IN THE STANDARD MODEL: THEORETICAL UPDATE

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A complete analysis of isospin breaking in $K \to 2\pi$ amplitudes, including both strong ($m_u \neq m_d$) and electromagnetic corrections at next-to-leading order in chiral perturbation theory, has been achieved recently. We discuss the implication of these effects, together with the previously known chiral loop corrections, on the direct CP-violating ratio $\varepsilon'/\varepsilon$. One finds $\text{Re} (\varepsilon'/\varepsilon) = (16.7 \pm 1.6) \cdot 10^{-4}$.

1 Introduction

The CP-violating ratio $\varepsilon'/\varepsilon$ constitutes a fundamental test for our understanding of flavour-changing phenomena. The experimental status has been clarified by the KTEV, $\text{Re} (\varepsilon'/\varepsilon) = (20.7 \pm 2.8) \cdot 10^{-4}$, and NA48, $\text{Re} (\varepsilon'/\varepsilon) = (14.7 \pm 2.2) \cdot 10^{-4}$, measurements. The present world average $^{3,4,5,6}$

$$\text{Re} (\varepsilon'/\varepsilon) = (16.7 \pm 1.6) \cdot 10^{-4}, \quad (1)$$

demonstrates the existence of direct CP violation in K decays.

The CP violating signal is generated through the interference of two different $K^0 \to \pi\pi$ decay amplitudes,

$$\varepsilon'/\varepsilon = e^{i\Phi} \frac{\omega}{\sqrt{2}} \left[ \text{Im} A_2 \over \text{Re} A_2 \right] \left[ \text{Im} A_0 \over \text{Re} A_0 \right], \quad (2)$$

In the limit of CP conservation, the isospin amplitudes $A_{0,2}$ are real and positive. $\varepsilon'/\varepsilon$ is suppressed by the small ratio $\omega = \text{Re} A_2 / \text{Re} A_0 \approx 1/22$. The strong S-wave rescattering of the two final pions generates a large phase-shift difference between the two amplitudes, making the phases of $\varepsilon'$ and $\varepsilon$ nearly equal: $\Phi \approx \delta_2 - \delta_0 + \pi/4 \approx 0$. Thus, unitarity corrections play a crucial role in $\varepsilon'/\varepsilon$. Moreover, the ratio $1/\omega$ amplifies any potential contribution to $A_2$ from small isospin-breaking corrections induced by $A_0$.

The CP-conserving amplitudes $\text{Re} A_I$, their ratio $\omega$ and $\varepsilon$ are usually set to their experimentally determined values. A theoretical calculation is only needed for $\text{Im} A_I$.

2 Theoretical Framework

Owing to the presence of very different mass scales ($m_\pi < M_K \ll M_W$), the gluonic corrections to the $S = 1$ process are amplified by large logarithms. The short-distance logarithmic corrections can be summed up using the Operator Product Expansion (OPE) and the renormalization group, all the way down from $M_W$ to scales $\mu < m_\pi$. One gets an effective Lagrangian, defined in the three-flavour theory,$^{7,8}$

$$\mathcal{L}_{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^{*} \sum_{i=1}^{10} C_i(\mu) Q_i(\mu), \quad (3)$$

which is a sum of local four-fermion operators $Q_i$, modulated by Wilson coefficients $C_i(\mu)$ which are functions of the heavy masses ($M > \mu$) and CKM parameters. These coefficients are known at the next-to-leading logarithmic order.$^{9,10}$ This includes all corrections of $O(\alpha_s^3 t^n)$ and $O(\alpha_s^{n+1} t^n)$, where $t \equiv \ln (M_1/M_2)$ refers to the logarithm of any ratio of heavy mass scales $M_1, M_2 \geq \mu$.

To a very good approximation, only two operators are numerically relevant for $\varepsilon'/\varepsilon$: the QCD penguin operator $Q_6$ governs $\text{Im} A_0$, while $\text{Im} A_2$ is dominated by the electroweak penguin operator $Q_8$. A naive vacuum insertion approximation to their hadronic matrix elements results in a large numerical cancellation, leading$^{11,12}$ to unphysical low values of $\varepsilon'/\varepsilon$ around $7 \times 10^{-4}$. The true Standard Model prediction is then very sensitive to the precise values of these two matrix elements.
Below the resonance region one can use 

\[ L \] 

symmetry considerations to define another effective field theory in terms of the QCD Goldstone bosons. Chiral perturbation theory (χPT) describes the pseudoscalar–octet dynamics, through a perturbative expansion in powers of momenta and quark masses over the chiral symmetry breaking scale \( \Lambda_\chi \sim 1 \text{ GeV} \). Chiral symmetry fixes the allowed operators. At lowest order, the most general effective bosonic Lagrangian with the same \( SU(3)_L \otimes SU(3)_R \) transformation properties as \( L^{eff} = L^{S=1}_\chi \) contains three terms:

\[ L^{\Delta S=1}_\chi = G_8 L_8 + G_{27} L_{27} + G_{\text{ew}} L_{\text{ew}}. \] (4)

\( L_{\text{ew}} \) gives the low–energy realization of \( Q_8 \), while \( Q_6 \) is included in the octet term.

\( L^{\Delta S=1}_\chi \) determines the \( K \rightarrow \pi\pi \) amplitudes at \( O(p^2) \). The calculation of the chiral couplings \( G_i \) from the short–distance Lagrangian (3) requires to perform the matching between the two effective theories. This can be done in the limit of an infinite number of quark colours, because the four–quark operators factorize into currents which have well–known chiral realizations. This is equivalent to the standard large–\( N_C \) evaluations of \( \langle Q_i \rangle \). Therefore, up to minor variations on some input parameters, the corresponding \( \varepsilon'/\varepsilon \) prediction, obtained at lowest order in both the \( 1/N_C \) and χPT expansions, reproduces the published results of the Munich and Rome groups.

3 Chiral Corrections

The large–\( N_C \) limit is only applied to the matching between the 3–flavour quark theory and χPT. The evolution from the electroweak scale down to \( \mu < m_c \) has to be done without any unnecessary expansion in powers of \( 1/N_C \); otherwise, one would miss large corrections of the form \( \frac{1}{N_C} \ln (M/m) \), with \( M \gg m \) two widely separated scales.¹⁵

Similarly, the long–distance rescattering of the two pions generates large logarithmic corrections, through chiral loops which are of higher order in both the momentum and \( 1/N_C \) expansions.² These next-to-leading contributions, which give rise to the large S–wave strong phases \( \delta_I \) (\( \delta_I = 0 \) at \( N_C \rightarrow \infty \)), were overlooked in the first large–\( N_C \) predictions¹¹,¹² giving a too small \( \varepsilon'/\varepsilon \). Larger values of \( \varepsilon'/\varepsilon \) were in fact obtained within models containing some kind of pion rescattering.¹⁷ The χPT framework allows us to incorporate rigorously these corrections in a model independent way.

The one–loop χPT analyses¹²,¹⁸ of \( K \rightarrow 2\pi \) show indeed that pion loops provide an important enhancement of the \( A_0 \) amplitude, associated with large infrared logarithms involving the light pion mass, and a sizeable reduction of \( A_2 \). These chiral corrections destroy the numerical cancellation between the \( Q_6 \) and \( Q_8 \) contributions, generating a large enhancement of the \( \varepsilon'/\varepsilon \) prediction.²

A complete one–loop calculation, including electromagnetic and isospin violation corrections, has been achieved recently.¹ The loop contributions are fully determined by chiral symmetry. The local corrections generated by the different higher-order chiral lagrangians have been computed at leading or-

| Scale | Fields | Eff. Theory |
|-------|--------|-------------|
| \( M_W \) | \( W, Z, \gamma, g, \tau, \mu, \epsilon, \nu_i \), \( t, b, c, s, d, u \) | Standard Model |
| \( M_K \) | \( \gamma; \mu, \epsilon, \nu_i \), \( \pi, K, \eta \) | \( \chi \)PT |

Figure 1. Evolution from \( M_W \) to \( M_K \).¹⁶
der in the $1/N_C$ expansion.

To account for isospin breaking, we can write
\[ \varepsilon' = -\frac{\varepsilon'_{\text{IB}} \omega_+}{\sqrt{2} |\varepsilon|} \left[ \frac{\text{Im} A_{\varepsilon}^{(0)}}{\text{Re} A_{\varepsilon}^{(0)}} (1 - \Omega_{\text{eff}}) - \frac{\text{Im} A_{\varepsilon}^{\text{emp}}}{\text{Re} A_{\varepsilon}^{(0)}} \right], \]
where the superscript $(0)$ denotes the isoscalar limit, $A_{\varepsilon}^{\text{emp}}$ is the electromagnetic penguin contribution to $A_2$, and $\omega_+ = \text{Re} A_2^+/\text{Re} A_0$ (with $\text{Re} A_2^+$ measured in $K^+ \to \pi^+\pi^0$) differs from $\omega = \text{Re} A_2/\text{Re} A_0 = \omega_+ (1 + f_5/2)$ by a pure $\Delta I = 5/2$ effect.

The quantity $\Omega_{\text{eff}} = \Omega_{\text{IB}} - \Delta_0 - f_5/2$ contains all isospin breaking effects to leading order. $\Delta_0$ accounts for isospin breaking corrections to $A_0$, while the more traditional parameter $\Omega_{\text{IB}}$ parameterizes the contributions to $\text{Im} A_2$ from other four-quark operators not included in $A_{\varepsilon}^{\text{emp}}$. Taking $\alpha = 0$, the isospin breaking is completely dominated by the $\pi^0 - \eta$ mixing contribution:\$\Omega_{\text{IB}}^{(0)} = 0.16 \pm 0.03$. Electromagnetic effects give sizeable contributions to all three terms, generating a destructive interference and a smaller final value for the overall measure of isospin violation in $\varepsilon'$:
\[ \Omega_{\text{eff}} = 0.06 \pm 0.08. \quad (5) \]

4 Discussion

Chiral loops generate an important enhancement ($\sim 35\%$) of the isoscalar $K \to \pi\pi$ amplitude and a sizeable reduction of $A_2$. This effect gets amplified in the prediction of $\varepsilon'/\varepsilon$, because at lowest order (in both $1/N_C$ and the chiral expansion) there is an accidental numerical cancellation between the $I = 0$ and $I = 2$ contributions. Since the chiral corrections destroy this cancellation, the final result is dominated by the amplitude $A_0$. The small value recently obtained\cite{1} for $\Omega_{\text{eff}}$ reinforces the dominance of the gluonic penguin operator $Q_6$. Taking this into account and updating all other inputs,\cite{2} the Standard Model prediction for $\varepsilon'/\varepsilon$ turns out to be
\[ \text{Re} (\varepsilon'/\varepsilon) = (19 \pm 2^{+9}_{-6} \pm 6) \cdot 10^{-4}, \quad (6) \]
in excellent agreement with the experimental measurement (1). The first error has been estimated by varying the renormalization scale $\mu$ between $M_{\rho}$ and $m_c$. The uncertainty induced by $m_s$, which has been taken in the range\cite{21} $m_s(2\text{GeV}) = 110 \pm 20 \text{MeV}$, is indicated by the second error.

The most critical step is the matching between the short and long-distance descriptions, which has been done at leading order in $1/N_C$. Since all next-to-leading ultraviolet and infrared logarithms have been taken into account, our educated guess for the theoretical uncertainty associated with subleading contributions is $\sim 30\%$ (third error).

The control of non-logarithmic corrections at the next-to-leading order in $1/N_C$ remains a challenge for future investigations. Several dispersive analyses\cite{22,23,24,25} and lattice calculations\cite{26} of $\langle Q_8 \rangle$ already exist (most of them in the chiral limit). Taking the chiral corrections into account, the results are compatible with the value used in (6). Unfortunately, the penguin matrix element is more difficult to compute. Two recent estimates in the chiral limit, using the so-called minimal hadronic approximation\cite{27} and $X$-boson approach\cite{25}, find large $1/N_C$ corrections to $\langle Q_0 \rangle$. It would be interesting to understand the physics behind those contributions and to study whether corrections of similar size are present for physical values of the quark masses. Lattice calculations of $\langle Q_6 \rangle$ are still not very reliable\cite{28} and give contradictory results\cite{26,29} (often with the wrong sign).

More work is needed to reduce the present uncertainty quoted in (6). This is a difficult task, but progress in this direction should be expected in the next few years.

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References

1. V. Cirigliano, G. Ecker, H. Neufeld and A. Pich, Eur. Phys. J. C33 (2004) 369; Phys. Rev. Lett. 91 (2003) 162001.
2. E. Pallante, A. Pich and L. Scimemi, Nucl. Phys. B617 (2001) 441; E. Pallante and A. Pich, Phys. Rev. Lett. 84 (2000) 2568; Nucl. Phys. B592 (2000) 294.
3. KTeV collab., Phys. Rev. D67 (2003) 012005; Phys. Rev. Lett. 83 (1999) 22.
4. NA48 collab., Phys. Lett. B544 (2002) 97; Eur. Phys. J. C22 (2001) 231; Phys. Lett. B465 (1999) 335.
5. NA31 collab., Phys. Lett. B206 (1988) 169; Phys. Lett. B317 (1993) 233.
6. E731 collab., Phys. Rev. Lett. 70 (1993) 1203.
7. F.J. Gilman and M.B. Wise, Phys. Rev. D20 (1979) 2392; D21 (1980) 3150.
8. A.J. Buras, hep-ph/9806471.
9. A.J. Buras, M. Jamin and M.E. Lautenbacher, Nucl. Phys. B408 (1993) 209; Phys. Lett. B389 (1996) 749.
10. M. Ciuchini et al., Phys. Lett. B301 (1993) 263; Z. Phys. C68 (1995) 239.
11. S. Bosch et al., Nucl. Phys. B565 (2000) 3; A.J. Buras et al., Nucl. Phys. B592 (2001) 55; A.J. Buras and M. Jamin, JHEP 01 (2004) 048.
12. M. Ciuchini et al., hep-ph/9910237.
13. S. Weinberg, Physica 96A (1979) 327.
14. J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 456; 517; 539.
15. W.A. Bardeen, A.J. Buras and J.-M. Gérard, Nucl. Phys. B293 (1987) 787; Phys. Lett. B192 (1987) 138, B180 (1986) 133.
16. A. Pich, hep-ph/9806303.
17. S. Bertolini et al., Rev. Mod. Phys. 72 (2000) 65; T. Hambye et al., Nucl. Phys. B564 (2000) 391.
18. J. Kambor et al., Nucl. Phys. B346 (1990) 17; Phys. Lett. B261 (1991) 496; Phys. Rev. Lett. 68 (1992) 1818; J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B521 (1998) 305; E. Pallante, JHEP 01 (1999) 012.
19. V. Cirigliano, J.F. Donoghue, and E. Golowich, Eur. Phys. J. C18 (2000) 83.
20. G. Ecker, G. Müller, H. Neufeld and A. Pich, Phys. Lett. B477 (2000) 88.
21. E. Gámiz et al., JHEP 01 (2003) 060; hep-ph/0408044; M. Jamin, J.A. Oller and A. Pich, Eur. Phys. J. C24 (2002) 237; K. Maltman and J. Kambor, Phys. Rev. D65 (2002) 074013; S.M. Chen et al., Eur. Phys. J. C22 (2001) 31; H. Wittig, hep-lat/0210025; C. Aubin et al., Phys. Rev. D70 (2004) 031504; M. Göckeler et al., hep-lat/0409312.
22. V. Cirigliano et al., Phys. Lett. B555 (2003) 71, B522 (2001) 245, B475 (2000) 351; Phys. Rev. D65 (2002) 054014; J.F. Donoghue and E. Golowich, Phys. Lett. B478 (2000) 172.
23. M. Knecht, S. Peris and E. de Rafael, Phys. Lett. B508 (2001) 117, B457 (1999) 227; S. Peris and E. de Rafael, Phys. Lett. B490 (2000) 213.
24. S. Narison, Nucl. Phys. B593 (2001) 3.
25. J. Bijnens et al., hep-ph/0309216; JHEP 10 (2001) 009, 06 (2000) 035, 01 (1999) 023.
26. J.I. Noaki et al., Phys. Rev. D68 (2003) 014501; T. Blum et al., Phys. Rev. D68 (2003) 114506; D. Bečirević et al., Nucl. Phys. B (Proc. Suppl.) 119 (2003) 619.
27. S. Peris, hep-ph/0310063; T. Hambye, S. Peris and E. de Rafael, JHEP 05 (2003) 027.
28. M. Golterman and E. Pallante, Phys. Rev. D69 (2004) 074503; JHEP 10 (2001) 037, 08 (2001) 023.
29. T. Bhattacharya et al., hep-lat/0409046; D. Pekurovsky and G. Kilcup, Phys. Rev. D64 (2001) 074502.