QUARK GLUON PLASMA BAGS AS REGGEONS

K. A. Bugaev

Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

Within an exactly solvable model I discuss the influence of the medium dependent finite width of quark gluon plasma (QGP) bags on their equation of state. It is shown that the large width of the QGP bags not only explains the observed deficit in the number of hadronic resonances, but also clarifies the reason why the heavy QGP bags cannot be directly observed as metastable states in a hadronic phase. I show how the model allows one to estimate the minimal value of the width of QGP bags being heavier than 2.5 GeV from a variety of the lattice QCD data and to get the minimal resonance width at zero temperature of about 600 MeV. The Regge trajectories of large and heavy QGP bags are established both in a vacuum and in a strongly interacting medium. It is shown that at high temperatures the average mass and width of the QGP bags behave in accordance with the upper bound of the Regge trajectory asymptotics (the linear asymptotics), whereas at low temperatures (below a half of the Hagedorn temperature $T_H$) they obey the lower bound of the Regge trajectory asymptotics (the square root one). Thus, for temperatures below $T_H/2$ the spin of the QGP bags is restricted from above, whereas for temperatures above $T_H/2$ these bags demonstrate the typical Regge behavior consistent with the string models.

1 Introduction

Recently several ground shaking results in statistical mechanics of strongly interacting matter are obtained [2, 3, 4, 5, 6]. All of them are derived within the exactly solvable models. In [2, 3, 4, 5, 6] the surface tension of large QGP bags was included into the statistical description of the QGP equation of state (EOS). Such an inclusion not only allowed one to formulate the analytically solvable statistical models for the QCD tricritical [2] and critical [6] endpoint, but also to develop the finite width model of QGP bags [3, 4, 5] and to bring up the statistical description of the QGP EOS to a qualitatively new level of realism by establishing the Regge trajectories of heavy/large QGP bags both in a vacuum and in a strongly interacting medium [4, 5] using the lattice QCD data.

Here I would like to summarize the main results obtained in [2, 3, 4, 5, 6] and discuss the theoretical and experimental perspectives to study the properties of the strongly interacting matter EOS.

2 From the MIT Bag Model EOS to Finite Width Model

From its birth and up to now the MIT bag model [7] remains one of the most popular model EOS to describe the confinement phenomenon of the color degrees of freedom. It gives a very simple picture of a color confinement by considering the freely moving massless quarks and gluons inside the bubble with negative pressure, $-B_{bag}$, (for vanishing baryonic charge). Then the total pressure of a bag is

$$
p_{bag} = \sigma T^4 - B_{bag}, \quad \varepsilon_{bag} = T \frac{dp_{bag}}{dT} - p_{bag} = 3\sigma T^4 + B_{bag},
$$

where $3\sigma$ is the usual Stefan-Boltzmann constant which can be trivially found for a given number of elementary degrees of freedom. The non-trivial feature of the bag model EOS is that the vacuum pressure, $-B_{bag}$, generates positive and large contribution to the bag energy density $\varepsilon_{bag}$.

In addition to a simplicity of the EOS [1] its other attractive features are based on the fact that the bag model also leads [8] to the Hagedorn mass spectrum of bags which was the first signal about the new physics above the Hagedorn temperature $T_H$. The easiest way to understand a special role of the Hagedorn mass spectrum, i.e. an exponentially increasing with mass $m$ density of hadronic states $\rho_H(m) \sim \exp\left(\frac{m}{T_H}\right)$ for $m \gg T_H$, is to consider the microcanonical ensemble of the test particles being in a thermal contact with the resonance of the exponential spectrum [9]. Then one can easily show that the exponential spectrum behaves as the perfect thermostat and perfect particle reservoir, i.e. it imparts its temperature $T_H$ to particles which are in thermal contact with it and forces them to be in chemical equilibrium [10]. Therefore there is simply no reason to study such a system in the canonical ensemble or in grand canonical ensemble, i.e. to bring it into the contact with an external thermostat that has other temperature than $T_H$, since two thermostats of different
temperatures cannot be in equilibrium. Perhaps these properties can explain the fast chemical equilibration of hadrons in an expanding fireball \[10\].

The first successful statistical model explaining the deconfinement phase transition (PT) from hadronic matter to QGP, gas of bags model (GBM) \[8\], is based on the MIT bag model and it interprets such a PT as a formation of the infinitely large bag. Further development in this direction led to many interesting findings \[11\] \[12\] \[13\], including an exact analytical solution for finite systems with a PT \[11\], but the most promising results were obtained only recently. The most hopeful of them are an inclusion of the quark gluon bags surface tension into statistical description \[2\] \[6\] and taking into account the finite width of large/heavy QGP bags \[3\] \[4\] \[5\]. Thus, the surface tension of QGP bags introduced in the quark gluon bag with surface tension model (QGBSTM) allows one to simultaneously describe the 1-st and 2-nd order deconfinement PT with the cross-over and generate the tricritical (QGBSTM1) \[2\] or critical endpoint (QGBSTM2) \[6\] at the vanishing value of the surface tension coefficient as required for the PT of liquid-vapor type \[14\] \[15\]. The finite width model (FWM) \[3\] \[4\] \[5\] naturally resolved two conceptual problems which, so far, were ignored by other statistical approaches for almost three decades: on one hand, the volume dependent width of large QGP bags easily explains a huge deficit in the number of observed hadronic resonances \[10\] with masses above 2.5 GeV predicted by the Hagedorn model \[11\] and used, so far, by the GBM and all its followers; and, on the other hand, the FWM shows that there is an inherent property of the strongly interacting matter EOS, the subthreshold suppression, which prevents the appearance of large QGP bags and strangelets inside of the hadronic phase even as metastable states in finite systems which are studied in nuclear laboratories. The latter effect explains the negative results of searches for strangelets and for not too heavy QGP bags, say with the mass of 10 − 15 GeV, in various physical processes.

The most convenient way to study the phase structure of any statistical model similar to the GBM or QGBSTM is to use the isobaric partition \[2\] \[6\] and find its rightmost singularities. Hence, I assume that after the Laplace transform the FWM grand canonical partition \[Z(V, T)\] generates the following isobaric partition:

\[
\hat{Z}(s, T) \equiv \int_0^\infty dV \exp(-sV) Z(V, T) = \frac{1}{\left[s - F(s, T)\right]},
\]

where the function \(F(s, T)\) contains the discrete \(F_H\) and continuous \(F_Q\) mass-volume spectrum of the bags

\[
F(s, T) \equiv F_H(s, T) + F_Q(s, T) = \sum_{j=1}^n g_j e^{−v_j s} \phi(T, m_j) = + \int dV \int dm \rho(m, v) \exp(−sv) \phi(T, m).
\]

The density of bags of mass \(m_k\), eigen volume \(v_k\) and degeneracy \(g_k\) is given by \(\phi_k(T) = g_k \phi(T, m_k)\) with

\[
\phi_k(T) \equiv \frac{g_k}{2\pi^2} \int_0^\infty p^2 dp \exp \left[−\frac{(p^2 + m_k^2)^{1/2}}{T}\right] = g_k \frac{m_k^2 T}{2\pi^2} K_2\left(\frac{m_k}{2}\right).
\]

The mass-volume spectrum \(\rho(m, v)\) is the generalization of the exponential mass spectrum introduced by Hagedorn \[11\]. Similar to the GBM, the QGBSTM1 and the QGBSTM2, the FWM bags are assumed to have the hard core repulsion of the Van der Waals type which generates the suppression factor proportional to the exponential \(\exp(−sv)\) of bag eigen volume \(v\).

The first term of Eq. (3), \(F_H\), represents the contribution of a finite number of low-lying hadron states up to mass \(M_0 \approx 2.5\) GeV \[4\] which correspond to different flavors. This function has no \(s\)-singularities at any temperature \(T\) and can generate only a simple pole of the isobaric partition, whereas the mass-volume spectrum of the bags \(F_Q(s, T)\) is chosen to generate an essential singularity \(s_Q(T) \equiv p_Q(T)/T\) which defines the QGP pressure \(p_Q(T)\) at zero baryonic densities \[2\]. Very recently our group discovered \[6\] absolutely new way to generate the rightmost singularity of the QGP which is a simple pole too. The latter is achieved by matching the deconfinement PT curve in the baryonic chemical potential and temperature plane with the curve of vanishing surface tension coefficient \[6\].

Here I use the parameterization of the spectrum \(\rho(m, v)\) introduced in \[3\]. It assumes that

\[
\rho(m, v) = \frac{\rho_1(v)}{\Gamma(v)} \frac{N_T}{m^{\alpha+\frac{3}{2}}} \exp\left[\frac{m - (m - B v)^2}{2\Gamma^2(v)}\right], \quad \text{with} \quad \rho_1(v) = f(T) v^{-b} \exp\left[−\frac{\sigma(T)}{v}\right].
\]

In fact, the FWM divides all hadrons into two groups depending on their width: the long (short) living hadrons belong to \(F_H\) (to \(F_Q\)). As one can see from \[3\] the mass spectrum has a Hagedorn like parameterization and the Gaussian attenuation around the bag mass \(B v\) \((B\) is the mass density of a bag of a vanishing width) with the volume dependent Gaussian width \(\Gamma(v)\) or width hereafter. I will distinguish it from the true width defined as \(\Gamma_R = \alpha \Gamma(v)\) \((\alpha \equiv 2\sqrt{\ln 2})\). It is necessary to stress that the Breit-Wigner attenuation of a resonance mass
cannot be used in the spectrum (5) because in case of finite width it would lead to a divergency of the mass integral in (2) or temperatures above \( T_H \).

The normalization factor is defined as
\[
N_\Gamma^{-1} = \int_{M_0}^{\infty} \frac{dm}{\Gamma(v)} \exp \left[ -\frac{(m - Bv)^2}{2v^2} \right].
\] (6)

Such a choice of mass-volume spectrum (5) is a natural extension of early attempts [11,12,13] to explore the QGP bags as Reggeons. For large values of the invariant mass squared, one can show that for heavy free bags (\( m \gg \sigma \)), the Gaussian in (5) acts like a Dirac property that heavy resonances have to develop the large mean width \( \Gamma \). Thus, for \( \Gamma \gg 1 \) leads to the linear Regge trajectory of heavy free QGP bags for large values of the invariant mass squared.

3 Subthreshold Suppression of Large and Heavy QGP Bags

Consider first the free bags. For large bag volumes \( (v \gg M_0/B > 0) \) the factor (3) can be found as \( N_\Gamma \approx 1/\sqrt{2\pi} \). Similarly, one can show that for heavy free bags \( (m \gg BV_0, V_0 \approx 1\text{ fm}^3 \) [3], ignoring the hard core repulsion and thermostat)
\[
\rho(m) = \int_{V_0}^{\infty} \frac{dv}{B} \frac{\rho_1^0(v)}{m^{\kappa+\frac{1}{2}}} \exp \left[ \frac{m}{T_H} \right].
\] (7)

It originates in the fact that for heavy bags the Gaussian in (5) acts like a Dirac \( \delta \)-function for either choice of \( \Gamma_0 \) or \( \Gamma_1 \). Thus, the Hagedorn form of (7) has a clear physical meaning and, hence, it gives an additional argument in favor of the FWM. Also it gives an upper bound for the volume dependence of \( \Gamma(v) \): the Hagedorn-like mass spectrum (4) can be derived, if for large \( v \) the width \( \Gamma \) increases slower than \( v^{1-\kappa/2} = v^{2/\beta} \).

Similarly to (7), one can estimate the width of heavy free bags averaged over bag volumes and get \( \Gamma(v) \approx \Gamma(m/B) \). Thus, for \( \Gamma_1 \) the mass spectrum of heavy free QGP bags must be the Hagedorn-like one with the property that heavy resonances have to develop the large mean width \( \Gamma_1(m/B) = \gamma \sqrt{m/B} \) and, hence, they are hardly observable. Applying these arguments to the strangelets, I conclude that, if their mean volume is a few cubic fermis or larger, they should survive a very short time, which is similar to the results of Ref. [23] predicting an instability of such strangelets.

Note also that such a mean width is essentially different from both the linear mass dependence of string models [24] and from an exponential form of the nonlocal field theoretical models [25]. Nevertheless, as it will be demonstrated while discussing the Regge trajectories, the mean width \( \Gamma_1(m/B) \) leads to the linear Regge trajectory of heavy free QGP bags for large values of the invariant mass squared.

Next let’s calculate \( F_Q(s, T) \) for the spectrum (5) performing the mass integration. There are, however, two distinct possibilities, depending on the sign of the most probable mass:
\[
\langle m \rangle = Bv + \Gamma^2(v)\beta, \quad \text{with} \quad \beta \equiv T_H^{-1} - T^{-1}.
\] (8)

If \( \langle m \rangle > 0 \) for \( v \gg V_0 \), one can use the saddle point method for mass integration to find the function \( F_Q(s, T) \)
\[
F_Q^+(s, T) \approx \left[ \frac{T}{2\pi} \right]^{\frac{1}{2}} \int_{V_0}^{\infty} \frac{dv}{\langle m \rangle^\kappa} \exp \left[ \frac{(s - sT)v}{T} \right]
\] (9)

and the pressure of large bags
\[
p^+ \equiv T \left[ \beta B + \frac{\Gamma^2(v)}{2v} \beta^2 \right], \quad \text{for} \quad \Gamma(v) = \Gamma_1(v) \Rightarrow p^+ = T \left[ \beta B + \frac{1}{2} \gamma^2 \beta^2 \right]
\] (10)
To get (9) one has to use in (3) an asymptotic form of the $K_2$-function $\phi(T, m) \simeq (mT/2\pi)^{3/2} \exp(-m/T)$ for $m \gg T$, collect all terms with $m$ in the exponential, get a full square for $\langle m - \langle m \rangle \rangle$ and make the Gaussian integration.

Since for $s < s_0^Q(T) \equiv p^+(v \to \infty)/T$ the integral (9) diverges at its upper limit, the partition (2) has an essential singularity that corresponds to the QGP pressure of an infinite large bag. One concludes that the width $\Gamma$ cannot grow faster than $v^{1/2}$ for $v \to \infty$, otherwise $\rho^+(v \to \infty) \to \infty$ and $F_Q^+(s, T)$ diverges for any $s$. Thus, for $(m) > 0$ the phase structure of the FWM with $\Gamma(v) = \Gamma_1(v)$ is similar to the QGBSTM1 [2], but it can be also reformulated to have the phase structure of QGBSTM2 [3] by proper choice of the surface tension dependence and value of the index $\tau > 2$. Comparing the $v$ power of the exponential pre-factor in (3) to the continuous volume spectrum of bags of the QGBSTM1 and QGBSTM2, one finds that $a + b \equiv \tau$.

The volume spectrum of bags $F_Q^+(s, T)$ (9) is of general nature and has a clear physical meaning. Choosing different $T$ dependent functions $\langle m \rangle$ and $\Gamma^2(v)$, one obtains different equations of state.

It is possible to use the spectrum (9) not only for infinite system volumes, but also for finite volumes $V \gg V_0$. In this case the upper limit of integration should be replaced by finite $V$ (see Refs. [11,12] for details). This will change the singularities of partition (2) to a set of simple poles $s_0^Q(T)$ in the complex $s$-plane which are defined by the same equation as for $V \to \infty$. Similarly to the finite $V$ solution of the GBM [11,12], it can be shown that for finite $T$ the FWM simple poles may have small positive or even negative real part which would lead to a non-negligible contribution of the QGP bags into the total spectrum $F(s, T)$ (3). In other words, if the spectrum (3), was the only volume spectrum of the QGP bags, then there would exist a finite (non-negligible) probability to find heavy QGP bags ($m \gg M_0$) in finite systems at low temperatures $T < T_c$. Therefore, using the results of the finite volume GBM and SMM, one can conclude that the spectrum (2) itself cannot explain the absence of the QGP bags at $T < T_c$ and, hence, an alternative explanation of this fact is required.

Such an explanation corresponds to the case $(m) \leq 0$ for $v \gg V_0$. From (8) one can see that for the volume dependent width $\Gamma(v) = \Gamma_1(v)$ the most probable mass $\langle m \rangle$ inevitably becomes negative at low $T$, if $0 < B < \infty$. In this case the maximum of the Gaussian mass distribution is located at resonance masses $m = \langle m \rangle \leq 0$. This is true for any argument of the $K_2$-function in $F_Q(s, T)$ (3). Since the lower limit of mass integration $M_0$ lies above $\langle m \rangle$, then only the tail of the Gaussian mass distribution may contribute into $F_Q(s, T)$. A thorough inspection of the integrand in $F_Q(s, T)$ shows that above $M_0$ it is strongly decreasing function of resonance mass and, hence, only the vicinity of the lower limit of mass integration $M_0$ sizably contributes into $F_Q(s, T)$. Applying the steepest descent method and the $K_2$-asymptotic form for $M_0T^{-1} \gg 1$ one obtains

$$F_Q^+(s, T) \simeq \left[ \frac{T}{2\pi} \right]^{3/2} \int_{V_0}^{\infty} d\rho_1(v) N_T \Gamma(v) \exp \left[ \frac{\langle p^- - \sqrt{8\pi} \rangle}{M_0^2} [M_0 - \langle m \rangle + a \Gamma^2(v)/M_0^2] \right], \quad \text{with} \quad p^- \bigg|_{v \gg V_0} = T \frac{\beta M_0 - (M_0 - Bv)^2}{2T^2(v)}$$

(11)

where $p^-$ defines the pressure of QGP bag.

It is necessary to stress that the last result requires $B > 0$ and it cannot be obtained for a weaker $v$-growth than $\Gamma(v) = \Gamma_1(v)$. Indeed, if $B < 0$, then the normalization factor (9) would not be $1/\sqrt{2\pi}$, but would become $N_T \approx [M_0 - \langle m \rangle] \Gamma^{-1}(v) \exp \left[ \frac{(M_0 - Bv)^2}{2T^2(v)} \right]$ and, thus, it would cancel the leading term in pressure (11). Note, however, that the inequality $(m) \leq 0$ for all $v \gg V_0$ with positive $B$ and finite $p^-(v \to \infty)$ is possible for $\Gamma(v) = \Gamma_1(v)$ only. In this case the pressure of an infinite bag is

$$p^-(v \to \infty) = -T \frac{B^2}{2\pi}.$$

Also it is necessary to point out that the only width $\Gamma(v) = \Gamma_1(v)$ does not lead to any divergency of the bag pressure in thermodynamic limit. This is clearly seen from Eqs. (10) and (11) since the multiplier $T^2(v)$ stands in the numerator of the pressure (10), whereas in the pressure (11) it appears in the denominator. Thus, if one chooses the different $v$-dependence for the width, then either $p^+$ or $p^-$ would diverge for the bag of infinite size.

The new outcome of this case with $B > 0$ is that for $T < T_H/2$ [3,4,5] the spectrum (11) contains the lightest QGP bags having the smallest volume since every term in the pressure $p^- \bigg|_{11}$ is negative. The finite volume of the system is no longer important because only the smallest bags survive in (11). Moreover, if such bags are created, they would have mass about $M_0$ and the width about $\Gamma_1(V_0)$, and, hence, they would not be distinguishable from the usual low-mass hadrons. Thus, the case $(m) \leq 0$ with $B > 0$ leads to the subthreshold suppression of the QGP bags at low temperatures, since their most probable mass is below the mass threshold $M_0$ of the spectrum $F_Q(s, T)$. Note that such an effect cannot be derived within any of the GBM-kind models proposed earlier. The negative values of $\langle m \rangle$ that appeared in the expressions above serve as an indicator of a different physical situation comparing to $\langle m \rangle > 0$, but have no physical meaning since $\langle m \rangle \leq 0$ does not enter the main physical observable $p^-$.
along with the QGP pressure, one can estimate the width of these bags directly from Eqs. (10) and (11). To demonstrate the new possibilities let’s now consider several examples of the QGP EOS and relate them to the above results. First, let’s study the possibility of getting the MIT bag model pressure \( p_{bag} \equiv \sigma T^4 - B_{bag} \) by the stable QGP bags, i.e. \( \Gamma(v) \equiv 0 \). Equating the pressures \( p^+ \) and \( p_{bag} \), one finds that the Hagedorn temperature is related to a bag constant \( B_{bag} \equiv \sigma T_H^4 \). Then the mass density of such bags \( \frac{\langle m \rangle}{v} \) is identical to

\[
B = \sigma T_H(T + T_H)(T^2 + T_H^2),
\]

(13)

and, hence, it is always positive. Thus, the MIT bag model EOS can be easily obtained within the FWM, but, as was discussed earlier, such bags should be observable.

The key point of the width estimation is based on the fact that the low \( T \) pressure (12) resembles the linear \( T \) dependent term in the LQCD pressure reported by several groups (for the full list of references see [4]). In [4] it is argued that the ansatz

\[
B(T) = \sigma T_H^2(T^2 + TT_H + T_H^2),
\]

(14)

not only gives the simplest possibility to fulfill all the necessary requirements to the mass density of QGP bags, but it also reproduces the LQCD pressure. Moreover, comparing ansatz (14) with the mass density (13) obtained for the pure MIT bag model pressure, one can see that they differ only by a term \( \sigma T_H T^3 \) which at low \( T \leq 0.5 T_H \) is a negligible correction to (14). Therefore, for low \( T \) the ansatz (14) looks quite reasonable because in this region it corresponds to the mass density of the most popular EOS of modern QCD phenomenology.

Using (14) it was possible to find out that the true width is independent for the number of quark flavors and of the color group number and is \( \Gamma_R(V_0, T = 0) \approx 1.22 \sqrt{T_0^2 T_H^4} \alpha \approx 600 \text{ MeV} \) and \( \Gamma_R(V_0, T = T_H) = \sqrt{12} \Gamma_R(V_0, T = 0) \approx 2000 \text{ MeV} \). These estimates clearly demonstrate that there is no way to detect the decays of such short living QGP bags even, if they are allowed by the subthreshold suppression.

4 Asymptotic Behavior of the QGP Bag Regge Trajectories

The behavior of the width of hadronic resonances was extensively studied almost forty years ago in the Regge poles method what was dictated by an intensive analysis of the strongly interaction dynamics in high energy hadronic collisions. A lot of effort was put forward [26, 27, 28] to elucidate the asymptotical behavior of the resonance trajectories \( \alpha(S) \) for \( |S| \rightarrow \infty \) (\( S \) is an invariant mass square in the reaction). Since the Regge trajectory determines not only the mass of resonances, but their width as well, it is worth to compare these results with the FWM predictions. Note that nowadays there is great interest in the behavior of the Regge trajectories of higher resonances in the context of the 5-dimensional string theory holographically dual to QCD [29] which is known as AdS/CFT.

In what follows I use the results of Ref. [28] which is based on the following most general assumptions: (I) \( \alpha(S) \) is an analytical function, having only the physical cut from \( S = S_0 \) to \( S = \infty \); (II) \( \alpha(S) \) is polynomially restricted at the whole physical sheet; (III) there exists a finite limit of the phase trajectory at \( S \rightarrow \infty \). Using these assumptions, it was possible to prove [28] that for \( S \rightarrow \infty \) the upper bound of the Regge trajectory asymptotics at the whole physical sheet is

\[
\alpha_u(S) = -g_u^2 [-S]^{\nu}, \quad \text{with} \quad \nu \leq 1,
\]

(15)

where the function \( g_u^2 > 0 \) should increase slower than any power in this limit and its phase must vanish at \( |S| \rightarrow \infty \).

On the other hand, in Ref. [28] it was also shown that, if in addition to (I)-(III) one requires that the transition amplitude \( T(s, t) \) is a polynomially restricted function of \( S \) for all nonphysical \( t > t_0 > 0 \), then the real part of the Regge trajectory does not increase at \( |S| \rightarrow \infty \) and the trajectory behaves as

\[
\alpha_l(S) = g_l^2 [-S]^{\frac{2}{3}} + C_l,
\]

(16)

where \( g_l^2 > 0 \) and \( C_l \) are some constants. Moreover, (16) defines the lower bound for the asymptotic behavior of the Regge trajectory [28]. The expression (16) is a generalization of a well known Khuri result [30]. It means that for each family of hadronic resonances the Regge poles do not go beyond some vertical line in the complex spin plane. In other words, it means that in asymptotics \( S \rightarrow +\infty \) the resonances become infinitely wide, i.e. they are moving out of the real axis of the proper angular momentum \( J \) and, therefore, there are only a finite number of resonances in the corresponding transition amplitude. At first glance it seems that the huge deficit of heavy hadronic resonances compared to the Hagedorn mass spectrum supports such a conclusion. Since there is a finite number of resonance families [31] it is impossible to generate from them an exponential mass spectrum and, hence, the Hagedorn mass spectrum cannot exist for large resonance masses. Consequently, the GBM and its followers run into a deep trouble. However, it was possible to show [4] that the FWM with negative value of the most probable bag mass \( \langle m \rangle \leq 0 \) can help to resolve this problem as well.
Note that the direct comparison of the FWM predictions with the Regge poles asymptotics is impossible because the resonance mass and its width $\Gamma(v)$ are independent variables in the FWM. Nevertheless, one can relate their average values and compare them to the results of Ref. [28]. To illustrate this statement, let’s estimate a resonance belonging to the trajectory (18).

The energy plane is identical to that one of resonances belonging to the trajectory (18) with which was used for the free QGP bags, I conclude that the location of the FWM heavy bags in the complex plane (19) of the resonances described by the Regge trajectory (18) and applying absolutely the same logic hadrons [28]!

The most remarkable output of such a conclusion is that the medium dependent FWM mass and width of the extended QGP bags obey the upper bound for the Regge trajectory asymptotic behavior obtained for point-like hadrons [28].

Next I consider the second way of averaging the mass-volume spectrum with respect to the resonance mass

$$m(v) \equiv \int dm \int \frac{d^3k}{(2\pi)^3} \rho(m,v) \; m \; e^{-\sqrt{m^2 + \mathbf{k}^2}} = \frac{\int dm \int \frac{d^3k}{(2\pi)^3} \rho(m,v) \; e^{-\sqrt{m^2 + \mathbf{k}^2}}}{\int dm \int \frac{d^3k}{(2\pi)^3} \rho(m,v)}$$

which is technically easier than averaging with respect to the resonance volume. The latter can be found in [4].

Using the results of Sect. 3 one can find the mean mass [21] for $T \geq 0.5 T_H$ (or for $\langle m \rangle \geq 0$) to be equal to the most probable mass of bag from which one determines the resonance width:

$$\langle m \rangle \approx \langle v \rangle \quad \text{and} \quad \Gamma_R(v) \approx 2\sqrt{2\ln 2} \Gamma_1 \left[ \frac{\langle m \rangle}{B + \gamma^2 \beta} \right] = 2\gamma \sqrt{\frac{2\ln 2 \langle m \rangle}{B + \gamma^2 \beta}}.$$  

These equations lead to a vanishing ratio $\frac{1}{\langle m \rangle} \sim \langle m \rangle^{-\frac{1}{2}}$ in the limit $\langle m \rangle \to \infty$. Comparing with (22) with the mass and width (19) of the resonances described by the Regge trajectory (18) and applying absolutely the same logic which was used for the free QGP bags, I conclude that the location of the FWM heavy bags in the complex energy plane is identical to that of one of resonances belonging to the trajectory (18) with

$$\langle m \rangle \approx |S|^{\frac{1}{2}} \quad \text{and} \quad a_r \approx -4\gamma \sqrt{\frac{\ln 2}{B + \gamma^2 \beta}}.$$  

The most remarkable output of such a conclusion is that the medium dependent FWM mass and width of the extended QGP bags obey the upper bound for the Regge trajectory asymptotic behavior obtained for point-like hadrons [28].

The extracted values of the resonance width coefficient along with the relation (14) for $B(T)$ allow us to estimate $a_r$ as

$$a_r \approx -4 \sqrt{\frac{2T T_H}{2T - T_H} \ln 2}.$$  

Using the formalism of [28], it can be shown that at zero temperature the free QGP bags of mass $m$ and mean resonance width $a \Gamma_1(v)|_{T=0} \approx a \gamma_0 \sqrt{m/B_0}$ precisely correspond to the following Regge trajectory

$$a_r(S) = g_r^2 \left[ S + a_r(-S)^{\frac{1}{2}} \right] \quad \text{with} \quad a_r = \text{const} < 0,$$

with $\gamma_0 \equiv \gamma(T = 0)$ and $B_0 \equiv B(T = 0)$ from [13]. Indeed, using $S = |S| e^{i \phi_r}$ in (18), and expanding the second term on the right-hand side of [18] and requiring $\text{Im} \{a_r(S)\} = 0$, one finds the phase of physical trajectory (one of four roots of one fourth power in (18)), which is vanishing in the correct quadrant of the complex $S$-plane, and going to the complex energy plane $E = \sqrt{S} \equiv M_r - i \frac{\Gamma_r}{2}$, one can also determine the mass $M_r$ and the width $\Gamma_r$

$$\phi_r(S) \rightarrow \frac{a_r \sin \frac{3}{2} \pi}{|S|^{\frac{1}{2}}} \rightarrow 0^{-}, \quad M_r \approx |S|^{\frac{1}{2}} \quad \text{and} \quad \Gamma_r \approx -|S|^{\frac{1}{2}} \phi_r(S) = \frac{|a_r||S|^{\frac{1}{2}}}{\sqrt{2}} = \frac{|a_r|M_r^{\frac{1}{2}}}{\sqrt{2}},$$

of a resonance belonging to the trajectory (18).

Comparing the mass dependence of the width in (19) with the mean width of free QGP bags (17) taken at $T = 0$, it is natural to identify them

$$a_{r\text{free}} \approx -\alpha \gamma_0 \sqrt{\frac{2}{B_0}} = -4 \gamma_0 \sqrt{\frac{\ln 2}{B_0}},$$

and to deduce that the free QGP bags belong to the Regge trajectory (18). Such a conclusion is in line both with the well-established results on the linear Regge trajectories of hadronic resonances [31] and with theoretical expectations of the dual resonance model [32], the open string model [33, 24], the closed string model [33] and the AdS/CFT [29]. Such a property of the FWM also gives a very strong argument in favor of both the volume expectations of the dual resonance model [32], the open string model [33, 24], the closed string model [33] and the AdS/CFT [29].
This expression shows that for $T \to T_H/2+0$ the asymptotic behavior \[18\] breaks down since the resonance width diverges at fixed $|S|$. I think such a behavior can be studied at NICA (Dubna) and/or FAIR (GSI, Darmstadt) energies. This can be seen from the following estimates. From \[24\] it follows that $a_0^2(T = T_H) \approx 22.18 T_H$ and $a_0^2(T \gg T_H) \approx 11.09 T_H$. In other words, for a typical value of the Hagedorn temperature $T_H \approx 190$ MeV \[24\] gives a reasonable range of the invariant mass $|S|^\frac{2}{3} \gg a_0^2(T = T_H) \approx 4.21$ GeV and $|S|^\frac{2}{3} \gg a_0^2(T \gg T_H) \approx 2.1$ GeV for which Eq. \[18\] is true. But then at $|S|^\frac{2}{3} \approx a_0^2(T = T_H) \approx 4.21$ GeV the asymptotic behavior \[18\] should be broken down and, hence, one may see the resonances widening at fixed $S$ values and decreasing $T$.

Now it is possible to find the spin of the FWM resonances $J = \text{Re} \alpha_r \langle m \rangle^2 \approx g^2_\rho \langle m \rangle - \frac{a_0^2}{T}$, which has a typical Regge behavior up to a small correction. Such a property can also be obtained within the dual resonance model \[32\], within the models of open \[33\] and closed string \[33, 24\], and within AdS/CFT \[29\]. These models support the relation between the spin and mass and justify it. Note, however, that in addition to the spin value the FWM determines the width of hadronic resonances. The latter allows one to predict the ratio of widths of two resonances having spins $J_2$ and $J_1$ and appearing at the same temperature $T$ to be as follows

$$
\Gamma_R \left[ \frac{\langle m \rangle_{J_2}}{(B + \gamma^2 \beta)} \right] \Gamma_R \left[ \frac{\langle m \rangle_{J_1}}{(B + \gamma^2 \beta)} \right]^{-1} \approx \sqrt{v_{J_2}} \left[ \sqrt{v_{J_1}} \right]^{-1} \approx \sqrt{\langle m \rangle_{J_2}} \left[ \sqrt{\langle m \rangle_{J_1}} \right]^{-1} \approx \left[ \frac{J_2}{J_1} \right]^\frac{2}{3},
$$

which, perhaps, can be tested at LHC.

Now let’s analyze the low temperature regime, i.e. $T \leq 0.5 T_H$. Using previously obtained results from \[21\] one finds

$$
\overline{m}(v) \approx M_0,
$$

i.e. the mean mass is volume independent. Taking the limit $v \to \infty$, one gets the ratio $\frac{\Gamma(v)}{\overline{m}(v)} \to \infty$ which closely resembles the case of the lower bound of the Regge trajectory asymptotics \[16\]. Similarly to the analysis of high temperature regime, from \[109\] one can find the trajectory phase and then the resonance mass $M_r$ and its width $\Gamma_r$

$$
\phi_r(S) \to -\pi + \frac{2|C_l||\sin(\arg C_l)|}{|S|^\frac{2}{3}}, \quad M_r \approx |C_l||\sin(\arg C_l)| \quad \text{and} \quad \Gamma_r \approx 2|S|^\frac{2}{3}.
$$

Again comparing the averaged masses and width of FWM resonances with their counterparts in \[27\], one finds a similar behavior in the limit of large width of resonances. Therefore, I conclude that at low temperatures the FWM obeys the lower bound of the Regge trajectory asymptotics of \[25\].

The above estimates demonstrate that at any temperature the FWM QGP bags can be regarded as the medium induced Reggeons which at $T \leq 0.5 T_H$ (i.e. for $\langle m \rangle \leq 0$) belong to the Regge trajectory \[16\] and otherwise they are described by the trajectory \[15\]. Of course, both of the trajectories \[16\] and \[15\] are valid in the asymptotic $|S| \to \infty$, but the most remarkable fact is that, to my knowledge, the FWM gives us the first example of a model which reproduces both of these trajectories and, thus, obeys both bounds of the Regge asymptotics. Moreover, since the FWM contains the Hagedorn-like mass spectrum at any temperature, the subthreshold suppression of QGP bags removes the contradiction between the Hagedorn ideas on the exponential mass spectrum of hadrons and the Regge poles method in the low temperature domain! Furthermore, the FWM opens a possibility to apply the Regge poles method to a variety of processes in a strongly interacting matter and account, at least partly, for some of the medium effects.

5 Conclusions

Here I present the novel statistical approach to study the QGP bags with medium dependent width. It is a further extension of the ideas formulated in \[34\]. I argue that the volume dependent width of the QGP bags $\Gamma(v) = \gamma v^2$ leads to the Hagedorn mass spectrum of heavy bags. Such behavior of a width allows us to explain a huge deficit of heavy hadronic resonances in the experimental mass spectrum compared to the Hagedorn model predictions. The key point of our treatment is the presence of Gaussian attenuation of bag mass.

Then it is shown that the Gaussian mass attenuation also allows one to “hide” the heavy QGP bags for $T \leq 0.5 T_H$ by their subthreshold suppression. The latter occurs due to the fact that at low temperatures the most probable mass of heavy bags $\langle m \rangle$ becomes negative and, hence, is below the lower cut-off $M_0$ of the continuous mass spectrum. Consequently, only the lightest bags of mass about $M_0$ and of smallest volume $V_0$ may contribute into the resulting spectrum, but such QGP bags will be indistinguishable from the low-lying hadronic resonances with the short life-time. On the other hand the large minimal width, about 600 MeV, of bags being heavier than $M_0$ and large than $V_0$ prevents their experimental behavior for $T > 0.5 T_H$, even, if they are allowed by the subthreshold suppression. Thus, the FWM naturally explains the absence of directly observable QGP bags and strangelets in the high energy nuclear and elementary particle collisions even as metastable states in hadronic phase.
Using the formalism of [28] it was shown that the average mass and width of heavy/large free QGP bags belong to the linear Regge trajectory \([15]\). Similarly, it was found that at high temperatures the average mass and width of the QGP bags behave in accordance with the upper bound of the Regge trajectory asymptotics \([15]\) (linear trajectory), whereas at low temperatures they obey the lower bound of the Regge trajectory asymptotics \([16]\) (square root trajectory). Such results create a new look onto the large and/or heavy QGP bags as the medium induced Reggeons and provide us with an alternative to the AdS CFT picture on the QGP bags. I would like to stress that it can be used not only to describe the deconfinement or cross-over and the corresponding phases, but for the metastable states of strongly interacting matter as well. Also as shown above such a coherent picture not only introduces new time scale into the heavy ion physics, but also naturally explains the existence of the tricritical or critical QCD endpoint due to the vanishing of the surface tension coefficient. Therefore, I am sure that further development of such a rich direction will lead to the major shift of the low energy paradigm of heavy ion physics and will shed light on the modification of the QCD (tri)critical endpoint properties in finite systems.

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References

[1] R. Hagedorn, Suppl. Nuovo Cimento 3, 147 (1965).
[2] K. A. Bugaev, Phys. Rev. C 76, 014903 (2007); Phys. Atom. Nucl. 71, 1585 (2008); arXiv:0711.3169
[3] K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Europhys. Lett. 85, 22002 (2009); arXiv:0801.4869
[4] K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Rev. C 79, 054913 (2009).
[5] K. A. Bugaev arXiv:0809.1023 [nucl-th].
[6] K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, arXiv:0904.4420 [hep-ph].
[7] A. Chodos et al., Phys. Rev. D 9, 3471 (1974).
[8] J. I. Kapusta, Phys. Rev. D 23, 2444 (1981).
[9] L. G. Moretto, K. A. Bugaev, J. B. Elliott and L. Phair, Europhys. Lett. 76, 402 (2006); arXiv:hep-ph/0511180 and K. A. Bugaev, J. B. Elliott, L. G. Moretto and L. Phair, arXiv:hep-ph/0504011
[10] C. Greiner et al., J. Phys. G 31, S725 (2005); C. Greiner, P. Koch-Steinheimer, F. M. Liu, I. A. Shovkovy and H. Stöcker, arXiv:nucl-th/0703079
[11] K. A. Bugaev, Phys. Part. Nucl. 38, (2007) 447; Acta Phys. Pol. B 36, 3083 (2005).
[12] K. A. Bugaev and P. T. Reuter, Ukr. J. Phys. 52, (2007) 489.
[13] I. Zakout, C. Greiner, J. Schaffner-Bielich, Nucl. Phys. A 781, 150 (2007).
[14] M. E. Fisher, Physics 3, 255 (1967).
[15] L. D. Landau and E.M. Lifshitz, Statistical Physics, (Fizmatlit, Moscow, 2001).
[16] W. Broniowski, W. Florkowski and L. Y. Glozman, Phys. Rev. D 70, 117503 (2004).
[17] K. A. Bugaev, L. Phair and J. B. Elliott, Phys. Rev. E 72, 047106 (2005); K. A. Bugaev and J. B. Elliott, Ukr. J. Phys. 52 (2007) 301.
[18] for a review see J. B. Elliott, K. A. Bugaev, L. G. Moretto and L. Phair, arXiv:nucl-ex/0608022
[19] J. P. Bondorf et al., Phys. Rep. 257, 131 (1995).
[20] K. A. Bugaev, M. I. Gorenstein, I. N. Mishustin and W. Greiner, Phys. Rev. C 62, 044320 (2000); nucl-th/0007062; Phys. Lett. B 498, 144 (2001); nucl-th/0103075 and references therein.
[21] P. T. Reuter and K. A. Bugaev, Phys. Lett. B 517, 233 (2001).
[22] L. G. Moretto et al., Phys. Rev. Lett. 94, 202701 (2005); PoS CPOD2006:037 (2006).
[23] S. Yasui and A. Hosaka, Phys. Rev. D 74, 054036 (2006).
[24] see, for instance, I. Senda, Z. Phys. C 55, 331 (1992).
[25] A. C. Kalloniatis, S. N. Nedelko and L. Smekal, Phys. Rev. D 70, 094037 (2004) and references therein.
[26] for more references see M. Ida, Prog. Theor. Phys. 40, 901 (1968).
[27] R. W. Childers, Phys. Rev. Lett. 21, 868 (1968).
[28] A. A. Trushkovsky, Ukr. J. Phys. 22, 353 (1977).
[29] see, for instance, A. Karch et al., Phys. Rev. D 74, 015005 (2006).
[30] N. N. Khuri, Phys. Rev. Lett. 18, 1094 (1967).
[31] P. D. B. Collins, An Introduction to Regge Theory and High-Energy Physics, (Cambridge University Press, Cambridge, 1977).
[32] G. Veneziano, Nuovo Cim. 57, 190 (1968).
[33] E. V. Shuryak, Prog. Part. Nucl. Phys. 62, 48 (2009).
[34] D. B. Blaschke and K. A. Bugaev, Fizika B 13, 491 (2004); Prog. Part. Nucl. Phys. 53, 197 (2004).