Model Predictive Control considering Reachable Range of Wheels for Leg / Wheel Mobile Robots

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Abstract. Obstacle avoidance is one of the important tasks for mobile robots. In this paper, we study obstacle avoidance control for mobile robots equipped with four legs comprised of three DoF SCARA leg/wheel mechanism, which enables the robot to change its shape adapting to environments. Our previous method achieves obstacle avoidance by model predictive control (MPC) considering obstacle size and lateral wheel positions. However, this method does not ensure existence of joint angles which achieves reference wheel positions calculated by MPC. In this study, we propose a model predictive control considering reachable mobile ranges of wheels positions by combining multiple linear constraints, where each reachable mobile range is approximated as a convex trapezoid. Thus, we achieve to formulate a MPC as a quadratic problem with linear constraints for nonlinear problem of longitudinal and lateral wheel position control. By optimization of MPC, the reference wheel positions are calculated, while each joint angle is determined by inverse kinematics. Considering reachable mobile ranges explicitly, the optimal joint angles are calculated, which enables wheels to reach the reference wheel positions. We verify its advantages by comparing the proposed method with the previous method through numerical simulations.

1. Introduction
In recent years, leg mobile robots are actively researched for operation in, for example, disaster sites and extraterrestrial exploration [1][2]. The robots are equipped with leg mechanism configured by multiple links and joints. Thus, the robots can climb a step and walk a rough road by those mechanisms [3]. By extending legs, the stability of robot is improved and the turnover of robot is avoided [4]. Moreover, by shrinking legs, the width of robot is reduced and the robot can pass through a narrow space which cannot be passed if legs are extended. However, the leg mobile robot moves by leg; moving velocity is slow and energy efficiency is low.

Contrarily, wheel mechanisms have advantages on moving velocity and energy efficiency. However, the robot equipped with wheel mechanism has trouble to climb a step or drive a rough road. There exist leg / wheel mechanism which is equipped with both leg and wheel mechanisms having the advantages of both mechanisms while supplementing the disadvantages of them. The robots equipped with this mechanism have been actively researched [5][6].

Planar Road Strider (Fig. 1) [7] is a kind of leg / wheel mobile robots equipped with three DoF SCARA leg / wheel mechanism. This robot is a horizontal plane model of the robot equipped with this mechanism and was developed for researching optimization program of wheel positions. Thus, this robot cannot move legs up and down, but the robot can change wheel positions to...
Rear Legs
Front Legs
Note PC
Servo Motors
Laser Range Sensors

Figure 1. Planar Road Strider (PRS); three DoF SCARA leg is equipped with wheel and body.

Table 1. Parameters of each leg.

| (i, j) | $x_{ij}^0$ [m] | $y_{ij}^0$ [m] | $r_{ij}^0$ [rad] | $r_{ij}^1$, $r_{ij}^2$ [rad] | $r_{ij}^3$ [rad] |
|-------|----------------|----------------|------------------|-----------------------------|-----------------|
| FL (0, 0) | 0.125 | 0.125 | $\frac{3\pi}{4}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| FR (0, 1) | 0.125 | −0.125 | $\frac{3\pi}{4}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| RL (1, 0) | −0.125 | 0.125 | $\frac{3\pi}{4}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| RR (1, 1) | −0.125 | −0.125 | $\frac{3\pi}{4}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ |

arbitrary positions on horizontal plane. This robot can conduct translational and rotational motions because it independently controls each wheel for steering and driving, while it can change the width by changing the wheel positions.

Obstacle avoidance is one of the important tasks for moving autonomous mobile robots in actual environment. It is widely researched for various control objects [8][9]. In [10], collision and obstacle avoidance for multiple robots are achieved. In [11], obstacle avoidance considering size of obstacle and orientation angle is realized for omnidirectional robots. The obstacle avoidance considering advantages of changing the width is important [12]. When the robot equipped with leg /wheel mechanism avoids obstacles, it can realize efficient movement by shrinking legs near obstacles. Moreover, if there is no obstacles, the robot can improve stability by extending legs.

In our past study, we proposed obstacle avoidance control by model predictive control (MPC) applying passage width constraints [13]. This method achieved obstacle avoidance by passage width constraints using internal common tangent between a leg and an obstacle and external common tangent between two obstacles. The size of obstacles and wheel positions are considered by those constraints. However, this method does not ensure existence of joint angles which achieves the reference wheel position calculated by MPC because its reachable range is not explicitly considered. In addition, the wheel position for longitudinal direction has not been considered. Thus, the wheel positions might not attain target positions by constraints of the mechanism. Moreover, the reachable range is narrow because this method does not consider longitudinal direction. So, the robot might not pass through a narrow space where it can pass through if the original reachable range is considered.

In this paper, we achieve motion of the longitudinal direction of leg by including constraints of reachable range of leg / wheel mechanism to MPC. This constraint reflects the reachable area for longitudinal and lateral directions. If the target position is located in this range, each joint angle attaining the target is calculated by inverse kinematics. Thus, the controls of joint angles and longitudinal direction are achieved. In this paper, we verify efficiency of the proposed method by comparing it with the conventional method through numerical simulations.
2. Control objects

In this paper, Planar Road Strider (Fig. 1) which is a kind of the leg / wheel mobile robots, is controlled.

Figure 2 depicts the model of this robot, where $X - Y$ coordinates is the inertial coordinate system fixed on field and $x - y$ coordinates is the robot coordinate system. The state equation is defined by

$$\frac{dx}{dt} = u,$$

where $x$ is the state vector and $u$ is the input vector defined by

$$x(t) = [X(t), Y(t), x^{00}(t), x^{10}(t), x^{11}(t), y^{00}(t), y^{01}(t), y^{10}(t), y^{11}(t)]^T,$$

$$u(t) = [U_X(t), U_Y(t), \mu_x^{00}(t), \mu_x^{01}(t), \mu_x^{10}(t), \mu_x^{11}(t), \mu_y^{00}(t), \mu_y^{01}(t), \mu_y^{10}(t), \mu_y^{11}(t)]^T.$$

$X$ and $Y$ are position of center of gravity (CoG) on $X - Y$ coordinates and $U_X$ and $U_Y$ are input velocity of CoG. $\phi$ is orientation angle and $U_\phi$ is input angular velocity. Moreover, $x^{ij}$ and $y^{ij}$ are third joint position of each leg on $x - y$ coordinates and $\mu_x^{ij}$ and $\mu_y^{ij}$ are input velocity of third joint position on $x - y$ coordinates. Superscript 00, 01, 10 and 11 denote Front Left (FL), Front Right (FR), Rear Left (RL) and Rear Right (RR), respectively. In this paper, we assume that the orientation angle $\phi$ is regulated to be zero by feedback control to linearize the state equation.

The leg / wheel mechanisms are explained. Figure 3 depicts the model of the front left leg, $\theta_0^{ij}, \theta_1^{ij}$ and $\theta_2^{ij}$ are joint angles. Moreover, movable scope of each joint angle is $\pm \pi/2$ rad. The third joint position $x^{ij}$ and $y^{ij}$ and steering angle $\delta^{ij}$ are calculated by

$$x^{ij} = x_0^{ij} + L_0 \cos \left(\theta_0^{ij} + r_0^{ij}\right) + L_1 \cos \left(\theta_0^{ij} - \theta_1^{ij} + r_1^{ij}\right),$$

$$y^{ij} = y_0^{ij} + L_0 \sin \left(\theta_0^{ij} + r_0^{ij}\right) + L_1 \sin \left(\theta_0^{ij} - \theta_1^{ij} + r_1^{ij}\right),$$

$$\delta^{ij} = \theta_0^{ij} - \theta_1^{ij} - \theta_2^{ij} + r_3^{ij},$$

where $x_0^{ij}$ and $y_0^{ij}$ are first joint position on $x - y$ coordinates, $r_0^{ij}, r_1^{ij}$ and $r_3^{ij}$ are offset angles of each joint and $L_0, L_1$ and $L_2$ are length of each link. Table 1 shows the parameters of the leg / wheel mechanism.
Figure 4 depicts the system block diagram of proposed method. In this paper, the velocity \( U_x, U_y \) and third joint positions \( x^{ij}, y^{ij} \) are calculated by MPC, where each joint angle is not considered for linearization of equation (1). Each joint angle is calculated by inverse kinematics using the velocity and each third joint position. In this study, the robot is controlled to achieve the reference velocity and joint angle calculated by MPC and inverse kinematics.

### 3. Calculation of joint angles by inverse kinematics

#### 3.1. Inverse kinematics for \( \theta_0^{ij} \) and \( \theta_1^{ij} \)

The mechanism between first and third joint positions is regarded as 2 link manipulator. Thus, the equations of inverse kinematics for \( \theta_0^{ij} \) and \( \theta_1^{ij} \) are

\[
\theta_0^{ij} = \pm \left( \tan^{-1} \left( \frac{y^{ij} - y_0^{ij}}{x^{ij} - x_0^{ij}} \right) \right) - \cos^{-1} \left( \frac{x^{ij} - x_0^{ij}}{2L_0 \sqrt{(x^{ij} - x_0^{ij})^2 + (y^{ij} - y_0^{ij})^2}} \right) - \frac{\pi}{4}, \tag{7}
\]

\[
\theta_1^{ij} = \pm \left( \cos^{-1} \left( \frac{L_0^2 + L_1^2 - (x^{ij} - x_0^{ij})^2 - (y^{ij} - y_0^{ij})^2}{2L_0 L_1} \right) \right) - \frac{\pi}{2}, \tag{8}
\]

where it is necessary to reverse the sign of each angle for each leg.

#### 3.2. Calculation of \( \theta_2^{ij} \) considering velocity vector field by inverse kinematics [14]

The wheel position \( x_0^{ij} \) and \( y_0^{ij} \) are calculated by

\[
x_0^{ij} = x^{ij} + L_2 \cos \left( \theta_0^{ij} - \theta_1^{ij} - \theta_2^{ij} + r_2^{ij} \right), \tag{9}
\]

\[
y_0^{ij} = y^{ij} + L_2 \sin \left( \theta_0^{ij} - \theta_1^{ij} - \theta_2^{ij} + r_2^{ij} \right), \tag{10}
\]

where \( r_2^{ij} \) is offset angle shown in Table 1. The equation of steering angle calculated by velocity of wheel position is

\[
\delta^{ij} = \frac{U_y + \dot{y}_0^{ij} + U_\phi \dot{x}_0^{ij}}{U_x + \dot{x}_0^{ij} - U_\phi \dot{y}_0^{ij}}, \tag{11}
\]

where \( \dot{x}_0^{ij} \) and \( \dot{y}_0^{ij} \) are time-derivatives of \( x_0^{ij} \) and \( y_0^{ij} \). \( \theta_2^{ij} \) calculated by equation (6) and (11) is

\[
\theta_2^{ij} = \theta_0^{ij} - \theta_1^{ij} + r_2^{ij} - \tan^{-1} \left( \frac{U_y + \dot{y}_0^{ij} + U_\phi \dot{x}_0^{ij}}{U_x + \dot{x}_0^{ij} - U_\phi \dot{y}_0^{ij}} \right), \tag{12}
\]

where \( \dot{x}_0^{ij} \) and \( \dot{y}_0^{ij} \) are time-derivatives of \( x^{ij} \) and \( y^{ij} \). Thus, \( \theta_2^{ij} \) considering velocity vector field is calculated by equation (12).

### 4. Model predictive obstacle avoidance control

#### 4.1. Model predictive control

Model predictive control (MPC) is a kind of optimal control methods, which can predict future motion of the control object and calculate the optimal input.

In this paper, the index function \( J \) is

\[
J = \Delta x^T (t + T) S \Delta x (t + T) + \int_{t}^{t+T} \left( \Delta x^T (\tau) Q \Delta x (\tau) + u^T (\tau) R u (\tau) \right) d\tau, \tag{13}
\]
where we define $\Delta \mathbf{x}(t) := \mathbf{x}(t) - \mathbf{x}_r(t)$ and $\mathbf{x}_r(t)$ is the reference trajectory. Moreover, $T$ is length of horizon and $\mathbf{S}$, $\mathbf{Q}$ and $\mathbf{R} \in \mathbb{R}^{10 \times 10}$ are positive definite matrices. The terminal cost evaluates the difference between the state and the reference trajectory at $t + T$. Moreover, the stage cost evaluates the difference between the state and the reference trajectory and the magnitude of input from $t$ through $t + T$.

We explain constraint for MPC. The constraints for input of CoG and legs are

$$
\| [U_x(t), U_y(t)]^T \|_1 \leq u_{g,\text{max}},
$$

$$
\| [\mu_x^{ij}(t), \mu_y^{ij}(t)]^T \|_1 \leq u_{l,\text{max}},
$$

where $\| \cdot \|_1$ is 1-norm, $u_{g,\text{max}}$ is maximum velocity of CoG and $u_{l,\text{max}}$ is maximum velocity of each leg. Moreover, the passage width constraints are considered for avoiding obstacles and explained in section 4.3. The computational model of MPC is the discrete time system of equation (1).

### 4.2. Linear approximation of constraints

A quadratic problem with nonlinear constraints might have multiple optimal solutions, however, a quadratic problem with linear constraints has a single optimal solution. Moreover, the calculation cost of the problem with linear constraints is smaller than the cost of the problem with nonlinear constraints.

In this study, the constraints are approximated to linear constraints so that the optimization for MPC becomes a convex problem in order to reduce the calculation cost and achieve a single optimal solution. The solver used is CVXGEN [15], which can generate a custom code to solve convex problem with linear constraints.

### 4.3. Passage width constraints [13]

The third joint positions and the size of obstacles are considered for avoiding obstacles by passage width constraints. For that propose, two kinds of passage width constraints are used.

The robot searches an obstacle that distance between third joint and the obstacle is shorter than a threshold. Obstacles are considered for the nearest legs to wheels. Let us explain these constraints using an front left leg. For each obstacle, security disk is defined with the radius including the size and the margin. Moreover, the circle of the wheel mobile range from the third joint position to wheel position is defined; the center is the third joint position and the radius is third link length $L_2$. Figure 5 depicts a mobile range and a proximate obstacle for front left leg. If those circles do not overlap, the wheel does not collide with obstacles.

#### 4.3.1. Internal common tangents

Let us explain the passage width constraint using internal common tangent. Figure 6 depicts relationship between front left leg and an obstacle for this constraint. Two internal common tangents between two circles for this leg and the corresponding obstacle are calculated as depicted in Fig. 6. The passage width constraint is defined by

$$
(Y(t + \Delta t) + y(t + \Delta t)) - (Y(t) + y(t)) \leq b_{in}(t) \left( (X(t + \Delta t) + x(t + \Delta t)) - (X(t) + x(t)) \right),
$$

Figure 5. Wheel mobile range and selection of obstacle nearest leg.
where $\Delta t$ is periodic time of MPC and $b_{in}(t)$ is a slope of the internal common tangent. The size of obstacles and leg position are considered by applying equation (16) to MPC. Similar relations are applied to other legs.

4.3.2. External common tangents  Obstacle avoidance is achieved by passage width constraint using internal common tangent where the obstacle nearest to each leg is considered by this constraint. If neighboring obstacles exist, the constraint does not change. The slope of internal common tangent changes rapidly when the target obstacle is switched to a neighboring obstacle. It might cause instability on motion of the robot. In order to suppress the effect of it, in this paper, a passage width constraint using external common tangent between neighboring obstacles is used.

Figure 7 depicts relationship between front left leg and neighboring obstacles for this passage width constraint. $X_{ex}(t)$ and $Y_{ex}(t)$ are coordinates of a tangent point between a circle and an external common tangent and $b_{ex}(t)$ is slope of this tangent. Moreover, $d_y(t)$ is $Y$-axis distance between the third joint position and the external common tangent. The passage width constraint using external common tangent is defined by

$$
(Y(t + \Delta t) + y(t + \Delta t)) - (Y_{ex}(t) - d_y(t)) \leq b_{ex}(t)((X(t + \Delta t) + x(t + \Delta t)) - X_{ex}(t)).
$$

(17)

Similar relation is applied to other legs.

5. Approximation of reachable range for wheels

5.1. Reachable range

Figure 8 depicts an example of reachable range of wheel position and Fig. 9 depicts an example of the range of third joint position. The blue area is the reachable range of $\delta = -\pi/6$ and the red area is the range of $\delta = \pi/2$. Those areas are calculated by equation (6), (9) and (10). In Fig. 8, the area changes by the steering angle, but in Fig. 9, the area does not change, because the reference steering angle is realized by $\theta_2$ calculated by equation (12).

In this study, the reachable range of third joint position is considered, because this position is the element of state vector and considering the constant range is easier than considering the variable range. Moreover, this range is described by nonlinear equations; this range is approximated by plural linear constraints. In order to achieve it, constant and variable constraints are used.

5.2. Constant constraints for reachable range

Figure 10 depicts original and approximated reachable ranges. The green area is the original area and the orange area is the approximated area using constant constraints. $P_i(i = 0, .., 5)$ are intersections between those reachable ranges.
Figure 8. Reachable range of wheel position (blue area is $\delta = -\pi/6$ and red area is $\delta = \pi/2$).

Figure 9. Reachable range of third joint position (blue area is $\delta = -\pi/6$ and red area is $\delta = \pi/2$).

Let us explain $x$-axis and $y$-axis minimum positions. $x$-axis minimum position is equivalent to that of $P_0$ which is the intersection point between the reachable range and the first link at $\theta_0 = \pi/2$ because the intersection between the boundary of this constraint and variable constraint exists on the original reachable range. $y$-axis minimum position is equivalent to that of $P_1$ which is the $y$-axis maximum position of the concave area.

Let us explain $x$-axis and $y$-axis maximum positions. $x$-axis maximum position is equivalent to that of $P_2$ which is the intersection between the reachable range and the boundary of $y$-axis minimum constraint. $y$-axis maximum position is equivalent to that of $P_3$ which is the intersection between the reachable range and the boundary of $x$-axis minimum constraint. In Fig. 10, $P_4$ is the intersection between the original reachable range and the boundary of $x$-axis maximum constraint and $P_5$ is the intersection between this range and the boundary of $x$-axis maximum constraint. The maximum constraint of the relationship of $x$ and $y$ is the straight line from $P_4$ to $P_5$. The orange area is depicted by those linear constant constraints in Fig. 10.

5.3. Variable constraints for reachable range
The outside range of inside arc is not approximated by constant constraints in Fig. 10. Thus, this range is approximated by the variable constraint.

For achieving it, the tangent of original reachable range is used. Figure 11 depicts area using a variable constraint. The tangent point for the variable constraint is intersection between the first link, and original reachable range and the equation of constraint is defined by

$$|y^{ij} - y_c^{ij}| \geq -\cot (\theta_0^{ij} + r_0^{ij}) \left( |x^{ij} - x_c^{ij}| \right) \quad \text{if} \quad \theta_0^{ij} + r_0^{ij} > 0,$$

$$|x^{ij}| \geq |x_c^{ij}| \quad \text{if} \quad \theta_0^{ij} + r_0^{ij} = 0,$$

$$|y^{ij} - y_c^{ij}| \leq -\cot (\theta_0^{ij} + r_0^{ij}) \left( |x^{ij} - x_c^{ij}| \right) \quad \text{if} \quad \theta_0^{ij} + r_0^{ij} < 0,$$

where

$$x_c^{ij} = x_0^{ij} + (L_0 - L_1) \cos \left( \theta_0^{ij} + r_0^{ij} \right),$$

$$y_c^{ij} = x_0^{ij} + (L_0 - L_1) \sin \left( \theta_0^{ij} + r_0^{ij} \right).$$

Figure 12 depicts the approximated reachable range using constant and variable constraints. The intersections between boundaries of the constant and variable constraints exist in original
reachable range. Thus, the range included in the original reachable range is obtained. When third joint position calculated by MPC exists on the approximated reachable range, each joint angle is calculated by equation (7), (8) and (12). The approximated reachable range is smaller than the original reachable range, however, we think that it does not strongly affect the performance because the third joint position can move to any point on which the approximated reachable range covers the most mobile region.

6. Simulation
6.1. Simulation condition
Table 2 shows the parameters for numerical simulations and Table 3 shows each obstacle position and size of margin. The initial state of robot is defined by

\[ x_r(t) = [X_0, Y_0, x_0, y_0, x_0, -y_0, -x_0, y_0, -x_0, -y_0]^T. \] (23)

The reference trajectory of robot is defined by

\[ x_r(t) = [X_0 + V_g t, Y_0, x_0, y_0, x_0, -y_0, -x_0, y_0, -x_0, -y_0]^T. \] (24)

Figure 13 depicts the location for the robot, obstacles and reference trajectory on CoG.

In conventional method [13], the reachable range is not considered. Thus, each \( x \)-axis third joint position is fixed to \( \pm 0.22 \) m and the movable scope of \( y \)-axis third joint position
### Table 2. Parameters of simulation.

| Parameter                                      | Value         |
|-----------------------------------------------|---------------|
| Initial gravity position \((X_0, Y_0)\) [m]   | \((0.5, 2.5)\) |
| Initial 3rd joint position \((x_0, y_0)\) [m] | \((0.18, 0.27)\) |
| Reference velocity \(V_g\) [m/s]              | 0.35          |
| Length of horizon \(T\) [s]                   | 0.5           |
| Max. input of gravity position \(u_{g,max}\) [m/s] | 0.45         |
| Max. input of third joint position \(u_{l,max}\) [m/s] | 0.1          |
| Discrete-times for simulation and MPC [s]     | 0.01 and 0.1  |

### Table 3. Obstacle positions.

| Position [m] | Margin [m] |
|--------------|------------|
| \((2.00, 2.40)\) | 0.2        |
| \((3.20, 2.80)\) | 0.2        |
| \((4.00, 3.00)\) | 0.2        |
| \((4.50, 2.34)\) | 0.2        |

Figure 13. Simulation condition.

is \(0.07 \text{ m} \leq |y^j| \leq 0.262 \text{ m}\). Each joint angle is calculated by equation (7), (8) and (12). Moreover, the trajectory of CoG equal to equation (24), however, the target value of each \(y\)-axis joint position is \(\pm 0.262 \text{ m}\) which is the max value of movable scope. The state is updated achieved by discrete state equation considering each joint angle.

6.2. Simulation results

Figure 14 depicts the simulation results by the proposed method. (a) depicts the movement locus of robot and the green, blue, red and orange lines are trajectories of each wheel. (b) depicts the extended figure of (a). In (b), the green ranges are original reachable ranges of each third joint, the orange ranges are approximated reachable ranges and the red lines are internal common tangents of each leg. (c) depicts the input of CoG, (d) depicts the \(x\)-axis third joint positions and (e) depicts the \(y\)-axis third joint positions. Moreover, (f) depicts the first joint angles, (g) depicts the second joint angles and (h) depicts the third joint angles. Each abscissa axis between (c) and (h) is the \(x\)-axis position of CoG. Figure 15 depicts simulation results by the conventional method [13] where obstacle avoidance with lateral motion on legs is considered. The notation of each figure in Fig. 15 is same to Fig. 14. In Fig. 14(b), the blue lines are the mobile ranges of each third joint position.

In Fig. 14(a), the robot avoids multiple obstacles by the proposed method. Moreover, in Fig. 14(d) and Fig. 14(e), the third joint positions move in all direction. However, in Fig. 15(a) and Fig. 15(c), the robot does not avoid those obstacles by the conventional method and the robot stops. In Fig. 15(e), the \(y\)-axis third joint positions of FR and RL are shrunken to the utmost limit of the constraints. The slope of internal common tangent for FR is 0.348 and the slope for RL is 0.227 at the time. The slope for FR is larger than slope for RL, so the passage width becomes narrow with moving forward. The robot cannot still shrink legs, so the robot stops. It is confirmed in Fig. 15(b).

The robot does not stop by the proposed method. The range of moving the third joint position is extended by the approximated reachable range explained in section 5. The robot avoids becoming narrow passage width where it cannot pass through; the robot avoids stopping
Figure 14. Simulation results by proposed method.
Figure 15. Simulation results by conventional method.
and passes through this space. Moreover, the slope for FR is 0.297 and the slope for RL is 0.300 at the time.

In Fig. 14(a) and Fig. 15(a), the wheel positions change rapidly. For achieving the reference velocity vector field, the reference steering angle rotates ±π and the sign of wheel velocity is reversed. In this paper, the effect of this switching is not considered. However, by this switching, motors might be broken down and the target movement might not be achieved. Thus, we have to avoid this switching in the future work.

7. Conclusion

In this paper, the aim is model predictive control considering longitudinal and lateral directions. In order to achieve it, the reachable range of the third joint position is considered. Thus, the aim is realized by applying the reachable range to MPC. The efficiency of the proposed method is shown through simulations of avoiding neighboring obstacles.

However, the third joint angle changes rapidly for achieving the reference steering angle. It might become a cause of breakdown of motors. Thus, the future works are avoidance of the rapid change and verification of proposed method by the experimental robot.

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References

[1] S Kitano, S Hirose, G Endo and E F Fukushima 2013 Development of lightweight Sprawling-type Quadruped Robot TITAN-XIII and its Dynamic Walking Int. Con. on Intelligent Robots and System (Tokyo: IEEE) pp 6025-30
[2] H Kazemi, V J Majd and M M Moghaddam 2013 Modeling and robust backstepping control of an underactuated quadruped robot in bounding motion Robotica 31 issue 3 pp 423-39
[3] G Zhang, X Rong, C Hui, Y Li and B Li 2015 Torso motion control and toe trajectory generation of a trotting quadruped robot based on virtual model control Advanced Robotics DOI: 10.1080/01691864.2015.113889
[4] S Gay, J S Victor and A Ijspeert 2013 Learning Robot Gait Stability using Neural Networks as sensory Feedback Function for Central Pattern generators Int. Con. on Intelligent Robots and Systems pp 194-201
[5] N H Wilcox, T Litwin, J Biesiadecki, J Matthews, M Heverly and J Morrison 2007 ATHLETE: A Cargo Handling and Manipulation Robot for the Moon Journal of Field Robotics 24 issue 5 pp 421-34
[6] S H Lim and J Teo 2014 A Multi-Objective Evolutionary Approach to optimize the Morphology of a Six Articulated-Wheeled Robot Int. Con. on Artificial Intelligence in Engineering and Technology pp 25-30
[7] Y Hagimori, K Nonaka and K Sekiguchi 2015 Sequential Model Predictive Control for Leg/Wheel Mobile robot using Single Leg Model Int. Automatic Control Con. pp 522-7
[8] A T Rashid, A A Ali, M Frasca and L Fortuna 2013 Path planning with obstacle avoidance based on visibility binary tree algorithm Robotics and Autonomous System 61 issue 12 pp 1440-9
[9] Z Shiller, S Sharma, I Stern and A Stern 2013 Online obstacle avoidance at high speeds Int. Journal of Robotics bf 32 pp 1030-47
[10] I Mas and C Kitts 2012 Obstacle Avoidance Policies for Cluster Space Control of Nonholonomic Multirobot Systems IEEE/ASME Transactions on Mechatronics 17 no 6 pp 1068-79
[11] T. Suzuki, M. Takahashi 2012 Obstacle Avoidance Considering Robot’s Size for an Autonomous Omni-Directional Mobile Robot by Simultaneous Control of Translational and Rotational Motions Transactions of the Japan Society of Mechanical Engineers Series (C) (in Japan) 76 no 772 pp 385-94
[12] N Takahashi and K Nonaka 2012 Model Predictive Obstacle Avoidance and Wheel Allocation of Mobile robots using Embedded CPU Journal of System Design and Dynamics 6 no 4, pp 447-65
[13] N. Suzuki, K. Nonaka and K. Sekiguchi 2015 Model Predictive Obstacle Avoidance Control with Passage Width Constraints for Leg/Wheel Robots The IEEE Multi-Con. on Systems and Control pp 330-5
[14] K. Nonaka 2008 Nonlinear Tracking Control and Leg Optimal Allocation for Wheeled Mobile Robots with 3DOF SCARA Legs Int. Con. on Motion and Vibration Control
[15] J. Mattingley and S. Boyd 2012 CVXGEN: a code generator for embedded convex optimization Optimization and Engineering 13 no 1 pp 1-27