MI-based optimal jamming waveform design

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Abstract. In order to improve the estimation performance of radar system, previous studies on radar waveform design mostly focused on maximizing the mutual information between the target and the echo to design the transmitted waveform. However, these studies neglect the existence of jammer in the actual electromagnetic environment, so this paper introduces a jammer to design the corresponding jamming waveforms according to whether there are signal-related clutter or not, which weakens the estimation performance of radar system. Simulation results show that the energy allocation strategies of the two jamming waveforms are different.

1. Introduction
Intelligent jammer can self-adaptively perceive the surrounding situation in complex electromagnetic environment, make the best jamming decision in the process of perception, and automatically generate the best jamming signal. Effective jamming waveform not only affects the signal processing ability of radar, but also suppresses the measurement accuracy and resolution of the system. Therefore, the study of jamming waveform design method has important practical application value for the engineering realization of electronic warfare. MI is a useful measure of information theory [2]. Jammer can minimize MI by designing jamming waveform, thus weakening the estimation performance of radar system. At present, there are few literatures about jamming waveform design [2-5]. Lulu Wang proposes a method to optimize MI-based jamming waveform, which reduces the estimation accuracy of radar system, obtains the optimal jamming power allocation strategy under different jamming power conditions, and designs a MI-based robust jamming waveform to optimize the worst performance of jammer [2-3]. Chenguang Shi studies the design of task-related adaptive radar jamming waveform based on LPI, obtains the MI criterion of parameter estimation, and uses trapezoidal fuzzy number to describe the detection threshold of the whole system based on reliability theory [4]. Nowadays, the existence of jammer cannot be ignored in electronic warfare. Therefore the signal models corresponding to signal-free and signal-related clutters are established in this paper. Then the MI-based optimal jamming waveforms are designed according to the established signal models, which can minimize the estimation performance of radar system. Simulation results show that the waveform energy allocation strategies of these two kinds of jamming waveforms are different.

2. Signal Model
The signals that can be detected at the radar receiver include target echo signal, clutter signal, jamming signal and noise signal. The model of radar receiving signal is established as shown in Figure 1 [6]. Figure 1a is a signal model without signal-related clutter, in which s(r) is a radar transmitted
signal, \( g(t) \) is a target impulse response, \( n(t) \) is an internal noise signal, \( j(t) \) is an jamming signal, and \( y(t) \) is a target echo signal. Figure 1b is a signal model with signal-related clutter, which has one more signal-related clutter response \( w(t) \) than Figure 1a.

![Figure 1. Radar receiving signal model](image)

**Figure 1. Radar receiving signal model**

The target echo signal without signal-related clutter is:

\[
y(t) = s(t) * g(t) + j(t) + n(t) \tag{1}
\]

The target echo signal with signal-related clutter is:

\[
\tilde{y}(t) = s(t) * g(t) + n(t) \tag{2}
\]

In Formulas (1) and (2), * denotes convolution.

The time domain convolution is equal to the frequency domain multiplication, so the frequency domain expressions for the cases of clutter and clutter-free are:

\[
Y(f) = S(f) \cdot G(f) + J(f) + N(f) \tag{3}
\]

and

\[
\tilde{Y}(f) = S(f) \cdot G(f) + S(f) \cdot W(f) + J(f) + N(f) \tag{4}
\]

respectively.

In Formulas (3) and (4), \( S(f) \), \( G(f) \), \( W(f) \), \( J(f) \) and \( N(f) \) denotes the spectrum response of \( s(t) \), \( g(t) \), \( w(t) \), \( j(t) \) and \( n(t) \).

### 3. Jamming waveform design

In order to weaken the estimation performance of radar system, the MI-based optimal jamming waveforms are designed in this section, which can optimize the jammer strategy in the case of non-signal-related clutter and signal-related clutter.

#### 3.1. Jamming waveform design in the case of no signal-related clutter

In the case of no signal-related clutter, the expression of MI is as follows:

\[
\text{MI} = T_s \int_{-BW}^{BW} \ln \left[ 1 + \frac{\left| G(f) \right|^2 \left| S(f) \right|^2}{T_s \left( J(f) + N(f) \right)} \right] df \tag{5}
\]

In the formula (5), \( T_s \) is the duration of the target echo. Then the optimization problem is:

\[
\min_{J(f)} \text{MI}(J(f)) \tag{6}
\]

s.t. \( \int_{-BW}^{BW} J(f) df \leq P \tag{7} \)

In the above formula, \( BW \) is the frequency bandwidth of the jamming waveform spectrum and \( P \) is the power constraint condition of the jamming waveform spectrum. The solution of the optimization problem is as follows:

\[
J(f) = \max \left[ 0, A(f) \left( B - C(f) \right) \right] \tag{8}
\]

where
\[ A(f) = \frac{\|G(f)\|^2 \|S(f)\|^2}{2T_s \cdot N(f) + \|G(f)\|^2 \|S(f)\|^2} \quad (9) \]

and

\[ C(f) = \frac{T_s \|N(f)\|^2 + \|G(f)\|^2 \|S(f)\|^2 N(f)}{\|G(f)\|^2 \|S(f)\|^2} \quad (10) \]

\( B \) is a constant which can be solved by the power constraint of the jamming waveform:

\[ \int_{f_{aw}} \max\left[0, A(f) \left( B - C(f) \right) \right] df \leq P \quad (11) \]

3.2. Jamming waveform design in the case of signal-related clutter

In the case of signal-related clutter, the expression of MI is as follows:

\[ MI = T_s \int_{f_{aw}} \ln \left[ 1 + \frac{\|G(f)\|^2 \|S(f)\|^2}{T_s \left( W(f) \|S(f)\|^2 + J(f) + N(f) \right)} \right] df \quad (12) \]

Then the optimization problem is:

\[ \min_{J(f)} \quad MI(J(f)) \quad (13) \]

s.t. \( \int_{f_{aw}} J(f) df \leq P \quad (14) \)

The solution of the optimization problem is as follows:

\[ \hat{J}(f) = \max\left[ 0, \hat{A}(f) \left( \hat{B} - \hat{C}(f) \right) \right] \quad (15) \]

where

\[ \hat{A}(f) = \frac{\|G(f)\|^2 \|S(f)\|^2}{2T_s \cdot \left( N(f) + W(f) \|S(f)\|^2 \right) + \|G(f)\|^2 \|S(f)\|^2} \quad (16) \]

and

\[ \hat{C}(f) = \frac{T_s \|N(f) + W(f)\|S(f)\|^2 + \|G(f)\|^2 \|S(f)\|^2 \left( N(f) + W(f) \|S(f)\|^2 \right)}{\|G(f)\|^2 \|S(f)\|^2} \quad (17) \]

\( \hat{B} \) is a constant which can be solved by the power constraint of the jamming waveform.

\[ \int_{f_{aw}} \max\left[0, \hat{A}(f) \left( \hat{B} - \hat{C}(f) \right) \right] df \leq P \quad (18) \]

4. Simulation and results

In this paper, simulation analysis is carried out to verify the effectiveness of the two designed optimal jamming waveforms. Figure 2 shows the prior information needed to design the optimal jamming waveform in the case of no signal-related clutter, including radar transmitted waveform, target spectrum response and noise power spectral density (PSD). The radar transmitted waveform distributes its power over the whole frequency band, and has a peak at -0.3. The target spectrum distributes its main power in the frequency bands near the frequency of -0.3, -0.1 and 0.2, and the noise PSD is a straight line in the whole frequency band. Figure 3 is designed jamming waveform in the case of no signal-related clutter. The jamming waveform assigns its power to the frequency bands with strong spectral response of the target and radar transmitted waveform. Figure 4 shows the prior information needed to design the optimal jamming waveform in the case of signal-related clutter. It adds a clutter spectrum based on Figure 2, which assigns its power to the frequency bands far from the frequency of 0.2. Figure 5 is designed jamming waveform in the case of signal-related clutter. The jamming waveform mostly distributes its power in the frequency bands with strong target response and weak clutter response. This phenomenon is interesting. The designed jamming waveform will
distribute its power in the frequency band with insufficient clutter response and compensate for it.

Figure 2. Prior information in the case of no signal-related clutter

Figure 3. Optimal jamming waveform in the case of no signal-related clutter

Figure 4. Prior information in the case of signal-related clutter
Figure 5. Optimal jamming waveform in the case of signal-related clutter

5. Conclusion
Firstly, the signal models without signal-related clutter and with signal-related clutter are established, and then the MI-based optimal jamming waveforms are designed according to these two different signal models. Simulation results show that the two designed jamming waveforms can effectively weaken the estimation performance of radar system, but their waveform energy allocation strategies are different, and we find that when there is signal-related clutter, there will be jamming to compensate for clutter.

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