Large Leptonic Dirac CP Phase from Broken Democracy with Random Perturbations

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Abstract

A large value of the leptonic Dirac CP phase can arise from broken democracy, where the mass matrices are democratic up to small random perturbations. Such perturbations are a natural consequence of broken residual $S_3$ symmetries that dictate the democratic mass matrices at leading order. With random perturbations, the leptonic Dirac CP phase has a higher probability to attain a value around $\pm \pi/2$. Comparing with the anarchy model, broken democracy can benefit from residual $S_3$ symmetries, and it can produce much better, realistic predictions for the mass hierarchy, mixing angles, and Dirac CP phase in both quark and lepton sectors. Our approach provides a general framework for a class of models in which a residual symmetry determines the general features at leading order, and where, in the absence of other fundamental principles, the symmetry breaking appears in the form of random perturbations.

1. Introduction

Flavor mixing has been observed in both quark and lepton sectors. Theoretically, there are two basic approaches in explaining the mixing patterns. One is top-down by assigning some full flavor symmetry to constrain the fundamental Lagrangian. However, the full flavor symmetries have to be broken, otherwise the up- and down-type fermions would be subject to the same flavor structure in their mass matrices and hence we obtain the same mixing matrices, $V_u = V_d$, leading to a trivial physical mixing matrix, $V_{\text{CKM}} = V_u^\dagger V_d = I$. If the mixing pattern is really determined by symmetry, it has to be a residual symmetry that survives symmetry breaking. The reverse of the top-down logic is the bottom-up phenomenological mass matrix approach [1–5]. By reconstructing the residual symmetries in both up- and down-type fermion sectors [6,7], the full flavor symmetry can be obtained as a product group [8–10].
In both approaches, the residual symmetry takes the role of predicting the mixing pattern [11–14]. In some sense, the residual symmetry takes the same role as the custodial symmetry in the gauge sector [15]. The electroweak $SU(2)_L \times U(1)$ gauge theory can predict the existence of four gauge bosons but not the weak mixing angle $\theta_w$. In the Standard Model, the weak mixing angle $\sin \theta_w = g'/\sqrt{g^2 + g'^2}$ is a function of the gauge couplings $g$ and $g'$ whose values cannot be predicted by gauge symmetry. The custodial symmetry can make correlation between physical observables, $\cos \theta_w = M_W/M_Z$. Likewise, residual symmetries can predict the correlation among mixing angles and the Dirac CP phase, also known as sum rule of mixing angles [16–18] in addition to the mass sum rules [19,20]. The custodial symmetry is essentially a residual symmetry. In this sense, the concept of residual symmetry and phenomenological mass matrix approach applies universally for all the observed mixing among fundamental particles.

The mixing patterns in the quark and lepton sectors are quite different. While the mixings in the quark sector are small, the lepton sector has large mixing angles. This seems hard to understand at the first glance. Considering the fact that the observed quark and lepton mixings are combined effects of mixings in both up- and down-type fermions, $V_{\text{CKM}} = V_u^\dagger V_d$ for quarks and $V_{\text{PMNS}} = V_\ell^\dagger V_\nu$ for leptons, a unified picture might be possible if a similar mixing pattern appears in both up- and down-type quark mass matrices but in only one of leptons and neutrinos, $V_u \approx V_d \approx V_\ell$ or $V_\nu$. Then the quark mixing $V_{\text{CKM}}$ is close to unity matrix while the neutrino mixing $V_{\text{PMNS}} \approx V_\ell$ or $V_\nu$ has large mixing angles [4,5,15,21,22].

Approximate democratic matrix is known as an interesting possibility to explain the large hierarchy in quark masses and the small CKM mixing angles when applied to both up- and down-type quarks. If one applies this hypothesis to the lepton sector for both charged leptons and neutrinos, with the help of residual $S_3$ symmetries, one may get small lepton mixing angles which is strongly excluded experimentally. However, it was pointed out that if we assume almost diagonal mass matrix for neutrinos we obtain large lepton mixing angles [4,5]. A natural consequence is that $V_{\text{CKM}}$ has only 1-2 mixing while $V_{\text{PMNS}}$ can have at least two large mixing angles at leading order, as we would elaborate in Sec. 2. The democratic matrix can also explain the mass hierarchies among charged fermions, with $m_1 = m_2 = 0$. To accommodate nonzero fermion masses and to get a better fit for observed mixing angles, one needs deviations from the democratic matrix which break the residual $S_3$ symmetries. With residual $S_3$ symmetries broken, there is no fundamental principle to regulate the deviations. A natural approach is to assume that small random perturbations make the mass matrix different from the democratic form. This approach is different from the anarchy model, where the mass matrix can be totally free of any constraint [23,24]. We will elaborate our predictions for both neutrino (see Sec. 3 and Sec. 4) and quark mixings (see Sec. 5).

2. Democratic Mass Matrix Hypothesis – Preliminary

The democratic pattern of mass matrix can be realized by applying two independent residual $S^L_3$ and $S^R_3$ symmetries to the left- and right-handed fermions. Then the fermion mass matrix in a natural
basis looks like

\[ M_f = \frac{M_0}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{2.1} \]

where \( M_0 \) characterizes the mass scale [4, 5, 25–51]. Its diagonalization involves two mixing matrices for the left- and right-handed fermions, \( M_f = V_L D_f V_R^\dagger \) where \( D_f = \text{diag}\{m_1, m_2, m_3\} \) is the diagonalized mass matrix and \( V_L (V_R) \) is the mixing matrix of left-handed (right-handed) fermions.

The democratic mass matrix form (2.1) applies for all fermions, except the neutrinos. This can be naturally realized with \( SO(3)_L \times SO(3)_R \) flavor symmetries [5]. The three generations of left- and right-handed fermions form triplets under \( SO(3)_L \) and \( SO(3)_R \) transformations, respectively. Similarly there are two triplet flavons \( \phi_L \) and \( \phi_R \), correspondingly. Then, two invariants can be written down to form a Yukawa term

\[ \sum_{ij} y_{ij} \langle \bar{\psi}_L, \phi_L \rangle \langle \phi_R, \psi_R \rangle. \tag{2.2} \]

The \( SO(3)_L \times SO(3)_R \) flavor symmetry would break down to the residual \( S^L_3 \times S^R_5 \) if the triplet Higgs obtains equal vacuum expectation values for the three components, \( \langle \phi_L, R \rangle \propto (1, 1, 1) \), leading to the democratic mass matrix (2.1) for charged fermions. For neutrinos, its mass term can be given by Weinberg-Yanagida operator [52, 53] which contains two left-handed fermions, \( L_{iL}, L_{iL} \). Note that these two fermions belong to the same \( SO(3)_L \) triplet whose product decomposes as \( 3 \times 3 = 1 + 3 + 5 \). Of the three decomposed representations, the triplet \( 3 \) has anti-symmetric Clebsch-Gordan coefficients which is not consistent with the Majorana neutrino mass matrix. The other two, \( 1 \) and \( 5 \), can give symmetric Majorana neutrino mass matrix. The singlet contribution is proportional to a unit matrix and for the \( 5 \) representation we adopt two scalar multiplets with

\[ \Sigma_L^{(1)} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad \Sigma_L^{(2)} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.3} \]

to give diagonal neutrino mass matrix. Altogether, the \( 1 \) and \( 5 \) representations give a diagonal neutrino mass matrix with the three mass eigenvalues being free.

The concrete form of \( V_L \) is determined by \( M_f M_f^\dagger = V_L D_f^2 V_L^\dagger \). Note that \( M_f M_f^\dagger \) also takes the same form as (2.1), but with \( M_0 \) replaced by \( M_0^2 \). By diagonalizing \( M_f M_f^\dagger \), we can obtain the mixing matrix \( V_L \)

\[ V_L^\dagger = \begin{pmatrix} e^{i\alpha_1} \\ e^{i\alpha_2} \\ e^{i\alpha_3} \end{pmatrix} \begin{pmatrix} c_T & s_T e^{i\phi} & 0 \\ -s_T e^{-i\phi} & c_T & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = RT V_0, \tag{2.4} \]

which is the most general form of the solution. For convenience, we denote the mixing angle among the states of degenerated mass eigenvalues \( m_1 = m_2 = 0 \) as \( (c_T, s_T) \equiv (\cos \theta_T, \sin \theta_T) \), to which a Dirac-type CP phase \( \phi \) is attached. In addition, there are three free rephasing degrees of freedom \( \alpha_i \) with \( i = 1, 2, 3 \) that are attached to the three mass eigenvalues. It is interesting to observe that the democratic mass matrix leads to hierarchical mass eigenvalues.
Since the same democratic mass matrix applies to both up- and down quarks, the CKM matrix naturally has suppressed 1-3 and 2-3 mixings,

\[ V_{\text{CKM}} = T_u T_d^\dagger = \begin{pmatrix} c_{T,u} & s_{T,u} e^{i\phi_u} & 0 \\ -s_{T,u} e^{-i\phi_u} & c_{T,u} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{T,d} & -s_{T,d} e^{i\phi_d} & 0 \\ s_{T,d} e^{-i\phi_d} & c_{T,d} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \] (2.5)

For cleanness, we have omitted the two rephasing matrices \( R_u \) and \( R_d \). The combined 1-2 mixing takes the form as

\[ \cos \theta_{12} = |c_{T,u} c_{T,d} + s_{T,u} s_{T,d} e^{i(\phi_u - \phi_d)}| \quad \text{and} \quad \sin \theta_{12} = |c_{T,u} s_{T,d} - s_{T,u} c_{T,d} e^{i(\phi_u - \phi_d)}|, \] (2.6)

while \( \theta_{13} = \theta_{23} = 0 \). This naturally explains why the mixing in the quark sectors are small.

For the neutrino sector, (2.4) is already the form for the PMNS matrix,

\[ V_{\text{PMNS}} = \begin{pmatrix} \frac{c_{T,\ell}}{\sqrt{2}} & \frac{s_{T,\ell} e^{i\phi_\ell}}{\sqrt{6}} & 0 \\ \frac{s_{T,\ell} e^{-i\phi_\ell}}{\sqrt{2}} & \frac{c_{T,\ell}}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \] (2.7)

assuming the neutrino mass matrix is diagonal, i.e., \( V_\ell = I \). Comparing with the standard parametrization of the PMNS matrix we can obtain

\[ \sin \theta_r = \frac{2s_{T,\ell}}{\sqrt{6}}, \quad \tan \theta_a = \sqrt{2} c_{T,\ell}, \quad \tan \theta_s = \sqrt{3} s_{T,\ell} e^{i\phi_\ell}, \quad \phi_\ell = \frac{3c_{T,\ell} - s_{T,\ell} e^{i\phi_\ell}}{3c_{T,\ell} + s_{T,\ell} e^{i\phi_\ell}}. \] (2.8)

The mixing angles have been denoted as \( (\theta_a, \theta_r, \theta_s) \equiv (\theta_{23}, \theta_{13}, \theta_{12}) \) for the atmospheric, reactor, and solar angles, respectively, to make their physical meaning explicit. Since there are two model parameters but four physical quantities in (2.8), two correlations (also called as sum rules) would emerge,

\[ t_a^2 = 2 - 3s_r^2, \quad \cos \delta_D = \frac{1 - t_a^2}{1 + t_a^2} \frac{c_r^2}{s_r \sqrt{2 - 3s_r^2}}. \] (2.9)

Essentially the atmospheric angle \( \theta_a \) and the Dirac CP phase \( \delta_D \) can be predicted as functions of the reactor and solar angles, \( \theta_r \) and \( \theta_s \), which is in the same spirit as residual symmetries [11–13].

Due to the degenerate mass eigenvalues \( m_1 = m_2 = 0 \), the Dirac CP phase \( \delta_D \) and the mixing \( \theta_{12} \) have no physical consequence. Nevertheless, we can still obtain some insight by playing these sum rules with the experimentally measured values as a preliminary exercise. Since the reactor angle is small, \( s_r \approx 1/6 \), the atmospheric angle naturally lives in the second octant, \( \theta_a \approx 54^\circ \). On the other hand, the fact that \( \cos \delta_D \) cannot actually exceed 1 puts a natural limit on the solar mixing angle

\[ \cos \delta_D \leq 1 \quad \Rightarrow \quad t_s^2 \geq \frac{c_r^2 - s_r \sqrt{2 - 3s_r^2}}{c_r^2 + s_r \sqrt{2 - 3s_r^2}} \approx 1, \] (2.10)

with \( s_r \ll c_r \). Namely, the solar angle \( \theta_s \) would also reside in the second octant to allow a physical Dirac CP phase and a small reactor angle, which is not consistent with our current knowledge on the neutrino mixing angles. It is necessary to introduce deviation to the democratic mass matrix (2.1) as we will discuss in the following section. Deviation is also necessary for eliminating the two vanishing mass eigenvalues, \( m_1 = m_2 = 0 \).
3. Random Perturbations

As we argued above, to account for a realistic mass hierarchy with nonzero mass eigenvalues and the measured mixing patterns, the democratic mass matrix can only be approximate one. We would effectively break the residual $S^L_3$ and $S^R_3$ symmetries. Since these two $S_3$ symmetries are already residual ones, there is no reason to expect anything that can still regulate the deviations. A natural scheme is that these deviations are totally random. In addition, the deviations should not be too far away from the zero’s order which can also fit the measured neutrino and quark mixings quite well. Consequently, the deviations should be small random perturbations.

Generally, the mass matrix (2.1) becomes

$$M_f = \frac{M_0}{3} \begin{pmatrix}
1 + \epsilon_{11} e^{i\phi_{11}} & 1 + \epsilon_{12} e^{i\phi_{12}} & 1 + \epsilon_{13} e^{i\phi_{13}} \\
1 + \epsilon_{21} e^{i\phi_{21}} & 1 + \epsilon_{22} e^{i\phi_{22}} & 1 + \epsilon_{23} e^{i\phi_{23}} \\
1 + \epsilon_{31} e^{i\phi_{31}} & 1 + \epsilon_{32} e^{i\phi_{32}} & 1 + \epsilon_{33} e^{i\phi_{33}}
\end{pmatrix},$$

by introducing a complex perturbation $\epsilon_{ij} e^{i\phi_{ij}}$ to the $(ij)$ element of the mass matrix, $M_{f,ij} = \frac{M_0}{3} + \delta M_{ij}$. The flat measure of the deviations looks like $d^2 M_{ij} = dM_{ij}^{(r)} dM_{ij}^{(i)} \propto \epsilon_{ij} d\epsilon_{ij} d\phi_{ij}$ where $M_{ij}^{(r)}$ and $M_{ij}^{(i)}$ are the real and imaginary parts of the deviations, respectively. We assign $0 \leq \epsilon_{ij} \leq \epsilon_{\text{max}}$ and $0 \leq \phi_{ij} \leq 2\pi$ for a random scattering. In Fig. 1, we show the predicted neutrino mixing for

$$\epsilon_{\text{max}} = 0.2 \text{ (medium)} \text{ and } 0.4 \text{ (thick). For comparison, we also show the prediction of mixing angles and the Dirac CP phase from the anarchy mass matrix} \text{ [24] as thick curves with different colors.}$$

The quasi democratic matrix can naturally explain the hierarchical structure in fermion masses. At the leading order, the three mass eigenvalues are $m_1 = m_2 = 0$ and $m_3 = M_0$. Introducing random perturbations can break the degeneracy between $m_1$ and $m_2$ and at the same time broaden the $m_3$ peak. The larger $\epsilon_{\text{max}}$, the broader mass peaks. In Fig. 1 we show predictions for both $\epsilon_{\text{max}} = 0.2$ and 0.4. An even larger $\epsilon_{\text{max}}$ would lead to more diverse mass eigenvalues and hence more unrealistic mass hierarchy. For comparison, the prediction of the anarchy mass matrix scheme is modulated by
the mass eigenvalues themselves, \((m_1^2 - m_2^2)^2(m_2^2 - m_3^2)^2(m_3^2 - m_1^2)\) \(m_1 m_2 m_3 dm_1 dm_2 dm_3\), which is symmetric under the interchange of any two mass eigenvalues. In other words, there is no preference for the mass hierarchy in the anarchy mass matrix. The quasi democratic mass matrix leads to a much more realistic prediction of the mass hierarchy.

It is interesting to observe that the predictions for the mixing angles and the Dirac CP phase are quite stable against changing the perturbation size. In other words, our prediction of the CP phases are not that sensitive to the cut-off we set on the perturbation size. This stability feature has already been observed and quantified in the anarchy model \([24]\).

The predicted mixing angles have wide distributions. However, the predictions of the three mixing angles clearly have different features. The atmospheric angle \(\theta_a\) tends to peak around the maximal value, \(\theta_a \approx 45^\circ\) while its tail is highly suppressed on both sides, naturally explaining the measured large value of it. In comparison, the reactor angle \(\theta_r\) tends to have a fat tail for small values, which is in accordance with its measured value \(\theta_r \approx 8.4^\circ\) \([54]\). Both \(\theta_a\) and \(\theta_r\) have asymmetric distributions and an upper bound around \(60^\circ\). On the other hand, the solar angle \(\theta_s\) has a symmetric probability distribution with peak at long tail on both sides. Hence, we would expect a natural hierarchy among the mixing angles, \(\theta_r \lesssim \theta_s \lesssim \theta_a\), if all three are in the first octant. This overall picture is quite different from the prediction of the anarchy model where \(\theta_a\) and \(\theta_s\) have exactly same symmetric distribution while \(\theta_r\) has an asymmetric distribution with peak in the first octant. The democratic mass matrix with random perturbations can provide a better prediction than the anarchy model \([24]\).

Most notably, introducing random perturbations to the democratic mass matrix produces two prominent peaks in the probability distribution of the Dirac CP phase around maximal values, \(\delta_D \approx \pm 90^\circ\), one of which is in perfect agreement with the current global fit \([54]\). In comparison, the anarchy model has no preference for any value of \(\delta_D\) which appears as a flat curve in the right panel of Fig. 1.

Fig. 2: Illustrative plot of the random perturbations with the blue patches denoting the region with small CP violation and the green patches with large CP violation in each perturbative deviations.
We use the illustrative Fig. 2 to show how these two CP peaks appear as a natural consequence of the random perturbations. For each element of the fermion mass matrix, there are two regions with small CP violation (one of which labeled as blue sector) and two regions with large CP violation (one of which labeled as green sector). For simplicity, we consider only these two extreme cases. Since we have omitted the intermediate regions, there is no need to require probability conservation, $P_s + P_l \leq 1$. The physical Dirac CP phase $\delta_D$ comes from the interplay of CP phases in all matrix elements. If a random large CP appears in every element of the mass matrix, it is very difficult for the physical Dirac CP phase to be small which can only happen when all the CP phases $\phi_{ij}$ and the matrix element size $\epsilon_{ij}$ are highly fine tuned to cancel each other. Consequently, a small Dirac CP phase usually appears when all the matrix elements have small CP. The probability for small $\delta_D$ is of the order $P_s^9$. In contrast, it is enough to have large Dirac CP phase if one of the matrix element has a large CP. So the probability for large $\delta_D$ can be roughly estimated as $9P_l$. The factor that $P_s < 1$ naturally leads to $P_s^9 \ll 9P_l$, in other words, there is much larger probability for maximal Dirac CP phase than the vanishing one.

Fig. 3: The predicted CP phase distributions with randomly selecting which input phase $\phi_{ij}$ can be nonzero and take randomly sampled values. For the curve labeled with $n$, $n$ of the 9 input phases are selected.

To make it explicit, we show predictions of the Dirac CP phase distributions with randomly selected input phases $\phi_{ij}$ in Fig. 3. For each curve, we randomly select $n$ of the 9 input phases. The value of the selected phase $\phi_{ij}$ is randomly sampled in the range $[0, 2\pi]$ while those unselected are set to be zero. We can clearly see the trend that with more input CP phases randomly sampled, the Dirac CP phase is easier to have large value.

4. Comparison with Measurements

In principle, the prediction of our model should come from unbiased sampling of the mass matrix element as we have shown in the previous section with even distributions according to the flat
measure $\epsilon_{ij}d\epsilon_{ij}d\phi_{ij}$. If the flavor mixing pattern is really determined by dice, the values of the mixing parameters are determined once for all and our current knowledge from experimental measurements would not affect their probability distributions. Anyhow, to make the features of our predictions more explicit, we show some interesting features by constraining the most precisely measured reactor angle $\theta_r$, as shown in Fig. 4.

Fig. 4: The prediction of the charged lepton masses (Left), the neutrino mixing angles (Middle), and the leptonic Dirac CP phase $\delta_D$ (Right) by random perturbations of the democratic mass matrix. We show randomly sampled phases $0 \leq \phi_{ij} \leq 2\pi$ and magnitude $0 \leq \epsilon_{ij} \leq 0.2$ (medium line) or $0 \leq \epsilon_{ij} \leq 0.4$ (thick line), while the reactor angle $\theta_r$ is constrained to be in the $3\sigma$ range of the current global fit, $8.44^\circ \pm (3 \times 0.16)^\circ$. For comparison, the prediction of mixing angles and the Dirac CP phase from the anarchy model for complex mass matrix, $d\sigma^2_{s}d\epsilon_{s}d\sigma^2_{a}d\delta_D$, is shown as thick curves with different colors.

The most significant effect of constraining the reactor angle $\theta_r$ appears in the predictions of mixing angles and the Dirac CP phase. In particular, the distributions of $\theta_a$ and $\theta_s$ shrink a lot and hence they are more predictive. As consistent to the zeroth order sum rule (2.9), the atmospheric angle $\theta_a$ now resides in the second octant which can be readily tested by future measurements. For the solar angle $\theta_s$, although it is still symmetric around the maximal value, its distribution also significantly shrinks while the true value $\theta_s \approx 34.5^\circ$ is still covered with sizable probability. On the other hand, the CP probability only has weak preference for maximal CP violation now. The tendency of having large $\cos \delta_D$ for a small reactor angle $\sin \theta_r \approx 1/6$ as demonstrated by (2.9) cancels the preference of large CP induced by the democratic mass matrix with random perturbations as explained in the previous section. However, the CP phase distribution in Fig. 4 is not worse than the anarchy model prediction.

5. Quark Mixing

The democratic mass matrix with random perturbations can not only explain the neutrino mixing but also the quark mixing. As demonstrated in Sec. 2, the quark mixing has naturally suppressed $\theta_{13}$ and $\theta_{23}$ while the $\theta_{12}$ can take any value, see (2.5). To make it exact, the measure of the $T_u$ and $T_d$ transformations are

$$dT_u dT_d = d\theta_{T,u} d\phi_u d\theta_{T,d} d\phi_d.$$  

By randomly sampling $\theta_{T,u}$, $\phi_u$, and $\phi_u - \phi_d$, we obtain a distribution of $\theta_{12}$, shown as a purple solid line in Fig. 5. At leading order, the $\theta_{12}$ tends to have large value with peak at $90^\circ$. Note that this is just a preliminary result as explained in Sec. 2.
and the prediction can be improved by random perturbations as we discuss below.

Fig. 5: The prediction of the quark mixing angles (Left) and the quark CP phase (Right) by random perturbations to the democratic mass matrix for both up and down quarks. While the phases $\phi_{ij}$ of perturbations are randomly sampled in the whole range $[0, 2\pi]$, the deviation size are randomly distributed in the range $0 \leq \epsilon_{ij} \leq 0.2$ (medium) or $0 \leq \epsilon_{ij} \leq 0.4$ (thick). For comparison, the prediction of $\theta_{12}$ at leading order is shown as a solid purple line in the left panel.

Introducing random perturbations to the democratic mass matrix (2.1) can naturally produce nonzero $\theta_{13}$ and $\theta_{23}$ as shown in Fig. 5. Since there is degeneracy between the two lightest mass eigenvalues, namely there is no difference between them, $\theta_{13}$ and $\theta_{23}$ have exactly the same probability distribution. The spread of of $\theta_{13}$ and $\theta_{23}$ distributions is closely related to the perturbation size. This is closely related to the quark mass distribution which takes exactly the same form as the charged lepton mass distribution in Fig. 1. In other words, the hierarchical mixing in the quark sector is closely related to the quark mass hierarchy. On the other hand, $\theta_{12}$ has wide symmetric distribution around the maximal value $45^\circ$ which is already significantly improved. In addition, the distribution of $\theta_{12}$ is actually independent of $\theta_{13}$ and $\theta_{23}$ as indicated by the zeroth-order analysis in Sec. 2, in contrast to the lepton case. For the CKM CP phase, its distribution is almost flat with weak preference of zero value. Although sampling multiple input CP phases in a single mass matrix can enhance the probability of large physical CP phase, the combined prediction from two mass matrices cancels any preference.

For comparison, the anarchy model prediction of quark mixing, $U_{\text{CKM}} = U_{u}^\dagger U_{d}$, takes exactly the same form as the anarchy model prediction of neutrino mixing, $U_{\text{PMNS}} = U_{\ell}^\dagger$, shown in the middle panel of Fig. 1 as thick curves with different colors. This is because left and right rotations can not affect the Haar measure, $dU_{\text{CKM}} = dU_{u} = dU_{d}^\dagger$. Since all Dirac fermions are subject to a $3 \times 3$ complex mass matrix with the same $U(3)$ Haar measure for their mixing, $dU_{d} = dU_{u}^\dagger = dU_{\ell}^\dagger$, and hence $dU_{\text{CKM}} = dU_{\text{PMNS}}$, which we have also checked by numerically sampling the up and down quark mixings. The democratic mass matrix with random perturbations gives a much better prediction than the anarchy model, especially for $\theta_{13}$ and $\theta_{23}$.

For quark mass matrices, the predictions of broken democracy can depend on the energy scale at which the symmetry is broken, because renormalization group running of couplings can alter and amplify the symmetry breaking effects. This is in contrast with the case of neutrino masses and mixings, where the effects of running are small due to the smallness if the symmetry breaking Yukawa couplings.
6. Conclusions

We have considered a broken democracy model with residual $S_3$ symmetries to dictate democratic mass matrices for both up and down quarks as well as for charged leptons. This naturally explains why the CKM matrix has only a sizable 1-2 mixing while the PMNS matrix can has two large mixing angles. In addition, this assignment also leads to massless quarks and charged leptons in the first two generations. To account for the measured values of neutrino mixing angles and the nonzero fermion masses, the residual $S_3$ symmetries have to be broken. Since the residual symmetry is already the one that survives symmetry breaking by definition, there is no other fundamental principle but random deviations to regulate the mass matrices after the residual $S_3$ symmetries are broken. With the general features fixed by residual symmetries at leading order, the random deviations can only be perturbative. Our broken democracy model with residual $S_3$ naturally leads to a large leptonic Dirac CP phase with two peaks around $\pm \pi/2$, in addition to the realistic mixing patterns and mass hierarchy in both quark and lepton sectors.

We have assumed in this paper that the neutrino mass matrix is diagonal ($V_\ell = I$). The neutrino masses are given by the Weinberg-Yanagida operator $[52, 53]$, $(\overline{L_\alpha} \tilde{H}^*) (\tilde{H}^\dagger L_\beta)/M$ where $L_\alpha$ are lepton doublets and $\tilde{H} \equiv \epsilon H^*$ is the CP conjugation of the Higgs doublet $H$, in the standard model, which gives us two $S_3^L$-invariant mass matrices such as

\[
\begin{pmatrix}
a & 0 & 0 \\
0 & a & 0 \\
0 & 0 & a
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
0 & b & b \\
b & 0 & b \\
b & b & 0
\end{pmatrix}.
\]

(6.1)

However, even if we choose the first matrix for the neutrinos, we have a problem. This is because this choice predicts the degenerate masses for the neutrinos which is strongly disfavored by the observations. Therefore, we need a large violation of the $S_3^L$ symmetry. As shown in Sec. 2, this problem may be solved in a model based on the $SO(3)_L \times SO(3)_R$ symmetry [5].

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References

[1] S. Weinberg, “The Problem of Mass,” Trans. New York Acad. Sci. 38, 185 (1977).
[2] H. Fritzsch, “Calculating the Cabibbo Angle,” Phys. Lett. 70B, 436 (1977).

[3] H. Fritzsch, “Quark Masses and Flavor Mixing,” Nucl. Phys. B 155, 189 (1979).

[4] M. Fukugita, M. Tanimoto and T. Yanagida, “Atmospheric neutrino oscillation and a phenomenonological lepton mass matrix,” Phys. Rev. D 57, 4429 (1998) [arXiv:hep-ph/9709388].

[5] M. Tanimoto, T. Watari and T. Yanagida, “Democratic mass matrices from broken $O(3)(L) \times O(3)(R)$ flavor symmetry,” Phys. Lett. B 461, 345 (1999) [arXiv:hep-ph/9904338].

[6] C. S. Lam, “Determining Horizontal Symmetry from Neutrino Mixing,” Phys. Rev. Lett. 101, 121602 (2008) [arXiv:0804.2622 [hep-ph]].

[7] C. S. Lam, “The Unique Horizontal Symmetry of Leptons,” Phys. Rev. D 78, 073015 (2008) [arXiv:0809.1185 [hep-ph]].

[8] D. Hernandez and A. Y. Smirnov, “Lepton mixing and discrete symmetries,” Phys. Rev. D 86, 053014 (2012) [arXiv:1204.0445 [hep-ph]].

[9] D. Hernandez and A. Y. Smirnov, “Discrete symmetries and model-independent patterns of lepton mixing,” Phys. Rev. D 87, no. 5, 053005 (2013) [arXiv:1212.2149 [hep-ph]].

[10] A. Esmaili and A. Y. Smirnov, “Discrete symmetries and mixing of Dirac neutrinos,” Phys. Rev. D 92, no. 9, 093012 (2015) [arXiv:1510.00344 [hep-ph]].

[11] D. A. Dicus, S. F. Ge and W. W. Repko, “Generalized Hidden $Z_2$ Symmetry of Neutrino Mixing,” Phys. Rev. D 83, 093007 (2011) [arXiv:1012.2571 [hep-ph]].

[12] S. F. Ge, D. A. Dicus and W. W. Repko, “$Z_2$ Symmetry Prediction for the Leptonic Dirac CP Phase,” Phys. Lett. B 702, 220 (2011) [arXiv:1104.0602 [hep-ph]].

[13] S. F. Ge, D. A. Dicus and W. W. Repko, “Residual Symmetries for Neutrino Mixing with a Large $\theta_{13}$ and Nearly Maximal $\delta_D$,” Phys. Rev. Lett. 108, 041801 (2012) [arXiv:1108.0964 [hep-ph]].

[14] R. M. Fonseca and W. Grimus, “Classification of lepton mixing matrices from finite residual symmetries,” JHEP 1409, 033 (2014) [arXiv:1405.3678 [hep-ph]].

[15] S. F. Ge, “Unifying Residual $Z_2^{23} \otimes Z_2^1$ Symmetries and Quark-Lepton Complementarity,” [arXiv:1406.1985 [hep-ph]].

[16] S. K. Agarwalla, S. S. Chatterjee, S. T. Petcov and A. V. Titov, “Addressing Neutrino Mixing Schemes with DUNE and T2HK,” [arXiv:1711.02107 [hep-ph]].

[17] S. T. Petcov, “Discrete Flavour Symmetries, Neutrino Mixing and Leptonic CP Violation,” [arXiv:1711.10806 [hep-ph]].

[18] P. Pasquini, “Review: Long-baseline oscillation experiments as a tool to probe High Energy Models,” [arXiv:1802.00821 [hep-ph]].

[19] J. Barry and W. Rodejohann, “Neutrino Mass Sum-rules in Flavor Symmetry Models,” Nucl. Phys. B 842, 33 (2011) [arXiv:1007.5217 [hep-ph]].
[20] J. Gehrlein, A. Merle and M. Spinrath, “Predictivity of Neutrino Mass Sum Rules,” Phys. Rev. D 94, no. 9, 093003 (2016) [arXiv:1606.04965 [hep-ph]].

[21] Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi and T. Fukuyama, “Universal texture of quark and lepton mass matrices and a discrete symmetry $Z_3$,” Phys. Rev. D 66, 093006 (2002) [hep-ph/0209333].

[22] A. S. Joshipura, “Universal 2-3 symmetry,” Eur. Phys. J. C 53, 77 (2008) [hep-ph/0512252].

[23] L. J. Hall, H. Murayama and N. Weiner, “Neutrino mass anarchy,” Phys. Rev. Lett. 84, 2572 (2000) [arXiv:hep-ph/9911341].

[24] N. Haba and H. Murayama, “Anarchy and hierarchy,” Phys. Rev. D 63, 053010 (2001) [arXiv:hep-ph/0009174].

[25] H. Harari, H. Haut and J. Weyers, “Quark Masses and Cabibbo Angles,” Phys. Lett. 78B, 459 (1978).

[26] Y. Koide, “Charged Lepton Mass Matrix With Democratic Family Mixing,” Z. Phys. C 45, 39 (1989).

[27] M. Tanimoto, “New Quark Mass Matrix Based on the BCS Form,” Phys. Rev. D 41, 1586 (1990).

[28] P. Kaus and S. Meshkov, “Generational Mass Generation and Symmetry Breaking,” Phys. Rev. D 42, 1863 (1990).

[29] H. Fritzsch and J. Plankl, “Flavor Democracy and the Lepton - Quark Hierarchy,” Phys. Lett. B 237, 451 (1990).

[30] H. Fritzsch and D. Holttmannspotter, “The Breaking of subnuclear democracy as the origin of flavor mixing,” Phys. Lett. B 338, 290 (1994) [arXiv:hep-ph/9406241].

[31] G. C. Branco and J. I. Silva-Marcos, “Predicting $V$ (CKM) with universal strength of Yukawa couplings,” Phys. Lett. B 359, 166 (1995) [arXiv:hep-ph/9507299].

[32] H. Fritzsch and Z. Z. Xing, “Lepton mass hierarchy and neutrino oscillations,” Phys. Lett. B 372, 265 (1996) [arXiv:hep-ph/9509389].

[33] Z. Z. Xing, “Implications of the quark mass hierarchy on flavor mixings,” J. Phys. G 23, 1563 (1997) [arXiv:hep-ph/9609204].

[34] A. Mondragon and E. Rodriguez-Jauregui, “The Breaking of the flavor permutational symmetry: Mass textures and the CKM matrix,” Phys. Rev. D 59, 093009 (1999) [arXiv:hep-ph/9807214].

[35] H. Fritzsch and Z. Z. Xing, “Large leptonic flavor mixing and the mass spectrum of leptons,” Phys. Lett. B 440, 313 (1998) [arXiv:hep-ph/9808272].

[36] H. Fritzsch and Z. Z. Xing, “Mass and flavor mixing schemes of quarks and leptons,” Prog. Part. Nucl. Phys. 45, 1 (2000) [arXiv:hep-ph/9912358].
[37] N. Haba, Y. Matsui, N. Okamura and T. Suzuki, “Are lepton flavor mixings in the democratic mass matrix stable against quantum corrections?,” Phys. Lett. B 489, 184 (2000) [arXiv:hep-ph/0005064].

[38] G. C. Branco and J. I. Silva-Marcos, “The Symmetry behind extended flavor democracy and large leptonic mixing,” Phys. Lett. B 526, 104 (2002) [arXiv:hep-ph/0106125].

[39] M. Fujii, K. Hamaguchi and T. Yanagida, “Leptogenesis with almost degenerate majorana neutrinos,” Phys. Rev. D 65, 115012 (2002) [arXiv:hep-ph/0202210].

[40] H. Fritzsch and Z. Z. Xing, “Democratic neutrino mixing reexaminied,” Phys. Lett. B 598, 237 (2004) [arXiv:hep-ph/0406206].

[41] W. Rodejohann and Z. Z. Xing, “Flavor democracy and type-II seesaw realization of bilarge neutrino mixing,” Phys. Lett. B 601, 176 (2004) [arXiv:hep-ph/0408195].

[42] T. Teshima, “Flavor mass and mixing and S(3) symmetry: An S(3) invariant model reasonable to all,” Phys. Rev. D 73, 045019 (2006) [arXiv:hep-ph/0509094].

[43] Z. Z. Xing, D. Yang and S. Zhou, “Broken S3 Flavor Symmetry of Leptons and Quarks: Mass Spectra and Flavor Mixing Patterns,” Phys. Lett. B 690, 304 (2010) [arXiv:1004.4234 [hep-ph]].

[44] S. Zhou, “Relatively large theta13 and nearly maximal theta23 from the approximate S3 symmetry of lepton mass matrices,” Phys. Lett. B 704, 291 (2011) [arXiv:1106.4808 [hep-ph]].

[45] R. Jora, J. Schechter and M. N. Shahid, “Naturally perturbed S3 neutrinos,” Int. J. Mod. Phys. A 28, 1350028 (2013) [arXiv:1210.6755 [hep-ph]].

[51] Z. G. Si, X. H. Yang and S. Zhou, “Broken S3L × S3R Flavor Symmetry and Leptonic CP Violation,” Chin. Phys. C 41, no. 11, 113105 (2017) [arXiv:1706.03991 [hep-ph]].

[52] S. Weinberg, “Baryon and Lepton Nonconserving Processes,” Phys. Rev. Lett. 43, 1566 (1979).
[54] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola and J. W. F. Valle, “Status of neutrino oscillations 2017,” [arXiv:1708.01186 [hep-ph]].