Scalar-tensor theories of gravity, neutrino physics, and the $H_0$ tension

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Abstract. We use Planck 2018 data to constrain the simplest models of scalar-tensor theories characterized by a coupling to the Ricci scalar of the type $F(\sigma)R$ with $F(\sigma) = N_{pl}^2 + \xi\sigma^2$. We update our results with previous Planck and BAO data releases obtaining the tightest constraints to date on the coupling parameters, that is $\xi < 5.5 \times 10^{-4}$ for $N_{pl} = 0$ (induced gravity or equivalently extended Jordan-Brans-Dicke) and $(N_{pl}\sqrt{8\pi G})^{-1} < 1.8 \times 10^{-5}$ for $\xi = -1/6$ (conformal coupling), both at 95% CL. Because of a modified expansion history after radiation-matter equality compared to the ΛCDM model, all these dynamical models accommodate a higher value for $H_0$ and therefore alleviate the tension between Planck/BAO and distance-ladder measurement from SNe Ia data from 4.4σ at best to 2.3σ. We show that all these results are robust to changes in the neutrino physics. In comparison to the ΛCDM model, partial degeneracies between neutrino physics and the coupling to the Ricci scalar allow for smaller values $N_{\text{eff}} \sim 2.8$, 1σ lower compared to the standard $N_{\text{eff}} = 3.046$, and relax the upper limit on the neutrino mass up to 40%.

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1 Introduction

The distance-ladder measurement of the Hubble constant from supernovae type Ia (SNe Ia) [1, 2] and from strong-lensing time delays [3–5] disagrees with the value inferred from the fit of the cosmic microwaves background (CMB) anisotropies [6, 7]. In particular, the value inferred using the Planck legacy data for a flat ΛCDM cosmological model, $H_0 = (67.36 \pm 0.54)$ km s$^{-1}$Mpc$^{-1}$ [7], is in a 4.4σ tension with the most recent measurement from the SH0ES team [8], that is $H_0 = (74.03 \pm 1.42)$ km s$^{-1}$Mpc$^{-1}$, determined using Cepheid-calibrated SNe Ia with new parallax measurements from HST spatial scanning [2] and from Gaia DR2 [9], in a 3.2σ tension with the strong-lensing time delay determination from the H0LiCOW collaboration [5], that is $H_0 = (73.3^{+1.7}_{-1.8})$ km s$^{-1}$Mpc$^{-1}$, and in a 5.3σ tension with the combined SH0ES + H0LiCOW value $H_0 = (73.8 \pm 1.1)$ km s$^{-1}$Mpc$^{-1}$ [5].

Although it does not seem possible to completely solve the discrepancy between CMB and local measurements by considering unaccounted systematic effects [10–12], revisions of the determination of the Hubble rate based on the Cepheid calibration [13–15], and from SNe Ia calibrated using the tip of the red giant branch method [16] point to values which are still higher than the Planck measurements, i.e. $\sim 70$ km s$^{-1}$Mpc$^{-1}$, but to a smaller extent (see however [17–20] for more recent developments).

Alternatively, the $H_0$ tension can be addressed by evoking new physics beyond the ΛCDM concordance model, either in the late or early time Universe [21]. Late-time resolutions include a phantom-like dark energy (DE) component [22, 23], a vacuum phase transition...
interacting dark energy [29–31], or quintessence in a non-flat Universe [32]. However, these resolutions are tightly constrained [1, 23, 30, 33] by late-time observational data, especially those from baryon acoustic oscillations (BAO) [34–36].

With the highly precisely determined angular scale of the last-scattering surface (LSS) [37], it has been suggested that a smaller value of the comoving sound horizon at baryon drag $r_s$ can provide a higher value of $H_0$ without spoiling the CMB angular power spectrum measurements and without changing the BAO observables [21, 38]. An example of such an early-time modification is the extension to additional light relics, eventually interacting with hidden dark sectors [39–46]. Another promising early-time solution, is an exotic early dark energy (EDE) component that remains subdominant for the majority of the cosmological evolution of the Universe and injects a small amount of energy in a very narrow redshift window [47–51]. Note that features in the primordial power spectrum [52, 53] and modifications of the recombination history [54, 55] are not able to increase $H_0$.

Another interesting possibility is to alleviate the $H_0$ tension through a modification of General Relativity (GR) [56–65], which can include early- and late-time modifications of the expansion history with respect to ΛCDM. Scalar-tensor theories of gravity that involve a scalar field non-minimally coupled to the Ricci scalar naturally change the effective relativistic degrees of freedom in the radiation-dominated epoch and the background expansion history from ΛCDM, thus leading to a higher CMB-inferred $H_0$ [56, 57, 60]. This has been shown for the extended Jordan-Brans-Dicke (eJBD) model in Refs. [56, 57] where the cosmological parameters estimation has been carried out using data from the Planck 2013 and 2015 release respectively. The robustness of a higher $H_0$ in these theories has been proved in Ref. [60], where a more general form of the non-minimal coupling (NMC) to gravity has been considered. In this paper, we confirm earlier results for the particular cases of eJBD and a conformally coupled (CC) scalar field using the most recent cosmological data.

Since the mechanism that drives a higher inferred $H_0$ relies on the radiation-like behavior of the scalar field at early times, it is interesting to investigate to what extent these simple scalar-tensor theories are degenerate with effective number of relativistic species due to neutrinos and to any additional massless particles produced well before recombination $N_{\text{eff}}$. The current tight constraints from the latest Planck 2018 data $N_{\text{eff}} = 2.89 \pm 0.19$ ($N_{\text{eff}} = 2.99 \pm 0.17$ including BAO) at 68% CL [7] can be changed in modified gravity theories as previously shown in the context of $f(R)$ gravity in [66, 67].

While changing $N_{\text{eff}}$ can lead to a higher value for $H_0$ compared with the value inferred in the ΛCDM model from the CMB anisotropies measurements, in the extension of ΛCDM model with non-zero total neutrino mass $m_\nu$, lower values of $H_0$ correspond to higher values of $m_\nu$ and higher values of $H_0$ correspond to lower $m_\nu$. For instance, the constraint from Planck 2018 data in combination with BAO data is $m_\nu < 0.12$ eV at 95% CL and combining with the measurement from the SH0ES team the limit further tightens to $m_\nu < 0.076$ eV [7, 68]. In scalar-tensor theories of gravity, it is possible to keep fixed the angular diameter distance at decoupling or even increase it in order to recover a higher $H_0$ while increasing the total neutrino mass value [69]. Given that neutrino oscillations are the only laboratory evidence of physics beyond the Standard Model of Particle Physics [70], it is natural to ask ourselves either how the cosmological constraints on neutrino physics can be relaxed in these simple models of scalar-tensor theories compared to the ΛCDM concordance model and how much constraints become larger in the NMC models including the neutrino parameters.
Future CMB experiments, such as the Simons Observatory \cite{simonsobservatory} \footnote{https://simonsobservatory.org/}, CMB-S4 \cite{cmb-s4} \footnote{https://cmb-s4.org/}, and future LSS surveys from DESI \footnote{http://desi.lbl.gov/} \cite{desi}, Euclid \footnote{http://sci.esa.int/euclid/} \cite{euclid, eucliddata}, LSST \footnote{http://www.lsst.org/} \cite{lsst}, SKA \footnote{http://www.skatelescope.org/} \cite{skatelescope, ska} will help to improve the constraints on these extended cosmologies \cite{extendedcosmology} and limit the degeneracy of neutrino parameters $N_{\text{eff}}$ and $m_\nu$ with scalar-tensor theories \cite{scalar-tensor}.

The paper is organized as follows. Sec. 2 is devoted to a description of the models considered. We present our results in Sec. 3. We start in Sec. 4 by discussing the updated constraints on the eJBD and CC model in light of new CMB and BAO data, then we further improve the analysis by extending the cosmological parameters to the ones describing the neutrino sector in Secs. 5.1-5.2-5.3. We draw our conclusions in Sec. 6. In App. A, we collect all the tables with the constraints on the cosmological parameters obtained with our MCMC analysis.

2 The model: non-minimally coupled scalar-tensor theory

As far as cosmological tests are concerned, one of workhorse models to test deviations from GR is the eJBD \cite{eJBD, eJBD2} theory, which has been extensively studied \cite{eJBDanalysis, eJBDanalysis2, eJBDanalysis3, eJBDanalysis4, eJBDanalysis5, eJBDanalysis6, eJBDanalysis7, eJBDanalysis8, eJBDanalysis9, eJBDanalysis10}. NMC eJBD theory is perhaps the simplest extension to GR within the more general Horndeski theory \cite{Horndeski}:

$$S = \int d^4 x \sqrt{-g} \left[ \frac{F(\sigma)}{2} R - \frac{g^{\mu \nu}}{2} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) + \mathcal{L}_m \right],$$

(2.1)

where $F(\sigma) = N_{\text{pl}}^2 + \xi \sigma^2$, $R$ is the Ricci scalar, and $\mathcal{L}_m$ is the Lagrangian density for matter fields. As in \cite{eJBDanalysis2}, we do not consider a quintessence-like inverse power-law potential (see for instance \cite{quintessence1, quintessence2, quintessence3, quintessence4}), but we restrict ourselves to a potential of the type $V(\sigma) = \lambda F^2(\sigma)/4$ for which the scalar field is effectively massless. Our choice reduces to eJBD theory after the redefinition $\sigma^2 = \phi/(8\pi \xi)$, $\xi = 1/(4\omega_{\text{BD}})$, and setting $N_{\text{pl}} = 0$.

For a flat Friedmann-Lemaître-Robertson-Walker (FLRW) Universe with $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$, the background Friedmann equations in the Jordan frame are \cite{friedmann}:

$$3FH^2 = \rho_m + \frac{\dot{\sigma}^2}{2} + V(\sigma) - 3H \dot{F},$$

$$-2F\dot{H} = \rho_m + \dot{\sigma}^2 + \ddot{F} - H \dot{F}.$$  

(2.2)\hspace{1cm}(2.3)

These equations are supplemented by the Klein-Gordon equation that governs the evolution of the scalar field:

$$\Box \sigma - V_\sigma + F_\sigma R = 0.$$  

(2.4)

The coupling between gravity and the scalar degree of freedom induces a time varying Newton’s gravitational constant $G_N$, which is given by $G_N = 1/(8\pi F)$. This quantity is usually denoted by the cosmological Newton’s gravitational constant, as opposed to the one that is actually measured in laboratory Cavendish-type experiments which is rather given by \cite{friedmann}:

$$G_{\text{eff}} = \frac{1}{8\pi F} \frac{2F + 4F_\sigma^2}{2F + 3F_\sigma^2}.$$  

(2.5)

\footnote{https://simonsobservatory.org/} \footnote{https://cmb-s4.org/} \footnote{http://desi.lbl.gov/} \footnote{http://sci.esa.int/euclid/} \footnote{http://www.lsst.org/} \footnote{http://www.skatelescope.org/}
Deviations from GR for a theory of gravitation are parameterized by the so called post-Newtonian (PN) expansion of the metric \cite{98}. In such an expansion, the line element can be expressed as:

\[ ds^2 = -(1 + 2\Phi - 2\beta_{\text{PN}}\Phi^2)dt^2 + (1 - 2\gamma_{\text{PN}}\Phi)dx^2, \]

where we have retained only the two non-null contributions to the PN expansion in the case of NMC theories, that is \cite{97}

\[
\gamma_{\text{PN}} = 1 - \frac{F_\sigma^2}{F + 2F_\sigma^2}, \quad \beta_{\text{PN}} = 1 + \frac{FF_\sigma}{8F + 12F_\sigma^2} \frac{d\gamma_{\text{PN}}}{d\sigma}. \tag{2.7}
\]

Solar-system experiments agree with GR predictions, for which both \(\gamma_{\text{PN}}\) and \(\beta_{\text{PN}}\) are identically equal to unity, at a very precise level. Measurements of the perihelion shift of Mercury constrain \(\beta_{\text{PN}} - 1 = (4.1 \pm 7.8) \times 10^{-5}\) at 68% CL \cite{98} and Shapiro time delay constrains \(\gamma_{\text{PN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}\) at 68% CL \cite{99}.

In this paper we restrict ourselves to two simple models, both of which only contain one extra parameter with respect to the \(\Lambda\)CDM model: induced gravity (IG) described by \(N_{pl} = 0\) and \(\xi > 0\), and a conformally coupled scalar field (CC) for which \(\xi = -1/6\) and the free parameter is \(N_{pl} > M_{pl}\). For both models, the effective value of the Newton’s gravitational constant \(G_N\) decreases with time, whereas the scalar field \(\sigma\) increases for \(\xi > 0\), e.g. for IG, and it decreases for \(\xi < 0\), e.g. for CC\(^1\). We have \(\gamma_{\text{PN}} < 1\) and \(\beta_{\text{PN}} \leq 1\) for \(\xi > 0\) (\(\beta_{\text{PN}} = 1\) for IG), whereas \(\gamma_{\text{PN}} < 1\) and \(\beta_{\text{PN}} > 1\) for \(\xi < 0\).

In order to connect the present value of the field \(\sigma_0 \equiv \sigma(z = 0)\) to the gravitational constant, we impose the condition \(G_{\text{eff}}(\sigma_0) = G\), where \(G = 6.67 \times 10^{-8}\ \text{cm}^3\ \text{g}^{-1}\ \text{s}^{-2}\) is the gravitational constant measured in a Cavendish-type experiment. This corresponds to

\[
\tilde{\sigma}_0^2 = \frac{1 + 8\xi}{\xi(1 + 6\xi)} \tag{2.8}
\]

for IG and

\[
\tilde{\sigma}_0^2 = \frac{18\tilde{N}_{pl}^2(\tilde{N}_{pl}^2 - 1)}{1 + 3\tilde{N}_{pl}^2}, \tag{2.9}
\]

for the CC case. For simplicity, we denote quantities normalized to \(M_{pl} \equiv 1/\sqrt{8\pi G}\) with a tilde.

### 3 Methodology and datasets

We present in this section the constraints obtained with a Markov Chain Monte Carlo (MCMC) analysis by using the publicly available code \texttt{MontePython}\(^2\) \cite{101, 102} connected to our modified version of the code \texttt{CLASS}\(^3\) \cite{103, 104}, i.e. \texttt{CLASSig} \cite{56}. Mean values and uncertainties on the parameters reported, as well as the contours plotted, have been obtained using \texttt{GetDist}\(^4\) \cite{105}.

\(^1\)We refer the interested reader to Refs. \cite{56, 57, 60} for a detailed analysis of the background dynamics in these models, see also Ref. \cite{100} for the generalization to anisotropic homogeneous cosmological models of the Bianchi I type.

\(^2\)https://github.com/brinckmann/montepython_public

\(^3\)https://github.com/lesgourg/class_public

\(^4\)https://getdist.readthedocs.io/en/latest
As baseline, we vary the six cosmological parameters for a flat ΛCDM concordance model, i.e. $\omega_b$, $\omega_c$, $H_0$, $\tau$, $\ln(10^{10}A_s)$, $n_s$, plus one extra parameter related to the coupling to the Ricci curvature. For IG ($N_{pl} = 0$, $\xi > 0$), we sample on the quantity $\zeta_{IG} \equiv \ln(1 + 4\xi)$, according to [56, 57, 89] in the prior range $[0, 0.039]$. For CC ($N_{pl} > M_{pl}$, $\xi = -1/6$), we sample on $\Delta \tilde{N}_{pl} \equiv \tilde{N}_{pl} - 1$, according to [60], with prior range $[0, 0.5]$. In our updated analysis in Sec. 4 we assume 3 massless neutrino with $N_{eff} = 3.046$, but a more general neutrino sector is considered in the rest of the paper.

We quote constraints on the variation of the Newton’s gravitational constant between the radiation era and the present time $\frac{\delta G}{G} = \frac{G(z = 0) - G_{ini}}{G_{ini}} = \frac{\sigma_{0}^2 - \sigma_{ini}^2}{\sigma_{0}^2} \leq 0$, (3.2)

$$\frac{\dot{G}}{G} = \frac{-2\dot{\sigma}_0}{\sigma_0} \leq 0,$$

$$\frac{G}{G} (z = 0) = \frac{1 + 6\xi}{1 + 8\xi} \leq 1,$$

(3.3) (3.4)

for IG and

$$\frac{\delta G}{G} (z = 0) = \frac{\xi}{N^2_{pl}} \frac{\sigma_{ini}^2 - \sigma_{0}^2}{\xi \sigma_{0}^2} = \frac{\sigma_{0}^2 - \sigma_{ini}^2}{6N^2_{pl} - \sigma_{0}^2} \leq 0,$$

(3.5)

$$\frac{\dot{G}}{G} (z = 0) = \frac{-2\xi \dot{\sigma}_0}{N^2_{pl} + \xi \sigma_{0}^2} = \frac{2\dot{\sigma}_0}{6N^2_{pl} - \sigma_{0}^2} \leq 0,$$

(3.6)

$$\frac{G}{G} (z = 0) = \frac{1}{4} \left( 3 + \frac{M^2_{pl}}{N^2_{pl}} \right) \sigma_{0}^2 \leq 1,$$

(3.7)

for the CC case, where $\delta G_{IG}/G_{IG}$, $\dot{G}_{IG}/G_{IG} = 0$ and $G_{IG}/G = 1$ correspond to the predictions in GR.

We also list the $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\Lambda CDM}$, where negative values indicate a better-fit to the datasets with respect to the ΛCDM model.

### 3.1 Datasets

We constrain the cosmological parameters using several combination of datasets. Our CMB measurements are those from the Planck 2018 legacy release (hereafter P18) on temperature, polarization, and weak lensing CMB anisotropies angular power spectra [106, 107]. The high-multipoles likelihood $\ell \geq 30$ is based on Plik likelihood. We use the low-$\ell$ likelihood combination at $2 \leq \ell < 30$: temperature-only Commander likelihood plus the SimAll EE-only likelihood. For the Planck CMB lensing likelihood, we consider the conservative multipoles range, i.e. $8 \leq \ell \leq 400$. We marginalize over foreground and calibration nuisance parameters of the Planck likelihoods [106, 107] which are also varied together with the cosmological ones.
Baryon acoustic oscillation (BAO) measurements from galaxy redshift surveys are used as primary astrophysical dataset to constraint these class of theories providing a complementary late-time information to the CMB anisotropies. We use the Baryon Spectroscopic Survey (BOSS) DR12 [36] consensus results on BAOs in three redshift slices with effective redshifts $z_{\text{eff}} = 0.38, 0.51, 0.61$ [108–110].

Finally, we consider the combination with a Gaussian likelihood based on the determination of the Hubble constant from from Hubble Space Telescope (HST) observations (hereafter R19), i.e. $H_0 = (74.03 \pm 1.42) \text{ km s}^{-1}\text{Mpc}^{-1}$ [8].

4 Updated Planck 2018 results

The results in this section update those obtained in Ref. [57] for IG and in Ref. [60] for CC, based on the Planck 2015 data (P15) [111, 112] in combination with an older compilation of BAO data, i.e. DR10-DR11 [34, 35, 114].

First, we discuss the IG case. We find that the constraint on the coupling parameter $\xi$ obtained from the CMB alone is almost half of the bound obtained with P15 which was $\xi < 0.0017$ at 95% CL. With the full high-$\ell$ polarization information and the new determination of $\tau$ we obtain $\xi < 0.00098$ at 95% CL. Adding the BAO data, we obtain $\xi < 0.00055$ at 95% CL, which is 25% tighter compared to the limit obtained with P15 in combination with BAO DR10-11, i.e. $\xi < 0.00075$ and half of the one obtained with P13 in combination with BAO DR10-11, i.e. $\xi < 0.0012$, see the left panel of Fig. 1. As we can see from Tab. 1, BAO data strongly constrain the model and are useful to break the degeneracy in the $H_0 - \xi$ parameter space.

We now discuss the CC case. The coupling to gravity is constrained to $N_{pl} < 1.000028 \ M_{pl}$ at 95% CL for P18 and $N_{pl} < 1.000018 \ M_{pl}$ at 95% CL in combination with BAO data. These constraints update the ones obtained with P15 in combination with D10-DR11 BAO data.

Figure 1. Left panel: marginalized joint 68% and 95% CL regions 2D parameter space using current versus previous releases of Planck data and BOSS BAO data from [56, 57]. Right panel: marginalized joint 68% and 95% CL regions 2D parameter space using P18 (gray) in combination with BAO (blue) and BAO + R19 (red) for the IG model.
Figure 2. Left panel: marginalized joint 68% and 95% CL regions 2D parameter space using P18 (P15) data in combination BAO in blue (red). Right panel: marginalized joint 68% and 95% CL regions 2D parameter space using P18 (gray) in combination with BAO (blue) and BAO + R19 (red) for the CC model.

\( N_{pl} < 1.000038 \, \text{M}_{\text{pl}} \) at 95% CL in Ref. [60]. As in the IG case, we still have a degeneracy between \( H_0 \) and the coupling to gravity \( N_{pl} \) as visible from Fig. 2.

Consistently with the constraints on the coupling parameters \( \xi \) and \( N_{pl} \), we find also tighter limits on the variation of the Newton’s gravitational constant (3.2)-(3.5) and its derivative (3.3)-(3.6) at present time. For IG, we have:

\[
\frac{\delta G_N}{G_N}(z = 0) > -0.016 , \quad \frac{\dot{G}_N}{G_N}(z = 0) > -0.66 \times 10^{-13} \, \text{yr}^{-1}
\]

(4.1)

for P18 + BAO at 95% CL, updating those obtained in Ref. [57] with P15 + BAO DR10-11, i.e.

\[
\frac{\delta G_N}{G_N}(z = 0) > -0.027 , \quad \frac{\dot{G}_N}{G_N}(z = 0) > -1.4 \times 10^{-13} \, \text{yr}^{-1}.
\]

(4.2)

For CC, we obtain the following 95% CL bounds for P18 + BAO:

\[
\frac{\delta G_N}{G_N}(z = 0) > -0.017 , \quad \frac{\dot{G}_N}{G_N}(z = 0) > -0.25 \times 10^{-23} \, \text{yr}^{-1}.
\]

(4.3)

Note that while the constraints on \( \delta G_N/G_N(z = 0) \) hardly change for different coupling \( F(\sigma) \), the limits on \( \dot{G}_N/G_N(z = 0) \) strongly depend on the details of the model \(^1\), but are anyway much tighter than those obtained by the Lunar Laser Ranging experiment [113]. Tab. 1 also reports the values of the post-Newtonian parameters as derived parameters from our samples: whereas for IG \( \gamma_{\text{PN}} \lesssim \mathcal{O}(10^{-3}) \), for CC the bounds we derive on \( \gamma_{\text{PN}} , \beta_{\text{PN}} \) are now tighter than those in the Solar System.

We note that the value of the initial (final) scalar field is small and sub-Planckian \( \sigma_{\text{ini}} = (0.22 \pm 0.10) \, \text{M}_{\text{pl}} \) \( (\sigma_0 = (0.0089 \pm 0.0040) \, \text{M}_{\text{pl}}) \) as opposed to the IG case where the evolution is super-Planckian.

\(^{\dagger}\)The same behaviour of the constraints on the variation of the Newton’s constant and its derivative has been observed changing the potential \( V(\sigma) \) for IG, see Ref. [57].
We have tested that our results are stable when we switch to a flat potential $V(\sigma) = \Lambda$ for both IG and CC. We show in Fig. 3 the consistency of the posterior distributions for $H_0$, $\xi$, and $N_{pl}$ with current cosmological data (P18 + BAO). The comparison with $V(\sigma) = \Lambda$ for IG updates the results obtained in [57] with Planck 2015 which showed that the cosmological parameters were stable when varying the index $n$ of a power-law potential $V(\sigma) = \lambda \sigma^n/4$ with $n \geq 0$. The stability of the results for the CC case when switching to $V(\sigma) = \Lambda$ is a new result, although, in analogy with what happens in IG, not totally unexpected since our data constrains the deviations $\mathcal{O}(\sigma^2/N_{pl}^2)$ of $F^2$ from a flat potential. We refer the interested reader to Ref. [65] for an extended analysis of $V(\sigma) = \Lambda$ for $F(\sigma) = M_{pl}^2 [1 + \xi (\sigma/M_{pl})^n]$ with $n = 2, 4$ with flat priors on $\sigma_{\text{ini}}^1$.

### 4.1 Implications for the $H_0$ tension

We find a higher value for the Hubble parameter for IG, i.e. $H_0 = (69.6^{+0.8}_{-1.7})$ km s$^{-1}$Mpc$^{-1}$, and for CC, i.e. $H_0 = (69.0^{+0.7}_{-1.2})$ km s$^{-1}$Mpc$^{-1}$, compared to the $\Lambda$CDM case, i.e. $H_0 = (67.36 \pm 0.54)$ km s$^{-1}$Mpc$^{-1}$, for P18.

The addition of BAO drives the value for $H_0$ to lower values, for IG to $H_0 = (68.78^{+0.53}_{-0.78})$ km s$^{-1}$Mpc$^{-1}$ and for CC to $H_0 = (68.62^{+0.47}_{-0.60})$ km s$^{-1}$Mpc$^{-1}$. Note however that these values are larger than the corresponding $\Lambda$CDM value, i.e. $H_0 = (67.66 \pm 0.42)$ km s$^{-1}$Mpc$^{-1}$.

Once we include R19, we obtain $H_0 = (70.1 \pm 0.8)$ km s$^{-1}$Mpc$^{-1}$ at 68% CL, $\xi = 0.00051^{+0.00043}_{-0.00046}$ at 95% CL for IG, $H_0 = (69.64^{+0.65}_{-0.73})$ km s$^{-1}$Mpc$^{-1}$ at 68% CL, $N_{pl} < 1.000031 M_{pl}$ at 95% CL for CC. Fig. 1 shows how the degeneracy between $H_0$ and $\xi$ can easily accommodate for larger $H_0$ value with respect to the $\Lambda$CDM concordance model reducing the $H_0$ tension from 4.4$\sigma$ to 2.4$\sigma$. We find that $H_0$ is about $\sim 0.5$ km s$^{-1}$Mpc$^{-1}$ higher in the IG compared to the CC case for every choice of datasets combination.

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1See also Ref. [64] for similar finding in NMC scalar-tensor gravity with a coupling $F(\sigma) = M_{pl}^2 + \xi \sigma^2$ in the parameter space range for $\xi < 0$. 

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**Figure 3.** Left panel: marginalized joint 68% and 95% CL regions 2D parameter space $H_0 - \xi$ using P18 + BAO data for IG with $V(\sigma) = \lambda F(\sigma)^2/4$ (red) and $V(\sigma) = \Lambda$ (blue). Right panel: marginalized joint 68% and 95% CL regions 2D parameter space $H_0 - N_{pl}$ using P18 + BAO data for CC with $V(\sigma) = \lambda F(\sigma)^2/4$ (red) and $V(\sigma) = \Lambda$ (blue).
Figure 4. Differences with respect to the ΛCDM with \( N_{\text{eff}} = 3.046 \) with IG (top panels) for \( \xi = 0.0008, 0.0016 \) (solid, dashed) and \( N_{\text{eff}} = 2.846, 3.046, 3.246 \) (red, green, blue), and CC (bottom panels) for \( N_{\text{pl}} = 1.00003, 1.00004 \) \( M_{\text{pl}} \) (solid, dashed) and \( N_{\text{eff}} = 2.846, 3.046, 3.246 \) (red, green, blue). \( D_{\ell} \equiv \ell (\ell + 1) C_{\ell} / (2\pi) \) are the band-power angular power spectra.

5 Degeneracy with the neutrino sector

5.1 Effective number of relativistic degrees of freedom

The presence of extra relativistic degrees of freedom in the Universe increases the expansion rate during the radiation-dominated era and shifts the epoch of matter-radiation equality, the shape of the matter power spectrum, and the history of recombination (see Refs. [115, 116] for a review). The extra radiation is usually parameterized by \( \Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046 \) which takes into account that neutrino decoupling was not quite complete when \( H_0 \) played [117–120].

In the context of the \( H_0 \) tension, a larger value of \( N_{\text{eff}} \) can attenuate the discrepancy on the \( H_0 \) value and reduce the comoving sound horizon at baryon drag. However, the tension is only reduced (\( \sim 2\sigma \)) and it appears again when BAO data is included [7, 38].

There is an interplay (negative correlation) between the contribution of extra radiation with respect to the standard ΛCDM scenario from the \( N_{\text{eff}} \) and from the scalar field coupling\(^1\) which acts as an additional source of radiation in the early Universe. Decreasing the effective number of extra relativistic species to \( N_{\text{eff}} = 2.846 \) we obtain deviations of the CMB anisotropies angular power spectra to the ΛCDM model of the same order of the ones obtained with \( \xi \) halved and \( N_{\text{eff}} = 3.046 \), see Fig. 4.

We see that, preferring lower values of \( N_{\text{eff}} \), the dataset allows for larger values for \( \xi \) compared to the case with \( N_{\text{eff}} = 3.046 \) fixed. We go from \( \xi < 0.00098 \) to \( \xi < 0.0019 \) at 95\% CL with P18 alone and from \( \xi < 0.00055 \) to \( \xi < 0.00078 \) once we include BAO, see Tab. 3.

The mean of \( N_{\text{eff}} \) moves around 1\(\sigma \) toward lower values with respect to the ΛCDM case with a similar error. For IG, we get at 68\% CL \( N_{\text{eff}} = 2.79 \pm 0.20 \) for P18 compared to \( N_{\text{eff}} = 2.89 \pm 0.19 \) in ΛCDM and \( N_{\text{eff}} = 2.85 \pm 0.17 \) in combination with BAO compared to \( N_{\text{eff}} = 2.99 \pm 0.17 \) in ΛCDM. In Fig. 5 (central panel), we show the enlarged \( H_0 - N_{\text{eff}} \)

\(^1\)See Refs. [66, 67] for an application in the context of \( f(R) \) gravity.
Figure 5. Marginalized joint 68% and 95% CL regions 2D parameter space using the P18 (gray) in combination with BAO (blue) and BAO + R19 (red) for the IG+N_{eff} model. In the central panel, we include the $H_0 - N_{eff}$ contours for the ΛCDM in green. In the right panel, we include the $H_0 - \xi$ contours for the IG with $N_{eff} = 3.046$ in green.

Figure 6. Marginalized joint 68% and 95% CL regions 2D parameter space using the Planck legacy data (gray) in combination with DR12 (blue) and DR12 + R19 (red) for the CC+N_{eff} model.

parameter space in IG compared to the ΛCDM concordance model (green contours) where it is possible to reach higher value of $H_0$ without increasing $N_{eff}$ in presence of a modification of gravity.

In the CC model, an analogous correlation in the $N_{eff} - N_{pl}$ parameter space is found, see Fig. 6. The constraints on $N_{pl}$ are larger, from $N_{pl} < 1.000028$ M_{pl} to $N_{pl} < 1.000057$ M_{pl} at 95% CL with P18 alone and from $N_{pl} < 1.000018$ M_{pl} to $N_{pl} < 1.000019$ M_{pl} at 95% CL once we include BAO, see Tab. 4.

While for the combination P18 + BAO we find a higher value for the Hubble parameter for IG, i.e. $H_0 = (68.78^{+0.53}_{-0.58})$ km s^{-1}Mpc^{-1}, and for CC, i.e. $H_0 = (68.62^{+0.47}_{-0.66})$ km s^{-1}Mpc^{-1}, compared to the ΛCDM+N_{eff} case, i.e. $H_0 = (67.3 \pm 1.1)$ km s^{-1}Mpc^{-1}, the addition of R19 data leads to a closer posterior distribution for $H_0$ among the three cases, i.e. $(70.1 \pm 0.8)$ km s^{-1}Mpc^{-1} for IG, $(69.6 \pm 0.7)$ km s^{-1}Mpc^{-1} for CC, and $H_0 = (70.0 \pm 0.9)$ km s^{-1}Mpc^{-1} for ΛCDM+N_{eff}. We find a similar posterior distribution also for IG+N_{eff} (CC+N_{eff}), i.e. $(70.3 \pm 0.9)$ km s^{-1}Mpc^{-1} $(70.1 \pm 0.9)$ km s^{-1}Mpc^{-1}, see Fig. 5.

The addition of BAO data reduces the degeneracy $H_0 - \xi (-N_{pl})$ increasing the one between $N_{eff} - \xi (-N_{pl})$ and $H_0 - N_{eff}$. In order to reduce comoving sound horizon to
5.2 Neutrino mass

The changes in the background evolution caused by neutrino mass, under standard assumptions and for a fixed set of standard cosmological parameters, are confined to late times. In particular, the neutrino mass impact the angular diameter distance and $z_\Lambda$ (the redshift of matter-to-cosmological-constant equality) (see Refs. [115, 116, 121–125] for a review on neutrino mass in cosmology).

In the standard ΛCDM scenario, a larger value of $m_\nu$ results in a lower Hubble rate inferred from the CMB, exacerbating the $H_0$ tension. However, there is partial correlation between the equation of state of dark energy (DE) $w$ and the total neutrino mass $m_\nu$, as first noticed by [122]. When $m_\nu$ is increased (or more generally $\Omega_\nu$), $\Omega_m$ can be kept unchanged, by simultaneously decreasing $w$, in order to keep the angular diameter distance at decoupling fixed. In this case, the impact of neutrino mass on the background is confined to variations of $z_\Lambda$ and of the late-time ISW effect.

Cosmological bounds on the neutrino masses can therefore be relaxed by using a DE component rather than a cosmological constant. Vice versa, cosmological constraints on the DE parameters become larger in comparison to cosmologies with massless neutrinos or with the standard minimal assumption of $m_\nu = 0$. The same conclusions have been obtained in the context of Galileon gravity [69].

We show in Fig. 7 the combined effect on the CMB anisotropies of varying both $\xi$ and $m_\nu$ in the IG model. For a fixed value of the coupling parameter $\xi = 0.0008$, the differences with respect to the ΛCDM concordance model are reduced by increasing the value of the neutrino mass $m_\nu$ from 0.1 eV to 0.3 eV. On the late-time observables, i.e. the weak lensing CMB anisotropies and the linear matter power spectrum, the partial degeneracy between

![Figure 7](image-url)
Figure 8. Differences with respect to the ΛCDM with \( m_\nu = 0 \) eV with IG (top panels) for \( \xi = 0.0008, 0.0016 \) (solid, dashed) and \( m_\nu = 0.1, 0.2, 0.3 \) eV (red, green, blue), and CC (bottom panels) for \( N_{\text{pl}} = 1.00003, 1.00004 \) Mpc (solid, dashed) and \( m_\nu = 0.06, 0.1, 0.2 \) eV (red, green, blue).

Figure 9. Marginalized joint 68\% and 95\% CL regions 2D parameter space using P18 (gray) in combination with BAO (blue) and BAO + R19 (red) for the IG+\( m_\nu \) model. In the central panel, we include the \( H_0 - N_{\text{eff}} \) contours for the ΛCDM in green.

modified gravity and the neutrino mass is still present but with differences concentrated on small scales, see Fig. 8.

In this case the constraints on the coupling parameter \( \xi \) become tighter compared to the case with \( m_\nu = 0 \), i.e. from \( \xi < 0.00098 \) to \( \xi < 0.00094 \) at 95\% CL for P18\(^1\). The CMB anisotropies data prefer to relax the upper bound on the neutrino mass which becomes \( m_\nu < 0.31 \) eV at 95\% CL for P18 29\% larger to the ΛCDM case \( m_\nu < 0.24 \) eV. Including the BAO data, the total neutrino mass is constrained to \( m_\nu < 0.17 \) eV at 95\% CL, 42\% larger to the ΛCDM case \( m_\nu < 0.12 \) eV, and we find \( \xi < 0.00076 \) at 95\% CL, see Tab. 5. The addition of R19 data leads to \( H_0 = (70.1 \pm 0.8) \) km s\(^{-1}\)Mpc\(^{-1} \) with an upper bound on the total neutrino mass \( m_\nu < 0.19 \) eV at 95\% CL, 2.5 times larger than the limit based on the ΛCDM model, with a 2σ detection of the coupling parameter \( \xi = 0.00065 \pm 0.00057 \) at 95\%.

\(^1\)We assume one massive and two massless neutrinos.
Figure 10. Marginalized joint 68% and 95% CL regions 2D parameter space using P18 (gray) in combination with BAO (blue) and BAO + R19 (red) for the CC+$\nu$ model.

Figure 11. Samples of the P18 + BAO chains in the $H_0$ – $N_{\text{eff}}$ ($H_0$ – $m_{\nu}$) plane, colour-coded by $\xi$ for the IG+$N_{\text{eff}}$+$m_{\nu}$ model. Dashed blue contours show the constraints for IG+$N_{\text{eff}}$+$m_{\nu}$ with P18 alone. Solid red contours show the constraints for the $\Lambda$CDM+$N_{\text{eff}}$+$m_{\nu}$ model. The gray bands denote the local Hubble parameter measurement from R19 [8].

CL, see Fig. 9.

Analogously, for CC the constraint on $N_{\text{pl}}$ becomes tighter compared to the case with $m_{\nu} = 0$, i.e. $N_{\text{pl}} < 1.000026$ $M_{\text{pl}}$ for P18 and $N_{\text{pl}} < 1.000024$ $M_{\text{pl}}$ for P18 + BAO at 95% CL, see Fig. 10. Also, for this model, the upper bound on the neutrino mass becomes 30% larger compared to the $\Lambda$CDM case, see Tab. 6.

5.3 Joint constraints on $N_{\text{eff}}$ and neutrino mass

Finally, we consider the case where both $N_{\text{eff}}$ and $m_{\nu}$ are allowed to vary. Despite the larger parameter space and the larger limits on the parameters, the models do not accommodate higher values of the Hubble parameter compared to the 7- and 8-parameters case analysed before, see Figs. 11-12. Moreover, the total neutrino mass is almost uncorrelated with the Hubble parameter. In this case, the modified gravity parameters $\xi$ and $N_{\text{pl}}$ are always compatible at 2$\sigma$ with the GR limit due to the larger parameter space and are given by (see Tabs. 7-8):

$$\xi < 0.0018 \ (95\% \ \text{CL}), \quad N_{\text{eff}} = 2.74 \pm 0.22, \quad m_{\nu} < 0.26 \ \text{eV} \ (95\% \ \text{CL})$$
for IG and  

\[ N_{pl} < 1.000050 \, M_{pl} \text{ (95\% CL)}, \quad N_{\text{eff}} = 2.73 \pm 0.21, \quad m_\nu < 0.26 \, \text{eV (95\% CL)} \]

for the CC case. When we include BAO data, we obtain  

\[ \xi < 0.0012 \text{ (95\% CL)}, \quad N_{\text{eff}} = 2.77 \pm 0.20, \quad m_\nu < 0.19 \, \text{eV (95\% CL)} \]

for IG and  

\[ N_{pl} < 1.000042 \, M_{pl} \text{ (95\% CL)}, \quad N_{\text{eff}} = 2.75 \pm 0.21, \quad m_\nu < 0.17 \, \text{eV (95\% CL)} \]

for the CC case. Adding also R19, we obtain  

\[ \xi < 0.0013 \text{ (95\% CL)}, \quad N_{\text{eff}} = 3.08 \pm 0.20, \quad m_\nu < 0.19 \, \text{eV (95\% CL)} \]

for IG and  

\[ N_{pl} < 1.000040 \, M_{pl} \text{ (95\% CL)}, \quad N_{\text{eff}} = 3.14 \pm 0.20, \quad m_\nu < 0.14 \, \text{eV (95\% CL)} \]

for the CC case.

6 Conclusions

We have studied the simplest class of scalar-tensor theories of gravity where Newton’s constant is allowed to vary in time and space. These non-minimally coupled theories are characterized by a coupling to the Ricci scalar \( R \) of the type \( F(\sigma) = N_{pl}^2 + \xi \sigma^2 \), which contain the induced gravity \([126–128]\) model for \( N_{pl} = 0 \) and \( \xi > 0 \), and the conformal coupling model \([60]\) for \( \xi = -1/6 \). These models contribute to the radiation density budget in the radiation-dominated epoch and change the background expansion history compared to the \( \Lambda \)CDM concordance model naturally alleviating the \( H_0 \) tension.

Compared to previous constraints \([57, 60]\), the improvement of Planck 2018 polarization data lead to tighter results, i.e. \( \xi < 0.00098 \) and \( N_{pl} < 1.000028 \, M_{pl} \) both at 95\% CL. When
BAO data from BOSS DR12 are added, we obtain tighter limits, i.e. \( \xi < 0.00055 \) and \( N_{\text{pl}} < 1.000018 \, M_{\text{pl}} \). For \( \xi = -1/6 \), P18 and BAO data lead to constraints on the post-Newtonian parameters which are tighter than those derived within the Solar System.

It is interesting to note that for CMB and BAO data these models allow for values of \( H_0 \) larger than \( \Lambda \)CDM. Using P18 data, we find \( H_0 = (69.6^{+0.8}_{-1.7}) \, \text{km s}^{-1}\text{Mpc}^{-1} \) (\( H_0 = (69.0^{+0.7}_{-1.2}) \, \text{km s}^{-1}\text{Mpc}^{-1} \)) for the IG (CC) case compared to \( H_0 = (67.36 \pm 0.54) \, \text{km s}^{-1}\text{Mpc}^{-1} \) for \( \Lambda \)CDM, alleviating the tension from \( 4.4\sigma \) to \( 2.7\sigma \) (\( 3.2\sigma \)). Including BAO and R19, we obtain \( H_0 = (70.06 \pm 0.81) \, \text{km s}^{-1}\text{Mpc}^{-1} \) (\( H_0 = (69.64^{+0.65}_{-0.73}) \, \text{km s}^{-1}\text{Mpc}^{-1} \)) for IG (for CC). The value for \( H_0 \) we find is similar to what can be obtained in other models which aim to solve the \( H_0 \) tension, but the models considered here have just one extra parameter as \( \Lambda \)CDM+\( \nu \)eff. Similar values of \( H_0 \) can also be found for NMC beyond the IG and CC cases considered here [65].

We have extended our analysis to a general neutrino sector by allowing the effective number of relativistic species \( N_{\text{eff}} \) and the neutrino mass \( m_\nu \) to vary. Both \( N_{\text{eff}} \) and \( m_\nu \) are partially degenerate with the deviations from GR, as happens in other modified gravity models [66, 67, 69]. Whereas \( N_{\text{eff}} \) and the scalar field act as an additional source of radiation in the early Universe, at late times the background contribution to \( \Omega_m \) due to \( m_\nu \) can be compensated from the scalar field in order to keep the angular diameter distance at decoupling fixed, see Figs. 4-7-8. We have shown however that these are only partial degeneracies which could be broken by combination of observations at different redshifts.

In case with \( N_{\text{eff}} \) (Sec. 5.1) the limit on \( \xi \) becomes \( \sim 94\% \) (\( \sim 42\% \)) larger with P18 (P18+BAO) while the mean on the number of neutrinos moves around 1\( \sigma \) towards lower values compared to the \( \Lambda \)CDM case without significantly degrading its uncertainty, i.e. \( N_{\text{eff}} = 2.79 \pm 0.20 \) (\( N_{\text{eff}} = 2.85 \pm 0.17 \)). For CC the limit on \( N_{\text{pl}} \) becomes \( \sim 104\% \) (\( \sim 6\% \)) larger with P18 (P18+BAO) and analogously to IG we find \( N_{\text{eff}} = 2.73^{+0.25}_{-0.22} \) (\( N_{\text{eff}} = 2.81 \pm 0.19 \)).

The upper bound on the neutrino mass (Sec. 5.2) is \( \sim 29\% \) (\( \sim 42\% \)) is also degraded with P18 (P18+BAO) compared to the \( \Lambda \)CDM case, i.e. \( m_\nu < 0.31 \, \text{eV} \) (\( m_\nu < 0.17 \, \text{eV} \)), whereas the constraint on \( \xi \) is slightly tighter with CMB data alone in order to relax the constraint on \( m_\nu \). Analogously, for CC the limit on the neutrino mass is \( \sim 17\% \) (\( \sim 33\% \)) larger with P18 (P18+BAO) compared to the \( \Lambda \)CDM case. When both \( N_{\text{eff}} \) and \( m_\nu \) are allowed to vary, we see that the constraints on \( \xi \) and \( N_{\text{pl}} \) degrade by a factor two compared to the case with \( N_{\text{eff}} = 3.046 \) and \( m_\nu = 0 \, \text{eV} \) also in presence of BAO data, i.e. \( \xi < 0.0012 \) and \( N_{\text{pl}} < 1.000042 \, M_{\text{pl}} \) at 95\% CL. For the data used, the combination of the modification to gravity in our models to non-standard neutrino physics does not lead to higher values of \( H_0 \) compared to the case with standard assumptions in the neutrino sector.

**Acknowledgments**

We wish to thank Maria Archidiacono and Thejs Brinckmann for useful suggestions on the use of MontePython. MBa, MBr, FF, DP acknowledge financial contribution from the contract ASI/INAF for the Euclid mission n.2018-23-HI.0. FF and DP acknowledge financial support by ASI Grant 2016-24-HI.0. AAS was partially supported by the Russian Foundation for Basic Research grant No. 18-52-45016.
## A Tables

### A.1 Updated Planck 2018 results

| Parameter          | P18          | P18 + BAO       | P18 + BAO + R19 |
|--------------------|--------------|-----------------|-----------------|
| $\omega_b$        | 0.02244 ± 0.00014 | 0.02239 ± 0.00013 | 0.02246 ± 0.00013 |
| $\omega_c$        | 0.1198 ± 0.0012 | 0.1201 ± 0.0011 | 0.1200 ± 0.0011 |
| $H_0$ [km s$^{-1}$Mpc$^{-1}$] | 69.6$^{+0.8}_{-1.7}$ (2.7$\sigma$) | 68.78$^{+0.53}_{-0.75}$ (3.5$\sigma$) | 70.06 ± 0.81 (2.4$\sigma$) |
| $\tau$            | 0.0551$^{+0.0065}_{-0.0078}$ | 0.0545$^{+0.0063}_{-0.0071}$ | 0.0554$^{+0.0064}_{-0.0073}$ |
| $\ln (10^{10}A_s)$ | 3.047$^{+0.014}_{-0.015}$ | 3.046 ± 0.013 | 3.049 ± 0.013 |
| $n_s$             | 0.9680$^{+0.0044}_{-0.0052}$ | 0.9662 ± 0.0038 | 0.9688 ± 0.0037 |
| $\Omega_G$        | < 0.0039 (95% CL) | < 0.0022 (95% CL) | 0.0020$^{+0.0009}_{-0.0010}$ |
| $\xi$             | < 0.00098 (95% CL) | < 0.0055 (95% CL) | 0.00051$^{+0.00044}_{-0.00046}$ (95% CL) |
| $\gamma_{PN}$     | > 0.9961 (95% CL) | > 0.9978 (95% CL) | 0.9980$^{+0.0009}_{-0.0069}$ |
| $\delta G_N/G_N$ (z=0) | > −0.029 (95% CL) | > −0.016 (95% CL) | −0.0149 ± 0.0068 |
| $10^{13}G_N/G_N$ (z=0) [yr$^{-1}$] | > −1.16 (95% CL) | > −0.66 (95% CL) | −0.61 ± 0.28 |
| $G_N/G_N$ (z=0)   | > 0.9981 (95% CL) | > 0.9989 (95% CL) | 0.9989$^{+0.00050}_{-0.00045}$ |
| $\Omega_m$        | 0.2940$^{+0.0015}_{-0.0013}$ | 0.3013$^{+0.0072}_{-0.0062}$ | 0.2903 ± 0.0068 |
| $\sigma_s$        | 0.8347$^{+0.0130}_{-0.0079}$ | 0.8308$^{+0.0067}_{-0.0096}$ | 0.840 ± 0.010 |
| $r_s$ [Mpc]       | 146.37 ± 0.40 | 146.63 ± 0.34 | 146.03 ± 0.57 |
| $\Delta \chi^2$   | 2.2          | 0.8            | −1.7            |

Table 1. Constraints on main and derived parameters (at 68% CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the IG model.

| Parameter          | P18          | P18 + BAO       | P18 + BAO + R19 |
|--------------------|--------------|-----------------|-----------------|
| $\omega_b$        | 0.02244 ± 0.00015 | 0.02241 ± 0.00013 | 0.02250 ± 0.00013 |
| $\omega_c$        | 0.1197 ± 0.0012 | 0.11990 ± 0.00094 | 0.1195 ± 0.0010 |
| $H_0$ [km s$^{-1}$Mpc$^{-1}$] | 69.6$^{+0.7}_{-1.2}$ (3.2$\sigma$) | 68.62$^{+0.47}_{-0.60}$ (3.6$\sigma$) | 69.64$^{+0.65}_{-0.73}$ (2.8$\sigma$) |
| $\tau$            | 0.0554$^{+0.0064}_{-0.0081}$ | 0.0551$^{+0.0058}_{-0.0076}$ | 0.0562$^{+0.0066}_{-0.0077}$ |
| $\ln (10^{10}A_s)$ | 3.048$^{+0.013}_{-0.016}$ | 3.047$^{+0.011}_{-0.013}$ | 3.050$^{+0.013}_{-0.015}$ |
| $n_s$             | 0.9684 ± 0.0047 | 0.9668 ± 0.0039 | 0.9707 ± 0.0040 |
| $N_{pl}$ [Mpc$^3$] | < 1.000028 (95% CL) | < 1.000018 (95% CL) | < 1.000031 (95% CL) |
| $\gamma_{PN}$     | > 0.999972 (95% CL) | > 0.999982 (95% CL) | > 0.999969 (95% CL) |
| $\beta_{PN}$      | < 1.00000023 (95% CL) | < 1.00000015 (95% CL) | < 1.00000025 (95% CL) |
| $\delta G_N/G_N$ (z=0) | > −0.026 (95% CL) | > −0.017 (95% CL) | > −0.029 (95% CL) |
| $10^{13}G_N/G_N$ (z=0) [yr$^{-1}$] | > −3.8 × 10$^{-9}$ (95% CL) | > −2.5 × 10$^{-9}$ (95% CL) | > −4.2 × 10$^{-9}$ (95% CL) |
| $G_N/G_N$ (z=0)   | > 0.999986 (95% CL) | > 0.999991 (95% CL) | > 0.999985 (95% CL) |
| $\Omega_m$        | 0.2995$^{+0.011}_{-0.009}$ | 0.3023 ± 0.0061 | 0.2928 ± 0.0064 |
| $\sigma_s$        | 0.832$^{+0.011}_{-0.007}$ | 0.8299$^{+0.0060}_{-0.0088}$ | 0.8364$^{+0.0089}_{-0.011}$ |
| $r_s$ [Mpc]       | 146.71$^{+0.46}_{-0.33}$ | 146.82$^{+0.37}_{-0.28}$ | 146.53$^{+0.51}_{-0.42}$ |
| $\Delta \chi^2$   | 2.2          | 0.8            | −1.7            |

Table 2. Constraints on main and derived parameters (at 68% CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the CC model.
### A.2 Degeneracy with the neutrino sector: $N_{\text{eff}}$

|                          | P18       | P18 + BAO | P18 + BAO + R19 |
|--------------------------|-----------|-----------|-----------------|
| $\omega_b$               | 0.02227$^{+0.00018}_{-0.00021}$ | 0.02225 ± 0.00019 | 0.02250 ± 0.00019 |
| $\omega_c$               | 0.1161 ± 0.0031 | 0.1172 ± 0.0030 | 0.1210 ± 0.0029 |
| $H_0$ [km s$^{-1}$Mpc$^{-1}$] | 69.2$^{+1.5}_{-2.4}$ (2.3$s$) | 67.9$^{+1.0}_{-1.2}$ (3.5$s$) | 70.28 ± 0.92 (2.2$s$) |
| $\tau$                   | 0.0547 ± 0.0078 | 0.0526 ± 0.0069 | 0.0549 ± 0.0072 |
| ln $(10^{10}A_s)$         | 3.038 ± 0.016 | 3.035 ± 0.015 | 3.050 ± 0.016 |
| $n_s$                    | 0.9617$^{+0.0049}_{-0.0088}$ | 0.9600$^{+0.0045}_{-0.0079}$ | 0.9707 ± 0.0069 |
| $\zeta_G$                | < 0.0076 (95$\%$ CL) | < 0.0031 (95$\%$ CL) | < 0.0040 (95$\%$ CL) |
| $N_{\text{eff}}$         | 2.79 ± 0.20 | 2.85 ± 0.17 | 3.11 ± 0.19 |

### Table 3. Constraints on main and derived parameters (at 68$\%$ CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the IG+$N_{\text{eff}}$ model.

|                          | P18       | P18 + BAO | P18 + BAO + R19 |
|--------------------------|-----------|-----------|-----------------|
| $\omega_b$               | 0.02223 ± 0.00022 | 0.02215 ± 0.00022 | 0.02257 ± 0.00018 |
| $\omega_c$               | 0.1151 ± 0.0033 | 0.1162 ± 0.0031 | 0.1213 ± 0.0030 |
| $H_0$ [km s$^{-1}$Mpc$^{-1}$] | 67.9 ± 1.4 (3.1$s$) | 67.1 ± 1.2 (3.7$s$) | 70.10 ± 0.92 (2.0$s$) |
| $\tau$                   | 0.0539$^{+0.0060}_{-0.0074}$ | 0.0544$^{+0.0063}_{-0.0074}$ | 0.0561$^{+0.0063}_{-0.0075}$ |
| ln $(10^{10}A_s)$         | 3.034 ± 0.017 | 3.035 ± 0.016 | 3.053$^{+0.014}_{-0.016}$ |
| $n_s$                    | 0.9598 ± 0.0084 | 0.9606 ± 0.0071 | 0.9736 ± 0.0062 |
| $N_{\text{pl}}$ [Mpc]    | < 1.000057 (95$\%$ CL) | < 1.000019 (95$\%$ CL) | < 1.000032 (95$\%$ CL) |
| $N_{\text{eff}}$         | 2.73$^{+0.25}_{-0.22}$ | 2.81 ± 0.19 | 3.16 ± 0.19 |

### Table 4. Constraints on main and derived parameters (at 68$\%$ CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the CC+$N_{\text{eff}}$ model.
### A.3 Degeneracy with the neutrino sector: \( m_\nu \)

| \( \omega_b \) | P18 | P18 + BAO | P18 + BAO + R19 |
|----------------|------|-----------|-----------------|
| \( \omega_c \) | 0.1205 \( \pm 0.0013 \) | 0.1203 \( \pm 0.0011 \) | 0.1203 \( \pm 0.0012 \) |
| \( H_0 \) [\( \text{km s}^{-1}\text{Mpc}^{-1} \)] | 68.5 \( \pm 1.8 \) (2.4\( \sigma \)) | 68.66 \( \pm 0.8 \) (3.4\( \sigma \)) | 70.12 \( \pm 0.81 \) (2.4\( \sigma \)) |
| \( \tau \) | 0.0567 \( \pm 0.0065 \) | 0.0564 \( \pm 0.0068 \) | 0.0572 \( \pm 0.0063 \) |
| \( \ln (10^{10} A_s) \) | 3.052 \( \pm 0.16 \) | 3.051 \( \pm 0.16 \) | 3.054 \( \pm 0.16 \) |
| \( n_s \) | 0.9668 \( \pm 0.0053 \) | 0.9672 \( \pm 0.0038 \) | 0.9700 \( \pm 0.0038 \) |
| \( \xi_{IG} \) | < 0.0037 (95% CL) | < 0.0030 (95% CL) | 0.0026 \( \pm 0.00010 \) |
| \( m_\nu \) [eV] | < 0.31 (95% CL) | < 0.17 (95% CL) | < 0.19 (95% CL) |
| \( \xi \) | < 0.00094 (95% CL) | < 0.00076 (95% CL) | 0.00065 \( \pm 0.00057 \) (95% CL) |
| \( \gamma_{PN} \) | > 0.9963 (95% CL) | > 0.9970 (95% CL) | 0.9974 \( \pm 0.0013 \) |
| \( \delta G_N / G_N \) (z=0) | > -0.027 (95% CL) | > -0.022 (95% CL) | -0.0190 \( \pm 0.0075 \) |
| \( 10^{13} G_N / G_N \) (z=0) [yr\(^{-1} \)] | > -1.1 (95% CL) | > -0.93 (95% CL) | -0.78 \( \pm 0.031 \) |
| \( G_N / G \) (z=0) | > 0.9981 (95% CL) | > 0.9985 (95% CL) | 0.9987 \( \pm 0.00064 \) |
| \( \Omega_m \) | 0.3068 \( \pm 0.0118 \) | 0.3029 \( \pm 0.0076 \) | 0.2905 \( \pm 0.0068 \) |
| \( \sigma_8 \) | 0.818 \( \pm 0.014 \) | 0.821 \( \pm 0.014 \) | 0.832 \( \pm 0.013 \) |
| \( r_s \) [Mpc] | 146.18 \( \pm 0.78 \) | 146.31 \( \pm 0.71 \) | 145.56 \( \pm 0.78 \) |
| \( \Delta \chi^2 \) | 3.0 | 0.2 | -3.3 |

**Table 5.** Constraints on main and derived parameters (at 68% CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the IG+\( m_\nu \) model.

| \( \omega_b \) | P18 | P18 + BAO | P18 + BAO + R19 |
|----------------|------|-----------|-----------------|
| \( \omega_c \) | 0.1209 \( \pm 0.0016 \) | 0.1201 \( \pm 0.00097 \) | 0.1197 \( \pm 0.0010 \) |
| \( H_0 \) [\( \text{km s}^{-1}\text{Mpc}^{-1} \)] | 68.0 \( \pm 1.4 \) (3.0\( \sigma \)) | 68.31 \( \pm 0.66 \) (3.7\( \sigma \)) | 69.62 \( \pm 0.71 \) (2.8\( \sigma \)) |
| \( \tau \) | 0.0563 \( \pm 0.0065 \) | 0.0564 \( \pm 0.0068 \) | 0.0576 \( \pm 0.0077 \) |
| \( \ln (10^{10} A_s) \) | 3.051 \( \pm 0.013 \) | 3.047 \( \pm 0.013 \) | 3.054 \( \pm 0.013 \) |
| \( n_s \) | 0.9674 \( \pm 0.0053 \) | 0.9681 \( \pm 0.0043 \) | 0.9720 \( \pm 0.0041 \) |
| \( N_{pl} \) [Mpc\(^3\)] | < 1.000026 (95% CL) | < 1.000024 (95% CL) | 1.000019 \( \pm 0.000017 \) (95% CL) |
| \( m_\nu \) [eV] | < 0.28 (95% CL) | < 0.16 (95% CL) | < 0.13 (95% CL) |
| \( \gamma_{PN} \) | > 0.999926 (95% CL) | > 0.999924 (95% CL) | 0.999919 \( \pm 0.000011 \) (95% CL) |
| \( \beta_{PN} \) | < 1.0000021 (95% CL) | < 1.0000020 (95% CL) | < 1.0000030 (95% CL) |
| \( \delta G_N / G_N \) | > -0.024 (95% CL) | > -0.023 (95% CL) | -0.0181 \( \pm 0.0099 \) |
| \( 10^{13} G_N / G_N \) (z=0) [yr\(^{-1} \)] | > -3.6 \( \times 10^{-9} \) (95% CL) | > -3.3 \( \times 10^{-9} \) (95% CL) | ( -2.7 \( \pm 1.5 \)) \( \times 10^{-9} \) |
| \( G_N / G \) (z=0) | > 0.999987 (95% CL) | > 0.999988 (95% CL) | 0.9999904 \( \pm 0.0000054 \) |
| \( \Omega_m \) | 0.3098 \( \pm 0.015 \) | 0.3047 \( \pm 0.0067 \) | 0.2935 \( \pm 0.0064 \) |
| \( \sigma_8 \) | 0.814 \( \pm 0.010 \) | 0.820 \( \pm 0.013 \) | 0.831 \( \pm 0.012 \) |
| \( r_s \) [Mpc] | 146.52 \( \pm 0.47 \) | 146.58 \( \pm 0.31 \) | 146.26 \( \pm 0.55 \) |
| \( \Delta \chi^2 \) | 3.0 | 0.0 | -1.5 |

**Table 6.** Constraints on main and derived parameters (at 68% CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the CC+\( m_\nu \) model.
A.4 Degeneracy with the neutrino sector: \((N_{\text{eff}}, m_\nu)\)

|        | P18          | P18 + BAO      | P18 + BAO + R19 |
|--------|--------------|----------------|-----------------|
| \(\omega_b\) | 0.02218 ± 0.00022 | 0.02220\(^\pm\)0.00019 | 0.02250 ± 0.00020 |
| \(\omega_c\) | 0.1162 ± 0.0034  | 0.1164 ± 0.0031  | 0.1208 ± 0.0030  |
| \(H_0\) [km s\(^{-1}\)Mpc\(^{-1}\)] | 67.7\(^{+2.0}_{-2.4}\) (2.6\(\sigma\)) | 67.6 ± 1.2 (3.5\(\sigma\)) | 70.25 ± 0.92 (2.2\(\sigma\)) |
| \(\tau\) | 0.0556\(^{+0.0083}_{-0.0083}\) | 0.0554\(^{+0.0073}_{-0.0073}\) | 0.0576\(^{+0.0063}_{-0.0063}\) |
| \(n_s\) | 3.039\(^{+0.016}_{-0.018}\) | 3.039 ± 0.016  | 3.056 ± 0.016  |
| \(\zeta_{\text{IG}}\) | 0.9577 ± 0.0086 | 0.9582 ± 0.0076 | 0.9710 ± 0.0071 |
| \(m_\nu\) [eV] | < 0.0070 (95\% CL) | < 0.0047 (95\% CL) | < 0.0053 (95\% CL) |
| \(N_{\text{eff}}\) | 2.74 ± 0.22 | 2.77 ± 0.20 | 3.08 ± 0.20 |

| \(\xi\) | < 0.0018 (95\% CL) | < 0.0012 (95\% CL) | < 0.0013 (95\% CL) |
| \(\gamma_{\text{PN}}\) | > 0.9931 (95\% CL) | > 0.9954 (95\% CL) | > 0.9948 (95\% CL) |
| \(\delta G_N/G_N\) (z=0) | > -0.050 (95\% CL) | > -0.334 (95\% CL) | > -0.038 (95\% CL) |
| 10\(^{13}\)\(\delta G_N/G_N\) (z=0) [yr\(^{-1}\)] | > -2.0 (95\% CL) | > -1.4 (95\% CL) | > 1.6 (95\% CL) |
| \(G_N/G\) (z=0) | > 0.9966 (95\% CL) | > 0.9977 (95\% CL) | > 0.9974 (95\% CL) |
| \(\Omega_m\) | 0.303\(^{+0.022}_{-0.019}\) | 0.3035 ± 0.0081 | 0.2904 ± 0.0069 |
| \(\sigma_8\) | 0.814\(^{+0.025}_{-0.019}\) | 0.815\(^{+0.015}_{-0.012}\) | 0.833\(^{+0.013}_{-0.011}\) |
| \(r_s\) [Mpc] | 148.6 ± 1.9 | 148.6 ± 1.8 | 145.3 ± 1.6 |
| \(\Delta\chi^2\) | 1.1 | 0.5 | -2.5 |

Table 7. Constraints on main and derived parameters (at 68\% CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the IG+N\(_{\text{eff}}\)+\(m_\nu\) model.

|        | P18          | P18 + BAO      | P18 + BAO + R19 |
|--------|--------------|----------------|-----------------|
| \(\omega_b\) | 0.02217 ± 0.00022 | 0.02222 ± 0.00020 | 0.02257 ± 0.00018 |
| \(\omega_c\) | 0.1158 ± 0.0034  | 0.1158 ± 0.0032  | 0.1212 ± 0.0031  |
| \(H_0\) [km s\(^{-1}\)Mpc\(^{-1}\)] | 66.7 ± 1.8 (3.2\(\sigma\)) | 67.2 ± 1.1 (3.8\(\sigma\)) | 69.96 ± 0.93 (2.1\(\sigma\)) |
| \(\tau\) | 0.0554\(^{+0.0064}_{-0.0067}\) | 0.0556\(^{+0.0063}_{-0.0067}\) | 0.0577\(^{+0.0069}_{-0.0068}\) |
| \(n_s\) | 3.039 ± 0.017  | 3.039 ± 0.016  | 3.057 ± 0.016  |
| \(N_{\text{pl}}\) [Mpc\(^3\)] | < 1.000050 (95\% CL) | < 1.000042 (95\% CL) | < 1.000040 (95\% CL) |
| \(m_\nu\) [eV] | < 0.26 (95\% CL) | < 0.17 (95\% CL) | < 0.14 (95\% CL) |
| \(N_{\text{eff}}\) | 2.73 ± 0.21 | 2.75 ± 0.21 | 3.14 ± 0.20 |
| \(\gamma_{\text{PN}}\) | > 0.99950 (95\% CL) | > 0.9958 (95\% CL) | > 0.9960 (95\% CL) |
| \(\beta_{\text{PN}}\) | < 1.000041 (95\% CL) | < 1.000035 (95\% CL) | < 1.000033 (95\% CL) |
| \(\delta G_N/G_N\) (z=0) | > -0.046 (95\% CL) | > -0.404 (95\% CL) | > -0.037 (95\% CL) |
| 10\(^{13}\)\(\delta G_N/G_N\) (z=0) [yr\(^{-1}\)] | > -6.7 × 10\(^{-9}\) (95\% CL) | > -5.7 × 10\(^{-9}\) (95\% CL) | > -5.5 × 10\(^{-9}\) (95\% CL) |
| \(G_N/G\) (z=0) | > 0.99975 (95\% CL) | > 0.999979 (95\% CL) | > 0.999980 (95\% CL) |
| \(\Omega_m\) | 0.310\(^{+0.019}_{-0.018}\) | 0.3056 ± 0.0074 | 0.2939 ± 0.0064 |
| \(\sigma_8\) | 0.808\(^{+0.024}_{-0.019}\) | 0.814\(^{+0.015}_{-0.011}\) | 0.833 ± 0.012 |
| \(r_s\) [Mpc] | 149.3\(^{+1.8}_{-2.1}\) | 149.2 ± 1.9 | 145.4 ± 1.7 |
| \(\Delta\chi^2\) | 3.0 | 0.4 | -0.6 |

Table 8. Constraints on main and derived parameters (at 68\% CL if not otherwise stated) considering P18 in combination with BAO and BAO + R19 for the CC+N\(_{\text{eff}}\)+\(m_\nu\) model.
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