Four Quark $cn$-$n\bar{c}$ States in $\tilde{U}(12)$-Scheme and $X(3872)/Y(3940)$

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Abstract

The properties of four quark $cn$-$n\bar{c}$ states are investigated as $cn$ di-quark and $n\bar{c}$ di-antiquark system in $\tilde{U}(12)$-classification scheme of hadrons, recently proposed by us. We consider the negative-parity di-quark and di-antiquark in ground states, which form with the ordinary positive-parity ones the respective linear representations of chiral symmetry. The masses of ground-states are predicted by using Joint Spring Quark Model, and the observed properties of $X(3872)$ and $Y(3940)$ are consistent, respectively, with those of the $J^{PC} = 1^{++}$ and $2^{++}$ states from the negative-parity di-quark and di-antiquark. Their narrow-widths are explained from an orthogonality of spinor wave functions. The properties of ground-state $cs$-$s\bar{c}$ system are also predicted in this scheme.

Key words: $X(3872)$, $Y(3940)$, $\tilde{U}(12)$ group, quark model
PACS: 13.39.Ki, 12.40.Yx

1 Introduction

The discovery of $X(3872)[1]$ and $Y(3940)[2]$ by Belle presented new problems on hadron spectroscopy. Because of their decay-properties, they are considered to be non-$c\bar{c}$ states, and there are some possible explanations for the origin of these states: $D^0\bar{D}^{*0}$ molecule state[3] and four-quark $cn$-$n\bar{c}$ state[4]. In this letter, we consider the four-quark explanation of $X(3872)/Y(3940)$, where the relevant particles are considered as the $cn$ di-quark and $n\bar{c}$ di-antiquark...
system. This explanation is criticized from the observed decay widths. The \( cn\bar{c} \) system decays to \( DD^* \) and \( J/\psi \rho \) (or \( J/\psi \omega \)). They are so-called OZI super-allowed processes, and proceed without no \( q\bar{q} \)-pair production or annihilation. Their widths are expected to be so large as a few hundred MeV or more. On the other hand the observed widths of \( X(3872) \) and \( Y(3940) \) are very small, \( \Gamma_X < 2.3 \text{MeV}[1] \) and \( \Gamma_Y = 87 \pm 22 \pm 26 \text{MeV}[2] \), respectively.

A few years ago[5], we have proposed a covariant level-classification scheme of hadrons based on \( \tilde{U}(12) \) spin-flavor group. It is a relativistic generalization of the \( SU(6)_{SF} \)-scheme in non-relativistic quark model(NRQM), and the squared-mass spectra of hadrons including light constituent quarks are classified as the representations of \( \tilde{U}(12) \). (Concerning the \( \tilde{U}(12) \)-scheme and its group theoretical arguments, see our ref.[6].) The essential difference from \( SU(6)_{SF} \) is a spinor WF. Each spinor index corresponding to light constituent quark in composite hadron WF is expanded by free Dirac spinors \( u_{r,s}(\nu) \) called urciton spinors. At the rest frame \( u_{+,s}(0) \) has the upper two-component which corresponds to the Pauli spinor appearing in NRQM, while the \( u_{-,s}(0) \) has the lower two-component representing the relativistic effect. The \( r \) index of \( u_{r,s} \) represents the eigen-value of \( \rho_3 \), and the corresponding freedom is called \( \rho \)-spin, while the ordinary Pauli-spin freedom is called \( \sigma \)-spin. In the \( \tilde{U}(12) \)-scheme, the \( u_{-} \) is supposed to appear as new degrees of freedom for light constituents, being independent of \( u_{+,s} \), and we predict the existence of a number of extra states out of the \( SU(6)_{SF} \)-framework for meson and baryon systems. These extra states are called chiral states, since \( u_{-} \) is obtained from \( u_{+} \) through chiral \( \gamma_5 \) transformation. The \( \tilde{U}(12) \) representation is considered to be actually realized in light-quark baryon spectra. The ground-state \( qqq \) baryons, \( N \)-octet and \( \Delta \)-decouplet, have positive-parity. Experimentally, the next low-lying states, \( N(1440) \), \( \Lambda(1600) \), \( \Delta(1600) \), \( \cdots \), have also positive-parity, contrarily to the expectation of NRQM. In \( \tilde{U}(12) \)-scheme this parity-doubling problem is resolved in its representation. The \( qqq \)-baryons and antibaryons in ground states form \( 364 \), which includes two \( 56 \) baryons with positive-parity in terms of \( SU(6)_{SF} \). The extra \( 56 \) has \( \rho_3 \) WF, \((+, -, -)\). The masses and strong decay widths of positive-parity baryons, \( N(1440) \), \( \Lambda(1600) \) and \( \Sigma(1660) \), are consistent with this extra \( 56[7] \), while the ordinary radially excited states have large widths of several hundred MeV and are expected not to be observed as resonant particles.

The heavy-light \( Q\bar{q} \) mesons in ground states are classified as \( 12^* \), which includes the chiral \( J^P = 0^+ / 1^+ \) states with positive-parity as well as the ordinary \( 0^- / 1^- \) states. They form linear representations of chiral symmetry[6,8]. The chiral states have \( (\rho_3^Q, \rho_3^\bar{q}) = (+, -) \), and they are expected to appear in smaller mass region as different states from the \( P \)-wave states. The masses

\[ \text{The other (and main) reason of this tiny } \Gamma_X \text{ is explained from its small phase space of the decays to } D^0 D^{*0} \text{ and } J/\psi \rho (\to \pi \pi). \]
As was explained in the previous section, in \( \bar{U}(12)\)-scheme the negative-parity \( cn \) di-quark(\( \bar{n}\bar{c} \) di-antiquark), called chiral state, appears in ground-state. They are denoted as \( \Gamma_{cn}^{P} \) (\( \Gamma_{\bar{n}\bar{c}}^{P} \)), which has negative \( \rho_{3}^{g} \) (\( \rho_{3}^{g} \)), while the ordinary positive-parity di-quark(di-antiquark), called Pauli state, is denoted as \( \Gamma_{cn}^{P} \) (\( \Gamma_{\bar{n}\bar{c}}^{P} \)), which is composed of constituents both with positive \( \rho_{3} \). Their explicit forms are given in Table 1.

### Table 1

| \( (\rho_{3}^{g}, \rho_{3}^{g}) \) | \( J^{P} \) | \( \Gamma_{cn}^{P} \) (Pauli state) | \( (\rho_{3}^{g}, \rho_{3}^{g}) \) | \( J^{P} \) | \( \Gamma_{\bar{n}\bar{c}}^{P} \) (chiral state) |
|---|---|---|---|---|---|
| \(+, +\) | \( 0^{+} \) | \((1 - iv \cdot \gamma) \frac{-\epsilon}{2\sqrt{2}}C^{\dagger}\) | \(+, -\) | \( 0^{-} \) | \((1 - iv \cdot \gamma) \frac{\epsilon}{2\sqrt{2}}C^{\dagger}\) |
| \(+, +\) | \( 1^{+} \) | \((1 - iv \cdot \gamma) \frac{iC \epsilon}{2\sqrt{2}}C^{\dagger}\) | \(+, -\) | \( 1^{-} \) | \((1 - iv \cdot \gamma) \frac{iC \epsilon}{2\sqrt{2}}C^{\dagger}\) |

We have pointed out in our previous work[7,9] a kind of phenomenological conservation rule applicable to chiral states, called \( \rho_{3}\)-line rule. In transitions of chiral states, the \( \rho_{3} \) quantum number along each spectator-quark line is approximately conserved. The violation of \( \rho_{3} \) is suppressed by the factor \( P/2M \), where \( P(M) \) is the final hadron momentum(mass), (not by \( P/2m_{q} \), where \( m_{q} \) is the constituent quark mass).

There is an interesting possibility that the observed \( X(3872) \) and \( Y(3940) \) are chiral states, which are made from negative-parity \( cn \) di-quark and \( \bar{n}\bar{c} \) di-antiquark, and that their narrow-width properties are explained by the \( \rho_{3}\)-line rule, since the relevant decays are quark-rearrangement processes and proceed only through the spectator-quark interaction. The purpose of this letter is to investigate the masses and properties of strong decays of ground \( cn\bar{n}\bar{c} \) states in \( \bar{U}(12)\)-scheme, and to consider whether the \( X(3872) \) and \( Y(3940) \) are explained as chiral \( cn\bar{n}\bar{c} \) states or not.

## 2 Di-quark and Di-antiquark Spinor Wave Function

As was explained in the previous section, in \( \bar{U}(12)\)-scheme the negative-parity \( cn \) di-quark(\( \bar{n}\bar{c} \) di-antiquark), called chiral state, appears in ground-state. They are denoted as \( \Gamma_{cn}^{P} (\Gamma_{\bar{n}\bar{c}}^{P}) \), which has negative \( \rho_{3}^{g} (\rho_{3}^{g}) \), while the ordinary positive-parity di-quark(di-antiquark), called Pauli state, is denoted as \( \Gamma_{cn}^{P} (\Gamma_{\bar{n}\bar{c}}^{P}) \), which is composed of constituents both with positive \( \rho_{3} \). Their explicit forms are given in Table 1.
As is seen from Table 1, aside from $C^\dagger$, the $\Gamma_c^{\bar{n}}$ is the same as the WF of $c\bar{n}$-meson $H_{c\bar{n}}$ in heavy quark effective theory. That is, $\Gamma_c^{\bar{n}} = H_{c\bar{n}}C^\dagger$. The $0^+/1^+$ di-quark WF correspond to $0^-/1^-$ meson WF, and the $0^-/1^-$ chiral di-quark WF correspond to the $0^+/1^+$ chiral meson WF. The chiral $0^+/1^+$ mesons become heavier[6,8] than the $0^-/1^-$ states through spontaneous chiral symmetry breaking($\chi$SB), which is described by the Yukawa coupling to the scalar $\sigma$-meson nonet $s = s^I\lambda^I/\sqrt{2}$ in the framework of $SU(3)$ linear $\sigma$ model[10]. In $Qq$ di-quark systems, because of the $C^\dagger$ factor, the chiral $0^-/1^-$ states are expected to have smaller masses than the $0^+/1^+$ Pauli states. The mass shift $\delta M^{\chi,P}$ in $\chi$SB is given by

$$\delta M^{\chi,P} = g_\sigma s_0 \langle \Gamma^{\chi,P}_{c^{\dagger}}} \bar{C} \Gamma^{\chi,P}_{c^{\dagger}}} \bar{C} \rangle = g_\sigma s_0 \langle H^{\chi,P}_{c^{\dagger}} \bar{C} \Gamma^{\chi,P}_{c^{\dagger}}} \bar{C} \rangle = -g_\sigma s_0 \langle H^{\chi,P}_{c^{\dagger}} \bar{C} \rangle, \quad (1)$$

where $\Gamma^{\chi,P}_{c^{\dagger}} = \gamma_4 (\Gamma^{\chi,P}_{c^{\dagger}})'^{\gamma_4}$, and the change of sign comes from $(C^\dagger)^2 = -1$. The $s_0$ is vacuum expectation value of $s$ and $g_\sigma$ is the Yukawa coupling constant. The chiral mass splitting $\Delta M^{\chi}(\equiv \Delta M^{\chi,P} - \Delta M^{\chi}) = 2g_\sigma s_0$ is predicted with $242$MeV in our previous work[6], which is estimated by using the masses of $D_s(2317)$ and $D_s(2460)$ as inputs.

Concerning $cn-\bar{n}\bar{c}$ system, we expect three combinations of $\Gamma^{\chi,P}_{c^{\dagger}}-\Gamma^{\chi,P}_{c^{\dagger}}$ : The $\Gamma^{P}_{c^{\dagger}}-\Gamma^{P}_{c^{\dagger}}$ denoted as Pauli-Pauli states, the $\Gamma^{P}_{c^{\dagger}}-\Gamma^{\chi,P}_{c^{\dagger}}$ (or $\Gamma^{\chi,P}_{c^{\dagger}}-\Gamma^{P}_{c^{\dagger}}$) denoted as Pauli-chiral (or chiral-Pauli) states, and the $\Gamma^{P}_{c^{\dagger}}-\Gamma^{\chi,P}_{c^{\dagger}}$ denoted as chiral-chiral states. Among them the chiral-chiral states have the smallest masses, the chiral-Pauli states are heavier by $\Delta M^{\chi}$, and the Pauli-Pauli states are further heavier by $\Delta M^{\chi}$. For the transitions between chiral and Pauli states, the amplitudes of pion emission are expected to be very large, since the pseudoscalar Nambu-Goldstone bosons have the spinor WF $i\gamma_5/2$, which changes $\rho_3$ quantum number of the interacting quark. The Pauli-Pauli states decay to Pauli-chiral states through pion emission, and the Pauli-chiral states do to chiral-chiral states. These decays are $S$-wave, and the widths are expected to be so large as several hundred MeV or a GeV. Accordingly, the Pauli-Pauli states and Pauli-chiral states are not observed as resonances, but as non-resonant backgrounds.

This is quite similar mechanism to the situation in $70_G$ baryons in $\tilde{U}(12)$-scheme. They are not observed as resonances because of their large widths of $\pi$ (or $K$) decays to $56_E$ baryons, except for $\Lambda(1405)[7]$. We focus our interests on chiral-chiral states in the following. Their masses are predicted by considering spin-spin interaction in the next section.
3 Oscillator WF and Ground-State Mass Spectra

(Oscillator WF) In order to predict the mass spectra of ground states, it is necessary to estimate quantitatively the spin-spin interaction. The space WF of $cn$-$\bar{n}\bar{c}$ system are required for this purpose, and we use the joint spring quark model (JSQM)\cite{11}, where the $cn$ di-quark ($\bar{n}\bar{c}$ di-antiquark) is in color $3^*(3)$, and they are connected by spring potential as shown in Fig. 1. Correspondingly, the Lagrangian of the four quark system is given by

$$L = \frac{m_c}{2}(\dot{x}_1^2 + \dot{x}_2^2) + \frac{m_n}{2}(\dot{x}_3^2 + \dot{x}_4^2) - \frac{k}{2}(s_1^2 + s_2^2 + s_3^2 + s_4^2) - \frac{k}{2}(r_1^2 - r_2^2), \tag{2}$$

where $x_{1,2} = X + r_1 + s_{1,2}$ and $x_{3,4} = X + r_2 + s_{3,4}$. The definition of each space coordinate is given in Fig. 1. The $m_c(m_n)$ are constituent quark mass of $c(n)$ and $k$ is the spring constant. The analytic formulas for eigen-modes are somewhat complicated, so we show only numerical results.

$$\mathbf{s} = (x_1 - x_2 - x_3 + x_4)/\sqrt{2}, \quad \mathbf{R} = 0.785(x_1 - x_4) + 0.083(x_2 - x_3),$$
$$\mathbf{U} = 0.365(x_1 - x_4) - 0.862(x_2 - x_3), \tag{3}$$

where we take $m_c = M_{J/\psi}/2 = 1.548\text{GeV}$ and $m_n = m_\rho/2 = 0.385\text{GeV}$. The $k$ is determined from the inverse Regge slope of $n\bar{n}$ meson system, $\Omega_{n\bar{n}} = \sqrt{32m_nk} = 1.14\text{GeV}^2$, as $k = 0.107\text{GeV}^3$\cite{12}. We may regard $\mathbf{s}$ as the relative coordinate in di-quark or di-antiquark, $\mathbf{R}(\mathbf{U})$ approximately as relative coordinate between $c$ and $\bar{c}(n$ and $\bar{n})$. The internal space WF is given by
\[ \langle \delta^{(3)}(x_i - x_j) \rangle = -C_{\text{Casimir}} \langle 0 \rangle \]

\[ \frac{\langle \alpha_{\text{eff}} m_i m_j \rangle \delta^{(3)}(x_i - x_j) \sigma^{(i)} \cdot \sigma^{(j)} \rangle}{\langle 0 \rangle} \]

\[ \begin{array}{|c|c|c|}
\hline
\text{pair} & \text{mass} & \text{shift} \\
\hline
 c_1 - n_2, \bar{n}_3 - \bar{c}_4 & 0.0085 \text{GeV}^3 & \frac{2}{3} \text{MeV} \\
 n_2 - \bar{n}_3 & 0.0045 \text{GeV}^3 & \frac{1}{3} \text{MeV} \\
 c_1 - \bar{n}_3, n_2 - \bar{c}_4 & 0.0067 \text{GeV}^3 & \frac{1}{3} \text{MeV} \\
 c_1 - \bar{c}_4 & 0.0123 \text{GeV}^3 & \frac{1}{3} \text{MeV} \\
\hline
\end{array} \]

Table 2

Strength of spin-spin interaction between the constituents.

\[ f_s(s) f_R(R) f_U(U) = \left( \frac{\beta_s \beta_R \beta_U}{\pi^3} \right)^\frac{3}{4} e^{-\frac{\beta_s^2 s}{2}} e^{-\frac{\beta_R^2 R}{2}} e^{-\frac{\beta_U^2 U}{2}}, \]

where \( \beta_s = 0.129 \text{GeV}^2 \), \( \beta_R = 0.254 \text{GeV}^2 \) and \( \beta_U = 0.115 \text{GeV}^2 \).

4 Strong Decays

(Overlapping of spinor WF) The decay mechanism of the relevant quark-rearrangement processes is unknown, however, it may be reasonable to assume that the decay matrix elements are proportional to the overlappings of the
$$\begin{array}{|c|c|c|c|}
\hline
\text{type} & J^{PC} & \Delta M_{SS} & \text{Mass} & \text{decay channels} \\
\hline
\Gamma_{c\bar{n}}^{0^-} & 0^+ & -65 & 3824 & (J/\psi\rho), \eta_c\pi, \, D \bar{D} \\
\frac{1}{\sqrt{2}}(\Gamma_{c\bar{n}}^{0^-} + \Gamma_{c\bar{n}}^{1^-}) & 1^- & -27 & 3862 & J/\psi\pi, \eta_c\rho, \, D \bar{D} \\
\frac{1}{\sqrt{2}}(\Gamma_{c\bar{n}}^{0^-} - \Gamma_{c\bar{n}}^{1^-}) & 1^+ & -17 & 3872 & J/\psi\rho, \, D^0 \bar{D}^{*0} \\
\Gamma_{c\bar{n}}^{1-} & 0^+ & -22 & 3867 & (J/\psi\rho), \eta_c\pi, \, D \bar{D} \\
\Gamma_{c\bar{n}}^{1-} & 1^- & 0. & 3889 & J/\psi\rho, \eta_c\rho, \, D \bar{D}^*(D\text{-wave}) \\
\Gamma_{c\bar{n}}^{1-} & 2^+ & +43 & 3932 & J/\psi\rho, \, D \bar{D}^{(*)}(D\text{-wave}) \\
\hline
\end{array}$$

Table 3

Masses of ground-state $c n-\bar{n}c$ system (in MeV). The mass of $X(3872)$ is used as input. The $\Delta M_{SS}$ are mass-shifts from spin-spin interaction. Possible decay channels of $I=1$ states are shown. For $I=0$ states, $\rho(\pi)$ is replaced with $\omega(\eta)$. The brackets of $(J/\psi\rho)$ for two $0^{++}$ states, of which masses are smaller than $M_{\psi} + m_{\rho}$, mean that this channel is possible as $J/\psi\pi\pi$ through the wide width of $\rho$-meson. The $\Gamma_{c\bar{n}}$ are related with $\Gamma_{c\bar{n}}$ by the charge conjugation transformation, $\Gamma_{c\bar{n}} \to C^{\dagger}\Gamma_{c}\bar{n}C$. The $C^{\dagger}\Gamma_{c\bar{n}}^{0^{+},1^{+}} C = \pm \Gamma_{c\bar{n}}^{0^{+},1^{+}}$, while the $C^{\dagger}\Gamma_{c\bar{n}}^{0^{-},1^{-}} C = \pm \Gamma_{c\bar{n}}^{0^{-},1^{-}}$. These relations are used to define the charge conjugation parity of total $c n-\bar{n}c$ system.

Initial and final WFs. The overlapping of spinor WFs, denoted as $A_{\pm}$, are given for the decays to $c \bar{n} + n \bar{c}$ and $\bar{c} \bar{c} + n n \bar{c}$, respectively, by

$$A_{\pm}(c \bar{n} + n \bar{c}) = \langle \frac{\Gamma X_{c\bar{n}}}{\bar{H} D(v_D)}(iv_D \cdot \gamma) \Gamma^X_{c\bar{n}} \Gamma^X_{c\bar{n}} \rangle$$

$$= \langle \frac{\Gamma X_{c\bar{n}}}{\bar{H} D(v_D)} \Gamma^X_{c\bar{n}} \Gamma^X_{c\bar{n}} \rangle$$

$$A_{\pm}(\bar{c} \bar{c} + n n \bar{c}) = \langle \frac{\Gamma X_{c\bar{n}}}{\bar{H} D(v_D)}(iv_D \cdot \gamma) \Gamma^X_{c\bar{n}} \Gamma^X_{c\bar{n}} \rangle$$

where $D \bar{D}$ and $J/\psi\rho$ are taken as examples of $c \bar{n} + n \bar{c}$ and $\bar{c} \bar{c} + n n \bar{c}$. The $\bar{H}(v)$ is the spinor WF of the final meson with velocity $v_{\mu}(\equiv P_{\mu}/M)$, and $\bar{H}$ is the one at the rest frame with velocity $v_{0\mu} = (0, 0, 0, i)$. They are related by Lorentz booster $B(v)$, $B(v) = c h \theta \pm \rho_1 \rho_2 \cdot \sigma \, s h \theta$, as $\bar{H}(v) = B(v) \bar{H} B(v)$, where the $c h \theta, s h \theta = \sqrt{\frac{E + M}{2M}}$. In Eq. (6), the change of $\rho_3$ on each quark-line in the decay occurs only through $\rho_1$ term in $B(v)$ and $\bar{B}(v)$, which is proportional to $s h \theta$. It is related with the momentum $P$ of the final meson as $c h \theta s h \theta = \frac{P}{2M}$, which is much smaller than unity for the relevant decays because of their small phase space. Thus, the $\rho_3$ quantum number along every the spectator-quark line is approximately conserved.[7]

In the chiral-chiral states, the $n$ in $c n$ di-quark and the $\bar{n}$ in $\bar{n}c$ di-antiquark are in negative $\rho_3$ state. Thus, their decay amplitudes to the Pauli mesons, of which $\rho_3$ of constituents are all positive, are doubly suppressed with the factor $(\frac{P}{2M})^2$. In $\bar{U}(12)$-scheme, the WF of vector mesons[14], $J/\psi, \rho, \omega, D^*$ and $D^*$, are commonly given by $\bar{H} = (1 + iv_0 \cdot \gamma) \frac{iv_0 \gamma}{2\sqrt{2}}$, where the $\rho_3$ of constituents are all positive, $(\rho^0_3, \rho^0_3) = (+, +)$. On the other hand, the WF of pseudoscalar
mesons, π and η, which are Nambu-Goldstone bosons, are \( \overline{H} = \frac{iq}{m} \), which has doubly-negative \( \rho_3 \) component as \((\rho_3^0, \rho_3^\pm) = \frac{1}{\sqrt{2}} ( (+, +) + (-, -) ) \) (, while the WF of \( D \) and \( \eta \) is \( \overline{H} = (1 + iv \cdot q) \frac{q_n}{m} \), which has \((\rho_3^0, \rho_3^\pm) = (++, +) \)). Thus, in the relevant decays of the chiral-chiral states, the transition amplitudes to \( J/\psi \rho \) \((\ln \mathcal{D}) \) (in the relevant decay modes of the chiral-chiral states, the transition amplitudes to \( J/\psi \pi \) \((\ln \mathcal{D}) \) (in the relevant mass region.

Thus, when the initial states have no \( \pi(\eta) \) decay modes, they have narrow widths, and are expected to be observed as resonant particles. The possible decay channels of the ground six states are shown in Table 3. Among them, only two states with \( J^{PC} = 1^{++} \) and \( 2^{++} \) have no \( \pi(\eta) \) decay modes and are considered to be detected as resonances. Concerning these two states the decays to \( \eta_c \pi(\eta) \) are also forbidden from the conservation of heavy quark spin[15]. This theoretical expectation is consistent with the present experimental situation: Only two states, \( X(3872) \) and \( Y(3940) \), are observed as the exotic candidates in the relevant mass region.

(Decay widths) We can also calculate the overlappings of space WFs, denoted as \( \tilde{F}_x \). They are given by

\[
\tilde{F}_x(c\bar{n} + n\bar{c}) = \int d^3p_1 d^3p_2 f_D(p_D) f_D(p_D) \cdot f_s(p_s) f_R(p_R) f_U(p_U),
\]

\[
\tilde{F}_x(c\bar{c} + n\bar{n}) = \int d^3p_1 d^3p_2 f_\psi(p_\psi) f_\rho(p_\rho) \cdot f_s(p_s) f_R(p_R) f_U(p_U),
\]

where the \( p_s \) etc. are internal momenta, the \( f_s(p_s) \)'s are WFs in momentum representation, \( f_s(p_s) = (\frac{1}{\pi\beta_3})^\frac{3}{2} e^{-\frac{p^2}{2}} \). Their numerical values are \( \tilde{F}_x(c\bar{n} + n\bar{c}) = 0.96 \text{GeV}^{-\frac{3}{2}} \) and \( \tilde{F}_x(c\bar{c} + n\bar{n}) = 1.05 \text{GeV}^{-\frac{3}{2}} \), respectively. We neglect the difference between them, and use a common value \( \tilde{F}_x \simeq 1 \text{GeV}^{-\frac{3}{2}} \).

The width \( \Gamma \) of the decay \( X(\text{or} Y) \rightarrow A + B \) is given by

\[
\Gamma = a |P| (E_A E_B M \tilde{F}_x^2) |A_s|^2
\]

with one dimension-less parameter \( a \), where \( M \) is the mass of initial 4 quark state, and \( E_A(E_B) \) is the energy of the final meson \( A(B) \). The \( |P| \) is the momentum of final meson. The explicit forms of \( A_s \) and \( |A_s|^2 \), and the corresponding decay widths for the relevant decay modes of \( X(3872) \) and \( Y(3940) \) are given in Table 4. The parameter \( a \) is fixed with \( a = 18.8 \), which is determined by the total width of \( Y(3940) \), \( \Gamma_Y = 87 \text{MeV} \), as input. The partial decay rates of \( Y(3940) \) are fairly consistent with the present experiments. By taking the \( I=0 \) component \( \cos \theta_Y \simeq 1 \), the main decay mode of \( Y(3940) \) is considered to be \( J/\psi \omega \). Experimentally the \( Y(3940) \) is ob-
\[ A_s \]

| channel | \( A_s \) | \( |A_s|^2 \) | \( \Gamma \) in MeV |
|---------|---------|----------|---------|
| \( Y(3940) \rightarrow J/\psi\omega \) | \(-i\epsilon^2 \epsilon \bar{e} \sqrt{2} \theta \omega \) | \( (\frac{P}{2m_\omega})^4 \) | 60.3 \cos^2 \theta_Y |
| \( Y(3940) \rightarrow D\bar{D} \) | \(-n_i^j \epsilon_i \bar{e} \sqrt{2} P_2 \) | \( (\frac{P}{2M_D})^4 \) | 20.1 |
| \( Y(3940) \rightarrow D\bar{D}^* \) | \( i(n^D \times \epsilon \bar{e}^{D^*}) \epsilon_i \bar{e} \sqrt{2} P_2 \) | \( (\frac{P}{2M_D})^4 \) | 6.6 |

\( X(3872) \rightarrow J/\psi \rho \) | \(-\frac{i}{\sqrt{2}} (\epsilon \times \bar{e}^p) \cdot \bar{e} \sqrt{2} \theta \rho \) | \( (\frac{P}{2m_\rho})^4 \) | \( 1.2 \sin^2 \theta_X \) |
| \( X(3872) \rightarrow J/\psi \omega \) | | \( (\frac{P}{2m_\omega})^4 \) | \( 0.2 \cos^2 \theta_X \) |
| \( X(3872) \rightarrow D^0 \bar{D}^{*0} \) | \( \epsilon_i n_i^j \epsilon \bar{e}^{D^*} \sqrt{2} P_2 \) | \( (\frac{P}{2M_D})^4 \) | \( 0(1.5\times10^{-4}) \) |

Table 4

Decay widths \( \Gamma \) of \( Y(3940) \) and \( X(3872) \). The parameter \( a \) (see, Eq. (8)) is fixed with \( a = 18.8 \) through \( \Gamma_{Y(3940)}^{\text{tot}} = 87 \text{MeV} \) as input. The overlaps of spinor WF, \( A_s \), and their spin-averaged squares \( |A_s|^2 \) are also given. The \( \epsilon_{ij}(\epsilon) \) is the \( J=2(1) \) polarization-tensor of \( Y(X) \). The \( n^D, \bar{n}^{D^*} \) are unit vectors to the directions of \( D, \bar{D}^* \) momenta. For the \( \Gamma \) of \( \rightarrow D\bar{D}^*, \bar{D}^* \) and \( \rightarrow D\bar{D} \) are also included. The \( \cos \theta_{X,Y} (\sin \theta_{X,Y}) \) are \( I=0(1) \) component of the flavor WFs of \( X,Y \). The \( \Gamma(X \rightarrow J/\psi \rho) \), of which decay mode is above the threshold, is given only by multiplying \( (p_\omega)/(p_\rho) \) with \( \Gamma(X \rightarrow J/\psi \omega) \).

\( X(3872) \) is not observed in \( J/\psi \omega \), not in \( D\bar{D} \) and \( D\bar{D}^* \). The \( \Gamma \) of \( X \) are largely dependent upon the treatment of the effective momenta of final mesons. In the case of \( X \rightarrow J/\psi \rho \), it is replaced with the effective momentum \( \langle p_\rho \rangle \), defined by \( \langle p_\rho \rangle = \frac{\sqrt{2} m_\rho \Gamma_\rho}{m_\rho^2 - s + m_\rho \Gamma_\rho} \), where \( \Gamma_\rho \) is the decay width of \( \rho \). The results are shown with parentheses. However, if we take \( \sin^2 \theta_X \approx 1/6 \), their decay properties are also consistent with the present experiments, \( \Gamma_X^{\text{tot}} < 2.3 \text{MeV}[1] \) and \( \Gamma(X \rightarrow J/\psi \rho) \approx 1.0 \pm 0.4 \pm 0.3[16] \).

5 Concluding Remarks

The \( X(3872) \) and \( Y(3940) \) have the properties of the four quark \( cn-n\bar{c} \) states with \( J^{PC} = 1^{++} \) and \( 2^{++} \), respectively, which come from \( 0^{-} -1^{-} \) and \( 1^{-} -1^{-} \) combinations of \( cn \) chiral di-quark and \( n\bar{c} \) chiral di-antiquark in \( \hat{U}(12) \)-scheme. Their mass difference is consistently explained by the spin-spin interaction between constituents in JSQM. Their narrow widths are explained from a phenomenological selection rule, coming from an orthogonality of spinor WFs, called \( \rho_3 \)-line rule. Only these two states are expected to be observed as resonant particles in ground states, and the other states have very large widths and are not observed as resonances but as non-resonant backgrounds.

In order to check this interpretation, we present the predictions for the properties of ground \( cs-s\bar{c} \) states in Table 5. Their masses are estimated simply by adding \( 2(M_{D^*} - M_{D^{*0}}) \approx 210 \text{MeV} \) to the corresponding \( cn-n\bar{c} \) states. Their
| State       | Mass (MeV) | Width (MeV)               |
|-------------|------------|---------------------------|
| $X_{cs-s\bar{c}}^{1++}$ | $\sim 4080$ | $\Gamma_{\text{tot}} < 1\text{MeV}$ |
| $Y_{cs-s\bar{c}}^{2++}$ | $\sim 4150$ | $\Gamma(Y^{2++} \rightarrow D_s\bar{D}_s) = 23\text{MeV}$  
$\Gamma(Y^{2++} \rightarrow D_s\bar{D}_s^*) = 8\text{MeV}$  
$\Gamma(Y^{2++} \rightarrow J/\psi\phi) = 11\text{MeV}$ |

Table 5
The predictions for the ground $cs-s\bar{c}$ states. The $J^{PC} = 1^{++}$ and $2^{++}$ states, denoted, respectively, as $X_{cs-s\bar{c}}^{1++}$ and $Y_{cs-s\bar{c}}^{2++}$, are expected to be observed. Their masses and the partial decay widths of $Y_{cs-s\bar{c}}^{2++}$ are given. Its total width is simply estimated by the sum as $\Gamma_{\text{tot}}^{Y_{cs-s\bar{c}}^{2++}} = 23 + 8 + 11 = 42\text{MeV}$.

decay widths are estimated by using Eq. (8) with the common parameter $a$. The mass of $X_{cs-s\bar{c}}^{1++}$ is expected to be quite close to the $D_s\bar{D}_s$ threshold and its decay is strongly suppressed. The $J/\psi\phi$ is an open channel. Thus, the main decay mode is considered to be electromagnetic ones of order keV. The main decay mode of $Y_{cs-s\bar{c}}^{2++}$ is $D_s\bar{D}_s$. In order to check the four quark nature of this state, it is necessary to observe $J/\psi\phi$ decay.

The authors would like to express their sincere gratitudes to the members of the $\sigma$-group for useful comments and encouragements. M. I. is very grateful to professor M. Oka for valuable comments and discussions.

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