Modification of $Z^0$ leptonic invariant mass in ultrarelativistic heavy ion collisions as a measure of the electromagnetic field

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An extraordinary strong magnetic field, $eB_0 \approx 10^{18}$ Gauss, is expected to be generated in non-central ultrarelativistic heavy ion collisions and it is envisaged to induce several effects on hot QCD matter including the possibility of local parity and local charge conjugation and parity symmetry violations. A direct signature of such e.m. fields and a first quantitative measurement of its strength and lifetime are still missing. We point out that both the mean value of leptonic invariant mass of $Z^0$ boson, reconstructed by its decaying lepton pairs, and the relative width are modified in relativistic heavy ion collisions due to the presence of strong initial e.m. fields. We propose a measurement of the leptonic invariant mass of $Z^0$ as a novel probe of the strength of the $B_0$. Both shifts could be up to about few hundred MeV and are found to depend on the integral of $B_0$ over the time duration quadratically (approximate). Hence it provides a novel and clear probe of electromagnetic fields, which can be tested experimentally.

I. INTRODUCTION

The ultrarelativistic heavy ion collisions (uRHICs) experiments conducted at both the BNL Relativistic Heavy Ion Collider (RHIC) [1, 2] and the CERN Large Hadron Collider (LHC) [3] have created a new state of matter with deconfined quarks and gluons, the quark-gluon plasma (QGP). The QGP is found to be the most perfect fluid created in nature [4–6]. Heavy ion collisions also provide the possibility to probe the local parity (P) as well as charge conjugation and parity (CP) symmetry violation processes in Quantum chromodynamics (QCD) that may be generated by the metastable local domains of gluon fields with a non-zero winding number [7–9]. The most promising way to probe the P and CP violations in QCD is to measure the chiral magnetic effect (CME) [10–16], where a strong magnetic field with a long lifetime is required in order to generate a signal large enough.

A huge electromagnetic field can be generated in non-central ultrarelativistic heavy ion collisions. However, there are a lot of inherent uncertainties in the calculation of the time evolution of the magnetic field in heavy ion collisions due to the uncertainty of the electrical conductivity of QGP [17–19], the poor knowledge of the properties of the initial non-equilibrium stage as well as the complexity of numerically solving magnetohydrodynamics (MHD). This inspired the search for a direct probe of the strong e.m. fields by measuring the directed flow $v_1 = (p_x/p_T)$ splitting between positively and negatively charged hadrons [20, 21], especially heavy meson pairs $(D^0, \bar{D})$ [22–24] or the leptons decayed from $Z^0$ boson [24]. Here we propose a new probe of electromagnetic fields via the leptonic invariant mass distribution of $Z^0$ boson reconstructed from its decaying lepton pairs, whose final momenta should be affected by the presence of e.m. fields. It should be significantly easier to measure the invariant mass distribution of $Z^0$ boson than the $v_1(p_T)$ splitting between its decaying leptons of opposite charge. Hence the measurement of the invariant mass distribution of $Z^0$ would open up a more accessible experimental probe that as we discuss in this Letter can be directly linked to the time integral of the magnetic field.

The paper is organized as follows: In Sec. II we discuss the parametrization of e.m. fields and a description of the coordinate and momentum distributions of both $Z^0$ boson and its decaying lepton pairs. In Sec. III we present several numerical results including the invariant mass distribution of $Z^0$ reconstructed by its decaying lepton pairs in the presence of e.m. fields and the dependence of the shifts of both the invariant mass of $Z^0$ and its width on the configuration of e.m. fields. Summary and conclusions are given in Sec. IV.

II. ELECTROMAGNETIC FIELDS AND LEPTONS FROM $Z^0$

To study the effect of e.m. fields on the $Z^0$ invariant mass reconstructed by its decaying lepton pairs, we adopt a general parametrization of the configurations of e.m. fields used in several studies [13–15, 25, 26]:

$$eB_y(x, y, \tau) = -B(\tau)\rho_B(x, y)$$

$$(1)$$

$$\rho_B(x, y) = \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)$$

$$(2)$$

$$B(\tau) = eB_0/(1 + (\tau/\tau_B)^3),$$

$$(3)$$

where $B_0$, $\sigma_x$ and $\sigma_y$ are usually given by the estimates of e.m. fields in the vacuum in AA collisions at $t = 0$ [27]. The above gives the transverse coordinate dependence and time evolution of $B_y$. The electric field...
\( eE_x \) is then determined by solving the Faraday’s Law \( \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t; \)

\[
e E_x(t, x, y, \eta_S) = \rho_B(x, y) \int_0^{\eta_S} d\chi B\left( t \cosh\chi \right) \frac{t}{\cosh\chi}. \tag{4}
\]

where the invariant time \( \tau \) and space-time rapidity \( \eta_S \) are related to \( t \) and \( z \) by \( \tau = \sqrt{t^2 - z^2} \) and \( \eta_S = \frac{1}{2} \ln \left( \frac{t+z}{t-z} \right) \).

We note that the above configurations of e.m. fields may not apply to space with a large magnitude of \( \eta_S \) and transverse coordinate \( \rho = \sqrt{x^2 + y^2} \), where one should solve the full Maxwell equations with complex boundary conditions. However, we can safely adopt the above configurations of e.m. fields at small magnitude of \( \eta_S \) and \( \rho \) considering initial transverse coordinates of particles are mostly centered in the overlapping region making the detailed behavior of the e.m. fields at large \( \rho \) irrelevant.

We will focus on 5.02 TeV PbPb collisions at 20-30\% centrality for the numerical calculations, which corresponds to impact parameter \( b = 7.5 \) fm. However, the conclusions should be general based on our physical arguments. The parameters in this colliding system are given by the binary nucleon-nucleon collisions of colliding nucleons. The local temperature \( T \) is related to the drag coefficient \( \gamma \eta \) energy of leptons \( E \), and the local transport number \( T \) by \( D_p = \gamma ET \), and \( \xi \) is a real number randomly sampled from a normal distribution with \( \langle \xi_i \rangle = 0 \) and \( \langle \xi_i \xi_j \rangle = \delta_{ij} \). \( D_p \) is related to the transverse momentum broadening rate \( \hat{\gamma} \) due to elastic collisions between leptons and the medium quarks by \( D_p = \hat{\gamma} / 4 \) [32]. Since the small angle scattering cross section for lepton-quark scattering is:

\[
\frac{d\sigma}{dq^2} \approx \frac{1}{2} \frac{2\pi\alpha^2}{q^4}, \tag{9}
\]

\( \gamma \) will be:

\[
\hat{\gamma} = \sum_q \int_{s^*} s^* \frac{dq^2}{q^2} \rho^q \sigma^q = \int_{s^*} s^* \frac{dq^2}{q^2} = \frac{12\zeta(3)}{\pi} \left( \frac{\alpha_e}{\mu^2} \right)^2 \ln s^*, \tag{10}
\]

where \( \alpha_e \) is the fine structure constant in QED, \( \zeta(3) \approx 1.202 \), \( \rho^q \) is the number density of quarks of each flavor, \( s^* \approx 5.6ET \) is the average center of mass energy of lepton-quark scattering through one photon exchange, and \( \mu^2 = \frac{1}{2}(3 + 2N_c \sum_q \sigma^q) e^2T^2 = 10\pi\alpha_e T^2 \) is the Debye screening mass for the exchange photon from quark and lepton loops.

To quantitatively characterize the effect of lepton-quark scattering or the e.m. fields on the invariant mass of \( Z^0 \) boson, we define two quantities:

\[
\Delta(M) = \langle M_f \rangle - \langle M_i \rangle \tag{11}
\]

\[
\Delta\sigma = \sigma_f - \sigma_i = \sqrt{\frac{\sum(M_f - \langle M_f \rangle)^2}{N-1}} - \sqrt{\frac{\sum(M_i - \langle M_i \rangle)^2}{N-1}}, \tag{12}
\]

where \( f \) and \( i \) stand for the invariant mass of \( Z^0 \) reconstructed by lepton pairs in vacuum and with the effect of lepton-quark scattering or the e.m. fields, and \( N \) is the number of \( Z^0 \) boson used in calculation.

After the evolution of leptons in QGP due to lepton-quark scattering described by Eq. (8), the results show \( \Delta(M) = -1.9 \) MeV and \( \Delta\sigma \leq 0.2 \) MeV, which is a small number compared to the experimental uncertainty on \( M_0 \) and \( \Gamma \) and the modification due to e.m. fields as we will show below. We thus do not include this in the following discussions of the effects of e.m. fields. However, it should be noted that this effect is stronger in more central collision because the lifetime is longer and the temperature of QGP is higher, while the effect of e.m. fields should be smaller because the magnetic field decreases in more central collisions.

III. NUMERICAL RESULTS

A. The effect of lepton-quark scattering

Before discussing the effect of external e.m. fields on the \( Z^0 \) boson invariant mass distribution, it should be noted that this distribution can also be modified due to the lepton-quark scattering in QGP [31]. To consider this effect, we employ the standard Langevin equations:

\[
dx_i = \frac{p_i}{E} dt, \tag{7}
\]

\[
dp_i = -\gamma p_i dt + \xi_i \sqrt{2D_p dt}, \tag{8}
\]

where the momentum diffusion coefficient \( D_p \) is related to the drag coefficient \( \gamma \eta \) energy of leptons \( E \), and the local transport number \( T \) by \( D_p = \gamma ET \), and \( \xi \) is a real number randomly sampled from a normal distribution with \( \langle \xi_i \rangle = 0 \) and \( \langle \xi_i \xi_j \rangle = \delta_{ij} \). \( D_p \) is related to the transverse momentum broadening rate \( \hat{\gamma} \) due to elastic collisions between leptons and the medium quarks by \( D_p = \hat{\gamma} / 4 \) [32]. Since the small angle scattering cross section for lepton-quark scattering is:

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B. Relating the leptonic invariant mass and width of $Z^0$ to e.m. fields strength

In Fig. 1 we show the time evolution of $B(\tau)$ with different sets of $eB_0$, $\tau_B$ and $a$, where $eB_0$, $\tau_B$ and $a$ increase from 73$m^2_{e}/5$ to 73$m^2_{e}$, from 0.05 fm/c to 0.4 fm/c and from 1 to 3, respectively.

The results of the invariant mass distribution of $Z^0$ are shown in Fig. 2, where the solid black line shows the initial distribution of $Z^0$ invariant mass in vacuum, which has a Breit-Wigner form as in Eq. (6). The red line shows the distribution of the $Z^0$ invariant mass reconstructed from lepton pairs after interacting with e.m. fields with $eB_0 = 73m^2_{e}$, $\tau_B = 0.4$ fm/c and $a = 1$. This set of parameters is found to reproduce the directed flow splitting between $D^0$ and $\bar{D}^0$ with $d\Delta v_1/dy = 0.49 \pm 0.17$ (stat.) $\pm 0.06$ (syst.) as measured by the ALICE experiment [33]. It is seen by the red dashed line that such an e.m. field would strongly increase the width $\sigma_{v_0}$ of the distribution of $Z^0$ invariant mass by about 300 MeV and decrease the mean value $\langle M_{Z^0} \rangle$ by about 250 MeV. We have varied $eB_0$, $\tau_B$ and $a$ by a factor of two, respectively. The results are shown by the navy, purple and green lines in Fig. 2, where it is seen that the width increases as well but not as much as the red line. The large uncertainty of ALICE measurements on the $v_1$ splitting of $D^0$ does not allow a determination of the e.m. field. Currently it is still to be clarified whether $\Delta v_0^D$ is determined only by the e.m. fields [24]. Therefore to have a comprehensive study of the effect of e.m. fields on the invariant mass of $Z^0$ reconstructed by lepton pairs, we vary $eB_0$, $\tau_B$ and $a$ in $B(\tau)$ to find some general pattern relating $\Delta(\langle M_{Z^0} \rangle)$ and $\Delta(\sigma_{v_0})$ to the strength and time dependence of the magnetic field. We vary $eB_0$ by a factor of 5 and the life time $\tau_B$ by a factor of 8 and the power law parameter $a$ by a factor of 3 respectively, while keeping other parameters unchanged.

In Fig. 3, we show how $p_T$ integrated $\Delta(\langle M \rangle)$ and $\Delta\sigma$ of midrapidity ($|y| \leq 0.5$) $Z^0$ boson induced by e.m. fields.

Because the invariant mass of $Z^0$ boson is symmetric with charge conjugation, $\Delta(\langle M \rangle)$ should be proportional to $(eB_0)^2$ in the leading order. More specifically, supposeing one $Z^0$ boson at rest with mass $M$ decays into lepton pairs whose momenta $p$ and $-p$ change by $\Delta p_1$ and $\Delta p_2$ due to e.m. fields, then the invariant mass will change
\[ \Delta M = M_f - M = \sqrt{(E(p + \Delta p_1) + E(-p + \Delta p_2))^2 - (\Delta p_1 + \Delta p_2)^2} - M \approx \frac{(\Delta p_1 - \Delta p_2)^2 + 4p \cdot (\Delta p_1 - \Delta p_2)}{2M}. \]  

with \( E(p) = \sqrt{m_i^2 + p^2} \). The negative value of \( \Delta(M) \) implies thus \( p \cdot (\Delta p_1 - \Delta p_2) < 0 \), noting that in general \( \Delta p_1 \neq \Delta p_2 \).

The time integral \( \int_{\tau_0}^{\tau_1} d\tau eB(\tau) \) should be a good quantity to qualify the effect of e.m. fields, where \( \tau_0 \) is the production time of lepton pairs that is about 0.08 fm/c and \( \tau_1 \) is the effective time when charged particles escape e.m. fields, which is about 6-8 fm/c in semi-peripheral collisions. We found that both \( \Delta(M) \) and \( \Delta\sigma \) can be simply fitted as \( k(\int_{\tau_0}^{\tau_1} d\tau eB(\tau))^2 \) with \( k_M = -5.17 \times 10^{-3} \) for the mass, shown as the red dash-dotted line in Fig. 3, and for the width \( k_\sigma = 6.44 \times 10^{-3} \), as the blue dashed line.

In Fig. 4, we extend the study to \( \tau_B \) dependence of the \( p_T \) integrated \( \Delta(M) \) and \( \Delta\sigma \) of midrapidity \( (|y_o| \leq 0.5) \) \( Z^0 \) boson induced by e.m. fields.

In the time dependence. The red squares and blue circles in Fig. 5 show that \( \Delta\langle M \rangle \) changes from -246 MeV to -27.9 MeV, and \( \Delta\sigma \) from 305 MeV to 44.1 MeV, with \( a \) increasing from 1 to 3. Moreover, as shown by the red dash-dotted and blue dashed lines in Fig. 5, \( \Delta\langle M \rangle \) is fitted well with \( k_M = -2.69 \times 10^{-3} \) and \( n_M = 2.33 \) which however stays still quite close to 2, while the parameters \( k_\sigma \) and \( n_\sigma \) used in fitting \( \Delta\sigma \) as a function of \( a \) are found again and quite remarkably to be the same as the other two cases, hence the quadratic relation remain a solid general relation.

In principle one may think to correlate the shifts of the mass \( \langle M \rangle \) and \( \sigma \) of \( Z^0 \) with the splitting in the directed flow \( d\Delta v_1/dy_z |_{y_z=0} \) of the leptons of opposite charge as has been studied in [24], however it has to be noticed that the latter depends only on the \( d\Delta p_x/dy_z \) while the invariant mass distribution depends on all the vector components of the shift, according to Eq. (13).

We have carried on a first study that finds a significant correlation, but only when \( d\Delta v_1/dy_z |_{y_z=0} > 0.05 \), i.e. the \( \Delta p_x \) remains dominant, but the correlations weakens when it has smaller positive and negative values. A more detailed analysis about this aspect will be published later.

We also notice that \( \langle \Delta p_1 - \Delta p_2 \rangle \) should be zero due to \( P \) symmetry, but if one looks at the \( y_z \) dependence of \( \langle \Delta p_1 - \Delta p_2 \rangle \), it will be proportional to \( y_z B \) in the leading order. Eq. (13) implies thus that \( \Delta\langle M \rangle \) should be proportional to \( y_z^2 (d\Delta p_1/dy_z - d\Delta p_2/dy_z)^2 \) at small \( |y_z| \). Therefore we have also performed an initial study for the case \( \tau_0 = 0.4 \text{ fm/c} \) and \( a = 1 \) finding at small \( y_z \) an additional \( y_z^2 \) dependence of both the mass and the width of \( Z^0 \). However the increase of \( \Delta\langle M \rangle \) is about one order of magnitude smaller than the one observed in at \( y_z = 0 \), while the \( \Delta\sigma \) can acquire an additional increase that is comparable to the one found at zero rapidity. We will report about these further aspects in an upcoming longer paper.
C. The centrality dependence of the leptonic invariant mass and width of $Z^0$ in the presence of the e.m. fields

Finally we present the shifts of $Z^0$ leptonic invariant mass and its width induced by e.m. fields as a function of centrality. To do this one needs to calculate the space extension, the strength and the time evolution of the magnetic field. Given the evolution of the first two with centrality it should follow from the initial geometry, while the time dependence of the magnetic field is the main quantity we aim to constraint, we consider the case where the time evolution is independent of centrality. This can serve as a baseline to interpret the future experimental results vs the centrality dependence.

On the centrality dependence of the strength and the space extension of the magnetic field, we estimate it using its value in the vacuum in AA collisions at $t = 0$ as well [27]. The results are shown in Fig. 6, where it is seen that both $\Delta(M)$ and $\Delta\sigma$ increase monotonically from -64 MeV to -340 MeV and from 79 MeV to 423 MeV respectively, as the centrality increases from 5% to 45%. The pattern comes as a balance between the increase with the impact parameter of the maximum initial value of the magnetic field and the decrease of the space and time extension of the fireball and of the magnetic field as driven by the evolution of the geometry.

At centrality around 40% the two effects become of equal magnitude and the mass and width modifications are nearly independent of centrality. Therefore we estimate that this is the centrality where the effects should be maximal; if experimentally the maximum is reached at smaller centrality it would be a signature that the lifetime of the magnetic field decreases with centrality already at smaller centrality.

![Graph showing centrality dependence of $Z^0$ boson](image)

FIG. 6. (Color online) The centrality dependence of $p_T$ integrated $\Delta(M)$ and $\Delta\sigma$ of midrapidity ($|y_\perp| \leq 0.5$) $Z^0$ boson induced by e.m. fields.

IV. CONCLUSIONS AND DISCUSSIONS

This Letter points out a new effect that should be observable in relativistic heavy ion collisions: the modification of both the mean value and the width of the $Z^0$ leptonic invariant mass due to the strong initial electromagnetic field, more specifically a decrease of the invariant mass of $Z^0$ that can be as large as few hundred MeV and the increase of the width by a similar magnitude. Using a wide range of reasonable parametrization for the electromagnetic field, and carrying out a comprehensive study of the modification of the invariant mass of $Z^0$, we find that the decrease of the invariant mass $\langle M_{Z^0} \rangle$ is proportional to $\left( \int_{\tau_0}^{\tau} d\tau B(\tau) \right)^n$ with $n$ that has a very weak dependence on the specific behavior of the $B_\parallel(\tau)$ and has a range of $n_M = 2.16 \pm 0.16$. Even more remarkable is that the increase in the width $\Delta\sigma_{Z^0} = k_\sigma \left( \int_{\tau_0}^{\tau} d\tau B(\tau) \right)^2$ with $k_\sigma = 6.44 \times 10^{-3}$ for all the configurations explored. Moreover, the shifts of both the invariant mass and the width of reconstructed $Z^0$ boson expected to depend also on the rapidity of $Z^0$ quadratically and are expected to further increase the width of the invariant mass distribution. These modifications on the invariant mass distribution of reconstructed $Z^0$ boson due to electromagnetic fields provide a clear probe of electromagnetic fields, which can be tested by experiments at LHC. The main effect pointed out is novel and quite relevant in itself considering that a modification of the invariant mass of the $Z^0$ in AA collisions has never been pointed out before, it appears to be a powerful tool to have a measure of the time integral of the magnetic field produced in relativistic heavy ion collisions. In the future it could be also complemented by the recent suggestions to measure the splitting of the directed flow of $D^0$ and $D^0$ and $l^\pm$ [22, 24, 34], that instead is found to be proportional to $\tau_0 B_\perp(\tau_0) - \tau_1 B_\perp(\tau_1)$. The scope of such studies is even more wide because a determination of the e.m. field can trigger a breakthrough in the ongoing search for the CME, CMW and CVE effects [10–16] as well as on the splitting of the $\Lambda$ polarization [35–37].

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