Flavor violating decays of the Higgs bosons in the THDM-III

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Abstract

We calculate the branching ratios for the decays of neutral Higgs bosons ($h^0, H^0, A^0$) into pairs of fermions, including flavor violating processes, in the context of the General Two Higgs Doublet Model III.

1 Introduction

The mechanism that generates the fermion masses is perhaps hidden behind the structure of the Yukawa couplings. In the Standard Model (SM) the form of these couplings is not well understood and neither are their origin nor the underlying principles (the flavor problem) [1], [2], [3]. Phenomenologically, the SM parameterizes the values of the Yukawa couplings, but, theoretically, we are in the darkness. This is one of the reasons why many physicists agree that the SM should be considered as an effective theory that remains valid up to an energy scale of $O$(TeV), and eventually will be replaced by a more fundamental theory. A way to picture this could be to consider that the SM is the top of an iceberg, so that a great deal still remains to be explored, in order to understand what lies underneath.

One simple extension of the SM adds a new Higgs doublet, and it is known as the Two Higgs Doublet Model (THDM) [4], [5]; which constitutes precisely the structure that is required in the construction of the Minimal Supersymmetric SM (MSSM) [6]. The direct consequences of this extension are: an increase in the scalar spectrum, a more generic pattern of Flavor Changing Neutral Currents (FCNC), including FCNC at tree level, which are highly excluded in low energy experiments and turn out to be a potential problem. In the earlier versions (THDM I-II), this problem was solved by
imposing a discrete symmetry that restricts each fermion to be coupled at most to one Higgs doublet.

In the so called version III of the model, FCNC are kept under control by imposing a certain form for the Yukawa matrices that reproduce the observed fermion masses and mixing angles. This more general model allows a rich phenomenology through extra terms that come from the Yukawa couplings, and that is present even at tree level. The use of texture forms permits to establish a direct relation from the elements of the matrix with the mixing parameter used in calculating the branching ratios, without dropping terms proportional to the lighter fermion masses in advanced. Specifically, considering an hermitian Yukawa matrix of the 6-textures type one gets the Cheng-Sher Ansatz (the flavor violation couplings are given as proportional to the square root of the mass product) for the flavor mixing couplings which is widely used in literature.

In this work, we departure from a 4-texture form of the Yukawa matrices. We are interested in employing this version (THDM-III) to evaluate the branching ratios of the neutral Higgs bosons decaying into fermion pairs, with the idea of studying the model predictions and its test at future Large Hadron Collider (LHC) data.

Hence, considering that the flavor violating interactions arise from the Yukawa couplings, our starting point is an Hermitian 4-texture ansatz in order to construct the mass matrices, which have been found to be in agreement with the observed data. The relation from the matrix elements to the mixing parameter used to calculate FCNC processes in the Higgs sector is developed in. Some Higgs phenomenology have been addressed in the literature, mainly aimed to describe the light Higgs boson.

The paper is organized as follows. In Sec. II we discuss the structure of the Yukawa lagrangian of the THDM-III, and present the couplings \( \phi^0 f_i f_j \), \( \phi^0 = h^0, H^0, A^0 \) by using an hermitian four-texture form for the mass matrices; we also show how the Yukawa couplings induce the LFV Higgs decays. Then, in Sec.III we present the analytical formulae for the couplings of the three neutral Higgs bosons with fermions and we compare them with the MSSM and for the flavor conserving modes with those of the SM case. We also present the formulae used for the calculations of the branching ratios. The results are presented in Sec. IV and some conclusions are left for Sec. V.
2 Structure of the Yukawa sector in the Two Higgs Doublet Model Type III

The couplings of the Higgs boson to the fermions are given by the Yukawa lagrangian, which in general can be written as follows

\[ L_Y = \sum_{a,i} Y_i^a F_L^a \Phi_a f_i^R + \text{h.c.}, \]  

(1)

where \( F_L \) denotes the fermion doublet (left-handed), \( f_R \) is the fermion singlet (right-handed), and \( \Phi_a \) are the two Higgs doublets \( (a = 1, 2) \). Considering the three generations, the coefficient \( Y_i^a \) can be expressed as a 3 \( \times \) 3 matrix and tagged as \( Y_{i,l}^a, Y_{i,u}^a, Y_{i,d}^a \), for leptons, \( u \) and \( d \) type quarks. Here we consider massless neutrinos.

What we know so far about the Yukawa terms is that they should reproduce the fermion masses, the mixing angles of the CKM matrix and keep the FCNC under control, as they are restricted by experimental data. From the form of the lagrangian, it is natural to consider that the flavor violation or the mixtures between families, could arise directly from the form of the Yukawa terms, which, in general, are not diagonal. In this work we are interested in the study of the Higgs boson decays \( H \rightarrow f_i f_j \), as a possible signal of fermion flavor violation for the complete spectrum of neutral Higgs bosons \( h^0, H^0, A^0 \) of the THDM-III.

In particular, in the Yukawa sector for the THDM-III, both Higgs doublets may couple with the two types of fermions, \( i.e., \) up and down, so that we have two different Yukawa terms for each doublet, \( Y_a \) with \( a = 1, 2 \). Thus, the Yukawa lagrangian is given by:

\[ \mathcal{L}_Y^l = Y_{l1}^l \bar{L}_L^l \Phi_1 u_R^l + Y_{l2}^l \bar{L}_L^l \Phi_2 u_R^l; \]  

(2)

\[ \mathcal{L}_Y^q = Y_{u1}^q \bar{Q}_L^u \tilde{\Phi}_1 d_R^u + Y_{u2}^q \bar{Q}_L^u \tilde{\Phi}_2 d_R^u + Y_{d1}^q \bar{Q}_L^d \Phi_1 d_R^l + Y_{d2}^q \bar{Q}_L^d \Phi_2 d_R^l, \]  

(3)

where the first equation corresponds to the leptonic sector and the second one to the quark sector. \( \tilde{\Phi}_i = i \sigma_2 \Phi_i^* \) and \( Y_{1,2}^{u,d,l} \) denote the \( (3 \times 3) \) Yukawa matrices.

After SSB (Spontaneous Symmetry Breaking), one can derive the fermion mass matrices from eqs. (2) and (3), namely

\[ M_f = \frac{1}{\sqrt{2}} (v_1 Y_{1}^f + v_2 Y_{2}^f), \quad f = u, d, l, \]  

(4)
Here, we are taking into account the fact that, working with a \textit{hierarchical ansatz} for the mass matrix and by means of equation (1), the simplest case is to consider that both Yukawas $Y_{f1,2}$ possess the same structure (without anomalous cancellation of any of the elements of the matrices), particularly we use an hermitian 4-texture form, and because of eq. (4) the complete mass matrix inherits this structure. The mass matrix is diagonalized through the bi-unitary matrices $V_{L,R}$, though each Yukawa matrices are not diagonalized by this transformation. The diagonalization is performed in the following way

$$\bar{M}_f = V^\dagger_{fL} M_f V_{fR}. \quad (5)$$

The fact that $M_f$ is hermitian, under the considerations given above (hermitian Yukawa matrices), directly implies that $V_{fL} = V_{fR}$, and the mass eigenstates for the fermions are given by

$$u = V^\dagger_u u', \quad d = V^\dagger_d d', \quad l = V^\dagger_l l'. \quad (6)$$

Then eq. (4) in this basis takes the form

$$\bar{M}_f = \frac{1}{\sqrt{2}} (v_1 \tilde{Y}^f_1 + v_2 \tilde{Y}^f_2) \quad (7)$$

where $\tilde{Y}^f_i = V^\dagger_{fL} Y^f_i V_{fR}$ and for the quark case we may write

$$\tilde{Y}^d_1 = \frac{\sqrt{2}}{v \cos \beta} \tilde{M}_d - \tan \beta \tilde{Y}^d_2$$

$$\tilde{Y}^u_2 = \frac{\sqrt{2}}{v \sin \beta} \tilde{M}_u - \cot \beta \tilde{Y}^u_1 \quad (8)$$

In the lepton case we perform the usual substitution $d \rightarrow l$.

By using the redefined fields eq. (6), as the physical states, and considering the Yukawas in this basis, eq. (5), we rewrite the THDM-III Yukawa Lagrangian in terms of the mass eigenstates. The interactions of the neutral Higgs bosons ($h^0, H^0, A^0$) with quark pairs acquire the following form:

$$\mathcal{L}^Q_Y = g \left( \frac{m_d}{m_W} \right) \bar{d}_i \left[ \cos \alpha \delta_{ij} + \frac{\sqrt{2}}{g \cos \beta} \sin(\alpha - \beta) \left( \frac{m_W}{m_d} \right) (\tilde{Y}^d_2)_{ij} \right] d_j H^0$$
\[ + \frac{g}{2} \left( \frac{m_{d_i}}{m_W} \right) \bar{d}_i \left[ - \sin \alpha \cos \beta \delta_{ij} + \frac{\sqrt{2}}{g} \frac{\cos(\alpha - \beta)}{\cos \beta} \left( \frac{m_W}{m_{d_i}} \right) \tilde{Y}_{2d}^{d_i} \right] d_j h^0 \]

\[ + \frac{ig}{2} \left( \frac{m_{d_i}}{m_W} \right) \bar{d}_i \left[ - \tan \beta \delta_{ij} + \frac{\sqrt{2}}{g} \sin(\alpha - \beta) \left( \frac{m_W}{m_{d_i}} \right) \tilde{Y}_{1d}^{d_i} \right] \gamma^5 d_j A^0 \]

\[ + \frac{g}{2} \left( \frac{m_{u_i}}{m_W} \right) \bar{u}_i \left[ \sin \alpha \sin \beta \delta_{ij} - \frac{\sqrt{2}}{g} \sin(\alpha - \beta) \left( \frac{m_W}{m_{u_i}} \right) \tilde{Y}_{1u}^{u_i} \right] u_j H^0 \]

\[ + \frac{g}{2} \left( \frac{m_{u_i}}{m_W} \right) \bar{u}_i \left[ - \cos \beta \delta_{ij} + \frac{\sqrt{2}}{g} \sin(\alpha - \beta) \left( \frac{m_W}{m_{u_i}} \right) \tilde{Y}_{1u}^{u_i} \right] u_j h^0 \]

\[ + \frac{ig}{2} \left( \frac{m_{u_i}}{m_W} \right) \bar{u}_i \left[ - \cot \beta \delta_{ij} + \frac{\sqrt{2}}{g} \sin(\alpha - \beta) \left( \frac{m_W}{m_{u_i}} \right) \tilde{Y}_{1u}^{u_i} \right] \gamma^5 u_j A^0, (9) \]

where \( i = 1, 2, 3 \), with \( d_1 = d, d_2 = s, d_3 = b, u_1 = u, u_2 = c, u_3 = t \). The charged leptonic lagrangian is obtained by substituting down quarks \( d_i \) by \( l_i \), where \( l_1 = e, l_2 = \mu, l_3 = \tau \).

So, we observe that eq. (9) includes FCNC couplings at tree level, which are highly restricted by experiment. Then, in order to suppress them we consider that the corresponding Yukawa matrices in eq. (9), have the form of an hermitian 4-zero texture type, with a hierarchy of the form:

\[ Y_f^i = \begin{pmatrix} 0 & C_f^i & 0 \\ C_f^{i*} & B_f^i & B_f^{i*} \\ 0 & B_f^{i*} & A_f^i \end{pmatrix}. \]

As we said earlier, it is assumed that each of the Yukawa matrices has the same form, hermitian 4-texture ansatz, which is inherited to the mass matrix, having

\[ M_f = \begin{pmatrix} 0 & C_f & 0 \\ C_f^{*} & B_f & B_f^{*} \\ 0 & B_f^{*} & A_f \end{pmatrix}. \]

Thus, from eq. (4) we see that the three matrices have the same hierarchy and can be parameterized in the same manner.

Accordingly, the \( V_{L,R} \) matrices are constructed as the product of two matrices, one of which contains the complex phases\(^1\). Furthermore, as is

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\(^1\)The complete process can be found in Ref. [8] and the explicit form of these matrices is given in Refs. [11] and [13].
given in [13] we impose the condition \( m_{f_1} \ll m_{f_2}, m_{f_3} \mid A^f \mid (f = u, d, l) \), with \( A^f = m_{f_3} - \beta^f m_{f_2} \) and \( \beta^f \) is a number within the interval \([0, 1]\).
Therefore, the couplings \( \tilde{Y}^d, l \) and \( \tilde{Y}^u \), that appear in eq. (9) acquire a simple structure given by:

\[
(\tilde{Y}^d, l)_{ij} = \frac{\sqrt{m_i^d, l}}{v} \tilde{\chi}^d, l_{ij} \\
(\tilde{Y}^u)_{ij} = \frac{\sqrt{m_i^u m_j}}{v} \tilde{\chi}^u_{ij}
\] (10)

Then we keep most of the FCNC processes under control provided that \( |\tilde{\chi}^f_{ij}| \leq O(10^{-1})\).

In order to carry out the phenomenological study, we rewrite the lagrangian (3) in terms of the parameter of the model (\( \tilde{\chi}^f_{ij} \)). We display here only the leptonic part of the Yukawa lagrangian:

\[
\mathcal{L}_Y^l = \frac{g_i}{2} \left[ \left( \frac{m_i}{m_W} \right) \cos \alpha \cos \beta \delta_{ij} + \sin(\alpha - \beta) \left( \frac{\sqrt{m_i m_j}}{m_W} \right) \tilde{\chi}^l_{ij} \right] l_j H^0 \\
+ \frac{g_i}{2} \left[ - \left( \frac{m_i}{m_W} \right) \sin \alpha \cos \beta \delta_{ij} + \cos(\alpha - \beta) \left( \frac{\sqrt{m_i m_j}}{m_W} \right) \tilde{\chi}^l_{ij} \right] l_j h^0 \\
+ \frac{ig_i}{2} \left[ - \left( \frac{m_i}{m_W} \right) \tan \beta \delta_{ij} + \frac{1}{\sqrt{2} \cos \beta} \left( \frac{\sqrt{m_i m_j}}{m_W} \right) \tilde{\chi}^l_{ij} \right] \gamma^5 l_j A^0.
\] (11)

We consider that the model parameter is complex in general, \( \tilde{\chi}^f_{ij} = \chi^f_{ij} \exp(i \vartheta_{ij}) \); with the real part \( \chi^f_{ij} = |\tilde{\chi}^f_{ij}| \) and the effect of the phase \( \vartheta_{ij} \) would be included as a variation of \(-1\) to \(1\) on \( \chi^f_{ij} \).

Having obtained the couplings in terms of the model parameter, we are ready to calculate the Higgs branching ratios.

3 Branching Ratios of the Neutral Higgs Bosons

Within the SM, we do not have flavor violation decays at tree level. The SM width for the decay of the Higgs boson to fermions at tree level is given
\[ \Gamma(\phi^0 \to ff) = \frac{N_c}{8\pi} \left( \frac{g m_f}{2m_W} \right)^2 \beta^2 m_{\phi^0}, \]

where \( N_c \) is 1 (3) for leptons (quarks). The term in parentheses stems from the Feynman vertex in the amplitude matrix, while the kinematic term is given as \( \beta^2 = 1 - 4m_f^2/m_{\phi^0}^2 \). For the SM Higgs \( \eta = 3 \), and when we consider the THDM this value also holds for \( \phi^0 = h^0, H^0 \), whereas \( \eta = 1 \) for \( \phi^0 = A^0 \).

In the case of the THDM-III the decay width for the case of different fermions includes modified kinematic factor which involves the three particle masses and is given as follows:

\[ \Gamma(\phi^0 \to \bar{f}_i f_j) = \frac{N_c}{8m_{\phi^0}^2} \left( \frac{g}{2m_W} \right)^2 \xi_{ij}^2 \left[ m_{\phi^0}^2 - (m_i + (-)^n m_j)^2 \right] \]
\[ \times \left[ (m_i^2 - m_j^2 - m_{\phi^0}^2)^2 - 4m_j^2 m_{\phi^0}^2 \right]^{1/2} \tag{12} \]

where \( n \) is even for \( h^0, H^0 \) and odd for \( A^0 \). Thus, we notice that, for the pseudoscalar \( A^0 \), the kinematic term in equation (12) involves a minus sign. The Yukawa coupling \( \xi_{ij} \), only affects the Higgs-fermion processes as shown in the Table 1.

In order to evaluate the Higgs branching ratios, we need to include the dominant decay modes, in addition to the fermionic ones. In the next subsection we display these expressions for each of the Higgs bosons, as taken from [5], [16], [17].

### 3.1 Decays of the light Higgs boson, \((h^0)\)

For \( h^0 \) we shall include the modes \( h \to WW^*, ZZ^* \) for \( m_h < 2m_{W,Z} \) and \( h \to WW, ZZ \) when kinematically allowed. The decay width into a real \( W \) and virtual \( W^* \) boson is given by

\[ \Gamma(h^0 \to WW^*) = \frac{g^4 m_{h^0}}{512\pi^3} \sin^2(\alpha - \beta) F(m_W/m_{h^0}). \tag{13} \]
The factor 4 appears because we consider that $W^* \to tb$ is allowed for $m_h > m_t + m_b + m_W$.

While the decay width $h \to ZZ^*$ is given by

$$\Gamma(h^0 \to ZZ^*) = \frac{g^4 m_{h^0}^5}{2048 \pi^3} \sin^2(\alpha - \beta) \left[ \frac{7 - \frac{40}{3} \sin^2 \theta_W + \frac{100}{9} \sin^4 \theta_W}{\cos^4 \theta_W} \right] \times F(m_Z/m_{h^0})$$

(14)

where

$$F(x) = -\left(1 - x^2\right)\left(\frac{47}{2} x^2 - \frac{13}{2} + \frac{1}{x^2}\right) - 3(1 - 6x^2 + 4x^4) \ln(x)$$

$$+ 3 \left[ -8x^2 + 20x^4 \right] \sqrt{1 - 4x^2} \cos^{-1}\left(\frac{3x^2 - 1}{2x^3}\right)$$

Once we reach the $WW$ or $ZZ$ thresholds, we need to consider the decay widths to the pairs of real vector bosons:

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Table 1: The fermionic vertex of the neutral Higgs bosons to pair of fermions, $\xi_{ij}$, for different models. $i,j$ stand for flavors.
\[ \Gamma(h^0 \rightarrow WW, ZZ) = \frac{g^2 m_{h^0}^3}{k_h 64\pi m_W^2} \sin^2(\beta - \alpha) \sqrt{1 - x_{(W,Z)}} \]
\[ \times [1 - x_{(W,Z)} + \frac{3}{4} x_{(W,Z)}^2] \tag{15} \]

The factor \( k_h = 1 \) for \( h^0 \rightarrow WW \) and \( k_h = 2 \) for \( h^0 \rightarrow ZZ \); with \( x_{(W,Z)} = \frac{4 m_W^2}{m_{H^0}^2} \).

We also take into account the decay mode into gluon pair, which has a decay width:

\[ \Gamma(h^0 \rightarrow gg) = \frac{\alpha_s^2 g^2 m_{h^0}^3}{128\pi^3 m_W^2 \sin^2 \beta} \sum_q \tau_q [1 + (1 - \tau_q) f(\tau_q)]^2 \tag{16} \]

where the sum is over all the quarks, \( \tau_q = \frac{4 m_q^2}{m_{H^0}^2} \), and

\[ f(\tau_q) = \begin{cases} 
\sin^{-1}(\sqrt{1/\tau_q})^2 & \text{if } \tau_q \geq 1 \\
\frac{1}{4}[\ln(\eta_+/\eta_-) - i\pi]^2 & \text{if } \tau_q < 1
\end{cases} \]

with \( \eta_{\pm} \equiv (1 \pm \sqrt{1 - \tau_q}) \). We include only top-contribution in the sum because it is the dominant one, since the other quarks have much smaller masses.

### 3.2 Decays of the heavy \( CP \)-even neutral Higgs boson, \( (H^0) \)

In this case, the corresponding widths for the decay into a pair of vector bosons are given by:

\[ \Gamma(H^0 \rightarrow WW, ZZ) = \frac{g^2 m_{H^0}^3}{k_H 64\pi m_W^2} \cos^2(\beta - \alpha) \sqrt{1 - x_{(W,Z)}} \]
\[ \times [1 - x_{(W,Z)} + \frac{3}{4} x_{(W,Z)}^2] \tag{17} \]

Again, factor \( k_H = 1 \) for \( H^0 \rightarrow WW \) and \( k_H = 2 \) for \( H^0 \rightarrow ZZ \). Here also, \( x_{(W,Z)} = \frac{4 m_W^2}{m_{H^0}^2} \). In fact, we can write some of the \( H^0 \) widths using the equations given in section (3.1), considering the respective mass values.
for the light and heavy neutral Higgs bosons. Namely, for $H \to WW^*, ZZ^*$, we have:

$$\Gamma(H^0 \to WW^*, ZZ^*) = \Gamma(h^0 \to WW^*, ZZ^*) \cot^2(\alpha - \beta).$$

(18)

The expression for the gluonic decay also has the same form as for $h^0$, but changing the masses and the dependence on the angles $\alpha - \beta$ of the top-Higgs vertex, namely:

$$\Gamma(H^0 \to gg) = \Gamma(h^0 \to gg) \tan^2 \alpha.$$ (19)

Now, in this case we also have the possibility of the Higgs decay into $H^0 \to h^0h^0, A^0A^0$, which have a width given by:

$$\Gamma(H^0 \to hh) = \frac{g^2 m_Z^2 f_h^2}{128 \pi m_{H^0} \cos^2 \theta_W} (1 - \frac{4m_h^2}{m_{H^0}^2})^{1/2}$$

(20)

where $h = h^0$ or $h = A^0$ and in each case $f_h$ consists of the following mixing angle factors

$$f_h = \begin{cases} 
\cos 2\alpha \cos(\beta + \alpha) - 2 \sin 2\alpha \sin(\beta + \alpha), & h = h^0 \\
\cos 2\alpha \cos(\beta + \alpha), & h = A^0 
\end{cases}$$

3.3 Decays of the CP − odd Higgs boson, ($A^0$)

In the case of the CP − odd Higgs boson, the changes are more evident. For the decay into gluon pairs we have:

$$\Gamma(A^0 \to gg) = \frac{\alpha_s^2 g^2 m_A^3 f_0}{128 \pi m_{A^0} \cos^2 \theta_W} \cot^2 \beta \left| \sum_i \tau_i f(\tau_i) \right|^2$$

(21)

and we also have the possibility of $A^0 \to Zh^0$. So, we need to include this mode:

$$\Gamma(A^0 \to Zh^0) = \frac{g^2 \lambda^{1/2} \cos^2(\beta - \alpha)}{64 \pi m_A^3 \cos^2 \theta_W} \left[ m_Z^2 - 2(m_{A^0}^2 + m_{h^0}^2) + \frac{(m_{A^0}^2 - m_{h^0}^2)^2}{m_Z^2} \right]$$

(22)

with $\lambda^{1/2} \equiv [(m_1^2 + m_2^2 - m_3^2) - 4m_A^2m_{A^0}^2]^{1/2}$. 

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We also notice that there are no tree level couplings of $A^0$ to vector boson pairs, as a consequence of assuming $CP$-conservation in the Higgs sector.

4 Results in the THDM-III

Although, we are working within a toy model we may get some hints for its possible application or connection to a more fundamental theory, by performing phenomenological analysis of Higgs decays. More specifically, we use this model to evaluate the branching ratios for the three neutral Higgs bosons.

In this THDM, the angles $\alpha$ (the mixing angle in the $CP$-even Higgs sector), and $\beta$ (which is the ratio of the vacuum expectation value of the two doublets), are free parameters. Unlike the case of the MSSM, where one can fix $\alpha$ in terms of $\tan \beta$ and $m_{A^0}$. However, we consider three different scenarios that depend on how these angles are related to each other:

| scenarios | $\beta - \alpha$ | Details |
|-----------|------------------|---------|
| 1         | $\pi/2$          | we obtain for $h^0$ the SM-like decays. |
| 2         | $0$              | we obtain for $H^0$ no flavor violation. |
| 3         | $\pi/3$          | we take an angle in between these extreme cases. |

Observing the form of the coefficient, $\xi_{ij}$ for these scenarios we found that for scenario 1, $\xi_{ij}^{h^0}$ becomes SM-like and $|\xi_{ij}^{H^0}|^2 = |\xi_{ij}^{A^0}|^2$. Whereas, for scenario 2, $\xi_{ij}^{H^0}$ becomes SM-like and $|\xi_{ij}^{h^0}|^2 = |\xi_{ij}^{A^0}|^2$, as we can see from Table 2.

4.1 Light Higgs Boson ($h^0$)

In order to constrain the value of the model parameter, $\chi_{ij}$, we consider the contributions that the model may induce for the specific decay $h^0 \rightarrow b\bar{b}$, see Table 3. We let the parameter $\chi_{ij}$ vary from $-1$ to $1$. We notice that for $\chi = -0.2$, the corrections to $h^0 \rightarrow b\bar{b}$ are minimal. One can see that the largest difference from MSSM-like couplings, i.e. $\chi_{ij} = 0$, occurs at $\chi = 1$.

As we mentioned, for the first scenario, where $\alpha - \beta = -\pi/2$, there is no flavor violation, so, by taking $\tan \beta = 1.5$, we see in Fig. 11 that the SM
\[ \beta - \alpha = \pi/2 \]

\[ \beta - \alpha = 0 \]

| \( \xi_{h_{u;ij}} \) | \( m_u \delta_{ij} \) | \( m_u \cot \beta \delta_{ij} - \frac{\sqrt{m_u m_u}}{\sqrt{2 \sin \beta}} \chi_{ij} \) |
| \( \xi_{d_{d;ij}} \) | \( m_d \delta_{ij} \) | \( m_d \tan \beta \delta_{ij} + \frac{\sqrt{m_d m_d}}{\sqrt{2 \cos \beta}} \chi_{ij} \) |
| \( \xi_{H_{u;ij}} \) | \( -m_u \cot \beta \delta_{ij} + \frac{\sqrt{m_u m_u}}{\sqrt{2 \sin \beta}} \chi_{ij} \) | \( m_u \delta_{ij} \) |
| \( \xi_{d_{d;ij}} \) | \( m_d \tan \beta \delta_{ij} - \frac{\sqrt{m_d m_d}}{\sqrt{2 \cos \beta}} \chi_{ij} \) | \( m_d \delta_{ij} \) |

Table 2: Explicit form of the coefficients \( \xi_{ij} \) for the two extreme scenarios, 1 and 2.

| \( \tan \beta \) | \( \chi_{ij} = 1.0 \) | \( \chi_{ij} = 0.5 \) | \( \chi_{ij} = 0.0 \) | \( \chi_{ij} = -0.2 \) | \( \chi_{ij} = -0.5 \) | \( \chi_{ij} = -1.0 \) |
| \( \tan \beta = 5 \) | 0.63958 | 0.88958 | 0.91325 | 0.91404 | 0.91294 | 0.90933 |
| \( \tan \beta = 10 \) | 0.69383 | 0.90235 | 0.91848 | 0.91815 | 0.91607 | 0.91165 |
| \( \tan \beta = 15 \) | 0.70424 | 0.90474 | 0.91944 | 0.91891 | 0.91607 | 0.91207 |
| \( \tan \beta = 20 \) | 0.70793 | 0.90558 | 0.91978 | 0.91917 | 0.91684 | 0.91221 |
| \( \tan \beta = 30 \) | 0.71059 | 0.90618 | 0.92002 | 0.91936 | 0.91698 | 0.91231 |
| \( \tan \beta = 50 \) | 0.71256 | 0.90649 | 0.92014 | 0.91945 | 0.91705 | 0.91237 |

Table 3: Branching ratios for \( h^0 \) to a pair of b-quarks for different values of the parameter \( \chi_{ij} \) and \( \tan \beta \). Observe that we obtain the MSSM decay in the value of \( \chi_{ij} = 0 \).

Branching ratios are reproduced \[18, 19\].

In Figs. 2 and 3 the dependence of the branching ratios on the light Higgs mass \( h^0 \) are shown, for \( \alpha = \beta \) and \( \alpha - \beta = -\pi/3 \); taking \( \chi_{ij} = -0.2 \), for \( \tan \beta = 5 \) and \( \tan \beta = 20 \). These representative values are taken because, the former case, is a small value \( i.e. 1 \leq \tan \beta \leq 10 \), here the dependence of the branching ratios vary more strongly. In the later case: large \( \tan \beta \), the value is chosen where the behavior of the branching ratios becomes, in general, quite independent of \( \tan \beta \), for different channels of the light Higgs boson.
Figure 1: Branching ratios for $h^0$ to pair of fermions in the scenario 1, taking $\tan \beta = 1.5$.

In order to achieve maximal flavor violation for the decay $h^0 \to sb$, the parameter space must corresponds to $\chi_{ij} \to 1$, and in this scenario the dependence on $\tan \beta$ is very mild, even though for small values of $\tan \beta$ the flavor violation is enhanced. If these flavor violation decay is highly restricted by the experiment, the parameter space should be in the region where $\chi_{ij}$ is close to zero, as can be seen in the first graph of figure 4. We are setting the Higgs mass value at 120GeV and considering the best scenario for this decay, $\beta = \alpha$.

A summary of maximal FV modes is shown in Table 4.

| Channel  | $BR(h^0 \to ff')$ | $\tan \beta$ | $m_{h^0}$ | Scenario |
|----------|-------------------|---------------|------------|----------|
| $ct$ - channel | $\sim 10^{-3}$ | 5 | $\sim 220 - 500$ | 2 |
| $sb$ - channel | $\sim 10^{-3}$ | 5 | $\sim 50 - 140$ | 3 |
| $\mu\tau$ - channel | $\sim 10^{-4}$ | 5 | $\sim 50 - 150$ | 3 |

Table 4: Maximal BR for the flavor violating decays of the light Higgs boson, $(h^0)$ for $\chi_{ij} = -0.2$. 
Figure 2: Branching ratios for $h^0$ to pair of fermions, where $\beta = \alpha$ for two values of $\tan \beta = 5, 20$ and $\chi_{ij} = -0.2$.

Figure 3: Branching ratios for $h^0$ to pair of fermions, where $\beta - \alpha = \pi/3$ for two values of $\tan \beta$, 5 and 20; with $\chi_{ij} = -0.2$. 
4.2 Heavy neutral Higgs bosons ($H^0$)

Because for $H^0$, the flavor violation couplings vanish for $\alpha - \beta = 0$, we only show results for scenarios 1 and 3, for $\tan \beta = 5$ and $\tan \beta = 20$, displayed on Figs. 5 and 6 respectively. In this case, the vector boson decay channel is open for almost the entire mass range, in fact it is the main decay mode, leaving the fermionic modes as a minor contribution of the decay rate. However, there is a region where the fermionic decay becomes important, when the top-threshold is reached.

In this case we also explore the parameter space to visualize the dependence of the flavor violation decay $H^0 \rightarrow sb$ on the parameters. In this case, there is a strong dependence, mostly on large values of $\tan \beta$ and a model parameter close to 1: $\tan \beta > 10$ and $\chi_{ij} \rightarrow 1$. On the other hand, the flavor violating decays are reduced in the region of parameter space where $\tan \beta$ is small and $\chi_{ij}$ is close to zero as can be seen in the second graph of figure 4. Here we have fixed the heavy Higgs mass at 300GeV and taking the more favored scenario for the enhanced of these decays, $\beta - \alpha = \pi/2$. A summary of maximal FV modes for $H^0$ is shown in Table 5.
Figure 5: Branching ratios for $H^0$ to pair of fermions, where $\beta - \alpha = \pi/2$ for two values of $\tan \beta = 5, 20$ and $\chi_{ij} = -0.2$.

Table 5: Maximal flavor violating decays for the light Higgs boson, $H^0$ for $\chi_{ij} = -0.2$.

| Channel       | $BR(H^0 \to ff')$ | $\tan \beta$ | $m_{H^0}$    | Scenario |
|---------------|--------------------|---------------|--------------|----------|
| $ct$ - channel| $\sim 10^{-3}$     | 5             | $\sim 220 - 600$ | 1        |
| $sb$ - channel| $\sim 10^{-4}$     | $5, 20$       | $\sim 50 - 650$ | 1        |
| $\mu \tau$ - channel | $\sim 10^{-5}$ | $5, 20$       | $\sim 50 - 1000$ | 1        |

4.3 Heavy neutral CP-odd Higgs boson ($A^0$)

For the pseudoscalar $A^0$, the fact that there is no coupling to gauge bosons makes the flavor violating signal more stable. The fermionic final states are the main decays even for the large mass range, (since we are not in the context of the MSSM, no possible decays into sparticles are taken into account). Another important issue about the pseudoscalar is that its fermion
Figure 6: Branching ratios for $H^0$ to pair of fermions, where $\beta - \alpha = \pi/3$ for two values of $\tan \beta = 5, 20$ and $\chi_{ij} = -0.2$.

couplings do not depend on the mixing angle $\alpha$, making its branching ratio almost independent of the chosen scenario, nevertheless there is a slightly difference coming from the width $\Gamma(A^0 \rightarrow ZH^0)$.

By fixing the two values of $\tan \beta$, as done before, and for the three scenarios: $\beta - \alpha = \pi/2$, $\beta = \alpha$ and $\beta - \alpha = \pi/3$ we obtain what is depicted on Figs. 7, 8 and 9. As inferred from these figures, the signals for flavor violating decays would in general be clearer and higher for $A^0$. In particular, the branching ratio $A^0 \rightarrow ct$ is very high in the mass range $290 < m_{A^0} < 2m_t$, where it becomes the dominant decay mode, reaching a branching ratio larger than about 50%, as shown in Fig. 7.

In order to explore the dependence of the flavor violating decays for $A^0$ on the parameter space, we plot in figure 10 the branching ratio $A^0 \rightarrow sb$ and $A^0 \rightarrow ct$ as function of $\chi_{ij}$ for different values of $\tan \beta$, obtaining. In left plot of Fig. 10 the regions with large $\tan \beta$ and with $\chi_{ij} \sim -1$ or $\chi_{ij} \sim 0$ (with $\chi_{ij} > 0$), have a large branching ratio for $A^0 \rightarrow sb$; while for the decay
Figure 7: Branching ratios for $A^0$ to pair of fermions, where $\beta - \alpha = \pi/2$ for two values of $\tan\beta = 5, 20$ and $\chi_{ij} = -0.2$.

$A^0 \to ct$, as shown on right plot of Fig. 10, the two regions correspond to small $\tan\beta \sim 1$ and $\chi_{ij} \sim \pm 1$. A summary of maximal FV modes for $A^0$ is shown in Table 6.

| Channel                  | $BR(A^0 \to ff')$ | $\tan\beta$ | $m_{A^0}$ (GeV) | Scenario |
|--------------------------|-------------------|--------------|-----------------|----------|
| ct channel               | $\sim 0.5$        | 5            | $\sim 210 - 350$ | 1        |
| sb channel               | $\sim 10^{-1}$    | 20           | $\sim 950$      | 1        |
| $\mu\tau$ channel       | $\sim 0.03$       | 20           | $\sim 650$      | 1        |

Table 6: Maximal flavor violating decays for the pseudoscalar Higgs boson, $A^0$ for $\chi_{ij} = -0.2$.

5 Conclusions

The THDM-III, enable us to study, in a more general way, the effects of the Yukawa couplings. In particular, the flavor violating decays could be re-
Figure 8: Branching ratios for $A^0$ to pair of fermions, where $\beta = \alpha$ for two values of $\tan \beta = 5, 20$ and $\chi_{ij} = -0.2$.

restricted at low energies by the specific form of 4-zero textures. In such case, the Yukawa matrices give the correct mass spectrum and mixing angles to the fermion sector. Using this model, we explore the branching ratios at tree level of the neutral Higgs sector over the whole range of the neutral Higgs masses. The model parameter is restricted in order to keep the flavor conserving decay modes near the MSSM case, and to avoid FCNC at low energies, though having in mind that as long as no experimental evidence is produced the parameter space is only mildly restricted.

For the specific parameter values $\chi (= -0.2)$ and $\tan \beta$, we have obtained maximal flavor changing decays at tree level for the $h^0$ of the order of $10^{-3}$ for $ct$ – channel in scenario 2 and similar results for $sb$ – channel in scenario 3, values of the order of $10^{-4}$ were obtained for $\mu\tau$ – channels in scenario 3, for $\tan \beta = 5$ in all three cases. The values of these branching ratios decays are reduced for $\tan \beta = 20$ down to the order of $10^{-4}$ for the quark channels, end even lower values for lepton violation decay, which are reduce to about $10^{-5}$.
Figure 9: Branching ratios for $A^0$ to fermion pairs, where $\beta - \alpha = \pi/3$ for two values of $\tan \beta = 5, 20$ and $\chi_{ij} = -0.2$.

In the case of $H^0$ we obtained maximal flavor changing decays of the order of $10^{-3}$ for $ct$–channel, $10^{-4}$ for $sb$–channel, and $10^{-5}$ for $\mu\tau$–channel, all cases in scenario 1 and $\tan \beta = 5$.

Finally, for the pseudoscalar $A^0$, we have obtained significant flavor violating decays rates (as large as 50%) for the $A^0 \rightarrow ct$ and as high as 30% for $A^0 \rightarrow sb$, for specific values of the parameter space, particularly in scenario 1. While the former decay is enhanced at $\tan \beta = 5$, the other two flavor violation decays, $sb$ and $\mu\tau$, are larger in the case of $\tan \beta = 20$ and $m_{A^0} \sim 1TeV$.

From this analysis we also conclude that the value for the parameter space $\chi_{ij} = -0.2$, does not lead to large flavor violating decay rates in the cases of the two $CP$–even Higgs bosons; while, in the other hand, increases flavor violating decay rates for the pseudoscalar $A^0$. 
Figure 10: Dependence of the decay $A^0 \to sb, ct$ on $\chi_{ij}$ for different values of $\tan \beta$, where $m_{A^0} = 300\text{GeV}$, in scenario 1.

We have explored the complete parameter space of this model in order to determinate the areas where $h^0, H^0, A^0$ reach maximal branching ratios. Studying these modes at future colliders (LHC) could be important to find new Higgs signals and explore the origin of flavor [20].

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