Five-phase multilevel inverter with output voltage waveform optimized using soft-computing methods

W V Yong¹, W T Chew¹, S L Ong² and J H Leong¹

¹Faculty of Electrical Engineering Technology, Universiti Malaysia Perlis, Malaysia
²Faculty of Electronic Engineering Technology, Universiti Malaysia Perlis, Malaysia

nick.unimap@gmail.com

Abstract. In this paper, genetic algorithm (GA) and particle swarm optimization (PSO) are employed to optimize the switching angles for 5-phase 15-level asymmetrical cascaded H-bridge multilevel inverter (ACHBMLI). Both optimization algorithms are applied in selective harmonic minimization pulse width modulation (SHMPWM) based switching-angle calculation technique that aims to maintain the desired fundamental harmonic while minimize the undesired harmonics. The performance of GA- and PSO-SHMPWM are compared in term of switching angles, objective function, and cumulative distribution function. For fair comparison, both GA- and PSO-SHMPWM are implemented and analyzed in MATLAB with the same population size and total number of iterations. Furthermore, a PSIM simulation model of 5-phase 15-level ACHBMLI has also been developed to validate the effectiveness of GA- and PSO-SHMPWM.

1. Introduction

Multilevel inverter (MLI) can be considered as one of the potential direct current (DC) to alternating current (AC) converter for renewable energy application. It offers several advantages such as lower switching losses and voltage stress on power switches compared to conventional DC to AC converter such as two-level pulse-width modulation (PWM) inverter [1]. Besides, MLI able to generate a near sinusoidal staircase output voltage waveform without the need of bulky and lossy filters.

MLI topologies can be classified into diode-clamped multilevel inverter (DCMLI), flying capacitor multilevel inverter (FCMLI), and cascaded H-bridge multilevel inverter (CHBMLI) [2-6]. Among these three major MLI topologies, CHBMLI required the least components for generating the desired output voltage waveform. CHBMLI can be classified into two categories, which is symmetrical and asymmetrical. Asymmetrical CHBMLI able to generate higher number of voltage level with the same number of power semiconductor switches compared to symmetrical CHBMLI.

Several PWM methods to control the MLI have been reported in the past. Sinusoidal PWM and space vector PWM are the common high-frequency PWM method to control the MLI. However, high switching loss is the main drawback in both methods. Selective harmonic minimization pulse-width modulation (SHMPWM) is the low frequency PWM method that can be employed to obtain the optimum switching angles without having the problem of high switching loss. SHMPWM can maintain the fundamental component and minimize the low-order harmonics at the same time. In this method, a set of nonlinear transcendental equations that consist of trigonometric terms are required to be solved. Newton Raphson (NR) is a common numerical method used to solve the equations [7-8]. However, NR is highly dependent on good initial switching angles guesses and it provides solution for a narrow range of modulation index. Hence, NR is not effective in SHMPWM to determine the optimum switching angles. Another method to optimize the MLI switching angles is by employing soft-computing (SC) approach. The advantage of the SC technique is that it can find optimal solution without the need of a
good guess of initial switching angles. Genetic algorithm (GA) and particle swarm optimization (PSO) are the two famous SC techniques employed in the MLI switching-angle optimization [9]. To the best knowledge of authors, to date, there is no related work that compares GA and PSO optimized switching angles in 5-phase 15-level binary asymmetrical CHBMLI. Hence, this paper aims to explore and compare the performance of GA and PSO based SHMPWM techniques in switching-angle optimization for 5-phase 15-level asymmetrical CHBMLI.

2. Five-phase fifteen-level asymmetrical cascaded H-bridge multilevel inverter

Asymmetrical CHBMLI can be constructed by using multiple H-bridge (HB) modules connected in series. Each HB module consists of a DC source and four power semiconductor switches. Figure 1 shows the 5-phase 15-level asymmetrical CHBMLI. Each phase consists of 3 HB module and the voltage of the DC sources increase in a geometrical progression factor of 2, i.e. $V_{DC}$ at 1st HB (top), $2V_{DC}$ at 2nd HB (middle), and $4V_{DC}$ at 3rd HB (bottom). It can synthesize a 15-level output voltage waveform as shown in figure 2. The highest phase voltage level that can be produced is $+7V_{DC}$, which is the sum of all the DC source voltages in each phase ($V_{DC}+2V_{DC}+4V_{DC}$).

![Figure 1. 5-phase 15-level asymmetrical CHBMLI](image)
3. Switching-angle optimization using GA- and PSO-SHMPWM

Switching angles applied to MLI are required to be computed to obtain desired fundamental voltage while minimizing the undesired low order harmonics. The output voltage waveform shown in figure 2 can be represented mathematically in a Fourier Series as:

\[ V_{\text{out}}(\omega t) = \sum_{h=1,3,5,...}^{\infty} \frac{4V_{\text{DC}}}{h\pi} \left[ \sum_{i=1}^{7} \cos(h\theta_i) \right] \sin(h\omega t) \]

where \( h \) is the odd order number of harmonics, \( V_{\text{DC}} \) is the magnitude of DC source, \( \theta \) is the switching angles and \( i \) is the number of switching angles. The switching angles need to satisfy the condition of \( 0^\circ \leq \theta_1 \leq \ldots \leq \theta_i \leq 90^\circ \). From equation (1), the \( h^{\text{th}} \) harmonic can be expressed as:

\[ V_h = \frac{4V_{\text{DC}}}{h\pi} \left[ \cos(h\theta_1) + \cos(h\theta_2) + \ldots + \cos(h\theta_i) \right] \]

Figure 2. Output phase voltage waveform
The nonlinear transcendental equation, also known as selective harmonic minimization (SHM) equation can be written based on equation (2) as:

\[
V_1 = \frac{4V_{dc}}{\pi} \left[ \cos(\theta_1) + \cos(\theta_2) + \ldots + \cos(\theta_i) \right] \\
V_3 = \frac{4V_{dc}}{3\pi} \left[ \cos(3\theta_1) + \cos(3\theta_2) + \ldots + \cos(3\theta_i) \right] \\
V_7 = \frac{4V_{dc}}{7\pi} \left[ \cos(7\theta_1) + \cos(7\theta_2) + \ldots + \cos(7\theta_i) \right] \\
V_9 = \frac{4V_{dc}}{9\pi} \left[ \cos(9\theta_1) + \cos(9\theta_2) + \ldots + \cos(9\theta_i) \right] \\
V_{11} = \frac{4V_{dc}}{11\pi} \left[ \cos(11\theta_1) + \cos(11\theta_2) + \ldots + \cos(11\theta_i) \right] \\
V_{13} = \frac{4V_{dc}}{13\pi} \left[ \cos(13\theta_1) + \cos(13\theta_2) + \ldots + \cos(13\theta_i) \right] \\
V_{17} = \frac{4V_{dc}}{17\pi} \left[ \cos(17\theta_1) + \cos(17\theta_2) + \ldots + \cos(17\theta_i) \right] \\
\]

where \( V_1, V_3, V_7, V_9, V_{11}, V_{13}, \) and \( V_{17} \) are fundamental harmonic, 3rd harmonic, 7th harmonic, 9th harmonic, 11th harmonic, 13th harmonic, and 17th harmonic, respectively. The equation of modulation index, \( M_{\text{idx}} \) is given by:

\[
M_{\text{idx}} = \frac{V_i}{V_{i(\text{max})}} 
\]

where \( V_{i(\text{max})} \) is the maximum fundamental harmonic. Equation (4) can be rewritten as:

\[
V_i = V_{i(\text{max})} M_{\text{idx}} 
\]

In this paper, The \( V_3, V_7, V_9, V_{11}, V_{13}, \) and \( V_{17} \) will set to zero to minimize these harmonics. This is because in 5-phase application, the quintuple harmonics will be eliminated naturally. Equation (6) can be expressed by combining equation (3) and equation (5):

\[
\frac{V_{i(\text{max})} M_{\text{idx}} \pi}{4V_{dc}} = \left[ \cos(\theta_1) + \cos(\theta_2) + \ldots + \cos(\theta_i) \right] \\
0 = \left[ \cos(3\theta_1) + \cos(3\theta_2) + \ldots + \cos(3\theta_i) \right] \\
0 = \left[ \cos(7\theta_1) + \cos(7\theta_2) + \ldots + \cos(7\theta_i) \right] \\
0 = \left[ \cos(9\theta_1) + \cos(9\theta_2) + \ldots + \cos(9\theta_i) \right] \\
0 = \left[ \cos(11\theta_1) + \cos(11\theta_2) + \ldots + \cos(11\theta_i) \right] \\
0 = \left[ \cos(13\theta_1) + \cos(13\theta_2) + \ldots + \cos(13\theta_i) \right] \\
0 = \left[ \cos(17\theta_1) + \cos(17\theta_2) + \ldots + \cos(17\theta_i) \right] 
\]
GA and PSO are implemented to solve the SHM equation. GA is a biological evolutionary algorithm that depends on the biological process such as mutation, crossover, and selection, whilst PSO is a population-based algorithm that mimics the behavior of bird flock and fish swarm in food searching or migration. Figures 3(a) and (b) shows the pseudo code of the GA and PSO algorithm, respectively. In GA- and PSO-SHMPWM, an objective function (OF) shown in equation (7) is required to maintain the desired fundamental harmonic while minimize the undesired harmonics.

\[
OF = \left( p \times \frac{V_1^* - V_1}{V_t} \right)^g + \frac{1}{h_1} \left( 50 \times \frac{V_h}{V_t} \right)^r + \frac{1}{h_2} \left( 50 \times \frac{V_h}{V_t} \right)^r + ... + \frac{1}{h_n} \left( 50 \times \frac{V_h}{V_t} \right)^r
\]

(7)

where \(V_1^*\) is desired fundamental harmonics.

\[
OF = \left( p \times \frac{V_1^* - V_1}{V_t} \right)^g + \frac{1}{h_1} \left( 50 \times \frac{V_h}{V_t} \right)^r + \frac{1}{h_2} \left( 50 \times \frac{V_h}{V_t} \right)^r + ... + \frac{1}{h_n} \left( 50 \times \frac{V_h}{V_t} \right)^r
\]

4. Results and discussion

The GA- and PSO-SHMPWM switching-angle optimization algorithms have been implemented in MATLAB. For fair comparison, the population size and total number of iterations in both algorithms are set as 100 and 500, respectively. The algorithms are executed over a wide range modulation index from 0.00 to 1.00 with step of 0.01. 5 runs are carried out to obtain the optimal switching angles to reduce the probability of local optimal trap. Figures 4(a) and (b) show the switching angle trajectories obtained by GA and PSO, respectively. Clearly, GA and PSO able to produce optimal sets of switching angles over a wide range of modulation index. Figures 5(a) and (b) compare the OF and cumulative

Figure 3. Pseudo code: (a) GA and (b) PSO
distribution function (CDF) achieved by GA and PSO, respectively. As shown in figure 5(a), the PSO results in lower OF than GA from \( M_{idx} = 0.68 \) to \( M_{idx} = 0.71 \) and \( M_{idx} = 0.75 \) to \( M_{idx} = 0.76 \). Unlike GA, the PSO capable of achieving a minimum OF of \( 10^{-26} \) and below over nearly 6% of the modulation range as evidently shown in figure 5(b). Hence, it can produce higher accuracy of optimal switching angles than GA.

![Figure 4. Switching angles trajectories: (a) GA and (b) PSO](image)

![Figure 5. GA versus PSO simulation results: (a) OF and (b) CDF](image)

A 5-phase 15-level asymmetrical CHBMLI is modelled using PSIM to validate the performance of the MLI controlled using GA and PSO optimized switching angles. The DC voltage sources in the model is configured in such that the total \( V_{DC} \) is 84.00 V and the load is pure resistive load. The model is tested at \( M_{idx} \) of 0.72. Fast Fourier Transform (FFT) is carried out to obtain the harmonic spectrum of the waveform. Figures 6(a) and (b) show the phase voltage waveform and its harmonic spectrum of the MLI controlled using GA- and PSO-SHMPWM, respectively. The amplitude of the fundamental harmonic for both GA- and PSO-SHMPWM are approximately 76.82 V, which matches the \( M_{idx} \) tested on the MLI. This can be proved by using equation (4). Also, the undesired harmonics (3rd, 7th, 9th, 11th, 13th, and 17th) in the phase voltage waveform are successfully minimized. Figures 7(a) and (b) show the line-to-line voltage waveform and its harmonic spectrum of the MLI controlled using GA- and PSO-SHMPWM, respectively. As shown in the harmonic spectrum, the quintuple harmonics such as 5th harmonic and 15th harmonic, are eliminated. This shows the feasibility of the MLI controlled using GA- and PSO-SHMPWM.
| Output Voltage Waveform | Harmonic Spectrum |
|------------------------|------------------|
| ![Output Voltage Waveform](a) | ![Harmonic Spectrum](a) |

**Figure 6.** Phase voltage waveform and harmonic spectrum: (a) GA and (b) PSO

| Output Voltage Waveform | Harmonic Spectrum |
|------------------------|------------------|
| ![Output Voltage Waveform](b) | ![Harmonic Spectrum](b) |

| Line-to-line Voltage Waveform | Harmonic Spectrum |
|-----------------------------|------------------|
| ![Line-to-line Voltage Waveform](a) | ![Harmonic Spectrum](a) |

**Figure 7.** Line-to-line voltage waveform and harmonic spectrum: (a) GA and (b) PSO
5. Conclusion

In this paper, the performance of GA- and PSO-SHMPWM for optimizing the switching angles in 5-phase 15-level asymmetrical CHBMLI have been analyzed and compared. The results show that GA and PSO can be applied in the SHMPWM to compute the optimal switching angles over a wide range of modulation index. In the simulated OF and CDF, it shows that PSO-SHMPWM is able to produce higher accuracy of optimal switching angles compared to GA-SHMPWM. For phase and line-to-line voltage harmonic spectrum, both GA- and PSO-SHMPWM is able to yield optimized switching angles which can maintain the fundamental harmonic while minimize the undesired harmonics.

Acknowledgments

This work was supported by the Ministry of Education Malaysia through the Fundamental Research Grant Scheme (FRGS/1/2020/TK0/UNIMAP/02/54).

References

[1] Majid T F Waqar A K Jiangbiao H Nathan W and Mostafa A 2020 Fast Online Diagnosis of Open-circuit Switching Faults in Flying Capacitor Multilevel Inverters Chinese Journal of Electrical Engineering 6 53.
[2] Hendi M and Venny Y 2020 New Topology Multilevel Inverter Type Diode Clamped Five Level Single Phase IOP Conf. Ser.: Mater. Sci. Eng. 807.
[3] Nandhakumar A Mugilraj S and Mohanrajan N D 2021 IOP Conf. Ser.: Mater. Sci. Eng. 1084 012099.
[4] Anzari M Meenakshi J and Sreedevi V T 2014 Simulation of a transistor clamped H-bridge multilevel inverter and its comparison with a conventional H-bridge multilevel inverter International Conference on Circuits, Power and Computing Technologies [ICCPCT-2014] 958.
[5] Grandi G Loncarski J and Dordevic O 2015 Analysis and Comparison of Peak-to-Peak Current Ripple in Two-Level and Multilevel PWM Inverters IEEE Transactions on Industrial Electronics 62 2721.
[6] Reddy K R et al 2017 Cascaded Multi-level Inverter Topology Developed from a Modified H-bridge Electric Power Components and Systems 0 1191.
[7] Youssef M Z Woronowicz K Aditya K Azeez N A and Williamson SS 2015 Design and development of an efficient multilevel DC/AC traction inverter for railway transportation electrification IEEE Transactions On Power Electronics 31 3036.
[8] Haghdar K and Shayanfar H A 2018 Selective Harmonic Elimination with Optimal DC Sources in Multilevel Inverters using Generalized Pattern Search IEEE Transactions On Industrial Informatics 14 3124.
[9] Taghizadeh H and Hagh M T 2008 Harmonic elimination of multilevel inverters using particle swarm optimization IEEE International Symposium on Industrial Electronics 393.