Determination of load capacity for flexible spline of harmonic drive

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Abstract. Harmonic gear drives are mechanical drives with engaging flexible and circular splines having substantial features that determine the area of their application. These features include large gear ratio in one stage, light weight of a drive relative to the transmitted load, possibility of motion transmission through a sealed wall, a flexible spline and wave generator. Due to the nature of transmission, a flexible spline is permanently deformed by a rotating wave generator. There are alternating stresses arising in it. Alternating stresses in a flexible spline often lead to its fatigue failure. The paper presents the analysis of stresses in the flexible spline, and dependences to determine these stresses are considered. Correction factors are introduced to the considered dependencies. These factors account for the influence of the gear rim on the flexible spline rigidity as well as the influence of certain gear dimensions ratio. These factors were obtained on the basis of experiments. As a result, the analysis of flexible gears load capacity dependence on their dimensions and key differentiator, being the thickness of a flexible spline under the gear rim, was conducted.

1. Introduction
The criteria of efficiency for most harmonic gear drives (HGD) are fatigue resistance of a flexible spline and limiting load on a wave generator. To prevent fatigue failure of a spline, it must be designed correctly by determining its inner diameter depending on the load, i.e. the transmitted torque. It is also important to correctly assign remaining dimensions of the flexible spline, such as the size of the gear rim, shell thickness, thickness of the spline under the gear rim and others. It is necessary to consider both stress in the flexible spline and the load on the harmonic gear drive generator, as far as limiting load excess results in generator skip and loss of transmission efficiency. When analyzing the efficiency of the harmonic gear drive on the criterion of fatigue resistance of a flexible spline, the stress from its deformation by a wave generator [1] and the stress arising from the load on the teeth [2] are examined, as well as the distribution of these voltages [3]. Also a spatial model of the flexspline, which takes into account its interaction with the wave generator, is considered [4]. In addition, the theoretical studies of the influence of various parameters on the limiting moment of the harmonic gear drive are considered [5].

On the basis of many studies of the performance of HGD conducted by Ivanov M. N. [6], Shuvalov S. A [7] and Poletuchy A. I. [8], recommendations for determining the size of the flexible spline, depending on the transmitted load, are developed. However the influence of some parameters that determine the load capacity of the flexible spline of the HGD has not been thoroughly considered. These parameters include, first of all, the thickness of the flexible spline under the toothed crown. Since it is this thickness which determines both the amount of stress in the flexible spline in the HGD and the load on the wave generator.
2. Task description
Designing a flexible spline of HGD which satisfies the requirements of fatigue resistance, it is essential to determine its design and specify dimensions. Then determination of the stress in the flexible spline, and analysis of the dependence of these stresses on the transmitted load and geometric parameters of the spline are performed. Based on this study it is possible to give recommendations for the design of the spline. A flexible spline of a "glass" type in Figure 1 is considered to be as the most widely spread and rational design. Figure 1 represents the design and main dimensions of the flexible spline as well as load F from the wave generator.

![Figure 1. Flexible gear – glass.](image)

On designing the flexible spline, initially its inner diameter is determined depending on the transmitted load \([2]\). Next, the stresses in the most dangerous area which is a gear rim are to be defined. These stresses depend on many factors influencing the fatigue resistance of a flexible spline. The influence of these factors should be taken into account, and, therefore, it is essential to introduce correction factors in theoretical formulas to calculate stresses. These correction factors must be determined in practical tests of flexible splines under the harmonic gear drive in operation. To determine these coefficients, special tests of flexible gears for fatigue resistance without the load \([1]\) and under the load \([2]\) were performed. After having obtained these factors and their dependence on the examined parameters of the flexible spline of HGD, it is essential to define stresses in the flexible spline. The value of these stresses should be analyzed in terms of parameters having a significant effect on these stresses.

3. Theory
The flexible spline in operation produces the following stresses:
- stresses \(\sigma_{h,c}\) occurring in the flexible spline due to its deformation by the wave generator;
- stresses \(\sigma_{t}\) occurring in the working spline due to the teeth load;
- stresses \(\tau_{m}\) occurring in the spline due to the torque T.

The equivalent stresses are determined according to the known relationship
\[
\sigma = \sqrt{\left(\sigma_{h,c} + \sigma_{t}\right)^2 + 4\tau_{m}^2}
\]

Let us introduce the correction factor to determine the values of stress \(\sigma_{h,c}\) through the stresses \(\sigma_{h,sm}\) in the smooth ring with the thickness \(\delta_1\) being equal to the thickness of the flexible spline under the gear rim.
where $\sigma_{h,sm}$ are the rated stresses in the smooth ring with the thickness under the gear rim $\delta_1$. The equivalent stresses are determined according to the known relationship

$$\sigma_{h,sm} = \frac{E\delta_1}{2} \frac{R - r}{(R + \delta_1/2)(r + \delta_1/2)}$$

(3)

where $E$ is the module of material elasticity; $\delta_1$ is the thickness of the flexible spline under the gear rim; $R$ is the radius of the inner surface of the spline before deformation; $r$ is the radius of the inner surface of the deformed spline.

The values of the factor $K_h$ were determined by the dependence

$$K_h = \frac{\sigma_{1,sm}}{\sigma_{1,sm}}$$

(4)

where $(\sigma_{1,sm})_{h}$ is the material fatigue limit of the smooth ring being defined experimentally on smooth samples; $(\sigma_{1,sm})_{h}$ is the fatigue limit for the flexible spline being determined by testing splines without teeth load, but only under the wave generator spline deformation [1]. In these tests flexible splines were only subjected to the action of a generator without the teeth load. As a result of the tests and data processing there was the dependence of the factor $K_h$ on the geometrical parameters of the flexspline obtained

$$K_h = 9.57 - 1.44 \frac{\delta_1}{m} - 3.68 \frac{L}{R} - 0.29 \frac{r_1}{R} - 5.58 \frac{b}{R} + 0.62 \frac{\delta_1}{m} + 3.19 \frac{L b}{R}$$

(5)

where $m$ is the module of meshing; $R$ is the radius of internal cylindrical surface of the flexspline; $\delta_1$ is the thickness of the flexspline under the gear rim; $L$ is the length of the flexspline; $b$ is the width of the gear rim; $r_1$ is the fillet radius from the gear rim to the smooth element.

Determination of stresses $\sigma_t$ arising in a flexible spline from the load on teeth was conducted through the nominal stress $\sigma_{t,n}$ and a revising factor $K_{teeth}$ available from experiments.

$$\sigma_t = K_{teeth} \sigma_{t,n}$$

(6)

Nominal stresses $\sigma_{t,n}$ are determined according to [3, 7] by the dependence

$$\sigma_{t,n} = \mu \frac{\pi T \sin 2\alpha C}{2m^2 Z K_L K_z} \left( h + 0.5 \delta_1 \right) \frac{6m}{\delta_1} K_J$$

(7)

where $\mu$ is the gear rim longitudinal load distribution factor; $T$ is the transmitted torque; $\alpha$ is the profile angle of the flexible spline; $C$ is the specific teeth rigidity; $m$ is the module of meshing; $Z$ is the number of teeth; $K_L$ is the ratio of length $L$ of the flexible spline to its inner diameter; $K_z$ is the factor to determine the share of meshing teeth in one zone; $h$ is the height of teeth; $K_J$ is the gear rim rigidity factor. A revising factor $K_{teeth}$ was obtained based on the tests of flexible splines to the fatigue resistance. Flexible splines were tested under the on-load operating drive. Flexible splines of the inner surface diameter of 80 mm were examined. Splines of various geometrical parameter combinations were subjected to testing. After processing the test results, the value of the factor $K_{teeth}$ was obtained.

$$K_{teeth} = 0.21 \frac{\delta_1}{m} + 7.84 \frac{L}{R} + 28.44 \frac{b}{R} - 2 \frac{\delta_1}{\delta_1} - 0.616 \frac{\delta_1}{m} - 16.25 \frac{L b}{R} - 6.79$$

(8)

According to the paper [4] the rated torsion stresses are defined as

$$\tau_{sm} = \frac{T}{K_{ny} 2\pi \left( R + \delta_1/2 \right)^2 \delta_2}$$

(9)
where $K_m$ is the factor that takes account of the number of teeth meshed simultaneously. Similarly to the load stresses on teeth, the maximum local shear stresses $\tau_{m,n}$ from the torque were determined:

$$\tau_{m,n} = K_m \tau_{m,n}$$

(10)

where the revising factor $K_m$ was defined by experiments being similar to the previous revising factors. Its value is defined as

$$K_m = 18.21 + 0.72 \frac{\delta_1}{m} - 8.94 \frac{L}{R} - 43.05 \frac{b}{R} + 1.2 \frac{r}{R} + 22.95 \frac{L}{R}$$

(11)

After substitution of the stress values in the formula (1) we obtain

$$\sigma = \sqrt{(\sigma_{h,m}K_h + \sigma_{m,n}K_m)^2 + 4(\tau_{m,n}K_m)^2}$$

(12)

4. Experimental results

On the basis of the obtained dependencies the calculation of the maximum stresses in the gear rim zone of the flexible spline was performed. The revising factors obtained in earlier experimental studies are used in these dependences. Flexible splines with the following parameters were considered in calculations: parameters of the flexible splines under consideration ($d = 80 \text{ mm}$; $L = 80 \text{ mm}$; $b = 16 \text{ mm}$). Different thicknesses $\delta_1$ of the flexible spline under the gear rim were considered. The results of the calculations are shown in Figure 2. The values of $\delta_1$ are given in Table 1.

| Curve № | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------|---|---|---|---|---|---|---|---|---|----|
| $\delta_1 \text{ mm}$ | 0.583 | 0.729 | 0.875 | 1.02 | 1.167 | 1.313 | 1.459 | 1.705 | 1.75 | 1.896 |
5. Conclusion

The analysis of the results allows one to draw recommendations on the design of flexible splines of HGD.

1. Thickness $\delta_1$ is the most significant dimension which affects stresses in the flexible spline under the gear rim.
2. It is practical to take the minimum values of thickness $\delta_1$ in the gear with accepted parameters ($d = 80\, \text{mm}; L = 80\, \text{mm}; b = 16\, \text{mm}$) under the load up to 30 Nm, and the largest possible values $\delta_1$ are practical under the load of more than 30 Nm (Table 1).

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