Asymptotics of neutron Cooper pair in weakly bound nuclei

Y. Zhang, M. Matsuo, and J. Meng

1Graduate School of Science and Technology and Department of Physics, Faculty of Science, Niigata University, Niigata 950-2181, Japan
2State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China
3School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China
4Department of Physics, University of Stellenbosch, Stellenbosch, South Africa

Abstract

Asymptotic form of neutron Cooper pair penetrating to the exterior of nuclear surface is investigated with the Bogoliubov theory for the superfluid Fermions. Based on a two-particle Schrödinger equation governing the Cooper pair wave function and systematic studies for both weakly bound and stable nuclei, the Cooper pair is shown to be spatially correlated even in the asymptotic large distance limit, and the penetration length of the pair condensate is revealed to be universally governed by the two-neutron separation energy $S_{2n}$ and the di-neutron mass $2m$.

PACS numbers: 21.10.Gv 21.10.Pc, 21.60.Jz

e-mail: yzhangjcnp@pku.edu.cn
The separation energy of the constituent particle, i.e., the minimum energy needed to remove particle(s) out of a system, influences strongly surface properties of the system. A characteristic example in nuclear physics is the neutron halo \cite{1–3}, a dilute neutron distribution extended far outside the nuclear surface. This exotic structure is found near the drip-line, i.e., in the most neutron-rich isotopes where the neutron separation energy is reduced by more than one order of magnitude compared with that in naturally abundant nuclei. It has been considered that the halo is formed by the last neutrons (the most weakly bound ones) penetrating deeply into the classically forbidden exterior of the nuclear potential. However, how they penetrate is a non-trivial question since nucleons are correlated due to the two-body interaction. In particular, the neutron pair correlation or the attraction between the weakly bound neutrons should be taken into account as they play decisive roles in the formation of halo \cite{4–11}.

To answer the question, one may consider wave function of a ”Cooper pair” formed by the last two neutrons. It can be generally defined by $\Psi_{\text{pair}}(r_1 \uparrow, r_2 \downarrow) = \langle \Phi_{N-2} | \psi(r_1 \uparrow) \psi(r_2 \downarrow) | \Phi_N \rangle$, where $|\Phi_{N[N-2]}\rangle$ is the pair-correlated ground state under interest with even neutron number $N$ and $N - 2$, while $\psi(r_1 \uparrow)$ and $\psi(r_2 \downarrow)$ are neutron annihilation operators at positions $r_1$ and $r_2$ with opposite spins. One needs to know the behaviors of this Cooper pair wave function in the asymptotic region $r_1, r_2 \to \infty$ far outside the nuclear surface.

The correlation of the halo neutrons is often studied for light-mass two-neutron halo nuclei by means of the three-body models \cite{5, 8, 10, 12} and the cluster models \cite{13, 14}, which suggest that the two halo neutrons are correlated spatially — often referred to as the di-neutron correlation. The asymptotic behavior is discussed in the Faddeev three-body approach using the hyperspherical coordinates \cite{9, 15}. However, these models assume a core plus very weakly bound two neutrons, and the analyses are limited to light-mass drip-line nuclei.

In this Letter, in contrast, we investigate the asymptotics and the correlation of the neutron Cooper pair on a more general ground, i.e., by using the selfconsistent mean-field model combined with the Bogoliubov quasiparticle approach for the pair correlation, which can be applied to essentially all the self-bound nuclei. In fact, the Hartree-Fock-Bogoliubov (HFB) models \cite{16, 17} and the relativistic Hartree-Bogoliubov models \cite{7, 18}, are successful in describing not only tightly bound nuclei, but also neutron-rich nuclei with small separation.
energies if they are formulated in the coordinate space \([6, 19–21]\). Examples include two-neutron halo nuclei such as \(^{11}\text{Li}\) \([6]\) and the giant halo, involving several neutrons, predicted e.g. in \(N > 82\) Zr isotopes \([4, 22, 23]\). We note also that the di-neutron correlation in the Cooper pair wave function is predicted in the HFB models applied to medium and heavy mass neutron-rich and stable nuclei \([24, 25]\) with separation energies \(\sim 2 – 10\) MeV. On these backgrounds, we investigate in this study how the asymptotics of the neutron Cooper pair vary as a function of the neutron separation energy.

The Bogoliubov’s quasiparticle method adopted in the HFB model is essentially the same as those applied to various Fermion systems with superfluidity caused by \(^1\text{S}\) short-range attractive interactions \([26–28]\). The ground state of a pair correlated nucleus is approximated as a variational vacuum \(|\Phi_0\rangle\) of independent quasiparticle states. The quasiparticles have two-component wave function \(\phi_i(x) = [\phi_{1,i}(x), \phi_{2,i}(x)]^T\) with \(x = r\sigma\), and obey the HFB equation

\[
\begin{pmatrix}
    h - \lambda & \Delta \\
    -\Delta^* & -h^* + \lambda
\end{pmatrix}
\begin{pmatrix}
    \phi_{1,i}(x) \\
    \phi_{2,i}(x)
\end{pmatrix}
= E_i
\begin{pmatrix}
    \phi_{1,i}(x) \\
    \phi_{2,i}(x)
\end{pmatrix},
\]

known also as the Bogoliubov-de Gennes equation in general \([27]\). The single-particle Hamiltonian, \(h = t + U\), includes the kinetic operator \(t\) and the selfconsistent mean field potential \(U\). The Fermi energy is \(\lambda\) and the pair potential \(\Delta\). The Cooper pair wave function in this approach is given as \(\Psi_{\text{pair}}(x_1, x_2) = \langle \Phi_0 | \psi(x_1)\psi(x_2) | \Phi_0 \rangle\), and expressed as

\[
\Psi_{\text{pair}}(x_1, x_2) = \frac{1}{2} \sum_i \phi_{1,i}(x_1)\phi_{2,i}^*(x_2) - (x_1 \leftrightarrow x_2) \tag{2}
\]

in terms of a sum of the quasiparticle wave functions.

A crucial step to explore the asymptotic form of \(\Psi_{\text{pair}}(x_1, x_2)\) is to note that it obeys "two-particle Schrödinger equation",

\[
[t(1) + t(2) + v(1, 2)]\Psi_{\text{pair}}(x_1, x_2) = 2\lambda\Psi_{\text{pair}}(x_1, x_2) \tag{3}
\]

for \(r_1, r_2 \to \infty\), with \(v(1, 2)\) the two-body force between neutrons.

The derivation of this equation is as follows. Operating the single-particle Hamiltonian \(h(1) + h(2)\) on \(\phi_{1,i}(x_1)\phi_{2,i}^*(x_2)\) in Eq.(2) and using Eq. (1), one finds

\[
[h(1) + h(2) - 2\lambda]\Psi_{\text{pair}}(x_1, x_2) = -\Delta(1) \sum_i \phi_{2,i}(x_1)\phi_{2,i}^*(x_2) - \Delta(2) \sum_i \phi_{1,i}(x_2)\phi_{1,i}(x_1). \tag{4}
\]
Secondly, it can be shown that the r.h.s. of Eq. (4) will be $-\Delta(x_2,x_1)$ in the asymptotic limit. Here we use the completeness relation of the Bogoliubov quasiparticle wave functions, 

$$
\sum_i [\varphi_{1,i}(x)\varphi_{1,i}^*(x') + \varphi_{2,i}(x)\varphi_{2,i}(x')] = \delta_{xx'},
$$

and the known asymptotic behavior \[15, 20\]

$$
\varphi_{1,i}(x) \to \begin{cases} 
  e^{-\kappa_1 r}/r & (E_i < |\lambda|) \\
  e^{\pm i\kappa_1 r}/r & (E_i \geq |\lambda|)
\end{cases},
\varphi_{2,i}(x) \to e^{-\kappa_2 r}/r,
$$

(5)

for $r \to \infty$ with

$$
\kappa_1 = \sqrt{\frac{2m|E_i + \lambda|}{\hbar^2}}, \quad \kappa_2 = \sqrt{\frac{2m|E_i - \lambda|}{\hbar^2}},
$$

(6)

which leads to $|\varphi_{1,i}(x)| \gg |\varphi_{2,i}(x)|$. We can neglect the terms $\propto \sum_i \varphi_{2,i}(x_1)\varphi_{2,i}(x_2)$ in Eq. (4) for $r_1, r_2 \to \infty$. Finally, with the definition of the pair potential $\Delta(x_2,x_1) = v(x_1,x_2)\Psi_{\text{pair}}(x_1,x_2)$ and asymptotically vanishing potential $U \to 0$, we obtain Eq. (3).

The structure of Eq. (3) is identical to the Schrödinger equation for two interacting particles with the total energy $E = 2\lambda < 0$. We note that the two-particle Schrödinger equation, known to hold for the strong coupling limit or the Bose-Einstein condensate (BEC) regime of the BCS-BEC crossover phenomenon \[29, 30\], also holds in the asymptotic limit far outside the surface. In the following we consider the Cooper pair wave function in the $^1S$ channel ($x_1 = r_1 \uparrow, x_2 = r_2 \downarrow$). We also assume the spherical symmetry of $|\Phi_0\rangle$.

Adopting the di-neutron coordinate system (the relative coordinate $r = r_1 - r_2$ and the c. m. coordinate of the di-neutron $R = (r_1 + r_2)/2$), a solution of Eq. (3) in a separable form can be obtained as,

$$
\Psi_{\text{pair}}(r_1, r_2) = \sum_L \int d\mathcal{E}_{\text{e}} \phi_{\text{e}}^{L}(r)\Phi_{\text{d, e}}^{L}(R)P_L(\cos \theta_{\text{Re}}).
$$

(7)

This solution is expressed by the relative wave functions $\phi_{\text{e}}^{L}(r)$ with the angular momentum $L$, obeying

$$
\left[ -\frac{\hbar^2}{2\mu} \Delta_r + v(r) \right] \phi_{\text{e}}^{L}(r)Y_{LM}(\hat{r}) = e\phi_{\text{e}}^{L}(r)Y_{LM}(\hat{r})
$$

with the reduced mass $\mu = \frac{1}{2}m$ and the relative energy $e$, and the c. m. wave function $\Phi_{\text{d, e}}^{L}(R)Y_{LM}(\hat{R})$ of the di-neutron, behaving at $R \to \infty$ as

$$
\Phi_{\text{d, e}}^{L}(R) \to \exp(-\kappa_{\text{d, e}}R)/R
$$

(8)

with the exponential constant $\kappa_{\text{d, e}} = \sqrt{2M(2|\lambda| + e)/\hbar}$ for the di-neutron with the mass $M = 2m$ and the energy $E_{\text{d}} = -2|\lambda| - e$.

The $nn$ system has no bound state with $e < 0$, but in the $^1S$ channel it has a virtual state due to the large scattering length $a = -18.5$ fm \[31\]. As a result, at small $r$ and $e \sim 0$
the $^1S$-wave function $\phi_{e}^{L=0}(r)$ has a large amplitude and it depends only very weakly on $e$. Provided that $C_{e}^{L=0} \neq 0$ and $C_{e}^{L=0}$ is smooth as a function of $e$, the asymptotic Cooper pair wave function \((7)\) is then dominated by the $L = 0$ and $e = 0$ component. Thus we have

$$
\Psi_{\text{pair}}(r_1, r_2) \to C_{e}^{L=0} \phi_{e}^{L=0}(r) \exp(-\kappa_{d,0}R)/R
$$

for $R \to \infty$ and small $r$, where the exponential constant is

$$
\kappa_{d,0} = \sqrt{\frac{2M|\lambda|}{\hbar^2}} = \sqrt{\frac{8m|\lambda|}{\hbar^2}}.
$$

The asymptotic form Eqs. (9) and (10) indicates the penetration of a di-neutron correlated spatially at short relative distances, and its penetration length is controlled only by di-neutron mass $M = 2m$ and the two-neutron separation energy $S_{2n} = 2|\lambda|$. The amplitudes $C_{e}^{L>0}$ and $C_{e}^{L>0}$ in Eq. (7) influence the behavior at larger $r$, and they may depend on detailed conditions, e.g. the pair wave function inside the nuclear surface, the quasiparticle spectra, and the Fermi energy.

In the following, we will examine the asymptotic behavior of the Cooper pair wave function by performing the selfconsistent HFB calculation with the Skyrme functional for even-even $^{44-76}$Ca, $^{60-88}$Ni, $^{92-138}$Zr and $^{120-150}$Sn covering from stable to neutron-rich drip-line nuclei. The Skyrme parameters are respectively SkM* [32] for Ca, SLY4 [33] for Ni and Sn, and SkI4 [34] for Zr. The pairing force is a density-dependent contact interaction, with the force strength $v_{0} = -458.4$ MeV fm$^{-3}$ and the energy cut-off $E_{\text{cut}} = 60$ MeV, which reproduces the $^1S$ scattering length [35, 36]. The HFB equation (1) is solved by mesh diagonalization in the radial coordinate space [37]. Compared with previous HFB calculations (e.g., Refs. [6, 7, 21–24, 38]), we use a larger box size $R_{\text{box}} = 100$ fm and a larger angular momentum space $l_{\text{max}} = 72$ to describe the asymptotic behaviors of the neutron pairing.

First, let us discuss a very neutron-rich nucleus $^{138}$Zr, which is predicted to have giant halo structure [22, 23, 38]. Figure II (a) and (b) show the single-particle levels and quasiparticle spectra of neutrons in $^{138}$Zr respectively. The quasiparticle spectra are presented in terms of the pair number density $\tilde{n}_{lj}(E)$ for the quasiparticle states with angular quantum numbers $l = 0 \sim 5$, $j = l \pm 1/2$ [38]. This nucleus has a very shallow Fermi energy $\lambda = -0.22$ MeV in our calculation, and thus all the quasiparticle levels turn out to be continuum states above the threshold $|\lambda|$ as seen in Fig. II (b).

The neutron pair condensate (the pair density) $\tilde{\rho}(R) \equiv \langle \Phi_0 | \psi(R \uparrow) \psi(R \downarrow) | \Phi_0 \rangle = \Psi_{\text{pair}}(R, R)$, which is nothing but the Cooper pair wave function at contact configuration,
FIG. 1: (a) Single-particle levels and (b) quasiparticle spectra of neutrons in $^{138}$Zr. The quasiparticle spectra are presented in terms of the pair number density $\tilde{n}_{lj}(E)$ [38]. The Fermi energy is denoted by the dashed curve. (c) Neutron pair condensate $\tilde{\rho}(R)$ in $^{138}$Zr. The solid curve is the total pair condensate and the dotted curves are partial contributions $\tilde{\rho}_{lj}(R)$ from quasiparticle states with $l = 0 \sim 5$ ($j = l \pm 1/2$), and $l = 10, 20, 30, 40$ ($j = l - 1/2$).

is shown in Fig. 1 (c). It has a huge extended tail with a very gentle exponential slope due to the shallow Fermi energy. A large number of partial waves reaching very high orbital angular momenta ($l \sim 10, 20, 30, 40$ at $R = 10, 20, 30, 40$ fm, respectively) have coherent contributions of comparable magnitudes to build up the total pair condensate $\tilde{\rho}(R)$. It is in contrast with the naive single-particle picture, in which the bound single-particle orbits located near the Fermi surface, $3p_{1/2}, 3p_{3/2}$ and $2f_{7/2}$ in the present case, would be dominant. Moreover it is consistent with the spatially correlated Cooper pair predicted in our analytic evaluation Eq. (9) since the $l$-coherence up to a large value $l_{\text{corr}}$ implies an angular correlation at small relative angles $\lesssim \theta_{\text{corr}} \sim \mathcal{O}(1/l_{\text{corr}})$ between two neutrons or equivalently the spatial correlation at short relative distance $r \lesssim R\theta_{\text{corr}} \sim \mathcal{O}(R/l_{\text{corr}})$ [24].

A more direct evidence for the analytic expression, Eqs. (9) and (10), can be seen in the slope of the exponential tail. By fitting $R^2 \tilde{\rho}(R) = Ae^{-\tilde{\kappa}R}$ at $R = 35 - 40$ fm, we obtain the exponential constant $\tilde{\kappa} = 0.218$ fm$^{-1}$, which is in good agreement with the analytic value $\kappa_{d,0} = 0.207$ fm$^{-1}$ calculated with Eq. (10).
FIG. 2: The asymptotic exponential constant $\kappa$ of the neutron pair condensate $\tilde{\rho}(R)$ obtained from the HFB calculation for $^{44-76}$Ca, $^{60-88}$Ni, $^{92-138}$Zr, $^{120-150}$Sn, plotted with filled symbols as a function of $\sqrt{\lambda}$, where $\lambda$ is the Fermi energy. The results for $^{48}$Ca, $^{78}$Ni, $^{132}$Sn, $^{110}$Zr and $^{122}$Zr are not included due to the absence of pairing. The open symbols denote the estimate $\kappa_{qp}$, which is evaluated for the lowest quasiparticle state (see text for details).

The asymptotic exponential constants $\kappa$ obtained by fitting to the results for $^{44-76}$Ca, $^{60-88}$Ni, $^{92-138}$Zr, and $^{120-150}$Sn are plotted in Fig. 2. We find that all the exponential constants $\kappa$ follow quite well the expected relation $\kappa = \sqrt{8m|\lambda|}/\hbar$ even though the Fermi energy varies from $\lambda = -10.3$ MeV ($^{44}$Ca) to $-0.22$ MeV ($^{138}$Zr).

We emphasize that the above results are non-trivial if one treats the problem from the viewpoint of the independent quasiparticle basis. Noting asymptotic forms of the quasiparticle wave function, given in Eq. (5), one may assume that the asymptotics of the pair condensate $\tilde{\rho}(R) \sim \sum_i \varphi_{1,i}(R)\varphi_{2,i}(R)$ is dominated by quasiparticle states with the lowest quasiparticle energy. This assumption gives an estimate $\tilde{\kappa}_{qp} = \kappa_{1,\text{min}} + \kappa_{2,\text{min}}$ with $\kappa_1$ and $\kappa_2$ evaluated for the lowest discrete quasiparticle energy $E_{i,\text{min}}$ as in Eq. (6). If there is no bound quasiparticle state (the case of shallow Fermi energy), the lowest quasiparticle state is the one at the threshold $E_{i,\text{min}} = |\lambda|$ for unbound continuum states, and the estimate would be $\tilde{\kappa}_{qp} = \kappa_{2,\text{min}} = \sqrt{4m|\lambda|}/\hbar$ [20]. These estimates are also plotted in Fig. 2. In the case of $^{138}$Zr, the above estimate gives $\tilde{\kappa}_{qp} = \sqrt{4m|\lambda|}/\hbar = 0.146$ fm$^{-1}$, but this is about 30% smaller than the actual value $\tilde{\kappa} = 0.218$ fm$^{-1} \approx \sqrt{8m|\lambda|}/\hbar$. Clearly a superposition of unbound continuum states is necessary [38]. We can justify this statement also by noting that the summation in Eq. (2) over the unbound continuum states with $E \geq |\lambda|$ can be
FIG. 3: Same as Fig. 1 but for $^{92}$Zr. In the inset of panel (c), the partial pair condensate of $d_{5/2}$ states (thin solid curve) is further separated into those from the discrete $2d_{5/2}$ (dashed curve) and continuum states (dotted curve).

evaluated approximately as [37]

$$R^2 \tilde{\rho}(R) \propto \int_{|\lambda|}^{\infty} \sin(\kappa_1(E)R)e^{-\kappa_2(E)R}dE$$

$$\sim K_2\left(\sqrt{\frac{8m|\lambda|}{\hbar^2}}R\right) \sim \exp\left(-\sqrt{\frac{8m|\lambda|}{\hbar^2}}R\right),$$

(11)

for $R \to \infty$. Here $K_2(z)$ is the modified Bessel function.

Finally we remark that the microscopic content of the asymptotic Cooper pair varies with the separation energy although the asymptotic exponential constant $\tilde{\kappa} = \sqrt{8m|\lambda|}/\hbar$ is universal for both neutron-rich and stable nuclei with shallow and deep Fermi energies. An example is a stable nucleus $^{92}$Zr ($\lambda = -6.6$ MeV). As shown in Fig. 3 (a) (b) there exist several bound quasiparticle states in this case. The $2d_{5/2}$ orbit is the one with the lowest quasiparticle energy that would be occupied by the last two neutrons in the independent single-particle picture.

The pair condensate $\tilde{\rho}(R)$ and its quasiparticle compositions $\tilde{\rho}_{lj}(R)$ of $^{92}$Zr are shown in Fig. 3 (c). Apart from the steep asymptotic exponential slope due to the large separation
energy, we find coherent high-\( l \) contributions in the exponential tail, as in the case of \(^{138}\)Zr. In addition we see another feature, not seen in \(^{138}\)Zr, that there is a significant contribution from the \( d_{5/2} \) states. It comes from the discrete quasiparticle state \( 2d_{5/2} \) as shown in the inset of Fig. 3(c). We note that the contribution of this quasiparticle state, \( \varphi_{1,i}(R \uparrow)\varphi_{2,i}^*(R \downarrow) \) in Eq.(2), has an asymptotic exponential constant \( \tilde{\kappa}_{\text{qp}} = \kappa_{1,\text{min}} + \kappa_{2,\text{min}} = 1.128 \text{ fm}^{-1} \), which is almost identical to the value \( \tilde{\kappa} = \sqrt{8m|\lambda|/\hbar} = 1.131 \text{ fm}^{-1} \) since \( E_i \ll |\lambda| \) (See also the open symbols lie on the line \( \tilde{\kappa} = \sqrt{8m|\lambda|/\hbar} \) for \( |\lambda| \gtrsim 4 \text{ MeV} \) in Fig. 2). Thus this quasiparticle contribution remains effectively for physically relevant range of large \( R \). In this case, the asymptotic Cooper pair wave function may be generally written as

\[
\Psi_{\text{pair}}(r_1, r_2) \sim C' \phi_0(r)e^{-\kappa_{d,0}R/R} + \frac{1}{2} \sum_{i_m} \left[ \varphi_{1,i_m}(r_1 \uparrow)\varphi_{2,i_m}^*(r_2 \downarrow) - (x_1 \leftrightarrow x_2) \right] \] (12)

where the second sum runs over the lowest quasiparticle states.

Equation (12) can cover from stable to drip-line nuclei. The second term represents the independent quasiparticle behavior, which survives if \( E_{i,\text{min}} \ll |\lambda| \) or \( \Delta \ll |\lambda| \), valid for nuclei close to the stability line (\( \Delta \) being the pairing gap). As the Fermi energy \( \lambda \) approaches zero, i.e. in the case of \( E_{i,\text{min}} \sim |\lambda| \) or \( \Delta \gtrsim |\lambda| \), the asymptotic Cooper pair wave function is dominated by the first term, representing the spatially correlated di-neutron penetration.

In summary, the neutron Cooper pair never looses its spatial correlation when it penetrates into the asymptotic region \( R \to \infty \). The asymptotic form of the neutron pair condensate is \( \tilde{\rho}(R) \sim e^{-\tilde{\kappa}R} \) with the exponential constant \( \tilde{\kappa} = \sqrt{2(2m)S_{2n}/\hbar} \), which is characterized by the two-neutron separation energy \( S_{2n} = 2|\lambda| \) and the di-neutron mass \( 2m \), irrespective of whether the Fermi energy \( |\lambda| \) is small or large. It implies that there is no theoretical upper bound on the penetration length of the pair condensate, since it scales as \( 1/\tilde{\kappa} \propto 1/\sqrt{S_{2n}} \). The spatial correlation in the asymptotic Cooper pair emerges explicitly in the weakly bound nuclei satisfying \( |\lambda| \lesssim \Delta \) (\( S_{2n} \lesssim 2\Delta \)), while for large separation energies the independent particle features coexist.

In the present analysis we have employed the fact that the \(^1S\) interaction has a large scattering length. If the interaction is weaker, the spatial correlation will be weakened accordingly. We remark also that the above results can be generalized to the surface penetration in any \( S\)-wave paired Fermi systems.

We thank T. Nakatsukasa, K. Washiyama, K. Yabana, and K. Yoshida for useful discus-
sions. This work was partly supported by the Major State 973 Program 2013CB834400; the National Natural Science Foundation of China under Grants No. 11335002, No. 11005069, and No. 11175002; the Research Fund for the Doctoral Program of Higher Education under Grant No. 20110001110087; and the Grant-in-Aid for Scientific Research (No. 21340073, No. 23540294 and No.24105008) from the Japan Society for the Promotion of Science.

[1] I. Tanihata, et al., Phys. Rev. Lett. **55**, 2676 (1985).
[2] B. Jonson, Phys. Rep. **389**, 1 (2004).
[3] I. Tanihata, H. Savajols, and R. Kanungo, Prog. Part. Nucl. Phys. **68**, 215(2013).
[4] P. G. Hansen and B. Jonson, Europhys. Lett. **4**, 409 (1987).
[5] G. F. Bertsch and H. Esbensen, Ann. Phys. (NY) **209**, 327,(1991).
[6] J. Meng and P. Ring, Phys. Rev. Lett. **77**, 3963 (1996).
[7] J. Meng, H. Toki, S.-G. Zhou, S. Q. Zhang, W. H. Long, and L. S. Geng, Prog. Part. Nucl. Phys. **57**, 470 (2006).
[8] M. V. Zhukov, B. V. Danilin, D. V. Fedorov, J. M. Bang, I. J. Thompson, and J. S. Vaagen, Phys. Rep. **231**, 151 (1993).
[9] A.S. Jensen, K. Riisager, D.V. Fedorov, and E. Garrido, Rev. Mod. Phys. **76**, 215 (2004).
[10] F. Barranco, P. F. Bortignon, R. A. Broglia, G. Colò, and E. Vigezzi, Eur. Phys. J. A **11**, 385 (2001).
[11] T. Myo, S. Aoyama, K. Katō, and K. Ikeda, Prog. Theor. Phys. **108**, 133 (2002).
[12] K. Hagino and H. Sagawa, Phys. Rev. C **72**, 044321 (2005).
[13] Y. Kanada-En’yo, Phys. Rev. C **76**, 044323 (2007).
[14] T. Myo, Y. Kikuchi, K. Katō, H. Toki, and K. Ikeda, Prog. Theor. Phys. **119**, 561 (2008).
[15] D. V. Fedorov, A. S. Jensen, and K. Riisager, Phys. Rev. C **49**, 201 (1994).
[16] P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, (Springer-Verlag, Berlin, 1980).
[17] M. Bender, P.-H. Heenen, and P.-G. Reinhard, Rev. Mod. Phys. **75**, 121 (2003).
[18] D. Vretenar, A. V. Afanasjev, G. A. Lalazissis, and P. Ring, Phys. Rep. **409**, 101 (2005).
[19] J. Dobaczewski, H. Floerch, and J. Treiner, Nucl. Phys. A **422**, 103 (1984).
[20] A. Bulgac, preprint FT-194-1980, Bucharest, 1980, *nucl-th/9907088*.
[21] J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Dechargé,
Phys. Rev. C 53, 2809 (1996).

[22] J. Meng and P. Ring, Phys. Rev. Lett. 80, 460 (1998).

[23] M. Grasso, S. Yoshida, N. Sandulescu, and N. Van Giai, Phys. Rev. C 74, 064317 (2006).

[24] M. Matsuo, K. Mizuyama and Y. Serizawa, Phys. Rev. C 71, 064326 (2005).

[25] N. Pillet, N. Sandulescu, and P. Schuck, Phys. Rev. C 76, 024310 (2007).

[26] N. N. Bogoliubov, Sov. Phys. JETP 7, 41 (1958).

[27] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).

[28] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 80, 1215 (2008).

[29] A. J. Leggett, In *Modern Trends in the Theory of Condensed Matter*, edited by A. Pękalski and J. A. Przystawa, Lecture Note in Physics 115, (Springer-Verlag, Berlin, 1980); A. J. Leggett, J. Phys. Colloques 41, C7-19 (1980).

[30] P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985).

[31] G. F. de Téramond and B. Gabioud, Phys. Rev. C 36, 691 (1987).

[32] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.-B. Häkansson, Nucl. Phys. A 386, 79 (1982).

[33] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A 635, 231 (1998).

[34] P.-G. Reinhard and H. Flocard, Nucl. Phys. A, 584, 467 (1995).

[35] M. Matsuo, Phys. Rev. C 73, 044309 (2006).

[36] M. Matsuo and Y. Serizawa, Phys. Rev. C 82, 024318 (2010).

[37] Y. Zhang, M. Matsuo, and J. Meng, in preparation.

[38] Y. Zhang, M. Matsuo, and J. Meng, Phys. Rev. C 86, 054318 (2012).