Unsteady magnetohydrodynamics flow about a stagnation point on a stretching plate embedded in porous medium

Nor Syazwani Mohd Azmi, Siti Khuzaimah Soid, Ahmad Sukri Abd Aziz and Zaileha Md Ali

Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 UiTM Shah Alam, Selangor, Malaysia

E-mail: khuzaimah@fskm.uitm.edu.my

Abstract. This paper presents an unsteady two-dimensional laminar boundary layer flow of an incompressible, viscous and electrically conducting fluid in a vicinity of a stagnation point towards a stretching plate embedded in porous medium with variable free stream velocity. The governing nonlinear partial differential equations of mass, momentum and energy are reduced to a system of nonlinear ordinary differential equations by using suitable similarity variables. The ordinary differential equations are tackled numerically by using Runge-Kutta Fehlberg fourth-fifth order method with shooting technique. The effect of the significant parameters on the fluid temperature and velocity are graphically presented and discussed. The numerically computed results are in good agreement with the available published results in the literature. It is found that the velocity increases with the increasing unsteadiness parameter where the velocity ratio parameter $\lambda = 0.5$ but it decreases for $\lambda = 1.5$.

1. Introduction

Boundary layer flow over a stretching plate has various applications in manufacturing processes such as the aerodynamic extrusion of plastic sheets, the cooling of metallic plates, the extrusion of polymers, etc. [1]. The basic understanding of the thermal transport and momentum are very crucial because the products’ quality in the processes of extrusion depend significantly on the stream flow and heat transfer features over a stretching plate [2]. Magnetohydrodynamics (MHD) flow has wide engineering applications such as MHD pumps, MHD bearing, MHD power generators, etc. [3]. The importance of magnetic field is to control the flow field and correspondingly the heat transfer rate. Jusoh et al. [4], Abbas et al. [5], Soid and Ishak [6] and Lok et al. [7] have solved numerically the MHD boundary layer flow over stretching/shrinking plate with various physical situations. Fang et al. [8] produced the exact solution for MHD flow with slip condition. There exists a unique solution for any combination of the slip, the magnetic and the mass transfer.

Hiemenz [9] produced an exact solution for the steady stagnation point flow and then Homann [10] continued his work to axisymmetric situation. The steady axisymmetric stagnation point was studied by Howarth [11]. Later, many authors have considered this kind of problems including Chiam [12], Lok et al. [13] and Pop et al. [14].

The flow through a medium that contains pores has become core of some applications in engineering field. It is vital to the variety of technical problems like stream through blood rheology, packed beds, environmental pollution and sedimentation [15]. Flow through medium has gained many researchers [16–18] attention.
This study investigates the unsteady magnetohydrodynamics flow about a stagnation point on a stretching plate in the presence of variable free stream velocity embedded in porous medium inspired by Sharma and Singh [19] and Mukherjee and Prasad [20].

2. Mathematical Formulation
Consider an unsteady two-dimensional flow in the vicinity of the stagnation point over a stretching plate with time dependent free stream embedded in porous medium. $U$ is the free stream velocity, $U_w$ is the velocity of the stretching plate and $T_w$ is temperature of the plate. The stretching plate is placed along the $x$-axis and as shown in figure 1. Assume that a constant magnetic field of strength $B_0$ is applied in the positive $y$-direction to the stretching plate.

![Figure 1. Physical model of boundary layer flow on a stretching plate.](image)

The governing equations of continuity, momentum and energy are [19,20]:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} - \frac{\nu (u - U)}{k} - \frac{\sigma B_0^2 (u - U)}{\rho}, \]
\[ \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \kappa \frac{\partial^2 T}{\partial y^2}, \]

where $u$ and $v$ are the velocity components along $x$ and $y$-directions, respectively, $\nu$ is the kinematic viscosity, $\rho$ is the fluid density, $C_p$ is the fluid specific heat capacity at constant pressure, $T$ is the fluid temperature, $\kappa$ is the thermal conductivity, $k$ is the permeability of the porous medium and $\sigma$ is the fluid electrical conductivity.

The boundary conditions are:

\[ y = 0: \quad u = U_w(x,t) = \frac{cx}{1 + ct}, \quad v = 0, \quad T = T_w. \]
where \( b \) and \( c \) are positive constants, \( \alpha \) is a constant, \( T_w \) is the surface temperature and \( T_\infty \) is the free stream temperature. The mathematical problem is simplified by introducing the similarity variables:

\[
\eta(y,t) = \sqrt{-\frac{c}{\nu(1+\alpha t)}} \, y, \quad \psi(x,y,t) = \sqrt{\frac{\nu c}{1+\alpha t}} \, x f(\eta), \quad \theta = \frac{T-T_\infty}{T_w-T_\infty} \quad \text{where} \quad T_w - T_\infty = \frac{1}{(1+\alpha t)^2},
\]

(6)

where \( \eta \) is the similarity variable, \( \psi \) is the stream function defined as \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \) which identically satisfy equation (1), \( f(\eta) \) is the dimensionless stream function and \( \theta(\eta) \) is the dimensionless temperature. The mathematical problem defined by equations (2) and (3) with boundary conditions (4) and (5) are transformed into a set of ordinary differential equations:

\[
f''(\eta)+\left(f(\eta)+\frac{S\eta}{2}\right)f''(\eta)-\left(f'(\eta)-S\right)f'(\eta)+\lambda^2-\lambda S-\left(\frac{1}{D}+M\right)(f'(\eta)-\lambda)=0,
\]

(7)

\[
\theta''(\eta)+Pr\left(f(\eta)+\frac{1}{2}S\eta\right)\theta'(\eta)+2PrS\theta(\eta)=0,
\]

(8)

where \( S = \alpha/c \) is the unsteadiness parameter, \( \lambda = b/c \) is the velocity ratio parameter \( 1/D = \alpha(1+\alpha t)/kc \) is the porosity parameter, \( M = \alpha B_0^2 (1+\alpha t)/\rho c \) is the magnetic parameter and \( Pr = \mu/\kappa \) is the Prandtl number.

The corresponding boundary conditions are:

\[
f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \quad \text{at} \quad \eta = 0,
\]

(9)

\[
f'(\infty) = \lambda, \quad \theta'(\infty) = 0 \quad \text{as} \quad \eta \to \infty.
\]

(10)

Parameters of physical interest are the skin friction coefficient \( C_{f\infty} \) and the local Nusselt number \( Nu_x \), which can be defined from the following relations:

\[
C_{f\infty} = \frac{\mu}{\rho U_x^2} \frac{\partial U}{\partial y} \bigg|_{y=0}, \quad Nu_x = -\frac{x}{T_w-T_\infty} \frac{\partial T}{\partial y} \bigg|_{y=0}
\]

(11)

where \( \mu \) is the dynamics viscosity. Using the non-dimensional variables (6), the skin friction coefficient and the local Nusselt number are given by:

\[
C_{f\infty} = Re^{-\frac{1}{2}} f''(0), \quad Nu_x = -Re^{-\frac{1}{2}} \theta'(0)
\]

(12)

where \( Re = U_w x / \nu \) is the local Reynolds number.

### 3. Results and Discussion

The nonlinear ordinary differential equations (7)-(8) along with the boundary conditions (9)-(10) were solved numerically using “dsolve” in Maple software. The procedure is based on Runge-Kutta-Fehberg fourth-fifth order method. This method uses a fourth order Runge-Kutta algorithm in tandem with a fifth order Runge-Kutta algorithm [22]. Comparison of the numerical results \( f''(0) \) with
previously published data in table 1 are in good agreement with Mahapatra and Gupta [21], Pop et al. [1] and Sharma and Singh [19].

Table 1. Value of $f''(0)$ for various values of $\lambda$ with $S=1/D=M=0$.

| $\lambda$ | Mahapatra and Gupta [21] | Pop et al. [1] | Sharma and Singh [19] | Present results |
|-----------|--------------------------|----------------|------------------------|-----------------|
| 0.1       | -0.9694                  | -0.9694        | -0.969386              | -0.9693861555  |
| 0.2       | -0.9181                  | -0.9181        | -0.9181069             | -0.9181071393  |
| 0.5       | -0.6673                  | -0.6673        | -0.667263              | -0.6672636688  |
| 2.0       | 2.0175                   | 2.0174         | 2.01749079             | 2.0175028017   |
| 3.0       | 4.7293                   | 4.7290         | 4.7292695              | 4.7292823559   |

Figures 2-10 describe the characteristics of the flow. Figure 2 shows that the velocity profile increases as the unsteadiness $S$ increases. It is observed that the higher velocity profile has the higher value of the magnitude of $f''(\eta)$ at an increasing distance $\eta$ from the sheet as shown in figure 2. The velocity profile gets slower thereafter and reaches the boundary condition earlier.

Figure 3 portrays that when the ratio velocity $\lambda = 1.0$, there is no formation of boundary layer as the fluid velocity equals to the surface velocity. Figure 4 illustrates the effect of unsteadiness $S$ on the velocity profile. Increasing the value of $S$ has the tendency to decrease the fluid velocity. It is noted that for $\lambda > 1.0$, an inverted boundary layer is found. Thus, the decrease in the velocity profile.

Figure 5 depicts the influence of unsteadiness $S$ on the temperature profile. Increase in $S$ causes the fluid temperature to increase as well as its boundary layer thickness. It is observed that larger value of $S$ produces the temperature overshoot effect. It indicates flow of heat from the fluid to the wall and free stream. Figure 6 displays the effect of porosity $1/D$ on the velocity profile. Increases in $1/D$ reduces the velocity profile. The porous media slows down the fluid flow. Figure 7 shows that for higher value of $1/D$, the temperature distribution increases, accompanied by the increase of the thermal boundary layer.

Figure 8 shows an increase in the magnetic parameter $M$ reduces the velocity profile. The applied magnetic field produces a force called Lorentz force that resists the motion of the fluid resulted in the decrease of the fluid velocity. Figure 9 exhibits the temperature profile for the variation of magnetic parameter $M$. The temperature of the fluid increases as $M$ increases. Therefore, the thermal boundary layer thickness also increases.

Figure 2. Velocity profile $f''(\eta)$ for various values of unsteadiness parameter, $S$ with $\lambda = 0.5$, $Pr = 0.5$, $1/D = 0.1$ and $M = 0.1$.

Figure 3. Velocity profile $f''(\eta)$ for various values of unsteadiness parameter, $S$ with $\lambda = 1.0$, $Pr = 0.5$, $1/D = 0.1$ and $M = 0.1$. 
Figure 4. Velocity profile $f'(\eta)$ for various values of unsteadiness parameter, $S$ with $\lambda = 1.5$, $Pr = 0.5$, $1/D = 0.1$, and $M = 0.1$.

Figure 5. Temperature profile $\theta(\eta)$ for various values of unsteadiness parameter, $S$ with $\lambda = 0.5$, $Pr = 0.72$, $1/D = 0.1$, and $M = 0.1$.

Figure 6. Velocity profile $f'(\eta)$ for various values of porosity parameter, $1/D$ with $\lambda = 0.1$, $Pr = 0.5$, $S = 0.3$, and $M = 0.1$.

Figure 7. Temperature profile $\theta(\eta)$ for various values of porosity parameter, $1/D$ with $\lambda = 0.1$, $Pr = 0.5$, $S = 0.3$, and $M = 0.1$.

Figure 8. Velocity profile $f'(\eta)$ for various values of magnetic parameter, $M$ with $\lambda = 0.1$, $Pr = 0.5$, $S = 0.1$, and $1/D = 0.1$.

Figure 9. Temperature profile $\theta(\eta)$ for various values of magnetic parameter, $M$ with $\lambda = 0.1$, $Pr = 0.5$, $S = 0.1$, and $1/D = 0.1$.

Figure 10. Temperature profile $\theta(\eta)$ for various values of Prandtl number, $Pr$ with $\lambda = 1.5$, $M = 0.1$, $S = 0.1$, and $1/D = 0.1$. 
Figure 10 illustrates the effect of the Prandtl number on the temperature profile. As increases, the thermal boundary layer thickness decreases. Fluid with higher Prandtl number has lower thermal conductivity which reduces conduction and thereby the thermal boundary layer thickness. Also, the heat transfer rate increases at the plate.

4. Conclusion
The effect of unsteadiness $S$, velocity ratio $\lambda$, porosity $1/D$, magnetic $M$ parameters and the Prandtl number $Pr$ on the unsteady MHD flow about a stagnation point on a stretching sheet in the presence of variable free stream embedded in porous medium were studied numerically. The results obtained revealed that the velocity profile increases with the increasing value of unsteadiness parameter for $\lambda = 0.5$ but the opposite behavior is observed for $\lambda = 1.5$. The temperature increases for higher value of unsteadiness, porosity and magnetic parameters. As porosity and magnetic parameters increase, the velocity profile decreases. Finally, an increase in the Prandtl number reduces the temperature distribution.

Acknowledgement
The authors acknowledge the financial support of Universiti Teknologi MARA under the Lestari Fund 600-IRMI/DANA 5/3/LESTARI (0140/2016). The authors also wish to express their deepest thanks to the reviewer for the valuable comments and suggestions.

References
[1] Pop S R, Grosan T and Pop I 2004 *Tech. Mech.* **25** 100–106
[2] Abel M S and Nandeppanavar M M 2009 *Commun. Nonlinear Sci Numer Simulat* **14** 2120–2131
[3] Ahmed N and Das K K 2013 *Sci. Res.* **3** 230–239
[4] Jusoh R, Nazar R and Pop I 2017 *Int. J. Mech. Sci.* **124–125** 166–173
[5] Abbas Z, Naveed M and Sajid M 2016 *J. Mol. Liq.* **215** 756–762
[6] Soid S K and Ishak A 2015 *Int. Symp. Math. Sci. Comput. Res.* 355–360
[7] Lok Y Y, Ishak A and Pop I 2011 *Int. J. Numer. Methods Heat Fluid Flow* **21** 61–72
[8] Fang T, Zhang J and Yao S 2009 *Commun. Nonlinear Sci. Numer. Simul.* **14**(11) 3731–3737
[9] Hiemenz K 1911 *J. Dinglers Polytech* **326** 321–324
[10] Homann F 1936 *Z. Angew. math. Phys.* **16** 153–164
[11] Howarth L 1951 *Philos. Mag. Ser. 7* **42**(335) 1433–1440
[12] Chiam T C 1994 *J. Phys. Soc. Japan* **63**(6) 2443–2444
[13] Lok Y Y, Amin N and Pop I 2006 *Int. J. Non. Linear. Mech.* **41**(4) 622–627
[14] Pop I, Lok Y Y, Ingham D B and Amin N 2009 *Int. J. Numer. Methods Heat Fluid Flow* **19** 459–483
[15] Kumar H 2011 *Therm. Sci.* **15** 187–194
[16] Pandey A K and Kumar M 2017 *Alexandria Eng. J.* **56** 55–62
[17] Aly E H *Powder Technol.* **301** 760–781
[18] Nayak M K, Dash G C and Singh L P 2016 *Propuls. Power Res.* **5**(1) 70–80
[19] Sharma P R and Singh G 2008 *Thammasat Int. J. Sci. Technol.* **13**(1) 11–16
[20] Mukherjee B and Prasad N 2013 *Int. J. Innov. Technol. Research* **1**(4) 321–326
[21] Mahapatra T R and Gupta A S 2002 *Heat Mass Transf.* **38** 517–521
[22] Timberlake T K and Mixon J W 2015 *Classical Mechanics with Maxima* (New York: Springer)