Abstract
We extend an effective Lagrangian embodying broken scale and chiral symmetry to include explicit chiral symmetry breaking and an additional chiral invariant term which allows for an axial coupling constant greater than unity. We also include a chiral Lagrangian for the isotriplet vector mesons which leads to a renormalization of the pion field. The properties of nuclear matter and nuclei, low energy $\pi N$ scattering and the behavior of quantities such as the pion mass and axial coupling at finite density are discussed.
1 Introduction

Since spontaneously broken chiral symmetry is thought to be a fundamental feature of low energy effective Lagrangians, it is remarkable that it is difficult to describe nuclear matter and nuclei with such a Lagrangian. For example, it has long been known [1] that the standard Lagrangian of the linear sigma model, supplemented by the repulsion of the \( \omega \) meson, does not yield a normal saturating equation of state for nuclear matter. In order to describe nuclear matter, the Lagrangian can be modified by generating the vector meson masses through coupling to the scalar \( \sigma \) field or by including one-loop vacuum corrections. However, it is found [2, 3, 4] that such models fail to reproduce the observed properties of nuclei.

In a previous paper [2], hereinafter referred to as I, we were able to satisfactorily describe nuclear matter and finite nuclei with an effective Lagrangian which incorporated broken scale symmetry in addition to spontaneously broken chiral symmetry, as suggested by quantum chromodynamics (QCD). In order to obtain good phenomenology it was necessary to generate the vector meson masses by coupling to the glueball field, \( \phi \), and to discard the conventional “Mexican hat” potential of the linear sigma model, \( \frac{1}{4} \lambda (\sigma^2 + \pi^2 - f_\pi^2)^2 \). In fact by introducing an \( (\omega_\mu\omega^\mu)^2 \) term in the Lagrangian and adjusting the coupling, we obtained results of a quality comparable to those of the standard Walecka model with non-linear \( \sigma^3 \) and \( \sigma^4 \) terms [4]. In our model the scalar masses are 1.5 GeV and approximately 0.5 GeV for the states which are predominantly \( \phi \) and \( \sigma \), respectively. Here the \( \sigma \) is the chiral partner of the pion and also provides intermediate range attrac-
tion. This view of the $\sigma$ meson finds support in the work of the Brooklyn group [6] using the Nambu-Jona-Lasinio model to study correlated two-pion exchange. They find that for the spacelike momentum transfers of interest here, the $T$ matrix can be represented by an effective low-mass sigma meson which is the chiral partner of the pion, while in the timelike region a physical, low-mass scalar particle is not obtained. On the other hand Furnstahl et al. [7] eliminate the scalar chiral partner of the pion by using the non-linear model and consider that a separate low-mass scalar generates the intermediate range attraction. They also incorporate broken scale invariance and obtain an effective Lagrangian which gives a good account of nuclear matter and finite nuclei. (The experimental situation in the scalar sector unfortunately remains confused [8].)

The purpose of the present paper is to further test our Lagrangian by explicitly studying pions, since they made no contribution in I. This requires some extension of the Lagrangian. Firstly, explicit chiral symmetry breaking is needed to endow the pion with a mass. Secondly, in I the axial coupling constant, $g_A$, was unity, rather than the actual value of 1.26; this defect can be corrected by introducing an additional chiral invariant term in the Lagrangian, as pointed out by Lee [9]. Thirdly, if the isotriplet vector and axial vector mesons ($\rho$ and $a_1$) are included in a linear chiral model, along with the scalar and pseudoscalar mesons, $\pi - a_1$ mixing ensues [10] and this we explicitly take into account. The theoretical development is given in Sec. 2, while in Sec. 3 we give results for nuclei, $\pi N$ scattering and the predicted density dependence of quantities such as $g_A$ and the pion mass. Our conclusions are presented in Sec. 4.
2 Theory

We begin by recalling the effective Lagrangian employed in I. It was written in the form

\[ L_1 = L_0 - V_G , \]  

where we have schematically separated out the scale and chiral invariant part, \( L_0 \), from the potential \( V_G \) which induces the breaking of scale and chiral invariance. We write \( L_0 \) in terms of the nucleon field \( N \), the chiral-invariant combination of sigma and pi fields, \( \sigma \) and \( \pi \), the glueball field \( \phi \) and the field of the omega vector meson \( \omega_\mu \) which is a chiral singlet. Specifically

\[ L_0 = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi \cdot \partial^\mu \pi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} \omega_\mu \omega^\mu + \frac{1}{2} \omega_\mu \omega^\mu G_{\omega \phi} \phi^2 + \left[ (G_4)^2 \omega_\mu \omega^\mu \right]^2 + \bar{N} \left[ \gamma^\mu \left( i \partial_\mu - g_\omega \omega_\mu \right) g(\sigma + i \pi \cdot \tau \gamma_5) \right] N . \]

Here the field strength tensor is defined in the usual way, \( \omega_\mu \omega^\nu = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) and we have included a term quartic in the vector meson field (note that since the coefficient of this term is positive, the violation of causality discussed in Ref. [11] does not arise here). In principle the scale invariant \( \omega \) mass term could be generated by coupling to the \( \phi \) or the \( \sigma \) field, however the latter choice does not yield a good phenomenology for nuclei [2, 3, 4] so we fix on the form \( \omega_\mu \omega^\mu \phi^2 \). The vacuum \( \omega \) mass is then \( m_\omega = G_\omega \phi_0 \), where \( \phi_0 \) denotes the vacuum glueball field (similarly \( \sigma_0 \) will be used for the sigma field).

For the potential \( V_G \) we used [12]

\[ V_G(\phi, \sigma, \pi) = B \phi^4 \left( \ln \frac{\phi}{\phi_0} - \frac{1}{4} \right) - \frac{1}{2} B \delta \phi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2} \]

\[ + \frac{1}{2} B \delta \zeta^2 \phi^2 \left( \sigma^2 + \pi^2 - \frac{\phi^2}{2 \zeta^2} \right) . \]
where \( \zeta = \frac{\phi_0}{\sigma_0} \) and \( B \) and \( \delta \) are parameters. Here the logarithmic terms contribute to the trace anomaly: in addition to the standard contribution from the glueball field \([13, 14]\) there is also a contribution from the \( \sigma \) field. Specifically the trace of the “improved” energy-momentum tensor is

\[
\theta^\mu_\mu = 4V_G(\Phi_i) - \sum_i \Phi_i \frac{\partial V_G}{\partial \Phi_i} = 4\epsilon_{\text{vac}} \left( \frac{\phi}{\phi_0} \right)^4 ,
\]

(4)

where \( \Phi_i \) runs over the scalar fields \( \{ \phi, \sigma, \pi \} \) and the vacuum energy, \( \epsilon_{\text{vac}} = -\frac{1}{4}B\phi_0^4(1 - \delta) \). Guidance on the value of \( \delta \) is provided by the QCD trace anomaly which is proportional to the beta function. At the one loop level, with \( N_c \) colors and \( n_f \) flavors, this is given by

\[
\beta(g) = -\frac{11N_c g^3}{48\pi^2} \left( 1 - \frac{2n_f}{11N_c} \right) + \mathcal{O}(g^5) ,
\]

(5)

where the first number in parentheses arises from the (antiscreening) self-interaction of the gluons and the second, proportional to \( n_f \), is the (screening) contribution of quark pairs. This suggests a value of \( \delta = 4/33 \) for the present case with \( n_f = 2 \) and \( N_c = 3 \) and we use this here, as in I, although nuclear properties are not sensitive to the precise value. The third term in the expression for \( V_G \) in Eq. (3) is needed to ensure that in the vacuum \( \phi = \phi_0 \), \( \sigma = \sigma_0 \) and \( \pi = 0 \), i.e. it provides for spontaneously broken chiral symmetry.

### 2.1 Explicit Chiral Symmetry Breaking

In order to generate a finite pion mass it is necessary to include explicit chiral symmetry breaking terms in the Lagrangian. To restrict the possibilities we recall \( SU(2) \) symmetry breaking at the quark level which is generated by

\[ \mathcal{V}_{SB} = \hat{m}(\bar{u}u + \bar{d}d) , \]

where \( \hat{m} \) is an average mass. This gives a contribution
$\langle V_{SB} \rangle$ to the trace of the energy-momentum tensor, since the operator is of dimension 3. We require our hadronic potential to have similar dimension. The double commutator of $V_{SB}$ with the axial charge $[Q^{5a}, [Q^{5b}, V_{SB}]] = V_{SB}\delta_{ab}$. We require an analogous relation to be satisfied in operator form for our hadronic potential.

From Eq. (2) the axial current and charge are

$$ A^a_\mu = -\bar{N}\gamma_\mu \gamma_5 \frac{1}{2}\tau^a N + \pi^a \partial_\mu \sigma - \sigma \partial_\mu \tau^a ; \quad Q^{5a} = \int A^a_0(x) d^3x . \quad (6) $$

It is straightforward to verify that there are three forms for the symmetry breaking which satisfy our criteria and we write the general potential

$$ V_{SB} = -\epsilon_1 \sigma \phi^2 - \epsilon_2 \sigma (\sigma^2 + \pi^2) + \epsilon_3 \bar{N}N . \quad (7) $$

In order to ensure that in the vacuum $\phi = \phi_0, \sigma = \sigma_0$ and $\pi = 0$ we need to add additional non-symmetry breaking terms to $V_{SB}$; we also subtract the vacuum value of these pieces. It is convenient to set $\epsilon_1' = \epsilon_1\sigma_0\phi_0^2, \epsilon_2' = \epsilon_2\sigma_0^3$ and work with the ratio of the fields to their vacuum values, $\chi = \phi/\phi_0$ and $\nu = \sigma/\sigma_0$. Then

$$ V_{SB}' = -\frac{1}{4}\epsilon_1' \chi^2 \left[ 4\nu - 2 \left( \frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) - \chi^2 \right] $$

$$ -\frac{1}{4}\epsilon_2' \left[ (4\nu - 6\chi^2) \left( \frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) + 3\chi^4 \right] + \epsilon_3 \bar{N}N - \frac{3}{4}(\epsilon_1' + \epsilon_2') . \quad (8) $$

Notice that $V_{SB}'$ gives a (relatively small) contribution to the trace of the energy-momentum tensor. Specifically the contribution to the vacuum energy is

$$ \epsilon_{vac}' = -(\epsilon_1' + \epsilon_2') = -(f_\pi m_\pi)^2 , \quad (9) $$
where for the latter equality, involving the pion decay constant and mass, we have used the relations given in Subsec. 2.4 below. This is the standard result for the vacuum energy arising from the symmetry breaking term.

Using the Lagrangian $L_1 - V_{SB}'$, it is appropriate to give Lagrange’s equations for nuclei or nuclear matter at this point, since $\pi = 0$ in the mean field approximation. For the nucleon, the Dirac equation is of standard form [15], but now the effective mass is $M^* = g\sigma + \epsilon_3 = M\nu + \epsilon_3(1 - \nu)$. The equations for the fields $\chi(r) = \phi(r)/\phi_0$, $\nu(r) = \sigma(r)/\sigma_0$ and $\omega_0(r)$ can be written

\[
\phi_0^2D\chi - 2[B_0(2 - \delta) + \epsilon'_1 - 3\epsilon'_2]\chi + 2[B_0\delta - 3\epsilon'_2]\nu
\]
\[
= 4B_0[\chi^3(\ln \chi - \delta \ln \nu) - \chi + \delta \nu] + B_0\delta[\chi(\nu^2 - \chi^2) + 2(\chi - \nu)]
\]
\[
-m^2_\omega\omega_0^2\chi - \epsilon'_1(2\chi\nu - \chi^3 - \chi\nu^2 + 2\chi) - 3\epsilon'_2[\chi^3 - \chi\nu^2 - 2\chi + 2\nu],
\]
\[
\sigma_0^2\nu - (2B_0\delta + \epsilon'_1 - 3\epsilon'_2)\nu + 2(B_0\delta - 3\epsilon'_2)\chi
\]
\[
= g\sigma_0\rho_s - B_0\delta\left[\frac{\chi^4}{\nu} - \chi^2\nu + 2(\nu - \chi)\right] - \epsilon'_1(\chi^2 - \chi^2\nu + \nu)
\]
\[
-3\epsilon'_2(\nu^2 - \chi^2\nu + 2\chi - \nu),
\]
\[
D\omega_0 - m^2_\omega\omega_0 = -g\omega\rho_B + m^2_\omega(\chi^2 - 1)\omega_0 + 4(G_4)^4\omega_0^3,
\]

where the densities $\rho_s = \langle \bar{N}N \rangle$ and $\rho_B = \langle N^\dagger N \rangle$ can be expressed in terms of the components of the nucleon Dirac spinors in the usual way [15]. In eq. (10) we have made the definitions $D \equiv \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$ and $B_0 \equiv B\phi_0^4$. The terms linear in the fields, *i.e.*, the kinetic energy and mass terms, have been separated out on the left of these equations. In the case of nuclear matter the fields are constant and so the derivatives are zero. We note that the mass matrix is not diagonal and in order to solve the equations for nuclei it is necessary to go to a representation which is diagonal in the limit that

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$r \to \infty$, i.e. $\chi, \nu \to 1$. This is discussed in the Appendix A.

The energy-momentum tensor can be used to obtain the total energy of the system in the standard way \[16\]. Subtracting constants so that the energy is measured relative to the vacuum, we obtain

$$E = \sum_{\alpha} \epsilon_{\alpha} (2j_{\alpha} + 1) - 2\pi \int_0^\infty dr r^2 \left\{ g_{\sigma} \nu \rho_s + g_{\omega} \omega_0 \rho_B ight.$$ 
$$\left. + 2B_0 \left[ \chi^4 (ln \chi - \delta ln \nu + \frac{1}{4}) - \frac{1}{4} \right] + \frac{1}{2} B_0 \delta \left[ \chi^2 (2\nu^2 - 3\chi^2) + 1 \right] \right. 
$$
$$\left. - m_{\omega}^2 \chi^2 \omega_0^2 - 2(G_4)^4 \omega_0^4 + \frac{1}{2} \epsilon_1' \left[ \chi^2 - 2\nu + 2\nu^2 - 1 \right] \right.$$
$$\left. + \frac{1}{2} \epsilon_2' \left( 6\nu^2 \chi^2 - 3\chi^4 - 2\nu^3 - 1 \right) \right\}. \tag{11}$$

In the first term on the right the $\epsilon_{\alpha}$ are the Dirac single particle energies and $j_{\alpha}$ is the total angular momentum of the single particle state. In nuclear matter this term becomes $4 \sum_{k} \left( g_{\omega} \omega_0 + \sqrt{k^2 + M^2} \right)$ and the integral in Eq. (11) is trivial since the fields are constant.

Finally, for nuclei the photon field is included in the standard way \[15\] and, since the contribution from the $\rho$ field is small, it is handled in the simplest manner as in I. These fields are suppressed in the above equations.

### 2.2 Axial Coupling Constant

Eqs. (2) and (8) yield, in the vacuum, $g_{\sigma_0} = M - \epsilon_3$ as the form of the Goldberger-Treiman relation \[17, 21\] which corresponds to a nucleon axial coupling constant $g_A = 1$. It has been pointed out by Lee \[9\] that this defect can be corrected by introducing an additional chiral invariant term in the Lagrangian. This has also been discussed by Akhmedov \[18\] in the context of the standard chiral model, which, as we have remarked, is unable
to reproduce the observed properties of finite nuclei.

It is straightforward to motivate the additional term. With the definitions

\[ \Sigma = \sigma + i \pi \cdot \tau \quad ; \quad \Pi = \sigma + i \pi \cdot \tau \gamma_5 , \]  

one observes that

\[ \overline{N} \gamma_\mu \Pi^\dagger (\partial^\mu \Pi) N = \overline{N}_R \gamma_\mu \Sigma^\dagger (\partial^\mu \Sigma) N_R + \overline{N}_L \gamma_\mu \Sigma (\partial^\mu \Sigma^\dagger) N_L , \]  

(13)

where \( N_{L,R} \) are the nucleon solutions of left- and right-handed chirality. Under global \( SU_L(2) \times SU_R(2) \) transformations \( \Sigma \rightarrow L \Sigma R^\dagger, N_L \rightarrow L N_L \) and \( N_R \rightarrow R N_R \), so the expression (13) is clearly invariant.

Simply Hermitizing the expression (13) yields \( \overline{N} \gamma_\mu \partial^\mu (\Pi^\dagger \Pi) N \) and, since the nucleon vector current is conserved, this can be written as a total divergence which gives no contribution to the action. A more interesting possibility \([9]\) is

\[ L_2 = \frac{D}{4\phi^2} \overline{N} \gamma_\mu i \left[ (\partial^\mu \Pi^\dagger) \Pi - \Pi^\dagger (\partial^\mu \Pi) \right] N \]

\[ = \frac{D}{\phi^2} \overline{N} \gamma_\mu \frac{1}{2} \tau \cdot [\pi \times \partial^\mu \pi + \gamma_5 (\sigma \partial^\mu \pi - \pi \partial^\mu \sigma)] N , \]  

(14)

where we have inserted a constant \( D \) and a factor of \( \phi^{-2} \) to ensure scale invariance. Since in a mean field calculation of nuclei or nuclear matter \( \pi = 0 \), \( L_2 \) will not modify the results obtained in I. It will, however, contribute to the axial vector coupling constant, \( g_A \), and the \( \pi NN \) coupling, \( g_{\pi NN} \), so that \([4, 18]\) the Goldberger-Treiman relation \( g_{\pi NN} \sigma_0 = g_A M - \epsilon_3 \) is obtained.

Before giving further details, the renormalization of the pion field needs to be addressed.
2.3 $\pi - a_1$ Mixing

The inclusion of the isotriplet vector and axial vector mesons ($\rho$ and $a_1$), along with the scalar and pseudoscalar mesons, in a linear chiral model results in significant complication. The details are discussed by Ko and Rudaz [19] (see also Gasiorowicz and Geffen [10]) and their vector meson Lagrangian is given in Appendix B. When the vector mesons are included, the principle of vector meson dominance in the currents is implemented by requiring that, apart from explicit symmetry breaking terms, the Lagrangian be invariant under local $SU_L(2) \times SU_R(2)$ save for the $\rho$ and $a_1$ mass terms. This requires the replacement of the ordinary derivatives of the $\sigma$ and $\pi$ fields in Eq. (2) by covariant derivatives:

$$\partial_\mu \sigma \to \Delta_\mu \sigma = \partial_\mu \sigma + f a_\mu \cdot \pi,$$
$$\partial_\mu \pi \to \Delta_\mu \pi = \partial_\mu \pi + f \rho_\mu \times \pi - f \sigma a_\mu,$$  \hspace{0.5cm} (15)

where $f$ is the gauge coupling constant for the vector fields. This induces mixing between the $\pi$ and the $a_1$ which results in a renormalization of the pion field. Considering just the kinetic energy terms for the $\pi$ and $a_1$, the mass term for the $a_1$ and the mixing, we have

$$\frac{1}{2} \partial_\mu \pi \cdot \partial^\mu \pi - \frac{1}{4} a_{\mu\nu} \cdot a^{\mu\nu} + \frac{1}{2} m_a^2 a_\mu \cdot a^\mu - f \sigma \partial_\mu \pi \cdot a^\mu$$
$$\rightarrow \frac{1}{2} Z^* \partial_\mu \pi \cdot \partial^\mu \pi - \frac{1}{4} a'_\mu \cdot a'^{\mu\nu} + \frac{1}{2} m_a^* a'_\mu \cdot a'^\mu,$$  \hspace{0.5cm} (16)

where $a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, the transformed field is

$$a'_\mu = a_\mu - \frac{f \sigma}{m_a^*} \partial_\mu \pi,$$  \hspace{0.5cm} (17)
and

\[ Z_\pi^* = 1 - (f\sigma/m_\pi^*)^2 . \]  

(18)

Here, and in the following, an asterisk indicates that the values of the \( \sigma \) and \( \phi \) fields in nuclear matter are used; thus \( m_\pi^* \) denotes the mass of the \( a_1 \) in matter. Guided by the observation in I that nuclei prefer the \( \rho \) mass to be generated by coupling to the glueball field rather than the \( \sigma \) field, we take the constants \( b \) and \( c \) of Ref. [19] to be equal so that the masses in matter are

\[ m_{a1}^2 = m_\rho^2 + (2c + 1)(f\sigma)^2 ; \quad m_\rho^2 = m_\rho^2 \left( \frac{\phi}{\phi_0} \right)^2 , \]  

(19)

where the absence of an asterisk on the mass implies the vacuum value. Here we have implicitly assumed that the fields are replaced by their average values in nuclear matter, \( i.e. \) used the mean field approximation, in which case \( Z_\pi^* \) is a renormalization and the physical pion field is defined by \( \pi' = \sqrt{Z_\pi^*}\pi \).

The redefinition (17) of the \( a_1 \) field in the Lagrangian (43) of Appendix B and in the \( a_1N \) coupling, \( -\frac{f}{2}\bar{N}\gamma^\mu\gamma_5\tau N \cdot a_\mu \), leads to additional terms involving the \( \sigma \) and \( \pi \) fields. We can neglect those which are \( O(\pi^4) \) or \( O(\partial\sigma\partial\pi)^2 \) since they will not contribute here. The additional terms we need to consider are

\[ \mathcal{L}_3 = \frac{1}{2}(Z_\pi^*-1) \left[ \partial_\mu\pi \cdot \partial^\mu\pi + \sigma^{-1}\bar{N}\gamma^\mu\gamma_5\tau \cdot N\partial_\mu\pi \right] + \frac{f^2\sigma}{m_{a1}^2}\partial^\mu\sigma \pi \cdot \partial_\mu\pi , \]  

(20)

where we have included the renormalization of Eq. (16). Again, these terms will not affect nuclear matter at the mean field level.
We must also replace the derivatives in Eq. (14) by covariant derivatives yielding the modified form
\[ L'_2 = \frac{D}{\phi^2} \bar{N} \gamma_\mu \frac{1}{2} \tau \cdot \left[ Z^*_\pi \pi \times \partial^\mu \pi + \gamma_5 \left( Z^*_\pi \sigma \partial^\mu \pi - \pi \partial^\mu \sigma \right) - (1 - Z^*_\pi) \sigma^{-1} \pi (\pi \cdot \partial^\mu \pi) \right] N. \] (21)
Thus our total Lagrangian is
\[ L_{\text{total}} = L_1 + L'_2 + L_3 - V_{SB}. \] (22)

2.4 Effective Couplings and the Pion Mass

The axial current, \( A_\mu \), arises from the breaking of local \( SU_L(2) \times SU_R(2) \) by the \( \rho \) and \( a_1 \) mass terms and can be written \[ A_\mu = -f^{-1} \left[ m_{\pi}^2 + f^2 (c\pi^2 - \sigma^2) \right] a_\mu - cf \left[ 2\sigma \pi \times \rho_\mu + 2(a_\mu \cdot \pi) \pi - \pi^2 a_\mu \right]. \] (23)
Using this, we can determine the axial vector coupling constant in medium, \( g_A^* \), by a tree-level calculation of \( \langle N(k-q)|A_\mu|N(k)\rangle \) and identification of the coefficient of \( -\bar{N} \gamma_\mu \gamma_5 \frac{1}{2} \tau N \) in the limit \( q \to 0 \). This gives
\[ g_A^* = Z^*_\pi \left( 1 + \frac{D\sigma^2}{\phi^2} \right). \] (24)
Since in the vacuum \( Z_\pi < 1 \), the presence of the new Lagrangian \( L'_2 \) is essential if one is to obtain \( g_A > 1 \).

Turning to the effective \( \pi NN \) coupling \( -ig^*_{\pi NN} \bar{N} \gamma_5 \tau \cdot \pi^r N \), where \( \pi^r \) denotes the renormalized (physical) pion field, we find
\[ g^*_{\pi NN} = \left( \sigma \sqrt{Z^*_\pi} \right)^{-1} \left[ g\sigma + M^*(g_A^* - 1) \right]. \] (25)
Here we have used the relation

\[ \tilde{N} \gamma_{\mu} \gamma_5 \tau \cdot \partial^{\mu} \pi N = -2iM^* \tilde{N} \gamma_5 \tau \cdot \pi N, \]  
(26)

for on mass-shell nucleons.

Using Eqs. (17) and (23), we have

\[ \langle 0 | A_{\mu}^b(0) | \pi^a(k) \rangle = i\sigma \sqrt{Z^*_\pi} k_\mu \delta_{ab}, \]  
(27)

so that the effective pion decay constant in medium is

\[ f^*_{\pi} = \sigma \sqrt{Z^*_\pi} \]  
(we use the standard nomenclature here while noting that there would be additional contributions to the calculation of the decay of an actual pion in medium). Then Eqs. (24) and (25) yield the Goldberger-Treiman [17] relation in medium

\[ g_{\pi NN}^* f^*_{\pi} = g_A^* M^* - \epsilon_3, \]  
(28)

which is a consequence of the chiral symmetry employed.

Finally we discuss the pion mass in symmetric (isospin zero) nuclear matter. In addition to the contributions that are generated by straightforward application of Lagrange’s equations to \( L_{\text{total}} \) of Eq. (22), one must include [20] the fluctuations in the axial density of nucleons that are induced by the presence of pions. This is represented by the propagator contribution in Fig. 1. We choose to evaluate the contribution to the pion mass at four-momentum \( q = 0 \) (rather than identifying the pole of the propagator at \( q = 0 \)) which yields \( g^2 \rho_s/(Z^*_\pi M^*) \). In this approximation the effective pion mass is given by

\[ m_{\pi}^* = \frac{1}{Z^*_\pi \sigma_0^2} \left[ \frac{\epsilon_1}{\nu} \chi^2 + \epsilon_2 \nu - \frac{\epsilon_3 \rho_s}{\nu (\nu + \epsilon_3/(g \sigma_0))} \right]. \]  
(29)
In the vacuum this evidently becomes
\[ m_{\pi}^2 = \frac{\epsilon_1' + \epsilon_2'}{Z_\pi \sigma_0^2}. \] (30)

We can now evaluate the pion-nucleon sigma term, which with our symmetry breaking is simply \( \langle N(p')|V_{SB}|N(p) \rangle \), evaluated at \( t = (p - p')^2 = 0 \). In the tree approximation, following the approach of Campbell [21], we obtain
\[ \Sigma(0) = \frac{2g\sigma_0 Z_\pi}{m_{\pi}^2 m_\gamma^2} \left[ 3m_{\pi}^2 \gamma + \frac{2\epsilon_1'}{Z_\pi \sigma_0^2 \phi_0} (\beta\sigma_0 - \gamma \phi_0) \right] + \epsilon_3, \] (31)
using the definitions of Appendix A.

3 Results

3.1 Nuclei

We shall examine the results obtained with and without the \((\omega_{\mu} \omega^\mu)^2\) term in the Lagrangian. For the latter case, the binding energy/particle (16 MeV) and density \( (\rho_B^{sat} = 0.148 \text{ fm}^{-3}) \) of equilibrium nuclear matter are sufficient to fix the parameters \( g_\omega \) and \( B_0 \) for various assumed values of the symmetry breaking constants \( \epsilon_1' \), \( \epsilon_2' \) and \( \epsilon_3 \). For the former, an additional parameter enters and we take \( G_4/g_\omega = 0.19 \). This is similar to the value used in I (where the glueball field was frozen, \( \chi = 1 \)) and is judged to give the best results for the nuclear properties; smaller values of \( G_4 \) produce little effect, whereas significantly larger values preclude a fit to equilibrium nuclear matter. As in I, two solutions are found and we pick that which yields the most reasonable effective mass, \( M^{*}_{\text{sat}} \), and compression modulus, \( K \). The parameters are listed in Table 1 for the cases where \( G_4 = 0 \) and \( G_4 > 0 \). These parameters actually
correspond to our favored values for the symmetry breaking, with $\epsilon'_1$ chosen
to give the pion mass (Eq. (30)) for $\epsilon'_2 = 0$, and $\epsilon_3 = -15$ MeV. However
the quantities listed are insensitive to the values of these parameters since
the scale of the symmetry breaking is small in comparison to the other scales
involved.

The first few quantities in Table 1 are similar to those discussed in I. Thus the vacuum energy is in rather good agreement with QCD sum rule
estimates \cite{22} of $\left(240 \text{ MeV}\right)^4$, as in I, while the $\omega$ coupling is comparable
to that needed for the nucleon-nucleon interaction \cite{22}. Since nuclei are not
sensitive to the ratio $\zeta = \phi_0/\sigma_0$ we fix the value such that the larger scalar
mass $m_\sigma = 1.5$ GeV in view of QCD sum rule estimates \cite{24} for a dominantly
 glueball state. The lower mass $m_\omega \simeq 0.5$ GeV. The saturation effective mass
ratio agrees with the recent QCD sum rule estimate \cite{24} of $0.73 \pm 0.09$. As
discussed in I, the lower effective mass for $G_4 > 0$ improves the spin-orbit
splittings in nuclei which are about 80% of the observed values, versus 65%
when $G_4 = 0$. Reference \cite{25} also determines the vector self-energy of a
nucleon in nuclear matter, $g_\omega \omega_0$, to be $0.31 \pm 0.06$ GeV. We obtain 0.22 GeV
with $G_4 = 0$ and 0.28 GeV for $G_4 > 0$ which suggests that the latter yields
the better phenomenology. As regards the compression modulus, $K$, current
estimates are in the range 200–300 MeV \cite{26} which again favors the case with
$G_4 > 0$. Defining the third derivative of the binding energy per particle as
$S = k_F^3 \frac{d^3}{dk_F^3} \left(\frac{E_A}{A}\right)$ (evaluated at equilibrium), we give in Table 1 the ratio $S/K$
since the data indicate \cite{27} a linear relation between this quantity and $K$.
For $G_4 > 0$ our point lies within the allowed error band, while for $G_4 = 0$ it
is slightly off the band.
For nuclei we need to fix $\sigma_0$ and the values which give the best fit are listed in Table 1; a reduction in these values leads to poor agreement, but a modest increase does not substantially change the results presented in this section. The vacuum pion renormalization, $Z_\pi$, and the gauge coupling, $f$ can then be determined. Since the latter is the $\rho NN$ coupling constant, it can be compared with the value of $5.3 \pm 0.3$ given by Dumbrajs et al. [28]; our results are of reasonable magnitude. The remaining quantities in Table 1 will be referred to below.

Since the single particle levels and charge density distributions are quite similar to those obtained in I, we shall content ourselves with discussing the bulk properties of oxygen, calcium and lead. These are given in Table 2; here the appropriate corrections for c.m. [29] and finite size effects [15] have been made. We also list the value of the sigma term from Eq. (31). With the choice $g_{\pi NN}^2/(4\pi) = 14.3$ from Höhler [30], the vacuum axial coupling constant, $g_A$, is determined and the values in Table 2 are seen to be quite reasonable. The parameter sets designated with a W in Table 2 correspond to the case with $G_4 > 0$. The cases labelled A correspond to no explicit symmetry breaking so, while $g_A$ is reasonable, the sigma term is necessarily zero; the remaining results are similar to those given in I. The other cases show the effect of various symmetry breaking parameters; here $\epsilon'_1$ and $\epsilon'_2$ are constrained by Eq. (30) to give the vacuum pion mass.

The symmetry breaking tends to decrease the binding energy and increase the radius or vice versa, but, as expected, the effects are fairly small. Simply choosing $\epsilon'_1 \neq 0$ (case WB) improves the radii at the expense of some loss of binding. Also the sigma term becomes non-zero and is reasonable in
comparison to the estimate of Gasser et al. of 45 MeV with \(\sim 15\%\) error. If we include \(\epsilon_2'\) (case WC), the radii are reduced and the sigma term becomes rather large, although the latter could be corrected by an appropriate choice of \(\epsilon_3\). We have also examined the choice \(\epsilon_2' = -\frac{1}{2} \epsilon_1'\), but this significantly increase \(K\) and correspondingly substantially reduces the binding energies. We also note that with \(\epsilon_2' \neq 0\), the \(\sigma^3\) term in \(V_{SB}\) would dominate over the potential \(V_G\) as \(\sigma \rightarrow \infty\), which would seem to be unphysical. We therefore favor setting \(\epsilon_2' = 0\). With this choice, we show in Table 2 the results obtained with various values of \(\epsilon_3\). A small negative value of \(\sim -15\) MeV appears to give the best results for nuclei and also for the sigma term. Set WF gives rather a good account of the radii with binding energies that are somewhat low, while set F improves the binding energies at the expense of the radii.

### 3.2 \(\pi N\) Scattering

The Lorentz-invariant scattering matrix is conventionally written in the form 
\[
\bar{N}(p_2)T_{ba}N(p_1),
\]
where
\[
T_{ba} = \left[ A^{(+)} + \frac{1}{2} (q_1 + q_2) B^{(+)} \right] \delta_{ba} + \left[ A^{(-)} + \frac{1}{2} (q_1 + q_2) B^{(-)} \right] i\epsilon_{bac} \tau^c. (32)
\]
Here \(q_1, a (q_2, b)\) specify the four momentum and isospin component of the incoming (outgoing) meson. It is straightforward, though tedious, to obtain the amplitudes in terms of the usual Mandelstam variables \(s = (p_1 + q_1)^2\), \(t = (q_1 - q_2)^2\) and \(u = (p_1 - q_2)^2\). The plus amplitudes receive contributions from nucleon exchange in the \(s\) and \(u\) channels and from the exchange of
scalar mesons in the $t$ channel. With on-mass-shell pions, we find

$$A^{(+)} = -\frac{2g}{\sigma_0 Z_\pi(t - m_\pi^2)(t - m_\pi^2)} \left\{ \frac{1}{2} Z_\pi(t - 2\gamma)(t + m_\pi^2) + \left( t - 2\gamma + 2\beta\zeta^{-1} \right) \left[ (2m_\pi^2 - \frac{1}{2}t) \frac{f^2 m_\pi^2 \sigma_0^2}{m_a^4} - \epsilon_1^2 \right] \right\} + \frac{g^2_{\pi NN}}{M} + \frac{g\epsilon_3}{Z_\pi \sigma_0 M} \right\}.$$

$$B^{(+)} = g^2_{\pi NN} \left( \frac{1}{u - M^2} - \frac{1}{s - M^2} \right).$$

Here $m_>$ and $m_<$ denote the scalar meson masses which are given in the Appendix A, along with the definitions for $\alpha$, $\beta$ and $\gamma$. Notice that $B^{(+)}$ is of standard form [16, 20].

For the minus amplitudes, nucleon exchanges in the $s$ and $u$ channels again contribute and in the $t$ channel we need to consider $\rho$ exchange. For the latter, the $\rho\pi\pi$ coupling follows from the Lagrangian quoted in Appendix B and is given by Ko and Rudaz [19] as

$$L_{\rho\pi\pi} = -\frac{f m_\rho^2}{Z_\pi m_a^2} \rho_\mu \cdot (\partial^\mu \pi^r \times \pi^r)$$

$$- \left( \frac{f^3 \sigma_0^2}{2Z_\pi m_a^4} + \frac{f Z_\pi \kappa_6}{2m_\rho^2} \right) \rho_{\mu\nu} \cdot (\partial^\mu \pi^r \times \partial^\nu \pi^r),$$

where $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$.

The $\rho NN$ vertex is

$$L_{\rho NN} = \frac{1}{2} f \bar{N} \gamma^\mu \tau \cdot \rho_\mu N.$$

In addition we have a four point $NN\pi\pi$ interaction in Eq. (21) which must be considered. The resulting amplitudes are

$$A^{(-)} = 0.$$
\[ B^(-) = -g_{\pi NN}^2 \left( \frac{1}{u - M^2} + \frac{1}{s - M^2} \right) + \frac{2Z_\pi - g_A^2 - 1}{2Z_\pi \sigma_0^2} \]
\[ - \frac{f^2}{Z_\pi m_a^2(t - m_\rho^2)} \left[ m_\rho^2 - t \left( \frac{f^2 \sigma_0^2}{2m_a^2} + \frac{\kappa_6 Z_\pi^2 m_a^2}{2m_\rho^2} \right) \right]. \]  

(36)

In terms of these amplitudes, the s-wave scattering lengths and effective ranges, \( a_{0}^{(\pm)} \) and \( r_{0}^{(\pm)} \), and the p-wave scatterings lengths, \( a_{1\pm}^{(\pm)} \), are

\[ a_{0}^{(\pm)} = \eta \left( A_{0}^{(\pm)} + m_\pi B_{0}^{(\pm)} \right) ; \quad a_{1\pm}^{(\pm)} = \frac{2}{3} \eta C_{0}^{(\pm)}, \]
\[ a_{1-}^{(\pm)} = 2\eta C_{0}^{(\pm)} - \frac{\eta}{4M^2} \left[ A_{0}^{(\pm)} - (2M + m_\pi)B_{0}^{(\pm)} \right], \]
\[ r_{0}^{(\pm)} = -\eta \left\{ 2C_{0}^{(\pm)} - \frac{(M + m_\pi)^2}{Mm_\pi} D_{0}^{(\pm)} \right. \]
\[ + \left. \frac{1}{2Mm_\pi} \left[ (1 - \frac{m_\pi}{2M}) A_{0}^{(\pm)} - \left( M + \frac{m_\pi}{2M} \right) B_{0}^{(\pm)} \right] \right\}, \]  

(37)

where \( C_{0} = [\partial (A + m_\pi B)/\partial t]_{t=0} \), \( D_{0} = [\partial (A + m_\pi B)/\partial s]_{t=0} \), with \( s \) and \( t \) taken to be the independent variables. Also \( \eta^{-1} = 4\pi(M + m_\pi)/M \). The subscript 0 implies that the amplitudes are to be evaluated at threshold: \( t = 0 \), \( s = (M + m_\pi)^2 \) and \( u = (M - m_\pi)^2 \).

In order to calculate the scattering amplitudes in the most general case, additional parameters are needed. The parameter \( D \) is determined from Eqs. (24) and (25) in the vacuum and is listed in Table 1. The quantity \( c \) in Table 1 is deduced from the value of \( Z_\pi \) and Eqs. (18) and (19) in vacuum. The parameter \( \kappa_6 \) listed in Table 1 is obtained by fitting the decay width, \( \Gamma(\rho \rightarrow \pi\pi) = 151.2 \pm 1.2 \text{ MeV} \) [32], using Eq. (34) with the \( \rho \) on the mass shell. The effective \( \rho\pi\pi \) coupling constant of 6.05 is of similar magnitude to the \( \rho NN \) coupling \( f \) of Table 1, which is in accord with the notion of \( \rho \) universality. Since significant cancellations may occur in calculating the scattering lengths and effective ranges, Eq. (37), we prefer to deduce \( A_0, B_0, \)
$C_0$ and $D_0$ from the experimental data [30] and these are listed in the first row of Table 3. The $\Delta$ resonance contributions to the amplitudes, constructed such that no double counting occurs, have been given by Höhler [30]. We list these in Table 3, along with the difference which we would hope to reproduce with our model. (For definiteness we note that the off-shell parameter $Z$ appearing in the $\pi N\Delta$ interaction was chosen to be $\frac{1}{2}$.) We first give results obtained with our simplest Lagrangian, $\mathcal{L}_1$. Since $g_{\pi NN} = g$ in this model, the results are quite poor. If we also include $\mathcal{L}_2$, this allows us to fit $g_{\pi NN}$ by adjusting $D$ (here $Z_\pi$ is still 1) and this gives much better agreement with the data. The remaining cases in Table 3 correspond to the full model and the notation follows that of Table 2. With no explicit symmetry breaking (cases A and WA), the sum of $A_0^{(+)}$ and $m_\pi B_0^{(+)}$ must cancel so that there is no contribution to $a_0^{(+)}$ in the chiral limit. In fact the sum should be negative, as it is for most of the cases displayed. The sensitivity to the symmetry breaking is not great, but there appears to be a weak preference for cases F and WF and we recall that these were also favored from the results of Table 1.

The question arises as to how well the scattering parameters test our model. In this connection it is useful to compare with the results of Matsui and Serot [16, 20] who used both the standard chiral model and QHD-II. In all models $g_{\pi NN}$ is taken from experiment so that the predictions for $B_0^{(+)}$ and $D_0^{(+)}$ are the same. Also $A_0^{(+)}$ will only differ from $-m_\pi B_0^{(+)}$ because of symmetry breaking terms, which are necessarily small. In Ref. [16, 20], $C_0^{(+)}$ is 2.0 (3.0) in the chiral (QHD-II) model where the scalar mass is 770 (520) MeV. Thus one distinct feature of the present model is that we get about the
right value for $C_0^{(+)}$ with $m_\pi \sim 500$ MeV. As regards the $(-)$ amplitudes, $D_0^{(-)}$ is fixed once $g_{\pi NN}$ is given and to a large extent $C_0^{(-)}$ is also; thus all models give much the same result. However in QHD-II, where the $\rho$ is included, $m_\pi B_0^{(-)} = 3.1$, whereas our value is smaller, due to the constant term and the more complicated form for the $\rho$ contribution, which brings us closer to the desired value.

### 3.3 Density Dependence of the Parameters

In our Lagrangian there are two scales, $\chi$ and $\omega$ which give the ratio of the glueball and sigma fields to their vacuum values. Their dependence on density is shown in Fig. 2 for the parameters of Table 1. The glueball field changes very little which implies that at saturation the $\omega$ and $\rho$ masses are reduced by only 3%. By contrast QCD sum rule estimates suggest a reduction of roughly $20 \pm 10\%$. Brown and Rho have also argued for a similar scaling. An effect of this magnitude would suggest scaling these vector masses with $\nu$ instead of $\chi$. Then, however, it is not possible to produce acceptable fits to the properties of nuclei, for instance the charge density distributions start to develop oscillations in disagreement with the data and these probably signal the onset of a situation where the ground state of nuclear matter is no longer uniform.

The pion decay constant in medium is given by $f_\pi^*/f_\pi = \nu \sqrt{Z_{\pi}^*/Z_{\pi}}$. The behavior of the pion renormalization, $Z_{\pi}^*/Z_{\pi}$, is shown in Fig. 3. The change in $Z_{\pi}^*$ in going from the vacuum to saturation is a 7–11% effect, depending on the parameters chosen, so that $f_\pi^*/f_\pi \simeq M^*/M$ as suggested by Brown.
and Rho [34]. Specifically the pion decay constant ratio at saturation is 0.74 ($G_4 = 0$) or 0.66 ($G_4 > 0$) which compare quite well with the effective masses given in Table 1. A similar level of agreement is obtained if one also includes a factor of $\sqrt{g_A'}/g_A$. The ratio of the axial coupling constants, given in Fig. 4, shows the expected reduction with density. The data indicate [36] that $g_A'$ in equilibrium nuclear matter should be close to 1, which is a little lower than our value of 1.1.

The quantities discussed thus far are insensitive to the value of the explicit symmetry breaking parameters. This is not the case for the effective pion mass, however, as we show in Fig. 5. At low densities, where the linear approximation is sufficient, the behavior is given by the expression [37, 38]

$$m^*_\pi/m_\pi = 1 - 2\pi a_0^{(+)} \rho_B/(m_\pi m_R),$$

where $m_R$ is the reduced mass of the pion-nucleon system. (This can also be obtained in leading order from Eqs. 29, 33 and 37.) The mass decreases for set WD since $a_0^{(+)}$ is positive, whereas is should be negative leading to an increased pion mass in matter. The pionic atom data indicate a 10% increase at saturation [38] and this suggests that $\epsilon_3$ should be zero or small and negative in agreement with the previous indications.

## 4 Conclusions

The effective Lagrangian of I, which embodied broken scale and spontaneously broken chiral symmetry as suggested by QCD, gave a good account of the properties of finite nuclei as well as nuclear matter. We have extended this Lagrangian by allowing for the explicit breaking of chiral symmetry and
by including an additional term which permits us to obtain an axial coupling constant, \( g_A \), which is close to the physical value of 1.26 in vacuum. We have also adjoined this Lagrangian to a chiral Lagrangian for the \( \rho \) and \( a_1 \) vector mesons \[19\] while maintaining reasonable properties for these particles. This required a renormalization of the pion field. The final Lagrangian yielded the Goldberger-Treiman relation at finite density, as well as in the vacuum.

The effect of explicit chiral symmetry breaking on nuclear matter and nuclei was, for the most part, small so that their properties continued to be reasonably well described. As in I, the presence of a term \((\omega_{\mu} \omega^\mu)^2\) in the Lagrangian improved the phenomenology. The low energy \( \pi N \) scattering parameters were found to be in good agreement with the data once the effect of the \( \Delta \) resonance was removed, since this is not included in our model. As regards the modifications of the various quantities in going from the vacuum to equilibrium nuclear matter density, the substantial reduction in the \( \rho \) and \( \omega \) masses predicted by QCD sum rules remains puzzling since the reduction we obtain is very slight. Further it appears that a more substantial reduction yields poor results for nuclei \[2, 3, 4\]. On the other hand, we find that the axial coupling is quenched to a value close to 1, as indicated experimentally, and the increase in the pion mass suggested by pionic atom data is obtained if the constant in the symmetry breaking potential \( \epsilon_3 \tilde{N}N \) is zero or very small and negative. The latter was favored by other data, particularly the pion-nucleon sigma term. Thus we conclude that our Lagrangian, in which chiral symmetry is realized in a linear fashion, embodies the essential physics necessary to describe quite a wide variety of low energy data.

We acknowledge partial support from the Department of Energy under
Appendix A: Scalar Field Mass Matrix

Setting $\sigma = \sigma_0 - \bar{\sigma}$ and $\phi = \phi_0 - \bar{\phi}$, we denote the $2 \times 2$ mass-squared matrix for the fluctuating parts of the scalar fields, $\bar{\sigma}$ and $\bar{\phi}$, by

$$
\begin{pmatrix}
2\alpha & -2\beta \\
-2\beta & 2\gamma
\end{pmatrix}.
$$

The matrix elements can be read off from the field equations (10):

$$
\alpha = \frac{2B_0\delta + \epsilon_0' - 3\epsilon_0''}{2\sigma_0^2}; \quad \beta = \frac{B_0\delta - 3\epsilon_0'}{\phi_0\sigma_0}; \quad \gamma = \frac{B_0(2 - \delta) + \epsilon_0' - 3\epsilon_0''}{\phi_0^2}.
$$

Defining $r = \sqrt{(\gamma - \alpha)^2 + 4\beta^2}$, the eigenvalues and eigenvectors are

$$
m_+^2 = \alpha + \gamma + r; \quad \psi_+ = a\bar{\sigma} - b\bar{\phi}
$$

$$
m_-^2 = \alpha + \gamma - r; \quad \psi_- = b\bar{\sigma} + a\bar{\phi},
$$

where $a/b = (\alpha - \gamma + r)/(2\beta) = 2\beta/(\gamma - \alpha + r)$ and $a^2 + b^2 = 1$.

Appendix B: The Lagrangian for the Vector Fields

Here we quote, for reference, the Lagrangian of Ko and Rudaz [19] for the vector and axial vector mesons, $\rho$ and $a_1$. They define left and right gauge
fields

\[ l_\mu = \frac{1}{2}(\rho_\mu + a_\mu) \cdot \tau \quad ; \quad r_\mu = \frac{1}{2}(\rho_\mu - a_\mu) \cdot \tau , \]

(41)

and field strength tensor

\[ l_{\mu \nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - if[l_\mu, l_\nu] , \]

(42)

with an analogous definition for \( r_{\mu \nu} \). The Lagrangian is then

\[
L_{\text{vector}} = -\frac{1}{4} \text{Tr}[l^2_{\mu \nu} + r^2_{\mu \nu}] + \frac{1}{2} m_0^2 \text{Tr}[l^2_\mu + r^2_\mu] \\
+ \frac{1}{4} b f^2 \text{Tr}[\Sigma \Sigma] \text{Tr}[l^2_\mu + r^2_\mu] - c f^2 \text{Tr}[l_\mu \Sigma r^\mu \Sigma^\dagger] \\
- i \frac{\kappa_6 f}{4 m_\rho^2} \text{Tr}[l_{\mu \nu} \Delta^\mu \Sigma \Delta^\nu \Sigma^\dagger + r_{\mu \nu} \Delta^\mu \Sigma^\dagger \Delta^\nu \Sigma] ,
\]

(43)

using the definitions of Eqs. \((12)\) and \((15)\). For simplicity we focus here on the minimal model of Ref. \[19\] which neglects a further term in the Lagrangian (\(\zeta_6 = 0\)). In the present work we have set \( b = c \) (see Table 1 for the values) and generated the dimensionful constants in a scale invariant way, \( i.e. \) replaced \( m_0^2 \) by \( m_\rho^2 (\phi/\phi_0)^2 \) and \( \kappa_6/m_\rho^2 \) by \( (\kappa_6/m_\rho^2)(\phi_0/\phi)^2 \).

We should briefly comment on the vector meson phenomenology with our chosen parameters. The predictions for the widths and the pion radius discussed in Ref. \[19\] are given in Table 4 for our parameters (note that the corrected formula \[39, 40\] is used for the \( a_1 \to \rho \pi \) decay width). In general the results are reasonable, although the \( a_1 \to \pi \gamma \) width is too large by a factor of two. This width is sensitive to the presence of a \( \zeta_6 \) term in the Lagrangian which leads to a renormalization of the \( \rho \) and \( a_1 \) fields. However, inclusion of this term to improve the phenomenology is beyond the scope of the present work.
References

[1] A.K. Kerman and L.D. Miller, “Field Theory Methods for Finite Nuclear Systems and the Possibility of Density Isomerism”, in *Proceedings of the Relativistic Heavy Ion Summer Study*, Berkeley report LBL–3675 (1974) p.73.

[2] E.K. Heide, S. Rudaz and P.J. Ellis, Nucl. Phys. A571 (1994) 713.

[3] R.J. Furnstahl and B.D. Serot, Phys. Rev. C47 (1993) 2338; Phys. Lett. B316 (1993) 12.

[4] R.J. Furnstahl, B.D. Serot and H.-B. Tang, preprint (1995) nucl-th/9511028.

[5] R.J. Furnstahl, C.E. Price and G.E. Walker, Phys. Rev. C36 (1987) 2590.

[6] L.S. Celenza, C.M. Shakin, W.-D. Sun and J. Szweda, Brooklyn College of the City University of New York preprint, #BCCNT 95/032/245; C.M. Shakin, W.-D. Sun and J. Szweda, ibid. #BCCNT 95/051/246.

[7] R.J Furnstahl, H.-B. Tang and B.D. Serot, Phys. Rev. C52 (1995) 1368 and work in progress.

[8] M. Svec, preprint (1995) hep-ph/9511203; N.A. Tornqvist and M. Roos, preprint (1995) hep-ph/9511210.

[9] B.W. Lee, *Chiral Dynamics* (Gordon and Breach, NY, 1972)
[10] S. Gasiorowicz and D.A. Geffen, Rev. Mod. Phys. 41 (1969) 531.

[11] G. Velo and D. Zwanziger, Phys. Rev. 188 (1969) 2218; G. Velo, Nucl. Phys. B65 (1973) 427.

[12] E.K. Heide, S. Rudaz and P.J. Ellis, Phys. Lett. B293 (1992) 259.

[13] J. Schechter, Phys. Rev. D21 (1980) 3393; A.A. Migdal and M.A. Shifman, Phys. Lett. B114 (1982) 445.

[14] H. Gomm and J. Schechter, Phys. Lett. B158 (1985) 449.

[15] C.J. Horowitz and B.D. Serot, Nucl. Phys. A368 (1981) 503.

[16] B.D. Serot and J.D. Walecka, Advances in Nuclear Physics, Vol. 16, ed. J.W. Negele and E. Vogt (Plenum, NY, 1986); B.D. Serot, Rep. Prog. Phys. 55 (1992) 1855.

[17] M. Goldberger and S.B. Treiman, Phys. Rev. 110 (1958) 1178.

[18] E.Kh. Akhmedov, Nucl. Phys. A500 (1989) 596.

[19] P. Ko and S. Rudaz, Phys. Rev. D50 (1994) 6877.

[20] T. Matsui and B.D. Serot, Ann. Phys. (NY) 144 (1982) 107.

[21] D.K. Campbell, Phys. Rev. C19 (1979) 1965.

[22] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.

[23] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. 149 (1987) 1.
[24] M.A. Shifman, Z. Phys. C9 (1981) 347; P. Pascual and R. Tarrach, Phys. Lett. B113 (1982) 495.

[25] D.B. Leinweber, X. Jin and R.J. Furnstahl, preprint (1995) nucl-th/9511031.

[26] J.P. Blaizot, Phys. Rep. 64 (1980) 171; N.K. Glendenning, Phys. Rev C37 (1988) 2733; M.V. Stoitsov, P. Ring and M.M. Sharma, Phys. Rev. C50 (1994) 1445.

[27] J.M. Pearson, Phys. Lett. B271 (1991) 12; see also S. Rudaz, P.J. Ellis, E.K. Heide and M. Prakash, Phys. Lett. B285 (1992) 183.

[28] O. Dumbrajs, R. Koch, H. Pilkuhn, G.C. Oades, H. Behrens, J.J. de Swart and P. Kroll, Nucl. Phys. B216 (1983) 277.

[29] J.W. Negele, Phys. Rev. C1 (1970) 1260; L.J. Tassie and F.C. Barker, Phys. Rev. 111 (1958) 940.

[30] G. Höhler, Pion-Nucleon Scattering, Landolt-Börnstein, New Series, ed. H. Schopper, Vol. I/9 b2 (Springer-Verlag, Berlin, 1983).

[31] J. Gasser, H. Leutwyler and M.E. Sainto, Phys. Lett. B253 (1991) 252.

[32] Particle Data Group, L. Montanet et al., Phys. Rev. Phys. Rev. D50 (1994) 1173.

[33] T. Hatsuda and S.H. Lee, Phys. Rev. C46 (1992) R34; X. Jin and D.B. Leinweber, preprint (1995) nucl-th/9510064.
[34] G.E. Brown and M. Rho, Phys. Rev. Lett. 66 (1991) 2720.

[35] C.E. Price, J.R. Shepard and J.A. McNeil, Phys. Rev. C41 (1990) 1234; C42 (1990) 247.

[36] B. Buck and S.M. Perez, Phys. Rev. Lett. 50 (1983) 1975; M. Rho, Ann. Rev. Nucl. Sci. 34 (1984) 531.

[37] V. Thorsson and A. Wirzba, Nucl. Phys. A589 (1995) 633.

[38] J. Delorme, M. Ericson and T.E.O. Ericson, Phys. Lett. B291 (1992) 379.

[39] V.L. Eletsky, P.J. Ellis and J.I. Kapusta, Phys. Rev. D47 (1993) 4084.

[40] C. Song, Phys. Rev. C47 (1993) 2861.

[41] N. Isgur, C. Morningstar and C. Reader, Phys. Rev. D39 (1989) 1357.
Table 1

Values of the parameters and derived quantities in nuclear matter.

| Quantity                   | $G_4 = 0$ | $G_4/g_\omega = 0.19$ |
|---------------------------|-----------|----------------------|
| $|\epsilon_{\text{vac}}|^{1/4}$ (MeV) | 236       | 221                  |
| $g_\omega$                | 10.5      | 12.4                 |
| $\zeta = \phi_0/\sigma_0$ | 1.3       | 1.4                  |
| $M_{\text{sat}}/M$        | 0.70      | 0.64                 |
| $K$ (MeV)                 | 388       | 317                  |
| $S/K$                     | 6.6       | 5.4                  |
| $\sigma_0$ (MeV)          | 110       | 102                  |
| $Z_\pi$                   | 0.71      | 0.83                 |
| $f$                       | 5.97      | 4.95                 |
| $D$                       | 1.36      | 1.10                 |
| $c$                       | 0.57      | 1.30                 |
| $\kappa_6$                | −1.52     | −1.90                |
Table 2

Bulk properties of nuclei, sigma term and vacuum axial vector coupling
 constant with various explicit symmetry breaking parameters.

| Case | $m_\pi$ (MeV) | $\epsilon'_2$ | $\epsilon_3$ | $g_A$ (MeV) | $\Sigma(0)$ (MeV) | BE/\(A\) (MeV) | $r_{ch}$ (fm) | Ca BE/\(A\) (MeV) | $r_{ch}$ (fm) | Pb BE/\(A\) (MeV) | $r_{ch}$ (fm) |
|------|----------------|---------------|---------------|-------------|-------------------|----------------|-------------|----------------|---------------|----------------|---------------|
| Expt. | 1.26 | 7.98 | 2.73 | 8.55 | 3.48 | 7.86 | 5.50 |
| A | 0 | 0 | 0 | 1.33 | 0 | 7.35 | 2.64 | 7.96 | 3.41 | 7.30 | 5.49 |
| F | 138 | 0 | −15 | 1.31 | 41 | 7.40 | 2.63 | 7.99 | 3.41 | 7.34 | 5.49 |
| WA | 0 | 0 | 0 | 1.33 | 0 | 7.02 | 2.68 | 7.75 | 3.45 | 7.33 | 5.51 |
| WB | 138 | 0 | 0 | 1.33 | 71 | 6.58 | 2.72 | 7.43 | 3.47 | 7.18 | 5.52 |
| WC | 138 | $\frac{1}{2}\epsilon'_1$ | 0 | 1.33 | 101 | 7.46 | 2.65 | 8.10 | 3.42 | 7.51 | 5.51 |
| WD | 138 | 0 | 50 | 1.38 | 120 | 6.36 | 2.73 | 7.26 | 3.48 | 7.06 | 5.53 |
| WE | 138 | 0 | −50 | 1.27 | 24 | 6.43 | 2.73 | 7.31 | 3.48 | 7.13 | 5.52 |
| WF | 138 | 0 | −15 | 1.31 | 56 | 6.58 | 2.72 | 7.45 | 3.47 | 7.20 | 5.52 |
Table 3
Low energy pion scattering parameters.

|                | $A_0^{(+)}$ | $m_\pi B_0^{(+)}$ | $C_0^{(+)}$ | $D_0^{(+)}$ | $A_0^{(-)}$ | $m_\pi B_0^{(-)}$ | $C_0^{(-)}$ | $D_0^{(-)}$ |
|----------------|-------------|-------------------|-------------|-------------|-------------|-------------------|-------------|-------------|
|                | (m⁻¹)       | (m⁻¹)             | (m⁻³)       | (m⁻³)       | (m⁻¹)       | (m⁻¹)             | (m⁻³)       | (m⁻³)       |
| Expt.          | 32.3±0.4    | −32.5±0.4         | 2.88±0.02   | 2.65±0.04   | −10.7±0.4   | 12.0±0.4          | −1.76±0.02  | −1.13±0.04  |
| $\Delta$ [30] | 7.0         | −5.4              | 0.98        | 0.65        | −10.7       | 10.3              | −0.64       | −0.85       |
| Difference     | 25.3        | −27.1             | 1.89        | 2.00        | 0.1         | 1.7               | −1.11       | −0.28       |
| $\mathcal{L}_1$| 10.7        | −10.7             | 1.18        | 0.79        | −           | 0.8               | −0.39       | −0.12       |
| $\mathcal{L}_1 + \mathcal{L}_2$| 26.4       | −26.4             | 1.76        | 1.97        | −           | 0.8               | −0.97       | −0.29       |
| Case A         | 26.4        | −26.4             | 1.64        | 1.97        | −           | 1.1               | −0.97       | −0.29       |
| Case F         | 25.9        | −26.5             | 1.79        | 1.97        | −           | 1.2               | −1.09       | −0.29       |
| Case WA        | 26.4        | −26.4             | 1.87        | 1.97        | −           | 1.1               | −0.97       | −0.28       |
| Case WB        | 25.8        | −26.5             | 2.01        | 1.97        | −           | 1.1               | −1.10       | −0.29       |
| Case WC        | 26.4        | −26.5             | 1.99        | 1.97        | −           | 1.1               | −1.10       | −0.29       |
| Case WD        | 26.6        | −26.5             | 2.00        | 1.97        | −           | 1.0               | −1.10       | −0.29       |
| Case WE        | 24.9        | −26.5             | 2.06        | 1.97        | −           | 1.3               | −1.10       | −0.29       |
| Case WF        | 25.6        | −26.5             | 2.02        | 1.97        | −           | 1.2               | −1.10       | −0.29       |
Table 4

Predictions for the vector mesons with our parameters.

| Quantity                                      | $G_4 = 0$ | $G_4/g_\omega = 0.19$ | Experiment |
|-----------------------------------------------|-----------|------------------------|------------|
| $\Gamma(a_1 \to \rho\pi)$ (MeV)              | 306       | 601                    | $\sim 400$|
| $a_1 \to \rho\pi$ D/S ratio                  | $-0.25$   | $-0.17$                | $-0.14 \pm 0.03$ [11] |
| $\Gamma(a_1 \to \pi\gamma)$ (MeV)           | 1.10      | 1.19                   | 0.64 ± 0.25 |
| $\Gamma(\rho^0 \to e^+e^-)$ (keV)           | 4.82      | 7.00                   | 6.77 ± 0.32 |
| $\langle r^2 \rangle^{1/2}_\pi$ (fm)         | 0.63      | 0.69                   | 0.66 ± 0.01 |

Figure Captions

Figure 1. Nucleon loop contribution to the pion propagator.
Figure 2. Ratio of glueball and sigma fields to their vacuum values, $\chi$ and $\nu$, as a function of density. The parameters F and WF are specified in the text. Equilibrium nuclear matter density is denoted by $\rho_{\text{sat}}$.
Figure 3. Ratio of the pion renormalization $Z_\pi^*$ in matter to the vacuum value $Z_\pi$ as a function of density.
Figure 4. Ratio of the axial coupling constant $g_A^*$ in matter to the vacuum value $g_A$ as a function of density.
Figure 5. Ratio of the pion mass $m_\pi^*$ in matter to the vacuum value $m_\pi$ as a function of density. The parameter sets are specified in the text.
Figure 3
