Poincaré Covariant Quark Models of Baryon Form Factors

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Abstract

Poincaré covariant quark models of the the nucleon, the $\Delta$ resonance and their excitations are explored. The baryon states are represented by eigenfunctions of the four-velocity and a confining mass operator, which reproduces the empirical spectrum up to $\sim 1700$ MeV to an accuracy of $\sim 6\%$. Models of constituent quark currents provide the relations between ground-state properties and transition amplitudes.
1. Introduction

The nonrelativistic quark model involves two underlying assumptions, neither one of which is required by its phenomenological success: (1) Constituent quarks have all the material properties of free particles, which happen to be confined by a potential. (2) The dynamics of the quarks is nonrelativistic. In the case of constituent quarks the relation of typical hadron sizes to the constituent quark masses is such that the Galilean approximation can provide no more than order of magnitude estimates. This limitation does not, however, affect applications to hadron spectroscopy, as such applications involve only the little group $SU(2)$.

The requirement of Poincaré covariance of the states and the current operators is essential for the consideration of form factors and transition amplitudes, and can be met in the context of relativistic particle dynamics. The success of quark models in accounting for hadron spectra does not depend on the assumption that constituent quark is a particle system. While states are represented by functions of three position, spin, flavor and color variables no additional particle assumptions are required. The principal features needed to account for empirical mass spectra, are the symmetry properties of the mass operator. For this no quark mass is required. Quark-mass parameters may however appear in the current-operator model as masses relate momenta to velocities. Here we explore a phenomenology, which does not assume a particle structure of hadronic quark currents. The principal purpose is to establish a basis for empirical relations between the electromagnetic structure of nucleons and baryon resonances. The conjecture is that this does not require any detailed assumptions about the material properties of constituent quark beside the fundamental symmetries.

In order to facilitate comparison with models based on the assumption of a constituent quark-particle structure we note that the particle structure determines free-quark currents, which satisfy all symmetry requirements in the absence of quark interactions. Confinement is implemented in this framework by a modification of the free-particle mass operator with appropriate symmetry and spectral features. The choice of a “form of dynamics” involves the choice of a “kinematic subgroup” for which the free and interacting unitary representations are identical. Modifications of the free-particle mass operator always destroy the Poincaré covariance of free-particle currents, which is restored by appropriate interaction currents.

In the “instant-form” of dynamics the center-of-mass position (the Newton-Wigner operator) remains a kinematic quantity. In this form all boosts are affected by the dynamics, which implies that in an impulse approximation the momenta of the contributing constituents cannot be related kinematically. This form of dynamics is appropriate when all relevant boosts are approximately Galilean. The “light front form” is unique in that it allows a consistent formulation of an impulse approximation, which retains the main qualitative features of the nonrelativistic impulse approximation for space-like momentum transfer. In it initial and final states are related by kinematic Lorentz transformations and the momenta of contributing constituents are related kinematically. In the impulse approximation a kinematic three-momentum transfer to the target is taken up by a single constituent. Calculations of form factors based on this approach have, however, not established that a quark-particle structure is either required or ruled out. In the “point form” of dynamics the full Lorentz group is the kinematic subgroup and all four translations depend on the dynamics. The point form has the advantage that Lorentz covariance is readily implemented.
by the operator structure and that translation covariance may easily be imposed on the matrix elements because the four components of the four-momentum commute. Only in this form is there a kinematic transformation to relative four-momenta and Lorentz covariant spinor wave functions. In general the four-momentum operator is specified by the mass operator and three kinematic variables. The choice of these variables determines the form of kinematics. In the point form the kinematic variables are the three independent components of the total four-velocity.

For a description of confined quark systems without quark particle structure one may start with a unitary Poincaré representation where the mass operator is a multiple of the identity. On the Hilbert space of states so defined quark dynamics is introduced by modification the mass operator. The harmonic oscillator model [4, 5, 12] provides a convenient prototype. The kinematics obtained in this manner is in the point-form. Quark currents may be specified by appropriate velocities related to the internal quark momenta. Once the eigenfunctions of the mass operator are known the unitary transformations, which relates different forms of kinematics are readily available. The symmetry requirements by themselves leave considerable freedom in the construction of models. Single-quark currents constructed with instant-form kinematics automatically include features that are interaction currents with point-form kinematics.

Here we introduce a simple confining mass operator which fits the empirical mass spectrum to an accuracy of 6% or better, up to $\sim 1700$ MeV. We use this spectroscopic model to formulate an exploratory approach to a phenomenology of quark currents. The simplest quark current is a function of spin and flavor, which depends on quark momenta only through the spectator constraints and hence the form of kinematics. This oversimplified model shares many well-known features with nonrelativistic quark models, while respecting all requirements of Poincaré covariance. Well-known gross features of the nucleon form factors determine the values and $Q^2$ dependence of all transition amplitudes. The model can be refined to include explicit dependence on orbital quark velocities.

Ultimately the question arises of the relation to quantum field theory. Point-form Hamiltonian dynamics may easily be related to constraint dynamics, in which all Poincaré transformations are implemented kinematically and states are represented by equivalence classes of functions with a semidefinite inner product. Covariant constraint dynamics [13] provides the bridge to Bethe-Salpeter formalisms [14] and models, which attempt to implement features of quantum field theory [15]. Euclidean Green functions, which satisfy reflection positivity [16] provide a basis for unitary representations of the Poincaré group. These issues are, however, beyond the scope of the present paper.

This paper is divided into 7 sections. In section 2 we define the Hilbert space of 3-quark wave functions. In section 3 the mass operator and its spectrum are described. General properties of current operators and current matrices are reviewed in section 4. The model for the kernels of the single-quark currents is constructed in section 5 to produce a rough description of the nucleon properties. The model so constructed is applied to inelastic transitions in section 6. Section 7 contains a summary.

2. The Hilbert Space of 3-Quark States

The states in the baryon spectrum are described by vectors in the little Hilbert space $H_\ell$, which is the representation space of the direct product of the little group, $SU(2)$, with flavor and
color \(SU(3)\). Concretely these states are realized by functions, \(\phi\), of three quark positions \(\vec{r}_i\), three spin variables \(\mu_i\) and three flavor variables \(f_i\), which are symmetric under permutations and invariant under translations \(\vec{r}_i \rightarrow \vec{r}_i + \vec{a}\). The translational invariance is realized by expressing the wave functions in terms of Jacobi coordinates

\[
\vec{r} := \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2), \\
\vec{\rho} := \sqrt{\frac{2}{3}} (\vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}).
\tag{2.1}
\]

Representations of the full Poincaré group obtain on the tensor product, \(\mathcal{H} := \mathcal{H}_\ell \otimes \mathcal{H}_c\), of the little Hilbert space \(\mathcal{H}_\ell\) with the Hilbert space \(\mathcal{H}_c\) of functions of the four-velocity \(v\), which is specified by 3 independent components. Translation are generated the four-momentum operator \(P = Mv\). Any confining self-adjoint mass operator \(M\), independent of \(v\), satisfies all relativistic symmetry requirements if it is invariant under rotations. At the level of spectroscopy alone there is no difference between relativistic and nonrelativistic quark models because the Galilean rest energy operator satisfies all the symmetry requirements of a mass operator.

The Poincaré invariant inner product of the functions representing baryon states is defined as

\[
(\Psi, \Psi) = \int d^4v 2\delta(v^2 + 1) \theta(v^0) \int d^3\kappa \int d^3k |\Psi(v, \vec{\kappa}, \vec{k})|^2,
\tag{2.2}
\]

where \(\vec{\kappa}\) and \(\vec{k}\) are the momenta conjugate to \(\vec{\rho}\) and \(\vec{r}'\). Summation over spin and flavor variables is implied. Under a Lorentz transformations \(v \rightarrow \Lambda v\) the vectors \(\vec{\kappa}\) and \(\vec{k}\) and the three quark spin variables \(\mu_1, \mu_2, \mu_3\) undergo Wigner rotations \(R_W(\Lambda, v)\),

\[
R_W(\Lambda, v) := B^{-1}(\Lambda v)AB(v).
\tag{2.3}
\]

Note that with canonical boosts

\[
R_W(\Lambda, v) := B^{-1}(\Lambda v)AB(v) = 1
\tag{2.4}
\]

for any rotationless Lorentz transformation \(\Lambda_v\) in the direction of \(\vec{v}\).

For heuristic constructions of impulse currents it is convenient to define internal four-momenta \(p\) and \(q\) by

\[
p := B(v)\{0, \vec{\kappa}\} q := B(v)\{0, \vec{k}\},
\tag{2.5}
\]

so that \(p^2 = |\vec{\kappa}|^2\) and \(q^2 = |\vec{k}|^2\).

The construction of unitary representations of the Poincaré group sketched here naturally leads to point-form kinematics. Once the eigenfunctions of the mass operator are known it is easy to realize unitary transformations to other forms of kinematics explicitly. Let \(\Psi_n(v, \vec{\kappa}, \vec{k})\) be eigenfunctions of \(\mathcal{M}\), with eigenvalues \(M_n\). Any state \(\Psi = \sum_n \Psi_n c_n\) can be represented by functions \(\Psi(\vec{P}, \vec{p}, \vec{q})\) normalized as

\[
(\Psi, \Psi) = \int d^3P \int d^3p \int d^3q |\Psi(\vec{P}, \vec{p}, \vec{q})|^2.
\tag{2.6}
\]

The unitary transformation \(\Psi(v, \vec{\kappa}, \vec{k}) \rightarrow \Psi(\vec{P}, \vec{p}, \vec{q})\) is specified by the variable transformation \(\{v, \vec{\kappa}, \vec{k}, n\} \rightarrow \{\vec{P}, \vec{p}, \vec{q}, n\}\) where \(\vec{p} = \vec{p}(v, \vec{\kappa})\) and \(\vec{q} = \vec{q}(v, \vec{k})\) are specified by eq. (2.3) and \(\vec{P} = M_n \vec{v}\) in each term of the sum over \(n\).
3. The Mass Operator and its Eigenfunctions

As suggested by the empirical baryon spectrum we shall choose the square of the zero order mass operator to be
\[ M_0^2 = 3 [\vec{\kappa}^2 + \vec{k}^2 + \omega^4 (\vec{\rho}^2 + \vec{r}^2)] , \] (3.1)
where \( \omega \) is a phenomenological parameter. Note that this mass operator does not contain a quark mass. The mass operator (3.1) commutes with the velocity \( v \) and the spin operator \( \vec{j} \), and is independent of \( v \). It is therefore Poincaré invariant as required, and completely symmetric under permutations of the coordinates.

To the zero-order mass operator \( M_0 \) we add a “hyperfine” correction, \( M' \) of the form
\[ M' = -C \sum_{i<j} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j , \] (3.2)
where \( C \) is a constant. Here \( \vec{\lambda}_i \) is the \( SU(3) \) flavor generator of the \( i \)th quark, and \( \vec{\sigma}_i \) is the corresponding spin matrix.

The flavor-spin structure of the hyperfine interaction (3.2) corresponds to the spin-flavor part of the interaction mediated by the exchange of the octet of light pseudoscalar bosons, which are the Goldstone bosons of the approximate chiral symmetry of QCD [17]. Here the main rationale for it is that leads to a very satisfactory description of the observed baryon spectrum up to \( \sim 1700 \) MeV as shown below. If all of the hyperfine interaction between the constituent quarks were assumed to arise mainly from exchange of a pseudoscalar boson octet [17], the constant \( C \) should be replaced by a radial function with vanishing volume integral, and which changes sign and takes the form of a pion exchange Yukawa function at large distances.

Taking \( C \) to be a constant implies an unrealistically long range for the hyperfine correction, and does not allow for the large empirical splitting of the \( SD \)-shell in the baryon spectrum. This deficiency may remedied, without loss of integrability, by adding an angular-momentum dependent correction of the following form to the mass operator:
\[ M'' = A [(\vec{r} \times \vec{k})^2 + (\vec{\rho} \times \vec{\kappa})^2] , \] (3.3)
where \( A \) is a constant. The inclusion of such a term makes it possible to extend the satisfactory description of the baryon spectra beyond 1700 MeV.

The eigenfunctions of the mass operator
\[ \mathcal{M} = M_0 + M' + M'' , \] (3.4)
are linear combinations of functions of the form
\[ \Psi_{N,L,[X]}(v, \vec{r}, \vec{k}, f_1, f_2, f_3, \sigma_1, \sigma_2, \sigma_3) = \phi_{N,L,[X]}(v, \vec{r}, \vec{k}) \Phi_{S,T,[X]}(f_1, f_2, f_3, \sigma_1, \sigma_2, \sigma_3) . \] (3.5)
Here \([X] = [3], [21], [111]\) are Young patterns, which label the symmetric, mixed and antisymmetric irreducible representations of the permutation group \( S_3 \) for the orbital wave function. The variables \( \sigma_i \) and \( f_i \) \( (i = 1, 2, 3) \) are quark spin and quark flavor labels. As we restrict the treatment here to the nucleon and the \( \Delta \) spectra the flavor indices \( f_i \) correspond to the usual isospin indices.
The functions $\phi_{N,L,[X]}(\vec{\kappa},\vec{k})$ are products of harmonic oscillator functions $\varphi_{n\ell m}$ of $\vec{\kappa}$ and $\vec{k}$, which are completely determined by the orbital angular momentum $L$, the principal quantum number $N \geq L$, and the symmetry character $[X]$. The explicit wave functions $\Psi$ for the baryon states in Table 1 are listed in Table 2.

In the unitarily equivalent instant-form representation one has

$$\Psi_{N,L,[X],S,T} (\vec{P}, \vec{p}, \vec{q}, f_1, f_2, f_3, \sigma_1, \sigma_2, \sigma_3) = \phi_{N,L,[X],S,T} (\vec{P}, \vec{p}, \vec{q}) \Phi_{S,T,[X]} (f_1, f_2, f_3, \sigma_1, \sigma_2, \sigma_3) .$$

In this form the orbital functions depend on $S$ and $T$.

The eigenvalues of the mass operator $M = M_0 + M' + M''$ are

$$\epsilon = \sqrt{6(N + 3)\omega + K(C, A)},$$

where $K$ is a hyperfine correction, which is listed for the states with $N \leq 2$, $L \leq 1$ up to $\sim 1700$ MeV in Table 1.

The mass operator, $M$, contains no quark mass parameter. The parameters $\omega$ and $C$ may be determined by the nucleon mass and the real part of the $\Delta(1232)$ pole position: $M(N) = \sqrt{18} \omega - 14C = 939$ MeV, and $M(\Delta(1232)) = \sqrt{18} \omega - 4C = 1211$ MeV. These equations yield the values $\omega = 311$ MeV and $C = 27.1$ MeV. The parameter $A$ in the correction term (3.3) is determined by the empirical difference of the $SD$ shell resonances $N(1440)$ and $N(1720)$ to be $A = 43$ MeV. The calculated resonance energies (averaged over the multiplets) obtained with these parameter values are listed in Table 1. The values so calculated deviate from the empirical values by about $\sim 6\%$ at most.

### 4. Current Operators

The quark current density operators $I^\mu(x)$ have to satisfy the following covariance conditions

$$U^\dagger(\Lambda) I^\mu(x) U(\Lambda) = \Lambda^\mu \nu I^\nu(\Lambda^{-1} x),$$

for arbitrary Lorentz transformations $\Lambda$. In the case of space-time translations this requirement takes the form

$$e^{iP \cdot a} I^\mu(x) e^{-iP \cdot a} = I^\mu(x + a).$$

Current conservation requires that

$$[P_\nu, I^\mu(0)] = 0 .$$

The current density operators are assumed to be operator valued tempered distributions. Under this assumption their Fourier transforms exist and are also operator valued tempered distributions:

$$\tilde{I}(Q) := \frac{1}{(2\pi)^4} \int d^4 x e^{-iQ \cdot x} I^\mu(x) .$$

The covariance relations

$$U^\dagger(\Lambda) \tilde{I}^\mu(Q) U(\Lambda) = \Lambda^\mu \nu \tilde{I}^\nu(\Lambda^{-1} Q),$$

and

$$[P_\nu, \tilde{I}^\mu(Q)] = Q^\nu \tilde{I}^\mu(Q),$$

6
follow from these requirements.

Let $|p, j, \sigma, \tau, \zeta\rangle \equiv |M, v, j, \sigma, \tau, \zeta\rangle$ be eigenstates of the four-momentum operator $P = Mv$ and the canonical spin, with $\sigma$ an eigenvalue of $j_z$ and $\zeta = \pm 1$ the intrinsic parity. It then follows from the translation covariance (4.6) that the matrix elements of $\tilde{I}^\mu(Q)$ and $I^\mu(0)$ are related by

$$\langle \kappa', \tau', \sigma', j', v', M' | \tilde{I}^\mu(Q) | M, v, j, \sigma, \tau, \kappa \rangle = \delta^{(4)}(M'v' - Mv - Q) \langle \kappa', \tau', \sigma', j', v', M' | I^\mu(0) | M, v, j, \sigma, \tau, \kappa \rangle .$$

(4.7)

It will be convenient to define a time-like unit vectors $u$, orthogonal to $v' - v$ by

$$u := \frac{v' + v}{\sqrt{-(v' + v)^2}} = \frac{v' + v}{2v^0} ,$$

(4.8)

where

$$v^0 := -u \cdot v = -u \cdot v' = \sqrt{1 + \eta} , \quad \eta := v^0 - 1 .$$

(4.9)

We may assume, without loss of generality, that the plane defined by $v'$ and $v$ is the $(t, z)$-plane.

The subgroup, $O(2)$, that leaves $v$, $v'$ invariant consists of the rotations $R_z(\varphi)$ about the $z$-axis and the reflection $P_y$ of the $y$-axis. Under these transformations the charge longitudinal components, $I^0(0)$ and $I_z(0)$ are scalars and the transverse current $I_{\pm 1} := \frac{1}{2}[(I_z(0) \pm iI_y(0)]$ transforms as an $O(2)$ vector:

$$U^\dagger[R_z(\varphi)]I^0(0)U[R_z(\varphi)] = I^0(0) , \quad U^\dagger(P_y)I^0(0)U(P_y) = I^0(0) ,$$

$$U^\dagger[R_z(\varphi)]I_z(0)U[R_z(\varphi)] = I_z(0) , \quad U^\dagger(P_y)I_z(0)U(P_y) = I_z(0) ,$$

$$U^\dagger[R_z(\varphi)]I_{\pm 1}(0)U[R_z(\varphi)] = e^{\pm i\varphi}I_{\pm 1}(0) , \quad U^\dagger(P_y)I_{\pm 1}(0)U(P_y) = I_{\mp 1}(0) .$$

(4.10)

States transform according to the rules

$$U^\dagger[R_z(\varphi)]|M, v, j, \sigma, \tau, \zeta\rangle = |M, v, j, \sigma, \tau, \zeta\rangle e^{i\sigma\varphi} ,$$

$$U^\dagger(P_y)|M, v, j, \sigma, \tau, \zeta\rangle = |M, v, j, -\sigma, \tau, \zeta\rangle (\zeta (-1)^{j-\sigma} .$$

(4.11)

It follows that the matrix elements of the scalar operators $P^0(0)$ and $I_z(0)$ vanish for $\sigma' \neq \sigma$ and the matrix elements of $I_{\pm 1}(0)$ vanish unless $\sigma' - \sigma = \pm 1$. Current conservation implies that $P \cdot I(0) = I(0) \cdot P$.

The irreducible representations of $O(2)$ are labeled by $|\sigma|$. The use of the Wigner-Eckart theorem for $O(2)$ is straightforward. The $O(2)$ covariant matrix elements are equal to invariant reduced matrix elements (form factors) multiplied by $O(2)$ Clebsch-Gordan coefficients, $C_{\sigma', j, \sigma}^{j, k, |\sigma|}$. 


\[ \langle \zeta', \sigma, j', v', M'| u \cdot I(0) | M, v, j, \sigma, \zeta \rangle = C_{\sigma,0,\sigma'}^{(\sigma', k, \sigma)} \langle \zeta', j', M'| I_0(v' \cdot v, |\sigma|) | M, j, \zeta \rangle, \]
\[ \langle \zeta', \sigma, j', v', M'| \frac{1}{2} (v' - v) \cdot I(0) | M, v, j, \sigma, \zeta \rangle = \frac{1}{2} C_{\sigma,0,\sigma'}^{(\sigma', k, \sigma)} \langle \zeta', j', M'| I_z(v' \cdot v, |\sigma|) | M, j, \zeta \rangle \sqrt{\eta}; \]
\[ \langle \zeta', \sigma, j', v', M'| I_\beta(0) | M, v, j, \sigma, \zeta \rangle = C_{\sigma',0,\sigma}^{(\sigma', k, \sigma)} \langle \zeta', j', M'| I_1(v' \cdot v, |\sigma|) | M, j, \zeta \rangle. \] (4.12)

where \( \beta = \pm 1 \). The non-vanishing Clebsch-Gordan coefficients are equal to \( \pm 1 \). We may choose
\[ C_{\sigma', k, \sigma}^{(\sigma', k, \sigma)} = 1 \quad \text{for} \quad \sigma \geq 0. \] (4.13)

The full current operator is determined by these reduced matrix elements.

The definition of the momentum transfer \( Q := M'v' - Mv \) implies that
\[ \frac{Q^2 + (M - M')^2}{4MM'} = \frac{\tilde{Q}^2}{(M' + M)^2} = \eta = v_z^2 = v_{z}'^2 = \frac{1}{4}(v' - v)^2, \] (4.14)

where
\[ \tilde{Q} := Q + (Q \cdot u) u = \frac{1}{2}(M' + M) (v' - v). \] (4.15)

It is customary to define Lorentz invariant “charge” and “longitudinal” current components, \( I_{CH}(0) \) and \( I_L(0) \), so that
\[ I(0) = I_{CH}(0)v' + I_L(0) \left[ \frac{v' - v}{2\sqrt{\eta}} \sqrt{1 + \eta} + \sqrt{\eta} u \right] + I_{\perp}(0). \] (4.16)

The transitions between the nucleons and the excited states are described by helicity amplitudes \( A_\lambda(Q^2) \) \( (\lambda = 1/2, 3/2) \), defined as matrix elements of the transverse current multiplied by a conventional invariant factor:
\[ A_\lambda(Q^2) := \frac{\sqrt{4\pi\alpha}}{2E_\gamma} \langle \zeta', \lambda, j', v', M'| I_1(0) | M, v, j, \lambda - 1, \zeta \rangle, \] (4.17)

where \( \alpha \) is the fine structure constant and
\[ E_\gamma := -v' \cdot Q + \frac{Q^2}{4M'M} = \frac{M'^2 - M^2}{2M'} \] (4.18)

is the photon energy for radiative decay.
5. Current Kernels and Nucleon Form factors

Current density operators $\tilde{I}^\mu(Q)$ can be represented by kernels $(\vec{k}', \vec{r}', v'|\mathcal{I}^\mu(Q)|v, \vec{r}, \vec{k})$ such that matrix elements $\langle f | \mathcal{I}^\mu(0)|a \rangle$ are related to the wave function $(3.5)$ by

$$
(f | \mathcal{I}^\mu(0)|i) = \int d^4Q\delta^4(M'v' - Mv - Q) \times \int d^3k' \int d^3k \int d^3\kappa \Psi_f(v', \vec{r}', \vec{k}')(\vec{k}', \vec{r}', v'|\mathcal{I}^\mu(Q)|v, \vec{r}, \vec{k}) \Psi_i(v, \vec{r}, \vec{k}).
$$

(5.1)

For the purpose of specifying impulse currents we define three formal quark-momentum transfers

$$
Q_1 := m(Q, v', v)(v' - v) - \frac{1}{2}\sqrt{\frac{2}{3}}(\vec{p}' - \vec{p}) + \sqrt{\frac{1}{3}}(\vec{q}' - \vec{q}),
$$

$$
Q_2 := m(Q, v', v)(v' - v) - \frac{1}{2}\sqrt{\frac{2}{3}}(\vec{p}' - \vec{p}) - \sqrt{\frac{1}{3}}(\vec{q}' - \vec{q}),
$$

$$
Q_3 := m(Q, v', v)(v' - v) + \sqrt{\frac{2}{3}}(\vec{p}' - \vec{p}),
$$

(5.2)

where $\vec{p} := p + (u \cdot p)u$ and $\vec{q} := q + (u \cdot q)u$. The scale factor $m(Q, v', v)$ introduced here plays the role of an effective quark mass. It should be emphasized that there is great latitude in the choice of this function. We specify the impulse current, $I^\mu_i$, by momentum constraints $Q_k = 0, \forall k \neq i$.

A requirement that the impulse constraints be kinematic significantly limits possible choices depending on the form of kinematics. With point-form kinematics the scale factor is restricted to functions $m(\eta)$. This leads to definite relations between nucleon elastic form factors and transition amplitudes which we will explore below. With $m(Q, v', v) = \frac{1}{2}\sqrt{Q^2/\eta}$ the impulse constraint (5.2) is kinematic with instant-form kinematics.

Because of the complete antisymmetry of the baryon wave functions it is sufficient to consider the current matrix elements of only one constituent, e.g. $i = 3$. It follows from the constraints $v \cdot p = 0$ and $v' \cdot p' = 0$ that

$$
p_\perp = \kappa_\perp = p'_\perp = \kappa'_\perp, \quad p^0 = p_z v_z / v^0, \quad p^0_\perp = p'_z v'_z / v^0, \quad p_z = v^0_\perp \kappa_z.
$$

(5.3)

It follows from the constraints $v \cdot p = 0$ and $v' \cdot p' = 0$ that

$$
p_\perp = \kappa_\perp = p'_\perp = \kappa'_\perp, \quad p^0 = p_z v_z / v^0, \quad p^0_\perp = p'_z v'_z / v^0, \quad p_z = v^0_\perp \kappa_z.
$$

(5.4)

The momentum constraints $Q_1 = Q_2 = 0$ imply that $Q_3 = 3m(v' - v)$, and

$$
(\vec{k}', \vec{r}', v'|\mathcal{I}^\mu|v, \vec{r}, \vec{k}) = 3\mathcal{I}^\mu_3(v', v)\delta[\vec{r}' - \vec{r} - \sqrt{\frac{6}{v^0}}(v' - \vec{v})]\delta(\vec{k}' - \vec{k})
$$

(5.5)

with $\mu = \{0, \perp\}$.

A very lean model for the current kernels is the following:
\[ \mathcal{I}_3^0(v', v) \equiv u \cdot \mathcal{I}_3(v', v) = \left[ \frac{1}{2} \lambda_3^{(3)} f_3 + \frac{1}{2} \sqrt{3} \lambda_8^{(3)} f_8 \right] \otimes 1 , \]

\[ \mathcal{I}_3(v', v) = \frac{\hat{\sigma}}{2} \times (\vec{v}' - \vec{v}) \left[ \frac{1}{2} \lambda_3^{(3)} g_3 + \frac{1}{2} \sqrt{3} \lambda_8^{(3)} g_8 \right] \otimes 1 . \quad (5.6) \]

With point-form kinematics it follows from

\[ \frac{1}{2} (|\vec{\kappa}|^2 + |\vec{\kappa}'|^2) = \frac{1}{4} |\vec{\kappa}' + \vec{\kappa}|^2 + \frac{6m^2(\eta)\eta}{(1 + \eta)} , \quad (5.7) \]

that all matrix elements of impulse currents are proportional to a function \( F_0(\eta) \), which for \( \eta \ll 1 \) can be approximated by the usual dipole form factor:

\[ F_0(\eta) := \exp \left( -\frac{6m^2(\eta)\eta}{\omega^2(1 + \eta)} \right) \approx \left( \frac{1}{1 + (3m^2(0)\eta/\omega^2)} \right)^2 , \quad (5.8) \]

with \( m^2(0) = \frac{5}{3} \omega^2 \). The dipole form obtains for all values of \( \eta \) with the choice

\[ m^2(\eta) = \omega^2 \frac{1 + \eta}{3\eta} \ln(1 + 5\eta) . \quad (5.9) \]

The isoscalar and isovector nucleon magnetic moments are respectively

\[ \mu_{IS} = \mu_p + \mu_n = g_8 , \quad \mu_{IV} = \mu_p - \mu_n = 5g_3 . \quad (5.10) \]

Agreement with the corresponding empirical values are obtained with \( g_8 = .88 \) and \( g_3 = .94 \).

For the magnetic moment of the \( \Delta^{++} \) and the \( N \to p \) transition moment the model yields

\[ \mu(\Delta^{++}) = \frac{3}{2}(g_8 + 3g_3) = 5.55 \text{ n.m.} \quad \mu(\Delta \to N) = 2\sqrt{2}g_3 = 2.65 \text{ n.m.} , \quad (5.11) \]

which may be compared to the corresponding empirical values 4.52 n.m. [19] and 3.1 n.m. [20] respectively.

The nucleon elastic form factors are then

\[ G_E^p(Q^2) = \frac{1}{2} [f_3 + f_8] F_0(\eta) , \quad G_E^n(Q^2) = \frac{1}{2} [f_3 - f_8] F_0(\eta) , \]

\[ G_M^p(Q^2) = \mu_p F_0(\eta) , \quad G_M^n(Q^2) = \mu_n F_0(\eta) , \quad (5.12) \]

with \( Q^2 = 4m_p^2\eta \). The magnetic form factors of the nucleons obtained with the quark mass (5.9) compare well with the empirical parametrization [18]. Observed features could be reproduced to any accuracy by assuming a suitable \( \eta \) dependence for the quark form factors \( f_3, f_8, g_3 \) and \( g_8 \).

Equivalent results may obviously be obtained with other forms of kinematics. For instance

\[ m^2(\hat{Q}^2, \eta) = \omega^2 \frac{1 + \eta}{3\eta} \ln(1 + \hat{Q}^2 / .71) , \quad (5.13) \]

yields again the dipole form for the elastic form factors, but differences will appear in the relations to transition amplitudes.
6. Transition Form Factors

Current conservation implies that the transition matrix elements must satisfy

\[(M'v' - Mv) \cdot I = \frac{1}{2}(M' + M)(v' - v) \cdot I + (M' - M)v^0 u \cdot I = 0.\] (6.1)

Since \((v' - v) \cdot I\) vanishes the second term must be cancelled by an appropriate interaction current. Non-vanishing longitudinal form factors depend on model dependent interaction currents. All transition form factors are functions of \(\eta\) multiplied by spin-flavor structure matrix elements. For each transition the dependence on \(Q^2\) is given by the general relation \(Q^2 = 4MM'\eta - (M' - M)^2\), which implies \(\eta = \eta_{rad} := (M' - M)^2/4M'M\) for real-photon transitions.

The helicity amplitudes \(A_{\lambda}(Q^2)\) are products of functions, which depend only on the spatial wave functions multiplied by spin-flavor amplitudes:

\[A_{\lambda}(Q^2) = \sqrt{4\pi\alpha} \sqrt{\frac{2M'\eta}{M'^2 - M^2}} F_{N,L}(\eta) A_{\lambda}(T, S, j).\] (6.2)

Here \(F_{0,0}(\eta) \equiv F_0(\eta)\) and

\[F_{2,2}(\eta) = \frac{\sqrt{2}}{3} F_{2,0}(\eta) = \sqrt{\frac{7}{3}} \frac{6m^2(\eta)\eta}{\omega^2(1 + \eta)} F_0(\eta), \quad F_{1,1}(\eta) := \frac{\sqrt{6m(\eta)}}{\omega} \sqrt{\frac{\eta}{1 + \eta}} F_0(\eta),\] (6.3)

The spin-isospin factors,

\[A_{\lambda}(T, S, j) := (L, S, 0, \lambda|j, \lambda) \frac{9}{2} \langle \tau, \lambda|\Phi^T_{S,T}\frac{1}{3}g_8 + \tau^{(3)}_z g_3 \rangle i\sigma_y^{(3)} \Phi_{L, \lambda - 1, \tau},\] (6.4)

for transitions to the states in Table 2 are tabulated in Table 3 (the spin-flavor matrix elements depend indirectly on the spatial wave function by the requirement that the baryon states be symmetric). The helicity amplitudes obtained with these expressions are compared to the corresponding empirical ones given in ref. \[20\] in Tables 4 and 5. The model helicity amplitudes were calculated using the model mass values in Table 1 in the kinematic expressions. The magnitudes of the model helicity amplitudes for photon decay are similar to those of the harmonic oscillator quark model \[21\], but the decrease with increasing \(Q^2\) values is much slower. Related to this is the fact that in the present point-form impulse approximation the \(N \rightarrow \Delta(1232)\) magnetic transition form factor falls off at a slower rate with \(Q^2\) than the dipole form factor, in disagreement with present data \[22\]. This disagreement is a robust feature of the point-form impulse approximation not shared by other forms of kinematics. For instance, \(N \rightarrow \Delta(1232)\) transition amplitudes obtained with \(5.13\) decrease faster than the corresponding nucleon form factor. The choice of the kinematics determines the relative role of impulse and interaction currents. Reduction of the present uncertainty of the empirical helicity amplitudes should provide a more definite indication of the preferred kinematics.

The present results share the qualitative feature of other quark models that the magnitudes of most helicity amplitudes are reasonable, the main exception being the spin-3/2 negative parity multiplet, where configuration mixing is needed for better overlap with the empirical values \[2\].
7. Summary

We have outlined a Poincaré covariant approach to electromagnetic form factors and transition amplitudes of baryons. The present, deliberately oversimplified, model was designed to elucidate the qualitative differences between baryon structure as described in terms of constituent quark and the structure of the few nucleon systems. Exact realization of Poincaré covariance is essential for the former. A few-nucleon system is essentially a system of free nucleons with a binding correction added to the free particle mass operator, whereas a system of confined quarks is described by a degenerate mass operator, with modifications that yield the required empirical mass splittings. In the description of confined quark the Poincaré representation with the degenerate mass operator plays a role similar to that of the free particle representation in the description of few-nucleon systems.

The point-form kinematics provides a particularly simple framework for the description of transition observables, when the momentum transfer ranges over both space- and time-like values.

A key element in the approach described above is a confining mass operator with a spin- and flavor dependent hyperfine term, the eigenfunctions of which are symmetrized products of orbital wave functions and spin-flavor functions. This mass operator provides a satisfactory account of the empirical spectra of the non-strange baryons, and may with minor adjustments be applied to the spectra of the strange \[17\] and heavy flavor hyperons as well \[23\]. The electromagnetic current model was constructed to implement the same qualitative features. The current matrix elements are integrals, which involve only the orbital wave functions multiplied by the spin-flavor matrix elements. The impulse approximation provides definite relations of transition amplitude to ground-state properties depending on the form of kinematics. It should be emphasized that the framework leaves considerable freedom in the construction of current models. A more elaborate version would involve dependence of the kernels \[5, 6\] on quark velocities as well as the spin and flavor variables \[24\]. Such dependence is required for the inclusion of a convection current, as well as for a realistic description of the axial vector structure of the baryons.

Additional features that are readily incorporated are a non-vanishing neutron charge form factor \[25\] – e.g. by including \(SU(3)_F\) breaking quark form factors in the charge operator in (5.5) – and a tensor component in the hyperfine term in the mass operator. A tensor component is required for a non-vanishing E2/M1 ratio for the \(\Delta(1232) \rightarrow N\) decay. Two– and three–quark current operators may of course also be added to the model, but there is no obvious need for such currents. The definition of single-quark currents is, of course, strongly model dependent.

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Table 1

The nucleon and $\Delta$-states up to $\sim 1700$ MeV. The column $\epsilon$ contains the eigenvalues of the mass operator (3.4). The average over the multiplet of the real part of the empirical pole positions is denoted EXP. The values predicted by the mass operator (3.4) are listed (in brackets) below the empirical values.

| Multiplet | $LS$ Multiplet | EXP (model value) | $\epsilon$ |
|-----------|----------------|------------------|-------------|
| 00[3]$_F$[21]$_F$[21]$_S$ | $\frac{1}{2}^+, N$ | 939 (940) | $\sqrt{18}\omega - 14C$ |
| 00[3]$_F$[3]$_F$[3]$_S$ | $\frac{3}{2}^+, \Delta$ | 1211 (1211) | $\sqrt{18}\omega - 4C$ |
| 20[3]$_F$[21]$_F$[21]$_S$ | $\frac{1}{2}^+, N(1440)$ | 1346 (1324) | $\sqrt{30}\omega - 14C$ |
| 11[21]$_F$[21]$_F$[21]$_S$ | $\frac{1}{2}^-, N(1535), \frac{3}{2}^-, N(1520)$ | 1508 (1554) | $\sqrt{24}\omega - 2C + 2A$ |
| 20[3]$_F$[3]$_F$[3]$_S$ | $\frac{3}{2}^+, \Delta(1600)$ | 1675 (1595) | $\sqrt{30}\omega - 4C$ |
| 11[21]$_F$[3]$_F$[21]$_S$ | $\frac{1}{2}^-, \Delta(1620); \frac{3}{2}^-, \Delta(1700)$ | 1620 (1718) | $\sqrt{24}\omega + 4C + 2A$ |
| 11[21]$_F$[21]$_F$[3]$_S$ | $\frac{1}{2}^-, N(1650); \frac{3}{2}^-, N(1700)$ | 1679 (1664) | $\sqrt{24}\omega + 2C + 2A$ |
| 2(20)2[3]$_F$[21]$_F$[21]$_S$ | $\frac{3}{2}^+, N(1720), \frac{5}{2}^+, N(1680)$ | 1693 (1582) | $\sqrt{30}\omega - 14C + 6A$ |
Table 2

Explicit wave functions for the baryon states in Table 2. The functions $\varphi_{nlm}$ are harmonic oscillator wave functions. The total angular momentum is denoted $J$ and the 3rd components of the spin and isospin are denoted $S_3$ and $T_3$ respectively. The subscripts $\pm$ on the spin-isospin states indicate are shorthands for the Yamanouchi symbols (112) and (121) respectively.

| $p, n, \frac{1}{2}^+$ | $\frac{1}{\sqrt{2}} \varphi_{000}(\vec{r}) \varphi_{000}(\vec{k}) \left\{ |\frac{1}{2}, T_3\rangle_+ |\frac{1}{2}, S_3\rangle_+ + |\frac{1}{2}, T_3\rangle_- |\frac{1}{2}, S_3\rangle_- \right\}$ |
|------------------|----------------------------------------------------------------------------------|
| $\Delta(1232), \frac{3}{2}^+$ | $\varphi_{000}(\vec{p}) \varphi_{000}(\vec{k}) |\frac{3}{2}, T_3\rangle |\frac{3}{2}, S_3\rangle$ |
| $N(1440), \frac{1}{2}^+$ | $\frac{1}{2} \left\{ \varphi_{200}(\vec{r}) \varphi_{000}(\vec{k}) + \varphi_{000}(\vec{r}) \varphi_{200}(\vec{k}) \right\}$ |
| | $\left\{ |\frac{1}{2}, T_3\rangle_+ |\frac{1}{2}, S_3\rangle_+ + |\frac{1}{2}, T_3\rangle_- |\frac{1}{2}, S_3\rangle_- \right\}$ |
| $N(1535), \frac{1}{2}^-$ | $\frac{1}{2} \sum_{ms} (1, \frac{1}{2}, m, s, J, S_3) \left\{ \varphi_{01m}(\vec{r}) \varphi_{000}(\vec{k}) \right\}$ |
| $N(1520), \frac{3}{2}^-$ | $\left[ |\frac{1}{2}, T_3\rangle_+ |\frac{1}{2}, s\rangle_+ - |\frac{1}{2}, T_3\rangle_- |\frac{1}{2}, s\rangle_- \right]$ |
| | $+ \varphi_{000}(\vec{r}) \varphi_{01m}(\vec{k}) \left[ |\frac{1}{2}, T_3\rangle_+ |\frac{1}{2}, s\rangle_+ + |\frac{1}{2}, T_3\rangle_- |\frac{1}{2}, s\rangle_- \right]$ |
| $\Delta(1600)$ | $\frac{1}{\sqrt{2}} \left\{ \varphi_{200}(\vec{r}) \varphi_{000}(\vec{k}) + \varphi_{000}(\vec{r}) \varphi_{200}(\vec{k}) \right\} |\frac{3}{2}, T_3\rangle |\frac{3}{2}, S_3\rangle$ |
| $\Delta(1620), \frac{1}{2}^-$ | $\frac{1}{\sqrt{2}} \sum_{ms} (1, \frac{1}{2}, m, s, J, S_3) \left\{ \varphi_{01m}(\vec{r}) \varphi_{000}(\vec{k}) |\frac{3}{2}, T_3\rangle |\frac{1}{2}, s\rangle_+ \right\}$ |
| $\Delta(1700), \frac{3}{2}^-$ | $+ \varphi_{000}(\vec{r}) \varphi_{01m}(\vec{k}) |\frac{3}{2}, T_3\rangle |\frac{1}{2}, s\rangle_- \left\}$ |
| $N(1650), \frac{1}{2}^-$ | $\frac{1}{\sqrt{2}} \sum_{ms} (1, \frac{3}{2}, m, s, J, S_3) \left\{ \varphi_{01m}(\vec{r}) \varphi_{000}(\vec{k}) |\frac{1}{2}, T_3\rangle_+ \right\}$ |
| $N(1700), \frac{3}{2}^-$ | $+ \varphi_{000}(\vec{r}) \varphi_{01m}(\vec{q}) |\frac{3}{2}, T_3\rangle_- \left\}$ |
| $N(1675), \frac{5}{2}^-$ | $|\frac{3}{2}, T_3\rangle_+ |\frac{1}{2}, s\rangle_+ + |\frac{1}{2}, T_3\rangle_- |\frac{1}{2}, s\rangle_- \left\}$ |
| $N(1720), \frac{3}{2}^+$ | $\frac{1}{2} |\frac{3}{2}, T_3\rangle \left\{ \varphi_{22m}(\vec{r}) \varphi_{000}(\vec{k}) + \varphi_{000}(\vec{r}) \varphi_{22m}(\vec{k}) \right\}$ |
| $N(1680), \frac{5}{2}^+$ | $\left\{ |\frac{1}{2}, T_3\rangle_+ |\frac{1}{2}, s\rangle_+ + |\frac{1}{2}, T_3\rangle_- |\frac{1}{2}, s\rangle_- \right\}$ |
The spin-isospin factors (6.4) for the helicity amplitudes for $p \rightarrow N^*$ and $p \rightarrow \Delta$ transitions. For transitions from the neutron to nucleon resonances the sign of the terms containing $g_3$ should be reversed.

| $N$ | $A_{1 \over 2}$ | $A_{3 \over 2}$ |
|-----|-----------------|-----------------|
| $\Delta(1232), {3 \over 2}^+$ | $-\sqrt{2} g_3$ | $-\sqrt{6} g_3$ |
| $N(1440), {1 \over 2}^+$ | $\frac{1}{2} [g_8 + 5g_3]$ | |
| $N(1535), {3 \over 2}^-$ | $-\sqrt{1 \over 12} [g_8 + 2g_3]$ | |
| $N(1520), {3 \over 2}^-$ | $\sqrt{1 \over 6} [g_8 + 2g_3]$ | 0.0 |
| $\Delta(1600), {3 \over 2}^+$ | $-\sqrt{2} g_3$ | $-\sqrt{6} g_3$ |
| $\Delta(1620), {1 \over 2}^-$ | $-\sqrt{1 \over 3} g_3$ | |
| $\Delta(1700), {3 \over 2}^-$ | $+\sqrt{2 \over 3} g_3$ | 0.0 |
| $N(1650), {1 \over 2}^-$ | $-\sqrt{1 \over 2\sqrt{3} [g_8 - g_3]}$ | |
| $N(1700), {3 \over 2}^-$ | $-\frac{1}{2\sqrt{15}} [g_8 - g_3]$ | $-\frac{3}{2\sqrt{5}} [g_8 - g_3]$ |
| $N(1675), {5 \over 2}^-$ | $\frac{1}{2} \sqrt{3 \over 5} [g_8 - g_3]$ | $\sqrt{3 \over 15} [g_8 - g_3]$ |
| $N(1720), {3 \over 2}^+$ | $-\sqrt{1 \over 10} [g_8 + 5g_3]$ | 0.0 |
| $N(1680), {5 \over 2}^+$ | $\sqrt{3 \over 20} [g_8 + 5g_3]$ | 0.0 |
Table 4

Comparison of the model helicity amplitudes for nucleon resonance decays (in units of GeV$^{-\frac{1}{2}}$) to the corresponding empirical values (Data) from ref. [20].

|       | $N^* \to p\gamma$ Data | $N^* \to n\gamma$ Data | $N^* \to p\gamma$ Model | $N^* \to n\gamma$ Model |
|-------|------------------------|------------------------|------------------------|------------------------|
| $N(1440)$ |
| $A_{1/2}$ | $-0.065 \pm 0.004$ | $+0.040 \pm 0.010$ | $+0.022$ | $-0.014$ |
| $N(1535)$ |
| $A_{1/2}$ | $+0.070 \pm 0.012$ | $-0.046 \pm 0.027$ | $+0.036$ | $-0.013$ |
| $N(1520)$ |
| $A_{1/2}$ | $-0.024 \pm 0.009$ | $-0.059 \pm 0.009$ | $-0.051$ | $+0.018$ |
| $A_{3/2}$ | $+0.166 \pm 0.005$ | $-0.139 \pm 0.011$ | $+0.0$ | $+0.0$ |
| $N(1650)$ |
| $A_{1/2}$ | $+0.053 \pm 0.016$ | $-0.015 \pm 0.021$ | $+0.001$ | $-0.024$ |
| $N(1700)$ |
| $A_{1/2}$ | $-0.018 \pm 0.013$ | $+0.001 \pm 0.050$ | $+0.001$ | $+0.092$ |
| $A_{3/2}$ | $-0.002 \pm 0.024$ | $-0.003 \pm 0.044$ | $-0.002$ | $+0.048$ |
| $N(1675)$ |
| $A_{1/2}$ | $+0.019 \pm 0.008$ | $-0.043 \pm 0.012$ | $+0.001$ | $-0.034$ |
| $A_{3/2}$ | $+0.015 \pm 0.009$ | $-0.058 \pm 0.013$ | $-0.002$ | $+0.048$ |
| $N(1720)$ |
| $A_{1/2}$ | $+0.018 \pm 0.030$ | $+0.001 \pm 0.015$ | $-0.052$ | $+0.035$ |
| $A_{3/2}$ | $-0.019 \pm 0.020$ | $-0.029 \pm 0.061$ | $+0.0$ | $+0.0$ |
| $N(1680)$ |
| $A_{1/2}$ | $-0.015 \pm 0.006$ | $+0.029 \pm 0.010$ | $+0.064$ | $-0.043$ |
| $A_{3/2}$ | $+0.133 \pm 0.012$ | $-0.033 \pm 0.009$ | $+0.0$ | $-0.0$ |
Table 5

Comparison of the model helicity amplitudes for $\Delta$ resonance decays (in units of GeV$^{-\frac{1}{2}}$) to the corresponding empirical values (Data) from ref. [20].

| $\Delta$ resonance | $\Delta \to N\gamma$ Data | $\Delta \to N\gamma$ Model |
|---------------------|---------------------------|---------------------------|
| $\Delta(1232)$      | $A_{1/2} = -0.140 \pm 0.005$ | $A_{1/2} = -0.089$ |
|                     | $A_{3/2} = -0.258 \pm 0.006$ | $A_{3/2} = -0.15$ |
| $\Delta(1600)$      | $A_{1/2} = -0.023 \pm 0.020$ | $A_{1/2} = -0.020$ |
|                     | $A_{3/2} = -0.009 \pm 0.021$ | $A_{3/2} = -0.034$ |
| $\Delta(1620)$      | $A_{1/2} = +0.027 \pm 0.011$ | $A_{1/2} = +0.038$ |
| $\Delta(1700)$      | $A_{1/2} = +0.104 \pm 0.015$ | $A_{1/2} = -0.054$ |
|                     | $A_{3/2} = +0.085 \pm 0.022$ | $A_{3/2} = +0.0$ |