One-loop Correction of the Tachyon Action in Boundary Superstring Field Theory

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Abstract

We compute one-loop correction to the string field theory action of the tachyon for unstable D-branes in the framework of the boundary superstring field theory. We would expect that the one-loop correction comes from the partition function of the two-dimensional world-sheet theory on the annulus. The annulus correction suggests that the genus expansion is, somehow, governed by the effective string coupling defined in terms of the tachyon $\lambda = g_s \exp(-T^2/4)$. 
1 Introduction

The open string tachyon condensation on non-BPS brane systems has attracted much interest recently. One framework of analysis is level truncation of the open string field theory (SFT) which lead to very good numerical agreements with expected values of vacuum energy and lower-dimensional D-branes tensions \[1, 2\]. Another framework is the boundary SFT (BSFT) [3]-[14]. It was argued that while in the SFT approach an infinite number of massive fields are involved in the condensation process, in the BSFT one can restrict to the tachyon field and study some aspects of the condensation, such as the tensions of the lower-dimensional D-branes, exactly [8, 9] (see also [13, 14]).

The main idea behind BSFT is that “the space of all two-dimensional world-sheet field theory might be a natural arena for string field theory”. In particular, the classical configuration space is the space of two-dimensional world-sheet theories on the disk which are conformal in the interior of the disk but have arbitrary boundary interactions. This idea has been first realized for bosonic string by making use of the Batalin-Vilkovisky formalism in [3]. This formalism provides a generic expression for the classical spacetime action in terms of the disk partition function of the two-dimensional world-sheet theory [4, 5]

\[ S = (1 - \beta^i \frac{\partial}{\partial \lambda^i}) Z, \] (1)

with $\beta^i$ the world-sheet beta function of the boundary coupling $\lambda^i$.

It has been recently shown that the Batalin-Vilkovisky formalism can also be applied to the superstring case [6, 7]. In the superstring case, it turns out that the classical spacetime action is exactly the disk partition function of the two-dimensional world-sheet theory, $S = Z$. We note however that, in the boundary string field theory, it has not been shown that the action provides a single cover of the moduli space, which was one of the nice things of the cubic open string field theory. Nevertheless, one hopes that this will somehow be guaranteed by the Batalin-Vilkovisky structure. At the same time, one hopes that it will also make sense to consider this action off-shell, as we are going to compute an off-shell contribution to the effective action.

Regarding the fact that the classical action comes from the disk computation, one might suspect that the corrections to the action would come from the surfaces with holes. In this sense, the next correction to the action should come from the two-dimensional surfaces with zero Euler number which could be either annulus (orientable) or Möbius strip (nonorientable). In fact, these corrections can be considered as the quantum effective action that one would get upon quantizing the classical BSFT action. The quantum effective action should contain contribution

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1 For review of tachyon condensation in the open string field theory, see [15].

2 I would like to thank J. de Boer for bringing my attention to this point and the point which is discussed in the following paragraph.
from all higher genus surfaces, so it can alternatively be computed directly on the
world-sheet.

This is the aim of this note to compute the partition function of the two-
dimensional world-sheet theory on the annulus and, thereby the one-loop correction
to the tachyon action in the framework of BSFT.

The world-sheet action we are going to study, is

\[ I = I_0 + I_{\text{boundary}}, \]  

with the standard NSR action in the bulk:

\[ I_0 = \frac{1}{4\pi} \int_{\Sigma} dz^2 \left( \partial X^\mu \partial X_\mu + \psi^\mu \partial \psi_\mu + \bar{\psi}^\mu \partial \bar{\psi}_\mu \right). \]  

The integral is over an annulus Σ, with inner and outer radii \( a \) and \( b \), respectively.

The boundary is described by superspace coordinates \((\phi, \theta)\), where \( \theta \) is the boundary Grassman coordinate. Given the boundary superfields \( \Gamma = \eta + \theta F \) and \( \mathbf{X} = X + \theta \psi \), the boundary action \( I_{\text{boundary}} \) is given by [17]:

\[ I_{\text{boundary}} = -\int_{\partial \Sigma} d\phi \frac{d\theta}{2\pi} (\Gamma D \Gamma + T(X) \Gamma), \]  

where \( D = \partial_\theta + \theta \partial_\phi \). Performing the integral over \( \theta \), we get

\[ I_{\text{boundary}} = -\int_{\partial \Sigma} d\phi \frac{d\phi}{2\pi} \left( F^2 + \bar{\eta} \eta + \bar{T} F + \psi^\mu \eta \partial_\mu T \right). \]  

The fermions \( \eta, \psi \) have anti-periodic boundary condition as is appropriate to the
NS sector. The boundary auxiliary fields, \( \eta \) and \( F \), can be integrated out and the boundary action reads:

\[ I_{\text{boundary}} = \frac{1}{4} \int_{\partial \Sigma} \frac{d\phi}{2\pi} \left[ T^2 + (\psi^\mu \partial_\mu T) \partial^{-1}_\phi (\psi^\nu \partial_\nu T) \right]. \]  

Here we will consider a case with the following tachyon profile

\[ T = T_0 + u_\mu X^\mu. \]  

The aim is to compute the partition function of the two-dimensional world-sheet
theory (3) with the boundary perturbation (4) and (5) and, thereby studying the one-loop correction to the string field theory action of the tachyon. We note however
that, although it is not clear how the boundary string field theory goes beyond the
disk approximation, we will follow the same strategy as the disk for the annulus
computation, namely, we assume that the relation between partition function and string action, \( S[u] = Z[u] \), holds in the annulus approximation.

We note also that, in general the partition function would be a function of the
annulus modulus, therefore we have to integrate over the annulus modulus. As a
result we have

\[ S[u] = \int_{\text{annulus modulus}} Z[a, b, u], \]
where
\[ Z[a, b, u] = \int DXD\psi e^{-I_0 - I_{\text{boundary}}}. \] (9)

The paper is organized as follows. In section 2, we will explicitly write down the form of Green’s function of the fields \( X \) and \( \psi \) on the annulus. In section 3, by making use of the explicit form of the Green’s functions, we will compute the partition function of the world-sheet theory given by (2) on the annulus and thereby, we shall study the spectrum of open strings with different boundary conditions on their end points. In section 4, we will compute the one-loop correction to the string field action of tachyon. In section 5, we shall give some comment on the results and the problem which this computation could have. In this paper we shall only consider the case of unstable D9-brane. One can easily generalize the result to the other unstable branes. An interesting question would be to generalize the results to the cases of the brane-antibrane system and those in the presence of a nonzero B-field as well.

2 The Green’s Function

In this section we are going to write down the explicit form of propagators of \( X \) and \( \psi \) in the annulus which are needed when we will be going to compute the partition function (9). In general we could have different boundary conditions for each boundary of the annulus. Therefore the general tachyon profile would be \( T = uX \) at boundary \( b \), and, \( T = vX \) at boundary \( a \). Actually, this means, we are considering an open string stretched between two D9-branes. In following we set \( b = 1 \). The Green’s function for \( X \) is given by \[ \text{18} \] (see also \[ \text{19} \]):

\[
G(z, w) = -\ln|z - w|^2 + \frac{2}{y} + \frac{xy\beta}{2} \left( \ln z\bar{z} - \frac{2}{y} \right) \left( \ln w\bar{w} - \frac{2}{y} \right) \\
+ \sum_{k=1}^{\infty} \frac{1}{k} C_k \left[ \left( \frac{z}{w} \right)^k + \left( \frac{w}{z} \right)^k + \left( \frac{\bar{z}}{\bar{w}} \right)^k + \left( \frac{\bar{w}}{\bar{z}} \right)^k \right] \\
+ \sum_{k=1}^{\infty} \frac{k + y}{k(k - y)} \frac{k - y}{k(k + y)} C_k \left[ \left( \frac{1}{z\bar{w}} \right)^k + \left( \frac{1}{\bar{z}w} \right)^k \right] \\
+ \sum_{k=1}^{\infty} \frac{k - y}{k(k + y)} (C_k + 1) \left[ (z\bar{w})^k + (\bar{z}w)^k \right],
\]

where \( y = u^2, \ x = v^2 \), and

\[
C_k = \frac{a^{2k}}{(k+y)(k+ax)} - \frac{a^{2k}}{(k-y)(k-ax)}, \quad \beta^{-1} = xy\ln a - x - \frac{y}{a}.
\]

\[ \text{3I would like to thank J. de Boer for his comment which led me to add section 4 in order to clarify the misinterpretation of the results in the early version of the paper.} \]
Setting $z = e^{i\phi}$ and $\omega = e^{i\phi'}$ one can compute the propagator of $X$ at the boundary $b = 1$. Collecting all terms together in (10), we get
\[
G_{\text{bos}}(\phi - \phi'; x, y)|_1 := \langle X(\phi)X(\phi') \rangle_{(x,y)}|_1
\]
\[
= 2 \sum_{k \in \mathbb{Z}} \frac{1}{|k| + y} e^{i k (\phi - \phi')} + 2 \sum_{k \in \mathbb{Z}} \frac{2|k|}{k^2 - y^2} C|k| e^{i k (\phi - \phi')} .
\]
(12)

Note that in the second summation the $k = 0$ case is singular, regularizing this term we get a term with the form of $2y \beta$.

Doing the same computation for the boundary at $a$, we find:
\[
\langle X(\phi)X(\phi') \rangle_{(a)}|_1 = \langle X(\phi)X(\phi') \rangle_{(ax)}|_1 .
\]
(13)

In order to find the propagator for the fermion we note the fact that, if the fermions $\psi$ were periodic, the contribution of the fermion would precisely cancel that of the bosons, so the full partition sum would be trivial. In fact the inverse derivative of the propagator of the fermion must be very similar to the bosonic one, the only difference being the overall sign and range of the index $k$ [9]. Therefore we get
\[
G'_{\text{fer}}(\phi - \phi'; x, y)|_1 := \langle \psi(\phi)\partial_{\phi'}^{-1} \psi(\phi') \rangle_{(x,y)}|_1
\]
\[
= -2 \sum_{k \in \mathbb{Z}+\frac{1}{2}} \frac{1}{|k| + y} e^{i k (\phi - \phi')} - 2 \sum_{k \in \mathbb{Z}+\frac{1}{2}} \frac{2|k|}{k^2 - y^2} C|k| e^{i k (\phi - \phi')} ,
\]
(14)

and for the boundary $a$, we have
\[
\langle \psi(\phi)\partial_{\phi'}^{-1} \psi(\phi') \rangle_{(a)}|_1 = \langle \psi(\phi)\partial_{\phi'}^{-1} \psi(\phi') \rangle_{(ax)}|_b .
\]
(15)

Moreover from (14) one can extract the boundary propagator for the fermions
\[
G_{\text{fer}}(\phi - \phi'; x, y)|_1 := \langle \psi(\phi)\psi(\phi') \rangle_{(x,y)}|_1
\]
\[
= 2i \sum_{k \in \mathbb{Z}+\frac{1}{2}} \frac{k}{|k| + y} e^{i k (\phi - \phi')} + 2i \sum_{k \in \mathbb{Z}+\frac{1}{2}} \frac{2k|k|}{k^2 - y^2} C|k| e^{i k (\phi - \phi')} ,
\]
(16)

similarly, from the equation (14) one can write the boundary propagator for the fermions at boundary $a$.

By writing the index $k$ in (14) as one half of an odd number, and rewriting the sum over odd integers as a difference of a sum over all integers and that over even integers, one finds
\[
G'_{\text{fer}}(\phi - \phi'; x, y, a)|_1 = G_{\text{bos}}(\phi - \phi'; x, y, a)|_1 - 2G_{\text{bos}}(\phi - \phi'; 2x \sqrt{a}, 2y, \sqrt{a})|_1 ,
\]
(17)
and the same for the boundary at $a$. 

4
3 Partition Function

In this section we shall compute the partition function on the annulus, by making use of the Green’s function given in the previous section. First we will assume a constant tachyon profile.

The partition function for a constant tachyon profile can be obtained by setting $u_{\mu} = 0$ in (7) and performing the path integral (9). This leads to

$$Z[a, 0] = e^{-\frac{1}{4} \int d\phi T^2_0} \int DXD\psi e^{-I_0} = V_0 e^{-\frac{a+1}{4} T^2_0},$$

where $V_0 = \int DXD\psi e^{-I_0}$. Therefore, we get

$$Z[T_0] = \int_0^1 \frac{da}{a} V_0 e^{-\frac{a+1}{4} T_0^2}.$$

In order to perform the integral over modulus we need to find $V_0$ as a function of modulus. Indeed, what we need to find is the vacuum amplitude on the annulus. For the bosonic open string theory this has been computed in [20]. Actually, we need to do the same computation for the superstring one. The procedure is the same as in [20], i.e. we note that the variation of the one-loop partition function with respect to the modulus is proportional to the expectation value of the energy-momentum tensor [21]

$$\frac{\partial \ln V_0[a]}{\partial a} = \frac{a}{(1 - a^2) \pi} \int d^2 z \left( \frac{1}{z^2} T_{zz} + \frac{1}{z^2} T_{\bar{z}\bar{z}} \right),$$

where

$$T_{zz} = -\frac{1}{2} \langle \partial X \partial X \rangle - \frac{1}{2} \langle \psi \partial \psi \rangle$$

is the energy-momentum tensor of the two-dimensional world-sheet theory. A straightforward computation, like what has been done for the bosonic case in [24], shows

$$V_0 \sim \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}}.$$

Plugging this result into (19), one finds

$$V(T_0) = T_0 \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}} e^{-\frac{a+1}{4} T_0^2} e^{-\frac{a+1}{4} T_0^2}.$$

Here, in order to emphasize the contribution of each boundary we separated the exponential into the two parts. We will use this result for fixing the integration constant when we will be computing the partition function for a non-constant tachyon profile. This could also be used for fixing the the normalization of the coordinates which is important to find the correct tachyon potential.
In order to compute the full action, we start with a tachyon profile such that \( T = uX \) at boundary \( b = 1 \) and \( T = vX \) at \( a \). So, the boundary action (6) reads:

\[
I_{\text{boundary}} = \frac{y}{8\pi} \int d\phi \left( X^2 + \psi \partial^{-1}_\phi \psi \right) \quad \text{at} \quad 1, \\
I_{\text{boundary}} = \frac{x}{8\pi} \int d\phi \left( X^2 + \psi \partial^{-1}_\phi \psi \right) \quad \text{at} \quad a.
\] (24)

Plugging the boundary action into (9) and differentiating with respect to \( y \) and \( x \), we get

\[
\frac{\partial \ln Z[x,y,a]}{\partial y} = -\frac{1}{8\pi} \int d\phi \langle X^2 + \psi \partial^{-1}_\phi \psi \rangle \quad \text{at} \quad 1, \\
\frac{\partial \ln Z[x,y,a]}{\partial x} = -\frac{1}{8\pi} \int d\phi \langle X^2 + \psi \partial^{-1}_\phi \psi \rangle \quad \text{at} \quad a.
\] (25)

Here the correlator needs to be regularized. In our case the natural prescription for defining the correlator is [9]

\[
\langle X^2 + \psi \partial^{-1}_\phi \psi \rangle = \lim_{\epsilon \to 0} \langle X(\phi)X(\phi + \epsilon) + \psi(\phi)\partial^{-1}_\phi \psi(\phi + \epsilon) \rangle.
\] (26)

We note that, this regularization has to do separately for both boundaries at \( a \) and \( b = 1 \). Using the correlators we have obtained in the previous section, (26) can be written as

\[
\langle X^2 + \psi \partial^{-1}_\phi \psi \rangle |_1 = \lim_{\epsilon \to 0} \left[ G_B(\epsilon; x, y) + G'_F(\epsilon; x, y) \right]_1 \\
= 2 \lim_{\epsilon \to 0} \left[ G_B(\epsilon; x, y, a) - G_B(\frac{\epsilon}{2}; 2x\sqrt{a}, 2y, \sqrt{a}) \right]_1, \\
\langle X^2 + \psi \partial^{-1}_\phi \psi \rangle |_a = \lim_{\epsilon \to 0} \left[ G_B(\epsilon; x, y) + G'_F(\epsilon; x, y) \right]_a \\
= 2 \lim_{\epsilon \to 0} \left[ G_B(\epsilon; x, y, a) - G_B(\frac{\epsilon}{2}; 2x\sqrt{a}, 2y, \sqrt{a}) \right]_a.
\] (27)

To evaluate this limit it is convenient to use the following expression for the bosonic propagator

\[
G_B(\epsilon; x, y, a) |_1 = -2\ln(1 - e^{i\epsilon}) - 2\ln(1 - e^{-i\epsilon}) + \frac{2}{y} - 2y \sum_{k=1}^{\infty} \frac{1}{k(k+y)} (e^{ik\epsilon} + e^{-ik\epsilon}) + \frac{2x}{y} \beta + 2 \sum_{k=1}^{\infty} \frac{2k}{k^2 - y^2} C_k (e^{ik\epsilon} + e^{-ik\epsilon}).
\] (28)

By making use of this form, we find

\[
\frac{1}{2} \langle X^2 + \psi \partial^{-1}_\phi \psi \rangle |_1 = -4\ln2 + f(y) + g(x, y, a) - \left( f(2y) + g(2x\sqrt{a}, 2y, \sqrt{a}) \right). \] (29)
where
\[
f(y) = \frac{2}{y} - 4 \sum_{k=1}^{\infty} \frac{y}{k(k+y)},
\]
\[
g(x, y, a) = \frac{2x}{y} \beta + 4 \sum_{k=1}^{\infty} \frac{2k}{k^2 - y^2} C_k.
\]

The same expression for the boundary \(a\) can be obtained by replacing \(y \rightarrow ax\) and \(x \rightarrow y/a\).

Now we have all information to compute the partition function. Using this information, the derivative of the partition function, \((25)\), for the case of \(y \neq ax\) reads
\[
\left. \frac{\partial \ln Z[x, y, a]}{\partial y} \right|_{y=1} = \frac{\partial}{\partial y} \ln \left| xy \ln a - \frac{y}{a} - x \right|^{-\frac{1}{2}} \times \prod_{k=1}^{\infty} \frac{(k+2y)(k+2ax) - (k-2y)(k-2ax)a^k}{[(k+y)(k+ax) - (k-y)(k-ax)a^{2k}]^2},
\]
\[
\left. \frac{\partial \ln Z[x, y, a]}{\partial x} \right|_{x=1} = \frac{\partial}{\partial x} \ln \left| xy \ln a - \frac{y}{a} - x \right|^{-\frac{1}{2}} \times \prod_{k=1}^{\infty} \frac{(k+2y)(k+2ax) - (k-2y)(k-2ax)a^k}{[(k+y)(k+ax) - (k-y)(k-ax)a^{2k}]^2}. \tag{31}
\]
Note that the derivatives of the \(\ln Z\) with respect to \(y\) and \(x\) have the same form and moreover, the expression is symmetric under exchanging of \(y \leftrightarrow ax\). This is the reason why in our computation we excluded the case of \(y = ax\). We will come back to this point later. One can now integrate and find the partition function. Using the \(\zeta\)-function regularization, we get
\[
Z[x, y, a] = Z_0[a] \frac{F(y)F(ax)}{\sqrt{|xy \ln a - \frac{y}{a} - x|}} \prod_{k=1}^{\infty} (1 + C_k) \prod_{r=\frac{1}{2}}^{\infty} (1 + C_r), \tag{32}
\]
where
\[
F(z) = 4^z \frac{\Gamma^2(z)}{2\Gamma(2z)}. \tag{33}
\]
Moreover, \(Z_0[a]\) is the integration constant which in general, as it is written explicitly, could be a function of the annulus modulus. It can be fixed by comparison with the partition function of a constant tachyon profile \((23)\). In fact, it can be seen that \(Z_0[a] = Z_0 a^{-1/2}\) with a numerical factor \(Z_0\). Thus the equation \((32)\) reads
\[
Z[x, y, a] = Z_0 \frac{F(y)F(ax)}{\sqrt{|axy \ln a - y - ax|}} \prod_{k=1}^{\infty} (1 + C_k) \prod_{r=\frac{1}{2}}^{\infty} (1 + C_r), \tag{34}
\]
We would like to note that, the partition function \((34)\) is a monotonically decreasing function of \(y\) and \(x\) as is needed in order to be identified with tachyon action. This
can be seen if we compare the expression (34) with that of the disk computation [1]

\[ Z_{\text{disk}}[y] = Z_0 y^{-\frac{1}{2}} F(y), \]  

(35)

Of course it is not the end of story. One has to integrate over the modulus of the annulus

\[ Z[x, y] = Z_0 \int_0^1 \frac{da}{a} \frac{F(y) F(ax)}{\sqrt{|axy \ln a - y - ax|}} \prod_{k=1}^{\infty} \frac{(1 + C_k)}{(1 + C_k)}, \]  

(36)

We would like to note that, having an annulus would naturally lead to a system with two parallel D-branes corresponding to two boundaries of the annulus. Depending on which boundary condition we would like to impose at the boundaries of the annulus, we could get different boundary conditions at the end points of the open strings. The different boundary conditions can be emerged by different possibilities which could be chosen for \( x \) and \( y \). In particular, setting \( y = x \) corresponds to those open strings with the same boundary condition at two end points. In this case, the partition function has the following expansion around \( y = 0 \)

\[ Z[x] = Z_0 \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}} \frac{1}{\sqrt{(1 + a)x}} \left[ 1 + \left( \frac{2(1 + a) \ln 2 + \frac{1}{2} \frac{a \ln a}{1 + a} \right) x + \cdots \right] \]  

(37)

which could be used to obtain the string field theory action up to two-derivative

\[ S = T_9 \int d^{10}X \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}} e^{-\frac{1+\frac{a}{a}}{4} T^2} \left[ 1 + \left( \frac{2(1 + a) \ln 2 + \frac{a \ln a}{2(1 + a)} \right) \partial^\mu T \partial_\mu T \right]. \]  

(38)

We note, however, that this is not the one-loop correction to the tachyon action we are looking for. In fact, it would be wrong if we read the one-loop correction to the tachyon potential using this action. In order to find the correction to the tachyon potential one should consider the partition function on the annulus for a single D9-branes which can be obtained under certain condition which has to be imposed for \( x \) and \( y \). This is what we are going to compute in the next section.

The spectrum of those open strings with different boundary conditions can also be obtained by setting, for example, \( x = 0 \). In this case the partition function (36) reads

\[ Z[y] = Z_{\text{disk}}[y] \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} (1 + C_k) \prod_{r=1}^{\infty} (1 + C_r). \]  

(39)

Using the expansion of (39) around \( y = 0 \), one can find the corresponding string action up to two-derivative

\[ S = T_9 \int d^{10}X \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}} e^{-\frac{1+\frac{a}{a}}{4} T^2} \left[ 1 + 2 \ln 2 \partial^\mu T \partial_\mu T \right]. \]  

(40)

\[ \text{we note that, since the modulus } a \text{ takes value between zero and one, the } \ln a \text{ term is always negative. As the result } (1 + a) - ax \ln a \text{ can not be zero. Therefore the partition function is not singular. I would like to thank T. Suyama for bringing my attention to this point.} \]
Finally we have the following expression for the partition function before and after tachyon condensation (for the bosonic string see [18])

\[
Z \sim \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}}, \quad \text{NN (} x = y \to 0 \text{)} ,
\]

\[
Z \sim \int_0^1 \frac{da}{a} (\ln a)^{-\frac{1}{2}} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}}, \quad \text{DD (} x = y \to \infty \text{)} ,
\]

\[
Z \sim \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 + a^{2k-1}}{1 + a^{2k}}, \quad \text{ND (} x = 0, y \to \infty \text{)} .
\] (41)

4 One-loop Tachyon Action

So far we have used the annulus partition function in order to study the spectrum of the open string stretched between two parallel D-branes. As we have already mentioned, having an annulus would naturally lead to a system with two branes. Nevertheless, under a certain circumstances one can also have a system with a single D9-branes. Regarding the fact that under exchanging of \( y \leftrightarrow ax \) we go from one boundary to the other one (see section 2), setting \( y = ax \) would mean that we are identifying two boundaries of the annulus and this, somehow, means that both boundaries represent a single D9-brane. In this case, we have \( C_k^{-1} = a^{-2k} \left( \frac{k+y}{k-y} \right)^2 - 1 \). Therefore, one finds

\[
\frac{\partial \ln Z[y, a]}{\partial y} = 2 \frac{\partial}{\partial y} \ln \left[ |y^2 \ln a - 2y|^{-\frac{1}{2}} \prod_{k=1}^{\infty} \frac{(k+2y)^2 - (k-2y)^2 a^k}{(k+y)^2 - (k-y)^2 a^{2k}} \right] ,
\] (42)

where the factor of two is because of the contribution of two boundaries. Doing the same computation as that in the previous section, we get

\[
Z[y] = Z_0 \int_0^1 \frac{da}{a} \frac{F^2(y)}{\sqrt{|y^2 \ln a - 2y|}} \frac{\prod_{k=1}^{\infty} (1 + C_k)}{\prod_{r=1}^{\infty} (1 + C_r)} .
\] (43)

By making use of this expression for the partition function on the annulus, the one-loop correction to the string field theory action for the tachyon can be obtained as follow

\[
S = T_9 \int d^{10} X e^{-\frac{1}{2} T^2} \int_0^1 \frac{da}{a} \frac{F^2 (\partial_{\mu} T \partial^{\mu} T)}{\sqrt{1 - \frac{\ln a}{2} \partial_{\mu} T \partial^{\mu} T}} \frac{\prod_{k=1}^{\infty} [1 + C_k (\partial_{\mu} T \partial^{\mu} T)]}{\prod_{r=1}^{\infty} [1 + C_r (\partial_{\mu} T \partial^{\mu} T)]} ,
\] (44)

where

\[
C_n (\partial_{\mu} T \partial^{\mu} T) = \frac{a^{2n}}{\left(n \partial_{\mu} T \partial^{\mu} T \right)^2} - a^{2n} \quad n = r, k .
\] (45)
In particular the one-loop correction to the tachyon potential is

\[ V_{\text{one-loop}}(T) = T_9 e^{-\frac{1}{2}T^2} \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}}. \] (46)

As we see, the annulus correction to the potential does not change the shape of the potential given by the disk computation \cite{9}, i.e. it has extremum at \( T = 0, \infty \). Nevertheless the unstable D9-brane tension has to be renormalized as the above integral diverges at \( a = 0 \). More precisely, setting \( \lambda = g_s e^{-T^2/4} \) \cite{8} and adding this result to that of disk computation \cite{1}, one finds

\[ V(T) = T_9 e^{-\frac{1}{2}T^2} \left( \frac{1}{\lambda} + \int_0^1 \frac{da}{a} \prod_{k=1}^{\infty} \frac{1 - a^{2k-1}}{1 - a^{2k}} \right). \] (47)

One can now absorb the infinite part of the modulus integral by renormalizing the effective string coupling

\[ V(T) = T_9 e^{-\frac{1}{2}T^2} \left( \frac{1}{\bar{\lambda}} + \Lambda \right). \] (48)

where \( \bar{\lambda} \) is the renormalized effective coupling and \( \Lambda \) is the finite part of the modulus integral.

5 Conclusion

In this letter we considered the superstring field theory on the annulus and computed the partition function of the world-sheet action on the annulus with the linear tachyon perturbation. By making use of the partition function, we have been able to write down the one-loop correction to the string field action of the tachyon. In this computation, we assumed that \( Z[x, y] = S[x, y] \), as we have in the string field theory on the disk. But it remains to be argued, if this assumption is correct beyond the disk approximation. Looking at the explicit derivation of the relation between string action and the partition function in the superstring case \cite{6, 7}, it is not a priori obvious that this should be the case, as their results depend on the explicit form of the correlation function of matter and ghost fields on the disk. Nevertheless, since the partition function we found is monotonically decreasing, hopefully setting \( S = Z \) comes true after all!

Using the one-loop computation, it seems that we would get the following perturbative expansion for the partition function

\[ Z = e^{-\frac{1}{2}T^2} \left( \frac{1}{\lambda} Z_1 + Z_0 + \cdots \right) \] (49)

where \( Z_\chi \) is obtained from the path integral on surface of Euler number \( \chi \) and no handles. This form has recently been suggested in \cite{8}. We would like to note that, in
the early version of this paper, the one-loop correction to the tachyon potential had
been extracted from equation (38) and therefore we had concluded that the genus
expansion is not governed by the effective string coupling defined in terms of the
tachyon (19), as has been discussed in [21]. In fact, as we mentioned in the previous
section, the correction to the tachyon action has to be read from (44) rather than
(38).

Note added: While typing the paper we received [22] where the same subject
has been studied, though, with a minor different result. This is because the authors
of [22] have missed the modulus dependence of the integration constant. Moreover
it seems that the $\zeta$-function regularization has not correctly applied to find the
partition function. In particular for the case of $y_a = y_b \equiv y$ in their notation, the
partition function is not monotonically decreasing (see equation (54) in [22]) and in
fact it grows like $4^y$ as $y \to \infty$. Of course, this is not what we would like to have as
we are going to identify the partition function with the boundary action.

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