Non-Universal Correction To $Z \to b\bar{b}$
And Flavor Changing Neutral Current Couplings

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ABSTRACT

A non-universal interaction, which involves only the heavy quarks ($t_L, b_L$) and $t_R$, modifies the neutral current couplings and induces flavor changing neutral currents (FCNC). The size of the FCNC effect depends crucially on the dynamics of the fermion mass generation. In this paper, we study the effect of the non-universal interaction on $Zb\bar{b}$, $Zb\bar{s}$, $Zd\bar{s}$ and $Zd\bar{b}$, by using an effective lagrangian technique and assuming the quark mass matrices in the form of a generalized Fritzsch ansatz. We point out that if fitting $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{Hadrons})$ to the LEP data within $1\sigma$, the induced FCNC couplings are very close to the allowed bounds of several rare decays.
Recently the CDF collaboration[1] at FNAL presented evidence for a top quark with a mass $m_t \sim 175$ GeV. Since $m_t$ is of the order of Fermi scale, the top quark couples strongly to the electroweak symmetry breaking sector and will play a key role in probing new physics beyond the standard model. This kind of new physics (i.e. non-universal interaction since it acts on only the top quark) can become manifest in top quark production processes at the hadron and next generation linear colliders. It can also affect the partial width of $Z \rightarrow b\bar{b}$ measured at LEP because the $SU(2)_L$ group places $(t_L, b_L)$ into a common doublet. The experimental observed value for the ratio $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{Hadrons})$ is slightly higher than the standard model expectation. This may be an indication of the non-universal interaction, if it is more than a statistical fluctuation.

It is known that a non-universal interaction will induce flavor changing neutral currents (FCNC) among the light fermions[2-5]. However, the size of the FCNC effect depends crucially on the quark mass mixing matrices. So one can not predict quantitatively the induced FCNC effect without specifying the mass matrices. At present it seems far too early to attempt an actual solution to the issue of mass generation. However, there has been a great amount of activities in looking for the relation between fermion masses and their mixing matrix elements, as commonly referred to as texture studies. One expects that a “successful” ansatz can provide clues to the dynamics of the fermion mass generation.

In recent years, most studies on the implication of fermion mass ansatz were focused on grand unification theories with and without supersymmetry. In this paper we take a phenomenological, model independent approach to new physics beyond the standard model, i.e., the effective lagrangian technique, and consider the implication of the fermion mass ansatz on the induced FCNC effect. Specifically, we will use one variation of the Fritzsch[6] ansatz to study the correlated
effects of new physics on $Zb\bar{b}$ and $Zb\bar{s}$, etc. We will point out that when fitting $R_b$ to the LEP data within 1σ, the induced FCNC couplings are very close to the allowed bounds of several rare decays. Our results show that the new physics associated with top quark may be revealed by the presence of FCNC processes.

We first discuss $Z \rightarrow b\bar{b}$. Following the general approach, we assume that anomalous, non-universal interaction is $SU(2)_L \times U(1)_Y$ invariant. Hence the $b$ quark will participate in any $t$ quark interactions when the left-handed doublet is involved. This can result in a modification of the $Zb\bar{b}$ vertex. We can parametrize the modification by introducing a parameter $\kappa_j$, which shifts the standard model tree level coupling, $g_j$, to effective coupling $g_j^{eff}$:

$$g_j^{eff} = g_j(1 + \kappa_j) , \quad (1.a)$$

where $j = L(R)$ denotes the left-(right) hand, and $g_j$ are the standard model coupling strengths of the neutral current,

$$g_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W ; \quad g_R = \frac{1}{3} \sin^2 \theta_W . \quad (1.b)$$

The contributions of the new physics to the $Z \rightarrow b\bar{b}$ width are proportional to $g_L^2$ and $g_R^2$. Since $g_L^2 \gg g_R^2$, we will neglect the modification to the right-handed interaction in this article. Defining $\delta \Gamma$ to be the purely non-universal correction of the new physics beyond the standard model to the $Z \rightarrow b\bar{b}$ width, $\Gamma_{b\bar{b}}$, we have

$$\frac{\delta \Gamma}{\Gamma_{b\bar{b}}} \simeq 2 \frac{g_L^2 \kappa_L}{g_L^2 + g_R^2} \simeq 2 \kappa_L . \quad (2)$$

Then the $R_b$ becomes

$$R_b \sim R_{b}^{SM} \left(1 + \frac{\delta \Gamma}{\Gamma_{b\bar{b}}}\right) \sim R_{b}^{SM} (1 + 2 \kappa_L) , \quad (3)$$
where the standard model value $R_b^{SM} = 0.2157$ for $m_t = 175$ GeV and $m_H = 300$ GeV. The experimental value of $R_b$ measured at LEP is $R_b = 0.2192 \pm 0.0018$[7], which is roughly within $2\sigma$ of the standard model expectation. A positive $\kappa_L$ would improve the situation.

In general, $\kappa_L$ can be viewed as functions of $q^2$[8], where $q$ is the 4-momentum of the Z-boson, and at LEP, $q^2 = m_Z^2$. Expanding $\kappa_L$ in terms of $q^2$, we have

$$\kappa_L = \kappa^0_L + q^2 - \text{dependent terms} \quad (4)$$

Gauge invariant operators describing $\kappa_L$ have been constructed explicitly in effective lagrangian with a non-linear[2] realization of $SU(2)_L \times U(1)_Y$. In this paper we use an effective lagrangian with a linear realization[9] of $SU(2)_L \times U(1)_Y$ for the discussion. The new physics effects are parametrized by a set of higher dimension operators $\mathcal{O}_i$, which are required to be invariant under the standard model gauge symmetry and contain only the standard model fields. The new physics effects on the light fermions are assumed to be negligible, so the higher dimension operators involve only $(t_L, b_L), \ t_R$, the gauge and scalar bosons. For dimension 6, there are two operators which generate directly [F.1] a $\kappa^0_L$ in eq.(4)[10],

$$\mathcal{O}^1 = i \left[ \phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi \right] \Psi_L^T \gamma^\mu \Psi_L ; \quad (5.a)$$

$$\mathcal{O}^2 = i \left[ \phi^\dagger \tau D_\mu \phi - (D_\mu \phi)^\dagger \tau \phi \right] \Psi_L^T \gamma^\mu \tau \Psi_L , \quad (5.b)$$

where $\phi$ is the doublet Higgs field of the standard model and $\Psi_L^T = (t, b)_L$. Let us introduce the effective lagrangian, $\mathcal{L}^{eff}$, containing higher dimension operators given in eqs.(5):

[F.1] We are not considering the operators which can affect $Zb\bar{t}$ indirectly by loop effects.
\[ \mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{SM}} + \frac{1}{\Lambda^2} (c_1 \mathcal{O}^1 + c_2 \mathcal{O}^2) , \]  

(6)

where \( c_i, \ i = 1, 2, \) are real parameters, which determine the strength of the contributions of the operators, \( \mathcal{L}^{\text{SM}} \) is the standard model lagrangian, \( \Lambda \) is the cutoff of the effective theory.

After the electroweak symmetry breaking, the anomalous couplings for \( Zb\bar{b} \) and \( Zb\bar{s} \), etc., from \( \mathcal{L}^{\text{eff}} \) are contained in

\[ \frac{g}{\cos \theta_W} \left( \frac{\bar{d}}{\bar{s}} \right)_T U_L^{(d)} \left( \begin{array}{cc} 0 & 0 \\ 0 & \delta_L \end{array} \right) U_L^{(d)} \left( \begin{array}{c} d \\ s \\ b \end{array} \right)_L Z^\mu , \]  

(7)

where

\[ \delta_L = \frac{v^2}{\Lambda^2} (c_1 + c_2) ; \]  

(8)

and \( v \simeq 250 \) GeV, \( U_L^{(d)} \) is unitary rotation matrix on the left-handed down quarks. The Cabibbo-Kabayashi-Maskawa (CKM) mixing matrix for the charged weak current is

\[ V = (U_L^{(u)})^\dagger U_L^{(d)} , \]  

(9)

where \( U_L^{(u)} \) is the unitary rotation matrix for the left-handed up quarks. Note that in the standard model, which corresponds to \( \mathcal{L}^{\text{eff}} \) in the limit \( \Lambda \to \infty \), the individual \( U_L^{(u)} \) and \( U_L^{(d)} \) are not measureable, but only \( V \) in eq.(9) is. Furthermore, the universality of the weak interaction in the standard model also guarantees the vanishes of the FCNC at tree level.

The relative size of the \( Zb\bar{b} \) to the FCNC couplings, \( Zb\bar{s}, \) etc., in eq.(7) depends on the rotation matrix \( U_L^{(d)} \). The elements of \( U_L^{(d)} \) can be evaluated once
the corresponding mass matrix is given. In the literature a widely used ansatz is the one suggested by Fritzsch[6] and its variations. The latter is given by

$$M(q) = \begin{pmatrix} 0 & x_q e^{i\alpha_q} & 0 \\ x_q e^{-i\alpha_q} & \omega_q & y_q e^{i\beta_q} \\ 0 & y_q e^{-i\beta_q} & z_q \end{pmatrix},$$

(10)

where $x_q$, $y_q$, $\omega_q$ and $z_q$ are real parameters and $q = u$ (d) denotes the up (down) type quarks. The original Fritzsch ansatz is given by putting $\omega_q = 0$, which predicts a too small top quark mass $m_t \leq 90$ GeV[11]. Here we consider one variation[12] which can have an acceptable top quark mass $m_t \leq 190$ GeV, and fits the current experimental data on the CKM matrix. In the variation[12], the rotation matrix, $U^{(q)} (= U^{(q)}_L = U^{(q)}_R)$ is given by

$$
\begin{pmatrix}
1 & -\left(\frac{m_1 m_2 (m_2 + w_q)}{m_2 m_3}\right)^{1/2} e^{-i(\alpha_q + \beta_q)} & -\left(\frac{m_1 m_2 (m_2 + w_q)}{m_2 m_3}\right)^{1/2} e^{i(\alpha_q + \beta_q)} \\
\left(\frac{m_1}{m_2}\right)^{1/2} e^{-i\alpha_q} & \left(\frac{m_1 m_2 (m_2 + w_q)}{m_2 m_3}\right)^{1/2} e^{-i\alpha_q} & \left(\frac{m_2 + w_q}{m_3}\right)^{1/2} e^{-i\alpha_q} \\
-\left(\frac{m_1 (m_2 + w_q)}{m_2 m_3}\right)^{1/2} e^{-i(\alpha_q + \beta_q)} & -\left(\frac{m_2 + w_q}{m_3}\right)^{1/2} e^{-i(\alpha_q + \beta_q)} & \left(\frac{m_2 + w_q}{m_3}\right)^{1/2} e^{-i(\alpha_q + \beta_q)}
\end{pmatrix},
$$

(11)

where $m_1$, $m_2$, and $m_3$ correspond to $m_u$, $m_c$ and $m_t$ for $q = u$, and $m_d$, $m_s$ and $m_b$ for $q = d$, and $w_u = m_c$, $w_d = 0$, $\alpha_q$ and $\beta_q$ are responsible for the CP violation phase in the CKM matrix.

In table I, we give the theoretical values of FCNC couplings and the corresponding experimental upper limits. One can see that if fitting $R_b$ to LEP data within $1\sigma$, the induced FCNC couplings are close to the allowed bounds of several rare decays[F.2]. For example, assuming a positive $\kappa_L$ and fitting $R_b$ to the ex-

[F.2] We realize that there are uncertainties in the numerical values of the rotation
perimental data within $1\sigma$, we have $|\tilde{\kappa}^{ds}_{L}| \geq (1.2 \sim 2.6) \times 10^{-5}$, which lies in the experimental limit of $K_L \to \pi \mu$.

In our calculations we have not considered the $q^2$ dependent terms in eq.(4), which are generally proportional to $m^2/\Lambda^2$ where $m$ is a typical mass of a process under consideration. The operators in eqs.(5) give rise to terms proportional to $v^2/\Lambda^2$. Therefore the momentum dependent terms are generally suppressed at low energies. We should point out that if a different ansatz from that in (11) is taken, the relative size of anomalous $Zb\bar{b}$ to $Zb\bar{s}$, etc., may be changed. Thus the future data on $Zb\bar{b}$ and $Zb\bar{s}$, etc., will provide an experimental test on various fermion mass ansatz.

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matrix elements caused by the uncertainties in the values of fermion masses, CKM mixing angles and the analytical approximation used in ref.[12].
Table Caption

Table I: Theoretical prediction on FCNC couplings, and corresponding experimental upper limits taken from Ref.[13]. $\tilde{\kappa}_L = U_L^{(d)} \text{diag}[0, 0, \delta_L] U_L^{(d)}$. The elements of $U_L^{(d)}$ are calculated by taking the central values of the down quark masses evaluated at $\mu = 1$ GeV, $m_s/m_b = 0.033$, $m_d/m_s = 0.051$. For $Z \to b\bar{b}$, $\tilde{\kappa}_L^{bb} = \delta_L$, and using definition of $\kappa_L$ in eq.(1.a) we have $\delta_L = g_L \kappa_L$, so $\frac{R_b - R_b^{SM}}{R_b^{SM}} = \frac{2\delta_L}{g_L}$.

| $|\tilde{\kappa}_L^{ij}|$ | Predictions | Limits and Processes |
|------------------------|-------------|----------------------|
| $|\tilde{\kappa}_L^{ds}|$ | $7.5 \times 10^{-3} \times \delta_L$ | $3 \times 10^{-4}$ ($K^0 - \bar{K}^0$ mixing) |
| $|\tilde{\kappa}_L^{ds}|$ | $7.5 \times 10^{-3} \times \delta_L$ | $2 \times 10^{-5}$ ($K_L \to \pi\mu\nu$) |
| $|\tilde{\kappa}_L^{db}|$ | $0.041 \times \delta_L$ | $4 \times 10^{-4}$ ($B_d - \bar{B}_d$ mixing) |
| $|\tilde{\kappa}_L^{bs}|$ | $0.182 \times \delta_L$ | $2 \times 10^{-3}$ ($B \not\to l^+l^-X$) |

Table I.
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