Fluxes of Atmospheric Neutrinos and Related Cosmic Rays
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The atmospheric neutrino beam simultaneously spans a range of pathlengths from ten to ten thousand kilometers, which correspond respectively to downward- and upward-going neutrinos. As with any neutrino oscillation experiment, also in this case the interpretation of the data depends on a detailed knowledge of the neutrino beam. The ingredients are the primary spectrum of cosmic-ray nucleons, the geomagnetic fields in which the charged particles propagate and the properties of interactions of hadrons in the atmosphere. In this talk I review the status of calculations in light of the recent evidence for neutrino oscillations from Super-Kamiokande.

1. Introduction

When cosmic ray protons and nuclei interact in the atmosphere, the secondary cascades include neutrinos from decay of pions, muons and kaons. Production of these neutrinos depends on the local zenith angle because of the competition between decay and interaction of the parent mesons in the tenuous atmosphere. A simple geometric construction shows that a trajectory from below with nadir angle $\theta$ has the same zenith angle $\theta$ on the other side of the earth. Therefore, since neutrinos with $E \ll 10^5$ GeV are virtually unattenuated by the earth, the flux of atmospheric neutrinos would be up-down symmetric in the absence of neutrino oscillations except to the extent that the isotropy of the primary cosmic rays is distorted by the geomagnetic field. Variation of the neutrino flux with azimuth is a consequence only of the geomagnetic field (the “East-West” effect) even in the presence of oscillations (because within a given band of zenith angle the distributions of neutrino pathlengths and energies are independent of azimuth). This fact allows an important check of the systematics of the Super-K analysis, which I discuss in §2.

Typical altitudes of production of the neutrinos are between 10 and 20 kilometers, so the distribution of neutrino pathlengths ranges from 10 km for vertically downward neutrinos (neutrinos that originate directly overhead) to $\sim 10^4$ km for upward neutrinos from below. The range of neutrino energies for contained or partially contained events is from sub-GeV to multi-Gev. For neutrino-induced upward, throughgoing muons it extends to $\sim 1000$ GeV. Thus the atmospheric neutrinos have a range of pathlength over neutrino energy $1 \leq L_{km} / E_{GeV} < 10^5$.

The $\pi \rightarrow \mu \rightarrow e$ decay chain is the predominant mode of production of atmospheric neutrinos in the sub-GeV to multi-GeV range. This leads to the basic prediction of

$$\frac{\nu_e + \bar{\nu}_e}{\nu_\mu + \bar{\nu}_\mu} \sim \frac{1}{2}$$  \hspace{1cm} (1)

for $E_\nu \leq 1$ GeV. The ratio decreases as energy increases because muons are increasingly likely to reach the surface before decaying. Comparison of decay length to energy-loss length in the earth leads to the conclusion that virtually all muons that reach the ground stop before decay (or capture) occurs. Therefore muons that reach the ground do not contribute the neutrinos with energy high enough to contribute even to the sub-GeV sample ($p_e > 100$ MeV/c).

In contrast with the expectation of Eq. (1), several experiments find

$$R = \frac{(\mu - like/e - like)_{data}}{(\mu - like/e - like)_{MC}} \approx 0.65,$$

(2)

which is equivalent to $(\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu) \approx 0.77$. Ingredients that enter the denominator of the ra-
tio of ratios in Eq. 2 include the calculated neutrino flux, the cross sections for neutrinos to interact in the quasi-elastic and various multi-prong channels and the detection and reconstruction efficiencies in the detector. The subject of this talk is the neutrino fluxes.

Three independent calculations [9–11] have been compared and analyzed [12] in order to identify and evaluate the sources of uncertainty in our knowledge of the atmospheric neutrino flux. Here I concentrate on comparison of the calculations of Honda et al. [10,13] with the “Bartol fluxes” [14,15], because these two have been used by the experimental groups [4,8,16] for analysis of their data. The calculations of Refs. [14] and [15] are extensions of the calculations of Ref. [9] respectively to high (> 3 GeV) and to low (< 200 MeV) energy. In addition, a more realistic treatment of the geomagnetic cutoffs is used [17], which reduces the calculated neutrino fluxes in the sub-GeV range by about 10 per cent. Honda et al. [13] also extended their calculation to high energy. At present the Bartol neutrino fluxes and the Honda et al. fluxes in the GeV range agree within 5% in magnitude as well as ratio. This level of agreement in magnitude is, however, smaller than the systematic uncertainties, as I will discuss in Sections 3 and 4.

In sections 3 and 4 I discuss the primary fluxes and the treatment of hadronic interactions, both of which influence the spectrum and shape of the neutrino spectra. In the Super-K analysis, the overall normalization is treated as a free parameter because of the large uncertainty in the normalization of the primary spectrum. There are recent measurements of the primary spectrum that should in principle allow one to reduce this source of uncertainty.

As emphasized by Perkins [18], muon fluxes high in the atmosphere are directly related to the neutrino fluxes, being produced by the same primary spectra and from the same interaction processes. In §5 I discuss how measurements of the flux of muons high in the atmosphere are being used to check the overall normalization and shape of the closely related neutrino flux. In the conclusion I list the various approximations common to the present calculations and how they might be expected to affect the results.

2. Geomagnetic effects

Propagation of a cosmic-ray nucleus through the geomagnetic field depends only on its gyradius and hence on the magnetic rigidity,

$$R = \frac{A \times pc}{(Ze)}$$

(3)

Here A and Z are the mass and charge of a nucleus of momentum-per-nucleon p. Low energy particles at low geomagnetic latitudes cannot reach the atmosphere to produce secondaries. Since energy per nucleon is the important quantity for production of secondaries, nuclei become relatively more important compared to protons at low geomagnetic latitudes because of the factor $A/Z \approx 2$ in Eq. 3.

Both neutrino flux calculations [13,14] use geomagnetic cutoffs obtained by the standard method of backtracking antiprotons through the geomagnetic field to determine the cutoffs for a particular location. For example, in the calculation of Ref. [17], which is used in Ref. [14], antiparticles are injected at 20 km altitude on an outward trajectory. If the trajectory reaches $30R_\oplus$ before it travels $500R_\oplus$ and without intersecting the surface of the earth, then it is assumed that positive particles of the same rigidity can reach the atmosphere from that direction.

For the location of Super-K we have compared the cutoffs used in Ref. [13] with those of Ref. [17] used for the calculation of Refs. [14,15]. The cutoff maps are very similar, but with some noticeable differences toward the east, where the cutoffs are slightly higher in Ref. [17].

At low geomagnetic latitudes such as Kamioka, average cutoffs are higher locally (i.e. for cosmic rays entering the atmosphere above the detector) than for the opposite hemisphere (i.e. for the cosmic rays entering the atmosphere on the other side of the earth, which give rise to upward-going events). The opposite is the case for a detector at a high geomagnetic latitude, such as Soudan. There the local cutoffs are negligible in the sense that essentially all cosmic-rays from above with sufficient energy to produce pions and contribute to the flux of neutrinos can reach the
atmosphere to interact. Upward events originate from the atmosphere over the entire hemisphere below each detector. Since the average over a full hemisphere is similar from any viewpoint, the upward/downward ratio should be greater than one at Kamioka but less than one at Soudan.

Fig. 1 illustrates the situation. The pair of curves labelled (A) shows the distribution of primary cosmic-ray energies that would contribute to the sub-GeV signal in Super-Kamiokande if there were no geomagnetic cutoff at all. The solid curve is for solar minimum and the dotted one for solar maximum. The middle pair (B) is the corresponding response from below, which would be similar if Super-K were moved to Soudan. The rightmost pair of curves (C) is the response for downward sub-GeV events at Super-Kamiokande. What is plotted is proportional to the event rate per logarithmic interval of primary energy, so in each case the area is proportional to the signal. Thus the upward/downward ratio at Super-K is \( B/C > 1 \). If Super-K were located at Soudan, the ratio would instead be \( B/A < 1 \). (The method used to simulate “sub-GeV” events is described in Ref. [3].)

As neutrino energy increases the up-down asymmetry from the geomagnetic effect diminishes. For this reason, the Super-Kamiokande group have emphasized the multi-GeV event sample in their search for neutrino oscillations [1, 19]. On the other hand, the full data set contributes to the evidence for oscillations. Therefore it is important to note [3] that the geomagnetic effects themselves provide a way of testing the integrity of the entire analysis chain that is independent of whether or not there are oscillations.

At the low geomagnetic latitude of Kamioka there is a pronounced east-west effect on the cosmic radiation. Cutoffs are significantly lower for positive particles from the west than from the east. For example, the trajectory of a 20 GeV antiproton injected toward the east from above Super-K at 70° from the zenith would be bent down by the geomagnetic field and intersect the surface of the Earth, while the same antiproton injected toward the west would escape from the geomagnetic field. In other words, the cutoff for protons with zenith angle 70° from the east at Super-K is > 20 GeV. For directions closer to the horizon the cutoff from the east approaches 50 GV. In contrast, for directions above the horizon from the west the cutoff is 5 to 10 GV at Kamioka.

The excess of primary cosmic rays from the west at Kamioka produces a corresponding east-west asymmetry of the low-energy neutrino flux and hence of the sub-GeV event rate. There is a much smaller, but still non-negligible asymmetry for the multi-GeV event sample [3]. Since the east-west effect is an azimuthal asymmetry, it is independent of oscillations; oscillation effects depend on neutrino pathlengths, which vary with zenith angle but are independent of azimuth.

Figure 2 [20] compares the azimuthal dependence of the Super-K data (0.4 < \( p_{\text{lepton}} < 2.0 \) GeV/c, single ring events in 22.5 kton fiducial volume) with expectation. The solid line uses the neutrino fluxes of Ref. [13] and the dashed line the calculation of Ref. [14, 15]. Although the fits are equally good (\( \chi^2 / \text{d.o.f.} \approx 1 \) for all four comparisons), the geomagnetic effect is somewhat more pronounced with the Bartol neutrino flux [14, 15] than with the flux of Honda et al. [3].
We have made some diagnostic tests to investigate the source of this difference and a similar difference between the two calculations that shows up in the zenith angle dependence of sub-GeV events. The difference arises in part from the difference in cutoffs mentioned above, but also from the difference in primary spectrum, as discussed below.

3. Primary spectrum

Both the normalization and the shape of the assumed primary spectrum have important consequences for the calculation of the neutrino fluxes. The normalization propagates directly through to the event rate. The assumed spectral index affects the shape of the neutrino energy spectrum in an obvious way, but it also affects the angular dependence through its interaction with the geomagnetic effects. Thus, a softer spectrum will lead to more pronounced geomagnetic effects because a larger fraction of the event rate comes from lower energy primaries, which are most affected by the geomagnetic field.

Fig. 3 is a summary of measurements of spectra of protons, helium and the CNO group of nuclei, compared with the primary spectra of Honda et al. [13] (solid lines) and the spectra used in Ref. [14] (dashed lines). From Fig. 1, it is apparent that $5 < E < 50$ GeV/nucleon is the most important region of the primary spectrum for sub-GeV events. The harder spectrum of Ref. [13] in this energy region, coupled with the geomagnetic field, contributes significantly to the fact that the geomagnetic effects are somewhat smaller in the neutrino spectra of Ref. [13] than in Ref. [14], as mentioned in the previous section.

A marked feature of the plot for hydrogen is the fact that the data of Webber [21] (shown by the open circles in Fig. 3) are significantly higher than those from the LEAP experiment [22] and other more recent experiments in the same energy region. The difference is outside the error bars, indicating a systematic effect. The most recent
result is from the BESS detector [23], and other recent experiments (Refs. [24,25]) are included in the BESS compilation. Generally (with the possible exception of the measurement of Ref. [26]), the interpretation of which is complicated by an unusually strong level of solar modulation) all the recent experiments are consistent with the LEAP results. In fits to their data the Super-K group have treated the overall normalization of their rates as a free parameter. The primary spectra and their potential consequences for interpretation of the data on neutrino interactions are discussed more fully in talks given at the Satellite Symposium [23,27,28].

4. Yields

It is important to note that the primary spectrum is not the only source of uncertainty in the normalization and shape of the energy spectrum of atmospheric neutrinos. Uncertainties in the yields of pions and kaons in interactions of hadrons with nuclei of the atmosphere are also important. Not all of phase space is covered in accelerator measurements with nuclear targets. For sub-GeV events the important range of beam energies is from a few GeV to several tens of GeV (see Fig. 1). In this energy range the atmospheric cascades are dominated by interactions of nucleons, and nearly all neutrinos are from the $\pi \rightarrow \mu \rightarrow e$ decay chain.

Existing measurements with beam energies around 20 GeV and light nuclear targets measure pions only above 3 or 4 GeV [29,30], and there are significant differences in how the lower energy pions are represented in the different neutrino flux calculations, as discussed in Ref. [12]. The pion multiplicities, and the momentum distributions as reflected by the spectrum-weighted moments for pion production, are highest in the calculation of [9,14,15]. This compensates to some extent for the higher assumed proton spectrum of Ref. [13] with the result that the calculated neutrino fluxes (comparing Refs. [14] and [13]) differ by less than either the primary spectrum or the yields.

Yields in a new calculation of Battistoni et al. [31] are intermediate between those of Refs. [1] and [10].

5. Muons

The same primary spectra and the same hadronic interactions determine both muon and neutrino fluxes. Therefore, comparison with measurements of muons high in the atmosphere offers a way to check directly the neutrino fluxes. The most important range of altitudes for pion decay is 10 to 25 kilometers, which corresponds to atmospheric depths of $\sim 20$ to $\sim 200$ g/cm$^2$.

Many of the same detectors referred to above in connection with recent measurements of the primary spectrum have also been used to measure the muon spectrum during ascent through the atmosphere and on the ground. The calculations of Refs. [14] (and [13]) compare reasonably well with the measurements of the MASS experiment [32], although there is a relative excess of muons below 1 GeV in the calculation. On the other hand, a recent comparison between [14] and the HEAT measurements of muons [33] showed better agreement in the shape of the spectrum but with an overall excess of the calculation relative to the data of as much as 50% in some bins.

Measurements on the ground and at float altitude necessarily have better statistics than data obtained during ascent. It is possible that some of the discrepancies referred to above could be a consequence of the short exposures during ascent. Both for MASS [12] and HEAT [13] there is a tendency for better agreement between calculation and measurement at float and at the ground than during ascent. This is an active area with further potential for reducing uncertainties in the flux of atmospheric neutrinos.

There are interesting possibilities with the muon measurements for probing details of the calculations. For one thing, muon fluxes at float altitude reflect directly the primary spectrum and the properties of pion production in single nucleon-nitrogen interactions with no intervening cascading.

A more interesting possibility arises from the fact that in some cases the same detector has been exposed at different locations with different geomagnetic cutoffs. The MASS experiment has been flown both in Northern Canada (essentially no cutoff) and from Ft. Sumner, NM where
the vertical cutoff is \( \approx 5 \) GV. The BESS detector has measured the muon charge ratio on the ground in northern Canada and in Japan. The low-energy behavior of the ratio is quite different in the two locations. Below \( \sim 1 \) GeV the \( \mu^+/\mu^- \) ratio decreases toward 1 in Japan, which can be understood as a consequence of the high local geomagnetic cutoff. There are two effects: First, with a high cutoff, heavy primaries are relatively more important because they have a higher rigidity for a given energy per nucleon. Protons produce more positive than negative pions (and hence more \( \mu^+ \)) and vice versa for neutrons. Thus, enhancing the contribution from nuclei, which carry the neutrons, suppresses the muon charge ratio slightly. Secondly, vertical \( \mu^+ \) at the ground have followed trajectories from slightly east of vertical (where the cutoffs are higher), whereas vertical \( \mu^- \) will have come slightly from the west where more of the primary spectrum reaches the atmosphere to produce secondaries. This enhances the negative relative to the positive muons and hence reduces the \( \mu^+/\mu^- \) ratio preferentially at low energy where the bending is more significant.

6. Conclusion

Present calculations \cite{13,14} include several approximations:

- They are one-dimensional; i.e. all neutrinos are assumed to follow the direction of the primary nucleon that produced them. This approximation has two effects:

  1. There should be some loss of particles that are produced at large angle. Given the momentum involved, as compared with the typical transverse momentum of produced pions, it is straightforward to check that this effect should be small for neutrino events in Super-K.

  2. Bending of charged particles in the atmosphere is not followed. This is perhaps the most important effect to check \cite{34} because it is systematic. As explained above, the vertical muon charge ratio is reduced when the cutoffs are high. There is a corresponding decrease in the \( \nu_e/\bar{\nu}_e \) ratio (and an increase in that part of the \( \nu_\mu/\bar{\nu}_\mu \) ratio that comes from muon decay).

- The superposition model has been used in Ref. \cite{14} for interactions of nuclei. Within the framework of a standard multiple scattering picture, this approximation can be shown to give a good account of the distribution of first interactions of each nucleon. It will, however, lead to some overestimate of the multiplicity of pions in the target fragmentation region.

- The cascades are propagated to sea-level all over the globe. In particular, the exact terrain over the detector (i.e. the mountain in the case of Super-K) has been neglected \cite{35}. This is negligible for muon neutrinos from pions, which decay high in the atmosphere. From the pathlength distributions of Ref. \cite{8}, it is possible to estimate the size of the effect of this approximation. To take an extreme case of a 4 km overburden (\(~\)NUSEX), the neutrinos from muon decay overhead are overestimated by about 10\%, leading to a \(~5\%\) overestimate of the calculated \( (\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu) \) ratio.

- The calculations are based on parametrizations of data in limited regions of phase space. Interpolations and extrapolations introduce some level of uncertainty. The yields of Ref. \cite {14} are at the high end of a spectrum, with \cite{11} the lowest and \cite{13} and \cite{31} in between.

At least two groups are embarking on three-dimensional calculations. The Italian group \cite{31} has published a short account of their plan with a
comparison of their one-dimensional results with those shown in Ref. [2]. They use FLUKA [36] with various hadronic interaction models at different energies. The authors of Refs. [9] and [14], together with Coutu, are also pursuing this goal. Although effects are generally expected to be small, in view of the importance of the experimental results, a greater level of detail in the calculations is warranted.

Acknowledgements. I am grateful to Ed Kearns for providing me with Fig. 2 [20] and to Todor Stanev for reading the manuscript and for collaboration on this work. I thank M. Goldhaber and M. Spiro for useful conversations, and M. Honda for very helpful exchanges of information about the calculations of Refs. [10,13].

REFERENCES

1. Y. Fukuda et al., Phys. Rev. Letters 81 (1998) 1562.
2. D. Ayres et al., Phys. Rev. D29 (1984) 902.
3. Paolo Lipari, Todor Stanev & T.K. Gaisser, Phys. Rev. D (to be published) astro-ph/9803093.
4. Y. Fukuda et al., Phys. Lett. B433 (1998) 9 (sub-GeV) and hep-ex/9805006 (to be published) (multi-GeV).
5. T.K. Gaisser & Todor Stanev, Phys. Rev. D57 (1998) 1977.
6. R. Becker-Szendy et al. (IMB Collaboration), Phys. Rev. D46 (1992) 3720 and references therein.
7. Y. Fukuda al. (Kamiokande Collaboration) Phys. Lett. B335 (1994) 237 and references therein.
8. W.W.M. Allison et al. (Soudan Collaboration), Phys. Lett. B391 (1997) 491.
9. Giles Barr, T.K. Gaisser & Todor Stanev, Phys. Rev. D39 (1989) 3532.
10. M. Honda, K. Kasahara, K. Hidaka & S. Midorikawa, Phys. Lett. B248 (1990) 193.
11. E.V. Bugaev & V.A. Naumov, Phys. Lett. B232 (1989) 391.
12. T.K. Gaisser, M. Honda, K. Kasahara, H. Lee, S. Midorikawa, V. Naumov & Todor Stanev, Phys. Rev. D54 (1996) 5578.
13. M. Honda, T. Kajita, K. Kasahara & S. Midorikawa, Phys. Rev. D52 (1995) 4985.
14. Vivek Agrawal, T.K. Gaisser, Paolo Lipari & Todor Stanev, Phys. Rev. D53 (1996) 1314.
15. T.K. Gaisser & Todor Stanev, Proc. 24th Int. Cosmic Ray Conf. (Rome) vol. 1 (1995) 694.
16. M. Ambrosio et al. (MACRO Collaboration), hep-ex/9807005 (throughgoing muons) and M. Spurio, hep-ex/9808001 (stopping muons).
17. Paolo Lipari & Todor Stanev, Proc. 24th Int. Cosmic Ray Conf. (Rome) vol. 1 (1995) 516.
18. D.H. Perkins, Astroparticle Physics 2 (1994) 249.
19. T. Kajita, to appear in Proc. of Neutrino98.
20. C. McGrew (Super-Kamiokande Collaboration), to appear in Proc. Int. Conf. on High Energy Physics (Vancouver, 1998) and Super-K paper on azimuthal dependence (forthcoming).
21. W.R. Webber, R.L. Golden & S.A. Stephens, Proc. 20th Int. Cosmic Ray Conf. (Moscow) vol. 1 (1987) 325.
22. E.S. Seo et al. (LEAP) Ap.J. 378 (1991) 763.
23. S. Orito, T. Sanuki et al. (BESS), to appear in proceedings of the conference "New Era in Neutrino Physics", Tokyo, June 11/12 (1998).
24. W.R. Menn et al. (IMAX) Proc. 25th Int. Cosmic Ray Conf. (Durban) vol. 3 (1997) 409.
25. M. Boezio et al. (CAPRICE), Ap. J. (to be published).
26. W.R. Webber et al. (MASS), Ap.J. 380 (1991) 230.
27. M. Honda, to appear in proceedings of the conference "New Era in Neutrino Physics", Tokyo, June 11/12 (1998).
28. T.K. Gaisser, to appear in proceedings of the conference "New Era in Neutrino Physics", Tokyo, June 11/12 (1998).
29. T. Eichten et al., Nucl. Phys. B44 (1972) 333.
30. J.V. Allaby et al., CERN Yellow Report No. 70-12 (unpublished).
31. G. Battiston et al., Proc. 5th TAUP Conf., Gran Sasso (1997).
32. M. Circella, C.N. De Marzo, T.K. Gaisser & Todor Stanev, Proc. 25th Int. Cosmic Ray Conf. (Durban), vol. 7 (1997) 117.
33. Stephane Coutu (HEAT Collaboration) to
appear in Proc. Int. Conf. on High Energy Physics (Vancouver, 1998).
34. Maurice Goldhaber has emphasized this point (private communication).
35. I thank M. Spiro for raising this question.
36. A. Fassò, A. Ferrari, J. Ranft and P.R. Sala, Proc. of the 3rd Workshop on Simulating Accelerator Radiation Environment, SARE-3, KEK-Tsukuba, May 7-9, 1997, (H. Hirayama, ed.) KEK Report 97-5, p. 32 (1997).