Low-energy multipole response in nuclei at finite temperature

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Abstract
The multipole response of nuclei at temperatures $T = 0 - 2$ MeV is studied using a self-consistent finite-temperature RPA (random phase approximation) based on relativistic energy density functionals. Illustrative calculations are performed for the isoscalar monopole and isovector dipole modes and, in particular, the evolution of low-energy excitations with temperature is analyzed, including the modification of pygmy structures. Both for the monopole and dipole modes, in the temperature range $T = 1 - 2$ MeV additional transition strength appears at low energies because of thermal unblocking of single-particle orbitals close to the Fermi level. A concentration of dipole strength around 10 MeV excitation energy is predicted in $^{60,62}$Ni, where no low-energy excitations occur at zero temperature. The principal effect of finite temperature on low-energy strength that is already present at zero temperature, e.g. in $^{68}$Ni and $^{132}$Sn, is the spreading of this structure to even lower energy and the appearance of states that correspond to thermally unblocked transitions.

Key words: hot nuclei, pygmy resonances, giant resonances, random phase approximation
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The multipole response of nuclei at energies close to the neutron emission threshold, and the possible occurrence of exotic low-energy modes of excitation, present a rapidly growing field of research \[1\]. For instance, in nuclei with a pronounced neutron excess, the phenomenon of pygmy dipole resonance (PDR), a low-energy mode that corresponds to oscillations of a weakly-bound neutron skin against the isospin saturated proton-neutron core, has been analyzed in a number of experimental and theoretical studies. In particular, recent $(\gamma, \gamma')$ and $(\alpha, \alpha'\gamma)$ experiments \[2, 3, 4\] have disclosed interesting features of the concentration of low-energy electric-dipole excitations. Exotic nuclear dynamics has also been explored in experiments with ra-
dioactive ion beams, extending the studies of low-lying dipole response toward nuclei with more pronounced neutron excess [5, 6].

Features of PDR in medium-heavy and heavy nuclei place constraints on the nuclear matter symmetry energy, and can also provide information on the size of the neutron skin [7, 8]. A concentration of electric-dipole strength at low energy is interesting not only as a unique structure phenomenon, but it could also play an important role in astrophysical processes. For instance, it could affect neutron capture rates in r-process nucleosynthesis [9, 10]. Reactions relevant for the r-process take place in stellar environment at finite temperature. Calculations of neutron capture rates on neutron-rich nuclei have typically included temperature only as a parameter of the Lorentzian function used to fold the E1 strength distribution [9, 11]. It would, of course, be useful to perform these calculations using a microscopic framework that provides a consistent description of nuclear ground-state properties and excitations at finite temperature. In particular, temperature induced modifications of low-energy dipole strength close to the neutron separation energy, e.g. the thermal unblocking of additional transitions, should be described in a consistent theoretical framework.

In this work we introduce a fully self-consistent finite-temperature RPA (random phase approximation) framework based on relativistic energy density functionals, and explore the evolution of multipole response as a function of temperature. The goal is to analyze how low-energy excitations evolve with temperature, the modification of the structure of PDR, and the possible occurrence of excitation modes that are not present in nuclei at zero temperature.

The multipole response of hot nuclei has been studied using a variety of theoretical approaches, including e.g. the RPA based on schematic interactions [12, 13, 14], linear response theory [15, 16], Hartree-Fock plus RPA with Skyrme effective interactions [17], RPA models extended with the inclusion of collision terms [18]. The decay of hot nuclei and thermal damping of giant resonances have also been explored in numerous studies, e.g. in Refs. [19, 20, 21, 22, 23, 24, 25], as well as sum rules at finite temperature [26, 27]. Finite temperature Hartree-Fock Bogoliubov (HFB) theory [28] has been used as the basis for the corresponding quasiparticle RPA [29, 30]. Experimental studies of giant resonances in hot nuclei are carried out by measuring high energy γ-rays from the decay of resonant states built on excited states of target nuclei [31, 32, 33, 34].

The self-consistent finite temperature relativistic random phase approximation (FTRRP A) is formulated in the single-nucleon basis of the relativistic mean-field (RMF) model at finite temperature (FTRMF). This model has been introduced in Ref. [35], based on the nonlinear Lagrangian NL3 [36]. In the theoretical frame-
work that we use in the present analysis relativistic effective interactions are imple-
mented self-consistently, i.e., both the FTRMF equations and the matrix equations of
FTRRPA are based on the same relativistic energy density functional with medium-
dependent meson-nucleon couplings \[37\]. Of course, the description of open-shell
nuclei necessitates a consistent treatment of pairing correlations like, for instance, in
the finite temperature HFB+QRPA framework \[29\]. However, in nuclei the phase
transition from a superfluid to a normal state occurs at temperatures \(T \approx 0.5−1\)
MeV \[28\,30\], whereas for temperatures above \(T \approx 4\) MeV contributions from states
in the continuum become large, and additional subtraction schemes have to be im-
plemented to remove the contributions of the external nucleon gas \[38\]. Therefore,
we consider the temperatures in the range \(T = 1−2\) MeV, for which the FTRMF
plus FTRRPA should provide an accurate description of the evolution of multipole
response.

The FTRRPA represents the small amplitude limit of the time-dependent relativistic
mean-field theory at finite temperature. Starting from the response of the time-depen-
dent density matrix \(\hat{\rho}(t)\) to an external field \(\hat{f}(t)\), the equation of motion
for the density operator reads

\[
i\partial_t \hat{\rho} = [\hat{h}[\hat{\rho}] + \hat{f}(t), \hat{\rho}] .
\] (1)

In the small amplitude limit the density matrix is expanded to linear order

\[
\hat{\rho}(t) = \hat{\rho}^0 + \delta \hat{\rho}(t),
\] (2)

where \(\hat{\rho}^0\) denotes the stationary ground-state density.

\[
\rho_{kl}^0 = \delta_{kl} n_k = \begin{cases} 1 + \exp\left(\frac{\epsilon_k - \mu}{kT}\right) & \text{for states in the Fermi sea (index } k, l) \\ 0 & \text{for unoccupied states in the Dirac sea (index } \alpha) \end{cases}
\] (3)

and includes the thermal occupation factors of single-particle states \(n_k\). The resulting
FTRRPA equations read

\[
\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar \omega \begin{pmatrix} X \\ Y \end{pmatrix},
\] (4)

where

\[
A = \begin{pmatrix} (\epsilon_m - \epsilon_i) \delta_{ii'} \delta_{mm'} \\ (\epsilon_{\alpha} - \epsilon_i) \delta_{\alpha\alpha'} \delta_{ii'} \end{pmatrix} + \begin{pmatrix} (n_{i'} - n_{m'}) V_{m'i'm'} & n_{i'} V_{m'i'\alpha'} \\ (n_{\alpha'} - n_{m'}) V_{\alpha'i'm'} & n_{\alpha'} V_{\alpha'i'\alpha'} \end{pmatrix},
\] (5)
and
\[
B = \begin{pmatrix}
(n_{i'} - n_{m'})V_{mm'ii'} & n_{i'}V_{nn'i'i'} \\
(n_{i'} - n_{m'})V_{nm'i'i'} & n_{i'}V_{nn'i'i'}
\end{pmatrix}.
\] (6)

The RRPA matrices A and B are composed of matrix elements of the residual interaction V, derived from an effective Lagrangian density with medium-dependent meson-nucleon couplings [37], as well as certain combinations of thermal occupation factors \( n_k \). The diagonal matrix elements contain differences of single particle energies \( \epsilon_m - \epsilon_i \), where \( \epsilon_i < \epsilon_m \). Because of finite temperature, the configuration space includes particle-hole (ph), particle-particle (pp), and hole-hole (hh) pairs. In addition to configurations of positive energy, the FTRRP A configuration space must also contain pair-configurations formed from the fully or partially occupied states of positive energy and the empty negative-energy states from the Dirac sea. The inclusion of configurations built from occupied positive-energy states and empty negative-energy states is essential for current conservation, decoupling of spurious states, and for a quantitative description of excitation energies of giant resonances [39].

The residual interaction is derived from the DD-ME2 effective Lagrangian [40], and single-particle occupation factors at finite temperature are included in a consistent way both in the FTRMF and FTRRP A. The same interaction is used both in the FTRMF equations that determine the single-nucleon basis, and in the matrix equations of the FTRRP A. The full set of FTRRP A equations is solved by diagonalization. The result are excitation energies \( E_\nu \), and the corresponding forward- and backward-going amplitudes \( X_\nu \) and \( Y_\nu \), respectively, that are used to evaluate the transition strength for a multipole operator \( Q_J \):
\[
B(J, E_\nu) = \left| \sum_{mi} (X_{mi}^J + (-1)^J Y_{mi}^J) \langle m||Q_J||i\rangle(n_i - n_m) \right|^2.
\] (7)

Discrete spectra are averaged with a Lorentzian distribution of arbitrary width (\( \Gamma = 1 \) MeV in the present calculation).

In Fig. 1 we display the FTRRP A strength distributions in \(^{56}\text{Ni}\), \(^{90}\text{Zr}\), \(^{132}\text{Sn}\), and \(^{208}\text{Pb}\) for the isoscalar monopole operator, at temperatures \( T = 0, 1, \) and \( 2 \) MeV. At zero temperature the transition strength distributions are clearly dominated by the pronounced collective isoscalar giant monopole resonance (ISGMR). Even by increasing the temperature to \( T = 2 \) MeV, the ISGMR gets only slightly modified in \(^{90}\text{Zr}\), \(^{132}\text{Sn}\), and \(^{208}\text{Pb}\). In the lighter nucleus \(^{56}\text{Ni}\), where the ISGMR is less collective and therefore more sensitive to modifications of the corresponding configuration space at finite temperature, at \( T = 2 \) MeV the overall strength distribution is lowered by
It is interesting to note that for all nuclei under consideration, in the temperature range \( T = 1 - 2 \) MeV additional transition strength appears at energies below 10 MeV. These low-energy transitions occur because of thermal unblocking of single-particle orbitals close to the Fermi level. The effect of finite temperature is especially pronounced for the neutron-rich nucleus \(^{132}\text{Sn}\), due to transitions from thermally populated weakly-bound neutron orbits. The low-energy strength is dominated by two rather pronounced peaks. The state at 5.45 MeV corresponds to the single-neutron transition from thermally unblocked orbitals: \(3p_{3/2} \rightarrow 4p_{3/2}\), whereas for the state at 7.02 MeV the dominant transition is between the neutron orbitals \(2f_{7/2}\) and \(3f_{7/2}\).

As already emphasized in the introduction, particularly relevant for possible astrophysical applications is a microscopic description of low-energy dipole response in hot nuclei. Fig. 2 displays the FTRRPA (DD-ME2) dipole strength distributions in Ni isotopes, at temperatures \( T = 0, 1, \) and 2 MeV. The transition matrix elements are evaluated for the isovector dipole operator. At zero temperature the main peak of the isovector giant dipole resonance (IVGDR) is located at \( \approx 18 \) MeV. For \(^{68}\text{Ni}\), in particular, additional low-energy PDR structure is predicted below 10 MeV even at zero temperature, and this result was recently confirmed in the experimental study reported in Ref. [6]. The IVGDR is somewhat modified at finite temperature. At \( T = 2 \) MeV the main peak is lowered by \( \approx 2 \) MeV in \(^{56}\text{Ni}\), whereas in \(^{62}\text{Ni}\) finite temperature reduces the fragmentation of the IVGDR strength in the interval \( \approx 17-20 \) MeV, and enhances the collectivity of the main resonance peak.

A very interesting finite temperature effect is predicted for the dipole strength distributions of \(^{60,62}\text{Ni}\), where no low-energy excitations are present at zero temperature. Already at \( T = 1 \) MeV novel transition strength develops below 10 MeV. For \(^{68}\text{Ni}\), in particular, at \( T = 2 \) MeV the main low-energy peak is located at 9.71 MeV, it exhausts 1.54\% of the Thomas Reiche Kuhn (TRK) sum rule, and is composed of 5 neutron and 4 proton single-particle transitions: \(\nu 2p_{3/2} \rightarrow \nu 2d_{5/2}\) (45.87\%), \(\nu 1f_{5/2} \rightarrow \nu 2d_{3/2}\) (13.92\%), \(\nu 2p_{3/2} \rightarrow \nu 3s_{1/2}\) (8.43\%), \(\nu 1f_{7/2} \rightarrow \nu 1g_{9/2}\) (3.58\%), \(\nu 2p_{1/2} \rightarrow \nu 2d_{3/2}\) (1.30\%), \(\pi 2p_{3/2} \rightarrow \pi 2d_{3/2}\) (7.89\%), \(\pi 2p_{3/2} \rightarrow \pi 2d_{5/2}\) (6.32\%), \(\pi 1f_{5/2} \rightarrow \pi 2d_{3/2}\) (3.62\%), and \(\pi 1f_{7/2} \rightarrow \pi 1g_{9/2}\) (1.21\%). The percentage in brackets corresponds to the contribution of a particular transition to the total FTRRPA amplitude \(\sum_{m}(X_{m}^{2} - Y_{m}^{2})(n_{i} - n_{m})\) for the state at 9.71 MeV. This rather rich RPA structure shows that the new low-energy state accumulates a small degree of collectivity due to thermal population of both neutron and proton single-particle levels around the Fermi surface.

In the case of \(^{62}\text{Ni}\), two pronounced low-energy peaks are calculated at temperature \( T = 2 \) MeV: the states at 9.78 MeV and 10.03 MeV, exhausting 1.2\% and
1.5% of the TRK sum rule, respectively. It is interesting to note that both states are dominated by proton transitions. For the state at 9.78 MeV: π2p₃/2 → π2d₅/2 (48.02%), π1f₅/2 → π2d₅/2 (20.99%), π2p₃/2 → π3s₁/2 (3.12%), ν₂p₃/2 → ν₂d₅/2 (17.06%), ν₁f₅/2 → ν₂d₅/2 (4.36%), ν₂p₃/2 → ν₂d₅/2 (1.22%). The structure of the state at 10.03 MeV appears to be less distributed, i.e. it is dominated by the transitions ν₂p₃/2 → ν₂d₅/2 (26.55%), ν₂p₃/2 → ν₃s₁/2 (63.60%), ν₁f₇/2 → ν₂d₅/2 (2.55%).

In ⁶⁸Ni, where the PDR structure is present already at T = 0, the increase in temperature leads to fragmentation of the PDR and its spreading to even lower energy. These examples nicely illustrate the variety of effects one can expect for the dipole response of hot nuclei, e.g. the appearance of novel modes of low-energy excitations dominated by proton transitions, and the thermal spreading of PDR structures in neutron-rich nuclei.

Finally, Fig. 3 shows the evolution of FTRRPA dipole transition strength with temperature for ¹³²Sn. This is an important example, because for this nucleus the occurrence of PDR was predicted in several theoretical studies [1], and recently confirmed in Coulomb dissociation experiments [5]. With the increase of temperature up to T ≈ 1 MeV the transition strength is virtually unchanged. Higher temperatures, however, have a pronounced effect on low-energy dipole strength. At temperatures ≈ 2 MeV, even though the main PDR peak at E ≈ 8 MeV does not change much, additional low-energy single-particle transitions appear in the interval E ≈ 2 – 7 MeV: E=3.75 MeV, ν₃p₃/2 → ν₃d₅/2 (99.65%); E=4.54 MeV, ν₂f₇/2 → ν₃d₅/2 (97.49%); E=4.92 MeV, ν₂f₇/2 → ν₂g₇/2 (99.58%); E=6.12 MeV, ν₂f₇/2 → ν₂g₉/2 (98.35%). At zero temperature the PDR state is calculated at E = 7.75 MeV, whereas at T = 2 MeV the PDR excitation energy is E = 7.77 MeV. In Table 1 we compare the structure of FTRRPA states for the main PDR peaks at temperatures T = 0 MeV and T = 2 MeV. The amplitudes of transitions dominant at T = 0 MeV get slightly reduced at higher temperature. Because of thermal unblocking of the ν₁g₉/2 orbit, at T = 2 MeV an additional non-negligible contribution to the PDR peak corresponds to the transition ν₁g₉/2 → ν₁h₁₁/2.

The results for the evolution of dipole response with temperature (c.f. Figs. 2 and 3) show that the principal effect of finite temperature on low-energy strength that is already present at zero temperature, e.g. in ⁶⁸Ni and ¹³²Sn, is the spreading of this structure to even lower energy and the appearance of states that correspond to thermally unblocked transitions. This could be important for calculation of astrophysical neutron capture rates relevant for r-process nucleosynthesis, because these reactions take place at finite temperature. As recently pointed out in Ref. [10], neutron capture rates computed with the Hauser-Feshbach model are sensitive to the
Table 1: FTRRPA transition amplitudes for the main PDR peaks in $^{132}\text{Sn}$ at temperatures $T = 0$ MeV and $T = 2$ MeV. Included are the contributions (in %) of dominant configuration to the total sum of FTRRPA amplitudes: $\sum m_i (X_{m_i}^2 - Y_{m_i}^2)(n_i - n_m)$.

| $T = 0$ MeV       | $T = 2$ MeV   |
|-------------------|---------------|
| $E = 7.75$ MeV    | $E = 7.77$ MeV|
| $\nu 3s_{1/2} \rightarrow 3p_{3/2}$ | 51.85         |
|                   | 50.37         |
| $\nu 2d_{3/2} \rightarrow 3p_{3/2}$ | 19.15         |
|                   | 17.22         |
| $\nu 2d_{3/2} \rightarrow 3p_{1/2}$ | 11.56         |
|                   | 10.43         |
| $\nu 3s_{1/2} \rightarrow 3p_{1/2}$ | 6.64          |
|                   | 5.48          |
| $\nu 1h_{11/2} \rightarrow 1i_{13/2}$ | 4.99          |
|                   | 4.44          |
| $\pi 1g_{9/2} \rightarrow 1h_{11/2}$ | 2.17          |
|                   | 2.43          |
| $\nu 1g_{9/2} \rightarrow 1h_{11/2}$ | -             |
|                   | 1.26          |

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Figure 1: Isoscalar monopole transition strength distributions in $^{56}$Ni, $^{90}$Zr, $^{132}$Sn, and $^{208}$Pb, calculated with the FTRRPA (DD-ME2) at temperatures $T = 0, 1,$ and $2$ MeV.
Figure 2: Isovector dipole strength distributions in Ni isotopes at temperatures $T = 0, 1, \text{ and } 2$ MeV, calculated with the FTRRPA (DD-ME2).
Figure 3: Evolution of electric dipole transition strength with temperature in $^{132}\text{Sn}$, calculated with the FTRRPA (DD-ME2).