Field theory for gravity at all scales

Michele Levi
Institut de Physique Théorique, CEA & CNRS, Université Paris-Saclay
91191 Gif-sur-Yvette, France
michele.levi@ipht.fr

We review here the main advances made by using effective field theories (EFTs) in classical gravity, with notable focus on those unique to the EFTs of post-Newtonian (PN) gravity. We then proceed to overview the various prospects of using field theory to study the real-world gravitational wave (GW) data, as well as to ameliorate our fundamental understanding of gravity at all scales, by going from the EFTs of PN gravity to modern advances in scattering amplitudes, including computational techniques and intriguing duality relations between gauge and gravity theories.

Keywords: Effective field theories; Theories of gravity; Duality of gauge and gravity theories; Scattering amplitudes; High precision calculations; Gravitational waves.

1. Introduction

In recent years there has been a growing recognition in the all-encompassing relevance of the field theoretic framework beyond the quantum realm of particle physics, in which much of it has been conceived. Most notably, for long, in the absence of a viable quantum theory of gravity, the overlap between field theory and classical gravity has been largely deemed nonexistent. In particular, the conceptual framework of effective field theories (EFTs), which is intimately tied with the notion of renormalization, was widely regarded as uniquely ingrained in the domain of quantum field theories (QFTs). In order to dispel the latter fallacy, however, suffice it to acknowledge that ‘renormalization’ can in fact be taken as synonymous with ‘resolution of physical scales’. Once this realization is made, it becomes clear that the framework of EFTs is evidently a universal one, and that it can be applied on any wide range of physical scales. Moreover, the EFT framework is a solid and powerful one, providing an inherent setup strategy and a robust toolbox, geared towards yielding high precision predictions to a desirable accuracy.

Indeed, just over a decade ago, an EFT approach to handle gravitational waves (GWs), emitted from inspiralling compact binaries, which are analytically treated with post-Newtonian (PN) gravity, was put forward by Goldberger et al.\textsuperscript{2–4}. This novel approach has resulted, in turn, further applications of EFTs in classical gravity. Notably, significant progress was demonstrated in the application of EFTs to higher dimensional gravity in the context of large extra dimensions\textsuperscript{5–9}. Further, also in the context of GWs, an investigation of an EFT treatment for the extreme mass ratio inspiral (EMRI) case of compact binaries, was initiated in Ref. 10. Moreover, within a decade the initial EFT approach to PN gravity has been able – in important sectors of the theory – to catch up and go beyond the spectacular state of the art\textsuperscript{11}, already accomplished within the traditional framework of General Relativity (GR).
In what follows we highlight the main meaningful unique advances made in the EFT approach to PN gravity, yet more importantly, we point to the main prospects on a broader scope of using field theory to study gravity, stretching from delivering highly demanding accurate predictions for real-world GW data, to confronting our fundamental grasp of QFTs and gravity theories at all scales.

2. From Gravitational Waves To Gravity At All Scales

Significant progress was made via the EFT approach to PN gravity, which is concerned with the analytical prediction of the inspiral portion of the GW signal. While this progress is evident and encouraging, it crucially serves to illustrate the great potential, still awaiting, in the use of field theory to study gravity, with many implications at various levels. First, there are clear anticipated developments still within the EFTs of PN gravity and the theoretical modeling of the GW signal. Further, even if still only in the context of GWs, modern advances in scattering amplitudes hold the promise to possibly enable an analytical treatment of the GW signal in its entirety, that is also in the strong field regime, beyond the inspiral phase. This may significantly improve upon the current phenomenological GW modeling, which relies heavily on the effective-one-body (EOB) formulation\cite{EOB}, by allowing for a smooth analytical model of the whole signal. Yet, more broadly, these advances from scattering amplitudes also importantly allow us to study gauge and gravity theories at a fundamental level, across the classical and quantum regimes.

Let us first review the main unique advancements accomplished within the EFTs of PN gravity so far. The most notable and extensive progress was realized in the treatment of the spinning sector, with several new higher order PN corrections. Next-to-leading order (NLO) effects in the conservative sector were first tackled in Refs.\ 13, 14, 15, following Refs.\ 16, 14, and further resolved and extended to the current state of the art, at the fourth PN (4PN) order for rapidly rotating compact objects, in Refs.\ 17, 18, 19, 20, 21, 22, 23, following Refs.\ 17, 24, 20, 18. Further, partial results at NLO in the radiative sector were presented in Ref.\ 25. Moreover, following Ref.\ 26, which also led to a new finite size correction of radiation reaction in classical electrodynamics\cite{ RCS}, and Refs.\ 28, 29, a leading order effect of radiation reaction with spins was obtained in Ref.\ 30. All in all, beyond the specific new PN results obtained with spins, there has been an improvement in the understanding of classical spins in gravity.

Another remarkable finding in the field was the uncovering of classical renormalization group (RG) flows of the Wilson coefficients, which characterize the effective theories, both at the one-particle level\cite{RG1}, and at the level of the composite system, for the multipole moments of the binary\cite{RG2}. These give rise to higher order PN logarithm corrections, which constitute unique predictions of the EFT framework. In addition, the 2PN order correction for a generic n-body problem was considered in Ref.\ 33, where automated computations were first advocated. Finally, the ‘EFTofPNG’ code for high precision Feynman calculations in PN gravity was cre-
ated, comprising the first comprehensive code in PN theory made public. The code consists of independent modules for easy adaptation and future development, where some prospects of building on this state of the art PN technology for various research purposes, were outlined in Ref. 35.

Within the methodology of the EFTs of PN gravity, there are several important directions, where development is required. The first is the solution of higher order PN equations of motion (EOMs) for accurate orbital dynamics. One interesting related way to find such closed form solutions, is the dynamical RG method, suggested in Ref. 36. Further, the treatment of non-conservative sectors with the EFTs of PN gravity has been rather limited, and so these various sectors should be tackled with a proper EFT formulation, and implementation for new PN results. Another objective to follow is the possible improvement or extension of the EFT formulation for spinning objects. For example, an alternative EFT formulation, which is ideally suited for the treatment of slowly rotating objects, based on a coset construction, was presented in Ref. 37. Further investigation is required in order to see whether these independent EFT formulations may possibly be integrated, and entail an even better understanding of classical spins in gravity. Next, we note the analysis of the mass- and spin-induced Wilson coefficients, and of the binary multipole coefficients, which also mostly remains to be approached via formal EFT matching. Finally, the continual public development of the ‘EFTofPNG’ code, or similar open codes, is required in order to keep up to date with the actual progress in PN theory. In that regard, it is important to note that classical theories of gravity, which modify GR in the IR, motivated by the cosmological constant problem and the dark matter puzzle, can be incorporated in a rather straightforward manner into the ‘EFTofPNG’ code, in order to study their analytical predictions for the GW signal from the compact binary inspiral, see, e.g., Refs. 42, 43, 44.

2.1. Advances and prospects for gravity in field theory

Ultimately, we would like to invoke the correspondence – at any level – between gauge and gravity theories, in order to push beyond the state of the art in high precision computation for concrete analytical predictions of the real-world GW signal, as well as to directly ameliorate our fundamental understanding of the foundations of these theories, in particular of gravity. These objectives can be considerably facilitated by turning to modern field theory advances in the domain of scattering amplitudes, see, e.g., Ref. 45. In fact, the field of EFTs in PN gravity, on which we have focused hitherto, and of scattering amplitudes, which are both directly concerned with baldly tackling demanding high precision computation driven by experiment, share more profound parallels beyond the obvious robust technical tools they entail, see, e.g., in Ref. 46. Both of these fields push to the exposure of the universal commonalities across classical and quantum field theories, and drive us to confront our fundamental grasp of the underlying foundations of these theories. One crucial difference to note, though, between these fields, is that where in
scattering amplitudes the computational outcome is directly related with the physical observables, the objects which are commonly computed in classical gravity are coordinate/gauge dependent quantities.

Let us then note, more specifically, some prospective avenues to deploy this broad correspondence of gauge theories and gravity. First, there is the standard working knowledge, which can be exchanged between the theories, e.g. multi-loop techniques in QFT, such as integration by parts (IBP), and high loop master integrals, along with other Feynman calculus and technology. This was nicely demonstrated, e.g., in Refs. 47, 48, where an analytic evaluation of a four-loop master integral was provided for the first time from the classical gravity context. Furthermore, modern scattering amplitudes advances, such as the BCFW on-shell recursion relations, and generalized unitarity methods, which imply that tree level data encodes all multiplicity at the integrand level, were put forward to extract classical higher order loop results for gravitational scattering. Such a scattering treatment may also enable to analytically tackle the strong field regime of the GW signal, and hence smoothly model analytically the entire signal, as we noted above.

Yet, of particular interest are the novel intriguing color-kinematics or BCJ duality relations, and the related double copy correspondence, see, e.g., review in Ref. 56, which were discovered in the context of high loop computations of amplitudes in supersymmetry and supergravity theories. Such relations were already known from string theory to hold at tree level, but formulated in terms of the novel generic double copy correspondence, these relations have been successfully used to study UV divergences of supergravity theories. This recent new perspective on gravity, viewing gravitons as double copies of gluons, suggests that even the simplest gauge theories and GR are in fact frameworks, which are intimately connected. Though this correspondence was uncovered in the perturbative context, it was also found, remarkably, that particular classical exact solutions in GR, are in fact double copies of exact “single copy” counterparts in corresponding gauge theories. Such classical solutions include all vacuum stationary solutions, most notably, Kerr black holes, as well as their higher dimensional generalizations, see Ref. 57 and references therein. Yet, the underlying origin of these perturbative relations, as well as their essential connection to the particular classical exact correspondence revealed, are yet to be uncovered.

In conjunction with the recent novel methods from scattering amplitudes, it is expected that this double copy correspondence can be used to advance the analytical calculations for the theoretical prediction of the GW signal, which may also help to shed more light on the nature and origin of this correspondence. Related with that end, an important time dependent case was studied in terms of the exact double copy of the Kerr-Schild form, of an arbitrarily accelerating point source. In this case the radiation current is double copied to the radiation stress-energy tensor in Fourier space, and both are related to the corresponding scattering amplitudes, which can be obtained from the amputated currents. Hence, the work in Ref. 57
made a first explicit connection between the classical double copy in an exact form, to that in the perturbative scattering amplitudes context. At this point it should also be stressed, that from the scattering amplitudes context, it is already known that gluon amplitudes can double copy to arbitrary combinations of amplitudes for gravitons, and the additional unobserved dilaton and B fields, depending on the choice of the polarization states in the gauge theory amplitudes. This was indeed an ambiguous issue in Ref. 58, which built on Ref. 57, and that subsequently Ref. 59 set out to address, with Ref. 60 following up successfully at NLO.

Finally, we note the topic of soft graviton theorems for scattering amplitudes, which were recently demonstrated to be equivalent to gravitational memory effects, as part of a triple equivalence of the IR structure of gauge and gravity theories among soft theorems, asymptotic symmetry, and memory effects. The latter may have observable signatures on the GW signal.

Acknowledgments

I am grateful to John Joseph Carrasco for his meaningful encouragement. I would like to thank Donato Bini for the warm hospitality throughout the MG15 meeting, and in particular on the session, where this review was presented. It is also a delightful pleasure to acknowledge Roy Kerr who graced us with his inspiring presence. My work is supported by the European Research Council under the European Union’s Horizon 2020 Framework Programme FP8/2014-2020 “preQFT” grant no. 639729, “Strategic Predictions for Quantum Field Theories” project.

References

1. L. Blanchet, Gravitational Radiation from Post-Newtonian Sources and Inspiraling Compact Binaries, Living Rev.Rel. 17 (2014).
2. W. D. Goldberger and I. Z. Rothstein, An Effective field theory of gravity for extended objects, Phys.Rev. D73 (2006) 104029.
3. W. D. Goldberger, Les Houches lectures on effective field theories and gravitational radiation, in Les Houches Summer School - Session 86: Particle Physics and Cosmology: The Fabric of Spacetime Les Houches, France, July 31-August 25, 2006, 2007.
4. W. D. Goldberger and A. Ross, Gravitational radiative corrections from effective field theory, Phys.Rev. D81 (2010) 124015.
5. Y.-Z. Chu, W. D. Goldberger, and I. Z. Rothstein, Asymptotics of d-dimensional Kaluza-Klein black holes: Beyond the Newtonian approximation, JHEP 03 (2006) 013.
6. B. Kol and M. Smolkin, Classical Effective Field Theory and Caged Black Holes, Phys. Rev. D77 (2008) 064033.
7. R. Emparan, T. Harmark, V. Niarchos, and N. A. Obers, World-Volume Effective Theory for Higher-Dimensional Black Holes, Phys. Rev. Lett. 102 (2009) 191301.
8. J. B. Gilmore, A. Ross, and M. Smolkin, Caged black hole thermodynamics: Charge, the extremal limit, and finite size effects, JHEP 09 (2009) 104.
9. R. Emparan, T. Harmark, V. Niarchos, and N. A. Obers, Essentials of Blackfold Dynamics, JHEP 03 (2010) 063.
10. C. R. Galley and B. Hu, Self-force on extreme mass ratio inspirals via curved spacetime effective field theory, Phys.Rev. D79 (2009) 064002.
11. M. Levi, arXiv:1807.01699 Effective Field Theories of Post-Newtonian Gravity: A comprehensive review, 2018.
12. A. Buonanno and T. Damour, Effective one-body approach to general relativistic two-body dynamics, Phys.Rev. D59 (1999) 084006.
13. R. A. Porto and I. Z. Rothstein, The Hyperfine Einstein-Infeld-Hoffmann potential, Phys.Rev.Lett. 97 (2006) 021101.
14. R. A. Porto and I. Z. Rothstein, Spin(1)Spin(2) Effects in the Motion of Inspiralling Compact Binaries at Third Order in the Post-Newtonian Expansion, Phys.Rev. D78 (2008) 044012, [Erratum-ibid. D81 (2010) 029904].
15. R. A. Porto and I. Z. Rothstein, Next to Leading Order Spin(1)Spin(1) Effects in the Motion of Inspiralling Compact Binaries, Phys.Rev. D78 (2008) 044013, [Erratum-ibid. D81 (2010) 029905].
16. R. A. Porto, Post-Newtonian corrections to the motion of spinning bodies in NRGR, Phys.Rev. D73 (2006) 104031.
17. M. Levi, Next to Leading Order gravitational Spin1-Spin2 coupling with Kaluza-Klein reduction, Phys.Rev. D82 (2010) 064029.
18. M. Levi and J. Steinhoff, Spinning gravitating objects in the effective field theory in the post-Newtonian scheme, JHEP 09 (2015) 219.
19. M. Levi, Binary dynamics from spin1-spin2 coupling at fourth post-Newtonian order, Phys.Rev. D85 (2012) 064043.
20. M. Levi and J. Steinhoff, Equivalence of ADM Hamiltonian and Effective Field Theory approaches at next-to-next-to-leading order spin1-spin2 coupling of binary inspirals, JCAP 1412 (2014) 003.
21. M. Levi and J. Steinhoff, Leading order finite size effects with spins for inspiralling compact binaries, JHEP 06 (2015) 059.
22. M. Levi and J. Steinhoff, Next-to-next-to-leading order gravitational spin-squared potential via the effective field theory for spinning objects in the post-Newtonian scheme, JCAP 1601 (2016) 008.
23. M. Levi and J. Steinhoff, arXiv:1607.04252 Complete conservative dynamics for inspiralling compact binaries with spins at fourth post-Newtonian order, 2016.
24. M. Levi, Next to Leading Order gravitational Spin-Orbit coupling in an Effective Field Theory approach, Phys.Rev. D82 (2010) 104004.
25. R. A. Porto, A. Ross, and I. Z. Rothstein, Spin induced multipole moments for the gravitational wave flux from binary inspirals to third Post-Newtonian order, JCAP 1103 (2011) 009.
26. C. R. Galley and M. Tiglio, *Radiation reaction and gravitational waves in the effective field theory approach*, Phys. Rev. D79 (2009) 124027.
27. C. R. Galley, A. K. Leibovich, and I. Z. Rothstein, *Finite size corrections to the radiation reaction force in classical electrodynamics*, Phys. Rev. Lett. 105 (2010) 094802.
28. C. R. Galley, *Classical Mechanics of Nonconservative Systems*, Phys. Rev. Lett. 110 (2013) 174301.
29. C. R. Galley, D. Tsang, and L. C. Stein, [arXiv:1412.3082](https://arxiv.org/abs/1412.3082) *The principle of stationary nonconservative action for classical mechanics and field theories*, 2014.
30. N. T. Maia, C. R. Galley, A. K. Leibovich, and R. A. Porto, *Radiation reaction for spinning bodies in effective field theory II: Spin-spin effects*, Phys. Rev. D96 (2017) 084065.
31. B. Kol and M. Smolkin, *Black hole stereotyping: Induced gravito-static polarization*, JHEP 02 (2012) 010.
32. W. D. Goldberger, A. Ross, and I. Z. Rothstein, *Black hole mass dynamics and renormalization group evolution*, Phys. Rev. D89 (2014) 124033.
33. Y.-Z. Chu, *The n-body problem in General Relativity up to the second post-Newtonian order from perturbative field theory*, Phys. Rev. D79 (2009) 044031.
34. M. Levi and J. Steinhoff, EFTofPNG: A package for high precision computation with the Effective Field Theory of Post-Newtonian Gravity, Class. Quant. Grav. 34 (2017) 244001.
35. M. Levi, [arXiv:1811.12401](https://arxiv.org/abs/1811.12401) *A public framework for Feynman calculations and post-Newtonian gravity*, 2018.
36. C. R. Galley and I. Z. Rothstein, *Deriving analytic solutions for compact binary inspirals without recourse to adiabatic approximations*, Phys. Rev. D95 (2017) 104054.
37. L. V. Delacrétaz, S. Endlich, A. Monin, R. Penco, and F. Riva, (Re-)Inventing the Relativistic Wheel: Gravity, Cosets, and Spinning Objects, JHEP 11 (2014) 008.
38. T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, *Modified Gravity and Cosmology*, Phys. Rept. 513 (2012) 1–189.
39. C. Deffayet and D. A. Steer, *A formal introduction to Horndeski and Galileon theories and their generalizations*, Class. Quant. Grav. 30 (2013) 214006.
40. C. de Rham, *Massive Gravity*, Living Rev. Rel. 17 (2014) 7.
41. A. Joyce, B. Jain, J. Khoury, and M. Trodden, *Beyond the Cosmological Standard Model*, Phys. Rept. 568 (2015) 1–98.
42. Y.-Z. Chu and M. Trodden, *Retarded Greens function of a Vainshtein system and Galileon waves*, Phys. Rev. D87 (2013) 024011.
43. C. de Rham, A. J. Tolley, and D. H. Wesley, *Vainshtein Mechanism in Binary Pulsars*, Phys. Rev. D87 (2013) 044025.
44. C. de Rham, A. Matas, and A. J. Tolley, *Galileon Radiation from Binary Sys-
tems, Phys. Rev. D87 (2013) 064024.
45. H. Elvang and Y. t. Huang, Scattering Amplitudes in Gauge Theory and Gravity (Cambridge University Press, 2015).
46. V. A. Smirnov, Analytic tools for Feynman integrals, Springer Tracts Mod. Phys. 250, 1 (2012).
47. S. Foffa, P. Mastrolia, R. Sturani, and C. Sturm, Effective field theory approach to the gravitational two-body dynamics, at fourth post-Newtonian order and quintic in the Newton constant, Phys. Rev. D95 (2017) 104009.
48. T. Damour and P. Jaranowski, Four-loop static contribution to the gravitational interaction potential of two point masses, Phys. Rev. D95 (2017) 084005.
49. R. Britto, F. Cachazo, and B. Feng, New recursion relations for tree amplitudes of gluons, Nucl. Phys. B715 (2005) 499–522.
50. N. E. J. Bjerrum-Bohr, J. F. Donoghue, and P. Vanhove, On-shell Techniques and Universal Results in Quantum Gravity, JHEP 02 (2014) 111.
51. F. Cachazo and A. Guevara, arXiv:1705.10262 Leading Singularities and Classical Gravitational Scattering, 2017.
52. A. Guevara, arXiv:1705.02314 Holomorphic Classical Limit for Spin Effects in Gravitational and Electromagnetic Scattering, 2017.
53. N. E. J. Bjerrum-Bohr, P. H. Damgaard, G. Festuccia, L. Plante, and P. Vanhove, General Relativity from Scattering Amplitudes, Phys. Rev. Lett. 121 (2018) 171601.
54. Z. Bern, J. J. M. Carrasco, and H. Johansson, New Relations for Gauge-Theory Amplitudes, Phys. Rev. D78 (2008) 085011.
55. Z. Bern, J. J. M. Carrasco, and H. Johansson, Perturbative Quantum Gravity as a Double Copy of Gauge Theory, Phys. Rev. Lett. 105 (2010) 061602.
56. J. J. M. Carrasco, Gauge and Gravity Amplitude Relations, in Theoretical Advanced Study Institute in Elementary Particle Physics: Journeys Through the Precision Frontier: Amplitudes for Colliders (TASI 2014): Boulder, Colorado, June 2-27, 2014, pp. 477–557, WSP, 2015.
57. A. Luna, R. Monteiro, I. Nicholson, D. O’Connell, and C. D. White, The double copy: Bremsstrahlung and accelerating black holes, JHEP 06 (2016) 023.
58. W. D. Goldberger and A. K. Ridgway, Radiation and the classical double copy for color charges, Phys. Rev. D95 (2017) 125010.
59. A. Luna, I. Nicholson, D. O’Connell, and C. D. White, Inelastic Black Hole Scattering from Charged Scalar Amplitudes, JHEP 03 (2018) 044.
60. C.-H. Shen, Gravitational Radiation from Color-Kinematics Duality, JHEP 11 (2018) 162.
61. A. Strominger, arXiv:1703.05448 Lectures on the Infrared Structure of Gravity and Gauge Theory, 2017.