Constraining nuclear matter parameters and neutron star observables using PREX-2 and NICER data

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We try to constraints some of the nuclear matter parameters such as symmetry energy ($J$) and its slope ($L$) from the recent inferred data of the PREX-2. Other nuclear matter parameters are adopted from [Phys. Rev. C 85 035201 (2012), Phys. Rev. C 90 055203 (2014)] papers and the linear correlation among them are checked by using the Pearson’s formula. We find the correlation between $J – L$, $K_r – J$ and $K_r – L$ with coefficients 0.85, 0.81 and 0.76 respectively. The neutron star properties such as mass and radius are calculated with 50 unified equation of states. The results are consistent with recently observed pulsars and NICER data except few exceptions. From the radii constraints, we find that the new NICER data allows a narrow radius range contrary to a large range of PREX-2 and the old NICER data leaving us an inconclusive determination of the neutron star radius.

I. INTRODUCTION

The neutron star (NS), a highly dense and asymmetric nuclear system having a central density 5–6 times the nuclear saturation density [1]. It has a unique internal structure, where all the four fundamental forces play an essential role. Study of the NS reveals that the internal structure is more complicated because new degrees of freedom like hyperons [2–9] and quarks are in the core [10–12]. To explore its properties, such as mass, radius and tidal deformability, etc., one has to consider the interaction between nucleons in the form of interaction Lagrangian. This provides the equation of state (EOS), the main ingredient for the calculation of the NS properties.

Different formalism have been developed to calculate the EOSs of the NS. The relativistic mean-field (RMF) [13–19], Skyrme-Hartree-Fock (SHF) [20–27], density-dependent RMF (DD-RMF) [28], and point couplings [29] formalism are quite successful. First, we focus on the nuclear matter (NM) system, where the Coulomb and surface interactions are neglected. The binding energy per particle of the NM system is $\sim –16$ MeV at the saturation density $\rho_0 \sim 0.148$ fm$^{-3}$ [30]. The characteristics EOS of the NM is calculated by using different force interactions [26]. There are some empirical/experimental data to constraint the NM EOSs as given in Refs. [19, 31]. Different NM quantities such as incompressibility, symmetry energy and its slope parameter etc. play important role to explore the NS properties [26, 32–37]. In this study, our motivation is to constraint these NM parameters using different force interactions [38–40].

Recently, the updated Lead Radius Experiment (PREX) has given the neutron skin thickness of $^{208}$Pb as $R_{\text{skin}} = 0.283 \pm 0.071$ fm [38]. Based on this data Patnaik et al. [41] tuned the G3 and IOPB-I parameter sets. The impacts of PREX-2 data on the NM and NS properties have been explicitly studied in the Ref. [39]. The inferred values of NM quantities such as symmetry energy ($J$), its slope parameter ($L$) are $38.1 \pm 4.7$ MeV and $106 \pm 37$ MeV, respectively. The inferred limits are systematically larger as compared with either theoretical or experimental values [35, 42–52]. Reed et al. has also been calculated the NS properties by combining old NICER and PREX-2 constraints. The predicted radius range is $13.25 < R_{1.4} < 14.26$ km. Recently, NICER also put a revised limit, which is inferred by combining the old NICER data, $\sim 2 M_\odot$ pulsars, and the tidal deformability constraints form GWs data, and different EOSs modeling. The new NICER radius range for the canonical star is $12.45 \pm 0.65$ km. In this study, we want to constraint the radius of the canonical NS using both NICER and Reed et al. data.

The paper is organized as follows: The formalism for the calculation of different NM quantities is given in Sec. II. The results and discussions on the NM and NS properties are provided in Sec. III. A summary of our work is enumerated in the Sec. IV.

II. FORMALISM

The energy density $\mathcal{E}(\rho, \alpha)$ of the NM system can be expanded in a Taylor series in terms of asymmetry factor $\alpha (=\frac{\rho_n - \rho_p}{\rho_n + \rho_p})$ [19, 47]:

$$\mathcal{E}(\rho, \alpha) = \mathcal{E}(\rho) + S(\rho)\alpha^2 + \mathcal{O}(\alpha^4),$$

(1)

where $\rho$, $\rho_n$, and $\rho_p$ are the total baryon, neutron, and proton densities, respectively. The $\mathcal{E}(\rho)$ is the energy density of the symmetric NM. The density dependence symmetry energy ($S(\rho)$) can be written as

$$S(\rho) = \frac{1}{2} \left( \frac{\partial^2 \mathcal{E}}{\partial \alpha^2} \right)_{\alpha=0}.$$

(2)

The value of $S(\rho)$ is the most uncertain property of the NM, and it has a large diversion at a high-density limit [53]. Many progress have been made to constrain the $S(\rho)$ starting from heavy-ion collision to NS [31, 54]. Here, we can expand the
\[ S(\rho) \text{ in a leptodermous expansion near the saturation density as follow [19, 55–58]}:\]
\[
S(\rho) = J + L\xi + \frac{1}{2}K_{\text{sym}}\xi^2 + \frac{1}{6}Q_{\text{sym}}\xi^3 + O(\xi^4), \tag{3}
\]
where \( \xi = \frac{\rho - \rho_0}{\rho_0} \), \( J \) is the symmetry energy at saturation density \( \rho_0 \) and the other parameters like slope \( (L) \), curvature \( (K_{\text{sym}}) \) and skewness \( (Q_{\text{sym}}) \) are given as follow:
\[
L = 3\rho \left. \frac{\partial S(\rho)}{\partial \rho} \right|_{\rho=\rho_0}, \tag{4}
\]
\[
K_{\text{sym}} = 9\rho^2 \left. \frac{\partial^2 S(\rho)}{\partial \rho^2} \right|_{\rho=\rho_0}, \tag{5}
\]
\[
Q_{\text{sym}} = 27\rho^3 \left. \frac{\partial^3 S(\rho)}{\partial \rho^3} \right|_{\rho=\rho_0}. \tag{6}
\]

Similarly, one can expand the asymmetric NM incompressibility \( K(\alpha) \) as
\[
K(\alpha) = K + K_\tau \alpha^2 + O(\alpha^4), \tag{7}
\]
where \( K \) is the incompressibility of the NM at the saturation density and
\[
K_\tau = K_{\text{sym}} - 6L - \frac{Q_0 L}{K}, \tag{8}
\]
with \( Q_0 = 27\rho^3 \frac{\partial^3 \xi}{\partial \rho^3} \) in symmetric NM at saturation density.

We use another quantity \( K' = -Q_0 \).

### III. RESULTS AND DISCUSSIONS

#### 1. Nuclear Matter Properties

In this section, we constrain the values of \( J \) and \( L \). Moreover, with the addition of NM properties, we also want to constraint the mass and radius of the NS with the help of recent NICER data. To calculate NM properties we take 224 RMF, 240 SHF, 7 DD-RMF and 18 PC parameter sets from the Dutra et al. [26, 29], which span a large parameter space. For the calculation of NS properties, we take 50 well-known EOSs from the Refs. [59, 60].

In Fig. 1, we plot the symmetry energy \( (J) \) as a function of the slope parameter \( (L) \) for the considered parameter sets. Recently, the PREX-II experiments put a limit on the skin thickness of \( ^{208}\text{Pb} \) is \( R_{\text{skin}} = 0.283 \pm 0.071 \) fm [38]. There is strong correlation have been observed between \( R_{\text{skin}} \) and \( L \) at saturation density [39, 61]. Reed et al. [39] inferred the values of \( J \) and \( L \) as \( 38.1 \pm 4.7 \) MeV and \( 106 \pm 37 \) MeV respectively.

To check the correlation between different NM parameters as calculated in Sub-Sec. III 1, we plot the correlation matrix as shown in Fig. 2. To check the linear correlation between pairs of quantities, we calculate the correlation coefficient using Pearson’s formula as used in Ref. [60]. We find that the correlation coefficient between \( J \) and \( L \) is found to be 0.85. Slightly weaker correlations are found between \( K_{\tau} - J \) and \( K_{\tau} - L \) with coefficients 0.81 and 0.76, respectively.
A. Neutron Star Properties

The NS is composed of neutrons, protons, and leptons. Inside the NS, the neutron decays to proton, electron, and antineutrino. This process is called $\beta$-decay. Both $\beta$-equilibrium and charge neutrality processes are required for the stability of the NS [30]. Therefore, the total EOS of the NS is the addition of baryons and leptons as given as [19, 62]

$$E_{NS} = E + \bar{E}_l, \quad P_{NS} = P + P_l,$$

where $E$ ($\bar{E}_l$) is the energy density of the NM (leptons) as given in Refs. [62–64]. The $l$ correspond to both electron ($e^-\nu^-\bar{\nu}$) and muon ($\mu^-\nu^-\bar{\nu}$). In Fig. 3, we plot 50 selected unified EOS taken from the Refs. [59, 60] for comparison. The mass and radius of the NS are calculated by solving Tolman-Oppenheimer-Volkoff equations [65, 66] with boundary conditions $P(0) = P_c$ and $P(R) = 0$ for a fixed central density. Each EOS gives a different maximum mass and radius. We calculate the mass-radius ($M$–$R$) profile of the NS for considered EOSs as shown in Fig. 4. The massive pulsars data such as PSR J1614-2230 [67], PSR J0348+0432 [68] and PSR J0740+6620 [69] are shown with different colour bars. Recently, the secondary component of the GW190814 event is observed in the mass range 2.50–2.67 $M_\odot$. Almost all EOSs satisfy old NICER data, as clearly visible in the figure. The old NICER radius range is 11.52–14.26 km which provides a wider limit. New NICER radius range is 11.8–13.1 km, which is a narrow band compared to old NICER data. Reed et al. radius range is 13.25–14.26 km which satisfy by some RMF and few SHF EOSs. If one carefully observe this three radius limits, there is a large uncertainty to constraint the radius of the NS. Future observation may put a tight constraint on the radius of the NS.

IV. SUMMARY

In this manuscript, we calculate the NM properties with different formalisms such as RMF, SHF, DD-RMF, and PC. The symmetry energy and its slope are constrained by the recently inferred data from the PREX-2 experiment. We find that a numbers of RMF parameter sets, almost all PC parameter sets, and few SHF and DD-RMF parameter sets satisfy the constraints given by Reed et al. We also obtain some correlations between $J - L$, $K_\tau - J$ and $K_\tau - L$ with correlation coefficients 0.85, 0.81 and 0.76 respectively.

For the NS case, we take 50 unified EOSs and calculate the mass and radius. Almost all EOSs satisfy $2 M_\odot$ constraints. None of the considered parameter sets satisfy the GW190814.

FIG. 3. (color online) EOSs are shown for RMF, SHF and DD-RMF sets.

FIG. 4. (color online) $M$–$R$ are shown for RMF, SHF and DD-RMF sets with different observational constraints. Both old NICER data shown with two violet boxes [72, 73] and new NICER data with dark red doubled head lines are also depicted [40] for canonical star. Recently radius constraint given by Reed et al. [39] is also shown with black headed line.
data. The old NICER radius range is reproduced by almost all considered EOSs as it spans over a broader radius region. But the new NICER data is reproduced by almost all SHF and DD-RMF with few of the RMF sets. From the PREX-2 and old NICER data, Reed et al. inferred the radius of the canonical star, which is shown with a double-headed black line. Only stiff EOSs satisfy the Reed et al. data because the radius range is a little bit higher as compared to new NICER data.

In conclusion, the PREX-2 data supports the EOSs, which are stiffer. Therefore, all the NM and NS properties inferred from PREX-2 data provide large value symmetry energy coefficient as compared to other theoretical/experimental data. In the case of the NS, the implied radius range is also pushed towards a higher value, which can only be supported by stiffer EOSs. But new NICER data provides the radius range, which supports by the softer EOSs. Therefore, a possible uncertainty has been developed to constraints the radius of the NS. We hope future experiments/observations may answer this question.

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