Reconstructing the electron in a fractionalized quantum fluid

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The low energy physics of the fractional Hall liquid is described in terms quasiparticles that are qualitatively distinct from electrons. We show, however, that a long-lived electron-like quasiparticle also exists in the excitation spectrum: the state obtained by the application of an electron creation operator to a fractional quantum Hall ground state has a non-zero overlap with a complex, high energy bound state containing an odd number of composite-fermion quasiparticles. The electron annihilation operator similarly couples to a bound complex of composite-fermion holes. We predict that these bound states can be observed through a conductance resonance in experiments involving a tunneling of an external electron into the fractional quantum Hall liquid. A comment is made on the origin of the breakdown of the Fermi liquid paradigm in the fractional hall liquid.

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The low energy excitations of an ordinary electron liquid resemble electrons. During the past two decades, there has been much interest in systems where strong interactions may cause a breakdown of the Fermi liquid paradigm. In the most dramatic instances, such a breakdown is signaled by the emergence of new quasiparticles that do not bear any resemblance to electrons, and even have quantum numbers which are a fraction of the electron quantum numbers. Is the electron irretrievably lost as a meaningful entity in such a \textquotedblleft fractionalized\textquotedblright liquid?

In this Letter, we shall investigate this question in the context of the fractional quantum Hall liquid (FQHL), formed when electrons are confined to two dimensions and subjected to a strong transverse magnetic field. The low-energy excitations of the FQHL carry a fractional charge. The question of our interest is whether an integral number of such fractionally charged entities can combine to produce an electron. The resolution requires a microscopic understanding of the strongly correlated FQHL state, which has been achieved in terms of composite fermions, bound states of electrons and an even number of quantized vortices. The ground state of an incompressible FQHL is accurately described as a state with an integral number of filled composite-fermion (CF) quasi-Landau levels (LL’s), and its low-energy excitations are “CF particles” (CFP’s; composite fermions in otherwise empty CF-quasi-LL’s) and “CF holes” (CFH’s; missing composite fermions in otherwise full CF-quasi-LL’s). A CFP or a CFH has a fractional charge excess or deficiency associated with it, consistent with general principles that tell us that incompressibility at fractional fillings results in fractional charge. It is obvious that an integral number of CFP's can have the same charge as an electron, but the key question is whether there exists a \textit{long-lived} multi-CF bound complex that has a non-zero overlap with the “electron quasiparticle,” namely the excitation obtained by adding an electron to the ground state. Alternatively, can the electron be viewed as a stable bound state of CFP’s? If so, what is that bound state? How can it be observed?

We will use below the spherical geometry, which takes \( N \) electrons confined to the surface of a sphere and exposed to a radial magnetic field \( B \) produced by a magnetic monopole of strength \( Q \), which is restricted to be an integer or a half odd integer according to Dirac’s quantization condition. The single particle eigenstates of an electron in this geometry are a generalization of the usual spherical harmonics, called monopole harmonics, denoted by \( Y_{Q,l,m} \), where \( l = |Q|, |Q| + 1, \ldots \) is the orbital angular momentum and \( m = -l, -l + 1, \ldots l \) is the \( z \) component of the orbital angular momentum. The different angular momentum shells are analogous to the Landau levels (LL’s) of the planar geometry. The degeneracy of the lowest Landau level shell (\( l = |Q| \)) is 2\( |Q| + 1 \) (without counting spin), and increases by two units in each successive shell. It will be assumed below that the magnetic field is sufficiently strong that electrons are confined to the lowest Landau level and are fully spin polarized (the spin degree of freedom is frozen). The only term remaining in the Hamiltonian is the Coulomb interaction, which determines the nature of the state in the lowest LL.

The CF theory postulates that electrons avoid one another most effectively by capturing an even number \((2p)\) of vortices to transform into composite fermions, which experience a reduced magnetic field, produced by a monopole of strength \( Q^* = Q - p(N - 1) \). The wave functions \( \chi \) for interacting electrons at \( Q \) are constructed from the electron wave functions \( \Phi^* \) at \( Q^* \) according to

\[
\chi = \mathcal{P}_{LLL} \Phi^* \Phi^\dagger \Phi^* \tag{1}
\]

where \( \mathcal{P}_{LLL} \) denotes projection into the lowest Landau level and \( \Phi^\dagger \) is the wave function of one filled Landau level. An explicit, lowest-Landau-level-projected form for \( \chi \) can be obtained by methods described in the literature. We shall consider \( N \) particles at a monopole strength

\[
Q = (p + 1/2n)N - (p + n/2), \tag{2}
\]
where \( p \) and \( n \) are integers, which is a finite size representation of the state at filling factor \( \nu = \lim_{N \to \infty} N/(2Q + 1) = n/(2pn + 1) \). It maps into a system of composite fermions at \( Q^* = (N - n^2)/2n \). Here, \( N \) composite fermions completely fill \( n \) CF-quasi LL’s, which can be seen by noting that the total number of states in the lowest \( n \) CF-quasi-LL’s is \( \sum_{j=0}^{n-1} (2Q^* + 2j + 1) = N \). The ground state is a filled shell state, with total orbital angular momentum \( L = 0 \), shown schematically in Fig. 1 (a) for \( \nu = 2/5 \). Its wave function is \( \chi_0 = P_{LLL}\Psi^0_{\mathbf{F}_n} \), where \( \Psi_n \) is the Slater determinant wave function of \( n \) filled Landau levels at \( Q^* \). It is known \( \mathbb{R} \) to provide an excellent description of the exact ground state at \( \nu = n/(2pn + 1) \).

An electron quasiparticle (EQP) is created by adding an electron to the system in the lowest Landau level creation operator \( \Psi^\dagger \) of \( \psi^\dagger \) c\(^\dagger\) projected thereon. Now, relative to \( n \) filled CF-quasi-LL’s, we have an excess of \( 2pn + 1 \) CFP’s or CFH’s. Thus, an EQP (a HQP) must be the bound state of \( 2pn + 1 \) CFP’s (CFH’s), consistent with the fact that a single CFP or CFH has a local charge of magnitude \( e/(2pm + 1) \).

\[
\chi_0 = \left[ \begin{array}{c} N_Q \left( \frac{2Q}{Q - m} \right) \right]^{1/2} \psi^\dagger \chi_{Q^0 - m} \ (4)
\]

\( \psi \) is the electron wave function, and \( \chi_{Q^0} \) is the wave function of \( n \) filled Landau levels at \( Q^* \). The (unnormalized) wave function of the EQP is given by

\[
\chi_0 = \left[ \begin{array}{c} N_Q \left( \frac{2Q}{Q - m} \right) \right]^{1/2} \psi^\dagger \chi_{Q^0 - m} \ (4)
\]

with \( u = \cos \frac{\theta}{2} e^{i\phi}/2 \), \( v = \sin \frac{\theta}{2} e^{-i\phi}/2 \), and \( N_Q = (2Q + 1)/4\pi \). With no loss of generality, we add an electron at the North Pole (\( \mathbf{H} = \mathbf{H}^\dagger \)); \( u = 1, v = 0 \) by application of

\[
\psi^\dagger = \sqrt{\frac{N_Q}{Q}} \chi_{Q^0} \ (5)
\]

A “hole quasiparticle” (HQP) at the North Pole is created similarly by application of the electron destruction operator \( \bar{\psi}^\dagger \).

\[
\chi_{Q^0} = \left[ \begin{array}{c} N_Q \left( \frac{2Q}{Q - m} \right) \right]^{1/2} \psi \chi_{Q^0} \ (4)
\]

The resulting (unnormalized) HQP wave function (with \( L = |M| = Q \)) is obtained by replacing one of the particle coordinates by \( \mathbf{H} \)

\[
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\]

A “hole quasiparticle” (HQP) at the North Pole is created similarly by application of the electron destruction operator \( \bar{\psi}^\dagger \).
verified that the lowest energy state with the desired angular momentum has \( n + 1 \) and \( n \) CFP’s in the lowest two unoccupied CF-quasi-LL’s, occupying the largest \( m \) orbitals. The corresponding wave function will be denoted by \( \chi^+ \). Similarly, the lowest energy state with the quantum numbers matching those of \( \chi_h \) is obtained by putting \( n + 2 \) and \( n - 1 \) CFH’s in the top two occupied CF-quasi-LL’s (for \( n = 2 \); for \( n = 1 \) there are three CFH’s in the only available CF-quasi-LL), again in the largest \( m \) orbitals; its wave function will be denoted \( \chi^- \).

Fig. 2 shows these bound complexes of CFP’s and CFH’s schematically for \( \nu = 2/5 \) (\( n = 2 \)). Of course, many more states with \( L = Q \) can be constructed, but they have at least one higher unit of CF-cyclotron energy compared to \( \chi^+ \) and \( \chi^- \), and are therefore separated by a finite gap from \( \chi^+ \) and \( \chi^- \) in the \( |M| = Q \) subspace. The wave function \( \chi^+ \) or \( \chi^- \) can be constructed straightforwardly from the analogous wave function of \( N_\pm \) electrons at \( Q^\pm \).

We have found that just like the ground state \( \chi_0 \), the CF complexes \( \chi_\pm \) are also very accurate representations of the exact eigenfunctions of the lowest energy state in the \( L = Q \) sector; the overlap between \( \chi_\pm \) and the corresponding exact states are 0.98-0.99 for up to \( N = 9 \).

To see how well \( \chi^+ \) and \( \chi^- \) represent the EQP and HQP, we calculate the overlaps \( O_+ = |\langle \chi^+ | \chi_0 \rangle|^2 / (\langle \chi^+ | \chi^+ \rangle \langle \chi_0 | \chi_0 \rangle) \) and \( O_- = |\langle \chi^- | \chi_0 \rangle|^2 / (\langle \chi^- | \chi^- \rangle \langle \chi_0 | \chi_0 \rangle) \), which are shown in Fig. 2 for several filling factors as a function of \( 1/N \). For small \( N \), \( \chi_0 \) and \( \chi_\pm \) are explicitly very accurate, so \( O_\pm \) are excellent approximations to the corresponding exact overlaps. That, we believe, remains true even in the thermodynamic limit, although there, strictly speaking, our calculation represents a prediction to be tested experimentally (see below). Our principal finding is that the overlaps approach a non-vanishing value in the limit \( N^{-1} \rightarrow 0 \), demonstrating that a rather complicated bound complex of CFP’s (CFH’s) has in it a non-zero content of an EQP (a HQP). The thermodynamic overlap decays rapidly with \( n \) along the sequence \( \nu = n/(2n + 1) \), as expected from the increasing complexity of the multi-CF bound state. It has been noted earlier that for \( \nu = 1/(2p + 1) \), where the wave functions for the CF ground state and the CFH are identical to those written earlier by Laughlin, \( O_- = 1 \).

The existence of an electron-like multi-CF bound complex is not merely a theoretical curiosity but has testable consequences for experiments that involve tunneling of an electron into an incompressible FQHL. An example is vertical interlayer transport in a bilayer system in the weak coupling limit (when the nature of the state in either layer is not affected by its proximity to the other layer), with each layer being at \( \nu = n/(2p + 1) \). For the tunneling Hamiltonian \( T = \sum d^2 r \Psi_1^\dagger (r) \Psi_2 (r) + H.c. \) (1 and 2 label the two layers; the tunneling amplitude \( T \) is taken to be energy independent in the relevant energy range), the tunneling current at zero temperature is proportional to \[ I(eV) \propto \int_0^{eV} dE \rho^{(+)} (E) \rho^{(-)} (eV - E) \] (8)

where \( eV \) is the bias voltage and \( \rho^{(+)} (E) \) and \( \rho^{(-)} (E) \) are the positive and negative frequency parts of the projected electron spectral function:

\[
\rho^{(\pm)} (E) = \sum_m \frac{|\langle m | \Psi^{(\pm)} (0) | 0 \rangle|^2}{\langle m | m \rangle \langle 0 | 0 \rangle} \delta (E - E_m^{N \pm 1} + E_0)
\]

\[
= \nu \pm N Q \sum_m O_\pm^{(m)} \delta (E - E_m^{N \pm 1} + E_0)
\] (9)

Here \( 0 \) is the exact ground state of \( N \) particles with eigenenergy \( E_0 \), and \( |m \rangle \) labels all exact eigenstates (only those with \( L = Q \) are relevant here) of the \( N \pm 1 \) particle systems with eigenenergy \( E_m^{N \pm 1} \). The symbols are defined as \( \nu_+ = \nu, \nu_- = 1 - \nu, \Psi^{(+)} = \Psi^\dagger, \Psi^{(-)} = \Psi \), and

\[
O_\pm^{(m)} = \frac{|\langle m | \Psi^{(\pm)} (0) | 0 \rangle|^2}{\langle m | m \rangle \langle 0 | \Psi^{(\pm)} (0) \rangle \langle \Psi^{(\pm)} (0) | 0 \rangle}.
\]

We have also made use of

\[
(0 | \Psi^{(\pm)} (0) \Psi (0) | 0) = N Q | \langle 0 | e^{+} \rangle Q Q Q | 0 \rangle = N Q \nu | 0 \rangle | 0 \rangle.
\]

We approximate the exact oscillator strengths \( O_\pm^{(0)} \) for the lowest energy states at \( L = Q \) by the CF overlaps \( O_\pm \) calculated earlier. Non-zero values for \( O_+ \) and \( O_- \) in the thermodynamic limit imply delta function peaks at \( E = E_\pm - E_0 \) in the electron spectral function, which in turn produces a sharp peak in the conductance at a voltage \( V \) given by \( eV = E_+ + E_- - 2E_0 \). For \( \nu = 1/3 \),
2/5, and 3/7, we have estimated $E_+ + E_- - 2E_0$ to be
$\sim 0.5e^2/\epsilon \ell$ by an extrapolation to $N^{-1} \rightarrow 0$, where
$\epsilon$ is the dielectric constant of the host semiconductor and
$\ell = \sqrt{\hbar c/eB}$ is the magnetic length. The “coherent”
peak is expected to be followed by a broad “incoherent”
peak where the tunneling electron couples into a quasi-
continuum of higher energy excited states.

The delta function peaks in the projected spectral
function indicate that the electron and hole quasipar-
ticles are long lived. Lower energy states do exist, but
the EQP’s or HQP’s cannot decay into them on account
of angular momentum conservation. In practice, disor-
der, left out in the above analysis, will impart a non-
zero width to the quasiparticle peak; it is not possible at
present to estimate the broadening for lack of a quanti-
tative understanding of the effect of disorder.

Tunneling experiments in bilayer systems have been
performed in the past \[1\]. The lack of tunneling at small
voltages and the presence of a broad conductance peak at
a finite voltage has been well understood \[13, 14, 17, 18\],
attributed to a Coulomb gap resulting from the strongly
correlated nature of the electron system in either layer.
It is natural to identify the observed peak as arising from
the incoherent part of the spectral function, given its lack
of sensitivity to the details of the correlations: the peak
is independent of the filling factor; it occurs for both
incompressible and compressible ground states \[14, 15\];
one may even model the liquid ground state as a Wigner
crystal to understand its origin \[13\]; and the energy gap
can be estimated from simple classical electrostatic con-
siderations \[15, 16\]. The coherent peak discussed in this
work, on the other hand, is crucially dependent on the
physics of the fractional quantum Hall effect and occurs
only for incompressible states, with a strongly $\nu$ de-
dependent oscillator strength. It is noteworthy that the theory
of Conti and Vignale \[17\], which models the bulk as a con-
tinuous elastic medium and the collective excitations as
bosons, predicts a coherent peak in the spectral function
for $\nu = 1/3$, with additional structure in the incoherent
peak.

An observation of a sharp coherent resonance in tun-
neling experiments will provide new insights into the na-
ture of the FQHL. There can be many reasons for its
absence in the earlier experiments. While the tunneling
from one layer to another takes place predominantly in
the bulk, the current is being injected and collected at a
lead connected to an edge of the sample. The passage of
tunnel current from the bulk to the lead not only exag-
gerates the effect of disorder, but also requires a reason-
ably high temperature (current flow in the bulk is ex-
ponentially suppressed at low temperatures). It is possible
that the combined broadening due to temperature and
disorder has suppressed the coherent peak in the pre-
vious experiments. Reducing disorder and the sample
area might help, as might a tri-layer FL-FQHL-FL (FL =
Fermi liquid) geometry, in which the tunneling electron
passes right through the FQHL layer.

Before ending, we make an observation on the above
results vis-à-vis the general question of the origin of “non-
Fermi liquid” behavior. The fundamental tenet of the
Fermi liquid theory is that electron-like quasiparticle is
long lived, i.e., there is a delta function peak (broad-
ened into a Lorenzian shape by disorder) in the electron
spectral function, with weight $Z$, called the renormaliza-
tion factor. One possible mechanism for the breakdown
of the Fermi liquid is the vanishing of $Z$. That is the
case in a well understood example of a non-Fermi liq-
uid, namely the Luttinger liquid in one dimension. The
FQHL provides a different paradigm. Here, the electron
like quasiparticle remains well defined, with a non-zero
$Z$, but the Fermi liquid description breaks down because
other excitations appear at lower energies, described in
terms of new elementary quasiparticles that are qualita-
tively distinct from electrons.

In summary, we have shown that an electron-like quasi-
particle exists in the fractional quantum Hall liquid, but
has a strikingly complicated structure. It is a complex
“atom” of an odd number of composite fermions, which
are themselves collective bound states of electrons and
quantized vortices of the wave function. A direct signa-
ture of this bound complex should appear as a conduc-
tance resonance in vertical tunneling transport in bilayer
systems.

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