Filtration and heat processes in lateral extrusion of plastically compressible materials

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Abstract. We consider the process of heat production and transfer in the lateral extrusion of plastically compressible material. Heating of the material occurs due to friction and due to energy dissipation during the irreversible deformation process. We assume that the material obeys to Green's elliptical yield condition and to the normality rule. We also consider the fluid filtration process in the pores of the material. It is assumed that the movement of the fluid is caused by change in density of the material during deformation. In the present paper we obtain exact solutions of the equations determining the mechanical behavior of the material at a steady flow in the case of small change in density. The approximate analytical solutions of the heat transfer problem and filtration problem in uncoupled formulation are also obtained.

1. Lateral extrusion process

One of the most common processes of intense plastic deformation of compact materials is equal channel angular pressing (ECAP). The mechanics of this process has been studied over the past three decades [1–8]. For non-compact materials (porous, powder, with defects of continuity), the mechanical behavior and boundary value problems describing the processes of intense plastic deformation turn out to be much more complicated. At present, only some exact solutions of problems are known, in which material compaction occurs, among other things, due to dilatancy effects (direct extrusion in axisymmetric and plane formulation) [9–14]. For lateral extrusion (angular extrusion), such solutions are not known. At the same time, the process of angular pressing seems to be a promising way of processing of non-compact materials. Because with ECAP, a stress state, which is close to pure shear, is realized. Consequently, the energy costs of deformation during ECAP can be lower than with hydrostatic compaction.

The thermal problem also requires separate consideration, because the intense plastic deformation is always accompanied by the heating of the material. And temperature effects in such processes are significant, since they determine the power of dissipative forces on surfaces of maximum friction.

It should be noted that for compact materials it is not difficult to determine the kinematics of the flow by experimental methods (for example knurled dividing grids, markers and other, more modern ones) at least at a sufficient distance from the channel angle. And the knowledge of kinematics for incompressible material allows one to reconstruct uniquely the source term of the heat and mass transfer equation as a grid function. Thus, the numerical solution of the corresponding heat transfer problem can be obtained with sufficient for engineering calculations accuracy. For non-compact
materials this possibility is absent radically, even if, in addition to kinematics, the density distribution is also established.

The thermoplastic problems are of fundamental complexity for numerical calculations in cases where the components of the strain rate tensor are unlimited inside or at the boundary of the computational domain, and, therefore, the source terms of the heat and mass transfer equation are singular. This is exactly what happens at the apex of the channel angle during ECAP process. And as the contribution of surface and volume sources to the total power of dissipation in this process is equivalent, it is impossible to ignore the power of plastic deformation without prior analysis. In this regard, we note the recent paper [15], in which, based on the classical solutions of Prandtl and Nadai for a thin plastic layer, a solution of the thermal problem was constructed. This solution takes into account both heat sources. In this rigid plastic problem, the strain rates were also unlimited near the surface of maximum friction.

As for filtration processes caused by the deformation of the porous medium, most of the known papers consider the frame material as viscoelastic continuum [16–17] or as viscous fluid. A number of studies have taken into account plastic deformation of the frame [18]. The results of numerical simulation have been obtained, for example, in the case of pressing in a closed die [19].

In the present paper, we have obtained exact solutions of the equations that determine the mechanical behavior of a plastically compressible material. These solutions have been obtained for a steady flow through an angular channel in the case of a small change in density. We supposed that the movement of material points occurs along arcs of the circles. This type of motion is consistent with the experimentally observed flow patterns. Also we have obtained approximate analytical solutions for the thermal problem and the filtration problem (in the case when the pores of the material are filled with fluid) in an uncoupled formulation.

The deforming of porous medium is considered when it passes through the junction zone of two flat slotted channels of the same section (see figure 1).

![Figure 1. Scheme of the process.](image)

In the case of the dry frame, a temperature field is to be found, and in the case of the frame saturated with fluid, a two-dimensional flow velocity field is to be found. The drain is organized on the cylindrical surface \( r = r_0 \). The limiting case is the presence of peculiarity \( r_0 = 0 \). It is assumed that there is no discontinuity of velocities at the inlet and outlet from the deformation zone. Following the principles of rigid plastic analysis, we assume that the deforming of the frame is solely irreversible and
it occurs in the sector of an annulus $D=[r_0, r_1] \times [0, \varphi_0]$. The angular coordinate starts from the surface of the outlet from the deformation zone and is directed against the movement of the material. At the outlet from the deformation zone the porous material with fluid residues is removed, so that the pressure in the fluid on the surface $\varphi=0$ does not differ from the atmospheric one.

2. Governing equations

It is assumed that the porous material obeys Green’s plastic potential [20]

$$\Phi = \left(\frac{\sigma}{\sigma_s}\right)^2 + \left(\frac{\tau}{\tau_s}\right)^2 - 1 = 0, \quad (1)$$

where $\sigma_s$, $\tau_s$ are plastic modules, $\sigma$ is a tensor of macroscopic Cauchy stresses in the frame. As a rule, $\sigma_s$, $\tau_s$ modules depend on the dimensionless density of the porous material. Besides, $\lim_{\rho \to 1} \sigma_s = +\infty$, $\lim_{\rho \to 1} \tau_s = \kappa$, where $\kappa$ is the shear yield strength of the frame. In this case the yield surface (1) within the limit becomes the von-Mises yield surface. In the present study we suppose that density of the material varies only insignificantly and, consequently plastic modules in (1) can be considered as constants.

The flow rule associated with the potential (1) is also accepted and given by

$$\varepsilon = \Lambda \frac{\partial \Phi}{\partial \sigma}, \quad (2)$$

where $\Lambda$ is the scalar plastic multiplier, $\varepsilon$ is macroscopic plastic strain rate tensor, $2\varepsilon = (\nabla \otimes \mathbf{v})^T + (\nabla \otimes \mathbf{v})$, $\nabla$ is Hamilton operator, $\mathbf{v}$ is the velocity vector of the material points of the frame.

The mass and inertial forces are neglected. The equilibrium configuration should satisfy the equation

$$\nabla \cdot \sigma = 0. \quad (3)$$

The continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (4)$$

where $t$ is time, $\rho$ is the dimensionless (relative) density of the porous material.

2.1. Filtration problem

The filtration velocity vector $\mathbf{v}^f$ is introduced in such a way that the total velocity of the fluid particles is $\left(\mathbf{v} + \mathbf{v}^f\right)$. Taking for both components of the mixture (the material of the frame and the fluid) condition of incompressibility, similarly to (4) one can write the equation for the fluid

$$\frac{\partial (1-\rho)}{\partial t} + \nabla \cdot \left[(1-\rho)(\mathbf{v} + \mathbf{v}^f)\right] = 0. \quad (5)$$

Rather a slow potential flow of Newtonian fluid in the frame is considered. Therefore, Darcy’s law is used (the linear filtration law)
\[ \mathbf{q} = (1 - \rho) \mathbf{v} = -\frac{k}{\mu} \nabla P, \]  

(6)

here \( \mathbf{q} \) is the flow vector, it is directly proportional to permeability \( k \) and inversely proportional to the viscosity of the fluid \( \mu \), \( P \) is pore pressure.

2.2. Heat transfer problem

Heat and mass transfer equation is

\[ c_\gamma \rho \left[ \frac{\partial T}{\partial t} + (\mathbf{v} \nabla)T \right] = \nabla \cdot (\lambda \nabla T) + \mathbf{\sigma} : \mathbf{\varepsilon}, \]

(7)

where \( c \) and \( \lambda \) are mass heat capacity and coefficient of thermal conductivity of the porous material, respectively, \( T \) is temperature, \( \rho \), as before, is the relative density of the porous material, \( \gamma \) is the density of the material frame (with null porosity). Mass heat capacity of the porous material is reasonably assumed to be constant. Besides, we consider the coefficient of the thermal conductivity also constant because the density changes insignificantly.

3. Model assumptions and boundary conditions

A stationary process is considered, so \( \frac{\partial \rho}{\partial t} = 0 \). It is assumed that the motion of the material points of the frame occurs along the arcs of the circle, the only non-zero component of the velocity vector is \( v_{\phi} < 0 \). The flow slows down, so \( \frac{\partial v_{\phi}}{\partial \varphi} < 0 \).

The boundary conditions are prescribed as follows:

- there is no back pressure at the outlet from the deformation zone, \( \sigma_{\phi\phi} \big|_{r = 0} = 0 \);
- linear velocity of the material points and the relative density at the inlet into the deformation zone are known to be \( v_{\phi} \big|_{\rho = \rho_0} = -v_0 = \text{const} \) and \( \rho \big|_{\rho = \rho_0} = \rho_0 = \text{const} \), respectively;
- at the outlet from the deformation zone the pore pressure is null, \( P \big|_{\rho = 0} = 0 \);
- azimuthal component of the filtration velocity vector at the inlet into the deformation zone is null, \( v_{\phi} \big|_{\rho = \rho_0} = 0 \), consequently \( \frac{\partial P}{\partial \varphi} \big|_{\rho = \rho_0} = 0 \);
- the condition of impermeability is set on the cylindrical surface \( r = r_0 \), \( v_{\theta} \big|_{r = r_0} = 0 \);
- the cylindrical surface \( r = r_1 \) is permeable for fluid, \( P \big|_{r = r_1} = 0 \);
- the temperature at the inlet into the deformation zone is known to be \( T \big|_{\rho = \rho_0} = T_0 \);
- there is no azimuthal component of the heat flow at the outlet, consequently \( \frac{\partial T}{\partial \varphi} \big|_{\rho = 0} = 0 \);
- the radial component of the heat flow on the contact surface is determined by the friction power, \( -\lambda \frac{\partial T}{\partial r} \big|_{r = r_1} = v_{\phi} \sigma_{\rho\phi} \);
- the radial component of the heat flow on the surface \( r = r_0 \) is null.
4. Solutions

The kinematics of the medium that satisfies equations (1) – (3) has the form

\[ v_{\varphi} = -v_{0} \frac{\cos^{1/2}(2\chi\varphi_{0})}{\cos^{1/2}(2\chi\varphi)}, \quad \varphi \in [0, \varphi_{0}], \quad \chi = \left[ \frac{4}{3} + \left( \frac{\sigma}{\tau_s} \right)^{2} \right]^{1/2}, \quad \text{const}, \]  

and the distribution of the frame density satisfies \( \rho v_{\varphi} = -\rho_{0}v_{0} \) equality.

The strain rate tensor has the form

\[ \varepsilon = \frac{1}{\rho} \frac{\partial v_{\varphi}}{\partial \varphi} \mathbf{e}_{\varphi} \otimes \mathbf{e}_{\varphi} - \frac{1}{2} \frac{v_{\varphi}}{r} \left( \mathbf{e}_{r} \otimes \mathbf{e}_{r} + \mathbf{e}_{\varphi} \otimes \mathbf{e}_{\varphi} \right). \]  

From (1) and (2) formulas the relation for Cauchy stress tensor is obtained as follows

\[ \sigma_{rr} = \sigma_{\varphi\varphi} = -\frac{r_{s}(1-2\chi^{2})}{\chi^{1+\chi^{2}\Omega^{2}}}, \quad \sigma_{\varphi\varphi} = -\frac{r_{s} \chi \Omega}{\sqrt{1+\chi^{2}\Omega^{2}}}, \quad \Omega = v_{0} \left( \frac{\partial v_{\varphi}}{\partial \varphi} \right)^{-1}. \]  

Taking into account equation (9), it can be concluded that the stressed state does not depend on the radial coordinate \( r \):

\[ \sigma_{rr} = \sigma_{\varphi\varphi} = -\frac{r_{s}(1-2\chi^{2})}{\chi^{1+\chi^{2}\Omega^{2}}}, \quad \sigma_{\varphi\varphi} = -\frac{r_{s} \chi \Omega}{\sqrt{1+\chi^{2}\Omega^{2}}}, \quad \Omega = v_{0} \left( \frac{\partial v_{\varphi}}{\partial \varphi} \right)^{-1}. \]  

Taking into account statement (8), pressure \( |\sigma_{\varphi\varphi}| \), necessary to support the deforming, is \( r_{s} \chi^{-1} \sin(2\chi\varphi_{0}) \). To compare, pressure \( |\sigma_{|} \), necessary to transfer the material into a plastic state at the uniaxial compression in the closed die, according to statement (10) is \( |\sigma_{|} = r_{s} \chi^{-1} \). This fact proves that the proposed method for compaction of the cellular plastics is power-efficient one.

The source-term in equation (7) then takes the form

\[ \sigma : \varepsilon = v_{0} r_{s} r^{-1} \cos^{2}(2\chi\varphi_{0}) \cos^{3+2}(2\chi\varphi). \]  

The power of friction force, acting on the contact surface \( r = r_{s} \), is given by

\[ v_{\varphi} \sigma_{\varphi\varphi} = -v_{0} r_{s} \cos^{2}(2\chi\varphi_{0}) \cos^{1+2}(2\chi\varphi). \]  

The ratio of integral power of the plastic dissipation to the integral power of friction force for the channel with the peculiarity \( r_{0} = 0 \) has the following form:

\[ \int_{0}^{\varphi_{0}} \int_{1}^{r_{b}} \sigma : \varepsilon r d r d \varphi = \int_{0}^{\varphi_{0}} \cos^{3+2} \xi d \xi \int_{0}^{2\pi\varphi_{0}} \cos^{1+2} \xi d \xi = \frac{\sin(2\chi\varphi_{0})}{\chi(2\chi\varphi_{0})^{2}} - 1, \]  

here \( E \) denotes the elliptic integral of the second kind.

The case of density low change of the porous material in the channel being here under study occurs under small values of \( 2\chi\varphi_{0} \). Therefore the ratio (11) has the value equals approximately to one. This means that the contribution of the volume heat source caused by plastic dissipation in the material to the forming of the temperature field is equal to the contribution of the surface heat source caused by friction.
4.1. Solution for heat transfer problem

The impact of the surface source is limited by small neighborhood of the contact surface. Being far from the contact surface, the radial component of the heat flow is rather small. Then equation (7) takes the form of ordinary differential equation with parameter $r$

$$\frac{\partial^2 T}{\partial \varphi^2} + \frac{c \gamma \rho v_0}{\lambda} r \frac{\partial T}{\partial \varphi} + \frac{\tau_s v_0 \cos^{1/2}(2 \chi \varphi_0)}{\lambda} r \cos^{-3/2}(2 \chi \varphi) = 0.$$ 

Therefore an approximate solution of the heat transfer problem has the form

$$AB^{-1} (T - T_0) = A r \int_0^\varphi e^{-Ar \xi} \cos^{-3/2}(2 \chi \xi) d \xi d \xi,$$

$$A = c \gamma \rho v_0 \lambda^{-1} = \text{const}, \quad B = \tau_s v_0 \lambda^{-1} \cos^{1/2}(2 \chi \varphi_0) = \text{const}.$$ 

On the contrary, being near the contact surface, the radial component of the heat flow dominates, and equation (7) can be written approximately in a parabolic form

$$c \gamma \rho v_0 \frac{\partial T}{\partial \varphi} + \frac{\lambda}{\xi} \frac{\partial^2 T}{\partial \xi^2} + \frac{\tau_s v_0 \cos^{1/2}(2 \chi \varphi)}{\cos^{1/2}(2 \chi \varphi_0)} = 0.$$ 

To obtain this, the curvature of the contact surface is neglected and the logarithmic coordinate $x = \ln(r / r_0)$ is introduced.

Then the approximate solution of the thermal problem (7) is of the form [21]

$$AB^{-1} (T - T_0) = \int_0^\varphi \cos^{-3/2}(2 \chi \xi) + \left[ \frac{A r \cos(2 \chi \xi)}{\pi} - \frac{Ar \xi^2}{4(\xi - \varphi)} \right] \exp \left[ -\frac{Ar \xi^2}{4(\xi - \varphi)} \right] d \xi.$$ 

The obtained approximate solutions (12) and (13) reasonably do not obey all boundary conditions of the original initial boundary value problem for the elliptic equation (7). It is useful to obtain data about the maximum heat value of the porous material when it passes through the channel under study. This information allows to define among others whether phase transition in the material takes place because of its heating. It is reasonable to consider that the maximum temperature is reached at the point of the channel which coordinate positions are $x = 0$, $\varphi = 0$. Then, using statement (13), one can obtain

$$\Delta T = \max_{x, \varphi} (T - T_0) = \frac{\tau_s \cos^{1/2}(2 \chi \varphi_0)}{2 c \gamma \rho \lambda} \int_0^\varphi \cos^{-3/2} \xi + \left[ \frac{2 \chi c \gamma \rho v_0 \xi \cos \xi}{\pi \lambda} \right]^{1/2} d \xi.$$ 

The function in the right-hand side of statement (14) non-monotone depends on the angle of the channel opening $\varphi_0$. On figure 2 one can observe the dependence plot of $\max_{x, \varphi} (T - T_0)$ on $\varphi_0$ under different values of process parameter $v_0 r_1$. The values of material constants typical for porous aluminum Alulight [22]: $c = 920 \, \text{Дж/кг} \cdot \text{К}$, $\lambda = 35 \, \text{Вт/м} \cdot \text{К}$, $\gamma = 2700 \, \text{кг/м}^3$, $\rho_0 = 0.35$, $\tau_s = 17.6 \, \text{МПа}$, $\sigma_s = 14.9 \, \text{МПа}$, $\chi = 0.7$. The considered maximum value of angle $\varphi_0$ corresponds to the rated increase of the relative density of the material when it passes through the channel a third.
Figure 2. Heat of the contact surface.

4.2. Solution for filtration problem
The exact solution of the filtration problem (5) – (6) in the case when the coefficient $k / \mu$ does not depend on the local relative density of the skeleton has the form [21]

$$P = n \frac{H}{k} [W(x, \varphi) + e^{-i} f(\varphi)],$$

$$W(x, \varphi) = \sum_{n=0} g_n \sin \left( \frac{\lambda_n \varphi}{\lambda_n x_0} \right) \left( e^{-i \lambda_n x_0} - \lambda_n \sin \left( \frac{\lambda_n (x_0 - x)}{\lambda_n x_0} \right) \right), \quad x_0 = \ln \frac{R}{r_0}$$

$$f(\varphi) = \cos \varphi \int_0^\varphi v_n \cos \varphi d\varphi + \sin \varphi \left( C - \int_0^\varphi v_n \sin \varphi d\varphi \right),$$

$$C = v_0 \cos^{-1} \varphi_0 + \tan \varphi_0 \int_0^\varphi v_n \cos \varphi d\varphi, \quad \lambda_n = \frac{\pi (2n + 1)}{2\varphi_0}, \quad g_n = \frac{2}{\varphi_0} \int_0^\varphi f(\varphi) \sin (\lambda_n \varphi) d\varphi.$$

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References
[1] Segal V M 1995 Materials processing by simple shear Mater. Sci. & Eng. A 197 pp 157–164
[2] Segal V M 2003 Slip line solutions, deformation mode and loading history during equal channel angular extrusion Mater. Sci. & Eng. A 345 pp 36–46
[3] Altan B S, Purcek G and Miskioglu I 2005 An upper-bound analysis for equal-channel angular extrusion J. Mater. Proc. Tech. 168 pp 137–146
[4] Bowen J R, Gholinia A, Roberts S M and Prangnell P B 2000 Analysis of the billet deformation behaviour in equal channel angular extrusion Mater. Sci. & Eng. A 287 pp 87–99
[5] Semiatin S L, DeLo D P and Shell E B 2000 The effect of material properties and tooling design on deformation and fracture during equal channel angular extrusion Acta Materialia 48:8
[6] Li S, Bourke M A M, Beyerlein I J, Alexander D J and Clausen B 2004 Finite element analysis of the plastic deformation zone and working load in equal channel angular extrusion Mater. Sci. & Eng. A 382 pp 217–236

[7] Kaushik A, Karaman I and Srinivasa A R 2009 Simulation of powder compaction using equal channel angular extrusion at room temperature: comparison of two constitutive theories Int. J. Struct. Changes in Solids – Mech. & Appl. 1:1 pp 211–226

[8] Zairi F, Aour B, Gloaguen J M, Nait-Abdelaziz M and Lefebvre J M 2008 Steady plastic flow of a polymer during equal channel angular extrusion process: experiments and numerical modeling Polymer Eng. & Sci. 48:5 pp 1015–1021

[9] Oh H-K and Lee J-K 1985 A study of the extrusion of sintered porous metal J. Mech. Work. Tech. 11:1 pp 53–69

[10] Monchiet V and Kondo D 2012 Exact solution of a plastic hollow sphere with a Mises – Schleicher matrix Int. J. Eng. Sci. 51 pp 168–178

[11] Thore P, Pastor F, Pastor J and Kondo D 2009 Closed-form solutions for the hollow sphere model with Coulomb and Drucker – Prager materials under isotropic loadings Comptes Rendus Mecanique 337 pp 260–267

[12] Durban D and Mear M E 1991 Asymptotic solution for extrusion of sintered powder metals Trans. ASME 58:2 pp 582–584

[13] Alexandrov S, Chesnikova O and Pirumov A 2007 An approximate solution for axisymmetric extrusion of porous material J. Tech. Plast. 32:1–2 pp 13–27

[14] Alexandrov S E and Druyanov B A 1990 Investigating the process of the steady extrusion of a compacted material J. Appl. Mech. & Tech. Phys. 31:4 pp 645–649

[15] Aleksandrov S and Miszuris W 2016 Heat generation in plane strain compression of a thin rigid plastic layer Acta Mechanica 227 pp 813–821

[16] Connolly J A D and Podladchikov Y Y 1998 Compaction-driven fluid flow in viscoelastic rock Geodinamica Acta 11:2–3 pp 55–84

[17] Tokareva M A 2015 Localization of solutions of the equations of filtration in poroelastic medium J. Siberian Federal University. Math. & Phys. 8:4 pp 467–477

[18] Showalter R E and Stefanelli U 2004 Diffusion in poro-plastic media Math. Methods in Appl. Sci. 27 pp 2131–2151

[19] Zhao J, Wang C-H, Lee D-J and Tien C 2003 Plastic deformation in cake consolidation J. Colloid & Interface Sci. 261 pp 133–145

[20] Green R J 1972 A plasticity theory for porous solids Int. J. Mech. Sci. 14 pp 215–224

[21] Polyanin A D and Nazaikinskii V E 2016 Handbook of Linear Partial Differential Equations for Engineers and Scientists (Boca Raton: CRC Press)

[22] Ashby M F, Evans A G, Fleck N A, Gibson L J, Hutchinson J W and Wadley H N G 2000 Metal Foams: A Design Guide (Boston: Butterworth-Heinemann)