The possibility of short-baseline neutrino oscillations due to the existence of one or more sterile neutrinos at the eV scale is a hot topic in current neutrino physics (see [1–4]). Besides the intrinsic interest in determining the existence of new phenomena and particles, the existence and properties of sterile neutrinos and active-sterile mixing could shed light on the physics beyond the Standard Model (see [5, 6]). The existence of light sterile neutrinos is also very important for astrophysics (see [7]) and cosmology (see [8, 9]), and the recent first Planck results [10] have generated interesting studies on the implications of cosmological data for light sterile neutrinos [11–17].

In this paper we extend the analysis of short-baseline electron neutrino and antineutrino disappearance data presented in Ref. [13] by taking into account also the more controversial indication of the LSND [19] experiment in favor of short-baseline $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ transitions and the recent ambiguous results of the MiniBooNE [20] experiment. We consider 3+1 and 3+2 extensions of the Standard Model in which there are, respectively, one or two sterile neutrinos at the eV scale [4] which generate short-baseline oscillations [18, 21, 39]. In the 3+1 scheme electron and muon neutrino and antineutrino appearance and disappearance in short-baseline experiments depend on:

A) One neutrino squared-mass difference, $\Delta m^2_{41} = m^2_4 - m^2_1$, where $m_k$ is the mass of the massive neutrino $\nu_k$ and $\Delta m^2_{21} \ll \Delta m^2_{31} \ll \Delta m^2_{41} \sim 1$ eV$^2$ in order to accommodate the observed oscillations of solar, reactor, atmospheric and accelerator neutrinos in the standard framework of three-neutrino mixing (see [40]). The probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions has the two-neutrino-like form

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 \left( \delta_{\alpha\beta} - |U_{\beta 4}|^2 \right) \sin^2 \left( \frac{\Delta m^2_{41} L}{4E} \right),$$

where $U$ is the mixing matrix, $L$ is the source-detector distance, and $E$ is the neutrino energy.

B) $|U_{e4}|^2$ and $|U_{\mu 4}|^2$, which determine the amplitude $\sin^2 2\theta_{e4} = 4|U_{e4}|^2|U_{\mu 4}|^2$ of $\nu_\mu \rightarrow \nu_e$ transitions, the amplitude $\sin^2 2\theta_{ee} = 4|U_{e4}|^2 (1 - |U_{e4}|^2)$ of $\nu_e$ disappearance, and the amplitude $\sin^2 2\theta_{\mu \mu} = 4|U_{\mu 4}|^2 (1 - |U_{\mu 4}|^2)$ of $\bar{\nu}_\mu$ disappearance.

Since the oscillation probabilities of neutrinos and antineutrinos are related by a complex conjugation of the elements of the mixing matrix (see [40]), the probabilities of short-baseline $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions in short-baseline experiments. In order to allow this possibility, one must consider a 3+2 scheme, in which, there are four additional effective mixing parameters in short-baseline experiments:
have added the new constraints on $\nu$ following three groups of experiments: 

1. The $\nu_\mu \rightarrow \nu_e$ appearance data of the LSND [19], MiniBooNE [20], BNL-E776 [41], KARMEN [42], NOMAD [43], ICARUS [44] and OPERA [45] experiments.

2. The $\nu_e$ disappearance data described in Ref. [18], which take into account the reactor [46, 48] and Gallium [49, 53] anomalies.

3. The constraints on $\nu_\mu$ disappearance obtained from the data of the CDHSW experiment [54] of the analysis [55] of the data of atmospheric neutrino oscillation experiments [3] from the analysis [56] of the MINOS neutral-current data [57] and from the analysis of the SciBooNE-MiniBooNE neutrino [58] and antineutrino [58] data.

With respect to the analysis presented in Ref. [33], we have added the new constraints on $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions obtained in the ICARUS [44] and OPERA [45] experiments. Following Ref. [39], we also added the constraints on $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ appearance obtained in the old BNL-E776 [41] experiment, which give a small contribution at $\Delta m^2$’s larger than about 2 eV$^2$, and the subleading effect of background disappearance in the analysis of MiniBooNE [20] data.

We consider the following three groups of experiments: 

- The $\nu_\mu \rightarrow \nu_e$ appearance data of the LSND [19], MiniBooNE [20], BNL-E776 [41], KARMEN [42], NOMAD [43], ICARUS [44] and OPERA [45] experiments.
- The $\nu_e$ disappearance data described in Ref. [18], which take into account the reactor [46, 48] and Gallium [49, 53] anomalies.
- The constraints on $\nu_\mu$ disappearance obtained from the data of the CDHSW experiment [54], from the analysis [55] of the data of atmospheric neutrino oscillation experiments [3] from the analysis [56] of the MINOS neutral-current data [57] and from the analysis of the SciBooNE-MiniBooNE neutrino [58] and antineutrino [58] data.

Table I shows the results of the 3+1-LOW fit of all the data above, including the three low-energy bins of the MiniBooNE experiment whose excess with respect to the background is widely considered to be anomalous because it is at odds with neutrino oscillations [33, 34]. We have considered also a 3+1-HIG fit of MiniBooNE data without the three anomalous low-energy bins. From Table I one can see that in both cases the oscillation fit of the data is much better than the no-oscillation fit, which has a disastrous p-value and is excluded in both cases at about 6$\sigma$. Although the best-fit values of the oscillation parameters are similar in the 3+1-LOW and 3+1-HIG fits, the goodness-of-fit of the 3+1-LOW case is significantly lower and the appearance-disappearance parameter goodness-of-fit is much lower. This result confirms the fact that the MiniBooNE low-energy anomaly is incompatible with neutrino oscillations, because it would require a small value of $\Delta m^2$ and a large value of $\sin^2 2\theta_{\mu\mu}$ [33, 34], which are excluded by the data of other experiments. Indeed, one can see from Fig. 1 that the best-fit 3+1-LOW averaged transition probability is far from fitting the three anomalous low-energy bins of MiniBooNE neutrino and antineutrino data. Therefore, we think that it is very likely that the MiniBooNE low-energy anomaly has an explanation which is different from neutrino oscillation [4] and the 3+1-HIG fit is more reliable than the 3+1-LOW fit. Moreover, the fact that both the global goodness-of-fit and the appearance-disappearance parameter goodness-of-fit are acceptable in the 3+1-HIG fit tells us that the fit is reliable. Hence, in Figs. 2 and 3 we present the allowed regions in the $\sin^2 2\theta_{\mu\mu} - \Delta m^2$, $\sin^2 2\theta_{ee} - \Delta m^2$, and $\sin^2 2\theta_{ee} - \Delta m^2$ planes, which are relevant, respectively, for $\nu_\mu \rightarrow \nu_e$ appearance, $\nu_e$ dis-

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2. We do not consider the IceCube data which could give a marginal contribution [56, 57], because the analysis is too complicated and subject to large uncertainties.

3. The interesting possibility of reconciling the low-energy anomalous data with neutrino oscillations through energy reconstruction effects proposed in [62, 63] still needs a detailed study.
appearance and (\nu_e^0) disappearance searches. One can see that the allowed region is well defined, with

\[ 0.82 < \Delta m_{41}^2 < 2.19 \text{eV}^2 \quad (3\sigma). \]  

Figure 2 shows also the region allowed by (\nu_\mu^0 \rightarrow (\nu_e^0 appearance data and the constraints on \sin^2 \theta_{\mu e}^\nu from (\nu_e^0 disappearance and (\nu_\mu^0 disappearance data. One can see that the combined disappearance constraint excludes a large part of the region allowed by (\nu_\mu^0 \rightarrow (\nu_e^0 appearance data, leading to the well-known appearance-disappearance tension [31, 36, 38, 39] quantified by the parameter goodness-of-fit in Tab. II. With respect to the results presented in Refs. [32, 34, 36, 38], the region at \Delta m_{41}^2 \approx 6 \text{eV}^2 which is allowed by (\nu_\mu^0 \rightarrow (\nu_e^0 appearance data is not allowed any more (at 3\sigma) by the global fit, mainly because of the old BNL-E776 data, which we included following the wise approach of Ref. [39]. This is consistent with the cosmological exclusion of this region.

It is interesting to investigate what is the impact of the MiniBooNE experiment towards the test of the LSND signal. With this aim we performed two additional 3+1 fits: a 3+1-noMB fit without MiniBooNE data and a 3+1-noLSND fit without LSND data. From Tab. II one can see that the results of the 3+1-noMB fit are similar to those of the 3+1-HIG fit and the exclusion of the case of no-oscillations remains at the level of 6\sigma. On the other hand, in the 3+1-noLSND fit the exclusion of the case of no-oscillations drops dramatically to \chi^2 \sim 2.1\sigma. Therefore, it

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & 3+2 & 3+2 & 3+1+1 & 3+1+1 \\
 & LOW & HIG & LOW & HIG \\
\hline
\chi^2_{\text{min}} & 284.4 & 256.4 & 289.8 & 259.0 \\
NDF & 252 & 246 & 253 & 247 \\
GoF & 8\% & 31\% & 6\% & 29\% \\
\Delta \chi^2 & 1.9 & 0.93 & 1.6 & 1.6 \\
\eta/\pi & 0.012 & 0.0097 & 0.011 & 0.012 \\
\Delta \chi^2_{\text{min}} & 4.1 & 1.6 & 0.013 & 0.018 \\
\eta/\pi & 0.0065 & 0.0091 & 0.0049 & 0.0052 \\
\hline
\end{tabular}
\caption{Results of the fit of short-baseline data taking into account all MiniBooNE data (LOW) and only the MiniBooNE data above 475 MeV (HIG) in the framework of 3+2 and 3+1+1 neutrino mixing. The notation is similar to that in Tab. I. The last two lines give the p-value (p-val) and the corresponding number of excluding \sigma’s (n\sigma) of the 3+1 scheme.}
\end{table}

4 This is due to the fact that without LSND the main indica-
is evident that the LSND experiment is still crucial for the indication in favor of short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions and the MiniBooNE experiment has been rather inconclusive.

Let us consider now the fit of short-baseline data in the framework of 3+2 mixing, which was considered to be interesting in 2010 when the MiniBooNE neutrino and antineutrino data showed a CP-violating tension. Unfortunately, this tension reduced considerably in the final MiniBooNE data and from Tab.\textsuperscript{11} one can see that there is little improvement of the 3+2 fit with respect to the 3+1 case, in spite of the four additional parameters and the additional possibility of CP violation. First, from Fig.\textsuperscript{11} one can see that the 3+2-LOW fit is as bad as the 3+1-LOW fit in fitting the three anomalous MiniBooNE low-energy bins.\textsuperscript{5} Moreover, comparing Tabs.\textsuperscript{11} and\textsuperscript{11} one can see that the appearance-disappearance tension in the 3+2-LOW fit is even worse than that in the 3+1-LOW fit, since the $\Delta \chi^2_{PG}$ is so much larger that it cannot be compensated by the additional degrees of freedom (this behavior has been explained in Ref.\textsuperscript{38}). Hence, as in the 3+1 case it is wise to neglect the three low-energy MiniBooNE anomalous bins and consider as more reliable the 3+2-HIG fit, which has an acceptable appearance-disappearance parameter goodness-of-fit. However, one must ask if considering the larger complexity of the 3+2 scheme is justified by the data. The answer is negative because, as one can see from Tab.\textsuperscript{11} the value of the $p$-value obtained by restricting the 3+2 scheme to 3+1 disfavors the 3+1 scheme only at 1.2$r$ in the 3+2-HIG fit.

\textsuperscript{5} One could fit the three anomalous MiniBooNE low-energy bins in a 3+2 scheme by considering the appearance data without the ICARUS and OPERA constraints, but the corresponding relatively large transition probabilities are excluded by the disappearance data.

\textsuperscript{6} See however the somewhat different conclusions reached in Ref.\textsuperscript{38}. 

\textsuperscript{4} See however the somewhat different conclusions reached in Ref.\textsuperscript{38}.
A puzzling feature of the 3+2 scheme is that it needs the existence of two sterile neutrinos with masses at the eV scale. We think that it may be considered as more plausible that sterile neutrinos have a hierarchy of masses. Hence, we considered also the 3+1+1 scheme in which $m_{5}$ is much heavier than 1 eV and the oscillations due to $\Delta m_{51}^{2}$ are averaged. Hence, in the analysis of short-baseline data the 3+1+1 scheme has one effective parameter less than the 3+2 scheme. The results of the 3+1+1-LOW and 3+1+1-HIG fits presented in Tab. II show that the 3+1+1-LOW is as bad as the 3+1-LOW and 3+2-LOW fits (see also the bad fit of the three low-energy MiniBooNE anomalous bins in Fig. I). On the other hand, the 3+1+1-HIG appearance-disappearance parameter goodness-of-fit is remarkably good, with a $\Delta \chi^{2}_{FG}$ that is smaller than those in the 3+1-HIG and 3+2-HIG fits. However, the $\chi^{2}_{\text{min}}$ in the 3+1+1-HIG is only slightly smaller than that in the 3+1-HIG fit and the high p-value of the 3+1 scheme does not allow us to prefer the more complex 3+1+1.

In conclusion, we have presented the results of the global analysis of all the available data of short-baseline neutrino oscillation experiments in the framework of 3+1, 3+2 and 3+1+1 neutrino mixing schemes. We have shown that the data do not allow us to reject the simplest 3+1 scheme in favor of the more complex 3+2 and 3+1+1 schemes. We have also shown that the low-energy MiniBooNE anomaly cannot be explained by neutrino oscillations in any of these schemes. Considering the preferred 3+1 scheme, we have updated the constraints on the oscillation parameters and we have shown that there is only one allowed region in the parameter space around $\Delta m_{31}^{2} \approx 1-2eV^{2}$. We have also shown that the crucial indication in favor of short-baseline $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ appearance is still given by the old LSND data and the MiniBooNE experiment has been inconclusive. Hence new better experiments are needed in order to check this signal. Let us finally emphasize that, besides the direct observation of short-baseline $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ transitions, it is crucial to observe also short-baseline $\bar{\nu}_{e}$ and $\nu_{\mu}$ disappearance. Since the reactor and Gallium anomalies indicate that $\bar{\nu}_{e}$ indeed disappear, it is important to search also for the disappearance of $\nu_{\mu}$.

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