The proper motion of sunspots umbra in the rising phase of Cycle 24

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Abstract

The differential rotation is the difference in the rate of rotation of each latitude in the Sun indicating that the Sun is not a solid body. The differential rotation is the result of the interaction between rotation and convection and causes dynamo circulation that affects the cycle of solar activity. In this research, we measured the coordinates of 304 sunspots umbra as tracer in the rising phase of the Solar Cycle 24 to obtain the differential rotation equation. Coordinates of sunspots were measured using AIA images at a wavelength of 4500 Å from SDO (Solar Dynamic Observatory) with JHelioviewer software. The areas of sunspots were measured using HMI (Helioseismic Magnetic Imager) images with ImageJ software. From the measurement, we derived the differential rotation equation and the relation of sidereal rotation and area of sunspots umbra. The differential rotation equation obtained in this research is \( \omega(B) = (14.27 \pm 0.01) - (0.78 \pm 0.10) \sin^2 B (\circ/\text{day}) \). There is a difference between the differential rotation equation of the northern and southern hemispheres that indicates the asymmetry between different hemispheres. We got the sidereal rotation of sunspots with area < 5 MH is 0.70% higher than sunspots with area > 15 MH.

1. Introduction

Differential rotation is the result of the interaction between rotation and convection and causes dynamo circulation that affects the solar activity cycle [1]. Differential rotation calculations provide an observational constraint on theoretical models of the rotating convection zone and the solar MHD dynamo which have an important role in generating magnetic fields and solar activity [2]. Differential rotation is a phenomenon of the Sun that is easily observed, but difficult to understand. Despite the long history of observations, solar differential rotation has yet to be adequately explained [3].

There are several methods to measure the differential rotation: spectroscopic, tracer, and helioseismology methods. In the spectroscopic method, we use spectral measurement and the Doppler equation to obtain the value of differential rotation [4]. In the tracer method, photosphere features such as sunspots and magnetic fields are used. Sunspots act as tracers of fluid dynamics [5]. Sunspots are easier to use as tracers than other features (magnetograms, filaments, coronal holes, etc.) because their intensity has high contrast to their surroundings, is easy to identify, and has a long lifetime. However, differential rotation only reaches the convection layer because the solar core has rotational characteristics close to rigid bodies.

The differential rotation obtained from sunspots as a tracer gives a higher rate than the spectroscopic method, due to the strong magnetic field in sunspots. The methods that use features with
strong magnetic fields (tracer method) will produce differential rotations at a higher rate than the methods that use features with weak magnetic fields (spectroscopic methods). Another method to measure the differential rotational rates is helioseismology. The sun oscillates simultaneously in millions of acoustic oscillation modes, called p-modes, which brings information about the solar interior to the surface [3]. Inversion of the measurement of the p mode frequency produces a map of rotation below the surface of the Sun that shows that the differential rotation extends deep into the convective zone and varies less with depth than latitude.

The objective of this research is to measure the proper motion of sunspots umbra to determine the differential rotation using images data from AIA (Atmospheric Imaging Assembly) instrument at a wavelength of 4500 Å onboard the SDO (Solar Dynamic Observatory) satellite. We also determine the relation of sidereal rotation rate and area of sunspots umbra using continuum images data from the HMI (Helioseismic and Magnetic Imager) instrument.

2. Method

To measure the proper motion of sunspots umbra, we used images from the AIA instrument at a wavelength of 4500 Å (white-light). Sunspot features are visible in 4500 Å wavelength compared to other wavelengths on the AIA instrument. The proper motion was obtained by measure the position of the umbra of the sunspots in Carrington Heliographic coordinates using Jhelioviewer software (https://www.jhelioviewer.org/). After obtaining the coordinates of the sunspots in latitude and longitude, we calculated the linear regression to find the longitudinal drift ($\frac{dL}{dt}$) in degrees/hour. The time cadence of the data that we used to find longitudinal and latitudinal drift is 8 hours. We converted that value to degrees/day to find the differential rotation equation. We used ImageJ (https://imagej.nih.gov/ij/), a Java-based software that is able to process and analyze images, to measure the area of sunspots umbra. The images that we used are continuum images from HMI (https://sdo.gsfc.nasa.gov). The format of the image is in jpg with resolution 4096x4096 pixels and the image size is around 4 – 5 MB.

The data in this research are sunspots during cycle 24 from 2010 to 2013. In 2010 and 2011, the solar activity was low so there were just a few sunspots that appeared on the solar disk. While in 2012 to 2013, the solar activity was high. Sunspot measurements are performed at an 8-hour interval every day. The sunspots that we used in the measurement have a lifetime of more than 24 hours and the shape of sunspots umbra doesn’t change quickly (stable). To avoid the appearance of asymmetrical sunspots on the edge of the Sun (Wilson effect), the sunspots that we measured are located less than 70° from the meridian. We checked the name and position of the active area of the sunspot groups from https://solarmonitor.org.

3. Results and discussions

3.1 Differential rotation equations

The differential rotation is expressed in the following equation [4]:

$$\omega\left(B\right) = a - b \sin^2 B - c \sin^4 B$$

(1)

where $\omega\left(B\right)$ is the sidereal rotation rate (°/day), a, b, and c are the differential rotation constants, and B is the average latitude. For low latitude features, such as sunspot, the last term is omitted [4]. The value of the sidereal rotation rate is obtained from the following formula.

$$\omega\left(B\right) = \frac{\left(14.184 \degree/day\times T\right) + \left(dL/dT\right)}{T}$$

(2)
where $14.184^\circ$/day is the value of Carrington velocity [3] and $T$ (day) is the duration or age of individual sunspot. Longitudinal drift ($dL/dt$) is the longitude drift of each individual sunspot that we got from linear regression between the longitude of sunspots and time.

We used linear regression between the sidereal rotation rate denoted by $\omega(B)$ of 304 sunspots and average latitude for each individual sunspot denoted by $\sin^2B$ to find the equation of differential rotation. We inserted the values of $a$ and $b$ from the linear regression into the equation $\omega(B) = a + b \sin^2B$. The error of each constant ($a$ and $b$) was obtained from the least square method.

For the measurement of 304 sunspots umbra, we obtained the differential rotation equation as follows:

$$\omega(B) = (14.27 \pm 0.01) - (0.78 \pm 0.1) \sin^2B \text{ (°/day)}$$

(3)

The differential rotation equations for sunspots on different hemispheres:

Sunspots in the northern hemisphere: $\omega(B) = (14.2661 \pm 0.0133) - (0.68 \pm 0.14) \sin^2B \text{ (°/day)}$ (4)

Sunspots in the southern hemisphere: $\omega(B) = (14.2714 \pm 0.0180) - (0.85 \pm 0.15) \sin^2B \text{ (°/day)}$ (5)

In equations 3, 4, and 5, the constant $b$ is negative, and it shows that the sidereal rotation rate decreases at high latitudes. The difference between differential rotation equations in the northern and southern hemispheres indicates the asymmetry. Differential rotation models with different line colors are shown in figure 1. The sidereal rotation rate of sunspots in the southern hemisphere has a higher rotation rate than in the northern hemisphere (figure 1). The value of constant $a$ that refers to equatorial rate in the southern hemisphere is larger of about $0.0053^\circ$/day or 0.745 m/s than in the northern hemisphere.

![Figure 1. Sidereal rotation rate for all sunspots (red line), sunspot in the northern hemisphere (blue line), and sunspot in the southern hemisphere (green line).](image)

However, because the differential rotation of the southern hemisphere has a greater value of constant $b$ or a large reduction in velocity, the sidereal rotation rate of the southern hemisphere will be lower than the northern hemisphere for sunspots at higher latitudes. At a latitude of $30^\circ$, sunspots in the northern hemisphere have a sidereal rotation rate of $14.096^\circ$/day (1973.4 m/s) and a sidereal rotation rate of $14.058^\circ$/days (1968.2 m/s) for sunspots in the southern hemisphere. The difference in the sidereal rotation rate in the two hemispheres indicates asymmetry of the north and south hemispheres or the complexity of the interaction of the magnetic field in the interior.

The rotation rate of a sunspot suggested that it corresponds to plasma rotation at the depth of the sunspot [6]. The MDI (The Michelson Doppler Imager) research results show that in deeper regions,
differential rotation becomes insignificant because the rotational rate increases at high latitudes. The sidereal rotation rate at the equator in this research has a value of 14.269°/day which corresponds to a depth of 1 \( R_\odot \). At a latitude of 40°, the sidereal rate of 13.948°/day corresponds to a deeper area of about 0.71 \( R_\odot \) from the core or 0.29 \( R_\odot \) from the surface. Thus, the roots of the sunspots are in a convection layer with a surface depth of 0.287 ± 0.003 \( R_\odot \) [3,7].

Table 1 shows the differential rotation from various studies. It can be seen that the differential rotation from the period before 2000 has a higher value than the current period [13] and this research which measures sunspots in Solar Cycle 24. When we compared our results with the differential rotation using sunspots as tracer, the differential rotation constant obtained from this research similar to large spots with an area of \( A_r < 10 \) MH (1 millionth Hemisphere or 1 MH = 3 \times 10^6 km²) which has a slower rate than other sunspots with smaller area [9].

Differential rotation measurements using magnetograms have a smaller rate than differential rotation using sunspots as tracers. Magnetograms have a weaker magnetic field than sunspots, so the root is estimated closer to the surface. Despite using different features, the constant \( a \) from this research has a fairly close value with differential rotation measurement using the magnetogram in the previous results [13]. While for constant \( b \), the value obtained in this study shows a lower one.

At the equator, the value of the sidereal rotation rate obtained by the previous result was 2001.4 m/s, very close to the sidereal rotation rate obtained from this research, 1997.7 m/s [13]. At a latitude of 40°, the sidereal rotation rate in this study has a value of 1952.7 m/s, higher than the previous result with the sidereal rotation rate value of 1894.6 m/s. There is also a comparison of differential rotations in the northern and southern hemispheres. In this research, the constants \( a \) and \( b \) obtained were greater in the southern hemisphere, but the previous result got the opposite value [13].

Table 1. Differential rotation equation from various study [3]

| Ref. | \( a \) (°/day) | ± (°/day) | \( b \) (°/day) | ± (°/day) | Period       | Note                                                                 |
|------|----------------|----------|----------------|----------|--------------|----------------------------------------------------------------------|
| [8]  | 14.342         | -2.952   |                |          | 1934-1944    | 136 sunspots with lifetime > 1 solar rotation                          |
|      |                |          |                |          |              | Sunspot with area < 5 (MH)                                            |
| [9]  | 14.510         | 0.005    | -2.861         | 0.079    | 1921-1982    | Sunspot with 5 < area < 10 (MH)                                       |
|      |                |          |                |          |              | Sunspot with area < 10 (MH)                                           |
|      | 14.402         | 0.009    | -2.609         | 0.079    | 1921-1982    | Magnetograms                                                          |
|      | 14.244         | 0.018    | -2.629         | 0.079    | 1921-1928    | Magnetograms                                                          |
|      | 14.382         | 0.019    | -1.999         | 0.133    | 1975-1991    | Magnetograms                                                          |
|      | 14.214         | 0.009    | -1.673         | 0.064    | 1984-1987    | 276 individual sunspots                                               |
|      | 14.5           | 0.74     | -2.5           | 0.43     | 1989-1993    |                                                                      |
|      | 14.296         | 0.006    | -1.847         | 0.056    | February 2011| Magnetograms                                                          |
|      | 14.292         |          | -1.584         |          | February 2011| Magnetograms in northern hemisphere                                   |
|      | 14.299         | -2.124   |                |          | February 2011| Magnetograms in southern hemisphere                                  |
|      | 14.27          | 0.01     | -0.78          | 0.10     | 2010-2013    | Sunspot umbra                                                         |
|      | 14.2661        | 0.0133   | -0.68          | 0.14     | 2010-2013    | Sunspot umbra in northern hemisphere                                |
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| Ref. | a (°/day) | ± (°/day) | b (°/day) | ± (°/day) | Period | Note |
|------|-----------|-----------|-----------|-----------|--------|------|
|      | 14.2714   | 0.0180    | -0.85     | 0.15      | 2010-2013 | Sunspot umbra in southern hemisphere |

Constants $a$ and $b$ in the differential rotation equation obtained from this research are lower than previous studies. This might be caused by Cycle 24 that had lower activity than the previous cycles. This low activity can be seen from the lower peak of Cycle 24 or the number of sunspots in the maximum phase of the lower Cycle 24. Low solar activity indicates that the magnetic field strength is low in Cycle 24. To get better results and confirm the low differential rotation value on Cycle 24, we need complete differential rotation data for Cycle 24, from the beginning of the cycle to the end of the cycle.

3.2 Relation between sidereal rotation rate and area of sunspots

Sunspots umbra areas were measured for sunspots in 2010-2011. We also measured 27 sunspots in 2012 that have the age of $t \geq 1$ week. Figure 3 shows the relation between the sidereal rotation rate with an area of sunspots. The relation obtained from linear regression is

$$\omega(B) = (14.21 \pm 0.01) - (4.39 \pm 2.01) \times 10^{-5} Ar \, \text{°/day}$$ (6)

where $\omega(B)$ is the sidereal rotation rate and $Ar$ is the area of the sunspot.

![Figure 2. Relation between sidereal rotation rate with area of sunspot.](image)

From figure 2, it is shown that small sunspot areas tend to have a higher sidereal rotation rate than large sunspots. This is consistent with a previous study [4] which plots the sidereal rotation rate for different sunspot sizes, and it appears that large sunspots have the lowest sidereal rotation rate. In Table 1, there are results of the previous study [9] that divides sunspots according to their size to find differential rotation values ranging from sunspots with $Ar < 5$ MH, 5-10 MH, and $Ar < 10$ MH. The sidereal rotation rate obtained for each range has a different value. By reviewing the constant $a$, sunspots with $Ar < 5$ MH have the highest rate of 14.51 °/day. Sunspots with size of 5-10 MH have a sidereal rotation rate of 14.40°/day and sunspots with sizes of $Ar < 10$ MH have sidereal rotation rate of 14.24°/day.

By converting the millionth Hemisphere (MH) to pixel units (1 pixel = 0.5 arc seconds), we got that 15 MH is equal to 335 pixels² and 5 MH is equal to 111.7 pixels². We obtained sunspots with the area $Ar > 15$ MH have average sidereal rotation rate 14.18 °/day (1984.78 m/s), while sunspots with
the area $A_r < 15 \, M_H$ have an average sidereal rotation rate of 14.21°/day (1988.84 m/s). The average rotational rate for sunspots with the area $A_r < 5 \, M_H$ is 14.28°/day (1998.64 m/s). Sunspots with the area $A_r < 5 \, M_H$ was 0.70% faster than sunspots with the area of $A_r > 15 \, M_H$. This value is quite close to the previous study [4] which states that sunspots with an area less than 5 MH rotate faster by about 0.8% than large sunspots with an area 15 MH.

Small sunspots have smaller magnetic field fluxes and move faster than large sunspots that have larger magnetic field fluxes [14]. The different depths of the magnetic flux ropes cause a difference in velocity. The rate of young sunspots corresponds to the sidereal rotation rate at a depth of 0.93 $R_\odot$ [3]. The recurrent sunspots correspond to the sidereal rotation rate at a depth of 0.80 $R_\odot$ or 0.98 $R_\odot$ [3]. Smaller sunspots rotate at a rate that matches the depth near the peak of the rotation rate (0.93 $R_\odot$). However, larger spots correspond to rates in deeper regions (perhaps as deep as 0.71 $R_\odot$, at the base of the convection zone) or just below the surface. It seems that large sunspots with strong magnetic fields do not lift quickly with magnetic buoyancy through the solar plasma as occurs in small sunspots with weak magnetic field fluxes [14]. Large sunspots may have greater viscosity shifts, but this hypothesis is still more numerical study [4].

4. Conclusions
We have measured 304 sunspots umbra in 2010 - 2013 to find the differential rotation equation and the relation of sidereal rotation and area of sunspots umbra. The differential rotation equation obtained from 304 umbra sunspots is $\omega(B) = (14.27 \pm 0.0133) - (0.68 \pm 0.14) \sin^2(B)$ (°/day). The sidereal rotation rate at the equator is 14.27°/day which corresponds to a depth of 1 $R_\odot$. At a latitude of 40°, the sidereal rotation rate is 13.9°/day, corresponds to a deeper area of around 0.71 $R_\odot$. There is a slight difference in the differential rotation equation between different hemispheres, which indicates asymmetry. The differential rotation equation for sunspots in the northern hemisphere is $\omega(B) = (14.2661 \pm 0.0133) - (0.68 \pm 0.14) \sin^2(B)$ (°/day) and for sunspots in the southern hemisphere is $\omega(B) = (14.2714 \pm 0.0180) - (0.85 \pm 0.15) \sin^2(B)$ (°/day). In general, sunspot roots are in the convection layer.

The average sidereal rotation rate for sunspots with an area of $A_r > 15 \, M_H$ is 14.18°/day (1984.8 m/s). Sunspots with an area of $A_r < 5 \, M_H$ have an average sidereal rotation rate of 14.28°/day (1998.6 m/s). Sunspot with an area of $A_r < 5 \, M_H$ has a higher velocity of 0.70% than sunspots with an area of $A_r > 15 \, M_H$. Smaller spots have smaller magnetic field fluxes and move faster than large sunspots that have larger magnetic field fluxes.

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