Efficient Network-aware Search in Online Social Bookmarking Applications

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Abstract

We consider in this paper top-k query answering in social tagging (or bookmarking) applications. This problem requires a significant departure from existing, socially agnostic techniques. In a network-aware context, one can (and should) exploit the social links, which can indicate how users relate to the seeker and how much weight their tagging actions should have in the result build-up. We propose an algorithm that has the potential to scale to current applications. While the problem has already been considered in previous literature, this was done either under strong simplifying assumptions or under choices that cannot scale to even moderate-size real-world applications. We first revisit a key aspect of the problem, which is accessing the closest or most relevant users for a given seeker. We describe how this can be done on the fly (without any pre-computations) for several possible choices - arguably the most natural ones - of proximity computation in a user network. Based on this, our top-k algorithm is sound and complete, while addressing the applicability issues of the existing ones. Moreover, it performs significantly better and, importantly, it is instance optimal in the case when the search relies exclusively on the social weight of tagging actions. To further reduce response times, we then consider directions for efficiency by approximation. Extensive experiments on real world data show that our techniques can drastically improve the response time, without sacrificing precision.

1. Introduction

Unprecedented volumes of data are now at everyone’s fingertips on the World Wide Web. The ability to query them efficiently and effectively, by fast retrieval and ranking
algorithms, has largely contributed to the rapid growth of the Web, making it simply irreplaceable in our everyday life.

A new dynamics to this development has been recently brought by the social Web, applications that are centered around users, their relationships and their data. Indeed, user-generated content is becoming a significant and highly qualitative portion of the Web. To illustrate, the most visited Web site today is a social one. This calls for adapted, efficient retrieval techniques, which can go beyond a classic Web search paradigm where data is decoupled from the users querying it.

An important class of social applications are the collaborative tagging applications, also known as social bookmarking applications, with popular examples including Del.icio.us, StumbleUpon or Flickr. Their general setting is the following:

- users form a social network, which may reflect proximity, similarity, friendship, closeness, etc,
- items from a public pool of items (e.g., document, URLs, photos, etc) are tagged by users with keywords, for purposes such as description and classification, or to facilitate later retrieval,
- users search for items having certain keywords (i.e., tags) or they are recommended items, e.g., based on proximity at the level of tags.

Collaborative tagging, and social applications in general, can offer an entirely new perspective to how one searches and accesses information. The main reason for this is that users can (and often do) play a role at both ends of the information flow, as producers and also as seekers of information. Consequently, finding the most relevant items that are tagged by some keywords should be done in a network-aware manner. In particular, items that are tagged by users who are “closer” to the seeker – where the term closer depends on model assumptions that will be clarified shortly – should be given more weight than items that are tagged by more distant users.

We consider in this paper the problem of top-$k$ retrieval in collaborative tagging systems. We investigate it with a focus on efficiency, targeting techniques that have the potential to scale to current applications on the Web, in an online context where the social network, the tagging data and even the seekers’ search ingredients can change at any moment. In this context, a key sub-problem for top-$k$ retrieval that we need to address is computing scores of top-$k$ candidates by iterating not only through the most relevant items with respect to the query, but also (or mostly) by looking at the closest users and their tagged items.

We associate with the notion of social network a rather general interpretation, as a user graph whose edges are labeled by social scores, which give a measure of the proximity or similarity between two users. These are then exploitable in searches, as they say how much weight one’s tagging actions should have in the result build-up. For example,

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1The most popular ones have user bases of the order of millions and huge repositories of data; today’s most accessed social Web application, which also provides tagging and searching functionalities, has more than half a billion registered users.
Figure 1: A collaborative tagging scenario and its social network.

even for tagging applications where an explicit social network does not exist or is not exploitable, one may use the tagging history to build a network based on similarity in tagging and items of interest. While we focus mainly on bookmarking applications, we believe that these represent a good abstraction for other types of social applications, to which our techniques could directly apply.

Example 1. Consider the collaborative tagging configuration of Figure 1. Users have associated lists of tagged documents and they are interconnected by social links. Each link is labeled by its (social) score, assumed to be in the $[0, 1]$ interval. Let us consider user Alice in the role of the seeker. The user graph is not complete, as the figure shows, and only two users have an explicit social score with respect to Alice. For the remaining ones, Danny, . . . , Jim, only an implicit social score could be computed from the existing links if a precise measure of their relevance with respect to Alice’s queries is necessary in the top-k retrieval.

Let us assume that Alice looks for the top two documents that are tagged with both news and site. Looking at Alice’s immediate neighbors and their respective documents, intuitively, $D_3$ should have a higher score than $D_4$, since the former is tagged by a more relevant user (Bob, having the maximal social score relative to Alice). If we expand the search to the entire graph, the score of $D_4$ may however benefit from the fact that other users, such as Eve or even Holly, also tagged it with news or site. Furthermore, documents such as $D_2$ and $D_1$ may also be relevant for the top-2 result, even though they were tagged only by users who are indirectly linked to Alice.

Under certain assumptions to be clarified shortly, the top-2 documents for Alice’s query will be, in descending score order, $D_4$ and $D_2$. The rest of the paper will present the underlying model and algorithms that allow us to build this answer.
Main related work. Classic top-$k$ retrieval algorithms, such as Fagin’s threshold algorithm [12] and the no random access (NRA) algorithm, rely on precomputed inverted-index lists with exact scores for each query term (in our setting, a term is a tag). Revisiting the setting in Figure 1 we would have two per-tag inverted lists $IL(\text{news}) = \{D4 : 7, D2 : 2, D1 : 2, D3 : 1, D6 : 1, D5 : 1\}$ and $IL(\text{site}) = \{D2 : 5, D4 : 2, D3 : 1, D6 : 1, D1 : 1, D5 : 1\}$, which give the number of times a document has been tagged with the given tag.

When user proximity is an additional ingredient in the top-$k$ retrieval process, a direct network-aware adaptation of the threshold algorithm and variants would need precomputed inverted-index lists for each user-tag pair. For instance, if we interpret explicit links in the user graph as friendship, ignoring the link scores, and only tagging by direct friends matters, Alice’s lists would be $IL_{\text{Alice}}(\text{news}) = \{D4 : 1, D6 : 1\}$ and $IL_{\text{Alice}}(\text{site}) = \{D3 : 1, D6 : 1\}$. Other 18 such lists would be required and, clearly, this would have prohibitive space and computing costs in a real-world setting. Amer-Yahia et al. [1] is the first to address this issue, considering the problem of network-aware search in collaborative tagging sites, though by a simplified flavor. The authors consider an extension to classic top-$k$ retrieval in which user proximity is seen as a binary function (0-1 proximity): only a subset of the users in the network are selected and can influence the top-$k$ result. This introduces two strong simplifying restrictions: (i) only documents tagged by the selected users should be relevant in the search, and (ii) all the users thus selected are equally important. The base solution of [1] is to keep for each tag-item pair, instead of the detailed lists per user-tag pair, only an upper-bound value on the number of taggers. For instance, the upper-bound for $(\text{news}, D4)$ would be 2, since for any user there are at most two neighbors who tagged $D4$ with $\text{news}$. This is called the GLOBAL UPPER-BOUND strategy. A more refined version, which trades space for efficiency, keeps such upper-bound values within clusters of users, instead of the network as a whole.

Only in Schenkel et al. [18], the network-aware retrieval problem for collaborative tagging is considered under a general interpretation, the one we also adopt in this paper. It considers that even users who are only indirectly connected to the seeker can be relevant for the top-$k$ result. Their CONTEXTMERGE algorithm follows the intuition that the users closest to the seeker will contribute more to the score of an item, thus maximizing the chance that the item will remain in the final top-$k$. The authors describe a hybrid approach in which, at each step, the algorithm chooses either to look at the documents tagged by the closest unseen user or at the tag-document inverted lists (a seeker agnostic choice). In order to obtain the next (unseen) closest user at any given step, the algorithm precomputes in advance the proximity value for all possible pairs of users. These values are then stored in ranked lists (one list per user), and a simple pointer increment allows to obtain the next relevant user.

Example 2. Consider the network of Fig. 1. With respect to seeker Alice, the list of users ranked by proximity would be $\{Bob : 0.9, Danny : 0.81, Charlie : 0.6, Frank : 0.4, Eve : 0.3, George : 0.2, Holly : 0.1, Ida : 0.1, Jim : 0.05\}$, with proximity between two users built as the maximal product of scores over paths linking them (formalized in Section 2.1).
The main drawbacks of [18] are scalability and applicability. Clearly, precomputing a weighted transitive closure over the entire network has a high cost in terms of space and computation in even moderate-size social networks. More importantly, keeping these proximity lists up to date when they reflect tagging similarity\(^2\) (as advocated in [18]), would simply be unfeasible in real-world settings, which are highly dynamic. (We revisit these considerations in Section 6.)

**Main contributions.** We propose an algorithm for top-\(k\) answering in collaborative tagging, which has the potential to scale to current applications and beyond, in an online context where network changes and tagging actions are frequent. For this algorithm, we first address a key aspect: accessing efficiently the closest users for a given seeker. We describe how this can be done on the fly (without any pre-computations) for a large family of functions for proximity computation in a social network, including the most natural ones (and the one assumed in [18]). The interest in doing this is threefold:

- we can support full scoring personalization, where each user issuing queries can define her own way to rank items, through parameters and score function choices,
- we can iterate over the relevant users in more efficient manner, since a typical network can easily fit in main-memory; this can spare the potentially huge disk volumes required by [18]'s algorithm (see Section 6), while also having the potential to run faster.
- social link updates are no longer an issue; in particular, when the social network depends on the tagging history, we can keep it up-to-date and, by it, all the proximity values at any given moment, with little overhead.

Based on this, our top-\(k\) algorithm TOPKS is sound and complete. We show that, when the search relies exclusively on the social weight of tagging actions, it is instance optimal in a large and important class of algorithms. Extensive experiments on real world data show that our algorithm performs significantly better than existing techniques, with up to 50% improvement (see Section 7).

For further efficiency, we then consider directions for approximate results. Our approaches present the advantages of negligible memory consumption (they rely on concise statistics about the user network) and reduced computation overhead. Moreover, these statistics can be maintained up to date with limited effort, even when the social network is built based on tagging history. Experiments show that approximate search techniques can drastically improve the response time, reaching around 25% of the running time of the exact approach, without sacrificing precision.

The main focus of our work is on the social aspects of top-\(k\) retrieval in collaborative tagging applications, and our techniques are designed to perform best in settings where tagging actions are mostly (if not exclusively) viewed through the lens of social relevance.

**Outline.** The rest of the paper is organized as follows. In Section 2 we formalize the top-\(k\) retrieval problem in collaborative tagging applications. We describe a key

\(^2\)Tagging similarity may indeed be a more pertinent proximity measure than friendship for top-\(k\) search in bookmarking applications.
aspect of our approach, the on-the-fly computation of proximity in Section 2.1. We then describe our top-k algorithm, first in an exclusively social form, in Section 3, and show it is instance optimal in Section 3.1. The general algorithm is then presented in Section 4. Two approaches for improving efficiency by approximation are given in Section 5. We discuss applicability and scalability issues in Section 6. Experimental results are presented in Section 7. We overview the related work in Section 8. We discuss future work and we conclude in Section 9.

2. General Setting

We consider a social setting in which we have a set of items (could be text documents, URLs, photos, etc) \( \mathcal{I} = \{i_1, \ldots, i_m\} \), each tagged with one or more distinctive tags from a dictionary of tags \( \mathcal{T} = \{t_1, t_2, \ldots, t_l\} \) by one or more users from \( \mathcal{U} = \{u_1, \ldots, u_n\} \). We assume that users form an undirected weighted graph \( G = (\mathcal{U}, E, \sigma) \) called the social network. In \( G \), nodes represent users and \( \sigma \) is a function that associates to each edge \( e = (u_1, u_2) \) a value in \((0, 1]\), called the proximity (or social) score between \( u_1 \) and \( u_2 \).

Given a seeker user \( s \), a keyword query \( Q = (t_1, \ldots, t_r) \) (a set of \( r \) distinct tags) and an integer value \( k \), the top-k retrieval problem is to compute the (possibly ranked) list of the \( k \) items having the highest scores with respect to the seeker and query.

We describe next the score model for this problem.

Extending the model for social tagging systems presented in [1], we also assume the following two relations for tags:

- **tagging**: Tagged\((v, i, t)\): says that a user \( v \) tagged the item \( i \) with tag \( t \),

- **tag proximity**: SimTag\((t_1, t_2, \lambda)\): says that tags \( t_1 \) and \( t_2 \) are similar, with similarity value \( \lambda \in (0, 1) \).

We assume that a user can tag a given item with a given tag at most once. We first model for a user, item and tag triple \((s, i, t)\) the score of item \( i \) for the given seeker \( s \) and tag \( t \). This is denoted \( \text{score}(i \mid s, t) \). Generally,

\[
\text{score}(i \mid s, t) = h(fr(i \mid s, t))
\]

(2.1)

where \( fr(i \mid s, t) \) is the overall term frequency of item \( i \) for seeker \( s \) and tag \( t \), and \( h \) is a positive monotone function.

The overall term frequency function \( fr(i \mid s, t) \) is defined as a combination of a network-dependent component and a document-dependent one, as follows:

\[
fr(i \mid s, t) = \alpha \times tf(t, i) + (1 - \alpha) \times sf(i \mid s, t).
\]

(2.2)

The former component, \( tf(t, i) \), is the term frequency of \( t \) in \( i \), i.e., the number of times \( i \) was tagged with \( t \). The latter component stands for social frequency, a measure that depends on the seeker.\(^3\)

\(^3\)The linear combination of Eq. (2.2) is one that is widely used when a local retrieval score and a global one are to be combined, e.g., in spatial search [7] or in social search [18]. However, any monotone combination of the two score components can be used in these approaches, as in ours.
If we consider that each user brings her own weight (proximity) to the score of an item, we can define the measure of social frequency as follows:

\[
  sf(i \mid s, t) = \sum_{v \in \{v \mid \text{Tagged}(v, i, t)\}} \sigma(s, v).
\]  

(2.3)

Then, given a query \( Q \) as a set of tags \((t_1, \ldots, t_r)\), the overall score of \( i \) for seeker \( s \) and query \( Q \),

\[
  \text{score}(i \mid s, Q) = g(\text{score}(i \mid s, t_1), \ldots, \text{score}(i \mid s, t_r)),
\]

is obtained using a monotone aggregate function \( g \) over the individual scores for each tag. In this paper, the aggregation function \( g \) is assumed to be a summation, \( g = \sum_{t_j \in Q} \text{score}(i \mid s, t_j) \).

**Extended proximity.** The above scoring model takes into account only the neighborhood of the seeker (the users directly connected to her). But this can be extended to deal also with users that are indirectly connected to the seeker, following a natural interpretation that user links (e.g., similarity or trust) are (at least to some extent) transitive. We denote by \( \sigma^+ \) an extended proximity, which is to be computable from \( \sigma \) for any pair of users connected by a path in the network. Now, \( \sigma^+ \) can replace \( \sigma \) in the definition of social frequency we consider before (Eq. (2.3)), yielding an overall item scoring scheme that depends on the entire network instead of only the seeker’s vicinity. We discuss shortly possible alternatives for \( \sigma^+ \) by means of aggregating \( \sigma \) values along paths in the graph. In the rest of this paper, when we talk about proximity we refer to the extended one.

For a given seeker \( u \), by her proximity vector we denote the list of users with non-zero proximity with respect to \( u \), ordered in descending order of these proximity values.

**Remark 1.** In Eq. (2.2), the \( \alpha \) parameter allows to tune the relative importance of the social component with respect to classic term frequency. When \( \alpha \) is valued 1, the score becomes network-independent. On the other hand, when \( \alpha \) is valued 0 the score depends exclusively on the social network.

**Remark 2.** Note that a network in which all the user pairs have a proximity score of 1 amounts to the classical document retrieval setting (i.e., the result is independent of the user asking the query).

**Remark 3.** Tag similarity can be integrated into Eq. (2.3), e.g., by setting a threshold \( \tau \) s.t. if \( \text{SimTag}(t, t', \lambda) \), with \( \lambda \) above \( \tau \), and \( \text{Tagged}(v, i, t') \), we also add \( \sigma(u, v) \) to \( sf(i \mid u, t) \). For the sake of simplicity this is ignored in this paper, but remains an integral part of the model.

**Remark 4.** Note that queries are not assumed to use only tags from \( T \). For any tag outside this dictionary, items will obviously have a score of 0.

### 2.1. Computing \( \sigma^+ \)

We describe in this section a key aspect of our algorithm for top-\( k \) search, namely on-the-fly computation of proximity values with respect to a seeker \( s \). The issue here is to facilitate at any given step the retrieval of the most relevant unseen user \( u \) in the
network, along with her proximity value $\sigma(s, u)$. This user will have the potential to contribute the most to the partial scores of items that are still candidates for the top-$k$ result, by Eq. (2.1) and (2.3).

We start by discussing possible candidates for $\sigma^+$, arguably the most natural ones, drawing inspiration from studies in the area of trust propagation for belief statements. We then give a wider characterization for the family of possible functions for proximity computation, to which these candidates belong.

Candidate 1 ($f_{mul}$). Experiments on trust propagation in the Epinions network (for computing a final belief in a statement) \cite{17} or in P2P networks show that (i) multiplying the weights on a given path between $u$ and $v$, and (ii) choosing the maximum value over all the possible paths, gives the best results (measured in terms of precision and recall) for predicting beliefs. We can integrate this into our scenario, by assuming that belief refers to tagging with a tag $t$. We thus aggregate the weights on a path $p = (u_1, \ldots, u_l)$ (with a slight abuse of notation) as

$$\sigma^+(p) = \prod_{i} \sigma(u_i, u_{i+1}).$$

For seeker Alice in our running example, we gave in the previous section (Example 2) the proximity values and the ordering of the network under this candidate for $\sigma^+$.

Candidate 2 ($f_{min}$). A possible drawback of Candidate 1 for proximity aggregation is that values may decrease quite rapidly. A $\sigma^+$ function that avoids this could be obtained by replacing multiplication over a path with minimal, as follows:

$$\sigma^+(p) = \min_{i} \{\sigma(u_i, u_{i+1})\}.$$ 

Under this $\sigma^+$ candidate, the values with respect to seeker Alice would be the following:

$\{Bob : 0.9, Danny : 0.9, Charlie : 0.6, Frank : 0.6, Eve : 0.5, George : 0.5, Harry : 0.5, Ida : 0.25, Jim : 0.25\}$.

Candidate 3 ($f_{pow}$). Another possible definition for $\sigma^+$ we consider relies on an aggregation that penalizes long paths, i.e., distant users, in a controllable way, as follows:

$$\sigma^+(p) = \lambda^{-\sum_{i} \frac{1}{\sigma(u_i, u_{i+1})}}.$$ 

where $\lambda \geq 1$ can be seen as a “drop parameter”; the greater its value the more rapid the decrease of proximity values. Under this candidate for $\sigma^+$, for $\lambda = 2$, the rounded values w.r.t seeker Alice would be $\{Bob : 0.46, Charlie : 0.31, Danny : 0.21, Eve : 0.077, Frank : 0.0525, George : 0.013, Ida : 0.003, Harry : 0.003, Jim : 0.0007\}$.

The key common feature of the candidate functions previously discussed is that they are monotonically decreasing over any path they are applied to, when $\sigma$ draws values from the interval $[0, 1]$. More formally, they verify the following property:

Property 1. Given a social network $G$ and a path $p = \{u_1, \ldots, u_l\}$ in $G$, we have $\sigma^+(u_1, \ldots, u_l) \geq \sigma^+(u_1, \ldots, u_{l-1})$. 

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We then define $\sigma^+$ for any pair of user $(s, u)$ who are connected in the network by taking the maximal weight over all their connecting paths. More formally, we define $\sigma^+(s, u)$ as

$$\sigma^+(s, u) = \max_p \{\sigma^+(p) \mid s \overset{p}{\rightarrow} u\}.$$  \hspace{1cm} \text{(2.4)}

Note that when the first candidate (multiplication) is used, we obtain the same aggregation scheme as in [17], which is also employed in [18] in the context of top-$k$ network aware search.

**Example 3.** In our running example, if we use multiplication in Eq. (2.4), for the seeker Alice, for $\alpha = 0$ (hence exclusively social relevance), by Eq. (2.2) we obtain the following values for social frequency: $
abla_{\text{Alice}}(\text{news}) = \{D_4 : 2.6, D_2 : 1.01, D_1 : 0.7, D_6 : 0.6, D_3 : 0.1, D_5 : 0.05\}$ and $\nabla_{\text{Alice}}(\text{site}) = \{D_4 : 1.11, D_2 : 1.1, D_3 : 0.9, D_6 : 0.6, D_1 : 0.05, D_5 : 0.05\}$.

We argue next that to all aggregation definitions that satisfy Property 1 and apply Eq. (2.4) a greedy approach is applicable. This will allow us to browse the network of users on the fly, at query time, visiting them in the order of their proximity with respect to the seeker.

More precisely, by generalizing Dijkstra’s algorithm [10], we will maintain a max-priority queue, denoted $H$, whose top element $\text{top}(H)$ will be at any moment the most relevant unvisited user. A user is visited when her tagged items are taken into account for the top-$k$ result, as described in the following sections (this can occur at most once). At each step advancing in the network, the top of the queue is extracted (visited) and its unvisited neighbours (adjacent nodes) are added to the queue (if not already present) and are relaxed. Let $\otimes$ denote the aggregation function over a path (one that satisfies Property 1). Relaxation updates the best proximity score of these nodes, as described in Algorithm 1:

**Algorithm 1: Relaxation**

```
if $\sigma^+(s, u) \otimes \sigma(u, v) > \sigma^+(s, v)$ then
    $\sigma^+(s, v) = \sigma^+(s, u) \otimes \sigma(u, v)$
end if
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It can be shown by straightforward induction that this greedy approach allows us to visit the nodes of the network in decreasing order of their proximity with respect to the seeker, under any function for proximity aggregation that satisfies Property 1.

We describe in the following section and in Section 4 how this greedy procedure for iterating over the network is used in our top-$k$ social retrieval algorithm. Without loss of generality, in the rest of the paper, consistent with social theories and with previous work on social top-$k$ search, proximity will be based on Candidate 1 (multiplication).

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4 Dijkstra’s classic algorithm [10] computes single-source shortest paths in a weighted graph without negative edges.
3. Top-k Algorithm for $\alpha = 0$

As the main focus of this paper is on the social aspects of search in tagging systems, we detail first our top-$k$ algorithm, TOPKS, for the special case when the parameter $\alpha$ is 0. In this case, $fr(i \mid s, t)$ is simplified as

$$fr(i \mid s, t) = sf(i \mid s, t).$$

For each user $u$ and tag $t$, we assume a precomputed projection over the Tagged relation for them, giving the items tagged by $u$ with $t$; we call these the user lists. No particular order is assumed for the items appearing in a user list.

We keep a list $D$ of top-$k$ candidate items, sorted in descending order by their minimal possible scores (to be defined shortly). An item becomes candidate when it is met for the first time in a Tagged triple.

As usual, we assume that, for each tag $t$, we have an inverted list $IL(t)$ giving the items $i$ tagged by it, along with their term frequencies $tf(t, i)$ in descending order of these frequencies. Starting from the topmost item, these lists will be consumed one item at a time, whenever the current item becomes candidate for the top-$k$ result. By $CIL(t)$ we denote the items already consumed (as known candidates), by $top_{-item}(t)$ we denote the item present at the current (unconsumed) position of $IL(t)$, and we use $top_{-tf}(t)$ as short notation for the term frequency associated with this item.

We detail mostly the computation of social frequency, $sf(i \mid u, t)$, as it is the key parameter in the scoring function of items. Since when $\alpha = 0$ we do not use metrics that are tag-only dependent, it is not necessary to treat each tag of the query as a distinct dimension and to visit each in round-robin style (as done in the threshold algorithm or in CONTEXTMERGE). It suffices for our purposes to get at each step, for the currently visited user, all the items that were tagged by her with query terms (one user list for each term).

For each tag $t_j \in Q$, by $unseen_{-users}(i, t_j)$ we denote the maximal number of yet unvisited users who may have tagged item $i$ with $t_j$. This is initially set to the maximal possible term frequency of $t_j$ over all items (value that is available at the current position of the inverted list of $IL(t_j)$, as $top_{-tf}(t_j)$).

Each time we visit a user $u$ who tagged item $i$ with $t_j$ we can (a) update $sf(i \mid s, t_j)$ (initially set to 0) by adding $\sigma^+(s, u)$ to it, and (b) decrement $unseen_{-users}(i, t_j)$.

When $unseen_{-users}(i, t_j)$ reaches 0, the social frequency value $sf(i \mid s, t_j)$ is final. This also gives us a possible termination condition, as discussed in the following.

At any moment in the run of the algorithm, the optimistic score $MAXSCORE(i \mid s, Q)$ of an item $i$ that has already been seen in some user list will be estimated using as social frequency for each tag $t_j$ of the query the following value:

$$top(H) \times unseen_{-users}(i, t_j) + sf(i \mid s, t_j).$$

In TOPKS, even though the social frequency does not depend on $tf$ scores, we will exploit the inverted lists and the $tf$ scores by which they are ordered, to better estimate score bounds. In particular, as detailed later, this allows us to achieve instance optimality.
Symmetrically, the pessimistic overall score, \( \text{MINSCORE}(i \mid s, Q) \), is estimated by the assumption that, for each tag \( t_j \), the current social frequency \( sf(i \mid s, t_j) \) will be the final one. The list of candidates \( D \) is sorted in descending order by this lowest possible score.

An upper-bound score on the yet unseen items, \( \text{MAXSCOREUNSEEN} \) is estimated using as social frequency for each tag \( t_j \) the value \( \text{top}(H) \times \text{top}_t tf(t) \).

When the maximal optimistic score of items that are already in \( D \) but not in its top-\( k \) is less than the pessimistic score of the last element in the current top-\( k \) of \( D \) (i.e., \( D[k] \)), the run of the algorithm can terminate, as we are guaranteed that the top-\( k \) can no longer change. (Note however that at this point the top-\( k \) items may have only partial scores and, if a ranked answer is needed, the process of visiting users should continue.)

We present the flow of \( \text{TOPKS} \) in Algorithm \ref{alg:topks}. Key differences with respect to CONTEXTMERGE’s social branch are (i) the on-the-fly computation of proximity values, in lines 1-7 and 29-31 of the algorithm, and (ii) the consuming of inverted list positions, when they become candidates, in lines 20-28. For clarity, we first exemplify a \( \text{TOPKS} \) run without the latter aspect (this would correspond to a CONTEXTMERGE run).

**Example 4.** Revisiting Example \ref{ex:topks}, recall that we want to compute the top-2 items for the query \( Q = \{ \text{news, site} \} \) from Alice’s point of view. To simplify, let us assume that \( \text{score}(i \mid u, t) = sf(i \mid u, t) \) and \( g \) is addition. We consider next how the algorithm described above runs.

At the first iteration of the line 8 loop in the algorithm, we visit Bob’s user lists, adding \( D3 \) to the candidate buffer. At the second iteration, we visit Danny’s user lists, adding \( D2 \) and \( D4 \) to the candidate buffer. At the third iteration (Charlie’s user list) we add \( D6 \) to the candidate list. \( D1 \) is added to the candidate list when the algorithm visits Frank’s user lists, at iteration 4. Recall that \( \text{top}_t tf(\text{news}) = 7 \) and \( \text{top}_t tf(\text{site}) = 5 \).

The 6th iteration of the algorithm is the final one, visiting George’s user lists, finding \( D2 \) tagged with news, site and \( D4 \) tagged with site. \( D4 \) and \( D2 \) are the top-2 candidates, with \( \text{MINSCORE}(D4, Q) = 2.61 \) and \( \text{MINSCORE}(D2, Q) = 2.21 \). The closest candidate is \( D6 \), with \( \text{MINSCORE}(D6, Q) = 1.2 \) and \( \text{MAXSCORE}(D6, Q) = 1.2 + 6 \times 0.1 + 4 \times 0.1 = 2.2 \). Also, \( \text{MAXSCOREUNSEEN}(Q) = 7 \times 0.1 + 5 \times 0.1 = 1.2 \). Finally, \( \text{MAXSCORE}(D6, Q) < \text{MINSCORE}(D2, Q) \) and since we have \( \text{MAXSCOREUNSEEN}(Q) < \text{MINSCORE}(D2, Q) \), the algorithm stops returning \( D4 \) and \( D2 \) as the top-2 items.

We discuss next the interest of consuming of inverted list positions, when these become candidates (illustrated in Example \ref{ex:topks2}). In lines 20-28, we aim at keeping to a minimum the worst-case estimation of the number of unseen taggers. More precisely, we test whether there are top-\( k \) candidates \( i \) (i.e., items already seen in user lists) for which the term frequency for some tag \( t_j \) of \( Q \), \( tf(t_j, i) \), is “within reach” as the one currently used (from \( IL(t_j) \)) as the basis for the optimistic (maximal) estimate of the number of yet unseen users who tagged candidate items with \( t_j \). When such a pair \( (i, t_j) \) is found, we can do the following adjustments:

1. refine the number of unseen users who tagged \( i \) with \( t_j \) from a (possibly loose) estimate to its exact value; this is marked when \( i \) is added to the CIL list of \( t_j \)
Algorithm 2: TOPKS$_{\alpha=0}$: top-$k$ algorithm for $\alpha = 0$

Require: seeker $s$, query $Q = (t_1, \ldots, t_r)$

1: for all users $u$, tags $t_j \in Q$, items $i$ do
2: $\sigma^+(s, u) = -\infty$
3: $sf(i \mid s, t_j) = 0$
4: set $IL(t_j)$ position on first entry; $CIL(t_j) = \emptyset$
5: end for
6: $\sigma^+(s, s) = 0$; $D = \emptyset$ (candidate items)
7: $H \leftarrow$ max-priority queue of nodes $u$ (sorted by $\sigma^+(s, u)$), initialized with $\{s\}$
8: while $H \neq \emptyset$ do
9: $u = \text{EXTRACT\_MAX}(H)$;
10: for all tags $t_j \in Q$, triples $\text{Tagged}(u, i, t_j)$ do
11: $sf(i \mid s, t_j) \leftarrow sf(i \mid s, t_j) + \sigma^+(s, u)$
12: if $i \not\in D$ then
13: add $i$ to $D$
14: for all tags $t_l \in Q$ do
15:       $\text{unseen\_users}(i, t_l) \leftarrow \text{top\_tf}(t_l)$ (initialization)
16: end for
17: end if
18: $\text{unseen\_users}(i, t_j) \leftarrow \text{unseen\_users}(i, t_j) - 1$
19: end for
20: while $\exists t_j \in Q$ s.t. $i = \text{top\_item}(t_j) \in D$ do
21: $tf(t_j, i) \leftarrow \text{top\_tf}(t_j)$ (the top_tf drop)
22: advance $IL(t_j)$ one position
23: $\Delta \leftarrow tf(t_j, i) - \text{top\_tf}(t_j)$
24: for all items $i' \in D \setminus CIL(t_j)$ do
25:       $\text{unseen\_users}(i', t_j) \leftarrow \text{unseen\_users}(i', t_j) - \Delta$
26: end for
27: add $i$ to $CIL(t_j)$
28: end while
29: for all users $v$ s.t. $\sigma(u, v) \in E$ do
30: RELAX($u, v$)
31: end for
32: if $\text{MinScore}(D[k], Q) > \text{max}_{l\geq k}(\text{MaxScore}(D[l], Q))$ and $\text{MinScore}(D[k], Q) > \text{MaxScore\_Unseen}$ then
33: break
34: end if
35: end while
36: return $D[1], \ldots, D[k]$
(line 27), and from this point on the number of unseen users will only change when new users who tagged $i$ with $t_j$ are found (line 18).

2. advance (at the cost of a sequential access) beyond $i$ in the inverted list of $t_j$, to the next best item; this allows us to refine (at line 25) the estimates $\text{unseen-users}(i', t_j)$ for all candidates $i'$ for which the exact number of users who tagged with $t_j$ is yet unknown.

(We found in the experimental evaluation (Section 7) that this aspect has the potential to drastically improve the cost of the search. Since tf-values in inverted lists fall quite rapidly in most practical settings, we witnessed significant cost savings, while using relatively few such list position increments.)

**Example 5.** Let us now consider how the choice of advancing in the inverted lists when possible influences the number of needed iterations. At first, $\text{top}_t \text{f}(\text{news}) = 7$, $\text{top}_t \text{item}(\text{news}) = D_4$, and $\text{top}_t \text{f}(\text{site}) = 5$, $\text{top}_t \text{item}(\text{site}) = D_2$.

The first iteration only introduces $D_3$ and thus we cannot advance in any of the two inverted lists. However, the discovery of $D_2$ and $D_4$ in step 2 allows us to fix their exact tf values and advance the inverted lists. The new positions are: $\text{top}_t \text{f}(\text{news}) = 2$, $\text{top}_t \text{item}(\text{news}) = D_1$, and $\text{top}_t \text{f}(\text{site}) = 1$, $\text{top}_t \text{item}(\text{site}) = D_6$. $D_6$'s discovery in iteration 3 allows us to advance further in the inverted lists. Finally, in step 4, the discovery of $D_1$ allows the algorithm to advance in the inverted lists to $\text{top}_t \text{f}(\text{news}) = 1$, $\text{top}_t \text{item}(\text{news}) = D_5$, and $\text{top}_t \text{f}(\text{site}) = 1$, $\text{top}_t \text{item}(\text{site}) = D_5$ (the only undiscovered item). This allows for some drastic score estimation refinements. We have the same top-2 candidates, $D_4$ and $D_2$ having $\text{MinScore}(D_4, Q) = 1.81$ and $\text{MinScore}(D_2, Q) = 1.21$. The closest item is again $D_6$ having $\text{MinScore}(D_6, Q) = \text{MaxScore}(D_6, Q) = 1.21$, since we know that we have visited all users who tagged $D_6$. $\text{MaxScoreUnseen}(Q) = 1 \times 0.3 + 1 \times 0.3 = 0.6$, since the maximal unseen document, $D_6$ is tagged only once with each tag. $\text{MaxScore}(D_6, Q) < \text{MinScore}(D_2, Q)$ and $\text{MaxScoreUnseen}(Q) < \text{MinScore}(D_2, Q)$ allows us to exit the loop, two steps before the unrefined algorithm, returning the exact top-2: $D_4$ and $D_2$.

We can prove the following property of our algorithm:

**Property 2.** For a given seeker $s$, TOPKS$_{\alpha=0}$ visits the network in decreasing order of the $\sigma^+$ values with respect to $s$.

As a corollary of Property 2 we have that TOPKS$_{\alpha=0}$ visit users who may be relevant for the query in the same order as CONTEXTMERGE [13]. More importantly, we prove in Section 3.1 that our algorithm visits as few users as possible, i.e., it is instance optimal with respect to this aspect. Moreover, the experiments show that TOPKS can drastically reduce the number of visited user lists in practice (see Section 7).

### 3.1. Instance Optimality of TOPKS$_{\alpha=0}$

We will use the same definition of instance optimality as in [12]. For a class of algorithms $A$, a class of legal inputs (instances) $D$, $\text{cost}(A, D)$ denotes the cost of running algorithm
\( \mathcal{A} \in \mathcal{A} \) on input \( \mathcal{D} \in \mathcal{D} \). An algorithm \( \mathcal{A} \) is said to be instance optimal for its class \( \mathcal{A} \) over inputs \( \mathcal{D} \) if for every \( \mathcal{B} \in \mathcal{A} \) and every \( \mathcal{D} \in \mathcal{D} \) we have \( \text{cost}(\mathcal{A}, \mathcal{D}) = O(\text{cost}(\mathcal{B}, \mathcal{D})) \).

Let \( c_{UL} \) be the abstract cost of accessing the user list - a process which involves the relatively costly operations of finding the proximity value of the user and retrieving the items tagged by the user with query terms - and let \( \text{users}(\mathcal{A}, \mathcal{D}) \) be the number of total user lists needed for establishing the top-\( k \) for algorithm \( \mathcal{A} \) on input \( \mathcal{D} \). Let \( c_{S} \) be the abstract cost of sequentially accessing the data in \( IL \), and let \( \text{seqitems}(\mathcal{A}, \mathcal{D}) \) be the total number of sequential accesses to \( IL \) for algorithm \( \mathcal{A} \) on input \( \mathcal{D} \). In practice, \( c_{UL} \gg c_{S} \) is a reasonable assumption, hence, for two algorithms \( \mathcal{A} \) and \( \mathcal{B} \), we have

\[
\frac{\text{users}(\mathcal{A}, \mathcal{D}) \times c_{UL} + \text{seqitems}(\mathcal{A}, \mathcal{D}) \times c_{S}}{\text{users}(\mathcal{B}, \mathcal{D}) \times c_{UL} + \text{seqitems}(\mathcal{B}, \mathcal{D}) \times c_{S}} \approx \frac{\text{users}(\mathcal{A}, \mathcal{D})}{\text{users}(\mathcal{B}, \mathcal{D})}.
\]

Therefore, for a fair cost estimate in practical social search settings, a reasonable assumption is to consider

\[ \text{cost}(\mathcal{A}, \mathcal{D}) = \text{users}(\mathcal{A}, \mathcal{D}). \]

Let us now define the class of “social” algorithms \( \mathcal{S} \) to which both \( \text{TOPKS}_{\alpha=0} \) and \( \text{CONTEXTMERGE} \) (when \( \alpha = 0 \)) belong. These algorithms correctly return the top-\( k \) items for a given query \( Q \) and seeker \( s \), they do not use random accesses to \( IL(t) \) indexes in order to fetch a certain \( tf \) value, and they do not include in their working buffers (e.g., candidate buffer \( D \)) items that were not yet encountered in the user lists. The last assumption could be seen as a “no wild guess” policy, by which the algorithm cannot guess that an item might be encountered in some later stages. This is a reasonable assumption in practice, as the number of items needed for computing a top-\( k \) result for a given seeker should in general be much smaller than the total number of items tagged by query terms.

The class \( \mathcal{D} \) of accepted inputs consists of the inputs that respect the setting described in Section 2.

**Theorem 1.** \( \text{TOPKS}_{\alpha=0} \) is instance optimal over \( \mathcal{S} \) and \( \mathcal{D} \), when the cost is defined as \( \text{cost}(\mathcal{A}, \mathcal{D}) = \text{users}(\mathcal{A}, \mathcal{D}). \)

The optimality proof is given in Appendix A.

### 4. Algorithm for The General Case

For the general case, in which \( \alpha \in [0, 1] \), we adapt the \( \text{CONTEXTMERGE} \) algorithm to include the on-the-fly processing of user proximities.

At each iteration, the algorithm can alternate, by calling \( \text{CHOOSEBRANCH()} \), between two possible execution branches: the social branch (lines 8-31 of Algorithm 2) and the textual branch, which is a direct adaptation of NRA.

As in the exclusively social setting of the previous section, we will read term frequency scores \( tf(t_j, i) \) from the inverted lists, on a per-need basis, either as in line 21 of \( \text{TOPKS}_{\alpha=0} \), or when advancing on the textual branch. Initially, all unknown \( tf \)-scores are assumed to be set to 0.
The optimistic overall score MAXSCORE(i, Q) of an item \(i\) that is already in the candidate list \(D\) will now be computed by setting \(fr(i | s, t)\), defined in Eq. (2.2), to

\[
fr(i | s, t) = (1 - \alpha) \times \text{top}(H) \times \text{unseen_users}(i, t) + (1 - \alpha) \times sf(i | s, t) + \alpha \times \max( tf(t, i), \text{top_tf}(t) ).
\]

The last term accounts for the textual weight of the score, and uses either the exact term frequency (if known), or an upper-bound for it (the score in the current position of \(IL(t)\)).

Symmetrically, for the pessimistic overall score MINSCORE(i, Q), the frequency \(fr(i | u, t)\) will be computed as

\[
fr(i | s, t) = (1 - \alpha) \times sf(i | s, t) + \alpha \times \max( tf(t, i), \text{partial_tf}(t) ),
\]

where \(\text{partial_tf}\) represents the count of visited users who tagged \(i\) with \(t_j\), which is used as lower-bound for \(tf(t_j, i)\) when this is not yet known.

The upper-bound for the score on the yet unseen items, MAXSCOREUnseen, is estimated using as overall frequency for each tag \(t_j\) the following value:

\[
fr(i | s, t) = \alpha \times \text{top_tf}(t) + (1 - \alpha) \times \text{top}(H) \times \text{top_tf}(t).
\]

We present the flow of the general case algorithm in Algorithm 3. Method \textsc{Initialize()} amounts to lines 1-6 of TOPKS\(_{\alpha=0}\), and method \textsc{ProcessSocial()} amounts to lines 8-31 of TOPKS\(_{\alpha=0}\) (modulo the straightforward adjustment for the count \(\text{partial_tf}\)).

The difference between the \(\alpha = 0\) case and the general case is the processing of the inverted lists (textual branch), which is done as in the NRA algorithm (see lines 7-13 of Algorithm 3). We discuss how the choice of the branch to be followed is done, by the \textsc{ChooseBranch()} subroutine, in Section 4.1.

### 4.1. Choosing between the social and textual branches

The TOPKS\(_{\alpha=0}\) algorithm, in which only the social branch matters, is instance optimal (see Theorem 1), with the cost being estimated as \(\text{users}(\text{TOPKS}_{\alpha=0}, D)\). As the NRA algorithm [12], when only the textual branch matters, TOPKS\(_{\alpha=1}\) is instance optimal, with the cost being estimated as \(\text{seqitems}(\text{TOPKS}_{\alpha=0}, D)\).

When \(\alpha\) is not one of the extreme values, under a cost function as a combination of the two above, of the form

\[
\text{users}(\text{TOPKS}_{\alpha=0}, D) \times c_{UL} + \text{seqitems}(\text{TOPKS}_{\alpha=1}, D) \times c_S,
\]

a key role for efficiency is played by \textsc{ChooseBranch()}.

In [12], the choice between the textual branch or the social one was done by estimating the maximum potential score of each, in round-robin manner over the query dimensions. For a query tag \(t_j\), the maximal contribution of the social branch would be estimated as \(\text{MAXSocial}(t_j) = (1 - \alpha) \times \text{max_tf}(t_j) \times \text{top}(H)\), where \(\text{max_tf}(t_j)\) is the maximum
Algorithm 3: TOPKS: top-k algorithm for the general case

Require: seeker $s$, query $Q = (t_1, \ldots, t_r)$

1: **Initialize**()
2: while $H \neq \emptyset$ do
3: 
4: 
5: 
6: 
7: 
8: 
9: 
10: 
11: 
12: 
13: 
14: 
15: 
16: 
17: 
18: 
19: return $D[1], \ldots, D[k]$
adapted in straightforward manner to a “conjunctive” interpretation: the pessimistic score should be maintained at 0 until the item’s scores – at least partial ones – are known for all tags.

5. Efficiency by Approximation

The algorithm described in the previous section is sound and complete, and requires no prior (aggregated) knowledge on the proximity values with respect to a certain seeker (e.g., statistics); this was also the assumption in [18]'s CONTEXTMERGE algorithm. Moreover, it is instance optimal in the exclusively social setting (our main focus in this paper) with respect to the number of visited users. While we improve the running time in both this setting and the general one (more on experimental results in Section 7), in practice, however, the search may still visit a significant part of the user network and their item lists before being able to conclude that the top-k answer can no longer change.

But if some statistics about proximity are known at query time (i.e., on how the values in a proximity vector variate from the most relevant user to the least relevant one), this may enable us to use more refined termination conditions, and thus to minimize the gap between the step at which the final top-k has been established and the actual termination of the algorithm. Indeed, the experiments we performed on Del.icio.us data showed that, in average, the last top-k change occurs much sooner, hence there is a clear opportunity to stop the browsing of the network earlier.

We take a first step in this direction, discussing two possible approaches for using score estimations based on proximity statistics, which trade accuracy for efficiency (in terms of visited users). More specifically, in Algorithm 3 the MAXSCORE, MAXSCOREUNSEEN and MINSCORE bounds have all used the safest possible values for the proximities of yet unseen users: either the top (maximum) value of the max-priority queue (top(H)) for the first two bounds, or its minimal possible value (zero) for the third one. In practice, however, any of these extreme configurations is rarely met. For illustration, we give in Figure 2 the proximity vectors for some randomly sampled users. Observe that these fall rapidly, and this may be the case in many real-world similarity or proximity networks.

Hence one possible direction for reducing the number of visited users is to pre-compute and materialize a high-level description (more or less complex, more or less accurate)
of users’ proximity vectors (of their distribution of values). This would allow us to use a tighter estimation for the remaining (unseen) users, instead of uniformly associating them the extreme score (top(H) or 0). In doing so, we may obviously introduce approximations in the final result, and our approximate techniques provide a trade-off between accuracy drop on one hand and negligible memory consumption and reduced running time on the other hand.

5.1. Estimating bounds using mean and variance

We first consider as a proximity vector description one that is very concise yet generally-applicable and effective, keeping for a given seeker two parameters: the mean value of the proximities in the vector and the variance of these values. We adopt here the simplifying assumption that the values in the seeker’s vector are independent, essentially interpreting the proximity vector as a random one.

At any step in the run of the algorithm, using the mean and variance, for the remaining (yet unvisited) unseen users \((i, t)\) for a given item \(i\) and tag \(t \in Q\), we can derive (a) lower bounds for the average of their proximity values, for MINSCORE estimations, or (b) upper bounds for the average of their proximity values, for MAXSCORE estimations. The guarantees of these bounds can be controlled (in a probabilistic sense) via a precision parameter \(\delta \in (0, 1]\), by which lower values lead to higher precision and 1 leads to a setting with no guarantees.

More precisely, let \(p\) be the current position in the proximity vector and let \(\sigma^+_{p:p}(s)\) be the vector containing the remaining (unseen) values of \(\sigma^+(s)\). Knowing the overall mean and variance of the entire proximity vector \(\sigma^+(s)\), and having the proximity values seen so far (denoted \(\sigma^+_{0:p}(s)\)), we can easily compute the average and variance of the remaining proximity values (those in \(\sigma^+_{p:p}(s)\)).

Then, the mean and variance of the average of unseen users \((i, t)\) randomly chosen proximity values from the remaining ones can be obtained as follows:

\[
\begin{align*}
\text{Exp}[\sigma^+_{p:p}, \text{unseen}\_users(i, t)] & = E[\sigma^+_{p:p}], \\
\text{Var}[\sigma^+_{p:p}, \text{unseen}\_users(i, t)] & = \frac{\text{Var}[\sigma^+_{p:p}]}{\text{unseen}\_users(i, t)}.
\end{align*}
\]

When the input query contains more than one tag, its size \(|Q|\) needs to be taken into account in the estimations. In order to avoid computational overhead, we uniformly chose a non-optimal per-tag probabilistic parameter \(\delta'\) that ignores per-tag score distributions, as follows:

\[
\delta' = 1 - (1 - \delta)^{1/|Q|}. \tag{5.1}
\]

EstMax\((p, \delta)\) represents, for each query tag, the upper bound of the expected value of the average of unseen users \((i, t)\) values drawn from \(\sigma^+_{p:p}(s)\), which holds with probability at least \(1 - \delta'\). Similarly, EstMin\((p, \delta)\) represents the lower bound of the expected

---

\(^6\)This is possible under independence assumptions that may not entirely hold, but turn out to be reasonable in practice (see Section[7]).
value of the average of unseen_users(i, t) values drawn from $\sigma^+(s)$, which holds with probability at least $1 - \delta'$. For estimating MINSCORE when $i \not\in CIL(t)$, the fact that we have no information about the difference between $tf(i, t)$ and partial_tf(t, i) (the users who tagged item $i$ with $t$ so far) means that we cannot assume that other users may have tagged $i$, so we keep this estimation as in the initial (exact) algorithm.

By using Chebyshev's inequality, these bounds can be computed as follows:

$$
\text{EstMax}(p, \delta) = E[\sigma^+(s)] + \sqrt{\frac{\text{Var}[\sigma^+(s)]}{\text{unseen_users}(i, t) \times \delta'}}
$$

$$
\text{EstMin}(p, \delta) = E[\sigma^+(s)] - \sqrt{\frac{\text{Var}[\sigma^+(s)]}{\text{unseen_users}(i, t) \times \delta'}}
$$

We give the score estimations, changed by generalizing the proximity estimations, in Table 1. We present in the experimental results the effect of this approximate approach on running time, showing significant overall improvement. In our experiments, even for $\delta = 0.9$, the returned top-$k$ answers had reasonable precision levels (around 90%).

We discuss in the next section another approach for tighter score estimates, using more detailed descriptions of proximity vectors. We conclude this section with a discussion on how these concise descriptions of proximity vectors could be maintained up-to-date in dynamic environments, in Section 5.3.

5.2. Estimating bounds using histograms

The advantage of the approach described the previous section is twofold: low memory requirements and estimation bounds that are applicable for any value distribution. However, it may offer estimation bounds that are too loose in practice, and hence not reach the full potential for efficiency of approximate score bounds. To address this issue, we can imagine – as a compromise between keeping only these two statistics and keeping the entire pre-computed proximity vector – an approach in which we describe the distribution at a finer granularity, based on histograms.

More precisely, for a seeker $s$, we denote this histogram as $h(\sigma^+(s))$. It consists of $b$ buckets, each bucket $b_i$, for $i \in \{1, \ldots, b\}$, containing $n_i$ items in the interval $(\text{low}_i, \text{high}_i]$ (the 0 values are assigned to bucket $b$). Then, the probability that there exists a proximity
value \( x \) greater than \( \text{low}_i \), knowing the histogram \( h(\sigma^+(s)) \), is

\[
Pr[x > \text{low}_i \mid h(\sigma^+(s))] = \sum_{j=1}^{n} \frac{n_j}{n}.
\]

At any step \( p \) in the run of the algorithm, we maintain a partial histogram denoted as \( h(\sigma^+_p(s)) \), obtained by removing from \( h(\sigma^+(s)) \) the \( p \) already encountered proximity values.

Similar to the previous approach, we can drill down the overall \( \delta \) parameter to a \( \delta' \) one for each query tag. Then, \( \text{EST}\text{MAX}(p, \delta) \) can be given by the minimal value in the partial histogram, such that the resulting estimation of \( \text{MAX}\text{SCORE}(i, t) \) holds with at least probability \( 1 - \delta' \). Conversely, \( \text{EST}\text{MIN}(p, \delta) \) is given by the maximal value in the partial histogram, such that the resulting estimation of \( \text{MIN}\text{SCORE}(i, t) \) holds with at least probability \( 1 - \delta' \).

In manner similar to Eq.\((5.1)\), we need to take into account the fact that a number of \textit{unseen} users \((i, t)\) such estimated values lead to an overall approximate estimation, for both \( \text{EST}\text{MIN} \) and \( \text{EST}\text{MAX} \). Therefore, each of these values is uniformly estimated using a stronger probabilistic parameter \( \delta''(i, t) \), depending on \textit{unseen} users \((i, t)\), as follows:

\[
\delta''(i, t) = 1 - (1 - \delta''_{\text{est}})^{1/\text{unseen}\text{-users}(i, t)}.
\]

Formally, having \( h(\sigma^+_p(s)) \) and \( \delta''(i, t) \), we estimate \( \text{EST}\text{MAX}(p, \delta) \) and \( \text{EST}\text{MIN}(p, \delta) \) as follows:

\[
\text{EST}\text{MAX}(p, \delta) = \min \{ \text{low}_i \mid Pr[x > \text{low}_i \mid h(\sigma^+_p(s))] \leq \delta''(i, t) \},
\]

\[
\text{EST}\text{MIN}(p, \delta) = \max \{ \text{low}_i \mid Pr[x > \text{low}_i \mid h(\sigma^+_p(s))] \geq 1 - \delta''(i, t) \}.
\]

The space needed for keeping such histograms is linear in the number of users and buckets. For instance, by setting the latter using the square-root choice, the memory needed is \( O(n^{3/2}) \). Also, as a consequence of the on-the-fly computation of proximity values, we can easily update the histogram of the seeker by merging the partial, “fresh” histogram obtained in the current run (until termination) with the remaining values from the existing (pre-computed) histogram.

### 5.3. Maintaining the description of the proximity vector

Since social tagging applications are highly dynamic in nature, we need to take into account the fact that the statistics we keep are likely to change quite often. While we can hope that mean, variance and even histogram descriptions are less subject to change than individual proximity values, we should still strive to maintain these statistics as fresh as possible. Recomputing them from scratch, at certain intervals, is an obvious option to consider, though one that may still be too expensive, knowing that we want to avoid keeping the \( n \times n \) materialized proximity matrix, as well as naïve re-computation of mean and variance pairs.
A more suitable alternative would be to rely on approximate techniques for maintaining a fully dynamic all-pairs shortest path information (APSP) in the network. Since our proximity metric relies on path multiplication, we can reformulate the computation of proximity values into a problem of computing shortest paths in a network with (a) the same set of vertices and edges, and (b) edge weights valued \( w(u, v) = -\log \sigma(u, v) \), where \( \sigma(u, v) \) is the user proximity from the original network.

A \((2+\epsilon)\)-approximate algorithm was given in [5], which handles fully-dynamic updates in a graph in \( \tilde{O}(e) \) (almost linear) time. It exhibits a query time of \( \tilde{O}(\log \log \log n) \) (the query returns an estimation of the shortest distance between two nodes), without the need of keeping a distance matrix. We could directly rely on this algorithm in the transformed \(-\log \sigma(u, v)\) graph. Mapping back the distances thus queried to our setting would give us an estimation \( \sigma^+_{est}(u, v) \) that verifies the inequality:

\[
\sigma^+(u, v) \geq \sigma^+_{est}(u, v) \geq \sigma^+(u, v)^{2+\epsilon}.
\]

For a given seeker \( s \), we could thus compute an approximation of its proximity vector in \( O(n \log \log \log n) \) time, and then compute the approximate statistics efficiently.

6. Scaling and performance

We argue in this section that, in a real-world setting, our algorithm TOPKS outperforms the one from existing literature both in terms of memory requirements and execution time. We discuss its practical impact in experiments in Section [7]

Let us consider, as an illustrating example, one of the most popular bookmarking applications, Del.icio.us, which currently has probably around \( 10^7 \) users. Unsurprisingly, this social network is quite sparse, with an average degree of about 100. If a similar graph configuration would be maintained when weights (the \( \sigma \) function) are associated to the edges of the network (e.g., based on tagging proximity or some other measure) the size of an index that would precompute the extended proximity value for each pair of connected users in the network (the \( \sigma^+ \) function) would be roughly of 700 terabytes (i.e., \((10^7)^2 \times 7 \) bytes, considering that 3 bytes are necessary for an user Id and 4 bytes are necessary for the float value of proximity). On the other hand, the weighted graph would require memory space of roughly 7 gigabytes (as \( 10^7 \times 100 \times 7 \) bytes), and could easily fit in the RAM space of an average commodity workstation. More, existing techniques for network compression [9] might allow us to reduce the space required to store the network by a factor of \( 10^{-15} \) while still supporting efficient updates and random access on compressed data.

The difference in memory requirements for the two alternatives becomes much more drastic when assuming a user base of the order of Facebook’s social network, which currently consists of roughly \( 7 \times 10^8 \) users (and is still growing at a fast pace). Precomputed lists for extended proximity go up to about 400 petabytes of memory space, while the

\footnote{We stress that, for the sake of generality, this is not assumed nor exploited in our algorithms, and is not accounted for in the experimental results for TOPKS (in both abstract cost and running time).}
Table 2: Computational costs for processing a query $Q$, when $\alpha = 0$.

| Algorithm   | Disk access | RAM access                           |
|-------------|-------------|--------------------------------------|
|             | RA  | SA | $n$ | $(|Q| - 1) \times n$ | $O(n \log n + e) + (|Q| - 1) \times n + n + e$ |

network itself requires only about 400 gigabytes. The space needed to store the network can further decrease to fit RAM capacity that moderate commodity servers can provide today, if considering the compression techniques mentioned previously.

We next discuss general performance aspects, which in practice may be as impacting as the memory and updatability advantages that our algorithm presents.

Let $n$ denote the number of users and let $e$ denote the number of edges in the network. We assume without loss of generality that the query consists of a single tag (for multiple-tag queries, all dimensions can share the results of a single $\sigma^+$ computation).

For our algorithm, let us assume that the social network resides in main memory, e.g., by means of adjacency lists: for each vertex, we have a list of its neighbors and their associated weights (we can safely assume the list comes presorted descending by weight). For one top-$k$ query execution, we will need at most $n + e$ operations to visit the entire network (we are guaranteed to take each vertex only once). For the proximity computation we can use a Fibonacci-heap based max-priority queue, since our graph is likely to be very sparse [16]. Each insertion into the heap takes $O(1)$ amortized time, each extraction takes $O(\log n)$ and each increase of a key (a relaxation step) takes $O(\log n)$, for an overall queue complexity of $O(n \log n + e)$.

CONTEXTMERGE requires no computations for proximity at query time. However, it uses disk accesses to read the precomputed proximity values: one random access to locate the seeker’s list and $n$ sequential disk accesses to read this list. (It suffices to do this just for one query term, and then keep and access a shared copy of this list in main memory.)

If we value the latency of a memory access as 1 and the one of a sequential disk access as $t$ (usually about five orders of magnitude slower than RAM access), with minor simplifications, our algorithm has the potential to perform better than CONTEXTMERGE when the following holds: $t > \log n + \frac{e}{n}$. So the network sparseness should verify the following inequality:

$$e < n \times (t - \log n),$$

which is a very plausible assumption in real applications.

A summary of this comparison on execution time is given in Table 2. Note that in this analysis we omitted initialization costs: the overhead necessary for CONTEXTMERGE to compute $\sigma^+$ values for all user pairs and the overhead to load in main-memory the social network, for our algorithm.
7. Experimental Results

Dataset and testing methodology. We have performed our experiments on a publicly available Del.icio.us dataset [20], containing 80000 users tagging 595811 items with 198080 tags. As this dataset does not give information regarding links between users, we have generated three similarity networks:

- **Item similarity network.** This network was constructed by computing the Dice coefficient of the common items bookmarked by any two users, resulting in a network of 49038 users and 3329540 links.

- **Tag similarity network.** This network was generated by computing the Dice coefficient of the common tags used by any two users. Since this computation results in a network that is too dense, we have filtered out the users who used less than 10 distinct tags in their tagging activity. The final networks thus contains 40319 users and 833544 links.

- **Item-tag similarity network.** This network was constructed by computing the Dice coefficient of the common items and tags bookmarked by any two users, resulting in a network containing 40353 users and 1849898 links.

We computed the top-10 and top-20 answers, generating a number of 20 two and three-tag semantically coherent queries, from tags that have a medium frequency (i.e., between 3000 and 5000 in our dataset). For each similarity network, 10 random users were also randomly chosen in the role of the seeker.

Testing was performed using two ranking functions (the $h$-function from our model). The first one is the standard tf-idf ranking function:

$$score(i \mid u, t) = fr(i \mid u, t) \times idf(t).$$

The second one is the BM15 ranking function used in [18]:

$$score(i \mid u, t) = \frac{(k1 + 1)fr(i \mid u, t)}{k1 + fr(i \mid u, t)} \times idf(t),$$

where inverse frequency $idf(t)$ is defined in standard manner as

$$idf(t) = \log \frac{|\mathcal{Z} - |\{i \mid Tagged(v, i, t)\}| + 0.5}{||\{i \mid Tagged(v, i, t)\}|| + 0.5}.$$

and the aggregation function $g$ is summation.

While these are two of the most commonly used ranking functions in IR literature, they have different properties when used in approximate approaches as the ones we describe. More precisely, since tf-idf is a linear function, both the maximal and minimal estimates over $fr$ scores lead to valid estimates for the overall scores. This is not necessarily the case for BM15: since it is a concave function, only the maximal overall score can be estimated. This was taken into account in the experiments.
We used a Java implementation of our algorithms, on a machine with a 2.8GHz Intel Core i7 CPU, 8GB of RAM, running Ubuntu Linux 10.04 and PostgreSQL 9.0.

As our focus is on optimizing the social branch of the top-

≈κretrieval, we report here our results for \( \alpha \in \{0, 0.1, 0.2, 0.3\} \). As [18], multiplication over the paths was chosen as the proximity aggregation function, as the best suited candidate for predicting implicit similarities.

**Remark.** The relevance of personalized query results is a topic that has been extensively treated ([19], [11], [15]). It is not our focus here, and we interpret the relevance of results as a consequence of the scoring functions \( g \) and \( h \). Moreover, the query itself could be viewed as the result of a transformation using techniques such as query expansion. The relevance of social search results was also extensively evaluated in [18], over Del.icio.us data, in a setting (including ranking model) similar to ours. We report, however, on two ground-truth experiments for evaluating the relevance of top-

≈κresults for \( \alpha = 0 \), at the end of this section.

**Efficiency results.** For the testing environment described previously, we report on efficiency for both exact and approximate algorithms, and on precision for the latter.

For efficiency, we report on two measures: the abstract cost of the algorithms and their wall clock running times. Abstract cost, which is the standard measure for early-termination algorithms that depend on database accesses, is computed as defined in Section 4.1, by choosing \( c_{UL} \), the cost of accessing a user lists, as valued 100 (a very conservative upper-bound), and \( c_S \), the cost of sequentially accessing an item in an inverted lists, valued 1. More formally,

\[
\text{cost}(A, D) = 100 \times \text{users}(A, D) + \text{seqitems}(A, D).
\]

We ignore differences in favor of TOPKS that are hard to account for, namely we do not distinguish between the user accesses by CONTEXTMERGE (which in a real setting would be to external memory) and the ones by TOPKS (which would be to main memory).

Figures 3, 4 and 5 present the comparison of abstract costs and running times for the BM15 and tf-idf ranking functions, for each of the three similarity networks. In each subfigure, the first pair of columns gives the abstract cost of [18]‘s CONTEXTMERGE algorithm, the second pair of columns the one of TOPKS, the third pair of columns the cost of TOPKS/MVar (approximate approach based on mean and variance of proximities, described in Section 5.1) and the fourth pair of columns the cost of TOPKS/Hist (approximate approach based on histograms, described in Section 5.2). For each algorithm, the average running times were recorded, and are represented by the black line in the plots (one dot indicates the average running time between the top-10 and the top-20). One can notice there that abstract cost closely captures the actual performance of the algorithms. However, running time optimization was not the focus of the present work, and many alternatives remain to be explored in that direction (e.g., tuning the database).\(^8\)

\(^8\)Note that we cannot compare with [1]’s approach, as it only extends classic top-k retrieval by inter-
Table 3: Comparison between CONTEXTMERGE and TOPKS_{α=0}.

| Network | CONTEXTMERGE | TOPKS_{α=0} |
|---------|--------------|--------------|
|         | users seqitems | users seqitems |              |
| item    | 21878 0       | 15588 65     |
| item-tag| 13028 0       | 6898 54      |
| tag     | 18718 0       | 15581 68     |

First, we can see that in general TOPKS drastically improves efficiency when compared to CONTEXTMERGE, in terms of both running time and abstract cost. For example, in the item-tag similarity network, when $\alpha = 0$, the running time and abstract cost are around 50% of that of CONTEXTMERGE.

Moreover, our approximate approaches lead to further improvements, which support the intuition that even limited statistics (such as mean and variance) can render the termination conditions more tight.

The abstract costs of TOPKS/MV ar and TOPKS/Hist in the figure were obtained for the probabilistic threshold $\delta = 0.9$. Even though this represents a quite weak guarantee, we found that it still yields a good precision/efficiency trade-off. For a better understanding of this trade-off, we show in Figure 6 the impact of $\delta$ on precision. When $\alpha > 0$, visiting the per-term inverted lists in parallel to the proximity vector helps in deriving tighter score bounds for unseen items, leading to a faster termination of the approximate approaches. These tighter score bounds also help in achieving better precision levels when $\alpha > 0$, as Figure 6 shows.

Furthermore, our branch choice heuristic in TOPKS (in both the exact and approximate variants) brings significant improvements overall (for instance, consider the difference between the cost savings for $\alpha = 0$ and $\alpha = 0.1$, in the tag similarity network). Finding even more effective heuristics for this aspect of the algorithm remains an interesting direction for future research.

We discuss next how the instance optimality of TOPKS_{α=0} reflects in the performance results. Table 3 reports the number of visited users by CONTEXTMERGE and TOPKS_{α=0} (columns users), for the three similarity networks. One can see that TOPKS_{α=0} achieves good savings (in terms of visited users), while relying only on very few sequential accesses in the inverted lists (column seqitems).

Finally, we consider the impact of the probabilistic parameter $\delta$ on precision and speedup in the approximate algorithms. We define precision as the ratio between the size of the exact result (by TOPKS) and the number of common items returned by the respective approximate approach and TOPKS, i.e.,

$$\text{precision} = \frac{|T_{\text{TOPKS/app}} \cap T_{\text{TOPKS}}|}{|T_{\text{TOPKS}}|},$$

interpreting user proximity as a binary function (0-1 proximity), by which only users who are directly connected to the seeker can influence the top-k result.
where $T_{\text{TOPKS/app}}$ is the set of items returned as top-$k$ by the approximate algorithms (either $\text{TOPKS/MVar}$ or $\text{TOPKS/Hist}$), and $T_{\text{TOPKS}}$ is the set of items returned by the exact algorithm.

The relative speedup is defined as

$$\text{speedup} = \frac{\text{cost}(\text{TOPKS}, D)}{\text{cost}(\text{TOPKS/app}, D)} - 1.$$  

We present in Figure 6 the results for both approximate approaches, TOPKS/MVar and TOPKS/Hist. For TOPKS/MVar, one can notice that $\delta$ has a limited influence on precision (with a minimum of 0.997 for $\delta = 1$), while ensuring reasonable speedup. The speedup potential is greater when using TOPKS/Hist and histograms, while reasonable precision levels are obtained (for instance, precision of around 0.805 when $\delta = 0.9$, for a speedup of around 2.5). For values of $\delta > 0.9$, we notice however a rapid drop in precision. The fact that MVar achieves better precision than Hist may seem counter-intuitive, since histograms give a more detailed description of proximity vectors. This difference in precision is due to looser bounds for MVar, as they directly influence the termination condition of the algorithm, result in a longer run and hence to better chances of returning a more refined top-$k$ results.

We also considered the influence of the $\alpha$ parameter on precision, while setting the probabilistic parameter to $\delta = 0.9$ (see Figure 7). We have measured both $\text{precision@10}$ (i.e., when requesting the top-10) and $\text{precision@20}$ for both TOPKS/MVar and TOPKS/Hist. We observed that the precision levels for TOPKS/MVar are quite stable for all values of $\alpha$. For TOPKS/Hist, the lowest values of precision are witnessed when $\alpha = 0$, but they stabilize to high values (above 0.97) for $\alpha > 0$.

Evaluating relevance We report now on two “ground-truth” experiments we have performed to test the bookmark prediction power of the exclusively social queries.

For the first experiment, we have selected $(\text{user}, \text{tag})$ pairs from users that have bookmarked between 5 and 10 items using tags that were used globally at least 1000 times. The objective of this experiment was to estimate the power of personalized results to predict items that are tagged using relatively popular tags.

Then, 1000 pairs were randomly selected. For each pair and for $k \in \{1, 2, 5, 7, 10\}$, we computed the following top-$k$ result, using as query the tags corresponding to the distinct user-ids: the network-unaware top-$k$, and, setting the user-id as the seeker, the personalized top-$k$ (for $\alpha = 0$) for each of the following aggregation functions: $f_{\text{mul}}$, $f_{\text{min}}$, and $f_{\text{pow}}$ with $\lambda \in \{1.1, 2\}$. For each personalized query, the items belonging to the seeker were ignored (so as not to influence positively the precision of the results).

An item was considered as “predicted” if it appeared in the resulting top-$k$ and was also tagged by the seeker userid with the query tags. We traced the proportion of pairs for which at least one such item has been predicted.

The results are presented in Figure 8. One can note that, for the item and item-tag similarity networks, personalization is considerably better at predicting bookmarked items than the “global” top-$k$, for all functions, except $f_{\text{min}}$ and, to a lesser extent, $f_{\text{pow}}$.
Figure 3: Abstract cost and running time comparison over the tag-similarity network and the $f_{mul}$ proximity function (red: top-10, yellow: top-20).

Figure 4: Abstract cost and running time comparison over the item-similarity network and the $f_{mul}$ proximity function (red: top-10, yellow: top-20).
Figure 5: Abstract cost and running time comparison over the item-tag similarity network and the $f_{mul}$ proximity function (red: top-10, yellow: top-20).

Figure 6: Precision rates versus speedup relative to TOPKS, when $\alpha = 0$.

Figure 7: Precision rates vs. $\alpha$, when $\delta = 0.9$. 
Figure 8: Predicting bookmarks tagged with semi-popular tags.

Figure 9: Predicting unpopular bookmarks.

(λ = 1.1). Moreover, the tag similarity network seems to not be such a good predictor, no matter the personalization function used, as the other two networks. This might indicate the fact that, in the case of tag similarity, one needs to go beyond simple set similarities and include more complex relationships between tags, like synonymy and polysemy.

For the second experiment, we have selected (user, item, tag) triples resulting from items that have been tagged only by few people in the network (between 5 and 10). The objective of this experiment was to estimate the power of personalizing results to predict items that are unpopular, i.e., the “long tail”. The tests and the measures tracked are identical to the setup of the first experiment.

The results are presented in Figure 9. They are similar to a good extent to those of the first experiment, with two main differences: (i) personalization fails completely in the tag similarity network, (ii) in the item and item-tag similarity networks personalization achieves considerably higher prediction performance than in the case of predicting items tagged with popular tags. This is because, in the case of long-tail items, the functions that are skewed towards the closest users, i.e., $f_{mul}$ and $f_{pow}$, $λ = 2$, will rank higher the items belonging to the closest users.
8. Other Related Work

The topic of search in a social setting has received increased attention lately. Studies and models of personalization of social tagging sites can be found in [19, 13, 11, 21]. Other studies have found that including social knowledge in scoring models can improve search and recommendation algorithms. In [8], personalization based on a similarity network is shown to outperform other personalization approaches and the non-personalized social search. A study on a last.fm dataset in [15] has found that incorporating social knowledge in a graph model system improves the retrieval recall of music track recommendation algorithms. An architecture for social data management is given in [2, 3], along with a framework for information discovery and presentation in social content sites. Another approach to rank resources in social tagging environments is CubeLSI [6], which uses a vector space model and extends Latent Semantic Indexing to include tags in the feature space of resources, in order to better match queries to documents. FolkRank [14] proposes a ranking model in social bookmarking sites, for recommendation and search, based on an adaptation of PageRank over the tripartite graph of users, tags and resources. It follows the intuition that a resource that is tagged with important tags by important users becomes important itself and, symmetrically, for tags and users. An alternative approach to social-aware search, using personalized PageRank, was presented in [4]. There, the same tripartite model of annotators, resources and annotations is used to compute measures of similarities between resources and queries, and to capture the social popularity of resources. However, none of these approaches incorporate the user-to-user relationships in their ranking model. In contrast, the social network is an integral part of the scoring model in our setting, if not the decisive one, while this network can have various semantics (e.g., tagging similarity, activity similarity or even trust).

The scoring model used in [18] is revisited in [22]. There, a textual relevance and a social influence score are combined in the overall scoring of items, the latter being computed as the inverse of the shortest path between the seeker and the document publishers. This model is also used in the context of top-k retrieval of spatial web objects [7], where a prestige-based relevance score is computed by combining the overall relevance of an object with its spatial distance.

9. Conclusions and Future Work

We considered in this paper top-k query answering in social bookmarking applications, proposing algorithms that have the potential to scale in real applications, in an online context where the social network, the tagging data and even the seekers’ search ingredients can change at any moment. Our solutions address the main drawbacks of previous approaches. With respect to applicability and scalability, we avoid expensive and hardly updatable pre-computations of proximity values, by an on-the-fly approach. We show that it is applicable to a wide family of functions for proximity computation in a social network. With respect to efficiency, we show that TOPKS is instance optimal in the exclusively social context and, via extensive experiments, that it performs significantly
better than the algorithm from previous literature. We also considered widely-applicable
approximate techniques, showing they have the potential to drastically reduce computa-
tion costs, while exhibiting high accuracy.

We see many directions for future work. As mentioned in the previous section, op-
timizing the branch choice heuristic is a promising direction that we plan to explore
further. Experimenting with other aggregation functions, probabilistic bounds using
statistics tailored to certain assumptions (e.g, for power-law distributions) or richer de-
scriptions for proximity vectors and term-frequencies are other important directions. We
are also investigating approaches for computing results in a distributed style, when one
has access to query results pertaining to various seekers, or when the same query is run
at various points in the network. Finally, we intend to adapt our approach to deal with
networks containing also negative links (e.g., trust / distrust networks).

References

[1] S. Amer-Yahia, M. Benedikt, L. Lakshmanan, and J. Stoyanovich. Efficient network
aware search in collaborative tagging sites. In VLDB, 2008.

[2] S. Amer-Yahia, J. Huang, and C. Yu. Building community-centric information
exploration applications on social content sites. In SIGMOD, 2009.

[3] S. Amer-Yahia, L. Lakshmanan, and C. Yu. Socialscope: Enabling information
discovery on social content sites. In CIDR, 2009.

[4] S. Bao, G. Xue, X. Wu, Y. Yu, B. Fei, and Z. Su. Optimizing web search using
social annotations. In WWW, 2007.

[5] A. Bernstein. Fully dynamic \((2 + \epsilon)\) approximate all-pairs shortest paths with fast
query and close to linear update time. In FOCS, 2009.

[6] B. Bi, S. Lee, B. Kao, and R. Cheng. An effective and efficient method for searching
resources in social tagging systems. In ICDE, 2011.

[7] X. Cao, G. Cong, and C. S. Jensen. Retrieving top-k prestige-based relevant spatial
web objects. In PVLDB, 2010.

[8] D. Carmel, N. Zwerdling, I. Guy, S. Ofek-Koifman, N. Har’el, I. Ronen, E. Uziel,
S. Yogevo, and S. Chernov. Personalized social search based on the user’s social
network. In CIKM, 2009.

[9] F. Chierichetti, R. Kumar, S. Lattanzi, M. Mitzenmacher, A. Panconesi, and
P. Raghavan. On compressing social networks. In KDD, 2009.

[10] E. W. Dijkstra. A note on two problems in connexion with graphs. Numerische
Mathematik, 1959.
A. Proof of Theorem 1

Proof. Since on each access to a user list, all items tagged by the respective user with any of the query terms are retrieved, the position in the proximity vector at any step in the run of the algorithm is not tag-dependent. So cost(\(A, D\)) is equal to the position \(p\) in the seeker’s proximity vector at the moment of \(A\)’s termination. Throughout the proof, we use the subscript \(p\) to denote the value of a given variable at step \(p\) in the execution of \(A\). We will use a proof argument similar in style to the one for NRA [12].

Let us assume that \(\text{TOPKS}_{\alpha=0}\) does not stop at position \(p\) (in the proximity vector) and that there exists an algorithm \(A \neq \text{TOPKS}_{\alpha=0}\) that does. Since \(\text{TOPKS}_{\alpha=0}\) does not stop at position \(p\), there exists an item \(r \notin \{D_p[1], \ldots, D_p[k]\}\) having \(\text{MAXCORE}_p(r, Q) > \text{MINSCORE}_p(D_p[k], Q)\), and \(\text{MINSCORE}_p(r, Q) \leq \text{MINSCORE}_p(D_p[k], Q)\).
\( \text{MinScore}_p(D_p[i], Q), \forall i \in \{1, \ldots, k\} \). If \( \text{MinScore}_p(r, Q) = \text{MinScore}_p(D_p[k], Q) \) then necessarily \( \text{MaxScore}_p(r, Q) \leq \text{MaxScore}_p(D_p[k], Q) \) (ties for pessimistic scores are broken by the optimistic ones, then arbitrarily for the optimistic scores).

In \( D \), we assume that at step \( p \) we have with \( \text{TOPK} = 0 \) in the current (unconsumed) position in each of the \( |Q| \) inverted lists \( IL(t_j) \) an item \( v_j \), necessarily not yet candidate. By definition, for any algorithm \( A \in S \), for any tag \( t_j \) of the input \( Q \), \( A \) is at most as advanced in the inverted list \( IL(t_j) \) as \( \text{TOPK} = 0 \). Without loss of generality, let us assume \( A \) is as advanced as \( \text{TOPK} = 0 \).

Towards a contradiction, showing that \( A \) is not sound over all possible inputs, we will construct an instance \( D' \), which is equal to \( D \) up to position \( p \). We consider the following two possible cases:

**Case 1:** \( A \) outputs \( r \) as one of the top-\( k \) item, i.e., there do not exist \( k \) items having a higher score than \( r \).

In \( D' \) will start from what \( A \) could have already read and used, including the items \( v_j = \text{top}_p(t_j) \) and the value \( \text{top}_p(H) \) (the proximity value of the \( p + 1 \) user).

\( D' \) will be such that \( \text{Score}(r, Q) = \text{MinScore}_p(r, Q) \), and \( \text{textscScore}(D_p[i], Q) = \text{MaxScore}_p(D_p[i], Q), \forall i \in \{1, \ldots, k\} \). Now, for each \( D_p[i] \), if \( \text{MaxScore}_p(D_p[i], Q) > \text{MinScore}_p(D_p[i], Q) \), i.e., we do not have \( D_p[i] \)'s final score at step \( p \), we assume the following in \( D' \). For each \( t_j \in Q \) for which \( tf(D_p[i], t_j) \) is unknown, we assume that we have \( D_p[i] \) in \( IL(t_j) \) after \( v_j \), with \( tf(D_p[i], t_j) = \text{top}_p tf(t_j) \). Also, for every \( t_j \in Q \) in the proximity vector, after \( p + 1 \), the next \( x_{ij} = \text{unseen}_users(D_p[i], t_j) \) values to \( \text{top}_p(H) \), making also \( D_p[i] \) present in each of these users’ lists for \( t_j \). By doing so, the exact score of each \( D_p[i], i \in \{1, \ldots, k\} \), is equal to the maximal possible one at step \( p \); after \( \text{max}_i, x_{ij} \) steps, all these \( k \) scores \( \text{Score}(D_p[i], Q) \) would be computed.

For item \( r \), for each \( t_j \in Q \) for which we do not have \( tf(r, t_j) \), since \( r \) must come later in \( IL(t_j) \) (after \( v_j \)), we can assume that \( tf(r, t_j) = \text{partial}_tf(r, t_j) \) (this makes \( \text{unseen}_users(r, t_j) = 0 \)). Also, for every \( t_j \in Q \) for which we do know \( tf(r, t_j) \), after the required \( \text{max}_i, x_{ij} \) proximity values set as described previously, we set the next \( \text{unseen}_users(r, t_j) \) in the proximity vector to 0, with each of these users having tagged \( r \) with \( t_j \). All this ensures that \( \text{MinScore}_p(r, Q) = \text{Score}(r, Q) \).

We can now contradict the correctness of algorithm \( A \), showing that \( \text{Score}(r, Q) < \text{Score}(D_p[i], Q) \) for all \( i \).

We have the following inequalities:

\[
\begin{align*}
\text{MinScore}_p(D_p[k], Q) & \geq \text{MinScore}_p(r, Q) \tag{A.1} \\
\text{MinScore}_p(D_p[k], Q) & \leq \text{MinScore}_p(D_p[i], Q), \forall i \tag{A.2} \\
\text{MinScore}_p(D_p[i], Q) & \leq \text{MaxScore}_p(D_p[i], Q), \forall i \tag{A.3}
\end{align*}
\]

If \( \text{MinScore}_p(r, Q) < \text{MinScore}_p(D_p[k], Q) \) then it follows from Eq. (A.1), (A.2), (A.3) that

\[
\text{Score}(r, Q) < \text{Score}(D_p[i], Q), \forall i.
\]

If \( \text{MinScore}_p(r, Q) = \text{MinScore}_p(D_p[k], Q) \) then, for each \( i \in \{1, \ldots, k\} \), if:
1. \( \text{MINSCORE}_p(D_p[k], Q) = \text{MINSCORE}_p(D_p[i], Q) \): we have \( \text{MAXSCORE}_p(r, Q) > \text{MINSCORE}_p(D_p[k], Q) \) and \( \text{MAXSCORE}_p(r, Q) \leq \text{MAXSCORE}_p(D_p[i], Q) \); it follows that \( \text{SCORE}(r, Q) < \text{SCORE}(D_p[i], Q) \).

2. \( \text{MINSCORE}_p(r, Q) < \text{MINSCORE}_p(D_p[k], Q) \): we have \( \text{MINSCORE}_p(D_p[k], Q) < \text{SCORE}(D_p[i], Q) \); it follows that \( \text{SCORE}(r, Q) < \text{SCORE}(D_p[i], Q) \).

Hence, in any possible configuration, \( r \) is not in the top-\( k \) result over \( D' \). But since \( D' \) and \( D \) are indistinguishable by algorithm \( A \), which stops at step \( p \) outputting \( r \) in the result, this contradicts \( A \)'s correctness.

Case 2: \( A \) does not output \( r \) as a top-\( k \) item, which means that \( A \) assumes that the final score of \( r \), \( \text{SCORE}(r, Q) \) is not in the top-\( k \) scores for \( D \).

\( D' \), undistinguishable from \( D \) up to position \( p \), will now be such that \( \text{SCORE}(r, Q) = \text{MAXSCORE}_p(r, Q) \) and \( \text{SCORE}(D_p[i], Q) = \text{MINSCORE}_p(D_p[i], Q) \), for each \( D_p[i] \in D_p \) s.t. \( D_p[i] \neq r \).

If \( r \)'s score at step \( p \) is not already the final one, i.e., \( \text{MAXSCORE}_p(r, Q) = \text{MINSCORE}_p(r, Q) \), we assume the following in \( D' \): for each tag \( t_j \in Q \) for which \( tf(r, t_j) \) is yet unknown, we assume that \( r \) comes later (after \( v_j \)) in \( \text{IL}(t_j) \), having \( tf(r, t_j) = \text{top}_p tf_p(t_j) \). Then, for every \( t_j \in Q \) we set in the proximity vector, after the \( p + 1 \) position, the next \( x_j = \text{unseen}_users(r, t_j) \) values to \( \text{top}_p(H) \), making also \( r \) present in each of these users' lists for \( t_j \).

By this, the exact score of \( r \) is equal to the maximal possible one at step \( p \); after \( \text{max}_j(x_j) \) steps, the score \( \text{SCORE}(r, Q) \) would be computed.

Symmetrically, for each each \( D_p[i] \in D_p \) s.t. \( D_p[i] \neq r \), and each \( t_j \in Q \) for which \( tf(D_p[i], t_j) \) is yet unknown, we assume that \( D_p[i] \) comes later (after \( v_j \)) in \( \text{IL}(t_j) \), having \( tf(D_p[i], t_j) = \text{partial}_p tf(D_p[i], t_j) \) (hence \( \text{unseen}_users(D_p[i], t_j) = 0 \)). Then, for every \( t_j \in Q \) for which we know \( tf(D_p[i], t_j) \), after the \( \text{max}_j(x_j) \) values set as described previously in the seeker’s proximity vector, we set the next \( y_{ij} = \text{unseen}_users(D_p[i], t_j) \) values to 0, making also \( D_p[i] \) present in each of these users' lists for \( t_j \). This construction ensures that, the exact score of each \( D_p[i] \) is equal to the minimal possible one at step \( p \); after \( \text{max}_i(y_{ij}) \) steps, all these scores \( \text{SCORE}(D_p[i], Q) \) would be computed.

Since we have that
\[
\text{SCORE}(r, Q) = \text{MAXSCORE}_p(r, Q) > \text{MINSCORE}_p(D_p[k], Q)
\]
and \( \text{MINSCORE}_p(D_p[k], Q) = \text{SCORE}(D_p[k], Q) \), given that for every item \( D_p[l] \), \( l > k \) s.t. \( D_p[l] \neq r \) we have \( \text{SCORE}(D_p[l], Q) < \text{MINSCORE}_p(D_p[k], Q) \), \( r \) should be among the top-\( k \) items in \( D' \). But since \( D' \) and \( D \) are indistinguishable by algorithm \( A \), which stops at step \( p \) without outputting \( r \) in the result, this contradicts \( A \)'s correctness.

In this proof, we have ignored \( \text{MAXSCORE}_p(\text{UNSEEN}(Q)) \) in the inequalities. The unseen items can be simulated by adding one virtual item \( i_v \) to \( D \), which does not exist and will never be encountered in user lists, with \( \text{MINSCORE}(i_v, Q) = 0 \) and \( \text{MAXSCORE}(i_v, Q) = \text{MAXSCORE}_p(\text{UNSEEN}(Q)) \). Then, the same proof argument applies to these items. \( \square \)
Figure 10: Abstract cost and running time comparison over the tag-similarity network and the $f_{\text{pow}}$, $\lambda = 1.1$ proximity function (red: top-10, yellow: top-20).

B. Other $\sigma^+$ functions

We present experimental results for the $f_{\text{pow}}$, $\lambda = 1.1$, in Figures 10, 11 and 12. While the results follow the same trend as those of $f_{\text{mul}}$, one can notice that the speedups achieved by the TOPKS variants are directly affected by the speed of the “drop” in proximity values. Generally, $f_{\text{mul}}$ values drop faster than those of $f_{\text{pow}}$. 
Figure 11: Abstract cost and running time comparison over the item-similarity network and the $f_{\text{pow}}$, $\lambda = 1.1$ proximity function (red: top-10, yellow: top-20).

Figure 12: Abstract cost and running time comparison over the item-tag similarity network and the $f_{\text{pow}}$, $\lambda = 1.1$, proximity function (red: top-10, yellow: top-20).