Cosmic vacuum and large-scale perturbations
in the concordant model

Arthur D. Chernin$^{a,b,c}$

$^a$Sternberg Astronomical Institute,
Moscow University, Moscow, 119899, Russia,
$^b$Tuorla Observatory, University of Turku, FIN-21500, Finland,
$^c$Astronomy Division, University of Oulu, FIN-90014, Finland

Abstract

The evolution of cosmological large-scale perturbations is described in terms of the concordant model based on the recent discovery of cosmic vacuum. It is demonstrated that the process is robustly controlled by a few epoch-independent physical parameters. The parameters guarantee that the initially weak perturbations become nonlinear before (but near) the termination of gravitational instability by cosmic vacuum. No fine tuning for the perturbation amplitude is needed.

PACS: 04.70.Dy; 04.25.Dm; 04.60-m; 95.35.+d; 98.80.Cq

Keywords: Cosmology: theory; Dark matter; Cosmic vacuum; Large-scale structure
1 Introduction

The existence of cosmic vacuum (Riess et al., 1998, Perlmutter et al., 1999) implies (Peebles, 1980, Zeldovich & Novikov, 1982, Heath, 1977, Carroll & Press, 1992) that the gravitational instability and the linear growth of large-scale perturbations in matter are effectively terminated by the antigravity of cosmic vacuum at the redshifts $z \leq 0.7 - 0.3$ when the cosmological dynamics becomes vacuum dominated. The perturbations must obviously become nonlinear before that moment. On the other hand, the nonlinear evolution starts not earlier than at $z \simeq 3 - 10$ (see again the references above). This means that the perturbations are finely tuned: their relative amplitude must reach a unity level within the narrow redshift interval from $z \simeq 10 - 3$ to $z \simeq 1$.

The accuracy of such fine-tuning may be seen from the standard relations for linear perturbation growth. The relative amplitude of density perturbation in cold dark matter grows as

$$\delta(t) \propto (1 + z)^{-1}, \quad z < z_{eq},$$

where $z_{eq}$ is the redshift at the moment when $\rho_R = \rho_D$, and $\rho_R, \rho_D$ are the densities of radiation and (dark) matter. For the earlier epoch of radiation domination,

$$\delta(t) \propto (1 + z)^{-2}, \quad z > z_{eq}.$$  

To reach a unity level, $\delta \sim 1$, within the redshift interval $10 - 1$, the amplitude $\delta(t)$ must be tuned to one part in $10^{16}$ at the epoch of light element production, to one part in $10^{20}$ at the electroweak energy ($\sim 1$ TeV) epoch, or to one part in $10^{60}$ at the Planck epoch.

These fine-tuning considerations (they may easily be refined to take into account all the details of gravitational instability process on various scales and at various epochs, etc.) show that the initial cosmological perturbations that give rise to the formation of the observed large-scale structure are not completely arbitrary. They are rather well organized to be in correspondence with the conditions of the whole cosmic dynamics. Whenever they are generated, the perturbations ‘know’ from the very beginning that they produce the structure before (but near) the termination epoch. The questions are:
Why do they know this in advance? And how exactly are they prepared to yield the condition?

Fine-tuning in cosmology was first discussed more than 30 years ago, when Dicke (1970) mentioned that the Universe must be ‘extremely finely tuned’ to account for the present observed balance between the kinetic energy of expansion $K$ and the gravitational potential energy of cosmic matter $U$. The balance is quantified in terms of $\Omega$, which is the energy ratio, $|U|/K$, and also the ratio of the total matter density, $\rho$, to the critical density, $\rho_c$. Thirty years ago, observational limits on $\Omega$ were described as $0.1 < \Omega_0 < 10$, and it was argued that this range implies a very narrow range at earlier epochs. It is estimated that $\Omega$ departs from unity by one part in $10^{16}$ at the epoch of light element production, or to one part in $10^{60}$ at the Planck epoch. Why was there such a remarkably fine balance between the kinetic and potential energies in the ‘initial conditions’ for the cosmic expansion? This argument is now known as the flatness problem, since the energy ratio is associated with the sign of the spatial curvature in the Friedmann model.

In an analysis of the flatness problem (Chernin, 2003), one and only one order-of-unity constant parameter is recognized behind the apparently fine-tuned evolving energy ratio. In this way, the real meaning of the problem is clarified and the problem is reduced to the question about the physical nature of the control parameter. In the present paper, I try a similar approach to the structure formation problem basing on the Friedmann standard model and the concordant dataset.

Note that inflation models (see a book by Linde, 1990) suggest elegant widely accepted solutions to both flatness problem and perturbation problem with the use of the near-Planckian physics in the very early universe. These hypothetical pre-Friedmann stages of cosmic evolution are out of the scope of my discussion here.

2 Physical parameters of the concordant model

The current standard cosmology is the Friedmann model together with the concordant dataset (see, for instance, a recent review by Peebles and Ratra, 2002). The observational data on cosmic vacuum ($V$), dark (D) matter, baryons (B) and radiation (R) are
represented in the model by four constant parameters $A_V, A_D, A_B, A_R$ that come from the Friedmann ‘thermodynamic’ equation:

$$A = [\kappa \rho a^{3(1+w)}]^{\frac{1}{1+3w}}. \quad (3)$$

Here $w = p/\rho$ is the constant pressure-to-density ratio for a given cosmic component in the co-moving volume; $w = -1, 0, 0, 1/3$ for vacuum, dark matter, baryons and radiation, respectively; $\rho$ is the corresponding energy density; $a(t)$ is the scale factor or/and the curvature radius of the isotropic 3-space; $\kappa = 8\pi G/3 = (8\pi/3)M_{Pl}^{-2}$; the Planck energy $M_{Pl} = 1.2 \times 10^{19}$ GeV. The units are used in which the Boltzmann constant $= c = \hbar = 1$.

Estimated with the concordant dataset, the four parameters prove to be numerically identical, on the order of magnitude (Chernin, 1968, 2002):

$$A_V \sim A_D \sim A_B \sim A_R \sim 10^{60 \pm 1} M_{Pl}^{-1}. \quad (4)$$

The identity of the parameters is hardly a simple arithmetical coincidence; it is rather a remarkable empirical fact in which a kind of epoch-independent internal symmetry between vacuum and cosmic matter reveals (Chernin, 2002).

The Friedmann ‘dynamical’ equation that incorporates these parameters,

$$\ddot{a}^2 = (A_V/a)^2 - 2 + A_D/a + A_B/a + (A_R/a)^2 - k, \quad (5)$$

contains also the fifth parameter which is the curvature discrete parameter $k = 1, 0, -1$, for positive, zero and negative 3-space curvature, correspondingly. This parameter cannot yet be determined directly from observation, so that flat and non-flat versions of the model must be considered, generally. Because of its simplicity, the flat model is usually used as the preferable one in the structure formation theory.

It is more important that the flat model is a good approximation as well for the dynamics and geometry of the non-flat models. Possible deviations from the flat model at the present epoch are strongly constrained by the concordant dataset. As for any other epochs, severe theoretical bounds on possible deviations from parabolic dynamics and flat 3-geometry are given by the concordant model (Chernin, 2003). The bounds come directly from Eqs.(4,5). Indeed, the maximal possible deviations from the flat
model are found to be near the present epoch; they can be quantified with the extremal (maximal or minimal) density ratio $\Omega_{ex}$:

$$\Omega_{ex} \simeq [1 - \frac{1}{2}k(A_V/A_D)^{2/3}]^{-1}. \quad (6)$$

In accordance with the symmetry relation of Eq.(4), $A_V \sim A_D$, and so $|\Omega - 1| \lesssim 1$ for any times.

It is Eq.(6) that enables to recognize the control physical parameter $A_V/A_D$ which is behind the phenomenon of flatness or the fine-tuned energy ratio, as is mentioned in Sec.1.

### 3 Linear perturbations

It was first mentioned by Zeldovich (1965) that the growth rate of linear perturbations in the flat model could be obtained from the Friedmann solution with the use of the variation of its two constants of integrations. The two constants are the curvature parameter $k$ (or the energy $E = K + U$, in the Newtonian treatment) and the time moment of zero radius $t_i$. In the unperturbed model, $k = E = t_i = 0$. If in a local perturbation volume $k \neq 0, t_i = 0$, the relative density perturbation increases as in Eqs.(1,2). If in the perturbation volume $k = 0, t_i \neq 0$, the relative density perturbation decreases as $\delta \propto t^{-1}$. The shape of the local perturbation volume does not matter in the linear analysis; as for its size, it must be larger than the Jeans length in the growing mode.

Having this in mind, let us look more closely at the growing, most interesting, mode of perturbations. The relative amplitude of density perturbation may be expressed via the density ratio $\Omega$:

$$\delta(t) = (\rho - \rho_c)/\rho_c = \Omega - 1. \quad (7)$$

Here $\rho$ is the density of the perturbation,

$$\kappa \rho = A_V^{-2} + A_D a^{-3} + A_B a^{-3} + A_B^2 a^{-4}, \quad (8)$$

described by the Friedmann equation (5) with $k \neq 0$; $\rho_c$ is the unperturbed ($k = 0$) density which is also the 'critical' density for the same equation. The relation of Eq.(7)
between the amplitude $\delta$ and density parameter $\Omega$ provides a clear analogy to the analysis of Sec.2.

From Eq.(5), one can find the time evolution of the perturbation amplitude in the linear approximation:

$$\delta(t) = \Omega - 1 = k(\kappa \rho a^2)^{-1} [1 - k(\kappa \rho a^2)^{-1}]^{-1}, \quad (9)$$

where the scale factor $a(t)$ is given by Eq.(5) with $k = 0$; actually the difference between the perturbed and unperturbed scale factors does affect the right-hand side of Eq.(9), in this approximation; same is for the density in the right-hand side of Eq.(9).

It is easy to see that the standard relations of Eqs.(1,2) are contained in Eqs.(8,9) as two limiting cases of matter and radiation domination. In addition, it is also seen from Eqs.(8,9) that the perturbation growth is terminated when the vacuum energy density becomes larger than the matter densities. Indeed, in the limit $z \ll z_V = (\rho_V/\rho_D(t_0))^{1/3} - 1$ when vacuum dominates completely,

$$\delta(t) \propto a^{-2} \propto \exp(-2t/A_V). \quad (10)$$

The extremal value of the amplitude $\delta(t)$ is reached near the termination at the redshift

$$z_{ex} = 2^{1/3} a(t_0)/(A_V^2 A_D)^{1/3} - 1 = (2\rho_V/\rho_D)^{1/3} - 1. \quad (11)$$

The maximal (positive, $k = 1$) and minimal (negative, $k = -1$) density contrast is given by the relation

$$\delta_{ex} = \frac{1}{2} k(A_V/A_D)^{2/3} [1 - \frac{1}{2} k(A_V/A_D)^{2/3}]^{-1}. \quad (12)$$

The extremal value of the amplitude $\delta$ is completely determined by two parameters: the discrete parameter $k$ and the constant ratio $A_V/A_D$, which is of the order of unity, according to Eq.(4). As a result, one has: $\delta_{ex} \simeq 1$ for $k = 1$ and $\delta_{ex} \simeq -1/3$ for $k = -1$.

It is easy to see that the density contrast in the dark matter distribution is not less than the total density contrast, $\delta_D \geq \delta$, at this moment.

Since the linear analysis becomes invalid near $z \simeq z_{ex}$, the numbers for $\delta_{ex}$ should be understood as approximate. But it is the very fact of nonlinearity that is most
important here: the perturbations do come to the nonlinear regime, and only after that the antigravity of vacuum wins over the gravity of matter.

It is worthwhile to note that Eqs.(7-12) can be re-written to describe (with a similar result) the perturbation evolution in an open \((k = -1)\) model. In this case, perturbations are characterized by \(k = 1\) and \(k = 0\).

4 Conclusions

As we see, the set of Eqs.(7-12) gives a self-consistent description of the perturbation evolution: the linear perturbations develop in the standard time rate and reach a unity-level amplitude to the appropriate epoch. The evolution is determined and robustly controlled by two constant physical parameters: these are the curvature discrete parameter \(k\) and the ratio \(A_V/A_D\). The both are unity in absolute value or about unity: \(|k| = 1, A_V/A_D \sim 1\). The first of them quantifies the perturbations, the second one comes from the symmetry relation between vacuum and matter. In combination, they guarantee a quantitative certainty to the picture of structure formation. At earlier stages of the perturbation evolution, the constants \(A_R\) and \(A_B\) are also in the play. No fine-tuning for the perturbation amplitude is evidently needed, in this picture at any stage. No specific assumptions about the epoch when the perturbations are generated are necessary either.

The relations of Sec.3 indicate that the amplitude of the growing density perturbations does not depend on their spatial scales; such a feature is known as the Harrison-Zeldovich spectrum. This (and not only this) indicates that the perturbations of Sec.3 are not arbitrary. As is obvious from Eqs.(7-9), they are prepared in a special way: each of the perturbation volume is a part of a uniform universe with the same set of parameters \(A_V, A_D, A_B, A_R\) as in the ‘unperturbed’ universe, but with its own curvature parameter \(k\) which is +1 for the areas of enhanced density and −1 for the areas of lower density. The whole distribution of the weak perturbations is represented by a linear superposition of such patches with different sizes and shapes over all the cosmic space.

Generally, the superposition may include not only the growing mode, but also the
decreasing mode of perturbations associated with the parameter $t_i$ which may be a function of the Lagrangian coordinates $\chi$. As is well-known, an arbitrary choice of initial conditions for perturbations assumes the existence of two independent arbitrary functions of $\chi$, one of which is scalar and the other is vector, to describe the density distribution and velocity field, correspondingly. The choice is essentially restricted by the parameters $k$ and $A_V/A_D$, in the description of Sec.3, while the function $t_i(\chi)$ remains arbitrary, and its dynamical effect is rapidly vanishing with time. The first two parameters determine completely the fate of the perturbations, independently of $t(\chi)$.

This is a possible answer to the question about the structure of the initial perturbations that is put in Sec.1.

As for the other question of Sec.1 ("Why do they know..."), the considerations above suggest that there is a link between the local space-time conditions in the universe and its global structure and behaviour. The perturbation analysis reveals only one of the aspects of the link. In this case, the local-global link is recognized in terms of a few constant physical parameters that seemingly run the cosmic grand design on its various space-time scales.

The parameters $k, A_V, A_D, A_B, A_R$ have a phenomenological status in the concordant model, and their numerical values are determined (or can be determined, in principle) from observations of the present-day universe. But their ultimate nature is in the fundamental physics that gave rise to the origin of the Friedmann universe with its cosmic species – vacuum, dark matter, baryons, radiation, etc. This is probably the electroweak-scale physics in which the symmetry relation of Eq.(4) is argued to have its roots (Chernin, 2002).
References

Carroll, S.M., Press, W.H., 1992, Annu. Rev. Atron. Astrophys. 30, 499.

Chernin, A.D., 1968, Nature 220, 250.

Chernin, A.D., 2002, New Astron. 7, 113.

Chernin, A.D., 2003, New Astron. (accepted); preprint: astro-ph/0112158.

Dicke, R.H., 1970, *Gravitation and the Universe*. Amer. Phil. Soc., Philadelphia.

Heath, D.J., 1977, MNRAS 179, 351.

Linde, A., 1990, *Particle physics and inflationary cosmology*. AIP, New York.

Peebles, P.J.E., 1980, *The Large Scale Structure of the Universe*, Princeton Univ. Press, Princeton.

Peebles, P.J.E., Ratra, B., 2002, preprint: astro-ph/0207347.

Perlmutter, S. et al., 1999, ApJ, 517, 565

Riess, A.G. et al., 1998, AJ, 116, 1009.

Zeldovich, Ya.B., 1965, Adv. Astron. Ap. 3, 241.

Zeldovich, Ya.B., Novikov, I.D., 1982. *The Structure and Evolution of the Universe*. (The Univ. Chicago Press, Chicago and London).