Recoverable, Abortable, and Adaptive Mutual Exclusion with Sublogarithmic RMR Complexity

Daniel Katzan
Tel Aviv University
Adam Morrison
Tel Aviv University

Abstract
We present the first recoverable mutual exclusion (RME) algorithm that is simultaneously abortable, adaptive to point contention, and with sublogarithmic RMR complexity. Our algorithm has \(O(\min(K, \log W, N))\) RMR passage complexity and \(O(F + \min(K, \log W, N))\) RMR super-passage complexity, where \(K\) is the number of concurrent processes (point contention), \(W\) is the size (in bits) of registers, and \(F\) is the number of crashes in a super-passage. Under the standard assumption that \(W = \Theta(\log N)\), these bounds translate to worst-case \(O(\frac{\log N}{\log \log N})\) passage complexity and \(O(F + \frac{\log N}{\log \log N})\) super-passage complexity. Our key building blocks are:

- A \(D\)-process abortable RME algorithm, for \(D \leq W\), with \(O(1)\) passage complexity and \(O(1 + F)\) super-passage complexity. We obtain this algorithm by using the Fetch-And-Add (FAA) primitive, unlike prior work on RME that uses Fetch-And-Store (FAS/SWAP).
- A generic transformation that transforms any abortable RME algorithm with passage complexity of \(B < W\), into an abortable RME lock with passage complexity of \(O(\min(K, B))\).

1 Introduction

Mutual exclusion (ME) [11] is a central problem in distributed computing. A mutual exclusion algorithm, or lock, ensures that some critical section of code is accessed by at most one process at all times. To enter the critical section (CS), a process first executes an entry section to acquire the lock. After leaving the critical section, the process executes an exit section to release the lock. The standard complexity measure for ME is remote memory references (RMR) complexity [4,7]. RMR complexity models the property that memory access cost on a shared-memory machine is not uniform. Some accesses are local and cheap, while the rest are remote and expensive (e.g., processor cache hits and misses, respectively). The RMR complexity measure thus charges a process only for remote accesses. There are various RMR definitions, modeling cache-coherent (CC) and distributed shared-memory (DSM) systems. The complexity of a ME algorithm is usually defined as its passage complexity, i.e., the number of RMRS incurred by a process as it goes through an entry and corresponding exit of the critical section.

For decades, the vast majority of mutual exclusion algorithms were designed under the assumption that processes are reliable: they do not crash during the mutual exclusion algorithm or critical section. This assumption models the fact that when a machine or program crashes, its memory state is wiped out. However, the recent introduction of non-volatile main memory (NVRAM) technology can render this assumption invalid. With NVRAM, memory state can remain persistent over a program or machine crash. This change creates the recoverable mutual exclusion (RME) problem [14], of designing an ME algorithm that...
Recovered, Abortable, and Adaptive ME with Sublogarithmic RMR Complexity

can tolerate processes crashing and returning to execute the algorithm. In RME, a *passage* of a process \( p \) is defined as the execution fragment from when \( p \) enters the lock algorithm and until either \( p \) completes the exit section or crashes. If \( p \) crashes mid-passage and recovers, it re-enters the lock algorithm, which starts a new passage. Such a sequence of \( p \)'s passages that ends with a crash-free passage (in which \( p \) acquires and releases the lock) is called a *super-passage* of \( p \).

RME constitutes an exciting clean slate for ME research. Over the years, locks with many desired properties (e.g., fairness) were designed and associated complexity trade-offs were explored \([27]\). These questions are now re-opened for RME, which has spurred a flurry of research \([8,10,12,15,19,22]\). In this paper, we study such questions. In a nutshell, we introduce an RME algorithm that is *abortable*, *adaptive*, and has sublogarithmic RMR complexity. Our lock is the first RME algorithm adaptive to the number of concurrent processes (or *point contention*) and the first *abortable* RME algorithm with sublogarithmic RMR complexity. It is also the first deterministic, worst-case sublogarithmic abortable lock in the DSM model (irrespective of recoverability). Our algorithm also features other desirable properties not present in prior work, as detailed shortly.

**Abortable ME & RME** An *abortable* lock \([18,25,26]\) allows a process waiting to acquire the lock to give up and exit the lock algorithm in a finite number of its own steps. Jayanti and Joshi \([22]\) argue that abortability is even more important in the RME setting. The reason is that a crashed process might delay waiting processes for longer periods of time, which increases the motivation for allowing processes to abort their lock acquisition attempt and proceed to perform other useful work.

Mutual exclusion, and therefore abortable ME (AME), incurs a worst-case RMR cost of \( \Omega(\log N) \) in an \( N \)-process system with standard read, write, and comparison primitives such as Compare-And-Swap (CAS) or LL/SC \([7]\). This logarithmic bound is achieved for both ME \([28]\) and AME \([17]\), and was recently achieved for a recoverable, abortable lock by Jayanti and Joshi \([22]\). However, while there exists an AME algorithm with sublogarithmic worst-case RMR complexity (in the CC model) \([8]\), no such abortable algorithm is known for RME. Moreover, Jayanti and Joshi’s \( O(\log N) \) abortable RME algorithm is suboptimal in a few ways. First, its worst-case RMR complexity is logarithmic only on a relaxed CC model, in which a failed CAS on a variable does not cause another process with a cached copy of the variable to incur an RMR on its next access to it, which is not the case on real CC machines. Their algorithm has linear RMR complexity in the realistic, standard CC model. Second, their algorithm is starvation-free only if the number of aborts is finite.

**Adaptive ME** A lock is *adaptive* with respect to *point contention* if its RMR complexity depends on \( K \), the number of processes concurrently trying to access the lock, and not only on \( N \), the number of processes in the system. Adaptive locks are desirable because they are often faster when \( K \ll N \). There exist locks with worst-case RMR cost of \( O(\min(\log N, K)) \) for both ME \([16]\) and AME \([17]\), but no adaptive RME algorithm is known (independent of abortability).

1.1 Overview of Our Results

In the following, we denote the number of crashes in a super-passage by \( F \) and the size (in bits) of the system’s registers by \( W \). We obtain three keys results, which, when combined, yield the first RME algorithm that is simultaneously abortable and adaptive, with worst-case \( O(\log W \ N) \) passage complexity and \( O(F + \log W \ N) \) super-passage complexity, in both CC and DSM models. Assuming (as is standard) that \( W = \Theta(\log N) \), this translates to worst-case
We show that $O\left(\frac{\log N}{\log^2 N}\right)$ passage complexity and $O(D + F + \log N)$ super-passage complexity. In contrast to Jayanti and Joshi’s abortable RME algorithm [22], our lock achieves sublogarithmic RMR complexity in the standard CC model and is unconditionally starvation-free. Our algorithm’s space complexity is a static (pre-allocated) $O(ND + W)$ memory words (which translates to $O\left(\frac{N\log^2 N}{\log^2 N}\right)$ if $W = \Theta(\log N)$). Jayanti and Joshi’s algorithm also uses static memory, but it relies on unbounded counters. The other sublogarithmic RME algorithms [10,12,19] use dynamic memory allocation, and may consume unbounded space.

Result #1: $W$-process abortable RME with $O(1)$ passage and $O(1 + F)$ super-passage complexity ($\S$ 3). Our key building block is a $D$-process algorithm, for $D \leq W$. It has constant RMR cost for a passage, regardless of if the process arrives after a crash. The novelty of our algorithm is that it uses the Fetch-And-Add (FAA) primitive to beat the $\Omega(D)$ passage complexity lower-bound. In contrast, the building blocks in prior RME work with worst-case sublogarithmic RMR complexity use the Fetch-And-Store (FAS, or SWAP) primitive and assume no bound on $D$, even though they are ultimately used by only a bounded number of processes in the final algorithm. By departing from FAS and exploiting the process usage bound, we overcome difficulties that made the prior algorithms’ building blocks [12,19] have only $O(D)$ RMR passage complexity.

These prior algorithms use a FAS-based queue-based lock as a building block. They start with an $O(1)$ RMR queue-based ME algorithm [9,23], in which a process trying to acquire the lock uses FAS to append a node to the queue tail, and then spins on that node waiting for its turn to enter the critical section. Unfortunately, if the process crashes after the FAS, before writing its result to memory, then when it recovers and returns to the algorithm, it does not know whether it has added itself to the queue and/or who is its predecessor (previously obtained from the FAS response). To overcome this problem, a recovering process reconstructs the queue state into some valid state, which incurs a linear number of RMRs. The recovery procedure is blocking (not wait-free), and multiple processes cannot recover concurrently. Overall, these prior building blocks have $O(D)$ passage complexity and $O(1 + FD)$ super-passage complexity. In contrast, our $D$-process abortable RME algorithm has $O(1)$ passage complexity and $O(1 + F)$ super-passage complexity, has wait-free recovery, and allows multiple processes to recover concurrently. While other $O(1)$ RME algorithms exist, they either assume a weaker crash model [13], rely on non-standard primitives that are not available on real machines [12,20], or obtain only amortized, not worst-case, $O(1)$ RMR complexity [3].

Result #2: Tournament tree with wait-free exit ($\S$ 4). In both ours and prior work [12,19], the main lock is obtained by constructing a tournament tree from the $D$-process locks. The tree has $N$ leaves, one for each process. Each internal node is a $D$-process lock, so the tree has height $O(\log_D N)$. To acquire the main lock, a process competes to acquire each lock on the path from its leaf to the root, until it wins at the root and enters the critical section. Our algorithm differs from prior tournament trees in a couple of simple ways, but which have important impact.

First: In our tree, a process that recovers from a crash returns directly to the node in which it crashed. This allows us to leverage our node lock’s $O(1 + F)$ super-passage complexity to obtain $O(H + F)$ super-passage complexity for the tree, where $H$ is the tree’s height. By taking $D = W = \Theta(\log N)$, our overall lock has $O(D + \log N)$ super-passage complexity and $O(\frac{\log N}{\log \log N})$ passage complexity. In contrast, prior trees perform recovery by having a process restart its ascent from the leaf. In fact, in these algorithms, there is no asymptotic benefit from returning directly to the node where the crash occurred. The reason is that node lock recovery in these trees has $O(D)$ complexity, so to obtain overall sublogarithmic complexity, we have to add a significant amount of work. Second, we depart from FAS and use the FAA primitive to beat the $\Omega(D)$ passage complexity lower-bound. In contrast, prior trees perform recovery by having a process restart its ascent from the leaf. In fact, in these algorithms, there is no asymptotic benefit from returning directly to the node where the crash occurred. The reason is that node lock recovery in these trees has $O(D)$ complexity, so to obtain overall sublogarithmic complexity, we have to add a significant amount of work.
Recoverable, Abortable, and Adaptive ME with Sublogarithmic RMR Complexity

--------

Table 1: Comparison of RME algorithms. (SP: super-passage, WF: wait-free, F*: total number of crashes in the system, FASAS: Fetch-And-Swap-And-Swap.) All algorithms satisfy starvation-freedom, wait-free critical-section re-entry, and wait-free exit (defined in § 2).

| Algorithm | Passage Complexity | Super-Passage Complexity | Primitives Used | Space Complexity | Additional Properties |
|-----------|--------------------|--------------------------|-----------------|-----------------|-----------------------|
| Goswami & Ramaraju [15, Section 4.2] with MCS [23] as base lock | $O(1)$ (no concurrent crashes) | $O(1)$ (no concurrent crashes) | CAS, FAS | $O(N \log N)$ | Abortable, adaptive, SP WF Exit |
| Jayanti & Joshi [21] | $O((\log N)/(\log \log N))$ (0 crashes) | $O((\log N)/(\log \log N))$ (no crashes) | CAS | $O(N \log N)$ | FCFS, SP WF Exit |
| Chan & Woelfel [8] | $O(1)$ (amortized) | same as passage complexity | CAS, FAS | Unbounded | Crash-adaptive |
| Jayanti, Jayanti, & Joshi [19] | $O(\log N)$ | $O((1 + F)/(\log \log N))$ | FAS | Unbounded | SP WF Exit |
| Jayanti, Jayanti, & Joshi [20] | $O(1)$ (in the DSM model) | $O(1)$ (in the CC model) | FASAS | $O(N)$ | SP WF Exit |
| Dhoked & Mittal [10] | $O(\log N)$ | $O((\log N)/(\log \log N))$ | CAS, FAS | Unbounded | Crash-adaptive |
| Mittal [10] | $O(\log N)$ | $O((\log N)/(\log \log N))$ | CAS, FAS | Unbounded | Crash-adaptive |

This work | $O(\min(K, \frac{\log N}{\log \log N}))$ | $O(\min(K, \frac{\log N}{\log \log N}) + F)$ | FAA, CAS | $O(\frac{\log N}{\log \log N})$ | Abortable, adaptive, SP WF Exit |

complexity, they take $D = \frac{\log N}{\log \log N}$, which means that node crash recovery costs the same as climbing to the node. Consequently, their overall super-passage complexity is multiplicative in $F$, $O((1 + F)/(\log \log N))$, instead of additive as in our tree.

Second: Our tree’s exit section is wait-free (assuming finitely many crashes). In contrast, in the prior trees, a process that crashes during its exit section might subsequently block. The reason is a subtle issue related to composition of RME locks. The model in these works [12,19] is that a process $p$ that crashes in its exit section must complete a crash-free passage upon recovery (i.e., re-enter the critical section and exit it again). Thus, $p$ must re-ascend to the root after recovering. Each node lock satisfies a bounded CS re-entry property, which allows $p$ to re-enter the node’s CS (i.e., ascend) without blocking—provided that $p$ crashed inside the node’s CS. However, this property does not apply if $p$ released the node lock (i.e., descended) before crashing. For such a node, $p$ simply attempts to re-acquire the node lock. Consequently, $p$ might block during its recovery, even though logically it is only trying to release the overall lock. We address this problem by carefully modeling the interface of an RME algorithm in a way that facilitates composition, which enables a recovering process to avoid re-acquiring node locks it had already released. Our overall algorithm thereby satisfies a new super-passage wait-free exit property.

Result #3: Generic RME adaptivity transformation (§ 5) We present a generic transformation that transforms any abortable RME algorithm with passage complexity of $B < W$ into an abortable RME lock with passage complexity of $O(\min(K, B))$, where $K$ is the number of processes executing the algorithm concurrently with the process going through the super-passage, i.e., the point contention. Applying this transformation to our tournament tree lock yields the final algorithm.

Summary of contributions and related work Table 1 compares our final algorithm to prior RME work. Dhoked and Mittal [10] use a definition of “adaptivity” that requires RMR cost to depend on the total number of crashes; we refer to this property as crash-adaptivity. Crash-adaptivity is thus orthogonal to the traditional notion of adaptivity [6]. Chan & Woelfel’s algorithm [8] uses FAA, but it is used to assign processes with tickets, which is different from our technique (§ 3). Their algorithm has only an amortized RMR passage complexity bound and its worst-case RMR cost is unbounded.
2 Model and Preliminaries

Model We consider a system in which $N$ deterministic, asynchronous, and unreliable processes communicate over a shared memory. The shared memory, $M$, is an array of $\Theta(W)$-bit words. (Henceforth, we refer to the shared memory simply as “memory”; process-private variables are not part of the shared memory.) The system supports the standard read, write, CAS, and FAA operations. $\text{CAS}(a,o,n)$ atomically changes $M[a]$ from $o$ to $n$ if $M[a] = o$ and returns true; otherwise, it returns false without changing $M[a]$. $\text{FAA}(a,x)$ atomically adds $x$ to $M[a]$ and returns $M[a]$'s original content.

A configuration consists of the state of the memory and of all processes, where the state of process $p$ consists of its internal program counter and (non-shared) variables. Given a configuration $\sigma$, an execution fragment is a (possibly infinite) sequence of steps, each of which moves the system from one configuration to another, starting from $\sigma$. In a normal step, some process $p$ invokes an operation on a memory word and receives the operation’s response. In a crash step, the state of some process $p$ resets to its initial state (but the memory state remains unchanged). An execution is an execution fragment starting from the system’s initial configuration.

Notation Given an execution fragment $\alpha$, if $\beta$ is a subsequence of $\alpha$, we write $\beta \subseteq \alpha$. If $e$ is a step taken in $\alpha$, we write $e \in \alpha$. If $e$ is the $t$-th step in an execution $E$, we say that $e$ is at time $t$. We use $[t,t']$ to denote the subsequence of $E$ whose first and last steps are at times $t$ and $t'$ in $E$, respectively.

RMR complexity The RMR complexity measure breaks the memory accesses by a process $p$ into local and remote references, and charges $p$ only for remote references. We consider two types of RMR models. In the DSM model, each memory word is local to one process and remote to all others, and process $p$ performs an RMR if it accesses a memory word remote to it. In the CC model, the processes are thought of as having coherent caches, with RMRs occurring when a process accesses an uncached memory word. Formally: (1) every write, CAS, or FAA is an RMR, and (2) a read by $p$ of word $x$ is an RMR if it is the first time $p$ accesses $x$ or if after $p$’s prior access to $x$, another process performed a write, CAS, or FAA on $x$.

Recoverable mutual exclusion (RME) Our RME model draws from the models of Golab and Ramaraju [15] and Jayanti and Joshi [21]. In the spirit of [15], we model the RME algorithm as an object exporting methods invoked by a client process. In the spirit of [21], we require recovery to re-execute the section in which the crash occurred, rather than restart the entire passage. An RME algorithm (or lock) provides the methods $\text{Recover}$, $\text{Try}$, and $\text{Exit}$. (In the code, we show the methods taking an argument specifying the calling process’ id.) If process $p$ invokes $\text{Try}$ and it returns TRUE, then $p$ has acquired the lock and enters the critical section (CS). Subsequently, $p$ exits the CS by invoking $\text{Exit}$. If $\text{Exit}$ completes, we say that $p$ has released the lock. The $\text{Recover}$ method guides $p$’s execution after a crash, which resets $p$ to its initial state. We assume $p$’s initial state is to invoke $\text{Recover}$, which returns $r \in \{\text{TRY}, \text{CS}, \text{EXIT}\}$. If $r = \text{TRY}$, $p$ invokes $\text{Try}$. If $r = \text{CS}$, $p$ enters the CS. If $r = \text{EXIT}$, $p$ invokes $\text{Exit}$.

A super-passage of $p$ begins with $p$ completing $\text{Recover}$ and invoking $\text{Try}$, either for the first time, or for the first time after $p$’s prior super-passage ended. The super-passage ends when $p$ completes $\text{Exit}$. A passage of $p$ begins with $p$ starting a super-passage, or when $p$ invokes $\text{Recover}$ following a crash step. The passage ends at the earliest of $p$ completing $\text{Exit}$ or crashing. We refer to an $L$-passage (or $L$-super-passage) to denote the lock $L$ that a
passage (or super-passage) applies to; similarly, we refer to a step taken in lock $L$'s code as an $L$-step. We omit $L$ when the context is clear. These definitions facilitate composition of RME locks. For instance, suppose that process $p$ is releasing locks in a tournament tree and crashes after releasing some node lock $L$. When $p$ recovers, it can invoke $L.Recover$, which will return $TRY$, and thereby learn that it has released $L$ and can descend from it—without the $Recover$ invocation counting as starting a new $L$-super-passage.

Well-formed executions formalize the above described process behavior:

▶ **Definition 1.** An execution is well-formed if the following hold for every lock $L$ and process $p$:

1. Recover invocation: $p$’s first $L$-step after a crash step is to invoke $L.Recover$.
2. Try invocation: $p$ invokes $L.Try$ only if $p$ is starting a new $L$-super-passage, or if $p$’s prior crash step was during $L.Try$.
3. CS invocation: $p$ enters the CS of $L$ only if $p$ receives $TRUE$ from $L.Try$ in its current $L$-passage, or if $p$’s prior crash step was during the CS.
4. Exit invocation: $p$ invokes $L.Exit$ only if $p$ is in the CS of $L$, or if $p$’s prior crash step was during $L.Exit$.

Henceforth, we consider only well-formed execution. We also consider only well-behaved RME algorithms, in which $Recover$ correctly identifies where a process crashes:

▶ **Definition 2.** An RME algorithm is well-behaved if the following hold, for every process $p$ and every well-formed execution:

1. $p$’s first complete invocation of $Recover$, and $p$’s first complete invocation of $Recover$ following a complete passage of $Exit$, returns $TRY$.
2. $p$’s first complete invocation of $Recover$ following a crash during $Try$ return $TRY$.
3. $p$’s first complete invocation of $Recover$ following a crash during the CS returns $CS$.
4. $p$’s first complete invocation of $Recover$ following a crash during $Exit$ returns $EXIT$.
5. A complete invocation of $Recover$ by $p$ during the CS returns $CS$.

Note: We consider $p$ to be in the $Try$ or $Exit$ section from the time it executes the first memory operation of that section and until it either crashes or executes the last memory operation of that section. Thus, $p$ is considered to be in the CS after it executes its final $Try$ memory operation.

**Fairness** We make a standard fairness assumption on executions: once $p$ starts a super-passage, it does not stop taking steps until the super-passage ends.

**Abortable RME** At any point during its super-passage, process $p$ can non-deterministically choose to abort its attempt, which we model by $p$ receiving an external abort signal that remains visible to $p$ throughout the super-passage (i.e., including after crashes) and resets once $p$ finishes the super-passages. Abortable RME extends the definition of a super-passage as follows. If $p$ is signalled to abort and its execution of $Try$ returns $FALSE$, then $p$ has aborted and the super-passage ends. (It is not mandatory for $Try$ to return $FALSE$, because an abort may be signalled just as $p$ acquires the lock.)

**$D$-ported locks** We model locks that may be used by at most $D$ processes concurrently as follows. In a $D$-ported lock, each process invokes the methods with a port argument, $1 \leq k \leq D$, which acts as an identifier. We augment the definition of a well-formed execution to include the following conditions:
5. **Constant port usage**: For every process $p$ and $L$-super-passage of $p$, $p$ does not change its port for $L$ throughout the super-passage.

6. **No concurrent super-passages**: For any $L$-super-passages $sp_i$ and $sp_j$ of processes $p_i \neq p_j$, if $sp_i$ and $sp_j$ are concurrent, then $p_i$’s port for $L$ in $sp_i$ is different than $p_j$’s port for $L$ in $sp_j$. (Two super-passages are not concurrent if one ends before the other begins.)

**Problem statement** Design a well-behaved abortable RME algorithm with the following properties.

1. **Mutual exclusion**: At most one process is in the CS at any time $t$.

2. **Deadlock-freedom**: If a process $p$ starts a super-passage $sp$ at time $t$, and does not abort $sp$, and if every process that enters the CS eventually leaves it, then there is some time $t' > t$ and some process $q$ such the $q$ enters the CS in time $t'$, or else there are infinitely many crash steps.

3. **Bounded abort**: If a process $p$ has abort signalled while executing $Try$, and executes sufficiently many steps without crashing, then $p$ complete its execution of $Try$.

The following properties are also desirable, and all but FCFS are satisfied by our algorithm:

4. **Starvation-freedom**: If the total number of crashes in the execution is finite and process $p$ executes infinitely many steps and every process that enters the CS eventually leaves it, then $p$ enters the CS in each super-passage in which it does not receive an abort signal.

5. **CS re-entry**: If process $p$ crashes while in the CS, then no other process enters the CS from the time $p$ crashes to the time when $p$ next enters the CS.

6. **Wait-free CS re-entry**: If process $p$ crashes in the CS, and executes sufficiently many steps without crashing, then $p$ enters the CS.

7. **Wait-free exit**: If process $p$ is executing $Exit$, and executes sufficiently many steps without crashing, then $p$ completes its execution of $Exit$.

8. **Super-passage wait-free exit**: If process $p$ is executing $Exit$, then $p$ completes an execution of $Exit$ after a finite number of its own steps, or else $p$ crashes infinitely many times. (Notice that $p$ may crash and return to re-execute $Exit$.)

9. **First-Come-First-Served (FCFS)**: If there exists a bounded section of code in the start of the entry section, referred to as the *doorway* such that, if process $p_i$ finishes the doorway in its super-passage $sp_i$ for the first time before some process $p_j$ begins its doorway for the first time in its super-passage $sp_j$, and $p_i$ does not abort $sp_i$, then $p_j$ does not enter the CS in $sp_j$ before $p_i$ enters the CS in $sp_i$.

Super-passage wait-free exit is a novel property introduced in this work. It guarantees that a process completes $Exit$ in a finite number of its own steps, as long as it only crashes finitely many times. Wait-free exit does not imply super-passage wait-free exit since it does not apply if the process crashes during $Exit$. Clearly, starvation-freedom implies deadlock-freedom, wait-free CS re-entry implies CS re-entry, and super-passage wait-free exit implies wait-free exit.

**Lock complexity** The *passage complexity* (respectively, *super-passage complexity*) of a lock is the maximum number of RMRs that a process can incur while executing a passage (respectively, super-passage). We denote by $F$ the maximum number of times a process crashes in an execution.
W-Port Abortable RME Algorithm

Here, we present our \( D \)-process abortable RME algorithm, for \( D \leq W \), which has \( O(1) \) passage RMR complexity and \( O(1 + F) \) super-passage complexity. The algorithm is similar in structure to Jayanti and Joshi’s abortable RME algorithm \cite{22}, in that it is built around a recoverable auxiliary object that tracks the processes waiting to acquire the lock. This object’s RMR complexity determines the algorithm’s complexity. Non-abortable RME locks implement such an object with a FAS-based linked list \cite{12,19}. Such a list has \( O(1 + FD) \) super-passage complexity—i.e., a crash-free passage incurs \( O(1) \) RMRs—but it is hard to make abortable. Jayanti and Joshi instead use a recoverable min-array \cite{16}. This object supports aborting, but its passage complexity is logarithmic, even in the absence of crashes.

Our key idea is to represent the “waiting room” object with a FAA-based \( W \)-bit mask (a single word), where a process \( p \) arriving/leaving is indicated by flipping a bit associated with \( p \)’s port. The key ideas are that (1) if \( p \) crashes and recovers, it can learn its state in \( O(1) \) RMRs simply by reading the bit mask and (2) the algorithm carefully avoids relying on any FAA’s return value. Our design thus obtains the best of both worlds: the object can be updated with \( O(1) \) RMRs as well as supports efficient aborting (with a single bit flip). The trade-off we make in this design choice is that we only guarantee starvation-freedom, but not FCFS. Unlike a min-array, the bit mask cannot track the order of arriving processes, as bit setting operations commute. We do, however, track the order in which processes acquire the lock, and thereby guarantee starvation-freedom.

Our algorithm guarantees starvation-freedom unconditionally, even if there are infinitely many aborts. This turns out to be a subtle issue to handle correctly (§ 3.2), and the Jayanti and Joshi algorithm is prone to executions in which a process that does not abort starves as a result of other processes aborting infinitely often (we show an example in § 3.2).

Since we assume \( W \)-bit memory words, we are careful not to use unbounded, monotonically increasing counters, which the Jayanti and Joshi lock does use. Our algorithm’s RMR bounds are in both the DSM and CC models, whereas the Jayanti and Joshi lock has linear RMR complexity on the standard CC model.

3.1 Algorithm Walk-Through

Figure 1 presents the pseudo code of the algorithm. We assume participating processes uses distinct ports in the range \( 0, \ldots, W - 1 \), so we refer to processes and ports interchangeably. For simplicity, we present the algorithm assuming dynamic memory allocation with safe reclamation \cite{24}. In this environment, a process can allocate and retire objects, and it is guaranteed that an allocation does not return a previously-retired object if some process still has a reference to that object. We show how to satisfy this assumption (with \( O(D^2) \) static, pre-allocated memory) in Appendix A.

Each process \( p \) has a status word, \( STATUS[p] \), and a pointer to a boolean spin variable, \( GO[p] \). (In the DSM model, a process allocates its spin variables from local memory, so that it can spin on them with \( O(1) \) RMR cost.) The lock’s state consists of a \( W \)-bit word, \( ACTIVE \), and a \( \Theta(W) \)-bit word, \( LOCK _ { STATUS} \). The \( LOCK _ { STATUS} \) word holds a tuple \( \langle taken, owner, owner _ { go} \rangle \), where \( taken \) is a bit indicating if the lock is acquired by some process. If \( taken \) is set, \( owner \) is the id (port) of the lock’s owner and \( owner _ { go} \) points to the owner’s spin variable.

The \( STATUS \) word of each process \( p \), initialized to \( TRY \), indicates in which section the process is currently at. This information is used by Recover to steer \( p \) to the right method when it arrives. The \( STATUS \) word changes when completing Try and entering the CS, when
aborting during Try, when exiting the CS and executing Exit, and when Exit completes. Note that the Exit method may be called as a subroutine during the Try section’s abort flow. In this case, its operations are considered part of the Try section (i.e., the subroutine call is to avoid putting a copy of Exit’s code in Try). To distinguish these subroutine calls from when a process invokes Exit to exit the CS, we add an abort argument to Exit, which is FALSE if and only if Exit is invoked to exit the CS (i.e., not as a subroutine).

In the normal (crash- and abort-free) flow, a passage of process p proceeds as follows. First, p allocates its spin variable, if it does not currently exist (lines 11–16). Then p flips its bit in the ACTIVE word, but only if p’s bit is not already set (lines 17–18). This check avoids corrupting ACTIVE when p recovers from a crash. Next, p executes a Promote procedure, which tries to pick some waiting process (possibly p) and make it the owner of the lock, if the lock is currently unowned (line 19). Finally, p begins spinning on its spin variable, waiting for an indication that it has become the lock owner (lines 20–25). Upon exiting the CS, p clears its bit in ACTIVE (again, only if the bit is currently set, to handle crash recovery) (lines 29–30). Then p executes Promote (line 11). Performing this call will have no effect, since p is still holding the lock, which may appear strange, but is required in order to support the abort flow, as explained shortly. Then, if p is indeed the lock owner (another check useful only in the abort flow), it releases the lock by clearing the taken bit in LOCK_STATUS (lines 12–15). Note that p leaves the owner and owner_go fields intact, for reasons described shortly. Finally, p executes Promote again, to hand the lock off to some waiting process (line 46). It then retires its spin variable, clears its GO pointer, and updates its STATUS to TRY, thereby completing Exit and thus its current passage and

```c
void Exit(int k, bool abort) {
  if abort = FALSE
    STATUS[k] = EXIT
  if k-th bit in ACTIVE is 1:
    FAA(ACTIVE, -2k)
    Promote(k)
    (taken, owner, owner_go) := LOCK_STATUS
    if taken = 1 and owner = k:
      CAS(LOCK_STATUS, (1, owner, owner_go),
      (0, owner, owner_go))
      Promote(⊥)
      GO[k] = ⊥
      Retire(GO[k])
      GO[k] := ⊥
      STATUS[k] := TRY
}
void Promote(int j) {
  if taken, owner, owner_go) := LOCK_STATUS
    if taken = 0:
      active := ACTIVE
      if active ≠ 0:
        j := next(owner, active)
        if j ≠ -1:
          CAS(LOCK_STATUS, (0, owner, owner_go),
          (1, j, GO[j]))
      (taken, owner, owner_go) := LOCK_STATUS
      if taken = 1:
        owner_go := TRUE
}

Figure 1 W-port abortable RME algorithm
```
super-passage (line 47–50).

If \( p \) receives the abort signal while spinning in Try, it sets its STATUS to ABORT, executes the Exit method as a subroutine, and returns FALSE (label 12–15). If \( p \) crashes during the execution of Exit, Recover will steer it to Try once it recovers, at which point it will again execute the Exit method and return FALSE. In the abort flow, the call to Exit does not modify \( p \)'s STATUS (the if is not taken, lines 37–35).

The main goal of Promote\((j)\) is to promote some waiting process to be the lock owner, if the lock is currently unowned. Promote tries to promote one of the waiting processes (as specified by ACTIVE). If there is no such process, then Promote tries to promote process \( j \) if \( j \neq \perp \), and does not promote any process otherwise (lines 53–60). A secondary goal of Promote is that it signals the (current or newly promoted) owner by writing to its spin variable (lines 61–63). Picking a process to promote from among the waiting processes is done in a manner that guarantees starvation-freedom. To this end, Promote picks the next id whose bit is set in ACTIVE, when ids are scanned starting from the previous owner's id (which, as described above, is written in LOCK_STATUS) and moving up (modulo \( W \)). (In the code, this is specified as next(owner, active).) Having picked a process \( q \) to promote, Promote tries to update LOCK_STATUS to \((1, q, GO[q])\) using a single CAS.

Finally, before completing, Promote checks again if the lock is owned by some process \( r \) (possibly \( r \neq q \)), and if so, signals \( r \) by writing TRUE to \( r \)'s spin variable.

The reason for executing Promote in Exit before releasing the lock, and not only afterwards, is to handle a scenario in which the lock owner \( q \) has released the lock and next\((q, ACTIVE) = p \), so any process \( r \) (possibly, but not necessarily, \( q \)) executing Promote tries to hand the lock to \( p \). If now \( p \) is signalled to abort, and did not also execute Promote before departing, deadlock would occur. By having \( p \) call Promote\((p)\), we guarantee that either (1) some process (possibly \( p \)) promotes \( p \), so \( p \)'s Exit call releases the lock before completing the abort; or (2) some process \( r \) (possibly, but not necessarily \( p \)), which does not observe \( p \) in ACTIVE, updates LOCK_STATUS from \((0, q, G)\) to \((1, q', G')\). In the latter case, our memory management assumption implies that LOCK_STATUS will not recycle to contain \((0, q, G)\) before every processes that has read \((0, q, G)\) from LOCK_STATUS executes its CAS. All such CASs, who are about to change \((0, q, G)\) to \((1, p, GO[p])\) thus fail, so the lock does not get handed to \( p \) and no deadlock occurs after it completes its abort.

### 3.2 Discussion: Guaranteeing Starvation-Freedom In the Presence of Infinitely Many Aborts

As discussed in § 3.1, a key idea in our algorithm is to invoke Promote even before releasing the lock, to handle the case in which the lock is about to be handed to an aborting process. While simple, this is a subtle idea, because a different (more straightforward) approach to dealing with this issue can lead to starvation. We explain the issue by describing and analyzing a starvation problem in Jayanti and Joshi’s abortable RME algorithm. The structure of our algorithm and of Jayanti and Joshi’s algorithm is similar, if one thinks of our ACTIVE word and their min-array as abortable objects which (1) maintain the set of waiting processes and (2) have some notion of the “next in line” waiting process, which becomes the lock owner. (Jayanti and Joshi refer to this object as a registry.) We describe the problem in the Jayanti and Joshi lock by contrasting its behavior with our algorithm’s.

Intuitively, starvation-freedom should follow from property (2) of the “waiting room” object, because every process executing Promote will eventually agree on the process \( p \) to promote, which would then become the lock owner. For this to be true, however, aborts need to be handled very carefully. Phrased in our terminology, in the Jayanti and Joshi
algorithm, a process \( p \) that receives an abort signal starts executing \( Exit \), where it removes itself from the “waiting room” object. Subsequently, if \( \text{LOCK\_STATUS} = (0, o, os) \), \( p \) tries (using a single CAS) to update \( \text{LOCK\_STATUS} \) from \((0, o, os)\) to \((0, p, \text{GO}[p])\). In other words, \( p \) tries to make it look as if it had acquired the lock and immediately released it. The motivation for this step is to fail any \textit{Promote} that is about to make \( p \) the lock owner, which if not handled, would result in deadlock.

This approach has the unfortunate side-effect of failing concurrent \( \text{Promotes} \) even if they are not about to make \( p \) the lock owner. This can lead to an execution in which aborting processes prevent the lock from being acquired, as described next.

Process \( p_1 \) arrives and enters the critical section. Process \( p_2, p_3, p_4 \) arrive and enter the waiting room. Now \( p_1 \) leaves the CS and executes \( Exit \), which (in Jayanti and Joshi’s algorithm) has a single \textit{Promote} call, after releasing the lock. Suppose the “waiting room” object indicates that \( p_2 \) should be the next lock owner. Now, \( p_1 \) stops in its \textit{Promote} call, just before CASing \( \text{LOCK\_STATUS} \) from \((0, p_1, \ast)\) to \((1, p_2, \ast)\). Next, \( p_3 \) aborts, executes the \( Exit \) code and successfully changes \( \text{LOCK\_STATUS} \) to \((0, p_3, \ast)\).

As a result, \( p_1 \)’s \textit{CAS} in \textit{Promote} fails. \( p_1 \) completes its \textit{Exit} section and then returns to the Try section, executes \textit{Promote}, and stops just before CASing \( \text{LOCK\_STATUS} \) from \((0, p_1, \ast)\) to \((1, p_2, \ast)\). Now, \( p_3 \) proceeds to the \textit{Promote} call in \textit{Exit}, stopping just before CASing \( \text{LOCK\_STATUS} \) from \((0, p_3, \ast)\) to \((1, p_2, \ast)\). We have reached a state in which \( p_4 \) is waiting, \( \text{LOCK\_STATUS} \) is \((0, p_3, \ast)\), \( p_1 \) is in its Try \textit{Promote} and \( p_3 \) is in its \textit{Exit} promote, both about to CAS \( \text{LOCK\_STATUS} \) from \((0, p_1, \ast)\) to \((1, p_2, \ast)\).

We continue as follows. Now \( p_4 \) receives the abort signal, proceeds to execute \( Exit \), and successfully changes \( \text{LOCK\_STATUS} \) from \((0, p_3, \ast)\) to \((0, p_4, \ast)\). Consequently, the CAS of both \( p_1 \) and \( p_3 \) fails, so \( p_1 \) enters the waiting room, whereas \( p_3 \) departs the algorithm, returns, and stops in the Try \textit{Promote} before CASing \( \text{LOCK\_STATUS} \) from \((0, p_4, \ast)\) to \((1, p_2, \ast)\). As for \( p_4 \), it enters the Exit \textit{Promote} and stops before CASing \( \text{LOCK\_STATUS} \) from \((0, p_4, \ast)\) to \((1, p_2, \ast)\). We have reached a similar situation as in the previous paragraph, and can therefore keep repeating this scenario indefinitely. Throughout, \( p_2 \) keeps taking steps in the waiting room, but will never enter the CS.

### 3.3 Proofs of RME Properties

Whenever a process \( p \) starts a super-passage in our algorithm, it allocates a fresh spin variable. To avoid unbounded space consumption, the memory used for spin variables eventually has to be recycled, i.e., an allocation by process \( p \) can return a variable it previously used. Our proofs assume that this recycling is done safely, namely, that an allocation of a new spin variable does not return an object that is currently being referenced by some process. (We show how to satisfy this assumption using \( O(D^2) \) static pre-allocated memory words in Appendix A.)

The above safe memory management assumption implies two properties that we use throughout the proofs. First, that if a process \( p \) is about to CAS \( \text{LOCK\_STATUS} \) in \textit{Promote}, and \( \text{LOCK\_STATUS} \) has changed between \( p \) last reading it and executing the CAS, then the CAS will fail. This holds because \( \text{LOCK\_STATUS} \) necessarily contains a different \textit{owner\_go} value. Second, that if \( p \) sets the spin variable of \( q \) to \textit{TRUE} and \( q \) has already started a new super-passage, then \( q \) will never read that \textit{TRUE} value. This holds because \( q \) allocates a different spin variable for its new super-passage.

Let \( p \) be a process in super-passage \( sp \) using port \( k \). For each passage in \( sp \), we say \( p \) is in its Try (respectively Critical or Exit) section from the time it executes the first memory operation of that section and until \( p \) crashes or executes the last memory operation of that section. Recall the subtle difference between \( p \) being in its Exit section and \( p \) executing the
Exit method. The latter can happen even if \( p \) is not in its Exit section, due to receiving an abort signal in its Try section and calling Exit as a subroutine. We say \( p \) is in its Exit section if and only if \( p \) invokes Exit with \( abort = False \).

While \( p \) is in its Try section, we define that \( p \) is in the waiting room from the time \( t \) where \( p \) starts the while loop in the Try section and until time \( t' \) where \( p \) finishes its Try section or crashes.

If \( p \) is in a super-passage \( sp \) with port \( k \), we define \( spin_{p,k,sp} \) as the only spin variable \( p \) is waiting on in its waiting room in this super-passage. We use the notation \&\( v \) for the memory address of \( v \).

We refer to our \( W \)-port abortable RME algorithm as Algorithm \( M \). We proceed to prove that Algorithm \( M \) satisfies the desired RME properties. All claims and proofs assume that executions are well-formed. Omitted claims and proofs appear in Appendix B.

\[ \text{Claim 3.} \quad \text{If at time } t, \text{ no process is in a super-passage with port } k, \text{ then } STATUS[k] = TRY. \]

\[ \text{Claim 4.} \quad \text{If process } p \text{ is in super-passage } sp \text{ with port } k \text{ and } p \text{ is in the Try section, then } STATUS[k] = TRY \text{ or } STATUS[k] = ABORT. \]

\[ \text{Claim 5.} \quad \text{If process } p \text{ is in super-passage } sp \text{ with port } k \text{ and } p \text{ is in the CS, then } STATUS[k] = CS. \]

\[ \text{Claim 6.} \quad \text{If process } p \text{ is in super-passage } sp \text{ with port } k \text{ and } p \text{ is in the Exit section, then } STATUS[k] = EXIT. \]

\[ \text{Lemma 7.} \quad \text{Algorithm } M \text{ is well-behaved.} \]

\[ \text{Claim 8.} \quad \text{If process } p \text{ is in super-passage } sp \text{ with port } k \text{ and is in the CS, then there was some time } t \text{ before } p \text{ first entered the CS in } sp, \text{ at which } LOCK_{\_STATUS} = (1, k, \&spin_{p,k,sp}). \]

\[ \text{Claim 9.} \quad \text{If process } p \text{ is in super-passage } sp \text{ with port } k, \text{ and at some time } t \text{ during } sp, \text{ } LOCK_{\_STATUS} = (1, k, \&spin_{p,k,sp}), \text{ then } LOCK_{\_STATUS} \text{ can only be changed by } p \text{ while executing the Exit procedure.} \]

\[ \text{Claim 10.} \quad \text{If process } p \text{ is in super-passage } sp \text{ with port } k \text{ and } p \text{ is in the CS, then } LOCK_{\_STATUS} = (1, k, \&spin_{p,k,sp}). \]

\[ \text{Lemma 11.} \quad \text{Algorithm } M \text{ satisfies mutual exclusion.} \]

\[ \text{Claim 12.} \quad \text{If process } p \text{ completes an execution of Promote without crashing in the interval } [t, t'] \text{ and } ACTIVE \neq 0 \text{ throughout } [t, t'], \text{ or if } p \text{ completes an execution of Promote(k) without crashing in the interval } [t, t'], \text{ then there is a time } t_0 \in [t, t'] \text{ at which } LOCK_{\_STATUS} = (1, *, *). \]

\[ \text{Claim 13.} \quad \text{If process } p \text{ is in super-passage } sp \text{ with port } k, \text{ } p \text{ sets the } k\text{-th bit in } ACTIVE \text{ to 1 at time } t, \text{ and subsequently reaches the waiting room at time } t' > t, \text{ then there is a time } t_0 \in [t, t'] \text{ at which } LOCK_{\_STATUS} = (1, *, *). \]
Claim 14. If \textit{LOCK\_STATUS} changes to $(1, k, \&\text{spin}_{p,k,sp})$ then there is some process $p$ in its super-passage with port $k$ and either (1) \text{spin}_{p,k,sp} eventually becomes TRUE, (2) $p$ aborts, or (3) there are infinitely many crashes in the execution.

Claim 15. If \textit{LOCK\_STATUS} changes to $(1, *, *)$ at time $t$, and if any process that enters the CS after $t$ eventually exits it, then there is a time $t' > t$ at which \textit{LOCK\_STATUS} changes to $(0, *, *)$ or there are infinitely many crashes in the execution.

Claim 16. If at some point of the execution \textit{LOCK\_STATUS} $= (0, *, *)$, and $ACTIVE \neq 0$, then \textit{LOCK\_STATUS} will change to $(1, *, *)$ or there are infinitely many crashes in the execution.

Claim 17. If a process $p$ is in its super-passage $sp$ with port $k$ and is in its waiting room, and $p$ does not abort, then \textit{LOCK\_STATUS} can change to $(1, *, *)$ at most $W$ times before changing to $(1, k, \&\text{spin}_{p,k,sp})$.

\textbf{Proof.} We define the \textit{distance of the $j$-th bit from the $i$-th bit}, denoted $d(i, j)$, as follows. If $j > i$ then $d(i, j) = j - i$. If $i > j$ then $d(i, j) = W - i + j$. Informally, $d(i, j)$ is the number of times we count up from $i$, modulo $W$, to obtain $j$. We number the bits in $ACTIVE$ from 1 to $W$ such that the least significant bit is numbered 1 and the most significant bit numbered $W$. Assume that $p$ is now in its waiting room in super-passage $sp$ with port $k$, and that \textit{LOCK\_STATUS} is $(1, o, os)$. Assume now that \textit{LOCK\_STATUS} changes again to $(1, o', os')$. The process that successfully CASed \textit{LOCK\_STATUS} previously reads $(1, o, os)$ from it, and then reads $a = ACTIVE$, implying that it sees the $k$-th bit set in $a$. Let $o' = next(o, a)$ be the next owner of the lock. It follows from the definition of $d(o, k)$ that either $o' = k$ or that $d(o', k) < d(o, k)$. Since the distance of any bit $i$ from any bit $j$ is at most $W$ and at least 0, the aforementioned change can happen at most $W$ times before $next()$ returns $k$.

Lemma 18. Algorithm $M$ satisfies starvation-freedom.

\textbf{Proof.} Assume there are finitely many crash steps in the execution. Assume process $p$ is in a super-passage $sp$ with port $k$, in which $p$ does not abort. Since there are finitely many crashes, then $p$ eventually reaches the waiting room in $sp$. From Claim 14, there is a time $t$ before $p$ reaches the waiting room at which \textit{LOCK\_STATUS} $= (1, *, *)$. Since there are finitely many crashes and every process does not stop taking steps while in a super-passage, every process that enters the CS after $t$ also reaches its Exit section. Thus, from Claim 15 there is a point $t' > t$ at which \textit{LOCK\_STATUS} changes to $(0, *, *)$. Because bit $k$ in $ACTIVE$ is set, Claim 16 implies that as long as $p$ is in its waiting room, at some point $t'' > t'$, \textit{LOCK\_STATUS} changes again to $(1, *, *)$. We can repeat this argument as long as $p$ is in its waiting room and does not abort. However, from Claim 17 after at most $W$ times, \textit{LOCK\_STATUS} becomes $(1, k, \&\text{spin}_{p,k,sp})$. At this point, by Claim 14 \text{spin}_{p,k,sp} eventually turns TRUE. Subsequently, $p$ enters the CS in a finite number of its own steps.

Theorem 19. If every execution of Algorithm $M$ is well-formed, then Algorithm $M$ satisfies mutual exclusion, bounded abort, starvation-freedom, CS re-entry, wait-free CS re-entry, wait-free exit, and super-passage wait-free exit. The passage complexity of Algorithm $M$ in both the CC and DSM models is $O(1)$ and the super-passage complexity is $O(1 + F)$. (Assuming, for the DSM model, that process memory allocations return local memory.) The space complexity of the algorithm is $O(D^2)$. 


4 Tournament Tree

A tournament tree lock, referred to as the main lock, is constructed by statically arranging multiple D-port RME algorithms, referred to as node locks, in a D-ary tree with N leaves (we assume \( D \leq W \)). Each leaf is uniquely associated with a process. To acquire the main lock, a process competes to acquire each lock on the path from its leaf to the root, until it wins at the root and enters the main lock’s CS. To release the main lock, the process descends from the root to its leaf, releasing each node lock on the path. In this section, we present our tournament tree algorithm.

Our algorithm has two distinguishing features: (1) that its super-passage RMR complexity is additive in \( F \), the number of crashes, and not multiplicative; and (2) that it satisfies super-passage wait-free exit (SP-WF-Exit), i.e., a process releasing the main lock is guaranteed to complete some execution of Exit after a finite number of its own steps (including crashes).

Our algorithm’s super-passage RMR complexity is \( O(FR + B \log_D N) \), where \( R \) and \( B \) are the recovery cost and passage complexity of the node lock, respectively. In comparison, prior trees have super-passage complexity of \( O(F(R + B \log_D N)) \). Obtaining our bound is simple: a process just needs to write its location in the tree to NVRAM, so that upon crash recovery, it can resume from there instead of starting to walk up or down the tree from scratch. We suspect that this simple optimization was not performed in prior tournament trees because their node lock has \( R = \log_D N = O(\frac{\log N}{\log \log N}) \) and \( B = O(1) \), so directly returning to the node at which the crash occurred does not asymptotically improve complexity. With our \( W \)-port RME algorithm, however, \( R = B = O(1) \), so being additive in \( F \) is asymptotically better, and would not be obtained using prior tournament trees.

The problem of obtaining SP-WF-Exit highlights the difficulty of composing recoverable locks. The issue is that a process in the main lock is composing critical sections of the node locks, which creates the problem of how recovery of the main and node locks interact. In the model of prior work [12,15], a process crashing in the main lock’s exit section attempts to re-acquire the main lock upon recovering. As a result, the process might now block in some node lock’s entry section, which violates SF-WF-Exit for the main lock. We address this problem by carefully modeling RME algorithms in a way that facilitates composition (§ 2). Instead of assuming how a process participates in the algorithm (i.e., cycling through entry, CS, exit), we model the RME algorithm as an object whose Recover procedure informs the process where it crashed in the super-passage. This approach allows client algorithms, composing the lock, to decide how to proceed. Our model allows a process returning to lock \( x \) after crashing in the main lock to realize that it had completed an \( x \)-super-passage and not start a new one. Consequently, our tournament tree avoids the problems described above and satisfies SP-WF-Exit.

We present detailed pseudo code and prove all of the algorithm’s properties. Omitted proofs appear in Appendix C.

4.1 Algorithm Walk-Through

Figure 2 shows the pseudo code of the algorithm. Each node has immutable parent and child pointers (as mentioned before, the tree structure is static). The parent of root is \( \perp \), as are all child pointers of a leaf node. Each process is statically assigned to a leaf based on its id (\( pid \)). Each node contains a D-port abortable RME lock.

Similarly to our \( W \)-port algorithm, each process \( p \) has a status word, \( STATUS[p] \), which is used by the main lock’s Recover procedure. Each process has a current_node pointer.

In Try, a process walks the path from its leaf to the root, acquiring each node lock along
Similarly to the lines 15–18. If an abort was signalled, is our K <B the super-passage. We show how to transform an abortable RME algorithm with passage complexity \( O(K + F) \) if \( K < B \) or \( O(B^* + F) \) otherwise, and space complexity \( O(S + N + B^2) \).

The transformation is essentially a fast-path/slow-path construction, where the fast path is our \( W \)-port abortable RME algorithm and the slow path is the original lock \( L \). A process

```
// initially all TRY.
STATUS : array of N status words
CURR_NODE : array of N nodes

// initially CURR_NODE[i] is the i-th leaf.

void Try(int pid) {
  if STATUS[pid] == ABORT:
    return FALSE
  node := CURR_NODE[pid]
  while STATUS[pid] \neq CS or node \neq root:
    if node is the j-th child of node.parent,
      then set k to j
    if node.Recover(k) = TRY:
      node.Try(k)
    if received abort signal:
      STATUS[pid] := ABORT
      Exit(pid, TRUE)
    return FALSE
  if node = root:
    break
  node := node.parent
  CURR_NODE[pid] := node
  STATUS[pid] := CS
return TRUE
}

void Exit(int pid, bool aborting) {
  if aborting == FALSE:
    STATUS[pid] := EXIT
  node := CURR_NODE[pid]
  while TRUE:
    if node is the j-th child of node, then set k to be j
    if node.Recover(k) \neq TRY:
      node.Exit(k, FALSE)
    if node = LEAF:
      break
    node := node.child(k)
  if STATUS[pid] = TRY:
    status := TRY
  if STATUS[pid] = EXIT:
    return EXIT
  if STATUS[pid] = CS:
    return CS
return TRY
}
```

Figure 2 The Tournament Tree

the way (lines 16–22). In each such lock, it uses a statically assigned port, corresponding to the number of the child from which it climbed into the node. After successfully acquiring the lock at node \( x \), process \( p \) writes \( x \) to current_node[\( p \)] (line 22). This allows \( p \) to return to \( x \) if it crashes, instead of having to start from scratch and climb the entire path again. The Exit flow is symmetric, with \( p \) releasing each lock along the path back to the leaf, and updating current_node[\( p \)] after each lock release (lines 31–39). In both entry and exit flows, \( p \) always execute node lock’s Recover procedure before entering that lock’s Try or Exit section. This allows \( p \) to behave correctly after crash recovery: on its way up (respectively, down) it will not execute Enter (respectively, Exit) on the same node lock twice (lines 13–14, respectively lines 34–35).

To support aborts, process \( p \) checks the abort signal after acquiring each node lock (lines 15–18). If an abort was signalled, \( p \) starts executing the main lock’s exit code to descend from the current node back to its leaf, releasing the node locks it holds along the way. (Similarly to the \( W \)-port algorithm, an aborting process execute Exit as a subroutine; it does not formally enter the main lock’s exit section). The algorithm correctly supports aborts because if an abort is signalled while \( p \) is in some node lock’s Try execution, it is guaranteed to complete in a finite number of its own steps. Subsequently, it will execute the main lock’s abort handling code in a constant number of its own steps.

5 Adaptive Transformation

We now present our generic adaptivity transformation, which transforms any abortable RME algorithm \( L \) whose RMR complexity depends only on \( N \) into an abortable RME algorithm whose RMR complexity also depends on the point contention \( K \), which is the number of processes executing the algorithm concurrently with the process going through the super-passage. We show how to transform an abortable RME algorithm with passage complexity \( B < W \), super-passage complexity \( B^* \), and space complexity \( S \), into an abortable RME algorithm with passage complexity \( O(\min(K, B)) \), super-passage complexity \( O(K + F) \) if \( K < B \) or \( O(B^* + F) \) otherwise, and space complexity \( O(S + N + B^2) \).

```
p attempts to capture port \(k = 0, \ldots, W - 1\) so it can use it in the fast path lock. Each such capture attempt is performed with CAS, and hence incurs an RMR. The idea is that if \(p\) fails to capture a port, then another process \(q\) succeeds. Therefore, if \(p\) fails to capture any port, the point contention is > \(W\). In this case, \(p\) gives up and enters the slow path. The fast path and slow paths are synchronized with a 2-port abortable RME lock, again implemented with our lock (§3).

We present detailed pseudo code and prove all of the algorithm’s properties. Omitted proofs appear in Appendix D.

### 5.1 Algorithm Walk-Through

Figure 3 presents the transformed algorithm’s pseudo code. The transformed algorithm uses three auxiliary abortable RME locks: a slow path lock, which is an N-process base lock being transformed into an adaptive lock, and fast path as well as 2_rme locks, both of which are instances of our D-port abortable RME (§3). The fast_path instance uses \(D = B\) and the 2_rme instance uses \(D = 2\).

The algorithm maintains \(K\_OWNERS\), an array of \(B\) words (initially all \(\bot\)) through which processes in the entry section try to capture ports to use in the fast-path lock (lines 5–17). Each process maintains a \(CURR\_K\) variable to store the next port the process attempts to capture, or its captured port (once it captures one). To capture a port, process \(p\) scans \(K\_OWNERS\), using CAS at each slot \(k\) in an attempt to capture port \(k\). If \(p\) captures port \(k\), it enters the fast-path lock using that port. Overall, if \(p\) reaches slot \(k\) in \(K\_OWNERS\), then \(k\) other processes have captured ports \(0, \ldots, k - 1\). If \(p\) reaches the end of \(K\_OWNERS\) and fails to capture a port, it enters the slow-path lock (lines 18–24). Regardless of which

```c
void Try(int pid) {
    if STATUS[pid] = ABORT:
        Exit(pid, TRUE)
    k := CURR_K(pid)
    while k < B {
        if K_OWNER[k] = pid
            or CAS(K_OWNER[k], \bot, pid):
                PATH[pid] = FAST
                if fast_path.Recover(k) = TRY
                    if fast_path.Try(k) = FALSE:
                        STATUS[pid] := ABORT
                        Exit(pid, TRUE)
            break loop
        k := k + 1
        CURR_K[pid] := k
        if PATH[pid] \neq FAST:
            PATH[pid] := SLOW
            if slow_path.Recover(pid) = TRY:
                if slow_path.Try(pid) = FALSE:
                    STATUS[pid] := ABORT
                    Exit(pid, TRUE)
            return FALSE
        if PATH[pid] = FAST:
            SIDE[pid] := RIGHT
            else \(PATH[pid] = SLOW\):
                SIDE[pid] := LEFT
            if 2_rme.Recover(SIDE[pid]) = TRY:
                if 2_rme.Try(SIDE[pid]) = FALSE:
                    STATUS[pid] := ABORT
                    Exit(pid, TRUE)
            return FALSE
            STATUS[pid] := CS
}
```

### Figure 3 Adaptive Transformation
lock \( p \) ultimately enters, it invokes that lock’s \textit{Recover} method first, to correctly handle the case in which \( p \) is recovering from a crash.

We use the 2-RME lock to ensure mutual exclusion between the owners of the fast-path and the slow-path. Once \( p \) acquires its lock, it enters the 2-RME lock from the right (respectively, left) if it is on the fast-path (respectively, slow-path). In the 2-RME lock, \( p \) takes on a unique right/left id, corresponding to its direction of entry. Once \( p \) acquires the 2-RME lock, it enters the CS (lines 29–33).

In the exit section, \( p \) releases the 2-RME lock (lines 44–46) and then the fast-path or slow-path lock, as appropriate (lines 49–57). After releasing the fast-path lock, \( p \) releases its port (line 54). These steps are done carefully to avoid having \( p \) return to the fast-path lock after crashing with the same port that is now being used by another process.

To handle aborts, if \( p \) receives a FALSE return value from some \textit{Enter} execution, it executes the transformed lock’s exit code (which, as a byproduct, releases \( p \)’s port if it has one). Subsequently, \( p \) completes the abort.

### 6 Putting It All Together & Conclusion

Let \( T \) be the RME algorithm obtained by instantiating our tournament tree (§ 4) with our \( W \)-port abortable RME algorithm (§ 3). Then \( T \)’s RMR passage complexity is \( O(\log_W N) < W \), super-passage complexity is \( O(\log_W N + F) \) and space complexity is \( O(NW \log_W N) \). We can therefore apply the transformation of § 5 to \( T \), obtaining our main result:

\textbf{Theorem 20.} There exists an abortable RME with \( O(\min(K, \log_W N)) \) RMR passage complexity, \( O(F + \min(K, \log_W N)) \) RMR super-passage complexity, and \( O(NW \log_W N) \) space complexity where \( K \) is the point contention, \( W \) is the memory word size, \( N \) is the number of processes, and \( F \) is the number of crashes in a super-passage.

Many questions about ME properties in the context of RME remain open, and we are far from understanding how the demand for recoverability affects the possibility of obtaining other desirable properties and their cost. Can the sublogarithmic RMR bounds be improved using only primitives supported in hardware, such as FAS and FAA? It is known that a weaker crash model facilitate better bounds \cite{13}, but is relaxing the crash model necessary? What, if any, is the connection between RME and abortable mutual exclusion? Both problems involve a similar concept, of a process “disappearing” from the algorithm, and for both problems, the best known RMR bounds (assuming standard primitives) are \( O(\frac{\log N}{\log \log N}) \). Can a formal connection between these problems be established?

### References

1. Y. Afek, H. Attiya, A. Fouren, G. Stupp, and D. Touitou. Long-Lived Renaming Made Adaptive. In \textit{PODC}, 1999.
2. Aghazadeh, Z. and Golab, W. and Woelfel, P. Making Objects Writable. In \textit{PODC}, 2014.
3. A. Alon and A. Morrison. Deterministic Abortable Mutual Exclusion with Sublogarithmic Adaptive RMR Complexity. In \textit{PODC}, 2018.
4. J.H. Anderson and Y.J. Kim. An Improved Lower Bound for the Time Complexity of Mutual Exclusion. \textit{Distributed Computing}, 15(4), 2002.
5. H. Attiya. Adapting to Point Contention with Long-Lived Safe Agreement. In \textit{SIROCCO}, 2006.
6. H. Attiya and A. Fouren. Algorithms Adapting to Point Contention. \textit{JACM}, 50(4), 2003.
7. H. Attiya, D. Hendler, and P. Woelfel. Tight RMR Lower Bounds for Mutual Exclusion and Other Problems. In \textit{STOC}, 2008.
A Memory Management

Here we show how to bound our algorithm’s space consumption. The basic idea is that when a process $p$ starts a super-passage and “allocates” a fresh spin variable, the allocation will be satisfied from a pre-allocated static array of $2D + 1$ spin variables maintained for $k$, the port used by $p$ in this super-passage. (Thus, the overall memory consumption is $O(D^2)$. ) The challenges are (1) how to identify an entry in port $k$’s array that is safe and not being referenced by any other process (these entries may have previously been used by other processes accessing port $k$, and so there may be active processes that hold references to them); and (2) how to find the entry with $O(1)$ RMRs.

To identify a safe entry, we use standard ideas from safe memory reclamation [24]. The high-level idea is that when process $p$ executing Promote reads $(*, k, v)$ from LOCK_STATUS, it announces that it now has a reference to $v$ by writing a pointer to $v$ in a new REFERENCED array. This announcement enables any process using port $k$ to avoid reusing this spin variable.
v, as long as p might still be referencing it. It could, however, be the case that p reads (b, k, v) and delays before writing v to REFERENCED, causing some process q using port k to decide to reuse v. To handle this problem, p must validate that LOCK_STATUS still contains v after writing the announcement. If the validation succeeds, p is considered to hold a reference to v. In such a case, p knows that both REFERENCED contains v and LOCK_STATUS = (b, k, v) held at the same point in time, so an allocation by some process q with port k that starts after p has a reference to v will not reuse v.

What if p's validation fails? We observe that in such a case, p can simply abandon its Promote operation. If p observes b = 1, then by Claim 2, the validation failing implies that q has exited the lock, so there is no need to signal q. Similarly, if p observes b = 0, the validation failing implies that p's later CAS is meant to fail §3.1.

Figure 4 shows the full algorithm: the code from Figure 1 extended with safe memory reclamation, as described above, as well as the procedures used to “allocate” a spin variable. In the rest of the section, we describe how this “allocation” is implemented in O(1) RMRs. We essentially use the lock-free allocation scheme of Aghazadeh et al. [2, Section 2.3], except that we make it wait-free using the aforementioned observation that a failed validation can be abandoned instead of retried.

Each port k is associated with a FREE queue that holds O(D) free variables that may be “allocated” safely; i.e., it is guaranteed that no other process holds a reference to these variables. Process p “allocates” a spin variable by calling the GetIndex procedure (line 71) which pops one spin variable from the FREE queue of its port (lines 49–52). When p finishes the super-passage (due to aborting or exiting the CS), it retires its current spin variable v (line 105). However, v can be pushed back into the FREE queue only after it is verified that no other process holds a reference to it. The key idea is to perform this verification lazily, by checking one entry of the REFERENCED array in each super-passage that uses port k (as part of the Retire call). To facilitate this, we maintain a reference counter in v. The reference counter of v is initialized to 1 when v is retired (line 27), and is incremented each time a reference to v is observed while scanning REFERENCED (lines 32–34). In addition, each such reference counter update is undone (by decrementing the counter) once D entries of REFERENCED have been scanned since the update was made (lines 35–39). All together, this means that if v's reference counter ever becomes 0, then no reference to it was observed in REFERENCED and v is put back into the port's FREE queue (lines 41–43).

The Retire procedure implements this algorithm by tracking all pending counter updates in two (per-port) queues of size D—a RETIRED queue for initial counter updates and an OBSERVED queue for counter increments—and pushing and popping one item from each queue in each invocation of Retire (lines 36–44). Since at any time, these queues can contain at most 2D spin variables out of the 2D + 1 spin variables associated with the port, we are guaranteed that a port's FREE queue always contains at least one spin variable when a super-passage starts.

It is straightforward to make the above scheme recoverable: all operations during allocation are done to private variables, so we can simply hold these variables, as well as the program counter of the GetIndex/Retire procedures, in the NVRAM, so that after a crash the process can resume its execution from the point it crashed. It is then safe to re-execute the operation (read/write) that was about to be performed, since the operation accesses a process-private variable, and all writes in those procedures are idempotent. The queues being used can also easily be made recoverable by constructing them in such a way that all writes are idempotent and again holding all variables in NVRAM and using the program counter to return to the point of the crash. For brevity, Figure 4 shows Retire and GetIndex without these details.
20 Recoverable, Abortable, and Adaptive ME with Sublogarithmic RMR Complexity

```c
ACTIVE: int // initially 0

GO: array of W status words // initially all TRY

// initially all ⊥.

LOCK_STATUS: struct {bool, port_id, pointer to Spin}

// initially (0, 0, ⊥).

struct Spin {
    int refcount
    bool val
    int port
}

FREE: array of W queues of Spin variables
// queue in index k is initialized with 2W + 1 Spin variables, each with refcount=0, val=FALSE, and port=⊥.

RETIRED: array of W queues of pointers to Spin vars
// initially each queue contains W ⊥ values

OBSERVED: array of W queues of pointers to Spin vars
// initially each queue contains W ⊥ values

COUNTER: array of W integers

REFERENCED: array of W pointers to Spin variables
// initially all ⊥.

Promote(k, ⊥)

while GO[k].value = FALSE:
    if got abort signal:
        STATUS[k] := ABORT
        Exit(k, TRUE)
        return FALSE
    if aborting = FALSE:
        STATUS[k] := EXIT
        GO[k] := GetIndex(k)
        return TRUE

void Exit(int k, bool aborting) {
    if aborting = FALSE:
        STATUS[k] := EXIT
        if k-th bit in ACTIVE is 1:
            FAA(ACTIVE, −2^k)
        else:
            FAA(ACTIVE, 2^k)
        Promote(k, k)
    Promote(k, ⊥)
    (taken, owner, owner_go) := LOCK_STATUS
    if taken = 1 and owner = k:
        CAS(LOCK_STATUS, (1, owner, owner_go),
            (0, owner, owner_go))
    Promote(k, ⊥)
    if (GO[k] ≠ ⊥):
        Retire(k, GO[k])
        GO[k] := ⊥
    STATUS[k] := TRY

void Promote(int k, int j) {
    (taken, owner, owner_go) := LOCK_STATUS
    REFERENCED[k] := owner_go
    if (taken, owner, owner_go) := LOCK_STATUS:
        if taken = 0:
            active := ACTIVE
        if active ≠ 0:
            j := next(owner, active)
            CAS(LOCK_STATUS, (0, owner, owner_go),
                (1, j, GO[j]))
            REFERENCED[k] := ⊥
    (taken, owner, owner_go) := LOCK_STATUS
    REFERENCED[k] := owner_go
    if (taken, owner, owner_go) := LOCK_STATUS:
        if taken = 1:
            owner_go.value := TRUE
        REFERENCED[k] := ⊥

Figure 4 W-port abortable RME algorithm with memory reclamation
```


\section*{B Proofs Omitted From Section 3.3}

\begin{itemize}
  \item Claim 21. If process \( p \) completes a super-passage \( sp \) with port \( k \), then \( p \) completes an execution of the \( \text{Exit} \) procedure and does not subsequently write to any memory location before \( sp \) ends.

  \textbf{Proof.} By definition, \( p \) finishes \( sp \) either by completing an \( \text{Exit} \) section, which means completing an execution of \( \text{Exit} \), or if \( p \)'s \( \text{Try} \) section returns \text{FALSE}. From the code, a \( \text{Try} \) section only returns \text{FALSE} immediately after completing \( \text{Exit} \).

  \item Claim 22. When a process \( p \) starts a super-passage \( sp \) with port \( k \), then \( GO[k] = \bot \), the \( k \)-th bit of \( \text{ACTIVE} \) is 0, and \( STATUS[k] = \text{TRY} \).

  \textbf{Proof.} By induction on the number of \( M \)-super-passages using port \( k \). The base case is the first \( M \)-super-passage that uses port \( k \). In this case, \( GO[k] \), the \( k \)-th bit of \( \text{ACTIVE} \) and \( STATUS[k] \) are initialized to their default values which are \( \bot \), 0, and \text{TRY}, respectively.

  For the inductive case, consider the last \( M \)-super-passage by some process \( p \) that used port \( k \). From Claim \( 21 \), when \( p \) completes that super-passage, it completes an execution of the \( \text{Exit} \) procedure, which resets \( GO[k] \), the \( k \)-th bit of \( \text{ACTIVE} \), and \( STATUS[k] \) to their initial values.

  \item Claim 3. If at time \( t \), no process is in a super-passage with port \( k \), then \( STATUS[k] = \text{TRY} \).

  \textbf{Proof.} Immediate from Claim \( 22 \).

  \item Claim 4. If process \( p \) is in super-passage \( sp \) with port \( k \) and \( p \) is in the \( \text{Try} \) section, then \( STATUS[k] = \text{TRY} \) or \( STATUS[k] = \text{ABORT} \).

  \textbf{Proof.} Recall that \( p \) is in the \( \text{Try} \) section from the first operation of \( \text{Try}(k) \) and until its last operation. If this is the first call to \( M\text{try}(k) \) in \( sp \), then from Claim \( 22 \), \( STATUS[k] = \text{TRY} \). From code inspection, \( STATUS[k] \) can only change to \text{ABORT} or to \text{CS} in the \( \text{Try} \) section. If \( STATUS[k] \) changes to \text{CS}, then by definition, \( p \) is no longer in the \( \text{Try} \) section. If \( STATUS[k] \) changes to \text{ABORT}, it can only later be changed by the \( \text{Exit} \) procedure. Since \( STATUS[k] = \text{ABORT} \), the first condition in \( \text{Exit} \) does not hold, and so \( STATUS[k] \) can only be changed back to \( \text{TRY} \) when the \( \text{Exit} \) procedure completes. Subsequently, \( p \) either completes the \( \text{Try} \) section by returning \text{FALSE} or crashes. If \( p \) crashes after setting \( STATUS[k] \) to \( \text{TRY} \), then upon recovery, \( p \) will change \( STATUS[k] \) back to \text{ABORT} and execute the \( \text{Exit} \) procedure again, since the abort signal remains signalled. This can repeatedly happen until eventually \( p \) does not crash and completes the \( \text{Try} \) section. Thus, \( STATUS[k] \) is always \( \text{TRY} \) or \( \text{ABORT} \) during this time period.

  \item Claim 5. If process \( p \) is in super-passage \( sp \) with port \( k \) and \( p \) is in the \( \text{CS} \), then \( STATUS[k] = \text{CS} \).

  \textbf{Proof.} By definition, \( p \) is in the \( \text{CS} \) from the time that \( STATUS[k] = \text{CS} \), or if \( p \) re-enters the \( \text{CS} \) after crashing inside it. Therefore, once \( STATUS[k] \) becomes \( \text{CS} \), \( p \) does not invoke \( M\text{try}(k) \) in \( sp \) again, and so \( STATUS[k] \) can only be changed when \( p \) exits the \( \text{CS} \) and invokes \( M\text{exit}(k) \). The first operation of \( \text{Exit} \) changes \( STATUS[k] \) to \( \text{EXIT} \), which by definition is when \( p \) exits the \( \text{CS} \).

  \item Claim 6. If process \( p \) is in super-passage \( sp \) with port \( k \) and \( p \) is in the \( \text{Exit} \) section, then \( STATUS[k] = \text{EXIT} \).

\end{itemize}
Proof. By definition, \( p \) is in the Exit section from when it changes \( STATUS[k] \) to \( EXIT \) in \( Exit \). This change only happens if \( p \) executes \( Exit \) when exiting the CS (i.e., not in the abort flow), since the execution is well-formed. Subsequently, \( STATUS[k] \) can only be changed again in \( sp \) by \( p \) to \( TRY \) at the end of \( Exit \). By definition, this is when \( p \) completes the Exit section and \( sp \) ends.

▶ Lemma 7. Algorithm \( M \) is well-behaved.

Proof. Immediate from Claims 3 and from the simple flow of the \( Recover \) procedure.

▷ Claim 23. If process \( p \) is in super-passage \( sp \) with port \( k \), and \( p \) reaches the waiting room, then \( p \) initializes \( GO[k] \) to point to a new spin variable, before the \( k \)-th bit of \( ACTIVE \) is set.

Proof. From Claim 22, when \( p \) starts the super-passage, \( GO[k] \) is \( \perp \), and the \( k \)-th bit of \( ACTIVE \) is 0. Thus, before \( p \) set the \( k \)-th bit to 1 using FAA, there is an execution of \( Try \) in which \( p \) observes \( GO[k] = \perp \) and so initializes \( GO[k] \).

▷ Claim 24. If process \( p \) is super-passage \( sp \) with port \( k \) sets \( GO[k] \) to some value other than \( \perp \) at time \( t \), then \( p \) does not change \( GO[k] \) to any value except \( \perp \) in \( sp \).

Proof. From the code, once \( p \) sets \( GO[k] \) to some value other than \( \perp \) in \( Try \), \( GO[k] \) can only change when \( p \) executes \( Exit \). \( p \) invokes \( Exit \) either due exiting the CS or due to aborting. If \( p \) exits the CS, then since the execution is well-formed, \( p \) does not execute \( Try \) again afterwards in \( sp \). If \( p \) aborts, then since the abort signal remains signalled in \( sp \), \( p \) never executes the step that initializes \( GO[k] \) in \( Try \) again.

▷ Claim 25. If process \( p \) reaches the waiting room in super-passage \( sp \) with port \( k \), then \( spin_{p,k,sp} \) exists, and if \( GO[k] \neq \perp \), then \( GO[k] = \&spin_{p,k,sp} \).

Proof. From Claim 23, \( p \) initializes \( GO[k] \) to \( spin_{p,k,sp} \) before reaching the waiting room in \( sp \). From Claim 24, \( GO[k] \) cannot change to any value except \( \perp \) in \( sp \).

▷ Claim 26. If process \( p \) is in super-passage \( sp \) with port \( k \) and \( p \) is in the CS, then \( spin_{p,k,sp} = TRUE \).

Proof. Since the execution is well-formed, the first time in \( sp \) at which \( p \) reaches the CS is when it sets \( STATUS[k] = CS \) in the \( Try \) section. This write can only happen if \( p \) finished the while loop in the waiting room, which occurs when \( *GO[k] = TRUE \). By Claim 25, this means that \( spin_{p,k,sp} = TRUE \). This is the only write to \( spin_{p,k,sp} \) in \( sp \), so this variable stays \( TRUE \) for the entire CS of \( p \) in \( sp \).

▷ Claim 8. If process \( p \) is in super-passage \( sp \) with port \( k \) and is in the CS, then there was some time \( t \) before \( p \) first entered the CS in \( sp \), at which \( LOCK STATUS = (1,k,\&spin_{p,k,sp}) \).

Proof. If \( p \) is in its CS, then \( spin_{p,k,sp} = TRUE \) by Claim 26. From the code, a spin variable can only be changed to \( TRUE \) in \( Promote \) by some process \( q \) (possibly \( q = p \)) that reads \( LOCK STATUS = (1,*,\&spin_{p,k,sp}) \). Similarly from the code, a process that writes \( LOCK STATUS = (1,*,\&spin_{p,k,sp}) \) must actually write \( LOCK STATUS = (1,k,\&spin_{p,k,sp}) \).
Claim 9. If process $p$ is in super-passage $sp$ with port $k$, and at some time $t$ during $sp$, $\text{LOCK\_STATUS} = (1, k, &\text{spin}_{p,k,sp})$, then $\text{LOCK\_STATUS}$ can only be changed by $p$ while executing the $\text{Exit}$ procedure.

Proof. From the code, $\text{LOCK\_STATUS}$ can only be changed from $(1, k, &\text{spin}_{p,k,sp})$ to some other value in $sp$ by the $\text{Exit}$ procedure invoked with a port argument of $k$. Since the execution is well-formed, such an invocation can only be performed by $p$.

Claim 10. If process $p$ is in super-passage $sp$ with port $k$ and $p$ is in the CS, then $\text{LOCK\_STATUS} = (1, k, &\text{spin}_{p,k,sp})$.

Proof. From Claim 8 there was a time $t$ before $p$ entered the CS at which $\text{LOCK\_STATUS} = (1, k, &\text{spin}_{p,k,sp})$. By Claim 9 this value can only be changed by $p$ executing the $\text{Exit}$ procedure. Since the execution is well-formed, $p$ can only change $\text{LOCK\_STATUS}$ in its $\text{Exit}$ section. Thus, if $p$ is in the CS, $\text{LOCK\_STATUS} = (1, k, &\text{spin}_{p,k,sp})$.

Lemma 11. Algorithm $M$ satisfies mutual exclusion.

Proof. Assume that two processes, $p_i \neq p_j$, are in super-passages, $sp_i$ and $sp_j$, with ports $k_i \neq k_j$. Assume towards a contradiction that both $p_i$ and $p_j$ are in the CS at the same time $t$.

From Claim 10 at time $t$, $\text{LOCK\_STATUS} = (1, k_i, &\text{spin}_{p_i,k_i,sp_i}) = (1, k_j, &\text{spin}_{p_j,k_j,sp_j})$, implying $k_i = k_j$, which is a contradiction.

Claim 12. If process $p$ completes an execution of $\text{Promote}$ without crashing in the interval $[t, t']$ and $\text{ACTIVE} \neq 0$ throughout $[t, t']$, or if $p$ completes an execution of $\text{Promote}(k)$ without crashing in the interval $[t, t']$, then there is a time $t_0 \in [t, t']$ at which $\text{LOCK\_STATUS} = (1, *, *)$.

Proof. There are two possible scenarios for $p$'s execution of $\text{Promote}$: When $p$ first reads $\text{LOCK\_STATUS}$, it either reads $\text{LOCK\_STATUS} = (1, *, *)$ or $\text{LOCK\_STATUS} = (0, *, *)$. In the first case, we are done. Otherwise, if $p$ reads $\text{LOCK\_STATUS} = (0, *, *)$, it tries to execute a CAS to change $\text{LOCK\_STATUS}$ from $(0, *, *)$ to $(1, *, *)$, either because $\text{ACTIVE} \neq 0$ or because $j \neq i$. $p$'s CAS either succeeds or fails due to another process changing $\text{LOCK\_STATUS}$ to $(1, *, *)$. In any case, there is a time $t_0 \in [t, t']$ at which $\text{LOCK\_STATUS} = (1, *, *)$.

Claim 13. If process $p$ is in super-passage $sp$ with port $k$, $p$ sets the $k$-th bit in $\text{ACTIVE}$ to 1 at time $t$, and subsequently reaches the waiting room at time $t' > t$, then there is a time $t_0 \in [t, t']$ at which $\text{LOCK\_STATUS} = (1, *, *)$.

Proof. Since $p$ reaches the waiting room, it completes an execution of $\text{Promote}$ in some interval $T \subseteq [t, t']$ without crashing. This execution occurs after the $k$-th bit of $\text{ACTIVE}$ is set. From the code, only $p$ can clear the $k$-th bit of $\text{ACTIVE}$, and it can do so only after reaching the waiting room. Thus, throughout the interval $T$, $\text{ACTIVE} \neq 0$. From Claim 12 $\text{LOCK\_STATUS} = (1, *, *)$ at some time $t_0 \in T$.

Claim 14. If $\text{LOCK\_STATUS}$ changes to $(1, k, &\text{spin}_{p,k,sp})$ then there is some process $p$ in its super-passage with port $k$ and either (1) $\text{spin}_{p,k,sp}$ eventually becomes $\text{TRUE}$, (2) $p$ aborts, or (3) there are infinitely many crashes in the execution.

Proof. Assume there are finitely many crash steps in the execution. Assume $\text{LOCK\_STATUS}$ is changed to $(1, k, &\text{spin}_{p,k,sp})$ by process $q$ (possibly $q = p$). From the code, this change occurs when $q$ executes the $\text{Promote}$ procedure. Because there are finitely many crashes,
q eventually completes a Promote call. (The reason is that since the execution is well-formed, if q crashes inside Promote, it returns to the same section of the lock and eventually executes Promote again.) When q completes Promote, it either observes \( \text{LOCK\_STATUS} = (1, k, &\text{spin}_p[k, sp]) \) and sets \( \text{spin}_p[k, sp] = \text{TRUE} \), or it reads some other value from \( \text{LOCK\_STATUS} \).

In the latter case, by Claim 21 \( \text{LOCK\_STATUS} \) was changed by p while executing the Exit procedure. The invocation of Exit can happen if p either aborts or exits the CS. If p exits the CS, Claim 20 implies that \( \text{spin}_p[k, sp] \) was set to \text{TRUE}.

\( \triangleright \) Claim 27. Consider process \( p \) in super-passage \( sp \) with port \( k \). If \( \text{LOCK\_STATUS} \) is changed to \((1, k, &\text{spin}_p[k, sp])\) at time \( t \), then at time \( t \), \( p \) has executed the FAA in Try and has not completed the first Promote in Exit in \( sp \).

**Proof.** Let \( q \) be the process that changes \( \text{LOCK\_STATUS} \) to \((1, k, \text{spin}_p[k, sp])\). There are two cases: If \( q = p \): From the code, \( p \) changes \( \text{LOCK\_STATUS} \) in Promote. Thus, from the code, time \( t \) is after \( p \)'s FAA in Try. Moreover, time \( t \) cannot occur in \( p \)'s invocation of Promote(\( \bot \)) in the Exit procedure. In this invocation, \( p \) can update \( \text{LOCK\_STATUS} \) to \((1, j, *)\) only if the \( j \)-th bit of \( \text{ACTIVE} \) is set. From the code, however, the \( k \)-th bit of \( \text{ACTIVE} \) is clear at this point. If \( q \neq p \): Let \( t_1 \) be the time when \( p \) executes the FAA in Try, \( t_2 \) be the time when \( q \) reads \( \text{ACTIVE} \) for the last time before changing \( \text{LOCK\_STATUS} \), then \( t_3 \) be the time when \( q \) changes \( \text{LOCK\_STATUS} \). We first show \( t_1 < t_3 \). Since \( q \) changes \( \text{LOCK\_STATUS} \) to \((1, k, \text{spin}_p[k, sp])\) then \( q \) observes the \( k \)-th bit of \( \text{ACTIVE} \) set. Thus, \( t_1 < t_2 < t_3 \). Next, if \( p \) never completes the first Promote in Exit in \( sp \), we are done. Otherwise, let \( t_4 \) be the time when \( p \) executes the FAA in Exit and \( t_5 \) be the time when \( p \) finishes executing Promote(k) in Exit. We show \( t_3 < t_5 \) by contradiction. We have \( t_2 < t_4 \), since \( q \) observes the \( k \)-th bit of \( \text{ACTIVE} \) set. Thus, \( t_1 < t_2 < t_4 < t_5 < t_3 \). Since \( q \)'s successful CAS of \( \text{LOCK\_STATUS} \) succeeds, and \( q \) reads \( \text{LOCK\_STATUS} \) before reading \( \text{ACTIVE} \), we have that \( \text{LOCK\_STATUS} = (0, *, *) \) throughout \([t_2, t_3]\). But \( p \) executed Promote(k) in \([t_4, t_5] \subset [t_2, t_3]\), and so \( p \)'s Promote(k) observes \( \text{LOCK\_STATUS} = (0, *, *) \) and so \( p \) should successfully CAS \( \text{LOCK\_STATUS} \) to \((1, k, &\text{spin}_p[k, sp])\) by \( t_5 \), which is a contradiction. \( \triangleleft \)

\( \triangleright \) Claim 15. If \( \text{LOCK\_STATUS} \) changes to \((1, *, *)\) at time \( t \), and if any process that enters the CS after \( t \) eventually exits it, then there is a time \( t' > t \) at which \( \text{LOCK\_STATUS} \) changes to \((0, *, *)\) or there are infinitely many crashes in the execution.

**Proof.** Assume there are finitely many crash steps in the execution. If \( \text{LOCK\_STATUS} \) changes to \((1, k, &\text{spin}_p[k, sp])\) at time \( t \), then by Claim 27, process \( p \) is in super-passage \( sp \) with port \( k \) and has not completed the first Promote in Exit in \( sp \). By Claim 14, after \( t \) either \( \text{spin}_p[k, sp] \) eventually becomes \text{TRUE} or \( p \) aborts. In either case, \( p \) eventually completes Exit. As mentioned, \( p \) does not complete Promote(k) in Exit before time \( t \). Thus, \( p \) changes \( \text{LOCK\_STATUS} \) to \((0, k, &\text{spin}_p[k, sp])\) after time \( t \), since that event is executed after Promote(k) returns. \( \triangleleft \)

\( \triangleright \) Claim 28. If at time \( t \), \( \text{ACTIVE} \neq 0 \) and \( \text{LOCK\_STATUS} \) changes to \((0, *, *)\), then \( \text{LOCK\_STATUS} \) eventually changes to \((1, *, *)\), or there are infinitely many crashes in the execution.

**Proof.** Assume there are finitely many crash steps in the execution. Let \( p \) be the process that changes \( \text{LOCK\_STATUS} \) to \((0, *, *)\) at time \( t \). Then \( p \) is executing the Exit procedure and still needs to execute Promote(\( \bot \)). Since the execution is well-formed, \( p \) eventually completes an execution of Promote(\( \bot \)). Let \( t' \) be the time where \( p \) completes its execution.
of \textit{Promote}(\_). If \textit{ACTIVE} \neq 0 throughout \([t, t']\), then by Claim 12 \textit{LOCK\_STATUS} changes to \((1, *, *)\) in \([t, t']\). Otherwise, if \textit{ACTIVE} changes to 0 during \([t, t']\), then there is some time \(t_0 \in [t, t']\) at which some process \(q\) executes \textit{FAA} in \textit{Exit}. Since there are finitely many crashes, \(q\) eventually completes an execution of \textit{Promote}(\(k\)) at some time \(t_0 > t_0 > t\). Thus, by Claim 12 \textit{LOCK\_STATUS} eventually changes to \((1, *, *)\) after \(t\).

\textbf{Claim 29.} If at time \(t\), \textit{LOCK\_STATUS} = \((0, *, *)\) and \textit{ACTIVE} changes from 0 to a non-zero value, then \textit{LOCK\_STATUS} eventually changes to \((1, *, *)\), or there are infinitely many crashes in the execution.

\textbf{Proof.} Assume there are finitely many crash steps in the execution. Since at time \(t\), \textit{ACTIVE} changes from 0 to a non-zero value, then some process \(p\) is executing \textit{Try} in its super-passage \(sp\) with port \(k\), and has not invoked \textit{Promote} (from \textit{Try}) at \(t\). Since there are finitely many crashes, \(p\) eventually completes such an execution of \textit{Promote} at some time \(t'\). If \textit{LOCK\_STATUS} changes to \((1, *, *)\) by \(t'\), we are done. Otherwise, we have that \(p\) completes an execution of \textit{Promote} in \([t, t']\) and \textit{ACTIVE} \neq 0 throughout this interval. By Claim 12 \textit{LOCK\_STATUS} changes to \((1, *, *)\) at some time \(t_0 \in [t, t']\).

\textbf{Claim 16.} If at some point of the execution \textit{LOCK\_STATUS} = \((0, *, *)\), and \textit{ACTIVE} \neq 0, then \textit{LOCK\_STATUS} will change to \((1, *, *)\) or there are infinitely many crashes in the execution.

\textbf{Proof.} Assume there are finitely many crash steps in the execution. Consider the earlier time \(t\) when \textit{ACTIVE} changed to a non-zero value. If \textit{LOCK\_STATUS} = \((0, *, *)\) at \(t\), then by Claim 29 \textit{LOCK\_STATUS} will change to \((1, *, *)\). Otherwise, it must be the case that \textit{LOCK\_STATUS} changes to \((0, *, *)\) at time \(t' > t\), while \textit{ACTIVE} \neq 0. Thus, by Claim 28 \textit{LOCK\_STATUS} will change to \((1, *, *)\).

\textbf{Theorem 19.} If every execution of Algorithm \(M\) is well-formed, then Algorithm \(M\) satisfies mutual exclusion, bounded abort, starvation-freedom, CS re-entry, wait-free CS re-entry, wait-free exit, and super-passage wait-free exit. The passage complexity of Algorithm \(M\) in both the CC and DSM models is \(O(1)\) and the super-passage complexity is \(O(1 + F)\). (Assuming, for the DSM model, that process memory allocations return local memory.) The space complexity of the algorithm is \(O(D^2)\).

\textbf{Proof.} Mutual exclusion follows from Lemma 11 Starvation-freedom follows from Lemma 18. From the code, \(p\) returns from \textit{Try} after a finite number of its own non-crash steps once the abort step is signalled, and so bounded abort is satisfied. Since \(M\) is well-behaved and \textit{Recover} is wait-free, then CS re-entry and wait-free CS re-entry are satisfied. Since \textit{Exit} is wait-free, wait-free exit is satisfied. Super-passage wait-free exit is satisfied since \textit{Exit} and \textit{Recover} are wait-free. A crash-free passage clearly incurs \(O(1)\) RMRs (assuming, in the DSM model, that the spin variable is allocated from local memory). Thus, the algorithm has \(O(1)\) passage complexity and \(O(1 + F)\) super-passage complexity in both CC and DSM models. The algorithm only uses a constant number of arrays of size \(D\) and \(O(D^2)\) statically pre-assigned boolean variables, so its space complexity is \(O(D^2)\).

\textbf{C Proofs Omitted From Section 4}

We refer to our main abortable RME algorithm as Algorithm \textit{Tree}(\(M\)), where \(M\) is some \(D\)-port RME algorithm that satisfies mutual exclusion, deadlock-freedom, bounded abort,
starvation-freedom, CS re-entry, wait-free CS re-entry and super-passage wait-free exit. We define a process \( p \) to be in its Try, Critical or Exit section of Tree\((M)\) similarly to the way defined in §3.3. We proceed to prove that Algorithm Tree\((M)\) satisfies the desired RME properties. All claims and proofs assume that executions are well-formed.

The main idea is to prove that if an execution of the Tree\((M)\) algorithm is well-formed, then the execution of every node lock \( L \) in the tree is also well-formed, and so every node lock maintains all of its properties. We then show that the properties of Tree\((M)\) follow from that.

**Lemma 30.** Algorithm Tree\((M)\) is well-behaved.

**Proof.** Analogous to the proof of Lemma 7, since both algorithms update the STATUS\([p]\) in the same way, and the Recover procedures of both algorithms are identical. ◀

Claim 31. Each execution of a \( D \)-port node lock \( M \) in the Tree\((M)\) algorithm is well-formed.

**Proof.** We need to show that for every node lock \( L \) in the tree, the projection of Tree\((M)\)'s execution on \( L \) is well-formed, i.e., that it satisfies the six properties of Definition 1. We show this using induction over the height of the tree.

The base case considers the leaves. Properties 5 (constant port usage) and 6 (no concurrent super-passage) hold trivially since the ports are statically assigned, and no two processes are assigned the same leaf with the same port. Let \( L \) be some leaf. We proceed by induction over \( E_L \), the subsequence of the execution occurring at \( L \). Assume that \( E_L \) is well-formed until time \( t \). Then:

- **Property 1** (Recover invocation): From the code, \( p \) always completes an execution of L.Recover before either invoking L.Try or L.Exit, or ascending/descending through \( L \)'s node.
- **Property 2** (Try invocation): Let \( t' \) be the first time after \( t \) at which some process \( p \) invokes L.Try. From the code of Tree\((M)\), \( p \) invokes L.Try only after completing L.Recover. If at \( t' \), \( p \) either invokes L.Try for the first time in \( E_L \), or for the first time after completing L.Recover since \( p \)'s last \( L \)-super-passage, then we are done. (This is the beginning of an \( L \)-super-passage by \( p \).) Otherwise, \( p \) has previously invoked L.Try in its current \( L \)-super-passage, which starts before \( t' \). Since the execution is well-formed up to \( t' \) and \( L \) is well-behaved, it follows that \( p \)'s prior crash step was in L.Try.
- **Property 3** (CS invocation): Let \( t' \) be the first time after \( t \) at which some process \( p \) enters \( L \)'s CS. From the code of Tree\((M)\), \( p \) enters \( L \)'s CS in one of two cases. First, if at \( t' \), \( p \) completes L.Try which returns TRUE. (If L.Try returns FALSE, since the execution is well-formed up to \( t' \) and \( L \) is well-behaved, it follows that an abort is signalled, and thus \( p \) does not advance to \( L \)'s CS.) In this case, we are done. Otherwise, \( p \) completes L.Recover which returns CS. Since the execution is well-formed up to \( t' \) and \( L \) is well-behaved, it follows that \( p \)'s prior crash step was in \( L \)'s CS.
- **Property 4** (Exit invocation): Let \( t' \) be the first time after \( t \) at which some process \( p \) invokes L.Exit. From the code of Tree\((M)\), \( p \) invokes L.Exit only after completing L.Recover that returns \( r \neq TRY \). First, observe that it cannot be the case the \( p \) invokes L.Exit after a crash-free execution from an invocation of L.Try that returns FALSE (i.e., after aborting while trying to acquire \( L \)). Since the execution is well-formed up to \( t' \) and \( L \) is well-behaved, L.Exit returns TRY in such a case, and so \( p \) does not invoke L.Exit. If \( p \) is in \( L \)'s CS, we are done. Otherwise, since the execution is well-formed up to \( t' \) and \( L \) is well-behaved, it follows that \( p \)'s prior crash step was in \( L \)'s Exit section.
Next, let $L$ be a non-leaf lock in the tree. Assume the claim is correct for all locks in the sub-tree of $L$.

- Property 5 (constant port usage): Immediate, since processes use statically assigned ports to access each lock in the tree.
- Property 6 (no concurrent super-passages): From the code, process $p$ accesses $L$ with port $k$ only if $p$ is in the CS of the $k$-th child of $L$. Since the execution of $L$’s $k$-th child is well-formed, it satisfies mutual exclusion. Thus, there are no two concurrent $L$-super-passages $sp_i$ and $sp_j$ of processes $p_i \neq p_j$ with the same port.
- Properties 1–4: Follow from similar reasoning as the leaf case.

▶ Lemma 32. Algorithm $\text{Tree}(M)$ satisfies mutual exclusion.

Proof. If some process $p$ is in the CS of $\text{Tree}(M)$, then it must be in the CS of the root lock. Claim 31 implies that the root lock satisfies mutual exclusion, so the claim follows. ▶

▶ Lemma 33. Algorithm $\text{Tree}(M)$ satisfies starvation-freedom.

Proof. Assume there are finitely many crash steps in the execution. Suppose, towards a contradiction, that process $p$ executes infinitely many steps without entering the CS of $\text{Tree}(M)$ and without receiving an abort signal. Since there are finitely many crashes, it follows that $p$ executes infinitely many steps in $L$.Try for some node lock $L$, without receiving an abort signal. This contradicts the fact that $L$ is starvation-free. ▶

▶ Lemma 34. Algorithm $\text{Tree}(M)$ satisfies bounded abort.

Proof. Immediate from the fact that each node lock $L$ satisfies bounded abort and wait-free exit. ▶

▶ Theorem 35. Let $M$ be some $D$-port RME algorithm that satisfies mutual exclusion, starvation-freedom, bounded abort, CS re-entry, wait-free CS re-entry, and super-passage wait-free exit. If every execution of Algorithm $\text{Tree}(M)$ is well-formed, then Algorithm $\text{Tree}(M)$ satisfies mutual exclusion, starvation-freedom, bounded abort, CS re-entry, wait-free CS re-entry, wait-free exit, and super-passage wait-free exit. The passage complexity of Algorithm $\text{Tree}(M)$ in both the CC and DSM models is $O(B \log_D N)$, where $B$ is the node lock passage complexity. The super-passage complexity of Algorithm $\text{Tree}(M)$ is $O(FR + B \log_D N)$, where $R$ is the recovery complexity of the node lock. The space complexity of Algorithm $\text{Tree}(M)$ is $O(S\frac{N}{D} \log_D N)$, where $S$ is the space complexity of the node lock.

Proof. Mutual exclusion follows from Lemma 32. Starvation-freedom follows from Lemma 33. Since $\text{Tree}(M)$ is well-behaved and its $\text{Recover}$ is wait-free, then CS re-entry and wait-free CS re-entry are satisfied. Because each node lock satisfies wait-free exit, both wait-free exit and super-passage wait-free exit are satisfied.

The height of the tree is $O(\log_n N)$ and a process ascends (respectively, descends) the height of the tree to complete $\text{Try}$ (respectively, $\text{Exit}$). Since each node lock $L$ has passage complexity $B$, the passage complexity of $\text{Tree}(M)$ is $O(B \log_D N)$.

In the worst case, following a crash, a recovering process returns to a node lock $L$ where it just completed a passage and executes the recovery code passage, so each crash incurs $O(1 + R)$ RMRs. Thus, the algorithm’s super-passage complexity is $O(FR + B \log_D N)$.

Since there are $O(\frac{N}{D})$ leaves in the tree, and the height of the tree is $O(\log_D N)$, there are $O(\frac{N}{D} \log_D N)$ nodes in the tree. The overall space complexity of the algorithm is therefore $O(S\frac{N}{D} \log_D N)$. ▶
D Proofs Omitted From Section 5

Let $M$ be an $N$-port abortable RME algorithm that satisfies mutual exclusion, deadlock-freedom, bounded abort, starvation-freedom, CS re-entry, wait-free CS re-entry, and super-passage wait-free exit. We refer to our transformation applied to $M$ as Algorithm $Adapt(M)$. We define a process $p$ to be in the Try, Critical or Exit section of $Adapt(M)$ analogously to definition in §3.3. We proceed to prove that Algorithm $Adapt(M)$ satisfies the desired RME properties. All claims and proofs assume that executions are well-formed.

The main idea is to prove that if an execution of the $Adapt(M)$ is well-formed then the execution of the three auxiliary locks used in the algorithm are also well-formed, and so they maintain their RME properties. The correctness of $Adapt(M)$ follows from that.

Lemma 36. Algorithm $Adapt(M)$ is well-behaved.

Proof. Analogous to the proof of Lemma 7, since both algorithms update the $STATUS[p]$ using the same flow, and the $Recover$ procedures of both algorithms are identical.

Claim 37. Any execution of the $slow_path$ lock in $Adapt(M)$ is well-formed.

Proof. We need to show that the projection of $Adapt(M)$’s execution over the $slow_path$ lock is well-formed, i.e., that it satisfies the five properties of Definition 1. Properties 5 (constant port usage) and 6 (no concurrent super-passage) hold trivially, since $slow_path$ is an $N$-port, and process ids are used as port numbers. We prove Properties 1–3 by induction over $E_a$, the subsequence of the execution occurring at the $slow_path$ lock. Assume that $E_a$ is well-formed until time $t$. Then:

- Property 1 (Recover invocation): From the code, $p$ always completes an execution of $slow_path.Recover$ before either invoking $slow_path.Try$ or $slow_path.Exit$. Moreover, if $p$ is in a $slow_path$-super-passage, then $PATH[p] = SLOW$, and so $p$ always invokes $slow_path.Recover$ after recovering from a crash.

- Property 2 (Try invocation): Let $t'$ be the first time after $t$ at which some process $p$ invokes $slow_path.Try$. From the code of $Adapt(M)$, $p$ invokes $slow_path.Try$ only after completing $slow_path.Recover$. If at $t'$, $p$ either invokes $slow_path.Try$ for the first time in $E_a$, or for the first time after completing $slow_path.Recover$ since $p$’s last $slow_path$-super-passage, then we are done. (This is the beginning of a $slow_path$-super-passage by $p$.) Otherwise, $p$ has previously invoked $slow_path.Try$ in its current $slow_path$-super-passage, which starts before $t'$. Since the execution is well-formed up to $t'$ and $slow_path$ is well-behaved, it follows that $p$’s prior crash step was in $slow_path.Try$.

- Property 3 (CS Invocation): Let $t'$ be the first time after $t$ at which some process $p$ enters $slow_path$’s CS. From the code of $Adapt(M)$, $p$ enters $slow_path$’s CS in one of two cases. First, if at $t'$, $p$ completes $slow_path.Try$ which returns $TRUE$. In this case, we are done. Otherwise, $p$ completes $slow_path.Recover$ which returns $CS$. Since the execution is well-formed up to $t'$ and $slow_path$ is well-behaved, it follows that $p$’s prior crash step was in $L$’s CS.

- Property 4 (Exit invocation): Let $t'$ be the first time after $t$ at which some process $p$ invokes $slow_path.Exit(FALSE)$. From the code of $Adapt(M)$, $p$ invokes $slow_path.Exit$ only if $PATH[p] = SLOW$ and after completing $slow_path.Recover$ that returns $r \neq TRY$. First, observe that it cannot be the case the $p$ invokes $slow_path.Exit$ after a crash-free execution from an invocation of $slow_path.Try$ that returns $FALSE$ (i.e., after aborting while trying to acquire $slow_path$), because then it invokes $Adapt(M)$’s $Exit$ procedure...
with an aborting argument of TRUE. If \( p \) is in slow\_path's CS, we are done. Otherwise, since the execution is well-formed up to \( t' \) and slow\_path is well-behaved, it follows that \( p \)'s prior crash step was in slow\_path's Exit section.

\[ \text{Claim 38.} \text{ Any execution of the fast\_path lock in Adapt}(M) is well-behaved.} \]

\textbf{Proof.} The proof of Properties 37-39 is analogous to the proof of the slow\_path lock. Constant port usage and no concurrent super-passages are satisfied since, from the code, if \( p \) invokes fast\_path.Recover\((k)\), fast\_path.Try\((k)\) or fast\_path.Exit\((k)\), it must be the case that \( K\_OWNERS[k] = p \). Further, if \( K\_OWNERS[k] = p \), it can only be changed to \( \bot \), and that can only happen by \( p \) in Exit, which can only happen after \( p \) finished its fast\_path.Exit\((k)\), implying \( p \) finished its fast\_path-super-passage with port \( k \) and will no longer access it.

\[ \text{Claim 39.} \text{ Any execution of the 2\_rme lock in Adapt}(M) is well-behaved.} \]

\textbf{Proof.} The proof of Properties 2\_rme is analogous to the proof of the slow\_path lock.

Property 6 (Constant port usage): Assume a process \( p \) sets \( \text{SIDE}[p] = \text{RIGHT} \), this means \( \text{PATH}[\text{pid}] = \text{FAST} \), which means there is some \( k \) such that \( K\_OWNERS[k] = p \) and \( \text{CURR}_K[p] = k \). These can only change in Adapt\((M)\)'s Exit section, after \( p \) has finished the 2\_rme Exit section. So if \( p \) crashes while accessing 2\_rme then it either executes the Adapt\((M)\) Entry section again, where it will again set \( \text{SIDE}[p] = \text{RIGHT} \), or Adapt\((M)\)'s CS or Exit section, where it will not change \( \text{SIDE}[p] \). Similarly, if \( p \) sets \( \text{SIDE}[p] = \text{LEFT} \) then there is no \( k \) such that \( K\_OWNERS[k] = p \) and \( \text{CURR}_K[p] = B \), causing \( p \) to skip the while loop in Adapt\((M)\)'s Try section if it executes it again in this super-passage. Thus \( p \) will not change \( \text{PATH}[\text{pid}] \), and consequently will also set \( \text{SIDE}[p] \) to \( \text{LEFT} \) again, and so constant port usage is satisfied.

Property 5 (No concurrent super-passages): In order for some process \( p \) to set \( \text{PATH}[\text{pid}] = \text{RIGHT} \) (respectively, LEFT), it must be in the CS of the fast\_path (respectively, slow\_path) lock, and since \( p \) sets it back to \( \bot \) before releasing the fast\_path (respectively, slow\_path), and both of them satisfy mutual exclusion, then no concurrent super-passages is satisfied.

\[ \text{Claim 40.} \text{ Any execution of each of the auxiliary locks in Adapt}(M) is well-behaved.} \]

\textbf{Proof.} Follows from Claims 37-39.

\[ \text{Lemma 41.} \text{ Algorithm Adapt}(M) satisfies mutual exclusion.} \]

\textbf{Proof.} If some process \( p \) is in the CS of Adapt\((M)\), then it must be in the CS of the 2\_rme lock. Claim 40 implies that the 2\_rme lock satisfies mutual exclusion, so the claim follows.

\[ \text{Lemma 42.} \text{ Algorithm Adapt}(M) satisfies starvation-freedom.} \]

\textbf{Proof.} Assume there are finitely many crash steps in the execution. Suppose, towards a contradiction, that process \( p \) executes infinitely many steps without entering the CS of Adapt\((M)\) and without receiving an abort signal. Since there are finitely many crashes, it follows that \( p \) executes infinitely many steps in the Try of some auxiliary lock \( M \), which contradicts the fact that these locks are starvation-free.
30 Recoverable, Abortable, and Adaptive ME with Sublogarithmic RMR Complexity

- **Lemma 43.** Algorithm $\text{Adapt}(M)$ satisfies bounded abort.

**Proof.** Assume there are finitely many crash steps in the execution. Assume $p$ is in its $\text{Try}$ section and receives an abort signal. After a finite number of its own steps, $p$ either returns or reaches $M.\text{Try}()$ for some auxiliary lock $M$. Since the execution of $M$ is well-formed, and $p$ has abort signalled, after a finite number of $p$’s steps, $M.\text{Try}()$ returns, and $p$ executes the Exit code of the $\text{Adapt}(M)$, which is wait-free.

- **Claim 44.** Denote by $\text{start}(p)$ the time at which $p$ starts its latest super-passage. If at time $t$, process $p$ fails to acquire port $i$, then there exist a time $t_i^p \in (\text{start}(p), t]$ such that at $t_i^p$ there is a set $X_i^p$ of $i$ processes, such that for each process $q \in X_i^p$ the following holds: $t_i^p > \text{start}(q)$ and $q$ successfully acquired a port number $\leq i$ by time $t$.

**Proof.** By induction. For $i = 0$, pick $t_0^p = t$ and $X_0^p = \{q\}$, where $q$ is the process that causes $p$’s CAS on port $0$ to fail. For the inductive step, assume the claim is true up to some $i$. Consider the process $q$ that causes $p$’s CAS on port $i + 1$ to fail. There are two possible cases:

1. $\text{start}(q) < t_i^p$. If $q \in X_i^p$ then $q$ has previously acquired a port $\leq i$. However, this contradicts the fact that $q$ has now acquired port $i + 1$ in the same super-passage. Thus, $q \notin X_i^p$. Then we define $t_{i+1}^p = t_i^p$ and $X_{i+1}^p = X_i^p \cup \{q\}$.
2. $\text{start}(q) \geq t_i^p$. The fact that $q$ acquired port $i + 1$ means that $q$ failed to acquire port $i$ before time $t$. Consider the time $t_i^p > \text{start}(q)$ and set $X_i^p$ from the induction hypothesis applied to $q$. We have $t_i^p \leq \text{start}(q) < t_i^p \leq t$. Also, $p \notin X_i^p$, since $p$ does not acquire any port by time $t$. Thus, we can define $t_{i+1}^p = t_i^p$ and $X_{i+1}^p = X_i^p \cup \{q\}$.

- **Claim 45.** If some process $p$ fails to acquire some port $i$, then the point contention this process experiences is at least $i$.

**Proof.** Immediate from Claim 44.

- **Theorem 46.** Let $M$ be some $N$-port RME algorithm that satisfies mutual exclusion, starvation-freedom, bounded abort, CS re-entry, wait-free CS re-entry, and super-passage wait-free exit, with passage complexity $B < W$, super-passage complexity $B^*$, and space complexity $S$. If every execution of Algorithm $\text{Adapt}(M)$ is well-formed, then Algorithm $\text{Adapt}(M)$ satisfies mutual exclusion, starvation-freedom, bounded abort, CS re-entry, wait-free CS re-entry, wait-free exit, and super-passage wait-free exit. The passage complexity of Algorithm $\text{Adapt}(M)$ in both the CC and DSM models is $O(\min(K, B))$, the super-passage complexity is $O(K + F)$ if $K < B$ and $O(B^* + F)$ otherwise. The space complexity of Algorithm $\text{Adapt}(M)$ is $O(S + N + B^2)$.

**Proof.** Mutual exclusion follows from Lemma 41. Starvation-freedom follows from Lemma 42. Bounded abort follows from Lemma 43. Since $\text{Adapt}(M)$ is well-behaved and its $\text{Recover}$ is wait-free, then CS re-entry and wait-free CS re-entry are satisfied. Because each auxiliary lock satisfies wait-free exit, both wait-free exit and super-passage wait-free exit are satisfied.

Every time $p$ starts a new super-passage, $\text{CURR}_K[p] = 0$, since it is reset in the previous super-passage’s Exit code (or is in the initial state). Thus, $p$ tries to acquire a port number in the range of 0 and $B$, and from Claim 45 it can fail at most $K$ times. If $p$ succeeds, then the RMR cost of acquiring and releasing the $\text{fast\_path}$ lock and $\text{2\_rme}$ lock is $O(1)$. If $p$ fails, it performs additional $O(B)$ RMRs to acquire and release the $\text{slow\_path}$ and $\text{2\_rme}$ locks. Overall, the total RMR passage complexity is $O(\min(K, B))$. 
If \( p \) crashes while trying to acquire the \textit{fast\_path} lock, then when recovering, it does not need to try to acquire a lock starting from 0 again, but it starts from the last port it tried to acquire before crashing. So if \( K < B \), \( p \) eventually succeeds in acquiring a port in \( O(k + F) \) RMRs. If \( K > B \), then it already performed \( O(B + F) \) RMRs, and will need to perform additional \( O(B^*) \) RMRs to acquire and release the \textit{slow\_path} and \textit{2\_rme} locks.

The space complexity of the \textit{Adapt}(M) is \( O(S + N + B^2) \) since the space complexity of the \textit{slow\_path} is \( O(S) \), the space complexity of the \textit{fast\_path} is \( O(B^2) \), the space complexity of the \textit{2\_rme} lock is \( O(1) \), and the transformation itself uses additional \( O(N) \) space.