Hypercharge and baryon minus lepton number in $E_6$

Junpei Harada

Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan
E-mail: harada@theo.phys.sci.hiroshima-u.ac.jp

Abstract: We study assignments of the hypercharge and baryon minus lepton number for particles in the $E_6$ grand unification model. It is shown that there are three assignments of hypercharge and three assignments of baryon minus lepton number which are consistent with the Standard Model. Their explicit expressions and detailed properties are given. In particular, we show that the $U(1)_{B-L}$ symmetry in $E_6$ cannot be orthogonal to the $SU(3)_R$ symmetry. Based on these investigations, we propose an alternative $SU(5)$ grand unification model.

Keywords: GUT, Beyond Standard Model.
1. Introduction

The supersymmetric (SUSY) grand unified theory (GUT) is one of the most attractive scenarios beyond the Standard Model (SM). Among all the possible SUSY GUT models, the minimal SUSY $SU(5)$ model has been most extensively studied in the past. However, recent experimental progress suggests that it is no longer the primary candidate and some modifications are obviously required for several reasons. The first reason is that the lower limit on the proton lifetime measured by the Super-Kamiokande [1] already excludes the minimal SUSY $SU(5)$ model [2]. The second reason is that small neutrino masses suggested by recent neutrino oscillation experiments [5] can be naturally explained by the seesaw mechanism [6], which requires right-handed neutrinos in addition to the minimal SM particles.

Although an extension to some non-minimal SUSY $SU(5)$ GUT models is one possible solution, it is worth investigating GUT models with alternative gauge groups. Extending the gauge group beyond $SU(5)$ is interesting since the theory includes the right-handed neutrinos and we have several ways of embedding the SM gauge group into the extended gauge group. For example, in the $SO(10)$ GUT there are two different assignments of hypercharge, which are related with each other by the $SU(2)_R$ symmetry. This fact leads to two different GUT models in the context of the $SU(5)$ group; the Georgi-Glashow model [7] or the flipped model [8] [9]. Recent studies show that the SUSY flipped $SU(5)$ model overcomes the difficulties in the minimal SUSY $SU(5)$ GUT [10].

The $E_6$ is an unique exceptional Lie group that has complex representations and gives an observed chiral structure at low-energies. The $E_6$ GUT [11] has been known to give, for example, the small Cabbibo-Kobayashi-Maskawa (CKM) mixing angles and bi-large neutrino mixings [12] [13]. Since $E_6$ is larger than $SO(10)$, there may be other different

\[^1\]There is still controversy in this statement. This issue is discussed in detail in refs. [3] [4].
assignments of hypercharge and baryon minus lepton number ($B - L$) in the $E_6$ grand unification. These possibilities lead to new alternative GUT models.

In this paper we thoroughly investigate assignments of the hypercharge and $B - L$ in the $E_6$ unification. We show that in the $E_6$ GUT there are three assignments of hypercharge and three assignments of $B - L$ that reproduce the SM. Their properties are very important for model building. We point out that these three assignments are related with each other by the $SU(2)$ subgroup of $SU(3)_R$, which is the subgroup of $E_6$. We also show that each assignment is orthogonal to the $SU(2)$ subgroup of $SU(3)_R$. In particular, we emphasize that the $U(1)_{B - L}$ symmetry in $E_6$ can not be orthogonal to the $SU(3)_R$ symmetry. This fact strongly restricts the $E_6$ grand unified theories with the gauged $U(1)_{B - L}$ symmetry. In those of 9($3 \times 3$) pairs of charge assignments, we show that 6 pairs are consistent with the SM. These observation indicate that $E_6$ has much potential for constructing alternative GUT models compared to the $SO(10)$ GUT.

We also propose a $E_6$-inspired alternative $SU(5)$ GUT model, “E-twisting flipped model”. The gauge group of this model is $SU(5) \times U(1)_V \times U(1)'_V$, which is the subgroup of $E_6$. Assignments of hypercharge and $B - L$ are different from those of the Georgi-Glashow model or usual flipped model. In this model, since the hypercharge is not a subgroup of $SU(5)$, the quasi-unification of strong, weak and hypercharge gauge couplings is predicted. While Dirac masses of quarks or leptons are naturally obtained through the renormalizable operators in the superpotential, the heavy Majorana masses of right-handed neutrinos are obtained via dimension five operators. Therefore, the conventional seesaw mechanism can work.

This paper is organized as follows. In section 2 we review $SO(10)$ GUT since it will be helpful when discussing $E_6$. Especially, we emphasize that as far as $SO(10) \supset U(1)_{B - L}$ is imposed, only the 16 representation, not 10, can be a candidate for the SM matters in both the Georgi-Glashow model and usual flipped model. In section 3 we investigate the hypercharge and $B - L$ assignments in the $E_6$ unification. We show that there are three assignments of hypercharge or $B - L$. Their explicit expressions and detailed properties are given. In section 4, as an example, we propose a $E_6$-inspired alternative $SU(5)$ GUT model. Section 5 is devoted to the conclusion.

2. Particle assignment in $SO(10)$

A crucial difference between the $SO(10)$ GUT and the $SU(5)$ GUT resides in the fact that the way of embedding the SM gauge group into the unified gauge group is unique or not. In the $SU(5)$ GUT it is unique, while in the $SO(10)$ GUT there are two different assignments of hypercharge. Since different assignments lead to different predictions, this problem is important for low energy physics. In this section we study this issue in detail.

In the minimal $SO(10)$ GUT, all matters in the SM and a right-handed neutrino belong to 16 for each family. Each spinor representation 16 is decomposed under a maximal subgroup $SO(10) \supset SU(5) \times U(1)_V$ as

$$16 = 5^* + 10_{-1} + 1_{-5},$$

\[ (2.1) \]
and furthermore under $SU(5) \supset SU(3)C \times SU(2)L \times U(1)_Z$ as
\[
5^* = (3^*, 1)_{1/3} + (1, 2)_{-1/2},
10 = (3, 2)_{1/6} + (3^*, 1)_{-2/3} + (1, 1)_1,
1 = (1, 1)_0. \tag{2.2}
\]

Notice here that $U(1)_Z$, which is the subgroup of $SU(5)$, is not identical with the $U(1)_Y$ hypercharge at this stage. Clearly, each $16$ includes two pairs of $(3^*, 1)$ or $(1, 1)$. This fact can be understood by considering the decomposition of $16$ under another maximal subgroup $SO(10) \supset SU(4)_PS \times SU(2)_L \times SU(2)_R$,
\[
16 = (4, 2, 1) + (4^*, 1, 2), \tag{2.3}
\]
and under $SU(4)_PS \times SU(2)_L \times SU(2)_R \supset SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$ as
\[
(4, 2, 1) = (3, 2, 1)_{1/3} + (1, 2, 1)_{-1},
(4^*, 1, 2) = (3^*, 1, 2)_{-1/3} + (1, 1, 2)_1. \tag{2.4}
\]

This decomposition shows that two pairs of $(3^*, 1)$ or $(1, 1)$ are $SU(2)_R$ doublets. One can also find that the quantum numbers of $U(1)_Z$ and $U(1)_Y$ are represented by the one of $U(1)_X$ and the third component $I_{3R}$ of $SU(2)_R$ as follows,
\[
V = -4I_{3R} - 3X,
Z = -I_{3R} + \frac{1}{2}X. \tag{2.5}
\]

Note that the $X$-charge can also be identified with $B - L$,
\[
B - L = X = -\frac{1}{5}(V - 4Z). \tag{2.6}
\]

Therefore, the $U(1)_{B-L}$ symmetry in $SO(10)$ is orthogonal to the $SU(2)_R$ symmetry.

From above equations, one can find that $SU(5)$ multiplets $5^*, 10$ and $1$ are formed as
\[
5^*_3 = (3^*, 1, I_{3R} = -1/2)_{-1/3} + (1, 2, 1)_{-1},
10_{-1} = (3, 2, 1)_{1/3} + (3^*, 1, I_{3R} = 1/2)_{-1/3} + (1, 1, I_{3R} = -1/2)_{1},
1_{-5} = (1, 1, I_{3R} = 1/2)_{1}. \tag{2.7}
\]

These arguments indicate that the way of embedding $SU(5)$ as $SO(10) \supset SU(5) \supset SU(3)_C \times SU(2)_L$ is not unique [12]. As far as the $U(1)_Y$ hypercharge is not considered, there is freedom of $SU(2)_R$ rotation. It is worth mentioning here that there still remains $SU(2)_R$ freedom of embedding $SU(5)$ as $SO(10) \supset SU(5) \times U(1)_Y \supset SU(3)_C \times SU(2)_L \times U(1)_{B-L}$. This is based on the following relation; $U(1)_{B-L} \perp SU(2)_R$.

Let us consider the hypercharge assignments in the $SO(10)$ GUT. The hypercharge $U(1)_Y$ is not identical with $U(1)_X$ and must be orthogonal to $SU(3)_C$ and $SU(2)_L$. This fact shows that the $U(1)_Y$ hypercharge is not orthogonal to $SU(2)_R$. In other words, the assignment of the hypercharge eliminates the $SU(2)_R$ freedom.
In order to give explicit expressions for the hypercharge, consider the following decomposition $SO(10) \supset SU(5) \times U(1)_V \supset SU(3)_C \times SU(2)_L \times U(1)_Z \times U(1)_V$. Therefore, the hypercharge $U(1)_Y$ must be a linear combination of $U(1)_Z$ and $U(1)_V$; $U(1)_Y \subset U(1)_Z \times U(1)_V$. There are two assignments of hypercharge that reproduce the SM\cite{8},

1. $Y/2 = Z$; Georgi-Glashow model \cite{7}

2. $Y/2 = -\frac{1}{5}(Z + V)$; flipped model \cite{8} [9].

These hypercharge assignments can also be expressed in terms of the third component of $SU(2)_R$ and the quantum number of $U(1)_X$ as

$$Y/2 = Z = -I_{3R} + \frac{1}{2}X,$$

(2.8)

for the Georgi-Glashow model and

$$Y/2 = -\frac{1}{5}(Z + V) = I_{3R} + \frac{1}{2}X,$$

(2.9)

for the flipped model\footnote{These expressions are useful for comparing two different hypercharge assignments. The formula between hypercharge and $B - L$ can be always reduced to a conventional one $Y/2 = I_{3R} + (B - L)/2$ by redefining the third component of $SU(2)_R$.}. These two assignments are different with each other in only the sign of the third component of $SU(2)_R$. This means that the $SU(5)$ group of flipped model is obtained from that of Georgi-Glashow model by the $\pi$ rotation in $SU(2)_R$\cite{12}. Namely the particle assignment of the flipped $SU(5)$ model is given from that of the Georgi-Glashow $SU(5)$ model by the interchange of $SU(2)_R$ doublets (flipping),

$$u^c \leftrightarrow d^c, \quad e^c \leftrightarrow \nu^c.$$

(2.10)

Although the way of embedding $SU(5)$ as $SO(10) \supset SU(5) \times U(1)_V \supset SU(3)_C \times SU(2)_L \times U(1)_Y$ is not unique, there remains no $SU(2)_R$ freedom anymore. Therefore, there are only two possibilities; $SU(5)$ must be $SU(5)_{GG}$ or $SU(5)_{flipped}$.

Finally, we investigate $10$ representations of $SO(10)$. As mentioned above, in the minimal $SO(10)$ GUT all matters in the SM and a right-handed neutrino belong to $16$ for each family. However, since GUT models in which $10$ includes the SM matters are possible, it is worth investigating $10$ of $SO(10)$. Each $10$ representation is decomposed under a maximal subgroup $SO(10) \supset SU(5) \times U(1)_V$ as

$$10 = 5_2 + 5^*_{-2},$$

(2.11)

and under $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Z$ as

$$5 = (3, 1)_{-1/3} + (1, 2)_{1/2},$$

$$5^* = (3^*, 1)_{1/3} + (1, 2)_{-1/2}.$$  

(2.12)
As before, consider the decomposition of $10$ under another maximal subgroup $SO(10) \supset SU(4)^P S \times SU(2)^L \times SU(2)^R$ as

$$10 = (6, 1, 1) + (1, 2, 2), \quad (2.13)$$

and under $SU(4)^P S \times SU(2)^L \times SU(2)^R \supset SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$ as

$$(6, 1, 1) = (3, 1, 1)_{-2/3} + (3^*, 1, 1)_{2/3},$$

$$(1, 2, 2) = (1, 2, 2)_0. \quad (2.14)$$

From these equations, one can see that $SU(5)$ multiplets $5$ and $5^*$ are formed as

$$5_2 = (3, 1, 1)_{-2/3} + (1, 2, I_{3R} = -1/2)_0,$$

$$5^*_{-2} = (3^*, 1, 1)_{2/3} + (1, 2, I_{3R} = 1/2)_0. \quad (2.15)$$

This indicates that if $5^*$ of $10$ is regarded as the SM matters, $U(1)_X$ cannot be identified with $U(1)_{B-L}$. We emphasize that as far as $SO(10) \supset U(1)_{B-L}$ is imposed, $10$ of $SO(10)$ must not be a candidate of the SM matters for both the Georgi-Glashow model and the flipped model. It is also noted here that even if $SO(10) \supset U(1)_{B-L}$ is not imposed, $10$ can not be a candidate of the SM matters in the flipped model.

3. Particle assignment in $E_6$

The $E_6$ GUT is one of the most attractive grand unified models and it is important for model building to investigate all the possible assignments of the hypercharge and $B - L$ in the $E_6$ unification. Although the $E_6$ GUT is more complicated than the $SO(10)$ GUT since the rank of $E_6$ is larger, the argument is essentially the same as for the $SO(10)$ GUT.

In the $E_6$ GUT, all matters in the SM (with a right-handed neutrino) and exotic matters belong to a fundamental representation $27$ for each family. Each $27$ representation is decomposed under a maximal subgroup $E_6 \supset SO(10) \times U(1)_V$, as follows

$$27 = 16_1 + 10_{-2} + 1_4. \quad (3.1)$$

These $SO(10)$ multiplets $16, 10$ and $1$ are decomposed under $SO(10) \supset SU(5) \times U(1)_V$ as

$$16 = 10_{-1} + 5^*_3 + 1_{-5},$$

$$10 = 5_2 + 5^*_{-2},$$

$$1 = 1_0. \quad (3.2)$$

The decomposition of $SU(5)$ multiplets under $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Z$ is given in the previous section. One sees that each $27$ includes three pairs of $(3^*, 1), (1, 2)$ or $(1, 1)$, where the numbers in parentheses are the dimensions of $SU(3)_C$ and $SU(2)_L$, respectively.

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3Here we mean “flipped model” as the model that is composed of $SU(5)$ multiplets $5^* = (u^c, e, \nu), 10 = (d^c, u, d, \nu^c)$ and $1 = e^c$. 

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3. Particle assignment in $E_6$
This can be understood by considering the decomposition of 27 under a maximal subgroup $E_6 \supset SU(3)_C \times SU(3)_L \times SU(3)_R$ as

$$27 = (3, 3, 1) + (3^*, 1, 3^*) + (1, 3^*, 3), \quad (3.3)$$

and under $SU(3)_C \times SU(3)_L \times SU(3)_R \supset SU(3)_C \times SU(2)_L \times SU(3)_R \times U(1)_{Y_L}$ as

$$(3, 3, 1) = (3, 2, 1)_{1/2} + (3, 1, 1)_{-1},$$

$$(3^*, 1, 3^*) = (3^*, 1, 3^*)_0,$$

$$(1, 3^*, 3) = (1, 2, 3)_{-1/2} + (1, 1, 3)_1. \quad (3.4)$$

These equations show that three pairs of $(3^*, 1), (1, 2)$ or $(1, 1)$ are $SU(3)_R$ triplets.

For later discussions, consider further decompositions under $SU(3)_C \times SU(3)_L \times SU(3)_R \supset SU(3)_C \times SU(2)_L \times SU(2)_{(R)} \times U(1)_{Y_L} \times U(1)_{Y_{(R)}}$ (the meaning of parentheses for "R" will be clarified soon) as

$$(3, 3, 1) = (3, 2, 1)_{1/2, 0} + (3, 1, 1)_{-1, 0},$$

$$(3^*, 1, 3^*) = (3^*, 1, 2)_{0, -1/2} + (3^*, 1, 1)_{0, 1},$$

$$(1, 3^*, 3) = (1, 2, 2)_{-1/2, 1/2} + (1, 1, 2)_{-1/2, -1} + (1, 1, 2)_{1/2} + (1, 1, 1)_{1, -1}, \quad (3.5)$$

where the numbers in parentheses are the dimensions of $SU(3)_C, SU(2)_L$ and $SU(2)_{(R)}$ respectively, and subscripts represent the quantum numbers of $U(1)_{Y_L}$ and $U(1)_{Y_{(R)}}$ respectively. Notice here that there are three $SU(2)$ subgroups of $SU(3)_R$. We define three $SU(2)$ subgroups of $SU(3)_R$ as follows $^4$: $(16, 10, 3^*, 1)$ and $(16, 5^*, 3^*, 1)$ are $SU(2)_R$ doublets, $(16, 10, 3^*, 1)$ and $(10, 5^*, 3^*, 1)$ are $SU(2)'_R$ doublets $^{[5]}$, $(16, 5^*, 3^*, 1)$ and $(10, 5^*, 3^*, 1)$ are $SU(2)_E$ doublets $^{[2]}$, where the numbers in parentheses are the dimensions of $SO(10), SU(5), SU(3)_C$ and $SU(2)_L$ respectively. The meaning of parentheses for "R" is clear now, namely, $SU(2)_{(R)}$ means $SU(2)_R, SU(2)'_R$ or $SU(2)_E$.

The quantum numbers of $U(1)_{V'}$, $U(1)_V$ and $U(1)_Z$ can be represented by those of $U(1)_{Y_L}, U(1)_{Y_{(R)}}$ and the third component $I_{3(R)}$ of $SU(2)_{(R)}$ as

$$V' = 2Y_L - 2Y_R,$$

$$V = -2Y_L - 4I_{3R} - 2Y_R,$$

$$Z = \frac{1}{3}Y_L - I_{3R} + \frac{1}{3}Y_R, \quad (3.6)$$

or

$$V' = 2Y_L + 3I'_{3R} + Y'_R,$$

$$V = -2Y_L + I'_{3R} + 3Y'_R,$$

$$Z = \frac{1}{3}Y_L - I'_{3R} + \frac{1}{3}Y'_R, \quad (3.7)$$

Note that the definition of $SU(2)$ subgroups of $SU(3)_R$ is not unique. We point out that there is another useful definition and it will be discussed later.
or

\[ V' = 2Y_L + 3I_{3E} + Y_E, \]
\[ V = -2Y_L + 5I_{3E} - Y_E, \]
\[ Z = \frac{1}{3}Y_L - \frac{2}{3}Y_E. \quad (3.8) \]

From these expressions, the SU(5) multiplets, which belong to a single 27 of \( E_6 \), are formed under \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{Y_L} \times U(1)_{Y_R} \) as

\[
(16,5^*) = (3^*,1,2(I_{3R} = -1/2))_{0,-1/2} + (1,2,1)_{-1/2,-1},
\]
\[
(16,10) = (3,2,1)_{1/2,0} + (3^*,1,2(I_{3R} = 1/2))_{0,-1/2} + (1,1,2(I_{3R} = -1/2))_{1,1/2},
\]
\[
(16,1) = (1,1,2(I_{3R} = 1/2))_{1,1/2},
\]
\[
(10,5) = (3,1,1)_{-1,0} + (1,2,2(I_{3R} = -1/2))_{-1,2,1/2},
\]
\[
(10,5^*) = (3^*,1,1)_{0,1} + (1,2,2(I_{3R} = 1/2))_{-1,2,1/2},
\]
\[
(1,1) = (1,1,1)_{-1}. \quad (3.9)
\]

and then under \( SU(3)_C \times SU(2)_L \times SU(2)'_R \times U(1)_{Y_L} \times U(1)'_{Y_R} \) as

\[
(16,5^*) = (3^*,1,1)_{0,1} + (1,2,2(I'_{3R} = 1/2))_{-1,2,1/2},
\]
\[
(16,10) = (3,2,1)_{1/2,0} + (3^*,1,2(I'_{3R} = 1/2))_{0,-1/2} + (1,1,2(I'_{3R} = -1/2))_{1,1/2},
\]
\[
(16,1) = (1,1,1)_{1,-1},
\]
\[
(10,5) = (3,1,1)_{-1,0} + (1,2,2(I'_{3R} = -1/2))_{-1,2,1/2},
\]
\[
(10,5^*) = (3^*,1,2(I'_{3R} = -1/2))_{0,-1/2} + (1,2,1)_{-1,2,1/2},
\]
\[
(1,1) = (1,1,2(I'_{3R} = 1/2))_{1,1/2}. \quad (3.10)
\]

and furthermore under \( SU(3)_C \times SU(2)_L \times SU(2)'_E \times U(1)_{Y_L} \times U(1)_{Y_E} \) as

\[
(16,5^*) = (3^*,1,2(I_{3E} = 1/2))_{0,-1/2} + (1,2,2(I_{3E} = 1/2))_{-1,2,1/2},
\]
\[
(16,10) = (3,2,1)_{1/2,0} + (3^*,1,1)_{0,1} + (1,1,1)_{1,-1},
\]
\[
(16,1) = (1,1,2(I_{3E} = -1/2))_{1,1/2},
\]
\[
(10,5) = (3,1,1)_{-1,0} + (1,2,1)_{-1,2,1/2},
\]
\[
(10,5^*) = (3^*,1,2(I_{3E} = -1/2))_{0,-1/2} + (1,2,2(I_{3E} = -1/2))_{-1,2,1/2},
\]
\[
(1,1) = (1,1,2(I_{3E} = 1/2))_{1,1/2}. \quad (3.11)
\]

As far as \( SU(3)_C \times SU(2)_L \) is concerned, there is \( SU(3)_R \) freedom. The relations among three \( SU(2) \) subgroups of \( SU(3)_R \) are given above.

Now we consider the hypercharge assignments in the \( E_6 \) unification. The hypercharge \( U(1)_Y \) must be a linear combination of \( U(1)_Z \), \( U(1)_V \) and \( U(1)_{V'} \); \( U(1)_Y \subset U(1)_Z \times U(1)_V \times U(1)_{V'} \). There are three assignments of hypercharge that are consistent with the SM \([4]\),

1. \( \frac{Y}{2} = Z \),

\[ \]
2. $\frac{Y}{2} = -\frac{1}{5}(Z + V)$,

3. $\frac{Y}{2} = -\frac{1}{20}(4Z - V - 5V')$.

The $U(1)_Y$ hypercharge for the first assignment or the second one is a subgroup of $SO(10)$, while the last $U(1)_Y$ is a subgroup of $E_6$. Their properties are very important for model building. Therefore, we express three assignments of hypercharge in terms of the quantum numbers of $U(1)_Y$ and the third component of $SU(2)_{(R)}$ as

$$\frac{Y}{2} = Z,$$
$$= \frac{1}{3}Y_L - I_{3R} + \frac{1}{3}Y_R,$$
$$= \frac{1}{3}Y_L - I'_{3R} + \frac{1}{3}Y'_R,$$
$$= \frac{1}{3}Y_L - \frac{2}{3}Y_E,$$ (3.12)

or

$$\frac{Y}{2} = -\frac{1}{5}(Z + V),$$
$$= \frac{1}{3}Y_L + I_{3R} + \frac{1}{3}Y_R,$$
$$= \frac{1}{3}Y_L - \frac{2}{3}Y'_R,$$
$$= \frac{1}{3}Y_L - I_{3E} + \frac{1}{3}Y_E,$$ (3.13)

or

$$\frac{Y}{2} = -\frac{1}{20}(4Z - V - 5V'),$$
$$= \frac{1}{3}Y_L - \frac{2}{3}Y_R,$$
$$= \frac{1}{3}Y_L + I'_{3R} + \frac{1}{3}Y'_R,$$
$$= \frac{1}{3}Y_L + I_{3E} + \frac{1}{3}Y_E.$$ (3.14)

These expressions show the properties of each assignment and the relations among them. The first hypercharge assignment $U(1)_Y$ is not orthogonal to $SU(2)_R$ or $SU(2)'_R$, but is $SU(2)_E$; $U(1)_Y \not\perp SU(2)_R$ or $SU(2)'_R$, but $U(1)_Y \perp SU(2)_E$. The second assignment is $U(1)_Y \not\perp SU(2)_R$ or $SU(2)_E$, but $U(1)_Y \perp SU(2)'_R$. The third assignment is $U(1)_Y \not\perp SU(2)'_R$ or $SU(2)_E$, but $U(1)_Y \perp SU(2)_R$. Thus, we conclude that in the $E_6$ GUT any assignments of hypercharge are orthogonal to the $SU(2)$ subgroup of $SU(3)_R$. One can also find the relations among three hypercharge assignments. Since differences of three hypercharge assignments are only in the sign of the third component of $SU(2)_{(R)}$, three assignments are related with each other by the $\pi$ rotation of $SU(2)_{(R)}$. Notice here that although the hypercharge assignment eliminates $SU(3)_R$ freedom, $SU(2)_{(R)}$ freedom still remain at this stage. This is different from the $SO(10)$ GUT case.
Now we come to the quantum number $B - L$ in the $E_6$ unification. In this paper we concentrate on the case in which the $U(1)_{B-L}$ symmetry is included in $E_6$ as a subgroup. First, consider the following subgroup $E_6 \supset SU(3)_C \times SU(3)_L \times SU(3)_R \supset SU(3)_C \times SU(2)_L \times U(1)_{Y_L} \times SU(3)_R$. From Eq. (3.4), $U(1)_{B-L}$ must not be identical with $U(1)_{Y_L}$. On the other hand, $U(1)_{B-L}$ must be orthogonal to $SU(3)_C$ and $SU(2)_L$. From these facts, we conclude that the $U(1)_{B-L}$ symmetry in $E_6$ must not be orthogonal to the $SU(3)_R$ symmetry;

$$U(1)_{B-L} \not\perp SU(3)_R.$$  \hspace{1cm} (3.15)

This relation is valid as far as $E_6 \supset U(1)_{B-L}$ is imposed. Therefore, the remaining $SU(2)_{(R)}$ freedom disappears. This strongly restricts $E_6$ grand unified models with the gauged $U(1)_{B-L}$ symmetry.

If $E_6 \supset U(1)_{B-L}$ is imposed, $U(1)_{B-L}$ must be a linear combination of $U(1)_Z$, $U(1)_V$ and $U(1)_{V'}$; $U(1)_{B-L} \subset U(1)_Z \times U(1)_V \times U(1)_{V'}$. There are three assignments of $B - L$ that reproduce the SM;

1. $B - L = -\frac{1}{5}(V - 4Z)$,

2. $B - L = \frac{1}{20}(16Z + V + 5V')$,

3. $B - L = -\frac{1}{20}(8Z + 3V - 5V')$.

The properties of these $B - L$ assignments are understood by expressing it in terms of the charges of $U(1)_{Y_L}$, $U(1)_{Y_{(R)}}$ and the third component of $SU(2)_{(R)}$ as

$$B - L = -\frac{1}{5}(V - 4Z),$$

$$= \frac{2}{3} Y_L + \frac{2}{3} Y_R,$$

$$= \frac{2}{3} Y_L - I_{3R} - \frac{1}{3} Y_{3R'},$$

$$= \frac{2}{3} Y_L - I_{3E} - \frac{1}{3} Y_E,$$  \hspace{1cm} (3.16)

or

$$B - L = \frac{1}{20}(16Z + V + 5V'),$$

$$= \frac{2}{3} Y_L - I_{3R} - \frac{1}{3} Y_R,$$

$$= \frac{2}{3} Y_L + \frac{2}{3} Y_{3R'},$$

$$= \frac{2}{3} Y_L + I_{3E} - \frac{1}{3} Y_E,$$  \hspace{1cm} (3.17)

or

$$B - L = -\frac{1}{20}(8Z + 3V - 5V').$$
\[
\begin{align*}
&= \frac{2}{3} Y_L + I_{3R} - \frac{1}{3} Y_R, \\
&= \frac{2}{3} Y_L + I'_{3R} - \frac{1}{3} Y'_R, \\
&= \frac{2}{3} Y_L + \frac{2}{3} Y_E.
\end{align*}
\] (3.18)

One can easily see that three \( B - L \) assignments are orthogonal to the \( SU(2) \) subgroup of \( SU(3)_R \) and relations among three assignments are similar to the case of the hypercharge, namely, they are related with each other by the \( \pi \) rotation of \( SU(2)_{(R)} \).

Since there are three assignments of hypercharge and three assignments of \( B - L \), \( 9(3 \times 3) \) pairs of charge assignment exist. In those of 9 pairs, 6 pairs are consistent with the SM. Since the case in which the hypercharge and \( B - L \) are orthogonal to the same \( SU(2) \) subgroup of \( SU(3)_R \) is not consistent with the SM, 3 pairs of \( U(1)_Y \) and \( U(1)_{B-L} \perp SU(2)_{(R)} \) are removed.

We summarize the 6 pairs of charge assignment which are consistent with the SM:

1. \( U(1)_Y \perp SU(2)_E \) and \( U(1)_{B-L} \perp SU(2)_R \),
2. \( U(1)_Y \perp SU(2)_E \) and \( U(1)_{B-L} \perp SU(2)'_R \),
3. \( U(1)_Y \perp SU(2)'_R \) and \( U(1)_{B-L} \perp SU(2)_R \),
4. \( U(1)_Y \perp SU(2)_R \) and \( U(1)_{B-L} \perp SU(2)'_R \),
5. \( U(1)_Y \perp SU(2)_R \) and \( U(1)_{B-L} \perp SU(2)_E \),
6. \( U(1)_Y \perp SU(2)'_R \) and \( U(1)_{B-L} \perp SU(2)_E \).

We express particle assignments for each case in terms of \( SO(10) \). The quantum numbers of each field are given in the table 1.

The first assignment is the most familiar one:

\[
\begin{align*}
16 &= (d^c + e + \nu) + (u^c + u + d + e^c) + \nu^c, \\
10 &= (D + E^c + N^c) + (D^c + E + N), \\
1 &= S.
\end{align*}
\] (3.19)

The second assignment \( [16] \) is given from the first one by the \( \pi \) rotation in \( SU(2)_E \):

\[
\begin{align*}
16 &= (D^c + E + N) + (u^c + u + d + e^c) + S, \\
10 &= (D + E^c + N^c) + (d^c + e + \nu), \\
1 &= \nu^c.
\end{align*}
\] (3.20)

The third assignment is given from the first one by the \( \pi \) rotation in \( SU(2)_R \), namely the flipped model:

\[
\begin{align*}
16 &= (u^c + e + \nu) + (d^c + u + d + \nu^c) + e^c, \\
10 &= (D + E + N) + (D^c + E^c + N^c), \\
1 &= S.
\end{align*}
\] (3.21)
The fourth one is given from the third one by the \( \pi \) rotation in \( SU(2)_E \);

\[
\begin{align*}
16 & = (D^c + E^c + N^c) + (d^c + u + d + \nu^c) + S, \\
10 & = (D + E + N) + (u^c + e + \nu), \\
1 & = e^c.
\end{align*}
\]  

(3.22)

The fifth one is given from the first one by the \( \pi \) rotation in \( SU(2)_R' \);

\[
\begin{align*}
16 & = (d^c + E^c + N^c) + (D^c + u + d + S) + \nu^c, \\
10 & = (D + e + \nu) + (u^c + E + N), \\
1 & = \nu^c.
\end{align*}
\]  

(3.23)

The sixth one is given from the fifth one by the \( \pi \) rotation in \( SU(2)_E \);

\[
\begin{align*}
16 & = (u^c + E + N) + (D^c + u + d + S) + e^c, \\
10 & = (D + e + \nu) + (d^c + E^c + N^c), \\
1 & = e^c.
\end{align*}
\]  

(3.24)

These possibilities certainly indicate that \( E_6 \) has much potential for constructing new alternative GUT models.

Finally, we comment on the electromagnetic symmetry \( U(1)_{em} \). In the SM, the electromagnetic symmetry \( U(1)_{em} \) is, of course, a linear combination of \( SU(2)_L \) and \( U(1)_Y \); \( U(1)_{em} \subset SU(2)_L \times U(1)_Y \). From Eqs. (3.12) - (3.18), one can find that \( Y/2 = -I_{3R} + (B - L)/2 = -I_{3R}' + (B - L)/2 \) for the first assignment of the hypercharge, \( Y/2 = I_{3R} + (B - L)/2 = -I_{3E}' + (B - L)/2 \) for the second one, and \( Y/2 = I_{3R}' + (B - L)/2 = I_{3E}' + (B - L)/2 \) for the third one. These relations and Eqs. (3.16) - (3.18) indicate \( U(1)_Y \subset SU(2)_{(R)} \times U(1)_{B - L} \) and \( U(1)_{B - L} \subset U(1)_{Y_L} \times U(1)_{Y_{(R)}} \). This observation may indicate that the origin of the electromagnetic symmetry \( U(1)_{em} \) is \( SU(3)_L \times SU(3)_R \); \( U(1)_{em} \subset SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_{(R)} \times U(1)_{B - L} \subset SU(2)_L \times SU(2)_{(R)} \times U(1)_{Y_L} \times U(1)_{Y_{(R)}} \subset SU(3)_L \times SU(3)_R \).\footnote{This relation becomes more simple form by redefining the \( SU(2) \) subgroups of \( SU(3)_R \); the \( SU(2) \) subgroup which is orthogonal to \( U(1)_Y \) is \( SU(2)_E \), the \( SU(2) \) subgroup which is orthogonal to \( U(1)_{B - L} \) is \( SU(2)_{R} \), and the remaining \( SU(2) \) subgroup is \( SU(2)_{R}' \). Using this definition the electromagnetic symmetry \( U(1)_{em} \) can be written as \( U(1)_{em} \subset SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_{(R)} \times U(1)_{B - L} \subset SU(2)_L \times SU(2)_{(R)} \times U(1)_{Y_L} \times U(1)_{Y_{(R)}} \subset SU(3)_L \times SU(3)_R \).}

4. Model

In this section, as a simple example, we propose a \( E_6 \)-inspired \( SU(5) \) GUT model, “E-twisting flipped model”. This model is based on the gauge group \( SU(5) \times U(1)_Y \times U(1)_Y' \), which is a subgroup of \( E_6 \). We also assume \( N = 1 \) supersymmetry. In this model, the hypercharge \( U(1)_Y \) is shared among \( SU(5) \) and \( U(1)_Y \). In this model, the hypercharge \( U(1)_Y \) is shared among \( SU(5) \) and \( U(1)_Y \), and is explicitly given by

\[
\frac{Y}{2} = -\frac{1}{20}(4Z - V - 5V'),
\]  

(4.1)
Table 1: The quantum numbers of left-handed fields that belong to 27 representation of $E_6$.

| fields | $SU(3)_C$ | $I_{3L}$ | $Y$ | $B - L$ | $Q_{em}$ |
|--------|-----------|----------|-----|---------|----------|
| $u$    | 3         | 1/2      | 1/3 | 1/3     | 2/3      |
| $d$    | 3         | −1/2     | 1/3 | 1/3     | −1/3     |
| $\nu^c$ | 3*       | 0        | −4/3| −1/3    | −2/3     |
| $d^c$  | 3*       | 0        | 2/3 | −1/3    | 1/3      |
| $\nu$  | 1         | 1/2      | −1  | −1      | 0        |
| $e$    | 1         | −1/2     | −1  | −1      | −1       |
| $\nu^c$ | 1       | 0        | 0   | 1       | 0        |
| $e^c$  | 1         | 0        | 2   | 1       | 1        |
| $D$    | 3         | 0        | −2/3| −2/3    | −1/3     |
| $E^c$  | 1         | 1/2      | 1   | 0       | 1        |
| $N^c$  | 1         | −1/2     | 1   | 0       | 0        |
| $D^c$  | 3*       | 0        | 2/3 | 2/3     | 1/3      |
| $N$    | 1         | 1/2      | −1  | 0       | 0        |
| $E$    | 1         | −1/2     | −1  | 0       | −1       |
| $S$    | 1         | 0        | 0   | 0       | 0        |

where notation is the same as in the previous section. This hypercharge $U(1)_{Y}$ is orthogonal to the $SU(2)_{R}$ symmetry. On the other hand, $B - L$ is given by

$$B - L = \frac{1}{20}(16Z + V + 5V'),$$

which is orthogonal to the $SU(2)'_{R}$ symmetry.

All matters in the SM and a right-handed neutrino in a family belong to

$$F_{5^*} = 5^*_{-2,-2} = (u^c, e, \nu),$$
$$T_{10} = 10_{-1,1} = (d^c, u, d, \nu^c),$$
$$X_{1} = 1_{0,4} = e^c,$$

where subscripts are the quantum numbers of $U(1)_{Y}$ and $U(1)_{Y'}$ respectively. At first glance, this particle assignments seems to be the same as those of usual flipped model. However, it is quite different. The $U(1)_{Y}$ hypercharge and $B - L$ of our model are not subgroups of $SO(10)$ such as usual flipped model, but of $E_6$. Since this model is given from the usual flipped model by the $\pi$ rotation in $SU(2)_E$ (E-twisting \cite{12}), we call this model as “E-twisting flipped model”. It is worth mentioning here that while $SU(5)$ multiplets $5^*, 10$ and $1$ of usual flipped model can form a single 16 representation of $SO(10)$, $SU(5)$ multiplets in our model cannot. They must be $(10,5^*)$, $(16,10)$ and $(1,1)$, where the numbers in the parentheses are the dimensions of $SO(10)$ and $SU(5)$. Therefore, we introduce the following exotic matters to cancel anomaly; $\tilde{T}_{5^*} \equiv 5^*_{3,1}, \tilde{T}_{5} \equiv 5_{2,-2}$ and $\tilde{T}_{1} \equiv 1_{-5,1}$. Although our model seems somehow complicated because of the existence of exotic matters, its structure is quite simple. Indeed, the SM matters and the exotic matters can form a single 27 representation of $E_6$. The exotic matters will decouple at
low-energies. We assign the chiral superfields of the SM matters and exotic matters the odd \( Z_2 \) parity to avoid unwanted operators. This \( Z_2 \) parity can be regarded as an extended \( R \)-parity. Indeed, this extended \( R \)-parity is not identical with the conventional one \( R = (-1)^{2S+3(B-L)} \), where \( B, L \) and \( S \) are the baryon, lepton and spin quantum numbers.

GUT symmetry breaking \( SU(5) \times U(1)_V \times U(1)_{V'} \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \) is induced by Higgs multiplets \( \Sigma_{10} \equiv 10_{-1,1} \) and \( \Sigma_{10^*} \equiv 10^*_{1,-1} \), where the electrically neutral components develop large vacuum expectation values (VEV), \( \langle \nu_{\Sigma} \rangle = \langle \nu_{\Sigma}^c \rangle \equiv M \).

On the other hand, the Higgs multiplets which give Dirac masses to the SM matters are \( \chi^5 \equiv 5_{-3,1} \) and \( H^5 \equiv 5_{-2,2} \). Notice here that \( \chi^5 \) and \( H^5 \) should be \((16,5^*)\) and \((10,5)\) respectively. This is different from the case of Georgi-Glashow model or usual flipped model. We introduce other Higgs multiplets to cancel anomaly; \( \phi_1 \equiv 1_{-5,1}, \phi_1 \equiv 1_{5,-1}, \chi^5 \equiv 5_{-3,-1}, \bar{T}^5_{5^*} \equiv 5^*_{-2,-2}, H^5 \equiv 5_{2,2} \) and \( \bar{T}^5_{5^*} \equiv 5^*_{-2,-2} \). We assign \( Z_2 \) parity even to the Higgs chiral superfields.

The \( Z_2 \) parity and \( SU(5) \times U(1)_V \times U(1)_{V'} \) invariant superpotential for the SM matters is

\[
W = f_u T_{10} F_5 \bar{\chi}_5^* + f_d T_{10} T_{10} H_5 + f_l F_5 \chi^5 H_5, \tag{4.4}
\]

where we omit the indices of generations for simplicity, and \( f \)'s are the Yukawa couplings. Thus, in this model, Dirac neutrino masses and up-type quark masses are the same each other; \( m_u = m_\nu_D \), which is in contrast with the Georgi-Glashow model. On the other hand, although down-type quarks and charged leptons acquire Dirac masses from the same Higgs VEV, there is no relation among their Yukawa couplings. In other words, although there is no bottom-tau unification, there are also no wrong relations of \( m_s = m_\mu \) and \( m_d = m_e \).

Right-handed Majorana masses are given by the following dimension five operator,

\[
W = \frac{f_R}{M_{pl}} T_{10} T_{10} \Sigma_{10} \Sigma_{10^*}. \tag{4.5}
\]

After GUT symmetry breaking, right-handed Majorana neutrinos acquire the following heavy masses,

\[
M_R \approx \frac{f_R \langle \Sigma \rangle^2}{M_{pl}}, \tag{4.6}
\]

which leads to small neutrino masses through the conventional seesaw mechanism.

The superpotential for exotic matters is

\[
W = y_D \bar{T}_5 \bar{F}_5 \phi_1 + \frac{y_S}{M_{pl}} \bar{T}_1 \bar{T}_1 \bar{T}_1 \phi_1 \phi_1. \tag{4.7}
\]

After Higgs fields \( \phi \) acquire VEVs, the exotic matters become super-heavy and decouple. Here we assume that VEVs of \( \phi \) is slightly smaller than \( M \).

Next we discuss the gauge coupling unification. First of all, notice that since the hypercharge assignment of our model is different from that of Georgi-Glashow model or flipped model, the gauge coupling flow is different from those of them. We normalize the charges and couplings in the following manner (where \( Z, V \) and \( V' \) are defined in
the previous section), \( \tilde{Z} \equiv \sqrt{3/5}Z \). The corresponding coupling is \( g_{\tilde{Z}} = g_5 \) at the \( SU(5) \) unification scale \( M_5 \). \( \tilde{V} \equiv 1/2\sqrt{5}V \) and \( g_{\tilde{V}} = 2\sqrt{10}g_V \). Note here that \( tr_r\tilde{V}^2 = tr_r\tilde{Z}^2 \) where \( r \) is any \( SO(10) \) representation. Therefore, we expect \( g_{\tilde{V}} = g_5 = g_{10} \) at the \( SO(10) \) unification scale \( M_{10} \). \( \tilde{V}^i \equiv 1/2\sqrt{5}V^i \) and \( g_{\tilde{V}^i} = 2\sqrt{6}g_{V^i} \). Note here that \( tr_r\tilde{V}^i r^2 = tr_r\tilde{V}^2 = tr_r\tilde{Z}^2 \) where \( r \) is any \( E_6 \) representation. We also expect \( g_{\tilde{V}^i} = g_{10} = g_6 \) at the \( E_6 \) unification scale \( M_6 \). We can find the following relation at the \( SU(5) \) unification scale,

\[
\frac{3}{5} \frac{1}{\alpha_Y} = \frac{1}{25} \frac{1}{\alpha_5} + \frac{3}{50} \frac{1}{\alpha_{\tilde{V}}} + \frac{9}{10} \frac{1}{\alpha_{V}}, \tag{4.8}
\]

where \( \alpha_i \) is defined by \( \alpha_i \equiv g_i^2/4\pi \) for any \( i \). Therefore, if \( 24/25\alpha_5^{-1} \sim 3/50\alpha_{\tilde{V}}^{-1} + 9/10\alpha_{V}^{-1} \) is satisfied at the \( SU(5) \) unification scale, the quasi-unification of strong, weak and hypercharge gauge couplings will realize. Since it seems natural that the unification scale of \( SO(10) \) and \( E_6 \) are not much higher than that of \( SU(5) \), this situation should be possible. We comment on the ratio \( K \equiv \alpha_1/\alpha_5 \) at the scale \( M_5 \), where \( \alpha_1 \) is defined by \( \alpha_1 \equiv 5\alpha_Y/3 \). The natural condition that \( M_5 < M_{10} < M_6 \) implies \( \alpha_{\tilde{V}}(M_5) < \alpha_5(M_5) \) and \( \alpha_{V}(M_{10}) < \alpha_5(M_{10}) \). From these relations and the infrared-free behavior of \( U(1)_{V^i} \), we find the following inequality \( \alpha_{V^i}, \alpha_{V^i} < \alpha_5 \) at the scale \( M_5 \). From both this relation and Eq. (4.8), we conclude that \( K < 1 \). This is certainly a required relation that MSSM predicts from the precise experimental values at \( M_Z \) scale.

Although the detailed quantitative discussion and other phenomenological aspects of this model are interesting and should be pursued, that is beyond the scope of this paper.

Final comment is on the Higgs superpotential. We need, of course, a Higgs superpotential to define the complete model. We can easily write down the most general superpotential which is allowed by the gauge symmetry. However, there are some unwanted operators in the most general superpotential in view of \( F \)-flatness conditions and/or doublet-triplet splitting problem. In order to avoid unwanted operators, it is necessary to impose some additional symmetry beyond \( SU(5) \times U(1)_Y \times U(1)_{V^i} \). Although there are some possibilities as an additional symmetry, we do not discuss here since they are model dependent. These possibilities will be investigated elsewhere.

5. Conclusion

In this paper we investigated assignments of the hypercharge and baryon minus lepton number for particles in the \( E_6 \) grand unification model. First we reviewed the \( SO(10) \) GUT and pointed out that as far as \( SO(10) \supset U(1)_{B-L} \) is imposed, only the \( \mathbf{16} \) representation, not \( \mathbf{10} \), can be a candidate for the SM matters. Next we studied the \( E_6 \) GUT and it was shown that there are three assignments of hypercharge and three assignments of \( B-L \) which are consistent with the SM. Their explicit expressions and detailed properties were given. We showed that three assignments of hypercharge or \( B-L \) are related with each other by the \( SU(2) \) subgroup of \( SU(3)_R \), which is the subgroup of \( E_6 \). We also pointed out that any charge assignments are orthogonal to the \( SU(2) \) subgroup of \( SU(3)_R \). In particular, we emphasized that the \( U(1)_{B-L} \) symmetry in \( E_6 \) can not be orthogonal to the
$SU(3)_R$ symmetry. This fact strongly restricts $E_6$ grand unified models with the gauged $U(1)_{B-L}$ symmetry. In those of 9 pairs of charge assignments, we showed that 6 pairs are consistent with the SM. These observation indicate that $E_6$ has much potential for constructing alternative GUT models.

We also proposed a $E_6$-inspired $SU(5)$ GUT model, “E-twisting flipped model”. The charge assignments of hypercharge and $B - L$ in this model are different from those of Georgi-Glashow model or usual flipped model. Since the hypercharge is not a subgroup of $SU(5)$, the quasi-unification of strong, weak and hypercharge gauge coupling is predicted. There are also no mass relations between down-type quarks and charged leptons.

Finally we would like to emphasize that it is worth investigating the alternative GUT models based on $E_6$ to overcome the difficulties of the minimal SUSY $SU(5)$ model.

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References

[1] Y. Hayato et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 83 (1999) 1529.
[2] H. Murayama and A. Pierce, Phys. Rev. D 65 (2002) 055009.
[3] B. Bajc, P. F. Perez and G. Senjanovic, Phys. Rev. D 66 (2002) 075007.
[4] D. Emmanuel-Costa and S. Wiesenfeldt, hep-ph/0302272.
[5] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81 (1998) 1562.
M. Apollonio et al. [CHOOZ Collaboration], Phys. Lett. B 466 (1999) 417.
S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 86 (2001) 5650.
Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 89 (2002) 011301.
K. Eguchi et al. [KamLAND Collaboration], Phys. Rev. Lett. 90 (2003) 021802.
[6] T. Yanagida, in Proceedings of the “Workshop on the Unified Theory and the Baryon Number in the Universe”, Tsukuba, Japan, 1979, edited by O. Sawada and A. Sugamoto, KEK Report No. KEK-79-18, p.95; M. Gell-Mann, P. Ramond and R. Slansky, in “Supergravity”, edited by D. Z. Freedman and P. van. Nieuwenhuizen (North-Holland, Amsterdam, 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
S. L. Glashow, in “Quarks and Leptons”, (Plenum Press, NY, 1980), p.707.
[7] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
[8] S. M. Barr, Phys. Lett. B 112 (1982) 219.
[9] J. P. Derendinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 139 (1984) 170.
I. Antoniadis, J. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B 194 (1987) 231.
J. Ellis, J. L. Lopez and D. V. Nanopoulos, Phys. Lett. B 371 (1996) 63.
[10] J. Ellis, D. V. Nanopoulos and J. Walker, Phys. Lett. B 550 (2002) 99.
D. V. Nanopoulos, hep-ph/0211122.
[11] F. Gursey, P. Ramond and P. Sikivie, Phys. Lett. B 60 (1976) 177.
Y. Achiman and B. Stech, Phys. Lett. B 77 (1978) 389.
[12] M. Bando and T. Kugo, *Prog. Theor. Phys.* **101** (1999) 1313.

[13] M. Bando, T. Kugo and K. Yoshioka, *Prog. Theor. Phys.* **104** (2000) 211; M. Bando and N. Maekawa, *Prog. Theor. Phys.* **106** (2001) 1257.

[14] See also, D. London and J. L. Rosner, *Phys. Rev. D* **34** (1986) 1530.

[15] E. Ma, *Phys. Rev. D* **36** (1987) 274.

[16] E. Ma, *Phys. Lett. B* **380** (1996) 283.