Sensitivity measurement for asymmetric two two-level atoms interacting with field obeys SU(1,1) Lie group via atomic inversion

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1. Introduction

The quantum optics model of the interaction between the atoms and field explicates new features of the quantum nature of the atoms as well as radiation field. This quantum nature affects on the properties of the emission spectrum from it. The properties of these emission spectra can be studied via quantum models which represent the fundamental issues. So, the essential issues in quantum mechanics have studied with the aid of many quantum models describing quantum effects in the atom–field interaction. One of the important models of the quantum optics models is the interaction between two two-level atoms and field especially that obeys SU(1,1) Lie group. This field is called SU(1,1) quantum system which generalizes to the generalized Jaynes–Cummings model (JCM) \cite{1–4}. The JCM considers a sample standard model of the interaction between the two-level atom and the single-mode quantized field considering the rotating wave approximation (RWA). Many generalized versions have been presented to modify the JCM in the literature. Most interesting physical and statistical properties have been studied with this model such as entanglement, atomic inversion, collapse and revival, quadrature squeezing, entropy squeezing and sub-Poissonian statistics. On the other hand, some of these quantum models have been realized experimentally \cite{5–7}. So, a study of the atom–field interaction considers one of the most prominent topics in quantum computation and communication, quantum information \cite{8–14}, and quantum cryptography \cite{15,16}. As is well known in recent years, the atomic inversion plays a main role in the modern quantum fields. For that reason, it is important to calculate the atomic inversion as on of the statistical and dynamics properties of the atom–field interaction. As known, there are three essential interesting models in the quantum optics field. These models are the atom–atom interaction \cite{17–24}, the field–field interaction \cite{25–33} and atom–field interaction \cite{34–46}. In this paper, our aim is to study the effects of two of these types of the interaction between atoms and field which are atom–atom and atom–field interactions. We consider that the field follows the SU(1,1) Lie group. The Hamiltonian operator considering the effects of the atom–atom interaction and the RWA is given as

\[
\hat{H} = \omega_0 \hat{K}_z + \sum_{j=1}^{2} \left[ \omega_j \hat{\sigma}_j^\dagger \hat{\sigma}_j + g_j \left( \hat{K}_- \hat{\sigma}_j^\dagger + \hat{K}_+ \hat{\sigma}_j \right) \right] + i \lambda \left( \hat{\sigma}_j^{\dagger} \hat{\sigma}_- - \hat{\sigma}_j \hat{\sigma}_j^{\dagger} \right),
\]

Here $\hbar = 1$, $\omega_j, j = 1, 2$ are the atomic transition frequencies, $j = 1$ for first atom and $j = 2$ for second atom. $\omega_0$ is the field frequency which obeys SU(1,1) Lie group. $g_j, j = 1, 2$ are the coupling constants between the atoms and the SU(1,1) quantum system. $\lambda$ is the coupling constant between the two atoms. $\hat{\sigma}_j^{\dagger}, j = 1, 2$ are the raising, lowering and inversion operators for the two-level atomic system which are satisfying the
commutation relations
\[ \left[ \hat{a}^{(j)}_\pm, \hat{a}^{(j)} \right] = \pm 2 \hat{a}^{(j)}_\pm, \quad \left[ \hat{a}^{(j)}, \hat{a}^{(j)}_\pm \right] = \hat{a}^{(j)}_\pm \delta_{ij}. \tag{2} \]

Notice that the SU(1,1) quantum system operators \( \hat{K}_\pm \) and \( \hat{K} \) follow the Lie algebra and satisfy the commutation relations \([47–50]\)
\[ \left[ \hat{K}_\pm, \hat{K}_\mp = \pm \hat{K}_\pm, \quad \left[ \hat{K}_\pm, \hat{K}_\mp = 2 \hat{K}, \tag{3} \right. \]
while the corresponding Casimir operator \( \hat{K}^2 \) defines as
\[ \hat{K}^2 = \hat{K}^2 - \frac{1}{2} \left( \hat{K}_+ \hat{K}_- + \hat{K}_- \hat{K}_+ \right). \tag{4} \]
The effects of the SU(1,1) operators on the eigenstate \(| m, k \rangle \) are as:
\[ \hat{K}_z | m, k \rangle = \left( m + k \right) | m, k \rangle, \]
\[ \hat{K}_+ | m, k \rangle = \sqrt{(m+1)(m+2k)} | m+1, k \rangle, \]
\[ \hat{K}_- | m, k \rangle = \sqrt{(m+1)(m+2k)} | m+1, k \rangle, \]
where \( m \) is any non-negative integer number, and \( k \) is the Bargmann index. Whereas \( \hat{K}_- | 0, k \rangle = 0 \).

Moreover for examples, the Casimir operators can be redefined by the boson annihilation \( \hat{a} \) and creation \( \hat{a}^\dagger \) operators, which satisfy \([\hat{a}, \hat{a}^\dagger] = 1\), such as
\[ \hat{K}_z = \frac{1}{2} \left( \hat{a}^\dagger \hat{a} + 1 \right), \quad \hat{K}_+ = \frac{1}{2} \hat{a} \hat{a}^\dagger, \quad \hat{K}_- = \frac{1}{2} \hat{a}^\dagger \hat{a}, \tag{6} \]
where the Bargmann index \( k \) takes either \( k = \frac{1}{2} \), for the even parity states, or \( k = \frac{3}{2} \), for odd parity states.

Also, the SU(1,1) Lie group represents the case of the intensity dependent forms as
\[ \hat{K}_z = \frac{1}{2} \left( \hat{a}^\dagger \hat{a} + 1 \right), \quad \hat{K}_+ = \sqrt{n} \hat{a}^\dagger, \quad \hat{K}_- = \hat{a} \sqrt{n}, \tag{7} \]
where \( \hat{n} = \hat{a}^\dagger \hat{a} \) is the number photon operator.

Notice that if there is a single atom and there is not atom–atom interaction \( (\lambda = 0) \), the Hamiltonian (1) was presented in \([48, 51]\). While, if there are symmetric two two-level atoms \( (g_1 = g_2) \) and there isn’t atom–atom interaction \( (\lambda = 0) \), the Hamiltonian (1) was presented in \([52]\). In this work, we present asymmetric two two-level atoms \( (g_1 \neq g_2) \) and the system includes the atom–atom interaction \( (\lambda \neq 0) \). So, the proposed quanti-

Our aim in this paper is to discuss sensitivity measurement for the asymmetric two two-level atoms interacting with the SU(1,1) quantum system to the detuning parameters and the initial atomic angles. This means that the non-classical properties of the proposed quantum model are studied via the atomic population inversion. So, the time-dependent wave function is obtained analytically using Heisenberg equations of motions in Section 2. While, the atomic population inversion is investigated and discussed in Section 3. Finally, the conclusions are epitomized in Section 4.

2. Analytical solution
To determine the atomic population inversion of the model (1), we should be calculated the time-dependent wave function via the Heisenberg equations of motion as:
\[ i \frac{d\hat{K}_z}{dt} = \left[ \hat{K}_z, \hat{H} \right] = \sum_{j=1}^{2} g_j \left( \hat{K}_+ \hat{a}_-^{(j)} - \hat{K}_- \hat{a}_+^{(j)} \right), \]
\[ i \frac{d\hat{a}_+^{(1)}}{dt} = \left[ \hat{a}_+^{(1)}, \hat{H} \right] = -2g_1 \left( \hat{K}_+ \hat{a}_-^{(1)} - \hat{K}_- \hat{a}_+^{(1)} \right) + 2\lambda \left( \hat{a}_+^{(1)} \hat{a}_-^{(2)} + \hat{a}_-^{(1)} \hat{a}_+^{(2)} \right), \]
\[ i \frac{d\hat{a}_-^{(2)}}{dt} = \left[ \hat{a}_-^{(2)}, \hat{H} \right] = -2g_2 \left( \hat{K}_+ \hat{a}_-^{(2)} - \hat{K}_- \hat{a}_+^{(2)} \right) - 2\lambda \left( \hat{a}_+^{(1)} \hat{a}_-^{(2)} + \hat{a}_-^{(1)} \hat{a}_+^{(2)} \right), \tag{8} \]
and consequently, we obtain
\[ i \frac{d}{dt} \left( \hat{K}_z + \frac{1}{2} \sum_{j=1}^{2} \hat{a}_+^{(j)} \right) = 0. \tag{9} \]
Let us define an operator
\[ \hat{H}_0 = \hat{K}_z + \frac{1}{2} \sum_{j=1}^{2} \hat{a}_+^{(j)}, \tag{10} \]
which is a constant operator of motion. Then the Hamiltonian operator should be rewritten as
\[ \hat{H} = \omega_0 \hat{H}_0 + \hat{H}_{\text{Int}}, \tag{11} \]
where the operator \( \hat{H}_{\text{Int}} \) is given by
\[ \hat{H}_{\text{Int}} = \sum_{j=1}^{2} \left( \frac{\delta_j}{2} \hat{a}_-^{(j)} + g_j \left( \hat{K}_+ \hat{a}_-^{(j)} + \hat{K}_- \hat{a}_+^{(j)} \right) \right) \]
\[ + i\lambda \left( \hat{a}_+^{(1)} \hat{a}_-^{(2)} - \hat{a}_-^{(1)} \hat{a}_+^{(2)} \right), \tag{12} \]
and the detuning parameters \( \delta_j \) are defined as \( \delta_j = 2\omega_j - \omega_0, j = 1, 2 \).

It is easy to prove that the Hamiltonian operators \( \hat{H}_0, \hat{H}_{\text{Int}} \) and \( \hat{H} \) are constants of motion. So, we consider that the initial wave function of the proposed system can be written as
\[ | \Psi(0) \rangle = (c_1 | +, + \rangle + c_2 | +, - \rangle + c_3 | -, + \rangle \]
\[ + c_4 | -, - \rangle) \otimes | \alpha, k \rangle, \tag{13} \]
where, \( | + \rangle \) and \( | - \rangle \) are the upper and lower states of the two-level atom, \( | +, - \rangle = | + \rangle | - \rangle \) represents the
product of the upper state, $|+\rangle$, for first atom by the lower state the second atom. $c_i, i=1, 2, 3, 4$ are constants which are arbitrary complex numbers satisfying the superposition condition $\sum_{m=1}^{4} |c_i|^2 = 1$. So, let us assume that

$$
c_1 = \cos \theta \cos \phi, \quad c_2 = \cos \theta \sin \phi, \quad c_3 = \sin \theta \cos \phi, \quad c_4 = \sin \theta \sin \phi$$

where, $\theta$ and $\phi$ are the atomic angles at $t=0$ which provide the possibilities that the two atoms are either in their ground state, excited state, or in the superposition state. In addition, the state vector $|\alpha, k\rangle$ is considered to be the initial state associated with Perelomov coherent state, which is defined as

$$
|\alpha, k\rangle = \sum_{m=0}^{\infty} Q_m|m,k\rangle,
$$

where $\alpha$ is the Perelomov parameter, and $|m,k\rangle$ is the eigenvector of the operator $K_i$ with eigenvalue $(m+k)$. Let us assume that the two atoms are fulfilled in exact resonance case, i.e., $\delta_1 = \delta_2 = 0$. The aim of this assumption is to overcome the difficulty of the proposed system. The time-dependent wave function at $t>0$, which is the most complete description for the proposed physical system, should be written as

$$
|\Psi(t)\rangle = \sum_{m=0}^{\infty} (A(m,t)|+\rangle + B(m,t)|-\rangle)|m,k\rangle
$$

For solving this system analytically, we can rewrite this system in the matrix formula as

$$
d\begin{pmatrix} A(m,t) \\ B(m,t) \\ C(m,t) \\ D(m,t) \end{pmatrix} = -i\begin{pmatrix} 0 & v_2 & v_1 & 0 \\ v_2 & \delta & i\lambda & \mu_1 \\ v_1 & -i\lambda & -\delta & \mu_2 \\ 0 & \mu_1 & \mu_2 & 0 \end{pmatrix} \begin{pmatrix} A(m,t) \\ B(m,t) \\ C(m,t) \\ D(m,t) \end{pmatrix}.
$$

Let us assume that

$$
\Omega(m) = \begin{pmatrix} 0 & v_2 & v_1 & 0 \\ v_2 & \delta & i\lambda & \mu_1 \\ v_1 & -i\lambda & -\delta & \mu_2 \\ 0 & \mu_1 & \mu_2 & 0 \end{pmatrix}
$$

is the coefficient matrix.

Notice that, we mainly deal with the ratio $g = g_2/g_1$ during the investigation, when $g = 1 (g \neq 1)$ it is called the symmetric (asymmetric) case. The explicit analytical expressions for the coefficients $A(m,t), B(m,t), C(m,t)$, and $D(m,t)$ for symmetric case $g=1$ was obtained in [52,53]. While in asymmetric case $(g \neq 1)$, the analytical solution of Equation (18) is determined exactly as follows [54]

$$
\begin{pmatrix} A(m,t) \\ B(m,t) \\ C(m,t) \\ D(m,t) \end{pmatrix} = \sum_{j=1}^{4} \exp(-i\Omega_j t)|\Omega_j\rangle \langle \Omega_j| \begin{pmatrix} c_1Q_m \\ c_2Q_{m+1} \\ c_3Q_{m+1} \\ c_4Q_{m+2} \end{pmatrix},
$$

where, $\Omega_j$ are the eigenvalues and $|\Omega_j\rangle$ are the eigenvectors corresponding to the eigenvalues $\Omega_j$ of the coefficient matrix $\Omega(m)$. Now, the eigenvalues and its eigenvectors are obtained analytically as follows

$$
\begin{align*}
\Omega_1 &= \frac{1}{2} \left( \Lambda - \sqrt{2\beta_1^2 + 2\beta_3^2 - \Lambda^2} \right), \\
\Omega_2 &= \frac{1}{2} \left( \Lambda + \sqrt{2\beta_1^2 + 2\beta_3^2 - \Lambda^2} \right), \\
\Omega_3 &= -\frac{1}{2} \left( \Lambda - \sqrt{2\beta_1^2 - 2\beta_3^2 - \Lambda^2} \right), \\
\Omega_4 &= -\frac{1}{2} \left( \Lambda + \sqrt{2\beta_1^2 - 2\beta_3^2 - \Lambda^2} \right),
\end{align*}
$$

and

$$
\begin{pmatrix} \Omega_j(\mu_2^2 + \nu_1^2 - \Omega_j(\Omega_j + \delta)) \\ v_2(\mu_2^2 - \Omega_j(\Omega_j + \delta)) - v_1(\mu_1\mu_2 - i\lambda\Omega_j) \\ v_2(\mu_2^2 - \Omega_j(\Omega_j + \delta)) - v_1(\mu_1\mu_2 - i\lambda\Omega_j) \\ v_2(\mu_2^2 - \Omega_j(\Omega_j + \delta)) - v_1(\mu_1\mu_2 - i\lambda\Omega_j) \end{pmatrix} = \chi_j,
$$

where

$$
\begin{align*}
\beta_1 &= \sqrt{\delta^2 + \lambda^2 + \mu_1^2 + \mu_2^2 + \nu_1^2 + \nu_2^2}, \\
\mu_j &= g_j \sqrt{(m+1)(m+2k)}/\sqrt{(m+2)(m+2k+1)}, \text{ and } j=1,2.
\end{align*}
$$
\[
\beta_2 = \sqrt{\mu_1^2 v_1^2 - 2 \mu_1 \mu_2 v_1 v_2 + \mu_2^2 v_2^2}, \\
\beta_3 = \delta (\mu_1^2 - \mu_2^2 - v_1^2 + v_2^2), \\
\Lambda = \frac{2}{3} \beta_1^2 + \frac{12 \beta_2^2 + \beta_1^4}{3 \xi} + \frac{1}{3} \xi, \\
\xi = 3 \left( \sqrt{\frac{36 \beta_1^2 \beta_2^2 - \beta_1^4 + \frac{22}{3} \beta_3^2}{\beta_1^2}} + \sqrt{\frac{36 \beta_1^2 \beta_2^2 - \beta_1^4 + \frac{22}{3} \beta_3^2}{\beta_1^2}} \right)^{2} - (12 \beta_2^2 + \beta_1^4)^{3} \\
\gamma = 2 \mu_1 \mu_2 v_1 v_2 (\Omega_1 (\delta + \Omega_1) - \mu_2^2) \\
+ v_1^2 (\mu_1^2 - \Omega_2 (\delta + \Omega_1))^2 + v_1^2 (\lambda_1^2 \Omega_2^2 + \mu_1^2 \mu_2^2) \\
\varepsilon_1 = \Omega_1^2 \left( \Omega_2^2 (\delta^2 + 2 \delta \Omega_2 + \lambda_2^2 + \Omega_2^2) + v_1^2 (\lambda_2^2) \\
+ v_2^2 - 2 \Omega_2 (\delta + \Omega_1) \right) + v_2^2 (\delta + \Omega_2)^2 + v_4^2 \\
\varepsilon_2 = \mu_2^2 \left( v_1^2 \left( v_2^2 + 2 \Omega_2^2 - \Omega_1 (2 \Omega_2 (\delta + \Omega_1) - \lambda_2) \\
+ 2 \Omega_2 (\delta + \Omega_2) \right) \right) + \mu_2^4 (v_2^2 + \Omega_2^2) \\
\varepsilon_3 = \mu_2^2 \left( (v_1^2 - \Omega_1 (\delta + \Omega_1))^2 + \mu_2^2 (v_1^2 + \Omega_2^2) \\
- 2 \mu_1 \mu_2 v_1 v_2 (\mu_2^2 + v_1^2 - \Omega_1 (2 \delta + 3 \Omega_2)) \right) \\
\chi_j = \sqrt{\frac{\gamma}{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}}, \quad j = 1, 2, 3, 4.
\]

Hereafter, to calculate the atomic population inversion as one of the statistical properties of the proposed quantum model, it is easily to evolve the density matrix \( \hat{\rho}(t) = |\Psi(t)\rangle \langle \Psi(t)| \). Therefore, we can use this density matrix to measure the sensitivity of the proposed quantum model to the detuning parameters and the initial atomic angles via the atomic population inversion in the next section.

### 3. Atomic population inversion

The atomic population inversion is one of important quantities related with atoms, which represents the difference between the probabilities of finding the atom in the upper atom state and in the lower atom state. This means that the atomic inversion gives us information on the predicted behaviour of the atom during the procession of the atom–field and atom–atom interactions. Our aim is to study the effects of the detuning parameters on the quantum model and the sensitivity measurement of the proposed quantum system to initial atomic angles. Note that we focus on studying the collapse and revival phenomenon which are usually appeared during the interaction between atom and quantum system or between atom and atom. To reach our aim, it requires to calculate the density matrix \( \hat{\rho}(t) \),

![Figure 1](image-url)
where \( j \) indicates to the number of the interested atom. This density matrix can be obtained by taking the trace over quantum system and one of atoms as follows

\[
\hat{\rho}^{(j)}(t) = \text{Tr}_{\text{qs}}(\hat{\rho}(t)|\Psi(t)\rangle \langle \Psi(t)|), \quad i,j = 1,2,
\]

\[
= \rho_{11}^{(j)}(t)|++\rangle\langle ++| + \rho_{12}^{(j)}(t) |+-\rangle\langle -| + \rho_{21}^{(j)}(t) |--\rangle\langle --| + \sum_{l=22}^{(j)}(t) |l--\rangle\langle l--|
\]

where,

\[
\rho_{11}^{(1)}(t) = \sum_{m=0}^{\infty} (|A(m,t)|^2 + |B(m,t)|^2), \quad (23)
\]

\[
\rho_{22}^{(1)}(t) = \sum_{m=0}^{\infty} (|C(m,t)|^2 + |D(m,t)|^2), \quad (24)
\]

\[
\rho_{12}^{(1)}(t) = \sum_{m=0}^{\infty} (A(m+1,t)C^*(m,t) + B(m+1,t)D^*(m,t))
\]

\[
D^*(m,t) = \rho_{21}^{(j)}(t),\quad (25)
\]

\[
\rho_{11}^{(2)}(t) = \sum_{m=0}^{\infty} (|A(m,t)|^2 + |C(m,t)|^2), \quad (26)
\]

\[
\rho_{22}^{(2)}(t) = \sum_{m=0}^{\infty} (|B(m,t)|^2 + |D(m,t)|^2), \quad (27)
\]

\[
\rho_{12}^{(2)}(t) = \sum_{m=0}^{\infty} (A(m+1,t)D^*(m,t) + B(m+1,t)C^*(m,t))
\]

\[
D^*(m,t) = \rho_{21}^{(2)}(t).
\]

The atomic inversion, \( W^{(j)}(t) \), for the \( j \)th atom is defined as

\[
W^{(j)}(t) = \rho_{11}^{(j)}(t) - \rho_{22}^{(j)}(t). \quad (29)
\]

To research of the conducts of the atoms during the atom–atom and atom–quantum system interactions, we plot the atomic inversion, \( W^{(1)}(t) \) for the first atom and \( W^{(2)}(t) + 2 \) for the second atom, against the scaled time \( T = g_1t \). Notice in Figure 1 that, the system is sensitive to the parameters of the initial state such that the atomic angles. So, to measure the sensitivity of the atomic inversion to the initial atomic angles, we fixed the values of \( \alpha = 0.5, g = 0.5, \delta = 2, \lambda = 2, k = 20 \) and focused on the variation of the initial atomic angles. In Figure 1(A), we put \( \theta = \phi = 0 \) which means that the states of both atoms are excited. As shown in this figure, the atomic population inversion discloses several periods of revivals and collapses. The amplitude of revivals decreases with time. In Figure 1(B), we consider that the initial state of the first atom is the superposition while the the state of the second atom is the excited

![Figure 2](image)

Figure 2. The atomic inversion function \( W(T) \) against the scaled time \((T = g_1t) \) at \( \alpha = 0.5, g = 0.5, \theta = \phi = 0, \text{and } \delta = \lambda = 2. \)
state i.e $\theta = \pi/4$ and $\phi = 0$. Note in this case, the amplitude of revivals for the first atom is very small compared with the amplitude of revivals for the second atom. Explanation that the effect of the quantum field, which obeys SU(1,1) Lie group, on the first atom is weak. In Figure 1(C), the atomic angles take values $\theta = 0$ and $\phi = \pi/3$. This means that the initial state of the first atom is the excited state while the state of the second atom is the superposition state. The amplitude of revivals for the first atom is small compared with the second atom. Also, amplitude of revivals decreases with time. In Figure 1(D), we consider that the initial state of both two atom are the superposition state ($\theta = \pi/4$ and $\phi = \pi/3$). We can see that the amplitude of revivals for the second atom is very small. Also the amplitude of revivals for the first atom is small compared with other cases. We note that in Figure 1: the number of revivals for the first atom is double the number of revivals for the second atom at the same time interval which is as a result to take $g = 0.5$. In addition, the amplitude of revivals decreases with increasing the atomic angles during the interval $[0, \pi/4]$. Whilst, the amplitude of revivals increases with increasing the atomic angles through out $[\pi/4, \pi/2]$.

To measure the sensitivity of the atoms in the proposed quantum model to the Bargmann index $k$ and Perelomov coherent parameter $\alpha$, we choose other parameters as $\alpha = 0.5, g = 0.5, \delta = 2, \lambda = 2, \theta = \phi = 0$. Firstly, we study the sensitivity of the atoms to the Bargmann index $k$, where the Perelomov coherent parameter $\alpha = 0.5$ is fixed in Figure 2. Observe that the fluctuation of the revivals increases with increasing the Bargmann index $k = 10, 20, 30, 40$. Also, the number of revivals increases slowly. Secondly, the Bargmann index $k = 20$ is fixed in Figure 3. While the Perelomov coherent parameter takes the values $\alpha = 0.2, 0.4, 0.6, and 0.8$. For small value of $\alpha = 0.2$ in Figure 3(A), the collapses are very small for first atom and disappear for second atom. In other figure, with increasing the Perelomov coherent parameter, the collapse regions increase and the revival regions compress. In addition, the amplitude of revivals for $\alpha = 0.8$ is smaller than the amplitude of revivals $\alpha = 0.4$ and $\alpha = 0.6$ as shown in Figure 3(B–D).

Finally, let us study the effects of the detuning parameters $\delta$ and $\lambda$ on the quantum model. We plot the atomic inversion with increasing the parameters $\delta$ and $\lambda$ as shown in Figures 4 and 5, respectively. Note in Figure 4(A–D) that the collapse regions increase and the revival regions compress with increasing the parameter $\delta$. Also the maximum values of revival amplitudes perturbed. While in Figure 5(A–D), we note that the collapse regions increase with increasing the detuning

Figure 3. The atomic inversion function $W(T)$ against the scaled time ($T = g_{1}t$) at $g = 0.5, k = 20, \theta = \phi = 0, and \delta = \lambda = 2$. 
Figure 4. The atomic inversion function $W(T)$ against the scaled time $(T = g_1 t)$ at $g = 0.5$, $\alpha = 0.5$, $k = 20$, $\theta = \phi = 0$, and $\lambda = 0$.

Figure 5. The atomic inversion function $W(T)$ against the scaled time $(T = g_1 t)$ at $g = 0.5$, $\alpha = 0.5$, $k = 20$, $\theta = \phi = 0$, and $\delta = 0$. 
parameter $\lambda$. Notice also, one or more a small revival becomes pronounced into the collapse regions. Furthermore, the density of fluctuations in the revival regions is irregular.

4. Conclusion

The time-dependent wave function for asymmetric two two-level atoms interacting with SU(1,1) quantum system was obtained analytically. Initially, the SU(1,1) quantum system prepared in the Perelomov coherent state and two atoms are in different states dependent on two atomic angles. The atomic population inversion was obtained and discussed with increasing the model parameters such as initial atomic angles, Perelomov coherent parameter, the Bargmann index and the detuning parameters. These effects can be summarized as: For asymmetric case, the number of revivals for the second atom is $g$ times of the number of revivals for the first atom at the same time interval. The amplitude of revivals decreases with increasing the atomic angles during the interval $[0, \pi/4]$. While, the amplitude of revivals increases with increasing the atomic angles through out $[\pi/4, \pi/2]$. The fluctuation of the revivals increases with increasing the Bargmann index. For increasing the Perelomov coherent parameter, the collapse regions increase, the revival regions compress, and the amplitude of revivals decreases. For increasing the detuning parameters $\delta$ and $\lambda$, the collapse regions increase and the revival regions compress. While for increasing the detuning parameter $\lambda$ only, one or more a small revival becomes pronounced into the collapse regions. Moreover, the density of fluctuations in the revival regions is irregular. Then, the proposed quantum model is sensitive to the variation in all proposed model parameters. In addition, there is non-classical behaviour in the presence the detuning parameters effects and the atom–atom interaction.

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References

[1] Jayness ET, Cummings FW. Comparison of quantum and semiclassical radiation theories with application to the beam maser. Proc. IEEE. 1963;51: 89–109.
[2] Cummings FW. Stimulated Emission of Radiation in a Single Mode. Phys Rev. 1965;140:A1051–A1056.
[3] Huang YX, Guo GC. A new measure for the Jaynes–Cummings model dynamics. Chin Phys. 1996;5: 901–910.
[4] Scully MO, Zubairy MS. Quantum optics. Cambridge: Cambridge University Press; 2005.
[5] Kaluzny Y, Goy P, Gross M, et al. Observation of self-induced Rabi oscillations in two-level atoms excited inside a resonant cavity: the ringing regime of superradiance. Phys Rev Lett. 1983;51:1175–1178.
[6] Rempe G, Walther H, Klein N. Observation of quantum collapse and revival in a one-atom maser. Phys Rev Lett. 1987;55:323–356.
[7] Boca A, Miller R, Birnbaum KM, et al. Observation of the vacuum Rabi spectrum for one trapped atom. Phys Rev Lett. 2004;93:233603.
[8] Nielsen MA, Chuang IL. Quantum computation and quantum information. Cambridge: Cambridge University Press; 2000.
[9] Benenti G, Casati G, Strini G. Principle of quantum computation and information. Singapore: World Scientific; 2005.
[10] Bouwmeester D, Ekert A, Zeilinger A, editors. The physics of quantum information. Berlin: Springer; 2000.
[11] Loss D, Di Vincenzo DP. Quantum computation with quantum dots. Phys Rev A. 1998;57:120–126.
[12] Levy J. Quantum-information processing with ferroelectrically coupled quantum dots. Phys Rev A. 2001;64: 052306.
[13] Bennett CH, Di Vincenzo DP. Quantum information and computation. Nature. 2000;404:247–255.
[14] Burkard G, Loss D. Cancellation of spin-orbit effects in quantum gates based on the exchange coupling in quantum dots. Phys Rev Lett. 2002;88:047903.
[15] Braunstein SL, Kimble HJ. Teleportation of continuous quantum variables. Phys Rev Lett. 1998;80:869–872.
[16] Milburn GJ, Braunstein SL. Quantum teleportation with squeezed vacuum states. Phys Rev A. 1999;60: 937–942.
[17] Bennet CH, Weisner SJ. Communication via one- and two-particle operators on Einstein–Podolsky–Rosen states. Phys Rev Lett. 1992;69:2881–2884.
[18] Abdel-Aty M, Abdalla MS, Sander BC. Tripartite entanglement dynamics for an atom interacting with nonlinear couplers. Phys Lett A. 2005;373:315–319.
[19] Immamoğlu A, Awschalom DD, Burkard G, et al. Quantum information processing using quantum dot spins and cavity QED. Phys Rev Lett. 1999;83:4204–4207.
[20] Zhang G-F, Li S-S. Thermal entanglement in a two-spin-qutrit system under a nonuniform external magnetic field. Eur Phys J D. 2006;37:123–127.
[21] Hu Z-N, Youn SH, Kang K, et al. Entanglement of a two-qubit system with anisotropic couplings in nonuniform magnetic fields. J Phys A Math Gen. 2006;39: 10523–10535.
[22] Abdalla MS, Lashin E, Sadiek G. Entropy and variance squeezing for time-dependent two-coupled atoms in an external magnetic field. J Phys B At Mol Opt Phys. 2008;41:015502.
[23] Sadiek G, Lashin E, Abdalla MS. Entanglement of a two-qubit system with anisotropic XYZ exchange coupling in a nonuniform field–time dependent external magnetic field. Phys B. 2009;404:1719–1728.
[24] El-Orany FAA, Abdalla MS. Variance squeezing and entanglement of the XX central spin model. J Phys A Math Theor. 2011;44:035302.
[25] Abdalla MS. Isotropic time-dependent coupled oscillators. Phys Rev A. 1987;35:4160–4166.

[26] Abdalla MS. Quantum treatment of the time-dependent coupled oscillators. J Phys A Math Gen. 1996;29: 1997–2012.

[27] Abdalla MS, Nassar MM. Quantum treatment of the time-dependent coupled oscillators under the action of a random force. Ann Phys. 2009;324:637–669.

[28] Abdalla MS, Ahmed MMA, Al-Homidan S. Quantum statistics of three modes coupled oscillators. J Phys A Math Gen. 1998;31:3117–3140.

[29] Abdalla MS. Quantum treatment of time dependent coupled oscillators. Int J Mod Phys B. 2002;16:2837–2855.

[30] Abdalla MS, Hassan SS, Obada A-SF. Multiphoton transition in the Tavis–Cummings model. Phys Rev A. 1986;34:4869–4874.

[31] Hassan SS, Abdalla MS, Obada A-SF, et al. Periodic squeezing in the Tavis–Cummings model. J Mod Opt. 1993;40:1351–1367.

[32] Abdalla MS, Perina J, Krepelka J. Statistical properties of multiphoton time-dependent three-boson coupled oscillators. J Opt Soc Am B. 2006;23:1146–1160.

[33] Abdalla MS, Perina J, Krepelka J. Non-classical effect of quantum non-demolition measurement in presence of parametric amplification. Opt Commun. 2009;282: 2878–2888.

[34] Tavis M, Cummings M. Exact solution for an N-molecule-radiation-field Hamiltonian. Phys Rev. 1968;170:379–384.

[35] Tavis M, Cummings M. Approximate solutions for an N-molecule-radiation-field Hamiltonian. Phys Rev. 1969; 188:692–695.

[36] Khalil EM, Abdalla MS, Obada A-SF, et al. Entropic uncertainty in two two-level atoms interacting with a cavity field in presence of degenerate parametric amplifier. J Opt Soc Am B. 2010;27:266.

[37] Khalil EM, Abdalla MS, Obada A-SF. Entropy and variance squeezing of two coupled modes interacting with a two-level atom: frequency converter type. Ann Phys. 2006;321:421–434.

[38] Abdalla MS, Khalil EM, Obada A-SF. Statistical properties of a two-photon cavity mode in the presence of degenerate parametric amplifier. Ann Phys. 2007;322:2554–2568.

[39] Khalil EM, Abdalla MS, Obada A-SF. Pair entanglement of two-level atoms in the presence of a nondegenerate parametric amplifier. J Phys B. 2010;43:095507.

[40] Abdalla MS, Obada A-SF, Abdel-Aty M. Von Neumann entropy and phase distribution of two mode parametric amplifier interacting with a single atom. Ann Phys. 2005;318:266–285.

[41] Abdalla MS, Krepelka J, Perina J. Effect of Kerr-like medium on a two-level atom in interaction with bimodal oscillators. J Phys B At Mol Opt Phys. 2006;39:1563–1577.

[42] Abdalla MS, Elech H, Perina J. Quantum treatment of atom–field interaction via the quadratic invariant. J Opt Soc Am B. 2012;29:719–728.

[43] Abdalla MS, Khalil EM, Obada A-SF. Exact treatment of the Jaynes–Cummings model under the action of an external classical field. Ann Phys. 2011;326:2486–2498.

[44] Khalil EM, Abdalla MS. Entropic uncertainty for two atoms interacting with a cavity field under the influence of two photons (off-resonance case). J Russ Laser Res. 2012;33:128–142.

[45] Abdalla MS, Khalil EM. Two coupled oscillators in the form of a frequency converter in interaction with a single atom via two-photon processes. J Russ Laser Res. 2012;33:336–348.

[46] Abdalla MS, Ahmed MMA. Some statistical properties for a spin-(1/2) particle coupled to two spirals. Opt Commun. 2012;285:3578–3586.

[47] Abdalla MS, Ahmed MMA. Entropy squeezing and entanglement of the interaction between SU(1,1) and SU(2) quantum systems. Opt Commun. 2011;284:1933–1940.

[48] Abdalla MS, Ahmed MMA. Some statistical properties for the interaction between SU(1,1) and SU(2) quantum systems. J Phys B At Mol Opt Phys. 2010;43:155503.

[49] Ng KM, Lo CF, Liu KL. Exact eigenstates of the intensity-dependent Jaynes–Cummings model with the counter-rotating term. Phys A. 2000;275:463–474.

[50] Masashi B. Decomposition formulas for Su(1,1) and Su(2) Lie algebras and their applications in quantum optics. J Opt Soc Am B. 1993;10:1347–1359.

[51] Abdalla MS, Khalil EM, and Obada A-SF, et al. Linear entropy and squeezing of the interaction between two quantum system described by su(1,1) and su(2) Lie group in presence of two external terms. AIP Adv. 2017;7:015013.

[52] Abdalla MS, Ahmed MMA, Obada A-SF. Quantum treatment for two two-level atoms in interaction with an SU(1,1) quantum system. J Russ Laser Res. 2013;34: 87–101.

[53] Abdalla MS, Khalil EM, Obada A-SF, et al. Quantum statistical characteristics of the interaction between two two-level atoms and radiation field. Eur Phys J Plus. 2015;130:227.

[54] Nahla AA, Ahmed MMA. Nonclassical properties of asymmetric two two-level atoms interacting with multiphoton quantized field. Int J Mod Phys. 2018;32:1850250.