Intra-Pulse Modulation Recognition for Fractional Bandlimited Signals Based on a Modified MWC-Based Digital Receiver

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This work was supported in part by the National Natural Science Foundation of China under Grant 61271354, and in part by the Henan Province Science and Technology Key Project of China under Grant 142102210431.

**ABSTRACT** In this paper, we present a modified modulated wideband converter (MWC)-based digital receiver for fractional bandlimited signals, and further propose an intra-pulse modulation recognition method in discrete time fractional Fourier domain (DTFrFD) for the intercepted signals. The proposed digital receiver can move the cross-channel signal to the baseband and since the nonzero part of its available spectrum is narrower in DTFrFD than in discrete time Fourier domain (DTFD), so a better separation can be achieved by the proposed digital receiver than by original MWC-based receiver. Then, with the data acquired from the digital receiver, we propose an intra-pulse modulation recognition method based on the optimal transformation order and the spectral kurtosis (SK) in DTFrFD for the six types of fractional bandlimited signals. In this algorithm, the optimal transformation order is tested to distinguish the encoded signals from the non-encoded signals, and then the SK is tested to determine the intra-pulse modulation type specially. The computational complexity of the proposed method is much lower than search-based methods. Meanwhile, since the SK of Gaussian signals is zero in DTFrFD, the proposed method shows better robustness than the original MWC discrete compressive sampling structure against SNR variation. Simulation results confirm the obtained analytical results.

**INDEX TERMS** Wideband digital receiver, modulated wideband converter, fractional Fourier transform, intra-pulse modulation recognition.

**NOMENCLATURE**

- AWGN: Additive white Gaussian noise
- BFSK: Binary frequency shift keying
- BPSK: Binary phase shift keying
- CS: Compressed sensing
- DSDFrFT: Digital simplified fractional Fourier transform
- DTFrFD: Discrete time fractional Fourier domain
- DTFrFT: Discrete time fractional Fourier transform
- DTFD: Discrete time Fourier domain
- DTSFrFD: Discrete time simplified fractional Fourier domain
- FrFD: Fractional Fourier domain
- FrFT: Fractional Fourier transform
- FT: Fourier transform
- LFM: Linear frequency modulation
- MWC: Modulated wideband converter
- NLFM: Nonlinear frequency modulation
- NS: Normal signal
- QPSK: Quadrature phase shift keying
- SFrFD: Simplified fractional Fourier domain
- SFrFT: Simplified fractional Fourier transform
- SK: Spectral kurtosis
- SNR: Signal to noise ratio

I. INTRODUCTION

Fractional bandlimited signals [1]–[4] such as linear frequency modulation (LFM) are widely used in the radar and ultra-wideband communication because of its ease in implementation, its versatility, and its suitability for...
Fractional Fourier transform (FrFT) can be interpreted as a signal decomposition in terms of a LFM basis as its kernel is constituted by LFM functions, it is the natural domain of non-stationary signals. Most research have focused on the MWC theorem expansions for the wideband digital receiver in DTFD from different perspectives [11], [16], [17], [18]–[21], but few have focused on the receiver in discrete time fractional Fourier domain (DFTFrFD). It is necessary to generalize a MWC-based signal receiver theorem in DFTFrFD.

Thus, in this paper, we present a modified MWC-based digital receiver for fractional bandlimited signals by extending the multichannel compressed sampling structure in [27], and further suggest an intra-pulse modulation recognition method in DFTFrFD for the fractional bandlimited signals which is intercepted by the new receiver. The proposed digital receiver consists of two mixing steps. At the first mixing, we multiply \( \frac{1}{T} \cos^2 \omega t \) with the original signal to establish the relationship between DTFrFT and DFTF. Then the spectrum of the mixed product is aliased by the random modulation at the second step. The production of the mixing signal is filtered by a low-pass filter, and then it is sampled under a low-sampling rate. The cross-channel signals can be mixed to the baseband and get better separation than by original MWC-based receiver.

Moreover, it is an important task for the proposed wideband digital receiver to recognize the intercepted fractional bandlimited signals, which could be used to choose the optimal algorithms to estimate the parameters of the signal. In practice, the signal waveform usually has lower signal power, which makes it difficult to classify the waveform directly. The conventional feature extraction and classification methods [28]–[38] work in Fourier domain and often perform unsatisfactorily because of its low concentration. Inspired by the property that the non-stationary signals enjoy both high concentration and absence of cross terms in fractional Fourier domain (FrFD), we propose a recognition method in DFTFrFD based on high-order cumulants to realize a classification of six types of signals including normal signal (NS), binary phase shift keying (BPSK), quadrature phase shift keying (QPSK), LFM, nonlinear frequency modulation (NLFM), and binary frequency shift keying (BFSK). First, we introduce the normalized second-order central moment (NSOCM) calculation method to directly obtain the optimal order of the received signal in DFTFrFD. The strength and advantage of the NSOCM method lies in its non-ergodic search mechanism, which can improve the computing efficiency to a great extent. The obtained optimal order based on NSOCM is able to effectively distinguish the encoded signals (BPSK, QPSK, and BFSK) and non-encoded signals (LFM and NLFM). Second, we use fourth central moment to calculate the spectral kurtosis (SK) in DFTFrFD which is used to complete the specific intra-pulse modulation for the received signal.

The main contributions of this paper are summarized as follows. First, we propose a modified MWC-based digital receiver architecture that can not only intercept the fractional bandlimited signals effectively in a low SNR, but also...
obtain better separation for cross-channel signals than by
original MWC-based receiver. Second, we propose an intra-
pulse modulation recognition method based on the optimal
transformation order and the SK for the fractional bandlim-
ited signals which is intercepted by the new receiver. Our
algorithm bears a relatively low complexity compared
with search-based methods and its detection performance
and recognition performance are higher than existing
algorithms.

The remainder of this paper is organized as follows: In
Section II, the basic preliminaries is introduced. The MWC
compressed sampling receiver for fractional bandlimited sig-
als is proposed in Section III. In Section IV, the intra-pulse
modulation recognition method for fractional bandlimited
signals is presented. In Section V, the detection performance
and recognition performance are simulated and analyzed.
Section VI is the conclusion

II. PRELIMINARIES

A. SIMPLIFIED FRACTIONAL FOURIER TRANSFORM(SFrFT)

The FrFT is an extension of the ordinary FT, which essen-
tially allows the signal in the time-frequency domain to be
projected onto a line of arbitrary angle [22]. SFrFT [39]
has the same effect as FrFT of order $p$ for filter design,
but it is simpler to implement digitally than the original
FrFT. And the first type of $p$th-order SFRFT is defined
as [39]:

$$\mathcal{F}_p (u) = (j2\pi)^{-\frac{1}{2}} \times \int_{-\infty}^{\infty} \exp (-jut + j^2 t^2 \cot \alpha/2) y (t) \, dt$$  

where $p$ is the transformation order, $\alpha$ is the rotation angle,
$\alpha = \frac{p \pi}{2}$. The inverse SFrFT is denoted as:

$$y (t) = (j/2\pi)^{\frac{1}{2}} \exp (-ju^2 \cot \alpha/2) \times \int_{-\infty}^{\infty} \exp (jut) \mathcal{F}_\alpha (u) \, dt$$

The digital simplified fractional Fourier transform (DSFrFT)

is given by [39]

$$F_\alpha (m) = (j2\pi)^{-\frac{1}{2}} \Delta t \sum_{n=-N}^{N} \exp \left(-j \frac{2\pi mn}{2N + 1} + j \frac{\alpha}{2} \cot (\Delta t)^2 \right) \cdot y (n)$$

where $y (n) = y (n \Delta t)$, $F_\alpha (m) = F_\alpha (m \Delta t)$, $m, n = -N, -N+1, \cdots, N$, $\Delta t$ and $\Delta u$ are the sample spacing in tempo-
ral domain and simplified fractional Fourier domain(SFrFD)
respectively. And $\Delta t \Delta u = \frac{2\pi}{(2N + 1)}$. We can also write
(1) in matrix form, expressed as

$$F_\alpha = c_1 O_F^T Y$$

where $c_1 = (j2\pi)^{\frac{1}{2}} \Delta t$, $O_F^T = (O_F^T)^*$, $*$ denotes
the transposed conjugate operator, and $O_F$ is a $(2N + 1) \times
(2N + 1)$ unitary matrix whose element $[O_F^T]_{mn}$ at the $m$ row,
$n$ column has the following form:

$$[O_F^T]_{mn} = \exp \left(-j \frac{2\pi}{2N + 1} (m - N - 1) (n - N - 1) \right) \cdot \exp \left[j (n - N - 1)^2 \cot \alpha (\Delta t)^2 /2 \right]$$

where $\lceil \cdot \rceil$ returns the nearest integer towards positive infinity.
With the change of $\alpha$, the frequency axis of the SFrFT is
located in different positions, and more abundant informa-
tion about the frequency characteristics of the signal can be
obtained compared to the FT.

B. FRACTIONAL BANDLIMITED SIGNALS AND ITS
SPECTRAL FEATURES IN SFrFD

A fractional bandlimited signal $f (t)$ has finite energy. The
SFrFT of $f (t)$ is zero outside the region $(-u_0 - u_\alpha, -u_0 + u_\alpha)$
under two sources is $0$, and the Poisson distribu-
tion is used to describe the rule that the radar pulses impinge
on the receiver in $\tau$ seconds can be expressed as

$$F_\alpha (u) = \begin{cases} F_\alpha (u), & u_0 - u_\alpha \leq |u| \leq u_0 + u_\alpha \\ 0, & \text{otherwise} \end{cases}$$

where $2u_\alpha$ is the fractional bandwidth of $f (t)$. According

To Parseval’s theorem, the bandlimited signal can also be
expressed as:

$$\int_{-\infty}^{\infty} |f (t)|^2 dt = \int_{-u_0}^{u_0} |F_\alpha (u)|^2 \, du + \int_{u_0}^{u_0} |F_\alpha (u)|^2 \, du$$

where $u_h = u_0 + u_\alpha$, and $u_l = u_0 - u_\alpha$.

III. PROPOSED MWC-BASED DIGITAL RECEIVER IN
DTrFpD

In this part, we explain how the receiver work and describe

the advantage of the proposed receiver over original MWC-based
digital receivers [18] when processing fractional bandlimited
signals.

The proposed digital receiver is designed for intercepting
the fractional bandlimited signals which are mainly produced
by a pulse working system. The complexity of the signal
environment where the digital receiver locates is usually
described with $\lambda$ pulses per second, and the Poisson distri-
bution is used to describe the rule that the radar pulses impinge
on the digital receiver. The probability for $I$ pulses impinging
on the receiver in $\tau$ seconds can be expressed as

$$P_I (\tau) = \frac{(\lambda \tau)^I e^{-\lambda \tau}}{I!}$$

Considering that the frequency interval $2 - 18 \text{GHz}$ which
the signals usually appear in practical can be divided into
several subbands. So it is appropriate to assume that there
are $\lambda = 10^5$ pulses per second in the surveillance frequency
interval whose bandwidth is about $1 \text{GHz}$. We design to detect
and acquire the signal pulses in a very short sampling time
by using the proposed receiver. According to (8), the prob-
ability for receiving single source is $9.05\%$, the probability
for receiving two sources is $0.45\%$, and the probability for
receiving more than two sources is very low in $1 \mu s$. If we
design $1 \mu s$ as the processing time unit, there would be only one signal detected and acquired in most cases [21].

The practical signal environment where the digital receiver locates is sparse distribution. Through the above analysis, the probability for the proposed digital receiver to deal with multi-mixed signals arriving simultaneously is very low. So, we can suppose there is only one signal received by the proposed receiver in the short sampling time.

A. SYSTEM DESCRIPTION

The received signal in discrete-time domain can be expressed as

$$x[n] = s[n] + \eta[n]$$

where $x[n]$ is the received signal, $s[n]$ is the incident signal, and $\eta[n]$ is the complex additive white Gaussian noise (AWGN) with zero mean and $\sigma^2$ variance. Let the received signal $x[n]$ be fractional bandlimited to the region $(u_l, u_h)$. And the Nyquist sampling rate is $f_{NYQ} = u_h / (\pi \sin \alpha)$.

The architecture of the proposed MWC-based digital receiver is shown in Fig. 1. Take the $m$th channel as an example. There are two mixing steps in each branch of the proposed multi-branch receiver. In the first step, the received signal $x[n]$ is multiplied by the signal $\exp\left((j/2) n^2 \cot \alpha\right)$, and the product is mixed with the random sign signal $\tilde{p}_m[n]$ in the second mixing step. The pseudo-random sequence $\tilde{p}_m[n]$ is a periodic function with period $T_p$, and there are $M_p = T_p f_{NYQ}$ elements per period. Then the production of the mixing signal is filtered by a low-pass filter $h[n]$ with a cutoff frequency $1/2T_s$ to obtain the filtered signal $w_m[n]$, and then $w_m[n]$ is sampled at sampling rate $f_s = 1/T_s$ to obtain the CS data $y_m[k]$, where $T_s$ denotes the down-sampling period. It is clear that the original MWC-based digital receivers is a special case at $\alpha = \pi/2$.

B. DISCRETE TIME SIMPLIFIED FRACTIONAL FOURIER DOMAIN (DTSFrFD) ANALYSIS

We define the mixing rate in DTSFrFD as $u_p = f_p \sin \alpha = \sin \alpha / T_p$, and design $u_p \geq B$ to avoid edge effects [17], here $B$ is the bandwidth of the incident signal $s[n]$ in $\alpha$-th order DTSFrFD. According to the spectrum distribution of the mixing function $\tilde{p}_m[n] \cdot \exp\left[(j/2) n^2 \cot \alpha\right]$, the coverage fractional “frequency” of the proposed receiver can be divided into $M_p$ sub-bands with bandwidths of $u_p$. The interval of the baseband in DTSFrFD is $U_p = [0, u_p]$. The DTFT of the mixing signal in the $m$th channel $x[n] \cdot \tilde{p}_m[n] \cdot \exp\left[(j/2) n^2 \cot \alpha\right]$ is:

$$X_m(\exp(j2\pi u \cdot \csc \alpha \cdot T_{NYQ}))$$

$$= \sum_{n=-\infty}^{\infty} \{x[n] \cdot \tilde{p}_m[n] \cdot \exp\left[(j/2) n^2 \cot \alpha/2\right]\} \cdot \exp(-j2\pi u \cdot \csc \alpha \cdot n \cdot T_{NYQ})$$

$$= \frac{1}{A_\alpha} \sum_{l=0}^{M_p-1} C_{il} \cdot \tilde{X}_\alpha\left(u - l \frac{2\pi}{M_p} \sin \alpha\right)$$

where $C_{il} = \sqrt{\frac{\sin \alpha - j \cot \alpha}{M_p}} \cdot C_{\alpha,il}$, $A_\alpha = \sqrt{\frac{1 - j \cot \alpha}{2\pi}}$, and $C_{\alpha,il}$ is the simplified fractional Fourier series coefficient of $\tilde{p}_m[n] \cdot \exp\left[(j/2) n^2 \cot \alpha/2\right]$, $\tilde{X}_\alpha(u) = \sum_{n=-\infty}^{\infty} x(n) \exp\left(-j\frac{1}{2} n^2 \cot \alpha + j n \cdot u \cdot \csc \alpha\right)$ is the DTSFrFT of $x[n]$.

According to Eq. (9), the two-step mixing produces a scale transformation and a relative $l \frac{2\pi}{M_p} \sin \alpha$ shift in DTSFrFD. Then, the mixing product is truncated by a low-pass filter with a cutoff fractional “frequency” $u_s/2$, where $u_s = f_s \sin \alpha$ is the fractional sampling rate for each channel. Consider $h(n)$ to be an ideal rectangle function in DTSFrFD and serves as a preceding anti-aliasing filter. The response of low-pass filter $h(n)$ in DTSFrFD is:

$$H(u \csc \alpha) = \begin{cases} 1, & u \in [-u_s/2, u_s/2] \\ 0, & \text{otherwise} \end{cases}$$

For simplicity, we assume $u_s = u_p$ to truncate the baseband signal which contains the full information of the original signal. The mixed product signal and its DTFT can be denoted by:

$$w_m[n] = \left(x[n] \cdot \tilde{p}_m[n] \cdot \exp\left[(j/2) n^2 \cot \alpha\right]\right) \ast h[n]$$

$$F(w_m[n]) (u \csc \alpha) = \sqrt{2\pi} F\left(x[n] \cdot \tilde{p}_m[n] \cdot \exp\left[(j/2) n^2 \cot \alpha\right]\right) \cdot (u \csc \alpha) \ast H(u \csc \alpha)$$

where $\ast$ denotes the convolution operator, and $F$ is the DTFT operator. Substituting Eq. (9) into Eq. (12), $F(w_m[n])$ can be simplified as:

$$F(w_m[n]) = \begin{cases} \frac{1}{A_\alpha} \sum_{l=0}^{M_p-1} C_{il} \cdot \tilde{X}_\alpha\left(u - l \frac{2\pi}{M_p} \sin \alpha\right), & u \in U_p \\ 0, & u \notin U_p \end{cases}$$
Then, the low-pass filtered signal $w_m[n]$ is down-sampled at a rate of $f_s = u_s \csc \alpha$ to obtain the baseband data $y_m[k]$, expressed as

$$y_m[k] = \{w_m[n]\}_{M_p} = \tilde{y}_m[k] + \bar{y}_m[k], \quad 0 \leq k \leq K$$

where $\{\cdot\}_{M_p}$ denotes the down-sampling operation. $\tilde{y}_m[k]$ is the signal component of interest for the $m$-th branch data, $\bar{y}_m[k]$ is the complex-valued AWGN component of the $m$-th branch data with zero mean and $\sigma^2/M_p$ variance, and $K = u_{NYQ}/u_p$ is the number of the data points in one branch. Therefore, the DTFT of the $m$th branch data $y_m[k]$, which is bandlimited to $U_s = [0, u_s]$, can be expressed as

$$F(y_m[k]) = \frac{1}{A_u} \sum_{l=0}^{M_p-1} C_l \cdot \tilde{X}_\alpha(u - l \frac{2\pi}{M_p} \sin \alpha), \quad u \in U_s$$

(15)

let

$$y_a(u) = \begin{bmatrix} F(y_1[k]) \\ F(y_2[k]) \\ \vdots \\ F(y_M[k]) \end{bmatrix}$$

(16)

We can rearrange (16) in matrix form, which is

$$y_a(u) = A \cdot z(u) = SFD \cdot z(u), \quad u \in U_s$$

(17)

where $y_a(u)$ is a column vector of length $M$, $S$ is a $M_p \times M_p$ random sign matrix. $F = \begin{bmatrix} \hat{F}_{L_0} & \cdots & \hat{F}_{-L_0} \end{bmatrix}_{M_p \times M_p}$ is a column subset of $\hat{F}$ which is an $M_p \times M_p$ discrete Fourier transform (DFT) matrix, and $M_p = 2L_0 + 1$. The $i$th column is denoted by:

$$\hat{F}_i = [\theta^{0i}, \theta^{1i}, \cdots, \theta^{(M_p-1)i}]^T, \quad 0 \leq i \leq M_p - 1$$

$$D = \text{diag}(d_{L_0}, \cdots, d_{-L_0})$$

is an $M_p \times M_p$ diagonal matrix with $d_l$ defined by:

$$d_l = \frac{1}{T_p} \int_0^{T_p} e^{-j \frac{2\pi}{T_p} l \theta} d\tau = \begin{cases} \frac{1}{M_p}, & l = 0 \\ \frac{1}{1 - \theta^l}, & l \neq 0 \end{cases}$$

$$\theta = e^{-\frac{2\pi}{M_p}}$$

Therefore, Eq.(17) can be rewritten as

$$\begin{bmatrix} F(y_1[k]) \\ F(y_2[k]) \\ \vdots \\ F(y_M[k]) \end{bmatrix} = \begin{bmatrix} S_{1,0} & \cdots & S_{1,M_p-1} \\ \vdots & \ddots & \vdots \\ S_{M,0} & \cdots & S_{M,M_p-1} \end{bmatrix} \begin{bmatrix} d_{L_0} \\ \vdots \\ d_{-L_0} \end{bmatrix} \begin{bmatrix} \tilde{X}_{\alpha}(u - L_0 \theta) \\ \vdots \\ \tilde{X}_{\alpha}(u + L_0 \theta) \end{bmatrix}$$

(18)

Furthermore, Assume the frequency of the signal $x(n)$ only exists in an unknown $h$th ($0 \leq h \leq M_p - 1$) sub-band in each branch of the proposed receiver. Then, the fractional spectrum information of other sub-bands in each branch can be ignored since there is very little fractional spectrum information of the signal of interest in those sub-bands, except for the $h$th sub-band. The DTFT of the $m$th branch and $m + 1$th branch of the CS data can be approximately expressed as

$$F(y_m[k]) = \frac{1}{A_u} C_{ih} \cdot \tilde{X}_\alpha(u - h \frac{2\pi}{M_p} \sin \alpha), \quad u \in U_s$$

$$F(y_{m+1}[k]) = \frac{1}{A_u} C_{ih} \cdot \tilde{X}_\alpha(u - h \frac{2\pi}{M_p} \sin \alpha), \quad u \in U_s$$

(18)

It is obviously that the DTFT of the $m$th branch and $m + 1$th branch of the CS data have the same form, there is not distortion of the phases of the multi-channel CS data in DTSFrFD, therefore, the multi-channel CS data can be superposed directly without phase correction in the proposed system.

An example is used to further describe the advantage of the proposed receiver over MWC-based receiver [18] when processing the LFM signal. Assume the received signal $x[n]$ is a cross-channel LFM signal with a bandwidth of $B$. Both the frequency spectrum of $x[n]$ and the fractional frequency spectrum is shown in Fig. 2, which is depicted as a rectangular band. The FT coefficients of $\tilde{p}_m[n]$ and the SFrFT coefficients of $\tilde{p}_m[n] \cdot \exp\left(\left(in^2 \cot \alpha \right)/2 \right)$ are also shown in Fig. 2, which are depicted as equidistant discrete spectral red lines. The frequency information and fractional frequency information after mixing are shown in Fig. 3.

Obviously, the LFM signal is sparse in DTSFrFD rather than in DTFD, and even if the signals satisfy the condition of the MWC-based receiver [18], the maximum bandwidth of signals in DTSFrFD is considerably narrower than that in DTFD, which is helpful to eliminate the cross-channel problem. Although the classic MWC-based receiver [18] can intercept such signals that show better sparsity in the DTSFrFD such as the signals in Fig. 2, the probability of successful intercept is much lower even with more hardware resources. It is not economical to use the classic MWC-based receiver [18] to intercept the fractional bandlimited signals.

Fig. 4 shows that the amplitude spectrums of the data for the intercepted LFM signal in DTSFrFD are presented with $SNR = 15dB$. The simulation parameters are shown in Section V-B. From Fig. 4, the baseband data contains the...
full spectrum information of the received signal, so we can directly process the data to acquire the parameter estimation of the original signal.

C. INTERCEPTION PERFORMANCE ANALYSIS

In this section, we formulate the interception performance of the proposed receiver and derive the interception criterion. Eq. (18) can be rewritten as

\[
\begin{bmatrix}
F(y_1[k]) \\
F(y_2[k]) \\
\vdots \\
F(y_M[k])
\end{bmatrix}
= \frac{2\pi}{M_p \cos \alpha}
\begin{bmatrix}
S_{1,0} & \cdots & S_{1,M_p-1} \\
\vdots & \ddots & \vdots \\
S_{M,0} & \cdots & S_{m,M_p-1}
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\begin{bmatrix}
e^{j\frac{2\pi}{M_p}} \\
e^{j\frac{2\pi}{M_p}(M_p-1)} \\
\vdots \\
e^{j\frac{2\pi}{M_p}(M_p-1)^2}
\end{bmatrix}
\begin{bmatrix}
\bar{X}_\alpha(u-L_0u_p) \\
\vdots \\
\bar{X}_\alpha(u+L_0u_p)
\end{bmatrix}
\]

\[
= \frac{2\pi}{M_p \cos \alpha} \text{PFZ} = \frac{2\pi}{M_p \cos \alpha} \text{AZ}, \quad u \in U_s
\]

where

\[
P = \begin{bmatrix}
S_{1,0} & \cdots & S_{1,M_p-1} \\
\vdots & \ddots & \vdots \\
S_{M,0} & \cdots & S_{m,M_p-1}
\end{bmatrix}
\]
expressed as

$$\eta \text{ is related with each other. As a result, Eq.(24) can be expressed as}
$$

$$(1)$$

\[
F = \begin{bmatrix}
1 & 1 & e^{\frac{2\pi i}{M_p}} & \cdots & e^{\frac{2\pi i}{M_p}(M_p-1)} \\
1 & e^{\frac{2\pi i}{M_p}} & \cdots & e^{\frac{2\pi i}{M_p}(M_p-1)} \\
\vdots & \vdots & \ddots & \vdots \\
1 & e^{\frac{2\pi i}{M_p}} & \cdots & e^{\frac{2\pi i}{M_p}(M_p-1)}^2 \\
\end{bmatrix}
\]

$$Z = \begin{bmatrix}
\tilde{X}_\alpha (u - L_0m_p) \\
\tilde{X}_\alpha (n) \\
\tilde{X}_\alpha (u + L_0m_p) \\
\end{bmatrix}, \quad A = P$$

According to Eq.(19), the interception model can be expressed as

$$F (y_i [k]) = \begin{cases}
\frac{2\pi}{M_p \cos \alpha} A_i \eta_i (n), & H_0 \\
\frac{2\pi}{M_p \cos \alpha} A_i F_i (s_i (n) + \eta_i (n)), & H_1
\end{cases}$$  \quad (20)

where $F (y_i [k])$ is the DTSFrFT of the compressed sampling sequence in the $i$ branch, $s_i (n) = [s_0 (n), s_1 (n), \ldots, s_{M_p-1} (n)]^T$ is the useful signal vector in the $i$ branch, $\eta_i (n) = [\eta_0 (n), \eta_1 (n), \ldots, \eta_{M_p-1} (n)]^T$ is the band-limited white noise vector with zero mean and $\sigma^2 / M_p$ variance. $A_i$ is the $i$th row vector of the compressed sampling matrix $A$. $H_0$ denotes that the useful signal $s_i (n)$ does not exist while $H_1$ means the opposite. The energy statistics of the compressed sampling sequence of the $i$th branch can be expressed as

$$E_i = \frac{1}{N} \sum_{n=1}^{N} |F (y_i [k])|^2$$  \quad (21)

Assuming that $H_0$ hypothesis is true, we randomly select one branch of MWC based on Eq.(19), i.e.

$$F (y_i [k]) = \frac{2\pi}{M_p \cos \alpha} A_i \eta_i (n) = \frac{2\pi}{M_p \cos \alpha} P_i F \eta_i (n)$$  \quad (22)

where $P_i$ denotes the $i$th row vector of periodic pseudo-random sequence matrix $P$. Substituting Eq.(22) into Eq.(21) yields

$$E_i = \frac{1}{N} \frac{2\pi}{M_p^2 \cos^2 \alpha} P_i F \left[ \sum_{n=1}^{N} |\eta_i (n)|^2 \right] F_i^H F_i^T$$  \quad (23)

where $(\cdot)^H$ denotes the conjugate transpose.

Let

$$R_\eta = \frac{1}{N} \sum_{n=1}^{N} |\eta_i (n)|^2$$  \quad (24)

as $\eta_i (n)$ is modulated by the pseudo-random sequence and low-pass filtered, the elements of the vector $\eta_i (n)$ are uncorrelated with each other. As a result, Eq.(24) can be expressed as

$$R_\eta = \text{diag} \left[ \frac{1}{N} \sum_{n=1}^{N} \eta_1^2 (n), \ldots, \frac{1}{N} \sum_{n=1}^{N} \eta_M^2 (n) \right]$$  \quad (25)

Defining $R_\eta = \frac{1}{N} \sum_{n=1}^{N} \eta_i^2 (n)$ the Eq.(25) can be rewritten as

$$R_\eta = RI$$  \quad (26)

where $I$ is the $M_p \times M_p$ identity matrix. According to Eq.(19),

$$FF^H = M_p I, \quad P_i P_i^H = M_p$$

so Eq.(23) can be simplified as

$$E_i = \frac{2\pi}{M_p^2 \cos^2 \alpha} P_i F R F_i^H F_i^T = \frac{2\pi}{N \cos^2 \alpha} \sum_{n=1}^{N} \eta_i^2 (n)$$  \quad (27)

Since the distribution of the band-limited AWGN $\eta_i (n)$ is

$$\eta_i (n) \sim \text{Normal} \left( 0, \frac{\sigma^2}{M_p} \right)$$  \quad (28)

Then the distribution of $E_i$ is

$$E_i = \frac{2\pi}{N \cos^2 \alpha} \sum_{n=1}^{N} \eta_i^2 (n) \sim \text{Normal} \left( \frac{\sigma^2}{\cos^2 \alpha} \left( M_p \cos^2 \alpha, \frac{2\sigma^4}{\cos^4 \alpha} \right) \right)$$  \quad (29)

In practice, we can use a known probability of false alarm $P_f$ to calculate each branch threshold $\gamma_i$. Since

$$P_f = Q \left( \frac{\gamma_i - N\sigma^2 / (M_p \cos^2 \alpha)}{\sqrt{2N\sigma^4 M_p^2 \cos^4 \alpha}} \right)$$  \quad (30)

where $Q (x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp (-x^2/2) dx$ Therefore, the threshold of each branch $\gamma_i$ can be expressed as

$$\gamma_i = \frac{2\pi \cdot \sigma^2}{M_p \cos^2 \alpha} \left( N + \sqrt{2NQ^{-1} (P_f)} \right)$$  \quad (31)

And the threshold for the all branches is

$$\gamma = \sum_{i=1}^{M} \gamma_i = \frac{2\pi \cdot M \sigma^2}{M_p \cos^2 \alpha} \left( N + \sqrt{2NQ^{-1} (P_f)} \right)$$  \quad (32)

where $M$ is the number of branches as shown in Fig.1. $M_p$ is the number of elements per period for the pseudo-random sequence $\tilde{p}_m [n]$.

**IV. PROPOSED INTRA-PULSE MODULATION RECOGNITION METHOD AND PROCEDURES**

**A. INTERCEPTED SIGNAL MODEL**

The received discrete time signal is composed of a modulated signal and noise. Its model is given by

$$x (n) = s (n) + \eta (n)$$

where $x (n)$ and $s (n)$ are received signal and modulated signal, respectively. $\eta (n)$ is assumed to be AWGN. The modulated signal $s (n)$ is given by

$$s (n) = A \exp \left( -j \left( 2\pi f_c n / f_{\text{Nyq}} + \phi (n) + \phi_0 \right) \right)$$

where $A$ is the amplitude, $f_c$ and $\phi_0$ are the carrier frequency and the initial phase, respectively. $f_{\text{Nyq}}$ is the Nyquist sampling rate. $\phi (n)$ is the phase function, which determines the modulation type of the signal. For simplicity and without loss of generality, we assume that $A$ is an invariant constant.
The different pulse compression waveforms considered in this paper are: NS, LFM, NLFM, BPSK, QPSK, BFSK. And \( \phi [n] \) for different modulation types are expressed as follows:

\[
\begin{align*}
\phi_{\text{NS}} [n] &= 0 \\
\phi_{\text{BPSK}} [n] &= \pi C_{\text{BPSK}} [n]
\end{align*}
\]

where the phase coding function \( C_{\text{BPSK}} [n] \) alternates between 0 and 1.

\[
\phi_{\text{QPSK}} [n] = \pi C_{\text{QPSK}} [n]
\]

where the phase coding function \( C_{\text{QPSK}} [n] \) alternates between 0, 0.5, 1, and 1.5.

\[
\phi_{\text{LFM}} [n] = \pi K_f n^2 / f_{\text{NYQ}}^2
\]

where \( K_f \) is the modulated rate.

\[
\phi_{\text{NLFM}} [n] = 2\pi a_0 + 2\pi a_1 n / f_{\text{NYQ}} + 2\pi a_2 (n / f_{\text{NYQ}})^2 + 2\pi a_3 (n / f_{\text{NYQ}})^3 + \phi_0
\]

where \( a_0, a_1, a_2, \) and \( a_3 \) are the frequency modulation coefficients.

\[
\phi_{\text{BFSK}} [n] = 2\pi (f_n - f_c) n / f_{\text{NYQ}}
\]

where \( f_n \) is the value of the hopping frequency.

**B. METHOD DESCRIPTION**

Our proposal consists of two steps. In the first step, we propose using the NSOCM to acquire the optimal transformation order of CS data in the DTSFrFD to distinguish the encoded signals and non-encoded signals roughly. The theoretical analysis in the next subsection will indicate that for the encoded signals (PSK and FSK) and NS signal, the optimal transformation order where \( X_n(u) \) reaches its extremum values is \( p = 1 \) while the values of the optimal order for the non-encoded signals (LFM and NLFM) are concentrated far away from 1. Therefore, the optimal order can be used to classify these signals into two categories. And in the second step, the SK of the CS data in DTSFrFD are used to complete the specific intra-pulse modulation for the received signal.

**C. THE OPTIMAL TRANSFORMATION ORDER ANALYSIS FOR THE 6 TYPES OF FRACTIONAL BANDLIMITED SIGNALS**

In the fractional Fourier domain (FrFD), support of signals change associated with the transform order and there exists an optimum transform order in which the energy of signals are maximally concentrated [4], [5]. When an signal is transformed by FrFT at its optimal order, transform kernel acts as a matched filter. Therefore, the optimal transformation order has the ability to maximize the absolute amplitude. The optimal transformation order \( p_{\text{opt}} \) corresponding to maximum magnitude obtained from the FrFT is given by

\[
p_{\text{opt}} = \max_p \left[ |F_p(t,u)|^2 \right]
\]

where \( F_p(t,u) \) is the FrFT of the signals in the \( p \)-order FrFD.

In this section, we calculate and compare the optimal order of 6 types of signals in DTSFrFD.

For the NS \( x_{\text{NS}}[n] \), its FT is

\[
|F| = 2\pi \sin(c (f - f_c) / f_{\text{NYQ}})
\]

as its spectrum in DTFD is an impulse, its optimal transformation order is \( p = 1 \) obviously.

For the LFM signal \( x_{\text{LFM}}[n] \), when the fractional rotation angle \( \alpha = \arccot(-K_f) \) [40], the amplitude spectrum of \( x_{\text{LFM}}[n] \) in SFrFD is

\[
|F_a| = A (j 2\pi 1/2 \Delta t \sin[c (\pi (mDelta Csc \alpha - f_c / f_{\text{NYQ}}))])
\]

i.e., for the received LFM signal, its DTSFrFT at the fractional rotation angle \( \alpha = \arccot(-K_f) \) is an impulse. Therefore, the fractional transformation order \( p = 2\alpha / \pi = \arccot(-K_f) \times 2 / \pi \) is the optimal transformation order in which the energy of LFM signals are most concentrated.

For the NLFM signal \( x_{\text{NLFM}}[n] \), the optimal transformation order of the NLFM signal is [41]

\[
p = 1 + \arctan(2\pi a_3 (n_1 + n_2) + 2\pi a_2) \frac{2\pi}{\pi}
\]

where \( n_1 \) and \( n_2 \) are the initial time and end time, respectively.

For the BFSK signal \( x_{\text{BFSK}}[n] \), the SFrFT of \( x_{\text{BFSK}}[n] \) can be calculated by

\[
F_a(m) = (j 2\pi 1/2 \Delta t \sum_{n=-N}^{N} \exp(-j2\pi mn/2N + 1) + jn^2 \cot(\alpha (\Delta t) / 2) X_{\text{FSK}}[n]
\]

\[
= (j 2\pi 1/2 \Delta t \sum_{n=-N}^{N} \exp(-j2\pi mn/2N + 1) + jn^2 \cot(\alpha (\Delta t) / 2) \cdot A \exp(-j(2\pi f_n / f_{\text{NYQ}} + 2\pi (f_n - f_c) n / f_{\text{NYQ}} + \phi_0))
\]

When the fractional order \( p = 1 \), the amplitude spectrum of \( x_{\text{BFSK}}[n] \) in DTSFrFD is

\[
|F_a| = A (j 2\pi 1/2 \Delta t \sin(c (\pi (mDelta u - f_n / f_{\text{NYQ}})))
\]

Therefore, the optimal transformation order for the BFSK signal is \( p = 1 \). Similarly, the optimal transformation order for the PSK signal (BPSK or QPSK) is also \( p = 1 \).

Increase the fractional order \( p \) from 0 to 2 with a step of 0.015, calculate the discrete time simplified fractional Fourier transform (DTSFrFT) \( X_{p}(u) \) of the six types of signals above for every \( p \), and extract the max amplitude \( X_{\text{pmax}}(u) \) in every \( p \)-order DTSFrFD. Traversing all the fractional order \( p \), and we can obtain the evolutions of the max normalized DTSFrFT amplitude \( X_{\text{pmax}}(u) \) of 6 types of modulation signals with respect to the fractional order \( p \) as shown in Fig.5 (simulation parameters are shown in Section V).

As shown in Figure 5, the values of the optimal order for the encoded signals in the DTSFrFD are concentrated around 1, while for the non-encoded signals, the fractional spectrum
Where \( m_0 = \int_{-\infty}^{\infty} u |X_a(u)|^2 du / \int_{-\infty}^{\infty} |X_a(u)|^2 du \) is the normalized first-order origin moment of \( X_a(u) \), and \( \omega_a = \int_{-\infty}^{\infty} u^2 |X_a(u)|^2 du / \int_{-\infty}^{\infty} |X_a(u)|^2 du \) is the normalized second-order origin moment of \( X_a(u) \). The NSOCM \( p_0 \cdot p_{a+1} \) represent the timewidth and frequency width of \( X_a(u) \), respectively. Hence, Eq. (33) becomes

\[
p_{\text{opt}} = \arg \min_{p} \{p_0 \cdot p_{a+1}\}, \quad 0 \leq p < 1
\]

The NSOCM product is given by

\[
p_0 \cdot p_{a+1} = p_0 \cdot p_1 + \frac{1}{4} \left( (p_0 - p_1)^2 - 4 \mu_0^2 \right) \sin^2 (\alpha \pi) + \frac{1}{2} \mu_0 (p_0 - p_1) \sin (2 \alpha \pi)
\]

where \( \mu_0 = (\omega_0 + \omega_1) / 2 + m_0 m_1 - \omega_{0.5} \) is the mixed second-order moment. Setting the first derivative of \( p_0 \cdot p_{a+1} \) with respect to the order equal zero, we obtain

\[
\tan (2 \alpha \pi) = \frac{4 \mu_0 (p_0 - p_1)}{4 \mu_0^2 - (p_0 - p_1)^2}
\]

For this case where \( \alpha \) is equal to the extreme point \( \alpha_e \), the product \( p_0 \cdot p_{a+1} \) reaches the extremum values. This result demonstrates that when \( p \) satisfies Eq. (34) as follows, the product \( p_0 \cdot p_{a+1} \) reaches its minimum.

\[
p = \frac{1}{2 \pi} \arctan \left( \frac{4 \mu_0 (p_0 - p_1)}{4 \mu_0^2 - (p_0 - p_1)^2} \right)
\]

Based on the theoretical analysis above, the calculation process of the optimal transformation order can be summarized into the specific procedures as follows:

1. Take the 0.5-th and 1-th order DTSFrFT of signal \( x[n] \) to obtain \( X_{0.5}, X_1 \).

2. Calculate the normalized first-order origin moments \( m_0 \) and \( m_1 \). The normalized second-order origin moments \( \omega_0, \omega_{0.5} \) and \( \omega_1 \), the mixed second-order moment \( \mu_0 \), and NSOCM \( p_0 \) and \( p_1 \) in accordance with the definition.

3. Obtain the optimal order \( p_{\text{opt}} \) of \( p \) by using Eq. (34) in the range of \([0,1]\).

**E. THE SK ANALYSIS OF THE CS DATA IN DTSFrFD**

The SK is a statistical tool which can indicate the presence of series of transients and their locations in the Fourier domain [46]. SK measures deviation from Gaussian distribution, the distribution has sharp peaks when the distribution has large kurtosis, conversely the distribution is flat. References [45], [47] extend the SK to fractional Fourier domain, and prove that the signal’s SK in the FrFD has the same optimal order with signal’s FrFT. And according to [48], the SK of Gaussian signals is zero in FrFD which helps suppressing Gaussian noise.

This subsection propose a DTSFrFT based on SK recognition method to complete the specific intra-pulse modulation.
for the received signal. The SK of the \( m \) th CS data in the optimal \( p \) th-order DTSFrFD is given by

\[
KS\left( X_p \left( m \right) \right) = \frac{E\left( X_p^2 \left( m \right) \right) - 2}{E\left( X_p \left( m \right) \right)^2} - 2 \tag{35}
\]

where \( X_p \left( m \right) \) is the optimal \( p \) th-order DTSFrFT, \( m \) is the variable in the \( p \) th-order DTSFrFD, \( E \left( \cdot \right) \) stands for the expected value operator, \( \ast \) denotes the transposed conjugate operator.

Table 1 gives the SK of CS data for LFM and NLFM signal using (35) with respect to different SNRs. It is obvious that the LFM signal’s SK is far greater than the SK of the NLFM signal, therefore, the SK in DTSFrFD can be used to differentiate LFM signal from NLFM signal. Moreover, As seen from these results of Table 1, when the different SNR varies from −6dB to 24dB with a step of 3dB, the SK still has a large value. That is because the SK of Gaussian noise is zero in FrFD which helps suppressing Gaussian noise.

| TABLE 1. SK of CS data (using (35)). |
|--------------------------------------|
|                                    |
| \( \text{LFM} \)                      |
| -6dB  | -3dB  | 0dB   | 3dB   | 6dB   | 9dB   |
| 35.23 | 34.74 | 34.62 | 35.42 | 34.87 | 35.08 |
| \( \text{NLFM} \)                     |
| -12dB | -15dB | -18dB | 21dB  | 24dB  |
| 34.98 | 34.76 | 35.96 | 35.11 | 35.27 |
| \( \text{NS} \)                       |
| -12dB | -15dB | -18dB | 21dB  | 24dB  |
| 16.87 | 16.54 | 16.85 | 16.29 | 16.83 |

Table 2 shows the SK of CS data for the NS signal and the encoded signals through simulations (simulation parameters are shown in Section V) using (35) when the SNR varies from −6dB to 24dB with a step of 3dB. It is obvious that the SK of the NS signal is far greater than the SK of other signals, therefore, the SK in DTSFrFD can be used to recognize NS signal. Similarly, since the SK of the BFSK signal is far greater than the SK of QPSK and BPSK signals, the BFSK signal can be differentiated from the QPSK and BPSK signals. In the same way, QPSK and BPSK signals can be distinguished by differences in SK values.

| TABLE 2. SK of CS data (using (35)). |
|--------------------------------------|
|                                    |
| \( \text{NS} \)                      |
| -6dB  | -3dB  | 0dB   | 3dB   | 6dB   | 9dB   |
| 76.20 | 76.08 | 75.85 | 75.53 | 75.24 | 75.41 |
| \( \text{BFSK} \)                    |
| 32.21 | 32.45 | 32.58 | 31.56 | 31.42 | 31.45 |
| \( \text{BPSK} \)                    |
| 15.08 | 15.96 | 14.87 | 14.92 | 14.22 | 13.75 |
| \( \text{QPSK} \)                    |
| 9.64  | 9.84  | 9.25  | 8.88  | 8.41  | 8.25  |
| \( \text{QPSK} \)                    |
| 12dB  | 15dB  | 18dB  | 21dB  | 24dB  |
| 74.99 | 74.68 | 74.02 | 74.21 | 74.32 |
| \( \text{BPSK} \)                    |
| 31.02 | 31.00 | 29.59 | 29.74 | 29.93 |
| \( \text{QPSK} \)                    |
| 13.91 | 13.56 | 12.95 | 12.75 | 11.84 |

\[KS\left( X_p \left( m \right) \right) = \frac{E\left( X_p^2 \left( m \right) \right) - 2}{E\left( X_p \left( m \right) \right)^2} - 2 \tag{35}\]

F. RECOGNITION PROCEDURES

The procedures of the proposed intra-pulse modulation recognition for fractional bandlimited signals based on the proposed digital receiver are shown in Fig. 6 which can be described as follows:

1) Rough recognition module for the encoded signals and the non-encoded signals. Calculate the optimal order of the input CS data by using the NSOCM calculation method. Then, compare the optimal order with the designed threshold \( p_{TH} \). If the optimal order is greater than \( p_{TH} \), the received signal can be classified as the encoded signals. Otherwise, the received signal is classified as non-encoded signals.

2) Specific recognition module for the non-encoded signals. Calculate the SK \( Q' \) of CS data. Then, compare \( Q' \) with the designed \( Q'_{TH} \). If \( Q' \) is more than \( Q'_{TH} \), the received signal can be classified as LFM signal. Otherwise, the received signal is classified as NLFM signal.

3) Specific recognition module for the encoded signals. Calculate the SK \( Q \) of CS data. Then, compare \( Q \) with the designed \( Q_{TH1} \). If \( Q \) is more than \( Q_{TH1} \), the received signal can be classified as NS signal. Otherwise, Compare \( Q \) with the designed \( Q_{TH2} \), if \( Q \) is more than \( Q_{TH2} \), the received signal can be classified as BFSK signal. Finally, Compare \( Q \) with the designed \( Q_{TH3} \). If \( Q \) is more than \( Q_{TH3} \), the received signal can be classified as BPSK signal. Otherwise, the received signal is QPSK signal.

V. NUMERICAL SIMULATION

A. DESIGN OF SIMULATION

Examples of numerical simulation are presented to evaluate the detection performance of the proposed modified MWC compressed sampling receiver and the intra-pulse modulation recognition performance of the proposed recognition method based on the new receiver.

The parameters of the proposed receiver are configured as follows. The Nyquist sampling rate of the received signal is \( f_{NYQ} = u_{NYQ} \csc \alpha = 2.2GHz \), where \( u_{NYQ} \) is the Nyquist sampling rate in DTSFrFD, \( \alpha \) is the fractional order which varies from \(-0.80 \times 10^{-9} \) to \(-0.62 \times 10^{-9} \) with a step of \(0.01 \times 10^{-9}\). The pseudo-random sequence is generated by a Bernoulli random binary \( \pm 1 \) sequence with \( M_p = 400 \), so the bandwidth of the baseband in DTSFrFD is \( u_p = u_{NYQ}/M_p \). An ideal low-pass filter with cutoff frequency \( u_p/2 \) in DTSFrFD is adopted and the down-sampling rate is \( u_s = u_p \). The total CS sampling rate is \( M_p u_s \). The sampling time is \( T = 10\mu s \). The number of the original sampling data is \( N = 22000 \). The number of the data for each branch is \( K = N/M_p = 55 \). The values of the parameters are listed in Table 3.

B. THE DETECTION PERFORMANCE OF THE PROPOSED MODIFIED MWC COMPRESSED SAMPLING RECEIVER

The detection performance of the proposed system is evaluated by the detection probability, and we demonstrate the simulations of the detection probability of the proposed energy
We use the LFM signal and the BPSK signal as the test subjects, which are typical frequency modulation signals and phase modulation signals. We simulate the system on the test subjects contaminated by AWGN. The original LFM signal in discrete-time domain is denoted by $x_{LFM}(n)$. The noisy signal is $x_{LFM}(n) + \omega(n)$. The signal to noise ratio (SNR) is defined as $10 \log(|x|/\|\omega\|)$. The SNR is given by the following:

$$x_{LFM}(n) = E \exp \left( j2\pi K_{lfm} f_{LFM}^2 / f_{NYQ}^2 \right) \cos (2\pi f_{LFM} n / f_{NYQ})$$

where $E$ is the amplitude of the signal which could be random or fixed. $K_{lfm} = 0.200 \times 10^9$ Hz/s is the signal modulation rate. The signal duration time is $T = 10\mu s$. So the bandwidth of $x(n)$ is $B = K_{lfm} \cdot T = 10$ MHz, $f_{NYQ}$

---

**FIGURE 6.** Procedures of the proposed intra-pulse modulation recognition method.

**TABLE 3.** List of the parameters of the proposed receiver.

| Parameters | DTSPFD | Notes |
|------------|--------|-------|
| $u_{NYQ}$ \(\cos \alpha\) | 2.2GHz | Nyquist rate of the signal |
| $M_p$ | 400 | Number of sign intervals in each $\beta_m [n]$ |
| $u_p$ | $u_{NYQ}/M_p$ | The bandwidth of baseband |
| $u_s$ | $u_s = u_p$ | Sampling frequency of a single channel |
| $M u_s$ | 54MHz | The overall sampling rate for the system |
is the Nyquist sampling rate of the signal. $f_1 = 1GHz$ is the initial frequency. The signal is both frequency bandlimited and fractional bandlimited with different bandwidths in the observation interval.

The original BPSK signal in discrete-time domain can be depicted as

$$x_{\text{BPSK}}(n) = A \exp \left\{ j2\pi f_0 n / f_{\text{NYQ}} + \pi C_{\text{BPSK}}(n) \right\}$$

where $A$ is the amplitude of the signal, $f_0 = 1GHz$ is the frequency carrier. $C_{\text{BPSK}}(n)$ is the phase coding function which alternates between 0 and 1.

Assume that the false alarm probability $P_f = 0.01$, the proposed detection threshold can be calculated by Eq.(32).

Fig. 7(a) and (b) depict the tradeoff of the SNRs and detection probability of the proposed receiver for LFM signal and BPSK signal respectively. The fractional order $\alpha$ is fixed to $-0.70 \times 10^{-9}$, and the number of sampling channels is $\{10, 15, 20\}$. From the results in Fig.7, the detection performance from the proposed receiver is preferable than the original method when SNR is under $-5\ dB$. And the detection probability approximates 100% when SNR is above $-5\ dB$. And it is common that the detection probability increases with increasing SNR and the number of sampling channels in both the proposed method and classic MWC discrete compressive sampling structure. Besides, when the orders $\alpha$ varies from $-0.80 \times 10^{-9}$ to $-0.62 \times 10^{-9}$ with a step of $0.01 \times 10^{-9}$, as can be seen from Eq.(32), the value of the intercept threshold is almost unchanged. Therefore, the interception performance is hardly affected by $\alpha$ when $\alpha$ is small enough.

In Fig. 8(a) and (b), the SNR is $\{-3, -8, -10\} dB$, the number of channels varies from 4 to 60 with a step of 2. It is observed that the detection probability has the same trend as the sampling channels, increasing the sampling channels leads to an increase of the detection probability and a smaller SNR correspond to more sampling channels. Occasionally, a low SNR may lead to failure of the detection. From the results we can conclude that the proposed method has better detection performance.
C. THE INTRA-PULSE MODULATION RECOGNITION PERFORMANCE OF THE PROPOSED RECOGNITION METHOD BASED ON THE NEW RECEIVER

In this section, the proposed recognition method based on the new receiver is measured by simulation signals. The purpose is to test the computational efficiency of the optimal order $p_{opt}$, the accurate rate of the identification results in different conditions.

1) CREATE SIMULATION SIGNALS

For all the six fractional bandlimited waveforms discussed above, there are different parameters that need to be set. For LFM, the initial frequency is $f_c = 1$ GHz, the bandwidth is $B = 20$ MHz, and the modulated rate is $K_f = 2$ MHz/µs. For NLFM, the frequency modulation parameters are $a_0 = 0$, $a_1 = 1.2 \times 10^8$, $a_2 = 0.7 \times 10^{12}$, $a_3 = 2.0 \times 10^{15}$. For NS, the carrier frequency is $f_c = 1$ GHz. For BFSK, the code rate varies from $f_1 = 1200$ MHz to $f_2 = 1210$ MHz. For QPSK, the carrier frequency is $f_c = 1$ GHz and a Frank code of length 16 is used to modulate the phase. For BPSK, the carrier frequency is $f_c = 1$ GHz and a Barker code of length 11 is used to modulate the phase. For more details, see Table 4.

The parameters of the proposed receiver are as described in section V-A.
2) COMPUTATIONAL EFFICIENCY OF THE OPTIMAL ORDER $p_{\text{opt}}$

Firstly, calculate the optimal order $p_{\text{opt}}$ of the CS data for the six types of signals above. Table 5 shows the optimal order $p_{\text{opt}}$ of the CS data for NS, BPSK, QPSK, LFM, NLFM, and BFSK signals through simulations without noise. In addition, the search-based method [43] is given for comparison with the NSOCM algorithm.

The optimal order calculated by NSOCM is very close to the search-based results, and the computation time of these methods is given in Table 6. It is obvious that the NSOCM calculation greatly reduces the computation time of search-based method. As seen from these results, the NSOCM algorithm can give a reasonable optimal order for the DTSFrFT and has the advantage of high computational efficiency.

As seen from Table 5, the values of the optimal order for the NS, BPSK, QPSK, BFSK signals are concentrated around 1. Therefore, the optimal order can be used to distinguish these signals from LFM and NLFM signals. And the range of the order threshold $p_{\text{TH}}$ can be expressed as $[0.97, 1.02]$.

Secondly, according to (35), calculate the SK of the CS data in the optimal order $p_{\text{opt}}$ with 500 Monte Carlo experiments for each signal when the SNR varies from -8 dB to 14 dB with a step of 2 dB. The results are given in Table 1 and Table 2, respectively. From Table 1, the range of the recognize threshold $Q_{\text{TH}}$, which is used to differentiate LFM signal from NLFM signal can be set as $[20, 30]$.

Thirdly, from Table 2, the range of the recognize threshold $Q_{\text{TH1}}$ which is used to distinguished the NS signal from the others can be set as $[45, 55]$. Similarly, the recognize threshold $Q_{\text{TH2}}$ is used to classify the BFSK signal from BPSK and QPSK signals while the range of $Q_{\text{TH2}}$ is $[20, 28]$. And finally, the range of the threshold $Q_{\text{TH3}}$ which can recognize the BPSK signal from QPSK signal is $[10, 10.7]$.

3) EXPERIMENT WITH SNR

The experiment research shows the relationship between the ratio of successful recognition (RSR) and SNR. The RSR serves as a recognition performance measure, with a higher ratio corresponding to an improved recognition performance. The probabilities are measured by the testing data of each kind of waveforms. For each signal, 100 Monte Carlo experiments are performed to calculate the RSRs for each SNR which is increased from -8 dB to 14 dB with a step of 2 dB. Figure 9 plots the RSR as a function of the SNR, and the overall probabilities are also calculated. The original MWC discrete compressive sampling structure [18] is given for comparison. It is clear that the RSRs for six types of radar signals have the same trend as the SNR. And when $\text{SNR} \geq 8$ dB, the RSRs of the original MWC discrete compressive sampling structure [18] are 100%, which rapidly decreases when $\text{SNR} \leq 8$ dB. At $\text{SNR} > 8$ dB, the RSR of the proposed system approaches 100%, with the decrease of SNR, the successful ratio can still be kept at a high level.

From Fig.9, we can also see that the RSRs of LFM and NLFM are greater than 90% when SNR is above $-3$ dB, the RSRs of BPSK and QPSK can reach 90% when SNR is above $-4$ dB, the RSRs of BFSK are greater than 90% when SNR is above 1 dB, and the RSRs of NS can reach 90% when SNR is above $-5$ dB. And all these results above are the under the condition that only one branch CS data of the proposed compressed sampling receiver are processed. Meanwhile, Fig.9 also shows the results under the condition that ten branches CS data of the proposed compressed sampling receiver are analyzed. And it is clear that the RSRs of the latter are significantly higher than the former at the same SNR, that is because the superposed multi-branch CS data can increase the OSNR [18]. And from the analysis in sec.III-B, since there is no distortion of the phases of the multi-branch CS data, therefore, the multi-branch CS data can be superposed directly without phase correction in the proposed system. Fig.9 validates the proposed recognition method based on superposed CS data outperforms the recognition system based on one branch of CS data under low SNRs. And it is obviously that our system performs better on the classification of each kind of waveform than [18], especially at low SNR and has better robustness against SNR variation.

VI. CONCLUSION

This paper introduces a MWC-based digital receiver architecture in DTSFrFD to intercept fractional bandlimited signals and propose an intra-pulse modulation recognition method in DTSFrFD for the fractional bandlimited signals which is intercepted by the new receiver. The original MWC-based receiver structures are confined to stationary signal and require a high SNR, the proposed structure in this paper overcomes the confine by taking advantage of the properties of FrFT, and the original MWC-based receiver structure is shown to be a special case of it. Meanwhile, an intra-pulse modulation recognition method based on the optimal transformation order and the SK in DTSFrFD is presented for such architecture. This algorithm shows better robustness than the original MWC discrete compressive sampling structure [18] against SNR variation, and it bears a relatively low complexity comparing with the search-based method.

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