Analytical Study of Half Scissor Like Elements Deployable Structures

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Abstract. The scissor based deployable structures have been widely adopted in various fields of engineering, architecture and technical application. This technique is currently used in prefab modular construction due to its advantages of fast assembled, ease to transport, save launching space, lightweight and embracing sustainable approach. Due to this, the deployable structures have attracted researchers around the world to introduce various innovative ideas in transforming the prefab modular construction industry towards future challenges. However, there is a crucial work at preliminary design stage for scissor based deployable structures since it is highly sensitive to the geometry compatible issue. Thus, the initial design is critical to ensure the stability of the deployable structures during folding, deployment and load bearing. To address this issue, this paper aims to simulate the kinematic behaviours of Half-Scissor Like Elements (H-SLEs) deployable mechanism for spatial deployable structure application under its self-weight. The kinematic equilibriums are formulated to simulate the structural behaviours of H-SLEs deployable mechanism under its own weight from 0° as beam to 90° as column. The results obtained indicated those length and deployment angles are two main parameters impact H-SLEs deployable mechanism under its own weight. Through these studies, it would be beneficial to the designer in predicting the H-SLEs deployable structures behaviours at preliminary design stage.

1. Introduction

The scissor mechanisms (SMs) based deployable structure with simple linear members connected, lightweight, compact, rapid assemblies and embracing sustainable principle currently gained popularity worldwide [1]. This deployable structures required deployable mechanism to connect each member to transmit force, motion [2] and implement morphological changes [3]. It behaves as mechanism during deployment and load bearing at restraint deployed configuration [4]. The SMs consists of two beams connected by pivot at an intermediate point [5] and hinged at their end nodes to connect others structures. Hence, this paper presents the Half-Scissor Like Elements (H-SLEs) mechanism deployable structures and analyse based on geometrical approach [6] and static analysis.
A reviewed on past scissor mechanisms deployable structures researches have introduced various innovative ideas and solutions based on problems diagnosed since first introduced by Pinero in 1961 [6]. Amongst that were additional cable restraints methods for system stabilization [7], sliding and flexible joints for catastrophic failures [8], strong pull for snap through action [9], flat slabs and curved structures for geometrical compatible issue [10], tension wire or membrane for stiffness problems [11], multi angulated pantographic structures for perimeter inconsistency [12], Modified SLEs for geometry shape [2], spatial scissor system for scissor shells polyhedral deployable structures [13] and etc. In 1989, self-stable scissor deployable structures were first introduced and nonlinear load–displacement curve was traced using finite element formulas and maximum member’s internal forces during deployment [1]. Besides, virtual experimental modal analysis was used to identify the effects of joint clearance size, gravity and link flexibility on the dynamic characteristics of deployable space structures [14]. The boundary controllers approach was used to dissipate the unwanted vibration and rotation energy of the deployable flexible beam [15]. During design stage, additional secondary inner SLEs and geometric incompatible principle also adopted to achieve system stability [16]. Therefore, analysis at preliminary design stage is importance to avoid failure issues during deployment and service periods.

The recent research development on scissor mechanisms spatial deployable structures is more focused on their geometry with expanded behaviours at deployment stage [17]. In 2006, [18] has successfully demonstrated deployable space structures with compact folding while maintaining the maximum expansion for building. The hinge joint dimension and misalignment will cause deployable structures imperfections [19]. Besides, slide radial joint [12], pin jointed support [20], fixed point rotation, hybrid of scissor hinge and common hinge [16] was used for deployable structures instability problems due to geometry issue. The additional constraint for compression uniform pantographic column was used to improve axial stiffness and postponed snap through buckling [21]. Besides, the scissor mechanisms deployable structure height and deployment angle are two parameters impact on structure instability measurement [5]. In addition, the transformable structures is a novel architecture towards better use of natural energy resources, better occupant comfort, space use and building function flexibility[22]. Due to this, scissor based deployable structures become a hot topic to be explored currently.

Therefore, this paper establishes a novel stability model to determine the structural behaviours of H-SLEs deployable mechanism spatial deployable structure under its own weight using kinematic equilibrium and investigate the main geometry parameters impact on the deployable structure performance. In principle, when the applied service loads exceed the design critical value, the structure may be destroyed and its shape changed or not precision due to the instability even not reach their material own yield limit [23]. When the degree of deployment exceeds the expected range, the deployable structure will automatically collapse or unstable under the influence of gravity. Therefore, in the design of scissor mechanisms spatial deployable structures, it is very important to predict in advance the structural behaviours for determined number of elements, deployment angle and geometric properties of the structure to meet the load requirements for actual working conditions to avoid structural instability.

2. Kinematic equilibrium of the H-SLEs deployable mechanism for spatial deployable structure

The H-SLEs deployable mechanism unit is connected by two beams with bolt connection at their one end node and hinges at the end node. The bolt connection position becomes the centre of symmetry consists of same bars length and bolt connection is at the mid-point between these bars end nodes. When the bottom node hinged to ground and the top node moves vertically up-right position, the planer structure (column) is formed and restraint is required to fix the bolt joint to become load bearing structure as showed in Figure 1.
2.1 The planer H-SLEs deployable mechanism analysis equilibriums

The motion path of the planer H-SLEs deployable mechanism can be determined by using geometric principle approach such as trigonometric which apply cosine rule in determined the transformation behaviour such as deployment angle ($\gamma$), bar length ($l$) and displacement ($\Delta$). When deployment takes place, the H-SLEs deployable mechanism can be divided into top and bottom part by centroid point “Q1” and geometrically symmetry ($Q_1PD_1 = Q_1R_1S$) as showed in Figure 2(b).

![Planer H-SLEs deployable mechanism](image)

**Figure 2.** Planer H-SLEs deployable mechanism

When the H-SLEs mechanism deploy with certain degree $\gamma$, point Q1 will displace in linear direction measured in length (mm). The motion of the H-SLEs deployable mechanism will displace in $x$ ($\Delta x$) and $y$ ($\Delta y$) axes, ($Q_1PD_1$). The bolt joint at $Q_1$ is free to rotate and the pulling force upward at point R will make this H-SLEs deployable mechanism deploy upward to form a column. The increment of deployment angle from $\gamma$ to $\gamma_1$ will further increase the displacement ($\Delta x$ to $\Delta x_1$, $\Delta y$ to $\Delta y_1$) as demonstrated from triangle RQP to R$_1$Q$_1$P. AT 90° vertical configuration, there is zero bending stiffness and once exceeds 90°, the H-SLEs deployable mechanism become geometrically unstable [23]. To obtain the displacements equations, consider the bottom part of the H-SLEs deployable mechanism represented by triangle PQ$_1D_1$ (Figure 2b). The deployment angle will cause the displacement $x$ and $y$ at the same time such as triangle PQ$_1D_1$, PQ$_2D_2$ which proportional to the deployment angle vary ranging $0° \leq \gamma° \leq 90°$. Thus, the displacements can be written as follow:

$$
\Delta_x = l_0 \sin \Delta \gamma
$$

(1)

$$
\Delta_y = l_0 - l_0 \cos \Delta \gamma
$$

(2)

where $\Delta_x$ is horizontal displacement, $\Delta_y$ is vertical displacement, $\Delta \gamma$ is deployment angle changes and $l_0$ is the original bar length. Based on (1) and (2), the motion path of the H-SLEs deployable mechanism at displacements $x$ and $y$ axes are dominated by deployment angle $\gamma$ and is proportional to the bar length $l_0$ changes.

The planer H-SLEs deployable mechanism is a symmetry characteristic (AB=BC) and by taking lower part, the self-weight is the transverse loading occurs along the horizontal member called beam. The internal forces will create the shear ($V$) and moment ($M$) due to the beam self-weights ($w$). The beam is assumed vertical, longitudinal plane of symmetry and the uniformly distributed load ($w$) is...
applied in that plane with the deployment angle varies ranging 0° ≤ γ ≤ 90°. By applying static equilibrium with cut section O-O at beam BC (Figure 1b), the shear forces (V) and bending moment (M) equations can be written as follow:

\[ V = w \cos \gamma \left( \frac{l}{2} - x \right) \quad (3) \]
\[ M = \frac{wx(\cos \gamma)^2}{2} \left[ l - x \right] \quad (4) \]

where \( w \) is uniformly distributed load, \( l \) is bar length, \( \gamma \) is deployment angle and \( x \) is distance measured from B to C along x-axis. Based on the shear force equation in (3), it represented that the internal forces due to beam self-weight exerted on BO must be equivalent to a shearing force \( V \) of magnitude and for bending moment equation in (4) is equivalent to half of uniformly distributed load with bar length proportional to deployment angle multiply by \( x \) distance from O to B.

Scissor mechanism deployable structures are highly sensitive to the member’s deformation within elastic limit and required sufficient stiffness so that excessive deflections do not have an adverse effect on the H-SLEs members (Beam). The elastic curve is the diagram with the longitudinal axis that passes through the centroid of each cross-sectional area of the beam. The bending action due to applied load may cause beam to deflect and deflection due to shear must be calculated if the beam’s dimension are not small compared with its length [24]. Within the elastic range, the expression relating to the curvature of a beam to the applied bending moment and flexural rigidity (\( EI \)) constant is expressed based on [24-25]. By substituting (4) into general term of double integration equation, we obtained the equation as follow:

\[ EI \frac{d^2 y}{dx^2} = \frac{wx}{2} \left[ l(\cos \gamma)^2 - x(\cos \gamma)^2 \right] \quad (5) \]

where \( w \) is beam self-weight, \( x \) is distance along x-axis, \( l \) is beam length and \( \gamma \) is deployment angle. Apply integration to (5) twice with respect to \( x \) then yields the equation of the deflection curve of the neutral plane of the beam given the slope and deflection equations as follow:

\[ EI \frac{d^2 y}{dx^2} = \frac{wx^2(\cos \gamma)^2}{4} - \frac{wx^3(\cos \gamma)^2}{6} + C_1 \quad (6) \]
\[ Ely = \frac{wx^3(\cos \gamma)^2}{12} - \frac{wx^4(\cos \gamma)^2}{24} + C_1x + C_2 \quad (7) \]

There are two unknowns \( C_1 \) and \( C_2 \) in (6) and (7) and by applying boundary condition, when \( dy/dx = 0 \), \( \gamma = \frac{l}{2} \) and \( x_c = l \), \( y_c = 0 \). Therefore,

\[ C_1 = -\frac{w l^3(\cos \gamma)^2}{24} \quad (8) \]
\[ C_2 = 0 \quad (9) \]

To obtain the deflection equation, substitute (8) and (9) into (7), we obtained

\[ Ely = \frac{wx^3(\cos \gamma)^2}{12} - \frac{wx^4(\cos \gamma)^2}{24} - \frac{wx^5(\cos \gamma)^2}{24} \quad (10) \]

These two equations can be checked for validation when the slope of the beam at distance \( x = l/2 \), \( dy/dx = 0 \) and simplify maximum deflection equation at mid-span when \( x = l/2 \). From (6), it is equivalent to zero and from (10),

\[ y_{\text{max}} = \frac{5wl^4(\cos \gamma)^2}{384EI} \quad (11) \]

The H-SLEs members possess its own mass and due to gravity effect, it will created internal forces and will be analyzed under its own weight at vertical downward direction parallel to the gravity. In term to obtain the reaction forces at support BC, taking moment at C as showed in Figure 1(b) and the reaction forces equilibriums can be written as follow:

\[ R_B = R_C = \frac{wl}{2} \quad (12) \quad R_{CX} = 0 \quad (13) \]

Based on the static equilibrium approach, the reaction forces for inclined configuration will produced the equal reactions at both supports and zero horizontal force.
3. **Circular ring beam analysis**

The circular ring beam reacts as tie-up function for six H-SLEs deployable mechanism in maintaining model prototype at circular shape will subjected to tension effect at top and bottom. At the intermediate level, the circular ring beam will react as restraint in J-hook position for system stabilization under compression effect. The ring beam positions were labelled as A (Top), B (Intermediate) and C (Bottom) as showed in Figure 3.

![Figure 3. Ring beams positions under tension and compression configuration](image)

![Figure 4. Plan view of tension forces for circular ring beam](image)

The circular ring beam analysis in tension effect has assumed their beam radius is greater than its radial thickness and beam deflection theory is applicable, deflections due to bending effect only, stress within the elastic limit and their circular shape is maintained even deformation occur [26]. The ring beam analysis in tension is based on the plan view as showed in Figure 4 and calculated as follow [26]:

\[
T = \frac{P}{2 \tan \beta} \quad (14) \quad ; \quad T_{\text{max}} = \frac{P}{2 \sin \beta} \quad \text{at mid-span } x = 0, 2\beta, 4\beta, \ldots \quad (15)
\]

where \( \beta \) is angle of 30° (\( \pi/180 \)), \( x \) is angular distance from the bottom of a circular ring and \( P \) is applied force. At mid-span of circular ring beams where at the position of \( x, 2\beta, 4\beta, \ldots \), and at the each load position (support position), the maximum moment can be calculated as follow [26]:

\[
M_{\text{max} + \text{ve}} = \frac{PR}{2 \left( \frac{1}{\sin \beta} - \frac{1}{\beta} \right)} \quad (16) \quad ; \quad M_{\text{max} - \text{ve}} = \frac{PR}{2 \left( \frac{1}{\beta} - \frac{1}{\tan \beta} \right)} \quad (17)
\]

where \( R \) is radius and \( \beta \) is (0 \( \leq x \leq \pi/6 \)). During tensile effect, the ring beam will suffer from radial displacement inward (\( \Delta_{RI} \)) at mid-span and outward (\( \Delta_{RO} \)) at each load position. It can be determined as follow [26]:

\[
\Delta_{RI} = \frac{PR^3}{4Ef \left( \frac{2}{\beta} - \frac{1}{\sin \beta} \right) \frac{\cos \beta}{\sin \beta^2}} \quad \text{at } x = 0, 2\beta, 4\beta, \ldots \quad (19)
\]

\[
\Delta_{RO} = \frac{PR^3}{2Ef \left( \frac{\beta + \sin \beta \cos \beta}{2(\sin \beta)^2} - \frac{1}{\beta} \right)} \quad \text{at each point load} \ldots \quad (20)
\]

The circular ring beam at the intermediate level of the H-SLEs deployable mechanism will suffer from compression effect due to it locate parallel to the bolt joint (Figure 3, point E) where two beam connected by bolt joint will push the ring beam in J-Hook to maintain the H-SLEs deployable mechanism in vertical configuration. In this analysis, the semi-circle arch approach [24] was used to derive the analysis equilibriums which simplified ring beam as semi-circle arch subjected to two point’s loads. The supports at both sides of the semi-circle arch are pinned because at the mid-depth, both sides will be restrained from the point load and the structure cannot move up and down. The analysis of free body diagram was showed in Figure 5.
The reaction forces are determined by using static equilibrium as follow:

\[ R_{Ay} = \frac{P_1 \cos \alpha (L - a) + P_2 \cos \alpha (a)}{L} \]  \hspace{1cm} (21) \hspace{1cm} R_{By} = 2P \cos \alpha - R_{Ay} \]  \hspace{1cm} (22)

The normal force at point 1 is obtained by resolving the forces to one side of 1 in a direction tangential to the arch at point 1. Then, considering forces to the left side of point 1 and taking tensile forces as positive,

\[ N_1 = -R_{Ay} \cos \beta - R_{Ax} \sin \beta \]  \hspace{1cm} (23)

The shear force at point 1 is found by resolving the forces to one side of 1 in a direction perpendicular to the tangent at 1. We shall take positive shear force as acting radially inwards when it is to the left side of the section. Therefore, considering forces to the left of 1,

\[ S_1 = -R_{Ay} \sin \beta + R_{Ax} \cos \beta \]  \hspace{1cm} (24)

Now, taking moment about 1 for forces to the left of the 1 and regarding a positive moment as causing tension on the underside of the arch,

\[ M_1 = -R_{Ay}(1-\cos \beta) - R_{Ax}(1\sin \beta) \]  \hspace{1cm} (25)

4. Simulation examples and results analysis

The H-SLEs deployable mechanism as linear deployable structures are employed to demonstrate the effect of the bar’s own weight for mechanism stability and investigate their folded, deployment and final deployed configuration (0° to 90°) behaviours based on theory formulations. The H-SLEs deployable mechanism can rotate around bolt connection at their midpoints and has symmetry geometric properties based on translational scissor unit principle [6]. The motions of these structures are within xy plane and the degree of deployment γ between the units and the horizontal plane is vary ranging from 0° ≤ γ ≤ 90° as depicted by Figure 2. The H-SLEs deployable mechanism profiles are cold-formed lipped C-section, C74x36x3x0.75mm thick and C100x50x12x1.6mm thick with 600 mm length, yield strength is 550 N/mm², elastic modulus is 210 kN/mm², and inertia moment Ic75 is 9.96x10⁴ mm⁴ and Ic100 is 5.6x10⁵ mm⁴. The circular ring beam is Y-12 bar with three differences circular ring beam diameter that are 1.42m (Restraint), 1.5m (Top) and 1.52m (Bottom). The vertical axial load is 24.11 kN and M10 bolt is used as connection for all joints.

4.1 Simulation examples and result analysis

The motion path as displacements x and y axes versus deployment angle γ (0° ≤ γ ≤ 90°) using equations (1) and (2) were presented in Figure 6. The displacements started at γ = 0°(0 mm) and intercept at γ =90°(600 mm) length (x-axis) and height (y-axis). The result obtained indicated that both displacements x and y are correlated which mean as deployment angle γ increase, the displacements Δx and Δy will also increase. In addition, the motion path of the H-SLEs deployable mechanism for both x and y axes is coincided and displacements are correlated to the deployment angle γ as denominator while the displacements measured in length are dependent parameter. The sagging curve pattern for displacement x-axis represented the shortening trend towards γ = 90° while for displacement y-axis represented hogging curve pattern represented elongation trend towards γ = 90°. The displacement in y-axis represented the changed in height while displacement in x-axis represented the changed in length due to deployment angle changed. Therefore, deployment angle and beam length are two main parameters impact the H-SLEs deployable mechanism motion path system.
The H-SLEs deployable mechanism under its self-weight at folded configuration (γ=0°) is assumed as beam element and will develop an internal shear force and bending moment vary along the beam length at longitudinal beam axis [24]. The simulation calculation with three differences deployment angles (0°, 45° and 90°) using equations (3) and (4) were showed in Figure 7. The result obtained indicated that the maximum shear and moment occurred at γ = 0°, then reduced with the increment of γ = 45° and zero at γ = 90°. This is because at γ = 0°, the load is perpendicular to their longitudinal axis and the loads distance is equal to the beam axis length. When deployment angle γ increased (Incline beam position), the loads distance was reduced and consequently reduced the shear and moment values. At fully deployed vertical configuration, the load perpendicular to their longitudinal axis does not exist and become zero shear and zero moment. Therefore, the beam longitudinal axis will influence the shear force and bending moment values and the deployment angle to change the beam position will reduce the beam length at longitudinal axis. Thus, the length (l) and deployment angle (γ) are two parameters will impact the shear force and bending moment of the H-SLEs deployable mechanism.

Figure 7. Shear force and bending moment diagram at deployment angle 0°, 45° and 90°

The simulation results of slope and deflection of H-SLEs deployable mechanisms using equations (6) and (10) from γ = 0° until γ = 90° was showed in Figure 8. Based on the results obtained, the maximum deflection and slope occurred at γ=0° which was 6.8x10⁻⁴ mm and 3.7x10⁻⁶ rad respectively. This results showed that self-weight also will produced deflection at mid-span and rotation at beam supports with a minimum values. The increment of deployment angle will reduced the slope and deflection values of the beam since these values are parallel to the longitudinal length such as at γ=45°, slope and deflection values reduced to1.9x10⁻⁶ rad and 3.4x10⁻⁴ mm respectively and zero slope and deflection at γ=90°. Thus, this has concluded that length (l) and deployment angle (γ) are two parameters will impact the slope and deflection of the H-SLEs deployable mechanism.
Figure 8. Slope and deflection of H-SLEs deployable mechanism at deployment angle 0°, 45° and 90°.

The H-SLEs deployable mechanism under its self-weight as simply supported beam will create internal forces react with two end supports at deployment angle ranging $0^\circ < \gamma < 90^\circ$. The reaction forces simulation calculation is based on equations (12) and (13) derived which considered the loads in vertical downward direction parallel to the gravity were tabulated in Table 1. The results obtained indicated that both support reaction forces were equal from $0^\circ < \gamma < 90^\circ$ and zero horizontal force ($H_c = 0$). This indicated that deployment angle ($\gamma$) not impacted the reaction forces for simply supported beam even at inclined position.

| $\gamma$ (°) | 0    | 10   | 30   | 50   | 70   | 90   |
|-------------|------|------|------|------|------|------|
| $R_{By}$ (N) | 5.935| 5.935| 5.935| 5.935| 5.935| 5.935|
| $R_{Cy}$ (N) | 11.87| 11.87| 11.87| 11.87| 11.87| 11.87|
| $R_{By} + R_{Cy}$ (N) | 11.87| 11.87| 11.87| 11.87| 11.87| 11.87|
| $w$ (N)     | 11.87| 11.87| 11.87| 11.87| 11.87| 11.87|
| $R_{ex}$ (N) | 0    | 0    | 0    | 0    | 0    | 0    |

4.3 Circular ring beam simulation results analysis

The simulation calculation of the circular ring beam under tension and compression effects was tabulated in Table 2. Based on the results obtained, the tension force developed was 20.88 kN due to 24.11 kN vertical load applied. The maximum bending moment occurred at force point (restraint point) which was 1.61 kNm while at mid-span, the bending moment was 0.81 kNm. This indicated that the restraint point will be more critical than mid-span since the moment occurred was the highest and need more emphasize during the design stage. The radial displacement outward was greater than inward which was 0.65 mm against 0.02 mm. The Y-12 reinforcement bar was able to resist 22.62 kN tension force. For circular ring beam under compression effect, the reactions forces for x and y axes were equal and the normal force tangential to the arch at point 1 ($N_1$) and shear force at point 1 ($S_1$) was -20.88 kN and -12.05 kN respectively. The Y-12 reinforcement bar was able to resist 20.94 kN compression force. By comparison, Y-12 reinforcement bar was behaves stronger in resist tension force compare with compression. Therefore, restraint ring beam under compression must be more emphasized compared with tension ring beam.

|                  | Tension Ring Beam | Results | Compression Ring Beam | Results |
|------------------|-------------------|---------|-----------------------|---------|
| Tension force, $T$ | 20.88 kN          | Reaction forces, $R_{Ay} = R_{By}$ | 20.88 kN |
| Moment, $M_{max(+ve)}$ | 0.81 kNm         | Horizontal forces, $R_Ax - R_Bx$ | 12.05 kN |
| Moment, $M_{max(-ve)}$ | 1.61 kNm         | Normal force at point 1, $N_1$ | -20.88 kN |
| Radial displacement inward, ($\Delta_{RI}$) | -0.02 mm         | Shear force at point 1, $S_1$ | -12.05 kN |
| Radial displacement outward, ($\Delta_{RO}$) | 0.65 mm          | Moment at point 1, $M_1$ | 0 kNm |
| Y-12 bar tension resistance | 22.62 kN          | Y-12 bar compression resistance, | 20.94 kN |
5. **Conclusion**

Based on the results findings, there are some conclusions that can be drawn for the studies as stated below:

1. The simulation results obtained indicated the deployment angle ($\gamma$) and length ($l$) are two importance parameters impact the H-SLEs deployable structure performance.
2. The H-SLEs deployable mechanism will have critical structural properties values at $\gamma = 0^\circ$.
3. The compression ring beam is more critical than tension ring beam.
4. The motion pathway indicated the displacement $y$-axis suffered from elongation (Hogging) while displacement $x$-axis suffered from shortening (Sagging).

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