The muon $g - 2$ anomaly and dark sector via the leptonic scalar portal

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We present a model in which dark matter (DM) interacts with the Standard Model (SM) via a scalar portal boson $a$ carrying both dark and SM leptonic numbers, and mediating a nondiagonal interaction between the electron and muon that allows $e \leftrightarrow \mu$ transitions. The model explains both the 4.2 $\sigma$ muon $g - 2$ anomaly and the relic density of dark matter with the parameter space for the $a$ mass and couplings that is directly accessible in the NA64 experiment at the CERN SPS. The $a$ could be produced in high-energy electron and/or muon scattering off a target nuclei in the reactions $e(\mu)Z \rightarrow \mu(e)Za$ followed by the prompt invisible decay $a \rightarrow \text{DM}$ particles and searched for in events with large missing energy. By using the NA64 results obtained in 2016-2018 on the search for dark matter and dark photons we present the first constraints on the $a$ that demonstrate the convincing potential of NA64 for probing the still unexplored parameter space.

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The recent precise determination of the anomalous magnetic moment of the positive muon $a_\mu = (g - 2)_\mu / 2$ from the experiment E989 at FNAL $^1$ confirmed the previous measurements of Ref.$^2$, and gives result which is about 4.2$\sigma$ higher than the Standard Model (SM) prediction

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

(1)

This result may signals the existence of new physics (NP) with mass below the electroweak scale ($\ll 100$ GeV). At present, one of the most attractive explanations of the anomaly suggests the existence of a sub-GeV gauge boson, which can be probed in a near future at a fixed-target experiment, see e.g. $^3$.$^4$.$^12$.

Another motivation for searches of NP in the low-mass range come from the dark matter (DM) sector. Despite many intensive searches at the accelerator and in non-accelerator experiments, still little is known about the origin and dynamics of the dark sector itself. One difficulty so far is that DM can be probed only through its gravitational interaction. Therefore, sensitive searches for possible portals that could transmit new feebie interaction between the ordinary and dark matter are crucial and, indeed, they have received significant attention in recent years $^{15}$.$^{17}$.

The goal of this work is to show that the $(g - 2)_\mu$ anomaly and dark matter puzzle give a hint for the existence of a new scalar $a$ in the sub-GeV mass range, $(m_a \gtrsim 100$ MeV) transmitting a flavour conserving nondiagonal interaction between the muon and electron. It is assumed that the $a$ decays predominantly invisibly, \[ \Gamma(a \rightarrow \text{invisible})/\Gamma_{\text{tot}} \approx 1, \] e.g. into dark sector particles, thus escaping stringent constraints placed today on the visible decay modes of the $a$ into SM particles from collider, fixed-target, and atomic experiments $^{15}$. The most stringent limits on the invisible $a$ in the sub-GeV mass range are obtained, so far, for the case of scalars $a$ coupled to electron and muon by the low-energy experiments searching for the decay $\mu \rightarrow ea$ $^{15}$, leaving a large area of the parameter space for the leptonic $a$ still unexplored. Therefore in this paper we assume that $m_a > m_\mu$.

Consider the interaction of complex scalar mediator $a(x)$ with electrons and muons, namely

\[ L_{a\mu e} = -h_{\mu e}\bar{\epsilon}_L a R\epsilon + h.c., \]

(2)

where $e_L = (\frac{(1 - \gamma_5)}{2})\epsilon_L$, $\mu_R = (\frac{(1 + \gamma_5)}{2})\mu$. The interaction (2) is invariant under the $L_\epsilon$, $L_\mu$ flavor global transformations $a(x) \rightarrow e^{i\alpha_\epsilon} a(x)$, $\mu(x) \rightarrow e^{i\alpha_\mu}(\mu(x))$, and $e(x) \rightarrow e^{i\alpha_\mu}(\epsilon(x))$. As a consequence for massless neutrino the interaction $a$ transmitted by the leptonic $a$ conserves both muon and electron lepton numbers. The interaction (2) leads to additional contributions to the electron and muon $(g - 2)$. One-loop contribution to $a_\mu$ is shown in Fig. and it reads $^{19}$

\[ \Delta a_\mu = \frac{h_{\mu e}^2 m_\mu^2}{16\pi^2 m_a^2} L, \]

(3)

\[ L = \frac{1}{2} \int_0^1 dx \frac{2x^2(1-x)}{(1-x)(1+\lambda^2 x) + (\epsilon\lambda)^2 x}, \]

(4)

where $\epsilon = \frac{m_e}{m_\mu}$ and $\lambda = \frac{m_\mu}{m_a}$. For electron magnetic magnetic moment we must replace $m_\mu$ to $m_e$ and $m_\mu$ to $m_\mu$.
in formulae. For \( m_a \gg m_\mu \) one can find that

\[
\Delta a_\mu = \frac{h^2 e}{48\pi^2 m_a^2},
\]

(5)

and

\[
\Delta a_e = \frac{h^2 e}{48\pi^2 m_a^2},
\]

(6)

respectively. As a consequence we find that

\[
\Delta a_e/\Delta a_\mu = \left(\frac{m_\mu}{m_a}\right)^2
\]

(7)

If we assume that the additional interaction explains the muon anomaly, then

\[
h^a_{\mu e} = (1.1 \pm 0.1) \times 10^{-3} \left(\frac{m_a}{m_\mu}\right).
\]

(8)

for \( m_a \gg m_\mu \). As it was introduced in the introduction in the rest of the paper we assume that \( m_a > m_\mu \). This assumption allows to prohibit the decay \( \mu \rightarrow e\alpha \) for which experimental data restrict rather strongly the coupling constant \( h^a_{\mu e} \). For our estimates we shall use the conventional point \( m_a = 3m_\mu \) resulting in

\[
h^a_{\mu e} = (3.3 \pm 0.3) \times 10^{-3}.
\]

(9)

for explaining the value.

The \( SU(2) \otimes U(1) \) invariant generalization of the interaction

\[
L_{\mu e, gen} = -\frac{h_{\mu e}}{H} (\bar{\nu}_e, e)_L H a_\mu R + h.c.
\]

(10)

where \( H \) is the vacuum expectation value of the Higgs isodublet \( H \). In the unitary gauge \( H = (0, \frac{h}{\sqrt{2}}, + < H >) \), where \( h \) is the Higgs field. Note that our complex scalar mediator \( a(x) \) is singlet of the \( SU(3) \otimes SU(2) \otimes U(1) \) SM gauge group. The interaction \( (10) \) is nonrenormalizable and it conserves both \( L_e \) and \( L_\mu \) flavour numbers in the approximation of massless neutrino. One can obtain the effective nonrenormalizable interaction \( (10) \) from the renormalizable interaction with vectorlike fermion \( E \), namely

\[
L_{E a_{\mu e}} = -(a_1 (\bar{\nu}_e, e)_L H + h_2 \bar{E} L H a_\mu R + h.c.) - M \bar{E} E
\]

(11)

Suppose the \( a(x) \) boson interacts with dark matter particles. Several models can be considered. In the first model the \( a(x) \) field interacts with two dark matter complex scalars \( s_1(x) \) and \( s_2(x) \) and the corresponding interaction has the form

\[
L_{a s_1 s_2} = g_{a s_1 s_2} a s_1 s_2 + h.c.
\]

(12)

Note that the coupling constant \( g_{a s_1 s_2} \) has the dimension of the mass. The interaction \( (12) \) is invariant under global transformations \( a \rightarrow e^{i\alpha_1}a, \text{ and } s_i \rightarrow e^{i\alpha_i}s_i, \) with \( i = 1, 2 \). As a consequence in the approximation of massless neutrino both \( L_e \) and \( L_\mu \) lepton flavors are conserved.

Consider another model, when the scalar \( a \) interacts with two light dark matter fermions \( \psi_1 \) and \( \psi_2 \) with the Lagrangian

\[
L_{a \psi_1 \psi_2} = g_{a \psi_1 \psi_2} a \bar{\psi}_1 \psi_2 + h.c.
\]

(13)

Again interaction \( (13) \) conserves both \( L_e \) and \( L_\mu \) lepton flavors. For the interactions \( (12) \) and \( (13) \) the \( a \) decays widths into \( s_1, s_2 \) and \( \psi_1, \psi_2 \) dark matter particles are

\[
\Gamma(a \rightarrow s_1 s_2) = \frac{g^2_{a s_1 s_2}}{4\pi} \frac{p_1}{m_a^2} \left(1 - \frac{(m_1 + m_2)^2}{m_a^2}\right)
\]

(14)

\[
\Gamma(a \rightarrow \psi_1 \psi_2) = \frac{g^2_{a \psi_1 \psi_2}}{4\pi} \frac{p_1}{m_a^2} \left(1 - \frac{(m_1 + m_2)^2}{m_a^2}\right)
\]

(15)

where \( p_1 = \left[m_1^2 - (m_1 + m_2)^2\right]/(m_1 + m_2)^2 \) is the momentum of the particle 1 in the rest frame of a particle a and \( m_1 \) and \( m_2 \) are the masses of particles 1 and 2. Here we assume that \( m_1 > m_1 + m_2 \). The decay width of \( a \) into \( \mu^+ \mu^- \) is determined by the formula

\[
\Gamma(a \rightarrow \mu^+ \mu^-) = \frac{h^2_{\mu e, \mu}}{8\pi} \left(1 - \frac{m_\mu e^2}{m_a^2}\right)
\]

(16)

Here \( p_\mu \) is the electron momentum in the center of mass frame. The annihilation cross sections of \( s_1, s_2 \) and \( s_1, s_2 \) into \( \mu e \) pair in the nonrelativistic approximation is s-wave and it is determined as

\[
\sigma_{an}(s_1 s_2 \rightarrow e^+ e^-) v_{rel} = \sigma_{an}(s_1 s_2 \rightarrow e^+ e^-) v_{rel} = |M|^2 \frac{p_{com}}{16\pi (m_1 + m_2)m_1 m_2}
\]

(17)

where

\[
|M|^2 = g^2_{a s_1 s_2} h^2_{\mu e} \left((m_1 + m_2)^2 - m_e^2 - m_\mu^2\right)/(m_a^2 - (m_1 + m_2)^2)^2
\]

(18)

and \( p_{com} \) is the momentum of electron in the centre of mass frame. For particular case \( m_1 = m_2 \gg m_\mu \) the annihilation cross section is

\[
\sigma_{an}(s_1 s_2 \rightarrow e^+ e^-) v_{rel} = \frac{h^2_{\mu e, \mu}}{8\pi (m_\mu e^2 - 4m_1^2)}
\]

(19)

For the second variant with fermions \( \psi_1, \psi_2 \) in nonrelativistic approximation the annihilation cross section \( \sigma_{an}(\psi_1 \psi_2 \rightarrow e^- \mu^+) v_{rel} \) is given by the formula

\[
|M|^2 = \frac{g^2_{a \psi_1 \psi_2}}{2} h^2_{\mu e} \left((m_1 + m_2)^2 - m_e^2 - m_\mu^2\right) m_1 m_2 v_{rel}^2
\]

(20)
For the particular case $m_1 = m_2 \gg m_\mu$ the cross section is

$$\sigma_{\text{an,tot}} = \frac{\hbar^2 m_\mu^2 g_{\psi_1 \psi_2}^2 m_1^2 v_{\text{rel}}^2}{8\pi (m_\mu^2 - 4m_1^2)^2}, \quad (21)$$

where $\sigma_{\text{an,tot}} = \sigma_{\text{an}}(\psi_1 \psi_2^* \rightarrow e^- \mu^+) + \sigma_{\text{an}}(\psi_1^* \psi_2 \rightarrow e^+ \mu^-)$. Thus, we see that in the nonrelativistic limit model with scalar dark matter particles has s-wave behaviour that at least in naive variant contradicts to the Planck data [20][12]. For the second model we have p-wave behaviour for the annihilation cross section that allows to escape Planck restrictions [20]. We assume that at the early Universe light dark matter is in equilibrium with ordinary matter. From the requirement that the current dark matter relic abundance is explained by our model we can estimate the coupling constant $g_{\psi_1 \psi_2}$ using standard formulae [21][25] for the dark matter density calculations. Here we make rough estimates for the second model with the p-wave annihilation cross section $\sigma_{\text{an,rel}} = O(1)$ pb and for the average relative velocity of annihilating dark matter particles $v_{\text{rel}} \sim c/\sqrt{3}$ which reproduce the observed dark matter density of the Universe [26]. Consider the simplest example with $m_1 = m_2 \gg m_\mu$. As a consequence of the formula (21) we find that

$$\frac{\hbar^2 g_{\psi_1 \psi_2}^2 m_\mu^2}{4\pi (m_\mu^2 - (m_1 + m_2)^2)^2} = O(10 \text{ pb}) \quad (22)$$

For the case $m_\mu = 3m_1$ we find

$$h_{\mu e} g_{\psi_1 \psi_2} \sim 10^{-3} \left( \frac{m_\mu}{\text{GeV}} \right) \quad (23)$$

In the assumption that the model explains muon $g - 2$ anomaly we find that $g_{\psi_1 \psi_2} \sim 0.1$ and it depends rather weakly on the $m_\mu$. As a consequence we obtain that $g_{\psi_1 \psi_2} \geq h_{\mu e}$ for $m_\mu \leq 10$ GeV and the mediator $a$ decays mainly invisibly into dark matter particles. So we find that our model can explain both the $(g - 2)_\mu$ anomaly and the dark matter relic abundance.

Let us briefly discuss constraints on the model from the existing data. Note, that as both $L_e$ and $L_\mu$ lepton numbers are conserved the muonium to antimuonium conversion, $\mu^- e^- \rightarrow \mu^- e^+$, is prohibited. If we assume that invisible $a$ boson decay is predominant, the interaction [2] results in LFV-like semi-visible $Z$-boson decays $Z \rightarrow e^+ e^- \pi^+$ and $Z \rightarrow \mu^- \mu^+ \pi^+$; $a \rightarrow invisible$ and $Z \rightarrow \mu^- \mu^+ a; a \rightarrow invisible$ and $Z \rightarrow \mu^- \mu^+ a; a \rightarrow invisible$. Here $e^+$ and $\mu^+$ are virtual electron and muon. For $m_{a} \ll m_{Z}$ the branching ratio $\Gamma(Z \rightarrow \mu^- \mu^+ a)/\Gamma(Z \rightarrow all) \sim h_{\mu e}^2/4\pi$. Assuming $m_{a} = 3m_\mu$ and $h_{\mu e} = 3.3 \times 10^{-3}$, one gets $\Gamma(Z \rightarrow \mu^- \mu^+ a)/\Gamma(Z \rightarrow all) \sim 10^{-8}$. This can be compared with the best experimental constraint $\Gamma(Z \rightarrow \mu^- \mu^+ a)/\Gamma(Z \rightarrow all) < 7.5 \times 10^{-7}$[18] which is much weaker. Assuming that for the missing mass $\Delta m_{\text{miss}} \lesssim 5$ GeV, which is the experimental resolution of the $Z$-mass peak [27], the decays $Z \rightarrow e^+ e^- a$ and $Z \rightarrow \mu^- \mu^+ a$ are indistinguishable, one could get $h_{\mu e} \lesssim 3 \times 10^{-2}$ for the sub-GeV $m_{a}$ region. Our model also predicts the $K \rightarrow \mu \nu \rightarrow e\nu a$ decay chain with the branching ratio $Br_{K \rightarrow e\nu a} \sim O(h_{\mu e}^2) \sim 2 \times 10^{-7}$. By using the experimental constraints $Br_{K \rightarrow e\nu a} < 6 \times 10^{-5}$ for the momentum range 220-230 MeV/c [28] and a phase-space spectrum for the $K \rightarrow e\nu a$ decay one can obtain modest bounds $h_{\mu e} \lesssim 7 \times 10^{-2}$ for the mass range $m_{a} \lesssim 200$ MeV. For $m_K - m_{a} \gtrsim 250$ MeV bound from $K \rightarrow e\nu a$ decay does not work due to kinematics constraints of Ref. [28].

The stronger limits on coupling $h_{\mu e}$ comes from anomalous magnetic moment of muon. By using Eq. (4) we obtain that at 3$\sigma$ level the contribution of new physics to $(g - 2)_\mu$ is $\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} \lesssim 428 \times 10^{-11}$. Using Eqs. (3)[6] one gets e.g for $m_{a} > m_{\mu}$ $h_{\mu e} \lesssim 1.42 \times 10^{-3}(m_a/m_{\mu})$ at 3$\sigma$ level. For $m_{a} = 3m_\mu$ we find that at 3$\sigma$ level $h_{\mu e} \lesssim 4.26 \times 10^{-3}$. Note that bound from $\Delta a_{\mu}$ is proportional to $m_{a}^{-1}$ and for large masses $m_{a} \gtrsim 1$ a few GeV the ATLAS bound from the $Z$-decays becomes more stronger.

Additional constraints also come from the NA64e experiment. For the sensitivity estimate we will use the results reported by NA64e on invisible decays of dark photons ($A'$) with the data samples of $n_{EOT} = 2.84 \times 10^{11}$ 100 GeV electrons on target (EOT) [29][31]. If the $a$ exists, it could be produced in the reaction $eZ \rightarrow \mu Z a; a \rightarrow invisible$ (24) of high-energy electrons scattering off nuclei of an active target of a hermetic NA64e detector, followed by the
prompt invisible $\alpha$ decay into DM particles, which carry away part of the beam energy. A more detailed description of the NA64e detector can be found in Refs. [30, 31]. Below, its main relevant features will be briefly mentioned. The detector schematically shown in Fig. 2 employed a 100 GeV pure electron beam, using the H4 beam-line of the CERN’s North Area with intensity of up to $\approx 10^7$ electrons per spill. The beam electrons impinging the target are measured by a magnetic spectrometer consisting of two successive dipole magnets and a low-material-budget tracker chambers $T1 - T4$ [32]. The beam electrons are tagged by detecting the synchrotron radiation (SR) emitted by them in the magnets with the SRD counter [33]. The active target is an electromagnetic calorimeter (ECAL), followed by a hermetic hadronic calorimeter (HCAL) consisting of three consecutive modules. The HCAL and the counters MU1-MU3, located between the modules, are used as an efficient veto against hadronic secondaries and also for identification of muons produced in the primary $e^-$ interactions in the final state.

The signature of the reaction [24] would be an event with a fraction of the beam energy deposited in the ECAL accompanied by a single muon outgoing from the target and passing the three HCAL modules, as shown in Fig. 2. In these searches a sample of $\approx 10^8$ rare dimuon events from form the QED production in the target, dominated by the hard bremsstrahlung photon conversion into the $\mu^+\mu^-$ pair on a target nucleus, $e^-Z \to e^-Z\gamma; \gamma \to \mu^+\mu^-$ was accumulated. Differently from the reaction [24] shown in Fig. 2 these events are accompanied by two muons in the final state passing though the HCAL modules. They exhibit themselves as a narrow strip in the measured distribution of events in the $(E_{ECAL};E_{HCAL})$ plane shown in Fig. 3 (region I), corresponding to the double MIP (minimum ionizing particle) energy deposition in the HCAL around $\approx 12$ GeV [30, 31]. Using these sample one could define the signal region for events from [24] to be ($E_{ECAL} < 50$ GeV; $E_{HCAL} \approx 6$ GeV) where the first cut (also used for the search for invisible decays of $\alpha$’s [30, 31]) is the one on the missing energy in the ECAL carried away by the $\alpha$ and the muon. While the second selection criterium corresponds to the requirement of the total energy deposited in three HCAL modules to be equal to the MIP energy from a single muon.

Intriguingly, five candidate events are observed in the signal region. The origin of these events is the subject of further detailed analysis. Here, we conservatively attribute these events to background, presumably due to the low-energy tail of the HCAL energy distribution for dimuon events. Taking this into account, the signal yield and detector signal acceptance were evaluated with a generic Monte Carlo simulations used for estimation of the NA64e sensitivity [34].

The combined 90% C.L. exclusion limits on the coupling $h_{\mu e}$ as a function of the $m_\alpha$ mass, are shown in Fig. 4. For the region $m_\alpha \lesssim 0.5$ GeV, NA64 bounds are more stringent than those derived from the $(g - 2)_\mu$ and ATLAS experiment, excluding part of the parameter space favoured by the muon anomaly.

In future searches the NA64e can be upgraded by a magnetic spectrometer downstream the HCAL allowing
for the measuring of both, the outgoing muon momentum and, in combination with the ECAL, the missing energy carried away by the $\tau$. This would significantly improve the search sensitivity, as the signature of the signal would be again a single muon from the target, plus a significant missing energy in the event. Another complementary search could be performed with the NA64$\mu$ detector at M2 muon beam of the CERN SPS [3, 6] by using the $a$ production in the reaction

$$\mu Z \rightarrow eZa; a \rightarrow \text{invisible} \quad (25)$$

of 100-160 GeV muon scattering on heavy nuclei. The main background for these searches came i) from an asymmetric dimuon production $eZ \rightarrow eaZ \gamma \rightarrow eZ \mu^+\mu^-$ when one of the muons is lost. The production of $\tau$ leptons in $eZ \rightarrow eZ \gamma \rightarrow eZ \tau^+\tau^-$ and corresponding background due to decays $\tau \rightarrow e(\mu)\nu\bar{\nu}$ is highly suppressed due to high mass of the $\tau$. ii) for reaction (25) the electron could be produced via the incoming from the muon decay chain $\mu \rightarrow e\nu\nu$ in the target or iii) production and decay of the leading pion $eZ \rightarrow eZ \pi^+X$; $\pi \rightarrow \mu\nu$ for the reaction [24]. The feasibility study to show that these and other possible background sources are either small or could be suppressed mainly due to a negligible admixture of muons in the primary $e^-$ beam and by applying the missing energy technique in combination with an active target measurements, successfully used for searches of invisible decays of dark photons [37, 38]. Further searches for the processes (24) and (25) with the increased by an order of magnitude number of events are promised to be background free and with a significantly improved sensitivity allowing to allow to probe decisively the $(g-2)_\mu$ and dark matter parameter space, as shown in Fig.1 The $a$ could be also searched for by using the missing momentum technique at the LDMX experiment with a sensitivity comparable to the NA64 one [33, 39].

Finally, note that our model also predicts contribution to the anomalous electron magnetic model $\Delta a_e = 0.6 \times 10^{-13}$. This value is a factor five less then the current error on $\Delta a_e = (4.8 \pm 3.0) \times 10^{-13}$ determined from the recent precise measurements of the fine-structure constant [40], and hopefully can be probed in the near future.

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[42] For $m_2 > m_1 + m_{\mu} + m_e$ the heaviest dark matter particle $s_2$ is unstable and it decays into the lightest dark matter particle $s_1$ and $\mu e$ pair, namely $s_2 \rightarrow s_1 \mu^+ e^-$. That in full analogy with the case of pseudo Dirac light matter allows to escape Planck restrictions. Here we shall not consider this possibility carefully.