Optimization of Measurement Device Independent 
Scarani-Acin-Ribordy-Gisin protocol

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Abstract

The measurement device independent (MDI) Quantum Key Distribution (QKD) is a practically implementable method for transmitting secret keys between respective partners performing quantum communication. SARG04 (Scarani-Acin-Ribordy-Gisin 2004) is a protocol tailored to struggle against photon number splitting (PNS) attacks by eavesdroppers and its MDI-QKD version is reviewed and optimized from secret key bitrate versus communication distance point of view. We consider the effect of several important factors such as error correction function, dark counting parameter and quantum efficiency in order to achieve the largest key bitrate versus longest communication distance.

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While Classical cryptography use two types of keys to encode and decode messages (secret or symmetric and public or asymmetric keys) Quantum cryptography uses QKD for transmitting secret keys between partners allowing them to encrypt and decrypt their messages. QKD principal characteristic is that it is practically implementable and has already been deployed commercially by several quantum communication providers such as SeQureNet in France, IQ Quantique in Switzerland, MagiQ Technologies in the USA and Quintessence-Labs in Australia. The second main feature of QKD is that it allows communicating parties to detect online eavesdroppers in a straightforward fashion.

In principle, QKD is unconditionally secure nevertheless its practical implementation has many loopholes and consequently has been attacked by many different ways exploiting some intermediate operation or another during secret key processing such as time-shift [1, 2], phase-remapping [3], detector blinding [4, 5], detector dead-time [6], device calibration [7], laser damage [8]...

This work is about optimization of SARG04 [9] MDI-QKD version protocol designed to fend off photon number splitting (PNS) attacks by considering important factors such as error correction function types, detector dark counting parameter and quantum efficiency. It is organized as follows: after reviewing the original four-state SARG04 protocol, we discuss its MDI version and describe the effects of various parameters on communication distance and secret key bitrate.

SARG04 protocol has been developed to combat PNS attacks that are targeted toward intercepting photons present in weak coherent pulses (WCP) that are used for communication. This stems from the fact, it is not possible presently to commercially exploit single photons in a pulse. However, progress in developing large scale methods targeted at using single photons in a pulse is advancing steadily.

SARG04 being very similar to BB84 [9] protocol, the simplest example of secret key sharing among sender and receiver (Alice and Bob), we review first the BB84 case below.

In the BB84 protocol framework, Alice and Bob use two channels to communicate: one quantum and private to send polarized single photons and another one classical and public (telephone or Internet) to send ordinary messages [10]. Alice selects two bases in 2D Hilbert space consisting each of two orthogonal states: $\oplus$ basis with $[0, \pi/2]$ linearly polarized photons, and $\otimes$ basis with $(\pi/4, -\pi/4)$ linearly polarized photons.

Four photon polarization states: $|\rightarrow\rangle, |\uparrow\rangle, |\nearrow\rangle, |\searrow\rangle$ are used to transmit quantum data
with $|↗⟩ = \frac{1}{\sqrt{2}}(|→⟩ + |↑⟩)$ and $|↘⟩ = \frac{1}{\sqrt{2}}(|→⟩ - |↑⟩)$.

A message transmitted by Alice to Bob over the Quantum channel is a stream of symbols selected randomly among the four above and Alice and Bob choose randomly one of the two bases $⨁$ or $⨂$ to perform photon polarization measurement.

Alice and Bob announce their respective choice of bases over the public channel without revealing the measurement results.

The raw key is obtained by a process called ”sifting” consisting of retaining only the results obtained when the used bases for measurement are same.

After key sifting, another process called key distillation [9] must be performed. This process entails three steps [9]: error correction, privacy amplification and authentication in order to counter any information leakage from photon interception, eavesdropping detection (with the no-cloning theorem [9]) and exploitation of announcement over the public channel.

The basic four-state SARG04 protocol is similar to BB84 but adds a number of steps to improve it and protect it against PNS attacks. The steps entail introducing random rotation and filtering of the quantum states. Before we describe it, we introduce some states and operators [11] using Pauli matrices $σ_X, σ_Y, σ_Z$:

- $R = \cos(\frac{\pi}{4})I - i \sin(\frac{\pi}{4})σ_Y$ is a $\pi/2$ rotation operator about $Y$ axis,

- $T_0 = I$ is the $(2×2)$ identity operator,

- $T_1 = \cos(\frac{\pi}{4})I - i \sin(\frac{\pi}{4})\frac{(σ_Z + σ_X)}{\sqrt{2}}$ is a $\pi/2$ rotation operator around the $(Z + X)$ axis,

- $T_2 = \cos(\frac{\pi}{4})I - i \sin(\frac{\pi}{4})\frac{(σ_Z - σ_X)}{\sqrt{2}}$ is a $\pi/2$ rotation operator around the $(Z - X)$ axis.

Alice prepares many pairs of qubits and sends each one of them to Bob after performing a random rotation over different axes with $T_l R^k$ where $l \in \{0,1,2\}$ and $k \in \{0,1,2,3\}$.

Upon receiving the qubits, Bob first applies:

- A random reverse multi-axis rotation $R^{-k'} T_{l'}^{-1}$,

- Afterwards, he performs a local filtering operation defined by $F = \sin(\frac{\pi}{8}) |0_x⟩⟨0_x| + \cos(\frac{\pi}{8}) |1_x⟩⟨1_x|$ where $\{0_x⟩, |1_x⟩\}$ are $X$-eigenstate qubits; they are also eigenvectors of $σ_X$ with eigenvalues $+1$, and -1 respectively. Local filtering enhances entanglement degree and the $\pi/8$ angle helps retrieve [12] one of the maximally entangled EPR
Bell \cite{B13, B14} states i.e. polarization entangled photon pair states given by: \( |\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|\rightarrow\uparrow\rangle \pm |\uparrow\rightarrow\rangle) \) and \( |\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|\rightarrow\rightarrow\rangle \pm |\uparrow\uparrow\rangle) \). They form a complete orthonormal basis in 4D Hilbert space for all polarization states of a two-photon system and the advantage of local filtering is to make Alice and Bob share pairs of a Bell state making the shared bits unconditionally secure \cite{B12}.

- After, Alice and Bob compare their indices \( k, l \) and \( k', l' \) via public communication, and keep the qubit pairs with \( k = k' \) and \( l = l' \) when Bob’s filtering operation is successful.

- They choose some states randomly as test bits, measure them in the \( Z \) basis, and compare their results publicly to estimate the bit error rate and the information acquired by the eavesdropper.

- Finally, they utilize the corresponding Calderbank-Shor-Steane (CSS) code \cite{B15} to correct bit and phase errors and perform a final measurement in the \( Z \) basis on their qubits to obtain the secret key.

Following Lo et al. \cite{B16} Mizutani et al. \cite{B17} modified the original SARG04 protocol by including an intermediate experimental setup run by Charlie, at mid-distance between Alice and Bob, consisting of Bell correlation measurements. The setup contains a half beam-splitter, two polarization beam-splitters to simulate photonic Hadamard and CNOT gates in order to produce Bell states, as well as photodiode detectors. This additional step will help discard non perfectly anti-correlated photons and thus reduce transmission error rates. In addition, Alice and Bob not only choose photon polarization randomly, they also use WCP amplitude modulation to generate decoy states in order to confuse the eavesdropper. The protocol runs as follows:

- Charlie performs Bell measurement on the incoming photon pulses and announces to Alice and Bob over the public channel whether his measurement outcome is successful or not. When the outcome is successful, he announces the successful events as being of Type1 or Type2. Type1 is coincidence detection events of \( AT \) and \( BR \) or \( BT \) and \( AR \). Type2 is coincidence detection events of \( AT \) and \( AR \) or \( BT \) and \( BR \) where \( AT, BT \) stand for detecting transmitted (\( T \)) photon events from Alice (\( A \)) or Bob (\( B \)) linearly
polarized at 45° whereas $AR, BR$ are for detecting reflected ($R$) photon events at -45°.

- Alice and Bob broadcast $k$ and $k'$, over the public channel. If the measurement outcome is successful with Type1 and $k = k' = 0, \ldots, 3$, they keep their initial bit values, and Alice flips her bit. If the measurement outcome is successful with Type2 and $k = k' = 0, 2$, they keep their initial bit values. In all the other cases, they discard their bit values.

- After repeating the above operations several times, Alice and Bob perform error correction, privacy amplification and authentication as described previously.

In the ideal case (no transmission errors, no eavesdropping) Alice and Bob should discard results pertaining to measurements done in different bases (or when Bob failed to detect any photon).

![Figure 1](image1.png)

**FIG. 1.** (Color on-line) Phase error probability $e_p$ versus bit error probability $e_b$ for Type1 and Type2 with different number of photons $(m, n)$ emitted by Alice and Bob.

In QKD, Alice and Bob should be able to determine efficiently their shared secret key as a function of distance $L$ separating them. Since, the secure key is determined after sifting and distillation, secure key rate is expressed in bps (bits per signal) given that Alice sends symbols to Bob to sift and distill with the remaining bits making the secret key.

For Type $i$ event, we define $e_{i,p}^{(m,n)}$ as the phase error probability that Alice and Bob emits $m$ and $n$ photons respectively, and Charlie announces a successful outcome with $Q_i^{(m,n)}$, the joint probability. Consequently the asymptotic key rate for Type $i$ is given as a sum over
partial private amplification terms of the form $Q_i^{(m,n)}[1 - h_2(e_{i,p}^{(m,n)})]$ and one error correction term $Q_i^{\text{tot}} f(e_i^{\text{tot}}) h_2(e_i^{\text{tot}})$ related to total errors as [17, 18]:

$$K_i(L) = Q_i^{(1,1)}[1 - h_2(e_{i,p}^{(1,1)})] + Q_i^{(1,2)}[1 - h_2(e_{i,p}^{(1,2)})] + Q_i^{(2,1)}[1 - h_2(e_{i,p}^{(2,1)})] - Q_i^{\text{tot}} f(e_i^{\text{tot}}) h_2(e_i^{\text{tot}}).$$  \hspace{1cm} (1)

The total probabilities $Q_i^{\text{tot}} = \sum_{m,n} Q_i^{(m,n)}$ and total error rates are given by $e_i^{\text{tot}} = \sum_{m,n} Q_i^{(m,n)} e_{i,b}^{(m,n)}/Q_i^{\text{tot}}$ where $e_{i,b}^{(m,n)}$ is the Type $i$ bit error probability and $h_2$ is the binary Shannon entropy [19] given by $h_2(x) = -x \log_2(x) - (1-x) \log_2(1-x)$. Moreover, the above asymptotic key rate is obtained in the limit of infinite number of decoy states [17].

Phase error probabilities are determined from bit error probabilities as depicted in fig. 1 for Type 1 and 2 and depending on photons $(m, n)$ emitted.

Since Charlie is in the middle between Alice and Bob, the channel transmittance to Charlie from Alice is the same as that from Bob. Considering that $L$ is the distance between Alice and Bob, the channel transmittance $\eta_T$ is obtained by replacing $L$ by $L/2$ resulting in: $\eta_T = 10^{-\alpha L/20}$.

For the standard Telecom wavelength [19] $\lambda = 1.55\mu m$, the loss coefficient with distance is $\alpha=0.21$ dB/km. The quantum efficiency and the dark count rate of the detectors are taken as $\eta = 0.045$ and $d = 8.5 \times 10^{-7}$, respectively as in the GYS [20] case.

We compare below the effect of a fixed error correction function with respect to a fixed value function.

The error correction function is given by Enzer et al. [21] as: $f_e(x) = 1.1581 + 57.200 x^3$

In figs 2,4,6 secret key rates for Type 1 and Type 2 events are displayed versus distance when $f_e$ function is considered as variable or fixed at a value of 1.33.

Improving quality of detection means that dark counting must be substantially reduced in order to avoid false ”clicks” (irrelevant event detection) of the detectors.

In figs 3,5 secret key rates for Type 1 and Type 2 events are displayed versus distance for different values of the dark count rate with error correction function $f_e$ freely varying.

Quantum yield is an important parameter that plays an important role in quantum communications.

In figs 7,9 secret key rates for Type 1 and Type 2 events are displayed versus distance for different values of the quantum yield $\eta$ with error correction function $f_e$ freely varying. The value of $\eta$ has been intentionally exaggerated in order to explore the range of communication
FIG. 2. (Color on-line) Key rate $K(L)$ for Type 1 events, in bps versus distance $L$ using the same parameters as in Ref. [20] $\eta=0.045$, $d = 8.5 \times 10^{-7}$, $\alpha = 0.21$ for different error correction function.

FIG. 3. (Color on-line) Key rate $K(L)$ for Type 2 events, in bps versus distance $L$ using the same parameters as in Ref. [20] $\eta=0.045$, $d = 8.5 \times 10^{-7}$, $\alpha = 0.21$ for different error correction function.

distances covered by it variation. It is interesting to note that the Quantum yields acts on communication distance and key bitrate simultaneously whereas dark count rate and error correction function changes affect solely communication distance.

Communication distances and secret key bitrates obtained in this work can be improved when we vary the error correction function, dark count rate and quantum efficiency. Insight into SARG04 protocol acquired by optimization leads to conclude that the most sensitive way to increase communication distance substantially is to decrease the dark count rate. The least sensitive parameter is the error correction function type and in spite of exaggerating the values of the quantum efficiency in order to probe the largest possible range of communication distances, the dark count rate parameter is the most promising. Conse-
FIG. 4. (Color on-line) Key rate $K(L)$ for Type 1 events, in bps versus distance $L$ using the same parameters as in Ref. [20] $\eta=0.045$, $\alpha = 0.21$ for different detector dark counting parameter $d$ and variable error correction function.

FIG. 5. (Color on-line) Key rate $K(L)$ for Type 2 events, in bps versus distance $L$ using the same parameters as in Ref. [20] $\eta=0.045$, $\alpha = 0.21$ for different detector dark counting parameter $d$ and variable error correction function.

Consequently future research efforts ought to be directed towards reducing it considerably. This improvement relies on developing special algorithms that will allow to discriminate between different events occurring around the photodetectors or developing materials with selective and specially engineered higher thresholds preventing false ”clicks” triggered by ”irrelevant” events.

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FIG. 6. (Color on-line) Key rate $K(L)$ for Type 1 events, in bps versus distance $L$ using the same parameters as in Ref. [20] $d = 8.5 \times 10^{-7}$, $\alpha = 0.21$ and $f_e$ variable for different values of the quantum yield parameter $\eta$.

FIG. 7. (Color on-line) Key rate $K(L)$ for Type 2 events, in bps versus distance $L$ using the same parameters as in Ref. [20] $d = 8.5 \times 10^{-7}$, $\alpha = 0.21$ and $f_e$ variable for different values of the quantum yield parameter $\eta$.

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