Some considerations on the present-day results for the detection of frame-dragging after the final outcome of GP-B

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Abstract – The cancellation of the first even zonal harmonic coefficient \( J_2 \) from the linear combination \( f^{(2L)} \) of the nodes \( \Omega \) of LAGEOS and LAGEOS II used in the latest tests of the Lense-Thirring effect cannot be perfect, contrary to what assumed so far. It is so also because of the uncertainties in the spatial orientation of the terrestrial spin axis \( \hat{k} \). As a consequence of above, the coefficient \( c_1 \) entering \( f^{(2L)} \), which is not a solve-for parameter being, instead, theoretically computed from the analytic expressions of the classical node precessions \( \dot{\Omega}_J \) due to \( J_2 \), is, on average, uncertain at a \( 10^{-8} \) level over multi-decadal time spans \( \Delta T \) comparable to those used in the data analyses performed so far. A further \( \approx 20\% \) systematic uncertainty, thus, occurs. The shift \( \Delta \rho_{LT} \) due to the gravitomagnetic frame-dragging on the station-spacecraft range \( \rho \) is numerically computed over \( \Delta T = 15 \text{d} \) and \( \Delta T = 1 \text{y} \). The need to look at such a directly observable quantity is highlighted, along with some critical remarks concerning the methodology used so far to measure the Lense-Thirring effect with the LAGEOS satellites. Suggestions for a different, more trustable and reliable approach are offered.

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Introduction. – In the Einsteiniang general relativity, which is a fully Lorentz-invariant theory of gravitation, matter-energy currents create an additional, magnetic-like component of the gravitational field [1] with respect to the static case. It is believed to play a relevant role in explaining relativistic jets ejected from active galactic nuclei [2,3]. The gravitomagnetic field of a rotating body affects orbiting test particles, precessing gyroscopes, moving clocks and atoms, and propagating electromagnetic waves with a variety of phenomena [4]. Some of them have been put to the test more or less recently. For an overview of such a phenomenology in the solar system, see, e.g., [5].

The Gravity Probe B (GP-B) experiment [6] officially came to an end, with the release of its final results [7] according to which the general relativistic gravitomagnetic gyroscope precession [8,9] would have been measured with a claimed accuracy of 19%. Such a figure is greater than the previously expected 1% level because of a number of unwanted systematic errors whose proper treatment required much additional efforts by the GP-B team [10]. Independent analyses by different teams will be important in critically assessing the reliability of the results of [7]. This is beyond the scope of the present paper. It is now even more important than before to critically scrutinize the competing tests of the gravitomagnetic Lense-Thirring orbital precessions [11] performed in the past years [12] with the LAGEOS and LAGEOS II SLR satellites in the gravitational field of the Earth, as originally proposed in [13]. Let us recall that the GP-B mission was a dedicated experiment in the terrestrial gravitational field which cost US$ 750 million and lasted 52 y, while the gravitomagnetic data analyses [14] of the LAGEOS spacecrafts, which were originally launched for different purposes, were much less expensive and comparatively less extended in time. According to Ciufolini, its accuracy would be 10% or better [15]; for recent articles establishing a comparison between GP-B and the previous LAGEOS-based results, see [16-18] in which it is basically argued that GP-B would have just reached the same results of the earlier tests with the LAGEOS spacecraft at a much higher cost and with an even worst, or, at most, comparable, accuracy.
Can the cancellation of the effect of the quadrupole mass moment of the Earth in the LAGEOS-based tests be perfect? – The following linear combination of the logarithm of the ascending nodes $f^{(2L)}$ of LAGEOS and LAGEOS II [19,20],

$$f^{(2L)} = \Omega^{(L)} + c_1 \Omega^{(L II)}, \tag{1}$$

was used in the tests of the Lense-Thirring effect performed so far with such artificial bodies orbiting the Earth. Frame-dragging was purposely not modeled [15], and time series of “residuals” of the nodes [21,22] of both satellites, combined according to eq. (1), were analyzed and subsequently fitted with a straight line plus other time-dependent signals. The coefficient $c_1$ entering eq. (1) is not one of the several solve-for parameters estimated in the data reduction process. Following an approach set forth in a different context [23], its value is theoretically computed as [20]

$$c_1 = -\frac{f^{(L)}}{\Omega^{(L II)}} \tag{2}$$

from the analytical expressions of the classical secular node precessions $\dot{\Omega}_2$ of both the LAGEOS satellites caused by the first even zonal harmonic coefficient $J_2$ of the expansion in multipoles of the Newtonian part $U_N$ of the terrestrial gravitational potential. This multipolar expansion of $U_N$ accounts for its departure from spherical symmetry because of the centrifugal deformation due to the Earth’s diurnal rotation [24]. Traditionally, eq. (2) has always been computed [20] so far from the well-known expression [24]

$$\dot{\Omega}_2 = \frac{3nJ_2R^2\cos I}{2a^2(1-e^2)^2}. \tag{3}$$

In eq. (3) $a$ is the semi-major axis of the satellite’s orbit, $e$ is its eccentricity, $I$ is its inclination to the reference $\{X,Y\}$-plane, assumed to be coincident with the Earth’s equator, $R$ is the terrestrial equatorial radius, and $n = \sqrt{GM/a^3}$ is the Keplerian mean motion of the satellite with respect to the Earth whose mass is denoted with $M$; $G$ is the Newtonian constant of gravitation. The orbital parameters of LAGEOS and LAGEOS II, referred to a geocentric inertial system, are shown in table 1.

The aim of eq. (1), with eq. (2), is to cancel out, by construction, such precessions. Since they are nominally 7 orders of magnitude greater than the Lense-Thirring ones,

$$\Omega_{LT} = \frac{2GS}{c^2a^3(1-e^2)^{3/2}}, \tag{4}$$

where $S$ is the Earth’s angular momentum and $c$ is the speed of light in vacuum, they represent a major source of systematic bias in determining them. For the LAGEOS satellites eq. (4) yields about 30 milliarcseconds per year (mas y$^{-1}$ in the following), so that the combined Lense-Thirring signal amounts to approximately 50 mas y$^{-1}$ according to eq. (1). In principle, such a removal of $J_2$ from eq. (1) is exact, or so it has always been considered until now. Indeed, in all the more or less realistic evaluations of the systematic error due to the geopotential existing in the literature [5,12,14], the part due to $J_2$ was always set to zero by definition and independently of $\sigma_{J_2}$. Thus, the focus was on the impact of the other, uncancellation even zonal harmonics of higher degree $J_\ell$, $\ell = 4, 6, 8, \ldots$, known with a certain level of uncertainty. Actually, the effect of $J_2$ on eq. (1) cannot be exactly zero because of a number of factors.

One of them relies on the fact that, for a given set of values of the satellites’ orbital parameters from which $c_1$ is computed by means of eq. (2) and eq. (3), the actual accuracy with which $c_1$ can be known is necessarily limited by the uncertainties with which the satellites’ Keplerian orbital elements of interest can be determined in the data reduction procedure. It was recently shown [25] that $\sigma_{a} \simeq 2$ cm and $\sigma_{I} \simeq 0.5$ mas yield $\Delta c_1 \simeq 10^{-8}$, corresponding to a further systematic uncertainty of about 20% in the Lense-Thirring signature. If, instead, one optimistically assumes $\sigma_{a} \simeq 2$ cm and $\sigma_{I} \simeq 10^{-30}$ mas, then $\Delta c_1 \simeq 8 	imes 10^{-9}$, which implies an additional 14% bias. On the contrary, $c_1$ was always released so far with a very limited number of significant digits; for example, in [12] we have $c_1 = 0.545$. As pointed out in [25], it would be incorrect to argue that the impact of $\Delta c_1$ would be negligible since it should be multiplied by the uncertainty in $J_2$. Indeed, the standard error propagation theory tells us that, in addition to the mixed, cross-correlated terms containing the products of the uncertainties, there are also

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1The longitude of the ascending node $\Omega$ is one of the angles determining the orientation in space of the satellite’s Keplerian ellipse.

2The term “residual” is, actually, improper for the node. Indeed, all the Keplerian orbital elements are not observable quantities. They can only be computed at various epochs from the corresponding state vectors in Cartesian coordinates which, in turn, are computed from the measured values of the direct observables.

3They were never explicitly specified in the analyses performed so far, by assuming for them some standard figures [12], approximately representative of the orbital configurations of the LAGEOS satellites.
the linear terms proportional to the uncertainties in each parameter. Moreover, in the LAGEOS tests both \( c_1 \) and \( J_2 \) are not estimated solve-for parameters. For the sake of definiteness, we will denote the values of \( c_1 \) obtained from eq. (3) with \( c_1^{(0)} \); table 1 yields \( c_1^{(0)} = 0.540976405 \).

Another issue, not yet considered in the literature, is that it is incorrect to assume a perfect alignment of the Earth’s spin axis, whose unit vector is denoted by \( \hat{k} \), and the reference Z-axis of the geocentric inertial system actually used. Indeed, on the one hand, the latter refers to a given reference epoch, typically J2000.0, while the time spans \( \Delta T \) over which the data of LAGEOS and LAGEOS II were analyzed necessarily cover 19 y or less: during such a temporal interval \( \hat{k} \) did not remain fixed in the inertial space due to a variety of physical processes [26]. Such changes, even if taken into account and modeled, are, of course, known only with a limited accuracy [27].

On the other hand, it is well known that another source of uncertainty in the location of \( \hat{k} \) is given by the polar motion [26] with respect to the Earth’s crust itself, known with an accuracy of about \( 4 \times 10^{-8} \) [26,28] over a time interval of just 1 y. Thus, it is important to quantitatively assess the further systematic error \( \Delta c_1 \) induced by the use of \( c_1^{(0)} \) with respect to values, denoted as \( c_1^{(\rho)} \), computed by taking into account the real spatial orientation of \( \hat{k} \). To this aim, a first step consists of computing the long-term node variations \( \Omega \) for a generic orientation of \( \hat{k} \). The acceleration experienced by a test body orbiting an oblate central mass rotating about a generic direction \( \hat{k} \) is [26]

\[
A_{J_2} = -\frac{3GMJ_2R^2}{2r^4}\{1 - 5(\hat{r} \cdot \hat{k})^2\hat{r} + 2(\hat{r} \cdot \hat{k})\hat{k}\}. \tag{5}
\]

Since its magnitude is quite smaller than the main Newtonian monopole, its effect on the particle’s orbital motion can be straightforwardly worked out with standard perturbative techniques. The Gauss equation for the variation of the node [29] allows to obtain the rate of change of \( \Omega \) averaged over one orbital revolution. It turns out to be

\[
\dot{\Omega}_{J_2} = \frac{3nJ_2R^2}{4a^2(1 - e^2)^2} \mathcal{F}(I, \Omega, \hat{k}), \tag{6}
\]

with

\[
\mathcal{F} = 2k_Z \cos 2I \cos(\hat{k}_X \sin \Omega - \hat{k}_Z \cos \Omega) + \cos I [k_X^2 + k_Y^2 - 2k_Z^2 + (k_Y^2 - k_X^2) \cos 2\Omega] - 2k_Xk_Y \sin 2\Omega]. \tag{7}
\]

It is an exact result in \( e \) and \( I \) in the sense that no a priori simplifying assumptions on their values were assumed; in general, it can also be useful in other contexts involving different central bodies and test particles [5]. It can be noticed that, according to eq. (6) and eq. (7), the long-term rate of change of \( \Omega \) consists of the sum of a genuine secular precession and of a harmonic, time-dependent signal involving \( \Omega \) and \( 2\Omega \). Moreover, eq. (7) reduces to

\[
\mathcal{F} = -2 \cos I \tag{8}
\]

for \( \hat{k}_X = \hat{k}_Y = 0, \hat{k}_Z = \pm 1 \), yielding the well-known secular precession of eq. (3). We will denote the value of \( c_1 \) computed from eqs. (6) and (7) by \( c_1^{(\rho)} \). In fig. 1 we plot the uncertainty in \( c_1 \) raising from having used just \( c_1^{(0)} \) over a temporal interval \( \Delta T = 19 \) y representative of the time spans actually used in real data analyses, and for a 10 mas uncertainty in the position of \( \hat{k} \).

It can be noticed that its impact is non-negligible since it is of the order of \( 4 \times 10^{-8} \), implying a further \( \gamma \approx 20\% \) systematic uncertainty in the gravitomagnetic signature.

**What was really measured in the LAGEOS-based tests?** – In SLR studies, the directly observable quantity is the range \( \rho \) between a spacecraft equipped with retroreflectors and a ground-based station [26]. It is straightforwardly computed by multiplying \( c \) by the time interval elapsed between the emission of the laser pulse sent to the orbiting target body and its subsequent reception after it was bounced back by the retroreflectors onboard the satellite. The precision of such measurements is nowadays at the mm level [26]. Post-fit range residuals

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4See also http://www.iers.org/nn_10398/IERS/EN/Science/EarthRotation/PolarMotion.html?nnn=true on the WEB.
for good targets like LAGEOS and LAGEOS II, obtained after the adjustment of a number of solved-for parameters pertaining to the satellites’ physical properties and orbital dynamics, and the measurement process itself, are as large as 1 cm or less in a Root-Mean-Square (RMS) sense [26]. They globally reflect the impact of all the unmodeled and mismodeled sources of errors like, e.g., some unknown or poorly modeled forces acting on the satellites. The post-fit range residuals are also a measure of the effectiveness of the orbit determination process in which the estimated values of some parameters may partly or totally absorb the effects of other parameters not included in the list of those to be adjusted, or of totally unmodeled forces themselves. In general, if one is interested in a certain dynamical feature, then it must be explicitly modeled in such a way that one or more dedicated solve-for parameters are estimated. Subsequently, the resulting covariance matrix can be examined to identify the correlations between various parameters. Clearly, the magnitude of post-fit range residuals can only be greater than, or as large as, the range measurement precision. Perfect models and/or total removal of all effects that have not been modeled would provide residuals as large as the measurement precision.

Extending such considerations to the frame-dragging tests made so far with the LAGEOS satellites, it must be remarked that, actually, the Lense-Thirring force was never modeled, so that it should be considered in the same way as a source of systematic error impacting, in case, the post-fit range residuals to a certain level. No dedicated solve-for parameters were ever estimated; thus, the gravitomagnetic signature might have been partly or totally absorbed in the estimation of the several other parameters in the data reduction process, and partially or totally removed from the range signature. If frame-dragging fully impacted the ranges as predicted by general relativity, there should be time series of post-fit range residuals with the characteristic signature of the gravitomagnetic force itself. See fig. 2 and fig. 3 displaying the numerically produced nominal Lense-Thirring effect on the station-satellite range for LAGEOS and LAGEOS II over a time interval of $\Delta t = 1$ y. On the other hand, the same set of data should be analyzed by explicitly modeling the Lense-Thirring effect in order to check if statistically significant differences with respect to the previous case would occur. This would be a crucial test of the ability to actually measure terrestrial gravitomagnetism by means of the LAGEOS and LAGEOS II SLR data. In fact, after more than 15 years since the first tests, such “gravitomagnetic” post-fit range residuals were never shown so far. It should be noticed that there is a contradiction between claiming sub-cm post-fit range residuals, obtained without modeling frame-dragging, and figs. 2 and 3 displaying signatures with RMS variances as large as 18.0 cm and 46.1 cm, respectively. Indeed, one should assume either that the gravitomagnetic signal, not modeled, was almost entirely removed or that it was almost canceled by the superposition of other unmodeled/mismodeled competing dynamical effects. After all, such a removal would not be implausible since, as shown by fig. 4 and fig. 5, the nominal size of the Lense-Thirring range perturbation is just at the level of cm on a timescale of $\Delta t = 15$ d. It is not clear, however, why all the other effects not modeled at all, or poorly modeled, should be exactly removed, or should cancel each other leaving just the completely unmodeled Lense-Thirring signal, which is precisely what one expects to find in the data. It is much more plausible that it is somewhat absorbed in some of the estimated parameters and removed from the residual signal to a certain extent. Somebody may argue that the removal of the Lense-Thirring signature can occur only if certain once-per-revolution empirical cross-track accelerations were estimated. First of all, it should be explicitly proven that they were actually not...
Some considerations on the present-day results for the detection of frame-dragging etc.

We remark that the LAGEOS-based tests are likely plagued by another source of intrinsic \textit{a priori} imprinting of general relativity itself in addition to those already pointed out [5]. Indeed, they always made use of a reference system whose materialization heavily relies upon SLR data, among which those from LAGEOS and LAGEOS II themselves play a fundamental role.

The considerations exposed here are, in principle, valid also for other performed or proposed tests of general relativity with the LAGEOS satellites [35], and also for those which should be implemented in the near future with the existing LAGEOS and LAGEOS II, and with the new LARES satellite [36], to be launched in late 2011 with a VEGA rocket.

Conclusions. – In conclusion, we can entertain reasonable doubts as to what it was actually seen in the tests with LAGEOS and LAGEOS II made so far, and what has been passed of as frame-dragging in them. Only the use of a completely different approach, more related to quantities that are actually measured, could afford to talk about clear and unambiguous tests of this subtle effect. Frame-dragging should be explicitly modeled and solved-for in the LAGEOS and LAGEOS II data reduction process; post-fit range residuals produced with and without a model for the Lense-Thirring effect should be displayed and analyzed; a different materialization of the reference system used so far, mostly based on the observations of LAGEOS and LAGEOS II themselves, should be adopted; it would be preferable that GR is explicitly modeled and solved-for in future dedicated global gravity field solutions combining data from several satellites. Otherwise, they should make clear why they do not implement the strategy advocated here which, after all, is standard practice in all branches of geodetic and astronomical studies.

Moreover, even accepting the strategy followed so far, the unavoidable uncertainties in our knowledge of the Earth’s rotation axis affect the necessarily imperfect calculation of the theoretical coefficient $c_1$ entering the linear combination of the nodes of LAGEOS and LAGEOS II. It does not allow to obtain an exact cancellation of the aliasing bias due to the first even zonal harmonic $J_2$ of the geopotential which, instead, would still be present at a $\simeq 20\%$ level of the Lense-Thirring signal. Let us recall that a further $10–20\%$ alias comes from the uncertainty in $c_1$ due to the errors in the satellites’ orbital parameters $a$ and $I$.

Thus, more work is still needed to really consider the LAGEOS-based attempt as a robust complement of the GP-B mission from the point of view of reliability, trustability and methodology. Although the LAGEOS-based tests had measured something that really relates to the Lense-Thirring effect, their overall uncertainty will probably make them less accurate than the GP-B experiment. Anyway, independent analyses of the data of the Stanford team by different groups are certainly required.

estimated in the dedicated LAGEOS data reductions. More importantly, it is impossible to \textit{a priori} decide in which of the estimated parameters the cancellation would actually occur. Suffice it to say that in much more “clean” scenarios like planetary astronomy, not plagued by the host of disturbances and non-gravitational effects of satellite geodesy, it is common practice to explicitly model the effects one is interested in and solve for one or more dedicated parameters just to avoid the risk that they may be partially or totally absorbed in the estimation of the initial state vectors. Interestingly, this has been done recently [31–34] even for hypothetical forces that, as the Pioneer Anomaly, if they really existed in Nature, would have caused signatures much greater than the accuracy of the observations themselves.

Fig. 4: (Colour on-line) Numerically integrated Lense-Thirring station-satellite range perturbation $\Delta \rho_{LT}$ for LAGEOS over $\Delta T = 15 \, \text{d}$. Its variance is 0.8 cm. We choose the ITRF2000 coordinates of the GRAZ station, from [30]: cut-off elevation angle of 20 deg.

Fig. 5: (Colour on-line) Numerically integrated Lense-Thirring station-satellite range perturbation $\Delta \rho_{LT}$ for LAGEOS II over $\Delta T = 15 \, \text{d}$. Its variance is 1.9 cm. We choose the ITRF2000 coordinates of the GRAZ station, from [30]: cut-off elevation angle of 20 deg.
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