An AFDM-Based Integrated Sensing and Communications

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Abstract—This paper considers an affine frequency division multiplexing (AFDM)-based integrated sensing and communications (ISAC) system, where the AFDM waveform is used to simultaneously carry communications information and sense targets. To realize AFDM-based sensing functionality, two parameter estimation methods are designed to process echoes in the time domain and the discrete affine Fourier transform (DAFT) domain, respectively. It allows us to decouple delay and Doppler shift in the fast time axis and can maintain good sensing performance even in large Doppler shift scenarios. Numerical results verify the effectiveness of our proposed AFDM-based system in high mobility scenarios.

Index Terms—Integrated sensing and communications, integrated waveform, affine frequency division multiplexing, discrete affine Fourier transform.

I. INTRODUCTION

Next-generation wireless systems (beyond 5G/6G) are expected to maintain reliable communications in high mobility scenario, improve spectral and energy efficiencies significantly and support ubiquitous connections of everything [1]. The integrated sensing and communications (ISAC) technique is one of the key enablers for beyond 5G/6G due to its ability to improve spectral and energy efficiencies and obtain information about the environment [2], [3].

Integrated waveform design is one of the cornerstones of ISAC. One approach to design the integrated waveform is to endow new functionality to traditional waveforms that have been used in radar or communications systems [4]–[7]. For example, the linear frequency modulation (LFM) waveform, a classical radar waveform, is used to embed communications information [4], [5]. Meanwhile, sensing functionality is added to the orthogonal frequency division multiplexing (OFDM) waveform that has been widely used in communications systems (LTE, 5G NR, WiFi, etc.) due to its robustness against multipath fading [6], [7]. Specifically, in [6], a symbol division-based method is proposed to suppress the sidelobes of radar image caused by random communications symbols and estimate parameters of targets, i.e., range and velocity. The symbol division operation is replaced with symbol conjugate multiplication by utilizing the cyclic cross-correlation (CCC) to avoid amplifying the noise background when symbols have non-constant modulus in the frequency domain [7]. However, the peak-to-sidelobe level ratio (PSLR) of radar images may deteriorate severely in the high mobility scenarios due to the couple of delay and large Doppler shift in the fast time axis.

Apart from traditional waveforms, some new waveforms are investigated for both sensing and communications to improve the overall performance of ISAC systems [8], [9]. The orthogonal time-frequency space (OTFS) waveform multiplexes information symbols in the delay-Doppler domain [8], [10]. From the perspective of communications, the OTFS has the ability to deal with large Doppler shift and obtain both time and frequency diversities in doubly selective channels. Moreover, it can achieve similar sensing performance with OFDM as shown in [11]. Meanwhile, a chirp-based multicarrier waveform is proposed, namely orthogonal chirp-division multiplexing (OCDM) [9], [12]. An important advantage of chirp-based waveform is the potential to support full duplex operation [13]. The OCDM multiplexes a set of orthogonal chirps that are complex exponentials with linearly varying instantaneous frequencies. Since each information is spanned the entire bandwidth, OCDM can achieve full diversity in frequency selective channels, which outperforms OFDM. The sidelobe level of resulting radar image is slightly increased compared with the OFDM waveform [14], [15]. However, OCDM can not achieve full diversity in doubly selective channels [16].

Recently, another chirp-based waveform, namely affine frequency division multiplexing (AFDM), is proposed by multiplexing information symbols in the discrete affine Fourier transform (DAFT) domain [16]–[18]. The AFDM waveform can adapt to the channel profile by optimizing its parameters such that all paths can be separated in the DAFT domain. As a result, AFDM can achieve full diversity in doubly selective channels. The communications performances as well as processing algorithms have been investigated for AFDM in [16]–[18]. Results in [16] show that compared with OTFS, AFDM has comparable communications performance in terms of bit error rate (BER) but with lower complexity and the advantage of less channel estimation overhead [16], [18]. Therefore, AFDM is considered as a potential candidate waveform for the ISAC system [13]. However, it remains interesting to investigate the sensing performance of the AFDM waveform.

In this paper, we propose an AFDM-based ISAC system, where the AFDM waveform is used to simultaneously carry communications information of downlink user and sense information about the targets. Two methods are designed to estimate the range and velocity parameters of targets using random information symbols. The first method extracts parameters in the time domain utilizing the CCC with low complexity. The
second method estimates parameters in the DAFT domain by compensating for the linear phase shift and estimating the cyclic shift of information symbols in the fast time axis. Benefiting from decoupling delay and Doppler shift in the fast time axis, the second method can enhance the maximum unambiguous Doppler shift and maintain good sensing performance in large Doppler shift scenarios. Numerical results show that our proposed AFDM-based system enjoys better PSLR performance than the OFDM-based system in high mobility scenarios.

II. SYSTEM MODEL

We consider an AFDM-based ISAC system, where the ISAC base station (BS) transmits the AFDM waveform to the downlink user and simultaneously receives echoes reflected by the targets around it, as shown in Fig. 1.

A. Communications Signal Model

In this subsection, we briefly review the basic concepts of AFDM proposed in [16]–[18]. Let \( x \) denote an \( N \times 1 \) vector of quadrature amplitude modulation (QAM) symbols. The \( N \) points inverse DAFT (IDAFT) is performed to map \( x \) to the time domain as

\[
s[n] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] e^{j2\pi \frac{m+n+c_m}{N}},
\]

(1)

where \( c_1 \) and \( c_2 \) are the AFDM parameters, and \( n = 0, \ldots, N-1 \). Then, a chirp-aperiodic prefix (CPP) is added with a length of \( N_{cp} \).

After transmission over the channel, discarding CPP and performing \( N \) points DAFT, the received sample matrix in the DAFT domain can be written in the matrix form as

\[
y = H_{\text{eff}} x + \tilde{w},
\]

(2)

where \( \tilde{w} \sim \mathcal{C}N \left( 0, \sigma_{\tilde{w}}^2 I \right) \) is an additive Gaussian noise vector and \( H_{\text{eff}} = \Lambda_{c_2} F \Lambda_{c_1} H_{\text{sym}} F^H \Lambda_{c_1}^H \) with \( F \) being the discrete Fourier transform (DFT) matrix [16], \( \Lambda_{c_i} = \text{diag} \left( e^{-j2\pi c_i n^2}, n = 0, \ldots, N-1 \right) \). \( H_{\text{sym}} \) being the matrix representation of the communications channel in the time domain, and \( (\cdot)^H \) denoting Hermitian transpose.

B. Radar Signal Model

This paper considers that the ISAC BS uses an AFDM frame with \( N_{\text{sym}} \) AFDM symbols to sense the targets around it. We derive the AFDM-based signal model of target echoes.

Let \( X = [x_0, \ldots, x_{N_{\text{sym}}-1}] \) and \( S = [s_0, \ldots, s_{N_{\text{sym}}-1}] \) denote the \( N \times N_{\text{sym}} \) blocks of QAM symbols and modulated signals in time domain, respectively. After adding CPP and parallel to serial conversion, the signal to be transmitted is given by \( \tilde{s} \in \mathbb{C}^{(N+N_{cp})N_{\text{sym}} \times 1}. \) Consider \( P \) targets. The scattering coefficient, range, velocity, time delay and Doppler frequency of the \( i \)-th target are denoted by \( h_i, R_i, v_{r,i}, \tau_i = 2R_i/c, f_{d,i} = 2v_{r,i}f_c/c \) with \( c \) and \( f_c \) being the speed of light and the frequency of carrier, respectively, and \( i = 1, \ldots, P \). The received target echo in the time domain is [19, Eq. (6)]

\[
r[n] = \sum_{i=1}^{P} \tilde{h}_i s[n-\tilde{l}_i] e^{j2\pi f_{d,i} n + \chi w_r[n]},
\]

(3)

where \( n = 0, \ldots, (N+N_{cp})N_{\text{sym}}-1 \). \( w_r[n] \sim \mathcal{C}N \left( 0, \sigma_{w_r}^2 \right) \) is the noise, \( \tilde{h}_i = h_i e^{-j2\pi f_{d,i} \tau_i} \), \( \tilde{l}_i = \tau_i/t_s \), \( f_{d,i} = f_{d,i}/t_s \), and \( t_s \) denotes the sampling interval. Following [6], [7], [16], it is assumed that \( N_{cp} \) is greater than the maximum delay \( l_{\text{max}} \). We define \( \nu_i = N_{f,i} = \frac{f_{d,i}}{2f_c} = \alpha_i + a_i \), where \( \nu_i \in [-\nu_{\text{max}}, \nu_{\text{max}}] \) is the Doppler shift normalized with respect to the subcarrier spacing \( \Delta f \). \( \alpha_i \in [-\alpha_{\text{max}}, \alpha_{\text{max}}] \) is its integer part, and \( a_i \in \left( -\frac{1}{2}, \frac{1}{2} \right) \) is its fractional part [16].

After serial to parallel conversion and discarding CPP, the received target echo can be expressed as

\[
R[n,k] = \sum_{i=1}^{P} \tilde{h}_i S[n-\tilde{l}_i,k] e^{j2\pi f_{d,i} n + \nu_i (N+N_{cp})} + W_r[n,k],
\]

(4)

where \( W_r \) denotes noise matrix, \( n = 0, \ldots, N-1 \), and \( k = 0, \ldots, N_{\text{sym}}-1 \). Performing \( N \) points DAFT on each column of \( R \), the DAFT domain samples matrix \( Y_r \) can be given by

\[
Y_r[m,k] = \frac{1}{N} \sum_{n=0}^{N-1} R[n,k] e^{-j2\pi \left( \frac{m+n+c_m}{N} \right)},
\]

(5)

where \( m = 0, \ldots, N-1 \) and \( k = 0, \ldots, N_{\text{sym}}-1 \).

As a result, motivated by [16, Eq. (26)], the input-output relation in DAFT domain can be written in the matrix form as

\[
Y_r = \sum_{i=1}^{P} \tilde{h}_i H_i X D_i + W_r,
\]

(6)

where \( D_i = \text{diag} \left( e^{-j2\pi f_{d,i} (N+N_{cp})} \right), k = 0, \ldots, N_{\text{sym}}-1 \), \( H_i[p,q] = 1/\sqrt{N} e^{j2\pi \left( N_{c_1} t_i^2 - q l_i + N_{c_2} q^2 - p^2 \right)} \), \( F_{i} \) and \( W_r \) denote \( \Lambda_{c_2} F_{i} W_{r} \). Different from [16], here \( F_{i} \) and \( W_r \) are normalized Doppler shift case, i.e., \( a_i = 0 \), there is only one non-zero element in each row of \( H_i \), i.e., the element of \( H_i \) at row \( p \) and column \( q \) can be written as

\[
H_{i}[p,q] = \begin{cases} 
1/\sqrt{N} e^{j2\pi \left( N_{c_1} t_i^2 - q l_i + N_{c_2} q^2 - p^2 \right)}, & q = (p + \text{locr}) N, \\
0, & \text{otherwise},
\end{cases}
\]

(7)
where \( \text{loc}_i = (2NCc_1i - \alpha_i)_{\text{N}} \) and \( (\cdot)_{\text{N}} \) is the modulo \( N \) operation. Hence, the input-output relation can be rewritten as

\[
Y_r[p,k] = \sum_{i=1}^{P} h_ie^{j2\pi f_i(N+\text{Nc}_i)k} \times e^{j2\pi(Nc_1i^2-q_i+l_i+Nc_2(q_i^2-p^2))} X[q_i,k] + \hat{W}_r[p,k], \tag{8}
\]

where \( p=0, \ldots, N-1 \), \( q_i = (p + \text{loc}_i)_{\text{N}} \) and \( k=0, \ldots, N_{\text{sym}}-1 \). In the fractional normalized Doppler shift case, there are \( 2k_{\text{r}}+1 \) non-zero elements and the peak is still at \( q=(p + \text{loc}_i)_{\text{N}} \) in each row of \( H_i \).

### C. Communications Processing

The communications processing algorithms for AFDM, i.e., channel estimation, equalization and symbol detection, have been studied in [16]–[18], [20]. In the AFDM-based ISAC system, the downlink user can exploit these methods to process received AFDM waveform and obtain the communications information. In this paper, we mainly focus on estimating the parameters of targets based on the AFDM waveform.

### III. AFDM-BASED PARAMETER ESTIMATION METHODS

In this section, we propose two parameter estimation methods based on the AFDM waveform to estimate the range and velocity of targets in the time domain and the DAFT domain, respectively.

#### A. Parameter Estimation in Time Domain

We exploit the Fast Cyclic Correlation Radar (FCCR) algorithm [7] to estimate the range and velocity of targets. Specifically, each column of the received signal matrix \( R \) and the transmit matrix \( S \) in the time domain undergoes the \( N \) points DFT, which results in two \( N \times N_{\text{sym}} \) matrices in the frequency domain. Then, the element-based conjugate multiplication is performed on the resulting matrices. The \( N \) points inverse DFT (IDFT) is computed for each column of matrix and the \( N_{\text{sym}} \) points DFT is calculated for each row of matrix. After these steps, a two-dimensional (2-D) range-Doppler matrix (RDM) is produced, which represents a 2-D radar image in range and Doppler.

As mentioned in [6], the processing gain \( G_p = NN_{\text{sym}} \), and the maximum unambiguous Doppler shift is given by \( \pm 1/(2T_{\text{AFDM}}) \), where \( T_{\text{AFDM}} \) denotes the duration of a AFDM symbol including CPP.

#### B. Parameter Estimation in DAFT Domain

We start from analyzing the input-output relation in the DAFT domain and then propose a method to estimate the range and velocity of targets in the DAFT domain. This method is derived utilizing the input-output relation in the integer normalized Doppler shift case, and numerical results show that it is also suitable for the fractional normalized Doppler shift case.

Following [16], \( c_1 \) is set as \( c_1 = \frac{2(s_{\text{max}}+k_{\text{r}})+1}{2\text{N}} \) and \( c_2 \) is set to be a rational number sufficiently smaller than \( \frac{1}{2\text{N}} \). Thus, if \( c_2 \) is small enough, the value of \( NC2(q_i^2 - p^2) \) will approach zero. Let \( q_i = (p + \text{loc}_i)_{\text{N}} = p + d_i \), i.e., \( d_i = (p + \text{loc}_i)_{\text{N}}-p \). Equation (8) can be rewritten as

\[
Y_r[p,k] \approx \sum_{i=1}^{P} \zeta_i e^{j2\pi f_i(N+\text{Nc}_i)k} e^{-j\frac{2\pi}{N}(l_i-p)} X[p+d_i,k] + \hat{W}_r[p,k], \tag{9}
\]

where \( \zeta_i = \tilde{h}_ie^{j2\pi(Nc_1i^2-d_i)} \). We can see from (9) that in each row of \( Y_r \), i.e., fixing \( p \), there is the linear phase shift between the information symbols \( X[p,\cdot] \) along the \( k \)-axis, which is caused by the Doppler shift. In the first column of \( Y_r \), i.e., fixing \( k \), there are the linear phase shift between the information symbols \( X[\cdot,k] \) and the cyclic shift of information symbols \( X[\cdot,k] \). The former is caused by the delay \( l_i \), and the latter is caused by the delay \( l_i \) and the integer part of normalized Doppler shift, i.e., \( \alpha_i \). Due to this couple of linear phase shift and cyclic shift along the \( p \)-axis, the sidelobes of the radar image obtained by the FCCR algorithm will be severely deteriorated when the Doppler shift is large. Numerical results will verify this conclusion.

To this end, we propose a parameter estimation method in the DAFT domain, which consists of four steps, to enjoy good performance in large Doppler scenarios. Specifically, in the first step, we compensate for the linear phase shift along the \( p \)-axis caused by the delay \( l_i \). The index of the cyclic shift of information symbols \( X \) and the linear phase shift along the \( k \)-axis caused by the Doppler shift are estimated by the matched filter in the DAFT domain and the DFT operation, respectively, in the second step. In the third step, the delay and Doppler shift are extracted from the resulting matrices using our derived relationships of delay and Doppler shift to the peak index. In the fourth step, the extracted integer and fractional normalized Doppler shifts are combined to enlarge the maximum unambiguous Doppler shift.

1) **Compensating for linear phase shift:** We first remove the effect of the linear phase shift in each column of \( Y_r \) by actively compensating for it. According to the above assumption that the CPP length \( \text{Nc}_p \) is greater than the maximum delay \( l_{\text{max}} \), the delay \( l \) is satisfied \( 0 \leq l < \text{Nc}_p \). For each delay \( l \), we generate a compensation matrix \( L_l = \text{diag} \left( e^{j2\pi l/p}, p=0, \ldots, N-1 \right) \) and multiply it by \( Y_r \), i.e., \( Z_l = L_l Y \) and

\[
Z_l[p,k] = \sum_{i=1}^{P} \zeta_i e^{j2\pi f_i(N+\text{Nc}_i)k} e^{-j\frac{2\pi}{N}(l_i-p)} X[p+d_i,k] + \hat{W}_r[p,k], \tag{10}
\]

This process results in \( \text{Nc}_p \) matrices.

2) **Matched filter in DAFT domain and DFT:** Next, we exploit the matched filter in the DAFT domain to estimate the index of the cyclic shift of information symbols \( X \). Specifically, for \( l=0, \ldots, \text{Nc}_p -1 \), each column of \( Z_l \) and \( X \) undergoes a \( N \) points DFT. Then, the element-based conjugate multiplication is performed. Finally, the \( N \) points IDFT is computed for each column and the \( N_{\text{sym}} \) points DFT is
calculated for each row\(^1\). The resulting matrix can be written in the matrix form as

\[
W_i = P^{\dagger} \left( (FL_i Y_{\pi})^* \otimes (FX) \right) F,
\]

where \((\cdot)^*\) and \(\otimes\) are conjugate and Hadamard product, respectively.

3) Extracting delay and Doppler shift: After the previous steps, we get \(N_{cp}\) matrices that contain the delay and Doppler information of \(P\) targets. Next, we investigate how to extract these information from these \(N_{cp}\) matrices. When \(l = l_i, e^{j \frac{2\pi}{N}(l-i)p} = 1\) for \(p = 0, \ldots, N - 1\), i.e., the effect of linear phase shift caused by the delay is removed, and the resulting matrix \(W_i\) will contain one or more peaks (depending on how many targets have this delay and different Doppler shift). We can use the existing constant false alarm rate (CFAR) algorithm to detect the existence of targets \([21]\). If the magnitude of peak of \(W_i, l = 0, \ldots, N_{cp} - 1\), exceeds the threshold, there is a target whose corresponding delay is \(l_i = l\). Let \(p_i\) and \(k_i\) denote the indexes of row and column of the peak exceeding the threshold in \(W_i\). As mentioned earlier, the index of the cyclic shift of \(X\) is caused by \(l_i\) and \(\alpha_i\). Therefore, we can get the estimation of integer part of normalized Doppler shift, given by

\[
\hat{\alpha}_i = 2N \alpha_i - (1 - \hat{p}_i)_N. \tag{12}
\]

Similar to the time domain method in Sec. III. A, the maximum unambiguous Doppler shift that \(k_i\) can represent is \(\pm 1/(2T_{ADFDM}) = \pm \frac{1}{2} \Delta f'\), where \(\Delta f' = 1/T_{ADFDM}\). We define \(\nu_k' = \frac{\nu}{N_{cp}} = \beta_i + b_i\), where \(\nu_k' \in [-\nu_{max}, N'_{max}]\) is the Doppler shift normalized with respect to the maximum unambiguous Doppler shift \(\Delta f'\). \(\beta_i\in [-\nu_{max}, \nu_{max}]\) is its integer part, and \(b_i \in [-\frac{1}{2}, \frac{1}{2}]\) is the fractional part. Consequently, the estimation of the fractional Doppler shift \(b_i\) is given by

\[
b_i = \frac{(k_i - 1 - N_{sym})/2}{N_{sym}}. \tag{13}
\]

4) Combining integer and fractional Doppler shift: Now, we have extracted the delay \(l_i\), the integer Doppler shift \(\alpha_i\) (normalized with \(\Delta f\)) and the fractional Doppler shift \(b_i\) (normalized with \(\Delta f'\)) from matrices \(W_i\), \(l = 0, \ldots, N_{cp} - 1\). If integer and fractional Doppler shifts can be combined, the maximum unambiguous Doppler shift will be significantly increased for a given \(T_{ADFDM}\). Note that the values of \(\alpha_i\) and \(b_i\) cannot be directly added because they are the integer and fractional parts of two different normalized Doppler shifts \(\nu\) and \(\nu_k'\), respectively. According to the definitions of \(\alpha_i\) and \(b_i\), we can get

\[
\hat{f}_{d,i} = (\hat{\alpha}_i + \bar{\alpha}_i) \Delta f = (\hat{\beta}_i + \bar{b}_i) \Delta f', \tag{14}
\]

and

\[
\bar{\alpha}_i = \frac{\Delta f'}{\Delta f} (\hat{\beta}_i + \bar{b}_i) - \bar{\alpha}_i = \frac{N}{N + N_{cp}} \hat{\beta}_i + \frac{N}{N + N_{cp}} \bar{b}_i - \bar{\alpha}_i. \tag{15}
\]

\(^1\)The result of DFT is circularly shifted by \(N_{sym}/2\) positions.
Table I

| Symbol | Parameter                  | Value   |
|--------|----------------------------|---------|
| $f_c$  | Carrier frequency          | 24 GHz  |
| $N$    | Number of subcarriers      | 4096    |
| $N_{sym}$ | Number of AFDM symbols    | 256     |
| $B$    | Signal Bandwidth           | 93.1 MHz|
| $\Delta f$ | Subcarrier spacing       | 22.729 kHz|
| $T$    | AFDM symbol duration       | 44 µs   |
| $N_{cp}$ | Chirp-periodic prefix length | 256 µs |
| $T_{cp}$ | Chirp-periodic prefix duration | 2.75 µs |
| $T_{AFDM}$ | Total AFDM symbol duration | 46.75 µs |
| $\Delta_R$ | Range resolution         | 1.61 m   |
| $\Delta_v$ | Velocity resolution       | 0.52 m/s |
| $G_f$  | Processing gain            | 60.2 dB  |

Figure 3. Calculated radar image for OFDM and AFDM waveforms with
SNR$_{sig}$=10 dB, P=1 and l=128.

(a) $f_d=0.1\Delta_f$

(b) $f_d=0.98\Delta_f$

Fig. 3. Calculated radar image for OFDM and AFDM waveforms with
SNR$_{sig}$=10 dB, P=1 and l=128.

our time-domain and DAFT-domain methods offer similar
PSLRs, which are slightly lower than that of the symbol
division method. When $f_d=0.98\Delta_f$, the PSLRs of our
time-domain method and the symbol division method deteriorate
severely. However, our DAFT-domain method still maintains
good PSLR performance.

Figure 4 illustrates the corresponding image SNR, which is
the ratio between the peak caused by target and the average
noise level in the 2-D radar image [6], versus the SNR of
received signal. The solid and dashed lines represent the cases of
$f_d=0.1\Delta_f$ and $f_d=0.98\Delta_f$, respectively. It can be observed
that in the case of $f_d=0.1\Delta_f$, the available image SNRs
decrease almost linearly with SNR$_{sig}$ for all methods for a
SNR$_{sig}$ below 0 dB. There appears a saturation of image
SNR starting approximately at the SNR$_{sig}$ of 10 dB for our
proposed methods. The symbol division method can output
higher image SNR at high SNR$_{sig}$ regions. In the case of
$f_d=0.98\Delta_f$, there appears a saturation of image SNR for all
methods, and the image SNRs severely decrease for our time-
domain method and the symbol division method. However, the
image SNR of our DAFT-domain method decreases slightly.
Furthermore, our two methods outperform the symbol division
method.

We compare the image SNR versus normalized Doppler shift $\nu$, as shown in Fig. 5. It can be seen that in the low
Doppler shift region ($\nu < 0.5$), all three methods output
similar image SNRs. As Doppler shift increases, the image
SNRs of our time-domain method and the symbol division
method decreases severely first and then increases. The min-
imum SNRs ($<0$ dB) are achieved at integer $\nu$. However,
our DAFT-domain method can always output an image SNR
greater than 50 dB, which verifies that our DAFT-domain
method can work in both integer and fractional normalized
Doppler shift scenarios.

Figure 6 shows radar image obtained by our DAFT-
domain method for multiple targets. There are $P=3$ point-
like targets with ranges $R=[400$ m, 402 m, 404 m] and velo-
cities $v_{rel}=[255$ m/s, 255 m/s, 256 m/s] (corresponding
$f_d=1.8\Delta_f$). Unlike the radar image obtained by the symbol
division method [6], our DAFT-domain method produces mul-
multiple radar image simultaneously. Each radar image contains the information of the targets having the same range. As shown in Figs. 6(a) and 6(b), there are two radar images that contain peaks exceeding the threshold. The latter contains two targets with different velocity. Hence, all three targets can be detected from these radar images.

Table II lists the velocity estimation of our time-domain and DAFT-domain methods with $P=1$ and $l=128$, where $f_d$ and $v_{rel}$ denote truth Doppler shift and corresponding velocity of target, respectively. $\hat{v}_1$ and $\hat{v}_2$ are estimated velocities of time-domain and DAFT-domain methods, respectively. We can see that when $f_d=0.45\Delta_f$, both methods can correctly estimate the velocity. When $f_d=1.4\Delta_f$ and $f_d=2.0\Delta_f$, the time-domain method outputs the wrong velocity due to ambiguity (its maximum unambiguous Doppler shift $= \pm 1/(2T_{AFDM}) = \pm 0.47\Delta_f$). However, the DAFT-domain method still works well.

V. Conclusion

In this paper, we proposed an AFDM-based ISAC systems. Two methods were proposed to estimate the range and velocity of targets in the time domain and the DAFT domain, respectively. The latter significantly increased the maximum unambiguous Doppler shift and still enjoyed good PSLR and image SNR performances in high mobility scenarios.

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Table II

| $f_d$     | Velocity $v_{rel}$ (230.1 km/h) | Velocity $\hat{v}_1$ (m/s) | Velocity $\hat{v}_2$ (m/s) |
|----------|---------------------------------|-----------------------------|-----------------------------|
| 0.45$\Delta_f$ | 63.9 m/s                       | 63.7 m/s                    | 63.7 m/s                    |
| 1.4$\Delta_f$ | 197.6 m/s                      | 63.7 m/s                    | 197.4 m/s                   |
| 2.0$\Delta_f$ | 284.1 m/s (1022.7 km/h)        | -65.3 m/s                   | 284.1 m/s                   |

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