Synthesizing quasi-bound states in the continuum in epsilon-near-zero layered materials

ABSTRACT

Bound states in the continuum (BIC) are highly confined, nonradiative modes that can exist in open structures, despite their potential compatibility and coupling with the radiation spectrum, and may give rise to resonances with arbitrary large lifetimes. Here, we study this phenomenon in layered materials featuring epsilon-near-zero constituents. Specifically, we outline a systematic procedure to synthesize quasi-BIC resonances at a given frequency, incidence angle, and polarization and investigate the role of certain critical parameters in establishing the quality factor of the resonances. Moreover, we also provide an insightful phenomenological interpretation in terms of the recently introduced concept of "photonic doping" and study the effects of the unavoidable material loss and dispersion. Our results indicate the possibility of synthesizing sharp resonances, for both transversely magnetic and electric polarizations, which are of potential interest for a variety of nanophotonics scenarios, including light trapping, optical sensing, and thermal radiation.

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Here, with focus on symmetric ENZ tri-layers, we revisit the above results and derive a systematic synthesis procedure for BIC-type high-Q resonances at a given frequency and angle of interest for both transversely magnetic (TM) and electric (TE) polarizations. Moreover, we carry out a series of parametric studies to illustrate the effects of certain relevant parameters. For the TM polarization, we also provide an interesting phenomenological interpretation based on the recently introduced concept of "photonic doping." Finally, we explore the effects of realistic (lossy, dispersive) implementations.

Referring to the two-dimensional (2D) geometry in Fig. 1 with all fields and quantities independent of y, we consider a symmetric structure made of three layers, stacked along the z-direction and of infinite extent in the x-y plane. Specifically, we assume the structure embedded in vacuum and identify a cladding region (the two exterior layers of thickness d1 and relative permittivity ε1) and a core region (the inner layer of thickness d2 and relative permittivity ε2). We assume that the materials are non-magnetic (i.e., relative permeability µ = 1) and, for now, neglect material dispersion and losses. Moreover, we consider a time-harmonic plane wave illumination with suppressed exp(−iωt) time dependence, angle of incidence θ, and TM (y-directed magnetic field) or TE (y-directed electric field) polarization. Accordingly, the relevant components of the incident wavevector can be expressed as

\[ k_x = k \sin \theta, \quad k_z = k \cos \theta, \]

(1)
expression in Eq. (6), we obtain a quartic equation in the unknown

$$t_2 = \frac{2\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4 t_1 (\varepsilon_1^2 k_{a1}^2 - \varepsilon_2^2 k_{a2}^2) + t_2^2 \varepsilon_1^2 \varepsilon_2^2 (k_{a1}^2 - \varepsilon_3^2 k_{a2}^2) + t_2^2 \varepsilon_1^2 (\varepsilon_1^2 k_{a2}^2 - \varepsilon_2^2 k_{a1}^2)}{\varepsilon_1^2 k_{a1}^2 (k_{a1}^2 - \varepsilon_2^2 k_{a2}^2) + \varepsilon_1^2 (\varepsilon_1^2 k_{a2}^2 - \varepsilon_2^2 k_{a1}^2)}.$$  

(2)

where

$$k_{a1,2} = k\sqrt{\varepsilon_{a1,2} - \sin^2 \theta}, \quad \text{Im}(k_{a1,2}) \geq 0$$  

(3)

are the longitudinal wavenumbers in the two materials, and for notational compactness, we have defined

$$t_{1,2} = \tan(k_{a1,2}d_{a1,2}), \quad a_{1,2} = \varepsilon_1 k_{a1} (i k_{a1}^2 + \varepsilon_2 k_{a2}) + \varepsilon_2 k_{a2} t_1 + i\varepsilon_2 k_{a1} t_1.$$  

(4)

In the ideal case of $\varepsilon_1 = 0$, it readily follows from Eq. (2) that the condition $t_2 = 0$ (corresponding to a Fabry–Pérot resonance in the core) trivially yields the coalescence of a zero and a pole, i.e., a perfect BIC, irrespective of cladding thickness $d_1$. This is consistent with results from previous studies. For finite values of the relative permittivities, by enforcing the zeroing of the numerator in Eq. (2), we obtain

$$t_2 = \frac{2\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4 t_1 (\varepsilon_1^2 k_{a1}^2 - \varepsilon_2^2 k_{a2}^2)}{\varepsilon_1^2 k_{a1}^2 (k_{a1}^2 - \varepsilon_2^2 k_{a2}^2) + \varepsilon_1^2 (\varepsilon_1^2 k_{a2}^2 - \varepsilon_2^2 k_{a1}^2)}.$$  

(5)

Then, by enforcing the pole condition in Eq. (2) and substituting the expression in Eq. (6), we obtain a quartic equation in the unknown $t_1$, which, recalling the branch-cut choice in Eq. (3), yields the following two classes of solutions:

$$t_1 = i, \quad t_2 = \frac{-2i\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4 k_{a2}}{\varepsilon_1^2 k_{a1}^2 + \varepsilon_2^2 k_{a2}^2}.$$  

(7a)

$$t_1 = -i\varepsilon_1 k_{a1} (\varepsilon_2 k_{a2} + k_{a1}) \quad \text{and} \quad t_2 = i.$$  

(7b)

By inspecting the first class of solutions in Eqs. (7a) and recalling the definitions in Eq. (4), we notice that, if the field is evanescent in the cladding region (i.e., $\varepsilon_1 < \sin^2 \theta$) and propagating in the core (i.e., $\varepsilon_2 > \sin^2 \theta$), the second equality can be, in principle, satisfied for real-valued frequencies. Hence, in order to work with arbitrary small incidence angles, we are led to select an ENZ-type cladding ($\varepsilon_1 \ll 1$), whereas the core material can be a regular dielectric. However, it is apparent that the first equality can never be satisfied exactly (since, for real-valued argument, the hyperbolic tangent is always smaller than one in magnitude), but only asymptotically (for $d_1 \to \infty$). This indicates that, for finite (albeit small) values of the relative permittivities, it is not possible to attain a perfect BIC. Acknowledging this limitation, we focus on suitably approximate solutions (with finite, but arbitrary large $Q$), which will be referred to as “quasi-BIC” (qBIC), which satisfy exactly the zero condition in Eq. (6).

To summarize, for a given operational frequency, incidence angle, material properties, and cladding thickness $d_1$, the relationship in Eq. (6) yields the required value of the core thickness $d_2$ to attain a qBIC condition with the $Q$-factor controlled by $d_1$ and $\varepsilon_1$. This enables a systematic synthesis of qBIC-type responses.

As for the second class of solutions in Eq. (7b), the role of the core and cladding is reversed. Once again, the second equality can be satisfied only asymptotically (for $d_2 \to \infty$), provided that the field is evanescent in the core region (i.e., $\varepsilon_2 < \sin^2 \theta$). However, unlike the previous case, the right-hand-side in the first equality is generally complex-valued, which implies that no solutions exist for real-valued frequencies. Therefore, we will not consider this last class of solutions and focus instead on that in Eq. (7a).
We now move on to considering the TE polarization, whose interaction with ENZ media is known to be weaker. In this case, the reflection coefficient for the structure in Fig. 1 (for incidence from either side) can be written as

\[ R_{TE} = \frac{2k_{z1}k_{z2}t_1(k_{z1}^2 - k_{z2}^2) + t_2\left[k_{z1}^2\left(k_{z1}^2 - k_{z2}^2\right) + t_1^2\left(k_{z2}^2 - k_{z1}^2\right)\right]}{t_1b_2 - 2k_{z1}k_{z2}(k_{z1}t_1 + ik_z)(k_{z1} - ik_z)t_1}, \]  

where

\[ b_{1,2} = k_{z1}(ik_{z2} \pm k_z) + t_1(k_zk_{z2} \pm ik_{z1}), \]

and all other symbols have already been defined previously. We note that, unlike the TM case, no perfect BIC solutions exist, even in the ideal case \( e_1 = 0 \). Ideally, in order to enhance the interaction with the TE polarization, and possibly attain a perfect BIC, we should consider (instead, or in addition) a dual, \( \mu\text{-near-zero} \) condition.

In the ENZ scenario of interest here, paralleling the reasoning above, we find two classes of solutions

\[ t_1 = i, \quad t_2 = -\frac{2ik_{z1}k_{z2}}{k_{z1}^2 + k_{z2}^2}, \]  

\[ t_1 = \frac{ik_{z1}(k_z + k_{z2})}{k_{z1}^2 + k_zk_{z2}}, \quad t_2 = i, \]  

for which qualitatively similar considerations hold as for the TM case above. Specifically, the first class in Eq. (10a) leads to a structure with an ENZ cladding. Also in this case, we look for approximate solutions by enforcing the zero condition only, viz.,

\[ t_2 = \frac{2k_{z1}k_{z2}t_1(k_{z1}^2 - k_z^2)}{k_{z1}^2(k_{z1}^2 - k_{z2}^2) + t_1^2(k_{z2}^2 - k_{z1}^2)}, \]

from which we can derive the core thickness \( d_2 \), while the cladding thickness \( d_1 \) controls the Q-factor. Even though, as previously mentioned, a perfect BIC is not attainable here, we still refer to this class of solutions as qBIC. Once again, the second class of solutions in Eq. (10b) cannot be generally satisfied for real-valued frequencies and is not pursued here.

Figure 2 shows some representative results for the synthesis of a qBIC at a given angular frequency \( \omega_0 \) and incidence angle \( \theta = 15^\circ \), by

![Diagram](image-url)
assuming $\varepsilon_1 = 0.005$ and $\varepsilon_2 = 2$. Specifically, for the TM polarization, we select $d_1 = \lambda_0$ (with $\lambda_0$ denoting the wavelength at the design frequency) and derive from Eq. (6) $d_2 = 0.356\lambda_0$. Figure 2(a) shows the corresponding reflection-coefficient magnitude as a function of the normalized frequency and the incidence angle, from which we observe a strong field enhancement in the core region with a typical Fabry–Pérot pattern, no apparent reflections, and full transmission; this is quite different from the response at a nearby frequency (also shown as a reference), from which a strong reflection and very low transmission are instead observed. Figures 2(c) and 2(d) show the corresponding results for the TE polarization. Here, assuming $d_1 = 1.5\lambda_0$, we obtain from Eq. (11) $d_2 = 0.401\lambda_0$. Once again, a qBIC signature is observed, though with a broader linewidth and weaker field enhancement, these characteristics are consistent with what also observed in ENZ slabs.

For a more quantitative assessment, we carry out a parametric study of the Q-factor, as a function of the thickness and relative permittivity of the cladding region. With this aim, for each combination of $d_1$ and $\varepsilon_1$, we re-calculate $d_2$ from Eqs. (6) and (11) and numerically compute the poles of the reflection coefficients in Eqs. (2) and (8). In view of passivity, this yields a complex-valued angular resonant frequency $\omega_0 = \omega_0^r - i\omega_0^i$ (with $\omega_0^i \geq 0$), from which we estimate

$$Q = \frac{\omega_0^r}{2\omega_0^i}.$$  

(6)

Figures 3(a) and 3(b) show, for both polarizations, a typical pole evolution with the complex angular frequency asymptotically approaching $\omega_0 = \omega_0^r$ (where the zero is placed by design) for increasing values of the cladding electrical thickness $d_1/\lambda_0$. Figures 3(c) and 3(d) show the corresponding Q-factors for three representative values of $\varepsilon_1$ in the ENZ regime. In both cases, we observe that the Q-factor increases exponentially with $d_1/\lambda_0$. In particular, for the TM polarization [Fig. 3(c)], very high Q-factors ($>1000$) can be attained even for relatively small cladding thicknesses ($d_1 \sim 0.5\lambda_0$). Moreover, as can be expected, reducing $\varepsilon_1$ (thereby approaching the perfect BIC condition) yields beneficial effects. On the other hand, for the TE polarization [Fig. 3(d)], the Q-factors are sensibly lowering, reaching values $\geq 100$ for relatively thick claddings ($d_1 \geq 2\lambda_0$), and the dependence on $\varepsilon_1$ is much weaker.

For the TM polarization, the above results also admit an intriguing interpretation in terms of the “photonic doping” concept that was recently put forward by Liberal et al.19 In essence, they showed that, for a 2D ENZ host medium and TM polarization, the presence of an arbitrary placed dielectric particle can dramatically change the optical response in a manner that resembles the concept of “doping” in solid-state physics. Quite interestingly, for an exterior observer, such response is equivalent to an effective magnetic permeability, which can be broad band tuned. Within this framework, the core region in our configuration (Fig. 1) can be interpreted as the “dopant,” and the analytic framework introduced in Ref. 19 can be straightforwardly extended to deal with regions of infinite extent. As a result, assuming an ENZ cladding, for an exterior observer, the tri-layer can be effectively replaced by a homogeneous slab of thickness $2d_1 + d_2$, the same ENZ relative permittivity as the cladding, and effective relative magnetic permeability

$$\mu_e = \frac{2\varepsilon_k^2 d_1 + \tan \left(\frac{k_{z2} d_2}{2}\right)}{k_{z2} (2d_1 + d_2)}.$$  

(12)

Accordingly, the reflection coefficient for the effective structure can be written as

$$R_{TM} = \frac{(\varepsilon_k^2 k_{z2}^2 - k_e^2)\tan \gamma}{2\varepsilon_k^2 k_{z2}^2 + (\varepsilon_k^2 k_e^2 + k_e^2)\tan^2 \gamma},$$  

(13)

where $k_e = k\sqrt{\varepsilon_1/\varepsilon_k - \sin^2 \theta_1} \tan \theta_1$, $\varepsilon_k \geq \varepsilon_2$, and $\tan \gamma = \tan \left(\gamma_{TM}(2d_1 + d_2)\right)$. Similar to what observed in Ref. 19, in view of the presence of the tangent term, the effective relative magnetic permeability in Eq. (12) can exhibit arbitrary values (either positive or negative) and, therefore, constitutes a key tuning parameter to design a qBIC resonance. Figure 4(a) shows a representative comparison between the responses of the actual and effective structures, from which we observe only a very slight frequency shift in the qBIC resonance, attributable to the finite (albeit very small) value of the cladding relative permittivity. Figure 4(b) shows instead the corresponding effective relative magnetic permeability, from which we observe the presence of a pole singularity nearby the qBIC frequency.

Finally, we take into account the isofar reflected effects of material dispersion and losses, which are unavoidable, especially in ENZ materials.24 In previous related studies, we consider 4H-SiC as a realistic low-loss ENZ material, characterized by a dispersion law

$$\varepsilon_{4H-SiC} = \varepsilon_\infty + \frac{1}{1 + \frac{(\varepsilon_{TO}^0/\varepsilon_\infty - \varepsilon_{TO}^0)^2}{\varepsilon_{TO}^0/\varepsilon_\infty + \gamma_\infty}},$$  

(14)

where $\varepsilon_\infty$ denotes the asymptotic (high-frequency) limit, $\gamma$ is a damping coefficient, and $\omega_{TO}^0$ are the longitudinal and transverse, respectively, optic phonon frequencies. Figure 5(a) shows a
representative dispersion law within a region of the thermal infrared spectrum with typical parameters taken from the literature. As can be observed, the ENZ condition is reached around 29 THz with an imaginary part \( \text{Im}(\varepsilon_{4\text{GHz-SiC}}) = 0.028 \).

For these parameters, we carry out the above described syntheses for a qBIC resonance at 29.05 THz and \( \theta = 15^\circ \), for both polarizations, assuming \( \varepsilon_2 = 2 \) and \( d_1 = 0.5x_0 = 5.163 \mu m \). Clearly, in view of the assumed losses, Eqs. (6) and (11) now yield complex values of the core thickness \( d_2 \) and simply discarding the imaginary part usually leads to poor-quality resonances. However, we found that re-adjusting the real part, by scanning a \( \pm 10\% \) neighborhood, is generally sufficient to bring the Q-factor to satisfactory levels. Figure 5(b) shows two synthesized reflection responses with Q-factors of 509 and 231 for the TM and TE polarizations, respectively. As shown in Fig. 5(c), at the designed qBIC angle and frequencies, we also attain sensible and sharp absorption peaks.

To sum up, with focus on symmetric ENZ tri-layers, we have presented a systematic synthesis approach for qBIC-type high-Q resonances at a given frequency, incidence angle, and polarization. Via parametric studies, we have highlighted the role of the critical parameters and, for the TM polarization, we have provided an insightful phenomenological interpretation in terms of photonic doping. Our results indicate that, even in the presence of realistic lossy materials, it is possible to attain sharp resonances (Q-factor of several hundreds) with sensible absorption peaks, for both polarizations. Overall, these outcomes appear of potential interest for applications including light trapping, optical sensing, and narrowband/directive thermal emission. Among the possible extensions that are under way, it is worth mentioning the incorporation of gain constituents in order to balance the loss effects, along the lines of previous studies on parity-time symmetric configurations. Within this framework, also of great interest is the study of cylindrical geometries, also taking into account the recent non-Hermitian extension of the photonic-doping concept.

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**AUTHOR DECLARATIONS**

**Conflict of Interest**

The authors have no conflicts to disclose.

**DATA AVAILABILITY**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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