The need for hypercritical accretion in massive black hole binaries with large Kerr parameters

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ABSTRACT

Recent measurements of the Kerr parameters of the black holes in M33 X-7 and LMC X-1 yield $a_* = 0.84 \pm 0.05$ and $a_* = 0.90^{+0.04}_{-0.09}$, respectively. We study massive binary evolution scenarios that can reproduce such high values for the Kerr parameters. We first discuss a model with Case C mass transfer leading to a common envelope and tidal synchronization of the primary before it collapses into a black hole. We also study a Case M evolution model (which involves tidally locked, rotationally mixed, chemically homogeneous stars in a close binary). Our analysis suggests that, regardless of the specific scenario, the observed Kerr parameters for the black holes in M33 X-7 and LMC X-1 had to be obtained through hypercritical mass accretion.

Key words: accretion, accretion discs – black hole physics – gravitation – binaries: close – X-rays: binaries.

1 INTRODUCTION

When a star accretes mass on to its surface, it releases energy in the form of photons. These in turn regulate the accretion rate to what is known, in the spherically symmetric case (or Bondi accretion; Bondi 1951), as the Eddington limit:

$$M_{\text{Edd}} = \frac{L_{\text{Edd}}}{c^2} = \frac{4\pi G M m_p}{\epsilon c^3} R,$$

where $L_{\text{Edd}} = \frac{4\pi G M m_p}{\epsilon c} \pi$ (1)

is the Eddington luminosity, with $m_p$ the mass of the proton, $\sigma_T$ the Thompson cross-section of the electron, $R$ and $M$ the radius and mass of the star and $\kappa_{ei}$ the opacity of the infalling material, which is likely to be ionized hydrogen (so $\kappa = 0.4 \text{ cm}^2 \text{ g}^{-1}$) and $\epsilon$ is the efficiency for converting mass into (photon) energy via the accretion process. In general, if the accretion rate grows, the luminosity increases and self-regulates the accretion rate to values below the Eddington limit. However, if the accretion rate grows to values which exceed this limit by a couple of orders of magnitude, then photons become trapped.

Chevalier (1981, 1989, 1990) and later Brown & Weingartner (1994) found that when the diffusion time-scale of the photons generated by the accretion process is longer than the dynamical timescale of the accreting material, the photons become trapped and a shock forms (at $r_{sh}$) inside the photon-trapping radius ($r_{ph}$). The shock diminishes the kinetic energy of the accreting material by a factor of ~50 and converts it into thermal energy. When the temperature reaches $T \sim 1 \text{ MeV}$, $e^- e^-$ pairs are created. These pairs annihilate into neutrino–antineutrino pairs which can easily escape and allow the transport of energy out of the accreting black hole (BH). This prevents the accretion luminosity from exceeding the photon $L_{\text{Edd}}$ as long as $r_{sh} < r_{ph}$. However, $M_{\text{Edd}}$ is exceeded by a factor of $\sim 10^4$ (Brown 1995). This is known in the literature as hypercritical mass accretion.

Mass accretion into a BH instead of on to a neutron star (NS), such as in the cases calculated by Chevalier (1981, 1989, 1990) and Brown & Weingartner (1994), might make an even stronger case for hypercritical accretion. This is because for a BH there is no surface on to which the infalling material can collide and radiate, but rather an event horizon through which matter passes uninhibitedly. Therefore the Eddington luminosity depends strictly on the efficiency to produce photons by the accreted material as it falls towards the event horizon. On top of this, most scenarios, at least in binaries, do not involve spherical mass accretion but rather accretion through a disc, so the Eddington limit might not even be the most accurate prescription.

On the other hand, the material being accreted has angular momentum which must be lost before it can reach the event horizon of the BH. Angular momentum loss from the accreting material is an important problem that will need to be addressed. This is beyond the scope of this paper. Here we will only point out the need of hypercritical mass accretion in models which evolve massive binaries into the observed BH binaries, such as those of Lee, Brown & Wijers (2002), or De Mink et al. (2008, 2009a,b), Moreno Méndez et al. (2011).

Lee et al. (2002) and later Moreno Méndez et al. (2011) have modelled the evolution of 15 Galactic BH binaries. They start from wide binaries (i.e. the initial orbital separation is $a_i \gtrsim 1500 R_\odot$) allowing the primary star in the binary to evolve as if it were a single star until it starts helium-shell burning. At this point, the primary
fills its Roche lobe (RL) and starts to transfer mass to the secondary. Mass transfer during He-shell burning is known in the literature as Case C mass transfer (e.g. Van den Heuvel 1994). The mass transfer to the less massive secondary star shrinks the orbit until a common envelope sets in. The secondary star spirals in while expelling the hydrogen envelope of the primary until the Roche lobe overflow (RLOF) is stopped and the two stars orbit each other in a much tighter orbit (a few R⊙). In such an orbit, and within a time-scale of 10^7–10^8 yr, the helium star becomes tidally synchronized before it collapses into a BH. Therefore the spin periods P_{spin} of the binary components coincide with the orbital period P_{orb}.

Assuming angular momentum is conserved during the collapse, knowing the orbital period allows a good estimate of the natal Kerr parameter of the BH, a_∗ = Jc/GM^2 (see e.g. fig. 1 of Brown, Lee & Moreno 2007), where J is the angular momentum and M is the mass of the collapsed object. Here we will assume that the star is rotating as a solid body at the moment of collapse. This provides an upper limit to the available angular momentum, which translates into an upper limit to the natal Kerr parameter.

Lee et al. (2002) and Moreno Méndez et al. (2011) estimated the Kerr parameters of 15 Galactic BH binaries. This was done by modelling the evolution of the orbital periods from their current to their pre-explosion conditions.

Measurements of the Kerr parameters on several Galactic BHs as well as LMC X-1 and M33 X-7 have been performed by fitting the X-ray continuum. This was done with a fully relativistic model of a thin disc around a Kerr BH whose plane is that of the binary and which assumes that the inner edge of the disc is at the innermost stable circular orbit. These measurements rely on the model of the disc and still need confirmation via other methods. However, at present, they are the only available data and we will assume they have been determined meaningfully.

Among the measurements of the Kerr parameters of Galactic BHs there are those of GRO J1655–40 (XN Sco 94), 4U 1543–47 (II Lupi) (Shafee et al. 2006) and more recently, Steiner et al. (2010) have measured the Kerr parameter of XTE J1550–564, utilizing the aforementioned method as well as by modelling the Fe Kα line shape. The match between the measurements of these three systems and the predictions suggests that the model of Lee et al. (2002) and Moreno Méndez et al. (2011) represents a viable approach to study the formation and evolution of the Kerr parameter in BH binaries.

Extending the model from the Galactic binaries to the massive binary in M33 X-7, Moreno Méndez et al. (2008) concluded that the measured Kerr parameter (Liu et al. 2008) in this system must have evolved from an initial low value, a_0 < 0.1, to its present state, a_∗ ≳ 0.84 ± 0.05, by accreting about 5 M⊙ hypercritically (in its 2–3 Myr lifetime; see Table 1 for the masses, orbital period and Kerr parameters).

This model overlooks the fact that, given the large masses of the components in M33 X-7, it is unlikely that they will go through Case C mass transfer. However, this model still provides an upper limit to the natal Kerr parameter of the BH. If late Case B (or any earlier mass transfer for that case) were to occur, a BH were still formed and the binary were to survive a merger, then the tidal locking might still occur. However, mass-loss during the late stages of the primary

### Table 1. The observed masses for BH and secondary star, orbital period and Kerr parameter, a_*, for LMC X-1 and M33 X-7, respectively.

| BH binary      | M_{BH} (M⊙) | M_{sec} (M⊙) | P_{orb} (d) | a_*         |
|---------------|-------------|-------------|-------------|-------------|
| LMC X-1       | 10.30 ± 1.34 | 30.62 ± 3.22 | 3.91        | 0.90^{+0.04}_{-0.09} |
| M33 X-7       | 15.65 ± 1.45 | 70.0 ± 6.9   | 3.45        | 0.84 ± 0.05 |

### Table 2. Notation used throughout the text. The lower two entries are examples of how subindices may be combined. ZAMS stands for zero-age main-sequence.

| Symbol | Symbol |
|--------|--------|
| Mass of the primary | P_{orb} Orbital period |
| Mass of the secondary | P_{sec} Spin period of primary |
| Mass at ZAMS | P_{sec} Spin period of secondary |
| Mass of the BH | P_{BH} Orbital period, pre-SN |
| Pre-SN mass | P_{1} Orbital period, post-SN |
| Post-SN mass | P_{2} Orbital period, post-SN |
| Mass observed | P_{now} Orbital, observed period |
| Mass at onset of He burning | P_{min} Minimum spin period for case M |
| Mass accreted | ΔM Mass lost during SN |
| Mass of primary at ZAMS | P_{pri} Post-SN period |

would cause angular momentum from the star and the orbit to be lost, therefore reducing the expected Kerr parameter of the BH as compared to that for Case C mass transfer.

It is important to point out that in the scenario for M33 X-7, as well as in those which will be discussed for LMC X-1, mass transfer from the donor to the BH cannot occur through RLOF. This mode of mass transfer is unstable for large mass ratios (currently q ≳ 5 and 3, respectively, where q = M_{sec}/M_{BH}, M_{sec} is the mass of the secondary star and M_{BH} is the mass of the BH, as outlined in Table 2; see e.g. Podsiadlowski, Joss & Hsu 1992, and references therein). The instability arises from the fact that during RLOF the RL of the donor shrinks faster (in a dynamical time-scale) than the star (which responds in a thermal time-scale). Hence, as the RL shrinks more mass is transferred resulting in a further decrease of the orbital separation and RL. This results in a runaway process leading to the formation of a common envelope, a spiral-in and a merger (see Tauris & van den Heuvel 2006, for a discussion on the subject).

A possible mode of mass transfer that may avoid a merger is through stellar winds. Mohamed & Podsiaiłowski (in preparation) have shown that wind mass transfer can be highly efficient. In their wind–RLOF scenario the wind fills the RL, and is focused and channeled on to the accretor through the inner Lagrangian point L1. The accretion rate can be as large as 70 per cent, as long as the wind velocity is less than the escape velocity from the RL surface. This is the case when the system is in a tight orbit and the wind is still accelerating when it reaches the RL surface. The donor stars in M33 X-7 and LMC X-1 are close to filling their RL (Orosz et al. 2011).

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1 It is usually accepted that angular momentum is efficiently transported in the interiors of massive stars. This is required to explain, for example, the slow spin rate of NSs (Heger et al. 2005).

2 Case C mass transfer and a common envelope phase, followed by tidal locking and conservation of angular momentum during the stellar collapse.

3 It is interesting to note that in the case of wind mass-loss from the donor with α ≃ 1/3, β = δ = 0 in equation (16.20) of Tauris & van den Heuvel (2006) (where α, β and γ represent the fractions of mass-loss due to wind of the donor, ejection from the accreting BH and from a coplanar circumbinary disc, respectively) a mass-transfer efficiency 0 < α = 1 − α ≃ 2/3 would leave the orbital separation constant.
2007, 2009), making this scenario a possibility (for early-type stars the velocity of the wind at 1.5 \( R_\odot \) is \( \sim v_\infty/2 \); see fig. 5 in Mueller & Vink 2008). Interestingly, evidence for such a mode of mass transfer comes from observations of another BH binary system, Cyg X-1. The massive donor is not filling its RL, but a component of the wind appears to be focused on to the BH (Sowers et al. 1998). Contrary to the RLOF case, wind RLOF is stable regardless of the value of \( q \) because the wind rather than the star fills the RL, thus leaving the star hydrostatically stable and confined to its RL. For donor stars as massive as those in M33 X-7 and LMC X-1, the wind mass-loss rate is a few solar masses per Myr \( (\sim 10^{-8} \ M_\odot \ \text{yr}^{-1}) \), large enough to transfer several solar masses during the main-sequence lifetime of the donor.

In Section 2 we discuss, as an example of the procedure with Case C mass transfer, LMC X-1. We first work through the model including hypercritical accretion and show that it is not possible to obtain the observed \( a_1 \) and orbital period. We later include hypercritical accretion.

De Mink et al. (2008, 2009a,b) have proposed an alternative scenario to form BH binaries with massive companions that undergo quasi-chemically homogeneous evolution (see e.g. Yoon, Langer & Norman 2006). We consider this binary evolutionary path as well. In Section 3 we show that the Case M scenario (tidally locked, rotationally mixed and chemically homogeneous stars) also requires a phase of hypercritical accretion to explain the measurements of the Kerr parameters in M33 X-7 and LMC X-1. We discuss our results, and the consequences of the relaxation of our assumptions on our results in Section 4. We show our conclusions in Section 5.

2 CASE C MASS TRANSFER BINARY EVOLUTION

2.1 LMC X-1 Without Hypercritical Accretion

The present-day Kerr parameter of the 10.30 \( M_\odot \) BH in LMC X-1 is \( a_1 = 0.90 \). Its orbital period is 3.91 d, which means the distance between BH and companion is roughly 36 \( R_\odot \). This is slightly more than twice the \((17 \pm 0.8) \ R_\odot \) of the companion. The companion is barely confined to its RL \( (R_{\mathrm{acc}} \sim 0.9 R_\mathrm{RL}) \), with \( R_{\mathrm{acc}} \) the radius of the companion and \( R_\mathrm{RL} \) the RL radius; Orosz et al. 2009). Given that at present \( M_{\mathrm{acc}} \sim 31 M_\odot \), we know this system cannot be much older than \( \sim 5 \) Myr (Gu et al. 2009).

If we do not allow hypercritical accretion the mass of the BH cannot be considerably altered since its formation. In fact the Eddington rate limits the accreted mass to the BH in these 5 Myr to \( M_{\mathrm{acc}} < 0.22 M_\odot \). As can be seen in fig. 6 of Brown et al. (2000), 0.22 \( M_\odot \) cannot even produce an increment in \( a_1 \) of 0.1 on a \( \sim 10 M_\odot \) BH.

The separation in the massive binary systems believed to be the progenitors of BH binary systems such as LMC X-1 and M33 X-7 has to be quite small. In particular the orbital separation is the smallest after the common envelope phase, before the collapse of the primary, which means that tides are very efficient at this point. This is the reason why such massive binary systems are believed to be tidally synchronized when the BH forms (e.g. Van den Heuvel & Yoon 2007).

Assuming that no angular momentum is lost during the stellar collapse phase, it is possible to calculate the upper limit of the natal Kerr parameter of the BH. This is the procedure adopted by Lee et al. (2002) to predict the Kerr parameters of XTE J1550–564, GRO J1655–40 and 4U 1543–47, and here we adopt the same method.

A natal Kerr parameter such as \( a_1 = 0.90 \) constrains the pre-explosion orbital period of the binary, \( P_1 \), to a value close to 0.3 d (see fig. 1 of Brown et al. 2007), which for the current masses would imply an orbital separation smaller than 7 \( R_\odot \). Even assuming a tremendous amount of mass is lost during the explosion \( (86 M_\odot \) which we will justify in the following discussion), this distance would not be larger than 9.5 \( R_\odot \). Similarly to the scenario pictured for M33 X-7 in Moreno Méndez et al. (2008), this is quite an unlikely situation. In fact the two stars in such an orbit would undergo a common envelope phase and end up as a merger. This is the strongest argument against the possibility that the observed Kerr parameter corresponds to the natal value.

Next, the formation of the BH must be such that the pre-explosion orbital period \( P_1 \sim 0.3 \) d is transformed to the post-explosion (present) orbital period \( P_2 = 3.91 \) d. The mass-loss in the Blaauw–Boersma (BB) explosion\(^4\) (Blaauw 1961; Boersma 1961) is the only mechanism to achieve this since not much mass (a few \( M_\odot \) since the companion is an O7/O8 III star) is lost from the system afterwards. Nevertheless, this is problematic. As stated in Moreno Méndez et al. (2008), if half the mass of the system is lost during the BB explosion the binary breaks apart i.e. using \( M_{\mathrm{BH}} \) for the BH mass and \( M_{\mathrm{acc}} \) for the mass of the companion, \( \Delta M = M_{\mathrm{BH}} + M_{\mathrm{acc}} \); (see Brown et al. 2001, for a more detailed derivation):

\[
\frac{P_{\text{breakup}}}{P_1} = \left( 1 + \frac{\Delta M}{M_{\mathrm{BH}} + M_{\mathrm{acc}}} \right)^2 = 4. \tag{2}
\]

Our scenario needs \( P_2/P_1 \sim 10 \) (or a mass-loss of \( \sim 86 M_\odot \)), \( P_{\text{breakup}} \), \( P_1 \) and \( P_2 \) representing binary breakup, pre- and post-explosion periods. As equation (2) shows, this cannot be achieved during the explosion, as such a mass-loss would break the system apart.

It is important to note that, given the geometry and the anticipated evolution of the binary system, we expect the rotational axes of both stars to be aligned with the orbital rotation axis. This is because, due to strong tides, the system is likely in a state of minimum energy. In this state the spin axes have aligned, the orbits are circular and the spin period of the primary and secondary stars are tidally synchronized, \( P_{\text{rot}} = P_{\text{sec}} = P_1 \) (see e.g. Zahn 1977).

The supernova (SN) explosion of a rapidly rotating star will likely be cylindrically symmetric along the rotational axis, and thus a large (and temporally long with respect to the spin period) asymmetry in the collapse (e.g. a neutrino flux) will likely produce a kick along this axial direction (see e.g. Spruit & Phinney 1998). Hence, at least as a first-order approximation, the SN kick will be perpendicular to the BB kick (and to the original orbital plane). Thus the SN kick is not only unable to counteract the BB kick but actually makes the total kick larger with respect to the CoM of the binary.

Therefore, the SN explosion must proceed with much less mass-loss in order to prevent the system from breaking apart. The SN must not expand the orbital period by more than \( \sim 4 \) times its pre-SN value (i.e. 1.2 d).

The amount of mass-loss (assuming a fast wind such that no mass is transferred to the BH) from the companion after the formation of the BH to expand the orbit from \( P_2 = 1.2 \) d to \( P_{\text{now}} = 3.91 \) d can be

\[4\text{When the primary star explodes, the binary loses mass asymmetrically with respect to its centre of mass (CoM). The CoM shifts towards the secondary. The orbital velocity of the two stars is suddenly too large for the new mass of the system. In order to conserve momentum the binary suffers a kick along the orbital plane.}\]
obtained from
\[
\left( \frac{P_{\text{now}}}{P_2} \right) = \left( \frac{M_2}{M_{\text{now}}} \right)^2
\]
(3)
(see Van den Heuvel 1994, for a derivation). Using \( M_{\text{now}} = 31 \, M_\odot \) for the final mass, we obtain a post-explosion mass \( M_2 = 56 \, M_\odot \). Given that a binary with two stars of zero-age main-sequence (ZAMS) mass above \( \gtrsim 60 \, M_\odot \) does not fit in a 0.3–d orbit, and that the necessary mass-loss to bring the orbit from 0.3 to 3.91 d is unrealistic, we must look for alternative evolutionary paths.

### 2.2 LMC X-1 with Hypercritical Accretion

The present-day period of the binary is \( P_{\text{now}} = 3.91 \) d which corresponds to a Kerr parameter of \( a_* \sim 0.1 \) in fig. 1 of Brown et al. (2007). Mass transfer after the formation of the BH can only occur from the now more massive companion towards the BH. Assuming conservative mass transfer implies that the orbit could only shrink from the time of the collapse until the present day. This means that \( a_* \sim 0.1 \) is an upper limit for the natal Kerr parameter of the BH. Most of the spin must be acquired through hypercritical mass accretion, that is, the original BH mass has to increase between 50 and 70 per cent (or some 4–5 \( M_\odot \)) from its natal value to acquire the observed \( a_* = 0.90_{-0.06}^{+0.06} \) value (see fig. 6 of Brown et al. 2000) in less than 5 Myr.

Assuming that the recircularization of the orbit does not alter the orbital period considerably, and that there is conservative mass transfer from the companion to the BH, we can reconstruct the post-explosion orbital period. \( P_2 \), starting from the current one, \( P_{\text{now}} \), in LMC X-1 (see e.g. equations 90–97 in Van den Heuvel 1994, for a derivation):
\[
P_2 = \left( \frac{M_{\text{BH,now}} \times M_{\text{sec,now}}}{M_{\text{BH,2}} \times M_{\text{sec,2}}} \right)^3 \times P_{\text{now}}
\]
\[
= \left( \frac{10.3 \, M_\odot \times 30.6 \, M_\odot}{6.3 \, M_\odot \times 34.6 \, M_\odot} \right)^3 \times 3.91 \, \text{d} = 11.8 \, \text{d},
\]
(4)
where \( 4 \, M_\odot \) are transferred or \( P_2 = 18.2 \) d if \( 5 \, M_\odot \) are transferred (where \( M_{\text{BH}} \) represents the BH mass, \( M_{\text{sec}} \) is the mass of the secondary and the subscripts 2 and now refer to post-explosion and present values). This restricts the natal \( a_* < 0.05 \). Nevertheless, if there were mass-loss during the formation of the BH, the pre-explosion period had to be somewhat smaller but taking into account the limits imposed by equation (2). Therefore, assuming half of the mass of the system was lost during the explosion, the lower limit on the orbital period is 3–4 d. This translates to a maximum natal Kerr parameter \( a_* \sim 0.15 \). We can argue that the BB explosion leading to the formation of the BH could not lose close to half of the mass of the system when it was formed. First of all, a BH with a mass \( \gtrsim 6 \, M_\odot \) was created. This implies that the SN shock wave had to be stalled at least until most of this material was accreted, otherwise this material would be lost from the binary. Eventually, the stalled shock subsided and the material was advected into the BH. As the BH formed and grew there were at least two mechanisms which could eventually launch an explosion if an accretion disc was able to form. The first scenario is the Blandford–Znajek (BZ) mechanism (Blandford & Znajek 1977). The second one involves neutrino pair annihilation (see e.g. MacFadyen & Woosley 1999). However, a disc might never form in this scenario given that the available angular momentum in this scenario is rather small.

For the sake of obtaining an idea of the available energy for an explosion after the BH is formed, let us discuss the first scenario. Using the formalism in Lee et al. (2002) we obtain \( E_{\text{BZ}} \sim 15 \) bethe \((1 \text{ bethe} = 10^{51} \text{ erg})\) for \( M_{\text{BH}} = 6 \, M_\odot \) with \( a_* \sim 0.15 \) but only about 4 bethes if \( a_* \sim 0.08 \) (we obtain this number in the following paragraph). These numbers are much lower than the hundreds of bethes available in the rotational energy of the Galactic BH binaries (Brown et al. 2007). This amount of energy may not be enough to produce a powerful explosion as only a fraction of this energy will be kinetic.

Another point to keep in mind is that for the secondary star to be \( M_{\text{sec,now}} \sim 31 \, M_\odot \), one can estimate that we need \( 35 \, M_\odot < M_{\text{sec,ZAMS}} < 40 \, M_\odot \) given that the star has not filled its RL, assuming a lifetime of 5 Myr and a mass-loss of \( \sim 10^{-6} \, M_\odot \, \text{yr}^{-1} \) (also, Orosz et al. 2009, by measuring the luminosity and temperature of the companion, estimate \( M_{\text{sec,ZAMS}} \sim 35 \, M_\odot \)). But this implies that the primary might have been \( 40 < M_{\text{pri,ZAMS}} < 50 \, M_\odot \). Using the empirical relation (Lee et al. 2002)
\[
M_{\text{He}} = 0.08(M_{\text{ZAMS}}/M_\odot)^{1.45} \, M_\odot,
\]
we can estimate that the mass of the He star was about \( M_{\text{He}} \sim 20 \, M_\odot \). So, forming a BH with \( M_{\text{BH}} \geq 6 \, M_\odot \) implies that, in equation (2), \( \Delta M \sim 14 \, M_\odot \). Therefore, the minimum pre-explosion orbital period is
\[
P_1 = [1 + 4/(6 + 35)]^{-1} \times 12 \, d = 6.7 \, d
\]
for this scenario. This equates to a natal Kerr parameter of \( a_* \sim 0.08 \).

Similar to the case of M33 X-7 (Moreno Méndez et al. 2008), the available information on LMC X-1 points to an explosion where little mass was lost when the BH was formed. This supports the scenario of a pre-explosion (and post-explosion) period roughly between 12 and 18 d, which has been shortened, by mass transfer to the BH, down to the presently observed 3.91 d. Such mass transfer had to be hypercritical, leading to a growth of the Kerr parameter from \( a_* \lesssim 0.1 \) to its currently observed 0.9 value.

### 3 CASE M BINARY EVOLUTION

As proposed in De Mink et al. (2008, 2009a,b), we now consider Case M evolution of massive binaries, i.e. tidally locked, rotationally mixed and chemically homogeneous stars. The radii of stars in such an evolutionary path barely change as they evolve. In the following calculations we assume that the radii of the stars do not change substantially from their ZAMS radii.\(^5\) On one hand, this will prevent them from filling their RLs. On the other hand, we assume they do not contract enough after the main sequence to substantially alter their spin.

#### 3.1 Without Hypercritical Accretion

##### 3.1.1 M33 X-7

In M33 X-7 we presently observe a \( M_{\text{sec}} \sim 70 \, M_\odot \) companion orbiting a \( M_{\text{BH}} \sim 15 \, M_\odot \) BH in a \( P_{\text{now}} \sim 3.45 \) d orbit (Liu et al.

\(^5\) It is interesting to note that Orosz et al. (2009) estimate a ZAMS mass of \( \sim 35 \, M_\odot \) for the secondary star, similar to what conservative mass transfer would require. However, as discussed earlier, this must occur through wind RLOF.

\(^6\) We assume that the radii of Case M stars do not change significatively during the whole evolution. Research is underway to establish whether the relaxation of this assumption may produce larger Kerr parameters.

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To form a massive binary with a Kerr parameter of \( a_*=0.84 \) and such an orbital period the system has to lose a substantial amount of mass at different stages. Next, we reconstruct the path that requires the lowest amount of mass-loss, i.e. the stars with the lowest ZAMS masses that can explain the observed masses and orbital period.

The binary has a \( P_{\text{orb},1}=0.4\) d orbit. This provides the observed \( a_*=0.84 \) (see fig. 3 of Brown, Lee & Moreno 2008) of the 15 M\( \odot \) BH. During the formation of the BH, the primary looses almost half the mass of the system, i.e. using equation (2), and the mass we estimate for the secondary on the following paragraph:

\[
\Delta M \sim M_{\text{BH}} + M_{\text{sec},2} = 15 M_{\odot} + 103 M_{\odot} = 118 M_{\odot},
\]

so

\[
P_{\text{orb},2} = \left[ 1 + 118 M_{\odot} / (15 + 103) M_{\odot} \right]^2 \times 0.4 \text{d} = 1.6 \text{d}.
\]

After the formation of the BH, the secondary looses \( (M_{\text{sec},2} - M_{\text{sec,now}}) \sim 33 M_{\odot} \); this changes the post-collapse \( P_{\text{orb},2} \sim 1.6 \text{d} \) orbital period to the present one (equation 3),

\[
P_2 = (103 M_{\odot} / 70 M_{\odot})^2 \times 1.6 \text{d} = 3.45 \text{d}.
\]

This implies that, prior to the formation of the BH, the primary star, with \( M_{\text{pri},1} = 15 M_{\odot} + 118 M_{\odot} = 133 M_{\odot} \), and the secondary, \( M_{\text{sec},1} = 70 M_{\odot} + 33 M_{\odot} = 103 M_{\odot} \), must orbit within \( \lesssim 14 R_{\odot} \). As stated above, these are the smallest masses necessary. If we chose smaller \( \Delta M \) during the collapse and a larger wind mass-loss from the secondary, the estimate becomes larger. It is evident that such stars do not fit in this orbit. A common envelope would merge them extremely rapidly. Note that this scenario has not yet accounted for mass-loss prior to the collapse of the primary into the BH. This would mean the stars were more massive and closer at ZAMS. From the above it is clear then that this is an unlikely scenario to reproduce the observed values of stellar masses, orbital period and Kerr parameter in M33 X-7.

### 3.1.2 LMC X-1

In the case of LMC X-1, the situation could be even tighter as the observed Kerr parameter is closer to \( a_* \sim 0.9 \) or a pre-collapse period close to \( P_{\text{orb},1} \sim 0.3 \) d. Then, following the procedure of Section 3.1.1,

\[
\Delta M \sim M_{\text{BH}} + M_{\text{sec},2} = 10 M_{\odot} + 56 M_{\odot} = 66 M_{\odot},
\]

so

\[
P_{\text{orb},2} = \left[ 1 + 66 M_{\odot} / (10 + 56) M_{\odot} \right]^2 \times 0.3 \text{d} = 1.2 \text{d}.
\]

After the formation of the BH, the secondary looses \( \sim 25 M_{\odot} \) which changes the post-collapse \( P_{\text{orb},2} \sim 1.2 \) d orbital period to the present one (equation 3),

\[
P_2 = (56 M_{\odot} / 31 M_{\odot})^2 \times 1.6 \text{d} = 3.9 \text{d}.
\]

This implies that the primary star, with \( M_{\text{pri},1} = 10 M_{\odot} + 66 M_{\odot} = 76 M_{\odot} \), and the secondary, with \( M_{\text{sec},1} = 31 M_{\odot} + 25 M_{\odot} = 56 M_{\odot} \), must orbit within \( \lesssim 10 R_{\odot} \).

Again, in this scenario the orbit is too tight to fit the two stars; from Table 3 we obtain that the actual (\( R_{\text{ZAMS}} \)) radii for rotationally mixed, chemically homogeneous stars of mass 70 M\( \odot \) are \( >10 R_{\odot} \) when the metallicity is \( Z \sim (0.2-0.1) Z_{\odot} \). These radii are very close to the orbital separation. This strongly suggests that even rotationally mixed chemically homogeneous stars could not orbit close enough to produce the large Kerr parameters at the time the BHs were formed without merging the two stars.

### 3.2 Hypercritical Accretion

Perhaps one of the most relevant points to note in favour of hypercritical accretion after Case M binary evolution is that in order to achieve the observed Kerr parameters one seems to need a spin period smaller than the critical rotation period of such stars at ZAMS (see Table 3). This is regardless of whether the stars fit or not in their RLs and their orbits. Nevertheless, this difficulty may be overcome by the shrinking of the core after the exhaustion of H. This, however, breaks the tidal lock with the companion, and unless the ratio of stellar radius to orbital separation changes substantially, tides (as well as winds) will eventually remove the gain in spin.

If we instead allow hypercritical accretion to occur into the BH, we can avoid the discrepancy between the current orbital period and Kerr parameter. None the less, there are still important constraints to be addressed regarding the allowed orbits. The most important points for LMC X-1 and M33 X-7 are the following.

(i) \( P_{\text{orb}} = P_{\text{spin}} \leq P_{\text{rim}} \), the tidally synchronized primary star must rotate faster than the minimum critical spin below which rotational mixing is not efficient enough to keep the star compact. Otherwise the star will expand, fill its RL, transfer mass, shrink the orbit and produce a common envelope and, most likely, end up in a merger.

(ii) The secondary star must be either a main-sequence star or must also be in a Case M evolutionary track. Otherwise we end up, again, with a merger and a larger but unobservable Kerr BH.

Contry to what was discussed in Section 3.1, in this scenario the stars are not required to rotate with enough angular momentum to produce the observed Kerr parameters. Instead, we demand approximately five solar masses to be transferred into the BHs (see fig. 6 of Brown et al. 2000) in both M33 X-7 and LMC X-1. Otherwise it is not possible to obtain at the same time the observed Kerr parameters and orbital periods.

Yoon et al. (2006) (fig. 3) note that the main requirement for rotational mixing inside a star is a spin period of at least 20–35 per cent of its critical rotation period (\( P_{\text{c}} \), i.e. where gravity on its equator is balanced by centrifugal acceleration. In tidally synchronized stars this means the orbital period \( P_{\text{orb}} \lesssim P_{\text{rim}} \approx 5P_{\text{c}} \) (where \( P_{\text{rim}} \) is the period necessary in order to have rotational mixing in the star). As we have mentioned above, this might be necessary for both stars.

From Table 3 it is clear that \( P_{\text{orb,now}} > P_{\text{rim}} \) for the secondary stars in both M33 X-7 and LMC X-1, this means that they cannot be currently tidally synchronized and rotationally mixed. The future fate of these systems will be very different depending on whether the secondary stars manage to stay in Case M or become tidally synchronized.

The orbital separation of the binary for the (ZAMS) stars in M33 X-7 to be tidally synchronized to the point where rotational mixing occurs would be between 33 and 38 R\( \odot \) (assuming \( M_{\text{sec}} = 75 M_{\odot} \) and \( M_{\text{pri}} = 85 M_{\odot} \), and for LMC X-1 it would be \( \sim 20 R_{\odot} \).
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(assuming \( M_{\ast} = 35 \, M_\odot \) and \( M_{\text{tot}} = 45 \, M_\odot \)). Comparing these values to those in Table 3 shows that these stars would barely be within their RLs (using Eggleton 1983, we obtain \( R_{\text{RL}} \sim 7.7 \, R_\odot \) for LMC X-1 with \( P_{\text{orb}} = 1.2 \, d \); \( R_{\text{RL}} \sim 14 \, R_\odot \) for M33X-7 with \( P_{\text{orb}} = 2 \, d \)).

These scenarios seem to be extremely tight, especially because no mass-loss has been considered yet, and that the stellar radii are very close to their RL radii. However, it is possible that they will produce binaries like those in M33 X-7 or LMC X-1.

Following these assumptions, and not accounting for the substantial mass and angular momentum loss during core He burning, a period of 1.2 d would give a maximum \( a_* < 0.3 \) for the BH in LMC X-1. A period of 2 d would produce a BH with a maximum \( a_* < 0.2 \) for M33 X-7. Since in this scenario, like in Case C mass transfer, we still face the problem of unstable mass transfer due to the high values of \( q \), it is clear that the observed Kerr parameters only seem attainable through a phase of wind RLOF with \( \sim 5 \, M_\odot \) accreted hypercritically by the BH.

4 DISCUSSION

To explain the observed Kerr parameters in massive BH binaries, here we have restricted our study exclusively to Case C mass transfer and Case M binary evolution without covering other possible evolutionary scenarios.

On one side of the initial-orbital-separation spectrum, Case C mass transfer spins the BH progenitor at the latest possible time before the stellar collapse (compared to Case A and B mass transfer). This allows a much shorter time for the stars to lose mass and angular momentum by means of stellar winds and/or tidal interactions. Most of the angular momentum can be retained until stellar collapse and contribute to the BH spin.

On the other extreme, i.e. for very short orbital periods, Case M spins the progenitor early in the evolution and maintains it tidally locked throughout most of its lifetime. Therefore, under the assumption that the stellar radii do not change too much during the evolution of chemically homogeneous stars, Case M appears as a natural candidate to produce fast-rotating collapsing stars.

Alternatively, Case A and B are scenarios in which mass transfer from the primary occurs early in the lifetime of the binary system. This means that mass and angular momentum can be efficiently lost in later phases of stellar evolution. Hence, a slowly rotating BH will be produced in such scenarios.

Therefore we consider the Kerr parameters estimated for Case C and Case M mass transfer to be upper limits. Even if we have not studied Case A and Case B mass transfer scenarios in detail, it seems very difficult to explain the observed Kerr parameters avoiding a hypercritical accretion phase.

In principle one may also suggest the interaction of stars in a triple or even multiple system. Perhaps the most difficult part to justify in a triple scenario is how to bring in the BH and the massive secondary into the currently observed tight orbit. After Case C mass transfer (where we have shown that the angular momentum is less likely to be lost), when the primary transfers mass and produces a common envelope which allows the first companion to spiral in and merge the price to pay is the loss of the envelope of the primary (see e.g. the discussion in Lee et al. 2002). Therefore, narrowing the orbital separation of the massive secondary may be difficult.

A likely scenario would be to wait for the massive secondary to expand, transfer mass to the BH and allow it to spiral in to a shorter orbital period. However, neither LMC X-1 nor M33 X-7 seems to have such evolved companion, in fact these seem to be still in the main sequence and within their RLs. In general, in the case of a multiple system, finding a way to reproduce the observed separation and Kerr parameter becomes a very complex exercise, and it is beyond what we can do at present.

It is also interesting to speculate what the final fate of these systems will be if they have evolved through Case M. The fact that they can no longer be both tidally locked and in Case M produces very different outcomes. After the SN and further wind mass-loss from the secondary, the orbit has grown larger making tidal locking less efficient, if the synchronization time-scale is longer than the remaining lifetime of the massive secondaries we could still expect them to be rotating rapidly. If the spin periods stay below \( P_{\text{spin}} \), they will likely stay in Case M. If the second SN does not disrupt the binaries they will eventually form BH–BH (or BH–NS if enough mass is lost during the remaining evolution) binaries. Like for the first-born BH, the natal Kerr parameters should be small, \( a_* < 0.3 \) for LMC X-1 and \( a_* < 0.2 \) for M33 X-7. Nevertheless these will not be able to increase given the lack of a mass donor. If this is not the case and the secondary stars are being spun down by tidal synchronization, they will eventually fill their RLs before the end of their main sequence. This will transfer mass to the BHs and narrow the orbits. Their fate will be a merger with their respective BHs. Given the large Kerr parameters of the BHs in these two systems, it could be expected that such an event will produce long gamma-ray bursts. In the end, a massive but non-observable Kerr BH will be the remnant.

Recently Valsecchi et al. (2010) have suggested a formation mechanism for M33 X-7 which involves Case A mass transfer. Their model does not include rotational mixing; however, it is compared with that of Hirschi, Meynet & Maeder (2005) which does include it in order to use the stellar angular momentum at the end of the main sequence calculated in this other model. The result nevertheless should be significantly different. Furthermore, as we have previously pointed out, most of the mass and angular momentum loss occurs at later stages. More important still, they obtain a system where the Wolf–Rayet (WR) star is synchronized with the massive companion in an orbital period longer than 3 d. Hence, the Kerr parameter of the resulting BH would not be larger than 0.1. This, contrary to what is claimed in the paper, is not consistent with the observed value. Hence, this model would still need wind RLOF and hypercritical accretion to explain the observed Kerr parameter.

As this paper prepares for press, a new paper, Axelsson et al. (2010), has appeared on the spin parameter of Cygnus X-1. The authors reach similar results as those from this paper; however, they conclude that the mechanism of Blondin & Mezzacappa (2007) to spin NSs up through a stationary accretion shock instability can produce the measured \( a_* = 0.48 \pm 0.01 \). In a future paper we will demonstrate that this mechanism would be extremely difficult to justify for BHs with massive companions such as M33 X-7, LMC X-1 or Cyg X-1 because of conservation laws.

5 CONCLUSIONS

LMC X-1 and M33 X-7 are two BH binaries where the masses of the components, the orbital period and the spin of the BH, have been well constrained observationally. We have shown that, if we model the evolution of the progenitors of such systems with the standard methods, we cannot explain the current state of affairs in such systems.

We have discussed that Case C mass transfer binary evolution provides an upper limit to the attainable natal spin of the BH (over Case A and Case B). By stretching this model to its limits, we are
able to estimate an upper limit to the Kerr parameters of LMC X-1 and M33 X-7 of $a_\ast < 0.15$, well below the observed values of 0.90 and 0.84, respectively.

We have further attempted to obtain an evolutionary model which may explain the observations through Case M evolution. This model requires a very finely tuned system in order to reproduce the observed BH binaries but does seem to provide the largest possible Kerr parameters ($a_\ast < 0.3$ for LMC X-1 and $a_\ast < 0.2$ for M33 X-7). However, like the rest of the models, Case M fails to come close to the observed values. It is interesting to study whether relaxing some of the assumptions on this model might provide larger Kerr parameters, but one must remember that we still have not considered mass-loss nor angular momentum loss at late evolutionary stages. In the end, this model may still suffer the fate of Case A and Case B where the mass-loss during He burning removes the outer layers of the star and with them, most of the angular momentum necessary to produce a substantial Kerr parameter.

Both Case C mass transfer and Case M models for binary evolution suggest that hypercritical accretion is necessary in order to explain the observations of the Kerr parameters of the BHs in LMC X-1 and M33 X-7. Nevertheless, given the current mass ratios, this implies that efficient wind mass transfer between the companion and the BH must occur.

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