Role of the explicit tensor correlation in neutron halo nuclei

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Abstract. We propose a new mechanism to explain the disappearance of the $N = 8$ shell gap in $^{10,11}\text{Li}$ with the tensor correlation. We first take into account the tensor correlation fully in the $^9\text{Li}$ core in the so-called tensor-optimized shell model. In $^{10,11}\text{Li}$, the tensor correlations in $^9\text{Li}$ are Pauli-blocked by additional two neutrons, which makes the $p$-state configurations pushed up in energy. As the final result including the pairing correlations, by solving the coupled $^9\text{Li}+n+n$ problem, the $N = 8$ shell gap naturally disappears in $^{10,11}\text{Li}$, such as the $s^2$ component with around 50% in $^{11}\text{Li}$.

1. Introduction

Studies of unstable nuclei have attracted much attention of physicists with the development of radioactive beams[1]. $^{11}\text{Li}$ is known to have a neutron halo structure known from the observed large radius and the small two-neutron separation energy. These exotic features suggest a large mixing of the $(1s)^2$ component for last two neutrons to the ordinary $(0p)^2$ component and the breaking of the neutron magic number $N = 8$ in $^{11}\text{Li}$[2]. However, the essential mechanism for this phenomenon is still unclear in $^{11}\text{Li}$ and neighboring nuclei.

The biggest puzzle from the theory side is a large $s$-wave component for the halo neutrons. If we interpret this fact in the shell model, the shell gap at $N = 8$ has to disappear. However, no theory is able to provide the disappearance of the $N = 8$ shell gap observed in $^{11}\text{Li}$ and neighboring nuclei. So far, there were many theoretical studies for $^{11}\text{Li}$[3, 4, 5, 6, 7, 8, 9, 10] and essentially all the theoretical works of $^{11}\text{Li}$ had to accept that the $1s_{1/2}$ single particle state is brought down to the $0p_{1/2}$ state without knowing its reason[5, 10]. It is therefore the challenge to understand the disappearance of the $N = 8$ shell gap, which is worked out in this paper by developing a framework to treat the tensor force explicitly[11, 12]. The halo structure of $^{11}\text{Li}$ is also related with the $1s$-state and the $0p$-state in $^{10}\text{Li}$. Several experiments suggest the $s_{1/2}$-states appears together with the $p_{1/2}$-states in energy[13, 14, 15].

In the nuclear force, the tensor force plays an important role in the nuclear structure[16, 17]. In many nuclear models, the dominant effect of the tensor force is renormalized into central and spin-orbit terms of the effective interactions. We consider that it is important to investigate the roles of the tensor force on the nuclear structure by explicitly describing the tensor correlation.
It is known that the $2p2h$ excitation is important to produce the strong tensor correlation explicitly in the wave function [16, 18, 19]. In the previous studies, we showed the role of the tensor correlation for He isotopes [19, 20, 21, 22]. It is found that the $(0s_{1/2})^{-2}(0p_{1/2})^2$ type of the $2p2h$ excitation of a proton-neutron pair has a special importance in describing the $sd$ coupling of the tensor force in the relative motion and the tensor correlation in He isotopes. This $2p2h$ excitation causes the Pauli-blocking in the $^4\text{He}+n(+n)$ systems for the $p_{1/2}$ orbit of last neutrons, which contributes to the $p$ wave doublet splitting in $^3\text{He}$ by about 30% [19, 22].

For $^{11}\text{Li}$, the similar blocking is expected and may produce the energy loss for $(0p)^2$ configuration of $^{11}\text{Li}$ [11, 12], in analogy with the pairing-blocking (Pauli-blocking induced by the pairing correlation [8, 23, 24]). It is worthwhile to study the blocking effect of the tensor correlation on the disappearance of the $N=8$ shell gap.

2. Models of $^{9}\text{Li}$ and $^{10,11}\text{Li}$

We explain the essential point of the model for Li isotopes to incorporate the tensor correlation, whose details were given in the Refs. [8, 11]. We start from $^{9}\text{Li}$, whose Hamiltonian is given as

$$H^{(9)\text{Li}} = \sum_{i=1}^{9} t_i - t_G + \sum_{i<j} v_{ij},$$

(1)

where, $t_i$, $t_G$, and $v_{ij}$ are the kinetic energy of each nucleon and the center-of-mass of the system, and the two-body $NN$ interaction. We use the effective interaction $\text{GA}'$ [19, 21] for $v_{ij}$, which has the explicit tensor force applying the $\text{AV8}'$ realistic potential from the $G$-matrix theory [25, 26]. We adjust the central force in this interaction to reproduce the observed binding energy and the matter radius of $^{9}\text{Li}$ [11]. This procedure does not change the characteristics of the tensor and pairing correlations. The wave function of $^{9}\text{Li}(3/2^-)$ is expressed by a multi-configuration $\sum_i a_i \Phi_i^{3/2^-}$, where we consider up to the $2p2h$ excitations within the $p$ shell in a shell model type wave function, the so-called tensor-optimized shell model (TOSM) [11, 21]. In TOSM, we adopt the spatially modified harmonic oscillator wave function (h.o.w.f.) as a single particle orbit and treat their length parameters $b_{0\text{h}}$, $b_{0p_{1/2}}$ and $b_{0p_{3/2}}$ as variational ones. We solve the variational equation of Hamiltonian for $^{9}\text{Li}$ in Eq. (1) and determine sets of amplitudes $\{a_i\}$ and length parameters.

For $^{11}\text{Li}$, the Hamiltonian is given in the orthogonality condition model [8],

$$H^{(11)\text{Li}} = H^{(9)\text{Li}} + \sum_{i=1}^{3} T_i - T_G + \sum_{i=1}^{2} V_{cm,i} + V_{nn},$$

(2)

where $H^{(9)\text{Li}}$, $T_i$ and $T_G$ are the internal Hamiltonian of $^{9}\text{Li}$ given in Eq. (1), the kinetic energies of each cluster and the center-of-mass of the three-body system, respectively. The $^{9}\text{Li}$-$n$ potential, $V_{cm}$, is given by a folding-type one [8]. For the potential $V_{nn}$ of the last two neutrons, $\text{AV8}'$ is used. We remove the Pauli forbidden states for the $^{9}\text{Li}$-$n$ relative motion, which depends on the configuration of $^{9}\text{Li}$, $\Phi_i^{3/2^-}$ [8, 11, 19]. We adjust the strength of the $^{9}\text{Li}$-$n$ potential to reproduce the observed two-neutron separation energy of $^{11}\text{Li}$ as 0.31 MeV and do not introduce any dependence for each partial wave.

The wave function of $^{11}\text{Li}$ with the spin $J$ is given as

$$\Psi^{(11)\text{Li}} = \sum_i A_i \left\{ [\Phi_i^{3/2^-}, \chi_i^J(nn)]^J \right\},$$

(3)

where $j$ is the spin of the last two neutrons. Since the amplitude $\{a_i\}$ in the $^{9}\text{Li}$ wave function depends on the relative coordinates between $^{9}\text{Li}$ and the last two neutrons from their coupling, we express $\{a_i\}$ by the function $\chi_i^J(nn)$. The wave function of $^{10}\text{Li}$ is similarly given in a coupled two-body problem of the $^{9}\text{Li}+n$ system.
Table 1. Properties of $^9$Li.

|                  | Theory | Expt. |
|------------------|--------|-------|
| $E$ [MeV]        | −45.3  | −45.3 |
| $\langle V_T \rangle$ [MeV] | −20.7  | —     |
| $R_{\text{matter}}$ [fm] | 2.31   | 2.32±0.02[27] |
| $R_{\text{charge}}$ [fm] | 2.23   | 2.217±0.035[28] |
|                  |        | 2.185±0.033[29] |

Table 2. Configuration mixing for $^9$Li in %.

|                | $0p0h\mid (0p_{3/2})_{01}\overline{0}(0p_{1/2})_{01}\mid (0s)_{10}^2(0p_{1/2})_{10}^2$ |
|----------------|-----------------------------------------------|
|                | 82.9                               | 9.0 | 7.2 |

3. Results
3.1. $^9$Li in TOSM
In Tables 1 and 2, we show the results of the $^9$Li in TOSM, where $b_0s$ and $b_0p_{3/2}$ are optimized as 1.45 fm and 1.8 fm. For $0p_{1/2}$-orbit, we adopt two bases with different length of $b_0p_{1/2}$ as 0.85 fm and 1.8 fm. This is to superpose the tensor and pairing correlations, since the tensor correlation is optimized with spatially shrunk excited nucleons[19, 21]. On the other hand, the pairing correlation is optimized with the spatially extended orbits. Among the $2p2h$ configurations, the large probabilities are given by $(0p_{3/2})_{01}^2(0p_{1/2})_{01}^2$ and $(0s)_{10}^2(0p_{1/2})_{10}^2$. The former one represents the $0p$ shell neutron pairing correlation and the latter represents the tensor correlation[11]. The subscripts 01 or 10 represent $J$ and $T$, spin and isospin for the two-nucleon pair, respectively. The charge radius $R_{\text{charge}}$ is found to be reproduced well.

3.2. Pauli-blocking effect in $^{11}$Li and $^{10}$Li
We consider the dynamical effect of the tensor and pairing correlations on the structures of $^{11}$Li. When we add two neutrons to $^9$Li, the $2p2h$ excitations induced by two correlations in $^9$Li are blocked by the last neutrons under the Pauli-principle. In Fig. 1, we show this Pauli-blocking effect on the $(0p)^2$ and $(1s)^2$ configurations of the last two neutrons in $^{11}$Li. For the $(0p)^2$ case, the $2p2h$ excitation in $^9$Li is suppressed[8, 11] and the energy of $^9$Li is lost inside $^{11}$Li. For the

![Pauli blocking](image)

Figure 1. Pauli-blocking effect in $^{11}$Li.
(1s)^2 case, the Pauli-blocking does not occur and \(^{9}\text{Li}\) gains its energy by the configuration mixing with the \(^{2}p2h\) excitations. Hence, the relative energy distance between two configurations of \(^{11}\text{Li}\) is expected to become small, which reduces the shell gap. The same blocking effect is expected in \(^{10}\text{Li}\). We examine whether our result provides a sufficient amount of the magicity breaking in \(^{11}\text{Li}\) and \(^{10}\text{Li}\).

3.3. \(^{11}\text{Li}\) and \(^{10}\text{Li}\) in a coupled \(^{9}\text{Li}+n+n\) model

We perform the coupled three-body calculation to obtain \(\Psi_J^{(11}\text{Li})\) in Eq. (3). In Fig. 2, our present calculation “Present” gives 46.9% of a large amount of the \((1s)^2\) probability \(P(s^2)\) for \(^{11}\text{Li}\), which explains the observations\cite{2}. The probabilities of \((p_{1/2})^2\), \((p_{3/2})^2\), \((d_{5/2})^2\) and \((d_{3/2})^2\) for the last two neutrons are obtained as 42.7%, 2.5%, 4.1% and 1.9%, respectively. For comparison, we calculate \(^{11}\text{Li}\) without core excitations of \(^{9}\text{Li}\) (“Inert Core”), namely, we adopt only the single \(0p0h\) configuration for \(^{9}\text{Li}\) without the Pauli-blockings explained in the middle panel of Fig. 1. In that case, \(^{11}\text{Li}\) has almost \(p\)-shell closed configuration of 90.6% with a small \(P(s^2)\) of 4.3%. In the present calculation, we estimate the energy difference between \((1s)^2\) and \((0p)^2\) configurations as –0.1 MeV, which is enough to couple the \((1s)^2\) configuration with the \((0p)^2\) one by the pairing correlation between the last two neutrons.

![Figure 2](image1.png) Figure 2. \(s^2\) component \(P(s^2)\) of \(^{11}\text{Li}\).

![Figure 3](image2.png) Figure 3. Two neutron correlation density \(\rho_{nn}(r_{\text{core-n}}, \theta)\) of \(^{11}\text{Li}\).

### Table 3. Various r.m.s. radius of \(^{11}\text{Li}\).

| \(R_{\text{matter}}\) [fm] | Present | Expt.        |
|-------------------------|---------|--------------|
| \(R_{\text{proton}}\)    | 2.34    | 2.88±0.11\cite{27} |
| \(R_{\text{neutron}}\)   | 3.73    | 3.21±0.17\cite{27} |
| \(R_{\text{charge}}\)    | 2.44    | 2.467±0.037\cite{28} |
| \(R_{n-n}\)              | 7.33    | 2.423±0.037\cite{29} |
| \(R_{e-2n}\)             | 5.69    |              |

### Table 4. The positions of the \(p\)-wave resonances in \(^{10}\text{Li}\) in units of MeV, where \(E_r\) and \(\Gamma\) are the resonance energy measured from the \(^{9}\text{Li}+n\) threshold and the decay width. The scattering lengths \(a_s\) of the \(s\)-states are in units of fm.

| \((E_r, \Gamma)(1^+)\) [MeV] | \((E_r, \Gamma)(2^+)\) [MeV] | \(a_s(1^-)\) [fm] | \(a_s(2^-)\) [fm] |
|-------------------------------|-------------------------------|-------------------|-------------------|
| 0.22, 0.09                    | 0.64, 0.45                    | -5.6              | -17.4             |
We show the results of the various r.m.s. radius of $^{11}\text{Li}$ in Table 3. Our result gives a large matter radius $R_{\text{matter}}$ for $^{11}\text{Li}$, which are enough to explain the observations because of a large value of $P(s^2)$ for halo neutrons. The observed charge radius ($R_{\text{charge}}$) is also reproduced well in our wave function. The enhancement of charge radius from that of $^9\text{Li}$ is mainly caused by the large distance between $^9\text{Li}$ and the paired two neutrons $R_{c-2n}$ obtained as 5.69 fm$^{11}$. This is a recoil effect and also related with the di-neutron correlation with a spatially localized two-neutron pair$^4$, shown in Fig. 3, where $r_{\text{core-n}}$ and $\theta$ are the core-neutron distance and the opening angle between two neutrons, respectively. In Fig. 3, the di-neutron type configuration (a large $r_{\text{core-n}}$ and a small $\theta$) gives a maximum value of the density, although the density of neutrons is widely distributed. The expectation value of $\theta$ is obtained as 65.3 degree. The detailed analysis of the density of halo neutrons is given by Kikuchi et al.$^{32}$. The present model also describes the $Q$-moments (in units of $e$ fm$^2$) of $^9\text{Li}$ and $^{11}\text{Li}$ as $-2.65$ and $-2.80$, respectively, while the experiments give $-2.74 \pm 0.10$ for $^9\text{Li}$ and $3.12 \pm 0.45$ as $|Q|$ for $^{11}\text{Li}$$^{33}$. Our values of $^9\text{Li}$ and $^{11}\text{Li}$ do not differ so much to each other. To explain this small difference, we consider the recoil effect in the $Q$-moment of $^{11}\text{Li}$. In our wave function of $^{11}\text{Li}$, last two neutrons almost form the $0^+$ state with around 99%. In that case, the $Q$-moment for the relative motion part of $^9\text{Li}-2n(c-2n)$ almost vanishes because of the non-zero rank operator of the $Q$-moment. This means that the recoil effect from the clusterization is negligible. Therefore, the $Q$-moment of $^{11}\text{Li}$ is caused mainly by that of $^9\text{Li}$ inside $^{11}\text{Li}$.

We further investigate the spectroscopic properties of $^{10}\text{Li}$. In Table 4, the dual $p$-state resonances coupled with the spin of $^9\text{Li}(3/2^-)$ are obtained near the $^9\text{Li}+n$ threshold energy, while the experimental uncertainty is still remaining including the spin assignment$^{15}$. For the $s$-wave states, their scattering lengths $a_s$ of the $^9\text{Li}+n$ system show negative values. In particular, the $2^-$ state show a large negative value of $a_s$, which is comparable to that of the $nn$ system ($-18.5$ fm) in the $^1S_0$ channel, and indicates the existence of the virtual $s$-state near the $^9\text{Li}+n$ threshold energy. Therefore the inversion phenomenon in $^{10}\text{Li}$ is reasonably explained in the present model.

We finally calculate the Coulomb breakup strength of $^{11}\text{Li}$ into $^9\text{Li}+n+n$ system to investigate the properties of the dipole excited states, and compare the strength with the new data from RIKEN$^{34}$. We use the Green’s function method combining with the complex scaling method to calculate the three-body breakup strength$^{35}$ using the dipole strength and the equivalent photon method. We cannot find three-body resonances to make a structure in the strength. In Fig. 4, the present model well reproduces the experiment, in particular, for low energy enhancement. Among the final state correlations, both of the core-$n$ and $nn$ interactions give comparable contributions to produce the low energy enhancement in the strength$^{12}$.
4. Summary
We have performed the systematic studies of $^{9,10,11}\text{Li}$ based on the extended three-body model, in which the tensor and pairing correlations are explicitly considered for $^9\text{Li}$. Two correlations in $^9\text{Li}$ were found to play an essential role to explain the anomalous structures of $^{11}\text{Li}$ and $^{10}\text{Li}$. The Pauli-blocking effect induced by the tensor and pairing correlations in $^9\text{Li}$ naturally explains the breaking of magicity and the halo formation in $^{11}\text{Li}$ and the inversion phenomena of $^{10}\text{Li}$, simultaneously. The present model further reproduces the recent observations of Li isotopes such as the charge radius, $Q$-moments and the Coulomb breakup strength of $^{11}\text{Li}$.

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