Next-to-leading order QCD corrections to $e^+e^- \rightarrow 3$ jets with massive quarks

A. Brandenburg*, W. Bernreuther, and P. Uwer

Institut für Theoretische Physik, RWTH Aachen, 52056 Aachen, Germany

We discuss a calculation of the next-to-leading order QCD corrections to the process $e^+e^- \rightarrow 3$ jets with massive quarks, and show numerical results for the three jet fraction and the differential two jet rate.

1. INTRODUCTION

In view of the large number of jet events at the $Z$ resonance collected both at LEP and SLC, it is desirable for precision tests of the standard model to use in the theoretical predictions next-to-leading order (NLO) partonic matrix elements that include the full quark mass dependence. This is particularly important for $b$ quark enriched samples, which can be obtained with high purity using vertex detectors.

The NLO QCD corrections to $e^+e^- \rightarrow 3$ jets are well-known for massless quarks [1–4]. The three, four, and five jet rates involving massive quarks have been computed to leading order in $\alpha_s$ already some time ago [5,6]. Recently, results for the NLO matrix elements including mass effects have been reported [7–11]. Knowing the NLO matrix elements including the mass dependence, one can try to extract the mass of the $b$ quark from three jet rates involving $b$ quarks at the $Z$ peak. This was suggested in [12], elaborated in [7,9], and experimentally pursued by the DELPHI collaboration [14,15]. Further applications include precision tests of the asymptotic freedom property of QCD by means of three jet rates and event shape variables measured at various center-of-mass energies, also far below the $Z$ resonance [16]. For theoretical predictions concerning the production of top quark pairs at a future $e^+e^-$ collider, the inclusion of the full mass dependence is of course mandatory.

We have computed the complete differential distributions for $e^+e^-$ annihilation into three and four partons via a virtual photon or $Z$ boson at order $\alpha_s^2$, including the full quark mass dependence [8,11]. This allows for order $\alpha_s^2$ predictions of oriented three jet events (for the massless case, see [17]), and of any quantity that gets contributions only from three and four jet configurations. In this talk we give an outline of our calculation (for details, see [11]), after which we present some numerical results.

2. OUTLINE OF THE CALCULATION

The calculation of an arbitrary quantity dominated by three jet configurations and involving a massive quark-antiquark pair to order $\alpha_s^2$ consists of two parts: First, the computation of the amplitude of the partonic reaction $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow Q\bar{Q}g$ at leading and next-to-leading order in the QCD coupling. Here $Q$ denotes a massive quark and $g$ a gluon. Second, the leading order matrix elements of the four-parton production processes $e^+e^- \rightarrow Z^*, \gamma^* \rightarrow ggQQ, QQq\bar{q}, QQ\bar{Q}Q\bar{Q}$ are needed. Here $q$ denote light quarks which are taken to be massless.

The infrared (IR) and ultraviolet (UV) singularities, which are encountered in the computation of the one-loop integrals, are treated within the framework of dimensional regularization in $d = 4 - 2\epsilon$ space-time dimensions. We remove the UV singularities by the standard $\overline{MS}$ renormalization. We have converted from the outset the on-shell mass of the heavy quark $Q$ into the corresponding running $\overline{MS}$ mass. It is known that far from threshold one thereby absorbs some large logarithms into the running mass.

After renormalization, the virtual corrections
to the differential cross section for $e^+e^- \rightarrow Q\bar{Q}g$ still contain IR singularities. These have to be cancelled by the singularities that are obtained upon phase space integration of the squared tree amplitudes for the production of four partons. Different methods to perform this cancellation have been developed (see \cite{15,20} and references therein). We use the so-called phase space slicing method elaborated in \cite{8}, which we modified to account for masses \cite{1}. The basic idea is to “slice” the phase space of the four parton final state by introducing an unphysical parton resolution parameter $s_{\text{min}}$, which is much smaller than all relevant physical scales. The parameter $s_{\text{min}}$ splits the phase space into a region where all four partons are “resolved” and a region where at least one parton remains unresolved. In the unresolved region soft and collinear divergences reside, which have to be isolated explicitly to cancel the singularities of the virtual corrections. This is considerably simplified due to collinear and soft factorizations of the matrix elements which hold in the limit $s_{\text{min}} \rightarrow 0$. (In the presence of massive quarks, the structure of collinear and soft poles is completely different as compared to the massless case.) After having cancelled these IR poles against the IR poles of the one-loop integrals entering the virtual corrections, one is left with a completely regular differential three-parton cross section which depends on $s_{\text{min}}$. The contribution to a three jet quantity of the “resolved” part of the four-parton cross section is finite and may be evaluated in $d = 4$ dimensions, which is of great practical importance. It also depends on $s_{\text{min}}$ and is most conveniently obtained by a numerical integration. Since the parameter $s_{\text{min}}$ is completely arbitrary, the sum of all contributions to any observable must not depend on $s_{\text{min}}$. Since the individual contributions depend logarithmically on $s_{\text{min}}$, it is a nontrivial test of the calculation to demonstrate that observables become independent of $s_{\text{min}}$ for small values of this parameter.

3. NUMERICAL RESULTS

We will now discuss results for some three jet observables involving massive quarks. All quantities are calculated by expanding in $\alpha_s$ to NLO accuracy. We consider here the JADE \cite{21} and Durham \cite{22} jet finding algorithms, although other schemes \cite{23} can also be easily implemented. First we demonstrate the independence of physical quantities on the parameter $s_{\text{min}}$ as $s_{\text{min}} \rightarrow 0$. We choose as an example the three jet fraction for $b$ quarks,

$$f_3^b(y_{\text{cut}}) = \frac{\sigma_3^b(y_{\text{cut}})}{\sigma_{\text{tot}}} \tag{1}$$

In \cite{1}, the numerator $\sigma_3^b$ is defined as the three jet cross section for events in which at least two jets containing a $b$ or $\bar{b}$ quark remain after the clustering procedure. This requirement ensures that the cross section stays finite also in the limit $m_b \rightarrow 0$. The contribution of the process $e^+e^- \rightarrow Z, \gamma^* \rightarrow q\bar{q}g^* \rightarrow q\bar{q}b\bar{b}$ to the three jet cross section with one tagged $b$ quark develops large logarithms $\ln(m_{TZ}^2)$ – which find no counterpart in the virtual corrections against which they can cancel – when the $b\bar{b}$ pair is clustered into a single jet. In principle, there are three ways to handle this problem: One may impose suitable experimental requirements/cuts to suppress contributions from events with two light quark jets and one jet containing a $b\bar{b}$ pair (the definition for

![Figure 1. The three jet fraction $f_3^b$ at NLO as defined in the text at $\sqrt{s} = \mu = m_Z$ as a function of $y_{\text{min}} = s_{\text{min}}/(\mu y_{\text{cut}})$ for the Durham algorithm at a value of the jet resolution parameter $y_{\text{cut}} = 0.03$ with $m_b = (m_Z) = 3$ GeV and $\alpha_s(\mu = m_Z) = 0.118$.](image)
σ^b_3 (chosen by us is an example for this), or one can improve the fixed order calculation by absorbing the large logarithm into a fragmentation function for a gluon into a b quark. The third possibility is simply to keep the large ln(m^2_b) term. This may however lead to an overestimate of the b quark mass effects in some observables. A detailed discussion of this issue will be presented elsewhere \[24\]. Fig. 1 shows the three jet fraction f^b_3 in the Durham scheme at NLO as a function of y_{min} = s_{min}/(s y_{cut}) at a fixed value of y_{cut} = 0.03 and \(\sqrt{s} = m_Z = 91.187\) GeV. The corresponding plot for the JADE scheme has a similar shape. For the renormalization scale we take in this plot \(\mu = \sqrt{s}\). As mentioned above, we use \(m^\text{\overline{MS}}_b(\mu)\) defined in the \(\overline{\text{MS}}\) scheme at the scale \(\mu\). The asymptotic freedom property of QCD predicts that this mass parameter decreases when being evaluated at a higher scale. (For low energy determinations of the b quark mass see e.g. 25, 26 and references therein.) With \(m^\text{\overline{MS}}_b(\mu = m_b) = 4.36\) GeV 25 and \(\alpha_s(m_Z) = 0.118\) as an input and employing the standard renormalization group evolution of the coupling and the quark masses, we use the value \(m^\text{\overline{MS}}_b(\mu = m_Z) = 3\) GeV. One clearly sees that \(f^b_3\) reaches a plateau for small values of \(y_{min}\). The error in the numerical integration becomes bigger as \(y_{min} \rightarrow 0\). In order to keep this error as small as possible without introducing a systematic error from using the soft and collinear approximations, we take in the following \(y_{min} = 0.5 \times 10^{-2}\) for the Durham algorithm and \(y_{min} = 10^{-2}\) for the JADE algorithm. At these values, the dominant \(s_{min}\)-dependent individual contributions from three and four resolved partons are about a factor of 4 (Durham) and 2.5 (JADE) larger than the sum. In Fig. 2 we plot \(f^b_3\) as a function of y_{cut}, again at \(\sqrt{s} = m_Z\). The QCD corrections to the LO result are quite sizable as known also in the massless case. The renormalization scale dependence (where \(\mu\) is varied between \(m_Z/2\) and \(2m_Z\)), which is also shown in Fig. 2, is modest in the whole y_{cut} range exhibited for the Durham algorithm. In Fig. 3 we take a closer look on the scale dependence of \(f^b_3\), now using the on-shell mass renormalization scheme. We vary the scale \(\mu\) between \(m_Z/16\) and \(2m_Z\) for a fixed value \(y_{cut} = 0.2 \times 10^{-3}\) and on-shell masses \(m^\text{pole}_b = 3\) GeV and \(m^\text{pole}_b = 5\) GeV. We see that the scale dependence of the LO result (which is solely due to the scale dependence of \(\alpha_s\) at this order) in the Durham algorithm amounts to about 100% in the \(\mu\) interval shown. The inclusion of the \(\alpha^2_s\) corrections reduces the scale de-
The double ratio $C$ as a function of the c.m. energy at $y_{\text{cut}} = 0.08$ for the JADE algorithm. The full curve and the points show the NLO result, the dashed curve shows the LO result. In both cases a running mass $m_b^{\overline{MS}}(\mu = \sqrt{s})$ evolved from $m_b^{\overline{MS}}(\mu = m_Z) = 3$ GeV is used. The dash-dotted curve shows the LO result for a fixed mass $m_b = 4.7$ GeV. The effect of the $b$ quark mass may be illustrated by looking at the double ratio

$$ C(y_{\text{cut}}) = \frac{f_3^b(y_{\text{cut}})}{f_3^{\text{incl}}(y_{\text{cut}})}, $$

where the denominator is the three jet fraction when summing over all active quark flavors, which is given to a very good approximation by the massless NLO result [27]. Similar double ratios have been studied in [7–9], and [12,13]. In Fig. 4 we plot $C$ as a function of the c.m. energy at $y_{\text{cut}} = 0.08$ for the JADE algorithm. The running of $\alpha_s$ is taken into account in the curves, where we again use as an input $\alpha_s(\mu = m_Z) = 0.118$. For the dashed (LO) and full (NLO) curve we use a running mass $m_b^{\overline{MS}}(\mu = \sqrt{s})$ evolved from $m_b^{\overline{MS}}(\mu = m_Z) = 3$ GeV. For comparison we also show the LO result for a fixed mass $m_b = 4.7$ GeV (dash-dotted curve), which is the corresponding value of the pole mass. One clearly sees that the effect of the $b$ quark mass gets larger for smaller c.m. energies.

Another interesting quantity to study mass effects is the differential two jet rate [28] defined as

$$ D_2(y) = \frac{f_2(y) - f_2(y - \Delta y)}{\Delta y}, $$

where $f_2(y)$ is the two jet fraction at $y = y_{\text{cut}}$ for a given jet algorithm. The advantage of $D_2$ over the three jet fraction $f_3$ lies in the fact that the statistical errors in bins of $D_2(y)$ are independent from each other since each event enters the distribution only once. To order $\alpha_s^2$, $D_2(y)$ can be calculated from the three- and four jet fractions using the identity

$$ 1 = f_2 + f_3 + f_4 + O(\alpha_s^3). $$

We define

$$ D(y) = \frac{D_2^b(y)}{D_2^{\text{incl}}(y)}, $$

where we – as in the case of the quantity $C$ – use the massless NLO result to evaluate the denominator. We plot our result for $D(y)$ in the Durham
scheme in Fig. 5, again for $\sqrt{s} = \mu = m_Z$. The effects of the $b$ quark mass are of the order of 5% or larger at small values of $y_{\text{cut}}$.

4. CONCLUSIONS

We have presented NLO results for a number of three jet observables involving massive quarks. It will be interesting to see how our predictions compare to detailed experimental analyses.

REFERENCES

1. R.K. Ellis, D.A. Ross, and A.E. Terrano, Nucl. Phys. B 178 (1981) 421.
2. K. Fabricius, I. Schmitt, G. Kramer, and G. Schierholz, Z. Phys. C 11 (1981) 315.
3. J.A.M. Vermaseren, K.J.F. Gaemers, and S.J. Oldham, Nucl. Phys. B 187 (1981) 301.
4. Z. Kunszt, Phys. Lett. B 99 (1981) 429.
5. B.L. Joffe, Phys. Lett. B 78 (1978) 277; G. Kramer, G. Schierholz, and J. Willrodt, Z. Phys. C 4 (1980) 149; E. Laermann and P. Zerwas, Phys. Lett. B 89 (1980) 225; H.P. Nilles, Phys. Rev. Lett. 45 (1980) 319.
6. A. Ali et al., Nucl. Phys. B 167 (1980) 454; A. Ballestrero, E. Maina, and S. Moretti, Phys. Lett. B 294 (1992) 425; Nucl. Phys. B 415 (1994) 265; A. Ballestrero and E. Maina, Phys. Lett. B 323 (1994) 53.
7. G. Rodrigo, Ph. D. thesis, Universitat de València, [hep-ph/9703359], Nucl. Phys. B (Proc. Suppl.) 54A (1997) 60.
8. W. Bernreuther, A. Brandenburg, and P. Uwer, Phys. Rev. Lett. 79 (1997) 189.
9. G. Rodrigo, A. Santamaria, and M. Bilenky, Phys. Rev. Lett. 79 (1997) 193.
10. P. Nason and C. Oleari, preprint CERN-TH/97-92, IFUM 566/FT, [hep-ph/9705292].
11. A. Brandenburg and P. Uwer, preprint PITHA 97/29, [hep-ph/9708350].
12. M. Bilenky, G. Rodrigo, and A. Santamaria, Nucl. Phys. B 439 (1995) 505.
13. G. Rodrigo, these proceedings.
14. J. Fuster, S. Cabrera, and S. Marti i Garcia, Nucl. Phys. B (Proc. Suppl.) 54A (1997) 39.
15. S. Marti i Garcia, J. Fuster, and S. Cabrera, these proceedings, [hep-ex/9708030].
16. S. Bethke, Nucl. Phys. B (Proc. Suppl.) 54A (1997) 314; O. Biegel, these proceedings.
17. J.G. Körner and G.A. Schuler, Z. Phys C 26 (1985) 559; Nucl. Phys. B 325 (1989) 557.
18. W.T. Giele and E.W.N. Glover, Phys. Rev. D 46, (1992) 1980.
19. S. Frixione, Z. Kunszt, and A. Signer, Nucl. Phys. B 467 (1996) 399.
20. S. Catani and M.H. Seymour, Phys. Lett. B 378 (1996) 287; Nucl. Phys. B 485 (1997) 291.
21. W. Bartel et al. (JADE collab.), Z. Phys. C 33 (1986) 23; S. Bethke et al. (JADE collab.), Phys. Lett. B 213 (1988) 235.
22. N. Brown and W.J. Stirling, Phys. Lett. B 252 (1990) 657, S. Catani et al., Phys. Lett. B 269 (1991) 432.
23. S. Bethke, Z. Kunszt, D.E. Soper, and W.J. Stirling, Nucl. Phys. B 370 (1992) 310.
24. W. Bernreuther, A. Brandenburg and P. Uwer, work in preparation.
25. M. Neubert, Phys. Rep. C 245 (1994) 259.
26. S. Narison, Phys. Lett. B 341 (1994) 73; C.T.H. Davies et al., Phys. Rev. Lett. 73 (1994) 2654; V. Gimenez, G. Martinelli, and C.T. Sachrajda, Phys. Lett. B 393 (1997) 124.
27. Z. Kunszt and P. Nason, “QCD” in Z Physics at LEP1, eds. G. Altarelli, R. Kleiss, and C. Verzagnassi, CERN Yellow Report 89-08 (1989), Vol. 1, p. 373.
28. M.Z. Akrawy et al. (OPAL collaboration), Phys. Lett. B 235 (1990) 389.

DISCUSSIONS

A.P. Contogouris

You stated three ways for treating large logarithms and that you followed one of them. Suppose you follow any of the other two. How much your higher corrections will change? Do you have any idea about their sensitivity, perhaps from another process?

A. Brandenburg

If one adds the contribution from $g^* \to b\bar{b}$ splitting "naively", i.e. keeps the large logarithm in the three jet cross section with one tagged $b$ quark as it is, the ratio $C(y_{\text{cut}})$ evaluated at $\sqrt{s} = \mu =$
$m_Z$ is, for large values of $y_{\text{cut}}$, up to 3% larger as compared to the result for this ratio with two tagged $b$ quark jets. The difference decreases for smaller values of $y_{\text{cut}}$. 