Consistency of the effective-field-theory approach to the nucleon-nucleon interaction problem revisited

Jambul Gegelia\textsuperscript{1,2} and Stefan Scherer\textsuperscript{1}

\textsuperscript{1}\textit{Institut für Kernphysik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany}
\textsuperscript{2}\textit{High Energy Physics Institute, Tbilisi State University, University St. 9, 380086 Tbilisi, Georgia}

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Abstract

It is argued that Weinberg’s approach to the nucleon-nucleon (NN) interaction problem within effective field theory provides a consistent power counting for renormalized diagrams. Within this scheme the NN potential is organized as an expansion in terms of small quantities like small external momenta and the pion mass (divided by the characteristic large scale of the effective theory). Physical observables to any given order in these small quantities are calculated from the solutions of the Lippmann-Schwinger (or Schrödinger) equation.

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I. INTRODUCTION

Weinberg’s work on constructing the nuclear forces from effective field theory (EFT) of the strong interactions [1, 2, 3] has triggered an intensive research activity during the last decade. According to Weinberg’s approach, in order to calculate any amplitude involving low-energy pions and nucleons, one first writes down the most general Lagrangian, draws all Feynman diagrams which contribute to the given process, and counts powers of small quantities (like small external momenta and the pion mass) assigned to these diagrams. For processes involving \( N > 1 \) nucleons, at any given order one finds an infinite number of diagrams. Weinberg observed that among these diagrams there is only a finite number of \( N \)-nucleon irreducible diagrams. Defining the sum of these irreducible diagrams as the potential, he suggested a systematic expansion by applying the power counting to the potential. Next, it is assumed that the renormalized coupling constants are natural, i.e., if a coupling constant \( C \) has dimension \((\text{mass})^{-d}\), then \( \tilde{C} = C Q^d \sim 1 \), where \( Q \) is the characteristic large scale of the EFT. Within this assumption the higher-order terms of the potential are suppressed by powers of small momenta or the pion mass divided by \( Q \). The contributions of reducible diagrams are taken into account by solving the Lippmann-Schwinger (LS) or Schrödinger equation.

The application of these ideas in practical calculations has encountered various problems. They originate from the fact that the approach outlined in Refs. [2, 3] does not exactly specify how to handle the problem of renormalization for the LS equation with nonrenormalizable potentials, i.e., when the iteration of the potential generates divergent terms with structures, which are not included in the original potential.

For definiteness let us consider the NN scattering problem, which has attracted much attention during the last few years (see, e.g., Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46]). The case of several nucleons is analogous. Since the NN potential of the EFT is nonrenormalizable in the traditional sense, for the renormalization of the solution of the LS equation one needs to take into account the contributions of an infinite number of higher-order counterterms, where the infinite number refers to both the loop and the chiral expansions. The freedom of choosing the finite parts of these counterterms is compensated by the running of the corresponding renormalized couplings. It has been argued that the coefficients of the divergent parts of the counterterms contributing in low-order calculations would set the scale of the renormalized couplings. As a consequence, even if these couplings were natural at some value of the renormalization scale, they would become unnaturally large for slightly different values of this parameter. This problem, in different variations, has been addressed as the inconsistency of Weinberg’s approach, and alternative power counting schemes have been suggested (see, e.g., Refs. [11, 12, 14, 35, 38]).

In principle, the parameters of the EFT are determined in terms of QCD. The numerical values of the couplings within a given renormalization scheme as well as their renormalization-group (RG) behavior are uniquely fixed. In practice, we are still far from obtaining the values of the coupling constants from QCD and we are unaware of their true RG behavior, but we keep in mind that, in principle, they do exist. In actual calculations one can use different power counting schemes and calculate the RG behavior of the coupling constants. Given two power counting schemes and provided that the same renormalization condition has been used, the difference between the two (from the point of view of the RG behavior of the coupling constants) can only be that one approximates the true RG behav-
ior better than the other. There cannot be a fundamental problem in one scheme (like the coupling constants blowing up for a small change of renormalization scale) while it is absent in the other. It is only possible that the RG behavior analysis can be trusted in one scheme and is unreliable in the other because, say, the beta function is calculated perturbatively while the perturbation theory diverges badly. To be more specific, if the renormalized couplings are well-behaved within the KSW power counting \[11\] or the new power counting of Ref. \[35\], then they are also well-behaved in Weinberg’s power counting. As mentioned above, the actual RG behavior of the renormalized coupling constants has nothing to do with power counting.

To the best of our knowledge the RG behavior of the coupling constants has never been calculated analytically within Weinberg’s scheme. In practice, the couplings are fitted to experimental data. The reasonable success of Weinberg’s approach in describing the experimental data \[2,30,43\] suggests that the coupling constants should be well-behaved. As will be demonstrated below, the estimates of the contributions into beta functions also suggest that there is no reason to expect that the renormalized couplings become unnaturally large.

The aim of the present paper is to argue that Weinberg’s approach can be rigorously completed (or rather worked out in details) in such a way that it provides a systematic power counting for renormalized diagrams, allowing one to calculate physical observables to any given order in small quantities. For similar discussions, see also Refs. \[7,16,24,25,26,36\].

II. GENERAL CONSIDERATIONS ABOUT EFT

The motivation for doing EFT of the strong interactions is the understanding that on the one hand there exists a “fundamental” theory of the strong interactions, namely QCD,\(^1\) which on the other hand cannot yet be directly applied in the low-energy region. Let us recall the central features which comprise a consistent approach to EFT calculations:

1. Write down the most general possible Lagrangian, consistent with assumed symmetry principles \[1,2,3\].

2. Consider all Feynman diagrams which contribute to the process in question.\(^2\)

3. Since loop diagrams diverge, they are renormalized by absorbing the infinite parts into the redefinition of the fields and parameters of the most general Lagrangian. According to Refs. \[1,2,3\], the renormalization points should be chosen of the order of small external momenta (but for logarithmic divergences this is not strictly necessary). The practical way of performing the renormalization systematically is to rewrite the Lagrangian in terms of renormalized fields and parameters. The expansion of bare quantities in terms of renormalized coupling constants then produces the main interactions and counterterms.

4. Define the power counting: each renormalized diagram is assigned a definite power of small quantities such as small external momenta or the pion mass. For simplicity we

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\(^1\) QCD itself is almost certainly a leading-order approximation of some effective theory \[47\].

\(^2\) Note that, strictly speaking, one needs to analyze an infinite number of diagrams to decide which of them contribute to any given order of perturbation theory.
use the notion “renormalized diagram” for the sum of the unrenormalized value of the basic graph and the sum of the counterterm graphs.

5. The existence of a consistent power counting depends on the applied renormalization scheme.\(^3\)

6. If the renormalized coupling constants are natural for a given renormalization condition then the higher orders of small parameters are suppressed. In order to calculate a physical quantity to some specified order, identify all renormalized diagrams up to the given order and sum them up.\(^4\) The result is, in general, renormalization-scheme independent only up to the given order. This general feature of quantum field theories is also characteristic for EFT.

7. For processes with more than one nucleon one finds an infinite number of diagrams at any given order. To sum them up one writes down equations for regularized diagrams (Lippmann-Schwinger equation for NN, Faddeev equations for NNN, etc.) and renormalizes the solution so that its perturbative expansion reproduces the sum of an infinite number of renormalized diagrams. When solving equations one usually also sums up some sets of higher-order contributions, but that should not affect the accuracy of the result provided that the power counting is at work, i.e., the contributions of higher-order diagrams are indeed suppressed.

In most cases it is technically impossible to solve the regularized equations analytically and thus one is not able to remove all divergences explicitly. Rather, one solves the equations for the regularized expressions numerically, matches the low-energy coupling constants to experimental data, and keeps the regularization parameter finite. This procedure is as reliable as the subtractive renormalization, if it is possible to choose the regularization parameter so that the difference between the properly renormalized (subtracting every single divergence and removing the regularization) and the fitted (with finite regulator) expressions of physical quantities is of order higher than the given accuracy. This is in fact the case provided that the renormalized couplings are natural and the cutoff parameter is kept of the order of the characteristic large scale of the theory.\(^5\)

8. Although the EFT contains an infinite number of renormalized low-energy coupling constants, in principle, they are fixed by QCD. In practice, at any given order only a finite number of parameters contribute, which can be estimated using experimental information. The so determined values can be used in calculations of other quantities, i.e., one can make predictions at the given order.

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\(^3\) For example, even in the purely mesonic sector of chiral perturbation theory using, say, a cutoff regularization with unsuitable renormalization condition would destroy the traditional consistent power counting formulated within dimensional regularization accompanied by the \(\overline{\text{MS}}\) scheme [45].

\(^4\) In this paper it is understood that up to some order stands for up to and including this order.

\(^5\) An estimate for this scale is provided by the mass of the lightest particle which has been integrated out.
III. THE NN SCATTERING PROBLEM IN EFT

Once the effective Lagrangian is known, a calculation of the NN scattering amplitude within EFT requires to specify the regularization. It is preferable to use a regularization which preserves the symmetries of the theory (for a chiral-symmetry-preserving “cutoff” regularization of the nonlinear sigma model, see [49]). Otherwise one needs to take special care of symmetry-breaking effects generated by the regularization. We will assume that some kind of regularization has been introduced and proceed without specifying its exact form, using the term “cutoff” for the regularization parameter. Next one rewrites the Lagrangian in terms of renormalized quantities. The nucleon-nucleon scattering amplitude up to order $n$ in small expansion parameters is, in principle, obtained by summing up an infinite number of renormalized diagrams in the framework of old-fashioned (time-ordered) perturbation theory. This can, at least formally, be done by substituting the following potential (parameterized in terms of renormalized couplings) in the Lippmann-Schwinger (LS) equation

$$V^{(n)}(E, p', p) = \sum_{i=0}^{\infty} h_i^{(i+1)n+i} C_{i}^{(j)}(p', p) + \sum_{j=0}^{m} \sum_{i=0}^{n} V_i^{(j)}(E, p', p), \quad (1)$$

where we have displayed both its chiral as well as loop expansions (expansion in $\hbar$). The $C$ terms stand for NN contact interactions and the $V$ terms for all other contributions. The chiral expansion of the $C$ part of the NN potential contains an infinite number of terms, because the EFT is not renormalizable in the traditional sense. At any fixed order in the loop expansion, the chiral expansion contains a finite number of terms. This number can be calculated by counting the so-called overall (or superficial) degree of divergence of the loop diagrams which are generated by iterating the LS equation, and demanding that all these divergences are cancelled by counterterm contributions. The $V$ part of the potential contains a finite number of terms, i.e. $m = m(n)$ is a finite number for any finite $n$ and its functional form is determined by the power counting. All terms in Eq. (1) having chiral orders larger than $n$ are contributions of counterterms, generated by the loop expansion of the bare couplings of the NN contact interaction terms. In general, the $V$ part of the potential depends on the energy $E$, while this dependence has been eliminated from the $C$ part using the equation of motion or a field transformation [3, 50].

The amplitude $T^{(n)}(E, p', p)$ is obtained by solving the LS equation

$$T^{(n)}(E, p', p) = V^{(n)}(E, p', p) + \int \frac{d^3 q}{(2\pi)^3} V^{(n)}(E, p', q) G(E, q) T^{(n)}(E, q, p), \quad (2)$$

where $G$ is the free two-nucleon propagator. It is understood that the loop integration in the LS equation is regularized. If the regularization is introduced on the level of the EFT Lagrangian, then the LS equation is automatically regularized. The counterterms of the Lagrangian are fixed by demanding that the divergences originating from both the loop

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6 Unfortunately, dimensional regularization does not seem to be useful when working with equations except for some simple cases, which can be solved exactly.

7 The solution of the LS equation will also contain the contributions of an infinite number of higher-order renormalized diagrams.

8 One can also consider energy-independent potentials using a unitary transformation [20, 30].
diagrams of the potential and the iteration of the LS equation are cancelled. The finite parts of the counterterms are fixed by the choice of the renormalization condition (see item 3. of Sec. [II]). The net result of loop diagrams and corresponding counterterms depends on the chosen renormalization condition. This renormalization-scheme dependence is exactly compensated by the running of the renormalized couplings so that the scattering amplitudes remain renormalization-scheme independent up to the given order of accuracy.

In practice it is not feasible to specify the infinite number of terms contributing to the potential $V^{(n)}$ of Eq. (1). However, as we will argue below, this is not necessary. Instead of the potential of Eq. (1) one rather considers

$$V^{(n)}(E, p', p) = \sum_{j=0}^{n} c^{(j)}(p', p) + \sum_{j=0}^{n} v^{(j)}(E, p', p)$$

$$\equiv \sum_{j=0}^{n} \sum_{i=0}^{\infty} h^{i, (j)}(p', p) + \sum_{j=0}^{n} \sum_{i=0}^{m} h^{i, V^{(j)}}(E, p', p).$$

(3)

The second (non-contact-interaction) part coincides with the second part of Eq. (1) but the first part (purely contact-interaction terms) contains only terms up to order $n$ in the chiral expansion. Note that, in general, $c^{(n)}_{i}(p', p) \neq C^{(n)}_{i}(p', p)$ for $i \neq 0$. One substitutes $v^{(n)}$ into the LS equation, solves numerically, and fits the available free parameters to data.

Let us have a closer look at the implications of such an approach. The counterterm contributions in Eq. (3) are fixed so that all divergences generated by iterations of the LS equation with $V^{(n)}$, the coefficients of which are of chiral order up to $n$, are cancelled. The remaining divergent parts are of order $n + 1$ or higher in the chiral expansion. Moreover, all cutoff-dependent terms of order $n$ or less which vanish in the removed-regularization limit (i.e. depend on inverse powers of the cutoff) can also be removed using the $c^{(j)}$. The difference between the $C^{(j)}_{i}$ and $c^{(j)}_{i}$ of Eqs. (1) and (3), respectively, occurs due to the fact that the $c^{(j)}_{i}$ contain terms which absorb the inverse powers of the cutoff and, more importantly, the $C^{(j)}_{i}$ contain additional terms which absorb divergences generated by iterating higher-order counterterms. Choosing the counterterm contributions as specified above, one obtains the amplitude which, to order $n$, coincides term by term with the amplitude which was obtained by iterating the potential of Eq. (1). The difference between the two results depends on the cutoff parameter $\Lambda$ and contains contributions like

$$\sim \left(\frac{q^2}{\Lambda^2}\right)^i \text{ with } i > n/2 \text{ as well as } \sim \left(\frac{q^2}{Q^{2i+j}}\right)^j \text{ with } i > n/2, \ j > 0,$$

(4)

where $q$ denotes a small external momentum or the pion mass and $Q$ the characteristic large scale of the EFT. To keep both types of contributions suppressed, one has to choose $\Lambda \sim Q$. For an example in the context of contact interactions plus a one-pion-exchange potential in the $^1S_0$ channel, see Ref. [36].

In subtractively renormalized EFT the contributions of an infinite number of counterterms of Eq. (3) should have been taken into account so that all positive powers of the cutoff are cancelled. Then one should and also could consider the removed regularization limit. On the other hand, if one substitutes Eq. (3) in the LS equation and fits the available

\[ To practicalize this scheme (in numerical calculations) one could use Wilson’s renormalization-group approach [51]. For an application to NN-potential models, see Refs. [52, 53, 54, 55]. \]
parameters to physical quantities, then one approximates the rigorous result of subtractively renormalized EFT up to the given accuracy only if one takes $\Lambda \sim Q$. Taking instead $\Lambda \rightarrow \infty$, the second type of contributions of Eq. (4) will dominate the amplitude instead of being discarded (i.e. subtracted), because the corresponding counterterms are not included. Therefore, although it might be interesting to consider such kind of models they have little to do with EFT. In this context, one can encounter problems which actually are not related with EFT, like being unable to describe a positive effective range or different regularizations leading to different results.

It is worth noting that by taking the potential of Eq. (3), solving the corresponding LS equation, fitting the parameters to some set of physical quantities, and substituting these parameters in any remaining physical quantity, one automatically performs the absorption of the corresponding divergences. One should therefore keep in mind that what is fitted to physical quantities are not the renormalized couplings of $c^{(j)}_0$ but rather the bare couplings of $c^{(j)}$ in Eq. (3). On the other hand, the couplings which contribute in the second (non-contact-interaction) part of the potential are the renormalized couplings.

One could argue that the above discussion, although applied to an infinite number of diagrams, is still perturbative. There might be nonperturbative contributions to the solutions of the LS equation which have trivial perturbative expansions. While this is certainly possible, the EFT provides a systematic power counting only for those parts of the solutions which, if expanded, reproduce perturbative diagrams term by term. One should handle the nonperturbative EFT problem in a way which is consistent with perturbative expansions. While this is a necessary condition we are not in a position to argue whether or not it is, in general, also sufficient.

IV. ESTIMATING RENORMALIZED COUPLING CONSTANTS

In this section we will estimate some contributions to the running of the renormalized couplings within Weinberg’s approach which have been pointed out as a possible source of the problem of unnaturally large couplings [6, 14]. To address the issue of the renormalization-group behavior of the running couplings let us consider the leading-order potential

$$ V_0(p', p) = C - \left( \frac{g_A^2}{4 F^2} \right) \frac{(q \cdot \sigma_1 q \cdot \sigma_2) (\tau_1 \cdot \tau_2)}{q^2 + M^2}, $$

where $q = p' - p$, $C$ stands for a contact-interaction contribution in a particular spin-isospin channel, $g_A = 1.267$ is the axial-vector coupling constant, $F = 92.4$ MeV the pion decay constant, and $M_\pi = 139.6$ MeV the charged-pion mass. Substituting $V_0$ into the LS equation and iterating, (divergent) loop diagrams are generated. The divergences have to be absorbed by contributions of the counterterms. The coefficients of the divergent parts of the counterterms are closely related to the running behavior of the corresponding renormalized couplings.

Let us consider an example which has previously been used to demonstrate a problem with the renormalization-group behavior of the renormalized couplings [6, 14]. We substitute the potential of Eq. (5) for the $^1S_0$ channel into the LS equation, iterate twice, and consider the contribution proportional to $C^2 g_A^2/4 F^2$. In dimensional regularization its divergent part reads [14]

$$ -\frac{1}{\epsilon} \frac{g_A^2 M^2 m_N^2}{256 \pi^2 F^2} C^2, $$

where
where $m_N$ is the nucleon mass. This divergence has to be cancelled by a contribution of a counterterm generated by the loop expansion of a $D_0M^2_\pi$ contact interaction term, where $D_0$ is the bare coupling. Equation (6) leads to following running of the renormalized coupling

$$D(\mu) = D(\mu_0) + \frac{g^2_m \pi^2}{256\pi^2 F^2} C^2 \ln \left( \frac{\mu}{\mu_0} \right).$$  \hfill (7)

If we take $D(\mu_0)$ negligible and $\ln(\mu/\mu_0) \sim 1$, then $D$ is dominated by second term in Eq. (7). Comparing the $DM^2_\pi$ term with the leading-order contact-interaction term, we obtain

$$\frac{DM^2_\pi}{C} = \frac{g^2_m \pi^2 M^2_\pi}{256\pi^2 F^2} C \approx C \frac{M^2_\pi}{15} \approx \frac{M^2_\pi}{(430 \text{ MeV})^2},$$  \hfill (8)

where we made use of the estimate of $C \approx 1/(110 \text{ MeV})^2$ of Ref. [11]. Equation (8) suggests that the coupling $D$ does not get enhanced and there is no need to promote it to the leading order.

Next we consider the ladder diagrams, obtained by iterating the one-pion-exchange potential. To be specific, let us take the $2n$ loop diagram contributing to the $^{3}S_1 - ^{3}D_1$ channel NN scattering. One can estimate the coefficient of the logarithmically divergent part of the $2n$-loop diagram as

$$\sim \frac{g^2_A}{4F^2} \left( \frac{1}{(4\pi)^{3/2} 4F^2 m_N} \right)^{2n} (q^2)^n \approx \frac{1}{(146 \text{ MeV})^2} \left( \frac{q^2}{(1010 \text{ MeV})^2} \right)^n,$$  \hfill (9)

where $q$ stands for a small momentum. The contribution to the renormalized coupling of the $(q^2)^n$ term corresponding to Eq. (9) for $\ln(\mu/\mu_0) \sim 1$ is

$$\sim \frac{1}{(146 \text{ MeV})^2} \left( \frac{1}{1010 \text{ MeV}} \right)^{2n}.$$  

From the above estimates we cannot conclude that the renormalized couplings of higher-order contact interactions would become unnaturally large due to the fact that the corresponding counterterms contribute to the renormalization of loop diagrams of lower order in the chiral expansion.

V. CONCLUSIONS

We have argued that Weinberg’s approach to nuclear physics problems in the framework of EFT is free of conceptual inconsistencies, once the power counting is applied to renormalized diagrams as opposed to unrenormalized diagrams and counterterm contributions separately. In order to solve the regularized equations and to subsequently renormalize the solutions, in general, it is necessary to take the contributions of an infinite number of counterterms into account. Although, except for a few simple cases, it seems to be impossible to carry out this program, in practical calculations one does not actually need to do so. For example, for NN scattering one solves the regularized LS equation and fits the parameters of the effective Lagrangian (up to the given order) to physical quantities. The difference between the so obtained amplitudes and the exactly renormalized amplitudes is of higher order, provided the renormalized couplings are natural and the cutoff parameter is chosen of the order of the characteristic large scale of the EFT.
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