Research Article

Regarding on the Results for the Fractional Clannish Random Walker’s Parabolic Equation and the Nonlinear Fractional Cahn-Allen Equation

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In this research, the \((\Psi, \Phi)\)-expansion scheme has been implemented for the exact solutions of the fractional Clannish Random Walker’s parabolic (FCRWP) equation and the nonlinear fractional Cahn-Allen (NFCA) equation. Some new solutions of the FCRWP equation and the NFCA equation have been obtained using this method. The diverse variety of exact outcomes such as intersection between rough wave and kinky soliton wave profiles, intersection between lump wave and kinky soliton wave profiles, soliton wave profiles, kink wave profiles, intersection between lump wave and periodic wave profiles, intersection between rough wave and periodic wave profiles, periodic wave profiles, and kink wave profiles are taken.

Comparing our developed answers and that got in previously written research papers presents the novelty of our investigation. The above techniques could also be employed to get exact solutions for other fractional nonlinear models in physics, applied mathematics, and engineering.

1. Introduction

Nonlinear fractional mathematical models (NLFMMs) are broadly implemented to express lots of significant phenomena and nonlinear dynamic applications in applied mathematics, mathematics, physics, engineering, signal processing, electromagnetics, communications, acoustics, genetic algorithms, viscoelasticity, robotics, electrochemistry, transport systems, material science, finance, image processing, stochastic dynamical systems, biology, plasma physics, chemistry, nonlinear control theory, and so many.

In accordance with determining the exact answers of NLFMMs, countless influential and well-organized schemes have been presented and industrialized, such as the variation of \((G'/G)\)-expansion scheme [1], modified \((G'/G)\)-expansion technique [2-5], the first integral technique [6], generalized Kudryashov technique [7], fractional subequation scheme [8, 9], improved fractional sub-equation scheme [10], generalized exponential rational task scheme [11], novel extended direct algebraic method [12], Sine-Gordon expansion technique [13], subequation scheme [14], Kudryashov technique [15], Jacobi elliptic task scheme [16], exp-task scheme [17], the Jacobi elliptic ansatz method [18], natural transform method [19], fractional iteration algorithm [20, 21], the unified method [22], the hyperbolic and exponential ansatz method [23], \((1/G')\)-expansion scheme [24], modified decomposition method [25], the quintic B-spline approaches [26], an efficient semi-analytical algorithm [27], the Jacobi elliptic function expansion (JEFE) method [28], the Lie symmetry technique [29], Hirota’s simple method [30, 31], the modified extended tanh expansion method [32], exponential finite difference method
and many more. Currently, Shehata and Amra [34] discovered a profoundly significant enlargement of the \((G'/G)\)-extension process, called the variation of \((G'/G)\)-expansion process to secure exact solutions of fractional nonlinear models. We instrument the variation of \((G'/G)\)-expansion way for making exact answers to the fractional nonlinear models in the existing effort to express the suitable and simplicity of the process. Therefore, we can effortlessly exchange the fractional nonlinear models into NPDE or NODE via appropriate conversion, with the purpose of everybody acquainted with fractional calculus lacking any trouble.

The chief advantage of the process implemented in this study over the other scheme is that it contributes additional novel exact answers, including added independent parameters, and we make a few novel results as well. The exact answers have vast significance in uncovering the fundamental device of the physical events. Apart from the dynamic relevance, the exact answers of fractional nonlinear models support the numerical solvers to compare their results’ accuracy and help them in the stability analysis. In our current effort, we instrument \((\Psi, \Phi)\)-expansion scheme for constructing exact answers of FCRWP and NFCA equations. We can therefore easily convert FCRWP and NFCA equations into nonlinear PDE or ODE via appropriate conversion, with the purpose of everybody acquainted with fractional calculus lacking any trouble.

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In the current object, the first subdivision offers the choice of the research as an introduction. The second subdivision covers a few analyze of the \((\Psi, \Phi)\)-expansion method. In the third subdivision, we will acquire answers of the FCRWP and NFCA equations by using the proposed technique. In the fourth subdivision, we will give some numerical simulation of the results obtained. In the last subdivision, we implement the conclusion.

2. Overview of the Technique

In this part of the study, detailed information on fractional calculus theory can be found at [35–37]. Here, we briefly analyzed the modified Lehman-Liouville derivative (MRLD) from the current fractional calculus recommended through Jumarie [38, 39] and provided a research
method. Let $S : [0, 1] \rightarrow \mathbb{R}$ be a continuous function and $\beta \in (0, 1)$. Jumarie has improved the fractional derivative of the order $\beta$ and $S$ could be clearly defined through [40] as

$$D^\beta_S S(\lambda) = \begin{cases} \frac{1}{\Gamma(-\beta)} \int_0^\lambda (\lambda - \chi)^{-\beta-1}[S(\chi) - S(0)]d\chi, & 0 > \beta, \\ \frac{1}{\Gamma(1-\beta)} \frac{d}{d\lambda} \int_0^\lambda (\lambda - \chi)^{-\beta}[S(\chi) - S(0)]d\chi, & 0 < \beta < 1, \\ (S^{(n)}(\lambda))^{(\beta-n)}, & n \leq \beta \leq n+1, \ n \geq 1. \end{cases}$$

We consider

$$P(\lambda, \lambda_x, \lambda_{xx}, \lambda_t, \lambda_{xt}, \lambda_{xxx}, \ldots) = 0,$$  

(2)

where $P$ is a polynomial in $\lambda$ as well as its derivatives. First, use the travelling variable:

$$\lambda = \lambda(x, t) = \lambda(\chi), \chi = p_3(x - Vt),$$

(3)

where $p_3$ and $V$ are a constant to be determined later. Using (3) into (2), We get the following ordinary differential equation:

$$R\left(\lambda, p_3 \lambda', p_3^2 \lambda'', -p_3 \lambda', p_3^3 \lambda''', -p_3^2 \lambda'''', \ldots\right) = 0.$$  

(4)

Second, considering the solving form:

$$\lambda(\chi) = \sum_{i=0}^M S_i \psi^i + \sum_{i=1}^M T_i \psi^{i-1} \Phi,$$  

(5)

where $\psi = (\Theta'/\Theta), \Phi = (\Omega'/\Omega)$, and $\Theta = \Theta(\chi), \Omega = \Omega(\chi)$ represent as

$$\Theta'(\chi) = -\Theta(\chi)\Omega(\chi), \quad \Omega'(\chi) = 1 - \Omega(\chi)^2.$$  

(6)

The above equations provide as follows:

$$\Theta(\chi) = \pm \sec h(\chi), \quad \Omega(\chi) = \tanh (\chi),$$  

(7)

$$\Theta(\chi) = \pm \csc h(\chi), \quad \Omega(\chi) = \coth (\chi).$$  

(8)
Third, a polynomial in $\Psi$ or $\Phi$ accomplished plugging equation (5) into equation (4). Defining the constant values of the corresponding power of $\Psi$ or $\Phi$ yields a system of equations, which might be defined to make $S_i$ and $T_i$. After getting $S_i$ and $T_i$ in (5), the answers of the studied model complete the intention of the answers of the proposed model.

3. Mathematical Analysis

This section presents solutions of two NLFMMs via the $(\Psi, \Phi)$-expansion technique.

3.1. Constructing the Solutions of the FCRWP Equation. We are considering the FCRWP equation [41].

$$\frac{\partial^\beta \chi}{\partial t^\beta} - \frac{\partial \chi}{\partial x} + 2 \chi \frac{\partial \chi}{\partial x} + \frac{\partial^2 \chi}{\partial x^2} = 0, \quad t > 0, \quad 0 < \beta, \quad (9)$$

With $\chi(x, 0) = S(x)$. By getting the variable $(x, t) = \chi(\chi)$, $\chi = x - Vt^\beta / \Gamma(\beta + 1)$ in equation (9), then, we have

$$-V\chi' - \chi' + 2\chi\chi' - \chi'' = 0. \quad (10)$$

The pole of equation (10) is set $N = 1$. Then, we find from equation (4)

$$\chi(\chi) = S_0\Psi^0 + S_1\Psi^1 + T_1\Psi^0 \Phi = S_0 + \frac{T_1}{\Omega} - (S_1 + T_1). \quad (11)$$

Assembling the coefficient of and solving the resulting system, then, we find

Cluster I:

$$V = V, S_0 = 0, S_1 = S_1, T_1 = 0. \quad (12)$$

Substituting the above values in equation (11), we get

$$\lambda_1(x, t) = -S_1 \tanh \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right). \quad (13)$$

Figure 3: The graphical representation of the solution $\lambda_6(x, t)$: (a) real three-dimensional shape, (b) complex three-dimensional shape, (c) real contour plot, and (d) imaginary contour plot.
\[ \lambda_2(x, t) = -S_1 \coth \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right). \]

Cluster II:

\[ V = 2S_0 - 1, S_0 = S_0, S_1 = 1, T_1 = 0. \]

Similarly, we get

\[ \lambda_3(x, t) = S_0 - \tanh \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right). \]

\[ \lambda_4(x, t) = S_0 - \coth \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right). \]

Cluster III:

\[ V = 3, S_0 = 2, S_1 = 2, T_1 = -1. \]

Similarly, we get

\[ \lambda_5(x, t) = 2 - \frac{1}{\tanh \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right)} - \tanh \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right). \]

\[ \lambda_6(x, t) = 2 - \frac{1}{\coth \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right)} - \coth \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right). \]

Cluster IV:

\[ V = 1, S_0 = 1, S_1 = 1, T_1 = -1. \]

Similarly, we get

\[ \lambda_7(x, t) = 1 - \frac{1}{\tanh \left( x - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right)}. \]

Figure 4: The graphical representation of the solution \( \lambda_t(x, t) \): (a) real three-dimensional shape, (b) complex three-dimensional shape, (c) real contour plot, and (d) imaginary contour plot.
3.2. Constructing the Solutions of the NFCA Equation. We are considering the NFCA equation \[42\].

\[
D^\beta_t \Phi - \Phi_{xx} + \Phi^3 - \Phi = 0, \quad t > 0, \quad 0 < \beta \leq 1. \tag{24}
\]

We introduce

\[
\Phi(x, t) = \Phi(\eta), \quad \eta = kx - \frac{Vt^\beta}{\Gamma(\beta + 1)}, \tag{25}
\]

where \(k\) and \(V\) are arbitrary constants. Implementing equations (25) and (24), then, we get

\[
-V\Phi' - k^2 \Phi'' + \Phi^3 - \Phi = 0. \tag{26}
\]

We find from equation (4). Then, we have

\[
\Phi(\eta) = S_0 \Psi^0 + S_1 \Psi^1 + T_1 \Psi^0 \Phi = S_0 + \frac{T_1}{\Omega} - (S_1 + T_1) \Omega. \tag{27}
\]

Collecting the coefficient of \(\Psi\) and \(\Phi\) and solving the resulting system, then, we find Cluster I:

\[
V = \frac{3}{4}, \quad k = \pm \frac{1}{2}, \quad S_0 = \frac{1}{2}, \quad S_1 = \frac{1}{2}, \quad T_1 = 0. \tag{28}
\]

Substituting the above values in equation (27), we get

\[
\Phi_1(x, t) = \frac{1}{2} \left\{ 1 - \tanh \left( kx - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right) \right\}. \tag{29}
\]

\[
\Phi_2(x, t) = \frac{1}{2} \left\{ 1 - \coth \left( kx - \frac{Vt^\beta}{\Gamma(\beta + 1)} \right) \right\}. \tag{30}
\]
Cluster II:

\[ V = \frac{3}{4}, \quad k = \frac{1}{2} \left( \pm \sqrt{-\frac{1}{2}} \right), \quad S_0 = \frac{1}{2}, \quad S_1 = -\frac{1}{2}, \quad T_1 = 0. \]  \[ (31) \]

Similarly, we get

\[ \Phi_3(x, t) = \frac{1}{2} \left\{ 1 + \tanh \left( kx - \frac{V t^\beta}{\Gamma(\beta + 1)} \right) \right\}. \]  \[ (32) \]

\[ \Phi_4(x, t) = \frac{1}{2} \left\{ 1 + \coth \left( kx - \frac{V t^\beta}{\Gamma(\beta + 1)} \right) \right\}. \]  \[ (33) \]

Cluster III:

\[ V = \frac{3}{4}, \quad k = \frac{1}{2} \left( \pm \sqrt{-\frac{1}{2}} \right), \quad S_0 = -\frac{1}{2}, \quad S_1 = \frac{1}{2}, \quad T_1 = 0. \]  \[ (34) \]

Similarly, we get

\[ \Phi_5(x, t) = -\frac{1}{2} \left\{ 1 + \tanh \left( kx - \frac{V t^\beta}{\Gamma(\beta + 1)} \right) \right\}. \]  \[ (35) \]

\[ \Phi_6(x, t) = -\frac{1}{2} \left\{ 1 + \coth \left( kx - \frac{V t^\beta}{\Gamma(\beta + 1)} \right) \right\}. \]  \[ (36) \]

Cluster IV:

\[ V = \frac{3}{4}, \quad k = \frac{1}{2} \left( \pm \sqrt{-\frac{1}{2}} \right), \quad S_0 = -\frac{1}{2}, \quad S_1 = -\frac{1}{2}, \quad T_1 = 0. \]  \[ (37) \]

Similarly, we get

\[ \Phi_7(x, t) = -\frac{1}{2} \left\{ 1 - \tanh \left( kx - \frac{V t^\beta}{\Gamma(\beta + 1)} \right) \right\}. \]  \[ (38) \]

\[ \Phi_8(x, t) = -\frac{1}{2} \left\{ 1 - \coth \left( kx - \frac{V t^\beta}{\Gamma(\beta + 1)} \right) \right\}. \]  \[ (39) \]

**Figure 6:** The graphical representation of the solution \( \Phi_3(x, t) \): (a) real three-dimensional shape, (b) complex three-dimensional shape, (c) real contour plot, and (d) imaginary contour plot.
Cluster V:

\[ V = \frac{3}{4}, \ k = \frac{1}{2} \left( \pm \sqrt{\frac{1}{2}} \right), \ S_0 = -\frac{1}{2}, \ S_1 = -\frac{1}{2}, \ T_1 = \frac{1}{2}. \] (40)

Similarly, we get

\[ \Phi_{11}(x, t) = \frac{1}{2} \left\{ 1 - \frac{1}{\tanh (kx - Vt^{\beta}/\Gamma(\beta + 1))} \right\}. \] (44)

\[ \Phi_{12}(x, t) = \frac{1}{2} \left\{ 1 - \frac{1}{\coth (kx - Vt^{\beta}/\Gamma(\beta + 1))} \right\}. \] (45)

Cluster VII:

\[ V = \frac{3}{8}, \ k = \frac{1}{4} \left( \pm \sqrt{\frac{1}{2}} \right), \ S_0 = -\frac{1}{2}, \ S_1 = -\frac{1}{2}, \ T_1 = \frac{1}{4}. \] (46)

Similarly, we get:

\[ \Phi_{11}(x, t) = -\frac{1}{2} + \frac{1}{4 \tanh (kx - Vt^{\beta}/\Gamma(\beta + 1))} + \tanh \left\{ kx - \frac{Vt^{\beta}}{\Gamma(\beta + 1)} \right\}. \] (47)

Figure 7: The graphical representation of the solution \( \Phi_4(x, t) \): (a) real three-dimensional shape, (b) complex three-dimensional shape, (c) real contour plot, and (d) imaginary contour plot.
4. Numerical Simulations

Shehata and Amra [34] have offered a process named the variation of \( (G'/G) \)-extension approach to look for the studied equation's exact structures and achieve many results shown in Section 3. Comparison between other methods, the studied approach is provided a more exact answer rather than the other approach. On the additional support, the auxiliary model employed in the integral method is different, and so the exact structures obtained are also other. Likewise, for any fractional nonlinear models, it could be determined that the studied approach is extremely more straightforward than the other schemes. In this paper, the integrable method applies to the studied fractional nonlinear models for the first time. We confirm that any other authors did not use the technique on the studied fractional nonlinear models. This part also will present numerous types of soliton of the created answers for various values of the constant coefficients. The kinds of the solitons are kink type soliton profiles, singular kink type soliton profiles, periodic wave soliton profiles, and so many. Also, we provide the contour graph of the attained answers which is constructed commencing binary variable tasks. One variable exemplifies on the horizontal axis and a second variable.
exemplifies on the vertical axis. The functional value exemplifies the color gradient and isolines. Contour graphs are technique to expression a 3D surface on a 2D plane. This kind of graph is broadly implemented in applied mathematics, mathematical physics, and engineering, where the contour lines normally demonstration elevation.

4.1. Graphical Representations of the Solution to the FCRWP Equation. In the current section, we have offered numerous mathematical simulations through the proposed procedure. To explain the dynamic performances of the answers acquired in Section 3.1. Figures 1–4 illustrate the graphical depictions of some selected computational results of the problem received utilizing the studied method. They are pictured below.

Figure 1 demonstrates the dynamic performance of $\lambda_1(x, t)$ using $\alpha = 0.5$. In particular, Figure 1 demonstrates the 3D shape and contour shape of $\lambda_1(x, t)$. This shape represents intersection between rough wave and kinky soliton wave profiles. The solution attributes of $\lambda_3(x, t)$ are displayed in Figure 2 using $\alpha = 0.9$. This shape represents intersection between lump wave and kinky soliton wave profiles. In Figure 3, we demonstrate the dynamic performance of $\lambda_6(x, t)$ using $\alpha = 0.01$. In particular, Figure 3 shows the three-dimensional shape and contour shape of the solution $\lambda_6(x, t)$. This shape represents soliton wave profiles. Figure 4 demonstrates the dynamic performances of $\lambda_8(x, t)$ by taking $\alpha = 0.001$. In particular, Figure 4 shows the three-dimensional shape and contour shape of the solution $\lambda_8(x, t)$. This shape represents kink wave profiles. The implemented mathematical simulations acknowledge that the answers are of periodic wave shapes and the rational, hyperbolic, trigonometric categorizations. Furthermore, through wisely observing at the construction of the acquired answers, it could be comprehended that the connecting fractional derivatives parameter of $\beta$ performs in the formulation of all the answers.

4.2. Graphical Representations of the Solution to the NFCA Equation. In the current section, we have implemented numerous mathematical simulations through the proposed process. To explain the dynamic performances of the
answers acquired in Section 3.2. Figures 5–9 illustrate the
graphical depictions of some selected computational results of the problem received utilizing the studied method. They
are pictured below.

Figure 5 demonstrates the dynamic performance of
\( \Phi_1(x, t) \) using \( \alpha = 0.3 \). In particular, Figure 5 demonstrates
the 3D figure and contour figure of \( \Phi_1(x, t) \). This shape
represents intersection between lump wave and kinky soli-
ton wave profiles. The solution attributes of \( \Phi_1(x, t) \) are
displayed in Figure 6 using \( \alpha = 0.1 \). Figure 6 demonstrates
the 3D figure and contour figure of \( \Phi_2(x, t) \). This shape
represents intersection between rough wave and periodic wave profiles. In Figure 7, we demonstrate the dynamic
performance of \( \Phi_3(x, t) \) using \( \alpha = 0.1 \). In particular,
the construction of the acquired answers, it could be com-
prehended that the connecting fractional derivative param-
eter of \( \beta \) performs in the formulation of all the answers.

5. Conclusion

In the current research, we successfully implement the
studied technique to demonstration that this scheme is
well-organized and virtually fine well-matched to imple-
ment in getting answers for FCRWWP equation and the
NFCA equation. The various of dynamical behaviours such as
intersection between rough wave and kinky soliton
wave profiles, intersection between lump wave and kinky
soliton wave profiles, soliton wave profiles, kink wave pro-
files, intersection between lump wave and periodic wave
profiles, intersection between rough wave and periodic
wave profiles, periodic wave profiles, and kink wave pro-
files are taken in the present study which present in
well-defined regions of mathematical physics. These solu-
tions will be useful for further studies in mathematical
physics which are shown in Figures 1–9.

In Figure 1, we represent intersection between rough wave
and kinky soliton wave profiles. Figure 2 represents intersec-
tion between lump wave and kinky soliton wave profiles.
Figure 3 shows soliton wave profiles. Figure 4 shows kink wave profiles. Figure 5 demonstrates intersection between lump
wave and kinky soliton wave profiles. Figure 6 demonstrates
intersection between rough wave and periodic wave profiles.
In Figure 7, we demonstrate intersection between lump wave
and periodic wave profiles. Figure 8 demonstrates periodic
wave profiles. Finally, we demonstrate singular kink wave pro-
files which are displayed in Figure 9. The most powerful aspect
of this method is that it can be applied to other fractional non-
linear models. Also, it has observed that the results obtained in
this study have been presented for the first time. It is impor-
tant to note that the solutions obtained can be applied to non-
linear models, plasma physics, optical physics, ion-acoustics
physics, optical engineering, soliton theory, nonlinear dyna-
mics, and other areas.

Data Availability

The data used to support the findings of this study are
included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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