CP Violation Induced by Heavy Majorana Neutrinos in the Decays of Higgs Scalars into Top-Quark, W- and Z-Boson Pairs

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ABSTRACT

We analyze the possibility of CP violation induced by heavy Majorana neutrinos in the decays of the Higgs particle into top-quark, W- and Z-boson pairs. In the framework of various “see-saw” models with interfamily mixings, we find that Majorana neutrinos may give rise to sizable CP-odd observables at the one-loop electroweak order. Numerical estimates of these CP-violating effects that may be detected in high-energy colliders are presented.

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The possible existence of heavy Majorana neutrinos plays an important rôle in addressing a number of outstanding questions in cosmology and astrophysics like the smallness in mass of the known neutrinos, the solar neutrino deficit, the baryon asymmetry in the universe, etc. Apart from the possibility of producing such heavy neutral leptons at high energy $ee$, $ep$, or $pp$ colliders, their presence may be manifested by detecting lepton-flavour violating decays of the $Z$ and the Higgs ($H^0$) particle or through their influence on the electroweak oblique parameters, $S$, $T$, $U$, or $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$.

Another interesting aspect, which will be investigated in this letter, is that quantum effects mediated by heavy Majorana neutrinos can induce sizable CP-odd signals in the production of top-quark ($t$), $W$- and $Z$-boson pairs originating from the decay of the Standard Model (SM) Higgs boson. Specifically, we shall investigate whether $CP$ asymmetries defined as

\[ A_{CP}^{(t)} = \frac{\Gamma(H^0 \rightarrow t_L \bar{t}_L) - \Gamma(H^0 \rightarrow t_R \bar{t}_R)}{\Gamma(H^0 \rightarrow t\bar{t})} \quad (1) \]

and

\[ A_{CP}^{(W)} = \frac{\Gamma(H^0 \rightarrow W^{+}_{(+1)} W^{-}_{(+1)}) - \Gamma(H^0 \rightarrow W^{+}_{(-1)} W^{-}_{(-1)})}{\Gamma(H^0 \rightarrow W^{+}_{(+1)} W^{-}_{(+1)}) + \Gamma(H^0 \rightarrow W^{+}_{(-1)} W^{-}_{(-1)})} \quad (2) \]

may be observable at LHC or SSC energies. The subscripts $L, R$, and $(\pm 1)$ in Eqs. (1) and (2) denote the two helicity states of the top quark and the helicities of the transverse degrees of the $W$ boson, respectively. The states $t_L$ and $W^{+}_{(+1)}$ are connected with the states $\bar{t}_R$ and $W^{-}_{(-1)}$ by $CP$ conjugation. Therefore, $A_{CP}^{(t)}$ and $A_{CP}^{(W)}$ represent genuine $CP$-violating parameters that can be determined experimentally. By analogy to Eq. (2), a $CP$ asymmetry, $A_{CP}^{(Z)}$, for the decay channel $H^0 \rightarrow ZZ$ can be constructed. It should be stressed that the $CP$ asymmetries $A_{CP}^{(t)}$, $A_{CP}^{(W)}$, and $A_{CP}^{(Z)}$ resulting from the Feynman graphs shown in Fig. 1 cannot be induced if the heavy neutrinos are of standard Dirac type. To be more specific, at one loop in the SM, the amplitude of $H^0 \rightarrow W^+W^-$ contains a term proportional to $\epsilon_{\mu\nu\rho\sigma}^a(k_+) \epsilon^a(k_-) k_+^\mu k_-^\rho$, where $k_\pm$ and $\epsilon(k_\pm)$ denote the four-momenta and polarization four-vectors of the $W^\pm$ bosons, respectively. However, the coefficient of this term vanishes for $k_+^2 = k_-^2$ and, in particular, for on-shell $W$ bosons.
The minimal class of models that predict heavy Majorana neutrinos can be obtained by simply adding \( n_G \) right-handed neutrinos, \( \nu^0_{Ri} \), to the field content of the SM \[18\], where \( n_G \) denotes the number of generations. After spontaneous breakdown of the \( SU(2) \otimes U(1) \) gauge symmetry, the Yukawa sector containing the neutrino masses reads

\[
- \mathcal{L}_Y^\nu = \frac{1}{2} (\bar{\nu}^0_L, \bar{\nu}^{0C}_R) M^\nu \begin{pmatrix} \nu^0_L \\ \nu^0_R \end{pmatrix} + H.c. ,
\]

where the neutrino mass matrix,

\[
M^\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix},
\]

(4)
takes the known “see-saw” form \[19\]. In Eq. (4), \( m_D \) and \( m_M \) are \( n_G \times n_G \)-dimensional matrices that cannot, in general, be diagonalized simultaneously. Note also that the \( B-L \) breaking mass terms, \( m_{Mi,j} \), can be assumed to be bare masses in the Lagrangian without violating the gauge symmetry of the SM. The symmetric matrix \( M^\nu \) can be brought into diagonal form by a \( 2n_G \times 2n_G \) unitary matrix \( U_\nu \) (i.e., \( \hat{M}^\nu = U^{\nu T} M^\nu U_\nu \)). This yields \( 2n_G \) physical Majorana neutrinos, \( n_i \), which are related with the weak eigenstates through the unitary transformations

\[
\begin{pmatrix} \nu^0_L \\ \nu^{0C}_R \end{pmatrix}_i = U_\nu^{\nu^*} n_{Lj}, \quad \begin{pmatrix} \nu^0_L \\ \nu^{0C}_R \end{pmatrix}_i = U_\nu^{\nu^*} n_{Rj} .
\]

(5)
The first \( n_G \) Majorana neutrinos, \( \nu_i \), are identified with the ordinary neutrinos, \( \nu_e, \nu_\mu, \) etc. (i.e., \( \nu_i = n_i \) for \( i = 1, \ldots, n_G \)), while the remaining \( n_G \) mass eigenstates represent heavy Majorana neutrinos that are predicted by the model (i.e., \( N_i = n_{i+n_G} \)). It is now straightforward to obtain for our purposes the relevant interactions of the Majorana neutrinos with the \( W^- \), \( Z^- \), and Higgs bosons in this minimal model. Adopting the notation of \[20\], these interactions are given in a renormalizable form by

\[
\mathcal{L}_{int}^W = - \frac{g_W}{2\sqrt{2}} W^-\mu \bar{n}_i B_{\mu j} \gamma_\mu(1 - \gamma_5) n_j + H.c. ,
\]

(6)

\[
\mathcal{L}_{int}^Z = - \frac{g_W}{4 \cos \theta_W} Z^- \bar{n}_i \gamma_\mu \left[i \text{Im} C_{ij} - \gamma_5 \text{Re} C_{ij}\right] n_j ,
\]

(7)

\[
\mathcal{L}_{int}^H = - \frac{g_W}{4M_W} H^0 \bar{n}_i \left[(m_{n_i} + m_{n_j}) \text{Re} C_{ij} + i \gamma_5 (m_{n_j} - m_{n_i}) \text{Im} C_{ij}\right] n_j ,
\]

(8)
where

\[ C_{ij} = \sum_{k=1}^{n_G} U_{k_i}^\nu U_{k_j}^{\nu*}, \]

\[ B_{li,j} = \sum_{k=1}^{n_G} V_{l,k}^I U_{k_j}^{\nu*}. \]

The matrices \( B \) and \( C \) obey a great number of useful identities that will help us to quantify our \( CP \) asymmetries. These identities read

\[ \sum_{l=1}^{2n_G} B_{l_1,i} B_{l_2,i}^* = \delta_{l_1 l_2}, \]

\[ \sum_{k=1}^{2n_G} C_{ik} C_{jk}^* = C_{ij}, \]

\[ \sum_{l=1}^{2n_G} B_{li,j} C_{ij} = B_{lj}, \]

\[ \sum_{k=1}^{2n_G} m_{n_i} C_{ik} C_{jk} = 0, \]

\[ \sum_{k=1}^{n_G} B_{ki,j}^* B_{ki,j} = C_{ij}. \]

In general, the mixings between light-heavy states, i.e., \( B_{iN} \) or \( C_{\nu N} \), can be constrained by a global analysis of low-energy experiments and LEP data

It is worth mentioning that the Lagrangians of Eqs. (6)–(8) violate the \( CP \) symmetry of the model. Analytically, Eq. (6) introduces \( CP \) violation by the known complex Cabbibo-Kobayashi-Maskawa-type matrix, \( B_{l_1,i} n_j \). The neutral-current interactions described by Eqs. (7) and (8) violate also the \( CP \) symmetry of the model, since the neutral particles \( H^0 \) and \( Z \) couple simultaneously to \( CP \)-even (\( \bar{n}_i n_j \) or \( \bar{n}_i \gamma_\mu n_j \)) and \( CP \)-odd (\( \bar{n}_i \gamma_5 n_j \) or \( \bar{n}_i \gamma_\mu \gamma_5 n_j \)) operators with two different Majorana neutrinos (i.e., \( n_i \neq n_j \)).

As a consequence of the latter, non-zero \( CP \)-odd parts are generated radiatively in the \( H^0 t\bar{t}, H^0 W^+ W^- \), and \( H^0 ZZ \) couplings, and thus \( CP \) violation is induced in the corresponding decays of the Higgs particle (see also Fig. 1).

After setting the stage, we are now in a position to calculate the \( CP \) asymmetry related to the \( H^0 \rightarrow t\bar{t} \) decay. Actually, we are looking for \( CP \)-odd correlations of the type \( \langle (\vec{s}_t - \vec{s}_\bar{t}) \cdot \vec{k}_t \rangle \) as given in Eq. (1). This kind of \( CP \)-odd observables, being odd under
$CPT$, should combine with the $CPT$-odd absorptive parts in the totally integrated decay rates. The important ingredient for a non-zero $A_{CP}^{(t)}$ are Eqs. (7) and (8), which violate the $CP$ symmetry of the model, as pointed out earlier. Thus, vacuum-polarization transitions between the $CP$-even Higgs and the $CP$-odd $Z$ boson, which are forbidden in the $SM$, can now give rise to a pseudoscalar part in the $H^0 t\bar{t}$ coupling (see also Fig. 1(a)). In this way, one obtains

$$A_{CP}^{(t)} = \frac{\alpha_W}{4} \text{Im} C_{ij}^2 \sqrt{\lambda_i \lambda_j} \left( \frac{\lambda_H - \lambda_i}{\lambda_H} \right) \frac{\lambda^1/2(\lambda_H, \lambda_i, \lambda_j)}{\lambda^1/2(\lambda_H, \lambda_i, \lambda_j)},$$

where

$$\lambda_i = \frac{m_{\nu_i}^2}{M_W^2}, \quad \lambda_H = \frac{M_H^2}{M_W^2}, \quad \lambda_t = \frac{m_t^2}{M_W^2},$$

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz.$$  \hspace{1cm} (17)

From Eq. (16) we see that two non-degenerate Majorana neutrinos with appreciable masses are at least required in order to get $A_{CP}^{(t)} \neq 0$. In general, the number of $CP$-odd phases that exist theoretically in the $SM$ with right-handed neutrinos is

$$N_{CP} = N_L (N_R - 1),$$

where $N_L$ and $N_R$ are the numbers of left-handed and right-handed neutrinos, respectively. Assuming now three generations, one has $\text{Im} C_{\nu_1, \nu_2}^2 \leq 10^{-2}$. However, the situation changes if one introduces an additional left-handed neutrino field, $\nu_L^0$, in the Lagrangian of the $SM$. If a non-trivial mixing between the two right-handed neutrinos is assumed, $M^\nu$ of Eq. (4) takes the form

$$M^\nu = \begin{pmatrix} 0 & a & b \\ a & A & 0 \\ b & 0 & B \end{pmatrix}.$$  \hspace{1cm} (19)

Obviously, the mass eigenvalues of $M^\nu$ should be in excess of $M_Z/2$ in order for the respective particles to escape detection at $LEP$ experiments. Employing Eq. (14), one can derive the helpful relations

$$\text{Im} C_{\nu_N}^2 = \sin \delta_{CP} |C_{\nu_N N_1}|^2.$$  \hspace{1cm} (20)
\[ \text{Im} C_{\nu N_1}^2 = -\frac{m_N}{m_{N_2}} \sin \delta_{CP} \left| C_{\nu N_1} \right|^2, \quad (21) \]
\[ \text{Im} C_{N_1N_2}^2 = \frac{m_\nu}{m_{N_2}} \sin \delta_{CP} \left| C_{\nu N_1} \right|^2. \quad (22) \]

In these models, the mixing \( \left| C_{\nu N_1} \right|^2 \) can be of order one. In Table 1, we present the numerical results for \( A_{CP}^{(t)} \) within the two models mentioned above. In particular, we see that \( CP \) asymmetries \( A_{CP}^{(t)} \approx 4 \times 10^{-2} \) are conceivable in such four-generation scenarios with Majorana neutrinos \([23,24]\). We emphasize again the fact that high-mass Dirac neutrinos cannot produce a non-zero \( CP \) asymmetry.

Similarly, heavy Majorana neutrinos can generate a \( CP \)-odd part in the \( H^0 W^+ W^- \) coupling through the triangle graph shown in Fig. 1(b). Since our model is free of anomalies \([23]\), one can uniquely search for \( CP \)-violating correlations of the form
\[ \epsilon_{\mu\nu\sigma} \varepsilon^{\mu \nu \sigma}_{(+1)}(\hat{\nu}) \varepsilon^{\nu \sigma}_{(+1)}(\hat{\nu}) = M_H \delta_{+} \left( \varepsilon_{(+1)}(\hat{\nu}) \times \varepsilon_{(+1)}(\hat{\nu}) \right), \quad (23) \]

In Eq. (23), we have assumed that the polarization vectors, \( \varepsilon^{\mu \nu \sigma}_{(+1)}(\hat{\nu}) \) and \( \varepsilon^{\nu \sigma}_{(+1)}(\hat{\nu}) \), describe two transverse \( W \) bosons with helicity +1. The presence of \( CP \)-odd terms in the transition amplitude of the decay \( H^0 \to W^+ W^- \) leads to the \( CP \) asymmetry
\[ A_{CP}^{(W)} = \frac{\alpha_W}{4} \left[ \text{Im}(B_{ik} B^*_{kj} C_{ij}) \sin \lambda_i \lambda_j (F_+ + F_-) + \text{Im}(B_{ik} B^*_{kj} C_{ij})(\lambda_i F_+ + \lambda_j F_-) \right], \quad (24) \]

where \( \alpha_W = (g_W^2/4\pi) \). The functions \( F_\pm \) in Eq. (24) receive absorptive contributions from three different kinematic configurations of the intermediate states that can become on-shell. A straightforward computation of all absorptive parts shown in Fig. 1(b) gives
\[ F_\pm(\lambda_i, \lambda_j, \lambda_{ik}) = \theta(M_H - m_{m_i} - m_{n_j}) \left[ \pm \frac{\lambda^{1/2}(\lambda_H, \lambda_i, \lambda_j)}{2\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W)} + G_\pm(\lambda_i, \lambda_j, \lambda_{ik}) \right. \]
\[ \times \ln \left( \frac{t^+(\lambda_i, \lambda_j, \lambda_{ik}) - \lambda_{ik}}{t^-(\lambda_i, \lambda_j, \lambda_{ik}) - \lambda_{ik}} \right) + \theta(M_W - m_{m_i} - m_{n_j}) \left[ \left( \frac{\lambda_H - 4\lambda_W}{\lambda_W} \right) \right. \]
\[ + \frac{\lambda_H}{4\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W)} + G_\pm(\lambda_i, \lambda_j, \lambda_{ik}) \ln \left( \frac{t^+(\lambda_i, \lambda_j, \lambda_{ik}) - \lambda_{ik}}{t^-(\lambda_i, \lambda_j, \lambda_{ik}) - \lambda_{ik}} \right) \]
\[ + \theta(M_W - m_{m_j} - m_{n_k}) \left[ \left( \frac{\lambda_H - 4\lambda_W}{\lambda_W} \right) \right. \]
\[ + \frac{\lambda_H}{4\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W)} + G_\pm(\lambda_i, \lambda_j, \lambda_{ik}) \ln \left( \frac{t^+(\lambda_i, \lambda_j, \lambda_{ik}) - \lambda_{ik}}{t^-(\lambda_i, \lambda_j, \lambda_{ik}) - \lambda_{ik}} \right), \quad (25) \]
where

\[ \lambda_{ll} = \frac{m_{ll}^2}{M_W^2}, \quad \lambda_W = 1, \quad \lambda_Z = \frac{M_Z^2}{M_W^2}, \quad (26) \]

\[ t^\pm(x, y) = -\frac{1}{2} \left[ \lambda_H - x - y - 2\lambda_W \mp \lambda^{1/2}(\lambda_H, x, y)\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W) \right], \quad (27) \]

\[ \bar{t}^\pm(x, y) = \lambda_H + x - \frac{1}{2} \lambda_H(\lambda_W + x - y) \pm \frac{1}{2} \lambda^{1/2}(\lambda_H, x, y)\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W), \quad (28) \]

\[ G_\pm(x, y, z) = \frac{x - y}{4\lambda_H} \pm \frac{2(\lambda_W + z) - x - y}{4(\lambda_H - 4\lambda_W)}. \quad (29) \]

The dominant contribution to \( A^{(W)}_{CP} \) comes from the \( n_in_j \) on-shell states, while absorptive parts arising from \( n_i\bar{l}_k \) or \( n_j\bar{l}_k \) states are vanishingly small. In Table 2, we exhibit numerical estimates for \( A^{(W)}_{CP} \) within the two representative “see-saw” scenarios discussed above. In order to measure \( A^{(W)}_{CP} \), one has to be able to discriminate the events \( W^{(+)1}W^{(-)1} \) and \( W^{(+)1}W^{(-)1} \) from the total number of \( W \) bosons produced by Higgs-boson decays. An estimate can be obtained from the following ratio:

\[ R^{(W)} = \frac{\Gamma(H^0 \rightarrow W^{(+)1}W^{(-)1}) + \Gamma(H^0 \rightarrow W^{(+)1}W^{(-)1})}{\Gamma(H^0 \rightarrow W^+W^-)} \]

\[ = \frac{8M_W^4}{M_H^4} \left( 1 - 4\frac{M_W^2}{M_H^2} + 12\frac{M_W^4}{M_H^4} \right)^{-1}. \quad (30) \]

Since one expects to be able to analyze \( 10^5-10^6 \) Higgs decays per year at \( SSC \) or \( LHC \) for a wide range of Higgs-boson masses, i.e., \( M_H = 300-800 \) GeV, it should, in principle, be possible to see \( R^{(W)} \) values of order \( 10^{-2}-10^{-3} \) and \( CP \) asymmetries of \( A^{(W)}_{CP} \approx 10\% \) in the \( SM \) with three (four) generations.

Similarly, heavy Majorana neutrinos induce, through the triangle graph of Fig. 1(c), a non-zero \( CP \) asymmetry in the decay \( H^0 \rightarrow ZZ \), which is found to be

\[ A^{(Z)}_{CP} = \frac{\alpha_W}{4} \left[ \text{Im}(C_{ij}C_{jk}C_{ki}) (\lambda_i F_1 + \lambda_j F_2) + \text{Im}(C_{ij}^*C_{jk}C_{ki})\sqrt{\lambda_i\lambda_j}(F_1 + F_2) \right. \]

\[ \left. - \left( \text{Im}(C_{ij}^*C_{jk}C_{ki})\sqrt{\lambda_i\lambda_k} + \text{Im}(C_{ij}C_{jk}^*C_{ki})\sqrt{\lambda_j\lambda_k}) (R + F_1 - F_2) \right) \right], \quad (31) \]

where

\[ R = \theta(M_H - m_{n_i} - m_{n_j}) \frac{1}{2} \ln \left( \frac{t^+(\lambda_i, \lambda_j) - \lambda_k}{t^-(\lambda_i, \lambda_j) - \lambda_k} \right) \]
\[ + \frac{1}{2} \ln \left( \frac{\bar{\lambda}^{-} (\lambda_{i}, \lambda_k) - \lambda_j}{\bar{\lambda}^{-} (\lambda_{i}, \lambda_k) - \lambda_i} \right) \]

The functions \( F_1, F_2, t^\pm, \) and \( \bar{t}^\pm \) are obtained from \( F_+, F_-, \) Eqs. (27) and (28), respectively, by making the obvious replacements, \( \lambda_W \rightarrow \lambda_Z \) and \( \lambda_{l_{k}} \rightarrow \lambda_k \), in the relevant formulae.

From Table 3 we see that \( A_{CP}^{(Z)} \) can be of the order of 10\%, which should be sizable enough to observe such a \( CP \)-violating effect at \( pp \) supercolliders.

In conclusion, we have shown that heavy Majorana neutrinos are of potential interest in accounting for possible \( CP \)-violating phenomena in the decays of Higgs bosons into \( t\bar{t}, W^+W^- \), and \( ZZ \) pairs. The minimal extension of the \( SM \) by right-handed neutrinos can naturally account for sizable \( CP \)-violating effects of the order of 10\% at high-energy colliders and thus provides an attractive alternative to complicated multi-Higgs-boson scenarios tuned to yield large \( CP \)-odd effects [14,15,16,26]. We must remark that the \( CP \) asymmetries \( A_{CP}^{(t)}, A_{CP}^{(W)}, \) and \( A_{CP}^{(Z)} \) may not be directly accessible by experiment. However, the \( CP \)-violating signals originating from such Higgs-boson decays will be transcribed to the decay products of the top-quark, \( W^- \), and \( Z \)-boson pairs. More realistic \( CP \)-odd projectors can be constructed, for instance, by considering angular-momentum distributions or energy asymmetries of the produced charged leptons and jets [14,15,27].

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[24] In fact, the model constructed here differs crucially from that of [23]. One should require a strong mixing between \( \nu^0_{L_4} \) and two right-handed neutrinos, \( \nu^0_{R_1} \) and \( \nu^0_{R_2} \), in order to describe \( CP \) violation. As a consequence of the absence of interfamily mixings, the Hill-Paschos model corresponds effectively to the case \( N_R = 1 \), which implies \( N_{CP} = 0 \) on account of Eq. (18).

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Figure and Table Captions

Fig. 1: Feynman graphs giving rise to a $CP$-odd part in the $H^0 t \bar{t}$, $H^0 W^+ W^-$, and $H^0 Z Z$ couplings.

Tab. 1: Numerical results of $A^{(t)}_{CP}/\text{Im} C^2_{N_1 N_2}$ ($A^{(t)}_{CP}/\text{Im} C^2_{\nu_4 N_1}$) in the context of inter-family “see-saw” models with three (four) generations. We use $m_{N_1}$ ($m_{\nu_4}$) = 100 GeV for the one heavy neutrino and $m_t = 150$ GeV.

Tab. 2: Numerical values of $A^{(W)}_{CP}/\text{Im} C^2_{N_1 N_2}$ and $A^{(W)}_{CP}/\text{Im} C^2_{\nu_4 N_1}$ for “see-saw” models with three and four generations, respectively. We use $m_l = 0$ for the charged leptons of the first three generations and $m_E = 100$ GeV \cite{28} for the charged lepton of the fourth generation.

Tab. 3: Numerical results of $A^{(Z)}_{CP}/\text{Im} C^2_{N_1 N_2}$ and $A^{(Z)}_{CP}/\text{Im} C^2_{\nu_4 N_1}$ for “see-saw” models with three and four generations, respectively.
Table 1

| $m_{N_2}$ [GeV] | $M_H = 400$ GeV | $M_H = 600$ GeV | $M_H = 800$ GeV |
|-----------------|-----------------|-----------------|-----------------|
| 150             | $-3.78 \times 10^{-3}$ | $-1.50 \times 10^{-3}$ | $-8.25 \times 10^{-4}$ |
| 200             | $-1.00 \times 10^{-2}$ | $-4.52 \times 10^{-3}$ | $-2.56 \times 10^{-3}$ |
| 250             | $-1.53 \times 10^{-2}$ | $-9.12 \times 10^{-3}$ | $-5.38 \times 10^{-3}$ |
| 300             | —                | $-1.49 \times 10^{-2}$ | $-9.34 \times 10^{-3}$ |
| 400             | —                | $-2.54 \times 10^{-2}$ | $-2.01 \times 10^{-2}$ |
| 450             | —                | $-2.48 \times 10^{-2}$ | $-2.63 \times 10^{-2}$ |
| 475             | —                | —                | $-2.92 \times 10^{-2}$ |
| 500             | —                | —                | $-3.19 \times 10^{-2}$ |
| 600             | —                | —                | $-3.68 \times 10^{-2}$ |
| 650             | —                | —                | $-3.14 \times 10^{-2}$ |
### Table 2

| $m_{N_2}$ [GeV] | $M_H = 400$ GeV | $M_H = 600$ GeV | $M_H = 800$ GeV |
|-----------------|-----------------|-----------------|-----------------|
|                 | $m_l$           | $m_E$           | $m_l$           | $m_E$           | $m_l$           | $m_E$           |
| 0 GeV           | 0               | 0               | 0               | 0               | 0               | 0               |
| 100 GeV         | -8.61 $10^{-3}$ | -3.98 $10^{-3}$ | -5.33 $10^{-3}$ | -2.48 $10^{-3}$ | -3.52 $10^{-3}$ | -1.65 $10^{-3}$ |
| 150             | -2.11 $10^{-2}$ | -1.13 $10^{-2}$ | -1.49 $10^{-2}$ | -7.76 $10^{-3}$ | -1.02 $10^{-2}$ | -5.23 $10^{-3}$ |
| 200             | -3.13 $10^{-2}$ | -1.94 $10^{-2}$ | -2.87 $10^{-2}$ | -1.64 $10^{-2}$ | -2.04 $10^{-2}$ | -1.13 $10^{-2}$ |
| 250             | 0               | 0               | -4.58 $10^{-2}$ | -2.83 $10^{-2}$ | -3.44 $10^{-2}$ | -2.02 $10^{-2}$ |
| 300             | 0               | 0               | -7.90 $10^{-2}$ | -5.65 $10^{-2}$ | -7.23 $10^{-2}$ | -4.76 $10^{-2}$ |
| 350             | 0               | 0               | -8.02 $10^{-2}$ | -6.14 $10^{-2}$ | -9.46 $10^{-2}$ | -6.55 $10^{-2}$ |
| 400             | 0               | 0               | -6.71 $10^{-2}$ | -5.29 $10^{-2}$ | -0.106          | -7.51 $10^{-2}$ |
| 450             | 0               | 0               | 0               | 0               | -0.117          | -8.48 $10^{-2}$ |
| 500             | 0               | 0               | 0               | 0               | -0.147          | -0.115          |
| 550             | 0               | 0               | 0               | 0               | -0.132          | -0.109          |
| 600             | 0               | 0               | 0               | 0               | 0               | 0               |
| 650             | 0               | 0               | 0               | 0               | 0               | 0               |
Table 3

| $m_{N_3}$ [GeV] | $M_H$ = 400 GeV | $M_H$ = 600 GeV | $M_H$ = 800 GeV |
|-----------------|-----------------|-----------------|-----------------|
|                 | $m_{N_1}(m_{\nu_4})$ = 150 GeV | $m_{N_1}(m_{\nu_4})$ = 250 GeV | $m_{N_1}(m_{\nu_4})$ = 350 GeV |
|                 | $m_{N_2}$ = 250 GeV | $m_{N_2}$ = 350 GeV | $m_{N_2}$ = 450 GeV |
| 3Gens           | 4Gens           | 3Gens           | 4Gens           |
| 300             | -1.60 $10^{-2}$ | -1.41 $10^{-2}$ |                 |
| 400             | -3.70 $10^{-2}$ | -3.23 $10^{-2}$ | -2.93 $10^{-2}$ |
| 450             | -4.40 $10^{-2}$ | -3.83 $10^{-2}$ | -5.32 $10^{-2}$ |
| 500             | -4.95 $10^{-2}$ | -4.29 $10^{-2}$ | -7.30 $10^{-2}$ |
| 600             | -5.74 $10^{-2}$ | -4.94 $10^{-2}$ | -0.103          |
| 650             | -6.03 $10^{-2}$ | -5.17 $10^{-2}$ | -0.115          |
| 1000            | -7.09 $10^{-2}$ | -6.02 $10^{-2}$ | -0.161          |

