Link Prediction and Unlink Prediction on Dynamic Networks

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Abstract—Link prediction on dynamic networks has been extensively studied and widely applied in various applications. However, temporal unlink prediction, which also plays an important role in the evolution of social networks, has not been paid much attention. Accurately predicting the links and unlinks on the future network greatly contributes to the network analysis that uncovers more latent relations between nodes. In this work, we assume that there are two kinds of relations between nodes, namely long-term relation and short-term relation, and we propose an effective algorithm called LULS for temporal link prediction and unlink prediction based on such relations. Specifically, for each snapshot of a dynamic network, LULS first collects higher-order structure as two topological matrices by applying short random walks. Then, LULS initializes and optimizes a global matrix and a sequence of temporary matrices for all the snapshots by using non-negative matrix factorization (NMF) based on the topological matrices, where the global matrix denotes long-term relation and the temporary matrices represent short-term relations of snapshots. Finally, LULS calculates the similarity matrix of the future snapshot and predicts the links and unlinks for the future network. Additionally, we further improve the prediction results by using graph regularization constraints to enhance the global matrix, resulting that the global matrix contains a wealth of topological information and temporal information. The conducted experiments on real-world networks illustrate that LULS outperforms other baselines for both link prediction and unlink prediction tasks.

Index Terms—Link prediction, unlink prediction, dynamic network, random walk, non-negative matrix factorization

I. INTRODUCTION

Link prediction is one of the fundamental problems that predicts whether two disconnected nodes in a network are likely to have a link [1]. It is useful in a wide variety of applications, such as recommendation [2], network reconstruction [3], and protein-protein interaction [4]. These networks always have dynamic nature, indicating that nodes and links can be added or removed with the evolution of networks [5]. It is inspiring and, in some respects, difficult to study networks at the level of individual edge formation or removal, especially in the dynamic scenario. Consequently, it still needs to be further studied on understanding the mechanisms by which such networks evolve at the level of individual edges.

Link prediction has been studied extensively in recent years [11], [19], [20], and there are two primary independent scenarios for predicting unknown links in the networks, forecasting of missing relationships and projection of future relationships. Although real-world networks have a highly dynamic structure, most of the previous link prediction studies focus on static networks [8]. In such scenario, it is anticipated that once two nodes are tied, they mostly remain tied (e.g., in the Facebook network, once two people are connected, they rarely break up). As a result, the link prediction problem focuses on identifying new links. On the other hand, for many networks, additional temporal information, such as the time of link creation and deletion as well as node addition and deletion, is available over a time interval, which is beneficial to understand the structure of the network. Inferring links in dynamic networks is more challenging because dynamic networks are often subjected to short-period changes due to noise. Furthermore, due to changes in the overall environment, some dynamic networks may endure long-term shifts [9].

Existing dynamic link prediction methods consider only either the structure of the network [8], [10], [11] or the temporal information [12], [13]. However, these families of approaches have several limitations. First, real-world networks are generally sparse and have partially observable links, resulting that the approaches based on only structure information may perform poorly. Second, the methods considering temporal information alone may lack insight provided by the other model. It is thus essential to use both topological and temporal features to comprehend complex behaviours of the dynamic network. Up to now, only a few studies [9], [14] have utilized both types of information together for the dynamic link prediction task. However, these methods fail to consider the intrinsic geometric structure of the data [15] resulting in lacking discriminative information for prediction.

Additionally, unlink prediction, which attempts to predict whether a previously occurred relationship will disappear in the future, is an important fundamental problem related to link prediction. Several studies [19], [17], [18] reveal that the probability of relationship to persist or to-be formed increases if a node pair has a high number of common neighbors as well as high transitivity through third parties. Furthermore, they also divulge that the probability of an edge to befall occurs between edges with low resemblance, fewer communal neighbors, and typically of low transitivity. These findings illustrate that the processes guiding link creations are negatively correlated with those guiding link removals, which signifies the importance of unlink prediction tasks. However, until now, the link prediction problem has been heavily studied [8], [19], [20], while its counterpart, unlink prediction problem, although reporting a high proportion of link changes [21], is rarely studied [22].

In this work, we assume that the relations in the dynamic
networks can be divided into two categories, long-term relation and short-term relation. The long-term relation represents a certain kind of stable relation, while the short-term relation denotes a sort of temporary relation. For instance, in the collaboration network, the authors in the same lab maintain a stable relation, and they may always collaborate on publishing papers. While the authors in the different labs or even focusing on different research fields have a temporary relation such that they only cooperate occasionally. As a result, these two kinds of relations determine the states of links and unlinks in the future network to a large extent.

Based on such assumption, we propose a new algorithm for the link prediction and unlink prediction with long-term relation and short-term relation, termed LULS. LULS utilizes the method of non-negative matrix factorization (NMF) that embeds nodes into two kinds of representation matrices containing both structural similarity and temporal information. More specifically, for each snapshot of network, LULS collects higher-order topological information as matrices by performing two random walk methods, which are light lazy random walk [23] and a new variant of random walk called modified light lazy random walk. Then, LULS initializes and optimizes a global matrix and a temporary matrix for each snapshot by using the method of NMF, where the global matrix represents the long-term relation that stores the temporal information and is shared by all the snapshots, while the series of temporary matrices denotes the short-term relation that only stores the topological information within the corresponding snapshot. Additionally, LULS also incorporates the geometric structure by using graph regularization constraints to enhance the global matrix. Therefore, the global matrix contains a wealth of both topological information and temporal information, which can further improve the prediction results. Afterwards, LULS calculates the similarity matrix of the future snapshot for the temporal predictions of links and unlinks based on the global matrix and temporary matrices. The experiments conducted on real-world networks illustrate that LULS outperforms other baselines on AUC for both tasks of link prediction and unlink prediction.

II. RELATED WORK

Link prediction in static network has been extensively studied [1]. On the other hand, for many real-world networks, additional temporal information is available overtime interval, and the network build from such data can be represent as a dynamic network. One feasible strategy to model the dynamic network is to explore the network topological properties, which indicates that any two nodes close to each other in the network are likely to form a link in the near future. For example, Rahman et al. [24] capture the topology of dynamic networks by graphlets, where graphlet transitions between different timestamps are coded in a feature vector and can be used by supervised learning. Gunes et al. [23] apply the ARIMA model on the series of node similarity scores to predict links in the next period based on the previous time series data. Moradabadi et al. [25] predict the future connections between pairs of nodes by using the previous snapshot as input. Besides, many approaches based on the matrix factorization approach has also been proposed [27], [10], [11], [14], [28]. Matrix factorization has the advantages of generating embedding which are simple to interpret. The main idea is to learn a latent low-dimensional vector representation for each node and the nodes close to each other in the low-rank space are similar. However, most of the real-world networks are sparse, so that the methods considering only the structural characteristics of the network may perform poorly.

Moreover, there are some approaches that utilizes temporal information, which reveals the relationship between the current snapshot and the previous snapshots, to predict links on dynamic networks [29], [10], [30], [12], [14], [31]. The main idea is the temporal smoothness, which assumes that the embeddings from the current snapshot should not deviate dramatically from the previous snapshots [29]. Although the approaches utilizing structural or temporal information alone have shown an encouraging performance, the combination of topological and temporal information always provides further insights that are missed with the use of single modeling. List [31] is proposed which exploits structural and temporal information and characterizes network information using time function to predict links on evolving networks. Step [9] is presented to exploit structural and temporal information for prediction and characterize network evolution using a global transition matrix to reflect different types of evolutionary patterns. Despite their good predictive performance, these approaches fail to consider the intrinsic geometric structure of the data, lacking discriminative information for prediction. Other approaches such as those based on graph neural network (GNN) [32], [33], [34] requires the node features, which may not always be available or may produce embedding with limited interpretability.

In additional to link prediction, unlink prediction also plays an important role in network evolution. For example, although some temporary relationships among people in an online social network are formed during a short-term, some of these relationships are likely to decay or even disappear in the future. Until now, there are only few studies address the problem. Preusse et al. [35] use various network structural features extracted from a knowledge network to predict the disappearance of links. Oliveira et al. [22] propose the method combining both the topology information and the information of individuals (semantic metrics) on evolving networks to predict the disappearance of links.

Comparing to the previous link prediction and unlink prediction methods, LULS has several advantages. First, to preserve the higher-order topological information as much as possible, LULS applies two random walk methods on each snapshot, so that after optimization, the global matrix and the temporary matrices include more and better topological patterns. Despite their good predictive performance, these approaches fail to consider the intrinsic geometric structure of the data, lacking discriminative information for prediction. Other approaches such as those based on graph neural network (GNN) [32], [33], [34] requires the node features, which may not always be available or may produce embedding with limited interpretability.

of the corresponding snapshot that accurately preserves the short-term relations between nodes. Therefore, the topological information of future snapshot can be precisely reconstructed. Last but not least, LULS also optimizes the global matrix by using graph regularization constraints which makes the global matrix preserve more topological information that further improves the prediction results.

III. PRELIMINARIES

In this section, we first give the problem definition, and then briefly introduce the random walk method. Important symbols used in this paper are listed in Table I.

| Symbol | Definition |
|--------|------------|
| \( G_t = (V_t, E_t) \) | the snapshot \( G_t \) with node set \( V_t \) and edge set \( E_t \) |
| \( W_t, H_t \) | the similarity matrices for \( G_t \) |
| \( U \) | the global representation matrix |
| \( V_t \) | the temporary representation matrix for \( G_t \) |
| \( m \) | the dimension of node representation |
| \( \theta \) | decay weight |
| \( \lambda \) | smoothness weight |
| \( \gamma \) | control the importance of constraint terms |
| \( k \) | the steps of random walk |

A. Problem Definition

Consider an undirected and unweighted graph \( G = (V, E) \), where \( V \) and \( E \) denote the set of observed nodes and edges, respectively. Let \( A \in [0, 1]^{V \times V} \) be the associated adjacency matrix, \( D \) be a diagonal matrix of node degrees, and \( I \) represents the identity matrix. A dynamic network is represented as a sequence of snapshots of graph \( G = \{G_1, G_2, \cdots, G_N\} \), where \( N \) is the final timestamp. We denote \( G_t = (V_t, E_t) \) as the graph at timestamp \( t \) \((1 \leq t \leq N)\). For simplicity, we assume the set of nodes do not change over snapshots, i.e., \( V_t = V_j \), for any \( t_i, t_j \in \{1, \cdots, N\} \), indicating that we ignore newly added and removed nodes. However, edges do appear and disappear in snapshots over the timestamps. Our goal is to predict the edges that will be added or removed in \( G_{N+1} \) based on the previous observed snapshots in \( G \).

Definition 1 (Dynamic Link Prediction): Given a sequence of snapshots of graph \( G = \{G_1, G_2, \cdots, G_N\} \), where \( N \) is the final timestamp, all the snapshots are with the same set of nodes but some edges may emerge or disappear along the sequence. For any pair of unconnected nodes \( u \) and \( v \) in \( G_N \), the link prediction task on dynamic network aims to predict whether \( u \) and \( v \) will have a link in \( G_{N+1} \).

Definition 2 (Dynamic Unlink Prediction): Given a sequence of snapshots of graph \( G = \{G_1, G_2, \cdots, G_N\} \), where \( N \) is the final timestamp, all the snapshots are with the same set of nodes but some edges may emerge or disappear along the sequence. For any edge \((u, v) \in E_N\), the unlink prediction task on dynamic network aims to predict whether edge \((u, v)\) will disappear in \( G_{N+1} \).

For example, as shown in Figure I, given a graph and its two snapshots at \( t = 1 \) and \( t = 2 \), we know all the changes of edges in both snapshots. For instance, nodes \( u \) and \( v \) are unconnected at timestamp \( t = 1 \) and then get linked at timestamp \( t = 2 \), while the edge between \( u \) and \( v \) presented at timestamp \( t = 1 \) disappears at timestamp \( t = 2 \). Then, we aim to predict all the to-be formed links (e.g., the edge between \( m \) and \( w \)) and to-be removed links (e.g., the edge between \( u \) and \( m \)) at timestamp \( t = 3 \) by using the link states at previous timestamps.

B. Random Walk Method

Random walk is one of the most popular methods to measure the importance of nodes in the graph w.r.t. the given node. Suppose that we start a standard random walk from the given node \( u \), \( p_0 \) is a one-hot vector in which the corresponding entry of \( u \) is 1 and the rest entries are 0. \( N_{rw} \) is the transition probability given as \( N_{rw} = D^{-1}A \). Then, the \( k \)-steps probability vector can be iteratively calculated by

\[
p^{(k)} = N_{rw}^T p^{(k-1)} = (N_{rw}^T)^k p^{(0)}.
\]

Let \( P^{(k)} \) be the matrix that combines \( p^{(k)} \) for all the nodes in the graph. Then, the \( i,j \)-th entry in \( P^{(k)} \) represents the probability that the random walker starts at node \( i \) to reach node \( j \) in \( k \) steps. In this work, we use \( P^{(k)} = P^{(k)} + (P^{(k)})^T \) as the similarity scores for all the pairs of nodes, where \( P_{ij} \) denotes the sum of the probabilities that a random walker jumps from node \( i \) to node \( j \) as well as from node \( j \) to node \( i \) within \( k \) steps.

IV. METHODOLOGY

In this section, we illustrate the detailed procedures of LULS that contains three main steps. First, for each snapshot \( G_t \), LULS calculates two matrices \( H_t \) and \( W_t \) by using local random walk methods to collect higher-order topological information. Then, LULS initializes a global representation matrix \( U \) and a temporal representation matrix \( V_t \) for each snapshot \( G_t \) to represent the long-term relations and short-term relations respectively, and optimizes \( U \) and \( V_t \) by applying the method of NMF according to \( H_t \) and \( W_t \) of each snapshot. Finally, LULS computes the similarity matrix \( R \) of the future snapshot based on the matrices \( V_1, V_2, \cdots, V_N \) and \( U \), and predicts the states of links and unlinks for the future snapshot. The whole framework is given in Algorithm I and we elaborate each step as follows.
Algorithm 1 The LULS Algorithm

Require: A series of adjacency matrices $A_1, A_2, \cdots, A_N$, the hyper-parameters $\lambda, \gamma$, and $\theta$

Ensure: Similarity matrix $R$

1: compute matrices $H_1, H_2, \cdots, H_N$
2: compute matrices $W_1, W_2, \cdots, W_N$
3: for $t = 1$ to $N$ do
4: randomly initialize $V_t$
5: end for
6: randomly initialize $U$
7: repeat:
8: for $t = 1$ to $N$ do
9: update $V_t$ according to Eq. (5)
10: end for
11: update $U$ according to Eq. (6)
12: until termination criteria are reached
13: compute $R$ according to Eq. (7)
14: return $R$

A. Collecting Topological Information

In the first step, for each snapshot $G_t$, LULS computes two matrices $H_t$ and $W_t$ to represent topological information. Note that the basic idea of NMF is to learn a low-dimensional vector for each node by factorizing the adjacency matrix. However, the adjacency matrix can not fully capture the higher-order relations between nodes. Moreover, since the real-world networks are always extremely large with millions or even billions of nodes, it is computationally expensive to apply global proximity methods to measure the strength between nodes. Therefore, a few variants of random walk, which can effectively collect the topological information, are proposed as solutions [23, 35].

In this work, we adopt light lazy random walk (LLRW) [23] and propose a new random walk variant called modified light lazy random walk (MLLRW) to collect the higher-order topological information. LLLRW and MLLRW are defined as follows:

1) Light Lazy Random Walk (LLRW).

$$N_{rw} = (D + \alpha I)^{-1}(\alpha I + A),$$

where $\alpha \in N^{0+}$ is a hyper-parameter. Compared to standard random walk, LLLRW retains some probability at the current node for the random walks. Moreover, LLLRW degenerates to standard random walk when $\alpha = 0$, and $\alpha = l$ indicates that the random walker performs $l$ loops on each node.

2) Modified Light Lazy Random Walk (MLLRW).

$$N_{rw} = \beta S + (1 - \beta)(D + \alpha I)^{-1}(\alpha I + A),$$

where $S$ is the diagonal matrix with degree centrality similarity between every node in the network, and $\beta \in [0, 1]$ is a hyper-parameter. MLLRW expects the ranking score of the nodes to be biased more to higher degree nodes, such that a random walker of MLLRW performs light lazy walk to one of the neighbors of the current node with probability $(1 - \beta)$ and jumps to any other node in the graph according to $S$ with probability $\beta$. Additionally, MLLRW degenerates to LLRW when $\beta = 0$.

Let $P_{LLRW}^{(k)}$ and $P_{MLLRW}^{(k)}$ represent the similarity matrices as introduced in Section III-B obtained by performing LLLRW and MLLRW over the graph, respectively. Then, we separately calculate $H_t = \hat{P}_{LLRW}^{(k)}$ and $W_t = \hat{P}_{MLLRW}^{(k)}$ for each snapshot $G_t$.

B. Node Representation

In the dynamic real-world network, nodes have long-term relations and short-term relations, and the temporal predictions of links and unlinks are determined by these two relations. Consequently, we leverage NMF to find the temporary representation matrices $V_1, V_2, \cdots, V_N$ and a global representation matrix $U$ such that the inner product of $V_t$ and $U$ can approximate the topological information $W_t$ of $G_t$. Note that, the global representation $U$ can be considered as long-term relation between nodes, while the temporary representations $V_1, V_2, \cdots, V_N$ evolved over time is regarded as short-term relations.

Specifically, let $m$ be the dimension of node representation, and for each $W_t$, we aim to factorize the matrix as $W_t \approx UV_t^T$, where the matrices $U \in \mathbb{R}_+^{N \times m}$ and $V_t \in \mathbb{R}_+^{N \times m}$ represent the global representation matrix and temporary representation matrix, respectively. Additionally, to ensure that the temporary representations of nodes between snapshots do not change significantly, we apply regularization to maintain the smoothness. Therefore, for the matrix $W_t$, the objective function is given as

$$\min_{U \geq 0, V_t \geq 0} J = \sum_{t=1}^{N} \theta^{N-t} \left\| W_t - UV_t^T \right\|_F^2 + \lambda \sum_{t=2}^{N} \theta^{N-t} \left\| V_t - V_{t-1} \right\|_F^2,$$

where $\theta \in [0, 1]$ is the decay weight and $\lambda$ is the smoothness weight.

Furthermore, to enhance the node representation $U$ that preserves more topological information, motivated by graph regularization technique [37], we define the similarity constraint term $M_t$ based on $H_t$ for each snapshot $G_t$ as follows

$$M_t = \frac{1}{2} \sum_{i, j} \| u_i - u_j \|^2 H_{i,j} = Tr(U^T L_t U),$$

where $H_{i,j}$ is the $ij$-th entry in $H_t$, $L_t = D_t - H_t$ is the Laplacian matrix, and $Tr(\cdot)$ is the trace matrix. Overall, the enhanced objective function is expressed as follows:

$$\min_{U \geq 0, V_t \geq 0} J = \sum_{t=1}^{N} \theta^{N-t} \left\| W_t - UV_t^T \right\|_F^2 + \gamma \sum_{t=1}^{N} M_t + \lambda \sum_{t=2}^{N} \theta^{N-t} \left\| V_t - V_{t-1} \right\|_F^2,$$

where $\gamma$ is a hyper-parameter to control the relevance importance of constraint terms.
C. Optimization

In this section, we present an iterative update algorithm to solve the optimization problem in Eq. (3). In each iteration, the algorithm updates each matrix in turn while fixing the other matrices. This procedure repeats until the matrices converge or the maximum number of iterations is reached. By removing the irrelevant items, i.e., \( \theta \), Eq. (3) can be simplified as:

\[
\min_{U \geq 0, V_t \geq 0} J = \sum_{t=1}^{N} \left\| W_t - UV_t^T \right\|_F^2 + \gamma \sum_{t=1}^{N} Tr(U^T L_t U)
\]

\[
+ \lambda \sum_{t=2}^{N} \left\| V_t - V_{t-1} \right\|_F^2.
\]

We first address the problem of optimizing \( V_t \) for each \( t \in [1, N] \). According to Eq. (3), the optimization problem is transformed as follows:

\[
\min_{V_t \geq 0} J = \sum_{t=1}^{N} Tr(\gamma(W_t - UV_t^T)(W_t - UV_t^T)^T)
\]

\[
+ \sum_{t=2}^{N} Tr(\lambda(V_t - V_{t-1})(V_t - V_{t-1})^T).
\]

Based on the non-negativity constraint of \( V_t \) following the standard constraint optimization theory, we introduce the Lagrangian multiplier \( \phi_t = [\phi_{ij}] \) and minimize the Lagrangian function \( L \), such that

\[
L = J + \sum_{t=1}^{N} Tr(\phi_t V_t^T).
\]

By computing the derivate of \( L \) with respect to \( V_t \), we have the following expression:

\[
\frac{\delta L}{\delta V_t} = -2W_t U + UV_t^T U^T + 2\lambda(V_t - V_{t-1}) + \phi_t.
\]

Next, by setting \( \frac{\delta L}{\delta V_t} = 0 \), we obtain the following equation for \( [V_t]_{ij} \):

\[
[-W_t U + UV_t^T U^T + \lambda(V_t - V_{t-1})]_{ij} (V_t)_{ij} = 0.
\]

Then, the update rule for \( V_t \) is derived as follows:

\[
V_t \leftarrow V_t \odot \sqrt{\frac{H_t U + \lambda(V_{t-1})}{UV_t^T U^T + \lambda(V_t)}},
\]

where \( \odot \) denotes the Hadamard product.

The global matrix \( U \) can be learned in a very similar to latent factor \( \hat{V}_t \). To handle non-negative constraints, we introduce the Lagrangian multiplier \( \psi = [\psi_{ij}] \) and minimize the Lagrangian function \( L \):

\[
L = J + \psi
\]

Then, the update rule for \( U \) is derived as follows:

\[
U \leftarrow U \odot \sqrt{\frac{\sum_{t=1}^{N} (W_t V_t + \gamma H_t U)}{\sum_{t=1}^{N} (UV_t^T V_t^T + \gamma D_t U)}}.
\]

D. Predicting Links and Unlinks

After optimizing \( V_1, V_2, \ldots, V_N \) and \( U \), LULS calculates the similarity matrix of future snapshot for the predictions of links and unlinks. The probability of link formation and link disappearance can be obtained by the inner product of the latent factors \( U \) and \( V_t \) for each \( t \in [1, N] \), such that

\[
R = \sum_{t=1}^{N} UV_t^T.
\]

Each \( ij \)-th entry in \( R \) denotes the proximity score for the pair of nodes \( i \) and \( j \). For the link prediction problem, the higher the similarity score between unconnected nodes in \( R \) the higher the probability of connect in the future. In contrast, for the unlink prediction problem, for two connected nodes in \( R \) if this pair of nodes have a lower similarity score, then they have a higher probability of disappearing in the future.

E. Complexity Analysis

In this section, we discuss the time complexity of LULS. Local spectral diffusion [6] depends on the number of nodes on the network, and the time complexity is \( O(N|V|^2) \), where \( l \) is the number of iterations to convergence. For NMF, the computation is dominated by matrix multiplications, i.e., the matrix multiplication between \( |V| \times |V| \) matrix and \( |V| \times m \) matrix. Therefore, the complexity involved for updating \( V_t \) and \( U \) is \( O(rN|V|^2m) \), where \( r \) denotes the number of iterations. Besides, the complexity of computing \( R \) is \( O(|V|^2m) \). Thus, the overall time complexity of our LULS model is \( O((rN + 1)|V|^2m + N|V|^2) \). Note that, since all matrices are sparse, the complexity between two sparse matrix is much smaller than \( O(|V|^2) \).

V. EXPERIMENTAL SETUP

A. Datasets

We use six real-world dynamic networks for evaluating the performance of LULS. The datasets contain Facebook Forum, Reality Mining, Dublin, Hep-Th, Facebook Messages, and Retweet. Table I summarizes the detailed information of these datasets.

- **Facebook Forum** [38]: The Facebook Forum network is the private messages exchanged between Facebook users from May to October in 2004, where the nodes are the users, and each edge is a message exchanged between a pair of users.
- **Reality Mining** [38]: Reality Mining network consists of individual’s mobile phone call events between a set of core users at Massachusetts Institute of Technology (MIT), where the vertices are users, and each edge is a phone call or voicemail between a pair of users.
TABLE II
STATISTICS OF THE DATASETS.

|                      | Facebook Forum | Reality Mining | Dublin | Hep-Th | Facebook Messages | Haggle |
|----------------------|----------------|----------------|--------|--------|-------------------|--------|
| Number of nodes      | 899            | 6416           | 6454   | 22908  | 274               | 18470  |
| Number of edges      | 7046           | 72250          | 26097  | 2444798| 274               | 2124   |
| Average degree       | 15.68          | 2.26           | 2.45   | 213.44 | 2.67              | 15.51  |
| Density              | 0.0175         | 0.0004         | 0.0012 | 0.0093 | 0.0087            | 0.0568 |
| Avg shortest-path distance | 2.3320       | 4.2367         | 6.6808 | 2.7220 | 3.0552            | 2.42   |
| Number of snapshots  | 6              | 7              | 8      | 11     | 8                 | 6      |

- **Dublin** [38]: Dublin network is a human contact network where the vertices represent individuals and the edges denote proximity.

- **Hep-Th**: This dataset is a collaboration network from high energy physics theory section on arXiv, where vertices are authors and an edge denotes a common publication for a pair of authors.

- **Facebook Messages**: This dataset is a Facebook-like social network originated from online community of the students at the University of California, where the nodes are users, and the edges are the messages exchanged between pairs of users.

- **Haggle**: This network reflects connections between people as measured by wireless devices carried by the participants. A node symbolizes a person, and an edge between two people indicates that they came into touch.

B. Evaluation Metric

In this paper, we apply AUC and average precision (AP) to evaluate the performance of LULS. Specifically, for each dataset, we use $G_1, G_2, \ldots, G_{N-1}$ as the training data and $G_N$ as the test data. Furthermore, the test data is divided into positive test samples and negative test samples. For the link prediction problem, the positive test set consists of the edges that appear in $G_N$ and do not present in $G_{N-1}$, while the negative test set consists of the edges that do not appear in $G_{N-1}$ and $G_N$. On the other hand, for unlink prediction problem, the positive test set contains the edges that appear in $G_{N-1}$ and $G_N$, while the negative test set consists of the edges that present in $G_{N-1}$ and disappear in $G_N$. To avoid the class imbalance, we randomly generate the same size of negative test set as that of positive test set for both link prediction and unlink prediction. In addition, the experiments are carried out five times independently and the average result is reported.

Generally, the AUC is described as the likelihood that the randomly selected actual link in the positive test set is assigned a higher score than a randomly selected link in the negative test set. Formally, if among $n$ comparisons, there are $n'$ times the the edges in the negative test set has a lower score than the edges in the positive test set and $n''$ times they have the same scores, the AUC scores are calculated as follows

$$AUC = \frac{n' + 0.5 \times n''}{n}.$$  

Note that, an algorithm has a better performance than pure chance when the value of AUC is bigger than 0.5.

AP combines recall and precision for ranking results. We calculate the precision after each true positive given a ranked list of predicted links. The average of these values gives the average precision for that link.

C. Baselines

We compare our method with the the-state-of-the-art methods as follows:

- **AA [20]**: AA assumes that two nodes are more likely to be linked together if they share more common neighbors.

- **DCN [39]**: This method uses a decay common neighbor to characterize the relationship between node pair.

- **TD [11]**: This method stacks all adjacency matrices of historical snapshots into a tensor with the time as the third dimension to improve the link prediction results.

- **TMF [14]**: This method uses matrix factorization techniques to characterize the network characteristics as a function of time.

- **GrNMF [40]**: This method directly approximates the link matrix over time $T$ using NMF by setting networks from 1 to $T-1$ as a regularizer.

Note that, the methods AA and DCN can be applied only to static network. Therefore, in the case of link prediction, these approaches are performed based on the links from all past time periods by combining them into a single link matrix. For the unlink prediction task, these approaches are conducted based on the link states over $G_{N-1}$.

VI. EXPERIMENTAL RESULTS

In the experiments, all the parameters of LULS have been manually tuned. Specifically, we set $m = 5$, $\theta = 0.4$, $\gamma = 1$ and $\lambda = 0.0001$ for Facebook Forum, Facebook Messages and Haggle networks. And for Reality mining and Dublin network, we set $\gamma = 0.0001$. For the random walks variants, we use $\alpha = 1$ for LLRW, and $\beta = 0.01$ for MLLRW. Besides, we use the random walk step $k = 4$ for the Facebook Forum, Reality mining, Facebook Messages and Hep-Th networks, and $k = 5$ for the Dublin network. Moreover, for evaluating the effectiveness of smoothness and constraint terms, we implement three versions of LULS as follows:

- **LULS$_1$**: $\lambda \neq 0$ and $\gamma \neq 0$;

- **LULS$_2$**: $\lambda \neq 0$ and $\gamma = 0$;

- **LULS$_3$**: $\lambda = 0$ and $\gamma = 0$. 

1http://networkrepository.com

2http://konekt.cc/networks


TABLE III
THE AUC SCORES FOR THE LINK PREDICTION TASK.

| Methods | Datasets | Facebook Forum | Reality Mining | Dublin | Hep-Th | Facebook Messages | Haggle |
|---------|----------|----------------|----------------|--------|--------|-------------------|-------|
| LULS1   | 0.9324   | 0.9715         | 0.9913         | 0.7351 | 0.7565 | 0.9881            |       |
| LULS2   | 0.9232   | 0.9764         | 0.9909         | 0.7089 | 0.9741 | 0.9870            |       |
| LULS3   | 0.8726   | 0.9710         | 0.9898         | 0.6974 | 0.9750 | 0.9835            |       |
| AA      | 0.3252   | 0.5192         | 0.8914         | 0.5612 | 0.3551 | 0.9230            |       |
| DCN     | 0.5313   | 0.5194         | 0.9620         | 0.5612 | 0.4601 | 0.6532            |       |
| TD      | 0.9059   | 0.9214         | 0.6495         | 0.6268 | 0.9122 | 0.5693            |       |
| TMF     | 0.8314   | 0.9284         | 0.6037         | 0.6985 | 0.7122 | 0.9127            |       |
| GrNMF   | 0.8792   | 0.9294         | 0.6542         | 0.6837 | 0.9474 | 0.9213            |       |

TABLE IV
THE AP SCORES FOR THE LINK PREDICTION TASK.

| Methods | Datasets | Facebook Forum | Reality Mining | Dublin | Hep-Th | Facebook Messages | Haggle |
|---------|----------|----------------|----------------|--------|--------|-------------------|-------|
| LULS1   | 0.8942   | 0.9570         | 0.9850         | 0.7012 | 0.9528 | 0.9810            |       |
| LULS2   | 0.8731   | 0.9604         | 0.9843         | 0.6926 | 0.9521 | 0.9775            |       |
| LULS3   | 0.8646   | 0.9602         | 0.9844         | 0.6845 | 0.9510 | 0.9842            |       |
| AA      | 0.5160   | 0.5130         | 0.8909         | 0.5547 | 0.7409 | 0.9430            |       |
| DCN     | 0.5646   | 0.4424         | 0.9563         | 0.5610 | 0.4619 | 0.6096            |       |
| TD      | 0.8870   | 0.9106         | 0.9028         | 0.6248 | 0.9357 | 0.7481            |       |
| TMF     | 0.8271   | 0.9177         | 0.5926         | 0.6954 | 0.7018 | 0.8020            |       |
| GrNMF   | 0.8444   | 0.8071         | 0.6018         | 0.6773 | 0.7355 | 0.8587            |       |

TABLE V
THE AUC SCORES FOR THE UNLINK PREDICTION TASK.

| Methods | Datasets | Facebook Forum | Reality Mining | Dublin | Hep-Th | Facebook Messages | Haggle |
|---------|----------|----------------|----------------|--------|--------|-------------------|-------|
| LULS1   | 0.8563   | 0.7970         | 0.7525         | 0.6923 | 0.7970 | 0.9345            |       |
| LULS2   | 0.8268   | 0.7968         | 0.7532         | 0.6567 | 0.8044 | 0.9334            |       |
| LULS3   | 0.8276   | 0.7661         | 0.7517         | 0.5051 | 0.8036 | 0.9316            |       |
| AA      | 0.546    | 0.5192         | 0.5437         | 0.5558 | 0.5101 | 0.6492            |       |
| DCN     | 0.4858   | 0.5014         | 0.5871         | 0.5437 | 0.5117 | 0.6096            |       |
| TD      | 0.7596   | 0.7651         | 0.5731         | 0.6154 | 0.7032 | 0.9367            |       |
| TMF     | 0.6780   | 0.7946         | 0.5801         | 0.6748 | 0.6961 | 0.8143            |       |
| GrNMF   | 0.6695   | 0.7848         | 0.6351         | 0.6645 | 0.7411 | 0.8445            |       |

TABLE VI
THE AP SCORES FOR THE UNLINK PREDICTION TASK.

| Methods | Datasets | Facebook Forum | Reality Mining | Dublin | Hep-Th | Facebook Messages | Haggle |
|---------|----------|----------------|----------------|--------|--------|-------------------|-------|
| LULS1   | 0.8120   | 0.7885         | 0.8231         | 0.7014 | 0.8162 | 0.9333            |       |
| LULS2   | 0.7948   | 0.8773         | 0.8238         | 0.6612 | 0.8329 | 0.9320            |       |
| LULS3   | 0.7877   | 0.8776         | 0.8210         | 0.5152 | 0.8280 | 0.9268            |       |
| AA      | 0.5462   | 0.5083         | 0.5437         | 0.5610 | 0.5089 | 0.6520            |       |
| DCN     | 0.4911   | 0.5011         | 0.5667         | 0.4619 | 0.4991 | 0.5741            |       |
| TD      | 0.5072   | 0.7620         | 0.7447         | 0.6272 | 0.7928 | 0.9377            |       |
| TMF     | 0.8280   | 0.7940         | 0.5612         | 0.6675 | 0.6961 | 0.8138            |       |
| GrNMF   | 0.7812   | 0.7845         | 0.6326         | 0.6651 | 0.7155 | 0.8341            |       |

A. Link Prediction

In this experiment, we evaluate the performance of LULS for link prediction. Table III and Table IV shows the performance of different approaches on six dynamic networks for the link prediction task. It can be observed that LULS models perform best in all the datasets, indicating that LULS can effectively integrate both temporal and structural information to extract significant node representation for the link prediction task. Precisely, our model has shown an impressive performance even on a very sparse graphs, e.g., Dublin network. Furthermore, all the approaches based on dynamic characteristics (i.e., LULS, TD, TMF, and GrNMF) consistently perform better than the approaches that ignore the temporal behaviour of the network (i.e., AA and DCN) in almost all the datasets. In addition, among LULS models, LULS1 is better than LULS2 and LULS3. Consequently, the smoothness and similarity constraint terms play important roles in link prediction.

B. Unlink Prediction

Next, we investigate the effectiveness of LULS for the unlink prediction task. In Table V and Table VI the AUC and AP scores show that LULS models significantly outperform the baseline methods in almost all the datasets. Precisely, our model have shown an impressive performance on Haggle network, suggesting that unlink prediction is more likely on dense graph than on sparse graphs. Similar to link prediction task, all the approaches which consider temporal information of the network outperform other methods. Moreover, among

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LULS models. LULS$_2$ is also better than LULS$_1$ and LULS$_3$, which indicates that the smoothness and similarity constraint terms are useful in unlink prediction. Additionally, comparing to link prediction, we can observe that the AUC scores of unlink prediction are lower, indicating that it is more difficult to predict unlinks than links in the future network.

VII. CONCLUSION

In this work, we address the problems of temporal link prediction and unlink prediction on dynamic networks. Assuming that there are two kinds of relations between nodes, namely long-term relation and short-term relation, we propose an effective algorithm called LULS for temporal link prediction and unlink prediction based on such relations. Specifically, LULS collects the topological information for each snapshot of a dynamic network and generates a global matrix and a sequence of temporary matrices to represent the long-term relation and short-term relation. Then, LULS utilizes the global matrix and the temporary matrices to predict the links and unlinks for the future network. The experiments conducted on six real-world networks show the superior results of LULS compared with the state of the art methods.

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