D-brane Interactions, World-sheet Parity and Anti-Symmetric Tensor

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Abstract

Using world-sheet parity we show that mixed D and N components of D-strings are dual to the anti-symmetric $B_{\mu\nu}$ field. The contribution of the latter is responsible for the reduction and even removal of all the interactions between two dissimilar D-branes.

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In the past year the study of D-branes and D-strings has attracted much attention since they may be used to obtain insight into non-perturbative structure underlying the strings. They have shown the properties one expects from the extended solutions to field theoeretic low energy limit of the (type-II) string theories mainly at long distances. It was through this kind of study that \textit{RR} charge of D-branes hidden in the simple topological defect, the brane, were discovered [1,2].

Attention has been concentrated mainly on the \textit{RR} charges, which the interaction of similar D-branes reveals it. A particular $D_p$-brane of dimension $p$ can emit and absorb quanta of the $RR \ p+1$ form. Therefore consideration of similar D-brane interactions manifests their $RR$-charge. On the other hand consideration of dissimilar branes hides their $RR$-charge and may reveal other properties hidden by similarity [3].

Recently we have found [4] that in certain conditions the gravitational attraction of D-branes can be balanced or even reversed. In this note we will show that the balancing force is due to the exchange of anti-symmetric $B_{\mu\nu}$ tensor which doesn’t participate in the interaction of similar branes because they form symmetric states.

To extract this information we exploit the anti-symmetric nature of the amplitude under world-sheet parity. This is done by insertion of the appropriate operator in the corresponding trace for the calculation of the amplitude.

We will show that the $DN$ modes of the string stretched between branes are dual to the $B_{\mu\nu}$ tensor fields of closed channel. The dependence of the effect of $B_{\mu\nu}$ on the relative angle and velocity is also discussed which will give us a handle for probing short distance behaviour of D-branes.

We impose the proper boundary conditions on the string with the ends on two parallel branes of dimensions $p, p'$ as given in [4]. The boundary conditions on the fermionic part is induced by world-sheet SUSY. We have two kinds of open D-strings $NS$ and $R$ type. The mode expansions of the bosonic and fermionic variables are:

\[
\begin{align*}
    NN \text{ components } & \quad X^\mu = p^\mu \tau + \sum_{n \in \mathbb{Z}} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma \quad \mu = 0, \ldots, p' \\
    DN \text{ components } & \quad = \sum_{r \in \mathbb{Z}+1/2} \frac{1}{r} \alpha_r^\mu e^{-ir\tau} \sin r\sigma \quad \mu = p' + 1, \ldots, p \\
    DD \text{ components } & \quad = Y^\mu \pi + \sum_{n \in \mathbb{Z}} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \sin n\sigma \quad \mu = p + 1, \ldots, 9.
\end{align*}
\]
for both $R$ and $NS$ types, and for the $\psi_\pm^\mu$, world-sheet fermions:

$$
\begin{align*}
\psi_+^\mu &= \sum_{n \in \mathbb{Z}} d_n^\mu e^{-i n_\tau + \sigma} \\
\psi_-^\mu &= \sum_{n \in \mathbb{Z}} d_n^\mu e^{-i n_\tau - \sigma} \\
\psi_+^\mu &= \sum_{n \in \mathbb{Z}} d_n^\mu e^{-i n_\sigma} \\
\psi_-^\mu &= \sum_{n \in \mathbb{Z}} d_n^\mu e^{-i n_\sigma}
\end{align*}
$$

(2)

for the $R$-sector and

$$
\begin{align*}
\psi_+^\mu &= \sum_{r \in \mathbb{Z} + 1/2} d_r^\mu e^{-i r_\tau + \sigma} \\
\psi_-^\mu &= \sum_{r \in \mathbb{Z} + 1/2} d_r^\mu e^{-i r_\tau - \sigma} \\
\psi_+^\mu &= \sum_{n \in \mathbb{Z}} d_n^\mu e^{-i n_\sigma} \\
\psi_-^\mu &= \sum_{n \in \mathbb{Z}} d_n^\mu e^{-i n_\sigma}
\end{align*}
$$

(3)

for the $NS$-sector.

Following the general approach for studying the interaction of D-branes [2] we consider a world-sheet corresponding to the exchange of a closed strings between the branes which is the same as a loop of D-string in the crossed channel. The focus of our attention is on the world-sheet parity. From the closed string point of view the world-sheet parity which can be introduced by $\sigma \leftrightarrow \pi - \sigma$ or equivalently $z \leftrightarrow -\bar{z}$, interchanges left moving and right moving modes.

The graviton and dilaton and $RR$ fields are even under this transformations, but $B_{\mu\nu}$ (anti-symmetric tensor field) is odd [2]. On the other hand $z \leftrightarrow -\bar{z}$ also acts as world-sheet parity on the open D-strings such that

$$
\begin{align*}
Y^\mu &\rightarrow -Y^\mu \quad ; \quad p^\mu \rightarrow p^\mu \\
\alpha_n^\mu &\rightarrow (-)^n \alpha_n^\mu \quad ; \quad \alpha_r^\mu \rightarrow (-)^{r-1/2} \alpha_r^\mu.
\end{align*}
$$

(4)

As is easily seen $DD, NN$ components of D-strings are even and $DN$ components are odd.

In the case of two similar parallel D-branes the system is symmetric under world-sheet parity and therefore the $B_{\mu\nu}$ field doesn’t contribute to the interactions, but when the D-branes are of different dimensions or not-parallel the $DN$ components of the open strings are present which give rise to a non-world-sheet parity invariant contribution manifested as the exchange of anti-symmetric $B_{\mu\nu}$ field in the closed channel. These observations are sufficient to show that the $DN$ modes in open string channel and anti-symmetric tensor field in the closed string channel are dual to each other.
They represent the contribution to amplitude originating from the odd fields under $z \leftrightarrow -\bar{z}$.

To make the point more clear we can gauge the theory with respect to world-sheet parity transformation which removes all the odd fields and keeps only the invariant (even) parts.

In the following we will use the above consideration to represent the world-sheet parity odd and even contributions to the amplitude and show that the share of graviton-dilaton and $B_{\mu\nu}$ enter with opposite sign.

The amplitude for the interaction in our discussions is given by:

$$ A = \int \frac{dt}{2t} \sum_{i,p} e^{-2\pi\alpha' t(p^2 + M_i^2)} $$

where $i$ indicates the modes of the open string and $p$ indicate momentum which has non-zero components in the first $p' + 1$ dimensions.

In order to calculate the "Trace" we have to insert the corresponding GSO projections for NS, R sectors separately to remove the tachyon contribution from the closed string channel. We can also introduce the world-sheet parity projection which separates $B_{\mu\nu}$ and $\{G_{\mu\nu}, \Phi\}$ (odd and even) contributions. Doing so we obtain the amplitude $A = A_+ + A_-$,

$$ A = V_{p'+1} \int \frac{dt}{2t} (8\pi^2\alpha')^{-(p'/2)} e^{-\frac{q^2}{2\pi^2\alpha'}} (\text{NS}_- - \text{R}_+) $$

where $\text{NS}_\pm$ and $\text{R}_\pm$ are given by

$$ \text{NS}_\pm = 2^{-2} \left( \frac{f_1}{f_3} \right)^{-8+\Delta} \left[ \left( \frac{f_2}{f_4} \right)^\Delta \pm \left( \frac{f_3}{f_4} \right)^\Delta \right] $$

$$ \text{R}_\pm = 2^{-2} \left( \frac{f_1}{f_2} \right)^{-8+\Delta} \left[ \left( \frac{f_3}{f_4} \right)^\Delta \pm \left( \frac{f_2}{f_4} \right)^\Delta \right] $$

where $q = e^{-\pi t}$ and $\pm$ stands for positive and negative parity of exchanged closed string respectively and $f_i$ are given in terms of $\Theta$ functions by:

$$ f_1 = (\frac{\Theta_1'}{2\pi})^{1/3} \quad f_2 = f_1 (\frac{2\pi \Theta_1}{\Theta_1'})^{1/2} $$

$$ f_3 = f_1 (\frac{2\pi \Theta_1}{\Theta_1'})^{1/2} \quad f_4 = f_1 (\frac{2\pi \Theta_1}{\Theta_1'})^{1/2} $$

Before extracting the contribution of the dilaton and graviton and $B_{\mu\nu}$ we note that when the number of symmetric D-string components and anti-symmetric components are equal the amplitude vanishes exactly. This occurs when

$$ p' + (8 - p) = p - p' \quad \text{or} \quad \Delta = 4. $$
As is shown in [4] the cancellation takes place order by order in all powers of $t$.

Taking the small $t$ limit of the integrand we can separate the massless closed string, the dilaton and graviton and $B_{\mu\nu}$, contributions which show the long range interactions:

$$A_{B_{\mu\nu}} = -\frac{1}{4} 2^{3-\Delta} \pi G_{9-p} (Y^2),$$  \hspace{1cm} (11)$$

$$A_{G,\Phi} = \frac{1}{4} 2^{3-\Delta} \pi G_{9-p} (8 - \Delta) G_{9-p} (Y^2).$$  \hspace{1cm} (12)$$

It is worth noting that since the branes are of different dimensions there is no RR interaction, besides the $B_{\mu\nu}$ charge density of the D-branes is the same as their mass density and equal to $(4\pi^2 \alpha')^{(3-p)/2}$.

Similar analysis works for branes at non-zero angle considered in [4]. For these branes complete cancellation occurs at $\Delta = 2, \theta = 1/2$ ($p, p+2$ perpendicular D-branes). Again in this case we have four world-sheet parity symmetric components and four anti-symmetric components. Although $\Delta = 2$ there are two other anti-symmetric components having their origin in the angle $\pi/2$ between the branes.

Since the cancellation takes place in all orders of $t$, we can smoothly take $Y$, the distance between branes, to be zero and obtain a composite brane with two different RR fields, dilaton and graviton and $B_{\mu\nu}$ field [5,6,7,8,9,10,11], one consisting of two perpendicular intersecting parts with $\Delta = 2$ and the other consisting of a p-brane with a $(p-4)$-brane sitting in it.

Similar results could be obtained in low energy limit from DBI action which governs D-brane low energy dynamics (it only concerns the exchange of massless fields between D-branes.)

$DBI$ action for bosonic varibles is given by [12,13]:

$$S = T_p \int d^{p+1} \eta \sqrt{det(h + F)}$$  \hspace{1cm} (13)$$

where $\eta$ is the world-brane coordinates and

$$h_{ab} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \eta^a} \frac{\partial X^\nu}{\partial \eta^b}$$  \hspace{1cm} (14)$$

is the induced metric on the brane. $F$ is the $U(1)$ gauge field strength:

$$F = B_{\mu\nu} \frac{\partial X^\mu}{\partial \eta^a} \frac{\partial X^\nu}{\partial \eta^b} + F_{ab}; \quad F = dA.$$  \hspace{1cm} (15)$$

The action in the flat back ground for weak field limit could be expanded as:

$$S = -T_p \int_{brane} Tr F^2 + O(F^3)$$  \hspace{1cm} (16)$$
which is the usual $U(1)$ gauge action. In order to study the dynamics of D-brane and $B_{\mu\nu}$ we must add the kinetic term of $B_{\mu\nu}$ to this action [11]:

$$S = \int_{\mathcal{M}} |dB|^2 - T_p \int_{brane} Tr F^2$$

(17)

where $\mathcal{M}$ is the space-time manifold, and the brane as a flat submanifold of it.

By using reparametrization invariance for $\eta$, the coordinates $X^\mu$ (space-time coordinates) could be chosen as:

$$X^\mu = \eta^a \delta^\mu_a + \Phi^A \delta^\mu_A$$

(18)

where $a = 1, \ldots, p$ and $A = p+1, \ldots, D = dim \mathcal{M}$. $\Phi^A|_{brane} = \Phi^A(\eta)$, are scalars respect to the brane, showing D-brane transverse fluctuations. $A_a(\eta)$ is $U(1)$ gauge field which represents the internal modes of brane.

In terms of this new coordinates the action (17) up to $O(\Phi^2)$ is written as:

$$S = \int_{\mathcal{M}} |dB|^2 - T_p \int_{brane} (B_{ab} - F_{ab})^2 - 2T_p \int_{brane} (B_{ab} - F_{ab})B_{aA}\partial_b\Phi^A.$$  

(19)

Hence $A_a$ and $B_{\mu\nu}$ field equations are (by using gauge freedom):

$$\left\{ \begin{array}{l}
\Box DB_{\mu\nu} = T_p F_{ab} \delta^\mu_a \delta^\nu_b \delta(bran0) \partial_a\partial_b \Phi^A = 0 \\
\end{array} \right.$$  

(20)

As we see $F_{ab}$ acts as a $B_{\mu\nu}$ source(charge) and also the action written above gives the $B_{\mu\nu}$ interaction of D-branes, which comes from the third term of action(the second term gives no contribution to interaction). This term vanishes in the case of two similar branes. It is proportional to number of DN modes because of $B_{aA}$ term. The same argument is held when we consider gravity (non-flat metric). So D-brane has $B_{\mu\nu}$ charge because of both its internal modes ($F_{ab}$) and also its transverse fluctuations($\partial_a\Phi^A$).

In string theoretic point of view these scalars and $U(1)$ fields can be interpreted as D and N components of D-strings respectively which are attached to one D-brane. As it is discussed in [11] D-strings (or D-branes) are $B_{\mu\nu}$ charged.

Extension of our analysis to moving branes shows that when branes have relative velocity $v$ exchange of $B_{\mu\nu}$ and graviton both give the non-zero velocity dependent potential [4]:

$$V_{NSNS} = -2V_0\left(\frac{4 - \Delta - v^2(2 - \Delta)}{1 - v^2}\right) ; \quad V_0 = 2^\Delta V_p(4\pi^2\alpha')^{3-(p+p')/2}G_{9-p}(Y^2).$$

(21)

which in $\Delta = 4$ non-relativistic limit it’s proportional to $v^2$, and potential is attractive. By parity arguments it is easily seen that graviton and dilaton contribution is proportional
to $\frac{1-v^2/2}{1-v^2}$. This point can be used to probe $B_{\mu\nu}$ behaviour of $D_p$-branes by $(p - 4)$ or $(p - 2)$ D-branes [14,15].

The particular cases of interest are:

In type IIA theory
- $D_0$ and $D_4$ branes
- $D_2$ and $D_4$ branes perpendicular
- $D_2$ and $D_6$ branes
- $D_4$ and $D_6$ branes perpendicular

In type IIB:
- $D_1$ and $D_5$ branes
- $D_1$ and $D_3$ branes at $\theta = \pi/2$
- $D_3$ and $D_5$ branes at $\theta = \pi/2$

SUGRA solutions with some of the above configurations have been found and we conjecture that the others also exist as p-brane solutions. Work in this direction is being pursued.

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References

[1] J. Polchinski, S. Chaudhuri, and C.V. Johnson, ”Notes on D-Branes,” [hep-th/9602052]
[2] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.
[3] G. Lifschytz, ”Comparing D-branes to Black-branes”, BRX-TH-394, [hep-th/9604156]
[4] H Arfaei, M.M. Sheikh-Jabbari, ”Different D-brane interactions”, IPM-96-163, [hep-th/9608167]
[5] M.J. Duff and J.X. Lu, ”Black and Super P-brane in Diverse Dimensions”, CERN-TH.6675/93, CTP/TAMU-54/92. M.J. Duff and J.X. Lu, Ramzi R. Khuri phys. Reports 295 (1995) 213.
[6] G.T. Horowitz and A. Strominger, Nucl. Phys. B360 (1991) 197.
[7] Miguel S. Costa, ”Composite M-branes”, [hep-th/9609181]
[8] Jerome P. Gauntlett, David A. Kastor, Jennie Traschen, ”Overlapping Branes in M-theory”, hep-th/9604179.

[9] G. Papadopolous, P.K. Townsend, ”Intersecting M-branes”, hep-th/9603087.

[10] A.A. Tseytlin, ”No Force Condition and BPS Combinations of P-brane in 11 and 10 dimensions”, hep-th/9609212.

[11] E. Witten, Nucl. Phys. B460 (1996) 335, hep-th/9510133.

[12] J. Dai, G. Leigh, J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073.

[13] G. Leigh, Mod. Phys. Lett. A4 (1989) 2767.

[14] M.R. Douglas, D. Kabat, P. Pouliot, and S.H. Shenker, ”D-branes and Short Distances in String Theory”, hep-th/9608024.

[15] S.H. Shenker, ”Another Length Scale in String Theory?” preprint RU-95-53, hep-th/9509132.