Quantifying the $\sigma_8$ tension with model independent approach

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One problem of the $\Lambda$CDM model is the tension between the $\sigma_8$ found in cosmic microwave background experiments and the smaller one obtained from large-scale observations in the late Universe. The $\sigma_8$ quantifies the relatively high level of clustering. Using Redshift Space Distortion data with Gaussian processes method, model-independent reconstructions of the growth history of matter in-homogeneity is studied. Markov Chain Monte Carlo fit for $\Lambda$CDM and $w$CDM models is performed with the original and the trained data. The trained data fit yields a closer $\sigma_8$ to the cosmological value predicted by Planck collaboration. Especially, with a 0.43$\sigma$ difference for the Matern kernels. The results raise the question of the tension and gives the possibility that in future measurements (such as J-PAS, DESI and Euclid) the tension may be resolved.

\textbf{I. INTRODUCTION}

One from latest breakthroughs in cosmology is the fact that our universe is not only expanding but also accelerating. This fact is proven from different data sets, such as Supernovae type Ia (SNIa) [1–9], cosmic chronometers [10–13] and Baryon Acoustic Oscillation (BAO) [14–22]. Assuming homogeneous and isotropic volume, the accelerated expansion is explained by the presence of Cold Dark Matter (CDM) in addition to the barionic matter and a cosmological constant $\Lambda$ [23–29]. The model is labeled as $\Lambda$CDM [30, 31].

The $\Lambda$CDM model suffers from well known problems [32, 33], such as the coincidence problem and the disagreement between the measured value of the vacuum energy density and the predicted one from Quantum Field Theory. Despite the good agreement with the majority of cosmological data [34], the model seems to be currently in tension with some recent measurements, such as the present value of the mass variance at $8h^{-1}$Mpc, namely the $\sigma_8$ tension [35–40]. There is 2$\sigma$ tension between the constraints from Planck on the matter density $\Omega_m^0$ and the amplitude $\sigma_8$ of matter fluctuations in linear theory and those from local measurements. Planck derives $\sigma_8 = 0.832 \pm 0.013$ [41], local measurements find smaller values: $0.78 \pm 0.01$ from Sunyaev-Zeldovich cluster counts [42], $0.783 \pm 0.025$ from DES [43] and $0.745 \pm 0.039$ from KiDS-450 weak-lensing surveys [44].

There are many claim how to solve the tension: from the observational point of view or from a new physics point of view [45–78]. Here, we focus on a model-independent parametrization for the $f\sigma_8$ data using the Gaussian process, and investigate its performance against the latest data, using kernel functions with some hyper-parameters is optimizing the data fit [79–94].

The plan of the paper is the following: Section II formulates the theoretical background for the standard models in cosmology. Section III summarizes about the foundations of the Gaussian Process Regression method. Section IV compares the observations and the trained data with the standard models. Finally, section V summarizes the results.

\textbf{II. GROWTH OF MATTER PERTURBATIONS}

The metric for a flat Friedmann Robertson Walker background reads:

$$ds^2 = -dt^2 - a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$ (1)

where $a(t)$ is the scale parameter of the universe. The scale factor and the redshift are connected: $a = 1/(1+z)$. The Friedmann equation for a flat universe with a $\Lambda$CDM background reads:

$$H(z)^2 = H_0^2 \left[\Omega_m^0(1+z)^3 + \Omega_\Lambda\right]$$ (2)

where $H = \dot{a}/a$ is the Hubble parameter, $\Omega_m^0$ is the current fraction of the matter density, $\Omega_\Lambda$ is fraction of the dark energy density and $z$ is the redshift.

For $w$CDM the Friedmann equation is generalized to:

$$H(z)^2 = H_0^2 \left[\Omega_m^0(1+z)^3(w+1) + \Omega_\Lambda\right].$$ (3)
It is useful to define the fraction:

$$E(z) := H(z)/H(0)$$

(4)

that reads the energy density relations without the Hubble parameters part. Precise large-scale structure measurements are helpful to distinguish different models and different histories for growth structure. The of growth structure is defined as:

$$\delta = \delta \rho_m/\rho_m.$$  \hspace{1cm} (5)

In the sub-horizon limit \((k \gg aH)\), the linear matter growth factor reads \([95]\):

$$\delta'' + \left[ \frac{1}{2} \left( \frac{E'(z)}{E(z)} \right)^2 - \frac{1}{1 + z} \right]\delta' = \frac{3(1 + z)}{2E(z)^2} \Omega_m^{(0)} \delta$$  \hspace{1cm} (6)

where prime denotes derivative with respect to the redshift \(z\). The analytic solution for the linear matter growth factor with wCDM background is being:

$$\delta(z) = \frac{1}{z + 1}.$$  \hspace{1cm} (7)

where \(2F_1\) is an hyperbolic function. These equations lead to the predicted evolution of the observable product \(f(a)\sigma_s(a)\), where \(f(a)\) is:

$$f(a) \equiv d \ln(a)/d \ln a,$$  \hspace{1cm} (8)

the growth of cosmological matter density perturbations. The robust observable reported by RSD surveys is the product:

$$f \sigma_s(z) = f(z)\sigma(z) = -(1 + z)\sigma_s \delta \rho_m(z).$$  \hspace{1cm} (9)

Therefore, for a given equation of states \(w\), the parameter \(\sigma_s\) and the energy fraction \(\Omega_m^{(0)}\) we can obtain the complete behavior of the function \(f \sigma_s(z)\).

Fig. 1 shows the data set we use in this work \([50]\). The Fig shows the predicted curve from Planck collaboration, with 5\(\sigma\) difference. Planck collaboration give a strong constraint on the values of the cosmological parameters. Because the error bars of the parameters are very small, the blue line in 1 is very thin.

### III. GAUSSIAN PROCESS METHOD

This section summarises the Gaussian Process (PG) algorithm. The GP reconstruct of a function from data without assuming a parametrization of the function \([79, \ 96]\). Gaussian Process method in cosmology is studied with different data sets. Having a data set \(D\):

$$D = \{(x_i, y_i)|i = 1, ..., n\},$$  \hspace{1cm} (10)

we can reconstruct in a model independent function \(f(x)\) which describes the data. In this case at any point \(x\), the value \(f(x)\) is a Gaussian random variable with mean \(\mu(x)\) and variance \(\text{Var}(x)\). The function values at any two different points are not independent from each other. Therefore, the covariance function \(\text{cov}(f(x), f(\tilde{x})) = k(x, \tilde{x})\) describes the corresponding correlations. The possibilities for the Kernel are wide. The current work use the Radial Basis Function (RBF):

$$k(x, \tilde{x}) = \sigma_f^2 \exp\left(-\frac{(x - \tilde{x})^2}{2l^2}\right),$$  \hspace{1cm} (11)

The Matern kernel with \(\nu = 7/2\):

$$k(x, \tilde{x}) = \sigma_f^2 \exp\left(-\sqrt{\frac{3|x - \tilde{x}|}{l}}\right)$$

$$\left(1 + \sqrt{\frac{3|x - \tilde{x}|}{l}} + \frac{14(x - \tilde{x})^2}{5l^2} + 7\sqrt{\frac{|x - \tilde{x}|^3}{15l^3}}\right),$$  \hspace{1cm} (12)

and Matern kernel with \(\nu = 9/2\):

$$k(x, \tilde{x}) = \sigma_f^2 \exp\left(-\frac{3|x - \tilde{x}|}{l}\right)$$

$$\left(1 + 3\frac{|x - \tilde{x}|}{l} + 27\frac{(x - \tilde{x})^2}{7l^2} + 18\frac{|x - \tilde{x}|^3}{7l^3} + 27\frac{(x - \tilde{x})^4}{35l^4}\right).$$  \hspace{1cm} (13)

\(\sigma_f\) and \(l\) are two hyperparameters which can be constrained from the observational data. With a Gaussian prior \(f|X, \sigma_f, l| \sim N(\mu, k(x, \tilde{x}))\), the log marginal likelihood reads:

$$\ln \mathcal{L} = -\frac{1}{2} (y - \mu)^T [k(x, \tilde{x}) + C]^{-1} (y - \mu)$$

$$- \frac{1}{2} \ln |k(x, \tilde{x}) + C| - \frac{n}{2} \ln 2\pi.$$  \hspace{1cm} (14)

The hyperparameters \(\sigma_f\) and \(l\) can now be optimized by maximizing equation (14). In order to calculate the trained data from the Gaussian Process method, we use the open source code Scikit-learn \([97]\).

As a consequence from the Gaussian Processes method, the "trained" data set is obtained. Fig 2 shows the trained data set for different Kernels. In order to compare the data sets, we define the deviation between the data by:

$$\chi^2_i = \frac{x_{di}^2 - x_{tdi}^2}{\sigma_{di}^2 + \sigma_{tdi}^2} + 2\pi \log (\sigma_{di}^2 + \sigma_{tdi}^2)$$  \hspace{1cm} (15)

where \(x_{di}\) and \(x_{tdi}\) are the eigenvalues of the data and the trained data, with their corresponding errors. The right hand side of Fig 2 shows the different \(\chi^2_i\). The total difference between the trained and the original data sets is defined as:

$$\chi^2 = \frac{1}{2N} \sum_i \chi^2_i.$$  \hspace{1cm} (16)
where $N$ is the number of data points we test. For the RBF kernel: $\chi^2 = 0.115$, and for the Matern functions: $\chi^2 = 0.124$. Those numbers show that the trained data and the original data are close one each other and the trained data may be used as a reference for the models we want to test.

IV. LIKELIHOOD ANALYSIS

In order to test the standard models with the new data sets, we use Likelihood Analysis. The $\chi^2$ between the models and the set is defined as:

$$\chi^2 = V^i C^{-1}_{ij} V^j$$

(17)

where $V^i = f \sigma_{8,i} - f \sigma_8(z_i; \Omega, w, \sigma_8)$. Here $f \sigma_{8,i}$ corresponds to each of the data points. $f \sigma_8(z_i; \Omega, w, \sigma_8)$ is the theoretical value for a given set of parameters values. Apart form the errors in the data set, there are three correlated points corresponding to WiggleZ, with the covariance matrix is given by:

$$C_{WiggleZ}^{ij} = 10^{-3} \begin{pmatrix} 6.4000 & 2.570 & 0.000 \\ 2.570 & 3.969 & 2.540 \\ 0.000 & 2.540 & 5.184 \end{pmatrix}$$

(18)
The total covariance matrix reads:

\[ C_{ij} = \text{diag} \left( \sigma_1^2, C_{ij}^{\text{WiggleZ}}, ..., \sigma_N^2 \right) \]  

(19)

We test the fit for two models: ΛCDM and wCDM. For ΛCDM we set \( w = -1 \) and therefore we left with 2 free parameters. The prior we choose is with a uniform distribution, where \( \Omega_m \in [0.05; 0.9] \), \( w \in [-2.5; 0.5] \) and \( \sigma_8 \in [0.1; 1.5] \). Regarding the problem of data fit, we use the open-source sampler \texttt{emcee} [98] with the \texttt{GetDist} [99] to present the results. We sample \( 10^6 \) samples with 20 walkers.

Fig 3 and Fig 4 shows the corner plot for ΛCDM and wCDM models respectively. Using different kernels yield different values of \( \sigma_8 \): \( 0.802 \pm 0.021 \) for the Matern with \( \nu = 7/2 \) functions. Moreover, the \( \Omega_m \) for the corresponding functions is reduces as well from \( 0.274 \pm 0.020 \) to the original observations to \( 0.25 \pm 0.02 \) for the Matern functions. The results suggest the possibility that there is no tension in between the Planck data and the RSD data.
for the $\sigma_8$ value. However the small tension for the $\Omega_m$ values raises the possibility for a real tension. It seems that only with future observations and with smaller error bars we may address this question completely.

wCDM have larger deviation for different kernels. The equation of state for the dark energy finds the best value close to $-0.5$ for some kernels. The best value for the $\sigma_8$ can approach $0.91\pm0.02$ for the Matern kernel ($\nu = 7/2$). But for the most kernels the results are closer to Planck data. Again it seems that from future observations we may address the question of the tension.

V. DISCUSSION

This paper analyze the latest $f\sigma_8$ data with model independent approach. $\Lambda$CDM is the best model that explain the expansion of the universe, both from early times and late times. Still there is a $2\sigma$ tension between the constraints from Planck on the matter density $\Omega_m^{(0)}$ and the amplitude $\sigma_8$ of matter fluctuations in linear theory and those from local measurements. Confronting these data with the growth rate obtained from $\Lambda$CDM and wCDM cosmology we find the tension can be reduced. From $1.63\sigma$ with the original observations, the tension is reduces to $1.28\sigma$ with the RBF kernel fit, or to $0.43\sigma$ with the Matern kernels.

The trained data fit yields a closer $\sigma_8$ to the cosmological value predicted by Planck collaboration. The result raises the question about the tension and gives the possibility that in future measurements the tension may be resolved, such as J-PAS [100], DESI [101] and Euclid experiments [102–104].

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