Spin pumping and magnetization dynamics in ferromagnet-Luttinger liquid junctions

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We study spin transport between a ferromagnet (FM) with time-dependent magnetization and a conducting carbon nanotube or quantum wire, modeled as a Luttinger liquid (LL). The precession of the magnetization vector of the ferromagnet due for instance to an outside applied magnetic field causes spin pumping into an adjacent conductor. Conversely, the spin injection causes increased magnetization damping in the ferromagnet. We find that, if the conductor adjacent to the ferromagnet is a Luttinger liquid, spin pumping/damping is suppressed by interactions, and the suppression has clear Luttinger liquid power law temperature dependence. We apply our result to a few particular setups. First we study the effective Landau-Lifshitz-Gilbert (LLG) coupled equations for the magnetization vectors of the two ferromagnets in a FM-LL-FM junction. Also, we compute the Gilbert damping for a FM-LL and a FM-LL-metal junction.

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I. INTRODUCTION

In the past years there has been a lot of interest in the field of spintronics, due especially to the many possible applications of spin-polarized transport. However, practical realization of devices in which spin can be well controlled and which allow for large stable spin currents is difficult. For instance, spin injection from magnetic to normal materials is reduced due to a large resistivity mismatch, while the spin-polarized currents obtained through optical pumping are small. Following the ideas of adiabatic charge pumping in mesoscopic systems, Y. Tserkovnyak et. al. proposed a new method of obtaining spin polarized currents through injection of spin from a ferromagnet with time dependent magnetization into a normal metal\cite{4,5,6}. The advantage of this method lies in the possibility of direct significant spin current injection. The authors also showed that spin pumping has important experimentally observable effects on the dynamics of the ferromagnet. In general, the dynamics of a magnetization vector in a ferromagnet is described by the LLG equation\cite{7}. This includes a term characterizing the precession of the magnetization vector in an external magnetic field, as well as damping terms which describe the reduction of the magnetization due to spin-flip processes, etc. The coefficient of the damping term is called “Gilbert damping”, and can be determined experimentally. The spin loss in the ferromagnet due to spin pumping, generates a renormalization of the coefficients in the LLG equation. Of most experimental interest is the renormalization of the Gilbert damping coefficient in the LLG equation. The magnitude of this effect is estimated in Refs.\cite{4} and\cite{5} for spin pumping from a FM into a normal metal.

In this work we focus on spin pumping from a ferromagnet into a Luttinger liquid. For this setup other theoretically interesting questions also arise, like what is the effect of the electron-electron interactions in the LL on the injection of spin, and how the interactions will affect the renormalization of parameters such as the Gilbert damping. We find that the spin injection and the renormalization of the Gilbert damping are suppressed due to interactions, and that the suppression has a power law dependence on temperature. The study of this setup is also important due to its possible applications; given the excellent transport properties of some LL’s, especially of carbon nanotubes, this setup would open the perspective of transporting the injected spin over large distances without important losses. The study of spin pumping into a Luttinger liquid is also of more general interest as a prototype for the effect of Coulomb interactions (i.e. charging effects) on spin injection.

In section II we compute the spin current pumped into a LL from a ferromagnet with a precessing magnetization. In section III we apply this result to study a FM–LL–FM junction assuming that the transport in the LL is ballistic. In section IV we compute the renormalization of the Gilbert damping parameter due to spin pumping from a FM into a finite size Luttinger liquid with diffusive transport. In section V we repeat this analysis for a FM–LL–metal junction assuming that the transport in LL is ballistic. We discuss the results and conclude in section VI.

II. SPIN PUMPING BETWEEN A FERROMAGNET WITH TIME DEPENDENT MAGNETIZATION AND A LUTTINGER LIQUID

We focus first on the spin current pumped from a FM into a LL as a result of the precession in the FM’s magnetization vector. We assume almost perfect backscattering, and a small amount of tunneling between the LL and the FM. The corresponding boundary conditions are \( F_R \) (\( \chi = 0 \)) = \( L \), \( F_L \) (\( \chi = 0 \)) = \( / e^l \), (\( \chi = 0 \), and similarly \( F_R \) (\( \chi = 0 \)) = \( F_L \) (\( \chi = 0 \)) = \( / e^l \) (\( \chi = 0 \)). Here the operators \( F_L = R \) correspond to the annihilation of a left/right moving electron with spin \( = \# \) in the LL/FM, while \( \# \) and \( \# \) respectively are the corresponding bosonized operators in the LL/FM sectors. The effective actions in bosonized variables for the LL and the FM are given by

\[
S_{LL} = \sum_{n} \left( g_n \gamma_{n}^{a} \gamma_{a}^{\dagger} \right) (n_{L}) \right) ;
\]

\[
S_{FM} = \sum_{n} \left( g_n \gamma_{n}^{a} \gamma_{a}^{\dagger} \right) (n_{L}) \right) ; \quad (1)
\]
where \( \theta = (\ast \#) = 2 \), \( g = 1 \) and \( \theta = g \) is the interaction parameter in the LL.

In terms of the fermionic operators the tunneling Hamiltonian can be described by

\[
H_{\text{tun}} = \sum_{\tau} \left[ i (\psi_{\tau}^\dagger F_{\tau}^\gamma \psi_{\tau}^\gamma + h.c.) \right. \\
+ \sum_{\tau} \left( \psi_{\tau}^\dagger F_{\tau}^\gamma \psi_{\tau}^\gamma \right) + i \sum_{\tau} \left( \psi_{\tau}^\dagger h x z \right)
\]

where \( \tau \) is the time dependent magnetization in the ferromagnet in units of \( c \).

The spin current operator corresponding to tunneling into the Luttinger liquid is

\[
I = \frac{d}{dt} \left( \int d\kappa \psi_{\kappa}^\dagger \psi_{\kappa} \right)
\]

where \( \kappa \) is the spin density in the LL at position \( \kappa \). Consequently this can be written as

\[
I = \frac{1}{2} \left( \int d\kappa \psi_{\kappa}^\dagger F_{\kappa}^\gamma \psi_{\kappa}^\gamma + h x z \right)
\]

where sums over repeated spin indices \( \kappa \) are implied.

We compute the spin current using the time dependent perturbation theory

\[
I(t) = \int dt C(t) \left( \psi_{\tau}^\dagger h x z \psi_{\tau}^\gamma \right)
\]

and for \( t = = k_B T \)

\[
\int C(\kappa) d\kappa / \int C(\kappa) d\kappa = \frac{\kappa}{2 k_B T}
\]

For temperatures of the order of \( 10 K \) and reasonable frequencies achievable experimentally we can safely assume that we are in the limit of \( t = = k_B T > 1 \), so that we can use the first expression above. Consequently the injected spin current has the same form as for non-interacting fermions, but with temperature dependent coefficients.

\[
I(t) = A_1 \frac{\kappa}{2} \left( \psi_{\tau}^\dagger h x z \psi_{\tau}^\gamma \right)
\]

where \( A_1 = \frac{1}{2} \psi_{\tau}^\dagger h x z \psi_{\tau}^\gamma \) and \( A_2 = \frac{\kappa}{2} \left( \psi_{\tau}^\dagger h x z \psi_{\tau}^\gamma \right) \).

This is the main result of this paper. In the following sections we will apply this result to a few particular setups.

**III. FERROMAGNET – LUTTINGER LIQUID – FERROMAGNET JUNCTION**

We first consider a ferromagnet – Luttinger liquid – ferromagnet junction. The magnetizations of the two ferromagnets
are time dependent, and spin currents are injected into the wire from both ends.

![Diagram](image)

**FIG. 1:** Ferromagnet-Luttinger liquid-ferromagnet junction

We assume that the two contacts are identical, and that the transport through the wire is ballistic, so that the spin current $\mathbf{I}$ and the spin chemical potential $\mathbf{\mu}_s$ are uniform throughout the wire.

The coupled Landau-Lifshitz-Gilbert equation for the magnetization vectors in the two ferromagnets can be written as:

$$\frac{d \mathbf{m}_i}{dt} = \mathbf{m}_i \times \mathbf{H}_i + \gamma \mathbf{m}_i \times \mathbf{m}_i \times \mathbf{I} \quad (10)$$

where $\gamma = g_0 B$ is the gyromagnetic ratio, $B$ is the Bohr magneton, $g_0$ is the dimensionless bulk Gilbert damping constant and $M_s$ is the ferromagnetic saturation magnetization. The index $i = 1, 2$ characterizes the two ferromagnets. Our convention is that spin current flows from FM 1 to FM 2. Thus the sign in front of the term proportional to the spin current in Eq. (10) corresponds to spin injection from FM 1 into the wire, while the + sign corresponds to spin current flowing from the wire into FM 2.

The total spin current flowing through the wire $\mathbf{I} = \mathbf{I}_1^2 + \mathbf{I}_2^2$ has two contributions. The terms $\mathbf{I}_0^1$ denote the spin currents pumped from the two ferromagnets solely as a result of the time dependence in the magnetization vectors. The general form for these currents has been derived in the previous section. As noted, their magnitudes depend only on the nature of the junctions, and are independent of setup details. The second contribution, the backscattered currents $\mathbf{I}_b^1$, are the result of a spin flow back to the ferromagnets due to spin accumulation in the Luttinger liquid. Since how much spin accumulates on the wire depends on various setup parameters, the backscattered currents also depend on various parameters such as the length of the wire, the spin flip time in the wire, etc. In general, as has been derived in Refs. [5] and [11],

$$\mathbf{I}_b^1 = \frac{1}{4} \mathbf{I}_1 + \mathbf{I}_2 \quad (\mathbf{s}, \mathbf{I}_0)$$

Here is a dimensionless parameter which measures the exchange coupling between the FM and the LL. Let

$$\mathbf{I} = \left( \frac{\mathbf{I}_1}{\mathbf{K}_{10}^0} + \frac{\mathbf{I}_2}{\mathbf{K}_{20}^0} \right) \delta \mathbf{K}_{10}^0$$

subject to identical magnetic fields $\mathbf{H}_1 = \mathbf{H}_2$, and linearizing the LLG equations we obtain

$$\mathbf{I} = \frac{1}{4} \mathbf{I}_1 + \mathbf{I}_2 \quad (\mathbf{s}, \mathbf{I}_0)$$

where we assumed $m_1^2 = 1$ and $m_2^2 = 1$. A tractable limit of the problem is the case of negligible exchange terms, $s = 0$. A detailed analysis of this situation is given in Appendix A. Assuming that the two ferromagnets are

$$\mathbf{I} = \frac{1}{4} \mathbf{I}_1^1 + \mathbf{I}_2^1 \quad (\mathbf{s}, \mathbf{I}_0)$$

$$\mathbf{I}_1^1 = \frac{1}{4} \mathbf{I}_1 + \mathbf{I}_2 \quad (\mathbf{s}, \mathbf{I}_0)$$

$$\mathbf{I}_2^1 = \frac{1}{4} \mathbf{I}_1 + \mathbf{I}_2 \quad (\mathbf{s}, \mathbf{I}_0)$$
The first term contain two terms, each of them being a superposition of precession and exponential decay. The first term ($m_1 + m_2$) is not affected in any way by the Luttinger liquid physics, while in the second term ($m_1$ and $m_2$) the time constants for precession and decay are modified due to the LL interactions. In particular, power law dependencies on temperature are added to these coefficients through the terms.

where $1_{i=2} = A_{1=2} = 2M_{\sigma i}$, and we assumed the initial conditions, $m_{1=2} = m_{1=2}$ and $m_{1=2} = 0$. We note that the solutions contain two terms, each of them being a superposition of precession and exponential decay. The first term ($m_{1} + m_{2}$) is not affected in any way by the Luttinger liquid physics, while in the second term ($m_{1}$ and $m_{2}$) the time constants for precession and decay are modified due to the LL interactions. In particular, power law dependencies on temperature are added to these coefficients through the terms.

$$m_{1=2} = \frac{m_{1} + m_{2}}{2} \cos \frac{H t}{1 + \gamma} \exp \left( \frac{m_{1} + m_{2}}{2} \cos \left( \frac{(0 + 2 \gamma)^{2} + (1 + 2 \gamma)^{2}}{(0 + 2 \gamma^{2} + (1 + 2 \gamma)^{2}} \right) \right)$$

$$m_{1=2} = \frac{m_{1} + m_{2}}{2} \sin \frac{H t}{1 + \gamma} \exp \left( \frac{m_{1} + m_{2}}{2} \sin \left( \frac{(0 + 2 \gamma)^{2} + (1 + 2 \gamma)^{2}}{(0 + 2 \gamma^{2} + (1 + 2 \gamma)^{2}} \right) \right)$$

IV. RENORMALIZATION OF THE GILBERT DAMPING CONSTANT DUE TO THE SPIN FLOW BETWEEN A FERROMAGNET AND A LUTTINGER LIQUID

As before, the LLG equation for the magnetization in the FM can be written as

$$\frac{d\mathbf{m}}{dt} = \mathbf{m} \times \mathbf{H} + \mathbf{m} \times \mathbf{\Omega} \mathbf{m} \times \mathbf{T}$$

where, similar to the previous section, the spin current $\mathbf{I} = \mathbf{I}_0 - \mathbf{I}_b$ is the total current flowing out of the ferromagnet. The two contributions $\mathbf{I}_0$ and $\mathbf{I}_b$ correspond to the pumped and backscattered spin currents, and are described by Eqs. (9), and (11) with $m_1 = m$, respectively. Equation (15) can be rewritten as

$$\frac{d\mathbf{m}}{dt} = \mathbf{I} \times \mathbf{H} + \mathbf{I} \times \mathbf{\Omega} \mathbf{m} \mathbf{I} \times \mathbf{T}$$

where $\mathbf{I}$ and $\mathbf{\Omega}$ are renormalized constants. Our goal is to compute the total current $\mathbf{I}$ and consequently determine the coefficients $\mathbf{I}$ and $\mathbf{\Omega}$ for various setups.

In this section we compute these renormalized constants for a ferromagnet in contact with a LL of finite size $L$, and characterized by a spin flip time $\gamma_{sf}$. We assume that the transport in the LL at long wavelengths is diffusive.

A few comments are in order: if $\gamma_{sf}$ is very large, i.e. if the spin flip is negligible, after a long enough time the spin accumulation in the LL would be large, such that the current injected from the FM would equal the backscattered current, and the total current will be zero. If $\gamma_{sf}$ is very small, any spin accumulation in the LL is relaxed instantaneously, the backscattered current is negligible and the total current is equal to the injected current.

For a real system neither situation may be correct, and we take the spin flip time $\gamma_{sf}$ to be finite. Along the lines of Ref. 4, the diffusion equation for the spin in the LL is

$$\mathbf{I} \times \mathbf{\Omega} = \mathbf{D} \mathbf{K} \times \mathbf{\Omega}$$

with the boundary conditions $\mathbf{K} = (2 \gamma = D) \mathbf{K}$, at $x = 0$, and of vanishing spin current, $\mathbf{K} = 0$, at $x = L$. Here $D$ is the diffusion coefficient.

The coupled equations for $\mathbf{K} = (2 \gamma = D) \mathbf{K}$ and $\mathbf{I}$ can be solved (for details see Appendix B). The renormalized LLG equation coefficients are:

$$\frac{d\mathbf{m}}{dt} = \mathbf{m} \times \mathbf{H} + \mathbf{m} \times \mathbf{\Omega} \mathbf{m} \times \mathbf{T}$$

where $\mathbf{B}_1 = (1 + \mathbf{T}) = (1 + T \mathbf{I}) = (1 + \mathbf{T} \mathbf{I})$ and $\mathbf{B}_2 = (1 - \mathbf{T}) = (1 + T \mathbf{I}) + (1 + \mathbf{T} \mathbf{I})(\mathbf{I} + \mathbf{T} \mathbf{I})$. And $\mathbf{K} = (2 \gamma = D) \mathbf{K} \times \mathbf{\Omega}$, where $k = 1 - \frac{1}{D} \gamma_{sf}$.

The effects of the Luttinger liquid physics are best observed by estimating the renormalization of the Gilbert damping, and focusing on its temperature dependence. The spin pumping causes a negligible renormalization to $\gamma_{sf}$, as $\gamma_{sf} \ll \gamma_{sf}$.
In order to extract the temperature dependence of this correction, for illustrative purposes, we focus on the particular limit $T \rightarrow 1$, other limits can be explored as well. In this case the Gilbert damping shows a power law dependence in temperature:

$$
\frac{\alpha}{M} = \frac{T}{\hbar} \left[ \frac{C_1 A_2 - C_2 A_1}{C_1 A_2 + C_2 A_1} \right];
$$

where the constant $C = 1/2 + t_0 A_0 = 4 \left[ (1 + \frac{2}{2}) = 16 \frac{2}{2} \right]$, and $t_0$ and $t_2$ are temperature independent coefficients defined in relation to $A_1 = \theta = t_2 (T = 0)$, and $A_2 = \theta = t_2 (T = 0)$. The renormalization of the Gilbert damping is significant if $\frac{C}{M}$ is smaller than the correction term in Eq. (19). This can be achieved experimentally for some clean materials in which $\frac{C}{M}$ is small, and the experimental signature of this effect would be a temperature power law dependence of the measured Gilbert damping coefficient.

V. FERROMAGNET – LUTTINGER LIQUID – METAL JUNCTION

We now consider a setup consisting of a ferromagnet with time dependent magnetization in contact with a Luttinger liquid of length $L$. The Luttinger liquid is connected at the other end to a normal metal. We assume that the length of the wire is shorter than the mean free path for spin flip so that the transport in the wire is ballistic.

As before we can write down equations for the current flowing through the wire. The current flowing from the FM to the LL is described by the same equations as in the previous section, while the current at the LL-metal junction is described by Ohm's law. We assume that the transport in the wire is ballistic so that the spin chemical potential $\sim 0$ and the spin current $\tilde{t}$ are constant throughout the wire. In the metal we assume that the spin dynamics is dominated by diffusion with $D_m$ being the diffusion coefficient, and $t_s$ being the spin flip time. The details of the derivation are given in Appendix C. We find

$$
0 = \frac{t_s}{\hbar} \left[ \frac{C_1 A_2 - C_2 A_1}{C_1 A_2 + C_2 A_1} \right];
$$

Here $C_1 = a_1 = (a_1^2 + a_2^2)$, $C_2 = a_2 = (a_1^2 + a_2^2)$,

$$
a_1 = \frac{1 + \frac{T}{T_2} (q + T_2 M) \frac{1}{4}}{1 + \frac{T}{T_2} (q + T_2 M) \frac{1}{4}}; 
$$

$$
a_2 = \frac{1 + \frac{T}{T_2} (q + T_2 M) \frac{1}{4}}{1 + \frac{T}{T_2} (q + T_2 M) \frac{1}{4}}; 
$$

and $M = \frac{D_m}{n_s} \coth \left( \frac{q}{D_m n_s} \right) = n_s \frac{D_m}{n_s} = \frac{N_m}{D_m} = \frac{S}{D_m}$. Also $N_m$ is the density of states in the metal, $D_m$ is the diffusion coefficient in the metal, and $S$ is the cross-section of the metal. Again we find that the corrections to $\alpha$ are negligible. We can further assume that the metal is a good spin sink, $\frac{n_s}{D_m}$ is very small, and the term proportional to $\frac{1}{M}$ can be dropped. Consequently we can write

$$
0 = \frac{C_m}{M} \frac{T}{0} \frac{C_m}{M} \frac{T}{0}.
$$

For illustrative purposes we consider two limits. First, if $T = 0$ and $D = t_s = (1 + T_1 = T_2)$ is temperature dependent. Second, if $T_0$,

$$
0 = \frac{T}{0} \frac{t_2}{0} 4 \frac{T_2}{0} 4.
$$

is renormalized by a term which shows a temperature power law dependence with an exponent twice as large than in the previous case. Here $\frac{T}{0}$ is temperature independent and is defined by $T_0 = T_0 (T = 0)$.

VI. CONCLUSIONS

We analyzed the injection of spin from a ferromagnet with time dependent magnetization into a Luttinger liquid. We found that the spin current injected into the LL has a similar form to the current that would be injected into a normal metal. However, the effect of the interactions in the LL is to suppress the spin pumping as a power law in temperature. We computed the total current flowing from a FM into a LL in a few cases: a FM-LL-LL junction, a FM connected to a finite size LL, and a FM-LL-metal junction. As a result of the LL physics, we found that in these setups the renormalization of the Gilbert damping is also suppressed from the normal metal case. It would be interesting to see if future experiments can identify junctions in which this power law suppression is observed. Also it would be interesting to test for spin currents injected through this method and transported over longer distances in good LL’s (e.g. carbon nanotubes).

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APPENDIX A: FM-LL-FM JUNCTION

If we consider the exchange coupling to be negligible, Eq. (13) describing the spin current simplifies to

$$\mathbf{I} = \frac{\mathbf{I}_1^1}{2} + \frac{\mathbf{I}_2^2}{2};$$

(A1)

We substitute this result for the spin current, into the corresponding coupled LLG Eqs. (15) for the ferromagnets’ magnetization vectors. We assume that in each FM the magnetization is $\mathbf{m}_1 = \mathbf{H}^1_1 + \mathbf{m}_1$, and $\mathbf{m}_1$ are small, so that we can write linearized equations in $\mathbf{m}_1$. We consider the case two magnetic fields parallel to each other in the two ferromagnets, $\mathbf{H}_1 \parallel \mathbf{H}_2$. We note that up to quadratic corrections, the coupled equations preserve the magnitudes of $\mathbf{m}_i$, i.e. $\mathbf{m}_1 \cdot \mathbf{d}\mathbf{m}_1/dt = 0$.

Thus the two coupled equations,

$$1 + \frac{A_2}{2M_s} \frac{d\mathbf{m}_1}{dt} = \mathbf{m}_1 \mathbf{H}^1 + 0 + \frac{A_1}{2M_s} \mathbf{m}_1 \frac{d\mathbf{m}_1}{dt},$$

$$1 + \frac{A_2}{2M_s} \frac{d\mathbf{m}_2}{dt} = \mathbf{m}_2 \mathbf{H}^2 + 0 + \frac{A_1}{2M_s} \mathbf{m}_2 \frac{d\mathbf{m}_2}{dt},$$

(A2)

can be linearized to yield

$$\begin{align*}
(1 + 2) \frac{d m_{x}^1}{dt} = & m_{x}^1 H_1 + (0 + 1) \frac{d m_{x}^1}{dt} + \frac{d m_{y}^1}{dt} + 2 \frac{d m_{z}^1}{dt}, \\
(1 + 2) \frac{d m_{y}^1}{dt} = & m_{y}^1 H_1 + (0 + 1) \frac{d m_{y}^1}{dt} + \frac{d m_{x}^1}{dt} + 2 \frac{d m_{z}^1}{dt}, \\
(1 + 2) \frac{d m_{z}^1}{dt} = & m_{z}^1 H_1 + (0 + 1) \frac{d m_{z}^1}{dt} + \frac{d m_{x}^1}{dt} + 2 \frac{d m_{y}^1}{dt}, \\
(1 + 2) \frac{d m_{x}^2}{dt} = & m_{x}^2 H_2 + (0 + 1) \frac{d m_{x}^2}{dt} + \frac{d m_{y}^2}{dt} + 2 \frac{d m_{z}^2}{dt}, \\
(1 + 2) \frac{d m_{y}^2}{dt} = & m_{y}^2 H_2 + (0 + 1) \frac{d m_{y}^2}{dt} + \frac{d m_{x}^2}{dt} + 2 \frac{d m_{z}^2}{dt}, \\
(1 + 2) \frac{d m_{z}^2}{dt} = & m_{z}^2 H_2 + (0 + 1) \frac{d m_{z}^2}{dt} + \frac{d m_{x}^2}{dt} + 2 \frac{d m_{y}^2}{dt};
\end{align*}$$

(A3)

where $s = A_1 + A_2 = 2M_s$ and we assumed that the two ferromagnets are identical, so that $M_{1} = M_{2} = M_s$. We can solve the above linear system of equations in the particular case $H_1 = H_2 = H_\perp$. Assuming that the initial configuration is $m_{x}^1 = m_1$, $m_{y}^1 = m_2$, and $m_{y}^1 = 0$, with $m_{1} = 1$ and $m_{2} = 1$, we obtain
APPENDIX B: FM-LL JUNCTION

As described in Section II, the injected current is

$$I_0 = A_1 m \frac{cm}{dt} + A_2 \frac{cm}{dt};$$

(1B)

If we denote the chemical potential of the spin accumulated in the LL by $\gamma_s$, the backscattered current due to the accumulation of spin (Eq. (11) can be written as

$$I_b = T_s \gamma_s;$$

(B2)

in the limit $k_B T$. The total current is given by $I = I_0 + I_b$, where $I_0$ and $I_b$ are given respectively by the equations (B1) and (B2).

Along the lines of Ref. 4 the diffusion equation for the spin in the LL is

$$\gamma_s = D \gamma_s^2 \gamma_s \frac{1}{2} \gamma_s t;$$

(B3)

with the boundary conditions $\gamma_s(x) = (2 \gamma = D) \Gamma$ at $x = 0$, and of vanishing spin current, $\gamma_s(x) = 0$, at $x = L$.

Correspondingly, the accumulated spin potential in the LL at the junction with the FM is $\gamma_s = \Gamma$, where $= (2 \gamma = D k) \coth(2L)$, $D$ is the diffusion coefficient in the wire, and $k = 1 + \gamma_s^2 = D \gamma_s$. Similar to Ref. 4. Since the frequency of precession of the ferromagnet is much smaller than the inverse spin flip time, we can take $k = 1 = D \gamma_s$, like in Ref. 4.

Consequently we obtain an equation for $\Gamma$

$$(l + \gamma_s \gamma_s \gamma_s = A_1 m \frac{cm}{dt} + A_2 \frac{cm}{dt} = I_0;$$

(B4)

whose solution is

$$\Gamma = B_1 I_0 + B_2 (I_0 m$$

$$= (B_1 A_1 + B_2 A_2) \frac{cm}{dt} = (B_1 A_1 + B_2 A_2) \frac{cm}{dt};$$

(B5)

where $B_1 = (l + \gamma_s \gamma_s \gamma_s = (1 + l + \gamma_s \gamma_s)^2)$ and $B_2 = (l + \gamma_s \gamma_s \gamma_s = (1 + l + \gamma_s \gamma_s)^2)$. Here we made the assumptions that the length of the magnetization vector $m$ is not changing with time and for simplicity we took it to be $\frac{\gamma_s}{m} = 1$; consequently $m \frac{cm}{dt} = 0$. The spin current is then incorporated in the LLG equation

$$\frac{cm}{dt} = m \frac{H}{s} + \frac{cm}{dt} M_s I_f;$$

(B6)

which can be rewritten as

$$\frac{cm}{dt} = m \frac{H}{s} + m \frac{cm}{dt};$$

(B7)

Here $m = 1 + (m = D_1 A_1 B_2 A_1)$. The spin current flowing from the ferromagnet to the LL is

$$I = I_0 + \frac{4}{4} \gamma_s m \gamma_s m = I_0;$$

(C1)

The current flowing from the LL into the metal is

$$I = I_0 + \frac{4}{4} \gamma_s m \gamma_s m = I_0;$$

(C2)

where $\gamma_s$ is the spin chemical potential in the wire, which in case of ballistic transport is uniform along the LL, and $\gamma_s$ is the spin chemical potential in the metal at point $B$. The two “spin conductances” $I_1$ and $I_2$ are defined similarly to the previous appendix, with the index 1 corresponding to the FM-LL interface, and the index 2 to the LL-metal interface. The spin current and the spin chemical potential $\gamma_s$ in the metal can be related by $d \gamma_s = \gamma_s \gamma_s$.

Here $m = \coth(2L)$, $L = \gamma_s \gamma_s \gamma_s$, $D_2$ is the density of states in the metal, $D_3$ is the diffusion coefficient in the metal, $\frac{4}{4}$ is the cross-section of the metal, and $\gamma_s = \gamma_s \gamma_s \gamma_s$. The corresponding equation for the spin current flowing through the junction is

$$a_1 I = a_2 I = I_0;$$

(C3)
with

\[ a_1 = \frac{h}{T_2} \left( 1 + \frac{T_1}{T_2} \right); \quad \text{(C4)} \]

and

\[ a_2 = \frac{1}{4} \left( 1 + \frac{T_2}{T_1} \right); \quad \text{(C5)} \]

Solving for \( \mathbf{I} \) we get

\[ \mathbf{I} = C_1 \mathbf{I}_0 + C_2 (\mathbf{I}_0 + \mathbf{m}) \]

\[ = (C_1 A_2 \quad C_2 A_1) \frac{\text{d}\mathbf{m}}{\text{d}t} \quad (A_1 C_1 + C_2 A_2) \mathbf{m} \quad \frac{\text{d}\mathbf{m}}{\text{d}t} \quad \text{(C6)} \]

where \( C_1 = a_1 = (a_1^2 + a_2^2) \) and \( C_2 = a_2 = (a_1^2 + a_2^2) \). We thus find in this case

\[ \mathbf{0} = [1 + ( -M_2 (C_1 A_2 \quad C_2 A_1)] \]

and

\[ \mathbf{0} = [1 + ( -M_2 (C_1 A_1 + A_2 C_2)] = [1 + ( -M_2 (C_1 A_2 \quad C_2 A_1)] \].