Singular solutions of Yang-Mills equations and bag model.

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Abstract

A model of quark confinement based on a singular solution of classical YM equation is proposed. Within the framework of this model we have calculated hadron masses that correspond to ground state configurations of quarks. Our results are in agreement with the experiment data with accuracy 3-7 percents for all hadronic masses except those of light pseudoscalar mesons.

1 Introduction.

Bag model is one of the first attempts to understand physics of hadrons in terms of quarks. In its simplest form it was formulated in the end of 60’s [1] and became well-known after the works of MIT group [2] in the middle of 70’s. So far many modifications of bag model were proposed [3]. It appeared that bag model gave rather satisfactory description of masses, magnetic moments and many other properties of hadrons [4]. However, in spite of these successes, bag model never was a very popular one. The origin is clear. In contrast to Standard model pretending for description
of hadron physics by proceeding from some fundamental principles, bag model is based on a pure phenomenological assumption that quarks are confined inside a certain sphere. Up to now it is absolutely unclear how to derive such an assumption from QCD and just so nowadays bag model is usually considered as a rather crude phenomenological model, and nothing more.

In the present paper we will try to show that, nevertheless, there exists a way to derive a kind of bag model from QCD.

Our basic assumption is that quark in zero approximation moves in a certain effective YM potential, that is a solution to classical YM equations with singularity on the sphere. Such solutions were discovered in 70's in papers [4]. More recently singular solution of YM equations was found as an analogue to Schwartzchild solution in general relativity [4]. In the latter paper a remarkable analogy between YM theory and general relativity established in the works [7] was used. (See [8] for further references). Later analogous solutions for Yang-Mills-Higgs equations were found [4]. Solutions with singularity on the torus and cylinder were investigated in [10].

Our model obviously can be derived from QCD by quantization in neighborhood of such a singular classical solution as zero approximation. Further corrections, in principal, can be also obtained in a systematic way. But in present paper we restrict ourselves to investigation of zero approximation. Namely, we will investigate the motion of quark in such field and evaluate mass spectrum of ground hadron states. We will show that our model gives quite satisfactory description of all ground hadron state except those of light pseudoscalar mesons. The discrepancy between our results and experiment data for masses of light pseudoscalar mesons is not a surprise. Indeed, it is known that many features of light mesons physics are related to spontaneous breaking of chiral symmetry. But the latter phenomenon is not taken into consideration within framework of our bag model at all. So one cannot hope to describe light meson mass spectrum within framework of the present simple version of our model. This problem needs further investigations.

The paper is organized as follows. In section 2 we describe spherically symmetric solutions of YM equations with singularity on the sphere. In section 3 we investigate motion of Dirac particle in such field. In section 4 we calculate the mass spectrum of ground hadron states by quantization in the neighborhood of this classical solution. In the last section we discuss results obtained.

2 Classical solutions of YM equations with singularity on the sphere.

Let us consider four dimensional SU(2) YM theory. Substituting the well-known Wu-Yang ansatz [11]

\[ A_0^a = 0, \ A_i^a = \varepsilon_{aij} \frac{x^j}{r^2}(1 - H) \]  

(1)
in YM equations, one get

\[ r^2 H'' = H(H^2 - 1) \]  \hspace{1cm} (2)

It can be proved that there are only two types of solutions of equation (2) that are singular on some sphere and regular at \( r = 0 \) (see Fig.1).

\[
H_1(r) \sim \frac{\sqrt{2}}{1 - r/R}, \quad r \to R - 0; \quad H_1(r) \sim 1 + c_1 \left(\frac{r}{R}\right)^2, \quad r \to 0 \]  \hspace{1cm} (3)

and of the second one are:

\[
H_2(r) \sim \frac{\sqrt{2}}{1 - r/R}, \quad r \to R - 0; \quad H_2(r) \sim -1 + c_2 \left(\frac{r}{R}\right)^2, \quad r \to 0 \]  \hspace{1cm} (4)

where \( R > 0 \) is an arbitrary constant, \( c_1 \approx 3.038 \) and \( c_2 \approx 9.448 \).

It appears that satisfactory agreement between experimental data and results obtained from our model can be achieved only for solution with asymptotics (4).
So in what follows we will suppose that functions $H(r)$ is defined by eqs. (2), (4). Starting from these singular solutions of $SU(2)$ YM equations, one can construct two different sets of singular solutions of $SU(3)$ YM equations that correspond to two non-equivalent embeddings of the algebra $SU(2)$ in the algebra $SU(3)$. Namely, up to unitary equivalence, the first set of such solutions can be defined as

$$A = \frac{1}{2} A^1 \lambda^1 + \frac{1}{2} A^2 \lambda^2 + \frac{1}{2} A^3 \lambda^3$$

and the second one as

$$A = \frac{1}{2} A^1 \lambda^2 + \frac{1}{2} A^2 \lambda^5 + \frac{1}{2} A^3 \lambda^7$$

where $\lambda^\alpha$, $\alpha = 1, 2 \cdot \cdot \cdot 8$, are Gell-Mann matrices and $A^a$, $a = 1, 2, 3$ are defined by formula (1).

3 Quark in Yang-Mills field with singularity on the sphere.

Let us consider solutions of Dirac equation

$$(i \gamma^0 \partial_t + i \gamma^j \nabla_j) \Psi = m \Psi$$

(7)

for the quark in potential (5) or (6). In the case (5) quark cannot be confined inside the sphere $r = R$ because the third component of quark field satisfies free Dirac equation. So in what follows we restrict ourselves to consideration of the case (6).

By virtue of spherical symmetry of the potential (6) Dirac Hamiltonian commutes with operators of angular momentum

$$J^a = \frac{i}{4} \varepsilon^{abc} [\gamma^b, \gamma^c] + I^a + l^a$$

where $(I^a) \equiv (\lambda^2, \lambda^5, \lambda^7)$ are color isospin operators and $l^a$ are orbital angular momentum ones,

$$l_i = -i \varepsilon_{ijk} x_j \partial_k$$

The solutions with definite total angular momentum and definite energy $E$ can be represented as

$$\Psi = \frac{1}{r} \left( \frac{B^+_1(r) \nabla_j \Omega_{JM}^{J+1/2} + D^+_1(r) n_j \Omega_{JM}^{J+1/2} + C^+_1(r) l_j \Omega_{JM}^{J+1/2}}{\sqrt{E - m} (B^-_1(r) \nabla_j \Omega_{JM}^{J-1/2} + D^-_1(r) n_j \Omega_{JM}^{J-1/2} + C^-_1(r) l_j \Omega_{JM}^{-J-1/2})} \right) e^{-iEt}$$

$$+ \frac{1}{r} \left( \frac{iB^+_2(r) \nabla_j \Omega_{JM}^{-J-1/2} + iD^+_2(r) n_j \Omega_{JM}^{-J-1/2} + iC^+_2(r) l_j \Omega_{JM}^{-J+1/2}}{\sqrt{E - m} (B^-_2(r) \nabla_j \Omega_{JM}^{J+1/2} + D^-_2(r) n_j \Omega_{JM}^{J+1/2} + C^-_2(r) l_j \Omega_{JM}^{1/2})} \right) e^{-iEt}$$

(8)
where $\Omega_{JM}^{J+1/2}$ are spherical spinors

\[
\begin{align*}
\Omega_{JM}^{J+1/2} &= \left( \sqrt{\frac{J+M+1}{2(J+1)}} Y_{J+1/2 M-1/2} \right) \\
\Omega_{JM}^{J-1/2} &= \left( \sqrt{\frac{J-M+1}{2(J+1)}} Y_{J+1/2 M+1/2} \right)
\end{align*}
\]

and

\[
\nabla_i^\Omega = r \frac{\partial}{\partial x_i} - x_i \frac{\partial}{\partial r}, \quad n_i = \frac{x_i}{r}
\]

One notes, that $J = 1/2, 3/2, \cdots$. Substituting (8) in (7), one gets:

\[
\begin{align*}
B_1^{+} &= \frac{1}{J + 1/2} \left( -(J - 1/2)(J + 3/2) \frac{1}{r} B_1^{+} + \frac{H}{r} D_1^{+} - (J - 1/2) \frac{1}{r} C_1^{-} - \mathcal{E}(C_1^{+} - (J - 1/2) B_1^{-}) \right) \quad (all \ J) \\
B_1^{-} &= \frac{1}{J + 1/2} \left( (J - 1/2)(J + 3/2) \frac{1}{r} B_1^{-} - \frac{H}{r} D_1^{-} - (J + 3/2) \frac{1}{r} C_1^{+} - \mathcal{E}(C_1^{-} + (J - 1/2) B_1^{+}) \right) \quad (J > 1/2) \\
D_1^{+} &= -(J + 1/2) \frac{1}{r} D_1^{+} + (J + 3/2) \frac{H}{r} B_1^{+} - (J - 1/2) \frac{H}{r} C_1^{-} + \mathcal{E}D_1^{-} \quad (all \ J) \\
D_1^{-} &= (J + 1/2) \frac{1}{r} D_1^{-} + (J + 3/2) \frac{H}{r} C_1^{+} - (J - 1/2) \frac{H}{r} B_1^{-} - \mathcal{E}D_1^{+} \quad (all \ J) \\
C_1^{+} &= \frac{1}{J + 1/2} \left( (J - 1/2)(J + 3/2) \frac{1}{r} C_1^{-} - \frac{H}{r} D_1^{+} - (J + 3/2) \frac{1}{r} B_1^{+} + \mathcal{E}(B_1^{-} + (J + 3/2) C_1^{+}) \right) \quad (J > 1/2) \\
C_1^{-} &= \frac{1}{J + 1/2} \left( -(J - 1/2)(J + 3/2) \frac{1}{r} C_1^{+} + \frac{H}{r} D_1^{-} - (J - 1/2) \frac{1}{r} B_1^{-} + \mathcal{E}(B_1^{+} - (J - 1/2) C_1^{-}) \right) \quad (all \ J)
\end{align*}
\]

where $\mathcal{E} = \sqrt{E^2 - m^2}$, and exactly the same system of equations for functions $B_2^{\pm}$, $C_2^{\pm}$, $D_2^{\pm}$. So initial Dirac equations are separated into two identical sets of equations. This leads to degeneration of all energy levels.

This is not surprises. Indeed, Dirac equations in external chromomagnetic field with property $(\tilde{A}(-x) = -\tilde{A}(x))$ are invariant under parity transformations. It is well-known [12] that this symmetry implies $N = 1$ supersymmetry of Dirac equations that, in turn, implies the doubling of energy levels. Thus all energy levels appear to be degenerate in parity. As a consequence, all hadrons must be degenerate in parity.
This is a puzzle of our model. In what follows, we postulate that quarks may be in states with only one parity.

States with definite parity correspond to solutions of Dirac eq.(7) for which either $B_1^\pm = C_1^\pm = D_1^\pm = 0$ or $B_2^\pm = C_2^\pm = D_2^\pm = 0$. For definiteness, we choose the second case.

If $J = 1/2$ then only four functions $B_1^+(r)$, $D_1^+(r)$, $C_1^+(r)$ and $D_1^-(r)$ survive in eqs.(9). Surprisingly, but four first order equations for the functions $B_1^+(r)$, $D_1^+(r)$, $C_1^+(r)$ and $D_1^-(r)$ reduce to one second order equation. Namely, if

$$
x_+ = D_1^+ + \sqrt{2} B_1^+, \quad x_- = D_1^+ - \sqrt{2} B_1^+
$$

then simple analysis of eqs.(9) shows that either

$$y_- ' + \left( \mathcal{E}^2 - \frac{2H^2}{r^2} - \frac{\sqrt{2}H'}{r} \right) y_- = 0 \quad (10)$$

or

$$y_+ ' + \left( \mathcal{E}^2 - \frac{2H^2}{r^2} + \frac{\sqrt{2}H'}{r} \right) y_+ = 0 \quad (11)$$

Eqs.(10) as well as eqs.(11) has only one square locally integrable solution at $r = R$. It automatically satisfies boundary condition (12)

$$y_\pm (R) = 0 \quad (12)$$

This means that quark cannot penetrate through the sphere $r = R$. Indeed, equations describing quark penetration through the potential barrier at $r = R$ must have at least two linear independent locally square integrable at $r = R$ solutions that correspond to two possible directions of quark motion (into the sphere and out of the sphere). Lowest eigenvalues $\mathcal{E} = \mathcal{E}_n/R$ of eqs.(10),(11) with function $H$ defined by eqs.(2),(4) are given in table 1.

Energy levels can be can be express via numbers $\mathcal{E}_n$ as

$$E_n(R) = \sqrt{\frac{\mathcal{E}_n^2}{R^2} + m^2} \quad (13)$$

Let us consider the case $J \geq 3/2$. In this case the equations (9) have four linearly independent solutions that are locally square integrable at $r = R$. Two of them vanish at $r = R$ whereas two other have asymptotics
Table 1: Eigenvalues $\mathcal{E} = \mathcal{E}_n/R$ for eq.(10) and (11). Data in the left and right columns correspond to eq.(10) and (11) respectively.

| $\mathcal{E}_n$ |  |
|-----------------|------------------|
| 7.800          | 1.997            |
| 10.920         | 7.288            |
| 14.074         | 10.885           |

where $C_1, C_2$ are constants.

This means that quarks with $J \geq 3/2$ are not confined inside the sphere $r = R$. In particular, normal component of the current $j^\nu_\nu = \bar{\psi} \gamma^\nu \psi$ on the surface $r = R$ is not vanished:

$$n_\nu j^\nu_{|r=R} \sim C_1 C_2$$  \hspace{1cm} (16)

In such situation the only possibility to confine quark with $J \geq 3/2$ sphere $r = R$ is to impose ("by hands", as in usual bag models) the boundary condition

$$n_\nu j^\nu_{|r=R} = 0$$  \hspace{1cm} (17)

Obviously eq.(17) implies $C_1 C_2 = 0$ and so either $C_1 = 0$ or $C_2 = 0$. In other words, we must forbid either asymptotics (14) or asymptotics (15). We choose the condition $C_1 = 0$.

Results of numerical evolution of lowest eigenvalues $\mathcal{E}$ of the system (9) with boundary condition $C_1 = 0$ are presented in table 2.
Let us consider "partition function"

\[ Z = \text{tr} e^{-iHT} \]

where \( H \) is QCD Hamiltonian. "Partition function" can be represented as

\[ Z = \int DA D\bar{\Psi} D\Psi e^{i \int_0^T dt \int d^3x (L_{YM}(A) + L_{ferm}(\bar{\Psi}, \Psi, A))} \]  

(18)

We assume that the main contribution in functional integral (9) is given by trajectories close to classical solution \( A_{cl} \) of YM equations defined by eqs.(2),(4), and (6). Applying stationary phase method, one gets in zero approximation:

\[ Z = \int_{\text{zero modes}} e^{-iE_{YM}(R)T} \prod_q \det \left[ \gamma^0 (i\gamma^0 \frac{\partial}{\partial t} + i\gamma^\mu \nabla(A_{cl}) - m_q) \right]_{\text{APBC}} \]  

(19)

where \( \text{APBC} \) means "anti-perodic boundary conditions" and \( E_{YM}(R) \) is classical energy of the field \( A_{cl} \). The determinant in (19) can be easy evaluated in terms of positive eigenvalues \( E_s(R, m_q) \) (defined by eq.(13)) of Hamiltonian

\[ i\gamma^0 \gamma^\mu \nabla(A_{cl}) - \gamma^0 m_q \]

that have been evaluated in section 3 (see, for instance, [13]). Substituting the result in (19), one obtains:

\[ Z = \sum_{1 \leq k_s \leq n_s} \int_{\text{zero modes}} e^{-i(E_{YM}(R) + \sum_q k_s E_s(R, m_q))T} \]  

(20)

where \( n_s = 4(2J + 1) \) is doubled multiplicity of the eigenvalue \( E_s \) (Remind, that multiplicity of the eigenvalue \( E_s \) with given \( J \) is \( 2(2J + 1) \). Additional factor 2 correspond to contribution of anti-quarks.).

### Table 2: Eigenvalues \( \mathcal{E} = \mathcal{E}_n/R \) for eq.(9)

| \( \mathcal{E}_n \) | \( J = 3/2 \) | \( J = 5/2 \) |
|------------------|-----------|-----------|
| 4.165           | 5.151     |
| 6.977           | 8.518     |
| 7.949           | 8.863     |
By virtue of scale invariance of YM equations the measure of integration in (20) comprises integration with respect to \( R \). So, applying again stationary phase method, one gets

\[
Z \sim \sum_{1 \leq k_s \leq n_s} e^{-i(E_{YM}(R_0) + \sum_q \sum_s k_s E_s(R_0, m_q))T} \tag{22}
\]

where \( R_0 \) is defined from an equation

\[
\frac{\partial}{\partial R} [E_{YM}(R) + k_s E_s(R, m_q)] \bigg|_{R=R_0} = 0 \tag{23}
\]

So hadronic masses are defined by the formula

\[
M^{\text{theor}} = E_{YM}(R_0) + \sum_{\text{quark}} E(R_0, m_q) \tag{24}
\]

The quantity \( E_{YM}(R) \) is divergent due to singularity of \( A_{cl} \) at \( r = R \). We assume that \( E_{YM}(R) \) become finite after renormalization. However, so far we have not elaborated renormalization procedure in our model. Instead, we simply postulate that

\[
E_{YM}(R) = BR^n \tag{25}
\]

where \( B \) is some constant. The choice \( n = 3 \) correspond to MIT model. In our model the choice \( n = 2 \) also seems natural because the main contribution in \( E_{YM}(R) \) is proportional to the area of this sphere.

Fortunately, it appears that hadronic masses depend on \( n \) very weakly. So concrete choice of this parameter is not important.

Results of calculation of hadronic masses, corresponding to ground state configurations of quarks, are presented in Tables 3 and 4. Parameters \( B, m_u = m_d, m_s, m_c \) and \( m_b \) are determined by minimizing of the quantity

\[
\Delta(B, m_u, m_s, m_c, m_b) = \sum_h [(M^{\text{exp}}_h - M^{\text{theor}}_h)/M^{\text{exp}}_h]^2
\]

where \( M^{\text{exp}}_h \) are masses of hadrons \( \Xi, \Lambda_c, \Lambda_b^0, D_c^\pm, D_{s_c}^\pm, B \) and \( \eta_c \) that are measured with the best accuracy in comparison with ones of other hadrons.

5 Discussion.

We see that our model gives rather good description for almost all hadron masses corresponding to ground states configurations of quarks except those of light pseudoscalar mesons. As we already have mentioned in Introduction, the description of
the light pseudoscalar mesons cannot be given without consideration of the problem of chiral symmetry breaking that is out of the scope of our model nowadays.

The accuracy 3-7 percents achieved in our model is maximal possible one for any constituent quark model in which interaction between quarks is not taken into account. Indeed, in any such model \( \Lambda \) and \( \Sigma \) particle must have the same mass. But really there exists approximately 7% difference between \( m_\Lambda \) and \( m_\Sigma \). The same is true for \( \Lambda_c \) and \( \Sigma_c \). So seven percents is, most likely, the maximal accuracy that can be achieved in any simple constituent quark model. In fact, this means that we cannot describe spectrum of hadron resonances in framework of our model now. Indeed, mass differences between hadron resonances are less, typically, than seven percents.

Our model meets some internal difficulties. In particular, we are obliged to impose "by hands" boundary condition (8) for quarks with \( J \geq 3/2 \). May be this difficulties (as many others) can be overcome if one consider instead of the solution (6) more general solution of SU(3) YM equation. Indeed, in the present paper we investigate, in fact, SU(2) QCD with quarks in adjoint representation. But gauge group of QCD is definitely SU(3) rather than SU(2). Most likely, many difficulties arising in our model are connected with this circumstance.

In the nearest future we plan to investigate singular solutions of SU(3) YM equations that cannot be reduced to any solution of SU(2) ones and also to develop perturbative theory on the background of such singular solutions. Results of the present paper show that there exists a good chance to obtain satisfactory description of mass spectrum and other properties of hadron in this way.

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| Particle(spin,mass(MeV)) | Quark configuration | Mass (n = 3) (MeV) | Mass (n = 2) (MeV) |
|--------------------------|---------------------|-------------------|-------------------|
| $p(n)$ ($J = 1/2, m = 940$) | $1S_u - 1S_u - 1S_d$ | 957 | 957 |
| $\Lambda (J = 1/2, m = 1116)$ | $1S_u - 1S_u - 1S_s$ | 1137 | 1137 |
| $\Sigma (J = 1/2, m = 1.189)$ | $1S_u - 1S_u - 1S_s$ | 1311 | 1311 |
| $\Xi (J = 1/2, m = 1320)$ | $1S_u - 1S_s - 1S_s$ | 2156 | 2149 |
| $\Lambda_c (J = 1/2, m = 2455)$ | $1S_u - 1S_u - 1S_c$ | 5601 | 5593 |
| $\bar{\Xi}_c (J = 1/2, m = 2465)$ | $1S_u - 1S_s - 1S_c$ | 2326 | 2318 |
| $\Xi_c (J = 1/2, m = 2465)$ | $1S_u - 1S_s - 1S_c$ | 2326 | 2318 |
| $\Sigma_c (J = 1/2, m = 2455)$ | $1S_u - 1S_u - 1S_c$ | 2326 | 2318 |
| $\Lambda_c (J = 1/2, m = 5641)$ | $1S_u - 1S_u - 1S_s$ | 1874 | 1883 |
| $D_s (J = 0, m = 1968)$ | $1S_u - 1S_s$ | 2029 | 2033 |
| $B (J = 0, m = 5278)$ | $1S_u - 1S_b$ | 5313 | 5320 |
| $\eta_c (J = 0, m = 2979)$ | $1S_c - 1S_c$ | 3011 | 3000 |
| $\rho (J = 1, m = 769)$ | $1S_u - 1S_u$ | 701 | 721 |
| $K^* (J = 1, m = 892)$ | $1S_u - 1S_s$ | 869 | 887 |
| $\phi (J = 1, m = 1019)$ | $1S_s - 1S_s$ | 1032 | 1047 |
| $Y (J = 1, m = 9460)$ | $1S_b - 1S_b$ | 9856 | 9830 |
| $\Delta^{++} (J = 3/2, m = 1232)$ | $1S_u - 1S_u - 1P_u$ | 1191 | 1159 |
| $\Omega^- (J = 3/2, m = 1672)$ | $1S_s - 1S_s - 1P_s$ | 1671 | 1646 |
| $\Omega_c^0 (J = 1/2, m = 2700(?)$) | $1S_s - 1S_s - 1S_s$ | 2491 | 2480 |

Table 4: Hadron mass spectrum evaluated by formulas (23)-(25). $1S_u - 1S_u - 1S_s$ means configuration in which two $u$ quarks and one $s$ quark in ground states with $J = 1/2$, etc; $1P_u$ and $1P_s$ means ground states of $u$ and $s$ quark with $J = 3/2$. One notes that configurations $1S_u - 1S_u - 1S_u$ and $1S_s - 1S_s - 1S_s$ are forbidden by Pauli principle. So just configurations $1S_u - 1S_u - 1P_u$ and $1S_s - 1S_s - 1P_s$ are true ground state configurations for $\Delta^{++}$ and $\Omega^-$. The value of spin in these cases automatically equal to 3/2.