Research article

Estimating lifetime malnourished period and its statistics based on the concept of Markov chain with reward

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**A R T I C L E   I N F O**

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**ABSTRACT**

Malnutrition among women, accessed through body mass index, has great consequences for achieving key national targets. This study introduces the concept of lifetime malnourished period (LMP): the number of years a woman would remain malnourished, either as underweight or overweight given that she is currently malnourished, and its measures of variation. Markov chain with rewards was used to compute the moments of LMP based on age-specific mortality rates and proportion of women of reproductive age that were either underweight or overweight using data from the 2013 Nigeria Demographic and Health Survey. Each of the two malnutrition status was treated as a Bernoulli-distributed reward with probability taken as the proportion of overweight or underweight women at specific age. Findings indicate that the average LMP for an underweight woman in Nigeria at age 15 years is 2.3 years but 5.8 for overweight. The remaining LMP for underweight is lower among women who attain higher level of education than for those with no or primary level of education with standard deviation reducing with age. Further, we found overweight women from the richest households and those from urban areas to have longer years of remaining in that state of health than their other counterparts, and that longevity contributes more to the variance in LMP for overweight than for underweight women.

1. Introduction

The nutritional status of individuals in a nation is a reflection of the country’s developmental and health challenges, and it can determine the nation’s accomplishment of key global objectives and targets. Battling the different forms of malnutrition has therefore remained one of the world’s most serious health challenges (Müller and Krawinkel, 2005; World Health Organization, 2002). There are several factors that cause malnutrition, most of which relate to inadequate dietary intake or severe and repeated infections, particularly in underprivileged populations. Malnutrition is closely linked to general standard of living, environmental condition, and availability of basic population’s needs such as food, housing and health care. As a health outcome, malnutrition exacerbates the burden of morbidity and mortality especially among women and young children (Gordon et al., 2004). In the case of women, several reviews have emphasized that due to their physiological composition and high nutrient requirements, they are vulnerable throughout their life cycle (Bitew and Telake, 2010; Tinker et al., 1995; Merchant and Kurtz, 1993). When women are malnourished, the impact extends beyond them, affecting not just their health, but also their children, households and, cumulatively, retards national development. The multifaceted effects of malnutrition whether in the form of under- and over-weight include poor pregnancy and birth outcomes, high vulnerability to infectious disease, lethargy and general body weakness leading to reduction in productivity, increased risk of maternal complications and it can, in the extreme case, lead to death (Müller and Krawinkel 2005; Denison et al. 2010; Gayawan et al. 2019).

Malnutrition, according to the World Health organization (WHO), is a state of deficiencies, excessive or imbalance in an individual’s intake of energy and/or nutrients (Blössner et al., 2005). Among adults, it is determined based on body mass index (BMI) calculated from an individual’s height and weight indices. The different forms of malnutrition are severe thinness, undernourished, overweight and obesity. Individuals with BMI < 16 kg/m² are said to be severely thin; those

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with BMI < 18.50 kg/m² are considered undernourished; those with BMI of 19 – 24.9 kg/m² are considered normal; BMI > 25 kg/m² but less than 30 kg/m² is classed to be overweight, while people with BMI > 30 kg/m² are said to be obese.

For the purpose of this study, we classify malnutrition into two groups: underweight, comprising severe thinness and undernourished, and overweight which comprises of overweight and obesity. There is no society that is entirely immune from the problem of malnutrition but what makes a difference is the rate and extent to which it is prevalent and the alternative courses of action applied to eradicate or reduce it. Nigeria has been known for large incidence of underweight population but of recent, there have been growing cases of overweight and obesity among women of reproductive age (Kandala and Stranges, 2014; Akersola and Gayawan, 2020). Report from the 2013 Nigeria Demographic and Health Survey (NDHS) has it that 11% of women of reproductive age in Nigeria are underweight, 17% are overweight while 8% are obese (NPC and ICF International, 2014).

Most studies on malnutrition in developing countries have been devoted to analysing the determinants or the geographical distributions (Mendez et al., 2005; Bitew and Telake, 2010; Kandala and Stranges, 2014; Gayawan et al., 2019; Akersola and Gayawan, 2020). The studies have established the nexus between geographical settings, socio-economic and environmental conditions and nutritional status of women and children. They are uniform in their conclusions that spatial-economic conditions have direct and indirect implications on the nutritional status of the population. However, till date, little has been done to provide information on the expected lifetime period a malnourished individual would remain in such condition given her present nutritional status, and the variation in lifetime malnourished period including the sources of this variation.

Our goal in this study is to introduce new perspectives to malnutrition studies. First, we introduce measures for lifetime malnutrition period (LMP): the number of years an individual would remain malnourished over her entire lifetime given that she is currently malnourished. These are obtained by integrating age-specific proportion of malnourished women taking into consideration the probabilities of survival at the specific ages. This is based on the mathematical framework of Markov chain with rewards, introduced by Howard (1960) to analyze Markov decision processes in the context of dynamic programming, and the present approach developed by Caswell (2011). These models describe the dynamics of a system, and associate a reward with each possible transition among the states of the chain (Paramonov and Hauka, 2010; van Daalen and Caswell, 2015). In a recent study, Caswell and Zarulli (2018) developed Markov chains with rewards as a general tool for analyzing healthy longevity, including their measures of variability. Our classification of malnutrition among the women is based on BMI as earlier described and our focus is, as also stated, on underweight and overweight women. Second, we aim at elucidating the variation in LMP in addition to the expectation among women of reproductive age based on their socio-economic variables. This inter-individual variation in LMP may arise from two sources: heterogeneity in individual characteristics (genetic heterogeneity, physiological differences, and environmental heterogeneity), and individual stochasticity. Individual stochasticity is the variation among individuals in the accumulation of random outcomes of vital rates such as mortality, growth, development, etc. (Caswell, 2009, 2014; van Daalen and Caswell, 2017). In this article, every individual is assumed to be subjected to the same rate at any given stage, so that there is no heterogeneity.

To quantify the effects of individual stochasticity, it is required to calculate some measures of dispersion for the LMP such as variance, standard deviation, coefficient of variation, and skewness of LMP. As with other analytical approaches, computations based solely on mean LMP provides no information on the variation in the malnourished period among members of the same cohort and thus, not revealing how unusually high or low values will be rampant among members of the same cohort. The coefficient of variation (CV) scales the standard deviation of the LMP relative to its mean and hence revealing the extent of variability in relation to the mean. Skewness in LMP measures the asymmetry of the distribution of malnutrition; positive skewness implies a distribution with a long positive tail, and a negative skewness implies the opposite.

2. Methodology

Our analytical approaches are based on those implemented by van Daalen and Caswell (2015, 2017) and Caswell and Zarulli (2018). We start by presenting the notations before proceeding to the Markov chain approach.

**Notation:** Matrices and vectors are denoted by uppercase and lowercase bold symbols respectively (e.g. A and a). By default, vectors are column vectors. Transpose of matrix B is denoted by Bᵀ. The vector 1 is a n × 1 vector of ones and Iₙ is the identity matrix of order n. D(ξ) is a diagonal matrix with vector ξ on the diagonal and zero elsewhere. The vec operator vec X transforms the columns of a m × n matrix X into a m × n × 1 column vector. The symbol ⊗ denotes element-by-element product or Hadamard. The transition matrices of Markov chains are written in column-to-row orientation, and are therefore column stochastic.

2.1. Markov chain with reward

The concept of Markov chain with reward have a long history in stochastic process theory (e.g., Howard, 1960; Benito, 1982; Puterman, 1994; Sheskin, 2016). More recently, some methods for application of this concept to health demography was introduced by Caswell (2011) and further developed in Caswell (2013) and Caswell and Zarulli (2018). These methods are applicable to age-structured and stage-structured models and to models where various types of temporal or environmental variations are incorporated.

Life cycle is described by a finite-state, discrete-time, absorbing Markov chain. Thus, the individual life cycle is described as an absorbing Markov chain in our analysis. In this study, we represent the human life cycle by transforming age-structured population projection matrices into a Markov chain; absorbing states represent death. r denotes the number of age classes. The Markov chain transition matrix is given as

$$P = \begin{bmatrix} \mathbf{U} & 0 \\ \mathbf{m} & 1 \end{bmatrix}$$

where \( \mathbf{U} \) is a \( r \times r \) matrix of transition probabilities among transient (i.e., living) states, and \( \mathbf{m} \) is a \( r \times 1 \) vector of mortality rates. \( \mathbf{U} \) contains the transition probabilities for living individuals, with the survival probabilities on the sub-diagonal and zero elsewhere. The matrix \( \mathbf{P} \) is column stochastic. Matrix \( \mathbf{U} \) is presented as:

\[
U = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 \\
\end{bmatrix}
\]

(2)

According to the probabilities in \( \mathbf{U} \), an individual experiences a sequence of transitions and a reward is associated with moving from one state to another or remaining in a state of the Markov chain. The reward \( r_j \) is collected by an individual moving from age \( j \) to age \( i \) (Caswell, 2011). The nutritional status of an individual can be considered as a reward, since it is used to classify individuals as either malnourished or not, a situation that can also change with age. Therefore, an individual may or may not be malnourished moving from state \( j \) to state \( i \). In this study, \( r_j \) depends on \( j \) but not on the transition made between \( j \) and \( i \). Thus, the nutritional outcome is a Bernoulli random variable with value 1 indicating that the individual is malnourished (underweight or over-
Fig. 1. Age-specific proportion of underweight and overweight women by socio-economic status based on 2013 NDHS.

Fig. 2. Statistics of lifetime malnourished period (underweight and overweight) as a function of age in Nigeria.
Table 1

Descriptive analysis of the demographic and biosocial characteristics of the respondents.

| Variables                  | Number of women (%) | Mean BMI (95% CI) Malnutrition prevalence (%) |
|----------------------------|---------------------|-----------------------------------------------|
|                            |                     | Normal | Underweight | Overweight |
| Total                      | 38334 (100)         | 23.1 (23.0, 23.1) | 64.3 | 10.4 | 25.3 |
| Place of residence         |                     |        |             |             |         |
| Urban                      | 15300 (39.9)        | 24.0 (23.9, 24.1) | 57.4 | 8.7  | 33.9 |
| Rural                      | 23034 (60.1)        | 22.5 (22.4, 22.5) | 68.9 | 11.6 | 19.5 |
| Region                     |                     |        |             |             |         |
| North Central              | 6178 (16.1)         | 23.6 (23.5, 23.7) | 64.6 | 6.4  | 28.9 |
| North East                 | 6468 (16.9)         | 22.3 (22.2, 22.4) | 65.4 | 14.7 | 19.9 |
| North West                 | 9519 (24.8)         | 21.9 (21.8, 22.0) | 70.3 | 14.4 | 15.3 |
| South East                 | 4382 (11.4)         | 23.7 (23.5, 23.8) | 63.3 | 7.3  | 29.4 |
| South South                | 5963 (15.6)         | 23.9 (23.8, 24.0) | 60.7 | 7.1  | 32.2 |
| South West                 | 5824 (15.2)         | 23.9 (23.8, 24.0) | 57.5 | 9.2  | 33.4 |
| Highest Education          |                     |        |             |             |         |
| No education               | 13480 (35.2)        | 22.0 (22.0, 22.1) | 69.9 | 13.7 | 16.4 |
| Primary                    | 7900 (18.3)         | 23.4 (23.3, 23.5) | 61.8 | 9.0  | 29.2 |
| Secondary                  | 14192 (37.0)        | 23.2 (23.1, 23.2) | 64.4 | 9.8  | 25.9 |
| Higher                     | 3662 (9.6)          | 25.7 (25.5, 25.9) | 48.2 | 3.7  | 48.1 |
| Wealth Index               |                     |        |             |             |         |
| Poorest                    | 6465 (16.9)         | 21.5 (21.4, 21.6) | 72.9 | 15.9 | 11.1 |
| Poorer                     | 7399 (19.3)         | 22.0 (21.9, 22.1) | 71.7 | 12.8 | 15.5 |
| Middle                     | 7866 (20.5)         | 22.7 (22.6, 22.8) | 67.5 | 10.4 | 22.1 |
| Richer                     | 8360 (21.8)         | 23.6 (23.5, 23.7) | 61.0 | 8.6  | 30.3 |
| Richest                    | 8244 (21.5)         | 25.1 (25.0, 25.2) | 51.3 | 5.8  | 42.9 |
| Employment Status          |                     |        |             |             |         |
| Unemployed                 | 14432 (37.6)        | 21.9 (21.9, 22.0) | 69.3 | 15.1 | 15.6 |
| Employed                   | 23700 (61.8)        | 23.8 (23.7, 23.8) | 61.2 | 7.6  | 31.2 |
| Missing                    | 202 (0.5)           | 22.5 (21.9, 23.1) | 71.3 | 10.9 | 17.8 |

Fig. 3. Variance and skewness of lifetime malnourished period calculated under the fixed reward and random reward models for underweight (a & b) and overweight (c & d) women.
weight as applicable) and 0 if otherwise. The probability function \( f(x) \) is illustrated in Eq. (3).

\[
f(x) = \begin{cases} 
1 & \text{with probability } f_i \\
0 & \text{with probability } (1 - f_i)
\end{cases}
\] (3)

where the probabilities \( f_i \) are age-specific malnourished rates. We assumed these probabilities to be the proportion of malnourished individuals in a given age while individuals in the absorbing state will accrue no reward (i.e. \( r = 0 \)), as the dead cannot be malnourished.

The reward matrices, \( R_i \), which give the moments of the reward for each transition, are required to calculate the statistical properties of lifetime malnourished period. Thus, \( R_i \) is a matrix of the \( k \)th moments of the rewards \( r_{ij} \). The first moment matrix is given by

\[
R_1 = \begin{bmatrix} 
  f_1 & f_2 & \cdots & f_t & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  f_1 & f_2 & \cdots & f_t & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  f_1 & f_2 & \cdots & f_t & 0 \\
\end{bmatrix}
\] (4)

Since the raw moments for Bernoulli distribution are equal irrespective of the order, the higher-order moments of reward under the assumption of Bernoulli are given by:

\[
R_3 = R_2 \circ R_1 
\] (5)

and if every individual experiences exactly the same age-specific malnourished rates, the reward is assumed fixed, thus, the moments of reward satisfy

\[
R_2 = R_1 \circ R_1 
\] (6)

\[
R_3 = R_1 \circ R_1 \circ R_1 
\] (7)

2.2. Lifetime accumulated reward

For each initial age, define \( \rho \) as a vector of dimension \((t + 1) \times 1\), of accumulated rewards. The accumulated malnourished period over the entire lifetime of an individual is the entry of the first age class (age 15). Then, the \( i \)th entry of \( \rho_k \) is the \( k \)th moment of the accumulation (i.e. malnourished period) over the remaining lifetime of an individual starting in \( i \) (i.e. age \( i \)). As the dead cannot be malnourished, the absorbing state accumulates no reward (i.e. the \( \rho_k \) corresponding to absorbing states are zero). A recursive formula for these moments was given by Caswell (2011), and an exact solution is given in van Daalen and Caswell (2015, 2017).

\[
\rho_k = (E[p]) \] (8)

Thus, the complete statistics of LMP can be obtained by finding the moment vectors \( \rho_k \), where \( \rho \) contains the first \( t \) entries of \( \rho \), corresponding to the transient (i.e. living) states. To this end, the matrix \( Z \) is defined as

\[
Z = \begin{bmatrix}
  1, & 0_{x \times 1}
\end{bmatrix}
\] (9)

The first three moments of the lifetime accumulated malnourished period satisfy the following system of equations:
\[ \hat{\rho}_1 = N \{ Z(P_o R_p) \}_{r+1} \]

\[ \hat{\rho}_2 = N \{ Z(P_o R_p) \}_{r+1} + 2(U \times \hat{R}_1)^\dagger \rho_1 \]

\[ \hat{\rho}_3 = N \{ Z(P_o R_p) \}_{r+1} + 3(U \times \hat{R}_2)^\dagger \rho_1 + 3(U \times \hat{R}_3)^\dagger \rho_2 \]  \hspace{1cm} (10-12)

where \( N = (I_r - U)^{-1} \) is the fundamental matrix of the Markov chain, \( I_r \) is a \( r \times r \) identity matrix and \( 1_{r+1} \) is a \( r+1 \) vector of ones. The entries of the first moment vector \( \hat{\rho}_1 \) correspond to the mean remaining lifetime malnourished period of each age class. Proofs of (10)-(12) are given in van Daalen and Caswell (2017), Theorem 1.

From these moment vectors (Eq. (10)-(12)), some descriptive statistics of the LMP, including the mean, variance, standard deviation, coefficient of variation, and skewness can be calculated thus:

\[ \text{Mean} : \hat{\rho}_1 \]

\[ V(\hat{\rho}) = \hat{\rho}_2 - (\hat{\rho}_1 \circ \hat{\rho}_1) \]

\[ SD(\hat{\rho}) = \sqrt{\hat{\rho}_2 - (\hat{\rho}_1 \circ \hat{\rho}_1)} \]

\[ CV(\hat{\rho}) = D(\hat{\rho}_1)^{3/2} SD(\hat{\rho}) \]

\[ Sk(\hat{\rho}) = D[V(\hat{\rho})]^{3/2} \rho_1^2 - 3 \hat{\rho}_1 \circ \rho_2 + 2 \hat{\rho}_1 \circ \hat{\rho}_1 \circ \hat{\rho}_1 \]  \hspace{1cm} (13-17)

2.3. Data

The motivating data for this study were obtained from the 2013 NDHS. The survey, executed by the National Population Commission between February and June 2013, was designed to provide population and health indicators at the national, zonal, and state levels. The data with those of other developing countries where similar surveys have been held are freely available upon request at the web page of The DHS Program (https://dhsprogram.com/). Details of the sampling procedure for the survey have been reported in NPC and ICF International (2014). As with similar surveys, the women respondents were restricted to those of reproductive age, that is, women aged 15 – 49 years, who were either permanent residents of the selected households or visitors present in the night before the survey. The Household Questionnaire was used to collect information on characteristics of the household dwelling unit and was further used to record height and weight measurements for the women and children. This record was used to calculate the BMI of the respondents. Data on BMI was available for 38,334 women aged 15 – 49 years. Other questions included in the survey which are used in this study include women’s year of birth, household wealth index, type of place of residence (urban or rural), women’s highest educational attainment, women’s employment status, and geographical region of residence. Our analyses are based on age-specific proportion of underweight and of overweight women from the survey classified by each of the variables listed. Fig. 1 presents...
the plots of age-specific proportion of underweight and overweight women that were used for the study based on their socio-economic status.

The probability of dying at each age was obtained from the female section of the general pattern of the United Nation model life table for developing countries (United Nations, 1982). Life expectancy for the year 2013, when the surveys were conducted, was obtained from the World Development Indicator published by The World Bank, and was used in determining the appropriate column to pick the mortality schedules from the life table. Given a life expectancy of 53 years for Nigerian women in 2013, the schedules were based on the \( Q(X) \) values of page 232 of the life table and the same probability values were used for all categories of women in the study.

3. Results and discussion

3.1. Descriptive analysis

Table 1 presents descriptive statistics of women included in the study based on levels of the categorical variables. Specifically, the Table presents the percentage distribution, mean BMI, and the distribution of the women based on their nutrition status (normal, underweight or overweight). Overall, about 10% (3,999/38,334) of the women were underweight while 25% (9,682/38,334) were overweight. The average BMI for all the women was 23.1 kg/m\(^2\) (95% CI: 23.0 - 23.1 kg/m\(^2\)). Of the 15,300 respondents who lived in urban areas, 8.7% were underweight while about 40% were overweight. However, of the respondents who lived in rural areas, about 12% and 20% were underweight and overweight respectively. Across the geo-political regions, the prevalence of underweight was highest among women from the North East region (14.7%; 954/6468), followed by North West (14.4%; 1372/9519) whereas it was lowest among those from the North Central region. On the other hand, the South West region had the highest prevalence of overweight women (33.4%; 1943/5824) but lowest among women from the North West region. Majority of the underweight women belong to those who had no formal education but almost half of women who attained higher level of education (48.1%) were overweight. A negative relationship between household wealth index and the prevalence of underweight was evident but the relation was positive with overweight. As for working status, findings indicate that underweight prevalence was highest 15.1% (2179/14432) among unemployed women while about 31% (7388/23700) of the working women were reported overweight.

3.2. The LMP pattern for Nigeria

We computed separately, the LMP and measures of dispersion for underweight and overweight women, starting with the entire country before proceeding to compute based on socio-economic conditions of the respondents. In order to ascertain which class of each variable has

Fig. 6. Statistics of lifetime malnourished period based on highest level of educational attainment for underweight women in Nigeria.
statistics of LMP equivalent to those of the country, statistics for the entire country were plotted alongside those for each of the variables considered.

Fig. 2 (a-d) presents the LMP estimates for Nigeria based on underweight and overweight as a function of age. Findings show that the mean LMP and standard deviaion (SD) for both underweight and overweight women decrease almost linearly as age increases. These results are in accord with the findings of Caswell and Zarulli (2018) if LMPs were to be considered as measures of health longevity, where the inter-individual variation in the expected years left decrease with advancement in age. Although, the mean for overweight is flat between ages 15 and 21 years, it reduces linearly thereafter. The mean remaining LMP was obtained to be about 6 years at age 15 years for overweight women but 2.3 years for underweight. The implications are that, at age 15, an overweight woman would spend more years in that state of health than an underweight woman would do; a difference of about 3.7 years. High values estimated for LMP for overweight women have serious implications for the fragile health care system of the country as literature has shown that overweight women have higher demand for medical care because they are more prone to haemorrhage, caesarean delivery, and various forms of infections (Thompson et al., 2001). The CV was 0.8 at age 15 years for both nutritional status, which thereafter increases rapidly for underweight women. The estimates for skewness for underweight women are higher than those for overweight women at all ages, and the difference increases with age. The increase in the estimates of CV and skewness with increasing age is also in line with Caswell and Zarulli (2018) where the estimates of these statistics for healthy longevity were found to increase with age.

3.3. Partitioning of total source of variation of LMP

The total variation in the nutritional status of individuals can be attributed to two components. First, the path through the stages of the life cycle of an individual. Two or more identical individuals subject to the same rate, at every stage of their life, will differ randomly in the pathway they follow through their life cycle (van Daalen and Caswell, 2017). In the age-classified model considered, the pathway is completely specified by an individual’s age at death. This constitutes the first component of the total variation which is referred to as variation in pathway. The second is the within pathway variation which accounts for the fact that individuals may or may not be malnourished at every stage of their life cycle. To decompose the total source of variation into these two components, the age-specific rewards are fixed at their mean values \( R_2 = R_1 \cdot e^{R_1} \) and \( R_3 = R_1 \cdot e^{R_1} \). The resulting variation obtained in this way is the variation in pathway. The within pathway variation is determined by subtracting the variation in pathway from the total source of variation. Fig. 3(a & b) compares the variance and skewness for fixed and random rewards for underweight women. Results show that variance at age 15 is about 0.71 under the fixed reward
but 2.67 based on random reward. That is, 27% of the overall variance is due to variation in pathways while about 73% is due to differences in reward pattern (i.e. variation within-pathways). Results in Fig. 3(c & d) show that variance of overweight women for fixed reward at age 15 years is about 16.28, lower than the 20.16 estimated for the random reward. Therefore, variation within pathway is about 19%, while the remaining 81% is due to variation between pathways. The variance between pathway for overweight women is amplified by the effect of differences in longevity. The implication is that length of life contributes more to the variance in LMP for overweight women than it does for the underweight women.

### 3.4. Estimates for underweight women

Fig. 4 (a-d) presents the statistics of LMP for underweight women based on household wealth index. As may be expected, the results indicate negative relationship between mean LMP and wealth index at all ages. Underweight women from poorest households would have longer LMP than women from the other quantiles. This is expected because poor women are likely to be engaged with energy consuming physical activities such as subsistence farming, fishing or other manual works, and would lack the requisite resources for balanced nutritional needs of the body, consistent with earlier findings from India (Ramesh and Jareena, 2009). The majority of these women might also have suffered from the aftermath of long history of poor nutrition, frequent illness and poor access to health facilities and hence, liable to spending more time in this condition (Akeresola and Gayawan, 2020). For all the wealth classes, the mean LMP reduces about linearly with age, indicating that the time an individual would spend in the state of being underweight reduces with advancement in age. The estimated national average corresponds to those of women from the middle wealth class. The estimates for CV and skewness are highest across the ages for women from the richest households. As for working status, findings indicate that the patterns for mean LMP and SD, though higher for non-working women, are similar for both group of women, and decline with age but the skewness and CV increase with age (Fig. 5 (a-d)). Women employment status naturally influences household income, which in turn reflects on the general household nutrition (Kennedy and Haddad, 1992; Bitew and Telake, 2010; Garret et al., 2008).

Like for the other findings, the mean and SD of LMP reduces linearly with age in the case of educational attainment (Fig. 6). The mean estimates for women with primary level of education across the ages reflects those obtained for the national average, but estimates for women with secondary and higher education are lower. Women with no formal education have the highest LMP, with a mean of about 3.4 years at age 15 against 1.2 for those with higher education. Thus, the higher the level of education, the lower the tendency of an underweight woman remaining in that state for a longer period. This is because an educated woman would know the right diet and health care for herself and her
family and she would use them for her advantage (Teller and Yimer, 2006; Yadav et al., 2015). Similarly, Fig. 7(a-d) shows that the mean remaining LMP and SD based on place of residence are similar in pattern, decreasing as age increases, with sharp decrease between age 15 and 20. At each age, women from rural areas have higher mean LMP than those in urban areas, and higher than the national average. Further, based on the country’s geopolitical regions, findings show that three regions: North Central, South East and South South have similar mean LMP throughout the reproductive ages while estimates for women from South West region follow the similar pattern of those three after about age 27 years. Estimates for women from the North East and North West regions are higher than the national average across the ages. The SD decline with age linearly for all the regions while CV and skewness rise with age in similar manner for all the regions (Fig. 8).

3.5. Estimates for overweight women

The results for overweight are similar to those obtained for underweight women, but mostly in the opposite direction. Estimates based on wealth index are presented in Fig. 9 (a-d). As may be expected, the mean remaining LMP are highest across ages for women from the richest households but this decreases sharply with age, like for the other categories, after age 27 years. The longer period that women from the richest households would spend in the overweight condition is no doubt the outcome of abundance of resources available to this women as studies from different regions of the world have confirmed that wealthiest households bear higher susceptibility to overweight/obesity (Bishwa-jit, 2017; Templin et al., 2019). The skewness is negative after age 17 years for the richest households. As for educational attainment (Fig. 10 (a-d)), the mean LMP is highest for women who attained higher educational level estimated to be around 9.3 years at age 15 years but 3.2 years for women with no education, a difference of about 6 years. The CV is less than 1 for all the different levels of education until around age 45 years. The skewness is highest for women with no education at all ages while it is negative after age 30 years for those with higher education. Throughout the reproductive age, the mean remaining LMP are highest for women who were gainfully employed but have about the same pattern with the estimates for the country (Fig. 11). Further, at age 15 years, overweight women residing in urban areas of the country would have about 8 years to remain in that condition compared with about 4.2 years for those from rural areas (Fig. 12). Urbanization often comes with less strenuous and sedentary jobs as well as access to more energy-dense foods causing more women to have positive energy balance resulting in overweight/obesity. In the context of rapid urbanization in Nigeria, largely driven by rural-urban migration, there is likelihood that the migrants would adapt to the new lifestyle and thus increase the burden of overweight/obesity in urban area (Ziraba et al.,

![Fig. 9. Statistics of lifetime malnourished period based on the wealth index for overweight women in Nigeria.](image-url)
Again, the skewness is negative from around age 22 years. Regarding geopolitical regions of the country, again, the remaining mean LMP for overweight women are lower than the national average, like in the case of underweight, for women from North East and North West regions but higher for those living in the South East, South West, South South and North Central regions (Fig. 13).

4. Conclusion

This study contributes by introducing the concept of LMP, a perspective that quantifies the number of years a woman would remain either overweight or underweight given her current malnutrition status, and its statistics using the concept of Markov chain with rewards. The model considers the dynamism of malnutrition as a woman moves from one age to another, and associates a reward for every stage with probability taken as the proportion of overweight or underweight women at each age and assumed to have a Bernoulli distribution. Though the reward system here was considered a binary process, studies by Caswell and Zarulli (2018) render it possible to consider other classifications of health or other outcomes of interest measured on several possible scales, thus allowing for different probability distributions to be considered.

The analyses reveal similar LMP patterns for underweight and overweight women though at every age, overweight women would spend more time in the condition than the underweight women would do. For both health conditions, the variabilities, as measured by the standard deviation, decline with age. However, both health conditions differ in the contribution of their variance components. A huge proportion of the variance in LMP for overweight women comes from the stochastic nature of having this condition but overweight women show very high variance in pathways, which in an age-classified model is determined by an individual’s length of life. Therefore, longevity contributes more to the variance in LMP for overweight than for underweight women. Furthermore, both health conditions respond in opposite direction to the socioeconomic variables considered. Our study found women who attained higher level of education, those from rich households and those residing in urban areas to have longer years of remaining overweight than their other counterparts while, on the other hand, women from the poorest households, those living in rural areas and those with little or no former education possess longer periods of remaining underweight, should they find themselves in those states of health. This is expected because maternal education influences nutritional outcomes through the effects of higher incomes and abundance of resources. Higher education opens better employment opportunities for women and leads to self-dependent, thereby improving their socioeconomic status. Economically advantaged women often adopt sedentary lifestyle partly due to the nature of their job and also consume more of energy-dense food which supply excessive energy to the body which, collectively, increase the burden of overweight. Intervention in the environment for physical activities particularly in urban areas and in work places can
benefit these women by creating healthier lifestyle practices. Contrariwise, the majority of women from poor households might have suffered from the aftermath of long history of poor nutrition, frequent illness and poor access to health facilities which in turn increase the burden of underweight. Any government intervention that could alleviate them from these conditions would improve their nutritional status and health.

Given that adequate nutrition is necessary for national development and well-being of the entire population, findings from the study can serve as a guide for policy makers and donor agencies towards intervention strategies for improving nutritional outcomes of women in Nigeria. For instance, immediate policy intervention could target women with longer periods of remaining in the state of health but without neglecting the other categories of women. The findings could also be useful in accessing the associations between available welfare packages and malnutrition among women.

Declarations

Author contribution statement

Seyifunmi M Owoeye, Bamidele M. Oseni: Analyzed and interpreted the data; Wrote the paper.

Ezra Gayawan: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

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The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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