Cosmology from quantum potential in a system of oscillating branes

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Recently, some authors proposed a new mechanism which gets rid of the big-bang singularity and shows that the age of the universe is infinite. In this paper, we will confirm their results and predict that the universe may expand and contract many times in a system of oscillating branes. In this model, first, N fundamental strings transit to N M0-anti-M0-branes. Then, M0-branes join to each other and build a M8-anti-M8 system. This system is unstable, broken and two anti-M4-branes, a compactified M4-brane, an M3-brane in additional to one M0-brane are produced. The M3-brane wraps around the compactified M4-brane and both of them oscillate between two anti-M4-branes. Our universe is located on the M3-brane and interacts with other branes by exchanging the M0-brane and some scalars in transverse directions. By wrapping of M3-brane, the contraction epoch of universe starts and some higher order of derivatives of scalar fields in the relevant action of branes are produced which are the responsible of generating the generalized uncertainty principle or GUP. By oscillating the compactified M4-M3-brane and approaching to one of anti-M4-branes, one end of M3-brane glues to the anti-M4-brane and other end remains sticking and wrapping around M4-brane. Then, by getting away of the M4-M3 system, M4 rolls, wrapped M3 opens and expansion epoch of universe begins. By closing the M4 to anti-M4, the square mass of some scalars become negative and they make a transition to a tachyonic phase. To remove these states, M4 rebounds, rolls and M3 wraps around it again. At this stage, expansion branch ends and universe enters to a contraction epoch again. This process is repeated many times and universe expands and contracts as due to oscillating of branes. We obtain the scale factor of universe in this system and find that it’s values only at \( t \to -\infty \) shrinks to zero. Thus, in our method, the big bang is replaced by the fundamental string and the age of universe is predicted to be infinite. Also, when tachyonic states disappear at the beginning of expansion branch, some extra energy is produced and leads to an increase in the velocity of opening of M3. In these conditions, our universe which is located on this brane, expands very fast and experiences an inflation epoch. Finally, by reducing the fields in eleven dimensional M-theory to the fields in four dimensional universe, we show that our theory matches with quantum field theory prescriptions.

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I. INTRODUCTION

Removing different singularities from cosmological models is one of important problems in physics that many scientists are trying to solve it. Recently, it has been shown that replacing classical trajectories or geodesics by their quantal (Bohmian) trajectories leads to the quantum Raychaudhuri equation (QRE) which prevents the formation of singularities [1]. The second order Friedmann equations obtained from the QRE contains a couple of quantum correction terms, the first of which can be known as cosmological constant while the second removes the big-bang singularity and shows that the age of our universe is infinite [2]. Then, it has been declared that the same result may be derived in a brane-anti-brane system; however, the origin of universe is N fundamental strings [3]. In this method, first, these strings are excited and transit to N pair of D0-anti-D0-branes. These branes glue to each other and form a system of D5-anti-D5-branes. This system is unstable, broken and a pair of universe-anti-universe in additional to a wormhole is created. Two universes in this system interact with each other via the wormhole and build a Blon [4,7]. Thus, there isn’t any big bang singularity in this system and total age of universe is equal to sum over the age of fundamental string, initial system of D5-anti-D5 and present shape of universe. It is observed that only in the case of infinite age of fundamental string, the scale factor of universe becomes zero which means that the age of universe is infinite [3].

In parallel, some models in loop quantum cosmology predict that universe contract and expand infinite times and thus the age of universe may be infinite [8]. Now, the main question arises that how we can unify these two types of theory in M-theory? We answer this question in a system of oscillating branes. In our model, at the beginning, N M0-anti-M0-branes are produced from decaying of N-fundamental strings. Then, these branes glue to each other and an M8-anti-M8 system is formed. The branes in this system interact with each other, annihilate and two anti-M4-branes, an M4-brane, an M3-brane plus one M0-brane are created. The M4-brane is compactified around a circle.

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in eleven dimension and interacts with other branes by exchanging M0-branes and scalars in transverse dimensions. The M3-brane which our universe is located on it, wraps around the M4-brane, universe contracts and generalized uncertainty principle on it is emerged. The M4-M3 system oscillates between anti-M4-branes and becomes close to one of them. At this stage, the M3-brane connects to the anti-M4-brane from one end and remains sticking and wrapping around the M4-brane from another end. Then, the M4 rebounds, rolls, wrapped M3 opens and universe expands. Eventually, the M4 approaches the anti-M4 and some scalars become tachyons. To solve this problem, M4 moves away from this anti-M4-brane, rolls and M3 wraps around it again. In these conditions, universe evolves and make a transition from expansion phase to contraction era. These contractions and expansions continue until infinite and thus the age of universe may be infinite.

The outline of the paper is as follows. In section II, we will construct the contraction branch of cosmology in a system of oscillating branes and show that the origin of universe is a fundamental string. In section III, we will consider expansion era of cosmology during rolling of M4-brane in this system and estimate the age of universe. In section IV, we will show that by vanishing tachyonic states, some extra energies is produced which leads to the inflation. In section V, we will indicate that the model matches with quantum field theory prescriptions. The last section is devoted to summary and conclusion.

II. FIRST CONTRACTION BRANCH OF UNIVERSE IN A SYSTEM OF OSCILLATING BRANES

In this section, we will show that all evolutions of universe from the birth to the expansion era can be considered in a system of anti-M4-M4-M3 brane. In this model, the formation of universe in first contraction branch is via the process (fundamental string → M0 + anti-M0 → M8 + anti-M8 → 2M4 + compact anti-M4 + wrapped M3 + M0 → contraction branch of universe). Also, in our model, some scalars make a transition to a tachyonic phase and causes the contraction branch to be terminated.

To begin, we explain the model of [2] in short terms. In this mechanism, we estimate an explicit form for \( \dot{H} = F(H) \), where \( F(H) \) is a function of Hubble parameter \( H \) that is derived from quantum Raychaudhuri equation. Using this function, we can calculate the age of our world:

\[
\dot{H} = F(H) \rightarrow T = \frac{1}{F_n(H_{\text{initial}})} \int dH \frac{1}{(H - H_{\text{initial}})^n} \rightarrow \infty
\]

where \( H_{\text{initial}} \) is the hubble parameter before the present era of universe. This equation shows that the age of universe is infinite. We will show that the same results can be obtained in string theory. In our model, the universe is located on an M3-brane which wraps around a compact M4 from one end and attached to anti-M4 from another end. By oscillating and rolling M4, M3 oscillates between wrapping and opening states and consequently, universe oscillates between contraction and expansion branches. To show this, we use of the mechanism in [3]. In this paper, it has been shown that a fundamental string can decay to a pair of D0-anti-D0-branes or a pair of M0-anti-M0-branes in additional to some extra energy (V) \[3\]

\[
S_{\text{extra-string}} = S_{D0} + S_{\text{anti-D0}} = S_{M0} + S_{\text{anti-M0}} + 2V(\text{extra})
\]

where, the actions of D0-brane and M0-branes have been defined as \[3, 9, 17\]:

\[
S_{D0} = S_{\text{anti-D0}} = -T_{D0} \int dt \text{Tr}(\Sigma_{m=0}^{\alpha}[X^{m}, X^{n}]^2)
\]

\[
S_{M0} = S_{\text{anti-M0}} = T_{M0} \int dt \text{Tr}(\Sigma_{M,N,L=0}^{10}[X^{M}, X^{N}, X^{L}], [X^{M}, X^{N}, X^{L}])
\]

Here, \( T_{D0} \) and \( T_{M0} \) are the brane tensions, \( X^{m} \) are transverse scalars, \( X^{M} = X_{\alpha}^{M}T^{\alpha} \) and

\[
[T^{\alpha}, T^{\beta}, T^{\gamma}] = f^{\alpha\beta\gamma}_{\eta}T^{\eta}
\]

\[
\langle T^{\alpha}, T^{\beta} \rangle = h^{\alpha\beta}
\]

\[
[X^{M}, X^{N}, X^{L}] = [X_{\alpha}^{M}T^{\alpha}, X_{\beta}^{N}T^{\beta}, X_{\gamma}^{L}T^{\gamma}]
\]

\[
\langle X^{M}, X^{M} \rangle = X^{M}_{\alpha}X^{M}_{\beta}\langle T^{\alpha}, T^{\beta} \rangle
\]

(5)
As can be seen from above equation, the relevant action of D0-branes contains two dimensional Nambu-Poisson bracket while the action of M3 has three one with the Li-3-algebra \([14, 17]\). Also, the actions of D0 and M0 have the following relations \([3, 9–12]\):

\[
S_{M0} = S_{D0} + V_{Extra}
\]

where

\[
V_{Extra} = -6T_{M0} \int dt \Sigma_{g=0}^{g=0} \epsilon_{MNL} \epsilon_{EFG} X^M X^N X^L X^E X^F X^G
\]

Here \(T_{D0} = 6T_{M0}(\frac{g^2}{l_s^2}) = \frac{1}{g_s l_s}\) is the brane tension and \(g_s\) and \(l_s\) are the string coupling and string length respectively.

At this stage, we want to obtain the relevant action for Dp-brane by summing over the actions of D0-branes. To this end, we use of following mappings \([3, 9–12]\):

\[
\Sigma_p^{\alpha} \Sigma_m^{\alpha} = 0 \rightarrow \frac{1}{(2\pi)^p} \int d^{p+1} \sigma \Sigma_m^{p+1} \Sigma_a^{0} \lambda = 2\pi l_s^2
\]

\[
[X^a, X^i] = i \lambda \partial_a X^i \quad [X^a, X^b] = i \lambda^2 F_{ab} \quad i, j = p + 1, ..., 9 \quad a, b = 0, 1, ..., p \quad m, n = 0, 1, ..., 9
\]

Now, we can obtain the relevant action of Dp-brane \([3, 9–12]\):

\[
S_{Dp} = -\Sigma_p^{\alpha} \Sigma_m^{\alpha} = 0 \rightarrow \int dt Tr(\Sigma_{m=0}^{g=0} [X^m, X^n]^2) = \Sigma_p^{\alpha} S_{D0}
\]

\[
= -T_{Dp} \int d^{p+1} \sigma Tr(\Sigma_{a=0}^{g=0} \Sigma_{i,j=0}^{g=0} \{ \partial_a X^i \partial_b X^j - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \})
\]

Also, to derive the relevant action for Mp-branes, we sum over the action of M0-branes and use of following mappings \([3, 9–17]\):

\[
\langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle = \frac{1}{2} \epsilon^{a e b c d} (\partial_a X^i) (\partial_a X^j) (T^a, T^b) = \frac{1}{2} (\partial_a X^i, \partial_a X^i)
\]

\[
\langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle = \frac{1}{2} \epsilon^{a e b c d} (\partial_a X^i) (\partial_a X^j) (T^a, T^b) = \frac{1}{2} (\partial_a X^i, \partial_a X^i)
\]

\[
\Sigma_m \rightarrow \frac{1}{(2\pi)^p} \int d^{p+1} \sigma \Sigma_{m-p-1}^{g=0} \{ \partial_a X^i, \partial_a X^i \}
\]

\[
F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab}
\]

Using above relations, the action of Mp-brane can be obtained as \([3, 14, 17]\):

\[
S_{Mp} = \Sigma_{a=0}^{g=0} S_{M0} = -\Sigma_{a=0}^{g=0} T_{M0} \int dt Tr(\Sigma_{m=0}^{g=0} ([X^a, X^b, X^c], [X^a, X^b, X^c]) =
\]

\[
-\Sigma_{a=0}^{g=0} T_{M0} \int d^{p+1} \sigma Tr(\Sigma_{a,b,c=e=0}^{g=0} \Sigma_{i,j,k=p+1}^{g=0} \{ \partial_a X^i, \partial_a X^i \}) - \frac{1}{4} ([X^i, X^j, X^k], [X^i, X^j, X^k]) +
\]

\[
\frac{1}{6} (F_{abc}, F_{abc})
\]

Now, we can build our model in M-theory. For this, first, a pair of M8-anti-M8-branes are constructed from joining M0-branes. The, these objects decay to two anti-M4-branes, one M4-brane, one M3-brane in additional one M0-brane:
\[ S_{tot} = \sum_{a=0}^{5} S_{M0} + \sum_{a=0}^{8} S_{anti-M0} = S_{M8} + S_{anti-M8} = \]
\[ 2\sum_{a=0}^{4} S_{anti-M0} + \sum_{a=0}^{6} S_{M0} + \sum_{a=0}^{3} S_{anti-M0} + S_{M0} = \]
\[ 2S_{anti-M4} + S_{M4} + S_{M3} + S_{M0} \quad (11) \]

In our method, the M4-brane is compactified around a circle in eleven dimensions and M3 wraps around it. Then, the M4-M3 system moves toward one of anti-M4-branes. Approaching anti-M4, M3 sticks to the anti-M4 from one end and remains sticking to M4 from another end. By getting away the M4, it rolls and M3 opens. This process repeats many times. During wrapping and compactifications, some components of gauge field should be replaced by scalars and by opening, some scalars converts to gauge field \[3, 9, 17\]. For this reason, we write following relations for mappings which includes both of states:

\[ [X^a, X^b, X^c] = \alpha \partial_a X^b + \beta F_{abc} \]
\[ [X^a, X^i/b, X^j/c] = \alpha (X^i \partial_a X^j + X^j \partial_a X^i) + \beta F_{abc} \]
\[ [X^i, X^j, X^k] = \varepsilon^{\alpha \beta \gamma} X^i_{\beta} X^j_{\gamma} X^k_{\alpha} \quad (12) \]

where \(a, b, c\) are the indexes on the branes and \(i, j, k\) are indexes in transverse directions. By wrapping and compactification of branes, some of brane’s indexes like \(a, b\) or \(c\) are replaced by \(i, j\). Also, \(\alpha\) and \(\beta\) are functions of time which increase and decrease during wrapping and opening phases. When, M3 wraps around M4 completely, \(\alpha\) should be maximum and as M3 opens completely, \(\beta\) is maximum. For this reason, we suggest that \(\alpha = \sin \omega t\) and \(\beta = \cos \omega t\) where \(\omega\) is the frequency of oscillation \((\omega = \frac{2\pi}{T})\) and \(T\) is the time of period. Using equations \((11)\) and \((12)\), we obtain:

\[ S_{M3-M4} = -T_{D3} \int d^4 \sigma Tr \left( \frac{\varepsilon^{\alpha \beta \gamma}}{\varepsilon^{ab}} \right) \{ \partial_a X^i \partial_b X^j + \alpha^2 \partial_a \partial_b X^i X^j + \beta^2 F_{abc}^{2}\}
+ \alpha^2 \partial_a \partial_b \partial_c X^i X^j + \alpha \partial_a \partial_b X^i X^j \partial_c X^k + \alpha F_{abc} (X^i \partial_a X^j + X^j \partial_a X^i) + \alpha^2 \partial_a \partial_b X^i \partial_c X^k F_{abc}
+ \varepsilon^{\alpha \beta \gamma} \varepsilon^{\alpha' \beta' \gamma'} X^i_{\alpha} X^j_{\beta} X^k_{\gamma} X^l_{\alpha'} X^m_{\beta'} X^n_{\gamma'} \} = S_{M3} + V_{int} \quad (13) \]

where \(V_{int} = -T_{D3} \int d^4 \sigma Tr \left( \frac{\varepsilon^{\alpha \beta \gamma}}{\varepsilon^{ab}} \right) \{ \alpha^3 F_{abc} + \alpha^2 \beta \partial_a \partial_b X^i \partial_c X^k F_{abc} \} \) is the interaction potential between M3 and M4. Using the above equation, we can derive the equation of motion for \(X^i\):

\[ \{ \partial_a^2 + (\alpha^2 + 2 \alpha \beta A_a) \partial_a^2 + \alpha^2 \beta A_a X^i \partial_b^2 + \alpha^2 X^i \partial_c X^k \partial_a^2 + \alpha A_a \partial_a^3 + \frac{\partial^2 V}{\partial x^2} \} X^i = 0 \]
\[ V = \varepsilon^{\alpha \beta \gamma} \varepsilon^{\alpha' \beta' \gamma'} X^i_{\alpha} X^j_{\beta} X^k_{\gamma} X^l_{\alpha'} X^m_{\beta'} X^n_{\gamma'} \quad (14) \]

At this stage, we substitute \(\partial_a = p_a\) and \(\frac{\partial^2 V}{\partial x^2} = m^2\) in above equation and obtain:

\[ \{ p_a^2 + (\alpha^2 + 2 \alpha \beta A_a) p_a^4 + \alpha^2 \beta A_a X^i p_a^2 + \alpha^2 X^i X^k X^l p_a^2 - \alpha A_a p_a^3 + m^2 \} X^i = 0 \quad (15) \]

If we compare above equations with usual equation for scalar fields,

\[ \{ \bar{p}_a^2 + m^2 \} X^i = 0 \quad (16) \]

we can define the momentum \(\bar{p}\) as:

\[ \bar{p}_a \sim (1 + \alpha^2 X^i X^k X^l)^{1/2} \bar{p}_a - (\alpha^2 + 2 \alpha \beta A_a)^{1/2} \bar{p}_a + \alpha^2 \beta A_a X^j p_a^3 \quad (17) \]

Thus our model produces the commutation relations in GUP:

\[ \{ x_a, \bar{p}_b \} = (1 + \alpha^2 X^i X^k X^l)^{1/2} \delta_{ab} - (\alpha^2 + 4 \alpha \beta A_a)^{1/2} \bar{p}_a + 3 \alpha^2 \beta A_a X^j p_a^2 \quad (18) \]
This is the GUP proposed in [18–22] which predicts maximum observable momenta besides the existence of minimal measurable length and is consistent with Doubly Special Relativity (DSR) theories, String Theory and Black Holes Physics. Thus, wrapping M3 around M4 leads to the non-commutative relations in GUP. Equation [18] gives the following uncertainty with using the argument used in [23–25]:

\[ \Delta x \Delta p \geq \frac{1}{2} \left[ (1 + \alpha^2 X^j X^k X^l)^{1/2} - (\alpha^2 + 4\alpha \beta A_a)^{1/2} p_a + 3\alpha^2 \beta A_a X^j X^k X^l \right] \]  

(19)

The solution of the above inequality as quadratic equation in \( \Delta p \) is [23–25]:

\[ \Delta p \geq \frac{2\Delta x + (\alpha^2 + 4\alpha \beta A_a)^{1/2}}{3\alpha^2 \beta A_a X^j} \left[ 1 - \sqrt{1 - \frac{6\alpha^2 \beta A_a X^j}{(2\Delta x + (\alpha^2 + 4\alpha \beta A_a)^{1/2})^2}} \right] \]  

(20)

Now, we assume that wrapped M3-brane could be modelled as (D - 1)-dimensional sphere of size equal to twice of Schwarzschild radius, \( r_s \). Thus the uncertainty in position of particle has its minimum value given by [23–25]:

\[ \Delta x = 2r_s = 2\lambda_D \left( \frac{G_D m}{c^2} \right)^{\frac{1}{D-2}} \]  

(21)

where \( \lambda_D = \left( \frac{16\pi}{(D-2)\Omega_D \pi} \right)^{\frac{1}{D-2}} \) and \( \Omega_D = \frac{2\pi D-1}{(2\pi)^{D-1}} \). We substitute the position defined by (21) in equation (20) and derive \( \Delta p \) as :

\[ \Delta p \geq \frac{4\lambda_D \left[ \frac{G_D m}{c^2} \right]^{\frac{1}{D-2}} + (4\alpha \beta A_a)^{1/2}}{3\alpha^2 \beta A_a X^j} \left[ 1 - \sqrt{1 - \frac{6\alpha^2 \beta A_a X^j}{(4\lambda_D \left[ \frac{G_D m}{c^2} \right]^{\frac{1}{D-2}} + (\alpha^2 + 4\alpha \beta A_a)^{1/2})^2}} \right] \]  

(22)

Applying definition of mass in equation [14], we calculate the explicit form of mass:

\[ m^2 = \frac{\partial^2 V}{\partial x^2} = \epsilon^{\alpha \beta \gamma} \epsilon^{\alpha' \beta' \gamma'} \frac{\partial^2 (X^i_{\alpha \beta} X^j_{\gamma \lambda} X^k_{\alpha' \beta'} X^l_{\gamma' \lambda'})}{\partial x^2} \]  

(23)

Substituting equation (23) in equation (22), we get:

\[ \Delta p \geq \frac{4\lambda_D \left[ \frac{G_D m}{c^2} \right]^{\frac{1}{D-2}} + (4\alpha \beta A_a)^{1/2}}{3\alpha^2 \beta A_a X^j} \left[ 1 - \sqrt{1 - \frac{6\alpha^2 \beta A_a X^j}{(4\lambda_D \left[ \frac{G_D m}{c^2} \right]^{\frac{1}{D-2}} + (\alpha^2 + 4\alpha \beta A_a)^{1/2})^2}} \right] \]  

(24)

This equation yields the following inequality for the scalars in transverse direction:

\[ A_a \leq \left[ \frac{4\lambda_D \left[ \frac{G_D m}{c^2} \right]^{\frac{1}{D-2}} + (4\alpha \beta A_a)^{1/2}}{6\alpha^2 \beta X^j} \right] \]  

(25)

As can be seen from the above inequality, as the M3 brane approaches the anti-M4-brane at \( t = \frac{T}{4} \), scalars on the M3 grow, the right hand of this equality becomes smaller than left hand and inequality violates. To avoid this violation and negative values under the square root, the square mass of some scalars becomes negative (\( m^2 \rightarrow -m^2 \)), they transit to tachyonic phases and contraction branch ends.
III. ESTIMATING THE AGE OF UNIVERSE IN A SYSTEM OF OSCILLATING BRANES

In previous section, we observed that as the M3-M4 becomes close to anti-M4, M3 attaches to it from one end and stays sticking from another end and the system faces some tachyonic states. To remove these states, M4 rebounds, rolls, M3 opens and expansion phase begins. During this new phase, gauge fields ($A_\mu$) on the brane grow, the left hand of equation (25) becomes bigger than right hand and inequality violates again. To avoid the negative values under the the $\sqrt{\rho}$, the square mass of some scalars becomes negative ($m^2 \to -m^2$) and they become tachyons. To solve this problem, M4 rebounds again, M3 wraps around it and contraction epoch begins. Now, the question arises that what is the age of universe? To answer this question, we should calculate the contribution of branes on four dimensional universe and write energy-momentum tensors. Using the action in [11], we can calculate the energy-momentum of M3 and put it equal to energy-momentum of universe:

$$\rho = \frac{3H^2}{k^2} = \frac{1}{2}(1 + \alpha^2 X^i)^2 + 2\alpha F_{abc} X^i (\dot{X}^i)^2 + \alpha^2 \partial_\alpha \partial_\beta X^i \partial_\alpha \partial_\beta X^i$$

$$+ \alpha \beta \partial_\alpha \partial_\beta X^i F_{abc} + \alpha^2 \beta \partial_\alpha \partial_\beta X^i \partial_\alpha \partial_\beta X^i F_{abc} + \beta^2 F_{abc}^2 + \beta^3 F_{abc}^3 + \varepsilon^{\alpha \beta \gamma \epsilon \gamma' \gamma''} X^i_\alpha X^j_\beta X^k_\gamma X^i\epsilon X^j\beta X^k\gamma$$

$$p = -\frac{1}{k^2}(3H^2 + 2\dot{H}) = \frac{1}{2}(1 + \alpha^2 X^i)^2 + 2\alpha F_{abc} X^i (\dot{X}^i)^2$$

$$- \alpha^2 \partial_\alpha \partial_\beta X^i \partial_\alpha \partial_\beta X^i - \alpha \beta \partial_\alpha \partial_\beta X^i F_{abc} - \alpha^2 \beta \partial_\alpha \partial_\beta X^i \partial_\alpha \partial_\beta X^i F_{abc}$$

$$- \beta^2 F_{abc} - \beta^3 F_{abc}^3 - \varepsilon^{\alpha \beta \gamma \epsilon \gamma' \gamma''} X^i_\alpha X^j_\beta X^k_\gamma X^i\epsilon X^j\beta X^k\gamma$$

Solving equations [15,16,25] and [26] simultaneously, we obtain the explicit form of $X^i, A^i$ and $a(t)$:

$$X^i \sim \sin(\dot{\omega} t) \quad \omega = \sqrt{\omega^2 + (\alpha^2 + 2\alpha) \omega^4 + \alpha^2 \beta \omega^6 + \alpha^5 \omega^2 + \omega^3 + m^2}$$

$$A^i \sim \left[ \sqrt{G_D} [\omega^2 + (\alpha^2 + 2\alpha) \omega^4 + \alpha^2 \beta \omega^6 + \alpha^5 \omega^2 + \omega^3 + m^2] [30 \sin^4(\dot{\omega} t) \cos^2(\dot{\omega} t) - 6 \sin^6(\dot{\omega} t)] + \sin \omega t \right]$$

$$\frac{1}{6\alpha^2 \beta \sin(\dot{\omega} t)}$$

$$a(t) \sim e^{\omega t} + f \int dt G(t)$$

$$G(t) \sim \left[ \sqrt{G_D} [\omega^2 + (\alpha^2 + 2\alpha) \omega^4 + \alpha^2 \beta \omega^6 + \alpha^5 \omega^2 + \omega^3 + m^2] \sqrt{\omega^2 + (\alpha^2 + 2\alpha) \omega^4 + \alpha^2 \beta \omega^6 + \alpha^5 \omega^2 + \omega^3 + m^2} \right]$$

$$3\alpha \beta \sqrt{[30 \sin^4(\dot{\omega} t) \cos^2(\dot{\omega} t) - 6 \sin^6(\dot{\omega} t)]} \times$$

$$[12 \sin^3(\dot{\omega} t) \cos^3(\dot{\omega} t) \cos(\dot{\omega} t) - 3 \cos(\dot{\omega} t) \sin^5(\dot{\omega} t)] + \alpha^2 \sin^2(\dot{\omega} t) \sin^2(\dot{\omega} t) + \cos(2\dot{\omega} t)$$

$$3 \alpha^2 \beta \sin^3(\dot{\omega} t)$$

As can be seen from these equations, during contraction branch ($0 < t < \frac{T}{2}$), the scalar fields ($X^i$) grow; while the gauge fields ($A^i$) decrease. However, by passing time and opening M3, universe enters into expansion phase ($\frac{T}{2} < t < \frac{T}{2}$), gauge fields grow and scalars decreases. Also, this equation shows that only in the case that time be infinite ($t \to -\infty$), the scale factor becomes zero. This means that the age of universe is infinite and thus our result is consistent with results of [2].

IV. CONSIDERING THE INFLATION ERA AT THE BEGINNING OF EXPANSION BRANCH

Until now, we have shown that by wrapping and opening M3, universe contracts and expands. Also, we have replace the big bang singularity by a fundamental string and indicate that the age of universe is infinite. Now, another question arises that how our universe undergoes an inflation phase at the beginning of expansion branch? To reply this question, we remind that at the end of contraction, some scalars gain negative square mass and transit to tachyons. To remove these states, contraction stops and universe enters to an expansion epoch. Also, the negative square mass of scalars ($-m^2 = -\frac{\partial^2 V}{\partial \phi^2}$) at the end of contraction should be converted to the positive square mass ($m^2 = \frac{\partial^2 V}{\partial \phi^2}$) at the beginning of expansion. Thus, the energy of system changes and some energy is released. Using equation [14], we can get:
In the dimensional universe, our model matches with known models in gravity. To this end, we use the following relations:

\[ m^2 - (-m^2) = 2m^2 \quad V = V_{\text{end of contraction}} = V_{\text{beginning of expansion}} \rightarrow \]

\[ \frac{\partial^2 V_{\text{beginning of expansion}}}{\partial x^2} - (-\frac{\partial^2 V_{\text{end of contraction}}}{\partial x^2}) = 2\frac{\partial^2 V}{\partial x^2} \rightarrow \]

\[ V_{\text{inf}} = V_{\text{beginning of expansion}} - V_{\text{end of contraction}} = 2V = 2\varepsilon^{a\beta\gamma} \varepsilon^{a'\beta'\gamma'} X_a^i X_{\beta'}^j X_{\gamma'}^k X_i^j X_k^i \quad (28) \]

This energy causes that the velocity of opening of M3 increases and our universe which is located on this brane, experience an inflation phase. Using equations (27) and (28) and assuming that inflation starts at the beginning of expansion \((t = \frac{1}{4})\), we can obtain the Hubble parameter:

\[ \rho_{\text{total}} = \rho + \rho_{\text{inf}} \]

\[ \rho_{\text{inf}} = \frac{3H_{\text{inf}}^2}{\kappa^2} = V_{\text{inf}} = 2V = 2\sin^6(\omega t) \rightarrow \]

\[ H_{\text{inf}} = \sqrt{\frac{2}{3}} \kappa \sin^3(\omega t) \]

\[ t = \frac{T}{4} + t_{\text{inf}} \rightarrow H_{\text{inf}} = \sqrt{\frac{2}{3}} \kappa \cos^3(\omega t_{\text{inf}}) \]

\[ H_{\text{tot}}^2 = H_{\text{inf}}^2 + H_{\text{inf}}^2 \quad H = G + \omega \]

\[ H_{\text{tot}}^2 = (G + \omega)^2 + \frac{2\kappa}{3} \cos^6(\omega t_{\text{inf}}) \quad (29) \]

We can test our model by calculating the magnitude of the slow-roll parameters and the tensor-to-scalar ratio \(r\) defined in [26] and comparing with previous predictions:

\[ \varepsilon = -\frac{\dot{H}_{\text{tot}}}{H_{\text{tot}}^2} = \frac{2\dot{G}(G + \omega) + \frac{12\kappa}{3} \omega \cos^5(\omega t_{\text{inf}}) \sin(\omega t_{\text{inf}})}{\left[(G + \omega)^2 + \frac{2\kappa}{3} \cos^6(\omega t_{\text{inf}})\right]^{3/2}} \]

\[ \eta = -\frac{\ddot{H}_{\text{tot}}}{2H_{\text{tot}} \dot{H}_{\text{tot}}} = \frac{\frac{12\kappa}{3} \omega \cos^5(\omega t_{\text{inf}}) \sin(\omega t_{\text{inf}})}{\left[(G + \omega)^2 + \frac{2\kappa}{3} \cos^6(\omega t_{\text{inf}})\right]^{3/2}} \times \frac{1}{2\dot{G}(G + \omega) + \frac{12\kappa}{3} \omega \cos^5(\omega t_{\text{inf}}) \sin(\omega t_{\text{inf}})} \]

\[ t_{\text{inf}} \ll T \quad \text{and} \quad \omega \sim \frac{1}{T^3} \rightarrow \omega t_{\text{inf}} \ll 1 \rightarrow \]

\[ \sin(\omega t_{\text{inf}}) \ll \cos(\omega t_{\text{inf}}) \quad \text{and} \quad \sin(\omega t_{\text{inf}}) \sim \frac{t_{\text{inf}}}{T} \sim 0 \quad \text{and} \quad \cos(\omega t_{\text{inf}}) \sim 1 \Rightarrow \]

\[ \varepsilon = \frac{1}{G^2} \ll 1 \quad \eta \sim \frac{1}{G} \ll 1 \Rightarrow \quad r = 16\varepsilon \sim \frac{16}{G^2} \ll 1 \quad (30) \]

This equation shows that slow parameters are very small and thus our model confirms the prediction of previous models for inflation era in [26]. Another interesting result that comes out from this equation is the value of the tensor-to-scalar ratio \(r\) which is very smaller than one and is in agreement with experimental data [27]. Thus, the extra energy which is produced during vanishing tachyons, leads to an increase in velocity of expansion and occurring inflation.

\[ \text{V. REDUCING THE MODEL TO QUANTUM FIELD THEORY PRESCRIPTIONS IN FOUR DIMENSIONAL UNIVERSE} \]

In this section, we will show that by reducing field theory in eleven dimensional M-theory to field theory in four dimensional universe, our model matches with known models in gravity. To this end, we use of following relations:

\[ \int d^4\sigma F_{abc}^2 = \int d^4\sigma (\partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab})(F_{abc}) = \]
\[ - \int d^4 \sigma (A_{bc} \partial_a F_{abc} - A_{ca} \partial_b F_{abc} + A_{ab} \partial_c F_{abc}) + \int d^4 \sigma \partial_a (O) \]

\[ \int d^4 \sigma F_{abc}^3 = \int d^4 \sigma (\partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab})(F_{abc}^2) = \]

\[ - \int d^4 \sigma (A_{bc} \partial_a F_{abc} - A_{ca} \partial_b F_{abc} + A_{ab} \partial_c F_{abc}) F_{abc} + \int d^4 \partial_a (O) \]

\[ \int d^4 \sigma A_{bc} F_{abc} \partial_a F_{abc} = - \int d^4 \sigma (\partial_d A_{bc} - \partial_c A_b) F_{abc} \partial_a F_{abc} = - \int d^4 \sigma (A_c \partial_b F_{abc} - A_b \partial_c F_{abc}) \partial_a F_{abc} \]

\[ \int d^4 \sigma \partial_a \partial_b X^i F_{abc} = - \int d^4 \sigma \partial_a X^i \partial_a F_{abc} + \int d^4 \sigma \partial_a O \]  

(31)

Substituting above relations in action [13], we obtain:

\[ S_{M3-M4} = -T_{D3} \int d^4 \sigma Tr(\Sigma_{a,b=0}^3 \Sigma_{i,j=4}^{\Sigma_i} \{ \partial_d X^i \partial_b X^j + \alpha^2 \partial_a \partial_b X^i \partial_a \partial_b X^i - \beta^2 A_{ab} \partial_a F_{abc} + \beta^3 A_a \partial_b F_{abc} \partial_c F_{abc} \}

- \alpha \beta \partial_a X^i \partial_b F_{abc} + \alpha^2 (X^i X^j \partial_b X^j + \partial_a F_{abc} + \alpha^2 \beta \partial_a \partial_b X^i \partial_b X^i \partial_a F_{abc}

+ \epsilon^{\alpha \beta \gamma} X^i X^j X^k \partial_a X_{\beta \gamma} \partial_b X_{\gamma} - \beta^3 A_{ab} \partial_a \partial_c F_{abc} - \alpha^2 \beta \partial_a X^i \partial_b X^i \partial_a \partial_b F_{abc} \}) \]  

(32)

At this stage, we can show that this action matches to action in four dimensional field theory by using following mappings:

\[ X^i \rightarrow \phi \quad A_{ab} \rightarrow h_{ab} \quad A_a \rightarrow e_a \]

\[ h_{ab} = \sqrt{-g} h^{ab} \quad g_{ab} = \eta_{ab} + h_{ab} \]

\[ F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab} \rightarrow \]

\[ F_{abc} = \partial_d h_{bc} - \partial_d h_{ca} + \partial_c h_{ab} \rightarrow \]

\[ \partial_a F_{abc} = \partial_a h_{bc} - \partial_a h_{ca} + \partial_c h_{ab} \rightarrow \]

\[ g^{bc} F_{abc} = \partial_a h^b_c + \ldots \rightarrow \sqrt{-g} R \]  

(33)

where \( \phi \) is the scalar field, \( h_{ab} \) is the graviton field and \( g_{ab} \) is the component of metric. Replacing strings and three form fields by scalars and elements of metric in four dimensions, we get:

\[ S_{\text{field theory}} = -T_{D3} \int d^4 \sigma Tr(\Sigma_{a,b=0}^3 \Sigma_{i,j=4}^{\Sigma_i} \{ \partial_d \phi \partial_b \phi + \alpha^2 \partial_a \partial_b \phi \partial_a \partial_b \phi - \beta^2 \sqrt{-g} R + \beta^3 e_a \sqrt{-g} R^2 \]

\[ - \alpha \beta \sqrt{-g} \partial_a \phi R + \alpha^2 \beta \partial_a \phi \partial_b \phi + \phi^2 \sqrt{-g} R + \alpha^2 \beta \partial_a \phi \partial_b \phi \sqrt{-g} R - \beta^3 e_a \sqrt{-g} \partial_a \partial_b \phi \partial_b R + \phi^6 - \alpha^2 \beta \sqrt{-g} \partial_a \phi \partial_b \phi \partial_b \phi \}) \]  

(34)

Now, we can rewrite the above action as follows:

\[ S_{\text{field theory}} = -T_{D3} \int d^4 \sigma \{ \sqrt{-g} F(R, \phi) + \partial_a \phi \partial_b \phi + V(\phi) \} \]  

(35)

where

\[ F(R, \phi) = (-\beta^2 - \alpha \beta \partial_a \phi + \alpha^2 \beta \partial_a \phi \partial_b \phi) R + \beta^3 e_a R^2 + \alpha^2 \beta \partial_a \phi \partial_b \phi \sqrt{-g} R - \beta^3 e_a \sqrt{-g} \partial_a \partial_b \phi \partial_b R + \phi^6 - \alpha^2 \beta \sqrt{-g} \partial_a \phi \partial_b \phi \partial_b \phi \]  

(36)

This equation is very the same of actions in \( F(R) \) gravity which has been discussed in [28]. This means that by redefining the quantum fields in M-theory and obtaining their relations by fields in four dimensional universe, the action of model matches the relevant action in quantum field theory prescription.
VI. SUMMARY AND DISCUSSION

In this research, we have reconsidered the results of [2] in a system of oscillating brane. We have discussed that the universe contracts and expands as due to interaction between branes. In our model, first, N fundamental string transit to N pairs of M0-anti-M0-brane. Then, these branes glue to each other and build a pair of M8-anti-M8-branes. This system is unstable, broken and two anti-M4-branes, an M4-brane, an M3-brane and an M0-brane are produced. M4-branes is compactified around a circle and M3 which our universe is located on it; wraps around it. The system of M4-M3 is located between anti-M4-branes and oscillate. As this system becomes close to one of anti-M4 branes, the M3 attaches to it from one end and stay sticking to another from another end. Also, the square mass of some scalars becomes negative and they make a transition to tachyonic states. To remove these states, M4 rebounds, rolls, M3 opens and expansion branch of universe begins. When M4 approaches to another anti-M4-brane, some other scalars gain negative square mass and new phase of tachyon is created. To solve this problem, M4 rebounds again, M3 wraps around it and new contraction branch starts. We compare thee energy-momentum tensor derived in this model with the energy-momentum tensor in our present stage of universe and obtain the scale factor. We notice that this scale factor, only in the case of $t \rightarrow -\infty$ becomes zero. This means that the age of universe may be infinite which is consistent with prediction of [2]. Also, we show that by disappearing tachyonic states, some energy is produced which leads to an acceleration in openning of M3 and expansion of universe. Finally, by reducing the quantum fields in eleven dimensional M-theory to ones in four dimensional universe, we observe that our model is consistent with usual field theory.

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