Probability Density Estimation Based Imitation Learning

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Abstract

Imitation Learning (IL) is an effective learning paradigm exploiting the interactions between agents and environments. It does not require explicit reward signals and instead tries to recover desired policies using expert demonstrations. In general, IL methods can be categorized into Behavioral Cloning (BC) and Inverse Reinforcement Learning (IRL). In this work, a novel reward function based on probability density estimation is proposed for IRL, which can significantly reduce the complexity of existing IRL methods. Furthermore, we prove that the theoretically optimal policy derived from our reward function is identical to the expert policy as long as it is deterministic. Consequently, an IRL problem can be gracefully transformed into a probability density estimation problem. Based on the proposed reward function, we present a “watch-try-learn” style framework named Probability Density Estimation Based Imitation Learning (PDEIL), which can work in both discrete and continuous action spaces. Finally, comprehensive experiments in the Gym environment show that PDEIL is much more efficient than existing algorithms in recovering rewards close to the ground truth.

1 Introduction

Recently, Reinforcement Learning (RL) has made remarkable success in many tasks, such as Go [Silver et al., 2017], Atari games [Mnih et al., 2013] and robot control [Abbeel and Ng, 2004; Andrychowicz et al., 2020]. In an RL system, in addition to the agent and the environment, there are four key elements: policy, reward signal, value function, and model [Sutton et al., 1998], among which the reward signal plays a key role in directly determining the overall performance of the RL system. More specifically, the reward signal defines the goal of the RL task, and guides the learning agent towards the desirable direction. However, in some real-world problems, the reward signal is difficult to receive or to measure explicitly. In such cases, alternative approaches such as Imitation Learning (IL) are required.

In IL, the learning agent receives demonstrations from an expert, and the goal is to recover a desired policy using the expert demonstrations through interactions with the environment. In general, IL methods can be categorized as Behavioral Cloning (BC) [Pomerleau, 1991; Ross and Bagnell, 2010; Bagnell et al., 2007] and Inverse Reinforcement Learning (IRL) [Russell, 1998; Ng et al., 2000]. In BC, the agent tries to mimic the expert’s action in each state, with the aim to match the expert as closely as possible. By contrast, IRL methods try to recover a reward function based on the demonstrations of the expert, and then use RL algorithms to train an agent based on the recovered reward function. In practice, BC methods are usually used to derive an initial policy for RL algorithms [Nagabandi et al., 2018; Rajeswaran et al., 2017], while IRL methods employ an iterative process alternating between reward recovery and RL.

However, BC methods have two major flaws. First, the prediction error may accumulate and agents in BC cannot recover the true action once mistakes occur. Second, BC methods are supervised learning methods and require a large amount of data for training. Furthermore, supervised learning methods assume that the demonstrations are i.i.d, which is unlikely to be the case in real-world tasks.

Most previous IRL methods are model-based and need to call a Markov decision process (MDP) solver multiple times during learning. Meanwhile, IRL methods are essentially cost function learning methods [Ng et al., 2000; Abbeel and Ng, 2004; Ziebart et al., 2008], aiming to minimize the distance between the expert and the learning agent. For example, Adversarial IL [Ho and Ermon, 2016] uses the KL divergence to estimate the cost function, while Primal Wasserstein Imitation Learning (PWIL) [Dadashi et al., 2020] uses the upper bound of the Wasserstein distance to estimate the cost, and Random Expert Distillation (RED) [Wang et al., 2019] uses the expert policy support estimation to estimate the cost.

In this paper, we focus on how to recover the reward function in IRL. The main contribution is that we propose a concise method to recover the reward function using expert demonstrations based on probability density estimation. Instead of being restricted by the concept of cost, we propose to recover the reward function directly, and the form of the reward function is also simpler and easier to calculate than previous IRL methods. We prove that, in theory, the

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Expert support estimation is another direction in IRL. The key idea is to encourage the agent to stay within the state-action support of the expert and several reward functions have been proposed, such as Soft Q Imitation Learning (SQIL) [Reddy et al., 2019], RED, Disagreement-Regularized Imitation Learning (DRIL) [Brantley et al., 2019], and PWIL. SQIL features a binary reward function, according to which when the agent selects an action in a state endorsed by the expert, the agent receive a reward of +1. Otherwise, the agent receive a reward of 0. RED uses a neural network to estimate the state-action support of the expert. DRIL relies on the variance among an ensemble of BC models to estimate the state-action support, and constructs a reward function based on the distance to the support and the KL divergence among the BC models. PWIL employs a “pop-outs” trick to enforce agent to stay within the expert’s support. In some sense, our method is similar to these methods as it employs a probability model to estimate the expert support as a component of the reward function.

4 Methodology
The objective of our work is to design a reward function that can make the resulting optimal policy equal to the expert policy. In this section, we first present the structure of our reward function, and then investigate why our reward function can make the optimal policy equal to the expert policy. Furthermore, we propose a revised version of the original reward function that can overcome its potential issue. Finally, the PDEIL algorithm that employs the reward function is introduced.

4.1 Reward Based on Probability Density Estimation
In traditional RL, the goal of the agent is to seek a policy that maximizes $J(\pi)$ in Equation (1). However, in an MDP/R environment, the agent cannot receive reward signals from the interactions with the environment. To accommodate this lack of reward signals, we need to design a reward function based on expert demonstrations. If the reward function can guarantee in theory that the corresponding optimal policy is equal to the expert policy, we can expect to recover the expert policy using an RL algorithm.

**Theorem 1.** Assume $\pi_e$ is a deterministic policy, then:

$$\forall \pi, \langle \pi_e(a|s), \pi_e(a|s) \rangle \geq \langle \pi(a|s), \pi(a|s) \rangle,$$

where $\langle \cdot, \cdot \rangle$ is the inner product; $\langle \pi(a|s), \pi(a|s) \rangle = \int_a \pi^2(a|s) da$ for continuous action spaces and $\langle \pi(a|s), \pi(a|s) \rangle = \sum_a \pi^2(a|s)$ for discrete action spaces. Moreover, if a policy $\pi$ becomes more deterministic, the value of $\langle \pi(a|s), \pi(a|s) \rangle$ will also increase, and $\langle \pi(a|s), \pi(a|s) \rangle$ can be used to measure the stochasticity of a policy (i.e., similar to the entropy of a policy [Haarnoja et al., 2018]).

**Proof.** When the action space is discrete, and $\pi_e$ is a deterministic policy, the expert agent only selects an action with probability 1 for each state, then:

$$\langle \pi_e, \pi_e \rangle = 1,$$
When the action space is continuous, the probability density we have:

\[ \pi \]

for all actions:

\[ 0 \leq \pi(a|s) \leq 1, \]

then:

\[ \langle \pi, \pi \rangle \leq \sum_{a \in A} \pi(a|s) = 1, \]

we have:

\[ \forall \pi, \langle \pi_e, \pi_e \rangle \geq \langle \pi, \pi \rangle \]

When the action space is continuous, the probability density function of a deterministic policy is a shifted Dirac function \((\delta(a-a_0))\) [Lillicrap et al., 2015]. Note that \( \int \pi_e^2(a|s)da \) is not integrable when \( \pi_e(a|s) = \delta(a-a_0) \), and \( \pi_e, \pi_e \rangle \) goes to infinity, so intuitively, \( \langle \pi_e, \pi_e \rangle \geq \langle \pi, \pi \rangle \).

Table 1 is an example of Theorem 1 when the action space is discrete. In Table 1, \( \langle \pi_1(a|s), \pi_1(a|s) \rangle = 1, \langle \pi_2(a|s), \pi_2(a|s) \rangle = \frac{1}{3} \). Since \( \pi_1 \) is a deterministic policy (it only selects action \( a_3 \)), and \( \pi_2 \) is a uniform random policy, therefore \( \langle \pi_1(a|s), \pi_1(a|s) \rangle \geq \langle \pi_2(a|s), \pi_2(a|s) \rangle \).

Table 1: An example of two different policies on the same state with a discrete action space.

|      | \( a_1 \) | \( a_2 \) | \( a_3 \) |
|------|----------|----------|----------|
| \( \pi_1(a|s) \) | 0        | 0        | 1        |
| \( \pi_2(a|s) \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) |

Figure 1 gives an example of Theorem 1 when the action space is continuous. It shows two alternatives to approach the shifted Dirac function: uniform distributions as Figure 1(a), and triangle distributions as Figure 1(b). When the height of the rectangle or triangle goes to infinity, the probability density function approaches \( \delta(a-a_0) \). Meanwhile, the higher the rectangle or triangle, the more deterministic the policy is. If we calculate the inner product of the policies in Figure 1, it is clear that more deterministic policies have greater inner product values.

Figure 1: An example of different policies on the same state with a continuous action space. In uniform policies, \( \langle \pi_1, \pi_1 \rangle = 0.5, \langle \pi_2, \pi_2 \rangle = 2, \langle \pi_3, \pi_3 \rangle = 1, \langle \pi_4, \pi_4 \rangle < \langle \pi_2, \pi_2 \rangle < \langle \pi_3, \pi_3 \rangle \); in triangle policies \( \langle \pi_1, \pi_1 \rangle = 0.5, \langle \pi_2, \pi_2 \rangle = \frac{2}{3}, \langle \pi_3, \pi_3 \rangle = \frac{1}{3}, \langle \pi_4, \pi_4 \rangle < \langle \pi_2, \pi_2 \rangle < \langle \pi_3, \pi_3 \rangle \).

Theorem 2. Assume:

\[ r(s, a) = \frac{\rho_s^{\pi_e}(s)}{\rho_s^*(s)} \pi_e(a|s) \]

\[ = \frac{\rho_s^{\pi_e}(s, a)}{\rho_s^*(s)}, \]

when the expert policy is a deterministic policy (i.e., Equation (2) is satisfied), the optimal policy \( \pi_e \) is identical to the expert policy under the optimal objective of Equation (1).

Proof. Since Equation (3), then:

\[ J(\pi) = \int_S \int_A \rho_s^*(s) \pi(a|s) r(s, a)da \]

\[ = \int_S \rho_s^*(s) \int_A \pi(a|s) \rho_s^{\pi_e}(s) \pi_e(a|s)da \]

\[ = \int_S \rho_s^*(s) \int_A \pi(a|s) \pi_e(a|s)da \]

\[ = \int_S \rho_s^*(s) \int_A \pi_e(a|s) \pi_e(a|s)da \]

\[ = \int_S \rho_s^*(s) \int_A \pi_e(a|s) \pi_e(a|s)da \]

Given the fact of Cauchy Inequality:

\[ (\int f(x)g(x)dx)^2 \leq \int f^2(x)dx \int g^2(x)dx \]

then:

\[ (\int_A \pi(a|s) \pi_e(a|s)da)^2 \leq \int_A \pi^2(a|s)da \int_A \pi_e^2(a|s)da \]

\[ \exists \text{ Equation (2), then:} \]

\[ (\int_A \pi(a|s) \pi_e(a|s)da)^2 \leq \int_A \pi^2(a|s)da \int_A \pi_e^2(a|s)da \]

\[ \leq (\int_A \pi_e^2(a|s)da)^2 \]

then:

\[ J(\pi) \leq J(\pi_e), \pi_e = \pi_e \]

According to Theorem 2, we can construct the reward function as Equation (3), and once the optimal policy based on this reward function is found, we can recover the expert policy. Since the distribution of population is not known \( a \) priori, \( \rho_s^{\pi_e}(s, a) \) and \( \rho_s^*(s) \) cannot be computed directly. The most common and intuitive solution is to estimate the two probability densities from corresponding samples. Consequently, the practical reward function can be written as:

\[ r(s, a) = \hat{\rho}_s^{\pi_e}(s, a), \]

(4)

where \( \hat{\rho}_s^{\pi_e}(s, a) \) can be estimated using the demonstrations of the expert, and \( \rho_s^*(s) \) can be estimated through the agent’s interactions with the environment. However, the reward function in the original form of Equation (4) has a major defect, which we will discuss in the following section.
4.2 Misleading Reward

The reward function in the form of Equation (4) has a potential issue, referred to as the misleading reward problem. It means that the agent relying on this reward function may get extremely wrong rewards in some cases. This problem happens when the agent reaches a state that it never encountered before. When the agent explores such states, where the values of $\hat{\rho}_s(s)$ are close to zero, the estimated rewards in these states are likely to have a large variance among different actions. Consequently, the agent will receive very high reward signals in these states compared with other ordinary states. These wrong rewards can inevitably mislead the agent, and make the RL algorithm fail to reach its intended target.

The cause of this misleading reward problem is the estimation error of $\hat{\rho}_s(s)$. When we use certain states as the samples to estimate the probability density, the probability density of other states may be estimated to be close to 0. Although this problem can be partially alleviated by increasing the number of states used to estimate $\hat{\rho}_s(s)$, more interactions will also be required, which is not sample-efficient.

To solve this problem, we make a trade-off between bias and variance in estimating the reward:

$$r(s, a) = \frac{2\hat{\rho}_{e,s}^e(s, a)}{\hat{\rho}_s^e(s) + \hat{\rho}_s^e(s)}$$

(5)

where $\alpha + \beta = 1$ and $0 \leq \alpha \leq 1$. The coefficient $\alpha$ plays the role of a variance controller: when $\alpha$ is close to 0, the estimator has high variance and low bias; when $\alpha$ is close to 1, the estimator has high bias and low variance.

Intuitively, $\alpha = 0.5$ indicates a reasonable balance between bias and variance, and a revised reward function is:

$$r(s, a) = \frac{2\hat{\rho}_{e,s}^e(s, a)}{\hat{\rho}_s^e(s) + \hat{\rho}_s^e(s)}$$

(6)

Figure 2 gives an illustration of the comparison between the reward functions in the form of Equation (4) and Equation (6). In Figure 2(b), we select two states where $\pi_e(a| s_1) = \pi_e(a| s_2)$ (the blue dot line), $\rho_\pi^e(s_1) = 1$, $\rho_\pi^e(s_2) = 0.1$, $\rho_\pi^e(s_2) = 0.05$.  

4.3 Algorithm

We present a novel IL algorithm based on the reward function in Equation (5) and the pseudo code of the algorithm is shown in Algorithm 1.

The framework of PDEIL consists of three major components, watching, trying and learning. In the watching part, the agent watches expert demonstrations, and uses these demonstrations to train $\hat{\rho}_{e,s}^e$ and $\hat{\rho}_s^e$. In the trying part, the agent interacts with the environment for some steps to make better understanding of the environment as well as to train and
In the learning part, the agent uses an RL algorithm to improve itself by further approximating the expert. In a complete training process, our agent watches once and iteratively performs the trying and learning operations several times as necessary, similar to the framework in [Zhou et al., 2019].

Probability density estimation is a classical problem that has been extensively investigated in statistics. For a discrete random variable, the most common way to estimate its probability is using a frequency table representing its distribution. For a continuous random variable, there are two different ways to estimate its probability density: the parametric density estimation and the nonparametric density estimation. Parametric density estimation methods model the overall distribution as a certain distribution family, such as Gaussian distributions, and the key is to determine the parameters of the distribution. Nonparametric density estimation methods are model-free and one of the most commonly used methods is the kernel density estimation. For a mixed random variable, such as $(s, a)$ where $s$ is continuous and $a$ is discrete, given the fact of $p(s, a) = p(a|s)p(s)$, this joint distribution estimation problem can be transformed into a continuous distribution estimation problem and a conditional probability estimation problem. For the conditional probability estimation, we can regard it as a classification task, and, for instance, use an SVM [Noble, 2006] classifier or other classifier methods to estimate $p(a|s)$.

5 Experiments

In the experimental studies, we aim to answer the following questions:

1. How does PDEIL perform in different environments?
2. Is PDEIL more efficient than other RL algorithms?
3. Is the recovered reward by PDEIL close to the ground truth reward?
4. Does the misleading reward problem really exist?

Our experiments were conducted in two environments in Gym, which is an open source platform for studying reinforcement learning algorithms. We chose two classical control environments, CartPole and Pendulum (see Figure 3), where CartPole is a discrete action space environment and Pendulum is a continuous action space environment. To evaluate PDEIL, we hid the original reward signals of these two environments during training. For CartPole, two Gaussian models were used to estimate $\rho^e_a$ and $\rho^e_s$ and an SVM model was used to estimate $\pi_e(a|s)$. PPO [Schulman et al., 2017] was used to update the policy in the learning steps. For Pendulum, we used three Gaussian models to estimate $\rho^{e,a}_s$, $\rho^{e}_a$, and $\rho^{e}_s$, while SAC [Haarnoja et al., 2018] was used to update the policy in the learning steps. Furthermore, we set $N = 100$, $T = 1000$ in both environments with $L = 10$ and 5000 on CartPole and Pendulum, respectively, according to some preliminary trials.

To answer Question 1, we applied PDEIL with various expert demonstrations with $\alpha = 0.5$. The performance of PDEIL in the two environments is shown in Figure 4. The results clearly indicate that PDEIL can recover desired policies that are reasonably close to the expert policies with a small amount of expert demonstrations in both discrete and continuous action spaces. We also find that PDEIL may occasionally experience some slight stability issue. For example, the performance of PDEIL with 5 episodes of expert demonstrations on Pendulum (the blue line in Figure 4(b)) was a bit fluctuated. We argued for that there are two possible reasons: i) the estimated reward function is biased when the agent is learning; ii) the optimization process for the neural network has some inherent instability.

To answer Question 2, we conducted extensive comparison among PDEIL, GAIL and BC algorithms with the same expert demonstrations. The trade-off parameter $\alpha$ was also fixed to 0.5 while the number of episodes of expert demonstrations was varied among 1, 2 and 5. In Figure 5, it is obvious that PDEIL is much more efficient than GAIL and BC. Although it seems that GAIL and PDEIL have a similar efficiency on CartPole, PDEIL is more stable and GAIL requires many more interactions with the environment in each learning step. Furthermore, PDEIL uses the Gaussian model for reward estimation while GAIL uses a neural network, which is more complicated and expensive in the training steps.

To answer Question 3, we collected the recovered rewards and the ground truth rewards from the original environment in the trying steps in PDEIL. We chose the Pendulum environment as the example for the sake of visual demonstration, as its ground truth reward is continuous. Each round of the experiment had 100 epochs from which the 13th to 16th epochs were picked as examples for illustration in Figure 6. It is clear that the correlation between the recovered reward and the ground truth reward gets increasingly stronger, which means that our proposed reward function can be expected to guide the RL algorithm to learn a competent policy.

To answer Question 4, in addition to the theoretical analysis in Section 4.2, we conducted further empirical study as
shown in Figure 7. The introduction of the trade-off parameter $\alpha$ is to control the variance of estimated reward. When $\alpha = 0$, we use the original reward function in Equation 4, and the misleading reward problem may occur. For example, the agent using the original reward function in the Pendulum environment (red line in Figure 7(b)) had a poor performance, which implied that the misleading reward problem occurred.

6 Conclusion and Discussion

In this work, we proposed a brand-new reward function in the scenario of IRL, which has a concise and indicative form. We also proposed an algorithm called PDEIL based on this reward function, featuring a "watch-try-learn" style. In PDEIL, to recover the reward function, the agent watches the expert demonstrations and performs interactions with the environment, and uses RL algorithms to update the policy.

It is expected that our work may reveal a new perspective for IRL by transforming the original IRL problem into a density estimation problem. We can prove that, with a perfect probability density estimator, the corresponding optimal policy is identical to the expert policy as long as it is deterministic. However, constructing a good probability density estimator can be challenging in some cases, for example, when the state is of high dimensionality (e.g., an image). Consequently, further enhancing the efficacy of PDEIL with more competent probability density estimators will be a key direction for our future work.

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