Axial form factor in the Chiral Quark Soliton Model

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Abstract

We calculate the axial form factor in the chiral quark soliton (semibosonized Nambu - Jona-Lasinio) model using the semiclassical quantization scheme in the next to leading order in angular velocity. The obtained axial form factor is in a good absolute (without additional scaling) agreement with the experimental data. Both the value at the origin and the \( q \)-dependence of the form factor as well as the axial m.s.radius are fairly well reproduced.

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Recently, including the $1/N_c$ rotational corrections (next to leading order) in the semi-classical quantization scheme, a natural solution for the problem of strong underestimation of the axial-vector coupling constant $g_A$ in the leading order in the semibosonized Nambu-Jona-Lasinio (chiral quark soliton) model has been found. These corrections lead to an enhancement of order $(N_c + 2)/N_c$ for $g_A$ and improve considerably the agreement with the experiment. Apparently one should expect that the $1/N_c$ corrections will not be small also for the axial form factor and it can change significantly the existing leading order results. It should be noted that the latter suffer from the fact that $g_A = G_A(0)$ is strongly underestimated which does not allow for an absolute agreement with experiment.

It is usually assumed that the axial form factor is scaled in the same way like the axial vector coupling constant $g_A$. This is, however, not necessarily true since the axial form factor contains a second contribution, which does not contribute to $g_A = G_A(0)$. Also in the calculation a low constituent mass $M = 363$ MeV is used, which differs from the value $M = 420$ MeV extracted from an overall fit to the static properties as well as to the electromagnetic form factors of the nucleon in the model. Therefore, it is worth to evaluate the axial form factor in the chiral quark soliton model using the semiclassical quantization scheme with the next to leading $1/N_c$ rotational corrections included, which is the aim of the present work.

We start with the general decomposition of the matrix element of the axial current $A_\mu^a = \Psi \gamma^0 \gamma_\mu \gamma_5 \frac{\tau_a}{2} \Psi$ in terms of the corresponding form factors:

$$\langle N(p', \xi') | \Psi \gamma^0 \gamma_\mu \gamma_5 \frac{\tau_a}{2} \Psi | N(p, \xi) \rangle = \bar{u}(p', \xi') \left[ G_A(q^2) \gamma_\mu + \frac{G_p(q^2)}{2M_N} q^\mu \right] \gamma_5 \tau_a \frac{E}{2} u(p, \xi), \quad (1)$$

where $q = p' - p$ and $\xi$ stands for spin and isospin. After some standard manipulations we can express the axial form factor as

$$G_A(q^2) = 3 \frac{M_N}{E} \int d^3 x \ e^{i\vec{q}\cdot\vec{x}} \left[ \langle p \uparrow | A_3^a(x) | p \uparrow \rangle - \frac{q_3 q_i}{q^2} \langle p \uparrow | A_i^a(x) | p \uparrow \rangle \right], \quad (2)$$

where $M_N$ and $E$ are the nucleon mass and energy $E = \sqrt{M_N^2 + \vec{q}^2}/4$, respectively, and $|p \uparrow\rangle$ is a proton state of spin up. It should be noted that eq.(2) contains matrix elements of the space components of the axial current $A_i^a$, and apparently only the first term in eq.(2) contributes to $g_A$.

For the evaluation of the matrix element of the axial current we use the simplest SU(2)-version of the chiral quark soliton model with up and down quarks, degenerated in mass. It is based on a semibosonized Nambu Jona-Lasinio lagrangean:

$$\mathcal{L} = \bar{\Psi} (-i\gamma^\mu \partial_\mu + m_0 + MU^\gamma_5) \Psi,$$

which includes auxiliary meson fields

$$U(x) = e^{i\vec{q} \cdot \vec{\pi}(x)/f_\pi},$$

constrained on the chiral circle. The model is non-renormalizable and one needs a cutoff to make it finite. This cutoff $\Lambda$ and the current quark mass $m_0$ are treated as parameters of the model and are both fixed in the mesonic sector to reproduce the physical pion mass $m_\pi$ and
the pion decay constant $f_\pi$. The third model parameter, the constituent quark mass $M$, can be related to the empirical value of the quark condensate but it still leaves a broad range for $M$. Actually, in order to obtain an overall good description of the baryonic properties a value around 420 MeV has to be used [11].

In the model the baryons appear as a bound state of $N_c$ (number of colors) valence quarks coupled to the polarized Dirac sea. Since the model lacks confinement the proper way to describe the nucleon is to consider [9] a correlation function of two $N_c$-quark currents with nucleon quantum numbers at large euclidean time-separation. Accordingly, the nucleon matrix element of the axial current $\Psi^\dagger \gamma^0 \gamma^\mu \gamma_5 \frac{T_a}{2} \Psi$, is represented by an euclidean functional integral [9] with lagrangean (3):

$$\langle N', \vec{p}' | \Psi^\dagger(0) \gamma^0 \gamma^\mu \gamma_5 \frac{T_a}{2} \Psi(0) | N, \vec{p} \rangle = \lim_{T \to \infty} \frac{1}{Z} \int d^3x d^3y e^{-i\vec{p}' \vec{x}' + i\vec{p} \vec{x}} \times \int D\Psi \int D\bar{\Psi} J_{N'}(\vec{x}', T/2) \Psi^\dagger(0) \gamma^0 \gamma^\mu \gamma_5 \frac{T_a}{2} \Psi(0) J_N^\dagger(\vec{x}, -T/2) e^{-\int d^4x \bar{\Psi} D(U) \Psi}.$$  (5)

Current $J_N$ is a composite $N_c$ quark operator [10] with nucleon quantum numbers $JJ_3, TT_3$:

$$J_N(\vec{x}, t) = \frac{1}{N_c!} \varepsilon^{\beta_1 \cdots \beta_{N_c}} \Gamma_{f_1 \cdots f_{N_c}} J_{J_3, TT_3} \Psi_{\beta_1 f_1}(\vec{x}, t) \cdots \Psi_{\beta_{N_c} f_{N_c}}(\vec{x}, t).$$  (6)

For the evaluation of the path integral in eq.(5) we follow the line of ref. [2,11]. Here we will only sketch the derivation. Integrating out the quarks in (5) it is easy to see that the result is naturally split in valence and sea parts. After that we integrate over the meson fields $U$ in saddle point approximation – large $N_c$ limit. It leads to a stationary localized meson configuration (soliton) of hedgehog structure

$$\bar{U}(x) = e^{i\vec{\tau} \cdot \vec{x} P(x)},$$  (7)

which minimizes the effective action

$$\text{Tr} \log D(U) = \text{Tr} \log \left[ \partial_\tau + h(U) \right],$$  (8)

where the one-particle hamiltonian $h$ is given by

$$h(U) = \frac{\vec{\alpha} \cdot \vec{\nabla}}{i} + \beta MU^{\gamma_5} + \beta m_0.$$  (9)

Since the hedgehog soliton field configuration $\bar{U}(x)$ does not preserve the spin and isospin, as a next step we make use of the rotational zero modes to quantize it [9]. It is done assuming a rotating meson hedgehog fields of the form

$$U(\vec{x}, \tau) = R(\tau) \bar{U}(\vec{x}) R^\dagger(\tau)$$  (10)

with $R(\tau)$ being a time-dependent rotation SU(2) matrix in the isospin space. It means that even in leading order in $N_c$ one has to go beyond the saddle point approximation extending the path integral in eq.(5) over all fields of the form (10) – a path integral over $R$. It is easy to see that for such an ansatz one can transform the effective action

$$\text{Tr} \log D(U) = \text{Tr} \log (D(\bar{U}) + i\Omega)$$  (11)
in order to separate the angular velocity matrix:

$$\Omega = -i R^\dagger(\tau) \dot{R}(\tau) = \frac{1}{2} \Omega_{a} \tau_{a}. \quad (12)$$

The dot stands for the derivative with respect to the euclidean time $\tau$. Similarly, the quark propagator in the background meson field $U$ can be rewritten as

$$\langle x | \frac{1}{D(U)} | x' \rangle = \langle x | R(\tau) \frac{1}{D(U) + i\Omega} R^\dagger(\tau') | x' \rangle. \quad (13)$$

Since the angular velocity is quantized according to the canonical quantization rule, it appears as $\Omega_{a} \sim \frac{1}{N_c}$. This allows one to consider $\Omega$ as perturbation and to evaluate any observable as a perturbation series in $\Omega$ which is actually an expansion in $\frac{1}{N_c}$. Actually, we make essentially use of the expansions

$$N_c \text{Tr} \log[D(\bar{U}) + i\Omega] = N_c \text{Tr} \log[D(\bar{U})] + \Theta_{\text{sea}} \int d\tau \Omega_{a}^2 + \ldots, \quad (14)$$

and

$$\frac{1}{D(U) + i\Omega} = \frac{1}{D(U)} - \frac{1}{D(U)} i\Omega \frac{1}{D(U)} + \ldots. \quad (15)$$

up to terms quadratic in $\Omega$ to arrive at a functional integral over the time dependent orientation matrices $R(\tau)$ with an action quadratic in angular velocities $[9]$. In large $N_c$ limit, the latter corresponds to the hamiltonian of the quantum spherica l rotator:

$$H_{\text{rot}} = \frac{J^2}{2\Theta}, \quad (16)$$

where $J_a$ is the spin operator of the nucleon and

$$\Theta = \Theta_{\text{sea}} + \Theta_{\text{val}} \sim N_c \quad (17)$$

is the total moment of inertia $[9,12]$, including both the valence and the sea quark contributions. It means that despite of the fact that in general the path integral over $R$ runs over all possible trajectories, in large $N_c$ limit the main contribution comes from trajectories close to those of the quantum rotator with hamiltonian (16). According to eq. (16) the quantization rule (in euclidean space-time) is given by

$$\Omega_{a} \rightarrow -i \frac{J_a}{\Theta}. \quad (18)$$

Due to the collective path integral over $R$ the order of the not-commuting collective operators

$$\Omega_{a}(R(\tau)) = -i \text{Tr}(R^\dagger(\tau) \dot{R}(\tau) \tau_{a}) \quad \text{and} \quad D_{ab}(R(\tau)) = \frac{1}{2} \text{Tr}(R^\dagger(\tau) \tau_{a} R(\tau) \tau_{b}), \quad (19)$$

is strictly fixed by the time ordering$^[3]$. For given spin $J, J_3$ and isospin $T, T_3$ the spin-flavor structure of the nucleonic solution can be expressed through the Wigner $D$ function

$^[3]$The details of this procedure can be found in ref. [11].
\[ |N, T_3 J_3\rangle (R) = (-1)^{T+T_3} \sqrt{2T + 1} D_{T_3, J_3}^{T=T_3} (R). \] (20)

In the above scheme, the matrix element of the space components of the axial current \( A^5_3 \) includes leading order terms \( \sim \Omega^5 \) as well as next to leading order ones \( \sim \Omega \) \((1/N_c)\). In Minkowski space-time it has the following structure:

\[
\langle N | A^5_3 (x) | N \rangle = -N_c \left\{ \left( \Phi^\dagger_{val} (x) \gamma^0 \gamma^k \gamma_3 \Phi_{val} (x) \right) - \sum_n \mathcal{R}^{\Omega_0}_\Lambda (\epsilon_n) \left( \Phi^\dagger_{n} (x) \gamma^0 \gamma^k \gamma_5 \gamma_0 \Phi_{n} (x) \right) \right. \\
+ \frac{i}{2 \Theta} \varepsilon^{ck3} \left[ \sum_{n \neq val} \text{sign}(\epsilon_n) \left( \Phi^\dagger_{val} (x) \gamma^0 \gamma^r \gamma_5 \gamma_0 \Phi_{n} (x) \right) \langle n | \tau_c | val \rangle \right] \\
- \sum_{n,m} \mathcal{R}^{\Omega_1}_\Lambda (\epsilon_n, \epsilon_m) \left( \Phi^\dagger_{m} (x) \gamma^0 \gamma^r \gamma_5 \gamma_0 \Phi_{n} (x) \right) \langle n | \tau_c | m \rangle \right\}. \] (21)

Here \( \Phi_n \) and \( \epsilon_n \) are the eigenfunctions and the eigenvalues of the hamiltonian \( \mathcal{H} \). In eq. (21) first and third terms are valence quark contributions in leading and next to leading order in angular velocity, respectively. They are finite and do not need any regularization. The other two terms represent the divergent Dirac sea part and need regularization. The regularization functions \( \mathcal{R}^{\Omega_0}_\Lambda \), \( \mathcal{R}^{\Omega_1}_\Lambda \) can be found in refs. [14,15].

Inserting the result (21) in eq.(2), after some straightforward calculations we obtain for the axial factor

\[ G_A(q^2) = \frac{M_N}{E} \int r^2 dr \left[ j_0(qr) A_0(r) - j_2(qr) A_2(r) \right], \] (22)

where both densities \( A_0(r) \) and \( A_2(r) \), split in valence and sea parts, contain leading and next to leading order terms in angular velocity \( \Omega \):

\[
A^{\Omega_0}_{0(2)} (r) = N_c \frac{1}{3 \sqrt{3}} \left\{ \left( \Phi^\dagger_{val} (r) | T^{\Omega_0}_{0(2)} | \Phi_{val} (n) \right) \\
+ \frac{1}{2} \sum_{n = all} \sqrt{2K_n + 1} \mathcal{R}^{\Omega_0}_\Lambda (\epsilon_n) \left( \Phi^\dagger_{n} (r) | T^{\Omega_0}_{0(2)} | \Phi_{n} (n) \right) \right\}, \] (23)

and

\[
A^{\Omega_1}_{0(2)} (r) = N_c \frac{1}{T} \frac{1}{9 \sqrt{2}} \left\{ \sum_{n \neq val} \frac{1}{\epsilon_n - \epsilon_{val}} \left( \Phi^\dagger_{val} (r) | T^{\Omega_1}_{0(2)} | \Phi_{n} (r) \right) \langle val | \tau^{(1)} | n \rangle \\
+ \frac{1}{2} \sum_{n,m = all} \mathcal{R}^{\Omega_1}_\Lambda (\epsilon_n, \epsilon_m) \left( \Phi^\dagger_{n} (r) | T^{\Omega_1}_{0(2)} | \Phi_{m} (r) \right) \langle n | \tau^{(1)} | m \rangle \right\}. \] (24)

The tensors \( T^{\Omega_0}_{0(2)} \) are defined as

\[ T^{\Omega_0}_{0(2)} = [\sigma^{(1)} \otimes \tau^{(1)}]^{0(1)}, \] (25)

and

\[ T^{\Omega_1}_{0(2)} = \sqrt{2\pi} \left[ \left[ \sqrt{2} \otimes \sigma^{(1)} \right]^{(1)} \otimes \tau^{(1)} \right]^{0(1)}. \] (26)
The solitonic solution $\bar{U}$ is found by solving numerically the corresponding equations of motion in an iterative self-consistent procedure [1]. To this end we use the method of Ripka and Kahana [13] for solving the eigenvalue problem in a finite quasi–discrete basis.

We also calculate the axial m.s. radius $\langle r^2 \rangle_A$ given by

$$\langle r^2 \rangle_A = - \frac{6}{G_A(0)} \frac{dG_A(q^2)}{dq^2} \bigg|_{q^2=0} = \frac{1}{G_A(0)} \int r^4 dr \ [A_0(r) + \frac{2}{5} A_2(r)] + \frac{3}{4M^2_N}, \quad (27)$$

and for convenience, parameterize also the calculated form factors using a dipole fit:

$$\frac{G_A(q^2)}{G_A(0)} = \left(1 + \frac{q^2}{M_A^2}\right)^{-2} . \quad (28)$$

Our results for the axial form factor are displayed on fig.1 for four different values of the constituent mass $M$, namely 360, 400, 420 and 440 MeV. Similar to $g_A$ [2] the main contribution to the axial form factor in the leading as well as in the next to leading order in angular velocity comes from the valence quarks and the Dirac sea contribution is almost negligible. Table I contains the corresponding values for the dipole mass and m.s.radius as well as for the axial vector coupling constant. Similar to the other nucleon properties [11] a value for constituent mass around 420 MeV is preferred. For this mass value the theoretical curve agrees fairly well with the experimental data without any scaling. The lowest value of 360 MeV is almost excluded. On fig.2 we also show the leading order results. As can be seen the two groups of curves differ in both magnitude and slope. Indeed, the extracted dipole masses for the axial form factors, calculated in leading order, are larger. In particular, for $M = 360$ MeV the dipole mass in the leading order still agrees with the experiment, whereas in the next to leading order it is out of the experimental window. It means that a simple scaling behavior of the axial form factor as it is assumed in ref. [5] is not valid. On fig.2 we present the separated $A_0$- and $A_2$-contributions to the axial form factor. As can be seen the $A_0$ part completely dominates the axial form factor at small momentum transfer.

In Table I we also present our results for the axial m.s.radius calculated from eq.(27) as well as from the dipole fit (in brackets) in comparison with the estimate from the experimental dipole fit. As can be seen the values directly calculated from the corresponding densities $A_0$ and $A_2$ are very close to those of the dipole fit which is an indication that even at small $q^2$ the dipole fit is a good approximation to the theoretical curves.

To summarize, we evaluate the axial form factor in the chiral quark soliton model taking into account the next to leading order $(1/N_c)$ rotational corrections in the semiclassical quantization scheme. These corrections provide not only the enhancement, needed to reproduce the experimental value of the axial vector coupling constant, but also change the slope of the axial form factor allowing for an almost perfect absolute (without additional scaling) description of the experimental data. The axial m.s.radii are also well reproduced.

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REFERENCES

[1] M. Wakamatsu and T. Watabe, *Phys.Lett.* **B312**(1993)184
[2] Chr. V. Christov, A.Blotz, K. Goeke, V. Yu. Petrov, P. V. Pobylitsa, M. Wakamatsu, T. Watabe, *Phys.Lett.* **B325**(1994)467
[3] A. Blotz, M. Praszalowicz and K. Goeke, *Phys.Lett.* **B317**(1993)195
[4] Chr.V.Christov, K.Goeke and P.V.Pobylitsa, *Phys.Rev.* **C51**(1995)
[5] T.Meissner and K.Goeke, *Z.Phys.* **A339** (1991) 513
[6] T.Meissner, A.Blotz, E.Ruiz Arriola and K.Goeke, Baryons in Effective Chiral Quark Model with Polarized Dirac Sea, Bochum preprint RUB-TP2-42/93, (submitted to *Rep.Prog.Theor.Physics*)
[7] Y.Nambu and G.Jona-Lasinio, Phys.Rev. **122** (1961) 354
[8] T.Eguchi, H.Sugawara, *Phys.Rev.* **D10** (1974) 4257; T.Eguchi, *Phys.Rev.* **D14** (1976) 2755
[9] D.I.Diakonov, V.Yu.Petrov and P.V.Pobylitsa, 21st LNPI Winter School on Elem.Part. Physics, Leningrad 1986; *Nucl.Phys.* **B306**(1988)809
[10] B.L.Ioffe, *Nucl.Phys.* **B188**(1981)317;**B191**(1981)591(E)
[11] Chr.V. Christov, A.Z. Górska, K. Goeke and P. Pobylitsa, *Nucl.Phys.* **A** (1995) (in print)
[12] H.Reinhardt, *Nucl.Phys.* **503B**(1989)825
[13] S.Kahana and G.Ripka, *Nucl.Phys.* **A429** (1984) 462
[14] N.J. Baker, et al, *Phys.Rev.* **D23** (1981) 2499
[15] T. Kitagaki, et al, *Phys.Rev.* **D28** (1983) 436
FIGURES

FIG. 1. Axial form factor in leading and next to leading order in angular velocity in comparison with the experimental data [14][15].

FIG. 2. Separated $A_0$- and $A_2$-contributions to the axial form factor.
TABLE I. Axial properties of the nucleon, calculated in the NJL model for four different values of the constituent mass $M = 360, 400, 420$ and $440$ MeV, compared with experimental values. The dipole masses for the form factors calculated in both leading and next to leading order are given. The m.s.radii obtained from the dipole fit (in brackets) is also presented.

| Constituent quark mass $M$ [MeV] | 360 | 400 | 420 | 440 | exp |
|----------------------------------|-----|-----|-----|-----|-----|
| $M_{A}^{N}$ [GeV]               | 0.910 ± 0.001 | 1.004 ± 0.001 | 1.040 ± 0.001 | 1.072 ± 0.001 |     |
| $M_{A}^{N+\Omega}$ [GeV]       | 0.849 ± 0.001 | 0.956 ± 0.001 | 0.995 ± 0.001 | 1.031 ± 0.001 | 1.05^{+0.12}_{-0.16} |
| $< r^2 >_{A}$ [fm$^2$]          | 0.70 (0.65)   | 0.51 (0.51)   | 0.46 (0.47)   | 0.43 (0.44)   | 0.42^{+0.18}_{-0.08} |
| $g_A$                            | 1.31          | 1.24          | 1.21          | 1.18          | 1.26          |
Axial form factor

\[ A_0 \text{-contribution} \]

\[ A_2 \text{-contribution} \]

Total

\[ M = 420 \text{MeV} \]

\[ Q^2 \text{[GeV}^2\text{]} \]