Theoretical analysis of pipeline with type I dent under the external force

Ying Wu, Pengwei Jin and Peng Zhang

Abstract
As an economical, efficient, and safe material transmission method, pipeline has an irreplaceable role in the petroleum chemical and natural gas industry. There will be a variety of mechanical damage in pipelines due to unknown or known factors, and the dent is the most typical form of mechanical damage. According to the service oil and gas pipeline with type I dent, a simplified calculation model of shell and tube was chosen. The rings and generators in the model were analyzed in the equilibrium state, and the internal pressure was taken into account. Finally, the analytical expression between external force and the indentation depth of type I dent was concluded, which can be used to analyze the response of the pressure pipe to the external force, and provides a theoretical guidance for maintaining pipeline in service.

Keywords
External force, type I dent, pipeline, theoretical analysis

Date received: 29 December 2016; accepted: 25 March 2017

Academic Editor: Filippo Berto

Introduction
Oil and gas are important components of the entire industrial system and the state’s strategic resources, and the safety of their storage and transportation process must be reliably protected. The consequences of failure of oil and natural gas pipelines are very serious, leading to pay more attention to the safety of pipelines by researchers worldwide. Defects on the pipeline are one of the most main causes of pipeline accidents, and the dent is one of the common types of defects on the pipeline.1–3 According to the US Department of Transportation report,4 pipeline mechanical damage is one of the most important causes of pipeline accidents. Therefore, it is important to study the extent of pipeline damage under external forces.

Dent defect in pipeline is a complex research topic, which involves the geometry and material science. Pipeline mechanical damage has been the focus of the pipeline industry since the mid-twentieth century. Mechanical damage to the pipeline in service was mainly caused by third-party construction. Mechanical damage includes wear, spalling, dents, and chisel marks.5,6 The dents are considered to be one of the most important damages due to the fact that the external forces such as sharp rock or excavation equipment produce a concentrated load or the distributed load on the outer wall of the pipe to produce plastic deformation, leading to a localized stress concentration.7–9 Dents on the pipeline in service will provide conditions for the formation of cracks. In addition, dents on the pipeline under the action of pressure will appear as cracks. The cracks will be further extended over time, and the dent will be tired. This process will bring great harm to the safe operation of the pipeline further. At present, some researchers for dent on pipeline have been conducted using different methods such as experiment, theoretical derivation, and numerical simulation.

Department of Civil Engineering and Architecture, Southwest Petroleum University, Chengdu, China

Corresponding author:
Pengwei Jin, Department of Civil Engineering and Architecture, Southwest Petroleum University, Chengdu 610500, China.
Email: swpujpw@163.com

Creative Commons CC-BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (http://www.creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
Wierzbicki and Suh\textsuperscript{10} indicated that most of the strength of pipeline was related to the deformation mode of the ring by carrying out experiment on buckling in pipelines. Wierzbicki and Suh\textsuperscript{10} have presented a simplified calculation model of shell and tube. The model consisted of a series of discontinuous rings and a group of trusses. However, it is necessary to obtain a relatively complete theoretical and analytical solution by some simplifying assumptions. Therefore, based on the simplified models, in our article, the rings and generators in the model of in-service pipeline have been analyzed in the equilibrium state, by taking the internal pressure into account. The analytical expression between external force and the indentation depth of type I dent has been concluded.

Establishing the pipeline model with type I dent

Model establishment

It is assumed that the type I dent is formed by pressing the pipe by vertical pipe axis of the major axis of round stick, as shown in Figure 1. So the type I dent is defined as the dents formed by the long axis of the round stick and the angle of the pipe axis at 90°.

The deformation of dent in pipeline is shown in Figure 2. As a result of local deformation of the pipeline to release the energy absorbed by the pipeline shell, the global deformation can be neglected for pipelines laid in their trench.\textsuperscript{11}

The actual deformation analysis shows that the local deformation region is composed of three regions: the middle of the dent area and the both sides of the triangle area as shown in Figure 2(a), where $R$ is the radius of pipeline. Figure 2(b) and (c) shows the AB profile and the overall deformation profile, respectively.

The length of the dent area $l$ keeps constant and is equal to the diameter of the round stick $2R_0$ during the generation of the dent. But the triangle area can be expanded, and the length of $\xi$ will increase with the dent depth $\delta_0$. As the round stick has applied load to the pipeline along the axial, the pipeline will generate elliptical deformation in the dent area, as shown in Figure 2(d).

Simplified shell model

To obtain a simplified shell model, we make the following hypothesis:

1. The local deformation region is composed of three regions: the middle of the dent area and the both sides of the triangle area as shown in Figure 2(a).
2. The cross-section of the shell deformation zone is regarded as a combination of plane and circular. There is no distortion and the pipeline is not oval beyond the dent-affected zone.\textsuperscript{10}
3. All the cross-sections will undergo rigid translation and torsion except for the crushing.

The shell calculation model is established according to the above assumptions, as shown in Figure 3(a) and (b). The rings and generators are loosely connected as shown in Figure 3(c).\textsuperscript{10}

It is required that the lateral displacements of the two one-dimensional model structures are the same to ensuring the coordinated deformation of the rings and generators. So the lateral deformation is coordinated, but the shear stress cannot be resisted.\textsuperscript{10} The rings are inextensible, $\varepsilon_{\theta\theta} = 0$. Thus, the crushing energy per unit width ring consists of the following two parts

$$
\dot{E}_{\text{crush}} = \frac{M_0 \kappa_0}{L} \sum_i M_i^0 \Omega_i^{(0)}
$$

where $M_0$ is fully plastic bending moment of the wall, $\kappa_0$ is circumferential curvature rate, $s$ is circumferential parameter of the ring, and $L = 2\pi R$ is circumference of the ring. The total crushing energy is obtained by integrating $\dot{E}_{\text{crush}}$

$$
\dot{E}_{\text{crush}} = 2 \int_0^L \dot{E}_{\text{crush}} ds + \int_0^L \dot{E}_{\text{crush}} dx
$$

where the first term on the right-hand side is the crushing energy of the two triangular deformations outside the indentation, the second term on the right-hand side is the crushing energy of the central dented zone, and $x$ is coordinate in axial direction.

The hinges were regarded as rigid-plastic beams in the model which can bend and the tensile deformation increases with the dent depth. By assembling the dissipation energy of all generators, we can get an expression for the total tube axial deformation energy

$$
\dot{E}_{\text{gen}} = \frac{2\pi R}{2\pi R} \int_0^\xi \dot{E}_{\text{gen}} dx + \int_0^\xi |N_0| dx ds
$$

where $N_0$ is fully plastic axial force.
The analysis of the pipeline with type I dent

Crushing of rings

According to the previous hypothesis, the ring is not stretched so that the sum of the half length of the smoothing zone and the two arcs length are constant and equal to the one half of the initial circumference of the ring, as shown in equation (4)

\[ s_1 + s_2 + s_3 = \pi R \]  

where \( s_1 = R_1 \phi \) is the segment arc length of cd; \( s_2 = R_2 (\pi - \phi) \) is the segment arc length of de; \( R_1, R_2 \) are radii of deformed tubes; and \( s_3 = (R_1 - R_2) \sin \phi \) is the length of the smoothing zone.

The rotational speed of two hinges can be known from equation (3)

\[ \Omega_1 = V_1 \left( \frac{1}{R_2} - \frac{1}{R_1} \right), \quad \Omega_2 = V_2 \frac{1}{R_2} \]  

where \( V_1 \) and \( V_2 \) are tangential velocity of two different curvature arcs after deformation of ring and are defined by

\[ V_1 = \frac{ds}{dt}, \quad V_2 = \frac{d}{dt}(s_1 + s_2) \]

As the hinges will move along the ring, the curvature of the two arcs \( s_1 \) and \( s_2 \) changes continuously with the deformation of the ring. The partial derivative between curvature and time is given by
\[ (\kappa_{\theta\theta})_1 = -\frac{\dot{R}_1}{R_1}, \quad (\kappa_{\theta\theta})_2 = -\frac{\dot{R}_2}{R_2} \]  
(7)

where \( \dot{R}_1 = (\partial/\partial t)(R_1) \) and \( \dot{R}_2 = (\partial/\partial t)(R_2) \). Substituting equations (5)–(7) into equation (1), we can obtain

\[
\dot{E}_{\text{crush}} = 2M_0 \left[ \frac{V_2}{R_2} + \frac{V_1}{R_2} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \frac{1}{\dot{R}_1} \right] + R_1 \phi \left( -\frac{\dot{R}_1}{R_1} \right) + R_2 (\pi - \phi) \left( -\frac{\dot{R}_2}{R_2} \right)
\]

Assuming that the position of the hinge in the present is \( \phi \), the parameter changes monotonically with time. The parameter is related to the radii \( R_1, R_2 \) and central deflection of vertex of the ring \( w \) by

\[ w(x, \alpha) = 2R - [R_1(1 - \cos \phi) + R_2(1 + \cos \phi)] \]
(9)

where \( w \) is indentation depth at midspan. According to the chain rule, the partial derivative of the time is replaced by the partial derivative of \( \phi \)

\[ \frac{d}{dt} = \left( \frac{d\phi}{dt} \right) \left( \frac{d}{d\phi} \right) \]
(10)

The product of the crushing energy can be regarded as instantaneous crushing force \( F_c \) and displacement rate

\[ \dot{E}_{\text{crush}} = F_c(w) \dot{w} \]
(11)

where \( \dot{w} = (\partial/\partial t)(w) \). From equations (8) and (11), equation (12) can be obtained

\[ 2M_0 \left[ \frac{V_2}{R_2} + \frac{V_1}{R_2} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \frac{1}{\dot{R}_1} \right] + R_1 \phi \left( -\frac{\dot{R}_1}{R_1} \right) + R_2 (\pi - \phi) \left( -\frac{\dot{R}_2}{R_2} \right) = F_c(w) \dot{w} \]
(12)

It appears that the asymmetric deformation mode of the ring is more in line with the actual situation when \( \phi_0 = \pi/2 \) and \( n = 1 \) through the analysis of equation (12) by Wierzbicki et al. According to the numerical simulation by Wierzbicki and Suh,\(^6\) the following simplified formula can be obtained

\[ F_c = \frac{8M_0}{R} \]
(13)

The displacement of the main generator of AB plane is shown in Figure 2(b). According to the large deformation analysis of the rigid-plastic beam, we assume that the velocity of the main generator varies linearly with \( x \) in the deformation zone outside the dented zone

\[ \dot{w}_c = \dot{w}_0 \left( 1 - R_0 - \frac{x}{\xi} \right) \]
(14)

\[ \dot{d} = \dot{d}_0 \left( 1 - R_0 - \frac{x}{\xi} \right) \]
(15)

We will move the origin to the junction area between the dented zone and the deformation zone in order to calculate simply. The range of \( x \) is \( 0 \leq x \leq \xi \), where \( \dot{d}_0 \) and \( \dot{d} \) are the maximum vertical displacement of the dented zone and the deformation zone; \( \xi \) is the length of the deformation zone outside the dented zone (see Figure 2); \( x = \xi \) is an instantaneous rotation point; and \( t = \partial(\partial t), t \) is time.

The total crushing energy of ring in the entire deformation zone is

\[ \dot{E}_{\text{crush}} = 2 \int_0^\xi \dot{F}_c \dot{d} dx + \int_0^{l} \dot{F}_c \dot{d}_0 dx \]
(16)

where the first term on the right-hand side is the crushing energy of the deformation zone outside the dented zone, and the second term on the right-hand side is the crushing energy of the deformation zone of the dented zone. According to the previous analysis, the crushing energy can be drawn as

\[ \dot{E}_{\text{crush}} = \frac{8M_0 \dot{d}_0}{R} (\xi + l) \]
(17)

where \( l \) is the length of the dented zone (see Figure 2).

**Deformation of generators**

The global strain rate is decomposed into three parts

\[ \int_0^{\xi} \dot{e}_{xx} dx + \int_0^l \dot{e}_{xx} dx = \int_0^{\xi} (\dot{e}_1 + \dot{e}_2) dx + \int_0^l (\dot{e}_1 + \dot{e}_2) + z \dot{\theta}_\theta \]
(18)

The first term on the right side of equation (18) represents the strain rate of the deformation zone outside the dented zone, and the second term represents the strain rate of the dimple zone. The strain rate \( \dot{e}_1 \) is the generator generated by the local scarring load, and each generator is different. The strain rate \( \dot{e}_2 \) is the result of uniform compression or tension of the pipe, and all generators are the same. The displacement rate \( z \dot{\theta}_\theta \) is generated by the overall rotation. The displacement rate change of the generator is mainly related to the normal coordinate \( z \). According to the medium deformation beam theory, the tensile strain is expressed as follows

\[ \int_0^{\xi} \dot{e}_{xx} dx + \int_0^l \dot{e}_{xx} dx = \int_0^{\xi} (\dot{e}_1 + \dot{e}_2) dx + \int_0^l (\dot{e}_1 + \dot{e}_2) + z \dot{\theta}_\theta \]
\[
\varepsilon = \frac{\sqrt{(dw)^2 + (dx)^2} - dx}{dx} = \sqrt{1 + \left(\frac{dw}{dx}\right)^2} - 1 = \frac{1}{2} \left(\frac{dw}{dx}\right)^2
\]  

(19)

The strain rate \( \dot{\varepsilon} \) is defined as:

\[
\dot{\varepsilon} = \frac{dw \, d\dot{w}}{dx}
\]

(20)

The contribution of the single generator to the dissipated power during the application of the load is given by the linear velocity field described by equation (14) and the first term in equation (18)

\[
2N_0 \left( \int_0^{\xi} \dot{\varepsilon} dx + \int_0^{l} \dot{\varepsilon} dx \right) = 2N_0 \frac{w_0 \dot{w}_0}{\xi}
\]

(21)

where \( w_0 \) is vertical displacement of intermediate ring in dent zone. The tensile deformation of the unit area will occur in the position of the axial force equal to the plastic axial force. This position is completely plastic deformation

\[
N = N_0 = \sigma_s h
\]

(22)

where \( \sigma_s \) is yield strain, \( h \) is pipeline wall thickness, and \( N_0 \) is dissipation stretch energy of tube shell unit area.

It is worth noting that the ring in the deformation is outside the dent, and the change of the generator curvature and circumferential curvature of the ring are very small. Therefore, we can ignore the pipe axial bending energy rate. The dissipated energy of the generator due to local loading is equal to the energy generated by the stretching of the pipeline deformation zone. According to the beam-like theory, the pure axial strain rate can be obtained as

\[
\varepsilon_2 = \frac{d\dot{u}}{dx}
\]

(23)

where \( u \) is axial displacement of a point on pipeline. It is easy to get the result by \( x \) integral. The \( z \) in equation (18) is only relevant to the angle \( \alpha \) in Figure 2 ( \( \alpha \) is defined in Figure 2). \( \alpha \) is used to determine the position of each generator before it is deformed.

The total dissipated energy of the generator is

\[
\dot{E}_{gen} = 2N_0 \int_0^{2\pi R} \left( \frac{w_0 \dot{w}_0}{\xi} + u_0 + R \cos \alpha \dot{\theta}_0 \right) ds
\]

(24)

where \( u_0 = \dot{u}|_{x-\xi} \) and \( \dot{\theta}_0 = \dot{\theta}|_{x-\xi} \).

**Analysis of internal pressure in pipeline**

Oil and gas pipeline will withstand a variety of different magnitudes of operating pressure in operation that it is internal pressure. When the round stick pushes the pipe shell down, the pressure, \( p \), will resist the pipe shell downward movement and try to push the shell up. The energy rate of resistance work by the internal pressure, \( p \), is

\[
\dot{E}_p = I \left\{ \int_0^{2\pi R} p \cdot \dot{w}_0(\alpha) \cos \alpha ds + \int_0^{\frac{\pi}{2}} \int_0^{R \sin \psi} p \cos \psi \cdot \dot{w}_c(\alpha) \cos \alpha ds dx \ight. \\
+ \int_0^{\frac{\pi}{2}} \int_0^{R \cos \psi} p(\alpha) \cos \psi \cdot \dot{w}_0(\alpha) \left( 1 - \frac{x}{\xi} \right) \cos \alpha ds dx \right.
\]

(25)

where \( p = p_{in} - p_{out} \); \( p_{in} \) is the internal pressure, which is generally operating pressure of oil and gas pipeline; \( p_{out} \) is the external pressure of pipeline; \( \psi \) is the angle of rotation of the pipe along the tube axis as the edge of the indentation deforms in Figure 2(b); \( \dot{w}_0 \) is the displacement rate of the dent zone; and \( \dot{w}_c \) is the displacement rate in the indent zone. \( \psi = 0 \) in the indent zone. In the deformation zone outside the indent zone, \( \psi \) is small in comparison with the ring angle and \( \cos \psi \approx 1 \). According to the geometric analysis (see Figure 2)

\[
w_0(\alpha) = \delta_0 - R(1 - \cos \alpha)
\]

(26)

And according to the previous analysis, the following expression is obtained

\[
\dot{E}_p = 2R^2 p(l + \xi) \delta_0 \delta_0 \left( 1 - \frac{\alpha}{\pi} \right)^2 / w_0 \delta_0 ds dx
\]

(27)

**Force–deformation relationship**

According to the principle of virtual work, the rate of external work can be balanced by the rate of internal energy dissipation

\[
\dot{E}_{ext} = \dot{E}_{int}
\]

(28)
where $\dot{E}_{ext}$ is rate of external work and $\dot{E}_{int}$ is rate of internal energy dissipation.

In the absence of torque, the power of the external energy change is expressed as follows

$$\dot{E}_{ext} = F \dot{\delta}_0 + 2M \dot{\theta}_0 + 2N \dot{u}_0$$  \hspace{1cm} (29)

where $F$ is (external) indenting force, $\delta_0$ is maximum indentation depth, $M$ is external bending moment, $\theta$ is rate of rotation, $N$ is external axial force, and $u_0$ is $u$ at $x = \xi$. The first term on the right-hand side is for the external force; the second term for the external bending moment and shell rotation rate of the product, the total bending moment to do the power; and the third term for the external axial force and axial velocity of the product. The second term for the external bending moment and shell rotation rate of the product, the total external force; the second term for the external bending moment and shell rotation rate of the product, the total external force; the second term for the external bending moment and shell rotation rate of the product, the total external force.

$$F = \frac{\pi}{4} \sigma_y h R \frac{\delta_0}{\xi} + \frac{2 \sigma_y h^2}{R} (\xi + l)$$

$$+ 2R^2 p \delta_0 (l + \xi - 2R_0) \int_0^\pi \frac{(1 - \frac{\xi}{\xi})^2}{\delta_0 - R + R \cos \alpha} d\alpha$$

\hspace{1cm} (30)

When the pipe radius $R$, wall thickness $h$, yield stress $\sigma_y$, round stick radius $R_0$, and the length of the middle of dent area $l$ are known, the relationship between $F$ and $\delta_0$ can be obtained from equation (30). Therefore, the relationship between $\xi$ and $\delta_0$ can be obtained from $\frac{dF}{d\xi} = 0$ and is given by

$$\xi = \sqrt{\frac{\pi h R^2 \sigma_y \delta_0}{8p R^3 \delta_0 \int_{\delta_0}^{(1-\frac{\delta_0}{R})^2} \frac{1}{R + a \cos \alpha} da + 8 \sigma_y h^2}}$$  \hspace{1cm} (31)

Substituting equation (31) into equation (30), we can obtain the expression of $F - \delta_0$.

### Results and discussion

Smith\textsuperscript{12} reported on carefully executed indentation tests on full scale and model tubes with almost identical length-to-diameter and diameter-to-thickness ratios. Wierzbicki and Suh\textsuperscript{10} compared the theoretical solutions, theoretical profile, and experimental data based on the results of these experiments, as shown in Figure 4.

The circle in Figure 4 represents the measured value of the leading generator, the solid line represents the present theoretical solution, and the triangle represents the corrected actual profile. From Figure 4, we can see that the three results are more consistent. Therefore, the hypothesis of the computational model of the tube is basically correct.

According to the analysis result of pipeline with type I dent, theoretical results from equations (30) and (31) are plotted in Figures 5 and 6 for two pipes. The pipe radius $R$ and shell thickness $h$ are 400 mm $\times$ 10 mm. Two pipe materials, X52 and X60, were chosen in the analyses with the yield stress of $\sigma_y = 360$ and 415 N/mm$^2$. The pressure is $p = 0$ and 5 MPa. The round stick radius $R_0 = R/5 = 80$ mm. The length of the middle of dent area $l = 2R_0 = 160$ mm.

It can be seen from Figures 5 and 6 that the indenting force increases with the increase in dent depth. The pressure has a large influence on the pipeline resistance to the external forces. The indenting force increases with the increase in pressure under the same dent depth.

### Conclusion

Through the theoretical analysis of pipeline with type I dent under the external force, the main conclusions are as follows:

1. According to the classical theory and pipeline simplified calculation model, the theoretical
2. An analytical expression between $F$ and the indention depth $d_0$ of type I dent was deduced.

The relationship between $F$ and $d_0$ is related to pipe radius, pipe wall thickness, yield stress, pressure, and indentation length.

Acknowledgements

Y.W. and P.Z. carried out the dent of pipeline studies and drafted the manuscript. P.J. has deduced the theoretical model. All authors read and approved the final manuscript.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was supported by grants from the National Natural Science Foundation of China (No. 50974105), Scientific research starting project of SWPU (No. 2015QHZ024), and State Administration of Work Safety (Sichuan-0016-2016AQ).

References

1. Wu Y, Zhang P and Xe Y. The stress analyses of the plain dent on pipeline based on FEM. Trans China Weld Inst 2013; 34: 57–60.

2. Wu Y, Wu L, Wu H-w, et al. Peak cycle stress analysis of plain dent on pipeline based on FE calculation. J Vibroeng 2014; 16: 1268–1275.

3. CSA Z662.1-11. Commentary on CSA standard Z662–03 oil and gas pipeline systems.

4. 7th report of the European gas pipeline incident data group, 2008, https://www.egig.eu/uploads/bestanden/d1244d38-8194-46e8-89f4-6b6258d05f3a

5. Macdonald KA, Cosham A, Alexander CR, et al. Assessing mechanical damage in offshore pipelines—two case studies. Eng Fail Anal 2007; 14: 1667–1679.

6. Firouzsalari SE and Showkati H. Thorough investigation of continuously supported pipelines under combined pre-compression and denting loads. Int J Pres Ves Pip 2013; 104: 83–95.

7. Firouzsalari SE and Showkati H. Investigation of free-spanned pipeline behavior due to axial forces and local loads. J Constr Steel Res 2013; 86: 128–139.

8. Noronha DB Jr, Martins RR, Jacob BP, et al. Procedures for the strain based assessment of pipeline dents. Int J Pres Ves Pip 2010; 87: 254–265.

9. Allouti M, Schmitt C, Pluvinage G, et al. Study of the influence of dent depth on the critical pressure of pipeline. Eng Fail Anal 2012; 21: 40–51.

10. Wierzbicki T and Suh MS. Indentation of tubes under combined loading. Int J Mech Sci 1988; 30: 229–248.

11. Liu JH and Francis A. Theoretical analysis of local indentation on pressured pipes. Int J Pres Ves Pip 2004; 81: 931–939.

12. Smith CS. Strength and stiffness of damaged tubular beam columns. In: Patrick JE, Dowling J and Agelidis N (eds) Buckling of shells in offshore structures. London: Granada, 1982, pp.1–24.

Appendix I

Notation

- $E_{\text{crush}}$: rate of total crushing energy
- $E_{\text{ext}}$: rate of external work
- $E_{\text{gen}}$: the total tube axial deformation energy
\( \dot{E}_{\text{int}} \) rate of internal energy dissipation
\( \dot{E}_p \) resistant work done by pressure
\( F \) (external) indent force
\( F_c \) ring crushing force
\( h \) pipeline wall thickness
\( l \) the length of the middle of dent area
\( L = 2\pi R \) circumference of the ring
\( M_0 \) fully plastic bending moment of the wall
\( N \) external axial force
\( N_0 \) fully plastic axial force
\( p \) internal pressure for land pipeline
\( p_{\text{in}} \) internal pressure
\( p_{\text{out}} \) external pressure
\( R \) pipe radius
\( R_0 \) round stick radius
\( R_1, R_2 \) radii of deformed tube
\( s \) coordinate in circumferential direction
\( s_1, s_2, s_3 \) length of each arc
\( t \) time
\( u \) axial displacement of a generic point
\( V_1, V_2 \) tangential velocity of each moving plastic hinge
\( (\cdot) = \partial()/\partial t \) differentiation with respect to time
\( \alpha, \beta, \gamma \) angles defined in Figure 2
\( \delta_0 \) the dent depth
\( \epsilon \) tensile strain
\( \kappa_{\theta} \) circumferential curvature rate
\( \xi \) the length of the triangle area
\( \sigma_y \) yield strain
\( \phi \) current position of lower plastic hinge
\( \omega \) vertical displacement of a generic point
\( \omega(\alpha) \) indentation depth at midspan
\( \Omega \) rate of relative rotation on both sides of hinge