A resonance mechanism of earthquakes

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It had been observed in [1] that there are periodic 4-6 hours pulses of \( \sim 200 \mu \text{Hz} \) seismogravitational oscillations (SGO) before 95% of powerful earthquakes. We explain this by beating between an oscillation eigenmode of a whole tectonic plate and a local eigenmode of an active zone which transfers the oscillation energy from the tectonic plate to the active zone causing the earthquake. Oscillation frequencies of the plate and ones of the active zone are tuned to a resonance by an additional pressure applied to the active zone due to collision of neighboring plates or convection in the upper mantle (plume). Corresponding theory may be used for short-term prediction of the earthquakes and tsunami.

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I. QUALITATIVE PICTURE

It is well known, that a stretching increases eigenfrequencies, recall a guitar string. On a contrary, a compression of an elastic object decreases the eigenfrequencies [2]. This way we may bring the frequency of a small plate in resonance with a frequency of a large plate. Therefore, the oscillation energy of a large tectonic plate may be transferred to the small compressed area through the beating phenomenon, which happens between two coupled oscillators with close frequencies. For instance, for two degenerate oscillators with coordinates \( x \) and \( y \) and interaction \( \delta xy \) [3]

\[
x = A \cos(\omega_y t + \varphi) \cos(\bar{\omega} t), \quad y = B \sin(\omega_y t + \varphi) \cos(\bar{\omega} t).
\]

Here the beating frequency \( \omega_y \ll \bar{\omega} \). General algebra of beating is presented in Appendix B. Further calculations of interaction of resonance modes for contacting small and large plates is cumbersome and will be published separately [4]. Here we present simple estimates.

The frequency of a transverse plane wave on a large thin plate depends on the pressure as [2]

\[
\omega = 2\pi \nu = k \sqrt{\frac{Dk^2 - Q}{\rho h}},
\]

where \( Q \) is the contracting force per unit length applied along the direction of the motion of the wave, \( \rho \) is the density and \( h \) is the thickness of the plate, \( D = \frac{E}{2(1-\nu^2)} \), \( E \) is the Young’s modulus, and \( \nu \) is the Poisson ratio. Solution for a spherical wave is presented in appendix C. We can use Eq. [1] to make a simple estimate of the effect of the contracting force. For the fundamental mode the wave vector \( k \sim 1/L \), where \( L \) is the size of the plate. For a tectonic plate \( E = 17.28 \times 10^{10} \text{ kg m}^{-1} \text{s}^{-2} \), \( \rho = 3380 \text{ kg m}^{-3} \), \( \nu = 0.28 \), and a typical frequency of a large tectonic plate vibration \( \nu = \sim 170 - 190 \mu \text{Hz} \). This gives us the required pressure to bring a small plate into resonance with a large plate (close to zero of eq. [1], \( Dk^2 = Q \))

\[
P \sim 10^{10} \left( \frac{h^2}{L^2} \right) \text{Pa}
\]

The linear size of active zone - the small plate - is \( L \sim 100 - 200 \text{ km} \). Large tectonic plates extend to \( L_t \sim 10000 - 100000 \text{ km} \) and are \( h \sim 200 - 300 \text{ km} \) thick on the continents, but much thinner, \( h = 30 - 100 \text{ km} \) on the oceans bottoms. A small ratio \( h/L \) may reduce the required pressure for the resonance up to two orders of magnitude. An upper estimate of an existing pressure may be given by a crushing pressure \( P \sim 10^9 \text{ Pa} \).

The resonance may also appear for a much smaller pressure if the tectonic plate is thick and the active zone is relatively thin (e.g. under ocean). Indeed, for small \( Q \) the plate frequency is \( \omega \sim (1/L^2) \sqrt{D/\rho h} \approx h/L^2 \) and the resonance condition is \( L^2/h \sim L^2_0/h_0 \). There also may be resonances between the fundamental frequency of the small plate and the higher SGO modes of the large tectonic plate \( (kL_t \sim n, n = 1, 2, ...) \). For small \( Q \) the tectonic plate frequency is \( \omega \sim (n^2/L_t^2) \sqrt{D/\rho h_0} \) and the resonance condition is \( L^2/h \sim L^2_0/(n^2) \). Finally, there may be a resonance between the different types of the modes on different plates which have different fundamental frequencies (e.g. transverse, longitudinal and surface (Rayleigh) modes). The lowest mode on the small plate may resonate with a higher frequency mode on the large plate. The increasing pressure on the small plate in this case produces scanning of the frequency until it comes into a resonance with one of the frequencies at the large plate.

Assume that initially the SGO live on a large plate. If the small and large plates are disconnected, their oscillations are independent. The additional pressure may bring the frequency of the small plate in resonance with the frequency of SGO of the large plate. Then even a weak interaction between the SGO processes on the...
plates results in forming the perturbed SGO mode of the pair of plates, manifesting the beating of the modes, causing the periodic migration of the energy from one plate to another. The sum of energies of oscillations of the plates remains constant. In absence of the exact resonance the energy transfer is not complete, but when the resonance becomes sharper, the energy transfer becomes fuller. When the energy of the large plate comes to the small plate, the amplitude of SGO on the small plate becomes enormous, causing the earthquake. Indeed, the ratio of averaged amplitudes $A_1/A_2 \sim L_1\sqrt{h_1}/L_2\sqrt{h_2}$.

This resonance interpretation of the earthquake mechanism may be useful for short-term earthquake predictions. If we have registered two pulses manifesting the moments when the migrating energy is situated on the large plate, then in a half period after the second pulse the whole energy will be already on the small plate, and may cause the earthquake. But even if we observed a single pulse and registered the moments of maximal amplitudes and the previous moment of minimal energy (amplitude) on the large plate, we are able to predict the moment when there will be maximum of the energy on the small plate, coincident with the next moment of minimal energy on the large plate. Note that the observations may be done on the large plate, very far from the active zone.

**Suggestion of experiment.** Mathematics can hardly provide reliable results for resonating eigenmodes of different plates under pressure, beating frequencies and transferred energy. However, a more reliable way to investigate this problem may be a laboratory experiment with plates of different sizes and shapes.

**The whip effect.** The described beating phenomenon may be viewed as a discreet analog of the celebrated whip effect, when, due to energy conservation, the amplitude is growing while the wave, running along the thinning channel is approaching the thin end of the whip. Indeed, that may happen on the ocean bottom, where the tectonic plate is thinning.

II. OBSERVATIONS

In the paper [1] the pulses of SGO were discussed as typical precursors of powerful earthquakes, arising with probability 95 %. The spectral nature of the SGO was demonstrated in [5]. Additional unpublished information was kindly provided to us by L. Petrova [6], who passed to us so-called spectral-time card constructed by herself based on monitoring of the SGO process preceding the earthquake 26 September 2005 in Peru (see Fig.1) . L. Petrova attracted our attention to some details on the cards, which may be considered as precursors of the earthquake, but were not interpreted yet properly. First, there are two “pulses” registered on SSB station (France) in the zones $\Delta_2^{SSB} = (190, 200) \times (55, 65); \Delta_3^{SSB} = (200, 210) \times (145, 165)$ introduced in the equation (4) below, separated by the time interval 96 hours. We believe that they are SGO -beating on the large plate. Secondly, we see between the pulses the shock at $T = 87$ h, causing generation of three oscillation modes registered on the INU-station (Japan). The signal may come from the small plate. Finally, the earthquake succeeded at the moment $T = 172$ h, in 48 hours after the second pulse, and 96 hours after the shock at the moment 87 h, see details in Appendix A.

![FIG. 1: Spectral-Time cards constructed by L. Petrova based on seismo-gravitational oscillations recorded on INU and SSB stations, in Japan and in France respectively, during the period 18-26 September 2005 preceding the strong Earthquake in North Peru. The horizontal axis for time, is graded in hours, the vertical axis , for the frequencies, is graded in $\mu$Hz.](image)

III. APPENDIX A: SPECTRAL -TIME CARDS

The spectral - time cards are obtained from the seismograms via averaging the amplitudes of the oscillations with certain frequency on the systems of 20 hours time - windows, selected by shifting an initial window by 30 minutes on each step. The seismograms underwent previously a double filtration with Potter filter: the high frequency one, with boundary period 6 h, and one more, with the window 1-6 h. The boundaries of the
domains on the cards, where the averaged amplitude of the SGO mode with certain frequency \( \nu \), at the given moment \( T \) of time, exceeds given value \( A \), form a system of isolines in the frequency/time coordinates \( \nu, T \).

The relief of the window averaged squared amplitude on the cards is graded by the isolines, with the step

\[
\delta A^2 = \frac{1}{10} \left[ A_{\text{max}}^2 - A_{\text{min}}^2 \right],
\]

and is painted accordingly between the isolines, with dull grey for the background value \( A_{\text{min}}^2 \) of the squared amplitude and white for the maximal value \( A_{\text{max}}^2 \). The results of monitoring represented on the cards correspond to the trains of SGO with maximal value \( A \).

value between the isolines, with dull grey for the background value \( A_{\text{min}}^2 \) of the squared amplitude and white for the maximal value \( A_{\text{max}}^2 \). The results of monitoring represented on the cards correspond to the trains of SGO with growing \( a \), constant \( b \) and decreasing \( c \) frequency \( \nu \).

- a) INU 88 \( \rightarrow \) 107, 220 < \( \nu \) < 235,
- b) SSB 50 \( \rightarrow \) 65, \( \nu \approx 195, \)
- c) SSB 70 \( \rightarrow \) 265 > \( \nu \) > 247,

and brief (6-20 hours) stationary SGO modes with high amplitudes in the (conventionally) rectangular zones on the cards in frequency - time coordinates as \( \Delta \nu, \Delta t \).

\[
\Delta_{\text{SSB}}^{\text{INU}} = (235, 245) \times (45, 55)
\]

\[
\Delta_{\text{SSB}}^2 = (190, 200) \times (55, 65)
\]

\[
\Delta_{\text{SSB}}^3 = (200, 210) \times (145, 165)
\]

One can also see on the SSB card two groups of stationary SGO modes with almost equal frequencies and visually similar relief in \( \Delta_{\text{SSB}}^2 \) and \( \Delta_{\text{SSB}}^3 \), which were identified by L. Petrova as “seismo-gravitational pulsations”. She also attracted our attention to a family of prolonged (up to 50 hours) SGO trains with growing frequency (1 \( \mu \text{Hz/hour} \)) on the intervals (50, 160)\( \text{INU} \) and almost total absence of the modes with growing frequency on the complementary intervals (0, 50)\( \text{INU} \) and (110, 145)\( \text{SSB} \). Vice versa, the modes with decreasing frequency are absent on the time interval (87, 120) on both cards. L. Petrova noticed that probably some important event (a shock?) succeeded at the moment 87, which excited three SGO modes on the interval (87, 120)\( \text{SSB} \), two of them also clearly seen on (87, 120)\( \text{INU} \). The extent of the clearly seen part of the middle train, measured on the middle line of the corresponding “ridge” on the interval (87, 120)\( \text{SSB} \) is about 24 hours, and the extents of the upper and lower modes are longer and shorter than the middle one by the intervals proportional to the difference of frequencies of the modes.

IV. APPENDIX B: ALGEBRA OF BEATINGS

The problem of beating of the seismogravitational modes has a trivial algebraic nature: it is modeled by a system of coupled oscillators with multiple eigenfrequency \( p \) which is perturbed such that the multiple eigenvalue is split into a starlet \( p_0^2 = p_0 + \delta \alpha^2 \) under a minor perturbation, which also transforms the initial eigenbasis \( \{ e_0^s \} \rightarrow \{ e_0^s \} \equiv U \{ e_0^s \} \), with an unitary generator which, in simplest case, is represented by an exponent of an antihermitian matrix \( U \equiv \exp(zB) \). Both the starlet and the generator of the basis rotation are usually found in terms of normal modes, and the perturbed evolution is represented as a linear combination of the normal modes

\[
u(t) = \sum_s A_s^0 \cos[(p_0 + \delta \alpha) t + \varphi_s] U \{ e_0^s \}
\]

The perturbed energy \( E_0(u) = \frac{1}{2} \left[ \| \dot{u} \|^2 + \sum_s |A_s^0|^2 |p_s^0|^2 \right] \) of the perturbed evolution \( u_{tt} + \alpha u = 0 \) and the unperturbed energy of the unperturbed evolution \( u_{tt} + \alpha u = 0 \) \( E_0(u) = \frac{1}{2} \| u \|^2 + |p_0| \| u \|^2 \) are conserved. The unperturbed values of energy \( E_0(P_0 u) \) of the projections \( P_0 u(t) \equiv \{ e_0^s \} \langle e_0^s, u(t) \rangle \) of the perturbed evolution onto the eigenvectors \( e_0^s \) of the unperturbed generator \( \alpha \), being averaged over properly selected time window, expose the beating phenomenon parametrized by the characteristics of the splitting starlet and the eigenbasis rotation:

\[
\frac{1}{2} \left[ T + \Delta/2 \right] E_0(P_0 u(t)) dt \approx \frac{p_0^2}{2} \sum_{n,m} A_n^m A_m^0 \cos[\delta(\alpha_n - \alpha_m)t + \varphi_n - \varphi_m] \langle e_0^s, e_n^s \rangle \langle e_0^s, e_m^s \rangle,
\]

see more in [4]. In the case when there are only splitting of the multiplicity two \( p_0 \rightarrow p_0 \pm \delta \) the beating is periodic, with the difference frequency \( 2\delta \).

The case of the tectonic plates is reduced to the above case of the connected oscillators via considering the boundary values of the solutions of the corresponding biharmonic wave equation \( p_{tt} + Lu = 0 \). The corresponding spectral problem requires considering the boundary form see [7] for the generator \( (Lu, v) - (u, \delta v) \) which is reduced to a boundary integral \( \int (\langle \delta u, v \rangle - \langle u, \delta v \rangle) dt \) and vanishes if appropriate boundary conditions with an hermitian matrix \( B \) are imposed: \( [N u - B D u]_\Gamma = 0 \) onto the boundary values.

Essential simplification of the original spectral problem is obtained while the Dirichlet-to-Neumann map, see [7], transforming the boundary values of the homogeneous problem \( Lu = \lambda p u \) \( N u = \mathcal{D} N (\lambda) u \) one to another is substituted by an appropriate finite-dimensional rational approximation \( \mathcal{D} N (\lambda) \rightarrow P \mathcal{D} N (\lambda) P^T \rightarrow \mathcal{D} N_e (\lambda) \), which corresponds to substitution of the original problem by a corresponding fitted solvable model [8].

V. APPENDIX C: WAVE EQUATION AND SEPARATION OF THE VARIABLES

The biharmonic equation for the transverse wave on a thin plate has the following form [2, 9]

\[
\rho \nu_{tt} + \mathbf{v}_i + D\Delta^2 v + \mathbf{Q} \Delta v = 0 \quad (5)
\]

\[
v \rightarrow i \omega^t u = D\Delta^2 u + i \omega^t \mathbf{B} u + \mathbf{Q} \Delta u = \omega^2 \rho u, \quad (6)
\]
Hereafter we neglect the liquid friction $\beta u_t$, which may be eliminated via an exponential factor $u \rightarrow \exp(-\beta t/2)u$ and a re-normalization of the frequency $\omega^2 \rightarrow \omega^2 - \frac{\beta^2}{4}$, while $\omega^2 - \frac{\beta^2}{4} > 0$. The dependence of the frequency of the plane wave on the wave vector is given by

$$\omega = 2\pi \nu = \sqrt{\frac{Dk^4}{\rho h} - \frac{Qk^2}{\rho h}}. \quad (7)$$

For a spherical plate the separation of the variables is possible if the constant pressure force $Q$ is applied in a spherically symmetric way. The wave equation can be factorized as

$$\left(\sqrt{D}\Delta + \frac{Q}{2\sqrt{D}} + \sqrt{\omega^2 - \frac{Q^2}{4D}}\right)u = 0, \quad (8)$$

and thus reduced to a pair of separate equations. Their solutions in the subspace $E$ of functions, independent on the angular variable, represented on the spherical plate $0 \leq r \leq L$ via Bessel functions

$$J_0 \left(\frac{\omega\sqrt{\rho h}}{\sqrt{D}} \right)^{1/2} \left[\sqrt{1 + \frac{Q^2}{4\omega^2 \rho h D}} - \frac{Q}{2\omega\sqrt{\rho h \sqrt{D}}} \right]^{1/2} r, \quad (9)$$

where $J_0$ is the standard Bessel function, and $I_0$ is a modified Bessel function of an imaginary argument $I_0(i\zeta) = J_0(iz)$. These solutions are regular at $r = 0$. However, large plate may be modelled by a circular plate with a circular hole in the centre. In this case we should add two other solutions, the Bessel functions $H_0$ and $K_0$. One also may use a model where a small plate is a sector inside a circular large plate. Solutions on the large plate in this case are Bessel function with the index $p$ determined by the angular size of the missing sector $\phi = \pi/p, p < 1$.

The eigenfunctions of the above biharmonic spectral problem on the small plate are obtained as a linear combinations of the Bessel functions $J$ and $I$, and the eigenvalues are calculated, depending on the contracting tension $Q$, by the substituting of the linear combination into relevant boundary conditions on the border $r = L$ of the plate. Comparison of the eigenvalues with eigenfrequencies of the large plate defines the condition of resonances.

We postpone all relevant mathematical details to the oncoming publication [4].

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