The $Q\bar{Q}$ Potential in the Color-Dielectric Formulation of the Transverse Lattice

Bob Klindworth and Matthias Burkardt

Department of Physics, New Mexico State University, Las Cruces, NM 88003, U.S.A.

ABSTRACT

The $Q\bar{Q}$ potential is calculated in the Color-Dielectric Formulation of Transverse Lattice QCD. In such a formulation an effective potential is used to enforce the $SU(N_c)$ symmetry of the link fields. This effective potential is truncated at fourth order. Surprisingly, there is striking rotational invariance in the ground state $Q\bar{Q}$ potential as well as in the first hybrid mode.

1. Introduction

Light front field theories have many advantages. First, high energy experiments often involve large momentum transfers. Therefore, it seems plausible that such experiments could be well-described in a frame moving at the speed of light. Second, light front wavefunctions can be regarded as correlation functions at equal light front time. From these wavefunctions it is comparatively easy to extract important physical observables like parton distribution functions, and these observables have a direct physical interpretation in terms of light-front coordinates.

The transverse lattice is a marriage of light front field theory with the very successful lattice gauge theory. One first defines the light front variables $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$ and keeps them continuous. The remaining two directions ($x^1, x^2$) are discretized. Like all light front theories, the transverse lattice lacks manifest rotational invariance. For this reason calculating the $Q\bar{Q}$ potential on the transverse lattice is a good way of testing the dynamics of our model.

In previous work we calculated the $Q\bar{Q}$ potential using 2+1 dimensional Transverse Lattice QCD. In such a theory the lattice gauge fields are matrices associated with the links between lattice sites. Ideally, we would like to work with an effective potential that has minima in $SU(N_c)$. However, as explained in Ref. [3], there are difficulties with the Light Front formulation of such a theory. Instead we use the Color-Dielectric formulation in which the link variables are assumed to be smeared variables. This framework allows us to cover a wider range of physical distances using relatively few degrees of freedom.

The results of the original 2+1 calculation exhibited surprising rotational invariance. This was quite surprising because we used a very crude effective potential, accepting only terms up to second order in the link fields. In the current work we repeat the analysis from Ref. [2], including terms up to fourth order in the effective potential and extending the calculation to 3+1 dimensions.
Fig. 1. Relevant fourth order interactions in the effective potential for the link fields. (a) canonical plaquette interaction, (b) local interaction with coupling $c_1$, (c) longitudinal non-local interaction with coupling $c_L$, (d) transverse non-local interaction with coupling $c_\perp$. In $2+1$ dimensions only (b) and (c) contribute.

2. The Effective Potential

The purpose of the effective potential is to constrain the link fields to be members of $SU(N_c)$. The terms in the effective potential up to fourth order are

$$V_{\text{eff}}(U) = \mu^2 \text{Tr}(U_i U_i^\dagger) + \frac{\lambda_1}{aN_c} \text{Tr}(U_i U_i^\dagger U_{i+1} U_{i+1}^\dagger) + \frac{\lambda_2}{aN_c} \text{Tr}(U_{i+1} U_{i+1}^\dagger U_i U_i^\dagger)$$

$$+ \frac{\lambda_3}{aN_c^2} \text{Tr}(U_i U_i^\dagger)\text{Tr}(U_i U_i^\dagger)$$

(1)

Our initial calculation truncated $V_{\text{eff}}$ at second order. The second and the fourth terms in this expansion do not contribute due to our Fock Space truncation. We know that our truncation is valid in the case of the ground state, so neglecting these terms seems plausible. It is hoped that this is also valid for the lowest excited states. Thus, the only fourth order term which contributes to our calculation is the nonlocal interaction.

$$V_{\text{eff}}(U) = \mu^2 \text{Tr}(U_i U_i^\dagger) + \frac{\lambda_2}{aN_c} \text{Tr}(U_{i+1} U_{i+1}^\dagger U_i U_i^\dagger)$$

It was found in Ref. [3] that the glueball spectrum could be fit to reliable Euclidean Lattice Monte Carlo data as long as $\mu^2$ and $\lambda_2$ lie along a scaling trajectory. In our calculation, we verified that points off of the scaling trajectory give rise to potentials which are not rotationally invariant.

3. Results in 2+1 Dimensions

With the effective potential fixed we calculated the $Q\bar{Q}$ Potential as before using the 2+1 dimensional Transverse Lattice. We used Discretized Light Cone Quantization.
Fig. 2. a.) Static $Q\bar{Q}$ potential in the color dielectric formulation of the $\perp$ lattice in 2+1 dimensions versus the distance $r \equiv \sqrt{x_1^2 + x_2^\perp}$ between the $Q$ and the $\bar{Q}$. Circles: ground state, triangles: first adiabatic hybrid state. The points represent calculations where the $Q\bar{Q}$ pair was separated by up to 4 lattice units in the transverse direction as well as by various separations in the longitudinal direction. In 3+1 dimensions, as long as the $\perp$ separation between the $Q$ and the $\bar{Q}$ is along one of the $\perp$ axis, the results are identical. b.) Same as a.), but for a 3 + 1 dimensional $\perp$ lattice. The transverse separations between the $Q\bar{Q}$ pair are $(n_x, n_y) = (1, 0), (2, 0), (3, 0), (1, 1)$ and $(2, 1)$.

The main differences when compared to the calculation in Ref. [2] are the addition of a “mass” term to the link fields and the inclusion of an attractive (Lorentz-) scalar interaction between adjacent links. The resulting potential has an anisotropy in the points with zero transverse separation resulting from the fact that the non-local coupling does not act on states whose transverse separation is less than two. Rotational invariance is restored by introducing a new term in the effective potential. In momentum space this interaction has the form,

$$V_{\text{contact}} = c_{\text{contact}} \frac{1}{\sqrt{k^+_+ k'^+_+}}$$

where $k^+_+$ and $k'^+_+$ are the incoming/outgoing momenta of the link fields which interact with the external charges. Note that this interaction acts only on the first and last link field in the “chain” connecting the $Q\bar{Q}$ pair since it involves the external charges. For the same reason, it does not conserve gluon momentum. This interaction has a Lorentz structure similar to the 4-point interaction between fermions and bosons that arises when the constrained component of the spinor field is eliminated.

In addition to studying the ground state potential we can also study the rotational invariance and qualitative shape of the first excited state. Surprisingly, this state also exhibits striking rotational invariance. This is surprising because higher Fock components, which we omit, could contribute significantly to the excited states.
4. Results in 3+1 Dimensions

Calculations in 3+1 dimensions introduce two new couplings: a plaquette coupling and a non-local coupling for states that turn a corner. The ground state potential is found to be approximately rotationally invariant along the trajectory $c_p - c_\perp = \text{const.}$, where $c_p$ is the plaquette coupling and $c_\perp$ is the new non-local coupling introduced by going to 3+1 dimensions. The ambiguity along this trajectory is fixed by demanding that longitudinally and transversely polarized excited (hybrid) states are degenerate. Rotational invariance is rather sensitive to the values of the plaquette coupling and the non-local coupling that turns a corner if they lie off of the trajectory described above. Roundness is not very sensitive to the values of the link field mass term, the contact coupling, and the straight non-local coupling. The optimal set of parameters, in our units is: $m^2 = 0.30$, $c_{\text{contact}} = -0.10$, $c_2 = -0.1375$, $c_\perp = 0$ and $c_p = 0.4875$.

The first excited state of the 3+1 dimensional potential exhibits a slight anisotropy. This is an expected result of our Fock Space truncation. However, the shape at intermediate to long distances is encouraging and it does agree rather well with results obtained on a Euclidean lattice.

5. Summary and Outlook

The static $QQ$ potential provides a test of the dynamics of our model. In all, the rotational invariance of the ground state potential is striking. However, the fact that rotational invariance is not extremely sensitive to all of the parameters in the effective potential suggests that other observables are needed in order to pin down these couplings unambiguously.

Alternatively, it is conceivable that one could determine the coefficients of the effective potential on the Euclidean lattice where one can make the transverse lattice spacing small. Not only could one be rather confident of the calculations done from such a theory, but one could also systematically determine the dependence of observables on high orders in the effective potential. Such work is currently underway.

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