Physical reality in asymmetrical double-slit experiments

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Abstract. The recent state-of-the-art double-slit experiments with single electrons and single photons seem to emphasize contradictable dilemma concerning the ontological physical reality in quantum physics. Because of the importance of this problem, we propose and perform another modified laser-beam asymmetrical double-slit experiment. In the results, a Feynman condition with closing mask allows to assess qualitatively the interference contributions of photons passing through one or another slit. Moreover, a definite "which-way" phenomenon has been identified with a high experimental confidence. This would be the simplest way without any disturbance of the photon beam to observe simultaneously both their path and momentum in consistency with the quantum statistical concept.

Key words. double-slit experiment—single-photon interference—single-electron interference—non-local physical reality—quantum statistics

PACS. 03.65.-w quantum mechanics—03.65.Sq foundation of quantum mechanics

1 Recent breakthroughs of physical reality

The problem of physical reality of a quantum substance has been discussed in the Young-Feynman double-slit experiment with single electrons, which Feynman considered a unique key to all mysteries of quantum physics [1]. Regarding wave-particle duality, the first Young double-slit experiment with a low intensity photon beam was performed a long time ago [2]. More recently double-slit experiments with single photons and quantum erasers have been conducted [3] which clearly show the self-interference of a single photon. Being reminiscent of Young’s optical interference, the first double-slit interference of electrons was observed in 1961 by Jonsson [4]. In a later experiment with low intensity electron beams, one could see self-interference of a single electron [5]. The Feynman thought experiments have recently turned out to be available [6,7], identifying single electrons as particles or waves by intervention of a moving mask opening one or both slits, which seem to reconfirm wave-particle incompatibility in the same measurement. Indeed, the problem of quantum physical reality remains a big puzzle. In particular, an early "which-way" experiment by the three-grating MZ interferometer (MZI) demonstrated an evidence of loss and revival of atomic interference correlated with photon scattering, which seems to support the ontological wave-particle duality against the Copenhagen interpretation [8]. Later, an idea for further developing MZI-experiment by double-slit alternative was proposed [9]. From the theoretical perspective, it was well-known that a causal interpretation by the de Broglie-Bohm (dBB)-theory of hidden parameters [10–12] leads to the search for Bohmian trajectories as an evidence of physical reality of microscopic substances. Along with the dBB-theory with hidden parameters, the recent modern Kaluza-Klein (K-K) theories with large extra-dimensions (ED) offer a new way to introduce the hidden parameters for interpretation of quantum theory. For example, being proposed in the ADD model [13], the large EDs are applied in 5D-AdS brane theories [14,15] to induce new physics which are proven to be able to link with quantum field theories (QFT). Regarding the modern K-K matter-induced approach, the space-time-matter theory (STM) [16–18] considers proper mass as a time-like ED which leads to different physical interpretations of both macroscopic cosmology and quantum mechanics [19,20]. Under the matter-induced general relativity, another time-space symmetry (TSS)-based cylindrical dynamical model is proposed for description of microscopic substances, such as an electron [21,22]. In a duality, the TSS-based model

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offers a derivation of the quantum Klein-Gordon-Fock equation from the geodesic sub-solution of the general relativity Ricci vacuum equation. Such a geodesic which can describe classical trajectories of relativistic point-like particles is reminiscent of the Bohmian trajectories. At variance with the dBB-theory, the TSS-based model doesn’t need any quantum potential.

Regarding the dBB-theory, a final verification of Bell’s inequality violation by Aspect removed the local reality [23, 24]. Since 2003, Leggett’s theory has been proposed for the nonlocal reality [25] which was violated in a new experiment by Zelinger et al. [26]. Such kinds of experiments removed a class of theoretical models with two photon correlation following Leggett’s inequality. However, this could not close all nonlocal realistic theories, in particular, the nonlocal reality caused by extra-dimensional physics remains a subject of intensive discussion [27]. Recently, Steinberg et al. [28] applied the concept of weak measurements proposed by Aharonov et al. [29] to measure the photon position in combination with averaged momentum over a selective photon ensemble for interpretation of Bohmian trajectories. Later, Steinberg et al. proved in another experiment that the weak-averaged Bohmian trajectories are not surreal, if the physical reality is nonlocal [30]. This doesn’t conflict with the Heisenberg uncertainty principle, as in the weak measurements one determines averaged Bohmian trajectories over selective statistical ensembles, but not individual trajectories (see [31]).

Another approach is a proposal to use asymmetrical slits for which-way identification of atomic particles in Fresnel or pre-Fraunhofer conditions where some asymmetry of diffraction patterns would be observed [32]. Only recent state-of-the-art technologies such as electron microscopes seem to allow a “which-way” experiment to be performed, using asymmetrical slits [33] in a pre-Fraunhofer condition, which may observe simultaneously both the path and self-interference fringes of single electrons against the principle of complementarity of conventional quantum mechanics, which seems to be described by evolution of quantum waves in a post-Fresnel condition calculated in case of light-massive particles passing both asymmetrical slits [32]. Following the causal interpretations of the dBB-theory, when a single electron as a particle, is fired through one of the two slits, it still generates a spherical front of harmonic waves in 3D-space or in a higher-dimensional time-space. Reaching the second slit, the wave front simulates the coherent sources at the exits of the double-slits which can create interference fringes on the screen at a distance from the slit diaphragm.

Concerning the asymmetrical double-slit method, our present study deals with the mystery of physical reality of the wave-particle duality by proposing and performing a simplified “which-way” double-slit experiment with monochromatic photons. The plan of this article is organized as follows: In Section 2 a “which-way” asymmetrical double-slit methodology with a laser beam is described; in Section 3 the main experimental samples are presented; in Section 4 the results are reviewed for discussion and some preliminary interpretations of physical reality under the light of Bohmian-like trajectories are conducted.

2 Subjects and Methods

2.1 Motivation

In principle, the double-slit experiments with electrons need more sophisticated instrumentation, while the original Young’s double-slit experiment was done with optical point sources. The optical theory of the symmetrical double-slit interferometer is classically well-done (see, e.g. [34]). The propagation of light through a double-slit aperture to a free space is generally described by the Kirchhoff equation. For a modeled rigid double-slits (with finite widths but infinite height) in the near-field condition, the plane waves of monochromatic photons from a point source take a standard form of the Fresnel solution, as an approximation of the Kirchhoff equation. For the objective of the present experiment, the far-field condition is applied, i.e. at an enough large distance from the slits for the wave function \( \psi(x, y, z) = \psi(x, L, t = A_0 U(x)e^{i\omega t} \) of the Fraunhofer equation, as another alternative approximation of the Kirchhoff equation. This leads to luminance distribution of the single-slit diffraction patterns along the horizontal axis on the screen as follows:

\[
I_m(x)dx = |u_m(x)|^2 dx \equiv I_0[|U_m(x)|^2] dx = I_0[|U_m|]^2 dx, \tag{1}
\]

where \( I_0[|U_m|]^2 \) is maximum luminance in the center. The spatial phase \( u_m(x) \) of the wave function reads:

\[
u_m = \frac{b_m x}{L \lambda}, \tag{2}\]

where \( b_m \) is slit-width of the double-slits, \( m = \{1, 2\} \) is a label of a slit; \( L \) is a distance between the slit-diaphragm and the screen; \( \lambda \) is the photon wavelength of a laser source. For the symmetrical double-slit diffraction of incoherent photons (without interference) the maximum of total luminance distribution \( I(x) \) is doubled as \( I(x) = \frac{1}{2} I_m(x) = \sum I_m(x) = \frac{\pi b_m x}{L \lambda} \).
which leads to a ratio of fringe numbers and its links with the slit-width ratio $R$ in case the separation of slits (6) and (8) as follows:

The number of fringes $n$ determined by a separation $D$ the interference fringes appear within each diffraction pattern as their horizontal envelop of $I_D(x)$. In a single-slit experiments, photon beams show only diffraction patterns $I_D(x)$ where the integer $i = 0$ corresponds to the center pattern of the diffraction pattern $I_D(x)$. The separation between the two first minima in the main diffraction pattern $D_{int}^m$, being inversely proportional to slit width $b_m$ as follows:

$$D_{int}^m = 2L\frac{\lambda}{b_m}.$$  

(4)

In a single-slit experiments, photon beams show only diffraction patterns $D_{int}^m$, but in a double-slit observation the interference fringes appear within each diffraction pattern as their horizontal envelop of $D_{int}^m$. The luminance distribution (1) along the horizontal axis $x$ on the screen is modified:

$$I_D(x) = 2\cos^2\left(\frac{\pi dx}{L\lambda}\right) \sum_{m=1}^{2} I_m(x),$$  

(5)

where $d$ is a separation between the two slits and the factor 2 is introduced to raise the fringe peaks twice higher for the intensity normalization (see [34], page 193). In particular, for symmetrical double-slit interference, $I_D(x) = 2I_0 \cos^2\left(\frac{\pi dx}{L\lambda}\right) \frac{\sin u}{u}^2$.

For an asymmetrical double-slit experiment ($b_1 > b_2$), it is assumed that when the laser beam points to a given slit $b_m$, an observer can expect which slit photons pass through. Consequently, the diffraction pattern gets a strict size corresponding to the slit width $b_m$, following Formula (4). When the laser beam spreads over both slits, it makes a picture of diffraction combined with interference. A modified Fraunhofer interpretation is suggested, namely, the photon, as a quantum, passing through only one of the two slits creates the diffraction pattern (4) different with $I_D(x)$.

Accordingly, for any quantities $L$ and $\lambda$, the ratio of diffraction patterns inversely depends on the ratio of the slit-widths as follows:

$$R = \frac{b_1}{b_2} \approx RD = \frac{D_{int}^m}{D_{int}^1}.$$  

(7)

Determined by a separation $d$ between the two slits, the spacing of interference fringes $\Delta F$ reads:

$$\Delta F = L\frac{\lambda}{d}.$$  

(8)

The number of fringes $n_m$ in each diffraction pattern $D_{int}^m$ are calculated by the ratio of the quantities from Formulas (6) and (8) as follows:

$$n_m = \frac{D_{int}^m}{\Delta F} \approx \frac{D_{int}^m}{D_{int}^1} \frac{d}{b_m}.$$  

(9)

In case the separation of slits $d = \text{const}$, then the fringe spacing $\Delta F$ is identical for all extended diffraction bands, which leads to a ratio of fringe numbers and its links with the slit-width ratio $R_{corr}$ after a correction:

$$R n = \frac{n_2}{n_1} = RD = R\left[1 + \left(\frac{1}{n_1} - \frac{1}{n_2}\right)\right] \equiv R_{corr}.$$  

(10)
Fig. 1. A/ Laser source and B/ a Layout of the Experiment ($L = 8.6 \text{ m}$, $X = 0.18 \text{ m}$)

Definition of horizontal Directions of Laser beam: Dirs. $(a)-(b)-(c)-(d)-(e)$.

Following (5), the condition of additional minimum and maximum orders due to interference in a Fraunhofer double-slit diffraction pattern are correspondingly as follows:

$$x_j(\text{min}) = \pm \left( j + \frac{1}{2} \right) \frac{L\lambda}{d}; \quad x_j(\text{max}) = \pm j \frac{L\lambda}{d},$$

(11)

where integer $j = 0, 1, 2, ..., n$. An ideal far-field Fraunhofer condition (when the Fresnel number $N_{Fr} = \frac{k^2}{\lambda L} \ll 1$) makes the interference fringes in double-slit diffraction bands spacing strictly in their right locations, without any displacing. In the present experiment, $N_{Fr} \approx 10^{-3}$ satisfies well the far-field condition, that facilitates the ability to carry-out approximation calculation of the diffraction patterns $D_{m\text{nt}}$ with details of interference fringes. Indeed, the latter would be well-controlled by analysis of the interference structure of each experimental pattern $D_{m\text{nt}}$, which is expected as an exact overlap of the two diffraction components of the photons passing this or other slit, regardless of how different slit-widths are. This expectation is well-proven in classical symmetrical double-slit interference (with $b_1 = b_2$), in which the minima remain clean (i.e. empty) in diffraction bands with interference $D_{m\text{nt}}$, exactly as this can be seen in the diffraction patterns $D_{m\text{nt}}$ of a single-slit experiment with the same slit-width $b_m$.

The idea of a "which-way" double-slit experiment with monochromatic photons aims to sample experimental data in kinds of photographic spectra with diffraction or interference by varying slit-widths. The purpose is to extract the two experimental indicators of the "which-way": i/ The different lengths of central diffraction patterns $D_{m\text{nt}} \approx D_{m\text{nt}}$, $m = 1, 2$; ii/ The numbers of interference fringes $n_m$ being distinguished in each of central diffraction patterns $D_{m\text{nt}}$.

Both indicators can be measured by visual observation. Because both indicators are in correlation, they are used in complementarity for "which-way" identification. Hereafter, the photographic spectra are numbered by abbreviation Spec.$(N,z)$ or Spec.$(N,z,n)$, where $N$ is Figure number, $z \equiv \{a,b,c...\}$ is alphabet label and $n$ is an optional sub-classification number.

2.2 Instrumentation

In the present study a traditional laser double-slit experiment with a geometrical modification, based on the nature of single photon interference (see Layout in Fig.1) is proposed. The present modification means the two slits are designed with different widths. Minimum resources for the experiment are:

2.2.1 Optical source

A red laser pointer Vesine VP101 ($\lambda = 650 \pm 30 \text{ nm}$) of power $W_{L\text{aser}} \leq 5 \text{ mW}$ serves the monochromatic optical source. Its original oval beam cross-section $S_{L\text{aser}} \equiv w \times h \leq (3.0 \times 4.0) \text{ mm}^2$ is used in controlling measurements. Then, the beam width is reduced to $w \approx 1.5 \text{ mm}$ by an additional non-metallic slit-mask in the main samples, which
fits the effective slit-widths $b_m$ and the distance $d \geq 0.75$ mm between the two slits to arrange the horizontal beam directions $\text{Dir.}(a) \div \text{Dir.}(e)$. In principle, an experiment with a relative low intensity of the laser beam can replace the single photon experiments. For example, for an ideal point source laser of intensity $W_m \approx 0.3$ (mW) $\equiv 10^{15}$ photons/sec., a photon can pass through a given slit without coincidence with photons passing through the other slit, because the typical time window of laser photons is roughly $10^{-15}$ sec. This condition ensures the self-interference of single photons. The probability of coincidences increases with intensity, which makes possible interference between photons from different slits. Because a typical slit width is almost for an order less than the width $w$ of the laser beam (being often an unideal point source) and without accounting for an uncertain particle position along the finite size of the beam height $h$, the effective intensity through a slit with $b_m \leq 0.2$ mm is already reduced to $W_{eff} \leq 0.3$ mW, which meets the condition for single photon self-interference. In the experimental layout in Fig.1 the laser source is fixed at a distance $X = 0.18$ m from the slit-diaphragm.

2.2.2 Asymmetrical slits

The diaphragms with slits are made from aluminum foil with two vertical slits with the height $h_{Slit} \approx (10 \div 12)$ mm. A diaphragm with two sets is used: i/ The sub-diaphragm $\text{DI}.a(R)$, (where $R \equiv \{b_1 : b_2\}$) is made with a fixed wide-slit of $b_1 = (0.15 \pm 0.02)$ mm, but with a narrow-slit having varying width along the slit height $h_{Slit}$, i.e. $0.03 < b_2 < 0.12$ mm (see above Fig.2); ii/ The sub-diaphragm $\text{DI}.b$ with two symmetrical wide-slits of fixed width $b_2 = b_1 \approx 0.15$ mm (see below Fig.2) which demonstrates a conventional double-slit experiment as a reference.

2.2.3 Other supporting devices

The screen is fixed at a distance $L = 8.6$ m from the diaphragm (see Layout of the experiment in Fig.1). There are supporting frames, a photo-camera and a linear meter for determining the sizes of spectra. A small aluminum mask is used to close one of the two slits in the Feynman condition of his "thought" double-slit experiment.

2.3 Methods

For the objectives of the "which-way" experiment, in Fig.1 there are five horizontal directions (a)-(b)-(c)-(d)-(e) of the laser beam pointing on the slits applied for observation of five following quantities:

(i) The first direction $\text{Dir.}(a)$ points to the left edge of the wide-slit for Diffraction patterns $D_1^{\text{dif}}$, when only the wide-slit ($b_1$) is active by minimizing the beam exposure on the other slit $b_2$;
(ii) The second direction $\text{Dir.}(b)$ points straightly to the wide-slit for Patterns with Interference fringes $D_1^{\text{int}}$ correlated with the main (central) diffraction pattern $D_1^{\text{dif}}$;
(iii) The third direction $\text{Dir.}(c)$ points to the middle position between the two slits for Central spectrum with mixing
$D^{int}$ and $D^{dif}$.

(iv) The fourth direction Dir.($d$) points straightly to the narrow-slit for Patterns with Interference fringes $D^{int}_2$ correlated with the main (central) diffraction pattern $D^{dif}_2$.

(v) The fifth direction Dir.($e$) points to the right edge of the narrow-slit for Diffraction patterns $D^{dif}_2$, when only the narrow-slit ($b_1$) is active by minimizing the beam exposure on the wide-slit ($b_2$). Note that in each laser beam direction, photons would pass through one or another slit, but they should aim at the same central symmetrical point due to parallel laser light, which makes the central symmetrical axes of both diffraction patterns $D^{dif}_1$ and $D^{dif}_2$ coincident.

In each of experimental diffraction patterns with interference $D^{int}_m$ one can count the number of fringes $n_m$ as an experimental quantity. Hereafter each of the abbreviations Dir.($x$), where $x \equiv \{a, b, c, d, e\}$ labels a given horizontal direction of the laser beam as shown in Fig.1, while the abbreviation Pos.($y$), $y \equiv \{a, b, c, d\}$ labels its position along the slit height $h_{slit}$ as shown in Fig.3.

The three runs of experimental samples are carried out: i/ Controlling samples for checking the effects of slit-width difference for "which-way" identification. As a result, an optimal position of the laser beam is chosen for the next main samples; ii/ The first main samples for checking the signal-background (S/B) relation in spectra. The S/B study can response to the "which-way" problem, at least qualitatively; iii/ The second main samples can search for a quantitative solution of the "which-way" problem. For each sample, an original spectrum is pictured in darkness to gain a good contrast, while its picture with sizing (by the linear meter) is taken with less quality under light, intending only for scaling spectrum lengths.

3 Measurements

3.1 Controlling samples

For verification of the methodology, the bare red laser source with its original beam size $S_{Laser} \approx 3.4 \times 10^{-2} \text{ (mm)}^2$ and the sub-diaphragm $D_{la}(R)$ with three different width-ratios $R$ of the double-slits are used at three beam exposure positions Pos.($3.a$), Pos.($3.b$) and Pos.($3.c$) along the slit-height $h_{slit}$ (see Column (I) in Fig.3). In addition, a sample with the symmetrical double-slits by the sub-diaphragm $D_{lb}$ is carried out as a reference at the exposure position Pos.($3.d$). In the results, four pairs of diffraction patterns with interference fringes $\{D^{int}_1, D^{int}_2\}$ are measured and their spectra are pictured. The samples, related to the varying narrow-slit are shown in Column (II) in Fig.3. Because the wide-slit keeps a width fixed, $b_1 \approx (0.15 \pm 0.02) \text{ mm}$, all the patterns related to the wide-slit (see Column (III) in Fig.3) look approximately identical like one of the symmetrical slits at the exposure position Pos.($3.d$) of the sub-diaphragm $D_{lb}$.

Based on an estimation of experimental as well as calculated uncertainties (see bellow), the experimental data are able to be compared with the expected calculation by Formulas (4) \((\text{v})\) (10). As a result of the controlling samples, the observable patterns $D^{int}_m$ and their counted numbers $n_m$ of interference fringes, as well as a strong correlation of those quantities with the laser beam direction, i.e. Dir.($b$) or Dir.($d$), serve the basic experimental data for verification of the proposed method of "which-way" identification. In particular, this leads to selection what position is optimal for avoiding additional systematical uncertainties in the next main samples N.1 and N.2 (see details in Section 4). As it is shown in Fig.3, due to high intensity of the laser source, some experimental spectra suffer from saturation near to their central maximum, which restricts full information of intensity distribution. Nevertheless, the interference fringes with saturation revealed brightest, become helpful in their 2D-spacing and ordering. It would serve as an important ontological indicator for "which-way" identification of interference mixing spectra.

3.2 Main samples N.1 - the Signal-Background relations

Hereafter, the main samples are presented, using the laser beam with a reduced cross-section $S_{Laser} \approx 1.5 \times 10^{-2} \text{ (mm)}^2$ which helps to eliminate diffraction backgrounds from one to another slit. Following the conclusion of Controlling samples, the laser exposure position Pos.($3.c$) of the sub-diaphragm $D_{la}(R)$ is selected as the most optimal in the present Main samples N.1. In the present main samples N.1, due to restriction of 2D-images for study on intensity distribution, one can combine the theoretical Fraunhofer approximation with 2D-scanning for experimental estimation of optical density of each spectrum, which leads to semi-quantitative assessment of relative contribution of the two mixing components of diffraction pattern, regarding one or another slit. When in a diffraction pattern $D^{int}_m$ one sees simultaneously the interference fringes, this confirms the wave feature of photons. Certainly, one can assume that the fringes behind one of the slits are not completely single-photon events, but they may be caused by interference between the photons passing through two slits, even though the number of photons scattered through the second slit is smaller. The major contribution of the photons passing through the first slit is labeled (S) as signal, while
the minor contribution of photons passing through the second slit is (B) as background. In case the signal intensity is dominant, i.e. $I_S \gg I_B$, for estimation of the background contribution (B) in signals (S), following a condition similar to the Young-Feynman thought experiment, one can use a mask closing a given $m$-slit, i.e. the wide-slit, to inactivate the major S-contribution of integrated intensity $I_1$ in $D^\text{int}_1$, or close the narrow-slit to inactivate its major S-contribution of $I_2$ in $D^\text{int}_2$, the minor B-contributions of the background diffraction patterns $D^\text{dif}_{\neq m}$ are revealed in this way. The mask is made from the same aluminum foil for closing a given slit $m$ to turn the image on the screen from $D^\text{int}_m$ to $D^\text{dif}_{\neq m}$. This is not an ideal mask-design to get perfect diffraction pictures without noise, but this mask still satisfies our purpose of counting the relative integral intensities. In particular, when the laser beam directs on the left to the wide-slit in Dir.(b), the major S-contribution of $I_1$ in the observed central pattern is dominant, then one can compare the B-spectrum Spec.(4.b.3) in $D^\text{dif}_2$ with the S-major pattern $D^\text{int}_1$ for $I_1/I_2$ analysis. Note, that in the present analysis, B-contribution of $I_2$ is not the whole intensity of the diffraction pattern $D^\text{dif}_2$, but being integrated in the same interval $\Delta x \equiv \{-x_1 \div +x_1\}$ of the length of the spectrum $D^\text{int}_1$. This is just an interval of overlap of both spectra, in which the mixing interference fringes cannot be separated. This shortened integration is applied due to two reasons: firstly, the ontological interpretation following Summation (5) and the geometry of the experimental layout as in Fig.1 make the central symmetrical axes of both patterns $D^\text{int}_m$ well-coincident; secondly, the intensities $I_1$ and $I_2$ of both components are compared relatively for each pair ($I_1/I_2$) in a given beam direction, to find their relative tendency when the laser beam changes its direction, but not for a purpose to measure their absolute quantities. Thus both interested intensities $I_m$ are calculated for an overlap of $D^\text{dif}_2$ with $D^\text{int}_1$ as follows:

$$I_m = \int_{-x_1}^{+x_1} I_m(x) dx = I_0[m] \int_{-x_1}^{+x_1} \left[ \frac{\sin \frac{u_m}{u_m}}{u_m} \right]^2 dx,$$

where $\pm x_1$ are location of the two first diffraction minima coming from (3) of the $I_1(x)$-distribution, which is indicated by arrows in Fig.4. In a similar manner, when the laser beam directs on the right to the narrow-slit in Dir.(d), the B-spectrum Spec.(4.d.3) in $D^\text{dif}_1$ would be compared with S-major pattern of $I_2$ in the interval $\Delta x$ of the pattern $D^\text{int}_1$. Such a way of comparison, called (Pr-1), offers an assessment of the low-limit of the background contribution. In particular, it seems to be satisfactory for the direction Dir.(b), when the contribution of $I_1$ is dominant in the pattern $D^\text{int}_1$ with interference fringes. In principle, when the background is significant to the signals, this procedure is not exact, because the geometry of the double-slits and the single-slit with the Feynman condition would not be identical, which would lead to systematical deviation. In general, it is safer to estimate the major S-contribution by a similar mask-closing of the second slit,
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Fig. 4. Main Samples for extraction of S-B contributions being carried out at Vertical Pos.(3.c) and in three Horizontal directions of Laser beam Dir.(b), Dir.(c) and Dir.(d).

Theoretical curves are Fraunhofer approximation. Procedure (Pr-2) with Feynman condition is applied. Clearly shown two first minimum gaps of the short pattern, being conserved well in mixing spectra.
accompanying, i.e. by adding a single-slit S-diffraction pattern $D_{1m}^{def}$ in Dir.(b) or $D_{2m}^{def}$ in Dir.(d). The two diffraction patterns in a given laser beam direction are then compared with each other for assessment of their relative contribution in the corresponding double-slit interference spectrum. In this procedure, called (Pr-2) the $S/B$-analysis is expected to be more quantitative, because the Feynman condition is more identical in both single-slit samples. Both experimental single-slit diffraction patterns $D_{1m}^{def}$ and $D_{2m}^{def}$ in each laser beam direction are presented besides the corresponding double-slit pattern $D_{m}^{int}$ in Fig.4. Also presented are the theoretical approximations of those experimental spectra by the Fraunhofer luminance distribution function (1). It is noticed that when the luminance is enough high, the central interference fringes reach saturation, thus their maximum luminance $I_0$ would not be seen correctly. Dealing with the problem, instead of experimental quantities, the presented theoretical approximation by Fraunhofer distributions (1) of luminance is acceptable. However, this indirect estimation would be considered as a semi-quantitative approach. Such a data set serves for $S/B$-estimation in each laser beam direction.

In brief, the more conservative procedure (Pr-2) is carried out in three steps. Firstly, it is done by comparison of maximum luminance of both S- and B-diffraction spectra $D_{1m}^{def}$ in which the maximum luminance of S-pattern, sinking in saturation, is restored by the theoretical distribution (1). Secondly, their integral intensities $I_S/I_B$ can be calculated by integration (12) in the central overlap interval $\Delta x$ of the two S- and B-diffraction patterns in each laser beam direction, as an experimental evidence. Thirdly, for checking the luminance of background, an additional comparison of the latter with the secondary diffraction maxima is carried out. In particular, near to the central major S-pattern Spec.(4.b.2) one can see the next much weaker second and third diffraction patterns with relative luminance as $I_0[1] : \delta_1[1] : \delta_2[1] = 1 : 0.045 : 0.016$, following the Fraunhofer theoretical calculation. The experimental maximal luminance of B-pattern Spec.(4.b.3) of $D_{2m}^{def}$ is found to be intermediate between luminances of the two secondary patterns: $\delta_2[1] \leq I_2 \leq \delta_1[1]$. Also, when the narrow slit is closed, in Spec.(4.b.2) of $D_{1m}^{def}$ their minima (being indicated by arrows) are empty, i.e. clean from S-photons of $I_1$, thus any $\Delta I_1(\pm x_1) \leq 0.016 x I_1$ with high confidence. When both slits open, as shown in Spec.(4.b.1) there very weak background dots appear, each in a minimum area (see in yellow circles). Their luminances are determined as $\Delta I_B \leq 0.016 x I_1[1]$ following a comparison relatively with the third diffraction pattern, i.e. $\delta_1[1] \leq 0.016 x I_0[1]$. This reconfirms that the contribution (if any) of B-spectrum from $I_2 \leq 0.03 x I_1$ in the interference spectrum Spec.(4.b.1). Another phenomenon observed by comparison of the single-slit Spec.(4.b.2) with the double-slit Spec.(4.b.1) is conservation of the minimum positions in the asymmetrical double-slit experiment. It is well-observable in the corresponding patterns $D_{1m}^{def}$ and $D_{m}^{int}$ that their first minimum gaps are in a correctly overlap with each other. Those gaps are quite clean, being almost empty (particularly, in the right yellow circle), except a tiny dot of some background in the left circle, as above-mentioned. This evidence implies that the conservation law of diffraction pattern sizes and the ordering of their interference fringes in symmetrical double-slit experiments would be naturally extended to asymmetrical double-slit interference.

In accordance with the procedure Pr-2 of S/B-intensity estimation, in the laser beam direction Dir.(b) the B-spectrum Spec.(4.b.3) of $D_{2m}^{def}$ consists $\sim (3.0 \pm 0.5\%)$ of the major S-spectrum Spec.(4.b.2) of $D_{1m}^{def}$ concerning the wide-slit contribution. Therefore, by this semi-quantitative assessment, the Spec.(4.b.1) consists of $\geq (95 \pm 3\%)$ as contribution of photons from the wide-slit and $\sim (2.5 \div 3.5\%)$ photons $I_2$ from the narrow-slit. The conservative estimation of the $S/B$-relation in Dir.(b) is $I_1 : I_2 = 0.97(\pm 0.03) : 0.03(\pm 0.01)$. Besides, another $S/B$-assessment by a direct comparison of the single-slit spectrum Spec.(4.b.3) of B-contribution with the pattern $D_{1m}^{int}$ of the interference spectrum Spec.(4.b.1) following the procedure (Pr-1), leads to almost the same result of (Pr-2), which serves an evidence that the closing mask for Feynman condition works well without disturbing the laser beam through the wide slit. In the same procedure Pr-2, in the direction Dir.(d) on the right, luminance of the B-spectrum Spec.(4.d.3) of $D_{1m}^{def}$ is compared with luminance of the major S-spectrum Spec.(4.d.2) of $D_{2m}^{def}$, as well as with luminance of its second minor diffraction pattern $\delta_2[2] = 0.045 x I_0[2]$. As a result, the $S/B$-relative integrated intensity $I_2 : I_1 \approx 0.75(\pm 0.09) : 0.25(\pm 0.05) \ a\ r \ s \ o \ u\ l \ t \ s$ where $I_1$ plays the role of background contribution of the B-spectrum Spec.(4.d.3) of $D_{1m}^{def}$ against $I_2$ from the major spectrum Spec.(4.d.2) of $D_{2m}^{def}$ concerning S-contribution of the narrow slit. For Dir.(c), the central direction of the laser beam points roughly to the middle between the two slits (see Layout Fig.1). Following the same Feynman procedure (Pr-2), the relative contribution $I_1 : I_2 = 0.65(\pm 0.08) : 0.35(\pm 0.06)$ is estimated.

As above-mentioned, in spectra with high intensity several central bright interference fringes of an interference spectrum $D_{m}^{int}$ reaches saturation which levels out a correct maximum luminance $I_0$. In opposite to the destructiveness of saturation, the smooth brightness of central interference fringes would be a positive indicator for identification of the shorter diffraction pattern of the wide slit in a mixing spectrum. Indeed, in the spectrum Spec.(4.d.2) of the diffraction pattern $D_{m}^{def}$ in the procedure (Pr-2) its central maximum luminance seems high but not yet saturated, which makes sense that any bright saturated dots in the central area of the mixing double-slit spectra would regard an overlap with the shorter band originated from the wide slit. In consistency with this expectation, in the center of both elongated interference spectra Spec.(4.c.1) and Spec.(4.d.1) in Fig.4, there are found seven smoothly-bright fringes exacting the number of fringes $n_1$ in the shortest interference spectrum Spec.(4.b.1) regarding the wide slit. It is understandable that being qualitative or even reaching quantitative, the presented S/B-estimation cannot separate.
individual photons passing through this or that slit. However, such an analysis serves as a strong argument for approaching to explicit information on the path of photons which is a decisive step towards the true "which-way" solution.

3.3 Main samples N.2 - "Which-way" identification without beam disturbance

For a direct observation of the "which-way" phenomena without disturbance of the photon beam, the red laser source pointing to the exposure position Pos.(3.c) of the sub-diaphragm DI.a(R) is applied again, and in all five horizontal directions Dir.(a)÷Dir.(e) of the laser beam (see Layout in Fig.1). In Fig.5 the experimental spectra, including both diffraction patterns $D_1^{\text{diff}}$ and $D_2^{\text{diff}}$, as well as both corresponding spectra with interference fringes $D_1^{\text{int}}$ and $D_2^{\text{int}}$ are shown. There the central diffraction patterns $D_1^{\text{diff}}$ and $D_1^{\text{int}}$ in spectra Spec.(5.a) and Spec.(5.b), correspondingly, are assumed to belong to photons passing through the wide-slit (with $b_1$) while in Spec.(5.d) and Spec.(5.e) the central patterns $D_2^{\text{int}}$ and $D_2^{\text{diff}}$, correspondingly, are considered as diffraction of photons passing through the narrow-slit, (with $b_2$). Indeed, in Fig.5 among interference fringes as the signals (S) in the central diffraction patterns of Spec.(5.b) or Spec.(5.d), some of them would be partly intensified by the photons passing through other slit as background (B) and would not easily be separated. Then the mixing interference spectrum in the direction Dir.(e) joins the full set as well. As above-mentioned in the main samples N.1, due to saturation in the central part of experimental images, the theoretical calculation by Fraunhofer distribution (1) is to be conducted and their curves are presented under each corresponding experimental spectrum. In Fig.5 one can see not only the curves of diffraction distribution, but also their detailed structural curves with interference fringes. In the same consideration as in the main samples N.1, one can observe seven interference fringes exacting the number of fringes $n_1$ of photons coming from the wide slit. Once again, reminiscing symmetrical double-slit interference, this semi-quantitative indicator supports conservation of length of the diffraction pattern $D_1^{\text{int}}$ in a mixing spectrum with diffraction pattern $D_2^{\text{int}}$, following strictly Formulas (4) and (6). The conservation law ensures a strict overlap of interference fringes in their spacing and ordering, which allows to analyze in detail of interference structure of the experimental samples under reference by theoretical curves of their corresponding intensity distribution. In particular, in Spec.(5.b) the background coming from the narrow-slit would mix in the major fringes of the photons passing through the wide-slit. The background, if there is any in Spec.(5.b), would be at a continuous level in an area of minima outside of the central diffraction pattern of $D_1^{\text{int}}$. Due to a relative good optical solution, this background is seen enough small in Spec.(5.b). Therefore, one can confirm that the interference $D_1^{\text{int}}$ in Spec.(5.b) is dominantly caused by the photons passing through the wide-slit. A similar interpretation would be extended to the interference picture $D_2^{\text{int}}$ in Spec.(5.d), indeed, the background B from the wide-slit is much greater, which makes the five (of seven) central fringes bright in $D_2^{\text{int}}$.

It is important to notice again that the gaps of the first minima in wide-slit’s diffraction pattern $D_1^{\text{diff}}$ in Spec.(5.a) are enough clean. Those locations are then serve to search for pure "which-way" interference fringes of photons passing strictly through the narrow-slit, as one can see in the next analysis and discussions.

3.4 Estimation of experimental uncertainties

Not only the experimental data having different statistical and/or systematical uncertainties, the calculation by theoretical formulas also suffered from some sources of systematical uncertainties in the input data, such as the widths $b_m$ and the distance $d$ between them, or the finite beam size of an unideal laser point source etc. A general assessment of the main experimental uncertainties as well as some systematical deviations in calculation are reviewed as follows:

i/ In principle, the numbers of fringes $n_m$ are integer. However, due to a limited photographic resolution, the edge fringe sometime is not clearly fixed, then an uncertainty roughly $\Delta n_m \leq 1.0$ is generally acceptable;

ii/ The experimental uncertainties of diffraction patterns $\Delta D_1^{\text{int}} \approx \Delta D_2^{\text{diff}}$ in Table 1 are within $\Delta D_m/D_m \leq 7\%$;

iii/ The experimental uncertainties come from a variation of slit’s position within the laser beam sizes and any laser incoherence $\Delta \lambda/\lambda \leq 5\%$;

iv/ The experimental uncertainties caused by determination of length quantities are: $\Delta L/L \approx 2\%$; $\Delta b_m/b_m \approx 10\%$;

v/ The relative uncertainties of the calculated ratio $R$ of diffraction patterns by Formulas (4) and (7) are within $(10. \div 15.)%$.

In general, the ratio of the experimental diffraction patterns in (ii) is basically consistent with ones of the calculated quantities in (v) within their uncertainties $\sim 15\%$. Finally, when the ratio of the two slits $R \geq 1.3$, the two observed diffraction patterns $D_1^{\text{int}}$ and $D_2^{\text{int}}$ are well identified from each other to classify which the slit they correlate to.
Fig. 5. Main Samples for extraction of "which-way" signals: A direct observation of pure interference fringes of photons passing through Narrow-slit (indicated by Arrows in Yellow ellipses): The curves are Fraunhofer approximation. Clearly shown a tendency of increasing luminance of the two fringes in minimum gaps of the short diffraction pattern while the laser beam direction changes from the left Dir.(a) to the right Dir.(e).

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- **a.1/** Diffraction Patterns
  - Wide Slit Dir.(a)

- **a.2/** Approximation
  - Direction (a) to the Left edge

- **b.1/** Interference Fringes
  - Wide-Slit Dir.(b)

- **b.2/** Approximation:
  - Direction (b) to Wide Slit

- **c.1/** Mixing
  - Interference Double Slit Dir.(c)

- **c.2/** Approximation:
  - Direction (c) to the Middle

- **d.1/** Interference Fringes
  - Narrow-Slit Dir.(d)

- **d.2/** Approximation:
  - Direction (d) to Narrow-Slit

- **e.1/** Diffraction Patterns
  - Narrow Slit Dir.(e)

- **e.2/** Approximation:
  - Direction (e) to the Right edge

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- **(I)-** Original Spectra
- **(II)-** Size Measurement
- **(III)-** as (I) Amplified

- **Blue curves:** $I_4$ contribution
- **Red curves:** $I_2$ contribution
Table 1. Double-slit measurements: inputs and results of controlling samples

| Exposure Position | $d$ (mm) | $b_1 : b_2$ (mm:mm) | $R_{corr}$ ⇒ $R$(Line) (cm) | $\Delta F$ (cm) | $n_1 : n_2$ | $D_{int}^1 : D_{int}^2$ (cm:cm) | $\pm \Delta A$ or $\pm \Delta A/A$ (%) |
|-------------------|---------|---------------------|-------------------------------|-----------------|-------------|--------------------------------|-------------------------------|
| (3.a)             | 0.60    | 0.16 : 0.05         | 3.6                          | 3.47            | 1.0         | 5 : 19                         | 4.7 : 19.4                   |
| (3.b)             | 0.70    | 0.15 : 0.07         | 2.3                          | 2.56            | 0.8         | 5 : 15                         | 4.3 : 14.3                   |
| (3.c)             | 0.75    | 0.15 : 0.10         | 1.6                          | 1.66            | 0.75        | 7 : 13                         | 5.2 : 11.3                   |
| (3.d)             | 0.85    | 0.15 : 0.16         | 0.9                          | 0.78            | 0.7         | 7 : 7                          | 5.0 : 4.7                    |

Fig. 6. Graphic presentation of the Controlling Samples from Table 1:

- Correlation between Experimental ratio $RD$ or $Rn$ with the fitted slit-width ratio $R$(Line).
- Four effective vertical positions of Laser beam are shown in abscissa. An optimal area for Laser beam (in Rectangular) around the position Pos.(3c) is selected for Laser beam exposure in the main samples.

4 Results and Discussion

Except some fixed input quantities, such as the distance $L \approx 8.6$ m or the wave length of the red laser $\lambda = 650$ nm, other input data and the results of controlling samples from Fig.3 are presented in Table 1. The estimated uncertainties $\Delta A$ or relative uncertainties $\Delta A/A$ are included in the last row in Table 1. Based on Table 1 a graphic presentation is conducted in Fig.6, in which there are two experimental ratio $RD$ between the lengths of diffraction patterns and $Rn$ between the numbers of their interference fringes are compared with the calculated linear line $R$(Line) in Col.(4) originated from $R \approx b_1/b_2$. The latter is treated under a correction in accordance with Formulas (10) as reported in Col.(3) of Table 1. The calculated $R$(Line) is determined by fitting approximation based on the given double-slit widths in believing that the design of the diaphragm $D_{1.a}$ is perfect. Indeed, the manufacture and size-measures of slit widths are not ideal, which leads to uncertainties of the "calculated" linear line $\Delta R/R_{corr} \approx 15\%$, being explicit in Fig.6. At the same time, the experimental ratios $RD$ and $Rn$ are measured with typical relative uncertainties $\approx 10\%$ which are not shown in Fig.6 but should be aware.

There at all four applied exposure Pos.(3.a) ÷ (3.d) a good correlation is seen between the ratio $RD$ and the ratio $Rn$ with the calculated ratio $R$(Line) of the two slit widths. It is in consistency as well with Formulas (7) and (10). There would be a small systematic deviation at the area near Pos.(3.b) due to a slight systematical decrease of the distance $d$ looking upward the slit’s height $h_{Slit}$. This increases the size of an interference fringe $\Delta F$ and due to quantum discreteness, the number of fringes $n_1$ turns from 7 to 5 units in Sample $D_{int}^1$ near to Pos.(3.b). However,
A tendency is approximated by Linear lines within the experimental uncertainties (constrained by four-wing stars), counting also uncertainty due to adjusting the horizontal directions of Laser beam.

The results of S/B-assessment in the main samples N1 are presented in Fig.7. Except the experimental samples from Fig.4, in addition, the graphics in Fig.7 includes as well the S/B-estimation of the two edged measurements of diffraction patterns $D_{int}^1$ in Dir.(a) and $D_{int}^2$ in Dir.(e), which are roughly approximated by 100% of S-contribution of $I_1$ or $I_2$ photons, correspondingly. As a result, one can find that the minor contribution of background pattern $D_{dif}^2$ in the major pattern $D_{int}^1$ is $\sim 2.5\%$, which implies that more than 95% of intensity in the major pattern with interference fringes $D_{int}^1$ comes from the photons, passing through the wide-slit ($b_1$). This seems to make the interference pattern $D_{int}^1$ in the direction Dir.(b) like a "which-way" solution, which implies that both wave and particle ontological properties exist objectively before the photon is detected at the screen. Indeed, the problem is that one can make a background assessment by a mask closing one of the two slits, but it is impossible to separate the background (B) contribution by its extraction from the total intensity of the major pattern, following the complementarity principle of quantum mechanics. Some minor mixing of $I_2$-photons is enough to disturb an ontological "which-way"solution.

The simplest linear approximation is applied to fit the experimental points with relatively large experimental uncertainties (roughly 15% – 25%) which are enhanced by some uncertainties in fixing each horizontal laser beam direction. Due to this reason, all uncertainties are expressed by four-wing stars in the graphics. The crossing point of two linear approximation lines systematically shifts to the right due to difference of effective photon beams passing through the two slits. All together, the graphics offers a clear tendency of a decrease of $I_1$-contribution from the wide slit and, correspondingly, an increase of $I_2$-contribution from the narrow slit, when the laser beam changes its orientation from the left to the right. From the main samples N1, the conservation law of diffraction pattern’s length and strict spacing and ordering of their interference fringes in a mixing asymmetrical double-slit spectrum are explicitly demonstrated in reminiscence of symmetrical double-slit interference. Besides, in comparison between three spectra in Dir.(b): Spec.(4.b.1), Spec.(4.b.2) and Spec.(4.b.3) in Fig.4, one can observe for the first time a weak dot in the left yellow circle in the pattern $D_{int}^1$ which would be thought as a hint of an interference fringe of $I_2$ from the narrow slit. Therefore, the results of the main samples N1 serve a decisively important basis for the next attempt of a pure "which-way" identification.

The final decision on the ontological physical reality of wave-particle duality of the photon can be conducted from the direct observation of a pure "which-way" phenomenon, as it is illustrated in Fig.5, where the arrows show the gaps in the first interference minima of the pattern $D_{dif}^1$ (see Spec.(5.a)). Due to a high coherence of the laser beam, the gaps are enough clean from the major contribution (S) of photons passing through the wide-slit. In the meantime, the above mentioned S/B analysis constrains a background $\sim (2.5 \div 3.5)\%$ from the narrow-slit to Sample...
Spec.(4,b,1) of $D^{{\text{int}}}$, so in the gaps of the diffraction pattern $D^{{\text{diff}}}$ in Spec.(5,a) the background from the narrow-slit is much less than 1% which guarantees a cleanness of the minima’s gaps with a confidence higher 99%. Looking at the corresponding locations of those ”clean” gaps (regarding to the photons from the wide-slit), in each of Spec.(5.c) or Spec.(5,d), one can find at least two intensive interference fringes each in a minimum gap as shown in the middle of each ellipse in the experimental spectra. It is observable that luminance of those fringes increases when the laser beam scans from the left to the right of the diaphragm which follows the increasing tendency of $I_1$-photons from the narrow slit, while it is against the decreasing tendency of $I_2$-photons from the wide slit, as shown in Fig.7. As a result, the interference fringes in the minimum gaps are naturally assigned to the contribution of the photons passing through the narrow-slit. Being carried out without any disturbance of the photon beam, those experimental samples demonstrate a simultaneous observation of both wave and particle behavior, which used to be incompatible from the point of view of the Copenhagen interpretation of quantum mechanics.

Due to the wave feature of the photon, it is impossible to fix its position, e.g. by making the width $b_2$ narrower, namely more precise than the wave length $\lambda$. Similarly, because its momentum variation depends on the diffraction angle, which is constrained by the size of an interference fringe $\Delta F$, but the latter can never be shorter than $\lambda$, neither. Therefore, a simultaneous observation of incompatible variables (the path and momentum) doesn’t mean a violation of Heisenberg uncertainty principle, but this emphasizes an ontological coexistence of the wave-particle duality of the photon. The wave feature ensures the statistical concept of quantum mechanics. Nevertheless, the particle feature of the photon requires a causal physical interpretation, even its trajectory can be described probably in a statistical average.

Regarding the causal interpretations of the dBB-theory [10–12] or the TSS-based model [22] by introduction of a classical sub-equation in duality to the conventional quantum equation, in similarity with electrons [33], the double-slit experiment with a laser beam would be explained qualitatively by the causal models of quantum physical reality of photon, which behaves at the same time as a classical point-like particle and, in a duality, as a source of quantum matter waves. Namely, while the photon selects a given slit, it emits periodically electro-magnetic waves transmitting with spherical wave fronts in the frame of the Huygens-Fresnel principle, including the direction through the second slit, which then interfere with the harmonical fluctuations accompanying the photon-particle transmitting through the first slit. The maxima of the interference (the crests of waves) will serve as the pilot waves for regulating the real trajectories of photon, being reminiscent of Bohmian (but relativistic) trajectories.

5 Conclusions

In the present asymmetrical double-slit experiment with monochromatic photons, both diffraction patterns and interference fringes are measured. For the main samples, not only contributions of the ”which-way” to the interference fringes are qualitatively determined using a Feynman condition with a closing mask, but also several interference fringes of the ”which-slit” they concern are identified explicitly. In particular, with experimental confidence better 99%, at least two interference fringes have been seen at the first minima of the wide-slit diffraction pattern. This phenomenon proves that at the same time and without disturbance of the photon beam, those interference fringes are assigned to the photons, passing through the narrow-slit. The possible simultaneous observation of both wave and particle features of the photon opens a new room to causal interpretations of physical reality on a microscopic scale. Following recent experimental observations of the weak-measured Bohmian trajectories, the asymmetrical double-slit experiments with electrons or with a laser beam offer new evidence of the microscopic nonlocal physical reality which would be understood by indirect or direct observation of individual physical quantities. Though the statistical concept of quantum mechanics is conserved, the explicit simultaneous observation of wave-particle duality requires a revision of the current interpretations of quantum mechanics, regarding the ontological physical reality of microscopic substances. Furthermore, the new asymmetrical double-slit experiments should be carried out with single photons or single electrons for a direct illustration of the self-interference of particles. Nevertheless, the present asymmetrical double-slit experiment with a laser beam would serve as a simple ”which-way” demonstration of the ontological physical reality of microscopic particles.

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