ARQ-Based Secret Key Sharing

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Abstract—This paper develops a novel framework for sharing secret keys using existing Automatic Repeat reQuest (ARQ) protocols. Our approach exploits the multi-path nature of the wireless environment to hide the key from passive eavesdroppers. The proposed framework does not assume the availability of any prior channel state information (CSI) and exploits only the one bit ACK/NACK feedback from the legitimate receiver. Compared with earlier approaches, the main innovation lies in the distribution of key bits among multiple ARQ frames. Interestingly, this idea allows for achieving a positive secrecy rate even when the eavesdropper experiences more favorable channel conditions, on average, than the legitimate receiver. In the sequel, we characterize the information theoretic limits of the proposed schemes, develop low complexity explicit implementations, and conclude with numerical results that validate our theoretical claims.

I. INTRODUCTION

Wireless communication, because of its broadcast nature, is vulnerable to eavesdropping and other security attacks. Therefore, pushing wireless networking to its full potential requires finding solutions to its intrinsic security problems. In this paper, we consider a physical layer-based scheme to share a secret key between two users (Alice and Bob) communicating over a fading channel in the presence of a passive eavesdropper (Eve). The private key can then be used to secure further exchange of information.

Arguably, the recent flurry of interest on wireless physical layer secrecy was inspired by Wyner’s wiretap channel [1], [2]. Under the assumption that Eve’s channel is a degraded version of Bob’s, Wyner showed that perfectly secure communication is possible by hiding the message in the additional noise level seen by Eve. The effect of slow fading on the secrecy capacity was studied later. In particular, by appropriately distributing the message across different fading realizations, it was shown that the multi-user diversity gain can be harnessed to enhance the secrecy capacity, e.g. [3], [8]. Another frame of work [4] proposed using the well-known Hybrid ARQ protocols to facilitate the exchange of secure messages between Alice and Bob.

This paper extends this line of work in two ways. First, by distributing the key bits over multiple ARQ frames, we establish the achievability of a vanishing probability of secrecy outages [4] at the expense of a larger delay. Interestingly, using this approach, a non-zero perfectly secure key rate is achievable even when Eve is experiencing a more favorable average signal-to-noise ratio (SNR) than Bob (unlike the scheme proposed in [4]). Second, we develop explicit constructions for secrecy ARQ coding that enjoy low implementation complexity. The proposed scheme utilizes the ARQ protocol to create an erasure wiretap channel and then uses known ideas from coset coding to construct optimal codes for this channel [5]–[7].

The rest of this paper is organized as follows. Our system model is detailed in Section II. Section III provides the information theoretic analysis of our model. Explicit secrecy coding schemes are developed in Section IV. In Section V, we present numerical results. Finally, Section VI summarizes our conclusions.

II. SYSTEM MODEL

Our model, shown in Figure 1, involves a legitimate receiver, Bob, with a feedback channel to the sender, Alice. Eve is a passive eavesdropper. We assume block fading channels that are independent of each other.
given by,
\begin{align*}
y(i, j) &= g_b(j) x(i, j) + w_b(i, j), \quad (1) \\
z(i, j) &= g_e(j) x(i, j) + w_e(i, j), \quad (2)
\end{align*}

where \( x(i, j) \) is the \( i \)th transmitted symbol in the \( j \)th block, \( y(i, j) \) is the \( i \)th received symbol by Bob in the \( j \)th block, \( z(i, j) \) is the \( i \)th received symbol by Eve in the \( j \)th block, \( g_b(j) \) and \( g_e(j) \) are the complex block channel gains from Alice to Bob and Eve, respectively. Moreover, \( w_b(i, j) \) and \( w_e(i, j) \) are the zero-mean, unit-variance additive white complex Gaussian noise at Bob and Eve, respectively. We denote the block fading power gains of the main and eavesdropper channels by \( h_b = |g_b(j)|^2 \) and \( h_e = |g_e(j)|^2 \). We do not assume any prior knowledge about the channel state information at Alice. However, Bob is assumed to know \( g_b(j) \) and \( g_e(j) \) to know both \( g_b(j) \) and \( g_e(j) \). We impose the following short-term average power constraint
\[
E(|x(i, j)|^2) \leq \bar{P}.
\quad (3)
\]

Our model only allows for one bit of ARQ feedback between Alice and Bob. Each ARQ epoch is assumed to be contained in one coherence interval (i.e., fixed channel gains) and that different epochs correspond to independent coherence intervals (the same assumptions as [4]). We denote the constant rate used in each transmission frame by \( R_0 \) bits/channel use. The transmitted packets are assumed to carry a perfect error detection mechanism that Bob (and Eve) used to determine whether the packet has been received correctly or not. Based on the error check, Bob sends back to Alice an ACK/NACK bit, through a public and error-free feedback channel. Eve is assumed to be passive (i.e., can not transmit); an assumption which can be justified in several practical settings. To minimize Bob’s receiver complexity, we adopt the memoryless decoding assumption implying that frames received in error are discarded and not used to aid in future decoding attempts.

III. INFORMATION THEORETIC FOUNDATION

In our setup, Alice wishes to share a secret key \( W \in \mathcal{W} = \{1, 2, \ldots, M\} \) with Bob. This key can be used for securing future data transmission. To transmit this key, Alice and Bob use an \((M, m)\) code consisting of: 1) a stochastic encoder \( f_m(.) \) at Alice that maps the key \( w \) to a codeword \( x^m \in \mathcal{X}^m \), 2) a decoding function \( \hat{y}^m \rightarrow W \) which is used by Bob to recover the key. The codeword is partitioned into \( a \) blocks each of \( n_1 \) symbols where \( m = a n_1 \). In this section, we focus on the asymptotic scenario where \( a \rightarrow \infty \) and \( n_1 \rightarrow \infty \).

Alice starts with a random selection of the first block of \( n_1 \) symbols. Upon reception, Bob attempts to decode this block. If successful, it sends an ACK bit to Alice who moves ahead and makes a random choice of the second \( n_1 \) and sends it to Bob. Here, Alice must make sure that the concatenation of the two blocks belong to a valid codeword. As shown in the sequel, this constraint is easily satisfied. If an error was detected, then Bob sends a NACK bit to Alice. Here, we assume that the error detection mechanism is perfect which is justified by the fact that \( n_1 \rightarrow \infty \). In this case, Alice replaces the first block of \( n_1 \) symbols with another randomly chosen block and transmits it. The process then repeats until Alice and Bob agree on a sequence of \( a \) blocks, each of length \( n_1 \) symbols, corresponding to the key.

The code construction must allow for reliable decoding at Bob while hiding the key from Eve. It is clear that the proposed protocol exploits the error detection mechanism to make sure that both Alice and Bob agree on the key (i.e., ensures reliable decoding). What remains is the secrecy requirement which is measured by the equivocation rate \( R_e \) defined as the entropy rate of the transmitted key conditioned on the intercepted ACKs or NACKs and the channel outputs at Eve, i.e.,
\[
R_e \triangleq \frac{1}{n} H(W|Z^n, K_b^b, G_b^b, G_e^b), \quad (4)
\]

where \( n \) is the number of symbols transmitted to exchange the key (including the symbols in the discarded blocks due to decoding errors, \( b = \frac{n}{m}, K_b^b = \{K(1), \ldots, K(b)\} \) denotes sequence of ACK/NACK bits, \( G_b^b \) and \( G_e^b \) are the sequences of channel coefficients seen by Bob and Eve in the \( b \) blocks, and \( Z^n = \{Z(1), \ldots, Z(n)\} \) denotes Eve’s channel outputs in the \( n \) symbol intervals. We limit our attention to the perfect secrecy scenario, which requires the equivocation rate \( R_e \) to be arbitrarily close to the key rate. The secrecy rate \( R_s \) is said to be achievable if for any \( \epsilon > 0 \), there exists a sequence of codes \( (2^{nR_s}, m) \) such that for any \( m \geq m(\epsilon) \), we have
\[
R_s \geq \frac{1}{n} H(W|Z^n, K_b^b, G_b^b, G_e^b) \geq R_s - \epsilon \quad (5)
\]

and the key rate for a given input distribution is defined as the maximum achievable perfect secrecy rate with this distribution. The following result characterizes this rate, assuming a Gaussian input distribution

**Theorem 1:** For the memoryless ARQ, the perfect secrecy rate for **Gaussian inputs** for a given transmit power \( P \) is given by:
\[
C_s = \max_{R_0, P \leq \bar{P}} \{\Pr(R_0 \leq \log(1+h_bP))E[R_0-\log(1+h_eP)]\}, \quad (6)
\]

where \( [x]^+ = \max(0,x) \). All logarithms in this paper are taken to base 2, unless otherwise stated.

**Proof:** Here, we only give a sketch of the proof of achievability. Due to space limitations, the converse will be deferred to the journal paper version. The proof is given for a fixed average power \( P \leq \bar{P} \) and transmission rate \( R_0 \). The key rate is then obtained by the appropriate maximization. Let \( R_s = C_s - \delta \) for some small \( \delta > 0 \) and \( R = R_0 - \epsilon \).

We first generate all binary sequences \( \{V\} \) of length \( mR \) and then independently assign each of them randomly to one of \( 2^{nR_e} \) groups, according to a uniform distribution. This ensures that any of the sequences are equally likely to be within any of the groups. Each secret message \( w \in \{1, \ldots, 2^{nR_e}\} \) is then assigned a group \( V(w) \). We then generate a Gaussian codebook consisting of \( 2^{n(R_0-\epsilon)} \) codewords, each of length \( n_1 \) symbols. The codebooks are then revealed to Alice, Bob,
and Eve. To transmit the codeword, Alice first selects a random group $v(i)$ of $n_1 R$ bits, and then transmits the corresponding codeword, drawn from the chosen Gaussian codebook. If Alice receives an ACK bit from Bob, both are going to store this group of bits and selects another group of bits to send in the next coherence interval in the same manner. If a NACK was received, this group of bits is discarded and another is generated in the same manner. This process is repeated till both Alice and Bob have shared the same key $w$ corresponding to $nR_e$ bits. We observe that the channel coding theorem implies the existence of a Gaussian codebook where the fraction of successfully decoded frames is given by

$$\frac{m}{n} = \Pr(R_0 \leq \log(1 + h_b P)),$$

as $n_1 \to \infty$. The equivocation rate at the eavesdropper can then be lower bounded as follows.

$$nR_e = H(W|Z^m, K^m, G_0^b, G_0^c)$$

(a) $= H(W|Z^m, G_0^a, G_0^e)
\leq H(W, Z^m|G_0^a, G_0^e) - H(Z^m|G_0^a)
= H(W, Z^m|G_0^a) - H(Z^m|G_0^a)
$$

(b) $\leq \sum_{j=1}^{a} H(X(j)|Z(j), G_b(j), G_c(j))
- H(X^m|W, Z^m, G_0^a, G_0^c)$

(c) $\geq \sum_{j \in N_m} [H(X(j)|G_b(j), G_c(j))
- I(X(j); Z(j)|G_b(j), G_c(j))
- H(X^m|W, Z^m, G_0^b, G_0^c)]$

$$\geq \sum_{j \in N_m} n_1 [R_0 - \log(1 + h_e(j) P) - \epsilon]
- H(X^m|W, Z^m, G_0^b, G_0^c)$

$$= n C_s - H(X^m|W, Z^m, G_0^b, G_0^c) - m \epsilon.$$

In the above derivation, (a) results from the independent choice of the codeword symbols transmitted in each ARQ frame which does not allow Eve to benefit from the observations corresponding to the NACKed frames, (b) follows from the memoryless property of the channel and the independence of the $X(j)$’s, (c) is obtained by removing all those terms which correspond to the coherence intervals $j \notin N_m$, where $N_m = \{ j \in \{1, \cdots, a \} : h_b(j) > h_e(j) \}$, and (d) follows from the ergodicity of the channel as $n, m \to \infty$. Now we show that the term $H(X^m|W, Z^m, G_0^a, G_0^c)$ vanishes as $n_1 \to \infty$ by using a list decoding argument. In this list decoding, at coherence interval $j$, the wiretapper first constructs a list $L_j$ such that $x(j) \in L_j$ if $(x(i), z(i))$ are jointly typical. Let $L = L_1 \times L_2 \times \cdots \times L_a$. Given $w$, the wiretapper declares that $x^m = (x^m)$ was transmitted, if $\hat{x}^m$ is the only codeword such that $\hat{x}^m \in B(w) \cap L$, where $B(w)$ is the set of codewords corresponding to the message $w$. If the wiretapper finds none or more than one such sequence, then it declares an error. Hence, there are two types of error events: 1) $E_1$: the transmitted codeword $x^m$ is not in $L$, 2) $E_2$: $\exists x^m \neq x_i^m$ such that $x^m \in B(w) \cap L$. Thus the error probability $\Pr(E_1 \cup E_2) = \Pr(E_1) + \Pr(E_2)$.

Based on the Asymptotic Equipartition Property (AEP), we know that $\Pr(E_1) \leq \epsilon_1$. In order to bound $\Pr(E_2)$, we first bound the size of $L_j$. We let

$$\phi_j(x(j)|z(j)) = \begin{cases} 1, & (x(j), z(j)) \text{ are jointly typical,} \\ 0, & \text{otherwise.} \end{cases}$$

Now

$$\mathbb{E}\{\|L_j\|\} = \mathbb{E}\left\{\sum_{x(j)} \phi_j(x(j)|z(j))\right\} \leq \mathbb{E}\left\{1 + \sum_{x(j) \neq x_i(j)} \phi_j(x(j)|z(j))\right\} \leq 1 + \sum_{x(j) \neq x_i(j)} \mathbb{E}\{\phi_j(x(j)|z(j))\} \leq 1 + 2n \log(1 + h_e(j) P - \epsilon) + \frac{\epsilon}{n_1}$$

Hence

$$\mathbb{E}\{\|L\|\} = \sum_{j=1}^{a} \mathbb{E}\{\|L_j\|\} = 2^{2^{nR_e} - 2}$$

(a) $\leq \mathbb{E}\{\|L\|2^{2R_e} - 2^{2R_e}\}
\leq 2^{2^{nR_e}} \sum_{j=1}^{a} \mathbb{E}\left\{\left[R_0 - \log(1 + h_e(j) P) - \epsilon\right]^{+} + \frac{\epsilon}{n_1}\right\}$

(b) $\leq -n \left(R_e - \frac{1}{2} \sum_{j=1}^{a} \mathbb{E}\left\{\left[R_0 - \log(1 + h_e(j) P) - \epsilon\right]^{+} + \frac{\epsilon}{n_1}\right\}\right)$,

$$= 2 \left(R_e - \frac{1}{2} \sum_{j=1}^{a} \mathbb{E}\left\{\left[R_0 - \log(1 + h_e(j) P) - \epsilon\right]^{+} + \frac{\epsilon}{n_1}\right\}\right),$$

$$Pr(E_2) \leq \mathbb{E}\left\{\sum_{x^m \in L, x^m \neq x_i^m} \Pr(x^m \in B(w))\right\} \leq \mathbb{E}\{\|L\|2^{nR_e}\} \leq 2^{2^{nR_e}} \sum_{j=1}^{a} \mathbb{E}\left\{\left[R_0 - \log(1 + h_e(j) P) - \epsilon\right]^{+} + \frac{\epsilon}{n_1}\right\}\leq 2^{2^{nR_e}} \sum_{j=1}^{a} \mathbb{E}\left\{\left[R_0 - \log(1 + h_e(j) P) - \epsilon\right]^{+} + \frac{\epsilon}{n_1}\right\}.$$
where (a) follows from the uniform distribution of the code-words in $B(w)$. Now as $n_1 \to \infty$ and $a \to \infty$, we get

$$Pr(\mathcal{E}_2) \leq 2^{-n(C_r - \delta - C_e + a \epsilon)} = 2^{-n(c \epsilon - \delta)},$$

where $c = Pr(h_b > h_e)$. Thus, by choosing $\epsilon > (\delta/c)$, the error probability $Pr(\mathcal{E}_2) \to 0$ as $n \to \infty$. Now using Fano’s inequality, we get

$$H(X^m | W, Z^n, G_b^n, G_e^n) \leq n \delta_n \to 0 \quad \text{as} \quad m, n \to \infty.$$ 

Combining this with (8), we get the desired result.

A few remarks are now in order

1) It is intuitively pleasing that the secrecy key rate in (6) is the product of the probability of success at Bob and the expected value of the additional mutual information gleaned by Bob, as compared to Eve, in those successfully decoded frames.

2) It is clear from (6) that a positive secret key rate is achievable under very mild conditions on the channels experienced by Bob and Eve. More precisely, unlike the approach proposed in [4], Theorem 1 establishes the achievability of a positive perfect secrecy rate by appropriately exploiting the ARQ feedback even when Eve’s average SNR is higher than that of Bob.

3) Theorem 1 characterizes the fundamental limit on secret key sharing and not message transmission. The difference between the two scenarios stems from the fact that the message is known to Alice before starting the transmission of the first block whereas Alice and Bob can defer the agreement on the key till the last successfully decoded block. This observation was exploited by our approach in making Eve’s observations of the frames discarded by Bob, due to failure in decoding, useless.

4) We stress the fact that our approach does not require any prior knowledge about the channel state information. The only assumption is that the public feedback channel is authenticated and only Bob can send over it.

5) The achievability of (6) hinges on a random binning argument which only establishes the existence of a coding scheme that achieves the desired result. Our result, however, stops short of explicitly finding such optimal coding scheme and characterizing its encoding/decoding complexity. This observation motivates the development of the explicit secrecy coding scheme in the next section.

6) The perfect secrecy constraint imposed in (5) ensures that an eavesdropper with unlimited computational resources cannot obtain any information about the key. In most practical scenarios, however, the eavesdropper is only equipped with limited computational power. The proposed scheme in the following section leverages this fact in transforming our ARQ secret sharing problem into an erasure-wiretap channel.

IV. EXPLICIT SECRECY CODING SCHEMES

Inspired by the information theoretic results presented earlier, this section develops explicit secrecy coding schemes that allow for sharing keys using the underlying memoryless ARQ protocol. The proposed schemes strive to minimize encoding/decoding complexity at the expense of a minimal price in performance efficiency. We proceed in three steps. The first step replaces the random binning construction, used in the achievability proof of Theorem 1 with an explicit coset coding scheme for the erasure-wiretap channel. As shown next, the erasure-wiretap channel is created by the ACK/NACK feedback and accounts for the computational complexity available to Eve. In the second step, we limit the decoding delay by distributing the key bits over only a finite number of ARQ frames. Finally, we replace the capacity achieving Gaussian channel code with practical coding schemes in the third step. Overall, our three-step approach allows for a nice performance-vs-complexity tradeoff.

The perfect secrecy requirement used in the information theoretic analysis does not impose any limits on Eve’s decoding complexity. The idea now is to exploit the finite complexity available at Eve in simplifying the secrecy coding scheme. To illustrate the idea, let’s first assume that Eve can only afford maximum likelihood (ML) decoding. Hence, successful decoding at Eve is only possible when

$$R_0 \leq \log(1 + h_e P), \quad (13)$$

for a given transmit power level $P$. Now, using the idealized error detection mechanism, Eve will be able to identify and erase the frames decoded in error resulting in an erasure probability

$$\epsilon = Pr(R_0 > \log(1 + h_e P)). \quad (14)$$

In practice, Eve may be able to go beyond the performance of the ML decoder. For example, Eve can generate a list of candidate codewords and then use the error detection mechanism, or other means, to identify the correct one. In our setup, we quantify the computational complexity of Eve by the amount of side information $R_e$ bits per channel use offered to it by a Genie. This side information reduces the erasure probability to

$$\epsilon_g = Pr(R_0 - R_e > \log(1 + h_e P)), \quad (15)$$

since now the channel has to supply only enough mutual information to close the gap between the transmission rate $R_0$ and the side information $R_e$. The ML performance can be obtained as a special case of (15) by setting $R_e = 0$.

It is now clear that using this idea we have transformed our ARQ channel into an erasure-wiretap channel, as in Figure 2.

![Figure 2. Erasure-wiretap channel equivalent model.](image-url)
In this equivalent model, we have a noiseless link between Alice and Bob, ensured by the idealized error detection algorithm, and an erasure channel between Alice and Eve. The following result characterizes the achievable performance over this channel.

**Lemma 1**: The secrecy capacity for the equivalent erasure-wiretap channel is

\[
C_e = \max_{R_0, P \leq \bar{P}} \left\{ R_0 \Pr(R_0 \leq \log(1 + h_e(P))) \right. \\
\left. \Pr(R_0 - R_e > \log(1 + h_e(P))) \right\},
\]

(16)

The proof follows from the classical result on the erasure-wiretap channel and is omitted here for brevity. It is intuitively appealing that the expression in Lemma 1 is simply the product of the transmission rate per channel use, the probability of successful decoding at Bob, and the probability of erasure at Eve. The main advantage of this equivalent model is that it lends itself to the explicit coset LDPC coding scheme constructed in [5]–[7]. In summary, our first low complexity construction is a concatenated coding scheme where the outer code is a coset LDPC for secrecy and the inner one is a capacity achieving Gaussian code. The underlying memoryless ARQ is used to create the erasure-wiretap channel matched to this concatenated coding scheme.

The second step is to limit the decoding delay resulting from the distribution of key bits over an asymptotically large number of ARQ blocks in the previous approach. To avoid this problem, we limit the number of ARQ frames used by the key to a finite number \( k \). The implication for this choice is a non-vanishing value for secrecy outage probability, which is the probability of Eve obtaining correctly all \( k \) frames. For example, if we encode the message as the syndrome of the rate \((k-1)/k\) parity check code then Eve will be completely blind about the key if at least one of the \( k \) ARQ frames is erased [5]–[7] (Here the distilled key is the modulo-2 sum of the key parts received correctly). The secrecy outage probability is

\[
P_{out} = \Pr\left( \min_{j \in \{1, \ldots, k\}} \log(1 + h_e(j)P) > R_0 - R_e \right),
\]

(17)

where \( h_e(1), \ldots, h_e(k) \) are i.i.d. random variables drawn according to the distribution of Eve’s channel. Assuming a Rayleigh fading distribution, we get

\[
P_{out} = \exp\left( -\frac{k}{\bar{P}} \left[ 2R_0 - R_e - 1 \right] \right).
\]

(18)

Under the same assumption, it is straightforward to see that the average number of Bernoulli trials required to transfer \( k \) ARQ frames successfully to Bob is given by

\[
N_0 = k \exp \left( \frac{2R_0 - 1}{\bar{P}} \right),
\]

(19)

resulting in a key rate

\[
R_k = \frac{R_0}{N_0} = \frac{R_0}{k} \exp \left( -\frac{2R_0 - 1}{\bar{P}} \right).
\]

(20)

Therefore, for a given \( R_e \) and \( P \), one can obtain a tradeoff between \( P_{out} \) and \( R_k \) by varying \( R_0 \). Our third, and final, step is to relax the assumption of a capacity achieving inner code. Now, we allow for practical coding schemes, including the possibility of uncoded transmission, with a finite frame length \( n_1 \). Simulation results are reported in the next section.

**V. NUMERICAL RESULTS**

Throughout this section we assume a Rayleigh fading channel, for both Bob and Eve, and focus on the symmetric scenario where the average SNRs experienced by both nodes are the same, i.e., \( \mathbb{E}(h_b) = \mathbb{E}(h_e) = 1 \). Under these assumptions, the achievable secrecy rate in (6) becomes

\[
C_s = \max_{R_0} \exp \left( -\frac{2R_0 - 1}{\bar{P}} \right) \cdot \left\{ R_0 - \frac{\exp(1/P)}{\log_e(2)} \left[ E_i(1/P) - E_i(2^{R_0}/P) \right] \right\}
\]

(21)

where \( E_i(x) = \int_x^\infty \exp(-t)/t \, dt \).

Figure 3 gives the variation of \( C_s \) and \( C_e \) with SNR under different constraints on the decoding capabilities of Eve. It is clear from the figure that \( C_e \) can be greater than \( C_s \). This can be the case for certain \( R_c \) and SNR values. For instance, in the case of \( R_c = 0 \), if Eve receives the transmitted packet with error, she discards it without any further attempts at decoding. The instantaneous secrecy rate becomes \( R_0 \), which is larger than that used in (6) \( C_s(i) = R_0 - \log_2(1 + h_e(i)P) \) where \( C_s(i), h_e(i) \) are the instantaneous secrecy rate, and Eve’s channel power gain, respectively. Averaging over all fading states, we can get a greater \( C_e \) than \( C_s \). It is worth noting that, under the assumptions of the symmetric scenario and the Rayleigh fading model, the scheme proposed in [4] is not able to achieve any positive secrecy rate.

Next, we turn our attention to the delay-limited coding constructions proposed in Section IV. Figures 4 and 5 show, for different \( R_0 \) and \( R_c \), the tradeoff between secrecy outage probability versus key rate for the proposed rate \((k-1)/k\) coset secrecy coding scheme assuming an optimal inner Gaussian channel coding. Figure 4 gives key rate corresponding to a desired secrecy outage probability, given some values for \( R_0 \) and \( R_c \). As is evident from Figure 3, the key rate required...
to obtain a certain outage probability gets smaller as $R_C$ increases. In Figure 6, we relax the optimal channel coding assumption and plot key rate for practical coding schemes or no coding, and finite frame lengths (i.e., finite $n_1$). The code used in the simulation is a punctured convolutional code derived from a basic $1/2$ code with a constraint length of 7 and generator polynomials 133 and 171 (in octal). We assume that Eve is Genie-aided and can correct an additional 50 erroneous symbols (beyond the error correction capability of the channel code). From the figure, we see that the key rate increases with increasing SNR and then drops after reaching a peak value. Note that we fix the transmission rate and make it independent of SNR. A low SNR means more transmissions to Bob and a consequent low key rate. As SNR increases, while keeping the transmission rate fixed, the key rate increases. However, increasing SNR at Eve’s receiver means an increased ability to correctly decode the codeword-carrying packets. This explains why the key rate curves peak and then decay with SNR. Note also that for a certain modulation and channel coding scheme, decreasing the packet size in bits lowers the key rate. Reducing the packet size increases the probability of correct decoding by Bob and, thus, decreases the number of transmissions. However, it also increases the probability of correct decoding by Eve and the overall effect is a decreased key rate.

VI. CONCLUSIONS

This paper developed a novel overlay approach for sharing secret keys using existing ARQ protocols. The underlying idea is to distribute the key bits over multiple ARQ frames and then use the authenticated ACK/NACK feedback to create a degraded channel at the eavesdropper. Our results establish the achievability of non-zero secrecy rates even when the eavesdropper is experiencing a higher average SNR than the legitimate receiver. It is worth noting that our approach does not assume any prior knowledge about the instantaneous CSI; only prior knowledge of the highest average SNR seen by the eavesdropper is needed. Moreover, we constructed a low complexity secrecy coding scheme by transforming our channel to an erasure wiretap channel which lends itself to explicit coset coding approaches. Our theoretical claims were validated via numerical examples that demonstrate the efficiency of the proposed schemes. The most interesting part of our work is, perhaps, the fact that it demonstrates the possibility of sharing secret keys in wireless networks via rather simple modifications of the existing infrastructure which, in our case, corresponds to the ARQ mechanism.

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