Estimation of reliable range of electron temperature measurements with sets of given optical bandpass filters for KSTAR Thomson scattering system based on synthetic Thomson data

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\textbf{ABSTRACT:} One factor determining the reliability of measurements of electron temperature using a Thomson scattering (TS) system is transmittance of the optical bandpass filters in polychromators. We investigate the system performance as a function of electron temperature to determine reliable range of measurements for a given set of the optical bandpass filters. We show that such a reliability, i.e., both bias and random errors, can be obtained by building a forward model of the KSTAR TS system to generate synthetic TS data with the prescribed electron temperature and density profiles. The prescribed profiles are compared with the estimated ones to quantify both bias and random errors.

\textbf{KEYWORDS:} Analysis and statistical methods; Data processing methods; Plasma diagnostics - interferometry, spectroscopy and imaging

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1 Introduction

Thomson scattering (TS) diagnostic system is widely used to measure both temperature and density of electrons in hot fusion-grade plasmas. Broadening of the spectral distribution of the Thomson scattered light is a function of electron temperature; whereas the intensity of the light contains the information of electron density [1]. The KSTAR TS system collects the scattered light with polychromators consisting of a set of five optical bandpass filters [2] as shown in figure 1. Reliably measurable ranges of temperature depend on the properties of the filters such as central wavelength, bandwidth and the number of the filters.

![Figure 1](image)

**Figure 1.** Examples of measured transmittance of the optical bandpass filters (+) with spectral distributions of Thomson scattered lights for various electron temperatures (lines) for (a) Core #4 and (b) Edge #4 of the KSTAR TS polychromators used during the 2016 Campaign.

Achievable fractional errors given a set of the filters together with TS system parameters have been estimated by considering measured signal level of background and scattered light with an electronic, i.e., amplifiers, noise [3–5]. Such fractional errors provide quantitative magnitude of random errors but do not present bias errors. With the aim of obtaining both bias (if finite)
and random errors, we have developed a forward model of the KSTAR TS system and generated synthetic TS data [6] with prescribed profiles of electron density and temperature. The forward model is based on the TS system parameters used during the 2016 KSTAR campaign.

Treating the synthetic TS data as if they were experimentally measured ones, we can obtain temperature and density of electrons by using a look-up table method [6, 7] which, then, are compared with the prescribed temperature and density. Figure 2 schematically describes such a procedure. This allows us to quantify both bias and random errors. In this paper, we investigate reliability of estimating various profiles of electron temperature with a fixed density profile.

2 Forward model of the KSTAR Thomson scattering system

Laser pulse with the wavelength of 1064 nm is tangentially injected from the L-port of KSTAR [8] and the Thomson scattered photons are collected via edge and core optics systems as shown in figure 3. These photons are passed to polychromators, where each polychromator contains five optical bandpass filters. Combinations of central wavelengths and bandwidths are different between core and edge polychromators as shown in figure 1(a) and (b), respectively. Photons passed through a bandpass filter are detected by an avalanche photodiode detector (APD: Hamamatsu S11519-30).

Since there are five bandpass filters in a polychromator, we have five APDs for each polychromator.

Each APD outputs an electronic signal proportional to the detected photons, and this signal can be modelled as [3, 6],

\[ V_{TS}^i = G n_e N_{laser} \frac{d\sigma_{TS}}{d\Omega} \Delta\Omega L T(\lambda_L) QE \int \frac{\phi^i(\lambda_s) S(\lambda_s; T_e, \theta, \lambda_L)}{\phi(\lambda_L)} \frac{\lambda_L}{d\lambda_s} \, d\lambda_s, \]

(2.1)

where superscript \( i \) denotes the \( i \)th channel of the optical bandpass filters in a given polychromator. \( G, n_e, N_{laser} \) and \( \frac{d\sigma_{TS}}{d\Omega} \) correspond to the APD gain factor, the electron density, the number of injected photons in a single laser pulse and the differential Thomson scattering cross-section, respectively. \( \Delta\Omega \) and \( L \) are the solid angle and the scattering length of the KSTAR TS system, respectively. Transmission coefficient is a function of wavelength, and this is captured via taking the absolute

![Figure 2](image.png) A schematic of the KSTAR TS forward model. Expected (synthetic) TS data are generated based on the KSTAR TS system specifications, and the synthetic data are analyzed as if they were actual experimental data. Then, results are compared with the prescribed profiles of electron density and temperature to estimate the reliability of the KSTAR TS system.
Figure 3. Top view of the KSTAR TS system. Nd:YAG laser pulse with the wavelength of 1064 nm is injected tangentially from the L-port, and the laser beam dump is located at the B-port. Thomson scattered light is collected via edge (red dashed lines) and core (black dashed lines) optics systems at the N-port.

coefficient at the laser wavelength $T(\lambda_L)$ ($\lambda_L = 1064$ nm is the laser wavelength) with the normalized filter transmittance function (or just simply filter function) $\phi'(\lambda_s)/\phi(\lambda_L)$, where $\lambda_s$ is the scattered wavelength. Figure 1(a) and (b) show such normalized filter functions (+). Note that the quantum efficiency $QE$ of the APD detector is also a function of the wavelength, but such a variation is captured by the filter function $\phi'(\lambda_s)$ as well; thus $QE$ in equation (2.1) is the quantum efficiency at the laser wavelength. $S(\lambda_S; T_e, \theta, \lambda_L)$ is a spectral distribution of the Thomson scattered light [1, 9] as a function of the scattered wavelength $\lambda_s$ at a certain electron temperature $T_e$, scattering angle $\theta$ and laser wavelength $\lambda_L$.

To be able to generate expected (or synthetic) KSTAR TS data based on equation (2.1), we need to use hardware specifications of the KSTAR TS system which are listed in table 1. The KSTAR TS system measures electron density and temperature at 12 spatial positions with the core optics system, i.e., $R = 1.81, 1.84, 1.87, 1.90, 1.96, 1.98, 2.02, 2.05, 2.08, 2.10, 2.13$ and $2.16$ m, and 15 spatial positions with the edge optics system, i.e., $R = 2.16, 2.17, 2.18, 2.19, 2.20, 2.21, 2.21,$

| Table 1. Specification of the KSTAR TS system used during the 2016 Campaign. |
|-----------------------------------------------|-----------------|-----------------|
| Laser Energy                                  | 2 J             | 2 J             |
| F/#                                          | 5.6             | 6.5             |
| Solid Angle ($\Delta \Omega$)                | 0.0250          | 0.0186          |
| Transmission Coefficient at $\lambda_L$ ($T$) | 0.57            | 0.57            |
| Quantum Efficiency $\lambda_L$ ($QE$)         | 0.58            | 0.58            |
Figure 4. Calculated (a) scattering angle $\theta$ and (b) scattering length $L$ as a function of the major radius $R$ for the KSTAR TS system used during the 2016 Campaign.

$R$ is the major radius. Note that a typical location of the magnetic axis in KSTAR is $R_0 \approx 1.80$ m. Depending on the spatial positions, scattering angle $\theta$ and scattering length $L$ are different, and they have been calculated based on the geometry of the KSTAR TS system (e.g. figure 3). Scattering angle and length as a function of the major radius $R$ are shown in figure 4.

Collected number of scattered photons as a function of the scattered wavelength denoted as $n_s$ just before passing a polychromator can be written as

$$n_s(\lambda_s) = n_e N_{\text{laser}} \frac{d\sigma_{\text{TS}}}{d\Omega} L T(\lambda_l) \frac{S(\lambda_s; T_e, \theta, \lambda_l)}{\lambda_l},$$  \hspace{1cm} (2.2)

and the number of photo-electrons $N_s$ from an APD detector can be written as

$$N_s(\lambda_s) = \frac{1}{F} n_s(\lambda_s) QE \frac{\phi'(\lambda_s)}{\phi(\lambda_l)},$$  \hspace{1cm} (2.3)

where $F$ is the noise figure to effectively decrease the number of collected photo-electrons [6]. The value of $F$ is selected to capture any differences between the ‘design’ specification and ‘experimental’ performance of the KSTAR TS system such as misalignment of the TS optics system.

Figure 5 shows examples of collected scattered photons $n_s(\lambda_s)$ and photo-electrons $N_s(\lambda_s)$ detected by the Core #4 polychromator ($R = 1.90$ m) when $T_e = 1.9$ keV and $n_e = 1.0 \times 10^{19}$ m$^{-3}$ with $F = 1$. Integrating photo-electrons with respect to the scattered wavelength $\lambda_s$ and multiplying it with the APD gain factor $G$ provides the electronic signal that we can read using a digitizer, i.e., equation (2.1). For the purpose of this paper, actual value of $G$ is irrelevant as we are interested in estimating reliability of electron temperature based on a look-up table method [6, 7] which cancels out $G$ as we take the ratio of signals from the APD detectors, i.e., five channels in a polychromator. We note that our forward model can act as a likelihood function in the frame of Bayesian probability theory [10].
Figure 5. Spectral distribution of collected scattered photons \( n_s(\lambda_s) \) (line) and photo-electrons \( N_s(\lambda_s) \) detected by five APD detectors (+) for Core #4 \((R = 1.90 \text{ m})\) when \( T_e = 1.9 \text{ keV} \) and \( n_e = 1.0 \times 10^{19} \text{ m}^{-3} \) with the noise figure \( F = 1 \).

3 Comparison between the prescribed \( T_e \) and the estimated \( T_e \)

Based on the KSTAR TS forward model described in section 2, we generate the KSTAR TS synthetic data with a typical electron density profile of L-mode discharge observed in KSTAR as shown in figure 6, while varying profiles of electron temperature. Depending on the total number of photo-electrons from an APD detector (e.g. figure 5), we add random Poisson noise to the signal. Using a look-up table method [6, 7], we estimate an electron temperature from the noise added synthetic data. Although the KSTAR TS system contains five bandpass filters and five APD detectors denoted as Ch.1 to Ch.5 for each polychromator, Ch.5 is excluded from the temperature estimation because Ch.5 is strongly affected by the stray light [6] as the laser wavelength, i.e., 1064 nm, is within the bandpass of this channel (see figure 1). In using the look-up table method, we have examined two cases: 1) using three filters (or channels), i.e., Ch.2, Ch.3 and Ch.4, and 2) using four filters (or channels), i.e., Ch.1, Ch.2, Ch.3 and Ch.4. We generate ten data sets from which mean and random error (standard deviation) are estimated for a given temperature profile where the randomness is added by the Poisson noise.

Comparisons between the prescribed and estimated electron temperatures for these two cases with various electron temperatures are shown in figure 7. The figure shows the results with \( F = 1 \) (red diamond) corresponding to the ‘design’ specification and with \( F = 10 \) (blue triangle) to show ‘experimental’ performance. \( F = 10 \) was found based on the experimentally obtained signal-to-noise ratio [6]. Points with the value of \( T_e = 0 \) in the figure indicate the failure of estimating \( T_e \) using the look-up table method. Note that the polychromator observing the most inner location, i.e., \( R = 1.81 \text{ m} \) uses ‘edge’ bandpass filters even if it is named as ‘core’ polychromators. Figure 8 shows the profiles of error ratio which is the ratio of the estimated standard deviation to the estimated
mean temperature with the same profiles of prescribed $T_e$ as in figure 7 for $F = 1$ (red diamond) and $F = 10$ (blue triangle). Error ratios for the points failed to estimate $T_e$ (points with $T_e = 0$ in figure 7) are not depicted here.

Discussing the results with $F = 1$ case (red diamonds in figures 7 and 8), i.e., based on the ‘design’ specification, for the core polychromator system whose combination of bandpass filters is shown in figure 1(a), we find that using four filters (Ch.1 to Ch.4) for estimating $T_e < 0.8$ keV results in underestimation, i.e., bias error, and relatively larger uncertainties compared to using three filters (Ch.2 to Ch.4) from figure 7(a)-(d). Since the bandpass of Ch.1 is farthest away from the laser wavelength, relatively low temperature generates very small amount of photo-electrons for Ch.1 resulting in a very small signal-to-noise ratio. On the other hand, using four channels gives us better temperature estimation for $T_e > 8$ keV as in figure 7(g)–(j). For such a large temperature, we now have a good signal-to-noise ratio for Ch.1, and this channel starts to provide additional valuable information for estimating temperature. For the edge polychromator system shown in figure 1(b), we find that as long as $T_e < 1$ keV, either using three filters or four filters provide us reasonably good estimation of temperatures. If $T_e > 1$ keV, using three filters results in the underestimation and larger uncertainties for estimating temperature compared to using four channels as in figure 7(g)-(j).

As expected, setting $F = 10$ (blue triangles in figures 7 and 8) results in generally worse performance due to decreased signal-to-noise ratio. For the core polychromator system, using three filters (Ch.2 to Ch.4) is in general more reliable than using four filters for $T_e < 2$ keV as shown in figure 7(a)–(f); while using four filters are slightly better for $5 < T_e < 8$ keV (see figure 7(g) and (h)). For $T_e > 8$ keV, we find that the measurements become unreliable. This is due to the fact that the effective number of detected photons becomes too small as the temperature increases, because larger temperature means wider spectral distribution with a fixed total area. For the edge polychromator system, it is better to use three filters for $T_e < 0.8$ keV (see figure 7(a)–(f)); while using four filters are better for $0.8 < T_e < 2$ keV (see figure 7(c)–(h)). Above 2 keV, measurements become unreliable.
Figure 7. Comparison between the prescribed (black line) and estimated electron temperature $T_e$ based on the KSTAR TS synthetic data with the noise figure $F = 1$ (red diamond) and $F = 10$ (blue triangle). Ten data sets are used to estimate a profile of $T_e$ for each prescribed $T_e$ profile. Means are marked with symbols, whereas error bars represent standard deviations.
Figure 8. Profiles of estimated error ratio, i.e., the ratio of estimated standard deviation to the estimated mean temperature where the profiles of prescribed $T_e$ are given in figure 7 with the noise figure $F = 1$ (red diamond) and $F = 10$ (blue triangle).
4 Conclusion

We have developed a forward model of the KSTAR TS system for the purpose of quantifying both bias and random errors in estimating electron temperature, where previous estimation of fractional errors provide just random errors. Our model can be used not only for the KSTAR TS system but also any other TS systems as the forward model is generic. For the examined KSTAR TS system with the design specification (noise figure $F = 1$), we find that three filters are to be used for the core polychromators when electron temperature is less than 0.8 keV, while using four filters are recommended when the temperature is larger than 8 keV. For the edge polychromators, choice of using three or four filters is irrelevant as long as the temperature is less than 1 keV, and using four filters is recommended for the temperature greater than 1 keV. With the experimental performance of the KSTAR TS system (noise figure $F = 10$), overall reliability of the TS system is found to be worse as expected. For the core polychromators, it is better to use three filters for the temperature less than 2 keV, whereas using four filters provides better results for the temperature between 5 and 8 keV. For the edge polychromators, using three filters are recommended for the temperature less than 0.8 keV and four filters for the temperature from 0.8 to 2 keV. The core (edge) polychromators do not provide reliable measurements above 8 (2) keV with the noise figure $F = 10$.

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