Hawking Radiation, Gravitational Anomaly, and Conformal Symmetry
– the Origin of Universality – *

SATOSHI ISO

High Energy Accelerator Research Organization (KEK),
and
Department of Particles and Nuclear Physics,
The Graduate University for Advanced Studies,
Tsukuba, Ibaraki 305-0801, Japan
satoshi.iso@kek.jp

The universal behavior of Hawking radiation is originated in the conformal symmetries of matter fields near the black hole horizon. We explain the origin of this universality based on (1) the gravitational anomaly and its higher-spin generalizations and (2) conformal transformation properties of fluxes.

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1. Introduction

Hawking radiation is the most prominent quantum effect to arise for quantum fields in a background space-time with an event horizon, and if we neglect the grey body factor induced by the effect of the scattering outside the horizon, the flux of the radiation is universally determined by the properties of black holes at the horizon.

We will explain that this universality can be understood in terms of the gravitational anomalies at the horizon and its higher-spin generalizations. In this method, it is important that matter fields in the black hole background behave as if they are a collection of two-dimensional conformal fields near the horizon in \((t, r^*_s)\) coordinates, where \(r^*_s\) is the tortoise coordinate. The conformal invariance near the horizon has been emphasized \(^{[1]}\) in understanding the black hole entropy by the near horizon Virasoro symmetries. We also show that the response of physical quantities to a conformal transformation gives the correct value of the Hawking radiation.

There are several derivations of Hawking radiation and all of them take into account the quantum effect in the black hole backgrounds in various ways. In the original derivation \(^{[2]}\), the radiation appears as a result of the particle interpretation of the quantum wave propagation in the black hole background. An understanding of

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the Hawking radiation in the field theory language was first given by Christensen and Fulling. In this calculation, the energy flux of the radiation is given by using the conservation law of the energy-momentum tensor and the quantum trace anomaly.

Recently Robinson and Wilczek showed that the energy flux of the Hawking radiation can be fixed by the value of the gravitational anomaly at the horizon. Here the universality of Hawking radiation is connected to the universality of the gravitational anomaly. This work was further generalized by us to obtain the flux of charge from charged black holes by considering the gauge anomaly in addition to the gravitational one. More recently the full thermal spectrum of the Hawking radiation was given by T. Morita, H. Umetsu and the present author by calculating the anomalies for the higher-spin currents.

In this review, we summarize these recent developments to understand the universal behavior of the Hawking radiation. In the next section, we first emphasize the emergence of the conformal symmetry for matter fields near the horizon. In section 3, we then explain how to calculate the expectation value of the energy-momentum tensor in two different methods. The first method is to calculate the response of the holomorphic energy-momentum tensor to a conformal transformation, and the second one is a method using the gravitational anomaly. In section 4, we generalize these two methods to higher-spin currents and obtain the full thermal spectrum of the Hawking radiation. Finally we conclude in section 5.

2. Near Horizon Conformal Symmetry

For simplicity, we consider a charged scalar field in a $d = 4$ Reisner-Nordstrom black hole background with a metric $ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2d\Omega^2$ and $f(r) = 1 - 2M/r + Q^2/r^2$. The scalar field is minimally coupled and the action is given by

$$S = \int d^4x \sqrt{-g}\{g^{\mu\nu}(\partial_{\mu} \phi + i e A_{\mu})\phi^* (\partial_{\nu} \phi - i e A_{\nu})\phi - m^2 \phi^* \phi - V(\phi)\}. \quad (1)$$

By decomposing the scalar field into partial waves as $\phi = \sum_{l,m} \phi_{lm}(r,t)Y_{lm}(\Omega)$, the action becomes an infinite set of two-dimensional fields in the $(t, r_*)$ coordinate as

$$S = \sum_{l,m} \int dt dr_* r^2(r_*) \left[ (\partial_t - i e A_t)\phi_{lm} \right]^2 - (\partial_{r_*} \phi_{lm})^2$$

$$- f(r(r_*)) \left( m^2 |\phi_{lm}|^2 + \frac{l(l+1)}{\rho^2} |\phi_{lm}|^2 + V(\Phi_{lm}) \right). \quad (2)$$

Here $r_*$ is the tortoise coordinate. When we consider a matter field with finite energy at $r = \infty$, the $d = 2$ kinetic term behaves regularly near the horizon while the potential term becomes negligible because of the damping coefficient $f(r)$ near the horizon. Hence the $d = 4$ scalar field can be considered as an infinite set of $d = 2$ conformal fields near the horizon in the $(t, r_*)$ coordinate. This is the most essential property to the universality of the Hawking radiation.

The emergence of the two-dimensionality and the conformal symmetries near the horizon is very general and the following derivations of the Hawking radiation are
generally applicable to higher dimensional cases. When the black hole is rotating, the system can be reduced to a charged black hole by the dimensional reduction and the analysis becomes identical to the case of the charged black holes. It is also true for any higher dimensional black holes or other black objects and the gravitational anomaly method has been applied to various cases.

Two comments are in order. First, in order to evaluate the Hawking fluxes at infinity, we need to extrapolate the fluxes near the horizon to infinity, and the thermal black-body radiation is distorted by the grey body factor by the effect of scattering of waves in the potential. There are two sources for the potential. The first is the original potential term in the action. The second is the $r$-dependence of the volume factor. Its main effect is to relate the integrated flux over the sphere in $d = 4$ with the $d = 2$ outgoing flux, but in addition to it, this gives an effective potential proportional to $1/r^3$. In this review, we neglect the effects of the potential (grey body factor) and mainly consider the universal behavior of radiation determined by the information at the horizon.

Next I would like to comment on another possible violation of the universality of Hawking radiation. The damping coefficient $f(r)$ of the potential terms in the action comes from the relation $\partial_r = f(r)^{-1}\partial_{r_*}$. Hence, if the action has higher-dimensional operators containing higher derivatives with respect to $r$, they become more dominant than the ordinary kinetic terms. For example, a term $M_{UV}^{-1}\partial^2_r \phi^2$ is enhanced near the horizon by a factor $f(r)^{-2}$ and becomes more relevant than the ordinary kinetic terms, if the enhancing factor dominates the low-energy suppression factor $(T_{BH}/M_{UV})^2$. Here $T_{BH}$ is the typical energy scale of Hawking radiation at infinity. This indicates that the local field theory based on a low-energy approximation is no longer valid near the horizon. This is an interesting issue, but we leave it for a future problem.

### 3. Universality of Hawking Energy Flux

In this section, we discuss why the energy flux can be universally determined only by the information at the horizon. We first obtain the Hawking energy flux by calculating the response of the holomorphic energy-momentum tensor to a conformal transformation from the Kruskal to the Schwarzschild coordinates. In the second subsection, we review the derivation of the Hawking energy flux based on the gravitational anomaly at the horizon.

#### 3.1. Response to conformal transformations

For simplicity we consider a neutral field in the Schwarzschild black hole with $f(r) = 1 - 2M/r$. The tortoise coordinate describes the region outside the horizon and is defined by $dr_* = dr/f(r)$. It behaves as $r_* \sim r$ at infinity and takes $-\infty$ near the horizon.

There are two important coordinate systems; the Schwarzschild light cone coordinates labeled by the outgoing $u = t - r_*$ and ingoing $v = t + r_*$ coordinates,
and the Kruskal coordinates defined by $U = -\exp(-\kappa u)$ and $V = \exp(\kappa v)$ where $\kappa = 1/4M$ is the surface gravity. The Schwarzschild coordinates are convenient to describe the physics at infinity, while the Kruskal coordinates are more appropriate to discuss the physics observed by an infalling observer near the horizon. Hawking radiation emanates from the future horizon $U \sim 0$ of the black hole, and the regularity condition imposed on the physical quantities observed by an infalling observer determines the boundary condition for the Unruh vacuum (with an additional condition for the ingoing modes).

Consider a reduced two-dimensional system describing one partial wave. In the approximations with the grey body factor neglected, the two-dimensional energy-momentum tensor for each partial wave is conserved $\nabla \mu T^\mu_\nu = 0$ and its trace receives the Weyl anomaly as $T^{\mu}_\mu = \frac{c}{24\pi} \frac{\hbar R}{2}$ where $c = 1$ for a boson and $c = 1/2$ for a fermion. In the curved space-time with $ds^2 = e^{\phi(u,v)} du dv$, these two equations enable us to define the holomorphic energy-momentum tensor as

$$t_{uu}(u) = T_{uu}(u,v) - \frac{c}{24\pi} \left( \frac{\partial^2 \phi}{\partial u^2} - \frac{1}{2} (\partial_u \phi)^2 \right),$$

which does not depend on the $v$ coordinate. This holomorphic energy-momentum tensor is no longer an ordinary tensor and under a coordinate transformation from $u$ to $U(u)$, it transforms as

$$t_{UU}(U) = \left( \frac{1}{\kappa U} \right)^2 \left( t_{uu}(u) + \frac{c}{24\pi} \{U, u\} \right), \quad \{U, u\} = \frac{U''}{U'} - \frac{3}{2} \left( \frac{U''}{U'} \right)^2.$$  

It is important to note that $t_{uu}$ and $T_{uu}$ coincide at $r = \infty$ because the conformal factor vanishes there. Hence the value of $t_{uu}(u)$ at $r = \infty$ gives the outgoing flux from the black hole. The value can be fixed by imposing the condition that the physical quantities should be regular for an infalling observer in the Kruskal coordinate. In order that $t_{UU}$ should behave regularly near the future horizon at $U \sim 0$, $t_{uu}$ must cancel the term $c\{U, u\}/24\pi$. Hence we get the value of the outgoing energy flux by the value of the Schwarzian derivative as

$$T_{uu} = -\frac{ch}{24\pi} \{U, u\} = \frac{ch\kappa^2}{48\pi}. \tag{5}$$

If we further assume that the ingoing flux vanishes at infinity $T_{vv} = 0$ for the Unruh vacuum, we get the asymptotic energy flux at $r = \infty$ as $T_i^r = T_{uu} - T_{vv} = c\hbar\kappa^2/48\pi$. This is equal to the integral of the thermal distribution

$$T_i^r = \int_0^\infty \frac{d\omega}{2\pi} \frac{h\omega}{e^{\beta\omega} \mp 1} = \frac{ch\kappa^2}{48\pi} \tag{6}$$

where the inverse temperature $\beta$ is given by $\beta = 2\pi/\kappa$.

The method can generalized to obtain an energy flux in a charged black hole. In this case, the conservation of energy-momentum tensor is modified to $\nabla \mu T^\mu_\nu = F_{\mu\nu} J^\mu$ in the presence of the electric field and the $U(1)$ current $J^\mu$, and the holomorphic energy-momentum tensor is changed accordingly. The energy flux
can be obtained by considering the response of the holomorphic energy-momentum tensor to the coordinate transformation and an additional gauge transformation.

### 3.2. Gravitational anomaly method

In this subsection we review the recent derivation of the Hawking radiation based on the gravitational anomaly at the horizon. The basic idea is the following. As we saw, each partial wave of a matter field behaves as a free field in the two-dimensional \((t, r_\ast)\) coordinates, and the outgoing (ingoing) modes correspond to the right (left) moving modes in \(d = 2\). Classically nothing can escape from the black hole and the ingoing modes at the horizon cannot affect the physics outside of the black hole. So they can be neglected classically. But if we neglect them the theory becomes chiral, and quantum mechanically it would break the gauge and general coordinate invariance through the quantum anomalies. Of course, the underlying theory is not anomalous and the classically irrelevant ingoing modes become relevant quantum mechanically. This gives the flux of the Hawking radiation.

In order to understand the Hawking radiation as stated above, it is suitable to consider the path integral formulation and investigate the Ward-Takahashi identities in the black hole background as in ref.\(^{4,5}\). In this formulation, the WT identities are written in terms of the consistent currents while the boundary conditions are imposed on the covariant currents, and we need to relate these two different types of currents. Here we instead take an equivalent but simpler method using only the covariant currents given by Banerjee and Kulkarni.\(^{10}\)

First consider the U(1) current in a charged black hole. If we neglect the ingoing modes at horizon, the U(1) current becomes anomalous. We further simplify the situation and assume that the ingoing modes are neglected everywhere. This can be justified if there is no scattering (no grey body factor) and by imposing the Unruh type boundary condition that the ingoing flux should vanish at infinity. Then we have the anomalous equation for the covariant form of the U(1) current

\[
\nabla_\mu J^\mu = \frac{\hbar}{4\pi} \epsilon_{\mu\nu} F^{\mu\nu}. \quad (7)
\]

In the static background, it can be easily solved as

\[
\langle J^\nu(r) \rangle = c_H + \frac{\hbar}{2\pi} (A_t(r) - A_t(r +)) = \frac{-hA_t(r_\ast)}{2\pi} = \frac{\hbar Q}{2\pi r_\ast}, \quad (8)
\]

Here the integration constant \(c_H\) gives the value of the covariant outgoing current at the horizon. The regularity condition at the future horizon imposes that \(c_H = 0\), otherwise the outgoing current \(J_U = -J_u/\kappa U\) observed by an infalling observer diverges at \(U = 0\). Hence the \(U(1)\) flux at infinity can be obtained as \(c_O = J^\tau(r = \infty) = \frac{-hA_t(r_\ast)}{2\pi} = \frac{\hbar Q}{2\pi r_\ast}\), where \(r_\ast\) is the position of the outer horizon.

The flux is determined by the gauge potential at the horizon. This universal behavior is obtained in the anomaly method because the r.h.s. of (7) is written as a total derivative and, by integrating (7), the current can be obtained by the boundary condition at the horizon only.
It can be generalized to the energy flux as well. We consider the covariant energy-momentum tensor with the covariant form of the gravitational anomaly:

\[ \nabla^\mu T_{\mu\nu} = F_{\mu\nu} J^\mu + \frac{\hbar}{96\pi} \epsilon_{\mu\nu\rho\sigma} \partial^\rho R. \]  

(9)

It is important to note again that the r.h.s. of this equation is written as a total derivative and the energy-momentum tensor can be solved as

\[ T^r_r = a_H + \int_{r_+}^r \partial_r \left[ c_O A_t + \frac{\hbar}{4\pi} A^2_t + \frac{\hbar}{96\pi} (f f'' - \frac{(f')^2}{2}) \right]. \]  

(10)

The regularity condition imposes \( a_H = 0 \), and the asymptotic outgoing energy flux can be determined by the value of the anomaly at the horizon as

\[ a_O = T^r_r (r = \infty) = \frac{\hbar Q^2}{4\pi r^4} + \frac{\hbar\pi}{12\beta^2}. \]  

(11)

4. Higher-spin Fluxes and Anomalies

In the previous section, we have calculated the energy flux based on two different methods. But the energy flux is a partial information of the thermal radiation which can be obtained as an integral \( \int_0^\infty \frac{\hbar \omega^{2n-1}}{2\pi e^{\beta \omega} + 1}. \) It would be desirable if we can obtain the full thermal spectrum of the Hawking radiation, or equivalently all the moments

\[ \int_0^\infty \frac{\hbar \omega^{2n-1}}{2\pi e^{\beta \omega} + 1}. \]  

(12)

In the following we show that they can be fully reproduced by the anomaly method or the conformal transformation method.

4.1. Conformal transformation properties of higher-spin currents

The energy-flux of the Hawking radiation was reproduced by calculating the Schwarzian derivative of the holomorphic energy-momentum tensor in \( n \). This method was generalized by us \( n+1 \) to obtain all the moments of \( n+2 \).

Here I will explain the case of a neutral scalar field, but generalizations to charged fields or fermion fields are straightforward. First consider a 4-th rank current constructed from a scalar field as \( J^{(1,3)} = - : \partial_u \phi \partial^3 u \phi(u) : \). This is regularized by the point splitting regularization, and a standard calculation shows that under a conformal transformation \( u \rightarrow U(u) = -e^{-\kappa u} \) it transforms as

\[ : \partial_u \phi \partial^3 u \phi(u) : = \kappa^4 U^2 : \partial_U \phi(U) \partial_U \phi(U) : + 3\kappa^4 U^3 : \partial_U \phi(U) \partial^2_U \phi(U) : \]

\[ + \kappa^4 U^4 : \partial_U \phi(U) \partial^3_U \phi(U) : - \frac{\hbar}{480\pi} \{ U, u \}_{(1,3)}. \]  

(13)

where the generalized Schwarzian derivative is given by

\[ \{ U, u \}_{(1,3)} = \frac{6U'''}{U'} - 20 \left( \frac{U'''}{U'} \right)^2 - 45 \left( \frac{U'''}{U'} \right)^4 + 90 \left( \frac{U'''}{U'} \right)^2 \left( \frac{U''}{U'} \right)^3 - 30 \left( \frac{U'''}{U'} \right)^2 \frac{U''}{(U')^2}. \]  

(14)
The regularity condition at the future horizon imposes again that the first three quantities in the r.h.s. of (13) should vanish at $U = 0$, and the spin-4 flux at infinity is given by $\langle J^{(1,3)}(r = \infty) \rangle = \hbar \{ U, u \}^{(1,3)}/480\pi = \hbar \kappa^4/480\pi$, which agrees with the $n = 2$ moment of the bosonic case [12]. Here note that another spin-3 current $\partial_u^2 \phi \partial_u^3 \phi(u)$ gives the same answer for the asymptotic flux and hence any linear combination of them does not change the final result, if appropriately normalized.

It can be straightforwardly generalized to any higher-spin currents, but instead of calculating them one by one, it is easier to calculate Schwarzian derivatives for a generating function of the higher-spin currents $J(u, u + a) = \partial_u \phi(u) \partial_u \phi(u + a) := \sum_{n=0}^{\infty} a_n n! \partial_u \phi(u)$. (15)

Under a transformation $u \to U(u)$, it transforms as $J(U(u), U(u + a)) = e^{\kappa a} (1 - U/\kappa U)^2 (J(u, u + a) - \hbar A_b(U, u))$. (16)

where $A_b(U, u)$ is given by

$$A_b(U, u) = -\frac{\partial_U U \partial_U U}{4\pi(U - U + a)^2} + \frac{1}{4\pi a^2} = -\frac{\kappa^2}{16\pi \sinh^2(\kappa a/2)} + \frac{1}{4\pi a^2}. (17)$$

The regularity condition at $U = 0$ determines the value of the flux as $\langle J(u, u + a) \rangle = \hbar A_b(U, u)$. By expanding it with respect to $a$, we get the right answer for the flux of general higher-spin currents: $\langle 1^{n-1} \partial_u \phi \partial_u^{n-1} \phi(u) \rangle = B_n \kappa^{2n}/8\pi n$, where $B_n$ is the Bernoulli number. $A_b(U, u)$ is nothing but the temperature dependent part of the thermal Green function of a scalar field.

4.2. Higher-Spin Gauge Anomaly

The gravitational anomaly method can be also generalized to the higher-spins and reproduce the correct thermal fluxes [2]. For this purpose, we need to obtain generalized equations of the gravitational anomaly [2] to higher-spin currents. The next simplest example is the conservation equation for the spin-3 current. For the spin-3 current constructed from a fermionic field, it is given by

$$\nabla_\mu J^{(3)\mu} \nu \rho = \left( -F_{\nu\mu} J^{(2)\rho} \mu - \frac{1}{16} \nabla_\nu (R J^{(1)} \rho) + (\nu \leftrightarrow \rho) \right) + \frac{1}{16} g_{\nu\rho} \nabla_\mu (R J^{(1)} \mu) + \frac{\hbar}{96\pi} (\epsilon_{\sigma\rho} \nabla_\sigma \nabla_\mu F_{\rho \nu} + \epsilon_{\rho\sigma} \nabla_\sigma \nabla_\mu F_{\nu \rho} - g_{\nu\rho} \epsilon_{\alpha\sigma} \nabla_\sigma \nabla_\mu F^{\mu\alpha}). (18)$$

The first line of r.h.s. is the classical violation in the presence of electric and gravitational background, while the second line is the quantum anomaly. In the present context the transformation generated by the spin-3 current is not gauged and it is not the anomaly associated with the gauge symmetry. But since it has the same quantum origin as the gravitational anomaly for the energy-momentum tensor,
we call it a higher-spin gauge anomaly here. An interesting property of the r.h.s. is that both of the classical and the quantum violations are written as total derivatives as is the case for the energy-momentum tensor. Hence the asymptotic flux from a black hole, which can be obtained by integrating the equation and imposing the regularity condition at the horizon, is determined only by the information at the horizon as

\begin{equation}
\kappa = -\frac{\hbar k^2 A_\ell(r_+)}{24\pi} - \frac{\hbar A_\ell(r_+)^3}{6\pi} = \frac{\hbar k^2 Q}{24\pi r_+} + \frac{\hbar}{6\pi} \left( \frac{Q}{r_+} \right)^3.
\end{equation}

Similar calculations can be done for higher-spin currents. It is interesting that the higher-spin anomalies can be also written as total derivatives.

Finally we summarize in four steps how to obtain the equation like (18):
(i) Regularize higher-spin currents covariantly on the light-cone with \( v \) fixed.
(ii) Regularize the higher-spin currents holomorphically.
(iii) Compare these two and obtain a relation like (3) for higher-spin currents.
(iv) Covariantize the relation.

With these four steps, we can obtain classical and quantum violations of the conservation equations for the higher-spin currents in the presence of the electric and gravitational backgrounds. For further details, please refer to ref. 6. Furthermore we can obtain the quantum violation of the classically traceless symmetric currents. For example, the spin-3 current has a trace anomaly

\begin{equation}
J^{(3)}_{\mu\nu} = \frac{\hbar (c_L + c_R)}{24\pi} \nabla_{\mu} F_{\nu}^\mu
\end{equation}

where \( c_L \) and \( c_R \) are left (right) central charges.

5. Conclusions and Discussions

Hawking radiation is determined only by the information at the horizon, if grey body factor is neglected. The universality can be explained in various ways, but it can be beautifully explained in the anomaly method. Namely the quantum anomaly is written as a total derivative and the regularity condition determines the value of the asymptotic flux by the value of the surface term of the anomaly at the horizon. This is reminiscent of the topological effects of instantons in the gauge theories. In the present case, the horizon plays a role of a boundary and the regularity condition there relates the asymptotic flux to the information at the horizon.

Another interesting issue is a possible violation of the universality, in the context of the information paradox, due to the enhancement of higher derivative terms very close to the horizon, as discussed in section 2. We would like to come back to this problem in future.

\begin{footnote}{The authors of ref. 12 state that the trace anomaly for spin 4 current is cohomologically trivial. It is interesting to see whether the higher-spin anomalies can be absorbed by redefining the currents.}
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