1. INTRODUCTION

Recent observations indicate that nearly all low- and intermediate-mass stars are born with massive circumstellar disks of gas and dust. Most young pre–main-sequence stars with ages of ∼1 Myr have gaseous disks with sizes of 100 AU or larger and masses of ∼0.01 M☉ (Beckwith 1999; Lada 1999). Many older main-sequence stars have dusty debris disks with sizes of 100–1000 AU (Aumann et al. 1984; Smith & Terrile 1984; Gaidos 1999; Habling et al. 1999; Song et al. 2001; Spangler et al. 2001). Current source statistics suggest that the percentage of stars with observable disks declines from ∼100% among the youngest stars to less than 10% for stars more than 1 Gyr old (Backman & Paresce 1993; Artymowicz 1997; Lada 1999; Lagrange, Backman, & Artymowicz 2000).

Models for the formation of our solar system naturally begin with a disk. In the 1700s, Immanuel Kant and the Marquis de Laplace proposed that the solar system collapsed from a gaseous medium of roughly uniform density (Kant 1755; Laplace 1796). A flattened gaseous disk—the protosolar nebula—formed out of this cloud. The Sun contracted out of material at the center of the disk; the planets condensed in the outer portions. Although other ideas have been studied since Laplace’s time, this picture has gained widespread acceptance. Measurements of the composition of the Earth, Moon, and meteorites support a common origin for the Sun and planets (e.g., Harris 1976; Anders & Grevesse 1989). Simulations of planet formation in a disk produce objects resembling known planets on timescales similar to the estimated lifetime of the protosolar nebula (Safronov 1969; Greenberg et al. 1978; Wetherill & Stewart 1993; Pollack et al. 1996; Alexander & Agnor 1998; Levison, Lissauer, & Duncan 1998; Kokubo & Ida 2000; Kortenkamp & Wetherill 2000; Chambers 2001).

The Kuiper belt provides a stern test of planet formation models. In the past decade, observations have revealed several hundred objects with radii of 50–500 km in the ecliptic plane at distances of ∼35–50 AU from the Sun (Jewitt & Luu 1993; Luu & Jewitt 1996; Gladman & Kavelaars 1997; Jewitt, Luu, & Trujillo 1998; Chiang & Brown 1999; Luu, Jewitt, & Trujillo 2000; Gladman et al. 2001). The total mass in these Kuiper belt objects (KBOs), ∼0.1 M☉, suggests a reservoir of material left over from the formation of our solar system (Edgeworth 1949; Kuiper 1951). However, this mass is insufficient to allow the formation of 500 km or larger KBOs on timescales of ∼5 Gyr (Fernández & Ip 1981; Stern 1995; Stern & Colwell 1997a; Kenyon & Luu 1998).

The Kuiper belt also provides an interesting link between local studies of planet formation and observations of disks and planets surrounding other nearby stars. With an outer radius of at least 150 AU, the mass and size of the Kuiper belt is comparable to the masses and sizes of many extrasolar debris disks (Backman & Paresce 1993; Artymowicz 1997; Lagrange et al. 2000). Studying planet formation processes in the Kuiper belt thus can yield a better understanding of evolutionary processes in other debris disk systems.

Making progress on planet formation in the Kuiper belt and the dusty disks surrounding other stars requires plausible theories that make robust and testable predictions. This paper reviews the coagulation theory for planet formation in the outer solar system (for reviews on other aspects of planet formation, see Mannings, Boss, & Russell 2000). After a short summary of current models for planet formation, I consider recent numerical calculations of planet formation in the Kuiper belt and...
describe observational tests of these models. I conclude with a discussion of future prospects for the calculations along with suggestions for observational tests of different models of planet formation.

2. BACKGROUND

Figure 1 shows the geometry of the outer part of our solar system. Surrounding the Sun at the center, four colored ellipses indicate the orbits of Jupiter (red), Saturn (cyan), Uranus (green), and Neptune (dark blue). The black ellipse plots the orbit of Pluto, which makes two orbits around the Sun for every three of Neptune. Roughly 20% of currently known KBOs, the Plutinos, have similar orbits. The black dots represent 200 KBOs randomly placed in the classical Kuiper belt, objects in roughly circular orbits outside the 3:2 resonance with Neptune. A few KBOs outside this band lie in the 2:1 orbital resonance with Neptune. The eccentric magenta ellipse indicates the orbit of one KBO in the scattered Kuiper belt (Luu et al. 1997). The total mass in classical KBOs is \( \sim 0.1 M_\odot \); the mass in scattered KBOs and KBOs in the 2:1 resonance may be comparable to but is not as well constrained as the mass in classical KBOs (Trujillo, Jewitt, & Luu 2001; Gladman et al. 2001).

Viewed edge-on, the orbits of the planets and the KBOs in our solar system lie in a disk with a height of \( \sim 20–30 \) AU and a radius of \( \sim 150–200 \) AU. Because a disk is the natural outcome of the collapse of a cloud with some angular momentum, this geometry formed the early basis of the nebular hypothesis. However, a cloud of gas and dust with the diameter of the Oort cloud, the mass of the Sun, and a modest rotation rate of \( \Omega \sim 10^{-3} \) yr\(^{-1} \) has too much angular momentum to collapse to the Sun’s present size. Building on previous realizations that a turbulent viscosity could move material inward and angular momentum outward through the protosolar nebula, von Weizäcker (1943, 1948) developed the basic physics of a viscous accretion disk and solved this angular momentum problem (see also Lüst 1952; Shakura & Sunyaev 1973; Lynden-Bell & Pringle 1974).

Most planet formation theories now begin with a viscous accretion disk (Kenyon 1999; Mannings et al. 2000 and references therein). The natural evolutionary timescale is the viscous timescale, which measures the rate at which matter diffuses through the disk,

\[
\tau_v \approx \frac{25,000 \text{ yr}}{\alpha} \left( \frac{A}{100 \text{ AU}} \right)^{5/4}.
\]  

This expression does not include a weak dependence on the mass of the central star. The viscosity parameter \( \alpha \) measures the strength of the turbulence relative to the local thermal pressure. Most studies of disks in interacting binaries and other objects indicate \( \alpha \sim 10^{-3}–10^{-2} \), which yields viscous timescales of 1–10 Myr at 100 AU (see Lin & Papaloizou 1995, 1996 for a review of the physics of accretion disks).

Another evolutionary timescale for the disk depends on an external source, the central star, instead of internal disk physics. Hollenbach et al. (1994) showed that high-energy photons from a luminous central star can ionize the outer skin of the gaseous disk and raise the gas temperature to \( \sim 10^4 \) K (see also Shu, Johnstone, & Hollenbach 1993; Richling & Yorke 1997, 1998, 2000). The thermal velocity of this gas is large enough to overcome the local gravity beyond \( \sim 10 \) AU for a 1 \( M_\odot \) central star. Material then leaves the disk, producing a bipolar outflow that may be observed in nearby star-forming regions (Bally et al. 1998; Johnstone, Hollenbach, & Bally 1998). Disk evaporation occurs on a timescale

\[
\tau_e \approx 10^7 \text{ yr} \left( \frac{M_\odot}{0.01 M_\odot} \right) \left( \frac{A}{10 \text{ AU}} \right) \left( \frac{\Phi_\odot}{10^{41} \text{ s}^{-1}} \right)^{-1/2},
\]

where \( \Phi_\odot \) is the flux of hydrogen-ionizing photons from the central star.

The evaporation time is sensitive to the spectral type of the central star. Early B-type stars with \( \phi_\odot \sim 10^{44} \text{ s}^{-1} \) can evaporate disks in \( \sim 1 \) Myr or less. The Sun has an observed flux of \( \Phi_\odot \sim 10^{38} \text{ s}^{-1} \) (Vernazza, Avrett, & Loeser 1981), which leads to a long evaporation time, \( \sim 3 \) Gyr, for a disk with \( A \sim 100 \) AU. However, young solar-type stars are 2–3 orders of magnitude brighter than the Sun at ultraviolet and X-ray wavelengths (e.g., Dorren, Guedel, & Guinan 1995 and references therein). The disk evaporation time for a young solar-type star is therefore \( \tau_e \approx 10–100 \) Myr for \( A \sim 10–100 \) AU.

The evaporation and viscous timescales provide a rough upper limit to the lifetime of a gaseous disk surrounding a solar-type
star. It is encouraging that both timescales are comparable to the disk lifetimes estimated from observations of gas and dust surrounding pre–main-sequence stars in the solar neighborhood, \( \tau_c \sim 1–10 \) Myr (Russell et al. 1996; Hartmann et al. 1998; Lada 1999; Brandner et al. 2000; Haisch, Lada, & Lada 2001). The observational timescales place strong constraints on planet formation models. Gas giants must form before the gas disappears. Rocky planets must form before the dust disappears. The observations constrain these timescales to 100 Myr or less.

Two theories—coagulation and dynamical instability—try to explain planet formation in a viscous disk. Coagulation theories propose that large dust grains decouple from the gas and settle to the midplane of the disk (Safronov 1969; Lissauer 1993). These grains may then collide to form successively larger grains (Weidenschilling 1980; Weidenschilling & Cuzzi 1993) or continue to settle into a very thin layer, which can become gravitationally unstable (Goldreich & Ward 1973). Both paths produce kilometer-sized planetesimals, which collide and merge to produce larger bodies (Weidenschilling 1984; Palmer, Lin, & Aarseth 1993). If the growth time is short compared to the viscous timescale in the disk, collisions and mergers eventually produce one or more “cores,” which accumulate much of the solid mass in an annular “feeding zone” defined by balancing the gravity of the planetary core with the gravity of the Sun and the rest of the disk (e.g., Chambers 2001; Rafikov 2001 and references therein). Large cores with masses of 1–10 \( M_{\oplus} \) accrete gas from the feeding zone (Pollack 1984; Pollack et al. 1996; Ikoma, Emori, & Nakazawa 2001). In our solar system, this model accounts for the masses of the terrestrial and several gas giant planets (Lissauer 1987; Lissauer et al. 1996; Weidenschilling et al. 1997; Levison et al. 1998; Bryden, Lin, & Ida 2000; Ida et al. 2000a; Inaba et al. 2001; Alexander, Boss, & Carlson 2001). Variants of this model, including orbital migration and other dynamical processes, can explain Jupiter-sized planets orbiting other solar-type stars (Weidenschilling & Marzari 1996; Lin & Ida 1997; Ward 1997; Ford, Rasio, & Sills 1999; Kley 2000). However, coagulation models barely succeed in making gas giant planets in 1–10 Myr, when observations suggest that most of the gas may be gone.

Dynamical instability models develop the idea that part of an evolving disk can collapse directly into a Jupiter-mass planet (e.g., Ward 1989; Cameron 1995; Boss 1997, 2000). When the local gravity overcomes local shear and pressure forces, part of the disk begins to collapse. Cool material flows into the growing perturbation and aids the collapse. Eventually, the perturbation reaches planet-sized proportions by accumulating all of the gaseous and solid material in the feeding zone. This model naturally forms large planets on timescales of \( 10^{-3}–10^3 \) yr, short compared to the evaporation or viscous timescales. However, dynamical instability models produce neither terrestrial planets in the inner disk nor icy bodies like Pluto in the outer disk. The disk mass required for a dynamical instability may also exceed the mass observed in pre–main-sequence disks (Beckwith 1999; Lada 1999).

The “minimum-mass solar nebula” is an important starting point to test these and other planet formation models (Hoyle 1946; Weidenschilling 1977a; Hayashi 1981; Lissauer 1987; Bailey 1994). The minimum mass is based on the near equality between the measured elemental compositions of the Earth, Moon, and meteorites (Anders & Grevesse 1989 and references therein) and the relative abundances of heavy elements in the Sun (see the discussion in Harris 1976; Alexander et al. 2001). This analysis leads to the hypothesis that the initial elemental abundances of the solar nebula were nearly identical to solar abundances. The surface density of the minimum-mass solar nebula follows from adding hydrogen and helium to each planet to reach a solar abundance and spreading the resulting mass uniformly over an annulus centered on the present orbit of the planet.

Figure 2 shows how the surface mass density varies with distance for the minimum-mass solar nebula. The arrows indicate the mass added to the terrestrial planets. The plot shows Venus, Earth, Jupiter, Saturn, Uranus, Neptune, and the Kuiper belt. When the material at the orbits of Venus and Earth is augmented to reach a solar abundance of hydrogen, the surface density for the gas follows the solid curve, \( \Sigma \approx \Sigma_0 (A/1 \text{ AU})^{-3/2} \), out to \( A \approx 10 \text{ AU} \) and then decreases sharply. The solid curve in Figure 2 has \( \Sigma_0 = 1500 \text{ g cm}^{-2} \); for comparison, Hayashi, Nakazawa, & Nakagawa (1985) concluded \( \Sigma_0 = 1700 \text{ g cm}^{-2} \) while Weidenschilling (1977a) proposed \( \Sigma_0 = 3200 \text{ g cm}^{-2} \). Following Hayashi (1981), the dot-dashed curve representing the mass density of solid material has

\[
\Sigma_0 = \begin{cases} 
7 \text{ g cm}^{-2} (A/1 \text{ AU})^{-3/2} & A \leq 2.7 \text{ AU}, \\
30 \text{ g cm}^{-2} (A/1 \text{ AU})^{-3/2} & A > 2.7 \text{ AU}.
\end{cases}
\]
The uncertainties in the coefficients are a factor of $\sim 2$. The change in the surface density of solid material at 2.7 AU corresponds to the region where ice condenses out of the gas in the Hayashi (1981) model. The location of this region depends on the disk structure (Sasselov & Lecar 2000).

The minimum-mass solar nebula was one of the great successes of early viscous accretion disk theories because steady state disk models often yield $\Sigma \propto A^{-3/2}$. The sharp decrease in the “observed” $\Sigma$ at 10–30 AU supports photoevaporation models where ionized hydrogen becomes unbound at $\sim 10$ AU (Shu et al. 1993). Current abundance measurements for the gas giants lend additional evidence: the gas-to-dust ratio appears to decrease with heliocentric distance in parallel with the surface density drop beyond 10 AU (Pollack 1984; Podolak, Young, & Reynolds 1985; Podolak & Reynolds 1987; Pollack et al. 1996). In the Kuiper belt, there may be two origins for the large drop in the observed surface density from a $\Sigma \propto A^{-3/2}$ model. Adding H and He to achieve a solar abundance at 30–40 AU increases the mass in the Kuiper belt by a factor of $\sim 30$. Material lost to orbital dynamics and to high-velocity collisions of objects in the belt may increase the current mass by another factor of 10–100 (e.g., Holman & Wisdom 1993; Davis & Farinella 1997), bringing the initial surface density in the Kuiper belt to $\sim 10^{-100}$ Myr at $\sim 40$ AU in a minimum-mass solar nebula (see below). The outermost gas giant, Neptune, must form on a similar timescale to accrete gas from the solar nebula before the gas escapes (e.g., Holman & Wisdom 1993; Davis & Farinella 1997), bringing the initial surface density in the Kuiper belt within range of the $\Sigma \propto A^{-3/2}$ line. If these estimates are correct, the total mass of the minimum-mass solar nebula is $M_{\odot}$ for an outer radius of $\sim 100$ AU, close to the median mass for circumstellar disks surrounding young stars in nearby regions of star formation (Lada 1999).

Figure 2 suggests that the Kuiper belt provides an important test of coagulation models. Forming objects with radii of $500–1000$ km requires $\sim 10–100$ Myr at $\sim 40$ AU in a minimum-mass solar nebula (see below). The outermost gas giant, Neptune, must form on a similar timescale to accrete gas from the solar nebula before the gas escapes (e.g., Holman & Wisdom 1993; Levison & Duncan 1993; Duncan, Levison, & Budd 1995; Malhotra 1996; Morbidelli & Valsecchi 1997). Neptune formation places another constraint on the KBO growth time; Neptune inhibits KBO formation at 30–40 AU by increasing particle random velocities on timescales of 20–100 Myr (Holman & Wisdom 1993; Levison & Duncan 1993; Duncan, Levison, & Budd 1995; Malhotra 1996; Morbidelli & Valsecchi 1997). Kenyon & Luu (1998, 1999a, 1999b) investigated how KBOs form by coagulation and compared their results with observations (see also Fernández 1997; Stern & Colwell 1997a, 1997b). The next section briefly describes the model results; § 4 compares these results with observations.

### 3. KUIPER BELT MODELS

#### 3.1. Coagulation Calculations

Safronov (1969) invented the current approach to planetesimal accretion calculations. In his particle-in-a-box method, planetesimals are a statistical ensemble of masses with a distribution of horizontal and vertical velocities about a Keplerian orbit (see also Greenberg et al. 1978; Ohtsuki, Nakagawa, & Nakazawa 1998; Wetherill & Stewart 1998; Spaute et al. 1991; Stern 1995; Kenyon & Luu 1998). Because $n$-body codes cannot follow the $10^{15}$ or more small planetesimals required in a typical coagulation calculation, the statistical approximation is essential. The model provides a kinetic description of the collision rate in terms of the number density and the gravitational cross section of each type of planetesimal in the grid. Treating planetesimal velocities as perturbations about a Keplerian orbit allows the use of the Fokker-Planck equation to solve for changes in the velocities due to gravitational interactions and physical collisions.

In our implementation of Safronov’s model, we begin with a differential mass distribution, $n(m)$, in concentric annuli centered at heliocentric distances, $r$, from a star of mass $M_*$ (Kenyon & Luu 1999a; Kenyon & Bromley 2001, 2002). The mass distribution has $N$ mass batches in each annulus; $\delta_i \equiv m_i/m$ is the mass spacing between batches. To provide good estimates of the growth time, our calculations have $\delta = 1.1–2.0$ (Ohtsuki & Nakagawa 1988; Wetherill & Stewart 1993), Poynting-Robertson drag (Burns, Lamy, & Soter 1979; Kary, Lissauer, & Greenzweig 1977b; Wetherill & Stewart 1993), and collective interactions such as dynamical friction and viscous stirring using a Fokker-Planck integrator (Stewart & Kaula 1980; Hornung, Pellat, & Barge 1985; Barge & Pellat 1990; Wetherill & Stewart 1993; Luciani, Namouni, & Pellat 1995; Ohtsuki 1999; Stewart & Ida 2000). The code reproduces previous calculations for accretion at 1 AU (Wetherill & Stewart 1993; Weidenschilling et al. 1997), collisional disruption of preexisting large KBOs at 40 AU (Davis & Farinella 1997), and $n$-body simulations of gravitational scattering at 1 AU (see Kenyon & Bromley 2001 and references therein).

During the early stages of planet formation, particle-in-a-box algorithms yield good solutions to the coagulation equation (Ida & Makino 1992; Kokubo & Ida 1996; Lee 2000; Malaspinas & Goodman 2001). Most published calculations have been made for a single accumulation zone to get a good understanding of the basic physics without spending a large amount of computer time (e.g., Greenberg et al. 1978; Ohtsuki et al. 2001; Wetherill & Stewart 1989; Stern 1996a; Kenyon & Luu 1998). Single-annulus calculations provide the basis for estimates of planetary growth rates as a function of heliocentric distance and initial disk mass (Lissauer 1987; Wetherill & Stewart 1993; Lissauer et al. 1996). Multiannulus calculations allow bodies in neighboring accumulation zones to interact and thus provide better estimates of planetary growth rates (Spaute et al. 1991; Kenyon & Bromley 2002). These codes enable calculations with additional physics, such as orbital migration,
which cannot be incorporated accurately into single-annulus codes. Once large objects form, one-on-one collisions become important; statistical estimates for collision cross sections and gravitational stirring in single- and multiannulus codes begin to fail. More detailed n-body calculations are then required to study the evolution of the largest objects.

In the following sections, I discuss published single-annulus models for KBOs and then outline new multiannulus calculations.

3.2. Single-Annulus Models

Our Kuiper belt models begin with an input cumulative size distribution

$$N_c \propto r^{-q},$$

(4)

with initial radii $r_i = 1$–80 m and $q_i = 3$. These particles are uniformly distributed in a single annulus with a width of 6 AU at 32–38 AU from the Sun. The total mass in the annulus is $M_0$; $M_0 \approx 10 M_\oplus$ for a minimum-mass solar nebula. All mass batches start with the same initial eccentricity $e_i$ and inclination $i_o = e_i/2$. The adopted mass density, $\rho_0 = 1.5$ g cm$^{-3}$, is appropriate for icy bodies with a small rocky component (Greenberg 1998). These bodies have an intrinsic tensile strength $S_p$, which is independent of particle size, and a total strength equal to the sum of $S_p$ and the gravitational binding energy (Davis et al. 1985; Davis, Ryan, & Farinella 1994). Kenyon & Luu (1999a, 1999b) describe these parameters in more detail.

Figure 3 shows the results of a complete coagulation calculation for $M_0 = 10 M_\oplus$, $e_0 = 10^{-3}$, and $S_0 = 2 \times 10^6$ ergs g$^{-1}$ (see also Stern 1996a; Stern & Colwell 1997a, 1997b). We separate the growth of KBOs into three regimes. Early in the evolution, frequent collisions damp the velocity dispersions of small bodies. Rapid growth of these bodies erases many of the initial conditions, including $q_i$ and $e_i$ (Kenyon & Luu 1998, 1999a, 1999b). These bodies slowly grow into 1 km objects on a timescale of 5–10 Myr ($M_i / 10 M_\oplus$) $^{-1}$. The timescale is sensitive to the initial range of sizes; because collisional damping is important, models starting with larger objects take longer to reach runaway growth. The linear growth phase ends when the gravitational range of the largest objects exceeds their geometric cross section. Gravitational focusing enhances the collision rate by factors of $(V_c/V_r)^2 \approx 10$–1000, where $V_c$ is the collision velocity and $V_r$ is the escape velocity of a merged object. The largest objects then begin “runaway growth” (e.g., Greenberg et al. 1978; Wetherill & Stewart 1993), during which their radii grow from $\sim 1$ to $\geq 100$ km in several Myr. During this phase, dynamical friction and viscous stirring increase the velocity dispersions of the smallest bodies from $\sim 1$ up to $\sim 40$ m s$^{-1}$. This velocity evolution reduces gravitational focusing factors and ends runaway growth. The largest objects then grow slowly to 1000+ km sizes on timescales that again depend on the initial mass in the annulus. Kokubo & Ida (1998) call this last phase in the evolution “oligarchic growth” to distinguish it from the linear and runaway growth phases (see also Ida & Makino 1993).

The shapes of the curves in Figure 3 show features common to all coagulation calculations (e.g., Wetherill & Stewart 1989; Stern & Colwell 1997a, 1997b; Weidenschilling et al. 1997; Davis, Farinella, & Weidenschilling 1999). Almost all codes produce two power-law size distributions. The merger component at large sizes has $q_f \approx 3$; the debris component at small sizes has $q_f = 2.5$ (Dohnanyi 1969; Tanaka, Inaba, & Nakazawa 1996). Dynamical friction produces a power-law velocity distribution in the merger component. The debris component has roughly constant velocity because it contains a small fraction of the initial mass. The transition region between the two components usually has a “bump” in the size distribution, where objects that can merge grow rapidly to join the merger population (Davis & Farinella 1997; Davis et al. 1999). Calculations for annuli closer to the Sun also yield a “runaway” population, a plateau in the size distribution of the largest objects (Wetherill & Stewart 1993). The objects in this plateau contain most of the mass remaining in the annulus (Wetherill & Stewart 1993; Weidenschilling et al. 1997). In our models, the largest 10–20 objects are not massive enough to produce a “runaway plateau” in the size distribution until the very late stages of the evolution (see below).

Our Kuiper belt calculations yield one result that is very different from coagulation calculations for annuli at less than 10 AU from the Sun. In all other published calculations, the
largest bodies contain most of the initial mass in the annulus. In the Kuiper belt, most of the initial mass ends up in 1 km objects. Fragmentation and gravitational stirring are responsible for this difference between calculations at 1–10 AU and at 40 AU. In our calculations, fragmentation produces a large reservoir of small bodies that damp the velocity dispersions of the large objects through dynamical friction. These processes allow a short runaway growth phase where 1 km objects grow into 100 km objects. Continued fragmentation and velocity evolution damp runaway growth by increasing the velocity dispersions of small objects and reducing gravitational focusing factors. Our models thus enter the phase of oligarchic growth earlier than models for planet growth at 1–10 AU. This evolution leaves ~1%–2% of the initial mass in 100–1000 km objects. The remaining mass is in 0.1–10 km radius objects. Continued fragmentation gradually erodes these smaller objects into dust grains that are removed from the Kuiper belt on short timescales, ~10^7 yr (Backman & Paresce 1993; Backman, Dasgupta, & Stencel 1995; Davis & Farinella 1997; Davis et al. 1999). Thus, in our interpretation, 100–1000 km radius objects comprise a small fraction of the original Kuiper belt.

Planet formation in the outer parts of a solar system is self-limiting. During the late stages of planetesimal evolution, large planets stir smaller objects up to the shattering velocity. This process leads to a collisional cascade, where planetesimals are ground down into smaller and smaller objects. Continued fragmentation, radiation pressure, and Poynting-Robertson drag then remove small particles from the disk faster than large objects can accrete. Because the shattering velocity depends on the tensile strength of a planetesimal, collisional cascades start sooner when planetesimals are weaker. The maximum mass of an icy object in the outer solar system thus depends on its strength (Fig. 4). At 35–140 AU, our calculations yield a linear relation between the maximum radius and the intrinsic strength of a planetesimal,

$$\log r_{\text{max}} \approx 2.45 - 0.09 \log \left( a / 35 \text{ AU} \right) + 0.22 \log S_0,$$

for planetesimals with $S_0 = 1–6$ (Kenyon & Luu 1999a).

The weak variation of $r_{\text{max}}$ with heliocentric distance is a new result based on calculations for this review. If planetesimals all have the same strength, the shattering velocity is independent of heliocentric distance. Once small planetesimals reach the shattering velocity, the largest objects do not grow. Because planetesimals at larger heliocentric distances are less bound to the central star, a massive planet at large $a$, stirs small planetesimals more effectively than the same planet in orbit at small $a$. Small planetesimals in the outer part of the disk thus require relatively less stirring to reach the shattering velocity than small planetesimals in the inner part of the disk. A less massive planet in the outer disk can stir planetesimals to the shattering limit as effectively as a more massive planet in the inner disk. Hence, larger objects form more easily in the inner disk than in the outer disk.

The initial mass $M_0$ is the main input parameter that establishes the formation timescale and the mass distribution of KBOs in the outer solar system. Figure 5 illustrates the time variation of the model parameter $r_5$, defined as the radius where the cumulative number of objects exceeds $10^5$ (Kenyon & Luu 1999a). Most surveys estimate $\sim 10^3$ KBOs with radii of 50 km or larger; the $r_5$ parameter thus provides a convenient way to compare theory with observations. Figure 5 shows that $r_5$ increases steadily with time during the linear growth phase. The number of 50 km radius KBOs increases dramatically during runaway growth and then approaches a roughly constant value during oligarchic growth. More massive models enter runaway growth sooner; the timescale for $r_5$ to reach 50 km is

$$\tau(r_5 = 50 \text{ km}) \approx 10 \text{ Myr } \left( M_0 / 10 M_\odot \right)^{-1}.$$  

More massive disks also produce more 50 km radius KBOs. Based on Figure 5, protosolar nebulae with less than 30% of the minimum mass produce too few 50 km radius KBOs; nebulae with more than $\sim 3$ times the minimum mass may produce too many.

The timescale for Pluto formation at 35 AU is also sensitive
3.3. Multiannulus Calculations

Multiannulus calculations address some of the limitations and uncertainties of coagulation models in a single accumulation zone (Spaute et al. 1991; Weidenschilling et al. 1997). By including long-range interactions between objects in neighboring annuli, a multiannulus code yields better treatment of velocity evolution and more accurate estimates for the accretion rates of large bodies. The improvement resulting from a multiannulus code scales with the number of annuli. Using more annuli allows a more accurate treatment of collision cross sections and velocity evolution (Kenyon & Bromley 2001, 2002).

To illustrate some results from our multiannulus code, I describe two calculations of large planetesimals in the Kuiper belt. The calculations begin with 0.1–1.0 km objects in 16 annuli at distances of 40–54 AU from the Sun. The planetesimals have an initial eccentricity $e_i = 2 \times 10^{-3}$ and a tensile strength $S_i = 2 \times 10^6$ ergs g$^{-1}$. The debris receives a small fraction $f_{KE} = 0.05$ of the impact kinetic energy. The calculations do not include gas drag or Poynting-Robertson drag. During the 1–5 Gyr of each calculation, drag forces have negligible impact on the evolution of objects with radii of 0.1 km or larger.

Figure 6 illustrates the time evolution of the size and horizontal velocity distributions for a model with fragmentation (Greenberg et al. 1984; Davis et al. 1985; Kenyon & Luu 1999a) and velocity evolution (Stewart & Ida 2000; Kenyon & Bromley 2001). During the first 20 Myr of this calculation, collisions damp the velocity dispersions of the smallest bodies. Planetesimals grow slowly from 1 to $\sim 10$ km. When objects are larger than $\sim 10$ km, gravitational focusing enhances collision rates. The largest objects then grow rapidly to sizes of $\sim 200$–$300$ km. Dynamical friction and viscous stirring heat up the orbits of the smallest objects. This evolution reduces gravitational focusing factors and ends runaway growth. A handful of large objects then grow slowly; their sizes reach $\sim 1000$ km at 70 Myr and $\sim 3000$ km at $\sim 120$ Myr.

The lower panel of Figure 7 illustrates the evolution of the
largest body in each annulus. Collisions are most rapid in the inner annuli; objects at 40 AU thus grow faster than objects at 50 AU. Runaway growth begins first at 40 AU (10–20 Myr), when objects at 55 AU have grown by less than a factor of 2. After 30 Myr, the largest objects at 40 AU have radii of \( \sim 100 \) km and then grow slowly to radii of \( \sim 1000 \) km during the oligarchic growth phase. This evolution is delayed at 50 AU. During runaway growth, objects at 50–55 AU grow from sizes of \( \sim 10 \) km at 30–50 Myr to \( \sim 100–200 \) km at 50–70 Myr. After \( \sim 100 \) Myr, the largest objects in all annuli grow slowly at roughly the same pace.

Objects grow much more slowly in models without fragmentation and velocity evolution (Fig. 7, top panel). During the first 300 Myr of the calculation, planetesimals grow slowly from \( \sim 1 \) to \( \sim 10 \) km. Because particle velocities are constant, gravitational focusing factors change little. Once particle sizes reach \( \sim 100 \) km, runaway growth begins in the innermost two annuli. A few large bodies rapidly accrete most of the mass in each annulus. At \( \sim 500 \) Myr, the largest body in the second annulus accretes the largest body in the first annulus and then consumes the rest of the bodies in annuli 1–5. Large bodies in annuli 6–8 begin runaway growth at \( \sim 700 \) Myr. A single large body in annulus 7 consumes all of the bodies in annuli 6–10. This process repeats for annuli 11–13 at \( \sim 1 \) Gyr, when a single large body in annulus 11 grows almost as large as the bodies in annuli 2 and 7. The remaining objects in annuli 14–16 probably form a fourth large object at \( \sim 1.3 \) Gyr; we terminated the calculation before this point.

Figure 8 compares the evolution of the largest bodies in each calculation. Collisional damping dominates the velocity evolution of small particles at 40–55 AU (see also Kenyon & Luu 1998, 1999a). Dynamical friction provides additional damping to the largest bodies. Smaller particle velocities produce larger gravitational focusing factors and more rapid growth rates. Models with velocity evolution thus enter the runaway growth phase earlier (\( \sim 10–30 \) Myr) than models without velocity evolution (\( \sim 300–500 \) Myr). During runaway growth, viscous stirring dom-
inates the velocity evolution of all particles. Larger particle velocities yield smaller gravitational focusing factors and smaller growth rates. By removing small particles from the grid, fragmentation reduces growth rates further. Thus, the largest bodies reach a maximum size that depends on the strength of the smallest bodies and the heliocentric distance (Fig. 4; see also Kenyon & Luu 1999a). In models without velocity evolution, gravitational focusing factors grow with the mass of the largest body. Thus, models without velocity evolution produce a few very massive objects. The orbital separation of these massive objects is roughly their gravitational range. In our calculations, this limit is \( \approx 2.4 \) Hill radii: \( R_H = \left( m_p / 3 M_0 \right)^{1/3} \), where \( m_p \) is the mass of the planet.

The results for KBO formation in these initial multiannulus calculations are encouraging. Successful KBO models need to form \( \approx 10^3 \) KBOs and at least one Pluto before Neptune attains its present mass at \( \approx 25 \) AU (e.g., Kenyon & Luu 1998, 1999a). If the gas in the solar nebula is depleted on timescales of 5–10 Myr, Neptune must form on similar timescales (Bryden et al. 2000). Some recent numerical calculations of gas accretion onto rocky cores can achieve this goal (Fernández & Ip 1984; Ip 1989; Pollack et al. 1996; Bryden et al. 2000). Although our Pluto formation timescale of 60–70 Myr is long compared to these constraints, single-annulus calculations starting from smaller bodies, 1–100 m in size, form Pluto and numerous KBOs on timescales of 10–20 Myr (Figs. 3 and 5; Kenyon & Luu 1999a). Scaling the single-annulus models suggests formation timescales of 5–20 Myr at 40–50 AU with a multiannulus code.

### 3.4. Long-Term Evolution

Several processes shape the long-term evolution of KBOs in the outer solar system (Holman & Wisdom 1993; Backman et al. 1995; Davis & Farinella 1997; Teplitz et al. 1999; Davis et al. 1999; Kuchner, Brown, & Holman 2002). Once Neptune attains its current mass and location, gravitational perturbations pump up orbital velocities and begin to remove KBOs of all sizes from the Kuiper belt. Gravitational stirring by the largest KBOs increases orbital velocities of smaller KBOs to the shattering limit. Once a collisional cascade begins, the largest objects do not grow significantly. Small objects are shattered and then removed from the Kuiper belt by radiation pressure and Poynting-Robertson drag.

To begin to understand how these processes have shaped the current population of KBOs, several groups have calculated the long-term collisional evolution of large objects in the Kuiper belt. Davis & Farinella (1997) used a single-annulus code to show that the observed population of KBOs with radii of 25–50 km or larger can survive disruptive collisions for 5 Gyr at 40–50 AU (Stern 1996b; Stern & Colwell 1997a, 1997b). These objects are thus remnants of the original population formed during the early evolution of the Kuiper belt. For reasonable values of \( S_\sigma \), smaller KBOs are collision fragments produced during the collisional cascade. Davis et al. (1999) confirmed these results. For \( t > 1 \) Gyr, the Davis et al. (1999) calculations yield a very steep power-law size distribution for the merger population, \( q_i \approx 11 \). This result differs from the results of single-annulus codes and the multiannulus result in Figure 6. The source of this difference is uncertain but may be due to different treatments of velocity evolution or fragmentation.

Figures 9 and 10 illustrate how the size distribution evolves at late times in our multiannulus calculations. The first model is a continuation of the calculation for Figure 6; the second model repeats this calculation for weak bodies with...
$S_o = 10^3$ ergs g$^{-1}$. The first 70–100 Myr of this second calculation closely follow the evolution of the first model. Runaway growth at 40–50 AU produces 10 km bodies in 20 Myr and 100 km bodies in 30 Myr. Slower oligarchic growth leads to 1000 km bodies at 70 Myr (40 AU) to 100 Myr (50 AU). At 70 Myr, the amount of debris produced from collisional erosion of the small bodies is negligible, $\sim$6% of the mass at the start of the calculation.

Models with strong, icy particles (Fig. 9; $S_o = 2 \times 10^4$ ergs g$^{-1}$) have a long oligarchic growth phase followed by a collisional cascade. The largest objects grow from $r \sim 1000$ km at $t = 70$ Myr to $r \sim 3000$ km at $t = 300$ Myr to $r \sim 6000$ km at $t = 1$ Gyr. This slow growth phase produces a power-law size distribution, with $q_i \approx 3.35$ for $r_i = 20–6000$ km. As the largest bodies reach sizes of 2000–6000 km, viscous stirring slowly increases the eccentricities of the smallest objects from $e_i \approx 0.01$ at 70 Myr to $e_i \approx 0.05$ at 300 Myr to $e_i \approx 0.09$ at $t = 1$ Gyr. Throughout most of this phase, collisions between small objects produce debris through cratering; this debris is $\sim$10% of the initial mass at 300 Myr and $\sim$70% of the initial mass at 1 Gyr. Cratering removes the bump in the size distribution for $r_i \sim 1$ km. Just before 1 Gyr, collisional disruption begins to deplete the population of 0.1–1 km bodies on timescales of $\sim$100 Myr. This evolution starts to produce a dip in the size distribution at $r_i \approx 1$ km.

Collisional cascades begin sooner in models with weak bodies (Fig. 10; $S_o = 10^3$ ergs g$^{-1}$). Cratering is not important in these models; most mass is lost through collisional disruption of small bodies. During the first 70 Myr of evolution, cratering is responsible for less than 1% of the mass loss; collisional disruption removes $\sim$6% of the initial mass. After 70 Myr, objects grow slowly as more and more material is lost to collisional disruption. The largest object has a radius of $\sim$2000 km at 300 Myr and $\sim$3000 km at 1 Gyr. The size distribution for the largest bodies follows a power law with $q_i \approx 3.15$ for $r_i = 20$ to 2000–3000 km. For smaller bodies, collisional disruption produces a pronounced dip in the size distribution at 0.3–1.0 km. The debris lost to bodies with $r_i < 0.1$ km (the smallest object in the grid) is $\sim$65% of the initial mass at 300 Myr and $\sim$85% of the initial mass at 1 Gyr.

These multiannulus calculations confirm some of the single-annulus results. The size of the largest object at 40–50 AU depends on the tensile strength of 0.1–10 km objects. Stronger small bodies allow the growth of larger large bodies (Fig. 4; Kenyon & Luu 1999a). We plan additional calculations to see whether the size-strength relation is similar to equation (5); preliminary results suggest a shallower relation. Gravitational stirring by 1000 km and larger objects in the grid leads to a collisional cascade, where cratering and collisional disruption remove small bodies from the grid (see also Davis & Farinella 1997; Davis et al. 1999; Stern & Colwell 1997a, 1997b). The duration of the collisional cascade is $\sim$100 Myr to $\sim$1 Gyr (see also Kenyon & Bromley 2001). Collisions convert $\sim$80%–90% of the initial mass in the grid to small particles with sizes of 100 m or smaller. Disruptive collisions and Poynting-Robertson drag can remove this material from the Kuiper belt on short timescales, $\sim$10–100 Myr (Backman et al. 1995; Stern 1996b; Teplitz et al. 1999).

During the late stages of our multiannulus calculations, the size distribution for the largest objects follows a power law with $q_i = 3.15–3.35$. Once the largest objects have radii of $\sim$1000 km or larger, the slope of the power-law size distribution is nearly invariant. We plan additional calculations to test the sensitivity of the slope to initial conditions and the fragmentation parameters.

4. OBSERVATIONAL TESTS OF COAGULATION MODELS

Observations provide powerful constraints on the KBO population. Sensitive imagers on large ground-based and space-based telescopes detect individual large objects directly. Current instrumentation yields direct detections of 50 km objects from the ground and 10 km objects from the Hubble Space Telescope (HST). Future large ground-based 30–100 m telescopes and the Next Generation Space Telescope will improve these limits by 1 order of magnitude or more. The population of smaller KBOs with radii of $\sim$1 km can be estimated indirectly from the frequency of short-period comets and from dynamical calculations. The population of KBOs with sizes smaller than $\sim$0.1 km can be derived only as an ensemble by measuring the surface brightness of the sky and eliminating other radiation sources. Despite confusion from the Galaxy and the local zodiacal, optical and far-infrared (far-IR) observations provide useful measures of the population of dust grains in the Kuiper belt.

These data allow broad observational tests of KBO formation models. The large sample of individual KBO detections provides a good measurement of KBO number counts, the number of KBOs per magnitude per square degree projected on the sky. For an adopted albedo $\omega$, for large KBOs, the number counts directly yield the KBO size distribution for objects with radii of 50 km or larger (Jewitt et al. 1998; Luu & Jewitt 1998; Chiang & Brown 1999; Gladman et al. 2001). The radial distribution of large KBOs follows from the size distribution and heliocentric distances derived from the orbit or from an adopted albedo (Dones 1997; Allen, Bernstein, & Malhotra 2001; Trujillo & Brown 2001; Trujillo et al. 2001). Surface brightness measurements constrain the size distribution of small KBOs. Far-IR data measure thermal emission from small grains in the Kuiper belt; optical and near-IR data measure scattered light. Deriving constraints on the size distribution from surface brightness data requires an assumption about the grain albedo $\omega$, which may differ from $\omega_o$.

4.1. Number Counts

The observed number counts of bright KBOs follow a simple relation

$$\log N = \alpha(R - R_o),$$

(7)
where \( N_c \) is the cumulative number of objects brighter than magnitude \( R \) (Gladman et al. 1998; Jewitt et al. 1998; Chiang & Brown 1999). Recent fits to the observations suggest \( \alpha = 0.65-0.70 \) and \( R_0 = 23.3-23.5 \) (Gladman et al. 2001; Trujillo et al. 2002). If the size distribution of KBOs is independent of heliocentric distance and if all KBOs have the same albedo, a power-law relation for the number counts implies a power-law size distribution,

\[
\log N_c = N_0 \left( \frac{r}{r_0} \right)^{-q},
\]

where \( N_0 \) is the cumulative number of objects with radius larger than \( r \) and \( q = 5\alpha \) (Jewitt et al. 1998; Chiang & Brown 1999; Gladman et al. 2001). Fits to the observations thus imply size distributions with \( q = 3.25-3.50 \). The characteristic radius \( r_0 \) is related to \( R_0 \); the scaling factor \( N_0 \) depends on the total mass in the Kuiper belt.

Kenyon & Luu (1999b) show that complete coagulation calculations produce power-law size distributions for large KBOs. For a wide range of input parameters, single-annulus models yield \( q = 2.75–3.25 \) for KBOs with radii of \( \approx 10–1000 \) km (see Table 2 of Kenyon & Luu 1999b). To construct predicted number counts, Kenyon & Luu (1999b) adopt \( \omega = 0.04 \) and the slope parameter, \( g = 0.15 \), in the standard two-parameter magnitude relation for asteroids (Bowell et al. 1989). An adopted heliocentric distance \( d \) and a random phase angle \( \beta \) from the Sun then specify the observed \( R \) magnitude for a KBO with radius \( r \). The slope parameter \( g \) relates the brightness of an asteroid at solar phase angle \( \beta \) to the brightness at opposition, \( \beta = 0^\circ \). Kenyon & Luu (1999b) assume that the KBO size distribution is independent of heliocentric distance, with 50% of the KBOs in a ring at 42–50 AU and the rest as Plutinos at 39.4 ± 0.2 AU. The resulting number counts are insensitive to the Plutino fraction and the outer radius of the ring.

The upper panel of Figure 11 compares predicted with observed number counts from several single-annulus calculations. Data are as indicated in the legend. Error bars for the measured points are typically a factor of 2–3 and are not shown for clarity. The curves plot predicted number counts for models with \( e_g = 10^{-3} \) and \( M_0 \approx 0.3 \) (dot-dashed curves), 1.0 (solid curves), and 3.0 (dashed curves) times the minimum-mass solar nebula. Models with different \( e_g \) are indistinguishable for \( R \leq 27 \) (Kenyon & Luu 1999b). The model luminosity functions agree well with current observations.

Multiannulus calculations also produce power-law size distributions for large KBOs (Figs. 6, 9, and 10). For several completed calculations, these models yield steeper slopes, \( q = 3.2–3.5 \), for the size distribution of objects with radii of \( 10–1000 \) km. These results are much closer to the observed slopes than the multiannulus calculations of Davis et al. (1999).

We plan additional multiannulus calculations to measure the scatter in the predicted slope of the size distribution. To construct an initial model for the number counts, I use radial distributions of KBOs derived from the coagulation code and adopt \( e = 0 \) and \( \beta = 0 \) for all sources. This model assumes that all sources are found at opposition and neglects bright KBOs closer than 40 AU.

The lower panel of Figure 11 compares observed number counts with predictions for several multiannulus calculations. The data are the same in both panels. The lines show predicted number counts for multiannulus models with an initial mass in solids of a minimum-mass solar nebula with \( e_g = 2 \times 10^{-3} \) and \( S_0 = 2 \times 10^{10} \) ergs g\(^{-1}\) (solid curve: 70 Myr; dashed curve: 1 Gyr) and \( S_0 = 10^7 \) ergs g\(^{-1}\) (dotted curve: 1 Gyr).

2002 PASP, 114:265–283
number counts follow a linear relation between log \( N \) and \( R \) (eq. [7]) with \( \alpha = 0.80 \pm 0.01 \) and \( R_0 = 22.45 \pm 0.05 \). After 1 Gyr, the slope of the number counts is \( \alpha = 0.65 \pm 0.02 \), much closer to the value derived from the data, \( \alpha = 0.65–0.70 \) (Gladman et al. 2001). The normalization derived for the models, \( R_0 = 21.95 \pm 0.10 \), is roughly a magnitude larger than the measured \( R_0 = 23.0–23.5 \). However, these models do not include loss of KBOs by dynamical interactions with Neptune. At 40–50 AU, these dynamical losses range from ~50% to ~80% of the initial mass in the Kuiper belt (e.g., Holman & Wisdom 1993; Levison & Stern 1995). Applying these losses to our 1 Gyr number count models yields \( R_0 \approx 22.70–23.70 \), passably close to the observed value.

### 4.2. KBOs and Olbers’s Paradox

Many KBOs are too faint to be detected as individual objects even with large telescopes. All together, these faint KBOs can produce a detectable diffuse background light. Optical and near-IR data measure the amount of scattered light from faint KBOs; far-IR and submillimeter data measure the amount of thermal emission. The KBO background light is smaller than diffuse emission from the local zodiac (Leinert et al. 1998) and has not been detected (Backman et al. 1995; Stern 1996b; Teplitz et al. 1999). Nevertheless, the upper limits on scattered and thermal emission provide interesting constraints on the population of small KBOs.

Measured optical and far-IR sky surface brightnesses demonstrate that KBO number counts cannot follow equation (7) to arbitrarily faint magnitudes. For equation (7) with \( \alpha > 0.4 \), the optical sky surface brightness of KBOs brighter than magnitude \( R \) is (Kenyon & Windhorst 2001)

\[
\mu_R = 41.03–2.5 \log \left( \frac{\alpha}{\alpha - 0.4} \right) + (1 - 2.5\alpha)(R - R_0). \tag{9}
\]

This surface brightness exceeds the measured sky surface brightness in the ecliptic plane,\(^1 \mu_R \approx 22 \text{ mag arcsec}^{-2}, \) at \( R \approx 45–55 \) for \( \alpha \approx 0.6–0.75 \) (Windhorst, Mathis, & Keel 1992; Windhorst et al. 1994; Windhorst, Keel, & Pascalelle 1998; Biretta et al. 2000; Kenyon & Windhorst 2001). For an adopted albedo \( \omega_c \), and temperature \( T_{\text{KBO}} \), the thermal background from small KBOs depends only on the optical surface brightness

\[
I_{\text{FIR}} \approx 9.5 \times 10^{17–0.4\mu_R} T_{\text{KBO}}^{-1} \left( \frac{1 - \omega_c}{\omega_c} \right) \text{ Jy sr}^{-1}. \tag{10}
\]

This result assumes that a small KBO emits less radiation than the maximum flux of a blackbody with temperature \( T_{\text{KBO}} \). For \( \omega_c \approx 0.5 \), small KBOs with \( \mu_R \approx 22 \text{ mag arcsec}^{-2} \) and \( T_{\text{KBO}} \approx 40 \text{ K} \) (Backman et al. 1995; Teplitz et al. 1999) have \( I_{\text{FIR}} \approx 4 \times 10^{7} \text{ Jy sr}^{-1} \). This limit exceeds the measured far-IR background of \( I_{\text{FIR}} \leq (1–2) \times 10^{5} \text{ Jy sr}^{-1} \) for wavelengths longer than ~10 \( \mu \text{m} \) (Fixsen et al. 1998; Hauser et al. 1998). The known, finite sky brightnesses at optical and far-IR wavelengths thus imply a turnover in the KBO number counts for \( R \geq 30 \) (Kenyon & Windhorst 2001).

Previous support for a turnover in the KBO number counts has relied on theoretical interpretations of available observations (see Weissman & Levison 1997). From numerical simulations, Levison & Stern (1995) show that KBOs can excite an eccentricity in the Pluto-Charon orbit. If perturbations from KBOs are the dominant source of the eccentricity, the measured \( e \) yields an upper limit to the number of KBOs with radii of 20–300 km. Orbital integrations of known Jupiter-family comets suggest an origin in the Kuiper belt (Duncan, Quinn, & Tremaine 1988; Levison & Duncan 1994; Duncan et al. 1995; Duncan & Levison 1997; Ip & Fernández 1997). If the Kuiper belt is the source of all Jupiter-family comets, the number of known Jupiter-family comets and lifetimes derived from the orbital integrations provide limits on the number of KBOs with radii of 1–10 km. These limits indicate that there are a factor of 10 fewer KBOs with radii of 1–100 km than suggested by a simple extrapolation of equation (7) to \( R \geq 27 \).

The coagulation calculations provide more theoretical support for a turnover in the number counts. Models with fragmentation predict two power-law size distributions, a merger population with \( q = 3 \) at large radii and a debris population with \( q = 2.5 \) at small radii (Figs. 6, 9, and 10; Stern & Colwell 1997a; Davis et al. 1999; Kenyon & Luu 1999a). The transition radius depends on the tensile strength of small objects. For \( S_0 \sim 10^{1–10} \text{ ergs g}^{-1} \), this radius is \( \sim 1–100 \text{ km} \) (Davis & Farinella 1997; Davis et al. 1999; Kenyon & Luu 1999a), which agrees with the turnover radius derived from dynamical constraints.

To place another constraint on the turnover radius, Kenyon & Windhorst (2001) construct a physical model for the surface brightness of small KBOs. They adopt a broken power-law size distribution,

\[
N_c(r) = \begin{cases} 
    n_0(r/r_0)^{-q_1} & r > r_0 \\
    n_0(r/r_0)^{-q_2} & r \leq r_0,
\end{cases} \tag{11}
\]

and assume that objects lie in a ring around the Sun with surface density \( \Sigma \propto A^{-7} \). The ring has an inner radius \( A_1 = 40 \text{ AU} \) and an outer radius \( A_2 = 50 \text{ AU} \). The optical counts set \( n_0 \) and \( q_1 \). For an adopted \( \omega_c \), \( \mu_R \) results from a sum over all objects projected into a box with an area of 1 arcsec\(^2\). For thermal emission, Kenyon & Windhorst (2001) adopt the Backman & Paresce (1993) relations to derive grain temperatures as a function of \( A \) and sum the thermal emission from all objects in a solid angle of 1 sr.
Kenyon & Windhorst (2001) demonstrate a clear turnover in the KBO number counts (Fig. 12). Small KBOs with radii of 1 μm to ~1 km must have a size distribution with $q \sim 3.4$ or less to satisfy the known limits on the sky surface brightness at optical and far-IR wavelengths.

Figure 12 shows how the optical and 100 μm surface brightness increase with fainter KBO $R$-band magnitude. Solid curves show results when all objects have $\omega_p = 0.04$; dot-dashed curves show how the surface brightness changes when the albedo varies smoothly from $\omega_p = 0.04$ for $r \geq 1$ km to $\omega_p = 0.5$ for $r \leq 0.1$ km. Larger albedos produce brighter optical surface brightnesses and a fainter far-IR surface brightness. For models with $q_p = 3.5$, KBOs with a small constant albedo have a limiting $\mu_p \sim 24.5$ mag arcsec$^{-2}$, fainter than the observed sky brightness. If small KBOs have $\omega_p \approx 3.5$ and a large albedo, the predicted $\mu_p$ exceeds the observed background at $R \sim 70$ mag. This limit corresponds to objects with $r \sim 0.03$ mm. In both cases, the far-IR surface brightness exceeds the measured sky brightness for $\lambda \leq 240$ μm at $R \approx 65$–70 mag. The predicted far-IR surface brightness lies below measured limits at longer wavelengths (see also Backman et al. 1995; Teplitz et al. 1999).

A direct detection of diffuse light from KBOs would begin to provide more stringent tests of coagulation models. Measurements of the variation of the diffuse light with ecliptic latitude or longitude would yield the scale height and orbital distribution of small KBOs. The sensitivity of archival deep HST WFPC2 images can improve constraints on the KBO optical background by a factor of 10. The Space Infrared Telescope Facility may improve the far-IR constraints by a similar factor. The Next Generation Space Telescope will provide direct detections of individual KBOs near the proposed knee in the size distribution at $R \approx 28$–31 mag and more accurate background measurements in the optical and near-IR. These and other facilities will yield better tests of model predictions for the size distribution of small KBOs.

4.3. Radial Distribution of KBOs

The radial distribution of KBOs provides direct constraints on several physical processes in the outer solar system. KBOs in the $2:1, 3:2, 5:3$ and other orbital resonances yield information on dynamical interactions between small bodies and gas giant planets (Holman & Wisdom 1993; Duncan et al. 1995; Hahn & Malhotra 1999; Kuchner et al. 2002). KBOs in the scattered Kuiper belt allow tests of models for the formation of the Oort comet cloud. KBOs in the classical Kuiper belt constrain the initial surface density and the formation history of large objects. Here I concentrate on the radial distribution of classical KBOs, where coagulation models can offer some insight into the observations.

The observed radial distribution of KBOs in the classical Kuiper belt is uncertain. Secular resonances with Neptune and Uranus truncate the inner edge of the classical Kuiper belt at ~41 AU (Duncan et al. 1995). Because the first surveys detected no KBOs outside 50 AU, Dones (1997) proposed an outer edge to the classical Kuiper belt at ~50 AU. Several large-angle surveys for KBOs provide support for an abrupt outer edge at 48–50 AU (Jewitt et al. 1998; Allen et al. 2001; Trujillo et al. 2001). Trujillo & Brown (2001) analyze discovery data for the apparent magnitude and heliocentric distance of all KBOs and derive an outer edge at 47 ± 1 AU. They conclude that plausible variations of the slope of the size distribution, the maximum radius, and the albedo cannot produce the observed edge. Gladman et al. (2001) note that recent, unpublished surveys identify distant KBOs more frequently than older surveys and conclude that the radial distribution of KBOs may continue smoothly beyond 48 AU.

Coagulation theory provides some explanations for possible origins of an outer edge to the observed radial distribution of classical KBOs. Because the formation timescale for large objects depends on the orbital period, the size of the largest object is a
sensitive function of heliocentric distance, \(a\). For \(t < 100\) Myr, multiannulus calculations yield \(r_{\text{max}} \propto a^{-3}\) (Fig. 7). This result implies a factor of 2 variation in the size of the largest object from \(a = 40\) AU to \(a = 50\) AU. After 100 Myr, the difference can be (1) enhanced if gravitational stirring by large objects at the inner edge of the Kuiper belt prevents the growth of objects farther out in the belt or (2) diminished if gravitational stirring by Neptune and other giant planets preferentially slows the growth of large objects at the inner edge of the belt.

The size distribution is an important factor in understanding the reliability of an edge in the observed radial distribution of KBOs. Monte Carlo simulations of the observations demonstrate that the edge is more robust for shallower input size distributions (Jewitt et al. 1998; Allen et al. 2001; Trujillo & Brown 2001; Gladman et al. 2001). For the \(q = 3\) power-law size distribution derived from single-annulus coagulation models, the edge is much more obvious than for the \(q = 3.25\) power law derived from multiannulus calculations. Several test calculations suggest that \(q\) grows with \(a\). If this conclusion holds with additional calculations, the coagulation models favor steeper size distributions at larger distances in the Kuiper belt. If this variation is real, the evidence for an outer edge to the Kuiper belt is more questionable.

Unless the tensile strength of objects decreases with \(a\), the variation of \(r_{\text{max}}\) with \(S_0\) from equation (5) is sufficient to yield a large variation in the radial distribution of KBOs. If \(S_0\) is independent of \(a\), \(r_{\text{max}}\) changes by less than 30% at 40–50 AU. Trujillo & Brown (2001) show that this small change cannot produce the observed lack of KBOs beyond 48 AU. A factor of 10 change of \(S_0\) at 40–50 AU can produce factor of 2–3 changes in \(r_{\text{max}}\). Because the magnitude of \(S_0\) for KBOs is not well known, quantifying changes of \(S_0\) with other variables in the model is pointless. Deriving tensile strengths of different comet families might help to quantify possible variations of \(S_0\) with \(a\) (see below).

To make an initial theoretical prediction for the radial distribution of classical KBOs based on the coagulation models, I use the number counts for multiannulus models from Figure 11 at 1 Gyr. The model assumes circular orbits but does not include collisional or dynamical evolution from 1 Gyr to the present. If this evolution is independent of \(a\), then the model provides a reasonable first approximation to the present situation in the outer solar system. For simplicity, I quote the result of this model as a ratio, \(N(40–47\text{ AU})/N(47–54\text{ AU})\). For a limiting magnitude \(R = 27\), the multiannulus model with \(S_0 = 2 \times 10^6\) ergs g\(^{-1}\) has \(N(40–47\text{ AU})/N(47–54\text{ AU}) = 3\); the model with \(S_0 = 10^5\) ergs g\(^{-1}\) has \(N(40–47\text{ AU})/N(47–54\text{ AU}) = 2\). Because of small number statistics, models with brighter limiting magnitudes produce unreliable results. Because deeper surveys sample more of the size distribution, the number ratio declines as the limiting magnitude increases.

The results of the coagulation models suggest some caution in the interpretation of the apparent edge in the radial distribution of KBOs beyond 47 AU. Factor of 2–3 declines in the apparent number of KBOs with \(a\) are a natural outcome of coagulation models when the input surface density follows a minimum-mass solar nebula. Larger changes are possible if the surface density declines more rapidly or with plausible changes to KBO properties as a function of \(a\). Larger surveys to \(R = 28\) or deeper surveys to \(R = 29–30\) should yield better statistics to discriminate among the possibilities.

A robust comparison between the models and observations is difficult because of uncertain observational biases and uncertain long-term dynamical evolution of the initial KBO population. Most KBO surveys concentrate on regions near the ecliptic plane, where the success rate is larger; distant KBOs may have a different inclination distribution from nearby KBOs (Brown 2001). The model estimates are smaller than the observed fraction, \(N(40–47\text{ AU})/N(47–54\text{ AU}) \approx 4–6\) (Trujillo & Brown 2001; Gladman et al. 2001). The model assumes no migration in \(a\) and no changes in \(i\) from 1 to 5 Gyr; dynamical models show that interactions with Neptune and other gas giant planets change \(a\), \(e\), and \(i\) on short timescales.

### 4.4. Orbital Elements of KBOs

The distributions of \(e\) and \(i\) yield information on the long-term dynamical evolution of KBOs. Numerical integrations of KBO orbits indicate that dynamical interactions with the gas giant planets dramatically change the orbital elements of objects in the outer solar system (e.g., Torbett & Smoluchowski 1990; Holman & Wisdom 1993; Duncan et al. 1995; Malhotra 1996; Levison & Duncan 1997; Morbidelli & Valsecchi 1997, Kuchner et al. 2002). This gravitational sculpting of the KBO population produces several dynamical KBO populations, including classical KBOs, Plutinos and other resonant KBOs, and scattered KBOs (e.g., Malhotra 1995; Gladman et al. 2001). Understanding how these phenomena produce the current \(e\) and \(i\) distributions of KBOs remains a major puzzle.

Coagulation calculations provide an important foundation for understanding the distributions of KBO orbital elements. Because the giant planets are also condensing out of the solar nebula, dynamical interactions between KBOs and gas giants are unimportant during the early stages of KBO growth. Collisional damping and dynamical friction thus set the early velocity evolution of the KBO population. These processes produce nearly circular orbits for large objects, \(e \approx 0.001\) for \(r \approx 100–1000\) km, and modestly eccentric orbits for smaller objects, \(e \approx 0.01\) for \(r \approx 10\) km (Figs. 9 and 10). Once the collisional cascade begins, viscous stirring dominates the velocity evolution. The orbits of all objects become more eccentric and more highly inclined. After \(\sim 1\) Gyr, large objects in the multiannulus calculations have \(e = 0.02\) for \(S_0 = 10^7\) ergs g\(^{-1}\) models and \(e \approx 0.1\) for \(S_0 = 2 \times 10^6\) ergs g\(^{-1}\) models. Both models have \(i e \approx 0.4\). These results indicate that KBOs probably have significant \(e\) and \(i\) without dynamical interactions with gas giant planets.

This conclusion is probably insensitive to initial conditions in
the Kuiper belt. Kenyon & Bromley (2001) show that 100–500 km objects can stir up velocities significantly on timescales of 1–5 Gyr in a minimum-mass solar nebula. Thus, large KBOs with sizes of 500–1000 km can stir up other KBOs to large $e$ and $i$ on a 1 Gyr timescale. If KBOs form in a low-mass solar nebula, stirring timescales are longer, $\sim$5 Gyr for 1000 km objects with 10% of the minimum mass and $\sim$50 Gyr for 1000 km objects with 1% of the minimum mass. Thus, our scenario for producing KBOs in a minimum-mass solar nebula leads to KBOs with large $e$ and $i$. Models that form massive KBOs in a low-mass solar nebula yield KBOs with low $e$ and $i$.

These results indicate that gravitational sculpting and the internal dynamics of KBOs are important in creating the current distributions of $a$, $e$, and $i$ for KBOs. Viscous stirring between large KBOs broadens the $e$ and $i$ distributions with time; gravitational sculpting by the gas giants broadens the $e$ and $i$ distributions and selects stable ranges of $e$ and $i$. Careful treatment of both processes is necessary to understand the current orbital elements of KBO populations.

5. DISCUSSION AND SUMMARY

The discovery of the Kuiper belt in the 1990s provides fundamental constraints on models for the formation and evolution of planets in the outer parts of our solar system. The observations imply $\sim$10$^5$ KBOs with radii of 50–500 km and a total mass of $\sim$0.1–0.2 $M_\oplus$ beyond the orbit of Neptune. The theoretical challenge is to understand the formation of large objects in a current reservoir of material that is $\sim$1% of the initial mass in the solar nebula. This goal assumes that KBOs formed locally and that the initial surface density of the solar nebula did not decease abruptly beyond the orbit of Neptune. Observations indicate typical disk radii of at least 100–200 AU in nearby pre-main-sequence stars, which suggests that the disk of our solar system originally continued smoothly beyond the orbit of Neptune. Testing the assumption of local KBO formation relies on future comparisons between observations and theory.

Coagulation calculations appear to meet the challenge posed by KBOs. Published numerical calculations demonstrate that the formation of a few Plutos and numerous 100–500 km KBOs in the outer parts of a solar system is inevitable (Stern 1995, 1996a; Stern & Colwell 1997a; Davis et al. 1999; Kenyon & Luu 1999a). For a variety of initial conditions, collisions between small bodies at 30–50 AU naturally produce larger objects. Once there is a range in sizes, dynamical friction efficiently reduces the orbital eccentricities of the largest objects. Large objects in nearly circular orbits grow quickly. At 30–50 AU, runaway growth can produce 100 km and larger objects on short timescales. These objects then grow slowly to radii of 1000 km or more.

The initial disk mass sets the timescale for Pluto formation in the outer parts of a solar system. Objects grow faster in more massive disks. For single-annulus calculations of planetesimals orbiting the Sun, the timescale to produce the first Pluto is

$$t_\text{p} \approx 20 \text{ Myr} \left( \frac{\Sigma_{55}}{0.2 \text{ g cm}^{-2}} \right)^{-1}.$$  \hspace{1cm} (12)

where $\Sigma_{55} \approx 0.2 \text{ g cm}^{-2}$ is the initial surface density of a minimum-mass solar nebula extrapolated into the Kuiper belt at $\sim$35 AU (Fig. 2; see also Stern & Colwell 1997a; Kenyon & Luu 1999a). This timescale depends weakly on the initial conditions. Growth is more rapid in a solar nebula with small initial eccentricities and with small initial bodies (Kenyon & Luu 1999a).

The growth timescale in the Kuiper belt is smaller than expected from coagulation calculations in the inner solar system. Lissauer et al. (1996) estimate a timescale to produce Moon-sized ($10^{26}$ g) objects as

$$t_\text{M} \approx 0.5 \text{ Myr} \left( \frac{1 \text{ g cm}^{-2}}{\Sigma(a)} \right) \left( \frac{a}{1 \text{ AU}} \right)^{3/2}.$$  \hspace{1cm} (13)

This relation implies timescales of $\sim$500 Myr at 35 AU and $\sim$1 Gyr at 45 AU. Our single-annulus models yield $t_\text{M} \sim 100$ Myr at 35 AU and $t_\text{M} \sim 600$ Myr at 70 AU. For calculations where the initial size distribution is composed of 1–10 km bodies, multiannulus models imply $t_\text{M} \sim 200$–300 Myr at 40–50 AU. Collisional damping causes the difference between our results and equation (13). In our calculations, collisional damping between small objects with radii of 1 m to 1 km reduces eccentricities by factors of 5–10. Dynamical friction couples the eccentricity reduction of the small bodies to the largest bodies. Because runaway growth begins when gravitational focusing factors are large, collisional damping in our Kuiper belt models leads to an early onset of runaway growth relative to models of the inner solar system where the collisional evolution of small bodies is not important.

Once large objects form in the outer part of a solar system, they stir up the velocities of small objects with radii of 10–100 km or less. Velocity stirring retards growth and produces debris. When the collision energy of small bodies is comparable to their tensile strength, the small bodies undergo a collisional cascade where planetesimals are ground down into smaller and smaller objects. This process produces numerous small grains that are ejected by radiation pressure ($\ll$1–3 $\mu$m grains) or pulled toward the Sun by Poynting-Robertson drag ($\gg$1–3 $\mu$m grains). These grains are lost on short timescales of 1 Myr or less. When the collisional cascade begins, most of the mass in the outer solar system is contained in small objects that are easy to fragment. The collisional cascade thus robs the larger bodies of material. Because collisional cascades start sooner in the evolution when bodies are weaker, the size of the largest object in a calculation depends on the tensile strength of the small planetesimals. Our models yield Earth-sized objects in $\sim$200–300 Myr at 40–50 AU.
the Kuiper belt for \( S_0 = 2 \times 10^4 \) ergs g\(^{-1}\) and Pluto-sized objects for \( S_0 = 10^2 - 10^3 \) ergs g\(^{-1}\).

The theoretical models thus resolve the dilemma of large objects in a low-mass Kuiper belt. Runaway growth of small objects at 40–50 AU in the solar nebula places \( \sim 5\%–10\% \) of the initial mass in large objects with radii of 50–500 km or larger. The collisional cascade converts \( 80\%–90\% \) of the initial mass into debris that is removed from the Kuiper belt on short timescales. Over the 4.5 Gyr lifetime of the solar system, gravitational interactions between KBOs and the gas giant planets can remove \( \sim 50\%–80\% \) of the remaining mass. Given the uncertainties, collisions and dynamics appear capable of removing more than \( 90\% \) of the original mass in the Kuiper belt.

The observed size distribution of KBOs provides strong observational tests of this picture. The final size distribution of a Kuiper belt calculation has three components. The merger component at large sizes is a power law with \( q_s \approx 3.0–3.5 \); the debris component at small sizes is a power law with \( q_s \approx 2.5 \). The collisional cascade depletes objects with intermediate sizes of 0.1–10 km. Depletion produces a dip in the size distribution for \( S_0 \approx 10^5 \) ergs g\(^{-1}\).

The observations of large KBOs generally agree with the power-law slope predicted for the merger component. The data are consistent with \( q_s = 3.3–3.5 \); the multiannulus models predict \( q_s = 3.15–3.35 \). If dynamical interactions and collisional evolution continue to remove KBOs from the 40–50 AU annulus after 1 Gyr, the predicted number of KBOs is within a factor of 2 of the observed number of KBOs. The multiannulus calculations produce more KBOs with radii of 1000 km or larger than are observed with current surveys. The predicted number of these large objects depends on \( S_0 \) and is therefore uncertain. The observed number of large objects is plagued by small number statistics. Future surveys will provide robust constraints on the population of large objects. Improved multiannulus coagulation calculations that include dynamical interactions with gas giant planets will improve the predictions.

Current constraints on the population of small KBOs are also consistent with model predictions. The data indicate a turnover in the KBO number counts, which implies a turnover in the size distribution for small objects. The derived turnover radius of 0.1–10 km is close to theoretical predictions. Better observations of the optical and far-IR surface brightnesses of the Kuiper belt can provide better estimates of the slope of the size distribution for KBOs with radii of 1 mm to 1 m. Observations with larger telescopes may detect the turnover radius directly.

Measuring the tensile strengths of comets provides an interesting test of this picture of KBO formation. In our models, the formation of Pluto by coagulation requires a tensile strength \( S_0 \approx 400 \) ergs g\(^{-1}\). Large tensile strengths, \( S_0 \approx 10^5 \) ergs g\(^{-1}\), allow the formation of large bodies, \( \sim 2000–3000 \) km, which have not been detected in the outer solar system. Because objects with radii of 2000–3000 km can form before Neptune reaches its current mass, the lack of large KBOs implies \( S_0 \approx 10^4 \) ergs g\(^{-1}\) in the coagulation theory. Estimates on the tensile strength derived from comet Shoemaker-Levy 9, \( S_0 \approx 10^3–10^4 \) ergs g\(^{-1}\) (e.g., Greenberg, Mizutani, & Yamamoto 1995), are close to the lower limit required to form Pluto. Theoretical estimates have a much larger range, \( S_0 \approx 10^2–10^6 \) ergs g\(^{-1}\) (Sirono & Greenberg 2000). As theoretical estimates improve and observations of disrupted comets become more numerous, these results can constrain the coagulation models.

The coagulation calculations demonstrate that planet formation in the outer parts of other solar systems is also inevitable. The mass of a minimum-mass solar nebula is comparable to the median disk mass derived for nearby pre–main-sequence stars (Beckwith 1999; Lada 1999; Mannings et al. 2000). The formation timescale for a 1000 km planet at 30–50 AU in one of these disks is therefore \( \sim 10–30 \) Myr. Although this planet cannot be observed directly, gravitational stirring leads to a collisional cascade and copious dust production. In the multiannulus models, dust is produced at a rate of roughly 0.1–1 Earth mass every 100 Myr (see also Kenyon & Bromley 2002).

Observations of nearby debris disk systems are consistent with dust produced in a planet-forming disk. The sizes of debris disks, \( \sim 10–100 \) AU, are similar to the radius of the Kuiper belt. The ages of the youngest debris disk systems are comparable to the Pluto formation timescale of \( \sim 10–20 \) Myr (LaGrange et al. 2000). If the timescale for Poynting-Robertson drag sets the residence time for 1 \( \mu \)m and larger dust grains in the disk, the instantaneous dust mass in the disk is \( \sim 0.1–1 \) lunar masses. This mass is comparable to the dust masses inferred from IR observations of debris disk systems such as \( \alpha \) Lyr and \( \beta \) Pic (Backman & Paresce 1993; Lagrange et al. 2000). Finally, the duration of the collisional cascade in our Kuiper belt models, \( \sim 100 \) Myr to \( \sim 1 \) Gyr, is similar to the estimated lifetimes of debris disk systems, \( \sim 500 \) Myr (Habing et al. 1999, 2001).

Kenyon & Bromley (2001) derive a similar predicted lifetime for debris disk systems from the coagulation equation (see also Kenyon 2000).

To make the connection between KBOs and debris disks more clear, Kenyon et al. (1999) investigate planet formation in the dusty ring of HR 4796A (Jayawardhana et al. 1998; Koerner et al. 1998; Augereau et al. 1999; Schneider et al. 1999; Greaves, Mannings, & Holland 2000). They show that a planetesimal disk with a mass of 10–20 times the mass of the minimum-mass solar nebula can form a dusty ring at \( 70 \) AU on 10–20 Myr timescales, comparable to the estimated age of HR 4796A. The model ring has a radial optical depth \( \sim 1 \), in agreement with limits derived from infrared images and from the excess infrared luminosity. Although the initial mass in this single-annulus calculation is large, multiannulus calculations suggest similar timescales with smaller masses.

Finally, multiannulus calculations are an important new tool in developing a robust model for planet formation. Current computer technology allows practical multiannulus calculations that cover roughly a decade in disk radius. We are thus...
1–2 orders of magnitude from constructing model grids of complete solar systems. Faster computers should resolve this difficulty in the next few years and allow us to consider the interfaces between (1) gas giants and terrestrial planets and (2) gas giants and the Kuiper belt. With some limitations, current multiannulus calculations promise predictions for the radial variation of the disk scale height (Kenyon & Bromley 2001) and the disk luminosity (Kenyon & Bromley 2002) as a function of stellar age, disk mass, and other physical parameters. Detailed comparisons between these predictions and observations of debris disks will yield interesting constraints on the physics of planet formation in other solar systems. Applying these results to our solar system will provide a better idea of how the Earth and other planets in our solar system came to be.

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