The Solar \textit{hep} Processes in Effective Field Theory

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By combining effective field theory with the standard nuclear physics approach (SNPA) we obtain a high-precision estimate of the $S$ factor for the solar \textit{hep} process. The accurate wave functions available in SNPA are used to evaluate the nuclear matrix elements for the transition operators that result from chiral perturbation theory (ChPT). All the contributions up to $N^3$LO in ChPT are included. The resulting parameter-free, error-controlled prediction is: $S(\text{hep}) = (8.6 \pm 1.3) \times 10^{-20}$ keV-b.

This brief report is based on the results of work done in collaboration with L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky and S. Rosati \cite{2}. A detailed exposition of the basic ideas underlying our approach can be found in \cite{1}.

1 Introduction

The \textit{hep} process in the Sun

\begin{equation}
^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu.
\end{equation}

produces the highest energy solar neutrinos, $E_{\nu}^{\text{max}}(\text{hep}) \approx 20$ MeV. The \textit{hep} neutrinos therefore can influence the interpretation of the results of a recent Super-Kamiokande experiment that have raised many important issues concerning the solar neutrino problem and neutrino oscillations \cite{4,5}. It is to be noted that the reliable estimation of the \textit{hep} cross section, indispensable for addressing these issues, is a long-standing challenge for nuclear physics \cite{6}. This is mainly because the leading one-body contributions are highly suppressed and furthermore the chiral filter mechanism – which allows us to accurately estimate many-body corrections – is ineffective for this process. For a detailed discussion, see Ref. \cite{2,3}.

The objective of our present work is to obtain a significantly improved estimate of the \textit{hep} rate using effective field theory (EFT). To this end, we adopt a strategy that exploits the known merits of the standard nuclear physics approach (SNPA) and heavy-baryon chiral perturbation theory (HBChPT) simultaneously. HBChPT, a well established low-energy EFT of QCD, is used to calculate the transition operators; all the operators up to next-to-next-to-next-to-leading order ($N^3$LO) will be considered. The evaluation of the corresponding nuclear matrix elements requires highly accurate nuclear wave functions. Although it is, at least in principle, possible to derive from HBChPT nuclear wave functions to a specified chiral order, we choose not to do so. Instead, we use realistic wave functions obtained in SNPA. The power of the proposed scheme is the ability to correlate the beta decay processes in the $A = 2, 3, 4$ nuclei. Thus, as explained in more detail below, if the single unknown constant in our EFT is fixed using one of these processes, then we can make totally parameter-free predictions for the remaining processes.
2 Theory and Results

The \textit{hep} process is dominated by the Gamow-Teller (GT) transition, and hence the reliability of the \textit{hep} rate estimate is essentially governed by precision with which one can calculate the GT amplitude. According to the chiral counting rule \cite{9}, the leading order contributions are due to the well-known one-body (1B) operators, and the first corrections arise from $N^3$LO two-body (2B) currents that are suppressed by $(Q/\Lambda_\chi)^3$ compared to the 1B. Here $Q$ stands for the typical three-momentum scale and/or the pion mass, and $\Lambda_\chi \sim 1$ GeV is the chiral scale. As stated, we consider here all the contributions up to $N^3$LO. It is worth emphasizing that three-body currents, which are $N^4$LO, can be legitimately ignored in our $N^3$LO calculation.

The $N^3$LO 2B currents consist of the one-pion-exchange (OPE) and nucleon-nucleon contact-term (CT) parts, $A_{2\text{B}} = A_{2\text{B}}^{\text{OPE}} + A_{2\text{B}}^{\text{CT}}$. With the low-energy constants fixed from $\pi N$ data \cite{13}, the OPE part is completely determined. On the other hand, the CT part contains one parameter, $\hat{d}_R$, whose direct evaluation from QCD is not available at present. Fortunately, it turns out that tritium $\beta$-decay, $\mu$-d capture and $\nu$–d scattering are sensitive to the same parameter, $\hat{d}_R$, and that they do not depend on any additional parameters up to $N^3$LO. Thus, any of these processes can give the renormalization condition to fix the value of $\hat{d}_R$. Here we choose to use the tritium $\beta$-decay rate, $\Gamma_\beta$, which is accurately known experimentally \cite{14}. Once $\hat{d}_R$ is fixed, our calculation involves no unknown parameters.

We calculate the matrix elements of the transition operators with state-of-the-art realistic nuclear wave functions. We employ the correlated-hyperspherical-harmonics (CHH) wave functions, obtained with the Argonne $v_{18}$ (Av18) potential (supplemented with the Urbana-IX three-nucleon potential for the $A \geq 3$ nuclei) \cite{7}. To control short-range physics in a consistent manner, we apply the regulator

$$ S_\Lambda(q^2) = \exp \left( -\frac{q^2}{2\Lambda^2} \right). $$

(2)

to all the nuclear systems in question. The cutoff parameter $\Lambda$ characterizes the energy-momentum scale of our EFT.

The value of $\hat{d}_R$ determined from the experimental value of $\Gamma_\beta$ is $\hat{d}_R = (1.00 \pm 0.07, 1.78 \pm 0.08, 3.90 \pm 0.10)$. Here and hereafter, parenthesized three numbers correspond to the three choices of $\Lambda$, $\Lambda = 500, 600$ and $800$ MeV, in this order. To see the role of the $\hat{d}_R$-term, it is informative to look at $\delta_{2\text{B}}$, the ratio of the 2B matrix element to that of 1B. With only the OPE part taken into account, we have $\delta_{2\text{B}}^{\text{OPE}} = (-1.1, -1.5, -2.0)$. The inclusion of the $\hat{d}_R$ term leads to $\delta_{2\text{B}}^{N^3\text{LO}} = \delta_{2\text{B}}^{\text{OPE}+\text{CT}} = (-0.60, -0.64, -0.73)$. Thus the $\hat{d}_R$-term drastically reduces the $\Lambda$-dependence of the 2B contribution; we see only $\sim 10\%$ variation for the entire range of $\Lambda$ under study. The $\Lambda$-dependence in the total GT amplitude becomes more pronounced due to a strong cancellation between the 1B and 2B terms, but this amplified $\Lambda$-dependence still remains within acceptable levels.

In addition to the $^3S_1$ contributions governed by the GT amplitude, there are also tiny $^1S_0$ and sizable $P$-wave contributions. The latter have little $\Lambda$-dependence ($< 2\%$), and responsible for about one-third of the total $S$ factor. Adding all the contributions, the $S$-factor at threshold reads

$$ S_{\text{hep}}(\text{hep}) = (8.6 \pm 1.3) \times 10^{-20} \text{ keV-b}, $$

(3)

where the “error” spans the range of the $\Lambda$-dependence for $\Lambda = 500$–800 MeV. This result is to be compared to the latest SNPA estimate in Ref. \cite{4}: $S = 9.64 \times 10^{-20} \text{ keV-b}$. 

2
3 Discussion

By determining the only parameter of the theory, $d^R$, from the experimental data on triton beta decay, we have succeeded in estimating the hep S-factor in a parameter-free and error-controlled manner. Our HBChPT calculation (up to $N^3$LO) gives a much more accurate estimate than hitherto available. The EFT results turn out to give support to those obtained in the latest SNPA calculation [7].

To decrease the uncertainty in Eq. (3), we need to reduce the $\Lambda$-dependence in the two-body GT term. According to a general tenet of EFT, the $\Lambda$-dependence should diminish when higher order terms are included. A preliminary study indicates that it is indeed possible to reduce the $\Lambda$-dependence significantly by including $N^4$LO corrections. In this connection, we remark that the hen process, $^3$He + n $\rightarrow$ $^4$He + $\gamma$, seems very interesting to look into. The hen process shares many features with the hep including the suppression of the 1B matrix element and the structure of the many-body currents. Accurate experimental data are available for the hen cross sections, but so far no theoretical calculations have succeeded in explaining the data quantitatively. Thus applying the same EFT technique to the hen process [15] is very interesting, and that will also provide a useful check of the formalism employed in our estimation of $S(hep)$.

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