Berry phase in a two-atom Jaynes–Cummings model with Kerr medium

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Abstract
The Jaynes–Cummings model (JCM) is an very important model for describing interaction between quantized electromagnetic fields and atoms in cavity quantum electrodynamics (QED). This model is generalized in many different directions since it predicts many novel quantum effects that can be verified by modern physics experimental technologies. In this paper, the Berry phase and entropy of the ground state for arbitrary photon number $n$ of a two-atom Jaynes–Cummings model with Kerr-like medium are investigated. It is found that there is some correspondence between their images, especially the existence of a curve in the $\Delta - \epsilon$ plane along which the energy, Berry phase and entropy all reach their special values. So it is available for detecting entanglement by applying Berry phase.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Berry phase [1] or geometric phase, which does not have classical correspondence, has become a focus point in modern physics. It describes a phase factor gained by the wavefunction after the system undergoes an adiabatic and cyclic evolution, which reflects the topological properties [2, 3] of the state space of the system and has untrivial connections with the character of the system [4], especially with the entanglement [5, 6]. Recently, the Berry phase was introduced into quantum computation to construct a universal quantum logic gate that may be robust to certain kinds of errors [7–11].

Cavity quantum electrodynamics (QED) is an important solid-state system for implementing quantum computation, and is studied extensively. In the theory of cavity QED, the Jaynes–Cummings [12] model (JCM) is recognized as the simplest and most effective model on the interaction between radiation and matter, which can be solved exactly. As an important theoretical model, JCM has led to many nontrivial predictions such as collapse–revival phenomenon [13], squeezing [14], antibunching [15, 16], chaos [17], and trapping states [18–20], etc. Furthermore, despite the simplicity of JCM, it is of great significance because recent technologies enabled scientists to experimentally realize this rather idealized model [21, 22] and to verify some of the theoretical predictions.

Stimulated by the success of the JCM, many people extend this model to explore new quantum effects. One simple way of extending is considering multiple atom and multiple mode fields instead of single atom and single mode fields [23, 24]. Another way is to consider the interactions between field and medium and fields themselves, such as a cavity filled with Kerr medium. Introducing Kerr nonlinearity into the system Hamiltonian will cause various nonlinear effects, so it attracts much attention of scientists [25–29]. One of the many applications of these nonlinear effects is to produce entangled states [30], which is extensively applied in quantum information, especially in quantum communication.

In this paper, we try to investigate a two-atom JCM in Kerr medium. First, we calculate the eigenvalues and eigenstates of the system. Then we evaluate the Berry phase of the ground state for arbitrary photon number $n$ in terms of the introduction of the phase shift operator, and for
comparing the phase with the entanglement we compute the von Neumann entropy as a measurement of entanglement. After these tedious computations, we compare the ground state energy, Berry phase and entropy, and find that there are tight connections between them.

2. Hamiltonian and ground state energy

The Hamiltonian of the system in the rotating wave approximation can be written as (assuming $\hbar = 1$)

$$H = \omega_0 a^\dagger a + \frac{\omega_0}{2} \sum_{j=1}^2 \sigma_j^\dagger + \varepsilon \sum_{j=1}^2 (a \sigma_j^\dagger + a^\dagger \sigma_j)$$

$$+ \chi (a^\dagger a a^\dagger a),$$

where $a^\dagger$ and $a$ denote the creation and annihilation operators of the single mode field, $\omega_0$ is the transition frequency of the field, $\omega_0$ is the atomic transition frequency, $\varepsilon$ is the coupling constant between these two atoms and field, $\chi$ represents the coupling of the fields induced by the Kerr medium. $\sigma_j = |e_j\rangle\langle g_j| - |g_j\rangle\langle e_j|$, $\sigma_j^\dagger = |e_j\rangle\langle e_j|$ and $|g_j\rangle$, with $|e_j\rangle$ and $|g_j\rangle$ being the excited and ground states of the $j$th atom, $j = 1, 2$. By the way, there exists a conserved quantity $K$ for the above Hamiltonian, which is

$$K = a^\dagger a + \frac{\sigma_1^\dagger + \sigma_2^\dagger}{2}.$$  

The basis of the subspace $(K = n + 2)$ is

$$|n, e_1, e_2\rangle, \quad |n+1, e_1, e_2\rangle, \quad |n+1, g_1, e_2\rangle, \quad |n+2, g_1, g_2\rangle.$$  

And in this basis, the Hamiltonian is written as (in an appropriate interaction picture)

$$H = \begin{pmatrix}
\Delta - \chi(2n + 2) & \varepsilon \sqrt{n+1} & \varepsilon \sqrt{n+1} & 0 \\
\varepsilon \sqrt{n+1} & -\chi & 0 & \varepsilon \sqrt{n+2} \\
-\chi & 0 & \varepsilon \sqrt{n+2} & \Delta - \chi(2n+2) \\
0 & \varepsilon \sqrt{n+2} & \varepsilon \sqrt{n+2} & \Delta - \chi(2n+2)
\end{pmatrix},$$

where $\Delta = \omega_0 - \omega_0$ is the detuning of the cavity field. The four eigenvalues $\lambda_j$ ($j = 1, 2, 3, 4$) and corresponding eigenstates $|\psi_j\rangle$ have been calculated analytically. However, it is useless to present their complicated formulae here, but $|\psi_j\rangle$ can be written simply as follows:

$$|\psi_j\rangle = c_j^1 |n, e_1, e_2\rangle + c_j^2 |n+1, e_1, e_2\rangle + c_j^3 |n+1, g_1, e_2\rangle + c_j^4 |n+2, g_1, g_2\rangle.$$  

When $n = 0$, the ground state energy, i.e. the lowest eigenvalue of the Hamiltonian for $n = 0$, as a function of detuning $\Delta$ and coupling constant $\varepsilon$, is shown in figure 1. We find in the figure that when $\varepsilon$ approaches 0 there exist two discontinuity points in the derivative of the energy, and the image of the function is symmetric to the line $\Delta = 2$ to some extent. As we will see, these two points will be singularities of Berry phase, and the symmetry of the energy function will also be inherited by the Berry phase.

3. Berry phase and entropy

Obviously, the whole system is quantized; to study the geometric properties of this system we resort to the method of [31] to evaluate the Berry phase of the system by introducing a phase shift operator:

$$R(t) = e^{-i\psi(t)a^\dagger a},$$

where $\psi(t)$ is changed from 0 to $2\pi$ adiabatically. Then the time independent eigenequation of the system: $H|\psi_j\rangle = \lambda_j |\psi_j\rangle$ is changed into $H'|\psi_j\rangle = \lambda_j' |\psi_j\rangle$, with $H' = R(t)HR(t) - iR(t)dR(t)/dt$ and $|\psi_j\rangle = R(t)|\psi_j\rangle$. Hence, the Berry phase can be evaluated according to the standard method as follows:

$$\gamma_j = \int_0^{2\pi} d\phi \left( \psi_j \frac{d}{d\phi} \psi_j \right) = \int_0^{2\pi} d\phi \left( \psi_j \left| R(t) \frac{d}{dt} R(t)^\dagger \right| \psi_j \right).$$

For our model, the Berry phase is given as

$$\gamma_j = 2\pi |c_j^1|^2 + (n+1) \left( |c_j^2|^2 + |c_j^3|^2 + (n+2) |c_j^4|^2 \right).$$

(7)

Apparently, $c_j^i$ ($i, j = 1, 2, 3, 4$) are functions of detuning $\Delta$ and coupling constant $\varepsilon$. So the Berry phase of the ground state can be controlled by $\Delta$ and $\varepsilon$. Figure 2 shows its image in the case $n = 0$.

Just as we have mentioned before, there are two singularities when $\varepsilon$ approaches 0 for the Berry phase, and
the image is centrosymmetric to some extent against the intersection curve of the Berry phase image and $2\pi$ plane where Berry phase is identically equal to $2\pi$, which is adjacent to the plane $\Delta = 2$. This result is similar to that of [32]. In the article [32], the authors calculate the Berry phase of the ground state of the Tavis–Cummings model, and it is also found that there is correspondence between the singularities of Berry phase and ground state energy as well as symmetry.

According to our computations, we present the figure of entropy as a function of detuning $\Delta$ and coupling constant $\epsilon$ when $n = 0$ in figure 3. Apparently, as the figures show, there are the same two points and symmetry corresponds to that of the Berry phase and energy.

To compare the Berry phase with the entanglement of the system, we consider the density operator of the system. Generally, when the system is in a pure state $|\psi\rangle$, its density operator $\rho_{\psi\psi} = |\psi\rangle \langle \psi|$.

According to our computations, we present the figure of entropy as a function of detuning $\Delta$ for each different value of $\epsilon$ and $\rho_{\psi\psi}$ is the reduced density operator of $\rho_{\psi\psi}$, and $\rho_{\psi\psi}$ represents the density operator of the system.

According to the image and our calculations, we find that for each different value of $\epsilon$, there exists a maximum value for the energy of the ground state when $\Delta$ satisfies the following equation:

$$\Delta = \frac{1}{2} + \sqrt{\frac{1}{2}} \sqrt{17 - 12\sqrt{2} + (12 - 8\sqrt{2})\epsilon^2(\epsilon \neq 0)}. \quad (9)$$

To our surprise, at these points where $\Delta$ and $\epsilon$ satisfy the above equation, the Berry phase is $2\pi$ and the entropy of the system reaches its relative minimum value when $\epsilon$ is near 0. The equation determines a curve in the $\Delta-\epsilon$ plane and because this curve reflects the main character of the ground state, we call it the characteristic curve of the ground state. The existence of the characteristic curves proves the tight connections between energy, Berry phase and entanglement.

We also considered the case $n \neq 0$, and find that the images of ground state energy, Berry phase and entropy versus $\Delta$ and $\epsilon$ are similar to the case $n = 0$, such as the symmetry against a line ($\Delta = 2n + 2$) to some extent, and the correspondence of singularities. To illustrate this, we represent the images of ground state energy, Berry phase and entropy when $n = 40$ in parts (b) of figures 1–3 respectively. Obviously, the main difference between them is the movement of the characteristic curve in the $\Delta-\epsilon$ plane and its equation reads ($\epsilon \neq 0$):

$$\Delta = \frac{1}{2}(2n + 1) + A$$

$$-\frac{1}{2}\sqrt{1 - 4(2n + 3)A + 8A^2 + (12 + 8n - 8A)\epsilon^2},$$

$$A = \sqrt{n^2 + 3n + 2}. \quad (10)$$

Figure 4 shows the characteristic curves for different values of $n$. We think this result may be due to the fact that the Berry phase and entropy are all functions of the ground state energy.

4. Conclusion

In conclusion, we calculated the Berry phase and entropy of a two-atom Jaynes–Cummings model with Kerr medium, and found that there are correspondences between their singularities and symmetry. Especially, there exists a class of curves in the $\Delta-\epsilon$ plane, along which the Berry phase and entropy all reach their special values like $2\pi$ for Berry phase. These results reflect the tight relations between the Berry phase and entanglement of the system, and may be caused by the fact that they are all functions of energy. Some physicists are trying to measure entanglement using Berry phase, and our results may be useful to them.

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