Study of non – Newtonian flow, heat transfer and entropy generation characteristics in trapezoidal corrugated channel

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Abstract. This study is focused on the thermo-hydraulic and entropy generation characteristics of a non-Newtonian fluid in a trapezoidal shaped wavy channel. The study has been conducted for different Reynolds number ranging from 25 – 100 and power law index varying from 0.5 – 1.5. The variation of average Nusselt number, pressure drop, thermal entropy generation, viscous entropy generation are being calculated. For the discretization of the governing equations, finite volume method is adopted in Ansys Fluent. Average Nusselt number value increases with an increase in the value of Reynolds number and decreases with an increase in power law index. On the other hand, pressure drop decreases on increasing the value of Reynolds number, whereas on increasing the value of power law index, pressure drop value showing a decreasing trend. Thermal entropy generation and viscous entropy generation follows almost the same trend but both having different order of magnitude. The trend shows that on increasing the value of Re both the entropy generations increases whereas on increasing the power law index, it decreases.

1. Introduction
The technological developments in the past shows us that more emphasis is given on to increase the output by supplying the same amount of input. Many technological modifications have been carried out in the industries related to process engineering, air conditioning, refrigeration, beverages industries, where the heat transfer is increased between the two fluids exchanging heat from each other or only between one fluid and heat source. Generally, two methods are adopted for increasing the heat transfer namely active and passive method. In active method changes in input is made whereas in passive method changes in the geometry of the flow passage is made in order to achieve an increased output.

Many studies have been reported on the performance of the wavy channels using numerical and experimental techniques. Rush et al. [1] experimentally concluded that an enhancement in the heat flow in corrugated channel is seen because of the formation of recirculation areas and these area depends upon geometry of channel and flow variables like Re. Yang and Chen [2] numerically found out thermo-hydraulic characteristics for a Newtonian fluid in V shaped wavy plate, Re ranging
between 2000 – 5500, they observed that heat transfer increment is directly proportional to the included angle in the V shaped wavy plate.

| Nomenclature                      | Greek symbols                      |
|-----------------------------------|------------------------------------|
| Re                                | Φ Length of corrugation parameter  |
| Nu                                | Φx                                 |
| Pr                                | μx                                 |
| Be                                | Fluid dynamic viscosity            |
| P                                 | (N m$^{-2}$)                        |
| T                                 | τ Rate of shear strain              |
| Cp                                | μa Shear stress                     |
| $x, y$                            | ρ Apparent viscosity               |
| u, v                              | λ Fluid density (kg m$^{-3}$)       |
| $k_t$                             | Wavelength of wavy part (m)        |
| a                                 | $s_{gen,thermal}$ Entropy generation per unit volume due to temperature gradient (W K$^{-1}$ m$^{-3}$) |
| E                                 | $s_{gen,viscous}$ Entropy generation per unit volume due to viscous effects (W K$^{-1}$ m$^{-3}$) |
| F                                 | $s_{gen}$ Total Entropy generation per unit volume (W K$^{-1}$ m$^{-3}$) |
| 2L                                | subscript                          |
| s                                 | i inlet                            |
| k$e$                              | W wavy wall                        |
| $\Delta p$                       | avg average                        |
| m                                 | $h$ Coefficient of heat transfer (W m$^{-2}$ K$^{-1}$) |

Wang and Chen [3] investigated numerically, the changes in $Nu_{avg}$ and $Nu_{Local}$ and coefficient of skin friction in sinusoidal channel using Newtonian fluid, and their variations on changing the parameters like sinusoidal wave amplitude, wavelength, Prandtl number and Reynolds number. Wang and Vanka [4] simulated numerically the steady Newtonian flow in a wavy passage for $Re$ upto 180 and they reported that the value of $Nu_{avg}$ is more for corrugated channel as compared to that of a straight channel and it increases as the state of flow changes from laminar to transition region. Bahaidarah et al. [5] numerically measured the changes in momentum and heat flow in a corrugated channel and compared the trend with straight channel for a Newtonian fluid having $Pr = 0.7$ and $Re$ in the range 25 – 400. They noticed that heat flow rate is same in both the channels at low $Re$ but at high $Re$ wavy channel shows more heat transfer. Mehta and Pati [6] numerically studied characteristics of heat flow in sinusoidal channel which is imposed to the sinusoidal heat flux for $Re$ ranging from 5 to 300, and they found out that for the lower amplitude of sinusoidal heat flux, average Nusselt number is highest. Mohammed et al. [7,8] made studies about variation of heat flow and other quantities, nearby entrance length of a corrugated channel for a forced convection in laminar regime. Pati et al. [9] made numerical studies and concluded that heat flow performance factor of the raccoon profile is always lower than serpentine wavy profile.

Metzner and Reed [10] performed studies on steady non-Newtonian fluids and provided correlation for the general Reynolds number in turbulent, laminar and transient regimes. Akbarzadeh et al. [11] made studies numerically to find the variations in heat flow, entropy generations, performance parameter and drop in pressure in three different wavy channels for a Newtonian fluid for $Re$ in the
range 400 – 1400. Guha and Pradhan [12] conducted numerical simulations on non-Newtonian fluid to study the forced convection on a horizontal plate. They concluded that Pr and index of power law are the two main factors that contributes to the change in pressure, temperature and velocity. Shubham et al. [13] studied the characteristics of heat flow for a non-Newtonian flow flowing in a sinusoidal corrugated profile channel and they reported that minimum pressure drop and maximum heat transfer occurs in case of pseudo-plastic fluids. Esfahani and Shahabi [14] studied entropy generation in a pipe flow using high Prandtl number fluid and the pipe wall subjected to different combinations of wall fluxes. Their studies show that maximum entropy generations are in the situation where heat flux is of decreasing nature along the length of the flow. Bejan [15,16] have given entropy generation minimization method for the convective heat flow which deals with the calculation to find the entropy generated because of heat transfer through finite temperature difference and viscous irreversibilities present in the fluid flow. It helps in making the process more efficient as one can reduce the factors that are mainly responsible for entropy generation.

As non-Newtonian fluids are majorly being used in the industries and less literature and studies are reported to illustrate the heat transfer and entropy generation characteristics. This study is an effort to fill this gap. The profiles of $Nu_{avg}$, dimensionless pressure drop, thermal and viscous entropy generations for non-Newtonian fluid comprising of both the pseudoplastic ($m < 1$) and dilatant fluid ($m > 1$), for $Re = 25$ to 100, $Pr = 6.93$ and the index of power law varying from 0.5 to 1.5 are reported in this study.

2. Problem definition

A 2D trapezoidal domain is considered for study, depicted in figure 1. Average distance between the two trapezoidal surface is 2L, starting and ending length of 3L and 5L, respectively are smooth adiabatic wall in nature. This starting and ending length are made in such a way that full development of flow can be experienced at entry of the wavy profile and no recirculation is formed at the exit of wavy section [3,7,8]. Length of 12L between the start and end length are isothermal in nature and trapezoidal in geometry. Amplitude of trapezoidal wavy profile is 0.4L, wavelength is 4L.

Figure 1. Schematic geometry of a trapezoidal channel.

In the above figure AB is the inlet, CD is the outlet, point E is the starting point of the wavy channel and point F is the ending point of it. Assumptions made for this study are:

I. Ostwald-de waele model (power law model) is used.
II. Flow is considered as incompressible, 2D and steady in nature.
III. Transverse heat conduction and viscous dissipation terms are neglected.
IV. No internal heat generation.

Ostwald-de waele model states that, shear stress and rate of shear strain are directly proportional for a given range of rate of shear strain. Mathematically, it can be written as:

$$\tau_{xy} = \mu_a \left( 1 - \frac{\dot{\gamma}_{xy}}{\dot{\gamma}_a} \right)$$

Where, $\mu_a$ is the apparent viscosity of the non-Newtonian fluid, mathematically it is expressed as:
\[ \mu_a = k_c \left( \dot{\gamma}_{xy} \right)^{m-1} \]  \hspace{1cm} (2)

In the above equation (2), by varying the value of \( m \) from less than 1 to greater than 1, one can move from shear thinning fluid (pseudoplastic) to shear thickening fluid (dilatant).

3. Mathematical equations
The governing equations are mentioned below:

- **Mass conservation**
  \[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (3)

- **Momentum conservation equations in x and y coordinates respectively**
  \[ \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  \hspace{1cm} (4)
  \[ \rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]  \hspace{1cm} (5)

- **Energy conservation equation**
  \[ \rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  \hspace{1cm} (6)

4. Boundary condition
Above mathematical equations are solved using the below boundary conditions:

- **Inlet of the channel.**
  \[ \frac{u}{u_{avg}} = \frac{2m+1}{m+1} \left( 1 - \left| 1 - \frac{m+1}{m} \right| \right), \quad v = 0, \quad T = T_i \]  \hspace{1cm} (7)

- **Channel wall.**
  \[ \frac{\partial T}{\partial y} = 0 \]  \hspace{1cm} (8)
  \[ T = T_W \text{ (such that } T_W > T_i) \]  \hspace{1cm} (9)
  
  Equation (8) is valid for straight wall and equation (9) is valid for corrugated part.

- **Channel outlet**
  \[ \frac{\partial u}{\partial x} = 0; \quad \frac{\partial v}{\partial x} = 0; \quad \frac{\partial T}{\partial x} = 0 \]  \hspace{1cm} (10)

5. Solution methodology
Governing equations are solved using the method of finite volume in Ansys Fluent using SIMPLE algorithm [17]. Second order upwind scheme is used for the discretization. The convergence criteria are taken as \( 10^{-6} \) for mass and momentum conservation, and \( 10^{-9} \) for energy conservation.
• Thermo-hydraulic transport characteristics analysis. Local and average Nusselt number can be calculated using below equations:

\[
Nu_{\text{local}} = \frac{-\frac{\partial T}{\partial y}L}{T_W - T_1} \quad \text{and} \quad Nu_{\text{avg}} = \frac{h_{\text{avg}} L \Phi}{K x}
\]

where, \( h_{\text{avg}} = \frac{1}{E - F} \int_F^E h_x \, ds \) (12)

• Entropy generation analysis. By this analysis one can know about the losses occurring while the process is taking place. Thermal and viscous entropy generations are calculated using entropy generation minimization technique [16].

\[
s_{\text{gen,thermal}} = \frac{k_f}{T} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] \quad \text{(13)}
\]

\[
s_{\text{gen,viscous}} = \frac{k_c}{T} \left( \frac{u_{\text{avg}}}{L} \right)^{m-1} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \quad \text{(14)}
\]

\[
s_{\text{gen}} = s_{\text{gen,thermal}} + s_{\text{gen,viscous}} \quad \text{(15)}
\]

\[
Be = \frac{s_{\text{gen,thermal}}}{s_{\text{gen,thermal}} + s_{\text{gen,viscous}}} \quad \text{(16)}
\]

Equation (16) shows the relation of Bejan number, which helps in finding the dominance of entropy generation because of temperature gradient in the total entropy generation. Dimensionless form of pressure difference, entropy generations are given below:

\[
\Delta p = \frac{P}{\rho u_{\text{avg}}} \quad \text{and} \quad N_{\text{gen,thermal}} = \frac{s_{\text{gen,thermal}} L^2}{k_f} \quad \text{and} \quad N_{\text{gen,viscous}} = \frac{s_{\text{gen,viscous}} L^2}{k_f} \quad \text{(17)}
\]

6. Grid independence test and meshing
Four grids are considered for performing grid independence test. The properties of the fluid used is such that Prandtl number is 6.93, \( Re = 100 \) and \( m = 1.6 \). Based on the percentage difference in average Nusselt number (Table 1) for the considered grid sizes, a grid size of 140×1400 is taken for the present study. Figure 2 shows the meshed computational domain. The mesh is created in such a way that it is fine near by surface and coarse in the core region because the gradients values of temperature, pressure and velocity are more near the walls.

Table 1. Average Nusselt number value and percentage difference at different grid size.

| Grid size | Nu_{avg}  | Percentage difference |
|-----------|-----------|-----------------------|
| 80×800    | 4.310725  | -                     |
| 100×1000  | 4.290502  | 0.4691                |
| 120×1200  | 4.282599  | 0.1841                |
| 140×1400  | 4.278881  | 0.0868                |

Figure 2. A sample meshing used in the present study.

7. Validation study
Numerical techniques used in the present study are validated using the value of \( Nu_{\text{local}} \) obtained by Wang and Chen [3] for Prandtl number 6.93, Reynolds number 100 and index of power law 1 using sinusoidal wavy channel whose dimensionless amplitude is 0.2. Another validation is done to compare the entropy generations values with the findings of Esfahani and Shahabi [14] as shown in figure 3(b).
Esfahani and Shahabi [14] have conducted the study for a high Pr fluid \((Pr = 13400)\) for a straight pipe of length 1m, diameter 0.025m and the velocity at inlet 0.02 m/s. Figure 3 shows the graphs obtained in the validation study, a close match can be seen in both the cases. Figure 3(a) depicts that \(Nu_{local}\) is maximum for wave one and decrement is seen on moving in flow direction and this decrement is significant upto fourth wave and after that this decrement becomes almost nil as formation of thermal boundary layer takes place.

8. Results and discussion

The velocity streamline contours are displayed in figure 4 for \(Re=25\) and \(Re=100\) for the values of \(m=0.5\) and \(m=1.5\). It can be observed that flow reversal is taking place in the diverging wall because of which a recirculation zone is created and the size of recirculation zone is directly proportional to \(Re\). For pseudoplastic fluids \((m<1)\), velocity is higher inside the channel as compared to dilatant fluids \((m>1)\) because the apparent viscosity value is higher in dilatant fluids, offering higher resistance to the flow.

![Figure 3](image1.png)

**Figure 3.** Comparison of present study with (a) Wang and Chen [3] results (b) Esfahani and Shahabi [14] results.

![Figure 4](image2.png)

**Figure 4.** Velocity streamline for (a)\(Re = 25, m = 0.5\) (b)\(Re = 25, m = 1.5\) (c)\(Re = 100, m = 0.5\) (d)\(Re = 100, m = 1.5\).
As the velocity in the region near the corrugated surface is lower compared to the velocity in the core region, therefore, the convection is dominant at the central region. Therefore, convective transport of energy is higher in the core, resulting in a lower temperature region in the central region. Because of lower velocity near the walls, convective transport of energy is lower, causing higher temperature in that region. Temperature contours are depicted in Figure 5 for \( Re=25 \) and \( Re=100 \) for the values of \( m=0.5 \) and \( m=1.5 \). It can be observed that as the nature of the fluids changes from shear thinning to shear thickening category, waviness of the contour displayed in blue colour, increases in the core region. The increase in waviness is seen because of the velocity pattern followed by the shear thickening fluids as a result of increase in the apparent viscosity.

![Temperature Contours](image)

**Figure 5.** Contour of temperature for (a) \( Re=25, \ m=0.5 \) (b) \( Re=25, \ m=1.5 \) (c) \( Re=100, \ m=0.5 \) (d) \( Re=100, \ m=1.5 \).

![Average Nusselt Number Variation](image)

**Figure 6.** (a) Average Nusselt number variation with respect to \( Re \) (b) variation of dimensionless pressure drop versus \( Re \) for different values of power law index.

**Figure 6(a)** depicts the plot of \( Nu_{avg} \) versus \( Re \) for various power law index, whereas **figure 6(b)** displays the dimensionless pressure drop. It is observed, as the value of \( Re \) increases, flow velocity increases which enhances convection and therefore \( Nu_{avg} \) increases. Also, as we move from pseudoplastic to dilatent fluids at the same \( Re \), \( Nu_{avg} \) decreases because of the increase in the value of apparent viscosity. Pressure drop shows a decreasing trend as value of Reynolds number is increased.
As the value of Reynolds number increases, it results in an increase in the velocity which helps in pushing the fluid forward. However, the viscosity is higher for $m > 1$ therefore, a higher-pressure drop is observed for shear thickening fluids. Entropy generation value shows an increment when an increase in temperature gradient is seen, since the temperature gradient is high for the pseudoplastic fluids therefore, its thermal entropy generation is higher. However, for higher value of Reynolds number, the size of recirculation region is increased which increases the mixing of fluid and decreases the temperature gradient as depicted in Figure 7. When index of power law is greater than unity, fluid viscosity increases. Therefore, the fluid gets sufficient time to transfer heat and this results in the decrement of temperature gradient. Figure 8 shows the contours of viscous entropy generation, where it is evident that by increasing the Reynolds number, viscous entropy generation increases as increase in the size of recirculation zone offers a kind of viscous resistance.

Figure 7. Thermal entropy generation contours for (a) $Re = 25$, $m = 0.5$ (b) $Re = 25$, $m = 1.5$ (c) $Re = 100$, $m = 0.5$ (d) $Re = 100$, $m = 1.5$.

Figure 8. Viscous entropy generation contours for (a) $Re = 25$, $m = 0.5$ (b) $Re = 25$, $m = 1.5$ (c) $Re = 100$, $m = 0.5$ (d) $Re = 100$, $m = 1.5$.

Figure 9 depicts the plot of dimensionless entropy generations, where figure 9(a) shows the plot of thermal entropy generation versus Reynolds number, whereas figure 9(b) shows the viscous entropy
generation trend with Reynolds number for different values of $m$. Both the graphs depict almost the same trend where an increment in the value of entropy generations can be seen on increasing the Reynolds number. For a constant Reynolds number, entropy generation value decreases due to an increase in the value of $m$. Bejan number contours shown in figure 10 help in determining the portion of thermal entropy generation in the total entropy generated and its value lies between 0 and 1. Where 0 means that viscous entropy generation is dominating; whereas a value of Bejan number equal to unity indicates thermal entropy generation is dominating. From the above contour, it is visible that with increment in the value of Reynolds number and power law index, thermal entropy generation dominance shows a decrement in the computational domain.

![Figure 9](image.png)

**Figure 9.** (a) Dimensionless thermal entropy generation variation with respect to Re (b) variation of dimensionless viscous entropy generation with respect to the Re for different values of index of power law.

![Figure 10](image.png)

**Figure 10.** Bejan number contours for (a)Re = 25, $m = 0.5$ (b)Re = 25, $m = 1.5$ (c)Re = 100, $m = 0.5$ (d)Re = 100, $m = 1.5$.

9. **Conclusions**

Some of the important findings from this study are listed below:
i. Average Nusselt number will be maximum for shear thinning fluids and for high Reynolds numbers.

ii. As Reynolds number increases, pressure drop decreases, shear thinning fluids have higher pressure drop compared to shear thickening fluids.

iii. Both thermal and viscous entropy generation increases on increasing the Re and is higher for shear thinning fluids.

iv. Bejan number shows that as Reynolds number and index of power law are increased, dominance of thermal entropy generation starts decreasing in the computational domain.

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