A REMARK ON THE CHOW RING OF SICILIAN SURFACES

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ABSTRACT. We propose a “Bloch type” conjecture for surfaces: if the cup product map in coherent cohomology is zero, then all intersections of homologically trivial divisors should be zero in the Chow group of zero–cycles. We prove this conjecture for Sicilian surfaces.

1. INTRODUCTION

For $X$ a smooth projective variety over $\mathbb{C}$, let $A^j(X)$ denote the Chow groups of codimension $j$ algebraic cycles on $X$ modulo rational equivalence. The intersection product makes a graded ring of $A^*(X) = \bigoplus_j A^j(X)$, the Chow ring of $X$.

In this note, we will be interested in the Chow ring of smooth projective surfaces $S$. What can be said about the image of the intersection product map

$$i_S : A^1(S) \otimes A^1(S) \to A^2(S)$$

For $K3$ surfaces, the image of $i_S$ is as small as possible: it is a free abelian group of rank 1 [5]. At the other extreme, for abelian surfaces the map $i_S$ is surjective (the same is true for the Fano surface of lines on a cubic threefold [6], and another example where this holds is given in remark 3.4 below). For surfaces $S \subset \mathbb{P}^3$, the rank of the image of $i_S$ can grow arbitrarily large [19].

There is a relation with the cohomology ring: if $i_S$ is surjective, then also the cup product map in coherent cohomology

$$H^1(S, \mathcal{O}_S) \otimes H^1(S, \mathcal{O}_S) \to H^2(S, \mathcal{O}_S)$$

is surjective [8]. The conjectural converse statement is studied in [17]. To complete the picture, we propose the following conjecture:

Conjecture 1.1. Let $S$ be a smooth projective surface, such that the cup product map

$$H^1(S, \mathcal{O}_S) \otimes H^1(S, \mathcal{O}_S) \to H^2(S, \mathcal{O}_S)$$

is zero. Then the intersection product map

$$j_S : A^1_{\text{hom}}(S) \otimes A^1_{\text{hom}}(S) \to A^2_{AJ}(S)$$

is also zero.

Here, $A^1_{\text{hom}}$ denotes homologically trivial cycles, and $A^2_{AJ}$ denotes the Albanese kernel. The point of conjecture [11] is that $A^2_{AJ}(S)$ is expected to be related to $H^2(S, \mathcal{O}_S)$ [6]. A particular

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case of conjecture 1.1 is that the map $j_S$ should be zero for any surface with irregularity $q(S) := h^{1,0}(S) = 1$.

We can prove conjecture 1.1 for so-called Sicilian surfaces. These surfaces (defined in [3], cf. also definition 2.11 below) form a 4–dimensional family of general type surfaces with $p_g(S) := h^{2,0}(S) = q(S) = 1$ and $K_S^2 = 6$.

**Theorem** (= theorem 3.1). Let $S$ be a Sicilian surface as in [3]. Then the intersection product map

$$j_S: A^1_{hom}(S) \otimes A^1_{hom}(S) \to A^2_{AJ}(S)$$

is zero.

This implies that the image of $i_S$ is “not so large” for Sicilian surfaces: it is supported on a divisor (corollary 3.3). The proof of theorem 3.1 is an easy application of O’Sullivan’s theory of symmetrically distinguished cycles on abelian varieties [20] (cf. also subsection 2.5 below).

**Conventions.** In this note, the word variety will refer to a reduced irreducible scheme of finite type over $\mathbb{C}$, and a surface will mean a 2–dimensional variety. For any variety $X$, we will denote by $A_j(X)$ the Chow group of dimension $j$ cycles on $X$. For a smooth $n$–dimensional variety $X$, we will write $A^j(X)$ for $A_{n-j}(X)$. For a smooth proper variety, $A^j_{hom}(X)$ and $A^j_{AJ}(X)$ will be used to indicate the subgroups of homologically trivial, resp. Abel–Jacobi trivial cycles. For a morphism between smooth varieties $f: X \to Y$, we will write $\Gamma_f \in A^*(X \times Y)$ for the graph of $f$, and $\Gamma_f \in A^*(Y \times X)$ for the transpose.

The contravariant category of Chow motives (i.e., pure motives with respect to rational equivalence as in [23], [18]) will be denoted $\mathcal{M}_{rat}$.

We will write $H^j(X)$ to indicate singular cohomology $H^j(X, \mathbb{Q})$.

2. **Preliminaries**

2.1. **Chow cohomology.** For a singular variety $X$, we follow the convention of [9] and write $A_*(X)$ for Chow groups and $A^*(X)$ for the operational Chow cohomology of [9, Chapter 17]. As proven in loc. cit., $A^*(X)$ is a contravariant functor from varieties to commutative rings, and for $X$ smooth the ring structure coincides with the usual intersection product. For $n$–dimensional quotient varieties $X = Y/G$ with $Y$ smooth and $G$ finite, the natural map induces isomorphisms

$$A^i(X) \xrightarrow{\sim} A_{n-i}(X) \quad \forall i$$

[9, Example 17.4.10]. The same is true for surfaces whose singularities are rational [27, Theorem 4.1], [15].

2.2. **Finite–dimensional motives.** We refer to [16], [2], [12], [18] for the definition of finite–dimensional motive. An essential property of varieties with finite–dimensional motive is embodied by the nilpotence theorem:

**Theorem 2.1** (Kimura [16]). Let $X$ be a smooth projective variety of dimension $n$ with finite–dimensional motive. Let $\Gamma \in A^n(X \times X)_{\mathbb{Q}}$ be a correspondence which is numerically trivial.
Then there is $N \in \mathbb{N}$ such that
$$\Gamma^N = 0 \in A^n(X \times X)_{\mathbb{Q}}.$$  

Actually, the nilpotence property (for all powers of $X$) could serve as an alternative definition of finite–dimensional motive, as shown by a result of Jannsen [12, Corollary 3.9]. Conjecturally, any variety has finite–dimensional motive [16]. We are still far from knowing this, but at least there are quite a few non–trivial examples.

2.3. The transcendental motive.

Theorem 2.2 (Kahn–Murre–Pedrini [14]). Let $S$ be a surface. There exists a decomposition
$$h^2(S) = t^2(S) \oplus h^2_{\text{alg}}(S) \text{ in } \mathcal{M}_{\text{rat}},$$
such that
$$H^*(t^2(S), \mathbb{Q}) = H^2_{tr}(S), \quad H^*(h^2_{\text{alg}}(S), \mathbb{Q}) = NS(S)_{\mathbb{Q}}$$
(here $H^2_{tr}(S)$ is defined as the orthogonal complement of the Néron–Severi group $NS(S)_{\mathbb{Q}}$ in $H^2(S, \mathbb{Q})$), and
$$A^*(t^2(S))_{\mathbb{Q}} = A^2_{AJ}(S)_{\mathbb{Q}}.$$  
(The motive $t^2(S)$ is called the transcendental part of the motive.)

Remark 2.3. It would be more precise to write $H^*(h^2_{\text{alg}}(S), \mathbb{Q}) = NS(S)_{\mathbb{Q}}(-1)$, taking into account the Tate twist. In this note, we will omit Tate twists from the notation.

2.4. Refined Chow–Künneth decomposition.

Theorem 2.4 (Vial [25]). Let $X$ be a smooth projective variety of dimension $n \leq 5$. Assume that $X$ has finite–dimensional motive, and that the Lefschetz standard conjecture $B(X)$ holds (in particular, the Künneth components $\pi_i \in H^{2n}(X \times X)$ are algebraic). Then there is a splitting into mutually orthogonal idempotents
$$\pi_i = \sum_j \pi_{i,j} \in A^n(X \times X)_{\mathbb{Q}},$$
such that
$$(\pi_{i,j})_* H^*(X) = \text{gr}_i \tilde{N}^* H^i(X).$$
(Here, $\text{gr}_i \tilde{N}^* H^i(X)$ denotes the graded quotient for the niveau filtration $\tilde{N}^*$ defined in [25].)

The motive $h^{1,0}(X) = (X, \pi_{i,0}, 0) \in \mathcal{M}_{\text{rat}}$ is well–defined up to isomorphism.

Proof. This is [25, Theorems 1 and 2]. The last statement follows from [25, Proposition 1.8] combined with [14, Theorem 7.7.3]. \qed

Remark 2.5. In dimension $n \leq 3$ (which will be the case when we apply theorem 2.4 in this note), the niveau filtration $\tilde{N}^*$ coincides with the coniveau filtration $N^*$ of [7].

Remark 2.6. In dimension $n = 2$, the motive $h^{2,0}(X)$ is isomorphic to the motive $t^2(X)$ of theorem 2.2.
2.5. Symmetrically distinguished cycles.

**Definition 2.7** (O’Sullivan [20]). Let $A$ be an abelian variety. Let $a \in A^\ast(A)$ be a cycle. For $m \geq 0$, let

$$V_m(a) \subset A^\ast(A^m)_\mathbb{Q}$$

denote the $\mathbb{Q}$–vector space generated by elements

$$p_*\left( (p_1)^\ast(a^{r_1}) \cdot (p_2)^\ast(a^{r_2}) \cdot \ldots \cdot (p_n)^\ast(a^{r_n}) \right) \in A^\ast(A^m)_\mathbb{Q}.$$ 

Here $n \leq m$, and $r_j \in \mathbb{N}$, and $p_i : A^n \to A$ denotes projection on the $i$–th factor, and $p : A^n \to A^m$ is a closed immersion with each component $A^n \to A$ being either a projection or the composite of a projection with $[-1] : A \to A$.

The cycle $a \in A^\ast(A)_\mathbb{Q}$ is said to be **symmetrically distinguished** if for every $m \in \mathbb{N}$ the composition

$$V_m(a) \subset A^\ast(A^m)_\mathbb{Q} \to A^\ast(A^m)_\mathbb{Q}/A^\ast\text{hom}(A^m)_\mathbb{Q}$$

is injective.

**Theorem 2.8** (O’Sullivan [20]). The symmetrically distinguished cycles form a $\mathbb{Q}$–subalgebra $A^\ast_{\text{sym}}(A)_\mathbb{Q} \subset A^\ast(A)_\mathbb{Q}$, and the composition

$$A^\ast_{\text{sym}}(A)_\mathbb{Q} \subset A^\ast(A)_\mathbb{Q} \to A^\ast(A)_\mathbb{Q}/A^\ast\text{hom}(A)_\mathbb{Q}$$

is an isomorphism. Symmetrically distinguished cycles are stable under pushforward and pull-back of homomorphisms of abelian varieties.

**Remark 2.9.** For discussion and applications of the theory of symmetrically distinguished cycles, in addition to [20] we refer to [24, Section 7], [26], [1], [10], [11].

**Proposition 2.10.** Let $A$ be an abelian variety of dimension $g$.

(i) There exists a Chow–K"unneth decomposition $\{\Pi_A^i\}$ that is self–dual and consists of symmetrically distinguished cycles. One has equality

$$(\Pi_A^{2i-j})_\ast A^i(A)_\mathbb{Q} = A^j(2)_\mathbb{Q}$$

where $A^j(2)_\mathbb{Q}$ denotes Beauville’s decomposition [4] on Chow groups with rational coefficients.

(ii) Assume $g \leq 5$, and let $\{\Pi_A^i\}$ be as in (i). There exists a further splitting in orthogonal projectors

$$\Pi_A^2 = \Pi_A^{2,0} + \Pi_A^{2,1} \text{ in } A^g(A \times A)_\mathbb{Q},$$

where the $\Pi_A^{2,i}$ are symmetrically distinguished and $\Pi_A^{2,i} = \pi_A^{2,i} \text{ in } H^{2g}(A \times A)$. Moreover, one has

$$(\Pi_A^{2,0})_\ast A^2(A)_\mathbb{Q} = (\Pi_A^{2,1})_\ast A^2(A)_\mathbb{Q} = A^2(2)_\mathbb{Q}.$$ 

**Proof.** (i) An explicit formula for $\{\Pi_A^i\}$ is given in [24, Section 7 Formula (45)].

(ii) The point is that $\Pi_A^{2,1}$ is (by construction) a cycle of type

$$\sum_j C_j \times D_j \text{ in } A^g(A \times A)_\mathbb{Q},$$
where $D_j \subset A$ is a symmetric divisor and $C_j \subset A$ is a curve obtained by intersecting a symmetric divisor with hyperplanes. This implies $\Pi_{A}^{2,1}$ is symmetrically distinguished. By assumption, $\Pi_{A}^{2}$ is symmetrically distinguished and hence so is $\Pi_{A}^{2,0}$.

For the “moreover” part, one notes that the projector $\Pi_{A}^{2,1}$ acts trivially on $A_{A}^{2}(A) \subset A_{A}^{2}(A)_{Q}$, for reasons of dimension.

\[\square\]

2.6. Sicilian surfaces.

Definition 2.11. A Sicilian surface is a minimal surface $S$ of general type satisfying:

1. $p_g(S) = q(S) = 1$ and $K_{S}^{2} = 6$;
2. There exists an unramified double cover $\hat{S} \to S$ with $q(\hat{S}) = 3$, and such that the Albanese morphism $\hat{\alpha}: \hat{S} \to A := \text{Alb}(\hat{S})$ is birational to its image $Z$, a divisor in $A$ with $Z_{3} = 12$.

Remark 2.12. Sicilian surfaces have an irreducible 4–dimensional moduli space [3, Theorem 6.1]. Sicilian surfaces can be characterized topologically; they form a connected component of the moduli space of surfaces of general type [3, Corollary 6.5]. Surfaces in the families $S_{11}$ and $S_{12}$ constructed in [3] are Sicilian surfaces.

We mention in passing the following result, which will not be used in the proof of the main result (theorem 3.1).

Theorem 2.13 (Peters [21]). Let $S$ be a Sicilian surface. Then $S$ has finite–dimensional motive. More precisely, let $A$ be the abelian threefold as in definition 2.11. Then the natural map

$$h^{2,0}(A) \to t^{2}(S) \text{ in } \mathcal{M}_{\text{rat}}$$

admits a right–inverse, and the natural map

$$t^{2}(S) \to h^{4}(A) \text{ in } \mathcal{M}_{\text{rat}}$$

admits a left–inverse.

3. Main Result

Theorem 3.1. Let $S$ be a Sicilian surface. The map induced by intersection product

$$j_{S}: A_{\text{hom}}^{1}(S) \otimes A_{\text{hom}}^{1}(S) \to A^{2}(S)$$

is the zero map.

Proof. As the image of $j_{S}$ is contained in $A_{A}^{2}(S)$ which is torsion free [22], it will suffice to prove that $j_{S} \otimes \mathbb{Q}$ is the zero map.

The next reduction step is to pass to the canonical model $S_{\text{can}}$. Let $f: S \to S_{\text{can}}$ the canonical morphism. There is a commutative diagram

$$\begin{array}{ccc}
A_{\text{hom}}^{1}(S) \otimes A_{\text{hom}}^{1}(S) & \xrightarrow{j_{S}} & A^{2}(S) \\
\uparrow (f^{*},f^{*}) & & \uparrow f^{*} \\
A_{\text{hom}}^{1}(S_{\text{can}}) \otimes A_{\text{hom}}^{1}(S_{\text{can}}) & \xrightarrow{j_{S_{\text{can}}}} & A^{2}(S_{\text{can}})
\end{array}$$
where \( A^2(S_{\text{can}}) \) denotes operational Chow cohomology. The vertical arrows are isomorphisms (for the left vertical arrow, this is because \( S_{\text{can}} \) has rational singularities, for the right vertical arrow this follows from the exact sequence of \([15, \text{Theorem 2.3}]\)). It thus suffices to prove that \( j_{S_{\text{can}}} \otimes \mathbb{Q} \) is the zero map.

As shown in \([3, \text{Theorem 6.1}]\), the surface \( S_{\text{can}} \) admits an inclusion as an ample divisor
\[
S_{\text{can}} \subset X = A/G ,
\]
where \( X \) is a Bagnera–de Franchis threefold (in the sense of \([3, \text{Section 5}]\)), and \( A \) is the abelian threefold of definition \([2.11]\) and \( G \cong \mathbb{Z}_2 \). Because \( q(X) = q(S) = 1 \), the cup product map
\[
H^1(X, \mathcal{O}_X) \otimes H^1(X, \mathcal{O}_X) \rightarrow H^2(X, \mathcal{O}_X)
\]
is the zero map. In view of the Hodge decomposition, this means that the composition
\[
H^1(X, \mathbb{C}) \otimes H^1(X, \mathbb{C}) \rightarrow H^2(X, \mathbb{C}) \rightarrow H^2(X, \mathcal{O}_X) ,
\]
which is the same as
\[
H^1(A, \mathbb{C})^G \otimes H^1(A, \mathbb{C})^G \rightarrow H^2(A, \mathbb{C})^G \rightarrow H^2(A, \mathcal{O}_A)^G ,
\]
is the zero map.

In terms of motives, this means that the composition
\[
\mathfrak{h}^1(A)^G \otimes \mathfrak{h}^1(A)^G \xrightarrow{\Delta_A^{sm}} \mathfrak{h}^2(A) \xrightarrow{\pi_2^{2,0}} \mathfrak{h}^2(0)(A) \quad \text{in } \mathcal{M}_{\text{hom}}
\]
is zero (where \( \Delta_A^{sm} \in A^6(A \times A \times A) \) is the “small diagonal”, and the motive \( \mathfrak{h}^{2,0}(A) \subset \mathfrak{h}^2(A) \) is as in proposition \([2.10]\)). In terms of correspondences, this means that the correspondence
\[
\Gamma := \Pi^2_0 \circ \Delta_A^{sm} \circ \left( \Pi^1_A \circ (\Delta_A + \Gamma_g) \right) \times \left( \Pi^1_A \circ (\Delta_A + \Gamma_g) \right) \in A^6((A \times A) \times A)_\mathbb{Q}
\]
is homologically trivial (i.e., it vanishes in \( H^{12}(A \times A \times A, \mathbb{C}) \) and hence also in \( H^{12}(A \times A \times A, \mathbb{Q}) \)). Here, we have written \( G = \{ \text{id}, g \} \cong \mathbb{Z}_2 \) (i.e., \( g \) is the non–trivial element of \( G \)), and \( \Pi^1_A, \Pi^2_0 \) are the projectors of proposition \([2.10]\).

The involution \( g \in \text{Aut}(A) \) is described explicitly in \([3, \text{Theorem 6.1}]\); it can be written as a group homomorphism \( \sigma \) followed by a translation \( t \) by a torsion element. In view of lemma \([3.2]\) below, the graphs \( \Gamma_g \) and \( \Gamma_\sigma \) are the same in the Chow group with rational coefficients. Therefore, we have equality
\[
\Gamma = \Pi^2_0 \circ \Delta_A^{sm} \circ \left( \Pi^1_A \circ (\Delta_A + \Gamma_\sigma) \right) \times \left( \Pi^1_A \circ (\Delta_A + \Gamma_\sigma) \right) \in A^6((A \times A) \times A)_\mathbb{Q} .
\]

But the right–hand side (being a composition of symmetrically distinguished cycles) is symmetrically distinguished. Therefore, theorem \([2.8]\) implies that
\[
\Gamma = 0 \quad \text{in } A^6((A \times A) \times A)_\mathbb{Q} .
\]

In particular, the action on Chow groups
\[
\Gamma_* : A^2(A \times A)_\mathbb{Q} \rightarrow A^2(A)_\mathbb{Q}
\]
is zero. On the other hand, let \( a, b \in A^1_{\text{hom}}(A)^G \) and consider the element
\[
a \times b \in \text{Im} \left( A^1_{\text{hom}}(A)^G \otimes A^1_{\text{hom}}(A)^G \rightarrow A^2(A \times A) \right) .
\]
Then (by construction of $\Gamma$) we have equality

$$\Gamma_*(a \times b) = 4 (\Pi_{A}^{2,0})_* (\Delta_{A}^{sm})* (a \times b) = 4 \cdot a \times b \quad \text{in } A^2(A)_Q.$$ 

(Here, for the last equality we have used that $a \times b \in A^2_{(2)}(A)$, as the Beauville decomposition of $A^*(A)$ is multiplicative.) The commutative diagram

$$
\begin{array}{ccc}
A^1_\text{hom}(A)^G \otimes A^1_\text{hom}(A)^G & \xrightarrow{\partial} & A^2_{A,J}(A)^G \\
\downarrow \cong & & \downarrow \cong \\
A^1_\text{hom}(X) \otimes A^1_\text{hom}(X) & \xrightarrow{\partial} & A^2_{A,J}(X) \\
\downarrow (\iota^*, \iota^*) & & \downarrow \iota^* \\
A^1_\text{hom}(S_{\text{can}}) \otimes A^1_\text{hom}(S_{\text{can}}) & \xrightarrow{\partial} & A^2_{A,J}(S_{\text{can}}),
\end{array}
$$

plus the fact that $\iota^*: A^1_\text{hom}(X) \to A^1_\text{hom}(S_{\text{can}})$ is an isomorphism (weak Lefschetz), now ends the proof.

In the above argument we have used the following, which is [13, Lemma 2.1]:

**Lemma 3.2 ([13]).** Let $A$ be an abelian variety of dimension $g$, and let $t \in \text{Aut}(A)$ be a translation by a torsion element. Then

$$\Gamma_t = \Delta_A \quad \text{in } A^0(A \times A)_Q.$$ 

□

**Corollary 3.3.** Let $S$ be a Sicilian surface. The image of the intersection product map

$$i_S: A^1(S) \otimes A^1(S) \to A^2(S)$$

is supported on a divisor.

**Proof.** Let $D_1, \ldots, D_r$ be generators of the Néron–Severi group of $S$. Given arbitrary divisors $D, D' \in A^1(S)$, let us write $D = \sum_{i=1}^r d_i D_i$, $D' = \sum_{j=1}^r d'_j D_j$ in $NS(S)$. This gives decompositions

$$D = \sum_{i=1}^r d_i D_i + D_0 \quad \text{and} \quad D' = \sum_{j=1}^r d'_j D_j + D'_0 \quad \text{in } A^1(S),$$

with $D_0, D'_0 \in A^1_\text{hom}(S)$.

It follows from theorem[3,1] that $D_0 \cdot D'_0 = 0$ in $A^2(S)$, and so

$$D \cdot D' = \sum_{i=1}^r \sum_{j=1}^r d_i d'_j D_i \cdot D_j + \sum_{i=1}^r d_i D_i \cdot D'_0 + \sum_{j=1}^r d'_j D_j \cdot D_0 \quad \text{in } A^2(S).$$

This implies that the image of $i_S$ is supported on the union $\bigcup_{j=1}^r D_r \subset S$. □
Remark 3.4. Theorem 3.1 applies in particular to the generalized Burniat type surfaces in the families $S_{11}$ and $S_{12}$ of [3] (as shown in loc. cit., these are Sicilian surfaces). It is instructive to contrast this with the behaviour of generalized Burniat type surfaces in the family $S_{16}$ (these have $p_g(S) = q(S) = 3$). Indeed, any surface $S$ in the family $S_{16}$ has a surjective cup product map

$$H^1(S, \mathcal{O}_S) \otimes H^1(S, \mathcal{O}_S) \to H^2(S, \mathcal{O}_S).$$

Moreover, $S$ has finite–dimensional motive [3, Theorem 4.13]. The main result of [17] then implies that

$$i_S: A^1(S) \otimes A^1(S) \to A^2(S)$$

is surjective (just as for abelian surfaces).

Remark 3.5. The argument of theorem 3.1 applies in a more general setting: it suffices that $S$ be a surface with $q(S) = 1$ obtained as the resolution of a nodal surface $S_{can}$, which can be embedded as ample divisor

$$S_{can} \subset X = A/G,$$

where $A$ is an abelian threefold, and $G$ a finite group acting by compositions of translations and group homomorphisms. It follows that theorem 3.1 is also true for the generalized Burniat type surfaces in the families $S_j$, $5 \leq j \leq 12$ of [3].

Remark 3.6. As Sicilian surfaces (and generalized Burniat type surfaces) are closely related to abelian varieties, it seems natural to ask whether they admit a multiplicative Chow–Künneth decomposition, in the sense of [24, Section 8]. I hope to return to this question later.

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