Renormalization group coupling flow of SU(3) gauge theory

QCDTARO Collaboration

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We present our new results on the renormalization group coupling flow obtained in 3 dimensional coupling space ($\beta_{11}, \beta_{12}, \beta_{\text{twist}}$). The value of $\beta_{\text{twist}}$ turns out to be small and the coupling flow projected on ($\beta_{11}, \beta_{12}$) plane is very similar with the previous result obtained in the 2 dimensional coupling space.

1. Introduction

Recently a lot of studies are devoted to search for improved actions which have much less lattice artifacts than the original Wilson action. Two approaches are mainly pursued to construct the improved actions, i.e., Wilson’s renormalization group (RG) method\cite{1} and Symanzik’s perturbation one\cite{2}. The RG approach has been further developed by P. Hasenfraz, Niedermayer\cite{3} and co-workers\cite{4}, which leads to the classical perfect action. Symanzik approach has become very promising in lattice simulations with the idea of the tadpole improvement\cite{5}.

The classical perfect action is an approximation to the quantum perfect action which is completely free from lattice artifacts. Although finding quantum perfect actions is a hard task, we have been trying to obtain them by using the Monte Carlo renormalization group (MCRG) method\cite{6}. In MCRG, we first block lattice configurations and then determine renormalized coupling constants which show the coupling flow under a certain blocking scheme.

Last year we obtained coupling flow in 2 dimensional space\cite{7}. To control truncation effects we continue our study by adding more coupling constants.

2. Action

The action we use here is written as

$$S = \sum_i \beta_i \sum ReTr(1 - \frac{1}{3} U_i),$$

where $i$ stands for the type of Wilson loops summarized in Table 1.

Although we consider actions up to 7 coupling constants in determining the coupling flow, here we mainly focus on a 3 dimensional coupling...
| \(i\) (Type of Wilson loop) | Path \((\nu \neq \mu \neq \rho \neq \gamma)\) |
|--------------------------|----------------------------------|
| 11                       | \(\nu, \mu, -\nu, -\mu\)       |
| 12                       | \(\nu, \mu, \mu, -\nu, -\mu, -\mu\) |
| 22                       | \(\nu, \nu, \mu, \mu, -\nu, -\nu, -\mu, -\mu\) |
| Chair                    | \(\nu, \mu, \rho, -\mu, -\nu, -\rho\) |
| Sofa                     | \(\nu, \mu, \rho, \rho, -\nu, -\mu, -\rho, -\rho\) |
| Twist                    | \(\nu, \mu, \rho, -\nu, -\mu, -\rho\) |
| 4Dtwist                  | \(\nu, \mu, \rho, \gamma, -\nu, -\mu, -\rho, -\gamma\) |

Space \((\beta_{11}, \beta_{12}, \beta_{\text{twist}})\),

\[
S = \beta_{11} \sum ReTr(1 - \frac{1}{3} U_{11}) + \beta_{12} \sum ReTr(1 - \frac{1}{3} U_{12}) + \beta_{\text{twist}} \sum ReTr(1 - \frac{1}{3} U_{\text{twist}}).
\] (2)

Last year we only considered two coupling constants \((\beta_{11}, \beta_{12})\). This year the term \(\beta_{\text{twist}}\) in Eq. (2) is newly added.

3. Coupling flow

3.1. Technique

There are several determination techniques of coupling constants. At the early stage of our study we used the demon method which needs an extra simulation (microcanonical simulation) to obtain values of the coupling constants. We now use the Schwinger-Dyson (SD) equation method which is computationally simple and needs no extra simulation.

Truncation effects may cause different results in the two methods. Indeed in 2 coupling space \((\beta_{11}, \beta_{12})\) we found 10% difference in the coupling constants. If the truncation effects would be negligibly small the result should be same in both cases.

3.2. Simulation

We employ the lattice of the size of \(8^4\). We generate configurations at certain \(\beta\) sets \((\beta_{11}, \beta_{12})\) then block the configurations. The \(\beta\) sets are chosen from the RG coupling flow obtained in 2 dimensional space in Ref. and the Iwasaki action.

At each set of \(\beta\) about 200 configurations separated by 10 sweeps are used for the study. On the blocked configurations we calculate values of Wilson loops and correlation between Wilson loops, then solve the SD equations.

3.3. Results

First we show a result of coupling constants after one blocking starting at \((\beta_{11}, \beta_{12}) = (11.0, -1.7)\). The result clearly shows an exponential decay with the length of Wilson loops, which indicates that contribution of coupling constants associated with large Wilson loops decreases rapidly. See Fig. 1.

We now turn to the 3 dimensional coupling space \((\beta_{11}, \beta_{12}, \beta_{\text{twist}})\). Figs. 2 and 3 show coupling flow projected on \((\beta_{11}, \beta_{12})\) and \((\beta_{11}, \beta_{\text{twist}})\) coupling space, respectively. The result projected on the \((\beta_{11}, \beta_{12})\) space is very similar with the previous result in 2 dimensional space and the value of coupling constant \(\beta_{\text{twist}}\) is very small compared to other two coupling constants, which means adding the coupling constant \(\beta_{\text{twist}}\) does not change the coupling flow very much. Fig. 4 shows the coupling flow drawn in 3 dimensional space. The renormalized trajectory may be read off joining the end points of the arrows.

4. Discussion

We have studied the coupling flow in 3 dimensional coupling space. If this coupling space is large enough to represent the real RG flow, the actions on the renormalized trajectory should show the good scaling behavior. We plan to measure several quantities on the obtained renormalized trajectory to check whether the actions are really "perfect". We also prepare to extend the analysis to include \(\beta_{\text{chair}}\).

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REFERENCES
Figure 1. $|\beta \times S|$ vs. length of Wilson loop, where $S$ is the multiplicity of the loop when updating one link.

1. K. Wilson, in Recent developments of gauge theories ed. G.‘tHooft et al. (Plenum, New York, 1980)
2. K. Symanzik Nucl. Phys. B226 (1983) 187, M. Lüscher and P. Weisz, Phys.Lett. B158 (1985) 250
3. P. Hasenfratz and F. Niedermayer, Nucl. Phys. B454 (1994) 785
4. T. DeGrand, A. Hasenfratz, P. Hasenfratz and F. Niedermayer, Nucl. Phys. B454 (1995) 587; B454 (1995) 615
5. G.P.Lepage and P.B.Mackenzie, Phys. Rev. D48 (1993) 2250
6. QCDTARO Collaboration, Phys. Rev. Lett. 71 (1993) 3063; Nucl. Phys. B (Proc. Suppl.) 34 (1994) 246
7. QCDTARO Collaboration, Nucl. Phys. B (Proc. Suppl.) 53 (1997) 938
8. M.Creutz, A.Gocksch, M.Ogilvie and M.Okawa Phys. Rev. Lett. 53 (1984) 875
9. T.Takaishi, Mod. Phys. Lett. 10 (1995) 503
10. T.Takaishi, Phys. Rev. D54 (1996) 1050
11. A. Gonzalez-Arroyo and M.Okawa, Phys. Rev. D 35 (1987) 672