Abstract

In the present paper the Yang-Mills theory in the first order formalism is studied. As a gauge theory it has more rich gauge algebra in comparison with the second order formalism. On classical level both formalisms are equivalent. It is proven that the covariant quantization of this theory leads to the statement about quantum non-equivalence with the quantization based on the second order formalism.

Keywords: Quantization of Yang-Mills fields, FP-method, BV-formalism.

PACS numbers: 11.10.Ef, 11.15.Bt

E-mail: lavrov@tspu.edu.ru
1 Introduction

Yang-Mills theories have always attracted (see, for example, the famous textbook by Weinberg [1] for qualitative discussions and presentations of numerous aspects of classical and quantum properties of Yang-Mills fields) and continue to attract [2, 3, 4, 5] the attention of researchers, since they play a key role in the mainstream of modern models of fundamental interactions. In contrast with the electrodynamics the Yang-Mills theories belong to non-abelian gauge theories. It caused the problem indicated firstly by Feynman [6] and related to the S-matrix unitarity in Yang-Mills theories when simple modification of quantization rules adopted in quantum electrodynamics is used. Later on the correct quantization of Yang-Mills theories has been found [7, 8].

Recently, there has been activity in the study of quantum properties of the Yang-Mills theory formulated in the first order formalism [9, 10, 11, 12] instead of standard approach based on the second order form [7]. On classical level both formulations are equivalent. It was really assumed in [11, 12] that all Green functions appeared in the standard quantization can be reproduced with the help of using more simple first order formulation. This proposal is closely related to quantum equivalence of both presentations of the Yang-Mills theory. In our knowledge the quantum equivalence of the Yang-Mills theory presented in the first and second orders was not studied before in the scientific literature.

In the present paper we study the Yang-Mills theory in the first order formulation. This formulation requires introduction additional antisymmetric second order tensor fields, $F^a_{\mu\nu}$. In turn it leads to appearing additional gauge symmetry. Corresponding gauge algebra contains additional gauge generators. Further quantization depends on properties of the gauge algebra to be closed/open and irreducible/reducible. Then the found structure of gauge algebra allows to construct a quantum action using, in general, the BV-formalism [13, 14]. Fixing a gauge via the standard BV procedure one obtains the full quantum action which is used to define the generating functional of Green functions in the form of functional integral. Quantum equivalence or non-equivalence of the Yang-Mills theory in two formulations can be studied by comparison of corresponding vacuum functionals. If the vacuum functional coincide then the quantum equivalence is realized. Otherwise one meets the quantum non-equivalence.

The paper is organized as follows. In Sec. 2 gauge symmetries of the Yang-Mills theory written in the first order formulation are studied. In Sec. 3 the full quantum action found as solution to the classical master equation of the BV-formalism with applying the standard gauge fixing procedure in linear Lorentz invariant gauges is constructed. In Sec. 4 the generating functional of Green functions in the form of functional integral over only fields of initial classical action is found. In Sec. 5 non-equivalence of two approaches for quantization of the Yang-Mills theory is proven. Finally, in Sec. 6 the results obtained in the paper are discussed.

In the paper the DeWitt’s condensed notations are used [17]. The right and left functional
derivatives with respect to fields and antifields are marked by special symbols "←" and "→" respectively. Arguments of any functional are enclosed in square brackets [], and arguments of any function are enclosed in parentheses, ( ).

2  Gauge symmetries

Standard formulation of the Yang-Mills fields, \( A^a_\mu \), operates with the second order action

\[
S^{(2)}[A] = -\frac{1}{4} F^{a}_{\mu \nu}(A) F^{a}_{\mu \nu}(A),
\]

(2.1)

where \( F^{a}_{\mu \nu}(A) \) is the field strength

\[
F^{a}_{\mu \nu}(A) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu,
\]

(2.2)

and \( f^{abc} \) are completely antisymmetric structure constants of the Lie algebra satisfying the Jacobi identity

\[
f^{abc} f^{cde} + f^{aec} f^{cbd} + f^{adc} f^{ced} = 0.
\]

(2.3)

The equations of motion read

\[
\frac{\delta S^{(2)}[A]}{\delta A^a_\mu} = D^a_\nu A^a_\mu F^{b}_{\nu \mu}(A) = 0,
\]

(2.4)

where \( D^a_\mu(A) \) is the covariant derivative

\[
D^a_\mu(A) = \delta^a_\mu \partial_\mu + f^{acb} A^b_\mu.
\]

(2.5)

The action (2.1) is invariant under the gauge transformations of \( A^a_\mu \),

\[
\delta \xi S^{(2)}[A] = 0, \quad \delta \xi A^a_\mu = D^a_\mu(A) \xi^b, \quad \delta \xi A^a_\mu = D^a_\mu(A) \xi^b,
\]

(2.6)

where \( \xi^a \) are arbitrary functions of space-time coordinates. Notice that under the gauge transformations the field strength tensor transforms by the tensor law,

\[
\delta \xi F^a_{\mu \nu}(A) = f^{abc} F^b_{\mu \nu}(A) \xi^c.
\]

(2.7)

Algebra of gauge transformations

\[
[\delta \xi_1, \delta \xi_2] A^a_\mu = D^a_\mu(A) \xi^b_3, \quad \xi^a_3 = f^{abc} \xi^b_1 \xi^c_2,
\]

(2.8)

is closed and irreducible.

The first order formulation of Yang-Mills fields is based on the action

\[
S^{(1)}[A, F] = \frac{1}{4} F^a_{\mu \nu} F^a_{\mu \nu} - \frac{1}{2} F^a_{\mu \nu}(A) F^a_{\mu \nu},
\]

(2.9)
where $F_{\mu\nu}^a$ are new antisymmetric tensor fields, $F_{\mu\nu}^a = -F_{\nu\mu}^a$. The equations of motion read
\[
\frac{\delta S^{(1)}[A,\mathcal{F}]}{\delta A^a_{\mu}} = D^{ab}_{\mu}(A)F^b_{\nu\mu} = 0, \quad \frac{\delta S^{(1)}[A,\mathcal{F}]}{\delta F^b_{\mu\nu}} = \frac{1}{2}(F^b_{\mu\nu} - F^b_{\nu\mu}(A)) = 0. \tag{2.10}
\]
From the second in (2.10) it follows $F^a_{\mu\nu} = F^a_{\nu\mu}(A)$. Substituting this result in the first of (2.10) one obtains (2.4). On classical level we have two equivalent descriptions of Yang-Mills fields.

The action (2.9) is invariant under the following gauge transformations
\[
\delta_\xi S^{(1)}[A,\mathcal{F}] = 0, \quad \delta_\xi A^a_{\mu} = D^{ab}_{\mu}(A)\xi^b_{\mu}, \quad \delta_\xi F^a_{\mu\nu} = f^{abc}F^b_{\mu\nu}\xi^c,
\]
which are considered as generalization of (2.6). Introduction of new fields leads to increasing the degrees of freedom and as consequence to existence of additional gauge symmetry. Indeed, the action (2.9) is invariant under the additional gauge transformations
\[
\delta_\xi S^{(1)}[A,\mathcal{F}] = 0, \quad \delta_\xi A^a_{\mu} = 0, \quad \delta_\xi F^a_{\mu\nu} = f^{abc}(F^b_{\mu\nu} - F^b_{\nu\mu}(A))\tilde{\xi}^c,
\]
where $\tilde{\xi}^a$ are arbitrary functions of space-time coordinates. The first order formulation of Yang-Mills fields is accompanying by the following gauge algebra
\[
\begin{align*}
\{\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}\}A^a_{\mu} &= D^{ab}_{\mu}(A)\xi^b_{\mu}, \quad \{\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}\}A^a_{\mu} = 0, \quad \{\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}\}A^a_{\mu} = 0, \\
\{\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}\}F^a_{\mu\nu} &= f^{abc}F^c_{\mu\nu}\xi^b_{\mu}, \quad \{\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}\}F^a_{\mu\nu} = f^{abc}(F^c_{\mu\nu} - F^c_{\nu\mu}(A))\tilde{\xi}^b_{\mu}, \\
\{\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}\}F^a_{\mu\nu} &= f^{abc}F^c_{\mu\nu}\tilde{\xi}^b_{\mu}.
\end{align*}
\tag{2.13}
\]
where
\[
\xi^a_3 = f^{abc}\xi^b_1\xi^c_2, \quad \tilde{\xi}^a_3 = f^{abc}\tilde{\xi}^b_1\tilde{\xi}^c_2, \quad \tilde{\xi}^a_3 = f^{abc}\tilde{\xi}^b_1\tilde{\xi}^c_2.
\tag{2.14}
\]
Due to the second in (2.10) the commutator $[\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}]F^a_{\mu\nu}$ can be written in the form
\[
[\delta_{\tilde{\xi}_1}, \delta_{\tilde{\xi}_2}]F^a_{\mu\nu} = 2f^{abc}\frac{\delta S^{(1)}[A,\mathcal{F}]}{\delta F^c_{\mu\nu}}\tilde{\xi}^b_{\mu},
\tag{2.15}
\]
proportional to the equations of motion which is typical for open algebras. But final answer to be a gauge algebra closed or open depends on fulfilment of the Jacobi identity without/with using the equations of motion. Direct calculations show that the Jacobi identity for gauge transformations (2.11), (2.12) is satisfied exactly. The gauge algebra (2.13) is closed and irreducible.

### 3 Quantum action

In construction of quantum action for Yang-Mills theory in the first order form we will follow prescriptions of the BV-formalism [13]. Minimal antisymplectic space is defined by the set of fields $\phi^A_{\text{min}}$ and antifields $\phi^*_a_{\text{min}}$
\[
\phi^A_{\text{min}} = (A^a_{\mu}, F^a_{\mu\nu}, C^a_1), \quad \phi^*_a_{\text{min}} = (A^*_g\mu, F^*_a_{\mu\nu}, C^*_a, C^*_1),
\tag{3.1}
\]
where $C^a$ and $C^a_1$ are the ghost fields corresponding to the gauge symmetries \textup{(2.11)} and \textup{(2.12)} respectively. Due to closedness and irreducibility of gauge algebra the minimal quantum action, $S_{\text{min}} = S[\phi_{\text{min}}, \phi^*_{\text{min}}]$, being a solution to the classical master equation\footnote{For any set of fields $\phi^A$ and antifields $\phi^*_A$ and any functionals $F$ and $G$ the antibracket is defined by the rule $(F,G) = F(\overleftarrow{\partial_{\phi^A}} \overrightarrow{\partial_{\phi^*_A}} - \overleftarrow{\partial_{\phi^*_A}} \overrightarrow{\partial_{\phi^A}})G.$}

\begin{equation}
(S_{\text{min}}, S_{\text{min}}) = 0,
\end{equation}

satisfying the boundary condition

\begin{equation}
S_{\text{min}} |_{\phi^*_\text{min}=0} = S^{(1)}[A, F]
\end{equation}
can be found in the form linear in antifields

\begin{equation}
S_{\text{min}} = S^{(1)}[A, F] + A^*_a X^a_\mu(\phi) + F^*_a Y^a_\mu(\phi) + C^*_a Z^a(\phi) + C^*_a U^a(\phi).
\end{equation}

The result reads

\begin{align*}
X^a_\mu(\phi) &= D^{ab}_\mu(A) C^b,
Y^a_\mu(\phi) &= f^{abc} F^b_{\mu\nu} C^c + f^{abc} (F^b_{\mu\nu} - F^b_{\nu\mu}(A)) C^c,
Z^a(\phi) &= -\frac{1}{2} f^{abc} C^b C^c,
U^a(\phi) &= f^{abc} \left( \frac{1}{2} C^b C^c + C^b C^c \right).
\end{align*}

Now the extended action $S = S[\phi, \phi^*]$ in full antisymplectic space

\begin{equation}
\phi^A = (A^a_\mu, F^a_{\mu\nu}, C^a, C^1, C^a_1, B^a, B^a_1), \quad \phi^*_A = (A^*_a, F^*_a, C^*_a, C^*_1, C^a_1, B^*_a, B^*_a_1),
\end{equation}

has the form

\begin{equation}
S = S^{(1)}[A, F] + A^*_a D^{ab}_\mu(A) C^b + F^*_a f^{abc} \left( F^b_{\mu\nu} C^c + (F^b_{\mu\nu} - F^b_{\nu\mu}(A)) C^c \right) - C^*_a f^{abc} C^b C^c + C^*_1 f^{abc} \left( \frac{1}{2} C^b C^c + C^b C^c \right) + C^*_a B^a + C^*_a_1 B^a_1
\end{equation}

and satisfies the classical master equation and the boundary condition

\begin{equation}
(S, S) = 0, \quad S |_{\phi^*_\text{min}=0} = S^{(1)}[A, F].
\end{equation}

Here $C^a_1$ and $C^1_1$ are the antighost fields to $C^a$ and $C^1_a$ and $B^a, B^a_1$ are auxiliary fields (Nakanishi-Lautrup fields). The gauge fixed action $S_{\text{eff}}[\phi]$ in the BV-formalism is defined as

\begin{equation}
S_{\text{eff}}[\phi] = S[\phi, \phi^* = \Psi \overleftarrow{\partial_{\phi}}],
\end{equation}

where $\Psi = \Psi[\phi]$ is an odd gauge fixing functional. For the closed and irreducible gauge algebra $S_{\text{eff}}[\phi]$ is usually refereed as the Faddeev -Popov action $S_{FP} [\phi]$. For the model under consideration the functional $\Psi$ corresponding to the case of non-singular Lorentz invariant and linear gauges can be chosen in the form

\begin{equation}
\Psi = \bar{C}^a \chi_a(A, B) + \bar{C}^a_1 \chi_1(A, F, B_1),
\end{equation}
where $\chi_a(A, B)$ and $\chi_{a1}(F)$ are gauge fixing functions,

$$\chi_a(A, B) = \partial_\mu A_\mu^a + \frac{\xi}{2} B_a^a, \quad \chi_{a1}(F, B_1) = \varepsilon_{\mu\nu} F_\mu^a + \frac{\xi_1}{2} B_1^a. \quad (3.12)$$

In (3.12) $\varepsilon_{\mu\nu}$ is the Lorentz invariant antisymmetric tensor, $\xi$ and $\xi_1$ are constant gauge parameters.

The Faddeev-Popov action for the model (2.9) in the gauges (3.12) reads

$$S_{FP}^{(1)}[\phi] = S^{(1)}[A, F] + \bar{C}_a \partial_\mu D_\mu^{ab} C^b + B_a^a \chi_a(A, B) + \bar{C}_1 f^{abc} (F^b - F^b(A)) C^c + \bar{C}_1 f^{abc} F^b C^c + B_1^a \chi_{a1}(F, B_1), \quad (3.13)$$

where the following notations

$$F_a^a = \varepsilon_{\mu\nu} F_{\mu\nu}^a, \quad F_a^a(A) = \varepsilon_{\mu\nu} F_{\mu\nu}^a(A) \quad (3.14)$$

are used.

The Faddeev-Popov action for the model (2.9) (3.12) has the form

$$S_{FP}^{(2)}[\phi] = S^{(2)}[A] + \bar{C}_a \partial_\mu D_\mu^{ab} (A) C^b + B_a^a \chi_a(A, B), \quad (3.15)$$

where the gauge fixing function $\chi_a(A, B)$ is the first in (3.12). The cases $\xi = 0$ for (3.15) and $\xi = 0, \xi_1 = 0$ for (3.13) correspond to singular gauges being useful for theoretical treatments of quantum properties while for practical quantum calculations the cases $\xi \neq 0$ and $\xi_1 \neq 0$ are more preferred.

4 Quantization

The generating functional of Green functions of fields $A_\mu^a$ and $F_{\mu\nu}^a$ for the Yang-Mills theory in the first order form reads

$$Z^{(1)}[j, J] = \int D\phi \exp \left\{ \frac{i}{\hbar} \left( S_{FP}^{(1)}[\phi] + jA + jF \right) \right\}, \quad (4.1)$$

where $j = \{j_\mu^a\}$ and $J = \{J_{\mu\nu}^a\}$ are external sources to fields $A_\mu^a$ and $F_{\mu\nu}^a$ respectively and the following abbreviations

$$jA = j_\mu^a A_\mu^a, \quad jF = J_{\mu\nu}^a F_{\mu\nu}^a \quad (4.2)$$

are used. Integrating over fields $B_a^a$ and $B_1^a$ reproduces the following presentation for $Z^{(1)}[j, J],$

$$Z^{(1)}[j, J] = \int DADFDCD\bar{C}DCD\bar{C}_1D\bar{C}_1 \exp \left\{ \frac{i}{\hbar} \left( S^{(1)}[A, F, C, \bar{C}, C_1, \bar{C}_1] + jA + jF \right) \right\}, \quad (4.3)$$

where

$$S^{(1)}[A, F, C, \bar{C}, C_1, \bar{C}_1] = S^{(1)}[A, F] + \bar{C}_a \partial_\mu D_\mu^{ab} (A) C^b + \bar{C}_a f^{abc} (F^c - F^c(A)) C^b + \bar{C}_a f^{abc} (F^c - F^c(A)) C^b + \frac{1}{\xi} (\partial_\mu A_\mu^a)^2 + \frac{1}{\xi_1} (F^a)^2. \quad (4.4)$$
Further integration over ghost and antighost fields allows to present the functional $Z^{(1)}[j, J]$ in the form

$$Z^{(1)}[j, J] = \int DA D\mathcal{F} \exp \left\{ \frac{i}{\hbar} \left( \frac{1}{4} F^a_{\mu\nu} F_{\mu\nu}^a - \frac{1}{2} F^a_{\mu\nu}(A) F_{\mu\nu}^a + \frac{1}{\xi} (\partial_{\mu} A_{\mu}^a)^2 + \frac{1}{\xi_1} (\mathcal{F}^a)^2 + jA + J\mathcal{F} \right) \right\} \Delta_{FP}[A] \Delta[\mathcal{F} - F(A)],$$

(4.5)

where $\Delta_{FP}[A]$ is the Faddeev-Popov determinant \[7\]

$$\Delta_{FP}[A] = \text{Det} \partial_{\mu} D_{\mu}^{ab}(A),$$

(4.6)

and $\Delta[\mathcal{F} - F(A)]$ is the functional determinant associated with the additional gauge symmetry,

$$\Delta[\mathcal{F} - F(A)] = \text{Det} f_{abc} (\mathcal{F}^c - F^c(A)).$$

(4.7)

Making use the change of integration variables in the functional integral (4.5)

$$\mathcal{F}_{\mu\nu}^a = E_{\mu\nu}^a + F_{\mu\nu}^a(A),$$

(4.8)

we obtain

$$Z^{(1)}[j, J] = \int DA \exp \left\{ \frac{i}{\hbar} \left( - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{\xi} (\partial_{\mu} A_{\mu}^a)^2 + jA + J\mathcal{F} \right) \right\} \Delta_{FP}(A) \Sigma[A, J],$$

(4.9)

where the functional $\Sigma[A, J]$ is defined by the following functional integral

$$\Sigma[A, J] = \int DE \exp \left\{ \frac{i}{\hbar} \left( \frac{1}{4} E_{\mu\nu}^a E_{\mu\nu}^a + \frac{1}{\xi_1} (E^a)^2 + \frac{2}{\xi_1} E^a F^a(A) + JE \right) \right\} \Delta(E).$$

(4.10)

From (4.10) it follows that $\Sigma[A, J]$ remains a non-trivial functional of $A_{\mu}^a$, $\Sigma[A]$, even when $J_{\mu\nu}^a = 0$,

$$\Sigma[A] = \int DE \exp \left\{ \frac{i}{\hbar} \left( \frac{1}{4} E_{\mu\nu}^a E_{\mu\nu}^a + \frac{1}{\xi_1} (E^a)^2 + \frac{2}{\xi_1} E^a F^a(A) \right) \right\} \Delta(E) \neq \text{const.}$$

(4.11)

The last fact plays a crucial role in solving the quantum equivalence problem for the Yang-Mills theory in the first and second order formulations.

5 Non-equivalence

The generating functional of Green functions of fields $A_{\mu}^a$ for Yang-Mills theory in the second order form is defined as

$$Z^{(2)}[j] = \int D\phi \exp \left\{ \frac{i}{\hbar} \left( S^{(2)}_{FP}[\phi] + jA \right) \right\}. $$

(5.1)

Integration over fields $C, \bar{C}, B$ leads to the well-known result \[7\]

$$Z^{(2)}[j] = \int DA \exp \left\{ \frac{i}{\hbar} \left( - \frac{1}{4} F_{\mu\nu}^a(A) F_{\mu\nu}^a(A) + \frac{1}{\xi} (\partial_{\mu} A_{\mu}^a)^2 + jA \right) \right\} \Delta_{FP}[A].$$

(5.2)
Now we are in position to study quantum (non)equivalence. To do this we have to compare vacuum functionals for the Yang-Mills theory formulated in the first and second order formalisms. From (5.2) and (4.5) it follows

\[ Z^{(2)}[0] = \int DA \exp \left\{ \frac{i}{\hbar} \left( -\frac{1}{4} F^a_{\mu\nu}(A) F^a_{\mu\nu}(A) + \frac{1}{\xi}(\partial_{\mu} A^a_{\mu})^2 \right) \right\} \Delta_F P[A], \]  

(5.3)

\[ Z^{(1)}[0,0] = \int DA \exp \left\{ \frac{i}{\hbar} \left( -\frac{1}{4} F^a_{\mu\nu}(A) F^a_{\mu\nu}(A) + \frac{1}{\xi}(\partial_{\mu} A^a_{\mu})^2 \right) \right\} \Delta_F P[A] \Sigma[A]. \]  

(5.4)

Due to the relation (4.11) the vacuum functionals do not coincide,

\[ Z^{(1)}[0,0] \neq Z^{(2)}[0]. \]  

(5.5)

It means the quantum non-equivalence of two schemes of quantizations.

The relation

\[ Z^{(2)}[j] = Z^{(1)}[j,0] \]  

(5.6)

used in [11, 12] to support calculations of Green functions within the standard quantum approach to the Yang-Mills theory with the help of the first order formulation, is broken

\[ Z^{(2)}[j] \neq Z^{(1)}[j,0] \]  

(5.7)

as it follows from (5.2) and (4.9).

6 Discussion

In the paper we have analyzed the Yang-Mills theory in the first order formulation. This formulation operates with two set of fields \( \{ A^a_{\mu} \} \) and \( \{ F^a_{\mu\nu} \} \) instead of \( \{ A^a_{\mu\nu} \} \) in the second order formalism. On classical level both formulations are equivalent. Equivalence on quantum level needs in special study. We have studied gauge symmetry of classical action in the first order formulation and found two types of gauge transformations. One of them can be considered as natural extension of gauge transformations of vector fields \( A^a_{\mu} \) in the second order formulation but the second type of gauge symmetry is specific for the first order formulation and is caused by the presence of second order tensor fields \( F^a_{\mu\nu} \). Then it was calculated and proved that the full gauge algebra is closed and irreducible. To construct full quantum action we have used the BV-formalism based on solution to classical master equation and gauge fixing procedure [13, 14]. With the help of full quantum action the generating functional of Green functions in the first order formulation has been constructed and presented in the form of functional integral over fields \( A^a_{\mu\nu} \) and \( F^a_{\mu\nu} \). It was found that the vacuum functionals constructed in the first and second order formulations do not coincide. It means quantum non-equivalence of the Yang-Mills theory considered in two possible formulations. In its turn it means that the action (2.9) corresponds to the gauge model of vector fields \( A^a_{\mu} \) and second order antisymmetric tensor fields \( F^a_{\mu\nu} \) which is differed the model of Yang-Mills fields (2.1).
Acknowledgments

The author thanks I.L. Buchbinder and I.V. Tyutin for useful discussions. The work is supported by Ministry of Education of Russian Federation, project FEWF-2020-0003.

References

[1] S. Weinberg, *The Quantum theory of fields*, Vol.II (Cambridge University Press, 1996).
[2] A.O. Barvinsky, D. Blas, M. Herrero-Valea, S.M. Sibiryakov and C.F. Steinwachs, *Renormalization of gauge theories in the background-field approach*, JHEP 1807 (2018) 035
[3] I.A. Batalin, P.M. Lavrov, I.V. Tyutin, *Multiplicative renormalization of Yang-Mills theories in the background-field formalism*, Eur. Phys. J. C78 (2018) 570.
[4] I.A. Batalin, P.M. Lavrov, I.V. Tyutin, *Gauge dependence and multiplicative renormalization of Yang-Mills theory with matter fields*, Eur. Phys. J. C79 (2019) 628.
[5] F.T. Brandt, J. Frenkel, D.G.C. McKeon, *Renormalization of six-dimensional Yang-Mills theory in a background gauge field*, Phys. Rev. D99 (2019) 025003.
[6] R.P. Feynman, *Quantum theory of gravitation*, Acta Phys. Pol. 24 (1963) 697.
[7] L.D. Faddeev, V.N. Popov, *Feynman diagrams for the Yang-Mills field*, Phys. Lett. B25 (1967) 29.
[8] B.S. DeWitt, *Quantum theory of gravity. II. The manifestly covariant theory*, Phys. Rev. 162 (1967) 1195.
[9] F.T. Brandt, D.G.C. McKeon, *Perturbative Calculations with the First Order Form of Gauge Theories*, Phys. Rev. D91 (2015) 105006.
[10] F.T. Brandt, J. Frenkel, D.G.C. McKeon, *Renormalization of a diagonal formulation of first order Yang-Mills theory*, Phys. Rev. D98 (2018) 025024.
[11] D.G.C. McKeon, J. Frenkel, S. Martins-Filho, F.T. Brandt, *Consistency Conditions for the First-Order Formulation of Yang-Mills Theory*, Phys. Rev. D101 (2020) 085013.
[12] D.G.C. McKeon, F.T. Brandt, J. Frenkel, S. Martins-Filho, *On Restricting First Order Form of Gauge Theories to One-Loop Order*, arXiv:2009.09553 [hep-th],
[13] I.A. Batalin, G.A. Vilkovisky, *Gauge algebra and quantization*, Phys. Lett. B102 (1981) 27.
[14] I.A. Batalin, G.A. Vilkovisky, *Quantization of gauge theories with linearly dependent generators*, Phys. Rev. D28 (1983) 2567.
[15] C. Becchi, A. Rouet, R. Stora, *The abelian Higgs Kibble Model, unitarity of the S-operator*, Phys. Lett. B 52 (1974) 344.
[16] I.V. Tyutin, *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Institute preprint No. 39 (1975), arXiv:0812.0580 [hep-th].
[17] B.S. DeWitt, *Dynamical theory of groups and fields*, (Gordon and Breach, 1965).