Physics Beyond Standard Model in Neutron Beta Decay

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Outline

1. Hamiltonians describing physics beyond Standard Model
2. Decay parameters
3. Selection of the data and fits
Historical outlook

Compare: T. D. Lee, C. N. Yang, Phys. Rev. 104, 254 (1956) and e.g. N. Severijns, M. Beck and O. Naviliat-Cuncic, Rev. Mod. Phys. 78, 991 (2006) [arXiv:nucl-ex/0605029].

\[ H_\beta = (\bar{p}n) (\bar{e} (C_S + C'_S \gamma_5) \nu) \]
\[ + (\bar{p} \gamma_\mu n) (\bar{e} \gamma_\mu (C_V + C'_V \gamma_5) \nu) \]
\[ + \frac{1}{2} (\bar{p} \sigma_{\lambda \mu} n) (\bar{e} \sigma_{\lambda \mu} (C_T + C'_T \gamma_5) \nu) \]
\[ + (\bar{p} \gamma_\mu \gamma_5 n) (\bar{e} \gamma_\mu \gamma_5 (C_A + C'_A \gamma_5) \nu) \]
\[ + (\bar{p} \gamma_5 n) (\bar{e} \gamma_5 (C_P + C'_P \gamma_5) \nu) \]
\[ + \text{H.c.} \]

Standard Model (SM): \[ C_A = C'_A = - \frac{V_{ud} G_F}{\sqrt{2}} g_A, \]
\[ C_V = C'_V = \frac{V_{ud} G_F}{\sqrt{2}} g_V \quad \text{for } g_A > 0 \text{ and all other 0.} \]
Historical outlook

Compare: G. Konrad, W. Heil, S. Baessler, D. Pocanic and F. Gluck, arXiv:1007.3027 [nucl-ex].

\[ \mathcal{H}_\beta = \frac{V_{ud} G_F}{\sqrt{2}} \sum_{j=S,V,A,T} \left\{ L_j (\bar{p} \Gamma_j n) \left( \bar{e} \Gamma_j \frac{1 - \gamma_5}{2} \nu \right) \right. \\
\left. + R_j (\bar{p} \Gamma_j n) \left( \bar{e} \Gamma_j \frac{1 + \gamma_5}{2} \nu \right) \right\} + \text{H.c.}, \]

\[ \Gamma_S = 1, \quad \Gamma_V = \gamma_\mu, \quad \Gamma_A = \gamma_5 \gamma_\mu, \quad \Gamma_T = \frac{i}{2 \sqrt{2}} [\gamma_\mu, \gamma_\nu]. \]

SM: \( L_A = -g_A, \quad L_V = g_V \) for \( g_A > 0 \) and all other 0.
\[ H_\beta = 4 \sum_{a,b = L,R} \left\{ a_{ab} \bar{e}\gamma_\mu P_a \nu^{(a)} \bar{u}\gamma^\mu P_b d 
right.
\]
\[ + A_{ab} \bar{e}P_a \nu^{(a)} \bar{u}P_b d 
right.
\]
\[ + \alpha_{aa} \bar{e} \frac{\sigma_{\mu\nu}}{\sqrt{2}} P_a \nu^{(a)} \bar{u} \frac{\sigma^{\mu\nu}}{\sqrt{2}} P_a d \right\} + \text{H.c.,} 
\]

\[ \nu^{(L)} = \sum_i U_{ei} P_L \nu_i, \quad P_L = \frac{1}{2} (1 - \gamma_5), \]
\[ \nu^{(R)} = \sum_i V_{ei} P_R \nu_i, \quad P_R = \frac{1}{2} (1 + \gamma_5). \]

Note: we work in the basis in which mass matrix of charged leptons is diagonal – after the P. Herczeg, Prog. Part. Nucl. Phys. 46 (2001) 413.

\[ \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu], \quad \text{SM: } a_{LL} = \frac{V_{ud} G_F}{\sqrt{2}} \text{ and all other 0.} \]
Neutron beta decay at $q^2 = 0$

P. Herczeg, Prog. Part. Nucl. Phys. 46 (2001) 413.

\[ g_V \bar{u}_p \gamma_\mu u_n = \langle p | \bar{u} \gamma_\mu d | n \rangle, \]
\[ g_A \bar{u}_p \gamma_\mu \gamma_5 u_n = \langle p | \bar{u} \gamma_\mu \gamma_5 d | n \rangle, \]
\[ g_S \bar{u}_p u_n = \langle p | \bar{u} d | n \rangle, \]
\[ g_T \bar{u}_p \sigma_{\mu\nu} u_n = \langle p | \bar{u} \sigma_{\mu\nu} d | n \rangle. \]

In the quark model with spherically symmetric wave functions of quarks

S. L. Adler et al., Phys. Rev. D11, 3309 (1975).

\[ g_V = 1, \quad g_S = -\frac{1}{2} + \frac{9}{10} g_A \approx 0.64, \]
\[ g_A \approx 1.27, \quad g_T = \frac{5}{3} \left( \frac{1}{2} + \frac{3}{10} g_A \right) \approx 1.47. \]
Decay Parameters

Given an amplitude $A$:

$$d\Gamma \sim \sum_{\lambda, \lambda'} \int d_{LIPS} \ (A_\lambda \ \rho_{\lambda, \lambda'} \ A^{*}_{\lambda'}) ,$$

where

$$\rho = U(R_{\bar{n}}) \begin{pmatrix} p_+ & 0 \\ 0 & p_- \end{pmatrix} [U(R_{\bar{n}})]^+ ,$$

$$p_+ + p_- = 1, \quad |\bar{n}| = 1, \quad \tilde{\lambda}_n = (p_+ - p_-) \bar{n}.$$

We are working at tree-level (except: calculation of $\langle E^{-1}_e \rangle$ – see next slides) and with approximate formulas.
\[
\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = p_e E_e E_\nu^2 \xi \left\{ 1 + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b \frac{m_e}{E_e} \right. \\
+ \left. \vec{\lambda}_n \cdot \left[ A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} + D \frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu} \right] \right\},
\]

\[E_\nu = E_0 - E_e, \quad N_{+/-} - \text{number of events, e.g. for } A:\]

\[N_{+/-} \sim \frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} \bigg|_{\vec{\lambda}_n \cdot \vec{p}_e \sim +/}, \quad A \sim \frac{N_+ - N_-}{N_+ + N_-}.\]

We will consider cases when \(a_{ab}, A_{ab}, \alpha_{aa}\) for \(a, b = L, R\) are real then \(D \equiv 0\) and time reversal symmetry is preserved.

**PDG 2010/2011 average:** \(D = (-4 \pm 6) \cdot 10^{-4}\).
e.g. N. Severijns, M. Beck and O. Naviliat-Cuncic, Rev. Mod. Phys. 78, 991 (2006) [arXiv:nucl-ex/0605029], G. Konrad, W. Heil, S. Baessler, D. Pocanic and F. Gluck, arXiv:1007.3027 [nucl-ex].

\[
\bar{a} = \frac{a}{1 + \langle W^{-1} \rangle b}, \quad \bar{A} = \frac{A}{1 + \langle W^{-1} \rangle b},
\]

\[
\bar{B} = \frac{B_I + B_{II} \langle W^{-1} \rangle}{1 + \langle W^{-1} \rangle b}, \quad \langle W^{-1} \rangle = m_e \langle E_e^{-1} \rangle.
\]

Note that \( b = 0 \) and \( B_{II} = 0 \) in SM and for some cases of physics beyond SM.

Our averages (\( E_e^{\text{min}} \) and \( E_e^{\text{max}} \) in general may be different for different experiments):

\[
\langle E_e^{-1} \rangle = \frac{\int_{E_e^{\text{min}}}^{E_e^{\text{max}}} dE_e \frac{d\Gamma}{dE_e} E_e^{-1}}{\int_{E_e^{\text{min}}}^{E_e^{\text{max}}} dE_e \frac{d\Gamma}{dE_e}}.
\]
Fermi function $F(E_e)$ (leading order QED correction) E. Fermi, Z. Phys. 88, 161 (1934) and e.g. P. Huber Phys. Rev. C84, 024617 (2011) [arXiv:1106.0687 [hep-ph]], M. Faber et al., Phys. Rev. C80 (2009) 035503 [arXiv:0906.0959 [hep-ph]], H. F. Schopper, in Weak interactions and nuclear beta decay, North–Holland Publishing Co., Amsterdam, 1966 – approximate $(d\Gamma/dE_e$ and $F(E_e))$ expressions:

$$\frac{d\Gamma}{dE_e} = (g_V^2 + 3g_A^2) \frac{G_F^2|V_{ud}|^2}{2\pi^3} p_e E_e (E_0 - E_e)^2 F(E_e),$$

$$F(E_e) = \frac{2\pi \alpha E_e / p_e}{1 - e^{-2\pi \alpha E_e / p_e}}.$$
Selection of the data

Compare: PDG 2010/2011, N. Severijns et al., Rev. Mod. Phys. 78, 991 (2006).

| PARAMETER | VALUE | ERROR  | $\langle W^{-1} \rangle$ | PAPER ID (PDG) |
|-----------|-------|--------|--------------------------|----------------|
| $a$       | 0.1054| 0.0055 | 0.655                    | BYRNE 02       |
|           | 0.1017| 0.0051 | 0.655                    | STRATOWA 78    |
|           | 0.091 | 0.039  | 0.604                    | GRIGOREV 68    |
| $A$       | 0.11966| 0.00166| 0.557                    | LIU 10         |
|           | 0.1189| 0.0007 | 0.534                    | ABELE 02       |
|           | 0.1160| 0.0015 | 0.582                    | LIAUD 97       |
|           | 0.1135| 0.0014 | 0.558                    | YEROZOLIMSKY 97|
|           | 0.1146| 0.0019 | 0.581                    | BOPP 86        |
| $B$       | 0.980 | 0.005  | 0.599                    | SCHUMANN 07    |
|           | 0.967 | 0.012  | 0.600                    | KREUZ 05       |
|           | 0.9801| 0.0046 | 0.594                    | SEREBROV 98    |
|           | 0.9894| 0.0083 | 0.554                    | KUZNETSOV 95   |
|           | 0.995 | 0.034  | 0.655                    | EROZOLIMSKII 70C|

not used: CHRISTENSEN 70 ($B = 1.00 \pm 0.05$)

We have used most of the $\langle W^{-1} \rangle$ calculated in

N. Severijns et al. Rev. Mod. Phys. 78, 991 (2006).
Why not neutron lifetime?

Figure (modified) from PDG 2010/2011.

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Parameters

Compare: P. Herczeg, Prog. Part. Nucl. Phys. 46 (2001) 413. For $a, b = L, R$:

\[
V_{ab} = \frac{a_{ab} \kappa_{a}}{a_{LL} g_{V}}, \quad S_{ab} = \frac{A_{ab} \kappa_{a} g_{S}}{a_{LL} g_{V}}, \quad T_{ab} = \frac{\alpha_{ab} \kappa_{a} g_{T}}{a_{LL} g_{V}},
\]

\[
\kappa_{L} = 1, \quad \kappa_{R} = \left(\frac{\sum'_{i} |V_{ei}|^2}{\sum'_{i} |U_{ei}|^2}\right)^{1/2},
\]

(summation runs only over kinematically allowed states).

Since we are working in the limit:

\[
\bar{u}_{p} \gamma_{5} u_{n} = 0 \text{ then } \bar{u}_{p} (1 \pm \gamma_{5}) u_{n} = \bar{u}_{p} (1 \mp \gamma_{5}) u_{n}
\]

and we have:

constrains on $S_{LL} \equiv$ constrains on $S_{LR}$,
constrains on $S_{RL} \equiv$ constrains on $S_{RR}$. 
Decay Parameters in SM

Note that $\lambda = \frac{g_A}{g_V} > 0$ in our convention, whereas $\lambda = \frac{g_A}{g_V} < 0$ in PDG 2010/2011 convention (for $g_A$ and $g_V$ real).

$$a = -\frac{\lambda^2 - 1}{3\lambda^2 + 1}, \quad A = -\frac{2\lambda(\lambda - 1)}{3\lambda^2 + 1}, \quad B = \frac{2\lambda(\lambda + 1)}{3\lambda^2 + 1},$$

PDG 2010/2011 average:

$$\lambda = 1.2701 \pm 0.0025 \text{ (error scaled by 1.9).}$$

Our fit:

$$\lambda = 1.2702 \pm 0.0023 \text{ (90\% C.L.)}$$

$$\pm 0.0029 \text{ (95.45\% C.L.)}$$

Note that these are also limits in the case: $V_{ab} = S_{ab} = T_{aa} = 0$ for $a, b = L, R$ except $V_{LR}$:

$$\lambda = \frac{g_A}{g_V} \frac{1 - V_{LR}}{1 + V_{LR}}.$$
Limits in the case: $V_{ab} = S_{ab} = T_{aa} = 0$ for $a, b = L, R$
except one of these: $V_{LR}$ or $V_{RR}$.

In the case of $V_{RR}$ we have $b = 0$ and $B_{II} = 0$. 
Limits in the case: $V_{ab} = S_{ab} = T_{aa} = 0$ for $a$, $b = L$, $R$
except one of these: $S_{LL}$, $S_{LR}$, $S_{RL}$, $S_{RR}$.

In the case of $S_{RL}$ and $S_{RR}$ we have $b = 0$ and $B_{ll} = 0$. 

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**Scalars**

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Physics Beyond Standard Model in Neutron Beta Decay
Limits in the case: \( V_{ab} = S_{ab} = T_{aa} = 0 \) for \( a, b = L, R \)
except one of these: \( T_{LL} \) or \( T_{RR} \).

In the case of \( T_{RR} \) we have \( b = 0 \) and \( B_{ll} = 0 \).
Prospects: calculate the density matrix of antineutrinos from $\beta^-$ decay of nucleus + detection in a near and in a far detector in the case of physics beyond SM:

$$\sigma_{\alpha \rightarrow \beta}(E, L) \sim \sum_{\lambda, \lambda'} \int d_{\text{LIPS}} \left( A_{\lambda}^\beta(E) \rho_{\lambda, \lambda'}^{\alpha}(E, L) (A_{\lambda'}^\beta(E))^* \right).$$
References and backup slides.
PDG 2010/2011 = K. Nakamura et al. (Particle Data Group), J. Phys. G37, 075021 (2010) and 2011 partial update for the 2012 edition, http://pdg.lbl.gov/

| PARAMETER | PAPER ID (PDG) | JOURNAL REFERENCE            |
|-----------|----------------|--------------------------------|
| a         | BYRNE 02       | J. Phys. G28 (2002) 1325       |
|           | STRATOWA 78    | Phys. Rev. D18 (1978) 3970     |
|           | GRIGOREV 68    | Sov. J. Nucl. Phys. 6 (1968) 239 |
| A         | LIU 10         | Phys. Rev. Lett. 105 (2010) 181803 |
|           | ABELE 02       | Phys. Rev. Lett. 88 (2002) 211801 |
|           | LIAUD 97       | Nucl. Phys. A612 (1997) 53     |
|           | YEROZOLIMSKY 97| Phys. Lett. B412 (1997) 240    |
|           | BOPP 86        | Phys. Rev. Lett. 56 (1986) 919  |
| B         | SCHUMANN 07    | Phys. Rev. Lett. 99 (2007) 191803 |
|           | KREUZ 05       | Phys. Lett. B619 (2005) 263    |
|           | SEREBROV 98    | Sov. Phys. ZETF 86 (1998) 1074 |
|           | KUZNETSOV 95   | Phys. Rev. Lett. 75 (1995) 794 |
|           | CHRISTENSEN 70 | Phys. Rev. C1 (1970) 1693      |
|           | EROZOLIMSKII 70C | Phys. Lett. 33B (1970) 351   |
\[ \lambda = \frac{g_A}{g_V} \]  Figure from PDG 2010/2011.

**Weighted Average**
-1.2701 ± 0.0025 (Error scaled by 1.9)

**Table**

| Experiment       | Value | Type  | Value |
|------------------|-------|-------|-------|
| LIU              | 10    | UCNA  | 2.0   |
| ABELE            | 02    | SPEC  | 4.0   |
| MOSTOVOI         | 01    | CNTR  | 0.1   |
| LIAUD            | 97    | TPC   | 1.1   |
| YEROZLIM...      | 97    | CNTR  | 7.9   |
| BOPP             | 86    | SPEC  | 2.6   |

\[ \chi^2 \]

\[ \chi^2 = \frac{(17.7)}{(Confidence Level = 0.0033)} \]
\[ \lambda = \frac{g_A}{g_V} \text{ on lattice} \]

H. Abele, Prog. Part. Nucl. Phys. 60, 1 (2008),
A. A. Khan et al., Phys. Rev. D 74, 094508 (2006) [arXiv:hep-lat/0603028]:

\[ \lambda = -1.26 \pm 0.08_{\text{stat.}} \pm 0.07_{\text{syst.}} \]