Effects of reheating on leptogenesis

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Abstract

We study the evolution of a cosmological baryon asymmetry in leptogenesis when the right-handed neutrinos are produced in inflaton decays. By performing a detailed numerical study over a broad range of inflaton-neutrino couplings we show that the resulting asymmetry can be larger by two orders of magnitude or more than in thermal leptogenesis, if the reheating temperature $T_{RH}$ is of the same order as the right-handed neutrino mass $M_1$. Hence, the lower limit on the baryogenesis temperature obtained in thermal leptogenesis can be relaxed accordingly.

1 Introduction

The observed cosmological baryon asymmetry can naturally be explained via decays of heavy right-handed neutrinos (RHN), a scenario known as leptogenesis \[1\]. In its simplest form, thermal leptogenesis, the baryon asymmetry is produced during the radiation dominated era and stringent limits on neutrino parameters are obtained. In particular, successful leptogenesis requires the reheating temperature of the universe to be larger than about $4 \times 10^8$ GeV. In the favoured strong-washout regime an even stricter lower limit on $T_{RH}$ of about $4 \times 10^9$ GeV is obtained. This may be in conflict with big bang nucleosynthesis (BBN) in supergravity (SUGRA) models due to the gravitino problem. There successful BBN is only possible if $T_{RH}$ is lower than about $10^{6-7}$ GeV\[2\].

An alternative production mechanism is considered in non-thermal leptogenesis models \[3, 4, 5, 6, 7, 8, 9, 10, 11, 12\] where one assumes that right-handed neutrinos are produced directly in the decays of some heavier particle. That particle could be the inflaton, the particle related to an inflationary phase in the very early universe. Supersymmetric \[4, 5, 8, 9, 10, 11\] and grand unified models \[3\] have been considered, the focus of all these studies being put on the underlying model of inflation in order to derive the coupling between the inflaton and the right-handed neutrino\[1]. Most of these models have in common that the decay width of

\[1\]There are recent attempts to couple the inflaton to the neutrino in a non direct way to allow for instant non-thermal leptogenesis \[13\].
the inflaton, $\Gamma_\Phi$, is much smaller than the decay width of the neutrino, $\Gamma_N$, i.e. $\Gamma_\Phi \ll \Gamma_N$. Hence, the neutrino decay instantaneously follows the inflaton decay and the reheating temperature $T_{RH}$ is much smaller than the RHN mass $M_1$. In such scenarios the produced baryon asymmetry can easily be evaluated without the need for a full numerical investigation. In this work we will also consider the case $T_{RH} \sim M_1$ and show that here a full numerical study by means of Boltzmann equations is needed. We will discuss quantitatively the dependence of the final baryon asymmetry on $\Gamma_\Phi$ and $\Gamma_N$ for a broad range of parameters. Furthermore, we will see how the bounds on neutrino parameters and the reheating temperature derived in thermal leptogenesis are relaxed.

In the next section we briefly review thermal leptogenesis in order to set the scene and introduce some notation that will be used in the rest of the paper. In section 3 we introduce our model for the inflaton-neutrino coupling and discuss the case that the decay width of the inflaton is much smaller than that of the neutrino. If that is not the case, a more detailed treatment in terms of Boltzmann equations is needed, which are introduced in section 4. Finally, in section 5 we present and discuss our results and their dependence on the parameters of the inflaton and the neutrino.

2 Thermal Leptogenesis

In the standard thermal leptogenesis scenario the right-handed neutrinos are produced dynamically by scattering processes in the thermal bath or are assumed to be initially in thermal equilibrium. Usually a hierarchical mass scheme is assumed, $M_3, M_2 \gg M_1$. Then the lightest right-handed neutrino, $N_1$, decays into a standard model lepton-Higgs pair, $N_1 \rightarrow H + l_L$ and $N_1 \rightarrow H^\dagger + l_L^\dagger$ generating a lepton asymmetry if $CP$ is not conserved in the decay. The decay rate of $N_1$ reads \cite{14}

$$\Gamma_N = H(M_1) \frac{K_1(z)}{K_2(z)},$$

where $K_1, K_2$ are Bessel functions and $H$ is the Hubble parameter. The parameter $K$,

$$K \equiv \frac{\Gamma_N(z = \infty)}{H(z = 1)} = \frac{\bar{m}_1}{m_*},$$

separates the regions of weak washout, $K \ll 1$ and strong washout, $K \gg 1$. Here, $\bar{m}_1$ is the effective light neutrino mass and $m_* \simeq 1.1 \times 10^{-3}$ eV. $\Gamma(z = \infty)$ is the decay rate in the rest frame of the particle, i.e. the decay width $\Gamma_{N,r} = H(M_1)K$.

The maximal CP asymmetry in the decays of $N_1$ is given by \cite{15} \cite{16}:

$$\epsilon_1^{\text{max}} = \frac{3}{16\pi} \frac{M_1 m_3}{v^2} \beta \approx 10^{-6} \left(\frac{M_1}{10^{10}\text{GeV}}\right) \left(\frac{m_3}{0.05\text{eV}}\right) \beta,$$

where $\beta \leq 1$, with the exact value depending on different see-saw parameters. This maximal CP asymmetry then yields the maximal baryon asymmetry $\eta_B^{\text{max}}$. This can
be produced in leptogenesis. Since $\epsilon_1^{\text{max}} \propto M_1$, requiring that the maximal baryon asymmetry is larger than the observed one, i.e. $\eta_B^{\text{max}} \geq \eta_B^{\text{CMB}}$, yields a lower bound on $M_1$ \[17]\).

$$M_1 > M_1^{\text{min}} = \frac{1}{d} \frac{16\pi}{3} \frac{v^2}{m_{\text{atm}}} \frac{\eta_B^{\text{CMB}}}{\kappa_f} \approx 6.4 \times 10^8 \text{ GeV} \left( \frac{\eta_B^{\text{CMB}}}{6 \times 10^{10}} \right) \left( \frac{0.05 \text{ eV}}{m_{\text{atm}}} \right) \kappa_f^{-1}. \tag{4}$$

Here $\kappa_f$ is the final efficiency factor which parametrizes the dynamics of the lepton asymmetry and neutrino production \[18\]. It is obtained by solving the relevant set of Boltzmann equations. In thermal leptogenesis its maximum value is by definition 1 for the case of a thermal initial abundance of right-handed neutrinos. In the factor $d = 3\alpha_{\text{sph}}/(4f) \approx 0.96 \times 10^{-2}$, $f = 2387/86$ accounts for the dilution due to photon production from the onset of leptogenesis till recombination and the factor $\alpha_{\text{sph}} = 28/79$ accounts for the partial conversion of the lepton asymmetry into a baryon asymmetry by sphaleron processes.

Since all this is assumed to happen in the thermal, radiation dominated phase of the universe, the lower bound on $M_1$ translates into a lower bound on the initial temperature of leptogenesis, which corresponds to a lower limit on the reheating temperature after inflation \[18\],

$$T_{\text{RH}} \approx M_1^{\text{min}} \geq 4 \times 10^8 \text{ GeV}. \tag{5}$$

Such a large reheating temperature is potentially in conflict with BBN in supersymmetric models, where upper bounds on the reheating temperature as low as $10^6$ GeV have been obtained in SUGRA models \[2\]. This and the dependence of the produced baryon asymmetry on the initial conditions in the weak washout regime are some of the shortcomings of thermal leptogenesis. Hence, it is worthwhile to study alternative leptogenesis scenarios. In the following we will discuss a scenario where the neutrinos are produced non-thermally in inflaton decays. As we shall see, the lower bound on the reheating temperature can be relaxed by as much as three orders of magnitude in the most interesting parameter range of strong washout.

### 3 Leptogenesis via inflaton decay

In the following we will assume that the inflaton $\Phi$ decays exclusively into a pair of the lightest right-handed neutrinos, $\Phi \rightarrow N_1 + N_1$. The decay width for this process can be parametrized as

$$\Gamma_{\Phi} \simeq \frac{|\gamma|^2}{4\pi} M_{\Phi}, \tag{6}$$

$\gamma$ being the inflaton-neutrino coupling. Further, we assume a hierarchical mass spectrum for the heavy neutrinos, $M_3, M_2 \gg M_1$, hence potential effects of $N_2$
and $N_3$ can be neglected.\footnote{Note, however, that the baryon asymmetry may also be generated by the second-lightest heavy neutrino in certain areas of parameter space\cite{19}.}

Neglecting potential contributions from preheating\cite{7, 20}, which are generically rather small anyway\cite{21, 22}, the decay considered above is kinematically allowed if $M_{\phi} \geq 2M_1$, which will always be the case in the following.

After the inflaton condensate has decayed away, the heavy neutrinos dominate the energy density of the universe, a scenario known as dominant initial abundance. When the right-handed neutrinos have become non-relativistic they decay in the standard way, thereby producing a lepton asymmetry, and reheat the universe since their decay products, standard model lepton and Higgs doublets, quickly thermalize.

The reheating temperature is usually computed assuming that the energy stored in the inflaton condensate is instantaneously transformed into radiation. This yields

$$T_{RH} = \left(\frac{90}{8\pi^3 g^*}\right)^{\frac{1}{4}} \sqrt{\Gamma_\phi M_{Pl}} = 0.06 \left|\gamma\right| \left(\frac{200}{g^*}\right)^{\frac{1}{4}} \sqrt{M_{\phi} M_{Pl}}, \quad (7)$$

where $g^*$ is the number of relativistic degrees of freedom at $T_{RH}$. Analogously, one can define a reheating temperature for the reheating process due to neutrino decays:

$$T_{RH}^N = \left(\frac{90}{8\pi^3 g^*}\right)^{\frac{1}{4}} \sqrt{\Gamma_{N_{\nu}} M_{Pl}} = M_1 \sqrt{K}. \quad (8)$$

In the decay chain considered the real physical reheating temperature is given by Eq. (8), since only after the neutrinos have decayed is the thermal bath of the radiation dominated universe produced. $T_{RH}$ from Eq. (7) will be used to parametrize the inflaton-neutrino coupling. Only in models where $\Gamma_\phi \ll \Gamma_{N_1}$, does $T_{RH}$ correspond to the real physical reheating temperature since then the neutrino mass $M_1$ is much larger than $T_{RH}$ and, therefore, the right-handed neutrinos decay instantaneously after having been produced in inflaton decays. The resulting lepton asymmetry in such scenarios can easily be evaluated\cite{3, 4}. After reheating the baryon asymmetry, defined here as the ratio of bayon number to photon density, is given by:

$$\frac{n_B}{n_\gamma} = \alpha_{sph} \epsilon_{1} \frac{n_{N_1}}{n_\gamma} = \alpha_{sph} \epsilon_{1} \frac{T_{RH}}{30M_1} \simeq 10^{-8} \frac{T_{RH}}{M_1} \quad (9)$$

Here, we have set $\kappa = 1$ since washout processes are completely negligible in this case. Furthermore, we have assumed that the energy density of the heavy neutrino is instantaneously converted into relativistic degrees of freedom, yielding a temperature for the thermal bath $\rho_R = \rho_{N_1} = (\pi^2/30)g_\ast T^4$. Again demanding that the baryon asymmetry is at least equal to the observed value one gets a constraint on the reheating temperature

$$T_{RH} \geq 10^{-2} M_1, \quad (10)$$
which corresponds to a lower limit on the inflaton-neutrino coupling $\gamma$ through Eq. (7).

4 The Boltzmann equations

In this work we shall consider a more general range of parameters. In particular we shall discuss in detail the case when the reheating temperature is of the same order as the heavy neutrino mass. Then, the simple approximation discussed above does not hold anymore, and the asymmetry has to be computed by solving a system of Boltzmann equations. We shall study in detail the dependence of the final efficiency factor $\kappa_f$ on the inflaton-neutrino coupling, again parametrized by the reheating temperature $T_{RH}$. We will see that in this model there is a strong correlation between the reheating temperature and the neutrino mass via the decay widths $\Gamma_{Nrf}$ and $\Gamma_\Phi$.

The relevant Boltzmann equations for the energy densities of the inflaton, the lightest of the heavy right-handed neutrinos, the $B-L$ asymmetry and the radiation energy density, respectively, read as follows:

\[
\begin{align*}
\dot{\rho}_\Phi &= -3H \rho_\Phi - \Gamma_\Phi \rho_\Phi \\
\dot{\rho}_N &= -3H \rho_N + \Gamma_\Phi \rho_\Phi - \Gamma_N (\rho_N - \rho_N^{eq}) \\
\dot{n}_{B-L} &= -3H n_{B-L} - \epsilon \Gamma_N (n_N - n_N^{eq}) - \Gamma_{ID} n_{B-L} \\
\dot{\rho}_R &= -4H \rho_R + \Gamma_N (\rho_N - \rho_N^{eq}) 
\end{align*}
\]

Here we consider only decays and inverse decays and neglect scattering processes of the right-handed neutrinos and the inflaton. Further, in the Boltzmann equation for $\rho_N$ we assumed that the right-handed neutrinos are non-relativistic. Surely this is an assumption which is not guaranteed and, depending on the mass of the inflaton, the produced heavy neutrinos can have energies much larger than their rest mass. But, this approximation works well for the computation of $\kappa_f$, which is our main purpose. The reason is that for all values of $K$ the final asymmetry is determined by right-handed neutrino decays when they are fully non-relativistic. Therefore the ultra-relativistic stage is not important for the final efficiency factor. An exact treatment would be relevant for the description of the evolution of the universe in the interval between inflation and the decay of the right-handed neutrinos as well as for the computation of the maximal value of the temperature, $T_{max}$, after inflation. As is shown in [23], $T_{max}$ can be much larger than the reheating temperature. However, this goes beyond the scope of this investigation.

In the actual numerical integration of these equations it is useful to use quantities in which the expansion of the universe has been scaled out. The relevant variables as well as the transformed Boltzmann equations and some numerical parameters are discussed in Appendix A. For definiteness we will always assume a

\[\text{Note that we are neglecting an inverse decay term } \sim \Gamma_\Phi \rho_\Phi^{eq} \text{ since the reheating temperature is assumed to be much smaller than the inflaton mass and hence the inflaton never comes into thermal equilibrium.}\]
neutrino mass $M_1 = 10^9$ GeV and an inflaton mass $M_\Phi = 10^{13}$ GeV and only vary the reheating temperature, i.e. the inflaton-neutrino coupling, and $K$ in the following.

5 Results and Discussion

In this section we shall present our results for the final efficiency factor $\kappa_f$, as shown in Fig. 1. In particular, we will discuss the dependence on the reheating temperature $T_{RH}$, i.e. the inflaton-neutrino coupling, and $K$.

5.1 General Observations

In Fig. 1 we have plotted our results for the final efficiency factor as a function of $K$ for various values of the reheating temperature as well as the standard results obtained in thermal leptogenesis \cite{17}. As one can see, for $T_{RH} \geq 5 \times 10^5$ GeV our results are in good agreement with the ones obtained in thermal leptogenesis in the strong washout regime, as one would naively expect. On the other hand, in the weak washout regime $\kappa_f$ is enhanced by up to two orders of magnitude. Further, the curves for $T_{RH} \geq 10^9$ GeV are in agreement with the results obtained in \cite{12}, where values $T_{RH} \leq M_1$ were not considered.

In the strong washout regime, i.e. for $K > 1$, the neutrino Yukawa coupling is large enough to keep the system close to equilibrium, thereby erasing any dependence on the initial conditions, as long as the physical reheating temperature is of order of the neutrino mass or larger, so that the right-handed neutrinos still can thermalize.
For reheating temperatures above $T_{RH} = 10^9$ GeV $\kappa_f$ for dominant initial $N_1$ abundance is in good agreement with those for zero and thermal initial abundance in the whole strong washout regime for $K \gtrsim 5$. When the reheating temperature becomes smaller than the neutrino mass, e.g. for $T_{RH} = 5 \times 10^8$ GeV one sees an agreement only for very strong washout, $K \gtrsim 40$. For $T_{RH} = 3.75 \times 10^8$ GeV, the washout effects are so weak that the final efficiency factor is larger than the one for thermal and zero initial $N_1$ abundance for all values of $K$.

In the weak washout regime, on the other hand, one can see that $\kappa_f$ for dominant initial $N_1$ abundance is quite independent of $T_{RH}$ and much larger than one, which is by definition the largest value for thermal initial $N_1$ abundance. This is due to the direct production of neutrinos via inflaton decays which leads to neutrino abundances larger than the equilibrium value.

### 5.2 Weak Washout Regime

In the weak washout regime, i.e. $K < 1$, $\kappa_f$ is much larger than in thermal leptogenesis, by a factor $\sim 10 - 100$ for all considered reheating temperatures and is almost independent of the reheating temperature. The physical reheating temperature, given by $T_{RH}^N = M_1 \sqrt{K}$, is smaller than the right-handed neutrino mass in the whole weak washout regime. Hence, the neutrinos decay strongly out of equilibrium.

The maximum value of $\kappa_f$ is reached at $K \sim 0.4$ and is almost independent of $T_{RH}$. This is due to the fact that the entropy produced in each neutrino decay, $\Delta S \sim M_1/T_{RH}^N$, corresponding to an increase of the number of photons, $\Delta N_\gamma \propto \Delta S$, becomes larger at small $K$, since the neutrinos decay later and hence carry a greater fraction of the energy density of the universe when they decay.

In figure 2a one can see that for $T_{RH} = 10^9$ GeV and $K = 10^{-2}$ the inflaton starts decaying when the scale factor $y$, defined in Appendix A, reaches $\sim 10^9$ while at $y \sim 10^{10}$ it has completely decayed. Figure 2b shows that when the inflaton starts decaying at $y = 10^9$ the energy density of the heavy neutrinos becomes constant and dominates the energy density of the universe. At $y \sim 5 \times 10^{10}$ the heavy neutrinos start decaying and the associated entropy release triggers a transition from a matter dominated to a radiation dominated universe as one can deduce from figure 2c where the rescaled radiation energy density becomes constant. The evolution of the efficiency factor is plotted in figure 2d. It remains constant at a value $\kappa \sim 80$ as long as the universe is (matter) dominated by the inflaton and gets reduced when the universe is $N_1$ (matter) dominated. After the neutrinos have decayed the efficiency factor reaches its final value of about $\kappa \sim 30$. In figures 2e and 2f we show the evolution of the temperature. The difference in the two plots is that in figure 2e we have taken into account only the contribution from radiation,

$$T = \left[ \frac{30 \rho_R}{\pi^2 g^*} \right]^{\frac{1}{4}}.$$  

This gives the temperature as long as the heavy neutrinos, produced in the inflaton
decays, are non-relativistic. When $M_\Phi \gg M_1$ the produced right-handed neutrinos are relativistic particles and their energy density contributes to the energy density of radiation that determines the temperature. This effect is included in figure 2f. Here, we have assumed that all the produced heavy neutrinos are relativistic particles and defined the temperature as:

$$T = \left[ \frac{30(\rho_R + \rho_N)}{\pi^2 g^*} \right]^{\frac{1}{4}}. \quad (13)$$

Figure 2: The evolution of $E_{\Phi}/E_{\Phi_1}, E_N/E_{\Phi_1}, R, \kappa, T$ and $T_{\text{tot}}$ is shown for $T_{RH} = 10^9$ GeV at $K = 10^{-2}$.
This, of course, is a rather rough approximation, since in the Boltzmann equations, Eqs. (11), the right-handed neutrinos are treated as non-relativistic particles. However, this shows that, cf. figure 2f, the maximal temperature achieved in the reheating process is much larger than the physical reheating temperature\(^\text{[23]}\), which in this case is \(T_{\text{RH}}^N \sim 7 \times 10^7\) GeV, as can be read off directly from figure 2c.

5.3 Strong Washout Regime

In the strong washout regime, i.e. \(K > 1\), the final efficiency factor \(\kappa_f\) is in perfect agreement with the results obtained in thermal leptogenesis, as long as \(T_{\text{RH}} \gtrsim M_1\). This is what one would expect, since all reactions involving \(N_1\) are in thermal equilibrium, hence the neutrinos rapidly thermalize and any information about the initial conditions is quickly lost. For reheating temperatures smaller than \(M_1\) this is not necessarily the case anymore, e.g. for \(T_{\text{RH}} = 5 \times 10^8\) GeV and \(K \lesssim 50\), the reactions involving \(N_1\) are not strong enough to bring them into thermal equilibrium, i.e. the neutrinos decay rather strongly out of equilibrium. Hence, the final efficiency factor is enhanced compared to thermal leptogenesis since washout processes are suppressed.

As an example, let us again consider the case \(T_{\text{RH}} = 10^9\) GeV but now with \(K = 500\) in some detail. As one can see in figure 3a, the inflaton again starts to decay at \(y \sim 10^9\) and has decayed completely at about \(y \sim 10^{10}\). The rescaled right-handed neutrino energy density, cf. figure 3b, becomes constant already at \(y \sim 3 \times 10^7\) and, due to the larger value of \(K\), the \(N_1\) start to decay much earlier than in the weak washout, already at \(y \sim 10^9\). Because of inverse decay processes, which in thermal equilibrium balance the decay processes, the decrease of the neutrino abundance is much slower than in the previous example. The neutrinos have fully decayed at \(y = 5 \times 10^{10}\), i.e. somewhat later than in the weak washout regime. The transition to a radiation dominated universe occurs at \(y \sim 10^9\) as can be seen in figure 3c. At this point the rescaled radiation energy density \(R/E_\Phi\) becomes constant. The stronger interactions of the heavy right-handed neutrino with the SM particles have a strong impact on the efficiency factor. It rises quickly to \(\sim 160\) and remains constant for a long time. When the energy of the right-handed neutrino becomes constant at \(y \sim 3 \times 10^7\) the efficiency factor decreases. This decrease even accelerates when the neutrinos start decaying, and once the neutrinos have entirely decayed away, the final efficiency factor reads \(\kappa_f \sim 4 \times 10^{-3}\). In figure 3c one can see that the temperature of the thermal bath of standard model particles rises quickly and then remains constant at \(T \sim 3 \times 10^9\). When \(E_N/E_\Phi\) becomes constant the temperature starts to decrease slowly and becomes inversely proportional to the scale factor once the universe is radiation dominated. The physical reheating temperature is now \(T_{\text{RH}}^N \sim 10^9\) GeV, approximately one order of magnitude larger than in the weak washout example we had considered previously.
Figure 3: The evolution of $E_\Phi/E_{\Phi_1}$, $E_N/E_{\Phi_1}$, $R$, $\kappa$, $T$ and $T_{\text{tot}}$ is shown for $T_{RH} = 10^9$ GeV at $K = 500$.

### 5.4 Behaviour for $T_{RH} \ll M_1$

The behaviour of the final efficiency factor for reheating temperatures $T_{RH} \ll M_1$ is somewhat different than for the values of $T_{RH}$ discussed above. As an example, let us discuss the case $T_{RH} \sim 10^8$ GeV.

As we can see in figure 3, $\kappa_f$ is now almost independent of $K$ and of order 10. Only in the limit $K \to 0$ does one obtain the same value for $\kappa_f$ as for $T_{RH} \gtrsim M_1$. For larger values of $K$ we see that washout is now completely negligible and that $\kappa_f$ remains almost constant even for very large $K$, where one only observes a small...
Figure 4: $\kappa_f$ for $M_1 = 10^9$ GeV (solid) and $M_1 = 10^7$ GeV (dashed)

decrease of $\kappa_f$. The effect of entropy production in $N_1$ decays is also somewhat weaker now. Indeed, for such a low reheating temperature the decay width of the inflaton is much smaller than the decay width of the lightest right-handed neutrino, $\Gamma_\Phi \ll \Gamma_N$. Hence, the $N_1$’s always decay strongly out-of-equilibrium and instantaneously after having been produced in inflaton decays. Therefore, the physical reheating temperature $T_{RH}^{N_1}$ becomes nearly independent of $K$ and is given directly by $T_{RH}$ since the time period of a neutrino dominated universe is negligibly short. For even lower reheating temperatures one expects neither an effect due to washout for $K > 1$ nor due to entropy production for $K < 1$ since the right-handed neutrino decay again follows instantaneously the inflaton decay. This is the scenario sketched at the beginning of section 3 which had been considered in the literature before.

5.5 Dependence of the Results on $M_\Phi$ and $M_1$

An obvious question is how the results presented so far depend on the masses of the inflaton, $M_\Phi$, and the right-handed neutrino, $M_1$. A change of the inflaton mass $M_\Phi$ can also be parametrized by a variation of the reheating temperature, cf. Eq. (7). Hence, for our purposes it is equivalent to a change in the inflaton-neutrino coupling $\gamma$, which we have already discussed above. The only limit is set by the kinematical lower bound on the inflaton mass, $M_\Phi > 2M_1$.

As an example for the dependence of the results on $M_1$, we have plotted the final efficiency factor $\kappa_f$ for $M_1 = 10^9$ GeV and $M_1 = 10^7$ GeV in fig. 4. As one can see, the differences between the two cases are negligible. This is due to the fact that the right-handed neutrinos always decay when they are non-relativistic. For $M_1 = 10^9$ GeV this happens instantaneously after the production in inflaton decays. For $M_1 = 10^7$ GeV, on the other hand, the $N_1$ abundance remains constant.
until the universe has cooled down to a temperature $\sim 10^7 \text{GeV}$. Then the non-relativistic neutrinos decay in the same way as for $M_1 = 10^9 \text{GeV}$. A variation of $M_1$ changes the point in time of the decay process and not the process itself and hence $\kappa_f$ is unchanged. This is completely analogous to the situation in thermal leptogenesis, where $\kappa_f$ is also independent of $M_1$, as long as $M_1 \lesssim 10^{13} \text{GeV}$.

### 5.6 Lower Bound on $T_{RH}^N$ and $M_1$

Finally, let us discuss the impact a dominant initial neutrino abundance has on the lower limits on the physical reheating temperature and the right-handed neutrino mass. Demanding successful leptogenesis in the standard thermal case leads to the lower limit \[ M_1 > M_1^{\text{min}}(K) \approx 6.4 \times 10^8 \text{GeV} \left( \frac{n_B^{\text{CMB}}}{6 \times 10^{10}} \right) \left( \frac{0.05 \text{eV}}{m_{\text{atm}}} \right) \kappa_f^{-1}(K). \] (14)

Hence, assuming a thermal initial abundance of right-handed neutrinos and in the limit $K \rightarrow 0$ the absolute lower limits on $M_1$ and the reheating temperature read

\[ T_{RH}, M_1 \gtrsim 4 \times 10^8 \text{GeV}. \] (15)

This was obtained at 3σ using $n_B^{\text{CMB}} = (6.3 \pm 0.3) \times 10^{-10}$ for the baryon asymmetry and $\Delta m^2_{\text{atm}} = (1.2 - 4.8) \times 10^{-3} \text{eV}^2$ for the mass square difference in atmospheric neutrino oscillations.

In the case of a zero initial neutrino abundance in thermal leptogenesis, the lower limits are reached at $K \simeq 1$ and read

\[ T_{RH}, M_1 \gtrsim 2 \times 10^9 \text{GeV}. \] (16)

In our case, the final efficiency factor in the limit $K \rightarrow 0$ is greatly enhanced, i.e. the lower limit $M_1^{\text{min}}$ gets relaxed accordingly. It will now not only depend on $K$, as in thermal leptogenesis, but also on the reheating temperature $T_{RH}$ which parametrizes the inflaton-neutrino coupling. The results are summarized in Fig. 5, where we have also shown the lower limits from thermal leptogenesis \[17\] for comparison.

A lower bound on $M_1$ again corresponds to a lower bound on the physical reheating temperature $(T_{RH}^N)^{\text{min}}$. Note that, since $(T_{RH}^N)^{\text{min}}(K) = M_1^{\text{min}}(K) \sqrt{K}$, the lowest value of $T_{RH}^N$ is achieved in the limit $\tilde{m}_1 \rightarrow 0$ where one obtains

\[ (T_{RH}^N)^{\text{min}} \sim 2.4 \times 10^6 \text{GeV}, \] (17)

which is two orders of magnitude lower than in the case of thermal leptogenesis.

The neutrino oscillation data favour the effective neutrino mass, i.e. $\tilde{m}_1 = K m_*$, to lie in the neutrino mass window $m_{\text{sol}} < \tilde{m}_1 < m_{\text{atm}}$, in the strong washout regime. In this range and for large reheating temperatures, $T_{RH} \gtrsim M_1$, we obtain the same lower limit as in thermal leptogenesis,

\[ (T_{RH}^N)^{\text{min}} \sim (4 \times 10^9 - 2 \times 10^{10}) \text{GeV}. \] (18)
Figure 5: The lower bound on $M_1$ is shown for thermal (short-dashed), zero (point-point-dashed) and dominant initial $N_1$ abundance for $T_{RH} = 10^9$ GeV (solid), $T_{RH} = 5 \times 10^8$ Gev (long-dashed), $T_{RH} = 3.75 \times 10^8$ GeV (point-dashed and $T_{RH} = 10^8$ GeV (point-dashed-dashed)).

For lower reheating temperatures, e.g. $T_{RH} = 5 \times 10^8$ GeV, one can see from figure 5, that the bound on $M_1$ is about one order of magnitude lower than in thermal leptogenesis, hence the lower limit on the physical reheating temperature now reads

$$ (T^N_{\text{RH}})_{\text{min}} \sim (4 \times 10^8 - 2 \times 10^9) \text{ GeV}. \quad (19) $$

Further lowering the reheating temperature, i.e. the inflaton-neutrino coupling, to $T_{RH} = 10^8$ GeV leads to an even weaker lower limit on the right-handed neutrino mass. As already discussed for the final efficiency factor, for such low reheating temperatures the results are almost independent of $K$. In the limit $K \to 0$ we recover for the physical reheating temperature the result obtained above, cf. Eq. (17).

In the more interesting strong washout regime favoured by neutrino oscillation data, the lower limit on the heavy neutrino mass reads,

$$ M_1 \gtrsim 4 \times 10^7 \text{ GeV} . \quad (20) $$

Correspondingly, the lower bound on the physical reheating temperature gets relaxed to

$$ (T^N_{\text{RH}})_{\text{min}} \sim 10^7 \text{ GeV} . \quad (21) $$

Hence, in the phenomenologically most interesting strong washout regime a dominant initial neutrino abundance produced in inflaton decays can be used to relax the rather stringent lower limit on the physical reheating temperature obtained in thermal leptogenesis by up to three orders of magnitude, provided the inflaton-neutrino coupling is small.
6 Conclusions

In this paper we have studied some aspects of non-thermal leptogenesis as an alternative to the standard thermal leptogenesis scenario. In particular, we investigated the interplay between inflation and leptogenesis by considering a decay chain where the inflaton first exclusively decays into heavy right-handed neutrinos which then decay into standard model lepton and Higgs doublets, thereby reheating the universe and creating the baryon asymmetry of the universe.

We have performed a full numerical study by means of a set of Boltzmann equations and have discussed the dependence of the final efficiency factor, corresponding to the maximal baryon asymmetry which can be produced, on the inflaton-neutrino coupling and the heavy neutrino Yukawa coupling. To that end, we have parametrized the inflaton-neutrino coupling in terms of the reheating temperature $T_{RH}$ defined in the standard way, which, however, should not be confused with the physical reheating temperature, since in our scenario the universe becomes radiation dominated once the heavy neutrinos and not the inflaton have decayed.

We have mainly discussed values of $T_{RH} \sim M_1$. This is in contrast to most scenarios considered before in the literature where $M_1 \gg T_{RH}$ is usually assumed. For those values the final efficiency factor is enlarged by a factor $\sim 10 - 100$ in the weak washout regime compared to the one obtained in thermal leptogenesis. In the strong washout regime, on the other hand, the final efficiency factor that one gets in thermal leptogenesis is reproduced, if $T_{RH} \gtrsim M_1$. Furthermore, we have seen that for $T_{RH} \ll M_1$ the right-handed neutrinos decay completely out-of-equilibrium and hence the final efficiency factor is almost independent of $K$, which parametrizes the neutrino Yukawa coupling. For such reheating temperatures the final efficiency factor is a factor $\sim 10$ larger than in thermal leptogenesis.

Increasing the efficiency of leptogenesis is particularly interesting in light of the rather stringent upper limits on the reheating temperature of $10^6 - 7$ GeV obtained in certain supersymmetric scenarios. Indeed, such an upper limit is in conflict with the lower limit on the reheating temperature of $4 \times 10^8$ GeV obtained in thermal leptogenesis.

Here, we could show that in the weak washout regime reheating temperatures as low as $\sim 10^6$ GeV are permissible in our non-thermal scenario, independently of the neutrino-inflaton coupling. In the phenomenologically more interesting neutrino mass window in the strong washout regime, reheating temperatures as low as $\sim 10^7$ GeV still allow for successful leptogenesis, as long as $T_{RH} \ll M_1$, i.e. as long as the neutrino-inflaton coupling is small.

Acknowledgments

We would like to thank J. Pradler and G. Raffelt for useful discussions and suggestions. Further, we are indebted to P. Di Bari for collaboration during early stages of this work as well as useful comments on the manuscript.
A Variable Transformation

When solving the Boltzmann equations it is convenient to use variables in which the expansion of the universe has been scaled out. In analogy to the procedure presented in [23], we shall use the following variables:

\[
\begin{align*}
E_{\Phi} &= \rho_{\Phi} a^3, \\
E_N &= \rho_N a^3, \\
\tilde{N}_{B-L} &= n_{B-L} a^3, \\
R &= \rho_R a^4,
\end{align*}
\]

(22)

where \(a\) is the scale factor of the universe. Moreover, it is convenient to write the Boltzmann equations as functions of the scale factor rather than time. More precisely, we shall use the ratio of the scale factor to its initial value,

\[
y = \frac{a}{a_I},
\]

(23)
as time variable. For definiteness, we shall use \(a_I = 1\). Then the expansion rate reads:

\[
H = \sqrt{\frac{8\pi(a_I E_{\Phi} y + a_I E_N y + R)}{3M_p^2 a_I^3 y^4}}.
\]

(24)

Further, instead of the temperature \(T\) we use the inverse temperature in units of the heavy neutrino mass,

\[
z = \frac{M_1}{T} = M_1 a_I \left[ \frac{\pi^2 g^*}{30R} \right]^{\frac{1}{4}} y.
\]

(25)

Then, the rescaled equilibrium energy density of \(N\) can be expressed as:

\[
E_N^{eq} = \rho_N^{eq} a^3 = \rho_N^{eq} a_I^3 y^3 = \frac{a^3 M_1^4 y^3}{\pi^2} \left[ \frac{3}{z^2} K_2(z) + \frac{1}{z} K_1(z) \right].
\]

(26)

In terms of these rescaled variables the Boltzmann equations, cf. Eqs. (11), are given by:

\[
\begin{align*}
\frac{dE_{\Phi}}{dy} &= \frac{\Gamma_{\Phi} E_{\Phi}}{H y}, \\
\frac{dE_N}{dy} &= \frac{\Gamma_{\Phi} E_{\Phi}}{H y} - \frac{\Gamma_N}{H y} (E_N - E_N^{eq}), \\
\frac{d\tilde{N}_{B-L}}{dy} &= -\frac{\Gamma_N}{H y} \left[ \epsilon_1 \left( \tilde{N} - \tilde{N}^{eq} \right) + \frac{n_{B-L}^{eq}}{n_{B-L}} \tilde{N}_{B-L} \right], \\
\frac{dR}{dy} &= \frac{\Gamma_N a_I}{H} (E_N - E_N^{eq}).
\end{align*}
\]

(27)

\footnote{Note that \(\tilde{N}_{B-L}\) is the particle density per comoving volume element for the asymmetry.}
As already mentioned in the main text, we fix the mass of right-handed neutrino and inflaton to $10^9$ GeV and $10^{13}$ GeV, respectively. The reheating temperature $T_{RH}$ is used to parametrize the inflaton-neutrino coupling, and $K$ parametrizes the Yukawa coupling of the heavy nuetrons.

For the initial energy density of the inflaton or the universe’s energy density we have from the condition $\Gamma_\phi = H(a_I)$:

$$\rho_I = \frac{3}{8\pi} M_\phi^2 M_{Pl}^2 .$$

(28)

Note that $N_{B-L}$ used in [17] is related to $\tilde{N}_{B-L}$, defined in Eq. (27), by the following relation:

$$N_{B-L} = \frac{n_{B-L}}{n_\gamma} = \left[ \frac{\pi^4}{30\zeta(3)} \right] \frac{n_{B-L}}{\rho_\gamma} T = \left[ \frac{\pi^4 g^* T^3}{30^3 \zeta(3)} \right] R^{-\frac{4}{3}} \tilde{N}_{B-L} ,$$

(29)

Defining the final efficiency factor as [17]

$$\kappa_f = -\frac{4}{3} \epsilon^{-1} \tilde{N}_{B-L} ,$$

(30)

we can compare the results for dominant initial $N_1$ abundance with those obtained for thermal and zero initial abundance in previous calculations [17].

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