Single laser modulated drive and detection of a nano-optomechanical cantilever

Vincent T. K. Sauer,1,2 Zhu Diao,1,3,a Jocelyn N. Westwood-Bachman,1,3 Mark R. Freeman,1,3 and Wayne K. Hiebert1,3,b

1National Institute for Nanotechnology, Edmonton, Alberta T6G 2M9, Canada
2Department of Biological Sciences, University of Alberta, Edmonton, Alberta T6G 2E9, Canada and Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta T6G 1H9, Canada
3Department of Physics, University of Alberta, Edmonton, Alberta T6G 2E1, Canada

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To reduce the complexity in a nano-optomechanical system a pump and probe scheme using only a single input laser is used to both coherently pump and probe the nanomechanical device. The system operates similarly to the traditional two laser system, but instead of using a constant power to probe the device and a separate, modulated laser to drive it with an optical gradient force, a single laser is utilized for both functions. A model of the measurement scheme’s response is developed which matches the experimental data obtained in the optomechanical Doppler regime and low cavity power limit. As such, the unconventional response still yields useful device information such as the resonant frequency of the device and its mechanical quality factor. The device is driven with low noise and its frequency is tracked using a phase-locked loop. This demonstrates its potential use for dynamic frequency measurements such as nanomechanical inertial mass loading. In such a system, the estimated mass resolution of the device is 6 zg and consistent with other detection methods. © 2017 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). [http://dx.doi.org/10.1063/1.4975347]

Nanomechanical beam resonators have been established as an effective method for detecting small amounts of mass with resolutions down to or below the mass of a single proton.1,2 This achieves mass sensitivity comparable to traditional mass spectrometers, but this has not yet been accomplished with commercial implementations.3–5 As a potential solution nano-optomechanical systems (NOMS) offer many advantages over other nanomechanical beam detection methods which include higher displacement sensitivities, very large frequency detection bandwidths and self-aligned/integrated transduction.6,7

To reduce the complexity and cost of NOMS resonator detection, and decrease the barrier of entry into the field for new and existing researchers, a single laser modulated drive and detection (SLMDD) system is modeled and demonstrated. Using this system, a nanomechanical cantilever device is driven with low noise and its frequency is tracked using a phase-locked loop. In this way, the system is demonstrated for use toward nanomechanical inertial mass loading experiments. Its estimated mass sensitivity is shown to be on par with conventional drive and detection methods, but the SLMDD method has a much greater upside pertaining to the flexibility of transduction design in a NOMS system and the ease of fabrication of such a device.

The general principles of NOMS detection involve an interaction between the mechanical resonator and a nanophotonic device such as a waveguide8–10 or optical cavity.11–16 The motion of the beam will modulate the optical properties of the nanophotonic structure which is detected as a power

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aNow at Department of Mathematics, Physics, and Electrical Engineering, Halmstad University, Halmstad, Sweden SE-301 18
bCorresponding author: wayne.hiebert@nrc-cnrc.gc.ca
modulation at the device output. The advantages of NOMS detection complement the effects of moving toward smaller and more sensitive nanomechanical mass sensors, which could have decreased device displacement and higher operating frequency.\textsuperscript{17}

The SLMDD setup operates similarly to a traditional NOMS pump/probe system, but instead of using a separate probe laser with a constant output power this probe laser is power modulated to coherently drive the nanomechanical resonator through an optical gradient force.\textsuperscript{6,18} This is distinct from dynamic backaction mechanical amplification where a single laser uses the relationship between optical radiation pressure and mechanical deformation of the cavity to transfer power to the mechanical mode.\textsuperscript{19,20} SLMDD has the advantage that the resonator can be driven at a broader range of laser detuning values. This occurs with varying efficiencies, however, since both the drive power and transduction efficiency are dependent on the cavity detuning and not maximized at the same value. This could be mitigated through increasing the laser power until the pumping limit is reached at the onset of mechanical non-linearity. Another benefit is the mechanical mode characteristics are decoupled from the drive. With this system, both the second laser source and the optical filter used in the traditional NOMS pump/probe method can be removed, but coherent drive is still achieved. While operating as a resonant sensor, the greatest frequency stability (which corresponds to the greatest sensor resolution) is achieved when the device is highly driven. The sensitivity increases proportionally to the drive amplitude divided by the amplitude of the noise limit, which in a good sensor system is the thermomechanical (TM) noise of the device.\textsuperscript{21–23} Just characterizing the mechanical frequency through an undriven method such as probing the TM response is not enough to reach the physical limits of sensor resolution.\textsuperscript{24} There is an added benefit of being able to drive and detect devices with a large free spectral range, such as in some photonic crystal optical cavities, where multiple optical modes may not be accessible within the same laser frequency band. The SLMDD schematic is shown in Fig. 1. It is similar to the setup described in Ref.\textsuperscript{25} with an additional electro-optic modulator (Lucent Technologies 2623NA) and erbium-doped fiber amplifier (Amonics AEDFA-23-B-FA) located after the laser source.

Using only a single laser, the drive amplitude modulation cannot be removed prior to the detector via an optical filter, and the measured signal exhibits a non-standard shape. The traditional high Q estimated, Lorentzian driven damped oscillator frequency response,\textsuperscript{26} which occurs in the experiment as it operates in the optomechanical Doppler regime, with weak optomechanical coupling (supplementary material), is superimposed on a weakly frequency-dependent background from the sinusoidally-modulated optical power. Since most nanomechanical sensor systems depend on tracking this characteristic frequency, the non-standard signal shape is of secondary concern as long as it is understood and can be tracked.

The output power of the system can be described as the product of the input power and the transmission coefficient of the device, $P_{\text{out}}(t) = P_{\text{in}}(t)T(t)$. In the SLMDD system, the $P_{\text{in}}$ consists

![FIG. 1. Schematic of the SLMDD system. A tunable diode laser is passed through an electro-optic modulator (EOM) and an erbium-doped fiber amplifier (EDFA) before being coupled to/from a nanophotonic chip. Polarization controllers (PC) are used to ensure proper polarization at the optical devices. A lock-in amplifier (LIA) controls the EOM output and reads the photodiode (PD) response.](image-url)
of a DC term and an AC term such that \( P_{in} = P_{dc} + P_{1f} \cos(\omega t) \). The transmission coefficient follows from the nanophotonic transduction of a nanomechanical beam.\(^{15,27}\) Full details are found in supplementary material. With the laser blue-detuned from the optical cavity, the response is separated into its frequency components at DC, the first harmonic and the second harmonic, respectively:

\[
P_{out,dc} = P_{dc} T_0 + \frac{P_{1f}}{2} A(\omega) \frac{\partial T}{\partial x} \cos \phi
\]

\[
P_{out,1f} = P_{1f} T_0 \cos(\omega t) + P_{dc} A(\omega) \frac{\partial T}{\partial x} \cos(\omega t - \phi)
\]

\[
P_{out,2f} = \frac{P_{1f}}{2} A(\omega) \frac{\partial T}{\partial x} \cos(2\omega t - \phi)
\]

In the equations above \( T_0 \) is the optical transmission coefficient when the mechanical resonator is stationary, and \( A(\omega) \) is the Lorentzian-shaped magnitude response of an under-damped harmonic oscillator which has a phase shift of \( \phi \). \( \frac{\partial T}{\partial x} \) is the transduction coefficient for the NOMS device, and \( \omega \) is the angular frequency.

The frequency response at the fundamental frequency of the nanomechanical beam, \( P_{out,1f} \), is the focus of the analysis. The measured signal will include the power modulated driving signal (the first term on the right hand side of Equation (2)) and the Lorentzian response of the nanomechanical under-damped harmonic oscillator (the second term with the shape of function \( A(\omega) \)). The addition of these vector signals with different phases leads to interference between them which causes a Fano-like lineshape. A graphical representation of this vector addition is shown in Fig. 2(a) with the real component corresponding to the in-phase signal, and the imaginary component corresponding to the quadrature signal. The response at \( 2f \) should exhibit the standard response of the nanomechanical beam. Interestingly, the DC response, \( P_{out,dc} \), also contains information about the mechanical resonance, which could be used in principle to monitor very high frequency mechanical resonances with low bandwidth.

Finally, the magnitude response at \( 1f \) can be simplified from Equation (2) to include device parameters such as the mechanical quality factor, \( Q = \frac{\omega_0}{2 \beta} \), and the fundamental frequency, \( \omega_0 \).

FIG. 2. (a) Vector representation of the combined signal, \( P_{out,1f} \), detected near the first harmonic frequency of a nanomechanical beam. The dashed circle represents the magnitude envelope of the damped driven mechanical oscillator. (b) The normalized complex response of the SLMDD device at various drive powers. The inset is the TM noise scatter plotted at a close-up scale with the units on the axes corresponding to mVpk Hz\(^{-1/2} \).
Here, $\beta$ is the attenuation rate of the envelope of the oscillator’s exponential decay. For ease of analysis let $u = P_{1f}T_0$ and $v = P_{dc}(\frac{\partial T}{\partial x})(G(m_{\text{eff}}T_0))$, where $G = F_0/P_{1f}$ is the gain factor converting optical power to optical force and $m_{\text{eff}}$ is the effective mass of the nanomechanical beam.

$$P_{\text{out},1f} = u \left[ 1 + \frac{v (v + 2(\omega_0^2 - \omega^2))}{(\omega_0^2 - \omega^2)^2 + (\omega_0\omega/Q)^2} \right] \tag{4}$$

$$\tan \theta = \frac{v (\omega_0\omega/Q)}{(\omega_0^2 - \omega^2)^2 + (\omega_0\omega/Q)^2 + v (\omega_0^2 - \omega^2)} \tag{5}$$

The setup is tested using a 3.9 $\mu$m long 220 nm wide and 165 nm thick nanomechanical cantilever which is side-coupled to a racetrack resonator optical cavity in an all-pass configuration. The device was fabricated with the assistance of CMC Microsystems using standard nanophotonic chip fabrication practices. The racetrack optical cavity has an optical $Q$ of $\sim 4800$, a linewidth of 0.32 nm, an FSR of 13.34 nm and a finesse of $\sim 42$. The laser detuning is approximately equal to half of the cavity linewidth. The DC optical power circulating in the racetrack is approximately 2.3 mW, with measurements having an AC optical power of approximately 140 $\mu$W$_{pk}$, 650 $\mu$W$_{pk}$ and 1.1 mW$_{pk}$ in the cavity. A full description of the device and measurement can be found in supplementary material.

The complex response of the system is plotted in Fig. 2(b). Here, the responses for the driven signals are normalized by the magnitude of their peak drive voltage. The response is expected to be circular as in the vector sketch in Fig. 2(a), but there is a background linear phase accumulation (see Fig. 3) across the driven sweep which causes the asymptotic line feature of the response. The elliptical response at larger signal magnitudes at higher drives are due to higher harmonics output by the EOM (see supplementary material). These occur and become appreciable when the voltage drive is large compared to the $V_{pk}$ of the EOM, and will become more pronounced if the EOM bias is not fully centered.

**FIG. 3.** The TM noise signal plotted alongside the SLMDD signal at 100 mV$_{pk}$ and 1 V$_{pk}$ AC drive voltages with the laser blue-detuned. The x-axis is identical for all plots with the y-axis at different scales. The phase response of each signal is shown on each respective plot. The insets show the extracted Lorentzian response with approximate displacement in nm on the y-axes. The TM noise is fit with the high Q estimated Lorentzian and the driven signals are fit to Equations (4) and (5).
Additionally, it is important to note that even though non-linear effects due to strong optomechanical phenomenon are negligible in certain experiments, they still may be present due to the opto-electronic equipment used, as above, or even the transduction scheme used. This would especially be true at large device displacements where the linear assumption of the transduction coefficient no longer holds, analogously to the force dependence on device displacement. This may affect the measured signal which is especially important to take into account when using such devices in a sensor environment.

As shown in Fig. 3 the SLMDD signals exhibit a non-Lorentzian shape but are still centered at the same location as their TM noise signals. As expected from Equation (4) there is a significant non-zero background component to the signal, and the magnitude rises and dips corresponding to the phase of the damped harmonic oscillator as it crosses its mechanical resonance (i.e. following the dashed circle in Fig. 2(a)).

The driven signals scale approximately linearly with the AC drive voltage. This can be explained by looking at Equation (4). The background value is scaled by $u = P_1 f T_0$, and this matches with the $P_1$ increase caused by the sinusoidal voltage response of the EOM sent to the EDFA. The shape of the peak is more complicated, but its relative height is related to $v = P_{dc} (\partial T/\partial x) (G/(m_{eff} T_0))$, while its width is related to $Q$. Extracting the harmonic oscillator signal characteristics from the SLMDD response (insets of Fig. 3) (supplementary material) the peak magnitudes scale supralinearly with the AC drive, in contrast to the DC background. This could occur if the optical gain factor, or force, on the nanomechanical beam increases at higher drive powers and beam deflections, or if the transduction coefficient increases and is no longer linear itself.

The mechanical device signal data is fit to Equations (4) and (5). The fundamental frequency and $Q$-factor of the device at various AC drive powers match closely with each other and the values extracted from its TM noise signal. For each excitation level (fitting both magnitude and phase), the fundamental frequencies match within error to 12.53 MHz. In addition, the extracted $Q$-factors from the driven devices are equal to approximately 8700. This is expected as the energy dissipated per cycle should remain approximately equal with our linearly driven devices.

To further distinguish this method from the dynamic backaction method the optical signal is also red-detuned from the cavity of a similar device. The difference in detuning causes the shape of the signal to invert as seen in Fig. 4(a). The magnitude response for both the red and blue laser detunings are approximately the same and are dependent on the optical driving power and transduction coefficient.

If one is looking for a typical response using a SLMDD approach, the analysis of supplementary material indicates that the response at $2f$ should yield a standard Lorentzian mechanical response shape. This does not occur. Measured in the experiment it yields a response shape similar to that at the fundamental frequency as is shown in Fig. 4(a). The $2f$ response occurs due to the higher drive harmonics present at the output of the EOM as illustrated in Figs. 4(b) and 4(c). These are due to a detuning of the EOM bias, along with drive values comparable to the $V_{pi}$ of the EOM, and not due to error in the model.

![Figure 4](image-url)

**FIG. 4.** (a) The responses at $f$ and $2f$ for a SLMDD red-detuned signal driven using an EOM with harmonic output modes. The 2nd harmonic response frequency is scaled by 1/2 to align with the fundamental response. The EOM output harmonics cause signal mixing at higher drive harmonics such that $2f$ responses are not Lorentzian shaped. (b) The normalized harmonic output of the EOM at 1 V pk and (c) 500 mV pk AC drive.
FIG. 5. The Allan Deviation for various drive powers taken at a 1 kHz measurement bandwidth.

The Allan deviation of the device is measured using a phase locked loop (PLL) (see supplementary material for details) with a 1 kHz bandwidth while under AC drive voltages of 5 mV\textsubscript{pk} to 1 V\textsubscript{pk}, as shown in Fig. 5. The Allan deviation can be used as a measure of the frequency fluctuations and is directly proportional to the ultimate mass resolution of the device, as \(\delta m = -2\left(\frac{m_{\text{eff}}}{\omega}\right)\delta \omega\). The background signal passes on the noise of the drive oscillator in a straightforward way and, as such, should add negligible noise to the SLMDD measurement scheme (assuming the mechanical resonator’s phase noise is much larger than the drive oscillator’s). Although the maximum phase change in Fig. 3 is reduced compared to the \(\pi\)-shift normally associated with a driven harmonic oscillator resonance, the corresponding phase noise is reduced by a similar factor – approximately the ratio of the SLMDD signal to the resonance signal (see supplementary material). Thus, SLMDD should maintain similar fractional frequency stability no worse than a factor of \(\pi\) as compared to the two-laser pump-probe technique of supplementary material.

The mass resolution improves with the drive power as the phase noise decreases as shown in Fig. 5. The negative slope corresponds to the white noise in the signal and the positive slope after the minimum corresponds to the random walk noise. At the highest drive powers the improvement in the phase noise saturates under the PLL conditions which are held constant, and hence so does the frequency stability. Using the lowest Allan Deviation to estimate the mass sensitivity gives a mass sensitivity of 6 zg. This is comparable to similar top-down devices driven and detected using more traditional methods.\textsuperscript{10,30}

In conclusion, a measurement scheme is described which uses only a single laser to coherently drive and detect a nano-optomechanical cantilever. The mathematical description fits closely with the measured data which confirms the signal mixing mechanism. The extracted physical characteristics of the nanomechanical beam from the SLMDD scheme match with those extracted from the fit of the TM noise. The device can still be implemented within a PLL to track the frequency response. Through doing so, the estimated mass resolution for the device was found to be comparable with traditional measurement techniques.

SUPPLEMENTARY MATERIAL

See supplementary material for a full description of the device fabrication and measurement, the device response and an estimate of the device phase noise.

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