NATURE OF DECOUPLING IN THE MIXED PHASE OF
EXTREMELY TYPE-II LAYERED SUPERCONDUCTORS

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Abstract

The uniformly frustrated layered $XY$ model is analyzed in its Villain form. A decouple pancake vortex liquid phase is identified. It is bounded by both first-order and second-order decoupling lines in the magnetic field versus temperature plane. These transitions, respectively, can account for the flux-lattice melting and for the flux-lattice depinning observed in the mixed phase of clean high-temperature superconductors.

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The high-$T_c$ superconductor Bi$_2$Sr$_2$CaCu$_2$O$_8$ is perhaps the best known example of an extremely type-II layered superconductor.\textsuperscript{1} In the absence of bulk pinning, the vortex lattice that exists in the mixed phase of this material melts across a first-order line in the temperature-magnetic field plane for magnetic fields applied perpendicular to the layers.\textsuperscript{2} The first-order line, $H_\perp = H_m(T)$, begins at the zero-field critical point, $T_c$, but it ends strangely in the middle of the phase diagram. The depinning line $T = T_{dp}(H_\perp)$, which marks the point at which the flux lattice depins itself through thermal excitations, appears to be unrelated to this melting line.\textsuperscript{3,4} Considerable theoretical work has accompanied such observations. For example, flux-lattice melting is observed in Monte Carlo (MC) simulations of the frustrated $XY$ model,\textsuperscript{5} as well as in simulations of Ginzburg-Landau theory.\textsuperscript{6} The Josephson coupling between layers is also predicted to diminish substantially at perpendicular fields, $B_\perp$, that exceed the dimensional (2D-3D) cross-over scale,\textsuperscript{7} $B_\perp^* \sim \Phi_0/\Lambda_0^2$. Here, $\Lambda_0$ denotes the Josephson penetration length. Last, it has been claimed that the layers decouple completely through a first-order transition at perpendicular field components $H_\perp$ many times larger than $B_\perp^*$.\textsuperscript{8} A theoretical explanation of the critical endpoint of the first-order melting line in clean high-$T_c$ superconductors is lacking, however.

In this paper, we obtain a schematic phase diagram for the uniformly frustrated $XY$ model composed of a finite number of weakly coupled layers, which can describe the mixed phase of extremely type-II layered superconductors.\textsuperscript{5,9–12} In contrast to previous work,\textsuperscript{6,7,8,12} the duality analysis\textsuperscript{13–16} that follows includes both gaussian and topological excitations\textsuperscript{17–19} within the intra-layer vortex lattices. We find a second-order melting line for weakly coupled two-dimensional (2D) vortex lattices in the limit of high perpendicular field.\textsuperscript{17,20} It is then argued that this line ends at an intersection with a first-order decoupling line\textsuperscript{8} in the middle of the $T-H_\perp$ plane (see Fig. 1). The theory potentially accounts for the same phenomenon that is observed in clean high-$T_c$ superconductors.\textsuperscript{2,3} We also demonstrate how the decoupling mechanism explicit in the duality analysis is intimately related to the entanglement of flux lines perpendicular to the layers. This is consistent with the supersolid scenario,\textsuperscript{19} with recent numerical studies of the anisotropic $XY$ model with frustration,\textsuperscript{21} and with very recent experimental determinations of the phase diagram in clean high-temperature superconductors.\textsuperscript{22}

Consider the application of a magnetic field to a layered superconductor in the extreme
type-II limit, $\lambda_L \to \infty$. Magnetic screening effects are then negligible and the theoretical description of the interior of the mixed phase reduces to a layered XY model in the presence of a uniform frustration.\textsuperscript{5,9–12} In particular, the thermodynamics is determined by the kinetic energy $E_{\text{XY}} = -\sum_{r,\mu} J_\mu \cos[\Delta_\mu \phi - A_\mu]$ as a function of the superconducting phase $\phi(r)$. Here, $J_x = J_y$ are the local phase rigidities, and $A_\mu = (0, b_\perp x, -b_\parallel x)$ is the vector potential. The magnetic induction parallel and perpendicular to the layers is related to the frustration, $\vec{b}$, through the respective identities $B_\parallel = (\Phi_0/2\pi d) b_\parallel$ and $B_\perp = (\Phi_0/2\pi a) b_\perp$. Here $a$ denotes the square lattice constant, which is of order the zero-temperature coherence length, while $d$ represents the spacing in between consecutive layers. Consider now the corresponding partition function, $Z[p] = \int \mathcal{D}\phi e^{-E_{\text{XY}}/k_B T} e^{i\sum p\phi}$, in the (dual) Villain form,\textsuperscript{13–16} which reads

$$Z[p] = \sum_{\{n_\mu(r)\}} \Pi_\nu \delta \left[\sum_\nu \Delta_\nu n_\nu |_r - p(r)\right] \exp \left[-\sum_r \left(\frac{1}{2\beta_\parallel} \vec{n}^2 + \frac{1}{2\beta_\perp} n^2_z + i \sum_\nu n_\nu A_\nu\right)\right]. \quad (1)$$

Above, $n_\mu(r)$ is an integer link-field on the layered lattice structure of points $r = (\vec{r}, l)$, with $\mu = \hat{x}, \hat{y}, \hat{z}$ and $\vec{n} = (n_x, n_y)$. Also, we set $\beta_{\parallel,\perp} = J_{\parallel,\perp}/k_B T$. Now decompose the parallel field $\vec{n}$ into transverse and longitudinal parts $\vec{n}(\vec{r}, l) = \vec{n}'(\vec{r}, l) - \vec{n}_{\perp}(\vec{r}, l) + \vec{n}_{\perp}(\vec{r}, l - 1)$, where the transverse and longitudinal fields, $\vec{n}'$ and $\vec{n}_{\perp}$, respectively satisfy the constraints $\vec{\nabla} \cdot \vec{n}' = 0$ and $\vec{\nabla} \cdot \vec{n}_{\perp} = n_z$ (for $p = 0$), with $\vec{\nabla} = (\Delta_x, \Delta_y)$.\textsuperscript{12,23} Take next the potential representation $\vec{n}_{\perp} = -\vec{\nabla} \Phi$ for the longitudinal inter-layer field. This yields the expression $\Phi(\vec{r}, l) = \sum_{\vec{r}'} G^{(2)}(\vec{r} - \vec{r}') n_z(\vec{r}', l)$ for the potential, where $G^{(2)} = -\nabla^2$ is the Greens function for the square lattice. We then obtain the form $Z[0] = Z_{\text{CG}} \cdot \Pi_l Z_{\text{DG}}[0]$ for the partition function (1), where

$$Z_{\text{CG}} = \sum_{\{n_z\}} y_0^{N[n_z]} \exp \left[-\frac{1}{2\beta_\parallel} \sum_l \sum_{\vec{r}_1, \vec{r}_2} q_l(\vec{r}_1) G^{(2)}(\vec{r}_1 - \vec{r}_2) q_l(\vec{r}_2) - i \sum_l \sum_{\vec{r}} n_z(\vec{r}, l) A_z(\vec{r})\right] \times \Pi_l \left(Z_{\text{DG}}[q_l]/Z_{\text{DG}}[0]\right) \quad (2)$$

is an inter-layer Coulomb gas (CG) factor. Here, $q_l(\vec{r}) = n_z(\vec{r}, l - 1) - n_z(\vec{r}, l)$ is the fluxon charge that collects onto layer $l$ and $y_0 = \exp(-1/2\beta_\perp)$ is the fugacity that is raised to the power $N[n_z] = \sum_{\vec{r},l} n_z^2(\vec{r}, l)$ per configuration.\textsuperscript{16} Note that a fluxon, $n_z(\vec{r}, l) = \pm 1$, represents a vortex ring that lies in between layers $l$ and $l + 1$ at the point $\vec{r}$.\textsuperscript{24} The
remaining factors

\[ Z_{DG}'[q_l] = \sum_{\{\vec{n}\}} \Pi_{\vec{r}} \delta[\vec{n} \cdot \vec{n}]_{\vec{r},l} - q_l(\vec{r}) \times \]

\[ \times \exp \left[ -\frac{1}{2\beta_{\|}} \sum_{\vec{r}} \vec{n}_l^2(\vec{r},l) - i \sum_{\vec{r}} \vec{n}(\vec{r},l) \cdot \vec{A}'(\vec{r}) \right] C_{sw}[q_l] \]  

above represent modified 2D XY models with uniform frustration corresponding to each layer, with a modified in-plane vector potential \( \vec{A}' = \vec{A} + i \beta_{\|}^{-1} [\vec{n}_-(\vec{r},l-1) - \vec{n}_-(\vec{r},l)] \) in the gauge \( \vec{n} \cdot \vec{A} = 0 \), and with an extra weight factor \( C_{sw}[q_l] = \exp\{-\sum_{\vec{r}}(2\beta_{\|})^{-1} [\vec{n}_-(\vec{r},l-1) - \vec{n}_-(\vec{r},l)]^2\} \). In physical terms, the CG factor (2) describes the Josephson coupling between layers, whereas the discrete gaussian (DG) model factors (3) describe the thermodynamics of pancake vortices within each layer. The last factor in expression (2), however, represents the renormalization of the Josephson coupling due to misalignments of pancake vortices between layers.\(^7,^8\) This important correction was omitted without proper justification in previous work.\(^12,^{23-25}\)

Consider now the weak-coupling limit, \( y_0 \to 0 \), in which case the \( n_z \) fluxons are dilute.\(^24\) The modified 2D XY model (3) can then be analyzed in the continuum limit, in which case we obtain the identification

\[ Z_{DG}'[q_l]/Z_{DG}[0] = \left\langle \exp \left[ i \sum_{\vec{r}} q_l(\vec{r}) \phi_{vx}(\vec{r},l) \right] \right\rangle_{J_\perp=0} \]  

with the vortex component of the phase correlations within an isolated layer \( l \),\(^13,^{14}\) i.e., \( \nabla^2 \phi_{vx} = 0 \). It is instructive to consider a single neutral pair of unit \( n_z \) charges that lie in between layers \( l' \) and \( l'+1 \), separated by \( \vec{r} \). The renormalization to the Josephson coupling that is encoded in the last factor of expression (2) then becomes the gauge-invariant product

\[ \Pi_{l}(Z_{DG}'[q_l]/Z_{DG}[0]) = C_{l}'(\vec{r})C_{l'+1}'(\vec{r}) \]  

of the corresponding phase autocorrelation functions,

\[ C_{l}'(\vec{r}) = \left\langle \exp \left[ i\phi_{vx}(0,l) - i\phi_{vx}(\vec{r},l) \right] \right\rangle_{J_\perp=0} , \]

within isolated layers. In the asymptotic limit, \( \vec{r} \to \infty \), this function has a magnitude of the form \( |C_{l}'(\vec{r})| = g_0(r_0/r_0')^{\eta_{sw}}(r_0/|\vec{r}|)^{\eta_{vx}} \) for \( |\vec{r}| \ll \xi_{vx} \), and a magnitude of the form \( |C_{l}'(\vec{r})| = g_0(|\vec{r}|/r_0')^{\eta_{sw}} \exp(-|\vec{r}|/\xi_{vx}) \) for \( |\vec{r}| \gg \xi_{vx} \),\(^13-15\) Here, \( \eta_{sw} = (2\pi\beta_{\|})^{-1} \) and \( \eta_{vx} \) are, respectively, the spin-wave and the vortex components of the correlation exponent inside
layer $l$, while $\xi_{vx}$ is the corresponding phase correlation length. Also, the length $r_0 = a_{vx}/2^{3/2}e^{\gamma_E}$ is set by the inter-vortex scale, $a_{vx} = (\Phi_0/B_\perp)^{1/2}$, and by Euler's constant, $\gamma_E$, while $r'_0 = a/2^{3/2}e^{\gamma_E}$. We therefore obtain the effective layered CG ensemble

\[ Z_{CG} \cong \sum_{\{n_z\}} y^{N[n_z]} \exp \left\{ -\frac{1}{2} \sum_l \sum_{\vec{r}_1 \neq \vec{r}_2} \left[ q_l(\vec{r}_1) \left[ \frac{\eta_{2D} \ln(r_0/|\vec{r}_1 - \vec{r}_2|)}{V_{\text{string}}^{[q_l]}(\vec{r}_1, \vec{r}_2)} \right] q_l(\vec{r}_2) \right] - i \sum_l \sum_{\vec{r}} n_z(\vec{r}, l) A_z(\vec{r}) \right\} \] (6)

that has been coarse grained up to the natural ultraviolet scale $a_{vx}$. In particular, the sums above are restricted to a square sublattice with lattice constant $a_{vx}$. This requires the introduction of an effective coarse-grained fugacity $y = g_0(a_{vx}/a)^2y_0$. At relatively small separation $|\vec{r}_1 - \vec{r}_2| \ll \xi_{vx}$, the fluxons experience a pure Coulomb interaction ($V_{\text{string}}^{[q_l]} = 0$) set by the 2D correlation exponent $\eta_{2D} = \eta_{sw} + \eta_{vx}$. At large separations $|\vec{r}_1 - \vec{r}_2| \gg \xi_{vx}$, on the other hand, the fluxons experience a pure ($\eta_{2D} = 0$) confining interaction $V_{\text{string}}^{[q_l]}(\vec{r}_1, \vec{r}_2) = |\vec{r}_1 - \vec{r}_2|/\xi_{vx}$ between those points $\vec{r}_1$ and $\vec{r}_2$ in layer $l$ that are connected by a string [see Eq. (3) and ref. 26].

We shall now see that the effective layered CG ensemble (6) can be employed to determine the macroscopic nature of the Josephson effect in the weak-coupling limit, $y \to 0$. Let us first identify the macroscopic intra-layer phase rigidity, $\tilde{J}_\parallel = k_B T/2\pi\eta_{2D}$. The above Coulomb gas ensemble indicates that fluxons ($n_z = \pm 1$) are in a plasma state at temperatures below the naive decoupling temperature $k_B T_* = 4\pi \tilde{J}_\parallel$ when quasi long-range intra-layer phase correlations are present, $\xi_{vx} = \infty$ and $V_{\text{string}}^{[q_l]} = 0$, while that they form a dilute gas of bound neutral pairs of size $\xi_{vx}$ at high temperatures when short-range intra-layer phase correlations exist, $\xi_{vx} < \infty$. We now quote the expression for the perpendicular phase rigidity of the XY model (2) in terms of fluxons:

$$\rho_{\perp}^s = N^{-1} \left\langle \left[ \sum_{\vec{r}, l} n_z(\vec{r}, l) \right]^2 \right\rangle k_BT,$$ (7)

where $N$ denotes the number of links between layers. It is obtained directly from the duality transformation (1) and from the definition $\rho_{\perp}^s = \frac{\partial^2}{\partial A_z^2} (G_{\text{cond}}/N)$ for this quantity. Under periodic boundary conditions, the low-temperature (plasma) phase thus sustains a macroscopic Josephson effect ($\rho_{\perp}^s \neq 0$) due the presence of free fluxons, whereas the high-temperature (dielectric) phase does not due to the absence of free fluxons ($\sum n_z = 0$).
Since the ordering temperature of a single layer is typically much less than $T^*$ (see below), we conclude that the only thermodynamic phases that are possible at weak coupling are a coupled superconductor at low temperatures and a decoupled “normal” state at high temperatures. This indicates that neither the Friedel scenario\(^{24,28}\) (decoupled superconducting layers) nor the line-liquid state\(^{9,12,23}\) (coupled normal layers) are likely to be thermodynamic states in the absence of disorder.

It is also important to determine the size of the local Josephson coupling. Consider the macroscopically decoupled phase at weak coupling, where short-range intra-layer phase correlations are present: $\xi_{vx} < \infty$. Inter-layer fluxon $(n_z)$ pairs are then bound by a confining string. Comparison of the CG ensemble (6) with the layered $XY$ model (1) yields Koshelev’s formula\(^{10}\)

$$\langle e^{i\phi_{l,l+1}} \rangle \cong y_0 \int d^2r C_l(\mathbf{r}) C_{l+1}^*(\mathbf{r}) e^{-ib_\parallel x/a^2}$$

(8)

for the local Josephson coupling (see refs. 15 and 16). Here, $\phi_{l,l+1}(\mathbf{r}) = \phi(\mathbf{r}, l+1) - \phi(\mathbf{r}, l) - A_z(\mathbf{r})$ is the gauge-invariant phase difference between consecutive layers, while $C_l(\mathbf{r})$ is the phase autocorrelator for layer $l$ in isolation [i.e., replace $\phi_{vx} \to \phi$ in Eq. (5)]. Eq. (8) implies that $\partial^2 \ln Z[0]/\partial B_\perp \partial B_\parallel = 0$ at $B_\parallel = 0$. We then have a null line tension for Josephson vortices, since $\varepsilon_\parallel = \varepsilon_\parallel|_{J_\perp = 0}$. In conclusion, we recover the previous result (7) that macroscopic Josephson coupling is absent in the weak-coupling limit if intra-layer phase correlations are short range. Next, observe that scaling considerations yield the form

$$\int d^2r C_l C_{l+1}^* = f_0 \xi_{vx}^2$$

for the integral on the right-hand side of Eq. (8), where $f_0$ is of order unity.\(^{10}\) Substitution into Eq. (8) then yields Koshelev’s formula

$$\langle \cos \phi_{l,l+1} \rangle \cong f_0 (\xi_{vx}/a)^2 y_0$$

(9)

for the local Josephson coupling in such case (see ref. 16). [Note that scaling implies the functional form $\xi_{vx} = a_{vx} e(\beta_\parallel)$ for the 2D correlation length.]

We can now analyze the $XY$ model (1) composed of a finite number of weakly-coupled layers with uniform frustration, $B_\perp$. Consider first the weak-coupling limit, $\langle \cos \phi_{l,l+1} \rangle \to 0$, which by Eq. (9) is reached at infinitely high perpendicular fields. It is well known that an isolated lattice of 2D vortices (3) melts at a temperature $k_B T_m^{(2D)} \cong J_\parallel/20$, above which quasi long-range positional correlation in the vortex lattice is lost.\(^{20}\) The transition
is driven by the unbinding of dislocation pairs and it is expected theoretically to be second-order.\textsuperscript{17} We shall now make the plausible assumption that the nature of phase coherence in the 2D vortex lattice is locked to the nature of the positional correlations, such that $\xi_{vX}$ diverges exponentially as the temperature approaches $T_{m}^{(2D)}$ in the disordered phase. By the previous analysis, we then conclude that the layers show a macroscopic Josephson effect at low temperature $T < T_{m}^{(2D)}$ signalled by a positive phase rigidity between layers (7), while that they are decoupled ($\rho_{s}^{\perp} = 0$) at high temperature $T > T_{m}^{(2D)}$. The former low-temperature phase is best described by 2D vortex lattices that display a Josephson effect. The decoupled high-temperature phase, on the other hand, corresponds to a liquid of intra-layer “pancake” vortices.\textsuperscript{7,8}

Consider next the weak-coupling regime, $\langle \cos \phi_{l,l+1} \rangle \ll 1$, at high perpendicular fields $B_{\perp} \gg B_{\perp}^{*}$ [see Eq. (9)]. Eq. (9) indicates that the selective high-temperature expansion breaks down ($\langle \cos \phi_{l,l+1} \rangle > 1$) in the decoupled phase at a temperature $T_{X}$ set roughly by the identification of length scales $\Lambda_{0} \sim \xi_{vX}(T_{X})$. We now observe that the layered XY model (1) without frustration can also be described by the Coulomb gas ensemble (6), but with the natural ultraviolet length scale replaced globally by $a_{vX} \rightarrow a$. By analogy with what is presently understood for the layered XY model without frustration,\textsuperscript{29} we conclude that a second-order transition should take place in the weak-coupling regime at a temperature $T_{m}$ that lies inside of the dimensional crossover window $T_{m}^{(2D)} < T < T_{X}$.

And what happens as the local Josephson coupling (9) approaches unity, which can be achieved by lowering the perpendicular field? The CG ensemble (6) is screened in the low-temperature Josephson-coupled phase, $T < T_{m}^{(2D)}$, for small effective fugacity. This implies that no phase transition is possible as a function of field there.\textsuperscript{13,14} Nevertheless, Eq. (9) clearly indicates that the selective high-temperature expansion, $y_{0} \rightarrow 0$, breaks down at perpendicular fields below $B_{\perp}^{*}$. In such case, a crossover into a flux-line lattice regime must therefore take place.\textsuperscript{7} At high temperatures $T > T_{X}$, on the other hand, the CG ensemble (6) is confining for small fugacity, $y \ll 1$. In particular, the string interaction (6) binds together dilute fluxon-antifluxon pairs into stable dipoles of dimension $\xi_{vX}$. In the limit of dense fluxons, $y \rightarrow 1$, these dipoles disassociate, however. This is due to the ineffectiveness of the string when the distance, $r_{s}$, between neighboring dipoles is small in comparison to the length of the string, $\xi_{vX}$. The system must therefore experience a (inverted) first-order
phase transition into a screened CG above a critical coupling $\langle \cos \phi_{l,l+1} \rangle_D$, since there is no diverging length scale. Monte Carlo simulation of the layered XY model with uniform frustration indicates that $\langle \cos \phi_{l,l+1} \rangle_D$ is constant and of order unity.\(^{11}\) By Eq. (9), we therefore expect a first-order decoupling transition at a perpendicular field

$$H_D = \left( \tilde{f}_0 \Phi_0 / a^2 \right) \left( \frac{1}{2} \beta_\perp / \langle \cos \phi_{l,l+1} \rangle_D \right)$$

of order $\beta_\parallel B_\perp^*$ for high temperatures, $T > T_x$. This results from the replacement $y_0 \rightarrow \frac{1}{2} \beta_\perp$ (see ref. 16). The phenomenology\(^3^0\) $J_\perp = E_J(T_c - T) / T_c$ for the Josephson energy in the vicinity of the zero-field transition at $T_c$ yields the dependence\(^2^,7^,8\) $H_D(T) = \gamma_2^{-2} H_{c2}(T)$ for the “cosine” XY model, where $H_{c2}(T) \sim (\Phi_0 / a^2)(T_c - T) / T$ is the mean-field perpendicular upper-critical field, and where $\gamma_2 \sim (\langle e^{i\phi_{l,l+1}} \rangle_D / f_0)^{1/2} \cdot (k_B T_c / E_J)^{1/2}$ is an effective anisotropy parameter. Similar results for the first-order decoupling transition (10) were obtained previously using the elastic medium description of vortex matter in layered superconductors.\(^7^,8\) The above discussion is summarized by the schematic phase diagram in Fig. 1.

We shall now give a physical interpretation of the results just obtained from the duality analysis. Consider two parallel vortices along the z-axis that exchange positions in between layers $l$ and $l + 1$. A moment’s thought determines that the exchange creates a (fluxon) vortex loop that lies in between those layers. Fluxon charge\(^2^4\) ($n_z$) can therefore entangle vortex lines aligned perpendicular to the layers. We conclude that entanglement is what actually drives the decoupling transitions shown in Fig. 1. This picture is consistent with (a) the classification of the coupled 2D vortex-lattice phase as a type of super-solid matter,\(^1^9\) with (b) recent Monte Carlo simulations of the anisotropic XY model with frustration that conclude that entanglement is what drives the first-order melting transition of the vortex lattice,\(^2^1\) and with (c) recent experimental work that also finds entanglement to be what controls the location of the critical endpoint of the vortex-lattice melting transition in clean high-temperature superconductors.\(^2^2\) This web of facts strongly supports the results obtained here.

We shall close by comparing the present theory for the uniformly frustrated layered XY model with known experimental results for the mixed phase of Bi$_2$Sr$_2$CaCu$_2$O$_8$, which is extremely type-II and layered. This system shows first-order melting in the absence of bulk pinning. In agreement with Fig. 1, the melting line ends in the middle of the phase
This suggests identifying it with the decoupling transition at $H_\perp = H_D$. Also, the depinning line, $T = T_{dp}$, is nearly vertical in such materials for perpendicular fields above $B^*_{\perp}$. This suggests identifying $T_{dp}$ with $T_m$ in Fig. 1. Finally, although Monte Carlo simulations of the frustrated XY model with anisotropy do observe a unique first-order vortex-lattice melting transition, no indication of the critical endpoint predicted here has been reported.\textsuperscript{11} Eq. (10) implies, however, that the critical anisotropy parameter, $\gamma' = (J_\parallel/J_\perp)^{1/2}$, is high: e.g., $\gamma'_c \sim 33$ for perpendicular fields $B_\perp = \Phi_0/56a^2$ at $T_m^{(2D)}$. The condition that the Josephson penetration length, $\Lambda_0 = \gamma'_ca$, be smaller than $L/2\pi$, where $L$ is the linear dimension of each layer, then indicates that extensive MC simulations\textsuperscript{21} are necessary in order to see the critical endpoint predicted here.\textsuperscript{15}

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Fig. 1. Shown is a schematic phase diagram for the uniformly frustrated XY model (1) made up of a finite number of weakly-coupled layers at $B_\parallel = 0$. A vestige of the vertical second-order line may extend down to lower fields in the form of a cross-over (compare with ref. 3). The Josephson temperature $T_J = J_\perp/k_B$ is assumed to be smaller than the scale of the figure. Notice that the inequality $T_J < T_m^{(2D)}$ required by the phase diagram indicates a minimum anisotropy parameter $\gamma' = (J_\parallel/J_\perp)^{1/2}$ equal to about four, since $k_B T_m^{(2D)} \simeq J_\parallel/20$. 
(NO BULK PINNING)

- COUPLED 2D VORTEX LATTICES
- DECOUPLED PANCAKE VORTEX LIQUID
- FLUX-LINE LATTICE
- DEFECTIVE FLUX-LINE LATTICE

- \( H_{c2} \)
- \( H \)
- \( T_m \)
- \( T_c \)

- FIRST-ORDER
- SECOND-ORDER
- CROSSOVER