Neveu-Schwarz Five-Branes And Matrix String Theory On K3

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The Matrix theory description of Type IIA string theory on a compact K3 surface as the theory of Neveu-Schwarz five-branes on $\tilde{K}3 \times S^1$ is analyzed. The full multiplet of space-time BPS states is identified in the five-brane world-volume as fluxes.
1. Introduction

The Matrix description of M-theory \[1\] on \(T^d\) is \(SYM_{d+1}\) on the dual torus \(\tilde{T}^d\) \[12\]. This definition is incomplete for \(d > 3\) since then the SYM theory is not renormalizable and more data needs to be added to describe the full theory. Compactifications on \(T^4\) and \(T^5\) have recently been proposed in terms of the five-branes of M-theory and Type II string theory \[4,5,6\]. A unified derivation for matrix theory compactification on tori has been given in \[7\]. The Matrix theory description of curved backgrounds has been given in \[8,9,10\] for non-compact ALE manifolds and in \[11,12,13\] for compact K3 surfaces. The former approach based on the quiver quantum mechanics of \[14,15\] must be modified for the compact case. Seiberg has proposed in \[6\] that the Matrix theory description of Type IIA string theory on K3 is given by the theory of Neveu-Schwarz (NS) five-branes compactified on \(\tilde{K}3 \times S^1\). In this paper we make this proposal more precise by identifying the expected space-time BPS states in the NS five-brane theories. In section 2 we consider Type IIB NS five-branes on \(\tilde{X} \times S^1\). We analyze possible bound states with D-branes and find fluxes in the five-brane theory corresponding to the bound states. In section 3 a similar treatment is given for Type IIA NS five-branes. In section 4 we give a Matrix interpretation to these wrapped brane theories. We show how T-duality acts on K3 and identify the \(SO(4,20,Z)\) multiplet of BPS states of Type IIA string theory on K3.

2. Type IIB Neveu-Schwarz five-branes on \(K3 \times S^1\)

In this section we consider the world-volume theory of \(N\) IIB NS five-branes compactified on a K3 surface \(\tilde{X} \times S^1\). S-duality \[6\] shows that at energies below the string scale \(M_s\) the flat volume theory is \((1,1)\) SYM\(_{5+1}\) with gauge group \(U(N)\) and gauge coupling

\[
\frac{1}{g^2} = M_s^2. \tag{2.1}
\]

When compactified on \(\tilde{X} \times S^1\) one is left with eight real unbroken supercharges. Excitations of this system can be described by bound states of wrapped D-branes with NS five-branes. In the \(g_s \to 0\) limit, where string modes decouple, these excitations can be interpreted as solitonic objects of the SYM\(_{5+1}\) on \(\tilde{X} \times S^1\) theory. A detailed understanding of these

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1 The relation between these manifolds will be made precise in section 4.
2 This problem has been considered before in \[10\] for different reasons.
bound states can be obtained by analyzing the S-dual description consisting of $N$ D5-branes compactified on $\tilde{X} \times S^1$. In [17,18,19] it was shown that the world-volume theory has the Chern-Simons coupling

$$I_{CS} = \int_{\tilde{X} \times S^1} C \wedge \text{ch}(F) \sqrt{\hat{A}(R)}. \quad (2.2)$$

$C$ is the sum of all RR vector potentials, $\text{ch}(F) = \text{Tr} \frac{iF^2}{2\pi}$ is the Chern-character and $\hat{A}(R)$ encodes the information of the curvature of the brane. From this coupling one can read off the charges under the different RR vector potentials in terms of the $\text{rank}(F)$, $c_1(F)$ and $c_2(F)$ of a vector bundle over $\tilde{X} \times S^1$. In [19] the moduli space of D-branes was identified with the moduli space of vector bundles on $\tilde{X} \times S^1$ with specified topological data. Therefore, BPS states are represented by harmonic forms on the moduli space of vector bundles on $\tilde{X} \times S^1$. More precisely a BPS configuration with charge vector

$$Q = (r, l, s) \in H^0(\tilde{X}, \mathbb{Z}) \oplus H^2(\tilde{X}, \mathbb{Z}) \oplus H^4(\tilde{X}, \mathbb{Z}) \quad (2.3)$$

is described by a $U(r)$ instanton bundle with Mukai vector

$$Q = v \equiv \left( r, c_1, \frac{1}{2}c_1^2 + r - c_2 \right). \quad (2.4)$$

The integers $(r, l, s)$ represent the net D5, D3 and D1 charge respectively. The dimension of the moduli space is positive only for states satisfying the BPS condition

$$v^2 \equiv l^2 - 2rs \geq -2, \quad (2.5)$$

where the inner product on $H^{2*}$ is given by the intersection form. Note that a wrapped D5-brane on $\tilde{X} \times S^1$ induces an effective negative D1-brane charge [20]. In the presence of wrapped states, when $c_1 \neq 0$, there is a further contribution to the induced negative D1-brane charge.

In the S-dual picture one has NS five-branes instead of D5-branes and $r, l, s$ correspond to the net NS five-brane charge, D3-brane charge and fundamental string winding number.

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3 This is justified since we will be interested in BPS states only.

4 Actually string duality requires these moduli spaces to be compactified by adding semi-stable coherent sheaves [19]. This distinction is unnecessary in the present paper.
respectively. The energy of a state with charge vector \((2.3)\) is given by the following BPS mass formula

\[
E = \sqrt{\left(\frac{1}{g_s^2} r \tilde{V} \Sigma_5^B M_s^6 + s \Sigma_5^B M_s^2 \right)^2 + \left( \frac{1}{g_s} |\Sigma_5^B| \tilde{\zeta} |M_s^4| \right)^2}. \tag{2.6}
\]

Here we have used the well known BPS mass formulas for wrapped branes and fundamental string windings in Type IIB string theory. \(\Sigma_5^B\) is the circumference of the circle and \(\tilde{V}\) is the volume of \(\tilde{X}\). The vector \(\tilde{\zeta}\) represents the periods of the Kähler class \(\tilde{J}\) and holomorphic two form \(\tilde{\Omega}\) on the cycle \(\gamma \in \tilde{X}\) which the D3-branes wrap

\[
\tilde{\zeta} = \left( \int_\gamma \tilde{J}, \int_\gamma \tilde{\Omega} \right). \tag{2.7}
\]

We will consider first the binding energy of \(r = N\) NS five-branes and \(n\) D1-branes and D5-branes respectively. Note that the theory has different topological sectors labeled by the instanton number \(c_2\). It will turn out later that the relevant sectors for matrix theory correspond to \(c_2 \geq N\). Here we consider for simplicity \(c_2 = N\) so that the net string winding number cancels. In the limit in which the brane physics decouples from the bulk one has the following energies

\[
E_1 = \lim_{g_s \to 0} \sqrt{\left(\frac{1}{g_s^2} N \tilde{V} \Sigma_5^B M_s^6 \right)^2 + \left( \frac{1}{g_s} s \Sigma_5^B M_s^2 \right)^2} - \frac{1}{2N} \tilde{V} \Sigma_5^B M_s^6 = \frac{n^2}{2N} \frac{\Sigma_5^B}{\tilde{V} M_s^2} \tag{2.8}
\]

for \(N\) NS five-branes and \(n\) D1-branes and

\[
E_5 = \lim_{g_s \to 0} \sqrt{\left(\frac{1}{g_s^2} N \tilde{V} \Sigma_5^B M_s^6 \right)^2 + \left( \frac{1}{g_s} s \Sigma_5^B M_s^2 \right)^2} - \frac{1}{2N} \tilde{V} \Sigma_5^B M_s^6 = \frac{n^2}{2N} \tilde{V} \Sigma_5^B M_s^6 \tag{2.9}
\]

for \(N\) NS five-branes and \(n\) D5-branes.

In a similar fashion to \[5\] one can identify the \(E_1\) excitation with the energy of the electric flux in the \(U(1)\) part of the \(U(N)\) curvature of the NS five-branes theory \[21\].

The energy of \(n\) units of electric flux around \(S^1\) is

\[
E_1 = \frac{n^2}{2N} \frac{\Sigma_5^B}{\tilde{V} M_s^2}, \tag{2.10}
\]

\[5\] Generally, the mass formula also depends on the periods of the NS two-form field. We will consider \(B = 0\) throughout the paper.

\[6\] Here we have to tune \(c_2 = n\) for the D5-brane vector bundle in order to cancel the induced D1-brane charge.
which matches with the binding energy \((2.8)\). As already noted in \([8]\) the binding energy \(E_5\) cannot be identified as an excitation in the \(SYM_{5+1}\).

We consider now the bound state of \(N\) NS five-branes and \(n\) D3-branes wrapped on \(\gamma \times S^1\). These configurations correspond to \(U(N)\) bundles with \(c_1 \neq 0\). The relevant topological sectors are labeled by the instanton number

\[
c_2 - \frac{1}{2} c_1^2 = N + s, \quad s \geq 0.
\] (2.11)

Equivalently, we can label them by the net fundamental string winding charge \(s\). In order to satisfy the BPS condition \((2.5)\) for arbitrary \(n\) and arbitrary cycles we have to consider sectors with high enough \(s\). The matrix theory interpretation will be clarified in section 4.

The energy is in this case

\[
E_3 = \sqrt{\left( \frac{1}{g_s^2} N \tilde{V} \Sigma^B_5 M_s^6 + s \Sigma^B_5 M_s^2 \right)^2 + \left( \frac{1}{g_s} n |\tilde{\zeta}| \Sigma^B_5 M_s^4 \right)^2} - \frac{1}{g_s^2} N \tilde{V} \Sigma^B_5 M_s^6 - s \Sigma^B_5 M_s^2.
\] (2.12)

In the limit \(g_s \to 0\) this becomes

\[
E_3 = \frac{n^2 |\tilde{\zeta}|^2 \Sigma^B_5 M_s^2}{2N \tilde{V}}.
\] (2.13)

In order to understand the flux associated with wrapped three-branes we return to the S-dual description. Consider a bound state of \(N\) D5-branes and \(n\) D3-branes wrapped on \(\gamma \times S^1\). As before we add background D1-branes wound around \(S^1\) to cancel the net D1 charge. This configuration is described by an \(U(N)\) field strength \(F\) such that the trace \(U(1)\) part \(F_1\) is an harmonic form and the non-abelian part \(F_0\) is anti self-dual \([19]\). These conditions define in fact an Einstein-Hermitian structure \([22,23]\). The low energy effective action for this sector of string theory is

\[
S = S_{bulk} + S_{D5} + S_{D3} + S_{D1}.
\] (2.14)

The relevant Chern-Simons couplings with the RR fields can be read off from \((2.2)\)

\[
\frac{i}{2\pi} \int_{R \times S^1 \times \tilde{X}} \text{Tr} F \wedge C^{(4)} + n \int_{R \times S^1 \times \gamma} C^{(4)}
\] (2.15)
where $C^{(4)}$ is the RR four-form potential with self-dual field strength $H^{(5)}_+$. This can be rewritten as

$$\frac{i}{2\pi} \int_{R \times S^1 \times \tilde{X}} \text{Tr} F \wedge C^{(4)} + n \int_{R \times S^1 \times \tilde{X}} C^{(4)} \wedge \omega$$

(2.16)

where $\omega$ is the Poincaré dual of $\gamma$. The equation of motion for $C^{(4)}$ is

$$d \star H^{(5)}_+ = \frac{i}{2\pi} \text{Tr} F \wedge \delta^{(4)} + n \omega \wedge \delta^{(4)}$$

(2.17)

where $\delta^{(4)}$ is a transverse delta four-form. Integrating (2.17) over $S^3 \times S^1 \times \eta$ with $\eta$ an arbitrary two cycle in $\tilde{X}$ we find

$$\int_{\eta} \left( \frac{i}{2\pi} \text{Tr} F + n \omega \right) = 0, \quad \forall \eta \subset \tilde{X}.$$ (2.18)

Therefore the $U(1)$ flux induced by the three-brane is

$$F_1 = \frac{2\pi in}{N} \omega_1.$$ (2.19)

The effective D1 charge is induced by the Chern-Simons coupling

$$-\frac{1}{8\pi^2} \int_{R \times S^1 \times \tilde{X}} \text{Tr} F^2 \wedge C^{(2)}.$$ (2.20)

Therefore the induced charge is given by the Chern-Weil formula

$$c_2 - \frac{1}{2} c_1^2 = \frac{1}{8\pi^2} \text{Tr} F^2.$$ (2.21)

The non-abelian flux $F_0$ is determined from (2.21)

$$\frac{1}{8\pi^2} \int_{\tilde{X}} \text{Tr} F_0^2 = N + s + \frac{n^2}{2N} \int_{\tilde{X}} \omega^2.$$ (2.22)

We conclude that D3 charged states appear in sectors of the theory characterized in the low energy limit by different instanton numbers. Formula (2.22) shows that the non-abelian instanton number can become fractional. This is not an inconsistency since it is the second Chern character (2.21) which must be integral. Equivalently, (2.22) is the fractional $SU(N)$ ’t Hooft flux in $U(N)$ gauge theories.

We will now show that the energy of the magnetic flux in the NS five brane theory coincides with the binding energy (2.13). We rotate the complex structure of $\tilde{X}$ so that
the cycle $\gamma$ is holomorphic. Then $\omega$ is a $(1, 1)$ form whose self-dual part is given by the projection along the Kähler class

$$\omega_+ = \frac{\int_\tilde{X} \omega \wedge \tilde{J}}{\int_\tilde{X} \tilde{J} \wedge \tilde{J}}.$$  

(2.23)

Note that for this complex structure

$$\tilde{\zeta} = \left( \int_{\tilde{X}} \omega \wedge \tilde{J}, 0 \right), \quad \tilde{V} = \frac{1}{2} \int \tilde{J} \wedge \tilde{J}.$$  

(2.24)

The energy of the flux is

$$E = -\frac{M^2 \Sigma^5}{8\pi^2} \int_{\tilde{X}} \text{Tr} (F \wedge *F) = \frac{M^2 \Sigma^5}{8\pi^2} \left( \frac{4\pi^2 n^2}{N} (\| \omega_+ \|^2 + \| \omega_- \|^2) + \| F_0 \|^2 \right)$$  

(2.25)

where $\| \cdot \|$ denotes $L^2$ norm. Using (2.22), (2.24) and

$$\int_{\tilde{X}} \omega \wedge \omega = \| \omega_+ \|^2 - \| \omega_- \|^2,$$  

(2.26)

we find the energy of the total $U(N)$ flux

$$\frac{n^2 |\tilde{\zeta}|^2 \Sigma^5 M^2_s}{2N \tilde{V}} + (N + s) \Sigma^5 B M^2_s.$$  

(2.27)

The first term is in precise agreement with the binding energy (2.13). The second term represents the energy of the fundamental strings dissolved in the five-branes as instantons. It appears here naturally since we have computed the energy of the total $U(N)$ flux.

3. **Type IIA NS five-branes on $K3 \times S^1$**

Here we consider the six dimensional theory of Type IIA NS five-branes on $\tilde{X} \times S^1$. This is related by T-duality on the circle to the Type IIB theories analyzed in the previous section. After T-duality, the Mukai vector $(r, l, s)$ corresponds to net NS five-brane charge, D2-brane charge and fundamental string momentum around the circle respectively. It is important to note that winding is mapped to momentum of the fundamental string. This suggests that wrapped NS five-branes on $\tilde{X} \times S^1$ induce a string momentum charge around the circle. The excitations of this system can be described in a similar fashion as bound states with D0, D2 and D4 branes in the free string limit.
These excitations can also be identified as fluxes in the NS five-branes theory. Along the moduli space of this theory, there are \( r \) tensor multiplets of \((2,0)\) supersymmetry. Each of them includes five scalars and a two-form \( B \), whose field strength is self-dual. It is important to note that one of the scalars is compact, and its radius is proportional to the M-theory circle yielding Type IIA string theory. The compact scalar \( \phi \) is dualized as in \cite{6} to a four-form \( A_4 \) with a five-form field strength \( F_5 \). Bound states of \( N \) NS five-branes with \( n \) D0-branes can be identified with \( n \) units of \( F_5 \) magnetic flux in \( \tilde{X} \times S^1 \). The corresponding energy is

\[
E_0 = \frac{n^2}{2N} \frac{1}{V \Sigma_5^4 M_s^4}
\]  

which is T-dual to \((2.8)\). Bound states of \( N \) NS five-branes with \( n \) D4-branes can be identified with \( n \) units of \( F_5 \) electric flux along the circle. Their energy

\[
E_4 = \frac{n^2}{2N} \frac{\tilde{V} M_s^4}{\Sigma_5^4}
\]  

is related by T-duality to \((2.9)\).

The flux associated with the wrapped D2-branes states cannot be completely described in Type IIA variables since the theory of non-abelian tensor multiplets is not known. It is however clear that the binding energy can be read off directly from \((2.13)\) after T-duality on the circle.

4. Matrix Theory on K3

Seiberg has proposed \cite{6} that the Matrix description of Type IIA on K3 is given by the NS five-brane theory compactified on \( \tilde{X} \times S^1 \), where \( \tilde{X} \) denotes the T-dual to the space-time K3 surface \( X \). In order to have a complete theory on the brane we send \( g_s \to 0 \) and keep the string scale \( M_s \) fixed \cite{6}. In \cite{7} it was shown that the DLCQ quantization of M-theory on a light-like circle of radius \( R \) is the Planck scaled, boosted, \( R_s \to 0 \) limit of another M-theory compactified on a space-like circle of radius \( R_s \). The definition of compactified M-theory then follows from T-duality on D0-branes \cite{24,7}. This logic can be applied to the present situation using a modified version of the T-duality transformation for K3 surfaces defined in \cite{25}. Since this maps a D0-brane charge on \( X \) to a D0-D4 bound state on \( \tilde{X} \),

\footnote{We want to keep the degrees of freedom corresponding to the M-theory membranes wrapped around the M-theory circle.}
we obtain naturally a sector of the theory characterized by instanton number \( c_2 = N \) as anticipated in section 2 [5]. The sectors with higher instanton number contain longitudinal matrix theory five-branes, [3], which can be identified with longitudinal heterotic solitons [26,27]. The space-time and auxiliary theory parameters can be related as follows. For Type IIA NS five-branes [5]

\[
\begin{align*}
\Sigma_5^A &= \frac{l_6}{RV} \\
M_s^2 &= \frac{R^2 V L_5}{l_p^9}
\end{align*}
\]

and

\[
\begin{align*}
\Sigma_5^B &= \frac{l_3}{RL_5} \\
M_s^2 &= \frac{R^2 V L_5}{l_p^9}
\end{align*}
\]

for Type IIB five-branes. These relations follow naturally from the analysis in [7]. \( V \) is the volume of the space-time K3, \( L_5 \) is the circumference of the circle which gives the Type IIA dilaton from M-theory and \( \Sigma_5 \) is the circumference of the circle which the NS five-branes wrap.

To identify the fluxes found previously with space-time states in the infinite momentum frame (IMF) one has to relate the geometry of \( X \) to that of \( \tilde{X} \). T-duality on K3 surfaces can be defined as follows [25]. Recall that the K3 cohomology lattice can be split as

\[
\Gamma^{4,20} \simeq \Gamma^{3,19} \oplus \Gamma^{1,1}.
\]

The first factor is identified with \( H^2(X, Z) \) and the second factor, generated by the null vectors \( w, w^* \), is identified with \( H^0(X, Z) \oplus H^4(X, Z) \) [28].

The space-like four plane which defines the conformal field theory decomposes similarly

\[
\Pi \simeq B' \oplus \Sigma'
\]

where

\[
B' = V w + w^* + B
\]

determines the volume and the B-field in a standard way [28], and \( \Sigma' \cap w^\perp /w \) is identified with the self-dual three plane in \( H^2(X, R) \). Here the field \( B \) is set to zero. The splitting

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8 The precise map between BPS states on \( X \) and BPS states on \( \tilde{X} \) can be defined as Fourier-Mukai transform [25]. To keep the discussion simple, we do not present the details.
follows naturally from compactifying the M-theory heterotic duality on a circle of radius $R$. This gives rise to the following relations

$$\alpha'_A g_A^2 = R^2, \quad \alpha'_A = \frac{l_6^3}{R}, \quad \frac{1}{\alpha'_H} = \frac{V}{g_A^2 \alpha'^2}. \quad (4.6)$$

In the heterotic theory, T-duality on the extra circle can be viewed as an automorphism of the Narain lattice interchanging $w \leftrightarrow w^*$. Using (4.6) and the usual T-duality formula

$$\tilde{R} = \frac{\alpha'_H}{R}, \quad (4.7)$$

it follows that

$$\tilde{V} = \frac{\alpha'^4}{V}, \quad \tilde{g}_A^2 = g_A^2 \frac{\alpha'^4}{V^2}. \quad (4.8)$$

Therefore the K3 volume and the IIA dilaton transform according to standard T-duality formulae. We conclude that T-duality on K3 can be defined as an automorphism of the cohomology lattice exchanging $w \leftrightarrow w^*$. This does not affect the second cohomology lattice $\Gamma^{3,19}$, hence the dual surfaces have the same complex structure. Note that this transformation is not uniquely defined by (4.8). In fact, it can be combined with any automorphism of $\Gamma^{3,19}$ to yield a valid symmetry transformation. In Matrix theory this freedom represents the redundancy of the description of the base theory.

The Kähler class is simply rescaled as

$$\tilde{J} = \frac{l_6^6}{R^2 V} J. \quad (4.9)$$

Given a two-cycle $\gamma \subset X$ the vectors of periods satisfy the dimensionless relation

$$\frac{\tilde{\zeta}^2}{\tilde{V}} = \frac{\zeta^2}{V}. \quad (4.10)$$

Following [3] we identify the bound state energies computed previously with BPS states of Type IIA string theory on K3 in the IMF. Since $E_i = \frac{RM^2}{2N}$ for particle-like states in the IMF one has

- Type IIB Bound states

$$M_1^2 = \left( \frac{1}{L_5} \right)^2$$
$$M_5^2 = \left( \frac{L_5 V}{l_6} \right)^2 \quad (4.11)$$
$$M_3^2 = \left( \frac{|\tilde{\zeta}|}{l_3} \right)^2$$
• Type IIA Bound state

\[ M_0^2 = \left( \frac{1}{L_5} \right)^2 \]
\[ M_4^2 = \left( \frac{L_5 V}{l_p^6} \right)^2 \]
\[ M_2^2 = \left( \frac{\zeta}{l_p^3} \right)^2 \]

(4.12)

Let us see what is the space-time interpretation of these states. \( M_1, M_0 \) correspond to KK momenta along the \( L_5 \) circle in the M-theory language and to D0-branes in the Type IIA language. \( M_5, M_4 \) correspond to a wrapped M-theory five-brane on \( K3 \times S^1 \) or to a D4-brane wrapped on \( K3 \). Finally, \( M_3, M_2 \) correspond to an M-theory membrane or a D2-brane wrapped on a cycle \( \gamma \) in \( K3 \). Since \( H_2(X, Z) \simeq \Gamma^{3,19} \) we are able to identify the expected \( SO(4,20,Z) \) multiplet of BPS states.

5. Discussion

In this paper we have identified fluxes in the NS five-brane theories corresponding to various bound states of D-branes with NS five-branes in the free string limit \( g_s = 0 \). Furthermore, using T-duality on \( K3 \), we have been able to give a Matrix theory description of Type IIA string theory on \( K3 \). It is worth mentioning that this identification makes sense only in the \( g_s = 0 \) limit in which the physics on the NS five-branes of Type IIA and Type IIB decouple from the bulk modes. Even though the bulk modes decouple, they give classical backgrounds to the five-brane theories. The moduli space of compactification data is therefore \( SO(4,20,Z) \backslash SO(4,20)/(SO(4) \times SO(20)) \). This makes manifest the moduli space of vacua of the space-time theory. In a very similar manner one can give a Matrix description of M-theory on \( K3 \) as the theory of the M-theory five-brane on \( \tilde{K}3 \times S^1 \). The analysis follows in a straightforward manner from the one we considered here.

We noted in section 2 that in order to describe wrapped states the instanton number of the vector bundle must be shifted. This is consistent with Douglas’s proposal \[8\]. There it was shown that in order to describe wrapped states with nonzero charge one has to consider non-regular quiver representations. As noted in \[10\] this is not necessary if the total D2-brane charge is zero, so that the original quiver can describe multiple wrapped brane states whose total charge is zero. It is nice to note how this conclusion can be reached directly from the NS five-brane theories.
These NS five-brane theories are not ordinary local quantum field theories since the base space is ambiguous. It would be interesting to identify states of the S-dual space-time theory, providing a Matrix definition of heterotic string on $T^4$. Partial evidence for this comes from the analysis of the zero modes of the NS five-brane wrapped on K3 $^{26,27}$ which yields the precise chiral structure of the heterotic string.

An important step towards proving that the Matrix theory description of M-theory compactifications is valid requires to check the degeneracy of BPS states in Matrix theory$^9$. Partial results along these lines were undertaken in $^{29}$ but many more checks need to be made. For the case we consider, the degeneracy of the fluxes of a single Type IIA NS five-brane theory was shown in $^{16}$ to exactly reproduce the degeneracy of BPS states of Type IIA on K3. It would be nice to generalize this result to arbitrary $N$.

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References

[1] T. Banks, W. Fischler, S. H. Shenker, L. Susskind, Phys.Rev. D55 (1997) 5112, hep-th/9610043.
[2] W. Taylor, Phys.Lett. B394 (1997) 283, hep-th/9611042.
[3] O. J. Ganor, S. Ramgoolan, W. Taylor, Nucl.Phys. B492 (1997) 191, hep-th/9611202.
[4] M. Rozali, Phys.Lett. B400 (1997) 260, hep-th/9702136.
[5] M. Berkooz, M. Rozali, N. Seiberg, hep-th/9704085.
[6] N. Seiberg, hep-th/9705221.
[7] N. Seiberg, hep-th/9710009.
[8] M. R. Douglas, hep-th/9612126.
[9] W. Fischler, A. Rajaraman, hep-th/9704123.
[10] D.E. Diaconescu, J. Gomis, hep-th/9707019.
[11] S. Govindarajan, hep-th/9705113.
[12] M. Berkooz, M. Rozali, hep-th/9705173.
[13] S. Govindarajan, hep-th/9707164.
[14] M.R. Douglas, G. Moore, hep-th/9603167.
[15] C. V. Johnson, R. C. Myers, Phys.Rev. D55 (1997) 6382, hep-th/9610140.
[16] R. Dijkgraaf, E. Verlinde, H. Verlinde, Nucl.Phys. B484 (1997) 543, hep-th/9504047.
[17] M.R. Douglas, hep-th/9512077.
[18] M. Green, J.A. Harvey, G. Moore, Class.Quant.Grav. 14 (1997) 47, hep-th/9605033.
[19] J.A. Harvey, G. Moore, hep-th/9609017.
[20] M. Bershadsky, V. Sadov, C. Vafa, Nucl.Phys. B463 (1996) 420, hep-th/9511222.
[21] R. Dijkgraaf, E. Verlinde, H. Verlinde, Nucl.Phys. B486 (1997) 89, hep-th/9604055.
[22] S. Kobayashi, Proc. Japan. Acad. Ser. A. Math. Sci., 58 (1982), 158.
[23] M. Lübke, Math. Ann. 260 (1982), 133.
[24] A. Sen, hep-th/9709220.
[25] K. Hori, Y. Oz, hep-th/9702173.
[26] J.A. Harvey, A. Strominger, Nucl.Phys. B449 (1995) 535, hep-th/9504047.
[27] P. K. Townsend, Phys.Lett. B354 (1995) 247, hep-th/9504095.
[28] P. S. Aspinwall, In: Mirror Symmetry II, B. Greene and S.-T. Yau, Eds., International Press, Cambridge, 1997, hep-th/9404151; TASI96(K3), hep-th/9611137.
[29] R. Gopakumar, hep-th/9704030.