Research on Deterministic Measurement Matrix of Power Line Carrier Compressed Sensing

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Abstract. To verify the advantages of deterministic matrix applied to power line carrier communication (PLCC) based on compressed sensing (CS). This article analyzed the research status of commonly used deterministic measurement matrices, and made simulation comparison. It is found that different types of deterministic measurement matrices generated based on chaotic mapping had higher reconstruction accuracy and higher reconstruction efficiency than Gaussian random matrix. Then, according to simulation results and the characteristics of PLCC signal, the Chebyshev sparse circulant (CSC) measurement matrix was designed by combining eighth-order Chebyshev chaotic and the idea of sparse and circulant. Actual circuit measurement shows that when compression rate was 40% and 60%, the reconstruction loss of CSC is 0.72dB and 0.49dB higher than that of Chebyshev chaotic measurement matrix and Chebyshev circulate measurement matrix, respectively. Obviously, the CSC measurement matrix designed in this paper can effectively improve the reconstruction accuracy.

Keywords: Power line carrier communication; Compressed sensing; Measurement matrix; Chaotic mapping; Chebyshev chaos.

1. Introduction
With the continuous improvement of power line carrier hardware facilities and the rapid development of related communication technology, the transmission rate of PLCC increases rapidly, and the traditional Nyquist sampling theorem can no longer meet the engineering requirements. CS [1] can realize the high probability reconstruction of sparse signals while sampling and compression are parallel, so it has become an important way for the real-time and efficient transmission of PLCC signal. The adaptability of measurement matrix that satisfies restricted isometry property (RIP) and the quality of CS reconstruction are the key factors that determine the efficiency of PLCC. The measurement matrix designed in combination with the characteristics of the PLCC signal can improve the compression rate and reduce the amount of data to be processed while ensuring that the information is complete after the sparse acquisition. Moreover, it can reconstruct the original signal quicker and more accurately, and further improve the information transmission completeness.

The measurement matrix at present mainly includes random matrices and deterministic matrices. Random matrices, such as Gaussian random matrix, generally have problems such as large resource occupation, difficulty in modularization, and poor adaptability to engineering applications. Relatively speaking the deterministic matrices have strong reproducibility and easy integration, so it has gradually become a research hotspot. However, there are few reports on the related research of measurement matrix designed for PLCC signal characteristics.
2. Research Status of Deterministic Matrices and Characteristic Analysis of Power Carrier Signal

2.1. Research Status of Deterministic Matrices
Research on deterministic matrices is currently focused on achieving reproducibility, reducing matrix complexity, reducing element types, increasing sparsity, etc.

In [2,3], chaotic sequences were generated through the Logistic chaotic mapping, and then they were sampled at equal intervals to generate a reproducible matrix. The matrix generated by the chaotic mapping only needs to determine the initial value, and the elements of the whole matrix are completely determined, so it is easy to be reproduced. However, for equal interval sampling of chaotic sequence, the number of elements to be generated is far greater than the actual demand, resulting in a waste of storage resources. In terms of reducing complexity, [4,5] generated sequences based on different methods, such as sub-circle classes, and then circulated to generate measurement matrix. The matrix generated by these methods are of low complexity, but the above methods of generating the first column of the matrix are not simple enough. [6,7] combined reproducibility with reducing generation complexity. Firstly, chaotic sequences were generated by using commonly used chaotic maps, then the first column of the measurement matrix was generated by sampling sequence at equal intervals. Finally, the column was circulated to generate the whole measurement matrix. In [8,9], the generated matrix elements were classified in different ways, and finally a bipolar matrix containing only two types of elements [−1,1] or a sparse matrix containing {0,1} was generated. This kind of measurement matrix greatly reduces the types of elements, increases the sparsity of the matrix. But it is slightly complicated to judge and classify the elements of the whole matrix.

The above are related to the chaotic matrix with reproducibility, the reduced complexity circulant matrix, the chaotic circulant matrix that combines the former two, and the sparse class matrix which reduces the types of elements and increases the sparse level. However, the design of measurement matrix which includes the above-mentioned aspects is rarely involved. Therefore, this article combined the advantages of the above-mentioned types of measurement matrices to design a comprehensive measurement matrix, and verified its good performance via experiments.

2.2. Characteristic Analysis of Power Carrier Signal
By collecting the actual indoor low-voltage PLCC signal, it is found that it conforms to the voltage flicker characteristics, which is expressed as follows

\[ y(t) = (1 + A \sin \beta \omega_0 t) \sin (\omega_0 t) \]  \hspace{1cm} (1)

Where flicker amplitude \( A \in [0.05,0.1] \), fundamental frequency multiple of flicker fluctuation \( \beta \in [0.1,0.5] \). The dyadic wavelet transform is used to decompose and calculate the power transmission signal, and the modulus maximum value with a certain degree of sparsity is obtained, which shows that the PLCC signal satisfies the CS condition in the wavelet domain.

3. Performance Analysis of Common Measurement Matrix and Design of CSC Measurement Matrix
When the measurement matrix is determined in CS, the changes of measurements and the sparsity seriously affect the reconstruction accuracy and reconstruction time. Therefore, in this section, Gaussian random matrix and chaotic matrix, circulant matrix, sparse matrix and binary matrix generated based on Logistic chaos are applied to orthogonal matching pursuit (OMP) algorithm [10], and the reconstruction success rate and reconstruction time of different types of matrices in the same reconstruction algorithm with sparsity and measurement values as variables are compared to verify the performance of the above deterministic matrix.
3.1. Comparison of Measurement Matrices Performance under Different Sparsity

The length $N$ of the sparse signal $x$ was set to 512, the measurements $M$ was set to 256, and the sparsity $K \in [10,200]$ takes values at intervals of 10. When the residual error between the reconstructed signal and the original signal $\varepsilon \leq 10^{-6}$, it is considered that the signal reconstruction is successful. The reconstruction time of each time starts from calculating the sparse coefficient to the end of the reconstructed signal. The signal was reconstructed 100 times and averaged under each sparsity. The simulation results are shown in Figure 1.

![Figure 1](image1.png)

**Figure 1.** Reconstruction success rate and time when sparsity is variable.

According to Figure 1, when the sparsity reaches 40, the reconstruction success rate of all matrices begins to decline, and the Gaussian random measurement matrix has the fastest decline rate and the worst performance. The performance of the determined chaotic, chaotic circulant, sparse and bipolar measurement matrix is better than the Gaussian matrix, and the circulant measurement matrix has the best performance.

3.2. Comparison of Measurement Matrices Performance under Different Measurements

The length $N$ of the sparse signal $x$ was set to 512, the sparsity $K$ was set to 50, and the measurements $M \in [80,400]$ takes values at intervals of 20. The comparison indicators are the same as those in section 3.1, and the comparison results are shown in Figure 2.

![Figure 2](image2.png)

**Figure 2.** Reconstruction success rate and time when sparsity is variable.

As shown in Figure 2, with the increase of measured value, the success rate of reconstruction of various deterministic matrices grows faster than that of Gaussian random matrix. When the measurements reach 300, the reconstruction success rate of each matrix is 100%. In terms of reconstruction time, the above deterministic matrices are obviously shorter than the Gaussian random matrix, and the sparse matrix has the smallest reconstruction time.

Based on Figure 1 and Figure 2, the performance of the above deterministic matrices is significantly improved compared with the Gaussian random matrix, and they have higher reconstruction accuracy.
and efficiency under the same conditions. However, the matrix mentioned above only involves the use of chaos mapping to improve the reproducibility of matrix or reduce the complexity of matrix generation and reduce element types, does not combine chaos, circulation and sparsity.

3.3. CSC Measurement Matrix Design and Experimental Verification

3.3.1. Design of CSC measurement matrix. In terms of the above, the performance of the measurement matrix can be improved by adding circulant to the deterministic matrix or making it sparse. Therefore, the CSC measurement matrix is designed by sampling the eighth-order Chebyshev chaos at intervals of 5 and combining the circulant and sparse ideas. The steps are as follows:

(1) Generate a Chebyshev chaotic sequence \( H \) with length \( N \times d \);

(2) Randomly select an element in the first \( d \) elements of the sequence \( H \) as the first element of the sequence \( Q \), then sample the sequence \( H \) with \( d \) as step size to form \( Q = \{q_1, q_2, \ldots, q_N\} \);

(3) Randomly select the \( w\% \) elements of \( Q \) and set them for zero, then we can get the sparse \( Q' \);

(4) An \( M \times N \) matrix is obtained by circulating \( Q' \) according to the step size 1 and normalize each column of the matrix to get CSC.

CSC measurement matrix combines the advantages of chaos, circulant and sparsity, so it takes up less storage space and is easy to be implemented in physical circuits.

3.3.2. RIP analysis

In [3], it is proved that the measurement matrix constructed by sampling the eight-order Chebyshev chaotic sequence with five as the step size satisfies the \( k \)-order RIP. According to the Johnson-Lindenstrauss theorem, when \( M > (c_1 k \cdot \ln(N/k)) \), the probability that the measurement matrix satisfies RIP is as follows:

\[
P \geq 1 - 2 \left( \frac{12}{\delta^2} \right) \exp \left( -c_2 \left( \frac{\delta}{2} \right)^2 M \right)
\]

Where \( \delta \in (0,1), c_1, c_2 \) are constants. It can be seen that the smaller \( c_1 \) is, the closer \( P \) is to 1. Therefore, the CSC measurement matrix satisfies RIP with a high probability.

3.3.3. Verify the performance of CSC. The CSC measurement matrix was used to sample and compress the modulus maxima of the data collected by the indoor low-voltage power transmission signal acquisition system according to Figure 3 after denoising, and OMP algorithm was selected for reconstruction.

![Figure 3. Indoor low-voltage power data acquisition system.](image)

The reconstruction losses of CSC, Chebyshev circulant and Chebyshev sparse matrix under different compression rates are compared, and the results are shown in Figure 4. The calculation of reconstruction loss is as shown in equation (2). To avoid randomness, we calculated the reconstruction loss 10 times at each compression rate and then averaged it.
\[
\begin{align*}
\text{err} &= s_r - s \\
\eta &= 20\log_{10}\left(\frac{\|s\|_2}{\|\text{err}\|_2}\right) (dB)
\end{align*}
\]

Where \(s_r\) denotes the reconstructed signal, \(s\) is the noised signal, \(\text{dev}\) represents the deviation of the reconstructed signal and the collected noised signal, and \(\eta\) denotes the reconstruction loss. The greater the reconstruction loss \(\eta\), the greater the difference between the reconstructed signal and the collected noisy original signal, that is, the smoother the reconstructed signal waveform after denoising.

\[\text{Figure 4. Comparison of reconstruction loss at various compression rates after denoising.}\]

It can be seen from Figure 4, CSC measurement matrix has higher reconstruction loss at each compression rate, while Chebyshev chaotic measurement matrix has poor robustness and large reconstruction loss fluctuation. When the compression rate is 40% and 60%, the reconstruction loss of CSC is 0.72dB and 0.49dB higher than that of Chebyshev chaotic measurement matrix and Chebyshev circulate measurement matrix, respectively. Therefore, the CSC measurement matrix designed in this article has higher reconstruction accuracy and more stable reconstruction effect.

**4. Conclusion**

This paper comprehensively analyzed the research status of commonly used deterministic measurement matrices at home and abroad based on the key point of measurement matrix design of CS in PLCC application. Then the reconstruction success rate and reconstruction time of chaotic matrix, circulant matrix, sparse matrix and bipolar matrix generated based on Logistic chaotic mapping were compared with Gaussian random matrix under different sparsity and different measurements. The results showed that under the above conditions, compared with the Gaussian matrix, the above-mentioned deterministic matrices have higher reconstruction power and less reconstruction time, which verified the advantages of the deterministic matrix in the application of PLCC CS. But it is also found that the above matrices do not satisfy the characteristics of robustness, circulant and high sparsity simultaneously. Therefore, the Chebyshev sparse CSC measurement matrix was designed by combining eighth-order Chebyshev chaotic and the idea of sparse and circulant. Then it was used to process the measured power line carrier signal and compared the reconstruction loss with the chaotic matrix, circulant matrix and sparse matrix which were also generated based on the eighth-order Chebyshev chaos. It is shown that when the compression rate was 40% and 60%, the reconstruction loss of CSC is 0.72dB and 0.49dB higher than that of Chebyshev chaotic measurement matrix and Chebyshev circulate measurement matrix, respectively. That is to say, the performance of the CSC measurement matrix designed in this paper is better.

In summary, the deterministic measurement matrix is easy to be realized in the physical circuit, which makes CS easier to be used in PLCC, thus improving the transmission efficiency and helping the smooth implementation of the smart grid. The next step of the research on the measurement matrix will mainly focus on making the matrix more reproducible, with fewer element types, and simpler to generate at the same time.
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