Charged particle acceleration by intermittent electromagnetic turbulence

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1. Introduction

Numerous observations of high energy particles (up to several 100 keV) [Sarafopoulos et al., 2001; Haaland et al., 2010] point to the presence of various mechanisms of charged particles heating and acceleration inside the magnetotail. These mechanisms may be divided into two main groups: quasi-stationary processes (e.g., magnetic reconnection in the downtail region resulting in acceleration due to the cross-tail electrostatic field $E_x$) and various non-stationary processes, where, in fact, inductive electric fields are responsible for particle acceleration. One can mention stochastic heating by electromagnetic turbulence (EMT) in the current sheet [Perrin et al., 2009; Artemyev et al., 2009], energization during the magnetic field dipolarization [Delcourt, 2002; Ono et al., 2009] and the acceleration due to the non-stationary magnetic reconnection [e.g., Hoshino, 2005; Retinò et al., 2008, and references therein]. Particles could gain only part of the dawn–dusk potential drop, which limits the efficiency of mechanisms from the first group to energies $<50–100$ keV. Therefore, to describe acceleration of relatively small particle populations up to several hundred keV, one needs to consider turbulent non-stationary mechanisms. On the one hand, regions in the magnetotail filled by turbulence are often strongly localized. On the other hand, turbulent acceleration is necessarily intrinsically coupled to the intensification of spatial transport, which again leads to a limitation in the maximum energy gain.

The intermittency level for magnetic field $B(t)$ can be estimated with structure function analysis $S_p = \Sigma_{n=0}^{|B(t_n + \Delta t) - B(t_n)|}$ [Frisch, 1995; Dudok de Wit and Krasnosel’skikh, 1996]. This function can be approximated as $S_p \sim \Delta^\zeta_p$, where $\zeta_p$ is the power law exponent. For Kolmogorov turbulence we have $\zeta_p \sim p$ [Frisch, 1995]. If the $\zeta_p$ curve deviates from the linear behavior, then we are dealing with intermittent turbulence [e.g., Dudok de Wit and Krasnosel’skikh, 1996, and references therein].

In this paper we propose the model of EMT with a variable level of intermittency (we define this level as the quantitative deviation of the power law exponent from the linear approximation $\zeta_p \sim p$). In the framework of this model we study efficiency of energization of charged particles and dependence of this energy gain on spatial transport.

2. Model of EMT

We developed the EMT model with a simple 2D geometry of the neutral plane of the magnetotail current sheet (see discussion in the last section). Only one component of the magnetic field $B_z$ is present and we consider the motion of particles in the $(x, y)$ plane. The EMT model consists of two parts: the stationary turbulence with the power law spectrum $B_{(1)}(x, y)$ and the field generated by an ensemble of non-stationary localized magnetic structures (magnetic clouds) $B_{(2)}(x, y, t)$. The first part provides the power law spectrum of the turbulence conforming with the observations and corresponds to quasi-stationary magnetic fluctuations. The second part corresponds to the small-scale stochastic dynamical processes in the magnetotail and additionally supports the charged particle energization. We consider this representation of the turbulence as a practical way to take into account some typical effects.
The distribution of amplitudes $A(x)$ over the spatial network ($n_x, n_y$) determines the behavior of EMT. For example, the random distribution should give EMT with the Kolmogorov structure function. To obtain $\zeta_p$, which is typical for the intermittent turbulence, we design a special distribution for $A(x)$. It is well known that a one-dimensional map $f_n = F(f_{n-1})$ can exhibit the property of intermittency. Manneville [1980] showed that, depending on $\beta$, $F(f_{n-1}) = f_{n-1} + f_{n-1}^\beta$ (mod1) demonstrates the multitude of various dynamic regimes. When $1 \leq \beta < 3/2$ the dynamics is normal, in the sense that the fluctuations of a random variable generated by the map are distributed according to Gaussian law. When $3/2 < \beta < 2$, the dynamics is transitional-anomalous, and when $\beta > 2$ the dynamics is strictly anomalous, described by the Levy statistics with the index $1/(\beta - 1)$. The latter regime corresponds to the map with intermittency. Here we modify this map to obtain not only a positive but also a negative values of $f_n$:

$$f_{n+1} = F(f_n) = \text{sgn}(f_n) \left( \left( f_n \right) + \left( f_n \right)^\beta + 1 \right) \text{ (mod2)} - 1 \right)$$

Here $f_n \in [-1, 1]$, $\beta \geq 1$. We used equation (3) to determine values of $A(x)$: $A(x) = A(0) + A(x) + A(1)(n_x, n_y) = F(A(0)(n_x, n_y)) + D$:

$$D = D(A(2)(n_x, n_y) - 2A(2)(n_x, n_y) + A(2)(n_x, n_y))$$

where the diffusion coefficient $D$ provides the connection between values of $A(x)$ at the net. The parameter $\beta$ allows to vary the level of the EMT intermittency. The parameter $d_0$ is defined in such a way that the energy $W(2) = \frac{1}{2\pi} \int_{L} L \frac{B(2)}{dr}$ is equal to $W(1)/5$ for all runs.

### 3. Model Verification

To obtain the structure function for the EMT model, we use the following technique. At the randomly chosen time $t_0$ the magnetic field $B = B(x, y) + B_0(x, y, t_0)$ is calculated along the line $x = y$ with a spatial step $\Delta x < 1/k_{\text{max}}$. The series $B_n(n)$ is the step number, $n = 1, 2, \ldots, N$. To obtain the structure function $S_p = \sum B_n(n + h) - B_n(n)$.

The approximation $S_p \sim h^{\zeta_p}$ gives the power law exponents $\zeta_p$ for various values of $\beta$ (Figure 2). The model parameter

![Figure 1. Scheme of the EMT model. Total field is a sum of (a) magnetic clouds component and (b) waves ensemble.](image)

![Figure 2. Power law exponent $\zeta_p$ as a function of its order $p$ for different values of $\beta$.](image)
The magnetic field observed by Interball Petrukovich controls the level of intermittency: an increase of 6\(R_t\) (700 \(\leq t \leq 10\) nT) were taken from the paper by Petrukovich [2005]. The straight line \(\zeta_p \sim \propto \) is normalized on \(\sim 1.5\) keV.

If we take \(R_t = 3.5\), then particles gain 150 keV (\(\varepsilon = 100\)) within the region \(R \sim (70–100)k_{\text{max}} \sim 6–8\ R_E\). Increase of proton fluxes on the energy \(\sim 100–500\) keV are often observed during active periods in the magnetotail [e.g., Ono et al., 2009; Haaland et al., 2010].

Another interesting effect obtained here is the existence of two regimes of particle acceleration. The dependence of \(\langle \varepsilon(t) \rangle\) on \(t\) from Figure 4 demonstrates that \(\langle \varepsilon(t) \rangle \sim t^*\) while \(t < t^*\) and \(\langle \varepsilon(t) \rangle \sim t\) if \(t > t^*\) (for \(\beta = 3.5\) \(t^* \approx 600\)).

The similar dependency of \(\langle \varepsilon(t) \rangle\) was found by Zelenyi et al. [2008] in the non-intermittent EMT model. The first regime \((\langle \varepsilon(t) \rangle \sim t^*)\) can be described by the theory of resonant acceleration, when each particle interacts with a single cloud (or wave) for a sufficiently long time. When particle gains enough energy, it began to wander between different waves (clouds) and acceleration becomes diffusive with \(\langle \varepsilon(t) \rangle \sim t\).

5. Conclusions

In this paper we propose a 2D model of EMT with a controlled level of intermittency. This model allows us to estimate the effect of intermittency on particle transport and acceleration in the vicinity of the neutral plane of the Earth’s magnetotail. Under the assumptions made, magnetic surfaces in the modelling box are strongly destroyed by the magnetic component of turbulence and charged particles can spend a sufficiently long time being trapped near the neutral plane. If the EMT is not strong enough to completely destroy the topology of field lines, particles should leave the neutral plane after a limited time interval [Speiser, 1967].

The relationship between spatial transport and acceleration in intermittent EMT is one of the most interesting results obtained in this paper. We show that charged particles can gain more energy within the same spatial domain \(R\) for more intermittent turbulence (for the same level of EMT energy). This result suggests a solution to the problem of

4. Efficiency of Particle Acceleration

This section is devoted to the numerical simulations of particle acceleration and transport in EMT for various levels of intermittency. The trajectories of \(10^4\) particles are integrated numerically. We use periodic boundary conditions. The time is normalized as \(t \rightarrow tq_B/mc\), coordinates as \(r \rightarrow r_{\text{max}}\) and energy \(\varepsilon = \frac{1}{2}(dr/dt)^2\) is normalized on \((q_B/mc_{\text{max}})^2\). We calculate an average energy of the particle ensemble \(\langle \varepsilon(t) \rangle\) and average spatial displacement \(R(t) = \langle \sqrt{(r(t) - r(0))^2} \rangle\) for various intermittency levels. Figure 4a demonstrates \(\langle \varepsilon(t) \rangle\): the increase of the intermittency level leads to the increase of energy gained by particles at a given time. Moreover, \(\langle \varepsilon(t) \rangle\) as a function of \(R(t)\) (Figure 4b) reveals a very important property of intermittent EMT: the energy gain by particles in the limited space region (for fixed \(R(t)\)) is larger for more intermittent turbulence. Therefore, the displacement \(R(t)\) grows with an increase of the intermittency level slower than \(\langle \varepsilon(t) \rangle\).

We estimate the efficiency of particle acceleration which can be provided by the model under consideration for typical magnetotail parameters. The magnitude of magnetic field fluctuation \(B_0 \sim 10\) nT \((q_B/mc \sim 1\) s\(^{-1}\)), the spatial scale of magnetic clouds \(1/k \sim 1000\) km and \(1/k_{\text{max}} \sim 500\) km, as a result \(q_B/mc_{\text{max}} \sim 500\) km/s and \(q_B/mc_{\text{max}}^2 \sim 1.5\) keV. If we take \(\beta = 3.0–3.5\), then particles gain 150 keV (\(\varepsilon = 100\)) within the region \(R \sim (70–100)k_{\text{max}} \sim 6–8\ R_E\). Increase of proton fluxes on the energy \(\sim 100–500\) keV are often

Figure 4. Average energy and displacement \(R\) of particles ensemble as a function of (a) time and (b) displacement \(R\) for various \(\beta\).
strong particle acceleration in spatially localized regions. Acceleration is necessarily associated with spatial transport for any of the EMT models [e.g., Perri et al., 2007; Zelenyi et al., 2008]. Thus, strong acceleration for non-intermittent turbulence is possible only if turbulence fills sufficiently large spatial regions. In contrast, intermittent EMT accelerates particles more effectively than it enhances spatial transport. 

[18] The main results of this work are obtained by means of numerical simulation. Although this method allows the prompt estimate of the influence of intermittency, further investigations of intermittency effects on particle dynamics are needed. By developing an analytical theory, it should be possible to verify the results obtained. The main effects of non-intermittent EMT were already reproduced in analytical estimates [see Zelenyi and Milovanov, 2004, and references therein]. Now these theories should be generalized for the intermittent case.

[19] In conclusion, we developed a 2D model of EMT with a controlled level of intermittency which, based on the comparison of the model structure functions with actual data, is shown to approximate observations with reasonable accuracy. We show that the increase in the intermittency level leads to the increase in the acceleration efficiency in a limited time interval and in a limited spatial domain.

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