Research Article

Dissipativity and Error Feedback Controller Design of Time-Delay Genetic Regulatory Networks

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Genetic regulatory networks (GRNs) play an important role in the development and evolution of the biological system. With the rapid development of DNA technology, further research on GRNs becomes possible. In this paper, we discuss a class of time-delay genetic regulatory networks with external inputs. Firstly, under some reasonable assumptions, using matrix measures, matrix norm inequalities, and Halanay inequalities, we give the global dissipative properties of the solution of the time-delay genetic regulation networks and estimate the parameter-dependent global attraction set. Secondly, an error feedback control system is designed for the time-delay genetic control networks. Furthermore, we prove that the estimation error of the model is asymptotically stable. Finally, two examples are used to illustrate the validity of the theoretical results.

1. Introduction

In the past few decades, research on gene regulatory networks’ modeling has attracted many biologists and mathematicians. There have been many research studies in this area, such as models based on Boolean networks [1], models based on Bayesian methods [2], and models based on differential equations [3, 4]. In particular, the gene regulation networks’ model based on ordinary differential equation technology is welcomed by researchers because of its simple mathematical technology and clear biological meaning.

The concept of dissipative system originates from the research of control theory. In a sense, it is a generalization of Lyapunov’s stability theory. There are fruitful research results on stability issues in various research fields, such as neural network systems [5–10], biological systems [11, 12], and nonlinear systems [13, 14]. However, the study of stability requires the existence of a balance point. In some special cases, the balance point of the system may not exist. At this time, we consider the dissipative characteristics of the system. In the dissipative problem, we need to find a positive invariant set, and we only need to consider the dynamic characteristics of the system in the invariant set. So far, the dissipative characteristics of gene regulatory network models based on ordinary differential equation technology have not been seen in related publications.

In addition, the control system has been widely used in all aspects of human society. In the ecosystem, a large number of phenomena can be observed through the setting of the controller, and different control schemes will produce different kinds of effects. In [15], the author gives some characteristics of the linear time-varying system under the control of time-varying feedback conjugate. In [16], the authors establish the stability criterion and saddle node bifurcation of gene regulatory networks that have nothing to do with coupling delay. In [17], the authors discuss a delayed fractional-order two-gene regulatory network model that can more accurately reflect the memory and genetic characteristics of the gene network.

Based on the enlightenment obtained from the above-mentioned literature research, we discuss the characteristics of time-delay gene regulation networks with external inputs. Compared with most previous related results, our novelty lies in the use of matrix measurement theory to carry out discussions. The matrix measurement strategy has the following two advantages. (1) Lyapunov function does not need to be constructed. (2) The result obtained by the matrix measurement is more accurate and less conservative than the result obtained by the matrix norm.
The rest of the paper is arranged as follows. In Section 2, we give the mathematical form of the gene regulatory network model and some basic assumptions. In Section 3, the global dissipativity of the model is discussed. In Section 4, we study the asymptotic stability of the error feedback control system. In Section 5, two examples are given to verify the validity of our conclusions.

Notations: in this paper, $R^n$ represents the $n$-dimensional Euclidean space. $A^T$ represents the transpose of matrix $A$. $x \in R^n \setminus \Omega$ means $x \in R^n$ but $x \notin \Omega$. For real symmetric matrices $A$ and $B$, $A \succ 0$ indicates that $A$ is positive definite. $A \succ B$ represents $A - B \succ 0$. $\lambda_{\max}(A)$ represents the maximum eigenvalue of matrix $A$.

2. Preliminaries

The gene regulatory networks’ model has the following classic form:

$$
\begin{align*}
\dot{m}_i(t) &= -a_im_i(t) + \sum_{j=1}^{n} w_{ij}(p_j(t - \sigma_j(t))) + q_i, \\
\dot{p}_i(t) &= -c_i p_i(t) + d_i m_i(t - \tau_i(t)), \quad i = 1, 2, \ldots, n,
\end{align*}
$$

where $m_i(t), p_i(t) \in R$ are the concentrations of the $i$th mRNA and protein, respectively, $a_i$ and $c_i$ are positive constants, representing the degradation rate of the $i$th mRNA and protein, respectively, $d_i > 0$ is the translation rate from the $i$th protein, $h_j(x) = \left(\langle x/\beta_j \rangle^H/1 + \langle x/\beta_j \rangle^H\right)$ is the regulatory functions of mRNA, where $H$ is the Hill coefficient, $\beta_j$ is a scalar, $\sigma_j(t)$ and $\tau_i(t)$ are transcriptional and translational delay with $0 < \sigma_j(t) \leq 0$, $0 < \tau_i(t) \leq \tau$, and $q_i = \sum_{j \in \mathcal{N}_i} a_{ij}$ with $\nu_j$ is the set of all the transcription factor $j$ which is a repressor of gene $i$, and

$$
\begin{align*}
w_{ij} &= \begin{cases} 
\alpha_{ij}, & \text{if factor } j \text{ activates gene } i, \\
0, & \text{if there is no connection between } j \text{ and } i, \\
-\alpha_{ij}, & \text{if factor } j \text{ represses gene } i.
\end{cases}
\end{align*}
$$

We rewrite model (1) in the following matrix form:

$$
\begin{align*}
\dot{m}(t) &= -A \cdot m(t) + B \cdot p(t) + q, \\
p(t) &= -C \cdot p(t) + D \cdot m(t - \tau(t)),
\end{align*}
$$

where $m(t) = [m_1(t), m_2(t), \ldots, m_n(t)]^T$, $p(t) = [p_1(t), p_2(t), \ldots, p_n(t)]^T$, $A = \text{diag}(a_1, a_2, \ldots, a_n)$, $B = [w_{ij}]_{n \times n}$, $C = \text{diag}(c_1, c_2, \ldots, c_n)$, $D = \text{diag}(d_1, d_2, \ldots, d_n)$, $\sigma(t) = [\sigma_1(t), \sigma_2(t), \ldots, \sigma_n(t)]^T$, $\tau(t) = [\tau_1(t), \tau_2(t), \ldots, \tau_n(t)]^T$, $h(p(t)) = [h_1(p_1(t)), h_2(p_2(t)), \ldots, h_n(p_n(t))]^T$, and $q = (q_1, q_2, \ldots, q_n)^T$.

Next, we give some basic assumptions and lemmas.

Assumption 1. The activation function $h_i(\cdot)$ meets the Lipschitz condition.

Remark 1. In most literature, $h_i(\cdot)$ satisfied $0 \leq (h_i(a) - h_i(b))/(a - b) < li$, for all $a, b \in R, l_i > 0, i \in \{1, 2, \ldots, n\}$. The condition can be reduced to $m_i^* \leq (h_i(a) - h_i(b))/(a - b) \leq m_i^*, \forall a, b \in R$, where $m_i^*$ and $m_i^*$ are constants. Furthermore, the condition is reduced to Assumption 1, i.e., there exists a constant $l_i > 0$ such that, for any $x, y \in R$, the following inequalities hold: $[h_i(x) - h_i(y)] \leq l_i |x - y|$, $l_i = \max\{|m_i^*|, |m_i^*|\}$. Clearly, Assumption 1 is more general and less conservative.

Let $(m^*, p^*)$ be the equilibrium point of (3), and it satisfies

$$
\begin{align*}
0 &= -A \cdot m^* + B \cdot p^* + q, \\
0 &= -C \cdot p^* + D \cdot m^*.
\end{align*}
$$

Let $x(t) = m(t) - m^*$ and $y(t) = p(t) - p^*$, and we obtain

$$
\begin{align*}
\dot{x}(t) &= -Ax(t) + Bf(y(t - \sigma(t))), \\
y(t) &= -Cy(t) + Dx(t - \tau(t)),
\end{align*}
$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T$, $y(t) = [y_1(t), \ldots, y_n(t)]^T$, $f(y(t)) = [f_1(y_1(t)), \ldots, f_n(y_n(t))]^T$, and $f_i(y_i(t)) = h_i(y_i(t) + p_i^*) - h_i(p_i^*)$.

In the field of gene therapy, the effect of external forces or stimuli on the biological system is very important. For example, mRNA vaccine is to transfer RNA to human cells after relevant modification in vitro for expression and produce protein antigen, which can lead the body to produce immune response to antigens, and then expand the immune ability of the body. Therefore, we consider the following time-delay GRNs with external inputs described by

$$
\begin{align*}
\dot{x}(t) &= -Ax(t) + Bf(y(t - \sigma(t))) + u, \\
y(t) &= -Cy(t) + Dx(t - \tau(t)) + v,
\end{align*}
$$

where $u = (u_1, u_2, \ldots, u_n)$ and $v = (v_1, v_2, \ldots, v_n)$ are control inputs for mRNA and protein, respectively.

The initial conditions of system (6) are

$$
x(s) = (\varphi_1(s), \ldots, \varphi_n(s)), \\
y(s) = (\psi_1(s), \ldots, \psi_n(s)), \quad s \in [-\rho, 0],
$$

$$
\rho = \max\{\tau, \sigma\}.
$$

By the definition of $f_i(\cdot)$ and Assumption 1, it satisfies

$$
|f_i(x)| \leq l_i |x|.
$$

Definition 1. If there exists a compact set $\Omega \subset R^n$, for any $\epsilon > 0$ and any initial value $\varphi(s), \psi(s) \in R^n \setminus \Omega, s \in [-\rho, 0]$, such that $\exists T > 0$, when $t \geq T, x(t, \varphi), y(t, \psi) \in \Omega$, where $\Omega$ is the $\epsilon$-neighborhood of $\Omega$, then, the time-delay GRNs (6) are said to be a global dissipative system. In this case, $\Omega$ is called a globally attractive set. If, for any initial value $\varphi(s), \psi(s) \in \Omega, s \in [-\rho, 0]$, there is $x(t, \varphi), y(t, \psi) \in \Omega, t \geq 0$, then the set is said to be positive invariant. Obviously, the globally attractive set is positive invariant.
Definition 2. If there exists a compact set $\Omega \subset R^n$, for any $\varepsilon > 0$ and any initial value $\varphi (s), \psi (s) \in R^n \setminus \Omega, s \in [-\rho, 0]$, there exist $M_1(\|\varphi\|), M_2(\|\psi\|) > 0$ and $\alpha, \beta > 0$ such that
\[
\inf_{x \in \Omega} \|x(t, \varphi) - x\| \leq M_1(\|\varphi\|) \exp[-\alpha t],
\]
\[
\inf_{y \in \Omega} \|y(t, \psi) - \psi\| \leq M_2(\|\psi\|) \exp[-\beta t].
\]
(9)

Then, the time-delay GRNs (6) are said to be a globally exponentially dissipative system.

Definition 3. For constant matrix $A = (a_{ij})_{n \times n} \in R^{n \times n}$, the matrix norms are given as
\[
\|A\|_1 = \max_{j} \sum_{i=1}^{n} |a_{ij}|,
\]
\[
\|A\|_2 = \sqrt{\max_{j} (A^T A)},
\]
\[
\|A\|_{\infty} = \max_{1 \leq i \leq n} |a_{ij}|.
\]
(10)

For $x \in R^n$, the vector norms are defined as follows:
\[
\|x\|_1 = \sum_{i=1}^{n} |x_i|,
\]
\[
\|x\|_2 = \left( \sum_{i=1}^{n} |x_i|^2 \right)^{1/2},
\]
\[
\|A\|_{\infty} = \max_{1 \leq i \leq n} |a_{ij}|.
\]
(11)

Definition 4. Suppose that $A = (a_{ij})_{n \times n}$ is a real matrix; then, the matrix measure of $A$ is defined as
\[
\mu_p(A) = \lim_{h \to 0} \frac{\|E_n + hA\|_p - 1}{h},
\]
(12)

where $E_n$ is an $n \times n$ identity matrix, $p = 1, 2, \infty$, and $\|::p$ is the corresponding induced matrix norm.

It can be calculated directly by Definition 4:
\[
\mu_1(A) = \max_j \left\{ a_{jj} + \sum_{i=1, i \neq j}^{n} |a_{ij}| \right\},
\]
\[
\mu_2(A) = \lambda_{\max} \left( \frac{A^T + A}{2} \right),
\]
\[
\mu_{\infty}(A) = \max_i \left\{ a_{ii} + \sum_{j=1, j \neq i}^{n} |a_{ij}| \right\}.
\]
(13)

Lemma 1 (see [18]). Let $r > 0, \xi > 0, \eta > 0, \epsilon > 0$, and $\tau > 0$, if the continuous function $V(t) \geq 0, t \in R$, and
\[
D^+ V(t) \leq r - \xi V(t) + \eta \|\nabla(t)\|, \quad t \leq t_0.
\]
(14)

There exists a constant $\sigma > 0$ such that $-\xi + \eta \sigma < 0$; then,
(1) When $\nabla(t_0) > (r/\sigma)$, the following formula is established:
\[
V(t) \leq \left( \frac{r}{\sigma} \right) \left( \frac{\nabla(t_0) - r}{\sigma} \right) e^{-\sigma (t-t_0)}, \quad t \geq t_0.
\]
(15)

(2) When $\nabla(t_0) \leq (r/\sigma)$, the following formula is established:
\[
V(t) \leq \frac{r}{\sigma}, \quad t \geq t_0.
\]
(16)

where $\nabla(t_0) = \sup_{t \in R} \nabla(s), \mu^*(\sigma)$ is the unique positive root of the transcendental equation $\mu - \xi + \eta e^{\sigma \tau} = 0$, and $D^+V(t) = \lim_{h \to 0} (V(t+h) - V(t))/h$ is the upper-right Dini derivative.

Lemma 2 (see [19]). For any $A, B \in R^{n \times n}$ and $p = 1, 2, \infty$, the matrix measure $\mu_p(\cdot)$ has the following properties:
(1) $-\|A\|_p \leq \mu_p(A) \leq \|A\|_p$;
(2) $\mu_p(aA) = a \mu_p(A)$, $\forall a > 0$;
(3) $\mu_p(A + B) \leq \mu_p(A) + \mu_p(B)$.

3. Global Dissipativity

In this section, we give the global dissipative properties of the studied mathematical model.

Theorem 1. If $f_i(\cdot)$ satisfies (8), there exist positive constant $\delta_1$, and the matrix measures $\mu_p(\cdot), p = 1, 2, \infty$, such that $\Pi_1 + \Pi_2 \leq -\delta_1 < 0$, where $\Pi_1 = \max \left\{ \mu_p(-A), \mu_p(-C) \right\}$ and $\Pi_2 = \max \left\{ \|B\|_p, \|D\|_p, I \right\} = \max_{1 \leq i \leq n} \{ l_i \}$. Then, the time-delay GRNs (6) are a globally dissipative system, and the set $\Omega_1 = \{ x, y \in R^n | \|x\|_p + \|y\|_p \leq (\|u\|_p + \|v\|_p/\delta_1) \}$ is a positive invariant set and a globally attractive set of (6).

Proof. Let
\[
V(t) = \|x(t)\|_p + \|y(t)\|_p.
\]
(17)

We calculate the upper-right hand Dini derivative of $V(t)$ along the solution of (6):
\[ D^V(t) \leq \lim_{h \to 0^+} \frac{\|x(t + h)\|_p - \|x(t)\|_p}{h} + \lim_{h \to 0^+} \frac{\|y(t + h)\|_p - \|y(t)\|_p}{h} \]
\[ = \lim_{h \to 0^+} \frac{\|x(t) + h\dot{x}(t) + o(h)\|_p - \|x(t)\|_p}{h} + \lim_{h \to 0^+} \frac{\|y(t) + h\dot{y}(t) + o(h)\|_p - \|y(t)\|_p}{h} \]
\[ \leq \lim_{h \to 0^+} \frac{\|x(t) + h(-A)x(t)\|_p + \|o(h)\|_p - \|x(t)\|_p}{h} + \lim_{h \to 0^+} \frac{\|y(t) + h(-C)y(t)\|_p + h\|Dy(t)\|_p + \|o(h)\|_p - \|y(t)\|_p}{h} \]
\[ + \lim_{h \to 0^+} \frac{\|E_n + h(-A)\|_p - 1}{h} \|x(t)\|_p + \lim_{h \to 0^+} \frac{\|E_n + h(-C)\|_p - 1}{h} \|y(t)\|_p \]
\[ + \|B\|_p \|f(y(t) - \sigma(t))\|_p + \|D\|_p \|x(t - \tau(t))\|_p + \|u\|_p + \|v\|_p \].

From (8), we can obtain
\[ \|f(y(t) - \sigma(t))\|_p \leq I \|y(t) - \sigma(t)\|_p. \] (19)

By (18) and (19), according to the definition of matrix measure, we deduce that
\[ D^V(t) \leq \mu_p (-A)\|x(t)\|_p + \mu_p (-C)\|y(t)\|_p \]
\[ + \|B\|_p \|y(t) - \sigma(t)\|_p + \|D\|_p \|x(t - \tau(t))\|_p + \|u\|_p + \|v\|_p. \] (20)

According to Lemma 1 and \[ \|\varphi(s)\|_p + \|\psi(s)\|_p > \left(\|u\|_p + \|v\|_p/\delta_1\right), \] \(s \in [-\rho, 0]\), we have
\[ V(t) \leq \frac{\|u\|_p + \|v\|_p}{\delta_1} + Me^{-\mu^* t}, \] (21)
where \(\mu^*\) is the unique positive root of equation \(\mu + \Pi_1 + \Pi e^{\rho\alpha} = 0\) and \(M = \sup_{-\rho < t < 0} \|V(s) - t\|_p \left(\|u\|_p + \|v\|_p/\delta_1\right)\). Then, (6) is a globally dissipative system, and the globally attractive set is
\[ \Omega_1 = \left\{ x, y \in \mathbb{R}^n | \|x\|_p + \|y\|_p \leq \frac{\|u\|_p + \|v\|_p}{\delta_1} \right\}. \] (22)

If \(\|\varphi(s)\|_p + \|\psi(s)\|_p > \left(\|u\|_p + \|v\|_p/\delta_1\right), s \in [-\rho, 0]\), by Lemma 2, we have \(V(t) \leq \left(\|u\|_p + \|v\|_p/\delta_1\right), \) for all \(t \geq 0\). Therefore, the set \(\Omega_1\) is also a positive invariant set of (6).

**Remark 2.** Obviously, from (22), we can easily get the following sets:

\[ \Omega_1^x = \left\{ x \in \mathbb{R}^n | \|x\|_p \leq \frac{\|u\|_p + \|v\|_p}{\delta_1} \right\}, \]
\[ \Omega_1^y = \left\{ y \in \mathbb{R}^n | \|y\|_p \leq \frac{\|u\|_p + \|v\|_p}{\delta_1} \right\}. \] (23)

**Corollary 1.** Let \(p = 2\), and other conditions are the same as Theorem 1. Then, the time-delay GRNs (6) are a globally exponentially dissipative system, and the globally attractive sets are \(\Omega_1^x = \left\{ x \in \mathbb{R}^n | \|x\|_p \leq \left(\|u\|_p + \|v\|_p/\delta_1\right) \right\}\) and \(\Omega_1^y = \left\{ y \in \mathbb{R}^n | \|y\|_p \leq \left(\|u\|_p + \|v\|_p/\delta_1\right) \right\}\), where \(o\) is the origin of coordinates. We have
\[ \|x(t) - \sigma\|_2 = \|x(t)\|_2 - \|\sigma\|_2 = \|x(t)\|_2 - \frac{\|u\|_2 + \|v\|_2}{\delta_1} \] (24)

Accordingly,
\[ \inf_{\sigma \in \Omega_1^x} \left(\|x(t) - \sigma\|_2 \right) \leq \|x(t)\|_2 - \frac{\|u\|_2 + \|v\|_2}{\delta_1} \leq M_1 e^{-\nu t}. \] (25)

Similarly,
\[ \inf_{\sigma \in \Omega_1^y} \left(\|y(t) - \sigma\|_2 \right) \leq \|y(t)\|_2 - \frac{\|u\|_2 + \|v\|_2}{\delta_1} \leq M_2 e^{-\nu t}. \] (26)
Thus, the time-delay GRNs (6) are a globally exponentially dissipative system and $\Omega^+_2$ and $\Omega^+_1$ are globally exponentially attractive sets of (6).

In Theorem 1, parameter $l_i$ is considered only as the maximum value information. If the information of each component $l_i$ is utilized, the conservativeness of the result can be decreased.

Denote

$$\bar{Q} = \begin{pmatrix} -A & 0 \\ 0 & -C \end{pmatrix},$$

$$\bar{B} = \begin{pmatrix} 0 & B \\ 0 & 0 \end{pmatrix},$$

$$\bar{D} = \begin{pmatrix} 0 & 0 \\ D & 0 \end{pmatrix},$$

$$L = \text{diag}[l_1, l_2, \ldots, l_n],$$

$$I = \begin{pmatrix} u \\ v \end{pmatrix}, \quad (27)$$

Then, (6) can be written in the form of matrix blocks:

$$\dot{z}(t) = \bar{Q}z(t) + \bar{B}F(y(t - \sigma(t)))z(t - \sigma(t)) + \bar{D}Gz(t - \tau(t)) + I,$$

where $z(t) = (x^T(t), y^T(t))^T$. □

**Theorem 2.** If $f_i(\cdot)$ satisfies (8), there exist positive constant $\delta_i$, and the matrix measures $\mu_p(\cdot)$, $p = 1, 2, \infty$, such that

$$-\delta_i = \mu_p(Q) + \|BL\|_p + \|DG\|_p < 0. \quad (28)$$

Then, the time-delay GRNs (6) are a globally dissipative system, and the sets

$$\Omega^+_2 = \left\{ x \in \mathbb{R}^n \mid \|x\|_p \leq \frac{\|I\|_p}{\delta_2} \right\},$$

$$\Omega^+_2 = \left\{ y \in \mathbb{R}^n \mid \|y\|_p \leq \frac{\|I\|_p}{\delta_2} \right\}, \quad (29)$$

are the positive invariant sets and the globally attractive sets of (6).

**Proof.** Consider the upper-right hand Dini derivative of $\|z(t)\|_p$ with respect to time along the trajectory of (6), and we obtain

$$D^+\|z(t)\|_p = \lim_{h \to 0^+} \frac{\|z(t + h)\|_p - \|z(t)\|_p}{h}$$

$$= \lim_{h \to 0^+} \frac{\|z(t) + hQz(t) + hBF(y(t - \sigma(t)))z(t - \sigma(t)) + hDGz(t - \tau(t)) + hI\|_p - \|z(t)\|_p}{h}$$

$$\leq \lim_{h \to 0^+} \frac{\|E_{2n} + hQ\|_p - 1}{h} \|z(t)\|_p + \|BF(y(t - \sigma(t)))\|_p \|z(t - \sigma(t))\|_p$$

$$+ \|DG\|_p \|z(t - \tau(t))\|_p + \|I\|_p$$

$$\leq \mu_p(Q)\|z(t)\|_p + \|BF(y(t - \sigma(t)))\|_p \|z(t - \sigma(t))\|_p$$

$$+ \|DG\|_p \|z(t - \tau(t))\|_p + \|I\|_p. \quad (30)$$
By the definition of $\| \cdot \|_p$ and (8), it is easy to acquire that $\| \bar{B} \dot{F}(y(t - \sigma(t))) \|_p \leq \| BL \|_p$ holds for $p = 1, \infty$. Meanwhile, it also holds for $p = 2$. In fact, for $LL - F(y(t - \sigma(t)))F(y(t - \sigma(t))) \geq 0$, we have $B(LL - F(y(t - \sigma(t)))F(y(t - \sigma(t))))B^T \geq 0$, i.e., $\bar{B}LLB^T \geq BF(y(t - \sigma(t)))F(y(t - \sigma(t)))B^T$. By Weyl’s monotonicity principle, we can obtain

$$\lambda_{\max}(\bar{B}LLB^T) \geq \lambda_{\max}(BF(y(t - \sigma(t)))F(y(t - \sigma(t))))B^T).$$

(31)

Since $\lambda(\bar{B}LLB^T) = \lambda(B^TLLB)$ and

$$\lambda(BF(y(t - \sigma(t)))F(y(t - \sigma(t))))=\lambda(B^TF(y(t - \sigma(t)))F(y(t - \sigma(t))))B).$$

(32)

4. State Estimation of Error Feedback Control System

In this section, we will discuss the global exponential stability of the following error dynamic system:

$$\begin{align*}
\dot{x}(t) &= -Ax(t) + Bf(y(t - \sigma(t))) + u, \\
\dot{y}(t) &= -Cy(t) + Dx(t - \tau(t)) + v,
\end{align*}$$

(38)

and the considered state estimator is of the following form:

$$\begin{align*}
\dot{x}(t) &= -Ax(t) + Bf(y(t - \sigma(t))) + u + \mathcal{U}, \\
\dot{y}(t) &= -Cy(t) + Dx(t - \tau(t)) + v + \mathcal{Y},
\end{align*}$$

(39)

where $\mathcal{U}$ be an error feedback control term with the form $\mathcal{U} = (K_1e_1, K_2e_2)^T$, $e_1 = \bar{x}(t) - x(t)$ and $e_2 = \bar{y}(t) - y(t)$. $K_1$ and $K_2$ are feedback control gains, $K_1 = \text{diag}[k_1, k_2, \ldots, k_n]$ and $K_2 = \text{diag}[k'_1, k'_2, \ldots, k'_n]$. Thus, the error dynamic system is given as

$$\begin{align*}
\dot{e}_1(t) &= -Ae_1(t) + Bf(y(t - \sigma(t))) + \mathcal{U}, \\
\dot{e}_2(t) &= -Ce_2(t) + De_2(t - \tau(t)) + \mathcal{Y}.
\end{align*}$$

(40)

In order to facilitate the discussion later, we write (40) for the matrix block form

$$\begin{align*}
\dot{e}(t) &= \bar{K}e(t) + \bar{B}F(e_2(t - \sigma(t)))e(t - \sigma(t)) + \bar{D}\bar{G}e(t - \tau(t)),
\end{align*}$$

(41)

where

$$e(t) = \begin{pmatrix} e_1^T(t) \\ e_2^T(t) \end{pmatrix} ,$$

$$\bar{K} = \begin{pmatrix} -A + K_1 & 0 \\ 0 & -C + K_2 \end{pmatrix} ,$$

(42)

$$F(e_2(t)) = \text{diag} \left\{ \frac{f_1(e_2^{(1)}(t))}{e_2^{(1)}(t)}, \ldots, \frac{f_n(e_2^{(n)}(t))}{e_2^{(n)}(t)} \right\} ,$$

$$\bar{F}(e_2(t)) = \begin{pmatrix} 0 & 0 \\ 0 & F(e_2(t)) \end{pmatrix} .$$

(43)
Theorem 3. If $f_i(\cdot)$ meets (8), there exist matrix $K_1$ and $K_2$ and matrix measure $\mu_p(\cdot), p = 1, 2, \infty$, such that $-\mu_p(K) > \|BL\|_p + \|DG\|_p > 0$. Then, the error dynamic system (40) is globally exponential stable under controller
\[
\begin{pmatrix}
\mathcal{U} \\
\mathcal{V}
\end{pmatrix} = \begin{pmatrix} K_1 e_1 \\
K_2 e_2 \end{pmatrix}.
\]

Proof. By inequality (30), we have
\[
D^+\|e(t)\|_p = \lim_{h \to 0} \frac{\|e(t + h)\|_p - \|e(t)\|_p}{h}
\]
\[
= \lim_{h \to 0} \frac{\|e(t) + hK_2(t - \sigma(t)) + h\hat{F}(e_2(t - \sigma(t))) + h\hat{G}(t - \tau(t))\|_p - \|e(t)\|_p}{h}
\]
\[
\leq \lim_{h \to 0} \frac{\|E_{2n} + hK_2\|_p - \|e(t)\|_p + \|\hat{F}(e_2(t - \sigma(t)))\|_p + \|\hat{G}(t - \tau(t))\|_p}{h}
\]
\[
\leq \mu_p(K)\|e(t)\|_p + \|\hat{F}(e_2(t - \sigma(t)))\|_p + \|\hat{G}(t - \tau(t))\|_p
\]
\[
\leq \mu_p(K)\|e(t)\|_p + (\|BL\|_p + \|DG\|_p)\|e(t)\|_p + \|\hat{G}(t - \tau(t))\|_p
\]
\[
\leq \mu_p(K)\|e(t)\|_p + (\|BL\|_p + \|DG\|_p)\sup_{t \leq s < t} \|e(s)\|_p.
\]

5. Numerical Simulations

In this part, we give a pair of examples to illustrate the validity of our theoretical results.

Example 1. Consider time-delay GRNs (6) with
\[
A = C = \text{diag}(2, 2, 2, 2), \\
D = \text{diag}(1, 1, 1, 1), \\
\begin{bmatrix}
2 \\
2.89 \\
1.98 \\
1.95
\end{bmatrix}
, \\
\begin{bmatrix}
1.28 \\
1.89 \\
1.98 \\
1.95
\end{bmatrix}
, \\
\begin{bmatrix}
0 & -0.4 & -0.4 & 0 & 0 \\
-0.4 & 0 & 0 & 0.4 & 0.4 \\
0 & 0.4 & 0 & 0 & 0 \\
0.4 & -0.4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.4 
\end{bmatrix}
\]

By computation, we can find that \(L_j^+ = 0\) and \(L_j^- = 0.65\), and hence,

\[
L^+ = \text{diag}(0, 0, 0, 0, 0), \\
L^- = \text{diag}(0.65, 0.65, 0.65, 0.65, 0.65). 
\]

In order to make the calculation simple, we choose \(A, C, \) and \(D\) to be a diagonal matrix. At this time, \(p\) takes any value in \(1, 2, \infty\), and we have \(\mu_p(A) = 2, \mu_p(C) = 2, \mu_p(D) = 2, \) and \(p = 1, 2, \infty.\) Since \(B\) is a nondiagonal matrix, it can be obtained through simple calculations:

\[
\|B\|_1 = 0.4, \|B\|_2 = 0.48, \text{ and } \|B\|_{\infty} = 0.4. \text{ Obviously, the parameters satisfy the conditions of Theorem 1, i.e., } \Pi_1 + \Pi_2 = -1 = -\delta_1 < 0. \text{ This means that system (6) is globally dissipative. From the vector norm, we get the following results:}
\]

\[
\|u\|_1 = 11.78, \\
\|v\|_1 = 9.06, \\
\|u\|_2 = 5.36, \\
\|v\|_2 = 4.0957, \\
\|u\|_{\infty} = 2.96, \\
\|v\|_{\infty} = 1.98. 
\]

When \(p = 1, \) \(\Omega_1 = \{x, y \in R^5 \mid \|x\|_1 + \|y\|_1 \leq 20.84\} \). When \(p = 2, \) \(\Omega_2 = \{x, y \in R^5 \mid \|x\|_2 + \|y\|_2 \leq 94.557\} \). When \(p = \infty, \) \(\Omega_{\infty} = \{x, y \in R^5 \mid \|x\|_{\infty} + \|y\|_{\infty} \leq 20.84\}. \) Figure 1 shows the time-dependent trajectory of mRNA and protein concentrations when the initial values are \(x(0) = (0.2, 0.5, 0.9, 1.5, 2.5)^T\) and \(y(0) = (0.8, 1, 1.8, 2.5, 0.6)^T\). The verification of Theorem 2 conditions is similar to the verification in Example 2, so we will not state it here.

**Example 2.** In this example, we consider the error dynamic system (40), in which the parameters are listed as follows:

\[
A = \text{diag}(3, 3, 3), \\
C = \text{diag}(4, 4, 4), \\
D = \text{diag}(1, 1, 1), \\
K_1 = \text{diag}(1, 1, 1), \\
K_2 = \text{diag}(1, 1, 1), \\
B = \begin{bmatrix}
0 & -0.4 & 0.4 \\
0.4 & 0 & 0.4 \\
0 & -0.4 & 0.4 
\end{bmatrix}, \\
\sigma(t) = 0.3|\cos(t)| + 0.2, \\
\tau(t) = 12.5|\sin(t)| + 0.5, \\
f_j(x) = \frac{x^2}{x^2 + 1}, \quad j = 1, 2, \ldots, n. 
\]

and \(f_j(x)\) is the same as Example 1. It is easy to obtain that

\[
L^+ = \text{diag}(0, 0, 0, 0, 0), \\
L^- = \text{diag}(0.65, 0.65, 0.65, 0.65, 0.65). 
\]
The above calculation results are easy to see that the conditions in Theorem 3 are satisfied. That is, $-\mu_p(\tilde{K}) > \|BL\|_p + \|\tilde{D}G\|_p > 0$ holds. This shows that system (40) is globally exponentially stable. Figure 2 shows the trajectory of error system (40) with time when the initial values are $e_1(0) = (0.8, 1.8)^T$ and $e_2(0) = (1.5, 0.6, 2.4)^T$.

6. Conclusion

This paper proposes a sufficient condition for the global dissipative properties of GRN with external inputs. For the proof of dissipative properties, a large number of documents adopt the method of constructing Lyapunov function. However, this paper adopts the method of matrix
measurement, which simplifies the calculation difficulty, and the result is satisfactory. At the same time, error feedback control is given to ensure the stability of the system. And, the limitation of this kind of control is very small, which can bring great convenience in practical application. In addition, the study of the influence of Brown process and Levy process on gene regulatory networks is a challenging problem, which requires more mathematical knowledge about stochastic processes, but it is a very meaningful topic.

**Data Availability**

All data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest.

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