Analytical Explanation of Electromagnetic Diffraction From Double-layered Dielectric Wedge

SHUVODIP MAJUMDAR©, (Graduate Student Member, IEEE), AND AMITABHA BHATTACHARYA©, (Senior Member, IEEE)
Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology Kharagpur, Kharagpur 721302, India
Corresponding author: Shuvodip Majumdar (shuvodipmajumdar@gmail.com)

ABSTRACT This article explains the phenomenon of electromagnetic diffraction from a double-layered dielectric wedge analytically. Mountain ridges with forest or snow cover, concrete building with wooden interior wall decoration or vertical garden, et cetera are good examples of double-layered dielectric wedges that often appear in indoor and outdoor propagation channel models. The analytical model presented here considers the contributions from the inner and outer wedges individually. Then the total diffracted field is calculated by superimposing one upon another. The problem becomes complex because the field diffracted from the inner wedge has a cylindrical wavefront, which gets the chance to reach air only after transmitting through the outer wedge’s dielectric material. Moreover, since the interface between air and the outer wedge dielectric is not planar, it adds to the complexity of the problem. The proposed analytical model has been validated by comparing its results with the results from finite-difference time-domain (FDTD) simulations.

INDEX TERMS Electromagnetic edge diffraction, dielectric wedge, Green’s function, uniform theory of diffraction, channel modeling, FDTD modeling.

I. INTRODUCTION Deterministic propagation channel modeling has retained its popularity among researchers for a long time. Especially, uniform theory of diffraction (UTD) based methods still come under consideration even when the propagation environment is complex [1], [2]. This motivated the authors to carry out the work presented in this paper. The following subsections under this section will familiarize the reader with the state of the art, contributions made in this communication, and the organization of the paper.

A. LITERATURE SURVEY Kouyoumjian and Pathak introduced UTD in 1974 [3], a significant improvement over its precursor geometrical theory of diffraction (GTD) [4]. In [3], [4], the wedges under consideration were made of perfect electric conductors (PEC). However, the practical problems demanded research on dielectric wedges too, because, in reality, most of the obstacles (e.g., building corners, hills) which block the line-of-sight path of propagation are made of dielectrics. Chamberlin, Luebbers, and Holm are among them who realized the demand and started developing heuristic UTD-based diffraction coefficients for dielectric wedges [5]–[7]. Later they were followed by many researchers around the globe who improved their coefficients in various aspects. Another group of researchers was also interested in dielectric wedges. However, they worked on the exact integral representation of the diffracted field given by Maliuzhinets [8] with a view to give it a UTD-like form [9]–[11] so that it can be used easily in ray models [12], [13].

On the other hand, in [14]–[19], extensive work on dielectric coated PEC wedges has been conducted following the approach proposed in [20]. In this approach, electric and magnetic physical optics (PO) surface currents, related to each observation domain, have been calculated following the equivalence theorem. Then, as per the observation region, appropriate currents have been used as sources in the radiation integral. After uniform asymptotic evaluation of the integrals, the diffracted field has been presented in the UTD framework.
The authors of [21], [22] also considered the situation of diffraction from dielectric coated metallic wedges and developed heuristic coefficients along with approximate UTD based ray solutions mainly for the situations where antennas are radiating near or on such wedges. Besides this, the approaches presented in [23]–[30] simulate the material coating on the PEC wedge using generalized impedance boundary condition (GIBC). Sommerfeld’s integral form has been used there to represent the total field, and ultimately by using the Maliuzhinets’ technique [8] or some modified version of it [25], [27], the diffracted field has been found out. In [31], a diffraction coefficient for double-layered dielectric wedges has been presented which also looks at the situation from outside and considers the material properties of the wedge in a combined manner.

The papers referred to in this literature survey do not form a complete list. However, on the issue of diffraction from layered wedges, they reflect the present state of the art and summarize all the different approaches adopted by various research groups.

B. CONTRIBUTIONS MADE IN THIS PAPER
The authors of this article observed that there is only one reference [31] where double-layered dielectric wedges have been considered. The rest of the literature [14]–[19], [21]–[30], considering layered wedges, has concentrated upon dielectric covered PEC wedges. Moreover, the existing approaches, summarized in the literature survey, focus on the wedge’s outer surface, and consider the situation in a composite manner. These existing methods do not investigate individual contributions from the inner and outer wedges. This is where the newly presented approach is novel and advantageous over the already existing approaches. In particular, the method by which the contribution from the inner wedge has been considered is the element of novelty in this paper. As a whole, this approach is advantageous because it offers an intriguing physical insight of the situation.

Since microwave power is expensive, better methods of prediction are always sought after. This analytical model takes care of the actual physical phenomena that take place during diffraction from such wedges; inclusion of these phenomena into the channel models will surely make the prediction better.

C. ORGANIZATION OF THE PAPER
The main content of this paper starts with section II, where the analytical model has been summarized. Sections III and IV provide all the relevant details on two main building blocks of the model: the Maliuzhinets’ diffraction coefficient and the appropriate Green’s function. Along with section IV, sections V and VI discuss the core of the analytical model and address the critical issues (e.g., the transmission of cylindrical wave through a bent dielectric interface, calculation of the ingredient parameters required for obtaining the field diffracted from the inner wedge). Section VII compares the diffracted fields only, and section VIII validates the results. The last section concludes the discussion.

II. THE ANALYTICAL MODEL FOR THE DOUBLE-LAYERED DIELECTRIC WEDGE
The situation of a double-layered wedge has been shown in Fig. 1 in two-dimension (2D) where medium 0 is air; medium 1 and medium 2 are two different dielectric materials. In the direction perpendicular to the page, the third dimension contains the edge of the wedge, which has been considered to be very large compared to the operating wavelength. Calculation of the diffracted field in this scenario is a bit complicated because of the contribution from two wedges: inner and outer. In Fig. 1, the situation has been shown for cylindrical wave incidence; the same situation with a plane wave incidence can also be imagined easily.

The field diffracted from the outer wedge can be obtained easily using the rigorous Maliuzhinets’ diffraction coefficient [8]. For the inner wedge, the incident field reaches its corner after transmitting through medium 1 and then it gets diffracted. Like the outer wedge corner, the inner wedge corner too acts like a virtual line-current source [4], [33], and produces the field diffracted from the inner wedge. After that, this diffracted field transmits to medium 0 and superimposes with the field diffracted from the outer wedge. Thus the total diffracted field is produced. This virtual line-current source at the edge of the inner wedge contains the influences from the incident field on the inner wedge corner and the diffraction characteristics of the inner wedge. Here, again the Maliuzhinets’ diffraction coefficient is used to define the virtual line-current source at the edge of the inner wedge. Hence, this coefficient is one of the building blocks of the method, and section III introduces it with all the necessary detailing.

Now, with a focus on the issue of diffraction from the inner wedge, the situation can be imagined as given in Fig. 2, where the source is the virtual line-current source encompassing the effects of the incident field on the inner wedge corner and the diffraction characteristics of the inner wedge.
Before working with the appropriate virtual line-current source, the other building block—the Green’s function has been tested with a simple line-current source in section IV. After obtaining a satisfactory result, the simple source has been replaced by the appropriate line-current source derived in section V. It is to be noted that the coordinate system, shown in Fig. 2, has been followed in the rest of the paper.

III. A SHORT INTRODUCTION TO THE RIGOROUS MALIUZHINETS’ DIFFRACTION COEFFICIENT

Following eq. 4.39 of [9], the rigorous Maliuzhinets’ diffraction coefficient can be expressed in a format given by

$$D(L, n; \phi, \phi') = M_1D_1 + M_2D_2 + M_3D_3 + M_4D_4. \quad (1)$$

The multiplying factors $M_i$ $(i = 1, 2, 3, 4)$ are prepared using Fresnel reflection coefficients and some meromorphic functions, which contain two important terms: $\theta_0$ and $\theta_n$. These two terms, given by

$$\sin(\theta^0_{0,n}) = \frac{\zeta}{Z^0_{0,n}} \quad \text{and} \quad \sin(\theta^h_{0,n}) = \frac{Z^h_{0,n}}{\zeta}, \quad (2)$$

are important because they contain the information regarding the polarization of the incident wave, the angle of incidence, and the constitutive parameters of the wedge faces. The former of (2) is for soft or TM polarization (indicated by $s$, where the electric field is parallel to the edge of the wedge), and the latter is for hard or TE polarization (indicated by $h$, where the magnetic field is parallel to the edge of the wedge), respectively. In (2), $\zeta$ is the free space impedance, and $Z^0_{0,n}$ are the directed (in the direction perpendicular to the medium interface) wave impedances for the appropriate wedge face and polarization. Details of all these terms are available in [9].

The components $D_l(l = 1, 2, 3, 4)$ of the diffraction coefficient in (1) are given by

$$D_l(L, n; \phi, \phi') = \frac{e^{-j\pi/4}}{2n\sqrt{2\pi}k} \cot \gamma(l)F(2kLn^2\sin^2\gamma(l)), \quad (3)$$

where

- $k$ is wave number;
- $F$ is the transition function defined in [3], [32];

for plane wave incidence;

$$\gamma^{(1,2)} = \left[\pi \pm (\phi - \phi')\right]/2n;$$

$$\gamma^{(3,4)} = \left[\pi \pm (\phi + \phi')\right]/2n.$$

It can be easily observed from Fig. 3 that $\phi$ and $\phi'$ are the angle of observation and angle of incidence, respectively, measured from the 0-face of the wedge.

IV. CALCULATION AND VALIDATION OF THE FIELD STRENGTH IN MEDIUM 0 WITH THE SOURCE LOCATED IN MEDIUM 1 USING GREEN’S FUNCTION FOR TM OR SOFT POLARIZATION

There is a Green’s function in section 5.5e of [34], which deals with the transmission of cylindrical waves in a semi-infinite dielectric media, as shown in Fig. 4. This Green’s function cannot be applied directly in the situation of Fig. 2 because the interface between the two dielectric mediums is not planar there. To deal with this problem and to make the Green’s function applicable, the space around the wedge has been divided into two regions (region 1 and region 2), as presented in Fig. 2. Now, each of the regions has one planar interface. Region 1 has the interface along the negative $y$-axis; region 2 has the interface along the negative $x$-axis, and the $z$-axis is along the direction perpendicular to the page, pointed towards the reader.

Since this section checks the effectiveness of the Green’s function in this changed situation with a bent interface, a simple electric line-current, given by (4), has been used here. Later, it has been replaced by an appropriate virtual...
line-current source as per the plan of action detailed in section II.

A. RECEIVER LOCATIONS IN MEDIUM 0
Like many other publications, including [33], here also it has been considered that the receivers are located on a circle of constant radius from the edge of the outer wedge. Starting from the 0-face, the receiver locations can be given by \((r, \phi)\). These location coordinates can be very easily converted to the Cartesian coordinate system given in Fig. 2 as \((x, y) = (−r \cos \phi, r \sin \phi)\).

B. RECEIVED ELECTRIC FIELD IN MEDIUM 0
Let us consider that the thickness of the outer layer is \(d\). Then following Fig. 2, the coordinate of the simple electric line-current source, given by

\[
\vec{I}_E = I_1 e^{j\omega t} \hat{a}_c,
\]

is \((-d, −d)\) because this is the position of the edge of the inner wedge. The \(e^{j\omega t}\) term has been omitted throughout considering that the observation is being done in steady-state at a particular point of time, say, at \(t = 0\).

Now as given in section 5.5e of [34], the Green’s function applicable here looks like

\[
G_0 = \frac{j}{4\pi} \int_{\infty}^{-\infty} e^{-jq(\eta)} \frac{1 + \Gamma(\eta)}{\sqrt{k_1^2 - \eta^2}} d\eta,
\]

where \(k_{1,0}\) is the propagation constant of medium 1 or medium 0; \(q(\eta)\) is given by

\[
q(\eta) = \eta(y − y′) + \sqrt{k_0^2 - \eta^2}x - \sqrt{k_1^2 - \eta^2}x′\]

for region 1 and

\[
q(\eta) = \eta(x − x′) + \sqrt{k_0^2 - \eta^2}y - \sqrt{k_1^2 - \eta^2}y′\]

for region 2; \(\Gamma(\eta)\) is

\[
\Gamma(\eta) = \frac{\sqrt{k_1^2 - \eta^2} - \sqrt{k_0^2 - \eta^2}}{\sqrt{k_1^2 - \eta^2} + \sqrt{k_0^2 - \eta^2}}.
\]

The electric field in medium 0 is given by

\[
\vec{E} = j\omega\mu_0 I_1 G_0 \hat{a}_c.
\]

Now \(G_0\) has been evaluated asymptotically following section 5.3f of [34] with \((x′, y′) = (-d, −d)\) and for every choice of receiver position \((x, y)\). The saddle-point \(\eta_s\), required for evaluating \(G_0\) can be obtained from \(dq/d\eta = 0\). Since \(dq/d\eta = 0\) is a non-linear equation, \(\eta_s\) can be obtained using the function fsolve of MATLAB [35].

Ultimately, \(G_0\) looks like

\[
G_0 = \frac{j}{2\pi} \sqrt{\frac{2\pi}{|d^2 q / d\eta|^2 \eta_s}} e^{-jq(\eta_s) − jq(\eta_s)/4\pi \text{sgn}(d^2 q / d\eta^2) \eta_s} \sqrt{k_1^2 - \eta_s^2 + \sqrt{k_0^2 - \eta_s^2}}.
\]

C. VALIDATION OF THE RESULTS
This section validates the method of calculating field strength in medium 0 when the source is located in medium 1 by comparing its results with FDTD simulation results.

gpMax is an open-source software that simulates electromagnetic wave propagation by solving the Maxwell’s equations using FDTD method [36]. The domain was designed in this simulator in 2D, as shown in Fig. 5 to simulate the situation of Fig. 2. The size of the domain is 3.6 m × 3.6 m, and the wedge size is 1.8 m × 1.8 m with \(\varepsilon_1 = 2.15\). In the direction perpendicular to the page, the third dimension contains the edge of the wedge, which has been considered of infinite length. The transmitter (Tx) and receiver (Rx) positions have also been indicated in Fig. 5.

![Simulation domain](image)

The comparison of results has been shown in Fig. 6. It can be observed from the figures that the analytical model works pretty well apart from a small region (approximately 20° in Fig. 6(a) and 10° in Figures 6(b) and 6(c), respectively) around the junction of region 1 and region 2. It can also be observed that the region of mismatch and the amount of mismatch comes down with the increase in the depth of medium 1, which puts the Tx further inside the wedge.

This theory has been applied here, in this paper, ignoring the mismatch mentioned above because the region of mismatch is very small. Besides this, the theory works perfectly in the probable shadow regions where the diffracted field is most important. The immediate next section describes how the effects of the incident field at the inner wedge corner and the diffraction characteristics of the inner wedge can be included in the line-current source.

V. THE VIRTUAL LINE-CURRENT SOURCE AT THE INNER WEDGE CORNER
Let us say that electric field \(\vec{E}_{li}\) is the incident field at the edge of the inner wedge. If the soft diffraction coefficient for the inner wedge is \(D_i\), and the observation point is located at a distance \(r_1\) from the edge of the inner wedge, following the conventional method [9], [33], the diffracted field can be obtained as

\[
\vec{E}_{di} = \vec{E}_{li} \frac{D_i}{\sqrt{r_1}} e^{-jkr_1}.
\]
The source, given in (4), radiates a cylindrical wave [37]. If $k_1 r_1 \rightarrow \infty$, the radiated electric field is given by

$$\vec{E}_r = \frac{-\omega \mu k_1 I_1}{2\sqrt{2\pi k_1 r_1}} e^{-j(k_1 r_1 - \frac{\pi}{4})} \hat{a}_z, \quad (12)$$

where $\mu$ and $k$ are permeability and propagation constant of the appropriate medium, respectively. Likewise, an electric line-current source $\vec{I}_d$ can be imagined, which will radiate an electric field same as presented in (11). Such a current source can be obtained by

$$\vec{I}_d = -\sqrt{\frac{8\pi k_1}{\omega \mu}} \vec{E}_1 \tilde{D}_i e^{-j\frac{\pi}{4}}. \quad (13)$$

This current source $\vec{I}_d$ contains the effects from $\vec{E}_1$, the incident electric field at the inner wedge corner, and $\tilde{D}_i$, the diffraction coefficient for the inner wedge. Calculation of $\vec{E}_1$ has been presented in the following subsections, and $\tilde{D}_i$ has been detailed in the next section.

**A. CALCULATION OF $\vec{E}_1$ WHEN THE INCIDENT WAVE IS PLANE**

If the incident wave in medium 0 is plane, the expression of $\vec{E}_1$ looks like

$$\vec{E}_1 = \vec{E}_{0i} T_{01} e^{-j(-d \cos \phi'_1 + d \sin \phi'_1)}, \quad (14)$$

where $\vec{E}_{0i}$ is the incident electric field at the outer wedge corner, $d$ is the thickness of medium 1 and $\phi'_1$ is the angle of incidence for the inner wedge. Once $\phi'_0$, the angle of incidence in medium 0, is known, $\phi'_1$ can be easily calculated from Snell’s law. In (14), $T_{01}$ represents the electric field transmission coefficient in medium 1 in a three-medium system. This has been calculated following article 2-9 of [38] and found to be

$$T_{01} = \frac{1 + R_{01}}{1 + R_{01} R_{12} e^{-j2k_{p1}d}}, \quad (15)$$

where $R_{m(m+1)}$ is the Fresnel reflection coefficient between two non-magnetic mediums $m^{th}$ and $(m + 1)^{th}$ and $k_{p1}$ is the wave vector for medium 1 directed perpendicular to the medium interface.

**B. CALCULATION OF $\vec{E}_1$ WHEN THE INCIDENT WAVE IS CYLINDRICAL**

Let us consider that an electric current source of strength $I_{d0} \hat{a}_z$, located at $(x', y')$, is radiating in medium 0. So the received electric field $\vec{E}_1$ can be calculated following the same method presented in section IV. The expressions will
be a little different because here, the source is located in medium 0, and the destination point \((-d, -d)\) is located in medium 1, but the method will be the same.

VI. THE DIFFRACTION COEFFICIENT FOR THE INNER WEDGE AND ITS INGREDIENT VARIABLES

Here, the rigorous Maliuzhinets’ diffraction coefficient in UTD-like form, as introduced in section III, has been used. In a commonly considered scenario, the wedge is made of a dielectric material, and it is surrounded by air. Since the situation considered here is not the same as the usual situation, some extra care and some new methods were required to deal with the variables of which \(D_1\) is a function. In the following subsections, these ingredient variables have been discussed.

A. THE TERM WHICH CONTAINS THE MATERIAL INFORMATION OF THE WEDGE, \(\theta_{0,n}\)

Here the dielectric constant of the inner wedge material (medium 2) is \(\varepsilon_2\), and it is surrounded by a material of dielectric constant \(\varepsilon_1\) (medium 1). Hence, following the same method given in [9], \(\theta_{0,n}\) have been obtained from the following expression:

\[
\sin \theta_{0,n} = \frac{Z_1}{Z_2} \sqrt{1 - \cos^2 \phi'_1 \frac{k_1^2}{k_2^2}},
\]

where \(\phi'_1\) indicates the angle of incidence for the inner wedge; \(Z_1\) and \(Z_2\) are the intrinsic impedances of medium 1 and medium 2, respectively.

B. OBSERVATION ANGLES, \(\phi_1\)

The observation angle \(\phi_1\) is one of the variables of which \(D_1\) is a function. In the single-layered wedge case, finding this angle of observation was easier because the diffraeted wave used to travel in a straight line from the wedge corner to the point of observation. Hence, this angular position of the straight line, measured from the 0-face of the wedge, was the angle of observation. Here, the diffracted ray changes its direction due to refraction, and hence, for every observation point in medium 0, some extra care is needed for calculating the observation angle. Then only this observation angle can be used in the expression of \(D_1\).

For example, let us take the observation point \((x_1, y_1)\) in medium 0, as per Fig. 7. The observation angle for \((x_1, y_1)\) is clearly \(\phi_{11}\). To reach at \(\phi_{11}\) the point of refraction \((p, 0)\) needs to be calculated first. The equation representing Snell’s law at the point of refraction is given as

\[
k_0 \sin \theta_{10} = k_1 \sin \theta_{11} = \frac{(p - x_1)}{\sqrt{(p - x_1)^2 + y_1^2}} = k_1 \frac{(-p - d)}{\sqrt{(-p - d)^2 + d^2}}.
\]

By solving this equation numerically using the function \texttt{fsolve} of MATLAB [35], \((p, 0)\) has been calculated here. After that, \(\phi_{11}\) can be obtained from

\[
\phi_{11} = \arctan \frac{d}{(-p - d)}.
\]

Following the same method, for \((x_2, y_2)\), the point of refraction \((0, q)\) can be obtained, and after that, the observation angle \(\phi_{12}\) can be calculated from

\[
\phi_{12} = \frac{3\pi}{2} - \arctan \frac{d}{(-q - d)}.
\]

C. ANGLE OF INCIDENCE, \(\phi'_1\)

Now, let us imagine that \((x_1, y_1)\) or \((x_2, y_2)\) is the source point, and \((-d, -d)\) is the destination point. Then this same calculation, presented in the above section, can be done, and \(\phi_{11}\) or \(\phi_{12}\) will give the values for the angle of incidence if the incident wave is cylindrical.

If the incident wave is plane, the method to obtain the incidence angle has already been discussed in section V-A.

D. DISTANCE TRAVELLED BY THE INCIDENT RAY AND THE DIFFRACTED RAY

Once the point of refraction has been calculated for a particular point of observation, say \((x_1, y_1)\), the distance travelled by the diffracted ray can be given by

\[
r_1 = \sqrt{(x_1 - p)^2 + y_1^2} + \sqrt{(p + d)^2 + d^2}.
\]

Following a similar approach, the distance travelled by the incident ray \(r_s\), from the source point to the inner wedge corner, can be calculated too.

VII. TOTAL DIFFRACTED FIELD FROM THE DOUBLE-LAYERED DIELECTRIC WEDGE FOLLOWING THE ANALYTICAL MODEL

This section presents the total diffracted field from the double-layered wedge and compares it with the individual contributions from the inner and outer wedges.

As mentioned in section IV-A, the receivers were deployed around the wedge at a distance of 1 m from the outer wedge corner. The distance between the Tx and the wedge corner is 1.7 m. At the receiver positions, the diffracted field values were calculated both due to the outer and inner wedges. For this calculation, it has been considered that medium 1 and
medium 2 have dielectric constants 2.15 and 5.31, respectively, and medium 2 has a conductivity of 0.21 S/m.

Now, the wedge can be illuminated in two ways: one-face illumination \((\phi' < (n - 1)\pi \text{ or } \phi' > \pi)\) and both-face illumination \(((n - 1)\pi < \phi' < \pi)\). Fig. 1 depicts a situation of one-face illumination, and when both of the faces are illuminated, the situation can be imagined as shown in Fig. 8.

![FIGURE 8. Both-face illumination case for the double-layered dielectric wedge.](image)

With one-face illumination, the theory described in section IV has been applied with the virtual line-current source, \(\vec{I}_d\), given in (13) for calculating the contribution from the inner wedge. Here, it is to be noted that this virtual line-current source contains the incident electric field at the inner wedge corner \((\vec{E}_{1i} \text{ incident at } \phi'_1)\) and the diffraction coefficient of the inner wedge \((D_i)\) in its expression where \(D_i\) depends upon \(\phi'_1\). Hence, the expression for the field diffracted from the inner wedge is given by

\[
\vec{E}_{di} = j\omega\mu_0\vec{I}_d G_0.
\]

After putting the values of \(\vec{I}_d\) and \(G_0\) from (13) and (10), respectively, into (22), it becomes

\[
\vec{E}_{di} = \vec{E}_{1i}D_i\sqrt{\frac{8\pi}{|d^2q/dq^2|_{0i}}} e^{-j\phi(0_i)-j\pi/4}[1+\text{sign}(d^2q/dq^2)]_{0i}.
\]

(23)

The same theory works with both-face illumination too, but here, two virtual line-current sources are needed to be considered. One of the current sources will depend upon the incident electric field coming through the 0-face and the diffraction coefficient of the inner wedge calculated with the corresponding angle of incidence. Likewise, another current source will take into account the incident electric field coming through the n-face and the corresponding diffraction coefficient. Then, at every observation point, a complex addition of diffracted fields originated due to the incident electric fields coming through both 0- and n-face will give the contribution from the inner wedge in a both-face illumination scenario.

The contribution from the outer wedge has been calculated as per eq. (4.39) of [9], which represents the rigorous Mal- luzhinets’ diffraction coefficient in UTD-like form. Finally, contributions from both inner and outer wedges have been added, and the result has been normalized with respect to \(\vec{E}_{0i}\).

In Fig. 9 and Fig. 10, this total diffracted field, normalized with respect to \(\vec{E}_{0i}\), has been presented along with the individual contributions from the inner and outer wedges. Fig. 9 is for a one-face illumination scenario, whereas Fig. 10 shows the comparison for a both-face illumination scenario.
VIII. VALIDATION OF THE ANALYTICAL MODEL THROUGH FDTD SIMULATION

This section compares the calculated responses obtained from the analytical model with the results from FDTD simulations. These simulations too, have been done in gprMax simulator [36].

In the simulator, the domain has been designed in 2D as shown in Fig. 11, where the wedge has an outer angle \((n\pi)\) of 270°. The third dimension, in the direction perpendicular to the page, contains the edge of the wedge, which has been considered to be of infinite length. The domain dimension is 3.6 m \(\times\) 3.6 m, and the wedge dimension is 1.8 m \(\times\) 1.8 m. The transmitter (Tx) and receiver (Rx) positions are same as considered in section VII, and that has been indicated in Fig. 11 too.

![Simulation domain in 2D with one wedge having a face length of 1.8 m.](image)

The simulations have been done with \(\varepsilon_r^1 = 2.15\) and \(\varepsilon_r^2 = 5.31\). A conductivity of 0.21 S/m has also been considered for the inner wedge. These values of material properties [39]–[43] have been chosen following the real-life examples of double-layered dielectric wedges (e.g., a concrete building with wooden interior wall decoration or vertical garden, snow or vegetation covered mountain ridges) to make these choices realistic.

Figures 12 and 13 present the comparison of results obtained from the analytical model and FDTD simulations. For Fig. 12, it is a one-face illumination case where the angle of incidence is 20°, and the outer layer’s thickness is 0.3 m. With 20° angle of incidence, the reflection shadow boundary (RSB) and the incident shadow boundary (ISB) fall at 160° (\(\phi_{RSB}\)) and 200° (\(\phi_{ISB}\)), respectively. Hence, the total field around the wedge can be obtained following Table 1 [33]. The diffracted field has been calculated following the method described in this paper. Apart from that, the incident and reflected fields for appropriate angular ranges have been calculated following geometrical optics. For \(0 < \phi < \phi_{RSB}\), the total received field at the observation points gets mostly contributed by either or both of the incident and reflected fields because they are substantially stronger than the diffracted field. Ultimately, a very close match between the simulated and calculated data has been obtained for \(0 < \phi < \phi_{ISB}\) in Figure 12. For the rest of the range of observation angle (\(\phi_{ISB} < \phi < n\pi\)) too, the agreement is good, where the total field is actually the diffracted field only.

![Comparison of total field around the wedge with \(\phi' = 20^\circ\), \(d = 0.3\) m, at operating frequency of 10 GHz.](image)

![Comparison of total field around the wedge with \(\phi' = 120^\circ\), \(d = 0.2\) m, at operating frequency of 10 GHz.](image)

**TABLE 1.** Rays to be considered for one-face illumination.

| Angular Range | Rays to be considered                  |
|---------------|----------------------------------------|
| \(0 < \phi < \phi_{RSB}\) | Incident, reflected and diffracted |
| \(\phi_{RSB} < \phi < \phi_{ISB}\) | Incident and diffracted              |
| \(\phi_{ISB} < \phi < n\pi\)  | Diffracted                            |

Fig. 13 presents the comparison for a both-face illumination scenario where the angle of incidence is 120°, and the thickness of the outer layer is 0.2 m. Since both the faces are being illuminated in this case, there is no ISB here, but it has two RSBs: RSB-1 at 60° (\(\phi_{RSB-1}\)) and RSB-2 at 240° (\(\phi_{RSB-2}\)). Here the total field can be calculated following
As a whole, both Fig. 12 and Fig. 13 prove that the analytical model presented in this communication is very effective.

IX. CONCLUSION AND FUTURE SCOPE

This paper presents and validates the analytical model of electromagnetic diffraction from a double-layered dielectric wedge. Since the UTD-based Maliuzhinets’ diffraction coefficient is one of the building blocks of this work, and UTD is a high-frequency technique, there is a limitation on the outer layer’s thickness of the wedge related to the operating frequency. This has been investigated, and it has been observed that this method works well when the outer layer’s (medium 1) thickness is around 10a or higher.

The analytical model presented in the paper has been realized in MATLAB R2020a, which involves equation solving by numerical methods. In a personal computer with Windows 10 operating system, Intel(R) Core(TM) i5-8250U processor, 8 GB RAM, this code takes 7.6 ms for computing the received electric field at an observation point after providing the necessary inputs (e.g., dielectric properties of the mediums, operating frequency, thickness of the wedge’s outer layer, location and strength of the source, location of the observation point).

It is important to note that this paper presents the validation of the analytical model in a two-tier fashion. In the first tier, section IV-C validates the base of the theory, and then section VIII validates the analytical model as a whole. This ensures that the physical phenomena that happen during diffraction from a double-layered dielectric wedge have been considered appropriately.

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SHUVODIP MAJUMDAR (Graduate Student Member, IEEE) was born in West Bengal, India, in 1989. He received the B.Tech. degree in ECE from the Maulana Abul Kalam Azad University of Technology, India, in 2011, and the M.Tech. degree in communication engineering from the University of Kalyani, West Bengal, in 2013. He is currently pursuing the Ph.D. degree with the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology Kharagpur, India. His research interests include electromagnetic diffraction and propagation channel modeling.

AMITABHA BHATTACHARYA (Senior Member, IEEE) was born in Kolkata, India, in 1964. He received the B.Tech. (E&ECE) degree from IIT Kharagpur, in 1986, the M.E. (E&TCE) degree from Jadavpur University, in 1994, and the Ph.D. (E&ECE) degree from IIT Kharagpur, in 1998. In 1986, he started his professional career by joining as a Junior Research Engineer in an ISRO sponsored research project at IIT Kharagpur, where he continued as a Senior Research Assistant in a DRDO sponsored research project, till 1991. In 1997, he joined SAMEER, Mumbai, and the Defence Laboratory, Jodhpur, as a Research Scientist. In 2000, he joined the teaching profession as an Assistant Professor with the Department of Electronics and Instrument, Indian School of Mines, Dhanbad, and with the faculty of Electronics and Electrical Communication Engineering Department, IIT Kharagpur, in 2007, where he is currently working as a Professor and involved in the teaching and research activities of the RF and Microwave Group of the E&ECE Department. He has published over 100 research publications in international journals and conferences and has written a textbook on Digital Communication. He has been the Principal Investigator of 27 sponsored research projects and consultancies, and he has conducted 18 sponsored short term courses around the country, mainly in the areas of electromagnetic environments. He has supervised seven Ph.D. thesis and thirty eight postgraduate thesis. His research interests include microwave imaging, high power microwaves, and microwave stealth technology.

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