Topical Review

FMR-related phenomena in spintronic devices

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Abstract
Spintronic devices, such as non-volatile magnetic random access memories and logic devices, have attracted considerable attention as potential candidates for future high efficient data storage and computing technology. In a heavy metal or other emerging material with strong spin–orbit coupling (SOC), the charge currents induce spin currents or spin accumulations via SOC. The generated spin currents can exert spin–orbit torques (SOTs) on an adjacent ferromagnet, which opens up new avenues for realization of magnetization dynamics and switching of the ferromagnetic layer for spintronic devices. In the SOT scheme, the charge-to-spin interconversion efficiency (SOT efficiency) is an important figure of merit for applications. For effective characterization of this efficiency, ferromagnetic resonance (FMR) based methods, such as spin transfer torque ferromagnetic resonance (ST-FMR) and spin pumping, are commonly utilized in addition to low frequency harmonic or DC measurements. In this review, we focus on ST-FMR measurements for the evaluation of the SOT efficiency. We provide a brief summary of the different ST-FMR setups and data analysis methods. We then discuss ST-FMR and SOT studies in various materials, including heavy metals and alloys, topological insulators, two-dimensional (2D) materials, interfaces with a strong Rashba effect, antiferromagnetic materials, 2D electron gas in oxide materials and oxidized nonmagnetic materials.

Keywords: magnetization dynamics, ferromagnetic resonance, spin transfer torque, spin–orbit torque

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, electrical manipulation of magnetization has been a central theme in modern spintronics [1–3]. Following the successful applications of magnetic tunnel junctions (MTJs) [4, 5] in the head reads of hard disk drives, new functional spintronic devices, such as non-volatile magnetic random access memories (MRAM) and magnetic logic devices [6, 7], are now being pursued enthusiastically by researchers. In practical spintronic applications, it is desirable to achieve efficient magnetization manipulation via electrical means with a higher operational speed and lower energy consumption. In order to evaluate the efficiency of the electrical manipulation of the magnetization and to understand the underlying physics in magnetic devices, various techniques based on magnetization dynamics under the application of currents have been developed.

The study of magnetization dynamics dates back to the observation of ferromagnetic resonance (FMR) in the first half of the 20th century [8, 9]. Originally, FMR was used to study magnetic films and the magnetization dynamics in such films in the context of external magnetic fields only. With the
discovery of spin transfer torques (STT) in 1996 [10, 11] and development of modern nano-fabrication techniques, which allow the device dimensions to be scaled down, electrical manipulation of nanomagnets using spin-polarized current injection gained popularity. After 2000, STT-driven magnetization switching using electric currents was experimentally observed in giant magnetoresistance (GMR) nanopillars and nanosized MTJ devices [12–14]. In order to gain insight into the magnetization dynamics of an individual nanomagnet or magnetic device under the application of an electric current, the spin transfer torque ferromagnetic resonance (ST-FMR) technique was developed and explored in GMR nanopillars and nanosized MTJs [15–18].

One of the drawbacks of the STT scheme is the requirement of an additional magnet to create spin-polarized currents. In 2004, the imaging of direct charge-to-spin current conversion in a single semiconductor GaAs due to the spin Hall effect (SHE) was observed [19]. Since these observations, pure spin current generation in non-magnets has attracted tremendous attention in spintronic devices [2]. Recently, it was demonstrated that the magnetization can be switched or driven into precession modes by pure spin currents only [20–24], which opened up a new route for realization of spintronic devices. In order to achieve highly efficient magnetization manipulation, a key challenge is to identify materials that exhibit a large charge-to-spin interconversion efficiency (i.e. spin Hall angle or spin–orbit torque (SOT) efficiency). The ST-FMR is an effective technique for evaluation of SOT efficiency. The first demonstration was reported in a Pt/NiFe bilayer planar spin Hall device in 2011 [25], which is based on the spin rectification of the anisotropic magnetoresistance (AMR) effect. Subsequently, ST-FMR has been used to evaluate SOT efficiencies and to understand the underlying physics in many emerging materials and is presented later.

This article provides a topical review of the ST-FMR technique and its applications in studying the SOT-related phenomena in diverse emerging materials. In section 2, we begin with the basic principle of magnetization dynamics. In sections 3 and 4, we introduce the ST-FMR setups and data analysis methods. In sections 5–7, we review the progress in SOT studies by using the ST-FMR technique in different material systems, such as heavy metals, topological insulators and other emerging material systems. We end with a summary and perspectives in section 8.

2. Magnetization dynamics

2.1. LLG equation

Under macrospin approximation, the time domain dynamics of the magnetization vector (m) of a ferromagnet in the presence of an effective magnetic field (Heff) can be described by the Landau–Lifshitz–Gilbert (LLG) equation [26],

\[
\frac{\partial \mathbf{m}}{\partial t} = (-\gamma \mu_0 \mathbf{m} \times \mathbf{H}_{\text{eff}}) + (\alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t})
\]  

(1)

where \(\gamma\), \(\mu_0\) and \(\alpha\) are the gyromagnetic ratio, vacuum permeability and the Gilbert damping parameter, respectively. The right-hand side of the LLG equation has two torque terms—the first term, \(-\gamma \mu_0 \mathbf{m} \times \mathbf{H}_{\text{eff}}\), is a precession term that rotates \(\mathbf{m}\) around \(\mathbf{H}_{\text{eff}}\), while the second term, \(\alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}\), is the damping (or energy dissipation) term which aligns \(\mathbf{m}\) along \(\mathbf{H}_{\text{eff}}\). The value of \(\alpha\) determines the rate at which \(\mathbf{m}\) damps towards \(\mathbf{H}_{\text{eff}}\). When the value of \(\alpha\) is large (small), \(\mathbf{m}\) damps towards \(\mathbf{H}_{\text{eff}}\) at a faster (slower) rate.

When a spin-polarized current (polarized along \(\hat{\sigma}\)) interacts with \(\mathbf{m}\), there are additional torques experienced by \(\mathbf{m}\) and the LLG equation is modified as [10, 11, 27]

\[
\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mu_0 \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \tau_{\text{DL}} + \tau_{\text{FL}}
\]  

(2)

where \(\tau_{\text{DL}}\) is of the \(\mathbf{m} \times (\hat{\sigma} \times \mathbf{m})\) symmetry and is the damping-like torque, while \(\tau_{\text{FL}}\) is of the \(\hat{\sigma} \times \mathbf{m}\) symmetry and is the field-like torque.

2.2. Ferromagnetic resonance (FMR)

In section 2.1, we discussed that \(\mathbf{m}\) precesses about \(\mathbf{H}_{\text{eff}}\) in the presence of \(\mathbf{H}_{\text{eff}}\). Due to energy dissipation (damping) in the ferromagnet, the amplitude of these precessions decays continuously. An external energy supplied to the ferromagnet in the form of an alternating magnetic field (HRF) can compensate this energy dissipation leading to forced precession of \(\mathbf{m}\). If the frequency of \(\mathbf{H}_{\text{RF}}\) is tuned to the natural precession frequency of \(\mathbf{m}\), the amplitude of the forced precession will be maximum due to resonance. This resonance is the FMR and the precession frequency during the resonance condition is the resonance frequency (\(f = \omega_{\text{res}} / 2\pi\)), \(f\) and \(H_0\) (the resonant field) are related by the Kittel’s relation,

\[
f = (\gamma / 2\pi) [H_0 (H_0 + 4\pi M_{\text{eff}})]^{1/2}
\]  

(3)

where \(4\pi M_{\text{eff}}\) is the effective demagnetization field of the ferromagnet. According to equation (3), the resonance condition can be matched by fixing \(H_0\) and tuning the frequency of \(H_{\text{RF}}\) or vice versa.

2.3. Spin transfer torque ferromagnetic resonance (ST-FMR)

In contrast to the magnetic field driven FMR, it is possible to excite FMR using spin torques generated by an alternating current, the ST-FMR. The ST-FMR technique was first used in a Pt/NiFe (Py) bilayer in the case of a spin Hall scheme [25], which is proven to be an effective method to evaluate the charge-to-spin conversion efficiency (i.e. spin Hall angle). We first provide a more general description of the ST-FMR process in the Pt/Py bilayer as an example.

As shown in figure 1(a), when an in-plane RF charge current (\(I_{\text{RF}}\)) is injected into the Pt/Py bilayer, non-equilibrium spins are generated at the Pt/Py interface due to spin–orbit coupling (SOC) in the Pt layer. Subsequently, these generated spins diffuse into the adjacent Py layer and exert SOTs (damping-like torque, \(\tau_{\text{DL}}\) and/or field-like torque, \(\tau_{\text{FL}}\)) on the Py local magnetization. In addition, the current in the Pt layer induced Oersted field can also exert the Oersted field torque (\(\tau_{\text{Oe}}\)) on the Py magnetization. Since the injected charge current is oscillatory, all the above torques are oscillatory and
hence trigger the Py magnetization into precession around the direction of the effective magnetic field, which leads to an oscillatory AMR in the ST-FMR device. Consequently, the oscillation of the AMR and the $I_{RF}$ in the ST-FMR device produce a mixing ST-FMR voltage $V_{\text{mix}}$ across the Pt/Py bilayer, which can be detected by a DC voltmeter or a lock-in amplifier. Since the ST-FMR signal is the spin rectification of the magnetoresistance in the device [28], the ST-FMR technique has a high enough sensitivity and can be used to study micro-sized or even nanosized magnetic devices.

Subsequently, we present a derivation of general expression for the ST-FMR signal $V_{\text{mix}}$. The RF current applied to the Pt/Py ST-FMR device is written as $I_{RF} = I \cos(\omega t)$, where $I$ is the amplitude of $I_{RF}$ with a frequency of $\omega/2\pi$. The device resistance is $R(t) = R_0 + \Delta R \cos^2 \theta(t)$, where $R_0$ is the resistance when $I_{RF}$ is perpendicular to the Py magnetization direction, $\Delta R$ is the AMR, $\theta(t)$ is the angle between $I_{RF}$ and Py magnetization, which is a function of time $t$ due to the magnetization precession, given by the equation $\theta(t) = \theta_{H} + \theta_{c} \cos(\omega t + \delta)$, where $\theta_{H}$ is the constant angle between $H_{ext}$ and $I_{RF}$, and $\theta_{c}$ is the cone angle of Py magnetization precession, which is a measure of the total torques on the Py magnetization and $\delta$ is usually much smaller than $\theta_{H}$. $\delta$ is the resonance phase between the driving force (i.e. damping-like torque $\tau_{DL}$, field-like torque $\tau_{FL}$ or Oersted field torque $\tau_{OE}$) and the magnetization response (i.e. the oscillation of AMR) [29, 30].

By using the Taylor’s expansion, we get $\cos \theta(t) = \cos \theta_{H} - \sin \theta_{H} \cdot \theta_{c} \cos(\omega t + \delta)$ and $R(t) = R_0 + \Delta R \left[ \cos^2 \theta_{H} - 2 \cos \theta_{H} \sin \theta_{H} \cdot \theta_{c} \cos(\omega t + \delta) \right]$. Hence, the product is

$$V(t) = I(t) \cdot R(t) = \left( I R_0 + I \Delta R \cos^2 \theta_{H} \right) \cos(\omega t) - I \Delta R \sin(2\theta_{H}) \cdot \theta_{c} \cos(2\omega t + \delta) / 2 - I \Delta R \sin(2\theta_{H}) \cdot \theta_{c} \cos(\cos \delta) / 2.$$  

The total voltage $V(t)$ is comprised of three terms at frequencies $\omega$, $2\omega$, and a time-independent term. The last DC term is the rectified ST-FMR signal, $V_{\text{mix}} = -I \Delta R \sin(2\theta_{H}) \cdot \theta_{c} \cos(\cos \delta) / 2$. The ST-FMR signal is determined by the combination of the amplitude of $I_{RF}$, AMR in the device, the angle of $H_{ext}$ with respect to $I_{RF}$, the cone angle of magnetization precession $\theta_{c}$ and the resonance phase $\delta$.

For in-plane alternative magnetic field driven magnetization dynamics of an in-plane magnetization dynamics of an in-plane magnetization [30, 31], the value of $\delta$ changes from $\pi$ to $0$ around the resonance field $H_{0}$ with a linewidth of $\Delta H$ (full width at half maximum) and $\delta = \pi/2$ at the resonance field $H_{0}$. Therefore, the $V_{\text{mix}}$ attributed to the in-plane alternative magnetic field has an antisymmetric Lorentzian line shape, which is the case for the $\tau_{DL}$ and/or $\tau_{FL}$ driven magnetization dynamics in a Pt/Py ST-FMR device as shown by the green curves in figure 1(b). Whereas for the $\tau_{DL}$ driven magnetization dynamics, there is an additional $90^\circ$ phase difference compared to $\tau_{FL}$ or $\tau_{OE}$ driven magnetization dynamics for an AMR based rectification system. Therefore, the driving force $\tau_{DL}$ is in-phase ($\delta = 0$) at the resonance field $H_{0}$, leading to a symmetric Lorentzian line shape arising from the damping-like torque in a Pt/Py ST-FMR device, as illustrated by the blue curves in figure 1(b). In section 4, we will present three quantitative analyses of the ST-FMR signal in detail.

3. ST-FMR setups

3.1. Traditional ST-FMR setup with a DC voltmeter

As described in section 2.3, the ST-FMR signal is a rectified DC voltage, which can be detected directly by a DC voltmeter. Figure 2 shows a schematic of the traditional ST-FMR setup with a DC voltmeter. It depicts a typical ST-FMR device with a microstrip of heavy metal (HM)/ferromagnet (FM) bilayer structure. A ground-signal-ground (G-S-G) coplanar
3.2. Precise ST-FMR setup with a lock-in amplifier

In the traditional ST-FMR setup, the noise level is usually in the order of ~100 nV because of the limitation of a DC voltmeter. This noise level reduces the signal-to-noise ratio substantially while studying some of the emerging material systems possessing smaller ST-FMR signals with the order of ~μV, such as Pt/Py systems [25, 32].

To elaborate on this limitation, consider the example of a typical CPW is illustrated in figure 4(a). The width of S is s and the gap between the S and G lines is w. The characteristic impedance of the CPW can be tuned by varying the s and/or w values. One can use a common CPW calculator software to select the proper combination of s and w values for the CPW design. For example, the s is about 60 μm and w is about 30 μm for the CPW design in [32]. Further, the HM/FM bilayer current channel should also be impedance matched, which can be tuned by changing the channel length L and width W (also see the current channel denoted by the red circles in figures 4(b) and (c)). For example, L is in the range of ~10–60 μm and W is in the range of ~10–30 μm in [32–34]. Finally, the Z_{in} of ST-FMR devices can be quantitatively calibrated by the microwave-network-analysis measurement using a vector network analyzer (VNA).

There are usually two kinds of CPW designs: the asymmetric and the symmetric CPW designs. The asymmetric CPW design is shown in figure 4(b), which was initially used for the HM/FM bilayer ST-FMR measurements [25]. However, there is a limitation with the asymmetric CPW. To elaborate on this limitation, consider the example of the Pt/Py ST-FMR device shown in figure 4(d). In addition to the current-induced in-plane Oersted field (H_{Oe}) on the Py magnetization, an imbalanced current-induced out-of-plane Oersted field H_{z} from the contacting electrodes can be produced as I_{RF} is transmitting along the CPW [32, 36]. Consequently, H_{z} exerts an in-plane torque (−m × H_{z}) on the FM current channel, the ST-FMR device should be designed to have an impedance (Z_{in}) of around 50 Ω. First, the CPW should be designed by considering impedance matching. One example of a typical CPW is illustrated in figure 4(a). The width of S is s and the gap between the S and G lines is w. The characteristic impedance of the CPW can be tuned by varying the s and/or w values. One can use a common CPW calculator software to select the proper combination of s and w values for the CPW design. For example, the s is about 60 μm and w is about 30 μm for the CPW design in [32]. Further, the HM/FM bilayer current channel should also be impedance matched, which can be tuned by changing the channel length L and width W (also see the current channel denoted by the red circles in figures 4(b) and (c)). For example, L is in the range of ~10–60 μm and W is in the range of ~10–30 μm in [32–34]. Finally, the Z_{in} of ST-FMR devices can be quantitatively calibrated by the microwave-network-analysis measurement using a vector network analyzer (VNA).
As shown in figure 4(c), the current-induced Oersted field angle evaluation, the symmetric CPW design was developed. Related influences in the ST-FMR measurements and spin Hall in the electrodes is balanced as $I_{RF}$ flows in the symmetric CPW [32]. Reprinted with permission from [32]. Copyright 2014, AIP Publishing LLC.

In order to diminish or eliminate the imbalanced $H_z$ and the related influences in the ST-FMR measurements and spin Hall angle evaluation, the symmetric CPW design was developed. As shown in figure 4(c), the current-induced Oersted field in the electrodes is balanced as $I_{RF}$ flows in the symmetric CPW, resulting in no $H_z$ component. Therefore, the symmetric CPW design is better than the asymmetric CPW for ST-FMR measurements.

4. Methods to evaluate the spin–orbit torque (SOT) and SOT efficiency

In section 2.3, a general derivation of the ST-FMR signal was provided. In this section, we will further present three quantitative analyses of the ST-FMR signal. The ST-FMR signals from a HM/FM bilayer device (see figure 1(b)) can be fitted by $V_{\text{mix}} = V_S F_S (H_{\text{ext}}) + V_A F_A (H_{\text{ext}})$, where $F_S (H_{\text{ext}}) = \Delta H^2 / \left[ (H_{\text{ext}} - H_0)^2 + \Delta H^2 \right]$ is a symmetric Lorentzian function of amplitude $V_S$ and $F_A (H_{\text{ext}}) = \Delta H (H_{\text{ext}} - H_0) / \left[ (H_{\text{ext}} - H_0)^2 + \Delta H^2 \right]$ is an antisymmetric Lorentzian function of amplitude $V_A$ [25, 32]. The symmetric component has a maximum value at the resonance field $H_0$ and is centered about $H_0$, whereas, the antisymmetric component has a dispersive curve with the value of zero at $H_0$. Both components have the same linewidth $\Delta H$ with respect to the magnetic field.

4.1. Ratio of $V_S/V_A$

It is noted that $V_S$ is proportional to the amplitude of spin currents $J_S$ (i.e. $\tau_{DL}$) and is written as $V_S \propto N J_S / (2 e \mu_0 M_s t)$ [18, 25, 37], where $M_s$ and $t$ are the saturation magnetization and thickness of the FM layer, respectively, and $h$ is the reduced Planck constant. While $V_A$ is correlated with the $H_{RF}$ induced Oersted field torque ($\tau_{RF}$), which is written as $V_A \propto H_{RF} [1 + (4 \pi M_{eff}/H_{ext})]^{1/2}$ [18, 25, 37], where $M_{eff}$ is the effective magnetization of the FM layer. Note that the field-like torque $\tau_{FL}$ is assumed to be negligible here. $H_{RF}$ is estimated by Ampere’s law as $H_{RF} = J_C d/2$, where $J_C$ is the charge current density in the HM layer and $d$ is the thickness of the HM layer. Note that the charge current flowing in the FM layer is assumed to be spatially uniform, therefore, there is no net Oersted field torque on the FM magnetization itself.

In the spin Hall scheme, the charge-to-spin conversion efficiency (i.e. spin Hall angle), defined as $\theta_{sh} = J_S / J_C$, can be found from the ratio of $V_S/V_A$ [25]. Using the above equations, one obtains the spin Hall angle as

$$\theta_{sh} = \left( V_S/V_A \right) \left( e \mu_0 M_s t / h \right) \left[ 1 + (4 \pi M_{eff}/H_{ext}) \right]^{1/2}.$$

This method is self-calibrated in the sense that the torques on the FM magnetization only arise from $J_C$ in the Pt layer and the spin Hall angle can be easily calculated without knowing the exact values of $I_{RF}$ and $H_{RF}$ in ST-FMR devices. The method works well to determine $\theta_{sh}$ under the assumption that the $V_A$ only attributes to $H_{RF}$. In other words, this method may give rise to a wrong estimate of $\theta_{sh}$ in the case of a non-negligible field-like torque $\tau_{FL}$, which might arise due to interfacial effects, such as the Rashba effect. Since $\tau_{FL}$ can also produce an antisymmetric Lorentzian line shape similar to $H_{RF}$, the value of $\theta_{sh}$ might be over- or underestimated from the method of $V_S/V_A$ [38, 39]. Therefore, one should be aware of no significant field-like torque from interfacial effects before using the analysis method of the ratio of $V_S/V_A$. 

Figure 4. (a) The schematic of a typical CPW, illustrating the ground-signal-ground (G-S-G) transmission line. ST-FMR devices with (b) asymmetric CPW and (c) symmetric CPW. (d) The Pt/Pt microstrip with the charge current $J_C$, spin current $J_S$, and current-induced in-plane Oersted field ($H_{RF}$). $H_z$ is the current-induced imbalanced out-of-plane Oersted field in the ST-FMR device with an asymmetric CPW design. Reprinted with permission from [32]. Copyright 2014, AIP Publishing LLC.
Note that for simplicity, hereafter, we refer to the spin Hall angle as the SOT efficiency, unless otherwise specified, in order to represent the charge-to-spin conversion efficiency in a variety of materials. After obtaining the SOT efficiency, we can easily evaluate the damping-like torque using \( \tau_{DL} = \theta_{sh} J C h / (2e M_s t) \).

4.2. \( V_{S'}\)–only

Since some material systems give rise to a significant \( \tau_{FL} \) to the adjacent FM magnetization, the SOT efficiency can be better determined by analyzing the symmetric component \( V_S \) only [33, 34, 40]. The damping-like torque, \( \tau_{DL} \), and the associated in-plane SOT efficiency, \( \theta_{sh} \), can be evaluated using \( V_S = -\frac{h c}{4} \cos \theta_{th} \frac{dR}{dh} (\tau_{FL} + \tau_{OE}) \left[ 1 + \frac{\sin M_s H_s}{\Delta} \right] F_{asy}(H_{ext}) \), where \( \Delta \) is the linewidth of the ST-FMR signal in the frequency domain, \( E \) is the microwave electric field across the ST-FMR device, and \( \sigma_S \) and \( \sigma \) are the spin Hall and longitudinal charge conductivities of the HM layer, respectively. In addition, the field-like torque, \( \tau_{FL} \), can be derived by

\[
V_A = -\frac{h c}{4} \cos \theta_{th} \frac{dR}{dh} (\tau_{FL} + \tau_{OE}) \left[ 1 + \frac{\sin M_s H_s}{\Delta} \right] F_{asy}(H_{ext})
\]

[33, 40]. In order to obtain \( \tau_{FL} \), the \( \tau_{OE} \) should be evaluated separately and excluded [33]. Similarly, the out-of-plane SOT efficiency, \( \tau_{1} \), can be evaluated accordingly.

Since the resistance of the HM/FM microstrip is written as \( R = R_0 + \Delta R \cos^2(\theta_{th}) \) due to the AMR effect (see section 2.3), we obtain \( \frac{dR}{dh} = -2 \Delta R \sin(\theta_{th}) \cos(\theta_{th}) \propto \sin(\theta_{th}) \cos(\theta_{th}) \). Consequently, we find \( V_{mix} \propto \cos^2(\theta_{th}) \sin(\theta_{th}) \) from the relation, \( V_{mix} \propto \frac{dt}{dh} \cos(\theta_{th}) \) [32]. Thus, it is observed that even though the change of AMR is maximum at \( \theta_{th} = 45^\circ \), the largest ST-FMR signals are achieved at \( \theta_{th} = 35^\circ \).

The comparison of the in-plane SOT efficiencies evaluated from the ratio of the \( V_S / V_A \) method and from the \( V_S \)-only method is presented in section 5.1 for Pt and in section 5.2 for Ta. These results suggest that the \( V_S \)-only method is a more general way for evaluating the SOT efficiency in materials with SOC. However, compared to the \( V_S / V_A \) ratio method, the \( V_S \)-only technique requires additional measurements to determine the \( dR / dh \) and \( \sigma \), and also requires quantitative determination of \( I_{RF} \) through an ST-FMR device by considering the device impedance and RF power losses using the microwave-network analysis measurement [32].

Note that the SOT efficiency or in-plane SOT efficiency in the entire review for a variety of materials is denoted as \( \theta_{sh} \), and the out-of-plane SOT efficiency is denoted as \( \theta_1 \) in order to be consistent.

4.3. Modulation of damping (MOD)

In the context of conventional STT, researchers found that the STT and magnetization fluctuation can be modulated in the vertical MTJs by applying a DC current bias [41, 42]. Recently, this method was used for HM/FM bilayer systems in the spin Hall scheme and the magnetization dynamics can be tuned by applying a DC current in the HM layer [43]. Based on the STT theory [41], the effective magnetic damping \( (\alpha) \) and thus the FMR linewidth \( (\Delta H) \) will increase or decrease depending on the relative direction of spin polarizations with respect to the magnetization direction. The relationship can be written as [25, 48]

\[
\Delta H = (\Delta H)_0 + (\Delta H)_{sh} = \frac{2\pi \alpha}{\gamma_f} + \left( \frac{\theta_{sh} \cos(\theta_{th})}{\cos(\theta_{th})} \right) J C h \theta_{th}
\]

where \( (\Delta H)_0 \) is the linewidth at zero DC bias, \( (\Delta H)_{sh} \) is the modulated linewidth due to the DC bias induced spin currents, and \( f \) is the RF current frequency. Therefore, one can perform the ST-FMR measurements to extract the \( \Delta H \) as a function of \( J_C \) as shown in figure 5. By linearly fitting the data, the SOT efficiency is obtained accordingly.

The modulation of damping (MOD) method is an extension of the ST-FMR measurements. So far, it has commonly been used to evaluate SOT efficiency in Pt and Ta [25, 44–47], which usually have considerably large ST-FMR signals. Recently, it has been utilized for the SOT efficiency evaluation of metallic antiferromagnetic materials [48]. However, for material systems possessing smaller ST-FMR signals, it is challenging to employ the MOD method to evaluate the SOT efficiency [40].

Table 1 summarizes the advantages and disadvantages of the above three SOT efficiency evaluation methods. The method of \( V_S / V_A \) is simple; however, it only works for material systems where the \( V_A \) only attributes to \( H_{RF} \) (i.e. there is no significant field-like torque). Therefore, this method has limitations for the SOT efficiency evaluation. However, the method of \( V_S \)-only can successfully address this issue and can be used as a more general method for extracting the SOT efficiency in a wide range of materials.

5. Spin Hall effect (SHE) in heavy metals and alloys

The SHE [49–52] is an electrical technique, which exploits the bulk SOC in a nonmagnetic material (NM) (such as HM) to convert charge currents into pure spin currents without any external magnetic field. The first theoretical predictions of SHE by Dyakonov and Perel dates back to 1971 [53] and, in
measurements [25, 32], several different techniques have been
driven magnetization switching or precession via pure spin
Hall magnetoresistance (SMR) measurements [58]. In the
following sub-sections, we first review the research progress of
the SOT efficiency evaluation in HMs and alloys using the
ST-FMR technique.

5.1 Spin Hall effect in Pt

Liu et al first utilized ST-FMR to evaluate the SOT efficiency
in the Pt (6 nm)/Py (4 nm) bilayer device at room temperature
[25]. The schematic of a Pt/Py bilayer and the main results are
shown in figure 7. The value of SOT efficiency in Pt is deter-
mined to be ~0.056. Further, they have also utilized the MOD
method as an independent check to confirm their results. By
taking advantage of the large SOT efficiency, they also demon-
strated SOT-driven magnetization switching in Pt/Co/Al₂O₃
trilayer structures [22].

Subsequently, the SOT efficiency in Pt has been quanti-
fied using different measurement methods by different
research groups. However, there have been significant disa-
greements in the reported values of the SOT efficiency in
Pt, ranging from ~0.012 to ~0.12 [32, 51, 52]. To figure out
the possible reasons for this discrepancy and determine the
intrinsic SOT efficiency value in Pt is of great importance.

For ST-FMR measurements, the measured SOT efficiency is
evaluated from the number of spins that are absorbed by the
FM layer. There are three aspects that affect the measured
SOT efficiency. The first is the spin diffusion in the Py layer
since the spins can transmit into Py with a characteristic
length (i.e. spin diffusion length), which is often ignored.

Wang et al [32] carried out Py thickness-dependent ST-FMR
measurements in Pt (6 nm)/Py (t = 2–10 nm) bilayers. They
found that θ_{sh} increases when the Py thickness increases as
shown in figure 8(a). By taking into account the spin diffusion
in the Py layer, the SOT efficiency of Pt is determined to be
~0.068 at room temperature [32]. The second aspect is
spin diffusion in the Pt layer. J_S and the measured SOT effi-
ciency θ_{sh} in a Pt film of thickness d should be reduced from the
bulk value (d = ∞) by J_S (d) = J_S (∞) [1 − sech (d/λ_S)]
and θ_{sh} = θ_{sh} [1 − sech (d/λ_S)], respectively [25, 32, 45,
59]. For example, the spin diffusion length, λ_S of Pt esti-
mated from a Pt/Py device is ~1.5 nm [32] (figure 8(b)). The
third aspect is the interface transparency as spin currents
transmit through the Pt and Py interface, which is discussed in
section 5.4.
Figure 8(c) shows a summary of SOT efficiencies as a function of $\lambda_S$ in Pt measured using different techniques from various research groups [25, 32, 58, 60–69]. A clear correlation between the SOT efficiency $\theta_{sh}$ and $\lambda_S$ is found, which is approximately an inverse relationship with $\theta_{sh} \sim \lambda_S^{-0.13}$ (denoted by the blue thick line) [32]. The $\lambda_S$ in a material is generally proportional to its electrical conductivity ($\sigma$) [32, 70]. Figure 8(d) shows an approximately linear relationship between the reported $\lambda_S$ and $\sigma$ in Pt films [32, 45, 55, 58–62, 64, 67, 71]. Therefore, the SOT efficiency is inversely related to the conductivity of the HMs. All of the data in figures 8(c) and (d) were measured at room temperature, except for those from [61, 71] which were measured at 10 K, as denoted by small black stars.

In addition, it was found that the $\theta_{sh}$ in Pt remains almost constant as the temperature decreases from 300 K to 13 K [32, 55]. It is possible that in ST-FMR measurements, spin pumping can also occur due to the inverse SHE by $J_C = \theta_{sh} (J_S \times \hat{\sigma})$.

Therefore, an additional DC voltage ($V_{SP}$) can also be produced and might contaminate the ST-FMR signals. The spin pumping contributions for Pt (6 nm)/Py ($t = 2–10$ nm) bilayers have been estimated in [32] using the following equations in the method in [40].

$$V_{SP} = \theta_{sh} eW\lambda_S R \frac{d}{2\lambda_S} \tanh \left( \frac{d}{2\lambda_S} \right) \times \text{Re} (g_{\uparrow \downarrow}^{\text{eff}}) \omega(\theta_c) \sin(\theta_H) \sqrt{H_0 / (H_0 + 4\pi M_{eff})},$$

$$\theta_c = \frac{1}{dR/d\theta_H} \frac{2}{R_{RF}} \sqrt{(V_S)^2 + (V_A)^2},$$

where $W$ is the channel width, $R$ is the device resistance, $d$ is the Pt thickness, $\text{Re}(g_{\uparrow \downarrow}^{\text{eff}})$ is the real part of the effective spin mixing conductance ($\approx 2 \times 10^{19}$ m$^{-2}$), $\theta_c$ is the maximum precession cone angle in the device plane, and $V_S$ and $V_A$ are the symmetric and antisymmetric components of the ST-FMR signal, respectively. It was found that the estimated spin pumping signals $V_{SP}$ are at least one order of magnitude smaller than the ST-FMR symmetric component $V_S$. It is known that $V_{SP}$ is proportional to the device resistance [57]. Since the resistance of the ST-FMR device is usually small (~$50 \Omega$ for the impedance matching), the spin pumping contributions in the ST-FMR measurements are usually much smaller than the ST-FMR signals.

In addition, the SOT efficiency in Pt/Py bilayers has been quantified using the $V_S/V_A$ ratio and $V_S$-only methods separately [32], and no clear difference between the SOT efficiencies from these two methods was found. This suggests that there is negligible $\tau_{FL}$ arising from the Pt/Py interface. However, it has also been reported that $\tau_{FL}$ might arise from the Rashba effect at the Pt/FM interface [20, 38, 72], which indicates that the characteristics of the interfaces, such as the interface quality and the degree of oxidation of the interfaces, could also affect the SOT efficiency. On the other hand, a Cu with negligible SOC or other insertion layers can be inserted between the HM and FM layer to eliminate or modify the interface SOT effect [46, 73, 74].

5.2. Spin Hall effect in Ta

In 2012, a large SOT efficiency and giant spin Hall induced magnetization switching were reported in a Ta (8 nm)/
Co$_{40}$Fe$_{40}$B$_{20}$ (CoFeB, 4 nm) bilayer at room temperature [21]. Figures 9(a) and (b) illustrate the ST-FMR measurements and results in a Ta/CoFeB bilayer. A very high SOT efficiency of $\sim 0.15$ was determined by the $V_S/V_A$ ratio method. The SOT efficiency shows a negative sign compared to the positive sign in Pt from various reports. The ST-FMR, SP, LSV and SMR represent spin torque ferromagnetic resonance, spin pumping, nonlocal measurement in lateral spin valves and the spin Hall magnetoresistance method, respectively. The blue thick line shows a correlation $\theta_a/\lambda_S \sim 0.13$ nm. (d) $\lambda_S$ in Pt as a function of its electrical conductivity ($\sigma_{Pt}$). The inset shows the measured SOT efficiency in various Pt films with different conductivities. The dashed lines serve as guide for the eye. Reprinted with permission from [32]. Copyright 2014, AIP Publishing LLC.

The interface effect between the Ta and CoFeB layer was not considered by Liu et al [21] in the Ta SOT efficiency evaluation. Thus, we compare SOT efficiency in Ta on Si/SiO$_2$ substrate/Co$_{40}$Fe$_{40}$B$_{20}$ (4 nm)/Ta (8 nm) and Si/SiO$_2$ substrate/Pt (4 nm)/Ta (8 nm) devices using ST-FMR measurements at different temperatures. A significant contribution of field-like torque, $\tau_{FL}$, is observed, which might arise from the interface between Ta and CoFeB layers. Figure 10(a) shows ST-FMR signals and fits (solid lines) in a CoFeB (4 nm)/Ta (8 nm) device for frequencies between 6 and 10 GHz at room temperature. The $H_{ext}$ is swept at an angle of $\theta_H = 35^\circ$. 

Figure 9. (a) Schematic of sample geometry for a Ta (8 nm)/CoFeB (4 nm) bilayer ST-FMR device. (b) ST-FMR signal at $f = 9$ GHz. From [21]. Reprinted with permission from AAAS. 

Figure 8. (a) Measured SOT efficiency $\theta_a$ for Pt/Py ($t$) devices with different Py thicknesses from ST-FMR measurements at 8 GHz and 300 K. (b) $\theta_a$ (blue squares) of Pt ($d$/Py (4 nm) devices as a function of Pt thickness at 300 K. (c) Summarized SOT efficiency $\theta_a$ as a function of $\lambda_S$ in Pt from various reports. The ST-FMR, SP, LSV and SMR represent spin torque ferromagnetic resonance, spin pumping, nonlocal measurement in lateral spin valves and the spin Hall magnetoresistance method, respectively. The blue thick curve shows a correlation $\theta_a/\lambda_S \sim 0.13$ nm. (d) $\lambda_S$ in Pt as a function of its electrical conductivity ($\sigma_{Pt}$). The inset shows the measured SOT efficiency in various Pt films with different conductivities. The dashed lines serve as guide for the eye. Reprinted with permission from [32]. Copyright 2014, AIP Publishing LLC.
with respect to the $I_{RF}$. The SOT efficiency in Ta is quantified by the $V_S/V_A$ ratio and $V_S$-only methods, respectively. As shown in figure 10(b), the in-plane SOT efficiency ($\theta_{sh}$) in the CoFeB/Ta device is ~0.109 using the $V_S/V_A$ ratio method at room temperature and it remains in the range of ~0.1–0.15 at most temperatures. The $\theta_{sh}$ value is similar to the previous report [21]. In addition, we perform the $V_S$-only method to determine $\theta_{sh}$ and the out-of-plane SOT efficiency ($\theta_\perp$) which characterizes the $\tau_{FL}$. Surprisingly, the $\theta_{sh}$ value from the $V_S$-only method is ~0.02 at room temperature as shown by the red circles in figure 10(b). This big inconsistency in $\theta_{sh}$ indicates a significant $\tau_{FL}$ (i.e. $\theta_\perp$) contribution in the CoFeB/Ta bilayer. As shown in figure 10(c), there is indeed a large $\theta_\perp$ of ~0.044 in a CoFeB/Ta bilayer at room temperature. This result is qualitatively similar to an earlier report of a larger $\tau_{FL}$ in CoFeB/Ta compared to $\tau_{DL}$ [75].

Similarly, we also characterize the Py (4 nm)/Ta (8 nm) bilayer device. As shown in figure 10(d), the $\theta_{sh}$ from the $V_S$-only method is ~0.01 at room temperature. The small inconsistency in $\theta_{sh}$ from the $V_S$-only method to $\theta_{sh}$ is not significant in the Py/Ta bilayer. Finally, $\theta_{sh}$ in Ta is ~0.01–0.02 obtained from the FM/Ta bilayer using ST-FMR at room temperature, which is consistent with other reports [61, 76–78]. The slightly different $\theta_{sh}$ from Py/Ta and CoFeB/Ta can be attributed to the different interface transparency [76, 79–81]. The $\theta_{sh}$ (from $V_S$-only) remains almost constant from 15 K to 300 K, while the $\theta_\perp$ shows a fast decrease at low temperature ranges, suggesting that $\theta_{sh}$ and $\theta_\perp$ might come from different origins. Moreover, we observed a similar $\theta_{sh}$ of ~0.01–0.015 in Py (20 nm)/Ta (6–25 nm) and CoFeB (15 nm)/Ta (15 nm) bilayers by spin pumping measurements at room temperature. These similar SOT efficiencies from both ST-FMR and spin pumping measurements suggest the Onsager reciprocity, which has been also demonstrated recently [82].

Figure 11 shows a summary of the SOT efficiencies in Ta films, ranging from 0.0037 to 0.26, measured using different techniques from different research groups [61, 75–77, 83–89]. The fit of $\theta_{sh}$ as a function of $\lambda_S$ indicates an approximately inverse relationship with $\theta_{sh}\lambda_S \sim 0.036$ nm (denoted by the blue thick line). However, it is observed that the correlation between $\theta_{sh}$ and $\lambda_S$ is not as clear as in the case of Pt as shown in figure 8(c), which might be due to multi-phases (such as $\alpha$-, $\beta$-phases or amorphous film) in different Ta thin films [21, 87].

### 5.3. Spin Hall effect in tungsten (W) and other alloys with heavy metal dopants

With respect to pure HMs, the largest SOT efficiency is ~0.3 observed in a $\beta$-phase W/CoFeB bilayer by ST-FMR measurements at room temperature [35]. This value is about two times larger than that in Ta, leading to efficient spintronic devices. Taking advantage of the highly efficient spin

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**Figure 10.** (a) ST-FMR measurements in a CoFeB (4 nm)/Ta (8 nm) device at different frequencies. The symbols are the measured data, and the solid lines are the fits by the sum of the Lorentzian functions. (b) The in-plane SOT efficiency and (c) the out-of-plane SOT efficiency in Ta as a function of temperature obtained from ST-FMR measurements on the CoFeB (4 nm)/Ta (8 nm) device. (d) The in-plane SOT efficiency as a function of temperature in the Py (4 nm)/Ta (8 nm) device.
oxide-semiconductor (CMOS) platform and Cu1 common metallization element in the complementary metal-SOT efficiency close to that of Pt. Because Cu is the most reasonable can be attributed to the multi-phases (such as α, β, as in the case of Pt as shown in figure 8(c). Similar to Ta, the shown in figures 13(a) and (b), the SOT efficiency of Cu1 Pt into the light metal Cu by ST-FMR measurements. As effects on SOT efficiency due to the systematic addition of [97] experimentally studied the μ– et al 97]. Ramaswamy [94] with heavy metal dopants have also been investigated – phases) in various W thin films [35, 90].

It was found that only 28% Pt in Cu1 increases as the Pt concentration increases from 0% to 40%. The θsh and λS were obtained at room temperature, except for that obtained by the LSV method which was measured at 10K as denoted in the figure.

current generation, SOT-driven magnetization switching has been demonstrated in a β-phase W-based three-terminal MTJ device [35]. The critical switching current density, Jc, is in the order of ~10⁶ A cm⁻² in W/CoFeB/MgO [90], which is almost one order of magnitude smaller than that in Pt.

Figure 12 shows a summary of the SOT efficiencies in W, ranging from 0.0043 to 0.95, measured using different techniques from different research groups [84, 85, 87, 89–93]. However, the correlation between θsh as in the case of Pt as shown in figure 8(c). Similar to Ta, the reason can be attributed to the multi-phases (such as α-, β-phases) in various W thin films [35, 90].

Besides the pure HMs, alloys consisting of a light metal with heavy metal dopants have also been investigated [94–97]. Ramaswamy et al [97] experimentally studied the effects on SOT efficiency due to the systematic addition of Pt into the light metal Cu by ST-FMR measurements. As shown in figures 13(a) and (b), the SOT efficiency of Cu₁₋ₓPtₓ increases as the Pt concentration increases from 0% to 40%. It was found that only 28% Pt in Cu₁₋ₓPtₓ can give rise to a SOT efficiency close to that of Pt. Because Cu is the most common metallization element in the complementary metal-oxide-semiconductor (CMOS) platform and Cu₁₋ₓPtₓ can have a large enough SOT efficiency, the Cu₁₋ₓPtₓ-based alloy makes it easier to integrate the spintronic devices into an existing Si fabrication technology. In addition, from further analysis of the ST-FMR data, it is found that the skew scattering contribution is significant for lower Pt concentrations, while the side-jump contribution is significant for higher Pt concentrations. This result is due to the different scaling of the contributions of skew scattering and side-jump with respect to Pt concentrations. As the Pt concentration increases, the longitudinal resistivity increases. In a simplistic picture, the skew scattering contribution is independent of this increased resistivity due to Pt, whereas the side-jump contribution is proportional to this increased resistivity [98, 99]. Thus, for higher Pt concentrations, the increase in the resistivity is larger, leading to the domination of the side-jump contribution. Furthermore, this result can be also correlated with an earlier theoretical report [100] and with experiments in the context of an anomalous Hall effect due to rare earth impurities in Gd [101].

5.4. Role of interface transparency of HM and FM in the SOT efficiency

It is possible for the spin loss to occur at the interfaces when the spins transmit into the FM layer from the spin generation sources [70, 79], and this can play an important role in the estimated SOT efficiency. Using the ST-FMR technique, Zhang et al [80] studied the transparency of Pt (6 nm)/Py (5.5 nm) and Pt (6 nm)/Co (5.2 nm), shown in figures 14(a) and (b), respectively. Using a standard ST-FMR analysis method, the authors found that the SOT efficiency in Pt/Co was ~0.11, much larger than ~0.05 in Pt/Py. They ascribed the discrepancy of the SOT efficiency to the interface transparency, which is correlated to the effective spin mixing conductance, Geff at the Pt and FM interface. They quantified the Geff to be 3.96 × 10¹⁹ m⁻² (for Pt/Co) and 1.52 × 10¹⁹ m⁻² (for Pt/Py), and the interface transparencies to the spin currents were ~0.65 (Pt/Co) and ~0.25 (Pt/Py), respectively. After taking into account the transparencies of these interfaces, they found that the intrinsic SOT efficiency in Pt has a much higher value of 0.17 ± 0.02 in Pt/Co and 0.20 ± 0.03 in the Pt/Py bilayer. Further, this result also suggests that the spins easily transmit through the Pt/Co interface and thus give a large measured SOT efficiency, which might be due to better matching of the electronic bands between the Pt and Co layer compared to the Pt and Py layers.

Pai et al [81] also conducted a systematic study of the interface transparency in Pt/Co and Pt/CoFe bilayers. They
found that the SOT efficiency can be modulated under different interface conditions and a much larger intrinsic SOT efficiency (~0.3) in Pt is obtained after considering the interface transparency between the Pt and FM layers. This value is much larger than the measured value of ~0.06 [25, 32], suggesting that there is a large spin memory loss at the interface. Therefore, it is understood that the measured SOT efficiency using the ST-FMR technique represents a
lower value because of the interface transparency. Therefore, modification or enhancement of the SOT efficiency via interface transparency engineering, such as inserting layers between the HM and FM [73, 74], manipulation of the film crystal structures [102] or using different combinations of HM and FM layers [80, 81], are interesting and important ongoing research activities.

5.5. Differences in the SOT efficiency between ST-FMR and the magnetization switching technique

From ST-FMR measurements, $\tau_{DL}$ and $\tau_{FL}$ can be separately extracted due to the $90^\circ$ phase difference in the magnetization dynamics (see section 2.3) [33, 40]. Subsequently, $\tau_{DL}$ can be used to evaluate the in-plane SOT efficiency $\theta_{IN}$. However, in the case of current-driven magnetization switching using SOT, it has been reported that both $\tau_{DL}$ and $\tau_{FL}$ contribute to the magnetization switching and the switching current density $J_C$ might be significantly reduced if there is an additional $\tau_{FL}$ [103–106], which can lower the energy barrier of magnetization switching. Therefore, the values of the SOT efficiency derived from SOT-induced magnetization switching measurements can be greater than those from ST-FMR measurements. In addition, during the ST-FMR measurements, the magnetization is uniformly aligned in the direction of an applied large external magnetic field. Hence, it is a coherent magnetization precession process. However, the current-induced magnetization switching proceeds through a thermally excited incoherent magnetization switching process [107, 108] (i.e. domain wall motion with a much lower energy barrier), which can also lead to a smaller $J_C$ and a larger $\theta_{IN}$ from magnetization switching measurements compared to ST-FMR measurements. Therefore, these aspects should be taken into account for SOT efficiency evaluation or comparison between ST-FMR and switching measurements.

6. SOT in topological insulators (TIs)

Topological insulators (TIs) are quantum materials that have a band gap just like a normal insulator but have topologically protected conducting edge states or surface states resulting from the inverted conduction and valence bands due to strong SOC [109–112]. These TI materials originated from a two-dimensional (2D) system known as 2D TI. The 2D TI was predicted [113] and experimentally observed in HgTe/CdTe quantum wells structures in 2007 [109]. As shown in figure 15(a), two conductive edge states are present in the edge of 2D TIs, each of which contributes one quantum of conductance $e^2/h$, where $h$ is the Planck’s constant. In the edge states, the spin polarization depends on the electron momentum.

A year later, in 2008, the first three-dimensional (3D) TI $\text{Bi}_2\text{Se}_3$ was experimentally discovered [114]. Since then, more 3D TIs such as $\text{Bi}_2\text{Se}_3$, $\text{Bi}_2\text{Te}_3$ and Sb$_2$Te$_3$, having a larger bandgap and single Dirac cone at the $\Gamma$ point, have been predicted and identified mainly by angle resolved photoemission spectroscopy (ARPES) experiments [110–112]. The 3D TIs possess spin-momentum-locked topological surface states (TSS) due to time reversal symmetry protection [115–117] and can be described by the Dirac Hamiltonian $H_D = v_F (\hat{\mathbf{z}} \times \hat{\mathbf{s}}) \cdot \mathbf{k}$, where $k$ is electron momentum, $\hat{\mathbf{z}}$ is the unit vector perpendicular to the TI films and $v_F$ is the Fermi velocity. This dispersion relationship reveals that on the TSS, the electron momentum and the spin polarization directions are strongly locked as shown in figure 15(b). As depicted in figure 15(c), in real space, as charge currents flow on the TSS, all the electron spins are expected to be fully polarized in the orthogonal direction to the electron moving direction due to the topological protection [118]. Therefore, a very efficient spin current generation and thus a giant SOT efficiency are expected in TIs due to TSS. For further information regarding TIs and related physics phenomena, the readers can refer to detailed reviews on TIs [116, 117]. In the following sections, we will review research works on 3D TIs in the context of SOT efficiency determination.

6.1. SOT efficiency in TIs

Research has been conducted to demonstrate the TSS and quantify the SOT efficiency in TI/FM heterostructures [33, 34, 40, 118–126]. In 2014, Mellnik et al [40] studied...
Bi$_2$Se$_3$ (8 nm)/Py (16 nm) bilayers using ST-FMR measurements at room temperature. Figures 16(a) and (b) show the schematic of the bilayer structure and the representative ST-FMR signal. Using the V$_S$-only analysis method, the in-plane SOT efficiency, $\theta_{sh}$, was determined to be $\sim$2.0–3.5 at room temperature, which is about one to two orders of magnitude larger than that in HMs reported previously. In addition, the out-of-plane SOT efficiency, $\theta_{\perp}$, arising from $\tau_{FL}$ has the similar order of amplitude as $\theta_{sh}$. The spin configuration is consistent with that expected in the TSS of TIs [115–117]. However, the exact mechanisms for this large SOT efficiency observed in Bi$_2$Se$_3$ was not clear at that time.

Subsequently, Wang et al [33] verified that the observed SOTs originate from the TSS in Bi$_2$Se$_3$ by temperature-dependent ST-FMR measurements on a Bi$_2$Se$_3$ (20 nm)/CoFeB (5 nm) heterostructure. By using the V$_S$-only analysis method, the in-plane SOT efficiency, $\theta_{sh}$, was determined to be $\sim$2.0–3.5 at room temperature, which is about one to two orders of magnitude larger than that in HMs reported previously. In addition, the out-of-plane SOT efficiency, $\theta_{\perp}$, arising from $\tau_{FL}$ has the similar order of amplitude as $\theta_{sh}$. The spin configuration is consistent with that expected in the TSS of TIs [115–117]. However, the exact mechanisms for this large SOT efficiency observed in Bi$_2$Se$_3$ was not clear at that time.

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Figure 16. (a) A schematic diagram of a Bi$_2$Se$_3$ (8 nm)/Py (16 nm) bilayer structure. The yellow and red arrows denote spin moment directions. (b) Measured ST-FMR signal at room temperature with $f = 8$ GHz and $\phi = 45^\circ$. A fixed microwave power of 5 dBm is absorbed by the device, corresponding to $h_{PE} = 7.7 \pm 1.1$ mA. The lines are the fits showing the symmetric and antisymmetric components. Reprinted from [40] (2014) with permission of Springer.

Figure 17. Temperature dependence of (a) in-plane SOT efficiency, $\theta_{sh}$, and (b) out-of-plane SOT efficiency, $\theta_{\perp}$, in Bi$_2$Se$_3$ (20 nm)/CoFeB (5 nm) for three devices: D1, D2 and D3. The SOT efficiencies are analyzed by two different methods: V$_S$-only and the V$_S$/V$_A$ ratio. Reprinted figure with permission from [33], Copyright 2015 by the American Physical Society.

Bi$_2$Se$_3$ (8 nm)/Py (16 nm) bilayers using ST-FMR measurements at room temperature. Figures 16(a) and (b) show the schematic of the bilayer structure and the representative ST-FMR signal. Using the V$_S$-only analysis method, the in-plane SOT efficiency, $\theta_{sh}$, was determined to be $\sim$2.0–3.5 at room temperature, which is about one to two orders of magnitude larger than that in HMs reported previously. In addition, the out-of-plane SOT efficiency, $\theta_{\perp}$, arising from $\tau_{FL}$ has the similar order of amplitude as $\theta_{sh}$. The spin configuration is consistent with that expected in the TSS of TIs [115–117]. However, the exact mechanisms for this large SOT efficiency observed in Bi$_2$Se$_3$ was not clear at that time.

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reduced as the Fermi level traversed through the Dirac point, which is possibly due to the inhomogeneity of $k_F$ and/or instability of the helical spin structure near the Dirac point.

6.2. Anisotropic SOT (efficiency) in Bi$_2$Se$_3$ MBE films

Bi$_2$Se$_3$ has a three-fold symmetry when viewed from the top (along the $z$-axis) as shown in figures 19(a) and (b). Therefore, the molecular beam epitaxy (MBE) grown Bi$_2$Se$_3$ films naturally have triangular islands as captured by an atomic force microscope in figure 19(c). It is found that the orientation of the triangular islands prefers a certain axis with twin structures due to the six-fold symmetric sapphire substrate. Although the anisotropy of electron transport has been observed in Bi$_2$Se$_3$ films previously [133], the anisotropy of SOT in Bi$_2$Se$_3$ has not been reported.

We have performed ST-FMR to study the anisotropic SOT characteristics in high quality MBE grown Bi$_2$Se$_3$ films shown in figure 19(c). Bi$_2$Se$_3$ Hall bars with different angle $\phi$, the angle between the channel current ($I$) direction with respect to the $x$-axis denoted in figure 19(c), were fabricated in the same sample. Similarly, using the same Bi$_2$Se$_3$...
considering the Bi\(_2\)Se\(_3\) twin structures. Thus, this oscillation \(\theta\) and returns to a low value but a high value at \(\theta = 0^\circ\). The oscillation period is also 60\(^\circ\). The angular dependent behaviors with the oscillation period of 60\(^\circ\). Furthermore, the in-plane SOT efficiency, \(\theta_{lp}\), in figure 19(f) also shows an oscillation behavior as the current \(I\) flows along different crystal axes (i.e. different angle \(\varphi\)). The oscillation period is also 60\(^\circ\), in which \(\theta_{lp}\) has a low value at \(\varphi = 0^\circ\) but a high value at \(\varphi = 30^\circ\), and returns to a low value at \(\varphi = 60^\circ\). This is consistent with the six-fold symmetry by considering the Bi\(_2\)Se\(_3\) twin structures. Thus, this oscillation of the SOT efficiency is attributed to the hexagonal warping in the Fermi surface of the Bi\(_2\)Se\(_3\) films\([129, 132, 134]\). An out-of-plane spin polarization in the TSS has been experimentally observed in Bi\(_2\)Se\(_3\)\([130, 131]\) due to the hexagonal warping effect, which shows a finite amplitude along the \(\Gamma-K\) direction but almost zero along the \(\Gamma-M\) direction in momentum space, leading to an oscillation period of 60\(^\circ\) in the out-of-plane spin polarization. If we assume that the total spin polarization is unity in TSS, the in-plane spin polarization should also show the same oscillation period of 60\(^\circ\), which is in line with our observation.

Note that since the width of the current channel of the Hall bars and ST-FMR devices is 20 \(\mu\)m, which is much larger than the triangle size of \(\sim 100-200\) \(\mu\)m in figure 19(c), our data only show the averaged transport results. In addition, there exist nonuniform microstructures in the Bi\(_2\)Se\(_3\) film, and the devices with different \(\varphi\) are located at different places in the same sample. Consequently, a fluctuation of the high (or low) values at each angle \(\varphi\) can be observed in figures 19(d)–(f). Note that the similar anisotropy behavior is expected in the range of \(\varphi = 180^\circ-360^\circ\) due to the crystal symmetry. Rather than rotating the crystal direction in different devices, the angular dependent ST-FMR measurements in one device can be combined to characterize the anisotropy of SOTs\([135]\), which will be discussed in section 7.2. Therefore, the ST-FMR is a suitable dynamic measurement method to study the anisotropy of SOTs.

6.3. TSS-dominated SOT

As mentioned earlier, the BS, 2DEG and TSS usually coexist in TIs such as Bi\(_2\)Se\(_3\). Therefore, it is important to understand the role of each channel on the SOT efficiency in TIs, so that one can maximize the SOT efficiency to realize highly efficient SOT-driven magnetization switching. Figure 20 shows a summary of the reported SOT efficiencies in various TIs with different thicknesses. The SOT efficiency increases significantly as TI films become thinner even though SOT efficiencies are evaluated using different techniques and in different TIs. Recently, the SOT efficiency of MBE grown Bi\(_2\)Se\(_3\) films with various thicknesses in the range of 5–20 QL have been quantified using ST-FMR measurements\([34]\). The optimum thickness range of Bi\(_2\)Se\(_3\) has been identified to be 5–8 QL to maximize the SOT effect in Bi\(_2\)Se\(_3\) devices. Figure 21(a) shows the schematic of the film stack used for the ST-FMR measurements. First, it was found that the resistivity, \(\rho_{BiSe}\), rapidly increases when the Bi\(_2\)Se\(_3\) thickness \(t_{BiSe}\) is less than 10 QL as shown in figure 21(b). On the other hand, the sheet carrier concentration \(n_{2D}\) shows an opposite trend, decreasing substantially below 10 QL as shown in figure 21(c). The carrier concentrations from BS \(n_{2D-Bulk}\), 2DEG \(n_{2D-2DEG}\) and TSS \(n_{TSS}\) were further separated and it was found that in the region of 5–8 QL, the value of \(n_{TSS}\) is larger compared to \(n_{2D-Bulk}\) and \(n_{2D-2DEG}\), indicating a TSS-dominated electrical transport in thin Bi\(_2\)Se\(_3\) films.

Figure 21(d) presents \(\theta_{lp}\) as a function of \(t_{BiSe}\) at room temperature by ST-FMR measurements on Bi\(_2\)Se\(_3\) \(t_{BiSe}/\) CoFeB \((7\text{ nm})\) bilayers. As \(t_{BiSe}\) is in the range of 5–8 QL, the Bi\(_2\)Se\(_3\) films exhibit a giant \(\theta_{lp}\) of \(\sim 1–1.75\) at room temperature, which further corroborates the TSS-dominated transport in thin Bi\(_2\)Se\(_3\) film regions. In addition, the interface SOT efficiency from TSS \(\lambda_{TSS}\) using an interface charge current density \(J_{C-TSS}\) \((\text{A cm}^{-1})\) in TSS were estimated for each \(t_{BiSe}\) as shown by the squares in figure 21(e). Furthermore, after subtracting the opposite 2DEG contribution, the intrinsic \(\lambda_{TSS}\) was calculated as 0.8 nm\(^{-1}\) for 7, 8 and 10 QL Bi\(_2\)Se\(_3\), as shown by the circles, which is similar to recently reported interface SOT efficiency values in Bi\(_{1–x}\)Sb\(_x\)\(_2\)Te\(_3\) \([120]\).

These results suggest that the BS and 2DEG dilute the TSS contribution and weaken the SOT efficiency in Bi\(_2\)Se\(_3\), and thus, by using moderately thin TI films (5–8 QL) one can obtain the TSS-dominated SOTs for device applications. In addition, TSS-dominated transport has also been reported recently\([123]\) by changing the Bi\(_2\)Se\(_3\) film thickness using a spin pumping technique, which is a reverse process (spin-charge conversion) of ST-FMR.

By taking advantage of this giant \(\theta_{lp}\), SOT-driven magnetization switching in the Bi\(_2\)Se\(_3\) (8 QL)/Py (6 nm) heterostructures was successfully achieved at room temperature without any external magnetic field\([34]\). This work makes
a substantial improvement to the working temperature of TI-based magnetization switching schemes from 1.9 K [119] to 300 K, demonstrating that a TI can be an excellent spin generator at room temperature. The critical switching current density $J_C$ in Bi$_2$Se$_3$ required for the magnetization switching is extremely low ($\sim 6 \times 10^5$ A cm$^{-2}$), which is almost two orders of magnitude smaller than that in HMs, such as Pt and Ta [20–22]. The much lower $J_C$ for magnetization switching using TIs is promising in order to address the outstanding scalability and power consumption issue in modern magnetic devices. Furthermore, the magnetization switching scheme demonstrated in [34] does not require an assistive magnetic field, which makes the TI/FM material systems easy to integrate into the well-established industrial technology for magnetic devices. In addition, the room temperature SOT-driven magnetization switching has also been observed recently in Bi$_2$Se$_3$/ferromagnetic CoTb heterostructures [136] and in the sputtered Bi$_x$Se$_{(1-x)}$/Ta/CoFeB/Gd/CoFeB heterostructures [137].

7. SOT beyond HMs and TIs

Beyond HMs and TIs, other novel material systems with strong SOC are emerging, which also have potential for future spintronic applications. The following sections discuss some representative examples, in which the ST-FMR technique was used to evaluate the SOTs (or SOT efficiency), to gain insight into the underlying physics of SOTs in these emerging systems.

7.1. Interface between two nonmagnetic materials

The Rashba SOC, which was first discussed in the 2DEG systems with spin degeneration lifting [138], can emerge at the interfaces in a variety of material systems these days [139]. The Rashba SOC at an interface is explained as follows. For an interface with inversion symmetry breaking in the direction perpendicular to the interface, an electric field $E$ is produced perpendicular to the interface. An electron moving within this interface with the electric field will experience an effective magnetic field $H$ due to the relativistic corrections, which causes a momentum-dependent spin splitting in the bands, analogous to the Zeeman splitting. In the scheme of Rashba SOC, the interaction between the spin polarization $\hat{\sigma}$ and electron momentum $k$ can be expressed by the Hamiltonian $H_R = \alpha_R (k \times \hat{z}) \cdot \hat{\sigma}$, where $\alpha_R$ is the Rashba coefficient and $\hat{z}$ is the unit vector perpendicular to the interface. Figures 22(a) and (b) schematically show the dispersion curves with spin splitting due to the Rashba SOC at the interface and the corresponding Fermi contours with spin polarization configuration. Figures 22(c) and (d) show the $\theta_{sh}$ as a function of Bi$_2$Se$_3$ thickness at room temperature. Each $\theta_{sh}$ represents the averaged value from three devices. Regions I, II and III, denoted by different colors, represent the charge-to-spin conversion dominated by different mechanisms. The inset shows the schematic of the band structure for each region. Regions I, II and III, denoted by different colors, represent the charge-to-spin conversion dominated by different mechanisms. The inset shows the schematic of the band structure for each region.
The Rashba SOC at the interface between the HM and FM layer, such as Pt/Co, has been studied previously [20, 72]. Recently, researchers have found that when one NM, such as Bi, Pb or Sb, contacts with other NM, such as Ag, their interface can also exhibit strong interface SOC [140–147]. In 2015, the charge-to-spin conversion induced by the Rashba–Edelstein effect was directly observed at the interface of Bi/Ag [146]. Using a spin-polarized positron beam, they found an opposite surface spin polarization between Bi/Ag/Al2O3 and Ag/Bi/Al2O3 samples. Subsequently, Jungfleisch et al [147] reported the Rashba–Edelstein-induced-spin driven ST-FMR in Bi/Ag/Py heterostructures. They evaluated the SOT efficiency of the Bi/Ag interface to be ~0.18 using the $V_S/V_A$ ratio method.

Another recent report shows a very large SOT efficiency at the interface between 10 QL Bi2Se3 TI materials and non-magnetic Ag thin layer material at room temperature using ST-FMR measurements [148]. As shown in figure 22(c), from first-principle calculations, it was found that there was a pair of large Rashba splitting bands emerging at the interface between Bi2Se3 and Ag. Moreover, the Rashba bands were located outside the TSS linear bands of the Bi2Se3 layer, and the Rashba bands had the same net spin polarization direction as the TSS of Bi2Se3. As shown in figure 22(d), due to the large interface Rashba SOC at the newly formed Bi2Se3/Ag interface, the measured SOT efficiency shows a significant enhancement as the Ag insertion layer thickness increases to ~2 nm and reaches a value of 0.5 for 5 nm Ag. The SOT efficiency in Bi2Se3/Ag (5 nm)/CoFeB is ~3 times higher than that from sole TSS in Bi2Se3/CoFeB at room temperature. The extracted $\alpha_R$ at the Bi2Se3/Ag interface has a significant large value about 2.83–3.83 eV Å and this value is similar to that of Bi/Ag reported previously [141]. Furthermore, Rashba-induced magnetization switching in Bi2Se3/Ag/Py with a low current density of $5.8 \times 10^4$ A cm$^{-2}$ has been demonstrated. As indicated in section 6.3, the SOT efficiency in Bi2Se3/Ag can be further enhanced by decreasing the Bi2Se3 thickness to eliminate the bulk contamination and the energy consumption will be further decreased for SOT-driven magnetization switching. These reports indicate that the nonmagnetic bilayer interface can be designed to provide a large Rashba spin splitting and thus giant SOT efficiency, which has the potential to be used as efficient spin current sources for future spintronic devices.

7.2. Transition-metal dichalcogenides (TMDs)

In recent years, 2D transition-metal dichalcogenide (TMDs) materials have attracted interest due to their novel physics and potential applications, which were reviewed in [149]. Because the TMDs display 2D characteristics, the thickness of 2D-TMDs can be reduced to as thin as a monolayer. Therefore, TMDs provide an interesting platform to observe exotic interface-related phenomena. Since 2016, 2D-TMDs have been studied as spin current sources for SOT devices [135, 150–153]. Spin-current-induced FMR has been observed in monolayer MoS2/Py structures [151], where the Py layer was deposited by either magnetron sputtering or e-beam evaporation. It was observed that both $\tau_{DL}$ and $\tau_L$ in the monolayer MoS2/Py system were present, with the ratio of $|\tau_{DL}/\tau_L|=0.19$. The observed SOTs were attributed to the interface between the MoS2 and Py layers. The ratio of $|\tau_{DL}/\tau_L|$ in MoS2/Py is much smaller than that recently reported in the Bi2Se3/Py (or CoFeB) systems [33, 40], whose value is in the range of 1.4–2. This implies that the damping-like torque $\tau_{DL}$ is much larger than $\tau_L$ in MoS2/Py systems. However, the real SOT efficiency of MoS2/Py bilayers was not evaluated in [151] and the real SOT efficiency should be estimated using the $V_S$-only method (or $\tau_{DL}$) described in section 4.2 for proper comparison with other material systems.

Semimetal WTe2 is also a layered TMD, which has strong SOC [154, 155] and has been reported to possess exotic topological properties [156]. Compared to other TMDs, such as MoS2, an interesting property of the single crystal WTe2 is the crystal symmetry. As shown in figure 23(a), the mirror symmetry exists only relative to the bc plane in WTe2, and there is no mirror symmetry in the ac plane. Therefore, there is no 180° rotation about the c-axis (perpendicular to the sample plane). Consequently, due to the lack of two-fold rotational symmetry about the c-axis, there is a possible existence of the out-of-plane damping-like SOT ($\tau_B$) in WTe2. Recently, MacNeill et al [135] have verified the presence of $\tau_B$ in WTe2/Py bilayer systems by ST-FMR measurements at room temperature, as shown in figures 23(b) and (c). It was found that when the charge current flows along the b-axis with mirror symmetry in a WTe2 (5.5 nm)/Py (6 nm) device, both the symmetric ($V_S$) and
antisymmetric ($V_A$) ST-FMR components at different in-plane magnetic field angle $\varphi$ follow the behavior of $\cos \varphi \times \sin(2\varphi)$, which is similar to that of the traditional HM/FM bilayers. However, as the charge current flows along the $a$-axis where mirror symmetry is broken, $V_A$ at different $\varphi$ exhibits a novel behavior of $\cos \varphi \times \sin(2\varphi) + \sin(2\varphi)$, indicating the presence of $\tau_B$ in WTe$_2$/Py, as shown in figure 23(d). Figure 23(e) shows the SOTs in WTe$_2$ with different thicknesses ranging from monolayer to ~16 nm [153]. The magnitude of $\tau_B$ shows a very weak WTe$_2$ thickness dependence, suggesting that it is of the interface origin.

Most of the studied interface systems so far only have broken inversion symmetry along their vertical structure, which leads to an in-plane damping-like torque or out-of-plane field-like torque due to the Rashba–Edelstein effect at the interface. On the contrary, the results in [135] demonstrate the in-plane rotational symmetry breaking induced out-of-plane damping-like torque in WTe$_2$, which is a novel interface-related SOT phenomenon and provides a platform for field-free SOT-driven magnetization switching of FM with perpendicular magnetic anisotropy (PMA).

### 7.3. Antiferromagnetic (AFM) materials

Antiferromagnetic (AFM) materials have zero net magnetization due to the antiparallel alignment of magnetic moments on the adjacent individual atoms. Therefore, the antiferromagnets are difficult to control by external magnetic fields. However, an injected charge-current-induced internal field can perturb the spin structures and thus magnetic ordering in AFM materials, as recently reported for different single AFM materials [157–159]. On the other hand, devices with AFM/FM heterostructures have also been utilized to realize the external-magnetic-field-free SOT-driven magnetization switching due to the presence of exchange bias [160–163], which breaks the magnetization up-down equivalence in the systems with PMA. Furthermore, a single AFM material itself can also work as a spin current source [48, 160, 164, 165], just like a HM layer. It has been found that the SOT efficiency of single AFMs, such as PtMn and IrMn, is of the same order as Pt [160, 166, 167].

Interestingly, it was also found that there is an anisotropy of the ST-FMR results from epitaxial PtMn [48] and IrMn [165] AFM films. For PtMn epitaxial films [48], the modulation of ST-FMR linewidth (MOD) is more significant...
along the $a$-axis than the $c$-axis in PtMn (10 nm)/Cu (1 nm)/Py (5 nm) samples as shown in figures 24(a) and (b), indicating a large SOT efficiency in $a$-axis PtMn films. A further $V_S$-only ST-FMR analysis method yields a SOT efficiency of ~0.048 for $c$-axis PtMn and ~0.089 for $a$-axis PtMn. In the case of epitaxial IrMn$_3$/Py bilayer devices [165], a similar anisotropic behavior was found from ST-FMR measurements of the SOT efficiency with respect to IrMn$_3$ crystal orientations. As shown in figure 24(c), the SOT efficiency is much larger (~0.2) in (0 0 1)-oriented single-crystalline IrMn$_3$ than that in either (1 1 1)-oriented or polycrystalline-oriented films. Moreover, after perpendicular field annealing, an enhanced SOT efficiency up to ~0.35 in (0 0 1)-oriented IrMn$_3$ thin films was demonstrated. Both works discussed that the observed anisotropic SOT efficiency in PtMn or IrMn$_3$ epitaxial films originates from the intrinsic SHE, which was further corroborated with the first-principle calculations.

7.4. 2DEG in oxide materials

Recent advances in film growth and synthesis techniques have enabled the growth of high quality oxide heterostructures with very smooth interfaces. This allows for engineering of novel electronic properties at the oxide interfaces [168]. One of the central oxide material systems is the interface between two wide-band-gap insulators, SrTiO$_3$ (STO) and LaAlO$_3$ (LAO) as a 2DEG is formed at the interface and possesses exotic properties including superconductivity [169, 170] and magnetism [171–173]. Further, due to the broken interfacial inversion symmetry, the 2DEG confined in the vicinity of a polar (LAO)/non-polar (STO) interface experiences a strong electric field directed perpendicular to the conduction plane [174]. Consequently, the LAO/STO interface possesses a strong Rashba SOC that leads to a strong coupling between the orbital and spin degrees of freedom. Due to the dielectric properties of STO, the strength of the Rashba SOC is tunable by applying an external gate voltage [175].
The presence of strong Rashba SOC in the $d$-bands of the 2DEG at the STO/LAO interface has been reported in various earlier studies [175–177]. Further, the Rashba SOC allows for generation of spin accumulation in the LAO/STO interface when a charge current flows in the 2DEG [138, 178]. Subsequently, the existence of current-induced SOTs at the STO/LAO interface has been verified experimentally by angular dependent magnetoresistance measurements [179]. Recently, Wang et al [180] have experimentally shown a giant room temperature charge-to-spin conversion efficiency (i.e. $\theta_{sh}$) in the STO/LAO/CoFeB structure using the ST-FMR technique shown in figure 25(a). The $\theta_{sh}$ value was estimated to be $\sim 6.3$ at room temperature, denoted by squares in figure 25(b), which is almost two orders of magnitude larger than that in HMs, such as Pt and Ta [21, 25]. However, the $\theta_{sh}$ decreases rapidly as the temperature decreases to $\sim 50$ K. Finally, it becomes negligible at $\sim 50$ K. From the temperature-dependent $\theta_{sh}$, it was suggested that inelastic tunneling via localized states, such as oxygen vacancies in the LAO band gap, accounts for the spin transmission through the LAO layer, as schematically shown in the inset of figure 25(b).

In addition to the ST-FMR measurements, a spin-to-charge conversion efficiency at the STO/LAO interface has also been reported by the spin pumping technique [181–183]. Additionally, a long spin diffusion length over 300 nm in the STO/LAO 2DEG channel has been reported [184, 185]. Therefore, oxide materials, such as STO/LAO heterostructures, can enable potential applications in oxide based spintronic devices.

### Figure 26.
A summary of the SOT efficiencies (left y-axis) and normalized power consumption of a variety of materials (right y-axis). The x-axis denotes the materials with their corresponding measurement temperature and reference. RT denotes room temperature. The Bi$_{1-x}$Se$_x$ represents the film grown by the magnetron sputtering, the STO/LAO represents the SrTiO$_3$/LaAlO$_3$ bilayer, and Cu (O) and W(O) represent the metal films with oxygen incorporation. The star represents an extremely small, normalized power consumption value ($\approx 5.5 \times 10^{-8}$) for (Bi,Sb)$_2$Te$_3$ at a temperature of 1.9 K. The effective conductivity $\sigma$ ($\sim 0.17 \, \mu\Omega^{-1} \cdot \text{cm}^{-1}$) of the Bi 4 (nm)/Ag (8 nm) bilayer, used for the power consumption estimation, is calculated using a parallel circuit model with the given conductivity values of $\sim 2.1 \times 10^{-4} \, \mu\Omega^{-1} \cdot \text{cm}^{-1}$ (for Bi) and $\sim 0.25 \, \mu\Omega^{-1} \cdot \text{cm}^{-1}$ (for Ag). The $\sigma$ ($\sim 0.0125 \, \mu\Omega^{-1} \cdot \text{cm}^{-1}$) of the Cu (O) layer, used for the power consumption estimation, is extracted from figure 1(b) in [187] using the given value of the pure Cu conductivity ($\sim 0.025 \, \mu\Omega^{-1} \cdot \text{cm}^{-1}$). Reproduced from [187]. CC BY 4.0.

### Figure 27.
A summary of the normalized power consumption as a function of SOT efficiencies for a variety of materials. ‘Metals and alloys’ represent Pt, Ta, W, and CuPt alloy and nonmagnetic oxidized materials, ‘topological insulators’ represent Bi$_2$Se$_3$ and (Bi,Sb)$_2$Te$_3$, ‘Rashba interface’ includes the STO/LAO interface and Bi/Ag interface, ‘Weyl semimetal’ represents WTe$_2$, and ‘antiferromagnet’ includes PtMn and IrMn$_3$.

### 7.5. Nonmagnetic oxidized materials
In conventional HMs, the reported largest SOT efficiency is $\sim 0.3$ observed in $\beta$-phase W [35]. However, the $\alpha$-phase W exhibits a much smaller SOT efficiency. Therefore, the SOTs can be controlled via HM microstructure engineering. Recently, it has been reported that the SOT efficiency can be
In this review, we first described the ST-FMR technique, including two ST-FMR setups and three ST-FMR analysis methods. As ST-FMR is a high frequency measurement in the GHz frequency range, several aspects should be carefully considered during ST-FMR measurements and data analysis, such as the symmetric CPW design, impedance matching, RF power absorption and device current correction as well as possible contamination considerations (field-like torque from the interface effect and spin pumping). Next, the latest SOT research progress using ST-FMR is presented in diverse materials including normal HMs and alloys, TIs, 2D materials, interfaces with strong Rashba SOC, AFM materials, 2DEG in oxide materials and nonmagnetic oxidized materials. We find that the ST-FMR measurement is a powerful technique to determine SOT efficiency including the recently observed out-of-plane damping-like SOT efficiency in a WTe2 Weyl semimetal. We hope that this review can further promote the ST-FMR technique in fundamental SOT research as well as exploiting novel materials for future SOT research and applications.

In figure 26, we have summarized the representative SOT efficiencies in various materials as well as the power consumption for switching a unit magnetization of an FM layer which is proportional to \(1/(\sigma \times (\theta_{ab}))\) [136]. Here, we assume that the switching time is the same in all the materials for simplicity. It shows that the TI materials generally show a larger SOT efficiency as well as low power consumption due to the efficient charge-spin conversion in the spin-momentum-locked surface states compared to HMs. Some other materials, apart from HMs and TIs, also exhibit a high performance in terms of both SOT efficiency and power consumption and thus are worth further studies.

Figure 27 shows the correlation of SOT efficiency and power consumption for different materials, obtained by reorganising and grouping the data in figure 26 based on their corresponding material system. Overall, we find that the TI materials (wrapped by the red dashed curve) show a better performance in terms of the SOT efficiency and the power consumption even though the values are scattered due to the influence of conducting bulk. The HMs and their alloys (wrapped by the dashed blue curve) possess a similar order of SOT efficiency and comparable power consumptions to the AFM systems but are inferior to the TI systems. Finally, the Weyl semimetal WTe2 seems to show a higher power consumption based on the single report in [135]. Since, Weyl semimetals are very interesting topological materials, more research is expected in the near future and one will thus obtain a more conclusive view about the performance of Weyl semimetals with regards to power consumption and SOT-driven switching efficiency.

Moving towards the real SOT applications, many recent reports have studied SOT devices in the nanosecond (ns) or even picosecond (ps) regime [104, 108, 189–192]. A very highly efficient SOT-driven magnetization switching has been realized recently [191], where the switching energy is only about 60 fJ for each bit writing with the current ~120 μA and time of 1.2 ns. The energy consumption for each bit writing is much smaller than the values known for STT-driven magnetization switching, which is between ~150 fJ to ~4 pJ [193–195]. Therefore, SOT devices are promising and potential candidates for future high efficient spintronic devices. Thus, the exploration of novel materials with even higher SOT efficiencies will be of great importance for future spin-based device applications and, for this purpose, the ST-FMR can be an easy and powerful technique.

So far, we have seen that the origins of the SOT can be from the bulk, surface and/or interface in a variety of materials. Based upon the origins of the SOT, the material systems will show different thickness-dependent behaviors. Figure 28 shows the schematic of the SOT efficiency as a function of thickness in representative TI material Bi2Se3, HM material Pt and Weyl semimetal WTe2. For HMs (blue curve), the dominant mechanism for the SOT is SHE, which is a bulk property. Therefore, the SOT efficiency in HMs usually increases and then saturates as the HM’s thickness increases. The thickness corresponding...
to the saturation point of SOT efficiency is correlated to the spin diffusion length in HMs. For the TIs (red curve), the TSS serve as the main mechanism for the SOT and should have an almost constant SOT efficiency against the TI thickness as reported in [34] (also see figure 21(e)). However, as mentioned in section 6.3, contamination from the conducting bulk might be unavoidable. Consequently, the SOT efficiency shows much larger values as the TI thickness becomes thinner in which the TSS becomes dominant. As reported in WTe2 [153], the out-of-plane damping-like SOT efficiency, due to a broken lateral mirror symmetry, remains almost constant with respect to the WTe2 thickness, which is claimed to arise from interface mirror symmetry, remains almost constant with respect to of-plane damping-like SOT efficiency, due to a broken lateral mirror symmetry, remains almost constant with respect to the WTe2 thickness, which is claimed to arise from interface mirror symmetry.

The weak thickness dependence is illustrated by the green curve in figure 28. Therefore, the thickness-dependent SOT efficiency (or SOT) measurements by ST-FMR or even other reliable techniques are very useful for gaining insight into the underlying physics of emerging materials.

The central physical effect used in the ST-FMR measurement can be further explored besides the GMR, tunneling magnetoresistance (TMR) or AMR rectification [196] in the present ST-FMR devices. The other magnetoresistance effects, such as the SMR, or Hall effects such as the anomalous Hall effect (AHE) and planar Hall effect (PHE), with both in-plane and out-of-plane measurement schemes, might also be developed in the near future to extend the ST-FMR technique to different material systems, especially for the materials with PMA. Meanwhile, the ST-FMR analysis theory should also be modified accordingly. Subsequently, the SOT efficiency from ST-FMR measurements can provide a much better comparison with those from other measurement techniques. Good examples of this are recent reports where the ST-FMR rectification signals were obtained based on the SMR effect in the Pt/yttrium iron garnet (YIG) magnetic insulator bilayer [197–199].

In addition, ST-FMR measurements on nontrivial topological materials, such as Weyl semimetals (WTe2 [154] or MoTe2 [200]), will be an interesting topic in the future. It would be a significant discovery if one could observe the Fermi-arc surface states in these materials, and perform ST-FMR measurements on these materials. The SOT efficiency (or SOT) measurements can provide a much better comparison with those from other measurement techniques. Good examples of this are recent reports where the ST-FMR rectification signals were obtained based on the SMR effect in the Pt/yttrium iron garnet (YIG) magnetic insulator bilayer [197–199].

Besides the applications in the SOT studies, it has been demonstrated recently that TMR-based ST-FMR devices have a very high efficiency in converting an RF signal to a DC signal, even better than the conventional Schottky diode [15, 204–206]. This high rectification efficiency of TMR-based ST-FMR devices can be utilized for wireless energy harvesting that is becoming more relevant to technologies such as the Internet of Things.

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