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Vaccination control measures of an epidemic model with long-term memristive effect

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COVID-19 is a drastic air-way tract infection that set off a global pandemic recently. Most infected people with mild and moderate symptoms have recovered with naturally acquired immunity. In the interim, the defensive mechanism of vaccines helps to suppress the viral complications of the pathogenic spread. Besides effective vaccination, vaccine breakthrough infections occurred rapidly due to noxious exposure to contagions. This paper proposes a new epidemiological control model in terms of Atangana Baleanu Caputo (ABC) type fractional order differ integrals for the reported cases of COVID-19 outburst. The qualitative theoretical and numerical analysis of the aforesaid mathematical model in terms of three compartments namely susceptible, vaccinated, and infected population are exhibited through non-linear functional analysis. The hysteresis kernel involved in AB integral inherits the long-term memory of the dynamical trajectory of the epidemics. Hyer–Ulam’s stability of the system is studied by the dichotomy operator. The most effective approximate solution is derived by numerical interpolation to our proposed model. An extensive analysis of the vigorous vaccination and the proportion of vaccinated individuals are explored through graphical simulations. The efficacious enforcement of this vaccination control mechanism will mitigate the contagious spread and severity.

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1. Introduction

COVID-19 is a topic that burdens all the public unrest. It is the highly transmissible pathogenic ailment that disrupts every aspect of human routine namely education, travel, employment, cargo, sports, and health care amenities [1]. The threatening global pandemic disease SARS-CoV-2 is an individual palpable-stranded RNA virus of the Nidovirales family causing mild to moderate sickness in human beings [2]. The major risk of contagion is by inhalation of infected droplets from an infected person through close contact or from the contaminated zone. The pathogens begin to incubate from 2 days to 2 weeks. Most infected people with mild symptoms recovered with gained immunity naturally from the disease and are known as Covids survivors. But some people undergo severe symptoms such as trouble breathing and infected lungs were hospitalized under intensive care. This contagious illness elicits severe air-tract pneumonia. Pneumonia would be such drastic that ends with blood clotting in the heart, brain, and lungs, cytokine storm, hyperlipidemia and even transplantation of the lungs [3,4]. A few other deadly critical symptoms include persistent pain around the chest, bluish
lips, inability to be active etc. Studies have reported the most common symptoms of COVID-19 as 99% fever, fatigue 70%, cough 59%, loss of appetite 40%, body ache 35%, breathing illness 31%, and phlegm 21% [4]. Self-hygienic practices of sanitized hands and usage of multi-layered masks have been strictly insisted on and monitored for preventive actions by the Government and Health Department. Some effective medical treatments such as tubing oxygen into nostrils, monoclonal antibodies, and antiviral medicines were given to lessen the virulence of the illness [3,4]. Vaccination is the most defensive mechanism boosting up human cells’ natural immunity to defy specific infections and makes the immune system intrinsignence. The horrendous Covid cases have alarmed the world to understand the importance of immunization. Immunizing artificially is an effective strategy to safeguard the community from dreadful diseases ahead of exposure to complications. The World Health Organization (WHO) ensures that vaccines become vaccinations recently with ameliorating supply to all people around the world [5]. 60.6% of the global population with 64.7% of Indian citizens inclusively were fully vaccinated for COVID-19. Statistical data, shows that nearly 53.2 crore people have been infected worldwide with COVID-19 as of June 2022 [6].

Modeling real time phenomena in terms of integer order differentiation briefly portray the reciprocations existing in various diverse systems [7,8], narrating the flow under study. The concept of non-integer calculus was introduced in the late 17th century as a simple academic overview of traditional derivatives and integrals [9,10]. Fractional-order integral equations play an important role in various applications of functional analysis. The fractional derivative is a net change of a quantity which geometrically accumulates the complete or vast neighborhood of the operation involved in the system. Qualitative, cogent, optimization, and significant contributions were done by various inquisitors in investigating fractional differential equations for hyper-chaotic behavior [11], traveling wave solutions of hydrodynamics [12], soliton transmission through optical fibers and ultra-short pulses in [13,14], stability analysis of solitary wave equations [15,16], [17,18], biological double-chain model of DNA [19], Schrödinger equation solved through conformable fractional derivatives in [20].

The recent fractional-order operator captivated jointly by Atangana, Baleanu in Caputo form is the ABC derivative [21] in 2016, which exhibits singular kernel via non-singular kernel detailed in [22]. The Mittag-Leffler kernel of ABC derivatives is the multifaceted function that inherits the long term memory effect [23,24] and relevant references therein [25–28]. The memristive behavior of the disease dynamics is well efficiently learned by recollecting the past behavior of the system from the initial state to the current state through the ABC type non-integer order derivatives of the parameters. In certain biological control systems of inoculation, the memristive effect briefs about the defensive effect of immunity before pharmaceutical interventions [29–32]. India reported 4.32 crore infections with 5.25 lakh deaths by June 2022 [6]. Several descriptions of fractional order conceptual model on Covid −19 disease with effective control measures such as isolation, environmental load, treatment, recovery, fear effects, and re-infections etc., have been found in literature [33–40]. Motivated by the aforesaid properties and effectiveness of ABC type kernel for its appropriate accuracy and crossover memory, this research explores a novel simple commensurate model [37], parameterizing the dynamical flow of real cases of COVID-19 ailment. The novelty of this paper accords the swift vaccination control to lessen the contagious spread. The major significant handouts of this study are summarized below:

- This paper explores a new epidemic model of the COVID-19 pandemic, in terms of ABC non-integer order derivatives with memory kernel.
- The novel approach to dealing with the vaccinated community and the effect of immunization on the spread of COVID-19 disease is investigated.
- The derivation of a unique solution to the framed model is disclosed in detail.
- Numerical Lagrange’s interpolation of double step Adams Bashforth method is recommended for desired approximate results.
- Graphical illustrations are made for distinct fractional order and vaccination rate.
- Figurative results encapsulated in this work show an aphoristic connection between epidemiological and mathematical parameters.

The rest of this article is structured as follows, Preliminary definitions and concepts of ABC fractional-order derivatives are narrated in Section 2, A novel vaccination control model on COVID-19 is formulated in Section 2, and Section 3 addresses the fundamental existence of the system through fixed point principles. Efficient numerical interpolation equations are derived in Section 4, Numerical analysis for real data on COVID-19 cases is disclosed in Section 5, Finally Section 6 sums up and discusses the epidemiological mechanisms of preventive control.

2. Preliminary definitions and concepts

A brief overview of basic notations and concepts required for understanding the non-integer order derivatives and integrals in ABC type is presented in this segment.

Let F be defined on the space, $\mathbb{C} [0,T], F' = (0,T) \subset \mathbb{R}, C[0,T] = (F,R)$ be functions with continuity. Define $T:[0,T] \rightarrow \mathbb{R}$ satisfies,

$$\max_{t \in [0,T]} \{ |S(t) + V(t) + I(t)| \}$$

where $S, V, I \in C[0, T]$.  

(1)
Definition 2.1 ([33]). The fractional Atangana Baleanu Caputo (ABC) derivative is given by,

\[
ABC_{\eta_0}^\eta f(t) = \frac{M(\eta)}{1-\eta} \int_0^t \frac{d}{dy} f(y) K_{\eta}(\frac{-\eta}{1-\eta}(t-y)^\eta) \, dy
\]

(2)
where \( M(\eta) = 1 \) is the normalization operator for \( \eta = 0 \) and 1.

Here \( K_{\eta} \) -hysteresis kernel of Mittag-Leffler type generalization of an exponential function is given by

\[
K_{\eta}(f) = \sum_{k=0}^{\infty} \frac{f^k}{K_{\eta} + 1}
\]

(3)

The corresponding integral is as follows,

\[
AB_{\eta}^\eta f(t) = \frac{1}{M(\eta)} f(t) + \frac{\eta}{M(\eta) \Gamma_{\eta}} \int_0^t (t-y)^{\eta-1} f(y) \, dy.
\]

(4)

The solution [41] of ABC type differential function \( ABC_{\eta_0}^\eta f(t) \) is given by,

\[
f(t) = f_0 \frac{1-\eta}{M(\eta)} f(t) + \frac{\eta}{M(\eta) \Gamma_{\eta}} \int_0^t (t-y)^{\eta-1} f(y) \, dy.
\]

(5)

2.2. Schauder’s contraction theorem [42]

For a non-empty closed and convex subset \( N \) of a Banach space \( B \). Let us assume a continuous map \( N \) onto a relatively compact \( f(N) \subset S \), then there exists a unique fixed point in \( N \), (i.e.) \( f(x) = x \) for \( x \in N \).

2.3. Krasnoselskii contraction theorem [42]

Let \( N \neq \phi \), be closed and convex subset of a Banach space \( (B, \|\|) \), \( T_1 \) and \( T_2 \) maps \( N \) into \( S \) such that,

(i) \( T_1x + T_2y \in N \) for all values \( x, y \in N \).

(ii) \( T_1 \) is continuous, \( T_1N \subset \) a compact set of \( B \).

(iii) A contraction map \( T_2 \), satisfying \( T_1x \, T_2x = x \).

2.4. Hyer–Ulam stable [43]

The system (1) is Hyer–Ulam’s stable if the largest eigenvalue of the spectral matrix is <1.

2.5. Definition [44]

A solution \( x = \varphi(t) \) of (1) is,

(i) attractive if the zero solution \( \varphi(t) = 0 \) such that \( \|x_0\| \leq \varepsilon \implies \lim_{t \to \infty} x_0 = 0 \).

(ii) Attractive and stable \( \Rightarrow \) asymptotic stability.

3. The framework of SVI mathematical model

The epidemiological disease dynamical system under investigation here is motivated and framed using the references therein [30–40] with defensive vaccine control measures. The contagious transmission has been explored by grouping the total considered community as susceptibles, infectives, and vaccinated population. The major contribution in this research is immunity slack and vaccine breakthrough infections.

Many vaccinated people gets infected due to loss of gained immunity being inoculated against the dreadful ailment, besides positive recoveries report boosted immunity against the pathogens. Every new infant birth are counted to be
susceptible ones, notable to be contagious are also been observed (see Table 1). The entire population is viewed as $N(t)$ $\equiv S + V + I$ with three different sub-individuals as,

$$ABC_{D_0^\eta}^{\eta}(t) = \begin{align*}
ABC_{D_0^\eta}^{\eta} S(t) &= (1 - \nu) a - \frac{\beta S I}{N} - \mu S \\
ABC_{D_0^\eta}^{\eta} V(t) &= \nu a - \frac{\alpha V I}{N} - \mu V + \gamma I \\
ABC_{D_0^\eta}^{\eta} I(t) &= \frac{\beta S I}{N} + \frac{\alpha V I}{N} - \mu I - \gamma I - \phi I
\end{align*}$$

where $0 < \eta < 1$, $ABC_{D_0^\eta}$ is the ABC derivative of order $\eta$, accompanied with initial values $S(0) = S_0 \geq 0$, $V(0) = V_0 \geq 0$, $I(0) = I_0 \geq 0$.

### 4. Uniqueness of solution

The non-linear functional analysis exhibits the point of the existence of any defined non-linear system through fixed point contractions. The detailed analysis of unique solution existence is guaranteed by fixed point mappings defined on Banach space [33], and [43]. By a theorem of fixed point mapping, the considered model (1) possesses at least a single solution in $[0, T]$.

Reformulation of (11) in the form of AB integral as expressed in (4) are given by,

$$\begin{align*}
S (t) &= S_0 + \frac{1 - \eta}{M (\eta)} L (t, S (t)) + \frac{\eta}{M (\eta)} \Gamma_\eta \int_0^t (t - y)^{\eta - 1} L (y, S (y)) \, dy = P_1 + P_2 \\
V (t) &= V_0 + \frac{1 - \eta}{M (\eta)} L (t, V (t)) + \frac{\eta}{M (\eta)} \Gamma_\eta \int_0^t (t - y)^{\eta - 1} M (y, V (y)) \, dy = Q_1 + Q_2 \\
I (t) &= I_0 + \frac{1 - \eta}{M (\eta)} L (t, I (t)) + \frac{\eta}{M (\eta)} \Gamma_\eta \int_0^t (t - y)^{\eta - 1} N (y, I (y)) \, dy = R_1 + R_2
\end{align*}$$

The most important application of nonlinear functional analysis is to prove the existence of any non-linear system using fixed point theorems. Applying Krasnoselski’s fixed point theorem, we prove the principal part of governing equations (7), $X(P_1, Q_1, R_1)$ as contraction maps and $Y(P_2, Q_2, R_2)$ as continuous compact integral parts, where

$$\begin{align*}
P_1 &= S_0 + \frac{1 - \eta}{M (\eta)} L (t, S (t)) , \quad P_2 = \frac{\eta}{M (\eta)} \Gamma_\eta \int_0^t (t - y)^{\eta - 1} L (y, S (y)) \, dy \\
Q_1 &= V_0 + \frac{1 - \eta}{M (\eta)} L (t, V (t)) , \quad Q_2 = \frac{\eta}{M (\eta)} \Gamma_\eta \int_0^t (t - y)^{\eta - 1} M (y, V (y)) \, dy \\
R_1 &= I_0 + \frac{1 - \eta}{M (\eta)} L (t, I (t)) , \quad R_2 = \frac{\eta}{M (\eta)} \Gamma_\eta \int_0^t (t - y)^{\eta - 1} N (y, I (y)) \, dy
\end{align*}$$

**Theorem 4.1.** The non-linear maps $X(P_1, Q_1, R_1):[0,T] \times R \times R \rightarrow R^3$ defined in (8) satisfy the Lipschitzian contractive condition for constants $K_P, K_Q, K_R > 0$.

**Proof.** Let the operators $X(P_1, Q_1, R_1):[0,T] \times R \times R \rightarrow R^3$, defined on a complete normed linear space with norm,

$$\| (S, V, I) \| = \max_{t \in [0, T]} \{ ||S (t) + V (t) + I (t)|| \}$$

where $S, V, I \in [0, T]$.

First, let us prove that $X(P_1, Q_1, R_1)$ is a contraction map.

\[ \text{Table 1} \]

| Parameters | Physical interpretation |
|-----------|-------------------------|
| $N$       | The Total population    |
| $S_0$     | Initial susceptible population |
| $V_0$     | Initial vaccinated population |
| $I_0$     | Initial infected population |
| $a$       | The Total recruitment rate |
| $\nu$     | The Rate of Initial vaccination |
| $\beta$   | The Transmission rate of susceptible |
| $\alpha$  | The Infection rate of vaccinated susceptible |
| $\gamma$  | Recovered with gained natural immunity against disease |
| $\mu$     | The Natural death rate |
| $\phi$    | Disease induced death rate due to severe infection |
For \( S(t) \) and \( \hat{S}(t) \), we have
\[
\| P(S, V, I)(t) - P(\hat{S}, \hat{V}, \hat{I})(t) \| = \left\| (1 - \nu) a - \frac{\beta_S}{N} s - \mu s - \psi s - (1 - \nu) a + \frac{\beta_\hat{S}}{N} \hat{s} + \psi \hat{s} \right\| = \left\| \frac{\beta_S}{N} (s - \hat{s}) - \mu (s - \hat{s}) - \psi (s - \hat{s}) \right\|.
\]
\[
\| P(S, V, I)(t) - P(\hat{S}, \hat{V}, \hat{I})(t) \| \leq \left\| \left( \frac{\beta_S}{N} \right) \| \| + \mu + \psi \right\| \| s - \hat{s} \|.
\]
\[
\| P(S, V, I)(t) - P(\hat{S}, \hat{V}, \hat{I})(t) \| \leq K_P \| s - \hat{s} \|, \text{ where } K_P = \left\| \left( \frac{\beta_S}{N} \right) \| + \mu + \psi \right\|.
\]

Similarly for the contractive transformations, \( Q, R \) we,
\[
\| Q(S, V, I)(t) - Q(\hat{S}, \hat{V}, \hat{I})(t) \| \leq K_Q \| V - \hat{V} \| \text{ where } K_Q = \left\| \left( \frac{\alpha}{N} \right) \| + \mu \right\|,
\]
\[
\| R(S, V, I)(t) - R(\hat{S}, \hat{V}, \hat{I})(t) \| \leq K_R \| I - \hat{I} \| \text{ where } K_R = \left\| \left( \frac{\beta_S + \alpha V}{N} \right) + \mu + \gamma + \phi \right\|.
\]

This conveys that, for the operator \( X(P_1, Q_1, R_1) \), we have
\[
\| X(S, V, I)(t) - X(\hat{S}, \hat{V}, \hat{I})(t) \| = \frac{1 - \eta}{M(\eta)} \max \left\| (S, V, I)(t) - (\hat{S}, \hat{V}, \hat{I})(t) \right\| \leq \frac{1 - \eta}{M(\eta)} K.
\]

with \( K = \max\{K_P, K_Q, K_R\} < 1 \) is Lipschitzian constant.

\( \Rightarrow X(S, V, I) \) is a non-expansive operator.

**To prove** \( Y(P_2, Q_2, R_2) \) **is Continuously compact:**

The compactness of the operator \( Y(P_2, Q_2, R_2) \) is depicted by the absolute modulus of all positively bounded continuous operators \( P, Q, R \) defined in (8) expressed by the non-negative non-zero constants, \( \epsilon_P, \epsilon_Q, \epsilon_R, \Omega_P, \Omega_Q, \Omega_R \), satisfying the boundedness inequalities,
\[
\begin{align*}
|P(t, S)| &\leq \epsilon_P \| S \| + \Omega_P, \\
|Q(t, V)| &\leq \epsilon_Q \| V \| + \Omega_Q, \\
|R(t, I)| &\leq \epsilon_R \| I \| + \Omega_R.
\end{align*}
\]

Let us consider a closed subset \( B \) of \( Z \) as \( \mathcal{B} = \{ (S, V, I) \in Z / \| (S, V, I) \| \leq \Lambda, \Lambda > 0 \} \).

For \( (S, V, I) \in B \), we have,
\[
\Rightarrow \| P(t, S) \| = \max_{t \in [0, \xi]} \left\| \frac{\eta}{M(\eta) \Gamma \eta} \int_0^t (t - y)^{\eta-1} P(y, S(y)) \ dy \right\| \leq \frac{\xi^n}{M(\eta) \Gamma \eta} \int_0^\xi (\xi - y)^{\eta-1} |P(y, S(y))| \ dy \leq \frac{\xi^n}{M(\eta) \Gamma \eta} \epsilon_P \Lambda + \Omega_P.
\]

In this manner, we have
\[
\| Q(t, V) \| \leq \frac{\epsilon_Q \Lambda + \Omega_Q}{M(\eta) \Gamma \eta}, \text{ and } \| R(t, I) \| \leq \frac{\epsilon_R \Lambda + \Omega_R}{M(\eta) \Gamma \eta}.
\]

Proceeding this operation, we derive the maximum norm of \( \| E(P_2, Q_2, R_2) \| \) as,
\[
\| E(P_2, Q_2, R_2) \| \leq \frac{\xi^n}{M(\eta) \Gamma \eta} \left[ (\epsilon_P + \epsilon_Q + \epsilon_R) \Lambda + \Omega_P + \Omega_Q + \Omega_R \right] = \xi^n, \text{ a positive constant.}
\]

Hence, \( \| E(S, V, I) \| \leq \xi^n \Rightarrow E \) is a uniformly bounded operator.
Next, for the values \( t_1 < t_2 \in [0,T] \), we have

To prove \( E \) is equi-continuous for \( t_1 < t_2 \in [0,T] \).

\[
|P_2(t_2, S) - P_2(t_1, S)|
\]

\[
= \frac{\eta}{M(\eta)\Gamma\eta} \left| \int_0^{t_2} (t - y)^\eta P(y, S(y)) \, dy - \int_0^{t_1} (t - y)^\eta P(y, S(y)) \, dy \right|
\]

\[
\leq \frac{\eta}{M(\eta)\Gamma\eta} \left( \int_0^{t_2} (t - y)^\eta - \int_0^{t_1} (t - y)^\eta \right) \left[ (c_F) A + \mathcal{D}_F \right]
\]

\[
\leq \frac{(c_F) A + \mathcal{D}_F}{M(\eta)\Gamma\eta} (t_2^\eta - t_1^\eta)
\]  

(10)

Also, the integrals \( Q_2, R_2 \) satisfy the inequality in (10),

\[
|Q_2(t_2, V) - Q_2(t_1, V)| \leq \frac{(c_F) A + \mathcal{D}_Q}{M(\eta)\Gamma\eta} (t_2^\eta - t_1^\eta)
\]

\[
|R_2(t_2, I) - R_2(t_1, I)| \leq \frac{(c_F) A + \mathcal{D}_R}{M(\eta)\Gamma\eta} (t_2^\eta - t_1^\eta)
\]

\( \Rightarrow \) \( \| E(P_2, Q_2, R_2(t_2) - E(P_2, Q_2, R_2(t_1)) \| \to 0 \), as \( t_1 \to t_2 \) independent of the operators \( (S, V, I) \).

\( \Rightarrow E(P_2, Q_2, R_2) \) is a completely continuous equi-continuity operator.

The Arzela’s theorem on uniform boundedness, postulates that the above stated completely continuous map \( E(P_2, Q_2, R_2) \) is relatively compact. The properties, Contraction and continuity of the operators \( X \) and \( Y \), impose the existence of a single unique solution followed by the Krasnoselski theorem.

**Theorem 4.2 (Attractive Solution [44])**. Theorem 4.2, signifies our COVID-19 vaccine control model (1) has a

(i) unique result if \( \frac{c_F K}{\rho M(\eta)\Gamma\eta} < 1 \), where \( K = \max\{K_F, K_Q, K_R\} \)

(ii) attractive solution if trivial solution \( \varphi(t) = 0 \) such that \( \|x_0\| \leq \varepsilon \Rightarrow \lim_{t \to \infty} x_0 = 0 \).

**Proof.** Let us define an operator \( W = (W_1, W_2, W_3) : Z \to Z \) using (9) as follows,

\[
W_1(S, V, I)(t) = S_0 + \frac{1 - \eta}{M(\eta)} P(t, S(t)) + \frac{\eta}{M(\eta)\Gamma\eta} \int_0^t (t - y)^\eta P(y, S(y)) \, dy
\]

\[
W_2(S, V, I)(t) = V_0 + \frac{1 - \eta}{M(\eta)} Q(t, V(t)) + \frac{\eta}{M(\eta)\Gamma\eta} \int_0^t (t - y)^\eta Q(y, V(y)) \, dy
\]

\[
W_3(S, V, I)(t) = I_0 + \frac{1 - \eta}{M(\eta)} R(t, I(t)) + \frac{\eta}{M(\eta)\Gamma\eta} \int_0^t (t - y)^\eta R(y, I(y)) \, dy
\]  

(11)

Accordingly for \( (S, V, I), (\hat{S}, \hat{V}, \hat{I}) \in Z \), and using (11) we claim,

\[
\left\| W_1(S, V, I)(t) - W_1(\hat{S}, \hat{V}, \hat{I})(t) \right\| = \left\| \frac{1 - \eta}{M(\eta)} P(t, S(t)) - P(t, \hat{S}(t)) \right\|
\]

\[
+ \frac{\eta}{M(\eta)\Gamma\eta} \int_0^t \left| P(y, S(y)) - P(y, \hat{S}(y)) \right| (t - y)^\eta P(y, S(y)) \, dy
\]

\[
\left\| W_1(S, V, I)(t) - W_1(\hat{S}, \hat{V}, \hat{I})(t) \right\| \leq \left\| \frac{1 - \eta}{M(\eta)} K_F \right\| \left\| S - \hat{S} \right\| + \frac{\xi^\eta}{M(\eta)\Gamma\eta} K_F \left\| S - \hat{S} \right\|
\]

\[
\leq \left( \frac{1 - \eta}{M(\eta)} + \frac{\xi^\eta}{M(\eta)\Gamma\eta} \right) K_F \left\| S - \hat{S} \right\|
\]

As \( S(t) \to \hat{S}(t) \), then \( \left\| S - \hat{S} \right\| \to 0 \), the above inequality becomes,

\[
\left\| W_1(S, V, I)(t) - W_1(\hat{S}, \hat{V}, \hat{I})(t) \right\| \leq \left( \frac{1 - \eta}{M(\eta)} + \frac{\xi^\eta}{M(\eta)\Gamma\eta} \right) K_F \leq 1, \text{ with }
\]

\[
\left\| W_1(S, V, I)(t) - W_1(\hat{S}, \hat{V}, \hat{I})(t) \right\| \left( 1 - \left( \frac{1 - \eta}{M(\eta)} + \frac{\xi^\eta}{M(\eta)\Gamma\eta} \right) K_F \right) \leq 0
\]
Proceeding this way, we have
\[
\left\| W_2 (S, V, I) (t) - W_2 \left( \mathcal{S}, \mathcal{V}, \mathcal{I} \right) (t) \right\| \left( 1 - \frac{1 - \eta}{M(\eta)} + \frac{\xi^n}{M(\eta) \Gamma^n} \right) K_Q \leq 0
\]
\[
\left\| W_3 (S, V, I) (t) - W_3 \left( \mathcal{S}, \mathcal{V}, \mathcal{I} \right) (t) \right\| \left( 1 - \frac{1 - \eta}{M(\eta)} + \frac{\xi^n}{M(\eta) \Gamma^n} \right) K_R \leq 0
\]

Hence the operator
\[
\left\| W (S, V, I) - W \left( \mathcal{S}, \mathcal{V}, \mathcal{I} \right) \right\| \leq \left( 1 - \frac{1 - \eta}{M(\eta)} + \frac{\xi^n}{M(\eta) \Gamma^n} \right) K \left\| (S, V, I) - (\mathcal{S}, \mathcal{V}, \mathcal{I}) \right\| (12)
\]

\( \Rightarrow \) The contraction map \( W \) inherits the properties of Schauder’s and Krasnoselski’s theorem and confirms the unique fixed point solution to our proposed model (5).

### Attractivity of the derived solution

The unique solution is attractive if the zero solution \((S, V, I) (t) = 0\) such that
\[
\| (S, V, I) \| \leq \varepsilon, \Rightarrow \lim_{t \to \infty} (S, V, I) (t) = 0.
\] (13)

Also the solution is attractive if the trivial solution \(\varphi(t) = 0\) such that \(\| x_0 \| \leq \varepsilon, \Rightarrow \lim_{t \to \infty} x_0 = 0.\)

Asymptotically stable.

4.3. Hyer–Ulam’s stability

Stability of the solution to modeled fractional differential equation system in Hyer–Ulam’s style is studied with differential inequality.

**Theorem.** The unique solution \((S, V, I)\) of the model (1) is stable of Hyer–Ulam’s kind [42] if the spectral radius of
\[
\begin{pmatrix}
  p & p & p \\
  q & q & q \\
  r & r & r
\end{pmatrix}
\]
lies strictly inside the unit circle (discrete dichotomy) [45], given by \(|p + q + r| < 1\).

\[
p = \left( 1 - \frac{1 - \eta}{M(\eta)} + \frac{\xi^n}{M(\eta) \Gamma^n} \right) K_p;
\]
\[
q = \left( 1 - \frac{1 - \eta}{M(\eta)} + \frac{\xi^n}{M(\eta) \Gamma^n} \right) K_Q;
\]
\[
r = \left( 1 - \frac{1 - \eta}{M(\eta)} + \frac{\xi^n}{M(\eta) \Gamma^n} \right) K_R.
\]

**Proof.** The system defined in the model (6) has a unique solution by theorem stated in definition 2.2.

On the contrary, let \((S, V, I)\) and \(\left( \mathcal{S}, \mathcal{V}, \mathcal{I} \right)\) be two different solutions of the same model, then we write with help of (12)
\[
\left\| W (S, V, I) - W \left( \mathcal{S}, \mathcal{V}, \mathcal{I} \right) \right\| \leq \left( 1 - \frac{1 - \eta}{M(\eta)} + \frac{\xi^n}{M(\eta) \Gamma^n} \right) K \left\| (S, V, I) - (\mathcal{S}, \mathcal{V}, \mathcal{I}) \right\|
\]

Linearization of the above norm for \((S, V, I)\) and \(\left( \mathcal{S}, \mathcal{V}, \mathcal{I} \right)\) in terms of matrices, we have \[
\left\| (S, V, I) - (\mathcal{S}, \mathcal{V}, \mathcal{I}) \right\| \leq \left| \begin{array}{ccc}
p & p & p \\
q & q & q \\
r & r & r
\end{array} \right| \left( \frac{S - \hat{S}}{S - \hat{S}} \right) \left( \frac{V - \hat{V}}{V - \hat{V}} \right) \left( \frac{I - \hat{I}}{I - \hat{I}} \right)
\]

The characteristic roots of the above matrix are \(x_1 = 0, x_2 = 0, x_3 = p + q + r\).

The spectral radius denoted by \(\rho = \max\{|x_i|, i = 1,2,3\} < 1\) exhibits a discrete dichotomy making the solution of model (1) as Ulam Hyer’s stable.

5. Numerical interpolation for the proposed model:

This section deliberates the numerical scheme used to solve the analyzed model. Nowadays, the Adams Bashforth approach has been incorporated efficiently to handle global complicated problems in engineering, science, and epidemics with references therein [11,12,17,35,37,40]. Numerical approximations are derived by using coupled Lagrangian interpolation of two-step and fundamental theorem of fractional calculus in our model (6) with the following results,
\[
S(t) - S(0) = \frac{1 - \eta}{M(\eta)} P(t, S(t)) + \frac{\eta}{M(\eta) \Gamma^n} \int_0^t (t - y)^{n-1} P(y, S(y)) dy
\] (14)
Setting \( t = t_{i+1} \), for \( i = 0,1,2\ldots \),

\[
S(t_{i+1}) - S(0) = \frac{1 - \eta}{M(\eta)} P(t_{i}, S(t_{i})) + \frac{\eta}{M(\eta) \Gamma(\eta)} \int_{0}^{t_{i+1}} (t_{i+1} - y)^{\eta-1} P(y, S(y)) \, dy.
\]

\[
= \frac{1 - \eta}{M(\eta)} P(t_{i}, S(t_{i})) + \frac{\eta}{M(\eta) \Gamma(\eta)} \sum_{j=0}^{i} \int_{j}^{t_{i+1}} (t_{i+1} - y)^{\eta-1} P(y, S(y)) \, dy.
\]

\[
(15)
\]

Interpolating the function \( P(t, S(t)) \) on \([t_j, t_{j+1}]\) we get,

\[
P(y, S(t)) \cong \frac{P(t_j, S(t_j))}{\delta} (t - t_{j-1}) + \frac{P(t_{j-1}, S(t_{j-1}))}{\delta} (t - t_j)
\]

\[
S(t_{i+1}) = S(0) + \frac{1 - \eta}{M(\eta)} P(t_{i}, S(t_{i}))
\]

\[
+ \frac{\eta}{M(\eta) \Gamma(\eta)} \sum_{j=0}^{i} \left( \frac{P(t_j, S(t_j))}{\delta} \int_{j}^{t_{i+1}} (t - t_{j-1}) (t_{i+1} - t)^{\eta-1} \, dt - \frac{P(t_{j-1}, S(t_{j-1}))}{\delta} \int_{j}^{t_{i+1}} (t - t_{j})(t_{i+1} - t)^{\eta-1} \, dt \right)
\]

\[
= S(0) + \frac{1 - \eta}{M(\eta)} P(t_{i}, S(t_{i})) + \frac{\eta}{M(\eta) \Gamma(\eta)} \sum_{j=0}^{i} \left( \frac{P(t_j, S(t_j))}{\delta} A_{j-1, \eta} - \frac{P(t_{j-1}, S(t_{j-1}))}{\delta} A_{j, \eta} \right)
\]

\[
(17)
\]

Computing the expression for \( A_{j-1, \eta} \) and \( A_{j, \eta} \) as follows,

\[
A_{j-1, \eta} = \int_{j}^{t_{i+1}} (t - t_{j-1}) (t_{i+1} - t)^{\eta-1} \, dt
\]

\[
= -\frac{1}{\eta}\left[ (t_{j+1} - t_{j-1}) (t_{i+1} - t_{j-1})^{\eta} - (t_{j} - t_{j-1})(t_{i+1} - t_{j})^{\eta} \right] - \frac{1}{\eta(\eta + 1)} \left[ (t_{i+1} - t_{j})^{\eta+1} - (t_{i+1} - t_{j+1})^{\eta+1} \right]
\]

\[
(18)
\]

\[
A_{j, \eta} = \int_{j}^{t_{i+1}} (t - t_{j}) (t_{i+1} - t)^{\eta-1} \, dt
\]

\[
= -\frac{1}{\eta}\left[ (t_{j+1} - t_{j}) (t_{i+1} - t_{j+1})^{\eta} \right] - \frac{1}{\eta(\eta + 1)} \left[ (t_{i+1} - t_{j})^{\eta+1} - (t_{i+1} - t_{j+1})^{\eta+1} \right]
\]

\[
(19)
\]

Substituting, \( t_j = j\delta \), in (26) and (27) then we have,

\[
A_{j-1, \eta} = -\frac{\delta^{\eta+1}}{\eta}\left[ (j + 1 - (j - 1))(i + 1 - (j + 1))^{\eta} - (j - (j - 1))(i + 1 - j)^{\eta} \right]
\]

\[
- \frac{\delta^{\eta+1}}{\eta(\eta + 1)} \left[ (i + 1 - (j + 1))^{\eta+1} - (i + 1 - j)^{\eta+1} \right]
\]

\[
= \frac{\delta^{\eta+1}}{\eta}\left[ -2(\eta + 1)(i - j)^{\eta} + (\eta + 1)(i + 1 - j)^{\eta} - (i - j)^{\eta+1} + (i + 1 - j)^{\eta+1} \right]
\]

\[
= \frac{\delta^{\eta+1}}{\eta(\eta + 1)} \left[ (i - j)^{\eta}(i - j + 1) - (i - j) + (i + 1 - j)^{\eta}(\eta + 1 + i + 1 - j) \right]
\]

\[
= \frac{\delta^{\eta+1}}{\eta(\eta + 1)} \left[ (i + 1 - j)^{\eta}(i - j + 2 + \eta) - (i - j)^{\eta}(i - j + 2 + 2\eta) \right]
\]

\[
A_{j, \eta} = \frac{-\delta^{\eta+1}}{\eta}\left[ (j + 1 - j)(i + 1 - (j + 1))^{\eta} \right] - \frac{\delta^{\eta+1}}{\eta(\eta + 1)} \left[ (i + 1 - (j + 1))^{\eta+1} - (i + 1 - j)^{\eta+1} \right]
\]

\[
= \frac{\delta^{\eta+1}}{\eta(\eta + 1)} \left[ -(\eta + 1)(i - j)^{\eta} - (i - j)^{\eta+1} + (i + 1 - j)^{\eta+1} \right]
\]
\[
\delta^{n+1} \left[ (i-j)^{\eta} (i-j + 1+\eta) \right] \\
\frac{(i-j)^{\eta} (i-j + 1+\eta)}{\eta (\eta+1)} 
\]

Substituting, (20) and (21) in (17) we get,

\[
S(t_{i+1}) = S(0) + \frac{1}{M(\eta)} \int_{t_{i}}^{t_{i+1}} P(t_{j}, S_{0}) \\
+ \frac{\eta}{M(\eta) \Gamma \eta} \sum_{j=0}^{\eta} \left( \frac{P(t_{j}, S(t_{j}))}{\Gamma \eta + 2} - \frac{P(t_{j-1}, S(t_{j-1}))}{\Gamma \eta + 2} \right) \delta^{\eta}[(i+1-j)^{\eta} + (i+j+1-\eta)] 
\]

\[
V(t_{i+1}) = V(0) + \frac{1}{M(\eta)} Q(t_{i}, V(t_{i})) \\
+ \frac{\eta}{M(\eta) \Gamma \eta} \sum_{j=0}^{\eta} \left( \frac{Q(t_{j}, V(t_{j}))}{\Gamma \eta + 2} - \frac{Q(t_{j-1}, V(t_{j-1}))}{\Gamma \eta + 2} \right) \delta^{\eta}[(i+1-j)^{\eta} + (i+j+1-\eta)] 
\]

\[
I(t_{i+1}) = 1(0) + \frac{1}{M(\eta)} R(t_{i}, I(t_{i})) \\
+ \frac{\eta}{M(\eta) \Gamma \eta} \sum_{j=0}^{\eta} \left( \frac{R(t_{j}, I(t_{j}))}{\Gamma \eta + 2} - \frac{R(t_{j-1}, I(t_{j-1}))}{\Gamma \eta + 2} \right) \delta^{\eta}[(i+1-j)^{\eta} + (i+j+1-\eta)] 
\]

Eqs. (22), (23), and (24) give the iterated results for approximate values of the parameters.

6. Numerical study and simulated comparisons

This section examines the COVID-19 outbreak reported cases in INDIA from February 15, 2020 to June 16, 2022 [6]. To validate the numerical scheme derived above using Adams Bashforth code, we consider Eqs. (22)–(24) to do simulations for three sets of real data marked from the whole country INDIA and its two southern states namely, TAMILNADU and KERALA. The simulations show the epidemic trends by inputting the reported positive case values for the epidemiological parameters as in [33]. In INDIA, we have 43,420,608 cases with 42,797,092 recoveries and 525,047 deaths reported as of June 16, 2022. The pandemic situation shows a rapid rise during April 19, 2020–Oct 28, 2020, March 05, 2021–May 08, 2021, and January 19, 2022—End of February 2022 which results in implementation of effective control measures. This
Fig. 1. Simulation flow of the model (6) reported in INDIA with vaccination 'v'.

The dynamics reported by the new model (6) with vaccination mediation 'v' for various arbitrary orders $\eta = 0.645$, 0.745, 0.845, 0.9, and 1 are plotted in Figs. 1–3, for the three regional datasets provided in Table 2. These figures depict a uniform decrease of susceptible people once they get vaccinated for the most minimal order of integration 0.645. There was an intense hike of infections marked which consistently converges in the wake of successive efficacious vaccination policies.

The Mittag-Leffler kernel involved in the ABC type fractional-order system is well-known for its long-term memristive behavior. The memory of the human immune system plays a vital role in the defensive strategy to combat the pathogens' virulent transference. All the plots in Figs. 1–3 are also plotted for $\eta = 1$ of integer order and found that the desired approximation is attained for the least arbitrary value of iteration. The simulated results validate our model is more consistent for the memristive kernel used in the compartments $S(t)$, $V(t)$, and $I(t)$ by marking instant abate of infections following intensified vaccinations.

Here, we have also plotted our compartmental model (6) without vaccination $v=0$, as the contradiction part of controlling the pandemic spread. The graphs in Fig. 4, exemplify the extraordinary effect of vaccine implementation on dreadful diseases. Without vaccination, our real data set in no time, abate the infectious people concerning to flatten the susceptible people. Hence we suggest a more consistent result on the effect of vaccination in controlling pandemics through these numerical analysis exhibited in Figs. 1–4.
Conclusion

In this manuscript, we developed a new epidemiological SVI model with vaccination involving ABC-type fractional integrals for the COVID-19 ailment in INDIA. Prevalence and singleness of solution are obtained for the designed model from Schauder and Krasnoselski fixed point theorems on Banach spaces. The Stability of the system is derived using spectral radius in Hyer–Ulam’s style. The iterative scheme is obtained by the Lagrangian interpolation double step Adams Bashforth method for the ABC type fractional order system. The memristive Mittag-Leffler kernel embeds a vast neighborhood of the disease dynamics with $\eta = 0.645, 0.845, 0.9,$ and 1. This study simulates the COVID-19 outbreak reported cases in INDIA from the month of February 15, 2020 to June 16, 2022. Figs. 1–3 depict that the Mittag-Leffler kernel closely interprets the reported Covid cases more efficiently for the least order derivative. The memory kernel describes the dynamical trajectory caused due to vaccination effect on Covid-19 transmission under study, for various fractional order '$\eta$' and vaccination ratio '$v$'. Simulated results in Figs. 1–3 imply the lessening of infectious people considerably after the implementation of potent vaccination. Meanwhile, the appropriate timely control step of vaccinating susceptibles help to eradicate the transmission chain and reduce the complexity of the virus. The obtained results would help to predict the control phenomena for various fractional-order dynamical systems in the future.
**Fig. 3.** Simulation flow of the model (6) in the KERALA state with vaccination ‘v’.

**Fig. 4.** Simulation dynamics of the model (6) without vaccination ‘v’ in the two mentioned states.
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