\[ \tau^- \rightarrow \eta \pi^- \pi^0 \nu_\tau \text{ and } \sigma(e^+ e^- \rightarrow \eta \pi^+ \pi^-) \] at low energies

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Abstract
We analyze the hadronization structure of \( \tau \rightarrow \eta \pi^- \pi^0 \nu_\tau \) decays. In the isospin limit only the vector current contributes to this process. We compute the relevant form factor within Resonance Chiral Theory, at leading order in the \( 1/N_C \) expansion, and considering only the contribution of the lightest vector resonances. The couplings in the resonance theory are constrained by imposing the asymptotic behaviour of vector spectral functions ruled by QCD. We reproduce the branching ratio of this mode and predict the low-energy behaviour of \( \sigma(e^+ e^- \rightarrow \eta \pi^+ \pi^-) \) using isospin symmetry.

1. Introduction

Hadronic decays of the \( \tau \) are a clean scenario to learn features about the hadronization of QCD currents in its non-perturbative regime \([1]\) and the decays involving the \( \eta \) meson are particularly interesting because of the selection rules that apply \([2]\) in the isospin limit. Within this approximation, it is seen that the \( \eta \) and the vector current, \( V_\mu \), have opposite G-parity than the \( \pi \) and the axial-vector current, \( A_\mu \). Then, the decay \( \tau \rightarrow \eta \pi^- \nu_\tau \) can only be produced in the SM as an isospin violating effect, since it has opposite G-parity to the participating vector current. The processes that we study here, \( \tau \rightarrow \eta(p_1)p^-\pi^0(p_2)\pi^0\nu_\tau \), can be considered as due only to \( V_\mu \) to a very good approximation.

The decay amplitude for these decays may be written as:

\[ M = -\frac{G_F}{\sqrt{2}} V_{ud} \bar{u}_\tau \gamma^\mu (1 - \gamma_5) u_\tau \mathcal{H}_\mu, \] with

\[ \mathcal{H}_\mu = i \varepsilon_{\mu \nu \rho \sigma} p_1^\nu p_2^\rho p_3^\sigma F(Q^2, s_1, s_2), \]

\[ Q_\mu = (p_1 + p_2 + p_3)_\mu, \quad s_i = (Q - p_i)^2. \] (3)

2. Theoretical framework

Since QCD is non-perturbative for \( Q^2 \lesssim M_\tau^2 \), it is not possible to go further in a model independent way. However, it would be desirable to keep as many QCD properties as possible. \( M_\tau \) is large enough to prevent the application of \( \chi PT \) \([3]\) for all the spectra \([4]\), however in the low-energy limit one should recover its results. The expansion parameter in \( \chi PT \) ceases to be valid at \( E \sim M_\rho \) and the inverse of the number of colours in QCD is a useful alternative to build the expansion upon \([5]\) and helps formulate a Lagrangian theory, \( R \chi T \), where the resonances that mediate hadronic \( \tau \) decays become active degrees of freedom \([6]\) and the known short-distance QCD behaviour \([7]\) has been demanded to the Green functions \([8, 9]\) and associated form factors.

For any phenomenological study it will be essential to provide the resonances with a proper energy-dependence width, that we obtain within \( R \chi T \) \([10]\). This program has been applied successfully to explain the phenomenology of many two- and three-meson \( \tau \) decay channels \([11, 12]\).
In the case at hand, \( \tau^- \to \eta \pi^- \pi^0 \nu_\tau \), only the vector current contributes in the isospin limit. Our formalism includes the Wess-Zumino term \[13\] as in \( \chi PT \). In addition, one considers the one-resonance mediated diagrams with an odd-parity term: the couplings of a vector and a pseudoscalar to the vector source \[8\] (\( c_i \) couplings) and those in which a vector resonance couples to three mesons \[14\] (\( g_i \) couplings). Two-resonance mediated diagrams are also accounted for in this negative parity sector \[8\] (the coefficients of the \( VVP \) operators are called \( d_i \)). 13 of the 18 Lagrangian couplings appear in the process under study.

Demanding the known ultraviolet behaviour of the imaginary part of the vector-vector correlator \[15\] one obtains relations among the couplings \[1\] that allow the theory to keep some predictive power. In this case, only 4 couplings remain unknown after this step. We read the value of the free combination of \( g_i \) couplings from Ref. \[12\] and rely on the value obtained in Ref. \[8\] of a \( d_i \) whose coefficient vanishes in the chiral limit (\( d_3 \)). Therefore, only 2 coefficients are still free.

### 3. Associated Phenomenology

We reproduce \[16\] the PDG branching ratio \[17\], \((1.39 \pm 0.10) \times 10^{-3}\), for natural values of both couplings \( (c_3 \text{ and } d_2) \) as it is displayed in Fig. 1.

In addition to this, we have fit these two couplings to Belle data \[18\] on the number of events per bin versus the invariant mass of the hadron system as it is represented in Fig 2. We have checked that the best fit is obtained with the value of \( d_2 \) assumed in Sect. 2. Moreover, we have also verified that the inclusion of a \( \rho' \) does not improve sensibly the fit and does not produce the bump structure that is observed near the endpoint.

Finally, using isospin symmetry, it is possible to relate the low-energy \( \sigma(e^+e^- \to \eta \pi^+ \pi^-) \) to \( \Gamma(\tau^- \to \eta \pi^- \pi^0 \nu_\tau) \) \[16\]. Then, our analysis for \( \tau^- \to \eta \pi^- \pi^0 \nu_\tau \) decays allows to predict the low-energy limit of this cross-section, as displayed in Fig. 3.

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\(^1\)Noteworthy, they are consistent with all phenomenological studies mentioned before.

\(^2\)One can proceed conversely and use the data on \( e^+e^- \) annihilation into hadrons to predict the corresponding semileptonic tau decays \[19, 20\].
4. Conclusions

We have studied the decay \( \tau^- \rightarrow \eta\pi\pi^0\nu_\tau \) within the framework of Resonance Chiral Theory guided by the large-\( N_C \) expansion of QCD, the low-energy limit given by \( \chi PT \) and the appropriate asymptotic behaviour of the vector form factor that helps to fix most of the initially unknown couplings. Indeed only two remain free after completing this procedure and having used information acquired in the previous related analyses. We reproduce the PDG branching ratio for this mode and study its spectral function assisted by Belle data.

These are flourishing days for this branch of Physics, where the data samples collected at CLEO, BaBar and Belle through the years have allowed very precise studies and current and forthcoming experimental results from them, BES-III, VEPP and hopefully super-B and super-\( \tau \)-charm factories will demand a dedicated effort both on the theory description and on the Monte Carlo event generation algorithms \[26\]. These hadron matrix elements will be implemented \[27\] in the Monte Carlo Generator for \( \tau \) decays TAUOLA \[28\]. Using isospin symmetry, we provide a prediction for the low-energy behaviour of \( \sigma(e^+e^- \rightarrow \eta\pi\pi^-) \) that may be of interest for the hadronic matrix element in the PHOKHARA \[29\] Monte Carlo generator.

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