Adaptive blind timing recovery methods for MSE optimization
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1 Introduction
A different sampling timing phase produces different channel responses in the presence of multipath channels. For finite length equalizers, which are always insufficiently long in practice for wireless multimedia broadcasting systems such as advanced television systems committee (ATSC) receivers, the mean squared error (MSE) performance of a fixed length minimum MSE (MMSE) equalizer depends on the sampled channel. Certain timing offsets yield channels relatively easy to equalize with baud-spaced equalizers and, consequently, the MSE performance of the MMSE equalizer of a given length is limited by the choice of timing phase offset. The problem of finding the optimal timing phase in the presence of long delay spread multipath distortion has been considered resolved with the introduction of fractionally spaced (FS) equalization [1]. FS equalizers not only equalize multipath channel distortion more effectively, but also plays a role of interpolation filter for the timing phase to produce the best MSE performance [1]. However, for long delay spread channels such as the ones ATSC digital television (DTV) receivers are facing, FS equalizers covering the entire range of multipath delays are often impractical due to hardware limitations. Therefore, most receivers prefer a baud-spaced linear equalizer combined with a decision feedback equalizer (DFE) operating at the baud rate. Consequently, the timing phase problem has resurfaced in ATSC receivers.

Most widely used timing recovery schemes are Gardner algorithm [2] and band-edge algorithm, or known as Godard algorithm, [3]. The band-edge algorithms has originated from the output energy maximization (OEM) of sampled received signals, i.e., finding timing phase maximizing the energy of the sampled signals. Since the sampled signals is mixed with inter-symbol-interference terms, the timing phase based on OEM is optimized for infinite length equalizers but not for a finite length equalizer. As we will show in this article, Gardner algorithm also belongs to this OEM category and, consequently, cannot produce optimized timing offset for a finite length equalizer. In general, it is difficult and costly task to optimize timing phase for a given finite length equalizer: joint optimization of timing and equalization has inherent latency problem and often requires frequent training signals.

Especially for ATSC receivers, the most important application area for baud timing recovery algorithms, several timing phase optimization techniques have been developed and applied. Most of these approaches use repetitive data segment syncs or periodically apply a
timing phase correction computed from the field sync, in parallel with commonly used timing acquisition algorithms such as Gardner, band-edge or variant of Gardner algorithms [4–6] algorithm. For example, a correlation function of three symbols (1 0 1) [7] or four symbols (1 1 -1 -1) [8,9] in segment training signals is used to generate the timing phase information, or the field sync sequence is used to generate the timing phase correction [10]. However, these data-aided timing phase acquisition approaches use only a fraction of the data (e.g., a four-symbol segment sync among 832 symbols in the data segment) to optimize timing offset.

In this article, we propose a non-data-aided (blind) timing acquisition method designed to approximate the optimal timing phase in the presence of multipaths. The timing phase offset generated by the proposed symbol timing recovery algorithm is located close to the optimal timing phase offset compared to the Gardner [2] or band-edge algorithms [3] without help of the equalized data without feedback from the equalizer. Hence, the proposed algorithm can be used with the data aided approaches in the place of the Gardner algorithm for ATSC receivers.

The purpose of this algorithm is to find the timing phase optimized for a single tap equalizer, the opposite extreme of the infinite length equalizer. This approach is called dispersion minimization (DM) approach [11] and produces better MSE performance for most finite equalizers than OEM timing, but an adaptive algorithm version of this DM algorithm has not been studied yet. We developed a baseband blind adaptive timing recovery algorithm that is closely related to this DM approach as Gardner is closely related to the OEM approach. Simulation results show that the proposed timing recovery algorithm enhances the performance of MMSE DFEs in comparison with Gardner timing.

In Section 2 we introduce OEM timing recovery approach and the relation to Gardner timing. In Section 3 a new blind timing recovery algorithm based on DM approach is proposed with a tutorial example showing the enhanced performance. Section 4 presents simulation results and Section 5 provides the conclusion.

2 Symbol timing offset of symbol timing recovery algorithms

Figure 1 describes a framework for timing recovery algorithms.

An identically independent source sequence \(s_k\) is converted to analog signal by a pulse shaping filter \(p(t)\)

\[
s(t) = \sum_{k=\infty}^{\infty} s_k p(t - kT)
\]

is distorted by a multipath channel

\[
c(t) = \sum_{i=0}^{N-1} \rho_i b(t - \tau_i) + \text{additive white Gaussian noise (AWGN)} w(t)
\]

Then, the received \(r(t)\) is matched filtered with \(g(t)\) and

\[
y(t) = r(t) \ast g(t) = \sum_{i=\infty}^{\infty} s_i h(t - kT) + w(t) \ast g(t),
\]

where \(h(t)\) is overall channel response combining the multipath channel \(c(t)\), pulse shaping filter \(p(t)\), and the matched filter \(g(t)\).

\[
h(t) = p(t) \ast c(t) \ast g(t),
\]

where \(\ast\) denotes convolution operation. The received analog time signal \(y(t)\) is sampled at the baud rate \(T\) with a timing phase offset \(\tau\) generated from a timing offset generation mechanism. Depending on the timing phase offset \(\tau\), we have a different discrete time domain channel. Denoting the discrete time impulse response sampled from \(h(t)\) with respect to the sampling phase \(\tau\) as a vector \(h_{\tau}\),

\[
h_{\tau} = [h(kT + \tau)]_{k=\infty}^{\infty}
\]

we have

\[
y(kT + \tau) = \sum_{i=\infty}^{\infty} s_i h_{\tau} [k - i] + w_{\tau},
\]

where \(w_{\tau}\) is sampled noise term.

Several optimization algorithms for adjusting timing phase offset \(\tau\) are developed. OEM approach to timing phase recovery involves choosing the timing phase to maximize the power of the sampled data, i.e.,

\[
\tau_{\text{OEM}} = \arg \max_{\tau} E[y(kT + \tau)]^2
\]

This approach consequently optimizes the MSE of the equalizers with infinite length, since the output energy usually contains inter-symbol interference (ISI) terms \(\sum_{i=\infty}^{\infty} s_i h_{\tau} [k - i]\), in the presence of multipath channels. An infinite length equalizer will deal with the ISI component to convert the ISI component to the signal component perfectly. For a finite or a relatively short equalizer, the OEM timing fails to achieve MMSE, since the remaining ISI degrades the MSE performance [11].
Godard’s band-edge algorithm [3] is a passband domain implementation of this approach. We now show that the widely used Gardner baseband timing recovery algorithm [2] given by

\[
\tau_{k+1} = \tau_k + \mu \varepsilon_k^G
\]

(8)

\[\varepsilon_k^G = \gamma (kT + \tau_k) \left[ \gamma \left( kT + \frac{T}{2} + \tau_k \right) - \gamma \left( kT - \frac{T}{2} + \tau_k \right) \right], \]

(9)

where \(\mu\) is a step size and \(\tau_k\) is the timing phase at time \(kT\), can be viewed as an approximated gradient descent implementation [12] of the OEM approach (7). The stochastic update equation [13] to achieve (7) is given by

\[
\tau_{k+1} = \tau_k + \frac{\mu}{2} \frac{d}{dt} |\gamma (kT + \tau)|_2^2 = \tau_k + \mu \gamma (kT + \tau) \frac{d}{dt} |\gamma (kT + \tau)|_2^2\]

(10)

Assuming that the timing phase changes slowly, the derivative term can be roughly approximated by

\[
\frac{d}{dt} |\gamma (kT + \tau)\mid_{\tau=\tau_k} \approx \gamma \left( kT + \frac{T}{2} + \tau_k \right) - \gamma \left( kT - \frac{T}{2} + \tau_k \right)
\]

(11)

Combining (10) and (11), we obtain Gardner algorithm (9) as an approximation of OEM algorithm. Hence, we can conclude that the Gardner algorithm, which is commonly used in symbol timing recovery circuits of ATSC DTV receivers, falls into the OEM timing recovery category. Consequently, as reported in [10], the Gardner algorithm does not perform optimally for ATSC receivers, in which the length of equalizers is always short when dealing with widely spread multipath channels.

In contrast, the DM timing recovery approach produces a peaky baud-spaced channel impulse response to offer better equalization performance for short equalizers. The DM timing [11] is optimized for a short (single-tap) equalizer. The DM timing phase is defined by minimization of the dispersion of sampled data,

\[
\tau_{DM} = \arg \max_{\tau} E \left[ |\gamma (kT + \tau)|^2 - \gamma \right]^2
\]

(12)

where \(\gamma\) is the dispersion constant [14] computed from the source signal (\(\gamma = 8/\sqrt{2T}\) for 8-PAM). This DM timing phase is optimized for one tap equalizer and located closer to the best timing phase offset for a finite length equalizer, minimizing equalizer output MSE better than other timing methods based on OEM [11]. In general, the baud-spaced channel produced by DM timing is easier to equalize with finite equalizers than the one produced by OEM timing. In the following section, we consider the adaptive solution of DM timing in the baseband.
Figure 3 Performance of proposed timing for no multipath channel. (a) Timing phases. (b) S-curve for proposed timing.
Figure 4 Performance of proposed timing for single echo 3 dB with 0.51 symbol delay. (a) Timing phases. (b) S-curve for proposed timing.
Figure 5 Discrete time channels for different timing phase.

Figure 6 MSE of MMSE equalizer length 20 under 30 dB SNR for various timing.
We consider a baseband adaptive solution for DM timing recovery. The stochastic update equation can be given as

$$ \tau_{k+1} = \tau_k + \mu \frac{d}{dt} \left[ |y(kT + \tau)|^2 - \gamma \right]_{\tau=\tau_k} $$

$$ = \tau_k + \mu y(kT + \tau_k) \left[ |y(kT + \tau)|^2 - \gamma \right] \frac{d}{dt} y(kT + \tau) |_{\tau=\tau_k} $$

(13)

(14)

With the same approximation of the derivative in the Gardner algorithm:

$$ \frac{d}{dt} y(kT + \tau) |_{\tau=\tau_k} \approx y \left( kT + \frac{T}{2} + \tau_k \right) - y \left( kT - \frac{T}{2} + \tau_k \right) $$

(15)

Hence, we define a new timing recovery algorithm with a new error function for timing recovery:

$$ \tau_{k+1} = \tau_k + \mu e^{DM}_k $$

(16)

$$ e^{DM}_k = y(kT + \tau_k) \left( |y(kT + \tau)|^2 - \gamma \right) \left[ y \left( kT + \frac{T}{2} + \tau_k \right) - y \left( kT - \frac{T}{2} + \tau_k \right) \right] $$

(17)

In comparison with the error function in Gardner algorithm, this new error function has an additional term related to dispersion, $|y(kT + \tau_k)|^2 - \gamma$. We expect this new timing algorithm to inherit the optimized MSE performance of DM timing. Figure 2 illustrates a possible implementation structure of a timing recovery circuit using the proposed timing algorithm.

The proposed timing successfully recovers the timing delay in the absence of multipaths (pure delay) as

3 Proposed timing recovery method

We consider a baseband adaptive solution for DM timing recovery. The stochastic update equation can be given as [13]

$$ \tau_{k+1} = \tau_k + \mu \frac{d}{dt} \left[ |y(kT + \tau)|^2 - \gamma \right]_{\tau=\tau_k} $$

$$ = \tau_k + \mu y(kT + \tau_k) \left[ |y(kT + \tau)|^2 - \gamma \right] \frac{d}{dt} y(kT + \tau) |_{\tau=\tau_k} $$

With the same approximation of the derivative in the Gardner algorithm:

$$ \frac{d}{dt} y(kT + \tau) |_{\tau=\tau_k} \approx y \left( kT + \frac{T}{2} + \tau_k \right) - y \left( kT - \frac{T}{2} + \tau_k \right) $$

(15)
shown in Figure 3. All timing recovery algorithms, OEM, DM, Gardner, and proposed one, produce the same timing phase offset. The timing phase in this case is the instant in which the main path has the peak, as shown in Figure 3a. The S-curve in Figure 3b confirms the capability of the proposed algorithm to converge to the correct timing phase in the absence of multipath.

Figure 4a shows the proposed timing phase for a single echo channel with a 3-dB echo and a 0.51 symbol delay,

\[ c(t) = \delta(t) + \sqrt{2} \delta(t - 0.51T), \]  

(18)

where we used a square-root raised filter with a roll-off factor of 11.5% as a pulse shaping filter. The proposed timing phase is located near the DM timing, while Gardner timing is located close to the OEM timing. The two timing phases, Gardner and the proposed one, are different and this difference produces the different channel shown in Figure 5. Note that the proposed timing produces a more *peaky* channel. This difference produces a difference in the MSE performance of the finite length MMSE equalizer, as shown in Figure 6 in the following simulation section.

Figure 6 plots the MSE performance of a finite length MMSE linear equalizer for normalized timing phase offsets spanning -0.5 to 0.5, i.e., \([-T/2, T/2]\). Since the effect channel lengths are about 12 taps in Figure 5, we have set equalizer length to 20 under 30 dB SNR. None of those timing offsets have achieve the MMSE, but DM timing and the proposed timing perform relatively better than OEM approaches (about 1 dB).

Although the proposed algorithm seems to outperform Gardner algorithm, the performance of the proposed algorithm depends on the length of equalizer. Figure 7 plots the MSE performance of the MMSE equalizers with various lengths for a fixed channel \(c(t) = \delta(t) + \delta(t - 0.51T)\) under 30 dB SNR. The proposed algorithm outperforms Gardner algorithm only for the equalizer length less than about 130. Unfortunately, the exact filter length determining the boundary is hard to obtain in general. However, we believe equalizers are always short in most practical situations.

### 4 Simulation results

We conducted a simulation to evaluate the overall MSE performance of the proposed timing for a receiver equipped with a DFE, perhaps the most widely used
Figure 9 MSE performance of the MMSE DFEs for different timing phases (Channel 1, SNR 20-30 dB).

Figure 10 MSE performance of the MMSE DFEs for different timing phases (Channel 2, SNR 20-30 dB).
equalization scheme for ATSC receivers. We have assumed perfect carrier phase offset recovery using many available blind methods [15] We used the two multipath channels as described in Table 1, a single echo channel and the Brazil channel B ensemble. We assigned 100 taps for the feed-forward filter of the DFE and 200 taps for the feedback filter. A blind adaptation strategy [12], which achieves a smooth transition from the infinite-impulse response constant-modulus algorithm to the decision-directed least mean square algorithm, was used to obtain the MMSE DFE coefficients. Figure 8 shows the cluster variance (CV) trajectories of the DFE for Channel 2 (Brazil B) with a different timing phase. We observe that the proposed timing phase outperforms the Gardner timing phase by about 2 dB after DFE convergence. Figures 9 and 10 show the MSE performance of the proposed timing with the DFE compared to Gardner timing for Channel 1 (single echo) and Channel 2 (Brazil B), respectively. For various values of SNR in the range 20-30 dB, the DFE with the proposed timing algorithm provides an increase of about 2 dB MSE and the gain tends to decrease slightly as the SNR decreases.

5 Conclusion
In this article, we described a blind timing method for ATSC DTV systems that produces better equalizer output MSE performance than other OEM-based timing methods such as Gardner timing. The proposed timing recovery algorithm can be considered as a baseband adaptive implementation of the DM timing approach. Simulation results confirmed the MSE enhancement of DFE output when equipped with the proposed timing algorithm.

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Competing interests
The author declares that they have no competing interests.

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