High Frequency Cutoff and Change of Radio Emission Mechanism in Pulsars

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Abstract Pulsars are the fast rotating neutron stars with strong magnetic field, that emit over a wide frequency range. In spite of the efforts during 40 years after the discovery of pulsars, the mechanism of their radio emission remains to be unknown so far. We propose a new approach to solving this problem for a subset of pulsars with a high-frequency cutoff of the spectrum from the Pushchino catalogue (the "Pushchino" sample). We provide a theoretical explanation of the observed dependence of the high-frequency cutoff from the pulsar period. The dependence of the cutoff position from the magnetic field is predicted. This explanation is based on a new mechanism for electron radio emission in pulsars. Namely, radiation occurs in the inner (polar) gap, when electrons are accelerated in the electric field that is increasing from zero level at the star surface. In this case acceleration of electrons passes through a maximum and goes to zero when the electron velocity approaches the speed of light. All the radiated power is located within the radio frequency band. The averaging of intensity radiation over the polar cap, with some natural assumptions of the coherence of the radiation, leads to the observed spectra. It also leads to an acceptable estimate of the power of radio emission.

Keywords pulsars: radio emission, spectra

1 Introduction

Pulsars are magnetized neutron stars that have a magnetosphere filled with an electron-positron plasma of about the GJ density (Smith 1977; Manchester & Taylor 1977; Beskin, et al. 1993). New discoveries of double pulsar system (Lane, et al. 2004) and intermittent pulsars (Kramer, et al. 2006; Lorimer, et al. 2012; Camilo, et al. 2012) give the direct observational support to that idea. It is thought that this plasma in the region of open magnetic field lines over the magnetic polar cap is generated by particles (through gamma quanta production) accelerating in a gap under the magnetosphere (Sturrock 1971; Ruderman & Sutherland 1975; Arons 1981; Beskin 2010). The acceleration of electrons occurs in the gap in the electric field that is longitudinal with respect to the magnetic field and induced by the rotation of the magnetized star. Directed coherent electromagnetic radiation of relativistic particles from the region of open lines creates the beacon effect that results in the pulses observed (the most popular explanation).

In explanation of radio emission of pulsars (see reviews (Malov 2004; Manchester 2009) and addition references in Malov & Machabeli 2009; Kontorovich 2009; Beskin & Philippov 2012)) the instabilities of plasma flow, beam instabilities and similar effects in the magnetospheric plasma\textsuperscript{1} have been discussed. Apparently, various mechanisms of radio emission are actually realized and may in certain circumstances succeed each other.

We show in this paper that for the observed pulsar radio emission a coherent radiation produced in a polar gap may be responsible, at least for pulsars of the

\textsuperscript{1}Note apart the plasma-beam (see as example Usov 1987; Kazbegi, et al. 1992), also the cyclotron, drift, modulation instabilities, Zakharov’s wave collapse and magnetic reconnection for GP, low-frequency "tails" of the synchrotron, Cherenkov, Doppler and curvature radiation in the relativistic electron-positron plasma.
frequency values in catalogue (Maron 2000) with new results that (Malofeev & Malov 1980). We don’t touch here the more high frequency and period have been found for some part of the spectra and the main result of this work.

In the well-known Ruderman-Sutherland model the strong electric field, non vanishing at the star surface, accelerates electrons so quickly that their radiation due to acceleration in the gap fully comes to the hard energy (X-ray) region with no radio emission. Only in an electric field slow rising from zero level on the star surface the radiation of accelerating electrons comes to the radio band.

Resulting radiation is limited by a cutoff frequency found in this study. It coincides with the high-frequency cutoff (Malofeev & Malov 1980; Malov 2004) in the pulsar spectra of the Pushchino sample. Such frequency limitation is due to the fact that the electron acceleration in the electric field, vanishing on the star surface, passes through a maximum and decreases as the electron velocity approaches the relativistic limit.

There is the all-new specification of our point of view and the main result of this work.

\[ w(z) = 10^{20} \text{ cm/s}^2 \]

Fig. 1 The dependence of the acceleration on the altitude \( z \) on the star surface

\[ 2 \text{ Acceleration of electrons in the speed up process} \]

We start from the equation for the electron Lorentz factor \( \Gamma(z) \) [Landau & Lifshitz 1994].

\[ d\Gamma(z)/dz = eE(z)/mc^2, \quad (1) \]

where \( z \) is the altitude of the electron above the star surface. The equation describes the change of electron energy in a nonuniform electric field \( E(z) = E_z(z) \). (Only component that is parallel to the strong magnetic field \( B \) directed on \( z \) axes is essential). Energy losses (either by curvature radiation nor by inverse Compton scattering) are insignificant for relevant for us values of Lorentz-factors \( \Gamma \) of order of some units. We also ignore, within the polar cap, the deviation of the magnetic field lines from orthogonality to the surface of the pulsar.

The low-frequency radiation occurs at the small altitudes when the accelerating electric field \( E(z) \) rises from zero level on the surface of the pulsar \( z = 0 \). The vanishing of the electric field on the star surface is related to a small electron work function [Jones 1986] of the surface. At altitudes \( z \ll h \), where \( h \) is the height of the gap, the field

\[ E(z) = E_0z/h \]

increases linearly [Arons 1981; Harding & Muslimov 1998; Dyks & Rudak 2000; Beskin 2010] with the altitude \( z \). The velocity \( V \) and the acceleration \( w \equiv \dot{z} \), expressed in terms of Lorentz factor \( \Gamma \), are equal to

\[ V = c\sqrt{\Gamma^2 - 1}/\Gamma, \quad \text{and} \]

\[ w(z) = eE(z)/m\Gamma^2 = e\Gamma^{-3}\Gamma' \]

where \( \Gamma' \equiv d\Gamma/dz \). In a linear field we have

\[ \Gamma(z) = \Gamma_0 + az^2, \quad a = eE_0/2mc^2h. \]

The acceleration increases from zero when \( z = 0 \), passes through a maximum (Fig.1) when

\[ \Gamma_m = 6/5, \quad V_m = c\sqrt{11}/6 \]

(not dependent on the field) and

\[ z_m^2 = 2mc^2h/5eE_0 \]

(for \( \Gamma_0 = 1 \)) and tends to zero at approaching of electrons to relativistic velocities. The value of particle acceleration at the maximum is

\[ w_m = (5/6)^3 c\sqrt{2eE_0/5mh}. \]
The height of the gap falls from these relations used henceforward the field of the form Muslimov & Tsygan

\[ E_0 \sim \Omega \cdot B h/c. \]

Physically, this estimate is quite obvious since on the scale of the gap the characteristic velocity due to rotation is \( \Omega \cdot h \). We omit here a factor of order of unity that will be partly considered below, which contains the difference \( E_0 \) on the position on the polar cap. We do not discuss here the influence of general relativity effects and dependence on the angle between the axis of rotation and magnetic axis of the pulsar Beskin 2011, which does not affect the estimates. (The exception is PSR B0531+21 which is close to an orthogonal rotator.) The estimate

\[ z_m \approx \sqrt{(P/1\ s) \cdot (10^{12} G/B) \cdot 1 \ cm} \]

confirms the legality of the conditions \( z << h \) which we use, as \( h \sim 10^4 \text{cm} \) for normal pulsars.

3 Radiation at acceleration in the gap

For the radiation field of accelerated electrons, we proceed from the retarded Lienard-Wiechert potentials Jackson 1962, Landau & Lifshitz 1994. In the problem considered the particle acceleration \( \mathbf{w} \) is directed along its velocity \( \mathbf{V} \). Then for the Fourier component of the wave magnetic field we have (at large distances from the radiating electron)

\[ \mathbf{H}_\omega = \frac{\mathbf{e}}{c^2 R_0} e^{ikz_0} \int_{-\infty}^{\infty} \frac{[\mathbf{w}(t), \mathbf{n}]}{(1 - n \mathbf{V}(t)/c)^2} e^{i(\omega t - k \mathbf{r}(t))} \, dt. \]

Here \( t = t(z) = \int_0^z \frac{dz'}{V(z')} \), \( \mathbf{r}(t) \) is the electron radius vector, \( R_0 \) is the distance from the origin on the star surface \( z = 0 \) to the field observation point, \( \mathbf{n} = \mathbf{k}/k \).

A connection of the time \( t \) with the electron coordinate \( z \) is given by the integral

\[ t(z) = \frac{1}{c} \int_0^z \frac{\Gamma(z') \, dz'}{\sqrt{\Gamma^2(z') - 1}}, \]

where \( \Gamma(z) \) is governed by (1).

To eliminate the logarithmic divergence at zero one should take into account in the difference \( \Gamma - 1 \) that the initial velocity \( V(0) = V_T \neq 0 \) and therefore \( \Gamma_0 \approx V_T^2/2e^2 \). Here \( V_T \ll c \) is the thermal velocity of electrons on the surface of the polar cap. It is convenient to represent the Fourier component \( \mathbf{H}_\omega \), which determines the emission spectrum, as an integral over coordinate:

\[ \mathbf{H}_\omega = \frac{e}{c^2 R_0} e^{ikz_0} [l, \mathbf{n}] L_\omega, \]

where \( l = \mathbf{V}/V, \mathbf{n} = \mathbf{k}/k, l \cdot \mathbf{n} = \cos \theta \) and

\[ L_\omega = \int_0^h \frac{w(z)}{V(z)} \left( 1 - \frac{\nu(z)}{c} \cos \theta \right)^2 e^{i\omega(t(z) - \frac{2}{c} \cos \theta)} \, dz. \]

Accordingly, we have for the spectral and angular radiation intensity density \( d\varepsilon_{\omega m}/d\omega do \) interesting us Jackson 1962, Landau & Lifshitz 1994

\[ d\varepsilon_{\omega m} = \frac{e^2}{4\pi^2 c^3} \sin^2 \theta \cdot |L_\omega|^2 \, d\omega do. \]

Here \( d\omega \) is the interval of frequencies, \( do \) is the interval of solid angles. It is seen from the integral \( L_\omega \) that due to rapid oscillations the field decreases exponentially for \( \omega > \omega_{CF} \), where the cutoff frequency is \( \omega_{CF} \approx \pi \sqrt{2eE_\nu/m} \) (Fig.2). This estimate can be obtained numerically from Eq.(5) and independently from the physical consideration, supposing that the electron movement becomes relativistic: \( e \int_0^z E(z) \, dz = mc^2 \) and \( \omega_{CF} = 2\pi c/z_{CF} \). We obtain \( \Gamma_{CF} = 2, V_{CF} = \sqrt{3}c/2, z_{CF} = 2mc^2h/eE_0 = 5z_m \). The coefficient \( a = 1/\lambda_{CF}^2 \), where \( \lambda_{CF} \) is the wavelength corresponding to the cutoff.

To move to the average spectra we take into account the dependence of the field from its position on the polar cap (cf. Dyks & Rudak 2000) of the form:

\[ E_0(r) = E_{max} \left( 1 - \frac{r^2}{R_{PC}} \right), \quad R_{PC} \approx R \sqrt{\frac{\Omega R}{c}} \]

\( (R \) is the star radius). Assuming for the maximum field at the center of the polar cap \( E_{max}/B = \Omega \cdot h/c \), we obtain the value of the cutoff frequency \( \omega_{CF}(0) = \pi \sqrt{2e\Omega B/mc} \), which is independent of the height gap. The cutoff frequency dependence on the pulsar magnetic field is shown in Fig.3.

At this frequency, the emission spectrum for the discussed mechanism must cutoff and be replaced by the next highest, forming a kink. The cutoff frequencies for a sample of normal pulsars had been found in
The emission spectrum of a particle at acceleration in the electric field linearly increasing with altitude $z$ above the surface of the star (the numerical calculation of (5) for $\theta = \pi/8$, $B = 10^{12} G$, $P = 1 s$)

$$\nu_{cf} \approx \sqrt{2} \cdot 10^9 Hz \left( \frac{B}{2 \cdot 10^{12} G} \right) \left( \frac{1 s}{P} \right),$$

(7) (this theory)

$$\nu_{cf} \approx 1.4 \cdot 10^9 Hz \left( \frac{1 s}{P} \right)^{0.46 \pm 0.18}.$$  

(Malofeev & Malov 1980)

(The tilde over frequency means the experimental value). The dependence $\tilde{\nu}_{cf}$ on $B$ and other parameters, (see as example (Dyks & Rudak 2000)) determining the electric field in the gap, is available for testing. It can also serve as a criterion for the correctness of this description. Below we use the cutoff frequency $\nu_{cf} = e/z_{cf}$ corresponding decrease of intensity radiation of a single electron to $e$ times (see Fig.2). The actual frequency of the cutoff may be less than the estimated cutoff frequency both due to the fact that the actual accelerating field in the gap is less than the adopted estimates and the cutoff corresponds to higher values of gamma-factor.

4 Average spectra

Now we consider the emission spectrum of a large number of electrons accelerated in the electric field of the gap. From the intensity of a single particle (taking into account that in the electric field, linearly increasing from zero on the star surface, the acceleration maximum is proportional to the square root of $E_0$)

$$I(r) = \frac{2e^2w^2(r)}{3e^3} \approx \frac{e^3E_0(r)}{mch},$$

(8)

and changing the spectrum in the Fig.2 by the step-function, we turn to the spectral density for $\omega < \omega_{cf}(r)$:

$$I(r,\omega)d\omega \approx I(r) \frac{d\omega}{\omega_{cf}(r)},$$

(9)

where $\omega_{cf}(r) = \pi \sqrt{\frac{2eE_0(r)}{mch}}$.

Accordingly, the frequency range of radiation of the single particle has the form

$$\omega^2 \leq \omega_{cf}^2(r) = \frac{2\pi^2eE_{max}}{mch} \left( 1 - \frac{r^2}{R_{ipc}^2} \right),$$

(10)
therefore for the spectrum (in an incoherent case) we obtain

\[ I(\omega) \propto 2\pi \int_0^{b(\omega)} r dr I(r, \omega) N, \]

\[ b(\omega) = R_{PC} \sqrt{1 - \frac{\omega^2}{\omega_{cf}^2(0)}}, \]  

(11)

where \( N \) is the number of emitting particles measured through the current across the polar cap. Integration over the polar cap gives

\[ I(\omega) \propto \int_0^{b(\omega)} r dr \sqrt{1 - \frac{r^2}{R_{PC}^2}} \propto 1 - \frac{\omega^3}{\omega_{cf}^2(0)}, \]  

(12)

\[ \omega_{cf}(0) \approx \pi \sqrt{\frac{2eE_{\max}}{mh}}, \quad \omega \leq \omega_{cf}(0), \]

i.e. the spectrum is flat and has a cut off at the frequency \( \omega_{cf}(0) \).

Let us now consider the coherence of the emission [(Ginzburg et al. 1969), Ginzburg 1971, Kuzmin & Solov’ev 1986, Golgreich & Keeley 1971]. In the discussed mechanism only thin a disk with \( z < z_{cf} \) emits and the disk thickness is less then any wavelengths emitted. Therefore, in our case there is no need to raise additional assumptions about formation in the longitudinal direction coherently emitting electron bunches, which represents up to now a significant difficulty in all theories of pulsar radio emission. However, we must assume that in the disk plane a fragmentation takes place to coherently emitting regions, just as it is supposed in explaining the drift of subpulses (Lyne & Graham-Smith 1998). For this case we have to restrict the discussion only by assumptions. Dividing polocap on coherent areas, at this stage we must restrict the discussion only by assumptions. Dividng polocap on the regions of order lambda, we obtain, in coherent case, the lower estimates for intensity of radiation due to the minimum square of the number of particles in each region.

Obviously, the number of blocks equals to \( N_{block} \approx (R_{PC}/\lambda)^2 \), and the number of particles in the block is \( N_{coh} \approx \pi \lambda^2 n_c \bar{z} \). Then for radiation spectrum we have

\[ I(\omega) \propto 2\pi \int_0^{b(\omega)} r dr \frac{1}{\lambda^2} \sqrt{E_0(r)/\lambda^{3/2} z^2} \approx \]  

(15)

\[ \propto \frac{1}{\omega^2} \int_0^{b(\omega)} \frac{r dr}{\sqrt{1 - r^2/R_{PC}^2}} = \frac{1}{\omega^2} \left( 1 - \frac{\omega}{\omega_{cf}(0)} \right). \]

where \( b(\omega) \) is given by Eq. (11) and the frequency \( \omega_{cf}(0) \) is determined accordingly to Eq. (12). We have obtained a power-law spectrum with spectral index 2

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\[ \text{Fig. 4} \quad \text{Formation of average spectrum of radio emission.} \]

The dotted line shows the change of the longitudinal electric field with the distance from the magnetic axis to the boundary of pulsar polar cap.
(Fig.5), which is typical for the majority of pulsars. Let us note, that Eq.(12) and (15) are only the asymptotics of the exact spectra. In reality there is a low frequency break (or turnover) at the frequency $\omega_{tr} \approx 0.1\omega_{cf}$ (Malov 2004). Such spectrum behaviour can be obtained in the discussed model and we have considered in detail the physical meaning of that break and its position in the separate paper (Kontorovich, Flanchik 2013). Near the cutoff frequency the emission spectrum can not be considered strictly as a power law. For a small number of measured values it may be perceived as a kink. The spectral index depends on transverse dimensions of coherent blocks, which it seems also reveals themselves in the geometry of the subpulse drift.

The coherence provides a reasonable estimate of the intensity of radio emission. The rough estimate of the power of radio emission has the form

$$I_R \sim N_{\text{block}} N_{\text{coh}}^2 I_1, \quad I_1 \approx \frac{e^3 E_0}{mc^2}$$

where $I_1$ is the radiation power of a single particle at the maximum of acceleration. Here $N_{\text{block}} \approx (R_{PC}/\lambda_{max})^2$, where $\lambda_{max} \sim 10^2 \text{cm}$ is a wavelength corresponding to the maximum in the spectrum of radio emission. Writing an estimate for the number of particles in a coherent block in the form $N_{\text{coh}} \approx \pi \lambda_{max}^2 n_e$, where $n_e \sim n_{GJ} = \Omega B/(2\pi ce)$ is the average density of particles near the surface, we have $I_R \sim (\pi R_{PC} \lambda_{max}^2 n_{GJ})^2 I_1$, whence the estimate $I_R \sim \lambda_{max}^2 \Omega^3 R^2 B^2/c^2$ results. For the parameters $B = 4 \cdot 10^2 \text{G}$, $P = 1 \text{s}$ it leads to $I_R \approx 10^{29} \text{erg/s}$, which agrees well with the data on the radio luminosity of pulsars. For the fast rotating pulsar B0531+21 with high magnetic field this estimate reaches $10^{32} \text{erg/s}$ taking into account its proximity to the orthogonal rotator.

Note, that the assumption about change of radiation mechanisms in the high frequency cutoff region allows in principle to explain (Kontorovich & Flanchik 2011) the main pulse disappearance of PSR B0531+21 at frequencies near 8 GHz (Hankins & Eilek 2007). Really, from our point of view at lower frequencies the radiation at longitudinal acceleration of subrelativistic electrons gives the principal contribution to the main pulse. This emission vanishes at the cutoff frequency that may lead to disappearance of the pulse. The radiation due to low-frequency tail of narrow directed aberrational relativistic mechanism remains. This anisotropic radiation does not fall to the main pulse window but reveals itself in the interpulse if its line of sight is more close to the magnetic axis.

Note also that the discussed mechanism of radio emission (for which the place of radiation is definite and occupies a thin layer near the surface of the star) may use for checking the new methods to determine the generation location by a dispersion delay of the signal in the magnetosphere (Hassall, et al. 2012). The accuracy of such methods is limited now by the lack of the adequate knowledge of magnetosphere properties.

5 Conclusions

We have found above that the radiation of accelerating electrons in the electric field, slow increasing from zero on the star surface, entirely comes to the radio spectral band. This is the main result of the work.

The considered approach allows us explain the radio emission for the pulsars from Pushchino sample, including the position of the spectrum cutoff. As a result, we obtain a power-law spectrum, which arise at averaging over the polar cap due to dependence of the accelerating electric field on its position. The density radiation in the gap is large without any assumptions about the gap as a cavity (cf. Kontorovich 2009). It explains also the part of the gamma-ray emission from pulsars and its correlation (Bilous, et al. 2011) with giant pulses (Kontorovich & Flanchik 2008).

The radiation with the linear acceleration was considered in (Melrose et al. 2009a) for acceleration in a so strong electric field on the star surface, when an electron reaches relativistic velocities in a time shorter then the period of the wave emitted. In this case the considered effects are absent.

The obtained results make it also possible to compare the theories of accelerating fields in the inner gap (Harding & Muslimov 1998; Dyks & Rudak 2000).
(that is the foundation of all physics of pulsars) with observations, transforming them from a "thing in itself" that is not available to direct observations, in the "thing for us".

**Acknowledgements** We are grateful to our colleagues from RI NAS of Ukraine and to participants of PRAO-2011, JENAM-2011 and NS-2011 conferences for discussions, to M. Azbel’, O. Ulyanov and V. Usov for useful comments and to N. Kisilova, Y. Schukin and V. Tsvetkova for their help in translation of the text. The authors are extremely grateful to reviewer for a kindly and qualified critique of this work.
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