New Cosmographic Constraints on the Dark Energy and Dark Matter Coupling

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Abstract

We consider some models describing interaction between the dark components and obtain an expression for the coupling constant which contains only the cosmographic parameters. It enables us on the one hand to find constrains on the coupling constants using observational data, and on the other hand, given fixed constraints on the coupling, to restrict number of numerous models describing the interaction in the dark sector.

Keywords: cosmographic parameters, interaction in the dark sector

Introduction

Typically, dark energy (DE) models are based on scalar fields minimally coupled to gravity, and do not implement the explicit coupling of the field to the background matter ¹². However there is no fundamental reason for this assumption in the absence of an underlying symmetry which would suppress the coupling. Given that we do not know the true nature of neither DE nor dark matter (DM) one cannot exclude that there exists a coupling between them. Whereas new forces between DE and normal matter particles are heavily constrained by observations (e.g. in the solar system and gravitational experiments on Earth), this is not the case for DM particles. In other words, it is possible

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Preprint submitted to Physics Letters B March 16, 2015
that the dark components interact with each other, while not being coupled to standard model particles. The study of the interaction of DE and DM is an important and promising research direction \[3, 4, 5\]. Moreover, disregarding the potential existence of an interaction between dark components may result in misinterpretations of observational data \[6\]. As the gravitational effects of DE and of DM are opposite (i.e., gravitational repulsion versus gravitational attraction), even a small change of their relative concentrations can have an effect on cosmological dynamics. Since there is no fundamental theoretical approach that may specify the functional form of the coupling between DE and DM, presently coupling models are necessarily phenomenological. Of course, one can always provide arguments in favor of a certain type of correlation. However, until the creation of a microscopic theory of dark components, the effectiveness of any phenomenological model will be defined only by how well it corresponds to observations.

Interaction between DE and DM is phenomenologically described by the following modification to the conservation equations

\[
\dot{\rho}_{\text{dm}} + 3H\rho_{\text{dm}} = Q, \quad (1) \\
\dot{\rho}_{\text{de}} + 3H(\rho_{\text{de}} + p_{\text{de}}) = -Q, \quad (2)
\]

where \(H\) is Hubble parameter, \(\rho_{\text{dm}}\) and \(\rho_{\text{de}}\) are densities of dark matter and dark energy components respectively. The function \(Q\) is known as the interaction function, and generally depends on the scale factor. In general, the coupling term \(Q\) can take any possible form

\[Q = Q(H, \rho_{\text{dm}}, \rho_{\text{de}}, t)\,.
\]

However, physically, it makes more sense that the coupling be time-independent. Among the time-independent options, preference is given to a factorized-\(H\) dependence

\[Q = 3Hq(\rho_{\text{dm}}, \rho_{\text{de}}).
\]

With this kind of factorization, the effects of the coupling on the dynamics of \(\rho_{\text{dm}}\) and \(\rho_{\text{de}}\) become effectively independent on the evolution of the Hubble
parameter $H$. The latter is related to the fact that the time derivatives that enter into the conservation equation (12) can be transformed in the following way:

$$\frac{d}{dt} \to H \frac{d}{d\ln a}.$$ 

In the simplest linear models the interaction is chosen to have one of the following forms

$$Q = 3 \delta H \rho_{dm}, \quad Q = 3 \delta H \rho_{de}, \quad Q = 3 \delta H (\rho_{dm} + \rho_{de}).$$ 

The present paper is aimed to find model-independent constraints on the coupling between DE and DM in different phenomenological models with interaction in the dark sector. Such constraints are formulated solely in terms of the cosmographic parameters values, which can be directly calculated basing on the results of certain cosmological observations (such as SNeIa, BAO, CMB.)

1. Cosmography background

Cosmography is an approach entirely based on the cosmological principle, stating that the Universe is homogeneous and isotropic on scales larger than a hundred megaparsecs. It allows to choose among whole possible variety of models describing the Universe a narrow set of homogeneous and isotropic models. The cosmological principle enables us to build the metrics of the Universe and make first steps towards interpretation of the cosmological observations. Cosmography is just kinematics of cosmological expansion. In order to build the key characteristic – time dependence of the scale factor $a(t)$ – one needs to take equations of motion (the Einstein’s equations) and make an assumption about material content of the Universe, which allow to construct the energy-momentum tensor. Cosmography is efficient because it allows to test any cosmological model which does not contradict the cosmological principle. Modifications of General Relativity or introduction of new components (such as dark matter or dark energy) evidently change the dependence $a(t)$ but it affects in no way the relations between the kinematic characteristics. In order to make
more detailed description of kinematics of cosmological expansion it is useful to consider the extended set of the parameters which includes higher order time derivatives of the scale factor 

\[ H(t) \equiv \frac{1}{a} \frac{da}{dt}; \quad (3) \]
\[ q(t) \equiv -\frac{1}{a} \frac{d^2 a}{dt^2} \left[ \frac{1}{a} \frac{da}{dt} \right]^{-2}; \quad (4) \]
\[ j(t) \equiv \frac{1}{a} \frac{d^3 a}{dt^3} \left[ \frac{1}{a} \frac{da}{dt} \right]^{-3}; \quad (5) \]
\[ s(t) \equiv \frac{1}{a} \frac{d^4 a}{dt^4} \left[ \frac{1}{a} \frac{da}{dt} \right]^{-4}; \quad (6) \]
\[ l(t) \equiv \frac{1}{a} \frac{d^5 a}{dt^5} \left[ \frac{1}{a} \frac{da}{dt} \right]^{-5}. \quad (7) \]

Dunajski and Gibbons \[10\] proposed an original way to test the cosmological models based on the General Relativity (GR) or its modifications. The procedure implies expressing the Friedmann equation in terms of directly measurable cosmographic parameters \( H, q, j, s, l \ldots \). In other words, the key idea is to treat the Friedmann equations as one algebraic constraint between the cosmographic parameters. This links the measurement of the cosmological parameters to a test of GR, or any of its modifications (which would lead to different constraints).

Let us consider a general case of Universe filled with two non-interacting components \( A \) and \( B \) with state equations

\[ p_A = w_A \rho_A, \quad p_B = w_B \rho_B. \quad (8) \]

Energy densities of the two non-interacting components satisfy the conservation equation separately

\[ \dot{\rho}_A + 3H(\rho_A + p_A) = 0. \]
\[ \dot{\rho}_B + 3H(\rho_B + p_B) = 0. \]

Taking into account the state equations \((8)\), one obtains

\[ \rho_A(a) = \rho_{0A} a^{-3(1+w_A)} \equiv \frac{A}{a^{\alpha+2}}, \]
\[ \rho_B(a) = \rho_{0B} a^{-3(1+w_B)} \equiv \frac{B}{a^{\beta+2}}, \]
where $A \equiv \rho_0 A, A \equiv \rho_0 A, \alpha \equiv 3w_A + 1, \beta \equiv 3w_B + 1$, and the first Friedmann equation can be rewritten in the form

$$\dot{a}^2 + k = \frac{A}{a^{\alpha}} + \frac{B}{a^{\beta}}$$  \hspace{1cm} (9)$$

Then we differentiate the equation (9) two times w.r.t. time to obtain

$$-2\frac{\ddot{a}}{a} = 2H^2 q = A\alpha a^{-\alpha-2} + B\beta a^{-\beta-2};$$

$$2\frac{\ddot{a}}{a} = 2H^2 j = A\alpha (\alpha + 1)a^{-\alpha-2} + B\beta (\beta + 1)a^{-\beta-2}.$$  \hspace{1cm} (10)$$

It is easy to solve the obtained system of two linear equations w.r.t. $A$ and $B$

$$A = \frac{2H^2(j - (\beta + 1)q)}{\alpha(\alpha - \beta)} a^{\alpha + 2};$$

$$B = \frac{2H^2(j - (\alpha + 1)q)}{\beta(\beta - \alpha)} a^{\beta + 2}.$$  \hspace{1cm} (11)$$

After substitution of (11) into (9) one obtains

$$\frac{k}{a^2 H^2} = \frac{2}{\alpha \beta} [q(\alpha + \beta + 1) - j] - 1.$$  \hspace{1cm} (12)$$

The third time derivative of the (9) leads to

$$\frac{2}{H} \left( \frac{a^{IV}}{a} - \frac{\dddot{a}}{a^2} \right) = 2H^2 (s + j) = -A\alpha (\alpha + 1)(\alpha + 2)a^{-\alpha - 2} - B\beta (\beta + 1)(\beta + 2)a^{-\beta - 2}.$$

Using the expressions (11) for constants $A$ and $B$, the equation above can be transformed to the form

$$s + qj + (\alpha + \beta + 3)j - q(\alpha + 1)(\beta + 1) = 0;$$  \hspace{1cm} (13)$$

For an example consider a Universe containing the cosmological constant $\Lambda$ and nonrelativistic matter (dust). The corresponding state equation parameters are $w_A = -1$ and $w_B = 0$, and respectively $\alpha = -2$ and $\beta = 1$. Then the relation (13) reduces to

$$s + 2(q + j) + qj = 0$$  \hspace{1cm} (14)$$

This relation is formally a fourth order ODE on the scale factor as a function of time. It is equivalent to the Friedmann equation, but it has an advantage to be
formulated entirely in terms of directly measurable quantities. Thus it provides the test of the model basing on which the latter equation was obtained.

The relation (12) in the considered case reduces to

\[ k = a^2 H^2 (j - 1). \]  

(15)

In particular if \( k = 0 \) (SCM) this relation reduces to a third order ODE

\[ j = 1 \]  

(16)

Although modern cosmology is mainly concerned with the parameters \( H \) and \( q \), one should however note unique importance of the jerk parameter, in particular for testing of the cosmological models. "It is a striking and slightly puzzling fact that almost all current cosmological observations can be summarized by the simple statement: The jerk of the Universe equals one" [10].

Let us perform an analogous procedure for the two-component Universe filled with non-relativistic matter and radiation which do not interact with each other. Now \( w_A = 1/3 \) and \( w_B = 0 \), and respectively \( \alpha = 2 \) and \( \beta = 1 \), then the relations (12) and (13) give

\[ k = a^2 H^2 (4q - j - 1); \]  

(17)

\[ s + 6(j - q) + jq = 0. \]  

(18)

The latter formulae reproduces well the results of the two limiting cases: a) a flat Universe filled only with non-relativistic matter has \( a(t) \propto t^{2/3} \), then by direct calculation \( q = 1/2 \), \( j = 1 \) and \( s = -7/2 \); b) in a flat Universe filled only with radiation \( a(t) \propto t^{1/2} \), then \( q = 1 \), \( j = 3 \) and \( s = -15 \).

2. Cosmographic constraints on the coupling constant

Now let us show that the approach described above will be very useful to test the models with interaction in the dark sector. Using observational data for
cosmographic parameters we can find constraints on the coupling constant. Let us consider a very simple model with the following conservation equations:

\[ \dot{\rho}_{\text{dm}} + 3H \rho_{\text{dm}} = Q, \quad (19) \]

\[ \dot{\rho}_{\text{de}} + 3H \rho_{\text{de}}(1 + w) = -Q, \quad (20) \]

where \( Q = 3H \delta \rho_{\text{dm}} \) is the source of interaction and \( w \) is the state equation parameter for dark energy. The positive small coupling constant \( \delta \) will eventually characterize how evolution of the matter energy density

\[ \rho_{\text{dm}} = \rho_{\text{dm}0} a^{-3(1-\delta)}, \quad (21) \]

will deviate from the standard case (\( \rho_{\text{m}0} \) is the present value of \( \rho_{\text{m}} \)).

To fit the observational data we must take \( \delta \leq 1/3 \). Otherwise the Universe will expand acceleratedly in the matter dominated era, which is prohibited by the observation of SNe Ia. Actually, we should expect \( \delta \ll 1 \) since so far there has been no report from observations about an anomalous dark matter expansion rate. Note that with \( \delta = 0 \) we get the standard expression for SCM.

The dark energy density in the consider model model takes on the form:

\[ \rho_{\text{de}} = \rho_{\text{de}0} a^{-3(1+w)} - \frac{a^{-3(1-\delta)} \delta \rho_{\text{dm}0}}{w + \delta}, \quad (22) \]

where the integration constant \( \rho_{\text{de}0} \) equals to the present value of the dark energy density.

Substituting the energy density dependence of the scale factor into the Friedmann equation, we get:

\[ H^2 = \frac{1}{3} \left( \rho_{\text{de}0} a^{-3(1+w)} - \frac{a^{-3(1-\delta)} \delta \rho_{\text{dm}0}}{w + \delta} + \rho_{\text{dm}0} a^{-3(1-\delta)} \right). \quad (23) \]

Note that here we consider a flat (\( k = 0 \)) Universe. Using the method described in the previous section we can write the equivalent Friedmann equation expressed in cosmographic parameters:

\[ (1 + 3w)(3\delta - 1) = 2(j - 3q(1 + w - \delta)). \quad (24) \]
As can be easily seen that when $\delta = 0$ and $w = -1$ equation take the form (16).

In order to express the coupling constant $\delta$ in terms of the cosmographic parameters we must know the second, the third and the fourth derivative of the scale factor, because we need three independent equations to resolve three independent constants. After the straightforward calculations we obtain

$$-2q(2 + 3w)(3\delta - 2) + j(-5 + 3(\delta - w)) - qj - s = 0,$$

(25)

or

$$\delta = \frac{-2q(2 + 3w) + j(5 + 3w) + jq + s}{3(j - q(2 + 3w))},$$

(26)

It can be seen that in the interaction-free case with $\delta = 0$ and $w = -1$ (cosmological constant) the above expression reproduces the equation (14).

For model with interaction term $Q = 3H\delta \rho_{de}$ the equivalent Friedmann equation expressed in cosmographic parameters has the form:

$$1 + 3(w + \delta) = -2(j - 3q(1 + w + \delta)),$$

(27)

for $\delta = 0$ and $w = -1$ this equation also take the form (16). And coupling constant $\delta$

$$-2q(2 + 3(w + \delta)) + j(2 + 3(w + \delta)) + qj + s = 0,$$

(28)

or

$$\delta = \frac{2q(2 + 3w) - j(5 + 3w) - qj - s}{3(j - 2q)}$$

(29)

for $\delta = 0$ and $w = -1$ we will return to the expression (14).

### 2.1. Model with a Decaying Vacuum Energy

Recently authors of [15] derive the parametrization of decaying vacuum energy directly from the first principle, namely from the quantum mechanics. On this approach authors obtain a Friedman equation as follow:

$$3 \frac{\dot{a}^2}{a^2} = \rho_m + \rho_{vac} = \rho_m + \Lambda_{bare} + \frac{3\beta}{t^2},$$

(30)

where matter density fulfils the conservation condition

$$\bar{T}^{\alpha\beta} = T^{\alpha\beta}_m + \Lambda(t)g^{\alpha\beta} = 0.$$

(31)
This condition for the flat homogeneous and isotropic Universe has the form

\[ \dot{\rho}_m + 3H\rho_m = -\dot{\Lambda}(t), \]  

(32)

where \( \Lambda(t) = \Lambda_{\text{bare}} + \frac{3\beta}{2} \).

For the model with vacuum decaying

\[ \left( \frac{H}{H_0} \right)^2 = \Omega_m + \Omega_{\Lambda_{\text{bare}}} + \frac{\beta}{H_0^2}T(z)^{-2} \]  

(33)

where

\[ T(z) = -\int_{\infty}^{z} \frac{dz}{(1 + z)H(z)} \]

is the age of the Universe up to the redshift \( z \). For estimation of the model parameters authors of the [15] use such parametrization of \( H(z) \) in which \( T(z) \propto H^{-1} \). Then

\[ \left( \frac{H}{H_0} \right)^2 = \Omega_m + \Omega_{\Lambda_{\text{bare}}} + \frac{\beta}{H_0^2}H^2. \]  

(34)

The acceleration equation has the form

\[ \dot{H} = \frac{\Lambda_{\text{bare}}}{2} - \frac{3}{4}(1 - \beta)H^2. \]  

(35)

After changing the variable \( t \to a \) and solving this equation we will find

\[ \left( \frac{H}{H_0} \right)^2 = \frac{\Omega_m a^{-3(1-\beta)}}{1-\beta} + \frac{\Omega_{\Lambda_{\text{bare}}}}{1-\beta}. \]  

(36)

\[ \Omega_{m0} + \Omega_{\Lambda_{\text{bare}0}} + \beta = 1. \]

As we can see when \( \beta = 0 \) the model reduces to the standard \( \Lambda \)CDM model.

Now we can obtain equation which is expressed only in cosmographical parameters

\[ s + (2 - 3\beta)(q + j) + qj = 0, \]  

(37)

note that when \( \beta = 0 \) we get relation [13] as for \( \Lambda \)CDM model. And \( \beta \) for this model has the form

\[ \beta = \frac{qj + 2(q + j) + s}{3(q + j)}. \]  

(38)

Also be seen that when \( w = -1 \) than \( \beta \) for this model is the same as \( \delta \) in the model with interaction term \( Q = 3H\delta \rho_m \) [26].
3. Observation data and testing models

In this section we will find constraints on the coupling constant by using recent observational data. We use values of cosmographic parameters which was find in [12]. In this work authors used the data of the SNeIa Union 2 compilation by the supernovae cosmology project [13]. They also adopt the HST measurements on 600 Cepheides, which impose a Gaussian prior on the Hubble parameter today of \( H_0 = 74.0 \pm 3.6 \) km/s/Mpc, and the measurements of the Hubble parameter \( H(z) \) at twelve different redshifts ranging from \( z = 0.1 \) to \( z = 1.75 \). All the values that we use are given in the table 1.

For first model with interaction that we considered in the section 2 we get \( 0.2865^{+0.3142}_{-0.3305} \). This value is the same as for \( \Lambda(t) \) model where we found that \( \beta = 0.2865^{+0.3142}_{-0.3305} \). For model where interaction term proportional \( \rho_{de} \) and \( w = -1 \) we get \( \delta = -0.0526^{+0.4803}_{-0.4803} \). These values are given in the table 2.

In Figures 1 and 2 we have given the state parameter dependence on the coupling constants in the two considered models.

4. Summary

We consider some models describing interaction between the dark components and obtain an expression for the coupling constant which contains only the cosmographic parameters. It enables us on the one hand to find constrains
Table 2: Table of $\delta$ for $Q = 3H\delta\rho_m$ and $Q = 3H\delta\rho_{de}$ models with $w = -1$ and $\beta$ for $\Lambda(t)$ model calculated using the best fits of cosmographic parameters $q_0$, $j_0$, $s_0$ and (table 1).

| Model     | $\delta$            |
|-----------|----------------------|
| $Q = 3H\delta\rho_m$ | $0.2865^{+3.3142}_{-3.3305}$ |
| $Q = 3H\delta\rho_{de}$ | $-0.0526^{+0.0693}_{-0.0469}$ |

Figure 1: The state parameter dependence on coupling constant $\delta$ in the $Q = 3H\delta\rho_m$ model.

Figure 2: The state parameter dependence on coupling constant $\delta$ in the $Q = 3H\delta\rho_{de}$ model.

on the coupling constants using observational data, and on the other hand, given fixed constraints on the coupling, to restrict number of numerous models describing the interaction in the dark sector. The method can be used to analyze any cosmological models with known dependence of the density on the scale factor. It was found that for models with couplings $Q = 3H\delta\rho_m$ and $Q = 3H\delta\rho_{de}$ the constant $\delta$ value lies in the theoretically permissible range.

5. Acknowledgement

We are grateful for useful discussions D. A. Yerokhin.
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