New methodology to determine the terminal height of a fireball

Manuel Moreno-Ibáñez, Josep M. Trigo-Rodríguez & Maria Gritsevich
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Background (I)

- **Asteroid**: diameter > 10 m.
- **Meteoroid**: diameter < 10 m.
- **Meteor**: meteoroid that interacts with the Earth atmosphere.
- **Fireball**: meteor that is able to get deep in the atmosphere. Brightness similar to Venus.
- **Great Fireball**: bigger fireball that is able to get to lower altitudes and can reach a brightness similar to the Moon.
- **Meteorites**: meteor that survives to its atmospheric flight and reaches the ground.
- **Micrometeorites**: small grains that get the atmosphere at low velocities and are deposited on the ground.

Adapted from Rendtel et al. (1995)
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**Ceplecha y McCrosky (1976)**

- Study of the Prairie Network to distinguish between ordinary and carbonaceous chondrites.
- The end height as the principal discriminating observational parameter in their discussion (*photometric mass*).
- Empirical Criterium:
  
  \[ PE = \log \rho_E + A \log m_\infty + B \log V_\infty + C \log(\cos Z_R) \]

- A, B and C obtained by Least–Squared fit on PN data.

**Wetherill and Revelle (1981)**

- Instead of using the average values as input parameters, they gathered all the unknowns into two new variables, \( \alpha \) and \( \beta \) (ballistic coefficient and mass loss parameter).
- Adjusting the equation to the registered values these new variables can be obtained.
- This describes in detail the meteoroid trajectory and allow to invent a classification for possible impacts.

**Stulov et al. (1995), Stulov (1997) and Gritsevich (2007)**

- Study of the Prairie Network to find similar meteorites to the Lost City meteorite.
- They used four criteria: End height agrees with the single-body theoretical value, calculated using *dynamic mass*, as well as with that of Lost City too within \( \pm 1.5 \)km, when scaled for mass, velocity and entry angle in accordance with classic meteor theory.
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Equations of motion (I)

The equations of motion for a meteoroid entering the atmosphere projected onto the tangent and to the normal to the trajectory

\[ M \frac{dV}{dt} = -\frac{1}{2} c_d \rho_a V^2 S + P \sin \gamma \]
\[ MV \frac{d\gamma}{dt} = P \cos \gamma - \frac{MV^2}{R} \cos \gamma - \frac{1}{2} c_L \rho_a V^2 S \]
\[ \frac{dh}{dt} = -V \sin \gamma \]

1. **Variation of the mass**

\[ H^* \frac{dM}{dt} = -\frac{1}{2} c_h \rho_a V^3 S \]

2. **Extra equations**

Isotermal atmosphere \( \rightarrow \rho = \exp \left( \frac{h}{h_0} \right) \)

Levin (1956) \( \rightarrow S / S_e = (M / M_e)^\mu \)

3. **Use of dimensionless parameters**

\( M = M_e m; \ V = V_e \nu; \ S = S_e s; \ h = h_0 y; \ \rho_a = \rho_0 \rho \)

Where index e indicates values at the entry of the atmosphere.

\( h_0 \) is the scale height \( (7.16 \text{ km}) \)
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Analytical solution of system:
Initial conditions : \( y = \infty ; \nu = 1 ; m = 1 \) \{ 

Equations of motion:

\[
m = \exp \left( -\frac{\beta (1 - \nu^2)}{1 - \mu} \right) \tag{1}
\]

\[
y = \ln 2\alpha + \beta - \ln \Delta, \quad \Delta = \bar{Ei}(\beta) - \bar{Ei}(\beta \nu^2) \tag{2}
\]

\[
\bar{Ei}(x) = \int_{-\infty}^{x} \frac{e^z}{z} \, dz
\]

In this methodology we gather all the unknown values of the meteoroid's atmosphere flight motion equations into two new variables (Gritsevich, 2009):

**Ballistic Coefficient**

\[
\alpha = \frac{1}{2} c_d \frac{\rho_0 h_0 S_e}{M_e \sin \gamma}
\]

**Mass loss parameter**

\[
\beta = (1 - \mu) \frac{c_h V_e^2}{2 c_d H^*}
\]
For quick meteors, a strong evaporation process takes place so $\beta$ becomes high ($\beta \gg 1$), the deceleration can be neglected and the velocity thus assumed constant. Stulov (1998, 2004) developed the following asymptotic solution:

$$v = 1, \quad m^{1-\mu} = 1 - 2\alpha \beta e^{-y}, \quad \ln 2\alpha \beta < y < \infty \quad [3]$$

However, the meteor velocity begins to decrease in a certain vicinity of $m=0$. In order to account for this change in velocity we combine the Eq.[1] (valid for arbitrary $\beta$ values) with the Eq. [3] suitable for high $\beta$ values:
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In order to test this methodology, we compare our derived terminal heights results with the fireball terminal heights registered by the Meteorite Observation and Recovery Project operated in Canada between 1970 and 1985 (MORP) (Halliday et al. 1996). We use previous $\alpha$ and $\beta$ values derived by Gritsevich (2009).

| Table 1b. The results of processing of observational data for the Canadian network fireballs. |
|---|
| Fireball No. | $V_e$, km/s | $\beta$ | $\gamma$ | $\sin\gamma$ | $M_0$, g | $M_0\gamma$, g |
| 18 | 18.5 | 24.13 | 1.475 | 0.862 | 0.572 | 5417.24 | 11466.23 | 5600 |
| 123 | 16.3 | 37.22 | 1.111 | 0.836 | 0.561 | 1651.41 | 3485.66 | 680 |
| 138 | 16.9 | 39.90 | 2.889 | 2.023 | 0.404 | 3669.82 | 7767.96 | 2400 |
| 141 | 32.1 | 70.19 | 6.403 | 1.243 | 0.767 | 0.09 | 0.19 | 17 |
| 144 | 23.5 | 137.13 | 1.301 | 0.471 | 0.953 | 6.38 | 13.51 | 140 |
| 169 | 22.9 | 50.62 | 1.575 | 0.601 | 0.337 | 2896.41 | 6109.70 | 12000 |
| 172 | 12.5 | 13.13 | 3.373 | 4.325 | 0.969 | 6816.28 | 14639.77 | 1200 |
| 177 | 15.8 | 77.63 | 1.777 | 1.424 | 0.631 | 121.19 | 256.52 | 300 |
| 187 | 16.4 | 83.76 | 3.815 | 2.254 | 0.603 | 83.17 | 176.06 | 340 |
| 194 | 14.5 | 34.47 | 0.757 | 0.720 | 0.389 | 5643.14 | 12630.23 | 9100 |
| 195 | 21.0 | 84.61 | 2.468 | 1.116 | 0.860 | 26.96 | 57.05 | 120 |
| 196 | 25.2 | 35.22 | 1.486 | 0.488 | 0.593 | 1588.83 | 3309.64 | 7000 |
| 204 | 13.0 | 10.13 | 3.620 | 4.284 | 0.537 | 88488.54 | 187204.65 | 2500 |
| 205 | 19.7 | 36.84 | 0.716 | 0.369 | 1.000 | 284.89 | 603.03 | 6300 |
| 207 | 17.9 | 24.46 | 0.776 | 0.484 | 0.927 | 1223.39 | 2699.66 | 3400 |
| 218 | 18.5 | 64.66 | 0.399 | 1.198 | 0.198 | 80.35 | 192.37 | 340 |
| 219 | 18.4 | 12.61 | 2.050 | 1.217 | 0.772 | 15813.00 | 33471.56 | 87000 |
| 223 | 27.7 | 18.33 | 1.600 | 0.403 | 0.326 | 66576.19 | 141046.66 | 29000 |
| 225 | 21.2 | 75.92 | 0.706 | 0.314 | 4.000 | 132.35 | 685.90 | 140 |
| 223 | 12.3 | 43.73 | 4.534 | 6.033 | 0.623 | 704.73 | 1481.07 | 150 |
| 231 | 27.9 | 52.72 | 1.371 | 0.352 | 0.304 | 1716.79 | 3639.96 | 4300 |
| 256 | 36.0 | 43.40 | 0.000 | 0.000 | 0.147 | 1.95 | 4.13 | 100 |
| 236 | 19.1 | 171.61 | 14.343 | 7.883 | 0.897 | 4.08 | 8.64 | 720 |
| 241 | 16.7 | 50.27 | 0.878 | 0.638 | 0.026 | 198.56 | 421.15 | 700 |

Halliday et al., 1996

Gritsevich, 2009
Adjusting the \((v_i, y_i)\) values of Eq. [2] to the trajectory observed \((v_i, y_i)\) values by means of a weighted least-squares method. Assigning manually the weighted factors may be quite complicated, so, since the height and velocity of a meteor decrease while it gets closer to the surface, the solution was proved to perform better if we take an exponential form of eq. [2] (see Gritsevich, 2008 for further details):

\[
2\alpha \exp(-y) - \Delta \exp(-\beta) = 0, \quad \Delta = \bar{E}i(\beta) - \bar{E}i(\beta v^2)
\]

\[
\bar{E}i(x) = \int_{-\infty}^{x} \frac{e^z \, dz}{z}
\]

We can derive these new variables \((\alpha, \beta)\) for each meteoroid by minimizing this expression.

\[
Q(\alpha, \beta) = \sum_{i=1}^{n} (F_i(y_i, v_i, \alpha, \beta))^2
\]

\[
F_i(y_i, v_i, \alpha, \beta) = 2\alpha \exp(-y_i) - \Delta_i \exp(-\beta) = 0
\]

From Eq.[3], at the point were \(m=0\) we have:

\[
h_I = h_0 y_t = h_0 \ln 2\alpha\beta \quad [5]
\]

If we reorder Eq.[4] we have:

\[
h_{II} = h_0 y_t = h_0 \ln \left( \frac{2\alpha\beta}{1 - e^{\beta(v_i^2-1)}} \right) \quad [6]
\]
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Results

\[ h_I = h_0 y_t = h_0 \ln 2\alpha \beta \]

\[ h_{II} = h_0 y_t = h_0 \ln \left( \frac{2\alpha \beta}{1 - e^{\beta(y_t^2 - 1)}} \right) \]

\[ \sigma = 4.11 \text{Km} \]

\[ \sigma = 1.52 \text{Km} \]
Differences could be due to the use of eq. [4] established for high $\beta$.

As suggested by Gritsevich et al. (2015), we would rather use the modification: $\beta \rightarrow \beta - 1.1$

$$h_{III} = h_0 y_t = h_0 \ln \left( \frac{2\alpha (\beta - 1.1)}{1 - e^{(\beta - 1.1)(v_t^2 - 1)}} \right)$$ [7]
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**Results (III)**

\[ h_I = h_0 y_t = h_0 \ln 2\alpha \beta \]

\[ \sigma = 4.11 \text{ Km} \]

\[ h_{II} = h_0 y_t = h_0 \ln \left( \frac{2\alpha \beta}{1 - e^{\beta (v_i^2 - 1)}} \right) \]

\[ \sigma = 1.52 \text{ Km} \]

\[ h_{III} = h_0 y_t = h_0 \ln \left( \frac{2\alpha (\beta - 1.1)}{1 - e^{(\beta - 1.1)(v_i^2 - 1)}} \right) \]

\[ \sigma = 0.75 \text{ Km} \]
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Discussion

For $\beta >5$ eq. [5] shall give good results. But the decrease in $v$ near the terminal point is not considered.

Eq.[6] shows a lineal tendency. Still discrepancies in all $\beta$ values.

The modification made in [7] leads to a good agreement between observed and theoretical data. However at low $\beta$, some discrepancies appear.

The small discrepancies at low $\beta$ shall be taken into account for future planetary defense applications. Meteoroids could reach lower height than those predicted.

The inverse problem is possible for non decelerated bodies if the terminal height is known $\rightarrow$ constraints in $\alpha$ and $\beta$.

Meteor height as a function of time $\rightarrow$ new problems may be scoped:
- Determination of luminous efficiency based on meteor duration.
- Critical Kinetic Energy to produce luminosity.

We use dimensionless parameters instead of the empirical set A,B, C coefficients of Ceplecha and McCrosky (1976). However $\alpha$ and $\beta$ keep the same variable dependency as the PE criterium.
| New methodology to determine the terminal height of a fireball | Conclusions |
|---------------------------------------------------------------|-------------|
| • We have derived the terminal heights for MORP fireballs using a new developed methodology. | • We have derived the terminal heights for MORP fireballs using a new developed methodology. |
| • This methodology had only been tested on several fully ablated fireballs with large $\beta$ values (Gritsevich and Popelinskaya, 2008). | • We were particularly interested in determining whether this new mathematical approach works equally for fully ablated fireballs and meteorite-producing ones. |
| • We were particularly interested in determining whether this new mathematical approach works equally for fully ablated fireballs and meteorite-producing ones. | • We introduced a new modification in the methodology which allows to get a higher accuracy. |
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| • We foresee a calculation of terminal height to be useful when the lower part of the trajectory was not instrumentally registered. | • It also brings critical knowledge into the problem when one needs to predict how long will be a total duration of the luminous flight or at which height a fireball produced by a meteoroid with given properties would terminate. |
| • It also brings critical knowledge into the problem when one needs to predict how long will be a total duration of the luminous flight or at which height a fireball produced by a meteoroid with given properties would terminate. | • Based on our investigations we can highly recommend the use of equation [7] also to solve inverse problem when terminal height and velocity are available from the observations, and parameters $\alpha$ and $\beta$ need to be derived. |
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mmoreno@ice.csic.es, trigo@ice.csic.es, maria.gritsevich@nls.fi

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