Squeezing enhanced nonlinearity sensing in dissipatively coupled optical systems

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In this manuscript, we propose a proposal to enhance the sensitivity of two cavities to its intrinsic nonlinearities by pumping one of the cavities. The system consists of two dissipatively coupled micro-ring cavities, one of which is driven by a squeezed laser. With a low pump rate, the spectrum of the system acquires a real spectral singularity. We find that this singularity is very sensitive to the pumping and nonlinearity of the system, and compared to the case of coherent drive, the sensitivity of the system to intrinsic nonlinearities at the singularity can be further increased by using the squeezed laser. Moreover, the scheme is robust against fabrication imperfections. This work would open a new avenue for quantum sensors, which could find applications in many fields, such as the precise measurement and quantum metrology.

I. INTRODUCTION

Hermiticity and real eigenvalues of the Hamiltonian in closed systems are the key postulate in the quantum mechanics. In recent years, it has been discovered that the axiom of Hermiticity can be replaced by the condition of parity-time ($\mathcal{PT}$) symmetry, leading to the foundations of non-Hermitian quantum mechanics \cite{1, 2}. Interestingly, the non-Hermitian Hamiltonians also exhibit entirely real eigenvalues when satisfying $[\mathcal{H}, \mathcal{PT}] = 0$, where $\mathcal{PT}$ is the joint parity-time operator. A more significant feature of such Hamiltonians is the breaking of the $\mathcal{PT}$ symmetry, in which the eigenspectrum switches from purely real to completely imaginary \cite{3–25}. This sudden $\mathcal{PT}$ phase transition is marked by the exceptional point (EP), associated with level coalescence, in which the eigenvalues and their corresponding eigenvectors simultaneously coalesce and become degenerate. Recently, the $\mathcal{PT}$ phase transition has been experimentally observed in various $\mathcal{PT}$ symmetric systems \cite{26–29}.

As a counterpart, the anti-parity-time ($\mathcal{APT}$) symmetry, namely the Hamiltonian of the system is anti-commutative with the joint $\mathcal{PT}$ operator (mathematically, $\{\mathcal{H}, \mathcal{PT}\} = 0$), has recently attracted great interest \cite{30–46}. In contrast to the $\mathcal{PT}$ symmetric system, the $\mathcal{APT}$ symmetric system does not require gain, but it can still exhibit EP with purely imaginary eigenvalues. This characteristic is of great significance for realizing non-Hermitian dynamics in the quantum domain without Langevin noise \cite{47}. Until now, several relevant experiments have been realized in different physical systems, including cold atoms \cite{37} , optics \cite{43}, magnon-cavity hybrid systems \cite{39}, electrical circuit resonators \cite{40}, and integrated photonics \cite{41, 42}.

Sensitivity enhancement based on EP has been demonstrated both theoretically and experimentally \cite{17–21, 45–55} in the particle detector \cite{17}, mass sensor \cite{51}, and gyroscope \cite{54}. It has been shown that if an EP is subjected to the strength $\epsilon$ of the linear perturbation, the frequency splitting (the energy spacing of the two levels) scales as the square root of the perturbation strength $\epsilon$ \cite{17–21, 45–55}. Recently, in the context of dissipatively coupled $\mathcal{APT}$ symmetric systems, a scheme was proposed to efficiently detect the nonlinear perturbations \cite{31}. This dissipatively coupled system has an imaginary coupling strength \cite{31}, resulting from the fact that the vacuum of the electromagnetic field can produce coherence in the process of spontaneous emission \cite{56}. Owing to this coherence, the system acquires a real spectral singularity which strongly suppresses the linewidth of a resonance spectrum, thereby drawing out a remarkable response. Particularly, near the coherence-induced singularity (CIS), the response $N$ behaves as $|N| \propto |U|^{-5/3}$, where $U$ quantifies the strength of the Kerr nonlinearity \cite{31}. Compared with EP-based sensors \cite{17–21, 45–55}, the sensitivity of the system to inherent nonlinearities has been greatly improved, and the protocol does not require any gains \cite{31}.

One of main limitations to the sensor sensitivity is quantum noises, which arise, for example, from random fluctuations due to the quantum nature of the electromagnetic vacuum field. In 1981, Caves clarified that the quantum noise could be sharply suppressed by replacing the ordinary vacuum states with the squeezed vacuum states \cite{57}. This has stimulated enormous interests in exploiting squeezed states to improve the measurement sensitivity of quantum interferometry beyond the standard quantum limit \cite{57–72}. Especially, in Ref. \cite{66}, the sensitivity of the detectors of the Laser Interferometer Gravitational-wave Observatory (LIGO) beyond the

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standard quantum limit has been enhanced by injecting the vacuum squeezed light into the interferometer.

Inspired by the LIGO experiment, in this manuscript, we theoretically propose a novel sensing protocol to enhance the detection of nonlinear effects. Our proposal is based on two dissipatively coupled micro-ring resonators, one of which is driven by the squeezed laser. The key point of our sensing protocol is that the spectrum of the dissipatively coupled system acquires a CIS at the low pump rate. In the vicinity of the CIS, the current sensing protocol exhibits a much larger nonlinear response compared with the previous work [31]. The advantages of this protocol mainly reflect in the following aspects: (i) it does not needs the APT symmetric prerequisite, making such configuration much more accessible, (ii) we only need to slightly increase the laser power to overcome the deleterious effects of the added fluctuation, and (iii) it opens up the potential of probing weak nonlinearities.

The remainder of this manuscript is organized as follows. In Sec. II, a physical model is introduced to describe the setup, and the dynamical equations for the system are derived. In Sec. III, we study the sensitivity of the system to weak nonlinearities with a squeezed-laser drive. In Sec. IV, we discuss the effect of the fabrication imperfections on the performance of the setup. In Sec. V, we discuss the experimental feasibility of the present scheme. Finally, the conclusions are drawn in Sec. VI.

II. MODEL AND DYNAMICAL EQUATIONS

The schematic diagram is sketched in Fig. 1. The setup consists of two dissipatively coupled micro-ring cavities, one of which is driven by a squeezed laser. In the rotating frame with respect to frequency \( \omega_p \) of the laser, the total Hamiltonian of the system reads \( (\hbar = 1) [31, 73] \),

\[
H = H_f + H_k + H_i + H_d,
\]

with

\[
\begin{align*}
H_f &= \Delta_a a^\dagger a + \Delta_b b^\dagger b, \\
H_k &= U(b^\dagger)^2, \\
H_i &= J(ab^\dagger + a^\dagger b), \\
H_d &= F(e^{-i\theta_p}b^\dagger b + e^{i\theta_p}b^\dagger b^\dagger).
\end{align*}
\]  

FIG. 1: Sketch of dissipative couplings between the cavity modes \( \tilde{a} \) and \( \tilde{b} \) through the shared dissipative environment. \( \gamma_a \) and \( \gamma_b \) denotes dissipation rate of the system \( a \) and \( b \), respectively. \( \sigma \) stands for the cooperative interactions between the two modes and the common reservoir. \( J \) is the direct coupling between the cavity modes. The \( \tilde{b} \) is the mode of cavity made of the Kerr-type nonlinear material with the third-order nonlinear susceptibility \( \chi^{(3)} \) and is driven by the pump laser with the amplitude \( F \).

Here \( H_f \) represents the free Hamiltonian of the uncoupled cavity modes \( \tilde{a} \) and \( \tilde{b} \), \( a^\dagger (b^\dagger) \) and \( a (b) \) are the creation and annihilation operators of the mode \( \tilde{a} (\tilde{b}) \), respectively. \( \Delta_j = \omega_j - \omega_p/2 \) \( (j = a, \tilde{b}) \) represents the detuning of the modes \( \tilde{a} \) and \( \tilde{b} \) with respect to the laser field, and the frequencies of the modes \( \tilde{a} \) and \( \tilde{b} \) are \( \omega_a \) and \( \omega_b \), respectively. The Hamiltonian \( H_k \) describes the Kerr nonlinearity of the mode \( \tilde{b} \), and the strength is denoted by \( U \). Note that the other nonlinearities such as the thermo-optical effect are not considered here, because the thermo-optical nonlinearity is extremely small in low temperature[74]. The Hamiltonian \( H_i \) describes the direct coupling between the modes with coupling strength \( J \), and \( H_d \) indicates that the mode \( \tilde{b} \) is nonlinearly driven by a squeezed laser with the amplitude \( F \), the frequency \( \omega_p \), and the phase \( \theta_p \). Throughout this manuscript, we set \( \theta_p = 0 \). Physically, a squeezed laser can be obtained by means of the degenerate parametric down-conversion [75–78]. On the other side, the dissipative environment can be roughly divided into two categories—one is that the modes are coupled independently to their local reservoirs, and the other is that a common reservoir interacts with both, as shown in Fig. 1.

A complete description of the two-mode system interacting with the dissipative environment is the master equation in the Lindblad form [56, 79],

\[
\frac{dp}{dt} = -i[H, \rho] + \gamma_a \mathcal{L}[a] \rho + \gamma_b \mathcal{L}[b] \rho + \sigma \mathcal{L}[c] \rho,
\]

where the second and third terms represent the intrinsic damping of the modes \( \tilde{a} \) and \( \tilde{b} \), respectively. The fourth term describes the cooperative interactions between the two modes and the common reservoir. The standard dissipative superoperator \( \mathcal{L}[\rho] \) is defined by \( \mathcal{L}[\rho] = 2\rho_{\tilde{a}a} \rho_{b\tilde{b}} - \rho_{\tilde{a}a} \rho_{b\tilde{b}} - \rho_{\tilde{a}a} \rho_{\tilde{a}a} \rho_{b\tilde{b}} - \rho_{b\tilde{b}} \rho_{\tilde{a}a} \rho_{b\tilde{b}}, \) and the jump operator \( c \) is a linear superposition of the annihilation operators \( a \) and \( b \), \( c \rightarrow \nu a + u e^{i\theta} b \). If the phase difference \( \theta \) of light propagation from one mode to another is a multiple of 2\( \pi \), the jump operator has the general form \( c \rightarrow \nu a + ub \) [79], where the coefficients \( \nu \) and \( u \) represent the cou-
plings of the two modes to the common reservoir, respectively. If the two modes are symmetrically coupled to the common reservoir, the operator $c$ is expressed as $c = (1/\sqrt{2})(a + b)$. The external damping rates induced by the common reservoir for the two modes are $\sigma \cdot \nu^2 = \kappa_a$ and $\sigma \cdot \nu^2 = \kappa_b$, respectively. The cooperative dissipations between the two modes is $\sigma \cdot \nu^2 = \sqrt{\kappa_a \kappa_b}$, where the $\sqrt{\kappa_a \kappa_b}$ represents the effect of quantum interference resulting from the cross coupling between the two modes. Without loss of generality, we assume that the parameters $\gamma_j$ and $\kappa_j$ ($j = a, b$) are the same for the whole system, i.e., $\gamma_a = \gamma_b = \gamma_0$, and $\kappa_a = \kappa_b = \Gamma$.

III. EFFECTIVE HAMILTONIAN AND THE SENSITIVITY OF THE SYSTEM TO WEAK KERR NONLINEARITIES

Starting from the Lindblad master equation in Eq. (3), we can obtain the mean value equations for the modes $\hat{a}$ and $\hat{b}$ via the relation $\langle \hat{\zeta} \rangle = \text{Tr}(\hat{\beta} \hat{\zeta})$ [56]

$$H_{\text{eff}} = \begin{pmatrix}
\Delta_a - i(\gamma_0 + \Gamma) & J - i\Gamma \\
J - i\Gamma & \Delta_b - i(\gamma_0 + \Gamma) + 2\hat{U} \\
0 & 0 \\
0 & -2F
\end{pmatrix},$$

where $\hat{U} = |\beta|^2$. It can be seen that the effective Hamiltonian (5) does not have the $\mathcal{APT}$ symmetry. Therefore, our system is easier to obtain than the previous schemes [31]. The effective Hamiltonian $H_{\text{eff}}$ has four eigenvalues forming two pairs and one pair is due to the appearance of $\alpha^*$ and $\beta^*$ in the dynamics.

In the limit of the weak drive amplitude $F$, we can bring the effective Hamiltonian into a block diagonal form, and we will study the block corresponding to $\alpha$ and $\beta$ in the following,

$$\hat{H}_{\text{eff}} = \begin{pmatrix}
\Delta_a - i(\gamma_0 + \Gamma) & J - i\Gamma \\
J - i\Gamma & \Delta_b - i(\gamma_0 + \Gamma)
\end{pmatrix},$$

where $\Delta_b = \Delta_b + 2\hat{U}$. Without loss of generality, we choose the parameter as follows, $\Delta_a = -\Delta_b = \delta/2$, $J = 0$, and $\hat{U} = 10^{-3} \Gamma$, similar to the parameters chosen in Ref. [31]. The eigenvalues of Eq. (6) are given by

$$\lambda_{\pm} = \hat{U} - i(\gamma_0 + \Gamma) \pm \frac{1}{2} \sqrt{4\hat{U}^2 - 4\Gamma^2 - 4\hat{U}\delta + \delta^2}$$

$$\approx -i(\gamma_0 + \Gamma) \pm \sqrt{\frac{\delta^2}{4} - \Gamma^2}. \quad (7)$$

With the intrinsic damping $\gamma_0$ of the mode approaching zero, one of the eigenvalues characterizing its dynamics tends to the real axis at $\delta = 0$. The dissipative coupling strength $i\Gamma$ can be viewed as an effective gain that offsets exactly the external dissipation of the coupled resonance.

The solid lines in Fig. 2 (a) and (b) show the real ($\omega$) and imaginary parts ($\gamma$) of the eigenvalues at $\hat{U} = 10^{-3} \Gamma$ [see Eq. (7)] as a function of the detuning $\delta$. For comparison, we numerically solve the eigenvalues of Eq. (5) at $F = 0.01 \Gamma$ and $\hat{U} = 10^{-3} \Gamma$ [see the circles and squares in Fig. 2 (a) and (b)]. We see that the numerical and analytical results are highly agreement at a weak drive amplitude $F$. This validates the approximations we made in the calculations. The spectrum of the dissipatively coupled system acquires a CIS in the limit $\delta = 0$ and $\gamma_0 = 0$, as marked by the star in Fig. 2 (b). The extreme condition $\gamma_0 = 0$ holds when none of the cavity modes suffers spontaneous emissions from the surroundings while interacting with the mediating bath. The CIS has prodigious sensing potential, allowing efficient detection of nonlinear effects in the configuration [31]. The physical origin of this peculiar behavior comes from an effective coupling induced between two modes in the presence of a shared
reservoir.

The CIS was exploited to measure the nonlinear effect with the coherent drive in Ref. [31]. Here, we elaborate a novel detection strategy with the squeezed-laser drive. Specifically, we apply this approach to detect the Kerr nonlinearity in the cavity. The key measurement quantity, in this case, is the steady-state response of the system to the parameter change of the nonlinearity. Solving the steady-state solutions of Eq. (4), we obtain

$$-i[\delta/2 - i\gamma]\alpha - \Gamma\beta = 0,$$

$$-\Gamma\alpha - i[-\delta/2 - i\gamma]\beta - 2U|\beta|^2\beta - 2iF\beta^* = 0,$$

$$i[\delta/2 + i\gamma]\alpha^* - \Gamma\beta^* = 0,$$

$$-\Gamma\alpha^* + i[-\delta/2 + i\gamma]\beta^* + 2iU|\beta|^2\beta^* + 2iF\beta = 0.$$

Eliminating $\alpha$ and $\alpha^*$, we get

$$\Gamma^2\beta + (i\delta/2 - \gamma)\beta - 2U|\beta|^2\beta - 2iF\beta^* = 0,$$

$$\Gamma^2\beta^* - (i\delta/2 - \gamma)\beta^* + 2iU|\beta|^2\beta^* + 2iF\beta = 0,$$

where $\gamma = \gamma_0 + \Gamma$. Defining $N = |\beta|^2$, the response $N$ satisfies a square relation

$$4U^2N^2 - \frac{2U\Theta\delta}{\gamma^2 + (\delta/2)^2}N + \frac{\Theta^2}{\gamma^2 + (\delta/2)^2} = 4F^2,$$

where $\Theta = -\Gamma^2 + \gamma^2 + (\delta/2)^2$. This response $N$ in Eq. (10) can be analytically obtained as,

$$N_\pm = \frac{1}{8U}\left[\Xi \pm \sqrt{\Xi^2 - 16(\Lambda - 4F^2)}\right],$$

where $\Xi = \frac{2\Theta\delta}{\gamma^2 + (\delta/2)^2}$ and $\Lambda = \frac{\Theta^2}{\gamma^2 + (\delta/2)^2}$.

Next, we demonstrate the role that the CIS plays in the sensing of nonlinearity. In Fig. 3, we numerically plot the response $N$ [see Eq. (10)] as a function of the detuning $\delta$ at three different strengths of the Kerr nonlinearity. The response shows a striking response to $U$ around $\delta = 0$ (corresponding to the CIS). A weaker nonlinearity begs a higher response. Especially, $\Theta$ becomes extremely small at the amplitude $\delta = 0$ and $\gamma_0 = 0$. Thus, the response $N$ [see Eq. (11)] is further simplified as,

$$N \approx \frac{F}{U}.$$  

Therefore, near the CIS, the response $N$ becomes drastically sensitive to variations in $U$. We define the sensitivity of the system to intrinsic nonlinearities as follows,

$$S = \left|\frac{dN}{dU}\right| = \left|\frac{F}{U^2}\right|.$$  

To reveal the advantage of our sensing protocol, we plot the sensitivity of the system to inherent nonlinearities as a function of the strength $U$ of the Kerr nonlinearity for two different laser field (see Fig. 4). The sensitivity of the system to intrinsic nonlinearities with the coherent drive is $S_0 = \left|\frac{2}{3}\left(\frac{k^2}{4\pi\sigma}\right)^{1/3}\right|$ [31]. The sensitivity $S$ of the sensor has been greatly improved compared to the sensitivity $S_0$, as shown in Fig. 4.
FIG. 4: The sensitivity of the system at the CIS to the Kerr nonlinearity $U$ vs $U$. The orange dashed line denotes the sensitivity of the system to the nonlinearities at the CIS under the coherent drive [31], while the blue solid line for the squeezed-laser drive. Other system parameters chosen are the same as in Fig. 2. Notice that the coordinate scales for $S$ and $S_0$ are different.

IV. THE EFFECT OF FABRICATION IMPERFECTIONS ON THE SENSITIVITY

The present scheme works for zero cavity-cavity couplings and $\Delta_\tilde{a} = \delta/2 = -\Delta_\tilde{b}$. In realistic scenarios, however, fabrication imperfections are unavoidable. In this section, we investigate the effects of fabrication imperfections on the performance of the scheme. Firstly, for a system consisting of two cavities with the non-zero coupling $J$ ($J \ll \Gamma$). The eigenvalues of Eq. (7) become $\tilde{\lambda}_\pm = -i\Gamma \pm \sqrt{(J-i\Gamma)^2 + \delta^2/4}$ at $\gamma_0 = 0$. We get a near-CIS around $\delta = 0$. We numerically plot the real [Fig. 5 (a)] and imaginary parts [Fig. 5 (b)] of the eigenvalues as a function of the detuning $\delta$. The small value of $J$ will lift the EP degeneracy, disrupting the performance of the EP-based sensor, as manifested in Fig. 5 (a) and (b). In contrast to the EP degeneracy, the CIS still exists [see Fig. 5 (b)]. Secondly, for a small mismatch $\epsilon$ in the magnitudes of the two frequency detunings, $\Delta_\tilde{a} = \delta/2, \Delta_\tilde{b} = -\delta/2 + \epsilon$ ($\epsilon \ll \delta$), we can get a similar conclusion. Furthermore, the mismatch $\epsilon$ results in a decrease in the response (see Fig. 3), and this decrease can be cancelled by slightly increasing the drive amplitude $F$ [see Fig. 5 (c)]. In this sense, the present work provides a new way for CIS-based sensor that is robust gainst defects or fabrication imperfections.

V. EXPERIMENTAL FEASIBILITY

Owing to recent progress in nanofabrication, our sensing protocol can be realized in experiments [80, 81]. Here, we consider a silicon integrated photonic apparatus comprising two micro-ring resonators, both of them are coupled to a one-dimensional (1D) waveguide, as depicted in Fig. 6. The two micro-ring resonators have a radius of 3.1 $\mu$m and the waveguide width of 0.4 $\mu$m. The gaps between the micro-ring resonators and the waveguide are 0.1 $\mu$m. In the setup, the silicon has a negligible intrinsic damping in the communication band ($\sim 1550$ nm). The resonant frequency can be tuned precisely by the electro-optic effects. Owing to the large distance between the two resonators, the direct coupling between them can be ignored. Thus, such a configuration constitutes a nice benchmark to test our protocol.

To make our protocol work, a driving laser of the frequency $\omega_p$ is applied to the mode $\tilde{b}$, the field amplitudes are given by $F = \sqrt{\kappa_j P_L/\hbar \omega_p}$ ($j = \tilde{a}, \tilde{b}$), where $P_L$ is the pump power. The Kerr nonlinear coefficient is given by [82]

$$U = \frac{\hbar c^2 \lambda_0 n_2}{n_0^2 V_{\text{eff}}},$$  \hspace{1cm} (14)

with

$$n_2 = \frac{3}{4n_0^2 c^2} \chi^{(3)},$$  \hspace{1cm} (15)

where $c$ and $V_{\text{eff}}$ are the speed of light in vacuum and
FIG. 6: Schematic of the dual-resonator system that demonstrates CIS. The two micro-ring resonators are dissipatively coupled through a 1D waveguide. The effective dissipative coupling strength between the two modes is $i\Gamma$. The mode $\tilde{b}$ denotes the mode in the cavity made of the Kerr-type nonlinear material and driven by the pump laser with the amplitude $F$.

the effective mode volume of the resonator, respectively. For a high $Q$-factor ($10^6 \sim 10^8$), $V_{\text{eff}}$ is typically between $10^2 \sim 10^4 \mu m^3$ [83], $n_0$ is the linear refractive index of the material with typical values $2 \leq n_0 \leq 4 \text{ cm}^2/\text{W}$, $n_2$ is the nonlinear refractive index of the material with typical values $10^{-14} \leq n_2 \leq 10^{-17} \text{ cm}^2/\text{W}$ [84], $\varepsilon_0$ is the vacuum dielectric constant, and $\chi^{(3)}$ is the third-order nonlinear susceptibility.

To bridge the theoretical protocol and experiments, we use experimentally feasible parameters [80], $\Gamma = \kappa_{\tilde{a},\tilde{b}} = 1334 \text{ GHz}$, to demonstrate the feature of the CIS and the sensing enhancement at the CIS. Figures 7 show the eigenvalues and the corresponding sensitivities. One can find that the CIS feature appears for $\delta = 0$ [see Fig. 7 (b)]. And from Fig. 7 (c) we can observe that the sensitivity is significantly enhanced by exploiting the CIS. This suggests that the sensing protocol works good in detecting the nonlinear perturbations.

VI. CONCLUSION

Taking the coherence-induced singularity (CIS) and the advantages of squeezed states into account, we proposed a sensing scheme to enhance the sensitivity of the system to weak nonlinearities in the system. The system consists of two dissipatively coupled micro-ring cavities, one of which is nonlinearly driven by the squeezed laser. At the low pumping rate, the spectrum of the dissipatively coupled system acquires a CIS, which exhibits high sensitivity to weak nonlinearities. The physical origin of this peculiar behavior lies in the effective coupling induced between two modes in the presence of a common reservoir. Compared to the case of the coherent drive [31], the pumping of squeezed laser increases drastically the sensitivity. We illustrate the sensing capabilities in a system consisting of a silicon integrated photonic apparatus with two micro-ring resonators and a 1D waveguide. Our scheme is robust against the fluctuations and open new avenue for weak nonlinearities. It is worth noting that our scheme does not require the $\mathcal{APT}$ symmetric prerequisite, and can be extended to a plethora of systems, including, laser-cooled atomic ensembles [30], magnon-photon systems [85–93], superconducting transmon qubits [94], optomechanical systems [95–98].

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