Air fleet endowment using methods of decision under certainty

R Balasa\textsuperscript{1,2}, M L Costea\textsuperscript{1,2}, A G Andrei\textsuperscript{1}, E I Apostol\textsuperscript{1,2} and A Semenescu\textsuperscript{1,3}

\textsuperscript{1}Politehnica University of Bucharest, 313 Splaiul Independentei, 060032 Bucharest, Romania
\textsuperscript{2}National Institute for Aerospace Research “Elie Carafoli” – INCAS Bucharest, Romania
\textsuperscript{3}Academy of Romanian Scientists, Ilfov 3, 050044, Bucharest, Romania

E-mail: balasa.raluca@incas.ro

Abstract. This study presents how to develop the information used during the decision-making process regarding the endowment of the Romanian air fleet with fighter aircraft, where after the analysis, we have as a result the appropriate model for the requirements and criteria for modernization and reorganization of the fleet. This analysis is performed by mathematical methods under certainty by the moments method or the Deutch Martin method. The first step of the method is to normalize the consequence matrix by linear transformations. It is taken into account that the four chosen criteria are of maximum. The moments method optimally generates clear results regarding the purchase of fighter aircraft that contribute to the development of the fleet.

1. Introduction
Accident prevention is one of the basic paradigms of safety management. If safety management is effective, then accidents should be non-existent. On the other hand, if accidents are occurring then effective safety management must be absent. Therefore, understanding how accidents occur is fundamental to determining how to prevent them. It would seem a very simple connection, but the reality is that accidents are very complex events, rarely the result of a single failure, and this complexity has made it difficult to understand how to cause accidents [1].

Over the years several authors have developed a lot of conceptual models. From the beginning, they seem as various as the problem they intend to solve, but a more detailed analysis shows that there are some common issues. There are linear models that suggest that one factor leads to the next factor and so on, a process that ultimately leads to the accident itself. Also, there are complex nonlinear models whose theory suggests that multiple factors act simultaneously and, through their combined influence, lead to the occurrence of the accident.

The domain of decision analysis models come between two extreme cases. This depends upon the degree of knowledge we have about the outcome of our actions. One “pole” on this scale is deterministic. The opposite “pole” is pure uncertainty. Between these two extremes are problems under risk. The main idea here is that for any given problem, the degree of certainty varies among managers depending upon how much knowledge each one has about the same problem. This reflects the recommendation of a different solution by each person. Probability is an instrument used to measure the likelihood of occurrence for an event. When probability is used to express uncertainty, the
Deterministic side has a probability of one (or zero), while the other end has a flat (all equally probable) probability [2].

It is known that modelling for decision making has two particular sides: the decision maker and the analyst (model-builder). The role of the analyst is to help the decision maker in the decision-making process, that person who cope with a lot of analytical methods.

Certainly, the most important task and also the difficult one for a manager is the decision making. Systems are formed with parts put together in a particular manner in order to pursue an objective. The relationship between the parts determines what the system does and how it functions as a whole. Therefore, the relationship in a system are often more important than the individual parts. In general, systems that are building blocks for other systems are called subsystems. A system that does not change is a static (i.e., deterministic) system. Many of the systems we are part of are dynamic systems, which are they change over time. We refer to the way a system changes over time as the system's behavior. And when the system's development follows a typical pattern, we say the system has a behavior pattern. Whether a system is static or dynamic depends on which time horizon you choose and which variables you concentrate on. The time horizon is the time period within which you study the system. The variables are changeable values on the system [3].

In deterministic models, a good decision is judged by the outcome alone. However, in probabilistic models, the decision-maker is concerned not only with the outcome value but also with the amount of risk each decision carries. As an example of deterministic versus probabilistic models, consider the past and the future: Nothing we can do can change the past, but everything we do influences and changes the future, although the future has an element of uncertainty. Managers are captivated much more by shaping the future than the history of the past [3]. Uncertainty is the fact of life and business; probability is the guide for a "good" life and successful business. The concept of probability occupies an important place in the decision-making process, whether the problem is one faced in business, in government, in the social sciences, or just in one's own everyday personal life. In very few decision-making situations is perfect information - all the needed facts - available. Most decisions are made in the face of uncertainty. Probability enters into the process by playing the role of a substitute for certainty - a substitute for complete knowledge [4].

Probabilistic Modelling is largely based on application of statistics for probability assessment of uncontrollable events (or factors), as well as risk assessment of your decision. The original idea of statistics was the collection of information about and for the State. The word statistics is not derived from any classical Greek or Latin roots, but from the Italian word for state. Probability has a much longer history. Probability is derived from the verb to probe meaning to "find out" what is not too easily accessible or understandable. The word "proof" has the same origin that provides necessary details to understand what is claim to be true [4].

2. The decision-making process

Unlike the deterministic decision-making process, in the decision-making process under uncertainty the variables are often more numerous and more difficult to measure and control. However, the steps are the same. They are:

1. Simplification
2. Building a decision model
3. Testing the model
4. Using the model to find the solution
   - It is a simplified representation of the actual situation
   - It need not be complete or exact in all respects
   - It concentrates on the most essential relationships and ignores the less essential ones.
   - It is more easily understood than the empirical situation and, hence, permits the problem to be more readily solved with minimum time and effort.
5. It can be used again and again for like problems or can be modified.
The mathematical models and techniques considered in decision analysis are concerned with prescriptive theories of choice (action) [5].

The main sources of errors in risky decision-making problems are:
- false assumptions,
- not having an accurate estimation of the probabilities,
- relying on expectations,
- difficulties in measuring the utility function,
- and forecast errors.

The domain of decision analysis models falls between two extreme cases. This depends upon the degree of knowledge we have about the outcome of our actions, as shown below:

| Complete Knowledge | Deterministic Model |
|--------------------|---------------------|
| Risky Situation    | Probabilistic Model |
| Ignorance          | Pure Uncertainty Model |

One “pole” on this scale is deterministic. The opposite “pole” is pure uncertainty. Between these two extremes are problems under risk. The main idea here is that for any given problem, the degree of certainty varies among managers depending upon how much knowledge each one has about the same problem. This reflects the recommendation of a different solution by each person. Probability is an instrument used to measure the likelihood of occurrence for an event. When you use probability to express your uncertainty, the deterministic side has a probability of one (or zero), while the other end has a flat (all equally probable) probability. For example, if you are certain of the occurrence (or non-occurrence) of an event, you use the probability of one (or zero). If you are uncertain, and would use the expression “I really don’t know,” the event may or may not occur with a probability of 50 percent.

This is the Bayesian notion that probability assessment is always subjective. That is, the probability always depends upon how much the decision maker knows. If someone knows all there is to know, then the probability will diverge either to one or zero. The decision situations with flat uncertainty have the largest risk.

For simplicity, consider a case where there are only two outcomes, with one having a probability of p. Thus, the variation in the states of nature is \( p \times (1-p) \). The largest variation occurs if we set \( p = 50\% \), given each outcome an equal chance. In such a case, the quality of information is at its lowest level. Due to statistics science the quality of information and variation are inversely related. That is, larger variation in data implies lower quality data (i.e., information). In this tutorial several techniques for decision making under risky, deterministic and uncertain situation are presented. These techniques will enable managers to challenge with nondeterministic outcomes of nature [3], [5].

2.1. Moments methods

All choices are made in relation to several criteria, even in the most common personal decisions, which leads to the consideration of the following type-problem: either \( V = \{ v_1, v_2, \ldots v_m \} \) a lot of objects (variants of decision) and \( C = \{ c_1, c_2, \ldots c_n \} \), a set of criteria for assessing objects in \( V \). The criteria in Care of equal importance. In order to solve the problem of the fleet endowment, the algorithm developed by S.B. Deutch and J.J. Martin will be applied, which is suitable for solving both
group decision-making and multidimensional problems. This method is also known as the "moment method" [6], [7].

The typical matrix of the multidimensional decision-making process, with input information is:

**Table 2. The typical matrix of the multidimensional decision-making process**

|     | $C_1$ | $C_2$ | ... | $C_n$ |
|-----|-------|-------|-----|-------|
| $V_1$ | $a_{11}$ | $a_{12}$ | ... | $a_{1n}$ |
| $V_2$ | $a_{21}$ | $a_{22}$ | ... | $a_{2n}$ |
| ...  | ...   | ...   | ... | ...   |

We have the following terms for the formula of maximus:
- $a_{ij}$ is the element of consequences matrix corresponding to $i$ mission and to $j$ criterion;
- $a_{j\text{max}}$ is the highest value from the consequences matrix corresponding to $j$ criterion;
- $a_{j\text{min}}$ is the lowest value from the consequences matrix corresponding to $j$ criterion, it is called normalized matrix attached to the consequences matrix.

The formula of maximum is applied to the matrix: $Max = \frac{a_{ij} - a_{j\text{min}}}{a_{j\text{max}} - a_{j\text{min}}}$ and the matrix will be obtained:

**Table 3. The matrix with the formula of maximum**

|     | $C_1$ | $C_2$ | ... | $C_n$ |
|-----|-------|-------|-----|-------|
| $V_1$ | $u_{11}$ | $u_{12}$ | ... | $u_{1n}$ |
| $V_2$ | $u_{21}$ | $u_{22}$ | ... | $u_{2n}$ |
| ...  | ...   | ...   | ... | ...   |
| $V_n$ | $u_{n1}$ | $u_{n2}$ | ... | $u_{nn}$ |

where $u_{ij} = (i = 1, 2, ...m; j = 1, 2, ..., n)$ are utilities.

The authors introduce for each line of the matrix a so-called "Moment-Line" $M_i^l$, calculated with the expression: $M_i^l = \frac{\sum_{j=1}^{n} j \times u_{ij}}{\sum_{j=1}^{n} u_{ij}}$.

Then for each column of the matrix, a so-called "Moment-Column" $M_j^c$, with the expression

$M_j^c = \frac{\sum_{i=1}^{m} i \times u_{ij}}{\sum_{i=1}^{m} u_{ij}}$.

The steps of the algorithm are the following:
1. It is made an arbitrary arrangement of the lines corresponding to the m variants;
2. The utilities are calculated according to the maximum formula;
3. The line-moments are calculated and the lines (variants) are rearranged in the ascending order of the line-moments;
4. The column-moments are calculated and the columns (criteria) are ordered in ascending order of the column-moments;
5. Resume the algorithm from Step 3 and continue until no more rearrangements of the columns are needed [7].

### 2.2. Decision making under certainty

We assume that it is wanted to equip the Romanian Air Force fleet with fighter aircraft. We have four models available and, in the end, we will choose the best option using the method of moments (Deutch Martin). A comparison will be made between the four models according to the following criteria:

- **C1-** Maximum speed
- **C2-** Maximum operating altitude
- **C3-** Maximum load
- **C4-** Military equipment

**Types of military aircraft:**
- **Mirage 2000** manufactured in France by Dassault, with a maximum speed of 2,2 Mach, service ceiling 16460 m, maximum load 6300 Kg and the armament (2 guns × 30 mm, 4 missiles × AAM);
- **F - 16 Block 15 Fighting Falcon** manufactured in the USA at Lockheed, with a maximum speed of 2 Mach, service ceiling 18000 m, maximum load 5000 Kg and military equipment consisting of 1 × 20 mm (an M61 cannon), 6/8 × AAM (6 guided missiles and 8 bombs);
- **F - 18C Hornet** produced in the USA at McDonnell Douglas, with a maximum speed of 1,8 Mach, a service ceiling of 15240 m, a maximum load of 8165 Kg and military equipment (1 × 20 mm, 4 × AAM);
- **MiG 21 Lancer C** produced in Russia by Mikoyan, with a maximum speed of 2,4 Mach, a service ceiling of 18200 m, a maximum load of 2000 Kg and military equipment consisting of 1 × 23 mm, 2/2 × AAM (2 K-13A missiles and 2 bombs) [8].

These values are in relation to the decisional criteria and written in a matrix that is called the matrix of consequences, as presented in the table below:

|     | C1  | C2   | C3  | C4  |
|-----|-----|------|-----|-----|
| **V1** | 2.2 | 16460| 6300| 6   |
| **V2** | 2   | 18000| 5000| 15  |
| **V3** | 1.8 | 15240| 8165| 5   |
| **V4** | 2.4 | 18200| 2000| 5   |
The moments method will be applied and we consider that the four criteria are equally important.

Step 1: Normalize the consequences matrix. We will use normalization by linear transformations. Since all the criteria are maximum, the formula according to which the normalized consequences will be calculated is

\[ r_{ij} = \frac{a_{ij} - a_{ij}^{\min}}{a_{ij}^{\max} - a_{ij}^{\min}}, \]

where \( a_{ij}^{\min} = \min \limits_{i \leq 4} a_{ij} \) and \( a_{ij}^{\max} = \max \limits_{i \leq 4} a_{ij} \).

In Table 5 we have normalized the consequences matrix:

**Table 5. Normalization of consequences matrix**

|     | C1  | C2  | C3  | C4  |
|-----|-----|-----|-----|-----|
| V1  | 0.66| 0.41| 0.69| 0.1 |
| V2  | 0.33| 0.93| 0.48| 1   |
| V3  | 0   | 0   | 1   | 0   |
| V4  | 1   | 1   | 0   | 0   |

Step 2: Calculate the moments corresponding to each line from the normalized matrix using the following formula:

\[ M_i^l = \frac{\sum \limits_{j=1}^{4} j \cdot r_{ij}}{\sum \limits_{j=1}^{4} r_{ij}}, \quad (\forall)i = 1, 4 \]

\[ M_1^l = \frac{1 \cdot 0.66 + 2 \cdot 0.41 + 3 \cdot 0.69 + 4 \cdot 0.1}{0.66 + 0.41 + 0.69 + 0.1} = \frac{4.01}{1.86} = 2.15 \quad (1) \]

\[ M_2^l = \frac{1 \cdot 0.33 + 2 \cdot 0.93 + 3 \cdot 0.48 + 4 \cdot 1}{0.33 + 0.93 + 0.48 + 1} = \frac{7.63}{2.75} = 2.77 \quad (2) \]

\[ M_3^l = \frac{1 \cdot 0 + 2 \cdot 0 + 3 \cdot 1 + 4 \cdot 0}{1} = 3 \quad (3) \]

\[ M_4^l = \frac{1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0 + 4 \cdot 0}{2} = 1.5 \quad (4) \]

Step 3: We have to rearrange the lines of the normalized matrix in ascending order of the values for the obtained moments.
**Table 6.** Normalized matrix of moments lines

|     | C1 | C2 | C3 | C4 |
|-----|----|----|----|----|
| V4  | 1  | 1  | 0  | 0  |
| V1  | 0.66 | 0.41 | 0.69 | 0.1 |
| V2  | 0.33 | 0.93 | 0.48 | 1  |
| V3  | 0  | 0  | 1  | 0  |

Step 4: Calculate the moments for each column of the new matrix using the following formula:

\[ M_j^c = \frac{\sum_{i=1}^{4} i \cdot r_{ij}}{\sum_{i=1}^{4} r_{ij}}, (\forall) j = 1,4. \]

\[ M_1^c = \frac{1 \cdot 1 + 2 \cdot 0.66 + 3 \cdot 0.33 + 4 \cdot 0}{1 + 0.66 + 0.33 + 0} = \frac{3.31}{1.99} = 1.66 \]  
(5)

\[ M_2^c = \frac{1 \cdot 1 + 2 \cdot 0.41 + 3 \cdot 0.93 + 4 \cdot 0}{1 + 0.41 + 0.93 + 0} = \frac{4.61}{2.34} = 1.97 \]  
(6)

\[ M_3^c = \frac{1 \cdot 0 + 2 \cdot 0.69 + 3 \cdot 0.48 + 4 \cdot 0}{0.69 + 0.48 + 1} = \frac{6.82}{2.17} = 3.14 \]  
(7)

\[ M_4^c = \frac{1 \cdot 0 + 2 \cdot 0.1 + 3 \cdot 1 + 4 \cdot 0}{0.1 + 1} = \frac{3.2}{1.1} = 2.9 \]  
(8)

Step 5: After we rearrange the columns of the new matrix in ascending order of the values for the obtained moments, we have Table 7 with following results:

**Table 7.** Normalized matrix of moments columns

|     | C1 | C2 | C4 | C3 |
|-----|----|----|----|----|
| V4  | 1  | 1  | 0  | 0  |
| V1  | 0.66 | 0.41 | 0.1 | 0.69 |
| V2  | 0.33 | 0.93 | 1  | 0.48 |
| V3  | 0  | 0  | 0  | 1  |
Step 6: The algorithm from Step 2 is resumed in a new iteration (repeat the operations until applied formulas are not suffering modifications, i.e. it remains the same values from the last obtained matrix).

Second iteration
Step 2: Calculate the moment line.

\[ M_1 = \frac{1*1 + 2*1 + 3*0 + 4*0}{2} = 1,5 \]  
\[ M_2 = \frac{1*0.66 + 2*0.41 + 3*0.1 + 4*0.69}{0.66 + 0.41 + 0.1 + 0.69} = \frac{4,54}{1.86} = 2,44 \]  
\[ M_3 = \frac{1*0.33 + 2*0.93 + 3*1 + 4*0.48}{0.33 + 0.93 + 1 + 0.48} = \frac{5,19}{2.26} = 2,29 \]  
\[ M_4 = \frac{1*0 + 2*0 + 3*0 + 4*1}{1} = \frac{4}{1} = 4 \]

Step 3: The normalized matrix lines will be rearranged in ascending order, as follows in Table 8:

| V4  | V2  | V1  | V3  |
|-----|-----|-----|-----|
| 1   | 0.33| 0.66| 0   |
| 1   | 0.93| 0.41| 0   |
| 0   | 1   | 0.1 | 0   |
| 0   | 0   | 0.69| 1   |

Step 4: Calculate the moments for each column of the new matrix.

\[ M_1 = \frac{1*1 + 2*0.33 + 3*0.66 + 4*0}{1 + 0.33 + 0.66} = \frac{3,64}{1.99} = 1,82 \]  
\[ M_2 = \frac{1*1 + 2*0.93 + 3*0.41 + 4*0}{1 + 0.93 + 0.41} = \frac{4,09}{2.34} = 1,74 \]  
\[ M_3 = \frac{1*0 + 2*1 + 3*0.1 + 4*0}{1 + 0.1} = \frac{2.3}{1.1} = 2.09 \]  
\[ M_4 = \frac{1*0 + 2*0.48 + 3*0.69 + 4*1}{0.48 + 0.69 + 1} = \frac{7.03}{2.17} = 3.23 \]
Step 5: The normalized matrix columns will be rearranged in ascending order, as follows:

**Table 9:** The normalized matrix columns

|   | C1 | C4 | C2 | C3 |
|---|----|----|----|----|
| V4| 1  | 0  | 1  | 0  |
| V2| 0.33| 1  | 0.93| 0.48|
| V1| 0.66| 0.1| 0.41| 0.69|
| V3| 0  | 0  | 0  | 1  |

*Third iteration*

Step 2: Calculate the moment line.

\[
M_1' = \frac{1 \times 1 + 2 \times 0 + 3 \times 1 + 4 \times 0}{2} = \frac{4}{2} = 2
\]  
(17)

\[
M_2' = \frac{1 \times 0.33 + 2 \times 1 + 3 \times 0.93 + 4 \times 0.48}{0.33 + 1 + 0.93 + 0.48} = \frac{7.04}{2.74} = 2.56
\]  
(18)

\[
M_3' = \frac{1 \times 0.66 + 2 \times 0.1 + 3 \times 0.41 + 4 \times 0.69}{0.66 + 0.1 + 0.41 + 0.69} = \frac{4.85}{1.86} = 2.60
\]  
(19)

\[
M_4' = \frac{1 \times 0 + 2 \times 0 + 3 \times 0 + 4 \times 1}{4} = \frac{4}{4} = 4
\]  
(20)

Step 3: The normalized matrix lines will be rearranged in ascending order, as is presented in Table 10:

**Table 10:** The normalized matrix lines

|   | C1 | C4 | C2 | C3 |
|---|----|----|----|----|
| V4| 1  | 0  | 1  | 0  |
| V2| 0.33| 1  | 0.93| 0.48|
| V1| 0.66| 0.1| 0.41| 0.69|
| V3| 0  | 0  | 0  | 1  |
It is noticed that the last table is identical to the previous one, so it is no longer possible to reorder the rows and / or the columns of the matrix.

This last ordering of the lines represents the best hierarchy of decisional variants. Therefore, the version V4 is optimal.

3. Analysis of results
Certainty is characterised by the types of decision situations in which influencing factors are known, they have a single condition, controllable, which determines the consequences of the decision to be known for sure. Our objective is to identify and characterise a common area of two processes: decision and selection.

For analysis, four types of fighter aircraft were compared for the development of the air fleet, based on the following criteria: maximum speed, maximum operating altitude, maximum load and military equipment. The assumption was made about these criteria and the optimal version is MiG 21 Lancer, which leads to a best choice from the Romanian fleet of fighter aircraft.

The decision maker, (in that case, the four types of fighter aircraft) has to hold up knowledge, data and information relevant to the analysed situation.

In this study case, the calculations show the following aspects (i.e. the selection):
1. The maximum speed (C1) subserves to a constantly development because is one of the most important criterion in different military missions. During the modernization we have as advantage the optimal flight time.
2. The armament (C4) is a maximum criterion and its advantage is that help to support high-end combat operations and sustain homeland defense.
3. Also, the other criteria provides a unique asymmetric advantage for the fleet through its ability to secure and maintain theater-wide air superiority. These attributes are vital to decisive strategies against highly capable adversaries.

4. Conclusions
To sum up, the use of some algorithms to optimize the decision-making process begins with choosing the variants for the statistical study of the characteristics of fighter aircraft. The moments method is a useful tool in the field of modernisation situations for finding solutions in all models available of military aircraft. Knowing the decision theory broadens the horizon of the decision maker allows him/her to better target the specific situation and to choose a scientific solution. This method can be applied to compare products, services, activities that have similar utility, establish their criteria and their weights, an essential part in the study of alternatives or variants. Using the moment method optimally generates clear results on air fleet endowment.

References
[1] Toft Y, Dell G, Klockner K K and Hutton A 2012 Models of Causation: Safety. In HaSPA (Health and Safety Professionals Alliance), The Core Body of Knowledge for Generalist OHS Professionals. Tullamarine, VIC. Safety Institute of Australia
[2] Wang J X 2015 Cellular Manufacturing: Mitigating Risk and Uncertainty, CRC Press
[3] Taghavifard M T, Khalili Damghani K and Tavakkoli Moghaddam R 2009 Decision Making Under Uncertain and Risky Situations, Society of Actuaries
[4] Ben-Haim Y 2001 Information-gap Decision Theory: Decisions under Severe Uncertainty, San Diego: Academic Press
[5] Hossein Arsham Tools for Decision Analysis: Analysis of Risky Decisions, http://home.ubalt.edu/ntsbarsh/opre640a/partIX.htm
[6] Preda I 1992 Teoria deciziilor statistice, București, Editura Academiei Române
[7] Barsan-Pipu N and Popescu I 2003 Managementul Riscului, Editura Universității „Transilvania” din Brașov
[8] ***Combat Aircraft “Flight International”, 13-19 July, 1994