Length-Scale Dependent Superconductor-Insulator Quantum Phase Transition in One Dimension

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We study the dissipation physics of one dimensional mesoscopic superconducting quantum interference device array by using the field-theoretical renormalization group method. We observe length scale dependent superconductor-insulator quantum phase transition at very low temperature and also observe the dual behaviour of the system for the higher and lower values of magnetic field. At a critical magnetic field, we also observe a critical behaviour where the resistance is independent of length.

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1. Introduction: The transition at zero temperature or at very low temperature belongs to the class of phase transition that is driven by the quantum fluctuations of the system [1]. The quantum fluctuations are controlled by the parameters of the systems such as the charging energies and Josephson couplings of the Josephson junctions array [2]. Here we present a field-theoretical renormalization group study to find the quantum dissipative phases of lumped superconducting quantum interference device (Squid). The quantum fluctuations of the system are controlled by the externally applied magnetic field and $\alpha (=\text{a ratio of quantum resistance to tunnel junction resistance})$.

In the Squid total current of device is modulated through the applied magnetic flux. Therefore the total current in the Squid is $I = 2I_{c}\sin(\theta)\cos\left(\frac{\Phi}{\Phi_0}\right)$, where $I_{c}$ is the critical current, $\Phi$ is the magnetic flux and $\Phi_0 (=\frac{\Phi}{2\pi})$ is the flux quantum, $\theta$ is the phase of superconducting order parameter. Similarly the Josephson coupling is also changing $E_{J} = 2E_{J0}\cos(\theta)\cos\left(\frac{\Phi}{\Phi_0}\right)$, where $E_{J0}$ is the bare Josephson coupling. So one can think that a lumped Squid system (Fig. 1A) can be described in terms of an array of superconducting quantum dots (SQD) but with a modulated Josephson coupling (Fig. 1B) and critical current [3, 4]. This effective mapping will help us to analyze (analytically) the system in detail. The experimentalist of Ref.3 and Ref.4 have also considered the mesoscopic Squid system as a modulated Josephson junctions array. We have been motivated from these well-accepted experimental findings [3, 4]. The plan of this manuscript is the following. Section (II) contains the analytical derivations and the physical explanation for the occurrence of quantum dissipative phase in the lumped Squid system. Conclusions are presented in section (III).

2. RENORMALIZATION GROUP STUDY FOR QUANTUM DISSIPATION PHASE IN THE MESOSCOPIC LUMPED SQUIDS

We now present the basic dissipation physics of mesoscopic Squid systems. The source of dissipation at very low temperature is due to the appearance of phase slip centers. Phase slip centers are of two kind: one is thermally activated phase slip center valid near to the superconducting transition temperature, well described by LAMH theory [5, 6, 7], and the other, quantum phase slip (QPS) center occurs at $T=0$ K or at very low temperature due to the quantum mechanical tunneling in different metastable states [8]. The most important type of fluctuation which occurs during this QPS process is that phase of the superconducting order parameter is changing by $\pm 2\pi$ at a point in the system and the amplitude of the order parameter vanishes at one point. Appearance of QPS in different low dimensional superconducting systems is a common phenomena, we have cited only those references which are related to our problem [1, 6, 7, 8]. At first we prove the appearance of thermally activated phase slip centres in lumped Squid system. We consider a Squid and use the Ginzburg-Landau theory to illustrate the physics of QPS. A mesoscopic Squid is superconducting ring (nano scale size) with two tunnel junctions. We therefore use the cylindrical polar coordinates. The superconducting order parameter inside the mesoscopic Squid is $\psi(\phi) = \psi_0 e^{in\phi}$, where $\phi$ is the azimuthal angle, $n$ is integer (winding number) and $\psi_0$ is constant. Here the Squid is pierced by the magnetic flux $\Phi$. The vector potential along the tangential direction is $A_\phi = \frac{\Phi}{2\pi R}$ ( $R$ is the radius of the Squid). The free energy corresponding to this wave-function and vector potential is

$$F_s(T) = F_s^0(T) + V_c \frac{\hbar^2}{2m^* R^2} |\Phi - n\Phi_0|^2 + \frac{c_1}{2\mu_0} \Phi^2. \hspace{1cm} (1)$$
First term is the free energy contribution in absence of magnetic flux. $m^*$ is the effective mass of the quantum system and $\mu_0$ is the permeability, $V_i$ is volume of the Squid, $c_1$ is a constant and the last term represents the vacuum magnetic field energy. Free energy is minimum when $\Phi = n\Phi_0$. Depending on the externally applied magnetic flux, system is in one of the metastable minima and a persistent current flows around the mesoscopic Squid to maintain the superconducting state. The system can jump from one metastable minimum to the other minimum to lower the energy. This is the source of dissipation and the decay of persistent current. Such an event corresponds to a change in the winding number (n), hence it is a phase slip. The rate of thermally activated phase slip is $\frac{1}{\tau_i} \sim e^{-\frac{E_b}{kT}}$, where $E_b \sim V_{q}\frac{\hbar^2|\vec{\theta}|^2}{2m^*|\vec{r}|}$. For the bulk Squid, $V_{q}$ is large and hence the appearance of phase slip centers is quite unusual. But for the mesoscopic system of nano-scale size such phase slip centers are likely possible. In the nano-wire, there are many evidences for the presence of thermally activated phase slip centers near the superconducting transition temperature $T_c$. The basic transport properties of low dimensional tunnel junction systems and nano-wire have many similarities [3, 4]. So we also expect the appearance of thermally activated phase slip centers for mesoscopic lumped Squid system. Temperature region close to the transition temperature of lumped Squid system is the region for thermally activated phase slip centers. Here we prove the appearance of QPS in SQD array with modulated Josephson coupling through an analysis of a minimal model. We consider two SQDs are separated by a Josephson junction. These two SQDs are any arbitrary SQDs of the array. Appearance of QPS is an intrinsic phenomenon (at any junction at any instant) of the system. The Hamiltonian of the system is

$$H = \sum_i n_i^2 \frac{2C}{E_i} - E_i|\cos(\frac{\pi\Phi}{\Phi_0})|\sum_i \cos(\theta_{i+1} - \theta_i) \tag{2}$$

where $n_i$ and $\theta_i$ are respectively the Cooper pair density and the superconducting phase of the $i$th dot and $C$ is the capacitance of the junction. The first term of the Hamiltonian present the Coulomb charging energies between the dots and the second term is nothing but the Josephson phase only term with modulated coupling, due to the presence of magnetic flux. We will see that this model is sufficient to capture the appearance of QPS in SQD systems. Hence this Hamiltonian has sufficient merit to capture the low temperature dissipation physics of SQD array. In the continuum limit, the partition function of the system is given by, $Z = \int D\theta(x, \tau) e^{-\frac{S_Q}{\hbar}(x, \tau)}$, where $S_Q = \int dx \int dt \frac{\hbar}{2m}\left[(\partial\theta(x, \tau))^2 + (\partial_\tau\theta(x, \tau))^2\right]$. The action is quadratic in scalar-field $\theta(x, \tau)$, where $\theta(x, \tau)$ is a steady and differentiable field, so one may think that no phase transition can occur for this case. This situation changes drastically in the presence of topological excitations for which $\theta(x, \tau)$ is singular at the center of the topological excitations. So for this type of system, we express the $\theta(x, \tau)$ into two components: $\theta(x, \tau) = \theta_0(x, \tau) + \theta_1(x, \tau)$, where $\theta_0(x, \tau)$ is the contribution from attractive interaction of the system and $\theta_1(x, \tau)$ is the singular part from topological excitations. We consider at any arbitrary time $\tau$, a topological excitation with center at $X(\tau) = (x_0(\tau), \tau_0(\tau))$. The angle measured from the center of topological excitations between the spatial coordinate and the x-axis $\theta_1(x, \tau) = \tan^{-1}\left(\frac{y-x_0}{x-x_0}\right)$. The derivative of the angle is $\frac{\partial\theta_1(x-X(\tau))}{\partial\tau} = \frac{1}{(x-X(\tau))^2}[-(\tau_0 - \tau_0), (x-x_0)]$ which has a singularity at the center of the topological excitation. Finally we get an interesting result when we integrate along an arbitrary curve encircling the topological excitations, $\int_C dx\frac{\partial\theta_1(x-X(\tau))}{\partial\tau} = 2\pi$. So we conclude from our analysis that when a topological excitation is present in the SQD array, the phase difference $\theta$, across the junction of quantum dots jumps by an integer multiple of $2\pi$. This topological excitation is nothing but the QPS in the $x, \tau$ plane. According to the phase voltage Josephson relation, $V_J = \frac{\Delta}{\pi}\frac{\hbar}{2e} (t \ is \ the \ time)$, a voltage drop occurs during this phase slip, which is the source of dissipation. This analysis is valid when $\Phi$ is away from $\frac{\pi}{2}$. Otherwise system is in the superconducting Coulomb blocked phase. Now the problem reduces to finding the quantum dissipation physics of SQD array with modulated Josephson couplings and critical current. There are a few interesting studies, following the prescription of Caldeira and Leggett [11], to uncover the quantum dissipation physics of low dimensional tunnel junctions system [12, 13, 14, 15]. Our starting quantum action is the same with Ref. [12, 13, 14, 15]. We will see the scaling analysis of RG equations derive from this action is sufficient to explain the experimental findings of Ref. [3-4].

$$S_1 = S_0 + \frac{\alpha}{4\pi T} \sum_m \omega_m |\theta_m|^2. \tag{3}$$

Here, $S_1$ is the standard action for the system with tilled wash-board potential [12, 13, 14, 15] to describe the dissipative physics for low dimensional tunnel junction system, $m$ is the Matsubara frequency. Where $S_0 = \int_0^\beta \frac{d\theta(x, \tau)}{d\tau} + V(\theta(\tau)), \alpha = \frac{R_Q}{R_0}$, Matsubara frequency $\omega_m = \frac{2\pi}{\beta} m$ and $R_Q (= 6.45k\Omega)$ is the quantum resistance and $R_0$ is the tunnel junction resistance, $\beta$ is the inverse temperature. Here $V(\theta) = -\frac{\Delta}{\pi}\cos(\frac{\pi\Phi}{\Phi_0}) + \frac{\Delta}{\pi}$. We would like to exploit the renormalization group (RG) calculation for weak potential. Without loss of generality, we do the analysis for $I = 0$ case as finite $I$ only inclined the potential profile. Since we are interested in the low energy excitations, we can ignore the contribution of $|\omega_m|^2$ compare to $|\omega_m|$. So in $S_0$, we only consider
the second term.

\[ S_0 = -\frac{I_e}{2e} |\cos(\frac{\pi \Phi}{\Phi_0})| \int_0^\beta \cos(\theta) d\tau = V_1 \int_0^\beta \cos(\theta) d\tau \]  

We can write the final action as

\[ S_1 = \frac{\alpha}{4\pi} \sum_m |\theta_m|^2 + V_1 \int_0^\beta \cos(\theta) d\tau \]  

The RG equation of the above action to study the low energy excitations is the following:

\[ \frac{dV_1}{dnb} = (1 - \frac{1}{\alpha})V_1. \]  

We have derived the above RG equation following the prescription of Ref. \[10]. Here \( b \) is a number ratio of the two energy scale. The time evolution of the coupling constant is \( V_1(t) = V_1(0)e^{(1-1/\alpha)t} \). We are mainly interested in the low energy theory of the system, suppose we consider the low energy frequency as \( \omega_m \) then the corresponding time is \( t = \ln(\frac{\omega_m}{\omega_1}) \). The coupling constant at maximum time reduces to \( V_1 = V_1(0)(\frac{\omega_m}{\omega_1})^{1-\frac{1}{\alpha}} \). We consider the lowest frequency allowed by the Matsubara allowed frequency quantization, i.e., \( \omega_m = 2\pi T \), so \( V_1(T) \propto T^{1-\frac{1}{\alpha}} \). When we consider a finite system of length, \( L (L > 1) \), then we might argue that the mode \( \theta(k) \) is quantized with \( \omega_1 = \frac{2\pi}{L} \), where \( \omega_1 \) is the lowest frequency and \( v \) is the velocity of low energy excitations. So the effective value of the coupling at the lowest frequency is \( V_1(L) \propto L^{1-\frac{1}{\alpha}} \). We observe that the potential \( V_1 \) increases for \( \alpha > 1 \) and decreases for \( \alpha < 1 \). So \( \alpha = 1 \) is the phase boundary. When \( \alpha > 1 \) owing to the strong dissipation effects particle comes to rest at one of the minima of the potential (local or particle like character), i.e., the system is confined in one of the metastable current carrying state. Hence the system is in the superconducting phase. It is interesting to observe that dissipation favors to stabilize the superconducting phase, \( \alpha < 1 \), implies weak dissipation has no effect on the potential. The phase fluctuation is large around the dots (nonlocal or wave like process). As a result, there is no phase coherent state in the system. Therefore there is no superconducting phase.

We also observe from our study that the applied magnetic flux has no effect on RG equation at the one-loop level. We consider the effect of magnetic flux at the phenomenological level, i.e., we replace \( \alpha \) by \( \alpha' = \alpha|\cos(\frac{\Phi_0}{\Phi_0})| \). When \( \Phi \) is zero or an integer multiple of flux quantum, the flux has no effect on the dissipation physics. For the larger values of \( \Phi \) (small \( \alpha' \)), make the quantum fluctuations in the system large thereby destroying the phase coherence of states. So the higher magnetic field drives the system from the superconducting phase to the insulating phase. This is consistent with the experimental findings \[3, 4\].

The analytical structure of our derive RG equation (Eq. 6) has some similarity with the RG equation of single impurity Luttinger liquid \[18\]. But the initial Hamiltonians of these two problems are quite different. In the strong potential, tunneling between the minima of the potential is very small. In the imaginary time path integral formalism tunneling effect can be described in terms of instanton physics. We will see that the strong coupling physics of our system can be described in terms of tunneling physics \[17, 18, 19\]. In the imaginary time path integral formalism the potential is inverted and therefore the particle can not reside at the maximum of the potential for long time and rolls down to one of the potential minima. It is convenient to characterize the profile of \( \theta \) in terms of its time derivative,

\[ \frac{d\theta(\tau)}{d\tau} = \sum_i e_i h(\tau - \tau_i), \]

where \( h(\tau - \tau_i) \) is the time derivative at time \( \tau \) of one instanton configuration. \( \tau_i \) is the location of the \( i \)th instanton, \( e_i = 1 \) and \( -1 \) are respectively the topological charge of instanton and anti-instanton. Integrated the function \( h \) from \( -\infty \) to \( \infty \), \[6 \int_{-\infty}^{+\infty} d\tau h(\tau) = \theta(\infty) - \theta(-\infty) = 2\pi \]. So the appearance of instanton (anti-instanton) is nothing but the appearance of QPS in the lumped Squid system and it leads to the dissipative phase of the system. Larkin et al. \[9\] have also supported the idea of QPS as a appearance of instanton (anti-instanton). So our system reduce to a neutral system consist of equal number of instanton and anti-instanton. Now our prime task is to present the partition function of the system. After a few steps, we will come to that stage. One can find the expression for \( \theta(\omega) \), after the fourier transform to the both sides of Eq. 7 and that yield

\[ \theta(\omega) = \frac{i}{\omega} \sum_i e_i h(i\omega)e^{i\omega\tau_i} \]

Now we substitute this expression for \( \theta(\omega) \) in the second term of Eq. 3, finally we get this term as \( \sum_{ij} F(\tau_i - \tau_j)e_i e_j \), where \( F(\tau_i - \tau_j) = \frac{2\pi}{\omega} \sum_m \frac{1}{|\omega_m|} \zeta \ln(\tau_i - \tau_j) \). We obtain this expression for very small values of \( \omega (\rightarrow 0) \). So effectively \( F(\tau_i - \tau_j) \) is representing the Coulomb interaction between the instanton and anti-instanton. This term is the main source of dissipation physics of the system. Following the standard prescription of imaginary time path integral formalism, we can write the partition function of the system \[17, 18, 19\].

\[ Z = \sum_{n=0}^{\infty} z^n \sum_{e_i} \int_0^{\beta} d\tau_n \int_0^{\tau_{n-1}} d\tau_{n-1}... \int_0^{\tau_2} d\tau_1 e^{-F(\tau_i - \tau_j)e_i e_j}. \]

Where \( z^n \) is the contribution from instanton, \( z = e^{-S_{\text{inst}}} (S_{\text{inst}} \approx \sqrt{\hbar TC} ) \). We may also write this expression as

\[ Z = \sum_{n=0}^{\infty} z^n \sum_{e_i} \int_0^{\beta} d\tau_n \int_0^{\tau_{n-1}} d\tau_{n-1}... \int_0^{\tau_2} d\tau_1 \]
\[
e^{-\frac{1}{\alpha T} \sum_q |q_n|^2 + \frac{1}{\alpha} \sum_i \epsilon_i q_i (\tau_i)}.
\]

Here \(q_\tau\) is the auxiliary field which arises during the functional integral. We again introduce \(\alpha'\) instead of \(\alpha\) to consider the effect of applied magnetic field. After extensive calculation, we get the partition function,

\[
Z = \int Dq(\tau) e^{-\frac{1}{\alpha' T} \sum_i \epsilon_i q_i (\tau_i) - \frac{1}{\alpha'} \sum_q |q_n|^2 + 2\pi \int_0^l dq \cos(q(\tau))}.
\]

\(q(\tau)\) is the auxiliary field. Following the method of previous paragraphs, we finally obtain the RG equation

\[
\frac{dz}{d\ln b} = (1 - \alpha') z
\]

The analytical structure of the quantum action and the RG equation is the same with weak potential, it implies the following mappings \(\alpha' \leftrightarrow \frac{1}{\alpha}\) and \(V_1 \leftrightarrow z\). Hence there is a duality in this problem between the weak and strong potential. One can also find the analytical expression for the variation of \(z\) as a function of temperature and length as we derive for weak coupling case, the only change being \(\alpha'\) replaced by \(1/\alpha'\). Fugacity depends on temperature and length scale as, \(z(T) \propto T^{\alpha - 1}, z(L) \propto L^{1 - \alpha'}.\)

In this complicated system, we estimate the behavior of resistance from the behavior of fugacity. It is expected to scale as \(z^2\), (because the major contribution of voltage/resistance occurs from the second order expansion of partition function, i.e., from the square of fugacity). In our study, resistance is evolving due to dissipation effect at very, low temperature (less than the superconducting Coulomb blocked temperature). According to our calculations, for large dissipation (\(\alpha' > 1\)), \(R(T) \propto R_{Q} T^{\beta_1}, \beta_1 > 0\). Therefore at very low temperature, the system shows the superconducting behavior. When \(\alpha' < 1\), the resistance of system \(R(T) \propto R_{Q} T^{-\beta_2}, \beta_2 > 0\). So at very low temperature, the resistance of the system shows Kondo-like divergence behavior and the system is in the insulating phase. According to our calculations, for large dissipation (\(\alpha' > 1\)), \(R(T) \propto R_{Q} L^{-\gamma_1}, \gamma_1 > 0\). Therefore the longer array system shows the less resistive state than shorter array in the superconducting phase of the system. When \(\alpha' < 1\), the resistance of system \(R(T) \propto R_{Q} L^{-\gamma_2}\), where \(\gamma_2 > 0\) (\(\beta_1, \beta_2, \gamma_1\) and \(\gamma_2\) are independent numbers). So the resistance at the insulating state is larger for longer array system than the shorter one. So we find the dual behavior of the resistance (voltage) for lower and higher values of magnetic field. When \(\alpha' = 1\), i.e., \(\Phi = (\Phi_0/\pi) \cos^{-1}(1/\alpha)\), the system has no length scale dependence superconductor-insulator transition at very low temperature. This is the critical behavior of system for a specific value of magnetic field. These theoretical findings are consistent with the experimental observations \[\[.\]

Fig. 2 shows the variation of resistance with temperature. At higher temperature, larger than \(T_c\). SQD array system is in the normal phase and the tunneling between the dots is the sequential tunneling (one after another, tunneling of Cooper pairs). We have shown in Ref. \[20\], that superconductivity occurs due to the cotunneling effect (higher order tunneling of Cooper pairs, virtual process). A presence of finite resistance at the superconducting phase (between \(T_1\) and \(T_2\) ) due to the dissipation effect and also for the presence of finite tunneling conductance (i.e., the finite resistance). Low resistance superconducting phase or insulating phase occurs at very low temperature, smaller than the superconducting Coulomb blocked temperature. Low resistance superconducting phase or insulating phase occurs at very low temperature (it will occur for few milliKelvin), smaller than the superconducting Coulomb blocked temperature. In experiment, they have measured up to 50 mK, so they have not found the decaying tendency of resistance at very low temperature. We have not considered the the classical phase (\(E_J >> E_C\) ) of the system. In this phase, one can also obtain the dissipative phase with phase slip centers, but one can loses the informations of intermediate quantum phases for the system \[20\]. Chakarvarty et al. \[13\] and Larkin et al. \[6\] had studied QPS for the classical phase. Currently Fristual et al. \[21\] have done some interesting work on collective Cooper pair transport in the insulating state of one and two dimensional Josephson junctions array. They have studied the current-voltage characteristics revealing thermally activated conductivity at small voltages and threshold voltage depinning. Our analytical approach to study the one dimensional mesoscopic squid system is quite different from them \[21\].

We have studied the clean system. Here we explain the effect of impurities in superconducting dots or in the tunnel barrier. Nonmagnetic impurities were found not to affect the Josephson supercurrent. This is in agreement with Anderson’s theorem \[22\] of dirty superconductor. In presence of paramagnetic impurities, there is an exchange interaction between the spin of conduction electrons with the magnetic impurity spin. The spin of the magnetic impurity polarize the spin of electrons and interferes with their tendency for pair formation in the singlet state. As one increase the probability (\(\Gamma\)) of scattering with spin flip. For highvalues of \(\Gamma\), the system enters into the gapless region and above a critical value ( \(\Gamma_c = \Delta/(0,0)\)), \(\Delta(0,0)\) being the order parameter for the superconductor at zero temperature in absence of impurity.) the superconductivity is destroyed \[23\]. For such situation, our system is in the Coulomb blocked insulating phase. The Josephson supercurrent is also zero for a clean dot when the applied magnetic flux of the system is half-integer multiple of flux quantum. At those values of magnetic flux, our system is in the Coulomb blocked insulating phase.
3. CONCLUSIONS

We have observed the length scale dependent superconductor-insulator quantum phase transition and the dual behaviour of the system for smaller and larger values of magnetic field, in a one dimensional mesoscopic lumped Squid systems. We have also observed a critical behaviour where the resistance is independent of length at a critical magnetic field. We find that weak and strong potential results are self dual. Our theoretical findings have experimental relevance of lumped Squids system [3, 4].

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FIG. 1: A. Schematic diagram of one dimensional array of small capacitance dc Squids. Each plaque is a squid with two Josephson junctions marked by the cross. Circle with plus sign represent the applied magnetic flux Φ. B. Equivalent representation of system A, where the dots are connected through tunnel junctions and the Josephson couplings of this system is tunable due to presence of magnetic flux Φ.

FIG. 2: Schematic phase diagram, variation of resistance with temperature. ST (see the text) is sequential tunneling regime and CT (see the text) is the Co-tunneling regime. $T_2$ (~ few Kelvin) is higher than superconducting transition temperature and $T_1$ (~ few milli Kelvin ) is the temperature region much less than the superconducting Coulomb blocked transition temperature.