Identifying the orbital angular momentum of light based on atomic ensembles

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Abstract – We propose a scheme to distinguish the orbital-angular-momentum state of the Laguerre-Gaussian (LG) beam based on the electromagnetically induced transparency modulated by a microwave field in atomic ensembles. We show that the transverse-phase variation of a probe beam with the LG mode can be mapped into the spatial intensity distribution due to the change of atomic coherence caused by the microwave. The proposal may provide a useful tool for studying higher-dimensional quantum information based on atomic ensembles.

Photon is a promising candidate for quantum information processes. Photons with the Laguerre-Gaussian (LG) mode possess both spin angular momentum (SAM) and orbital angular momentum (OAM) [1]. The SAM relates to the light polarization, while the OAM relates to the transverse angular phase of light in the form of exp(ilφ). The LG beam has attracted much attention in the last two decades [2,3], since the OAM states of light can be encoded as higher-dimensional quantum information [4]. A higher-dimensional quantum system can be engineered to a quantum repeater based on atomic ensembles [5], which plays a crucial role in realizing long-distance quantum communication. Recently, there have been many works about the propagation of LG modes through atomic ensembles, such as generating the entanglement of OAM states of photon pairs in a hot atomic ensemble [6], narrowing the electromagnetically induced transparency (EIT) spectrum linewidth by LG modes [7], modulating LG modes through a non-material lens in a vapor cell [8], transferring LG modes from control beam to probe beam in atomic gases [9] and transferring the frequency of LG modes in four-wave mixing [10]. On the other hand, measuring LG modes is the basic and important problem in the applications of OAM. There are also many works about identifying LG modes by interference [11,12], diffraction [13,14] and image reformatting [15]. Considering the interesting application of LG modes in the atomic ensembles, it is worth studying the identification of the OAM of light based on atomic ensembles. This may provide us with an interface for higher-dimensional quantum information between OAM and atomic ensembles. Here, we propose a scheme to achieve the identification of LG modes in atomic ensembles. This method also has other potential applications in the aspect that it is based on atomic ensembles. For example, it is possible to achieve efficient light storage and retrieval for complex images. As we know, the image storage and retrieval in atomic ensembles is affected by optical diffusion [16], which we always try to diminish [17]. Our method may be useful for reducing the effect of diffusion on the visibility of the reconstructed image [16].

EIT is a quantum interference phenomenon based on coherent population trapping. Since it was found by Harris and co-workers, this unique effect has been applied to many fields such as laser frequency stabilizations [18], slow light and light storage [19,20]. EIT is usually modeled by a three-level Λ atomic system with a control light and a probe light coupling one upper level with two ground levels. Recently, it has been shown that if a microwave field is applied as a perturbation to the two ground levels, EIT can be enhanced or suppressed by adjusting the relative phase of two light fields and the microwave field [21,22].
In this letter, we show that light carrying OAM can be identified by using the EIT system which involves two optical fields and one microwave field. We demonstrate that when a beam with different LG modes propagates through such system, its transverse phase can be manipulated by the experimental configuration and eventually mapped into the intensity profile. The proposal provides an efficient method to distinguish the OAM states of LG modes based on atomic ensembles.

Our scheme starts from a three-level Λ atomic system whose energy levels are shown in fig. 1(a). A strong control beam \( E_2 \) and a weak probe beam \( E_1 \) are used to couple \( |c\rangle \rightarrow |a\rangle \) and \( |b\rangle \rightarrow |a\rangle \) atomic transitions, respectively. A microwave field \( E_{\mu} \) is used to couple the two ground levels. This is a typical closed Λ system and has been intensively studied [22]. However, in the following analysis, we show that such system can be used to identify LG modes of the probe light. The LG\(_{p,l}\) mode has two parameters, \( p \) and \( l \), \( l \) is the azimuthal index giving an OAM \( l\hbar \) per photon, and \( p+1 \) is the number of radial nodes in the intensity distribution [3]. Typically, we set \( p = 0 \) for simplicity. The intensity distribution of the LG\(_{0,l}\) beam is shown in fig. 1(b). Figure 1(c) illustrates an experimental configuration to implement our proposal. A probe beam with a LG mode produced by a computer-generated hologram (CGH) is incident into the atomic cell with a small cross angle to the control beam. The atomic cell is placed inside a magnetic-shielding cavity and a microwave resonator. In our scheme, the control and probe beams are phase-locked to ensure the following coherent interaction process. The corresponding fields can be expressed as

\[
E_1 = \varepsilon_1 (r/w)^l e^{-(r/w)^2} \cos(\omega t - k_1 z + \phi_1 + l\varphi),
\]

\[
E_2 = \varepsilon_2 \cos(\omega t - k_2 z + \phi_2),
\]

\[
E_{\mu} = \varepsilon_\mu(z) \cos(\omega t + \phi_\mu),
\]

where \( \varepsilon, \omega \) and \( \phi \) are the amplitude, angular frequency and initial phase, respectively. \( k_1(k_2) \) is the wave number for probe field (control field), \( w \) is the waist of the probe beam, and \( l \) is the OAM number. For the \( E_1 \) field, we have neglected the term \( \exp \left[ \frac{i\mu k_2^2}{2(\mu + \frac{1}{2})} \right] \exp \left[ -i(2p + l) \tan^{-1}\left( \frac{2p + l}{\mu + 1/2} \right) \right] \) as this term has little influence on the result in this model.

To understand the probe beam propagation properties in such system, we need to know the atomic polarization seen by the probe beam. This can be obtained from the motion equation of the density matrix, i.e.,

\[
\dot{\rho} = -\frac{i}{\hbar} [H, \rho] - \frac{1}{2} \{\Gamma, \rho\},
\]

where \( H = \hbar \omega_0 \langle a | (a + \hbar \omega_b | b \rangle \langle b | + \hbar \omega_c \langle c | (c + \hbar \Delta \rho_{ab} E_1 | a \rangle \langle b | + \hbar \Delta \rho_{ac} E_2 | a \rangle \langle c | + \hbar \Delta \rho_{bc} E_{\mu} | b \rangle \langle c | + \hbar \Delta \rho_{cb} E_{\mu} | c \rangle \langle b | + \hbar \Delta \rho_{ac} E_2 | c \rangle \langle b | ) \) is the Hamiltonian of the microwave modified EIT system. \( \varphi_{ab} = \varphi_{ba} = \varphi \) \( \varphi_{ac} = \varphi_{ca} = \varphi' \) and \( \varphi_{bc} = \varphi_{cb} = \varphi \) are the matrix elements of the electric dipole moment. \( \Gamma \) is the relaxation matrix. To solve eq. (4), we introduce the slowly varying envelope as \( \rho_{ab} = \sigma_{ac} e^{i(k_1 z - \omega_1 t - il\varphi)}, \rho_{ac} = \sigma_{ac} e^{i(k_2 z - \omega_2 t)}, \rho_{cb} = \sigma_{cb} e^{i(k_1 z - \omega_1 t)}, \rho_{aa} = \sigma_{aa}, \) where \( \Delta k = k_1 - k_2 \) [23].
Under the rotating-wave approximation, eq. (4) is changed to

\[
\dot{\sigma}_{ab} = -\Gamma_{ab}\sigma_{ab} - i\Omega_1 (\sigma_{aa} - \sigma_{bb}) + i\Omega_2 \sigma_{cb} - i\mu_\epsilon e^{i(\omega_1 - \omega_2 - \omega_\epsilon)t - i\Delta k z + il\varphi}\sigma_{ac},
\]

\[
\dot{\sigma}_{ac} = -\Gamma_{ac}\sigma_{ac} - i\Omega_2 (\sigma_{aa} - \sigma_{cc}) + i\Omega_1 \sigma_{bc} - i\mu_\epsilon e^{-i(\omega_1 - \omega_2 - \omega_\epsilon)t + i\Delta k z + il\varphi}\sigma_{ab},
\]

\[
\dot{\sigma}_{cb} = -\Gamma_{cb}\sigma_{cb} - i\Omega_2 e^{i(\omega_1 - \omega_2 - \omega_\epsilon)t - i\Delta k z + il\varphi}x(\sigma_{cc} - \sigma_{bb}) + i\Omega_2 \sigma_{ab} - i\Omega_1 \sigma_{ac},
\]

where \(\Omega_1 = \frac{\nu_{ab} \epsilon_1}{\hbar}(r/w)^l e^{-(r/w)^2} e^{-i\phi_1}\), \(\Omega_2 = \frac{\nu_{ab} \epsilon_2}{\hbar} e^{-i\phi_2}\), and \(\Omega_\epsilon = \frac{\nu_{ab} \epsilon_0}{\hbar^2} e^{-i\phi_0}\) are the Rabi frequencies of the corresponding fields. At the steady state, assuming all atoms populate the ground state \(|b\rangle\) due to the strong control field, i.e., \(\rho_{ab} \approx 1\) and \(\rho_{aa} = \rho_{cc} \approx 0\), we obtain an analytical solution of the atomic coherence term related to the probe beam,

\[
\sigma_{ab} = \frac{i\Gamma_{ab}\Omega_1}{\Gamma_{ab} + |\Omega_1|^2} - \frac{i\Omega_2 \epsilon_\text{op}}{\Gamma_{ab} + |\Omega_2|^2},
\]

where \(\Gamma_{cb} = \gamma_{cb} + i\Delta\) and \(\Gamma_{ab} = \gamma_{ab} + i\Delta\). To be realistic, we need to consider the motion of atoms such that \(\Gamma\) should be modified. Without loss of generality, we can neglect the Doppler width of the two-photon transition and just take into account the Doppler width of the one-photon transition. Thus, the modified \(\Gamma\) can be expressed as,

\[
\Gamma_{ab} = \gamma_{ab} + k_1 u + i\Delta, \quad \Gamma_{ac} = \gamma_{ac} + k_2 u \quad \text{and} \quad \Gamma_{cb} = \gamma_{cb} + i\Delta, \quad \text{where} \ u \text{is the most probable speed of the atoms [22,24].}
\]

Once \(\sigma_{ab}\) is obtained, the atomic polarization governing the probe propagation can be determined, i.e., \(P(z,t) = c_\text{pop} \rho_{bc}(z,t) + c.c.\). From the Maxwell equations, we can then solve the propagation equation for the probe beam as

\[
\frac{\partial \Omega_1}{\partial z} = -i\eta \sigma_{ab},
\]

where \(\eta = \omega_1 N_\text{pop}^2/(2\epsilon_0 c \hbar)\) is the coupling constant, \(N\) is the atomic density, and \(\epsilon_0\) is the permittivity in vacuum. In this sense the propagation equation of probe beam with the LG mode passing through an EIT system controlled by a microwave field can be expressed as [22,24]

\[
\frac{\partial \Omega_1}{\partial z} = - \eta \Gamma_{ab}\Omega_1 \frac{\epsilon_\text{op}}{\Gamma_{ab} + |\Omega_2|^2} - i\frac{\eta \Omega_2 \epsilon_\text{op} e^{-i\Delta k z - i\varphi}}{\Gamma_{ab} + |\Omega_2|^2}.
\]

The first term on the right side of eq. (10) generally describes a \(\Lambda\)-system EIT effect, while the second term is from the process involved in the microwave field, which is an essential part to distinguish the LG modes in our scheme. Assuming the initial Rabi frequency of the probe beam is \(\Omega_{10} = \epsilon_{10} e^{-(r/w)^2} (r/w)^l\) at the position of \(z_0\), the transmitted probe field passing through the atomic cell with length \(L\) can be expressed as

\[
\Omega_1(z_0 + L) = \epsilon_{10} e^{-\alpha L} \left( \frac{r}{w} \right)^l e^{-(r/w)^2} \left[ e^{-i\Delta k z_0 - i\alpha L} \right] \frac{\eta \Omega_2 \epsilon_\text{op}}{\Gamma_{ab} + |\Omega_2|^2} \frac{1}{(-i\Delta k + \alpha)},
\]

where \(\alpha = \eta \Gamma_{ab} \epsilon_\text{op}\) is the absorption coefficient. Thus the intensity profile can be calculated by \(|\Omega_1(z_0 + L)|^2\).

To show how our theory can be used to identify the LG modes, we use the \(87\text{Rb D1 line} (5^2S_{1/2} \rightarrow 5^2P_{1/2}, \lambda = 795 \text{ nm})\) to numerically simulate the probe beam profile. In this case, the two EIT ground levels are the two hyperfine ground states \(5^2S_{1/2} (F = 1)\) and \(5^2S_{1/2} (F = 2)\). The corresponding parameters are set as \(\gamma_{ab} = 6\), \(\gamma_{bc} = 10^{-3}\), \(\epsilon_{10} = 0.1\), \(\Omega_2 = 1\), \(\Omega_\mu = 0.02\), \(\eta = 0.9\), \(L = 3 \text{ cm}\), \(k_1 u = 500\), \(\Delta k = 1.43 \text{ cm}^{-1}\), and \(w = 1 \text{ mm}\). The above values are common in an EIT system and can be easily obtained in the experiment. Figure 2 shows the intensity distributions of the probe beam behind the Rb cell for the different OAM numbers. Clearly, the transverse-phase information carried by the LG mode now is converted into the intensity distribution, and a different OAM number \(l\) corresponds to a different bright-area number. By counting the number of bright areas, an unknown LG mode can be easily recognized.

As we know, the OAM has both negative and positive values. For these mode beam identification, it is usually difficult to sort out in other ways [25]. The intensity distributions shown in fig. 2 do not provide any additional information about the sign of OAM. However, as the phase of control beam and probe beam is locked in our configuration, these two beams form a wave packet along the \(z\)-axis. The frequency of the wave packet equals the frequency difference of the two beams. So the total relative
Fig. 3: (Colour on-line) The spatial intensity profiles of the probe beam with the LG$_{0,2}$ mode vs. the different positions of the Rb cell. Panels (a) to (d) correspond to $z_0 = 5, 6.46, 7.93,$ and $9.4 \text{ cm}$, respectively.

phase ($\Delta k z - l \varphi$) will change linearly with the position of the Rb cell moving along the $z$-axis. In such a way, the OAM states with number $+l$ and $-l$ can also be distinguished in our scheme. Figure 3 shows the intensity profile variations of a LG$_{0,2}$ probe beam with the positions of the Rb cell moving along the $z$-direction. We can see that the total relative phase changing at different Rb cell positions causes a rotation of the intensity profile. For the typical $^{87}\text{Rb}$ D1 line, the frequency difference of the two ground hyperfine levels is $6.83 \text{ GHz}$, which corresponds to the beat wavelength $L' = 4.4 \text{ cm}$. Therefore the intensity profile is recovered as the Rb cell moves one beat wavelength as shown in fig. 3. Moreover, our theoretical analysis indicates that the rotating direction of the intensity pattern is related to the sign of the OAM. In other words, the reverse rotating direction of the intensity profile indicates that the LG mode carries an opposite OAM, i.e., LG$_{0,-2}$. Thus, we can facilely sort out the different order of the LG modes by moving the Rb cell and checking the rotating directions of the intensity profile. Furthermore, we use the absorption to detect the rotation direction of the phase in the LG beam, which is related to the atomic cell length and the beat wavelength. From eq. (11), a simple derivation shows that the spatial profile will have a good distinguishability when $L \neq nL'$, and $n$ is an integer.

Our scheme is also valid for the LG$_{p,l}$ mode with $p \neq 0$. We calculate the LG$_{2,l}$ mode as an example and illustrate the results in fig. 4. Obviously, three rings appear corresponding to the radial mode of the LG beam. And each ring has a specific bright area number related to its OAM number. Thus, the general LG$_{p,l}$ mode can also be identified. Interestingly, the intensity distribution is complementary between the adjacent rings. This is due to the intrinsic transverse-phase distribution of the LG mode.

It should be mentioned that diffraction and diffusion effects are not considered in our scheme. These effects are important in light storage experiments. However in our scheme, the probe light propagates through the atomic cell together with the control light and the cell length is usually short, so the above two effects should not play an important role. In this letter, we assume that the phase and amplitude of the microwave are constant. It is also interesting to note that a similar rotation occurs by changing the phase of the microwave. Moreover, the strength of the microwave could affect the distinguishability of the intensity profile.

Before drawing a conclusion, we should address the fact that our method for mode sorting is not as convenient as the other optical methods. However, we exploit an OAM sorting way based on the prospective atomic ensembles. It may be useful for quantum repeaters and higher-dimensional quantum communication. Furthermore, this method can be used for reducing the effect of diffusion on the visibility of the reconstructed image in light storage.

In summary, we have theoretically exhibited how to recognize the LG mode based on atomic ensembles. The microwave is used to perturb the atomic coherence of the two ground levels. As the transverse phase of the probe beam varies, the phase information is then converted to the transmission intensity change. So a different order of the LG modes can be distinguished by the transverse intensity distribution. Moreover we find that our scheme can be easily adapted to sort out the negative and positive modes of the LG beam by moving the position of the atomic cell. This novel approach to sort out the LG mode may provide potential applications in quantum memory and light storage with the EIT system. It may also be used in the phase contrast imaging with high modulation depth or in phase-to-amplitude conversion in an atomic system.

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