Joint Covering Congestion Rents in Multi-Area Power Systems Considering Loop Flow Effects

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Abstract—We consider the problem of how multiple areas should jointly cover congestion rents of internal and tie-lines in an interconnected power system. A key issue of our concern is the loop flow problem, which represents discrepancies between scheduled and actual power flow distributions because electric power does not always flow along the most direct paths of transactions. We employ generalized coordinated transaction scheduling (GCTS) for interchange scheduling, which can eliminate dispatch errors caused by loop flow effects and asymptotically converge to the joint economic dispatch (JED) under ideal assumptions. Subsequently, we propose approaches for each area to recover multipliers of the global GCTS model, as well as quantify and price its contribution to line congestions in a distributed manner. Thereby, all areas and interface bids can jointly cover congestion rents of internal and tie-lines with their merchandise surpluses and profits. Simulations demonstrate the effectiveness of the proposed approach of locational marginal prices (LMP) recovery and joint covering congestion rents.

Index Terms—Loop flow effects, congestion rent, generalized coordinated transaction scheduling, locational marginal price recovery.

I. INTRODUCTION

A. Motivation

In our efforts towards a carbon-neutral society, power system interconnections will play a more and more important role [1], [2], [3]: The geographical distributions of renewable energies and load centers are usually opposite, and we also need wide-area coordination and mutual support to guard against extreme weather conditions when wind or solar power is not available in a certain area [4], [5]. Therefore, it is necessary to develop effective methods for interchange scheduling to distribute resources in a larger area.

Fig. 1. Interconnected two-area power system with tie-line congested.

The design of interchange scheduling mechanisms relies on the structure of regional electricity markets [6]. In the U.S., the standard design of regional electricity markets is based on the economic dispatch model and locational marginal prices (LMP) [7], [8]. For a single-area electricity market, such a design based on the lossless DC optimal power flow (DCOPF) model leads to many attractive properties, with one of them being revenue adequacy: The merchandise surplus collected by each ISO through LMP is equal to the congestion rent, which can adequately support any feasible financial transmission right (FTR) auctions [9], [10]. Unfortunately, such an important property does not automatically hold for an interconnected multi-area power system. Take the two-area system in Fig. 1 as an example, assume one of the two tie-lines is congested and there is no other congestion. Area 2 imports active power from area 1. Namely, area 2 pays area 1 for this imported power, forming the cash flow. It is an open question how should the two areas jointly cover the congestion rent of that tie-line. In the implementation of coordinated transaction scheduling (CTS), New York Independent System Operator (NYISO) and ISO-New England (ISO-NE) split tie-line congestion rent by halves [5], which may not be the best solution under various operation conditions.

To answer the question above, we need to decompose the power flow of congested lines into contributions of different kinds of transactions and price each component properly. In particular, not only inter-regional power transactions will flow through tie-lines, a portion of internal transactions in area 1 and 2 will also wheel through the other area physically, contributing positively or negatively to the congestion. Such phenomena are referred to as loop flows, which are the unintended or unscheduled flows of electricity through a line or system.
according to the definition provided by the Federal Energy Regulatory Commission (FERC) [11]. Loop flow effects have already increased economic costs and caused security problems in electricity markets of the U.S. [12], Europe [13], and China [14]. Determining how to model the impact of the loop flow problem is hitherto an open question.

B. Literature Review

In the U.S., Southwest Power Pool (SPP) sets phase angle regulators (PARs) between its border with Western Interconnection [15]. And in Europe, European Commission tries to solve this problem by dividing pricing zones, optimizing the network topology, and setting PARs [16]. Note that loop flow may be caused by load forecast errors and inadequate access to data [17]. However, these works have not clearly explained how the loop flow is generated associated with independent modeling of each ISO without proper coordination, thus may not be able to eliminate the loop flow problem.

Research on the coordinated dispatch between areas can be divided into two types. First, distributed algorithms to solve the joint economic dispatch (JED) problem have been studied. Lagrange relaxation-based algorithm [18], consensus-based fully distributed algorithm [19], marginal equivalent decomposition algorithm [20], and critical region exploration method [21] have been developed. Traditional distributed algorithms may suffer from slow convergence and computation speed, whereas recent advances have shown much faster convergence [20], [21], [22].

Second, novel market mechanisms for multi-area coordinated dispatch have been studied in multiple ISOs. Midcontinent Independent System Operator (MISO)-PJM Interconnection (PJM), MISO-SPP, and NYISO-PJM have respectively made joint operation agreements on the market-to-market (M2M) coordination to manage flowgate congestion in real-time operations [23], [24]. Approaches of tie optimization (TO) [5] and CTS [25] have been developed, but they rely on the proxy model which may lead to substantial errors [26]. Take MISO-PJM as an example, they implemented M2M coordination in 2005 [27] and CTS in 2017 [28]. Paper [29] further puts forward generalized CTS (GCTS), which eliminates the need for proxy buses and can asymptotically converge to JED results under certain conditions. However, in all of these studies, how to jointly cover congestion rent in multi-area power systems has not been adequately addressed.

C. Contributions

This paper mainly focuses on the market mechanisms and discusses how multiple areas should properly and explicitly cover congestion rents. In particular, contributions of this paper include the following:

i) Post-optimization approaches are proposed for each area to recover global LMPs. The results are the same as LMPs calculated by the centralized GCTS model.

ii) Loop flow effects are decomposed into different categories. And contributions of each area and interface bids to line congestions are quantified.

iii) Congestion rents are covered proportionally according to the power flow contribution to line congestions.

The rest of this paper is organized as follows: Section II presents preliminary models for the interchange scheduling, including CTS and GCTS. Section III describes the framework of jointly covering congestion rents of both internal lines and tie-lines. Section IV presents a distributed LMP recovery method and its properties. Section V settles each area and interface bids and jointly covers congestion rents in the global system. And settlement properties are also presented. Finally, numerical results and concluding remarks are shown in Sections VI and VII.

II. INTERCHANGE SCHEDULING SCHEMES

Since loop flows are closely related to interchange scheduling mechanisms, we first review state-of-the-art market mechanisms CTS and GCTS in this section.

A. Coordinated Transaction Scheduling

Between the borders of many ISOs in the U.S., CTS gets implemented. Market participants bid to buy power at the proxy bus in one area and sell the same amount at the proxy bus in the other area. CTS schedules the interchange level between two neighboring areas with the following steps:

i) Compare LMPs of proxy buses in the two areas when the interchange level is zero. The direction of inter-regional transactions will be from the lower-LMP area to the higher-LMP area.

ii) Each area generates its supply/demand curve (ISO1 as the black curve and ISO2 as the blue curve in Fig. 2) as its proxy-bus LMP under different interchange levels.

iii) The coordinator, which is one of the two ISOs, collects these supply and demand curves, as well as interface bids from market participants. A modified demand curve (green curve in Fig. 2) is obtained by subtracting the aggregated interface bid curve (red curve) from the original demand curve.

iv) The interchange will be set as the intersection of the modified demand curve and the supply curve, if it does not exceed the interchange capacity limit, as in Fig. 2(a). Otherwise, the interchange level will be set as its capacity limit, as in Fig. 2(b).

We further give the CTS market coordination processes in Fig. 3. In each time interval, interchange scheduling, internal
dispatch of each area, and LMP calculation are operated sequentially. The core design of CTS is to incorporate external market participants to buy and sell at proxy buses of neighboring areas (See Fig. 4). In essence, it is market participants who decide the interchange level and facilitate inter-regional transaction scheduling. Hence, the neutrality of ISOs will be preserved. However, CTS may suffer from the modeling errors from proxy buses, which may lead to inefficient or infeasible tie-line schedules and exacerbate the loop flow problem [30].

B. Generalized Coordinated Transaction Scheduling

In this paper, we employ the GCTS scheme in [29] as the basic interchange scheduling tool. GCTS preserves the key design of CTS. However, it eliminates proxy buses and allows market participants to bid on any pair of boundary buses. Namely, each interface bid $\bar{R}_b$ is a trio consisting of

$$\bar{R}_b \triangleq \{ < B_{pm}, B_{qn} >, \Delta \pi_b, \bar{\pi}_b \},$$

where $< B_{pm}, B_{qn} >$ represents that interface bidder $b$ is to buy from bus $p$ of area $m$ and sell to bus $q$ of area $n$, constant $\Delta \pi_b$ is the anticipated price difference between nodes $p$ and $q$, and $\bar{\pi}_b$ is maximum power quantity.

Without loss of generality, we assume that boundary buses have no generators and loads, and each internal bus has one generator and one load. Note that a boundary bus with a generator can be split into a fictitious internal bus with a generator and a boundary bus without injection. Based on bids submitted by market participants in the form of (1), the market-clearing model of GCTS is

$$\min_{g_i} \quad C = \sum_{i=1}^{N} c_i^T g_i + \Delta \pi^T s$$

$$1^T g = 1^T d$$

$$\left[ S_{i,i} \right] \times (g_i - d_i) + \left[ S_{i,-B_i} \right] M_{-i} s \leq \left[ f_i \right]$$

$$g_i \leq \bar{g}_i \leq \bar{g}_i$$

$$0 \leq s \leq \bar{s}$$

$$\rho_i,$$

where decision variables are generators output $g$ and cleared quantities of interface bids $s$. Vectors $c_i, \Delta \pi, d_i, f_i, g_i, \bar{g}_i$ represent, respectively, prices of the generators, anticipated price difference of interface bids, load powers, capacity limits of transmission lines, limits of generators output of area $i$. Vector $f_i$ represents the capacity limits of tie-lines. Vector $s$ represents the capacity limits of interface bids. Vector $\bar{g}_i$ represents the nodal admittance matrix associated with boundary and internal buses in area $i$, and vector $B_{iq}$ represents the self-admittance matrix of area $i$. Note that those two matrices are composed of reciprocals of branch reactance. Vector $S_{i,i}$ represents the shift factor between the internal lines and nodes of area $i$. Vector $S_{i,-B_i}$ represents the shift factor between the internal lines and boundary nodes of all areas except area $i$. Vector $S_{i,-B_i}$ represents the shift factor between the tie-lines and internal nodes of area $i$.

Note that the market coordination processes of GCTS are the same as those of CTS (See Fig. 3). The internal dispatch model of GCTS is given as follows.

$$\min_{g_i} \quad c_i^T g_i$$

$$1^T g_i = 1^T d_i^R + 1^T M_i s$$

$$S_{i,i}(g_i - d_i^R) + S_{i,-B_i} M_{-i} s \leq f_i$$

$$g_i \leq \bar{g}_i \leq \bar{g}_i$$

$$\rho_i,$$

where $d_i^R$ represents the real-time loads of area $i$. Other entries in (8)–(12) are similarly defined as model (2)–(7). For simplicity, we assume that there is no real-time prediction error, i.e., $d_i^R = d_i$. Thus the optimal primal and dual variables for (2)–(7) are also optimal for (8)–(12) because they satisfy KKT conditions for the latter problem.

LMP is defined as the marginal increase in the overall system costs for the additional unit active power consumption at each bus. To calculate the LMPs of the buses in area $i$, we formulate the Lagrangian function $L$ of the GCTS model (2)–(7) as

$$L = C(g_i, s) - \lambda (1^T g - 1^T d_i)$$

$$+ \mu_i^T (S_{i,i}(g_i - d_i) + S_{i,-B_i} M_{-i} s - f_i)$$

$$\lambda, \mu_i,$$
\[ + \mu_i T (S_{i,i} (g_i - d_i) + S_{i,-j} M_{i,-s} - f_i) + \eta_i T (g_i - g_i) + \eta_i T ^* (g_i - g_i) + \eta_i T (-s) + \eta_i T (s - s) - \mu_i T (-B_{i,i} B_{i,i}^{-1} (g_i - d_i) - M_{i,s}) . \]

According to the Envelope theorem, we calculate the LMPs of area \( i \) as

\[ \nabla d_i C^+ = \frac{\partial L}{\partial d_i} = \lambda \times 1 - S_{i,i} \gamma T \mu_i + (B_{i,i} B_{i,i}^{-1}) \rho_i. \]  

(14)

Note that the second term of LMP represents marginal congestion price, which is consistent with the standard lossless DCOPF model [8].

The sensitivity of the global optimal cost with respect to the interface bid quantities cleared on the boundary of area \( i \) is equal to the LMP of boundary nodes, which is calculated as

\[ \nabla M_{i,s} C^* = \frac{\partial L}{\partial M_{i,s}} = \lambda \times 1 - S_{i,i} \gamma T \mu_i + \nabla M_{i,s} C^+ \times M_{i,s} . \]  

(15)

Vector \( S_{i,B_i} \) represents the shift factor between the internal lines and boundary nodes of area \( i \). Vector \( S_{i,B_i} \) represents the shift factor between the tie-lines and boundary nodes of area \( i \).

Further, the merchandise surplus of area \( i \), denoted by \( MS_i \), is obtained as

\[ MS_i = \nabla d_i C^+ (d_i - g_i) + \nabla M_{i,s} C^+ \times M_{i,s} . \]  

(16)

The arbitrage revenue of a given interface bid buying from area \( j \) and selling to area \( i \), denoted by \( R_{i,j} \), is obtained as

\[ R_{i,j} = \nabla M_{i,s_i,j} C^+ \times M_{i,s_i,j} - \nabla M_{i,s_i,j} C^+ \times M_{i,s_i,j} . \]  

(17)

In practice, ISOs may need more than one system for scheduling and pricing. As far as we can see, there are at least two reasons: 1) Regional markets are cleared independently, and interface bids do arbitrage on the boundary between areas. It is not trivial to connect current dispatch models to the joint optimal dispatch model of the super ISO [5]; 2) There is a time sequence problem for internal and interchange optimization problems. In general, interchange scheduling is optimized first, followed by internal dispatch. Note that load power may change between the interchange scheduling and internal dispatch, making it necessary to have more than one model to be solved sequentially.

For the GCTS model (2)\((\sim)(7)\), market participants are allowed to bid on any pair of boundary nodes. Thus we no longer need proxy buses. Cleared interface bids are matched with equivalent power injections \( M_s \) via (7). If we assume there is no load power deviation between the interchange scheduling and internal dispatches, it can be proven that its results are equal to the joint optimal dispatch of a “super ISO” [29]. Thus it is possible for us to focus on a single model of joint optimal dispatch as (2)\((\sim)(7)\).

III. FRAMEWORK

Loop flow occurs as power flow is strictly governed by physical laws, such as Kirchoff’s laws and Ohm’s law, and may deviate from the direct path of financial transactions between market participants. We take the three-area system in Fig. 5 as a general example to explain the causes of loop flow issues. When area \( i \) exports power to area \( j \) (e.g., \( s_{i,j} \)), physical power flows corresponding to that may wheel through internal lines of area \( m \) and tie-lines \( l_{i,m}, l_{j,m} \). Similarly, when area \( i \) schedules its interchange scheduling and internal dispatch, making it not be affected much when multiple time periods are considered. As long as the generator dispatch results for the dynamic scheduling model are obtained, we can recover LMPs and do further settlement.

**Fig. 5.** An illustrative example of loop flow effects.

**Step 1:** Recover LMPs by (14) and (15) in a distributed manner.

**Step 2:** Each area settles internal and interface bids, leading to its merchandise surplus.

**Step 3:** Distribute the congestion rent among areas and interface bids according to their contributions.

The second step above is a standard process. Next, we will further explain details of steps 1 and 3, respectively. Since our work is mainly about pricing and jointly covering congestion rent based on given optimization results, it will not be affected much when multiple time periods are considered. As long as the generator dispatch results for the dynamic scheduling model are obtained, we can recover LMPs and do further settlement.
IV. LMP RECOVERY IN A DISTRIBUTED MANNER

Multiple distributed optimization algorithms can be used to obtain the optimal primal solution to (2) in a distributed manner. However, for many of them, each ISO may not be able to obtain optimal dual variables automatically. In this section, we discuss how each area can recover LMPs in its territory that are the same as the global GCTS model with local and boundary equivalent information only. We first adopt linear cost functions for generators, and the case of quadratic cost functions will be discussed separately in Section C.

Namely, assumptions are made as follows:

i) We assume there is no degeneracy in the GCTS model. Degeneracy in each area’s internal dispatch problem, however, is still possible under certain conditions. When degeneracies in global problems happen, the method in [31], [32] can be applied.

ii) Each ISO has access to its internal network and equivalent network from all other areas. For example, ISO \( i \) considers a network as shown in Fig. 6.

For simplicity, we start with a simple case without any line congestion to illustrate the idea of distributed LMP recovery.

A. An Illustrative Example

Suppose there is no congestion and no artificial price gap over tie-lines between different areas caused by interchange scheduling. In that case, only one marginal unit decides the same LMP of the entire three-area system. Without loss of generality, we assume the marginal unit is in area \( i \). From ISO \( i \)’s point of view, it knows the vector of LMPs of all nodes which is equal to the bidding price of the marginal unit as shown in Fig. 7(a). For ISO \( j \) or \( m \), however, the LMP is not uniquely determined since all generators are reaching either their upper or lower limits (See Fig. 7(b)). That is to say, LMP can be selected as any value in the interval \([p_1, p_2]\) according to the price signal given by the external network, which does not contradict the local problems of ISO \( j \) and \( m \). Therefore, in this case, ISO \( i \) will share the price of its marginal unit to other areas so that they can perceive the vector of LMPs of all nodes, where all entries are equal to the price of the marginal unit in area \( i \). From the global point of view, when the LMP recovery approach is not adopted, different nodes have different prices even if the whole network is not congested.

Fig. 7. Marginal pricing when no degeneracy in the global system. (a) ISO\( i \), (b) ISO\( j \) and ISO\( m \).

B. General LMP Recovery Method

For more general cases with possible congestions, we need to calculate the global shift factor matrix in a distributed manner before recovering LMPs. Without loss of generality, we assume that the global phase angle reference bus is selected on the boundary. Entries in the shift factor matrix are classified into two types: intra-regional entries (e.g., transmission line \( l \) is an internal line in area \( i \) or a tie-line, and node \( n \) is in area \( i \)) and inter-regional entries (e.g., transmission line \( l \) is in area \( i \) and node \( n \) is in area \( j \)). Based on the network with boundary equivalent branches (See Fig. 6), ISO \( i \) will independently calculate the intra-regional shift factors without coordinating with other areas. Then the calculation of the intra-regional shift factor is the same as that in a single area. Details are given in Appendix A. For the inter-regional entry between line \( l \) in area \( i \) and node \( n \) in area \( j \) (See Fig. 8), it will be calculated with the following steps:

Step 1: Compute the vector of intra-area shift factor entries \( S_{l,Bj} \) between line \( l \) in area \( i \) and boundary nodes set of area \( j \), denoted by \( B_j \).

Step 2: Compute the coefficient matrix \( W_{Bj,n} = -B_{jj}^{-1} \) between boundary nodes and internal nodes of area \( j \). The matrix \( W_{Bj} \) maps the relationship between boundary equivalent power injections and internal power injections of area \( j \).

Step 3: Compute inter-regional shift factor entry \( S_{l,n} \) by

\[
S_{l,n} = S_{l,Bj} \times W_{Bj,n},
\]

This may encourage arbitrage on the boundary between different areas and is a signal of improper pricing [8].

Fig. 8. Calculate inter-regional shift factor entry \( S_{l,n} \) in a distributed manner.
where \( W_{Bi,n} \) is the column vector corresponding to boundary nodes of area \( j \) and node \( n \).

According to the three steps above, inter-regional shift factor entries will be obtained based on the equivalent network. It should be clarified that we do not need to compute and store all inter-regional shift factor entries. Whenever this entry is needed in the following LMP recovery, we compute it in an on-demand manner.

Next, we explain the general LMP recovery method. According to (14) and (15), LMPs and interface bid prices can be calculated as long as the following multipliers in the GCTS model are recovered:

i) multiplier \( \lambda \) associated with constraint (3).
ii) multipliers \( \mu_i, \mu_l \) associated with constraint (4).
iii) multiplier \( \rho_i \) associated with constraint (7).

We recover these multipliers by looking at marginal generators or marginal interface bids. Marginal prices in (14) and (15) are equal to prices of the marginal generators or marginal interface bid offers, denoted by \( c_{mi} \) and \( \Delta \pi_m \) respectively, which are known to local operators. Thus we can write a group of linear equations as

\[
\frac{\partial L}{\partial g_m} = 0 \Rightarrow \nabla g_m^* C^* = c_m, \quad (19)
\]

\[
\frac{\partial L}{\partial \pi_m} = 0 \Rightarrow \nabla \pi_m^* C^* = \Delta \pi_m, \quad (20)
\]

where \( L \) is the Lagrangian function and the subscript \( m \) represents the set of marginal generators or interface bids. Under the non-degenerate assumption, the number of linearly independent equations in (19) and (20) is equal to the number of variables \( \lambda, \mu_i, \mu_l, \rho_l \). Thus linear Eqs. (19) and (20) have a unique solution, which can be used to further recover marginal prices of internal and interchange offers as per (14) and (15). Note that many standard distributed algorithms can be adopted to solve linear Eqs. (19) and (20) [33], [34]. Thereby, we can obtain the solution to multipliers \( \lambda, \mu_i, \mu_l, \rho_l \) and each ISO will recover its vector of LMPs of all nodes through (14) and (15) based on those multipliers.

C. On Cases With Quadratic Cost Functions

In subsections A and B, we adopt linear cost functions for generators in the LMP recovery method. Now, we discuss the case of quadratic cost function. The cost function is shown as

\[
C_i = g_i^T Q_i g_i + b_i^T g_i, \quad (21)
\]

where \( Q_i, b_i \) represent coefficients of the quadratic cost function. Substituting right sides of (19) by the first-order derivative of quadratic cost functions, we obtain the equation as

\[
\nabla g_m^* C^* = 2Q_m g_m^* + b_m, \quad (22)
\]

where \( g_m^* \) is the power output of marginal generators obtained from the market-clearing results. Thus the right side of (22) is also a constant vector. We can still recover LMPs with (20) and (22) when adopting quadratic cost functions.

D. LMP Recovery Properties

The relationship among LMP recovery results, GCTS results, and JED results is analyzed in this subsection. Solving a centralized GCTS model to obtain all LMPs directly requires the information shared among all ISOs. Although distributed algorithms will solve this privacy problem, they do not reveal LMPs directly. Note that the LMP recovery method will obtain the same results as the centralized GCTS based on the generator outputs. Only marginal units and the equivalent boundary system need to be shared. We prove that the proposed LMP recovery results are equal to the GCTS results. The proof is further given in Appendix B.

Theorem 1: Under the assumption of no degeneracy in GCTS, the LMP recovered in a distributed manner obtained by (19) and (20) for cases with linear cost functions are equal to those forms (14) and (15) calculated with the global GCTS model.

It is noted that even if degeneracy happens in GCTS model, the proposed LMP recovery method still works. The only problem is that its results may differ from multipliers in GCTS, since they do not have unique solutions. It is similar for the case with quadratic cost functions. We drop the proof for brevity.

It has already been verified that GCTS results converge to JED results when interface price gaps converge to zero [29]. According to theorem 1, LMP recovery results also converge to JED results when there are sufficiently many interface bids, and price gaps over interfaces converge to zero. Thus, the LMP recovery method will not suffer from proxy bus problems.

V. JOINTLY COVERING CONGESTION RENT AMONG EACH AREA AND INTERFACE BIDS

After recovering global LMPs, each ISO settles internal generators, internal loads, and interface bids accordingly. In this section, we show how system operators and interface bids should jointly cover congestion rents of internal and tie-lines.

A. Jointly Covering Congestion Rent

Physically, tie-line congestions are caused by internal power injections in different areas. In GCTS, however, the external impact of internal transactions is fully captured by interface bids via (7). Consequently, internal transactions in one area will not contribute to tie-line or external line congestions. Thus congestion rents of tie-lines will be covered only by interface bids, whereas congestion rents of internal lines should be covered by internal transactions in the same area and interface bids together. Without loss of generality, the aforementioned three components are denoted by \( \psi_{i-j,m-n}, \psi_{i,i}, \) and \( \psi_{i,m-n} \) respectively, where the first subscripts \( i \) or \( i-j \) represents congestion rent of internal lines in area \( i \) or that of tie-lines between areas \( i \) and \( j \), and the second subscripts \( i \) or \( m-n \) shows this is the portion of congestion rent covered by the merchandise surplus of the system operator in area \( i \) or an interface bid between areas \( m \) and \( n \). Note that \( i-j \) may or may not be equal to \( m-n \). These three components of congestion rents can be calculated based on shift factors and shadow prices of line congestions [35]:

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Fig. 9. An illustrative example of jointly covering congestion rents in the three-area power system.

i) Contribution of an interface bid \( s_{m,n} \) between areas \( m \) and \( n \) to tie-lines \( i-j \) is calculated as

\[
\psi_{i,j,m-n} = \mu^T_{i-j} \times (S_{i-j,Bm} - S_{i-j,Bn}) s_{m-n},
\]

where \( \mu_{i-j} \) is the shadow price vector corresponding to the line capacity constraint of tie-lines \( i-j \). Based on the LMP recovery method, we obtain the shadow price from (19) and (20). Vector \( S_{i-j,Bm} \) is the shift factor between tie-lines \( i-j \) and the boundary node of area \( m \), calculated based on the global equivalent network. Other shift factor entries \( S_{i-j,Bm}, S_{i,i}, S_{i,Bm}, S_{i,Bn} \) in (23), (24), (25) are similarly obtained.

ii) Contribution of internal transactions in area \( i \) to its internal lines is calculated as

\[
\psi_{i,i} = \mu^T_i \times (S_{i,i} (g_i - d_i)),
\]

where \( \mu_i \) is the shadow price vector corresponding to line capacity constraints in area \( i \).

iii) Contribution of an interface bid \( s_{m,n} \) between areas \( m \) and \( n \) to internal lines in area \( i \) is calculated as

\[
\psi_{i,m-n} = \mu^T_i \times (S_{i,Bm} - S_{i,Bn}) s_{m-n}.
\]

Take the three-area power system in Fig. 5 as an illustrative example, and its settlement process is in Fig. 9. Entries \( \beta_i \) represents the congestion rent of internal lines in area \( i \). Entries \( \beta_{i-j} \) represents the congestion rent of tie-lines \( i-j \). Entries \( \gamma_i \) represents the congestion rent covered by area \( i \). Entries \( \gamma_{i-j} \) represents the congestion rent covered by the interface bid between areas \( i \) and \( j \). As shown in Fig. 9, there are

\[
\beta_i = \psi_{i,i} + \psi_{i,m-n} + \psi_{i,Bm} - \psi_{i,Bn},
\]

\[
\beta_{i-j} = 1^T \psi_{i-j,i-j} + 1^T \psi_{i-j,m-n} + 1^T \psi_{i-j,Bm} \]

\[
\gamma_i = \psi_{i,i},
\]

\[
\gamma_{i-j} = \psi_{i,j-i-j} + \psi_{i,m-i-j} + \psi_{m,i-j} + \psi_{j,m-i-j},
\]

where vector \( \psi_{i,j-i-j} \) includes contributions of all interface bids \( i-j \) to tie-lines \( i-j \), and other bold variables in (26), (27) are defined similarly. Entries \( \beta_j, \beta_{m-j}, \beta_{j-m}, \gamma_j, \gamma_m, \gamma_{i-m}, \gamma_{j-m} \) in Fig. 9 can be obtained similarly.

B. Properties

We next establish properties of the settlement process in Fig. 9 as theorems 2, 3, 4.

**Theorem 2:** Under the assumption of no degeneracy in GCTS, congestion rent of tie-lines and that of internal lines in each area are equal to the amount calculated by the joint economic dispatch problem:

\[
\beta_{i-j} = \mu^T_{i-j} f_{i-j}, \beta_i = \mu^T_i f_i, \]

where \( f_{i-j}, f_i \) represent the power flows of tie-lines \( i-j \) and internal lines in area \( i \), respectively. Please refer to Appendix C for the proof. Even if degeneracy occurs, as long as multipliers are obtained from the GCTS model, we can recover LMPs to settle each area and interface bids. Then this theorem still holds.

**Theorem 3:** Under the assumption of no degeneracy in GCTS, the congestion rent afforded by ISO is equal to its merchandise surplus. And for an interface bid buying from area \( j \) and selling to area \( i \), the congestion rent afforded by it is equal to its profit:

\[
\gamma_i = MS_i,
\]

\[
\gamma_{i-j} = R_{i-j} - \Delta \pi_{m,i-j} s_{i-j},
\]

where entry \( \Delta \pi_{m,i-j} \) represents the anticipated price difference between areas \( i-j \). Please refer to Appendix D for the proof. Similarly, when global LMPs are recovered, this theorem still holds under the degeneracy case.

It should be noticed that, in general, shift factors will change with the selection of phase angle reference bus, whereas shadow prices \( \mu \) and congestion rents do not. In the multi-area problem, the proposed scheme should not be affected by that either. In other words, all congestion rent components in (23)-(25) should be independent to the selection of phase angle reference bus. Next, we will elaborate that such a property holds in the proposed mechanism.

When we calculate \( \psi_{i-j,m-n} \), the difference term \( S_{i-j,Bm} - S_{i-j,Bn} \) in (23) represents the shift factor between tie-lines \( i-j \) and the interface bid \( s_{m-n} \). For interface bid \( s_{m-n} \), when every MW power injects at one node, a corresponding MW power withdraws at the other node. The compensation effects from the phase angle reference bus cancel out. Thus the calculation result of entry \( \psi_{i,j,m-n} \) is independent to the selection of slack bus. And the calculation result of the entry \( \psi_{i,m-n} \) in (25) is similar.

In order to make \( S_{i,i} \) and \( \psi_{i,i} \) in (24) independent to the selection of phase angle reference bus, interface bids will play the role of slack bus proportionally. Note that we have already assumed that there is no generator or load at the boundary node, meaning that the corresponding power injection is equal to zero. Take area \( i \) in Fig. 10 as an example, where areas \( j \) and \( m \) have been represented by the equivalent network. The interface bid \( s_{B1,B3} \) withdraws power from the boundary node \( B3 \), namely supplying power for area \( j \). This power is bought from the boundary node \( B1 \) in area \( i \), and it is physically provided by internal nodes of area \( i \) proportionally. It is similar for the
Under the assumption of no degeneracy in between internal line and boundary nodes. Please refer to Appendix E for the shift factor matrix \( \psi_{i,i} \) and node \( S_i \) for line (2-4) to based on (35).\( \psi_{i,i} \) represents the shift factor between the internal lines and its internal lines. of area \( i \) is independent to the common phase angle reference bus. That is to say, \( \psi_{i,i} \) in (24) is independent to the selection of phase angle reference bus. Next, we calculate entries of the shift factor matrix \( S_{i,i} \) between internal lines and nodes in area \( i \) as follows.

**Step 1:** Compute the shift factor entry \( S_{l,n} \) between internal line \( l \) and node \( n \) of area \( i \) with a phase angle reference bus.

**Step 2:** Compute the shift factor \( S_{l,Bi} \) between internal line \( l \) and boundary nodes of area \( i \), denoted by \( Bi \).

**Step 3:** Compute the weight coefficient vector \( B_{l,Bi}^{-1} \) based on network equivalence.

**Step 4:** Compute the shift factor entry of the matrix \( S_{i,i} \) by
\[
S_{l,n} = (S_{l,n} \bar{1} - S_{l,Bi})B_{l,Bi}^{-1}.
\]

Similar to (23) and (25), the difference term \( S_{l,n} \bar{1} - S_{l,Bi} \) in (35) also removes the compensation effects from the phase angle reference bus. Thus, the calculation results of the shift factor \( S_{1,i} \) and component \( \psi_{1,i} \) are independent to the selection of phase angle reference bus. Note that in [29], internal congestion rents are solely covered by the local ISO, and interface bids cover tie-line congestion rents. However, in [29] interface bids are settled at a different price from the LMP of boundary buses, which is not easy to understand. In this paper, by defining prices of interface bids with Eq. (15), which is equal to the LMP at the same boundary bus, and explicitly calculating components of congestion rents, formerly obscure interface bid prices become understandable: We settle interface bids according to the boundary bus LMPs, then subtract the internal and tie-line congestion rents covered by them. In the following theorem, we show that the proposed scheme is equivalent to that in [29].

**Theorem 4:** Under the assumption of no degeneracy in GCTS, we have
\[
\nabla^T C_i^r - \mu_i^T S_{i,B} M = \nabla^T M_{i,B}^C C^r M_i,
\]
where \( S_{i,B} \) represents the shift factor between the internal lines of area \( i \) and boundary nodes. Please refer to Appendix E for the proof. Similarly, after we recover the global LMPs, this theorem still holds under the degenerate case.

### C. Computational Burden Analysis

Computation of settlement processes in this research mainly includes the followings: i) calculating shift factor entries, ii) calculating multipliers by solving linear Eqs. (19) and (20), and iii) quantifying and pricing contributions of internal and interchange transactions on congested lines. First, shift factor entries are computed on-demand whenever needed in LMP recovery, which will not lead to a heavy computational burden. Second, although matrix inverse calculation is included in solving (19) and (20), the dimension of the coefficient matrix is small, which equals the number of marginal units. Thus, this calculation does not require much time. Third, we adopt (23)~(25) to obtain the joint covering congestion rent results, which can be calculated quickly by multiplying matrixes. Note that we do not have any convergence problem. Thus the CPU cost of the proposed settlement approaches can be negligible.

### VI. NUMERICAL EXPERIMENTS

The performance of the proposed method is evaluated with the two-area (4 nodes) power system in Fig. 11, the two-area (23 nodes) power system in Fig. 13, and the three-area (118 nodes) power system in Fig. 16, respectively. Simulations are carried out on a PC with an Intel Core i5 processor running at 3.00 GHz with 12 GB of memory.

#### A. Case 1: Two-Area (4 Nodes) Power System

We first use a two-area (4 nodes) system in Fig. 11 to solve the GCTS model and jointly cover the congestion rent among areas and interface bids. Parameters of generators and loads are listed in Table I. Without loss of generality, node 1 is selected as the common phase angle reference bus of two areas. The capacity of tie-line (1-3) is set as 10 MW, while capacities of other lines are set as infinity. Reactances of lines (1-2), (1-3), (3-4) are all set as 1.0 p. u.. We set a variable reactance \( x_{2,4} \) for line (2-4) to consider the network topology with or without a loop. When \( x_{2,4} \)
is set as infinity, there is no loop in the network. The profile of interface bids is shown in Table II, including bidding locations, maximum power quantities, and the anticipated bidding prices.

Next, we use the case of setting $x_{2-4}$ as infinity to explain the settlement process of joint covering congestion rent. When $x_{2-4}$ is set as 1.0 p.u., the process is similar and omitted here.

First, we solve the GCTS model. Market-clearing results are listed in Table III, including LMPs, generator outputs, and net revenue of each node. Clearing results of interface bids are listed in Table IV, including cleared quantities, tie-line flows, and marginal prices. The shadow price of the capacity constraint corresponding to the congested line (1-3) is 1.0 $/MWh. Congestion rent of the global system is 10.0 $/h.

Second, according to the market-clearing results of GCTS, area 1, area 2, and interface bids are settled. Take area 1 as an example. It collects 30 $/h from internal load $L_1$ and 10 $/h from interface bids $s_{1-3}$. And it pays 40 $/h for the internal generator $G_1$. The merchandise surplus of area 1 is 0 $/h. The global congestion rent is 10.0 $/h.

In this case, results of joint covering congestion rents among area 1, area 2, and interface bid are shown in Fig. 12 and Table V, which gives a clear answer to the example in Fig. 1.

We next compare the cases with and without loop flow effects. Like the example in Section IV-A, all line capacities here are set as infinity, so there is no line congestion. Therefore, the congestion rent of the global system will be 0 $/h if a “super ISO” governs two areas and properly considers the impacts between areas.

Market-clearing results with and without loop flow effects are given in Table VI. First, when each ISO does internal dispatch to obtain LMPs independently, LMPs of nodes 1 and 2 are 1 $/MWh, while LMPs of nodes 3 and 4 are 2 $/MWh. Then the interface bid $s_{1-3}$ withdraws 10 MW power from node 3, namely supplying 10 MW power for area 2. This power is bought from node 1 in area 1, and it is physically provided by node 2. In area 1, when the power injection at node 2 increases every MW power, the equivalent power withdrawal at node 1 will increase a corresponding MW power. In area 2, it is similar for nodes 3 and 4. Thus, shift factor entries between tie-line (1-3) and $s_{1-3}$, $G_1-L_1$, $G_2-L_2$ are 1, 0, 0, respectively. By multiplying the quantities of $s_{1-3}$, $G_1-L_1$, $G_2-L_2$ with shift factor entries, their contributions to tie-line (1-3) are 10 MW, 0 MW, 0 MW. Then, by multiplying the contributions with the shadow price of tie-line (1-3) 1.0 $/MWh, we obtain the congestion rents afforded by interface bid $s_{1-3}$, area 1, area 2 as 10 $/h, 0 $/h, 0 $/h, respectively.

In this case, results of joint covering congestion rents among area 1, area 2, and interface bid are shown in Fig. 12 and Table V, which gives a clear answer to the example in Fig. 1.
TABLE VI
MARKET-CLEARING RESULTS WITH AND WITHOUT LOOP FLOW EFFECTS

| Node | With loop flow effects | Without loop flow effects |
|------|------------------------|---------------------------|
| Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 1 | Bus 2 | Bus 3 | Bus 4 |
| LMP ($/MWh) | 1.0 | 1.0 | 2.0 | 2.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| Generator output (MW) | / | 90.0 | / | 0.0 | / | 90.0 | / | 0.0 |
| Net revenue ($/h) | 0.0 | -60.0 | 0.0 | 120.0 | 0.0 | -60.0 | 0.0 | 60.0 |

Fig. 13. The two-area power system in case 2.

TABLE VII
PROFILE OF INTERFACE BIDS IN CASE 2

| Indices | Sell to (Area-Bus) | Buy from (Area-Bus) | Upper limit of bidding quantity (MW) |
|---------|--------------------|---------------------|-------------------------------------|
| 1       | 2-18               | 1-7                 | 200                                 |
| 2       | 2-18               | 1-10                | 250                                 |
| 3       | 2-20               | 1-7                 | 300                                 |
| 4       | 2-20               | 1-10                | 350                                 |
| 5       | 1-7                | 2-18                | 200                                 |
| 6       | 1-10               | 2-18                | 250                                 |
| 7       | 1-7                | 2-20                | 300                                 |
| 8       | 1-10               | 2-20                | 350                                 |

TABLE VIII
GENERATOR OUTPUTS IN CASE 2

| Indices | Generator outputs (S0, S1 (MW)) | Generator outputs (S2, S3 (MW)) | Generator outputs (S4, S5 (MW)) |
|---------|---------------------------------|---------------------------------|---------------------------------|
| 1       | 332.4                           | 332.4                           | 332.4                           |
| 2       | 140.0                           | 113.3                           | 0.0                             |
| 3       | 92.6                            | 0.0                             | 0.0                             |
| 4       | 0.0                             | 0.0                             | 62.1                            |
| 5, 6, 7 | 0.0                             | 0.0                             | 0.0                             |
| 8       | 0.0                             | 221.3                           | 172.5                           |

TABLE IX
CLEARING RESULTS OF INTERFACE BIDS IN CASE 2

| Cases | S0 | S1 | S2 | S3 | S4 | S5 |
|-------|----|----|----|----|----|----|
| Cleared | 122.3 | 134.1 | 110.3 | 113.6 | 111.1 | 111.2 |
| quantities | 3 | 59.8 | 48.0 | 71.6 | 68.5 | 71.0 |
| of | 4 | 22.7 | 34.5 | 0.0 | 0.0 | 0.0 |
| interface | 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| bids | 6 | 0.0 | 0.0 | 29.3 | 32.6 | 13.0 |
| price ($/MWh) | 7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 8 | 0.0 | 0.0 | 59.1 | 55.8 | 26.6 | 26.5 |

TABLE X
CLEARING RESULTS OF INTERFACE BIDS IN CASE 2

| Tie-line flows | Bus 7 to 18 | 164.1 | 164.1 | 88.7 | 88.7 | 100.8 |
|---------------|-------------|-------|-------|------|------|-------|
| Bus 10 to 20 | 150.9 | 150.9 | 5.0 | 5.0 | 41.7 | 41.7 |
| Marginal prices ($/MWh) | 3.0 | 3.0 | 2.8 | 2.8 | 1.4 | 1.3 |
| Bus 10 | 3.0 | 3.0 | 0.3 | 0.3 | 9.8 | 9.9 |
| Bus 18 | 3.0 | 3.1 | 5.4 | 5.6 | 0.8 | 4.8 |
| Bus 20 | 3.0 | 3.1 | 8.0 | 8.0 | 4.7 | 8.0 |
| Total cost ($/h) | 1809 | 1841 | 3044 | 3071 | 2872 | 2894 |

merchandise surplus of the global system is equal to 60 $/h. Second, we recover the LMPs based on the proposed method in this paper, which are equal to 1 $/MWh. Then the merchandise surplus of the system is equal to 0 $/h. From these results, we observe that loop flow effects make the merchandise surplus not equal to the congestion rent under “super ISO”. And the proposed approaches in this paper have solved this problem.

B. Case 2: Two-Area (23 Nodes) Power System

We next use a power system of 23 nodes (See Fig. 13) to verify the effectiveness of the proposed approaches. The case is composed of IEEE 14, 9-bus system and two extra tie-lines. We provide parameters of generators and tie-lines in [36]. The profile of interface bids is described in Table VII. We adopt six cases (S0 ∼ S5) set as follows.

S0: Capacities of transmission lines are set as infinity, and bidding prices of interface bids are 0.0 $/MWh.
S1: Capacities of transmission lines are set as infinity, and bidding prices of interface bids are 0.1 $/MWh.
S2: Capacity of tie-line (10-20) is set as 5 MW, and capacities of other transmission lines are infinity. The bidding prices of interface bids are 0.0 $/MWh.
S3: Capacity of tie-line (10-20) is set as 5 MW, and capacities of other transmission lines are infinity. The bidding prices of interface bids are 0.1 $/MWh.
S4: Capacity of the internal line (9-10) is set as 5 MW, and capacities of other transmission lines are infinity. The bidding prices of interface bids are 0.0 $/MWh.
S5: Capacity of the internal line (9-10) is set as 5 MW, and capacities of other transmission lines are infinity. The bidding prices of interface bids are 0.1 $/MWh.

For this two-area system, network equivalence is applied individually. As a result, area 1 includes 16 nodes (12 internal nodes and 4 boundary nodes) and area 2 contains 11 nodes (7 internal nodes and 4 boundary nodes). Without loss of generality, node 18 on the boundary system is adopted as the common phase angle reference bus.

First, we solve the GCTS model of cases S0 ∼ S5 separately. Generator outputs are listed in Table VIII. Clearing results of interface bids are shown in Table IX, including cleared quantities of interface bids, tie-line flows, marginal prices, and total cost. Here, the cost includes both generation cost and the cost of clearing interface bids.

Second, we will verify the effectiveness of the LMP recovery method. When cost functions are linear, the distributed LMP recovery method reveals the same results as the GCTS solved in a centralized manner, which is shown in Fig. 14. Compared with obtaining LMPs from the centralized GCTS model, the
Fig. 14. Comparisons between LMP recovery and GCTS when cost functions are linear. (a) S0. (b) S1. (c) S2. (d) S3. (e) S4. (f) S5.

Fig. 15. Processes of jointly covering congestion rents in case 2.

LMP recovery method is based on marginal units and the shift factor entries obtained in a distributed manner so that the privacy of each area is better protected.

Third, after recovering the global LMPs, we further explain the approach of jointly covering global congestion rent. Settlement processes are shown in Fig. 15, and values of entries are further given in Table X. Settlement results of each area and interface bids under cases S0 ∼ S5 are shown in Table XI. For S0 and S1, there are no line congestions in the global system. Congestion rents covered by area 1, 2, interface bids 1-2 are all 0 $/h. For S2 and S3, congestion rent of tie-line (10-20) would be covered by interface bids 1-2. For S4 and S5, congestion rent of internal line (9-10) in area 1 would be covered by area 1 and interface bids 1-2 proportionally. Under cases S0 ∼ S5, the merchandise surplus of area 1 is equal to congestion rent afforded by area 1. Results of area 2 are similar. And profit of interface bids 1-2 is equal to congestion rent afforded by them.

C. Case 3: Three-Area (IEEE 118-Bus) Power System

Next, we extend pricing and settlement approaches to a three-area power system with more interfaces. We adopt the IEEE 118-bus test system [37] in Fig. 16, which consists of three areas. In this case, the anticipated price differences of interface bids are set as 0.1 $/MWh. Capacities of the congested internal line (11-12) and tie-line (30-38) are set as 10 MW.

Settlements of areas 1, 2, 3 and interface bids 1-2, 1-3, 2-3 are shown in Fig. 17. In this case, the congestion rent of tie-line (30-38) would be covered by interface bids 1-2, 1-3, 2-3 proportionally. Under cases S0 ∼ S5, the merchandise surplus of area 1 is equal to congestion rent afforded by area 1. Results of area 2 are similar. And profit of interface bids 1-2 is equal to congestion rent afforded by them.
VII. CONCLUSION

In this paper, approaches of joint covering congestion rents in the multi-area power system considering loop flow effects are illustrated. We develop the framework of quantifying and pricing the contribution of each area and interface bids to line congestions. When settling a multi-area power system, we first develop a distributed LMP recovery method to protect the privacy of each area. Calculation results of the distributed LMP recovery are equal to that of the centralized GCTS model. Then the contribution of each area and interface bids to line congestions is quantified and priced based on shift factors and shadow prices. Finally, congestion rents are jointly covered among all areas and interface bids considering loop flow effects.

In the future, pricing and settlement approaches will be studied further in the line expansion to improve long-term efficiency.

A. Calculating Shift Factors

We illustrate how to calculate the shift factor based on the DCOPF model. More details can be found in [7]. In DCOPF, DC power flow equations and line capacity constraints are

\[ B\theta = g - d, \]
\[ F\theta \leq f. \]

To calculate the line flow constraints without computing phase angles \( \theta \), we reformulate the constraints by introducing the shift factor matrix. Note that matrix \( B \) is singular. By setting the phase angle reference bus \( \theta_0 \) and eliminating the row and column that corresponds to it, we can obtain

\[
\begin{bmatrix}
B_{11} & B_{10} \\
B_{01} & b_{11}
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_0
\end{bmatrix}
= \begin{bmatrix}
g_1 - d_1 \\
g_0 - d_0
\end{bmatrix}
\Rightarrow
\begin{cases}
\theta_1 = B_{11}^{-1} B_{10} g_1 - d_1 \\
1^T g = 1^T d.
\end{cases}
\]

Shift factor matrix \( S \) between lines and buses in a single area is obtained as

\[ S = F B_{11}^{-1}. \]

B. Proof of Theorem 1

The key issue of recovering global LMPs is determining the marginal units under the pre-determined scheduling of generators and interface bids. To simplify the explanation, the GCTS model of (2)~(7) is reformulated as follows:

\[ \min_{g, s} C = c^T g + \Delta \pi^T s, \]
\[ h(g, s) = 0, \]
\[ f_l^g(g, s) \leq 0, \]
\[ g_s^g(g, s) = 0, \]
\[ g_s^s(s) \leq 0 \]
where (42) represents line capacity constraint, (43) represents line capacity limits, and (44), (45) represent the capacity limits of generators and interface bids. Multipliers of all constraints are listed on the right column.

After solving the GCTS model, binding constraints in (43), (44), (45) correspond to congested lines, non-marginal generators, and non-marginal interface bids, respectively. Those three sets are shown as:

\[ A_L(g^*, s^*) = \{ l : 1 \leq l \leq N_L, g_l^g(g^*, s^*) = 0 \}, \]
\[ A_G(g^*, s^*) = \{ g : 1 \leq g \leq N_G, g_g^g(g^*, s^*) = 0 \}, \]
\[ A_S(g^*, s^*) = \{ s : 1 \leq s \leq N_S, g_s^g(g^*, s^*) = 0 \}, \]
where \( N_L, N_G, N_S \) represent the number of congested lines, non-marginal generators, and non-marginal interface bids, respectively. Vectors \( g^*, s^* \) represent the optimal primal solution of the GCTS model.

With no degeneracy, market-clearing results satisfy the linear independence constraint qualification (LICQ) condition [38], which means the row vectors of gradient matrix \( D \) in (49) are
linear independent.

\[
D = \nabla h(g^*, s^*) \cup \{\nabla g_L^{g^*}(g^*, s^*)\}_{l \in A_L(g^*, s^*)} \\
\cup \{\nabla g_G^{g^*}(g^*, s^*)\}_{g \in A_G(g^*, s^*)} \\
\cup \{\nabla g_s^{g^*}(g^*, s^*)\}_{s \in A_S(g^*, s^*)}.
\]  

(49)

After performing the elementary transformation, the gradient matrix will be obtained by

\[
D' = \begin{bmatrix}
1_{1 \times N_m} & 0 \\
S_{NL \times N_m} & 0 \\
0 & I
\end{bmatrix} = \begin{bmatrix}
\hat{D}_{(1+N_s) \times N_m} & 0 \\
0 & I
\end{bmatrix},
\]

(50)

where \(N_m\) represents the number of marginal units, vector \(S_{NL \times N_m}\) represents the shift factor between congested lines and marginal units. Matrix \(D'_{(1+N_s) \times N_m}\) is corresponding to all congested lines and marginal units, which is used to calculate non-zero Lagrange multipliers.

Hence, first-order optimality conditions associated with marginal units are sufficient to calculate LMP under the assumption of DCOPF and no degeneracy. Thus LMP recovery results are equal to LMPs obtained from the GCTS model.

C. Proof of Theorem 2

Due to network equivalence, we further derive the right side of (30) as

\[
\mu_{i,j}^* f_{i,j} = \mu_{i,j}^* \left( -S_{i,j,Bi}B_{i,j}^{-1}(g_i - d_i) - S_{i,j,Bm}B_{i,m}^{-1}(g_m - d_m) \\
- S_{i,j,Bj}B_{j,m}^{-1}(g_j - d_j) \right)
\]

= \mu_{i,j}^* \left( \begin{bmatrix} S_{i,j,Bi} & S_{i,j,Bm} & S_{i,j,Bj} \end{bmatrix} M s \right)

= 1^T \psi_{i,j,i,j} + 1^T \psi_{i,j,i,m} + 1^T \psi_{i,j,m,j} = \beta_{i,j},
\]

(51)

\[
\begin{align*}
\mu_{i}^* f_i &= \mu_{i}^* \left( S_{i,i}(g_i - d_i) - S_{i,Bm}B_{i,m}^{-1}(g_m - d_m) \\
- S_{i,j,Bj}B_{j,m}^{-1}(g_j - d_j) \right) \\
&= \mu_{i}^* \left( \begin{bmatrix} S_{i,i} & S_{i,Bi} & S_{i,Bm} & S_{i,Bj} \end{bmatrix} M s \right)
\end{align*}
\]

= \psi_{i,i,i} + 1^T \psi_{i,i,i} + 1^T \psi_{i,i,m} + 1^T \psi_{i,m,i} = \beta_i.
\]

(52)

Thus, theorem 3 is verified.

D. Proof of Theorem 3

The market surplus of the area \(i MS_i\) is derived as

\[
MS_i = \nabla_{d_i}^T C^* (d_i - g_i) + \nabla_{M,s}^T C^* \times M s
\]

= \left( \lambda \times 1 - S_{i,i}^T \mu_i - S_{i,i}^T \mu_i + \rho_i \right)^T (d_i - g_i)
\]

+ \left( \lambda \times 1 - S_{i,Bi}^T \mu_i - S_{i,Bi}^T \mu_i + \rho_i \right)^T M s
\]

= -\mu_i^T S_{i,i}(d_i - g_i) - \mu_i^T S_{i,i}(d_i - g_i)
\]

+ \mu_i^T S_{i,Bi}B_{i,i}^{-1}(d_i - g_i) + \mu_i^T S_{i,Bi}B_{i,i}^{-1}(d_i - g_i)
\]

\times (d_i - g_i)
\]

= \mu_i^T S_{i,i}(g_i - d_i) = \gamma_i.
\]

(53)

The arbitrage revenue of a given interface bid buying from area \(j\) and selling to the area \(i R_{i,j}\) is derived as

\[
R_{i,j} = \nabla_{M,s}^T C^* \times M s_{i,j} - \nabla_{M,s}^T C^* \times M s_{i,j}
\]

= \left( \lambda \times 1 - S_{i,Bi}^T \mu_i - S_{i,Bi}^T \mu_i - S_{i,Bi}^T \mu_j \right)
\]

- \left( \lambda \times 1 - S_{i,Bi}^T \mu_i + \rho_i \right)^T M s_{i,j} - \left( \lambda \times 1 - S_{i,Bi}^T \mu_j \right)
\]

- \left( \lambda \times 1 - S_{i,Bi}^T \mu_j + \rho_j \right)^T M s_{i,j}
\]

= \Delta \pi_{m,s} s_{i,j} + \mu_i^T \left( S_{i,Bi} - S_{i,Bj} \right) s_{i,j}
\]

+ \mu_j^T \left( S_{i,Bj} - S_{i,Bi} \right) s_{i,j} + \mu_i^T \left( S_{m,Bi} - S_{m,Bj} \right) s_{i,j}
\]

= \Delta \pi_{m,s} s_{i,j} + \psi_{i,j,i} + \psi_{j,i,j} + \psi_{m,i,j} + \psi_{j,m,i} = \Delta \pi_{m,s} s_{i,j}
\]

\]

\]

Thus, theorem 4 is verified.

E. Proof of Theorem 4

According to [29], the sensitivity of the local optimal cost of area \(i\) with respect to cleared quantities of interface bids \(s\) is calculated as

\[
\nabla_s C_i^* = M^T \left[ \hat{B}_{ii} \hat{B}_{jj} \right]^{-1} \left[ \begin{bmatrix} B_{ii} & B_{ij} \end{bmatrix} \left[ \begin{bmatrix} \xi_i \xi_j \end{bmatrix} + \begin{bmatrix} H_i^T \mu_i \end{bmatrix} \right] \right],
\]

(55)

where \(\xi_i, \xi_j\) represent the LMPs of internal and boundary nodes of area \(i\), vector \(B_{ii}\) represents the nodal admittance sub-matrix between boundary buses in areas \(i\) and \(j\), vector \(B_{jj}\) represents the equivalent self-admittance of boundary buses of area \(i\). Other terms in (55) are similarly defined.

For area \(i\), from the optimality condition of the local economic dispatch model in [29], we have

\[
B_{ii} \xi_i + B_{ij} \xi_j + H_i^T \mu_i = 0.
\]

(56)

By substituting (55) and (56) into the left side of (36), we have

\[
\nabla_s^T C_i^* - \mu_i^T S_{i,B} M
\]

= \left( \begin{bmatrix} B_{ii} & B_{ij} \end{bmatrix} \left[ \begin{bmatrix} \xi_i \xi_j \end{bmatrix} + \begin{bmatrix} H_i^T \mu_i \end{bmatrix} \right] \right)^T \left[ \begin{bmatrix} B_{ii} & B_{ij} \end{bmatrix} \left[ \begin{bmatrix} \xi_i \xi_j \end{bmatrix} + \begin{bmatrix} H_i^T \mu_i \end{bmatrix} \right] \right]^{-1} M
\]

- \mu_i^T S_{i,B} M
\]
\[
\begin{align*}
&= \left( \begin{bmatrix} \bar{B}_{ij} \xi_i - \bar{H}_{ij}^T \mu_i \\ \bar{B}_{ji} \xi_j \end{bmatrix} \right)^T - \begin{bmatrix} \mu_j \bar{H} \\ 0 \end{bmatrix} \begin{bmatrix} \bar{B}_{ij} \bar{B}_{ij} \end{bmatrix}^{-1} M \\
&= \begin{bmatrix} \xi_i^T & 0 \end{bmatrix} \begin{bmatrix} \bar{B}_{ij} & \bar{B}_{ji} \\ \bar{B}_{ji} & \bar{B}_{jj} \end{bmatrix}^{-1} M_i \\
&= \xi_i^T M_i = \nabla^T M_i + C^* \times M_i, 
\end{align*}
\]

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