How to create entanglement among 100 spatial modes with a single photonic lattice

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(Dated: May 14, 2020)

Multimode entanglement is an essential resource for quantum networks. Continuous-variable encoding of quantum information in the optical domain has recently yielded large temporal and spectral entangled states instrumental for quantum computing and quantum communication. We introduce a protocol for the generation of spatial multipartite entanglement in a monolithic photonic device: the array of quadratic nonlinear waveguides. We theoretically demonstrate the generation of large multipartite entangled states in the spontaneous parametric downconversion regime. Our protocol is remarkably simple and robust as it does not rely on specific values of coupling, nonlinearity or length of the sample.

Optical networks play a key role in our everyday life as the substrate of a long-range communication grid: the internet. One goal of the blooming quantum technologies is the development of a quantum internet: an information web with unparalleled capabilities with respect to its current classical counterpart where information will be processed by quantum computers, transmitted in quantum secure channels, and routed towards quantum end nodes [14]. A must-have of the quantum internet is multipartite entanglement, where information is strongly correlated between the distributed nodes which compose the network [2]. In addition, multipartite entanglement is the resource of a number of protocols in quantum communication, quantum sensing and quantum computing [3–5]. Sources of multipartite entanglement are thus required in quantum networks and, particularly, light-based sources at telecom wavelengths are favored due to the current availability of large optical fiber networks.

Quantum information can be encoded in variables that can take a continuous spectrum of eigenvalues – continuous variables (CV) – [6]. In the optical domain, the fluctuations of the field quadratures can be used as carriers of quantum information [7]. A number of tabletop experiments have demonstrated CV quantum networks in the spatial, frequency and temporal domains [8–13]. The transfer to real technologies is however far from feasible with bulk-optics systems. Scalability, stability and cost are issues that only well-established technologies like integrated and fiber optics can overcome [14]. Generation of two-mode CV entanglement through bulk-integrated hybrid approaches has been explored [15] and, remarkably, the first demonstration of a fully-on-chip source of CV bipartite entanglement has been recently proposed [17]. Nevertheless, the extension of that bulk-optics-based scheme – sequential squeezing and entanglement – to larger number of modes is very demanding with current technology. In this paper we present a simple and practical protocol for the generation on chip of spatial multipartite entanglement of spontaneous parametric downconverted (SPDC) light based on a currently-available technology: the array of $\chi^{(2)}$ nonlinear waveguides (ANW) [18, 21].

Harnessing the ANW for any purpose including multipartite entanglement is challenging: the continuous interplay between nonlinearity and evanescent coupling prevents in general analytical solutions [22, 23]. This interplay also results in an extreme sensitivity to physical parameters such as the device length and linear and nonlinear coupling strengths. Here, we analytically demonstrate multipartite entanglement among the SPDC modes generated in the $(N+1)/2$ odd waveguides of an ANW with an odd number of waveguides $N$. Our method is based on efficient build-up of the zero supermode of the array when pumping with a flat profile. This protocol is remarkably simple and robust as it does not rely on specific values of coupling, nonlinearity or length of the sample. Below we introduce the ANW, calculate analytical solutions for a flat pump profile and use these solutions to demonstrate multipartite entanglement.

The ANW consists of $N$ of identical $\chi^{(2)}$ waveguides. In each waveguide, an input harmonic field at frequency $\omega_s$ is downconverted into a signal field at frequency $\omega_s$. Pump-signal waves phase matching is produced only in the coupling region and is set to produce degenerate SPDC light. The generated signal fields are then coupled through evanescent tails in contrast to the pump fields which are not coupled due to a higher confinement in the waveguides. The physical processes involved are described by the following system of equations [21]

$$\frac{d\hat{A}_j}{dz} = iC_0(f_{j-1}\hat{A}_{j-1} + f_j\hat{A}_{j+1}) + 2in_j\hat{A}_j^\dagger,$$  \hspace{1cm} (1)

where $\hat{A}_0 = \hat{A}_{N+1} = 0$, $f_0 = f_N = 0$ and $j = 1, \ldots, N$ is the individual mode index. $\hat{A}_j \equiv \hat{A}_j(z,\omega_s)$ is a monochromatic slowly-varying amplitude annihilation operator of signal (s) photons related to the mode propagating in the $j$th waveguide where $[\hat{A}_j(z,\omega), \hat{A}_j^\dagger(z',\omega')] = \delta(z-z')\delta(\omega-\omega')\delta_{j,j'}$. $n_j = g \alpha_{h,j}$ is the effective nonlinear coupling constant corresponding to the $j$th waveguide, with $g$ the nonlinear constant proportional to $\chi^{(2)}$ and the spatial overlap of the signal.

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and harmonic fields in each waveguide, and \( \alpha_{0,j} \) a strong coherent pump field. The nonlinear coupling constant \( \eta_j \) can be tuned by means of a suitable set of pump phase and amplitude at each waveguide. \( C_j = |C_0| f_j \) is the linear coupling constant between nearest-neighbor modes \( j \) and \( j+1 \), with \( C_0 \) the coupling strength and \( f_j \) the coupling profile. \( z \) is the coordinate along the direction of propagation.

We are interested here in studying CV features of the SPDC light generated in the ANW. The natural variables in this framework are the quadratures of the optical field. The quadratures \( \hat{x}_j, \hat{y}_j \), where \( \hat{x}_j = (A_j + A_j^\dagger) \) and \( \hat{y}_j = i(A_j^\dagger - A_j) \), are, respectively, the orthogonal amplitude and phase quadratures corresponding to a signal optical mode \( A_j \). The system of equations (1) can be rewritten in terms of the quadratures as \( \Delta(\xi)/d\xi = \Delta(z) \tilde{\xi} \), where \( \Delta(z) \) is a \( 2N \times 2N \) matrix of coefficients and \( \tilde{\xi} = (\hat{x}_1, \hat{y}_1, \ldots, \hat{x}_N, \hat{y}_N)^T \). In general, this equation – or Equation (1) – can be solved numerically for a specific set of parameters \( C_j, \eta_j \) and \( N \), or even analytically if \( N \) is small. Numerical or low-dimension analytical solutions do not provide much physical insight when increasing \( N \). Remarkably, we have identified a case where analytical solutions are available for any \( N \). This is the case of a flat pump profile, i.e. pumping all the waveguides with \( \eta_j = |\eta| \). In this case the eigenmodes of the system are propagation eigenmodes –supermodes– \ref{20}. These eigenmodes form a basis and are represented by an orthogonal matrix \( M \) with real elements \( M_{k,j} \). The individual and propagation supermode bases are related by \( \xi_{S,k} = \sum_{j=1}^N M_{k,j} \xi_j \). Figure 1 shows the supermodes of an array of 5 waveguides with a homogeneous coupling profile \( \vec{f} = (f_1, \ldots, f_N) = \vec{1} \). The supermodes are orthonormal \( \sum_{j=1}^N M_{k,j} M_{m,j} = \delta_{k,m} \), and the spectrum of eigenvalues is \( \lambda_k \). The solution of the propagation in this basis can be written as \( \xi_{S,k}(z) = S(z) \xi_{S,k}(0) \), where \( S(z) = \text{diag}\{S_1(z), \ldots, S_N(z)\} \) and

\[
S_k(z) = \begin{pmatrix}
\cos(F_k z) & -e^{-r_k} \sin(F_k z) \\
e^{r_k} \sin(F_k z) & \cos(F_k z)
\end{pmatrix},
\]

with \( r_k = (1/2) \ln[(\lambda_k + 2|\eta|)/(|\lambda_k - 2|\eta|)] \) and \( F_k = \sqrt{\lambda_k^2 - 4|\eta|^2} \). For typical coupling strengths and pump powers found in quadratic ANW the condition \( |\lambda_k| > 2|\eta| \) is fulfilled. This regime is the relevant one for entanglement since, as the nonlinear interaction surpasses the linear coupling, the SPDC light tends to be more and more confined in the waveguide where is created and then the ANW acts only as a group of individual squeezers \ref{20}. We consider \( F_k \in \mathbb{R} \) in the remainder of the article. Notably, the analytical solution of Equation (2) is general for any evanescent coupling profile \( \vec{f} \), any number of waveguides \( N \) and any propagation distance \( z \).

The quantum states generated in ANW are Gaussian. The most interesting observables are then the second-order moments of the quadrature operators, properly arranged in the covariance matrix \( V(z) \) \ref{27}. For a quantum state initially in vacuum, the elements of the covariance matrix \( V(z) \) can be straightforwardly obtained from Equation (2) using \( V(z) = S(z) S^T(z) \). The covariance matrix elements in the supermode basis are then

\[
V(x_{S,k}, x_{S,l}) = [\cosh(r_k) + \sinh(r_k) \cos(2F_k z)] e^{-r_k},
\]

\[
V(y_{S,k}, y_{S,l}) = [\cosh(r_k) - \sinh(r_k) \cos(2F_k z)] e^{+r_k},
\]

\[
V(x_{S,k}, y_{S,l}) = \sinh(r_k) \sin(2F_k z).
\]

The squeezing ellipses for each supermode vary along propagation. Maximum (respectively null) squeezing are obtained periodically at distances \( L_k = (2n-1)\pi/(2F_k) \) (respectively \( L_k' = n\pi/F_k \)) that are different for each \( k \)th supermode, with \( n \) any positive integer. The maximum value of squeezing available in the \( k \)th supermode is \( e^{-2r_k} = (\lambda_k - 2|\eta|)/(|\lambda_k + 2|\eta|) \).

Instrumentally for our protocol, waveguide arrays with an odd number of identical waveguides present a supermode with zero eigenvalue –the zero supermode– \ref{20}. This corresponds to the central supermode with \( k = (N+1)/2 \) and \( \lambda_1 = 0 \) as shown in Figure 1. The elements of the covariance matrix for the zero supermode are thus

\[
V(x_{S,1}, x_{S,1}) = V(y_{S,1}, y_{S,1}) = \cosh(4|\eta| z),
\]

\[
V(x_{S,1}, y_{S,1}) = \sinh(4|\eta| z).
\]

In contrast to the side supermodes \( k \neq l \), the zero supermode noise efficiently builds up at all propagation distances and, notably, for large coupling strength the zero supermode is quickly dominant over the side supermodes. The reason is that this is the only supermode which is phase-matched along propagation (\( \lambda_1 = 0 \)).

The covariance matrix Equation (3) in the supermode basis shows the total squeezing available in the downconverted signal fields. However, that basis is diagonal and no entanglement is available in that encoding. A simple change of basis takes Equation (3) to the individual

![FIG. 1. Sketch of the supermodes related to an array of linear waveguides with a homogeneous coupling profile \( \vec{f} = \vec{1} \) and \( N = 5 \). The horizontal axis stands for the individual modes. The propagation constants corresponding to each supermode are \( \lambda = \{\sqrt{3}C_0, C_0, 0, -C_0, -\sqrt{3}C_0\} \). \( k \equiv l = 3 \) is the zero supermode. \( k \neq l \) are the side supermodes.](image-url)
mode basis, corresponding to the individual waveguides output, obtaining

\[ V(\xi_i, \xi_j) = \sum_{k=1}^{N} M_{i,k} M_{j,k} V(\xi_{S,k}, \xi_{S,k}), \]

with \( \xi \equiv x \) or \( y \). Hence, the flat pump configuration generates quantum correlations—off-diagonal components of the covariance matrix—between any pair \( i \) and \( j \) of individual modes, and thus full inseparability among individual modes can be produced.

Measuring multipartite full inseparability in CV systems requires the simultaneous fulfillment of a set of conditions which leads to genuine multipartite entanglement when pure states are involved [29]. This criterion, known as van Loock - Furusawa (VLF) inequalities, can be easily calculated from the elements of the covariance matrix \( V \). Full \( N \)-partite inseparability is guaranteed if the following \( N - 1 \) inequalities are simultaneously violated

\[ V\{x_j(\theta_j) - x_{j+1}(\theta_{j+1})\} + V\{x_j(\theta_j + \frac{\pi}{2}) + x_{j+1}(\theta_{j+1} + \frac{\pi}{2}) + \sum_{m \neq j,j+1}^{N} G_m x_m(\theta_m + \frac{\pi}{2})\} \geq 4, \]

where \( \theta(j), \theta(j) + \frac{\pi}{2} \) are the measurement phase corresponding to the \( j \)th local oscillator (LO), and \( G_1, \ldots, G_N \) are \( N \) real parameters corresponding to electronic gains in multimode balanced homodyne detection (BHD) which are set by optimization. \( V[\sum_j \sigma_j \xi_j] = \sum_j \sigma_j V(\xi_j, \xi_j) + \sum_{i \neq j} \sigma_j \sigma_i V(\xi_j, \xi_i) + \sum_i \sigma_j \sigma_i V(\xi_i, \xi_j) \) with \( \sigma_j \) a set of real numbers. \( \theta \equiv (\theta_1, \ldots, \theta_N) \) and \( \vec{G} \equiv (G_1, \ldots, G_N) \) stand, respectively, for the LO phase and gain profiles. These two families of parameters can be set in order to minimize suitably the value of Equation (6). Below, we show a remarkably simple way of generating multipartite entanglement in ANW with an odd number of waveguides based on the efficient squeezing of the SPDC zero supermode.

Firstly, in order to gain insight about the multipartite entanglement generated with the flat pump configuration we tackle the limit of large coupling \((C_0 \rightarrow \infty)\). This limit is not physical as next-nearest-neighbor coupling should be in that case included in the model, but it gives us a clear insight on the dynamics of the system as the zero-supermode is then the dominant supermode generated in the array. In this limit an asymptotic lower bound on the violations of the VLF inequalities is obtained for the non-optimized case \( \vec{G} = \vec{0} \). The covariance matrix elements Equations (3) for an array with odd number \( N \) of waveguides in the limit of large coupling \((C_0 \rightarrow \infty)\) is significantly simplified to

\[ V(x_i, x_j) = V(y_i, y_j) \rightarrow \delta_{i,j} + 2M_{i,l} M_{j,l} \sin^2(2|\eta|z), \]

\[ V(x_i, y_j) \rightarrow M_{i,l} M_{j,l} \sin(4|\eta|z). \]

Applying this result into the general expression for the VLF inequalities Equation (4) without optimization \((G_{m \neq j,j+1} = 0)\) and using generalized quadratures with \( \theta_j = 0 \) and \( \theta_{j+1} = -\pi/2 \) or \( \pi/2 \), we obtain

\[ VLF_j(\infty, \vec{0}, z) = 4 - 2(M_{j,l}^2 + M_{j+1,l}^2) \]

\[ + (M_{j,l} \pm M_{j+1,l})^2 e^{4|\eta|z} + (M_{j,l} \mp M_{j+1,l})^2 e^{-4|\eta|z} \geq 4. \]

where the upper (lower) signs corresponds to \( \theta_{j+1} = -\pi/2 (\pi/2) \) and we have introduced the notation \( VLF \equiv VLF(C_0, \vec{G}, z) \) for the sake of clarity. The best scenario in terms of violation of these inequalities corresponds to the case \( M_{j,l} = M_{j+1,l} \), for which we obtain

\[ VLF_j(\infty, \vec{0}, z) = 4 - 4M_{j,l}^2(1 - e^{-4|\eta|z}) < 4 \quad \forall z > 0. \]

Particularly, the coefficients of the zero supermode in an array with homogeneous coupling profile \( \vec{f} = \vec{1} \) are given by \( M_{j,l} = \sin(\frac{2\pi l}{L})/\sqrt{L} \) (Figure 1). Hence, mapping the mode \( 2j - 1 \) into the label \( j \) of Equation (8), two solutions which maximize the violation of the separability conditions are obtained: i) the \( l \) odd elements of the zero supermode satisfy \( M_{2j-1,l} = -M_{2j+1,l} \) such that for a LO profile \((\theta_{2j-1}, \theta_{2j+1}) = (0, \pm \pi/2)\) multipartite entanglement is obtained among all the odd individual modes \( \{1, 3, 5, \ldots, N\} \), and ii) the odd elements of the zero supermode satisfy \( M_{2j-1,l} = M_{2j+1,l} \), thus for \((\theta_{2j-1}, \theta_{2j+1}) = (0, \pm \pi/2)\) the multimode state is decoupled in two multipartite entangled states: \( \{1, 5, 9, \ldots\} \) and \( \{3, 7, 11, \ldots\} \). Thus, the LO phase profile acts as an entanglement switch between two multimode entangled states. The individual modes propagating in the odd waveguides are fully inseparable in a measurement basis and separable in two parties—each with elements fully inseparable—in the other. The following degenerate violation of the inseparability conditions is obtained in both cases

\[ VLF_j(\infty, \vec{0}, z) = 4 \left( 1 - \frac{e^{-4|\eta|z}}{l} \right) < 4 \quad \forall z, l. \]

Hence, the strength of the violation of the VLF inequalities depends asymptotically only on the number of odd individual modes \( l \) which make up the zero supermode. Moreover, the use of an optimized gain profile \( \vec{G} \neq \vec{0} \) can only improve the
above result. We have found the following optimized violation of the VLF conditions which depends also on the parity of \( l \) (see supplemental material)

\[
\text{VLF}_j(\infty, \vec{G}, z) = \text{VLF}_j(\infty, \vec{0}, z) + \begin{cases} 
\frac{-2(2^4 - 4^4 + 1 - \eta^4)(\eta^4 + 1)^2}{l^2(1 - 2\eta)} - 2 & \forall z, l \text{ odd}, \frac{-2(2^4 - 4^4 + 1 - \eta^4)(\eta^4 + 1)^2}{l^2(1 - 2\eta)} - 2 \leq VLF_j(\infty, \vec{0}, z) \forall z, l \text{ even.}
\end{cases}
\]

The correction produced optimizing \( \vec{G} \) is zero for \( l = 2, 3, \) and negative for \( l > 3 \). It scales as \( t^{-1} \) in the limit of a large number of modes. Therefore, we have demonstrated that our protocol always produces multipartite entanglement in ANW in the limit of large coupling.

When finite coupling \( C_0 \) is taken into account, the side supermodes are present in the optimization of the VLF inequalities. This generates fluctuations around the value \( \text{VLF}_j(\infty, \vec{G}, z) \). Figure 2 (color) shows one, two, three and four inequalities for arrays with, respectively, \( l = 2, 3, 4 \) and 5 propagating modes obtained through minimization of Equations (10) with a suitable gain profile \( \vec{G} \) where we have used the analytical solutions of Equations (5),\( ^{30} \). The simultaneous violation of the inequalities (\( \text{VLF}_j < 4 \) in our notation) guarantees full inseparability, and since we deal with pure states the propagating signal modes are genuinely multipartite entangled. Interestingly, lower coupling strengths \( C_0 \rightarrow 0 \) can lead to higher entanglement at specific lengths due to the increased strength of the side supermodes. The solid gray lines correspond to violations in the limit of large coupling \( C_0 \rightarrow \infty \) and optimized gain profiles \( \vec{G} \) [Equation (11)] for each case. Figure 2 thus demonstrates the validity of Equation (11) as a mean value of the real VLF inequalities that can be used to assess the possible entanglement generated in the array.

Remarkably, Equation (11) shows that quantum correlations are exhibited at any \( z \) independently of the number \( l \) of modes involved. Figures 3 and 4 depict respectively the evolution of multipartite entanglement along propagation for large number of modes \( (l = 25, 50 \) and 100), and the relationship of the asymptotic value of VLF inequalities with the number of involved modes \( l \) for \( z \rightarrow \infty \). These figures demonstrate the scalability of our protocol. Thus, using our source of multipartite entanglement, fidelities of quantum teleportation in multimode networks better than what could be achieved in any classical scheme are expected\(^ {2} \). Noticeably, the asymptotic violation of the VLF inequalities in the double asymptotic limit \( (C_0, z \rightarrow \infty) \) given by \( \text{VLF}_j(\infty, \vec{0}, \infty) = 4(l - 1)/l \) is the same as that obtained in second harmonic generation (SHG) when a zero supermode is excited at the input of the ANW\(^ {31} \). Unlike the SHG case where only the zero supermode is present, here the \( k \neq l \) side supermodes are involved in the production of entanglement. They increase the violation of the inequalities along \( z \) through the use of optimized gains \( \vec{G} \) as shown in Equation (11). The asymptotic behavior exhibited in Figures 3 and 4 appears as a consequence of tracing over the fields present in the even channels\(^ {31} \).

Finally, we would like to underline that this configuration is very appealing for the generation of scalable multipartite entanglement since it relies on coupling \( C_0 \) and nonlinearity \( g \) within the array, but not on specific values of these parameters. Our protocol yields significant and useful entanglement over a wide range of number of modes. Furthermore, our method gives insight on further extension of the possibilities of the ANW for a resource-efficient generation of large entangled states. For example, trying to phasematch every supermode of the array. This would represent a compact, efficient and fully scalable way to produce multipartite entanglement. We are currently studying possible solutions in that direction using for example supermode quasi-phasematching\(^ {32} \). Another approach which can improve the entanglement and even generate large cluster states for measurement-based quantum computing is the optimization of pump and LO profiles. Suitable optimization of selected tuning parameters indeed turn the ANW in a versatile spatially-encoded entanglement synthesizer\(^ {23} \).

This work was supported by the Agence Nationale de la Recherche through the INQCA project (Grants No. PN-II-ID-JRP-RO-FR-2014-0013 and No. ANR-14-CE26-0038), the Paris Ile-de-France region in the framework of DIM SIRTEQ through the project ENCORE, and the Investissements d’Avenir program (Labex NanoSaclay, reference ANR-10-LABX-0035).
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