Quark-hadron continuity beyond Ginzburg-Landau paradigm

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Quark-hadron continuity is a scenario that hadronic matter is continuously connected to color superconductor without phase transitions as the baryon chemical potential increases. This scenario is based on Landau’s classification of phases since they have the same symmetry breaking pattern. We address the question whether this continuity is true as quantum phases of matter, which requires the treatment beyond Ginzburg-Landau description. To examine the topological nature of color superconductor, we derive a dual effective theory for \(U(1)\) Nambu-Goldstone (NG) bosons and vortices of the color-flavor locked phase, and discuss the fate of emergent higher-form symmetries. The theory has the form of a topological \(BF\) theory coupled to NG bosons, and fractional statistics of test quarks and vortices arises as a result of an emergent \(Z_3\) two-form symmetry. We find that this symmetry cannot be spontaneously broken, indicating that quark-hadron continuity is still a consistent scenario.

INTRODUCTION

One of the most fundamental questions in nuclear physics is to identify possible phases of quantum chromodynamics (QCD) [1–5]. Understanding the phase structures at finite baryon densities is relevant to the physics inside neutron stars and has been of interest to nuclear and astrophysicists [6]. Hadronic matter is expected to exhibit nucleon superfluidity at finite densities. At very high densities, color superconductivity [7, 8] appears with symmetric-pairing pattern among light three flavors, called color-flavor locked (CFL) phase [9]. As classical many-body physics, phases of matter is classified by the pattern of spontaneous symmetry breaking. Based on this view, it is proposed that nucleon superfluidity and the CFL phase are connected with smooth crossover, since they have the same symmetry breaking pattern: This is the quark-hadron continuity [10].

The question we would like to address here is whether this continuity holds beyond Ginzburg-Landau (GL) paradigm. Classification of quantum phases, i.e. zero-temperature phases of quantum many-body systems, requires beyond-GL description, because local order parameters cannot capture topological order. Importance of topology has been recognized in understanding gapped quantum phases [11, 12]. Microscopic picture of topological order is given by long-range entanglement [13–15], and its low-energy description has spontaneously-broken higher-form global symmetry [16]. An important consequence is that states with different topological order cannot be continuously connected and there should be a quantum phase transition between them. In recent years, the role of topology for gapless quantum systems is also gradually taken into account, and it potentially has an impact for understanding cuprate superconductors [17, 18].

The quark-hadron continuity is recently examined in the presence of superfluid vortices [19–21]. In the CFL phase, the minimal superfluid circulation of vortices is a fractional number 1/3 [22–24]. In addition to \(U(1)\) circulation, they also carry color holonomies. It is pointed out that this is a physical observable using color Wilson loops, and results in fractional statistics between test quarks and vortices [21]. This has a certain similarity with topologically ordered phase in condensed matter physics, which poses a doubt on quark-hadron continuity as quantum phases of matter [21].

In this paper, we carefully examine the role of topology in the CFL phase\(^1\). We first derive the low-energy effective field theory of the CFL phase starting from the gauged GL model. It describes Nambu-Goldstone (NG) bosons associated with the breaking of \(U(1)\) symmetry and superfluid vortices in a unified way. In particular, the effective theory correctly encodes the relation between the superfluid vortex and Wilson loop, which is responsible for the fractional statistics of colored test particles and vortices. We clarify that the \(Z_3\) fractional phase is a consequence of an emergent \(Z_3\) two-form symmetry in the effective theory, generated by color Wilson loops. The charged object under this symmetry is nothing but the CFL vortices. We show that this emergent two-form symmetry cannot be spontaneously broken, and thus emergent two-form gauge field is confined. This means that the CFL phase has a trivial topological structure, and we conclude that quark-hadron continuity scenario is alive.

\(^1\)Earlier works along this direction include [25, 26], in which the effects of color holonomies are not considered. In Ref. [27], Aharonov-Bohm (AB) scattering of quark off color magnetic fluxes is studied in the 2SC phase, in which vortices are not topologically stable. In Ref. [28], scattering of color-neutral particles off CFL vortices is discussed.
also as quantum phases of matter.

**SYMMETRY OF COLOR FLAVOR LOCKING**

We consider 3-flavor QCD with degenerate quark masses. The system has the global symmetry,

\[
SU(3)_f \times U(1) \frac{\mathbb{Z}_3 \times \mathbb{Z}_3}{\mathbb{Z}_3 \times \mathbb{Z}_3},
\]

(1)

where \(SU(3)_f\) is the vector-like flavor symmetry, \(U(1)\) is the quark-number symmetry, and two \(\mathbb{Z}_3\) factors in the denominators are introduced to remove the redundancies among \(SU(3)_f\), \(U(1)\), and \(SU(3)\) color gauge invariance [29–31]. In the presence of large chemical potential, quarks form the Fermi surface with large Fermi momentum. Since QCD has asymptotic freedom, the presence of this typical large energy scale suggests that the system is weakly coupled and semiclassical computation becomes reliable [9]. Within the one-gluon exchange, quark-quark interaction is attractive in the anti-symmetric channel, indicating the Cooper instability of Fermi surface. Motivated by this observation, it is quite useful to introduce the diquark operator \(\Phi\) using the quark field \(|c\rangle \frac{\mathbb{Z}_3 \times \mathbb{Z}_3}{\mathbb{Z}_3 \times \mathbb{Z}_3},\)

(2)

Here, \(c_i\) and \(f_i\) represent the color and flavor labels, respectively. The diquark field \(\Phi\) is in the anti-fundamental representation of \(SU(3)\) color and \(SU(3)_f\) flavor symmetry, and it has charge 2 under \(U(1)\).

Using this diquark field, the simplest effective Lagrangian is given by the gauged Ginzburg-Landau model

\[
S = \frac{1}{2g_{YM}^2}|G|^2 + \frac{1}{2}((\partial + ia_{SU(3)})\Phi|^2 + V_{eff}(\Phi^\dagger \Phi, \text{det}(\Phi)),
\]

(3)

where \(a_{SU(3)}\) is the \(SU(3)_c\) color gauge field, \(G\) is its field strength, and the effective potential \(V_{eff}\) depends only on the color-singlet order parameters, \(\Phi^\dagger \Phi\) and \(\text{det}(\Phi)\), and \(V_{eff}\) has the symmetry \([SU(3)_f \times U(1)]/[\mathbb{Z}_3 \times \mathbb{Z}_3]\). For simplicity of discussion, we neglect the effect of absence of Lorentz symmetry due to the chemical potential, but the extension will be straightforward. Let us now assume that \(V_{eff}\) has the minima at

\[
\Phi^\dagger \Phi = \Delta_0^2 1.
\]

(4)

Taking the determinant of both sides, we get \(|\text{det} \Phi| = \Delta_0^3\). In the gauge-invariant language [32, 33], classical vacua break the global symmetry spontaneously as

\[
\frac{SU(3)_f \times U(1)}{\mathbb{Z}_3 \times \mathbb{Z}_3} \rightarrow \frac{SU(3)_f \times \mathbb{Z}_6}{\mathbb{Z}_3 \times \mathbb{Z}_3} = \frac{SU(3)}{\mathbb{Z}_3} \times \mathbb{Z}_2.
\]

(5)

Picking up a classical vacuum with \(\text{det}(\Phi) = \Delta_0^3\), we can fix the gauge of \(SU(3)\) color group so that

\[
\Phi = \Delta_0 1.
\]

(6)

Since \(\Phi\) is in the bi-(anti-)fundamental representation of \(SU(3)_c \times SU(3)_f\), the symmetry breaking pattern in this fixed gauge looks as

\[
\frac{SU(3)_c \times SU(3)_f \times U(1)}{\mathbb{Z}_3 \times \mathbb{Z}_3} \rightarrow \frac{SU(3)}{\mathbb{Z}_3} \times \mathbb{Z}_6,
\]

(7)

where \(SU(3)_{c+f}\) is the diagonal subgroup of \(SU(3)_c \times SU(3)_f\). This is why it is called color-flavor locking [9].

**DERIVATION OF A UNIFIED THEORY OF **

**NG BOSONS AND CFL VORTICES**

In the CFL phase, there are massless NG bosons associated with the spontaneous breaking of \(U(1)\) baryon number symmetry, and we can construct the phenomenological Lagrangian by nonlinear realization. Because of the quark masses, CFL pions are massive and they can be neglected at low energies. Starting from the gauged GL theory with \(SU(3)_c\) color gauge group, we derive the effective low-energy theory that satisfy this requirement.

In order to correctly describe the possible low-energy excitations including higher dimensional object, it is important to take into account the topology of ground state manifold. Here, we take the gauge so that the diquark field \(\Phi\) is a diagonal matrix,

\[
\Phi = \Delta_0 \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix},
\]

(8)

where \(\phi_i\) is \(2\pi\) periodic scalar fields. This realizes (4), and hence indicates the symmetry breaking pattern (5). This choice of gauge is an analogue of maximal Abelian gauge in Yang-Mills theory with adjoint scalars [34]. In this gauge fixing, the local gauge redundancy becomes the Cartan subgroup of \(SU(3)_c\),

\[
\frac{U(1)_{\tau_3} \times U(1)_{\tau_8}}{\mathbb{Z}_2} \subset SU(3)_c.
\]

(9)

Here, \(U(1)_{\tau_3}\) and \(U(1)_{\tau_8}\) are \(U(1)\) groups generated by \(\tau_3 = \text{diag}[1, -1, 0]\) and by \(\tau_8 = \text{diag}[1, 1, -2]\), respectively. Since the rotations by \(\pi\) in \(U(1)_{\tau_3}\) and \(U(1)_{\tau_8}\) gives the same transformation matrix, \(\text{diag}[e^{i\pi}, e^{i\pi}, 1]\), the group structure is divided by \(\mathbb{Z}_2\). Let us denote the corresponding \(U(1)\) gauge fields by \(a_3\) and \(a_8\), and then the low-energy effective action (3) becomes

\[
S = \frac{1}{2g_0^2} \left(|d\phi_1 + a_3 + a_8|^2 + |d\phi_2 - a_3 + a_8|^2 + |d\phi_3 - 2a_8|^2\right).
\]

(10)
Here, we omit the kinetic term of gauge fields since they become heavy by Higgs mechanism, and $g_0 = \Delta_0^{-1}$.

Each scalar $\phi_i$ is not gauge invariant, and the only gauge-invariant combination is

$$\varphi = \phi_1 + \phi_2 + \phi_3,$$  \hspace{1cm} (11)

and this corresponds to the NG boson associated with the spontaneous breaking of $U(1)$ symmetry. Another important remark is that each Wilson loop of gauge-invariant combination is

$$W_3(C)^2, \ W_8(C)^2, \ W_3(C)W_8(C),$$  \hspace{1cm} (12)

where $W_3(C) = \exp \left( i \int_C a_3 \right)$ and $W_8(C) = \exp \left( i \int_C a_8 \right)$.

As a related fact, the normalization of gauge fields $a_3$, $a_8$ has to be modified from canonical choice of $U(1)$ gauge fields as

$$\int da_3 \in \pi \mathbb{Z}, \ \int da_8 \in \pi \mathbb{Z},$$  \hspace{1cm} (13)

with the constraint

$$\int da_3 = \int da_8 \mod 2\pi.$$  \hspace{1cm} (14)

We are interested in the role of vortex configurations in the CFL phase, and they are realized as the defect of the scalar field in the gauged GL description. For description of topological defects, it is convenient to take an Abelian scalar field in the gauged GL description. For description of the CFL phase, and they are realized as the defect of the periodic scalar field, and

$$S = \frac{1}{2g_0^2}(d\phi + ka) \wedge *(d\phi + ka),$$  \hspace{1cm} (15)

where $\phi$ is the $2\pi$ periodic scalar field, $a$ is the $U(1)$ gauge field, and $k \in \mathbb{Z}$ is the $U(1)$ charge. We can rewrite this theory by introducing the $\mathbb{R}$-valued 3-form field $h$ as

$$S = \frac{g_0^2}{8\pi^2}h \wedge *h - \frac{i}{2\pi} h \wedge (d\phi + ka).$$  \hspace{1cm} (16)

Solving equation of motion of $h$, we get $h = \frac{2\pi i}{g_0^2}*(d\phi + ka)$, and obtain the original action by substitution. Instead of integrating out $h$, we solve the equation of motion of $\phi$ first, and then we obtain that

$$h = db,$$  \hspace{1cm} (17)

with $U(1)$ two-form gauge field $b$. The action becomes

$$S = \frac{g_0^2}{8\pi^2}|db|^2 + \frac{k}{2\pi} h \wedge da.$$  \hspace{1cm} (18)

This is the dual action of the Abelian Higgs model with charge $k$.

Applying this procedure to the effective action (10) for the CFL phase, we obtain

$$S_{\text{eff}} = \frac{g_0^2}{8\pi^2} \sum_{i=1}^{3} |db_i|^2 + \frac{i}{2\pi} \sum_{i=1}^{3} \sum_{A=3,8} K_{iA} b_i \wedge da_A,$$  \hspace{1cm} (19)

where the matrix $K$ is given by

$$K = \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 0 & -2 \end{pmatrix}.\hspace{1cm} (20)$$

This is the low-energy effective gauge theory describing NG boson, vortices, and color Wilson lines$^4$. It has a structure of a topological $BF$ theory coupled with massless NG bosons. General properties of this theory will be discussed elsewhere [38].

**FRACTIONAL STATISTICS AND AN EMERGENT 2-FORM SYMMETRY**

The effective theory derived here encodes the relation between the color holonomies and superfluid circulations. In the dual description, we can define the vortex operator as the Wilson surface operator:

$$V_i(M_2) = \exp \left( i \int_{M_2} b_i \right),$$  \hspace{1cm} (21)

where $M_2$ is a vortex worldsheet. Using (19), one can show that the braiding statistics between the vortex $V_i$ and test quarks $W_A$ is given by$^5$

$$\frac{\langle V_i(M_2) W_A(C) \rangle}{\langle V_i(M_2) \rangle} = \exp \left[ 2\pi i K_{iA}^+ \text{link}(C, M_2) \right],$$  \hspace{1cm} (22)

where $\text{link}(C, M_2) \in \mathbb{Z}$ is the linking number of $C$ and $M_2$, and $K_{iA}^+$ is the Moore-Penrose inverse of $K$.

$$K^+ = \begin{pmatrix} \frac{1}{6} & -\frac{1}{2} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{3} \end{pmatrix}.\hspace{1cm} (23)$$

Now, let us recall that the physical Wilson loops consist only of $W_3^2$, $W_8^2$, and $W_3W_8$. We find that $W_3^2 = 1$, $W_8^2 = (W_3W_8)^{-1}$, and

$$\frac{\langle V_i(M_2) W_8(C)W_8(C) \rangle}{\langle V_i(M_2) \rangle} = \exp \left[ \frac{2\pi i}{3} \text{link}(C, M_2) \right].$$  \hspace{1cm} (24)

$^4$ Let us point out that the effective theory in (19) is 4-dimensional field theory and thus it is different from the 5-dimensional theory proposed in [21].

$^5$ The denominator necessary to cancel non-topological contributions due to the coupling of vortices with massless NG bosons.
This reproduces the observation made in a recent paper [21]. Equation (24) indicates the emergence of a $\mathbb{Z}_3$ two-form symmetry [16] in the CFL phase, where the generators are Wilson loops, $W^2_a$, and charged objects are CFL vortices, $V_i$. The explicit transformation of this two-form symmetry is given by

$$b_1 \mapsto b_1 + \frac{1}{3} \lambda, \quad b_2 \mapsto b_2 + \frac{1}{3} \lambda, \quad b_3 \mapsto b_3 - \frac{2}{3} \lambda,$$  \hspace{1cm} (25)

where $\lambda$ is a flat two-form $U(1)$ connection with $\int_{M_2} \lambda \in 2\pi\mathbb{Z}$ for $\partial M_2 = 0$. Under this transformation, the action changes as

$$\Delta S = \frac{i}{2\pi} \int \lambda \wedge (2 da_8) \in 2\pi i \mathbb{Z},$$  \hspace{1cm} (26)

using the fact that $\int \lambda \in 2\pi\mathbb{Z}$ and $\int da_8 \in \pi\mathbb{Z}$. Since $\exp(-\Delta S) = 1$, we have confirmed that this two-form transformation is the symmetry. Note that there is no one-form symmetry for $a$, unlike the case of the BF theory with level $k$. This is because $\text{dim}(\text{coker } K) \neq 0$, which is equivalent to the existence of massless NG modes.

**IMPLICATION FOR QUARK-HADRON CONTINUITY**

If the CFL is a superfluid phase with topological order, there should be an emergent higher-form symmetry, and it has to be spontaneously broken. We have seen that there exists an emergent $\mathbb{Z}_3$ two-form symmetry, whose charged objects are CFL vortices, $V_i$. However, these vortices show the logarithmic confinement, and $(V_i)$ vanishes as vortex world-sheets become larger. This implies that the $\mathbb{Z}_3$ two-form symmetry is unbroken. Consequently, there is no deconfined topological excitation and the emergent two-form symmetry does not change the topological structure of ground states. Therefore, it does not rule out the possibility that the CFL phase is continuously connected to the nucleon superfluidity.

This can be further supported by a general theorem of quantum field theory, without relying on the mean-field approximation. Since the $U(1)$ symmetry is spontaneously broken, interaction of low-energy Lagrangian should be written by the derivative of NG boson, $\frac{i}{2\pi} d \varphi = \frac{g_0^2}{4\pi^2} \ast d b_1 + d b_2 + d b_3$. If the vortex fluctuation is heavy enough, then the topological defect of $\varphi$ is negligible in the path integral, and $\frac{i}{2\pi} d \varphi$ is conserved $U(1)$ current, generating the $U(1)$ two-form symmetry. There is a subgroup $\mathbb{Z}_3 \subset U(1)$, which could be a different symmetry from Eq. (25). Incidentally, those two symmetries act in the same way on physical observable $\exp(i \int b_i)$ as $2\pi i/3$ phase rotations. A generalized version [16] of Coleman-Mermin-Wagner theorem [39, 40] states that $U(1)$ $p$-form symmetry cannot be broken in less than or equal to $p + 2$ dimension, and thus $U(1)$ two-form symmetry cannot be broken in our 4-dimensional spacetime. Consequently, its subgroup $\mathbb{Z}_3 \subset U(1)$ is unbroken. Since this symmetry has the same order parameter as the emergent $\mathbb{Z}_3$ two-form symmetry, it cannot be broken either in CFL phase. This suggests the *quark-hadron continuity beyond Ginzburg-Landau paradigm*.

**BREAKING SU(3)$_f$ FLAVOR SYMMETRY**

Let us consider the effect of explicit $SU(3)_f$ breaking. To see this, we assume that $V_{\text{eff}}$ has the minimum at $\Phi = \text{diag}(\Delta_1^2, \Delta_2^2, \Delta_3^2)$. After gauge fixing, the diquark field is

$$\Phi = \begin{pmatrix} \Delta_1 e^{i\varphi_1} & 0 & 0 \\ 0 & \Delta_2 e^{i\varphi_2} & 0 \\ 0 & 0 & \Delta_3 e^{i\varphi_3} \end{pmatrix},$$  \hspace{1cm} (27)

instead of (8) (see, e.g., [41]). The absence of $SU(3)_f$ symmetry is translated as $\Delta_i \neq \Delta_j$ for different $i$. Correspondingly, the dual effective action is changed as

$$S_{\text{eff}} = \frac{1}{8\pi^2} \sum_{i=1}^3 g_i^2 |d b_i|^2 + \frac{i}{2\pi} \sum_{i=1}^3 \sum_{A=3,8} K_{iA} b_i \wedge d a_A,$$  \hspace{1cm} (28)

with $g_i = 1/\Delta_i$.

To find the statistics, let us consider the equation of motion under the presence of $V_3(M_2)$ vortex, which again has $1/3$ circulation. Equations of motion of $a_3, a_8$ are

$$d b_1 = d b_2 = d b_3.$$  \hspace{1cm} (29)

Equations of motion of $b_1, b_2, b_3$ say

$$\frac{g_1^2}{4\pi^2} d \ast d b_1 = \frac{i}{2\pi} d (a_3 + a_8),$$

$$\frac{g_2^2}{4\pi^2} d \ast d b_2 = \frac{i}{2\pi} d (-a_3 + a_8),$$

$$\frac{g_3^2}{4\pi^2} d \ast d b_3 = \frac{i}{2\pi} d (-2a_8) - i \delta^+(M_2).$$  \hspace{1cm} (30)

where $\delta^+(M_2)$ is the two-form valued delta function whose support is $M_2$. As a result, for example, we find

$$\frac{\langle V_3(M_2) W_3(C)^2 \rangle}{\langle V_3(M_2) \rangle} = \exp \left( \frac{2\pi i g_3^2}{g_1^2 + g_2^2 + g_3^2} \text{link}(C, M_2) \right),$$  \hspace{1cm} (31)

which is not quantized to $\mathbb{Z}_3$ phase unless we require $g_1 = g_2 = g_3$ coming out of $SU(3)$ flavor symmetry. In the absence of $SU(3)_f$ symmetry, two-form symmetry generated by Wilson loops becomes an infinite group, in general. Since this may be regarded approximately as $U(1)$ two-form symmetry, the vortices should be confined by generalized Coleman-Mermin-Wagner theorem.
We have derived the effective gauge theory of CFL phase describing the NG bosons and vortices. The fractional statistics between vortices and colored test particles is shown to be a result of an emergent $Z_3$ two-form symmetry. Color Wilson loops are the generator of symmetry, and the charged objects are superfluid vortices. This emergent two-form symmetry is unbroken since the vortex-vortex interaction shows logarithmic confinement. This is also supported by the generalized Coleman-Mermin-Wagner theorem since we can find $Z_3$ two-form symmetry is a subgroup of emergent $U(1)$ two-form symmetry generated by $\frac{1}{2}\pi d\varphi$. Therefore, the symmetry breaking pattern of the CFL phase is the same as that of nucleon superfluidity not only for ordinary symmetries but also for higher-form symmetries. The effect of explicit $SU(3)_F$ breaking is also studied, and we checked that no higher-form symmetry is spontaneously broken. Our analysis indicates that the quark-hadron continuity scenario is consistent also as quantum phases of matter.

Our analysis suggests that there is some continuous local deformation of QCD Hamiltonian at finite densities that connects hadronic superfluid and CFL phase without quantum phase transition. It is important to point out, however, that we do not know if the chemical potential direction corresponds to this continuous deformation, so there may exist phase transition when we change the baryon chemical potential. Answer for this question requires the knowledge on dynamics of finite-density QCD, and one must go beyond the kinematical approach based on symmetry, anomaly matching, etc.

Lastly, let us make several comments. The current work is based on a Lagrangian in the mean field approximation, however whole analysis is translated into the language of generalized global symmetry. This indicates that the result of our analysis does not change under the effect of perturbative fluctuations. Vortices can appear as excited states (by rotation, for example). There are Majorana-fermionic excitations inside them [26, 42–44]. Roles and consequences of possible physics from those states inside neutron stars are to be understood.

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\[ \text{SUMMARY AND CONCLUSIONS} \]

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