On Gravitational Repulsion

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Abstract

The concepts of negative gravitational mass and gravitational repulsion are alien to general relativity. Still, we show here that small negative fluctuations - small dimples in the primordial density field - that act as if they have an effective negative gravitational mass, play a dominant role in shaping our Universe. These initially tiny perturbations repel matter surrounding them, expand and grow to become voids in the galaxy distribution. These voids - regions with a diameter of $40 h^{-1}$ Mpc which are almost devoid of galaxies - are the largest object in the Universe.

1. Strange and Not So Strange Negative Masses

Two (gravitational) negative masses attract each other. The equivalence principle requires that their inertial masses is negative as well. In this case the masses move away from each other. A negative and positive mass pair is even stranger. They repel each other. The positive mass moves away from the negative one but the negative mass accelerates towards the positive one. The distance between the masses does not vary while they are jettisoned together at a constant acceleration. Momentum and energy are conserved as
the negative momentum and kinetic energy of the negative mass balance the positive energy and momentum of the positive one.

This is so strange that in spite of the equivalence principle it is worthwhile to consider particles with a negative gravitational mass and a positive inertial mass. These are somewhat more reasonable. Two such masses attract each other and as a result they move towards each other. A (gravitational) negative mass repels a (gravitational) positive mass and now both masses move away from each other. This resembles, of course, electrically charged particles (with the corresponding change in the sign of the force). A test particle with an “effective” negative gravitational mass and a positive inertial mass can be easily simulated. A light object immersed in a denser fluid feels an “effective” repulsive gravitational force that mimics the gravitational repulsion felt by a test particle with a negative gravitational mass in the gravitational field of a positive mass.

This trivial example of a test particle raises the natural question. Are there sources of “effective” repulsive gravitational force? Are there objects with an “effective” negative gravitational mass? Surprisingly, the answer is yes. Moreover, these “effective” negative masses are responsible for the creation of the largest structures in the Universe!

2. Cosmological Negative Gravitational Masses

We live in an expanding universe which is close to its critical density (for simplicity we consider here an $\Omega = 1$ Universe). The Cosmic Microwave Background Radiation (CMBR) tells us that in the past the Universe was practically homogeneous. Our existence, the observations galaxies and even the CMBR itself reveals that the perfectly homogeneous FRW model is an idealization. Small primordial deviations, of order $10^{-5}$ at horizon crossing, were present. These fluctuations grew during the matter dominated era to become the very large deviations from homogeneity observed today.

We understand very well how does a cosmological positive density fluctuation evolve. This is not surprising, after all we live within one! A spherical overdense region behaves like a closed universe within an outer flat one. This closed universe expands until it reaches its maximal size and then it begins to collapse. The collapse continues until “virialization” when the kinetic

\footnote{Somewhat strangely this phase is often referred to as “recollapse” - however this region has never collapsed before!}
and gravitational energies are equal and the region has shrunk to half of its maximal physical size. This simple picture could be misleading at times. Gravitational collapse is unstable to non-spherical modes. Even if the spherical approximation is initially valid it will break down during the collapse and flat pancakes rather than round balls are more likely to form. Most observed cosmological objects, like galaxies or clusters, are indeed far from spherical.

Clearly there is an equal number of underdense and overdense regions. Still the fate of underdense regions was, somehow, largely ignored, and when it was discussed it was forgotten. We show that cosmological underdense regions behave like “effective” negative gravitational masses (with a positive inertial mass). They repel nearby positive masses and attract other underdense regions. Using this analogy we examine the evolution of cosmological underdense regions and we find that primordial negative density perturbations, small initial dimples in the density field, grow and become the observed voids in the galaxy distribution. Using recent galaxy redshift surveys we demonstrate that these voids are the largest objects in the Universe containing most of the volume of the Universe today.

Consider an idealized spherical underdense region. At some initial time $t_i$ (e.g. at horizon crossing) it is characterized by a comoving size $R_i$ and a negative fractional underdensity $\delta \rho / \rho = -\epsilon_i$. As long as the physical size is larger than the horizon the dimple is frozen with a constant underdensity and a constant comoving size. Once it crosses the horizon, the dimple behaves, in analogy with an overdense region, like an open universe within the outer flat universe. As an open universe it expands faster than the surrounding flat one. Matter within it expands away from the center of the dimple faster than the surrounding matter. This open universe also influences strongly the surrounding regions. A sphere surrounding the dimple contains less mass than an equivalent sphere elsewhere. Consequently a shell on its boundary expands faster than average, as if there is a negative mass repelling it. The dimple has an “effective” negative gravitational mass! The surrounding matter forms a high density ridge along the rim of the dimple. At a time $t_{sc}$ given by $\epsilon_i (t_{sc}/t_i)^{2/3} \approx (1 + z_i)/(1 + z_{sc}) \approx \delta_{\text{crit}}$, (where $\delta_{\text{crit}}$ ranges between 2.5 and 4.5 depending on the initial velocity distribution) the shell located just outside the dimple, which feels the strongest repulsion.

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2 This statement depends, of course, on gauge choice - but for practical purposes it yields a good description.
overtakes its outer neighboring shell \[2\]. Shell crossing occurs. The density of the rim becomes infinite and the model of an open universe within a flat one breaks down. At \(t_{sc}\) the comoving radius of the dimple \(R_{sc}\) is 1.7 times the initial comoving radius \(R_i\). The local density is then \(\approx 0.2\) of the average density.

After that a second phase begins in which shell crossing continues and the expansion of the underdense region settles quickly to a self similar solution \[3, 4, 5\]. The surrounding high density ridge expands, in comoving coordinates, as \(t^{2/9} \propto (1 + z)^{-1/3}\) and its comoving radius satisfies: \(R/R_{sc} = ((1 + z_{sc})/(1 + z))^{1/3}\). This expansion is much slower than the expansion during the earlier phase.

Pairs or more complicated systems of multiple dimples are more difficult to analyze. Generally they do not preserve their shape and mass as matter flows out from them. Consequently even the simple “two body problem” of two nearby underdense regions can be addressed only using numerical N-body simulations. Dubinski \textit{et al.}, \[6\] have considered several idealized configurations of interacting dimples. They found that two nearby dimples expand towards each other repelling the surrounding matter and creating a high density ridge between them. This ridge is later broken, its matter is repelled outwards in a direction perpendicular to the line connecting the two centers. At this stage the two underdense regions merge to a single one. This region continues to expand outwards repelling surrounding matter and becoming more and more spherical. This is a nice feature of underdense regions, making their analysis simpler. While a collapsing region is unstable to non-spherical perturbations, an expanding one becomes more spherical with time.

### 3. Voids in the Galaxy Distribution

Perhaps one of the most intriguing findings of dense and complete nearby redshift surveys has been the discovery of numerous large voids on scales of \(\sim 50h^{-1}\) Mpc in the galaxy distribution. Although the voids are a fundamental element of the large-scale structure (LSS) of the universe, the realization that they dominate the LSS is relatively recent \[7\]. Early surveys published during the 70’s, like the Coma/A1367 redshift survey \[8\] and the Hercules/A2199 redshift survey \[9\], gave the first indications for the existence of voids, each revealing a void with a characteristic diameter of \(\sim 20h^{-1}\) Mpc. Surprising as
these findings might have been, it was not before the discovery of the Boötes void [10] that the voids caught the attention of the astrophysical community (for a review about the early void explorations, see [1]).

The unexpectedly large void found in the Boötes constellation, confirmed to have a diameter of $\sim 60 h^{-1}$ Mpc [12], naturally brought up the question whether the empty regions we observe are a common feature of the galaxy distribution, or rather rare exceptions. Wide-angle yet dense surveys probing relatively large volumes of the nearby universe, established that the voids are indeed a common feature of the LSS, and as such must be incorporated into any valid model of it. The first slice from the Center for Astrophysics (CfA) redshift survey [13] revealed the picture of a universe where the galaxies are located on the surfaces of bubble-like structures, with diameters in the range $25 - 50 h^{-1}$ Mpc. The extensions of the CfA survey [7], complemented in the south hemisphere by the Southern Sky Redshift Survey (SSRS) and its extension, the SSRS2 [14, 15] have shown that not only large voids exist, but more importantly – that they occur frequently (at least judging by eye), suggesting a compact network of voids filling the entire volume.

Until recently no suitable algorithm was available to quantitatively study the properties of voids. Only some gross estimates were inferred from visual inspection of the existing redshift surveys. During the last two years we have developed the void finder algorithm [16, 17, 18] which identifies voids in redshift surveys and measures their size and underdensity. We have used the void finder to analyze the void distribution in the SSRS2 survey [16] and the IRAS survey [17] and we have verified the visual picture of a void filled universe in which galaxies are mostly located along walls surrounding voids (see Figs. 1 and 2). In both surveys we find that voids whose diameter is $\approx 40 Mpc h^{-1}$ with a typical density of galaxies of order 10% of the average density contain more than half of the volume of the universe. These voids are clearly the largest object observed in the Universe.

4. Voids, Negative Masses and the LSS

The observed correlation between voids in the galaxy distribution found by the void finder [17] and regions with low dark matter density found by potent [19] show clearly that the origin of the voids must be gravitational. The previous analysis of the evolution of an underdense regions suggests the following picture: Primordial underdense regions - dimples - act like
cosmological “negative” masses. These dimples are the seeds of the observed voids. While overdense regions collect more and more matter and shrink in both real and comoving sizes, dimples repel matter and expand. We can view the centers of the underdense regions as effective “negative” gravitational masses that repel matter. The repelled matter is aligned along walls located between the “negative” centers. Voids, with low galaxy and dark matter densities are centered on these “negative” masses and are surrounded by walls. Eventually the walls are torn apart, the voids merge and a network of larger voids on a large scale forms.

It is illuminating to consider underdense regions of a given comoving scale $\lambda$. These dimples cross the horizon at the moment when their physical size equals horizon’s size, with a typical amplitude $\epsilon_i(\lambda)$ (which is scale independent if the primordial spectrum is a scale independent one). The dimples grow in amplitude and in comoving size. By the time that they reach shell crossing their comoving size has increased by a factor of 1.7. With a typical density of 20% of the average density they definitely qualify as voids. These voids practically touch each other and fill the universe. Later the walls between the voids break down. The voids merge and new voids on a larger scale appear. The network of voids is replaced in an effective self-similar manner by a network of voids with a larger characteristic scale. At each moment dimples that reach shell crossing form the current prominent voids, these are destroyed latter forming larger voids and so on.

Denoting the current radius of the observed voids by $R_{\text{voids}}$ we find $\lambda_{\text{voids}} = R_{\text{voids}}/1.7$ and $\epsilon_i(\lambda_{\text{voids}})[1 + z_i(\lambda_{\text{voids}})] = \delta_{\text{crit}}$. Amazingly $\epsilon_i$ determined in this way approximately equals $10^{-5}$, as determined from extrapolation of CMBR observations \[3\]. This overall agreement is impressive. It demonstrates that gravitational repulsion, caused by dimples in the primordial density field - cosmological “negative” masses - create the voids, the largest structures in the Universe.

References

\[3\] A detailed analysis \[20\] shows that this estimate indicates a slightly larger primordial fluctuations than what other methods, such as a scale independent interpretation of the CMBR data or observations of rich clusters, yield. However, one can think of numerous refinements of both types of estimates.
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Figure 1: Voids in the IRAS 1.2-Jy survey. The colored areas mark the intersection of the SGZ = 30h$^{-1}$Mpc plane with the three-dimensional voids. Void 4 is the Local Void. The walls surrounding the voids are highlighted by drawing dark lines connecting nearby wall-galaxies. The dark area marks the ZOA, caused by the Galaxy.
Figure 2: Three-dimensional view of the voids in the *IRAS* survey. The ZOA, caused by the Galaxy, runs horizontally across the image. The area at the left, near the ZOA, with no voids, corresponds to the Great Attractor. The absence of voids from the lower, right-hand part of the image, is due to the Cetus wall and the Perseus-Pisces supercluster.