Ground State Baryons in Covariant Level-Classification Scheme

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In the covariant level-classification scheme of hadrons based on $\tilde{U}(12)_{SF} \otimes O(3, 1)_{L}$ symmetry, the ground state baryons and antibaryons are assigned to the $(12 \times 12 \times 12)_{\text{Sym}} = 364$-multiplet in the spin-flavor $\tilde{U}(12)_{SF}$ symmetry. This multiplet includes, in addition to the ordinary $56$-multiplet of the static $SU(6)_{SF}$, the extra chiral $56'$ and $70$-multiplets, called “chiralons,” which are expected to exist with masses in $1 \sim 2$ GeV region. Their electro-magnetic properties, magnetic moments and radiative decay amplitudes, are investigated. A longstanding puzzle, that the predicted value of the width $\Gamma(\Delta \rightarrow N\gamma)$ by the conventional treatment is inconsistently small with the experimental one, may be solved by the mixing effect of $56$-states with the chiral $56'$-states.

§1. Introduction

In the covariant level classification scheme of hadrons presented by one of the authors S. I. (referred to as II\textsuperscript{1}), the level spectra of hadrons including light quarks in lower mass region are classified as the representations of $\tilde{U}(12)_{SF} \otimes O(3, 1)_{L}$ group\textsuperscript{2}, which is a covariant generalization of $SU(6)_{SF} \otimes O(3)_{L}$ in the non-relativistic quark model (NRQM). The light quark baryons and anti-baryons in the ground state are assigned as the $(12 \times 12 \times 12)_{\text{Sym}} = 364$-multiplet in the spin-flavor $\tilde{U}(12)_{SF}$ group. This multiplet includes, in addition to the ordinary $56$ of the static $SU(6)_{SF}$, the extra $56'$ and $70$, which are predicted to exist with masses in $1 \sim 2$ GeV region and are out of the conventional framework. The essential reason for obtaining the extra multiplets is the inclusion of $\rho$-spin degree of freedom for the constituent quarks in addition to the ordinary Pauli- ($\sigma$-) spin, which correspond to the $\rho \otimes \sigma$ decomposition of Dirac $\gamma$-matrices. The $\rho$-spin freedom is necessary for covariant description of composite hadrons. The wave functions (WFs) of non-relativistic $56$ states correspond to the boosted Pauli-spinors, the multi-Dirac spinors consisting of Dirac spinors with positive $\rho_3$ spin. On the other hand the WFs of the extra $56'$ and $70$ states include the multi-Dirac spinors consisting of negative $\rho_3$-spin Dirac spinors, which is related to the ones with the positive $\rho_3$-component through chiral transformation. These extra states are called chiral states (“chiralons”).\textsuperscript{3}

In this talk we investigate the electromagnetic properties of ground state baryons, their magnetic moments and radiative decays. For this purpose, at first, we construct their covariant spinor-flavor WFs.
§2. Flavor-Spinor WF of Ground State Baryons

2.1. Spinor WF and chiral states

In order to make clear the physical background for the chiral states we describe the spinor WF of the ground baryons, neglecting the internal space-time variables. First we define the Dirac spinor for quarks \( W_q \) as BW spinors with single index by

\[
\psi_{q,\alpha}(X) = \sum_{P,r} [e^{iPX} W_{q,\alpha}^{(+)}(P) + e^{-iPX} W_{q,\alpha}^{(-)}(P)]
\]

\[
W_{q,\alpha}^{(+)}(P) = u_\alpha(P), \quad W_{q,\alpha}^{(-)}(P) = u_\alpha(-P).
\]

(2.1)

They take the following form at the hadron rest frame as

\[
W_{q,r}^{(+)}(P = 0) = \begin{pmatrix} \chi_r \\ 0 \end{pmatrix}, \quad \rho_3 = +, \quad W_{q,r}^{(-)}(P = 0) = \begin{pmatrix} 0 \\ \chi_r \end{pmatrix}, \quad \rho_3 = -. (2.2)
\]

It is to be noted that all Dirac spinors with positive and negative values of \( \rho_3 \) spin for quarks are required as members of complete set of expansion bases inside of hadrons. The spinor WF for ground states of baryons are given by tri-Dirac spinors as BW spinors with three indices as

\[
\Phi^{(B)}_{\alpha\beta\gamma}(P) = W_{q,\alpha}(P) W_{q,\beta}(P) W_{q,\gamma}(P)
\]

\[
\Phi_{\alpha\beta\gamma}(P = 0); (\rho_3^{(1)}, \rho_3^{(2)}, \rho_3^{(3)}) = (+, +, +) : \text{boosted Pauli states} \quad (2.3)
\]

\[
(+, +, -), (-, - ,-) : \text{“Chiral States”}
\]

The WF with \( (\rho_3^{(1)}, \rho_3^{(2)}, \rho_3^{(3)}) = (+, +, +) \) are the multi-boosted Pauli spinors, which reduce to the multi-NR Pauli spinors at the rest frame, while the WF with the other values of \( (\rho_3^{(1)}, \rho_3^{(2)}, \rho_3^{(3)}) \) describe the chiral states, which newly appear in the covariant classification scheme.

In our scheme hadrons are generally classified as the members of multiplet in the \( \tilde{U}_{SF}(12) \times O(3, 1) \) scheme. The light-quark ground state baryons are assigned to the representations \( (12 \times 12 \times 12)_{\text{Symm}} = 364 \) of the \( U(12)_{SF} \) symmetry. (See, Table I.) The numbers of freedom of spin-flavor WF in NRQM are \( (6 \times 6 \times 6)_{\text{Symm}} = 56 \) for baryons and \( 56^* \) for antibaryons: These numbers in COQM are extended to \( 364 = 182 \) (for baryons) + \( 182 \) (for anti-baryons).

Baryons: \( (12 \times 12 \times 12)_{\text{Symm}} = 364 = 182 + 182 \)

\[
\begin{array}{c|cc}
56 & \Delta^{3/2}_{1/2} & N^0_{1/2} \\
70 & \Delta_{1/2}^{(1)} & N^0_{3/2} \\
56^* & \Delta_{3/2}^{(2)} & N^0_{3/2} \\
\end{array}
\]

Table I. Quantum numbers of ground-state baryon multiplet in \( U(12)_{SF} \) symmetry
2.2. Flavor-Spin Decomposition of WF

The total baryon WF should be full-symmetric (except for the color freedom) under exchange of constituent quarks: The WF is denoted as $\Phi_{ABC}$, where $A = (\alpha, a)$ etc. and $\alpha (a)$ represents spinor (flavor) index. It is obtained as a product of the sub-space $\rho, \sigma$ and $F(\text{flavor})$ WF with respective symmetric properties: $\Phi_{ABC} = \langle ABC|F_{\rho\sigma}\rangle$. There are three ways to decompose the total symmetric WF, $|F_1F_2\rangle_S$, into the sub-space WF, $|F_1\rangle$ and $|F_2\rangle$:

$$|F_1F_2\rangle_S = |F_1\rangle_S|F_2\rangle_S, \quad |F_1\rangle_A|F_2\rangle_A, \quad \frac{1}{\sqrt{2}}(|F_1\rangle_\alpha|F_2\rangle_\alpha + |F_1\rangle_\beta|F_2\rangle_\beta), \tag{2.4}$$

where $|\rangle_S$, $|\rangle_\alpha(\beta)$ and $|\rangle_A$ represent the full-symmetric, $\alpha(\beta)$-type partial symmetric and full anti-symmetric subspace WF, respectively. Results are given in Table II.

The $|\sigma\rangle_S (|\sigma\rangle_{\alpha,\beta})$ corresponds to the total spin $\frac{3}{2}(\frac{3}{2})$ WF. The $|F\rangle_S (|F\rangle_{\alpha,\beta})$ corresponds to the $\Delta$-decouplet (N-octet). The $|F\sigma\rangle_S (|F\sigma\rangle_{\alpha,\beta})$ corresponds to the 56 (70) representation of static $SU(6)_{SF}$. The intrinsic parity is defined by 

$$\hat{P} = \prod_{i=1}^{3}(i) = \prod_{i=1}^{3}P_{3}(i). \quad |\rho, \frac{3}{2} (|= |++--\rangle) \text{ and } |\rho, -\frac{3}{2} (|= |---\rangle) \text{ have positive parity, while } |\rho, -\frac{3}{2} (|= |--\rangle) \text{ and } |\rho, \frac{3}{2} (|= |--\rangle) \text{ have negative one.}$$

| \text{positive-parity WF } |F_{rho\sigma}\rangle^P | \text{negative-parity WF } |F_{rho\sigma}\rangle^P |
|-----------------------------|-----------------------------|
| $E: 56$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_S$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_S$ |
| $\Delta_{1/2}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |
| $N_{1/2}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |
| $F: 56^\prime$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_S$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_S$ |
| $\Delta_{1/2}^\prime$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |
| $N_{1/2}^\prime$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |
| $G: 70$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_S$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_S$ |
| $\Delta_{1/2}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |
| $N_{1/2}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |
| $\Delta_{1/2}$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |
| $N_{1/2}$ | $|\rho, -\frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ | $|\rho, \frac{1}{2}\rangle_S|F\sigma\rangle_{\alpha,\beta}$ |

Table II. Flavor-spinor WF of ground-state $qqq_{12}H_3 = 364^*$-multiplet of $U(12)_{SF}$ symmetry.

As shown in Table II $364/2 = 182^*$ representation is decomposed, in static $SU(6)_{SF}$, into $56 + 56^\prime + 70$ representation.

The $\rho$-spin WF of $56 E^{\otimes}$, $|\rho, \frac{3}{2}\rangle_S$, consists of only positive-$\rho_3$ constituent quark spinor, and thus, it is related directly with non-relativisitc two-component Pauli-spin WF in NRQM. The $E^{\otimes}$ is its creation part. It is remarkable that there appear the extra $56^\prime$ $F$ and $70$ $G$ multiplets in the ground states.

The experimentally observed $56$-state (including $N(939)$ octet and $\Delta(1232)$ decouplet) is considered to be described by a superposition of non-relativisitve $E^{\otimes}$-WF and relativisitc $F^{\otimes}$-WF. We expect the existence of extra $56$ and $70$ states (chiral states), which have the WF orthogonal to the one of ordinary $56$. Among the experimentally observed baryons, $N^{\otimes}(1440)$, $\Delta^{\otimes}(1600)$ and $\Lambda^{\otimes}(1405)$, which have inconsistently light masses with the predictions by NRQM, are candidates for chirals.
2.3. WF in general frame and its conjugate

The WF in general frame are obtained by boosting the WF at rest frame, given in Table II, with the velocity \( v_\mu \equiv P_\mu / M \). For single quark,

\[
W(P) = B(v)W(0), \quad \bar{W}(P) = \bar{W}(0)\bar{B}(v)
\]

\[
B(v) = ch\theta + \rho_1\sigma_z sh\theta, \quad \bar{B}(v) = \gamma_4 B(v)\gamma_4 = ch\theta - \rho_1\sigma_z sh\theta \quad (\text{for } v \propto \hat{z}),
\]

\[
ch\theta = \sqrt{\frac{\omega + 1}{2}}, \quad sh\theta = \sqrt{\frac{\omega - 1}{2}}, \quad \omega = \frac{E}{M}, \quad |P| = 2ch\theta sh\theta.
\]

In the non-relativistic limit \(|v| \to 0\), the \( ch\theta = 1 \) and \( sh\theta = 0 \). The \( sh\theta \) represents the relativistic recoil effect.

The baryon WF with momentum \( P \), \( \Phi_{\alpha\beta\gamma}(P) \), is obtained by operating the booster \( B^{(i)} \) for respective constituents of \( \Phi_{\alpha\beta\gamma}(0) \), as

\[
\Phi_{\alpha\beta\gamma}(P) = B_{(1)}^{a}(v)B_{(2)}^{b}(v)B_{(3)}^{c}(v)\Phi_{\alpha'\beta'\gamma'}(0).
\]

In calculating the transition matrix elements of radiative decays, the Pauli-conjugate \( \bar{\Phi}(\equiv \Phi_{(1)}^{\dagger}\gamma_4^{(2)}\gamma_4^{(3)}) \) is necessary to be revised, in order to give correct charge for the WF with negative \( \rho_3 \)-spin, into “unitary conjugate” \( \bar{\Phi}_U \) defined by

\[
\bar{\Phi}_U(v) = \Phi(v)(-iv \cdot \gamma^{(1)})(-iv \cdot \gamma^{(2)})(-iv \cdot \gamma^{(3)}).
\]

\[
\bar{\Phi}_U(v) = \Phi^{(1)}(v)\Pi_{i=1}^{3}\bar{B}^{(i)}(v)
\]

\( \bar{\Phi}_U(v) \) coincides with \( \Phi^{(1)} \) at the rest frame \( v = 0 \). Because of \(-iv \cdot \gamma^{(i)}\), \( \bar{\Phi}_U(v)\Phi(0) \) is invariant under chiral transformation \( \Phi \to \Pi_{i=1}^{3}e^{i\alpha_i^0}\Phi \).

It should be noted that the above WF \( \Phi_{\alpha\beta\gamma}(v) \) are equivalent to those\(^*\) of manifestly covariant form given in ref.4).

§3. Electro-magnetic Property

3.1. Form of electromagnetic interaction

The electro-magnetic interaction of the constituent quark is given, in momentum representation, by

\[
j_{\mu}^{EM}(P',P)A(\mu)(q) = eQ(1)\left(\frac{1}{2M}(P_\mu + P'_\mu) + \frac{g}{2m_i}i\sigma^{(1)}_\mu q_\nu\right)A(\mu)(q)
\]

where \( j_{\mu}^{EM} = j_{\mu}^{con} + j_{\mu}^{spin} \). The first (second) term of Eq. (3-1) comes from the convection (spin) current \( j_{\mu}^{con} \) (\( j_{\mu}^{spin} \)). The photon \( A_\mu \) is supposed to couple to the first quark denoted by index (1). By using \( i\sigma_{ij}q_jA_i(q) = \sigma_kH_k(q) \) and \( i\sigma_{ij}q_jA_i(q) + i\sigma_{ij}q_jA_i(q) = -i\rho_1\sigma_iE_i(q) \), the second term is rewritten by

\[
\mu_i^{spin}A(\mu)(q) = \mu_i^{(1)}(\sigma_kH_k(q) - i\rho_1^{(1)}\sigma_kE_k(q)), \quad \mu_i^{(1)} = eQ(1)\frac{g}{2m_i}, \quad (3.2)
\]

\(^*\) The WF's are given in the form \( \Phi_{ABC} = \frac{\sqrt{2}}{2}\epsilon^{abc}D_{\alpha\beta\gamma}V_{[\alpha\beta\gamma]} + V_{\alpha\beta\gamma}V_{[\alpha\beta\gamma]} \), where the decoupled WF \( D_{\alpha\beta\gamma} \) (the singlet WF \( \epsilon^{abc}V_{[\alpha\beta\gamma]} \) ) has completely symmetric (antisymmetric) indices \( \alpha\beta\gamma \) and \( abc \) including \( \Delta^{G}_{1/2}, \Delta^{F}_{3/2} \) and \( \Delta^{G}_{1/2} \). While \( \epsilon_{abcd}N_{\alpha\beta\gamma}^{abc} \) has partially antisymmetric indices \( \alpha\beta \) and \( ab \) including \( N_{1/2}, N_{0}^{0} \) and \( N_{1/2}^{2/3} \).
where the $\sigma_kH_k$ term is the ordinary magnetic interaction appearing in NRQM, while the second term comes from the relativistic effect, where the $\rho$ and $\sigma$ spins couple to the electric field $E_k$ directly. It may be called “intrinsic electric dipole” interaction. Our effective current Eq. (3.1) is conserved in the symmetric limit ($M = M'$; $M(M')$ being the initial(final) meson mass in the relevant case of the transitions between ground state baryons).

### 3.2. Magnetic moment

By using Eq. (3.1) and the WF in table II, we can calculate the magnetic moments (m.m.),

$$m.m. = \langle F_0 \sigma | \sum_{i=1}^3 \mu_i^{(i)} | F_0 \sigma \rangle = \langle F_0 \sigma | 3\mu_1^{(1)} | F_0 \sigma \rangle,$$

(3.3)

at the rest frame, supposing the baryon being polarized along $z$-direction. The factor 3 comes from the full-symmetry of WF. The results are given in Table III (for octets), IV (for decuplets) and V (for singlet).

| $|N_{p}\rangle$ | $|\Delta\rangle$ | $|\Delta^\prime\rangle$ | $|\Delta^\prime\prime\rangle$ | $|\Sigma^+\rangle$ | $|\Sigma^-\rangle$ | $|\Xi^-\rangle$ | $|\Xi^\prime-\rangle$ | $|\Xi^\prime\prime-\rangle$ | $|\Omega^\prime\rangle$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\mu_+ = \mu_0$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ |
| Theor. | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 |
| Exp. | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 |

Table III. Magnetic moment of nucleon octets: Unit is the nuclear magneton $\mu_N = \frac{e}{2m_N}$. $\mu_{1/2} = -\mu_u/2$ is assumed. The values with underlines, which are used as inputs, give $\mu_u = 1.862\mu_N$ and $\mu_s = -0.613\mu_N$.

| $|N_{p}\rangle$ | $|\Delta\rangle$ | $|\Delta^\prime\rangle$ | $|\Delta^\prime\prime\rangle$ | $|\Sigma^+\rangle$ | $|\Sigma^-\rangle$ | $|\Xi^-\rangle$ | $|\Xi^\prime-\rangle$ | $|\Xi^\prime\prime-\rangle$ | $|\Omega^\prime\rangle$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\mu_+ = \mu_0$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ | $\mu_0 + \mu_2$ |
| Theor. | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 |
| Exp. | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 | 2.793 |

Table IV. Magnetic moment of $\Delta$ decuplets: Unit is the nuclear magneton $\mu_N$.

Here the $m.m.$ of $N(939)$ and $A(1116)$ are used as inputs, to determine $\mu_u = 1.862\mu_N$ and $\mu_s = -0.613\mu_N$ ($\mu_u = e/(2m_N)$ and $\mu_d = -\mu_u/2$ is assumed). These values correspond to $g_M/m_u = 1/0.336\text{GeV}$ and $g_M/m_s = 1/0.510\text{GeV}$.
Table V. Magnetic moment of a singlet

|   |   |
|---|---|
| A^+ | A^- |
| A^0_{56} | 0.106 |

Table VI. Form of WF of the relevant baryons:

|   |   |
|---|---|
| N(939), A(1116), Σ(1192) | c_1|N(F^0)| - s_1|N(E^0)| |
| N(1440) | c_1|N(F^0)| - s_1|N(E^0) | |
| Δ(1232) | c_1|Δ(F^0)| - s_1|Δ(E^0) | |
| Δ(1600) | - s_1|Δ(F^0)| - c_1|Δ(E^0) | |
| Λ(1405) | - s_1|N(F^0)| - c_1|N(E^0) | |

Table VII. Form of WF of the relevant baryons: c_1, (s_1) etc. are abbreviations of cosθ_1, (sinθ_1), and thus, c_1^2 + s_1^2 = 1.

In order to keep the successful values of m.m. in NRQM, we consider only the mixing among 56_\Lambda, 56_F, and 56E for N(939)-octet, neglecting the mixing with 70_G. For N(1440) the mixing with 70_E is also considered. For Λ(1405) we generally consider the mixing among four As, 3 octet Λ (in 56, 56', 70) and one singlet (in 70). The radiative decay amplitudes obtained by using these WFs are given in Table VII.
In the ideal case, case A, the WFs, corresponding to $c_1 = c'_1 = c''_1$ and $s_1 = s'_1 = s''_1$, are orthogonal to each other. In case B considering the possible mixing effects with radially excited states, we relax these relations.

($\Delta(1232) \rightarrow N\gamma$ and $\Delta(1600) \rightarrow N\gamma$) There is a longstanding problem in the radiative decay $\Delta(1232) \rightarrow N\gamma$. The decay width predicted by NRQM is small, and inconsistent with the experimental data.

In case A, $A_2 = -\frac{\sqrt{2}}{3\sqrt{3}} \left( (ch^3\theta + ch^2\theta s h\theta) (1 - \frac{4}{3} c_1^2 + \frac{4}{\sqrt{3}} c_1 s_1) \right)$ and $A_4 = -\frac{\sqrt{7}}{9} (ch^3\theta + ch^2\theta s h\theta)$ (in unit of $\frac{4\pi\alpha}{2\sqrt{3}} \sqrt{|q|}$). In non-relativistic limit, $ch\theta = 1$ and $sh\theta = 0$, and the helicity amplitudes take simple forms, $A_{NR}^2 = -\frac{\sqrt{2}}{3\sqrt{3}}$ and $A_{NR}^4 = -\frac{\sqrt{7}}{9}$.

In the actual case of $\Delta(1232) \rightarrow N(939)\gamma$, $sh\theta = 0.1366$, which represents the strength of relativistic recoil effect. In $A_2^2$ the second term proportional to $ch^2\theta s h\theta$ is dependent upon the mixing coefficients $c_1$ and $s_1$. The factor $(1 - \frac{4}{3} c_1^2 + \frac{4}{\sqrt{3}} c_1 s_1)$ can take the value from $-1$ to $\frac{5}{3}$, corresponding to $(c_1, s_1) = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$, respectively. In the latter case, the numerical values of $A_j$ are improved as

$$A_{2j}^{NR} = -0.187 \rightarrow A_{2j} = -0.236, \quad A_{4j}^{NR} = -0.108 \rightarrow A_{4j} = -0.126 \quad (3.6)$$

which are almost consistent with experimental data shown in Table VIII. Correspondingly the decay width is improved as $\Gamma_{NR}^{\Delta} = 379\text{keV} \rightarrow \Gamma_{theor}^{\Delta} = 581\text{keV}$, which is almost consistent with experimental value $\Gamma_{exp}^{\Delta} = 624 \sim 720\text{keV}$. By changing the mixing angle, $\Gamma_{theor}^{\Delta}$ can take the value from $353\text{keV}$ to $581\text{keV}$. Thus, the problem of the $\Delta \rightarrow N\gamma$ width may be solved by considering the relativistic effect. It is notable that, in case of maximum $\Gamma_{theor}^{\Delta}$ with $\frac{1}{2} s_1 c_1 - \frac{1}{\sqrt{3}} (c_1^2 - s_1^2) = 0$, the $\Delta(1600)$ decay amplitudes vanishes consistently with experiment. (In case B we show the result obtained by using $A_{2j}^2$ of $\Delta(1600)$ decay as input, which gives $(c_1' s_1 - c_1 s_1') = 0.11 \pm 0.09$.)
(N(1440) → Nγ) As shown in Table VII the effect of 70G-mixing represented by
s2 gives the ratio nγ/pγ = −1, which seems to be inconsistent with the experimental
value ∼ − 3 2. So we take s2 = 0. In case A, c′1s1 − s′1c1 = 0, and thus, both A 2
and A 1 vanish, being inconsistent with experimental data. In case B we take the value
c′1s1 − s′1c1 = −0.1728, which reproduces the data well.
(A(1405) → Aγ, Σ0γ) In case A of maximum ∆ → Nγ, (−c2 1 + s2 1 − 2 3s1c1) = 0,
and thus, the effect on A 2 from the mixing with 56E and 56F vanishes. So, we
consider only the mixing between octet A8,G and singlet A1,G in 70G, taking c2 = 1.
In case of A(1405) being purely A1,G (c3 = 1), (ΓAγ, ΓΣ0γ) = (98, 195)keV, while
in case of purely A8,G (s3 = 1), (0.3,17)keV. Both cases are inconsistent with the
experiment. By taking (c3, s3) = (0.478, 0.878), the experimental ΓAγ is reproduced,
and in this case ΓΣ0γ is predicted with 15.9keV, which seems to be consistent with
the present experimental values.

| Process                | λ  | A^theor_{case A}^ | A^theor_{case B}^ | A^{exp}(GeV^{-2}) | Γ^{tot}_{case A} | Γ^{tot}_{case B} | Γ^{exp}_{tot}(keV) |
|------------------------|----|-------------------|-------------------|-------------------|-----------------|-----------------|------------------|
| ∆(1232) → Nγ          |    | -0.236            | -0.126            | -0.255±0.008      | 581             | < 581           | 624~720          |
| ∆(1600) → Nγ          |    | 0                 | 0                 | -0.009±0.021      | 0               | 1~192          | 3.5~70           |
| p(1440) → pγ          |    | -0.065            | 0.044             | -0.065±0.004      | 0               | 150             | 150±18           |
| η(1400) → nγ          |    | 0                 | 0                 | 0.040±0.010       | 69              | 57±28          |
| A(1405) → Aγ          |    | -0.040            | able              | 0.040±0.005       | 27              | able           | 27±8             |
| A(1405) → Σ0γ         |    | -0.039            | to fit            | 0.041±0.005       | 16              | to fit         | 10±4             |

Table VIII. Predicted values of helicity amplitudes (in unit of GeV^{-2}) in comparison with experiments. Decay widths are also given (in unit of keV).

§4. Concluding Remarks

We have investigated the electromagnetic properties of ground state baryons in
the covariant level-classification scheme. The longstanding puzzle on reproducing the
large experimental ∆(1232) → Nγ amplitudes is solved by considering the mixing
with relativistic 56F states. This mixing explains simultaneously the vanishing
∆(1600) decay amplitudes. This fact strongly supports the validity of our covariant
level classification scheme. This scheme predicts the existence of many other chiral
states not describable in NRQM. The search for them is urgently required.

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