Problems and Progress in Covariant High Spin Description

Mariana Kirchbach and Víctor Miguel Banda Guzmán
Instituto de Física, Universidad Autónoma de San Luis Potosí, Av. Manuel Nava 6, Zona Universitaria, San Luis Potosí, SLP 78290, México
E-mail: mariana@ifisica.uaslp.mx, vmbg@ifisica.uaslp.mx

Abstract. A universal description of particles with spins \( j \geq 1 \), transforming in \((j,0) \oplus (0,j)\), is developed by means of representation specific second order differential wave equations without auxiliary conditions and in covariant bases such as Lorentz tensors for bosons, Lorentz-tensors with Dirac spinor components for fermions, or, within the basis of the more fundamental Weyl-Van-der-Waerden \((2,C)\) spinor-tensors. At the root of the method, which is free from the pathologies suffered by the traditional approaches, are projectors constructed from the Casimir invariants of the spin-Lorentz group, and the group of translations in the Minkowski space time.

1. Introduction
High-spin \( j \geq 1 \) fields, both massive and massless, always have counted to the main topics in field theories. In particle physics such fields describe hadron resonances whose spins vary from 1/2 to 17/2 for baryons, and from zero to six for mesons [1]. At hadron colliders, intermediate high spin resonances can be produced which subsequently decay in a pair of photons, a process of high interest in the literature [2]. In gravity, high-spin bosons can couple to the metric tensor and cause its deformation [3], and are besides this fundamental to the physics of rotating black holes [4]. Also gravitational interactions between high-spin fermions are of interest [5]. The traditional approaches to high-spin fields have been developed between 1939 and 1964 (see [6] for a recent review, and [7] for a standard textbook) by Fierz and Pauli (FP) [8], Rarita and Schwinger [9], Bargmann and Wigner (BW) [10], Joos [11] and Weinberg [12]. In the next section we briefly highlight these four methods, comment on their problems, and report in section 3 on a new approach, suggested in [13]-[15], before closing with a brief summary.

2. The traditional methods for high-spin description and their problems
The methods for high-spin description are distinct through the type of the representation spaces (reps) used to embed the spin of interest. Below we list various possibilities to describe spin-\( j \).

2.1. Multiple-spin valued representation spaces: the totally symmetric Lorentz tensors for bosons, and Lorentz-tensor–Dirac–spinors for fermions. The Fierz-Pauli and Rarita-Schwinger frameworks
Particles of spins, \( j > \frac{1}{2} \), have been so far frequently described in terms of \( so(1,3) \) representation spaces of multiple spins and alternating parities, corresponding to the totally symmetric (Sym)
Lorentz tensors of rank-$j$, 

$$ \text{Sym} \Phi_{\mu_1 \ldots \mu_j} \simeq \left( \frac{j}{2}, \frac{j}{2} \right), $$

(1)

for bosons (Fierz-Pauli framework [8]), or, the totally symmetric rank-$(j - 1/2)$ Lorentz tensors with Dirac spinor components, $\psi$,

$$ \text{Sym} \Psi_{\mu_1 \ldots \mu_{j-\frac{1}{2}}} \simeq \left( \frac{j - \frac{1}{2}}{2}, \frac{j - \frac{1}{2}}{2} \right) \otimes \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right], $$

(2)

for fermions (Rarita-Schwinger framework [9]), respectively. The particles have been associated with the degrees of freedom (d.o.f.) of the highest spins in the spaces under discussion, while the remaining degrees of freedom, corresponding to lower spins, had to be projected out by properly chosen auxiliary conditions for the sake of ensuring the correct number of physical d.o.f. required by spin-$j$. The wave equations in the Fierz-Pauli approach to high-spin-$j$ bosons read,

$$ (\partial^2 + m^2) \text{Sym} \Phi_{\mu_1 \ldots \mu_j} = 0, \quad \text{tr Sym} \Phi_{\mu_1 \ldots \mu_j \mu_j} = 0, \quad \partial^\mu \text{Sym} \Phi_{\mu_1 \ldots \mu_j} = 0, $$

(3)

while the Rarita-Schwinger approach to spin-$j$ fermions is cast as,

$$ (i\partial - m) \text{Sym} \psi_{\mu_1 \ldots \mu_{j-\frac{1}{2}}} = 0, \quad \gamma^\mu \text{Sym} \psi_{\mu_1 \ldots \mu_{j-\frac{1}{2}}} = 0, \quad \partial^\mu \text{Sym} \psi_{\mu_1 \ldots \mu_{j-\frac{1}{2}}} = 0. $$

(4)

2.2. Single-spin valued $(j, 0) \oplus (0, j)$ “bi-vector” representation spaces. The Joos-Weinberg framework

Alternatively, it has been considered that spin-$j$ particles can also be described in terms of single-spin valued representation spaces of the even number of $2(2j + 1)$ components,

$$ \psi^{(j)}_B \simeq (j, 0) \oplus (0, j), \quad B \in [1, 2(2j + 1)], $$

(5)

a scheme known as the Joos-Weinberg framework [11], [12]. Differently from the RS approach, the $2(2j + 1)$-componet wave function, column($\psi^{(j)}_1 \ldots \psi^{(j)}_{2j+1} \psi^{(j)}_{2j+1+1} \ldots \psi^{(j)}_{2(2j+1)}$), occasionally termed to as “bi-vector”, satisfies one sole differential equation, which is however of a high-order,

$$ \left( i^{2j} \left[ \gamma_{\mu_1 \mu_2 \ldots \mu_{2j}} \right]_{AB} \partial^{\mu_1} \partial^{\mu_2} \ldots \partial^{\mu_{2j}} - m^2 \delta_{AB} \right) \psi^{(j)}_B(x) = 0. $$

(6)

Here, $\left[ \gamma_{\mu_1 \mu_2 \ldots \mu_{2j}} \right]_{AB}$ are the elements of the generalized Dirac Hermitean matrices of dimensionality $[2(2j + 1)] \times [2(2j + 1)]$, which transform as Lorentz tensors of rank-$2j$. The complete sets of such matrices have been extensively studied in the literature for the purpose of constructing all the possible field bi-linear terms in the definitions of the generalized currents, both transitional and diagonal [16]. Though well elaborated, the Weinberg-Joos formalism has attracted comparatively less attention than the linear Rarita-Schwinger framework not only because of the difficult to tackle high order of the differential equations and the high dimensionality of the generalized Dirac matrices. In addition, bi-vectors as a rule can not be equipped neither with Lorentz-, nor with Dirac indexes, a reason for which their couplings to spinorial, vectorial etc targets have to be described through uncomfortable rectangular matrices.
2.3. Single-spin valued totally symmetric Weyl–Van–der–Waerden tensor-spinor representation spaces. The Bargmann-Wigner framework

Another option for spin-j description is provided by the totally symmetric product, of \( n = 2j \) copies of a Dirac spinor, \( \psi \simeq (1/2, 0) \oplus (0, 1/2) \), where \((1/2, 0)\) and \((0, 1/2)\) are the right- and left-handed Weyl–Van–der–Waerden two-component spinors corresponding to the two nonequivalent fundamental \( sl(2, C) \) representation spaces, also known as “spinor” (\( \xi \)), and “co-spinor” (\( \eta \)),

\[
\text{Sym} \psi_{b_1 \ldots b_n}^{(n)} = \text{Sym} \psi_{b_1} \ldots \psi_{b_n} \simeq \text{Sym} \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right]_1 \otimes \cdots \otimes \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right]_n ,
\]

where \( b_i = 1, 2, 3, 4 \), a scheme known as the Bargmann-Wigner method [10]. The Bargmann-Wigner rank-n spinor satisfies also a high-order differential equation which reads,

\[
(\gamma_\mu p^\mu - m)^{a_1 b_1} \ldots (\gamma_\mu p^\mu - m)^{a_n b_n} \text{Sym} \psi_{b_1 \ldots b_n} = 0 .
\]

All four schemes are known to be plagued by serious inconsistencies, which are however of distinct kinds. The Fierz-Pauli and the Rarita-Schwinger fields have multiple redundant components which are eliminated by the auxiliary conditions solely at the free-particle level. Upon couplings to external fields, the auxiliary conditions no longer serve their purpose, in consequence of which the wave fronts of the (classical) coupled solutions can propagate acausally (Velo-Zwanziger problem [17]). A further inconsistency has been revealed by Johnson and Sudarshan [18] who showed that the equal time anti-commutators between spin-3/2 fields, minimally coupled to the electromagnetic field, are not positive definite. In addition, the attempts to construct Lagrangians within these methods are becoming increasingly involved due to difficulties of incorporating the auxiliary conditions at the interaction level, a reason for which it is common to pursue alternative, though not fully covariant methods in data evaluations. Specifically in hadron collisions, where the calculations of the cross sections are carried out within the preferred Center of Mass frame, it is common to account for the contributions of intermediate states of high-spin-J to processes as, say, diphoton production, by parametrizing the corresponding polarized decay amplitude by a single constant and factorizing a rotational \( SO(3) \) Wigner function, \( d_m^{\lambda, \chi}(\theta) \), that encodes the angular dependence of the amplitude. Specifically in [2], the absence of high-spin Lagrangians has been compensated by suggesting effective Lagrangians that may be used to simulate some particular intermediate resonances of both low and high spins, through Monte Carlo generators.

On the other side, the Joos-Weinberg and the Bargmann-Wigner frameworks refer to a single wave equation without any need for auxiliary conditions, but they are of the order twice the particle’s spin. Differential equations of the order higher than 2 are as a rule difficult to tackle because they lead to higher-order field theories which suffer severe inconsistencies such as ghost solutions of the bad type, kinetic terms of wrong signs, states of negative norms, violation of unitarity etc. Furthermore, particles in such theories can propagate non-locally. All the experimentally verified fundamental theories in modern physics are based on Lagrangians of second order in the momenta. Only in effective theories with out integrated fields, high order Lagrangians may appear. One of the reasons behind the problems caused by high order differential equations is the so Ostrogradskian instability [19] which, at the level of, say, classical mechanics, for concreteness, predicts phase spaces of unstable orbits, and needs a special effort to be handled [20]. In the light of the above discussion, it is desirable to search for a method which would allow for a consistent description of any spin,

- by one sole wave equation of maximally second order in the derivatives, and derivable from a Lagrangian,
within Lorentz-, or, Weyl-Van-der-Waerden tensor-spinor bases.

Such a formalism for spin- in single-spin- \((j, 0) \oplus (0, j)\) reps has been designed in refs. [13],[14], [15], and is briefly highlighted below.

3. Progress in describing any spin- in \((j, 0) \oplus (0, j)\) in Lorentz-tensor-, Lorentz-tensor-Dirac-spinor-, or, Weyl-Van-der-Waerden tensor-spinor bases and wave equations of second order. The spin-Lorentz group projector method

The first goal of the references [13], [14], [15] has been to describe the pure spin \((j, 0) \oplus (0, j)\) states, be it through Lorentz-, or, Weyl-Van-der-Waerden spinor-tensors, this for the sake of constructing by simple index contractions vertexes which involve interactions of high-spins with gauge fields, such as the photon, and/or spinorial targets, such as the proton, and thus avoid the introduction of the cumbersome index-matching rectangular matrices, typical for the Joos-Weinberg formalism. In order to illustrate the essentials of the method, we here bring as representative examples the two simplest cases, beginning with the description of the \((1, 0) \oplus (0, 1)\) field as a totally anti-symmetric Lorentz tensor of second rank, \(B_{[\mu, \nu]}\), with the brackets denoting index anti-symmetrization. On these grounds, in a next step, pure spin-3/2 can be described by means of that very totally-antisymmetric second-rank Lorentz tensor 2 transforming in \((3/2, 0) \oplus (0, 3/2)\) sector contained as,

\[
B \otimes \psi \rightarrow \left[\left(\frac{3}{2}, 0\right) \oplus \left(0, \frac{3}{2}\right)\right] \oplus \left[\left(\frac{1}{2}, 1\right) \oplus \left(1, \frac{1}{2}\right)\right] \oplus \left[\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)\right]. \tag{9}
\]

The latter equation shows that pure spin-3/2 transforming in \((3/2, 0) \oplus (0, 3/2)\) can be described in terms of a 24 component wave function, denoted by \(\Psi^{[3/2, 0] \oplus [0, 3/2]}_{\mu, \nu}\), of two anti-symmetric Lorentz indexes, \([\mu, \nu]\), and a Dirac index, "\(\alpha\)", taking the values, \(a = 1, 2, 3, 4\), provided, one would be able to eliminate from them the redundant 16 components belonging to the \(so(1, 3)\) irreducible sectors, \(\left(\frac{1}{2}, 1\right) \oplus \left(1, \frac{1}{2}\right)\), and \(\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)\). Such an elimination can indeed be realized by the aid of momentum independent projectors constructed from the invariants of the spin-Lorentz group, a result due to [13],[14].

3.1. The spin-Lorentz group and static covariant projectors on irreducible representation spaces

The Lorentz group transforming the internal spin degrees of freedom, henceforth termed to as spin-Lorentz group, and denoted by, \(L\), is a subgroup of the complete Lorentz group, which acts besides on the spin- also on the external space-time degrees of freedom. The \(L\) generators, denoted by \(S_{\mu\nu}\), are quadratic \(d \times d\) constant matrices, where \(d\) fixes the finite dimensionality of the internal representation space, and encodes the spin value. For the special case of a pure spin, dimensionality and spin are related as \(d = 2(2j + 1)\), while for representations of multiple spins, relations of the type \(d = \sum_{j}(2j + 1)\), or, \(d = 2 \sum_{j}(2j + 1)\), can hold valid. The algebra of the spin-Lorentz group, termed to as homogeneous spin-Lorentz group (HSLG) reads [21],

\[
L : \quad [S_{\mu\nu}, S_{\rho\sigma}] = i(g_{\mu\rho}S_{\nu\sigma} - g_{\mu\sigma}S_{\nu\rho} + g_{\nu\rho}S_{\mu\sigma} - g_{\nu\sigma}S_{\mu\rho}). \tag{10}
\]

It has two Casimir invariants, here denoted in their turn by, \(F\) and \(G\), and defined as,

\[
F_{AB} = \frac{1}{4} [S_{\mu\nu}]_{AD} [S_{\mu\nu}]_{DB}, \quad G_{AB} = \frac{1}{8} \epsilon_{\mu\nu}^{\alpha\beta} [S_{\mu\nu}]_{AC} [S_{\alpha\beta}]_{CB}, \quad A, B, C, D, ..., = 1, ..., d, \tag{11}
\]
The two operators in (11) identify unambiguously any irreducible finite dimensional \( \mathcal{L} \) group representation space, here generically denoted, \( \psi_{(j_1,j_2)\oplus (j_2,j_1)} = \phi^R_{(j_1,j_2)} \oplus \phi^L_{(j_2,j_1)} \), where \( \phi^R_{(j_1,j_2)} \) and \( \phi^L_{(j_2,j_1)} \) are in their turn its left- and right-handed chiral components, according to,

\[
F \Psi_{(j_1,j_2)\oplus (j_2,j_1)} = c_{(j_1,j_2)} \Psi_{(j_1,j_2)\oplus (j_2,j_1)}, \quad c_{(j_1,j_2)} = \frac{1}{2} \left( K(K + 2) + M^2 \right),
\]

\[
G \phi^R_{(j_1,j_2)} = r_{(j_1,j_2)} \phi^R_{(j_1,j_2)}, \quad G \phi^L_{(j_2,j_1)} = r_{(j_2,j_1)} \phi^L_{(j_2,j_1)},
\]

\[
r_{(j_1,j_2)} = -r_{(j_2,j_1)} = i(K + 1)M, \quad K = j_1 + j_2, \quad M = |j_1 - j_2|.
\]

The idea of [13] has been to employ the Casimir invariant \( F \) in the construction of a momentum independent (static) projectors on the irreducible sectors of the Lorentz-tensor-spinor in (9) and to explore the consequences. The Lorentz projector, denoted by \( \mathcal{T}_F^{(3/2,0) \oplus (0,3/2)} \), that identifies the irreducible \((3/2,0) \oplus (0,3/2)\) representation space in (9), is constructed from \( F \) in (12) as,

\[
P_F^{(3/2,0) \oplus (0,3/2)} = \left( \frac{F - c_{(1,2)/2}}{c_{(3/2,0)} - c_{(1,2)/2}} \right) \left( \frac{F - c_{(1,2)/2}}{c_{(3/2,0)} - c_{(1,2)/2}} \right),
\]

\[
c_{(1,2)/2} = \frac{3}{4}, \quad c_{(3/2,0)} = \frac{15}{4}, \quad c_{(1,2)/2} = \frac{11}{4}.
\]

The equation (15) shows that the operator \( \mathcal{T}_F^{(3/2,0) \oplus (0,3/2)} \) has the property to nullify any irreducible representation space which is different from \((3/2,0) \oplus (0,3/2)\). Instead, for \((3/2,0) \oplus (0,3/2)\), it acts as the identity operator, meaning that \( \mathcal{T}_F^{(3/2,0) \oplus (0,3/2)} \) is a projector on this very space. In recalling the notation of the spin-3/2 wave function, \( \left[ \Psi^{(3/2,0) \oplus (0,3/2)}_{[\mu,\nu]} \right]_a \), we find

\[
\left[ \mathcal{T}_F^{(3/2,0) \oplus (0,3/2)} _{[\mu,\nu]} \right]_a \left[ \Psi^{(3/2,0) \oplus (0,3/2)}_{[\eta,\rho]} \right]_b = \left[ \Psi^{(3/2,0) \oplus (0,3/2)}_{[\mu,\nu]} \right]_a.
\]

The simplest way to bring in kinematics is to set \( \left[ \Psi^{(3/2,0) \oplus (0,3/2)}_{[\mu,\nu]} \right]_a \) on its mass shell,

\[
p^2 \left[ \mathcal{T}_F^{(3/2,0) \oplus (0,3/2)} _{[\mu,\nu]} \right]_{[\eta,\rho]} \left[ \Psi^{(3/2,0) \oplus (0,3/2)}_{[\eta,\rho]} \right]_b = m^2 \left[ \Psi^{(3/2,0) \oplus (0,3/2)}_{[\mu,\nu]} \right]_a.
\]

The symmetry of (17) is the direct product of the group of translations, \( T_{(1+3)} \), in \((1+3)\) space time, with the spin-Lorentz group, i.e. \( T_{(1+3)} \otimes \mathcal{L} \), some times termed to as the inhomogeneous spin-Lorentz group. In this way, it becomes possible to describe a particle residing in the pure spin-3/2 representation space, \((3/2,0) \oplus (0,3/2)\), of the \( so(1,3) \) algebra of the spin-Lorentz group, by a Lorentz-tensor-Dirac-spinor wave function, and by means of the one sole second-order differential equation in (17). It has been shown in [13] that within this scheme, one reproduces the precise electromagnetic multipole moments obtained within the canonical Joos-Weinberg method, where the field is described by an eight component "bi-vector" [22]. It has been furthermore checked that also the properties of the remaining two sectors, known from elsewhere, are correctly reproduced, as it should be. Moreover, in [13] the proof that the wave equation in (17) is free from the Velo-Zwanziger problem has been delivered by demonstrating that the wave fronts of its (classical) solutions propagate always causally within an electromagnetic environment. The above presentation should leave it clear that the scheme does not restrict to product spaces of the type given in (9) but extends to any product spaces containing the pure spin \((j,0) \oplus (0,j)\) of interest. In particular, it applies to bases constructed as direct products of four-vectors, or, to general Weyl-Van-der-Waerden tensor-spinors, as used in the Bargmann-Wigner framework [15]. Moreover, the scheme also extends to high- spins carried by two-spin valued representation spaces of the type, \((1/2, j - 1/2) \oplus (j - 1/2, 1/2)\), [14], in
which case the second order wave equation is not obtained from the mass shell-condition alone but from combining it by another covariant projector, \( P_{W^2}^{(j,m)} = (-\mathcal{W}^2/m^2 - j(j-1)p^2/m^2)/(2j) \), which projects besides on the mass shell, also on the highest of the two spin degrees of freedom. Here, \( \mathcal{W}^2 \) is the squared Pauli-Lubanski operator, the second Casimir invariant of the algebra of inhomogeneous spin-Lorentz group. Such a projector technique has first been employed by Aurilia and Umezawa in [23] at the free particle level, and independently, almost 40 years later by [24] at the interacting level. In the latter work the second order differential equation following from \( P_{W^2}^{(j,m)} \) has been extended by the most general terms allowed by relativity and containing \( \partial_\mu, \partial_\nu \) commutators. These terms, identically vanishing at the free particle level, provide upon gauging essential contributions proportional to the electromagnetic field strength tensor, \( F_{\mu\nu} \) and guarantee that the resulting wave equation is free from the Velo-Zwanziger problem for a \( g \) factor taking the value of \( g = 2 \). However, the solutions to second order equations, be them to the representation unspecific Klein-Gordon-, and Proca types, or to the representation specific equation in (17), describe arbitrary mixing of spin-\( j \) particles of opposite parities. This arbitrariness can be removed by the aid of the second Casimir invariant \( G \) in (11), the subject of the next subsection.

3.2. Separating left from right chiral degrees of freedom and identifying the parity states

In parallel to (15), also the \( G \) invariant of the spin-Lorentz group algebra can be employed in the construction of projector operators within any \((j_1, j_2) \oplus (j_2, j_1)\) representations space. Such operators, here denoted by \( P^{(j_1, j_2)}_G \) and \( P^{(j_2, j_1)}_G \), respectively, have the property to separate the reps under consideration into left (L)- and right (R)-handed degrees of freedom according to,

\[
P^{(j_1, j_2)}_G = \frac{1}{2} \frac{G + r_{(j_1, j_2)}}{r_{(j_1, j_2)}}; \quad P^{(j_2, j_1)}_G = -\frac{1}{2} \frac{G - r_{(j_1, j_2)}}{r_{(j_1, j_2)}}.
\]

Therefore, from (18) one observes that the projectors \( P^{(j_1, j_2)}_G \) and \( P^{(j_2, j_1)}_G \) decompose the \( 2(2j_1 + 1)(2j_2 + 1) \) degrees of freedom residing in \((j_1, j_2) \oplus (j_2, j_1)\) into the two independent right-handed, \((j_1, j_2)\), and left-handed, \((j_2, j_1)\), irreducible sectors, each having half, i.e. \((2j_1 + 1)(2j_2 + 1)\), of the independent degrees of freedom of the initial representation space. Then the two equations,

\[
\frac{p^2}{m^2} P^{(j_1, j_2)}_G P_F^{(j_1, j_2) \oplus (j_2, j_1)} \phi_{(j_1, j_2)} = \phi_{(j_1, j_2)}^R; \quad \frac{p^2}{m^2} P^{(j_1, j_2)}_G P_F^{(j_2, j_1) \oplus (j_2, j_1)} \phi_{(j_2, j_1)} = \phi_{(j_2, j_1)}^L,
\]

uniquely fix the chiral components of the representation space under consideration. The chiral components are defined by the symmetric and anti-symmetric combinations of wave functions of positive and negative parity states, a reason for which the parity states spanning the representation space of interest can be recovered without ambiguities from the chiral states. Specifically for the Dirac spinor, \( j_1 = 1/2, j_2 = 0 \), where the spin-Lorentz group generators are \( 1/2 \sigma_{\mu\nu} \), the \( G \) invariant calculates as, \(-3/4i\gamma^5\), thus defining the corresponding projector operators as \( P^{(1/2, 0)}_G = (1 - \gamma_5)/2 \), and \( P^{(0, 1/2)}_G = (1 + \gamma_5)/2 \), leading to the respective Dirac’s chiral spinors, \((u-v)/2\), and \((u+v)/2\).

4. Summary and conclusions

We suggested a universal recipe for calculating the states of particles with any high spin-\( j \), covariantly transforming according to \((j, 0) \oplus (0, j)\), and in terms of Lorentz-tensor-Dirac-spinors (16), or, Weyl–Van–der-Waerden tensor-spinors [15], thus facilitating vertex constructions by simple contractions of indexes, and avoiding rectangular-matrix insertions. To avoid the
problems of the high-order differential equations, we, similarly to the Fierz-Pauli method, approximated the kinematics through the mass-shell condition alone, and kept the reference frame specification of the reps (encoded by the boost) solely at the wave-function level (generated in any frame by construction), while dropping it from the wave equation. Boost incorporation into the wave equations necessarily rises their order. The wave equations in our method are derivable from a Lagrangian. To be specific, within the spin-Lorentz group projector method pure spin-3/2 transforming in the totally symmetric Lorentz tensor of second rank with Dirac spinor components, \( \Psi_{\mu,\nu}^{(3/2,0)\oplus(0,3/2)} \), has been described by means of the following Lagrangian [13],

\[
L_{\text{free}}^{(3/2,0)\oplus(0,3/2)} = \left( \partial^\mu \left[ \Gamma^{(3/2,0)\oplus(0,3/2)}_{\mu\nu\rho} \right] \right) A_{\alpha\beta} B_{\gamma\delta} + m^2 \left[ \bar{\psi}^{(3/2,0)\oplus(0,3/2)} \right] A_{\mu\nu} B_{\gamma\delta},
\]

with the \( \Gamma \)-tensor being defined as,

\[
\Gamma^{(3/2,0)\oplus(0,3/2)}_{\mu\nu\rho} = 4 \left[ P^{(3/2,0)\oplus(0,3/2)}_{\mu\nu\rho} \right] \left[ \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \right] \left[ \sigma^\mu \sigma^\nu \sigma^\gamma \sigma^\delta \right] + 4 \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta ,
\]

where \( \sigma_{\mu\nu} \) stands for the standard totally anti-symmetric Dirac tensor of second rank. Along this line, Lagrangians for any spin can be constructed on the cost of increasing the number of the Lorentz indexes. We tested this approach in the evaluation of Compton scattering processes in [13], [14] and found that, compared to the Joos-Weinberg bi-vector basis, it notably speeds up the calculations by allowing employment of the FeynCalc software.

Acknowledgments: We thank the Editors for their kind invitation to contribute to this volume of historical relevance, in honor of the important impact of the DFyC on physics in Mexico.

References

[1] Olive K A et al. Chinese Physics C 2015 38 090001
[2] Panico G, Vecchi L, and Wulzer A 2016 JHEP 1606 184
[3] Adamek J, Davero D, Durrer R and Kunz M 2016 Nature physics 12 346
[4] Campoleoni A, Gonzalez H A, Ohlak B and Riegler M 2015 E-print Arxiv: 1512.03353 [hep-th]
[5] Henneaux M, Gomez G L and Rahman R 2014 JHEP 1401 087
[6] Panico G, Vecchi L, and Wulzer A 2016 Proceedings of the Royal Society A 472 20160001
[7] Fierz M and Pauli W 1939 Proc. Roy. Soc. Lond. A 173 211
[8] Barut A and Raczka R 1980 Group representation theory and its applications, Vol. 2 (Moscow, Mir)
[9] Velo G and Zumino B 1969 Il Nuovo Cimento A 51 14
[10] JHEP 1318
[11] Weinberg S 1964 Phys. Rev. B 133 1318
[12] Delgado Acosta E G, Banda Guzmán V M and Kirbach M 2015 Eur. Phys. J. A 51 35
[13] Delgado Acosta E G, Banda Guzmán V M and Kirbach M 2015 Int. J. Mod. Phys. E 24(7) 1550060
[14] Fierz M and Pauli W 1939 Proc. Roy. Soc. Lond. A 173 211
[15] Sankaranarayanan A 1968 Il Nuovo Cimento LVI A 2 459
[16] Velo G and Zumino B 1969 Phys. Rev. B 188 2218
[17] Johnson K and Sudarshan E C G 1961 Ann. Phys. (N.Y.) 1 13 126
[18] Woodward R P 2003 Phys. Rev. A 67 016102
[19] Tai-Jun Chen et al 2013 JCAP 02 042
[20] Kim Y S and Noz M 1986 Theory and applications of the Poincaré group (D. Reidel Publ. Comp., Dordrecht)
[21] Delgado Acosta E G, Kirbach M, Napsuciale M and Rodriguez S 2012 Phys. Rev. D 85 116006
[22] Aurilia A and Umezawa H 1967 Nuovo Cim. A 51 14
[23] Napsuciale M, Kirbach M and Rodriguez S 2006 Eur. Phys. J. A 29 289