Acoustic topological interface states of one dimensional metamaterial propagating through a T-shaped junction

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Received May 13, 2021; revised May 26, 2021; accepted June 4, 2021; published online June 16, 2021

We design a supermolecular structure composed of two identical scatterers with opposite orientations in air. By adjusting the interval between them and rotating them, topological phase transitions occur. The combination of rotational and translational operations provides us with wide scope of interface states and multiple-choice to achieve interface states. Therefore, the interface states must exist at the interface between two sublattices with different topological phases. We investigate the subwavelength interface states propagating through a T-shaped junction theoretically, which consists of three one-dimensional waveguides. The results have promising prospects in developing acoustic double-channel transmission devices based on interface states. © 2021 The Author(s). Published on behalf of The Japan Society of Applied Physics by IOP Publishing Ltd

The discoveries of the quantum Hall effect1) and the quantum spin Hall effect2) opened a new chapter in the study of topological insulators. Due to unidirectional and robust transmission of topological states,3,4) topological insulators receive extensive attentions in condensed matter physics, including electronic system,5) photonic crystals,6–9) and phononic crystals (PCs).10–20) Specifically, acoustic topological insulators are expected to improve our ability to control propagation of sound, such as negative refraction,21) holography,22) antennas,23) and so on.

Generally, topological properties of band structures can be characterized by topological invariants, e.g. Chern number11–15) or Zak phase.24,25) In one-dimensional (1D) acoustic systems, topological properties of band structures are determined by Zak phases,26) which are successfully applied to predict the existence of topological interface states.

Two methods are used to obtain topological phase of 1D acoustic systems. One of the ways is to alter the interval of scatterers in a supermolecule, the other is to rotate scatterers. In 2018, Fung and Zhao et al. changed the interval between two scatterers to adjust inter-supermolecular and intra-supermolecular coupling strength, and gained topological properties of subwavelength bands in acoustic systems.27) In 2019, Liu and Chen et al. designed a two-channel Mie resonator based on maze-type metamaterials, and obtained topological interface states by adjusting the spacing of two scatterers.28) In 2020, we regulated topological interface states by rotating the two split-cylinders in a supermolecule.29)

Though two separate operations (altering the interval of scatterers and rotating scatterers) can modulate and improved controllability of topological interface states, there remain questions about 1D PCs. For example, can a mixing of these operations provide us with multiple choices to achieve alternative topological bandgaps? How do acoustic waves of topological interface stats modulated by a combination of rotational and translational operations propagate through a T-shaped junction? Addressing the two questions is meaningful to the practical applications of topological interface states.

To answer these questions, we mix these operations to investigate topological properties of 1D PCs. Moreover, topological interface states still exist at interface between two sublattices with different topological phases adjusted by combination of the two operations, and possess robustness against perturbation. Finally, the feasibility of a T-shaped junction of subwavelength topological interface states are investigated theoretically. The finite-element software COMSOL MULTIPHYSICS is utilized in all simulations.

A scatterer is composed of an external cylinder with a single split and an internal labyrinthine resonator, as shown in Fig. 1(a). The thicknesses of all walls of the scatterer are \( t = 0.16 \) mm. The width of each split of internal resonator is \( t_1 = 0.64 \) mm, and the width of the external single-slit cylinder is \( t_2 = 0.8 \) mm. The interlayer spacing of the internal resonator is \( w = 0.36 \) mm, and the spacing between the external layer and internal resonator is \( w_1 = 1 \) mm. The minimum and maximum radii are \( R_0 = 1.3 \) and \( R = 3.66 \) mm, respectively. The scatterer is placed at the center of a square air matrix as a unit cell, which is regarded as a hard boundary in our simulations. The mass density and the longitudinal wave velocity of air are \( 1.29 \) kg m\(^{-3}\) and \( 340 \) m s\(^{-1}\), respectively.

Two unit cells constitute a supermolecule (shown in Fig. 1(b)). Our scatterers are periodically arranged in the direction of the \( x \) axis, there is one direction of period which forms one dimensional PCs. The lattice constant of each unit cell is \( a = 1 \) cm, and thus lattice constant of a supermolecule is \( 2a \). The \( O \) (\( O' \)) is the center of the left (right) scatterer, and \( OO' \) is the interval between two scatterers, which is defined as a parameter \( D \). The vertical dashed black line \( MN \) divides a supermolecule into two equal squares. When we move two scatterers along the \( x \) (\( OO' \)) axis, we make sure that the distances from two scatterers in supermolecule to the vertical dashed line \( MN \) are equal to keep mirror symmetry, then we introduce a spacing parameter \( d = D - a \). When \( d > 0 \), the scatterers move in the outward direction in supermolecule; when \( d < 0 \), they approach to the center line \( MN \).

We rotate the external split-cylinders around \( O \) and \( O' \) points with respect to the \( y \) axis. We define \( \theta > 0 \) for the anticlockwise rotation and \( \theta < 0 \) for the clockwise action. To keep mirror symmetry, we need to make sure that the rotational angles of two scatterers in the supermolecule are equal in the opposite directions. For convenience, the angle \( \theta \) in the following discussion refers to the angle of the left scatterer.

Band structure of the PC with \( d = 0 \) cm and \( \theta = 0 \)° along \( x \) direction in momentum space is shown in Fig. 1(c). Six
Degenerate points are formed at two ends of the first Brillouin zone ($k_x = \pm \pi/2a$), which results from the zone-folding mechanism. To clearly demonstrate the resonant nature, the absolute pressure distributions of the unit cell are illustrated in Fig. 1(d). Clearly, the two resonant frequencies are located within bandgaps.

Now we use $\theta$ (rotational operation) and $d$ (translational operation) to change the band structures of the PCs to lift the double degeneracies, then achieve topological transitions. We keep $d$ unchanged ($d = 0$) and choose $\theta = -60^\circ$ and $60^\circ$. The band structures for the PCs with $\theta = -60^\circ$ and $\theta = 60^\circ$ are shown in Figs. 2(a1) and (a4), respectively. We keep $\theta$ unchanged ($\theta = 0^\circ$) and select $d = -0.16$ and $0.16$ cm. The band structures for the PCs with $d = -0.16$ and $0.16$ cm are exhibited in Figs. 2(b1) and (b4), respectively. In Fig. 2, the six double degenerate states at the two ends of the first Brillouin zone are lifted to open bandgaps. The parities of these eigenmodes are symmetric (even parity represented by the red “+”) and asymmetric (odd parity represented by the blue “−”) with respect to the center plane of supermolecule labeled as MN in Fig. 1(b). We mark the parity of each band at $k_x = 0$ and at $k_x = \pm \pi/2a$, respectively.

Zak phase $\phi$, as the topological invariant in 1D PCs, characterizes the topological phase of each band, and interprets the existence of topological interface states. It is related to the parities of the eigenmodes, which is written as

$$\frac{\phi}{\pi} = \frac{1}{2} [\eta(k_x = \pi/2a) - \eta(k_x = 0)],$$

where $\eta$ is the parity (±1) at the center and the end of the first Brillouin zone, respectively. In general, Zak phase is determined by the choice of the integral origin of the supermolecule. When the integral origin is located at the center of the supermolecule as marked by a red dot of line MN in Fig. 1(b), the quantized Zak phase is 0 or $\pi$ from equation above. Zak phases of all bands are marked on all bands as shown in Fig. 2. For the rotational (translational) operations, i.e., $\theta = -60^\circ$ and $60^\circ$ ($d = -0.16$ and $d = 0.16$ cm), we observe that all corresponding bands are inverted, which indicates a topological phase transition, by comparing Zak phases of six bands in Fig. 2(a1) [Fig. 2(b1)] with ones in Fig. 2(a4) [Fig. 2(b4)].

Xiao and Chan et al. proved that the sign of $\zeta^{(n)}$ in the $n$th bandgap is related to the topological interface states. The sign of $\zeta^{(n)}$ can be obtained from the following formula:

$$\zeta^{(0)} = (-1)^{n+1} \exp\left( i \sum_{m=1}^{n} \phi_{m} \right).$$

Here, the signs of $\zeta$ of the third and the fifth bandgaps are marked on the band structures in Fig. 2. For rotational operation, the signs of $\zeta$ of the third (fifth) bandgaps of PCs...
with $\theta = -60^\circ$ and $\theta = 60^\circ$ are opposite in Figs. 2(a1) and (a4); for translational operation, the signs of $\zeta$ of the third (fifth) bandgaps of PCs with $d = -0.16$ cm and $d = 0.16$ cm are also opposite in Figs. 2(b1) and (b4). The signs of $\zeta$ in the third (fifth) bandgaps of PCs with $\theta = -60^\circ$ and $d = -0.16$ cm have the opposite (same) sign in Figs. 2(a1) and (a2), and the signs of $\zeta$ in the third (fifth) bandgaps of PCs with $\theta = 60^\circ$ and $d = 0.16$ cm have the same (opposite) sign in Figs. 2(a1) and (b4).

It is known to us that topological interface states occur at the interface between two PCs with the different signs of $\zeta$.\textsuperscript{27–29} Then we have multiple choices to implement the topological interface states at the interfaces of finite-size superlattices. Besides the choices reported in previous works,\textsuperscript{27–29} now we can combine PCs with $\theta = -60^\circ$ and $d = \pm0.16$ cm ($\theta = 60^\circ$ and $d = \pm0.16$ cm) together to obtain topological interface states.

In Fig. 3, we study the evolution of these eigenstates at the end of the first Brillouin zone as parameter $\theta$ ($d$) is changed. When parameter $\theta$ increases from $-180^\circ$ to $180^\circ$, the third and the fifth bandgaps are opened, and closed and reopened, in Fig. 3(a). When $\theta < 0^\circ$, the signs of $\zeta$ of the third and fifth bandgaps are positive and negative in Fig. 3(a), respectively; while $\theta > 0^\circ$, the signs of those two bandgaps are in the inverse of the cases for $\theta < 0^\circ$. It is obvious that $\theta = 0^\circ$ is a topological transition point.

In Fig. 3(b), as parameter $d$ is tuned continuously from $-0.24$ to $0.24$ cm, the two bandgaps go through the same process of the cases modulated by $\theta$. We note that $d = 0$ cm is also the topological transition point. If $d < 0$, the signs of $\zeta$ of the third and the fifth bandgaps are negative; if $d > 0$, they are positive. In consequence, the topological interface states must be generated at the interface of a superlattice composed of two sublattices with opposite signs of $\zeta$. Then, by combination of two sublattices with different parameters at

![Fig. 2](image1.png)

**Fig. 2.** (Color online) Band structures and total acoustic pressure distributions regulated by rotating angle $\theta$ and distance $d$. (a1) and (a4) Band structures of PCs with $\theta = -60^\circ$, and $60^\circ$, respectively. (a2) and (a3) Corresponding eigenmodes from the first to the sixth bands at $k_x = \pi/2a$ in (a1) and $k_x = -\pi/2a$ in (a4), respectively. (b1)–(b4) Same as (a1)–(a4), but for PCs with $d = -0.16$, and $0.16$ cm, respectively. The even (odd) parity is represented by red “+” (blue “−”). The positive and negative $\zeta$ are marked by the light red and light blue regions in bandgaps, respectively.

![Fig. 3](image2.png)

**Fig. 3.** (Color online) Process of topological transitions as functions of $\theta$ and $d$. The red (blue) curves refer to the even (odd) parities. (a) and (b) Eigenfrequencies of eigenstates from the third to the sixth bands at $k_x = \pi/2a$. The third and fifth bandgaps are marked with different colors to show that they have different topological characteristics. Parameters $\theta$ and $d$ are from $-180^\circ$ to $180^\circ$ and from $-0.24$ to $0.24$ cm, respectively.
Observe both sides of interface, our strategy provides an alternative topological bandgap. For example, the topological interface state just appears in the fifth bandgap when parameters $\theta$ and $d$ have opposite signs; however, for parameters $\theta$ and $d$ having the same sign, the topological interface state only emerges in the third bandgap. The first bandgap has the same properties as the fifth bandgap, but it is too narrow to adjust conveniently. So we missed this bandgap.

According to the results above, we design an acoustic superlattice to obtain topological interface states, in which five supermolecules with parameter $\theta = -60^\circ$ are on the left side and five supermolecules with parameter $d = 0.16$ cm are on the right side, as shown in Fig. 4(a). Meanwhile, two perturbations are introduced into the two supermolecules near the interface in order to verify the robustness of topological interface state, i.e., $\Delta \theta = 12^\circ$ and $\Delta d = 0.04$ cm labeled as the red and green arrows in insets of Fig. 4(a), respectively. Without perturbation, the dispersion relations, transmission spectrum, and total pressure field distribution of the superlattice are displayed in Figs. 4(b), 4(d), and 4(f), respectively. As expected, the topological interface state (solid red line, 8892.1 Hz) emerges in the fifth bandgap (light red region) in Fig. 4(b). At the same time, a transmission peak (dashed red line, 8892.12492 Hz) appears in the same bandgap (light cyan region) in Fig. 4(d), which corresponds to the topological interface state of Fig. 4(b).

In the presence of perturbations, the dispersion relations, transmission spectrum, and total pressure field distribution of superlattice are also investigated, as shown in Figs. 4(c), 4(e), and 4(g), respectively. It is obvious that the transmission peak (8892.84734 Hz) corresponding topological interface state still exist in the fifth bandgap. Comparing the frequencies of the peaks in Figs. 4(b) and 4(c), there is a small frequency shift. Hence, we consider that the superlattice has strong robustness as perturbations are introduced. The similar result can be obtained if we exchange the signs the two parameters. However, if the two parameters have the same signs, e.g., $\theta, d < 0$ or $\theta, d > 0$, the topological interface state also appears in the third bandgap.

To answer our second question, we design a T-shaped junction, which consists of two horizontal 1D waveguides and a vertical 1D waveguide. The two horizontal 1D waveguides, are made of 10 supermolecules with $d = 0.16$ cm, and the vertical 1D waveguide is 10 supermolecules with $\theta = 60^\circ$. Here, our T-shaped junction possess three ports, which labeled as 1, 2, and 3, respectively. In our simulations, except for the plane wave radiation conditions at the three ends of the T-shaped junction, all the other boundaries are set as rigid boundaries.

In Fig. 5(a), the vertical port 1 with an amplitude of 1 Pa radiates plane waves, and the horizontal ports 2 and 3 are outlets. Incident plane waves, originating from the bottom, propagate upward, and enter into the left and right ports at the T-junction, respectively. The enlarged view of the pressure and velocity in the vicinity of the T-junction is shown in the inset (large black dashed rectangle), and the green arrows are the directions of velocities, given by the simulations, which exhibit that incident waves propagate in one direction only along the left and right horizontal waveguides. The phenomenon can be regard as a double-channel transmission process. This double-channel transmission spectra of the topological interface state at 6446 Hz in the third bandgap also occurs, and the amplitudes of the topological interface state at 6446 Hz at horizontal ports 2 and 3 are equal to 0.5 Pa approximately, as shown in Fig. 5(c).

In Fig. 5(b), the ports 2 and 3 in the horizontal direction are acoustic sources with an amplitude of 1 Pa, and the vertical port 1 is the exit. Acoustic waves from the top-left and top-right ports propagate in the inward directions, and merge at the cross, then spread together in one direction along the downward vertical waveguide, as shown in the inset of Fig. 5(b). The inset (large black dashed rectangle) exhibits zoomed-in image of the distributions of the pressure and velocity near to the T-shaped junction. From Fig. 5(d), we

\[ \begin{align*}
\text{(a)} & \quad \Delta \theta, \Delta d \\
\text{(b)} & \quad \text{Dispersion relations} \\
\text{(c)} & \quad \text{Transmission spectrum} \\
\text{(d)} & \quad \text{Total pressure field distribution} \\
\text{(e)} & \quad \text{Transmission spectrum} \\
\text{(f)} & \quad \text{Total pressure field distribution} \\
\text{(g)} & \quad \text{Transmission spectrum}
\end{align*} \]

Fig. 4. (Color online) Robustness of 1D PCs. (a) Superlattice with perturbations $\Delta \theta = 12^\circ$ and $\Delta d = 0.04$ cm by breaking the mirror symmetries near the interface. The perturbations $\Delta \theta$ and $\Delta d$ are characterized by the red and green arrows in two insets, respectively. The insets are expanded views of the two supermolecules on the left and right of the interface. The superlattice is composed of five supermolecules with $\theta = -60^\circ$ on the left side and five supermolecules with $d = 0.16$ cm on the right side, and the dashed red line is the interface. (b) and (c) dispersion relations of the superlattice are investigated, as shown in Figs. 4(c), 4(e), and 4(g), respectively. It is obvious that the transmission peak (8892.84734 Hz) corresponding topological interface state still exist in the fifth bandgap. Comparing the frequencies of the peaks in Figs. 4(b) and 4(c), there is a small frequency shift. Hence, we consider that the superlattice has strong robustness as perturbations are introduced. The similar result can be obtained if we exchange the signs the two parameters. However, if the two parameters have the same signs, e.g., $\theta, d < 0$ or $\theta, d > 0$, the topological interface state also appears in the third bandgap.

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observe a transmission peak corresponding to the topological interface state (6446 kHz) in the third bandgap reaches 2 Pa.

In this letter, we construct a complicated scatterer which composed of an external cylinder with a single split and internal labyrinthine resonator, and use it to form a supermolecule. Then we modulate the topological properties of band structures of PCs by single parameter $\theta$ or $d$, and achieve an alternative topological bandgap by the combination of $\theta$ and $d$. Besides, we obtain topological interface states at the interface between two PCs with distinct topological phases, and validate their robustness against defects because there occurs a small frequency shift. Last we design the 1D T-shaped junction, and implement acoustic double-channel transmission adjusted by the combination of $\theta$ and $d$. We expect that our works have the promising prospects in engineering acoustic double-channel transmission devices.

Acknowledgments This work was supported by the National Nature Science Foundation of China (Grants No. 11 264 022, No. 12074156), and the Yunnan Local College Applied Basic Research Projects (Grant No. 2017FH001-001).

Fig. 5. (Color online) Acoustic topological interface states of one dimensional metamaterial propagating through a T-shaped junction. (a) and (b) Total pressure field distributions corresponding to acoustic double-channel transmission. The red arrows indicate the directions of acoustic waves. Two zoomed-in images of the distributions of the pressure and velocity in the vicinity of the T-shaped junction are shown in (a) and (b), respectively; and the green arrows indicate the velocity directions of the topological interface state in the two insets, given by the simulations. (c) and (d) Transmission spectra in (a) and (b). The light yellow region is the area of the third bandgap. For acoustic double-channel transmission, the solid green line and the dotted red line in (c) indicate the transmission spectra of the output ports 2 and 3, respectively. The transmission spectrum of the output 1 in (b) is shown in (d) with the solid purple line.
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