A new interpretation of superposition, entanglement, and measurement in quantum mechanics

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Abstract

We present a new interpretation of the terms superposition, entanglement, and measurement that appear in quantum mechanics. We hypothesize that the structure of the wave function for a quantum system at the sub-Planck scale has a deterministic cyclic structure. Each cycle comprises a sequential succession of the eigenstates that comprise a given wave function. Between unitary operations or measurements on the wave function, the sequential arrangement of the current eigenstates chosen by the system is immaterial, but once chosen it remains fixed until another unitary operation or measurement changes the wave function. The probabilistic aspect of quantum mechanics is interpreted by hypothesizing a measurement mechanism which acts instantaneously but the instant of measurement is chosen randomly by the classical measuring apparatus over a small but finite interval from the time the measurement apparatus is activated. At the instant the measurement is made, the wave function irrevocably collapses to a new state (erasing some of the past quantum information) and continues from thereon in that state till changed by a unitary operation or a new measurement.

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**Introduction**

Modern quantum mechanics is an axiomatic system and is based on the following four postulates \[1\]:

**Postulate 1:** Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space in finite space) known as the state space of the system. The system is completely described by its state vector, which is a unit vector in the system’s state space.

**Postulate 2:** The evolution of a closed quantum mechanical system is described by a unitary transformation. That is, the state \( |\psi(t_1)\rangle \) of the system at time \( t_1 \) is related to the state \( |\psi(t_2)\rangle \) of the system at time \( t_2 \) by a unitary operator \( U \) which depends only on the times \( t_1 \) and \( t_2 \),

\[
|\psi(t_2)\rangle = U|\psi(t_1)\rangle.
\]

A more refined version of Postulate 2 is the linear Schrödinger equation, which describes the evolution of a closed quantum system in continuous time,

\[
i\hbar \frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle.
\]

Here \( \hbar \) is the reduced Planck’s constant, and \( H \) is a fixed Hermitian operator known as the quantum Hamiltonian operator of the closed system. \( H \) is a self-adjoint operator which acts on the state space and is related to the total energy of the system.

**Postulate 3:** Quantum measurements are described by a collection \( \left\{ M_m \right\} \) of measurement operators. These are operators, which act on the state space of the system being measured. The index \( m \) refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is \( |\psi\rangle \) just before the measurement, then the probability that result \( m \) occurs is given by

\[
p(m) = \langle \psi | M_m^\dagger M_m |\psi\rangle,
\]
and the state of the system after the measurement is,

$$\frac{M_m |\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}},$$

where the measurement operators satisfy the completeness condition,

$$\sum_m M_m^\dagger M_m = I.$$

The completeness condition expresses the requirement that the respective probabilities associated with each state of $|\psi\rangle$ must sum to one:

$$I = \sum_m p(m) = \sum_m \langle\psi|M_m^\dagger M_m|\psi\rangle.$$ ■

**Postulate 4:** The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through $n$, and system number $i$ is prepared in the state $|\psi_i\rangle$ then the joint state of the total system is $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_n\rangle$. ■

As is expected of axiomatic systems, the four postulates are independent of one another.

It is, of course, widely known that while quantum mechanics is the crown jewel of theoretical physics, it is non-intuitive and axiomatically different from classical physics. In quantum mechanics, while the wave function $|\psi\rangle$, introduced in postulate 2, is completely deterministic, its value, even using proxies, cannot be determined by a classical measurement system. A measurement made on a quantum system is governed by a separate postulate (postulate 3) and yields, in general, only partial non-deterministic information about the state of the system immediately prior to the measurement. It is not possible to know the complete state of an unknown quantum system by making multiple measurements on the same system or even on multiple available copies of the system. The very first measurement on a system irreversibly “collapses” the state of the system and it does that in a probabilistic manner. In fact, all the probabilities associated with quantum mechanics are
enshrined only in postulate 3 and not in the other three postulates. The wave function of postulate 2 is an abstract mathematical entity; its origin and any underlying structure at the sub-Planck level that might support it are unknown and open to further speculation or interpretation. Postulate 3 does not tell us what the underlying measurement process is, it only tells us what the measurement result will be only in a probabilistic sense. So the measurement process too is open to further speculation or interpretation. Note that measurement is a nonlinear phenomenon and it is not governed by the Schrödinger equation. Indeed, one is immediately struck by the contradictory nature of postulates 2 and 3. As David Albert notes [2]:

The dynamics and the postulate of collapse are flatly in contradiction with one another ... the postulate of collapse seems to be right about what happens when we make measurements, and the dynamics seems to be bizarrely wrong about what happens when we make measurements, and yet the dynamics seems to be right about what happens whenever we aren’t making measurements.

Lalloë neatly summarizes the dilemmas produced by the two postulates [3]:

Obviously, having two different postulates for the evolution of the same mathematical object is unusual in physics; the notion was a complete novelty when it was introduced, and still remains unique in physics, as well as the source of difficulties. Why are two separate postulates necessary? Where exactly does the range of application of the first stop in favor of the second? More precisely, among all the interactions - or perturbations - that a physical system can undergo, which ones should be considered as normal (Schrödinger evolution), which ones as a measurement (wave packet reduction)? Logically, we are faced with a problem that did not exist before [in classical physics], when nobody thought that measurements should be treated as special processes in physics.

Superposition and entanglement of quantum states are two other intriguing aspects of quantum mechanics. Superposition of states which results from
postulate 2 is the same as is generally understood in the mathematics of linear systems and in classical physics. Postulate 2 does not require one to view superposition where an electron is concurrently in spin-up and spin-down states. It is postulate 3 related to measurement which allows us to adopt the implausible view that an electron can be concurrently in spin-up and spin-down states rather than in some in-between state. Entanglement, viewed at one time as spooky action at a distance, comes from postulates 1, 2 and 4. It is no longer viewed with suspicion [4] since entangled particles are now routinely produced in experiments. John Bell [5] showed hidden variables are not needed to explain the phenomenon and Aspect, et al [6] provided experimental verification that entanglement is real. We have already noted that a quantum system can exist in a continuum of states until it is measured. But this is not the only bizarre consequence of a measurement. When two or more particles are entangled then a measurement on any one particle or a combined measurement on a subgroup of particles will cause a ‘jump’ to occur instantly on the remaining particles even if some or all of them are light years apart. Indeed, distance is irrelevant for entangled particles. A group of entangled particles have a distributed existence in the sense that the group behaves as a single entity even when spread out in space. Such bizarre behaviors have been extensively verified in experiments.

The postulates of quantum mechanics do not enlighten us as to the nature of the underlying structure of the wave function at the sub-Planck scale that would support entanglement, and which, in turn, would lead to a wave function that follows the postulates of quantum mechanics. So, once again, the underlying structure of the wave function is open to speculation or interpretation. The interpretation does not predict results; that is the job of the Schrödinger wave equation and the measurement postulate. However, for any interpretation to be acceptable it must be compatible with measured results; that is, it must put the mathematical model into correspondence with experience. The distinction between an axiomatic system and its interpretation has been elucidated by Hofstadter [7] with great clarity in his book Gödel, Escher, Bach. A given axiomatic system may have more than one interpretation.
Some earlier interpretations of quantum mechanics

While the formalism of quantum mechanics is widely accepted, there is no single interpretation of it that is agreeable to everyone. The disagreements essentially stem from the incompatibility that exists between postulates 2 and 3. Indeed, without postulate 3 telling us what we can observe, the equations of quantum mechanics would be just pure mathematics that would have no physical meaning at all. Note also that any interpretation can come only after an investigation of the logical structure of the postulates of quantum mechanics is made. We briefly digress to elaborate what we mean by an interpretation in the context of this paper.

Newtonian mechanics does not define the structure of matter. How we interpret or model the structure of matter is largely an issue separate from Newtonian mechanics. However, any model of the structure of matter we propose is expected to be such that it is compatible with Newton’s laws of motion in the realm where Newtonian mechanics rules. If it is not, then Newtonian mechanics, as we know it, would have to be abandoned or modified, or the model of the structure of matter would have to be abandoned or modified. One may also have a partial interpretation and leave the rest in abeyance till further insight strikes us and leads us to a complete or a new interpretation. As we know, our understanding (interpretation) of the structure of matter has undergone several changes (including our current understanding of the subatomic structure of matter as enshrined in the still evolving standard model of particle physics) without affecting Newton’s laws of motion. A question such as whether a particular result deduced from Newton’s laws of motion is deducible from a given model of material structure is therefore not relevant.

Likewise, as long as our interpretation (or model) of superposition, entanglement, and measurement does not require the axioms of quantum mechanics to be altered, and as far as we can determine, it does not, none of the predictions made by quantum mechanics should be incompatible with our interpretation. This assertion is important because we make no comments on the Hamiltonian, which captures the detailed dynamics of a quantum system. Quantum mechanics does not tell us how to construct the Hamiltonian.
In fact, real life problems seeking solutions in quantum mechanics need to be addressed in detail by physical theories built within the framework of quantum mechanics. The postulates of quantum mechanics provide only the scaffolding around which detailed physical theories are to be built.

Our interpretation and the postulates of quantum mechanics are two different but related things. Quantum mechanics leaves room for interpretation because the wave function is an abstract mathematical object. Neither its origin nor its underlying structure has been disclosed in the postulates of quantum mechanics. Furthermore, the mechanisms for superposition, entanglement, and measurement too have not been elucidated in quantum mechanics. Hence, as noted earlier, they too are open to interpretation. We have, in a sense, tried to provide a sub-Planck scale view of the wave function, superposition, entanglement, and measurement without affecting the postulates of quantum mechanics. The sub-Planck scale is chosen to provide us with the freedom to construct mechanisms for our interpretation that are not necessarily bound by the laws of quantum mechanics. In particular our interpretation does not have to satisfy the Schrödinger wave equation because quantum mechanics is not expected to rule in the sub-Planck scale. The high point of our interpretation is that it is able to explain the measurement postulate as the inability of a classical measuring device to measure at a precisely predefined time.

In passing we note that there already exist several interpretations of quantum mechanics, each with its own bizarre elements. Human experiences of the world are entirely based on macroscopic objects which behave according to the laws of classical physics. Each attempted interpretation of quantum mechanics has therefore been in a language that lacks the appropriate concepts. Perhaps the most widely preferred by physicists is the Copenhagen interpretation, possibly followed by Everett’s many world interpretation (a variant of which is favored by David Deutsch), and Bohm’s pilot wave interpretation (favorably commented upon by John Bell). Each known interpretation contains concepts or elements that are alien to our everyday experience. No one knows what the reality of a quantum system could be.
The Copenhagen interpretation

We are not aware of any systematic rendition of the Copenhagen interpretation in the scientific literature. As Jammer notes: “The Copenhagen view is not a single, clear-cut, unambiguously defined set of ideas but rather a common denominator for a variety of related viewpoints” [8]. The Copenhagen interpretation (also known as the ‘shut up and calculate’ interpretation) was provided by Niels Bohr and Werner Heisenberg around 1927. In doing so, they extended the probabilistic interpretation of the wave function proposed by Max Born [9]. Some interesting features included in the Copenhagen interpretation are: the position of a particle is essentially meaningless; the act of measurement causes an instantaneous collapse of the wave function and the collapsed state is randomly picked to be one of the many possibilities allowed for by the system’s wave function; the fundamental objects handled by the equations of quantum mechanics are not actual particles that have an extrinsic reality but “probability waves” that merely have the capability of becoming real when an observer makes a measurement. Even then it does not explain entanglement; this experimentally verified “nonlocality” is a mathematical consequence of quantum theory [10]. In the Copenhagen interpretation one cannot describe a quantum system independently of a measuring apparatus. Indeed, it is meaningless to ask about the state of the system in the absence of a measuring system. For example, the Copenhagen interpretation not only does not explain how an electron goes through the two slits in the double slit experiment, it categorically states that even to ask such a question is meaningless and that we should restrict our comments to the observed interference pattern on the screen [11]. The role of the observer is central since it is the observer who decides what he wants to measure [11]. Nevertheless, there is a clear demarcation between the quantum system being measured and the macroscopic measuring device (described by classical mechanics).

In Bohr’s view [11],

... there is no quantum world. There is only abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics [only] concerns what we can say about nature.
This view is very different from that of Einstein’s who believed that the job of physical theories is to ‘approximate as closely as possible to the truth of physical reality’ [11].

**Everett’s many world interpretation**

Hugh Everett III, in his doctoral dissertation of 1956 (see Ref. [12] for the journal version), proposed what he called the “relative state interpretation”, which is now generally known as the many world interpretation (the name was coined by Bryce DeWitt in the late 1960s). This interpretation is perhaps the most bizarre and yet perhaps the simplest (it is free of the measurement problem because Everett omits the measurement postulate) if one is willing to accept that we inhabit one of an infinite number of parallel worlds! Everett assumes that when a quantum system is faced with a choice such as a photon going through one or the other slit in the two-slit experiment, rather than the wave function entering a superposition, the entire world along with it splits into a number of worlds equal to the number of options available. These different worlds are identical to each other except for the different option chosen by the quantum system (such as the photon passing through the upper slit in one world and through the lower slit in another world). Until decoherence, that is, (spontaneous) interactions between a quantum system and its environment leads to the suppression of wave interference, sets in, the worlds overlap only in regions where interference is taking place. Decoherence causes the worlds to separate into non-interacting independent worlds. This allows Everett to avoid the non-intuitive problems related to measurement since there is no measurement process involved. When a world splits, observers in it will also split with it. Thus there will be other copies of the observers in parallel worlds, each of whom will see the specific outcome that appears in his respective split world [11].

Physicists at the time ridiculed Everett’s interpretation (he got his PhD alright and the work was published in Ref. [12]). It was said that “Bohr gave him the brush-off when Everett visited him in Copenhagen” [11]. He was so discouraged by the ridicule that he left physics, became a defense analyst, then a private contractor to the U.S. defense industry which made him a multimillionaire [11, 13]. Variants of the many-world interpretation
are the multiverse interpretation, the many-histories interpretation, and the many-minds interpretation.

**Bohm’s interpretation**

In Bohm’s interpretation [14], which appeared in 1952 and predates Everett’s interpretation, the whole universe is entangled and one cannot isolate one part of the universe from the other. Rather than interpret entanglement as some mysterious phenomenon, as in the Copenhagen interpretation, Bohm favors an interpretation which makes non-locality explicit. In his view, the interaction between entangled particles is not mediated by any conventional field known to physics (such as the electromagnetic field), but by a very special anti-relativistic quantum information field (pilot wave) that does not diminish with distance and that binds the whole universe together. This field is an all pervasive field that is instantaneous. This field is not physically measurable but manifests itself in terms of non-local correlations. The idea is not only interesting but entirely derivable from the Schrödinger equation. Consequently, in Bohm’s interpretation, for example, the electron is a particle with well-defined position and momentum at any instant. However, the path an electron follows is guided by the interaction of its own pilot wave with the pilot waves of other entities in the universe. A major supporter of Bohm’s interpretation was John Bell.

There are other interpretations which we omit from our discussion since a thorough review of them is not our intent. The widely different views from which the various interpretations follow are rather remarkable. In short, as yet there is no unique and fully satisfactory interpretation [11].

**A new interpretation**

Following the principle of Occam’s razor that “entities should not be multiplied unnecessarily” or the law of parsimony, our interpretation adopts the viewpoint that the sub-Planck scale structure of the wave function is such that the wave function is in only one state at any instant but oscillates be-
tween its various “superposed” component eigenstates. (There is no expenditure of energy in maintaining the oscillations.) To this we add a probabilistic measurement model which determines only the instantaneous eigenstate of the system at the instant of measurement. We essentially hypothesize a measurement mechanism which acts instantaneously but the instant of measurement is chosen randomly by the classical measurement apparatus over a small but finite interval from the time the measurement apparatus is activated. In particular, we regard measurement as the joint product of the quantum system and the macroscopic classical measuring apparatus. We do not explain how the collapse of the wave function occurs when a measurement is made. However, once a measurement is made, the wave function assumes the collapsed state. The notion of the collapse of the wave function upon measurement was introduced by Heisenberg in 1929 [11].

In our interpretation, superposed states appear as time-sliced in a cyclic manner such that the time spent by an eigenstate in a cycle is related to the complex amplitudes appearing in the wave function. Entangled states binding two or more particles appear in our interpretation as the synchronization of the sub-Planck level oscillation of the participating particles. Unlike the Copenhagen interpretation, in our interpretation it is not meaningless to ask about the state of the system in the absence of a measuring system. In essence, we seek an interpretation that can be related to the macroscopic world we live in and hence appears more intuitive to the human mind. We note that no rational interpretation of quantum mechanics in terms of images and concepts familiar to us from everyday experience in the macroscopic world has yet been found. Our’s is perhaps the closest yet. The basic sub-Planck model that underpins our interpretation is as follows. Consider a qubit (quantum bit) described by the wave function

\[ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \]

where the particle is in a superposed state comprising the eigenstates |0\rangle and |1\rangle, and \( \alpha, \beta \) are normalized complex constants such that \( |\alpha|^2 + |\beta|^2 = 1 \). The structure of the wave function \( |\psi\rangle \), outside the realm of quantum mechanics and in the realm of sub-Planck scale that we now propose, is illustrated in Figure 1. Here we have two eigenstates, |0\rangle and |1\rangle, and in a cycle time of \( T_c \) the state of the particle oscillates between states |0\rangle and |1\rangle. We assume \( T_c \) to be much smaller than Planck time (\( \ll 10^{-43} \) sec) to allow us to interpret the wave function independently of the Schrödinger equation. (The
implicit assumption here is that in some averaged sense, perhaps with some additional information, these oscillations will represent the wave function $|\psi\rangle$, say, analogous to the case of a volume of gas in classical mechanics, where random molecular motions, appropriately averaged, represent classical pressure, temperature, and density of a volume of gas. It is not necessary for us to know the value of $T_c$. We only assert that it is a universal constant.

Within a cycle, the time spent by the particle in state $|0\rangle$ is $T_0 = |\alpha|^2 T_c$ and in state $|1\rangle$ is $T_1 = |\beta|^2 T_c$ so that $T_c = T_0 + T_1$. Superposition is interpreted here as the deterministic linear sequential progression of the particle’s states $|0\rangle$ and $|1\rangle$. A measurement made on this particle will return the instantaneous state the particle was in at the instant of the measurement.

Our model of the measurement device is as follows. Let $\Delta t_m$ be the time interval during which some chosen measuring device makes a measurement. Here $\Delta t_m$ is assumed to be orders of magnitude greater than Planck time; otherwise its actual value is immaterial. Effectively, the device is assumed to make the measurement instantaneously at some instant, randomly chosen by the measuring device in the interval $\Delta t_m$. To avoid bias, we assume that the device can choose any instant in the interval $\Delta t_m$ with equal probability. Thus the source of indeterminism built into quantum mechanics via postulate 3 is interpreted here as occurring due to the classical measuring device’s inability to measure at a precisely predefined time. As soon as the measurement is made, the quantum system assumes the measured state “instantaneously” as dictated by postulate 3.

Since the measurement device need not operate with the $\{|0\rangle, |1\rangle\}$ basis, we need a rule that allows us to go from one basis to another. The rule turns
out to be rather simple. Let the measurement basis be \{|x\rangle, |y\rangle\} obtained by rotating the basis \{|0\rangle, |1\rangle\} anti-clockwise by the angle \(\theta\), then
\[|0\rangle = \cos \theta |x\rangle + \sin \theta |y\rangle \] and
\[|1\rangle = \cos \theta |y\rangle - \sin \theta |x\rangle \] and conversely
\[|x\rangle = \cos \theta |0\rangle - \sin \theta |1\rangle \] and
\[|y\rangle = \sin \theta |0\rangle + \cos \theta |1\rangle.\] In essence, Figure 2 shows how the vectors \{|0\rangle, |1\rangle\} will be observed by a measuring device in the basis \{|x\rangle, |y\rangle\} and the probabilities with which it will measure \(|x\rangle\) or \(|y\rangle\). Choosing a basis different from \{|0\rangle, |1\rangle\} means changing the values of \(t_1\) and \(t_2\) to \(t_x\) and \(t_y\) and correspondingly re-labeling the eigenstates to \(|x\rangle\) and \(|y\rangle\). Thus, we can easily verify that
\[|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]
\[= \alpha (\cos \theta |x\rangle + \sin \theta |y\rangle) + \beta (\cos \theta |y\rangle - \sin \theta |x\rangle) \]
\[= (\alpha \cos \theta - \beta \sin \theta) |x\rangle + (\alpha \sin \theta + \beta \cos \theta) |y\rangle.\]

Further, \(T_x\) and \(T_y\) corresponding to the time durations the system will be in state \(|x\rangle\) and \(|y\rangle\), respectively, with respect to \(T_c\) in \{|0\rangle, |1\rangle\} basis is given by
\[T_x = |\alpha \cos \theta - \beta \sin \theta|^2 T_c, \quad T_y = |\alpha \sin \theta + \beta \cos \theta|^2 T_c, \]
\[T_c = T_0/|\alpha|^2 = T_1/|\beta|^2.\]
Thus, if the measurement basis is \( \{|x\rangle, |y\rangle\} \), the system, when measured, will randomly collapse to \( |x\rangle \) or \( |y\rangle \) with probability \( T_x/T_c = |\alpha \cos \theta - \beta \sin \theta|^2 \) or \( T_y/T_c = |\alpha \sin \theta + \beta \cos \theta|^2 \), respectively.

Finally, our model of entanglement requires that any unitary operation that causes entanglement, say, between two particles, also synchronizes their sub-Planck level oscillations. This is shown in Figure 3 for the two-particle system Bell states,

\[
|\psi_1\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}, \\
|\psi_2\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}.
\]

Figure 3: Two-particle entangled systems; \( |\psi_1\rangle \) (left), \( |\psi_2\rangle \) (right).

Note that while entanglement results in a synchronous state for the two particles, the converse is not necessarily true. When a measurement is made on one of the entangled particles, both will collapse simultaneously. According to our model, the particles will collapse to the state they are in at the instant of measurement (such as \( |\psi_1\rangle \) or \( |\psi_2\rangle \) in Figure 3), which is in accord with the measurement postulate (postulate 3). We do not know how Nature might accomplish the required synchronization.

It is, of course, clear that our interpretation cannot violate the uncertainty principle since the latest measurement on a system collapses the system according to the measurement postulate. Thus there can be no direct correlation between any earlier results of measurement on the system, and the succeeding measurement.
Our interpretation, in essence, is yet one more underlying theory in which the nature and consistency of quantum theory can be investigated and clarified; it rests entirely on the notion of external observations because without it we have no means to ascribe a physical interpretation.

**Application of the basic model**

We now provide a few examples of quantum systems to show that our interpretation is consistent with the outcomes of measurements made on those systems at any instant.

**Measurement of a two-particle entangled system**

Recall that a measurement made on either particle in an entangled pair will automatically and instantaneously alter the state of the other particle. We are now confronted with two measurement possibilities: (1) measurement using commutating observables (such as of electron spin along the same axis); and (2) measurement using non-commutating observables (such as of electron spins along different axes).

1. **Commutating observables.** Consider an entangled pair of electrons, where \( |0\rangle \) and \( |1\rangle \), represent spin-up and spin-down, respectively. Then according to our model, if a measurement is made along the spin axis, the electrons will collapse to similar spins, with the spin state determined by the instant of measurement if the entangled pair is described by the Bell state \( |\psi_1\rangle \) in Figure 3. Likewise, it will collapse to opposite spins if the entangled pair is described by \( |\psi_2\rangle \) in Figure 3.

2. **Non-commutating observables.** Consider an entangled pair of electrons where the electrons have spin components along two axes, say, \( x \)-axis and \( y \)-axis (see Figure 4). Note that at any given instant the spin of both electrons will be along only one of the axes. Further, at any instant, such as \( \tau_1 \), \( \tau_2 \), \( \tau_3 \), and \( \tau_4 \) shown in Figure 4, the two electrons can be in only one of the four states: \( |00\rangle_x \), \( |11\rangle_x \), \( |00\rangle_y \), and \( |11\rangle_y \), respectively. The suffixes \( x \), \( y \) represent...
the \( x, y \) components, respectively, of \(|00\rangle\) and \(|11\rangle\).

Figure 4: Two-particle entangled system with non-commuting observables.

Now, if a measurement is made and the system collapses, say, to the \( x \)-component, then a subsequent measurement along the \( y \)-axis will return a null result.

In the more general case of a system of \( n \)-particles, if a combined measurement is made on \( m \leq n \) of those particles then those \( m \)-particles will collapse to one of their possible group states (the actual number of states at any given instant may vary from 1 to \( 2^m \)) on measurement while the remaining \( n - m \) particles will assume states which are consistent with the collapsed state of the \( m \)-particles.

**Quantum adder**

The quantum adder operation can be explained by our interpretation in a consistent manner. The initial input state of the required three particle system, where each particle represents a qubit, is given by \(|\psi_0\rangle = |000\rangle\). We now apply the Hadamard gate to the first two qubits to create the four possible inputs for the addition operation. Thus we have

\[
|\psi_1\rangle = (|000\rangle + |010\rangle + |100\rangle + |110\rangle)/2.
\]

To carry out the add operation we apply the Toffoli gate to the three qubits with the third qubit as target, followed by the \( C_{not} \) gate to the first two
qubits with the second qubit as target to get
\[ |\psi_2\rangle = (|000\rangle + |010\rangle + |110\rangle + |101\rangle)/2, \]
where the second qubit is the sum and the third qubit is the carry bit. Note that the carry bit in the adder is the result of an AND operation. The carry and AND are really the same thing. The sum bit comes from an XOR gate (that is, the C_{not} operation). Figure 5 captures the four possible eigenstates represented by \(|\psi_1\rangle\) and \(|\psi_2\rangle\) at the instants \(\tau_1, \tau_2, \tau_3,\) and \(\tau_4.\)

![Figure 5: Quantum adder input and output states; \(|\psi_1\rangle\) (left), \(|\psi_2\rangle\) (right).](image)

**Teleporting a qubit of an unknown state**

In our final example we show that the teleportation method of Bennett et al [15] is also correctly explained by our interpretation. Here Alice wishes to teleport a qubit, labelled by subscript 1, of unknown state \(|\phi\rangle = \alpha|0_1\rangle + \beta|1_1\rangle\), to Bob. In addition, there is an entangled pair of auxiliary qubits designated by subscripts 2 and 3 in the state \(|\chi\rangle = (|0_21_3\rangle - |1_20_3\rangle)/\sqrt{2}.\) Alice holds the qubit with subscript 2 in addition to the one with subscript 1 while Bob holds the qubit with subscript 3. Thus the initial state of the three qubit system is (see Figure 6 where the qubit subscripts (in this and subsequent Figures 7 and 8) have been omitted since they can be inferred from their position in the state \(|\ldots\rangle\)) given by

\[ |\phi\chi\rangle = |\psi_0\rangle = |\alpha|0_1\rangle(|0_21_3\rangle - |1_20_3\rangle) + \beta|1_1\rangle(|0_21_3\rangle - |1_20_3\rangle)/\sqrt{2}. \]
Alice now applies the $C_{not}$ gate (with qubit 2 as the target) to the qubits held by her. This changes the state of the three qubit system to (see Figure 7)

$$|\psi_1\rangle = [\alpha|01\rangle(|0213\rangle - |1203\rangle) + \beta|11\rangle(|1213\rangle - |0203\rangle)]/\sqrt{2}$$

Next, Alice applies the Hadamard gate to qubit 1 which puts the three qubit system in the state (see Figure 8)

$$|\psi_2\rangle = [|0102\rangle(\alpha|13\rangle - \beta|03\rangle) - |0112\rangle(\alpha|03\rangle - \beta|13\rangle) + \\
|1102\rangle(\alpha|13\rangle + \beta|03\rangle) - |1112\rangle(\alpha|03\rangle + \beta|13\rangle)]/2$$

![Figure 6: Initial state $|\psi_0\rangle$ of the teleportation system.](image)
Figure 7: State $|\psi_1\rangle$ of the teleportation system after $C_{\text{not}}$ operation.

Figure 8: State $|\psi_2\rangle$ of the teleportation system after Hadamard operation.
Finally, Alice makes a “combined” measurement on the two qubits she holds. Such a measurement gives access to some combined (or global) information on both qubits, but none on a single qubit, that is, no distinction between the two qubits can be established. Her measurement will lead the pair to collapse to one of the four possible states $|00\rangle$, $|01\rangle$, $|10\rangle$, or $|11\rangle$, while the third qubit, correspondingly, will immediately collapse to the state $\alpha|1_3\rangle - \beta|0_3\rangle$, $\alpha|0_3\rangle - \beta|1_3\rangle$, $\alpha|1_3\rangle + \beta|0_3\rangle$ or $\alpha|0_3\rangle + \beta|1_3\rangle$, respectively, since it is also entangled with qubits 1 and 2. Table 1 shows the measurement result Alice will get depending upon the instant the measurement actually occurred, along with the post-measurement state of qubit 3 held by Bob.

Table 1: Measurement outcomes in the teleportation algorithm.

| Measurement instant | State of qubits 1 & 2 after Alice’s measurement | State of qubit 3 after Alice’s measurement | Decoder to bring qubit 3 to state $\phi$ |
|---------------------|--------------------------------------------|------------------------------------------|--------------------------------------|
| $\tau_1, \tau_2$    | $|00\rangle$          | $\alpha|1\rangle - \beta|0\rangle$     | Y $|0_3\rangle \rightarrow -|1_3\rangle$ |
|                     |                            |                                          | $|1_3\rangle \rightarrow |0_3\rangle$ |
| $\tau_3, \tau_4$    | $|01\rangle$          | $\alpha|0\rangle - \beta|1\rangle$     | Z $|0_3\rangle \rightarrow |0_3\rangle$ |
|                     |                            |                                          | $|1_3\rangle \rightarrow -|1_3\rangle$ |
| $\tau_5, \tau_6$    | $|10\rangle$          | $\alpha|1\rangle + \beta|0\rangle$     | X $|0_3\rangle \rightarrow |1_3\rangle$ |
|                     |                            |                                          | $|1_3\rangle \rightarrow |0_3\rangle$ |
| $\tau_7, \tau_8$    | $|11\rangle$          | $\alpha|0\rangle + \beta|1\rangle$     | I $|0_3\rangle \rightarrow |0_3\rangle$ |
|                     |                            |                                          | $|1_3\rangle \rightarrow |1_3\rangle$ |

Alice communicates the classical result of her “combined” measurement ($|00\rangle$, $|01\rangle$, $|10\rangle$, or $|11\rangle$) to Bob (using classical means such as telephone, email, etc.). Bob then uses the decoder (a unitary transformation) listed in Table 1 corresponding to the state of qubits 1 & 2 conveyed to him by Alice to bring his qubit to state $|\phi\rangle = \alpha|0_3\rangle + \beta|1_3\rangle$. 

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Conclusion

We have presented a new interpretation of the terms superposition, entanglement, and measurement that appear in quantum mechanics. In this interpretation the structure of the wave function at the sub-Planck scale has a deterministic cyclic structure. Our attempt has been to provide minimal structure at the sub-Planck level for it to match observations. This structure is independent of the Schrödinger wave equation. Each cycle comprises a sequential succession of the eigenstates that comprise a given wave function.

In our interpretation, superposed states appear as time-sliced in a cyclic manner such that the time spent by an eigenstate in a cycle is related to the complex amplitudes appearing in the wave function. Entangled states binding two or more particles appear in our interpretation as the synchronization of the sub-Planck level oscillation of the participating particles.

The probabilistic aspect of quantum mechanics is interpreted by hypothesizing a measurement mechanism which acts instantaneously but the instant of measurement is chosen randomly by the classical measuring apparatus over a small but finite interval from the time the measurement apparatus is activated. At the instant the measurement is made, the wave function irrevocably collapses to a new state and continues from thereon in that state till changed by a unitary operation or a new measurement.

We hope our interpretation makes quantum theory more intelligible and intuitively acceptable to the human mind. As a working strategy, we have no issue with the Copenhagen interpretation, but at a psychological level humans are more comfortable with an interpretation they can intuitively relate to. Our interpretation creates a hypothetical mechanism at the sub-Planck level, which supposedly drives quantum mechanical phenomenon. We believe our interpretation has value because it appears to be compatible with measurements at the classical level. We are not physically equipped to perceive the quantum world and our measuring devices are classical. We can therefore only speculate what Nature is like.
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