Nonlinear Transports of Electrons on Liquid $^4$He in a 1.6 $\mu$m Channel

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Abstract. Two-dimensional electrons floating on the surface of superfluid $^4$He form a Wigner crystal at low temperatures accompanied with the commensurate deformation of the surface. The coupling of the Wigner crystal with the surface deformation gives rise to peculiar nonlinear phenomena in transport, such as the resonant Bragg-Cherenkov (BC) scattering of surface waves and the decoupling transition. Here we present results of transport measurement of a Wigner crystal in a quasi-one-dimensional channel 1.6 $\mu$m in width with a few electrons in the confined direction. We observe the clear nonlinear behaviors associated with the BC scattering and the decoupling transition, indicating that the electrons form a crystal-like structure with one or two rows of electrons in the confined direction.

1. Introduction
Electrons floating on the surface of liquid $^4$He form a clean two-dimensional (2D) electron system with a small density of typically $10^{11}$–$10^{13}$ m$^{-2}$[1, 2]. In such low density, the Coulomb interaction between electrons dominates over the kinetic energy, and electrons form a Wigner crystal at low temperatures. In the Wigner crystal phase, the electrons strongly couple to the surface by generating commensurate deformation of the surface called dimple lattice. The strong coupling gives rise to interesting nonlinear phenomena in transport such as the resonant scattering of surface waves due to the Bragg-Cherenkov (BC) mechanism[3, 4, 5] and the decoupling transition of the Wigner crystal from the dimple lattice[5, 6]. These nonlinearities are specific to the Wigner crystal coupled with the dimple lattice, and they do not take place when the crystal melts.

In a quasi-one-dimensional (Q1D) system, a Wigner crystal with long range positional order is formed only at zero temperature. At non-zero temperatures, significant thermal fluctuations due to long-wave thermal phonons would destroy the long range order. Thus, in a Q1D electron system, it is not clear whether the BC scattering occurs at finite temperatures because the presence of the lattice structure accompanied with a dimple lattice is essential for the BC scattering to take place. Here we study nonlinear behaviors in transport in a Q1D geometry with a few electrons in the confined direction. In this article, we report a device to implement a Q1D electron system, numerical calculations of distribution of electrons in the Q1D geometry, and nonlinear behaviors in transport properties.
2. Device
A Q1D electron system is implemented by confining electrons in a channel 1.6 μm in width using a device shown in Fig. 1. The device has a double-layered structure with a top and bottom aluminum electrodes separated by an insulating SiO$_2$ layer. A Q1D channel (center channel) of width $W = 1.6$ μm and length $L = 200$ μm is formed at the center of the device. The center channel connects two reservoirs of electrons composed of parallel channels 8 μm in width. The channels have a vertical groove of depth 1.0 μm, which are filled with liquid $^4$He by the capillary action. The top electrode called guard is biased by a negative $V_{\text{guard}}$. The bottom electrodes consist of source, drain, and gate, and they are positively biased by $V_S$, $V_D$, and $V_G$, respectively. Electrons are trapped and confined into the channels by the trapping potential produced by the combination of a negative voltage $V_{\text{guard}}$ and a positive voltage applied on the bottom electrodes (Fig. 1(d)). The trapping potential is varied by $V_{\text{guard}}$. A more negative $V_{\text{guard}}$ confines electrons stronger, hence an electron density smaller. Transport measurements are performed by superimposing an ac voltage $V_{\text{ac}}$ on $V_S$ with a frequency $f$ in a range of 10–30 kHz. $V_{\text{ac}}$ induces a current $I$ in the center channel, and the induced current is detected by the drain[5]. Data presented here were obtained at $V_S = V_D = +0.25$ V and $V_G = +0.45$ V.

![Figure 1](image)

(a) Illustration of the device for transport measurements of electrons confined in a 1.6 μm-wide channel. (b) Scanning electron microscope image of the device. (c) Magnified view of the center channel. (d) Schematic cross-section of the center channel. The trapping potential and the electron density can be adjusted by $V_{\text{guard}}$.

3. Numerical calculations of distribution of electrons in the channel
To discuss experimental results in detail, we should know distribution and density of electrons in the center channel for a given set of applied voltages. For this purpose, we made numerical calculations of the trapping potential and the distribution of electrons.

We consider ground state distribution of classical electrons with a continuous density distribution in the center channel. The length of the channel is assumed to be infinite, which reduces the three-dimensional electrostatic problems to two-dimensional calculations. $x$-coordinate is taken in the confined direction as shown in Fig. 2(a). The channel width $W$ is divided into $N = 200$ segments. The trapping potential $\phi(x)$ produced by voltages applied on the guard and the bottom electrodes is calculated using a finite element modeling (FEM) software under appropriate boundary conditions. After $\phi(x)$ is computed, an areal density of electrons $\rho(x_i)$ at an $i$-th segment is calculated by the Green’s function method[7] as follows. We first calculate the electric potential $G(x_i, x_j)$ at $x_j$ produced by a charge segment with unit areal density located at $x_i$. The potential of electrons at $x_j$ is then given by $\phi(x_j) = \phi(x_j) - e \sum_i \rho(x_i)G(x_i, x_j)$, where $e$ is elementary charge. In equilibrium, all electrons have a same potential $\phi(x_j) = \phi_0$, and therefore the density distribution is obtained.
by numerically solving the linear equations:

\[ \rho(x_i) = \frac{1}{\epsilon} \sum_j (\tilde{\phi}(x_j) - \phi_0) G(x_i, x_j)^{-1}, \]  

where \( G(x_i, x_j)^{-1} \) is inverse matrix of \( G(x_i, x_j) \). The similar calculations are also applied for obtaining distribution in the reservoir channel with a different width.

The distribution in the center channel is determined by balancing the potentials of electrons between the center and the reservoir channels. When \( V_{\text{guard}} \) is varied, the distributions in both channels change, which are determined again by equating the potential of electrons in both channels by keeping the number of electrons \( N_R \) in the reservoir constant. \( N_R \) is determined from an experimental value of the threshold guard voltage \( V_{0,\text{guard}} \), at which the current is pinched off due to the depletion of electrons in the center channel, as described below. Actual procedure to determine electron distribution in the center channel follows three steps.

Step 1: First, we calculate the potential of electrons in the reservoir channel at \( V_{0,\text{guard}} \). This is obtained by calculating \( \tilde{\phi}(x) \) at \( V_{0,\text{guard}} \) in the center channel, and the minimum in \( -\tilde{\phi}(x) \) corresponds to the potential of electrons in the reservoir. (Note that a more positive potential corresponds to a lower energy for an electron.) With this potential, we calculate a distribution in the reservoir channel and \( N_R \) using the Green’s function method.

Step 2: We calculate a distribution in the reservoir channel when \( V_{\text{guard}} \) is varied under a constant \( N_R \) using the Green’s function method. This calculation also gives the potential of electrons in the reservoir when \( V_{\text{guard}} \).

Step 3: Because of equilibrium, the potential of the electrons in the reservoir channel is same as that in the center channel. By equating the potential in the center channel to that in the reservoir, we calculate an electron distribution in the center channel using the Green’s function method.

The results of the calculations are shown in Fig. 2(b). The trapping potential is well described by a parabolic curve, \( -e\tilde{\phi}(x) = \frac{1}{2}m\omega^2x^2 - e\tilde{\phi}(0) \) with \( \hbar\omega \sim 1 \text{ K} \), where \( m \) is mass of an electron. As sweeping \( V_{\text{guard}} \) positively, the trapping potential decreases and \( \rho(x) \) at around the center increases. The linear density \( n_1 \) calculated by integrating \( \rho(x) \) in \( x \)-direction is shown in Fig. 2(c) as a function of \( V_{\text{guard}} \). \( n_1 \) increases monotonically as weakening the confinement.

**Figure 2.** (a) Geometry for calculating a trapping potential and a distribution of electrons. (b) Distributions of electrons (left axis) and trapping potentials (right axis) in the center channel. The calculations are performed for \( V_G = +0.45 \text{ V}, V_S = +0.25 \text{ V}, \) and \( V_{0,\text{guard}} = -0.26 \text{ V} \). Numbers indicate \( V_{\text{guard}} \). (c) The linear density (left axis) and \( \tilde{n} \) (right axis) as a function of \( V_{\text{guard}} \). The dots indicate the results of the numerical calculations and the solid line represents the smoothly interpolated curve.
For interpreting experimental results, we should know the crystal structure of electrons at a given density. The crystal structure at ground state was calculated by Piacente et al. for a classical electron system trapped in the parabolic potential \(\frac{1}{2}m\omega^2x^2\)\(^8\). Their calculations showed that the structure evolves from a single chain to double chains, followed by triple chains, \(\cdots\) with increasing linear density\(^8, 9\), and that the phase boundaries of the single-double chain and double-triple chain structures locate at \(\tilde{n} = 0.78\) and 1.75, respectively, where \(\tilde{n} = n_1(\frac{e^2}{\varepsilon_0 m \omega^2})^{1/3}\) is the dimensionless linear density (\(\varepsilon_0\) is vacuum permittivity). The phase boundaries in our case are indicated by the vertical lines in Fig. 2(c). We should note that quantum effect associated with confining the electrons in the trapping potential (\(\hbar \omega \sim 1\) K) is neglected here.

4. Nonlinear behaviors

Figure 3 the shows currents as a function of \(V_{ac}\) at 0.44 K measured at \(V_{guard} = -0.21\) V and \(-0.17\) V. The currents exhibit a clear plateaus associated with the Bragg-Cherenkov scattering. In addition, a sharp rise in current is observed at a large \(V_{ac}\), which is caused by the decoupling of the Wigner crystal from the dimple lattice. The linear densities estimated from the above numerical calculations are \(3.1 \times 10^6\) m\(^{-1}\) and \(5.3 \times 10^6\) m\(^{-1}\) at \(V_{guard} = -0.21\) V and \(-0.17\) V, respectively, at which the ground state corresponds to the single-chain and the double-chain structures. Our observations of the nonlinear behaviors indicates that the positional correlation is sufficiently developed for the dimple lattice to be formed at this temperature, although such Wigner crystals would have no long range order because of thermal fluctuation.

![Figure 3](image)

**Figure 3.** Current as a function of ac voltage measured at \(f = 30.0\) kHz. Data obtained at \(V_{guard} = -0.21\) V and \(-0.17\) V are shown.

References

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