We demonstrate that depolarization is not necessarily a signature of multiple scattering. Within a single scattering regime, we show that disordered systems can exhibit depolarization if two sources of disorder are present and have different individual polarization responses. To this end, we study a disordered system consisting of a randomly rough surface that separates vacuum from a medium with volume dielectric fluctuations. This scattering system depolarizes the incident light even within the single scattering regime and there exist directions of perfect depolarization which we characterize geometrically.

Polarimetric measurements are often used for the characterization of ordered systems such as thinfilms, metamaterials and plasmonic surfaces [1–3], or disordered systems such as colloidal suspensions [4–6] and rough surfaces [7, 8], or systems displaying both surface and volume disorder [9]. In particular, measurements of depolarization is often associated with the characterization of scattering media to assess the multiple scattering regime [10–14]. As an example, measurements of depolarization have been put forward as a technique for discriminating between surface and volume scattering and was tested experimentally on highly scattering samples [12–14]. The experimental evidences seem to indicate that volume scattering depolarizes more efficiently than surface scattering, a result which thus may be of interest for sample characterization. There is no doubt that multiple scattering can lead to strong depolarization. However, is depolarization necessarily a signature of multiple scattering? The common intuition, confirmed by single scattering theories for rough surfaces or volume dielectric fluctuations, for instance, seems to indicate that single scattering cannot lead to depolarization. The question above can be reformulated as: Does a scattering system exist which depolarizes in the single scattering regime?

In this Letter, we show that depolarization can occur in the single scattering regime in a system with two different sources of disorder, consisting of a heterogeneous medium bounded upwards by a randomly rough surface. In particular, we develop an expression for the degree of polarization accounting for the two types of disorder within the single scattering regime. Under the assumption of uncorrelated surface and volume disorders, we show that depolarization can occur due to the difference of polarization response between surface and volume scattering. Furthermore, we also find that the degree of polarization may vanish at some critical angles of scattering which we interpret in terms of simple geometrical arguments.

Surface and volume disorder — We consider the scattering system depicted in Fig. 1(a), consisting of a heterogeneous substrate with dielectric fluctuations bounded by a randomly rough surface. The surface profile function and the dielectric (volume) fluctuations are assumed to be realizations of two stochastic processes which here are assumed to be independent for simplicity. We refer to Refs. [15, 16] for a more detailed description of correlated surface and volume disorders, and for an analysis of the influence of such correlations on light scattering. The surface profile function $x_1 = \zeta(x_1)$ is assumed to constitute a stationary, zero-mean, isotropic, Gaussian random process that is a function of $x_1 = (x_1, x_2, 0)$. It is defined by $\langle \zeta \rangle = 0$ and the Gaussian surface-surface correlation function

$$\langle \zeta(x_1)\zeta(x_2) \rangle = \sigma_x^2 \exp \left( -\frac{|x_1-x_2|^2}{\ell_x^2} \right). \tag{1}$$

Here and in the following, the angle brackets denote an average over the ensemble of realizations of the stochastic process. The parameters introduced in Eq. (1) are the surface root-mean-square (rms) roughness $\sigma_x$, and the surface correlation length $\ell_x$. Similarly, the dielectric fluctuations are realizations of the continuous, stationary, zero-mean Gaussian stochastic process $\Delta \varepsilon(x_1)$ defined by $\langle \Delta \varepsilon \rangle = 0$, and the Gaussian dielectric-dielectric correlation function

$$\langle \Delta \varepsilon(x_1)\Delta \varepsilon(x_2) \rangle = \sigma_{\varepsilon}^2 \exp \left( -\frac{|x_1-x_2|^2}{\ell_{\varepsilon}^2} \right). \tag{2}$$

Here $\sigma_{\varepsilon}$ and $\ell_{\varepsilon}$ are the rms dielectric fluctuation and the in-plane dielectric correlation lengths, respectively. It will be assumed that the region above the surface $\zeta(x_1)$ has dielectric constant $\varepsilon_1$ while the region $x_3 < \zeta(x_1)$ is characterized by the dielectric function $\varepsilon_2 + \Delta \varepsilon(x_1)$. Hence, the dielectric function in the whole
When the scattering system is illuminated from medium 1 by a plane wave, respectively, we have assumed the dielectric fluctuations to vary only in the $(x_1, x_2)$-plane (i.e. constant along $x_3$). The scattering from such a configuration, referred to as a surface-like configuration, was recently studied in Ref. [16]. Furthermore, both the rms roughness, $\sigma_\ell$, and the depth of the fluctuating layer, $L$, are assumed to be smaller than the wavelength, i.e. $k_0\sigma_\ell \ll 1$ and $k_0 L \ll 1$, where $k_0 = 2\pi/\lambda$ with $\lambda$ the wavelength of the incident light.

Reflection amplitudes — A single-scattering theory of polarized light by systems with surface and volume disorder is developed in Refs. [15, 16]. The main result is that the scattered electric field $E^{(sc)}$ can be written as the sum of the field $E_k^{(sc)}$ scattered by the rough surface separating two homogeneous media of dielectric constants $\varepsilon_1$ and $\varepsilon_2$, and the field $E^{(sc)}_{E_r}$ scattered by the dielectric fluctuations bounded by the planar interface $x_3 = 0$, viz.

$$E^{(sc)} = E_k^{(sc)} + E^{(sc)}_{E_r}. \tag{4}$$

When the scattering system is illuminated from medium 1 by a plane wave of the form

$$E_0(x) = \sum_{\nu=p,s} \varepsilon_{0,\nu} \hat{e}_{1,\nu}(p_0) \exp[i k_1(p_0) \cdot x], \tag{5}$$

the Fourier amplitude of the scattered reflected field can be written in the form [16]

$$E^{(sc)}(p, x_3) = \sum_{\mu=p,s} \hat{e}_{1,\mu}(p) \times \sum_{\nu=p,s} R_{\mu,\nu}(p, p_0) \varepsilon_{0,\nu} \exp[i a_3(p_0) x_3]. \tag{6}$$

Here $\hat{e}_{0,\mu}$ and $\varepsilon_{0,\nu}$ are the amplitudes for the $p$- and $s$-polarized components of the incident field, respectively, $p_0$ is the in-plane wave vector of the incident wave, and $p$ is the in-plane wave vector defining the observation direction of the scattered field. In Eqs. (5) and (6), we have also introduced the wave vectors

$$k_{\pm}^i(p) = p \pm \alpha_i(p) \hat{e}_3 \tag{7a}$$

$$\alpha_i(p) = (\varepsilon_i k_0^2 - p^2)^{1/2}, \quad \text{Re}(\alpha_i) \geq 0, \quad \text{Im}(\alpha_i) \geq 0 \tag{7b}$$

and the polarization vectors

$$\hat{e}_{\pm,\mu}(p) = \hat{e}_{\mu}(p) = \hat{e}_3 \times \hat{p} \tag{7c}$$

$$\hat{e}_{\pm,\nu}(p) = \frac{\pm \alpha_{\nu}(p) \hat{p} - |p| \hat{e}_3}{\sqrt{k_0^2 |p|^2}} \tag{7d}$$

The different polarization and geometrical parameters are represented in Fig. 1(b). Furthermore, $R_{\mu,\nu}(p, p_0)$ in Eq. (6), is the reflection amplitude for a $\nu$-polarized scattered wave with in-plane wave vector $p$ given an incident unit $\mu$-polarized plane wave with in-plane wave vector $p_0$. It is the sum of surface and volume reflection amplitudes, defined as [16]

$$R_{\mu,\nu}(p, p_0) = s(p, p_0) R_{\mu,\nu}(p, p_0) \tag{8a}$$

$$R_{\nu,\mu}(p, p_0) = v(p, p_0) R_{\nu,\mu}(p, p_0) \tag{8b}$$

where

$$s(p, p_0) = \frac{i k_0^2}{2\varepsilon_2(p)} (\varepsilon_2 - \varepsilon_1) \xi(p - p_0) \tag{9a}$$

$$v(p, p_0) = \frac{i k_0^2}{2\varepsilon_2(p)} \Delta \xi(p - p_0) L, \tag{9b}$$

describe the surface and volume responses, respectively, and

$$R_{\nu,\mu}(p, p_0) = \frac{i}{2\varepsilon_2(p)} \left[ \delta_{\mu,\nu} \xi(p - p_0) + r_{\mu,\nu}(p_0) \hat{e}_{\mu,\nu}(p_0) \right] \tag{10a}$$

$$R_{\mu,\nu}(p, p_0) = \frac{i}{2\varepsilon_2(p)} \left[ \delta_{\mu,\nu} \xi(p - p_0) + r_{\mu,\nu}(p_0) \hat{e}_{\mu,\nu}(p_0) \right] \tag{10b}$$

are polarization coupling amplitudes. Here $\xi$ and $\Delta \xi$ denote the Fourier transforms of the surface profile function and of the dielectric fluctuation function, respectively. The functions $r_{\mu,\nu}^{(i)}(p)$ and $r_{\mu,\nu}^{(v)}$ are the Fresnel reflection and transmission amplitudes for a $\nu$-polarized plane wave incident from medium $i$ onto a planar surface, respectively.

Regarding depolarization, the crucial observation to make from the single scattering theory is that the surface and volume reflection amplitudes in Eq. (8) have different polarization coupling factor [see Eq. (10)] which are independent of any particular realization of the stochastic processes. The realization dependent part of the reflection amplitudes is entirely contained in the factors $s$ and $v$ which are independent of polarization and encode the speckle field.

Degree of polarization — There exist several quantities to measure depolarization such as the degree of polarization or the
depolarization index [17]. Here we have chosen to work with the degree of polarization adapted from the definition for time dependent sources given in Ref. [18]. For an incident plane wave in the state $|\mathbf{p}_0, \nu\rangle$, the degree of polarization for a wave scattered in direction $k_1^\prime(|\mathbf{p}|)$ is defined as

$$\mathcal{P}(p, p_0) = \left(1 - \frac{4}{\text{Tr}[J^{(v)}(p, p_0)]^2} \right)^{1/2},$$

(11)

where the elements of the Jones/coherency matrix are given by

$$J^{(v)}_{\mu\nu}(p, p_0) = \left\langle R_{\mu\nu}(p, p_0) R_{\nu\mu}^*(p_0, p) \right\rangle.$$

(12)

With the definition (11), the degree of polarization characterizes the scattered wave for a given state of polarization of the incident wave. Therefore, different incident states of polarization will, a priori, lead to different degree of polarization for the scattered wave. By inserting Eq. (8) into Eq. (11), we obtain

$$\det J^{(v)} = \left| \left\langle |s|^2 \right\rangle \left\langle |\nu|^2 \right\rangle \right|^2 \rho_{s,\nu}^2 - \rho_{s,\nu} \rho_{s,s} = \left| \left\langle |s|^2 \right\rangle \left\langle |\nu|^2 \right\rangle \right|^2 \rho_{s,\nu}^2 - \rho_{s,\nu} \rho_{s,s},$$

(13)

$$\text{Tr} J^{(v)} = \left| \left\langle |s|^2 \right\rangle \left\langle |\nu|^2 \right\rangle \right|^2 \rho_{s,\nu}^2 + \rho_{s,\nu}^2 \left| \left\langle |s|^2 \right\rangle \left\langle |\nu|^2 \right\rangle \right|^2 + \left| \left\langle |s|^2 \right\rangle \left\langle |\nu|^2 \right\rangle \right|^2 \rho_{s,\nu}^{2*}.$$ 

(14)

In deriving these results, we have used the fact that surface and volume disorders are uncorrelated, so that terms proportional to $\langle ss^* \rangle$ and $\langle vv^* \rangle$ vanish.

Results and discussion — First, let us assume that only one type of disorder is present in the scattering system. Then, either $s = 0$ or $v = 0$, which causes $\det J^{(v)}$ to vanish and, hence, $\mathcal{P}(v) = 1$ identically. Indeed, when only one of the two types of disorder is present, the polarization state of an outgoing wave is, in the single scattering regime, entirely given by $\rho_{s,\nu}$ and $\rho_{s,\nu}$ independently of the realization. From one realization to the next, the speckle pattern will of course change, but the ratio of the $p$- and $s$-polarized components of the scattered electric field (or any other two components in an orthogonal polarization basis) will remain the same for a given scattering direction. This confirms that a sample with pure surface or volume scattering does not depolarize in the single scattering regime.

Second, let us now consider the two types of disorder simultaneously. Equation (13) now predicts that in general $\det J^{(v)}$ is non-zero and, therefore, the degree of polarization is smaller than one. This is due to the difference in polarization response between the two types of disorder, i.e., $\rho_{s,\nu} \neq \rho_{s,\nu}$. Although each of the polarization responses remains unchanged from one realization to the next, they are weighted by independent random variables (namely $s$ and $v$), which will change the relative weights of the $p$ and $s$ components for a given scattering direction when the realization of disorder is changed. This shows that depolarization is not necessarily a signature of multiple scattering, but can also occur in a single scattering regime from the interference of fields that originate from (at least) two sources of disorder with different polarization response. Note that if the two sources of disorder have the same polarization response, then the degree of polarization is unity since the last factor in Eq. (13) vanishes. For instance, this occurs at normal incidence $[\theta_0 = 0^\circ]$. Moreover, for an $s$-polarized incident wave, $\mathcal{P}(s) = 1$ for all angles of incidence and scattering when the single scattering regime is assumed. Actually, it can easily be seen from Eq. (10) that $\rho_{s,\mu} = \rho_{s,\mu}$ (note that $1 + \rho_{s,\mu}^2 = 1$).

For the scattering system considered here, the case of par-
ticular interest is an obliquely incident $p$-polarized wave. Figures 2(a) and 2(b) show contour maps of the surface and volume $p$-to-$p$ polarization coupling factors for angles of incidence $(\theta_0, \phi_0) = (70^\circ, 0^\circ)$. The parameters, given in the caption of Fig. 2, were chosen so that the two types of disorder scatter light with the same strength, i.e., $\langle |s|^2 \rangle = \langle |v|^2 \rangle$. By comparing the results in Figs. 2(a) and 2(b), one can appreciate the differences between the surface and the volume polarization couplings; notice, in particular, the difference in sign. Due to their difference, the depolarization $P^{(p)}$ may become smaller than unity for some scattering directions [Fig. 2(c)]. Interestingly, the degree of polarization $P^{(p)}$ even vanishes at two points in the $p$-plane located symmetrically about the plane of incidence (the $p_1 p_3$-plane). To further investigate the conditions leading to $P^{(p)} = 0$, Fig. 2(d) presents $P^{(p)}$ as a function of the polar angle of scattering for a set of positive azimuthal angles of scattering. For the angles of incidence that we assume, $P^{(p)}$ vanishes for $(\theta, \phi) \approx (61.6^\circ, \pm 19.0^\circ)$. According to Eq. (11), this should happen for a critical in-plane wave vector of scattering, $p_*$, that satisfies

$$4 \det J^{(p)}(p_*, p_0) = \left[ \text{Tr} J^{(p)}(p_*, p_0) \right]^2.$$

From the polarization vectors defined in Eqs. (7c) and (7d) one can readily show that [see Fig. 2(o)]

$$\rho_{\xi,sp} = \rho_{s,sp} \equiv \rho_{sp}.$$  

Combining this result with the expressions in Eqs. (13) and (14), we can rewrite condition (15) as

$$4 |\rho_{sp}|^2 \left| \rho_{\xi,sp} - \rho_{c,sp} \right|^2 = \left[ |\rho_{\xi,sp}|^2 + |\rho_{c,sp}|^2 + 2|\rho_{sp}|^2 \right]^2,$$

where we have used that $\langle |s|^2 \rangle = \langle |v|^2 \rangle$. Figure 2(f) illustrates that at the critical points, the polarization coupling factors actually satisfy $\rho_{sp} = \rho_{\xi,sp} = -\rho_{c,sp}$, so that the condition (17) and (15) is trivially fulfilled, and the degree of polarization $P^{(p)}(p_*, 0_0)$ vanishes. Depending on the angles of incidence (characterized by $p_0$), the critical wave vectors $p_*$ may correspond to evanescent or radiative modes. We have experienced that the critical points appear in the radiative region for a sufficiently large polar angle of incidence (see the animation provided in Visualization 1).

An enlightening geometrical interpretation can be given for $p_*$. For such a critical wave vector, the two contributions to the scattered electric field originating from the surface and the volume are orthogonal. Indeed, for an incident $p$-polarized wave, the electric fields scattered by the surface or by the volume are proportional to, respectively,

\begin{align}
E^{(sc)}_{\xi}(p, x_3) &\propto \rho_{\xi,sp}(p_0, p_0) \hat{e}_{\xi,sp}(p) + \rho_{c,sp}(p, p_0) \hat{e}_{c}(p) \quad (18a) \\
E^{(sc)}_{\xi}(p, x_3) &\propto \rho_{c,sp}(p_0, p_0) \hat{e}_{\xi,sp}(p) + \rho_{s,sp}(p, p_0) \hat{e}_{s}(p). \quad (18b)
\end{align}

The scalar product of these two electric fields reads

$$E^{(sc)}_{\xi} \cdot E^{(sc)}_{\xi} \propto \rho_{\xi,sp}^2 + \rho_{c,sp}^2,$$

which vanishes at the critical wave vector since $\rho_{sp} = \rho_{\xi,sp} = -\rho_{c,sp}$. Hence, the interpretation of perfect depolarization is clear. For scattering into modes characterized by the critical wave vectors $p_*$, the polarization vectors associated with the surface and volume contributions to the scattered electric field are orthogonal. Therefore, they form a basis for polarization and are weighted by the uncorrelated random variables $s(p_* p_0)$ and $v(p_* p_0)$. In this basis, the two contributions to the scattered electric field are uncorrelated, which, indeed, is the definition of perfect depolarization [18]. It should be noted that whenever $\langle |s|^2 \rangle \neq \langle |v|^2 \rangle$, zero degree of polarization may not be possible to achieve. In such cases, only a minimum for $P^{(p)}$ can be reached at a critical wave vector if the values for $\langle |s|^2 \rangle$ and $\langle |v|^2 \rangle$ are sufficiently close.

In summary, we have demonstrated that depolarization can be observed in single scattering of light by a complex system. The mechanism responsible for depolarization in this case is the superposition of random scattering contributions emerging from two types of disorder with different polarization responses. We have illustrated this effect in the case of combined rough surfaces and volume dielectric fluctuations, but the mechanism put forward is more general and should hold for an arbitrary set of disorder contributions as long as their polarization responses are different. Other examples of systems where depolarization can take place even in the single scattering regime are films bounded by two (or more) randomly rough interfaces, or sets of particles of different species having different anisotropic dipolar responses. The analysis could be extended to the case of correlated surface and volume disorders. In particular, it could be of interest to study whether correlations can significantly enhance or attenuate the depolarization.

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