Research Article
Optimal Pricing Decision of Fashion Apparel considering Experiential Service and Design Enhancement

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Fashionable clothing is susceptible to seasonality, fashion popularity, and other factors. The decline in the fashion level for fashion apparel will cause its market value to continuously decrease, reducing market demand and creating a backlog of apparel inventory. Under such a circumstance, the apparel retailer chooses to maintain the fashion of the goods by providing experiential services or enhancing product design capabilities. This paper focuses on the discussions on the issue of whether experience service and design efforts are complements or substitutes. The major objective is to simultaneously determine the experience service investment and the optimal selling price to maximize the total profit. First, a Cobb–Douglas utility function is used to derive a demand function that depends on the price and fashion level. Four kinds of inventory models are further established to obtain optimal pricing and inventory ordering strategies. Second, an algorithm is presented to search for the optimal solutions of the proposed model. Finally, a numerical example is provided to perform a sensitivity analysis of the key parameters and to discuss specific managerial insights. The numerical examples show that both the experiential services and the enhanced fashion design can effectively reduce the apparel company’s inventory and increase profits. When the two strategies are combined, they will produce complementary or substitution effects, which depend on the deterioration rate of the fashion level of the apparel. If the deterioration rate is less than a critical value, the interaction of experiential services and design investment has a complementarity effect.

1. Introduction

China is a major country in apparel production. With the advent of the era of personalized consumption, consumers are increasingly demanding fashion in apparel. However, since fashion apparel is a typical seasonal product, how to embody product innovation by enhancing the fashion level of the apparel to gain more market competitive advantage has become an urgent problem faced by apparel companies. The life cycle of fashion clothing sales is short, and its fashion level largely depends on the fashion elements contained in the clothing, such as style, pattern, material technology, and decoration accessories. Generally, the higher the fashion level is and the more fashionable the clothing is, the higher the market share of the products is. When new fashion products are launched, they will be favored and sought after by many consumers who prefer fashion. However, as time goes by, consumers’ preference for original apparel gradually decreases, so the fashion level of clothing will gradually decrease with time, and the market value of the apparel is declining. This feature usually makes consumers have different attitudes for fashion at different times. That is, the fashion level of clothing will gradually decay with time, easily causing product backlog. According to a study, the apparel industry is generally threatened by high inventories. In the first half of 2016, 79 textile and apparel listed companies had a total inventory of $81.366 billion. On the one hand, the apparel retailer is paying more and more attention to providing virtual try-on, tie-in, fashion display, and other experiential services, so as to improve consumers’ value on the apparel and, thus, to improve the fashion level of the apparel and stimulate demand [1]. On the other hand, enhancing the fashion design of clothing and continuously meeting the changing needs of the market have gradually become a new requirement for the source design of fashion clothing. Therefore, experience and design do not exist in
isolation but are closely related, and they influence each other. The decay of clothing fashion over time has an important impact on consumers' perceived value, clothing demand, and profits of clothing companies. Therefore, how to improve the interactive participation of clothing "experience and design" through experience services and clothing design investment is a very interesting and practical problem.

The existing literature related to fashion apparel mainly focused on the quick-response (QR) and price reduction. As for the research on the QR of fast fashion apparel products, Sen [2] analyzed the American fashion industry and its supply chain, and they believed that the inventory of the quick-response retailers tracked the sales of stores in real time could be kept to the minimum. Choi et al. [3] pointed out that, in the fashion industry, quick-response is a timely and influential business strategy implemented by many international companies such as Zara and H&M. Choi [4] studied the QR expectations of retailers and manufacturers and analyzed the coordination strategies of wholesale pricing and price discount contracts in the quick-response fashion supply chain. Choi [5] studied the impact of the QR plans targeting inventory services on the profits of the apparel retailer, manufacturers, and fashion supply chain systems. Cachon and Swinney [6] analyzed two key features of the fashion supply chain. One is QR production capacity, and the other is enhanced product design capabilities. All of the abovementioned studies focus on the impact of QR on inventory without considering the impact of price changes on clothing inventory during the sales period. Since price reduction promotion is a commonly used method by the apparel retailer, the promotion strategy has received extensive attention from scholars. Cachon and Feldman [7] found that frequent discounts can attract more consumers considering the consumer buying behavior, and the discount strategy is often more effective than the fixed price strategy. Cachon and Swinney [6] considered the multitype clothing price reduction optimization problem of the retailer and used approximate dynamic programming algorithms to study the price reduction strategy of each product. Aviv et al. [9] studied that the seller who sells seasonal fashion goods can adopt dynamic pricing and quick inventory replenishment during the sales season to enable consumers to purchase fashion products at a higher price in advance. In view of the short life cycle of fashion products and the high uncertainty of market demand, Choi [10] considered a two-stage inventory strategy and used Bayesian methods to optimize the inventory and pricing strategies of fashion products based on actual demand data in the sales season. However, a few studies have explicitly considered clothing inventory decisions from the perspective of pricing and replenishment cycles.

Fashion clothing is a short life cycle product, and its market value is similar to the characteristics of perishable products [11]. Therefore, the research on inventory management and pricing of perishable products can be referenced for the study of the fashion apparel. Wang et al. [12] considered an inventory optimization model for perishable products whose deterioration rate followed the Weibull distribution, and the demand depends on the instantaneous inventory level. Sicilia et al. [13] constructed a perishable inventory control model, in which a time-dependent exponential function of demand is considered. Chen et al. [14] constructed a multiperiod joint pricing and inventory control model that does not allow stock shortage. Besides, some scholars studied the inventory and pricing decisions of perishable products from the perspectives of other factors, such as purchase rate, promotional efforts, and preservation investment. Maihami and Karimi [15] analyzed the influence of the repeated purchase rate of perishable products and the speed of product diffusion on the pricing and replenishment decisions of enterprises. Tsao and Sheen [16] considered a joint decision-making on inventory and pricing of perishable products under promotional efforts. Li et al. [17] considered perishable products under the constraints of fresh-keeping investment and service investment and studied the joint pricing, ordering, and fresh-keeping investment decision-making issues of nonimmediate deterioration items. Li et al. [18] studied the stochastic production inventory model of perishable products under the dual influence of price and service level and discussed the optimal joint dynamic pricing, service, and production strategies of enterprises under the influence of external random disturbances on inventory changes.

Unlike perishable goods whose quantity and quality are declining and changing with time, the market value of fashion apparel is declining over time, but the quantity is not. Tsao [19] studied the inventory control of fashion items under the condition of trade credit and partial overstocking. Considering the effects of the fashion level and reserve price on the performance of the apparel retailer, Chen et al. [11] analyzed the optimal pricing and order strategy of the apparel retailer under dynamic and fixed cases. The literatures mentioned above did not consider the impact of experiential services on the fashion level. Chen and Xu [1] studied the optimal pricing and inventory ordering strategy in the case of no discount and discount in the sales period with considering the experiential service. Among the studies mentioned above, the work of Chen and Xu [1] and Chen et al. [11] are closest to our paper. Both studies examine the inventory and pricing decisions of fashion clothes, but they all adopt a formalized demand function, which is a linear combination of price and fashion level. However, this assumption is not in line with the reality. The difference between fashion apparel and perishable goods is that the fashion level of an apparel is a quality attribute of the apparel. Consumers can visually observe its characteristics, which is an important reference for consumers to buy. Although the price of fashion clothing is also the main factor that affects consumers' purchasing behavior, different consumers have different preferences for the price and fashion level, which will lead to the practical problem that the actual demand is difficult to determine. Some scholars have used the utility theory to study the nonlinear demand function [20–24]. For example, Su [20] took product pricing and consumer waiting costs as important factors influencing consumer utility and studied the optimal pricing strategy for retailers facing different categories of consumers. Wang et al.
[21] discussed the effect of changes in quality information asymmetry on the price decisions by using the Cobb-Douglas utility function. However, the utility functions of Su [20] and Wang et al. [21] cannot reflect the characteristics of fashion clothing, that is, the consumer’s utility function is affected by the fashion level of apparel and changes dynamically with time. Besides, the purchase quantity and the disposable income of consumers are also two important influencing factors. In short, this paper comprehensively considers the influence of fashion level, sales price, purchase quantity, and disposable income on inventory decisions.

The existing literature has conducted a lot of research on the management of perishable products, but for the fashion of clothing products, there are few studies combining experiential services and enhanced fashion design. Based on the principle of consumer utility maximization, we derive a time-varying nonlinear demand function, which mainly depends on the price and fashion level and is a nonlinear combination of the two factors. We analyze four potential operating systems—a traditional system (without experiential service and design efforts), experiential service system, design efforts system, and fast fashion systems (with experiential service and design efforts). This paper focuses the discussions on the issue of whether experience service and design efforts are complements or substitutes. The major objective is to simultaneously determine the experience service investment and the optimal selling price to maximize the total profit. It is found that both the experiential services and the enhanced fashion design can effectively reduce the apparel company’s inventory and increase profits. When the two strategies are combined, they will produce complementary or substitution effects, which depend on the deterioration rate of the fashion level of the apparel. If the deterioration rate is less than a critical value, the interaction of experiential services and design investment has a complementarity effect.

The rest of the paper is structured as follows. In Section 2, the symbols and assumptions of the model are described. Section 3 constructs four inventory models. Section 3.1 studies the retailer’s profit model and optimal pricing strategy under the basic scenario (where the apparel retailer neither provides experiential services nor enhances design). Section 3.2 discusses the case where only experiential services are provided to improve the influence of the fashion level on operational decisions; Section 3.3 investigates the situation that an apparel enterprise enhances design efforts to improve the fashion level of an apparel; Section 3.4 examines the combination of providing experiential services and enhancing design innovation and analyzes whether the two strategies are complements or substitutes. Section 4 provides numerical results and sensitivity analysis. Finally, Section 5 provides conclusions.

2. Notation and Assumptions

2.1. Problem Description and Notation. We first provide a problem description. We model an apparel retailer that sells fashionable clothing products. The demand for fashionable clothing is determined by the selling price and fashion level of clothing during the selling period. A replenishment problem of a single fashion item is considered. The inventory system evolves as follows: Q units of items arrive at the inventory system at the beginning of each cycle. The inventory level \( I(t) \) is depicted in Figure 1 and decreases due to consumers’ demand during the replenishment cycle. Fashion apparel provides experiential service to consumers. Also, fashion apparel tries to enhance design efforts to improve the fashion level of the apparel, thus to raise the demand. The selling price and experiential service are the decision variables. We seek to answer the following questions:

1. Are experiential service and design efforts complements or substitutes?
2. How to make the optimal price and experiential service for the apparel retailer to maximize his total profit during the period?

Thus, we analyze a total of four potential operational systems. A traditional system, which represents that no experiential service or design effort is offered during the sales period, an experiential service system that employs experiential service but maintains the design efforts unchanged, a design efforts system that provides design efforts to improve the fashion level of goods but does not provide the experiential service, and a fast fashion system that employs both experiential service and design efforts.

The notations used in this paper are illustrated in Table 1.

2.2. Utility Function. When fashion products are released, they are favored by consumers, but as new products appear over time, consumers’ preference for original fashion clothes decreases, so the fashion level of fashion clothes will gradually decrease over time. This feature usually makes consumers buy clothes at different times to obtain different utilities, that is, the utility of consumers changes dynamically with the fashion level. Referring to the work by Wang et al. [21], a Cobb–Douglas utility function is introduced to characterize the dynamics of the utility as follows:

\[
U_t = A \left( \frac{D_t}{p} \right)^a \omega_k^b + k (Y - p D),
\]

where \( U_t \) is the utility of consumers at time \( t \) and \( \omega_k \) is the fashion level of apparel. The parameter \( a \) (0 < \( a \) < 1) represents the consumers’ preference for the quantities and price of fashion clothes. The parameter \( b \) (0 < \( b \) < 1) represents the consumers’ preference for the fashion level of the apparel. The notations \( D_t, p, \) and \( Y \) represent the market demand, the selling price, and the consumers’ disposable income, respectively. \( A \) and \( k \) are positive parameters. Considering that the utility of consumers changes with time, at a certain moment, consumers will choose to purchase the optimal quantity under the circumstances of a given price and fashion to maximize their utility.

The maximization of utility provides the necessary condition for the market demand.
and fashion level, respectively, and we have to buy fashion clothing at different time points is different. If other conditions remain unchanged, the maximum utility levels for consumers is given by

\[ U_t = k(1-a) \left( \frac{Aa}{k} \right)^{1-a} p^{-(1+a)/(1-a)} \omega_t^{b/(1-a)} \omega_t e^{-\eta t} + kY. \]  

(3)

Formula (3) shows that the rational consumer always purchases the optimal amount of fashionable clothing within a certain period of time to obtain the instantaneous maximum utility. Under the condition that other conditions remain unchanged, the maximum utility level for consumers to buy fashion clothing at different time points is different.

Next, we discuss the rationality of formula (3). First, we take the first partial derivative of (3) with respect to the price and fashion level, respectively, and we have

\[ \frac{\partial U_t}{\partial p} = -2k \left( \frac{Aa}{k} \right)^{1-a} p^{-(1+a)/(1-a)} \omega_t^{b/(1-a)} \omega_t e^{-\eta t} < 0, \]  

(4)

\[ \frac{\partial U_t}{\partial \omega_t} = k \frac{a}{b} \left( \frac{Aa}{k} \right)^{1-a} p^{-(2a)/(1-a)} \omega_t^{a+b/(1-a)} e^{-\eta t} > 0. \]  

(5)

From inequalities (4) and (5), it can be observed that the utility function decreases as the sales price increases, but increases as the fashion level increases. The results given above are consistent with common sense in the industrial practice, so the assumption about the utility function of (1) is reasonable.

2.3. Fashion Function. It can be noted from Section 2.2 that both the consumers’ utility function and the market demand for clothing are affected by the fashion level of the clothing. The complexity of this relationship arises a primitive question: how to characterize the dynamics of the fashion level? Motivated by the method used by Chen and Xu [1], this paper provides a fashion level function to describe the attenuation of the fashion level. The fashion level is given by

\[ \omega_t = \omega_0 e^{-\eta t}, \]  

(6)

where \( \omega_0 (> 0) \) is the initial fashion level and represents the fashion level of the garments when they were launched. \( \eta (> 0) \) is the deterioration rate of fashion level, which means the fashion level \( \omega_t \) decreases with time. We compare the influence of different initial fashion levels on the fashion level function. The fashion level decreases with time; however, the larger \( \omega_0 \) is, the greater the remaining fashion level of fashion clothing at the same time. This shows that the initial fashion level has a significant delay effect on the fashion of clothing. In order to meet consumers’ personalized demands for fashion apparel products, retailers can provide consumers with experiential services, such as try-on, fashion display, and apparel collocation, to enhance consumers’ psychological feelings about apparel fashion, so as to improve the initial fashion level of clothing to a certain extent.

Similarly, the fashion level deterioration rate \( \eta \) also has an effect on the fashion level. The smaller the \( \eta \) is, the slower the fashion level attenuation will be. A large part of the fashion level of an apparel depends on the fashion elements contained in the apparel, such as style, color, design, material technology, and decoration, and these fashion elements determine the size of the deterioration rate \( \eta \) of the fashion level of the apparel. Therefore, if retailers enhance product design and increase investment in the creative design of fashion elements through cooperation with suppliers, then the fashion level of an apparel can be reduced and relatively improved.

Substituting (6) into the demand equation (2) gives

\[ D(p, t) = \left( \frac{Aa}{k} \right)^{(1-a)} p^{-(1+a)/(1-a)} \omega_0^{b/(1-a)} e^{-\eta t}. \]  

(7)
Formula (7) shows that the demand for clothing depends nonlinearly on the selling price and fashion level.

3. The Model

3.1. The Traditional System. The traditional system refers to the scenario in which the apparel retailer neither directly provides the experiential service nor indirectly invests in promoting the apparel enterprise to enhance the design during the sales period. Under the traditional scenario, the fashion level function is

\[ \omega_t = \omega_0 e^{-at}. \]  (8)

Substituting (8) into the (1) and (2), the utility function and the demand function in this section can be expressed as follows:

\[ D_1 = \left( \frac{Aa}{k} \right)^{(1/1-a)} p^{-(1+a/1-a)} \omega_t^{(b/1-a)} , \]  (9)

\[ U_1 = \frac{k(1-a)}{a} \left( \frac{Aa}{k} \right)^{(1/1-a)} p^{-(2/1-a)} \omega_t^{(b/1-a)} + kY. \]

From Figure 1, we can see that the inventory level \( I(t) \) decreases due to consumers’ demand during the time interval \([0, T] \). The differential equation that represents the inventory status is given by

\[ \frac{dI(t)}{dt} = -D_1 = -p^{-(1+a/1-a)} \left( \frac{Aa}{k} \right)^{(1/1-a)} \omega_0^{(b/1-a)} e^{(-b/1-a)t} , \]

where \( c \) is per unit ordering cost and \( A \) is the fixed ordering cost.

(ii) The cumulative inventory-holding cost,

\[ H = \int_0^T I(t) \, dt = p^{-(1+a/1-a)}G_2, \]  (15)

where \( h \) is unit inventory-holding cost per time unit.

In addition, the retailer’s sale revenue (SR) equals to the selling price \( p \) times the sum of the demand during the replenishment period, that is,

\[ SR = pQ = p^{-(2a/1-a)}G_1. \]  (16)

Therefore, total profit (denoted by \( \pi_1 (p) \)) during the time interval \([0, T] \) is given by

\[ \pi_1 (p) = \max_{p \ge 0} [SR - H - C_1] = (p - c)p^{-(1+a/1-a)}G_1 + p^{-(2/1-a)}G_1 \]

\[ p^{-(1+a/1-a)}G_2 - B. \]

\[ \pi_1 (p) \] is a function of \( p \). Maximization of the formula (17) with respect to \( p \) yields

\[ p^{-(2/1-a)} + \frac{1}{1-a} \left( \frac{1}{cG_1 + G_2} \right) \]  (18)

By simplifying formula (17), the optimal pricing can be obtained:

\[ p^* = \frac{(1 + a)(cG_1 + G_2)}{2aG_1}. \]  (19)

According to the analysis mentioned above, the following proposition can be obtained.

**Proposition 1.** When the retailer’s pricing \( p \) is the optimal pricing \( p^* \), the corresponding order quantity \( Q \) is also the optimal order quantity \( Q^* \).

**Proof of Proposition:** 1 the retailer’s optimal order quantity \( Q^* \) satisfies \( \partial \pi_1 (p) / \partial Q = 0 \), \( Q \) is a function of \( p \), and we get \( \partial \pi_1 (p) / \partial p = (\partial \pi_1 / \partial Q) \partial Q / \partial p \). If \( p = p^* \), there is \( \partial \pi_1 (p) / \partial Q \big|_{p^*} = 0 \), and then, \( (\partial \pi_1 (p) / \partial Q) \partial Q / \partial p = 0, \partial Q / \partial p = -(1 + a/b) \left( \frac{Aa/k}{1-a} \right) \right) e^{-(b/1-a)t} < 0 \). Hence, \( \partial \pi_1 (p) / \partial Q \big|_{Q^*} = 0 \). These steps complete the proof. \( \square \)

3.2. The Experiential Service System. The experiential service system is based on the traditional system. Specifically, the retailer invests in experiential services, such as try-on, fashion display, and apparel collocation, in order to enhance the consumer’s perception of the value of clothing fashion, thereby enhancing the consumer’s psychological value. Referring to the research on fresh perishable goods preservation investment in Li et al. [17] and Dye [25], this section considers the effect of experiential service investment on improving the initial fashion level of clothing. We assume that the experiential service investment (denoted by \( s \)) is a
continuous variable, \( s \in (0, +\infty) \), which represents the experiential service investment of the apparel retailer in unit time. The more the investment per unit time, the greater the effort of retailers’ experiential service in unit time. In order to be consistent with the law of diminishing marginal returns, it is assumed that the effect of service investment on fashion can be expressed as \((2 - e^{-rt})\omega_0\), that is, the initial fashion degree increases with the increase of service investment, but the incremental value is smaller and smaller. Hence, \( \lim_{t \to +\infty} (2 - e^{-rt})\omega_0 = 2\omega_0 \), that is, the initial fashion degree ultimately reaches a limit value \(2\omega_0\). In other words, we assume that the experiential service effort can double the psychological recognition of the initial fashion. Therefore, the fashion level function in an experiential service system is given by

\[
\omega(t) = (2 - e^{-rt})\omega_0 e^{-at}.
\] (20)

Experiential service means that the apparel retailer incurs the extra cost. The experiential service cost is assumed as follows:

\[
C_s = rs^2 \int_0^T D(t) \, dt,
\] (21)

where the positive parameter \( \tau \) is the cost coefficient of the experiential service effort. By substituting formula (20) into (1) and (2), the demand function and consumer utility function in the case of providing experiential services are obtained as follows:

\[
D_2 = \left( \frac{Aa}{k} \right)^{(1-a)} \omega_0^{(b(1-a))} (2 - e^{-rt})^{(b(1-a))} e^{-(bq(1-a)t)},
\]

\[
U_2 = k \left( \frac{1-a}{a} \right) \left( \frac{Aa}{k} \right)^{(1-a)} \omega_0^{(2a(1-a))} p^{(b(1-a))} (2 - e^{-rt})^{(b(1-a))} e^{-(bq(1-a)t)} + kY.
\] (22)

According to \( (\partial U_2/\partial s) > 0 \), the abovementioned consumer utility increases with the experiential efforts provided by the apparel retailer. Similar to that in Section 3.1, within a single cycle \( t \in [0, T] \), we can obtain the inventory level \( I(t) \), order quantity \( Q \), sales revenue \( SR \), inventory cost \( H \), order cost \( C_1 \), and experience effort cost \( C_s \) in the case of providing experiential service:

\[
I(t) = p^{-1(1+a/1-a)} \left( \frac{Aa}{k} \right)^{(1-a)} \omega_0^{(b(1-a))} \left( \frac{1-a}{-bq} \right) (2 - e^{-rt})^{(b(1-a))} e^{-(bq(1-a)t)},
\]

\[
Q = I_0 = p^{-1(1+a/1-a)} (2 - e^{-rs})^{(b(1-a))} G_1,
\]

\[
SR = pQ = p^{-1(2a(1-a)/1-a)} (2 - e^{-rs})^{(b(1-a))} G_1,
\]

\[
H = h \int_0^T I(t) \, dt = p^{-1(1+a/1-a)} (2 - e^{-rs})^{(b(1-a))} G_2,
\]

\[
C_1 = cQ + B = cp^{-1(1+a/1-a)} (2 - e^{-rs})^{(b(1-a))} G_1 + B,
\]

\[
C_s = rs^2 \int_0^T D_2(p, s, t) \, dt = rs^2 p^{-1(1+a/1-a)} (2 - e^{-rs})^{(b(1-a))} G_1.
\] (23)

Therefore, the total profit (denoted by \( \pi_2(p, s) \)) during the time interval \([0, T]\) is given by

\[
\pi_2(p, s) = \max_{p \in [0, +\infty]} \{ SR - H - C_1 - C_s \},
\]

\[
= (p - c - rs^2) p^{-1(1+a/1-a)} (2 - e^{-rs})^{(b(1-a))} G_1
\]

\[
- p^{-1(1+a/1-a)} (2 - e^{-rs})^{(b(1-a))} G_2 - B.
\] (24)

The necessary conditions for the retailer to maximize the total profit within a single cycle are

\[
\frac{\partial \pi_2(p, s)}{\partial p} = 0,
\]

\[
\frac{\partial \pi_2(p, s)}{\partial s} = 0.
\] (25)

Maximization of formula (24) with respect to \( p \) yields

\[
\frac{\partial \pi_2(p, s)}{\partial p} = \left( \frac{1}{1-a} \right) p^{-(2a(1-a)/1-a)} (2 - e^{-rs})^{(b(1-a))}
\]

\[
\cdot \left[ (1+a)(G_2 + (c + rs^2)G_1) - 2aG_1p \right] = 0.
\] (26)

By simplifying formula (26), the optimal pricing can be obtained as follows:

\[
p_* = \left( \frac{1}{1-a} \right) \left( G_2 + (c + rs^2)G_1 \right).
\] (27)

Maximization of the formula (24) with respect to \( s \) yields

\[
\frac{\partial \pi_2(p, s)}{\partial s} = p^{-1(1+a/1-a)} (2 - e^{-rs})^{-1(1-a)}
\]

\[
\cdot \left[ (p - c - rs^2)G_1 - G_2 \cdot \frac{b}{1-a}re^{-rs} \right. \left. - 2rs(2 - e^{-rs})G_1 \right] = 0.
\] (28)

The optimal experiential service effort level \( s_* \) of the fashion apparel retailer satisfies the following equation:

\[
br e^{-rs} (p - c - rs^2)G_1 - G_2 = 2(1-a)rsG_1 (2 - e^{-rs}).
\] (29)

Since both the equations (27) and (29) are highly nonlinear, it is difficult to obtain accurate solutions. However, Newton’s iterative method can be used to find approximate solutions of equations in the real field, so we use Newton’s iterative method to solve the system of simultaneous binary nonlinear equations. The specific algorithm is given in Section 3.4. By solving the model, the optimal pricing and order quantity of the fashion apparel retailer can be obtained in the experiential service system.

3.3. The Design Innovation System. In the era of pursuing fashion and individuation, enhancing the design of fashion clothing can improve the fashion level of products, further
making the marginal value increasing the effect of consumers on the valuation of garments. At the same time, strengthening the design of fashion apparel will also affect the supplier’s production methods, which will further change the situation of coexistence and shortage of fashion products and ultimately reduce the level of apparel inventory. From the perspective of supply chain collaboration, the retailer can order products from the supplier based on the analysis of consumer purchase history data. Obviously, the initial fashion level of garments plays an important role in the retailer’s ordering decision. In order to obtain fashion products with higher fashion levels, clothing retailers can provide clothing manufacturers with product design innovation subsidies, thereby improving the fashion level of clothing and reducing the decay rate of fashion levels. A smaller fashion level deterioration rate means a bigger subsidy which is charged by the manufacturer to the apparel retailer. The design innovation cost $C_\eta$ is introduced as follows:

$$C_\eta = \lambda (\eta - \eta_1)^2 \int_0^T D dt,$$

where the parameter $\lambda > 0$ represents the cost coefficient of the design innovation cost. $\eta$ represents the deterioration rate corresponding to the standard design; $\eta_1$ is an exogenous variable controlled by the apparel retailer. The smaller the $\eta_1$ is, the higher the fashion level of clothing is and the more fashionable the apparel is, but the higher the cost of the design subsidy will be.

The fashion level function of the design innovation system is

$$\omega_t = \omega_0 e^{-\eta_1 t}.$$  

Substituting (31) into (1) and (2), the utility function and the demand function in this section can be expressed as follows:

$$D_3 = \left( \frac{Aa}{k} \right)^{(1/1-a)} p^{- (1+a)/1-a} \omega_0^{(b/1-a)} e^{(-b\eta_1/1-a) t},$$

$$= k (1 - a) \left( \frac{Aa}{k} \right)^{(1/1-a)} p^{- (2a/1-a) \omega_0^{(b/1-a)} e^{(-b\eta_1/1-a) t} + kY.}$$

$$U_3 = \frac{k (1 - a) (Aa)}{a} \left( \frac{Aa}{k} \right)^{(1/1-a)} p^{- (2a/1-a) \omega_0^{(b/1-a)} e^{(-b\eta_1/1-a) t} + kY.}$$

Similar to that in Section 3.1, the order quantity is

$$Q_3 = p^{- (1+a)/1-a} G_3,$$

Within a single cycle time $t \in [0, T]$, the profit function of the retailer can be obtained as follows:

$$\pi_3(p) = \max_{p \geq 0} \left\{ SR - H - C_1 - C_\eta \right\},$$

$$= \left( p - c + \lambda (\eta - \eta_1)^2 \right) p^{- (1+a)/1-a} G_3 - p^{- (1+a)/1-a} G_4 - B,$$

where

$$G_3 = (Aa/k)^{(1/1-a)} \omega_0^{(b/1-a)} (1 - a / -b\eta_1) (e^{(-b\eta_1/1-a)t})^{-1} - 1)$$

and

$$G_4 = h (Aa/k)^{(1/1-a)} \omega_0^{(b/1-a)} (1 - a / -b\eta_1)^2 ((-b\eta_1/1-a) T + 1).$$

$\pi_3(p)$ is a function of $p$. Maximization of the formula (34) with respect to $p$ yields the optimal pricing:

$$p^*_3 = \frac{(1 + a) (c G_3 + \lambda (\eta - \eta_1)^2 G_3 + G_4)}{2a G_3}$$

As can be seen from equation (35), $p^*_3$ is affected by $\eta_1$ of the fashion level through the enhanced design input, that is, the optimal price of the apparel retailer is affected by the deterioration rate which is determined by the apparel retailer according to the actual situation of the market.

3.4. Fast Fashion System. From the discussion of Sections 3.2 and 3.3, it can be observed that the apparel retailer not only provides experiential services to enhance its initial fashion but also increases design subsidy investment to reduce the consumer’s psychological perception of the decay of clothing fashion, both of which can affect the operation decision of the clothing retailer. In this section, we analyze the impact of combining experiential services and design efforts in a fashion system. A fast fashion system employs both experiential services and design efforts.

The total profit of the fast fashion system is denoted by $\pi_4$. Specifically, we seek to answer the following question: are experiential services and design efforts complements or substitutes? If they are complements, then investing experiential services in a design innovation system results in a superadditive benefit: the incremental value of a fashion system (the change in profit over a traditional system) is more than the combined incremental value of experiential services and design efforts employed in isolation, i.e.,

$$\pi_4 - \pi_1 > (\pi_2 - \pi_1) + (\pi_3 - \pi_1).$$

Simplifying this expression, experiential service and design efforts are complements if and only if $\pi_4 - \pi_2 > \pi_3 - \pi_1$. For notational convenience, let $\Delta \pi_4 = \pi_4 - \pi_2$ and $\Delta \pi_3 = \pi_3 - \pi_1$. If $\Delta \pi_4 > \Delta \pi_3$, then experiential service and design efforts are complements. If $\Delta \pi_4 < \Delta \pi_3$, then experiential service and design efforts are substitutes.

The utility function and the demand function in the fashion system can be expressed as follows:

$$D_4 = \left( \frac{Aa}{k} \right)^{(1/1-a)} p^{- (1+a)/1-a} \omega_0^{(b/1-a)} (2 - e^{-rs})^{(b/1-a)} e^{(-b\eta_1/1-a) t},$$

$$U_4 = \frac{k (1 - a) (Aa)}{a} \left( \frac{Aa}{k} \right)^{(1/1-a)} p^{- (2a/1-a) \omega_0^{(b/1-a)} e^{(-b\eta_1/1-a) t} + kY.}$$

$$\cdot (2 - e^{-rs})^{(b/1-a)} e^{(-b\eta_1/1-a) t} + kY.$$  

(37)

Considering the combined effect of experiential service and design efforts, the profit of the retailer is
\[
\pi_4(p, s) = \max_{p \in [0, a]} \{SR - H - C_i - C_y - C_a\},
\]
\[
= (p - c - ts^2 - \lambda(\eta - \eta_1)^2)p^{(1+\alpha/1-a)} - (2 - e^{-\eta_1})^{(b(1-a)/a)}G_4 - p^{(1+\alpha/1-a)} - (2 - e^{-\eta_1})^{(b(1-a)/a)}G_4 - B.
\]

The necessary conditions for the retailer to maximize the total profit within a single cycle are
\[
\frac{\partial \pi_4(p, s)}{\partial p} = 0,
\]
\[
\frac{\partial \pi_4(p, s)}{\partial s} = 0.
\]

Maximization of formula (38) with respect to \( p \) yields
\[
\frac{\partial \pi_2(p, s)}{\partial p} = (\frac{1}{1-a}) p^{(2a-1-a)} (2 - e^{-\eta_1})^{(a(1-a)/a)} - [(1+a)(G_2 + (c + ts^2)G_1)] - 2aG_1 p = 0.
\]

By simplifying formula (40), the optimal pricing can be obtained as follows:
\[
p^*_4 = \frac{(1+a)(G_4 + (c + ts^2 + \lambda(\eta - \eta_1)^2)G_3)}{2aG_3}
\]

The optimal experiential service effort level \( s^*_4 \) of the fashion apparel retailer satisfies the following equation:
\[
br e^{-\eta_1}(\frac{p - c - ts^2 - \lambda(\eta - \eta_1)^2}{G_3} - G_4) = 2(1-a)ts(2 - e^{-\eta_1})G_3.
\]

Since both the equations (41) and (42) are highly nonlinear, it is difficult to obtain accurate solutions. We use the Newton iteration method to search and solve the design algorithm of (41) and (42).

Step 1: let \( f_1 = \partial_p \pi_4(p, s) = 0 \) and \( f_2 = \partial_s \pi_4(p, s) = 0 \), and the abovementioned simultaneous equations are denoted by \( f(\vec{x}) = 0 \), where \( \vec{x} = (p, s)^T \), \( f(\vec{x}) = (f_1, f_2)^T \). The Jacobian matrix of \( f(\vec{x}) \) is denoted by \( \tilde{f}(\vec{x}) \).

Step 2: it is assumed that \( \vec{x}^{(i)} = (\vec{p}^{(i)}, \vec{s}^{(i)}) \) is the solution to step \( i \) iteration of \( \tilde{f}(\vec{x}) = 0 \); thus, \( \vec{x}^{(i+1)} = \vec{x}^{(i)} - [\tilde{f}(\vec{x}^{(i)})]^{-1} \tilde{f}(\vec{x}^{(i)}) \).

Step 3: it is begun with \( i = 1 \) and the initial trial value of \( \vec{p}^{(i)} = p^*_4, \vec{s}^{(i)} = s^*_4 \).

Step 4: \( \vec{x}^{(i)} = (\vec{p}^{(i)}, \vec{s}^{(i)}) \) and \( \vec{x}^{(i+1)} = (\vec{p}^{(i+1)}, \vec{s}^{(i+1)}) \) are calculated.

Step 5: If \( \max[|p^{(i)} - p^{(i+1)}|, |s^{(i)} - s^{(i+1)}|] < 10^{-6} \), then \( (p^*_4, s^*_4) = (p^{(i)}, s^{(i)}) \) is set and proceed to Step 6. Otherwise, \( i = i+1 \) is set and we go to Step 4.

Step 6: Using this algorithm, we obtain the optimal solution \((p^*_4, s^*_4)\). We can obtain and \( \pi_4^*(p, t_1) \) using that in (38).

4. Numerical Analysis

To illustrate the solution procedure and the results, let us apply the proposed algorithm to solve following numerical example. The results can be obtained by applying the software Matlab2016a. We set \( k = 0.5, a = 0.35, b = 0.65, c = 1.5, h = 0.5, r = 0.5, A = 120, \omega_0 = 0.8, T = 15, Y = 10000, \eta = 0.015, \) and \( r = 1, \lambda = 2 \).

4.1. Optimal Solutions. Figure 2 shows that the profit \( \pi_4 \) of the retailer is a strictly concave function of the selling price \( p \) in the traditional system. As shown in Figure 2, the global maximum can be obtained at the local maximum point, that is, there exists an optimal sales price. The scenario of the design innovation system is similar to that of the traditional system. As can be seen from Figure 3, the profit function \( \pi_2 \) is strictly concave in \((p, s)\). Therefore, we ensure that a local maximum is the global maximum. The scenario of the fashion system is similar to that of the experiential system. Hence, we can conclude that the maximum searched by the algorithm could be viewed as the optimal solution of the problem.

4.2. Impact of the Preference Combinations \((a, b)\). In the case of providing experiential services, in order to better explain the model, we specifically analyze the impact of experiential service efforts and prices on retailer decisions under different combination preferences. From Table 2, it can be observed that when consumers’ fashion preference for apparel \( b \) is greater than price preference \( a \), the retailer should invest more in experiential service efforts to improve the initial fashion of the apparel. At this time, the sales price is gradually increasing. On the contrary, consumers have little preference for clothing fashion, and retailers lack the incentive to invest in service.

4.3. Impact of the Deterioration Rate \((\eta)\). Next, we will analyze the impact of the fashion level deterioration rate \( \eta_1 \) on the retailer’s decision-making in the case of enhancing design. Note that \( \eta_1 \) is exogenous variables. Here, we adjust the value of \( \eta_1 \) such that it varies from 0.007 to 0.015 at a step of 0.001 and keep the values of other parameters unchanged, as shown in Table 3. We can see from Table 4 that when \( \eta_1 = \eta = 0.015 \), the design cost function is \( C_n = 0 \). It represents that the apparel retailer did not invest in fashion design, which is equivalent to the traditional system. With decrease in \( \eta_1 \), that is, the increase in fashion design reduces the deterioration rate of the fashion level of an apparel, the price and order quantity are correspondingly enhanced, and the consumer’s utility and retailer’s profit are also increased. However, when \( \eta_1 \) reaches 0.009, the order quantity of the retailer does not decrease, and the profits and consumers’ utility reach an inflection point and, then, start to decrease. This is because of the decrease in fashion deterioration rate benefits from the design that enhances fashion. When it is reduced to a certain extent, the cost of the effort to enhance the design rises rapidly, and the retailer shares part of the
design cost through the order volume, thus offsetting part of the sales revenue. Therefore, investment in fashion design needs to be within a certain range. The optimal design investment can reduce the deterioration rate of the fashion level and increase the profit at the same time. However, excessive emphasis on the fashion level will inevitably increase the investment in fashion design and indirectly affect the income of the apparel retailer.

We have analyzed the impact of experiential service investment and design efforts on apparel retailers’ operational decisions. Next, we analyze whether experiential service and design efforts are substituted or complementary.

Consider four different scenarios: the basic scenario (the case where the apparel retailer provides neither experiential services nor enhanced design), the experiential service scenario alone, the enhanced design scenario alone, and the experiential service and enhanced design combination due to the retailer’s revenue are affected by the fashion level of clothing, so this paper will discuss the impact of the fashion deterioration rate on the optimal profit and, then, analyze whether the experiential services and design efforts are an alternative relationship or a complementary relationship.

The sensitivity analysis of the deterioration rate results is shown in Table 4.

Table 4 shows that the profits $\pi^*_4$, $\pi^*_3$, and $\pi^*_2$ are always larger than the profit $\pi^*_1$ in the traditional system. This result indicates that investing in experiential services alone, product design innovation alone, or implementing both strategies can help apparel retailers increase profits. Besides, the profit $\pi^*_4$ of the fashion system is larger than the experiential one $\pi^*_3$ until the deterioration rate of the fashion level reaches a certain threshold $\eta_1 = 0.015$. In other words, when $\eta_1$ is small ($\eta_1 \in (0.01, 0.015)$), $\pi^*_4 > \pi^*_3$, the retailer will further improve profits by offering design efforts on the basis of providing experiential service. Experiential service and promotion efforts are complements. When $\eta$ is relatively large ($\eta \in [0.015, 0.02)$), $\pi^*_4 < \pi^*_2$, and this is in contrast to previous policies. Moreover, we conduct a sensitivity analysis of the deterioration rate of the fashion level, which is shown in Figure 4.

As we can see from Figure 4, when $\eta_1$ is relatively large ($\pi^*_4 < \pi^*_2$), $\Delta \pi_2 < \Delta \pi_3$. Thus, experiential services and
design efforts are substitutes. Moreover, the difference between $\Delta \pi_{31}$ and $\Delta \pi_{42}$ is increasing in $\eta_1$. When $\eta_1$ is large (costume designing is relatively poor), the substitution effect is dramatic. This occurs because the larger the deterioration rate $\eta_1$ is, the faster the fashion level $\omega(t)$ fading is. If the costume’s design is relatively poor, it is difficult for the apparel retailer to improve the fashion level of clothing by providing experiential services. These apparels are very outdated, resulting in backlog. Besides, when $\eta_1$ is relatively small ($\eta_1 > \eta_2$), $\Delta \pi_{42} > \Delta \pi_{31}$. Thus, experiential services and design efforts are complementary. Moreover, the difference between $\Delta \pi_{31}$ and $\Delta \pi_{42}$ is decreasing in $\eta_1$. When $\eta_1$ is small (the design of the clothing is very novel), the complementarity effect is dramatic. This occurs because the smaller the deterioration rate $\eta_1$ is, the slower the fashion level $\omega(t)$ fading is. If the design of the clothing is very novel, the clothing retailer can enhance the brand by providing experiential services, thereby attracting more consumers to purchase products and obtain more profits.

5. Conclusions

The decline in the fashion level of fashion for fashion apparel has caused its market value to continuously decrease, reducing market demand and creating a backlog of clothing inventory. The demand for fashionable clothing is determined by the selling price and fashion degree of clothing during the selling period. Under these backgrounds, the apparel retailer invests in experiential services, such as providing virtual and physical try-on, to improve the fashion level of apparel and, thus, to increase the demand. On the other hand, the fashion apparel adopts a design innovation strategy, that is, the apparel manufacturer improves the innovative design of the apparel according to the requirements of retailers in order to produce an apparel that meets a certain attenuation criterion. We present a modeling framework that allows us to capture and isolate the key aspects that define a fast fashion system: experiential service and design efforts. By employing this approach, we analyze four potential operating systems—a traditional system (without experiential service and design efforts), experiential service system, design efforts system, and fast fashion systems (with experiential service and design efforts). We characterize the properties of the optimal solution (the selling price and experiential service investment) in each case and propose an iterative algorithm. This paper focuses on discussing the issue of whether experiential service and design efforts are complements or substitutes. We find that when the deterioration rate of the fashion level is relatively small, the incremental value of a fashion system (the change in profit over a traditional system) is more than the combined incremental value of experiential service and design efforts employed in isolation. In other words, the interaction of experiential service and design efforts has a complementarity effect when the design of the clothing is relatively novel.

In the proposed inventory model, the price was assumed as a static variable. As we all know, price strategy, as an important factor in the success of the business of the modern firms, has a significant effect on the consumers’ demand. The static price strategy ignores the fashion attribute of fashion apparel, which has been declining over time. In fact, dynamic pricing is a powerful tool to manage the demand of different quality level products. Therefore, according to the time-varying characteristics of fashion clothing, how to implement the time-varying dynamic pricing strategy would be worth exploring in the future. Besides, this paper considers the deterministic situation that we may also consider the stochastic demand in the model. The pricing and experiential service investment policy in the supply chain of fashion items should also be considered. Last but not the least, it would be useful to test our results through qualitative research and case studies.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also form part of an ongoing study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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