A study of white dwarf shock detonation and type Ia supernova explosion

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Abstract. Type Ia supernovae are not only sources of the elements of “life”, but also “standard candles” for measuring distances in the Universe. To describe the process, a hydrodynamic model of white dwarfs that takes into account the nuclear combustion of carbon is constructed. It is closed by a stellar equation of state and supplemented by a Poisson equation for the gravitational potential. The results of a mathematical simulation of a white dwarf shock detonation and subsequent explosion of a type Ia supernova are described. The computational experiments have shown that the dynamics of the shock detonation is determined by the explosion energy and the ignition point. A study of scalability and energy efficiency of the software implementation are also performed.

1. Introduction

In modern astrophysics, type Ia supernovae (SNeIa) have always been the most popular phenomena of the stellar world for a number of well-substantiated reasons. As objects of observations, SNeIa still remain unsurpassed astronomical standards of brightness for measuring distances in a world of extremely distant galaxies, for (as it has been found recently) a possible correction of the present-day models of the Universe. Studies of the chemical composition of stellar populations of different ages have made it possible to establish that these supernovae are major generators of iron and nickel in the Universe, thus determining, to a great extent, the evolution of its chemical composition in time. The study of thermonuclear explosions of degenerate dwarfs has recently become a central problem of modern multidimensional numerical astrophysical gas dynamics. All this greatly stimulates a comprehensive study of scenarios of the formation and gas dynamics of explosions of degenerate dwarfs of various kinds considered as major causes of their formation.

Paper [1] is the first in the history of development of modern concepts of the nature of SNeIa. It was found in this paper that the burning of carbon in a degenerate CO dwarf leads to its detonation and complete dispersion of the iron-nickel product of such an explosion. The SNeIa observed in elliptic galaxies made us consider, in search for suitable progenitors, tight binary stars, such as cataclysmic systems. In such systems the degenerate component reaches a Chandrasekhar mass that is maximal for degenerate configurations, accreting the matter of its companion filling the Roche lobe [2]. The accretion rate necessary for stable burning of hydrogen and helium in the envelope of a degenerate dwarf and increasing of its mass to a limiting value was found in [3]. Another possibility of overcoming the Chandrasekhar mass
limit by degenerate dwarfs was established later [4]. It consists in the following assumption: the degenerate components of a tight binary system with separating components of less than three solar radii and a total mass of the components exceeding the limit can merge due to emission of gravitational waves by the system [5]. The surpassing of the limiting mass is still a sufficient condition for an explosion. However, the question whether this condition must be satisfied remains open. For some reasons, now an explosion of a degenerate dwarf whose mass is smaller than the limiting one seems possible for degenerate configurations. As noted above, SNeIa have long been considered absolute standards of brightness. This, in particular, made it possible to conclude that the expansion of the Universe is accelerating due to a decrease in the brightness of distant SNeIa (z = 1) by approximately one-half of the stellar magnitude, or one and a half times [6]. This was attributed to ”dark energy”. However, small but well-observed variations of brightness curves [7], [8] and SNeIa spectra [9] have set problems of their causes, the limits of their observed manifestations, and the physics of these supernovae [10]. Let us state a problem on the explosion of degenerate dwarfs. Although the merging of degenerate dwarfs as a scenario for SNeIa explosions seems simple and convincing, a detailed consideration of this phenomenon gives rise to some difficult and still unsolved problems. First, the Roche lobe filling with the lower mass component causes the appearance of a gas jet from point L1, which hits, as a rule, directly the second component, since their masses are comparable. The role of this jet in nuclear fuel detonation remains unknown. It is possible that the collapsing donor first turns into a massive disk near the accretor, which explodes only when it reaches a temperature or a critical mass sufficient for a nuclear fuel detonation.

In the second section, the numerical model used in the study is briefly described. In the third section, the results of a study of its parallel implementation are presented. The fourth section is devoted to some computational experiments. The fifth section provides conclusions to the paper.

2. Numerical Model

An overdetermined conservative form of the equations of gravitational gas dynamics is considered: laws of conservation of mass, angular momentum, total mechanical energy, and an equation for entropy supplemented by a Poisson equation for the gravitational potential. A stellar equation of state is used that consists of the pressure of a nondegenerate hot gas, the pressure due to radiation, and the pressure of a degenerate gas [11]. In the case of a degenerate gas, relativistic and nonrelativistic regimes are considered. A formulation of pressure and internal energy in terms of an entropy function makes it possible to easily calculate temperature variations without solving any nonlinear equation. In carbon burning in white dwarfs, the main way of obtaining heavy elements (such as nickel and iron) is to use an α-network [12]. Since we are primarily interested in explosion energy, we will consider a chain of reactions of the form $^{14}_{6}C \rightarrow ^{35}_{16}Ni$. Detailed information about the computational model can be found in [13].

The computational model is implemented with distributed calculations, where the hydrodynamic evolution of white dwarfs is calculated on a shared memory architecture (basic calculations). When the temperature and density reach some critical values, a new task is launched on a distributed memory architecture where the development of hydrodynamic turbulence is simulated, leading to supersonic nuclear burning of carbon (satellite calculations). Adaptive nested grids are used to discretize the computational domain, since the supernova explosion process at the stage of basic calculations takes place on different scales. For this, a regular root grid is constructed in the computational domain, with each cell being a nested grid. This discretization method allows simulating with a wide range of scales. To organize the satellite computations, a regular grid is used, which is a nested grid for the corresponding cells of nested grids of the basic calculations. For the basic and satellite calculations, the same numerical method adapted for nested grids is used to solve the equations of gravitational hydrodynamics.
To solve the Poisson equation on the root grid in the basic calculations, an algorithm based on fast Fourier transform is used. For nested grids, the successive overrelaxation method is used at the stage of basic calculations.

The basic calculations are performed on a shared memory architecture on adaptive nested grids and distributed using OpenMP technology within each single process. In our computational experiments we used a node with Intel Optane technology, which allows using 700 GB of RAM for one process. The satellite calculations are performed on a distributed memory architecture, with a software implementation based on a one-dimensional geometric decomposition of the regular computational domain using MPI, followed by a decomposition of the calculations into threads using OpenMP within each single process. A schematic of the calculations is shown in Fig. 1. This way of organizing the calculations allows considering in detail the carbon burning in the process of hydrodynamic evolution.

3. Parallel implementation
Since the algorithm is complicated in terms of linear algorithmic complexity of the code basis, let us study the speedup and scalability of a software implementation. We will consider the number of MPI processes for a given number of processors, cores, and a number of threads per core.

For the study we used a 48-core Intel Xeon Platinum 8268 processor, which is used for one of the nodes of the Siberian Supercomputer Center. The code speedup was studied on a $256^3$
calculation grid. For this, the calculation time was measured for different numbers of processes. There was a clear tendency towards a speedup when a larger number of processors was used, with the number of MPI processes remaining the same. An eightfold speedup was achieved with 32 processes split into two processors and 16 cores in a single-threaded mode without Hyper-Threading.

The code scalability was investigated on a $256P \times 256 \times 256$ calculation grid, where $P$ is the number of processes. Thus, for each MPI process the subdomain size is $256^3$. On the whole, the results are consistent with the conclusions formulated when analyzing the speedup. Note that the efficiency decreases when the sockets interact. The minimum scalability is 79 percent.

Only strong and weak scalability of a software implementation is usually studied. However, we will go further and investigate the energy efficiency of the thus obtained speedup and scalability values. For this we define energy efficiency ($E$) as a function of the number of MPI processes as follows:

$$E = \frac{E_1}{E_K},$$

where $E_1$ is the energy when using one core, and $E_K$ is the energy per one core when using $K$ cores. In fact, we introduce an analog of efficiency in terms of the energy consumption of an application when using a multi-core architecture. With the built-in functions of Intel processors for determining energy efficiency, we calculated the above-obtained values and tried to construct a function approximating the energy efficiency.

Some papers (see, for instance, [14]), analyzed energy efficiency versus the memory and cores used. We propose an approximation in the following form:

$$E(K) = \frac{7}{8 (\frac{K-1}{A} + 1)^B} + \frac{K - 1}{C} + \frac{1}{8},$$

where $E$ is the energy efficiency, $K$ is the number of cores, $A = 16$, $B = 8$ and $C = 800$ are variable coefficients. In Fig. 2 we show this approximation of real data.

![Figure 2. Energy efficiency.](image-url)
4. Numerical Simulation
For computational experiments, we consider a white dwarf with a mass equal to two solar masses and a temperature $T = 10^9$ K. At a distance of 20% of the radius we will consider a bubble with an explosion energy of $10^{51}$ Erg, which is static and moves in the direction of the center and away from the center at the sound speed. Fig. (3) presents the results of simulating an explosion of a static bubble, Fig. (4) the results of simulating a "move-out" bubble, and Fig. (5) those of simulating a "move-in" one. Note that the results of the above simulations are

![Figure 3](image)

**Figure 3.** Relative density distribution in the equatorial plane during a noncentral static explosion of type Ia supernova at 0 sec (a), 1 sec (b), 5 sec (c), and 10 sec (d).

rather close to each other and show the formation of a "C"-form of the remainder. The bubble motion practically does not affect the result of the explosion. It seems that the results of the thermonuclear supernova explosion depend on the explosion energy and the ignition point.

5. Conclusions
The results of a mathematical simulation of a type Ia supernova explosion on massively parallel supercomputers have been presented. To describe the process, a hydrodynamic model of white dwarfs was constructed taking into account nuclear carbon burning. The model was closed by a stellar equation of state and supplemented by a Poisson equation for the gravitational potential. The technology of basic and satellite calculations was used to simulate the hydrodynamics of the process and give a detailed description of turbulent carbon burning. With computational experiments it has been shown that the dynamics of shock detonation is determined by the explosion energy and the ignition point. A study of scalability and energy efficiency of the software implementation are also performed.
Figure 4. Relative density distribution in the equatorial plane during a noncentral move out explosion of type Ia supernova at 0 sec (a), 1 sec (b), 5 sec (c), and 10 sec (d).

Figure 5. Relative density distribution in the equatorial plane during a noncentral move in explosion of type Ia supernova at 0 sec (a), 1 sec (b), 5 sec (c), and 10 sec (d).
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