Chromatic and spectroscopic signatures of microlensing events as a tool for the gravitational imaging of stars

David Valls-Gabaud\textsuperscript{1,2}\textsuperscript{*}

\textsuperscript{1}UMR 7550, CNRS, Observatoire de Strasbourg, 11 rue de l’Université, 67000 Strasbourg, France
\textsuperscript{2}Royal Greenwich Observatory, Madingley Road, Cambridge CB3 0EZ

Accepted 1997 October 20. Received 1997 October 6; in original form 1997 May 15

Abstract

The detection of microlensing events from stars in the Large Magellanic Cloud and in the Galactic bulge raises important constraints on the distribution of dark matter and on galactic structure, although some events may be the result of a new type of intrinsic variability. When lenses are relatively close to the sources, we predict that chromatic and spectroscopic effects are likely to appear for a significant fraction of the microlensing events. These effects are due to the differential amplification of the limb and the centre of the stellar disc, and present a systematic dependence with wavelength and time that provides an unambiguous signature of a microlensing event (as opposed to a new type of intrinsic stellar variability). We present detailed predictions of the effects, using realistic model atmospheres. The observations of these effects provide a direct constraint on stellar atmospheres, allowing a three-dimensional reconstruction or imaging of its structure, a unique tool with which to test the current models of stellar atmospheres.

Key words: stars: atmospheres – stars: imaging – Galaxy: halo – Galaxy: stellar content – dark matter – gravitational lensing.

1 Introduction

Following the pioneering idea by Paczi\'nsky (1986), the detections of several microlensing events from stars in the Large Magellanic Cloud (Alcock et al. 1993, 1996; Aubourg et al. 1993) and of more than 150 from stars in the Galactic bulge (Udalski et al. 1993, 1994; Alcock et al. 1995, 1997a; Alard & Guibert 1997) have raised important constraints on the structure of the different components of our Galaxy. The associated data bases containing millions of light curves are also a gold mine for studies in stellar variability (see Ferlet et al. 1997 and references therein).

Yet doubts may reasonably be cast on the detections of at least some microlensing events. The two EROS (Expérience de Recherche d’Objets Sombres) candidates are variable stars: EROS 1 is a Be star (Beaulieu et al. 1995), while EROS 2 is an eclipsing binary (Ansari et al. 1995). Although spectra of other candidates taken after the event (Della Valle & Livio 1996) show no apparent signs of anomalies, the possibility remains that these events are just detections of a new type of variable star. The detection of ‘bumpers’ (Alcock et al. 1996), a hitherto unknown type of variable star, and the possibility that dwarf novae and cataclysmic variables mimic some events (Della Valle & Livio 1996) are particularly unsettling, even though the distribution of the events in the Hertzsprung-Russell diagram indicates that not all the events can be explained by such phenomena. In addition to continuous and follow-up monitoring, it seems important to find unambiguous proofs that the events detected can indeed be associated with microlensing events.

In this context it is interesting to note that the large rate observed towards the Galactic bulge may perhaps be accounted for if about 50 to 90 per cent of the events are a result of lenses within the bulge itself (Kiraga & Paczy\'nski 1994), particularly if the bar is oriented close to the line of sight. The same applies if many of the lenses are within the LMC (Wu 1994; Sahu 1994) or in the halo and bulge of M 31.

If this is the case, a significant number of microlensing events should present characteristic chromatic (Valls-Gabaud 1994; Witt 1995; Bogdanov & Cherepashuk 1995; Gould & Welch 1996) and spectroscopic signatures (Valls-
2 MICROLENSING OF STARS AS EXTENDED SOURCES

The amplification produced by a lens of mass $M_\text{L}$ at a distance $D_{\text{OL}}$ on a source assumed to be point-like at a distance $D_{\text{OS}}$ (Refsdal 1964; Liebes 1964) is $A_\ell(y) = (\gamma + 2) [\gamma (\gamma + 2)]^{-1}$, where $y$ is the dimensionless impact parameter at time $t_\ell$, and $\epsilon$ is the angular Einstein radius $\epsilon = 4GM_\text{L}/c^2D_{\text{OS}}$, $\tau = D_{\text{OL}}/\epsilon$ is the dimensionless impact parameter at time $t_\ell$, and time-scale of the event, with $v_\perp$ the transverse velocity of the lens. $\epsilon$ is the angular Einstein radius $\epsilon = 4GM_\text{L}/c^2D_{\text{OS}}^{1/2}$ where $D_{\text{OS}}$ is the distance separating the lens and the source. Since all scales involved are much larger than the wavelength of light, the geometrical optics approximation is valid, and the events should not show diffraction, interference or chromatic effects. Yet, given the colour–magnitude diagrams of the present surveys, one expects that between one-third and two-thirds of the events are associated with giant stars, so that the point-source approximation might be questionable.

If the angular radius $\theta_\theta$ of a star of radius $R_\star$ seen at a distance $D_{\text{OS}}$ is comparable with the angular Einstein radius $\epsilon$, then effects arising from the finite size of the stars will become apparent. Thus it is only when either the lens mass $M_\text{L}$ or the relative distance $D_{\text{LS}}/D_{\text{OS}}$ (or both) is small that these angles are similar. Although the conditions for this to be true might seem contrived, there will always be a number of microlensing events in which the lens will transit through the stellar disc (Gould 1994; Witt 1995). The fraction $f_\ell$ of all lensing events which have a maximum impact parameter smaller than the Einstein radius is simply $f_\ell = \langle \theta_\theta \rangle / \langle \epsilon \rangle$, where the average must be taken over the distribution of distances of both lenses and sources, and hence depends on the galactic model adopted.

Since the fraction increases as $M_\text{L}^{-1/2}$, if the lenses are substellar objects the fraction may reach unity for a large range of $D_{\text{LS}}$ distances. However, for lenses of 0.1 $M_\odot$ and typical fractional distances $D_{\text{LS}}/D_{\text{OS}}$ from 0.1 (budge) to about $5 \times 10^{-1}$ (M 31), the fraction of transit events goes from 31 to 16 pr cent for typical clump giants of 10 $R_\odot$. Since they constitute at least a third of the sources, the minimum fraction of transit events is around 10 per cent for the bulge and about 5 per cent in M 31, for 0.1 $M_\odot$ lenses. For these events, the size of the stars has to be taken into account.

3 CHROMATIC AND SPECTROSCOPIC SIGNATURES

The inclusion of the finite size of the sources was first analysed by Bontz (1979) and more recently by Nemiroff & Wickramasinghe (1994), Gould (1994) and Witt & Mao (1994) for the idealized case of a uniform brightness source, but little attention has been paid so far to the amplification produced by realistic models of stars. It is also well known that even in the Eddington approximation the difference in intensity between the centre and the limb of a star reaches almost one magnitude. Simmons et al. (1995b) used the greybody approximation to show a significant chromatic effect for wavelengths differing by a factor of 5. In the general case, then, the magnification $A_\ell$ of an extended source with specific intensity $f(\ell)$ is given by the integration over the extension $\Omega$ of the source of the amplification produced on an infinitesimal area on the source at an angular distance $\theta$ from the point lens $A_\ell(\theta) = \int_0^{\theta_{\text{os}}} W \frac{d\theta}{\theta_{\text{os}}} A_\ell(\theta) / \int_0^{\theta_{\text{os}}} d\theta \frac{d\theta}{\theta_{\text{os}}}$ (Refsdal 1964). Asuming that the star is spherically symmetric (i.e., neglecting rotational flattening, spots, etc., but cf. Heyrovsky & Loeb 1997 for the case of elliptical sources) and using polar coordinates in the source plane and the notation $\theta = \gamma + x^2 + 2\gamma x \cos \phi$, the actual magnification of the star becomes

$$A_\ell(\gamma) = \frac{1}{\pi R_\star} \int_0^{2\pi} \int_0^1 d\phi d\theta_\parallel(x) A_\ell(p_\parallel(x_\parallel)), (1)$$

where $x = r/R_\star$, $x_\parallel = \theta_{\text{os}}/\theta_\parallel$, $\gamma = \theta_\parallel \theta_{\text{os}}/\theta_\parallel$ is the angular separation between the lens and the centre of the star in units of the stellar angular radius, $A_\ell(x) = \theta(x)/\theta(0)$ is the normalized brightness profile, and $r_\parallel = 2\gamma^3 / d\cos \phi$ is the dimensionless effective radius at that wavelength. In the case in which $A_\ell(x)$ is constant, the maximum amplification reduces to $A_{\text{max}} = (1 + 4/\gamma)^{1/2}$ (Refsdal 1964). Note that for a star which is very extended in comparison with the Einstein radius, $x_\parallel \gg 1$ and the amplification is very small, while for $\gamma \gg 1$, $A_{\text{max}} \approx A_{p_\parallel}$, the point-source limit. The brightness profile $A_\ell$ is given by the solution of the radiative transfer equation,

$$\theta(\omega) = \int_0^\infty \int_0^1 S_\parallel(\omega) \exp \left[ -\frac{\tau_\parallel}{(1 - x^2)^{1/2}} d\tau_\parallel \right] (2)$$

where $\tau_\parallel$ is the emerging intensity at the centre of the star and $S_\parallel$ the source function. It is the explicit dependence with wavelength of the brightness profile $\theta$ and the differential amplification that produce both chromatic and spectroscopic effects (Valls-Gabaud 1994), which reflect the temperature gradient $T(\omega)$ in the stellar atmosphere.

Given a stellar atmosphere model, one derives the corresponding limb profiles for both the continuum and the lines. We have used the large grid of local thermodynamic equilibrium (LTE) models from Kurucz (1994) and fitted, for both the continuum and a selected set of lines, simple analytical expressions. The reason for this is twofold: first, the computational speed is substantially increased, and secondly, the information content of the limb darkening is limited and prevents the use of very detailed models (Bohm 1961). Following the pioneering studies of M line (1922), the limb profiles for the continuum are approximated by a
linear law \( B_{11}(x) = 1 - x(1 - \mu) \), where \( \mu = (1 - x)^{1/2} \). In the modelling of stellar atmospheres, however, non-linear laws have yielded significantly better fits and in particular the logarithmic law \( B_{log}(x) = 1 - x(1 - \mu) - y_1 \mu \) in \( \mu \) and the square-root law (Díaz-Cordovés, Claret & Giménez 1995) \( B_{sqr}(x) = 1 - x(1 - \mu) - y_1(1 - \sqrt{\mu}) \) seem to provide good fits to most of the computed profiles (Van Hamme 1993), although we stress that actual precise data is only available for the Sun. In order to predict the different light curves in the photometric passbands used in microlensing monitoring projects, the limb-darkening coefficients are integrated over these bands, and our results agree with those published by Claret & Giménez (1990) and Van Hamme (1993), for both the Johnson and Strömgren systems.

The important point to note is that since the limb-darkening coefficients obviously depend on wavelength because of the variation in opacity, different limb profiles will appear depending on the photometric band. This strong dependence with wavelength implies that the same microlensing event will present quite different light curves in different photometric bands, as shown in Fig. 1.

![Figure 1](https://academic.oup.com/mnras/article-abstract/294/4/747/1026074)

**Figure 1.** Chromatic signatures of the microlensing of giant red star of \( T_{\text{eff}} = 4000 \) K, \( \log g = 2.0 \) and solar metallicity, for the indicated geometrical parameters \( \gamma_0 = \gamma(t_o) \). The magnifications for the extended Johnson photometric broad bands are calculated assuming a square root non-linear limb-darkening profile. The lower panels give the relative magnification relative to the one in the Johnson V band, \( Q = A, A_\gamma \) which reflects the relative change in colour index \( \Delta(m_v - m_\gamma) \approx 1.086(1 - Q) \).

We can understand the behaviour of the light curves as a function of wavelength by noting that the effective radius \( R_\text{e} \), increases from 0.64 in the U band to 0.90 in the K band for the typical red giant star in Fig. 1. Thus, the longer the wavelength, the more uniform the star, while in the near UV stars are more sharply peaked, giving a larger amplification when the impact parameter is small. At large impact parameters, the strong decrease in brightness at shorter wavelengths takes over and the magnification becomes smaller, relative to larger wavelengths. Using the actual limb-darkening profiles, the linear M line law or the non-linear profiles do not make any significant difference \( (\Delta A/A < 0.01) \) in the final magnifications, and hence the use of the linear approximation is fully justified.

One should stress that all these predictions are based on theoretical models which have never been tested in such detail. There may well be further complications, such as (for example) at larger wavelengths, in the far-IR or sub-mm range, where the limb can actually be brighter (at least in the Sun). In extended envelopes, the increased opacity in the UV can produce a larger \( R_\text{e} \) in this domain, and decrease the contrast \( Q \), while in the X-ray domain, the coronal
emission is very patchy and a smooth, spherically symmetric distribution is meaningless. There may be also complications related to the presence of chromospheres, which change the temperature gradient and hence the limb profiles (see Loeb & Sasselov 1995 for an example in this context). The spectral lines will be more sensitive to this effect (see below). To first order, the chromatic effect does not depend on the colour of the stellar source, since the integrated intensity of the star does not appear in equation (1). However, there will be some dependence, since limb-darkening profiles do depend on the spectral type of the star as well as on the luminosity class. In general, the limb darkening increases when the gravity or temperature decreases and also when the metallicity increases, so the giants of the Galactic and M31 bulges are ideal probes.

The best way to measure the chromatic effect is to observe the event in photometric passbands as widely separated in wavelength as possible, ideally \( U \) and \( K \) which are the most discriminatory for giant stars. It is unfortunate that the current surveys use bands as close as \( V, R \) and \( I \), where the effect is predicted to be very small. As Fig. 1 shows, the relative magnifications \( Q \) (with respect to the \( V \) band) are smaller than 10 per cent, that is, the change in colour index \( \Delta(m_r - m_i) \approx 1.086(1 - Q) \) is less than 0.1 mag, hence it is also essential to do differential, rather than absolute, photometry to measure the effect. Present technology can achieve an accuracy of 0.001 mag (Youn et al. 1991), although a precision of 0.01 will be enough. Finally, the use of narrow-band Strömgren photometry is also recommended, since the Strömgren bands probe different opacity regimes where the source function changes strongly. The wavelength and time dependence of the light curves are unlike those produced by any intrinsic stellar variability, and hence constitute a unique signature of microlensing events.

The spectroscopic signature arises, once again, because line profiles change significantly across the disc, although in this case the variation will depend on the mechanism of formation of the line. For example strong resonant lines, like \( Ly \) or \( Ca II H + K, Mg II h + k \) present a decreasing relative depression with respect to the continuum, and hence appear brighter at the limb, while pure absorption lines disappear at the limb. Lines are far more sensitive to the actual gradient of temperature in the atmosphere, since they probe a larger range of depths, while others are more sensitive to pressure or turbulence. However, since the observations of the variability of the line profiles seem difficult with the present technology, a more appropriate observable is the equivalent width (EW) of the line. For very strong lines, narrow-band photometry in the appropriate range could also be used (Loeb & Sasselov 1995), but the availability of real-time spectra makes this less practical. To illustrate the predicted behaviour of the EWs during a microlensing event, Fig. 2 presents the predictions for the Balmer \( H\beta \) and CO 2.3-\( \mu m \) lines, the CO forming at lower temperature, much higher in the atmosphere, and is limb-brightened. It is immediately apparent that the amplitude of the spectroscopic effect may reach 20 per cent. In the case of \( H\beta \), which is limb-darkened, the EW first decreases when most of the flux originates from the limb, then increases when the lens is probing the central areas of the star. For the vibrational–rotational bands of CO (from 2.2 to 4.6 \( \mu m \)), the opposite behaviour is present, and the temperature gradient is probed at higher altitude in the atmosphere. Once again, the temporal variation of the lines during the event constitutes a unique signature of microlensing, unlike any variation produced by intrinsic stellar variability. Note also that the time-scale is no longer the Einstein ring-cross-
ing time, but rather the stellar-crossing time $\sim R_* f_r (1 - D_L / D_O)$ (see Fig. 2).

4 DISCUSSION

The measure of the chromatic and spectroscopic signatures described above allows for the possibility of the gravitational imaging of stars. Indeed, measures of the amplification in a given wavelength give the brightness distribution on the stellar disc at that wavelength, the solution of the integral equation (1). In addition, the source function at that wavelength can be recovered, solving equation 2, hence giving a three-dimensional picture. $\mathcal{I} (\theta, \tau_3) = \mathcal{I} (\theta, h)$ of the stellar atmosphere, provided the opacity sources contributing at that wavelength are known. This is unprecedented in astrophysics, where only the Sun and very few nearby stars are spatially resolved, either directly (e.g. Gilliland & Dupree 1996) or by interferometry (e.g. Baldwin et al. 1996; Burns et al. 1997).

A crucial technique for the inversion of equations (1) and (2) cannot fit within the scope of this paper, and will be dealt with elsewhere. We simply note here the important implications of stellar imaging. The predictions presented here are based on model atmospheres, in LTE or non-LTE, and the expected limb-darkening coefficients have seldom been compared with observations other than the solar ones. The reason is that the only way to measure these coefficients (besides the solar case) accurately is to use eclipsing binaries, which must have circular orbits, well-detached components, and produce total eclipses. Even then, the derived coefficients correlate with the assumed sizes of the stars (e.g. Tabachnik 1968), so the measure of limb-darkening coefficients in microlensing events is a unique opportunity to test stellar atmosphere models. Note, however, that the inclusion of non-linear terms produces significant differences in the resulting magnification, yet even the measure of the linear coefficients is important, for they provide an estimate of the contribution function of the continuum (basically the integrand of equation 2). This in turn gives a direct estimation of the temperature profile, something that is known in very few stars (e.g. Matthews et al. 1996) and that provides unprecedented constraints on the opacity sources. This technique, applied to the lines (ideally the profiles, but the EWs are still exceptionally important) may lead to the element abundance imaging of the star. In general, atmospheric diagnostics will be possible, by selecting lines sensitive to pressure or temperature, etc.

Unlike Doppler or Zeeman–Doppler imaging, the gravitational imaging is unbiased towards bright stars with large spots, although we note that the presence of spots will considerably complicate the inversion unless the periods are much longer than the time-scale of the event. Complications may also arise from blending (Kamionkowski 1995) and extinction. The ongoing real-time follow-up teams, PLANET (Planet and Lensing Anomalies Network: Albrow et al. 1997) and GMAN (Gravitational Microlensing Anomalies Network: Abe et al. 1997), could adapt their observations strategies to measure these signatures, by increasing the time sampling, widening the wavelength range covered and obtaining spectra with the highest possible resolution. A recent detection of a transit event (Alcock et al. 1997b) gives real-time photometry and spectroscopy of an event associated with an M 4 giant in the Galactic bulge. Both signatures may be present, although further analysis is required for confirmation.

In conclusion, the chromatic and spectroscopic signatures of microlensing events allow us not only to deduce properties of the lenses, but also, and maybe more importantly, to open an entirely new window on stellar atmospheres.

ACKNOWLEDGMENTS

I am grateful to W. V an Hamme and A. Claret for providing their latest limb darkening calculations for comparison with ours.

REFERENCES

Abe F. et al. (GMAN collaboration), 1997, in Ferlet R. et al., eds, V ariable stars and astrophysical returns of microlensing surveys. Editions Frontières, Gif-sur-Yvette, p. 75
Alard C., Gilbert J., 1997, A&A, 326, 1
Albrow M. et al. (PLANET collaboration), 1997, Ferlet R. et al., eds, V ariable stars and astrophysical returns of microlensing surveys. Editions Frontières, Gif-sur-Yvette, p. 135
Alcock C. et al. (MACHO collaboration), 1993, Nat, 365, 621
Alcock C. et al. (MACHO collaboration), 1995, ApJ, 445, 133
Alcock C. et al. (MACHO collaboration), 1996, ApJ, 461, 84
Alcock C. et al. (MACHO collaboration), 1997a, ApJ, 479, 119
Alcock C. et al. (MACHO collaboration), 1997b, ApJ, submitted (astro-ph 9702199)
Anarsi R. et al. (EROS collaboration), 1995, A&A, 299, L21
Aubourg E. et al. (EROS collaboration), 1993, Nat, 365, 623
Baldwin J. E. et al., 1996, A&A, 306, L13
Beaulieu J. P. et al. (EROS collaboration), 1995, A&A, 299, 168
Benetti S., Pasquini L., West R. M., 1995, A&A, 294, L37
Bogdanov M. B., Cherepashchuk A. M., 1995, Astron. Rep. 39, 873
Bogdanov M. B., Cherepashchuk A. M., 1996, Astron. Rep. 40, 713
Böhm K.-H., 1961, ApJ, 134, 264
Bonitz R. J., 1979, ApJ, 233, 402
Burns D. et al., 1997, MNRAS, 290, L11
Claret A., Giménez A., 1996, A&A, 230, 412
Coleman I. J., Simmons J. F. L., Newsam A. M. B., Bjorkmann J. E., 1997, in Ferlet R. et al., eds, V ariable stars and astrophysical returns of microlensing surveys. Editions Frontières, Gif-sur-Yvette, p. 147
Della Valle M., 1994, A&A, 287, L31
Della Valle M., Liivo, M., 1996, ApJ, 457, L77
Díaz-Cordovés J., Claret A., Giménez A., 1995, A&A & S, 310, 329
Ferlet R., Maillard J. P., Raban B., 1997, eds, V ariable stars and astrophysical returns of microlensing surveys. Editions Frontières, Gif-sur-Yvette
Gililand R. L., Dupree A. K., 1996, ApJ, 463, L29
Gould A., 1994, ApJ, 421, L71
Gould A., Welch D. L., 1996, ApJ, 464, 212
Heyrovsky D., Loeb A., 1997, ApJ, in press (astro-ph 9702097)
Kamionkowski M., 1995, ApJ, 442, L9
Kiraga M., Paczyński B., 1994, ApJ, 430, L101
Kurucz R. L., 1994, AAtlas CDROMs (E-mail: kurucz@cfa.harvard.edu)

© 1998 RAS, MNRAS 294, 747–752
Downloaded from https://academic.oup.com/mnras/article-abstract/294/4/747/1026074
on 26 July 2018
Lennon D. J., Mao S., Fuhrmann K., Gehren T., 1996, ApJ, 471, L23
Liebes S., 1964, Phys. Rev., 133, 835
Loeb A., Sasselov D., 1995, ApJ, 449, L33
Maoz D., Gould A., 1994, ApJ, 425, L67
Matthews J. M., Wehlau W. H., Rice J., Walker G. A. H., 1996, ApJ, 459, 278
Milne E. A., 1922, Phil. Trans. Roy. Soc., A 223, 201
Nemiroff R. J., Wickramasinghe W., 1994, ApJ, 424, L21
Paczynski B., 1986, ApJ, 304, 1
Refsdal S., 1964, MNRAS, 128, 295
Sahu K. C., 1994, Nat, 370, 275
Simmons J. F. L., Willis J. P., Newsam A. M., 1995a, A&A, 293, L46
Simmons J. F. L., Newsam A. M., Willis J. P., 1995, MNRAS, 276, 182
Tabachnick V. M., 1968, Astron. Zh., 45, 1048
Udalski A. et al. (OGLE collaboration), 1993, Acta Astron., 43, 289
Udalski A. et al. (OGLE collaboration), 1994, Acta Astron., 44, 1
Valls-Gabaud D., 1994, in Mücke J. et al., eds, Large scale structures in the universe. World Scientific, Singapore, p. 326
Van Hamme W., 1993, AJ, 106, 2096
Witt H. A., 1995, ApJ, 449, 42
Witt H. A., Mao S., 1994, ApJ, 430, 505
Wu X.-P., 1994, MNRAS, 4354, 66
Young A. T. et al., 1991, PASP, 103, 221