Abstract: The problem of tracking control for piezoelectric actuator (PEA) is investigated in this article. In consideration of that, the control precision of PEA is being restricted due to the hysteretic effect. For the purpose of improving the tracking performance, two issues are, respectively, taken into account – the compensation of the hysteretic effect and the controller design for the PEA system. To solve these problems, first, a new hysteresis observer based on sliding mode control theory is established. The design of the observer has considered the uncertainty in PEA modelling and the exogenous disturbance. Besides, according to Lyapunov theory, the hysteresis observer can asymptotically estimate the hysteretic character with \( H_{\infty} \) performance. Second, a fault-tolerant controller based on robust backstepping control theory is presented to make the PEA system to track the desire input. At last, numerical examples are given to verify the effectiveness of the proposed methods.

1 Introduction

PEA systems are widely used, because of its high stiffness, solid-state actuation, and fast responses, such as energy harvester [1], vibration control [2], and positioning system [3]. However, strong hysteretic effect, which is contained by piezoelectric materials, can degrade the control performance [4]. Specifically, the hysteretic effect can induce positioning error as high as 10–15% of the stage travel range [5].

Hysteretic effect is a kind of non-linearity with local memory. In order to analyse the hysteretic effect, many mathematical models have been proposed, which include Preisach model [6], Prandtl–Ishlinskii model [7], Bouc–wen model [8], and lines-segments hysteresis model [9]. Besides that, neural network technology has also been applied to describe the hysteretic effect [10]. For eliminating the non-linear influence which is caused by the hysteretic effect, feedforward compensations which is established by the inverse hysteresis models have been the focus and main content. Based on the lines-segment model, a non-smooth inverse model is presented in [11]. Since the Prandtl model is non-differentiable, an approximate inversion of the Prisach hysteresis operator is established in [12]. As Bouc–wen model can describe a variety of hysteretic effects with less parameters [13], one can easily obtain its inversion model. A number of studies have been made to compensate the hysteretic effect by using inverse Bouc–wen model. In [8], tracking control for piezoelectric stack APA120S is carried out by using inverse Bouc–wen model and proportion integral differential (PID) algorithm. The study of 3\textsuperscript{rd} freedom piezotube scanner is addressed in [14], in which the design of the compensator is based on the combination of multiplicative inverse Bouc–wen models.

However, since the creep and temperature effects cannot be ignored in high frequency, the application of inverse hysteresis models is limited [15]. To overcome these phenomena, the hysteresis observer has attracted considerable attention. As the hysteretic effect can be considered as a bounded disturbance, a hysteresis observer is proposed in [16], and a Kalman filter-based non-linearity observer is established in [17]. Luenberger-wise observer is studied in [15, 18, 19].

As for the tracking controller design for the PEA system, both feedback and feedforward control techniques have been studied. An adaptive backstepping controller is proposed in [20]. A PID sliding model controller is designed in [5], in which a \( \chi \)T parallel micromanipulator is used to verify the control algorithm, and PID control for PEA system can be seen in [8, 15, 16]. Motivated by the above discussion, an observer-based tracking control system for PEA is considered in this paper. Specifically, first, based on the sliding mode theory, a new hysteresis observer is proposed. The design of the observer has considered the modelling error and the external disturbances. Furthermore, by using the \( H_{\infty} \) theory and linear matrix inequalities (LMI) technology, the hysteresis observer can be proved estimating the hysteresis state with \( H_{\infty} \) performance. Second, a robust backstepping controller is established in the paper. By the reasonably designed controller, the PEA system can track on the reference input despite the observation error and the external disturbances.

This paper is organised as follows: in Section 2, the problem of PEA system is summarised; in Section 3, a hysteresis observer is introduced to compensate the hysteresis effect; in Section 4, an observer-based robust backstepping controller is developed; and in Section 5, the effectiveness of the control system is shown in simulations. Finally, this paper is concluded in Section 6.

2 Problem statements

According to the earlier research, a PEA dynamic model with Bouc–wen hysteretic effect can be stated as follows:

\[
\begin{align*}
mx(t) + bx(t) + kx(t) &= (ru - h) \\
\dot{h}(t) &= ad[\dot{u}(t) - \beta u(t)]h(t)\dot{h}(t)^{-1} - \gamma u(t)h(t)^{\alpha} 
\end{align*}
\]

where \( x(t) \) is the displacement (m) of the actuator; \( h(t) \) represents the hysteretic non-linear term; \( m \) (kg), \( b \) (Ns/m), \( k \) (N/m) denote the mass, damping coefficient, stiffness of the PEA, respectively; \( d \) is the ratio coefficient (m/V) of the displacement; the input voltage to the piezoelectric actuator is represented by \( u(t) \). \( \alpha, \beta, \gamma \) are the parameters used to determine the hysteretic loop’s magnitude and shape, and \( n \) controls the smoothness of the hysteretic loop.

In order to facilitate analysis, (1) can be rewritten as

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= \frac{k}{m}x_2(t) - \frac{b}{m}x_1(t) + kx(t) \\
v(t) &= H[u(t)] = du(t) - h(t)
\end{align*}
\]

where \( x_1(t) \) takes the place of \( x(t) \), \( x_2(t) = x(t) \), and \( H[u(t)] \) describes the hysteretic character of PEA. Without loss of
PEA system can be described by the following block diagram. For the sake of simplification, (2a) can be substituted by

\[ y(t) = C\tilde{x}(t) \]

where \( \tilde{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \), and

\[
A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = (1 \quad 0)
\]

A typical piezoelectric actuator stage is shown in Figs. 2a and b that shows a physical model of PEA.

### 3 Hysteresis observer design

As shown in Fig. 1, a PEA model contains two parts: a non-linear hysteretic model and a second-order linear system. The relationship between the input and the output of the hysteretic model can be characterised as a hysteretic loop. For instance, define \( u(t) = \sin(t) \), \( d = 1.5 \), \( a = 1 \), \( \beta = 2 \), \( \gamma = 1.8 \), and the non-linear relationship between the desire input and PEA output can be shown in Fig. 3.

In order to compensate the hysteretic effect shown above, a hysteresis observer is established in this paper. The block diagram of the observer–compensation system is depicted in Fig. 4.

In order to analyse and design the hysteresis observer, (1) can be rewritten as

\[
\begin{align*}
\dot{\tilde{x}}(t) &= AX(t) + Bu(t) + |u(t)|MX(t) + u(t)Q\tilde{x}(t) \\
y(t) &= C\tilde{x}(t)
\end{align*}
\]

where \( \tilde{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \) is the output of the hysteretic model. Then, (2a) can be rewritten as

\[
\begin{pmatrix} x_1 \dot{t} \\ x_2 \dot{t} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{m}{k} & -\frac{m}{b} & \frac{k}{m} & \frac{k}{m} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \dot{t} \\ u_2 \dot{t} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta \gamma & -\beta \gamma \\ 0 & 0 & -\gamma \beta & -\gamma \beta \end{pmatrix}, \quad \beta = 1, \quad \gamma = 0
\]

Partly inspired by [21], the hysteresis observer designed in this paper is given as

\[
\begin{align*}
\xi(t) &= TA\tilde{X}(t) + T\tilde{B}u(t) + [u(t)]T\tilde{M}\tilde{X}(t) + u(t)TQ\tilde{x}(t) + L(y(t) - C\tilde{x}(t)) \\
\dot{\tilde{X}}(t) &= \xi(t) + Ny(t)
\end{align*}
\]

where \( \xi(t) \) is the state vector of the hysteresis observer, \( \tilde{x}(t) \) is the estimated value of \( \tilde{x}(t) \), \( T, N \) are real constant matrices with appropriate dimensions to be designed. Apparently, \( \tilde{x}(t) \) contains the estimated value \( \hat{h}(t) \), which can be obtained by \( \hat{h}(t) = (0 \quad 0 \quad 1)\tilde{X}(t) \). The proposed hysteresis observer design is based on sliding model control and \( H_\infty \) control theory, and the design procedure is given as follows:

**Lemma 1:** For any matrices \( R \in \mathbb{R}^{a \times b} \), \( M \in \mathbb{R}^{b \times c} \) and \( Z \in \mathbb{R}^{d \times c} \), if \( \text{rank}(M) = c \), then for the following equation

\[
RW = Z
\]

the general solution of \( R \) can be obtained by

\[
R = ZW^T + U[I_b - WW^T]
\]
where $U$ is a given matrix with appropriate dimension [22].

**Remark 1:** It is easy to derive that $\text{rank}[^I, \overset{\sim}{C}] = 3$. By the definition of generalised inverse matrix, there always exists a set of $T$ and $N$ satisfying the following condition:

$$T + N\overset{\sim}{C} = I.$$  

Then, based on Lemma 1, the parameter matrices $T$ and $N$ can be given by

$$(T \quad N) + \left(\begin{array}{c|c}
[I] & \overset{\sim}{C} \\
\hline
[C] & \overset{\sim}{C}
\end{array}\right) + S \left(\begin{array}{c|c}
[I] & \overset{\sim}{C} \\
\hline
[C] & \overset{\sim}{C}
\end{array}\right)$$

where $S$ is a given matrix.

Define $e_o(t) = \overset{\sim}{X}(t) - \overset{\sim}{\dot{X}}(t)$ as the observation error, then from Remark 1, one can obtain

$$e_o(t) = (T + NC)\overset{\sim}{X}(t) - \overset{\sim}{\dot{X}}(t)$$

Based on (4) and (5), the dynamic equation of $e_o(t)$ can be obtained as

$$e_o(t) = (T\overset{\sim}{A} - LC)e_o(t) + e_m(t) + o(t)$$

where $o(t)$ denotes the external disturbance. $e_m(t)$ represents the entire modelling error, and it follows that $e_o^T(t)e_o(t) \leq [H\overset{\sim}{X}(t)]^T[H\overset{\sim}{X}(t)].$

**Definition 1:** For a dynamic equation $\rho(t) = D\rho(t) + o(t)$, where $\rho(t)$ represents the dynamic equation state vector, $o(t) \in L_2[0, T]$ denotes an exogenous disturbance, $D$ is a given matrix with appropriate dimensions. Then, the dynamic equation is asymptotically stable with $H_\infty$ disturbance attenuation level $\chi$, if the following conditions hold:

$$\int_0^t \rho^T(s)P\rho(s)ds \leq \rho^T(0)Q\rho(0) + \chi^2 \int_0^t o^T(s)o(s)ds$$

where $P$ and $Q$ are symmetric and positive-definite matrices.

**Lemma 2:** For any real matrices $X$, $Y$, with appropriate dimensions, it holds that

$$X^TY + Y^TX \leq \sigma^{-1}X^TNX + \sigma Y^TN^{-1}Y$$

where $N$ is a symmetric and positive-definite matrix, and $\sigma$ is a constant which is $> 0$ [23].

**Theorem 1:** Consider the hysteresis observer (5), if there exist positive constants $\lambda$, $\epsilon$, symmetric and positive-definite matrices $P_o$, $Q_o$ such that

$$(T\overset{\sim}{A} - LC)^TP_o + P_o(T\overset{\sim}{A} - LC) + \epsilon H^TH$$

$$+ \epsilon^{-1}P_oP_o + \frac{1}{\lambda}P_oP_o + Q < 0$$

Then, the hysteresis observer can asymptotically estimate the hysteretic non-linear term with $H_\infty$ performance.

**Proof:** Consider the following Lyapunov function candidate:

$$V_o(t) = e_o^T(t)P_o e_o(t)$$

Taking the time derivative of both the sides of (9) gives

$$V_o(t) = e_o^T(t)P_o e_o(t) + e_o^T(t)e_o(t)$$

$$= e_o^T(t)[(T\overset{\sim}{A} - LC)^TP_o + P_o(T\overset{\sim}{A} - LC)] e_o(t)$$

$$+ e_o^T(t)P_o e_o(t) + e_o^T(t)P_o e_o(t) + \omega^T(t)P_o e_o(t)$$

$$+ \frac{e_o^T(t)P_o e_o(t)}{\lambda} + \frac{1}{\lambda}e_o^T(t)P_o e_o(t)$$

$$+ \frac{1}{\lambda}e_o^T(t)P_o e_o(t) + \frac{1}{\lambda}e_o^T(t)P_o e_o(t)$$

(10)

Then, according to Lemma 2, it follows that

$$V_o(t) = e_o^T(t)[(T\overset{\sim}{A} - LC)^TP_o + P_o(T\overset{\sim}{A} - LC) + \frac{1}{\lambda}P_oP_o] e_o(t)$$

$$+ \frac{1}{\lambda}e_o^T(t)P_o e_o(t) - \lambda o(t)$$

$$+ \frac{1}{\lambda}e_o^T(t)P_o e_o(t)$$

$$+ \frac{1}{\lambda}e_o^T(t)P_o e_o(t)$$

(11)

According to the condition (8), one can get

$$V_o(t) \leq - e_o^T(t)Q_o e_o(t) + \epsilon \|o(t)\|^2$$

(12)

Integrating (12) from 0 to $t_f$ yields

$$\int_0^{t_f} e_o^T(t)Q_o e_o(t)dt \leq e_o^T(0)Q_o e_o(0) + \epsilon \int_0^{t_f} o^T(s)o(s)ds$$

(13)

Then, from Definition 1, the hysteresis observer can asymptotically estimate the state of (4) with $H_\infty$ performance. Hence, the hysteretic non-linear term $h(t)$ can be estimated. This completes the proof.

**Remark 2:** The condition (8) in Theorem 1 can be recast as an LMI solving problem by the following procedure: defining the changes of variables $J = P_oL$, (8) can be rewritten as

$$\begin{pmatrix}
(T\overset{\sim}{A})^TP_o + P_o(T\overset{\sim}{A}) - \epsilon I^T J^T - J^T C \\
\epsilon H + \epsilon H^T \\
\epsilon^2 I - \epsilon J \\
\end{pmatrix} < 0$$

(14)

Then, the gain matrices of the hysteresis observer can be obtained as $L = P_o^{-1}J$.

### 4 Robust backstepping controller design

In this section, a backstepping sliding mode controller is designed to make the output of PEA to track the reference inputs. Due to the uncertainty modelling error, (2a) can be rewritten as

$$\begin{pmatrix}
x_1(t) = x_1(t) \\
x_1(t) = A_1 x_1(t) + A_2 x_1(t) + ku(t) + F
\end{pmatrix}$$

(15)

where $A_1 = -k/m$, $A_2 = -b/m$. As a matter of convenience, we suppose that the observer can compensate the hysteretic effect. Without generality, the observation error and the modelling error can be contained by $F$, and it follows that $|F| \leq \bar{F}$.

**Theorem 2:** The dynamic system (15) can track the reference inputs $r(t)$ if the backstepping sliding mode controller is given by
where \( k_1, k_2, k_3, k_4 \) are real constants, \( z(t) \) is the track error which can be defined by \( z(t) = x(t) - r(t) \). \( z_i(t) = x_i(t) - r_i(t) + k_i z(t) \) is the virtual control input, \( \sigma(t) = k_i z_i(t) + z_i(t) \) denotes the switching term.

**Proof:** Consider the following Lyapunov function:

\[
V_i(t) = \frac{1}{2} z_i^2(t)
\]  

(17)

Taking the time derivative of both the sides of (17) gives

\[
\dot{V}_i(t) = z_i(t) \dot{z}_i(t) = z_i(t) \dot{z}_i(t) - k_i z_i^2(t)
\]  

(18)

Based on the definition of \( \sigma(t) \), one can obtain

\[
\sigma(t) = k_i z_i(t) + z_i(t) = (k_i + k_i) z_i(t) + z_i(t)
\]  

(19)

Thus, if \( \sigma(t) = 0 \), then both \( z_i(t) \) and \( z(t) \) will be equal to 0, which implies \( V_i(t) \leq 0 \). Take the Lyapunov candidate function of the form

\[
V_i(t) = V_i(t) + \frac{1}{2} \sigma^2(t)
\]  

(20)

Then, using (20) and differentiating \( V_i(t) \) with respect to time yield

\[
\dot{V}_i(t) = \dot{V}_i(t) + \sigma(t) \dot{\sigma}(t)
\]  

\[
\dot{V}_i(t) = z_i(t) \dot{z}_i(t) - k_i z_i^2(t) + \sigma(t) \dot{\sigma}(t)
\]  

\[
\dot{V}_i(t) = z_i(t) \dot{z}_i(t) - k_i z_i^2(t) + \sigma(t) \dot{k}_i [z_i(t)]
\]  

\[
\dot{V}_i(t) = z_i(t) \dot{z}_i(t) - k_i z_i^2(t) + \sigma(t) [k_i z_i(t)]
\]  

\[
\dot{V}_i(t) = z_i(t) \dot{z}_i(t) - k_i z_i^2(t) + \sigma(t) [k_i z_i(t)] - k_i z_i(t) + \dot{z}_i(t) - r_i(t) + k_i z_i(t)
\]  

\[
\dot{V}_i(t) = z_i(t) \dot{z}_i(t) - k_i z_i^2(t) + \sigma(t) [k_i z_i(t)] + A_i z_i(t) + r_i(t) + A_i z_i(t) + r_i(t) - k_i z_i(t)
\]  

\[
\dot{V}_i(t) = k_i \dot{z}_i(t) + F - r_i(t) + k_i z_i(t)
\]  

(21)

Substituting (16) into (21) gives

\[
\dot{V}_i(t) = z_i(t) \dot{z}_i(t) - k_i z_i^2(t) - k_i \dot{\sigma}^2(t)
\]  

(22)

\[
\dot{V}_i(t) = -k_i z_i^2(t) - k_i \dot{\sigma}^2(t) - k_i \dot{\sigma}(t)
\]  

Define

\[
\tilde{z}^T R \tilde{z} = [z_i(t) \ z_i(t)] \begin{bmatrix} k_i & k_i \ k_i & k_i \end{bmatrix} \begin{bmatrix} k_i & k_i \ k_i & k_i \end{bmatrix} \begin{bmatrix} z_i(t) \ z_i(t) \end{bmatrix}
\]  

(23)

which leads to

\[
\dot{V}_i(t) \leq -\tilde{z}^T R \tilde{z} - k_i \dot{\sigma}^2(t)
\]  

(24)

Then, one can obtain \( V_i(t) \leq 0 \) by choosing \( k_i, k_2, k_3, k_4 \). This completes the proof. \( \square \)

5 Experimental results

In this section, a numerical example is used to illustrate the effectiveness of the developed control system. The values of PEA are taken from the previous work [15], which are given in Table 1.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( m \)   | 2.17 kg | \( a \)   | 0.38  |
| \( b \)   | 4378.67 Ns/m | \( \beta \) | 0.0335 |
| \( k \)   | 3 x 10^4 N/m | \( \gamma \) | 0.0295 |
| \( d \)   | 9.013 x 10^{-7} m/V | \( n \) | 1 |

**Table 1** Values of the PEA

![Fig. 5 Hysteretic effect under the compensation](image)

For the design of the observer (5), according to Remark 1, \( T \) and \( N \) are obtained as

\[
T = \begin{bmatrix} 0.9 & 1 & 0.1 \ 0.9 & 0 & 0 \ 0 & 0.9 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0.1 \ 0 \ 0 \end{bmatrix}
\]

From Theorem 1, by solving the LIM (14), \( L \) can be obtained as

\[
L = 1 \times 10^3 \begin{bmatrix} 0 \ -6.912 \ 0 \end{bmatrix}
\]

Taking the reference input \( r(t) = 10^{-4} \sin 10t \mu m \), the compensation results are shown in Fig. 5. As shown in Fig. 5, the relationship between the desire input and the hysteresis model output is approximate linearity. This phenomenon verifies the effectiveness of the designed hysteresis observer.

For the backstepping controller (16), the parameters are chosen as follows: \( k_i = 10, k_2 = 0.1, k_3 = 1 \times 10^4, k_4 = 5 \), and \( F = 0.01 \). Then, the tracking performance is shown in Fig. 6.

As shown in Fig. 6, by the control system designed in this paper, the output of PEA system can track the reference input.
In this paper, we have dealt with the problem of tracking control for PEA system. First, a sliding mode-based observer is proposed. Though the compensation provided by the observer, the hysteresis effect in PEA system can be remarkably reduced. Then, according to backstepping control theory, an observer-based controller is established. The design of the controller has considered the observation error and the modelling error. At last, simulations have confirmed the effectiveness of the proposed control methods.

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8 References

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