Abstract

The virtual photon structure function $g_1(x,Q^2,P^2)$, which can be obtained in polarized $e^+e^-$ colliding-beam experiments, is investigated for $\Lambda^2 \ll P^2 \ll Q^2$, where $-Q^2 (-P^2)$ is the mass squared of the probe (target) photon. The analysis is made to next-to-leading order in QCD, in the framework of the QCD improved parton model with the DGLAP evolution equations. The non-leading corrections significantly modify the leading log result, in particular, at large $x$ as well as at small $x$. 
In the last several years, the nucleon spin structure functions $g_1^{p(n)}(x, Q^2)$ and $g_2^{p(n)}(x, Q^2)$ have been extensively studied by deep-inelastic scattering of polarized leptons on polarized nucleon targets. The information on the spin structure of photon would be provided by the resolved-photon process in polarized electron and proton collision in the polarized version of HERA \cite{1}. More directly, the spin-dependent structure function of photon can be measured by the polarized $e^+ e^-$ collision in the future linear colliders. For real photon ($P^2 = 0$) target, there exists only one spin structure function, $g_1^\gamma(x, Q^2)$, which is equivalent to the structure function $W_4^\gamma(x, Q^2)$ ($g_1^\gamma \equiv 2W_4^\gamma$) discussed some time ago in \cite{2}. The leading order (LO) QCD correction to $g_1^\gamma$ was first studied in \cite{3} and later in \cite{4, 5}. Recently the first moment of $g_1^\gamma$ attracted attention in the literatures \cite{4, 5, 6} in connection with its relevance to the axial anomaly, which also plays an important role in the QCD analysis of the nucleon spin structure function $g_1^{p(n)}$.

Now the next-to-leading order (NLO) QCD analysis for $g_1^\gamma$ is possible, since the required spin-dependent two-loop splitting functions in DGLAP evolution equations or, equivalently, the two-loop anomalous dimensions of the relevant operators have been calculated recently \cite{8, 9}. Actually there has been already an analysis of $g_1^\gamma$ for the case of the real photon target by Stratmann and Vogelsang \cite{10}.

In this paper we shall investigate the polarized virtual photon structure function $g_1^\gamma(x, Q^2, P^2)$ to the NLO in QCD, where $-Q^2 (-P^2)$ is the mass squared of the probe (target) photon (Fig.1). We examine $g_1^\gamma(x, Q^2, P^2)$ for the kinematical region: $\Lambda^2 \ll P^2 \ll Q^2$, where $\Lambda$ is the QCD scale parameter. The advantage in studying the virtual photon target is that we can calculate the whole structure function entirely up to NLO by the perturbative method. On the other hand, for the real photon target there appear the non-perturbative pieces in NLO \cite{11}, which we may dispose of by using, say, vector-meson-dominance model. The unpolarized virtual photon structure functions $F_2^\gamma(x, Q^2, P^2)$, and $F_L^\gamma(x, Q^2, P^2)$ were studied in LO \cite{12} and in NLO \cite{13}.

The analysis can be performed either in the framework of the QCD improved parton model \cite{14} using the DGLAP parton evolution equations or in the framework of the operator product expansion (OPE) supplemented by the renormalization group
(RG) method. In this paper we follow the former approach.

Let \( q_i(x, Q^2, P^2) \), \( G_i'(x, Q^2, P^2) \), \( \Gamma_i'(x, Q^2, P^2) \) be quark with \( i \)-flavor, gluon, and photon distribution functions with \( \pm \) helicities of the longitudinally polarized virtual photon with mass \(-P^2\). Then the spin-dependent parton distributions are defined as

\[
\Delta q^i \equiv q_+^i - q_-^i - \bar{q}_+^i + \bar{q}_-^i , \quad \Delta G^\gamma \equiv G_+^\gamma - G_-^\gamma , \quad \Delta \Gamma^\gamma \equiv \Gamma_+^\gamma - \Gamma_-^\gamma .
\]

In the leading order of the electromagnetic coupling constant, \( \alpha = e^2/4\pi \), \( \Delta \Gamma^\gamma \) does not evolve with \( Q^2 \) and is set to be \( \Delta \Gamma^\gamma(x, Q^2, P^2) = \delta(1 - x) \). For later convenience we use, instead of \( \Delta q^i \), the flavor singlet and non-singlet combinations of spin-dependent quark distributions as follows:

\[
\Delta q_S^\gamma \equiv \sum_i \Delta q_i^\gamma , \quad \Delta q_{NS}^\gamma \equiv \sum_i e_i^2 \left( \Delta q_i^\gamma - \frac{\Delta q_S^\gamma}{n_f} \right) ,
\]

where \( n_f \) is the number of flavors of active quarks. In terms of these parton distribution functions, the polarized virtual photon structure function \( g_1^\gamma(x, Q^2, P^2) \) is expressed in the QCD improved parton model as

\[
g_1^\gamma(x, Q^2, P^2) = \int_1^x \frac{dy}{y} \left\{ \Delta q_S^\gamma(y, Q^2, P^2) C_S^S \left( \frac{x}{y}, Q^2 \right) + \Delta G^\gamma(y, Q^2, P^2) C^G \left( \frac{x}{y}, Q^2 \right) 
+ \Delta q_{NS}^\gamma(y, Q^2, P^2) C^{NS} \left( \frac{x}{y}, Q^2 \right) \right\} + C^\gamma(x, Q^2) ,
\]

where \( C^S(C^{NS}), C^G \), and \( C^\gamma \) are the coefficient functions corresponding to singlet(non-singlet)-quark, gluon, and photon, respectively, and are independent of \( P^2 \).

The parton distributions \( \Delta q_S^\gamma, \Delta G^\gamma, \) and \( \Delta q_{NS}^\gamma \) evolve with \( Q^2 \). Their evolution equations are expressed in a compact matrix form, once we introduce a row vector \( \Delta q^\gamma = (\Delta q_S^\gamma, \Delta G^\gamma, \Delta q_{NS}^\gamma) \):

\[
\frac{d}{d \ln Q^2} \Delta q^\gamma(x, Q^2, P^2) = \Delta k(x, Q^2) + \int_x^1 \frac{dy}{y} \Delta q^\gamma(y, Q^2, P^2) \Delta P \left( \frac{x}{y}, Q^2 \right) ,
\]

where \( 1 \) The detailed derivation of this formula will be reported elsewhere [15].
where the elements of a row vector $\Delta k = (\Delta K_S, \Delta K_G, \Delta K_{NS})$ are polarized splitting functions from virtual-photon to singlet-quark, gluon, and non-singlet-quark, respectively. The $3 \times 3$ matrix $\Delta P(z, Q^2)$ is written as

$$\Delta P(z, Q^2) = \begin{pmatrix} \Delta P_{qq}^S(z, Q^2) & \Delta P_{qG}(z, Q^2) & 0 \\ \Delta P_{qG}(z, Q^2) & \Delta P_{GG}(z, Q^2) & 0 \\ 0 & 0 & \Delta P_{qS}^{NS}(z, Q^2) \end{pmatrix}. \quad (7)$$

with its element $\Delta P_{AB}$ representing a splitting function of $B$-parton to $A$-parton.

Once we get the information on the coefficient functions in Eq.(5) and parton splitting functions in Eq.(6), we can predict the behaviour of $g_1^\gamma(x, Q^2, P^2)$ in QCD. The NLO analysis is now possible since the spin-dependent one-loop coefficient functions and two-loop parton splitting functions are available \[8, 9\]. There are two methods to obtain $g_1^\gamma(x, Q^2, P^2)$ in NLO. In one method, we use the parton splitting functions up to two-loop level and we solve numerically $\Delta q^\gamma(x, Q^2, P^2)$ in Eq.(6) by iteration, starting from the initial quark and gluon distributions in the virtual photon at $Q^2 = P^2$. The interesting point of studying the virtual photon with mass $-P^2$ is that when $P^2 \gg \Lambda^2$, the initial parton distributions of the photon are completely known up to NLO in QCD. Then inserting the solved $\Delta q^\gamma(x, Q^2, P^2)$ into Eq.(6), and together with the known one-loop coefficient functions we can predict $g_1^\gamma(x, Q^2, P^2)$ in NLO.

The other method, which is more common than the former, is by making use of the inverse Mellin transformation. We follow the latter method in this paper. First we take the Mellin moments of Eq.(6),

$$\int_0^1 dx x^{n-1} g_1^\gamma(x, Q^2, P^2) = \Delta q^\gamma(n, Q^2, P^2) \cdot C(n, Q^2) + C_n^\gamma \quad (8)$$

where we have defined the moments of an arbitrary function $f(x)$ as

$$f(n) \equiv \int_0^1 dx x^{n-1} f(x). \quad (9)$$

Henceforce we omit the obvious $n$-dependence for simplicity. We expand the splitting functions $\Delta k(Q^2)$ and $\Delta P(Q^2)$ in powers of the QCD and QED coupling constants as

$$\Delta k(Q^2) = \frac{\alpha}{2\pi} \Delta k^{(0)} + \frac{\alpha \alpha_s(Q^2)}{(2\pi)^2} \Delta k^{(1)} + \cdots \quad (10)$$
\[ \Delta P(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \Delta P^{(0)} + \left[ \frac{\alpha_s(Q^2)}{2\pi} \right]^2 \Delta P^{(1)} + \cdots , \]  
and introduce \( t \) instead of \( Q^2 \) as the evolution variable \[ \dagger \], 
\[ t \equiv \frac{2}{\beta_0} \ln \frac{\alpha_s(P^2)}{\alpha_s(Q^2)} . \]

Then, taking the Mellin moments of the both sides in Eq.(6), we find that \( \Delta q^\gamma(t)(= \Delta q^\gamma(n, Q^2, P^2)) \) satisfies the following inhomogeneous differential equation \[ \ddagger \]:

\[
\frac{d\Delta q^\gamma(t)}{dt} = \frac{\alpha}{2\pi} \left\{ \frac{2\pi}{\alpha_s} \Delta k^{(0)} + \left[ \Delta k^{(1)} - \frac{\beta_1}{2\beta_0} \Delta k^{(0)} \right] + O(\alpha_s) \right\} \\
+ \Delta q^\gamma(t) \left\{ \Delta P^{(0)} + \frac{\alpha_s}{2\pi} \left[ \Delta P^{(1)} - \frac{\beta_1}{2\beta_0} \Delta P^{(0)} \right] + O(\alpha_s^2) \right\} 
\]

where we have used the fact the QCD effective coupling constant \( \alpha_s(Q^2) \) satisfies

\[
\frac{d\alpha_s(Q^2)}{d\ln Q^2} = -\beta_0 \frac{\alpha_s(Q^2)^2}{4\pi} - \beta_1 \frac{\alpha_s(Q^2)^3}{(4\pi)^2} + \cdots 
\]

with \( \beta_0 = 11 - 2n_f/3 \) and \( \beta_1 = 102 - 38n_f/3 \). Note that the \( P^2 \) dependence of \( \Delta q^\gamma \) solely comes from the initial condition (or boundary condition) as we will see below.

We look for the solution \( \Delta q^\gamma(t) \) in the following form:

\[
\Delta q^\gamma(t) = \Delta q^\gamma(0)(t) + \Delta q^\gamma(1)(t) 
\]

where the first (second) term represents the solution in the LO (NLO). First we discuss about the initial condition of \( \Delta q^\gamma \). In the framework of OPE, the twist-2 operators \( R_n^i \) \( (i = S, G, NS, \gamma) \) are relevant for the polarized photon structure function \( g_1^\gamma \). The expressions of \( R_n^i \) are found, for example, in Ref.\[ 3 \]. For \(-p^2 \equiv P^2 \gg \Lambda^2 \) we can calculate perturbatively the photon matrix elements of the hadronic operators \( R_n^i \) \( (i = S, G, NS) \). Choosing the renormalization point \( \mu^2 \) to be close to \( P^2 \), we get, to the lowest order \[ \ddagger \],

\[
\langle \gamma(p) \mid R_n^i(\mu) \mid \gamma(p) \rangle = \frac{\alpha}{4\pi} \left( -\frac{1}{2} K_n^{0,i} \ln \frac{P^2}{\mu^2} + A_n^i \right), \quad i = S, G, NS 
\]

where we have used the fact the QCD effective coupling constant \( \alpha_s(Q^2) \) satisfies
where $K_{0}^{0,i} = (K_{0}^{i})^{0}i$ are one-loop anomalous dimension matrices between the photon and hadronic operators. In fact the $K_{0}^{0,i}$-terms and $A_{0}^{i}$-terms represent the operator mixing between the photon operators and hadronic operators in LO and NLO, respectively, and the operator mixing implies that there exists quark and gluon distributions in the photon. When we renormalize the photon matrix elements of the hadronic operators at $\mu^2 = P^2$, we obtain
\[
\langle \gamma(p) \mid R_{n}^{i}(\mu) \mid \gamma(p) \rangle|_{\mu^2 = P^2} = \frac{\alpha}{4\pi} A_{n}^{i}
\]
which shows that, at $\mu^2 = P^2$, quark distribution exists in the photon, not in the LO but in the NLO. Thus we have
\[
\Delta q^{(0)}(0) = 0, \quad \Delta q^{(1)}(0) = \frac{\alpha}{4\pi} A_{n}^{i}.
\]
Explicitly, $A_{n}$ is given by
\[
A_{n} = 6 \left( < e^2 >, 0, < e^4 > - < e^2 >^2 \right) A_{n}^{qG}
\]
where $< e^2 >= \sum_{i} e_{i}^{2} / n_{f}$ and $< e^4 >= \sum_{i} e_{i}^{4} / n_{f}$, and $A_{n}^{qG}$ is the finite part of the gluon matrix element of the flavor-singlet quark operator, whose expression in the \overline{MS} scheme is given by \[8\],
\[
A_{n}^{qG} = 2 n_{f} \left\{ \frac{n-1}{n(n+1)} \sum_{j=1}^{n} \frac{1}{n(j)} + \frac{4}{(n+1)^{2}} - \frac{1}{n^{2}} - \frac{1}{n} \right\}.
\]
With these initial conditions, we obtain for the solution $\Delta q^{\gamma}(t)$ of Eq.(13) to the NLO (see [15] for details):
\[
\Delta q^{\gamma}(t)/\left[ \frac{\alpha}{8\pi \beta_{0}} \right] = \frac{4\pi}{\alpha_{s}(t)} K_{n}^{0} \sum_{i} P_{i}^{n} \left\{ \frac{1}{1 + \frac{\lambda_{i}}{2\beta_{0}}} \right\} \left\{ 1 - \frac{\alpha_{s}(t)}{\alpha_{s}(0)} \right\}^{1+\frac{\lambda_{i}}{2\beta_{0}}}
\]
\[
+ \left\{ K_{n}^{1} \sum_{i} P_{i}^{n} \frac{1}{\lambda_{i}/2\beta_{0}} + \frac{\beta_{1}}{\beta_{0}} K_{n}^{0} \sum_{i} P_{i}^{n} \left( 1 - \frac{1}{\lambda_{i}/2\beta_{0}} \right) \right\} \left\{ 1 - \frac{\alpha_{s}(t)}{\alpha_{s}(0)} \right\}^{1+\frac{\lambda_{i}}{2\beta_{0}}}
\]
\[
- K_{n}^{0} \sum_{j,i} P_{j}^{n} \frac{1}{2\beta_{0}} \left( \frac{1}{\lambda_{j}/2\beta_{0}} - \frac{1}{\lambda_{i}/2\beta_{0}} \right) - 2\beta_{0} A_{n} \sum_{i} P_{i}^{n} \left\{ 1 - \frac{\alpha_{s}(t)}{\alpha_{s}(0)} \right\}^{1+\frac{\lambda_{i}}{2\beta_{0}}}
\]
\[
+ \left\{ K_{n}^{0} \sum_{i,j} P_{i}^{n} \frac{1}{2\beta_{0}} \left( \frac{1}{\lambda_{j} - \lambda_{i}} + \frac{\beta_{1}}{\beta_{0}} K_{n}^{0} \sum_{i} P_{i}^{n} \frac{1}{\lambda_{i}/2\beta_{0}} \right) \right\} \left\{ 1 - \frac{\alpha_{s}(t)}{\alpha_{s}(0)} \right\}^{1+\frac{\lambda_{i}}{2\beta_{0}}}
\]
\[
+ 2\beta_{0} A_{n}.
\]
Here the moments of the splitting functions are related to the anomalous dimensions of operators as follows \cite{8, 9}:

\[
\Delta P^{(0)} = -\frac{1}{4} \hat{\gamma}_n^0, \quad \Delta P^{(1)} = -\frac{1}{8} \hat{\gamma}_n^{(1)}
\]

(22)

\[
\Delta k^{(0)} = \frac{1}{4} K_n^0, \quad \Delta k^{(1)} = \frac{1}{8} K_n^1
\]

(23)

and we have introduced the projection operators given by

\[
\Delta P^{(0)} = -\frac{1}{4} \hat{\gamma}_n^0 = -\frac{1}{4} \sum_{i=+,-,NS} \lambda_i^n P^n_i, \quad i = +, -, NS
\]

(24)

\[
P^n_i P^n_j = \begin{cases} 0 & i \neq j, \\ P^n_i & i = j, \end{cases} \quad \sum_i P^n_i = 1
\]

(25)

where \(\lambda_i^n\) are the eigenvalues of the matrix \(\hat{\gamma}_n^0\). In fact, the parton distributions \(\Delta q_\gamma(t)\), do depend on the initial conditions \(\Delta q_\gamma(0) = (\alpha/4\pi) A_n\) but the structure function \(g_\gamma(x, Q^2, P^2)\) itself is independent of \(\Delta q_\gamma(0)\) in NLO in QCD, which will be discussed in detail in \cite{15}.

When the coefficient functions are given, up to one-loop level, by,

\[
C^S(n, Q^2) = < e^2 > \left\{ 1 + \frac{\alpha_s(Q^2)}{4\pi} B_S^n \right\}
\]

(26)

\[
C^G(n, Q^2) = < e^2 > \left\{ 0 + \frac{\alpha_s(Q^2)}{4\pi} B_G^n \right\}
\]

(27)

\[
C^{NS}(n, Q^2) = 1 + \frac{\alpha_s(Q^2)}{4\pi} B_{NS}^n
\]

(28)

\[
C^\gamma(n, Q^2) = \frac{\alpha}{4\pi} 3n_f < e^4 > B_\gamma^n
\]

(29)

then we obtain from Eq.(8) the following formula for the moments of \(g_\gamma(x, Q^2, P^2)\) in the NLO:

\[
\int_0^1 dx x^{n-1} g_\gamma^i(x, Q^2, P^2)
\]

\[
= \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \left[ \sum_{i=+,-,NS} \tilde{P}_i^n \frac{1}{1 + \lambda_i^n/2\beta_0} \frac{4\pi}{\alpha_s(Q^2)} \left\{ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0+1} \right\} \right]
\]

\[
+ \sum_{i=+,-,NS} A_i^n \left\{ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0} \right\}
\]

\[
+ \sum_{i=+,-,NS} B_i^n \left\{ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n/2\beta_0+1} \right\} + C^n + O(\alpha_s)
\]

(30)
where \( \tilde{P}_n^i, A_n^i, B_n^i \) and \( C_n \) are given by

\[
\tilde{P}_n^i = K_0^0 P_n^i C_n(1,0),
\]

\[
A_n^i = -K_0^0 \sum_j \frac{P_n^{\gamma_0(1)} P_n^i}{2\beta_0 + \lambda_n^j - \lambda_n^i} C_n(1,0) \frac{1}{\lambda_n^j / 2\beta_0} - K_0^1 \frac{\beta_1}{\beta_0} P_n^i C_n(1,0) \frac{1 - \lambda_n^i / 2\beta_0}{\lambda_n^i / 2\beta_0} + K_n^1 P_n^i C_n(1,0) \frac{1}{\lambda_n^i / 2\beta_0} - 2\beta_0 A_n P_n^i C_n(1,0)
\]

\[
B_n^i = K_0^0 \sum_j \frac{P_n^{\gamma_0(1)} P_n^i}{2\beta_0 + \lambda_n^j - \lambda_n^i} C_n(1,0) \frac{1}{1 + \lambda_n^j / 2\beta_0} + K_0^0 P_n^i \left( \frac{< e^2 > B_S^n}{< e^2 > B_G^n} \right) \frac{1}{1 + \lambda_n^i / 2\beta_0}
\]

\[
C_n = 2\beta_0 \left\{ 3n_f < e^4 > B_S^n + A_n \cdot C_n(1,0) \right\}
\]

and the column vector \( C_n(1,0) = (< e^2 >, 0, 1)^T \) represents tree-level coefficient functions. The expressions of Eqs. (30) and (31)–(34) are actually the same in form as the ones obtained before by one of the authors and Walsh for the case of the virtual photon structure function \( F_2^\gamma \) [13]. All the quantities necessary to evaluate \( \tilde{P}_n^i, A_n^i, B_n^i, \) and \( C_n \) are now known, which will be reported in [15]. Especially we have at hand the results of the two-loop anomalous dimensions [8, 9] and one-loop coefficient functions \( B_l^n \) \((l = S, G, NS, \gamma)\) in Eqs. (26)–(29) [18, 19, 8, 9] which were calculated in the \( \overline{\text{MS}} \) scheme.

In the case of real photon target, the polarized structure function \( g_1^\gamma(x, Q^2) \) satisfies a remarkable sum rule which holds true in all order of \( \alpha \) and \( \alpha_S \) [5, 6, 20],

\[
\int_0^1 g_1^\gamma(x, Q^2) dx = 0.
\]

Now we can ask what happens to the first moment of the virtual photon structure function \( g_1^\gamma(x, Q^2, P^2) \). This can be studied by taking the \( n \rightarrow 1 \) limit of Eq. (30).

Note that we have the following eigenvalues for the one-loop anomalous dimension matrix \( \tilde{\gamma}_{n=1}^0 \):

\[
\lambda_{+n=1} = 0, \quad \lambda_{-n=1} = -2\beta_0, \quad \lambda_{NSn=1} = 0
\]
Physically speaking, the zero eigenvalues $\lambda_{+}^{n=1} = \lambda_{NS}^{n=1} = 0$ correspond to the conservation of the axial-vector current at one-loop order. The other eigenvalue $\lambda_{-}^{n=1} = -2\beta_0$, which is negative, is rather an artifact of continuation of the gluon anomalous dimension to $n = 1$, since no gauge-invariant twist-2 gluon operator exists for $n = 1$. However, in the QCD improved parton model approach, there is no reason why the $n = 1$ moment of the polarized gluon distribution should not be considered \[21\]. In the $n \to 1$ limit, we expect in Eq.(30)

\[
\tilde{P}_i^n \frac{1}{1 + \lambda_i^n / 2\beta_0} \to 0 \quad (i = +, -, NS)
\]

$A_+^n \to \text{finite}$, $A_-^n \to 0$, $A_{NS}^n \to \text{finite}$

$B_+^n \to 0$, $B_-^n \to \text{finite}$, $B_{NS}^n \to 0$. \quad (37)

However, $A_+^n$, $A_{NS}^n$, and $B_-^n$ are multiplied by the following vanishing factors

$$
\left\{ 1 - \left( \frac{\bar{g}^2(Q^2)}{\bar{g}^2(P^2)} \right)^{\lambda_i^n / 2\beta_0} \right\}, \left\{ 1 - \left( \frac{\bar{g}^2(Q^2)}{\bar{g}^2(P^2)} \right)^{\lambda_{NS}^n / 2\beta_0} \right\}, \left\{ 1 - \left( \frac{\bar{g}^2(Q^2)}{\bar{g}^2(P^2)} \right)^{\lambda_{-}^n / 2\beta_0 + 1} \right\},
$$

respectively, and thus the terms proportional to $\tilde{P}_i^n$, $A_i^n$, and $B_i^n$ in Eq.(30) all vanish in the $n = 1$ limit. Note that these vanishing factors are specific to the case of the virtual photon target, and that such factors do not appear when the target is real photon. Thus we find

\[
\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} c_{n=1} + O(\alpha_s) . \quad (39)
\]

Now due to the relation $B_{\gamma}^n = \frac{2}{n_f} B_{G}^n$, we have from Eq.(34)

\[
C_{n=1} = 12\beta_0 < e^4 > (B_{G}^n + A_{qG}^n)|_{n=1} . \quad (40)
\]

It is noted that the combination $(B_{G}^n + A_{qG}^n)$ is renormalization-scheme independent \[22\]. The results in the MS scheme \[8, 9, 19\] are

\[
B_{G}^{n=1} = 0, \quad A_{qG}^{n=1} = -2n_f \quad (41)
\]

The same results have been obtained by Kodaira \[23\] in the framework of OPE and RG method. He set $B_{G}^{n=1} = 0$, observing that there is no gauge-invariant twist-2
gluon operator for $n = 1$ and obtained $A_{gG}^{n=1} = -2n_f$ from the Adler-Bell-Jackiw anomaly. In the end, we have for the sum rule of the virtual photon structure function $g_1^\gamma$,

$$
\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \sum_{i=1}^{n_f} e_i^4 + \mathcal{O}(\alpha_s), \quad \text{for } \Lambda^2 \ll P^2 \ll Q^2 .
$$

(42)

Finally it should be pointed out that we can further pursue the QCD corrections of order $\alpha_s$ to the first moment of $g_1^\gamma$, which will be discussed in detail in [15]. Here we just write down the result which turns out to be:

$$
\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \sum_{i=1}^{n_f} e_i^4 \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)
- \frac{2}{\beta_0} \beta_1 \sum_{i=1}^{n_f} e_i^4 \left(\frac{\alpha_s(P^2)}{\pi} - \frac{\alpha_s(Q^2)}{\pi}\right) + \mathcal{O}(\alpha_s^2).
$$

(43)

This result is perfectly in agreement with the one obtained by Narison, Shore and Veneziano in ref.[7], apart from the overall sign for the definition of $g_1^\gamma$.

The polarized virtual photon structure function $g_1^\gamma(x, Q^2, P^2)$ is recovered from the moments, Eq.(30) by the inverse Mellin transformation. In Fig.2 we have plotted, as an illustration, the result for $n_f = 3$, $Q^2 = 30\text{GeV}^2$, $P^2 = 1\text{GeV}^2$ and the QCD scale parameter $\Lambda = 0.2\text{GeV}$. The vertical axis corresponds to

$$
g_1^\gamma(x, Q^2, P^2)/\frac{3\alpha}{\pi} n_f < e^4 \ln \frac{Q^2}{P^2}.
$$

(44)

Here we have shown three cases; the box (tree) diagram contribution,

$$
g_1^{(Box)}(x, Q^2, P^2) = (2x - 1) \frac{3\alpha}{\pi} n_f < e^4 \ln \frac{Q^2}{P^2},
$$

(45)

the LO and NLO results in QCD. We observe that the NLO QCD corrections are significant at large $x$ as well as at low $x$. Other examples with different $Q^2$ and $P^2$ will be reported in the forthcoming paper [15]. We have not seen any sizable change for the normalized structure function (44).

Now let us consider the real photon case $P^2 = 0$. The structure function can be decomposed as

$$
g_1^\gamma(x, Q^2) = g_1^\gamma(x, Q^2)|_{\text{pert.}} + g_1^\gamma(x, Q^2)|_{\text{non-pert.}}.
$$

(46)
The second term can only be computed by some non-perturbative method such as lattice QCD, or estimated by vector-meson-dominance model. The first term, the point-like piece, can be calculated in a perturbative method. Actually, it can be reproduced formally by setting $P^2 = \Lambda^2$ in Eq. (30). In Fig. 3, we have plotted the point-like piece of $g_1^\gamma$ of the real photon. The LO QCD result coincides with the previous result obtained by one of the authors in [3]. The NLO result is qualitatively consistent with the analysis made by Stratmann and Vogelsang [10].

In the present paper, we have investigated polarized virtual photon structure function $g_1^\gamma(x, Q^2, P^2)$ in the NLO in QCD for the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$. The analysis has been made in the framework of the QCD improved parton model with the DGLAP parton evolution equations. We have shown that the behavior of $g_1^\gamma(x, Q^2, P^2)$ can be predicted up to NLO in QCD without any free parameter. In fact the NLO QCD corrections are significant at large $x$ and also at small $x$. The first moment of $g_1^\gamma$ for the virtual photon is non-vanishing in contrast to the case of real photon where the first moment vanishes.

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Figure Caption

Figure 1
Deep inelastic scattering on a virtual photon in polarized $e^+ e^-$ collision, $e^+ e^- \rightarrow e^+ e^- + \text{hadrons}$. The mass squared of the “probe” (“target”) photon is $-Q^2 (-P^2)$ ($\Lambda^2 \ll P^2 \ll Q^2$).

Figure 2
The polarized virtual photon structure function $g_1^\gamma(x, Q^2, P^2)$ to the next-to-leading order (NLO) in units of $\frac{3\alpha}{\pi} n_f < e^4 > \ln \frac{Q^2}{\Lambda^2}$ for $Q^2 = 30\text{GeV}^2$, $P^2 = 1\text{GeV}^2$ and the QCD scale parameter $\Lambda = 0.2\text{GeV}$ with $n_f = 3$. We also plot the leading order (LO) result (long-dashed line) and the box diagram contribution (short-dashed line).

Figure 3
The point-like piece of the polarized real photon structure function $g_1^\gamma(x, Q^2)$ to the next-to-leading order (NLO) in units of $\frac{3\alpha}{\pi} n_f < e^4 > \ln \frac{Q^2}{\Lambda^2}$ for $Q^2 = 30\text{GeV}^2$, and the QCD scale parameter $\Lambda = 0.2\text{GeV}$ with $n_f = 3$. We also plot the leading order (LO) result (long-dashed line) and the box diagram contribution (short-dashed line).
$e^-(e^+)$
\[\iff\]
\[\iff\]
$q^2 = -Q^2 < 0$

$e^+(e^-)$
\[\iff\]
\[\iff\]
$p^2 = -P^2 < 0$

Fig. 1
$Q^2 = 30 \text{ GeV}^2$
$P^2 = 1 \text{ GeV}^2$
$n_f = 3$

![Graph showing $g_1^\gamma(x, Q^2, P^2)/(3n_f<e^4>/\pi)\ln Q^2/P^2$ vs. $x$ for $Q^2 = 30 \text{ GeV}^2$, $P^2 = 1 \text{ GeV}^2$, and $n_f = 3$. The graph includes curves for Box, LO, and NLO approximations.](image)
$Q^2 = 30 \text{ GeV}^2$

$n_f = 3$

$g_1^\gamma(x, Q^2) < e^4 Q^2 > / m^2 \ln Q^2 / \Lambda^2$

Fig. 3