CP-violating Phases in M-theory and Implications for EDMs

Gordon Kane¹, Piyush Kumar², and Jing Shao¹

¹Michigan Center for Theoretical Physics, Ann Arbor, MI 48109 USA

²Department of Physics, University of California, Berkeley, CA 94720 USA & Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720

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We demonstrate that in effective theories arising from \( \mathcal{N} = 1 \) fluxless compactifications of M-theory on a \( G_2 \) manifold with low energy supersymmetry, CP-violating phases do not appear in the soft-breaking Lagrangian except via the Yukawas. Such a mechanism may be present in other string compactifications as well; we describe properties sufficient for this to occur. CP violation is generated via the Yukawas since the soft trilinear matrices are generically not proportional to the Yukawa matrices. Within the framework considered, the estimated theoretical upper bounds for electric dipole moments (EDM) of the electron, neutron and mercury are all within the current experimental limits and could be probed in the near future.

I. INTRODUCTION

The null measurements of the electric dipole moments (EDMs) of the neutron [1], and recently, heavy atoms like Thallium (\(^{205}\)Tl)[2, 3] and Mercury (\(^{199}\)Hg) [4, 5], have put very strong constraints on the amount of CP violation from new physics beyond the Standard Model (SM). The precision of these measurements is expected to significantly improve in a few years. If an excess above the SM prediction is observed, it requires the presence of new physics beyond the SM. However, since the EDMs, even if observed, are already “small”, this strongly suggests that the new physics must be such that it has an underlying mechanism to naturally suppress EDMs.

In general versions of supersymmetric extensions of the Standard Model, new sources of CP violation can arise from complex phases of the soft supersymmetry breaking parameters. These phases are therefore tightly constrained to be small [6, 7] (or to have cancellation [8, 9, 10, 11]) for TeV scale superpartners. Thus, from a theoretical perspective, the existence of such small phases has to be explained by some underlying mechanism. Many studies of supersymmetric models from a low-energy phenomenological perspective focus on the mediation mechanism and only parameterize the supersymmetry breaking. Explaining small soft CP-violating phases, which requires a dynamical understanding of supersymmetry breaking, is especially challenging as this is not available in such a framework. Without a specification of the supersymmetry breaking mechanism, this problem exists in both gravity and gauge-mediated models of supersymmetry breaking in general.

Put differently, whenever supersymmetry is treated as a general TeV-scale effective theory both the values and phases of the soft-breaking masses are treated as arbitrary, and EDMs are typically much larger than experimental values. Many people have argued that such large EDMs are implied or required from supersymmetry, and that this is a problem for supersymmetry. Such arguments ignore the fact that any underlying theory will predict and relate phases. This implies that the underlying theory of which low energy supersymmetry is a low energy limit has a structure that suppresses or relates the low scale phases.

Substantial progress has been made towards understanding dynamical supersymmetry breaking, especially in recent years. In this work, we will be interested in dynamical mechanisms of supersymmetry breaking with low superpartner masses which can be naturally embedded in the framework of an underlying microscopic theory like String/M-theory. In particular, we study the effective four-dimensional theory resulting from fluxless \( \mathcal{N} = 1 \) compactifications of M-theory with chiral matter [12]. These are especially interesting because a hierarchy between the Electroweak and Planck scale is generated, and all geometric moduli are stabilized, at the same time [13]. We find that the supersymmetry breaking and mediation dynamics is such that it naturally gives rise to vanishing CP-violating phases from supersymmetry breaking at leading order, providing an excellent starting point to explain suppressed EDMs. The mechanism is a non-trivial generalization of an old idea [14] (and more recently [15]), and may also apply to other classes of string compactifications where moduli are stabilized in a de Sitter vacuum.

Although the CP-violating phases from supersymmetry breaking vanish at leading order, there could still be significant contributions to CP violation in the flavor-diagonal sector in principle. First, in the M-theory framework, the trilinear matrices are typically not proportional to the Yukawa matrices after moduli are stabilized, which in general leads to non-trivial CP-violating phases in the trilinear A-terms in the basis of quark and lepton mass eigenstates and therefore generates non-zero EDMs [7, 16]. The estimated upper bounds on EDMs are all within the current experimental limits. For some val-
ues of parameters, some upper bounds on the EDMs are close to the experimental limits. As will be clear, two features - large sfermion masses and trilinears, and hierarchical Yukawa textures, both natural within the M-theory framework, are important for getting viable but interesting EDMs results. In addition, we argue that even though higher order corrections to the Kähler potential exist, they do not give rise to new CP-violating phases. Finally, it should be remarked that the EDM results are robust anytime the trilinears dominantly acquire their phases from Yukawas and the mass spectrum is as dictated by the M-theory framework.

It is worth mentioning that the solution to the supersymmetric CP problem here is largely independent of any particular solution to flavor issues as long as they satisfy a certain criterion. As will be seen, the only feature of the flavor structure used in computing results for EDMs is that the sfermion mass matrices for visible matter in the super-CKM basis are approximately flavor diagonal at low energies. Therefore, the results hold true for any proposed solution to the flavor problem which is consistent with the above feature. For the M-theory framework in particular, this approximately flavor diagonal structure arises due to the presence of $U(1)$ symmetries under which the chiral matter fields are charged. The spontaneous breaking of these symmetries may introduce small non-diagonal components as long as it occurs at a scale sufficiently below the Planck scale. In the M-theory framework, large sfermion masses $\geq 10$ TeV already mitigate the FCNC problems. In addition, small off-diagonal components (after going to the super-CKM basis) suppressed by an order of magnitude or more could arise from the approximate flavor-diagonality of the Kähler metric mentioned above and/or due to family symmetries which could be present in the underlying theory. This would then probably be consistent with all FCNC observables. This paper focusses on CP violation arising in the flavor-diagonal sector, which is present in general even if FCNC problems, arising from off-diagonal terms in the squark mass matrices in the super-CKM basis, are solved; hence it is largely decoupled from flavor physics. A more detailed discussion of flavor issues will appear elsewhere.

The plan of the paper is as follows. In section II we review the basic mechanism of supersymmetry breaking and its implications for soft CP-violating phases at leading order. In section III we discuss the connection between yukawa textures and imaginary parts of the diagonal trilinear matrix components. After a brief discussion of EDMs using the low energy effective Lagrangian and the present experimental limits in section IV we compute the detailed predictions for EDMs within this framework in section V. In section VI we generalize the results in section II to include more general situations and list conditions which need to be satisfied to naturally suppress CP-violating phases. We conclude in section VII. The appendices deal with some technical details of the computations.

II. SMALL CP-VIOLATING PHASES FROM SUSY BREAKING

In fluxless compactifications of M-theory[13], the moduli superpotential is entirely generated non-perturbatively, and hence, exponentially suppressed relative to the Planck scale. This is crucial for both stabilizing the moduli $z_i$ and generating the hierarchy. The strong gauge dynamics resides in a three-dimensional submanifold of the internal manifold which generically does not intersect the three-dimensional submanifold where the supersymmetric standard model particles live as these three-manifolds are embedded inside a seven dimensional internal manifold. For simplicity, we consider two non-abelian asymptotically free gauge groups with at least one of them assumed to contain light charged matter fields $Q$ and $\bar{Q}$ (with $N_f < N_c$), and an associated meson $\hat{\phi} = (\bar{Q}Q)^{1/2}$ in the low energy. The strong gauge dynamics in the hidden sector stabilizes the moduli of the $G_2$ manifold, and dynamically generates a supersymmetry breaking scale with $O(10)$ TeV gravitino mass. The supersymmetry breaking is then mediated to the visible sector through gravitational ($m_p$ suppressed) interactions.

A. Superpotential and Kähler Potential

For concreteness, we give a brief review of the effective action of the fluxless compactifications of M-theory. First, the superpotential can be separated into two parts:

$$W = \hat{W} + Y'_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma$$

(1)

where $\hat{W}$ depends only on the moduli $z_i = s_i + it_i$ and the meson $\phi$. Here $C_{\alpha}$ are the matter fields in the minimal supersymmetric standard model (MSSM) with $\alpha$ being higgs, quark or lepton chiral superfields. $Y'_{\alpha\beta\gamma}$ denote the superpotential Yukawa couplings. The effective Yukawa couplings (still not fully normalized) in the MSSM are given by $Y_{\alpha\beta\gamma} = e^{K/2} Y_{\alpha\beta\gamma}$. The connection to the usual convention in the MSSM can be made by taking the first index to be the Higgs fields, the second to be the quark doublets, and the third to be the quark singlets, for example, $Y_{H_u Q_{u_i}} = Y_{H_u Q_{u_i}}$. In the M-theory framework, an elegant way to generate Yukawa couplings $Y_{\alpha\beta\gamma}$ is from membrane instantons[18][19], which also depend holomorphically on the moduli $z_i$ in general as they measure the volume of the manifold which the instanton wraps. It appears natural to generate a hierarchical Yukawa texture from such effects.

The first term $\hat{W}$ in (1) is the moduli superpotential, and is generated non-perturbatively from gaugino condensation[20]

$$\hat{W} = A_1 (\det(\phi^2))^n e^{-b_1 f_{\text{hid}}} + A_2 e^{-b_2 f_{\text{hid}}}$$

(2)

Here $b_{1,2}$ are the beta function coefficients of the two hidden sector gauge groups and $f_{\text{hid}}$ are the corresponding
gauge kinetic functions given by \( f_{\text{hid}} = \sum_{i=1}^{N} N_i \phi_i \). In the first term in the superpotential, we have included the meson field \( \phi = (\tilde{Q}Q^I)^{1/2} \). The parameter \( a \) in the superpotential is a constant depending on \( N_c \) and \( N_f \), and is not important for our discussion.

The Kähler potential can be written as

\[
K = \hat{K} + \tilde{K}_{\alpha\beta} \bar{C}^{\alpha} C^\beta + \left( Z_{\alpha\beta} \bar{C}^{\alpha} C^\beta + \text{h.c.} \right)
\] (3)

Here \( \hat{K} \) is the moduli Kähler potential and \( \tilde{K}_{\alpha\beta} \) is the Kähler metric of matter fields \( C^{\alpha} \). \( Z_{\alpha\beta} \) is expected to be non-zero only for Higgs field \( H_{u,d} \), which is needed to generate \( \mu \) and \( B \) terms. In these compactifications, charged chiral matter fields with different flavors are localized at isolated conical singularities [21]. These charged matter fields, in addition to being charged under the relevant non-abelian gauge group, are also charged under \( U(1) \) factors which arise from the Kaluza-Klein reduction of the three-form in eleven dimensional supergravity on two-cycles present in the internal manifold [22]. These \( U(1) \)'s survive at low energies as good symmetries to all orders in perturbation theory and hence must be respected (up to exponentially suppressed non-perturbative effects). Importantly, it turns out that conical singularities associated to different flavors cannot carry the same charges under the \( U(1) \)'s in a given basis [17] (at least in local models). This forbids the existence of off-diagonal terms in the Kähler potential of the form \( \bar{C}^{\alpha} C^\beta, \alpha \neq \beta \). Off-diagonal components may be introduced if these symmetries are spontaneously broken, but these will be suppressed as long as this occurs sufficiently (an order of magnitude or more) below the Planck scale.

Thus, the Kähler metric is expected to be approximately flavor diagonal, i.e. \( \tilde{K}_{\alpha\beta} \approx \hat{K}_{\alpha\beta} \delta_{\alpha\beta} \) at the high scale. As argued in [23], the Kähler potential for localized matter fields \( C_{\alpha} \) in the 11D frame is canonical, i.e. \( \bar{C}_\alpha C_\alpha \) due to the absence of local moduli. Going to the Einstein frame implies that there is an overall dependence on the internal volume \( V_X \), but this still preserves the approximate diagonality. A flavor-diagonal Kähler metric will lead to flavor-diagonal soft scalar mass parameters. Note that the above features hold at the high scale (\( \sim \) GUT scale). RG effects will in general also lead to small flavor off-diagonal contributions to scalar mass parameters at the electroweak scale.

The Kähler potential for moduli fields contains two pieces

\[
\hat{K} = -3 \ln(V_X) + \frac{2}{V_X} \text{Tr} \left( \phi^\dagger \phi \right)
\] (4)

Here \( V_X \) is the volume of the \( G_2 \) manifold in units of the eleven-dimensional length scale \( l_{11} \). The second term originates from the Kähler potential for vector-like matter fields \( Q \) and \( \tilde{Q} \) in the hidden sector, which generally takes the form [23]

\[
\tilde{K} = \frac{1}{V_X} \left( Q^\dagger Q + \tilde{Q}^\dagger \tilde{Q} \right)
\] (5)

By using the D-term equations \( Q^\dagger Q = \tilde{Q}^\dagger \tilde{Q} \) and the definition of the meson field \( \phi \), it can be rewritten in terms of \( \phi \) as given in second term in Eq. (4). Of course, there could be additional (higher order) corrections, these will be discussed in Section VII. Now for the simple case \( N_f = 1 \), we can replace \( \det(\phi^\dagger) \) by \( \phi^2 \) and \( \text{Tr}(\phi^\dagger \phi) \) by \( \phi \phi \) in Eq. (2) and (4) respectively.

After supersymmetry is spontaneously broken by the strong gauge dynamics, soft supersymmetry breaking terms in the visible sector are generated which take the following form as usual

\[
\mathcal{L}_{\text{soft}} = \frac{1}{2} (M_\alpha \lambda + \text{h.c.}) - m_{\alpha\beta} \bar{C}^{\alpha \beta} \tilde{C}^{\alpha \beta}
\]

\[
- \frac{1}{6} A_{\alpha\beta\gamma} \bar{C}^{\alpha} \tilde{C}^{\beta} \tilde{C}^{\gamma} + \frac{1}{2} \left( B_{\alpha\beta} \bar{C}^{\alpha} \tilde{C}^{\beta} + \text{h.c.} \right)
\] (6)

where \( \bar{C}^{\alpha} \) are the canonically normalized chiral matter fields. The trilinear \( A_{\alpha\beta\gamma} \) can often be factorized as \( A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma} \). In the following, we will be careful in distinguishing between trilinears \( \hat{A} \) and \( A \).

B. CP-violating Phases

Now we turn to analyzing the CP-violating phases in the soft Lagrangian. In order to study the dependence of the soft parameters on complex phases, it is crucial to understand the structure of the superpotential in the relevant supersymmetry breaking vacuum. In the superpotential \( W \) in Eq. (2), \( A_1, A_2, z_i \) and \( \phi \) are complex variables in general. However, (see Appendix A) the relative phases of the superpotential \( \alpha_{\text{CP}} \)'s in a given basis [17] (at least in local models). The second term in the Kähler potential is a constant depending on \( \phi \). Therefore, we have \( F_1 = \text{real} \times e^{i \gamma_1} \). The Kähler potential, \( K \), as seen from Eq. (4), only depends on real fields \( s_i \) which determine \( V_X \) and the combination \( \phi \phi \), so does not contain any explicit phases.

The structure of the F-terms \( F_I = \hat{K}^{IJ} F_J = \hat{K}^{IJ} (\partial_J \hat{W} + (\partial_J K) \hat{W}) \) where \( I, J \) run over both \( z_i \) and \( \phi \) in general, can be computed as follows. For \( J \) corresponding to M-theory geometric moduli \( z_i \), it is easy to see \( \partial_J K \) is real and \( F_J = \text{real} \). For \( J \) corresponding to meson moduli \( \phi \), \( (\partial_J K) \hat{W} = \text{real} \times e^{i \gamma_1} \), where \( \gamma_1 \) is the phase of \( \phi \). Also, since \( W \) depends holomorphically on \{\( z_i, \phi \)\} as in the first line in Eq. (2), one again finds \( \partial_J \hat{W} = \hat{W} / \phi = \text{real} \times e^{i \gamma_0} \). Therefore, we have \( F_J = \text{real} \times e^{i \gamma_0} \) for \( J \) corresponding to meson moduli \( \phi \). Based on these observations, it is not difficult to find that \( F^I = \text{real} \) or \( F^I = \text{real} \times e^{i \gamma_0} \) depending on \( I \) equals to \( z_i \) or \( \phi \) respectively. This leads to interesting implications for the soft supersymmetry breaking parameters.
First, the tree-level gaugino masses are given by:

$$M_{\alpha}^{\text{tree}}(\mu) = \frac{g_2^2(\mu)}{8\pi} \left( \sum_I e^{\hat{K}^I/F_I} \partial_I f^{\text{vis}}_\alpha \right)$$  \hspace{1cm} (7)

Since $f^{\text{vis}}_\alpha$ only depends on the geometric moduli $z_i$ with integer coefficient, and as we have found, the auxiliary component $F_I$ of $z_i$ are real, there are no phases generated for the tree-level gaugino masses. In the M-theory framework, the tree-level gaugino masses are suppressed relative to the gravitino mass [12, 23], and the one-loop anomaly mediated contribution has to be included, which is given by [25]

$$M_{\alpha}^{\text{AMSB}} = -\frac{g_2^2}{16\pi^2} \left( b_\alpha e^{\hat{K}/F_I} \bar{W} - b_\alpha e^{K/2} F_I \hat{K}_I \right)
+ 2 \sum_i C_i^\alpha e^{\hat{K}/F_I} \partial_I \ln \hat{K}_i.$$  \hspace{1cm} (8)

This contribution includes terms proportional to either $\bar{W}$ or $F_I \partial_I \hat{K}$ or $F_I \hat{K}_I$. Since the Kähler potential is a real function of $z_i$, $\partial_i \hat{K}$ and $\partial_i \hat{K}_I$ are real. In addition, the Kähler potential only depends on $\hat{\phi}$, which implies that the derivative with respect to $\phi$ is proportional to $\hat{\phi} \sim e^{-\gamma i}$. Therefore, all these terms are real, which gives rise to real anomaly mediated gaugino masses. Hence, the gaugino masses have no observable phase in the above framework.

The trilinear $A$-terms (with the Yukawa couplings factored out) are given in general by [26]:

$$A_{\alpha\beta\gamma} = e^{\hat{K}/F_I} \partial_I \left[ \ln \left( e^{\hat{K}/Y_{\alpha\beta\gamma}} \hat{\phi} \right) \right]$$  \hspace{1cm} (9)

where $I, J$ run over both $z_i$ and $\phi$. It should be noted that in order to be able to factor out the Yukawa matrices the matter Kähler metric has to be diagonal. This is a good approximation in the M-theory framework as we have discussed. Since the moduli Kähler potential $\hat{K}$ and the visible sector Kähler metric $\hat{K}$ are real functions of $z_i$ and $\phi$ and superpotential takes the form in Eq. [1], it is straightforward to check that the contractions $F_I \partial_I \hat{K}$, $F_I \partial_I \hat{K}_I$ and $F_I \partial_I \ln Y'$ are all real, implying that no CP phases are generated in the trilinear $A$-terms through supersymmetry breaking. However, there could be phases in the full trilinear couplings $\hat{A}$ coming from the Yukawa couplings, as we shall discuss in the next section.

Finally, we move on to the $\mu$ and $B$ terms. We focus on the case where the superpotential contribution to the overall high scale $\mu$ parameter vanishes. This can be easily guaranteed by a symmetry [13]. In this case, $\mu$ and $B$ parameters of $O(m_{3/2})$ can be generated by the Giudice-Masiero mechanism [27] via the parameter $Z_{\alpha\beta}$ in Eq. [4]. The general result for $\mu$ and $B$ can be written in terms of $Z, F_I \partial_I \hat{K}, F_I \partial_I \hat{K}_I$ and $F_I \partial_I \ln Z$ [26], all of which have the same phase $\gamma_Z$ from $Z_{\alpha\beta}$ (complex in general). Therefore, $\mu$ and $B$ can have a phase $e^{\gamma_Z}$. However, this phase is not physical since it can be eliminated by a $U(1)_{PQ}$ rotation [26].

### III. CP-VIOLATING PHASES FROM YUKAWAS

Although the CP-violating phases from supersymmetry breaking are absent or small as found above, there is an additional contribution to CP violation if the trilinear $A$ parameters are not aligned with the Yukawas. This can be easily seen as follows. Since the Yukawa matrix generically contains $O(1)$ phases in order to explain the observed CKM phase, the unitary matrices needed to go to the super-CKM basis (in which the Yukawa matrices are real and diagonal) also contain some phases. Therefore, the rotation by itself can induce CP-violating phases even if the $A$ or $A'$ matrices are initially completely real as long as $A$'s are not proportional to Yukawas in the flavor eigenstate basis (or equivalently $A$'s are flavor non-universal and non-diagonal). This implies in particular that the diagonal components of trilinear $\hat{A}$ will contain CP phases in the super-CKM basis, giving rise to possibly important contributions for EDMs.

In the M-theory framework, the Yukawa couplings $Y_{\alpha\beta\gamma}$ depend holomorphically on the geometric moduli $z_i$ in general which get non-zero $F$-term vevs. Hence, from [9] we find that the second term in the expression for trilinears gives rise to an $O(1)$ misalignment between the Yukawas and the trilinears. If the Yukawa couplings depend on moduli or other hidden sector fields which do not break supersymmetry, then the trilinears can be naturally aligned with the Yukawas [15]. However, within M-theory, this does not seem to be a generic situation; hence we will consider the conservative case in which the trilinears are misaligned with the Yukawas.

In the remainder of this section, we will estimate the diagonal CP phases in the trilinear $\hat{A}$ in the super-CKM basis since they are directly related to the EDM observables. We consider flavor non-universal and non-diagonal trilinear $A$-matrices (in the flavor eigenstate basis) at the GUT scale with real $O(1)$ matrix elements. To set the conventions, we write down the soft trilinear terms explicitly

$$L_{\text{soft}} = A^u_{ij} Y^u_{ij} \bar{Q}_L h_u u_R + A^d_{ij} Y^d_{ij} \bar{Q}_L h_d d_R
+ A^e_{ij} Y^e_{ij} \bar{L}_L h_d e_R.$$  \hspace{1cm} (10)

where $A^u,d,e$ are the trilinear matrices in the gauge eigenstate basis of matter fields.

In the M-theory framework, chiral matter fields are localized on singular points inside the compact $G_2$ manifold [18, 29, 30, 31]. Although a detailed understanding of Yukawas within M-theory is not yet available, a hierarchical Yukawa texture seems well motivated. From a phenomenological point of view, therefore, we consider the following Yukawa texture:

$$Y_{ij}^u \sim \epsilon_{ij}^u, \quad Y_{ij}^d \sim \epsilon_{ij}^d, \quad Y_{ij}^e \sim \epsilon_{ij}^e.$$  \hspace{1cm} (11)

This kind of texture can also be realized naturally by the localization of matter fields in extra dimensional models [32, 33, 34, 35] or by a spontaneously broken flavor.
symmetry (Froggatt-Nielsen mechanism) \[36\]. Then, the fermion mass hierarchy is given by:

\[
\begin{align*}
\frac{m_u}{m_d} &\sim |\epsilon_1^u \epsilon_1^d| / |\epsilon_2^u \epsilon_2^d|, \\
\frac{m_e}{m_d} &\sim |\epsilon_1^e \epsilon_1^d| / |\epsilon_2^e \epsilon_2^d|,
\end{align*}
\]

It is straightforward to check that the observed fermion mass hierarchy can be accommodated by a set of properly chosen \(\epsilon_i\) with the hierarchy \(|\epsilon_1| \lesssim |\epsilon_2| \lesssim |\epsilon_3|\). The above Yukawa couplings can have \(\mathcal{O}(1)\) phases in order to explain the CP phase in the CKM matrix. To simplify the discussion, we eliminate the phases in the diagonal elements by a redefinition of the quark and lepton fields. Therefore, the diagonal elements \((\hat{A}^u)^{ij}_{11,22,33}\) with \(\psi = u, d, e\) are all real at the GUT scale.

First, we point out that the renormalization group (RG) corrections to the trilinear couplings typically mix the phases between different flavors. This will lead to phases in the diagonal elements of the trilinear matrices. It can be understood from the RG equation for Yukawa couplings and trilinear couplings, e.g. for \(Y^u\) and \(\hat{A}^u\), which are given by

\[
\begin{align*}
\frac{dY^u}{dt} &\sim \frac{1}{16\pi^2} Y^u \left[ 3 \text{Tr}(Y^u Y^u) + 3Y^u Y^u + Y^d Y^d \right]
\end{align*}
\]

only terms involving Yukawas are explicitly shown. From the above equations, we notice that the phases in \(\hat{A}^u\) evolve during the RG running. To illustrate this, one can examine the following term which contributes to the running of \(\hat{A}^u_{11}\):

\[
\begin{align*}
\frac{d\hat{A}^u_{11}}{dt} &\sim \frac{5}{16\pi^2} \hat{A}^u_{13} Y^{u*}_{31} Y^{u}_{13} + \frac{4}{16\pi^2} Y^{u*}_{13} Y^{u*}_{31} \hat{A}^u_{31}
\end{align*}
\]

From the equation, one can see that the phases of \(Y^u_{31}, Y^u_{13}, \hat{A}^u_{13}\) and \(\hat{A}^u_{31}\) can enter that of \(\hat{A}^u_{11}\) through RG effects although it was real at the high scale. The magnitude of this correction can be significant since the magnitude of the right-hand side of the above equation is proportional to \(1/\text{ln}^2 |Y^u_{31}|^2\) given the factorizable Yukawa matrices as in Eq. \([11]\). This indicates that the RG corrections to \(\hat{A}^u_{11}\) can give \(\mathcal{O}(1)\) phases. This is also true for other elements in the trilinear matrices and Yukawas. The only exception is for the third generation \(Y^u_{33}\) and \(\hat{A}^u_{33}\), for which the largest RG corrections come from the terms involving only \(Y^u_{33}\) and \(\hat{A}^u_{33}\) with additional flavor mixing terms typically suppressed by \(\epsilon_2^2/\epsilon_3^2\). Since \(\hat{A}^u_{33}\) has the same phase as \(Y^u_{33}\), the corresponding A-term \(A^u_{33} = \hat{A}^u_{33}/Y^u_{33}\) remains real up to corrections of the order \((\epsilon_2/\epsilon_3)^2\).

Starting from the Yukawa matrices \(Y^{u,d,e}_{ij}\) defined in the flavor eigenstate basis, the super-CKM basis can be achieved by unitary rotations of the matter fields so that the Yukawa matrices are real and diagonal. In the super-CKM basis, the trilinear couplings become

\[
\begin{align*}
\hat{A}^u_{\text{CKM}} &\sim (V_L^u)^{*}_{ij} A^u_{ik} Y^u_{kj} (V_R^u)^{kj}_{i} \text{ (15)}
\end{align*}
\]

where \(\psi = u, d, e\). Given the hierarchical Yukawa matrices in Eq. \([11]\), the unitary transformation matrices are given by

\[
\begin{align*}
(V_L^u)_{ij} &\sim (V_L^u)_{ji} \sim \epsilon_1^u \epsilon_1^d, \text{ for } i < j \\
(V_R^u)_{ij} &\sim (V_R^u)_{ji} \sim \epsilon_1^e \epsilon_1^u, \text{ for } i < j
\end{align*}
\]

One can now perform the same transformation for the trilinear terms to get the diagonal elements in the super-CKM basis, which can be schematically written as

\[
\begin{align*}
\hat{A}^u_{\text{CKM}} &\sim \epsilon_1^u \epsilon_1^d \sum_{i,j=1,2,3} \xi_{ij} A^u_{ij}, \text{ (17)} \\
\hat{A}^u_{\text{CKM}} &\sim \epsilon_2^u \epsilon_2^d \sum_{i,j=2,3} \eta_{ij} A^u_{ij}, \text{ (18)}
\end{align*}
\]

\[
\begin{align*}
\hat{A}^u_{\text{CKM}} &\sim \epsilon_3^u \epsilon_3^d \sum_{i,j=33} \eta_{ij} A^u_{ij}, \text{ (19)}
\end{align*}
\]

where \(\xi_{ij}\) and \(\eta_{ij}\) are possibly \(\mathcal{O}(1)\) coefficients arising from the Yukawa matrices, and are complex in general. In the above equations, we have neglected subleading terms suppressed by the fermion mass hierarchy. Since the off-diagonal components in \(A_{ij}\) can be \(\mathcal{O}(1)\) within our framework, the summations in Eqs. \([17]\) and \([18]\) can be of \(\mathcal{O}(1)\) in magnitude with \(\mathcal{O}(1)\) phases. The \(A_{33}\) component, however, does not mix with other components and is proportional to \(Y^u_{33}\), so no phase is generated at leading order for \(A^u_{33}\).

Therefore, we conclude that the first two diagonal components of the complete trilinear coupling in the super-CKM basis can contain order one phases, while the third diagonal component is real up to small corrections, i.e.

\[
\begin{align*}
\text{Im}(\hat{A}^u_{\text{CKM}})_{11} &\sim A_0 Y^u_{11}, \\
\text{Im}(\hat{A}^u_{\text{CKM}})_{22} &\sim A_0 Y^u_{22}, \\
\text{Im}(\hat{A}^u_{\text{CKM}})_{33} &\sim A_0 \left(\frac{\epsilon_2}{\epsilon_3}\right)^2 Y^u_{33}
\end{align*}
\]

where \(A_0\) is the characteristic magnitude of the trilinear A-terms. Here we do not distinguish between \(q, u, d\) and use \(\epsilon_i\) for the average value. For later purposes, it is convenient to take \(\epsilon_2/\epsilon_3 \approx 0.1\), which provides a right order of magnitude estimate and is also compatible with the quark mass hierarchy. This result has important implications for EDM predictions, which we compute in section \(V\).

IV. ELECTRIC DIPOLE MOMENTS AND THE EXPERIMENTAL LIMITS

Before starting our calculation of EDMs, we briefly summarize some general results relevant for the calculation of EDMs. In the minimal supersymmetric standard
models, the important CP-odd terms in the Lagrangian are:

\[
\delta \mathcal{L} = - \sum_{q=u,d,s} m_q \bar{q}(1 + i \theta_q \gamma_5)q + \theta_{G} \frac{\alpha_s}{8\pi} G \tilde{G} \\
\frac{i}{2} \sum_{f=u,d,s} (\bar{q}_f \gamma^\mu \sigma_{\mu\nu} \gamma_5 q + \bar{q}_f \gamma^\alpha G^{\alpha}_{\mu\nu} \sigma_{\mu\nu} \gamma_5 q) \\
- \frac{i}{6} \bar{q}_f \gamma^\mu \sigma_{\mu\nu} \sigma_{\rho\sigma} \gamma_5 \theta \epsilon \gamma_5 q,
\]

where \( \theta_{G} \) is the QCD \( \theta \) angle. The terms in the second line in (21) are dimension five operators, which are generated by CP violation in the supersymmetry breaking sector and evolved down to \( \sim 1 \) GeV. The coefficients \( d_{q}^{E,C} \) correspond to quark electric dipole moment and chromo-electric dipole moment (CEDM) respectively. The last line in (21) contains the gluonic dimension six Weinberg operator. The CP-odd four-fermion interactions are not important here, and so have not been included above.

Now let us briefly summarize the EDM results for electrons, neutrons and mercury in terms of the coefficients of these operators. The electron EDM in minimal supersymmetric models is given by:

\[
d_{e}^{E} = d_{e}^{+} + d_{e}^{0} + d_{e}^{BZ}
\]

where \( d_{e}^{+} \) and \( d_{e}^{0} \) are one-loop contributions from the neutralino and chargino while \( d_{e}^{BZ} \) is the two-loop Barr-Zee type contribution. It should be noted that what is actually measured is the atomic EDM \( d_{T1} \), which receives contributions mainly from the electron EDM and the CP-odd electron-nucleon couplings [60].

\[
d_{T1} = -585 \times d_{e}^{E} - 8.5 \times 10^{-19} e\text{cm}(C_{S} \text{TeV}^{2}) + \cdots
\]

where \( C_{S} \) is the coefficient of the operator \( \bar{e}i\gamma_5 e\bar{N}N \). The \( C_{S} \) coefficient could be generated from a new scalar particle coupled to quarks and leptons through a CP-odd higgs like coupling [60]. However, this is independent of CP-odd interactions originating from the soft terms. Given the current experimental limit \( |d_{T1}| < 9 \times 10^{-25} e\text{cm} \), we obtain an upper limit on electron EDM

\[
|d_{e}^{E}| < 2 \times 10^{-27} e\text{cm}
\]

For the neutron, there exist several different approaches to compute the corresponding EDM. In the following discussion, we shall follow a simple approach, i.e., the naive dimensional analysis (NDA) [47, 48, 49]. The neutron EDM can be calculated as:

\[
d_{n} = \frac{4}{3} d_{d} - \frac{1}{3} d_{u}.
\]

In this expression, the quark EDMs can be estimated via NDA as:

\[
d_{q} = \eta^{E} d_{q}^{E} + \eta^{C} \frac{e}{4\pi} d_{q}^{C} + \eta^{G} \frac{e\Lambda}{4\pi} d_{q}^{G}.
\]

with \( d_{q}^{E,C} = d_{q}^{\tilde{E}}(E,C) + d_{q}^{\tilde{C}}(E,C) + d_{q}^{\tilde{G}}(E,C) \). The QCD correction factors are given by \( \eta^{E} = 1.53, \eta^{C} \sim \eta^{G} \sim 3.4 [9] \), and \( \Lambda \sim 1.19 \text{ GeV} \) is the chiral symmetry breaking scale. The current experimental limit on neutron EDM is given by

\[
|d_{n}| < 3 \times 10^{-26} e\text{cm}
\]

The current theoretical estimate for the mercury EDM induced by dimension 5 operators is given by [50]:

\[
d_{H_{q}} = -7.0 \times 10^{-3} e(d_{q}^{E} - d_{q}^{C} - 0.012d_{q}^{G}) + 10^{-2} \times d_{e}
\]

where we have included the contribution from the strange quark CEDM [51]. The recent experimental result on Mercury EDM [5] significantly tightens the bound

\[
|d_{H_{q}}| < 3.1 \times 10^{-29} e\text{cm}
\]

In the Standard Model, the primary source of hadronic EDMs comes from the QCD \( \theta \) term in (21). This gives the following results [46, 52, 53, 54]:

\[
d_{n} \sim 3 \times 10^{-16} \theta e\text{cm}
\]

\[
d_{D} \sim -1 \times 10^{-19} \theta e\text{cm}
\]

\[
|d_{H_{q}}| \sim O(10^{-18} - 10^{-19}) \theta e\text{cm}
\]

(23)

On the other hand, the electron EDM is induced by the SM electroweak interactions, which is typically of order \( 10^{-38} e\text{cm} [55, 56] \). The results in (23) together with the suppressed leptonic EDMs provide a correlation pattern for the \( \theta \) induced electric dipole moments. The current upper bound on the neutron EDM implies \( \theta < O(10^{-10}) \), which leads to the strong CP problem. Once EDMs are observed for \( n, H_{g} \) and \( T_{I} \) it will be essential to separate the strong and weak contributions, by combining data on different nuclei and \( d_{e}^{E} \).

V. PREDICTIONS FOR EDMs

For an explicit computation of the EDMs, it is important to specify the general structure of supersymmetry breaking parameters, in particular the structure of the trilinear parameters (especially the imaginary part of the diagonal components), as well as that of the scalar and gaugino masses, since all of these appear in the final expression for the EDMs. This is the subject of this section.

Within the M-theory framework, the general structure of supersymmetry breaking parameters is as follows. For the choice of microscopic parameters with a vanishingly small positive cosmological constant, the gravitino mass naturally turns out to be in the range 10-100 TeV [13]. The gravitino mass is essentially \( \sim O(10^{-10}) \), and \( \Lambda \sim 1.19 \text{ GeV} \) is the chiral symmetry breaking scale. The current experimental limit on neutron EDM implies \( \theta < O(10^{-10}) \), which leads to the strong CP problem. Once EDMs are observed for \( n, H_{g} \) and \( T_{I} \) it will be essential to separate the strong and weak contributions, by combining data on different nuclei and \( d_{e}^{E} \).
where masses we consider gauginos with masses typical turn out to be of $\mathcal{O}(m_{3/2}) \sim \mathcal{O}(10)$ TeV. The third generation squarks, however, could be significantly lighter because of the RG effects.

As we have discussed in Section II within the M-theory framework it is natural to expect that the Kähler metric for visible matter fields is approximately diagonal in the flavor indices. Then, the scalar mass matrix turns out to be roughly diagonal with suppressed off-diagonal contributions. The estimates for the EDMs then depend on the overall scale of the squark masses. So, for concreteness we consider gauginos with masses $\lesssim 20$ TeV, and $\mu, B\mu$ and trilinear parameters of the same order as scalar masses. Some contributions to EDMs depend primarily on third generation sfermion masses, so we also mention the situation when third generation scalars are much lighter, i.e. $\mathcal{O}(1)$ TeV.

We now estimate the contribution to the EDMs of the electron, neutron and mercury from dimension 5 and 6 operators (Eq. (21)) in the M-theory framework. As we have seen in the Section III, the CP-violating phases appear only in the trilinear $A$ parameters. After renormalization group evolution and the super-CKM rotation of the trilinear matrices, these phases appear in the off-diagonal elements in the squark mass matrices, leading to imaginary parts of the following mass-insertion parameters:

$$ (g_q^{ii})_{LR} = \frac{v_d((\hat{A}_{SCKM}^{t})^{ii} - \mu Y_d^R R_q)}{(m_{3/2}^q)^{ii}} (24) $$

where $R_{u(d)} = \cot \beta (\tan \beta)$ and $v_{u(d)} = v \sin \beta (v \cos \beta)$. As explained above, $\hat{A}_{SCKM}$ is in general a $3 \times 3$ matrix in the Super-CKM basis and its diagonal components contain CP-violating phases. Thus, these insertion parameters contribute to EDMs through the dimension 5 and 6 operators in (21).

A. Leading Contributions

The dimension five electric and chromo-electric couplings can be generated at leading order in the M-theory framework at one-loop through the vertices $f f \tilde{\chi}_i^0$, $f f \tilde{\chi}_i^\pm$ and $q g g$ as can be seen in Fig. 1.

First consider the quark CEDM which contributes to both the mercury and neutron EDMs. Since there exists a hierarchy between gauginos and squarks in the M-theory framework, one can expand using the small ratio $r \equiv m_{3/2}^q/m_q$, where $m_i$ is the corresponding neutralino, chargino or gluino mass in the diagram. One then obtains the following result

$$ d_q^C \sim \frac{g_s \alpha_s m_q^2}{4\pi} \text{Im}(A_{SCKM}^{\chi^0}) r^2 G(r) \quad (25) $$

where $A_{SCKM}^{\chi^0}$ is the diagonal element of the corresponding trilinear matrix (factoring out the Yukawa coupling) in the super-CKM basis. In the expression, the function $G(r) = C(r) + r C'(r)$ for gluinos and $G(r) = B(r) + r B'(r)$ for charginos and neutralinos. The function $B(r)$ and $C(r)$ are loop functions defined in the Appendix. One can see that $d_q^C$ decreases rapidly as $m_q^4$ when the squark masses increase. However, the function $G(r)$ behaves differently for different particles ($g$, $\tilde{\chi}^\pm$, $\tilde{\chi}^0$) in the loop. Due the gaugino and squark mass hierarchy, $r$ is small. From Fig. 2 we can see that $C(r) + r C'(r)$ is enhanced in the small $r$ region compared to other functions which remain small. Therefore, the gluino contribution dominates the quark CEDM. For the quark EDM, it is given by a similar expression as (25) but now the quantity $G(r)$ is determined only by $A(r)$ and $B(r)$. In particular, $G(r)$ is determined solely by $B(r)$ for $\tilde{g}$ and $\tilde{\chi}^0$ in the loop, and by a combination of $A(r)$ and $B(r)$ for $\tilde{\chi}^\pm$ in the loop. Since $A(r) + r A'(r)$ and $B(r) + r B'(r)$ are much smaller than $C(r) + r C'(r)$ as seen from Figure 2 the quark EDM contributions to the neutron EDM are negligible compared to that of the quark CEDM contributions. Therefore, we only need to calculate the quark CEDM, for which the gluino diagram gives the dominant contribution as explained above. Since $A_{SCKM}^{\chi^0} \sim m_{3/2}$ in the M-theory framework, one obtains

$$ d_q^C \sim 10^{-28} \cdot \left( \frac{m_q}{1 \text{MeV}} \right) \left( \frac{m_{3/2}}{600 \text{GeV}} \right) \left( \frac{20 \text{TeV}}{m_{3/2}} \right)^3 \text{e cm} \quad (26) $$

Based on the quark EDM and CEDM, the neutron EDM can be computed from (22):

$$ d_n^{DA} \sim 3 \times 10^{-28} \cdot \left( \frac{m_{3/2}}{600 \text{GeV}} \right) \left( \frac{20 \text{TeV}}{m_{3/2}} \right)^3 \text{e cm} \quad (27) $$

Similarly, the mercury EDM is

$$ |d_{He}| \sim 10^{-30} \cdot \left( \frac{m_{3/2}}{600 \text{GeV}} \right) \left( \frac{20 \text{TeV}}{m_{3/2}} \right)^3 \text{e cm} \quad (28) $$

FIG. 1: One-loop contributions to fermion (C)EDMs.
does not contribute as seen from Eq. (B7). This is because it requires gaugino-higgsino mixing in order to have chirality flipping; but this mixing is negligibly small in the M-theory framework. The neutralino contribution, on the other hand, gives rise to a non-zero contribution because of a dependence on the selectron mixing parameters (which contains CP-violating phases) in its couplings (see Eqn. (B8)). Given the fact that the higgsino coupling to electron and selectron is suppressed by the small electron Yukawa coupling, and the wino does not couple to right-handed fermions and sfermions, the dominant contribution is from the diagram with $\tilde{\chi}_0^2$ (almost pure bino in the M-theory framework), which can be calculated using Eq. (B11) in the Appendix. Thus, the electron EDM is given by:

$$d^E_e \sim \left( \frac{m_{\tilde{q}}}{200\text{GeV}} \right) \left( \frac{20\text{TeV}}{m_\tilde{q}} \right)^3 \times 10^{-31} \text{e cm} \quad (29)$$

### B. Two-loop Contributions

So far, we have considered the one-loop contribution to quark and electron EDMs (and/or CEDMs). In addition, there are two-loop Barr-Zee type contributions such as the one in Fig. 3. In general, the Barr-Zee type diagrams can involve squarks, charginos or neutralinos in the inner loop, and higgs bosons (neutral or charged) and/or gauge bosons in the outer loop (the two-loop diagram considered in split supersymmetry is not relevant here, since there the CP violation is not from trilinear couplings, but instead from the chargino sector). Since only the trilinear couplings contain CP-violating phases in our framework, we consider those diagrams with squarks running in the inner loop as seen in Figure 3. One might wonder whether there are any two-loop diagrams that would contribute if there were phases in the gaugino masses or $\mu$ term such as in the split supersymmetry scenario. Since the higgsino in the M-theory framework is very heavy with mass $\mu \sim m_{3/2}$ and hence decoupled from the low energy theory, the only diagram which might contribute is the one in Fig. 4. However, it turns out that the CP phases in the two W-chargino-neutralino couplings cancel out (up to a small correction due to the heavy higgsino) in the final result giving no EDM contribution.

When the mass splitting between the two third generation squarks is not particularly large, the diagram to the quark CEDM can be estimated as (see Appendix C):

$$d^C_{BZ} f \approx \frac{g_s \alpha_s}{64 \pi^3} \frac{m_f R_f \mu}{M_A^4} \sum_{q=\tilde{t},\tilde{b}} y_q^2 \text{Im}(A^q_{\text{SCKM}}) F'(r_q) \sim 10^{-32} \cdot R_f \left( \frac{m_f}{1\text{MeV}} \right)^2 \left( \frac{20\text{TeV}}{m_{\tilde{q}}} \right)^2 \text{e cm} \quad (30)$$

where $R_f = \cot \beta (\tan \beta)$ for $I_3 = 1/2(-1/2), \, r_q \equiv m_q^2/M_A^2$ with $m_q$ third generation squark mass and $M_A$ the pseudoscalar mass of $A_0$. Here we have used (20) for $\text{Im}(A^q_{\text{SCKM}})$. For simplicity, we also take $\mu \sim M_A \sim m_{\tilde{t},\tilde{b}} \sim m_\tilde{q}$. It can be seen that the result of the Barr-Zee diagram to quark CEDM (similar for EDM) is negligibly small. One of the reasons is that CP violation in the third generation is suppressed by about two orders of magnitude as in (20). Similarly, for the electron EDM the result is:

$$d^E_{BZ} \sim 10^{-33} \cdot \left( \frac{20\text{TeV}}{m_\tilde{e}_3} \right)^2 \tan \beta \text{ e cm} \quad (31)$$

which is again quite suppressed. This contribution may be enhanced for large $\tan \beta$ as seen from above. The M-theory framework, however, generically predicts $\tan \beta = O(1)$. 

![FIG. 3: Two-loop Barr-Zee type diagrams contributing to fermion (C)EDMs.](image)

![FIG. 4: Two-loop Barr-Zee type diagrams which do not involve sfermion in the loop.](image)
The neutron EDM could also get a contribution from the dimension six pure gluonic operator (Weinberg operator), which can be generated from the two loop gluino-top-stop and gluino-bottom-sbottom diagrams. For the case where CP violation only comes from the soft trilinear couplings, the result can be estimated by [58]

\[ d^G \approx -3\alpha_s \left( \frac{g_s}{4\pi} \right)^3 \frac{1}{m_{\tilde{q}}^2} \sum_{q=t,b} \text{Im}(A_q^{\text{SCKM}}) z_q H(z_1, z_2, z_q) \]

where \( z_i = m_\phi^2 / m_{\tilde{q}}^2 \) for \( i = 1, 2 \), and \( z_q = m_\phi^2 / m_{\tilde{q}}^2 \) for \( q = t,b \). The two-loop function \( H(z_1, z_2, z_t) \) is given in [58]. This gives a contribution to the neutron EDM \( d^G \sim 10^{-30} \text{ cm} \) for \( m_{t,b} \approx 20 \text{ TeV} \), \( m_{\tilde{t}} = 600 \text{ GeV} \) and \( A_q = 20 \text{ TeV} \). Thus the neutron EDM from the Weinberg operator is smaller than the one-loop CEDM contribution. However, when the masses of the third generation squarks and trilinears are around 1 TeV, the contribution to the neutron EDM can be significantly larger, and be comparable to the one-loop result.

To summarize our results, we have calculated the EDMs arising from the CP-violating phases in the trilinear terms in a general framework with light gauginos and heavy sfermions, and the results are within current experimental bounds. We find that the one-loop diagram is typically the dominant contribution to EDMs. However, in contrast to the situation in which gaugino and sfermion masses are comparable, the one-loop diagram with gluino is enhanced over the one with neutralino. This leads to a larger ratio between neutron EDM and electron EDM of \( \gtrsim 10^3 \). In typical supersymmetric models with gaugino and sfermion masses of the same order, this ratio is \( \lesssim 10^2 \) [59]. This seems to be a robust feature of models in which the gauginos are suppressed relative to the squarks and the trilinears and the trilinears are not proportional to Yukawas. Finally, it is easy to see that the mercury EDM provides the most stringent limit on the squark masses. For squark masses around 10 TeV, the mercury EDM will increase to \( \sim 10^{-29} \text{ cm} \), which could be tested in the near future with better experimental precision. Basically, we have found that the upper bounds on the EDMs in the M-theory framework result from two of its generic features - large scalar masses and trilinears \( (O(10) \text{ TeV}) \), and CP-violating phases only in the trilinear couplings and those arising only from the Yukawas.

**VI. HIGHER-ORDER CORRECTIONS AND GENERALIZATIONS**

In Section [1], we found that there are no CP-violating phases from supersymmetry breaking at leading order in the framework of M-theory compactifications considered. It is therefore important to check if corrections to the Kähler potential and superpotential lead to further contributions to CP-violating phases in the soft parameters and in turn to the EDMs. Although the detailed form of possible corrections is not known in M-theory, some general arguments can nevertheless be made, which strongly suggest that higher order corrections still naturally suppress CP-violating phases.

The corrections to the soft parameters may arise in general from corrections in the superpotential and the Kähler potential. In the zero flux sector, which we have considered, the superpotential may receive additional non-perturbative corrections from strong gauge dynamics or from membrane instantons. These corrections are naturally suppressed compared to the existing terms if the arguments in the corresponding exponentials are just \( O(1) \) larger than that in the existing terms. Even if they are comparable, the mechanism of dynamical alignment of the phases can still be applied and leads to no additional phases.

More importantly, large corrections could arise in the Kähler potential for the hidden sector comprising the moduli and hidden matter fields, such as terms with higher powers of \( \phi \). However, the field \( \phi \) is composed of elementary quark fields \( Q, Q^\dagger \) which are charged under the hidden gauge group. Therefore, higher order corrections must be functions of \( Q^\dagger Q \) or \( Q^\dagger \bar{Q} \) in order to be gauge invariant. When written in terms of \( \phi \), these corrections are always functions of \( \phi^\dagger \phi \). This structure is important for our claim of small CP-violating phases since it does not introduce any new phases in the soft parameters. In addition, the crucial fact that the Kähler potential is a real function of \( z_i \) is also expected to be true for the perturbative corrections. The dependence on \( z_i \) is a reflection of the shift symmetry of the axion \( t_i \rightarrow t_i + \delta \), which is only broken by non-perturbative effects. Thus, although it is very hard in general to compute the form of corrections in M-theory, the above result should be quite robust as it only relies on symmetries.

One could try to generalize the results obtained within the M-theory framework so that they could be applied to other string compactifications as well. Consider an \( N = 1 \) string compactification to four dimensions which can be described by \( N = 1 \) supergravity at energies lower than the compactification scale. The nature of the hidden sector dynamics is such that there are gauge-singlet...
scalar fields $X_i$ and moduli fields $T_j$, both of which break supersymmetry in general. These additional scalar fields $X_i$ could arise from matter fields for example. The following conditions, if satisfied, would lead to no CP-violating phases in the soft supersymmetry breaking parameters:

- The moduli superpotential is a polynomial in $X_i$ and exponential in $T_j$ of the form:
  \[
  \hat{W} = \hat{W}_0 + \sum_{n=1}^{N_W} \hat{W}_n = d_n(X_i) \exp(b_n^i T_j),
  \]
  where $\hat{W}_0$ is a constant and $d_n$s are polynomial function of $X_i$, and $b_n^i$s are real.

- The Kähler potential is a real function of $T_j + \bar{T}_j$ and $\bar{X}_i X_i$
  \[
  K = K(T_i + \bar{T}_i, \bar{X}_i X_i)
  \]

- The gauge kinetic functions are given by $f_a = k_a^i T_j$ with real constants $k_a^i$.

- The holomorphic (unnormalized) Yukawa couplings have the following structure:
  \[
  Y_{\alpha\beta\gamma} = y(X_i) e^{\sum_j c_{\alpha\beta\gamma}^j T_j} h_{\alpha\beta\gamma}
  \]

The dynamical alignment of the phases among different fields which do not break supersymmetry, which we do not include here explicitly. The gauge kinetic functions are given by $f_a = k_a^i T_j$ with real constants $k_a^i$. The holomorphic (unnormalized) Yukawa couplings have the following structure:

\[
Y^T_{\alpha\beta\gamma} = y(X_i) e^{\sum_j c_{\alpha\beta\gamma}^j T_j} h_{\alpha\beta\gamma}
\]

where $y(X_i)$ is a polynomial function of $X_i$ and $c^{\alpha\beta\gamma}$s are real. These parameters could also vanish in specific cases. In addition, the Kähler potential and superpotential can also depend on the scalar fields which do not break supersymmetry, which we do not include here explicitly.

The dynamical alignment of the phases among different terms in the superpotential is expected to hold as explained in Section [1] and Appendix [A]. Thus, the structure of $W$ guarantees that $F T_i$'s remain real. The requirement that the kähler potential be a function of $\bar{X}_i X_i$ and that the superpotential and holomorphic (unnormallized) Yukawas be a polynomial function of $X_i$ makes sure that the contractions $F^{X_i} \partial X_i K$ and $F^{X_i} \partial X_i Y$ are real. Therefore, both gaugino masses and trilinear couplings are real up to an overall phase which can be rotated away. The phases of $\mu$ and $B$ can also be rotated away if they are generated from the Giudice-Masiero mechanism. The conditions listed above can be seen as a non-trivial generalization of the ones discussed in [14] [15]. They can be naturally satisfied in a general moduli stabilization framework in which additional matter fields provide a dynamical $F$-term uplifting mechanism for the vacuum energy.

### VII. CONCLUSIONS

In this paper, we have discussed CP violation in theories arising from fluxless M-theory compactifications with low energy supersymmetry and all moduli stabilized. We have found that the structure of the low-energy theory is such that CP-violating phases arise dominantly from phases in the quark and lepton Yukawas but not from soft supersymmetry breaking. The trilinear couplings pick up CP-violating phases from the Yukawa couplings as the trilinear matrices are not proportional to the Yukawa matrices. Given a model of Yukawas, therefore, one can estimate the effects of these phases of Yukawas on the trilinear couplings, and therefore on the EDMs. Since hierarchical Yukawa textures are well motivated within this framework, we compute EDMs for such textures. The other relevant feature of the low energy theory is that the scalar masses and trilinears are naturally of $O(m_{3/2})$ due to the absence of sequestering generically. Moreover, these scalar masses and trilinears are naturally of $O(10 \text{ TeV})$ since this is the natural scale of the gravitino mass within the M-theory framework. These two features naturally give rise to small CP-violating effects consistent with experimental limits.

We have estimated the electron, neutron and mercury electric dipole moments utilizing the above features, and found that the estimated upper bounds of the EDMs are all within current experimental limits. The estimated upper bound for the mercury EDM is close to the current experimental limit and could be probed in the near future. A robust prediction of the framework is the existence of a hierarchy of about three orders of magnitude between the neutron and electron EDMs. This essentially results from the mass hierarchy between gauginos and scalars as predicted within M-theory [13], and provides an additional means to test the framework.

It should be emphasized that our results for EDMs are based on the result that the CP-violating phases are entirely from the Yukawas, and therefore, any experimental result which indicates other significant sources of phases would contradict and rule out this approach. We also discuss effects of possible corrections to the Kähler potential and superpotential, and the generalization to other string compactifications.

Note that our results are largely independent of a full solution to the flavor problem. Our results have been derived using the fact that the squark matrices are approximately flavor diagonal at low energies which is naturally predicted within the M-theory framework, as explained in Section [IA]. Therefore, any solution to the flavor problem consistent with the above feature is consistent with our results.

The quark Yukawas give the CKM phases, and the lepton Yukawas the PNMS phases. The latter can provide the phases needed for baryogenesis via leptogenesis consistent with the above framework, for example as described in [60]. So even with no phases from the soft supersymmetry breaking this framework can give a complete description of all known CP violation.
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APPENDIX A: DYNAMICAL ALIGNMENT OF PHASES IN THE SUPERPOTENTIAL

The dynamical alignment of phases is crucial for solving the SUSY CP problem in the M-theory framework. It means that all terms in the superpotential dynamically align to acquire the same phase in the vacuum. To show that, we start with the generic moduli superpotential

\[ W = \sum_{n=1}^{N_{\text{f}}} |W_n| e^{\gamma_n} \quad (A1) \]

The superpotential depends on geometric moduli \( z_i \) as well as other chiral superfields \( \phi_i \) in general. It is generically a polynomial and/or exponential function of these fields (for example, see Eq. \( A4 \)), and so \( \gamma_n \) are functions of \( \text{Im}(z_i) \) and the phase of \( \phi_i \), which are collectively denoted by \( \psi_i \). The scalar potential in \( N = 1 \) supergravity is written as:

\[ V = e^K \left( K^{ij} D_i W D_j W - 3|W|^2 \right) \quad (A2) \]

In the following, we consider the Kähler metric to be diagonal for simplicity, but the results are independent of this assumption. Then we have:

\[ V \sim \sum_i K^{ij} |D_i W|^2 - 3|W|^2, \quad (A3) \]

where

\[
|W|^2 = \sum_n |W_n|^2 + 2 \sum_{n \neq m} |W_n||W_m| \cos(\gamma_n - \gamma_m),
\]

\[
|D_i W|^2 = \sum_n |D_i W_n|^2 + 2 \sum_{n \neq m} |D_i W_n||D_i W_m| \times \cos(\gamma_n - \gamma_m). \quad (A4)
\]

The second line in Eq. \( A4 \) can be obtained by utilizing the fact that the superpotential is generically a polynomial and/or exponential function of fields. For fields \( \phi_i \) occurring polynomially, one has \( \partial_i W_n \sim W_n/\phi_i = \text{real} \times e^{i\gamma_n - i\gamma_i} \), as well as \( \partial_i K W_n = \text{real} \times e^{i\gamma_n - i\gamma_i} \).

Similarly, for fields \( z_i \) occurring exponentially in the superpotential, one has \( \partial_i W_n = \text{real} \times e^{i\gamma_n} \) as well as \( \partial_i K W_n = \text{real} \times e^{i\gamma_n} \), giving rise to the expressions in Eq. \( A4 \) above. This then implies that the first and second terms in Eq. \( A3 \) contain the same functional dependence on \( \gamma_n \).

Therefore, the minimization of the scalar potential with respect to \( \psi_i \) gives

\[
\sum_{n \neq m} A_{nm} \sin(\gamma_n - \gamma_m) \partial_w [\gamma_n(\psi_i) - \gamma_m(\psi_i)] = 0 \quad (A5)
\]

where \( A_{nm} \) is a real matrix whose explicit form is not important here. Since \( \gamma_n \) is a combination of several phases, \( \partial_w(\gamma_n - \gamma_m) \) is non-zero for some \( i \) in general. Therefore, the solution to the system of equations in Eq. \( A5 \) is:

\[
\sin(\gamma_n - \gamma_m) = 0 \quad (A6)
\]

Generically, the number of degree of freedom \( \psi_k \) is more than the number of \( \gamma_n \), and the above equations can be solved by \( \gamma_n = \gamma_m \) (mod \( \pi \), leading to a minus sign) for any \( n \) and \( m \). As an example, for the superpotential in \( A2 \), there is only one independent equation corresponding to the combination \( ((b_1 - b_2)\bar{N} \cdot \bar{t} + a \gamma_\phi) \) but \( N + 1 \) degree of freedom corresponding to \( N \) axion \( t_i \) and the phase of the meson \( \gamma_\phi \). This means all \( W_n \) share the same phase, which proves that the phases are dynamically aligned in the vacuum. Of course, if the number of dynamical phases is less than the number of independent equations, then there could be relative phases between different terms in the superpotential. However, that is not generic.

APPENDIX B: THE LEADING ONE-LOOP CONTRIBUTIONS TO EDM

The fermion EDMs can be generated at one-loop in supersymmetric models with CP-violating phases in the soft supersymmetry breaking sector. Within the framework considered in this paper, the CP-violating phases only reside in the trilinear terms and therefore appear in the mass mixing terms of the left- and right-handed sfermions. Therefore, the main contribution to the quark EDM and CEDM comes from diagrams involving gluinos because of the large gauge coupling. For the electron EDM, the dominant contribution comes from the diagram involving neutralinos. This is because the diagrams with charginos in the loop require CP-violating phases in the chargino sector which do not arise within the M-theory framework considered.

Let us first consider the diagrams contributing to quark EDM and CEDM with gluino running in the loop

\[
edm_q^{(E)} = -\frac{2e\alpha_s}{3\pi} \sum_{k=1}^{2} \text{Im}(\Gamma_{qk}^{1k}) \frac{m_\tilde{g}}{m_{\tilde{q}_k}} Q q B \left( \frac{m_\tilde{g}^2}{m_{\tilde{q}_k}^2} \right) \quad (B1)\]

\[
edm_q^{(C)} = \frac{g_s\alpha_s}{4\pi} \sum_{k=1}^{2} \text{Im}(\Gamma_{qk}^{1k}) \frac{m_\tilde{g}}{m_{\tilde{q}_k}} C \left( \frac{m_\tilde{g}^2}{m_{\tilde{q}_k}^2} \right) \quad (B2)\]
where $\Gamma_{1k}^k = D_{q_{1k}} D_{q_{1k}}^*$ and $D_q$ is the $2 \times 2$ matrix which diagonalizes the squark mass matrix $m_q^2$

$$D_q^* m_q^2 D_q = \text{Diag}(m_{\tilde q_1}^2, m_{\tilde q_2}^2).$$  \hspace{1cm} (B3)

More explicitly

$$\begin{align*}
\hat q_L &= D_{q_{11}} \tilde q_1 + D_{q_{12}} \tilde q_2 \\
\hat q_R &= D_{q_{21}} \tilde q_1 + D_{q_{22}} \tilde q_2.
\end{align*}$$  \hspace{1cm} (B4)

Here $B(r)$ and $C(r)$ are loop functions defined as:

$$\begin{align*}
B(r) &= \frac{1}{2(r-1)^2} \left( 1 + r + \frac{2r \ln (r)}{1-r} \right) \\
C(r) &= \frac{1}{6(r-1)^2} \left( 10r - 26 + \frac{2r \ln (r)}{1-r} - 18 \ln (r) - 1 \right).
\end{align*}$$

In the above equations, we assume no flavor mixing in the squark mass matrices as argued in the main body of the paper. Using the fact that $\text{Im}(\Gamma_{11}^q) = -\text{Im}(\Gamma_{12}^q)$, we have:

$$d_{q}^{E,(E)} \approx \frac{-2e \alpha_s Q_q \text{Im}(m_q^2)_{LR}}{3(3\pi)} r^2 (B(r) + r B'(r))$$  \hspace{1cm} (B5)

Similarly

$$d_{q}^{E,(C)} \approx \frac{g_s \alpha_s}{4 \pi} \frac{\text{Im}(m_q^2)_{LR}}{m_q^2} r^2 (C(r) + r C'(r))$$  \hspace{1cm} (B6)

In the calculation above, we assume the mass splitting of squarks is small compared to the squark mass. This is usually true since we are only interested in the up and down squarks. When $r = m_B^2 / M_{\tilde Q}^2 \ll 1$, one finds that $C(r) \gg A(r), B(r)$. It is easy to see that $d_{q}^{E,(C)} \gg d_{q}^{E,(E)}$. For other diagrams which involve neutralino and charginos, the structure is very similar. However, they are much smaller than $d_{q}^{E}$ and can be neglected.

Now let us turn to the one-loop diagrams contributing to the electron EDM

$$\begin{align*}
d_{e}^{x,+} &= \frac{e \alpha_{em}}{4 \pi \sin^2 \theta_W} \sum_{k=1}^{2} \text{Im}(\Gamma_{ei}) \frac{m_{\tilde e^+}}{m_{\tilde e^+}} A \left( \frac{m_{\tilde e^+}}{m_{\tilde e^+}} \right) \hspace{1cm} (B7) \\
d_{e}^{x,0} &= \frac{e \alpha_{em}}{4 \pi \sin^2 \theta_W} \sum_{k=1}^{2} \text{Im}(\Gamma_{eik}) \frac{m_{\tilde e^0}}{m_{\tilde e^0}} B \left( \frac{m_{\tilde e^0}}{m_{\tilde e^0}} \right) \hspace{1cm} (B8)
\end{align*}$$

where $\Gamma_{ei} = U_{\tilde e_i}^T V_{\tilde e_i}$, and

$$\eta_{eik} = \left[ -\sqrt{2} \{ \tan \theta_W (Q_e - T_{3e} X_{1i} + T_{3e} X_{2i}) D_{e1k}^* + \kappa_e X_{bi} D_{e1k}^* \} \sqrt{2} \tan \theta_W Q_e X_{1i} D_{e2k} - \kappa_e X_{bi} D_{e1k} \right]$$

Here we have

$$\kappa_e = \frac{m_e}{\sqrt{2} m_W \cos \beta}$$  \hspace{1cm} (B9)

The loop function $A(r)$ is given by

$$A(r) = \frac{1}{2(1-r)^2} \left( 3 - r + \frac{2 \ln (r)}{1-r} \right)$$  \hspace{1cm} (B10)

In the above equations, $U(V), X$ and $D_e$ are the conventional chargino, neutralino and selectron mixing matrices. It is easy to see that the chargino diagram do not contribute to the electron EDM in the framework considered, since there is no CP-violating phases in the chargino sector. In the absence of the neutralino mixing, the expression of $d_{e}^{E}$ can be significantly simplified

$$d_{e}^{E} \approx \frac{-e \alpha_{em} \eta_{q}^{(2)}}{4 \pi \sin^2 \theta_W} \frac{\text{Im}(m_q^2)_{LR}}{m_q^2} r_1^2 [B(r_1) + r_1 B'(r_1)]$$  \hspace{1cm} (B11)

where $r_1 = m_{B_{\tilde e}}^2 / m_{\tilde e}^2$ with $m_{\tilde e}$ denoting the average mass of the selectrons. In the above result, the higgsino contribution is neglected since it is suppressed by the small $Y^\tau_{\tilde e}$.

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**APPENDIX C: BARR-ZEE DIAGRAM**

As we have discussed in subsection [V B], we are concerned with the Barr-Zee diagram with the third generation squarks, i.e. $\tilde t$ and $\tilde b$, running in the inner loop. Here we give the detailed derivation of Eq. (30) and (31). We start with the general results of EDM and CEDM for the Barr-Zee diagram [39]

$$\begin{align*}
d_{F}^{E} &= Q_f \left[ \frac{3e \alpha_{em} R_{1m_f}}{32 \pi^3} \frac{M_A}{m_f^2} \sum_{q=t,b} \xi_q \right] Q_q \left[ F(r_1) - F(r_2) \right] \\
d_{F}^{C} &= \frac{g_s \alpha_s}{64 \pi^3} \frac{R_{1m_f}}{M_A^2} \sum_{q=t,b} \xi_q \left[ F(r_1) - F(r_2) \right] \hspace{1cm} (C1)
\end{align*}$$

where $M_A$ is the mass of pseudoscalar higgs $A_0$, $r_{1,2} = m_{H_{\tilde t_{1,2}}^2} / M_{A_{\tilde t}}^2$, $R_f = \cot \beta (\tan \beta)$ for $I_{3} = 1/2(-1/2)$ and $F(z)$ is the two-loop function

$$F(z) = \int_{0}^{1} dx \frac{x(1-x)}{z} \ln \left[ \frac{x(1-x)}{z} \right].$$  \hspace{1cm} (C2)

The CP-violating couplings are given by

$$\begin{align*}
\xi_t &= - \frac{\sin 2 \theta_t m_t \text{Im}(\mu e^{i \delta_t})}{2 v^2 \sin^2 \beta} \\
\xi_b &= - \frac{\sin 2 \theta_b m_b \text{Im}(A_{\mu} e^{i \delta_b})}{2 v^2 \sin \beta \cos \beta} \hspace{1cm} (C3)
\end{align*}$$

where $\theta_{t,b}$ are the stop and sbottom mixing angles, and $\delta_q = \text{Arg}(A_q + R_q \mu^*)$. The mixing angle of the squark sector is given by

$$\tan (2 \theta_q) = - \frac{2 m_q [\mu R_q + A_q^*]}{M_Q^2 - M_{\tilde Q}^2 + \cos 2 \beta M_Z^2 (T_q^u - 2 e_q s_w^2)}$$  \hspace{1cm} (C4)
Therefore, Eq. (C3) becomes
\[\xi_t \approx \frac{y_t^2 A_t^* + \mu \cot \beta \text{Im}(\mu e^{i\delta_t})}{M^2_t - \delta_t^2} \]
\[\xi_b \approx \cot \beta \frac{y_b^2 A_b^* + \mu \tan \beta \text{Im}(A_b e^{-i\delta_b})}{M^2_b - \delta_b^2} \]
\[\text{(C5)}\]

Using Eq. (C3) and (C4), we can rewrite Eq. (C1) as
\[d_f^2 \approx \frac{3e\alpha_e m_{f}}{32\pi^3} \text{Im} \left[ \frac{4y_t^2}{9} \mu (A_t + \mu^* \cot \beta) F'(r_1) \right] + \frac{y_b^2}{9} A_b (A_b^* + \mu \tan \beta) \cot \beta F'(r_2) \]
\[d_f^2 \approx \frac{g_s \alpha_s R_f m_f}{64\pi^3} \text{Im} \left[ \frac{y_t^2}{9} \mu (A_t + \mu^* \cot \beta) F'(r_1) \right] + \frac{y_b^2}{9} A_b (A_b^* + \mu \tan \beta) \cot \beta F'(r_2) \]
\[\text{(C6)}\]

where \(r_1 \equiv m_t^2/M_A^2\) and \(r_2 \equiv m_b^2/M_A^2\) with \(m_{t,b}\) being the average masses of the stops and sbottoms respectively.

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