Higgs Mass from Compositeness at a Multi-TeV Scale

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based on current work with Hsin-Chia Cheng and Bogdan A. Dobrescu
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Introduction

- Hierarchy problem.
- One solution: no light fundamental scalar!
- Composite Higgs that no longer exists above the compositeness scale.
- No new physics at LHC yet. Are we at a crossroads?
- Small hierarchy may still exist.
- New strong dynamics at the compositeness scale.
- Usually predicts a heavy Higgs due to large quartic couplings, unless the Higgs mass is protected by some symmetry.
The Nambu-Jona-Lasinio Model

Consider some theory at scale $\Lambda$ with an effective four-fermion vertex

$$\mathcal{L}_\Lambda = \bar{\psi}_L i \partial \psi_L + \bar{\psi}_R i \partial \psi_R + \frac{g^2}{\Lambda^2} (\bar{\psi}_L \psi_R)(\bar{\psi}_L \psi_R).$$  \hfill (1)

The four-fermion vertex may come from some spontaneously broken gauge theory by integrating out the heavy gauge bosons.

Eq. (1) can be rewritten in the following form with an auxiliary field $H$

$$\mathcal{L}_\Lambda = \bar{\psi}_L i \partial \psi_L + \bar{\psi}_R i \partial \psi_R + (g \bar{\psi}_L \psi_R H + \text{h.c.}) - \frac{\Lambda^2}{2} H^\dagger H. \hfill (2)$$

(I will follow the appendix of arXiv:hep-ph/0203079 (C. T. Hill & E. H. Simmons).)
The Nambu-Jona-Lasinio Model

- Evolving down to scale $\mu$ with the fermion bubble approximation

- which generates kinetic and quartic terms of the $H$ field and also gives a correction to the mass term

$$
\mathcal{L}_\mu = \bar{\psi}_L i \phi \psi_L + \bar{\psi}_R i \phi \psi_R + (g \bar{\psi}_L \psi_R H + \text{h.c.})
+ Z_H |\partial_\nu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2} (H^\dagger H)^2
$$

- where

$$
Z_H = \frac{g^2 N_c}{(4\pi)^2} \log \left( \frac{\Lambda^2}{\mu^2} \right), 
\ m_H^2 = \Lambda^2 - \frac{2g^2 N_c}{(4\pi)^2} (\Lambda^2 - \mu^2), 
\lambda_0 = \frac{2g^2 N_c}{(4\pi)^2} \log \left( \frac{\Lambda^2}{\mu^2} \right).
$$
The Nambu-Jona-Lasinio Model

- When $\mu \to \Lambda, Z_H \to 0$, which means $H$ is no longer a physical degree of freedom.
- If we normalize the kinetic term of $H$, then the couplings blow up at $\Lambda$.

$$\mathcal{L}_\mu = \bar{\psi}_L i \not{\partial} \psi_L + \bar{\psi}_R i \not{\partial} \psi_R + (\xi \bar{\psi}_L \psi_R H + \text{h.c.})$$

$$+ | \partial_\nu H |^2 - \tilde{m}_H^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2$$  \hspace{1cm} (5)$$

$$\xi^2 = \frac{g^2}{Z_H} = \frac{16\pi^2}{N_c \log (\Lambda^2/\mu^2)} , \hspace{1cm} \tilde{m}_H^2 = m_H^2/Z_H ,$$

$$\lambda = \frac{\lambda_0}{Z_H^2} = \frac{32\pi^2}{N_c \log (\Lambda^2/\mu^2)} = 2\xi^2 .$$ \hspace{1cm} (6)$$

- One can think of $H$ as a composite particle of the fermions, while $\Lambda$ is the compositeness scale, at which the couplings are strong.
The Nambu-Jona-Lasinio Model

\[ m_H^2 = \Lambda^2 - \frac{2g^2 N_c}{(4\pi)^2} (\Lambda^2 - \mu^2). \]  

- \( m_H^2 < 0 \) if \( g \) is large enough. (Spontaneous symmetry breaking!)
- If the theory is spontaneously broken, \( \lambda = 2\xi^2 \) implies
  \[ m_h = 2m_f. \]  
- The results are subject to change when effects of other interactions are included.
Top Condensation

- The Higgs field is a low energy condensate $\langle \bar{t}t \rangle$ triggered by some new fundamental interaction at a higher scale $\Lambda$.

- Instead of the fermion bubble approximation, the full one-loop RG equations are used. [Phys. Rev. D 41, 16471660 (1990), (Bardeen, Hill, Lindner)]

- To get the right Electroweak VEV, top quark is too heavy unless the compositeness scale is extremely large. (Need the top Yukawa coupling to be very large at $\Lambda$ and be $\approx 1$ at weak scale.)
  - $\Lambda = 10^5$ GeV $\Rightarrow m_{\text{top}} \approx 360$ GeV.
  - $\Lambda = 10^{19}$ GeV $\Rightarrow m_{\text{top}} \approx 220$ GeV.

- $m_h \gtrsim m_{\text{top}}$.

- It doesn’t work!
Top Condensation Seesaw

- option 1: Give up.
- option 2: Modify the theory until it works!
- Minimal modification: add a new vector-like top partner.
- A number of papers at the end of last century
  - arXiv:hep-ph/9712319 (Dobrescu, Hill)
  - arXiv:hep-ph/9809470 (Chivukula, Dobrescu, Georgi, Hill)
  - arXiv:hep-ph/9908391 (Dobrescu)
- With the top seesaw mechanism, one can have a large \( \gg 1 \) Yukawa coupling while keeping the correct top mass \( 173 \text{ GeV} \).
- We found that by imposing an approximate \( U(3)_L \) symmetry, the Higgs mass has a rather restricted range and we can easily obtain a 126 GeV Higgs.
Introducing a new vector-like quark

We introduce a new $SU(2)_W$-singlet vector-like quark, $\chi$ of electric charge $+2/3$.

$\psi^3_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$, $\chi_L$, $t_R$, $\chi_R$ form bound states due to some strong interactions at scale $\Lambda$, which approximately preserves $U(3)_L \times U(2)_R$ flavor symmetry.

We label the composite scalars collectively as $\Phi$, which is a $3 \times 2$ matrix

$$\Phi = \begin{pmatrix} \Phi_t \\ \Phi_\chi \end{pmatrix}, \quad (9)$$

$$\Phi_t \sim t_R \begin{pmatrix} \psi^3_L \\ \chi_L \end{pmatrix}, \quad \Phi_\chi \sim \chi_R \begin{pmatrix} \psi^3_L \\ \chi_L \end{pmatrix}. \quad (10)$$

Yukawa couplings of the fermions and composite scalars:

$$\mathcal{L}_{\text{Yukawa}} = -\xi \begin{pmatrix} \psi^3_L \\ \chi_L \end{pmatrix} \Phi \begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{H.c.} \quad (11)$$

The lighter mass eigenstate is the physical top, which can be “light” because of the seesaw mechanism.
Effective potential of the scalar sector

- The Yukawa couplings give rise to the following potential for $\Phi$:

$$V_\Phi = \frac{\lambda_1}{2} \text{Tr}[(\Phi^\dagger \Phi)^2] + \frac{\lambda_2}{2} \left(\text{Tr}[\Phi^\dagger \Phi]\right)^2 + M_\Phi^2 \Phi^\dagger \Phi . \quad (12)$$

- We introduce additional explicit $U(2)_R$ breaking effects in the mass term which distinguish $t_R$ and $\chi_R$.

$$V_{U(2)} = \delta M_{tt}^2 \Phi_t^\dagger \Phi_t + \delta M_{\chi\chi}^2 \Phi_\chi^\dagger \Phi_\chi + (M_{\chi t}^2 \Phi_\chi^\dagger \Phi_t + \text{H.c.}) \quad (13)$$

- SM gauge invariant mass terms at scale $\Lambda$

$$\mathcal{L}_{\text{mass}} = -\mu_{\chi t} \bar{\chi}_L t_R - \mu_{\chi\chi} \bar{\chi}_L \chi_R + \text{H.c.} \quad (14)$$

map to tadpole terms for the $SU(2)_W$-singlet scalars below $\Lambda$

$$V_{\text{tadpole}} = -(0, 0, C_{\chi t}) \Phi_t - (0, 0, C_{\chi\chi}) \Phi_\chi + \text{H.c.} \quad (15)$$

$$C_{\chi t} \approx \frac{\mu_{\chi t}}{\xi} \Lambda^2 , \quad C_{\chi\chi} \approx \frac{\mu_{\chi\chi}}{\xi} \Lambda^2 . \quad (16)$$

- $U(3)_L \times U(2)_R$ is broken down to $U(2)_L \times U(1)_R$ (with approximate $U(3)_L$).
Two doublets + two singlets

- Rewrite the scalar potential

\[
V_{\text{scalar}} = \frac{\lambda_1 + \lambda_2}{2} [(\Phi_t^\dagger \Phi_t)^2 + (\Phi_X^\dagger \Phi_X)^2] + \lambda_1 |\Phi_t^\dagger \Phi_X|^2 + \lambda_2 (\Phi_t^\dagger \Phi_t)(\Phi_X^\dagger \Phi_X) + M_{tt}^2 \Phi_t^\dagger \Phi_t + M_{XX}^2 \Phi_X^\dagger \Phi_X + (M_{xt}^2 \Phi_X^\dagger \Phi_t + \text{H.c.})
\]

\[
- (0, 0, 2C_{xt}) \text{Re} \Phi_t - (0, 0, 2C_{XX}) \text{Re} \Phi_X .
\]

(17)

- Write \( \Phi_t \) and \( \Phi_X \)

\[
\Phi_t = \begin{pmatrix} H_t \\ \phi_t \end{pmatrix} , \quad \Phi_X = \begin{pmatrix} H_X \\ \phi_X \end{pmatrix} .
\]

(18)

- For certain values of parameters, all 4 scalars will have vacuum expectation values. (We will have \( M_{tt}^2 > 0 \), \( M_{XX}^2 < 0 \).)

\[
\langle H_t \rangle = \begin{pmatrix} v_t \\ 0 \end{pmatrix} , \quad \langle H_X \rangle = \begin{pmatrix} v_X \\ 0 \end{pmatrix} , \quad \langle \phi_t \rangle = \frac{u_t}{\sqrt{2}} , \quad \langle \phi_X \rangle = \frac{u_X}{\sqrt{2}} .
\]

(19)
Chiral symmetry breaking scale

- We need to have the correct electroweak VEV, $v = 246$ GeV.

\[ v_t^2 + v_\chi^2 = v^2, \quad u_t^2 + u_\chi^2 = u^2. \] (20)

- Chiral symmetry breaking scale

\[ f = \sqrt{u^2 + v^2}. \] (21)

- We expect $\Lambda \sim 4\pi f$ for $f$ to be natural.

- $T$ parameter constraint requires $v \ll f$, which requires tuning!

- The $U(3)_L$ symmetry does not contain a custodial $SU(2)$ symmetry.

- No new physics at LHC so far, some tuning in the electroweak scale is probably inevitable.
Choosing a particular basis

- Perform an $U(2)_R$ transformation to go to a basis where $v_t = 0$ and $v_\chi = v$. (no more $\tan \beta$!)

- Also define angle $\gamma$ so that

\[ u_t = u \sin \gamma, \quad u_\chi = u \cos \gamma \quad (22) \]

- Short-hand notation $s_\gamma = \sin \gamma$.

- In the limit $s_\gamma \to 0$, the tadpole terms will vanish and the Higgs field becomes massless.
Neglecting the mixing of the charm and up quarks with $t$ and $\chi$, the mass terms of the heavy charge-2/3 fermions quarks are given by

$$-rac{\xi}{\sqrt{2}} (t_L, \chi_L) \begin{pmatrix} 0 & v \\ us_\gamma & uc_\gamma \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{H.c.}$$ (23)

The mass of the top quark is suppressed by $s_\gamma$ so that $\xi$ can be much larger than one.

$$m_t \approx \frac{\xi v s_\gamma}{\sqrt{2}} \quad \Rightarrow \quad s_\gamma \approx \frac{y_t}{\xi}.$$ (24)

We can obtain the correct top mass while keeping the compositeness scale relatively small.

The heavier eigenstate is the “top partner” and has mass

$$m_t' \approx \frac{\xi f}{\sqrt{2}}.$$ (25)
Light Higgs

- two doublets + two singlets

\[ \Phi_t = \begin{pmatrix} H_t \\ \phi_t \end{pmatrix}, \quad \Phi_\chi = \begin{pmatrix} H_\chi \\ \phi_\chi \end{pmatrix}. \tag{26} \]

- Three NGBs that will become the longitudinal modes of \( W \) and \( Z \).
- 4 CP-even neutral scalars, 3 CP-old neutral scalars and 1 charged scalar.
- The lightest mass eigenstate of the 4 CP-even neutral scalars is a PNGB of the approximate \( U(3)_L \) symmetry. It is the 126 GeV “Higgs”.
- Keeping the leading order terms in \( v^2/f^2 \) and \( s_\gamma \),

\[ M_h^2 \approx \frac{\lambda_1}{2\xi^2} \left( 1 + \frac{\lambda_1 m_{t'}^2}{\xi^2 M_{H^\pm}^2} \right)^{-1} y_t^2 v^2. \tag{27} \]

- With \( 0.4 \lesssim \frac{\lambda_1}{2\xi^2} \lesssim 1 \), we have \( M_h \lesssim 185 \text{ GeV} \).
Using RGE to estimate $\lambda_1/(2\xi^2)$ and $\lambda_2/\lambda_1$

- The Yukawa coupling $\xi$ and the quartic couplings $\lambda_1$, $\lambda_2$ are related.

$$V_{\text{quartic}} = \frac{\lambda_1}{2} \text{Tr}[(\Phi^\dagger \Phi)^2] + \frac{\lambda_2}{2} (\text{Tr}[\Phi^\dagger \Phi])^2.$$  \hspace{1cm} (28)

- In the fermion loop approximation, $\lambda_1 = 2\xi^2$, $\lambda_2 = 0$.

- In one loop RG running, the ratios of couplings $\lambda_1/(2\xi^2)$ and $\lambda_2/\lambda_1$ are quickly driven to some approximate fixed points.

- The Evolutions of $\lambda_1/(2\xi^2)$ and $\lambda_2/\lambda_1$ are quite insensitive to the value of $\xi$. 
Using RGE to estimate $\lambda_1/(2\xi^2)$ and $\lambda_2/\lambda_1$

Boundary conditions: $\lambda_1/(2\xi^2) = 1$, $\lambda_2/\lambda_1 = 0$ at $\Lambda$.
Choosing different boundary conditions for $\xi$, $\xi(\Lambda) = 5, 20$.
One loop RG running predicts that

$$\frac{\lambda_1}{2\xi^2} \approx 0.4 \quad , \quad \frac{\lambda_2}{\lambda_1} \approx -0.2 .$$

(29)
Using RGE to estimate $\frac{\lambda_1}{(2\xi^2)}$ and $\frac{\lambda_2}{\lambda_1}$

- Do we trust the results from 1-loop RGEs? No.
- Coupling is strong, higher loop contributions may be large.
- To avoid excessive tuning, the chiral symmetry breaking scale is not far below the compositeness scale.
- If we assume a smooth evolution, the ratios of couplings are expected to lie in between their initial values and the infrared fixed point values:

$$0.4 \lesssim \frac{\lambda_1}{2\xi^2} \lesssim 1 , \quad -0.2 \lesssim \frac{\lambda_2}{\lambda_1} \lesssim 0 .$$

(30)
$U(3)_L$ breaking from electroweak interactions

- So far we assume that the only explicit $U(3)_L$ breaking comes from the tadpole terms.
- Other explicit $U(3)_L$ breaking effects can feed into the mass and quartic terms through loops.
- We can parameterize the $U(3)_L$ breaking terms as

$$\Delta V_{\text{breaking}} = \frac{\kappa_1}{2} [(H_t^\dagger H_t)^2 + (H_x^\dagger H_x)^2 + 2(H_t^\dagger H_x)(H_x^\dagger H_t)] + \frac{\kappa_2}{2} (H_t^\dagger H_t + H_x^\dagger H_x)^2$$
$$+ \kappa_1' [H_t^\dagger H_t \phi_t^\dagger \phi_t + H_x^\dagger H_x \phi_x^\dagger \phi_x + (H_t^\dagger H_x \phi_x^\dagger \phi_t + \text{H.c.})]$$
$$+ \kappa_2' (H_t^\dagger H_t + H_x^\dagger H_x)(\phi_t^\dagger \phi_t + \phi_x^\dagger \phi_x)$$
$$+ \Delta M_{tt}^2 H_t^\dagger H_t + \Delta M_{xx}^2 H_x^\dagger H_x + (\Delta M_{xt}^2 H_x^\dagger H_t + \text{H.c.}) . \quad (31)$$

- In leading order of $s_\gamma$ and $v^2/f^2$,

$$\Delta M_h^2 \approx \left( \kappa_1 + \kappa_2 - \frac{5}{2}(\kappa_1' + \kappa_2') - \frac{\Delta M_{xx}^2}{f^2} \right) v^2 . \quad (32)$$

- This can screw up the prediction of Higgs mass!
In our model, the additional $U(3)_L$ breaking effects come from the $SU(2)_W \times U(1)_Y$ gauge interactions.

We assume this contribution is cut off by $M_\rho$, presumably the mass of some vector state in the theory.

Contributions to mass terms and quartic couplings ($\mu \sim m_t \approx \xi f / \sqrt{2}$)

$$\Delta M^2_{\chi\chi} = \Delta M^2_{tt} = \frac{9g_2^2 + 3g_1^2}{64\pi^2} M_\rho^2, \quad \Delta M^2_{\chi t} = 0,$$  \hspace{1cm} (33)

$$\frac{\kappa_1(2)}{\lambda_1(2)} \approx 2 \frac{\kappa'_1(2)}{\lambda_1(2)} \approx \frac{3(3g_2^2 + g_1^2)}{16\pi^2} \ln \left( \frac{M_\rho}{\mu} \right).$$  \hspace{1cm} (34)

The contributions to mass terms and quartic couplings both reduce the Higgs mass.

With not too large $M_\rho (\lesssim 5f)$, we can still get the correct Higgs mass.
Numerical study!

- We want to verify the Higgs mass prediction with a numerical study.
- Our model contains the following parameters:
  \[ \xi, \lambda_1, \lambda_2, M_{tt}^2, M_{\chi\chi}^2, M_{\chi t}^2, C_{\chi t}, C_{\chi\chi}, M_{\rho}. \] (35)
- Choose the \( v_t = 0 \) basis and write in terms of \( M_{H^\pm} \) and the VEVs,
  \[ \xi, \lambda_1, \lambda_2, M_{H^\pm}, v, f, s_\gamma, M_{\rho}. \] (36)
- \( v = 246 \text{ GeV} \), use top mass to solve for \( s_\gamma \),
  \[ \xi, \lambda_1/(2\xi^2), \lambda_2/\lambda_1, f, M_{H^\pm}/f, M_{\rho}/f. \] (37)
- To calculate the Higgs mass, we match the theory with SM at scale \( m_t' \approx \frac{\xi f}{\sqrt{2}} \), compute \( \lambda_h \) and evolve it down to the weak scale.
Parameter space

▶ What are the expected ranges of the 6 parameters?

\[
\xi, \frac{\lambda_1}{2\xi^2}, \frac{\lambda_2}{\lambda_1}, f, \frac{M_{H^\pm}}{f}, \frac{M_\rho}{f}.
\]

(38)

▶ T parameter constraint requires \( f \gg v \). We consider \( f \) up to 10 TeV to avoid excessive fine tuning.

▶ The states in the theory should have masses below the cutoff scale \( M_{H^\pm}, M_\rho \lesssim 4\pi f \).

▶ Using 1-loop RGE, we expect \( 0.4 \lesssim \lambda_1/(2\xi^2) \lesssim 1 \) and \( -0.2 \lesssim \lambda_2/\lambda_1 \lesssim 0 \).

▶ \( \xi \) is expected to be roughly between 2.5 and 5. Use \( \xi = 2\pi/\sqrt{3} \approx 3.6 \) as the standard reference value.
We can have a 126 GeV Higgs!

- Plot Higgs mass as a function of the dimensionful parameters. ($\lambda_2 = 0$)
- $M_h = 126$ GeV can be obtained with reasonable parameters of our model.
The heavy fermion $t'$ can give a large contribution to the T parameter, which is related to the fact that $U(3)_L$ does not contain a custodial $SU(2)$ symmetry.

$$T = \frac{3}{16\pi^2\alpha v^2} \left[ s_L^4 m_{t'}^2 + 2 s_L^2 (1 - s_L^2) \frac{m_{t'}^2 m_t^2}{m_{t'}^2 - m_t^2} \ln \left( \frac{m_{t'}^2}{m_t^2} \right) - s_L^2 (2 - s_L^2) m_t^2 \right],$$  

(39)

It can be rewritten in terms of $\xi$ and $f$ as

$$T \approx \frac{3}{16\pi^2\alpha f^2} \left[ \frac{v^2 \xi^2}{2} + 4 m_t^2 \ln \left( \frac{\xi f}{\sqrt{2} m_t} \right) - 2 m_t^2 \right].$$  

(40)
68% bound \( \rightarrow T \lesssim 0.1 \) corresponds to \( f \gtrsim 4.3 \) TeV (for \( \xi = 3.6 \)).

95% bound \( \rightarrow T \lesssim 0.15 \) corresponds to \( f \gtrsim 3.5 \) TeV (for \( \xi = 3.6 \)).
The Higgs mass in the leading order is sensitive to $\lambda_1/(2\xi^2)$, $M_{H^\pm}/f$ and $M_\rho/f$.

Fix $f = 4$ TeV, $\xi = 3.6$, $\lambda_2 = 0$. The dependence on these parameters is mild.

A larger Higgs mass occurs for larger $M_{H^\pm}/f$, $\lambda_1/(2\xi^2)$ and smaller $M_\rho/f$.

$M_h \lesssim 175$ GeV.
Lower bound on Higgs mass

- $M_h \text{ min}$ as a function of $f$ for $\xi = 3.6$, allowed by the condition $\lambda_h > 0$ at scale $m_{h'} \approx \xi f / \sqrt{2}$.
- Higgs mass is restricted by $80 \lesssim M_h \lesssim 175$ GeV.
The dependence on $\xi, f, \lambda_2$ are mild.

The dependence of Higgs mass on $\xi, f, \lambda_2$ is mild.
Heavy state spectrum

- Use Higgs mass (126 GeV) to fix $M_\rho$, plot the required $M_\rho$ that gives the correct Higgs mass.

- $f = 4$ TeV, $\xi = 3.6$, $\lambda_2 = 0$ ($m_{t'} = 10.2$ TeV).
- The lightest CP-odd neutral scalar is also a PNGB.
Heavy state spectrum

\[ f = 4\text{ TeV}, \; \xi = 3.6, \; \lambda_2 = 0. \]

\[ \text{Too heavy for LHC!} \]
Phenomenology?

- New states (apart from the 126 GeV Higgs) are too heavy to be probed at the LHC.
- But they can be probed at a $\mathcal{O}(100)$ TeV hadron collider!
- Higgs couplings are very close to SM values, approximately given by SM values times a factor of $\cos(v/f) \approx 1 - v^2/(2f^2)$.
- 0.2% deviation for $f = 4$ TeV, probably even beyond the reach of a future $e^+e^-$ collider.
- A precise determination of the $T$ parameter would help probe or constrain this model.
Conclusion

▶ The Top Seesaw Model is a modification of Top Condensation by introducing a new vector like top partner.

▶ It addresses the origin of both electroweak symmetry breaking and top Yukawa coupling.

▶ The Higgs mass is related to the top mass and has a rather restricted range, $80 \lesssim M_h \lesssim 175$ GeV, and one can easily obtain a 126 GeV Higgs.

▶ Constraint from $T$-parameter requires the chiral symmetry breaking scale to be much higher than the electroweak scale, which requires tuning.

▶ What if LHC doesn’t find anything?

▶ Modifications that embeds custodial symmetry can bring down the chiral symmetry breaking scale and predict interesting phenomenology at the LHC.

sidenote: I also worked on Stop searches using kinematic variables.