Plastic energies in layered superconductors

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We estimate the energy cost associated with two pancake vortices colliding in a layered superconductor. It is argued that this energy sets the plastics energy scale and is the analogue of the crossing energy for vortices in the continuum case. The starting point of the calculation is the Lawrence-Doniach version of the Ginzburg-Landau free energy for type-II superconductors. The magnetic fields considered are along the $c$-direction and assumed to be sufficiently high that the Lowest Landau level approximation is valid. For Bi-2212, where it is known that layering is very important, the results are radically different from what would have been obtained using a three-dimensional anisotropic continuum model. We then use the plastic energy for Bi-2212 to successfully explain recent results from Hellerqvist et al. on its longitudinal resistance.

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There has been much recent experimental interest in the dependence of the longitudinal resistivity in the HTC-superconductor on field and temperature. In a recent paper, we produced a theory for some aspects of this data which involved knowing the crossing energy for two vortices (flux lines). Our work was essentially an extension of the hydrodynamic theory of Marchetti, Nelson and Cates. It gave a reasonable fit to the longitudinal resistivity data of Safar et al. on YBCO crystals. However, it was clear from our calculation that the length scale over which the crossing took place was of the order of the spacing between the superconducting Cu-O planes. Our calculation had been done using anisotropic continuum Ginzburg-Landau theory, and so neglected any layering effects. The crossing energy that we calculated for the three-dimensional anisotropic case can be thought of as the plastic energy cost of making non-trivial topological deformations of the vortices, and we shall refer to it as $U_{\text{pl}}$.

In this paper we calculate the energy associated with two pancakes colliding within a layer: this is an estimate of the plastic deformation energy for vortices in a strongly layered superconductor and sets the timescales for deformation of the vortices within the system. Intuitively one might have expected that neighbouring pancake vortices in a layer would have exchanged positions by moving around each other. However, within the Lowest Landau Level (LLL) approximation which we use, Dodgson and Moore have shown that the lowest energy barrier for this to occur is when the vortices collide and pass through each other.

We start from a Lawrence-Doniach Ginzburg-Landau free energy functional. We find that including the layering leaves the results for the plastic energy of YBCO essentially unchanged but gives markedly different results for Bi-2212. This suggests that treating YBCO as a 3D continuum anisotropic superconductor within GL theory is reasonable, but that in Bi-2212 the effect of the Cu-O layers must be included — as was to be expected as the coupling between the layers in Bi-2212 is known to be very weak.

To fix notations, we briefly describe the modified Ginzburg-Landau theory for a layered superconductor, where $\psi$, is our spatially dependent order parameter. We begin with the continuum free energy functional,

$$ \frac{F[\psi(\mathbf{r})]}{k_B T_c} = \int d^3 r \left( \alpha(T) |\psi|^2 + \beta \frac{\psi^4}{2} + \sum_{\mu=1}^{3} \frac{|(-i\hbar \partial_\mu - 2eA_\mu)\psi|^2}{2m_\mu} + \frac{B^2}{2\mu_0} \right). $$

(1)

Here $\alpha(T)$ is the temperature-dependent variable, $\beta$ is the coupling constant, and $m_\mu$ are the effective masses. In the cases we consider the masses in the $ab$-plane are taken as equal and are denoted by $m_{ab}$, and the mass in the $c$-direction is written as $m_c$. The temperature dependence of $\alpha(T)$ is taken to be of the form, $\alpha(T) = (T - T_c)\alpha'$. We also assume the LLL approximation (see Refs. 9 for justification of it). In this approximation the derivative terms in the direction perpendicular to the planes reduce to $e\hbar B/m_{ab}|\psi|^2$, so it is convenient to write the temperature variable as $\alpha_H = \alpha + e\hbar B/m_{ab}$. This is zero along the $H_{c_2}$ line, negative below and positive above. According to T€esani{c} et al. the LLL approximation can be trusted when $H > \hbar c_2/f$, where $f = 3$.

In our previous calculation we chose to consider a system that contained only two vortices, but with different types of periodic boundary conditions. In this paper we instead generalise to the layered case the procedure of Dodgson
and Moor, who allowed only the two vortices that are crossing to move and kept all the other vortices fixed in their triangular lattice positions. The advantage of using a background of fixed vortices is that the resulting crossing energy, $U_{\text{int}}$, is an upper-bound on the true crossing energy. Furthermore, it has been shown that the corrections to this bound seem to be very small, at least for the continuum limit.

The general form for the order parameter in the $ab$-plane within the LLL approximation is:

$$
\psi(x, y) = f(\zeta)e^{-\pi|\zeta|^2B/(2\Phi_0)} \quad \text{where} \quad \zeta = x + iy,
$$

and $f(\zeta)$ can be written in the product form,

$$
f(\zeta) \propto \Pi_i(\zeta - \zeta_i)
$$

where the $\zeta_i$ are the (complex) positions of the vortices. The minimum potential free energy configuration for this system is the triangular lattice of lattice spacing $\sqrt{3}/2t^2 = \Phi_0/B$. We will label the order-parameter that describes this configuration by $\psi_0(x, y)$.

We are interested in two vortices being displaced along the $x$-axis from their lattice positions at $\zeta = \pm l/2$ and colliding at the origin $x = y = 0$. Using the formalism of Ref. 8 we have for the order parameter at height $z$

$$
\psi(x, y, z) = \psi_0(x, y)(\zeta + a(z))(\zeta - a(z))
$$

where $a(0) = 0$. (See Fig. 1.) Our other boundary condition is that the displaced vortices lie on top of each other. By symmetry this should occur at $z = 0$. The previous results for the continuum case of Dodgson and Moore, with a rigid lattice background of vortices led to a crossing energy of $2.3\hbar\Phi_0|\alpha_H|^{3/2}/(\sqrt{2m_e}\beta B)$, which is a rigorous upper bound to the crossing energy. Our earlier calculations using only two vortices with periodic boundary conditions led to an energy of $1.46\hbar\Phi_0|\alpha_H|^{3/2}/(\sqrt{2m_e}\beta B)$, which is somewhat lower, possibly due to errors from the finite size of the system. In this paper we use the rigid lattice approach, as the equations are more readily physically interpreted, and easier to solve.

The resulting effective Ginzburg-Landau free energy functional within the LLL approximation for a layered structure can then be written as,

$$
F = d \sum_n \int \left( -|\alpha_H||\psi_n|^2 + \frac{\beta}{2} |\psi_n|^4 + \frac{\hbar^2}{(2m_e d^2)} |\psi_{n+1} - \psi_n|^2 \right) \, dx \, dy
$$

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The extra free energy, $\Delta F$, associated with the two displaced vortices as compared with the original lattice is found by substituting Eq. (3) into Eq. (2) and then doing the integral over the $(x, y)$ directions.

$$
\Delta F = \frac{2d|\alpha_H|^2\Phi_0}{\sqrt{3}\beta A B} g(\eta) \quad \text{where} \quad g(\eta) = \sum_{n=-N}^{N} \left( \frac{4}{\sum_{m=0}^{2N}} a^2_m c_m a^2_{m+1} \right) + \frac{I_1}{\eta^2} \left( a^2_{n+1} - a^2_n \right)^2
$$

where $a_n$ is the value of $a(z)$ in the $n$th layer of which there are $(2N + 1)$. The factor $\beta A \simeq 1.16$ is the Abrikosov factor. The quantity $\eta = d/\xi_c$, where $\xi_c = \hbar/\sqrt{2m_e|\alpha_H|}$ — the ‘$c$-axis correlation length’ measures the importance of layering effects. The formula for $\xi_c$ in terms of $\alpha_H$ should be modified very close to the $H_{c2}$ line a regime which we therefore shall not consider. When $\eta$ is large, layering effects are important; $\eta \to 0$ is the continuum limit. The constants $c_m$ and $I_1$ are determined by integration in the $xy$-plane, $c_0 = 0.7903$, $c_2 = -5.201$, $c_3 = -32.7365$, $c_4 = 100.8232$, and $I_1 = 23.5502$. The $\alpha$ and $\beta$ terms can be respectively thought of as the ‘potential’ and ‘kinetic’ contributions to the plastic energy.

In order for the vortices to collide we require that at the central layer ($n = 0$) the displaced vortices lie on top of each other. By symmetry this should occur at $a_0 = 0$. (See Fig. 1) Our other boundary condition is that $a_{\pm\infty} = \pm l/2$, that is we require the vortices to return to their equilibrium positions in the original lattice.

The energy barrier for vortices to collide is then found by minimising Eq. (4), with the boundary conditions imposed. The results unlike in the continuum case cannot be written in terms of a reduced temperature variable. They are now
field, temperature and layer thickness $d$ dependent. The easiest way to get an idea of how the plastic energy in the layered case varies is to compare it with the plastic/crossing energy calculated for the rigid lattice background in the continuum case.

$$U_{\text{pl}}^{\text{disc}} / U_{\text{pl}}^{\text{cont}} = 0.43 \eta g(\eta)$$

(7)

where $U_{\text{pl}}^{\text{disc}}$ is the crossing energy found in the discrete case and $U_{\text{pl}}^{\text{cont}}$ in the continuum case. The form of this function is shown in Fig. 2. Putting in typical system parameters for YBCO: $|dB/dT| = 2T/K$, $m_{ab}/m_c = \varepsilon^2 = 1/59$, $T_c = 93K$, and a spacing between Cu-O planes of 11.4Å (where we have treated the closely spaced pair of Cu-O layers as a 'single' layer) we find that $\eta \sim 2$, for values of field and temperature in the vicinity of the irreversibility line. As can be seen from Fig. 2 for $\eta \sim 2$ the plastic energy calculated according to the continuum approximation and that calculated allowing for layering are approximately the same, indicating that for YBCO the continuum approximation is satisfactory, certainly for all the regions of the phase diagram in which we would expect the LLL approximation to be valid. The material for which layering effects would be expected to be more important is Bi-2212, where it is known from experiments that the coupling between the Cu-O layers is very weak. Our calculation bears this out; using system parameters of $|dB/dT| = 0.5T/K$, $T_c = 86$, $\varepsilon = 1/55$ (there appears to be a factor of 1/3 uncertainty in the value of $\varepsilon$) and $d = 30.9Å$, we find $\eta \sim 25$ and that the $U_{\text{pl}}^{\text{disc}} / U_{\text{pl}}^{\text{cont}} \sim 8$. Thus the continuum estimate appears to seriously underestimate the plastic energy for this weakly coupled layered system.

Analysis of $g(\eta)$ shows that for the values of $\eta$ appropriate to Bi-2212 we can greatly simplify the formula for the plastic energy, Eq. (6). The displacement of the pancake vortices for these values of $\eta$, is completed within three layers (see Fig. 3), and is such that the ‘kinetic energy’ term in Eq. (6) is negligible in comparison to the ‘potential energy’ term, which tends to the constant $c_0$. Hence when $\eta$ is large

$$U_{\text{pl}}^{\text{disc}} \simeq \frac{2d|\alpha_H|^2\Phi_0}{\sqrt{3}\beta_A B} c_0.$$  

(8)

In the three dimensional continuum case we were able to use the formula of Cates et al. which says that the length scale over which the system is coherent along the field direction, $\delta_c$ can be written as

$$\delta_c \sim l_c \exp[U_{\text{pl}}^{\text{cont}} / kT].$$  

(9)

The distance $l_c$ is the distance along a flux line that is traveled before encountering another flux line, and is related to the equilibrium flux line spacing by $l_c = a_0^2/l_p$, where $l_p$ is a persistence length which in our case is equal to the c-axis correlation length $\xi_c$. (There is uncertainty in the numerical constant multiplying $l_c$ but it should be of O(1)). We motivated Eq. (9) in Ref. 13 in some detail but it can also be readily understood using an explanation of Marchetti and Nelson. If we concentrate on the dynamics of one flux line in a flux liquid and label the c-direction by time then $1/l_c$ becomes an effective attempt frequency to cross or collide, which occurs with probability $\exp[-U_{\text{pl}}^{\text{cont}} / kT]$, leading immediately to Eq. (9). Crossing of flux lines always produces a loss of phase coherence. In the layered case we can no longer consider ‘crossing energies’. However, the arguments detailed above follow through once we consider the plastic deformation energy rather than the crossing energy. The importance of the plastic energy for flux line motion in the liquid state is shown in Blatter et al.

We are now in a position to compare with the longitudinal resistivity measurements of Hellerqvist et al. on Bi-2212. Using a single crystal of size $2 \times 2 \times 0.030mm$ (smallest dimension in the c-direction) they measured the dynamic resistance $\partial V / \partial I$ as a function of bias current in a magnetic field along the c-direction. Their data of $\rho_c$ against $T$ has a smooth peaked behavior with the position of the peak shifting to higher temperatures as the field is lowered. By fitting to a power law of the form $\partial V / \partial I \propto I^\alpha$ they find linear $I - V$ behavior for $T > T_{\text{peak}}$ but for $T < T_{\text{peak}}$ the exponent $\alpha$ is positive and varies smoothly with temperature and field.

The apparent disappearance of the longitudinal linear resistivity at the peak suggest that the sample has become phase coherent along the c-direction. To locate the point at which the longitudinal resistivity vanishes we set $\delta_c$ to equal the system size, $L = 0.03mm$. Analysis of the Hellerqvist et al. data, shows that the field-temperature dependence of the longitudinal resistivity peaks is well fitted by the formula $L \simeq \delta_c$ (see Fig. 4). At high fields and low temperatures we begin to see deviations from the experimental data — but this is no surprise, as these points are so far away from the $H_{c2}$ line that it is unlikely that the LLL approximation remains valid.

We conclude by observing that the onset of vanishing longitudinal resistance is well-explained by equating the plastic energy length scale $\delta_c$ to the system size. No phase transition need be invoked to explain the experimental data.
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FIG. 1. Trajectory of two lines colliding in the $xy$-plane.

FIG. 2. The ratio $U_{pl}^{disc}/U_{pl}^{cont}$ versus $\eta = d/\xi_c$. As $\eta \to 0$ the ratio tends to unity while it is linear for large $\eta$.

FIG. 3. Configuration for vortices colliding in Bi-2212. The positions of the vortices are only defined in the layers.

FIG. 4. Comparison of experiment (points) and theory (solid line) for the line marking the disappearance of the longitudinal resistivity in Bi-2212.
Fixed vortices

Equilibrium positions of crossing vortices

Displaced vortices
