Wideband dual sphere detector of gravitational waves

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We present the concept of a sensitive and broadband resonant mass gravitational wave detector. A massive sphere is suspended inside a second hollow one. Short, high-finesse Fabry-Perot optical cavities read out the differential displacements of the two spheres as their quadrupole modes are excited. At cryogenic temperatures one approaches the Standard Quantum Limit for broadband operation with reasonable choices for the cavity finesse and the intracavity light power. A molybdenum detector of overall size of 2 m, would reach spectral strain sensitivities of $2 \cdot 10^{-23}$ Hz$^{-1/2}$ between 1000 Hz and 3000 Hz.

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Resonant mass detectors of gravitational waves (GW) are commonly indicated as narrowband devices. In currently operating cylindrical bar detectors \textsuperscript{[8–11]}, all equipped with resonant transducers, the bandwidth, even in the limit of quantum limited performance of the final displacement readout, would not open up for more than about the frequency interval between the resulting two mechanical modes of resonance, currently about 30 Hz.

This stems however from the noise performance of the employed readout systems. A general analysis of the problem, valid for any linear detector, has been given in ref. \textsuperscript{[2]}. Secondary resonant masses are linked to the main resonant mass to efficiently couple the signal amplitude to the final readout, but then the bandwidth is limited to a fraction of the main resonator frequency. Such a coupling is poorer the smaller the total number $n$ of resonators, and correspondingly the bandwidth decreases with $n$. To open the bandwidth one would have to use multimode systems \textsuperscript{[8–11]} with $n \geq 3$, but to date only two-(mechanical) mode systems have worked their way into operating detectors, giving a fractional bandwidth $\Delta f/f \ll 0.1$.

However, if a single mechanical resonator were driven only by its thermal noise and by a signal force the signal to noise ratio (SNR) would be independent of frequency, and thus the band would open up provided enough signal amplitude can be coupled to the final readout. The possibilities offered nowadays by optomechanical systems are such that the interplay between the back-action of the radiation pressure and the photon counting noise in a high finesse, high power Fabry-Perot cavity would allow enough signal coupling to get broadband operation at the SQL \textsuperscript{[8–11]}.

We have been attracted by the possibilities offered by optical readout systems, as vigorously developed for interferometric GW detectors, and more recently applied in connection with cryogenic bar GW detectors \textsuperscript{[8–11]}. We take a Fabry-Perot optical cavity as the motion sensor. In a system under development \textsuperscript{[7]} the length of the sensor cavity is compared to that of a second cavity, separately kept, which acts as reference. We do not take into account here the noise introduced by the reference cavity, assuming for simplicity that it is negligible. With a sensor cavity length of the order of centimeters there is no loss of signal strength for finesse $F$ as high as the highest attainable with current technology, $F = 10^6$, for GW in the kHz range. So we have considerable freedom to vary the finesse and the light power $P$ incident on the cavity, in search for optimal conditions at a chosen frequency, which do not demand unreasonable values for these parameters.

Let us then turn to the primary mechanical resonator, whose motion is directly related to the incoming GW. We take into consideration both solid and hollow spheres as resonant systems of interest. They are very attractive for a number of reasons, and in fact they have received significant attention in the literature of the last few years \textsuperscript{[13–16]}. Spherical detectors are omnidirectional, have a more efficient coupling to the GW field relative to cylindrical bars, both in the first and in the second quadrupole mode, and enable a deconvolution of the GW signal if they are equipped with five (or more) suitable motion sensors \textsuperscript{[13–16]}. For instance, a chirp signal from a merging compact binary can be fully deconvolved with a spherical detector \textsuperscript{[13]}. Two spheres would make up for a complete observatory, in which all parameters characterizing the incoming wave (e.g. velocity, direction, polarization) can be resolved \textsuperscript{[14,15]} -see also \textsuperscript{[16]}. Located close to an interferometric detector, a spherical detector could be used for searches of stochastic background \textsuperscript{[17]}. Such capabilities make of the spherical detector a conceptually unique device.

However, the sensitive spherical detectors proposed in the past suffer from the above discussed bandwidth limitation. As an example, a hollow sphere of CuAl$^{109}$, 4 m in diameter and 0.3 m thick, cooled at sub-Kelvin tem-
temperatures and equipped with resonant transducers and a quantum limited readout, gives a spectral strain noise as low as $6 \cdot 10^{-24} \text{ Hz}^{-1/2}$ [1], but only in two bands of 35 Hz and 135 Hz, respectively around the first and second quadrupole resonances at 350 Hz and at 1350 Hz.

Let us then consider a spherical detector with non resonant optical readout. We need to integrate the two mirrors of each Fabry-Perot sensing cavity in two separate systems, which must be cold, massive and of high mechanical $Q$ factor, otherwise the thermal noise would be unacceptably large. We are thus led to the concept of a GW detector based on a massive dual sphere system of resonators: a hollow sphere which encloses a smaller solid sphere, see Fig. 1. Motion sensors in this system will be optical Fabry-Perot cavities formed by mirrors coated face to face to the inner surface of the hollow sphere and to the solid sphere, in either a PHC [12] or a TIGA [8] layout.

![FIG. 1. A dual sphere GW detector with Fabry-Perot cavities as motion sensors.](image)

The main sources of noise are: thermal noise in the large detector masses, back-action noise introduced by the radiation pressure, and photon counting noise. Given the optical figures, the evaluation of all three contributions for our design is straightforward, as the spectrum of the resonant frequencies of the two spheres is known [10,11] once their material(s) and dimensions are fixed.

Assuming the same material is used for both spheres, and that the inner one of radius $a$ fills up almost completely the interior of the other (external radius $R$, internal radius $\geq a$), the first quadrupole resonance of the outer hollow sphere is at the lowest frequency, while that of the inner solid sphere is 2-3 times higher. The frequency region in between is of particular interest: the GW signal drives the hollow sphere above resonance and the solid sphere below resonance. The responses of the two resonators are then out of phase by $\pi$ radians and therefore the differential motion, read by the optical sensors, results in a signal enhancement. In this region only a small number of non-quadrupole resonances occur, which are not GW active. The pattern repeats for the two second quadrupole modes at higher frequency and so on. At a few specific frequencies above the first quadrupole resonance of the solid sphere, the GW signal drives the hollow sphere above resonance and the frequency region in between is of particular interest: the outer hollow sphere is at the lowest frequency, while that of the inner one of radius $R$, in internal radius $\geq a$), the first quadrupole resonance of the solid sphere, under the combined effect of the response to GW of all the quadrupole modes, their responses subtract and the sensitivity is reduced and eventually lost in a few narrow bands. In this higher frequency region, in addition to such loss of response, several resonances from the GW-inactive modes appear. While the spectral sensitivity would be still of some interest, we prefer for brevity to not discuss it here.

Let us assume that we only sense radial displacements and that the spherical symmetry of the resonators is not broken; suspending a solid spherical resonator has shown to alter only marginally the spectrum of resonances [8]. Using the notation of references [10] and [11] the response to a GW of the solid sphere at its surface and of the hollow at its inner surface (i.e. at the radius $a$) are respectively given by expressions of the type

$$u(\omega) = -\frac{1}{2} \sum_{n=0}^{\infty} b_n A_{n2}(a) \omega^2 \tilde{h}(\omega) L_{n2}(\omega),$$

where $A_{n2}(a)$ are radial function coefficients, $b_n$ are the coefficients in the orthogonal expansion of the response function of each sphere, $L_{n2}(\omega)$ is the Lorentzian curve associated to the mode $\{n2\}$, the $n$-th quadrupole harmonic, and $\tilde{h}(\omega) \equiv \tilde{h}_{ij}(\omega)_{n,i,j}$ is the Fourier amplitude of the GW strain at the sensing point direction, defined by the unit radial vector $n$ relative to the system’s center of mass. Of course all these quantities must be calculated for either sphere.

Each sensor output is affected by thermal and back-action displacement noise spectral densities, which must be formed for both spheres:

$$S_{uu}^{\text{th}+\text{ba}}(\omega) = \sum_{nl} \frac{2l+1}{4\pi M} |A_{nl}(a)|^2 |L_{nl}(\omega)|^2 \left[ \frac{2kT \omega^2}{Q_{nl} \omega} + \frac{2l+1}{4\pi M} |A_{nl}(a)|^2 \sum_j |P_l(n \cdot n_j)|^2 S_{FF}^{ba} \right],$$

where $k$ is Boltzmann’s constant, $T$ the sphere’s thermodynamic temperature, and $M$ the sphere’s mass, whether solid or hollow. $S_{FF}^{ba}$ is the back action force spectral density, $P_l$ a Legendre polynomial, and $n_j$ the spherical coordinates of the optical cavities ($n \equiv n_1$). The sum over $j$ accounts for the fact that each sensor is additionally affected by the back-action noise forces exerted by the others. The back-action noise force is given by

$$S_{FF}^{ba}(\omega) = \frac{4}{c^2 \pi^2} (1 - \zeta)^2 \nu T \frac{P}{F} \frac{1}{1 + \left( \frac{2F \nu T}{\pi c} \right)^2},$$

where $\nu T$ is the light frequency, $c$ the speed of light, $P$ the light power incident on the cavity and $\zeta^2$ the fraction of light reflected by the cavity at its resonance.
with a sound velocity $v_s$ written as $v_s = 6.2$ km/s and a density $\rho = 10^{-3}$ kg/m$^3$. The fabrication of the dual sphere may proceed from Mo powders, which can be pressed, sintered to 95%-density and hot formed to custom shapes. This procedure allows mechanical Q of 2-10$^7$ at temperatures $\leq 4$ K [29], necessary to approach the SQL as above discussed. The thermal properties of molybdenum are such that no particular difficulties are expected to arise when cooling down such large masses [29]. Building on the experience developed for vibrationally insulating masses of few tons, as in bar detectors, we are confident that a suitable design can be made for suspending few tens of tons.

Hot pressed, sintered beryllium is even more interesting, as $v_s = 13$ km/s and $\rho = 1.9 \cdot 10^3$ kg/m$^3$; it has already been used in large sizes (over 1.3 m) and its thermal properties make it viable as well. The low-temperature mechanical Q is going to be investigated [22]. Another interesting material is sapphire ($v_s = 10$ km/s, $\rho = 4 \cdot 10^3$ kg/m$^3$) which is already known to show $Q > 10^8$ at $T < 10$ K. Sapphire, acting as substrate of the mirrors, would also minimize thermaelastic effects at low temperatures [22]. Sapphire drawback mainly resides on the difficulty of growing large enough crystals and/or joining together several pieces while preserving the high mechanical Q. We note that the $\rho v_s^5$ factor for beryllium is a factor of 2 greater than for sapphire.

A molybdenum detector with $R = 0.95$ m and $a = 0.57$ m, with a small gap in between to place the motion sensing optical cavities, would give an interestingly low strain spectral noise in a rather wide frequency band in the kHz region, see Fig. 2. Here we plot the SQL spectral strain noise, when the radiation pressure noise is matched to the shot noise at 1.3 kHz: this requires an input light power of 7 W and $Q/T \geq 2 \cdot 10^8$ K$^{-1}$. The spectral strain noise is also shown for a lower light power $P = 1$ W, $Q/T = 2 \cdot 10^7$ K$^{-1}$ possibly more amenable to cryogenic operation at $T \approx 1$ K, but still giving an interesting performance. Note that the spectral sensitivity is contaminated by the thermal noise peaks of the non-quadrupole resonances. The problem of unwanted narrow resonances in the sensitive frequency band is also present in the case of interferometric detectors and methods have been devised to filter them out [22-25].

Figure 3 shows a comparative plot of the sensitivities of various GW detectors to come: initial VIRGO [24], the cryogenic interferometer LCGT [24] and LIGO II operated in the narrowband dual-recycled mode [25]. For the proposed dual sphere we show the SQL operation of the SQL molybdenum system of Fig. 2 with the non-quadrupole resonances suppressed for clarity. The large drop in sensitivity, indicated by the prominent spike towards the right end of the figure is due to signal cancellation at a frequency $\omega_*$ for which $u_{\text{hollow}}(\omega_*) = u_{\text{solid}}(\omega_*)$, which causes the denominator in eq. (4) to vanish. The presence of such a frequency $\omega_*$ is expected on the ba-
sis of the intrinsic structure of eq. [4], and therefore it must be taken into consideration when one chooses materials and dimensions. We also show in Fig. 3 the sensitivity of a beryllium system operated at SQL for 1300 Hz, which requires an input light power of 12 W and \( Q/T = 2 \times 10^8 \) K\(^{-1}\); the gain in sensitivity is due to the larger \( \rho v_s^5 \) factor. As it can be seen, the dual sphere system well compares with the best foreseen GW detectors, especially in the high frequency region where e.g. relatively small mass 10\( M_\odot \) BH-BH mergers are expected [28].

In the end, the system we propose may still look like a two-mode system in that the most useful band is obtained between the two first quadrupole resonances of the two spheres. However the concept we propose allows one to choose such two frequencies with a lot of freedom, and in fact to open considerably the band with respect to systems which make use of resonant secondary masses to get the two-mode operation.

We have noted that crucial practical issues for the realization of such a massive system, especially in respect to fabrication, suspensions, cooling, can be dealt with; however the level of practicability of the concept may still strongly depend on supportive research.

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