k_1)a^{ii}_b(k_2)|in\rangle \times N(p - k_1)N(q - k_2), \quad (7)$$

where $|in\rangle$ is the wave function of the incoming projectile, $C$ is a constant, $N(p - k)$ is the probability that the incoming gluon with transverse momentum $k$ acquires transverse momentum $p$ after scattering and, hereafter, we use the notation $\int_{k_1} \equiv \int \frac{d^2k_1}{4\pi}$. This scattering probability is of course determined by the distribution of target fields. In this equation we have assumed that the scattering of the two gluons is independent and that the target wave function is translationally invariant, so that the momentum transfer is the same to the gluon in the amplitude and complex conjugate amplitude. The last assumption does not allow one to discuss the correlation mechanisms proposed in [7, 8] within this framework.

Also note that we have not indicated the rapidity variable on the gluon creation and annihilation operators. Within the glasma graph calculation the gluon production is rapidity independent. Rapidity dependence becomes significant only when the rapidity difference between the observed hadrons becomes large, $\Delta \eta \sim 1/\alpha_s$. The origin of this independence is that the CGC hadronic wave function is approximately boost-invariant. In fact, only the rapidity independent mode of the gluon field is large in the wave function of the fast hadron, and only the creation operators of this one rapidity mode are relevant to the discussion of correlations.

Thus, the creation and annihilation operators entering the above equation are the original gluon operators integrated over rapidity,

$$a^i_a(k) \equiv \frac{1}{\sqrt{\pi}} \int_{|\eta|<Y/2} \frac{d\eta}{2\pi} a^i_a(\eta, k). \quad (8)$$

Here the rapidity interval $Y/2$ is arbitrary, but large enough to contain the rapidities of both observed gluons. The operators defined this way satisfy the standard commutation relations in the transverse momentum space:

$$[a^i_a(k), a^j_b(p)] = (2\pi)^2 \delta^i_a \delta^j_b \delta(k - p). \quad (9)$$

The integral over momenta $k_1, k_2$ in eq. (7) contains a contribution from the region $k_1 = k_2$. If the wave function $|in\rangle$ exhibits Bose enhancement, there is enhanced probability that the two gluons have equal momenta. This excess in the initial state will then translate into final state correlations. Note that this effect is suppressed by the squared number of colors $1/N_c^2$, since Bose enhancement is only operational for bosons with identical quantum numbers.

We thus have to understand what is the nature of the projectile state $|in\rangle$, and in particular we need to calculate

$$D(k_1, k_2) \equiv \langle in|a^{ij}_a(k_1)a^{ij}_b(k_2)a^i_a(k_1)a^i_b(k_2)|in\rangle. \quad (10)$$
Averaging over the projectile state in the standard CGC approach involves two elements. One needs to calculate the average over the soft degrees of freedom, as well as that over the valence color charge density. Conventionally this is done in the spirit of the Born-Oppenheimer approximation, namely first one averages over the soft gluon degrees of freedom at fixed valence color charge density, and subsequently averages over the valence density distribution.

The wave function of the soft fields for fixed valence color charge density for a dilute projectile is a simple coherent state

$$|in\rangle_\rho = \exp \left\{ i \int_k b_\alpha^i(k) \left[ a_\alpha^{i\dagger}(k) + a_\alpha^i(-k) \right] \right\} |0\rangle,$$

with the Weizsäcker-Williams field $b_\alpha^i(k) = g \rho_\alpha(k) \frac{\phi^i\dagger}{2\pi^2}$.

The averaging over the soft degrees of freedom leads to the well known expression for the observable in terms of the charge density:

$$D(k_1, k_2)_\rho = b_\alpha^i(k_1) b_\alpha^{i\dagger}(k_2) b_\alpha^i(-k_1) b_\alpha^{i\dagger}(-k_2).$$

Since at fixed $\rho$, the soft gluon state is a coherent state, this expression does not seem to exhibit Bose enhancement. This is however misleading, since averaging over $\rho$ is part of the quantum averaging over the initial state wave function $|in\rangle$. It is therefore instructive to reverse the conventional order of averaging, and average over the valence degrees of freedom first. The result of such a procedure is a density matrix on the soft gluon Hilbert space. The subsequent averaging over this density matrix is a density matrix on the soft gluon Hilbert space.

The soft gluon density matrix of course depends on the weight for the valence color charge density. For illustrative purposes we choose the same Gaussian weight used in the glasma graph calculation, the McLerran-Venugopalan model $^{[15]}$,

$$\langle \cdots \rangle_\rho = \mathcal{N} \int D[\rho] \cdots e^{-\int_k \frac{g^2}{2\pi^2} \rho_\alpha(k) \rho_\alpha(-k)},$$

where $\mathcal{N}$ is the normalization factor.

Thus the density matrix of the soft gluons is given by

$$\hat{\rho} = \mathcal{N} \int D[\rho] e^{-\int_k \frac{g^2}{2\pi^2} \rho_\alpha(k) \rho_\alpha(-k)} \times e^{\int_k b_\alpha^i(k) \phi^i\dagger(-k) \phi^i(-k) - \int_k b_\alpha^{i\dagger}(k) \phi^i\dagger(-k) \phi^i(-k)}$$

where we have defined $\phi_\alpha^i(k) = a_\alpha^{i\dagger}(k) + a_\alpha^i(-k)$. The integral over $\rho$ can be performed with the result

$$\hat{\rho} = e^{-\int_k \frac{g^2}{2\pi^2} \rho_\alpha(k) \rho_\alpha(-k)} \times \left\{ \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{m=1}^{n} \frac{g^2 \mu^2(p_m)}{p_m^4} \phi^i_{\alpha_m}(p_m) \phi^{i\dagger}_{\alpha_m}(p_m) \right\} \times \langle 0 | \prod_{m=1}^{n} p_m^{i\dagger} \phi^i_{\alpha_m}(p_m) |0\rangle \times e^{-\int_k \frac{g^2}{2\pi^2} \rho_\alpha(k) \rho_\alpha(-k)}$$

The interesting correlator is given by

$$D(k_1, k_2) = \text{tr}[\hat{\rho} a_\alpha^{i\dagger}(k_1) a^{i\dagger}_\alpha(k_2) a_\alpha^i(k_1) a^{i\dagger}_\alpha(k_2)].$$

Using the following auxiliary formula

$$\hat{\rho} a_\alpha^i(p) = -\hat{\rho} a_\alpha^{i\dagger}(-p) = g^2 \mu^2(p) \frac{p^i p^j}{p^4} [\hat{\rho}, \phi^j_{\alpha}(p)],$$

it is a matter of straightforward algebra to show that

$$\text{tr}[\hat{\rho} a_\alpha^{i\dagger}(k) a^{i\dagger}_\alpha(p)] = \frac{(2\pi)^2 \delta^{(2)}(k - p) g^2 \mu^2(p) p^i p^j}{p^4},$$

$$\text{tr}[\hat{\rho} a_\alpha^i(k) a^{i\dagger}_\alpha(p)] = \frac{(2\pi)^2 \delta^{(2)}(k - p) g^2 \mu^2(p) p^i p^j}{p^4}.$$
and then find
\[
D(k_1, k_2) = S^2(N_e^2 - 1)^2 \left\{ g^4 \mu^2(k_1) \mu^2(k_2) \frac{1}{k_1 k_2} \right. \]
\[
+ \frac{1}{S(N_e^2 - 1)} \left[ \delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2) \right] \frac{g^4 \mu^2(k_1)}{k_1^2} \}.
\]

In order to get eq. (19), we have made substitutions of the type $(2\pi)^2 \delta^{(2)}(k_1 - k_1) \rightarrow S$, where $S$ is the transverse area of the projectile. This regularization amounts to taking into account the discreteness of the transverse momentum spectrum of confined gluons.

The first term in this equation is the "classical" term equal to the square of the number of particles. The second term is the typical Bose enhancement term, suppressed with respect to the first "classical" term by the total number of degrees of freedom (color and area). The third term is specific to the density matrix at hand and, as explained in [7], appears due to reality of the gluon field scattering amplitude.

This establishes our point that the soft glue density matrix exhibits Bose enhancement, so that the likelihood of finding two gluons with the same transverse momentum is higher than average. Note that this effect is naturally subleading in $N_e$ as the enhancement is only effective if both gluons are in the same color state.

Returning now to eq. (7), it is clear that this initial state correlation must also appear as a correlation between final state particles. This is especially true if the projectile wave function has an intrinsic saturation momentum $Q_s$, and if the transverse momenta of the final state particles are chosen to be close to it. In such a situation the production amplitude is dominated by the contribution from $k_1, k_2 \sim Q_s$ and the scattering probabilities $N(p - k_1), N(q - k_2)$ do not sizeably change the distribution. Thus the initial correlation is transmitted to the final state with minimal distortion, provided fragmentation and final state effects in $p$-p and $p$-A collisions are small as expected. On the other hand, when $|p|, |q| \gg Q_s$, the initial correlation is smeared out by the large momentum transfer from the target, and the correlation in the final state should disappear. These qualitative features are of course borne out by the numerical calculations of [10].

One interesting question naturally follows on from the above discussion. Fermions in the initial state wave function surely experience Pauli blocking. One therefore may expect negative correlation between final state hadrons that originate from quarks or antiquarks in the initial state. Such correlation should exhibit anticollimation rather than collimation, and therefore a valley rather than a ridge at $\Delta \phi = 0$. Whether such a valley extends to large relative rapidities between the observed particles is a question that should be explored. Quark-antiquark pairs are present in the hadronic wave function within the CGC approach at the next to leading order in $\alpha_s$ via splitting of gluons. Since the gluonic wave function is boost invariant, the same is true for the quark and antiquark distribution. However, the main question here is whether the fluctuations around some "mean field" are not too large to mask the correlations in rapidity event by event. Another way of saying it, is to recall that in our discussion of gluons only a single rapidity independent mode of the quantum gluon field was large in the CGC wave function. As a result any correlation extended over large intervals in rapidity. Whether a similar effect dominates the quark wave function has to be investigated. Work on these questions is ongoing [10].

Perhaps an even more pressing question to understand is whether such valleys can be observed experimentally, given that the quark contribution is suppressed by $\alpha_s$ relative to that of gluons. Here we see two possible avenues. One point is that, as opposed to gluon contribution to correlations, the quark contribution is not symmetric under $\Delta \phi \rightarrow -\Delta \phi$. It thus can generate a non-vanishing $v_3$ coefficient within the CGC approach. Such mechanism will be quite different from the one explored in [17] based on the idea of local anisotropy suggested in [6]. Another possibility is to trigger on final states which predominantly arise from quarks. For example it may be interesting to study correlations between two D-mesons (or B-mesons), since open charm (or beauty) should have a relatively larger component coming from fragmentation of quarks, rather than that of gluons [10].

We finish with a remark which is somewhat peripheral to our main subject. Note that eq. (15) gives an explicit form of the soft gluon density matrix in the incoming hadron. Using similar methods one can calculate the density matrix in the final state [18]. This will contain the complete information about the distributions of particles in the final state. For example, it should encode in a concise way the negative binomial distribution calculated in [19]. The knowledge of the density matrix will also open a way of directly calculating the entropy produced in high-energy collisions in the CGC approach [18].

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