Massive Gauge Bosons in Yang-Mills Theory 
without Higgs Mechanism

Xin-Bing Huang*

Shanghai United Center for Astrophysics (SUCA),

Shanghai Normal University, No.100 Guilin Road, Shanghai 200234, China

Abstract

Two kinds of Yang-Mills fields are found upon the concepts of mass eigenstate and nonmass eigenstate. The Yang-Mills fields of the first kind were proposed by Yang and Mills, which couple to the mass eigenstates with the same rest mass, whose gauge bosons are massless. I find that there are second kind of Yang-Mills fields, which are constructed on a five-dimensional manifold. Only the nonmass eigenstates couple to the Yang-Mills fields of the second kind, which are the nonmass eigenstates as well and composed of mass eigenstates of gauge bosons. The mass eigenstates of the Yang-Mills fields of the second kind live in the four-dimensional spacetime, the corresponding gauge bosons of which may be massive. The $SU(2) \times U(1)$ gauge fields of the second kind are studied carefully, whose gauge bosons, which are the mass eigenstates, are the $W^\pm$, $Z^0$ and photon fields. The rest masses of $W^\pm$ and $Z^0$ obtained are the same as that given by the Glashow-Salam-Weinberg model of electroweak interactions. It is discussed that this model should be renormalizable.

PACS numbers: 11.15.-q, 11.10.Kk, 12.60.-i

*huangxb@shnu.edu.cn
55 years ago, Yang and Mills constructed the gauge field theory of non-Abelian group, which has become the most fundamental content in quantum field theory. Upon the principle that physical laws should be covariant under the local isospin rotation they proposed the $SU(2)$ Yang-Mills theory [1]. But they could not obtain the massive gauge bosons then. About 10 years later, an ingenious trick called the Higgs mechanism was independently invented by Higgs and Englert and Brout [2], who introduced a scalar field and the spontaneous symmetry broken mechanism of vacuum by fixing a vacuum expectation value of the scalar field and make the intermediate vector bosons obtain masses.

Based on the Yang-Mills fields and the Higgs mechanism, Glashow, Salam and Weinberg etc. proposed a renormalizable theory unifying the weak and electromagnetic interactions, namely $SU_L(2) \times U_Y(1)$ gauge theory [3]. Although this electroweak theory had predicted the masses of intermediate vector bosons, which were confirmed by experiments, there are still several unconfirmed predictions or conflicting phenomena in it. e.g. Firstly, experimenters have not found any hints of the Higgs boson till now; Secondly, a lot of recent experiments imply that the neutrinos should be massive and be mixed [4]. Here I discuss a model to give the massive gauge bosons in Yang-Mills theory without Higgs mechanism.

In this letter, the signature of spacetime metric $\eta_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ is $(+, -, -, -)$, and the spacetime coordinates are described by the contravariant four-vector $x^\mu$ ($\hbar = c = 1$ is adopted). In Ref.[5], the rest mass operator

$$\hat{m} = -i\partial_z$$

(1)

is defined by introducing an extra parameter $z$ besides of the spacetime coordinates $x^\mu$. From the mathematical point of view, $z$ and $x^\mu$ establish a five-dimensional manifold. The definition of the rest mass operator leads to a theorem that a field $\mathcal{F}(x, z)$ is massless if and only if $\mathcal{F}(x, z)$ is $z$-independent [5]. Hence the massless gravitational

\[\ddagger\]

I use $\partial_z \equiv \frac{\partial}{\partial z}$, $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$ and $\partial_\alpha \equiv \frac{\partial}{\partial x^\alpha}.$
field, the electromagnetic field and $SU(3)$ gauge fields in QCD are all $z$-independent, who live in the $z = 0$ brane of five-dimensional manifold.

The Lagrangian of a nonmass eigenstate $\Phi(x, z)$ of free spin-$\frac{1}{2}$ fields is of the form\footnote{$L_{1n}$, $L_{1m}$ denote the Lagrangian of one nonmass eigenstate or one mass eigenstate respectively. $L_{2n}$, $L_{2m}$ have the similar meanings.}

$$L_{1n} = \bar{\Phi}(x, z) \left( i \gamma^\mu \partial_\mu + i \partial_z \right) \Phi(x, z) ,$$

(2)

here $\bar{\Phi} \equiv \Phi^\dagger \gamma^0$ is called the spinor adjoint to $\Phi$. I indicated that the mass eigenstate of a spin-$\frac{1}{2}$ field satisfies $\Phi(x, z) = e^{imz} \phi(x)$ in Ref.[5], where $m$ is the rest mass. Therefore one can obtain the Lagrangian of the mass eigenstate of a free spin-$\frac{1}{2}$ field from (2), that is

$$L_{1m} = \bar{\phi}(x) \left( i \gamma^\mu \partial_\mu - m \right) \phi(x) ,$$

(3)

where $\bar{\phi} \equiv \phi^\dagger \gamma^0$ is the spinor adjoint to $\phi$.

Let’s consider a quantum field system in which two different nonmass eigenstates $\Psi_1(x, z)$ and $\Psi_2(x, z)$ of free spin-$\frac{1}{2}$ fields form an isospin doublet as follows

$$\Psi(x, z) = \left( \begin{array}{c} \Psi_1(x, z) \\ \Psi_2(x, z) \end{array} \right) .$$

(4)

So the Lagrangian of two nonmass eigenstates of free spin-$\frac{1}{2}$ fields is

$$L_{2n} = \bar{\Psi}(x, z) \left( i \gamma^\mu \partial_\mu + i \partial_z \right) \Psi(x, z) .$$

(5)

Here Let’s first consider a special case: if $\Psi_1(x, z)$ and $\Psi_2(x, z)$ are mass eigenstates with the same rest mass, then $\Psi_1(x, z) = e^{imz} \psi_1(x)$, and $\Psi_2(x, z) = e^{imz} \psi_2(x)$, where $m$ is the rest mass. I can therefore obtain $\Psi(x, z) = e^{imz} \psi(x)$ by defining

$$\psi(x) = \left( \begin{array}{c} \psi_1(x) \\ \psi_2(x) \end{array} \right) .$$

(6)

Hence the Lagrangian of a quantum field system where two mass eigenstates $e^{imz} \psi_1(x)$ and $e^{imz} \psi_2(x)$ form an isospin doublet is acquired from (5), namely

$$L_{2m} = \bar{\psi}(x) \left( i \gamma^\mu \partial_\mu - m \right) \psi(x) .$$

(7)
The largest inner gauge symmetry group in this system is obviously $SU(2) \times U(1)$. The total Lagrangian of this system reads

$$L_{2mt} = i \bar{\psi} \gamma^\mu (\partial_\mu - ig' T \cdot B_\mu) \psi - e \bar{\psi} \gamma^\mu A_\mu \psi - m \bar{\psi} \psi - \frac{1}{4} \tilde{F}_{\mu\nu} \cdot \tilde{F}^{\mu\nu} - \frac{1}{4} \tilde{E}_{\mu\nu} \tilde{E}^{\mu\nu},$$

where $e, g'$ are the coupling constants of $U(1)$ and $SU(2)$ gauge fields respectively, and the dot “$\cdot$” denotes a scalar product in the isospace. In this case, $T \cdot B_\mu$ means

$$T \cdot B_\mu = T^1 B^1_\mu + T^2 B^2_\mu + T^3 B^3_\mu,$$

where $T^a, a = 1, 2, 3$ are the generators of $SU(2)$ group, which are written as

$$T^a = \frac{1}{2} \tau^a,$$

where $\tau^a$ are the traceless matrices

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

known as the Pauli matrices. They obey the commutation relations

$$[\tau^a, \tau^b] = 2i \sum_{c=1}^{3} \varepsilon_{abc} \tau^c.$$

Here $\varepsilon_{abc}$ is the totally antisymmetry tensor in 3-dimensions.

In Yang-Mills theory [1], $T \cdot B_\mu$ is called the $SU(2)$ gauge field, and its field strength tensor is of the form

$$\tilde{F}^{\mu\nu} \cdot T = \partial_\mu (B_\nu \cdot T) - \partial_\nu (B_\mu \cdot T) - ig' [B_\mu \cdot T, B_\nu \cdot T].$$

Hence the field strength $\tilde{F}^{\mu\nu}$ satisfies

$$\tilde{F}^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + g' B_\mu \times B_\nu.$$

I use $A_\mu$ to denote the $U(1)$ gauge field. The field strength of the $U(1)$ gauge field is defined by

$$\tilde{E}^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$
We are very familiar with above $SU(2)$ gauge fields and $U(1)$ gauge field which have been the fundamental content of quantum field theories. The gauge bosons are massless. From a viewpoint of the rest mass operator, above $SU(2)$ gauge fields couple to the mass eigenstates with the same rest mass. I call this Yang-Mills fields the first kind. The $SU(3)$ gauge fields in $QCD$ obviously belong to this kind.

From now on I will study another kind of Yang-Mills fields carefully. To make the gauge invariance explicit, let’s formally introduce the extra dimension $x^4$ as follows

$$x^4 = -x_4 = z, \quad \gamma^4 = -\gamma_4 = 1.$$  

Hence the Lagrangian $L_{2n}$ is rewritten as

$$L_{2n} = i\bar{\Psi}(x, z)\gamma^\alpha \partial_\alpha \Psi(x, z), \quad \alpha = 0, 1, 2, 3, 4.$$  

Since two different nonmass eigenstates $\Psi_1(x, z)$ and $\Psi_2(x, z)$ form an isospin doublet, I can consider a local isospin rotation logically similar to what Yang and Mills did in their original paper. That is

$$\Psi'(x, z) = S(x, z)\Psi(x, z),$$  

where $S(x, z)$ is a $2 \times 2$ matrix. To make sure that the probability density $\bar{\Psi}(x, z)\Psi(x, z)$ is invariant under above rotation (18), the matrix $S(x, z)$ must be unitary with unit determinant

$$S^\dagger(x, z)S(x, z) = 1.$$  

All the matrices satisfy this condition generate the group $SU(2)$, which is a non-Abelian Lie group. The transformation (18) directly means that

$$\bar{\Psi}'(x, z) = \bar{\Psi}(x, z)S^\dagger(x, z).$$  

The matrix $S(x, z)$ can be written in the form

$$S(x, z) = \exp \left( i \sum_{a=1}^{3} \frac{\tau^a}{2} \Theta^a(x, z) \right).$$
To discuss the gauge invariance, here I introduce the gauge-invariant derivative

$$\hat{D}_\alpha = \partial_\alpha - ig_1 T \cdot W_\alpha(x,z),$$  \hspace{1cm} (22)

where $g_1$ is the coupling constant of $SU(2)$ gauge fields, and

$$T \cdot W_\alpha(x,z) = \sum_{a=1}^{3} T^a W^a_\alpha(x,z).$$  \hspace{1cm} (23)

Invariance requires that

$$(\partial_\alpha - ig_1 T \cdot W^\prime_\alpha) \Psi' = S(\partial_\alpha - ig_1 T \cdot W_\alpha) \Psi.$$  \hspace{1cm} (24)

Combining (18) and (24), I obtain the gauge transformation on $W_\alpha$:

$$T \cdot W^\prime_\alpha = S T \cdot W_\alpha S^{-1} + \frac{i}{g_1} S (\partial_\alpha S^{-1}).$$  \hspace{1cm} (25)

In analogy to the procedure of obtaining gauge invariant field strengths in electromagnetic case, I define now

$$F_{\alpha\beta} \cdot T = \sum_{a=1}^{3} F^a_{\alpha\beta} T^a = \hat{D}_\alpha(T \cdot W_\beta) - \hat{D}_\beta(T \cdot W_\alpha)$$

$$= \partial_\alpha(W_\beta \cdot T) - \partial_\beta(W_\alpha \cdot T) - ig_1 [W_\alpha \cdot T, W_\beta \cdot T]$$

$$= (\partial_\alpha W_\beta) \cdot T - (\partial_\beta W_\alpha) \cdot T + g_1 \sum_{abc} W^a_\alpha W^b_\beta \varepsilon_{abc} T^c$$

$$= (\partial_\alpha W_\beta - \partial_\beta W_\alpha + g_1 W_\alpha \times W_\beta) \cdot T.$$  \hspace{1cm} (26)

Therefore the isovector of field strengths is

$$F_{\alpha\beta} = \partial_\alpha W_\beta - \partial_\beta W_\alpha + g_1 W_\alpha \times W_\beta.$$  \hspace{1cm} (27)

One easily shows from the equation (25) that

$$F^\prime_{\alpha\beta} \cdot T = S F_{\alpha\beta} \cdot T S^{-1}.$$  \hspace{1cm} (28)

I obtain a gauge invariant Lagrangian by performing the trace over the isospin indices:

$$\mathcal{L}_{SU(2)} = -\frac{1}{2} \text{Tr}\{ (F_{\alpha\beta} \cdot T) (F^{\alpha\beta} \cdot T) \}$$

$$= -\frac{1}{4} F_{\alpha\beta} \cdot F^{\alpha\beta} = -\frac{1}{4} \sum_{a=1}^{3} F^a_{\alpha\beta} F^{a\alpha\beta}. \hspace{1cm} (29)$$
Considering the couplings between fermions and gauge bosons and the self-couplings of gauge bosons, one can get the complete Lagrangian as follows

\[
\mathcal{L}_{2nt} = \mathcal{L}_{2n} + \mathcal{L}_{int} + \mathcal{L}_{SU(2)}
\]

\[
= i\bar{\Psi}\gamma^{\alpha}(\partial_{\alpha} - ig_1T \cdot W_{\alpha})\Psi - \frac{1}{4}F_{\alpha\beta} \cdot F^{\alpha\beta}.
\]  

(30)

In order to build a foundation for setting up an electroweak model without Higgs mechanism, I discuss the \(SU(2) \times U(1)\) gauge fields in this letter. The \(U(1)\) gauge field that couples to a nonmass eigenstate has been studied in my preceding paper [5]. The Lagrangian (17) shows me that the maximal gauge groups for this quantum field system are \(SU(2) \times U(1)\). I have introduced the \(SU(2)\) gauge fields in this system, now I put in the \(U(1)\) gauge field. Let us multiply the nonmass eigenstates \(\Psi(x, z)\) by a local phase \(e^{i\Theta(x, z)}\), namely

\[
\Psi'' = e^{i\Theta(x, z)}\Psi, \quad \bar{\Psi}'' = e^{-i\Theta(x, z)}\bar{\Psi}.
\]  

(31)

According to the discussion in Ref.[5], I introduce the \(U(1)\) gauge field of the second kind, that is \(X_\alpha(x, z)\). Under the transformation of (31), \(X_\alpha(x, z)\) transforms as

\[
X''_\alpha(x, z) = X_\alpha(x, z) + \frac{1}{g_2}\partial_\alpha\Theta(x, z),
\]  

(32)

here \(g_2\) is the coupling constant of \(U(1)\). The strength tensor of \(U(1)\) gauge field is of the form

\[
E_{\alpha\beta}(x, z) = \partial_\alpha X_\beta(x, z) - \partial_\beta X_\alpha(x, z),
\]  

(33)

which is invariant under the transformations of (31) and (32). Therefore the total Lagrangian including the \(U(1)\) gauge field of the second kind is written as

\[
\mathcal{L}_{total} = \mathcal{L}_{2n} + \mathcal{L}_{int} + \mathcal{L}_{U(1)} + \mathcal{L}_{SU(2)}
\]

\[
= i\bar{\Psi}\gamma^{\alpha}(\partial_{\alpha} - ig_2X_\alpha - ig_1T \cdot W_{\alpha})\Psi
\]

\[
- \frac{1}{4}E_{\alpha\beta}E^{\alpha\beta} - \frac{1}{4}F_{\alpha\beta} \cdot F^{\alpha\beta}.
\]  

(34)

Obviously the total Lagrangian is also invariant under the transformations of (31) and (32). The gauge covariance requires that \(X_\alpha\) and \(T \cdot W_\alpha\) are all nonmass eigenstates.
Till now I merely constructed the $SU(2) \times U(1)$ gauge fields on a five-dimensional manifold, which is quite the same as the gauge fields proposed by Yang and Mills. Yes, from the five-dimensional point of view, the gauge bosons are massless in above discussed $SU(2) \times U(1)$ gauge fields since there are no mass term in the total Lagrangian (34). But, things will be quite different when I discuss them from the viewpoint of $z = 0$ brane.

I have pointed out that the gravitational field, the electromagnetic field and $SU(3)$ gauge fields in QCD are living in the $z = 0$ brane of five-dimensional manifold. Also I have proved that the $z$-independent electromagnetic field, gravitational field and $SU(3)$ gauge fields only couple to the mass eigenstates. Therefore I can find that the mass eigenstates coupled by the gravitation, the electromagnetic field and the gluon fields are also living in the $z = 0$ 4-dimensional brane.

It is indicated that the nonmass eigenstate is composed of mass eigenstates [5]. To discuss the physical properties of the mass eigenstates who compose the gauge fields $X_\alpha$ and $W_\alpha$, I write out the spacetime component and $z$-related component of gauge fields separately. Therefore

$$X_\alpha(x, z) \equiv (X_\mu(x, z), X_z(x, z)) ,$$

$$W_\alpha(x, z) \equiv (W_\mu(x, z), W_z(x, z)) .$$

(35) (36)

To list the components of $W_\alpha(x, z)$ manifestly, I rewrite (36) as

$$W^a_\alpha(x, z) \equiv (W^a_\mu(x, z), W^a_z(x, z)) , \quad a = 1, 2, 3 .$$

(37)

After that, the strength tensor $E_{\alpha\beta}(x, z)$ is correspondingly divided into three parts

$$E_{\mu\nu}(x, z) = \partial_\mu X_\nu(x, z) - \partial_\nu X_\mu(x, z) ,$$

$$E_{\mu z}(x, z) = -E_{z\mu}(x, z) = \partial_\mu X_z(x, z) - \partial_z X_\mu(x, z) ,$$

$$E_{zz}(x, z) = \partial_z X_z(x, z) - \partial_z X_z(x, z) \equiv 0 .$$

(38) (39) (40)

Surely one can also get the decomposition of the strength tensor $F_{\alpha\beta}(x, z)$ as follows

$$F_{\mu\nu}(x, z) = \partial_\mu W_\nu - \partial_\nu W_\mu + g_1 W_\mu \times W_\nu ,$$

(41)
\[ F_{\mu z}(x, z) = -F_{z\mu}(x, z) = \partial_\mu W_z - \partial_z W_\mu + g_1 W_\mu \times W_z, \quad (42) \]

\[ F_{zz}(x, z) \equiv 0. \quad (43) \]

Now let’s consider the movement of gauge bosons. Firstly, the interaction term \( L_{\text{int}} \) in total Lagrangian (34) shows that the movement of gauge bosons is decided by the momentum of \( \Psi \) and \( \bar{\Psi} \). The \( \Psi \) and \( \bar{\Psi} \) are nonmass eigenstates, who are composed of mass eigenstates that are living in the \( z = 0 \) brane and moving along the \( z = 0 \) brane, hence gauge bosons must move along the \( z = 0 \) brane. Secondly, once the gauge bosons are produced, they are constrained by gravitation, which is living in the \( z = 0 \) brane. Consequently

\[ W_z(x, z) = 0, \quad X_z(x, z) = 0. \quad (44) \]

Hence the equations (39) and (42) reduce to

\[ E_{\mu z}(x, z) = -E_{z\mu}(x, z) = -\partial_z X_\mu(x, z), \quad (45) \]

\[ F_{\mu z}(x, z) = -F_{z\mu}(x, z) = -\partial_z W_\mu(x, z). \quad (46) \]

The spacetime components \( X_\mu(x, z) \) and \( W_\mu(x, z) \) are nonmass eigenstates, which are linear combinations of mass eigenstates. I define the mass eigenstates of bosons \( W^\pm \) by

\[ W^+_\mu(x, z) = \frac{1}{\sqrt{2}} \left( W^1_\mu(x, z) - i W^2_\mu(x, z) \right), \quad (47) \]

\[ W^-_\mu(x, z) = \frac{1}{\sqrt{2}} \left( W^1_\mu(x, z) + i W^2_\mu(x, z) \right). \quad (48) \]

When the mass eigenstates of \( W^\pm \) are expressed by

\[ W^+_\mu(x, z) = e^{im_w z} W^+_\mu(x), \quad W^-_\mu(x, z) = e^{im_w z} W^-_\mu(x), \quad (49) \]

\( m_w \) being the rest mass of \( W^\pm \), the nonmass eigenstates \( W^1_\mu(x, z) \) and \( W^2_\mu(x, z) \) are manifestly given by

\[ W^1_\mu(x, z) = \frac{1}{\sqrt{2}} \left( W^+_\mu(x, z) + W^-_\mu(x, z) \right) \]

\[ = \frac{1}{\sqrt{2}} e^{im_w z} \left( W^+_\mu(x) + W^-_\mu(x) \right), \quad (50) \]
and
\[ W_\mu^2(x, z) = \frac{i}{\sqrt{2}} \left( W_\mu^+(x, z) - W_\mu^-(x, z) \right) \]
\[ = \frac{i}{\sqrt{2}} e^{imzw} \left( W_\mu^+(x) - W_\mu^-(x) \right). \tag{51} \]

The boson fields \( W_\mu^0(x, z), W_\mu^-(x, z), Z_\mu(x, z) \) and photon field \( A_\mu(x) \), which are mass eigenstates, constitute a complete Hilbert space. From Ref.[5], I know that
\[ Z_\mu(x, z) = e^{imzw} Z_\mu(x), \tag{52} \]
here \( m_Z \) is the rest mass of boson \( Z^0 \). The nonmass eigenstates \( W_\mu^3(x, z) \) and \( X_\mu(x, z) \) are the linear combinations of \( Z_\mu(x, z) \) and \( A_\mu(x) \), namely
\[ W_\mu^3(x, z) = \sin \theta_W A_\mu + \cos \theta_W Z_\mu(x, z) \]
\[ = \sin \theta_W A_\mu + \cos \theta_W e^{imzw} Z_\mu, \tag{53} \]
\[ X_\mu(x, z) = \cos \theta_W A_\mu - \sin \theta_W Z_\mu(x, z) \]
\[ = \cos \theta_W A_\mu - \sin \theta_W e^{imzw} Z_\mu, \tag{54} \]
where \( \theta_W \) is the Weinberg angle.

The nonmass eigenstates \( W_\alpha^a(a = 1, 2, 3) \) in \( T \cdot W_\alpha \) must have the same rest mass because of two reasons: Each \( W_\alpha^a \) plays the quite equal role in gauge field \( T \cdot W_\alpha \); The model must be \( SU(2) \) gauge invariant. Consequently
\[ m_{W_\mu^1} = m_{W_\mu^2} = m_{W_\mu^3}. \tag{55} \]

In Ref.[5], it is indicated that the rest mass squared of nonmass eigenstate of vector fields can be calculated, that is
\[ m_{V_\mu}^2 = \sum_{j=1}^{n} a_j a_j^* m_j^2 \tag{56} \]
is right if and only if
\[ V_\mu = \sum_{j=1}^{n} a_j [V_\mu]_j = \sum_{j=1}^{n} a_j e^{im_jz} [V_\mu]_j. \tag{57} \]
Then one can easily obtain the rest masses of $W_1^\mu$ and $W_2^\mu$ from (50) and (51) respectively

$$m^2_{W_1^\mu} = m^2_{W_2^\mu} = m^2_W,$$

(58)

also get the rest mass of $W_3^\mu$ from (53)

$$m^2_{W_3^\mu} = m^2_Z (\cos \theta_W)^2.$$

(59)

Therefore combining (55), (58) and (59), I obtain the following relation

$$m_W = m_Z \cos \theta_W.$$

(60)

In the total Lagrangian (34), the kinetic terms of Fermions and the interaction terms will be discussed carefully in my forthcoming paper [6], in this letter I only discuss the self-couplings of $SU(2) \times U(1)$ gauge fields, namely the terms $\mathcal{L}_{U(1)} + \mathcal{L}_{SU(2)}$ in (34). It has been pointed out that the gauge bosons in my model merely propagate along the $z = 0$ brane, therefore $W_z(x, z) = 0, X_z(x, z) = 0$. In this case, substituting (54) into (45) yields

$$E_{z\mu} = -im_Z \sin \theta_W e^{im_Z z} Z_{\mu}.$$

(61)

Substituting (50), (51) and (53) into (46) respectively, I obtain

$$F_{z\mu}^1 = \frac{i}{\sqrt{2}} m_W e^{im_Z z} (W^+_\mu + W^-_\mu),$$

(62)

$$F_{z\mu}^2 = -\frac{1}{\sqrt{2}} m_W e^{im_Z z} (W^+_{\mu} - W^-_{\mu}),$$

(63)

$$F_{z\mu}^3 = im_Z \cos \theta_W e^{im_Z z} Z_{\mu}.$$

(64)

Substituting (61), (62), (63) and (64) into $\mathcal{L}_{U(1)} + \mathcal{L}_{SU(2)}$, I find that the self-coupling terms of $SU(2) \times U(1)$ gauge fields become

$$\mathcal{L}_{U(1)} + \mathcal{L}_{SU(2)}$$

$$= -\frac{1}{4} E_{\alpha\beta} E^{\alpha\beta} - \frac{1}{4} F_{\alpha\beta} \cdot F^{\alpha\beta}$$
\[
\frac{1}{4} E_{\mu\nu} E^{\mu\nu} - \frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{2} (E_{z\mu} E^{z\mu} + F_{z\mu} \cdot F^{z\mu})
\]

\[
= -\frac{1}{4} E_{\mu\nu} E^{\mu\nu} - \frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{2} \left( E_{z\mu} E^{z\mu} + F_{z\mu}^{1} F^{1z\mu} + F_{z\mu}^{2} F^{2z\mu} + F_{z\mu}^{3} F^{3z\mu} \right)
\]

\[
= -\frac{1}{4} E_{\mu\nu} E^{\mu\nu} - \frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} + \frac{1}{2} m_Z e^{2i m_Z z} Z_\mu Z^\mu + m_W e^{2i m_W w} W^+ W^- .
\]

It is indicated that all the mass eigenstates coupled by the gravitation, the electromagnetic field and the gluon fields are living in the \( z = 0 \) brane. Hence, expressed by the mass eigenstates in the \( z = 0 \) brane, the self-coupling terms of \( SU(2) \times U(1) \) gauge fields reduce to

\[
\mathcal{L}_{U(1), z=0} + \mathcal{L}_{SU(2), z=0}
\]

\[
= -\frac{1}{4} E_{\mu\nu} E^{\mu\nu} - \frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + m_W^2 W^+ W^- ,
\]

where \( F_{\mu\nu} \equiv \{ F_{\mu\nu}^1, F_{\mu\nu}^2, F_{\mu\nu}^3 \} \), and \( E_{\mu\nu} \) is formulated by

\[
E_{\mu\nu}(x) = \partial_\mu X_\nu(x) - \partial_\nu X_\mu(x) ,
\]

and \( F_{\mu\nu} \) is given by

\[
F_{\mu\nu}(x) = \partial_\mu W_\nu(x) - \partial_\nu W_\mu(x) + g_1 W_\mu(x) \times W_\nu(x) ,
\]

in which \( W_\mu(x) \equiv \{ W_\mu^1(x), W_\mu^2(x), W_\mu^3(x) \} \). The four-dimensional fields \( X_\mu(x) \) and \( W_\mu(x) \) are composed of mass eigenstates which are constrained in the \( z = 0 \) brane. From (50), (51), (53) and (54), one can easily obtain the expressions of them in the following

\[
W_\mu^1(x) = \frac{1}{\sqrt{2}} \left( W_\mu^+(x) + W_\mu^-(x) \right) ,
\]

\[
W_\mu^2(x) = \frac{i}{\sqrt{2}} \left( W_\mu^+(x) - W_\mu^-(x) \right) ,
\]

\[
W_\mu^3(x) = \sin \theta_W A_\mu(x) + \cos \theta_W Z_\mu(x) ,
\]

\[
X_\mu(x) = \cos \theta_W A_\mu(x) - \sin \theta_W Z_\mu(x) .
\]

Obviously they have the same forms as the definitions of gauge bosons of \( SU(2) \times U(1) \) gauge fields in the Glashow-Salam-Weinberg model [7]. The fields \( W_\mu^+(x), W_\mu^-(x), \)
$Z_\mu(x)$ and $A_\mu(x)$ in above expressions are mass eigenstates that are constrained in the $z = 0$ brane.

The $SU(2) \times U(1)$ gauge fields of the second kind merely couple to the nonmass eigenstates, which are the nonmass eigenstates as well, hence cannot be observed directly. The nonmass eigenstates of gauge fields are composed of the mass eigenstates that are constrained in the $z = 0$ brane. When I reexpress the Lagrangian $\mathcal{L}_{U(1)} + \mathcal{L}_{SU(2)}$ by the mass eigenstates of gauge bosons who live in four-dimensional spacetime, I find that the fields $W_\mu^+(x), W_\mu^-(x)$ and $Z_\mu(x)$ can be treated as massive gauge bosons from the four-dimensional point of view since their mass terms automatically appear in the four-dimensional Lagrangian (66).

The mass terms of gauge bosons who are living in four-dimensional spacetime aren’t inserted by hands, which is produced automatically. From five-dimensional point of view, the $SU(2) \times U(1)$ gauge fields of the second kind are massless, which are the usual gauge fields that we are very familiar with, since there are no mass term in the total Lagrangian (34). Therefore, the gauge fields in this model should be renormalizable.

let me explicitly explain this model again: The general equation of a nonmass eigenstate of spin-$\frac{1}{2}$ fields is built on a five-dimensional manifold, therefore the gauge fields of the second kinds, who couple to the nonmass eigenstates of spin-$\frac{1}{2}$ fields, are constructed on a five-dimensional manifold as well. But the nonmass eigenstates are composed of mass eigenstates, which are physically observable. The mass eigenstates are merely coupled by the electromagnetic field, the gravitation and the gluon fields, who are living in the $z = 0$ brane. Hence the initial momentum of the nonmass eigenstates of spin-$\frac{1}{2}$ fields are along the $z = 0$ brane, which decides that the gauge fields of the second kind must propagate along the $z = 0$ spacetime as well. When the gauge fields of the second kind are reexpressed by their mass eigenstates that living in the four-dimensional spacetime, it is found that the gauge bosons who are mass eigenstates can be treated as massive vector fields.
References

[1] C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

[2] P. W. Higgs, Phys. Rev. Lett. 13, 508 (1964); P. W. Higgs, Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964).

[3] S. L. Glashow, Nucl. Phys. 22, 579 (1960); J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127, 965 (1962); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[4] For recent reviews on neutrino masses and mixing angles, see Z. Z. Xing, Int. J. Mod. Phys. A 19, 1 (2004); R. D. McKeown and P. Vogel, Phys. Rept. 394, 315 (2004); M. C. Gonzalez-Garcia and Y. Nir, Rev. Mod. Phys. 75, 345 (2003); M.-C. Chen and K. T. Mahanthappa, Int. J. Mod. Phys. A 18, 5819 (2003).

[5] X.-B. Huang, “Nonmass Eigenstates of Boson and Fermion Fields”, [arXiv:hep-th/0906.2441].

[6] X.-B. Huang, “An Electroweak Model without Higgs Mechanism”, in preparation.

[7] W. Greiner and B. Müller, Gauge Theory of Weak Interactions, (3rd. edition), (Springer-Verlag, 2000).