Constraining non-minimally coupled tachyon fields by the Noether symmetry

Rudinei C de Souza and Gilberto M Kremer

Departamento de Física, Universidade Federal do Paraná, Curitiba, Brazil
E-mail: kremer@fisica.ufpr.br

Received 8 January 2009, in final form 19 May 2009
Published 8 June 2009
Online at stacks.iop.org/CQG/26/135008

Abstract
A model for a homogeneous and isotropic Universe whose gravitational sources are a pressureless matter field and a tachyon field non-minimally coupled to the gravitational field is analyzed. The Noether symmetry is used to find expressions for the potential density and for the coupling function, and it is shown that both must be exponential functions of the tachyon field. Two cosmological solutions are investigated: (i) for the early Universe whose only source of gravitational field is a non-minimally coupled tachyon field which behaves as an inflaton and leads to an exponential accelerated expansion and (ii) for the late Universe whose gravitational sources are a pressureless matter field and a non-minimally coupled tachyon field which plays the role of dark energy and is responsible for the decelerated–accelerated transition period.

PACS numbers: 98.80.–k, 98.80.Cq, 95.35.+d

1. Introduction
Currently, it is well accepted by the scientific community that the evolution of the Universe started with an exponential accelerated expansion dominated by an entity called inflaton followed by a decelerated period dominated by matter fields and that the Universe has entered into a new accelerated period governed by a dark energy field.

Search for models that could explain satisfactorily the inflationary period as well as the present accelerated expansion is an object of intense interest of the researchers in cosmology. Models which explain the accelerated expansion of the Universe are based on scalar (see e.g. [1]) or fermion (see e.g. [2]) fields that are minimally or non-minimally coupled to the gravitational field and which can simulate the inflaton in the primordial era and the dark energy in the present period.

1 Author to whom any correspondence should be addressed.
Within the context of scalar fields, the models with tachyon fields minimally coupled to the gravitational field received considerable attention from researchers. The tachyon field has its roots in string theory, but it can be introduced in a simple manner as a generalization of the Lagrangian density of a relativistic particle, i.e. $\mathcal{L} = -m\sqrt{1 - \dot{q}^2}$ → $\mathcal{L}_\phi = -V(\phi)\sqrt{1 - \partial^\mu\phi\partial_\mu\phi}$, in the same way as the quintessence could be considered as a generalization of the Lagrangian density for a non-relativistic particle, namely $\mathcal{L} = \dot{q}^2/2 - V(q) → \mathcal{L}_\phi = \partial^\mu\phi\partial_\mu\phi/2 - V(\phi)$. In this work, the role of the tachyon field within a cosmological framework is discussed and the search for any relationship of the tachyon field to its root in string theory is not considered.

In [3], the inflationary period of the Universe was investigated by using tachyon fields minimally coupled to the gravitational field for different self-interaction potential densities, in the form of power law, exponential and hyperbolic functions of the tachyon field. Tachyon fields with such kinds of potential densities were also used in order to describe the present acceleration period of the Universe in [4] where the tachyon field behaves as dark energy. Furthermore, in work [5] a tachyon field with an exponential potential density which can play the role of the inflaton and dark energy was also studied. Although in the majority of these papers the tachyon field is minimally coupled to the gravitational field, in [6] a non-minimally coupled tachyon field with potential densities and coupling functions given by power-law functions was investigated.

In all the above-quoted works the potential densities were proposed in an ad hoc way, with the aim of determining desired cosmological solutions. The objective of the present work is to analyze a generic model for a homogeneous and isotropic Universe whose gravitational sources are a matter field and a tachyon field non-minimally coupled to the gravitational field. The present model appears to be interesting because it can describe in a reasonable way what is observed, since via the Noether symmetry approach we can find a unique form for the potential and coupling which describe the inflationary and the decelerated–accelerated period. The forms of the potential density and coupling function are determined from the Noether symmetry applied to the Lagrangian density which describes the model. Thus, in this paper Noether’s symmetry approach works as a criterion for the selection of the couplings and potentials instead of proposing their forms in an ad hoc way. By this proceeding, we can restrict the possibilities of choosing the functions which partially fix the potentials and couplings. Moreover, the existence of this symmetry guarantees conserved quantities, which provide a motion constant that can help to integrate the field equations. Constraining scalar fields by the Noether symmetry is a subject of several papers in the literature (see e.g. [7]). In a recent paper [8], the authors applied the Noether symmetry for a fermion field non-minimally coupled with the gravitational field.

The evolution equations of the Universe follow from Einstein’s field equations and the Klein–Gordon equation for the coupled tachyon field which are solved for a given potential density and coupling function obtained from the Noether symmetry. It is shown that a non-minimally coupled tachyon field could play the role of an inflaton describing the exponential accelerated expansion in the early Universe and it could behave as dark energy describing the decelerated–accelerated transition period of the late Universe.

This work is organized as follows. In section 2, Einstein and Klein–Gordon equations are derived from a point-like Lagrangian obtained from the action for a non-minimally coupled tachyon field and for a Friedmann–Robertson–Walker metric. The purpose of section 3 is the determination of the possible forms of the self-interaction potential density and of the coupling function from the Noether symmetry. The search of an inflationary cosmological solution is the subject of section 4 whereas in section 5 the decelerated–accelerated transition period is analyzed. Final remarks and conclusions in section 6 close this work. From now on
the signature \((+, -, -, -)\) is adopted, the natural units \(8\pi G = c = \hbar = 1\) whereas the Ricci scalar in terms of the cosmic scale factor \(a(t)\) is given by
\[
R = 6 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right).
\]

2. Point-like Lagrangian and field equations

Let \(\phi\) denote a rolling tachyon field with a Lagrangian density \(L_{\phi} = -V(\phi)\sqrt{1 - \partial_{\mu}\phi\partial^{\mu}\phi}\), where \(V(\phi)\) is the self-interaction potential density. The action for a tachyon field non-minimally coupled to the gravitational field is written as
\[
S = \int \sqrt{-g} \, d^4 x \left\{ F(\phi) R - V(\phi) \sqrt{1 - \partial_{\mu}\phi\partial^{\mu}\phi} \right\} + S_m, \tag{1}
\]
where \(S_m\) is the action of a pressureless matter field, \(R\) denotes the Ricci scalar and \(F(\phi)\) represents an arbitrary \(C^2\) function of the tachyon field which is related to the coupling of the tachyon field with the gravitational field.

A homogeneous and isotropic Universe described by the Friedmann–Robertson–Walker metric is considered:
\[
ds^2 = dt^2 - a(t)^2 \left[ dr^2 + r^2(\text{d} \theta^2 + \sin^2 \theta \, d\phi^2) \right], \tag{2}
\]
where \(k = 0, \pm 1\).

The point-like Lagrangian which follows from (1) through a partial integration and by considering a homogeneous scalar field \(\phi = \phi(t)\) reads as
\[
L = 6a\dot{a}F + 6a^2 \dot{\phi} \frac{dF}{d\phi} - 6kaF + a^3 V \sqrt{1 - \dot{\phi}^2} + \rho_m^0. \tag{3}
\]

Above, \(\rho_m^0\) is a constant value of the energy density of the matter field referred to as an initial state and the dot denotes a derivative with respect to time.

The Klein–Gordon equation for the coupled tachyon field is obtained from the point-like Lagrangian (3) through the use of the Euler–Lagrange equation for \(\phi\), yielding
\[
\ddot{\phi} + 3H\dot{\phi} + \frac{1}{V} \left[ \frac{dV}{d\phi} - 6 \left( \dot{H} + 2H^2 + \frac{k}{a^2} \right) \sqrt{1 - \dot{\phi}^2} \frac{dF}{d\phi} \right] = 0, \tag{4}
\]
where \(H = \dot{a}(t)/a(t)\) denotes the Hubble parameter.

Likewise, from the Euler–Lagrange equation for the cosmic scale factor \(a\) applied to the point-like Lagrangian (3) one can obtain the acceleration equation, namely
\[
\frac{\ddot{a}}{a} = -\frac{\rho + 3p}{12F}. \tag{5}
\]

In the above equation, \(\rho = \rho_\phi + \rho_m\) and \(p = p_\phi\) are the energy density and the pressure of the sources of the gravitational field respectively. The energy density and the pressure of the tachyon field are given by
\[
\rho_\phi = \frac{V}{\sqrt{1 - \dot{\phi}^2}} - 6H \frac{dF}{d\phi} \dot{\phi}, \tag{6}
\]
\[
p_\phi = -V \sqrt{1 - \dot{\phi}^2} + 2 \left( \frac{dF}{d\phi} \dot{\phi} + 2H \frac{dF}{d\phi} \dot{\phi} + \frac{d^2F}{d\phi^2} \dot{\phi}^2 \right), \tag{7}
\]
respectively. Note that the matter field is considered as a pressureless fluid, i.e. \(p_m = 0\).
One can obtain Friedmann’s equation

\[ H^2 = \frac{\rho}{6F} - \frac{k}{a^2} \]  

(8)

by imposing the fact that the energy function associated with the point-like Lagrangian vanishes, i.e.

\[ E_{\mathcal{L}} = \frac{\partial \mathcal{L}}{\partial \dot{a}} \dot{a} + \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \dot{\phi} - \mathcal{L} \equiv 0. \]  

(9)

The above expression turns out to be another independent field equation when one considers a Friedmann–Robertson–Walker metric and a homogeneous scalar field in action (1).

3. Constraints from the Noether symmetry

Let \( X \) be the following infinitesimal generator of symmetry:

\[ X = \alpha \frac{\partial}{\partial a} + \beta \frac{\partial}{\partial \phi} + \left( \frac{\partial \alpha}{\partial a} \dot{a} + \frac{\partial \alpha}{\partial \phi} \dot{\phi} \right) \frac{\partial}{\partial \dot{a}} + \left( \frac{\partial \beta}{\partial a} \dot{a} + \frac{\partial \beta}{\partial \phi} \dot{\phi} \right) \frac{\partial}{\partial \dot{\phi}}, \]  

(10)

where \( \alpha \) and \( \beta \) are only functions of \((a, \phi)\). Furthermore, let \( L_x \) denote Lie’s derivative of the point-like Lagrangian \( \mathcal{L} \) with respect to the vector \( X \) which is defined in the tangent space.

Noether’s symmetry is satisfied by the condition \( L_x \mathcal{L} = 0 \), i.e. \( X \mathcal{L} = 0 \) which implies

\[ \alpha \left( 6a^2 F + 12a \dot{a} \dot{\phi} \frac{dF}{d\dot{\phi}} + 3a^2 V \sqrt{1 - \dot{\phi}^2} - 6k F \right) + \beta \left( 6a \dot{a} \frac{dF}{d\dot{\phi}} + 6a^2 \dot{\phi} \frac{d^2 F}{d\dot{\phi}^2} \right) \]

\[ + a^2 \left( \frac{dV}{d\phi} \sqrt{1 - \dot{\phi}^2} - 6ka \frac{dF}{d\phi} \right) + \left( \frac{\partial \alpha}{\partial a} \dot{a} + \frac{\partial \alpha}{\partial \phi} \dot{\phi} \right) \left( 12a \dot{a} F + 6a^2 \dot{\phi} \frac{dF}{d\phi} \right) \]

\[ + \left( \frac{\partial \beta}{\partial a} \dot{a} + \frac{\partial \beta}{\partial \phi} \dot{\phi} \right) \left( 6a^2 \dot{a} \frac{dF}{d\phi} - \frac{a^3 \dot{\phi} V}{\sqrt{1 - \dot{\phi}^2}} \right) = 0. \]  

(11)

After some rearrangements equation (11) depends explicitly on \( \dot{a}, \dot{\phi} \), their powers and combinations with \( \sqrt{1 - \dot{\phi}^2} \); hence their coefficients must vanish, yielding

\[ \left( \alpha + 2a \frac{\partial \alpha}{\partial a} \right) F + \left( \beta + a \frac{\partial \beta}{\partial a} \right) a \frac{dF}{d\phi} = 0, \]  

(12)

\[ 2a \frac{\partial \alpha}{\partial \phi} + \left( 2a + a \frac{\partial \alpha}{\partial a} + a \frac{\partial \beta}{\partial \phi} \right) \frac{dF}{d\phi} \frac{d^2 F}{d\phi^2} = 0, \]  

(13)

\[ \frac{\partial \alpha}{\partial \phi} \frac{dF}{d\phi} = 0, \quad \frac{\partial \beta}{\partial a} = 0, \quad \frac{\partial \beta}{\partial \phi} = 0, \]  

(14)

\[ 3a V + a \beta \frac{dV}{d\phi} = 0, \]  

(15)

\[ \alpha F + a \beta \frac{dF}{d\phi} = 0. \]  

(16)

Equations (12)–(16) are obtained by imposing the fact that the coefficients of \( \dot{a}^2, \dot{\phi}, \dot{\phi}^2, \dot{a} \phi/\sqrt{1 - \dot{\phi}^2}, \dot{\phi}^2/\sqrt{1 - \dot{\phi}^2} \) and \( \sqrt{1 - \dot{\phi}^2} \) vanish. Moreover, it was supposed that \( \dot{a} \) and \( \dot{\phi} \) do not vanish during the time evolution of the Universe described by this model and that the
restriction $\dot{\phi}^2 \neq 1$ holds. Observe that the simplifications related to $a$—which were done to obtain equations (12)–(16)—are possible because it was considered that $a \neq 0$.

The analysis of the system of equations (12) through (16) proceeds as follows. First, from equations (14) and (14) one infers that $\beta \equiv \beta_0$ must be constant. Next, from equation (14), two cases must be considered, namely $dF/d\phi = 0$ and $dF/d\phi \neq 0$.

Equation (16) cannot be satisfied simultaneously with the rest of the system for the case $dF/d\phi \neq 0$. When we take the case $dF/d\phi = 0$, the system is solved if $F = 0$, which excludes the gravitational field from action (1). This means that if the Noether symmetry is satisfied, the case $k \neq 0$ is ruled out. But when one considers $k = 0$—a flat Universe—for the point-like Lagrangian (3), the condition $L_x \mathcal{L} = 0$ furnishes system (12)–(16) with equation (16) and the system can present an interesting solution. Hence, the following analysis will be done for a homogeneous, isotropic and spatially flat Universe and it will be considered in all the remaining equations that $k = 0$. Below, the two cases ($dF/d\phi \neq 0$ and $dF/d\phi = 0$) will be examined separately for the unique situation allowed by the Noether symmetry, i.e. for $k = 0$.

(i) First case: $dF/d\phi = 0$. From the system of equations (12) and (13), it follows that $\alpha$ do not depend on $\phi$ and is proportional to $1/\sqrt{a}$. Further, one concludes that the only possibility of fulfilling equation (15) is by taking $V = 0$. This case is not interesting because the energy density and the pressure of the tachyon field vanish identically.

(ii) Second case: $dF/d\phi \neq 0$. For $dF/d\phi \neq 0$ it follows from (14) that $\alpha$ is only a function of $a$ and the only possibility for equation (15) to be satisfied is that $\alpha = \alpha_0 a$ and $V = \lambda \exp(-\xi \phi)$, (17) where $\alpha_0$, $\lambda$, and $\xi = 3\alpha_0/\beta_0$ are constants. Finally, from the system of equations (12) and (13) one can get the coupling function $F = \gamma \exp(-\xi \phi)$, (18) where $\gamma$ is another constant.

The constant of motion associated with the Noether symmetry is

$$\Sigma_0 = a \frac{\partial \mathcal{L}}{\partial a} + \beta \frac{\partial \mathcal{L}}{\partial \dot{\phi}},$$

which, for the symmetry found, leads to

$$\frac{\Sigma_0}{6\gamma \beta_0} = - \left[ \left( \frac{\kappa}{\sqrt{1 - \dot{\phi}^2}} + \frac{\xi^2}{3} \right) \dot{\phi} + \frac{\xi}{3} H \right] a^2 \exp(-\xi \phi),$$

where $\kappa = \lambda/6\gamma$. This is the form of the constant of motion associated with the Noether symmetry which was found for the dynamics described by the general action (1). Such a constant of motion is a new conserved quantity which does not have any relation to some known a priori quantity.

Note that the non-interacting and pressureless matter field is proportional to $a^3$ and it appears as a constant in the point-like Lagrangian (3). Furthermore, it does not have any influence on equations (11)–(14) which result from the symmetry condition. Then the constant of motion associated with the symmetry is the same for the cases with and without a matter field.

4. Inflationary period

In the inflationary period the role played by a matter field is negligible, so one can get rid of it by letting $\rho_{0_m} = 0$. In this case one has to solve the system of coupled differential equations that follow from equations (4) and (8), namely
\[
\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{\xi}{\kappa}(\dot{H} + 2H^2)\sqrt{1 - \dot{\phi}^2} - \xi = 0, \tag{21}
\]
\[
H^2 - \xi H\dot{\phi} - \frac{\kappa}{\sqrt{1 - \dot{\phi}^2}} = 0, \tag{22}
\]

by using equations (6), (17) and (18).

For the determination of an analytical solution of the coupled system of differential equations (21) and (22), one makes use of the constant of motion associated with the Noether symmetry (20), which after differentiation with respect to \(\phi\) becomes an ordinary differential equation for \(\phi\) that can be written as
\[
\left(\frac{3\kappa}{\xi^2\sqrt{1 - \dot{\phi}^2}} + \frac{3}{2}\right)\dot{\phi} \pm \sqrt{\left(\frac{\kappa}{\xi^2\sqrt{1 - \dot{\phi}^2}} + \frac{\dot{\phi}^2}{4}\right)} = 0, \tag{23}
\]
thanks to (22) and \(\partial H/\partial \phi = 0\).

By considering \(\xi = -i\xi_0\), where \(\xi_0\) is a real constant, the solution of the differential equation (23) is given by
\[
\phi(t) = i(k_1 t + k_2), \tag{24}
\]
where \(k_1\) and \(k_2\) are real constants. This solution implies the following relationship for the constants:
\[
\kappa = \lambda = \frac{1}{18}\left(\frac{\xi_0}{k_1}\right)^2 \sqrt{1 + k_1^2}\left(1 + \sqrt{1 + 18k_1^2} + 9k_1^2 + 9k_1^4\right). \tag{25}
\]

Now, the substitution of the time evolution of the tachyon field (24) into the expressions for the potential density (17) and coupling (18) leads to
\[
V(t) = \lambda \exp[-\xi_0(k_1 t + k_2)], \quad F(t) = \gamma \exp[-\xi_0(k_1 t + k_2)], \tag{26}
\]
respectively. By requiring that the potential density must decay with time, one infers from equation (26) that \(\xi_0 k_1 > 0\).

Finally, one can obtain from equation (22), by integrating the time evolution of the cosmic scale factor,
\[
a(t) = \exp[K(t - t_0)], \quad \text{where} \quad K = \frac{\kappa}{\sqrt{(1 + k_1^2)}} + \frac{\xi_0 k_1}{4} + \frac{\xi_0 k_1}{2}. \tag{27}
\]
Hence, this solution describes an inflationary period, where the cosmic scale factor increases exponentially with time.

The time evolution of the energy density and pressure of the tachyon field can be obtained from equations (6), (7), (24) and (26), yielding
\[
\rho_\phi(t) = \left(\lambda + 6\xi_0\gamma k_1\right) \exp[-\xi_0(k_1 t + k_2)], \tag{28}
\]
\[
p_\phi(t) = -\lambda \sqrt{1 + k_1^2} \left(2k_1 k_1^2 - 2\xi_0\gamma k_1 (2K - \xi_0 k_1)\right) \rho_\phi(t). \tag{29}
\]
From the above equations, one concludes that (i) the pressure is proportional to energy density and is negative and (ii) the inflationary period comes to an end at a finite time since the energy density and pressure of the tachyon field tend to zero after that time.
5. Decelerated–accelerated transition period

The analysis of the case where the sources of the gravitational field are the tachyon and the pressureless matter fields proceeds by changing the variable and introducing the red-shift \( z \) through the relationships

\[
    z = \frac{1}{a} - 1, \quad \frac{d}{dt} = -H(1+z) \frac{d}{dz}. \tag{30}
\]

In terms of the red-shift, the Klein–Gordon (4) and acceleration (5) equations become

\[
    H^2(1+z)^2\varphi'' + \left[ H + H'(1+z) \right] H(1+z)\varphi' = \left[ 1 - H^2(1+z)^2\varphi^2 \right] + \frac{\lambda}{\kappa} \xi H \left[ H'(1+z) - 2H \right] \sqrt{1 - H^2(1+z)^2\varphi^2 + \xi} \right], \tag{31}
\]

\[
    4\gamma HH' \exp(-\xi\varphi)(1+z) = \rho_m + \rho_\phi + p_\phi, \tag{32}
\]

where the energy densities of the matter and tachyon fields and the pressure of the tachyon field read as

\[
    \rho_m(z) = \rho_m^0(1+z)^3, \tag{33}
\]

\[
    \rho_\phi(z) = \exp(-\xi\varphi) \left\{ \frac{\lambda}{\sqrt{1 - H^2(1+z)^2\varphi^2}} - 6\xi\gamma H^2(1+z)\varphi' \right\}, \tag{34}
\]

\[
    p_\phi(z) = -\exp(-\xi\varphi) \left\{ \frac{\lambda}{\sqrt{1 - H^2(1+z)^2\varphi^2}} + 2\xi\gamma \left[ H^2(1+z)^2\varphi'' - (H'(1+z) - H)H(1+z)\varphi' - \xi H^2(1+z)^2\varphi^2 \right] \right\}, \tag{35}
\]

respectively. In the above equations, the prime denotes the derivative with respect to the red-shift \( z \).

Searching for exact solutions of the coupled system of differential equations (31) and (32) is a very hard job, and a numerical solution of the system of equations will be analyzed afterwards.

For the determination of the numerical solutions of the system of differential equations (31) and (32), one has to specify initial conditions for \( \varphi(z), \varphi'(z) \) and \( H(z) \) at present time, i.e. at \( z = 0 \). As usual, the energy densities are replaced by the density parameters \( \Omega_\phi(z) = \rho_\phi(z)/\rho(z) \) and \( \Omega_m(z) = \rho_m(z)/\rho(z) \), where \( \rho(z) = \rho_\phi(z) + \rho_m(z) \) is the total energy density of the sources of the gravitational field. Furthermore, one introduces the dimensionless quantities \( H_\phi(z) = H(z)/\sqrt{\rho(0)}, \varphi_\phi(z) = \sqrt{\rho(0)}\varphi(z), \xi_\phi = \xi/\sqrt{\rho(0)} \) and \( \lambda_\phi = \lambda + \rho(0). \) Here, the present values of the energy densities adopted are \( \Omega_\phi(0) = 0.72 \) and \( \Omega_m(0) = 0.28 \) (see e.g. [9]).

One expects that the present value of the coupling is \( F = c^3/16\pi G \) or \( F = 1/2 \) in natural units. Hence, it follows from (18) that \( \varphi_\phi(0) = \ln(2\gamma)^{1/\kappa}. \) Besides, since in the late time the Universe is accelerating, the negative pressure of the tachyon field must be responsible for its acceleration. To guarantee this behavior, one imposes the condition \( \dot{\varphi}^2(0) \ll 1. \) It follows from equation (8) that \( \rho_\phi(0) \approx V(0), \) so that \( \varphi_\phi(0) = \ln(\lambda_\phi/0.72)^{1/\kappa}, \) which together with the previous expression leads to the relationship \( \gamma = \lambda_\phi/1.44. \)

From the above considerations among the three free parameters \( \xi_\phi, \lambda_\phi \) and \( \gamma, \) only two are linearly independent. To find the numerical solutions of the system of differential equations, \( \lambda_\phi = 1 \) and three values for the parameter which is related to the strength of the coupling,
namely $\xi_* = 0.05, 0.10, 0.20$, were chosen. Furthermore, the desired conditions at $z = 0$—by taking into account that $\dot{\varphi}^2(0) \ll 1$—are

$$H_*(0) = \sqrt{1/3}, \quad \varphi_*(0) = -\ln(0.72)^{1/\xi_*}, \quad \varphi'_*(0) = 10^{-3}. \quad (36)$$

In the left frame of figure 1, the density parameters of the matter and tachyon fields are plotted as a function of the red-shift in the range $0 \leq z \leq 2$. One observes from this figure that the coupling parameter $\xi_*$ has an influence on the decay of the density parameter of the tachyon field and the corresponding increase of the matter field, it being more efficient as the value of $\xi_*$ decreases. This behavior can be explained by noting that the increase of the coupling parameter $\xi_*$ increases the energy transfer from the tachyon field to the gravitational field. Furthermore, as was expected, the limiting case $\xi_* \to 0$ tends to the $\Lambda CDM$ model.

The deceleration parameter $q = 1/2 + 3p/2\rho$ is plotted as a function of the red-shift $0 \leq z \leq 2$ in the right frame of figure 1, whereas in table 1 the values taken from the curves are compared with the predicted values of the present deceleration parameter $q(0)$ and the transition red-shift $z_t$, where the decelerated period turns over an accelerated period. From table 1, one infers that all values are almost within the uncertainty of the predicted values. Furthermore, by increasing the value of the coupling the red-shift transition $z_t$ decreases whereas the deceleration parameter $q(0)$ increases. It is noteworthy to draw attention to the fact that for high values of the red-shift the coupling is not important, since the deceleration parameters for different values of the coupling parameter $\xi_*$ tend to a common value.

It is also interesting to analyze the evolution with the red-shift of the coupling function $F(z)$ for different values of the coupling parameter $\xi_*$. The plot of $F(z)$ versus $z$ is given in the left frame of figure 2 where one can infer that the coupling parameter $\xi_*$ has a prominent role in the behavior of $F$. Indeed, the variation of the coupling function $F$ is less accentuated for small values of the coupling constant $\xi_*$. This behavior was expected due to the expression of
Figure 2. Left frame: coupling function $F$ versus the red-shift $z$. Right frame: ratio of pressure to energy density of the tachyon field $\omega_\phi$ versus the red-shift $z$.

$F$ given by equation (23). Another point is that for different values of the coupling parameter $\xi_*$, the coupling function tends to an asymptotic value by increasing values of the red-shift. The behavior of the potential density $V$ with the red-shift is similar to the coupling function $F$, since it has the same exponential dependence on $\phi$.

In the right frame of figure 2, the ratio of the pressure to the energy density of the tachyon field $\omega_\phi = p_\phi/\rho_\phi$ as a function of the red-shift $z$ is plotted. One observes from this figure that $\omega_\phi \to -1$ when $z \to 0$ for small values of the coupling parameter $\xi_*$, i.e. for small values of $\xi_*$ the tachyon field approaches the $\Lambda$CDM model. For high values of the red-shift $\omega_\phi \to 0$, i.e. the tachyon field tends to a pressureless matter field.

The difference between the apparent and absolute magnitudes of a source is defined in terms of the luminosity distance and given by

$$\mu_0 = m - M = 25 + 5 \log \left[ (1 + z) \int_0^z \frac{dz'}{H(z')} \right].$$

It is plotted in figure 3 as a function of the red-shift, where the circles denote observational data of super-novae of type Ia (see [12]). Only one curve for the coupled model, namely for $\xi_* = 0.2$, was plotted since there is no sensible difference with the $\Lambda$CDM model when $\xi_* \to 0$. From this figure, one may observe that the values of $\mu_0$ for the coupled model does not differ from the $\Lambda$CDM model for small values of the red-shifts. There exists only a small depart of the two curves for higher values of the red-shift.

6. Final remarks

In the previous section, the conditions chosen for the scalar field were (i) $F(0) = \gamma \exp[-\xi_\phi(0)] = 1/2$ which presently corresponds to the value of the gravitational constant and (ii) that the tachyon field $\phi$ rolls very slowly in the late accelerated period so that $\dot{\phi}^2 \ll 1$.

With these conditions, action (1) becomes

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2} - V \right\} + S_m,$$

for the present time. So one recovers the gravitational action that describes the gravity as is observed today. Moreover, one has a potential term with cosmological constant behavior which accelerates the actual expansion of the Universe.

The action (38), which results from the limit of small derivatives of the field $\phi$, coincides with the actions analyzed in detail for the models proposed in [14, 15]. Then the limiting
situation (38) leads to a constant gravitational coupling for a constant field $\psi$. In this limit the present model is similar to the models in the quoted references which present this limiting property, they being in accordance with the experimental data. The models proposed in the works [14, 15] can produce an exponential solution for the scale factor, characterizing an accelerated expansion. One should hope this same behavior from the field equations of the present model once there is a relation between the two models for the limit of small derivatives, and this actually happens. One may infer this kind of solution by observing that for the late time, when the small derivatives are imposed for the present model, the solutions describe a progressing accelerated Universe while the energy density of the matter decays and the energy density of the tachyon field increases. Furthermore, the present model can describe the inflationary period with the same function of coupling and potential which describes the present acceleration by taking the limit of small derivatives, but one should note that in the inflationary case the limit of small derivatives is not taken into account.

It is also interesting to analyze the evolution equation for the energy density of the coupled tachyon field $\rho_\psi$. Differentiating the Friedmann equation (8) with respect to time and taking into account the acceleration equation (5) and the expression for the coupling function (18) yield

$$\dot{\rho} + 3H(\rho + p) = -\xi\dot{\psi}\rho,$$

hence

$$\rho_\psi + 3H(\rho_\psi + p_\psi) = -\xi\dot{\psi}\rho_\psi,$$

(39)

thanks to $\rho = \rho_m + \rho_\psi$, $p = p_\psi$ and $\rho_m + 3H\rho_m = 0$. From equation (39)2, one can understand the role played by the coupling constant $\xi$ on the energy transfer from the tachyon field to the gravitational and matter fields.

7. Conclusions

As was explained before, the application of the Noether symmetry to the Lagrangian density is a very important tool, since it guarantees the conservation laws and restricts the possible expressions for the potential density and for the coupling function of a tachyon field. Here it was shown that according to the Noether symmetry, only exponential functions of the
tachyon field are possible expressions for the potential density and for the coupling function of non-minimally coupled tachyon fields.

The cosmological solutions found are of two kinds. The first one refers to a non-minimally coupled tachyon field as the only gravitational source of the early Universe which behaves as an inflaton and leads to an exponential accelerated expansion which ends in a finite time. In the second one, the gravitational sources of the Universe are a non-minimally coupled tachyon field and a pressureless matter field where the tachyon field comports as dark energy and is responsible for the decelerated–accelerated transition period of the late Universe. In the latter case it was shown that the coupling constant has influence on the density and deceleration parameters and also on the luminosity distance, since the coupling constant is connected with the energy transfer from the tachyon field to the gravitational and matter fields.

Acknowledgments

The authors acknowledge the support from CNPq (Brazil).

References

[1] Peebles P J E and Ratra B 2003 *Rev. Mod. Phys.* 75 559
Szydłowski M, Kurek A and Krawiec A 2006 *Phys. Lett.* B 642 171
Binder J B and Kremer G M 2006 *Gen. Rel. Grav.* 38 857

[2] Ribas M O, Devecchi F P and Kremer G M 2005 *Phys. Rev.* D 72 123502
Saha B 2006 *Phys. Rev.* D 74 124030
Ribas M O, Devecchi F P and Kremer G M 2008 *Europhys. Lett.* 81 19001
Chimento L P, Devecchi F P, Forte M and Kremer G M 2008 *Class. Quantum Grav.* 25 085007

[3] Abramo L R W and Finelli F 2003 *Phys. Lett.* B 575 165
Liu D and Li X 2004 *Phys. Rev.* D 70 123504
Kremer G M and Alves D S M 2004 *Gen. Rel. Grav.* 36 2039
Steer D A and Vernizzi F 2004 *Phys. Rev.* D 70 043527
Campuzano C and Del Campo S 2006 *Phys. Lett.* B 633 149
Herrera R, Del Campo S and Campuzano C 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)009
Xiong H and Zhu J 2007 *Phys. Rev.* D 75 084023
Balart L, Del Campo S, Herrera R, Labraña P and Saavedra J 2007 *Phys. Lett.* B 647 313

[4] Padmanabhan T 2002 *Phys. Rev.* D 66 021301
Hao J and Li X 2002 *Phys. Rev.* D 66 087301
Bagla J S, Jassal H K and Padmanabhan T 2003 *Phys. Rev.* D 67 063504
Jassal H K 2004 *Pramana* 62 757
Del Campo S, Herrera R and Pavón D 2005 *Phys. Rev.* D 71 123529
Das A, Gupta S, Saini T D and Kar S 2005 *Phys. Rev.* D 72 043528
Panotopoulos G 2006 arXiv:astro-ph/0606249v2
Ren J and Meng X 2008 arXiv:astro-ph/0610266v2

[5] Cárdenas V H 2006 *Phys. Rev.* D 73 103512

[6] Srivastava S K 2005 arXiv:gr-qc/0509074v4

[7] Capozziello S and Lambiase G 2000 *Gen. Rel. Grav.* 32 295

[8] De Souza R C and Kremer G M 2008 *Class. Quantum Grav.* 25 225006
[9] Fukugita M and Peebles P J E 2004 Astrophys. J. 616 643
[10] Virey J M et al 2005 Phys. Rev. D 72 061302
[11] Riess A G et al 2004 Astrophys. J. 607 665
[12] Riess A G et al 2007 Astrophys. J. 659 98
[13] Spalinski M 2008 J. Cosmol. Astropart. Phys. JCAP04(2008)002
[14] Fiziev P P 2002 arXiv:gr-qc/0202074v4
[15] Fiziev P P and Georgieva D A 2003 Phys. Rev. D 67 064016