Vertical and rotational motion of mahogany seed

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Abstract. Starting with a set of basic assumptions and with the application of well-known Newtonian physics, a theoretical model has been established for the flight of the mahogany winged seed. Using a high-speed camera, we successfully confirmed that the mahogany winged seed attains a vertical and rotational terminal velocity. From our model the mahogany seed has a terminal speed of 1.45 m/s. The experimental value of the terminal velocity on the average is 1.47 m/s (only about 1% error). The experimental value of the angular velocity was found to be around 54.33 rad/s, about 14% error compared to the predicted terminal velocity of 47.5 rad/s. The high predictable nature of a mahogany’s terminal velocity can facilitate the biologist’s study of mahogany mass seed dispersal.

1. Introduction

It is an observed fact that not all falling objects travel straight downward under the influence of gravity. Leaves, paper and seeds all follow complicated trajectories as they fall. This certain phenomenon is greatly utilized by nature; one good example can be seen in trees in their seed dispersal.

Seed dispersal is an important process in plant population dynamics. There are a lot of tree species among them are Triplaris caracasana, Swietenia mahogany, and Acer macrophyllum that have developed rather special way of dispersing seeds in the environment. One of the trees that exhibit this seed dispersal behavior, Swietenia mahogany, can be found around the University of San Carlos Talamban campus. The seed of these trees will be the subject of this paper.

The objective of this paper is to model the aerodynamics of mahogany winged seeds to discover whether there is a systematic limit to the vertical and rotational terminal velocity. Studying seed’s terminal velocity is important. In the work of Green [1] he pointed out that modeling seed dispersal by wind for tropical species of seeds requires that one understand the magnitude of the variation in terminal velocity; that is, the first step in understanding the dispersal of the seeds of these trees is to acquire knowledge on how aerodynamic forces and the mahogany morphology act to regulate its rate of descent.
2. Theory

2.1 Physical model of mahogany
In order to have any successful model of the flight of mahogany seed a lot of sophisticated formalism should be considered such as Kutta-Zhukovsky formalism or Navier-Stoke theorem. Mahogany seed complex structure is apparently the reason that suggests the use of advanced aerodynamic theories. In the study of C. L. Ladera and P. A. Pineda [2] on the flight of samara, a three-winged seed that disperses comparable to mahogany, they have shown, using reasonable assumptions that their simplified model agreed with their experimental results.

In this paper we use the same assumptions, but modified the differential equations to cater the different wing structure of the mahogany seed. The assumptions are: when a mahogany seed descends to the ground its axis of rotation is seen to remain essentially vertical coincident with its main symmetry axis all along, that is, the mahogany wings behave as a rigid body which rotates with respect to the vertical axis, another simplification is that the camber of arching of the wings can be neglected [3]. Also, it is assumed that the speed of the wind is constant. The differential equations that govern the translation and rotational motion of the mahogany seed are discussed next.

2.2. Model of the vertical and rotational motion of mahogany seed
Essentially, as the mahogany leaf falls, it begins to rotate on its own around its center of mass, which is the seed, by the action of the air sweeping through its wing. Figure 1 refers to the diagram of the forces acting on the mahogany seed. The downward force is the weight \(mg\). While falling to the ground a force pushes upward on the mahogany seed called the aerodynamic vector force \(L\). We can write the vertical forces in differential form as,

\[
m \frac{d^2y}{dt^2} = mg - L_v,
\]

where \(y\) is the acceleration while \(L_v\) is the magnitude of the vertical components of the lift force respectively.

To describe the kinematics of the falling mahogany seed, equation 1 was solved. Knowledge on the physical quantities involved in the vertical component of lift was determined beforehand. Experiments in the wind tunnel show that the magnitude of lift may be, within a good precision, considered proportional to the square air-speed \(v\) of the wing \([1, 2]\).

\[
L = K_L v^2,
\]

where \(K_L\) is the lift coefficient depending on many factors.

Figure 1. a) The mahogany seed is modeled as a flat slab \(ww'\), with an inclination angle \(\beta\) with respect to the z-axis. The lift force \(L\) is inclined the angle \(\alpha\) with respect to the z-axis. \(V_{air}\) is the velocity of the wind and \(\varepsilon\) is the angle of the deflected wind b) Horizontal component of the lift force \(L\), a small angle \(\phi\) is the tilt of the wing. The angle \(\theta\) is the angular displacement.
Let \( \rho \) be the air density, \( l \) the length of the mahogany wing and \( \alpha, \beta, \theta, \text{and} \epsilon \) are the angles defined in the model depicted in Figure 1a and 1b. In the wing’s reference frame, the angles are defined as follows; \( \beta \) is the angle with respect to the vertical axis. In the full dynamical analysis of fluids the angle \( \epsilon \) from the deflected wind to the vertical axis also affects the magnitude of the lift. This further can be explained by the momentum theory [3].

The factors in lift of coefficient can be determined with the proportionality that follows; the lift is proportional to the density \( \rho \) of air and it is also proportional to the cross-sectional area of the wings [2, 3].

The wings of the mahogany seeds are actually like an “actuator disk”, a circular region which retards the passage of air over its entire surface. In the work of Green [1] the area they considered was the area of the disk swept by the wing of samara. For a given angle of inclination \( \beta \) the relevant area is not the cross-section area of the wing but the area \( \pi (l \sin \beta)^2 \), which is exactly the same area that would be involved if rotating helicopter wings is involved. Considering the change in momentum of the air passing over the wing, the lift is also proportional to the \( \sin \epsilon \), \( \epsilon \) being the deflection angle. The coefficient of the lift \( K_v \), can now be written as,

\[
K_v = \rho \pi (l \sin \beta)^2 \sin \epsilon. \tag{3}
\]

The vertical component of the lift force is given by the cosine of equation 2 in which we can write equation now as,

\[
m \frac{d^2 y}{dt^2} = mg - \rho \pi (l \sin \beta)^2 (\sin \epsilon)(\cos \alpha) v^2. \tag{4}
\]

For the rotational motion of the falling mahogany, Newton 2\textsuperscript{nd} law is given by

\[
\tau = I \frac{d^2 \theta}{dt^2}, \tag{5}
\]

\( I \) being the moment of inertia, \( \theta \) is the relevant coordinate (refer to figure 1b) and \( d^2 \theta/dt^2 \) is the angular acceleration.

The rotational aerodynamic torque \( \tau_{rot} \) and the drag torque \( \tau_{drag} \) are two opposing torques that determines the rotation. The wing of mahogany seed shows a tilt \( \phi \) considered as the angle of attack of the air flow as it rotates. The tilt produces small horizontal forces that generate the rotational torque [2]. Thus, the horizontal component of the lift causes a rotational aerodynamic torque as a result of the interaction of the air with the wings as the mahogany seed descends. From the center of the wing to the axis of rotation it was considered the mean value of the lever arm \( b_o \). The rotational torque is given by,

\[
\tau_{rot} = b_o L_H. \tag{6}
\]

The horizontal component \( L_H \) can be shown to be given as,

\[
L_H = \pi \rho (l \sin \beta)^2 (\sin \epsilon)(\cos \phi) v^2. \tag{7}
\]

The drag force results to an opposing torque to the rotational torque as a result of air drag on the upper surfaces of the wings as the mahogany rotate [3]. The differential of the drag on an infinitesimal length of the surface of the mahogany wing was considered which was integrated over all the length of the wing to give us the drag torque as,

\[
\tau_{drag} = \frac{1}{8} C_d \rho (l \sin \beta)^3 \omega^2, \tag{8}
\]

where \( a \) is the width of the wing, \( \omega \) is the rotational speed of the wing and \( C_d \) is the drag coefficient.

The net torque of the mahogany with the use of Newton 2\textsuperscript{nd} law of rotational motion is given as,

\[
\tau_{rot} - \tau_{drag} = I \frac{d^2 \theta}{dt^2}, \tag{9}
\]

The rotational differential equation of the physical model is given by replacing the equation above with the rotational torque and drag torque and by dividing it with the moment of inertia. The result is,
\[
\frac{d^2 \theta}{dt^2} = Cv^2 - D \left( \frac{d\theta}{dt} \right)^2,
\]

where the coefficient C and D are given by the following,

\[
C = \frac{\pi b_o \rho (\sin \beta)^2 (\sin \epsilon)(\sin \phi)}{I}
\]

and

\[
D = \frac{C_d \rho (al^3) (\sin \beta)^3}{8I}.
\]

For the vertical motion, the solution of equation 4 was obtained using Mathematica\textsuperscript{TM}. The equation of motion and velocity are,

\[
y = y_o + \frac{1}{A} \log(\cosh(\sqrt{A}B))
\]

\[
v = (\sqrt{A/B}) \tanh(\sqrt{A}B),
\]

where \(A = \left(\frac{l^2}{m}\right)C\), \(B=9.8\), and \(C = \pi \rho (\sin \beta)^2 \sin \epsilon \cos \alpha\). \(y_o\) is the initial position.

The non-linear differential equation for rotational motion given in equation 10 was solved using numerical method. Runge and Kutta algorithm in Scilab\textsuperscript{TM} was utilized.

The constants A, B, C and D were calculated using the field collected sample measurement of mahogany for each required parameters. The average mass was \((5.17 \pm 0.01) \times 10^{-4} kg\), average wing length of \(8.73 cm\), air density of \(1.05 kg/m^3\), wing width \(a = 24 mm\), the inclination \(\beta = 48^\circ\), mean of the torque arm \(b_o = 30 mm\), moment of inertia \(I = (1.14 \pm 0.01) \times 10^{-6} kg \cdot m^2\), tilt \(\phi = 2^\circ\), assumed lift force angle \(\alpha = 42^\circ\), assumed deflection angle \(\epsilon = 90^\circ\) and drag coefficient of \(C_d = 0.2\) [2].

The solutions of the differential equation of our physical model using the required parameters were graphed and are given in figure 2.

![Figure 2](image-url)

\textbf{Figure 2.} Left) Speed descent of a modelled mahogany seed, as it falls to the ground. The predicted terminal is close to 1.45 m/s. This terminal velocity was attained about 0.55 s after it falls. Right) Shows the predicted angular speed of the single-winged mahogany seed as a function of time. The predicted terminal angular speed is around 48.7 rad/s attained about 5 s after it falls.

\section{Methodology}

The experimental set-up as shown in figure 3 are composed of a high speed camera that is connected to a pre-made LabVIEW\textsuperscript{TM} program that can change the threshold of the camera and crop the range of...
interest in which the camera would capture, then eventually export the position-time data to Microsoft Excel so it could be conveniently analyzed. Mahogany samples were then collected then spray painted with white so it would stand out on the black background that was hanged on a wall that is to be captured by the camera. The selection of the range was then done and the proper thresholding was done on the LabVIEW™ program, after which the sample was then dropped at a certain measured height simultaneously as the camera starts recording the position-time data of the sample with a capture rate of 30 frames per second. The data generated by this set-up is used to describe the vertical component of the mahogany’s flight.

The set-up shown in figure 3B was used to be able to describe the rotational motion of the mahogany’s flight utilizing the same concept as the one on figure 3A but only having the cameras line of sight at worm’s view to the descent of the mahogany sample. One can then determine the average period of the spinning motion of the leaf from the images captured.

**Figure 3.** A) Shows the experimental set-up used to measure the terminal velocity of mahogany seed. The high-speed camera was used to gather the position-time data. B) Shows the set-up to calculate the angular frequency of mahogany seed. From the image collected the period of mahogany rotational motion was crudely measured by taking note of the time it takes to complete one revolution.

4. **Results and discussion**

The position of the mahogany winged seed was plotted against its time of descent, as shown in figure 4. The theoretical points (broken line) approached a final uniform regime from a short non-linear transient.
Figure 4. The theoretical graph (broken line) was super imposed with the experimental velocity (solid line) time graph of a sample of mass 0.47 g and length of 79 mm. An oscillating part is observed in the terminal regime of the motion.

Our data provides unifying properties of the mahogany winged seed’s descent. From our model the mahogany seed reaches ground with a mean speed close to 1.45 m/s, the actual value depending on its physical parameters (mass, wing, inclination and the like). Hence, the samples showed a little variation on the value of each terminal velocity. The value of the experimental terminal velocity on the average was found to be around 1.47 m/s. This result agreed with our theoretical model shown in figure 2(Left) (only about 1% error). In figure 4, ‘small oscillating’ departure are observed with respect to the theoretical curve. This can be explained from the small wobbling of mahogany axis as it falls. We have assumed that the mahogany falls to the ground with its axis exactly along a vertical however a real mahogany seed can wobble a bit so a refined model must account to these small wobbling of its axis as it falls.

With the known initial condition $\omega(0)=0$, Runge Kutta method was used to obtain the solution of the nonlinear equation which gives us a predicted terminal angular speed of 47.5 rad/s. As shown in figure 2(Right) this terminal angular speed is reached about 5 s after the mahogany falls. From our crude method of determining the angular frequency, we have calculated a mean angular speed of $\omega_t = 54.33$ rad/s (this is about 14% error) from the data in Table 1. This is a result of the poor quality of the experimental technique used to determine the angular frequency and its large variation as shown in table 1.

Table 1. Rotational terminal velocity with its respective mahogany seed’s mass and length.

| Mahogany Sample Number | (Mass±0.0001)kg | (Length±0.001)m | Angular velocity (rad/s) |
|------------------------|-----------------|-----------------|------------------------|
| 20                     | 0.0053          | 0.087           | 61.83                  |
| 26                     | 0.0059          | 0.100           | 41.40                  |
| 28                     | 0.0063          | 0.094           | 43.39                  |
| 31                     | 0.0051          | 0.042           | 62.83                  |
| 39                     | 0.0035          | 0.050           | 62.20                  |
5. Conclusion
We have confirmed that the vertical motion of the mahogany winged seed attained a terminal velocity. This is a confirmation of our model simulation which predicted a terminal velocity of around 1.5 m/s. We also have modeled the rotational motion of the mahogany seed as it falls and predicted a terminal angular speed of about 48 rad/s. The experiment with a sample of mahogany gives a mean angular velocity of about 54.3 rad/s. This is about 14% error compared to the predicted value and is due to the poor quality of the experimental technique used to determine the angular frequency. The vertical terminal velocity has been experimentally obtained, with good agreement with our theoretical value. For the rotational motion of mahogany a better experimental technique should be used to measure the angular frequency. In addition, a refined model must take into account the small wobbling of its axis at it falls.

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