Effect of alpha value change on thrust quadcopter Qball-X4 stability testing using backstepping control

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Abstract. Quadrotor or commonly referred to quadcopter or drone, has 4 kinds of movements. One of those movements is the impulse of the movement. In this study, a QBall-X4 quadcopter controller is using a backstepping control system to achieve movement that can reach the height when doing thrust. The results showed that the backstepping method can adjust the height and stabilize the roll angle, pitch and yaw, by adjusting alpha value (a stabilizer constant). The more precisely the alpha value of the system is more stable and the response to reach steady state is faster, with small errors. At setpoint 0 to 3 condition an error of 0.0216.

1. Introduction
A quadcopter is an aircraft that has a simple control mechanism that has the potential to take off, hover, fly maneuver, and land even in small and narrow areas [1]. In the use of quadcopter for various purposes, the stability of hover in quadcopter is very important and must be possessed by quadcopter for optimum utilization [2].

In the study of hover stability arrangement on quadcopter using backstepping controls showed good results [3], where the control method was able to control the stability of hover so as to overcome the various disorders given [4]. In the previous study, the backstepping block control method worked well enough when there was no external interference [5], in which the controller was able to control the micro quadcopter motion for a defined waypoint with a small error tracking averages [6]. Therefore to implement it, the quadcopter and actuators are used so that the controller is expected to achieve the stability of the roll and pitch angles and reach the z position height during hover as done during simulation.

2. Quadcopter model
Quadcopter Q-ball X4 is an unmanned helicopter combined with four motors whose patterns are crossed [7]. Quadcopter produces lift by the value of all four motors [8].
2.1. Linearization of quadcopter dynamics models

This section will explain about linearization of dynamics quadcopter [9]. When the quadcopter is in hover condition. The yaw angle is 0 rad and the angular speed of roll, pitch, and yaw are close to 0 rad/s [10].

The state space equation for the linear model of the roll and pitch dynamics can be expressed:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{1}{J} \\
1 & 0 & -\omega
\end{bmatrix} \begin{bmatrix}
\phi \\
\theta \\
v
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\omega
\end{bmatrix} = \Delta u_2
\] (2)

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{1}{J} \\
1 & 0 & -\omega
\end{bmatrix} \begin{bmatrix}
\phi \\
\theta \\
v
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\omega
\end{bmatrix} = \Delta u_2
\] (3)

The linear model of dynamics position which obtained on the axis x and y in the state space is as follows:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{4K}{J} \\
0 & 0 & -\omega
\end{bmatrix} \begin{bmatrix}
x \\
y \\
v
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\omega
\end{bmatrix} u
\] (4)

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{4K}{J} \psi \\
0 & 0 & -\omega
\end{bmatrix} \begin{bmatrix}
x \\
y \\
v
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\omega
\end{bmatrix} u
\] (5)

The value of that parameter is obtained from [11], which written in Table.1 if the v state variable used to present the actuator dynamic.

| Table 1. The Parameter value from quadcopter dynamic modelling. |
|---------------------------------------------------------------|
| Parameter | Symbol | Value |
| Mass      | \(m\)  | 3.499 kg |
| Gravity   | \(g\)  | 9.81 kg/m\(^2\) |
| Inertia Moment on X axis | \(J_{xx}\) | 0.03 kg.m\(^2\) |
| Inertia Moment on Y axis | \(J_{yy}\) | 0.03 kg.m\(^2\) |
| Inertia Moment on Z axis | \(J_{zz}\) | 0.04 kg.m\(^2\) |
Table 1. Cont.

| Parameter                          | Value               |
|------------------------------------|---------------------|
| Distance of rotor from the         | 0.2 m               |
| center of mass                     |                     |
| Drag force                         | 3.13×10⁻⁵           |
| Trust Force                        | 7.5×10⁻⁷            |
| Actuator Bandwidth                 | 15 rad/s            |
| Constant thrust force              | 120 N               |

3. Backstepping controls for altitude edge subsystem [4]

To find the altitude on the quadrotor relate to the axis z. State \( \dot{x}_5 \) and \( \dot{x}_6 \) which represents the quadrotor altitude of the earth, is taken from the quadrotor model in Equations (4 and 5). The block diagram of the hover height adjustment system on the quadrotor can be seen in Figure 2.

Figure 2. The block diagram of the hover height adjustment system on the quadrotor (z).

The last step to look for control signals \( U_1 \) for altitude (altitude /z) in the same way that is:

3.1. Determine the tracking error to look for errors from altitude (z)

\[
\begin{bmatrix} \dot{y} \\ \dot{y} \\ \dot{y} \\ v_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 4K_T \psi & 0 \\ 0 & 0 & -\omega & 0 \\ 0 & 0 & 0 & \omega \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \dot{y} \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega \end{bmatrix} u
\]

\( e_7 = zds - x_5 \) (8)

where \( z_{ds} : z \) (altitude ref); \( x_5 : z \) (altitude /z)

3.1.1. Using Lyapunov function to test which stability \( V(e_7) \) is definite and positive -\( V(e_7) \) is definite negative

\[
V(e_7) = \frac{1}{2} e_7^2
\]

\[
\dot{V}(e_7) = \frac{1}{2} e_7 \dot{e}_7
\]

\[
\dot{V}(e_7) = \frac{1}{2} e_7 (Z_{ds} - \dot{e}_5)
\]

\[
\dot{V}(e_7) = e_7 (Z_{ds} - x_7)
\]

To obtain \( e_7 \) which is stable then inserted a virtual input control with \( a_7 > 0 \)

\[
x_6 = \dot{Z}_{ds} + a_7 e_7
\]

From Eq. (12) is obtained

\[
\dot{V}(x_6) = e_7 \dot{Z}_{ds} - x_6
\]

\[
\dot{V}(e_7) = e_7 \dot{Z}_{ds} - (x_6 + a_7 e_7) = -a_7 e_7^2
\]

\[
\dot{V} < 0
\]
Next to get $e_8$, by changing the variable

$$e_8 = x_6 - ZDs - a_7e_7$$  \(17\)

### 3.1.2. Lyapunov function augmented

$$V(e_7e_8) = \frac{1}{2}(e_7^2 + e_8^2)$$  \(18\)

$$\dot{V}(e_7e_8) = e_7\dot{e}_7 + e_8\dot{e}_8$$  \(19\)

So in can signal overall control as follows:

$$U_2(roll) = \frac{1}{b_1}(e_7 - a_1x_{10}x_{12} - a_1x_{10}\Omega - a_4(e_2 + a_1e_1) - a_2e_2)$$  \(20\)

$$U_3(pitch) = \frac{1}{b_2}(e_3 - a_3x_{0}\Omega - a_3(x_{0} + a_3e_3) - a_4e_4)$$  \(21\)

$$U_4(yaw) = \frac{1}{b_3}(e_6 - a_3x_{0}\Omega - a_3(x_{0} + a_3e_3) - a_6e_6)$$  \(22\)

### 4. Simulation results

#### 4.1. Open loop quadcopter testing

In quadcopter can occur thrust when the quadcopter is visually flying float and silent not attached ground or upward force that experienced quarotor equal to gravity. Figure 3 shows the nominal moment or rotational speed equal to zero.

The simulation test of open loop quadcopter system shown in Figure 4 with a pulse with mathematical equation at $z = 0.3m$ (10%) at 10 seconds, roll angle = 0.1 rad / s at 12 seconds, pitch angle = 0.0001rad / s at the 14th second, and the yaw = 0.1 rad / s angle at the 16th second indicates that the system is not capable of overcoming the noise that can be seen in the response graph where the height value (z) begins to fall at the 12th second which is about 0.6 m. Time value constant $\tau = 1.55$ seconds and settling time 6.2 seconds.

Further testing of the same open loop quadrotor system is shown in Figure 5 with a disruption at $z = 0.6 m$ (20%) at 10 seconds, roll angle = 0.2 rad / s at 12 seconds, pitch angle = 0.2 rad / s at seconds 14, and the yaw = 0.2 rad / s angle at the 16th second indicates that the system is not capable of overcoming the disturbance which can be seen in the response graph where the height value (z) drops at 12 seconds by 2.2 m. Value $\tau = 1.55$ seconds and settling time 6.2 seconds.
4.2. Change alpha when testing quadcopter using backstepping control

The effect of alpha on the height response (z) can be seen in Table 2. The greater the alpha value of the system is getting stable and the α7 is most affected in the settings.

In the backstepping control there is a positive (positive) definite (α) function of the stabilizer or its value must be greater than zero, in this study there are 8 alpha α1 and α2 to adjust the angle of roll, α3 and α4 to adjust the pitch angle, α5 and α6 to adjust the angle yaw, and α7 and α8 to set the height (z).

From the table above can be seen the higher the alpha value of the system is getting stable and the time value constant (τ) is getting smaller. From the above data when α = 10 value τ = 0.6 seconds, α = 200 value τ = 0.5 seconds, and α = 315 value τ = 0.4791 seconds.

Figure 4. Elevation response (z), rolling angle, pitch, and yaw open loop.

Figure 5. Elevation response (z), rolling angle, pitch, and yaw on open loop.

Figure 6. Elevation response, rolling angle, pitch, and yaw.
Table 2. Effect of alpha value change on thrust.

| No | a1 | a2 | a3 | a4 | a5 | a6 | a7 | a8 | z   | Response        |
|----|----|----|----|----|----|----|----|----|-----|----------------|
| 1  | 0.1| 0.1| 0.1| 0.1| 0.1| 0.1| 0.1| 0.1| 124.18| Unstable       |
| 2  | 1  | 1  | 0.9999| 0.9999| 1  | 1  | 1  | 1  | 7.908 | Overshoot      |
| 3  | 10 | 1  | 0.9999| 0.9999| 1  | 1  | 1  | 1  | 7.908 | Overshoot      |
| 4  | 10 | 1  | 0.9999| 0.9999| 1  | 1  | 1  | 1  | 7.908 | Overshoot      |
| 5  | 10 | 1  | 0.9999| 0.9999| 10 | 1  | 1  | 1  | 7.908 | Overshoot      |
| 6  | 200| 1  | 0.9999| 0.9999| 200| 1  | 315| 1  | 3.0156| Stable         |
| 7  | 200| 1  | 0.9999| 0.9999| 200| 1  | 315| 1  | 3.0156| Stable         |
| 8  | 200| 1  | 0.9999| 0.9999| 200| 1  | 300| 1  | 3.0164| Stable         |
| 9  | 200| 1  | 0.9999| 0.9999| 200| 1  | 310| 1  | 3.0158| Stable         |
| 10 | 10 | 1  | 0.9999| 0.9999| 10 | 1  | 10 | 1  | 3.491 | A little overshoot|
| 11 | 100| 1  | 0.9999| 0.9999| 10 | 1  | 10 | 1  | 3.491 | A little overshoot|
| 12 | 100| 1  | 0.9999| 0.9999| 100| 1  | 10 | 1  | 3.491 | A little overshoot|
| 13 | 100| 1  | 0.9999| 0.9999| 100| 1  | 100| 1  | 3.491 | A little overshoot|
| 14 | 100| 1  | 0.9999| 0.9999| 100| 1  | 200| 1  | 3.0245| somewhat stable |
| 15 | 200| 1  | 0.9999| 0.9999| 200| 1  | 200| 1  | 3.0245| somewhat stable |

5. Conclusions
The simulation result shows that setting using backstepping with proper alpha value determines the stability of roll angle (φ), pitch (θ), yaw (ψ) close to zero and to set the height (z) so that the error is very small that is 0.0156 m. The greater the alpha value, the quicker the response to achieve steady state stability of roll angle (φ), pitch (θ), yaw (ψ) close to zero and to set the height (z) so that the error is.

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