Generating two-mode squeezing with multimode measurement-induced nonlinearity

M Riabinin, P R Sharapova, T J Bartley and T Meier

Department of Physics, University of Paderborn, Warburger Straße 100, D-33098 Paderborn, Germany

Keywords: quantum optics, quantum state engineering, measurement induced nonlinearity

Abstract

Measurement-induced nonclassical effects in a two-mode interferometer are investigated theoretically using numerical simulations and analytical results. We demonstrate that for certain parameters measurements within the interferometer lead to the occurrence of two-mode squeezing. The results strongly depend on the detection probability, the phase inside the interferometer, and the choice of the input states. The appropriate parameters for maximized squeezing are obtained. We analyze the influence of losses and confirm that the predicted effects are within reach of current experimental techniques.

1. Introduction

In continuous-variable quantum optics, quadrature squeezing and non-Gaussian entanglement are two important properties of nonclassical light which are required for the implementation of many quantum computation and communication protocols [1]. The most common method used to generate quadrature squeezed light is to exploit a nonlinear interaction in the medium [2], for example by parametric downconversion (PDC) or four-wave mixing (FWM). In the low-gain regime, the amount of squeezing in such processes is proportional to the intensity of the pump fields [3]. In some cases, however, it may be desirable to generate squeezing and entanglement without a strong pump. An alternative is to use measurement-induced nonlinearities (MINL), whereby nonlinear effects can be acquired by applying detection [4, 5].

Early experimental work from Lvovsky and Mlynek showed that combining measurement-induced nonlinearity with single-photon ancilla states, through a process they termed ‘quantum catalysis,’ a number of nonclassical properties can be induced [6]. Since then, combining single-photon ancilla states with single photon measurement has been used for further exotic state generation and manipulation [7–19]. In the context of quadrature squeezing, it was shown that, depending on the interaction parameters, the state in the single output mode may be squeezed [19]. Although the amount of squeezing is limited to 1.25 dB, the appearance of single-mode quadrature squeezing from conditional interference of a single photon and a weak coherent state is not immediately intuitive. The question therefore arises whether, when expanding to more modes, two-mode squeezing [20–26] can be induced using a similar scheme and more generally, whether other classes of multimode entangled states can be generated. Such studies are interesting in the context of identifying the resource requirements for generating multimode non-Gaussian states, which are required for a wide range of continuous-variable quantum information protocols [1, 27].

In this work, we present a detailed theoretical investigation of measurement-induced nonlinearity generated in a four-mode system. We consider a two-mode interferometer in which the single-photon measurements occur within the interferometer itself. In the resulting two output channels, we analytically describe and numerically optimize the acquired nonclassical effects conditional on certain detection events. We show that the implemented detection modifies the photon statistics and leads to the generation of two-mode squeezing (TMS) in the system.

This paper is organized as follows: In section 2 we present our theoretical description of the scheme. In section 3 we present and discuss the analytical results and numerical simulations which demonstrate squeezing for optimized parameters in the case of photon-number-resolved detection. In section 4 we consider the case of...
click detection. In section 5 we visualize the generated states with their Wigner functions. In section 6 we consider the influence of losses. We close with a brief summary in section 7. Additional analytical results are provided in the appendix.

2. Theoretical model

The scheme we consider is shown in figure 1. We consider a Mach–Zehnder interferometer, into which various quantum states can be injected. We consider the specific case of input states which do not exhibit (single-mode) quadrature squeezing, namely a single photon state $|1\rangle_1 = \hat{a}_1^\dagger |0\rangle$ in channel 1 and a coherent state of mean photon number $|\alpha|^2$, $|\alpha|^2 = \exp \left( -\frac{1}{2} |\alpha|^2 \right) \sum_n \frac{|\alpha|^n}{n!} (\hat{a}_1^\dagger)^n |0\rangle$ in channel 2 respectively, as shown in figure 1. The state following interference of the single photon and the coherent state at the final beam splitter is entangled, and exhibits interesting photon statistics [28, 29]. However, the output state does not exhibit two-mode quadrature squeezing, for any beamsplitter parameter chosen. Two-mode squeezing can nevertheless be induced by certain outcomes of measurements made within the interferometer.

The whole interferometer acts on the input light as a series of transformations. First, the light passes through BS$_1$. Afterwards it is split up into the four channels at the beam splitters BS$_2$ and BS$_3$. Detection is possible in the channels 3 and 4, governed by the detection operator $\hat{D}$. The explicit form of $\hat{D}$ depends on the type of detector used, as described in the following section. After detection, a phase shift $\hat{P}_2(\phi)$ is implemented in the upper channel before the final beam splitter BS$_4$. Altogether, the resulting transformations defining the relation between the input and output density matrices can be written as:

$$\tilde{\rho} = \mathcal{Tr}_{A4} [\hat{B}(t_4, t_3) \hat{B}(t_1) \rho_{in} \hat{B}^\dagger(t_3) \hat{B}^\dagger(t_4) \hat{D}]$$

$$\rho_{out} = \hat{B}(t_4) \hat{P}_2(\phi) \hat{P}_4^\dagger(\phi) \hat{B}^\dagger(t_4)$$

(1)

In this formalism, the evolution of the input states through the interferometer in figure 1 is characterized by a set of (lossless) beamsplitters $\text{BS}_i$ of transmission $T_i = t_i^2 = \cos^2 (\theta_i)$ and reflection $R_i = r_i^2 = 1 - T_i = \sin^2 (\theta_i)$ coefficients. In general, the BS operator $\hat{B}(t_i) = \exp [i\theta_i (\hat{a}_1 \hat{a}_2^\dagger + \hat{a}_2 \hat{a}_1^\dagger)]$ with $t_i = \cos (\theta_i)$ defines a linear transformation of the creation (and annihilation) operators between the input $\hat{a}^\dagger = [\hat{a}_1^\dagger, \hat{a}_2^\dagger]^T$ and the output $\hat{a}^\dagger = [\hat{a}_1^\dagger, \hat{a}_2^\dagger]^T$ modes [30]:

$$\hat{B}(t) \hat{a}_1^\dagger \hat{B}^\dagger(t) = \hat{a}_1^\dagger \cos (\theta) - i \hat{a}_2^\dagger \sin (\theta) = t \hat{a}_1^\dagger - ir \hat{a}_2^\dagger,$$

$$\hat{B}(t) \hat{a}_2^\dagger \hat{B}^\dagger(t) = \hat{a}_2^\dagger \cos (\theta) - i \hat{a}_1^\dagger \sin (\theta) = t \hat{a}_2^\dagger - ir \hat{a}_1^\dagger.$$  

(2)

The density matrix of the quantum state can be written as a function of creation and annihilation operators $\rho = \rho(\hat{a}, \hat{a}^\dagger)$. The transformation of the density matrix at each BS can be obtained by using the input/output relations, equation (2), for each operator. The output density matrix $\rho_{out}$ is obtained from input $\rho_{in}$ by applying the BS transformation operator $\hat{B}(t)$, i.e. $\rho_{out} = \hat{B}(t) \rho_{in} \hat{B}^\dagger(t)$. The action of BS$_2$ and BS$_3$ is considered together by the operator $\hat{B}(t_2, t_3) = \hat{B}(t_2) \hat{B}(t_3)$ which describes a transformation of the operators $\hat{a}_1^\dagger$ and $\hat{a}_2^\dagger$ into output modes $\hat{a}_1^\dagger$ in four channels:
\[ \hat{B}(t_2) \hat{a}_2^\dagger \hat{B}(t_2) = t_2 \hat{a}_2^\dagger - i \omega_2 \hat{a}_2, \]
\[ \hat{B}(t_3) \hat{a}_4^\dagger \hat{B}(t_3) = t_3 \hat{a}_4^\dagger - i \omega_4 \hat{a}_4, \]
where \( \hat{B}(t) = \exp \{ i \theta(t \hat{a}_2 \hat{a}_3^\dagger + \hat{a}_2^\dagger \hat{a}_3) \}, \)
\( \hat{B}(t_2) = \exp \{ i \theta(t_2 \hat{a}_4 \hat{a}_3^\dagger + \hat{a}_4^\dagger \hat{a}_3) \} \), and, as before, \( t_i = \cos \theta_i \).

### 2.1. Detectors

In this work we consider two types of detectors: click detectors which measure the absence or presence of photons but provide no information about the photon number and photon-number-resolving (PNR) detectors [31].

PNR detection in one channel can be described by the projection of the state on the chosen Fock state with \( n \) photons in the \( i \)-th channel \( |n\rangle; |\psi_a \rangle = |n\rangle \langle n| \psi_a \rangle \) where \( |\psi_a \rangle \) is the state before detection. The probability of such an event is \( P_{\text{det}} = \langle \psi_a | \langle n| \psi_a \rangle \) for simplicity, we consider only single-photon PNR detection, i.e., projection onto the single photon state \( |1\rangle \) or the vacuum state \( |0\rangle \). In this work, therefore, for the two detectors in channels 3 and 4, see figure 1, we consider four different outcomes: (i) both detectors measure one photon, (ii) only the detector in channel 4 registers a photon, (iii) only the detector in channel 3 registers a photon, and (iv) both detectors measure vacuum:

\[
\begin{align*}
|\psi_{(3\&4)} \rangle &= |1\rangle \langle 1| \otimes |1\rangle \langle 1| |\psi_a \rangle \\
|\psi_{(4)} \rangle &= |0\rangle \langle 0| \otimes |1\rangle \langle 1| |\psi_a \rangle \\
|\psi_{(3)} \rangle &= |1\rangle \langle 1| \otimes |0\rangle \langle 0| |\psi_a \rangle \\
|\psi_{\text{none}} \rangle &= |0\rangle \langle 0| \otimes |0\rangle \langle 0| |\psi_a \rangle ,
\end{align*}
\]
where \( |\psi_a \rangle \) is the state on the four channels after BS₂ and BS₃ before detection.

By contrast, click detectors do not resolve the number of photons and must therefore take into account all possible photon-number contributions. The action of click detectors can be described in terms of the positive operator valued measure (POVM) operators \( \hat{f}_i(\cdot) = |0\rangle \langle 0| \) and \( \hat{f}_i(\cdot) = I - \hat{f}_i(\cdot) = \sum_0^\infty |n\rangle \langle n| \) which describe the absence and presence of a click, respectively. The two detectors are again described by four possible projection operators:

\[
\begin{align*}
\hat{f}_{3\&4} &= \hat{f}_4(\cdot) \otimes \hat{f}_3(\cdot) \\
\hat{f}_4 &= \hat{f}_4(\cdot) \otimes \hat{f}_3(\cdot) \\
\hat{f}_3 &= \hat{f}_3(\cdot) \otimes \hat{f}_4(\cdot) \\
\hat{f}_\text{none} &= \hat{f}_3(\cdot) \otimes \hat{f}_3(\cdot).
\end{align*}
\]

To obtain a density matrix after detection \( \rho' \), in the click detection case we apply the POVM operators to the density matrix before detection \( \rho \) and take the partial trace over the detecting channels \( \rho' = T_{3\&4}(\rho \hat{f}_\text{event}). \) The detection probability is given by \( P_{\text{det}}(\text{event}) = \text{Tr}(\rho' \hat{f}_\text{event}) \), where \( \hat{f}_\text{event} \) is the POVM for a particular measurement event.

For both PNR and click detection, each measurement outcome leads to an unnormalized density matrix. Therefore, we define a new normalized detection operator \( \hat{D} = \hat{f}_\text{event} / P_{\text{det}}(\text{event}) \).

### 2.2. Generating two-mode squeezing

Single-mode squeezing is defined as the reduction of the quadratic variance below the shot noise level \( \Delta^2 X_i < \frac{1}{4} \) [32, 33], with generalized quadratures defined by \( X_i(\xi) = \frac{1}{2}(e^{-i\xi}a + e^{i\xi}a^\dagger) \) and \( \sigma_i(\xi) = \frac{1}{2}(e^{-i\xi}a^\dagger - e^{i\xi}a) \) where \( \xi \) is a quadrature phase and the variance is defined as \( \Delta^2 X = \langle X^2 \rangle - \langle X \rangle^2 \). TMS between modes \( a \) and \( b \) is connected with the mutual variance of quadratures and is described by the joint quadrature operators [34]:

\[
\begin{align*}
C_1 &= \frac{1}{\sqrt{2}} (X_a + X_b) = \frac{1}{\sqrt{8}} (e^{-i\xi}(a + b) + e^{i\xi}(a^\dagger + b)) \\
C_2 &= \frac{1}{\sqrt{2}} (X_a^* + X_b^*) = \frac{1}{i\sqrt{8}} (e^{-i\xi}(a + b) - e^{i\xi}(a^\dagger + b)) \\
P_1 &= \frac{1}{\sqrt{2}} (X_a - X_b) = \frac{1}{\sqrt{8}} (e^{-i\xi}(a - b) + e^{i\xi}(a^\dagger - b)) \\
P_2 &= \frac{1}{\sqrt{2}} (X_a^* - X_b^*) = \frac{1}{i\sqrt{8}} (e^{-i\xi}(a - b) - e^{i\xi}(a^\dagger - b)),
\end{align*}
\]

Similarly to the single-mode case, two-mode light is squeezed if one of the variances in equation (6) is lower than the shot noise level e.g.: \( \Delta^2 C_i < \Delta^2 C_i^{(0)} = \frac{1}{4} \). The condition
Δ^2C_1 = \frac{1}{2}\Delta^2X_1^a + \frac{1}{2}\Delta^2X_1^b + \text{Cov}[X_1^a, X_1^b] < \frac{1}{2}\frac{\Delta^2X_1^a}{\Delta^2X_1^b}$ can be satisfied either if the two modes are uncorrelated and, simultaneously, one or both of them are individually squeezed, or when nonclassical correlations between the modes (entanglement) exist. Two-mode squeezing is defined as a reduction of the variance in comparison to the shot noise level $S_i = 10\log_{10}(\Delta^2C_i/\Delta^2C^0_i)$. In the main text we focus on the quadratures $C_1$ and $C_2$, $P_1$ and $P_2$, however, exhibit similar behavior and analytical results for $\Delta^2P_1$ and $\Delta^2P_2$ are provided in the Appendix.

2.3. Role of detection

To demonstrate the significance of the detectors for the generation of two-mode squeezing in the circuit depicted in figure 1, we first investigate a simplified setup without detection, i.e., the beam splitters BS$_2$ and BS$_3$ transmit the light with $T_2 = T_3 = 1$. Considering the input state $|1\rangle_1 \otimes |\alpha\rangle_2$, the variances can be calculated analytically: $\Delta^2C_1 = \Delta^2C_2 = \frac{1}{2} + \sin(\frac{1}{2}(t_1 + t_4 t_4 - t_2^2 t_4 + t_4^2 t_4)).$ This result depends neither on the mean number of photons of the coherent state $\alpha$ nor on the quadrature phase $\xi$ and has a minimum of $\Delta^2C_1 = \frac{1}{4}$ (0 dB). This means that in the case of only linear elements and non-squeezed input states, the output light is not squeezed as well.

2.3.1. Numerical optimization routine

Adding detectors to the scheme may generate two-mode squeezing. To demonstrate this in general, we perform a numerical optimization to find the minimum of the variance $\Delta^2C_1$ in order to maximize squeezing. We use the following algorithm: chose some detection event $d = D_j$, constrain the probability to be higher than some minimum value $P_{\text{crit}}$, fix phases $\phi$ and $\xi$, and then minimize the variance over all BS parameters:

$$\text{minimize: } \Delta^2C_1(T_1, \phi, \xi, d)$$
$$\text{subject to: } \phi = \phi_0,$$
$$\xi = \xi_0,$$
$$d = D_j,$$
$$P_{\text{det}} \geq P_{\text{crit}}$$
(7)

The constraint on the probability is implemented to avoid cases where squeezing may be generated with vanishing detection probability. In principle, arbitrary values for $P_{\text{crit}}$ may be chosen; this will be determined by the parameters of an experiment.

Although the quantities $\Delta^2C_1$ and $\Delta^2C_2$ exhibit smooth continuous behavior over all parameters, it is still numerically difficult to find a global minimum of these four-variable functions. The straightforward approach with evaluating variances over a multidimensional grid and choosing their minimal values is computationally expensive. One way to improve the situation is to use a gradient descent-based algorithms. In this work we apply the gradient-based algorithm ‘Adam’ [35] with its TensorFlow library implementation [36]. To speed up the convergence of the algorithm, different starting points were chosen.

3. Photon-number-resolved detection

For the case of PNR detection, analytical expressions for the output states and the detection probabilities can be obtained for the cases of single- and both-channel detection which are given in equations (8)–(11). For the case of single-channel PNR detection the state is given by:

$$P_{\text{out}} = |\psi_{\text{single}}\rangle \langle \psi_{\text{single}}|$$
$$|\psi_{\text{single}}\rangle = N_{\text{single}}(\gamma_0 + \gamma_1 \hat{a}_1 + \gamma_2 \hat{a}_2^\dagger) |\alpha_1\rangle |\alpha_2\rangle$$
$$N_{\text{single}} = P_{\text{single}}^{\frac{1}{2}} \exp\left[-\frac{1}{2}(|\alpha_3|^2 + |\alpha_4|^2)\right],$$
(8)

with coefficients

$$\gamma_0 = i\hbar r_3$$
$$\gamma_1 = \alpha_{in} \gamma_3 (r_1 t_2 t_4 - t_1 t_2 r_3 e^{i\phi})$$
$$\gamma_2 = -i\alpha_{in} \gamma_3 (r_1 t_2 t_4 + t_1 t_2 r_3 e^{i\phi})$$
$$\alpha_1 = i\alpha_{in} (\hat{a}_1 t_2 + \hat{a}_2 t_4 e^{i\phi})$$
$$\alpha_2 = \alpha_{in} (\hat{a}_1 t_2 t_4 - \hat{a}_2 t_4 e^{i\phi})$$
$$\alpha_3 = \alpha_{in} t_2$$
$$\alpha_4 = -\alpha_{in} \gamma_3 r_3,$$

where $|\alpha_1, \alpha_2\rangle = |\alpha_1\rangle_1 \otimes |\alpha_2\rangle_2$ is the product of two coherent states and $\alpha_{in}$ is the initial coherent state.
Following a detection event, the state after detection is not normalized. To normalize this state we use the detection probability; or the state given by equation (8) this probability is given by:

\[ \tilde{P}_{\text{single}} = \exp \left[ -|\alpha_{\text{in}}|^2(T_2R_2 + R_1R_3) \right] \times R_1[|\alpha_{\text{in}}|^2T_2^2 + T_1(1 + |\alpha_{\text{in}}|^2T_3 - 2T_2) + |\alpha_{\text{in}}|^2T_2^2(T_2 - T_1)^2]. \]  

(9)

For the case where both channels register a detection event the output state is given by:

\[ \rho_{\text{out}} = |\psi_{\text{both}}\rangle \langle \psi_{\text{both}}| \]

\[ |\psi_{\text{both}}\rangle = N_{\text{both}}(\tilde{\gamma}_0 + \tilde{\gamma}_1\hat{a}_1^\dagger + \tilde{\gamma}_2\hat{a}_2^\dagger)|\alpha_1, \alpha_2\rangle \]  

(10)

where

\[ N_{\text{both}} = \tilde{P}_{\text{both}}^{-1} \exp \left[ -\frac{1}{2}(|\alpha_3|^2 + |\alpha_4|^2) \right] \]

\[ \tilde{\gamma}_0 = \alpha_{\text{in}}^2T_3^2(2T_1^2 - 1) \]

\[ \tilde{\gamma}_1 = \gamma_1\alpha_3 \]

\[ \tilde{\gamma}_2 = \gamma_2\alpha_3 \]

and the probability of realizing the state in equation (10) is:

\[ \tilde{P}_{\text{both}} = \exp \left[ -|\alpha_{\text{in}}|^2(T_2R_2 + R_1R_3) \right] \times |\alpha_{\text{in}}|^2T_3^2(1 + |\alpha_{\text{in}}|^2T_3 - 2T_2) + |\alpha_{\text{in}}|^2T_2^2(T_2 - T_1)^2] \]

\[ + R_1[-2T_1 + |\alpha_{\text{in}}|^2T_2^2(-2T_2 + 3T_3)]. \]  

(11)

The formulas for the output density matrices for the single-detector-click and the both-detectors-click for PNR detection cases share a similar form, but with different sets of coefficients \( \gamma \) and \( \tilde{\gamma} \), see equations (8) and (10). However, the probabilities for detecting one and two photons in the system are different, see equations (9) and (11).

3.1. Special case: neglecting BS4

To analyze the analytical expressions above, we start by considering a particular case of a simplified interferometer with fixed parameters \( T_1 = \frac{1}{2} \), \( T_2 = 2T_1 \equiv T \) and \( T_4 = 1 \). Using equations (A3) and (A4) from the Appendix, the analytical results for variances for the single PNR detection case take the form

\[ \Delta^2 C_1 = \frac{1}{(1 + x)^2} \left\{ \frac{1}{4} + \frac{x}{2} + \frac{x^2}{2} + \frac{x}{8} \{-\cos(2\xi) + \right. \]

\[ + \cos(2\xi - 2\phi) + 2\sin(2\xi - \phi) + 2x\sin(\phi) \left\} \right. \}

\[ \Delta^2 C_2 = \frac{1}{(1 + x)^2} \left\{ \frac{1}{4} + \frac{x}{2} + \frac{x^2}{2} + \frac{x}{8} \{\cos(2\xi) - \right. \]

\[ - \cos(2\xi - 2\phi) - 2\sin(2\xi - \phi) + 2x\sin(\phi) \left\} \right. \}

\[ x = T\alpha^2, \]  

(12)

with the probability of detection given by

\[ \tilde{P}_{\text{single}} = \frac{1 - T}{2} \exp \left[ -|\alpha|^2(1 - T) \right] \left( 1 + |\alpha|^2 \frac{2T}{3} \right). \]  

(13)

Analytical results for \( \Delta^2 P_1 \) and \( \Delta^2 P_2 \) take similar form and provided in the appendix (A5). Further for simplicity, we consider only quadratures \( C_1 \) and \( C_2 \) since the other two exhibit similar behavior. For phases \( \xi \) and \( \phi \), squeezing of quadrature \( C_1 \) is maximized when \( \phi = \pi/2 \) and \( \xi = \pm \pi \). The resulting two-mode squeezing as a function of detection beam splitter transmissivity \( T \) and coherent state amplitude, along with the detection probability, are shown in figures 2(a) and (b), respectively.

As can be seen, the maximum squeezing is -1.25 dB and it can be achieved for all values of \( \alpha \) with the appropriate choice of beamsplitter transmissivity \( T \). This amount of squeezing is identical to the single-mode
case in [19], and the fact that the maximum can always be achieved, independent of $\alpha$, is also similar behavior. However, for larger values of $\alpha$ the probability is strongly reduced. The maximum amount of squeezing as function of the detection probability in this case is shown in figure 2(c). It clearly shows that the maximum
squeezing that can be obtained is limited to -1.25 dB, and that the largest probability with which the maximum squeezing can be obtained gradually decreases for $\alpha > 1$.

3.2. General case

3.2.1. Phase dependence

In general, the maximum amount of squeezing is sensitive to the interplay of phases $\phi$ and the quadrature phase $\xi$. The dependence of the squeezing on each phase, for $|\alpha|^2 = 1$ and maximized over all beam splitter parameters for the quadrature $C_1$ for a single detector registering a photon, is shown in figure 3. To perform the optimization, we use the procedure described in section 2.3.1 with a probability constraint of $P_{\text{crit}} = 0.1$. As can be seen from figure 3, to observe the maximum squeezing, which is equal to -1.25 dB, the phases should be set to $\phi = \pi/2$, $\xi = \pi/2 + \pi$. For the case where both detectors register a single photon, the maximized squeezing has a dependence very similar to that of figure 3, however, a smaller amount of squeezing, the maximum TMS is equal to -0.96 dB, can be generated. The maximum possible value of -1.25 dB squeezing is not achieved in this case, since the probability to do so does not exceed $P_{\text{crit}} = 0.1$ for $|\alpha|^2 = 1$.

3.2.2. Probability considerations

Since we explore a probabilistic effect it is important to understand which amount of squeezing can be achieved for different probability constraints. Figure 4 shows the squeezing maximized over all $T_i$ and phases $\phi$ and $\xi$ as a function of the probability constraint for different values of $\alpha$. For a single-channel detection event, see figure 4(a), a value of $\alpha \approx 1$ gives the optimal result, i.e., the maximum squeezing with the largest probability. For the case that both detectors click, see figure 4(b), we find that some minimal $\alpha \approx 0.4$ is required to be able to achieve the maximum squeezing at all and that the optimal $\alpha$ is increased to about 2. So for both cases in order to realize the maximal squeezing with a reasonable probability $\alpha$ should not be too small but should also not be very large as the maximal probability decreases with increasing $\alpha$.

4. Click detection

We now consider the case where click detections, rather than those with photon number resolution, are used. Analogously to our previous analysis, one can derive analytical expressions for the output states and the detection probabilities for the case of click detection in a single channel and in both channels, which are presented in equations (14)–(17). For the single-channel click detection event, the state is given by

$$
|\psi_k\rangle = (k\gamma_0 + \gamma_1 a_1^+ + \gamma_2 a_2^+)|\alpha_1, \alpha_2\rangle,$$

$$
\rho_{\text{out}} = \frac{e^{-|\alpha|^2 - |\alpha|^2}}{P_{\text{single}}} \sum_{k=1}^{\infty} \frac{1}{k!} |\alpha|^4 e^{-2|\alpha|^2} |\psi_k\rangle \langle \psi_k|.
$$

Figure 3. The squeezing of the quadrature $C_1$ maximized over all beam splitter parameters $T_i$ as a function of the phases $\phi$ and $\xi$. The single PNR detection case with a critical probability $P_{\text{crit}} = 0.1$ is considered and the coherent state has $\alpha = 1$. 

$\xi_1$
and the probability of generating this state is

\[
P_{\text{single}} = e^{-|\alpha|^2}\left(\sum_{n_2,n_4=0}^{\infty} \frac{\hat{\alpha}_2^{n_2-1}\hat{\alpha}_3^{n_4-1}}{\sqrt{n_2!n_4!}}(g_2 n_2\hat{\alpha}_2 + g_4 n_4\hat{\alpha}_4)^2\right).
\]

where

\[
g_2 = \gamma_3 t_2, \quad g_3 = \gamma_0, \quad g_4 = \gamma t_3, \quad \hat{\alpha}_2 = \alpha_{in} t_2, \quad \hat{\alpha}_3 = \alpha_4, \quad \hat{\alpha}_4 = i\alpha_{in} n t_3.
\]

For click detection in both channels, the output state is given by

\[
\rho_{\text{out}} = \frac{P_{\text{both}}}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} |\alpha_3|^{2m-2}|\alpha_4|^{2n-2}m!n!}
\]

\[
|\psi_{m,n}\rangle = (m\tilde{\gamma}_3 + n\tilde{\gamma}_4 + \tilde{\gamma}_3\hat{\alpha}_4^\dagger + \tilde{\gamma}_4\hat{\alpha}_3^\dagger)|x_1, x_2\rangle
\]

where

\[
\tilde{\gamma}_3 = \alpha_{in} t_3^2 r_3, \quad \tilde{\gamma}_4 = -\alpha_{in} t_4^2 r_4.
\]
and the probability of realizing this state is

\[
\begin{split}
P_{\text{both}} &= 1 - e^{-|a|^2} \left( \sum_{n_2,n_3,n_4=0}^{\infty} \left| \frac{\alpha_2^{m-1} \alpha_3^{n-1} \alpha_4^{n_4-1}}{\sqrt{n_2!n_3!n_4!}} \times (g_2 n_2 \alpha_3 \alpha_4 + g_3 n_3 \alpha_2 \alpha_4 + g_4 n_4 \alpha_2 \alpha_3) \right|^2 \right. \\
&\quad \left. - \sum_{n_2,n_3,n_4=0}^{\infty} \left| \frac{\alpha_2^{m-1} \alpha_3^{n-1} \alpha_4^{n_4-1}}{\sqrt{n_2!n_3!n_4!}} \times (g_2 n_2 \alpha_2 \alpha_4 + g_3 n_3 \alpha_2 \alpha_4 + g_4 n_4 \alpha_2 \alpha_3) \right|^2 \right) \\
&\quad + \sum_{n_2,n_3,n_4=0}^{\infty} \left| \frac{\alpha_2^{m-1} \alpha_3^{n-1} \alpha_4^{n_4-1}}{\sqrt{n_2!n_3!n_4!}} \times (g_3 n_2 \alpha_4 + g_4 n_4 \alpha_2) \right|^2 \right) \tag{17}
\end{split}
\]

where

\[
\begin{align*}
g_1 &= -\eta r_2 \\
\alpha_1 &= \alpha_2.
\end{align*}
\]

For the click detection case, the amount of squeezing can be calculated numerically using quantum states given by equations (14)–(17) in the Fock basis. In the numerical evaluations the considered states are limited to a maximal photon number of 14, which is sufficient to obtain converged results for the range of \(\alpha\) values considered in this section.

4.1. Phase dependence

The maximized squeezing with the probability constraint of \(P_{\text{crit}} = 0.1\) as function of the phases \(\phi\) and \(\xi\) is shown in figure 5. Compared to the PNR detection, with click detection the maximally achievable squeezing is reduced slightly. With click detectors, the phase dependence of the maximized squeezing is quite similar to the case of PNR detection, see figure 3, and also the maximum squeezing is obtained for the phases \(\phi = \pi/2, \xi = \pi/2 \pm \pi\) which is equal to -1.11 dB for a single detector click and -0.86 dB (not shown in figure) when both detectors click. These values are only slightly smaller than for the PNR detection case since we consider a coherent state with \(\alpha = 1\) for which the contributions from higher photon numbers are small.

4.2. Probability considerations

Figure 6 presents the quadrature squeezing \(C_1\) maximized over all \(T_i\) with the fixed phases \(\phi = \pi/2, \xi = \pi/2 \pm \pi\) as a function of the probability constraint for different values of \(\alpha\). When a single detector clicks, see figure 6(a), the region where squeezing exceeding -1 dB can be achieved with a reasonable probability increases with increasing \(\alpha\). However, already for \(\alpha > 0.8\) the maximum squeezing decreases with increasing \(\alpha\). Thus, similar to the case of PNR detection, also for click detection a trade-off between squeezing and the detection probability is obtained.
When both detectors click, see figure 6(b), much larger values of $\alpha$ are required to get significant squeezing with reasonable probability, since two photons are removed. In this sense, the optimal situation is reached for some $\alpha > 1.6$. Although we have not computed squeezing for larger values of $\alpha$, in order to keep the numerical requirements within reasonable limits, our analysis suggests that squeezing can be obtained for all $\alpha$, however the probability is likely to be very small.

5. Visualizing the generated states

In addition to quadrature squeezing, the structure of the output light can be revealed from the photon number distribution between the two channels $P_{n_1,n_2}$. For each input value of $\alpha$ the maximum squeezing of $-1.25$ dB can be achieved by several combinations of interferometer parameters. As an example, the photon number distribution for maximal squeezing in the single PNR detection case with parameters $\alpha = 1.0, S = -1.25$ dB, $T = [0.68, 0.82, 0.38, 1.0], \phi = 3\pi/2, \xi = \pi/2, P_{\text{det}} = 0.3$ is shown in figure 7(a). Furthermore, figure 7(b) shows the dependencies of the squeezing in the two quadratures $C_1$ and $C_2$ as function of the quadrature phase. The squeezing has a sinusoidal dependence on the phase and the results for the two quadratures are phase shifted by $\pi/2$ with respect to each other, as would be expected. From this plot it is clear that the state generated is not a minimum uncertainty state, since $C_1 + C_2 > 0$ (on a dB scale). In addition, for these parameters, the Reid criterion [37] for identifying the EPR-type correlations is met. This indicates the generation of EPR-correlated states in our two-mode interferometer.

In addition to the photon number distribution, the Wigner function can give information on the phase-space distribution of the state. As an example we compare the reduced Wigner functions, see equation (A7) in the Appendix, of the two-mode squeezed vacuum state (TMSV) $|\text{TMSV}\rangle = \sqrt{1 - |z|^2} \sum_{n_1,n_2} \sqrt{n_1!n_2!} |n_1\rangle |n_2\rangle$ and the state generated in our interferometer, see figure 8. We choose the parameter $z = 0.143$ to obtain the same amount of squeezing $S_x = -1.25$ dB in both cases. In the interferometer we consider a single PNR detection.
with the same parameters as in figure 7. The reduced Wigner functions in figures 8 (a), (b), (e), and (f) are presented in variables of a single mode, whereas shape of the reduced Wigner functions \( W_{PP,12} \) and \( W_{XX,12} \) in figures 8 (c), (d), (g), and (h) is responsible for the squeezing between the two channels. When these Wigner functions take elliptic (squeezed) form, they visualize the squeezing; the more the light is squeezed the narrower the ellipses are. Moreover, one can observe that the Wigner function in figure 8 (g) is shifted relative to the origin due to the presence of a coherent state in the generated light, contrary to the TMSV in figure 8 (c).

6. Influence of losses

For the considered setup, losses related to absorption and scattering are expected to be the largest contribution. To model losses in our scheme, we place additional beam splitters in both channels between BS1 and BS4 and consider losses before and after detection, i.e., before and after BS2 and BS3, as shown in figure 9.

Non-vanishing reflectivities of the additional beam splitters correspond to the removal of a certain fraction of photons from our circuit. The coefficients \( R_{1}^{\Sigma} \) and \( R_{2}^{\Sigma} \) are the total reflection coefficients of the additional beam splitters (losses) placed before and after detection, respectively. They are defined as the sum of the top and bottom reflection coefficients: \( R_{1}^{\Sigma} = R_{1}^{\text{top}} + R_{1}^{\text{bottom}} \) and \( R_{2}^{\Sigma} = R_{2}^{\text{top}} + R_{2}^{\text{bottom}} \).

We perform numerical simulations where the coefficients \( R_{1}^{\Sigma} \) and \( R_{2}^{\Sigma} \) are varied under the condition: \( R_{1}^{\Sigma} = R_{2}^{\Sigma} \) both before and after detection. For low losses, \( R_{1,2}^{\Sigma} \in [0, 0.1] \), the dependence of squeezing on the total reflection is shown in figure 10. This loss regime is compatible with state of the art implementations of
integrated interferometers, in which internal circuit losses of a few percent are feasible [38]. Together with losses at the beam splitters and the detectors [39] we consider 10% as a realistic upper boundary for losses in each channel. For instance, including 5% loss before and after detection ($R_b^{\Sigma} = R_a^{\Sigma} = 0.05$), squeezing is reduced from $-1.25$ dB to $-1.0$ dB. It is worth to note that losses before detection reduce the squeezing much more significantly than losses after detection, perhaps due to the different photon numbers before and after detection. For instance, 5% loss before detection ($R_b^{\Sigma} = 0.05$, $R_a^{\Sigma} = 0$) reduces squeezing from $-1.25$ dB to $-1.01$ dB, however, including 5% loss only after detection ($R_b^{\Sigma} = 0$, $R_a^{\Sigma} = 0.05$), squeezing is reduced from $-1.25$ dB to $-1.21$ dB.

7. Conclusions

We present theoretical and numerical investigations of a linear two-mode interferometer with nonlinear detection operations. With a single-photon Fock state and a coherent state as the two input states to the interferometer, we analyze the influence of detection on two-mode squeezing for the cases of photon-number-resolving and click detection. It is demonstrated that by applying detection it is possible to generate two-mode squeezing. The largest amount of squeezing that can be generated is $1.25$ dB, independent of the amplitude of the coherent state, although varying this, along with other parameters of the interaction, has a significant influence on the success probability, namely that the correct measurement outcome is obtained. To investigate the
feasibility of observing the predicted effects in experiments, we analyze the influence of losses and show that squeezing is degraded only weakly for not too high losses.

It is interesting to note that the amount of two-mode squeezing this interaction generates is identical to the single-mode case [19]. This suggests that this interaction, when the parameters are correctly chosen, produces not only coherence between the photon-number terms required for single-mode squeezing, but also correlations between the photon-number terms when considering a two-mode state. It remains to be seen if further non-Gaussian operations acting on the modes can increase this squeezing further.

Acknowledgments

Financial support of the Deutsche Forschungsgemeinschaft (DFG) through project number 231 447 078 (TRR 142, project C06) is gratefully acknowledged. P. R. Sh. thanks the state of North Rhine-Westphalia for support by the Landesprogramm für geschlechtergerechte Hochschulen. We also thank the PC² (Paderborn Center for Parallel Computing) for providing computing time.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A: Analytical results

A.1. No detection

The output state of light for the circuit without detection ($T_2 = T_3 = 1$) is given by

$$|\psi\rangle = (\gamma_{01}\hat{a}_1^+ + \gamma_{02}\hat{a}_2^+)|\alpha_{01}, \alpha_{02}\rangle$$

$$\gamma_{01} = \eta_4 - \eta_3$$

$$\gamma_{02} = i\eta_4(\eta_4 + \eta_3)$$

$$\alpha_{01} = i\alpha_{in}(\eta_4 + \eta_4)$$

$$\alpha_{02} = \alpha_{in}e^{i\theta}(\eta_4 - \eta_3)$$

where $|\alpha_{01}, \alpha_{02}\rangle = |\alpha_{1}\rangle_1 \otimes |\alpha_{2}\rangle_2$ is a product of two coherent states in different channels. For this state, the variance $\Delta^2C_q = \Delta^2C_2$ can be calculated analytically and is given by

$$\Delta^2C_q = \Delta^2C_2$$

Figure 10. Dependence of squeezing on losses in the channels. $R^T_1$ and $R^T_2$ are total reflection coefficients of loss BS placed before and after detection respectively defined as the sum of the top and the bottom reflection coefficients: $R^T_1 = R_{1\text{top}} + R_{1\text{bottom}}$ and $R^T_2 = R_{2\text{top}} + R_{2\text{bottom}}$, where $R_{\text{top}} = R_{\text{bottom}}$. 

where $|\alpha_{1}, \alpha_{2}\rangle = |\alpha_{1}\rangle_1 \otimes |\alpha_{2}\rangle_2$ is a product of two coherent states in different channels. For this state, the variance $\Delta^2C_q = \Delta^2C_2$ can be calculated analytically and is given by

$$\Delta^2C_q = \Delta^2C_2$$
\[ \Delta^2 C_1 = \langle C_1^2 \rangle - \langle C_1 \rangle^2 = \frac{1}{2} \left( \sin \phi \left( \frac{1}{2} (t_4 t_4 - t_1 t_2) - t_1^2 t_4 - t_2^2 t_4 \right) \right). \] (A2)

**A.2. Two-mode variance**

Variances of the quadratures can be calculated as

\[ \Delta^2 C_1 = \frac{1}{4} \left( \text{Re}(e^{-i\phi}(\langle a^2 \rangle + \langle b^2 \rangle + 2\langle ab \rangle)) + \langle a d \rangle + \langle b d \rangle - 1 \right) - \frac{1}{2} \left( \text{Re}(e^{-i\phi}(\langle a \rangle + \langle b \rangle)) \right)^2 \]

\[ \Delta^2 C_2 = -\frac{1}{4} \left( \text{Re}(e^{-i\phi}(\langle a^2 \rangle + \langle b^2 \rangle + 2\langle ab \rangle)) - \langle a d \rangle - \langle b d \rangle + 1 \right) - \frac{1}{2} \left( \text{Im}(e^{-i\phi}(\langle a \rangle - \langle b \rangle)) \right)^2 \]

\[ \Delta^2 P_1 = \frac{1}{4} \left( \text{Re}(e^{-i\phi}(\langle a^2 \rangle + \langle b^2 \rangle - 2\langle ab \rangle)) + \langle a d \rangle + \langle b d \rangle - 1 \right) - \frac{1}{2} \left( \text{Re}(e^{-i\phi}(\langle a \rangle - \langle b \rangle)) \right)^2 \]

\[ \Delta^2 P_2 = -\frac{1}{4} \left( \text{Re}(e^{-i\phi}(\langle a^2 \rangle + \langle b^2 \rangle - 2\langle ab \rangle)) - \langle a d \rangle - \langle b d \rangle + 1 \right) - \frac{1}{2} \left( \text{Im}(e^{-i\phi}(\langle a \rangle - \langle b \rangle)) \right)^2 \] (A3)

For the PNR detection case the analytical formulas for average values of the operators used in equation (A3) take forms:

\[ \langle a \rangle = N^2 (\gamma_0 \gamma_4 \alpha_1 + \gamma_0^* \gamma_4^* \alpha_4^* + \gamma_0 \gamma_4 \alpha_4 + \gamma_0^* \gamma_4^* \alpha_1^*) + 2 \gamma_0 \gamma_4 \alpha_4 \alpha_1 + 2 \gamma_0^* \gamma_4^* \alpha_1 \alpha_4^* + 2 \gamma_0 \gamma_4 \alpha_4 \alpha_1 + 2 \gamma_0^* \gamma_4^* \alpha_4 \alpha_1^* + 2 \gamma_0 \gamma_4 \alpha_4 \alpha_1^* + 2 \gamma_0^* \gamma_4^* \alpha_4 \alpha_1 \]

\[ \langle b \rangle = N^2 (\gamma_0 \gamma_4 \alpha_2 + \gamma_0^* \gamma_4^* \alpha_5^* + \gamma_0 \gamma_4 \alpha_5 + \gamma_0^* \gamma_4^* \alpha_2^*) + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_2 + 2 \gamma_0^* \gamma_4^* \alpha_2 \alpha_5^* + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_2^* + 2 \gamma_0^* \gamma_4^* \alpha_5 \alpha_2 \]

\[ \langle a^2 \rangle = N^2 (\gamma_0^2 \alpha_1^2 + \gamma_0^* \gamma_4 \alpha_1 \alpha_4^* + \gamma_0 \gamma_4 \alpha_4 \alpha_1^* + 2 \gamma_0 \gamma_4 \alpha_4 \alpha_1 + 2 \gamma_0^* \gamma_4^* \alpha_1 \alpha_4 + 2 \gamma_0 \gamma_4 \alpha_4 \alpha_1 + 2 \gamma_0^* \gamma_4^* \alpha_4 \alpha_1^* + 2 \gamma_0 \gamma_4 \alpha_4 \alpha_1^* + 2 \gamma_0^* \gamma_4^* \alpha_4 \alpha_1 \]

\[ \langle b^2 \rangle = N^2 (\gamma_0^2 \alpha_2^2 + \gamma_0^* \gamma_4 \alpha_2 \alpha_5^* + \gamma_0 \gamma_4 \alpha_5 \alpha_2^* + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_2 + 2 \gamma_0^* \gamma_4^* \alpha_2 \alpha_5 + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_2 + 2 \gamma_0^* \gamma_4^* \alpha_5 \alpha_2^* + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_2^* + 2 \gamma_0^* \gamma_4^* \alpha_5 \alpha_2 \]

\[ \langle a d \rangle = N^2 (\gamma_0^2 \alpha_1 \alpha_2 + \gamma_0^* \gamma_4 \alpha_1 \alpha_5^* + \gamma_0 \gamma_4 \alpha_5 \alpha_1^* + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_1 + 2 \gamma_0^* \gamma_4^* \alpha_1 \alpha_5 + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_1 + 2 \gamma_0^* \gamma_4^* \alpha_5 \alpha_1^* + 2 \gamma_0 \gamma_4 \alpha_5 \alpha_1^* + 2 \gamma_0^* \gamma_4^* \alpha_5 \alpha_1 \]

\[ \langle b d \rangle = N^2 (\gamma_0^2 \alpha_2 \alpha_5 + \gamma_0^* \gamma_4 \alpha_2 \alpha_1^* + \gamma_0 \gamma_4 \alpha_1 \alpha_2^* + 2 \gamma_0 \gamma_4 \alpha_1 \alpha_2 + 2 \gamma_0^* \gamma_4^* \alpha_2 \alpha_1 + 2 \gamma_0 \gamma_4 \alpha_1 \alpha_2 + 2 \gamma_0^* \gamma_4^* \alpha_1 \alpha_2^* + 2 \gamma_0 \gamma_4 \alpha_1 \alpha_2^* + 2 \gamma_0^* \gamma_4^* \alpha_1 \alpha_2 \]

where parameters \( \gamma_i (\gamma_0) \), \( \alpha_i (\alpha_0) \), and \( N \equiv N_{\text{single(bboth)}} \) are defined in equations (8) and (10) for cases where single-detector (both-detectors) measures one photon.

**A.3. Variances \( \Delta^2 P_1 \) and \( \Delta^2 P_2 \) for the special case with neglected BS4**

For the simplified interferometer with fixed parameters \( T_1 = \frac{1}{2}, T_2 = T_3 = T \) and \( T_2 = 1 \) and single PNR detection case, quadrature variances can be calculated using equations (A3) and (A4) and take form:
\[ \Delta^2 P_1 = \frac{1}{(1 + x)^2} \left\{ \frac{1}{4} + \frac{x}{2} + \frac{x^2}{2} + \frac{x}{8} \left[ \cos(2\xi) + \cos(2\xi - 2\phi) - 2 \sin(2\xi - \phi) - 2 \sin(\phi) \right] \right\} \\
\Delta^2 P_2 = \frac{1}{(1 + x)^2} \left\{ \frac{1}{4} + \frac{x}{2} + \frac{x^2}{2} + \frac{x}{8} \left[ \cos(2\xi) - \cos(2\xi - 2\phi) + 2 \sin(2\xi - \phi) - 2 \sin(\phi) \right] \right\} \\
x = T\alpha^2, \quad (A5) \]

A.4. Wigner function for two-mode state

We use the following definition of the Wigner function for the two-mode state [40, 41]:

\[ W_p(\alpha, \beta) = 4 \text{Tr}[\rho \hat{D}_1(2\alpha) \hat{D}_2(2\beta) \hat{P}_1 \hat{P}_2], \quad \text{(A6)} \]

where \( \alpha = \frac{1}{2}(X_1 + iP_1) \) and \( \beta = \frac{1}{2}(X_2 + iP_2) \) are two complex variables corresponding to modes 1 and 2, respectively, and \( \hat{D}_1(\alpha) = \exp(\alpha a_1^\dagger - \alpha^* a_1) \) and \( \hat{P}_j = \exp(\text{im} a_j^\dagger a_j) \) with \( j = 1, 2 \) are the displacement and parity operators for modes 1 and 2, respectively. In order to visualize the quantum state we integrate the four-dimensional Wigner function over two variables and define four reduced Wigner functions as:

\[ W(X_1, P_2) = \int W(X_1, P_1, X_2, P_2) dX_1 dP_1 \]
\[ W(X_1, P_1) = \int W(X_1, P_1, X_2, P_2) dX_2 dP_2 \]
\[ W(P_1, P_2) = \int W(X_1, P_1, X_2, P_2) dX_1 dX_2 \]
\[ W(X_1, X_2) = \int W(X_1, P_1, X_2, P_2) dP_1 dP_2, \quad \text{(A7)} \]

where each function is normalized according to \( \int \int W(x, y)^2 dxdy = 1. \)

**ORCID iDs**

M Riabinin  
https://orcid.org/0000-0002-5915-2717

P R Sharapova  
https://orcid.org/0000-0003-2115-1847

T J Bartley  
https://orcid.org/0000-0002-4145-846X

T Meier  
https://orcid.org/0000-0001-8864-2072

**References**

[1] Andersen U L, Leuchs G and Silberhorn C 2010 Las. Phot. Rev. 4 4337
[2] Klyshko D N 1988 Photons and Nonlinear Optics Gordon and Breach Science (New York: Publishers)
[3] Andersen U L, Gehring T, Marquardt C and Leuchs G 2016 Phys. Scr. 91 055001
[4] Schel S, Nemoto K, Munro W J and Knight P L 2003 Phys. Rev. A 68 032310
[5] Kalman O, Kiss T and Jex I 2018 J. Russ. Laser Res. 39 382
[6] Lvovsky A I and Mlynek J 2002 Phys. Rev. Lett. 88 250401
[7] Usuga M A, Müller C R, Wittmann C, Marek P, Filip R, Marquardt C, Leuchs G and Andersen U L 2010 Nat. Phys. 6 767
[8] Marek P and Filip R 2010 Phys. Rev. A 81 022302
[9] Müller C R, Wittmann C, Marek P, Filip R, Marquardt C, Leuchs G and Andersen U L 2012 Phys. Rev. A 86 010305(R)
[10] Marek P 2013 Phys. Rev. A 88 040508
[11] Miyata K, Ogawa H, Marek P, Filip R, Yonezawa H, Yoshikawa J and Furusawa A 2016 Phys. Rev. A 93 022301
[12] Yuka M, Miyata K, Yonezawa H, Marek P, Filip R and Furusawa A 2013 Phys. Rev. A 88 053816
[13] Marek P, Filip R and Furusawa A 2011 Phys. Rev. A 84 033802
[14] Ourjoumtsev A, Jeong H, Tualle-Brouri R and Grangier P 2007 Nature 448 784
[15] Ferreyrol F, Barbieri M, Blandino R, Fossier S, Tualle-Brouri R and Grangier P 2010 Phys. Rev. Lett. 104 123603
[16] Xiang G Y, Ralph T C, Lund A P, Walk N and Pryde G J 2010 Nat. Photonics 4 316
[17] Sanaka K, Resch K J and Zeilinger A 2006 Phys. Rev. Lett. 96 083601
[18] Resch K J, O’Brien J L, Weinhold T J, Sanaka K, Lanyon B P, Langford N K and White A G 2007 Phys. Rev. Lett. 98 203602
[19] Bartley T J, Donati G, Spring J B, Jin X, Barbieri M, Datta A, Smith B J and Walmsley I A 2012 Phys. Rev. A 86 043820
[20] Magana-Loaiza O S et al 2019 npj Quantum Information 5 80
[21] Han Y, Xue L and Chen B 2020 Quantum Inf. Process. 19 135
[22] Poporalek S et al 2019 Nat. Commun. 10 2694
[23] Diniz E C, Rossatto D Z and Villas-Boas C J 2018 Quantum Inf. Process. 17 202
[24] Lawrie B J, Lett P D, Marino A M and Pooser R C 2019 ACS Photonics 6 1307
[25] Rojas-Rojas S, Barriga E, Muñoz C, Solano P and Hermann-Avigliano C 2019 Phys. Rev. A 100 023841
[26] Larsen M V, Guo X, Breum C R, Neergaard-Nielsen J S and Andersen U I 2019 Quantum Information and Measurement (QIM) V: Quantum Technologies, OSA Technical Digest (Optical Society of America) (https://osapublishing.org/abstract.cfm?URI=QIM-2019-T5A.20) paper T5A.20
[27] Ra Y S, Dufour A, Walschaers M, Jacquard C, Michel T, Fabre C and Treps N 2020 Nat. Phys. 16 144
[28] Biagi N, Costanzo I S, Bellini M and Zavatta A 2020 Phys. Rev. Lett. 124 033604
[29] Meyer-Scott E, Tiedau J, Harder G, Shalm L K and Bartley T J 2017 Sci. Rep. 7 41622
[30] Loudon R 2000 The Quantum Theory of Light 3rd edition (New York, NY: Oxford University Press)
[31] Silberhorn C 2007 Contemp. Phys. 48 143
[32] Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press)
[33] Loudon R and Knight P L 1987 J. Mod. Opt. 34 709
[34] Schnabel R 2017 Phys. Rep. 684 1
[35] Kingma D P and Ba J 3rd International Conference on Learning Representations (CLR) 2015
[36] Abadi M et al 2016 12th USENIX Symposium on Operating Systems Design and Implementation (OSDI 16) (Savannah, GA, USA, November 2–4, 2016) pp 265–283 (https://usenix.org/sites/default/files/osdi16_full_proceedings.pdf)
[37] Reid M D 1989 Phys. Rev. A 40 913
[38] Sharapova P R, Luo K H, Herrmann H, Reichelt M, Meier T and Silberhorn C 2017 New J. Phys. 19 123009
[39] Ferrari S, Schuck C and Pernice W 2018 Nanophotonics 7 11
[40] Cahill K E and Glauber R J 1969 Phys. Rev. 177 1882
[41] Seyfarth U, Klimov A B, de Guise H, Leuchs G and Sanchez-Soto L I 2020 Quantum 4 317