Entangled two cavity modes preparation via a two-photon process

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We propose a scheme for entangling two field modes in two high-Q optical cavities. Making use of a virtual two-photon process, our scheme achieves maximally entangled states without any real transitions of atomic internal states, hence it is immune to the atomic decay.

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Entanglement is one of the most characteristic features of quantum systems and lies at the heart of the difference between the quantum and classical multi-particle world. It is the phenomenon that enables quantum information processing and computing [1]. Beyond these and other related applications, complex entangled states, such as the GHZ triplets of particles [2] can be used for tests of quantum non-locality [3]. Moreover, the relaxation dynamics of larger entangled states sheds light on the decoherence process and on the quantum-classical boundary [4]. There are a lot of proposal devoted to the preparation of quantum entangled states, among them the idea for photon down-conversion process [3], with trapped ions [5], for cavity quantum electrodynamics [6], with macroscopic objects [7] or for an optical fibre [8] has been realized experimentally.

In the latter case, the entanglement results from the nonlinear interaction between the two modes in an optical fibre. This is closely connected to the recent advance of enhancing nonlinear coupling via electromagnetically induced transparency (EIT) mechanism [9]. Measured value of the $\chi^{(3)}$ parameter are up to six order of magnitude larger than usual [10]. This has opened the door toward the application of this kind of nonlinear process to quantum information processing even for the very low photon-number case [11]. In fact, there are several proposals for exploiting huge Kerr non-linearities to perform computation and quantum teleportation [12,13] or for quantum non-demolition measurements [14]. Apart from the Kerr nonlinearity, the experimental achievement of atomic Bose-Einstein condensation (BEC) also provide us a chance to create many particle entanglement with nonlinear interactions [15–22]. All these show that the non-linear interaction between different quantum modes is a valuable resource for quantum information processing.

In this paper, we present a new theoretical scheme for entangling two quantum modes in two high-Q optical cavities. Through a virtual two-photon process, an effective non-linear interaction between the two modes can be established. By making use of the virtual two-photon process, our new protocol significantly reduces the effect of atomic spontaneous emission during the entanglement preparation process.

Our system consists of two optical cavities as in Ref. [23] and an atom system surrounded by the two optical cavities. The axis of the two cavities are perpendicular each to other, the internal structure of the atom is depicted in Figure 1.
FIG. 1. A 4-level atom interacting with two cavity quantum fields. Both cavity fields are detuned from atomic resonance by $\Delta = \Omega - \omega_i$, and $\delta = 2\Omega - \omega_c$.

The atom is assumed to make two-photon transitions of frequency $\omega_a$, between the nondegenerate states $|g\rangle$ with energy $\omega_a = 0$ and the excited state $|e\rangle$. The transitions are mediated by two intermediate degenerate levels $|i_1\rangle$ and $|i_2\rangle$ (with frequency $\omega_i$): the frequencies for transitions $|g\rangle \leftrightarrow |i_1\rangle'(or |i_2\rangle')$ and $|i_1\rangle'(or |i_2\rangle') \leftrightarrow |e\rangle$ are $\Omega - \Delta$ and $\Omega + \Delta - \delta$, respectively. With these notations, the system can be described by

$$
H = \hbar \Omega_a a^\dagger a + \hbar \Omega_b b^\dagger b + \hbar g_c (|g\rangle \langle i_1 a^\dagger + |i_2\rangle c^\dagger c + h.c.) + \hbar g_c (|i_2^\dagger b^\dagger + |i_2\rangle e^\dagger e + h.c.) + \hbar \omega_i (|i_1\rangle + \hbar \omega_c e^\dagger e + \hbar \omega_i |i_2\rangle |i_2^\dagger),
$$

(1)

where $a(b)$ and $a^\dagger(b^\dagger)$ are the annihilation and creation operator for the cavity mode $a(b)$ with frequency $\Omega_a (\Omega_b)$, respectively, $g_c$ is the coupling constant of the atom to the cavity mode $a(b)$ driving the transition $|g\rangle \leftrightarrow |i_1\rangle$ or $|i_1\rangle \leftrightarrow |e\rangle$ ( $|g\rangle \leftrightarrow |i_2\rangle$ or $|i_2\rangle \leftrightarrow |e\rangle$ ). We will not consider the position dependance of the cavity-atom coupling $g_c(\gamma^2)$, a good approximation in the Lamb-Dicke limit. For the reason of simplicity, we assume $\Omega_a = \Omega_b = \Omega$ hereafter. Our Scheme works in the following limit: 1)Both the cavity mode $a$ and $b$ are strongly detuned, i.e., $\Delta = \Omega - \omega_i >> g_c, \Delta >> \delta$ and $\delta >> |g_c|^2/\Delta$; and 2) the cavity decay rate $\kappa << |g_c|^2/\Delta$ as required for the high-Q optical cavity. Because these transitions $|g\rangle \leftrightarrow |i_1\rangle , |i_1\rangle \leftrightarrow |e\rangle$ ( $|g\rangle \leftrightarrow |i_2\rangle$ and $|i_2\rangle \leftrightarrow |e\rangle$ ) driven by the two cavity modes are far off-resonant, we may adiabatically eliminate the intermediate states $|i_1\rangle$ and $|i_2\rangle$ independently, the Hamiltonian (1) then takes the following form

$$
H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b + \hbar \lambda (|g\rangle \langle g| a^2 + |e\rangle \langle e| a^2)
+ \frac{\hbar \omega_A}{2} (|e\rangle \langle e| - \langle g| g\rangle)
+ \hbar \lambda (|g\rangle \langle e| b^2 + |e\rangle \langle b| b^2),
$$

(2)

with $\omega = \Omega + 2|g_c|^2/\Delta, \Delta = \Omega - \omega_i, \lambda = |g_c|^2, \omega_A = \omega_c - \omega_A$. This is the Hamiltonian which is broadly used to describe the two-photon process, and has received an extensive study during the last decades. For instance, the experimental realization of a two-photon cascade micromaser [26], the generation of Squeezing amplification [27] and the creation of entangled states [28]. Our proposal works with a new mechanism different from that by making use of very high Kerr coupling, the coupling between the two cavity modes in our protocol, to be discussed below, results from virtual two-photon processes.

In the limit $\delta >> \lambda$, i.e., $(2\omega - \omega_A) >> |g_c|^2/\Delta$, the two-photon process is on off-resonance, we may adiabatically eliminate the atom from the system, the Hamiltonian (2) then takes the following form in the interaction picture

$$
H_{eff} = \hbar \frac{|\lambda|^2}{\delta} (a^2 a^\dagger + b^2 b^\dagger + a^{2\dagger} b^2 + b^{2\dagger} a^2),
$$

(3)

in derivation of the Hamiltonian (3), the atom in its ground state $|g\rangle$ initially is assumed. The two mode states will be defined in terms of the usual two-mode Fock states $|m, n\rangle = |m\rangle_a \otimes |n\rangle_b$, with $m(n)$ photons in mode $a(b)$. First we consider a simple case that there are only two photons in the mode $a$ while the cavity mode $b$ in vacuum initially. The Hamiltonian (3) for this simple case is equivalent to

$$
H_{eff} = 2\hbar \frac{|\lambda|^2}{\delta} (|E\rangle \langle E| + |E\rangle \langle E| + \sigma^+_a \sigma^-_b + \sigma^+_b \sigma^-_a),
$$

with definition $|E\rangle = |2\rangle_x, (x = a, b)$, and $\sigma^+_x(-)$ is the pauli operator for mode $x$. It shows that after the interaction time $T_0 = \delta \pi/8|\lambda|^2$, the two cavity modes evolve to a maximal entangled state $\frac{1}{\sqrt{2}}(|0E\rangle + |E0\rangle)$ while leave the atom in its ground state $|g\rangle$. We would like to note that the non-linear interaction term $|a^{2\dagger} b^2 + h.c.|$ is different from the Kerr nonlinear interaction $a^{2\dagger} a^2$ and $a^{\dagger} a^\dagger b^\dagger b$ [29,30], as the latter one is in the form of the square of the free Hamiltonian, and hence either $a^\dagger a$ or $b^\dagger b$ is a constant of motion.

We have performed extensive numerical simulations with the full Hamiltonian Eq. (1). Ignoring the atomic spontaneous emission and the cavity decay, we find the above analytical insights to be completely accurate, i.e., we indeed get the maximal entangled state (see Figure 2). In fact, we find that the approximated Hamiltonian Eq.(3) is quite an good approach to the full Hamiltonian Eq.(1).

The top panel in Figure 2 shows selected results for the dependence of the population of state $|0, 2\rangle$ (dotted line) and $|2, 0\rangle$ (dashed line) on time, while the lower panel display the von Neumann entropy taken in base 2. An initial state $|0, 2\rangle$ and system parameters $\Delta = 20g, \delta = 5g$ are chosen for this plot. The maximal entangled state may be obtained at time $t = 785$ with perfect Fidelity ($>>99.9\%$). The Fidelity is defined as the overlap between the output state and the desired state.

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FIG. 2. Populations of states $|0, 2\rangle$ and $|2, 0\rangle$, and the von Neumann entropy versus time, this figure is plotted for the case when the adiabatic limit satisfied. The constant line in the top panel denotes the probability of the atom being in its ground state, disregard where the photons are.

Similar results are found for the initial state $|4, 0\rangle$, it is illustrated in Figure 3. The difference is that there are three components $|4, 0\rangle$, $|2, 2\rangle$ and $|0, 4\rangle$ in the output state, their population are shown in the top panel in Figure 3 by c,b,a, respectively. It is interesting to note that there is a time point (in Fig. 3, top panel) when line c and line a overlap, which corresponds to the system in $\frac{1}{\sqrt{2}}(|0, 4\rangle + |4, 0\rangle)$. And at this point the entropy as plotted in the lower panel is 1, it equals the entropy of a maximally entangled state for two-qubit. A recent study on entangled two modes show that the amount of entanglement present in a given state depends on how one defines one’s systems [31]. This means we could redefine our two modes such that the state $\frac{1}{\sqrt{2}}(|0, 4\rangle + |4, 0\rangle)$ represents a maximally entangled state.

It is surprising to find that the same dynamics as in the adiabatic limit persist even when adiabatic elimination is not valid. As an example, in Figure 4 and Figure 5, we display results for $\Delta = 8g, \delta = 3g$. Apparently, the atom as the interaction agent for the two cavity modes is enough for establishing an effective interaction between them.

Now, we discuss effects of the dissipation or decoherence due to both the atomic decay and the cavity loss. As with any proposal for quantum information processing, ultimately its success depends on being able to complete many coherent dynamics during the decoherence time. In principle, as long as a) $\frac{\lambda^2}{3} \gg \kappa$ and b) $\frac{\lambda^2}{3} \gg \Gamma$, we could expect essentially the same results as illustrated in Figure 2 and Figure 3. As there are no real transitions of atomic states in our proposal, it makes this scheme immune to the atomic spontaneous emission or atomic decay, so the restriction b) make no sense in this scheme. On the other hand, the condition a) is difficult to achieved because the two-photon process is relatively weak due to large off-resonant detunings for all its intermediate states. In Figure 6 and Figure 7, the effects of cavity decay on dynamics of the proposed system is illustrated. As known, the decoherence time for a state $|m, n\rangle$ depends on the total number of photons, and as Figure 6 and Figure 7 show, relative good results are found when the cavity loss rate $\kappa$ is small.
κ as well as the von Neumann entropy, with cavity decay rate \( \kappa = 0.005g \), the other parameters chosen are the same as in Figure 4.

![Figure 4](image1)

**FIG. 4.** The same as in Figure 5, but with cavity decay rate \( \kappa = 0.005g \).

FIG. 6. Populations for state \(|2, 0\rangle\) and \(|0, 2\rangle\)(top panel), as well as the von Neumann entropy, with cavity decay rate \( \kappa = 0.005g \), the other parameters chosen are the same as in Figure 4.

![Figure 5](image2)

**FIG. 5.** Populations for state \(|2, 0\rangle\) and \(|0, 2\rangle\)(top panel), as well as the von Neumann entropy, with cavity decay rate \( \kappa = 0.005g \), the other parameters chosen are the same as in Figure 4.

Finally, we want to stress that the requirement for the intermediate states degenerate is not necessary, in fact, our proposal works in the same manner when the detuning \( \Delta_i \) defined by \( \Delta_i = \Omega - \omega_i \) have a different sign, i.e., \( \Delta_1 = -\Delta_2 \). As all existing cavity QED-based quantum computation protocols, optical cavity with high-Q and atom with small decay rate remain challenging because of the technological limit of the Fabry-Perot optical cavity [32–35].

In conclusion, we have proposed a new protocol for preparing the maximally entangled state in two high-Q optical cavities. As the photons act as the information carrier, cavity with very high-Q factor is highly desired. We have explained the scheme in terms of the virtual two-photon process induced nonlinear interaction. In addition, our protocol can also be explained in terms of entanglement distribution by separable states [36]. This new protocol has advantages that its successful implementation involves no real transitions of atomic states, it makes this proposal immune to the atomic spontaneous emission or atomic decay.

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