Fossil gas and the electromagnetic precursor of supermassive binary black hole mergers

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ABSTRACT
Using a 1D height integrated model, we calculate the evolution of an unequal mass binary black hole with a coplanar gas disc that contains a gap due to the presence of the secondary black hole. Viscous evolution of the outer circumbinary disc initially hardens the binary, while the inner disc drains on to the primary (central) black hole. As long as the inner disc remains cool and thin at low \( \dot{M}_{\text{ext}} \) (rather than becoming hot and geometrically thick), the mass of the inner disc reaches an asymptotic mass typically \( \sim 10^{-3} - 10^{-4} M_\odot \). Once the semimajor axis shrinks below a critical value, angular momentum losses from gravitational waves dominate over viscous transport in hardening the binary. The inner disc then no longer responds viscously to the inspiraling black holes. Instead, tidal interactions with the secondary rapidly drive the inner disc into the primary. Tidal and viscous dissipation in the inner disc lead to a late time brightening in luminosity, \( L \propto t^{-5/4} \) minus, where \( t_{\text{minus}} \) is the time prior to the final merger. This late time brightening peaks \( \sim 1 \) d prior to the final merger at \( \sim 0.1 L_{\text{Edd}} \). This behaviour is relatively robust because of self-regulation in the coupled viscous–gravitational evolution of such binary systems. It constitutes a unique electromagnetic signature of a binary supermassive black hole merger and may allow the host galaxy to be identified if used in conjunction with the Laser Interferometric Space Antenna localization.

Key words: accretion, accretion discs – black hole physics – gravitational waves – binaries: general – galaxies: active – galaxies: nuclei – quasars: general.

1 INTRODUCTION

There is now strong evidence that supermassive black holes (SMBHs) inhabit the spheroids of most galaxies (Magorrian et al. 1998; Tremaine et al. 2002). Galaxy mergers, which are a natural consequence of hierarchical assembly of galaxies, should cause the SMBHs in each galaxy to form a binary in the newly assembled host galaxy (Begelman, Blandford & Rees 1980). These binary black holes (BBHs) are believed to coalesce due to an uncertain combination of dynamical friction, three body interactions, interaction with viscous circumbinary discs and gravitational wave (GW) losses. Observation of the GW that these systems produce is one of the primary scientific objectives of the proposed Laser Interferometer Space Antenna (LISA). In addition to their usefulness as probes of general relativity in the strong field regime, GWs can also be used to measure the luminosity distance to high accuracy, i.e. \( \delta D_L/D_L \) at the per cent level in many cases (Hughes 2002; Holz & Hughes 2005).

The high accuracy that these ‘standard sirens’ offer would allow these GW sources to constrain cosmological models if the host galaxy can be identified, so that a source redshift can be measured. By itself, LISA observations will provide sky localization errors that are typically \( \sim 10^{-4} \) deg2 (see for instance Holz & Hughes 2005; Kocsis et al. 2007; Kocsis, Haiman & Menou 2008; Lang & Hughes 2008; Arun et al. 2009), in which there may be \( \sim 10^4 \) galaxies.1 Hence, much recent work has focused on finding an unambiguous electromagnetic counterpart to a BBH merger (Schutz 1986) to help identify its host galaxy. In addition, the known BH masses and spins from GWs, an observed electromagnetic counterpart would also provide strong constraints on models of accretion discs and quasars/active galactic nuclei (AGNs).

At present, most studies of these electromagnetic counterparts to BBH mergers have focused on the post-merger phase. At the time of

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1 The 3D localization of LISA would give \( \sim 10 \) candidate galaxies per arcmin−2 (Holz & Hughes 2005; Kocsis et al. 2006).
merger, a fraction of the rest mass of the two BHs is radiated as GW and the merged BH receives a kick between 100 and 4000 km s\(^{-1}\) (Bekenstein 1973; Favata, Hughes & Holmo 2004; Baker et al. 2006; González et al. 2007; Herrmann et al. 2007; Boyle, Kesden & Nissanke 2008). If a circumbinary gas disc surrounded the two BHs (pre-merger), the sudden decrease in BH mass and the kick provided by the BH may produce an observable electromagnetic signature (Bode & Phinney 2007; Lippai, Frei & Haiman 2008; O’Neill et al. 2009; Schnittman & Krolik 2008; Shields & Bonning 2008; Bode & Phinney 2009). On a longer time-scale, the circumbinary disc would accrete into the newly merged BH, leading to AGN activity (Milosavljević & Phinney 2005) or perhaps an offset quasar (Madau & Quataert 2004).

There are potential difficulties associated with identifying these electromagnetic signatures of BBH mergers. The signatures associated with kicks are likely to be weak and difficult to distinguish from AGN variability, if they are a few per cent of the background disc emission (Kocsis & Loeb 2008; O’Neill et al. 2009; Schnittman & Krolik 2008). Delayed AGN and offset AGN activity, while significantly more promising, might have a substantial delay (Milosavljević & Phinney 2005; Loeb 2007).

The subject of this work is to explore electromagnetic emission associated with the pre-merger phase. Various authors (Armitage & Natarajan 2005; Milosavljević & Phinney 2005; MacFadyen & Milosavljević 2008) have studied the physics of such viscously driven BBH mergers. These studies suggest that the interaction between the disc and the BHs can induce a small eccentricity in the disc and the binary (Armitage & Natarajan 2005; MacFadyen & Milosavljević 2008; Cuadra et al. 2009). In addition, some of this gas may form accretion discs around the two BHs leading to binary AGN activity (Dotti et al. 2007).

In this paper, we build on the work of Armitage & Natarajan (2002) and show that although the inner disc drains when the binary is well separated, a small amount of remnant or fossil gas can remain around the primary BH, giving rise to a distinct electromagnetic signature. The amount of fossil gas is very small (∼10\(^{-3}\)M\(_\odot\)), but even this tiny amount of gas is sufficient to drive the luminosity of the binary to ∼0.1L\(_{\text{edd}}\) one day before merger.

The plan of this work is as follows. In Section 2, we summarize the basic scenario. We then present the relevant physics in Section 3. In Section 4, we argue that a small amount of fossil gas could exist in the inner disc between the primary and secondary BHs and that this fossil gas would be tidally driven into the primary on a very short time-scale near the final merger, leading to a bright electromagnetic precursor. Finally in Section 5, we discuss some of the caveats of this model and directions for future work.

### 2 BASIC PICTURE

We illustrate the basic scenario schematically in Fig. 1. A secondary BH with mass M\(_{\text{sec}}\) and a coplanar accretion disc orbits a more massive primary BH at a radius r\(_{\text{esc}}\). We expect the disc to be aligned to the orbital plane of the two BHs because of inclination damping. Ivanov, Papaloizou & Polnarev (1999) showed that in the case of a BH orbiting a more massive BH with an associated accretion disc, the inner accretion disc aligns with the orbit of the binary independent of the viscosity. As a result, the accretion disc is truncated into two parts: an inner accretion disc which surrounds the primary and a circumbinary disc which surrounds the binary. Flow between the inner disc and outer disc is prevented by the presence of the secondary BH at least in one dimension. The inner accretion disc has an inner radius, which is typically the innermost stable circular orbit (ISCO), and an outer radius where it is truncated due to tidal interactions with the secondary BH. Similarly, the outer disc is also truncated.

A similar model has previously been studied by Armitage & Natarajan (2002). However, we will include several improvements in our treatment. First we include more realistic physics for the evolution of the viscous disc, i.e. we use a self-consistent α-disc instead of a fixed viscosity law. This significantly changes how the disc drains and we believe that it provides a more realistic estimate of the luminosity evolution of the disc prior to merger. Secondly, we focus on the luminosity of the late time electromagnetic precursor and its simple shape, which might aid its identification.

### 3 BASIC EQUATIONS

We now present the governing equations for our model. These are standard in the literature (Lin & Papaloizou 1979a,b; 1986; Hourigan & Ward 1984; Ward & Hourigan 1989; Rafikov 2002; Chang 2008; van de Ven & Chang 2009), but we state them here for completeness.

The equation of continuity is (Frank, King & Raine 1992)

\[
\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial (r \Sigma v_r)}{\partial r} = 0, \tag{1}
\]

where \( r \) is the radial coordinate and \( v_r \) is the radial component of the velocity. The equation of angular momentum conservation is (Pringle 1981; Rafikov 2002)

\[
\frac{\partial (\Sigma r^2 \Omega)}{\partial t} + \frac{1}{r} \frac{\partial (r v_r \Sigma r^2 \Omega)}{\partial r} = -\frac{1}{2 \pi r} \left( \frac{\partial T_{\text{visc}}}{\partial r} - \frac{\partial T_\Omega}{\partial r} \right), \tag{2}
\]

where \( \Omega = \sqrt{GM_{\text{BH}}(r)/r^3} \) is the orbital frequency, \( M_{\text{BH}} \) is the mass of the primary BH and

\[
T_{\text{visc}} = -2\pi^2 v \Sigma \frac{\partial \Omega}{\partial r}, \tag{3}
\]

is the viscous torque, where \( v \) is the viscosity. We have adopted the standard \( \alpha \) prescription for the viscosity ( Shakura & Sunyaev 1973):

\[
v = \frac{2\alpha P_g}{3\Omega \rho}, \tag{4}
\]

with proportionality to \( P_g \), the mid-plane gas pressure. We chose to rely on the so-called ‘\( \beta \) model’, i.e. stress proportional to gas pressure as opposed to total pressure, to ensure viscous and thermal stability in our 1D model. For most of our calculations, we use \( \alpha = 0.1 \).\(^2\) A simple \( \alpha \)-prescription applied to radiation pressure dominated discs is both viscously and thermally unstable (Lightman & Eardley 1974; Piran 1978). Recent work by Hirose, Krolik & Blaes (2009) suggests that the thermal instability is avoided in radiation pressure dominated situations because stress fluctuations lead the associated pressure fluctuations. However, the viscous instability of

\(^2\) For simplicity, we do not make a distinction between the centre of mass of the binary and the location of the primary BH in our analysis, even though the secondary is not much less massive than the primary in some of our models.

\(^3\) The exception is the suite of calculations presented in Table 1, where we have taken \( \alpha = 0.01 \) and 0.1
radiation pressure dominated discs might remain. Adopting the ‘β model’ for the viscous stress will only change the evolution at high $\dot{M}_{\text{ext}}$ and small radius (note the discussion in Section 5) and does not affect our key results: once the binary is at small separation, the discs’ evolution is dominated by GW losses and not by internal viscosity.

Using equation (1), we simplify (2) to be

$$\Sigma \frac{\partial (r^2 \Omega)}{\partial t} + \nu \Sigma \frac{\partial (r^2 \Omega)}{\partial r} = -\frac{1}{2\pi r} \left[ \frac{\partial T_{\text{visc}}}{\partial r} - \frac{\partial T_d}{\partial r} \right].$$  \hfill (5)

Solving for $v$, in equation (5) and plugging it into (1), we find

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2\pi r} \frac{\partial}{\partial r} \left( \frac{\partial (r^2 \Omega)}{\partial r} \right)^{-1} \left[ \frac{\partial}{\partial r} (T_{\text{visc}} - T_d) \right].$$  \hfill (6)

The orbiting secondary BH exerts a torque density on the disc, $\partial T_d/\partial r$, of the form (Lin & Papaloizou 1979b; Goldreich & Tremaine 1980; Ward 1997; Armitage & Natarajan 2002; Chang 2008):

$$\frac{\partial T_d}{\partial r} \approx \frac{4}{9} f \text{sgn}(r - r_{\text{sec}}) \frac{G^2 M_{\text{BH}}^2 \Sigma r}{r_c^2(r - r_{\text{sec}})^2},$$  \hfill (7)

where $f$ is a constant which is $\approx 0.1$ following Armitage & Natarajan (2002).

Plugging equations (3) and (7) into (6), we find

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{3r^{1/2}}{\partial r} \left( \nu \Sigma r^{1/2} \right) \right]$$

$$- \frac{4}{9\pi} f \text{sgn}(r - r_{\text{sec}}) \Omega^2 q^2 \Sigma \left( \frac{r}{r - r_{\text{sec}}} \right)^4 \right].$$  \hfill (8)

where $q = M_{\text{sec}}/M_{\text{BH}}$ is the mass ratio between the secondary and primary BHs.

The radial motion of the secondary BH, assuming a circular orbit, is governed by tidal torquing of the disc and the torque due to GWs:

$$\frac{1}{2} M_{\text{sec}} \Omega_{\text{sec}} r_{\text{sec}} \frac{\partial r_{\text{sec}}}{\partial t} = \frac{4}{9} f G M_{\text{BH}} q^2 \int S \left( \frac{r}{r - r_{\text{sec}}} \right)^4 \Sigma \text{d}r - T_{\text{GW}},$$  \hfill (9)

where $S = \text{sgn}(r - r_{\text{sec}})$ is the sign of the torque and $T_{\text{GW}}$ is the torque exerted by GW losses (Peters 1964),

$$T_{\text{GW}} = \frac{\sqrt{8}}{5} M_{\text{BH}} c^2 \left( \frac{r_{\text{sec}}}{r_{\text{sec}}} \right)^{7/2} \left( \frac{M_{\text{sec}}}{M_{\text{BH}}} \right)^2 \sqrt{\frac{M_{\text{sec}} + M_{\text{BH}}}{M_{\text{BH}}}},$$  \hfill (10)

where $r_{\text{sec}} = 2GM_{\text{BH}}/c^2$ is the Schwarzschild radius.

Finally we assume vertical energy balance at every radius between heating and radiative cooling, i.e. $F = D(r)$, where $F$ is the local radiative flux and $D$ is the local dissipation rate. In a one-zone model, the local radiative flux is

$$F = \frac{4}{3} \frac{\sigma T^4}{\kappa \Sigma}.$$  \hfill (11)
where $T$ is the central temperature and $\kappa$ is the opacity, which we assume to be constant (Thomson) for simplicity. More detailed models would include variations in opacity with radius in the disc. We assume that the disc emits as a blackbody. The heating is given by a combination of viscous dissipation and tidal dissipation. Viscous dissipation is given by the standard formula (Frank et al. 2002)

$$D_{\text{visc}} = \frac{9}{8} \pi G M_{\text{BH}} \frac{\Omega_{\text{sec}}}{r^3},$$

where $D_{\text{visc}}$ is the local viscous dissipation rate per unit area.

We now consider the energy dissipation from tidal torques. We assume that the torque on the disc raised by the satellite, $T_d$, is mediated by the excitation of spiral density waves and locally damped. The time-scale associated with this tidal torque is the angular momentum of the satellite, $L_{\text{sec}} = M_{\text{sec}} \Omega_{\text{sec}} r_{\text{sec}}^2$ divided by the torque or $t_{\text{side}} = L_{\text{sec}}/T_d$. The power associated with such a torque is then

$$E_{\text{side}} = L_{\text{sec}} \Omega_{\text{sec}}/t_{\text{side}} = T_d \Omega_{\text{sec}},$$

where $E_{\text{sec}} = GM_{\text{BH}} M_{\text{sec}}/r_{\text{sec}}$ is the orbital energy of the secondary. Because we assume local damping of spiral density waves, the corresponding binding energy liberated locally (assuming circular orbits) is in proportion to the spiral density waves which mediate this interaction. Hence the local tidal dissipation rate is given by

$$D_{\text{tide}} = \frac{1}{2\pi} E_{\text{side}} \frac{|dT_d/dr|}{d|dT_d/dr|},$$

As we will show, these tidal interactions are especially important at late times when GW losses drive the secondary inward on a circular orbit. A tiny fraction ($M_d, M_{\text{sec}} \sim 10^{-10}$, where $M_d = 2 \pi \int_0^{r_{\text{sec}}} \Sigma r^2 dr$ is the mass of the inner disc) of the angular momentum losses is offset by a gain from the tidal interaction with the inner disc. The tidal-GW evolution allows this fraction of the prodigious gravitational luminosity $L_{\text{GW}} \sim 10^{39} \text{erg s}^{-1}$, to be released electromagnetically by heating the inner disc gas, which is enough to power the inner accretion disc close to its Eddington luminosity.

4 RESULTS

We solve equations (8) and (9) using standard explicit finite-difference methods (Press et al. 1992). We choose 200 grid points logarithmically spaced between $r = r_{\text{ISCO}} = 3 r_g$ (i.e. the ISCO) and $10^{2} r_g$. We set a zero-torque boundary condition at the inner radius and impose an outer boundary condition such that there is a constant external feeding rate, $M_{\text{ext}}$. As a test, we initially solve these equations for no satellite and with inflow boundary conditions and find that we recover the steady state $\alpha$-disc solution with an error of $\lesssim 1$ per cent.

The first two cases we consider are a $10^{7} M_{\odot}$ BH with a $q = 0.1$ and a $q = 0.3$ secondary, respectively. We start the secondary at an initial radius of $r_{\text{out}} = 10^{2} r_g (\lesssim 0.01 \text{ pc})$. We begin with a low-mass ($10^{2} M_{\odot}$) disc which extends from the ISCO to $r_{\text{outer}} = 10^{2} r_g$. This disc has an assumed profile of $\sim r^{-3.5} \sqrt{1 - r_{\text{ISCO}}/r}$, which corresponds to a gas pressure supported $\alpha$-disc with a constant (Thomson) opacity. We clear a region around the secondary’s initial radius, $0.5 r_{\text{ISCO}} < r < 2 r_{\text{ISCO}}$, to model the initial clearing of a gap around the secondary BH. At the outer edge of the grid, we consider an outer $M_{\text{ext}}$ of either $M_{\text{ext}} = 0.1 M_{\odot} \text{ yr}^{-1}$ or $0.01 M_{\odot} \text{ yr}^{-1}$.

The GW luminosity can maximally be $c^2/G \sim 3 \times 10^{59} \text{ erg s}^{-1}$, but the GW luminosity when the disc is still present is closer to $10^{55} \text{ erg s}^{-1}$.

Figure 2. Evolution of the disc mass inside the secondary’s orbital radius, $r_{\text{sec}}$ for a $10^{7} M_{\odot}$ primary and $10^{6} M_{\odot}$ secondary, and external mass accretion rates of $10^{-1} M_{\odot} \text{ yr}^{-1}$ ($M_{\text{ext}} \approx 0.01 M_{\odot}$; top set of curves) and $10^{-2} M_{\odot} \text{ yr}^{-1}$ ($M_{\text{ext}} \approx 0.1 M_{\odot}$; bottom set of curves). Three scenarios in each case, with initial disc masses of $10^{5}$, $10^{4}$ and $10^{3} M_{\odot}$ are represented by the curves from top to bottom. Note that the asymptotic disc mass is independent of the initial disc mass.

As an initial condition, we choose a low initial disc mass ($10^{5} M_{\odot}$) and assume that it is distributed in a steady state $\alpha$-disc (corresponding to a very low accretion rate for a steady-state disc of $M = 5.5 \times 10^{-3} (\alpha/0.1)^{-1/3} M_{\odot} \text{ yr}^{-1}$). We choose such a low initial disc mass so that our models can be integrated on a reasonable time-scale. The reason is that the Courant condition, which is set by the viscous time of the disc near the ISCO, limits the time-step to very low values unless the disc has a very low mass. Fortunately, our results do not depend on this initial condition. The mass of the inner disc, $M_{\text{in}}$, approaches an asymptotic value which is independent of its initial value. With larger initial disc masses, the inner disc would have initially drained on a faster time-scale, but the final disc mass would remain the same.

We illustrate this point in Fig. 2 where we show models with different disc masses of $10^{7}, 10^{6}$ and $10^{5} M_{\odot}$ (which corresponds to a steady state mass accretion rates of $M = 1.5 \times 10^{-17}$, $5.5 \times 10^{-16}$ and $1.1 \times 10^{-15} (\alpha/0.1)^{1/3} M_{\odot} \text{ yr}^{-1}$) for two different external mass accretion rates: $M_{\text{ext}} = 10^{-1} M_{\odot} \text{ yr}^{-1}$ (top set of curves) and $M_{\text{ext}} = 10^{-2} M_{\odot} \text{ yr}^{-1}$ (bottom set of curves). Note that the mass of the inner disc approaches an asymptotic value that is independent of its initial value. The reason for this is that the inner disc drains until its viscous time is comparable to the merger time. At late times, however, when the evolution of the system is dominated by GWs, the inner disc can no longer viscously drain on the time-scale of the merger. Hence, the inner disc mass is essentially frozen. The frozen mass of the inner disc at late times is given by the mass of the inner disc when the system transitions from viscosity-dominated evolution to GW-dominated evolution, which occurs typically at $r_{\text{GW}} \sim 500 r_g$ (Armitage & Natarajan 2002; Haiman, Kocsis & Menou 2009 also see Section 4.2). In the remainder of this work, we use this fact and start with a low disc mass to save computational cost.

We should also note that like the inner disc, the outer disc mass is also not set by its initial value, albeit for a different reason. Rather, the exterior mass inflow quickly increases the outer disc mass.
mass wiping out any memory of its initial value. This is perhaps unsurprising as the outer disc, forced to accept a fixed mass inflow rate, adjusts itself to a quasi-steady state. Hence, the outer disc mass (at any \( r_{\text{ext}} \)) for any of the curves in Fig. 2 is not determined by the initial mass, but rather by the exterior mass inflow rate. In addition, this asymptotic inner disc mass scales linearly with the external mass accretion rate. This seemingly simple relation is a puzzle and we currently do not have an adequate argument as to the origin of this relationship between the inner disc and external accretion rate.

4.1 Viscous evolution of the BBH and the outer disc

We first consider the case of a \( 10^7 \, M_\odot \) BH with a \( 10^6 \, M_\odot \) secondary and an outer mass inflow rate of \( \dot{M}_{\text{ext}} = 0.1 \, M_\odot \, \text{yr}^{-1} \) (Fig. 3) or \( \dot{M}_{\text{ext}} = 0.01 \, M_\odot \, \text{yr}^{-1} \) (Fig. 4). Figs 3 and 4 show four different cuts of the disc properties for a \( 10^7 \, M_\odot \) primary BH with a \( 10^6 \, M_\odot \) secondary BH (whose position is marked with an ‘\( \times \)’), for an external accretion rate of \( 10^{-1} \, M_\odot \, \text{yr}^{-1} \). We plot the surface density \( \Sigma \) (solid lines), viscous time \( t_{\text{visc}} \) (dashed line, matched to right axis), and merger time of the secondary \( t_{\text{merge}} \) (dotted horizontal line, match to the right-hand side). (a) After the system has reached an initial quasi-stationary state where the outer disc has accumulated some mass due to accretion and has begun to influence the secondary, the inner disc has a viscous time similar to the merger time. (b) After the semimajor axis has decreased by an order of magnitude due to tidal–viscous interaction with the outer circumbinary disc. Note that the mass of the inner disc has declined significantly compared to (a), while the outer disc is approximately an \( \alpha \)-disc with a cut-off around the radial position of the secondary. (c) The semimajor axis has decreased by yet another order of magnitude, and the inner disc mass has reached its asymptotic value. Angular momentum losses are now dominated by GWs. (d) The secondary tidally forces the remaining mass in the inner disc to accrete rapidly into the central BH. It is this final forced feeding of the central BH which leads to the electromagnetic precursor (see Figs 5a and b).
from our 1D simulations to illustrate the different regimes through which the binary and the disc evolve with time. In each cut, we plot $\Sigma$ (solid line matched to the left axis) and $t_{\text{visc}}$ (dashed line matched to the right axis). We also show for each cut, the merger time of the BBH $t_{\text{merge}}$ as a horizontal dotted line matched to the right axis. In (a), the initially low-mass outer disc has accreted sufficient mass to influenced the secondary and move it inward slightly. We have selected to show this as the first plot as the satellite-disc system has reach a quasi-stationary state where the viscous time of the inner disc is same as the merger time of the binary. This quasi-stationary state is similar to much of its subsequent evolution and described below. In (b), the secondary’s radius has decreased by an order of magnitude due to its tidal interaction with the viscously evolving outer disc. Meanwhile, the inner disc has drained substantially.

In (c), the binary has shrunk by another order of magnitude in radius due to both tidal-viscous evolution and GW losses, with the latter becoming dominant at small radii. Note that the inner disc density profile has begun to invert, i.e. the surface density rises with radius as opposed to falling. This is due to the inability of the inner disc to viscously respond to the increasingly rapid infall of the secondary. In (d), a large surface density spike has built up near the outer edge of the inner disc. This forcing of the inner disc on to the primary BH causes it to brighten with a maximum luminosity, $L_{\text{peak}}$, reached at a time, $t_{\text{peak}}$, before the final merger.

A crucial aspect of the evolutionary sequence shown in Figs 3 and 4 is that, during the early phases (plots a and b; before GWs dominate), the viscous time, $t_{\text{visc}} = r^2/\nu$ (dashed lines) near the outer edge of the inner disc adjusts to become comparable to the
evolving merger time, $t_{\text{merge}} = |r_a/v_{\text{esc}}|$ (shown as a horizontal dotted line in plots a and b). This quasi-stationary state implies that the evolutionary time for the inner disc is the same as the merger time of the binary. This adjustment occurs because a more massive disc would drain much more rapidly until it drains until the point where its viscous time equals the lifetime of the system. Similarly, a less massive disc would remain static until the external forcing from the secondary decreases the disc’s outer radius and increases the local surface density to the point where the local viscous time matches the merger time. Hence the inner disc approaches an asymptotic state that just depends on the merger time of the binary. The binary merger time itself depends on the external feeding rate until GWs take over. This self-regulated evolution explains lack of any difference of the inner disc mass on the initial disc mass shown in Fig. 2. Note that once GWs dominate (plots c and d), $t_{\text{visc}}$ does not adjust to $t_{\text{merge}}$, the disc can no longer respond viscously to the rapidly infalling secondary and its dynamics becomes tidally dominated as shown vividly in panels (d) of Figs 3 and 4. The dissipation of tidal energy and the forcing of disc material to smaller and smaller radii is the primary means by which energy is produced in this late stage.

As Figs 3 and 4 show, the general evolutionary sequence of a BBH system can be summarized as follows. Initially the BBH starts out at large radii, where the tidal–viscous interaction with the outer gas disc shrinks the semimajor axis of the BBH. Meanwhile, the inner disc continually drains. Eventually, the BBH shrinks until $r < r_{\text{GW}}$, where GWs become the principle mechanism by which angular momentum is lost. Shortly afterwards, the inner disc can no longer respond viscously to the inspiralling secondary and it reaches its asymptotic mass, $M_{\text{asy}}$. The inner disc is forced to smaller and smaller radii by the inspiraling secondary until it completely accretes on to the central BH. At this time, $t_{\text{peak}}$ before the final merger, the inner disc has a peak luminosity, $L_{\text{peak}}$. We have run a suite of models at different $\alpha$, $M_{\text{ext}}$, $M_{\text{BH}}$ and $q$ to explore the parameter space of $M_{\text{d, in}}$, $L_{\text{peak}}$ and $t_{\text{peak}}$. These results are summarized in Table 1 and discussed below.

### 4.2 The electromagnetic precursor of the BBH merger

As Table 1 indicates, the peak luminosity of the electromagnetic precursor can be appreciable even though the asymptotic mass of the inner disc seems insignificant. The precursor’s light curve is shown as a function of time prior to merger in Fig. 5, where we plot the luminosity due to tidal and viscous dissipation as a function of the time before merger, $t_{\text{min}}$ (measured from when the BHs finally merge, which is not the same as the instantaneous merger time $r_{\text{esc}}/|v_{\text{esc}}|$) for the two cases of $M_{\text{BH}} = 10^6 M_{\odot}$ (a) and $10^7 M_{\odot}$ (b). The results shown in Fig. 5a are for the two mass inflow rates, $0.1 M_{\odot}$ yr$^{-1}$ (solid line) and $0.01 M_{\odot}$ yr$^{-1}$ (dashed line), and for $q = 0.1$ and $q = 0.3$ (top and bottom plots, respectively). The electromagnetic precursor manifests itself as a sudden brightening to $\approx 0.1-1 L_{\text{Edd}}$ during the last few days before merger. There are a few notable characteristics of the light curve. First, the light curve of the precursor has a very characteristic power-law shape. The time of the peak $t_{\text{peak}}$ depends only on the masses of the two BHs. Finally, the peak luminosity and temperature depends only on the mass of the inner disc.

The light curve follows a characteristic power-law shape, which we now show is a property of the gravitational radiation (GR)-driven decay of the secondary on the inner disc. Tidal interactions between

| $M_{\text{BH}}$ | $q$ | $M_{\text{ext}}$ | $\alpha$ | $M_{\text{asy}}$ | $t_{\text{peak}}$ | $L_{\text{peak}}$ |
|----------------|-----|----------------|---------|----------------|----------------|--------------|
| $10^6$         |     | $10^6$         |         |               | $10^{43}$      |              |
| 0.1            | 0.1 | 0.1            | 1.2     | 0.07           | 1.9            |              |
|                | 0.01| 0.1            | 12.0    | 0.07           | 3.6            |              |
| 0.01           | 0.1 | 0.1            | 0.12    | 0.07           | 0.2            |              |
| 0.1            | 0.1 | 0.1            | 1.2     | 0.07           | 0.38           |              |
|                | 0.01| 0.1            | 0.04    | 0.04           | 0.5            |              |
| 0.01           | 0.1 | 0.1            | 0.008   | 0.04           | 0.05           |              |
|                | 0.01| 0.1            | 0.04    | 0.027          |               |              |
| $10^7$         |     | $10^6$         |         |               | $10^{44}$      |              |
| 0.1            | 0.1 | 0.1            | 6.6     | 0.7            | 2.0            |              |
|                | 0.01| 0.1            | 460     | 0.7            | 14             |              |
| 0.01           | 0.1 | 0.1            | 0.73    | 0.7            | 0.23           |              |
| 0.1            | 0.1 | 0.1            | 6.4     | 0.7            | 2.0            |              |
|                | 0.01| 0.1            | 0.62    | 0.4            | 0.19           |              |
| 0.01           | 0.1 | 0.1            | 0.062   | 0.4            | 0.019          |              |
|                | 0.01| 0.1            | 0.75    | 0.4            | 0.2            |              |
| $10^8$         |     | $10^6$         |         |               | $10^{45}$      |              |
| 0.1            | 1   | 0.1            | 20      | 7              | 6.5            |              |
|                | 0.01| 1              | 90      | 7              | 28             |              |
| 0.1            | 0.1 | 1              | 2.9     | 7              | 0.9            |              |
|                | 0.01| 1              | 18      | 7              | 5.7            |              |
| 0.3            | 1   | 0.1            | 2.4     | 4              | 0.67           |              |
|                | 0.01| 1              | 15      | 4              | 4.2            |              |
| 0.1            | 0.1 | 3.3            | 4       | 0.09           |               |              |
|                | 0.01| 3.3            | 4       | 0.9            |               |              |

The secondary BH and the accretion disc drive the outer edge of the disc to smaller radii, and the dissipation of the induced density waves leads to heating of the disc. The amount of energy dissipated at any time is

$$L \approx \frac{GM_{\text{BH}} M_{\text{d, in}}}{r_{\text{esc}}} t_{\text{merge}}^{-1},$$

(15)

where $M_{\text{d}}$ is fixed as we are in a regime with the merging time much shorter than the viscous time. This mass is set primarily by the viscosity of the disc, i.e. $\alpha$, and the external $M_{\text{ext}}$. It directly influences the luminosity of the inner disc in this GW-driven phase of the merger. The merging time-scale due to GWs is

$$t_{\text{merge}} = \frac{5}{8} \frac{a}{c} \frac{M_{\text{BH}}}{M_{\text{d, in}}} \left( \frac{a}{r_{\text{g}}^3} \right)^{\frac{3}{2}},$$

(16)

Combining equations (15) and (16), we find

$$L \propto t_{\text{min}}^{-5/4}.$$  

(17)

This power-law shape is a good approximation to the light curve in the more detailed calculations in Figs 5(a) and (b). The associated effective temperature is $T_{\text{eff}} \propto [L/(r_a t_{\text{peak}}^{1/2})]^{1/4} \propto t_{\text{min}}^{-7/16}$.

Fig. 6 shows the corresponding multi-temperature blackbody spectra at $t_{\text{min}} = 1$ d and 10 d for $M_{\text{BH}} = 10^7 M_{\odot}$ (left-hand panel) and $10^8 M_{\odot}$ (right-hand panel). These spectra are also shown for different mass ratios, $q = 0.1$ (thick lines) and $q = 0.3$ (thin lines), and for different outer accretion rates, $M_{\text{ext}} = M_{\text{asy}}$ (solid lines) and $0.1 M_{\text{asy}}$ (dashed lines). As this plot demonstrates, the majority of the emission prior to merger will be in the extreme UV and soft X-ray. As a result, the electromagnetic precursor may
Figure 5. Bolometric luminosity (thick curves) and mass of the inner disc (thin curves) as a function of time before merger, $t_{\text{minus}}$. (a) Evolution for a $10^7 M_\odot$ primary with a $10^6 M_\odot$ secondary (upper panel) and a $3 \times 10^6 M_\odot$ secondary (lower panel). We plot this evolution for $\dot{M}_{\text{ext}} = 10^{-1} M_\odot \text{yr}^{-1}$ (solid lines) and $\dot{M}_{\text{ext}} = 10^{-2} M_\odot \text{yr}^{-1}$ (dashed lines). (b) Similar evolution for a $10^8 M_\odot$ primary with a $10^7 M_\odot$ secondary (upper panel) and a $3 \times 10^7 M_\odot$ secondary (lower panel) and for $\dot{M}_{\text{ext}} = 1 M_\odot \text{yr}^{-1}$ (solid lines) and $\dot{M}_{\text{ext}} = 10^{-1} M_\odot \text{yr}^{-1}$ (dashed lines). Note that the precursor approximately follows a $t^{-5/4}$ power law until the peak.

Figure 6. Multi-temperature blackbody spectra at $t_{\text{minus}} = 1$ d for $M_{\text{BH}} = 10^7 M_\odot$ (left) and at $t_{\text{minus}} = 10$ d for $M_{\text{BH}} = 10^8 M_\odot$ (right) in the evolutionary scenarios of Fig. 5. Thick lines represent a mass ratio $q = 0.1$ and thin lines represent $q = 0.3$. Note the gap in the emission between the low-energy component, which arises from the circumbinary disc, and the high-energy component, which arises from the tidally forced inner disc.

be difficult to detect from the ground or in the presence of obscuration. We have presumed no reprocessing of the inner disc emission by the outer disc. If present, this may increase the luminosity of the source in more easily detectable wavelength, i.e. optical or near-IR, with possibly significant time delays. Large amplitude spiral density waves from the strong tidal forcing may contribute to non-thermal high-energy emission. One notable aspect of these spectra is a gap in the emission between the low-energy part, which is due to emission from the circumbinary disc, and the high-energy part, which is due to emission from the tidally forced inner disc.

The final two points are clear from examining Figs 5(a) and (b). The time-scale of maximum luminosity only depends on the masses of the two BHs as one can see by comparing the cases $q = 0.1$ and $q = 0.3$ in Figs 5(a) and (b). For instance for the $M_{\text{BH}} = 10^7 M_\odot$ ($10^8 M_\odot$) case in Figs 5(a) and (b), the characteristic shape and time of the peak of the light curve are the same for the two different $\dot{M}_{\text{ext}}$, but the normalization is different. The normalization of the
light curve depends on the mass of the inner disc, which in turns depends on the outer mass accretion rate and $\alpha$ (see Table 1).

We will note, however, that there is a great deal of uncertainty in regards to the absolute value of this mass. For instance, the mass in the inner disc can be dramatically affected if additional sources of mass inflow exist beyond the viscous condition considered here. For instance, if the gap between the inner disc and outer disc is not completely clear, then a small trickle of mass from the outer disc to the inner disc could change the asymptotic inner disc mass significantly. To quantify this point, at the transition radius between viscous and GW-driven evolution, the mass drainage rate of the inner disc is

$$M = \frac{M_{in}}{t_{merge}(r_{GW})} \sim 10^{-10} - 10^{-9} M_\odot \text{yr}^{-1},$$

where $t_{merge} = r_{sec}/v_{sec}$ is the instantaneous merger time. This very small mass inflow rate implies that any additional source of mass whose time-averaged input rate over $t_{merge} \sim 10^{-9} \text{yr}$ equals or exceeds equation (18) would greatly modify the inner disc evolution.

There are many such sources of additional mass. For instance the mass loss from a few massive stars such as found in the Galactic Centre (Quataert 2004; Loeb 2004) or an old population of stars as in the nucleus of M31 (Chang et al. 2007), could easily enhance the mass of the inner disc. In addition, our 1D models presume that the gap between the inner and circumbinary disc is clean and that no mass flows between the two. While this may not be an unreasonable assumption given the comparable masses of the two BHs, 2D simulations of gaps induced by protoplanets in a protoplanetary disc suggest that $\sim 10$ per cent of $M_{ext}$ in the outer disc can flow across the gap on to the inner disc (Lubow & D’Angelo 2006; MacFadyen & Milosavljević 2008). Mass leakage across the gap in our case is likely to be much smaller than what it is in the protoplanetary case, but even a small amount of mass will make a substantial difference to the evolution of the inner disc. The calculated asymptotic disc mass may thus be considered to be a conservative lower limit on the mass of the inner disc.

5 DISCUSSION AND CONCLUSIONS

We have calculated models of BBH evolution subject to torques from a circumbinary accretion disc and gravitational radiation losses. In general, the viscous time inside the orbit of the secondary BH is much less than the time-scale for tidal–viscous interaction with the circumbinary disc to shrink the orbit of the binary. At first glance, this suggests that the inner disc should completely drain away. However, we have shown that the inner accretion disc maintains a small, but non-trivial residual mass (Fig. 2), the value of which is set when GWs take over (at $r_{GW} \sim 500 r_g$) as the primary mechanism shrinking the orbit of the binary. This inner disc, which is then tidally forced into the primary by the secondary as it spirals inward, brightens significantly during the last day prior to merger. The bolometric luminosity rises as $L_{edd}^{3/4}$, peaking at one day with a peak luminosity of $\sim 0.1L_{edd}$ for a representative $10^6 M_\odot$ BH (see Table 1 for results for various $\alpha$, $M_{BH}$, $M_{ext}$ and $q$).

Such an electromagnetic signature is very important for the identification of the host galaxies of BBH mergers, but our study is subject to several caveats. One such caveat is related to the persistence of a thin accretion disc at very low accretion rates. Studies of X-ray binaries suggest that below an accretion rate of $10^{-4}$–$10^{-2} M_{edd}$, accretion discs transition from a thin radiatively efficient accretion disc to a thick radiatively inefficient accretion flow (Gallo, Fender & Pooley 2003; Fender, Belloni & Gallo 2004). If this were the case in our scenario prior to GW-driven evolution, the inner disc would completely drain into the primary BH and hence the characteristic electromagnetic signature we have identified would be absent. We emphasize, however, that the inner disc asymptotically approaches a gas-supported, optically thick disc solution, which in itself is stable at very low accretion rates. Thus, it is not a priori obvious whether the transition to a geometrically thick disc must occur in all cases.

The second caveat is our use of the ‘\beta model’ as our viscosity prescription. For most of the parameter range which we explore, this model is perfectly adequate as the disc is dominated by gas pressure. However, at late stages when the tidal forcing by the secondary is dominant, radiation pressure may begin to play an important role especially if the luminosity of the disc begins to approach the Eddington luminosity. We expect this would force the peak luminosity to be capped at the Eddington luminosity, but be more extended time wise compared to the sharp peaks shown in Fig. 5.

A third caveat is our initial conditions, namely a secondary in a circular orbit around an initially low-mass disc. The initial eccentricity of a forming BH pair is not known, but recent work have argued that the eccentricity should be low, i.e. circular orbits, (Dotti et al. 2007, 2009) if the merger is mediated at least in part by a circumnuclear gas disc. Indeed, Dotti et al. (2009) have shown that even an initially retrograde orbit of the secondary can be ‘flipped’ by a sufficiently massive gas disc prior to merger. Note that this need not be the case if the merger is mediated by stellar scattering (Berentzen et al. 2009) or secular (e.g. Kozai) processes (Blaes, Lee & Socrates 2002). We also note that these simulations assume quite a bit of circumnuclear gas, far more than the amount of gas which orbit our primary BH initially.

This brings us to our final caveat. In our calculations, we have presumed an initially low-mass gas disc. However, the circumnuclear gas disc may be much larger than what we have assumed. We have argued in this paper that the asymptotic mass of the gas disc only depends on the mass accretion rate at large radii and not on the initial mass of the disc. This is not true if the initial mass of the disc is smaller than the asymptotic mass (very unlikely) or if the initial mass of the circumnuclear disc is larger than the secondary. In our calculations and the work of Haiman et al. (2009), the time-scale associated with the decay of the binary is $\sim M_{sec}/M$. This implies that the outer circumnuclear disc acquires a mass similar to the mass of the secondary. Hence, if the initial mass of the circumnuclear disc is comparable to or larger than the secondary, we would expect the evolution to proceed faster. This would lead to a larger asymptotic mass of the inner disc due to larger starting mass and faster evolution.

Our calculated mass for the inner disc represents a conservative lower limit. When the secondary transitions from a viscosity-driven evolution to a GW-driven evolution, the inner disc mass corresponds to a mass accretion rate of $M = M_{d}/t_{merge} \sim 10^{-8} M_\odot \text{yr}^{-1}$. For any time-averaged mass source of cold gas present over $t_{merge} \sim 10^6$–$10^7$ yr that is greater than this, the mass of the inner disc would be larger than what we have calculated. Such sources of mass may include stellar winds, gas flow across the gap, etc.

We expect the perturbation to the GW signal from the tidal forcing of the inner disc to be minor in the sense that it would not be detectable with LISA. In the final year of inspiral, the inner disc has reached its asymptotic mass of $\sim 10^{-2} - 10^{-1} M_\odot$ (see Figs 5a and b). In this stage, the inner disc modifies the torque from GWs by a factor of $M_{d}/M_{sec} \sim 10^{-10}$. Such a tiny mass correction is well below the best achievable precision on mass measurements.
for the secondary with LISA (e.g. Hughes 2002; Lang & Hughes 2008).

The tidal forcing of an accretion disc by a spiraling binary is a somewhat unusual situation in astrophysics and the possibility that it occurs in the mergers of BBHs motivates further exploration of the physics of this scenario. One interesting possibility is that the precursor rebrightening event that we have identified could be time-variable due to perturbations from the orbiting secondary. This is analogous to the case of superhumps, where perturbations from a low-mass secondary drive the disc in a cataclysmic variable system to be mildly eccentric (Lubow 1991). A similar eccentricity growth has been suggested for the outer disc of BBHs by MacFadyen & Milosavljević (2008) and Cuadra et al. (2009). Time-variability on the binary’s period (or some multiple) would allow for an unambiguous association of the electromagnetic signal with the GW signal measured from future GW detectors such as LISA. The emergence of a time-variable signal from the tidally forced accretion of this fossil gas is worthy of further investigation.

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REFERENCES

Armitage P. J., Natarajan P., 2002, ApJ, 567, L9
Armitage P. J., Natarajan P., 2005, ApJ, 634, 921
Arun K. G. et al., 2009, Classical Quantum Gravity, 26, 094027
Baker J. G., Centrella J., Choi D.-I., Koppitz M., van Meter J. R., Miller M. C., 2006, ApJ, 653, L93
Begelman M. C., Blandford R. D., Rees M. J., 1980, Nat, 287, 307
Bekenstein J. D., 1973, ApJ, 183, 657
Berentzen I., Preto M., Berczik P., Merritt D., Spurzem R., 2009, ApJ, 695, 455
Blaes O., Lee M. H., Socrates A., 2002, ApJ, 578, 775
Bode N., Phinne y, 2007, APS Meeting Abstracts
Bode J. N., Phinne y, 2009, in Am. Astron. Soc. Meeting Abstracts Vol. 213, Observability of Circumbinary Discs Following Massive Black Hole Mergers
Boyle L., Kissen M., Nissak S., 2008, Phys. Rev. Lett., 100, 151101
Chang P., 2008, ApJ, 684, 236
Chang P., Murray-Clay R., Chiang E., Quataert E., 2007, ApJ, 668, 236
Cuadra J., Armitage P. J., Alexander R. D., Begelman M. C., 2009, MNRAS, 393, 1423

Dotti M., Colpi M., Haardt F., Mayer L., 2007, MNRAS, 379, 956
Dotti M., Ruszkowski M., Paredi L., Colpi M., Volonteri M., Haardt F., 2009, MNRAS, 396, 1640
Favata M., Hughes S. A., Holz D. E., 2004, ApJ, 607, L5
Fender R. P., Belloni T. M., Gallo E., 2004, MNRAS, 355, 1105
Frank J., King A., Raine D. J., 2002, Accretion Power in Astrophysics, 3rd edn. Cambridge University Press, Cambridge, p. 398
Gallo E., Fender R. P., Pooley G. G., 2003, MNRAS, 344, 60
Goldreich P., Tremaine S., 1980, ApJ, 241, 425
González J. A., Hannam M., Sperhake U., Brügmann B., Husa S., 2007, Phys. Rev. Lett., 98, 231101
Haiman Z., Kocsis B., Menou K., 2009, ApJ, 700, 1952
Herrmann F., Hinder L., Shoemaker D., Laguna P., Matzner R. A., 2007, ApJ, 661, 430
Hirose S., Krol J. H., Blaes O., 2009, ApJ, 691, 16
Holz D. E., Hughes S. A., 2005, ApJ, 629, 15
Hourigan K., Ward W. R., 1984, Icarus, 60, 29
Hughes S. A., 2002, Phys. Rev. D, 66, 102001
Ivanov P. B., Papaloizou J. C. B., Polnarev A. G., 1999, MNRAS, 307, 79
Kocsis B., Loeb A., 2008, Phys. Rev. Lett., 101, 041101
Kocsis B., Frei Z., Haiman Z., Menou K., 2006, ApJ, 637, 27
Kocsis B., Haiman Z., Menou K., Frei Z., 2007, Phys. Rev. D, 76, 022003
Kocsis B., Haiman Z., Menou K., 2008, ApJ, 684, 870
Lang R. N., Hughes S. A., 2008, ApJ, 677, 1184
Lightman A. P., Eardley D. M., 1974, ApJ, 187, L1
Lin D. N. C., Papaloizou J., 1979a, MNRAS, 188, 191
Lin D. N. C., Papaloizou J., 1979b, MNRAS, 186, 799
Lin D. N. C., Papaloizou J., 1986, ApJ, 307, 395
Lippai Z., Frei Z., Haiman Z., 2008, ApJ, 676, L5
Loeb A., 2004, MNRAS, 350, 725
Loeb A., 2007, Phys. Rev. Lett., 99, 041103
Lubow S. H., 1991, ApJ, 381, 259
Lubow S. H., D’Angelo G., 2006, ApJ, 641, 526
MacFadyen A. I., Milosavljević M., 2008, ApJ, 672, 83
Madau P., Quataert E., 2004, ApJ, 606, L17
Magorrian J. et al., 1998, AJ, 115, 2285
Milosavljević M., Phinney E. S., 2005, ApJ, 622, L93
O’Neill S. M., Miller M. C., Bogdanović T., Reynolds C. S., Schnittman J. D., 2009, ApJ, 700, 859
Peters P. C., 1964, Phys. Rev., 136, 1224
Piran T., 1978, ApJ, 221, 652
Press W. H., Flinnery T. S., Flannery B. P., 1992, Numerical Recipes. Cambridge Univ. Press, Cambridge
Pringle J. E., 1981, ARA&A, 19, 137
Quataert E., 2004, ApJ, 613, 322
Rafikov R. R., 2002, ApJ, 572, 566
Schnittman J. D., Krolik J. H., 2008, ApJ, 684, 835
Schutz B. F., 1986, Nat, 323, 310
Shakura N. I., Syunyaev R. A., 1973, A&A, 24, 377
Shields G. A., Bonning E. W., 2008, ApJ, 682, 758
Tremaine S. et al., 2002, ApJ, 574, 740
van de Ven G., Chang P., 2009, ApJ, 697, 619
Ward W. R., 1997, Icarus, 126, 261
Ward W. R., Hourigan K., 1989, ApJ, 347, 490

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