Conversion electrons used to monitor the energy scale of electron spectrometer near tritium endpoint – a simulation study

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Abstract
Measurements of the endpoint region of the tritium $\beta$–decay spectrum provides good possibility to determine neutrino mass. This, however, needs a perfect monitoring of the spectrometer energy scale. A parallel measurement of electron line of known energy – in particular the $^{83m}$Kr conversion K-line – may serve well to this purpose. The $^{83}$Rb decaying to $^{83m}$Kr seems to be a very suitable radioactive source due to its half-life of 86.2 day. In this work, we determine the amount of $^{83}$Rb which is necessary for a successful monitoring.

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1 Introduction
In any spectroscopic experiment, good knowledge of spectrometer energy scale is the condition sine qua non for a successful performance. First, we must assure that the energy is really that one which we think it is (as the instruments tell us it is); this is called calibration. Second, this condition must be fulfilled during the whole duration of the measurement; this is the aim of the energy monitoring.

One of the most reliable ways of monitoring is to measure some spectrum line (preferably a well separated one) of a well known energy. When measuring the tritium spectrum by a “main” spectrometer it is reasonable to use a smaller monitoring spectrometer – measuring the K-conversion line of $^{83m}$Kr – connected to the same controls as the “main” one. As the source, $^{83}$Rb decaying to $^{83m}$Kr may be utilized. (Note that the idea to calibrate and/or monitor the tritium spectrometer using $^{83m}$Kr is nothing new – see e.g. [1] and references cited therein.) Another possibility might be to measure photoelectrons from Co initiated by $^{241}$Am 26 keV $\gamma$-rays.

As stated before, we need the stable energy scale. In other words, if any change appeared in it, we need to recognize it immediately. In this work we determine how much krypton activity (and then, how much $^{83}$Rb activity, since the krypton and rubidium decays are in balance with each other) is necessary to fulfill such a task in a real situation. We use the data which are expected for the prepared experiment KATRIN and are published in [2, 3]. We simulate $^{83m}$Kr conversion lines with intentionally wrong (mutually shifted) energies and then apply statistical tests to find the minimum shift which makes the spectra distinguishable. Repeating this for various activities and/or exposure times we get relations which enable one to optimize the monitoring.

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2 Method

The shape of the conversion line is described by a Lorentzian curve

\[ K(E) = \frac{1}{2\pi} \frac{\Gamma}{(E - E_{\text{peak}})^2 + (\Gamma/2)^2}, \]

where \( E_{\text{peak}} \) and \( \Gamma \) are the line energy and FWHM, respectively.

This line shape is measured by an integrating spectrometer, the response function of which is

\[
RpF(E, qU) = \begin{cases} 
0 & 0 \leq E \leq qU \\
1 - \sqrt{1 - \frac{E - qU}{\beta_S}} & qU \leq E \leq qU + \Delta E \\
1 & qU + \Delta E \leq E 
\end{cases}
\]

where \( B_S, B_A \) are the spectrometer magnetic field settings at the source and in the analyzing plane, and \( \Delta E \) is the spectrometer resolution. The registered spectrum is then given by summing

\[ S(E) = \int_{0}^{\infty} RpF(T, E) K(T) \, dT. \]

This integral may be partially simplified

\[ S(E) = \int_{E}^{E+\Delta E} RpF(T, E) K(T) \, dT + \frac{1}{2} - \frac{1}{\pi} \arctan \left( \frac{2(E + \Delta E - E_{\text{peak}})}{\Gamma} \right). \]

The first part must be calculated numerically. However, the middle part of the response function, Eq. 2, may be well approximated by a straight line. For such a case we get an analytical formula

\[
\int_{E}^{E+\Delta E} RpF(T, E) K(T) \, dT = \frac{1}{2\pi\Delta E} \times \left\{ \frac{\Gamma}{2} \ln \left( \frac{(E - E_{\text{peak}} + \Delta E)^2 + (\Gamma/2)^2}{(E - E_{\text{peak}})^2 + (\Gamma/2)^2} \right) \right.
\]

\[ -2(E - E_{\text{peak}}) \left[ \arctan \left( \frac{2(E - E_{\text{peak}} + \Delta E)}{\Gamma} \right) - \arctan \left( \frac{2(E - E_{\text{peak}})}{\Gamma} \right) \right]. \]

Using both the numerical integration and this formula, we can simply check the precision of our numerical procedures. We did so for the values of the \(^{83}\text{m}\text{Kr} \) conversion K-line \((E_{\text{peak}}=17823.8 \text{ eV}, \Gamma=2.8 \text{ eV})\)[4], the spectrometer resolution of \( \Delta E=2 \text{ eV} \), and set of 5 values of \( E \) (17810 to 17830 eV with the step of 5 eV). Using the simplest possible integration method – summation with the step of 0.01 eV – the results agreed with the analytical ones of Eq. 5 up to more than 5 digits.

3 Calculations

To get reasonable estimates of the sensitivity we performed a lot of simulations of the \(^{83}\text{m}\text{Kr} \) K-line spectra. Moreover, the simulations were done assuming similar conditions under which such a spectrum was really measured[5] in June 2005 in Mainz. This assures that the results might be directly applicable in practice.

In our simulations we did not cover the whole measured energy interval [5] but the most sensitive part only, usually between 17819 and 17825 eV. Since the linewidth fitted in [5] was
\( \Gamma_{fit} = 3.08 \text{ eV} \), we repeated the simulation for both this value and the value \( \Gamma = 2.8 \text{ eV} \) of [4]. (In all cases, the realistic magnetic fields setting of \( B_S/B_A \approx 8000 \) was used.) The difference between the two sets of results turned out to be negligible. In the following, then, we present the results obtained with \( \Gamma = 2.8 \text{ eV} \) only.

In one run, always ten spectra were simulated having identical amplitudes and linewidths but mutually shifted by a fixed energy interval \( \delta E \). An example of such simulated spectrum is at the Fig. 1. Then the statistical tests [6] were applied to the spectra set. (We remind the reader here that those tests compare every spectrum with each other to verify whether they were measured under the same conditions.) We seek the smallest \( \delta E \) for which the tests are able to distinguish two neighbouring spectra. Thus we repeat the above procedure with decreasing \( \delta E \) until the neighbouring spectra cease to be distinguishable.

We accept the spectra as distinguishable if – in the particular run – most of the corresponding [6] \( \chi^2 \)-values (per one d.o.f.) are greater than 1.5 and/or most of the corresponding [6] Student-test-values are in absolute value greater than 1. (The rather high value of \( \chi^2 = 1.5 \) was chosen due to the following reason: having \( \chi^2 \geq 1.5 \), we can be almost sure that the spectra compared are really different, i.e. the difference is not due to statistics.)

Here is one thing which must be kept in mind. The tests [6] check only “measurements were or were not done under the same conditions”. So a change of detection efficiency would also lead to the ‘not same conditions’ signal thus imitating a spectrum shift. Fortunately, the realization of the tests (the software) contains a branch performing a normalization of the spectra being compared to each other. If the ‘not same conditions’ signal is caused by a detection efficiency change, this normalization releases it while if the cause is an energy scale change, it does not. This allows us to discern the false and true effects. And this is also the reason why we included several low-energy points (four leftmost ones in Fig. 1) into simulations. They are out of the region sensitive to energy shift but they are sensitive to amplitude change which strongly helps to the normalization procedure.

![Figure 1: An example of simulated K-line of \(^{83m}\)Kr. The \(^{83m}\)Kr activity of 190 kBq and exposure of 10 s per point is assumed. Analyzing energy is the quantity \( qU \) of Eq. (2). The reason to include the four leftmost points is given later in the text.](image)
4 Results

First, we simulated the situation corresponding to the measured spectrum [5], i.e. the \(^{83m}\)Kr activity \(A=1.9 \ \text{kBq}\) and exposure of 350 s per point. The 28 points in the most sensitive region (in particular from 17819 to 17824.75 eV with the step of 0.25 eV) were used.

Under these conditions, we are able to distinguish the spectra which were shifted by about 100 meV or more which is not too satisfactory. Fortunately, there are possibilities to improve it.

First, we were increasing the activity up to hundredfold larger than the original one. As expected, smaller and smaller shifts could be resolved. The results are given in Fig. 2. The recognizable shift of less than 10 meV is excellent. However, the overall measurement time of about \(2\frac{3}{4}\) hour is rather long for practical monitoring.

![Figure 2: Dependence of the minimum detectable spectra shift on the \(^{83m}\)Kr activity.](image)

As the next step we performed the simulations for various (shorter) exposures. As for the activities, we used 95 and 190 kBq; this corresponded to 50\(\times\) and 100\(\times\), respectively, the activity [5] and is realizable without difficulties. The results are in Fig. 3. It turns out that quite satisfying sensitivity to shift of about 30 meV is reached after the exposure of some 10 minutes. A longer exposure brings very little gain.

Finally, we tried to reduce the number of measured points in a hope to further reduce the necessary measurement time. We took only 15 points instead of 28 in the same region as before (i.e. we applied the step of 0.5 eV) and repeated all the previous simulations. The dependence on activity (Fig. 2) practically did not change. For the exposure dependence, however, approximately twofold exposure per point was needed to achieve the same sensitivity as before. Then the overall time needed to measure the spectrum remains the same.

5 Conclusions

The measurement of \(^{83m}\)Kr conversion K32–line is a very good method to monitor the energy scale of electron spectrometer near the tritium endpoint. The necessary activity of \(^{83}\)Rb is reasonable and there are no difficulties to produce it. However, for practical utilization there
still remains the question of long–term stability of the $^{83}$Rb/$^{83m}$Kr composition \[5, 7\] (i.e. of spontaneous releasing of the krypton into vacuum) to be solved.

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**References**

[1] R.G.H. Robertson, T.J. Bowles, G.J. Stephenson, Jr., D.L. Wark, J.F. Wilkerson: Phys. Rev. Lett. 67 (1991) 957.

[2] A. Osipowicz et al. (KATRIN Collaboration), \texttt{arXiv:hep-ex/0109033} (2001).

[3] KATRIN Collaboration, “KATRIN Design Report”, Report FZKA 7090, NPI Řež EXP-01/2005, MS-KP-0501. \texttt{http://www-ik.fzk.de/katrin/publications/documents/FZKA7090.pdf}

[4] A. Picard et al.: Z. Physik A342 (1992) 71.

[5] J. Kašpar: private communication.

[6] O. Dragoun, M. Ryšavý, A. Špalek: Nucl. Instr. Meth. A391 (1997) 345.

[7] D. Vénos, A. Špalek, O. Lebeda, M. Fišer: Appl. Radiat. Isot. 63 (2005) 323.