QCD Analysis of Inclusive $\Delta S = 1, 2$ Transitions: The $|\Delta I| = 1/2$ Rule*

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Abstract

The interplay of QCD in $\Delta S = 1, 2$ non-leptonic weak transitions can be rigorously analyzed, at the inclusive level, by studying the 2–point functions associated with the corresponding $\Delta S = 1, 2$ effective Hamiltonians. The next-to-leading order calculation of the se correlators shows a huge ($\sim 100\%$) gluonic enhancement of the $|\Delta I| = 1/2$ channel, providing a qualitative understanding of the $|\Delta I| = 1/2$ rule within QCD.

1. Introduction

The origin of the empirically observed enhancement of strangeness-changing non-leptonic weak amplitudes with isospin transfer $|\Delta I| = 1/2$ is a long-standing question in particle physics. The short-distance analysis of the product of weak hadronic currents results in an effective $\Delta S = 1$ Hamiltonian

$$H_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_i C_i(\mu^2) Q_i,$$

which is a sum of local four-quark operators $Q_i$, constructed with the light ($u, d, s$) quark fields only, modulated by Wilson coefficients $C_i(\mu^2)$ which are functions of the heavy ($t, Z, W, b, c$) masses and an overall renormalization scale $\mu$.

In the absence of strong interactions, $C_2(\mu^2) = 1$ and all other Wilson coefficients vanish. The operator $Q_2$ can be decomposed as $Q_2 = (Q_++Q_-)/2$, where $Q_- \equiv Q_2 - Q_1$ is a pure $|\Delta I| = 1/2$ operator and $Q_+ \equiv Q_2 + Q_1$ induces both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ transitions. The standard electroweak model gives then rise to $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ amplitudes of nearly equal size, while experimentally the ratio between both amplitudes is a factor of twenty. To solve this big discrepancy, QCD effects should be enormous.

The leading $\alpha_s$ corrections indeed give, for $\mu$-values around 1 GeV, an enhancement by a factor two to three of the $Q_-$ Wilson coefficient with respect to the $Q_+$ one. Moreover, the gluonic exchanges generate the additional $|\Delta I| = 1/2$ operators $Q_i$ ($i=3,4,5,6$), the so-called “Penguins”. Nevertheless, this by itself is not enough to explain the experimentally observed rates, without simultaneously appealing to a further enhancement in the hadronic matrix elements of at least some of the isospin–1/2 four-quark operators.

The evaluation of hadronic matrix elements is unfortunately very difficult, since it involves non-perturbative dynamics at low energies. The problem gets, moreover, complicated by the $\mu$-dependence of the matrix elements, which should exactly cancel the corresponding renormalization-scale dependence of the Wilson coefficients. In order to get meaningful results, a full QCD calculation is required; this is a highly non-trivial task.

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The problem becomes much easier at the inclusive level, where the properties of $\mathcal{H}_{\text{eff}}^{\Delta S=1}$ can be analyzed through the 2–point function

$$
\Psi(q^2) \equiv i \int dx e^{iqx} \langle 0 | \mathcal{H}_{\text{eff}}^{\Delta S=1} (x) \mathcal{H}_{\text{eff}}^{\Delta S=1}(0)^\dagger | 0 \rangle = \left( \frac{G_F}{\sqrt{2}} \right)^2 |V_{ud}|^2 \sum_{i,j} C_i(\mu^2) C_j^*(\mu^2) \Psi_{ij}(q^2).
$$

(2)

This vacuum-to-vacuum correlator can be studied with perturbative QCD methods, allowing for a consistent combination of Wilson coefficients $C_i(\mu^2)$ and 2–point functions of the 4–quark operators, $\Psi_{ij}$, in such a way that the renormalization scheme and scale dependences exactly cancel (to the computed order). The associated spectral function,

$$
\frac{1}{\pi} \text{Im} \Psi(q^2) = (2\pi)^3 \sum_{\Gamma} \int d\Gamma \left| \langle 0 | \mathcal{H}_{\text{eff}}^{\Delta S=1} | \Gamma \rangle \right|^2 \delta^4(q - pr),
$$

(3)

is a quantity with definite physical information; it describes in an inclusive way how the weak Hamiltonian couples the vacuum to physical states $\Gamma$ of a given invariant mass. General properties like the observed enhancement of $|\Delta I| = 1/2$ transitions can be then rigorously analyzed at the inclusive level.

A detailed analysis of two-point functions associated with $\Delta S = 1$ and $\Delta S = 2$ operators was presented in Ref. [1], where the $O(\alpha_s)$ corrections to the correlators $\Psi_{ij}$ were calculated. The next-to-leading order (NLO) corrections to the $|\Delta I| = 1/2$ 2–point functions were found to be very large, confirming the QCD enhancement obtained in a previous approximate calculation [2]. Those results were, however, incomplete because the NLO corrections to the Wilson-coefficients of “Penguin” operators were still not known.

The recent calculation of $\mathcal{H}_{\text{eff}}^{\Delta S=1}$ at NLO [3, 4] has allowed us to improve the results of Ref. [1], matching matrix elements and Wilson coefficients consistently at NLO [3]. Previously missing contributions from evanescent operators have been also incorporated [3]. In order to have a check of the results, the calculation has been performed in two different renormalization schemes for $\gamma_5$ (naively anticommuting $\gamma_5$ and ’t Hooft–Veltman), and the scale- and scheme-independence of the final physical quantities has been verified.

2. Approximate results

The full calculation of $\Psi(q^2)$ is rather involved due to the fact that there are several operators which mix under renormalization. One needs to compute, at the four-loop level, all possible 2–point functions $\Psi_{ij}$; i.e. a $6 \times 6$ (12 × 12 at intermediate steps to include the contributions of evanescent operators) matrix correlator which must be renormalized in matrix form, and later convoluted with the NLO Wilson coefficients as indicated in Eq. [2]

It is possible to obtain some simplified results by using two different approximations which eliminate the mixing among operators, while keeping at the same time the important physical effects [3]:

i) If “Penguins” are neglected, the operators $Q_{\pm}$ are multiplicatively renormalizable. The corresponding scheme- and scale-independent spectral functions $\Phi_{\pm \pm}(s) \equiv C_2^i(\mu^2) \frac{1}{\pi} \text{Im} \Phi_{\pm \pm}(s, \mu^2)$ are found to be [3]:

$$
\Phi_{++}(s) \sim \frac{8}{15} \frac{s^4}{(4\pi)^6} \alpha_s(s)^{-4/9} \left[ 1 - \frac{3649}{1620} \frac{\alpha_s(s)}{\pi} \right],
$$

(4)

$$
\Phi_{--}(s) \sim \frac{4}{15} \frac{s^4}{(4\pi)^6} \alpha_s(s)^{8/9} \left[ 1 + \frac{9139}{810} \frac{\alpha_s(s)}{\pi} \right].
$$

(5)

ii) The interesting “Penguin” operator $Q_6$ can be isolated, by noting that in the large $N_c$ limit ($N_c =$ number of colours) the anomalous dimension matrix $\gamma_{ij}$ of the set of operators $Q_i$ becomes zero, but for $\gamma_{66}$; i.e. in this limit there is no mixing among operators and only $Q_6$ gets renormalized. The $O(\alpha_s^2)$ correction can also be easily computed in this limit [3]:

$$
\Phi_{66}(s) \sim \frac{3}{5} \frac{s^4}{(4\pi)^6} \alpha_s(s)^{18/11} \left[ 1 + \frac{117501}{4840} \frac{\alpha_s(s)}{\pi} 
+ 470.72 \left( \frac{\alpha_s(s)}{\pi} \right)^2 \right].
$$

(6)

The NLO corrections to the $|\Delta I| = 1/2$ correlators turn out to be very big and positive, while for $\Phi_{++}$ the correction is moderate and negative. Taking $\alpha_s(s)/\pi \approx 0.1$, we find a moderate suppression of $\Phi_{++}$ by roughly 20%, whereas $\Phi_{--}$ acquires a huge enhancement of the order of 100%. The correction is even bigger for the “Penguin” correlator $\Phi_{66}$: 240% at NLO and 700% at next-to-next-to-leading order! The perturbative calculation blows up in the $|\Delta I| = 1/2$ sector, clearly showing a dynamical gluonic enhancement of the $|\Delta I| = 1/2$ amplitudes.
3. Exact results

Following the notation of Refs. [3], the Wilson-coefficient functions can be decomposed as $C_I (s) = z_i (s) + \tau y_i (s)$, where $\tau \equiv -(V_{ud} V_{us}^*) / (V_{us} V_{us}^*)$. The coefficients $z_i (s)$ govern the real part of the effective Hamiltonian, while $y_i (s)$ parametrize the imaginary part and govern, e.g., the measure for direct CP-violation in the $K$-system, $\varepsilon' / \varepsilon$. We can then form two different scale- and scheme-invariant spectral functions,

\[
\tilde{\Phi}_z (s) = \sum_{i,j} z_i (\mu^2) \frac{1}{\pi} \text{Im} \Psi_{ij} (s, \mu^2) z_j (\mu^2), \quad (7)
\]

\[
\tilde{\Phi}_y (s) = \sum_{i,j} y_i (\mu^2) \frac{1}{\pi} \text{Im} \Psi_{ij} (s, \mu^2) y_j (\mu^2), \quad (8)
\]

corresponding to $z_i$ and $y_i$ respectively.

Since we are mainly interested in the size of the radiative corrections, let us write $\tilde{\Phi}_{z,y} (s)$ as

\[
\tilde{\Phi}_{z,y} (s) = \tilde{\Phi}_{z,y}^{(0)} (s) + \tilde{\Phi}_{z,y}^{(1)} (s), \quad (9)
\]

where the superscripts $(0)$ and $(1)$ refer to the leading and next-to-leading order respectively. The exact results obtained [3] for the ratios $\tilde{\Phi}_z^{(1)}/\tilde{\Phi}_z^{(0)}$ and $\tilde{\Phi}_y^{(1)}/\tilde{\Phi}_y^{(0)}$ are plotted in Fig. 1, for $\Lambda_{\overline{MS}}^{(3)} = 200, 300$, and 400 MeV.

![Figure 1](image_url)

**Figure 1.** The ratios $\tilde{\Phi}_z^{(1)}/\tilde{\Phi}_z^{(0)}$ and $\tilde{\Phi}_y^{(1)}/\tilde{\Phi}_y^{(0)}$.

From Fig. 1, we can see that in the region $Q = 1 - 3$ GeV, and for a central value $\Lambda_{\overline{MS}}^{(3)} = 300$ MeV, the radiative QCD correction to $\tilde{\Phi}_z$ ranges approximately between 40% and 120%, whereas in the case of $\tilde{\Phi}_y$, we find a correction of the order of 100%–240%. As explicitly shown by the approximate results of the previous section, the large $\alpha_s$ corrections correspond to the $|\Delta I| = 1/2$ part of the effective weak Hamiltonian. In fact, the corrections to the $|\Delta I| = 3/2$ correlator are exactly given by Eq. [3] (“Penguins” only give $\Delta I = 1/2$ contributions), and therefore are quite moderate.

In the case of $\Delta S = 2$ transitions, there is only one 4–quark operator. Since the $\Delta S = 2$ and $|\Delta I| = 3/2$ operators belong to the same representation of the (flavour) $SU(3)_L \otimes SU(3)_R$ group, the NLO corrections to the $\Delta S = 2$ correlator are also exactly given by Eq. [3].

4. Summary

The short-distance behaviour of the $\Delta S = 1$ correlators clearly shows a dynamical enhancement of the $|\Delta I| = 1/2$ channel, as a consequence of the interplay of gluonic corrections. The structure of the radiative corrections also allows for a deeper understanding of the underlying dynamical mechanism [3]: large corrections appear wherever quark-quark correlations can contribute. This explains why the phenomenological description of the $|\Delta I| = 1/2$ rule in terms of intermediate effective diquarks [3] was so successful.

A full QCD calculation has been possible because of the inclusive character of the defined 2–point functions. Although only qualitative conclusions can be directly extracted from these results, they are certainly important since they rigorously point to the QCD origin of the infamous $|\Delta I| = 1/2$ rule.

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