THE DECAYS $\nu_H \rightarrow \nu_L \gamma$ AND $\nu_H \rightarrow \nu_L e^+ e^-$ OF MASSIVE NEUTRINOS

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Abstract: If, as recently reported by the Super-Kamiokande collaboration, the neutrinos are massive, the heaviest one $\nu_H$ would not be stable and, though chargeless, could in particular decay into a lighter neutrino $\nu_L$ and a photon by quantum loop effects. The corresponding rate is computed in the standard model with massive Dirac neutrinos as a function of the neutrino masses and mixing angles. The lifetime of the decaying neutrino is estimated to be $\approx 10^{44}$ years for a mass $\approx 5 \times 10^{-2}$ eV.

If kinematically possible, the $\nu_H \rightarrow \nu_L e^+ e^-$ mode occurs at tree level and its one-loop radiative corrections get enhanced by a large logarithm of the electron mass acting as an infrared cutoff. Thus the $\nu_H \rightarrow \nu_L e^+ e^-$ decay largely dominates the $\nu_H \rightarrow \nu_L \gamma$ one by several orders of magnitude, corresponding to a lifetime $\approx 10^{-2}$ year for a mass $\approx 1.1$ MeV.

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1 Introduction

Evidence for the transmutation between the two neutrino species $\nu_\mu \leftrightarrow \nu_\tau$ has been recently reported by the Super-Kamiokande collaboration[1]. As a consequence, neutrinos could have non-degenerate tiny masses, and mixing among different lepton families becomes likely, in analogy with the Cabibbo-Kobayashi-Maskawa flavor mixing in the quark sector [2].

We assume that the neutrino “flavor” eigenstates $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are linear combinations of the three neutrino mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$ of nonzero and non-degenerate masses $m_1$, $m_2$ and $m_3$ respectively according to

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix} \equiv U_{lep} \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
\]

(1)

where the $3 \times 3$ matrix $U_{lep}$ is unitary.

The effective weak interactions of the leptons can now be written as

\[
\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} L_\lambda^\dagger L_\lambda,
\]

(2)

where the charged current $L_\lambda$ is

\[
L_\lambda = \sum_\ell \sum_{i=1}^3 U_{\ell i} \bar{\nu}_i \gamma_\lambda (1 - \gamma_5) \ell.
\]

(3)

Here $\ell$ stands for $e^-$, $\mu^-$, $\tau^-$ and $\nu_i$ (with $i = 1, 2, 3$) are the three neutrino mass eigenstates.

Although the neutrinos are chargeless, a heavy neutrino $\nu_H$ can decay into a lighter neutrino $\nu_L$ by emitting a photon; this decay is entirely due to quantum loop effects. Now, if kinematically possible, the mode $\nu_H \rightarrow \nu_L e^+ e^-$ largely dominates, because it is governed by a tree diagram and its radiative corrections get enhanced, as we will see, by a large logarithm.

Neutrino oscillation measurements provide constraints usually plotted in the $(\sin^2 2\theta_{12}, \Delta m_{21}^2 = |m_2^2 - m_1^2|)$ plane, where $\theta_{12}$ is one of the three Euler angles of the rotation matrix $U_{lep}$.

For practical purposes, we shall assume for $U_{lep}$ the following form [3]:

\[
U_{lep} = \begin{pmatrix}
\cos \theta_{12} & -\sin \theta_{12} & 0 \\
\frac{1}{\sqrt{2}} \sin \theta_{12} & \frac{1}{\sqrt{2}} \cos \theta_{12} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \sin \theta_{12} & \frac{1}{\sqrt{2}} \cos \theta_{12} & \frac{1}{\sqrt{2}}
\end{pmatrix};
\]

(4)

$\theta_{23} \approx 45^0$ is suggested by the Super-Kamiokande data and $\theta_{13} \approx 0^0$ comes from the CHOOZ data [1, 4] which give $\theta_{13} \leq 13^0$, and also from the Bugey experiment [4], whereas $\theta_{12}$ is arbitrary. Although $\theta_{12}$ is likely small $\approx 0^0$, the maximal mixing $\theta_{12} \approx 45^0$ may also be possible allowing $\nu_e \leftrightarrow \nu_\mu$ (as suggested by the LSND experiment [1, 5, 6]).

2 The decay $\nu_H \rightarrow \nu_L \gamma$

The first calculations of radiative neutrino decays have been reported in [5] and [6].

In the most general renormalizable gauge (conventionally called $R_\xi$), six Feynman diagrams contribute to the process $\nu_H(p) \rightarrow \nu_L(p) \gamma(q)$, where the photon can be real ($q^2 = 0$) or virtual ($q^2 \neq 0$);
the latter is necessary when we consider the one-loop radiative corrections to $\nu_H \to \nu_L e^+ e^-$. They can be grouped into two sets: four in Figs. 1a-d:

![Fig. 1a](image1a.png)

![Fig. 1b](image1b.png)

![Fig. 1c](image1c.png)

![Fig. 1d](image1d.png)

and two in Figs. 2a-b:

![Fig. 2a](image2a.png)

![Fig. 2b](image2b.png)

Each one is gauge-dependent but it turns out that the $\xi$ dependence cancels out for each group of diagrams separately, yielding the overall gauge independence of the physical process.

We shall give the results in the 't Hooft-Feynman gauge $\xi = 1$.

For each diagram, the corresponding amplitude $A$ is written in terms of the effective vertex $\Gamma_\mu$

$$i A_{\nu_H \to \nu_L \gamma} = (-i e) \left( \frac{ig}{2\sqrt{2}} \right)^2 \sum_\ell U_{L\ell} U_{H\ell}^* \bar{u}(p) \Gamma_\mu(\ell) u(P) \varepsilon^*_\mu(q),$$

(5)
where the $u$’s are the (Dirac) spinors of the two neutrinos, $\varepsilon_\mu$ is the photon polarization, $e$ the charge of the electron and $g$ the $SU(2)_L$ coupling constant. One has $G_F/\sqrt{2} = g^2/8M_W^2$.

The ultraviolet divergences are handled via the procedure of dimensional regularization, going to $n = 4-\epsilon$ dimensions.

The mass $m$ of the lightest (outgoing) neutrino is always neglected, such that the results depend on the mass $M$ of the incoming neutrino, the mass $M_W$ of the $W$ gauge boson, and the masses $m_\ell$ of the internal fermions, which will always appear in the dimensionless ratio

$$r_\ell = \frac{m_\ell^2}{M_W^2}. \quad (6)$$

After expressing the amplitude for each diagram in terms of two-dimensional parametric integrals, we restrict ourselves in this section to the case of a real outgoing photon, for which, due to $q^\mu\varepsilon^\mu = 0$ and to the conservation of the electromagnetic current, only the magnetic form factor proportional to $i\sigma_{\mu\nu}q^\nu$ in the effective vertex contributes (see for example [7][8]). The integration over the Feynman parameters is made simpler by neglecting $M^2/M_W^2$ in the denominators.

### 2.1 Computation of the six diagrams

- **Diagram 1a** The corresponding effective vertex $\Gamma_{1a}^\mu$ writes

$$\Gamma_{1a}^\mu = \frac{1}{8\pi^2}(1 + \gamma_5) \int_0^1 dx \int_0^{1-x} dy \frac{N_{1a}^\mu}{D_1(\ell)} \quad (7)$$

where

$$D_1(\ell) = M_W^2[(1 - x) + r_\ell x] - M^2xy - q^2y(1 - x - y) \quad (8)$$

and ($\gamma$ is the Euler constant $\gamma \approx 0.577$)

$$N_{1a}^\mu = \left\{ \left[ 2(1 - x)(1 - y) + y]M^2 - 2[(1 - x)(1 - y) + y^2] q^2 \right. \\
+ 6D_1(\ell) \left[ \frac{2}{\epsilon} + \ln 4\pi - \gamma - \frac{1}{2} - \ln \frac{D_1(\ell)}{\Lambda^2} \right] \right\} \gamma^\mu \\
+ 2M \left\{ y(1 - 2y)P^\mu + [2y^2 - (1 - x)(1 + 2y)]p^\mu \right\}. \quad (9)$$

In (9) and in the rest of the paper, $\Lambda$ is an arbitrary scale coming from the dimensional regularization.

By translational invariance, $\Gamma^\mu$ depends only on the four-momentum transfer $q^\mu$ and not on $P^\mu$; the latter may be projected onto the basis formed by three independent four-vectors $i\sigma_{\mu\nu}q_\nu$, $q^\mu$, and $\gamma^\mu$, using the following relation valid for $m = 0$:

$$2\not\!\!\!\!\!\!p(1 + \gamma_5)P^\mu u(P) = \not\!\!\!\!\!\!p(1 + \gamma_5)(i\sigma_{\mu\nu}q_\nu + M\gamma^\mu + q^\mu) u(P). \quad (10)$$

This yields

$$N_{1a}^\mu = iM\sigma_{\mu\nu}q_\nu[x - 1 - y(1 - 2x)] + \not\!\!\!\!\!\!q^\mu[1 - x + 3y - 2xy - 4y^2] \\
+ \left\{ M^2[1 - x - 2y(1 - 2x)] - 2q^2[y^2 + (1 - x)(1 - y)] \right. \\
+ 6D_1(\ell) \left[ \frac{2}{\epsilon} + \ln 4\pi - \gamma - \frac{1}{2} - \ln \frac{D_1(\ell)}{\Lambda^2} \right] \right\} \gamma^\mu. \quad (11)$$
The $x, y$ integrations for the pure magnetic term yield the contribution of diagram 1a to the decay amplitude $\nu_H \to \nu_L \gamma$

$$A_{1a} = A_0 \sum_\ell U_{H\ell} U_{L\ell}^* F_{1a}(\ell),$$

(12)

where

$$A_0 = \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} \pi(p) M(1 + \gamma_5)i\sigma^{\mu\nu} q_\nu u(P) \epsilon^*_\mu(q).$$

(13)

One gets

$$F_{1a}(\ell) = \frac{r_\ell^2 (1 - 3r_\ell) \ln r_\ell}{2(r_\ell - 1)^4} + r_\ell \left[ \frac{7}{12(r_\ell - 1)} + \frac{2}{(r_\ell - 1)^2} + \frac{1}{(r_\ell - 1)^3} \right] - \frac{7}{12}. $$

(14)

The singularities of $F_{1a}(\ell)$ at $r_\ell = 1$ are fake: $F_{1a}(\ell) = -5/12$ for $r_\ell = 1$.

Formula (14) is in agreement with similar calculations [7] for $\mu^- \to e^- \gamma$ at the limit $r_\ell \to 0$, where only the linear term in $r_\ell$ was kept and the logarithmic term neglected.

If $m$ were not neglected, the $M(1 + \gamma_5)$ term in (13) would be simply replaced by $M(1 + \gamma_5) + m(1 - \gamma_5)$. If we keep $M^2xy$ in $D_1(\ell)$, we will still obtain explicit analytic forms for the $F$’s but the results will be complicated and not illuminating.

- **Diagram 1b**

Writing in a similar way

$$\Gamma_{1b}^\mu(\ell) = \frac{1}{8\pi^2} (1 + \gamma_5) \int_0^1 dx \int_0^{1-x} dy \frac{N_{1b}}{D_1(\ell)},$$

(15)

one finds

$$N_{1b}^\mu = r_\ell \left\{ M(1 - y) [(2y - 1) P^\mu + (1 - 2x - 2y) P^\mu] + D_1(\ell) \left[ \frac{2}{\epsilon} + \ln 4\pi - \gamma - \frac{1}{2} - \ln \frac{D_1(\ell)}{\Lambda^2} \right] \gamma^\mu \right\}. $$

(16)

The use of (10) transforms the above expression into

$$N_{1b}^\mu = r_\ell \left\{ i M\sigma^{\mu\nu} q_\nu x(y - 1) + g q^\mu (y - 1)(1 - x - 2y) + \left( M^2 x(y - 1) + D_1(\ell) \left[ \frac{2}{\epsilon} + \ln 4\pi - \gamma - \frac{1}{2} - \ln \frac{D_1(\ell)}{\Lambda^2} \right] \right) \gamma^\mu \right\}, $$

(17)

and, after performing the parametric integration of the purely magnetic term one obtains

$$F_{1b}(\ell) = \frac{r_\ell^2 (r_\ell - 2) \ln r_\ell}{2(r_\ell - 1)^4} + r_\ell \left[ -\frac{1}{3(r_\ell - 1)} - \frac{1}{4(r_\ell - 1)^2} + \frac{1}{2(r_\ell - 1)^3} \right]. $$

(18)

The singularities of $F_{1b}(\ell)$ at $r_\ell = 1$ are again only apparent; in fact $F_{1b}(\ell) = -1/8$ for $r_\ell = 1$.

The computations proceed along the same way for the other diagrams.
• Diagram 1c

\[ N_{1c}^\mu = i M \sigma^{\mu\nu} q_\nu (x + y - 1) + q q^\mu (1 - x - y) + \left( M^2 (x - 1) + m_\ell^2 \right) \gamma^\mu \]  

(19)
gives after the integrations over \( x \) and \( y \)

\[ F_{1c}(\ell) = \frac{-r_\ell^2 \ln r_\ell}{2(r_\ell - 1)^3} + r_\ell \left[ \frac{1}{4(r_\ell - 1)} + \frac{1}{2(r_\ell - 1)^2} \right] - \frac{1}{4} \]  

(20)

• Diagram 1d

\[ N_{1d}^\mu = m_\ell^2 \gamma^\mu \]  

(21)
yields

\[ F_{1d}(\ell) = 0. \]  

(22)

• Diagram 2a  \hspace{1em} \text{Calling}

\[ D_2(\ell) = M^2_W x + m_\ell^2 (1 - x) - M^2 x y - q^2 y (1 - x - y) \]  

(23)
one has

\[ N_{2a}^\mu = 2 i M \sigma^{\mu\nu} q_\nu (y - 1) + 2 q q^\mu (1 - y) (x + 2 y) + \left\{ -2 m_\ell^2 + 2 q^2 (y - 1) (x + y) - 2 D_2(\ell) \left[ \frac{2}{\epsilon} + \ln 4 \pi - \gamma - \frac{1}{2} - \ln \frac{D_2(\ell)}{\Lambda^2} \right] \right\} \gamma^\mu, \]  

(24)
and

\[ F_{2a}(\ell) = \frac{r_\ell (2 r_\ell - 1) \ln r_\ell}{(r_\ell - 1)^4} + r_\ell \left[ \frac{2}{3(r_\ell - 1)} - \frac{3}{2(r_\ell - 1)^2} - \frac{1}{(r_\ell - 1)^3} \right] - \frac{2}{3}. \]  

(25)

• Diagram 2b

\[ N_{2b}^\mu = r_\ell \left\{ i M \sigma^{\mu\nu} q_\nu [x (1 + y) - 1] + q q^\mu [1 - x (1 + y) - 2 y^2] \right. \]

\[ \left. + \left\{ -m_\ell^2 + M^2 x + q^2 y (x + y - 1) - D_2(\ell) \left[ \frac{2}{\epsilon} + \ln 4 \pi - \gamma - \frac{1}{2} - \ln \frac{D_2(\ell)}{\Lambda^2} \right] \right\} \gamma^\mu \right\} \]  

(26)
yields

\[ F_{2b}(\ell) = \frac{r_\ell (2 - r_\ell) \ln r_\ell}{2(r_\ell - 1)^4} + r_\ell \left[ \frac{-5}{12(r_\ell - 1)} + \frac{3}{4(r_\ell - 1)^2} - \frac{1}{2(r_\ell - 1)^3} \right]. \]  

(27)

### 2.2 Cancelation of the ultraviolet divergences

All terms that are \( \ell \)-independent do not contribute to the amplitude because of the unitarity of \( U_{lep} \); this is in particular the case of the (divergent) terms \( (2/\epsilon + \ln 4 \pi - \gamma - 1/2) \) in the diagrams 1a and 2a. The only two remaining divergent diagrams are 1b and 2b; however the coefficients of their \( (\ell \)-dependent) divergent terms exactly cancel, ensuring the finiteness of the final result.
2.3 Result for the total amplitude of $\nu_H \rightarrow \nu_L \gamma$

Dropping the constants $(-7/12), (-1/4), (-2/3)$ in (14), (20) and (25) which, being $\ell$-independent, do not contribute to the decay amplitude (see above), we obtain for the sum of the six contributions $\sum_\ell U_{H\ell} U_{L\ell}^* [F_{1,a-d}(\ell) + F_{2,a,b}(\ell)]$ the expression

$$A_{\nu_H \rightarrow \nu_L \gamma} = \frac{3}{4} A_0 \sum_\ell U_{H\ell} U_{L\ell}^* \frac{r_\ell}{(1-r_\ell)^3} \left[1 - r_\ell^2 + 2 r_\ell \ln r_\ell \right],$$

(28)

where $A_0$ has been defined in (13). Our result (28) agrees with formula (10.28) for the function $f(r)$ in reference [8] (where the three irrelevant constants mentioned above are kept).

The corresponding decay rate is

$$\Gamma_0 \equiv \Gamma_{\nu_H \rightarrow \nu_L \gamma} = \frac{G_F^2 M^5}{192\pi^3} \left(\frac{27\alpha}{32\pi}\right) \left|\sum_\ell U_{H\ell} U_{L\ell}^* \frac{r_\ell}{(1-r_\ell)^3} \left[1 - r_\ell^2 + 2 r_\ell \ln r_\ell \right]\right|^2.$$  

(29)

With the assumptions about $U_{lep}$ and the corresponding mixing angles mentioned in the introduction, one finds for $M \approx 5 \times 10^{-2} \text{ eV}$

$$\Gamma_{\nu_H \rightarrow \nu_L \gamma} \approx 10^{-44}/\text{year}.$$  

(30)

This is to be compared with the experimental lower limit found in [9].

The detectability of this decay and its relevance for astronomy has been emphasized for example in [10].

3 The decay $\nu_H \rightarrow \nu_L e^+ e^-$

If kinematically allowed, this decay is governed at tree level by the diagram of Fig. 3, and at the one-loop level by ten diagrams: the six previously considered in Figs. 1, 2 where the photon, now off-mass-shell, decays into an electron-positron pair, and the four box diagrams of Fig. 4 in which the $W^+ - W^-$ pair is converted into the $e^+ - e^-$ pair.

![Fig. 3]

The tree amplitude

$$A_{\text{tree}} = \frac{G_F}{\sqrt{2}} U_{He} U_{Le} \bar{\nu}(k_-) \gamma^\mu (1-\gamma_5) u(P) \bar{\nu}(p) \gamma_\mu (1-\gamma_5) v(k_+)$$

(31)
can be recast, by a Fierz transformation and using the unitarity of $U_{lep}$, into

$$A_{tree} = (-1)^2 G_F \frac{1}{\sqrt{2}} \sum_{j=\mu,\tau} U_{Hj}^* U_{Lj} \bar{u}(p) \gamma^\mu (1 - \gamma_5) u(P) \bar{v}(k_-) \gamma_\mu (1 - \gamma_5) v(k_+).$$  (32)

As for the one-loop corrections, a careful examination of all the terms in (11), (17), (19), (21), (24) and (26) for the six vertices $\Gamma_{1a-d}(\ell)$, $\Gamma_{2a,b}(\ell)$ shows that the dominant behavior comes from the $q^2$ term in (24) corresponding to Fig. 2a; it exhibits a $\ln r_\ell \to \infty$ for $r_\ell \to 0$ contribution, reflecting mass singularities (or infrared divergences) of the loop integrals.

We can track down this divergent behavior by examining the integration limits $x = 0$ and $x = 1$ of the denominators $D_{1,2}(\ell)$. When $r_\ell = 0$, an infrared-like divergence occurs if the numerators $N_{2\mu}(\ell)$ lack an $x$ term to cancel the $x = 0$ integration limit of the $x M^2_W$ term in the denominator $D_2(\ell)$. This happens with the $2y(y - 1)q^2$ term of $N_{2\mu}(\ell)$ in (24).

This infrared-like divergence, which arises when there are two massless ($r_\ell = 0$) internal fermions in the loop, has been noticed a long time ago in the computation of the neutrino charge radius [11]. Compared to $\ln r_\ell$, all other terms are negligible because they are strongly damped by powers of $r_\ell^n \ln r_\ell$, where $n > 0$ and $r_\ell < 10^{-3}$. Thus Fig. 2b is damped by $r_\ell \ln r_\ell$, and the four diagrams of Fig. 1 are all strongly damped since an infrared-like divergence cannot occur here: the $x = 1$ integration limit of the $(1 - x) M^2_W$ in the denominator $D_1(\ell)$ is systematically canceled by the $(1 - x)$ coming from the integration over the $y$ variable. Explicit $x,y$ integrations of all six vertices $\Gamma^n_{1a-d}(\ell)$, $\Gamma^n_{2a,b}(\ell)$ confirm these features.

Similar considerations show that the box diagrams of Fig. 4 share the same power suppression $r_\ell^n \ln r_\ell$ as the five other diagrams of Figs. 1a-d and Fig. 2b. The origin of this $r_\ell$ power suppression in all one-loop diagrams except Fig. 2a can be traced back to the fact that they involve two ($W, \Phi$) propagators; only Fig. 2a and Fig. 2b have one, but the latter nevertheless gets an $r_\ell$ suppression from the $\Phi$-fermion couplings.

Fig. 4
To summarize, at one-loop, only the 2\(y(y-1)q^2\) term in (24) yields an infrared-like divergence \(\propto \ln r_\ell\) while all other terms get damped by powers of \(r_\ell\).

The leading \(q^2\ln r_\ell\) term of Fig. 2a in the \(\nu_H - \nu_L - \gamma\) vertex cancels the photon propagator 1/\(q^2\) in Fig. 5 and yields an effective local four-fermion coupling proportional to \(G_F\). The leading contribution to the one-loop radiative corrections to the \(\nu_H \to \nu_L e^+ e^-\) tree amplitude is accordingly found to be

\[ A_{\text{rad}} = \frac{G_F}{\sqrt{2}} \frac{e^2}{24\pi^2} \sum_{\ell} U_{\ell H}^* U_{\ell L} \ln r_\ell \overline{\varphi}(p) \gamma^\mu (1 - \gamma_5) u(P) \overline{\varphi}(k_-) \gamma_\mu v(k_+), \tag{33} \]

which can be put, using again the unitarity of \(U_{\text{lep}}\) into a form similar to \(A_{\text{tree}}\) in (32):

\[ A_{\text{rad}} = \frac{G_F}{\sqrt{2}} \frac{e^2}{24\pi^2} \sum_{j=\mu,\tau} U_{Hj}^* U_{Lj} \ln \frac{m_j^2}{m_e^2} \overline{\varphi}(p) \gamma^\mu (1 - \gamma_5) u(P) \overline{\varphi}(k_-) \gamma_\mu v(k_+). \tag{34} \]

Fig. 5

The sum \(A_{\text{tree}} + A_{\text{rad}} = B\) is now easy to manipulate when we consider the interference between \(A_{\text{tree}}\) and \(A_{\text{rad}}\) in \(|B|^2\) for the decay rate.

\[ B = \frac{G_F}{\sqrt{2}} \overline{\varphi}(p) \gamma^\mu (1 - \gamma_5) u(P) \overline{\varphi}(k_-) \gamma_\mu (g_V - g_A \gamma_5) v(k_+), \tag{35} \]

with

\[ g_V = \sum_{j=\mu,\tau} U_{Hj}^* U_{Lj} \left( 1 + \frac{\alpha}{3\pi} \ln \frac{m_j}{m_e} \right), \]
\[ g_A = \sum_{j=\mu,\tau} U_{Hj}^* U_{Lj}. \tag{36} \]

From the amplitude \(B\), we compute [12] the decay rate \(\Gamma_1 \equiv \Gamma_{\nu_H \to \nu_L e^+ e^-}\) and find

\[ \frac{d\Gamma_1}{dq^2} = \frac{G_F^2}{192\pi^3} \frac{\sqrt{q^2(\sqrt{q^2 - 4m_e^2})}}{q^4 M^3} \left( M^2 - q^2 \right)^2 \left\{ (g_V^2 + g_A^2) [q^2(M^2 + 2q^2) + 2m_e^2(M^2 - q^2)] + 6m_e^2 q^2 (g_V^2 - g_A^2) \right\}, \tag{37} \]

from which one gets

\[ \Gamma_1 = \int_{4m_e^2}^{M^2} dq^2 \frac{d\Gamma_1}{dq^2} = \frac{G_F^2 M^5}{192\pi^3} \left\{ \frac{g_V^2 + g_A^2}{2} G(x) + (g_V^2 - g_A^2) H(x) \right\}, \tag{38} \]
where \( x = m_e^2/M^2 \), and \( G(x), H(x) \) are the phase-space functions given by

\[
G(x) = \left[ 1 - 14x - 2x^2 - 12x^3 \right] \sqrt{1 - 4x} + 24x^2 (1 - x^2) \ln \frac{1 + \sqrt{1 - 4x}}{1 - \sqrt{1 - 4x}},
\]

\[
H(x) = 2x(1 - x)(1 + 6x) \sqrt{1 - 4x} + 12x^2(2x - 1 - 2x^2) \ln \frac{1 + \sqrt{1 - 4x}}{1 - \sqrt{1 - 4x}}.
\] (39)

To this leading logarithmic radiative correction expressed by \( \approx \alpha \ln r \) in (36,38), we may also add the non-leading (simply \( \alpha \), without \( \ln r \)) electromagnetic correction to the \( e^+ e^- \) pair. This non-leading correction can be obtained from the one-loop QCD correction to the well known \( e^+ e^- \rightarrow \) quark-pair cross-section, or the \( \tau \rightarrow \nu_{\tau} + \) quark-pair decay rate found in the literature [12]; the only necessary change is the substitution \( \alpha_s \leftrightarrow 3\alpha/4 \). Thus, in addition to \( \Gamma_1 \), we have the non-leading contribution \( \Gamma_2 \)

\[
\Gamma_2 = \frac{G^2_F M^5}{192 \pi^3} \left( \frac{3\alpha}{4\pi} \right) G(x) K(x, x);
\] (40)

the function \( K(x, x) \) is tabulated in Table 14.1 of [12].

We emphasize that \( K(x, x) \) is a spectacular increasing function of \( x \), acting in the opposite direction to the decreasing phase-space function \( G(x) \).

The present direct experimental limit on the mass of \( \nu_{\tau} \) is [13] \( m_{\nu_{\tau}} \leq 18.2 \, \text{MeV} \); if we take, for example, the mass of the heavy decaying neutrino to be 1.1 MeV, its lifetime is found to be \( \approx 10^{-2} \) year.

Other stronger limits (below 1 MeV) mainly come from cosmological arguments [14],[15].

Finally we note that the virtual weak neutral \( Z^0 \) boson replacing the virtual photon in Fig. 4 also contributes to \( \nu_H \rightarrow \nu_L e^+ e^- \). However it can be safely discarded, being strongly damped by \( q^2/M_Z^2 \) due to the \( Z^0 \) propagator.

4 Conclusions

The recent observation by the Super-Kamiokande collaboration of a clear up–down \( \nu_{\mu} \) asymmetry in atmospheric neutrinos is strongly suggestive of \( \nu_{\mu} \rightarrow \nu_X \) oscillations, where \( \nu_X \) may be identified with \( \nu_{\tau} \) or even possibly a sterile neutrino. These results have many important physical implications. In particular, neutrino oscillations mean that neutrinos have a non-vanishing mass, which, according to the new data, may be at least as heavy as \( 5 \times 10^{-2} \) eV. If a neutrino \( \nu_H \) has indeed a mass, it may not be stable against decay and could in principle decay into a lighter neutrino, \( \nu_L \), through a cross-family electroweak coupling. We have studied two such decay modes, \( \nu_H \rightarrow \nu_L \gamma \) and \( \nu_H \rightarrow \nu_L e^+ e^- \), and found that the latter, which, in contrast to the former, arises at tree level and gets further enhanced by large radiative one-loop corrections, is by far the dominant process and may therefore be detectable provided that \( \nu_H \) has a mass \( > 2 m_e \). A positive evidence for such decay modes would give a clear signal of the onset of ’new physics’.

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