Conditions for the generation of spin and charge currents in bulk spin Hall devices

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Abstract – We investigate the out-of-equilibrium stationary states of spin Hall devices on the basis of the least dissipation principle in the two-spin-channel model. The variational technique allows the values of the free parameters of the Hall device to be derived. The free parameters are the chemical potentials, the transverse Hall currents and the charge accumulations in the two spin channels. We show that, for a bulk paramagnet with spin-orbit interaction, in the case of the Hall bar geometry the principle of minimum dissipated power prevents the generation of transverse spin and charge currents, while in the case of the Corbino disk geometry, transverse currents, can be produced. More generally, we show that electric charge accumulation prevents the stationary spin to charge current conversion from occurring inside the device.

The interest for low-power-consumption electronic devices is one of the main motivations for the research in spintronics. In this context, considerable efforts have been devoted to the study of the spin Hall effect (SHE) and the inverse spin Hall effect (ISHE) [1–10]. In the Dyakonov-Perel (DP) model [1,6], SHE is a generalization of the usual Hall effect in which the spin-orbit interaction plays the role of an internal magnetic field that depends on the electronic spin states. In particular, SHE describes the spin accumulation produced in the direction perpendicular to an injected current. The inverse effect ISHE refers to the experimental configuration in which a spin current—or a non-uniform spin-accumulation—generates a transverse electric field. These effects open the way to the implementation of new devices, that allow the conversion of spin current into charge current [11]. Establishing the conditions under which this conversion, that takes place in a stationary state, obeys the thermodynamic laws is a problem of crucial importance currently.

Both SHE and ISHE can be analyzed by the spin-Hall transport equations, first proposed by Dyakonov and Perel [1] in order to describe paramagnetic conductors with spin-orbit interaction. A similar effect is well known for bulk ferromagnetic conductors in terms of an anomalous Hall effect (AHE) [12]. After the discovery of giant magnetoresistance at the turn of the millennium, SHE was also considered at an interface in the context of spin injection [2,3]. In parallel, the concept of pure spin current was proposed [13,14] in order to describe spin accumulation occurring in a part of the device that does not drive electric current. The presence of a spin current was found in a great number of systems and situations [5,7,9,15,16]. However, for convenience, the experimental protocol usually consists in measuring a voltage and not current (with the exception of ref. [15]). The conditions for the existence of a current instead of a voltage is an open problem crucial in the energetics of the system since that current, in contrast to spin accumulation, entails heat dissipation in usual materials.

The goal of this work is to calculate the minimum power dissipation in different kinds of spin Hall devices, in order to deduce the conditions for the existence of transverse spin and charge currents. The study is based on the DP model described in the framework of the two-spin-channel approach. We will show that, under some general assumptions, the principle of minimum dissipated power prevents the generation of transverse spin and charge currents in the case of the Hall bar geometry, while a transverse pure spin current can be generated in the case of the Corbino disk geometry.
In the framework of the two-channel model, the system is defined by an ensemble of two populations of electric carriers: those with spins $\uparrow$ and $\downarrow$; the spin-polarization axis is defined by a unit vector $\vec{p}$ perpendicular to the current. We assume that the temperature is constant everywhere. The transport equations are then described by Ohm’s law applied to each spin channel:

$$\vec{J}_{\pm} = -\sigma_{\pm} \nabla \mu_{\pm},$$

(1)

where $\sigma_{\pm}$ is the conductivity tensor and $\mu_{\pm}$ is the electrochemical potential. We assume for convenience that the system is a two-dimensional thin layer so that $\sigma_{\pm}$ is a $4 \times 4$ matrix, which is isotropic in the absence of spin-orbit coupling (see ref. [17] for the case of a ferromagnetic material). In a Cartesian coordinate system $(\vec{e}_x, \vec{e}_y)$ we have $\sigma_{xx\pm} = \sigma_{yy\pm} = \sigma_{\pm}$.

The Dyakonov-Perel (DP) equations are obtained by the application of the relevant symmetries to the spin-dependent conductivity tensor $\sigma_{\pm}$. Since, in the DP model, the effect of the spin-orbit coupling is equivalent to that of a local magnetic field perpendicular to the current, the cross-coefficients obey the Onsager reciprocity relations [18,19] $\sigma_{xy\pm} = -\sigma_{yx\pm} = \sigma_{\mp\pm}$ (where $\sigma_{\mp\pm}$ is the Hall conductivity due to this local spin-orbit field). Furthermore, if the local spin-orbit field due to the spin polarization $\uparrow$ is along the direction $\vec{e}_z$ the spin-orbit field due to the spin polarization $\downarrow$ is along the direction $-\vec{e}_z$, so that the same reciprocity relations also impose $\sigma_{x\sigma\pm} = -\sigma_{y\sigma\pm}$. In the Cartesian reference frame, the conductivity tensor reads

$$\sigma_{\pm} = \begin{pmatrix} \sigma_{\uparrow} & \sigma_{\sigma\uparrow} & 0 & 0 \\ -\sigma_{\sigma\uparrow} & \sigma_{\downarrow} & 0 & 0 \\ 0 & 0 & \sigma_{\sigma\downarrow} & -\sigma_{\sigma\downarrow} \\ 0 & \sigma_{\sigma\downarrow} & \sigma_{\sigma\downarrow} & \sigma_{\downarrow} \end{pmatrix}. $$

(2)

The resistivity matrix is given by the inverse matrix $\rho_{\pm} = \sigma_{\pm}^{-1}$, so that it is defined by the two coefficients $\rho_{\parallel} = \sigma_{\parallel}/(\sigma_{\parallel}^2 + \sigma_{\perp}^2)$ and $\rho_{\perp\parallel} = -\sigma_{\perp\parallel}/(\sigma_{\parallel}^2 + \sigma_{\perp}^2)$. Due to the possible accumulation of electric charges at the edges, the density of spin-dependent carriers $n_{\pm}$ is not necessarily constant, and the corresponding diffusion terms should be taken into account in the electrochemical potential $\mu_{\pm}$ [20]:

$$\mu_{\pm} = \frac{kT_F}{q} \ln \left( \frac{n_{\pm}}{n_0} \right) + V + \mu_{\pm}^{ch},$$

(3)

where $T_F$ is either the Fermi temperature in the case of a metal or the temperature of the thermostat in the case of a semi-conductor (non-degenerated), $k$ is the Boltzmann constant, $V$ is the electric potential, $q$ is the charge of the carriers, and $n_0$ is the density of electric carriers corresponding to electric neutrality. The term $\mu_{\pm}^{ch}$ takes into account all other contributions to the chemical potential (e.g., due to spin injection, Mott relaxation, magneto-optical excitation, etc).

Each term of the electrochemical potential gives rise to a force: the thermodynamic force proper of the diffusion process $\nabla n_{\pm}$, the spin-dependent electric field $\vec{E}_{\pm} \equiv \vec{E} - \nabla \mu_{\pm}^{ch}$ [21], and the spin-pumping force $\Delta \mu_{\pm}^{ch} = \mu_{\pm}^{ch} - \mu_{\pm}$ [22,23]. Introducing eq. (3) and eq. (2) into eq. (1) we have

$$\vec{J}_{\pm} = -\sigma_{\pm} \vec{E}_{\pm} + D_{\pm} \nabla n_{\pm} \pm \vec{p} \times (\sigma_{\sigma\pm} \vec{E}_{\pm} + D_{\sigma\pm} \nabla n_{\pm}),$$

(4)

where the signs $+$ and $-$ correspond to channels $\uparrow$ and $\downarrow$, respectively. We have introduced the longitudinal and transverse spin-dependent diffusion coefficients $D_{\pm} = kT_F \sigma_{\sigma\pm}/(q n_{\pm})$ and $D_{\sigma\pm} = kT_F \sigma_{\pm}/(q n_{\pm})$.

The current of electric charges is defined by the sum $\vec{J} = \vec{J}_{\uparrow} + \vec{J}_{\downarrow}$, and the spin current is defined by the difference $\vec{J}_s = \vec{J}_{\uparrow} - \vec{J}_{\downarrow}$. Equations (4) are equivalent or generalize those derived in refs. [3,4]. For the sake of simplicity, we restrict the analysis to the DP model without spin-flip scattering, and $\mu_{\uparrow}^{ch}$ and $\mu_{\downarrow}^{ch}$ are assumed constant in space. In addition we assume that the conductivities of the two spin channels are equal: $\sigma_{\uparrow} = \sigma_{\downarrow} = \sigma$. Equations (4) can then be compared to the Dyakonov-Perel equations [21]. We now apply the Kirchhoff-Helmholtz principle which states that the current distributes itself so as to minimize heat dissipation, for a given bias voltage and boundary conditions. In other terms, the stationary states are those states at which dissipation is a minimum compatible with the ensemble of constraints applied to the system [24,25]. In the case of Hall devices, there are two external constraints that impose non-equilibrium. On the one hand the injection of the electric field along one direction of the Hall device (along the $\vec{e}_x$ direction in the case of the Hall bar, and along $\vec{e}_y$ in the case of the Corbino sector) leads to the electric current. On the other hand, the application of the magnetic field leads to a charge accumulation $\delta n = n - n_0$ at the edges [21]. The density of the electric charges is governed by Gauss’s law which can be viewed as a constraint to the values of these quantities.

Beside, we do not impose any specific boundary conditions at the edges: the system is free to give any values to the parameters $n_{\uparrow}, n_{\downarrow}, J_{\uparrow}, J_{\downarrow}$ or $\delta n$, and $n_0$ everywhere inside the device, including at the edges $y = \pm l$ or $\theta = \{0, \alpha\}$ (see fig. 1).

The power dissipated by the system is $P_T = J_{\uparrow} \cdot \vec{v} \mu_{\uparrow}^T + J_{\downarrow} \cdot \vec{v} \mu_{\downarrow}^T$. In view of the linear relation (1) it is a function of the partial derivatives of the electrochemical potentials $\partial_j \mu_{\pm}$ (where $j = \{x, y\}$) (that contain drift and diffusion terms). The power can equivalently be expressed as a function of the currents $J_{\pm}$ or the thermodynamic forces, through eqs. (1). The Coulomb equation $\text{div}(\nabla V) = -q \delta n / \mathcal{E}$ (where $\mathcal{E}$ is the permittivity and $\delta n = \delta n_{\uparrow} + \delta n_{\downarrow}$) the electric-charge accumulation in the two channels) and the expression $-2 \nabla V = -\nabla \mu_{\uparrow} + kT_F \nabla \ln(n_{\uparrow}) - \nabla \mu_{\downarrow} + kT_F \nabla \ln(n_{\downarrow})$ (deduced from eq. (3)) lead to the Poisson law $\nabla^2 (\mu_{\uparrow} + \mu_{\downarrow}) = -kT_F \nabla^2 \ln(n_{\uparrow}) - kT_F \nabla^2 \ln(n_{\downarrow}) + 2q \delta n / \mathcal{E} = 0$. In order
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The spin Hall angles $\theta_{SH}$ are defined as the angles between $-\nabla \mu^s$ and the injected current $J_1$.

to find the minimum of the Joule power $P_J$ compatible with the constraints, we introduce the corresponding Lagrange multipliers $\lambda(x, y), \beta(y)$ such that the function to minimize reads

$$
\mathcal{F}[\nabla \mu^s, n_\pm, \lambda, \beta] = \int_0^L \int_0^L \left[ \sigma_\uparrow (\partial_x \mu^s)^2 + \sigma_\downarrow (\partial_y \mu^s)^2 + \sigma_\downarrow (\partial_x \mu^s)^2 \sigma_\uparrow (\partial_y \mu^s)^2 - \lambda(x, y) \left( \nabla^2 (\mu^s + \mu^c) - kT_F \nabla^2 \ln(n^s) \right) \right] dxdy,
$$

where $J_0^0 = \vec{j}_e \cdot \vec{e}_r$ is the constant charge or spin current density injected in the device along the $x$-direction, with the different options: $i = \{c, s\}$ (with $i = c$ associated to a charge current and $\epsilon_s = 1$, $i = s$ associated to a spin current and $\epsilon_s = -1$), and $\nabla = (\partial_x, \partial_y)$. The expression of the last constraint is derived from eq. (1). In the following equations, $\pm$ or $\mp$ account for the two spin channels $\uparrow, \downarrow$. Minimization of $\mathcal{F}$ in the case of an imposed charge current ($i = c$) leads to the following conditions:

$$
\nabla \partial_x \mu^c \mathcal{F} = 0 \Rightarrow \partial_x \mu^c = -\frac{1}{2} \beta(y) - \frac{1}{2 \sigma^c_\uparrow} \partial_x \lambda,
$$

$$
\nabla \partial_y \mu^c = 0 \Rightarrow \partial_y \mu^c = -\frac{1}{2} \sigma^c_\downarrow \beta(y) - \frac{1}{2 \sigma^c_\downarrow} \partial_y \lambda,
$$

and

$$
\nabla n^s \mathcal{F} = \frac{kT_F}{qn^s} \nabla^2 \lambda - 2 \frac{q}{e} \lambda = 0.
$$

The solution for the chemical potential in the case $i = c$, derived from (6) and (7) is (see Supplementary material Supplementarymaterial.pdf (SM))

$$
\lambda_D^2 \nabla^2 \mu^c = 2 \mu^c(x, y) + \beta \left( x \pm \frac{\sigma_\uparrow}{\sigma^c_\uparrow} y \right) + C^c_1,
$$

where $\lambda_D = \sqrt{\frac{e kT_F}{q^2 n}}$ is the Debye-Fermi length, $C^c_1$ is a constant and $\beta(y) = \beta$ is the Lagrange multiplier introduced above. Note that eq. (8) can be interpreted as a screening equation for the electric charges, with the term $\beta(x \pm \frac{\sigma_\uparrow}{\sigma^c_\uparrow} y)$ as a source term. We assume $\lambda_D \ll l$, where $l$ is the typical length of the device along the direction $\vec{e}_r$.

The corresponding solution of eq. (8) in the bulk is then

$$
\mu^c(x, y) = -\left( x \pm \frac{\sigma_\uparrow}{\sigma^c_\uparrow} y \right) \beta J_0^0 - \frac{C^c_1}{2}
$$

with the mean resistivity $\bar{\rho} = \frac{\sigma_\uparrow}{\sigma^c_\uparrow} \rho_{xy} = \frac{1}{\rho} + \frac{1}{\rho_{xy}})^{-1}$, and, according to eq. (3), the constant is $C^c_1 = \bar{\rho} \rho_{xy}$. In the case $i = s$ of an imposed spin current ($\epsilon_s = -1$ in eq. (5)), we find

$$
\lambda_D^2 \nabla^2 \mu^s = 2 \mu^s(x, y) + \beta \left( x \pm \frac{\sigma_\uparrow}{\sigma^c_\uparrow} y \right) + \bar{C}^c_1,
$$

where $\bar{C}^c_1$ is a constant which gives, in the bulk ($l \gg \lambda_D$),

$$
\mu^s(x, y) = \mp \left( x \pm \frac{\sigma_\uparrow}{\sigma^c_\uparrow} y \right) \beta J_0^0 - \frac{\bar{C}^c_1}{2}
$$

Inserting eq. (9) or eq. (11) into eq. (1) we obtain

$$
\vec{j}_e \cdot \vec{e}_r = J_{\uparrow 1} = 0
$$

or $J_{\downarrow 1} = J_{\downarrow 2} = 0$. For $i = c$: $J_{\uparrow 1} = (\bar{\rho}/\rho_{xy}) J_{yx}$; and for $i = s$: $J_{s \uparrow 1} = (\bar{\rho}/\rho_{xy}) J_{sx}$. The corresponding minimal power dissipation density is in both cases $P_{min} = \bar{\rho} (\rho_{xy})^2$.

The simple but fundamental result that $J_{\uparrow 2} = 0$ inside the device has been disregarded in the spin Hall literature [1–10,12,16] because it is a property of the sole stationary states. It is due to the fact that the spin-orbit force is responsible for the accumulation of electric charges in each spin channel [21]. However, as will be shown below, the result in eq. (12) is no longer valid in the case of the Corbino disk.

The principal characteristic of the spin Hall effect is the spin accumulation due to spin-orbit coupling. From eq. (9) ($i.e.$, the case $i = c$) we have, when $\sigma_\uparrow = \sigma_\downarrow$,

$$
\Delta \mu^c(y) = -\frac{2 \sigma_\uparrow}{\sigma} \beta \bar{\rho} J_0^0 + \Delta \mu^{ch},
$$

where the constant $\Delta \mu^{ch}$ accounts for the other possible contributions to the spin accumulation. The spin Hall angles $\theta_{SH1}$ are defined by the relations $\tan \theta_{SH1} \equiv -\partial_y \mu^c/\partial_x \mu^c = \mp \sigma_\uparrow/\sigma$ (see fig. 1). This spin accumulation occurring in bulk paramagnets with strong

Fig. 1: (Colour online) Schematic views of two equivalent spin Hall devices: (a) Hall bar and (b) Corbino sector. The boundary conditions at the edges ($i.e.$, the values of $\mu^c_1, n^s_2$ and $J_{\uparrow 2}$ or $J_{\downarrow 2}$ at $y = \pm l$ or $\theta = \{0, \alpha\}$) are kept free before the minimization. The spin Hall angles $\theta_{SH}$ are defined as the angles between $-\nabla \mu^s$ and the injected current $J_2$. 

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spin-orbit coupling is well known and has been observed in various systems [6].

We discuss now a different device, in which the Hall bar is deformed according to a conformal transformation (x, y) \rightarrow e^{i \theta} y = re^{i \theta}$, where $r, \theta$ are the polar coordinates [26]. We then obtain a Corbino sector (fig. 1(b)) or the Corbino disk (fig. 2). In the case of the Corbino angular sector of angle $\alpha$, with inner radius $r_1$ and outer radius $r_2$ shown in fig. 1(b), charge accumulation is allowed. The function to minimize reads now

$$\mathcal{F}^{Cor}[\nabla \mu_2^n, n_z^\pm, \lambda, \beta] = \int_{r_1}^{r_2} \int_0^\alpha \left[ \sigma_\uparrow (\partial_r \mu_\uparrow)^2 + \sigma_\downarrow (\partial_r \mu_\downarrow)^2 - \lambda(\theta, r) \nabla^2 (\mu_\uparrow + \mu_\downarrow) - kT_F \nabla^2 \ln(n_\uparrow) - kT_F \nabla^2 \ln(n_\downarrow) + \frac{2q \delta n}{E} \right] \nabla \mu \cdot \hat{r} d\theta d\theta$$

where $i = \{c, s\}$ and $I_\uparrow = I_\uparrow + \epsilon_\downarrow I_\downarrow$. The polar gradient takes the form $\nabla = (\partial_r, \partial_\theta/r)$ and the Laplacian is $\nabla^2 = \frac{1}{r^2} \partial_r (r^2 \partial_r) + \frac{1}{r^2 \sin^2 \theta} \partial_\theta^2$. $I_\uparrow = \alpha h J_\uparrow$ is the charge or spin current injected radially (see fig. 1(b), $h$ is the thickness of the layer, and $J_\uparrow = J_\uparrow$ is the radial current density. The radial current is such that $r J_\uparrow = -\sigma_\uparrow r \partial_r \mu_\uparrow + \sigma_\uparrow r \partial_r \mu_\downarrow$. Proceeding as in the previous case, we obtain the result for $i = c$

$$\mu_c^\uparrow(r, \theta) = -\left( \ln \left( \frac{r}{r_1} \right) \pm \frac{\sigma_\uparrow \theta}{\sigma_\uparrow} \right) \frac{I_\uparrow}{\alpha h} - \frac{C_1}{2}$$

that leads to $J_\uparrow = 0$. The result is the same as the one obtained for the Hall bar.

However, if we consider a perfect Corbino disk with $\alpha = 2\pi$, it is equivalent to a Hall bar in which the two edges are in perfect electric contact so that charge accumulation is not allowed. The variable $n$ is fixed to zero $n_z = 0$ everywhere, and the Poissson constant reduces to the harmonic equation $\nabla^2 \mu_\downarrow = 0$, whose solution

$$\mu_\downarrow^{Cor}(r, \theta) = -\frac{I_\uparrow}{2\pi \sigma_\uparrow h} \ln \left( \frac{r}{r_1} \right).$$

In that case, both longitudinal and transversal stationary currents are present for the two spin channels: $J_{\uparrow 1} = J_{\downarrow 1}/(2\pi h)$ and $J_{\uparrow 2} = \pm \sigma_\uparrow \sigma_\downarrow J_{\downarrow 2}$. The current lines form two opposite spirals for the two channels $\uparrow$ and $\downarrow$ (see fig. 2). The currents $J_{\uparrow 1}$ form the same spin Hall angles $\theta_{SH\uparrow}$, such that $\tan(\theta_{SH\uparrow}) = \pm \sigma_\uparrow \sigma_\downarrow$, with $-\nabla \mu_\downarrow$, as for the previous cases, but the system dissipates more than $P_{min}$. From the experimental point of view, the most

Fig. 2: (Colour online) Schematic view of the Corbino disk with the current lines corresponding to spin $\uparrow$ and spin $\downarrow$, and $-\nabla \mu_\downarrow$.

direct way to evidence the current $\vec{J}_\uparrow$ is to compare the resistances in the case of a perfect Corbino disk $R^{Cor}$ and the resistance $R^o$ of the same scratched Corbino disk (i.e., a Corbino sector of angle $\alpha \approx 2\pi$). The ratio of the measured resistances is equal to the ratio of the powers (see SM):

$$\frac{R^{Cor}}{R^o} = \frac{P^{Cor}}{P^o} = 1 + \left( \frac{\sigma_\uparrow}{\sigma_\downarrow} \right)^2.$$

Before concluding, it is necessary to clarify the role of spin-flip scattering in this context. Indeed, the spin-flip scattering has not been taken into account in this first variational approach of the problem because it needs an important technical development which is beyond the aim of the present work. However, it is important to point-out that the discussion presented here (transverse voltage vs. transverse current) is not qualitatively changed if spin-flip scattering is taken into account. Indeed, like for giant magnetoresistance, the spin-flip scattering leads to an exponential-like decrease of the spin accumulation over the distance of the spin-diffusion length. A demonstration based on the drift-diffusion equations can be found in ref. [21].

In conclusion, we have shown that the spin to charge current conversion can be performed in the case of the Corbino geometry as far as charge accumulation is not allowed. The conversion equation $\vec{J}_c = (-\sigma/\sigma_\sigma) \vec{J}_s \times \vec{p}$ is then verified (see SM). In contrast, the spin to charge current conversion at the stationary state cannot be performed if charge accumulation occurs, in particular for the usual Hall bar geometry or for the scratched Corbino disk (keeping free the boundary conditions at the edges). In those configurations, the above conversion relation between $\vec{J}_c$ and $\vec{J}_s$ is not verified inside the spin Hall device. These predictions could be checked experimentally by measuring the ratio of the resistances $1 + (\sigma_\uparrow/\sigma_\downarrow)^2$ of the same scratched and non-scratched Corbino disk.
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