I. INTRODUCTION

Multiscale adiabatic dynamics of classical particles in oscillating and static fields is simplified within the oscillation-center (OC) approach, which allows separating fast quiver motion of the particles from their slow translational motion \([1, 2, 3]\). Hence the average forces are embedded into the properties of the OC, yielding a quasiparticle with a variable effective mass \(m_{\text{eff}} \equiv \frac{m}{1 - v^2/c^2}\). In each given case, \(m_{\text{eff}}\) can be Taylor-expanded at nonrelativistic energies so as to appear as an effective potential \(\Psi \equiv \frac{U - mc^2}{m_{\text{eff}} c^2}\), e.g., ponderomotive \([6, 7, 8, 9]\) or diamagnetic \([10]\). Yet the nonrelativistic limit must permit also an independent calculation of \(\Psi\). For linear oscillations, the generalized effective potential was derived in Ref. \([3]\). However, a comprehensive method of finding \(\Psi\) for nonlinear quiver motion has not been proposed.

The purpose of this work is to calculate, from first principles, the generalized effective potential \(\Psi\) for a nonrelativistic classical particle undergoing arbitrary oscillations in high-frequency or static fields. We proceed by finding eigenmodes in the particle-field system: hence \(\Psi\) is obtained like in the dressed-atom approach \([11, 12]\), but from nonlinear classical equations. Specifically, we show that the ponderomotive potential extended to a nonlinear oscillator has multiple branches near the primary resonance. Also, for a pair of natural frequencies in a beat resonance, \(\Psi\) scales linearly with the internal actions and is analogous to the dipole potential for a two-level quantum system. Therefore excited quasiclassical objects permit uniform manipulation tools, particularly, one-way walls.

The work is organized as follows. In Sec. II, we obtain the general form of the effective potential \(\Psi\). In Sec. III we derive the equations for oscillation modes. In Sec. IV we calculate the ponderomotive potential from the infinitesimal frequency shift of the oscillating field coupled to a particle at a primary resonance, both linear and nonlinear; see also Appendix A. In Sec. V we find \(\Psi\) near a beat resonance and show the analogy with the dipole potential. In Sec. VI we explain how \(\Psi\) allows one-way walls. In Sec. VII we summarize our main results.

II. GENERALIZED EFFECTIVE POTENTIAL

Consider a particle which exhibits slow dynamics in canonical variables \((\mathbf{r}, \mathbf{P})\) superimposed on fast oscillations in angle-action variables \((\vartheta, J)\) (const), such that zero \(J\) corresponds to purely translational motion. The OC Lagrangian is then written as \([4]\)

\[
L_0 = -m_{\text{eff}} c^2 \sqrt{1 - v^2/c^2},
\]

where \(v \equiv \mathbf{v}\), and \(m_{\text{eff}}(\mathbf{r}, \mathbf{v}; J)\) is the effective mass, the dependence on \(J\) being parametric \([24]\). Thus the complete Lagrangian, which describes also the oscillations, equals \(L_0 + \vartheta \cdot J\), so the corresponding Hamiltonian \(\mathcal{H}\) matches that of the oscillation center: \(\mathcal{H} = \mathbf{P} \cdot \mathbf{v} - L_0\).

Assume nonrelativistic dynamics, i.e., \(v \ll c\) and \(\delta m \equiv m_{\text{eff}} - m \ll m\), where \(m\) is the true mass. Hence

\[
L_0 = \frac{1}{2} mv^2 - U, \quad U = \delta mc^2,
\]

so the Hamiltonian reads

\[
\mathcal{H} = \frac{P^2}{2m} + \Psi, \quad \Psi = U - \frac{1}{2m} (\partial_r U)^2,
\]

where \(\mathbf{P} = m \mathbf{v} - \partial_r U\), and \(\Psi(J = 0) = 0\).

Following Ref. \([4]\), one can find \(\Psi\) from the relativistic particle trajectory in given fields; however, a general nonrelativistic approach is also possible. Formally, field modes can be understood as particle degrees of freedom. Then, like in the dressed-atom approach \([11, 12]\), the effective potential can be found from eigenfrequencies \(\varpi \equiv \vartheta \cdot \partial_J \mathcal{H}\) of the particle-field system:

\[
\Psi = \int \varpi \cdot dJ.
\]

In this case, \(\Psi\) depends on \(\mathbf{r}\) and \(\mathbf{P}\) only parametrically, through \(\varpi(\mathbf{r}, \mathbf{P}, J)\).

The canonical frequencies can be redefined such that \(\varpi \rightarrow \varpi + \text{const}\), adding a constant to \(\Psi\). Albeit arbitrarily large, this contribution does not affect the motion equations, so we abandon the requirement that \(\Psi\) must
remain small compared to $m c^2$; hence actual physical frequencies can be used for $\omega$. Particularly, for uncoupled modes one gets $\Psi = \Psi_0$,

$$\Psi_0 = \Omega \cdot J + \omega \cdot I,$$

(5)

where we used Eq. (4) and $(\Omega, J)$, $(\omega, I)$ for the unperturbed frequencies and actions of the particle and the field, correspondingly. For an unbounded field ($I \to \infty$), the second term in Eq. (4) is infinite. As it is fixed though, the force on a particle is determined only by $\Omega \cdot J$ and the finite modification of the effective potential due to coupling,

$$\Phi = \Psi - \Psi_0,$$

(6)

where $\Phi$ also can be found from Eq. (4), as shown below.

### III. PARTIAL MODE DECOMPOSITION

Suppose weakly nonlinear oscillations $\xi(t)$, both of the particle [3] and of external fields [25], so their Lagrangian reads $\tilde{L}(\xi, \dot{\xi}) = \tilde{L}_0 + \tilde{L}_{\text{int}}$, where $\tilde{L}_0$ is a perturbation to a bilinear form $L_0$ [26],

$$\tilde{L}_0 = \frac{1}{2} (\dot{\xi} \cdot \tilde{M} \xi) - (\xi \cdot \tilde{R} \xi) - \frac{1}{2} (\xi \cdot \tilde{Q} \xi).$$

(7)

Here $\tilde{M}, \tilde{R}, \tilde{Q}$ are $N \times N$ real matrices; $\tilde{M}$ and $\tilde{Q}$ are symmetric, $\tilde{R}$ is antisymmetric, and rank $\tilde{M} = N \equiv \text{dim} \xi$. At zero $\tilde{R}, \tilde{L}_0(\xi, \dot{\xi})$ can be diagonalized to yield

$$\tilde{L}_0 = \sum_{j=1}^{N} L_j, \quad L_j = \frac{1}{2} M_j \xi_j^2 - \frac{1}{2} Q_j \xi_j^2,$$

(8)

where $L_j$ describe individual modes $\xi_j$ [27]. Then

$$\tilde{D} \xi_j = \delta \xi_j \tilde{L}_{\text{int}}, \quad \tilde{D}_j = M_j d_j^2 + Q_j,$$

(9)

$\delta$ and $d_j$ standing for the variation and time derivatives. Yet, such decomposition does not hold in the general case, so we redefine eigenmodes, following Ref. [28].

Extend the configuration space by introducing

$$\langle \ell | = (-\pi \tilde{M}^{-1} \xi), \quad | r \rangle = (\xi, \tilde{M}^{-1} \pi)^T$$

(10)

as the new, “left” and “right”, coordinate vectors, where $\pi = \tilde{M} \xi - \tilde{R} \dot{\xi}$ is the old canonical momentum. Then

$$\tilde{L}_0 = \frac{1}{4} (\langle \ell | \tilde{M} | r \rangle - \langle \ell | \tilde{M} | r \rangle^*) + \frac{1}{2} \langle \ell | \tilde{S} | r \rangle,$$

(11)

where we omitted a full time derivative and introduced

$$\tilde{M} = \begin{pmatrix} \tilde{M} & 0 \\ 0 & \tilde{M} \end{pmatrix}, \quad \tilde{S} = \begin{pmatrix} \tilde{R} \tilde{M} \\ \tilde{F} \tilde{M} \end{pmatrix},$$

(12)

with $\tilde{F} = \tilde{R} \tilde{M}^{-1} \tilde{R} - \tilde{Q}$. Thus the resulting equations are

$$\langle \ell | \tilde{M} + \langle \ell | \tilde{S} = 0, \quad | \tilde{M} | r \rangle - \tilde{S} | r \rangle = 0,$$

(13)

both equivalent to

$$\tilde{M} \xi - 2 \tilde{R} \dot{\xi} + \dot{\Phi} \xi = 0.$$  

(14)

Eq. (14) has $2N$ eigenmodes $\xi_j = \hat{\xi}_j e^{-i \nu_j t}$, with $\nu_j$ hence assumed real and nonzero; therefore, for each $\xi_j$, there also exists a mode $\xi_{-j} = \xi_j$, and $\hat{\xi}_j$ are generally not orthogonal. The corresponding eigenmodes of Eqs. (13) are

$$\langle \ell_j | = e^{i \nu_j t} \langle \tilde{\ell}_j |, \quad | r_j \rangle = e^{-i \nu_j t} | \tilde{r}_j \rangle,$$

(15)

with vector amplitudes

$$\langle \tilde{\ell}_j | = (-i \nu_j \hat{\xi}_j - \hat{\xi}_j^* \tilde{R} \tilde{M}^{-1} \hat{\xi}_j), \quad | \tilde{r}_j \rangle = (\xi_j, -i \nu_j \hat{\xi}_j - \tilde{M}^{-1} \tilde{R} \hat{\xi}_j)^T,$$

(16)

and $M_{jk} \equiv \langle \tilde{\ell}_j | \tilde{M} | \tilde{r}_k \rangle = -2i \rho_{jk}$, where

$$\rho_{jk} = \frac{1}{2} \hat{\xi}_j \cdot [\nu_j + \nu_k] \tilde{M} - 2i \tilde{R} \cdot \hat{\xi}_k.$$  

(18)

The matrix $\hat{\rho}$ is diagonal for distinct $\nu_j$, as seen from Eq. (14), or can be diagonalized when some of the frequencies coincide [28]; thus,

$$M_{jk} = -2i \rho_{jk}, \quad \rho_{jk} \equiv \nu_j (\hat{\xi}_j^* \cdot \tilde{M} \hat{\xi}_k),$$

(19)

$\rho_j = -\rho_{-j}$. (Hence modes with $\nu_j = 0$ are orthogonal to the others and can be considered separately, as implied below.) Therefore any $\langle \ell |$ and $| r \rangle$ are decomposed as

$$\langle \ell | = \sum_{j=-N}^{N} \ell_j \langle \tilde{\ell}_j |, \quad | r \rangle = \sum_{j=-N}^{N} r_j | \tilde{r}_j \rangle,$$

(20)

where the primes stand for skipping $j = 0$, and

$$\ell_j = \frac{i}{2 \rho_j} \langle \tilde{\ell}_j | \tilde{M} | \tilde{r}_j \rangle, \quad r_j = \frac{i}{2 \rho_j} (\tilde{r}_j | \tilde{M} | r \rangle).$$

(21)

Since $\langle \ell |$ and $| r \rangle$ are real, one has $r_j = \ell_j^* \equiv \psi_j \sqrt{2}$ and $\psi_j = \psi_j^*; \text{ hence Eq. (5), but with }$

$$L_j = \frac{i \rho_j}{2} \left( \psi_j^* \psi_j - \psi_j \psi_j^* \right) - \rho_j \nu_j |\psi_j|^2.$$  

(22)

The resulting equations for individual modes are

$$\tilde{D}_j \psi_j = \delta \psi_j \tilde{L}_{\text{int}}, \quad \tilde{D}_j = \rho_j (\nu_j - i d_j),$$

(23)

similar to reduced Eqs. (9). Particularly, at zero $\tilde{L}_{\text{int}},$

$$\psi_j = \sqrt{2 J_j} e^{-i \theta_j}, \quad \dot{\theta}_j = \nu_j, \quad J_j = \text{const}.$$  

(24)

On the other hand, $L_j = (\dot{\theta}_j - \nu_j) J_j$; thus $\dot{\theta}_j, L_j = J_j$ is also the action corresponding to the angle $\theta_j$:

$$J_j = |\psi_j|^2,$$

(25)

so $\nu_j$ is the canonical frequency. Then the mode energy is $\nu_j J_j$ (thus $\rho_j > 0$ for stable modes with $\nu_j > 0$, henceforth implied), and Eq. (5) is recovered.

In the next sections, we apply Eqs. (23) to find eigenmodes for nonzero $\tilde{L}_{\text{int}}$, with $\psi$ becoming partial oscillations. Hence the effective potential modification $\Phi$ [Eq. (4)] is obtained from Eq. (4).
### IV. PRIMARY RESONANCE

#### A. Linear oscillator

First, we calculate $\Psi$ for a linear coupling between a pair of modes $\psi_1$ and $\psi_2$, say,

$$\hat{L}_{\text{int}} = \sigma_1 \psi_1 \bar{\psi}_2 + \sigma_2 \bar{\psi}_1 \psi_2,$$

(26)

where $\sigma = \text{const}$. In this case, Eqs. (23) yields

$$\hat{D}_1 \psi_1 = \sigma \psi_2, \quad \hat{D}_2 \psi_2 = \sigma \psi_1;$$

(27)

hence a quadratic equation for the eigenfrequencies $\omega$,

$$\rho_1 \rho_2 (\omega - \nu_1) (\omega - \nu_2) = |\sigma|^2,$$

(28)

from which $\Psi = \bar{\omega}_1 J_1 + \bar{\omega}_2 J_2$ is obtained.

As a particular case, consider interaction of a particle internal mode having frequency $\Omega$ and action $J = \rho |\psi|^2$ with an external oscillating field $E = E e^{-i\omega t}$ having frequency $\omega$ and action $I = \rho |E|^2$. Given that the field occupies a volume $V \to \infty$, the frequency shifts $\delta \Omega$ and $\delta \omega$ are infinitesimal, Eq. (28) yielding

$$\delta \Omega = \frac{|\sigma|^2}{(\omega - \Omega)\rho E}, \quad \delta \omega = \frac{|\sigma|^2}{(\omega - \Omega)\rho E}.$$ (29)

Since $\rho E \propto V$, one has $\delta \Omega J \ll \delta \omega I$, whereas

$$\delta \omega I = \frac{|\sigma|^2}{(\omega - \Omega)\rho} |\bar{E}|^2$$ (30)

is nonvanishing. Then one gets

$$\Psi = \Omega J + \Phi_0, \quad \Phi_0 = -\frac{1}{4} \alpha |\bar{E}|^2,$$ (31)

where $\Phi_0$ is the so-called ponderomotive potential (for the general expression see Appendix A), an insignificant constant $\omega I$ is removed, and $\alpha = 4 |\omega|^2 / (\Omega - \omega)^{-1}$. Since

$$\Phi_0 = \frac{\kappa^2 |\bar{E}|^2}{\omega - \Omega},$$ (32)

where $\kappa^2 \equiv |\sigma|^2 / \rho > 0$, the effective potential becomes infinite at the linear resonance [Fig. 1(a)]. However, nonlinear effects remove this singularity, as we show below.

#### B. Nonlinear oscillator

Consider the effective potential near a nonlinear resonance, with a Duffing oscillator as a model system. Then

$$\hat{L}_{\text{int}} = \sigma \psi E^* + \sigma^* \bar{\psi} E + \frac{1}{2} \beta |\psi|^4,$$ (33)

where $\beta = \text{const}$, yielding

$$\hat{D}_1 \psi = \sigma \psi E + \beta |\psi|^2 \psi, \quad \hat{D}_E E = \sigma \psi.$$ (34)

Separate the driven motion from free oscillations, $\psi = X e^{-i(\omega + \delta \omega) t} + Y e^{-i(\Omega + \delta \Omega) t}$, so

$$-\delta \omega \rho E \bar{E} = \sigma X,$$ (35)

$$\Omega - \omega \rho X = \sigma^* \bar{E} + 2 \beta |Y|^2 X + \beta |X|^2 X,$$ (36)

$$-\delta \Omega \rho Y = \bar{\beta} |Y|^2 Y + 2 \beta |X|^2 Y.$$ (37)

From Eq. (37), it follows that $\delta \Omega \sim \beta |XY|^2$, which we assume, for simplicity, small compared to $\Phi_0 \sim \delta \omega I$; hence $\Psi \approx \Omega J + \Phi_0$, where $\Phi = \int_0^\infty \delta \omega dI$ is the modified ponderomotive potential. The field frequency shift is found from the cubic equation $(1 + \zeta h^2) h = 1$ for $h = \delta \omega I / \Phi_0$; $\zeta = \beta \Phi_0 / (\gamma \rho^2)$, $\Phi_0 = |\sigma |^2 \rho / (\gamma \rho)$, $\gamma = \omega - \Omega + \zeta (J)$, $\zeta = 2 \beta / \rho^2$, and the above condition of negligible $\delta \Omega J$ reads $\zeta / (\omega - \Omega) \ll 1$. Thus

$$\Phi = \frac{\gamma^2 \rho^2}{\beta} W(\zeta), \quad W(\zeta) = \int_0^\zeta h(\zeta) d\zeta,$$ (38)

where $h(\zeta)$ has three branches [Fig. 1(b)]. One of those is unstable [29]; hence two branches of the ponderomotive potential, $\Phi_1$ and $\Phi_2$. Their asymptotic form flows from $W_{1,2}(\zeta \to 0) \approx \zeta$ and $W_{1,2}(\zeta \to \infty) \approx 2 \zeta^3 / (3 \beta)$ [Fig. 1(c)]:

$$\Phi_{1,2} \approx \Phi_0 = \frac{\kappa^2 |\bar{E}|^2}{\gamma}, \quad \Phi_1 \approx \frac{2 \kappa \rho |\bar{E}|^2 |\gamma \beta|}{\beta},$$ (39)

and Eq. (32) is recovered from $\Phi_{1,2}^\pm$ in the limit of small $\zeta$ and $\epsilon = 0$ [Fig. 1(d)]. On the other hand, $W_{1,2}(\zeta \sim \infty) \approx \frac{3}{2} |\zeta|^{2/3}$, so $\Phi_{1,2} \to 3 |\sigma |^2 / (3 |\beta|^{2/3} / (2 \beta)$ at the resonance, i.e., the singularity vanishes.

### V. BEAT RESONANCE

Suppose that $\hat{L}_{\text{int}}$ also contains a term cubic in $\psi$, say,

$$\delta \hat{L}_{\text{int}} = \epsilon \psi_1 \bar{\psi}_2 \bar{\psi}_3 + \epsilon^* \psi_1 \psi_2 \bar{\psi}_3,$$ (40)

where $\epsilon = \text{const}$. That would normally yield a potential small compared to $\Phi_0$, yet maybe except when

$$\nu_1 \approx \nu_2 + \nu_3.$$ (41)

Below we study the beat resonance [11] neglecting the quadratic $\hat{L}_{\text{int}}$, so the envelopes $\psi_j = \psi_j e^{i\nu_j t}$ satisfy

$$-i \rho_1 \dot{\psi}_1 = \epsilon \psi_2 \psi_3 e^{-i\Delta t},$$ (42)

$$-i \rho_2 \dot{\psi}_2 = \epsilon \bar{\psi}_1 \bar{\psi}_3 e^{i\Delta t},$$ (43)

$$-i \rho_3 \dot{\psi}_3 = \epsilon \bar{\psi}_1 \bar{\psi}_2 e^{i\Delta t},$$ (44)

where $\Delta \equiv \nu_1 + \nu_2 - \nu_3$ is the detuning frequency.

#### A. Linear coupling

Suppose that $\Delta$ is large enough, so Eqs. (42)–(44) can be solved by averaging. Then split $\psi_j$ into driven and
where $\Gamma \equiv \rho_1 \rho_2 \rho_3 / |\epsilon|^2 > 0$. Then, from Eqs. (41), one gets $\Psi = \Psi_0 + \Phi$,

$$\Phi = J_1 J_2 + J_1 J_3 - J_2 J_3 / (\nu_1 - \nu_2 - \nu_3) \Gamma.$$  

(48)

Particularly, if the third mode corresponds to a macroscopic oscillating field tuned close to the beat resonance

$$\omega \approx \Omega_1 - \Omega_2,$$  

(49)

then $J_{1,2} \ll I \propto |E|^2$. Hence the “hybrid” ponderomotive potential that is obtained is simultaneously proportional to $|E|^2$ and the internal actions $J_j$:

$$\Phi = \kappa^2 \Delta (J_2 - J_1) |E|^2,$$  

(50)

where $\Delta = \omega - (\Omega_1 - \Omega_2)$, and $\kappa^2 \equiv |\epsilon|^2 / (\rho_1 \rho_2) > 0$.

B. Nonlinear coupling

At $\Delta \lesssim \delta \nu_j$, the internal mode equations

$$-i\rho_1 \psi_1 = e^{\epsilon \psi_2 \bar{E}} e^{-i \Delta t}, \quad -i\rho_2 \psi_2 = e^{\epsilon \psi_1 \bar{E}} e^{i \Delta t}$$  

(51)

do not allow averaging, so the derivation is modified as follows. First, consider strictly periodic oscillations, in which case $\psi \equiv (\bar{\psi}_1, \bar{\psi}_2)^T$ rewrites as

$$\bar{\psi} = T_\Lambda \bar{\psi}_0 \bar{x},$$  

where $T_\Lambda = \text{diag}(e^{-i\nu_1 t}, e^{i\nu_2 t})$, and

$$\bar{\psi}_0 = \bar{T}_\phi \text{diag}(\rho_1, \rho_2)^{1/2}, \quad \phi = \arccos(\epsilon \bar{x}) + \pi.$$  

Then

$$i \dot{x}_1 = \frac{\epsilon}{2} x_2 - \frac{\Delta}{2} x_1, \quad i \dot{x}_2 = \frac{\epsilon}{2} x_1 + \frac{\Delta}{2} x_2,$$  

(52)

where $\bar{x} = 2\kappa |\bar{E}|$. Eqs. (52) yield two eigenmodes at frequencies $\pm \frac{\epsilon}{2} \Lambda$, $\Lambda = (\epsilon^2 + \Delta^2)^{1/2}$, so $\bar{x} = \bar{U} \bar{T}_\Lambda \bar{x}$, where

$$\bar{U} = \left[ \begin{array}{cc} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{array} \right].$$  

(53)

Thus the corresponding equations read

$$-\eta_1 \delta \nu_1 X_1 = e^{\epsilon \psi_2 \bar{E}} e^{-i \Delta t}, \quad -\eta_2 \delta \nu_2 Y_1 = e^{\epsilon \psi_1 \bar{E}} e^{i \Delta t},$$  

(54)

where insignificant oscillating terms are neglected. Since $\delta \nu_j \ll \Delta$, one has

$$\delta \nu_1 = -J_2 + J_3 / \Gamma \Delta, \quad \delta \nu_2 = J_3 - J_1 / \Gamma \Delta, \quad \delta \nu_3 = J_2 - J_1 / \Gamma \Delta,$$  

(45)

Fig. 1: (a) Resonant ponderomotive potential $\Phi_0$ [Eq. (22)] in units $\Phi_0 \equiv \kappa^2 |\bar{E}|^2 / \omega - \Omega$ in units $\omega$. (b) Solid - squared normalized frequency shift $h = \delta \omega f / \Phi_0$ vs. $\zeta = \beta \Phi_0 / (\gamma \rho)^2$, $\zeta (J = 0) \propto (\omega - \Omega)^{-3}$; dashed - approximations $h^2 = 1$ and $h^2 = -\zeta^{-1}$. (c) $W(\zeta)$ [Eq. (38)], consists of branches $W_1 > 0$ and $W_2 < 0$. (d) Effective potential $\Phi$ in units $\Phi_e \equiv \bar{\rho} \omega^2 / \beta = \Phi(0) vs. \omega - \Omega$ in units $\omega_e \equiv \beta^{3/2} |\bar{E}|^2 / \rho$, assuming $J = 0$ and $\beta > 0$: solid - Eq. (38) (consists of branches $\Phi_1 > 0$ and $\Phi_2 < 0$), dashed - Eqs. (39).

Thus the corresponding equations read

$$-\eta_1 \delta \nu_1 X_1 = e^{\epsilon \psi_2 \bar{E}} e^{-i \Delta t}, \quad -\eta_2 \delta \nu_2 Y_1 = e^{\epsilon \psi_1 \bar{E}} e^{i \Delta t},$$  

$$-\eta_3 \delta \nu_3 X_3 = e^{\epsilon \psi_2 \bar{E}} e^{-i \Delta t}, \quad -\eta_4 \delta \nu_4 Y_3 = e^{\epsilon \psi_1 \bar{E}} e^{i \Delta t},$$  

(54)

$$\eta_1 = \delta \nu_2 + \delta \nu_3 - \Delta, \quad \eta_2 = \delta \nu_1 - \delta \nu_3 - \Delta, \quad \eta_3 = \delta \nu_1 - \delta \nu_2 - \Delta.$$  

(46)

(47)
Therefore $\chi_{z}$ are independent linear modes with frequencies $\omega_{z}$, and conserved actions $J_{z} = \rho_{z}|\chi_{z}|^{2}$ (Sec. III), and Eq. (11) yields

$$
\Psi = \frac{1}{2} (\Omega_{1} + \Omega_{2}) (J_{+} + J_{-}) + \frac{\Lambda}{2} (J_{+} - J_{-}).
$$

(58)

At $\Delta \gg \varepsilon$, $J_{z} = J_{1,2}$ for $\Delta < 0$ and $J_{z} = J_{2,1}$ for $\Delta > 0$, so Eq. (50) is recovered by Taylor expansion of Eq. (58).

### C. Quantum analogy

The above classical particle is the limit of a quantum system with plentiful states coupled to the field simultaneously and $\Omega_{1,2}$ being the unperturbed transition frequencies [Fig. 2(a)]. Yet, with $\rho_{j} \rightarrow \hbar$, the equations (51) are also equivalent to those describing a two-level system [13, 14, 30], with the unperturbed eigenfrequencies $\Omega_{1,2}$ and the Rabi frequency $\Omega_{R} = \varepsilon$ [Fig. 2(b)]. Hence Eq. (53) yields as well the dipole potential for a two-level quantum object, e.g., a cold atom [11, pp. 454-461], [12]:

$$
\Psi = \frac{\hbar}{2} (\Omega_{1} + \Omega_{2}) + \frac{\hbar}{2} (n_{+} - n_{-}) \sqrt{\Delta^{2} + \Omega_{R}^{2}},
$$

(59)

where $n_{j} = |\langle j | \rangle^{2}$ are the occupation numbers ($\langle j | \rangle \rightarrow \hbar \rho_{j}$), satisfying $n_{+} + n_{-} = 1$. Similarly, Eq. (50) is equivalent to the dipole potential at weak coupling [30, 31]:

$$
\Phi = \frac{\hbar \Omega_{R}^{2}}{4 \Delta} (1 - 2n_{1}).
$$

(60)

[At $n_{1} = 0$, Eq. (60) is further analogous to Eq. (32), with the resonance at the transition frequency $\Omega_{1} - \Omega_{2}$.] This parallelism originates from Eq. (25) yielding Schrödinger-like equations for $L_{\text{int}}$ quadratic in internal mode amplitudes. Therefore, for any classical particle governed by a hybrid potential, an analogue is possible which is governed by a quantum dipole potential, and vice versa. Hence the two types of objects permit uniform manipulation techniques, as also discussed in Sec. VII.

### VI. ONE-WAY WALLS

As an effective potential, $\Psi$ can have properties distinguishing it from true potentials. Particularly, it can yield asymmetric barriers, or one-way walls [15, 16, 17, 18, 19, 20, 21, 22, 23], allowing current drive [32, 33, 34] and translational cooling [17, 22, 35, 36]. We explain these barriers as follows.

Suppose a ponderomotive potential of the form [32] [or, similarly, (58)]. Given $\Omega = \Omega(z)$, with, say, $\partial_{z} \Omega < 0$, the average force $F_{z} = -\partial_{z} \Psi$ is everywhere in $+z$ direction, except at the exact resonance where the effective potential does not apply [Fig. 2(a)]. Hence particles can be transmitted when traveling in one direction but reflected otherwise, even assuming uniform $E(z)$ [32, 33, 34]. In Ref. [13], such dynamics was confirmed for cyclotron-resonant rf fields, and, in Ref. [10], a similar scheme employing abrupt $E(z)$ was proposed.

Hybrid potentials (Sec. VII) permit yet another type of one-way walls. Assume uniform $\Omega_{1}$ and $\Delta > 0$; then $\Psi$ [Eqs. (50), (58)] is repulsive for cold particles ($J_{1} < J_{2}$) but attractive for hot particles ($J_{1} > J_{2}$). Thus, if particles incident, say, from the left are preheated (via nonadiabatic interaction with another field), they will be transmitted, whereas those cold as incident from the right will be repelled [Fig. 2(b)]; hence the asymmetry.

In agreement with the parallelism shown in Sec. V C, similar one-way walls for cold atoms have been suggested [17, 18, 19, 20, 21, 37] and enjoyed experimental verification [22, 23]. However, the new result here is that quasiclassical particles like Rydberg atoms and molecules can, in principle, be manipulated in the same manner, despite their involved eigenspectrum is different (Fig. 2).

### VII. CONCLUSIONS

We propose a method to calculate the generalized effective potential $\Psi$ for a nonrelativistic classical particle undergoing arbitrary oscillations in high-frequency or static fields. We derive $\Psi$ from the oscillation eigenfrequencies in the particle-field system [Eq. (1)], like in the dressed-atom approach [11, 12, 13, 14] but from nonlinear classical equations [Eqs. (23)]. Specifically, we show that the ponderomotive potential [Eqs. (32), (A6); Fig. 1(a)] extended to a nonlinear oscillator has multiple branches near the primary resonance [Eq. (58), Fig. 1(d)]. Also, for a pair of natural frequencies in a beat resonance, $\Psi$ scales linearly with the internal actions [Eqs. (50), (58)] and is analogous to the dipole potential [Eqs. (50), (58)] for a two-level quantum system (Fig. 2). Thus cold quantum particles and highly-excited quasiclassical objects permit uniform manipulation tools, particularly, station-
FIG. 3: Schematic showing two types of one-way walls (arrows denote particle transmission and reflection). (a) The one-way wall is due to a ponderomotive potential near a primary resonance [Eqs. (31), (32)] with uniform $\frac{\partial}{\partial z} \Omega < 0$, so $F_z = -\frac{\partial}{\partial z} \Psi > 0$ for all $z$, except at the exact resonance where the effective potential does not apply. [The inclined asymptote corresponds to $\Psi = \Omega J + \text{const.}$.] (b) The one-way wall is due to a hybrid potential [Eqs. (50), (58)] with uniform $\Omega_j$ and $\Delta > 0$: particles incident from the left receive $J_1 > J_2$, so they see an attractive $\Psi$; those incident from the right have $J_1 < J_2$, so they see a repulsive $\Psi$.

VIII. ACKNOWLEDGMENTS

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APPENDIX A: GENERAL EXPRESSION FOR THE PONDEROMOTIVE POTENTIAL

Consider generalization of the ponderomotive potential [31, 32] to multiple internal oscillations and vector field

$$\mathbf{E} = \sum_{\mu} \mathbf{E}_\mu, \quad \mathbf{E}_\mu = e_\mu \vec{E}_\mu, \quad (A1)$$

composed of modes $\mu$ with polarizations $e_\mu$. From Sec. IV A, it is known that the internal energy $\delta \Omega \cdot \mathbf{J}$ yields a negligible contribution to $\Psi$. Thus

$$\Phi_0 = \sum_{\mu} \delta \omega_\mu I_\mu, \quad (A2)$$

where the infinitesimal frequency shifts $\delta \omega_\mu$ of the field are found as follows. Use [3]

$$\mathbf{L}_{\text{int}} = \frac{1}{2} \text{Re} [\mathbf{E}^* \cdot \mathbf{d}], \quad (A3)$$

where insignificant oscillating terms are removed, and $\mathbf{d}$ is the particle induced dipole moment. From Eq. (23),

$$-\delta \omega_\mu \rho_\mu \vec{E}_\mu = \frac{1}{4} e_\mu^* \cdot \mathbf{d}. \quad (A4)$$

Multiply Eq. (A4) by $\vec{E}_\mu^*$ and substitute $\mathbf{d} = \hat{\alpha} \mathbf{E}$, where $\hat{\alpha}$ is the polarizability tensor; then

$$\delta \omega_\mu I_\mu = -\frac{1}{4} \vec{E}_\mu^* \cdot \hat{\alpha} \mathbf{E}. \quad (A5)$$

Hence summation over all field modes yields the known expression [38, 39]

$$\Phi_0 = -\frac{1}{4} (\mathbf{E}^* \cdot \hat{\alpha} \mathbf{E}), \quad (A6)$$

which holds for any internal modes contributing to $\hat{\alpha}$ [3].

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