Towards Precise Determinations of the CKM
Matrix without Hadronic Uncertainties*

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Abstract
We illustrate how the measurements of the CP asymmetries in $B_0^{d,s}$-decays together with a
measurement of $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ or $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ and the known value of $|V_{us}|$ can
determine all elements of the Cabibbo-Kobayashi-Maskawa matrix essentially without any hadronic
uncertainties. An analysis using the ratio $x_d/x_s$ of $B_d - \bar{B}_d$ to $B_s - \bar{B}_s$ mixings is also presented.

1. Setting the Scene
An important target of particle physics is the
determination of the unitary $3 \times 3$ Cabibbo-Kobayashi-
Maskawa matrix which parametrizes the charged
current interactions of quarks:

$$J^{cc}_{\mu} = (\bar{u}, \bar{c}, \bar{t}) L \gamma_{\mu} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

(1)

It is customary these days to parametrize these matrix
by the four Wolfenstein parameters ($\lambda, A, \varrho, \eta$). In
particular one has

$$|V_{us}| = \lambda \quad |V_{cb}| = A \lambda^2$$

(2)

and

$$V_{ub} = A \lambda^3 (\varrho - i \eta) \quad V_{td} = A \lambda^3 (1 - \varrho - i \eta)$$

(3)

Here following [1] we have introduced

$$\varrho = \varrho (1 - \lambda^2/2) \quad \eta = \eta (1 - \lambda^2/2).$$

(4)

which allows to improve the accuracy of the Wolfenstein
parametrization.

From tree level K decays sensitive to $V_{us}$ and tree
level B decays sensitive to $V_{cb}$ and $V_{ub}$ we have:

$$\lambda = 0.2205 \pm 0.0018 \quad |V_{ub}| = 0.039 \pm 0.004$$

(5)

$$R_b \equiv \sqrt{\varrho^2 + \eta^2} = (1 - \lambda^2/2) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.36 \pm 0.14$$

(6)

corresponding to

$$\left| \frac{V_{ub}}{V_{cb}} \right| = 0.08 \pm 0.03$$

(7)
$R_b$ is just the length of one side of the rescaled unitarity triangle in which the length of the side on the $\bar{q}$ axis is equal unity. The length of the third side is governed by $| V_{td} |$ and is given by

$$R_t \equiv \sqrt{(1 - \bar{q})^2 + \bar{q}^2} = \frac{1}{\lambda} | V_{td} | V_{cb}$$  \hspace{1cm} (8)$$

In order to find $R_t$ one has to go beyond tree level decays.

As we have seen at this conference a large part in the errors quoted in (6), (7) results from theoretical (hadronic) uncertainties. Consequently even if the data from CLEO II improves in the future, it is difficult to imagine at present that in the tree level $B$-decays a better accuracy than $\Delta | V_{cb} | = \pm 2 \cdot 10^{-3}$ and $\Delta | V_{cb}/V_{cb} | = \pm 0.01 (\Delta R_t = \pm 0.04)$ could be achieved unless some dramatic improvements in the theory will take place.

The question then arises whether it is possible at all to determine the CKM parameters without any theoretical (hadronic) uncertainties. Examples are $B^0 - \bar{B}^0$ mixing, $x_K$ and $\varepsilon'/\varepsilon$. Let us in this connection recall the expectations from a "standard" analysis of the unitarity triangle which is based on $\varepsilon_K$, $x_d$ giving the size of $B^0 - \bar{B}^0$ mixing, $| V_{cb} |$ and $| V_{td}/V_{cb} |$ with the last two extracted from tree level decays. As a recent analysis [7] shows, even with optimistic assumptions about the theoretical and experimental errors it will be difficult to achieve the accuracy better than $\Delta \theta = \pm 0.15$ and $\Delta \eta = \pm 0.05$ this way. Therefore in what follows we will only discuss the four finalists in the field of weak decays which essentially are free of hadronic uncertainties.

2. Finalists

2.1. CP-Asymmetries in $B^0$-Decays

The CP-asymmetry in the decay $B_d^0 \rightarrow \psi K_S$ allows in the standard model a direct measurement of the angle $\beta$ in the unitarity triangle without any theoretical uncertainties [3]. Similarly the decay $B^0 \rightarrow \pi^+ \pi^-$ gives the angle $\alpha$, although in this case strategies involving other channels are necessary in order to remove hadronic uncertainties related to penguin contributions [3]. The determination of the angle $\gamma$ from CP asymmetries in neutral B-decays is more difficult but not impossible [3].

Also charged B decays could be useful in this respect [3]. We have for instance

$$A_{CP}(\psi K_S) = - \sin(2\beta) \frac{x_d}{1 + x_d^2},$$

$$A_{CP}(\pi^+ \pi^-) = - \sin(2\alpha) \frac{x_d}{1 + x_d^2},$$

where we have neglected QCD penguins in $A_{CP}(\pi^+ \pi^-)$. Since in the usual unitarity triangle one side is known, it suffices to measure two angles to determine the triangle completely. This means that the measurements of $\sin 2\alpha$ and $\sin 2\beta$ can determine the parameters $\theta$ and $\eta$. The main virtues of this determination are as follows:

- No hadronic or $\Lambda_{MS}$ uncertainties.
- No dependence on $m_t$ and $V_{cb}$ (or A).

2.2. $K_L \rightarrow \pi^0 \nu \bar{\nu}$

$K_L \rightarrow \pi^0 \nu \bar{\nu}$ is the theoretically cleanest decay in the field of rare K-decays. $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is dominated by short distance loop diagrams involving the top quark and proceeds almost entirely through direct CP violation. The last year calculations [6,8,10] of next-to-leading QCD corrections to this decay considerably reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression [1]. Typically the uncertainty in $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is of $\pm 10\%$ in the leading order is reduced to $\pm 1\%$. Since the relevant hadronic matrix elements of the weak current $\bar{s} \gamma_\mu (1 - \gamma_5) d$ can be measured in the leading decay $K^+ \rightarrow \pi^0 e^+ \nu_e$, the resulting theoretical expression for $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is only a function of the CKM parameters, the QCD scale $\Lambda_{MS}$ and $m_t$. The long distance contributions to $K_L \rightarrow \pi^+ \nu \bar{\nu}$ are negligible. We have then:

$$Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 1.50 \cdot 10^{-5} \eta^2 \left| V_{cb} \right|^4 x_t^{1.15}$$

where $x_t = m_t^2/M_W^2$ with $m_t \equiv m_t(m_t)$. The main features of this decay are:

- No hadronic uncertainties
- $\Lambda_{MS}$ and renormalization scale uncertainties at most $\pm 1\%$.
- Strong dependence on $m_t$ and $V_{cb}$ (or A).

2.3. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is CP conserving and receives contributions from both internal top and charm exchanges. The last year calculations [6,8,10] of next-to-leading QCD corrections to this decay considerably reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression [1]. Typically the uncertainty in $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is of $\pm 20\%$.
in the leading order is reduced to $\pm 5\%$. The long distance contributions to $K^+ \to \pi^+ \nu \bar{\nu}$ have been considered in [13] and found to be very small: two to three orders of magnitude smaller than the short distance contribution at the level of the branching ratio. $K^+ \to \pi^+ \nu \bar{\nu}$ is then the second best decay in the field of rare decays. Compared to $K_L \to \pi^0 \nu \bar{\nu}$ it receives additional uncertainties due to $m_t$ and the related renormalization scale. Also its QCD scale dependence is stronger. Explicit expressions can be found in [10, 12]. The main features of this decay are:

- Hadronic uncertainties below 1%
- $\Lambda_{\overline{MS}}$, $m_t$ and renormalization scales uncertainties at most $(5 - 10)\%$.
- Strong dependence on $m_t$ and $V_{cb}$ (or $A$).

2.4. $B^o - \overline{B^o}$ Mixing

Measurement of $B^o_d - \overline{B^o_d}$ mixing parametrized by $x_d$ together with $B^o_s - \overline{B^o_s}$ mixing parametrized by $x_s$ allows to determine $R_t$:

$$R_t = \frac{1}{\sqrt{R_{ds}}} \frac{1}{x_s \lambda} \tag{12}$$

with $R_{ds}$ summarizing SU(3)-flavour breaking effects. Note that $m_t$ and $V_{cb}$ dependences have been eliminated this way and $R_{ds}$ contains much smaller theoretical uncertainties than the hadronic matrix elements in $x_d$ and $x_s$ separately. Provided $x_d/x_s$ has been accurately measured a determination of $R_t$ within $\pm 10\%$ should be possible. The main features of $x_d/x_s$ are:

- No $\Lambda_{\overline{MS}}$, $m_t$ and $V_{cb}$ dependence.
- Hadronic uncertainty in SU(3)-flavour breaking effects of roughly $\pm 10\%$.

Because of the last feature, $x_d/x_s$ cannot fully compete in the clean determination of CKM parameters with CP asymmetries in B-decays and with $K_L \to \pi^0 \nu \bar{\nu}$. Although $K^+ \to \pi^+ \nu \bar{\nu}$ has smaller hadronic uncertainties than $x_d/x_s$, its dependence on $\Lambda_{\overline{MS}}$ and $m_c$ puts it in the same class as $x_d/x_s$.

3. $\sin(2\beta)$ from $K \to \pi \nu \bar{\nu}$

It has been pointed out in [13] that measurements of $Br(K^+ \to \pi^+ \nu \bar{\nu})$ and $Br(K_L \to \pi^0 \nu \bar{\nu})$ could determine the unitarity triangle completely provided $m_t$ and $V_{cb}$ are known. In view of the strong dependence of these branching ratios on $m_t$ and $V_{cb}$ this determination is not precise however [13]. On the other hand it has been noticed recently [2] that the $m_t$ and $V_{cb}$ dependences drop out in the evaluation of $\sin(2\beta)$. Introducing the "reduced" branching ratios

$$B_+ = \frac{Br(K^+ \to \pi^+ \nu \bar{\nu})}{4.64 \cdot 10^{-11}} \quad B_L = \frac{Br(K_L \to \pi^0 \nu \bar{\nu})}{1.94 \cdot 10^{-10}} \tag{13}$$

one finds

$$\sin(2\beta) = \frac{2r_s(B_+, B_L)}{1 + r_s^2(B_+, B_L)} \tag{14}$$

where

$$r_s(B_+, B_L) = \sqrt{(B_+ - B_L) - P_0(K^+)} \quad \tag{15}$$

so that $\sin(2\beta)$ does not depend on $m_t$ and $V_{cb}$. Here $P_0(K^+) = 0.40 \pm 0.09$ [14] is a function of $m_c$ and $\Lambda_{\overline{MS}}$ and includes the residual uncertainty due to the renormalization scale $\mu$. Consequently $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ offer a clean determination of $\sin(2\beta)$ which can be confronted with the one possible in $B^o \to \psi K_S$ talked above. Any difference in these two determinations would signal new physics. Choosing $Br(K^+ \to \pi^+ \nu \bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$ and $Br(K_L \to \pi^0 \nu \bar{\nu}) = (2.5 \pm 0.25) \cdot 10^{-11}$, one finds [12]

$$\sin(2\beta) = 0.60 \pm 0.06 \pm 0.03 \pm 0.02 \tag{16}$$

where the first error is "experimental", the second represents the uncertainty in $m_c$ and $\Lambda_{\overline{MS}}$ and the last is due to the residual renormalization scale uncertainties. This determination of $\sin(2\beta)$ is competitive with the one expected at the B-factories at the beginning of the next decade.

4. Precise Determinations of the CKM Matrix

Using the first two finalists and $\lambda = 0.2205 \pm 0.0018$ [14] it is possible to determine all the parameters of the CKM matrix without any hadronic uncertainties [13]. With $a = \sin(2\alpha)$, $b = \sin(2\beta)$ and $Br(K_L) = Br(K_L \to \pi^0 \nu \bar{\nu})$ one determines $g$, $\eta$ and $|V_{cb}|$ as follows [15]:

$$\bar{\eta} = 1 - \eta \bar{r}_+(ar{b}) \quad \eta = \frac{r_-(a) + r_+(b)}{1 + r_0^2(b)} \tag{17}$$

$$|V_{cb}| = 0.039 \sqrt{\frac{0.39}{\eta}} \left[ \frac{170 \text{ GeV}}{m_t} \right]^{0.575} \left[ \frac{Br(K_L)}{3 \cdot 10^{-11}} \right]^{1/4} \tag{18}$$

where

$$r_\pm(z) = \frac{1}{z}(1 \pm \sqrt{1 - z^2}) \quad z = a, b \tag{19}$$

We note that the weak dependence of $|V_{cb}|$ on $Br(K_L \to \pi^0 \nu \bar{\nu})$ allows to achieve high accuracy for this CKM element even when $Br(K_L \to \pi^0 \nu \bar{\nu})$ is not measured precisely.

As illustrative examples we consider in table 1 three scenarios. The first four rows give the assumed
The result of this exercise is shown in table 2. We observe that even with rather optimistic assumptions on the accuracy of \( R_t \), this determination of CKM parameters cannot fully compete with the previous one. Again the last two rows give the results when \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) is replaced by \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \).

5. Final Remarks

- Precise measurements of all CKM parameters without hadronic uncertainties are possible.
- Such measurements are essential for the tests of the standard model. Of particular interest will be the comparison of \( |V_{cb}| \) determined as suggested here with the value of this CKM element extracted from tree level semi-leptonic B-decays. Since in contrast to \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) and \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \), the tree-level decays are to an excellent approximation insensitive to any new physics contributions from very high energy scales, the comparison of these two determinations of \( |V_{cb}| \) would be a good test of the standard model and of a possible physics beyond it.

Precise determinations of all CKM parameters without hadronic uncertainties along the lines presented here can only be realized if the measurements of CP asymmetries in B-decays and the measurements of \( Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) \), \( Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \) and \( x_d/x_s \) can reach the desired accuracy. All efforts should be made to achieve this goal.

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