High-Resolution Digital SQUID Magnetometer using Sub-Flux Quantum Feedback

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Abstract. Digital SQUIDs with the single flux quantum (SFQ) feedback have attracted much attention because of the feasibility of realizing a wide dynamic range and high slew rate for digital magnetometers. In order to realize higher resolution, we have studied a digital SQUID with sub-flux quantum feedback. Combining the sub-flux quantum feedback to a feedback loop and the direct SFQ feedback to a main loop triggered by carry SFQ pulses from an up/down counter. In this study, we discuss the method of multi-flux quantum generation and compact design of the SFQ up/down counter for high-speed operation of the digital SQUID magnetometer.

1. Introduction

Superconducting quantum Interference Device (SQUID) is known to be one of the most sensitive magnetometer. For linearizing the flux-voltage transfer function of the SQUID, a flux-locked loop (FLL) is used and the dynamic range can be increased. Digital SQUID with single flux quantum $\Phi_0$ feedback is attracting attention because of its high slew rate and wide dynamic range [1], using the single-flux quantum logic [2]. However, the flux feedback of the digital SQUID cannot be lower than the minimum unit of the flux quantum $\Phi_0$ and the magnetic field noise of magnetometer using a digital SQUID is 3-5 orders of magnitude higher than that of a magnetometer using an analog SQUID [3, 4] because the flux quantum feedback causes the quantization noise of the digital SQUID. One way to realize highly sensitive magnetometers with a digital SQUID is to increase the the operational speed of the digital SQUID and employ sub-flux quantum (sub-$\Phi_0$) feedback. The digital SQUID is a kind of the oversampling delta-type (\textDelta-type) analog-to-digital converters (ADCs) [5] and the sub-$\Phi_0$ feedback was already used in the oversampling delta ADCs for reducing the quantization noise.

We have designed a highly sensitive magnetometer that maintain a wide dynamic range using a digital SQUID with sub-$\Phi_0$ feedback[6, 7]. To produce an FLL feedback circuit with a sub-$\Phi_0$ feedback resolution, we used an additional feedback circuit with a magnetic coupling coil to implement the digital SQUID.

In this study, we discuss the method of multi-flux quantum generation and compact design of the SFQ up/down counter for high-speed operation of the digital SQUID magnetometer.
2. Design of Digital SQUID with Sub-Flux Quantum Feedback

Figure 1 shows (a) Schematic of a digital SQUID magnetometer circuit connecting to a pick-up coil, for attaining the sub-$\Phi_0$ flux feedback with a quasi-one-junction SQUID (QOS) current comparator [8, 9], (b) schematic of floating Josephson transmission lines (FJTLs) with floating double flux quantum (FDFQ) drivers [10, 11] working as a multi-flux generators (MFGs) and (c) circuit diagram of an one-bit up/down counter gate, T$^2$-FF [12].

The signal current coupled to a superconducting main loop comprises three inductances: $L_1$, $L_2$ and $L_3$. The SFQ FLL operation is performed by the QOS comparator, followed by a DFF and a NOT. The SFQ pulses are stored in a superconducting feedback loop, comprising $L_f$ and $L_c$, and are coupled to the main superconducting loop ($L_1$, $L_2$ and $L_3$) through a mutual inductance $M$. Because the loop current in the feedback loop continually increases with increasing input current, any circuit without a direct SFQ feedback in the main superconducting loop will suffer restriction of its dynamic range by the critical current of the Josephson junctions (JJs) in the feedback loop. To avoid this problem, we added an up/down counter [3], as shown in Fig. 1(a), connects the outputs S and $\bar{S}$ from DFF and JNOT gates. Direct SFQ feedback to the main loop is attained using the SFQ outputs $F$ and $\bar{F}$ from the up/down counter; this helps to maintain the magnetic flux in the main loop within a certain range. At the same time, $F$ and $\bar{F}$ trigger MFQs to generate parallel SFQ pulses, each contains $2^n$ SFQ pulses, to the feedback loop for compensating the direct SFQ feedback in the main loop. An SFQ pulse is output from the $F$ terminal when the T$^2$-FF output of the most significant bit (MSB) generates carry-up signals and an SFQ pulse is output from the $\bar{F}$ terminal when the T$^2$-FF output of the MSB generates $\bar{F}$.

![Figure 1](image_url)

**Figure 1.** (a) Schematic of a digital SQUID magnetometer circuit connecting a pick-up coil, for attaining the sub-$\Phi_0$ flux feedback with a QOS current comparator [8, 9], (b) schematic of FJTLs with FDFQ drivers [10, 11] and (c) circuit diagram of an one-bit up/down counter gate, T$^2$-FF [12].
carry-down signals.

The sub-$\Phi_0$ feedback resolution is expressed as

$$\Phi_{fb} = \frac{\Phi_0}{2^n}. \quad (1)$$

The value of the integer $2^n$ is roughly determined using $L_\ell$, $L_c$ and $L_2$ and can be changed by changing the inductance of $L_c$ without changing $M_f$. The flux resolution of the SQUID main loop is determined by the magnetic flux resolution of the QOS comparator $\Delta \Phi_{QOS}$ and can be expressed as $\Phi_{res} \approx \frac{\Phi_0}{2^2}$, where $\Delta \Phi_{QOS}/\Phi_0 = 0.725[7]$. Using the oversampling ratio $\alpha = 2^m(=f_{\text{clk}}/f_0)$, where $f_0$ is the Nyquist frequency of the magnetic signal and $f_{\text{clk}}$ is the clock frequency of the SFQ logic circuit, the flux noise $S_{\Phi}^2$ and the SQUID loop slew rate $\Phi_{fb} = |\partial \Phi_{fb}/\partial t|_{\text{max}}$ can be obtained. To construct a magnetometer using the digital SQUID discussed here, it is necessary to connect a pickup loop with a large effective flux capture area to the inductance of $L_{\text{in}}$, shown in Fig. 1(a). For a washer-type inductor $L_1$, the inductance in an $\ell$-turn integrated coil is given approximately by $L_{\text{in}} = \ell^2 L_1$, where $\ell$ is an integer. Using the effective area $A_{p,\text{eff}}$ of a pick-up coil $L_p(=L_{\text{in}}=\ell^2 L_1)$ and the coupling constant $k_{\text{in}}$ between $L_{\text{in}}$ and $L_1$, the magnetic field noise and the magnetic field slew rate for a magnetometer using a digital SQUID are given by

$$S_{B,\text{min}}^{1/2} = \frac{S_{\Phi}^2}{A_{\text{eff},\text{max}}} = \frac{\Phi_{\text{res}}}{A_{\text{eff},\text{max}} \sqrt{f_{\text{clk}}}} = \frac{2\ell \Phi_{\text{res}}}{k_{\text{in}} A_{p,\text{eff}} \sqrt{f_{\text{clk}}}} \approx \frac{2\ell L_p}{k_{\text{in}} A_{p,\text{eff}} 2^{m+1+n} \sqrt{f_0}} \Phi_0 \quad (2)$$

$$\dot{B}_{\text{fb}} = \frac{\Phi_{fb}}{A_{\text{eff},\text{max}}} = \frac{\Phi_{fb} f_{\text{clk}}}{k_{\text{in}} A_{p,\text{eff}}} = \frac{2\ell \Phi_{fb} f_{\text{clk}}}{k_{\text{in}} A_{p,\text{eff}}} = \frac{2^{m+2-n} \Phi_0 f_0}{k_{\text{in}} A_{p,\text{eff}}}. \quad (3)$$

From Eq.(3), large $n$, $m$ and $A_{p,\text{eff}}$ are useful for obtaining a digital SQUID with lower magnetic field noise. On the other hands, from Eq.(4), large $f_{\text{clk}}$, that is large $m$, and small $n$ and $A_{p,\text{eff}}$ are useful for obtaining a digital SQUID with higher slew rate. So there is trade-off between lower magnetic field noise and higher slew rate, but always higher clock frequency $f_{\text{clk}}$ is preferable. A SFQ pulse generator (PG), is previously used for generating a series SFQ pulse train, has big impact for obtaining higher clock frequency $f_{\text{clk}}$ using large $n$. Because of repulsive nature of SFQ pulses, generation time using PG is linearly increased by increasing $n$. So, for realizing higher clock frequency operation with large $n$, we needed to consider use of parallel SFQ generation methods with MFGs.

### 3. Performance of Digital SQUID with Sub-Flux Quantum Feedback

A layout of the proposed digital SQUID was designed based on the CONNECT cell library [13] and its dynamic performance was simulated using device parameters extracted from the layout design using a superconducting circuit simulator (JSIM) [14]. Newly designed T$^2$-FF showed a critical margin of 26 % and a bias margin of 18 % from simulations. Propagation delay of carry (borrow) signals is 18 ps for T$^2$-FF gate, is much faster than those of 83 ps for previous designed up/down counter using TFF, NDRO and two JAND gates. So using T$^2$-FF gates for the up/down counter we would expect small circuit size and also low power consumption.

The simulation results for the digital SQUID are shown in Fig. 2(a) and (b). We confirmed correct operation at a clock frequency of 8 GHz with a feedback resolution of $\Phi_0/2^4$, as shown in Fig. 2(a). The loop current through $L_\ell$ was found to be between -2 and 7 $\mu$A even if the input varied. The loop current through $L_2$ was also found to be between -2 and 0 $\mu$A except current spikes caused by direct SFQ feedbacks.

Fig. 2(b) shows the enlarged views of time scale for $I_L$ of the previous design using PGs and JTLs (upper curve) and the new design using multi-flux generators with FJTLs (lower curve).
Figure 2. Simulation results for a digital SQUID with a clock frequency of 8 GHz and a feedback resolution of $\Phi_0/2^4$ with the multi-flux generators, FJTLs and the up/down counter using T²-FFs. (a) Relationship between the input current $I_{in}$ the feedback loop current $I_{L_f}$ the main SQUID loop current $I_{L_2}$ and count number of the up/down counter. (b) Enlarged view of time scale for $I_{L_2}$ using pulse generators (PGs) and multi-flux generators (MFGs) for digital SQUID with clock frequency of 4 GHz and $n = 3$.

Upper curve shows the direct SFQ feedback to the main loop, followed by the SFQ pulse train, containing eight SFQ pulses and then the QOS operation. This takes 200 ps and we could not increase $n$ without decreasing clock frequency $f_{clk}$. Lower curve shows also the direct feedback to the main loop, followed by the eight SFQ pulses input finished within 70 ps and then the QOS operation. This takes only 120 ps resulting 8 GHz operation.

Fig. 3 shows operating period and frequency as a function of $n$ for the digital SQUID magnetometer using MFQ and T²-FF, and PG for generating series of SFQ pulses. We can also use higher $n$ with keeping clock frequency using MFG for generating parallel SFQ pulses. Finally we expected performance of magnetometer using digital SQUID with $n = 4$. As already mentioned, operation clock frequency $f_{clk}$ is reached up to 8 GHz for digital SQUID with $n = 4$. Fig. 4 shows the clock frequency dependence of the magnetic field noise and the magnetic field slew rate for $n = 4$, calculated using parameter $A_{p,eff} = 164$ mm². It is seen that the higher $f_{clk}$ results in lower magnetic field noise in the magnetometer and larger magnetic field slew rate. From the simulation result, we could also confirm that, with signal bandwidth of 0.977 MHz, the magnetic field noise and the magnetic field slew rate were 1.77 fT/$\sqrt{\text{Hz}}$ and 317 mT/s for the operation at $f_{clk}=8$ GHz.

The most promising application of the digital SQUID will be the SQUID transient Electromagnetics (TEM). Required specifications are the magnetic field noise less than 10 fT/$\sqrt{\text{Hz}}$, the bandwidth more than 5 MHz, the slew rate more than 5 mT/s and the dynamic range more than 190 dB [15]. Expected performance confirmed using the simulations is already satisfied by the digital SQUID discussed in this study.
Figure 3. Operation period and frequency as a function of $n$ for the digital SQUID magnetometer using MFG and $T^2$-FF, and PG for generating series of SFQ pulses.

Figure 4. Magnetic field noise and magnetic field slew rate as a function of operation clock frequency $f_{\text{clk}}$ for the digital SQUID magnetometer with $n = 4$ using MFG and $T^2$-FF.

Conclusion
In this study, we designed the digital SQUID with sub-$\Phi_0$ feedback with the multi-flux generators for SFQ feedback operation and $T^2$-FF gates for the up/down counter. We successfully confirmed the correct digital SQUID operation with a clock frequency of 8 GHz by the analog simulation. This implied much high magnetic field resolution for the digital SQUID with sub-$\Phi_0$ feedback for using high-$J_c$ Nb process.

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