Tool-path planning method for kinematics optimization of blade machining on five-axis machine tool

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Received: 29 December 2021 / Accepted: 23 April 2022 / Published online: 27 May 2022
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Abstract
Planning tool paths on free-form surfaces is a widely discussed issue. However, satisfying all the requirements of blade machining using traditional path-planning methods remains a challenge. Herein, a new iso-parametric path-planning strategy, based on a novel parameterization method and combined with conformal transformation theory, is proposed. The presented strategy adapts to the curvature of blade surfaces, improving the kinematic performance of the machining process, reducing the complexity of multi-axis coordinated motion control, and improving the machining quality. A comparison between the proposed and three traditional methods is also discussed. The influence of the tool path on the kinematic performance of the machine tool is quantitatively examined based on the kinematic models of two different machines. Finally, the results of a deep-cutting milling experiment conducted to verify the improvement of the machining quality due to kinematic optimization are described. The proposed method provides a more reasonable path-planning approach for blade machining on a five-axis machine tool, which can significantly reduce the cost of blade machining, minimize the risks of blade failure, and enhance the large-scale automated production of blades.

Keywords Tool-path planning · Aero-engine blade · Kinematic optimization · Conformal transformation · Free-form surface

1 Introduction
Blades are a vital component of turbomachinery, which play an important role in fields as diverse as aerospace, shipbuilding, and new energy. They are one of the most widely used free-form surface components and, owing to their complex curvature, their surface quality and profile accuracy are critical to the performance and lifespan of turbomachines [1]. The precision machining of blades requires a robot or machine tool with at least five degrees of freedom (DOF) to ensure that the machine tool axis coincides with the vector normal to the blade surface at all times, so that constant contact force and effective removal are guaranteed [2]. Nowadays, primarily due to their high machinability and precision, five-axis machine tools are the first choice for the milling [3], grinding [4], and polishing of blades.

Tool-path planning plays a key role in five-axis blade machining. As with any other typical free-form surface machining approach, blade tool-path planning can be achieved with many existing techniques, among which the most common are the iso-parameter [5], iso-planar [6–8], pocketing [9], and iso-cusp height [10–13] methods. The first two methods can be summarized as extending the path along the direction of a constant parameter and offsetting it along another parameter direction such that adjacent parallel paths can be connected end-to-end to generate a continuous machining path. In the iso-parameter method, the shape of the parameter plane is defined with parameters $u$ and $v$ and can be regularized to be filled by iso-parameter paths to achieve boundary conformation. Conversely, the iso-planar path is generated with parameters $x$, $y$, and $z$ in the Cartesian coordinate system. This method is not designed to be boundary conformed, but its path space is easier to control. The pocketing method takes the surface boundary as the first path and gradually offsets it inward to generate other paths, making it naturally boundary conformed. The iso-cusp height method...
method maintains a constant cusp height between neighboring tool contact paths. This method, which is not boundary conformed, is designed, similar to many other methods, to satisfy specific machining requirements, including optimized machining efficiency [14], collision avoidance [15], and machine tool kinematic and dynamic characteristics [16].

Generally, tool-path planning can be divided into four steps, namely, determining the path direction, planning the tool posture, setting the path space, and setting the step size. Research on path optimization has traditionally focused on the last three steps, based on the requirements of various machining methods. The path direction is often selected depending on the surface characteristics. As the shape of free-form surfaces is widely varied, no path-planning method can adapt to all surfaces, and it can be difficult to quantitatively optimize the path direction. However, for free-form parts with specific shape features and machining requirements, a path that conforms to their shape characteristics can significantly improve the machining results.

Blades are one such type of component, with their surface delimited by a flat upper surface (back, pressure surface), a lower surface (abdomen, suction surface), and the curved leading (inlet) and trailing (exhaust) edges (LTEs), where the machining difficulty is concentrated. The profile accuracy of the LTEs has a significant impact on the aerodynamic performance of the blade. Moreover, because of the harsh blade working environment, any stress concentration caused by machining defects—such as overcutting, undercutting, or surface corrugation—significantly reduces the service life of the blade and increases the risk of accidents.

More than 80% of blade failures originate from machining defects [17, 18]. However, due to their complex curvature and low rigidity, LTEs are also the most difficult areas to machine. Researchers have conducted studies on contact force control [19], process parameter adjustment [20], and trajectory parameter optimization [21, 22] to improve LTE machining accuracy. However, the influence of tool-path planning on the machining of the blade LTEs is rarely discussed.

Zhang et al. [23] proposed an optimized removal strategy for the LTEs to strengthen the control of the contact force and removal, suggesting that frequent and large-scale adjustments of the tool posture would reduce machining accuracy and efficiency. Therefore, the longitudinal path (along the length direction of the blade) was found to be more suitable for LTE machining than the transverse path (along the airfoil direction). However, the study did not explain how to generate longitudinal paths that conformed to LTE features, nor did it quantitatively analyze the impact of each of the two examined path types on the kinematic performance of the machine tool. Xuan and Yonglin [24] divided the main body of the blade into four regions based on the curvature and generated corresponding paths, which is similar to the path-planning methods used for die-type free-form surfaces. However, Huai et al. [25] suggested that regional division should not be implemented for blade surfaces because machining traces at the boundaries between regions could cause stress concentrations. Zhang et al. [26] suggested that to ensure their adequate aerodynamic performance, blades should be machined with a boundary-conformed tool path.

In summary, tool paths for the machining of blade LTEs should satisfy the following three requirements.

- Be boundary conformed to avoid the gap of the fractal area.
- Have a uniform path space to avoid stress concentration.
- Conform with the surface characteristics to reduce tool posture changes.

Traditional path-planning methods struggle to simultaneously satisfy all of the above requirements. Therefore, in this paper, a new iso-parameter path-planning method, in which the paths are generated based on a novel parameterization method combined with conformal transformation theory, is proposed. The resulting paths can pass through points with similar curvature characteristics on each blade section, avoiding frequent tool posture adjustments along the LTEs, thereby improving the kinematic performance of the blade machining. The kinematic models of different tool paths are used to quantify their influence on the machine tool performance, and the impact of kinematic optimization on the machining results is experimentally verified. The proposed method aims to reduce the difficulty of blade machining and the possibility of machining defects by tool-path optimization, thereby enhancing the large-scale automated production of blades.

2 Methodology

2.1 Traditional tool-path planning methods

Planning tool paths on free-form surfaces is a widely discussed subject that is commonly achieved through the iso-parameter, iso-planar, and pocketing methods. Figure 1 illustrates these three traditional path-planning methods. An iso-planar path consists of the intersections between the workpiece surface and a set of parallel cutting planes, the path space being easily controllable. However, the path is not designed to be boundary conformed, and it can be difficult to adapt to the surface curvature. Iso-parameter and pocketing paths are generated on a parameter plane, thus being inherently boundary conformed. The coordinates of the blade data points on the parameter plane can be calculated based on the chord length accumulation.
For data point $p_{i,j}$, the parameter $v_{i,j}$ can be calculated as:

$$v_{i,j} = \frac{l_i}{L},$$

where $l_i$ is the vertical distance from the section plane where point $p_{i,j}$ lies to the starting end plane, and $L$ is the total length of the blade. The parameter $u_{i,j}$ is the cumulative chord length of the section airfoil, which can be calculated using [27]:

$$\begin{align*}
\bar{u}_{0,j} &= 0 \\
\bar{u}_{i,j} &= u_{i-1,j} + \sum_{n=i}^{n} \| p_{i,j} - p_{i-1,j} \|_2 \\
\bar{u}_{n,j} &= 1
\end{align*}$$

Based on the three-dimensional and parameter coordinates of the data points, C~2 continuous reconstruction surfaces can be established using the B-spline fitting method, such that any point on the parameter plane uniquely corresponds to a three-dimensional coordinate point on the reconstruction $B$-spline surface. Consequently, the path on the parameter plane can be mapped on the reconstruction surface.

These traditional path-generation methods only consider a uniform distribution of paths along the surface. However, the factors restricting path planning for machining are far more complex. A common disadvantage of traditional paths is that they cannot conform to the curvature of the surface. Figure 2 shows the surface normal vectors along four types of paths. By definition, at points of high curvature on the blade surface, the normal vector rapidly changes. In most machining methods, a specific angle is required between the tool axis and the surface normal to control the removal volume and shape. Thus, frequent changes of the normal vector demand equally frequent adjustments of the tool posture, which require large concomitant movements of the machine tool, resulting in poor kinematics and machining accuracy.

### 2.2 Parameterization based on conformal transformation

In most types of path-planning methods, the path is often extended along one direction and offset in another direction to generate a trajectory pattern that evenly covers the curved surface. For example, as illustrated in Fig. 1a (right),
in the longitudinal iso-planar method the path is extended along the \( Y-Z \) plane and offset in the \( X \) direction. Consequently, every point on a given path has the same parameter \( x \). Moreover, traditional path-planning methods define a path in terms of one principal parameter, for instance, parameter \( u \) is the principal parameter of the longitudinal iso-parameter path and parameter \( v \) is the principal parameter of the transverse iso-parameter path. Thus, the reason traditional path-planning methods cannot adapt to the curvature of the surface is that there is no connection between their principal parameter and the surface curvature. Conversely, a path based on a parameter related to the curvature can completely solve this problem.

Conformal transformation is a mathematical algorithm to map an area from one plane to another, maintaining the detailed shape of infinitesimal elements \([28]\), and it is widely used in surface parameterization, fluid analysis \([29, 30]\), and airfoil design \([31, 32]\). As a conformal transformation, the Joukowski transformation can map a circle from the complex plane, \( Z_c \), into a blade section airfoil on another complex plane, \( \varsigma \), as follows:

\[
\varsigma = a \rho \theta \exp(\theta), \tag{5}
\]

where \( \theta \) is the polar angle and \( \rho(\theta) \) is the polar radius of the quasi-circle, which is close to being a constant. Substituting Eq. (5) into Eq. (4):

\[
\begin{align*}
\begin{cases}
\begin{align*}
x &= a \left( \rho + \frac{1}{\rho} \right) \cos \theta \\
y &= a \left( \rho - \frac{1}{\rho} \right) \sin \theta
\end{align*}
\end{cases}
\end{align*}
\]

Variables \( \rho \) and \( \theta \) can be solved as follows:

\[
\sin^2 \theta = \frac{1}{2} \left( h + \sqrt{h^2 + \frac{2}{a^2}} \right), \tag{6}
\]

where \( h = \frac{-x^2 - y^2 + 4a^2}{4a^2} \).

As shown in Fig. 3, using Eq. (7), an airfoil in the rectangular coordinate system can be converted into a quasi-circle in the polar coordinate system. Each value of the quasicircular polar angle, \( \theta \), uniquely corresponds to a data point on the airfoil and is related to the curvature, making it a
good principal parameter for the longitudinal path planning of the blade surface.

Figure 4 shows the relationship between the curvature, $k$, and the principal parameter of different path-planning methods. Asterisks mark the trailing edges of the blade section profiles, and dashed lines indicate the principal parameter and the corresponding path generated on the surface. There is no correlation between parameters $u$ or $x$ and the curvature, and their paths irregularly traverse different curvature areas of each section, so the paths ignore the curvature characteristics of the surface and bypass the large curvature area, resulting in a rapid change of the normal vector along the path. In contrast, the quasi-circle polar angle, $\theta$, shows a strong correlation with the curvature. The paths pass through points with similar curvature characteristics on each section. For example, the highlighted path passes through the trailing edges of all the sections. Consequently, it is not affected by the curvature of the airfoil and always passes through the LTE along the direction of small curvature.

### 2.3 Quasi-circle polar angle path-planning method

A new iso-parameter tool-path planning method can be generated using the quasi-circular polar angle, $\theta$, as the principal parameter. The procedure can be summarized as follows:

1. Calculate parameter $v$ using Eq. (1).
2. Normalize the section airfoils by taking the furthest two points in each section as the leading and trailing edges, and assigning them coordinates $(-0.5, 0)$ and $(0.5, 0)$, respectively, through coordinate transformation and scaling.
3. Calculate the quasi-circle polar angle, $\theta$, corresponding to each point using Eq. (7).
4. Compute the quasi-circle parameter $u_\theta = \theta/2\pi$, so that this parameter can be normalized in the interval $[0, 1]$ as a knot vector.
5. Use the B-spline surface fitting method to generate a reconstruction surface as follows:

$$
S(u_\theta, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,k}(u_\theta)N_{j,l}(v)d_{ij},
$$

where $d_{ij}(i = 0, 1, \cdots, n, j = 0, 1, \cdots, m)$ is the control vertex mesh, $N_{i,k}(u_\theta)$ is the $k$-th B-spline basis function defined by the knot vector $U = (u_{\theta 0}, u_{\theta 1}, \cdots, u_{\theta n+k})$; and $N_{j,l}(v)$ is the $l$-th B-spline basis function defined by the knot vector $V = (v_0, v_1, \cdots, v_m+l)$.
6. Generate iso-parameter paths on the parametric plane, extending a path along the $v$ direction and offsetting it along the $u_\theta$ direction.
7. Map the parameter paths to the reconstruction surface.
The proposed method is graphically explained in Fig. 5, which depicts the corresponding flowchart in Fig. 5a and a schematic diagram of some key steps in Fig. 5b. As shown, after describing the surface shape in terms of discrete coordinate points, the path-planning procedure comprises three stages: parameterization, path generation, and path optimization. It is worth pointing out that, in contrast to traditional approaches, the main advantage of the proposed method is that it uses the quasi-circular polar angle as the principal parameter to construct the spline surface in the parameterization stage, resulting in a path that can satisfactorily adapt to the blade surface curvature.

Figure 6 shows an example of the path and corresponding surface normal vectors resulting from the proposed method. The path is boundary conformed, evenly distributed, and adapts to the curvature, with little bending or distortion. Compared with traditional paths, as shown in Fig. 2, there is no rapid change in the direction of surface normal vectors along the LTEs. Consequently, the machine tool will not need to frequently adjust its posture, which enhances the machining stability, reduces the complexity.
of motion control, and improves the kinematics of the machine tool.

3 Kinematic analysis and comparison

Free-form surfaces have curvature changes in multiple directions and are prone to interference problems during machining. In addition, a fixed angle should be maintained between the machine tool axis and the vector normal to the surface at the contact point. Therefore, to appropriately adjust the position and posture of a tool used for blade surface machining, at least five DOF are required, including three translation (3 T) and two rotational (2R) degrees of freedoms, as shown in Fig. 7a. By assigning different actuators to each DOF, five-axis machine tools may adopt many configurations. However, as shown in Fig. 7b, regardless of the configuration, when the tool bypasses a large curvature area, it needs to perform a huge concomitant movement.

In this section, the kinematic constraint equations of two five-axis machine tools are established. The influence of tool paths on the machine tool kinematics is quantitatively evaluated using the inverse kinematics solution. If the drive axis displacement changes smoothly at a lower speed when the tool is machining along a path, the machine tool kinematic performance is significantly improved and the machining exhibits a higher stability and motion control accuracy.

3.1 Kinematics constraint equations of two different machine tools

In kinematic analysis, the machine tool can be simplified as a multi-body system that only focuses on the kinematics chain. By establishing a fixed coordinate system for each body, the relative motion between bodies can then be described through coordinate transformation matrixes. The motion constraint equations of different machine tools can be obtained by arranging the coordinate transformation matrixes based on the low-order body array.

Figure 8 shows a picture and the structural diagram of machine tool I, which has a serial configuration. The system consists of an X-direction guide (1), Y-direction guide (2), polishing tank (3), rotating platform (4), tool rotating platform (6), and Z-direction guide (7). The blade (5) is fixed to the rotating platform (4) and can rotate with it to provide C-direction rotation, \( \gamma \), around the Z-axis. The X-direction guide (1), Y-direction guide (2), and Z-direction guide (7) provide X-direction displacement, \( \tilde{x} \), Y-direction displacement, \( \tilde{y} \), and Z-direction displacement, \( \tilde{z} \), respectively. The tool rotating platform (6) provides B-direction rotation, \( \tilde{\beta} \), around the Y-axis.

The kinematic constraint equation of the machine tool I can be expressed as follows:
where \((x, y, z)\) are the coordinates of the contact point in the blade coordinate system, \((x_t, y_t, z_t)\) is the position of the origin, \(O_r\), of the rotating platform coordinate system in the machine global coordinate system, \(R\) is the distance between the end of the tool and \(O_r\), and \(r\) is the tool radius. The axes movements can be obtained from the inverse kinematic solution of the tool position and posture, as follows:

\[
\begin{bmatrix}
    -r \\
    0 \\
    -R \\
    1
\end{bmatrix} = \begin{bmatrix}
    \cos(\beta) & 0 & -\sin(\beta) & 0 \\
    0 & 1 & 0 & 0 \\
    \sin(\beta) & 0 & \cos(\beta) & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    1 & 0 & 0 & -x_t \\
    0 & 1 & 0 & -y_t \\
    \sin(\beta) & 0 & \cos(\beta) & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    1 & 0 & 0 & -\tilde{z} - z_t \\
    0 & 1 & 0 & \tilde{y} \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    \cos(\gamma) & -\sin(\gamma) & 0 & 0 \\
    \sin(\gamma) & \cos(\gamma) & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}.
\]

(9)

The X-direction guide \((5)\) and Y-direction guide \((1)\) provide X-direction displacement, \(\tilde{x}\), and Y-direction displacement, \(\tilde{y}\), respectively. The 3-RPS parallel motion platform \((3)\) locks the rotation degree of freedom around the \(Y\)-axis, allowing only \(Z\)-direction displacement, \(\tilde{z}\), and \(A\)-direction rotation around the \(X\)-axis.

The kinematic constraint equation for machine tool II can be expressed as follows:

\[
\begin{bmatrix}
    x_r \\
    y_r \\
    z_r
\end{bmatrix} = \begin{bmatrix}
    1 & 0 & 0 & -x_t \\
    0 & 1 & 0 & -y_t \\
    \sin(\beta) & 0 & \cos(\beta) & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
    \cos(\beta) & 0 & \sin(\beta) & 0 \\
    0 & \cos(\beta) & -\sin(\beta) & 0 \\
    -\sin(\beta) & 0 & \cos(\beta) & 0 \\
    0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    x_p \\
    y_p \\
    z_p \\
    1
\end{bmatrix} = \begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}.
\]

(11)

where \((x, y, z)\) are the coordinates of the contact point in the blade coordinate system, \((x_r, y_r, z_r)\) is the position of the origin of the blade coordinate system, \(O_b\), with respect to the motion platform coordinate system, \((x_p, y_p, z_p)\) is the position of the origin of the motion platform coordinate system, \(O_m\), with respect to the fixed platform coordinate system, \((x, y, z)\) is the position of the tool nose in the machine global coordinate system, and \(\tilde{z}\) and \(\tilde{a}\) are coupled by the linear motion of three parallel drive axes, which can be calculated as follows:

\[
L_1 = \sqrt{\left(\frac{3}{2} \cdot r_b \cdot \cos(\tilde{a}) - \frac{1}{2} \cdot r_b \cdot r_p \right)^2 + \left(-r_b \cdot \sin(\tilde{a}) + \tilde{z} + \tilde{a}\right)^2}
\]

\[
L_2 = L_3 = \sqrt{\left(r_b - r_p \right)^2 + \left(\frac{3}{2} \cdot r_b \cdot \sin(\tilde{a}) + \tilde{z} - \tilde{a}\right)^2}
\]

(12)

where \(r_p\) and \(r_b\) are the distances from the center of the motion and fixed platforms, respectively, to the hinge point.

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**Fig. 9** Machine tool II with a hybrid configuration
Kinematic analysis of the tool paths

The displacements of the machine tool drive axes for traditional paths, as computed from the inverse kinematic solution, are shown in Fig. 10. Although their configurations differ, the two machines show similar kinematic characteristics. In the areas of high curvature where the surface normal vector changes rapidly, the kinematic characteristics deteriorate. The displacements of multiple drive axes related to the adjustment of the tool posture also change rapidly over a small range, which is significantly different from the displacement over flat areas. When machining highly curved areas, the drive axes need not only high-speed motions but also rapid acceleration. Multi-axis dynamic coordination is required to ensure the accuracy of the tool movements, which increases the difficulty of control and the probability of machining defects. The transverse and pocket paths extend along the direction of the blade, and unavoidable tool posture adjustments are required. Similarly, two longitudinal paths extending along the length direction still cannot avoid this problem in key areas, as the paths cannot adapt to the surface curvature.
In contrast, with the proposed method, this problem is completely solved. Figure 11 shows the machine tool drive axes displacements corresponding to the proposed path. Even around the trailing-edge area—with its complex curvature—the axes displacements change smoothly with no rapid variations. Compared with the traditional paths near the trailing-edge area, the axes rotations and motions reduce by 90% and 60%, respectively.

When the feed rate is constant at 500 mm/min, the axes motion speeds are as shown in Fig. 12. Because the kinematic characteristics of the pocketing path are similar to those of the transverse path, they are not compared in the figure. Among the remaining three traditional methods, the longitudinal iso-parametric path is the least affected by tool posture adjustment. The maximum rotation and motion speeds for the proposed path are only 1.2% and 10%, respectively, of those for the longitudinal iso-parametric path and concomitant movements around the LTEs are reduced by 98%, all of which improves the kinematic characteristics of the machining when using the proposed path-planning method.

Supposing a speed limit of 100 mm/s for the linear axis, and of 30°/s for the rotary axis, the maximum speeds that the machines can reach are as shown in Fig. 13. For the traditional paths, the tool feed speed is restricted when machining around the LTE, reducing the efficiency and causing uneven removal. On the other hand, for the proposed path, the feed speed is always close to the limit of the linear axis and is not restricted by the concomitant motion axis, displaying minimum feed speeds four times higher than those of the traditional longitudinal iso-parametric path, which results in excellent kinematic efficiency.

4 Experimental results

The kinematic analysis in Sect. 4 showed that the proposed path-planning method could significantly improve the kinematic characteristics of blade machining. In this section, the experimentally verified machining improvement due to kinematic optimization is discussed. A five-axis machine tool was used to mill aero-engine blades following different paths with deep cutting and a high feed speed. The machining results were evaluated based on the accuracy of the milling traces and the uniformity of the removal depth.

We imported several paths into the UG NX software application in the form of discrete points to generate CAM machining files. The built-in post-machining system of the machine tool compiled these files to generate control codes. The cutting depth was set to 0.2 mm and the feed speed was constant at 500 mm/min. The considerable cutting depth and high feed speed placed a higher demand on the machine tool motion control, making it easier to find machining defects during the experiment. The path-planning methods evaluated included transverse, longitudinal iso-parametric, and longitudinal iso-planar, plus the...
one proposed herein. One path close to the LTE and prone to result in machining defects was selected for each path-planning method. The proposed path was evenly distributed on the surface in a parallel raster form. The experiment was repeated three times. The milling process and traces are as shown in Fig. 14.

To control the influence of other factors on the machining accuracy, the machining and blade experiment were completed in one clamping. The chord error was set to 0.001 mm—which was much smaller than the cutting depth—to eliminate interpolation errors. Path planning and blade machining were based on the same model, so there was no path matching error. Since the influence of path matching, interpolation, and clamping errors were eliminated, the machining defects can be attributed to inaccurate multi-axis coordinated motion. Poor kinematics is the primary reason it is difficult for the machine tool motion control system to achieve multi-axis coordinated motion accurately.

A Keyence ultra-depth-of-field microscope and a Taylor contact stylus profiler were used to inspect the milling traces and measure the removal depth, as shown in Fig. 15.

The inspection and measurement results obtained from the traditional paths evaluated are presented in Fig. 16. Figure 16a shows the results at four locations along the transverse path. The cutting depths at points 1 and 3, which are on the flat faces of the blade, are close to 200 μm, thus

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**Fig. 13** Speed constraints for different tool paths

**Fig. 14** Milling process and traces. (a) Milling process. (b) Milling traces of the proposed path. (c) Milling traces of the traditional paths

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**Fig. 15** Milling traces of the proposed path. **Fig. 16** Milling traces of the traditional paths
satisfying the precision requirements. The cutting depths of points 2 and 4, which are located in the LTEs, are inaccurate. At point 2 the cutting depth varies from 206 to 158 μm, which corresponds to a clear undercutting. At point 4 the cutting depth changed between 198 and 267 μm, which is an overcutting. The cutting depth at 20 measuring positions along the traces of three experiments is also shown in the figure. Overcutting and undercutting also occurred at positions 8, 9, 10, 16, 17, 19, and 20, which are all located in high curvature areas. Figure 16b shows the results at four locations along a longitudinal iso-parameter trace that bypasses the sharp trailing edge, from point 1 on one side of the trailing edge to point 4 on the other side. The undercutting problem occurs at positions 2 and 3, which are very close to the trailing edge, resulting in depths of 167 and 138 μm, respectively, both below the expected 200 μm. From the cutting depth measurement at 20 measuring positions along the traces, it can also be observed that the depths at measuring positions 9, 10, 11, and 12 reveal great uncertainty, since undercutting occurred in the first two experiments, and overcutting occurred in the third experiment. This also shows that the source of the error is not some kind of regular mismatch but is caused by the insufficient kinematic precision of the machine tool. Contrasting these results with the drive axes displacements in Fig. 10, there is a close correlation between machining defects and rapid changes in displacement. In the transverse and longitudinal iso-parameter paths, the Z-axis—which controls the milling depth—bears the largest concomitant motion, moving rapidly around the LTEs and leading to overcutting and undercutting defects on both paths. It can be shown that axes with poor kinematics are more prone to machining defects.

This inference is further confirmed in Fig. 16c, which shows the measurement results for the longitudinal iso-planar path near the leading edge. Because the leading edge of the experimental blade has a much lower curvature than the trailing edge, no considerable overcutting or undercutting problems were found in the three milling traces. The cutting depth at 20 measuring positions along the traces is mostly consistent across the three experiments. However, at point 1, there is an evident path distortion, with the milling trace bulging to the right side of the picture in the area near the inflection point, which is inconsistent with the expected smooth machining path. Contrasting the drive axes displacements for the iso-planar path in Fig. 10, the Y-axis bears the largest concomitant motion, which is also the axis that causes path distortion. This further proves that the different kinematic performance caused by different path-planning methods have a huge impact on the machining quality.

Figure 17 shows the measurement results for the path generated with the proposed method. In order to demonstrate the stability of the machining, milling traces are spread over the entire blade. Three long-distance measurements were taken along the upper surface, lower surface, and leading edge of the blade using the Taylor contact stylus profiler. The resulting measurements show that the cutting depth of the milling traces at different areas is consistently close to the expected depth of 200 μm. As shown by the four representative locations selected within the measurement range, no overcutting or undercutting problems were found. Depth measurements were also taken at 20 positions along the milling trace near the trailing edge, where machining defects are most likely to occur, revealing high consistency across the three experiments.

Table 1 presents the averages and standard deviations of the cutting depth measurement results for different path-planning methods. The average cutting depth of the transverse path is close to the expected value, but the standard deviation is large due to overcutting and undercutting in areas of high curvature, i.e., the machining uniformity is poor. The average and standard deviation of the cutting depth for the longitudinal iso-parametric path are both poor, and the milling results of the path through the trailing edge are unsatisfactory. The mean and standard deviation for the longitudinal iso-planar path located near the leading edge are comparatively satisfactory. But, the corresponding path in the trailing edge is so severely overcut that it was not included in the comparison. The milling traces left near the trailing edge using the proposed method have the closest-to-expected mean and lowest standard deviation. Compared with the traditional longitudinal iso-parametric path near the trailing edge, the maximum deviation of the cutting depth and the standard deviation are reduced by 90% and 70%, respectively.
Fig. 16 Experimental results for the traditional methods. (a) Transverse path. (b) Longitudinal iso-parameter path. (c) Longitudinal iso-planar path.
In synthesis, the experimental results demonstrated that the kinematics optimization achieved through the proposed path-planning method can improve the quality of the blades processed using a five-axis machine tool and reduce machining defects. This improvement was attributed to the fact that the proposed path is able to adapt to the curvature characteristics of the blade, which could be applied to different machine tool configurations and methods. The proposed path reduces the difficulty of blade machining, so that a low-precision five-axis machine tool without a force servo control system can avoid machining defects in the LTEs, thereby reducing the cost of blade machining and the risks of blade failure.

### 5 Conclusions

A new iso-parametric path-planning strategy for a blade free-form surface based on a novel parameterization method combined with conformal transformation was proposed. The proposed method could adapt to the curvature characteristics of the blade surface, thus simultaneously satisfying the three requirements of blade machining—that is, it should be boundary conformed, have uniform distribution, and small tool posture changes. Compared with traditional paths, the proposed path was smoother and straighter, without bending or twisting in the LTE area, thus avoiding frequent adjustments of the tool machining posture.

The influence of different tool paths on the kinematic performance was quantitatively examined based on the kinematics models of two machine tools with different configurations. Compared with the traditional iso-parameter longitudinal path—which had the best kinematic performance of the traditional paths—the maximum rotation speed of the proposed path in the LTE area was reduced by 98%, the concomitant motion speed was reduced by 98%, which in turn reduced the maximum linear speed by 90%. The feed speed of the tool could reach the axis speed limit without mechanical restriction, reaching four times that of the traditional path in the LTE area.

A large cutting depth milling experiment was conducted to verify machining quality improvements due to the kinematics optimization. Several machining defects were found on the milling traces of the traditional paths, due to
insufficient accuracy of the multi-axis coordinated motion caused by poor kinematic performance. Under the same machining conditions, the proposed path did not show any machining defects and reduced the maximum deviation of cutting depth by 90% and the standard deviation by 70%.

The proposed method provides a more reasonable path-planning method for blade machining based on kinematics optimization. This improvement can be attributed to the fact that the proposed path could adapt to the curvature characteristics of the blade, which can be applied to different machine tool configurations and machining methods. The proposed path reduces the difficulties involved in blade machining, so that low-precision five-axis machine tools without a force servo control system can avoid machining defects in the LTE area, thereby reducing the cost of blade machining and the risk of blade failure. This is of great significance for the large-scale automated production of blades.

Author contribution Zhongyang Lu: methodology, visualization, data curation, validation, writing, and editing; Xu Yang: conceptualization, editing, and supervision; Ji Zhao: supervision.

Funding This study was supported by a grant from the National Natural Science Foundation of China (Grant No. 51135006, 52175538).

Declarations

Conflict of interest The authors declare no competing interests.

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