High-energy density implications of a gravitoweak unification scenario

Roberto Onofrio$^{1,2,\ast}$

$^1$Dipartimento di Fisica e Astronomia 'Galileo Galilei', Università di Padova, Via Marzolo 8, Padova 35131, Italy

$^2$ITAMP, Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

(Dated: December 16, 2014)

Abstract

We discuss how a scenario recently proposed for the morphing of macroscopic gravitation into weak interactions at the attometer scale affects our current understanding of high-energy density phenomena. We find that the Yukawa couplings of the fundamental fermions are directly related to their event horizons, setting an upper bound $y_f \leq \sqrt{2}$ for their observability through gauge interactions. Particles with larger Yukawa couplings are not precluded, but should interact only gravitationally, providing a natural candidate for dark matter. Furthermore, the quantum vacuum contribution to the cosmological constant is reduced by several orders of magnitude with respect to the current estimates. The expected running of the Newtonian gravitational constant could provide a viable alternative scenario to the inflationary stage of the Universe.

PACS numbers: 04.80.Cc, 12.10.Kt, 04.60.Bc

\*Electronic address: onofrior@gmail.com
Recently, we have proposed a conjecture in which weak interactions should be considered as the microscopic counterpart of macroscopic gravitation \[1\]. This conjecture relies upon a quantitative relationship between the Fermi constant of weak interactions \(G_F\) and a renormalized Newtonian universal gravitational constant \(\tilde{G}_N\) which satisfy

\[G_F = \sqrt{2} \left( \frac{\hbar}{c} \right)^2 \tilde{G}_N.\]  

This expression holds provided that we choose \(\tilde{G}_N = 1.229 \times 10^{33} \text{G}_N = 8.205 \times 10^{22} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}\). Models for a gravitoweak unification have been discussed in the past using formal arguments \[2–7\], and a possible running of the Newtonian gravitational constant in purely four-dimensional models has been recently proposed \[6, 8\]. In \[1\], we have focused the attention on consequences of this conjecture at low and intermediate energies, with particular emphasis on anomalous gravitational contributions to bound states with size in the femtometer to picometer distance range. In following contributions, we have developed a quantitative attempts to explain the so-called “proton radius puzzle” arising from high-precision spectroscopy on muonic hydrogen \[9\]. We discuss here features of this morphing with implications for scattering states such as in high-energy experiments at colliders and in cosmic rays observatories, and finally comment on its interplay with the cosmological constant problem. This contribution should be considered as part of a more extended program, pioneered by Amelino-Camelia and collaborators, whose aim is to develop quantum gravity with a strong phenomenological content, allowing for a faster constructive feedback from Nature on the otherwise virtually infinite ways to unfold this subfield if no connection to observable reality is actively pursued \[10\].

The starting point of our analysis is that if the relevant parameter describing gravity in the microworld is the renormalized Newtonian gravitational constant \(\tilde{G}_N\), then the Schwarzschild radii of the elementary particles become quantities of phenomenological interest, as they are boosted by 33 orders of magnitude with respect to the standard scenario using \(G_N\). For instance, the Schwarzschild radius of the top quark should become

\[R_s^{(t)} = 2\tilde{G}_N m_t/c^2 = 5.67 \times 10^{-19} \text{ m},\]

a lengthscale not far from those currently explored at the highest energy colliders. Insights on the physical consequences of these large Schwarzschild

\[1\] This relationship differs by a factor 2 from Eq. 2 in \[1\] since the Planck mass has been chosen as the one corresponding to the equality between the Compton wavelength and the Schwarzschild radius, i.e. 
\[\hbar/(M_P c) = 2G_N M_P/c^2,\] the factor 2 in the Schwarzschild radius having being instead omitted in \[1\].
radii may be gained by considering a dimensionless parameter $\eta_f = R^{(f)}_s/\tilde{\Lambda}_P$, allowing to compare the Schwarzschild radius of a generic fundamental fermion $R^{(f)}_s = 2\tilde{G}_N m_f/c^2$ to the lengthscale of quantum vacuum fluctuations of space-time, i.e. the renormalized Planck length $\tilde{\Lambda}_P = (2\hbar\tilde{G}_N/c^3)^{1/2} = 8.014 \times 10^{-19}$ m. Considering that the renormalized Planck energy $\tilde{E}_P$ coincides with the Higgs vacuum expectation value $v$ as commented in [1], $\tilde{E}_P = (\hbar c^5/(2\tilde{G}_N))^{1/2} = v$, we have the following chain of relationships

$$\eta_f = \frac{R^{(f)}_s}{\tilde{\Lambda}_P} = \left(\frac{2\tilde{G}_N}{\hbar c}\right)^{1/2} m_f = \left(\frac{\sqrt{2} c G_F}{\hbar^3}\right)^{1/2} m_f = \frac{m_f c^2}{v} = y_f \sqrt{2},$$

where we have introduced the Yukawa coupling coefficient $y_f$ such that the mass of a fundamental fermion is written as $m_f = y_f v/(\sqrt{2} c^2)$. We assume that the parameter $\eta_f$ determines the possibility for a fundamental fermion to communicate information via gauge bosons only if its horizon is smaller than the renormalized Planck length, that is $\eta_f \leq 1$. In this framework it becomes natural that the Yukawa coefficients are small, as $R^{(f)}_s < \tilde{\Lambda}_P$ implies $y_f < \sqrt{2}$ for all particles observable through their non-gravitational effects. If the Schwarzschild radius exceeds the Planck length, i.e. $y_f > \sqrt{2}$, a particle should behave like a black hole, therefore gauge bosons emitted and reabsorbed via virtual vacuum effects will be unable to propagate beyond the horizon, and only gravitational effects may be detectable, such as the ones currently attributable to dark matter.\(^2\)

This has direct implications for high-energy physics experiments since the discovery of new massive states of mass larger than 246.22 GeV/c\(^2\), if using high-energy physics detectors relying on non-gravitational interactions, should be precluded. Moreover, even existing

\(^2\) If the Compton wavelength of a gauge boson is much larger than the Schwarzschild radius of the particle emitting or absorbing the gauge boson, the latter does not play any role for the boson-exchange, as in usual quantum field theory in flat spacetime, and the range of the mediated interaction is established by the Compton wavelength, for instance infinite range in the case of the photon, attometer range in the case of the $W^\pm$ and $Z^0$ bosons, so gauge bosons do not have any horizon limitation for their virtual propagation. In the opposite situation, instead, we expect the emission of virtual gauge bosons to be strongly suppressed and limited to a range determined by the Schwarzschild radius of the involved particle, rather than the Compton wavelength of the boson itself, as well-known for the simplest, celebrated case of photons which allowed to define black holes themselves. The possibility of emitting gauge bosons via Hawking radiation is not considered here since for particles with mass $m > v/c^2$ the temperature of the Hawking radiation is not consistently defined (or, equivalently, the Hawking radiation may be reliably estimated only for energy scales much smaller than the one of quantum gravity, which in our setting means for Hawking temperatures $T_H << v/K_B$). On the stability of black holes at the Planck scale see [11–13].
fundamental fermions will be affected by this constraint at energies large enough. The minimum distance between two colliding particles is related to the center-of-mass energy $E_{cm}$, in the relativistic limit, as $\delta x \simeq \frac{hc}{E_{cm}}$, i.e. the corresponding de Broglie wavelength. If this distance is smaller than the Schwarzschild radius the particles will be confined within the horizon and afterward will not be able to propagate beyond it apart from subleading Hawking effect propagation via quantum tunneling. This may happen in spite of the fact that the energy-momentum dispersion relationship is violated, something absolutely possible in a quantum gravity regime in which we do not expect necessarily on-shell states. Therefore, for collisions between fundamental particles with center-of-mass energy larger than $E_{s}^{(f)} = \frac{hc}{R_{s}^{(f)}}$ we expect inhibition of scattering at large momentum transfers of order $q^2 \simeq \left(\frac{\hbar}{R_s}\right)^2$.

As seen in Table I, this progressively affects particles of decreasing mass, so the first to be hit by this inhibition should be the top quark. This could be seen, in the cleanest environment provided by an electron-positron collider (or an equivalent $\mu^+\mu^-$ machine) in the 2.2 TeV range, as a sudden drop in the normalized cross-section $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ from its expected value (without QCD corrections) of 5 in the range $175 \text{ GeV} < E_{cm} < 2.2 \text{ TeV}$, to a value of $11/3$ for $E_{cm} > 2.2 \text{ TeV}$, since the top quark is no longer an active degree of freedom. Other possible implications are in the high energy component of the cosmic rays and in high-energy neutrinos observatories, with limitations to very high-energy flux. For instance, muons with energy $E_{\mu \text{thr}} = E_{s}^2/(2m_p c^2) \simeq 7 \times 10^{12} \text{ GeV}$ will reach their event horizon limit when interacting with protons, and therefore we expect inhibition of their measurable flux for energies higher than the threshold energy $E_{\mu \text{thr}}$. We also notice, see last column in Table I, that the Schwarzschild energy density of fermions is inversely proportional to the square of their mass, so lightest particles have higher Schwarzschild energy density than heaviest ones. This is suggestive of interpreting the weak decays of fundamental fermions as induced by gravitational collapses to states with progressively larger gravitational binding energy.

The discussion above should be complemented by considering, for charged fermions, the Reissner-Nordström radius defined as $R_q = (q^2 G_N/(4\pi\epsilon_0 e^4))^{1/2}$. The condition for not emitting gauge bosons for a charged particle is obtained whenever the Planck length is smaller than either the Schwarzschild or the Reissner-Nordström radii. The Reissner-Nordström radius is equal to $R_s = q \times 4.83 \times 10^{-20} \text{ m}$ where $q$ is expressed in elementary charge.
that electromagnetic interaction for a particle endowed with an elementary charge unit, $M > 2/3$ if $q = 0$, the corresponding value of the Schwarzschild radius, its ratio to the renormalized Planck length $\tilde{\Lambda}_P$, the value of the energy $E_s$ at which the Schwarzschild radius becomes equal to the de Broglie wavelength, and the Schwarzschild energy density of the fermion $\rho_s^{(f)}$, assuming that it is confined in a spherical volume $V_s^{(f)}$ of radius $R_s^{(f)}$, with energy in units of TeV and distance in units of attometer. The top quark, due to its large mass and proximity to the renormalized Planck scale, is the most promising one to look for effects related to the event horizon in collider physics.

units, so $q = (0, \pm 1/3, \pm 2/3, \pm 1)e$ for fundamental fermions. This gives a condition on the fermion mass, and in particular, it turns out that $R_s > R_q$ for a mass $M > qM_0/2$, where $M_0$ is the mass for which the renormalized gravitational interaction equals the electromagnetic interaction for a particle endowed with an elementary charge unit, i.e. such that $\tilde{G}_N M_0^2 = c^2/(4\pi\epsilon_0)$, corresponding to $M_0 = [c^2/(4\pi\epsilon_0\tilde{G}_N)]^{1/2}$ = 31 GeV. This means that $R_s > R_q$ for quarks with charge $q = \pm 1/3$ if $M > 5.16$ GeV/$c^2$, for quarks with $q = \pm 2/3$ if $M > 10.33$ GeV/$c^2$, and for charged leptons with charge $q = \pm 1$ if $M > 15.5$ GeV/$c^2$, where $q$ expressed in elementary charge units. The relevant event horizon is $R_s$ for the top quark alone, while all other quarks and charged leptons have $R_s < R_q$, and the neutrinos should obviously have $R_q = 0$.

We now sketch some considerations on the interplay between the renormalized Newtonian

| $m_f$         | $R_s^{(f)}$ (m) | $R_s^{(f)}/\tilde{\Lambda}_P = y_f/\sqrt{2}$ | $E_s^{(f)} = hc/R_s^{(f)}$ (TeV) | $\rho_s^{(f)} = m_f c^2/V_s^{(f)}$ (TeV/am$^3$) |
|---------------|----------------|---------------------------------------------|---------------------------------|----------------------------------------------|
| $e$ 0.511 MeV/c$^2$ | $1.66 \times 10^{-24}$ | $2.07 \times 10^{-6}$ | $7.45 \times 10^5$ | $2.65 \times 10^{10}$ |
| $\mu$ 105.66 MeV/c$^2$ | $3.44 \times 10^{-22}$ | $4.29 \times 10^{-4}$ | $3.60 \times 10^3$ | $6.20 \times 10^5$ |
| $\tau$ 1.777 GeV/c$^2$ | $5.78 \times 10^{-21}$ | $7.21 \times 10^{-3}$ | 214.29 | $2.19 \times 10^3$ |
| $u$ 2.75 MeV/c$^2$ | $8.95 \times 10^{-24}$ | $1.12 \times 10^{-5}$ | $1.38 \times 10^5$ | $9.16 \times 10^8$ |
| $d$ 5.5 MeV/c$^2$ | $1.79 \times 10^{-23}$ | $2.23 \times 10^{-5}$ | $6.92 \times 10^4$ | $2.29 \times 10^8$ |
| $s$ 95 MeV/c$^2$ | $3.09 \times 10^{-22}$ | $3.86 \times 10^{-4}$ | $4.01 \times 10^3$ | $7.67 \times 10^5$ |
| $c$ 1.25 GeV/c$^2$ | $4.06 \times 10^{-21}$ | $5.08 \times 10^{-3}$ | 304.63 | $4.43 \times 10^3$ |
| $b$ 4.70 GeV/c$^2$ | $1.53 \times 10^{-20}$ | $0.019$ | 81.02 | $3.14 \times 10^2$ |
| $t$ 174.2 GeV/c$^2$ | $5.67 \times 10^{-19}$ | $0.707$ | 2.186 | 0.228 |

TABLE I: Impact on the Schwarzschild radius $R_s$ of the renormalized Newtonian constant of gravitation $\tilde{G}_N$. In each row we report the fundamental fermions with known mass (no neutrinos), the corresponding value of the Schwarzschild radius, its ratio to the renormalized Planck length $\tilde{\Lambda}_P$, the value of the energy $E_s$ at which the Schwarzschild radius becomes equal to the de Broglie wavelength, and the Schwarzschild energy density of the fermion $\rho_s^{(f)}$, assuming that it is confined in a spherical volume $V_s^{(f)}$ of radius $R_s^{(f)}$, with energy in units of TeV and distance in units of attometer.
constant $\tilde{G}_N$ and the cosmological constant. As already outlined in [1] the identification of the Fermi constant with a renormalized Newtonian universal constant via fundamental constants $\hbar$ and $c$ allow to identify Fermi and Planck scales as identical, $\tilde{E}_P = v$, where $\tilde{E}_P$ and $v$ are respectively the renormalized Planck constant and the vacuum expectation value of the Higgs field, sometimes called the Fermi scale, then avoiding any hierarchy issue. Another longstanding issue of the current interface between cosmology and high-energy physics - the contribution of quantum fluctuations to the cosmological constant - is also mitigated. With the new definitions of the Planck energy and the Planck length $\tilde{\Lambda}_P = \hbar c / v$, the vacuum energy density at the Planck scale is written as

$$\rho_P = \frac{\tilde{E}_P}{\tilde{\Lambda}_P^3} = \frac{c^7}{4\hbar G_N^2} = \frac{v^4}{(hc)^3} = 0.48 \text{ TeV}/\text{am}^3,$$

which represents the ratio between the vacuum expectation energy of the Higgs field $v$ and a confinement volume with size equal to the associated Compton wavelength $\hbar c / v$. Due to the larger value of $\tilde{G}_N$, this results in a vacuum density at the Planck scale $1.5 \times 10^{66}$ smaller than the one estimated with the currently assumed Planck scale using the macroscopic, measurable value of the Newtonian constant. This is still a large contribution both with respect to a zero value for a null cosmological constant and with respect to the value inferred from the interpretation of the data on the SNIa events in terms of an accelerating Universe (see however [17, 18] for a critical analysis of this interpretation). Nevertheless, the issue of the largeness of the cosmological constant contribution due to the quantum fields at least now coincides with the issue of the largeness of the vacuum density energy due to the Higgs field [16, 19, 20]. It is possible, as remarked in [16], that a large nonminimal coupling between the Higgs and the space-time curvature of the form $\xi R \phi^2$, with $\xi$ the related coupling constant, $R$ the Ricci scalar and $\phi$ the Higgs field, may conspire to end up with an effective cosmological constant much smaller then the one evaluated above (see [21], [22], and [23] for first attempts to determine bounds of curvature-Higgs couplings in astrophysics, gravitation, and high-energy physics, respectively).

Finally, we compare our insights with two relevant directions already pursued, emphasizing analogies and differences. First, intriguing scenarios in which the Newtonian gravitational constant becomes instead zero at small distances have been proposed [24, 26]. These models remove any issue of compatibility of structureless, point-like particles with a finite horizon since the Schwarzschild radius goes to zero along with the Newtonian constant.
While predictions and their observation will be required to rule out either this asymptotic freedom scenario or the one presented in this paper, we notice that if setting a null Newtonian gravitational constant at small distances an explanation of the tentative acceleration of the Universe seems rather unnatural, since at earlier times the retaining effect of gravity would have been weaker than at later times. Vice versa, in our setting tentative accelerating stages of the Universe such as inflation could be at least partially attributed to a progressive decrease of the Newtonian gravitational constant as the expansion goes on, with the decreased gravitational retention mimicking an effective acceleration. Second, in a series of pioneering papers, Markov has discussed the possibility of an upper bound to the mass of elementary particles (so-called maximons) and its cosmological consequences. This direction has been later pursued by Kadyshevsky and collaborators. While sharing some features, in our proposal the Planck scale is shifted down at the Fermi scale, and we do not interpret this scale as an upper bound to the mass of elementary particles, rather as a bound to the mass of particles with other than gravitational interactions.

In conclusion, we have performed a preliminary analysis of what we consider the most prominent high-energy implications of a Newtonian gravitational constant increasing at small distances and morphing into what we call weak interactions at a Planck scale coinciding with the Fermi scale. Among the appealing features emerging from this scenario are the understanding of the observed small Yukawa couplings in terms of event horizons, the absence of a hierarchy problem, and the related mitigation of the cosmological constant problem to the Higgs scale. We have also discussed predictions for the non-observability at LHC energies of particles with mass larger than the Higgs VEV, in particular the inhibition of gauge boson-mediated scattering from existing particles above well-defined thresholds corresponding to the Schwarzschild energy. In this scenario the top quark has a privileged status since it should be the first to be affected by an inhibition of hard scattering. Particles with mass larger than 246 GeV/c² should only be observable through their gravitational effects, either contributing significantly to dark matter, or via detection of gravitational waves emitted during their collisions, provided that the propagation of gravitons is not subjected to the limitations of the horizon. On a final note, we observe that by attributing a finite size to elementary particles makes possible to discuss them in terms of energy density rather than energy in absolute terms. This makes even more natural the need for a consistent merging, via the quantum, between elementary particle physics and genuine general relativity, the lat-
ter being quintessentially related to the interplay between high-energy density phenomena and spacetime.

[1] R. Onofrio, Mod. Phys. Lett. A 28, 1350022 (2013).
[2] F. W. Hehl and B. K. Datta, J. Math. Phys. 12, 1334 (1971).
[3] F. W. Hehl, P. von der Heyde, G. D. Kerlick and J.M. Nester, Rev. Mod. Phys. 48, 393 (1976).
[4] N. A. Bataakis, Class. Quant. Grav. 3, L99 (1986).
[5] Yu. M. Loskutov, JETP 80, 150 (1995).
[6] S. Capozziello, M. De Laurentis, L. Fabbri and S. Vignolo, Eur. Phys. J. C 72, 1908 (2012).
[7] S. Alexander, A. Marcianó and L. Smolin, arXiv:1212.5246.
[8] X. Calmet, S. D. H. Hsu and D. Reeb, Phys. Rev. D 77, 125015 (2008).
[9] R. Onofrio, EPL 104, 20002 (2013); Int. J. Mod. Phys. D 23, 1450005 (2014).
[10] G. Amelino-Camelia, Living Rev. Relativ. 16, 5 (2013).
[11] V. K. Mal’tsev and M. A. Markov, Quantum mini-objects in general relativity, Preprint PO160INR, Moscow (1980).
[12] T. S. Bunch, J. Phys. A 14, 739 (1981).
[13] V. F. Muchanov, Pis’ma Zh. Eksp. Teor. Fiz. 86, 63 (1986).
[14] J. H. MacGibbon, Nature 329, 24 (1987).
[15] V. P. Frolov, M. A. Markov and V. F. Mukhanov, Phys. Rev. D 41, 383 (1990).
[16] I. L. Shapiro and J. Solà, Phys. Lett. B 475, 236 (2000).
[17] S. Sankar, Gen. Rel. Grav. 40, 269 (2008).
[18] T. Mattsson, Gen. Rel. Grav. 42, 567 (2010).
[19] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[20] A. Okopińska, Acta Phys. Pol. B 14, 235 (1983).
[21] R. Onofrio, Phys. Rev. D 82, 065008 (2010).
[22] R. Onofrio, Eur. Phys. J. C 72, 2006 (2012).
[23] M. Atkins and X. Calmet, Phys. Rev. Lett. 110, 051301 (2013).
[24] M. A. Markov, Usp. Fiz. Nauk 164, 67 (1994) [Physics-Uspekhi 37, 57 (1994)].
[25] A. Bonanno and M. Reuter, Phys. Rev. D 62, 043008 (2000).
[26] O. Lauscher and M. Reuter, Phys. Rev. D 65, 025013 (2001).
[27] B. F. L. Ward, Mod. Phys. Lett. A 17, 2371 (2002).
[28] B. F. L. Ward, JCAP 02, 011 (2004).
[29] B. F. L. Ward, Int. J. Mod. Phys. A 20, 3128 (2005).
[30] M. A. Markov, Zh. Eksp. Teor. Fiz. 51, 878 (1967) [Soviet Phys. JETP 24, 584 (1967)].
[31] M. A. Markov, Phys. Lett. 94A, 427 (1983).
[32] M. A. Markov, Phys. Lett. A 172, 331 (1993).
[33] V. G. Kadyshevsky, Nucl. Phys. B 141, 477 (1978).
[34] V. G. Kadyshevsky and M. D. Mateev, Phys. Lett. B 106, 139 (1981).
[35] V. G. Kadyshevsky and M. D. Mateev, Nuovo Cimento A 87, 324 (1985).
[36] M. V. Chizhov, A. D. Donkov, V. G. Kadyshevsky and M. D. Mateev, Nuovo Cimento A 87, 350 (1985); 373 (1985).