Optimal selection and application analysis of multi-temporal differential interferogram series in StaMPS-based SBAS InSAR

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ABSTRACT

The optimal selection of multi-temporal differential interferogram series is an important step to monitor ground subsidence using the Stanford method for persistent scatterers (StaMPS)-based small baseline subset (SBAS) interferometric synthetic aperture radar (InSAR). A deformation model and its two solution methods, least squares and singular value decomposition, present the composing mode and optimal selection of multi-temporal differential interferogram series and show that their quality and quantity affect the accuracy of monitored deformation information of SBAS InSAR. Using ENVISAT ASAR images covering urban areas of Beijing, China, a different number of optimal multi-temporal differential interferogram series are formed to monitor urban ground subsidence by the StaMPS-based SBAS InSAR method. The comparison and verification of test results indicate that the quality and quantity of multi-temporal differential interferogram series substantially impact the singularity and degree of ill condition of the deformation model, locations of the selected slowly decorrelating filtered phase (SDFP) pixels, and monitored annual mean subsidence velocities. The number of multi-temporal differential interferogram series under the optimal quality is 1–2 for a month in urban ground subsidence monitoring using StaMPS-based SBAS InSAR, a higher quantity of differential interferograms of the optimal quality is not always better.

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INTRODUCTION

The Stanford method for persistent scatterers (StaMPS) is a software package that implements persistent scatterers interferometric synthetic aperture radar (PS InSAR), small baseline subset (SBAS) InSAR, and combined multi-temporal (CMT) InSAR to monitor the different types of surface deformations in mining areas, cities and seismic fault zones over a large-scale and for a long time (Hooper, Segall, & Zebker, 2007). StaMPS-based SBAS InSAR creates a series of differential interferograms from multi-temporal radar images (multi-temporal differential interferogram series) to derive deformation magnitudes and velocities, and uses single look differential interferograms to select slowly decorrelating filtered phase (SDFP) pixels as highly coherent pixels to estimate deformation information at the highest possible resolution (Hooper, Zebker, Segall, & Kampes, 2004; Tiwari, Dwivedi, Dikshit, & Singh, 2016; Xue, Meng, Peng, Kai, & Guan, 2015). Moreover, it uses newer 3D phase unwrapping algorithms to unwrap the wrapped phase of the series of differential interferograms, rather than unwrapping individual differential interferograms (Cuenca, Hooper, & Hanssen, 2011).

Thus far, overseas and domestic research on SBAS InSAR focused on improving the identification and selection of highly coherent pixels (Costantini, Falco, Malvarosa, & Minati, 2009; Ferretti, Novali, Zan, Prati, & Rocca, 2008), increasing the accuracy and efficiency of phase unwrapping (Heshmat, Tomioka, & Nishiyama, 2011; Huang & Wang, 2014), effectively separating error phase components (Li, Fielding, & Cross, 2006), constructing and robustly estimating deformation models (Jiang, Liu, & Tao, 2016; Zhai, Liu, Tao, & Xin, 2017) and extending new applications (Costantini, Mouratidis, Schiavon, & Francesco, 2016; Esmaeili, Motagh, & Hooper, 2017; Liu et al., 2014). However, in practical applications, it was found that the quality and quantity of multi-temporal differential interferogram series seriously affected the monitored surface deformation results of StaMPS-based SBAS InSAR. In other words, on setting different threshold conditions of spatio-temporal baselines, the quality and quantity of multi-temporal differential interferogram series generated from the same SAR dataset would be different, which would result in considerably different monitored deformation magnitudes and velocities in the same study area and time period. To address this issue, Hooper (2006) considered the effect of spatio-temporal baselines and proposed the concept of integrated correlation coefficient to obtain the multi-temporal differential...
interferogram series with the maximum total correlation and best quality. Akbari and Motagh (2012) used the correlation matrix of multi-temporal differential interferogram series as the weight matrix to solve the deformation model of SBAS InSAR. Lauknes, Zebker, and Larsen (2010) controlled the threshold condition of spatio-temporal baselines and formed the multi-temporal differential interferogram series with the maximum total correlation and best quality in their applications of SBAS InSAR. All of the above studies considered the effect of spatio-temporal baselines and selected the multi-temporal differential interferogram series with the best quality among numerous interferometric pairs. However, they did not consider whether the quantity of multi-temporal differential interferogram series was appropriate for monitoring surface deformation using SBAS InSAR, which resulted in much trouble to the users, especially beginners of SBAS InSAR. Some users could even select more than 100 differential interferometric pairs to monitor a relatively slow surface deformation in just over a year. In fact, the setting of threshold condition of the integrated correlation coefficient and spatio-temporal baselines appeared to be random and chaotic, and the effect of different qualities and quantities of multi-temporal differential interferogram series on the monitored surface deformation results of the SBAS InSAR had not been compared or analyzed in the existing literature.

In the present study, we systematically investigate the deformation model of SBAS InSAR and its solution methods, and present the composing mode and optimal selection of multi-temporal differential interferogram series. On the basis of these theories, 29 Environmental Satellite (ENVISAT) Advanced Synthetic Aperture Radar (ASAR) single look complex (SLC) images over urban areas of Beijing, acquired between January 2007 and October 2010, are used to generate multiple optimal multi-temporal differential interferogram series under different threshold conditions of the integrated correlation coefficient and spatio-temporal baselines. The degree of ill condition and singularity of their deformation models are analyzed based on the eigenvalue analytical theorem and condition number. Five ground subsidence monitoring tests are conducted with 41, 48, 62, 85 and 108 multi-temporal differential interferogram series to obtain the distributions and 1D line of sight (LOS) velocities of ground subsidence in the same study area and time period. The comparison and analysis of research results indicate that, with the increase of quantity of multi-temporal differential interferogram series of the optimal quality, the deformation model changes from singularity to non-singularity, and the degree of ill condition may be decreased but cannot be avoided in practical applications. The quantity and density of selected SDFP pixels are not very different from each other, but their overlapping locations are low with a slight difference. The spatial distribution patterns of ground subsidence monitored by five types of multi-temporal differential interferogram series are similar, while quantitative comparisons show that 1D LOS velocities are relatively different with a maximum difference of \(-44\) mm/yr between 41 and 108 multi-temporal differential interferogram series. The standard deviation, leveling and global positioning system (GPS) measurements prove that the ground subsidence velocities obtained from the 48 and 62 multi-temporal differential interferogram series are more reasonable and reliable.

**Study area and data**

Beijing is located in the northern part of the North China Plain. Geographically, Beijing extends from longitudes \(115^\circ25'/117^\circ30'E\) and latitudes \(39^\circ26'/41^\circ03'N\) with a total area of approximately 16,808 km\(^2\). Beijing has a long history of ground subsidence, which has led to damages to urban infrastructure including buildings, railways, highways, subways and underground facilities. By the end of 2009, the maximum annual subsidence and cumulative subsidence of Beijing reached 138 and 1163 mm, respectively. Therefore, the accurate mapping and time-series analysis of ground subsidence of Beijing are critical for early hazard warnings and sustainable urbanization. Many institutions (Chen, B., Gong, H., Li, X., Lei, K., Zhang, Y., Li, J., … Dang, Y. (2011); Hu, Wang, Sun, Hou, & Liang, 2014; Li, Zhang, Li, & Luo, 2013; Liu, Jia, & Chu, 2012) have investigated Beijing’s ground subsidence by different measurement methods and techniques, including GPS, InSAR and leveling.

In this study, a total of 29 C-band ENVISAT ASAR images are used to investigate the optimal selection method of multi-temporal differential interferogram series and their practical application to Beijing’s ground subsidence monitoring. These images were captured with descending orbits, VV polarization modes, a track number of 490, an incidence angle of approximately 23° in the imaging center, and a pixel spacing of 7 m in the slant range and 4 m in the azimuth range. External digital elevation model (DEM) data from Shuttle Radar Topography Mission-3 SRTM DEM at a spatial resolution of 90 m provided by the American National Aeronautics and Space Administration (NASA) and National Imagery and Mapping Agency (NASA) are firstly resampled to the SAR image coordinate system to give them a consistent coordinate system and resolution (Tao, Gao, Liu, & Wang, 2017), and then used to simulate and remove the topographic phase
contribution in data processing. Orbital corrections are performed with the help of precise orbits obtained from the European Space Agency (ESA). Figure 1 shows the study areas, and Table 1 presents some basic information on 29 images.

**Methodology**

Deformation model of SBAS InSAR and its solution method

Let us consider a set of $N + 1$ SAR images of the same study area, acquired at the ordered times $t_0, t_1, \ldots, t_N$. One image is selected as the common master image, and the other images are co-registered with it in order to have a common reference grid. The image pairs meeting the threshold condition of integrated correlation coefficient and spatio-temporal baselines are selected as interferometric pairs, and complex-conjugate multiplication is performed to generate Minterferograms. The topographic phase is then removed from each of the Minterferograms by using an external DEM, following which the Mmulti-temporal differential interferogram series of SBAS InSAR are formed. Note that the external DEM can be not only 90 m SRTM DEM, but also 30 m ASTER GDEM, 30 m SRTM DEM and other DEMs. However, the accuracy of 30 m ASTER GDEM is not as high as 90 m SRTM DEM (Tao et al., 2017), and we cannot obtain 30 m SRTM DEM. So we use 90 SRTM DEM in our study.

Once the Mmulti-temporal differential interferogram series are generated, the next step is the selection of initial candidate SDFP pixels using the amplitude difference dispersion index as the criterion. The phase values of these candidates in the differential interferograms contain the deformation phase, atmospheric delay phase, orbit error phase,

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**Figure 1.** Study area of this research marked in the red rectangle and overlaid on the administrative map of Beijing. The blue rectangle is the coverage of the common master image acquired on January 26, 2009.

**Table 1.** Orbit and date of 29 ENVISAT ASAR images.

| Number | Orbit   | Date       | Number | Orbit   | Date       | Number | Orbit   | Date       |
|--------|---------|------------|--------|---------|------------|--------|---------|------------|
| 1      | 25,594  | 22/01/2007 | 11     | 33,610  | 04/08/2008 | 21     | 41,125  | 11/01/2010 |
| 2      | 26,095  | 26/02/2007 | 12     | 34,111  | 08/09/2008 | 22     | 41,626  | 15/02/2010 |
| 3      | 27,097  | 07/05/2007 | 13     | 34,612  | 13/10/2008 | 23     | 42,127  | 22/03/2010 |
| 4      | 27,598  | 11/06/2007 | 14     | 35,614  | 22/12/2008 | 24     | 42,628  | 26/04/2010 |
| 5      | 28,099  | 16/07/2007 | 15     | 36,115  | 26/01/2009 | 25     | 43,129  | 31/06/2010 |
| 6      | 30,103  | 03/12/2007 | 16     | 37,117  | 06/04/2009 | 26     | 43,630  | 05/07/2010 |
| 7      | 30,604  | 07/01/2008 | 17     | 37,618  | 11/06/2009 | 27     | 44,131  | 09/08/2010 |
| 8      | 31,105  | 11/02/2008 | 18     | 39,121  | 24/08/2009 | 28     | 44,632  | 13/09/2010 |
| 9      | 31,606  | 17/03/2008 | 19     | 39,622  | 28/09/2009 | 29     | 45,133  | 18/10/2010 |
| 10     | 33,109  | 30/06/2008 | 20     | 40,123  | 02/11/2009 |        |         |            |
residual DEM error phase and noise phase. The residual DEM error phase consists of the spatially uncorrelated look angle (SULA) error phase and spatially correlated look angle (SCLA) error phase. The noise phase and SULA error phase are estimated and filtered by applying a low-pass and adaptive filter, the phase stability indicator for each of the initial candidate SDFP pixels is estimated, and the final SDFP pixels are selected (Aobpaet, Cuenca, Hooper, & Trisirisatayawong, 2013).

The phase values of the final SDFP pixels are wrapped to modulo $2\pi$ and need to be corrected by adding the appropriate number of $2\pi$ cy’cles, a step known as phase unwrapping, which will provide the original phase difference of SDFP pixels in the differential interferograms. The SCLA error phase, atmospheric delay phase, and orbit error phase are simultaneously estimated and removed from the unwrapped phase of SDFP pixels, and the rest only contains the deformation phase (Cuenca, 2012). The deformation phase $\delta \varphi_i (r, x)$ for an SDFP pixel located at $(r, x)$ in the $i$th differential interferogram can be expressed as:

$$\delta \varphi_i (r, x) = \varphi (t_B, r, x) - \varphi (t_A, r, x) = -\frac{4\pi}{\lambda} [d(t_B, r, x) - d(t_A, r, x)], i = 1, 2, \cdots, M$$

(1)

where $d(t_A, r, x)$ and $d(t_B, r, x)$ are the cumulative deformations along the LOS direction at $t_A$ and $t_B$ ($t_A < t_B$), respectively, with respect to the reference first scene $t_0$, assuming that $d(t_0, r, x)$ is zero; $\lambda$ is the central wavelength of the transmitted signal; and $\varphi (t_A, r, x)$ and $\varphi (t_B, r, x)$ represent the deformation phase of the two images acquired at $t_A$ and $t_B$, respectively.

Let $\varphi^T = [\varphi (t_1), \cdots, \varphi (t_N)]$ be the vector of the $N$ unknown LOS cumulative deformation phase values with respect to $t_0$, $\delta \varphi^T = [\delta \varphi_1, \cdots, \delta \varphi_N]$ be the vector of the $N$ known deformation phase values derived from the multi-temporal differential interferogram series, and $IM^T = \begin{bmatrix} IM_1, \cdots, IM_M \end{bmatrix}$ and $IS^T = \begin{bmatrix} IS_1, \cdots, IS_N \end{bmatrix}$ correspond to the acquisition time index of the master and slave images, respectively.

Assuming the master $IM$ and slave $IS$ images to be chronologically ordered, i.e. $IM_i < IS_i$, where $i = 1, 2, \cdots, M$, we have:

$$\delta \varphi_i = \varphi (IS_i) - \varphi (IM_i), i = 1, 2, \cdots, M$$

(2)

Accordingly, we derive a system of $M$ equations in $N$ unknowns of the SDFP pixel located at $(r, x)$ from the $M$ multi-temporal differential interferogram series (Berardino, Fornaro, Lanari, & Sansosti, 2002):

$$\delta \varphi = B \hat{\varphi}$$

(3)

where $\hat{\varphi}$ are the optimal estimates of deformation phase values of the $M$ differential interferograms of the series. $\varphi$ are the optimal estimates of LOS cumulative deformation phase values. $B$ is a known matrix consisting of 0, 1 and $-1$. For instance, if the first differential interferogram of the series is formed by the third image as the slave image and the first image as the master image, $\delta \varphi_1 = \varphi (t_2) - \varphi (t_0)$. In the same manner, assuming $\delta \varphi_2 = \varphi (t_3) - \varphi (t_1)$, then

$$B = \begin{bmatrix}
0 & 1 & 0 & 0 & \cdots \\
-1 & 0 & 1 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
\end{bmatrix}_{M \times N}$$

The error equation of Equation (3) can be expressed as:

$$V_{\delta \varphi} = B \hat{\varphi} - \delta \varphi$$

(4)

Unfortunately, $M$ differential interferogram series may not belong to a SB subset, and $B$ exhibits a rank deficiency; therefore, $B^T B$ is a singular matrix. In other words, if there are $L$ different SB subsets, $R(B^T B) = N - L + 1$. The inverse matrix of $B^T B$ in Equation (5) does not exist. $\hat{\varphi}$ of Equations (3) and (4) can be estimated by the singular value decomposition (SVD) method (Chen et al., 2011; Nie, 2016). $B$ is decomposed as follows:

$$B = U S V^T$$

(6)

where $U$ and $V$ are the left-singular and right-singular matrix of $B$. $S$ can be expressed as:

$$S = \begin{bmatrix}
\Sigma & 0 \\
0 & 0
\end{bmatrix}$$

(7)

where $\Sigma = \operatorname{diag}(\sigma_1, \cdots, \sigma_{N-L+1})$ and $\sigma_1, \cdots, \sigma_{N-L+1}$ are the non-zero singular values of $B$.

Let $S^+ = \begin{bmatrix}
\Sigma^{-1} & 0 \\
0 & 0
\end{bmatrix}$. The pseudo inverse of $B$ is $B^+ = VS^+ U^T$, and $\hat{\varphi}$ can be estimated by:

$$\hat{\varphi} = B^+ \delta \varphi = VS^+ U^T \delta \varphi$$

(8)

According to the LS or SVD method, we can obtain $\hat{\varphi}^T = \begin{bmatrix} \hat{\varphi} (t_1), \hat{\varphi} (t_2), \cdots, \hat{\varphi} (t_N) \end{bmatrix}$. Then, the deformation magnitude $\delta \varphi$ and deformation
velocity \( v \) during each imaging period can be estimated as:

\[
\delta \mathbf{R}^T = \left[ -\frac{\lambda}{4\pi} (\hat{\phi}(t_1) - \hat{\phi}(t_0)), -\frac{\lambda}{4\pi} (\hat{\phi}(t_2) - \hat{\phi}(t_1)), \ldots, -\frac{\lambda}{4\pi} (\hat{\phi}(t_N) - \hat{\phi}(t_{N-1})) \right]
\]

\( \delta \mathbf{R}^T = \left[ -\frac{\lambda}{4\pi} \frac{\phi(t_1) - \phi(t_0)}{t_1 - t_0}, -\frac{\lambda}{4\pi} \frac{\phi(t_2) - \phi(t_1)}{t_2 - t_1}, \ldots, -\frac{\lambda}{4\pi} \frac{\phi(t_N) - \phi(t_{N-1})}{t_N - t_{N-1}} \right]
\]

\( \delta \mathbf{R}^T = \left[ -\frac{\lambda}{4\pi} \frac{\phi(t_1) - \phi(t_0)}{t_1 - t_0}, -\frac{\lambda}{4\pi} \frac{\phi(t_2) - \phi(t_1)}{t_2 - t_1}, \ldots, -\frac{\lambda}{4\pi} \frac{\phi(t_N) - \phi(t_{N-1})}{t_N - t_{N-1}} \right]
\]

(9)

(10)

Composition and optimal selection of multi-temporal differential interferogram series

Let us consider that each scene of \( N + 1 \) SAR images covering the same study area can interfere with at least one image among the other \( N \) SAR images. The number of multi-temporal differential interferogram series, say \( M \), should satisfy the following inequality (Berardino et al., 2002):

\[
(N + 1)/2 \leq M \leq \left\lceil \frac{N(N + 1)}{2} \right\rceil
\]

For example, 29 SAR images are used for our study, and the value of \( M \) widely varies from 15 to 406.

The composing mode of \( M \) multi-temporal differential interferogram series in StaMPS-based SBAS InSAR is shown in Figure 2. A solid double-ended arrow indicates that two corresponding SAR images can satisfy the spatio-temporal baseline threshold condition and can form a differential interferogram. A dashed double-ended arrow indicates that two corresponding SAR images cannot meet the short spatio-temporal constraints or form a differential interferogram of the \( M \) multi-temporal differential interferogram series.

The correlation of two SAR images is mainly affected by the temporal baseline (\( T \)), spatial perpendicular baseline (\( B_\perp \)), Doppler centroid frequency baseline (\( F_{DC} \)), and thermal noise (\( \gamma_{thermal} \)). The integrated correlation coefficient between two SAR images is defined as:

\[
y = y_{temporal} \cdot y_{spatial} \cdot y_{doppler} \cdot y_{thermal}
\]

\[
y \approx \left(1 - f\left(\frac{T}{T_c}\right)\right) \left(1 - f\left(\frac{B_\perp}{B_{\perp c}}\right)\right) \left(1 - f\left(\frac{F_{DC}}{F_{DC c}}\right)\right) \gamma_{thermal}
\]

(12)

where

\[
f(x) = \begin{cases} x, & x \leq 1 \\ 1, & x > 1 \end{cases}
\]

(13)

\( y_{temporal} \) is the temporal correlation, \( y_{spatial} \) is the spatial correlation, and \( y_{doppler} \) is the correlation in the Doppler centroid frequency. The superscript \( c \) denotes the critical parameter values; for \( T, B_\perp, \) and \( F_{DC} \) greater than their critical values, two SAR images exhibit almost complete decorrelation. A higher \( y \) value implies a higher correlation (better quality) of two SAR images (Hooper, 2006; Hooper et al., 2012). Assuming a constant value for \( \gamma_{thermal} \), the \( y \) values between each scene of \( N + 1 \) SAR images and the other \( N \) SAR images are calculated sequentially using

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(11)

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(12)

---

(13)

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Figure 2. Composing mode of multi-temporal differential interferogram series.
Equations (12) and (13), and the minimum integrated correlation coefficient \( \gamma_{\text{min}} \) is selected as a threshold value according to our experience. If the \( \gamma \) value between two SAR images is greater than \( \gamma_{\text{min}} \), a differential interferogram will be formed in accordance with the composing mode shown in Figure 2. In particular, for a scene image, all values are not greater than \( \gamma_{\text{min}} \), and the corresponding image with the largest \( \gamma \) value between the two will be selected to form a differential interferogram to ensure that each scene image can interfere with at least one other image. We can simultaneously use \( \gamma_{\text{min}}, T^c \), and \( B^c \) to control the composition of \( M \) multi-temporal differential interferogram series in a practical application.

From the above, we can see that, in the case of a stable number of SAR images, the \( \gamma_{\text{min}} \) value has a dominant effect on the quality and quantity of multi-temporal differential interferogram series, and a larger the \( \gamma_{\text{min}} \) value corresponds to better quality, lower quantity, and increased isolated subset; \( B \) may have a rank deficiency, and \( B^T B \) may be a singular matrix. The SVD method is needed to solve the deformation model to obtain deformation magnitudes and deformation velocities. This method has the capability of global optimal data processing and can maintain the information of original data to the maximum extent. However, it does not have strong interpretability or the stable processing ability of noise data (Yin, Yang, Wang, & Wang, 2011).

When the \( \gamma_{\text{min}} \) value decreases, the quality of multi-temporal differential interferogram series decreases and the quantity increases, all the differential interferograms series belong to the same subset, \( B \) may be a column full-rank matrix, and \( B^T B \) may be a non-singular matrix with different degrees of ill condition. The ill condition seriously affects the quality of the LS method in calculation and statistics. In calculation, the ill condition causes the increase of condition number of \( B^T B \) in the LS estimation, a small change of data leads to a substantial change of solutions, and the numerical calculations are very unstable. In statistics, the ill condition of \( B^T B \) means a greater difference in the maximum and minimum eigenvalues of \( B^T B \), the mean square error (MSE) becomes very large, LS estimations differ greatly from the corresponding true values, and the accuracy is very poor (Liu, Zhao, Zhang, Wang, & Tao, 2012).

The condition number \( k \) of \( B^T B \) can be defined as follows:

\[
k = \frac{\lambda_{\text{max}}(B^T B)}{\lambda_{\text{min}}(B^T B)}
\]

where \( \lambda_{\text{max}}(B^T B) \) and \( \lambda_{\text{min}}(B^T B) \) represent the maximum and minimum eigenvalues of \( B^T B \), respectively. If \( \lambda_{\text{min}}(B^T B) \) is very small or approximately equal to zero, the condition number \( k \) is generally larger; then, the matrix is ill conditioned. In general, we suggest that the matrix is not ill conditioned when \( k \) is greater than zero and less than one hundred, the matrix is moderately or strongly ill conditioned when \( k \) ranges from one hundred to one thousand, and the matrix is seriously ill conditioned when \( k \) is greater than one thousand.

The MSE of the estimated \( \phi \) is defined as follows:

\[
\text{MSE}(\phi) = \sigma_0^2 \text{tr}[(B^T B)^{-1}] = \sigma_0^2 \sum_{i=1}^{N} \frac{1}{\lambda_i}
\]

where \( \sigma_0^2 \) is the unit weight variance estimation, and \( \lambda_i \) is the \( i \)th eigenvalue of \( B^T B \).

Therefore, the quality and quantity of multi-temporal differential interferogram series affect the quality of \( \phi \) and the properties of \( B \), which ultimately affect the accuracy of \( \phi, \delta \bar{E}, \nu \) and the other estimated deformation information.

**SAR data processing method**

Since the 29 ASAR images are level-0 raw data, the imaging process is firstly carried out to obtain level-1 SLC images by StaMPS, following which the study area shown in Figure 2 is selected. The \( \gamma \) values between each image and the other images are calculated sequentially according to Equations (12) and (13) (Assuming \( \gamma_{\text{thermax}} \) equals 1). The image acquired on 26 January 2009 that maximizes the sum of \( \gamma \) values is chosen as a common master image, and the other images and external DEM are co-registered with it. The researchers have given many co-registration approaches for InSAR, which accuracy can reach 1/10 pixels or even higher (Chen, Zhang, & Zhang, 2016; Natsuaki & Hirose, 2013). In our following experiment, we determine whether the co-registration accuracy is reliable or not by visual interpretation and checking the simulated DEM elevation values after resampling to SAR image coordinate system in StaMPS-based SBAS software. Different \( \gamma_{\text{min}}, T^c \), and \( B^c \) are set to generate a different number of multi-temporal differential interferogram series. The ill condition and singularity of their deformation models are analyzed. We selected five series of 41, 48, 62, 85 and 108 multi-temporal differential interferograms to monitor the urban ground subsidence in Beijing using the StaMPS-based SBAS method. The five monitored results are compared with each other in terms of the number and location of the selected SDFP pixels as well as the pattern and 1D LOS velocities of ground subsidence; the results are then compared with the results of leveling and GPS measurements. The primary data processing procedure is shown in Figure 3.
Results

Optimal selections of multi-temporal differential interferogram series

The baselines (differences) of Doppler centroid frequency of 29 ASAR images are very small (the maximum is 27 Hz). Therefore, only considering the spatial perpendicular baselines smaller than 1100 m ($B_c$ equal to 1100 m) and the temporal baselines shorter than 500 days ($T$ equal to 500 days), a different number of multi-temporal differential interferogram series are formed with different $\gamma_{\text{min}}$ values. Table 2 summarizes the quantities and properties of the series for different $\gamma_{\text{min}}$ values. Figure 4 shows the temporal and spatial baseline combinations of two multi-temporal differential interferogram series.

In Table 2, $\gamma_{\text{min}}$ represents the minimum correlation of differential interferograms of the multi-temporal differential interferogram series. $M$ is the quantity of the multi-temporal differential interferogram series; $L$ is the quantity of subsets; singularity represents whether the matrix $B^TB$ is singular; and $k$ and $\lambda_{\text{min}}$ are the condition number and minimum eigenvalue of $B^TB$, respectively.

Table 2 and Figure 4 validate that $\gamma_{\text{min}}$ determines the quality and quantity of multi-temporal differential interferogram series under a fixed $B_c$ and $T_c$. As $\gamma_{\text{min}}$ decreases, the quality decreases and quantity increases, many isolated subsets are coupled.

Table 2. The quantities and properties of multi-temporal differential interferogram series under different $\gamma_{\text{min}}$ (the minimum integrated correlation coefficient).

| $\gamma_{\text{min}}$ | $M$ | $L$ | Singularity | $k$ | $\lambda_{\text{min}}$ |
|-----------------------|-----|-----|-------------|-----|------------------------|
| 0.80                  | 30  | 4   | Yes         | –   | –                      |
| 0.77                  | 41  | 2   | Yes         | –   | –                      |
| 0.75                  | 48  | 1   | No          | 488 | 0.0144                 |
| 0.70                  | 62  | 1   | No          | 585 | 0.0159                 |
| 0.65                  | 85  | 1   | No          | 607 | 0.0207                 |
| 0.60                  | 108 | 1   | No          | 527 | 0.0275                 |
| 0.55                  | 139 | 1   | No          | 530 | 0.0309                 |
| 0.50                  | 175 | 1   | No          | 320 | 0.0632                 |
| 0.45                  | 210 | 1   | No          | 341 | 0.0652                 |
| 0.40                  | 227 | 1   | No          | 245 | 0.0990                 |
| 0.35                  | 257 | 1   | No          | 160 | 0.1693                 |
| 0.25                  | 319 | 1   | No          | 70  | 0.4121                 |
into the same subset, and $B^TB$ is changed from singularity to non-singularity. When $B^TB$ is non-singular, the condition number and minimum eigenvalue will change with the $\gamma_{\text{min}}$ value. In general, the $k$ value will decrease and the degree of ill condition will weaken as the $\gamma_{\text{min}}$ value decreases. When $\lambda_{\text{min}}$ equals 0.25, the ill condition no longer exists. However, in the actual process of data processing, it is impossible to set $\gamma_{\text{min}}$ to equal 0.25 and form a series of 319 multi-temporal differential interferograms to monitor ground subsidence; a moderate or strong ill condition always exists and is difficult to avoid.

**Subsidence information extraction of study area**

In order to study the influence of different multi-temporal differential interferogram series on the final subsidence monitoring results, 41, 48, 62, 85 and 108 series are selected to monitor the ground subsidence in Beijing. The selected study area is approximately 54.8 km × 21.5 km in area, and the corresponding lines and pixels of images are 12,000 × 2760. Table 3 summarizes some basic information on the selected SDFP pixels for the five series. Figure 5 shows the distribution of the selected SDFP pixels for 48 and 85 multi-temporal differential interferogram series.

As can be seen from Table 3 and Figure 5, the quantity and density of the selected SDFP pixels are at least 116,680 and 99/km$^2$, respectively, and the quantity and density of the selected SDFP pixels are at least 116,680 and 99/km$^2$, respectively, and the

![Figure 4. Composing mode of spatio-temporal baselines of 30 and 62 multi-temporal differential interferogram series: (a) 30 series and (b) 62 series.](image)

**Table 3.** Quantity, density and amount of common pixels of the selected SDFP pixels for the five multi-temporal differential interferogram series.

| $M$ | Quantity | Density (SDFP/km$^2$) | Amount of common pixels |
|-----|----------|------------------------|-------------------------|
| 41  | 137,333  | 117                    |                         |
| 48  | 139,229  | 118                    |                         |
| 62  | 138,832  | 118                    | 3372                    |
| 85  | 116,680  | 99                     |                         |
| 108 | 133,910  | 113                    |                         |
maximum quantity and corresponding density are 139,229 and 118/km². Moreover, the SDFP pixels selected by the five series are very dense in the urban area (subsidence area) and can completely meet the need of phase unwrapping. Therefore, the influence of different series on the quantity and density of the selected SDFP pixels can be neglected. However, the number of selected SDFP pixels for 48 and 62 multi-temporal differential interferogram series has a small difference of 397, and the number of common SDFP pixels selected for the two series is only 40,741. Furthermore, the number of common

Figure 5. Distribution of the SDFP pixels selected by 48 and 85 multi-temporal differential interferogram series: (a) 48 series; (b) 85 series; (c) the overlay of the SDFP pixels selected by 48 and 85 series.
SDFP pixels selected for the five different series is only 3372. Hence, different series seriously affect the locations of the selected SDFP pixels and thereby affect the final subsidence information monitored by StaMPS-based SBAS InSAR, which implies that the influence of the series on the locations should be considered. Figure 6 shows the locations of the 3372 common SDFP pixels.

The wrapped phase of the SDFP pixels is unwrapped, and error phase components are simultaneously estimated and removed from the unwrapped phase to ensure that the rest contain only the deformation phase. Then, the deformation model is established and solved to obtain the deformation magnitudes, velocities, etc. Figure 7 shows the subsidence distribution and velocities of the SDFP pixels between 22 January 2007 and 18 October 2010 monitored by five multi-temporal differential interferogram series. Table 4 presents the distributions of 3372 common SDFP pixels obtained for the five series at every interval of subsidence velocity.

As can be seen from Figure 7 and Table 4, the spatial distribution of the ground subsidence monitored by the five series is consistent and mainly distributed on the east of Chaoyang district, the junction of Haidian district and Changping district, and the southwest of Shunyi district. However, the monitored annual mean LOS subsidence velocities greatly differ from each other with distributed intervals of (−127, 18 mm/year), (−126, 20 mm/year), (−106, 16 mm/year), (−94, 14 mm/year) and (−83, 13 mm/year), respectively. The maximum difference of the largest monitored LOS subsidence velocity reaches −44 mm/year between 41 and 108 multi-temporal differential interferogram series. In order to compare and analyze this difference in detail, 20 common SDFP pixels are randomly selected (shown in Figure 6(a,b) with the purple pushpins), and their subsidence velocities obtained by five series are counted and put them in

**Figure 6.** Distribution of the 3372 common SDFP pixels: (a) Distribution of the 3372 common SDFP pixels and (b) overlaid on subsidence map monitored by 62 series.
numerical order according to subsidence velocity of 41 multi-temporal differential interferogram series. The statistical results are shown in Table 5 and Figure 8.

Table 4. Distribution of 3372 common SDFP pixels obtained by five multi-temporal differential interferogram series.

| Intervals of subsidence velocity (mm/year) | M = 41 | 48 | 62 | 85 | 108 |
|------------------------------------------|-------|----|----|----|-----|
| \([-127, -100)\]                          | 6     | 5  | 1  | 0  | 0   |
| \([-100, -80)\]                           | 24    | 16 | 9  | 3  | 0   |
| \([-80, -50)\]                            | 107   | 107| 89 | 72 | 0   |
| \([-50, -20)\]                            | 383   | 314| 367| 72 | 0   |
| \([-20, -10)\]                            | 326   | 315| 330| 391| 0   |
| \([-10, 0)\]                              | 438   | 501| 474| 354| 465 |
| \([-10, 20)\]                             | 2088  | 2114| 2102| 2104| 2092|

From 22 January 2007 to 18 October 2010, a SDFP pixel should have only one value of annual mean LOS subsidence velocity. However, it can be found from Table 5 and Figure 8 that the monitored subsidence velocities for the five series vary widely without any regular pattern. For example, in the 5th common SDFP pixel, the monitored subsidence velocity varies from \(-97\) mm/year to \(-48\) mm/year with a maximum difference of \(-49\) mm/year. For the 62 and 85 series, the minimum difference of the monitored subsidence velocity is 0 mm/year in the 11th, 12th, 14th and 17th common SDFP pixels, and the maximum is \(-14\) mm/year in the 4th SDFP pixel. Therefore, the difference of the monitored subsidence velocities for the five series is not caused by the...
selected reference pixel of phase unwrapping but has a strong relationship with the quality and quantity of multi-temporal differential interferogram series. In practical tests, the pixel on the geographical coordinates (115°58'16"E,39°57'31"N) near Mentougou, shown in Figure 6(a,b), is selected as the reference pixel of phase unwrapping.

Discussion

The standard deviation of mean LOS velocity

To compare the precision of the monitored subsidence velocities for the five series, the standard deviations of mean LOS velocities are calculated, analyzed and shown in Figures 9 and 10.

As can be seen from Figures 9 and 10, the maximum standard deviations of mean LOS velocities for the five multi-temporal differential interferogram series are all

| Number | Subsidence velocities (mm/year) | Number | Subsidence velocities (mm/year) |
|--------|---------------------------------|--------|---------------------------------|
|        | 41                              | 11     | 41                              |
| 1      | −120                            | 11     | −38                            |
| 2      | −109                            | 12     | −31                            |
| 3      | −107                            | 13     | −29                            |
| 4      | −101                            | 14     | −25                            |
| 5      | −97                             | 15     | −19                            |
| 6      | −85                             | 16     | −18                            |
| 7      | −78                             | 17     | −8                             |
| 8      | −63                             | 18     | −6                             |
| 9      | −51                             | 19     | −5                             |
| 10     | −46                             | 20     | −2                             |

Table 5. LOS velocities of ground subsidence of 20 common SDFP pixels.

Figure 8. LOS velocities of ground subsidence and their difference of 20 common SDFP pixels: (a) LOS velocities of ground subsidence and (b) difference of velocities between two adjacent series.
less than 19 mm/year, and most of them are concentrated between 0 and 10 mm/year. Although the mean LOS subsidence velocities for the five multi-temporal differential interferogram series are quite different, their standard deviation differences are not great.

Validation with leveling
Liu, Jia, and Chu (2012) introduced ground subsidence monitoring systems and methods for Beijing, and gave the subsidence velocities of three leveling benchmarks in Wangjing, Tianzhu, and Baxianzhuang. These three leveling benchmarks were measured by first order leveling using the high accuracy electronic level, and the accidental standard deviation per kilometer is 0.4 mm and the total standard deviation per kilometer is 1.0 mm. Hu, Wang, Sun, Hou, and Liang (2014) used the leveling benchmarks in Wangjing and Tianzhu to validate the use of the SBAS InSAR technology to monitor the ground subsidence of Beijing. Owing to the lack of the geographical latitudes and longitudes of the leveling benchmark in Baxianzhuang, Table 6 only shows a comparison between two subsidence velocities derived from the leveling benchmarks in Wangjing (shown in Figure 6(a,b)) and Tianzhu (shown in Figure 6(a,b)) and from their nearest SDFP pixels.

Figure 9. The standard deviations of mean LOS velocity for the five multi-temporal differential interferogram series: (a) 41 series; (b) 48 series; (c) 62 series; (d) 85 series; (e) 108 series.
The leveling-measured subsidence velocity of the benchmark in Wangjing was along the vertical direction from 4 September 2009 to 10 October 2010, and the corresponding subsidence velocity of its nearest SDFP pixel monitored by StaMPS-based SBAS InSAR was along the LOS direction from 28 September 2009 to 18 October 2010. The leveling-measured subsidence velocity of the benchmark in Tianzhu was also along the vertical direction from 15 January 2010 to 14 October 2010, and the corresponding subsidence velocity of its nearest SDFP pixel monitored by StaMPS-based SBAS InSAR was also along the LOS direction from 11 January 2010 to 18 October 2010.

In Table 6, “vertical velocity” and “LOS velocity” represent the leveling-measured subsidence velocity along the vertical and LOS direction of the benchmarks in Wangjing and Tianzhu, respectively. “InSAR-monitored LOS velocity” represents the subsidence velocity along the LOS direction of the nearest SDFP pixels monitored using StaMPS-based SBAS InSAR by our five multi-temporal differential interferogram series.

As can be seen from Figures 6(b) and 7 as well as Table 6, the benchmark in Wangjing was located at the edge of the Chaoyang subsidence area, and some
small subsidence areas were distributed nearby. The monitored subsidence velocities for our five series were all (−10, 10 mm/year), and no subsidence occurred at the nearest SDFP pixel of the benchmark in Wangjing. Two types of subsidence velocities cannot agree with each other very well, mainly because, first, the location of leveling benchmark cannot be consistent with its nearest SDFP pixel, which is occasionally far from each other, and second, leveling-measured subsidence velocity can only reflect the subsidence velocity of a “point,” whereas InSAR-monitored subsidence reflects the subsidence velocity of a “surface”, which is 4 m × 7 m in this case. In the marginal region of the subsidence, the subsidence velocity of this “leveling point” in the “surface” may be affected by the surrounding non-subsidence areas, and the subsidence velocities monitored by our five series may become smaller, greater than zero or equal to zero. The benchmark in Tianzhu belonged to the Shunyi subsidence area, for which the subsidence velocities monitored by our five series were (−20, −10 mm/year), and the monitored subsidence velocities of the nearest SDFP pixel for five series were quite smaller than the leveling measured.

Validation with GPS

To further quantitatively verify accuracy of the ground subsidence monitored by the five series using StaMPS-based SBAS InSAR, a comparative analysis of differences between SBAS InSAR and GPS derived results is performed. Two GPS points shown in Figure 6(a,b) are used in validation. These two GPS stations were surveyed by Trimble NET R9 dual-frequency GNSS receivers with Zephyr Geodetic antennas, both had stable monuments and provided daily receiver-independent exchange observation. The RINEX GPS observation files were processed with the GAMIT/GLOBK 10.6 software. Hectore time-series analysis software was used to estimate the annual cycle, half-year cycle and noise model to estimate the accurate velocity of the site with typical formal position uncertainties of 4 mm in vertical direction. Table 7 shows a comparison between two subsidence velocities derived from the GPS points and from their nearest SDFP pixels.

As can be seen from Figures 6 and 7 as well as Table 7, the GPS point named Chao was located at the Chaoyang subsidence area, the monitored subsidence velocities for our five series were all (−50, −20 mm/year). The GPS point named Dsqi belonged to the Changping subsidence area, for which the subsidence velocities monitored by our five series were (−51, −10 mm/year), and the monitored subsidence velocities of the nearest SDFP pixel for five series were quite smaller than the GPS measured, two types of subsidence velocities cannot also agree with each other very well.

Conclusions

This study systematically investigated the composing mode and optimal selection of multi-temporal differential interferogram series of StaMPS-based SBAS InSAR, leading to the following conclusions:

(1) The quality of multi-temporal differential interferogram series affects the quality of differential interferometric phases of the deformation model, the quantity and composing mode affect the properties of the coefficient matrix, and they ultimately affect the solution accuracy of deformation velocities or other deformation information obtained using SBAS InSAR. In a practical application, the setting of $\gamma_{\text{min}}$, $T^c$, and $B^c$ can ensure that the total correlation of multi-temporal differential interferogram series is the best and that the total quality is optimal. However, as the quantity increases, the overall quality of differential interferometric phases of the deformation model decreases, $B^T B$ is changed from singularity to non-singularity, the condition number decreases, the minimum eigenvalue increases, and consequently, the degree of ill condition weakens, but a moderate or strong ill condition always exists and cannot be avoided.

(2) The spatial distribution of the ground subsidence monitored by different multi-temporal differential interferogram series is consistent, but the monitored annual mean LOS subsidence velocities greatly differ from each other. Generally, under the condition of optimal quality, as the quantity increases, both the maximum of monitored subsidence velocity and the velocity of the common SDFP pixels decrease. Moreover, the difference of subsidence velocities of the common SDFP pixels is not equal, and no regularity could be discerned from them.

(3) The mean LOS subsidence velocities for the five multi-temporal differential interferogram series are quite different, but their standard deviation differences are not great, and the StaMPS-based SBAS InSAR-monitored subsidence velocities of the
nearest SDFP pixels for five series are all quite smaller than the leveling and GPS measured results. Comprehensive comparison, the monitored results of 48 and 20 multi-temporal differential interferogram series are relatively better. Considering the extended time span of the 29 ASAR images (approximately 45 months) and the characteristics of slow ground subsidence in Beijing, the suitable number of multi-temporal differential interferogram series of the optimal quality is 1–2 for a month.

(4) In the practical application of StaMPS-based SBAS InSAR to monitor surface deformation, a higher quantity of multi-temporal differential interferogram series of the optimal quality is not always better, and the quantity should be determined by the phase stability of the selected SDFP pixels, type of SAR images, characteristics of the surface deformation, time span of the deformation, etc.

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