The Upper Limit of Magnetic Field Strength in Dense Stellar Hadronic Matter

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It is shown that in strongly magnetized neutron stars, there exist upper limits of magnetic field strength, beyond which the self energies for both neutron and proton components of neutron star matter become complex in nature. As a consequence they decay within the strong interaction time scale. However, in the ultra-strong magnetic field case, when the zeroth Landau level is only occupied by protons, the system again becomes stable against strong decay.

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The study of the effect of strong quantizing magnetic fields on dense neutron star matter has become extremely interesting and also important after the recent discovery of a few strongly magnetized neutron stars, known as magnetars [1,2,3,4]. These strange stellar objects are supposed to be relatively young (age \( \sim 10^4 \) yrs.) neutron stars and are also believed to be the possible sources of anomalous X-ray pulses and soft gamma ray emissions (AXP and SGR). If the magnetic fields are strong enough, particularly at the central region, then most of the physical properties of these strange stellar objects and also various physical processes taking place at the core region should change significantly. The strong magnetic field at the core region affects the equation of state of dense neutron star matter [13]. It is seen that in the case of a compact neutron star, the possibility of phase transition to quark matter at the core region is totally forbidden if the magnetic field strength exceeds \( 10^{15} \) G [7]. The elementary processes, e.g. weak and electromagnetic reactions and decay processes are found to be strongly influenced by such intense magnetic fields. As the cooling of neutron stars are mainly due to the emission of neutrinos through weak processes (URCA or modified URCA), the presence of strong quantizing magnetic field should significantly affect the thermal evolution of neutron stars [7]. The presence of strong magnetic field affects chiral properties of both QED and QCD matter and the corresponding vacuum, in low as well as in \( 3 + 1 \) dimensions. The strong quantizing magnetic field acts like a catalyst to generate the mass dynamically [8,9,10,11,12].

Motivated by a recent study of Shabad and Usav [13], in the present article we shall investigate the possibility of upper limit of magnetic field strength in neutron star matter beyond which the hadronic matter becomes unstable. We shall follow some of our recent studies where we have developed a relativistic formalism of Landau theory of Fermi liquid for dense neutron star matter in presence of strong quantizing magnetic field with the exchange of \( \sigma - \omega - \rho \) mesons [14,15]. We have noticed that the densities of both proton and neutron components, which are in \( \beta \)-equilibrium are functions of magnetic field strength. This is a consequence of variation of weak interaction rates with magnetic field. In this article we shall follow the formalism developed recently in ref. [13] to obtain the complex self-energies for both neutron and proton of dense neutron star matter. The detail calculations are available in the last reference.

In this study, we have considered the elementary processes as shown in fig.(1). In this figure, we are particularly interested in the exchange diagram of \( n-p \) scattering processes with the transfer of \( \sigma-\omega-\rho \) mesons and direct diagram with \( \rho \) meson exchange. The self-energies of neutron and proton components become complex in nature because of these two diagrams. Now the underlying technique to obtain complex self-energies is to evaluate the Landau-Fermi liquid interaction function (also known as the quasi-particle interaction function) from two particle forward scattering matrix for these two diagrams. The self-energies for proton and neutron components are then obtained by evaluating the momentum integrals of respective quasi-particle interaction functions (here we have assumed that the temperature \( T = 0 \) for neutron star matter and therefore the upper limits of both proton and neutron momentum integrals are the corresponding Fermi momentum). Now it is easy to see from reference [13] that in the evaluation of proton and neutron self energies we needed the following traces of projection operators:

1. For \( \sigma \)-meson exchange case, we need \( \text{Tr}[\Lambda_{pn}(x,p)\Lambda_{pn}(x',p')] \). Which is found to be complex in nature.
2. In the case of \( \omega \)-meson exchange, it is necessary to evaluate the trace \( \text{Tr}[\Lambda_{pn}(x,p)\gamma^\mu\Lambda_{pn}(x',p')\gamma_\mu] \). It is seen that only \( \mu = 0 \) and \( z \) give non-zero contributions and both of them are complex in nature.
3. In the case of neutral \( \rho \)-meson exchange, the contribution is exactly same as that of \( \omega \)-meson case, except a factor of 1/2.
4. Finally in the case of charged \( \rho \) meson exchange (\( n-p \) direct scattering diagram) we have to evaluate \( \text{Tr}[\Lambda_{pn}(x,p)\gamma^\mu]\text{Tr}[\Lambda_{pn}(x',p')\gamma_\mu] \). In this case also only for \( \mu = 0 \) and \( z \), the traces are non-zero and are found to be again complex in nature.

The projection operators are given by

\[
\Lambda_{np}(x,p) = u^+_n \bar{u}^+_p + u^+_n \bar{u}^+_p \tag{1}
\]
and

$$\Lambda_{pn}(x, p) = u_p^\dagger \bar{u}_n^\top + u_p^\dagger \bar{u}_n^\top$$

(2)

with

$$u^\dagger = \frac{1}{[E_\nu(E_\nu + m)]^{1/2}} \begin{pmatrix}
    (E_\nu + m)I_{\nu;p_p}(x) \\
    0 \\
    p_z I_{\nu;p_p}(x) \\
    -i(2\nu q B_m)^{1/2} I_{\nu-1;p_p}(x)
\end{pmatrix}$$

(3)

and

$$u^\dagger = \frac{1}{[E_\nu(E_\nu + m)]^{1/2}} \begin{pmatrix}
    0 \\
    (E_\nu + m)I_{\nu-1;p_p}(x) \\
    (E_\nu + m)I_{\nu-1;p_p}(x) \\
    -p_z I_{\nu-1;p_p}(x)
\end{pmatrix}$$

(4)

are the Dirac spinors, with the symbols $\uparrow$ and $\downarrow$ for up and down spin states respectively and

$$I_\nu = \left(\frac{q B_m}{\pi}\right)^{1/4} \frac{1}{(\nu!)^{1/2}} 2^{-\nu/2} \exp \left[ -\frac{1}{2} q B_m \left( x - \frac{p_y}{q B_m} \right)^2 \right]$$

$$H_\nu \left( (q B_m)^{1/2} \left( x - \frac{p_y}{q B_m} \right) \right)$$

(5)

with $H_\nu$ the well-known Hermite polynomial of order $\nu$ and $L_y$, $L_z$ are length scales along $y$ and $z$ directions respectively. Here $\nu$ is the Landau quantum number proton proton and can have any integer value including zero. For neutron as neutral particle, we have used the standard Dirac spinor solutions. As we have found in ref. [15] that with these spinor solutions, the traces as mentioned before become complex in nature for the diagrams we have considered.

Here we have chosen the gauge $A^y = (0, 0, x B_m, 0)$, so that the constant magnetic field $B_m$ is along $z$-axis. Now the Landau levels for the protons will be populated if the magnetic field strength $B_m$ exceeds the quantum critical value $B^{(c)(p)}_m = m_p^2/q_p \approx 1.6 \times 10^{20}$ G, where $m_p$ and $q_p$ are proton mass and charge respectively. In the relativistic region the quantum critical value is the typical strength of the magnetic field at which the proton cyclotron quantum exceeds the corresponding rest mass energy or equivalently the de Broglie wavelength for proton exceeds the corresponding Larmor radius. Now for $B_m > B^{(c)(p)}$, with the relation $p_F^2 \geq 0$, the maximum value of Landau quantum number at $T = 0$ is given by

$$[\nu_{\text{max}}^{(p)}] = \frac{\left( \mu_p^2 - m_p^2 \right)}{2q_p B_m}$$

(6)

which is an integer but less than the actual value of right hand side and $\mu_p$ is the proton chemical potential. The external magnetic field will therefore behave like a classical entity if the strength is less than the quantum threshold value and in this region one has to use the plane wave solution with standard form of Dirac spinors for both proton and neutron components. Then it is possible to obtain the equation of state of dense neutron star matter in a straightforward manner following reference [16]. Further, it is obvious from eqn.(6) that for a given proton density, the upper limit for $\nu$ can become zero for some large magnetic field strength. In that case only the zeroth Landau level will be occupied by protons. For such strong magnetic field since only allowed value of Landau quantum number for proton is $\nu = 0$, then using the relation that $H_{-n} = 0$ for integer $n > 0$, the spin up component (eqn.(3)) reduces to

$$u^\dagger = \frac{1}{[E_0(E_0 + m)]^{1/2}} \begin{pmatrix}
    (E_0 + m)I_{0;p_p}(x) \\
    0 \\
    p_z I_{0;p_p}(x) \\
    0
\end{pmatrix}$$

(7)

whereas the down spin component (eqn.(4)) becomes a null column matrix. It’s physical meaning is that for such a strong magnetic field all the protons will come down to the zeroth Landau level with the spins aligned along the direction of magnetic field. The crucial point which should be noted here is that in this case if one evaluates the traces as mentioned before they are no longer be complex in nature, they will be absolutely real quantities, which actually means that the self-energies will be real for both proton and neutron components. To explain the point a
little more detail, we have plotted in fig.(2) the density of proton as a function of quantum critical limit for magnetic field strength. Since the neutron star matter is in $\beta$-equilibrium and rates of the weak interaction processes (URCA and modified URCA) are strongly dependent on the strength of magnetic field, the equilibrium density of proton will be a function of magnetic field strength. Which means that the proton or neutron density can not be arbitrary for $B_m > B_m^{(\nu)}(p)$.

Let us now consider fig.(2) and try to understand different regions as indicated in the diagram. At the bottom, since $B_m < B_m^{(\nu)}(p)$, it is the classical region. The Landau levels for protons are not populated in this region, they are represented by plane wave solutions with standard Dirac spinors. As a result the dense neutron star matter will be stable in this region. This region is marked by stable classical region.

Next we consider the other stable region at the top of this figure. In this region the strength of magnetic field is such that the maximum value of Landau quantum number of protons is zero. The Dirac spinor for proton is given by eqn.(7), which makes the self energies absolutely real. As a consequence the matter is again stable against strong decay. Since the quantum mechanical effect of strong magnetic field is very much important in this region, we call it as stable quantum region.

We now consider the region which in our opinion is the most important part of this figure and the aim of this article is to show that for dense stellar hadronic matter such a region in density-magnetic field space actually exists in magnetized neutron stars. Within this region the spinor solutions for proton are given by eqns.(3) and (4) and as a consequence the self energies of both proton and neutron components are complex in nature. Which makes the system in this region unstable. Both proton and neutron decay within the strong interaction time scale as shown in fig.(3).

This region started from a point with $B_m = B_m^{(\nu)}(p)$ and the proton density at this point is the lowest allowed value inside a magnetized neutron star with quantum critical strength of magnetic field. Since the energy of the system decreases as more and more protons occupy the low lying Landau levels, the density of proton increases with the increase of magnetic field. It occurs within the neutron star by the conversion of more and more neutrons to protons through weak processes in presence of strong magnetic field.

Finally, the region at the top right corner is forbidden both classically and quantum mechanically. Since in this region the magnetic field strength is greater than the quantum critical value, this part can not be treated as classical. Whereas, quantum mechanically the density of protons in $\beta$-equilibrium can not be so low. Therefore the hadronic matter within strongly magnetized neutron stars can not occupy this region, We have marked it by forbidden region.

The horizontal line, separating the stable classical zone and the unstable region is for $\nu = \nu_{max}$, whereas within the unstable region, $\nu_{max}$ can have all possible integer values beginning with unity.

Hence we finally conclude that in a strongly magnetized neutron star, if the magnetic field strength is $> 1.6 \times 10^{20}\text{G}$, the quantum critical value for protons, the matter can not be stable and therefore this is the upper limit for a neutron star magnetic field with stable hadronic matter.

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FIG. 1: Magnetic field (expressed as $B_m/B_m^{(c)}$) plotted against the density of proton matter (in fm$^{-1}$).

FIG. 2: Variation of proton and neutron life times (in sec. in unit of $10^{22}$) with proton density (expressed in terms of normal nuclear density $n_0 = 0.17 \text{fm}^{-3}$). The upper curve is for $B_m = 10^{18} \text{G}$, middle one is for $B_m = 10^{16} \text{G}$ and the lower one is for $B_m = 10^{14} \text{G}$. 