Supersymmetry breaking, R-symmetry, and conformal dynamics

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Abstract

We study $N = 1$ global and local supersymmetric theories with a continuous global $U(1)_R$ symmetry. We discuss conditions for supersymmetry (SUSY) breaking and vacuum structures of R-symmetric SUSY models. Especially we find the conditions for R-symmetry breaking and runway vacua in global supersymmetric theories. We introduce explicit R-symmetry breaking terms into such models in global and local supersymmetric theories. Such explicit R-symmetry breaking terms can lead to a SUSY preserving minimum. We classify explicit R-symmetry breaking terms by the structure of newly appeared SUSY stationary points as a consequence of the R-breaking effect, which could make the SUSY breaking vacuum metastable. Based on the generic argument, we propose the scenario that conformal dynamics causes approximate R-symmetry and metastable SUSY breaking vacua. At a high energy scale, the superpotential in our model is not R-symmetric, and has a supersymmetric minimum. However, conformal dynamics suppresses several operators along the renormalization group (RG) flow toward the infrared fixed point. Then we can find an approximately R-symmetric superpotential, which has a metastable SUSY breaking vacuum, and the supersymmetric vacuum moves far away from the metastable supersymmetry breaking vacuum. Furthermore, we find that conformal dynamics also leads approximate R-symmetry in softly SUSY breaking theories, even in more complicated models such as the duality cascade. We investigate the RG flow of SUSY breaking terms as well as supersymmetric couplings in the duality cascade of softly broken supersymmetric theories. It is found that the magnitudes of SUSY breaking terms are suppressed in most regimes of the RG flow through the duality cascade and approximate R-symmetry is realized at a low energy scale. We also show the possibility that cascading would be terminated by the gauge symmetry breaking, which is induced by the so-called B-term. Finally, we find some models to arrive at standard-model-like models and to cause gauge symmetry breaking corresponding to electro-weak symmetry breaking.
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1 Introduction

Supersymmetric extension of the standard model is a promising candidate for the physics around TeV scale. Supersymmetry (SUSY) can stabilize the huge hierarchy between the weak scale and the Planck scale, and supersymmetric models with R-parity have the lightest superparticle which is a good candidate for the dark matter. In addition, the minimal SUSY standard model realizes the unification of three gauge couplings at a scale $M_{GUT} \sim 2 \times 10^{16}$ GeV. That may suggest some underlying unified structure in the nature.

In our real world, the SUSY must be broken with certain amount of the gaugino and scalar masses. The dynamical SUSY breaking has a strong predictability of the structure of such SUSY particle masses. It was shown by Nelson and Seiberg (NS) [1] that a global $U(1)_R$ symmetry is necessary for a spontaneous F-term SUSY breaking at the ground state of generic models with a global SUSY. The spontaneous $U(1)_R$ symmetry breaking predicts an appearance of massless Goldstone mode, R-axion. [1]

Recently, it has been argued by Intriligator, Seiberg and Shih (ISS) [2] that the SUSY breaking vacuum we are living can be metastable for avoiding the light R-axion and also obtaining gaugino masses, and that such situation can be realized by a tiny size of explicit $U(1)_R$ breaking effects, whose representative magnitude is denoted by $\epsilon$. In the limit $\epsilon \to 0$, there would be no SUSY vacuum. However, explicit R-symmetry breaking terms with a tiny, but finite size of $\epsilon$ can lead to a SUSY minimum. Such newly appeared SUSY minimum could be far away from the SUSY breaking minimum, which is found in the R-symmetric model without explicit R-symmetry breaking terms. Furthermore, such R-symmetry breaking terms would not have significant effects on the original SUSY breaking minimum, because R-symmetry breaking terms are tiny. The distance between the original SUSY breaking minimum and the newly appeared SUSY preserving minima may be estimated as $O(1/\epsilon)$ in the field space. Thus, if R-symmetry breaking terms, i.e., the size of $\epsilon$, are sufficiently small, the original SUSY breaking minimum would be a long-lived metastable vacuum.

On the other hand, an introduction of gravity into SUSY theories requires that the SUSY must be a local symmetry, i.e., supergravity. In supergravity, the structure of the scalar potential receives a gravitational correction,

\[1\] See for recent works on R-symmetry breaking, e.g. Refs. [2] [8] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] and references therein.
and also the background geometry of our spacetime is determined by the equation of motion depending upon the vacuum energy. In the above global SUSY model with metastable SUSY breaking vacuum, some fields have large vacuum values at the SUSY preserving vacuum. In such a case, supergravity effects might be sizable. Another important motivation to consider supergravity is to realize the almost vanishing vacuum energy. The global SUSY model always has positive vacuum energy at the SUSY breaking minimum. Supergravity effects could realize almost vanishing vacuum energy.

F-flat conditions have supergravity corrections. Thus, the supergravity model with global $U(1)_R$ symmetry would have aspects different from the global SUSY model. Furthermore, adding R-symmetry breaking terms would have different effects between global and local SUSY theories. In section 2, we study in detail generic aspects of global and local SUSY theories with R-symmetry and generic behaviors caused by adding explicit R-symmetry breaking terms. We reconsider the above argument for the dynamical SUSY breaking and its metastability by comparing global and local SUSY theories, based on [14].

The important keypoint is to realize the almost vanishing vacuum energy. That is impossible in the SUSY breaking vacuum of global SUSY models, and that is a challenging issue in supergravity models. The vacuum energy may be tuned to vanish, e.g., by the constant superpotential term, which is a sizable R-symmetry breaking term. That would affect all of vacuum structure such as metastability of SUSY breaking vacua and presence of SUSY preserving vacua. Here we study this vacuum structure by using several concrete models, where we start R-symmetric models and add certain classes of R-symmetry breaking terms such that the vanishing vacuum energy is realized.

In section 2.1, we study a generic aspect of R-symmetric models within the framework of global SUSY, such as spontaneous SUSY breaking, R-symmetry breaking, flat directions and runaway directions. In section 2.2, we consider the generalized O'Raifeartaigh (OR) model [15] following [2]. We introduce explicit R-breaking terms into the model and analyze in detail the newly appeared SUSY vacua as a consequence of the R-symmetry breaking effects. We also examine the stability of the original SUSY breaking vacuum under such R-breaking terms.

In section 2.3, we consider supergravity models with R-symmetry. We extend the argument by NS to the local SUSY theories and study the supergravity OR model. In this section, we also show a special SUSY stationary point, which does not obey the NS condition, and the associated
SUSY breaking vacuum in a certain class of R-symmetric supergravity models. We introduce explicit R-breaking terms into the supergravity OR model in section 2.3 and classify them.

In section 2.5, we study the case with R-symmetry breaking terms (A-type) which might not cause a metastability of SUSY breaking minimum, because corresponding SUSY vacua disappear when we set the vacuum energy at the SUSY breaking minimum vanishing. On the other hand, in section 2.6, we show that another class of R-symmetry breaking terms (B-type) makes SUSY breaking minimum metastable. Sec. 2.7 is summary of section 2. In Appendix A, we show some general features of R-axion masses, and find that the special SUSY solution exhibited in section 2.3 is at best a saddle point solution.

In section 3, we argue that conformal dynamics can realize such a metastable SUSY breaking vacuum in global SUSY, based on [16]. We start with a superpotential without R-symmetry. However, we assume the conformal dynamics. Because of that, certain couplings are exponentially suppressed. Then, we could realize an R-symmetric superpotential or an approximately R-symmetric superpotential with tiny R-symmetry breaking terms. It would lead to a stable or metastable SUSY breaking vacuum. We study this scenario by using a simple model. Also, we study 5D models, which have the same behavior.

The application of conformal dynamics provides several interesting aspects in supersymmetric models. For example, contact terms like \( \int d^4 \theta |X|^2 |Q|^2 \) are suppressed exponentially by conformal dynamics in the model that the chiral superfield \( X \) belongs to the hidden conformal sector and the chiral superfield \( Q \) belongs not to the conformal sector, but to the visible sector. Such conformal suppression mechanism, i.e. conformal sequestering, is quite important to model building for SUSY breaking [17, 18, 19, 20, 21, 22]. When \( X \) contributes to SUSY breaking sizably, the above contact terms, in general, induce flavor-dependent soft SUSY breaking terms, soft sfermion masses and the so-called A-terms, and they lead to flavor changing neutral current (FCNC) processes, which are strongly constrained by current experiments. However, conformal sequestering can suppress the above contact terms and flavor-dependent contributions to soft SUSY breaking terms. Then, flavor-blind contributions such as anomaly mediation [23] would become dominant.
This situation could be also realized in our models.

As we discuss in section 3, conformal dynamics realizes approximate R-symmetry. On the other hand, even if explicit soft SUSY breaking terms are included, R-symmetry tends to be recovered by conformal dynamics in almost cases. We discuss softly SUSY broken theories in section 4, and study how explicit R-symmetry breaking terms are suppressed by conformal dynamics. For example, gaugino mass is one of explicit R-symmetry breaking and soft SUSY breaking terms. It becomes exponentially suppressed as a gauge coupling approaches an infrared (IR) fixed point. A-terms, which are trilinear couplings of scalar fields, are also suppressed. We find that approximate R-symmetry can be also realized in softly broken SUSY theories by conformal dynamics.

Furthermore, it happens in more complicated models, such as the duality cascade we study in section 5, that R-symmetry is recovered approximately at a low-energy scale. The duality cascade is a successive chain of the Seiberg dualities \cite{24,25} from the ultraviolet (UV) region to the IR region and reduces the rank of gauge groups one after another. This leads to more complicated and interesting renormalization group (RG) flows of dual field theories. We discuss the duality cascade of softly SUSY broken theories in section 5, based on Ref.\cite{26}. First, we review the duality cascade and show the unique RG flows, based on \cite{27,28}. After that, we study the RG flows of soft SUSY breaking terms by using the spurion method \cite{29,30,31,32,33,34} and show that R-symmetry breaking and SUSY breaking terms are strongly suppressed as gauge couplings and yukawa couplings approach toward IR fixed points. However we find that B-term, which is a quadratic term, remains to be a finite value at a low energy scale.

Moreover, several models have been proposed to realize supersymmetric standard models (SSM) as well as their extensions at the bottom of the cascade \cite{35,36,37}. Those models are quite interesting and have opened possible candidates for high energy theories. We consider the model with soft SUSY breaking terms and try to construct models which possibly become realistic models at the bottom of the cascade. In our discussion, we assume that SUSY is softly broken at the beginning of the cascade. Then, we study RG flows of SUSY breaking terms as well as supersymmetric couplings. Finally, we find some models to arrive at standard-model-like (SM-like) models and to cause gauge symmetry breaking corresponding to electro-weak (EW)
symmetry breaking.

In section 5.1 we review briefly the RG flow of supersymmetric couplings in the duality cascade. In section 5.2 we study RG flows of SUSY breaking terms in the duality cascade. In section 5.3 we study symmetry breaking due to the B-term by using illustrative examples. In section 5.4 we give a simple example whose fields contents are similar to the minimal supersymmetric standard model (MSSM) or its extensions. Section 5.5 is conclusion in section 5. Section 6 is devoted to summary. In Appendix B and C we give a short introduction of the spurion method and the supergraph formalism [38].

2 Generic arguments about SUSY breaking

We study generic arguments about vacuum structures based on [1] and [14] in global and local SUSY. R-symmetry plays a key role in SUSY breaking. In section 2.1 we discuss vacuum structures of R-symmetric SUSY models in global SUSY.

2.1 R-symmetry in global supersymmetric theory

2.1.1 The Nelson and Seiberg argument

First, we review briefly the argument by Nelson and Seiberg [1] in R-symmetric global SUSY models. Let us consider the global SUSY model with \( n \) chiral superfields \( Q_I (I = 1, \ldots, n) \) and their superpotential \( W(Q_I) \). In the case of global SUSY, F-flat conditions are determined by

\[
W_I = 0, \tag{1}
\]

where \( W_I = \partial Q_I W \). Hereafter we use a similar notation for derivatives of functions \( H(X) \) by fields \( X \) as \( H_X \). The conditions (1) are \( n \) complex equations for \( n \) complex variables, and these can have a solution for generic superpotential.

Now, we consider global SUSY models with a continuous global \( U(1)_R \) symmetry and a nonvanishing superpotential. Since the superpotential has the R-charge 2, there exists at least one field with a nonvanishing R-charge. Suppose that the \( n \)-th field \( Q_n \) is such a field with the nonvanishing R-charge,
$q_n \neq 0$. Then, in the following field basis

\[ \chi_i = \frac{Q_i}{Q_n^{q_i/q_n}}, \quad (q_{\chi_i} = 0), \]

\[ Y = Q_n, \quad (q_Y = q_n \neq 0), \tag{2} \]

where \( i = 1, 2, \ldots, n - 1 \), the superpotential can be written as

\[ W_{NS} = Y^{2/q_Y} \zeta(\chi_i). \tag{3} \]

Then the F-flat conditions (1) become

\[ (2/q_Y) Y^{2/q_Y} \zeta^{-1}(\chi_i) = 0, \tag{4} \]

\[ Y^{2/q_Y} \partial_{\chi_j} \zeta(\chi_i) = 0. \tag{5} \]

When we look for an R-symmetry breaking vacuum, \( \langle Y \rangle \neq 0 \), these conditions are equivalent to

\[ \zeta(\chi_i) = 0, \quad \partial_{\chi_j} \zeta(\chi_i) = 0. \tag{6} \]

These are \( n \) complex equations for \( n - 1 \) complex variables, that is, these are over-constrained conditions. These cannot be satisfied at the same time for a generic function \( \zeta(\chi_i) \), and the SUSY can be broken. This is an observation by Nelson and Seiberg \[1\] that the existence of an R-symmetry is the necessary condition for a dynamical SUSY breaking, and is also the sufficient condition if the R-symmetry is spontaneously broken, \( \langle Y \rangle \neq 0 \).

However, R-symmetry is often unbroken, because the scalar potential, which is obtained from the superpotential (3) and the Kähler potential \( K(|Y|, \chi_i, \bar{\chi}_i) \), is found to have the global minimum at \( Y = 0 \), unless the Kähler potential \( K(|Y|, \chi_i, \bar{\chi}_i) \) is non-trivial. Thus, SUSY is not broken dynamically with the NS superpotential (3).

In the following section, we find out the condition for SUSY breaking and R-symmetry breaking concretely. We discuss models with chiral superfields by \( Q_I \) and their R-charges are denoted by \( R[Q_I] = q_I \). Furthermore, we have \( R[W] = 2 \) and \( R[W_I] = 2 - q_I \). All of chiral superfields are classified by their R-charges into three classes, \( X_a \ (a = 1, \ldots, N_X) \), \( \phi_\alpha \ (\alpha = 1, \ldots, N_\phi) \), and \( \Phi_i \ (i = 1, \ldots, N_\Phi) \). R-charges of \( X_a \), \( \phi_\alpha \), and \( \Phi_i \) are given as \( R[X_a] = 2 \), \( R[\phi_\alpha] = 0 \), and \( R[\Phi_i] \neq 0, 2 \), respectively. These are shown in the following table.
| chiral fields ($Q_I$) | R-charge ($R[Q_I] = q_I$) | The number ($N[Q_I]$) |
|---------------------|--------------------------|-------------------|
| $X_a$               | $R[X_a] = 2$             | $N[X_a] = N_X$    |
| $\phi_\alpha$      | $R[\phi_\alpha] = 0$   | $N[\phi_\alpha] = N_\phi$ |
| $\Phi_i$            | $R[\Phi_i] \neq 0, 2$   | $N[\Phi_i] = N_\Phi$ |

### 2.1.2 R-symmetric vacua

We find that spontaneous R-symmetry breaking is the sufficient condition for SUSY breaking in the last subsection. However, this does not mean R-symmetric SUSY breaking vacua cannot exist. In this subsection, we look for R-symmetric vacua. If R-symmetric vacua exist, the vacuum expectation values (VEVs) of R-charged fields must vanish. This means that the following condition

$$< X_a >= < \Phi_i >= 0$$

must be satisfied and $W_I$ must also vanish except for $W_{X_a}$,

$$W_{\Phi_i} = W_{\phi_\alpha} = 0.$$  

(8)

If the R-symmetric vacua are supersymmetric, F-flat conditions for $X_a$, which depend on only $\phi_i$, should be satisfied, i.e.,

$$W_{X_a}(\phi_\alpha, X_a, \Phi_i)|_{X_a = \Phi_i = 0} = 0.$$  

(9)

These are $N_X$ equations with $N_\phi$ variables. In the case that the number of equations $N_X$ is less than the number of variables $N_\phi$, the F-flat conditions can be solved generally. Based on the Nelson-Seiberg argument and the above result, the classification in the following table can be realized:

| $N_X \leq N_\phi$ | $N_X > N_\phi$ |
|-------------------|----------------|
| R-symmetric SUSY vacua exist. | SUSY is always broken. |

### 2.1.3 R-symmetry breaking vacua

In this subsection, we look for the sufficient condition for R-symmetry breaking. In this case, the VEV of at least one of $X_a, \Phi_i$ is nonzero, so we define

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2In subsection 2.1.4, we discuss models with runaway directions where the relation $N_X > N_\phi$ is satisfied but SUSY is restored by the limit that a R-charged field $X_a$ or $\Phi$ goes to infinite.
Y which is one of $X_a, \Phi_i$ with nonzero VEV. Under this assumption, all $W_I$ are described as

$$W_I = Y^{\frac{2-\Theta_I}{q}} f_I(Q_I/Y^{\frac{\Theta_I}{q}}),$$

(10)

where we define $q = R[Y]$ and each $f_I$ is a function which depends on $(N_X + N_{\Phi} + N_{\phi} - 1)$ variables. We define $Y$ as $Q_1 (q = q_1)$, and define $z_J$ as $Q_J/Y^{\frac{\Theta_J}{q}}$ for $J = 2, \ldots, N$ where $N$ satisfies $N = N_X + N_{\Phi} + N_{\phi}$.

In the following arguments, we assume that Kähler potential is canonical,

$$K = \sum_{I=1}^{N} |Q_I|^2.$$  

(11)

Then the scalar potential is written by

$$V = \sum_{I=1}^{N} |W_I|^2.$$  

(12)

Under the assumption and conditions, this scalar potential is described as

$$V = \sum_{I=1}^{N} |Y|^{\frac{2(2-q_J)}{q}} |f_I(z_J)|^2$$

$$= \sum_{\alpha=1}^{N_{\Phi}} |Y|^\frac{4}{7} |f_{\phi_{\alpha}}|^2 + \sum_{i=1}^{N_X} |Y|^\frac{2(2-q_{\Phi_i})}{q} |f_{\Phi_i}|^2 + \sum_{a=1}^{N_X} |f_{X_a}|^2.$$  

(13)

The scalar potential $V$ is monotonous about $|Y|$ or $1/|Y|$, as long as $\Phi_i$ do not include fields with $2 - q_{\Phi_i} < 0$, because all $f_I$ do not depend on $Y$. This leads classifications of R-symmetric models based on the assignments of fields. We define $\hat{Q}_I$ and $\tilde{Q}_I$, such that $\hat{Q}_I$ are the fields which couple with $X_a$, and $\tilde{Q}_I$ are the R-charged fields which couple with $X_a$.

For example, we consider the case that all R-charges of $\Phi_i$ are less than 2. In this case, the potential is described as

$$V = \sum_{I=1}^{N} |Y|^n_I |f_I(z_J)|^2,$$

(14)

3 The phase direction of $Y$ is the Goldstone mode of $U(1)_R$ symmetry breaking.

4 $\hat{Q}_I$ and $\tilde{Q}_I$ include $X_a$, and $\tilde{Q}_I$ do not include $\phi_{\alpha}$. 

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where $f_I$ describe $f_{\phi_\alpha}$, $f_{\Phi_i}$, and $f_{X_a}$. In the case that $q$ is positive (negative), each $n_I$ is positive (negative) or vanishing. If R-symmetry breaking vacua exist, the stationary condition for $Y$ must be satisfied,

$$\frac{\partial V}{\partial |Y|} = \sum_{\alpha=1}^{N_\phi} \frac{4}{q} |Y|^{q-1} |f_{\phi_\alpha}|^2 + \sum_{i=1}^{N_\Phi} \frac{4 - 2q\Phi_i}{q} |Y|^{\frac{2(q-2\Phi_i)}{q}-1} |f_{\Phi_i}|^2 = 0. \quad (15)$$

We assume that $Y$ is non-vanishing, so that the unique solution is

$$f_{\phi_\alpha}(z_J) = f_{\Phi_i}(z_J) = 0 \text{ for all } \phi_\alpha, \Phi_i. \quad (16)$$

This means that the F-flat conditions for all fields except for $X_a$ should be satisfied. The F-flat conditions are $(N - N_X)$ equations,

$$W_{\phi_\alpha} = W_{\Phi_i} = 0 \text{ for all } \alpha, i. \quad (17)$$

If we can find the solutions of these equations, the scalar potential along the slice $W_{\phi_\alpha} = W_{\Phi_i} = 0$ is described as

$$V(\tilde{z}_J) = \sum_{a=1}^{N_X} |f_{X_a}|^2, \quad (18)$$

where $\tilde{z}_J$ is defined as $\tilde{z}_J \equiv \tilde{Q}_J / Y^{2q}$. The fields $\tilde{z}_J$ must satisfy the $N[\tilde{z}]$ stationary conditions of this potential, where $N[\tilde{z}]$ stands for the number of $\tilde{z}_J$. In order that all equations can be solved generally, the number of equations must not be larger than the number of the variables. The number of the equations is $(N - N_X + N[\tilde{z}])$ and there are $(N - 1)$ complex variables. Eventually, we find the condition for R-symmetry breaking as $(N - N_X + N[\tilde{z}]) < N$, i.e.,

$$0 < N_X - N[\tilde{z}]. \quad (19)$$

When this relation is satisfied, R-symmetry can be spontaneously broken and $(N_X - N[\tilde{z}])$ complex fields are flat directions. The relation (19) corresponds to the condition that F-flat conditions for $X_a$ have no solution. This is because F-flat conditions for $X_a$ are $N_X$ equations with $N[\tilde{z}]$ variables, so the $N_X$ equations can not be solved if the relation (19) is satisfied. However, $U(1)_R$-invariant operators which consist of $\tilde{Q}_I$ only appear in $W_{X_a}$. Here we define $\omega_p \ (p = 1, \ldots, N_\omega)$ as $U(1)_R$-invariant independent operators which appear in $W_{X_a}$. For example, $\omega_p$ include $\phi_\alpha$ and operators which consist of
\( \hat{Q}_I \), such as \((X_a/X_b) (a \neq b)\) on which \(W_{X_a}\) depends. In other words, all \(\hat{Q}_I\) cannot be fixed by \(\partial \hat{z} V = 0\). \(N[\hat{z}]\) is not larger than \(N_\omega\). This fact limits \((19)\) to
\[
0 < N_X - N_\omega. \tag{20}
\]

On the other hand, under the condition \(N_X \leq N_\omega\), R-symmetry breaking does not happen, so that there is an R-symmetric SUSY breaking vacuum. However, we consider models with only positive R-charge fields, so there is a possibility that \(\hat{Q}_I = 0\) gives singular values to \(W_{X_a}\).

To obtain \((20)\), we limit the assignment of R-charges, but we find the same condition for satisfying both \((16)\) and \(\partial \hat{z}_J V = 0\) in the model with superfields whose R-charges are more than 2. Moreover, the redefinition in \((13)\) leads to scalar potentials with both positive and negative power terms of \(|Y|\). There would be another possibility that we would find a stationary point where \(Y\) is also stabilized at a finite value. However, in the model with \(\omega_p\) satisfying \((20)\), a global minimum value of the scalar potential, \(V_{min}\), is given by
\[
V_{min} = V_2(\omega_{p_{min}}) = \sum_{a=1}^{N_X} |f_{X_a}(\omega_{p_{min}})|^2, \tag{21}
\]
where \(V_2(\omega_p)\) is defined as \(\sum_{a=1}^{N_X} |f_{X_a}(\omega_p)|^2\) and \(V_2(\omega_{p_{min}})\) is a minimum value of \(V_2(\omega_p)\). \(\omega_{p_{min}}\) satisfy \(\partial \hat{z}_J V = 0\) as long as the equations \((17)\) with any \(\omega_p\) are satisfied by the other fields.\(^5\) This is because the part of the scalar potential \(V_2\) only depends on \(\omega_p\), and \(W_{\phi_\alpha}\) and \(W_{\Phi_i}\) depend on not only \(\omega_p\) but also the other fields. The F-flat conditions, \(W_{\phi_\alpha} = W_{\Phi_i} = 0\), can be satisfied by the fields which do not appear in \(W_{X_a}\).

On the other hand, models with \(N_\omega \geq N_X\) have solutions for F-flat conditions for \(X_a\) and eventually we find SUSY vacua in the limit \(Y \rightarrow \infty\) or \(Y \rightarrow 0\). In fact, we find runaway supersymmetric vacua in models with \(N_\omega \geq N_X\) as we discuss in the following subsection.

### 2.1.4 Runaway vacua

In this section, we study runaway directions in R-symmetric models generally.\(^6\) We classify \(\Phi_i\) to \(\Phi^+_{i_+}, \Phi^-_{i_-}, \Phi_k\). The R-charges of \(\Phi^+_{i_+}\) and \(\Phi^-_{i_-}\)

\(^5\) The limit \(|Y| \rightarrow 0\) or \(Y \rightarrow \infty\) becomes a solution for \((17)\). The functions \(f_I(\hat{z}_J)\) whose the coefficients \(Y^\frac{2-q_I}{2-q}\) become infinite in the limit also need to vanish by the fields except for \(\omega_p\).

\(^6\)See also Ref.\([13]\).
satisfy $R[\Phi^+_i] = q_i > 2$ and $R[\Phi^-_i] < 0$. The fields $\Phi_k$ for $k = 1, \ldots, (N_\Phi - N[\Phi^+_i] - N[\Phi^-_i])$ describe the fields with R-charges $q_k$ ($0 < q_k < 2$). Based on the above argument, the potential is described as

$$V = \sum_{a} |f_{X_a}|^2 + \sum_{i_+} \left( \frac{1}{|Y|} \right)^{2q_+ - 2} |f_{\Phi_{i_+}}|^2 + \sum_{i_-} (|Y|) \frac{2^{q_+ - 2}}{|q_+|} |f_{\Phi_{i_-}}|^2$$

$$+ \sum_{k} |Y| \frac{2^{q_k - 2}}{|q_k|} |f_{\Phi_k}|^2 + \sum_{\alpha} (|Y|) \frac{4}{|q_\alpha|} |f_{\phi_\alpha}|^2.$$  \hfill (22)

We assume that $q$ is positive. In this case, if we can find the direction that all $f_I(z_J)$ vanish except for $f_{X_a}$ and $f_{\Phi_{i_+}}$, the scalar potential along such a direction becomes

$$V = \sum_{a} |f_{X_a}|^2 + \sum_{i_+} \left( \frac{1}{|Y|} \right)^{2q_+ - 2} |f_{\Phi_{i_+}}|^2.$$  \hfill (23)

This scalar potential in the limit $|Y| \to \infty$ can be described as

$$V \to \sum_{a} |f_{X_a}|^2.$$  \hfill (24)

This means that $Y$ is a runaway direction. Especially, if $f_{X_a} = 0$ for all $a$ can be satisfied, SUSY is restored by the limit $|Y| \to \infty$.

On the other hand, when we assume that $Y$ is chosen as one of $\Phi^-_{i_-}$, the potential on the slice $f_{\Phi^+_i} = 0$ is given by

$$V = \sum_{a} |f_{X_a}|^2 + \sum_{i_-} \left( \frac{1}{|Y|} \right)^{2q_- - 2} |f_{\Phi_{i_-}}|^2$$

$$+ \sum_{k} \left( \frac{1}{|Y|} \right)^{2q_+ - 2} |f_{\Phi_k}|^2 + \sum_{\alpha} (|Y|) \frac{4}{|q_\alpha|} |f_{\phi_\alpha}|^2.$$  \hfill (25)

In the limit $|Y| \to \infty$, the potential is described as

$$V \to \sum_{a} |f_{X_a}|^2.$$  \hfill (26)
This limit also corresponds to $V \to 0$ when $f_{X_a} = 0$ for all $a$ are satisfied. The condition for the solution existing corresponds to $0 \geq (N_X - N_\omega)$.

Finally we conclude that there are some runaway directions in models with the fields whose R-charges are negative and/or more than 2. The scalar potential in the limit where at least one VEV of R-charged field is infinite, is given by $\sum |W_{X_a}|^2$. Especially, the limit restores SUSY in models satisfying $N_X \leq N_\omega$.

### 2.1.5 Example

We show illustrating examples which describe the above generic arguments. The fields $X_a$ ($a = 1, \ldots, n$) have R-charge 2, and $\Phi$, $\overline{\Phi}$ and $\phi$ denote fields with R-charge 1, $-1$ and 0 fields respectively. Based on the generic argument, we expect that the global minimum of the scalar potential exists along the slice $W_\phi = W_\Phi = W_{\overline{\Phi}} = 0$ although that may correspond to a runaway direction. It depends on $n$ whether the minimum preserves SUSY or not.

We consider the renormalizable superpotential as follows,

$$W = \sum_{a=1}^{n} \left( f_a(\phi) X^a + \lambda_a X^a \Phi \overline{\Phi} \right) + \frac{1}{2} m(\phi) \Phi^2, \quad (27)$$

where $m(\phi)$ is linear, and $f_a(\phi)$ are quadratic functions of $\phi$.

The derivatives of $W$ are given by

$$W_{X_a} = f_a(\phi) + \lambda_a \Phi \overline{\Phi},$$

$$W_\Phi = \sum_{a=1}^{n} \lambda_a X^a + m(\phi) \Phi,$$

$$W_{\overline{\Phi}} = \sum_{a=1}^{n} \lambda_a X^a \Phi,$$

$$W_\phi = \sum_{a=1}^{n} \frac{\partial f_a(\phi)}{\partial \phi} X^a + \frac{1}{2} \frac{\partial m(\phi)}{\partial \phi} \Phi^2. \quad (28)$$

\[7\] If we define $Y$ as a field with negative R-charge, the limit $Y \to \infty$ also gives the vanishing superpotential, $W \to 0$. If we consider supergravity effects, we would find that the limit $Y \to \infty$ restores SUSY in the models where covariant derivatives $W_I + K_I W$ also go to $W_I$ in the limit. We need classify R-charge assignments to discuss the supergravity effect. This is our future work.
First, we consider the example with \( n > 3 \), which corresponds to the case that a global minimum of the scalar potential is given by \( V_2 = |W_{X_a}|^2 \) along the slice \( W_\phi = W_\Phi = W_\overline{\Phi} = 0 \). The scalar potential \( V_2 (\bar{Q}) \) is obtained as

\[
V_2 (\Phi, \overline{\Phi}, \phi) = \sum_{a=1}^n |f_a (\phi) + \frac{1}{2} \lambda_a \Phi \overline{\Phi}|^2,
\]

where the fields \( \{ \bar{Q} \} \) correspond to \( \{ \Phi, \overline{\Phi}, \phi \} \). In this case, the F-flat conditions for all \( X_i \), \( W_{X_a} = 0 \), can not be solved. We look for the solutions for \( \partial \bar{Q} V_2 = 0 \). \( V_2 \) depends on two \( U(1)_R \)-invariant operators, \( \phi (\equiv \omega_1) \) and \( \Phi \overline{\Phi} (\equiv \omega_2) \).

We find that one solution for \( \partial \bar{Q} V_2 = 0 \) is \( \Phi = \overline{\Phi} = 0 \) and the stationary condition for \( \phi \), \( \partial \phi V_2 = 0 \), is also satisfied as follows,

\[
\Phi = \overline{\Phi} = 0,
\]

\[
\sum_{a=1}^n \left. \frac{\partial f_a (\phi)}{\partial \phi} X_a \right|_{\phi = \phi^*} = 0.
\]

(30)

In this case, all \( \bar{Q} \) are fixed by the stationary condition of \( V_2 \). When equation (30) is satisfied, the F-flat conditions, \( W_\phi = W_\Phi = W_\overline{\Phi} = 0 \), are also satisfied by \( X_a \),

\[
\sum_{a=1}^n \left. \frac{\partial f_a (\phi)}{\partial \phi} X_a \right|_{\phi = \phi^*} = 0,
\]

(31)

where the scalar potential \( V \) is estimated as \( \sum_{a=1}^n |f_a|^2 \), and the VEVs of \( (n-1) \) fields \( X_a \) are flat directions.

Furthermore, we find the other solution for \( \partial \bar{Q} V_2 = 0 \) as follows. The stationary conditions for \( V_2 \), \( \partial_\phi V_2 = \partial_\phi V_2 = \partial_\phi V_2 = 0 \), are satisfied by \( \omega_1 (\equiv \phi) \) and \( \omega_2 (\equiv \Phi \overline{\Phi}) \) satisfying the following equations

\[
\Phi \overline{\Phi} = -\sum_{a=1}^n \frac{\lambda_a f_a (\phi)}{\sum_{a=1}^n |\lambda_a|}.
\]

\[
\sum_{a=1}^n \left. \frac{\partial f_a (\phi)}{\partial \phi} W_{X_a} \right|_{\phi = \phi^*} = 0.
\]

(32)

On the other hand, \( \Phi \) must vanish in order that the F-flat conditions for \( \Phi \) and \( \overline{\Phi} \), \( W_\phi = W_\overline{\Phi} = 0 \) are satisfied, as long as \( m (\phi) \) is non-vanishing. When
\( \omega_1 \) and \( \omega_2 \) are fixed by (32), the limit \( \Phi \to 0 \) \((\Phi \to \infty)\) gives the F-flat conditions and the fields \( X_a \) satisfy the following equations,

\[
\sum_{a=1}^{n} \lambda_a X^a = 0,
\]
\[
\frac{\partial f_a(\phi)}{\partial \Phi} X^a = 0.
\]

In this runaway direction, \( V \) is given by

\[
V \to \sum_{a=1}^{n} (|f_a|^2 - |\lambda_a|^2|\Phi|^2|\Phi|^2),
\]

where the VEVs of \((n - 2)\) fields \( X_a \) are flat directions. Eventually, the either of these two solutions corresponds to the global minimum of the scalar potential \( V \).

Second, we consider the model with \( n = 3 \). In this case, the number of \( X_a \) \((a = 1, 2, 3)\) is equal to the number of \( \bar{\tilde{Q}}_1 \) \((\phi, \Phi, \overline{\Phi})\) which couple with \( X_a \). However, the vacuum structure is the same as in the case with \( n > 3 \) because \( V_2 \) depends on only \( U(1)_R \)-invariant operators, \( \phi \) and \( \Phi \overline{\Phi} \). Based on the condition (20), the model with \( n = 3 \) is classified as models satisfying \( N_X > N_\omega = 2 \).

Now we consider the case with \( n \leq 2 \). Based on the generic argument, there is a runaway direction in this case. In fact, we discuss the model with \( n = 2 \). When the fields, \( \phi, \Phi \overline{\Phi}, \) and \( X_a \) are fixed by,

\[
\Phi \overline{\Phi} = -\frac{f_1(\phi)}{\lambda_1},
\]
\[
\sum_{a=1}^{2} \lambda_a X_a = 0,
\]
\[
f_2(\phi)\lambda_1 - \lambda_2 f_1(\phi) = 0,
\]
\[
\sum_{a=1}^{2} \frac{\partial f_a(\phi)}{\partial \phi} X^a = 0,
\]

where the derivatives of \( W \) are given by \( W_{\overline{\Phi}} = W_{X_a} = 0, W_\phi = (1/2)\partial_\phi m\Phi^2 \) and \( W_\Phi = m(\phi)\Phi \). When the field \( \Phi \) vanishes, the field \( \overline{\Phi} \) becomes infinite.
The direction $\Phi \to \infty$ corresponds to a runaway direction where SUSY is restored.

In addition to the above runaway direction, the model with $n = 2$ has a R-symmetric vacuum on the slice $\Phi = \overline{\Phi} = 0$.

We give a comment about the case $N[\Phi] = 0$. The renormalizable superpotential is obtained by $\lambda_a = 0$ in (27)

$$W = \sum_{a=1}^{n} f_a(\phi) X^a + \frac{1}{2} m(\phi) \Phi^2. \quad \text{(36)}$$

In this case, the derivatives of $W$ are given by

$$W_{X_a} = f_a(\phi),$$
$$W_{\Phi} = m(\phi) \Phi,$$
$$W_{\phi} = \sum_{a=1}^{n} \frac{\partial f_a(\phi)}{\partial \phi} X^a + \frac{1}{2} \frac{\partial m(\phi)}{\partial \phi} \Phi^2. \quad \text{(37)}$$

We can not find a runaway direction, and the global minimum is obtained along the slice $W_{\Phi} = W_{\phi} = 0$. The two F-flat conditions lead to the following solution,

$$\Phi = 0,$$
$$\sum_{a=1}^{n} \frac{\partial f_a(\phi)}{\partial \phi} X^a = 0. \quad \text{(38)}$$

The field $\phi$ is fixed by $\partial_{\phi} V_2 = 0$, so that the global minimum in the model with $n > 1$ corresponds to SUSY breaking vacua with $(n - 1)$ flat directions and the global minimum in the model with $n \leq 1$ describes SUSY vacua.

### 2.1.6 Short summary

We summarize the results we find in the previous subsections and classify models according to the number of fields with R-charge 2.

$$[1] \quad N_X \leq N_\phi$$

The F-flat conditions for $X_a$ can be solved generally, so R-symmetric and supersymmetric vacua can exist. If we find a SUSY breaking vacuum, it is
metastable. When models have fields with negative R-charge and/or more than 2 R-charge, there are runaway directions.

\[ 2 \] \( N_\phi < N_X \leq N_\omega \)

\( N_\omega \) denotes the number of the \( U(1)_R \)-invariant independent operators of the fields which appear in the derivatives of \( W \) by the fields \( X_a \). We can find R-symmetric SUSY breaking vacua in this model. However, R-symmetry cannot be broken, as long as there is no field whose R-charge is negative or more than 2. On the other hand, there are runaway directions in the model with fields whose R-charges are either negative and/or more than 2. The limit that an R-charged field goes to infinite restores SUSY, so that R-symmetric SUSY breaking vacua are metastable. This does not conflict with the NS argument.

\[ 3 \] \( N_\omega < N_X \)

There is no supersymmetric vacuum in this model with any assignment of R-charges. It is possible that R-symmetry is also broken generally. Eventually, there are SUSY breaking minima with \( (N_X - N_\omega) \) flat directions, where the VEVs of R-charged fields correspond to the flat directions. Furthermore, the minimum of the partial potential \( V_2 = \sum_a |W_{X_a}|^2 \) is a global minimum of the full potential \( V \). The relation \( N_\omega < N_X \) corresponds to the condition that there is no solution satisfying all F-flat conditions for \( X_a \), so that this condition for R-symmetry breaking corresponds to the condition for SUSY breaking.

We consider R-symmetric models to discuss SUSY breaking generally, and we find that we can realize SUSY breaking vacua, if we consider models which satisfy \( N_X > N_\omega \). The O’Raifeartaigh model [15], which we study in the next subsection, is well-known as one of those models.

It is not a sufficient condition for spontaneous SUSY breaking that supersymmetric models have R-symmetry. However, recently it is proved that stable SUSY breaking vacua can not exist at tree level in models even without R-symmetry which have general polynomial superpotential and canonical Kähler potential [11, 39]. SUSY breaking vacua always have flat directions,

\(^8\)We assume that the condition, \( N_\phi \leq N_\omega \), is always satisfied in our models.
as far as avoiding tachyonic modes. This result and our argument indicate that it is appropriate that we concentrate on the O’Raifeartaigh model to discuss SUSY breaking generally, as we actually do later.

2.1.7 The generalized O’Raifeartaigh model

The O’Raifeartaigh model [15] is a good example of R-symmetric SUSY models, where SUSY is spontaneously broken. Its generalization is shown in Ref. [2] as the generalized OR model, which has the following superpotential,

\[ W_{\text{OR}} = \sum_a g_a(\phi_i) X_a, \] (39)

where \( a = 1, 2, \ldots, r \) and \( i = 1, 2, \ldots, s \), and the numbers of fields are constrained as \( r > s \). Their R-charges are assigned as \( q_{X_a} = 2 \) and \( q_{\phi_i} = 0 \), and \( g_a(\phi_i) \) is a function of \( \phi_i \). Based on the discussion in the previous subsections, this generalized OR model is a minimal model to cause SUSY breaking. In this model, \( F \)-flat conditions for \( X_a \) are just given by

\[ \partial_{X_a} W = g_a(\phi_i) = 0. \] (40)

These are \( r \) complex equations for \( s \) complex variables, that is, these are over-constrained conditions for \( r > s \). Therefore, there is no SUSY solution satisfying (40) for generic functions \( g_a(\phi_i) \) with \( r > s \). The superpotential of the generalized OR model (39) is a specific form of the NS superpotential (4). In the generalized OR model, SUSY is always spontaneously broken independently of whether R-symmetry is spontaneously broken or not, or the fields \( X_a \) develop nonvanishing vacuum expectation values or not.

The simplest OR model is the model with \( r = 1 \) and \( s = 0 \), and has the superpotential

\[ W_{(\text{OR})_1} = f X_1, \]

where \( f \) is a constant. Obviously, SUSY is spontaneously broken in this model, because \( W_{X_1} = f \). The basic O’Raifeartaigh model corresponds to the model with \( r = 2 \) and \( s = 1 \), and \( g_1(\phi) = f + \frac{1}{2} h \phi^2 \) and \( g_2(\phi) = m \phi \), and has the following superpotential,

\[ W_{(\text{OR})_{\text{basic}}} = (f + \frac{1}{2} h \phi^2) X_1 + m \phi X_2. \] (41)

\(^9\)The flat directions are also discussed generally in Ref. [11].
The model has only a SUSY breaking pseudo-moduli space,

$$\phi = X_2 = 0, \quad X_1 : \text{undetermined},$$  \hspace{0.5cm} (42)

with $W_{X_1} = f$ as a global minimum of the potential. When integrating out heavy modes $X_2$ and $\phi$, we obtain $W_{(OR)}$ as an effective superpotential. However, the flat direction along $X_1$ is lifted at the one-loop level by integrating out $\phi$, and the SUSY breaking vacuum in the quantum corrected OR model is given by

$$\phi = X_2 = X_1 = 0.$$  \hspace{0.5cm} (43)

These simple models suggest that the tadpole term of $X_a$ is important for SUSY breaking. Indeed, we can show by simple discussion that non-vanishing terms of $g_a(\phi_i)$ at $\phi_i = 0$ are sources of SUSY breaking. We assume that $g_a(\phi_i)$ are non-singular functions. Then, we can always rewrite the superpotential (39) as

$$W_{OR} = \sum_a f_a X_a + \sum_a g_a(\phi_i) X_a$$

$$= \tilde{f} \tilde{X}_1 + \sum_a \tilde{g}_a(\phi_i) \tilde{X}_a, \quad (\tilde{g}_a(0) = 0),$$  \hspace{0.5cm} (44)

where $f_a = g_a(0)$, $\tilde{g}_a(\phi_i) = g_a(\phi_i) - f_a$, $\tilde{X}_a = U_{ab} X_b$, $\tilde{g}_a(\phi_i) = \tilde{g}_a U_{ab}^\dagger$ and $U_{ab}$ is a constant unitary matrix defined by $f_a U_{ab}^\dagger = \tilde{f}_b = (\tilde{f}, 0, \ldots, 0)$. In the following, we will frequently use this basis of fields and omit the tildes to simplify the notation. In this basis, the F-flat conditions for $X_a$, Eq. (40), are written by

$$W_{X_a} = g_a(\phi_i) - \delta_{a1} f = 0.$$  \hspace{0.5cm} (45)

Together with $W_{\phi_i} = \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0$, we find that, if $f = 0$, there is a solution $X_a = \phi_i = 0$ and SUSY is not broken. Then it is obvious in the field basis (41) that a nonvanishing $f$ is the source of dynamical SUSY breaking in the generalized OR model.

In the generalized OR model with the above field basis, the field $X_1$ plays a special role, while each of $X_a$ ($a \neq 1$) has the qualitatively same character as others $X_b$ ($b \neq 1$). Thus, the simple model with $r = 2$ and $s = 1$, and the superpotential,

$$W_{(OR)2} = (f + g_1(\phi)) X_1 + g_2(\phi) X_2,$$
shows qualitatively generic aspects of the generalized OR model. Its scalar potential is written as

\[ V = |f + g_1(\phi)|^2 + |g_2(\phi)|^2 + |W_\phi|^2, \]

and stationary conditions are obtained as

\[
\begin{align*}
V_{X_1} &= W_\phi g_1'(\phi) = 0, \\
V_{X_2} &= W_\phi g_2'(\phi) = 0, \\
V_\phi &= W_\phi W_\phi + (f + g_1(\phi))g_1'(\phi) + g_2(\phi)g_2'(\phi) = 0,
\end{align*}
\]

where \( g'_a(\phi) = dg_a(\phi)/d\phi \) and \( W_\phi = \sum_a X_a g'_a(\phi) \). Unless \( W_\phi \) does not vanish, we would have over-constrained conditions, i.e., \( g'_1(\phi) = g'_2(\phi) = 0 \) for generic functions. Thus, in general, the solution of the above stationary conditions corresponds to

\[
W_\phi = X_1 g'_1(\phi) + X_2 g'_2(\phi) = 0, \\
(f + g_1(\phi))g_1'(\phi) + g_2(\phi)g_2'(\phi) = 0.
\]

(46)

The latter is the condition to fix \( \phi \). For a fixed value of \( \phi \), a ratio between \( X_1 \) and \( X_2 \) is fixed by the former condition, but the linear combination

\[ X_1 g'_2(\phi) - X_2 g'_1(\phi), \]

remains undetermined. That is the pseudo-flat direction, and would be lifted by loop effects. Similarly we can discuss models with several fields \( X_a \) and \( \phi_i \) \((r > s)\).

2.2 Explicit R-symmetry breaking and metastable vacua

In order to have Majorana gaugino masses in addition to soft scalar masses, the R-symmetry must be broken spontaneously or explicitly at the SUSY breaking minimum we are living. On the other hand, as shown in the previous section, the NS argument requires an exact R-symmetry for the dynamical SUSY breaking. Then, an appearance of an unwanted massless Goldstone mode, an R-axion, is inevitable in such R-symmetry breaking minimum. Does this mean the dynamical SUSY breaking is phenomenologically disfavored?
Recently, it has been argued by Intriligator, Seiberg and Shih [2] that our world must reside in a metastable state, in order to avoid the above conflict between gaugino masses and the massless R-axion. The argument is as follows. Consider a theory with an approximate R-symmetry which has a small R-symmetry breaking parameter $\epsilon$. In the limit $\epsilon \to 0$, the R-symmetry becomes exact, and the theory possesses a SUSY breaking ground state due to the NS argument. For a nonzero but tiny parameter $\epsilon$, this SUSY breaking minimum still remains as a local minimum of the potential, although there appear SUSY ground states somewhere in the field space due to explicit R-symmetry breaking effects. As long as the parameter $\epsilon$ is small enough, the separation between the SUSY breaking minimum and the supersymmetric vacua is large, and the former can be a long-lived metastable vacuum. These facts were exhibited by ISS based on the O'Raifeartaigh model as a simple example of dynamical SUSY breaking model with R-symmetry. Indeed, such O'Raifeartaigh-type model can be realized in some region of the moduli space of SUSY Yang-Mills theories [40].

Here following the discussion by ISS we study generic aspects of explicit R-symmetry breaking terms, and SUSY preserving vacua. We also classify explicit R-symmetry breaking terms in global SUSY models. In addition, we discuss metastability.

The simplest R-symmetry breaking term is the constant term $W_R = c$, but the constant term does not play any role in global SUSY theory. Thus, we do not discuss about adding the constant term in this section. It is obvious that when we add any R-symmetry breaking term $W_R(Y, \chi)$ to the NS superpotential (3), that can relax over-constrained conditions and F-flat conditions can have SUSY solutions.

The generalized OR model has richer structure in explicit R-symmetry breaking terms. To see such structure, we consider the generalized OR model with three types of typical R-symmetry breaking terms, i) a function including only $\phi_i$ fields $W_R = w(\phi)$, ii) a function including only $X_a$ $X_a \neq 1$, $W_R = w(X_a)$, and iii) a function including only $X_1$, $W_R = w(X_1)$. The first type of R-symmetry breaking terms $W_R = w(\phi)$ do not change F-flat conditions for $X_a$, i.e., $\partial_{X_a} W = f \delta_{a1} + g_a(\phi_i) = 0$. Hence, there is no SUSY solution.

For the second type of R-symmetry breaking terms $W_R = w(X_a)$ ($a \neq 1$),
F-flat conditions are obtained as

\[
\begin{align*}
W_{X_1} &= f + g_1(\phi_i) = 0, \\
W_{X_a} &= g_a(\phi_i) + w_{X_a}(W_a) = 0 \quad \text{for } a \neq 1, \\
W_{\phi_i} &= \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0.
\end{align*}
\]

Thus, if \( w_{X_a}(W_a) \neq 0 \) for all of \( X_a \), over-constrained conditions can be relaxed and a SUSY solution can be found. If all of \( \phi_i \) vanish, we have \( g_1(\phi_i) = 0 \) and the condition \( W_{X_1} = 0 \) can not be satisfied. Hence, the SUSY minimum, which appears by adding \( W_R = w(X_a) \) \((a \neq 1)\), corresponds to the point, where some of \( \phi_i \) develop nonvanishing vacuum expectation values.

For the third type of R-symmetry breaking terms \( W_R = w(X_1) \), F-flat conditions are obtained as

\[
\begin{align*}
W_{X_1} &= f + g_1(\phi_i) + \partial_{X_1} w(X_1) = 0, \\
W_{X_a} &= g_a(\phi_i) = 0 \quad \text{for } a \neq 1, \\
W_{\phi_i} &= \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0.
\end{align*}
\]

If \( r = s + 1 \), the over-constrained conditions can be relaxed. In this case, the point \( \phi_i = 0 \) for all of \( i \) can be a solution for \( W_{X_a} = 0 \) for \( a \neq 1 \). Furthermore, the conditions,

\[
f + \partial_{X_1} w(X_1) = 0, \quad \sum_a X_a \partial_{\phi_i} g_a(\phi_i) = 0,
\]

should be satisfied.

When R-symmetry breaking terms include \( X_1 \) and \( X_a \) \((a \neq 1)\), over-constrained conditions can be relaxed and a solution for F-flat conditions would correspond to \( \phi_i \neq 0 \) for some of \( \phi_i \).

The SUSY breaking minimum is found in the generalized OR model without explicit R-symmetry breaking terms, as discussed in the previous subsection. As discussed above, SUSY vacua can appear, when we add the definite form of explicit R-symmetry breaking terms to the generalized OR model. Thus, the previous SUSY breaking minimum is a metastable vacuum, if such R-symmetry breaking effects are small around the SUSY breaking minimum and the SUSY breaking vacuum itself is not destabilized by such R-symmetry breaking terms.
As an illustrating example, we consider the basic OR model \((4.1)\) with explicit R-symmetry breaking terms. ISS introduced an explicit R-symmetry breaking term in the superpotential \(W = W_{(OR)basic} + W_R\), where

\[
W_R = \frac{1}{2} \epsilon m X_2^2. \tag{48}
\]

In this case, there appears a SUSY minimum,

\[
\phi = \sqrt{-\frac{2f}{\epsilon h}}, \quad X_2 = -\frac{1}{\epsilon} \phi, \quad X_1 = \frac{m}{\epsilon h},
\]

which is far away from the (local) SUSY breaking minimum \((43)\) for a sufficiently small \(\epsilon \ll 1\). In addition, the SUSY breaking minimum is not destabilized by the above R-symmetry breaking term \((48)\). Then the original SUSY breaking vacuum \((43)\) becomes metastable which can be parametrically long-lived for \(\epsilon \ll 1\).

Instead, if we consider the following R-breaking term \[(41)\]

\[
W_R = \frac{1}{2} \epsilon m X_1^2, \tag{49}
\]

the newly appeared SUSY point is found as

\[
\phi = X_2 = 0, \quad X_1 = -\frac{f}{\epsilon m}.
\]

In this case, the pseudo-moduli space \((42)\) disappears at the tree level. However, the SUSY breaking point \((43)\) remains as a local minimum due to the one-loop mass for \(X_1\), but becomes metastable. Then the situation is similar to the above example. We easily find that any R-breaking terms which consist of only \(\phi\) do not restore SUSY.

Now, let us study whether the SUSY breaking minimum, which is found without R-symmetry breaking terms, is destabilized by adding R-symmetry breaking terms. We consider the generalized OR model with \((r = 2, s = 1)\), i.e., \(W_{(OR)2}\), whose stationary conditions \((46)\) are studied in the previous subsection. Their solutions are denoted by \(X_a = X_{a}^{(0)}\) and \(\phi = \phi^{(0)}\). First, we add a small R-symmetry breaking term, \(W_R = \epsilon w(X_2)\), which depends only on \(X_2\). Then, the scalar potential is written as

\[
V = |f + g'_1(\phi)|^2 + |g_2(\phi) + \epsilon w'(X_2)|^2 + |W_\phi|^2,
\]

\(^{10}\)See also Ref. \[42\].
where \( W_\phi = X_1 g'_1(\phi) + X_2 g'_2(\phi) \). In addition, we assume that the stationary conditions of \( V \) are satisfied by \( X_a = X_a^{(0)} + \delta X_a \) and \( \phi = \phi^{(0)} + \delta \phi \), and that all of \( \delta X_a \) and \( \delta \phi \) are of \( \mathcal{O}(\epsilon) \). For example, the stationary condition along \( \phi, V_\phi = 0 \), gives the following condition,

\[
\left( \sum_a |g'_a(\phi^{(0)})|^2 + \sum_a \left( \bar{f}_a + g_a(\phi^{(0)}) \right) g''_a(\phi^{(0)}) \right) \delta \phi + \epsilon g'_2(\phi^{(0)}) w'(X_2^0) = 0,
\]

where we have used the stationary conditions (46) at \( X_a = X_a^{(0)} \) and \( \phi = \phi^{(0)} \).

This is the equation to determine \( \delta \phi \). The stationary condition along \( X_1, V_{X_1} = 0 \), reduces to

\[
g'_1(\phi^{(0)}) \delta W_\phi = 0,
\]

where

\[
\delta W_\phi = \sum_a g'_a(\phi^{(0)}) \delta X_a + \sum_a X_a^{(0)} g''_a(\phi) \delta \phi.
\]

Thus, this shows a relation among \( \delta X_a \) and \( \delta \phi \) unless \( g'_1(\phi^{(0)}) = 0 \). On the other hand, the stationary condition along \( X_2, V_{X_2} = 0 \), leads to the following equation,

\[
\epsilon w''(X_2^{(0)}) g_2(\phi^{(0)}) = 0.
\]

This is not an equation among \( \delta X_a \) and \( \delta \phi \), but implies that the stationary condition is destabilized unless \( w''(X_2^{(0)}) \bar{g}_1(\phi^{(0)}) = 0 \). In the above basic O’Raifeartaigh model, we have \( g_1(\phi^{(0)}) = 0 \). Thus, the SUSY breaking minimum is not destabilized by adding the mass term of \( X_2, w(X_2) = \frac{1}{2} m X_2 \), i.e., \( w''(X_2) \neq 0 \) at \( X_2 = 0 \).

Now, let us add an R-symmetry breaking term, \( W_R = \epsilon w(X_1) \), which depends only on \( X_1 \). Similarly, we can examine stationary conditions of the scalar potential,

\[
V = |f + g'_1(\phi) + \epsilon w'(X_1)|^2 + |g_2(\phi)|^2 + |W_\phi|^2.
\]

The stationary conditions along \( X_2 \) and \( \phi \) give an equation to determine \( \delta \phi \) and a relation among \( \delta X_a \) and \( \delta \phi \). However, the stationary condition along \( X_1, V_{X_1} = 0 \), leads to

\[
w''(X_1^{(0)}) \left( \bar{f} + \bar{g}_1(\phi^{(0)}) \right) = 0.
\]
If this condition is not satisfied, the stationary condition at the SUSY breaking vacuum is destabilized. Indeed, the basic O’Raifeartaigh model has $f + g_1(\phi) = f$ at $\phi = 0$. Thus, when we add the mass term of $X_1$, $w(X_1) = \frac{1}{2}mX_1^2$, i.e., $w'' \neq 0$, the SUSY breaking minimum becomes destabilized at the tree level as shown above. Note that this kind of destabilization would be related to the existence of the flat direction (47) in the OR model with global SUSY.

The above discussion shows that adding generic R-symmetry breaking terms can destabilize the SUSY breaking minimum, which is found in the model without such explicit R-symmetry breaking terms. In order to realize metastability of the original SUSY breaking minimum, we need a certain type of R-symmetry breaking terms. Alternatively, loop-effects would be helpful not to destabilize the original SUSY breaking minimum by R-symmetry breaking terms.

### 2.3 R-symmetry in supergravity

In the previous section, based on the argument by ISS, we have shown that a certain type of explicit R-symmetry breaking terms can restore SUSY, and the original SUSY breaking vacuum can become metastable when a certain (but not generic) class of explicit R-symmetry breaking terms are added and/or loop effects stabilize the original SUSY breaking minimum. The metastable minimum can be parametrically long-lived if the coefficient of the R-breaking term is sufficiently small with which the SUSY ground state is far from the metastable state in the field space.

This argument has been performed in a decoupling limit of gravity. As we find in the above discussion, however, we have to treat a large distance between some separated minima in the field space. This may imply that large vacuum values of some fields might be involved in the analysis, where supergravity effects could become sizable. Moreover, in global SUSY, the SUSY breaking minima always have a positive vacuum energy with the magnitude of the SUSY breaking scale, which never satisfies the observation that the vacuum energy almost vanishes. In such a sense, we would be forced to consider supergravity.

Note that, even in supergravity, it is often a hard task to tune the vacuum energy at the stationary points of the scalar potential to be almost vanish-

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11 Such flat direction would be lifted by supergravity effects.
ing. This might require a large R-symmetry breaking effect specialized to supergravity, i.e., a constant term in the superpotential \[43\]. The existence of such a special R-symmetry breaking term could also affect the ISS argument of metastability. Loop effects have contributions to the vacuum energy. Here we assume that such loop effects are subdominant, and we tune our parameters such that we realize \( V \approx 0 \) at the tree level. Hereafter we use the unit with \( M_{Pl} = 1 \), where \( M_{Pl} \) denotes the reduced Planck scale.

### 2.3.1 Nelson-Seiberg argument

In this subsection, we study the NS argument within the framework of supergravity theory. In the case of supergravity, F-flat conditions (1) are modified as

\[
D_I W \equiv W_I + K_I W = 0,
\]

where \( K \) denotes the Kähler potential, \( K(|Y|, \chi_i, \bar{\chi}_i) \). In the field basis (2) with the superpotential (3), these are written as

\[
D_{\chi_i} W = Y^{2/q_Y} (\zeta_i + K_i \zeta) = 0,
\]

\[
D_Y W = (2/q_Y + Y K_Y) Y^{2/q_Y - 1} \zeta = 0.
\]

Then, we find the following two candidates of R-breaking SUSY solutions in supergravity,

\[
\zeta_i = 0, \quad \zeta = 0, \quad \text{(50)}
\]

and

\[
D_{\chi_i} \zeta = \zeta_i + K_i \zeta = 0, \quad 2/q_Y + Y K_Y = 0. \quad \text{(51)}
\]

The first conditions (50) contain \( n \) complex equations for \( n - 1 \) complex variables, and the situation is the same as the case of global SUSY (6), that is, the solution does not exist for a generic function \( \zeta \). On the other hand, the second conditions (51) are \( n \) complex equations for \( n \) complex variables which can have a solution. This corresponds to a SUSY stationary point specialized to R-symmetric supergravity.

In this subsection, we analyze the special SUSY stationary solution (51) which appears due to purely the supergravity effect and does not obey the NS condition. Then, in the following we assume that there is a solution for

\[
2/q_Y + Y K_Y = 0. \quad \text{(52)}
\]
For instance, if the Kähler potential is given by
\[ K = \sum_{nY=1}^{c_nY} |Y|^{2nY} + \hat{K}(\chi_i, \bar{\chi}_i), \] (53)
the condition (52) becomes
\[ \frac{2}{qY} + \sum_{nY=1}^{c_nY} nYc_nY|Y|^{2nY} = 0. \]

Then, we need at least one negative value of \( \{c_nY, qY\} \) to have a solution. In the simplest minimal case with \( c_{nY>1} = 0 \) (and then \( K_{YY} = c_1 > 0 \)), a negative charge, \( qY < 0 \), is required.

A nontrivial point of this solution is that this SUSY stationary point is always tachyonic as we can see from the arguments in Appendix A. In addition, we can find a SUSY breaking minima along the direction \( D_{\chi_i}\zeta = 0 \) (the first condition in Eq. (51)), if we assume that \( \chi_i \) receives a heavy SUSY mass \( m_{\chi_i}^2 \gg m_{3/2}^2 \) by the condition \( D_{\chi_i}W = 0 \). This is a reasonable assumption because \( \chi_i \) has a vanishing R-charge and \( \zeta(\chi_i) \) in \( W \) is assumed to be a generic function.

The scalar potential along \( D_{\chi_i}\zeta = 0 \) is found to be
\[ v(Y) = V\big|_{D_{\chi_i}f=0} = e^K(K_{YY}^{-1}|2/qY + K_{YY}|^2 - 3|Y|^2)|Y|^{2(2/qY - 1)}|\zeta|^2. \]

Again, for the minimal Kähler potential (53) with \( c_1 = 1 \) and \( c_{nY>1} = 0 \), the stationary condition
\[ \partial_Y v(Y) = e^{K(\chi_i),\bar{(\chi)}_i)}e|Y|^2|Y|^{2/qY - 2}(2/qY + |Y|^2) \times \left(|Y|^4 + 2(2/qY - 1)|Y|^2 + (2/qY)^2 - 2/qY\right) = 0, \]
leads to solutions
\[ |Y|^2 = -2/qY, \] (54)
and
\[ |Y|^2 = 1 - 2/qY \pm \sqrt{1 - 2/qY}. \] (55)

The first solution (54) corresponds to the SUSY saddle point and the second solutions (55) are SUSY breaking minima. We can find this kind of SUSY breaking minima in a similar way for more generic Kähler potential.
We can study the same system in a different viewpoint. We redefine the field \( Y \) as
\[
T = -\frac{2}{aq_Y} \ln Y,
\]
where \( a \) is a real constant. In this basis, the Kähler potential and the superpotential (3) is written as
\[
K = K(T + \bar{T}, \chi_i, \bar{\chi}_i),
\]
\[
W = e^{-aT} \zeta(\chi_i).
\]
This type of Kähler and superpotential appear in the four-dimensional effective theory derived from superstring theory, where \( T \) may be a modulus field associated to some compactified dimensions. In such a case, the Kähler potential is typically given by
\[
K = -n_T \ln(T + \bar{T}) + \hat{K}(\chi_i, \bar{\chi}_i),
\]
where \( n_T \) is a fractional number, and the \( T \)-dependence of the superpotential (57) may originate from nonperturbative effects such as string/D-brane instanton effects and gaugino condensation effects, where the corresponding gauge coupling is determined by the vacuum value of \( T \). In this case, the scalar potential along \( D\chi_i\zeta = 0 \) is given by
\[
v(T) = V|_{D\chi_i,f=0} = e^K \left( K_{TT}^{-1}(K_T - a)^2 - 3 \right) |e^{-aT} \zeta|^2,
\]
and then the stationary condition
\[
\partial_t v(t) = -e^{K(\chi_i,\bar{\chi}_i)} e^{-at} e^{-aT - n_T - 1} \times \left( at + n_T \right) \left( (a^2/n_T) t^2 + 2a(1 - 1/n_T) t + n_T - 3 \right) = 0,
\]
results in a SUSY saddle point \( t = -n/a \) and SUSY breaking minima
\[
t = -(n_T/a)(1 - 1/n_T) \pm (n_T|a|/a^2) \sqrt{5/n_T + 1/n_T^2},
\]
where \( t = T + \bar{T} \).
In the literature, there are examples of the models which have this kind of vacuum structure of the potential. Typical superstring models have several moduli $T_i$ with the Kähler potential $K = \ln \prod_i (T_i + \bar{T}_i)^{-\alpha_{11}}$. The superpotential induced by some nonperturbative effects is given by

$$W = \sum_n A_n e^{\sum_i a_i^n T_i},$$

where $A_n$ and $a_i^n$ are constants. If the number of the moduli is the same as or larger than the number of the nonperturbative terms appearing in the superpotential [44], we can define an R-symmetry. A particular linear combination of $T_i$’s corresponds to $T$ in Eq. (57) which is determined by the condition that all the remaining combinations corresponding $\chi_i$’s receive a heavy mass by the SUSY condition $D\chi_i W = 0$. This is possible for certain values of $a_i^n$. For the two moduli with double nonperturbative terms, i.e., racetrack models, a detailed analysis was carried out in Ref. [45].

We stress that the analysis of the SUSY breaking minimum as well as the SUSY saddle point in this subsection is based on the assumption that all the other fields $\chi_i$ than $Y$ or $T$ are stabilized by $D\chi_i W = 0$, that is, by the SUSY masses [46]. We comment that these stationary solutions have a nonvanishing and negative vacuum energy. We need to uplift the SUSY breaking minimum to a Minkowski vacuum in order to identify this minimum as the one we are living. For such purpose, we need another sector which provides the uplifting energy and is well sequestered in order not to spoil the original structure of dynamical SUSY breaking. Such sector can be realized by a dynamically generated F-term [47, 48] for which the discussions in the following sections would be important.

In summary, there is a possibility of special SUSY stationary solution in R-symmetric supergravity with a generic superpotential. However, it is always a saddle point at best and we find SUSY breaking minima with lower vacuum energy. This may imply that the NS argument for a dynamical SUSY breaking is qualitatively correct also in this case, although there is a SUSY solution. Furthermore, the NS argument still holds in supergravity as long as the Kähler potential satisfies $2/q_Y + Y K_Y \neq 0$ for any value of $Y$ in the field basis (2). For instance, in typical models with $q_Y > 0$ and $K = |Y|^2$, we always find $2/q_Y + Y K_Y > 0$. 

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2.3.2 Generalized O’Raifeartaigh model in supergravity

Now we consider the generalized OR model (39) in supergravity. The F-flat conditions (40) for \(X_a\) become

\[
D_{X_a} W = \partial_{X_a} W + (\partial_{X_a} K) W = \sum_b M_{ab}(X_c, \phi_i) (g_b(\phi_i) + \delta_{b1} f) = 0, \tag{58}
\]

where

\[
M_{ab}(X_c, \phi_i) = \delta_{ab} + K_{X_a} X_b.
\]

We define its determinant as

\[
\Delta \equiv \det M_{ab} = 1 + \sum_a K_{X_a} X_a. \tag{59}
\]

If there is no solution for \(\Delta = 0\), the matrix \(M_{ab}\) has an inverse matrix and consequently the F-flat conditions (58) are reduced to the same ones as Eq. (40) in the global SUSY,

\[
g_a(\phi_i) + \delta_{a1} f = 0,
\]

which does not allow a solution for \(r > s\) in general. However, in the limit \(f \to 0\) in the tilde basis (44), these equations are satisfied at \(\phi_i = 0\). Thus, the constant \(f\) represents the typical size of SUSY breaking effects and \(g_a(\phi_i)\) as the global SUSY case. We comment that the situation changes if there exists a solution of \(\Delta = 0\). Actually, the condition \(\Delta = 0\) is an analogue of the second condition in Eq. (51). Then, we can carry out a similar analysis as in the previous subsection also for this OR model. That is straightforward and is omitted here. Note that the condition \(\Delta = 0\) is never satisfied for a minimal Kähler potential,

\[
K = \sum_a |X_a|^2 + \sum_i |\phi_i|^2. \tag{60}
\]

In the following, we just assume that there is no solution for \(\Delta = 0\).

We comment that, even in supergravity, the scalar potential is positive, \(V > 0\), in the generalized OR model (44) with the minimal Kähler potential (60). In this case, the scalar potential is written as

\[
V = e^K \left[ (\bar{g}_a + \delta_{a1} \bar{f}) \{\delta_{ab} + (|X_c|^2 - 1) \bar{X}_a X_b\} (g_b + \delta_{b1} f) + |X_a D_{\phi_i} g_a|^2 \right].
\]
For any vacuum values of $X_a$, we can always rotate their basis as

$$U_{ab} X_b = \hat{X}_a = (0, \ldots, 0, \hat{X}_c, 0, \ldots, 0),$$

by a unitary matrix $U(X_a)$, and in this basis we can write

$$e^{-K} V = \left\{ (|\hat{X}_c|^2 - 1/2)^2 + 3/4 \right\} |\hat{g}_c|^2 + \sum_{a \neq c} |\hat{g}_a|^2 + \sum_i |\hat{X}_c D\phi_i \hat{g}_c|^2 > 0,$$

where $\hat{g}_a = (U^\dagger)_{ab} (g_b + \delta_{b1} f)$. Note that $\hat{g}_a$ are now $X_a$-dependent functions. As discussed above, the conditions, $\hat{g}_a(\phi) = 0$, can not be satisfied at the same time. Thus, the vacuum energy must be positive, $V > 0$. Since typical magnitudes of $\hat{g}_a(\phi)$ would be of $O(f)$, we would estimate $V \sim f^2$. To realize the almost vanishing vacuum energy $V \approx 0$ at this SUSY breaking minimum, we need a negative and sizable contribution to the vacuum energy, which can be generated by R-symmetry breaking effects, e.g., the constant term in the superpotential.

We would find the features like this in the models whose superpotentials do not have quadratic terms of $X_a$, $m_{ab} X^a X^b$

$$W = X_a g^a(\phi, \Phi_I) + \hat{w}(\phi_i, \Phi_I), \quad (61)$$

where the R-charges of $\Phi_I$ are not zero or 2. This is because $\Phi_I = 0$ for all $I$ could be a solution of the F-flat conditions for $\Phi_I$. This leads the same situation as the above because of $\hat{w}(\phi_i, \Phi_I) = 0$. However, this argument is formed on the slice, $\Phi_I = 0$, and it is not easy to discuss whether vacua with negative vacuum energy exist in the directions, $\Phi_I \neq 0$.

### 2.4 Explicit R-symmetry breaking in supergravity

Here we study explicit R-symmetry breaking terms in supergravity and examine whether SUSY solutions can be found by adding explicit R-symmetry breaking terms to the NS model and the generalized OR model. In the previous section, we have pointed out that there is a SUSY stationary point when the condition (52) or the condition $\Delta = 0$ is satisfied. In the following sections, we consider the models, where such conditions are not satisfied, and SUSY is broken in the NS and generalized OR models even within the framework of supergravity like global SUSY theory.
First we consider the NS model with explicit R-symmetry breaking terms $W_R = w(Y, \chi_i)$. The total superpotential is written as,

$$W = Y^{2/\eta} \zeta(\chi_i) + w(Y, \chi_i).$$

In this case, F-flat conditions of supergravity theory, $D_Y W = D_{\chi_i} W = 0$, do not lead to over-constrained conditions for any non-vanishing function $w(Y, \chi_i)$. It is remarkable that within the framework of supergravity theory the constant term $W_R = c$ breaks R-symmetry and even such term is enough to relax the over-constrained conditions.

### 2.4.1 Generalized O’Raifeartaigh model

Let us study more explicitly the generalized OR model with explicit R-symmetry breaking terms $W_R = w(X_a, \phi_i)$. The total superpotential is written as,

$$W = f X_1 + \sum_{a=1}^{r} g_a(\phi_i) X_a + w(X_a, \phi_i).$$

First, we consider the case with the constant R-symmetry breaking term, $W_R = c$. In this case, F-flat conditions are written explicitly as

\begin{align*}
D_{X_a} W &= f \delta_{a1} + g_a(\phi_i) + K_{X_a} \left( f X_1 + \sum_{a=1}^{r} g_a(\phi_i) X_a + c \right) = 0, \\
D_{\phi_i} W &= \sum_a X_a \partial_i g_a(\phi_i) + K_{\phi_i} \left( f X_1 + \sum_{a=1}^{r} g_a(\phi_i) X_a + c \right) = 0.
\end{align*}

The former conditions are not always over-constrained for $c \neq 0$. Furthermore, the vacuum expectation value of $W$ and at least $(r - s)$ vacuum values of $K_{X_a}$ are required to be non-vanishing. Otherwise, the former conditions become over-constrained for generic functions $g_a(\phi_i)$. Furthermore, when $K_{X_a}$ for $a \neq 1$ does not vanish, all vacuum values of $\phi_i$ can not vanish to satisfy $D_{X_a} W = g_a(\phi_i) + K_{X_a} W = 0$. Thus, a SUSY solution can be found by adding $W_R = c$. This solution corresponds to the AdS SUSY minimum, because non-vanishing \langle W \rangle is required and the scalar potential at this point is evaluated as $V = -3e^K |W|^2 < 0$. The values of the constant $c$ and \langle W \rangle must be sizable, because this AdS SUSY point disappears in the limit that $c \to 0$. 

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or $\langle W \rangle \to 0$. Magnitudes of $c$ and $\langle W \rangle$ are expected to be comparable with $f$ when $K_{X_a} = O(1)$. Hence, we can find the new type of SUSY solution, which can not be found in global SUSY theory. However, that requires large values of $c$ and $\langle W \rangle$, and may have sizable effects on the previous SUSY breaking minimum, which is found in the generalized OR model without R-symmetry breaking terms.

Similarly, we can discuss the case that R-symmetry breaking terms include only $\phi_i$ fields, i.e., $W_R = w(\phi_i)$. In this case, F-flat conditions along $X_a$, $D_{X_a}W = 0$, are written as

$$D_{X_a}W = f \delta_{a1} + g_a(\phi_i) + K_{X_a} \left( f X_1 + \sum_{a=1}^{r} g_a(\phi_i) X_a + w(\phi_i) \right) = 0.$$ 

Thus, the situation is quite similar to the case with $W_R = c$. To have a SUSY solution, it is required that $\langle W \rangle$, $\langle w(\phi_i) \rangle$ and at least $(r-s)$ vacuum values of $K_{X_a}$ must be non-vanishing. Sizes of $\langle W \rangle$ and $\langle w(\phi_i) \rangle$ are expected to be comparable with $f$.

Finally, we consider the case that R-symmetry breaking terms include $X_a$ fields, $W_R = w(X_a, \phi_i)$. In this case, F-flat conditions along $X_a$, $D_{X_a}W = 0$, are written as

$$D_{X_a}W = f \delta_{a1} + g_a(\phi_i) + \partial_{X_a} w(X_a, \phi_i) + K_{X_a} W = 0.$$ 

When $K_{X_a} W$ is sufficiently small, the above F-flat conditions correspond to F-flat conditions in global SUSY theory. In such a case, we have a SUSY solution when $w(X_a, \phi_i)$ depend on at least $(r-s)$ $X_a$'s. Otherwise, the global SUSY solution can not be found, but a SUSY solution with $\langle w(X_a, \phi_i) \rangle \neq 0$ and $\langle W \rangle \neq 0$ can be found within the framework of supergravity theory. Such situation is similar to the case with $W_R = c$.

We have discussed that the NS model and generalized OR model with R-symmetry breaking terms have SUSY solutions with $\langle W \rangle \neq 0$ in supergravity theory. If the SUSY breaking minimum, which is found without R-symmetry breaking terms, is not destabilized by the presence of R-symmetry breaking terms, the previous SUSY breaking minimum would correspond to a SUSY breaking metastable vacuum. However, a sizable vacuum value of superpotential is required unless $\partial_{X_a} w(X_a, \phi_i) \neq 0$ for at least $(r-s)$ $X_a$ fields. Such large superpotential (even if that is a constant term) would affect the stability of the previous SUSY breaking minimum.
Furthermore, we have another reason to have a large size of \( \langle W \rangle \) at the previous SUSY breaking minimum. At the previous SUSY breaking minimum, the vacuum energy is estimated as \( V \sim |f|^2 > 0 \) for \( \langle W \rangle = 0 \). To realize the almost vanishing vacuum energy, \( V \approx 0 \), we need a non-vanishing value of \( \langle W \rangle \), which are comparable with \( f \). In this case, supergravity effects at the previous SUSY breaking minimum are not negligible. This purpose to realize \( V \approx 0 \) has the implication even for the case that R-symmetry breaking terms include more than \( (r-s) \) \( X_a \) fields. In this case, we can find a (global) SUSY solution even for \( \langle W \rangle = 0 \). However, realization of \( V \approx 0 \) requires a sizable vacuum value of \( \langle W \rangle \), although values \( \langle W \rangle \) at the SUSY breaking minimum and SUSY preserving minimum are not the same. Hence, it is quite non-trivial whether one can realize a metastable SUSY breaking vacuum with \( V \approx 0 \) in supergravity theory, which has a SUSY minimum. We will study this possibility concretely by using simple classes of the generalized OR models in the following sections. We will concentrate ourselves to the minimal Kähler potential (60) in most cases of the following discussions.

### 2.4.2 Classification of R-breaking terms in supergravity

In this subsection and the following sections, we consider minutely the previous discussions about the explicit R-symmetry breaking in the supergravity framework by examining concrete examples. We introduce the explicit R-symmetry breaking terms \( W_R \) into the above supergravity OR model,

\[
W_R = c(\phi_i) + \frac{1}{2} \sum_{a,b} m \epsilon_{ab}(\phi_i) X_a X_b + \cdots ,
\]

(62)

where \( c(\phi_i) \) and \( \epsilon_{ab}(\phi_i) \) are generic functions of \( \phi_i \) including \( \phi \)-independent constants, and the ellipsis denotes the higher order terms in \( X_a \). Note that, as mentioned before, only the \( \epsilon_{ab}(\phi_i) \) terms are relevant to the recovery of SUSY vacua in the case of global SUSY. Now we have the total superpotential, \( W = W_{OR} + W_R \). The F-flat conditions (45) are modified as

\[
D_{X_a} W = \sum_b M_{ab} (g_b(\phi_i) + \delta_{a1} f_1 + \sum_{c,d} M_{bc}^{-1} \epsilon_{cd}(\phi_i) X_d + \Delta^{-1} K_{X_b} W_R ) = 0 .
\]

(63)

Here we find that all the terms in \( W_R \) including \( c(\phi_i) \) are accompanied by \( X_a \) in the above F-flat conditions and then have a possibility for restoring SUSY, contrary to the case of global SUSY explained in the previous subsection.
Most notably, just a constant superpotential

\[ W_R = c, \]  

i.e., \( c(\phi_i) = c \) and \( \epsilon_{ab}(\phi_i) = 0 \), can restore SUSY. In this case with the minimal Kähler potential \( (60) \), we find a solution for Eq. \( (63) \) as

\[ \bar{X}_a = -c^{-1} \Delta g_a(\phi_i), \]  

where \( \Delta = 1 + \sum |X_a|^2 \) defined in Eq. \( (59) \) is real and positive. From Eq. \( (65) \), \( X_a \) can be written in terms of \( \phi_i \), and then \( \Delta \) is given by

\[ \Delta = \frac{|c|^2 \pm |c| \sqrt{|c|^2 - 4 \sum |g_a(\phi_i)|^2}}{2 \sum |g_a(\phi_i)|}, \]

which should be a real number. Therefore, in order for the SUSY solution \( (65) \) to be valid, the constant superpotential \( c \) must satisfy the condition

\[ 4 \sum |g_a(\phi_i)|^2 \leq |c|^2, \]

where \( \langle \phi_i \rangle \) are solutions of \( D\phi_i W = 0 \) under the condition \( (65) \).

Because \( X_1 \) is distinguished in the superpotential \( (44) \), we divide the generic R-breaking terms \( (62) \) into two parts:

\[ W_R = W_R^{(A)} + W_R^{(B)}, \]

where

\[ W_R^{(A)}(X_{a\neq 1}; \phi_i) = c(\phi_i) + \frac{1}{2} \sum_{a,b\neq 1} m \epsilon_{ab}(\phi_i) X_a X_b + \cdots, \]

\[ W_R^{(B)}(X_1; X_{a\neq 1}, \phi_1) = \sum_{a\neq 1} m \epsilon_{a1}(\phi_1) X_a X_1 + \frac{1}{2} m \epsilon_{11}(\phi_1) X_1^2 + \cdots. \]

The ellipses denote the higher order terms in terms of \( X_{a\neq 1} \) in \( W_R^{(A)} \), and those of \( X_1 \) and \( X_{a\neq 1} \) in \( W_R^{(B)} \). Without loss of generality, we can assume that \( \epsilon_{11}(0) \) is real and positive among \( \epsilon_{ab}(0) \), which is referred as \( \epsilon \) in Sec. 2.6.
2.5 Type-A breaking: Polonyi-like models

In this section, we study the effect of R-breaking terms \( \mathbf{67} \), which we call the A-type breaking, that is, the total superpotential is written by

\[
W = W_{OR} + W_R^{(A)}.
\]

Because this type of breaking terms does not contain \( X_1 \), we find the Polonyi model \([49]\]

\[
W \big|_{X_{a \neq 1} = 0, \phi_i = 0} = W_{\text{Polonyi}} \equiv f X_1 + c,
\]

in the hypersurface \( X_{a \neq 1} = 0, \phi_i = 0 \) of the scalar potential, where \( c = c(0) \). This hypersurface would be a stationary plane in the \( X_{a \neq 1} \)- and the \( \phi_i \)-directions if \( \partial_{\phi_i} g_{a \neq 1}(0) \) are sufficiently large, which correspond to SUSY masses for \( X_{a \neq 1} \) and \( \phi_i \) on that plane.

Moreover, if \( m_i^1 \) and/or \( h_{ij}^1 \) in

\[
g_1(\phi_i) = m_i^1 \phi_i + h_{ij}^1 \phi_i \phi_j + \cdots,
\]

are nonvanishing, the Polonyi model in this hypersurface can be affected/modified by a tree-level SUSY mass and/or a one-loop SUSY breaking mass for \( X_1 \). Then, we further classify the A-type breaking models into two cases, \( g_1(\phi_i) = 0 \) and \( g_1(\phi_i) \neq 0 \).

2.5.1 Decoupled case: \( g_1(\phi_i) = 0 \)

In the case with \( g_1(\phi_i) = 0 \), the superpotential of the A-type breaking models is written as

\[
W = f X_1 + \sum_{a \neq 1} g_a(\phi_i) X_a + c(\phi_i) + \frac{1}{2} \sum_{a, b \neq 1} m_{\epsilon ab}(\phi_i) X_a X_b + \cdots
\]

\[
= c + f X_1 + \frac{1}{2} \mu_{AB} \Phi_A \Phi_B + \cdots,
\]

where \( \Phi_A = (X_{a \neq 1}, \phi_i) \) with the index \( A = (a \neq 1, i) \). The SUSY mass matrix \( \mu_{AB} \) is given by the R-breaking components, \( \mu_{a \neq 1, b \neq 1} = m_{\epsilon ab}(0) \), \( \mu_{ij} = \partial_{\phi_i} \partial_{\phi_j} c(0) \) and the R-symmetric components, \( \mu_{a \neq 1, i} = 2 \partial_{\phi_i} g_a(0) \). After the unitary rotation which makes \( \mu_{AB} \) diagonal, the above superpotential takes the form of

\[
W = c + f X_1 + \frac{1}{2} \mu_A \Phi_A^2 + \cdots,
\]

(71)
where $\mu_A$ represents the eigenvalues of $\mu_{AB}$. Because of the SUSY mass $\mu_A$, the field $\Phi_A$ would be integrated out without affecting the low energy dynamics of $X_1$, because $X_1$ is completely decoupled in the present case\footnote{We may have to assume that the Kähler mixing is also zero or negligible between $X_1$ and the others.}

Then, the effective action for $X \equiv X_1$ is just determined by the Polonyi superpotential (69), where the phase of $c$ and $f$ can be eliminated by the $U(1)_R$ rotation and the rephasing of $X_1$. Assuming the minimal Kähler potential (60) for simplicity, the effective scalar potential is minimized by a real vacuum value $X = \bar{X} = x$ satisfying the stationary condition

$$V_X = e^G G_X (G_{XX} + G_X^2 - 2) = 0,$$

where $G = K + \ln |W|^2$ and

$$G_{XX} + G_X^2 - 2 = f W^{-1} (x^3 + f^{-1} c x^2 - 2 f^{-1} c),$$
$$G_X = f W^{-1} (x^2 + f^{-1} c x + 1).$$

(72) \hspace{1cm} (73)

The F-flat condition for $X$ corresponds to $G_X = 0$, and the SUSY breaking stationary point is determined by the condition $G_{XX} + G_X^2 - 2 = 0$.

As we declared, we persist in obtaining a vanishing vacuum energy at the SUSY breaking minimum. Then in addition to the stationary condition $G_{XX} + G_X^2 - 2 = 0$, we set $V = e^G (G_{XX} |G_X|^2 - 3) = 0$. In this case, we have to take a definite value of the constant $c$ and find two solutions

$$(x, f^{-1} c) = (\sqrt{3} - 1, 2 - \sqrt{3}),$$

(74) \hspace{1cm} (75)

and

$$(x, f^{-1} c) = (-\sqrt{3} - 1, 2 + \sqrt{3}).$$

The mass eigenvalues of $(\text{Re} X, \text{Im} X)$ are computed as $(2\sqrt{3} f^2, (4 - 2\sqrt{3}) f^2)$ for the first solution (74), and $(-2\sqrt{3} f^2, (4 + 2\sqrt{3}) f^2)$ for the second one (75), at this SUSY breaking Minkowski stationary point where $W = f$. Then, only the first solution (74) can be a minimum of the potential, while the second one (75) is a saddle point. We comment that $\phi_i$ and $X_{\alpha \neq 1}$ directions would not possess tachyonic masses at these points for sufficiently large SUSY mass $\mu_A$ compared with the SUSY breaking mass $f$. Therefore, the candidate for our present universe, where the SUSY is broken with (almost) vanishing vacuum energy, is the first solution (74).
In addition to a SUSY breaking solution satisfying $G_{XX} + G_X^2 - 2 = 0$, we have a SUSY solution $G_X = 0$ due to the R-breaking effect $c \neq 0$, that is,

$$x_\pm = \frac{1}{2}(-f^{-1}c \pm \sqrt{(f^{-1}c)^2 - 4}),$$  \hspace{1cm} (76)$$

if the R-breaking constant $c$ satisfies

$$|f^{-1}c| \geq 2.$$  \hspace{1cm} (77)

Note that this condition (77) corresponds to Eq. (66) in the previous general argument for the generalized OR model. The mass eigenvalues of $(\text{Re}X, \text{Im}X)$ are computed as $W_\pm^2(x_\pm^2 - 2)(x_\pm^2 + 1)$ and $W_\pm^2(x_\pm^2 - 1)(x_\pm^2 + 2)$ at this SUSY AdS stationary point where

$$|W_\pm| = |fx_\pm + c| = \frac{1}{2} \left| f(f^{-1}c \pm \sqrt{(f^{-1}c)^2 - 4}) \right| > 0,$$

and then we obtain

$$V = -3e^G = -3e^{x_\pm^2}|W_\pm|^2 < 0.$$  

Remark that, in the vanishing (one of) R-breaking limit, $c \to 0$, the condition (77) is not satisfied, and the SUSY solution (66) disappears. In the other words, this SUSY solution is a consequence of the R-breaking constant term $c$ in the superpotential. Due to the appearance of this SUSY solution, there is a possibility that the SUSY breaking point determined by $G_{XX} + G_X^2 - 2 = 0$ becomes a metastable vacuum as in the case of global SUSY explained previously.

However, this is not the case. Interestingly, if we tune the R-breaking constant superpotential $c$ as $f^{-1}c = 2 - \sqrt{3}$ so that the solution (74) with the vanishing vacuum energy is realized, the condition (77) is not satisfied and the SUSY stationary solution (76) disappears. In such a sense, the constant R-breaking term $c$ does not lead to a metastability of SUSY breaking Minkowski minimum (74).

Next, we consider the SUSY stationary solutions outside the Polonyi slice $X_{a \neq 1} = 0, \phi_i = 0$. For the superpotential (71), the F-flat directions are determined by

$$D_{\Phi_A} W = K_A W + \mu_A \Phi_A + \cdots = 0,$$
$$D_{X_1} W = f + K_{X_1} W = 0,$$
which can be satisfied by distinguishing a single field $\Phi_B \neq 0$ for $^3B$ as

$$
W = -K^{-1}_B \Phi_B(\mu_B + \cdots) = -K^{-1}_{X_1}f \quad \text{(for } ^3B),
$$

$$
\Phi_A = K_A = 0 \quad \text{(for } A \neq B),
$$

(78)

where the ellipsis represents the higher order terms of $\Phi_B$. The first line gives two complex equations for two complex variables $X_1$ and $\Phi_{^3B}$, which have a solution in general.

For example, if the Kähler potential is minimal (60), all the parameters in the superpotential are real and there is no higher order terms of $\Phi_B$ (no ellipses in the above expressions), then the solution for Eq. (78) is found as

$$
\Phi^2_B = -2 \left( \frac{c}{\mu_B} + \frac{f^2}{\mu^2_B} + 1 \right) > 0, \quad \Phi_{A \neq B} = 0.
$$

(79)

For this value of $\Phi_B$, the remaining condition $D_{X_1}W = 0$ is satisfied by

$$X_1 = f/\mu_B.$$

Note that the number of these SUSY points is $n_X + n_\phi - 1$ because the solution (79) is valid for every choice of $B = (b \neq 1, j)$. In order for the solution (79) to be valid, the parameter $\mu_B$ must satisfy

$$\mu^2_B + c\mu_B + f^2 \leq 0.$$

This leads to the same condition (77) for the R-breaking constant term $c$ as in the Polonyi-type SUSY solution.

In summary, the A-type breaking terms (67) can restore SUSY in the generalized OR model (39) or equivalently (44) in general. However, if we tune the R-breaking constant term in the superpotential so that the SUSY breaking minimum has a vanishing vacuum energy, i.e., (74), the SUSY solutions (76) and (79) disappear. Therefore, in this sense, the A-type R-symmetry breaking terms do not lead to a metastability of the SUSY breaking (Minkowski) vacuum aside from a possibility of the existence of more complicated SUSY solutions than (79).

### 2.5.2 Generic case: $g_1(\phi_i) \neq 0$

Now we turn on a nonvanishing $g_1(\phi_i)$ as in Eq. (70). With this term, the tree-level (field dependent) mass matrices in the $\phi_i = 0$ plane contain the
following contributions,

\[
V_{X_1X_1}|_{\phi_i=0} = |m_1^i|^2 + \cdots ,
V_{\phi_i\phi_j}|_{\phi_i=0} = m_1^i m_1^j + 4 h_1^{ik} \bar{h}_1^{\bar{j}k} |X_1|^2 + \cdots ,
V_{\phi_i\phi_j}|_{\phi_i=0} = h_1^{ij} \bar{f} + \cdots ,
V_{X_1\phi_i}|_{\phi_i=0} = 2 h_1^{ij} \bar{m}_1^i X_1 + \cdots ,
\]

(80)

where the ellipses represent the original terms involving \(X_{a\neq 1}\), those coming from \(c(\phi_i)\), and the supergravity corrections. Here the doubled indices are summed up. The Kähler covariant derivatives of the superpotential in the hypersurface \(\phi_i = 0, X_a \neq 1 = 0\) are given by

\[
D_{X_1} W |_{\phi_i=0} = f + K_{X_1} W, \quad D_{X_{a\neq 1}} W |_{\phi_i=0} = 0, \quad D_{\phi_i} W |_{\phi_i=0} = m_1^i X_1 .
\]

From the third equation, we find that \(\phi_i\) can not be integrated out prior to \(X_1\) by the F-flat condition \(D_{\phi_i} W = 0\) unlike before. This is because, with the nonvanishing \(m_1^i\), the source field \(X_1\) for SUSY breaking shares a common SUSY mass with \(\phi_i\) as shown in Eq. (80).

In this case, the purely \(X_1\)-direction is no longer special in the scalar potential. We have to treat \(X_1\) and \(\phi_i\) at the same time. The analysis is quite complicated, and then we consider the case with \(m_1^i = 0\) in the following, where \(g_1(\phi_i)\) starts from the quadratic term in \(\phi_i\), and the \(h_1^{ij}\) can be integrated by their F-flat conditions \(D_{\phi_i} W = 0\) resulting \(\phi_i = 0\). We will comment about the case with \(m_1^i \neq 0\) in Sec. 2.6.2 together with more general R-breaking terms. The components of the mass matrices (80) are now reduced to

\[
V_{\phi_i\phi_j}|_{\phi_i=0} = 4 h_1^{ik} \bar{h}_1^{\bar{j}k} |X_1|^2 + \cdots , \quad V_{\phi_i\phi_j}|_{\phi_i=0} = h_1^{ij} \bar{f} + \cdots .
\]

From the second equation, we observe that some linear combinations of \(\text{Re} \phi_i\) and \(\text{Im} \phi_j\) become tachyonic in the \(\phi_i = 0\) plane if \(|h_1^{ij} \bar{f}|\) dominate the SUSY mass for \(\phi_i\). The \(X_1\)-dependence in the first one indicates that a SUSY breaking mass of \(X_1\) is generated at the one-loop level, which is proportional to \(h_1^{ij}\).

Therefore, the effective potential after integrating out \(\phi_i\) and \(X_{a\neq 1}\) is given by

\[
V = V^{(0)} + V^{(1)}, \quad V^{(0)} = e^G (G^{XX} |G_X|^2 - 3), \quad V^{(1)} = m_X^2 |X|^2 ,
\]

(81)
where $X \equiv X_1$, $G = K + \ln |W|^2$, and the effective superpotential $W = W_{\text{Polonyi}}$ is shown in Eq. (69). The one-loop mass $m_X$ is determined by $h_{ij}^{(1)}$ as well as $f$, which would be considered as an independent parameter in the effective action. The stationary condition $V_X = 0$ results in

$$X \simeq 2fc/m_X^2,$$

for $c \sim f \sim m_X \ll 1$ in the unit with $M_{Pl} = 1$, and the vanishing vacuum energy at this minimum requires

$$c = f/\sqrt{3} + \mathcal{O}(f^3/m_X^2).$$

The SUSY is broken at this Minkowski minimum with $D_XW = f + \mathcal{O}(f^2)$ and $W = f/\sqrt{3} + \mathcal{O}(f^2)$.

### 2.6 Adding type-B breaking: Metastable universe

In the previous section, we have analyzed the generalized OR model with the explicit R-symmetry breaking terms (68) which do not involve the source field $X_1$ for the dynamical SUSY breaking. In this section, we study more general case with the R-breaking terms (67) including $X_1$, i.e.,

$$W = W_{\text{OR}} + W_{\text{R}}^{(A)} + W_{\text{R}}^{(B)}.$$

In the type-B breaking terms (67), the first term with $\epsilon_{\alpha \neq 1,1}(0)$ gives the common SUSY mass for $X_1$ and $X_{\alpha \neq 1}$ in the $\phi_i = 0$ plane. Then the situation is similar to the case with a nonvanishing $m_i^1$ in Eq. (70), that is, we can not integrate out $X_{\alpha \neq 1}$ prior to $X_1$, and we will include this case also in Sec. 2.6.2.

By setting $\epsilon_{\alpha \neq 1,1}(0) = 0$, the superpotential in the hypersurface $\phi_i = X_{\alpha \neq 1} = 0$ is given by

$$W = fX + \frac{1}{2}m\epsilon X^2 + c + \cdots,$$

where $X \equiv X_1$, $\epsilon = \epsilon_{11}(0)$ and the ellipsis stands for the higher order terms in $X$. 

43
2.6.1 Decoupled case: \( g_1(\phi_i) = 0 \)

As in the previous section, we first consider the case with \( g_1(\phi_i) = 0 \), where \( X_1 \) is decoupled from the others in the superpotential. In this case the hypersurface \( \phi_i = X_{a \neq 1} = 0 \) would be stable in the \( \phi_i \)-, \( X_{a \neq 1} \)-direction as in Sec. 2.5.1. The effective theory in this slice is described by the superpotential \( \langle 82 \rangle \).

With the minimal Kähler potential \( \langle 60 \rangle \), real parameters \( f, c, m \) and no higher order terms (ellipsis) in the superpotential \( \langle 82 \rangle \) for simplicity, the SUSY breaking and SUSY stationary conditions are respectively given by Eqs. \( \langle 72 \rangle \) and \( \langle 73 \rangle \). In the limit \( \epsilon \to 0 \) of Eq. \( \langle 82 \rangle \), the SUSY breaking solution is given by Eq. \( \langle 74 \rangle \). Then we can find the deviation of \( X \) from this point assuming \( \epsilon \ll 1 \) and \( m \sim c^{1/3} \sim f^{1/2} \). We find a SUSY breaking minimum with a vanishing vacuum energy at

\[
X_{SB} = X_0 + \delta X, \quad X_0 = \sqrt{3} - 1, \quad \delta X = \frac{em}{2f} + \mathcal{O}(\epsilon^2), \quad \langle 83 \rangle
\]

where the constant superpotential term \( c \) is tuned as

\[
c = (2 - \sqrt{3})f + (2\sqrt{3} - 3)em + \mathcal{O}(\epsilon^2). \quad \langle 84 \rangle
\]

On the other hand, a SUSY solution,

\[
X_{SUSY} \simeq -\frac{2f}{em}, \quad \langle 85 \rangle
\]

arises as a consequence of the B-type R-breaking term represented by the parameter \( \epsilon \), although the vacuum energy is set to be vanishing at the SUSY breaking minimum. This is unlike the case of SUSY solutions \( \langle 76 \rangle \) and \( \langle 79 \rangle \) caused by the introduction of A-type R-breaking terms \( \langle 67 \rangle \). The shift of SUSY breaking minimum \( \delta X \) in Eq. \( \langle 83 \rangle \) is rewritten as

\[
\delta X/X_0 \simeq \frac{1}{\sqrt{3} - 1} \frac{1}{X_{SUSY}},
\]

and we find

\[
|X_{SUSY}| > \frac{1}{\sqrt{3} - 1} \sim \mathcal{O}(1),
\]

in order for the shift \( \delta X \) to reside in a perturbative region, \( |\delta X/X_0| < 1 \).

This means that the vacuum value of \(|X|\) at the newly appeared SUSY vacuum must be larger than the Planck scale \( M_{Pl} = 1 \), where the supergravity
Figure 1: Parameter region (white) of $\mu_B$ and $\epsilon$ allowing the SUSY solution (86). All the parameters are assumed to be real and the constant term $c$ is fixed by the vanishing vacuum energy condition (84) at the SUSY breaking minimum (83). In the shaded region, the SUSY solution (86) is not allowed and the SUSY breaking solution (83) does not become metastable due to the R-breaking effect parameterized by $\epsilon$. We find no allowed region in the limit $\epsilon \to 0$ which corresponds to the solution (79).

calculation might not be valid. It would be possible that the potential is lifted for $|X| > 1$ by the effect of quantum gravity, the above SUSY vacuum is washed out and the SUSY breaking minimum remains as a global minimum. If the supergravity approximation is valid even for $|X| > 1$ by any reason, we obtain a constraint on the R-breaking parameter $\epsilon$ as

$$\epsilon < 2(\sqrt{3} - 1)|f/m|,$$

from the above condition.

We also find a SUSY minimum outside the hyperplane $\phi_i = X_{a \neq 1} = 0$, which is a generalization of Eq. (79), given by

$$\Phi_B^2 = -\frac{2}{\mu_B} \left\{ \mu_B + c + \frac{f^2}{\mu_B^2} \left( \frac{c}{\mu_B^2} - \frac{\epsilon m}{2(\mu_B - \epsilon m)} \right) \right\} \geq 0,$$

$$\Phi_{A \neq B} = K_{A \neq B} = 0, \quad X = \frac{f}{\mu_B - \epsilon m},$$

(86)
where we assumed the minimal Kähler potential (60), and the absence of the higher order terms of $X$ in the superpotential for concreteness. In the limit $\epsilon \to 0$, this solution is reduced to (79). In contrast to (79), the above solution (86) does not disappear in all of the parameter region, even after the vacuum energy at the SUSY breaking minimum is set to zero as in Eq. (83). Such parameter region of $\mu_B$ and $\epsilon$ allowing the SUSY solution is shown in Fig. 1. In the shaded region, the SUSY solution (86) is not allowed and the SUSY breaking solution does not become metastable due to the R-breaking effect represented by $\epsilon$. Note that we find no allowed region along the $\epsilon = 0$ axis, which corresponds to the case of the solution (79).

2.6.2 Generic case: $g_1(\phi_i) \neq 0$

Finally we introduce nonvanishing $g_1(\phi)$. As in subsection 2.6.2, we first consider the case with $m_i^2 = 0$ in Eq. (70). In this case we can still integrate $\phi_i$ and $X_{a\neq 1}$ by use of $D\phi_i W = D_{X_{a\neq 1}} W = 0$ resulting in $\phi_i = X_{a\neq 1} = 0$.

The remnant of these heavy fields would be the one-loop mass $m_X$ for $X_1 = X$ in Eq. (81). The effective scalar potential is in the same form as Eq. (81) but the effective superpotential $W$ in $G = K + \ln |W|^2$ is now replaced by Eq. (82). For $\epsilon \ll c \sim f \sim m_X \ll 1$ in the unit with $M_{Pl} = 1$, we can obtain a SUSY breaking Minkowski minimum

$$X_{SB} = \frac{2fc}{m_X^2} (1 + \mathcal{O}(\epsilon^2)), \quad (87)$$

where the R-breaking constant

$$c = f/\sqrt{3} + \mathcal{O}(f^3/m_X^2; \epsilon^2),$$

is determined by the vanishing vacuum energy condition.

The SUSY ground state in the hyperplane $\phi_i = X_{a\neq 1} = 0$ which originates from the R-breaking parameter $\epsilon$ is the same as Eq. (85), and the above breaking minimum becomes metastable. Unlike (83), the SUSY breaking minimum (87) is not affected by the R-breaking term at $\mathcal{O}(\epsilon)$ due to the one-loop mass $m_X$, that is, the SUSY minimum (85) is independent of the SUSY breaking minimum (87) at this order. There might exist SUSY points analogous to Eq. (85) outside the hypersurface $\phi_i = X_{a\neq 1} = 0$ also in this case, but the solution would be more complicated due to the nonvanishing $h_{ij}^1$ in Eq. (70).
Finally we comment about the case with $m_i^1 \neq 0$ in Eq. (70). In this case, as mentioned in Sec. 2.5.2, the field $X_1$ has a SUSY mass with the same magnitude as those of $\phi_i$’s as shown in Eq. (80). Then the field $X_1$ in the field basis (44) is no longer special. In this generalized OR model with most general R-breaking terms, the total superpotential would be written as

$$W = fX_1 + \sum_{a=1} g_a(\phi_i)X_a + c(\phi_i) + \frac{1}{2} \sum_{a,b=1} m_{ab}(\phi_i)X_aX_b + \cdots$$

$$= c + fX_1 + \frac{1}{2} \mu_{IJ}\Phi_I\Phi_J + \cdots,$$

where $\Phi_I = (X_a, \phi_i)$, $I = (a, i)$ and the ellipses denote the higher order terms in $\Phi_I$. The SUSY mass matrix $\mu_{IJ}$ is given by the R-breaking components, $\mu_{ab} = m_{ab}(0)$, $\mu_{ij} = \partial_{\phi_i} \partial_{\phi_j} c(0)$ and the R-symmetric components, $\mu_{ai} = 2\partial_{\phi_i} g_a(0)$. Note that $\mu_{1i} = 2\partial_{\phi_i} g_1(0) = 2m_i^1$. After the unitary rotation which makes $\mu_{IJ}$ diagonal, the above superpotential takes the form of

$$W = c + fU_{1I}\Phi_I + \frac{1}{2} \mu_I\Phi_I^2 + \cdots,$$

where $U_{IJ}$ is the rotation matrix and $\mu_I$ represents the eigenvalues of $\mu_{IJ}$. The F-flat conditions, $D_I W = W_I + K_I W = 0$, allow a solution in general and SUSY would not be broken for $m_i^1 \sim f$.

### 2.7 Conclusion in section 2

In section 2.1 we considered $N = 1$ global supersymmetric models with a continuous global $U(1)_R$ symmetry. We discussed the features of models with SUSY breaking vacua and runaway directions. For example, models with fields whose R-charges are negative and/or more than 2 R-charge have runaway directions. Furthermore, models which satisfy $N_X > N_\omega$ have no solution for F-flat conditions, so that the condition $N_X > N_\omega$ is a sufficient condition for SUSY breaking. The generalized O’Raifeartaigh model satisfies the condition.

In section 2.2 we studied the effect of explicit R-symmetry breaking terms in detail. In global supersymmetric models, based on the argument by ISS, we have shown that a specific type of explicit R-symmetry breaking terms can restore SUSY, and the original SUSY breaking vacuum can become metastable when a certain (but not generic) class of explicit R-symmetry breaking terms are added and/or loop effects stabilize the original SUSY breaking minimum.
We also considered $N = 1$ local supersymmetric models with a continuous global $U(1)_R$ symmetry in section 2.3. We have executed similar analyses in R-symmetric supergravity models. First we examined the general argument by NS in supergravity and found that it also holds with local SUSY except for the nontrivial case where the Kähler potential allows solution for the second condition in Eq. (51). We presented concrete examples of this exception. These models lead to AdS SUSY stationary solutions and associated SUSY breaking vacua with lower vacuum energy. We found the general argument that this class of SUSY solutions corresponds to at best a saddle point, referring to Appendix A.

In section 2.4, 2.5, and 2.6, we studied the generalized OR model in supergravity with explicit R-symmetry breaking terms. We analyzed the structure of newly appeared SUSY stationary points as a consequence of the R-breaking effect and classified them. We have shown that these SUSY solutions disappear for type-A breaking terms (67), when we tune the R-breaking constant term in the superpotential such that the original SUSY breaking minimum has a vanishing vacuum energy. In this sense, the introduction of explicit R-breaking terms do not always lead to a metastability of the SUSY breaking vacuum. On the other hand, the introduction of type-B breaking terms (68) could cause a metastability of SUSY Minkowski minimum. We examined a parameter region which yields metastable vacuum in some concrete examples.

3 Metastable supersymmetry breaking vacua from conformal dynamics

In section 2, we have argued that an approximately R-symmetric superpotential with tiny R-symmetry breaking terms is favored to avoid conflicts with experimental results. In this section, we suggest the models to realize the tiny R-symmetry breaking terms and cause metastable SUSY breaking vacua effectively.

We start with a superpotential without R-symmetry. Based on the Nelson-Seiberg argument [1], SUSY would not be broken in this situation. However, we assume the conformal dynamics. Because of that, certain couplings are exponentially suppressed at a low-energy scale. Then, we could realize an R-symmetric superpotential or an approximate R-symmetric superpotential.
with tiny R-symmetry breaking terms. It would lead to a stable or metastable SUSY breaking vacuum. We study this scenario by using a simple model. Also, we study 5D models, which have the same behavior.

As the other good point in our model, contact terms between the hidden conformal sector and the visible sector are suppressed exponentially by conformal dynamics. As we discuss in the introduction, such conformal suppression mechanism, i.e. conformal sequestering, is quite important to model building for SUSY breaking [17, 18, 19, 20, 21, 22]. The suppression can lead the situation that flavor-blind contributions such as anomaly mediation [23] would become dominant.

This section is organized as follows. In section 3.1, we give a 4D simple model to realize our conformal scenario. In section 3.2, we study 5D models, which have the same behavior. Section 3.3 is short summary of our models.

### 3.1 4D conformal model

Our model is the $SU(N)$ gauge theory with $N_f$ flavors of chiral matter fields $\phi_i$ and $\tilde{\phi}_i$, which are fundamental and anti-fundamental representations of $SU(N)$. The flavor number satisfies $3N \geq N_f \geq \frac{3}{2}N$, and that corresponds to the conformal window [24, 25], that is, this theory has an IR fixed point [50]. The Novikov-Chifman-Veinstein-Zaharov (NSVZ) beta-function of physical gauge coupling $\alpha = g^2/8\pi^2$ is

$$
\beta^{NSVZ}_\alpha = -\frac{\alpha^2}{1-N\alpha}(3N-N_f+N_f\gamma_{\phi}),
$$

where $\gamma_{\phi}$ is the anomalous dimension of $\phi_i$ and $\tilde{\phi}_i$ [51, 52]. Since the IR fixed point corresponds to $\beta^{NSVZ}_\alpha = 0$, around that point the matter fields $\phi_i$ and $\tilde{\phi}_i$ have anomalous dimensions $\gamma_{\phi} = -(3N-N_f)/N_f$, which are negative.

In addition to the fields $\phi_i$ and $\tilde{\phi}_i$, we introduce singlet fields $\Phi_{ij}$ for $i, j = 1, \cdots, N_f$. The gauge invariance allows the following superpotential at the renormalizable level,

$$
W = h\phi_i\Phi_{ij}\tilde{\phi}_j + f\text{Tr}_{ij}\Phi_{ij} + \frac{m}{2}\text{Tr}_{ik}\Phi_{ij}\Phi_{jk} + \frac{\lambda}{3}\text{Tr}_{i\ell}\Phi_{ij}\Phi_{jk}\Phi_{k\ell}.
$$

Here we have preserved the $SU(N_f)$ flavor symmetry. Even if the $SU(N_f)$ flavor symmetry is broken, e.g. by replacing $f\text{Tr}_{ij}\Phi_{ij}$ by $f_{ij}\Phi_{ij}$, the following discussions would be valid. For simplicity, we assume that all of couplings,
$h$, $f$, $m$, $\lambda$, are real, although the following discussions are available for the model with complex parameters, $h$, $f$, $m$ and $\lambda$. We can add the mass terms of $\phi_i$ and $\tilde{\phi}_j$ to the above superpotential. We will comment on such terms later, but at the first stage we study the superpotential without the mass terms of $\phi_i$ and $\tilde{\phi}_j$.

If $m = \lambda = 0$, the above superpotential corresponds to the superpotential of the Intriligator-Seiberg-Shih (ISS2) model [40].\footnote{The flavor number does not satisfy $3N \geq N_f \geq \frac{3}{2}N$ which corresponds to the conformal window in the ISS2 model [40]. However, we call the model we introduce here the ISS2 model.} The ISS2 model corresponds to the generalized OR model we discuss in the section 2: $\Phi_{ij}$ corresponds to the R-charge 2 field described as $X_a$, and $\phi_i$, $\tilde{\phi}_i$ corresponds to the R-charge 2 field described as $\phi$ in the generalized OR model. As we discuss later, the ISS2 model always causes SUSY breaking without explicit R-symmetry breaking terms.

We consider that our theory is an effective theory with the cut off $\Lambda$. We assume that dimensionless parameters $h$ and $\lambda$ are of $O(1)$ and dimensionful parameters $f$ and $m$ satisfy $f \approx m^2$ and $m \ll \Lambda$. We denote physical couplings as $\hat{h} = (Z_\phi Z_{\tilde{\phi}} Z_\Phi)^{-1/2} h$, $\hat{f}_{ij} = (Z_\Phi)^{-1/2} f_{ij}$, $\hat{m} = (Z_\Phi)^{-1} m$ and $\hat{\lambda} = (Z_\Phi)^{-3/2} \lambda$, where $Z_\phi, Z_{\tilde{\phi}}, Z_\Phi$ are wavefunction renormalization constants for $\phi, \tilde{\phi}, \Phi$, respectively.

The F-flat conditions are obtained as

$$
\partial_{\Phi_{ij}} W = h\phi_i \tilde{\phi}_j + f \delta_{ij} + m \Phi_{ij} + \lambda \Phi_{jk} \Phi_{ki} = 0, \quad (90)
$$

$$
\partial_{\phi_i} W = h \Phi_{ij} \tilde{\phi}_j = 0, \quad (91)
$$

$$
\partial_{\tilde{\phi}_j} W = h \phi_i \Phi_{ij} = 0. \quad (92)
$$

These equations have a supersymmetric solution for generic values of parameters, $h, m, \lambda$. To see such a supersymmetric solution, following [40] we decompose $\phi, \tilde{\phi}$ and $\Phi$ as

$$
\Phi = \begin{pmatrix} Y \\ Z^T \\ X \end{pmatrix}, \quad \phi = \begin{pmatrix} \chi \\ \rho \end{pmatrix}, \quad \tilde{\phi}^T = \begin{pmatrix} \tilde{\chi} \\ \tilde{\rho} \end{pmatrix}, \quad (93)
$$

where $Y$, $\chi$ and $\tilde{\chi}$ are $N \times N$ matrices, $X$ is an $(N_F - N) \times (N_F - N)$ matrix, $Z$, $\tilde{Z}$, $\rho$ and $\tilde{\rho}$ are $(N_F - N) \times N$ matrices. Let us consider the slice with

\footnote{The gaugino condensation contribution can be included in explicit R-symmetry breaking terms in [40].}
\( Z = \tilde{Z} = \rho = 0 \) in the field space, where the first derivatives of \( W \) reduce to

\[
W_{\Phi_{ij}} = \begin{pmatrix} f \delta_{ij} + h \chi_i \tilde{\chi}_j + m Y_{ji} + \lambda Y_{jk} Y_{ki} & 0 \\ 0 & f \delta_{ij} + m X_{ji} + \lambda X_{jk} X_{ki} \end{pmatrix},
\]

(94)

\[
W_{\phi_i}^T = \begin{pmatrix} h Y_{ij} \tilde{\chi}_j \\ 0 \end{pmatrix}, \\
W_{\tilde{\phi}_j} = \begin{pmatrix} h \chi_i Y_{ij} \\ 0 \end{pmatrix}.
\]

(95)

Here, we have used the same indices for \( \Phi_{ij}, \phi_i, \tilde{\phi}_j \) and their submatrices. Thus, the fields \( X_{ij} \) and the others are decoupled in the F-flat conditions, \( W_{\Phi_{ij}} = W_{\phi_i} = W_{\tilde{\phi}_j} = 0 \). The F-flat condition \( W_{\Phi_{ij}} = 0 \) for \( X_{ij} \) has a solution as \( X_{ij} = x_s \delta_{ij} \) with

\[
x_s = \frac{-m \pm \sqrt{m^2 - 4f \lambda}}{2\lambda}.
\]

(96)

The F-flat conditions \( W_{\Phi_{ij}} = W_{\phi_i} = W_{\tilde{\phi}_j} = 0 \) for \( Y_{ij}, \chi_i \) and \( \tilde{\chi}_j \) have the following solution,

\[
f \delta_{ij} + h \chi_i \tilde{\chi}_j = 0, \quad Y_{ij} = 0.
\]

(97)

In addition, the D-flat conditions correspond to \( |\chi_i| = |\tilde{\chi}_i| \).

There is another solution, \( \chi_i = \tilde{\chi}_j = 0 \) and \( Y_{ij} = x_s \delta_{ij} \). However, only the above solution (97) survives at the IR region, as \( \hat{m} \) and \( \hat{\lambda} \) become to vanish as we will see later. Thus, we concentrate to the solution (97). At any rate, the superpotential (89) does not have R-symmetry, and there is a supersymmetric minimum.

The above aspect is the behavior of this model around the energy scale \( \Lambda \). Now let us study the behavior around the IR region. We assume that the gauge coupling is around the IR fixed point, i.e. \( \beta_{\alpha} \approx 0 \), and that \( \phi_i \) and \( \tilde{\phi}_i \) have negative anomalous dimensions \( \gamma_{\phi} \). In addition, we assume that the physical Yukawa coupling \( \hat{h} \) is driven toward IR fixed points. The beta-function of \( \hat{h} \) is obtained as

\[
\beta_{\hat{h}} = \hat{h}(\gamma_{\phi} + \gamma_{\tilde{\phi}} + \gamma_{\hat{\phi}}).
\]

(98)

The condition of the fixed point leads to \( 2\gamma_{\phi} + \gamma_{\Phi} = 0 \). Since \( \gamma_{\phi} < 0 \), we obtain a positive anomalous dimension for \( \Phi_{ij} \). Then, physical couplings, \( \hat{f}, \hat{m} \) and \( \hat{\lambda} \), are suppressed exponentially toward the IR direction as

\[
\hat{f}(\mu) = \left( \frac{\mu}{\Lambda} \right)^{\gamma_{\phi}} \hat{f}(\Lambda), \quad \hat{m}(\mu) = \left( \frac{\mu}{\Lambda} \right)^{2\gamma_{\phi}} \hat{m}(\Lambda),
\]

\[
\hat{\lambda}(\mu) = \left( \frac{\mu}{\Lambda} \right)^{3\gamma_{\phi}} \hat{\lambda}(\Lambda).
\]

(99)
Thus, the mass parameter $\hat{m}$ and 3-point coupling $\hat{\lambda}$ are suppressed faster than $\hat{f}$. If we neglect $\hat{m}$ and $\hat{\lambda}$ but not $\hat{f}$, the above superpotential becomes the superpotential of the ISS2 model, and there is a SUSY breaking minimum around $\Phi_{ij} = 0$ because of the rank condition.

Let us see more explicitly. We concentrate ourselves to the potential of the fields $X_{ij}$, because $X_{ij}$ contribute to SUSY breaking in the ISS2 model. Furthermore, we consider their overall direction, i.e. $X_{ij} = x\delta_{ij}$, and we use the canonically normalized basis, $\hat{x}$. Then, the above superpotential leads to the following scalar potential,

$$ V_{\text{SUSY}} = (N_f - N)|\hat{f} + \hat{m}\hat{x} + \hat{\lambda}\hat{x}^2|^2. $$

(100)

In addition, around $\hat{x} = 0$, SUSY is broken and that generates one-loop effective potential of $\hat{x}$. Around $\hat{x} = 0$, the mass term $m_x^2|\hat{x}|^2$ in the one-loop effective potential would be important. Hence, we analyze the potential, $V = V_{\text{SUSY}} + m_x^2|\hat{x}|^2$, and we use $m_x^2$, which has been calculated in [40], i.e.

$$ m_x^2 = \frac{\hat{h}^3\hat{f}}{8\pi^2} N(N_f - N)(\log 4 - 1). $$

(101)

Note that $m_x^2$ is suppressed toward the IR region like $\hat{f}$. We consider only the real part of $\hat{x}$. The stationary condition $\partial_{\hat{x}} V = 0$ is written as

$$ (\hat{f} + \hat{m}\hat{x} + \hat{\lambda}\hat{x}^2)(\hat{m} + 2\hat{\lambda}\hat{x}) + m_x^2\hat{x} = 0. $$

(102)

At a high energy scale corresponding to $Z_\Phi = O(1)$, we have $|\hat{f}|, |\hat{m}|^2 \gg m_x^2$, because $m_x^2$ is smaller than $\hat{f}$ by a loop factor. The potential and the stationary condition are controlled by $|\hat{f}|, |\hat{m}|^2, \hat{\lambda}$, but not $m_x$. Thus, there is no (SUSY breaking) minimum around $x = 0$, but we have a supersymmetric minimum

$$ \hat{x}_s = \frac{-\hat{m} \pm \sqrt{\hat{m}^2 - 4\hat{f}\hat{\lambda}}}{2\hat{\lambda}}. $$

(103)

However, toward the IR direction, $\hat{m}^2$ becomes suppressed faster than $m_x^2$. Then, the couplings $\hat{f}$ and $m_x^2$ are important in the potential. Around $\hat{x} = 0$, the stationary condition (102) becomes

$$ \hat{f}\hat{m} + m_x^2\hat{x} + \cdots = 0, $$

(104)
that is, the stationary condition is satisfied with
\[ \hat{x}_{sb} \approx -\frac{\hat{f} \hat{m}}{\hat{m}_x^2}. \]  (105)

At this point, SUSY is broken, and this point becomes close to \( \hat{x}_{sb} = 0 \) toward the IR. Around \( \hat{x} = 0 \), the size of mass is estimated by \( m_x \), because the other terms are suppressed. Hence, the SUSY breaking metastable vacuum corresponding to \( \hat{x} \sim 0 \) appears at the IR energy scale, where \( \hat{m}^2 \ll m_x^2 \).

Moreover, the previous SUSY vacuum (103) moves to a point far away from the origin \( \hat{x} = 0 \), because it behaves like
\[ \hat{x}_s = \frac{-\hat{m} \pm \sqrt{\hat{m}^2 - 4f^2\hat{\lambda}}}{2\hat{\lambda}} \sim \left( \frac{\Lambda}{\mu} \right)^{\gamma_{\phi}}. \]  (106)

Both breaking scales of the \( SU(N) \) gauge symmetry and supersymmetry at the metastable SUSY breaking point \( \hat{x} = 0 \) are determined by \( O(\hat{f}(\mu)) \). Thus, such an energy scale is estimated as \( \mu_{IR} \sim \hat{f}(\mu_{IR}) \), i.e.
\[ \mu_{IR} \sim \left( \frac{\hat{f}(\Lambda)}{\Lambda^{\gamma_{\phi}}} \right)^{1/(2-\gamma_{\phi})}, \]  (107)

and at this energy scale conformal renormalization group flow is terminated.

So far, we have assumed that the mass term of \( \phi_i \) and \( \tilde{\phi}_i \), \( m_{\phi_i} \phi_i \tilde{\phi}_i \) vanishes. Here, we comment on the case with such terms. The physical mass \( \hat{m}_\phi \) becomes enhanced as
\[ \hat{m}_\phi(\mu) = \left( \frac{\mu}{\Lambda} \right)^{2\gamma_{\phi}} \hat{m}_\phi(\Lambda), \]  (108)

because of the negative anomalous dimension \( \gamma_{\phi} \). At \( \mu \sim \hat{m}_\phi(\mu) \), the matter fields \( \phi_i \tilde{\phi}_i \) decouple and this theory removes away from the conformal window. Thus, if \( \hat{m}_\phi(\mu) > \mu_{IR} \), the conformal renormalization group flow is terminated at \( \mu_D \sim \hat{m}_\phi(\mu_D) = (\mu_D/\Lambda)^{2\gamma_{\phi}} \hat{m}_\phi(\Lambda) \).

We have studied the scenario that conformal dynamics leads to metastable SUSY breaking vacua. As an illustrating example of our idea, we have used the simple model. Our scenario could be realized by other models.

### 3.2 5D model

There would be an AdS dual to our conformal scenario. Indeed, we can construct simply various models within the framework of 5D orbifold theory.
Renormalization group flows in the 4D theory correspond to exponential profiles of zero modes like $e^{-c_i R_y}$, where $R$ is the radius of the fifth dimension\(^\text{15}\) and $y$ is the coordinate for the extra dimension, i.e. $y = [0, \pi]$ and $c_i$ is a constant. The parameter $c_i$ corresponds to anomalous dimension in the 4D theory, and each field would have a different constant $c_i$. In 4D theory, values of anomalous dimensions are constrained by concrete 4D conformal dynamics. However, constants $c_i$ do not have such strong constraints, although they would correspond to some charges. Hence, 5D models would have a rich structure and one could make model building rather simply. Here we show a simple 5D model. We consider the 5D theory, whose 5-th dimension is compactified on $S^1/Z_2$. Two fixed points on $S^1/Z_2$ correspond to $y = 0$ and $y = \pi$. We introduce three bulk fields $X$, $\phi_1$, $\phi_2$. They correspond to chiral multiplets of bulk hyper-multiplets and zero modes of their partners in hyper-multiplets $X^c$, $\phi_1^c$, $\phi_2^c$ are projected out by the $Z_2$ orbifold projection. We assume that zero mode profiles of $X$, $\phi_1$ and $\phi_2$ behave along the $y$ direction as $e^{-c_i R_y}$, $e^{-c_1 R_y}$ and $e^{-c_2 R_y}$, respectively. We integrate $y$ and obtain their kinetic term coefficients $Y_i$ of 4D effective theory, that is, the field corresponding to the zero mode profile $e^{-c_i R_y}$ has the following kinetic term coefficient \[53, 54\]

\[
Y_i = \frac{1}{c_i} \left(1 - e^{-2c_i \pi R} \right).
\]

(109)

In the limit $c_i \to 0$, $Y_i$ becomes $2\pi R$. Their superpotential is not allowed in the bulk, but is allowed on the boundary.

Suppose that the following superpotential is allowed only on the $y = \pi$ boundary,

\[
\int dy \delta(y - \pi) W^{(\pi)},
\]

(110)

\[
W^{(\pi)} = f e^{-c_X R_y} X + me^{-2c_X R_y} X^2 + he^{-3c_X R_y} X^3
+ m_{12} e^{-(c_1 + c_2) R_y} \phi_1 \phi_2
+ m_2 e^{-2c_2 R_y} \phi_2^2
+ \sum_{i,j} h_{ij} e^{-(c_X + c_i + c_j) R_y} X \phi_i \phi_j.
\]

(111)

Here we have assumed extra $Z_2$ symmetry, under which $X$ has the even $Z_2$ charge and $\phi_1$ and $\phi_2$ have the odd $Z_2$ charge. That allows the mass term $m_{11} \phi_1^2$, but we have assumed it vanishes by the same reason as why we did\(^\text{15}\) Assume that the radion is stabilized.

---

\(^\text{15}\) We assume that the radion is stabilized.
not add the mass term \( m_{ij}\phi_i\tilde{\phi}_j \) in the superpotential \( (89) \). We assume that \( f \approx m^2 \approx m_{12}^2 \approx m_2^2 \) and \( h, h_{ij} = O(1) \). We take
\[ c_1 = 0, \quad c_2 = c_X, \quad (112) \]
and \( c_X > 0 \) with \( c_X\pi R = O(1) \) and \( e^{-c_X\pi R} \ll 1 \). The 4D superpotential \( \hat{W} \) becomes
\[ \hat{W} = e^{-c_X\pi R}(f X + m_{12}\phi_1\phi_2 + h_{111}X\phi_1^2) + e^{-2c_X\pi R}\Delta W. \quad (113) \]
When we neglect \( \Delta W \), the superpotential \( W \) corresponds to the O’Raifeartaigh model \[15\], that is, SUSY is broken. Such a minimum is metastable and there is a SUSY minimum, when we take into account \( \Delta W \) \[14\]. The O’Raifeartaigh model with the following superpotential,
\[ W_0 = \hat{f}X + \hat{m}_{12}\phi_1\phi_2 + \hat{h}_{111}X\phi_1^2, \quad (114) \]
leads to the SUSY breaking minimum of scalar potential \( V = |\hat{f}|^2 \) at \( \phi_1 = \phi_2 = 0 \) and arbitrary \( X \), that is, it has the pseudo-flat direction. One-loop effects lift up this pseudo-flat direction, and the field \( X \) has the mass \( m_X \),
\[ m_X^2 = O \left( \frac{1}{4\pi^2 \hat{m}_{12}^2} \right), \quad (115) \]
around \( X = 0 \). In the case with \( h_{11} = O(1) \) in the superpotential \( (113) \), we would have a rather small mass \( m_X \) by the suppression factor \( e^{-c_X\pi R} \). To have larger mass \( m_X \), we can assume the following superpotential \( W^{(0)} \) at \( y = 0 \) as
\[ \int dy\delta(y)W^{(0)} = \int dy\delta(y)h^{(0)}_{111}X\phi_1^2. \quad (116) \]
In this case, the 4D superpotential becomes
\[ \hat{W} = (h^{(0)}_{111} - h_{111}e^{-c_X\pi R})X\phi_1^2 + e^{-c_X\pi R}(f X + m_{12}\phi_1\phi_2) + e^{-2c_X\pi R}\Delta W. \quad (117) \]
This leads to the metastable SUSY breaking minimum around \( X = 0 \) and the field \( X \) can have a larger mass around \( X = 0 \) than the previous model, because the coupling \( h^{(0)}_{111} \) has no suppression factor like \( e^{-c_X\pi R} \). The SUSY breaking source \( F^X \) is quasi-localized around \( y = 0 \).

We can construct more various models for approximately R-symmetric superpotential with metastable SUSY breaking vacua in 5D theory.
3.3 Short summary

We have studied the scenario that conformal dynamics leads to approximately R-symmetric superpotential with a metastable SUSY breaking vacuum. We have shown a simple model to realize our scenario. We can make 5D models with the same behavior. Since in our 4D scenario, metastable SUSY breaking vacua are realized by conformal dynamics, such a SUSY breaking source would be sequestered from the visible sector by conformal dynamics. We indicate that conformal dynamics leads the suppressions of cross terms between the visible sector and the hidden sector $\Phi_{ij}$. Our model, which have conformal dynamics in the hidden sector, also leads to the situation that anomaly mediation is dominant, so that the suppression to avoid large FCNC processes is realized.

In our scenario, at a high energy scale, there would be only SUSY minimum and at low energy metastable SUSY breaking vacuum would appear. To realize the initial condition such that a metastable SUSY breaking is favored at a high energy scale, finite temperature effects would be important, because finite temperature effects might favor a metastable SUSY breaking vacuum.

We suggest that conformal dynamics leads to approximately R-symmetric superpotential at low energy. In fact, conformal dynamics can realize this situation even in softly SUSY breaking theories. For example, gaugino mass, that is one of soft SUSY breaking terms and also a explicit R-symmetry breaking term, is strongly suppressed according to gauge coupling approaching toward an IR fixed point. We show how explicit R-symmetry breaking terms are suppressed in softly SUSY broken theories in section 4.

We introduce the RG flow of SUSY breaking parameters in softly SUSY breaking theories in section 4. We discuss models with conformal dynamics and soft SUSY breaking terms, and we do not specify the origin of SUSY breaking there. Finally, we find the effects of SUSY breaking and explicit R-symmetry breaking are suppressed strongly by conformal dynamics. The suppression can be also realized in more complicated models, such as the Klebanov-Strassler (KS) model we show in section 5 so that this result leads to the other scenario that the visible sector, which includes SSM, has conformal dynamics. In section 5, we introduce the model with conformal dynamics, which is expected to realize SSM at a low-energy scale.
4 RG flow of soft SUSY breaking terms in conformal dynamics

Here, we study the RG flow of SUSY breaking terms in softly broken super-symmetric theories and show that approximate R-symmetry is realized. In this study, all soft SUSY breaking terms are free parameters, and a gauge coupling and yukawa couplings have IR fixed points. The soft SUSY breaking terms, such as gaugino masses and A-terms, also break R-symmetry explicitly.

It is convenient to use the spurion method [29, 30, 31, 32, 33, 34] to derive RG equations of soft SUSY breaking terms from those for supersymmetric couplings. In the appendix B and C, we discuss the justification of the spurion method based on superfield perturbation and symmetry.

4.1 A gauge theory with conformal fixed point in a softly SUSY broken theory

Now we consider a $SU(N)$ gauge theory with $N_f$ flavors of fundamental and anti-fundamental matter fields $(Q, \bar{Q})$ and vanishing superpotential $W = 0$. $N_f$ satisfies $\frac{3}{2}N \leq N_f \leq 3N$, so that the gauge coupling has an IR fixed point as we discuss in section 3.1. We find that explicit R-symmetry breaking terms in the superpotential goes to zero by the conformal dynamics in section 3.1. Here our model does not have superpotential, so a explicit R-symmetry breaking term is only gaugino mass, which is one of soft SUSY breaking terms. The soft SUSY breaking term is given by as follows,

$$L_{SQCD}^{soft} = -\int d^4\theta (m_{ij}^2 \theta^2 \bar{\theta}^2) Q^i Q^j - \int d^4\theta (\bar{m}_{ij}^2 \theta^2 \bar{\theta}^2) \bar{Q}^i \bar{Q}^j - \int d^2\theta (M_{1/2} \theta^2) \frac{1}{g^2} Tr(W^\alpha W_\alpha) + h.c. \tag{118}$$

where $i$ denotes the flavor index ($i = 1, \ldots, N_f$), $M_{1/2}$, $m_{ij}^2$ and $\bar{m}_{ij}^2$ denote soft SUSY breaking terms: gaugino mass, squared scalar masses respectively.

We use the spurion method to analyze the RG flows of the soft SUSY breaking terms, so we give a brief review on the spurion method. The more precise argument about the spurion method is given in the appendix [C]. We also discuss the dual gauge theory, whose gauge symmetry is $SU(N_f - N)$...
with $N_f$ flavor pairs and superpotential is given by the yukawa couplings, in the next section.

Let us consider a generic gauge theory with a gauge coupling $g$, a gaugino mass $M_{1/2}$, yukawa couplings $y_{ijk}$, corresponding A-terms $a_{ijk}$ and soft scalar masses $m_i$.

$$L_{soft} = - \int d^4\theta (m^2_{ij} \theta^2 \bar{\theta}^2) \Phi^i \Phi^j - \int d^2\theta (M_{1/2} \theta^2) \frac{1}{g^2} Tr(W^\alpha W_\alpha) - \int d^2\theta \frac{1}{6} (h_{ijk} \theta^2) \Phi^i \Phi^j \Phi^k + h.c. \quad (119)$$

We define the following superfield couplings

$$\tilde{\alpha} = \alpha \left(1 + M_{1/2} \theta^2 + \frac{M^2_{1/2}}{2} + (2|M_{1/2}|^2 + \Delta_g) \theta^2 \bar{\theta}^2\right), \quad (120)$$

$$\tilde{y}_{ijk} = y_{ijk} - a_{ijk} \theta^2 + \frac{1}{2} (m^2_i + m^2_j + m^2_k) y_{ijk} \theta^2 \bar{\theta}^2, \quad (121)$$

where $\alpha$ is defined as $\alpha \equiv g^2/(8\pi^2)$ and $\Delta_g$ is written as $\Delta_g = \frac{F(\alpha)}{\alpha} \left[ \sum_i T_i m^2_i - T_G |M_{1/2}|^2 \right]. \quad (122)$

Then, beta-functions of superfields $\tilde{\alpha}$ and $\tilde{y}_{ijk}$ ($\bar{\tilde{y}}_{ijk}$) including soft SUSY breaking terms are obtained from those of $\alpha$ and $y_{ijk}$ ($\bar{y}_{ijk}$), $\beta_\alpha(\alpha, y_{ijk}, \bar{y}_{ijk})$ and $\beta_{y_{ijk}}(\alpha, y_{ijk}, \bar{y}_{ijk})$ by replacing $\alpha$ and $y_{ijk}$ ($\bar{y}_{ijk}$) by $\tilde{\alpha}$, $\tilde{y}_{ijk}$ ($\bar{\tilde{y}}_{ijk}$), i.e.,

$$\mu \frac{d\tilde{\alpha}}{d\mu} = \beta_\alpha(\tilde{\alpha}, \tilde{y}_{ijk}, \bar{\tilde{y}}_{ijk}), \quad \mu \frac{d\tilde{y}_{ijk}}{d\mu} = \beta_{y_{ijk}}(\tilde{\alpha}, \tilde{y}_{ijk}, \bar{\tilde{y}}_{ijk}). \quad (123)$$

That implies that the beta-function of the gaugino mass $M_{1/2}$ is obtained as

$$\mu \frac{dM_{1/2}}{d\mu} = \left( M_{1/2} \alpha \frac{\partial}{\partial \alpha} - a_{ijk} \frac{\partial}{\partial y_{ijk}} \right) \left( \frac{\beta_\alpha}{\alpha} \right) \equiv D_1 \left( \frac{\beta_\alpha}{\alpha} \right). \quad (124)$$

The RG equation for the soft scalar mass $m_i$ of a chiral superfield $\phi_i$ is also easily obtained as

$$\mu \frac{dm^2_i}{d\mu} = \gamma_i(\tilde{\alpha}, \tilde{y}_{ijk}, \bar{\tilde{y}}_{ijk}) |_{\theta=\bar{\theta}}. \quad (125)$$

\footnote{We show the function $F(\alpha)$ in appendix C}
These equations are found to be consistent with the equations for the $\theta^2\bar{\theta}^2$ components of Eqs. (123). Explicitly, the RG equations are written down as

$$\mu \frac{d m^2_i}{d \mu} = D_2 \gamma_i ,$$

$$D_2 = D_1 D_1 + (|M_{1/2}|^2 + \Delta_g) \alpha \frac{\partial}{\partial \alpha}$$

$$+ \frac{1}{2} (m^2_i + m^2_j + m^2_k) \left( y_{ijk} \frac{\partial}{\partial y_{ijk}} + \bar{y}_{ijk} \frac{\partial}{\partial \bar{y}_{ijk}} \right).$$

Let us turn back to the RG flows of the soft SUSY breaking terms in SQCD given by (118). We find an IR attractive fixed-point of the gauge coupling, by solving the equation that the beta-function given by (88) is equal to zero. We consider the perturbation around the fixed point as $\alpha = \alpha^* + \delta \alpha$, where $\delta \alpha \ll 1$. The beta-function of $\delta \alpha$ around the fixed point is written as

$$\mu \frac{d \delta \alpha}{d \mu} = \left( \frac{\partial \beta_\alpha}{\partial \alpha} \right)_{\alpha=\alpha^*} \delta \alpha \equiv \Gamma \delta \alpha .$$

Because this fixed point is the IR attractive, that leads to $\Gamma > 0$. Then, the spurion method leads immediately to the RG flow of the gaugino mass, that is, the gaugino mass is renormalized as

$$M_{1/2}(\mu) = M_{1/2}(\mu_0) \left( \frac{\mu}{\mu_0} \right)^\Gamma .$$

This is because the spurion method tells that

$$\delta \tilde{\alpha} = \alpha^* M_{1/2} \theta^2 - F(\alpha^*) \sum_i T_i m^2_i \theta^2 \bar{\theta}^2$$

also decrease exponentially toward the IR direction.

Thus the gaugino mass $M_{1/2}$ is found to be exponentially suppressed around the IR fixed point. Eventually we find that the explicit R-symmetry breaking term, gaugino mass, is also suppressed by conformal dynamics, and we can also show that the sum $\sum_i T_i m^2_i$ is exponentially suppressed in this theory.

Furthermore, it is straightforward to extend this discussion to the theory with a gauge coupling and yukawa couplings and to show that the gaugino mass $M_{1/2}$ and the A-term $a_{ijk}$ as well as the sums $\sum_i T_i m^2_i$ and $m^2_i + m^2_j + m^2_k$ are exponentially suppressed.
4.2 The dual gauge theory in a softly SUSY broken theory

We have shown that SQCD with $W = 0$, gaugino mass and soft scalar masses recovers approximate R-symmetry at low energy. Here we discuss soft SUSY breaking terms in the dual gauge theory. Based on the Seiberg’s argument [24, 25], the gauge symmetry and the superpotential are given by $SU(\tilde{N}) = SU(N_f - N)$ with $N_f$ flavor pairs $(q, \overline{q})$ and $W = y_{ij}q_iM_{ij}\overline{q}_j$. The soft SUSY breaking terms are described as

$$L_{\text{soft}}^d = -\int d^4\theta (m_{ij}^2 \theta^2 \overline{\theta}^2) q^i q^j - \int d^4\theta (\overline{m}_{ij}^2 \theta^2 \overline{\theta}^2) \overline{q}^i \overline{q}^j$$
$$- \int d^2\theta (\overline{M}_{1/2} \theta^2) \frac{1}{g'^2} Tr(W^\alpha W_\alpha) - \int d^2\theta \frac{1}{6} (a_{ij} \theta^2) \overline{q}^i M^{ij} q^j + h.c. \quad (131)$$

We define $\tilde{\alpha}$ and $\tilde{y}_{ij}$ in the dual gauge theory as follows,

$$\tilde{\alpha}' = \alpha' \left(1 + \tilde{M}_{1/2} \theta^2 + \overline{\tilde{M}}_{1/2} \overline{\theta}^2 + (2|\tilde{M}_{1/2}|^2 + \Delta_g) \theta^2 \overline{\theta}^2\right), \quad (132)$$
$$\tilde{y}_{ij} = y_{ij} - a_{ij} \theta^2 + \frac{1}{2}(m_i^2 + m_j^2 + m_{ij}^2) y_{ij} \theta^2 \overline{\theta}^2, \quad (133)$$

where $\alpha'$ is defined as $\alpha' \equiv g'^2/(8\pi^2)$ and $\Delta_g$ is written as

$$\Delta_g = -\frac{F_d(\alpha')}{\alpha'} \left[\sum_i T_i m_i^2 - T_G |\tilde{M}_{1/2}|^2\right]. \quad (134)$$

We obtain the RG equations of $\tilde{M}_{1/2}$ by replacing $M_{1/2}$ and $y_{ijk}$ in (124) by $\tilde{M}_{1/2}$ and $\tilde{y}_{ij}$. The RG equations of $a_{ij}$ are also obtained as follows based on the last section and appendix C.

$$\mu \frac{d a_{ij}}{d \mu} = \frac{1}{2}(\gamma_i + \gamma_j + \gamma_{ij}) a_{ij} - (D_1 \gamma_i + D_1 \gamma_j + D_1 \gamma_{ij}) y_{ij}. \quad (135)$$

In the dual-side, the gauge coupling and the yukawa couplings have IR-fixed points, and in fact this behavior corresponds to our model in section 3.1.

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17We show the function $F_d(\alpha')$ in appendix C.
It is also found that these RG equations lead to very interesting properties of the soft SUSY breaking parameters at the vicinity of an IR attractive fixed point [34, 56, 57] as the last section. Deviations of the gauge coupling and the yukawa coupling from their fixed point values, \( \delta \alpha' = \alpha' - \alpha'^* \) and \( \delta y_{ij} = y_{ij} - y^*_{ij} \), decrease exponentially. Then the spurion method tells that both of

\[
\delta \tilde{\alpha}' = \alpha'^* \tilde{M}_{1/2} \theta^2 + \alpha'^* \tilde{M}_{1/2} \bar{\theta}^2 - F(\alpha'^*) \sum_i T_i m_i^2 \theta^2 \bar{\theta}^2, \tag{136}
\]

\[
\delta \tilde{y}_{ij} = -a_{ij} \theta^2 + \frac{1}{2} (m_i^2 + m_j^2 + m_{ij}^2) y^*_{ij} \theta^2 \bar{\theta}^2, \tag{137}
\]

also decrease exponentially toward the IR direction. Therefore, the gaugino mass \( \tilde{M}_{1/2} \) and the A-term \( a_{ij} \) are found to be suppressed\(^\text{18}\) and the soft scalar masses satisfy the IR sum rules given by \( \sum_i T_i m_i^2 = 0 \) and \( m_q^2 + m_{\bar{q}}^2 + m_M^2 = 0 \).

We may also understand this as follows. When we use the one-loop anomalous dimensions, we can show that at the fixed point the gauge coupling and yukawa coupling are related as \( y^* = C g'^* \), where \( C \) is a constant determined by group-theoretical factors [58]. At the fixed point, this relation is realized as the relation between superfield couplings as \( |\tilde{y}|^2 / (8\pi^2) = C^2 \tilde{\alpha}' \), and their \( \theta^2 \bar{\theta}^2 \)-terms lead to [59, 60]

\[
m_q^2 + m_{\bar{q}}^2 + m_M^2 = |\tilde{M}_{1/2}|^2. \tag{138}
\]

Since the gaugino mass \( \tilde{M}_{1/2} \) is exponentially damping toward the conformal fixed point, the sum \( m_q^2 + m_{\bar{q}}^2 + m_M^2 \) is also exponentially damping as mentioned above.

Finally, we find that conformal dynamics realizes approximate R-symmetry at a low energy scale because of the suppressions of gaugino mass and A-term even in softly SUSY broken theories. However, a few parameters, such as B-term which is a quadratic term of scalar, are not controlled by the dynamics. As we discuss later, these parameters cause gauge symmetry breaking and disturb the RG flow of the gauge coupling.

In section 5 we discuss more complicated model with conformal dynamics, which have two gauge couplings and nonzero superpotential. In that case, we also find the feature that approximate R-symmetry is realized at low energy, and the B-term frightens the gauge symmetries. We discuss not

\(^{18}\) That implies that the ratio \( a_{ij} / y_{ij} \) is also suppressed exponentially, because the yukawa coupling \( y_{ij} \) has a fixed point.
only the aspect, but also the other applications of conformal dynamics for phenomenology.

5 The Duality cascade of softly broken supersymmetric theories

As we discuss in section 4, conformal dynamics restores R-symmetry. We find that this suppression can be realized in a more complicated model with SUSY breaking parameters. We discuss the duality cascade with soft SUSY breaking terms based on [26].

The duality cascade is caused by conformal dynamics, and has interesting RG flow of gauge couplings and yukawa coupling [27, 28]. After a brief review on the duality cascade of rigid SUSY theory, we discuss RG flows of soft SUSY breaking terms in the duality cascade, and show that the dynamics also leads the suppression of soft SUSY breaking terms and leads to approximate R-symmetry. However, B-term, which is a quadratic scalar term, stays in a finite value at low energy.

Conformal dynamics plays important roles in various aspects of (supersymmetric) field theories and particle phenomenology. For example, in Ref. [56, 57, 61, 62], conformal dynamics in the visible sector leads not only realistic hierarchies of quark and lepton masses, but also the alignment of A-terms to avoid large FCNC processes. At the same time, sfermion masses are exponentially suppressed toward the IR fixed point. In the later of this section, we study the model with conformal dynamics in the visible sector. This model is based on the duality cascade, and has unique RG flows. Finally, we suggest models with conformal dynamics in the visible sector, which have gauge symmetries and matters of SSM. Then we find that the models cause gauge symmetry breaking corresponding to EW symmetry breaking.

We show the dynamics of the duality cascade in the section 5.1. Conformal fixed points and conformal field theories (CFTs) are essential in Seiberg duality [24, 25]. That leads to more complicated and interesting RG flows of dual field theories, that is, the duality cascade [27, 28], which is a successive

19 A similar dynamics would be useful to control a large radiative correction on Higgs soft masses [63].
chain of the dualities from UV region to the IR region and reduces the rank of gauge groups one after another. Furthermore, the AdS/CFT (gravity/gauge) correspondence [64] suggests that the cascading theories would be realized in supergravity theory with a warped background, that is, the Klebanov-Strassler warped throat. In the supergravity description, the energy scale of the field theory corresponds to the distance from a tip of the throat. The duality cascade process means that the charges of D-branes disappear as the probe brane gets closer to the tip. The investigation of the duality cascade from the string/supergravity viewpoint is a highly non-trivial check for the gravity/gauge correspondence.

In section 5.2, we study more about the duality cascade in the model with soft SUSY breaking terms, based on [26]. As we comment in the introduction, several models have been proposed to realize supersymmetric standard models as well as their extensions at the bottom of the cascade [35, 36, 37] recently. To explain how we obtain the standard model like theories with fewer ranks from the infinitely many string vacua, which would generally have gauge groups with large ranks, those models are quite interesting and have opened possible candidates for high energy theories. Those models are exactly supersymmetric. At any rate, supersymmetry is broken in Nature even if supersymmetric theory is realized at high energy. Thus, if the cascading theories are relevant to the particle physics at the weak scale, supersymmetry should be broken at a certain stage, e.g. at the top or bottom of the cascade (high or low energy) or between them (intermediate energy). Here we assume that SUSY is softly broken at the beginning of the cascade. Then, we study RG flows of SUSY breaking terms as well as supersymmetric couplings.

This section is organized as follows. In section 5.1, we review briefly the RG flow of supersymmetric couplings in the duality cascade. In section 5.2, we study RG flows of SUSY breaking terms in the duality cascade. In section 5.3, we study symmetry breaking due to the B-term by using illustrative examples. In section 5.4, we give a simple example whose fields contents are similar to the MSSM or its extensions. Section 5.5 is conclusion and discussion of section 5.
5.1 RG flow in duality cascade of rigid supersymmetric theories

Here, we give a brief review on the RG flow in the duality cascade of rigid supersymmetric theories [27, 28]. We consider the gauge group \( SU(kN) \times SU((k-1)N) \) and we denote their gauge couplings, \( g_k \) and \( g_{k-1} \). Also, our model has two chiral multiplets \( Q_r (r = 1, 2) \) in the bifundamental representation of \( SU(kN) \times SU((k-1)N) \), i.e. the fundamental representation for \( SU(kN) \) and the anti-fundamental representation for \( SU((k-1)N) \), and two chiral multiplets \( \bar{Q}_s (s = 1, 2) \) in the anti-bifundamental representation.

Then we introduce the following superpotential, \( W = h \text{tr det} (Q_r \bar{Q}_s) = h \left[ (Q_1)^a_\alpha (Q_2)^b_\beta (\bar{Q}_2)^b_\alpha (\bar{Q}_1)^a_\beta - (Q_1)^a_\alpha (Q_2)^b_\beta (\bar{Q}_2)^b_\alpha (\bar{Q}_1)^a_\beta \right] \), (139)

where the indices \( \alpha \) and \( \beta \) are group indices for \( SU(kN) \) and the indices \( a \) and \( b \) are group indices for \( SU((k-1)N) \).

Now, we study the RG flow of gauge couplings \( g_k \) and \( g_{k-1} \) and the quartic coupling \( h \) and their fixed points. The fields \( Q_r \) and \( \bar{Q}_s \) have the same anomalous dimension, which we denote by \( \gamma_Q \). In the NSVZ scheme [51], beta-function of the gauge coupling \( g \) in generic gauge theory is written as

\[
\mu \frac{d \alpha}{d \mu} = \beta_\alpha = -\tilde{F}(\alpha)[3T_G - \sum_i T_i(1 - \gamma_i)],
\]

(140)

where \( \alpha = g^2/(8\pi^2) \) and

\[
\tilde{F}(\alpha) = \frac{\alpha^2}{1 - T_G \alpha}.
\]

(141)

Here, \( T_i \) and \( \gamma_i \) denote Dynkin indices and anomalous dimensions of the chiral matter fields, while \( T_G \) denotes the Dynkin index of the adjoint representation. For example, we have \( T_G = N \) for the \( SU(N) \) gauge group and \( T_i = 1/2 \) for the fundamental representation of the \( SU(N) \) gauge group. Using this scheme, beta-functions of the gauge couplings \( g_k \) and \( g_{k-1} \) are written as

\[
\beta_{\alpha_k} = -\tilde{F}(\alpha_k)N[k + 2 + 2(k - 1)\gamma_Q],
\]

(142)

\[
\beta_{\alpha_{k-1}} = -\tilde{F}(\alpha_{k-1})N[k - 3 + 2k\gamma_Q].
\]

(143)

In addition, we can write the beta-function of \( \eta = h\mu \) as

\[
\beta_\eta = \eta(1 + 2\gamma_Q).
\]

(144)
Suppose that both $SU(kN)$ and $SU((k - 1)N)$ sectors are within the conformal window\cite{24}, i.e. $3k/2 \leq 2(k - 1) \leq 3k$ and $3(k - 1)/2 \leq 2k \leq 3(k - 1)$. Then, we have two fixed points \cite{50,24,25},

$$A: \quad k - 3 + 2k\gamma_Q = 0, \quad \alpha_k = \eta = 0,$$  \hspace{1cm} (145)

and

$$B: \quad k + 2 + 2(k - 1)\gamma_Q = 0, \quad \alpha_{k-1} = \eta = 0.$$  \hspace{1cm} (146)

The anomalous dimension $\gamma_Q$ is a function of the couplings. We represent a value of the gauge coupling $g_{k-1}$ ($g_k$) at the first (second) fixed point by $g_{k-1}^*$ ($g_k^*$).

At the vicinity of the first fixed point $A$ given by (145) with $g_{k-1} \approx g_{k-1}^*$ and $0 < \alpha_k, \eta \ll 1$ (region I), it is found that $\beta_{\alpha_k} \approx 0$, $\beta_{\alpha_{k-1}} < 0$ and $\beta_{\eta} > 0$,

that is, $\alpha_k$ increases and $\eta$ decreases toward the IR direction. Thus, the theory would flow to the other fixed point $B$ given by (146) toward the IR direction. On the other hand, around the fixed point $B$ with $g_k \approx g_k^*$ and $0 < \alpha_{k-1}, \eta \ll 1$ (region II), it is found that $\beta_{\alpha_k} \approx 0$, $\beta_{\alpha_{k-1}} > 0$ and $\beta_{\eta} < 0$.

Hence, the quartic operator $h \text{tr} \det_{r,s}(Q_rQ_s)$ is relevant and the coupling $\eta$ increases toward the IR, while $\alpha_{k-1}$ shrinks.

We could examine the RG flows of the gauge couplings $\alpha_k$ and $\alpha_{k-1}$, if we admit using the anomalous dimension $\gamma_Q$ obtained in the 1-loop level. For a sufficiently large $N$, the anomalous dimension $\gamma_Q$ is given as

$$\gamma_Q = -N(k\alpha_k + (k - 1)\alpha_{k-1}).$$  \hspace{1cm} (147)

In Fig. 2, the RG flows in the coupling space $(\alpha_k, \alpha_{k-1})$ obtained in the NSVZ scheme are shown in the case of $k = 5$. Here we rescale the couplings as $N\alpha \rightarrow \alpha$. The points $A$ $(0, 0.05)$ and $B$ $(0.175, 0)$ represent the fixed points. The renormalized trajectory (R.T.) connecting these fixed points is shown by the bold line.

The flows in the region I are subject to the conformal dynamics around the UV fixed point $A$, while the flows in the region II are subject to that around the IR fixed point $B$. The convergence in the region I is not strong, since the fixed point coupling $\alpha_{k-1}^*$ is not so strong in the case of $k = 5$. It is seen in Fig. 2 that the R.T. bends on the way and the behavior of the R.T. changes quickly there. Thus the RG property on the R.T. in Fig. 2 may be characterized well as that in the region I or II.

The theory around the fixed point $B$ is strongly coupled and would be well-described by its Seiberg dual \cite{24,25}, which has the gauge group $SU((k-
Figure 2: RG flows in the coupling space \((\alpha_k, \alpha_{k-1})\) in the case of \(k = 5\). The points A and B represent the UV and IR fixed points respectively. The renormalized trajectory connecting these fixed points is shown by the bold line.

1) \(N \times SU((k - 2)N)\) and two bifundamental chiral multiplets \(q_r\) and two anti-bifundamental chiral multiplets \(\bar{q}_s\) and another kind of chiral multiplets \(M_{rs}\), which correspond to \(Q_r\bar{Q}_s\) and belong to the adjoint representation for \(SU((k - 1)N)\) and singlet for \(SU((k - 2)N)\). This dual theory has the following superpotential,

\[
W = y \, \text{tr} \, \bar{q}_r M_{rs} q_s + m \, \text{tr} \, \text{det}_{r,s} M_{rs}. \tag{148}
\]

The second term is the mass term of \(M_{rs}\), which corresponds to \(h \, \text{tr} \, \text{det}_{r,s} (Q_r\bar{Q}_s)\). The mass \(m\) would be related with the coupling \(h\) as

\[
h(\Lambda_k)\Lambda_k \sim \frac{m(\Lambda_k)}{\Lambda_k}, \tag{149}
\]

where \(\Lambda_k\) is a typical energy scale of \(SU(kN)\) gauge theory such as the energy scale, where the theory enters the conformal regime, i.e. \(g_k(\Lambda_k) \approx g_k^*\). The \(\beta\)-function of \(\alpha_{k-2}\) is written in a way similar to those of \(\alpha_{k-1}\) and \(\alpha_k\). In addition, the \(\beta\)-function of \(y\) is written as

\[
\beta_y = \frac{y}{2}(\gamma_M + 2\gamma_q), \tag{150}
\]

\(\uparrow\) In the followings, we will ignore the irrelevant mesons \(M_{rs}^0\) which are singlet for \(SU((k - 1)N)\).
where $\gamma_M$ is the anomalous dimension of $M_{rs}$. The dual theory has a non-trivial fixed point, $g_{k-2} = g_{k-2}^*$ and $y = y^*$ when $g_{k-1} = 0$, where $g_{k-2}^*, y^* \neq 0$. At the fixed point, it is satisfied that $\gamma_M = -2\gamma_q$, that is $M_{rs}$ has the same conformal dimension as $Q_r \bar{Q}_s$. Thus, at the vicinity of the fixed point, $g_{k-2} = g_{k-2}^*, y = y^*$ and $g_{k-1} = 0$, both operators, $h \text{ tr det}_{r,s}(Q_r \bar{Q}_s)$ and $m \text{ tr det}_{r,s} M_{rs}$ are relevant, and the mass $m/\mu$ and coupling $h\mu$ increase toward the IR direction. Because the fields $M_{rs}$ become heavy, we integrate out them and the effective superpotential results in

$$W = \tilde{h} \text{ tr det}_{rs} q_r \bar{q}_s,$$

(151)

where $\tilde{h} = -y^2/m$. The operator $\tilde{h} \text{ tr det}_{rs} q_r \bar{q}_s$ is irrelevant and the coupling $\tilde{h}$ decreases toward the IR direction. Thus, the low energy effective theory is the same as the starting theory except replacing the gauge group $SU(kN) \times SU((k-1)N)$ by $SU((k-1)N) \times SU((k-2)N)$. This RG flow would continue and the low-energy effective theory would become the $SU((k-n)N) \times SU((k-n-1)N)$ gauge theory with the quartic superpotential $W = \tilde{h} \text{ tr det}_{rs} q_r \bar{q}_s$ until the theory becomes outside of the conformal window. The RG flow toward the IR is illustrated as

\[
\begin{align*}
(g_k \approx 0, g_{k-1} \approx g_{k-1}^*, \eta \approx 0) & \quad \downarrow \\
(g_k \approx g_k^*, g_{k-1} \approx 0, \eta \approx 0) & \quad \leftrightarrow \quad (g_{k-2} \approx g_{k-2}^*, g_{k-1} \approx 0, y \approx y^*, m/\mu \approx 0) \\
(g_k \approx g_k^*, g_{k-1} \approx 0, \eta \gg 1) & \quad \leftrightarrow \quad (g_{k-2} \approx g_{k-2}^*, g_{k-1} \approx 0, y \approx y^*, m/\mu \gg 1) \\
& \quad \downarrow \text{dual} \\
& \quad \downarrow \text{integrating out } M_{rs} \\
& \quad (g_{k-2} \approx g_{k-2}^*, g_{k-1} \approx 0, \tilde{\eta} \approx 0).
\end{align*}
\]

At the end of cascade we would obtain the $SU(2N) \times SU(N)$ gauge theory. The $SU(2N)$ gauge sector has the $2N$ flavors and the quantum deformed moduli space $25, [65]$, $\Delta W = X (\text{det}_{\text{all}} M_{rs} - BB - \Lambda^{4N})$, where $X$ is a Lagrange multiplier superfield, $B$ and $\bar{B}$ are baryon and anti-baryon superfields, which are singlets under $SU(N)$. If we assume that only $B$ and $\bar{B}$ develop their VEVs, then baryons and mesons become massive. Thus the effective theory becomes the pure $SU(N)$ supersymmetric Yang-Mills theory and finally the theory is confined.

\[21\text{See also } [60].\]
5.2 RG flow of soft SUSY breaking terms in the duality cascade

Here, we study the RG flow of SUSY breaking terms in softly broken supersymmetric theories. We assume all soft SUSY breaking terms are free parameters as in section 4. Applying the spurion method to the cascading theory, we investigate the RG flow of soft SUSY breaking terms through the duality cascade. We consider the $SU(kN) \times SU((k-1)N)$ gauge theory with two pairs of chiral matter fields $Q_r$ and $\bar{Q}_s$ and their superpotential \text{(139)}. The beta-functions of their gaugino masses, $M^{(k)}_{1/2}$ and $M^{(k-1)}_{1/2}$, are written as

$$
\mu \frac{dM^{(k)}_{1/2}}{d\mu} = -N(k + 2 + 2(k-1)\gamma_Q)H'(\alpha_k)\alpha_k M^{(k)}_{1/2}$$
$$-2(k-1)NH(\alpha_k)\frac{\partial \gamma_Q}{\partial \alpha_k} \alpha_k M^{(k)}_{1/2}$$
$$-2(k-1)NH(\alpha_k)\frac{\partial \gamma_Q}{\partial \alpha_{k-1}} \alpha_{k-1} M^{(k-1)}_{1/2}, \quad (152)
$$

$$
\mu \frac{dM^{(k-1)}_{1/2}}{d\mu} = -N(k - 3 + 2k\gamma_Q)H'(\alpha_{k-1})\alpha_{k-1} M^{(k-1)}_{1/2}$$
$$-2kNH(\alpha_{k-1})\frac{\partial \gamma_Q}{\partial \alpha_{k-1}} \alpha_{k-1} M^{(k-1)}_{1/2}$$
$$-2kNH(\alpha_{k-1})\frac{\partial \gamma_Q}{\partial \alpha_k} \alpha_k M^{(k)}_{1/2}, \quad (153)
$$

where $H(\alpha) = \tilde{F}(\alpha)/\alpha \approx \alpha$ and $H'(\alpha) = dH/d\alpha$.

As in Section 5.1, we start the RG flow at the energy scale $\Lambda$ from the vicinity of the fixed point $A$, i.e. $(g_k, g_{k-1}, \eta) = (0, g^*_k, 0)$. Around the fixed point $A$, we have $k - 3 + 2k\gamma_Q \approx 0$. As long as $g_{k-1}$ is large and stable, the second term in \text{(153)} reduces the gaugino mass $M^{(k-1)}_{1/2}$ exponentially as the energy scale $\mu$ decreases. On the other hand, we find $\beta_{M^{(k)}_{1/2}} < 0$ because $k + 2 + 2(k-1)\gamma_Q > 0$ and $H(\alpha_k) \approx \alpha_k \approx 0$. Thus, the gaugino mass $M^{(k)}_{1/2}$ increases as the energy scale $\mu$ decreases. However, such increase of $M^{(k)}_{1/2}$ is not drastic during the weak coupling region of $\alpha_k$.

Next, the theory moves from the vicinity of the fixed point $(g_k, g_{k-1}, \eta) = (0, g^*_k, 0)$ toward another fixed point, $(g_k, g_{k-1}, \eta) = (g^*_k, 0, 0)$, where $k + 2 + 2(k-1)\gamma_Q \approx 0$. Around the latter fixed point, we find $\beta_{M^{(k-1)}_{1/2}} > 0$ because

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\[ k - 3 + 2k\gamma_Q < 0, \quad H(\alpha_{k-1}) \approx 0 \text{ and } M_{1/2}^{(k)} \text{ becomes irrelevant as below. That is, the gaugino mass } M_{1/2}^{(k-1)} \text{ decreases perturbatively as the energy scale } \mu \text{ decreases.} \]

On the other hand, the gaugino mass \( M_{1/2}^{(k)} \) would be suppressed exponentially in turn due to the second term in (152), as going toward the IR fixed point. However we note that the third term cannot be neglected, when \( \alpha_k M_{1/2}^{(k)} \) is reduced to be comparable with \( \alpha_{k-1} M_{1/2}^{(k-1)} \). Then the gaugino mass \( M_{1/2}^{(k)} \) does not follow a simple exponential suppression. Rather it converges to a certain value determined by \( \alpha_{k-1} \) and \( M_{1/2}^{(k-1)} \) obtained at the renormalized scale.

If we admit using the one-loop anomalous dimension, then the RG behavior discussed above could be explicitly examined. Here we shall look into the theory on the renormalized trajectory given in Fig. 2. In Fig. 3, the gaugino masses \( M_{1/2}^{(k-1)}(\mu) \) and \( M_{1/2}^{(k)}(\mu) \) of the theory with \( k = 5 \) are plotted with respect to the scale parameter \( \ln(\mu/\mu_0) \). At the scale \( \mu_0 \), the gauge couplings are chosen as \( (\alpha_k, \alpha_{k-1}) = (0.0128, 0.04) \), which is a point on the renormalized trajectory rather close to the fixed point A in Fig. 2. The initial values at \( \mu = \mu_0 \) are taken to be 1.0 for both gaugino masses.

![Figure 3: RG running of the gaugino masses](image)

It is seen that \( M_{1/2}^{(k-1)} \) is reduced as discussed, but turns to be negative due to the third term in (153), since \( M_{1/2}^{(k)} \) glows slightly first. In the region
II, the gaugino mass $M_{1/2}^{(k)}$ turns out to be suppressed strongly, while $M_{1/2}^{(k-1)}$ changes perturbatively. In Fig. 4, the log-plot of the gaugino mass $M_{1/2}^{(k)}$ is shown by the bold line. It is also seen that the suppression behavior deviates from the exponential one in the end and $M_{1/2}^{(k)}$ converges to a line. Indeed, the convergence point of $\alpha_k M_{1/2}^{(k)}$ could be estimated as

$$\alpha_k M_{1/2}^{(k)} \sim -\alpha_{k-1} M_{1/2}^{(k-1)}.$$  \hfill (154)

![Figure 4: RG running behaviors of the scalar mass $\ln m_Q^2$ and the gaugino mass $2 \ln M_{1/2}^{(k)}$ are shown by dotted lines and the bold line, respectively.](image)

Similarly, we examine the RG running of the soft mass squared $m_Q^2$. At the vicinity of the fixed points, $m_Q^2$ is also expected to be exponentially suppressed as discussed in section 4.1. However existence of two gauge couplings makes the situation more complicated. The RG equation for $m_Q^2$ is given as

$$\mu \frac{d m_Q^2}{d \mu} = \gamma_Q(\bar{\alpha}_k, \bar{\alpha}_{k-1})|_{\theta_2 \bar{\theta}_2}.$$ \hfill (155)

Here, let us use the one-loop anomalous dimension given by (9). Then the RG equation is reduced to be

$$\mu \frac{d m_Q^2}{d \mu} = -k \alpha_k (2 |M_{1/2}^{(k)}|^2 + \Delta_k) - (k - 1) \alpha_{k-1} (2 |M_{1/2}^{(k-1)}|^2 + \Delta_{k-1}),$$ \hfill (156)
where

\[
\Delta_k = H(\alpha_k) \left[ 3k|M_{1/2}^{(k)}|^2 - 2(k - 1)m_Q^2 \right], \tag{157}
\]

\[
\Delta_{k-1} = H(\alpha_{k-1}) \left[ 3(k - 1)|M_{1/2}^{(k-1)}|^2 - 2km_Q^2 \right]. \tag{158}
\]

In Fig. 4, the RG evolution of $m_Q^2$ of the same theory on the renormalized trajectory is shown by dotted lines. The initial values are taken as $\ln m_Q^2 = 0, 2.5, 5.0$ just for the illustration. In the region I, we may neglect subleading terms of $\alpha_k$ and also $M_{k-1}$, since it is suppressed. Then, Eq. (156) is approximated to be

\[
\mu \frac{dm_Q^2}{d\mu} \simeq 2k(k - 1)(\alpha_{k-1}^* m_Q^2 - 2k\alpha_k |M_{1/2}^{(k)}|^2. \tag{159}
\]

This equation tells us that $m_Q^2$ is not just suppressed but converges as

\[
m_Q^2 \to \frac{1}{(k - 1)(\alpha_{k-1}^*)^2 \alpha_k |M_{1/2}^{(k)}|^2}, \tag{160}
\]

since running of $\alpha_k |M_{1/2}^{(k)}|^2$ is rather slow. In the case of $k = 5$, the fixed point coupling $g_{k-1}^*$ is not large and the convergence is not so strong. In the region II, running of $M_{1/2}^{(k)}$ changes to exponential suppression. However, similarly to the behavior in the region I, it converges in the IR limit as

\[
m_Q^2 \to \frac{1}{k(\alpha_k^*)^2 \alpha_{k-1} |M_{1/2}^{(k-1)}|^2}. \tag{161}
\]

We summarize the RG flow of the gaugino masses and soft scalar masses as the theory moves from the fixed point $(g_k, g_{k-1}, \eta) = (0, g_{k-1}^*, 0)$ toward the fixed point $(g_k, g_{k-1}, \eta) = (g_k^*, 0, 0)$. At the first stage, i.e., the perturbative regime of $\alpha_k$, the gaugino mass $M_{1/2}^{(k-1)}$ is suppressed, while $M_{1/2}^{(k)}$ and the soft scalar mass squared $m_Q^2$ increase perturbatively. In entering the conformal regime of $\alpha_k$, both $M_{1/2}^{(k)}$ and $m_Q^2$ begin exponential damping, while $M_{1/2}^{(k-1)}$ runs perturbatively. In the IR limit, the gaugino mass $M_{1/2}^{(k)}$ and the soft scalar mass squared $m_Q^2$ are found to converge to certain values determined by $\alpha_{k-1}$ and $M_{1/2}^{(k-1)}$. Hence, these parameters evolve to be of the same order and are fixed in the IR limit irrespectively of their initial values.
In addition to the gaugino masses $M_{1/2}^{(k)}$, $M_{1/2}^{(k-1)}$ and scalar mass $m_Q$, the SUSY breaking terms corresponding to the superpotential (139) may be important, that is,

$$ W = h(1 - A_h \theta^2) \text{ tr } \det_{r,s}(Q_r \bar{Q}_s). $$

(162)

The RG flow behavior of the coupling $\mu h A_h$ is drastic following the anomalous dimensions of $Q_r$ and $\bar{Q}_s$. Both RG flows of $\mu h$ and $\mu h A_h$ are almost the same. That implies that their ratio $A_h$ does not change drastically.

The theory around the fixed point $(g_k, g_{k-1}, \eta) = (g^*_k, 0, 0)$ would be well-described by its dual theory with the gauge coupling $g_{k-2}$ and the yukawa coupling $y$. The dual theory has the gaugino mass $M_{1/2}^{(k-2)}$, soft scalar masses of $q_r$, $\bar{q}_s$ and $M_{rs}$ as $m_q$ and $m_M$, the A-term $a$ and the B-term $b$. The latter two terms are associated with the superpotential (148) and are written as

$$ W = y(1 - A_y \theta^2) \text{ tr } \bar{q}_r M_{rs} q_s + m(1 - B \theta^2 \text{ tr det } M_{rs}). $$

(163)

Here, we denote $a = y A_y$ and $b = m B$. The exact matching relations of soft terms between dual theories are not clear, but we assume that $M_{1/2}^{(k)}(\Lambda_k) \sim M_{1/2}^{(k-2)}(\Lambda_k)$ and all of soft scalar masses are of the same order at $\Lambda_k$. Furthermore, we assume that all of $A_h$, $A_y$ and $B$ are of the same order at $\Lambda_k$.

When the gauge coupling $g_{k-2}$ approaches toward its non-trivial fixed point, the gaugino mass $M_{1/2}^{(k-2)}$ and soft scalar mass squared $m_q^2$ are also exponentially suppressed. This behavior is similar to that of $M_{1/2}^{(k)}$ and $m_Q^2$ discussed previously. Moreover, in the dual theory the yukawa coupling $y$ approaches to the fixed point $y^*$. In this case, a small deviation $\delta y = y - y^*$ is exponentially damping as (128). The spurion method leads that the A-term coupling $A_y$ is also suppressed exponentially. On the other hand, the RG behavior of $B$ is rather similar to one of $A_h$. It is found that both RG flow behaviors of the mass $m/\mu$ and $b/\mu$ are almost the same and they are determined by large anomalous dimensions of $M_{rs}$. However, their ratio $B$

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22Note that the RG flow of $\eta$ has no fixed point with a finite value of $\eta$. In our case, the RG flow of $A_h$ will be ruled by gauge couplings and gaugino masses which can be finite values. In the region II, $A_h$ will be affected by mainly $\alpha_{k-1}^{n_k} M_{1/2}^{(k-1)}$ in the dimensionful parameters.
does not change drastically\(^{23}\).

In the dual theory, not only \(m_q^2\) but also the sum of soft scalar masses squared, \(m_q^2 + m_{\bar{q}}^2 + m_M^2\), is also suppressed in the conformal region. That implies that each of \(m_q^2\) and \(m_M^2\) is suppressed when \(m_q^2 = m_{\bar{q}}^2\), which is the relation we are assuming. However, we cannot neglect the effects through \(SU(N(k-1))\) gauge interaction such as the discussions of convergence points, \(^{154}\) and \(^{161}\), in the original \(SU(Nk) \times SU(N(k-1))\) theory.

The gaugino mass \(M_{1/2}^{(k-2)}\), the A-parameter \(A_y\) and the scalar masses squared \(m_q^2\) and \(m_M^2\) in the dual theory are not just suppressed out, rather converge to certain values given by \(\alpha_{k-1} M_{1/2}^{(k-1)}\) in the IR limit again. It is straightforward to solve the RG equations, if we admit using the one-loop anomalous dimensions of \(q\) and \(M\) just as performed above. However, we shall avoid to present a similar analysis here. It may be explicitly seen that both \(m_q^2\) and \(m_M^2\) as well as \(|A_y|^2\) converge the values of the same order given by \(\alpha_{k-1} M_{1/2}^{(k-1)}\)^2. The meson field \(M\) belongs to the adjoint representation of \(SU(N(k-1))\) group and suffers from the effects through \(SU(N(k-1))\) gauge interaction more. Therefore, \(m_M^2\) is found to be positive and larger than \(m_q^2\) in the IR\(^ {24}\).

When the supersymmetric mass \(m\) of the chiral fields \(M_{rs}\) becomes large, we integrate out these fields. Then, the low energy theory becomes the \(SU((k-1)N) \times SU((k-2)N)\) gauge theory with two pairs of bifundamental and anti-bifundamental fields and the quartic superpotential, \(W = \tilde{h} \det q_r \bar{q}_s\). The theory has soft SUSY breaking terms, i.e. the gaugino masses, \(M_{1/2}^{(k-1)}\) and \(M_{1/2}^{(k-2)}\), and soft scalar mass \(m_q\). In addition, we have the SUSY breaking term corresponding to the superpotential \(W = \tilde{h} \det q_r \bar{q}_s\), that is,

\[(164)\]

\[W = \tilde{h}(1 - \theta^2 A_{\tilde{h}}) \text{tr} \det q_r \bar{q}_s.\]

The size of \(A_{\tilde{h}}\) may be of the order of \(B\) or \(A_y\) at the decoupling scale of \(M_{rs}\). If these SUSY breaking terms are smaller than other mass scales such as the energy scale \(\mu\) and the supersymmetric mass \(m\), the above cascade continues as rigid SUSY theory in Section 5.1. Through the cascade, the gaugino masses and soft scalar masses are damping except the perturbative regime.

\(^{23}\)Note that the RG flow of \(m/\mu\) has no fixed point with its fine value. In this IR region, \(B\) will be affected by mainly \(\alpha_{k-1} M_{1/2}^{(k-1)}\) in the dimensionful parameters.

\(^{24}\)Soft masses for singlet mesons \(M_0^{(k)}_{rs}\) may be driven to be negative because of the yukawa couplings.
where the theory moves from the fixed point \((g_k, g_{k-1}, \eta) = (0, g_k^{*}, 0)\) toward the fixed point \((g_k, g_{k-1}, \eta) = (g_k^{*}, 0, 0)\). When we integrate out \(M_{rs}\), which are charged under the \(SU((k-1)N)\) gauge group, threshold corrections would appear. For example, the gaugino mass \(M_{1/2}^{(k-1)}\) would receive such threshold corrections \(\Delta M_{1/2}^{(k-1)}\), which would be estimated by \(\Delta M_{1/2}^{(k-1)} = \mathcal{O}(\alpha_{k-1}B)\). That would be small, because \(\alpha_{k-1}\) is small. At any rate, if the cascade continues, the total gaugino mass \(M_{1/2}^{(k-1)}\) would be suppressed at the next stage such as the gaugino mass \(M_{1/2}^{(k)}\) is suppressed at the stage discussed above.

As discussed above, the cascade would continue unless SUSY breaking terms are comparable with other mass scales such as the energy scale \(\mu\) and the supersymmetric mass \(m\). Gaugino masses and SUSY breaking scalar masses would be suppressed through the cascade except the regime I, where the gaugino mass \(M_{1/2}^{(k)}\) would increase. On the other hand, the SUSY breaking parameters, \(B\) and \(A_h\), would not be suppressed like the others. Note that the B-term corresponds to the off-diagonal entries of mass squared matrix of the fields \(M_{rs}\), that is, eigenvalues of mass squared would be written by \(|m|^2 + m^2_M \pm |mB|\). A large value of \(|B|\) would induce a tachyonic mode. Then, the scalar component of superfields \(M_{rs}\) may develop their VEVs and the gauge symmetry \(SU((k-1)N)\) may be broken. Also, through this symmetry breaking, the matter fields \(q_r\) and \(\bar{q}_s\) may gain mass terms due to the yukawa coupling with \(M_{rs}\). Then, the duality cascade would be terminated when mass parameters, \(|m|^2\), \(|mB|\) and \(m^2_M\), are comparable. This type of ending of the duality cascade could happen only in the softly broken supersymmetric theories and such symmetry breaking would be important. Thus, we will study such breaking more explicitly in the next section. Similar symmetry breaking would be realized not only in the “magnetic dual theory”, but also in the original “electric theory” with the quartic A-term \((\ref{162})\). If the quartic A-term is comparable with SUSY breaking scalar masses \(m_Q\), the origin of scalar potential of \(Q\) would be unstable and similar symmetry

\footnote{Similarly, the singlet meson fields \(M^0_{rs}\) may develop their VEVs depending on values of their various mass terms. Their VEVs induce mass terms of dual quarks. If such masses are large enough, the dual quarks would decouple and the flavor number would reduce to be outside of the conformal window. Then, the cascade could end. In addition, scalar components of \(q_r\) and \(\bar{q}_s\) may develop their VEVs depending on values the A-terms and their soft scalar masses as well as other parameters in the scalar potential. Their VEVs break gauge symmetry and the cascade would end.}
breaking would happen. Such gauge symmetry breaking with reducing the flavor number may correspond to the symmetry breaking by VEVs of $M_{rs}$ with inducing dual quark masses.

Whether $M_{rs}$ include tachyonic modes depends on values of $|m|^2 + m^2_M \pm |mB|$, i.e. their initial conditions as well as matching conditions. In a certain parameter region, the scalar fields $M_{rs}$ may include tachyonic modes and symmetry breaking may happen. In the other parameter regions, the cascade would continue like the rigid supersymmetric theory. For example, when the magnitude of SUSY breaking terms is much smaller than the supersymmetric mass $m$ and the energy scale $\mu$, the cascade would continue in almost the same way as the rigid supersymmetric theory. Then, it would end with the pure $SU(N)$ supersymmetric Yang-Mills theory with non-vanishing gaugino mass.

5.3 Symmetry breaking

In the previous section, we have pointed out the possibility that a tachyonic mode in the meson fields $M_{rs}$ would appear because of soft SUSY breaking terms and its VEV would break the symmetry. Here, we study this aspect more explicitly.

5.3.1 $SU(kN) \times SU((k-1)N)$ model

First, we study the $SU((k-2)N) \times SU((k-1)N)$ theory, which is dual to the $SU(N) \times SU((k-1)N)$ theory. As discussed in the previous section, the dual theory includes the meson fields $M_{rs}$, which have the supersymmetric mass $m$, the SUSY breaking soft scalar masses $m_M$ and the B-term $mB$. That is, their scalar potential $V$ is written by

$$V = (|m|^2 + m^2_M) \sum_{rs} |M_{rs}|^2 + (mB(M_{11}M_{22} - M_{12}M_{21}) + h.c.) + V_D^{(k-1)},$$

$$V_D^{(k-1)} = \frac{1}{2}g_{k-1}^2 D_{(k-1)}^2,$$

where $D_{(k-1)}$ denotes the D-term of the $SU((k-1)N)$ vector multiplet. Here, we have assumed the $SU(2)$ invariance for the $(r,s)$ indices of $M_{rs}$.

The eigenvalues of mass squared matrix are given by

$$|m|^2 + m^2_M \pm |mB|.$$  

(166)
If $|m|^2 \gg |m_M|^2, |m_B|$, the theory is almost supersymmetric and the duality cascade would continue. (Note that $m$ is the supersymmetric mass and the others are masses induced by SUSY breaking.) However, if the masses $m$ include a negative eigenvalue, there appears a tachyonic mode at the origin of the field space $M_{rs}$. Note that the D-flat direction corresponds to the VEV direction, where $M_{rs}$ are written by diagonal elements, that is, Cartan elements. That implies that when a negative eigenvalue is included in $m$, such a direction would be unbounded from below in the tree-level scalar potential. Thus, the meson fields $M_{rs}$ would develop their VEVs, whose order would be equal to the cut-off scale of the $SU((k-2)N) \times SU((k-1)N)$ theory, i.e. $\Lambda_k$. The VEVs of adjoint fields $M_{rs}$ break the gauge group $SU((k-1)N)$ to a smaller group and induce mass terms of $q_r$ and $\bar{q}_s$ through the Yukawa couplings $y_{qr}M_{rs}\bar{q}_s$.

**5.3.2 $\prod_i SU(N_i)$ quiver model**

The above analysis can be extended to the $\prod_i SU(N_i)$ quiver gauge theory with their bifundamental matter fields. We consider a subsector of the quiver theory, that is, the $SU(N_1) \times SU(N_2) \times SU(N_3)$ gauge theory with bifundamental matter fields, $(N_1, \tilde{N}_2, 1)$ and $(1, N_2, \tilde{N}_3)$ as shown in Fig. 5. The $SU(N_1)$ and $SU(N_3)$ sectors would have other types of bifundamental matter fields, but we neglect them. In addition, for simplicity we consider

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26In each non-abelian gauge group, for example, we need vector-like matter fields in order to cancel anomaly. However, we assume that the theory is anomaly-free at every stage.
the case with $N_1 = N_3$. Here, we dualize the $SU(N_2)$ sector. Then, there appear the dual matter fields $q$ and $\bar{q}$ with the representations $(\tilde{N}_1, \tilde{N}_2, 1)$ and $(1, \tilde{N}_3 N_3)$, where $\tilde{N}_2 = N_1 - N_2$. In addition, the meson field $M$ with the representation $(N_1, 1, \tilde{N}_3)$ appears and has yukawa couplings among $q$ and $\bar{q}$. The supersymmetric mass term of the meson field in the superpotential is not allowed, i.e. $m = mB = 0$. In this case, only the SUSY breaking soft scalar mass $m_M$ as well as the D-term potentials appears in the scalar potential of the meson field $M$. Thus, the scalar potential is simple. The scalar mass squared $m_M^2$ tends to converge to a positive value as discussed in the previous section. Thus, the symmetry breaking may not happen by the VEV of $M$ in this theory.

When $N_1 = N_3 = 2$, supersymmetric mass terms of meson fields in the superpotential would be allowed. Alternatively, when the model includes anti-meson fields $\bar{M}$, the supersymmetric mass term would be allowed in the superpotential. In these models, the corresponding B-terms would also be allowed. Furthermore, in the latter model, there are D-flat directions, i.e. $M = \pm \bar{M}$. In this case, the scalar potential would be written by

$$V = (|m|^2 + m^2_M)|M|^2 + (|m|^2 + m^2_M)|\bar{M}|^2 + (mBM\bar{M} + h.c.) + V^{(N_1)}_{D} + V^{(N_3)}_{D},$$

(167)

where $V^{(N_1)}_{D}$ and $V^{(N_3)}_{D}$ are D-term scalar potentials for the $SU(N_1)$ and $SU(N_3)$ vector multiplets. This potential at the tree level is unbounded from below along the D-flat direction $M = \pm \bar{M}$ if

$$2|m|^2 + m^2_M + m^2_{\bar{M}} < 2|mB|.$$  

(168)

In addition, the meson fields include a tachyonic mode when

$$(|m|^2 + m^2_M)(|m|^2 + m^2_M) < |mB|^2,$$

(169)

or

$$(|m|^2 + m^2_M)(|m|^2 + m^2_M) > |mB|^2$$ and $2|m|^2 + m^2_M + m^2_{\bar{M}} < 0.$  

(170)

Thus, various phenomena could happen depending on values of mass parameters, $m$, $m_M$, $m_{\bar{M}}$ and $mB$, that is, the unbounded-from-below direction, the symmetry breaking without the unbounded-from-below direction or no symmetry breaking. Indeed, this situation is quite similar to what happens in the Higgs scalar potential of the MSSM.
5.4 Illustrating model

Here we give a simple example of theories, whose field contents are similar to the MSSM or its extensions and where symmetry breaking would happen.

We consider the gauge group $U(3) \times USp(6)_L \times USp(6)_R \times U(1)$ and three families of bifundamental fields, $\tilde{Q}_L : (3, 6, 1, 0)$, $\tilde{Q}_R : (\bar{3}, 1, 6, 0)$, $\tilde{L}_L : (1, 6, 1, -1)$ and $\tilde{L}_R : (1, 1, 6, 1)$ and the superpotential

$$W = \hat{h}\tilde{Q}_L \tilde{Q}_R \tilde{L}_L \tilde{L}_R.$$  (171)

We expect that first the gauge couplings of $USp(6)_L \times USp(6)_R$ would approach to their non-trivial fixed point. Then, the $USp(6)_L \times USp(6)_R$ sector is dualized, that is, the gauge group is $U(3) \times USp(2)_L \times USp(2)_R \times U(1)$ as shown in Fig. 6. Note that $USp(2) \simeq SU(2)$. In addition we would have matter fields, $\hat{Q}_L : (\bar{3}, 2, 1, 0)$, $\hat{Q}_R : (3, 1, 2, 0)$, $\hat{L}_L : (1, 2, 1, 1)$ and $\hat{L}_R : (1, 1, 2, -1)$. Also, we would have several “meson fields” $\hat{M} : (3, 1, 1, -1)$ and $\hat{M} : (\bar{3}, 1, 1, 1)$, which have mass terms $m\hat{M}\hat{M}$ and Yukawa couplings with $\hat{Q}_L, \hat{Q}_R, \hat{L}_L$ and $\hat{L}_R$, but they can be integrated out because of heavy mass terms $m\hat{M}\hat{M}$. Then, we obtain the superpotential

$$W = \hat{h}\hat{Q}_L \hat{Q}_R \hat{L}_L \hat{L}_R.$$  (172)
Next, we expect that the gauge coupling of $SU(3)$ approaches to the conformal fixed point. Then, the $U(3)$ sector is dualized. The gauge group is $U(3) \times USp(2)_L \times USp(2)_R \times U(1)$ and we would have matter fields, $Q_L : (3, 2, 1, 0)$, $Q_R : (3, 1, 2, 0)$, $L_L : (1, 2, 1, 1)$ and $L_R : (1, 1, 2, -1)$ as well as several “Higgs fields” $H : (1, 2, 2, 0)$. The superpotential is obtained as

$$W = y_Q Q_L Q_R H + y_L L_L L_R H + m H H.$$  \hfill (173)

Note that the operator $y_L L_L L_R H$ corresponds to $y_L L_L L_R H$. However, the gauge symmetry $U(3) \times USp(2)_L \times USp(2)_R \times U(1)$ allows the mass terms $m H H$. Thus, we assume that such terms would be induced and we have added such terms. Then, if SUSY breaking terms induce a tachyonic mode of $H$, the symmetry $USp(2)_L \times USp(2)_R$ would be broken.

In this model, $USp(2)_L$ and $USp(2)_R$ symmetry breaking would happen at the same time. Although the left-right asymmetry is required for a realistic model, it would be difficult to generate such left-right asymmetry in this model. Some modification is necessary for a realistic model. At any rate, this model is an illustrating model for symmetry breaking. Such symmetry breaking by SUSY breaking terms in the duality cascade may be important, e.g. to realize the standard model at the bottom of the cascade.

5.5 Conclusion in section 5

We have studied the RG flow of softly broken supersymmetric theories showing the duality cascade. We find that conformal dynamics realizes approximate R-symmetry in the duality cascade as the discussion in section 4 as follows. Gaugino masses, A-term and scalar masses are suppressed in most regime of the RG flow although they increase in a certain perturbative regime. After exponential damping, the gaugino mass $M^{(k)}_{1/2}$ corresponding to the strongly coupled sector converges to a certain value, which is determined by the gauge coupling $\alpha_{k-1}$ and the gaugino mass $M^{(k-1)}_{1/2}$ in the weakly coupled sector. The scalar mass would also converge to the same order value. At the next stage of the cascade, the strongly and weakly coupled sectors are interchanged with each other and the gaugino mass $M^{(k-1)}_{1/2}$ would be suppressed. Thus, through the sequential cascade, the magnitude of gaugino masses and scalar masses would be suppressed.

The B-term may be important. In a certain parameter region, the B-term would induce tachyonic modes of $M_{rs}$ and symmetry breaking would happen. Such an aspect would be important to realistic model building.
The RG flow of SUSY breaking terms in the cascading theory is quite non-trivial as the RG flow of gauge couplings. The gravity dual of the cascade rigid supersymmetric theory has been studied extensively. However, our analysis implies that the dilaton is also running as $e^{-\phi} \sim \alpha_k^{-1} + \alpha_{k-1}^{-1}$ [27], but the supergravity solution of the D3-brane does not admit this running behavior and most of the supergravity dual theories concentrate on the constant dilaton backgrounds. In this sense, the suppression of the gaugino masses would be a quite different mechanism from the suppression due to the warp factor as already pointed out in [66, 67, 68, 69]. The region of RG flow in our study might be outside of the supergravity approximation, but it would be quite interesting to study the gravity dual side corresponding to the RG flow of SUSY breaking terms including the dilaton running.

We have considered the scenario that supersymmetry is broken at high energy and investigated the RG flow of SUSY breaking terms. Alternatively, we could consider another scenario that supersymmetry would be broken at some stage of the cascade. For example, supersymmetry is broken dynamically through the cascade and such breaking is mediated to the visible sector. Such a study would also be important.

6 Summary

In section 2, we studied SUSY breaking in local and global supersymmetric theory. The NS argument suggests that $U(1)_R$ symmetry plays an important role in building the models which cause SUSY breaking. We discussed the features of R-symmetric models which have SUSY breaking vacua in section 2.1. We suggest that it is the sufficient condition for SUSY breaking that the number of fields with R-charge 2 is larger than the number of $U(1)_R$-invariant independent operators which couple with the fields with R-charge 2. In the model, there is no supersymmetric vacuum and the minimum of the potential is given by the F-terms of the fields with R-charge 2. On the other hand, there are runaway directions in the model with fields whose R-charges are negative and/or more than 2.

In section 2.2 we studied the effect of explicit R-symmetry breaking terms in global supersymmetric models. Based on the argument by ISS, we have shown that certain classes of explicit R-symmetry breaking terms can restore SUSY, and the original SUSY breaking vacuum can become metastable.

In section 2.3 we discussed SUSY breaking in local supersymmetric mod-
els. It is a challenging issue in supergravity models to realize the almost vanishing vacuum energy. The vacuum energy may be tuned to vanish, e.g., by the constant superpotential term, which is a sizable R-symmetry breaking term. That would affect all of vacuum structure such as metastability of SUSY breaking vacua and presence of SUSY preserving vacua. We studied this vacuum structure by using the generalized O’Rraifeartaigh model added certain classes of R-symmetry breaking terms such that the vanishing vacuum energy is realized. We found that the metastability of SUSY breaking vacua can be avoided by limiting classes of R-symmetry breaking terms.

In section 3, we argued that conformal dynamics causes SUSY breaking vacua based on the argument in section 2. We find that fields with R-charge 2 play a key role in SUSY breaking and their quadratic and trilinear couplings destabilize the SUSY breaking vacua in explicit R-symmetry breaking models. Conformal dynamics can realize the suppression of unpleasant terms, so that it causes long-lived metastable SUSY breaking vacua. This is because conformal dynamics makes approximate R-symmetry recovered at a low energy scale. We found that approximate R-symmetry can be realized even in softly SUSY breaking theories in section 4. In fact, gaugino mass and A-term, which are soft SUSY breaking and explicit R-symmetry breaking terms, are suppressed corresponding to a gauge coupling and yukawa couplings going to IR fixed points. These suppressions appear even in more complicated, such as the duality cascade. Conformal dynamics allows Seiberg dual near an IR fixed point, and we call sequential Seiberg dual the duality cascade. In section 5, we found that gaugino masses and A-term are suppressed as long as the cascade continues. However, B-term remains to be a finite value at a low energy scale, so there is a possibility that the B-term causes gauge symmetry breaking corresponding to EW symmetry breaking. We suggest a scenario that a SM-like model appear at the bottom of the duality cascade, based on the above arguments.

Conformal dynamics plays important roles in various aspects of (super)symmetric field theories and particle phenomenology, and other good aspects of models with conformal dynamics are also suggested. For example, conformal dynamics realizes not only suppressions of flavor-dependent SUSY breaking terms [17], [18], [19], [20], [21], [22], [70], but also realistic hierarchies of quark and lepton masses [57], [61], [62]. Our models, which we introduce in section 3, 4 and 5, are also expected to realize the interesting aspects for phenomenology. However, we have not yet discussed application of our model, especially in section 5, for phenomenology explicitly. This is our future work.
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A Supersymmetric masses involving R-axion

In this appendix, we show some general results for the SUSY masses for the scalar component of an R-axion multiplet. For this analysis, it is convenient to redefine the R-charged superfield $Y$ by

$$R = \frac{2}{q_Y} \ln Y,$$

where $R$ can be interpreted as the R-axion superfield. (Note that $R = -aT$ in Eq. (56).) In this basis, the Kähler potential and the superpotential (3) is written as

$$K = K(R + \bar{R}, \chi_i, \bar{\chi}_i),$$
$$W = e^R \zeta(\chi_i).$$

(174)

From Eq. (174), we find $W^{-1} \partial^m W = 1$ where $m = 1, 2, \ldots$, and obtain

$$G_{RR} = K_{RR} + W^{-1} W_{RR} - (W^{-1} W_R)^2 = K_{RR} = K_{RR},$$
$$G_{\chi_i, R} = K_{\chi_i, R} + W^{-1} W_{\chi_i, R} - (W^{-1} W_{\chi_i})(W^{-1} W_R) = K_{\chi_i, R} = K_{\chi_i, \bar{R}}.$$

Substituting these into the general formulae for the second derivatives at the SUSY point,

$$V_{IJ}^{D_{K}W=0} = e^G (G^{MN} G_{MI} G_{NJ} - 2 G_{IJ}),$$
$$V_{IJ}^{D_{K}W=0} = -e^G G_{IJ},$$

we find

$$V_{RR}^{D_{K}W=0} = V_{RR}^{D_{K}W=0} = -K_{RR} m^2_{3/2};$$
$$V_{\chi_i, \bar{R}}^{D_{K}W=0} = V_{\chi_i, \bar{R}}^{D_{K}W=0} = -K_{\chi_i, \bar{R}} m^2_{3/2};$$

(175)

(176)
where \( m^2_{3/2} = e^G \) is the gravitino mass square.

From Eq. (175), the mass squared eigenvalues of \((\text{Re} \, R, \text{Im} \, R)\) can be calculated as 0 and \(-2m^2_{3/2}\) with the canonical kinetic terms normalized by \( K_{RR} > 0 \). The first massless eigenmode corresponds to the R-axion scalar associated to the spontaneously broken global \( U(1)_R \) symmetry. The second negative-definite eigenvalue indicates that the special SUSY solution \((51)\) is at best a saddle point solution. Note that the gravitino mass \( m_{3/2} \) is nonvanishing at this point and the vacuum energy is negative. We also find from Eq. (176) that the mixing-mass between \( R \) and \( \chi_i \) is vanishing if the Kähler (kinetic) mixing is vanishing, \( K_{\chi_i, R} = 0 \). In this case, the R-axion direction is completely separated from the other fields \( \chi_i \), that is, the above mass eigenvalues of R-axion multiplet become exact in this case.

Finally we comment that the second derivatives (175) and (176) are all vanishing at the SUSY point \((50)\) where \( m_{3/2} = 0 \). In this case, both the real and the imaginary scalar component of R-axion multiplet remain massless. Note that Eq. (51) may also allow a solution even in this case if \( \zeta \) is not a generic function.

\section*{B Supergraph formalism}

We discuss superfield propagators and Feynman rules for supergraphs, based on Ref.\[38\]. Let us consider the Lagrangian of \( \Phi \), chiral superfield, as follows.

\[
L = \int d^4 \theta \left\{ \frac{1}{2} \left( \Phi \Phi^\dagger \right) M \left( \frac{\Phi}{\Phi^\dagger} \right) \right\}
\]

\[
M = \left( \begin{array}{cc} - \frac{m}{4} D^2 & 1 \\ 1 & - \frac{m}{4} D^2 \end{array} \right) + L_{\text{int}}
\]  

(177)

We define the generating function for superfield Green functions,

\[
Z[J, J^\dagger] = \langle 0 | \text{exp}[i \int d^4 \theta d^4 x \{ J(z) \left( - \frac{1}{4} D^2 \right) \Phi(z) \\
+ J^\dagger(z) \left( - \frac{1}{4} D^2 \right) \Phi^\dagger(z) \} ] | 0 \rangle
\]  

(178)
, where \( z \) is defined as \( z = (x, \theta, \bar{\theta}) \) and we use the below relation,

\[
\frac{\delta}{\delta J(x, \theta, \bar{\theta})} J(x', \theta', \bar{\theta}) = -\frac{1}{4} \overline{D}^2 \delta^4(\theta - \theta') \delta^4(x - x'). \tag{179}
\]

\( D \) and \( \overline{D} \) are covariant derivative, and also satisfy the identities,

\[
\int d^4x d^2\theta d^4x' d^2\theta' f(z) = \int d^4x d^2\theta (-\frac{1}{4} \overline{D}^2 f(z)). \tag{180}
\]

We define \( Z_0[J, J^\dagger] \) as the generating functional for free superfield Green’s functions, and \( Z[J, J^\dagger] \) is described as

\[
Z[J, J^\dagger] = \exp\{i \int d^4x L_{int} \left( \frac{\delta}{\delta J}, \frac{\delta}{\delta J^\dagger} \right) \} Z_0[J, J^\dagger]. \tag{181}
\]

We find \( Z_0[J, J^\dagger] \),

\[
Z_0[J, J^\dagger] = \exp\{-\frac{i}{2} \int d^4x d^4x' d^2\theta d^4x' d^2\theta' (J J^\dagger) \Delta_{GRS}(z, z') \left( J J^\dagger \right) \}, \tag{182}
\]

where \( \Delta_{GRS} \) is the propagator introduced by Grisaru, Roček and Segel \[38\]:

\[
\Delta_{GRS}(z - z') = \frac{1}{\Box - m^2} \left( \frac{-m^2}{1} \frac{1}{-\frac{m^2}{4} \overline{D}^2} \right) \delta(z - z'). \tag{183}
\]

We consider renormalizable superpotential, \( W(\Phi) = \frac{m^2}{2} \Phi^2 + \frac{1}{6} \Phi^3 \), so that \( L_{int} \) is given by

\[
L_{int} \left( \frac{\delta}{\delta J}, \frac{\delta}{\delta J^\dagger} \right) = \int d^4x d^2\theta \left( \frac{1}{i} \overline{D}^2 D^2 \frac{\delta}{\delta J} \right) + h.c. \ , \tag{184}
\]

where we use

\[
\overline{D} D^2 \overline{D} = 16 \Box \overline{D}^2 \tag{185}
\]

\[
D^2 \overline{D}^2 D^2 = 16 \Box D^2.
\]
The vertexes can be obtained from the following formula by using (180),

\[
\int d^4x d^4\theta L_{\text{int}} \left( \frac{\delta}{\delta J^1} \frac{\delta}{\delta J^1} \right) J(z_1)J(z_2)J(z_3) = -\frac{\lambda}{6} \int d^4x d^2\theta \left( \frac{\delta}{\delta J(z)} \right)^3 J(z_1)J(z_2)J(z_3)
\]

\[
= -\lambda \int d^4x d^4\theta \delta^8(z_1 - z) \left\{ -\frac{1}{4} D_2^2 \delta^8(z_2 - z) \right\} \left\{ -\frac{1}{4} D_3^2 \delta^8(z_3 - z) \right\}.
\]

(186)

On the other hand, the propagator of vector superfield is described as

\[
\Delta_V(z, z')^{AB} = -\frac{1}{\Box} \delta^{AB} \delta(z - z').
\]

(187)

Now we find out Feynman rules for supergraph. Each vertex includes a factor of \(-\frac{1}{4} D^2\) or \(-\frac{1}{4} D^2\) acting on each chiral (or antichiral) superfield, but we omit one \(-\frac{1}{4} D^2\) (or \(-\frac{1}{4} D^2\)) and integrate \(\int d^4x d^4\theta\). The amputated one-particle-irreducible graphs in the effective action should have each amputated external line, so \(-\frac{1}{4} D^2\) (or \(-\frac{1}{4} D^2\)) should be omitted at a vertex for each external chiral (or antichiral) superfield.

The Feynman rules and the GRS propagators give a loop graph with n-th vertexes an expression of the following form:

\[
(D_1^2)^{l_1} (D_1) \delta^4(\theta_1 - \theta_2)(D_2^2)^{l_2} (D_2) \delta^4(\theta_2 - \theta_3)\ldots(D_n^2)^{l_n} (D_n) \delta^4(\theta_n - \theta_1),
\]

(188)

where \(l_i, k_i\) are 0 or 1, and we use (185). It is integrated over \(d^4\theta_1\ldots d^4\theta_n\). We find the n-th point Green functions are always given by the form of

\[
\int d^4\theta \Pi^n_{i=1} d^4x G_n(x_1, \ldots, x_n)f(\Phi, D\Phi, ..).
\]

(189)

The function \(f\) depends on superfields and covariant derivatives, so that this result leads to non-renormalization of the superpotential, since the form of each loop graph is \(\int d^4\theta\).

Furthermore, we discuss power counting rules to look into the degree of the divergence. All renormalizable vertexes go as \(D^4 \sim p^2\), and external chiral lines go as \(1/D^2 \sim 1/p\). Loops go as \(d^4p/D^4 \sim p^2\), because 4 \(D\) are used to cancel a loop’s delta function, such as \(\delta^4(\theta_n - \theta_1)\) in (188). Eventually, the degree of the divergence, \(\omega\), is given by

\[
\omega = 2L - 2P - C + 2V - E = 2 - C - E,
\]

(190)
where \( L, P, C, V \), and \( E \) denote the number of loops, propagators \((\Delta_{GRS})_{11,22}\), chiral propagators \((\Delta_{GRS})_{12,21}\), vertexes, and chiral external lines respectively, and we use \( L = 2P - V + 1 \). This result means that we find the divergence is at most logarithmic.

SUSY breaking terms can be incorporated into the superspace perturbation by using spurion superfields we introduce in the Sect.\( \text{C} \). In the spurion formalism, the spurion superfields, which include the SUSY breaking terms as the components, are treated as external fields in perturbation. This does not destroy the above argument, so that the absence of quadratic divergence is ensured.

\( \text{C} \) Spurion Technique

We discuss the spurion method, based on Ref. [31, 34]. The soft SUSY breaking terms, which does not cause quadratic divergence, are restricted to the following,

\[
L_{\text{soft}} = - \int d^4 \theta (m_{ij} \theta^i \theta^j) \Phi^i \Phi^j - \int d^2 \theta (\frac{1}{g^2} Tr(W^\alpha W_\alpha)) \\
- \int d^2 \theta \frac{1}{2} (\mu_{ij} \theta^2) \Phi^i \Phi^j - \int d^2 \theta \frac{1}{6} (h_{ijk} \theta^2) \Phi^i \Phi^j \Phi^k + \text{h.c.}
\]  

(191)

It is possible that we check these are soft SUSY breaking terms according to appendix app:superfield. First, we concentrate on the contributions of yukawa couplings, A-term \( h_{ijk} \) and scalar mass terms \( m_i^2 \). Based on the non-renormalization of the superpotential, the effective Lagrangian in this case is given by,

\[
L_{\text{eff}} = \int d^4 \theta \bar{Z}_i(\theta, \bar{\theta}) \Phi^i \Phi_i + \int d^2 \theta \frac{1}{2} (m_{0ij} - \mu_{ij} \theta^2) \Phi^i \Phi^j \\
+ \int d^2 \theta \frac{1}{6} (y_{0ijk} - h_{0ijk} \theta^2) \Phi^i \Phi^j \Phi^k + \text{h.c.}
\]  

(192)

Since the renormalization of (anti-)chiral superfields must be also (anti-)chiral, \( \bar{Z}_i(\theta, \bar{\theta}) \) should be described as the following form,

\[
\bar{Z}_i(\theta, \bar{\theta}) = Z_i(\bar{\theta})^\dagger (1 - m_i^2 \theta^2 \bar{\theta}^2) Z_i(\theta).
\]  

(193)

\( ^{27} \)D-term does not receive radiative corrections, as far as gauge symmetry is preserved.
Under this description, the renormalized yukawa coupling superfields can be defined as

\[ Y_{ijk}(\theta) = y_{ijk} - h_{ijk}\theta^2 = Z_i^{-1}(\theta)Z_j^{-1}(\theta)Z_k^{-1}(\theta)(y_{0ijk} - h_{0ijk}\theta^2). \]  

These correspond to field redefinitions of (anti-)chiral superfields. \( Y_{ijk}(\theta) \) are treated as external fields in perturbation. Furthermore, we find that the superfield propagators in the softly broken theories are modified from \( \Delta_{GRS} \),

\[ \Delta_{soft}^{ij} = (1 + \frac{1}{2}m_i^2\theta^2\bar{\theta}^2)\Delta_{GRS}^{ij}(1 + \frac{1}{2}m_j^2\theta^2\bar{\theta}^2). \]

In other words, this modification means that yukawa couplings are modified as follows,

\[ \tilde{y}_{ijk} = Y_{ijk} + \frac{1}{2}(m_i^2 + m_j^2 + m_k^2)y_{ijk}\theta^2\bar{\theta}^2. \]

Eventually, the dependence of \( \tilde{Z}_i(\theta, \bar{\theta}) \) is given by,

\[ \tilde{Z}_i(\theta, \bar{\theta}) = \tilde{Z}_i(\tilde{y}_{ijk}, \bar{\tilde{y}}_{ijk}). \]

We define \( Z_i \) as the wave function renormalization factors of \( \Phi_i \) in supersymmetric models without soft terms, so that \( \tilde{Z}_i \) satisfy \( \tilde{Z}_i(\tilde{y}_{ijk}, \bar{\tilde{y}}_{ijk}) = Z_i(y_{ijk}, \bar{y}_{ijk}) \).

We can also discuss gauge coupling and gaugino mass, according to the above argument. Before the discussion about the soft term, we review the RG flow of gauge coupling. The holomorphic gauge coupling \( S \) is renormalized only at 1-loop \[71\]. The RG equation for the holomorphic coupling is given by

\[ \mu \frac{dS}{d\mu} = \frac{1}{16\pi^2}(3T_G - \sum_i T_i), \]

where \( T_G = C_2(G) \) and \( T_i = T(R_i) \) for the gauge representation \( R_i \) of the chiral superfield \( \Phi_i \). The relation between the holomorphic gauge coupling and the physical gauge coupling \( \alpha \) is given by

\[ 8\pi^2(S + S^\dagger) - \sum_i T_i \ln Z_i = F_\beta(\alpha) \]

\[ F_\beta(\alpha) = \frac{1}{\alpha} + T_G \ln \alpha + \sum_{n>0} a_n \alpha^n, \]  

87
where \( \alpha = \frac{g^2}{8\pi^2} \) and \( a_n \) are scheme dependent constants. The NSVZ scheme corresponds to the case of \( a_n = 0 \) for all \( n \). Eventually, we find the relation (199) gives the exact beta function,

\[
\mu \frac{d}{d\mu} \alpha = \beta_\alpha = \frac{1}{F_g'(\alpha)} \{ 3T_G - \sum_i T_i(1 - \gamma_i) \},
\]

(200)

where \( \gamma_i \) denotes the anomalous dimension for \( \Phi_i \),

\[
\gamma_i = -\mu \frac{d \ln Z_i}{d\mu}.
\]

(201)

Now we discuss the soft SUSY breaking term, gaugino mass, according to the discussion about the scalar masses and A-term. In softly broken theory, the holomorphic gauge coupling is modified as follows,

\[
\tilde{S} = \frac{1}{g_h^2}(1 - 2M_{1/2}\theta^2).
\]

(202)

Furthermore, (197) and (199) gives the following description,

\[
8\pi^2(\bar{S} + \tilde{S}^\dagger) - \sum_i T_i \ln \tilde{Z}_i = F_g(\tilde{\alpha}).
\]

(203)

where \( \tilde{\alpha} \) is defined as follows,

\[
\tilde{\alpha} = \alpha(1 + M_{1/2}\theta^2 + \overline{M}_{1/2}\overline{\theta}^2 + (2|M_{1/2}|^2 + \Delta_g)\theta^2\overline{\theta}^2).
\]

(204)

and \( \Delta_g \) is given by

\[
\Delta_g = \frac{1}{\alpha F_g'(\alpha)} \{ \sum_i T_i m_i^2 - (\alpha^2 F_g'(\alpha))' |M_{1/2}|^2 \}.
\]

(205)

On the other hand, we find the dependence of \( \tilde{Z}_i \) by applying the argument about yukawa coupling,

\[
\tilde{Z}_i(\theta, \overline{\theta}) = \tilde{Z}_i(\alpha, \tilde{y}_{ijk}, \overline{\tilde{y}}_{ijk}).
\]

(206)

Finally, we find RG equations of soft SUSY breaking terms we introduce in Sect.4. We define the beta functions of gauge coupling and yukawa coupling as Sect.4,

\[
\mu \frac{d\tilde{\alpha}}{d\mu} = \beta_\alpha(\tilde{\alpha}, \tilde{y}_{ijk}, \overline{\tilde{y}}_{ijk}), \quad \mu \frac{d\tilde{y}_{ijk}}{d\mu} = \beta_{y_{ijk}}(\tilde{\alpha}, \tilde{y}_{ijk}, \overline{\tilde{y}}_{ijk}).
\]

(207)
The beta-function of the gaugino mass $M_{1/2}$ is obtained as
\[ \mu \frac{dM_{1/2}}{d\mu} = \left( M_{1/2} \alpha \frac{\partial}{\partial \alpha} - a_{ijk} \frac{\partial}{\partial y_{ijk}} \right) \left( \frac{\beta_{\alpha}}{\alpha} \right) \equiv D_1 \left( \frac{\beta_{\alpha}}{\alpha} \right). \] (208)

The RG equation for the A-term is
\[ \mu \frac{dh_{ijk}}{d\mu} = \frac{1}{2} (\gamma_i + \gamma_j + \gamma_k) h_{ijk} - (D_1 \gamma_i + D_1 \gamma_j + D_1 \gamma_k) y_{ijk}. \] (209)

The RG equation for the soft scalar mass $m_i$ of a chiral superfield $\phi_i$ is obtained as
\[ \mu \frac{dm_i^2}{d\mu} = \gamma_i (\tilde{\alpha}, \tilde{y}_{ijk}) \big|_{\theta^2 \bar{\theta}^2}. \] (210)

This leads the following,
\[ \mu \frac{dm_i^2}{d\mu} = D_2 \gamma_i, \] (211)
\[ D_2 = D_1 \tilde{D}_1 + (|M_{1/2}|^2 + \Delta_g) \alpha \frac{\partial}{\partial \alpha} \]
\[ + \frac{1}{2} (m_i^2 + m_j^2 + m_k^2) \left( y_{ijk} \frac{\partial}{\partial y_{ijk}} + \bar{y}_{ijk} \frac{\partial}{\partial \bar{y}_{ijk}} \right). \] (212)

The above results, such as the coupling superfields given by (204) and (196) are also supported by the symmetry argument [33, 72]. Once we suppose that the coupling superfields are dynamical, then the softly broken theories have a global $U(1)_{\Phi_i}$ symmetry corresponding to each chiral superfield $\Phi_i$,
\[ \Phi_i \rightarrow e^{T_i} \Phi_i, \]
\[ \tilde{Z}_i \rightarrow e^{-T_i} \tilde{Z}_i e^{-T_i} \]
\[ Y_{ijk} \rightarrow e^{-T_i} Y_{ijk} \]
\[ S \rightarrow S - \frac{T}{8\pi^2} \sum_i T_i, \] (213)

where $T_i$ is also a chiral superfield. $Y_{ijk} \tilde{Z}_i^{-1} Y_{ijk}^\dagger$ is given as an invariant form. This leads the deformation of yukawa coupling in (196). The symmetry also gives the deformation of gauge coupling in (204).
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