The medium effect of magnetic moments of baryons on the neutron star under strong magnetic fields

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Abstract. We investigate the effect of density dependent magnetic moment of baryons in strong magnetic fields for neutron star with hyperons and kaons. Under strong magnetic fields, all charged particles experience Landau quantization, so that chemical potentials, equation of state and mass-radius relations depend on magnetic fields strength. In particular, since magnetic fields are combined with magnetic moments of baryons, we calculate the neutron star with hyperons and kaon condensation, and investigate the effect of density dependent magnetic moment of baryon octet obtained by the quark-meson coupling model. The effect is maximally $0.1M_\odot$ for hyperonic matter, but in kaonic matter, its effect can be ignored.

1. Introduction
Pulsars have been known to have magnetic fields and the order of magnetic fields is about $B \sim 10^{12-13}$ G in the surface of the stars. However, very strong magnetic fields were observed at the surface of anomalous x-ray pulsars (AXP) and soft gamma ray repeaters (SGR), called magnetars, which show $B \sim 10^{14-15}$ G [1]. The strength of magnetic fields can be explained by the magnetic flux conservation proposed by Landau. In this conservation, since $< B > R^2$ is conserved during supernovae collapse, a neutron star as the remnant of supernovae can have very strong magnetic fields when we assume that the magnetic fields of a supernovae is about $B \sim 10^{4-5}$ G and its radius is about $R \sim 10^6$ km. However, the magnitude of magnetic fields in the interior of the neutron star still has much uncertainty.

In the interior of neutron stars, according to the scalar virial theorem, the magnetic field strength can reach to $B \sim 10^{18}$ G. Such strong magnetic fields may affect the structure of a neutron star such as the populations of particles, the equation of state (EoS) and mass-radius relations. Many studies for neutron stars with strong magnetic fields have been reported by several papers, which included the electromagnetic interaction, the Landau quantization of charged particles, and anomalous magnetic moments (AMMs) of baryons [2, 3, 4, 5, 6, 7, 8]. But roles of relevant particles’ AMMs in a strong magnetic field are still uncertain because properties of the AMMs in nuclear matter are not fully scrutinized yet.

Thus in this paper, we investigate the role of density dependent magnetic moment of baryons in neutron star with hyperons and kaon condensation under strong magnetic fields. The magnetic
moments of baryons are calculated by using the quark-meson coupling model, in which quarks inside MIT bags directly interact with mesons. So, with the ratio of magnetic moments of baryons, we study the effect of medium dependence of magnetic moment for various magnetic fields.

2. Theory

The Lagrangian density of the QHD model for dense matter in the presence of strong magnetic fields, which is introduced by the vector potential \( A^\mu \) due to magnetic fields, can be represented in terms of octet baryons, leptons, five meson fields and kaons. Thus, the total Lagrangian is separated as \( \mathcal{L}_{\text{tot}} = \mathcal{L}_b + \mathcal{L}_l + \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{kaon}} \) where subscripts, \( b \) and \( l \), represent octet baryon and leptons\((e^-, \mu^-)\), respectively. Thus,

\[
\mathcal{L}_b + \mathcal{L}_l + \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{int}} = \sum_b \bar{\psi}_b \left[ i \gamma_\mu \partial^\mu - g_{b\gamma} \gamma_\mu A^\mu - M^*_b (\sigma, \sigma^*) - g_{\omega b} \gamma_\mu \omega^\mu - g_{\phi b} \gamma_\mu \phi^\mu \right] \psi_b + \sum_l \bar{\psi}_l \left[ i \gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l \right] \psi_l
\]

\[
+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) + \frac{1}{2} \partial_\mu \sigma^* \partial^\mu \sigma^* - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} - \frac{1}{4} W_{\mu\nu} W^{\mu\nu}
\]

\[
+ \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu} + \frac{1}{2} \phi_\mu \phi^\mu - \frac{1}{4} R_{\mu\nu} R^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \rho_\mu \rho^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (2)
\]

The effective mass of a baryon, \( M^*_b \), is simply given by \( M^*_b = M_b - g_{sb}\sigma - g_{sb}\gamma^1 \sigma^*, \) where \( M_b \) is the free mass of a baryon in vacuum. The \( \sigma \), \( \omega \) and \( \rho \) meson fields describe interactions of nucleon-nucleon (\( N - N \)) and nucleon-hyperon (\( N - Y \)). Interaction of \( Y - Y^* \) is mediated by \( \sigma^* \) and \( \phi \) meson fields. \( U(\sigma) \) is the self interaction of the \( \sigma \) field given by \( U(\sigma) = \frac{1}{4} g_2 \sigma^2 + \frac{1}{4} g_3 \sigma^4 \).

\( W_{\mu\nu}, R_{\mu\nu}, \Phi_{\mu\nu}, \text{ and } F_{\mu\nu} \) represent the field tensors of \( \omega, \rho, \phi \) and photon fields, respectively. The AMMs of baryons interact with an external magnetic field in the form of \( \kappa_{b\mu\nu} F^{\mu\nu} \) where \( \sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu] \) and \( \kappa_b \) is the strength of AMM of a baryon, i.e. \( \kappa_p = 1.7928 \mu_N \) for a proton in a vacuum where \( \mu_N \) is the nuclear magneton. The \( \kappa_b \) of a baryon can be calculated by using quark model, so that we employ the quark-meson coupling model to obtain density dependent magnetic moments of baryons [9, 10]. Thus, in our work, \( \kappa \) depends on density, but \( \mu_N \) does not.

The Lagrangian for kaons is given by

\[
\mathcal{L}_{\text{kaon}} = D^\mu_{K} \bar{K} D^\mu_{K} - m_K^2 K \bar{K}, \quad (3)
\]

where the covariant derivative \( D_{K} \) is given by

\[
D_{K} = \partial_{\mu} + ig_{K} A_{\mu} + ig_{\omega K} \omega_{\mu} + ig_{\phi K} \phi_{\mu} + ig_{\rho K} \tau_{\mu} \rho_{\mu} \quad (4)
\]

and the effective mass of a kaon is

\[
m_K^* = m_K - g_{\sigma K} \sigma - g_{\sigma^* K} \sigma^*. \quad (5)
\]

The equation of motion for meson fields are given by

\[
m_{\sigma}^2 \sigma + \frac{\partial U(\sigma)}{\partial \sigma} = g_{sb} \sum_b \rho_{s}^b + g_{\sigma K} \frac{m_K^*}{\sqrt{m_K^{*2} + |q_K^-| B}} \rho_{K^-},
\]

\[
m_{\sigma^*}^2 \sigma^* = g_{\sigma^* b} \sum_b \rho_{s}^b + g_{\sigma^* K} \frac{m_K^*}{\sqrt{m_K^{*2} + |q_K^-| B}} \rho_{K^-},
\]

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\[
\begin{align*}
\rho_2 \omega &= g_{\omega} \sum_b \rho_b - g_{\omega} K \rho K^{-}, \\
\rho_2 \phi &= g_{\phi} \sum_b \rho_b - g_{\phi} K \rho K^{-}, \\
\rho_2 \rho &= g_{\rho} \sum_b I_b \rho_b - g_{\rho} K \rho K^{-},
\end{align*}
\] (6)

where \(\rho_s\) and \(\rho_v\) are respectively the scalar and the vector densities.

The chemical potentials of baryons, leptons and a s-wave kaon are obtained as
\[
\begin{align*}
\mu_b &= E_b^f + g_{\omega} \omega_0 + g_{\phi} \phi_0 + g_{\rho} I_3 \rho_0, \\
\mu_l &= \sqrt{k^2_f + m^2_l + 2\nu |q_l| B}, \\
\mu_K &= \sqrt{m^2_K + |q_K| B - g_{\omega} K \omega_0 + g_{\phi} K \phi - g_{\rho} K \rho_0},
\end{align*}
\] (7)

where \(E_b^f\) is the Fermi energy of a baryon and \(k_f\) is the Fermi momentum of a lepton. For charged particles, the \(E_b^f\) is written as
\[
E_b^{f,2} = k_b^f + (\sqrt{m_b^2 + 2\nu |q_b| B - s\kappa_b B}).
\] (8)

We exploit three constraints for calculating properties of a neutron star: baryon number conservation, charge neutrality, and chemical equilibrium. The meson field equations are solved with the chemical potentials of baryons, leptons and kaon under the above three constraints. Total energy density is given by \(\varepsilon_{\text{tot}} = \varepsilon_m + \varepsilon_K + \varepsilon_f\), where the energy density for matter fields is given by
\[
\varepsilon_m = \sum_b \varepsilon_b + \sum_l \varepsilon_l + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\rho^2 \rho^2 + U(\sigma),
\] (10)

and the energy density due to the magnetic field is given by \(\varepsilon_f = B^2 / 2\). The total pressure can also be written as
\[
P_{\text{tot}} = P_m + \frac{1}{2} B^2,
\] (11)

where the pressure due to matter fields is obtained from \(P_m = \sum \mu_i \rho_i^f - \varepsilon_m\). The relation between mass and radius for a static and spherical symmetric neutron star is generated by calculating the Tolman-Oppenheimer-Volkoff (TOV) equations with the equation of state (EoS) above.

3. Results and discussion
We use the parameter set in Ref. [11] for the coupling constants, \(g_{\sigma N}, g_{\omega N}\) and \(g_{\rho N}\), where \(N\) denotes the nucleon. For the coupling constants of hyperons in nuclear medium, \(g_{\omega Y}\) is determined by the quark counting rule, and \(g_{\sigma Y}\) is fitted to reproduce the potential of each hyperon at saturation density, whose strengths are given by \(U_\Lambda = -30\) MeV, \(U_\Sigma = 30\) MeV and
Figure 1. The equation of state and the relation between mass and radius for \( npH \) for various magnetic fields. The parameter for magnetic fields, \( B^*_0 \), is written in unit of \( 10^5 \). Thick lines are obtained from density dependent magnetic moments of baryons and thin lines are from constant magnetic moments.

\[ U_\Xi = -15 \text{ MeV}. \]

For density-dependent AMM values of baryons, we use the values obtained from our previous calculation done by the MQMC model [9]. Since the magnetic fields may also depend on density, we take density-dependent magnetic fields used in Refs. [2, 8]

\[ B(\rho/\rho_0) = B^\text{surf} + B_0 [1 - \exp\{-\beta (\rho/\rho_0)\gamma\}], \]  

(12)

where \( B^\text{surf} \) is the magnetic field at the surface of a neutron star, which is taken as \( 10^{15} \text{ G} \) from observations and \( B_0 \) represents the magnetic field saturated at high densities.

In the present work, we use \( \beta = 0.02 \) and \( \gamma = 3 \) for varying magnetic fields. Since the magnetic field is usually written in a unit of the critical field for the electron \( B^c_e = 4.414 \times 10^{13} \text{ G} \), the \( B \) and the \( B_0 \) in Eq. (12) can be written as \( B^* = B/B^c_e \) and \( B_0 = B_0/B^c_e \). Here, we regard the \( B^*_0 \) as a free parameter and investigate medium effects of AMMs in a neutron star for three different magnetic fields given by \( B^*_0 = 1 \times 10^5 \), \( 2 \times 10^5 \), and \( 3 \times 10^5 \). In this work, \( npH \) is hyperonic matter with baryon octet, and \( npH K \) is for the case of adding kaon condensation.

3.1. \( npH \)

In Fig. 1, the equation of state (EoS) and the relation between mass and radius are shown. One can first see the effect of magnetic fields. For charged particles, Landau quantization appears as \( m^*_b = m^2 + 2\nu eB \) in the energy spectra. Here the Landau level, \( \nu \), runs \( \nu = 0, 1, 2, \cdot \cdot \). This means the effect of Landau level causes the effective mass of a charged particle to increase when \( \nu \) is not equal to zero. Thus chemical potential of a charged particle increase with increasing effective mass. Finally, since the pressure is determined by chemical potential, the EoS becomes stiff with increasing magnetic fields.

For the effect of density dependent magnetic moment of baryons, one cannot see the effect up to \( B^*_0 = 1 \times 10^5 \). In very high magnetic fields, the density dependent effect appears, but it is not so large.

In right panel, the mass-radius relations are obtained by TOV equations. The mass-radius data are compared with data of XTE J1739-285 and Ter 5 I. In the case without magnetic fields, the mass and radius curve cannot satisfy the both constraints. However, when magnetic fields are turned on, the result with \( B \approx 2 \times 10^{18} \text{ G} \) (\( B^*_0 = 0.5 \) in the panel) can explain large maximum mass about \( 2M_\odot \) and satisfy the constraint from XTE J1739-285. The effect of density dependent magnetic moments does not appear in low magnetic fields region. Though the effect appears in high magnetic fields, its effect show us about maximally \( 0.1M_\odot \).
3.2. npHK
In the star with hyperons and kaon condensation, the populations of kaons depend on the strength of magnetic fields very strongly. In the chemical potential of kaons, magnetic fields cause effective mass of a kaon to increase. Since kaon condensation is s-wave Bose-Einstein condensation, the lower chemical potential of a kaon is needed to make s-wave kaons condensed. However, if magnetic fields are very strong, kaon condensation has to be suppressed by magnetic fields because of above reason. Thus, in Fig. 2, one can see the suppression of kaon condensation by magnetic fields. Thus, when we test higher magnetic fields, kaon condensation cannot take place. The limit of kaon condensation is up to about $B_0^* = 8 \times 10^4$.

Fig. 3 shows us the EoS and mass-radius relation of npHK. In this region, magnetic fields affect kaon condensation strongly. But, the effect of density dependent magnetic moments cannot appear within this strength of magnetic fields. So, in this model, we can conclude the kaon condensation in neutron star with hyperons under strong magnetic fields may not be affected by the effect of density dependent magnetic moments of baryons.

4. Summary
We investigate the effect of the density-dependent AMM of baryons in neutron star under strong magnetic fields by using the QHD model, which includes baryon octet, leptons and kaon condensation. By exploiting the density-dependent AMM values of baryons obtained from...
the MQMC model, we calculate the populations of particles, EoS, and the mass-radius relations for varying magnetic fields. The strength of magnetic field is expressed as EM interaction of all charged particles and their AMM of baryon octet.

In the populations of particles, all charged particles experience Landau quantization and its effect depends severely on the strength of magnetic fields. The increase of the magnetic fields enhances the chemical potentials of all charged particles. Thus, the increasing chemical potentials cause the EoS to be stiff because the pressure also increases with chemical potential. Thus, the stiff EoS makes the maximum mass of neutrons star larger.

The mass-radius relations of neutron star obtained from magnetic fields are compared with observational data. The mass-radius relation by magnetic fields satisfy the constraint by XTE J1739-285. In $npH$, the effect of density dependent AMM appears in very high magnetic fields, causing the increase of maximum mass of the star with about 0.1 $M_\odot$ in $B_0^\ast = 3 \times 10^5$. However, the density dependent magnetic moment could not affect kaon condensation because the kaon condensation are suppressed by strong magnetic fields.

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