Implications of maximum acceleration on dynamics

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Considering the corrected Unruh temperature as well as the entropic force perspective of gravity, we are able to derive the modification of the Newton’s law of gravity. In addition, we also investigate the effects of the highest achievable acceleration correction to the Unruh temperature on the Poisson equation. Moreover, we address modifications to the Newtonian cosmology as well as the Friedmann equations corresponding to the upper bound for acceleration.

I. INTRODUCTION

Since the discovery of black holes thermodynamics in 1970’s\textsuperscript{[1–6]}, physicists have been speculating that there should be some deep connection between gravity and thermodynamics. Indeed, thermodynamics also gets us some motivations for obtaining various spacetimes\textsuperscript{[7–9]}. According to the black hole thermodynamics, a black hole has an entropy proportional to its horizon area and a temperature proportional to its surface gravity, and the entropy and temperature together with the mass of the black hole satisfy the first law of thermodynamics. The pioneering work on the direct connection between gravity and thermodynamics was done by Jacobson\textsuperscript{[10]} who disclosed that the hyperbolic second order partial differential Einstein equation can be derived by applying the fundamental relation $\delta Q = T dS$ together with proportionality of entropy to the horizon area of the black hole. This profound connection between the first law of thermodynamics and the gravitational field equations has been extensively observed in various gravity theories\textsuperscript{[11–16]}. Applying the thermodynamics laws to dynamical and static horizons, one may obtain the gravitational field equations and the Friedmann equations in a wide range of gravity theory\textsuperscript{[17–33]}. The deep connection between horizon thermodynamics and gravitational dynamics, help to understand why the field equations should encode information about horizon thermodynamics. These results prompt people to take a statistical physics point of view on gravity.

The great step towards understanding the statistical origin of gravity, put forward by Verlinde\textsuperscript{[34]} who claimed that the laws of gravity are not fundamental and in particular they emerge as an entropic force caused by the changes in the information associated with the positions of material bodies. According to Verlinde, the tendency of a system to increase its entropy leads to emergence of gravity and spacetime\textsuperscript{[34]}. Moreover, if the distance between test particle and holographic screen is of order of the Compton wavelength of test mass, then it is assumed that particle is completely attracted by the system\textsuperscript{[34]}. In addition, applying the first principles, namely, the holographic principle and the equipartition law of energy, Verlinde derived Newton’s law of gravitation, the Poisson equation, and in the relativistic regime the Einstein field equations. Similar argument was also done by Padmanabhan who observed that the equipartition law of energy for the horizon degrees of freedom combined with the thermodynamic relation $S = E/2T$ lead to Newton’s law of gravity\textsuperscript{[35]}. Introducing a new proposal for the entropic force scenario\textsuperscript{[36]}, Cai et. al., have obtained the Newtonian cosmology and Friedmann equations using the entropic force scenario.

It was also argued that entropic force scenario is in conflict with the observation of ultracold neutrons in the gravitational field of Earth\textsuperscript{[37]}, a result which makes this hypothesis doubtful\textsuperscript{[38]}. However, subsequent studies have shown that the mentioned experiment can not necessarily reject the entropic origin of gravity\textsuperscript{[39–41]}, motivating physicists to investigate its relation with generalized entropy formalism\textsuperscript{[41]}, conservative force notion\textsuperscript{[42]}, and the quantum fluctuations of fields\textsuperscript{[43]}.

It is worth noting that in the entropic force approach towards gravity, the entropy expression of the holographic screen, the equipartition law of energy and the Unruh temperature formula on the holographic screen play crucial role. Any modification of any of these quantities may lead to modified versions of the gravitational field equations as well as Friedmann equations. For example, the entropy-area relation can be modified from the inclusion of quantum effects, motivated from the loop quantum gravity\textsuperscript{[44–51]}. It was shown that by employing the modified entropy-area relation, one can derive corrections to Newton’s law of gravitation as well as modified Friedmann equations by adopting the viewpoint that gravity can be emerged as an entropic force\textsuperscript{[52]}. Moreover, inspired by the Debye model for the equipartition law of energy in statistical thermodynamics, it was shown that by modification of the equipartition law of energy in the very low temperature and adopting the viewpoint of the entropic force, Einstein field equations and Poisson equation can be modified as well\textsuperscript{[53]}. Interestingly, it was also shown that the origin of the modified Newtonian dynamics (MOND) theory can be understood from Debye entropic gravity perspective\textsuperscript{[53, 54]}. It has also been shown that if one considers either quantum statistics or non-extensive statistical mechanics instead of the class-
cal statistics, then one can find theoretical origins for the MOND theory \cite{53, 50}. It has also been shown that the entropic force hypothesis can be used to obtain various gravitational theories and their corresponding cosmology \cite{57, 52}. It is worthwhile mentioning that none of these attempts study the effects of corrections to the Unruh temperature on systems.

There are various arguments which claim that there is an upper bound for the acceleration of systems \cite{93–99}. This upper bound can have different values depending on the physical situations of the system \cite{100}. These various proposed values for the upper bound are comparable with the value of the Planck acceleration which is of order $10^{51} m/s^2$. It has also been shown that the quantum value of maximum acceleration is in relation with the rate of the universe expansion during the Planck era \cite{101}. Moreover, this upper bound can modify Unruh temperature \cite{102}. It is useful to note here that there are also another corrections to Unruh temperature found in \cite{103–108}. Taking into account the fact that the Unruh temperature plays a crucial role in the entropic force scenario \cite{94}, one can ask which modifications to the Newtonian cosmology and Friedmann equations are allowed by the modified Unruh temperature given in \cite{102}? Moreover, whether or not this modified Unruh temperature leads to corrections to the Newton’s law of gravity and the Poisson equation?

In the present paper, we would like to address the above questions by taking into account the maximum acceleration correction to Unruh temperature on the holographic screen. The rest of this paper is organized as follows. In the next section, we show that correction to the Unruh temperature modifies Newton’s law of gravity and the Poisson equation. Modifications to the Newtonian cosmology and the Friedmann equations caused by the corrected Unruh temperature are explored in section III. We summarize our results in section IV.

II. MODIFICATION TO NEWTON LAW’S OF GRAVITY AND POISSON EQUATION

Consider a particle with acceleration $a$ moving in a spacetime with line element $ds^2$. It has been shown that if the particle acceleration value is bounded by an upper value ($a_m$), then the line element will be modified as \cite{93, 94}

$$ds^2 = \left(1 - \frac{a^2}{a_m^2}\right) ds^2. \quad (1)$$

Using the above correction to the line element, Benedetto and Feoli have proven that the Unruh temperature is also affected by the maximum acceleration ($a_m$) as \cite{102}

$$T = \frac{T_U}{\sqrt{1 - \frac{a^2}{a_m^2}}}. \quad (2)$$

where

$$T_U = \frac{\hbar a}{2\pi \hbar c B}, \quad (3)$$

is the well-known Unruh temperature \cite{6}. It is apparent that if we do not consider the maximum acceleration limitation (or equally $a_m \to \infty$), then we have $ds^2 \to ds^2$ and $T \to T_U$, desired results. Finally, it is worth to mention that different values of $a_m$ have been proposed in various physical situations (see \cite{100} and references therein).

We consider a system with total relativistic energy $E = Mc^2$ and finite boundary which forms a closed surface, and plays the role of storage device for information, i.e. a holographic screen \cite{34}. We also assume that the holographic principle holds, i.e. the number of surface degrees of freedom ($N_S$) is equal to those of the system bulk ($N_b$) leading to $N_S = N_b \equiv N$. Now, the equipartition law of energy can be employed to find \cite{34}

$$E = \frac{1}{2} N k_B T, \quad (4)$$

where

$$N = \frac{A}{\ell_p^2}, \quad \ell_p = \sqrt{\frac{G \hbar}{c^3}}, \quad (5)$$

and $\ell_p$ is the Planck length. It is also useful to note here that the mass $M = E/c^2$ is located in the center of the holographic screen \cite{34}.

A. Correction to the Newton’s law of gravity

We consider a spherical holographic screen with radius $r$ as the boundary of the system. The area of this sphere is $A = 4\pi r^2$. Combining Eqs. (2) and (5) and using relation $E = Mc^2$, one can obtain

$$\frac{a}{\sqrt{1 - \frac{a^2}{a_m^2}}} = \frac{GM}{r^2}, \quad (6)$$

which can also be rewritten as

$$a = \frac{a_m}{\sqrt{1 + \left(\frac{2m}{a_N}\right)^2}}, \quad (7)$$

where $a_N \equiv \frac{GM}{r^2}$ is the ordinary Newtonian acceleration. It is easy to obtain that if $a_N \ll a_m$ ($a_m \ll a_N$), then we have $a \approx a_N$ ($a \approx a_m$) meaning that $a_N \leq a \leq a_m$. Thus, modification to the acceleration may play role in the highly accelerated systems for which $a_N$ is at least comparable with $a_m$. It is also useful to mention here that by using Eqs. (2) and (6), one can get $T = \frac{\hbar a}{2\pi \hbar c B}$.
as the Unruh temperature of a holographic screen with radius \( r \) felt by an observer with acceleration \( a_N \).

In this way we obtained the correction to the Newton’s law of gravity resulting from the corrections to the Unruh temperature. Let us compare the result obtained here with the modified Newtonian dynamics (MOND) resulting from Deby entropic gravity [53]. It was argued that by adopting Debye correction to the equipartition law of energy in the framework of entropic gravity scenario, it is possible to understand the theoretical origin of the MOND theory [34, 53]. According to the MOND theory, the Newton’s law of gravity is modified as,

\[
a a (a_0) = \frac{GM}{r^2},
\]

in order to explain the flat rotational curves of spiral galaxies [109–111]. Here \( \mu = 1 \) for usual-values of accelerations and \( \mu = \frac{a}{a_0} (\ll 1) \) if the acceleration ‘\( a \)’ is extremely low, lower than a critical value \( a_0 = 10^{-10} \text{ m/s}^2 \) [53]. At large distance, at the galaxy out skirt, the kine
tatical acceleration ‘\( a \)’ is extremely small, smaller than \( 10^{-10} \text{ m/s}^2 \); i.e., \( a \ll a_0 \), hence the function \( \mu (\frac{a}{a_0}) = \frac{a}{a_0} \). Consequently, the velocity of star on circular orbit from the 
galaxy-center is constant and does not depend on the distance; the rotational-curve is flat, as it is observed.

Therefore, we conclude that the MOND theory is the modification of Newton’s law of gravity for small acceleration (temperature), \( a \ll a_0 = 10^{-10} \text{ m/s}^2 \), while the modification derived in Eq. (6), resulting from Deby entropic gravity [53]. It was argued that by adopting Debye correction to the equipartition law resulting from the corrections to the Unruh temperature by adopting the the viewpoint of Refs. [34, 53], to show that \( a \ll a_0 \), it is possible to understand the theoretical origin of the MOND theory [53].

B. Corrections to the Poisson equation

Since for a system of mass \( M \), enclosed by surface \( A \), we have \( E = M c^2 \), Eq. (1) can be written as [34]

\[
M = \frac{ck_B}{2G\hbar} \int T dA. \tag{9}
\]

Moreover, for a system with the acceleration \( \ddot{a} \), it is natural to define the Newtonian potential as

\[
\ddot{a} = -\ddot{\nabla} \phi. \tag{10}
\]

Following Refs. [34, 53], we can insert Eq. (10) into Eq. (8) to reach at

\[
T_U = \frac{\hbar |\ddot{\nabla} \phi|}{2\pi c k_B}, \tag{11}
\]

for the Unruh temperature. Now, using Eqs. (2) and (11), in order to rewrite Eq. (9), one can follow the approach of Refs. [34, 53] to show that

\[
M = \frac{1}{4\pi G} \int \nabla \cdot \left[ \nabla \phi D(x) \right] dV = \int \rho(\vec{x}) dV, \tag{12}
\]

where \( \rho(\vec{x}) \) is the energy density and \( D(x) = \frac{1}{\sqrt{1-x}} \), with \( x = (\ddot{\nabla} \phi)^2/a_m^2 \). The above equation can be rewritten as

\[
\nabla \cdot [\ddot{\nabla} \phi D(x)] = 4\pi G \rho. \tag{13}
\]

This is the modified Poisson equation resulting from corrected Unruh temperature by adopting the the viewpoint that gravity is an entropic force. For \( \ddot{\nabla} D(x) \approx 0 \), we get \( D(x)^2 \ddot{\phi} = 4\pi G \rho \) as the corrected Poisson equation. Finally, it is useful to note that the standard Poisson equation \( (\ddot{\nabla}^2 \phi = 4\pi G \rho) \) is recovered for \( x \ll 1 \) \((\ddot{\nabla} \phi \ll a_m)\).

III. MODIFIED NEWTONIAN AND FRIEDMANN COSMOLOGY

We consider a homogenous and isotropic universe described by a FLRW metric as

\[
ds^2 = -dt^2 + R^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]. \tag{14}
\]

where \( R(t) \) is scale factor, and \( k = -1, 0, 1 \) corresponds to the open, flat and closed universes, respectively [112]. The apparent horizon of this spacetime, which is a marginally trapped hypersurface with vanishing expansion, is given by

\[
R_A = R(t) r_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \tag{15}
\]

where \( r_A \) is the co-moving radius of the apparent horizon, and can be considered as a proper causal boundary for this spacetime [22, 23, 113, 115]. Moreover, \( H = \dot{R}/R \), where dot denotes derivative with respect to time, is the Hubble parameter. Now, consider a situation in which the FLRW universe is filled by an energy-momentum source of \( T^\mu_\nu = \text{diag}(\rho, p, p, p) \), where \( \rho \) and \( p \) denote the energy density and pressure of the cosmic fluid, respectively, which obeys the energy-momentum conservation law as

\[
\dot{\rho} + 3H(\rho + p) = 0. \tag{16}
\]

Since the total mass \( (M) \) and the active gravitational (Tolman-Komar) mass \( (M) \) confined by the volume \( V \) are evaluated as [36]

\[
M = \int (T_{\mu\nu} u^\mu u^\nu) dV, \tag{17}
\]
and
\[ \mathcal{M} = 2\int (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu})u^\mu u^\nu dV, \]  
(18)
respectively, simple calculations lead to
\[ M = \rho V \]  
(19)
and
\[ \mathcal{M} = (\rho + 3p)V, \]  
(20)
for the above definitions of mass [36]. In order to obtain the above relations, we assumed \( \rho \) and \( p \) are functions of time, an assumption which comes from the homogeneity and isotropy of universe compatible with the FLRW model of universe [31].

A. Modified Newtonian Cosmology

Consider the apparent horizon as the holographic surface, we have
\[ a = -\frac{d^2 R_A}{dt^2} = -\ddot{R} r_A, \]  
(21)
for the acceleration of a radially co-moving observer at \( r_A \). Since Eq. (2) is the backbone of our calculations in this subsection and the following subsection, our results are valid for systems in which \( |a| < a_m \) parallel to the \( |\dot{R}| \sim \frac{a}{r_A} \) condition.

Now, inserting Eq. (21) into Eq. (6), and using Eq. (19), one obtains the modified Newtonian cosmology as
\[ \frac{\ddot{R}}{R} \left[ 1 - \left( \frac{\dot{R} r_A}{a_m} \right)^2 \right]^{1/2} = -\frac{4\pi G}{3} \rho. \]  
(22)
It is apparent that the Newtonian cosmology is recovered in the absence of the maximum acceleration limitation (or equally at the \( a_m \to \infty \) limit).

For \( |\dot{R}| \ll \frac{a_m}{r_A} \), we can easily expand this result to find
\[ \frac{\ddot{R}}{R} \left[ 1 + \frac{1}{2} \left( \frac{\dot{R} R_A}{a_m} \right) \right]^2 = -\frac{4\pi G}{3} \rho, \]  
(23)
where we neglected the higher order terms and used the \( R_A = R(t)r_A \) relation to obtain this equation. Therefore, the first modification to the Newtonian cosmology, due to the maximum acceleration limitation, is in the form of \( O\left[\left(\frac{a_m}{r_A}\right)^3\right] \), and it only plays role in highly accelerated systems.

B. Modified Friedmann equations

Here, we are going to find the effects of corrected Unruh temperature [2] on the Friedmann equations. In order to achieve this goal, we use Eq. (20) instead of Eq. (19), and follow the recipe which led to Eq. (22). This procedure leads to
\[ \frac{\ddot{R}}{R} \left[ 1 - \left( \frac{\dot{R} r_A}{a_m} \right)^2 \right]^{-1/2} = -\frac{4\pi G (\rho + 3p)}{3}. \]  
(24)
It is easy to check that the standard second Friedmann equation is obtainable in the appropriate limit of \( A \to \infty \). In fact, the corrected term will be tangible whenever \( \dot{R} r_A \) is comparable with \( a_m \). In the \( \frac{\dot{R} r_A}{a_m} \ll 1 \) limit, this equation is reduced to
\[ \frac{\ddot{R}}{R} + \frac{1}{2} \left( \frac{\dot{R}}{R} \right)^3 \left[ \frac{R_A}{a_m} \right]^2 = -\frac{4\pi G (\rho + 3p)}{3}. \]  
(25)
Indeed, the second term in the lhs is the first order correction to the acceleration equation due to the maximum acceleration limitation.

Using Eq. (19), the rhs of this equation can be written as
\[ -\frac{4\pi G (\rho + 3p)}{3} = \frac{4\pi G d(\rho R^2)}{3R} \frac{dR}{dR}. \]  
(26)
Combining this result with Eq. (24), we can get the modified Friedmann equation as
\[ \frac{1}{R^2} \int \frac{d\dot{R}^2}{\sqrt{1 - \left( \frac{\dot{R} r_A}{a_m} \right)^2}} + \frac{\beta}{R^2} = \frac{8\pi G}{3} \rho, \]  
(27)
where \( \beta \) is the integration constant. Since the first Friedmann equation of the standard cosmology should be recovered at \( a_m \to \infty \) limit, one easily finds \( \beta = k \), which finally leads to
\[ H^2 + \frac{1}{2a_m^2 R^2} \int \left( \frac{\dot{R}}{R} \right)^2 R_A^2 d\dot{R}^2 + \frac{k}{R^2} = \frac{8\pi G}{3} \rho. \]  
(28)
in which \( H \equiv \dot{R}/R \) is the Hubble parameter. We considered the \( \frac{\dot{R} r_A}{a_m} \ll 1 \) limit, and expanded the integral function up to the first order of approximation to obtain this result. Therefore, this equation together with Eq. (24) cover the effects of maximum acceleration limitation on the evolution of FLRW universe. It is worth noting that the effects of the obtained corrections to the Friedmann equations may be seen in highly accelerated systems.
Adopting the Cai et al.’s version of the entropic force scenario \cite{19} and taking into account the maximum achievable acceleration correction to the Unruh temperature \cite{20,21}, we showed that the Newton’s law of gravity is modified as well. In addition, following the entropic force scenario and considering a continuous distribution of density $\rho$, we investigated the effects of the corrected Unruh temperature on the Poisson equation.

The effects of the maximum acceleration correction to the Unruh temperature on the Newtonian cosmology have also been addressed. Finally, we obtained the corrections to the Friedmann equations, and set the obtained constant ($\beta$) by considering the original Friedmann equations (standard cosmology) limit.

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