Cosmological constant as a finite temperature effect

I. Y. Park

†Department of Applied Mathematics, Philander Smith College
Little Rock, AR 72223, USA
inyongpark05@gmail.com

Abstract

We reexamine the cosmological constant problem in a finite temperature setup and propose an intriguing possibility of carrying out perturbative analysis by employing a renormalization scheme in which the renormalized Higgs mass (or resummed mass, to be more precise) taken to be on the order of the CMB temperature. Finite-temperature-induced complexification of the effective potential is observed and its interpretation is given. It is shown that the cosmological constant problem is avoided. A peculiarity of the perturbation series in the new scheme is noted.

keywords: cosmological constant problem, finite temperature, quantization of gravity, bare and renormalized perturbation theories
1 Introduction

The cosmological constant (CC) problem [1] (see, e.g., [2–5] for reviews\(^1\)) arises from the loop effects of Standard Model (SM) particles, such as the Higgs particle. Since the electroweak scale is much higher than, say, the temperature of the cosmic microwave background (CMB), it has been a standard practice to apply the zero-temperature setup to formulate and tackle the problem. However, since the CC, as vacuum energy, is an infrared effect (in the sense explained below) and thus governed by the low energy sector, the infrared structure of the theory must be important and its meticulous description is desirable. The fact that the CC is a vacuum energy also implies that quantization and renormalization of gravity must be involved in its systematic treatment. In particular, the resolution of the problem would require renormalization of the vacuum energy. In this work we show that the finite temperature effects allow, when properly taken into account in the quantum gravitational setup, one to avoid the CC fine-tuning problem.

The vacuum energy is defined as a minimum of the effective potential. In vacuum energy computation both the ultraviolet (UV) and infrared (IR) structures play roles. To some extent the UV and IR contributions to the vacuum energy are intertwined. The relevance of the UV structure is evident since renormalization procedure is involved. The relevance of the infrared structure is subtler. It is well demonstrated in Casimir energy analysis (see, e.g., the account in [13]). The point is that for proper evaluation of vacuum energy it is necessary to employ an infrared regulator, such as a finite-size box in the Casimir case in momentum cutoff regularization. In the body we employ dimensional regularization with additional finite renormalization, and pay a close attention to the infrared structure introduced by the presence of the temperature.

Are there indications that the finite temperature effects may a priori be important for CC analysis, despite the fact that the electroweak scale is much higher than the CMB temperature? First of all, it should definitely be possible, and is natural, to obtain zero-temperature results as a vanishing-temperature limit of the corresponding finite-temperature results. A hint of

\(^1\)The inspiring review [5] is largely based on the previous works by the author and his collaborators (see, e.g., [6–8]) that were further developed in [9–11]. One of the main themes of these works is the so-called running vacuum. These works are further commented on in the conclusion and [12].
an indication comes from energy scalings in zero- and finite temperature loop analyses. In zero temperature, a loop analysis typically yields logarithmic factors such as \( \ln \frac{m}{\mu} \), where \( m \) is the renormalized mass of the field and \( \mu \) the renormalization scale. For the benefit of convergence, it is necessary to choose \( \mu \sim m \). (Or one may consider renormalization group equations to conduct certain resummations.) By the same token it will be desirable to take \( \mu \sim m \sim T \) once the temperature enters. The question is whether there is an additional, more quantitative rationale for enforcing the scaling. The answer is affirmative: in the present work this scaling is achieved in the course of improving the perturbative analysis by optimal perturbation theory (OPT)\(^2\) [17] as well as standard thermal resummation: we show that there is an OPT procedure that enforces the scaling.

In the body we reformulate the CC problem as a zero-temperature limit of the finite temperature counterpart. A potential obstruction to any finite-temperature perturbative analysis is the well-known infrared problem. (Reviews can be found in [18–22].) For a high temperature, say, that of the QCD era, a well-known example is the ‘Linde problem’ [23], which has been an active topic of research; see [24] [25] and references therein. An effective field theory approach combined with lattice computation was proposed to deal with the problem [26]. The focus of the present work is a low temperature, the temperature of CMB. (Nevertheless, some of the textbook results obtained for a high temperature can be borrowed for reasons to be explained.) We show that the finite-temperature effects are in fact crucial: in particular, once the convergence property of the perturbation series is improved through a variant OPT, the optimized renormalized mass of the Higgs turns out to be essentially the temperature,\(^3\) thus removing the root of the CC fine-tuning problem.

In essence, what is being proposed is an intriguing possibility of carrying out perturbative analysis by employing - with proper justifications, namely, the scaling and our variant OPT - a renormalization scheme in which the renormalized Higgs mass is taken to be on the order of the CMB temper-

\(^2\)OPT is based on the variational principle. Other thermal physics techniques based on the variational principle include screened perturbation theory [14] [15] and 2PI formalism [16].

\(^3\)It has been revealed in the recent works of [27–30] that quantum corrections can qualitatively change the classical solution. In the present work we see a similar novelty: the finite temperature non-perturbative effects dictate, small temperatures notwithstanding, the renormalized mass.
ature. In a general renormalization scheme, there is hardly anything that stops one from adopting a renormalized mass of an arbitrary value - other than the question of why one would make such a choice. Variant OPT is introduced to address this question. Also, our point of contention is that such a choice accommodates a wider range of physics and at the same time leads to interesting predictions in cosmology. More on this in section 2 and the conclusion.

A cautionary remark is in order. The issue is that of renormalization schemes. There are two widely-used renormalization schemes: onshell and “offshell” scheme, such as $\overline{\text{MS}}$. The former was predominantly used in the past whereas more recent literature employs the latter. (Among textbooks, the former is used in [31]; many other textbooks, such as [32] [33] [13], employ the latter.) In the onshell scheme, the mass appearing in the Lagrangian is taken to be the physical mass whose more general definition is the pole of the 2-point Green’s function. In contrast, the mass in the $\overline{\text{MS}}$ scheme (or any scheme of renormalized perturbation theory; more generally) is a renormalized mass. The renormalized mass is initially viewed arbitrary: it gets determined at the end by requiring the pole of the Green’s function to match with the physical value, 125 GeV in the case of Higgs field. We take the renormalized mass (or the resummed mass, more precisely; see the body) of the Higgs to be on the order of the temperature, with the value of the physical mass intact as 125 GeV. We do so by making use of the freedom associated with choice of the value of the renormalized mass in renormalized perturbation theory. While the scaling argument above is a qualitative justification for the temperature-order renormalized mass, our OPT procedure provides a quantitative justification.

With the renormalized mass determined, the following task still remains: the zero-temperature theories, such as the zero-temperature Standard Model, have been quite successful. There, the renormalized masses turn out to be close to the pole masses, usually within a few percent. In the case of the SM Higgs field, for instance, the renormalized mass is close to 125 GeV, the pole mass value. If one now wants to take the renormalized mass to be around the CMB temperature, which is much smaller than the pole mass, one must yet maintain compatibility with the zero-temperature analysis: the resulting perturbation theory should preserve the success of the original zero-

\footnote{The perturbative analysis employing an “offshell” scheme often goes under the name of renormalized perturbation theory.}
temperature theory. This is the main task that we undertake in section 2; briefly, what underpins the compatibility at a fundamental level is the fact that the theory is defined by the bare Lagrangian. In section 3, after examining vast freedom in choosing renormalization conditions, we invoke renormalization group invariance of physical quantities to affirm this. We note that a certain resummation is behind the invariance. We also note that the small renormalized mass introduces a peculiarity in the renormalized perturbative series worth examining further.

To be specific, the cosmological constant problem is examined by taking an Einstein-scalar with a Higgs-type potential. A variant optimal perturbation theory is implemented in the recently proposed quantum-gravitational framework. The optimized renormalized mass, i.e., the renormalized mass determined by the variant optimal perturbation theory, of the scalar field turns out to be on the order of the temperature. Since the CMB temperature is on the same order of magnitude as the measured value of the cosmological constant in today’s Universe, the temperature-order CC implies that the cosmological constant problem is avoided. More rigorously, the temperature-order CC shifts the cosmological constant problem to compatibility of the consequent perturbative analysis. The compatibility is guaranteed essentially by invariance of physical quantities under renormalization scheme change. (Several explicit checks are suggested in the conclusion.) We point out the resummation behind the invariance.

In our OPT finite-temperature induced complexification of the effective potential is observed. In other words this effect is introduced through our variant OPT. It is thus additional to the well-known presence of the imaginary part in the zero-temperature effective potential. In particular, the vacuum expectation value of the scalar field itself becomes complex. We interpret the complexity to be instability - associated with the finite temperature - toward zero temperature.

With the renormalized mass being of the order of the temperature, the CC problem is avoided: the CMB temperature in terms of eV is $6.6 \times 10^{-4}$ eV. Approximating this as $10^{-12}$ GeV, the vacuum energy contribution associated with the thermal mass of a Higgs-type field is $\sim 10^{-48}$ GeV$^4$. This is on the same order as the observed CC value $\sim 10^{-48}$ GeV$^4$.

The rest of the paper is organized as follows. Invariance of physical quantities under renormalization scheme change (as well as renormalization group
flow) originates from the fact that the theory is defined by the bare Lagrangian. The invariance will play a central role in the proposed resolution of the CC problem. In section 2 we explore the freedom in adopting a renormalization scheme and do so for a zero-T setup to avoid inessential complications. We take a massive scalar theory with a quartic potential and discuss the perspectives of both bare and renormalized perturbation theories. In section 3 we carry out one-loop computation of the effective potential in a finite temperature setup. We do this for a Higgs-type scalar in a flat spacetime first, and subsequently extend the analysis to the graviton sector. By employing an OPT-induced renormalization scheme, we obtain the main result of the one-loop effective potential given in (30). In section 4 we end with summary, comments on potential implications of the present result for cosmology, and future directions.

2 Freedom in renormalization scheme

The renormalization scheme that we are employing in the main analysis is such that the Higgs renormalized mass is taken to be far smaller than the physical (i.e., pole) mass. This is not necessarily a problem (and with the help of bare perturbation theory one will see that it indeed is not). It will nevertheless be useful to examine the potential consistency issue with the new scheme.

The gist of the conventional analysis is as follows. After UV regularization one subtracts out the infinite part and fix the finite part of the vacuum energy. In \( \overline{\text{MS}} \) scheme one removes essentially the \( \frac{1}{\epsilon} \) part, and this fixes the finite part. At this point the renormalized mass is still undetermined: it is determined by matching the pole expression of the 2-point function with the physical value of the field, 125 GeV for a Higgs field. In section 3, we put forth an OPT that leads to the renormalized mass of the order of the temperature. With this one is to conduct the consequent perturbative analysis. In the proposed new scheme it is the value of the renormalized mass, instead of the finite part, that is fixed first, through our OPT, to be on the order of the temperature. Matching with the physical mass determines the finite part. This may raise a question on consistency of the resulting perturbative analysis, since the new renormalized mass is much smaller than...
the actual physical mass. Although in nature the modification amounts to finite renormalization and thus must not affect the physics, it will be useful to take a close look at how the perturbative analysis modifies in the new scheme.

The best formal framework for assuring the consistency is bare perturbation theory. However, it will be useful to have a qualitative discussion of the reasoning behind the new scheme and its potential ramifications before quantitatively proving the consistency of the new scheme. Let us play with informal and intuitive modifications of the standard $\overline{\text{MS}}$ scheme of renormalized perturbation theory. We mainly consider a zero-T setup, since the issue at hand is already present at zero temperature. In the standard renormalized perturbation theory, say, with dimensional regularization and $\overline{\text{MS}}$ scheme, the finite part is pre-fixed and then the renormalized mass is determined at the end by matching the pole expression with the physical value of, say, 125 GeV. The renormalized mass turns out to be within a few percent of 125 GeV. For the sake of argument, let us suppose that the renormalized mass is determined to be 120 GeV. The precise value doesn’t matter for the point to be made. Let us conduct the following ‘Gedankenexperiment’ with the new scheme, i.e., the scheme in which the renormalized mass is fixed prior to the finite part denoted by $c_m$. (One of the purposes of the Gedankenexperiment is to show without the technicalities of bare perturbation theory that the new scheme points toward interesting predictions in the case of a finite-T setup; see below). One may take the renormalized mass $m$, say, $m = 110$ GeV in the new scheme, and repeat the renormalization analysis. Since the difference between 120 GeV and 110 GeV is minor, the perturbation series must remain valid and the finite constant $c_m$ will turn out to be close to the value prefixed in the standard approach. (Since $c_m$ is zero in $\overline{\text{MS}}$ scheme, its value in the new scheme will be small.) Now one keeps lowering the value of the renormalized mass $m$. One can be certain that the series will definitely be valid at least for a certain range of the renormalized mass values. The real question is whether or not one can modify the renormalized mass to a degree required to solve the CC problem. By employing bare perturbation theory we will see below that this is obviously the case. One can conduct a similar Gedankenexperiment involving a finite-T setup. Here one can play a

\footnote{We stress that if one’s theory is such that the renormalized mass happens to turn out to be much smaller, unlike in the SM Higgs sector, than the physical value, the new scheme would hardly require any justification.}
similar game with two parameters (or three, including $c_m$), the renormalized mass and temperature. Let us consider the temperature to be around that of the electroweak (EW) era. The scheme leads to potentially very interesting predictions: the CC will turn out to be determined by the temperature, just the way it is determined in section 3. Then since the temperature should be time-dependent (checks are necessary but the time-dependence of the temperature in the expanding Universe should be generic), the result will lead to time-dependence of the CC.$^6$ More generally, one will have cosmology substantially different [34] from the standard one.

Let us turn to bare perturbation theory and assure that the key to preserving the success of the zero-temperature lies in vast freedom in choosing subtraction schemes and/or renormalization conditions. The system that we employ for this task is a real scalar theory with a quartic potential:

$$S = -\int d^4x \left[ \frac{1}{2} \partial_\mu \zeta \partial^\mu \zeta + \frac{1}{2} m^2 \zeta^2 \right] - \int d^4x \frac{\lambda}{4!} \zeta^4. \quad (1)$$

Invariance of the physical quantities under renormalization scheme change is based on the following well-known relation

$$\mathcal{L}_R(m, g_\mu, c_m) = \mathcal{L}_c(m_0, g_0) \quad (2)$$

where $\mathcal{L}_R$ ($\mathcal{L}_c$) denotes renormalized (classical) Lagrangian. The theory is defined by the bare Lagrangian, i.e., the Lagrangian on the right-hand side. Let us again focus on the mass sector. Although one introduces $m$ and $c_m$, it is only a convenient “splitting” of a single parameter $m_0$. One can obtain the pole mass expression to the desired order in bare perturbation theory. For example, let us consider

$$G_0(k, m_0) = \frac{1}{k^2 + m_0^2 - \Sigma(k, m_0)} \quad (3)$$

where $\Sigma(k, m_0)$ is a series in $\hbar$. The series is convergent; it is convergent irrespective of the aforementioned splitting of $m_0.$$^7$

$^6$Section 4.2 of [35] demonstrated the time-dependence of the CC in the zero-T setup.

$^7$In general, a perturbation series in quantum field theory is an asymptotic one. However, the point to be stressed here is that whether the series is asymptotic or genuinely convergent does not depend on one’s renormalization scheme, e.g., how one splits $m_0$: if the series under consideration is convergent for one renormalization scheme it will be so for other renormalization schemes as well.
To be entirely heuristic, let us consider the self-energy at one-loop; one finds
\[ \Sigma^{(1)} = -\frac{\lambda}{32\pi^2} m_0^2 \left( \frac{m_0^2}{4\pi\mu^2} \right)^{-\varepsilon} \Gamma(1 - D/2). \] (4)
where \( \varepsilon \equiv \frac{4-D}{2} \) and \( D \) denotes the spacetime dimension. To remove the divergence at this point, one can split \( m_0 \) into \( m \) and the finite part \( c_m \) according to one’s choice of renormalization scheme. One can compute the series to a desired loop-order and subsequently split \( m_0 \). As this example clearly demonstrates, the splitting can be (quite arbitrarily) done after obtaining the series to a desired order without having to worry about the convergence. How the splitting is done is part of one’s renormalization scheme, which amounts to redistributions of various values in the convergence series, i.e., certain resummation.

Considering higher loops, the corresponding set of finite constants \( c^{(n)} \) where \( n \) denotes the loop order can be introduced as a matter of principle. Of course, it would be pleasant if things could work with a ‘minimal’ set of \( c^{(n)} \), and this is what happens, e.g., in the SM analysis with the usual renormalization schemes. As we further comment below, one of the lessons of the present work is that one can cover a wider range of physics with a possibly slightly extended set of \( c^{(n)} \) that are not prefixed.

Although the bare perturbation theory component above assures the validity of our renormalization scheme, renormalized perturbation theory has its advantages in practical computations, thus it will be useful to examine the convergence issue in that framework. We do this by considering two-loop renormalization of the propagator. The two-point proper vertex is defined as
\[ \Gamma^{[2]} = k^2 + m^2 - \Sigma(k, m, c_m) \] (5)
where Σ again denotes self-energy. At one-loop, Σ can be computed by considering the diagrams in Fig. 1: the one-loop two-point divergence introduces the counter-term:

\[-\frac{m^2}{4} \frac{\lambda}{(4\pi)^2} \left( \frac{1}{\varepsilon} + c_m \right) \zeta^2\]

where \(\varepsilon \equiv \frac{4-D}{2}\) as before and \(D\) denotes the spacetime dimension; \(c_m\) is a constant to be determined by one’s subtraction scheme. For instance, the modified minimal subtraction (MS) corresponds to setting \(c_m = 0\). Fixing the renormalized mass according to the OPT principle of minimal sensitivity (PMS) (as we will in section 3) is in contrast to the usual practice in zero-temperature: there, one fixes \(c_m\) by one’s subtraction scheme, and then the renormalized mass by the pole mass condition,

\[k^2 + m^2 - \Sigma(k) \bigg|_{k^2 = -m_P^2} = 0\]

where \(m_P\) denotes the physical pole mass. In the new scheme, it is the coefficient \(c_m\) that is determined by the pole mass condition, while the renormalized mass is prefixed (by variant OPT in section 3).

As stated in the introduction, the advantage of the new scheme is obvious: it realizes the scaling mentioned in the introduction and thereby allows one to avoid the CC fine-tuning problem. To see the disadvantage, let us note that the pole mass condition in the new scheme yields

\[c_m m^2 \sim m_P^2 - m^2\]

and thus implies a larger value of \(c_m\), compared with the standard approach where the pole mass condition typically leads to \(m^2 \simeq m_P^2\). Note, however, that it is the combination \(m^2 c_m \sim m_P^2\), but not \(c_m\) alone, that appears in the counter-term (6). In general, for mass-related quantities it will not be possible at tree-level to achieve suitable agreement with experimental values, since the renormalized mass is pre-fixed: it will be necessary to go to one-loop where one has the freedom of adjusting the finite parts. The two-loop-relevant diagrams are given in Fig. 2. The circle in the last diagram in Fig. 2 represents the counter-term for the one-loop four-point diagram (not explicitly shown). Adding all up, the total two-loop self-energy \(\Sigma^{(2)}\) is

\[\Sigma^{(2)} = m^2 \lambda^2 \mu^{4\varepsilon} \left[ \frac{1}{2\varepsilon^2} + \frac{1}{4\varepsilon} (-1 + c_m + 3c_\lambda) - \frac{p^2}{24m^2\varepsilon} + \cdots \right]\]
where $c_\lambda^8$ denotes the finite part of the counter-term of the one-loop four-point amplitude and $p$ the momentum entering through one end of the diagrams. The $\varepsilon$-pole terms will have to be removed by two-loop counter-terms. Let us focus on the finite parts. The constant $c_m - m^2 c_m$, more precisely - will also appear in the finite parts represented by the ellipses.\footnote{The new scheme concerns only the renormalized mass: for the other quantities, such as the coupling constants, one can just employ the standard schemes.} By imposing the pole mass condition and solving it, say, interactively for $c_m$, it will be possible to determine its two-loop correction. Since $\Sigma^{(2)}$ has an additional $\lambda$ and $\hbar$ compared with one-loop, the presence of $m^2 c_m$ cannot disturb the series in any significant way. (It is bare perturbation theory that ultimately confirms this.) After all, the new perturbation can be viewed as finite renormalization. We will point out later that a certain resummation is behind this finite renormalization.

To be thorough, let us take the counter-term, $\sim c_m^{(1)} m^2 \zeta^2$ and examine its feedback to the CC in renormalized perturbation theory to see how things are organized in this sector. (Note that we have now denoted $c_m$ by $c_m^{(1)}$ to distinguish the two-loop finite parts from those at one-loop.) The counter-term $c_m^{(1)} m^2 \zeta^2$ will yield, upon self-contracting the $\zeta$'s, a contribution to the CC at two-loop. Such a contribution does not cause any harm to the two-loop analysis in [12], as we now show. The contraction, being divergent, necessitates the corresponding CC renormalization at two-loop by introducing a CC counter-term $\delta \Lambda$. The renormalization procedure introduces $c_\Lambda^{(2)}$. It is

\footnote{Since the issue under consideration belongs to the mass, one may well set $c_\lambda = 0$ in the spirit of the $\overline{\text{MS}}$. Similarly, the two-loop analogues of $c_m, c_\lambda$ (and the finite part associated with the wavefunction renormalization) may be set to zero. In other words, too much freedom in choosing the finite parts can be a burden: the freedom remaining after determining $c_m$ can be fixed just as in a convenient subtraction scheme, such as the $\overline{\text{MS}}$, to facilitate ‘next’ steps, e.g., solving the renormalization group equations.}
natural to take $c_A^{(2)}$ to be small. Needless to say that an additional factor of $\hbar$ dampens the two-loop result. One must choose its value in a manner consistent with the one-loop result and known CC value. The presence of the counter-term implies that it will be useful to carry out the resummation analogous to the one invoked when treating the mass term of a massive theory as a vertex. As well known, one gets the usual massive propagator after resumming the mass vertex corrections to the massless propagator. The resummation needed for the present case will of course be a finite-T analogue.

3 Variant OPT-induced renormalized mass

The crux of the variant optimal perturbation theory (OPT) can be captured by considering a scalar system in a flat spacetime. Since the UV divergences originate locally from a short distance, they are insensitive to global geometry. For this reason, the zero-temperature UV regularization can be employed in a finite-temperature theory. As for quantities depending on the infrared structure, the prime example of which is vacuum energy, one must consider in principle the actual background. The difference between using the curved background and the flat one lies in the finite parts. (However, the finite parts are adjusted by the renormalization conditions anyway; we refer to [36] for further discussion.)

In thermal field theory, convergence of perturbative analysis is improved by resummation. The convergence can be further enhanced with a touch of non-perturbative techniques, such as OPT. The OPT implemented in this work is a relatively minor, but nonetheless crucial, variation of the widely-studied one. In the widely-used OPT, an artificial mass term is subtracted out after adding. This is one way of ensuring artificial-mass independence of the full closed results. In our case, the renormalized mass itself serves as the OPT parameter to be fixed by the OPT principle of minimal sensitivity. As well known in the context of the variational principle in quantum mechanics, there is no unique way of implementing the principle. For instance, the more variational parameters one introduces, the more accurate the approximation generally becomes. Our OPT is one that has an advantage of achieving the goal of avoiding the CC problem through improved infrared convergence.
The gravity-scalar system that we consider is

\[
S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \, R - \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta + V(\zeta) \right)
\]

(10)

where \( \kappa^2 = 16\pi G \) with \( G \) being Newton’s constant. The potential \( V(\zeta) \) is

\[
V(\zeta) = \frac{\lambda}{4!} \left( \zeta^2 + \frac{6}{\lambda} \nu^2 \right)^2.
\]

(11)

Note the notation change as compared with section 2: the mass has been denoted by \( \nu \). One conceptual hurdle is the justification of the complete-square form of the potential instead of the usual \( V = \frac{1}{2} \nu^2 \zeta^2 + \frac{1}{4!} \lambda \zeta^4 \). The value of CC depends, of course, on whether one uses the complete-square form or the form without the constant piece. A shift of potential by a constant is immaterial in flat spacetime quantum field theory. The same is not true, however, in the quantum gravitational context. Whether one should use the complete-square form or the more usual form is not part of the CC problem. A closely related question, whose answer is not currently known [31], is why the minimum value of the classical Higgs potential should be taken to be zero. It is an independent problem that must ultimately be answered experimentally. Our goal here is to show that in the setup dictated largely by convergence of thermal perturbation theory, the fine tuning-problem is not present; this goal can be achieved more conveniently with the complete-square form.

To set the stage for the refined BFM [35–38], we shift the fields as

\[
g_{\mu\nu} \rightarrow h_{\mu\nu} + \bar{g}_{\mu\nu} \quad \zeta \rightarrow \hat{\zeta} + \bar{\zeta}
\]

(12)

with

\[
\bar{g}_{\mu\nu} \equiv g_{c\mu\nu} + \varphi_{\mu\nu} \quad \bar{\zeta} \equiv \zeta_c + \xi
\]

(13)

where \( g_{c\mu\nu}, \zeta_c \) denote the classical solutions, \( \varphi_{\mu\nu}, \xi \) the background fields, and \( h_{\mu\nu}, \hat{\zeta} \) the fluctuation fields. The zero-temperature loop analysis is based on the following two-point functions (see [38] for the conventions). For the metric,

\[
<h_{\mu\nu}(x_1)h_{\rho\sigma}(x_2)> = \tilde{P}_{\mu\nu\rho\sigma} \tilde{\Delta}(x_1 - x_2)
\]

(14)

where the tensor \( \tilde{P}_{\mu\nu\rho\sigma} \) is given by

\[
\tilde{P}_{\mu\nu\rho\sigma} \equiv \frac{\tilde{\kappa}^2}{2} \left( \bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} + \bar{g}_{\mu\sigma} \bar{g}_{\nu\rho} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{g}_{\rho\sigma} \right);
\]

(15)
where $\kappa^2 \equiv 2\kappa^2$ and satisfies
\[
\tilde{P}_{\mu\nu\kappa_1\kappa_2} \tilde{P}^{\kappa_1\kappa_2}_{\rho\sigma} = \kappa^2 \tilde{P}_{\mu\nu\rho\sigma}.
\] (16)

$\tilde{\Delta}(x_1 - x_2)$ is Green’s function for a (massless) scalar theory in the background metric $\tilde{g}_{\mu\nu}$:

\[
< \tilde{\zeta}(x_1) \tilde{\zeta}(x_2) > = \tilde{\Delta}(x_1 - x_2).
\] (17)

The explicit form of $\tilde{\Delta}$ for a massive scalar theory will be given and utilized in section 3.2 where the curved space analysis of the matter-involving sector is conducted. With these the finite temperature can be computed in the standard way.

In passing, the following can be said for breaking of gauge invariance in a finite temperature setup. In gravity theory, introducing a finite temperature should be viewed as part of specifying the background. Once temperature enters, the diffeomorphism is lost in a manner that can be dubbed as generalized spontaneous symmetry breaking. The roles played by this concept of generalized spontaneous symmetry breaking can be found, e.g., in [39] [40] [41].

### 3.1 Flat spacetime analysis

As stated in the beginning, the crux of our OPT is captured by considering a scalar system in a flat spacetime. We employ the $\overline{\text{MS}}$ subtraction scheme in the present section. We will come back to the deviation from the $\overline{\text{MS}}$ scheme in section 3.3.

The starting point of the OPT-improved thermal resummation can be taken as the following renormalized action

\[
S(\zeta) = -\int d^4x \frac{1}{2} \partial_\mu \zeta \partial^\mu \zeta - \int d^4x \left( \frac{1}{2} M^2 \zeta^2 + \frac{\lambda}{4!} \zeta^4 \right) - \int d^4x \frac{3\nu^4}{2\lambda}
\] (18)

with

\[
M^2(T) \equiv \nu^2 + \frac{\lambda}{24} T^2
\] (19)

We take the expression $M^2(T) \equiv \nu^2 + \frac{\lambda}{24} T^2$ for the purpose of computing the loop contributions, namely, to take the thermal resummation into account.
Later, we use $M^2 \equiv \nu^2$ for the classical part of the one-loop effective potential eq. (25).\footnote{One may consider using the expression $M^2(T) \equiv \nu^2 + \frac{\lambda}{24} T^2$ even for the classical part of the potential in eq. (25). This would amount to finite renormalization of the mass term. This possibility will be further examined in [12].} Shift the field
\begin{equation}
\zeta \to \hat{\zeta} + \tilde{\zeta}
\end{equation}
where $\hat{\zeta}, \tilde{\zeta}$ denote the fluctuation field and background field, respectively. Since we are interested in the potential as opposed to the action, the background field $\tilde{\zeta}$ can be treated as a constant. Then the potential can be effectively computed by considering the field-dependent mass term
\begin{equation}
M^2(T) \to \tilde{M}^2(T, \tilde{\zeta}) = \nu^2 + \lambda \frac{T^2}{2} \tilde{\zeta}^2
\end{equation}
and integrating out the fluctuation field $\hat{\zeta}$. A remark is in order before proceeding to determination of the OPT-induced renormalized mass. We stated earlier that although we consider a CMB-order temperature, the high-energy expansion can be utilized. The temperature being high or low is relative to the mass and we will show below that our OPT implies $\tilde{M} \sim \hbar^{1/2} T$ ($\hbar$ will be kept implicit). One can now tell why the high-energy expansion is justified: since the auxiliary mass $\tilde{M}$ satisfies $\tilde{M}/T \sim \hbar^{1/2}$, the intermediate analysis corresponds to that of high temperature: $\frac{\tilde{M}}{T} << 1$.

The two-loop calculation of the effective potential was conducted long ago, e.g., in [42], [43], and [44]. For our goal, it is necessary to keep tract of the field-independent terms as well. Also, the $M$- and $\tilde{M}$- dependence is important. Let us focus on the one-loop potential; after carefully following these terms, one gets
\begin{equation}
V_{\text{opt}}(\tilde{\zeta}) = \frac{3\nu^4}{2\lambda} - \frac{\pi^2 T^4}{90} - \frac{\tilde{M}^4}{32\pi^2} \ln \frac{\bar{\mu}e^{\gamma_E}}{4\pi T} + \frac{1}{24} \tilde{M}^2 T^2 \\
+ \frac{1}{2} \nu^2 \tilde{\zeta}^2 - \frac{1}{12\pi} \tilde{M}^3 T + \frac{1}{4!} \lambda \tilde{\zeta}^4 + \mathcal{O} \left( \frac{\tilde{M}^6}{T^2} \right)
\end{equation}
where $\bar{\mu} \equiv \mu \left( \frac{4\pi}{e^{\gamma_E}} \right)^{1/2}$ is a scaling parameter of dimensional regularization (with the \textit{MS} scheme). The field equation associated with (25), $\frac{\partial}{\partial \tilde{\zeta}} V_{\text{opt}} = 0$, yields
\begin{equation}
\frac{\lambda}{6} \tilde{\zeta}^2 + \nu^2 - \frac{\lambda}{(4\pi)^2} \tilde{M}^2 \ln \frac{\bar{\mu}e^{\gamma_E}}{4\pi T} + \frac{1}{24} \lambda T^2 - \frac{\lambda}{8\pi} \tilde{M} T = 0
\end{equation}
up to terms of two-loop order. The solution is
\[ \tilde{\zeta}^2(M) \simeq -\frac{6\nu^2}{\lambda} - \frac{T^2}{4} + \frac{3}{8\pi^2} \left( -3\nu^2 + M^2 \right) \ln \left( \frac{\bar{\mu}e^{\gamma_E}}{4\pi T} \right) + \frac{3T\sqrt{(M^2 - 3\nu^2)}}{4\pi} + \cdots. \] (24)

With this, one gets the following onshell potential:
\[ V_{\text{opt}} = \frac{3\nu^4}{2\lambda} - \frac{\pi^2 T^4}{90} - \frac{\tilde{M}^4}{32\pi^2} \ln \frac{\bar{\mu}e^{\gamma_E}}{4\pi T} + \frac{1}{24} \tilde{M}^2 T^2 + \frac{1}{2} \nu^2 \tilde{\zeta}^2(M) - \frac{1}{12\pi} \tilde{M}^3 T + \frac{1}{4!} \tilde{\lambda}^4(M) + \mathcal{O}\left( \frac{\tilde{M}^6}{T^2} \right). \] (25)

The following PMS condition for \( \nu \)
\[ \frac{\partial V_{\text{opt}}}{\partial \nu} = 0 \] (26)

admits\(^{11}\)
\[ \nu = 0 \] (27)
as a solution. This implies
\[ M^2 = \frac{1}{24} \lambda T^2 \] (28)

Interestingly, this is the same as the result obtained in [45] (see also [46] and [47]) by considering renormalization group and choosing appropriate renormalization conditions.\(^{12}\) The other branches of the solutions have undesirable features. For instance, in those branches the small \( \tilde{M} \)-expansion is not justified. The condition (27) yields
\[ \tilde{\zeta}^2 = -\frac{T^2}{4} + i \frac{\sqrt{3\lambda}}{8\pi} T^2 + \cdots \] (29)

\(^{11}\)Strictly speaking, one will have to keep \( \nu \) approaching zero to be compatible with renormalization group consideration. (This is a relatively minor point and does not, of course, qualitatively change the conclusions.) See further comments in the note added at the end of the Conclusion.

\(^{12}\)Two cautionary remarks: firstly, although we have been loosely speaking that the optimized renormalized mass is on the order of the temperature, it is really \( M^2 \) that is on the order of the temperature. (However, the other branches of the solutions of (26) have that property, and in this sense one may say that the solution is generically on the order of the temperature.) Secondly, in the earlier arXiv version (v1) of this work, there was an algebraic error. Although they were relatively minor, they rendered the coefficient in (28) different from 1/24.
Note the novelty that $\tilde{\zeta}^2$ takes a complex value. Once the potential (25) is evaluated with this value, one gets

$$V_{\text{opt}} = -\frac{1}{90}\pi^2 T^4 + \cdots$$

which then allows one to avoid the fine-tuning problem.

The complete two-loop offshell form of the potential will be presented in \[12\]. There one again encounters the novelty: the potential develops an imaginary part, signaling instability of the vacuum toward zero temperature.\footnote{Strictly speaking, the potential itself remains real even at two-loop. However, this is because the source of the imaginary parts, $\tilde{M}^3$, does not contribute even at two-loop. Due to the expected contribution of $\tilde{M}^3$ term (and the terms with higher odd-integer powers of $\tilde{M}$ that should appear in higher-loop computations), it is expected that the complexity of the potential will become manifest at three-loop and on.}

More on this in the conclusion.

### 3.2 Curved space analysis

Whereas what we referred to as the second-layer perturbation in \[36\] is necessary for the pure gravity sector computation, there exists, for the matter sector, a powerful shortcut based on the first-layer perturbation, the “one-stroke” method. In the present work the first-layer perturbation will be used exclusively. From the results obtained, it becomes evident that the qualitative conclusion of the flat space analysis remains unchanged.

**Graviton sector**

Let us recall the zero-temperature case first. In \[35\] and \[36\], we conducted the computation in a brute-force manner by employing the second-layer perturbation and viewing the classical CC as the graviton mass term. As shown, e.g., in \[36\], the result is a divergent CC term

$$\sim \int \sqrt{-g}.$$  \hspace{1cm} (31)

Strictly speaking, for a flat background, the one-loop results vanish in dimensional regularization in the absence of the classical CC treated as the graviton mass term. For consistency with the observed value of the CC, the
classical (i.e., renormalized) CC will have to be set to $\sim T^4$ in the renormalization program described below, and thus will not qualitatively affect the proposed resolution of the CC problem.

The two-loop vanishes at zero temperature, due to the tracelessness of $P_{\mu\rho\sigma\tau}$ and (16). The results of $n$-loop with $n > 2$ will presumably vanish in dimensional regularization due to the structures of the vacuum diagrams. Let us examine things in more detail. An arbitrary $n$-loop graph with $n \geq 2$ contains a product of vertices that can be written as

$$< h_{\alpha_1\alpha_2} h_{\alpha_3\alpha_4} h_{\alpha_5\alpha_6} \ldots h_{\beta_1\beta_2} h_{\beta_3\beta_4} h_{\beta_5\beta_6} \ldots > \tag{32}$$

where the upper and lower indices are fully contracted. Contractions of the fields lead to

$$\tilde{P}_{\mu_1\nu_1,\rho_1\sigma_1} \tilde{P}_{\mu_2\nu_2,\rho_2\sigma_2} \ldots \tag{33}$$

where again, all the indices are contracted one way or another with $\tilde{g}^{\mu\nu}$'s. Whenever a pair of $\tilde{P}$'s have a pair of the indices contracted, the explicit expression for $\tilde{P}$ given in (15) can be used. One can also use (16) to reduce the total number of $\tilde{P}$'s until only one $\tilde{P}$ remains at the end. Since all of the indices must be contracted, the final expression must be $\sim \tilde{P}_{\mu\nu}^{\mu\nu}$. Thus the overall tensor structure gets simplified. As for the loop integrals, the two-loop diagrams, i.e., the sunset- and “8” - types of diagrams vanish in dimensional regularization due to masslessness of a graviton. We expect the same to be true for higher loops. However, these diagrams will have non-vanishing contributions if either a CC term is present and treated as a graviton mass term or temperature enters. Further analysis of the finite temperature case will be presented in [12].

**Matter-involving sector**

The matter-sector diagrams can be subdivided, depending on whether or not they involve a graviton loop. The matter-involving vertices do not, unlike the pure graviton vertices, come with $\frac{1}{\kappa^2}$. Since the graviton propagator comes with $\kappa^2$, the diagrams leading in $\kappa$ are those with a matter loop, which are our focus.

There exists a highly effective “one-stroke” method of computing the matter-involving part of the effective action, in which the flat spacetime analysis can
be entirely carried over. For this, note the explicit form of $\tilde{\Delta}(x_1 - x_2)$ can be written as
\begin{equation}
\tilde{\Delta}(x_1 - x_2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{\sqrt{-\tilde{g}(x_1)}} \frac{e^{ik(x_1-x_2)}}{i(k_{\mu}k_{\nu}\tilde{g}^{\mu\nu}(x_1) + m^2)},
\end{equation}

Defining “flattened” momentum and coordinates as
\begin{equation}
K_\alpha \equiv \tilde{e}_\mu^\alpha k_\mu, \quad X_\beta \equiv \tilde{e}_\nu^\beta x^\nu, \quad \tilde{e}_\alpha^{\mu} \tilde{e}_\beta^{\nu} \tilde{g}_{\mu\nu} = \eta_{\alpha\beta},
\end{equation}
where the underlined indices are flattened, one gets the flattened propagator:
\begin{equation}
\tilde{\Delta}(X_1 - X_2) = \int \frac{d^4K}{(2\pi)^4} \frac{e^{iK_\alpha(x_1-x_2)\eta_{\alpha\beta}}}{i(K_\alpha K_\beta \eta_{\alpha\beta} + m^2)}.
\end{equation}

When computing a diagram, one can pull out all of the background fields and contract the fluctuation fields. The propagators can then be transformed to the above. Afterward, the steps become parallel to those corresponding to the flat cases. The matter part of the effective action can thus be computed exactly in the same manner in which it is computed in the flat case.

### 3.3 Consistency of new subtraction scheme

With the renormalized mass around the CMB temperature, one should make sure that that framework preserves the success of the zero-temperature theory such as that seen in the zero-temperature SM. The new perturbation has been
illustrated in section 2 by taking a simple scalar theory. In the case of the SM, each sector in the SM, i.e., the Higgs, gauge, and fermion, the renormalization process should be modified from that of the standard renormalization scheme, e.g., \( \overline{\text{MS}} \). More specifically, in the modified scheme, the pole mass value must be realized by adjusting the finite parts of the divergent integrals that are chosen differently from those in the standard renormalization scheme: in analysis with \( \overline{\text{MS}} \), the pole mass condition is part of the \( \overline{\text{MS}} \) scheme, and the renormalized mass is not determined prior to the pole mass condition. It is the pole mass condition that determines the renormalized mass. In contrast, in the new scheme the renormalized mass is determined, as demonstrated in section 3.1, by the OPT. The finite parts are to be determined by the physical pole mass condition. What it implies is that it is necessary to go to the one-loop level to achieve the accuracy of amplitudes obtained in \( \overline{\text{MS}} \). In order to match the values of, e.g., the tree amplitudes computed in \( \overline{\text{MS}} \), one must insert one-particle-irreducible diagrams to the internal lines. (We anticipated that one-loop insertion will normally be sufficient. Since one-loop contributions will be dominant even over tree-level, it will be useful to examine two-loop contributions to see how fast two-loop, compared to one-loop, decreases. We suggest this as a future task in the conclusion.) The insertions amount to a certain resummation. The situation is generally illustrated in Fig. 3. In light of this, it is also worth noting that the \( T^2 \)-scaling of the renormalized mass was previously obtained in [45] by choosing appropriate renormalization conditions.

4 Conclusion

Since some of the Standard Model particles, such as the Higgs, are massive, the matter contributions to the CC are naively expected to be larger than that of the graviton. The variant OPT reveals, however, that the ultimate determining factor of the CC is the temperature. We believe that this identifies the cosmological constant problem at its root. This leads to necessity of employing a new renormalization scheme. In the new scheme, one needs to go a few orders higher and perform a certain resummation to achieve the same level of proximity to the values of the physical observables as in the zero-temperature standard schemes. Given what it brings, this seems to a relatively small price to pay.

The finding in the present work that a non-traditional renormalization
scheme is needed to tackle the cosmological constant problem is in line with the observation, e.g., in [7]. The overarching umbrella for this and [6,8–11] is the so-called running vacuum with time-varying and dynamical energy, a view that we find very natural. In these works, the cosmological constant problem was coherently tackled from different angles and various cosmological implications of such a vacuum were discussed. (Even the relevance of temperature to the cosmological constant problem was discussed to some extent.) Several different regularization methods and their pertinent renormalization schemes were analyzed. We believe that the present work lends support, by providing independent motivation to consider a non-standard renormalization scheme, to those approaches. It is also interesting to note that the present finite temperature contribution to the stress-energy tensor seems to bear a certain resemblance to the ‘stress-energy tensor of cosmic interaction’ obtained in [49].

Before commenting on potential implications of the new scheme for cosmology, let us discuss some checks that are desirable for solidifying the new renormalization scheme in the context of renormalized perturbation theory. One peculiarity of the present scheme worth a closer look is that the one-loop part should be bigger than the classical part. This is because the renormalized mass is taken to be far smaller than the physical value. This is the point illustrated in Fig. 3: since the pole mass in the new scheme will match poorly with the physical value at the classical level, one should go to at least one-loop, which must be large enough to offset the deficit of the classical value. It should be useful to look into things in two directions: Firstly, it will be illuminating to explicitly check how the new scheme will play out in an explicit SM example. Secondly, it will be useful to examine in the new scheme the rate at which the two-loop decreases compared to one-loop. In the case of a scalar theory, one should be able to check this with reasonable amount of effort. We will report on progress in [34].

We end with three ramifications of our results and further directions. The results obtained in this work suggest that, earlier in the thermal history of the Universe, the value of the CC should have been larger. In other words the CC becomes time-dependent through the temperature, and the present small value must be due to the age of the Universe [31]. More quantitatively, the following is the basis for this anticipation. In [35] it was shown that there exists a time-dependent solution that approaches the minimal value of the potential. By using the one-parameter family of the potentials labeled by the
temperature, one can repeat the analysis with the present finite temperature potential. One can then analyze the resulting quantum-level action, and it should be possible, at the time-dependent solution level, to establish a CC that decreases to a small value. One may introduce a renormalized CC, $\Lambda_{\text{ren}}$. To be consistent with the fact that the observed value of the CC is small, one will have to take $\Lambda_{\text{ren}}$ to be small. It will be of interest to pursue this line of study. Another ramification, not unrelated to the first, is that the large value of the CC becomes natural when the temperature is on the order of the EW scale. (This may be something profound, and have implications for the hierarchy problem in the SM.) Since the Universe was at higher temperatures in the previous eras, it will be a meaningful endeavor to explore whether one could come up with a streamlined description covering the entire temperature range, say, from the near-Planck era to the present. With the recent progress in the OPT literature, the present results indicate toward an affirmative answer. Lastly, as we will report in [12], the potential is expected to develop a imaginary part. We interpret this as an indication of the vacuum decay from a finite temperature to zero temperature. Just as an analogous decay induced by a bounce worm hole solution plays an important role in black hole information [40] [48], the present vacuum is likely to have interesting cosmological implications that deserve further study.

Note added: As a matter of fact, a novel renormalization condition had been adopted (with MS scheme) in the work by Hamada et al. [50] - which I became aware of sometime after publication of the present work - wherein bare perturbation theory was employed. Their scheme can be translated, in renormalized perturbation theory, into one that is in line with the present scheme. The renormalization condition employed therein amounts to setting the renormalized mass approximately to zero. More specifically, their renormalization condition in bare perturbation theory is spelled out in eqs. (5) - (7). Let us focus on the one-loop case eq. (6). The momentum was evaluated at $k = 0$ as a way of narrowing down to the potential, as opposed to kinetic, term. Let us consider a zero-temperature Higgs type scalar theory with the renormalized mass denoted by $m$ (as in section 2). To one-loop order, one gets

$$\Sigma = -\frac{\lambda m^2}{32\pi^2} \left( \frac{2}{D-4} - 1 + \gamma_E + \cdots \right).$$  \hspace{1cm} (37)
On the other hand one can write, by employing $\overline{\text{MS}}$ subtraction scheme,

$$m_0^2 = m^2 - \frac{\lambda m^2}{2(4\pi)^2} \left( \frac{1}{D - 4} - 1 + \gamma_E \right). \quad (38)$$

Combining these two equations one sees

$$m_0^2 - \Sigma = m^2. \quad (39)$$

With eq. (5) in [50] - which is approximately valid, the renormalization condition eq. (5) above implies

$$m^2 \simeq 0. \quad (40)$$

The lesson of this and the present renormalization condition is that a vast amount of the freedom in choosing renormalization conditions can (and must) be utilized to accommodate a wider range of physics.
References

[1] S. Weinberg, “The Cosmological Constant Problem,” Rev. Mod. Phys. 61, 1-23 (1989) doi:10.1103/RevModPhys.61.1

[2] T. Padmanabhan, “Cosmological constant: The Weight of the vacuum,” Phys. Rept. 380, 235-320 (2003) doi:10.1016/S0370-1573(03)00120-0 [arXiv:hep-th/0212290 [hep-th]].

[3] E. Bianchi and C. Rovelli, “Why all these prejudices against a constant?,” [arXiv:1002.3966 [astro-ph.CO]].

[4] J. Martin, “Everything You Always Wanted To Know About The Cosmological Constant Problem (But Were Afraid To Ask),” Comptes Rendus Physique 13, 566-665 (2012) doi:10.1016/j.crhy.2012.04.008 [arXiv:1205.3365 [astro-ph.CO]].

[5] J. Solà Peracaula, “Cosmological constant and vacuum energy: old and new ideas,” J. Phys. Conf. Ser. 453, 012015 (2013) doi:10.1088/1742-6596/453/1/012015 [arXiv:1306.1527 [gr-qc]].

[6] I. L. Shapiro and J. Solà Peracaula, “On the scaling behavior of the cosmological constant and the possible existence of new forces and new light degrees of freedom,” Phys. Lett. B 475, 236-246 (2000) doi:10.1016/S0370-2693(00)00909-3 [arXiv:hep-ph/9910462 [hep-ph]].

[7] I. L. Shapiro and J. Solà Peracaula, “Scaling behavior of the cosmological constant: Interface between quantum field theory and cosmology,” JHEP 02, 006 (2002) doi:10.1088/1126-6708/2002/02/006 [arXiv:hep-th/0012227 [hep-th]].

[8] J. Solà Peracaula, “Dark energy: A Quantum fossil from the inflationary Universe?,” J. Phys. A 41, 164066 (2008) doi:10.1088/1751-8113/41/16/164066 [arXiv:0710.4151 [hep-th]].

[9] J. Solà Peracaula, A. Gómez-Valent and J. de Cruz Pérez, “First evidence of running cosmic vacuum: challenging the concordance model,” Astrophys. J. 836, no.1, 43 (2017) doi:10.3847/1538-4357/836/1/43 [arXiv:1602.02103 [astro-ph.CO]].
[10] J. Solà Peracaula, “Brans–Dicke gravity: From Higgs physics to (dynamical) dark energy,” Int. J. Mod. Phys. D 27, no.14, 1847029 (2018) doi:10.1142/S0218271818470296 [arXiv:1805.09810 [gr-qc]].

[11] C. Moreno-Pulido and J. Solà Peracaula, “Running vacuum in quantum field theory in curved spacetime: renormalizing $\rho_{\text{vac}}$ without $\sim m^4$ terms,” Eur. Phys. J. C 80, no.8, 692 (2020) doi:10.1140/epjc/s10052-020-8238-6 [arXiv:2005.03164 [gr-qc]].

[12] I. Y. Park, “Quantization of Gravity and Finite Temperature Effects,” Particles 4, no.4, 468-488 (2021) doi:10.3390/particles4040035 [arXiv:2109.01647 [hep-th]].

[13] M. D. Schwartz, “Quantum field theory and the Standard Model,” Cambridge university press (2014)

[14] F. Karsch, A. Patkos and P. Petreczky, “Screened perturbation theory,” Phys. Lett. B 401, 69-73 (1997) doi:10.1016/S0370-2693(97)00392-4 [arXiv:hep-ph/9702376 [hep-ph]].

[15] J. O. Andersen and M. Strickland, “Mass expansions of screened perturbation theory,” Phys. Rev. D 64, 105012 (2001) doi:10.1103/PhysRevD.64.105012 [arXiv:hep-ph/0105214 [hep-ph]].

[16] J. P. Blaizot, E. Iancu and A. Rebhan, “Approximately selfconsistent resummations for the thermodynamics of the quark gluon plasma. 1. Entropy and density,” Phys. Rev. D 63, 065003 (2001) doi:10.1103/PhysRevD.63.065003 [arXiv:hep-ph/0005003 [hep-ph]].

[17] P. M. Stevenson, “Optimized Perturbation Theory,” Phys. Rev. D 23, 2916 (1981) doi:10.1103/PhysRevD.23.2916

[18] J. I. Kapusta and C. Gale, “Finite-Temperature Field Theory: Principles and Applications,” Cambridge university press (2006)

[19] M. Le Bellac, “Thermal Field Theory,” Cambridge university press (2000)

[20] J. P. Blaizot, “Quantum Fields at Finite Temperature ”from tera to nano Kelvin”,” Soryushiron Kenkyu Electron. 119, 11-108 (2011) doi:10.24532/soken.119.2_11 [arXiv:1108.3482 [hep-ph]].
[21] M. Laine and A. Vuorinen, “Basics of Thermal Field Theory,” Lect. Notes Phys. 925, pp.1-281 (2016) doi:10.1007/978-3-319-31933-9 [arXiv:1701.01554 [hep-ph]].

[22] E. Senaha, “Symmetry Restoration and Breaking at Finite Temperature: An Introductory Review,” Symmetry 12, no.5, 733 (2020) doi:10.3390/sym12050733.

[23] A. D. Linde, “Infrared Problem in Thermodynamics of the Yang-Mills Gas,” Phys. Lett. B 96, 289-292 (1980) doi:10.1016/0370-2693(80)90769-8.

[24] J. Ghiglieri, A. Kurkela, M. Strickland and A. Vuorinen, “Perturbative Thermal QCD: Formalism and Applications,” Phys. Rept. 880, 1-73 (2020) doi:10.1016/j.physrep.2020.07.004 [arXiv:2002.10188 [hep-ph]].

[25] Q. Du, M. Strickland, U. Tantary and B. W. Zhang, “Two-loop HTL-resummed thermodynamics for $\mathcal{N} = 4$ supersymmetric Yang-Mills theory,” JHEP 09, 038 (2020) doi:10.1007/JHEP09(2020)038 [arXiv:2006.02617 [hep-ph]].

[26] E. Braaten and A. Nieto, “Effective field theory approach to high temperature thermodynamics,” Phys. Rev. D 51, 6990-7006 (1995) doi:10.1103/PhysRevD.51.6990 [arXiv:hep-ph/9501375 [hep-ph]].

[27] I. Y. Park, “Quantum-corrected Geometry of Horizon Vicinity,” Fortsch. Phys. 65, no.12, 1700038 (2017) doi:10.1002/prop.201700038 [arXiv:1704.04685 [hep-th]].

[28] A. J. Nurmagambetov and I. Y. Park, “Quantum-induced trans-Planckian energy near horizon,” JHEP 05, 167 (2018) doi:10.1007/JHEP05(2018)167 [arXiv:1804.02314 [hep-th]].

[29] A. J. Nurmagambetov and I. Y. Park, “Quantum-gravitational trans-Planckian energy of a time-dependent black hole,” Symmetry 11, no.10, 1303 (2019) doi:10.3390/sym11101303 [arXiv:1909.10054 [hep-th]].

[30] A. J. Nurmagambetov and I. Y. Park, “Quantum-gravitational trans-Planckian radiation by a rotating black hole,” [arXiv:2007.06070 [hep-th]], to appear in Fortsch. Phys.
[31] S. Weinberg, “The quantum theory of fields”, vol I,II, Cambridge university press (1995)

[32] G. Sterman, “An introduction to quantum field theory”, Cambridge university press (1993)

[33] M. Peskin and D. Schroeder, “An introduction to quantum field theory”, Addison-Wesley publishing company (1995)

[34] I. Y. Park, “Implications of an OPT-motivated renormalization scheme for cosmology” (tentative title), work in progress.

[35] I. Y. Park, “One-loop renormalization of a gravity-scalar system,” Eur. Phys. J. C 77, no. 5, 337 (2017) doi:10.1140/epjc/s10052-017-4896-4 [arXiv:1606.08384 [hep-th]].

[36] I. Y. Park, “Revisit of renormalization of Einstein-Maxwell theory at one-loop,” PTEP 2021, no.1, 013B03 (2021) doi:10.1093/ptep/ptaa167 [arXiv:1807.11595 [hep-th]].

[37] I. Y. Park, “Lagrangian constraints and renormalization of 4D gravity,” JHEP 04, 053 (2015) doi:10.1007/JHEP04(2015)053 [arXiv:1412.1528 [hep-th]].

[38] I. Park, “Foliation-Based Approach to Quantum Gravity and Applications to Astrophysics,” Universe 5, no.3, 71 (2019) doi:10.3390/universe5030071 [arXiv:1902.03332 [hep-th]].

[39] I. Y. Park, “Reduction of BTZ spacetime to hypersurfaces of foliation,” JHEP 01, 102 (2014) doi:10.1007/JHEP01(2014)102 [arXiv:1311.4619 [hep-th]].

[40] I. Y. Park, “Foliation-based quantization and black hole information,” Class. Quant. Grav. 34, no.24, 245005 (2017) doi:10.1088/1361-6382/aa9602 [arXiv:1707.04803 [hep-th]].

[41] I. Y. Park, “Boundary dynamics in gravitational theories,” JHEP 07, 128 (2019) doi:10.1007/JHEP07(2019)128 [arXiv:1811.03688 [hep-th]].

[42] R. R. Parwani, “Resummation in a hot scalar field theory,” Phys. Rev. D 45, 4695 (1992) doi:10.1103/PhysRevD.45.4695 [arXiv:hep-ph/9204216 [hep-ph]].

27
[43] P. B. Arnold and O. Espinosa, “The Effective potential and first order phase transitions: Beyond leading-order,” Phys. Rev. D 47, 3546 (1993) doi:10.1103/PhysRevD.47.3546 [arXiv:hep-ph/9212235 [hep-ph]].

[44] S. Chiku and T. Hatsuda, “Optimized perturbation theory at finite temperature,” Phys. Rev. D 58, 076001 (1998) doi:10.1103/PhysRevD.58.076001 [arXiv:hep-ph/9803226 [hep-ph]].

[45] J. P. Blaizot and N. Wschebor, “Massive renormalization scheme and perturbation theory at finite temperature,” Phys. Lett. B 741, 310-315 (2015) doi:10.1016/j.physletb.2014.12.040 [arXiv:1409.4795 [hep-ph]].

[46] J. L. Kneur and M. B. Pinto, “Scale Invariant Resummed Perturbation at Finite Temperatures,” Phys. Rev. Lett. 116, no.3, 031601 (2016) doi:10.1103/PhysRevLett.116.031601 [arXiv:1507.03508 [hep-ph]].

[47] J. L. Kneur and M. B. Pinto, “Renormalization Group Optimized Perturbation Theory at Finite Temperatures,” Phys. Rev. D 92, no.11, 116008 (2015) doi:10.1103/PhysRevD.92.116008 [arXiv:1508.02610 [hep-ph]].

[48] I. Y. Park, “Black hole evolution in a quantum-gravitational framework,” PTEP 2021, no.6, 063B03 (2021) doi:10.1093/ptep/ptab045 [arXiv:1912.07413 [hep-th]].

[49] G. Ryskin, “The emergence of cosmic repulsion,” Astropart. Phys. 62, 258-268 (2015) doi:10.1016/j.astropartphys.2014.10.003 [arXiv:1810.07516 [physics.gen-ph]].

[50] Y. Hamada, H. Kawai and K. y. Oda, “Bare Higgs mass at Planck scale,” Phys. Rev. D 87, no.5, 053009 (2013) [erratum: Phys. Rev. D 89, no.5, 059901 (2014)] doi:10.1103/PhysRevD.87.053009 [arXiv:1210.2538 [hep-ph]].