Analysis of algebraic reasoning ability of cognitive style perspectives on field dependent field independent and gender

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Abstract: The purpose of this study is to analyse algebraic reasoning ability using the SOLO model as a theoretical framework to assess students' algebraic reasoning abilities of Field Dependent cognitive (FD), Field Independent (FI) and Gender perspectives. The method of this study is a qualitative research. The instrument of this study is the researcher himself assisted with algebraic reasoning tests, the problems have been designed based on NCTM indicators and algebraic reasoning according to SOLO model. While the cognitive style of students is determined using Group Embedded Figure Test (GEFT), as well as interviews on the subject as triangulation. The subjects are 15 female and 15 males of the sixth semester students of mathematics education, STKIP Sebelas April. The results of the qualitative data analysis is that most subjects are at the level of unistructural and multi-structural, subjects at the relational level have difficulty in forming a new linear pattern. While the subjects at the extended abstract level are able to meet all the indicators of algebraic reasoning ability even though some of the answers are not perfect yet. Subjects of FI tend to have higher algebraic reasoning abilities than the subject of FD.

1. Introductions

Research on algebra and algebraic reasoning has been studied by several researchers including some studies showing that there are many Algebraic Perspectives [1], including the thought that algebra is considered as general arithmetic, a means for solving a particular problem and the study of relationships and structures. Algebra can be seen as general arithmetic and also to study the functions and relationships among other variables. Algebraic reasoning is important to encourage students to understand mathematics beyond specific calculations and procedural usage of formulas. Kaput and Blanton [2].

Algebra is the language to investigate and communicate most of Mathematics problems. Algebra is a way of thinking, a collection of concepts, and skills that enable students to generalize, model, and analyse mathematical situations [3]. Furthermore, in the Principles and Standards of School Mathematics [3], there are four stated goals related to algebra: (1) understand the patterns, the relationships, and the functions, (2) representing and analysing mathematical situations and structures by using algebraic symbols, (3) Using mathematical models to represent and understand quantitative relationships and (4) analysing the changing in various contexts. In addition, suggested algebra is the way to express generalizations about numbers, quantities, relationships and functions [4], suggested at the algebra school level is described as: (1) manipulating and transforming statements in symbolic form, (2) generalization of rules on numbers and patterns, (3) study of abstract structure and system of computation and relation, and (4) rules in transformation and solving the equations.
Further Watson and van Amerom suggested several different perspectives on algebra, they are: (1) algebra as a generalized arithmetic, (2) algebra as a problem-solving tool, (3) algebra as the study of relationships, and (4) algebra as the study of structures [5]. Algebraic reasoning underlies all mathematical thinking, including arithmetic, as it allows us to explore mathematical structures [3]. Everyone has the capacity to think algebraically because algebraic reasoning is essentially the way humans interact with the world in searching of patterns, pay attention to important aspects of patterns, and then generalize from simple to unfamiliar situations. According to [6], Algebraic reasoning is a process whereby students generalize mathematical ideas from a collection of facts, prepare their generalizations through statements and express the statement in an increasingly formal and age-appropriate way.

Algebraic reasoning is a key word for students in an effort to improve mathematics and science skills [7]. Research on algebraic reasoning has been done by [8] whose research proved that algebra is difficult for most students. Algebraic reasoning involves of forming generalizations from experience by numbers and calculations, forming these ideas by using meaningful symbol systems, and exploring the concept of patterns and functions [9].

From some of the above opinion it can be concluded that algebraic reasoning is a process whereby students do activities to find patterns of a certain mathematical problems or contextual situations, create relationships between quantities and make their generalizations through formal symbolic representations and manipulations. Thus, algebraic reasoning is closely related to the ability to understand patterns and make generalizations, which are generalizations and modelling using equations as the key to algebraic reasoning. In this study to determine the algebraic reasoning level by using the SOLO model (Structure of the Observed Learning Outcome) [10]. In the algebra field, the SOLO model has been used to describe the solving of equation problem on elementary students [10]. The SOLO model is used to reflect four levels: unistructural, multistructural, relational and extended abstract. Sorted by the level of difficulties from lowest to complex.

The levelling of algebraic reasoning abilities based on the NCTM 2000 indicators [3] are: (1) understanding patterns, (2) making algebraic symbols, (3) creating a mathematical model, and (4) analysing the change. Here is the algebra reasoning course based on the NCTM indicator: 

Unistructural: Understand pictorial patterns and form symbols that use the information and diagrams presented.

Multistructural: The ability to understand patterns and create algebraic symbols by looking at a consecutive given process that identifies the recursive relationship between the term and its sequence.

Relational: Generalize the linear relationship of the pattern symbolically from algebraic symbols then create a mathematical model based on all the information provided.

Extended abstract: The ability to analyse the linear pattern in a broader case. That is, the use of a linear relationship to form the rules of a new linear pictorial pattern; Try to make guesses and verify allegations deductively, all the information provided must be integrated to produce algebraic and pictorial statements.

The students’ algebraic reasoning abilities can be viewed from a variety of dimensions; one of them is the cognitive style. According to Brown E, et al The cognitive style refers to a person's characteristics in responding, processing, storing, thinking, and using information in response to a task or various types of environmental situations [11]. The process of algebraic reasoning in problem solving needs to get the attention of teachers and lecturers to help students develop algebraic reasoning abilities. Cognitive style is a dimension to review students' algebraic reasoning abilities.

The cognitive style is the individual choice and the habitual approach to organizing and representing information, which then influences the way one perceives and responds to events and ideas [12]. A number of researchers have proposed various dimensions of cognitive style, such as independent field-dependent field cognitive style. According to Witkin, the cognitive style of the dependent field and the independent field was first proposed [13], the dependent field cognitive field (FD) and the independent field (FI) is a concept that refers to an individual's ability to recognize or trace an embedded figure complex background [14].
Individuals with cognitive style FD and FI have different personality traits. Individual FI has greater cognitive abilities. They are usually autonomous, impersonal and manipulative [15]. Lack of awareness of social problems their stimulus values are usually self-oriented, selfish and individualistic. Rayner and Riding [11] added that FI Students set goals for themselves, dependent on intrinsic reinforcement, and tend to devise their own strategies for learning, while Individual FD has a better ability of interpersonal relationships. They have a tendency to relate well with others and are often portrayed as warm, accommodating, affective or empathic [16]. Students of FD depend on social, gregarious and willing to make a good impression, suitable and sensitive to social environment.

Gender is the congenital sex that is influenced by social and cultural factors. Gender is also one of the factors that influence how to acquire mathematical knowledge. According to Witkin and Goodenough1981 gender is often mentioned as one of the factors that affect the cognitive style of FD and FI in humans. However, the existence of the FD and FI cognitive style in relation to gender has its own controversy. This makes the difficulties experienced by each person is different. So, from the description, it is necessary to do an analysis to determine the ability of algebraic reasoning from the perspective of cognitive style FD, FI and gender. The purpose of this study was to analyse the ability of algebraic reasoning using the SOLO model as a theoretical framework to assess students' algebraic reasoning abilities from the perspective of Field Dependent (FD), Field Independent (FI), and gender.

2. Methods
This study is a qualitative research. The instrument of this study is the researcher as the main instrument assisted by algebraic reasoning test (TPA), while the cognitive style of students is determined using Group Embedded Figure Test (GEFT) to group the subject into Low Dependent Field (FDL), Strong Field Dependent (FDK), Low Field Independent (FIL), and Strong Field Independent (FIK) then subjects are grouped also by gender as well as interviews on the subject as triangulation. The subjects are 15 male and 15 females of the sixth semester students of mathematics education, STKIP Sebelas April Sumedang. The problems have been designed based on the NCTM indicator and algebra ranking according to the SOLO model. Each question consists of a situation or a story and some related questions. The questions represent four levels of reasoning: the unistructural, multi-structural, relational, and extended abstract stages. The second phase, a clinical interview was conducted to collect the qualitative data. A clinical interview session was conducted after a written test of eight subjects.

3. Result and Discussion
From the test result of algebraic reasoning ability, a GEFT test was given first to see the cognitive style of FD and FI of 15 male subjects and 15 female subjects which then grouped into subject of FDL, FDK, FIL, and FIK. The levelling of Students' algebraic reasoning is based on the SOLO model according to Collis, Romberg, & Jurnak [17], there are four stages of algebraic reasoning, they are: unistructural, multi-structural, relational, and extended abstract.

Based on Table 1, the findings show that the majority of 23 subjects (76.7%) are at the level of unistructural and multi-structural, 13 female subjects and 10 male subjects from cognitive style of FDL, FDK, FIL, and FIK. Overall on algebraic reasoning ability, the subjects on this level have the ability to investigate and understand the pattern and then get the next pattern, making the algebraic symbol by looking at the given process, identifying the recursive relationship between the term and its sequence. Most are able to solve various numerical problems involving specific cases, explaining patterns on successive forms, generalize problems in algebraic form. While those who cannot solve this problem have difficulty in making generalizations using linear equations, they were failed in relating the linear patterns given in the question because of the lack of algebraic concepts, especially about what is known (variables) and linear equations.
Table 1. The results of data analysis of algebraic reasoning ability of cognitive style and gender perspective

| Subject | Unistructural level | Multi-structural level | Relational level | extended abstract level |
|---------|---------------------|-----------------------|------------------|------------------------|
| FDL Female | √ | − | − | − |
| FDL Male | √ | √ | − | − |
| FDK Female | √ | √ | √ | √ |
| FDK Male | √ | √ | − | − |
| FIL Female | √ | √ | √ | √ |
| FIL Male | √ | √ | − | − |
| FIK Female | √ | √ | √ | √ |
| FIK Male | √ | √ | √ | √ |

At the relational level, the analysis data shows that there are 13 subjects (43.3%), 9 female subjects and 5 male subjects from cognitive style of FDK female, FIL female, FIK female and FIK male. Subjects at this level have algebraic reasoning ability: to understand patterns, to use algebraic symbols, to generalize the linear relationships of patterns symbolically from algebraic symbols and to make a mathematical model based on all the information provided. Subjects at this level have difficulty in forming new linear patterns or making new linear equations.

At the extended abstract level, the analysis data shows that 5 subjects (16.7%), 3 female subjects and 2 male subjects from cognitive style of FDK female, FIK female and FIK male. At this level, the subject is able to meet all the algebraic reasoning indicators according to NCTM although there is still space for improvement to analyse the linear pattern in the wider case. That is, the use of linear relationships, forming rules for new linearly pictorial patterns, attempting to make conjectures and verifying allegations deductively, analysing changes in various contexts. Researchers found that subjects with high ability are more capable of finding a recurrent linear and identify the linear relationships between variables. They are able to coordinate all the information provided to generalize algebraic patterns, the ability to use the linear pattern concepts in more abstract situations such as forming new linear pattern rules they create themselves, and using their own consistent methods to find the solutions.

The study of algebra is consistent with the results of research showing that there are some teachers’ perspectives on what is important in teaching school algebra [1]. Teachers also consider algebra more about the use and manipulation of symbols than about application, communication or reasoning of algebraic ideas.

The results of the study of algebraic reasoning ability in terms of FD, FI and gender cognitive styles are presented in Table 2.

Table 2. The result of the percentage distribution of cognitive style of FD, FI, and gender

| Subject | N | % |
|---------|---|---|
| FD Male | 4 | 13,3 |
| Female | 9 | 30 |
| FI Male | 11 | 36,7 |
| Female | 6 | 20 |
| Total | 30 | 100 |

Based on Table 2, it shows that female subjects are higher in FD while the male subjects are higher in FI. This finding is in accordance with [18] research results that the proportion of male respondents is higher in FI while the proportion of women respondents is higher in FD. Subjects of FI tend to have higher algebraic reasoning abilities than students in the subject of FD. The subject of FI has self-reliance but lack of awareness of social problems; their stimulus values are usually self-oriented,
selfish and individualistic. This is consistent with the findings of [15] that Individual FI has greater cognitive abilities. Their restructuring is usually autonomous, impersonal and manipulative. Unlike the case with FD subjects who have better skills for interpersonal relationships, they are more able to socialize with friends, the environment, and usually not a stiff person. So, each of the cognitive styles has its advantages and disadvantages.

4. Conclusion

From the results and discussion, the conclusion is as follows: (1) Most subjects are at the level of unistructural and multi-structural, there are 23 subjects (78%), they are 13 female subjects and 10 male subjects who came from cognitive style FDL, FDK, FIL, and FIK, (2) at the relational level based on the results of data analysis there are 13 subjects (43.3%), 9 female subjects and 5 male subjects, almost all algebraic reasoning indicators could be achieved, they only failed at analysing the changes, (3) at the extended abstract level based on the analysis of 5 subjects (16.7%), at this level the subjects are able to meet all the indicators of algebraic reasoning, (4) female subjects tend to be higher in FD while male subjects are higher in FI, and (5) FI subjects tend to have higher algebraic reasoning abilities than students in the subject of FD.

The researcher suggests teachers and researchers from the results of this study to pay attention to the following things in teaching, namely: (1) the use of the model or approach of learning must be appropriate in accordance with the characteristics of the material and also the characteristics of students, (2) the use of learning models should be varied, and (3) the use of teaching materials to help students to understand algebra materials and be more active when learning activities take place.

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