Zero Color Magnetization in QCD Matter

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We show that all spatial gluon connected correlation functions in SU(N) or SO(N) QCD vanish at finite temperature and zero momentum in lattice Landau or Coulomb gauges, due to the proximity of the Gribov horizon. These observations also apply to QCD with two colors and an even number of flavors at large chemical potential. These nonperturbative results may have consequences on the nature of the thermal magnetic mass and the character of the magnetic color superconductivity.

1. At high temperature or density, the physics of color charge screening and oscillation in QCD is closely related to the behaviour of the gauge-fixed gluonic correlation functions \([1]\). Finite temperature QCD perturbation theory has proven to be useful but limited to leading orders due to the persistence of infrared singularities \([2]\). Finite density QCD perturbation theory is believed to be infrared tamed due to Landau-damping \([3]\).

Lattice Yang-Mills simulations of the spatial Wilson loops show an area law above the critical temperature \([4]\), an indication that magnetic gluons suffer little change across the deconfining temperature. The electric gluons appear to be screened as is evident in the behaviour of the Polyakov loop correlation functions across the critical temperature \([4]\).

In this note we show that a lattice regularized version of QCD at finite temperature, yields zero for the connected magnetic correlation functions in the infinite volume limit, both in Landau and Coulomb gauges, extending an early observation by Zwanziger \([5]\) at zero temperature. This result applies to both SU(N) and SO(N) gauge groups irrespective of the center \(Z_N\). In short, in Landau and Coulomb gauges, the proximity of the Gribov horizon \([6]\) in the infrared directions puts strong bounds on the mean color magnetization \([6]\). Fortunately, no such bounds can be made for the connected electric correlation functions, which are expected to enjoy perturbative screening.

In general, our arguments do not extend to QCD with finite chemical potential, as the measure on the gauge-fixed configurations is no longer positive. An exception is QCD with two colors and an even number of flavors for which positivity is recovered. In this case, both the electric and magnetic gluon connected correlation functions vanish at zero momentum in Landau gauge, an indication that nonperturbative physics permeates the problem at even large chemical potential.

2. Our arguments will follow closely those given by Zwanziger \([6]\), and we refer the reader to them for more details. In short, in the minimal Landau and Coulomb gauges the fields are selected by the two conditions \([6]\):

\[ \triangle \cdot A = 0 \quad K(A) \geq 0 \] (1)

where \(A\) is the classical lattice connection

\[ A_\mu(x) = U_\mu(x) - U_\mu^+(x) \] (2)

which is Lie group valued. \(\triangle\) is the lattice divergence. The \(U_i\)’s are SU(N) valued link variables from \(x\) to \(x + e_\mu\) on a D-dimensional asymmetric lattice \(V = \beta \times L^{D-1}\) with periodic boundary conditions.

In Landau gauge, we may follow \([6]\) and use the positivity of the Faddeev-Popov kernel \(K\) to bound the zero frequency component of the lattice gluon field \(A\). Using plane-waves, the bounds are \([6]\):

\[ |A_4| = \frac{1}{V} \sum_x A_4(x) \leq 2 \tan(\pi/\beta) \] (3)

\[ |A_i| = \frac{1}{V} \sum_x A_i(x) \leq 2 \tan(\pi/L) \] (3)

for actually any SU(N) gauge group. The present bounds hold for SO(N) as well. While the mean of the spatial color magnetization vanishes as \(L \to \infty\), its temporal analogue does not at finite temperature since \(\beta = 1/T\) is kept fixed.

Bounds on the free energy follow from the partition function

\[ Z(H) = \int d[A] \rho(A) \exp \left( \sum_x H \cdot A(x) \right) \] (4)

where \(A\) is selected by the conditions \([6]\), \(\rho(A)\) is the QCD measure \([10]\), and \(H^a = (H^a_4, \vec{H}^a)\) a constant colored magnetic field. At finite \(\beta\) the measure is positive, a requirement for the following bounds to hold. The free energy \(w(H_4, \vec{H}) = \ln Z(H)/V\) depends separately on \(H_4^a\) and \(\vec{H}^a\). A rerun of the arguments in \([6]\) shows that

\[ 0 \leq w(H_4, 0) \leq 2 \tan(\pi/\beta)|H_4| \]

\[ 0 \leq w(0, \vec{H}) \leq 2 \tan(\pi/L) \sum_i |H_i| \] (5)
As $L \to \infty$, the second inequality implies $w(0, \vec{H}) = 0$. The first inequality remains unaffected. The system responds to a constant $H^a_i$ but not to a constant $\vec{H}^a$. It follows that all spatial gluon connected correlation functions vanish at zero three-momentum in lattice Coulomb gauge, and zero four-momentum in lattice Landau gauge. In the latter case, the limit is understood in the screening sequence as $\omega \to 0$ then $|\vec{k}| \to 0$, with $k = (\omega, \vec{k})$. It is important that the temporal gluon correlation functions are not bounded. Indeed, screening of colored gluons with a finite electric mass takes place in the high temperature phase [3].

The vanishing of the zero-momentum spatial gluon propagator in Landau gauge, takes the following form in the infinite volume limit

$$D_{ii}(0) = \lim_{|k| \to 0} \lim_{\omega \to 0} \int d^D x \ e^{i k \cdot x} \langle A_i(x) A_i(0) \rangle = 0 \quad (6)$$

in the unrenormalized case. For $D < 4$, the renormalization constants are finite and (6) holds in the continuum limit. For $D = 4$, the diverging character of the renormalization constants prevent us from making similar statements for the renormalized propagator in the continuum limit. For $|\vec{k}| \to 0$, [3] suggests a continuum behaviour analogous to $|\vec{k}|^2/((|k|^4 + m^4_M)$, that is screening masses that are moved to infinity at zero momentum, in line with Gribov’s suggestion in the vacuum [2]. Given the infrared problem noted in QCD perturbation theory [3], we expect $m_M \sim g^2 T$.

Recently, lattice simulations of the gluon propagator in Landau gauge have been carried out on a symmetric lattice in three-dimensions [11], and on an asymmetric lattice in four-dimensions [12]. The results in three-dimensions and for small momenta are in agreement with the original suggestion by Zwanziger [3], and consistent with our results. The fit to the lattice data in [11] was forced to $e^{-m_M z}$, while the analogue of the Gribov propagator at finite temperature suggests a fit to

$$\sum_{z_\perp} D_{ii}(z_\perp, z) \sim e^{-m_M z/\sqrt{2}} \cos \left( \frac{m_M z}{\sqrt{2}} + \frac{\pi}{4} \right) \quad (7)$$

This may explain the $z$-dependence observed in the reading of the magnetic mass in [11]. The pre-exponent in (7) indicates a change in sign in the propagator at large distance. Some subtleties related to the extrapolation to small momenta on a finite lattice have been discussed in [3].

3. The extension of the present observations to finite chemical potential runs into the difficulty that the fermion determinant is no longer real, making the measure $\rho(A)$ in [3] complex. However, the case $N_c = 2$ and even $N_f$ is an exception. In general, for $N_c = 2$ the fermion determinant admits an extra symmetry under charge conjugation (assuming that the continuum Dirac operator is recovered on a fine lattice). If $\lambda$ is an eigenvalue, so is $-\lambda$ by chiral symmetry, and $\lambda^*$ by the invariance under $\tau_3 C^{-1} K$ where $K$ is complex conjugation and $C$ is charge conjugation. Not all eigenstates are four-fold bunched. The purely real or purely imaginary eigenstates are only two-fold bunched. Hence the fermion determinant is real. For even $N_f$ it is even positive. In this case, most of the arguments presented in [3] applies for both the temporal and spatial correlations of the gluon connected correlation functions at $\beta = L$ (symmetric lattice). The precedent arguments apply to the spatial correlation functions for $\beta \neq L$ (asymmetric lattice).

In a recent argument [3] it was suggested that at large chemical potential $\mu$, color magnetic superconductivity can be sustained by Landau-damping. Below the light cone, the spatial gluon correlator follows from QCD perturbation theory in the form $1/(|k^2 - i m^2_\omega/|\vec{k}|)$ with $m_E \sim g_{\mu}$. The present arguments suggest that below the light cone the propagator for the spatial gluons may be analogous to the one suggested by Gribov, e.g. $|\vec{k}|^2/((|k|^4 + m^4)$, with permanent confinement of magnetic gluons in the infrared. We expect $m_s \leq m_E$. For $N_c = 2$ and $N_f$ even perturbation theory fails below the light cone at even large chemical potential. Could this affect the perturbative arguments for QCD with three colors at large chemical potential [3]?

4. We have suggested that the proximity of the Gribov horizon at finite temperature, and also for a special case of finite chemical potential, forces a vanishing of the color magnetization in QCD in both Landau and Coulomb gauges. On the lattice, this observation holds for any SU(N) or SO(N) gauge group, making the issue of the center $Z_N$ not important for this problem. In the continuum, this forces the spatial and connected gluon correlations to vanish at zero three-momentum, suggesting magnetic screening masses that become infinite in the infrared. These observations are relevant to the current arguments related to the onset of a magnetic mass in QCD at high temperature. They also suggest that QCD perturbation theory in the magnetic sector may not apply even at large chemical potential.

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