Three-phase coupled seismic analyses of unsaturated/saturated grounds

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ABSTRACT

The role of pore air pressure on seismic behavior of unsaturated sandy soil is discussed through recent numerical simulations with three-phase and simplified two-phase coupled analyses. Equations governing the dynamic deformation of unsaturated soil were briefly shown based on porous media theory and constitutive models. The validity of three-phase coupled analysis and the applicability of simplified two-phase coupled analysis are discussed through simulations of cyclic triaxial test, seismic behaviors of horizontal ground and embankments.

Keywords: unsaturated soil, seismic response, three-phase coupled analysis, two-phase coupled analysis

1 INTRODUCTION

Some Asian countries share a similar natural environment and face the same threats from natural disasters such as earthquake, typhoon, heavy rainfall, flood, and landslide. What makes a bad situation even worst is that these natural disasters very often did not come alone, which is known as combined disasters. For example, heavy rain and earthquake are likely to occur sequentially in a relatively short period. A three-phase coupled analysis will be a promising tool to discuss dynamic behaviors of unsaturated/saturated grounds such as slopes and embankments during combined disasters.

Coupled analyses of unsaturated soil response have been presented by Meroi et al. (1995), Zienkiewicz et al. (1999), Uzuoka et al. (2001), Khoei et al. (2004), Mehrabadi et al. (2007) and Marasini & Okamura (2015) without determining the pore air pressure explicitly. They assumed that the pore air pressure was zero, or the pore air pressure was the same as pore water pressure. These methods can be called two-phase coupled analysis considering unsaturated soil properties such as soil water characteristics curve and compressibility of pore air. However, in the present study, pore air pressure plays an important role in the dynamic response of unsaturated soil. (Kazama et al. 2006, Unno et al. 2006, Okamura & Soga 2006) and hence should be explicitly considered in the formulation of dynamic problems. Recently, pore air pressure has been treated as a primary variable in dynamic analyses performed by Schrefler & Scotta (2001), Ravichandran (2009), Ravichandran & Muralletharan (2009), Khoei & Mohammadnejad (2011), Uzuoka & Borja (2012), Noda & Yoshikawa (2015), Shabbutah-Khan et al. (2015), Yerro et al. (2015), Ghorbani et al. (2016), Matsumaru & Uzuoka (2016), Yoshikawa et al. (2016), Zhang & Muralletharan (2017, 2018). However, in this paper the role of pore air pressure on seismic behavior of unsaturated sandy soil is discussed through recent numerical simulations with three-phase and simplified two-phase coupled analyses. First equations governing the dynamic deformation of unsaturated soil were briefly shown based on porous media theory and constitutive models. Next the validity of three-phase coupled analyses and the applicability of simplified two-phase coupled analysis are discussed through simulations of cyclic triaxial test, seismic behaviors of horizontal ground and embankments.

2 NUMERICAL METHOD

2.1 Governing equations

We used soil-water-air coupled analysis code (Uzuoka & Borja 2012, Matsumaru & Uzuoka 2016) to simulate the seismic behavior of unsaturated ground. The equations governing the dynamic deformation of unsaturated soil are derived based on porous media theory (de Boer 2000, Schreifer 2002) and constitutive models. The governing equations consist of the momentum balance equations of overall three-phase material, and the mass and momentum balance equations (continuity equations) of pore water and air derived with the following assumptions. 1) Soil particle
is incompressible. 2) Exchange of mass among phases is neglected. 3) Material time derivative of relative velocities and advection terms of pore fluids to the soil skeleton are neglected. 4) Isothermal condition is assumed. The momentum balance equation of the overall three-phase material is derived as

$$\rho \ddot{a}' = \text{div} \left\{ \sigma' - (s''p'' + s''p'')I \right\} + \rho \dot{b}$$  \hspace{1cm} (1)$$

where \( \rho \) is the overall density of three-phase material, \( a' \) is the acceleration vector of solid phase, \( \sigma' \) is the Cauchy effective stress tensor, \( s'' \) is the degree of water saturation and \( s'' \) is the degree of air saturation, \( p'' \) is the pore water pressure and \( p' \) is the pore air pressure, \( I \) is the unit tensor and \( b \) is the body force vector. The stress and strain are positive in tension and the pore fluid pressures are positive in compression. The Cauchy effective stress tensor is so called average skeleton stress (Gallipoli et al. 2003) which is equivalent to Bishop’s effective stress with \( s'' \) in place of parameter \( \chi \). The mass and momentum balance equations of the pore water and air are derived as

$$\begin{align*}
\left( \frac{n s'' \rho_{w}^{\text{in}}}{K_{w}} - n \rho_{w}^{\text{in}} c \right) \frac{D'}{Dt} p_{w}^{\prime} + n \rho_{w}^{\text{in}} c \frac{D'}{Dt} p_{w}^{\prime} \\
+ s'' \rho_{a}^{\text{in}} \text{div} \nu' \\
+ \text{div} \left[ \left( \frac{k_{w}^{	ext{av}}}{g} \right) \left( - \text{grad} p_{w}^{\prime} + \rho_{w}^{\text{in}} b - \rho_{a}^{\text{in}} a' \right) \right] = 0
\end{align*}$$

$$2.2 \text{ Finite element formulation and time integration}$$

Weak forms of the equations (1) - (3) were implemented in a finite element formulation. Newmark implicit scheme was used for time integration. The primary variables are the second-order material time derivatives of displacement of soil skeleton, pore water pressure and pore air pressure. The weak forms are linearized and solved by Newton-Raphson method iteratively at each time step. The linearized forms of the weak forms were derived as

$$\begin{align*}
D \delta w'[\Delta a'] + D \delta w'[\Delta p'] + D \delta w'[\Delta \ddot{p}'] &= -\delta w'(k) \\
D \delta w''[\Delta a''] + D \delta w''[\Delta p''] + D \delta w''[\Delta \ddot{p}''] &= -\delta w''(k) \hspace{1cm} (4)
\end{align*}$$

where \( \delta w', \delta w'' \) and \( \delta w'' \) are the weak forms of the equations (1) - (3) respectively, \( D \delta w'[\Delta a'] \) is directional derivative of \( \delta w' \) with respect to \( \Delta a' \), \( \delta w''(k) \) is the residual at the iteration step of \( k \). Iteration was continued until the norm of the residual vectors became less than the convergence tolerance of \( 1.0 \times 10^{-8} \). In the finite element formulation, Galerkin method and isoparametric 8-node elements were used. The soil skeleton displacement and the fluid pressures were approximated at 8 nodes and 4 nodes respectively to satisfy the discrete LBB conditions for the locally undrained case at infinitesimal deformation.

Implicit stress integration and consistent tangent modulus at infinitesimal strain (e.g. Simo & Taylor 1985) were used to achieve the convergence of global iteration of (4).

3 \text{ SIMULATION OF CYCLIC TRIAXIAL TEST}$$

Numerical simulations of undrained cyclic triaxial tests (Unno et al. 2008) with unsaturated sandy soils were performed using the porous media theory and simplified elastoplastic constitutive model (Unno et al. 2013, Uzuoka et al. 2014). The constitutive model (Matsumaru & Uzuoka 2016) was basically same as the model for saturated sand because suction effect on yield function assumed to be not significant for unsaturated sand with relatively low suction. The effect of suction was taken into account by skeleton stress only. The effect of pore air pressure on cyclic behavior of unsaturated sandy soil was discussed through the simulations with three-phase analysis, simplified two-phase analysis with \( p'' = p' \) under nearly undrained condition, and simplified two-phase analysis with \( p'' = 0 \).

Under the condition that pore air bubbles are included into pore water, we can assume \( p'' = p' \). Substituting \( p'' = p' \) to equations (2) and (3), the simplified two phase formulation is obtained. In this formulation the bulk modulus \( K' \) of fluid including pore air and water can be calculated as

$$\frac{1}{K'} = \frac{s''}{K''} + \frac{s''}{K'}$$  \hspace{1cm} (5)$$
The initial suction was taken into account to the initial mean effective stress for easy comparison with the test results and simulations with three-phase formulation. In the case of simplified two-phase analysis with \( p^a = 0 \), we simply neglect the equation (3). The matric suction \( p^c \) can be calculated by \( p^c = -p^w \).

Cyclic triaxial tests which continuously measured pore air and water pressure were performed by coauthors (Unno et al. 2006, Unno et al. 2008). The details of the tests were described in the literature. Toyoura sand which is uniform sand was used as our testing material. The initial degree of water saturation was from about 40\% to 100\% by controlling air pressure during isotropic consolidation process. The cyclic shear was applied to the specimen under undrained air and water conditions. The input axial strain was the sinusoidal wave with multi-step amplitudes whose single amplitudes were 0.2, 0.4, 0.8, 1.2, 1.6 and 2.0\% with every ten cycles. The frequency of the sinusoidal wave was 0.005 Hz. This loading rate is slow enough to achieve a stable condition of the air and water pressure in the specimen.

The material parameters of simplified elastoplastic constitutive equation of soil skeleton were calibrated with the test results in saturated case. Figure 1 shows the time histories of suction, mean skeleton stress, pore water pressure, pore air pressure and void ratio from tests and simulations with three-phase coupled analysis for the case of initial water saturation of about 90\%. In the test results (denoted “Test” in the figures), the pore water and air pressures increased during cyclic shear, and the suction became almost zero. At the same time the mean skeleton stress also became almost zero and the shear stiffness was lost. This means that the unsaturated specimen liquefied completely at this time. Although this behavior is similar to liquefaction of saturated sand, the void ratio of unsaturated soil decreased unlike saturated sand. This volumetric change in unsaturated specimen is due to negative dilatancy and high compressibility of pore air. In the simulated results (denoted “Model” in the figures), the model well reproduced the test results in the case with lower initial suction. Therefore, the simplified constitutive equation of soil skeleton without suction dependent plastic hardening was applicable to predict liquefaction of unsaturated sand in the framework of three-phase porous media theory.

Figure 2 and 3 show the results with simplified two-phase analyses with \( p^w = p^a \) and with \( p^a = 0 \) respectively. The simulated results with two-phase analysis with \( p^w = p^a \) were similar to those of three-phase. This means that the simplified two-phase analysis with \( p^w = p^a \) can be applicable in the case with lower initial suction. On the other hand, the simulated results with simplified two-phase analyses with \( p^a = 0 \) underestimated pore water pressure generation, and could not reproduce liquefaction behavior. This means that the simplified two-phase analysis with \( p^a = 0 \) cannot be applicable even in the case with lower initial suction.

4 SIMULATION OF UNSATURATED HOMOGENEOUS HORIZONTAL GROUND

Ueda et al. (2018) carried out a series of centrifuge model tests and effective stress analyses in order to clarify the effect of desaturation on the seismic behavior of homogeneous horizontally layered ground. In the effective stress analyses, simplified two-phase
analyses, where equivalent bulk modulus is used for a mixture of pore water and air assuming that suction is zero, were also performed in addition to three-phase analyses strictly considering pore water and pore air pressures. The governing equations were basically the same as that described in Section 2. The constitutive model for skeleton stress was a strain space multiple mechanism model for saturated soil (Iai et al. 2011).

Figure 4 shows the experimental model ground with transducers. The model ground was made from uniform fine Silica sand with the relative density of 60%. In the model tests, unsaturated model ground with the degree of saturation of 90% was created by controlling the water injection speed from the bottom of a rigid box during the saturation process. Sinusoidal wave with the frequency of 1.0 Hz and the peak acceleration of 200 gal was input at the bottom under the centrifugal acceleration of 50g.

Figure 5 shows the time histories of excess pore water pressures at P1, P3 and P5 in Fig. 4 by the experimental results and three-phase analysis. The simulation with three-phase analysis reproduced the experimental responses of excess pore water pressures. Figure 6 shows the results by the simplified two-phase analysis with $p^w = p^a$. The simulation with two-phase analysis also reproduced the experimental results. The simulations clarified that the simplified two-phase analyses with $p^w = p^a$ can be applied to the seismic behavior of unsaturated sandy ground when the degree of saturation is relatively high such as 90%.

5 SIMULATION OF SOIL COLUMN NEAR GROUND SURFACE

Figure 7 shows the finite element model of an unsaturated/saturated one-dimensional soil column with the surface sand layer and the base rock layer to simulate weathered surface soil on a mountain slope. The surface sand was assumed to be a simplified
elastoplastic material, and the base rock was assumed to be a linear elastic material. Material parameters of surface sand were the same as Inagi sand (Matsumaru & Uzuoka 2014). Before a seismic analysis, static self-weight analysis and seepage analysis were performed in order to determine the initial stress and moisture conditions after rainfall. At the bottom of soil column, the soil displacement was fixed in all directions and pore water pressure was zero. At the surface of soil column, pore air pressure was set to be an atmosphere pressure. Continuous rainfall with an intensity of 20 mm/h was applied on the surface for 12 hours to reproduce a realistic initial conditions before the earthquake. In the seismic analysis, the input acceleration was applied at the bottom. This acceleration record was Kaihoku-kyo wave, one of seismic design waves (Japan Road Association). The coefficients in Newmark implicit time integration were 0.6 and 0.3025. The time increment was 0.002 seconds in the seismic analysis. We used Rayleigh damping with a factor of 0.002 for the initial stiffness.

Figure 8 shows the distributions of mean skeleton stress (negative in compression), matric suction and degree of water saturation after rainfall. From the surface to the elevation of 3.5m the suction is constant and the degree of water saturation is about 55% due to the rainfall infiltration from the surface. Figure 9 shows the time histories of mean skeleton stress (negative in compression) at the element of e40 an e31 in Fig. 7 by three-phase analysis and two-phase analysis with \( p_a = 0 \). The decrease in the mean skeleton stress with three-phase analysis is larger than that with two-phase analysis with \( p_a = 0 \). Under a realistic conditions of unsaturated/saturated soil layers near ground surface, the simplified two-phase coupled analysis with \( p_a = 0 \) can underestimate the decrease in mean skeleton stress.

6 SIMULATION OF UNSATURATED EMBANKMENT

Numerical analysis focusing on the dynamic behavior of unsaturated embankments was performed with the three-phase and two-phase coupled analysis based on the porous media theory (Matsumaru & Uzuoka 2014). In the analyses, a simplified elastoplastic constitutive model for unsaturated sandy soil was employed in the numerical method. Simulations of the 1-g shaking table test of a model embankment affected by seepage water confirmed the applicability of the numerical method. Furthermore, the advantages of the three-phase coupled analysis were discussed by comparing with the two-phase coupled analysis and the three-phase analysis in which the water and air in soil were separated by using large specific water capacity.

Figure 10 shows the finite element model of the unsaturated experimental embankment model. The embankment was made of Inagi sand with the fine contents of 23.6% and the 50% diameter of 0.134 and the uniformity coefficient of 9.29. In the finite element mode, the soil displacement at the bottom boundary is fixed in all directions and the lateral boundaries are vertical rollers. The bottom and lateral boundaries are impermeable and a part of the surface on the embankment is permeable with zero water pressure. As for pore air pressure, the right and bottom boundaries are impermeable, while air drainage is allowed at the surface of the embankment. The parameters of the constituitive models were determined from the simulations of cyclic triaxial tests of Inagi sand in unsaturated conditions. Before shaking, water was poured into the embankment by maintaining the water level at 200 mm in the water tank prepared at the back
of the embankment. The shaking was conducted after confirming the rise of water level at the toe of the embankment. The wave used for shaking was a sinusoidal wave of 5 Hz with the multiple amplitudes of 200 and 400 gal.

In order to discuss the advantage of the three-phase coupled analysis, three cases of simulations were conducted. Case1 is three-phase analysis mentioned in section 2. Case2 is also three-phase analysis, in which the water and air in soil were separated by using large specific water capacity. Case3 is two-phase analysis considering soil particle and water phase, not considering air phase. Two-phase analysis is used widely for the problem of the liquefaction of the ground caused by earthquakes. Figure 11 shows the SWCC used in the simulations of Case1 and Case2. The SWCC in Case1 was determined to reproduce the wetting curve of the water retention test in large suction and drying curve in small suction. On the other hand, the SWCC in Case2 does not coincide with the water retention curves. The parameters were determined to change the degree of saturation largely from the residual to maximum at the suction \( p^* = 0 \).

Figure 12 shows the distributions of mean skeleton stress before shaking. Compared to the initial stress in Case1, the stress in Case2 and Case3 is small. In Case2 and Case3, the zone where the mean skeleton stress is smaller than 1 kPa spreads from the toe of the embankment to the top. However, the zone was not shaped in the simulation of Case1. The mean skeleton stress is defined by using net stress, suction and the degree of saturation. Particularly, the distributions of the degree of saturation before shaking were different between Case1 and Case2, which means that the degree of saturation above the water line is smaller in Case2 than that in Case1. For this reason, the mean skeleton stress would be smaller in Case2. In Case3, the mean skeleton stress is the smallest among three cases because the effect of the suction could not be considered due to no-consideration of the pore air pressure.

Figure 13 shows the time histories of the vertical displacement at the top of the embankment, by magnitude of shaking of 200 gal and 400 gal for all cases. The simulated results in Case1 almost coincided with the experiments and could explain the procedures of the embankment failure. Compared to the displacement in Case1, the displacements of Case2 and Case 3 are larger, which means that the simulated results of both cases differ more largely from the experimental ones. Particularly in the simulation of Case3, displacement occurred by 200 gal shaking though in the experiment it did not occur during the shaking. These differences were due to the differences in the initial mean skeleton stress and generation in pore water and air pressures. In the cases of unsaturated embankment, the conventional two-phase coupled
analyses without considering air phase and SWCC can overestimate the seismic deformation of unsaturated embankment.

7 CONCLUSIONS

The validity of three-phase coupled analyses and the applicability of simplified two-phase coupled analysis were discussed through simulations of cyclic behaviors of triaxial tests, seismic behaviors of horizontal ground and embankments.

Simulation of cyclic triaxial tests showed that the simplified two-phase analysis with $p^w = p^p$ can be applicable in the case with lower initial suction. On the other hand, the simplified two-phase analysis with $p^w = 0$ cannot be applicable even in the case with lower initial suction.

The simulation of unsaturated homogeneous horizontal ground showed that the simplified two-phase analyses with $p^w = p^p$ can be applicable to the seismic behavior of unsaturated sandy ground when the degree of saturation is relatively high such as 90%.

The simulation of soil column near ground surface showed that the simplified two-phase coupled analysis with $p^w = 0$ can underestimate the decrease in mean skeleton stress.

The simulation of unsaturated embankment showed that the conventional two-phase coupled analyses without considering air phase and SWCC can overestimate the seismic deformation of unsaturated embankment due to the underestimation of initial mean skeleton stress.

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