Aharonov-Bohm Effect and Disclinations in an Elastic Medium

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In this work we investigate quasiparticles in the background of defects in solids using the geometric theory of defects. We use the parallel transport matrix to study the Aharonov-Bohm effect in this background. For quasiparticles moving in this effective medium we demonstrate an effect similar to the gravitational Aharonov-Bohm effect. We analyze this effect in an elastic medium with one and $N$ defects.

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I. INTRODUCTION

The first approach to the theory of defects in an elastic media was done by the Italian school with the development of the theory of dislocations in 1900. After those initial approaches, many theories and experiments were put forward in order to describe and to observe defects in solids\textsuperscript{11}. In recent years a series of articles, inspired by the pioneering ideas of Kröner\textsuperscript{2} and Bilby et al.\textsuperscript{3}, developed a geometric theory of defects. In this framework the elastic solids with topological defects can be described by Riemann-Cartan geometry\textsuperscript{4,5,6,7}.

In this formulation we use the techniques of differential geometry to describe the strain and stress induced by the defect in an elastic medium. All this information is contained in the geometric quantities (metric, curvature tensor, etc.) that describes the elastic medium with defects. The boundary conditions imposed by the defect, in the elastic continuum medium, are accounted for by a non-Euclidean metric. In the continuum limit, the solid can be viewed as a Riemann-Cartan manifold. In general, the defect corresponds to a singular curvature or torsion (or both) along the defect line\textsuperscript{8}, where the curvature and the torsion of the manifold are associated to the topological defects, disclinations and dislocations, respectively.

There are some advantages of this geometric description of defects in solids. In contrast to the ordinary elasticity theory, this approach provides an adequate language for continuous distribution of defects. The problem of the description of quantum (or classical) dynamics of particles (or quasiparticles) in the elastic medium is reduced to a problem in a curved/torsioned space. In this framework, the influence of the defects on the motion of electrons and phonons, for example, becomes reasonably easy to analyze, due to the fact that the boundary conditions imposed by the defects are incorporated into the geometry. In this geometric point view, the quasiparticles motion experience an effective non-Euclidean metric in an elastic media. This change in the effective metric experienced by the quasiparticles in this medium is caused by the stress and strain provoked in the elastic medium by topological defects. The classical motion of quasiparticles in the elastic background is described by geodesics in the effective metric.

In this work we use the geometric theory of defects to describe a medium with disclinations. We are interested in the study of phonon scattering in the presence of defects. In this context, phonons are represented by geodesics in the space that describes a deformed medium. Recently, the scattering of phonons by defects in solids was investigated making use of the geometric theory of defects\textsuperscript{8,9,10}. In these previous analyses, the calculus of geodesics was used to investigate the scattering of phonons in the presence of topological defects. In the present work we use parallel transport of vectors around the defects to investigate the Aharonov-Bohm effect in the scattering of phonons in the presence of a disclination and of multiple disclinations.

The geometric theory of defects shows the equivalence between three-dimensional gravity with torsion and the theory of defects in solids. The defect acts as a source of a "gravitational" distortion field. The metric describing the medium surrounding the defect is then a solution to the three-dimensional Einstein-Cartan equation. In a metric gravitational theory, a gravitational field is related to a nonvanishing Riemann curvature tensor. However, the presence of localized curvature can produce effects on the geodesic motion and also produce effects on parallel transport in regions where the curvature vanishes. A known example of this is provided when a particle is transported along a closed curve, which encircles an idealized cosmic string\textsuperscript{11}. This situation corresponds to the gravitational analogue\textsuperscript{12} of the electromagnetic Aharonov-Bohm effect\textsuperscript{13}.

These effects are of nonlocal origin and may be viewed as a manifestation of the nontrivial topology of the space-time of the cosmic string. It is worth calling attention to the fact that, differently from the electromagnetic Aharonov-Bohm effect, which is essentially a quantum
effect, its gravitational analogue appears also at a purely classical level. Thus, in summary, the gravitational analogue of the electromagnetic Aharonov-Bohm effect, in this context, is the following: particles constrained to move in a region where the Riemann curvature tensor vanishes may exhibit a gravitational effect arising from a region of nonzero curvature from which they are excluded.

In the present case, the space that describes the defects is characterized by conical singularities in the curvature tensor. The conical singularity gives rise to curvature concentrated on the disclination axis. In this way, a medium that contains a disclination is described by a nontrivial curvature concentrated on the defect axis. In this way, the quasiparticle moving in the elastic medium with a disclination is moving in a region of null curvature. We investigate the existence of the Aharonov-Bohm effect in the movement of a quasiparticle in the presence of a disclination based in this physical observations. The solid state analog of the Aharonov-Bohm effect in semiconductors with dislocations has been investigated recently. In this case, it was shown that the occurrence of Aharonov-Bohm interference for particles moving around charged dislocations in a semiconductor gives rise to magnetooconductance oscillations in the macroscopic sample. Related to this, the existence of a geometrical phase in screw dislocations has been also investigated.

Lately, Kataanæv has demonstrated that a geometric theory of defects is an equivalent description to nonlinear elasticity theory. He also showed that in the linear limit of his nonlinear geometric elasticity theory its solutions recovers the usual solutions of defects in a linear elasticity theory. Other alternative approaches to study this problem have been proposed which use either a gauge field or a gravity-like approach. Similar analogies hold in (2 + 1) dimensional gravity, where the spacetime geometry of a point particle can be understood in terms of distortions.

Our objective in this work is to demonstrate that an effect similar to the gravitational Aharonov-Bohm effect occurs in the scattering of phonons in an elastic medium with disclinations. This letter is organized as follows, in the next section we consider the defects in an elastic medium. In the third section, we study the holonomy transformations associated to the deficit angle investigation in a space with disclinations. In the fourth section, we extend our analysis to a global characterization of multiple defects in a medium. In the last section, we present the concluding remarks.

II. DEFECTS IN AN ELASTIC MEDIUM

Defects can appear from thermal fluctuations, from extreme boundary conditions imposed or by the action of external fields. A class of defects is called topological associated to some broken continuous symmetry and can be characterized by a core region where the order is destroyed and by an external region where the elastic properties are not changed. The physical systems can present broken continuous symmetry in which the elastic variables describe variations from a initial configuration. When we use a continuum approach we do not have information about microscopic details, however we can find a qualitative description. Usually, we use continuum models to represent a specific material. It could be based in information obtained by phenomenological constitutive models. We make an idealization to describe an elastic solid where the action of the microscopic degrees of freedom is summed up into few material parameters known as the elastic constants.

Recently, it was shown a quantitative agreement between the standard elasticity theory and the geometric theory of defects. The results obtained by a linear approximation from the geometric theory agree with those of elasticity theory, showing that there is a good concordance between the treatments.

We consider a wedge dislocation as an infinite elastic medium without the z-axis. This kind of defect, is constructed by a cut of an infinite wedge with the angle -2πθ, which can be removed or inserted. We start from the metric below which describes a wedge dislocation in the framework of the geometric theory of defects,

\[ ds^2 = \left( \frac{r}{R} \right)^{(2\beta-1)} \left( dr^2 + \frac{\alpha^2 r^2}{\beta^2} d\phi^2 \right) + dz^2, \]  

with

\[ \beta = -\theta \sigma + \sqrt{(\theta \sigma)^2 + 4(1 + \theta)(1 - \sigma)^2} \frac{2(1 - \sigma)}{2(1 - \sigma)}, \]

where σ is the Poisson ratio and θ is an angle associated with the formation angle of the defect. This Poisson ratio is related with elastic properties in real materials, and is determined by the strain along the normal direction. It can be observed experimentally. Note that, if we use in the metric the following transformation

\[ \xi = \frac{\alpha}{\beta} \left( \frac{1}{R} \right)^{\beta^{-1}} r^\beta, \]  

we obtain the conical metric

\[ ds^2 = \frac{1}{\alpha^2} d\xi^2 + \xi^2 d\phi^2 + dz^2, \]

that can also transformed by \( \rho = \xi/\alpha \) into

\[ ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2. \]

This result is very interesting because it resembles the space of the cosmic string. The nonzero components of the Riemannian curvature tensor of the metric above is given by

\[ R^2_{12} = R^1_2 = R^2_2 = 2\pi \frac{(1 - \alpha)}{\alpha} \delta^{(2)}(\rho), \]
which indicates that this metric corresponds to a locally flat medium with a conical singularity at the origin. Here, $\delta^2(\rho)$ is the two dimensional delta function in flat space. From the last expressions it follows that if $\alpha \in (0, 1)$ the defect carries positive curvature and if $\alpha \in (1, \infty)$ the defect carries negative curvature.

The metric given by (11) can be obtained by a linear approximation in $\theta$, where $\beta \approx 1 + \frac{\theta |2-\sigma|}{2(1-\sigma)}$ [13],

$$ds^2 = \left(1 + \theta \frac{1-2\sigma}{1-\sigma} \ln \frac{r}{R}\right) d\sigma^2 + r^2 \left(1 + \theta \frac{1-2\sigma}{1-\sigma} \ln \frac{r}{R} + \theta \frac{1}{1-\sigma}\right) d\theta^2. \quad (7)$$

This metric correspond the usual metric which is described by linear elasticity theory.

III. HOLONOMY TRANSFORMATION

Our study of the analogue of gravitational Aharonov-Bohm scattering of quasiparticles by a topological defect is done by computing the holonomy matrix for some specific closed paths. When a vector is parallel propagated along a loop in a manifold $M$, the curvature of the manifold causes the vector, initially at $p \in M$, to appear rotated with respect to its initial orientation in tangent space $T_pM$, when it returns to $p$. The holonomy is the path dependent linear transformation $T_pM \rightarrow T_pM$ responsible for this rotation. Positive and negative curvature manifolds, respectively, yield deficit or excess angles between initial and final vector orientation under parallel transport around such loops. This global property of the manifold can be used as a means of global classification of spacetimes, as pointed out by Rothman, Ellis and Murugan [32].

In recent works [8, 4, 10], the geodesics of particles in the presence of topological defects has been calculated. In the eikonal approximation, or geometric optics, elastic deformations described by the wave equation move along extremals. By analogy with motion of photons in electrodynamics it is natural to assume that extremals are trajectories of phonons in elastic media with defects. In this way analysis of extremals yields a complete picture of the scattering of phonons on disclinations. Here, the approach adopted by us consists in the utilization of the holonomy matrix to describe the phonon scattering in a disclimated medium. In this work we use the holonomy matrix to investigate in the classical level the analog of the Aharonov-Bohm effect. Essentially, holonomy is a matrix related to the notion of parallel transport of objects like vectors, spinors and tensors along curves in a given $n$-dimensional manifold. Associated to a path $\gamma$, connecting points $A$ and $B$ in the manifold, the parallel transport matrix is given by the path-ordered exponen-

$$U(\gamma) = \mathcal{P} \exp \left(-\int_{A}^{B} \Gamma_{\mu} dx^\mu \right), \quad (8)$$

where $\Gamma_{\mu}$ is the $n$-adic connection on the manifold and $\mathcal{P}$ is the path ordered product. When the path $\gamma$ is closed, $U(\gamma)$ is known as the holonomy matrix. Notice the similarity of this equation with the Dirac phase factor $\Phi(\gamma) = \mathcal{P} \exp \left(ig \oint_{\gamma} A_{\mu} dx^\mu \right)$, the observable in the Aharonov-Bohm effect.

Holonomy has had an important role in the loop formulation of gauge theories [34, 38], of quantum gravity [36] and is a very convenient tool to obtain topological properties of specific geometries. For instance, Burges [31] and Bezerra [32] examined the effects of parallel transport of vectors and spinors both around a point-like solution and a cylindrically symmetric cosmic string pointing out to a gravitational analogue of the Aharonov-Bohm effect. More recently, holonomy has been used to study quantum computation in a geometric approach [40].

The metric (11) is described by a dual 1-form basis $e^a$ defined in terms of the dreinbeins $e^a_\mu$ by: $e^a = e^a_\mu dx^\mu$, where

\begin{align}
e^1 &= dz, \quad (9a) \\
e^2 &= \left(\frac{r}{R}\right)^{(\beta-1)} dr, \quad (9b) \\
e^3 &= \left(\frac{r}{R}\right)^{(\beta-1)} \frac{\alpha r}{\beta} d\phi. \quad (9c) \\
\end{align} (9d)

In this way the metric is written as $ds^2 = e^1 e^1 + e^2 e^2 + e^3 e^3$. We need to determine the 1-form connection $\omega^a_b$ in order to find the holonomy transformations for the "space-time" of a defect. The 1-form connection satisfies Cartan’s structure equation, $T^a = de^a + \omega^a_b \wedge e^b$, where $T^a$ is the torsion 2-form. Using the fact that the correspondent geometry is torsion-free, we find the following 1-form connections

$$\omega^3_2 = -\omega^3_3 = \alpha d\phi, \quad (10)$$

the other terms are null. The connection 1-forms transform in the same way as the gauge potential of a non-Abelian gauge theory. The above connections leads to the following spin connection

$$\Gamma_{\phi} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\alpha \\ 0 & \alpha & 0 \end{pmatrix}. \quad (11)$$

Note that this matrix $\Gamma_{\phi}$ can be written in terms of the generator of rotation $J_{12}$ about the $z$-axis

$$\Gamma_{\phi} = i\alpha J_{12}. \quad (12)$$

Let us analyze the parallel transport of vectors around closed curves that contains the singularity. In our case
the unique contribution to the holonomy is provided by the azimuthal spin connection. Thus, we may write:

\[ U(\gamma_\phi) = \mathcal{P} \exp \left( - \oint \Gamma_\phi d\phi \right). \] (13)

Making the expansion of this expression and noticing that we are always able to write the exponents of upper order in the \( \Gamma_\phi \) and \( \Gamma_\phi^2 \) terms, the above equation can be written in the following way

\[ U(\gamma_\phi) = 1 - \frac{\Gamma_\phi}{\alpha} \sin(2\pi \alpha) + \left( \frac{\Gamma_\phi}{\alpha} \right)^2 [1 - \cos(2\pi \alpha)]. \] (14)

Alternatively one can express the holonomy in a matrix form

\[ U(\gamma_\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(2\pi \alpha) & \sin(2\pi \alpha) \\ 0 & -\sin(2\pi \alpha) & \cos(2\pi \alpha) \end{pmatrix}. \] (15)

Considering the linear approximation of the term \( \beta \) we can rewrite the holonomy matrix in terms of the elastic parameters

\[ U(\gamma_\phi) \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \left( \frac{2\pi (2 - \sigma) - 1}{(1 - 2\sigma)} \right) & \sin \left( \frac{2\pi (2 - \sigma) - 1}{(1 - 2\sigma)} \right) \\ 0 & -\sin \left( \frac{2\pi (2 - \sigma) - 1}{(1 - 2\sigma)} \right) & \cos \left( \frac{2\pi (2 - \sigma) - 1}{(1 - 2\sigma)} \right) \end{pmatrix} \] (16)

where

\[ \beta \approx 1 + \frac{(1 - 2\sigma)}{2(1 - \sigma)}. \] (17)

The holonomy group measures the deviation of the space from global flatness.

The deficit angle \( \chi \) is obtained when we compare final and initial positions of the parallel transported vector and it is given by

\[ \cos \chi_a = U_{a1}, \] (18)

where \( a \) is the tetradic index. The terms of non-vanishing angular deviations occurs when \( a = 1 \) and \( a = 2 \), so we have

\[ |\chi| = |2\pi \alpha + 2\pi j|, \] (19)

where \( j \) is an integer. When \( \alpha \to 0 \) we must have \( \chi = 0 \), so we choose \( j = 0 \) which leads to

\[ |\chi| = \frac{2\pi |\alpha|}{\beta(2 - \sigma) - 1} \left( \frac{2\pi (2 - \sigma) - 1}{(1 - 2\sigma)} \right). \] (20)

The above expression shows that \( \chi \neq 0 \), if we parallel transport a vector around a closed path the final vector does not coincides with the original vector. This physical effect could be understood as a gravitational analogue of the Aharonov-Bohm effect which is denominated elastic Aharonov-Bohm effect.

### IV. GLOBAL CHARACTERIZATION OF THE MULTIPLE DEFECTS

In this section we use the previous holonomy matrix to investigate the Aharonov-Bohm effect for phonon scattering in a medium with \( N \) parallel disclinations. We use the global characterization process to obtain the phase acquired by the vector when it is parallel transported in a closed path around the defects [11]. As an application of the holonomy transformation we study the space configuration from the global point of view of \( N \) defects located, at points \( a_j, j = 1, 2, \ldots, N \). In order to do this we use the result of the previous section that only defects enclosed by the circle contribute to the phase factor acquired by a vector parallel transported along the circle, in the background of the multiple defects. If we parallel transport a vector \( \vec{X} \) around a circle enclosing a chiral magnetic string we get the following result after this process:

\[ \vec{X}^{(1)} = U_1 \vec{X}, \] (21)

where \( U_1 \) is obtained from

\[ U_k = \exp[-2\pi \alpha_k J_1], \] (22)

taking \( k \) equal to one.

Now, let us consider a system of two defects, the defect 1 at \( a_1 = 0 \) (origin) and defect 2 at \( a_2 \). If we take a vector \( \vec{X} \) and carry it along a circle around defect 2, the resulting vector is given by \( U_2 \vec{X} \). We then carry this resulting vector parallel to a circle around defect 1. Then, we get the following result

\[ \vec{X}^{(2)} = \vec{b}_{1,2} + U_1 U_2 \vec{X}, \] (23)

where \( \vec{b}_{1,2} = U_1(1 - U_2) \vec{a}_2 \).

If we consider a system of three defects, we have

\[ \vec{X}^{(3)} = \vec{b}_{1,2,3} + U_1 U_2 U_3 \vec{X}, \] (24)

where \( \vec{b}_{1,2,3} = U_1(1 - U_2) \vec{a}_2 + U_1 U_2(1 - U_3) \vec{a}_3 \). It is easy to generalize this result for a system of \( N \) defects, located, respectively, at \( \vec{a}_1, \vec{a}_2, \ldots, \vec{a}_N \). The vector \( \vec{X}^{(N)} \) obtained after the parallel transport of a vector \( \vec{X} \) is given by the expression

\[ \vec{X}^{(N)} = \vec{b}_{1,2,\ldots,N} + U_1 U_2 \ldots U_N \vec{X}, \] (25)

where \( \vec{b}_{1,2,\ldots,N} = U_1(1 - U_2) \vec{a}_2 + U_1 U_2(1 - U_3) \vec{a}_3 + \cdots + U_1 U_2 \ldots U_{N-2}(1 - U_N) \vec{a}_{N-1} U_{N-1}(1 - U_N) \vec{a}_N \), and \( U_N \) is given by Eq. \((22)\) with \( k = N \). Then, a vector \( \vec{X} \) parallel transported in the field of \( N \) defects acquires a phase factor given by \( U_1 U_2 \ldots U_N \) and from the global point of view, the system of defects behaves like a simple string with matching conditions given Eq. \((25)\).

Let us consider a single defect that behaves like this system. So, if we parallel transport a vector \( \vec{X} \) around
a circle, in whose center we have an equivalent defect of
deficit angle characterized by $\alpha_g$, we get the following vector after this process
\[
\vec{X}^{N+1} = U \vec{X}
\] (26)
where $\vec{a}$ is the position of defect string that is equivalent to the system of defects.

Equating the geometrical phases acquired by a vector $\vec{X}$ in both cases, we have
\[
U_1U_2 \cdots U_{N-1}U_N = U.
\] (27)
Taking the trace of Eq. (27) we obtain
\[
\cos \phi = \cos(\sum_{j=1}^{N-1} \phi_j) \cos \phi_N - \sin(\sum_{j=1}^{N-1} \phi_j) \sin \phi_N
\] (28)
where $\phi_j = 2\pi \alpha_j$ and $\phi = 2\pi \alpha$. Using some trigonometric manipulation we can write the expression (28) in the following form
\[
\cos \phi = \cos(\sum_{j=1}^{N} \phi_j)
\] (29)
This is a relation between the deficit angles of the resulting space and the deficit angle of the defect involved. We have thus demonstrated that, in the global point of view, the $N$ topological defects can be seen as an equivalent defect with deficit angle given by
\[
\phi = \sum_{j=1}^{N} \phi_j.
\] (30)
Since the space outside the region of multiple defects is locally flat, we can describe the analytic solution purely in terms of space patches with Euclidean metric, but connected by some matching conditions which are given by
\[
\begin{pmatrix}
  z' \\
  x' \\
  y'
\end{pmatrix} =
\begin{pmatrix}
  1 & 0 & 0 \\
  0 & \cos(\phi) & \sin(\phi) \\
  0 & -\sin(\phi) & \cos(\phi)
\end{pmatrix} \begin{pmatrix}
  z \\
  x \\
  y
\end{pmatrix},
\] (31)
that relates points $(\vec{X})$ and $(\vec{X}')$ along the edges. Equation (31) is the exact expression for the holonomy, for circles in the space of multiple defects. Only the defect surrounded by the circle contributes to the change in this vector.

The existence of locally flat coordinates in this space permits us to consider Eq. (31) as a parallel transport matrix. So we can say that when a vector is carried out along a circle in this space-time it acquires a phase that depends on $\alpha_i$, which prevents it from being equal to the unit matrix. This effect is due to the non-trivial topology of the space under consideration. This is the gravitational analogue Aharonov-Bohm effect for $N$ linear defects in solids.

V. CONCLUDING REMARKS

We have shown that the geometric theory of defects can be applied in order to investigate classical effects due to influence of the topology. We have analyzed in this work the scattering of phonons in a disclinated media using loop variables, which allowed us to obtain various physical and geometric properties in a specific background. We have considered a geometric point view in order to describe the motion of quasiparticles experiencing an effective non-Euclidean metric in an elastic medium associated with the presence of disclinations.

We have known that a single dislocation produces a dilatation that breaks the translational symmetry of the crystaline lattice, producing a deformation of the geodesics around the defect. We have demonstrated that a single disclination modifies the medium, and provokes a break of rotational symmetry which is noted in the vector parallel transport framework.

In our final result, which is given by Eq. (31), we notice that the change in the phase of the quantum wave function for a particle, incorporates the effect of the effective deficit angle associated with the topological defects. We also studied the analogue of the gravitational Aharonov-Bohm effect for scattering of quasiparticles by topological defects. This was done by computing the holonomy matrix for some specific closed paths. We have used the matrix of parallel transport to study the Aharonov-Bohm effect in this background. We have demonstrated that an effect similar to the gravitational Aharonov-Bohm effect occurs for quasiparticles moving in this effective metric. We have analyzed this effect in an elastic medium with one and with $N$ defects via global characterization. In this way, we have demonstrated that the scattering of phonons by one or more disclinations produces a global effect in an elastic medium, that we denominated elastic Aharonov-Bohm effect, due to deformations caused by defects in this medium.

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