Trajectory Planning for Rotational Chain Shell Magazine

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Abstract. To ensure the stability and reliability of the rotational chain shell magazine moving system, the trajectory planning such as cubic curve, quintic curve, T-shaped velocity curve and S-shaped velocity curve were considered in this paper. The effectiveness and feasibility of each trajectory planning method for the control system of the rotational chain shell magazine were verified by the combination simulation technology of MSC.ADAMS and MATLAB. The simulation results show that the S-shaped velocity curve satisfies the requirement of the smoothness and continuity of the trajectory of the magazine. It ensures the continuity of angular velocity and acceleration, and reduces the buffeting and impact effectively that other curves can't satisfy. It is of great significance for the stability and reliability of the rotational chain shell magazine.

1. Introduction
Trajectory planning refers to expressing the acceleration, velocity and displacement of motion as functions of time, so as to calculate the reference data sets such as acceleration, velocity and position that satisfy the constraints of system dynamics. Trajectory planning is an indispensable part of motion control system. In motion control system, the accuracy of trajectory can directly affect the accuracy and performance of motion system, which is of great significance for the efficiency and stability of motion system [1]. Typical trajectory planning methods include polynomial trajectory planning, T-shaped trajectory planning, S-shaped trajectory planning, and spline function planning. Different trajectory planning algorithms have different smoothing effect, transition time, and sophistications. Liu [2] analyzed and compared the motion performance of the cubic curve and quintic curve in the trajectory planning of industrial robots. The results show that the trajectory planning effect of the quintic curve is better than that of the cubic curve. Zhu [3] proposed a new approach for four-order S-shaped motion profile algorithms, and the algorithms proposed had been verified by the three-mover haul platform. The experiments state that the velocity and acceleration curves of this new algorithm are smoother than that of traditional motion profile algorithm at the case without the large amounts of the curve data. The algorithms improve the flexibility of the system effectively. Wang [4] proposed trapezoidal velocity curve to optimize the curves of velocity and acceleration of the robot, improving the motion properties of parallel robots.

The rotational chain shell magazine motion control system is composed of trajectory planning algorithm and motion control algorithm. The trajectory planning algorithm is used to smooth the position instruction, so that there is no flexible impact during the movement of the magazine. The motion control algorithm is used to realize the actual rotation angle of the magazine following the trajectory planning value [5]. In the previous research, scholars focused on solving the problem of motion control algorithm, a few people deeply explored the trajectory planning part, while ignoring the connection between the
trajectory planning parameters and the dynamic characteristics of the controlled object, which result in the decrease of motion accuracy. ZOU [6] proposed some key problems of the autoloader control system in a large caliber howitzer. And he studied them in-depth by combining the theories of multi-body dynamics, sliding mode control, fuzzy mathematics, adaptive control, control system reliability and so on. However, the expected trajectory planning is not involved in the studies above.

Aiming at the defects of the trajectory planning of the rotational chain shell magazine, the theoretical analysis of proposed velocity planning methods such as cubic curve, the quintic curve, T-shaped velocity curve and S-shaped velocity curve is carried out. Using the joint simulation technology of MSC.ADAMS and MATLAB, the continuity and smoothness of the angle, angular velocity and acceleration curve of each trajectory planning method are observed, and the advantages and disadvantages of several planning methods are compared intuitively.

2. Rotational chain shell magazine

The rotational chain shell magazine is an important part of the large caliber artillery automatic loading system. As shown in figure 1, each cartridge is strung into a barrel chain, and the projectiles are stored in each cartridge. Under the control system, the cartridge is driven by the sprocket drive chain to the designated position.

![Figure 1. Over view of a rotational chain shell magazine.](image)

Figure 1. Over view of a rotational chain shell magazine.

Based on the above analysis, a dynamic model is constructed in MSC. ADAMS. The topological graph of rotational chain shell magazine shown in figure 2, \( h_1 \) fixes the shelf of magazine to the ground, \( h_2 \) fixes the rail to the shelf of magazine, \( h_3 \) is the revolute joint between the chain wheel and the shelf of magazine, \( h_4 \) is the contact between supporting wheels and the rail, \( h_5 \) is the contact between rollers and the rail, \( h_6 \) is the contact between rollers and the chain wheel, \( h_7 \) is the revolute joint between supporting wheels and cartridges, \( h_8 \) is the revolute joint between chains and rollers, \( h_9 \) is the revolute joint between chains and cartridges.

The contact stiffness is calculated according to Hertz's contact law [7]. The contact impact force is equivalent to the spring damping model. When the penetration is \( \delta \), the normal contact force \( F_n \) can be expressed as:

\[
F_n = k \delta^n + \text{step}(\delta, 0, 0, d_{\text{max}}, c_{\text{max}}) \frac{d\delta}{dt}
\]  

(1)

In the formula, \( k \) is the contact stiffness; \( n \) is the non-linear exponent; \( d_{\text{max}} \) is the penetration depth when the damping reaches the maximum value \( c_{\text{max}} \), and the value depends on the material of the two
colliding parts; \( \text{step}(\delta, 0, 0, d_{\text{max}}, C_{\text{max}}) \) is the calculation function of damping \( c \), indicates that when the penetration value \( \delta \) changes from 0 to \( d_{\text{max}} \), the damping \( c \) changes from 0 to \( C_{\text{max}} \) correspondingly. The friction force in the model is calculated by Coulomb's friction law

3. Trajectory planning methods
The point-to-point trajectory planning algorithm can be understood as determining a smooth curve \( \theta(t) \) moving from a known starting point to an ending point within a prescribed time \( T \). At present, the trajectory planning method mostly adopts polynomial trajectory planning. Yet in some practical problems, smooth angle, velocity and acceleration curves cannot be obtained. Although increasing the number of polynomials can improve the trajectory accuracy, the calculation amount will also increase accordingly and the "Runge" phenomenon will be easier to appear. They will lead to the increase of position error [8].

In this paper, the point-to-point trajectory planning is carried out by using cubic curve, quintic curve, T-shaped velocity curve and S-shaped velocity curve

3.1. Cubic curve
When the motion system only has position constraints and velocity constraints, cubic curves can be used for trajectory planning [9].

In order to satisfy the system motion trajectory as a smooth motion curve, at least four constraints are required, namely two endpoint position constraints and two endpoint velocity constraints. The endpoint position constraint refers to the angle corresponding to the start point and the end point respectively. The value of \( \theta(t) \) at time \( t_0 \) is the starting angle \( \theta_0 \), and the value at time \( t_f \) is the ending angle \( \theta_f \):

\[
\theta(t_0) = \theta_0; \quad \theta(t_f) = \theta_f
\]  

In order to meet the continuity requirement of the system motion speed, the endpoint speed constraint needs to be set. The corresponding constraint conditions are the velocity at the starting point \( \dot{\theta}_0 \) and the velocity at the termination point \( \dot{\theta}_f \):

\[
\dot{\theta}(t_0) = \dot{\theta}_0; \quad \dot{\theta}(t_f) = \dot{\theta}_f
\]  

From the four constraints given above, a cubic polynomial can be uniquely determined:

\[
\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3
\]  

By taking equation (2), (3) into (4), four unknown parameters \( a_0 - a_3 \) can be determined, thereby determining the cubic curve from the starting point to the ending point that satisfies the requirements of continuous stationary motion.

3.2. Quintic curve
To satisfy the continuity of the acceleration of the system, the quintic curve is considered [10]. Based on the constraints of endpoint position and velocity, the acceleration constraints of endpoints are added. The corresponding constraints are the acceleration at the starting point \( \ddot{\theta}_0 \) and the acceleration at the ending point \( \ddot{\theta}_f \):

\[
\ddot{\theta}(t_0) = \ddot{\theta}_0; \quad \ddot{\theta}(t_f) = \ddot{\theta}_f
\]  

According to the six constraints of position, velocity and acceleration, the unique quintic multinomial can be obtained by:

\[
\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5
\]
By substituting equation (2), (3), and (5) into (6), six unknown parameters $a_0 - a_5$ can be determined, thereby a quintic curve from the starting point to the ending point that satisfies the requirements of continuous stationary motion can be determined.

3.3. T-shaped velocity curve

The T-shaped velocity curve means that the planned velocity curve is trapezoidal. It has the characteristics of simple and convenient algorithm as well as easy implementation of control strategy [11]. As shown in figure 3, the T-shaped velocity curve consists of three sections: uniform acceleration section, uniform velocity section and uniform deceleration section. The constraints are the starting time $t_0$, the ending time $t_f$, the starting angle $\theta_0$, the ending angle $\theta_f$ and the velocity in the specified uniform speed range $v$.

![T-shaped velocity curve](image)

Figure 3. T-shaped velocity curve.

Set the time corresponding to the uniform acceleration section (the uniform deceleration section) is $t_b$, the velocity of the uniform velocity section is $v$ and the acceleration of the uniform acceleration section (the uniform deceleration section) is $a$, then

$$v = at_b$$

Equation (8) is derived from the displacement equation:

$$\frac{1}{2}at_b^2 + v(t_f - t_0 - 2t_b) + \frac{1}{2}at_b^2 = \theta_f - \theta_0$$

After specifying the value of the uniform velocity the acceleration (deceleration) time can be obtained by bringing equation (7) into (8):

$$t_b = t_f - t_0 - \frac{\theta_f - \theta_0}{v}$$

To facilitate the analysis of the planned trajectory, setting $v_{lim}$ as:

$$v_{lim} = \frac{\theta_f - \theta_0}{t_f - t_0}$$

Compare $v$ with $v_{lim}$, if $v_{lim} > 0$, the following speed planning situation can be obtained by:

- If $v_{lim} < v < 2v_{lim}$, the planned acceleration (deceleration) time is $t_b = t_f - t_0 - \frac{\theta_f - \theta_0}{v}$.
- If $v = 2v_{lim}$, the planned acceleration (deceleration) time is $t_b = \frac{1}{2}(t_f - t_0)$, and there is no uniform motion section.
- If $v = v_{lim}$, there is only the uniform motion section.
- If $v < v_{lim}$, there is no solution section.
• If \( v > 2v_{\text{max}} \), there is no uniform velocity section.

3.4. S-shaped velocity curve

The required constraints of the S-shaped velocity curve are the starting time \( t_0 \), the ending time \( t_f \), the starting angle \( \theta_0 \), the ending angle \( \theta_f \), the speed of the specified uniform velocity segment \( v_{\text{max}} \) and the maximum acceleration \( a_{\text{max}} \). The velocity and acceleration constraints need to be established according to the dynamic constraints of the controlled object.

In terms of the known constraints, the jerk \( j \) (derivative of acceleration) can be calculated by equation (11):

\[
j = \frac{a_{\text{max}}^2 v_{\text{max}}}{(t_f - t_0)v_{\text{max}}^2} \left( a_{\text{max}}^2 - a_{\text{max}}^2 - v_{\text{max}}^2 \right)
\]

(11)

The S-shaped velocity curve, usually in the form shown in figure 4, is of a seven-segment type, which is defined as [12]:

- First segment: Increasing the acceleration from 0 to acceleration \( a_{\text{max}} \) with a constant jerk \( j \).
- Second segment: Keeping acceleration at a constant acceleration \( a_{\text{max}} \).
- Third segment: Reducing the acceleration from \( a_{\text{max}} \) to 0 with a constant negative jerk \(-j\).
- Fourth segment: Moving at a constant speed \( v_{\text{max}} \).
- Fifth segment: Reducing the acceleration from 0 to \(-a_{\text{max}}\) with a constant negative jerk \(-j\).
- Sixth segment: Keep deceleration at a constant acceleration \(-a_{\text{max}}\);
- Seventh segment: Increasing the acceleration from \(-a_{\text{max}}\) to 0 with a constant positive jerk \( j \).

![S-shaped velocity curve](image)

**Figure 4. S-shaped velocity curve.**

4. Simulation and result analysis

Based on MSC.ADAMS, the rotational chain shell magazine dynamical model is constructed, the parameters are as follows: single cartridge moment of inertia \( J = 3.22 \text{kg} \cdot \text{m}^2 \); coefficient of static friction \( \mu_s = 0.08 \); coefficient of dynamic friction \( \mu_k = 0.05 \). The control model is established under MATLAB, by setting the simulation parameters \( a_{\text{sc}} t_0 = 0, t_f = 1s, \theta_0 = 0, \theta_f = 60^\circ, v = 73.5 \cdot \text{s}^{-1}, a_{\text{max}} = 400 \cdot \text{s}^{-2}, v_{\text{max}} = 90 \cdot \text{s}^{-1} \).

Establish co-simulation solver, the four kinds of trajectory planning algorithms are simulated. The magazine rotating to \( 60^\circ \) within 1s is the control target set in this paper.

Since the rotational chain shell magazine is in the form of chain transmission and the number of drum magazines is large, the meshing impact force and the motion transmission are not stable during the movement. Therefore, it is impossible to obtain a smooth angular acceleration curve, only the curve
trends can be observed. This is caused by the structure of the model and inevitable in simulation. A set of better-performing curves from multiple sets of simulation results is shown in the following figures:

Figure 5. Magazine rotation angle vs. time.

Figure 6. Magazine rotation angular velocity vs. time.

As seen in figure 5, four trajectory planning methods all satisfy the position constraint requirements, and the curve smooth transition satisfies the continuity requirement. According to figure 6, the four trajectory planning methods satisfy the angular velocity constraint. Among them, the S-shaped velocity curve has the best smooth continuity of angular velocity, followed by the quintic curve. The T-shaped velocity curve has no smooth transition at the two connection points of the uniform acceleration stage, the uniform deceleration stage and the uniform velocity stage. The angular velocity trends of the cubic curve is the worst. It can be seen from the figure 7 that the angular acceleration of the cubic curve is abruptly changed at the start time and the end time; the angular acceleration of the T-shaped velocity curve is abruptly changed at the speed connection point; the angular acceleration curves of the quintic curve and S-shaped velocity curve are smooth and meet the requirement of continuity.

The cubic curve only satisfies the constraint of position and angular velocity, but the angular acceleration is discontinuous at the beginning and the end of the curve, as shown in figure 5, figure 6 and figure 7(a). At the initial point $t = 0$, the sudden change of angular acceleration produces an impact, which makes the initial angular velocity of the magazine jitter. Similarly, the angular acceleration does not reach 0 at the terminal point $t = 1s$, which causes an impact.

The quintic curve satisfies the constraints of position, angular velocity and angular acceleration, as shown in figure 5, figure 6 and figure 7(b). The angular acceleration curve, as a sinusoidal function, is continuous at the initial moment and the termination moment. Therefore, the abrupt change of the angular acceleration at the two moments of the cubic curve is eliminated.

As shown in figure 5, figure 6 and figure 7(c), the T-shaped velocity curve meets the constraints of position, angular velocity and angular acceleration. But the angular acceleration mutates at $t = 0.1838s$ and $t = 0.8126s$, the two connection points of the uniform acceleration stage, the uniform deceleration stage and the uniform speed stage, resulting in flexible impact. The angular velocity curve does not have a smooth transition in the connection points with tremble.

The S-shaped velocity curve satisfies the position, angular velocity and angular acceleration constraints as shown in figure 5, figure 6 and figure 7(d). The angular acceleration has no abrupt change at the eight different speed stages of $t = 0, t = 0.1838s, t = 0.2250s, t = 0.3333s, t = 0.6666s$, and
The angular velocity curve continuously and smoothly transitions at the velocity connection points. It can be seen that S-shaped curve solves the problem of sudden change of acceleration in T-shaped curve and avoids flexible impact in magazine movement. The S-shaped velocity curve can satisfy the constraints of acceleration and is more suitable for the magazine motion system than the quintic curve.

Figure 7. Magazine rotation angular acceleration vs. time.

In summary, the S-shaped velocity curve is selected to plan the trajectory of the chain-type rotary warehouse. It satisfies the requirement of the smoothness and continuity of trajectory, ensures the continuity of angular velocity and acceleration, reduces buffeting and impact effectively, that other curves can't satisfy.

5. Conclusion
The theoretical analysis of trajectory planning methods such as cubic curve, the quintic curve, T-shaped velocity curve and S-shaped velocity curve is carried out in this paper. The trajectory curves of each trajectory planning method for the control system of the rotational chain shell magazine are verified. The advantages and disadvantages of each trajectory planning method are compared by the combination simulation technology of MSC.ADAMS and MATLAB. The simulation results show that the trajectory of the magazine meets the requirement of continuity and smoothness with the S-shaped velocity curve, which ensures the continuity of angular velocity and acceleration. The angular acceleration curves of cubic curve and T-shaped velocity curve are not discontinuous, which cause impact. The quintic curve is unable to track high precision systems. Therefore, choosing the S-shaped velocity curve as the trajectory planning method of great significance for the stability and reliability of the rotational chain shell magazine.

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