The plasmasphere boundary layer

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Abstract. The plasmapause region is a site of geophysically important interactions between the plasmas of the hot plasma sheet and of the cool plasmasphere. We believe that from the standpoint of the multiple physical processes at work, this region should be called the Plasmasphere Boundary Layer, or PBL. Within such a conceptual framework, applied both in future textbook discussions of the plasmasphere and in scientific communications, much progress can be made on longstanding questions about the physics involved in the formation of the plasmapause and in the cycles of erosion and recovery of the plasmasphere.

1 Introduction

In space, boundary layers tend to form at the interfaces between plasmas that have distinctly different properties, either when considered as fluids or in terms of kinetic descriptions (e.g., Hughes, 1995; Roth et al., 1993; Lakhina et al., 2000). In geospace, such layers tend to be sites of significant wave-particle energy and momentum exchange, cross-boundary energy transfer, and particle precipitation into the Earth’s ionosphere and atmosphere. Familiar examples are the plasma sheet boundary layer (PSBL) and the low latitude boundary layer (LLBL). Analogous phenomena are a boundary layer in the troposphere just above the ground and one in the ocean, immediately below the sea surface, where air bubbles are engulfed in water within a layer whose thickness depends on the wind velocity and amplitude of the surface waves.

Curiously, the plasmapause region has not been described as a boundary layer, in spite of being observed at locations where the cool (\(\approx 1\) eV) dense (\(\approx 400\) el/cc) plasmasphere overlaps with or is otherwise in close proximity to the hot (\(\approx 100\) eV-100 keV) tenuous (\(\approx 1\) el/cc) plasmas of the plasmatrough or the plasmasheet and ring current (e.g., Frank, 1971; Thomsen et al., 1998). Furthermore, the term “boundary layer” has not come into use in spite of widespread belief in, and at least limited evidence of, a shielding effect whereby night-side juxtapositions of hot and cold plasmas give rise to unbalanced charge densities and therefore polari-
represents the plasmapause, either in a developed state or in the process of formation.

Within the framework of this paradigm, the unsteady nature of magnetospheric convection is commonly accounted for by parameterizing the high latitude convection field by some measure of disturbance activity and then tracking the displacements of an assumed initial plasmasphere boundary. Because of the inherent slowness of cold plasma bulk motions in the magnetosphere, the plasmapause is not assumed to coincide with the instantaneous LCE unless a quasi-steady state is reached, in which case the current LCE and plasmapause are expected to coincide. To account for anticipated effects of the hot plasmas of the plasma sheet and ring current on the evolving shape of the plasmasphere, simple modifications of the high latitude potential distribution are commonly introduced.

One can understand the appeal of this paradigm as a pedagogic device. It offers a plausible explanation of the reductions in plasmasphere size during periods of enhanced convection. Crude estimates of the intensity of the solar-wind-induced electric field can ostensibly be made, based upon some measure of the distance to a stagnation point in the flow pattern (somewhat by analogy to the way in which the standoff distance to the magnetopause can be estimated). Furthermore, the paradigm is consistent with accumulated evidence that strong plasmasphere erosion effects, leading to outlying or sunward-extending density structures, regularly occur in the afternoon-dusk sector (e.g., Chappell, 1974; Higel and Wu, 1976; Carpenter et al., 1993).

The shortcomings of this approach stem from its focus on the plasmasphere as a body of essentially zero energy particles and the use of ad hoc modifications of the model high-latitude electric field to represent hot/cold plasma interactions in the plasmapause region. These aspects of introductory material have had the unfortunate effect of deflecting the attention of the larger research community away from questions about specific physical processes which may (in concert with the dynamo sources underlying the main flow and the magnetopause) be estimated. Furthermore, the paradigm is consistent with accumulated evidence that strong plasmasphere erosion effects, leading to outlying or sunward-extending density structures, regularly occur in the afternoon-dusk sector (e.g., Chappell, 1974; Higel and Wu, 1976; Carpenter et al., 1993).

2 Examples of research on the PBL

Support for use of the PBL concept may be found in theoretical and experimental works (some made many made years ago) concerning the stability of the plasmapause boundary.
and of the plasmasphere in general. These works are particularly important in the light of evidence that the plasmapause density profile becomes highly structured in the aftermath of disturbances and during extended recovery periods (e.g., Oya and Ono, 1987; Kooms, 1989; Horwitz et al., 1990; Carpenter et al., 1993, 2000; Moldwin et al., 1995).

The occurrence of energy transfer at the PBL has long been evident through ionospheric consequences such as Stable Auroral Red (SAR) arcs (e.g., Kozyra et al., 1997) and local peaks in ionospheric electron temperature (e.g., Brace and Theis, 1974; Afonin et al., 1997). Substantial energy transfer between the hot electron plasma sheet and the ionosphere has been found to occur in the region of diffuse aurora that extends from the night side into the dawn sector outside the nominal plasmapause (e.g., Thomsen et al., 1998). The loss to the ionosphere of plasma sheet electrons at energies below 30 keV is believed due to wave-particle interactions, but there are ongoing questions about the details of the process (e.g., Lyons, 1997; Koskinen, 1997).

Several recent experimental developments have brought new attention to the PBL. One is the operation since May 2000 of the EUV instrument on the IMAGE satellite, which is providing for the first time global images of the plasmasphere (Sandel et al., 2001, 2003). These images, acquired over multi-hour periods on successive (≈14-hour) polar orbits, represent a quite new basis for study of plasmasphere erosion and the formation of sunward-extending density plumes, subjects that are in their essence boundary layer phenomena. Another is the use of GPS satellite signals to obtain maps in time series of total electron content (TEC) (Coster et al., 1992) and hence of the time-varying distribution of plasma structures in the ionosphere and overlying plasmasphere (Foster et al., 2002). These maps, in conjunction with scatter radar, are helping to elucidate the complexities of what are now called Sub-Auroral Polarization Streams (SAPS) (Foster et al., 2004), as well as the related phenomenon of latitudinally narrow westward flows called Polarization Jets (PJ) or Sub Auroral Ion Drifts (SAIDs) (e.g., Galperin et al., 1973; Anderson et al., 1991, 1993), all of which develop at the outer limits of the plasmasphere and are apparent consequences of the motions of the equatorial boundary of the plasma sheet.

Still other recent developments include the use of energetic neutral atom ENA detectors and the FUV instrument on IMAGE (e.g., Mitchell et al., 2001; Mende et al., 2003), the former to investigate the ring current and its location with respect to the plasmasphere and the latter to detect energetic proton precipitation. For example, such precipitation appears to be induced as ring current particles encounter a plasmasphere plume and are scattered through the action of electromagnetic cyclotron waves (e.g., Spasojević et al., 2004).

The CLUSTER mission has provided its share of new results on PBL structure, reinforcing earlier findings from ISEE-1, CRRES, and synchronous satellites that the plasmapause is far from the one-step “knee” that the LCE scenario would anticipate under “steady-state” geomagnetic activity conditions. The WHISPER and EFW experiments onboard the four CLUSTER spacecraft have shown that the plasmapause region is most of the time very structured; small scale (< 50 km) and large scale (> 0.5 R_E) density irregularities are generally observed in this intermediate region separating the low density plasmatrough and high density plasmasphere (Darrouzet et al., 2004; Décréau et al. 2004).

To add further complexity to our now rapidly unfolding picture of the plasmasphere and the PBL, Carpenter et al. (2002) using the Radio Plasma Imager (RPI) on IMAGE have found the PBL to be a “rough” radar target, one that does not return the discrete echo patterns predicted by simple theoretical plasma density models (e.g., Angerami and Thomas, 1964; Lemaitre, 1976; Bailey et al., 1990; Green et al., 2000). Furthermore, there is evidence from RPI of a type of density structure that seems to permeate the plasmasphere, namely field-aligned irregularities with cross-B scale sizes from 200 m to 10 km and densities within ≈10% of background. These irregularities, preliminary evidence of which was obtained in the era of the ISIS series topside sounders (e.g., Muldrew, 1963, 1969; Loftus et al., 1966) appear to play the important role of guiding sounder-produced waves along the geomagnetic field lines, thus facilitating investigation of plasma density distributions within the natural coordinate system of the magnetosphere (e.g., Reinisch et al., 2001; Fung et al., 2003).

3 Thoughts about future discussions of the PBL

What are some of the outstanding questions concerning the PBL? One of the most important but least often asked questions is: what is the physical mechanism or combination of mechanisms by which the plasmapause is formed? Virtually all models, including the most sophisticated ones, tell us where the plasmapause is or should be at a given time, but it is assumed that the plasmapause is formed naturally as an integral effect of the cross-B plasma flows associated with calculated potential distributions. If in these cases an initial plasmasphere configuration is either assumed or taken from observations, there is no explanation of the physical process or processes that gave rise to that initial configuration and the boundary is simply tracked to its later position(s). In some sense the ionosphere is given credit for establishing the plasmapause through its capacity to gradually fill flux tubes at all relevant latitudes in the presence of a yes-no condition on whether flux-tube recipients move so as to enclose the dipole.

There is evidence that plasma entrainment by convection electric fields can be a major factor in establishing the differing density levels on either side of an apparent separatrix between flow regimes, an example being the dusk-side transition between “dayside” and “nightside” plasma trough density levels (Carpenter et al., 1993). However, there is very little reason to believe that the physics involved in forming the main plasmapause are limited to the adiabatic compression or expansion of plasmas in the neighborhood of a pre-existing PBL, and that they do not include local processes...
involving field aligned currents, turbulence, and instabilities. For example, as noted above, field aligned currents driven by pressure gradients in the ring current ion population are found to give rise to intense, poleward directed electric fields in the ionosphere and to associated fast azimuthal bulk plasma flows (e.g., Anderson et al., 2001 and references cited therein). Patchy auroral activity at the low latitude edge of the diffuse aurora in the premidnight sector has been considered as possible evidence of a shear-flow instability associated with the fast azimuthal flows (e.g., Kelley 1986). A belt-like region of the outer plasmasphere tends to become structured in the aftermath of a convection event, such that irregularities with peak to valley ratios of 3:1 and more appear, as well as evidence of MHD turbulence (e.g., Carpenter and Lemaire, 1997 and references cited therein). The plasmapause region itself appears to be a major spawning ground for density irregularities (e.g., Carpenter et al., 1993), giving the impression that dense plasma elements can be detached from or shed by the plasmasphere, perhaps by analogy to the manner in which icebergs are “calved” from a glacier. Instabilities are clearly at work in this interface region between hot and cold plasmas. Which instabilities are they likely to be, and what particular geophysical roles do they play? These are challenging questions, ones that we hope will draw increasing attention in the future. For our present purposes, we limit ourselves to a few remarks, as follows:

Recent comprehensive studies of the type 1 and 2 quasi-interchange modes induced in stratified plasmas by the gravitational force have confirmed that a plasma density in hydrostatic equilibrium becomes convectively unstable and develops into a field-aligned expansion when the centrifugal force is enhanced. André (2003) showed that the so called translation motion, corresponding to type 2 quasi-interchange mode, originally introduced by Newcomb (1961), becomes unstable before the “pure interchange” mode (type 1 in Newcomb’s categorization) that has dominated discussions in magnetospheric physics since the seminal paper of Gold (1959). The role in plasmapause formation of quasi-interchange instabilities associated with super-corotational plasma flow in the post-midnight sector has been discussed by Lemaire and Gringauz (1998), Lemaire (2001), and Pierrard and Lemaire (2004).

Proponents of instabilities as factors in plasmapause formation must deal with the fact that simple MHD models predict large-scale plasmasphere configurations that are not unlike those that have been observed in the dawn sector and in the outer dayside magnetosphere (e.g., Elphic et al., 1996). However, as remarked above, an MHD model prediction of a plasmasphere configuration does not imply that the evolving properties of the plasmapause density profile have been explained. Furthermore, in the global scale plasmasphere observations from IMAGE there is much that is not predicted by the simple MHD theory (e.g., Spasojević, 2003; Sandel et al., 2003; Goldstein et al., 2004). This should help to stimulate new assessments of the physics of plasmasphere erosion.

Many fascinating problems come to mind as one considers the PBL and the challenges that it presents. What role does the PBL play in terms of the distribution of global ULF wave activity? How do the dynamos associated with bulk plasma motions interact with one another? The assumption of strict corotation of the inner plasmasphere with the Earth has been shown to be incorrect (e.g., Sandel et al., 2003), and it is not clear that the Earth imposes strict corotation on the overlying region at high latitudes (e.g., Hines, 1960). How closely coupled are low altitude plasma flows near the polar cap boundary with their counterparts at high altitude near the dayside magnetopause? Near-equatorial electron density measurements beyond synchronous orbit from ISEE 1 (Carpenter et al., 1993) and Geotail (Matsui et al., 1999) suggest that flow of plasma eroded from the plasmasphere into the magnetopause region may at some stages of convection activity be inhibited (e.g., Carpenter et al., 1993).

4 Suggestions for the textbooks of the future

We suggest that future introductions to the plasmasphere and its boundary layer begin with the phenomenology of the PBL as revealed by, as examples: global views obtained by the EUV instrument on IMAGE (e.g., Sandel et al., 2003); WHISPER data on density structure near CLUSTER perigee (e.g., Darrouzet et al., 2004; Décréau et al., 2004), electron content data obtained through scatter radar and ground measurements of GPS signals (TEC) (e.g., Foster et al., 2002); equatorial electron density data along CRRES satellite orbits (e.g., LeDocq et al., 1994; Carpenter et al., 2000). From these the student should be able to appreciate the huge size of the region as well as the complexity and variability of its boundaries and internal structure. The traditional MHD paradigm discussed above could then be presented as background, with attention both to the early physical insights that it provided as well as to the substantial shortcomings noted above. The concept of the PBL should be introduced, including the challenge to understand the as yet poorly understood physical processes at work in the plasmapause region. In this connection, it would not be inappropriate to quote some prescient remarks made in 1978 by G. Morfill (1978) in an article on the location and formation of the plasmapause:

“It is clear...that particle drift in the equatorial region of the magnetosphere is not a simple well-defined problem of solving the equation of motion with given force terms, but that irreversible processes (e.g., pitch-angle scattering), bulk properties (e.g., pressure gradients) and perturbation forces (e.g. gravity and centrifugal forces) all play a role in determining the location and stability of the plasmapause.”

At the end of his article, Morfill speculated that shielding of the convection electric field might be an important factor in explaining differences between various modeling results, and went on to say “If this situation is correct, and a great deal of work, both experimental and theoretical is necessary before we can be certain, then we have yet another fascinating boundary problem in plasma physics to solve.”
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