Simulation of water flow in a conduit using radiotracer-axial dispersion model

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Abstract. A simple radiotracer experiment has been carried out to simulate water flow in a conduit made of PVC plastic with a diameter of 2 in. (50.8 mm). The water was supplied from a water tank with a volume of 500 liters. During the experiment, the flow rate of water in the conduit was maintained constant at a speed of 0.2 m.s⁻¹ by adjusting valve at the farthest end of the conduit. Br-82 isotope solution in a concentration of 1 mCi.ml⁻¹ was injected instantaneously into the conduit. Injected isotope follows the bulk flow of water in the conduit. A scintillation NaI(Tl) radiation detector which was placed on the conduit at a distance of 8 m from injection point captures the radiation energy of injected radiotracer when passing it and generate residence time distribution (RTD) curve which represent isotope particle spent in the conduit. To get meaningful information, a mathematical simulation so-called the axial dispersion model has been developed to evaluate the RTD curve with Peclet number, Pe, as a model parameter. The best fitting of the RTD curve model onto the experimental one was achieved when the Pe is equal to 13. The calculated fitting error was 0.003, very extremely low as expected. This result indicated that the axial dispersion model was able to quantify water flow and simulated Peclet number indicated that convection flow is more dominant compared to the radial flow.

1. Introduction
From a micro-scale point of view, transportation of fluids in a pipe, either in small or large diameter is complex processes. Bulk flow of fluid axially may generate fluid flow radially due to mass and momentum transfer [1]. As long as there is no thermal energy difference, energy transfer in fluid flow is regardless. Although controlling and measuring fluid flow is fundamental in engineering, however, flow gauge installed on a pipe can only give specific information on how much fluids pass through a cross-sectional area [2]. There are many kinds of sensor flowmeter applied such as electromagnetic flowmeter [3], piezoelectric sensor [4], differential pressure type flowmeter [5], ultrasonic flowmeter [6] and optical fiber sensor [8]. For scientists and engineers knowing the distribution of fluid flow in a pipe is more essential than metering flow itself because it gives an insight of knowledge to understand the mixing level of fluids particles either in single-phase or multiphase systems [9-12].

One of the optional methods to study the distribution of fluid particles in a dynamic system is by the implementation of computer modeling or simulation. The simulation requires differential equations as a mathematical model to describe material transport in the system. The solution of a differential equation is usually solved using numerical methods. However, to validate the model experimental work is recommended to be carried out. Tracers either as chemical tracers [13-19] dan or radioactive tracers [20-26] are commonly used for tracer experiments in laboratories and fields. In practice, a large number of chemical tracers are needed for experiments because the mode of measurement is frequently carried out.
by sampling. In contrast, a small amount of radioactive tracers or radiotracer is needed for the same purpose because radiation energy of radioactive traces can penetrate the wall, therefore, the measurement can be carried out from outside of the system. This advantage shows the state of the art of radiotracer for many cases of measurement.

Radiotracer principle is based on the injection of isotope into a system under investigation to generate residence time distribution (RTD) data. The successful application of the radiotracer experiment strongly depends on the selection of radioactive material or isotopes as a radiotracer. They have been mentioned elsewhere that the selection of isotopes for the radiotracer experiment is based on compatibility of physico-chemical property of radiotracer to the traced bulk materials, energy strength of radiation source, and the half-life of isotopes [27,28]. Also, the injection of radiotracer must not disturb or modify the hydrodynamic of the system due to the chemical reaction of an isotope with the bulk-flow or with the wall of the containment [29]. Among available radioactive material, gamma emitter isotopes are the best to be selected as radiotracer and for this study, bromine-82 isotope has been chosen for radiotracer experiment. Its high specific gamma energy makes it easily detected even in small amounts without ambiguity.

The purpose of this study is to simulate water flow in the pipe conduit containing water flow. The governing equation based on the axial dispersion process is introduced to model the flow. The analytical solution for this equation is available but the determination of the model parameter should be calculated numerically. On the one hand, the model is useful to describe the flow, on the other hand, the model should be validated by radiotracer experimental data. It is therefore that the other purpose of this study is to validate the model. The validation is judged by best fitting for certain model parameters, namely the Peclet number.

2. Governing equation
In radiotracer flow modeling, two models are commonly used, namely the tanks-in-series model dan the axial dispersion model [27]. Although both models are usually used for predicting the mixing level of fluid flow in a system, however, their basic idea of flow is completely different. As this study is concerned with the axial dispersion model, therefore the governing equation of water flow is based only on this model. Those who are interested in the tanks-in-series model are encouraged to search from other publications [9].

The axial dispersion model is basically derived from a mass balance concept that contains flow due to dispersion and flow due to convection (bulk flow). It is therefore the complete differential equation for the steady-state condition in a tubular reactor at the position of $\Delta z$ is [30]

$$\left[(-D \frac{\partial C}{\partial Z} + uC) \pi R^2 \right] \Delta t = \left[(-D \frac{\partial C}{\partial Z} + uC) \pi R^2 \right]_{z+\Delta z} + \pi R^2 \Delta Z \Delta C \tag{1}$$

When divided by $\pi R^2 \Delta t \Delta Z$, and by taking the limit approach for $\Delta z \to 0$, the above equation is simplified to

$$\frac{\Delta C}{\partial t} = D \frac{\partial^2 C}{\partial Z^2} - u \frac{\partial C}{\partial Z} \tag{2}$$

When is expressed into dimensionless quantity for which $\theta = t/\bar{t} = tu/L$, and $Z = (ut + x)/L$ the Eq. (2) has become

$$\left(\frac{1}{P_e}\right) \frac{\partial^2 C}{\partial Z^2} - u \frac{\partial C}{\partial Z} = \frac{\Delta C}{\partial \theta} \tag{3}$$
Where
\[ C = \frac{C(t)}{C(0)} \] is radiotracer concentration, dimensionless.
\[ P_e = \frac{uL}{D} \] is Peclet number, dimensionless.
\[ u = \text{flow velocity, } \text{ms}^{-1} \]
\[ Z = \frac{x}{L} \] is distance, dimensionless
\[ \theta = \frac{t}{\bar{t}} \] is time, dimensionless

Equation (3) is a second-order differential equation. To solve these equation boundary conditions are introduced. As the physical system is a conduit, open-open boundary conditions are applied for which
\[ \frac{dc}{dz} = 0 \text{ at } z = 0, \text{ and} \]
\[ \frac{dc}{dz} = 0 \text{ at } Z = L \]

Equation (6) is an RTD function that describes particle distribution of the axial dispersion model with model parameter is Peclet number, \( P_e \).

3. Experiment
The radiotracer experiment was carried out at the laboratory. A conduit made of PVC pipe with a diameter of 3-inch containing flowing water has been prepared as the system of experimentation. The water flow was supplied from a water tank that is connected to the pipeline. During the experiment, the water tank is maintained at full level. The water flow is adjusted by a hand valve which is located at another end of the pipeline in such a way that the constant flow velocity of 0.2 m/min is maintained. The hand valve is connected to the storage tank for collecting contaminated water through a plastic conduit. Bromine-82 isotope [gamma energy: 550 (70%) keV and 1320 keV (27%)] with activity around 1 mCi in volume of 1 mL is prepared for injection. A sodium iodide NaI(Tl), a radiation detector was installed above the pipeline at a distance of 8 meters from the injection point. The injection point is a small hole in the pipeline which is covered by a rubber band. The radiation detector is connected to the laptop computer into which a dedicated software for radiotracer measurement has been installed. Before injection, a dummy test was carried out to ensure that the real injection is assured smoothly and safely. When all equipment and instrumentation are working properly, injection is then carried out instantaneously using a disposable plastic syringe. The injected radiotracer will follow the bulk traced to water in the conduit pipeline. The radiation detector records all radiation intensity emitted from injected radiotracer. The maximum recorded radiation intensity is achieved when the injected radiotracer is at the point closest to the radiation detector. The data-logger which is integrated into the laptop computer was controlled by radiotracer software to save all recorded data of measurement automatically. Whenever the experiment finished, the instrumentation is switched off and the water flow system is then closed. During the experimentation, all flow parameters, such as water pressure and water flow were kept constant. This is important for radiotracer data analysis. The schematic radiotracer experiment is shown in Fig. 1.
4. Result and discussion

Before going further to discuss the experimental result in detail, lets us remind ourselves that RTD data in the form of a curve obtained from the injection of radiotracer into a flow system give us qualitative information on the mixing level of fluid particles in the system. This information can be recognized from the shape and spread out of the RTD curve. When the curve is tall and narrows the mixing level is low and vice versa. Moreover, the RTD curve cannot be defined exactly by any differential or integral equations. To get meaningful information on the mixing level, the quantitative analysis by using a mathematical model is recommended to be performed. In this study, the mathematical model used is the axial dispersion model which is theoretically based on the dispersion of fluid particles in the axial direction.

The experiment is one of the efforts for validating the proposed model. Our task is now to generate an RTD model that is similar to the RTD data. Tanks to discrete mathematic that make it possible to solve many physical problems numerically. Once the RTD model is generated completely, it is then fitted to the RTD data to validate the model. The best-fitting is judged by a minimum numerical error between RTD data and the RTD model.

Experimental data in the time domain obtained from the injection of radiotracer into a conduit is presented in Fig. 2. The data shows a distribution function that presents the spent time of each fluid particle in the system. This typical data of measurement is commonly called residence time distribution (RTD). The form of the RTD curve strongly depends on the dynamic flow of fluid in the conduit due to molecular diffusion, turbulence and non-uniformity of particle velocity [31]. The first step to analyze the RTD data is by observing the form of the data structure, especially its fluctuation. Fluctuation may come from a combination of flow dynamic and random process of the radioactivity of injected radiotracer.

Although radioactivity property of radiotracer is always naturally occurred, reducing radioactivity due to decay processes does not necessarily be corrected because the half-life of injected radioisotope ($^{82}$Br) is longer compared to the time required to do a complete experiment. In addition to this, the correction due to background radioactivity is also not necessarily be performed because the RTD data can clearly be distinguished from its background. To analyze the RTD curve, the background intensity that is come before and after curve may be eliminated, because these data do not give any information related to dynamic flow in the system. In this case, the RTD data is the area that is limited to the starting from the point of the rising curve and until completely down to the background point. As can be seen from Fig. 2, the RTD curve represents an area, it is therefore the analysis of such a curve is based on applied mathematics or area analysis.

![Figure 1. Schematic radiotracer experiment.](image)
4.1 Mean residence time calculation

RTD data obtained from measurement is the only real data that give invaluable information about the flow dynamics of the system. One important quantity to be considered is the mean residence time (MRT). This quantity is needed to be calculated because it represents the first moment or center of the weight of a curve. As we might have known that injected radiotracer, even in small amounts, contains uncountable isotope particles. When injected into the system, these particle is distributed and follow the bulk traced flow in it. Each particle will reside at a different time in the system. By MRT which is formulated as Eq. (7), this particle distribution is simplified to become as if it is a single particle movement.

\[
\bar{t} = \frac{\int_{0}^{\infty} t c(t) \, dt}{\int_{0}^{\infty} c(t) \, dt} = \frac{\sum_{i=1}^{N} t c(t) \Delta t_i}{\sum_{i=1}^{N} c(t) \Delta t_i} \tag{7}
\]

It is worth to note that the requirement for fully developed flow conditions in the pipe conduit must be fulfilled to get an accurate result of MRT calculation. In radiotracer practitioners and mentioned in the limited publication, the condition of fully developed flow is achieved when the detector is placed at least 200 times of pipe conduit diameter [27]. In addition, although it has been mentioned previously, the constant flow is kept constant, the numerical approximation of MRT calculation using Eq. (7) is 22.03 s.

4.2 Flow simulation

In this study, flow simulation or flow modeling is an effort to describe unseen features of water flow dynamics in a pipe conduit. The radiotracer experimental work is considered as a tool for validation of the axial dispersion model that is being proposed. The governing equation for the axial dispersion model consists of two components, namely a convective component arising from bulk motion of fluid and a diffusive component due to random motion of the fluid in responding to the turbulent flow [32]. The analytical solution of the governing equation for the open-open boundary condition is expressed in Eq. (6). Peclet number, \( P_e \), in Eq. (6) which describes the ratio of flow due to convection to the flow due to diffusion is the only model parameter for the axial dispersion model. When \( P_e \to 1 \), the flow prefers to follow well-mixed low and when \( P_e \to \infty \), the flow is preferred to follow plug flow [27].

A solution of governing equation based on the theoretical point of view, some assumption points are needed. The assumptions that have been made are as follows [32]: (1) steady-state condition is achieved and is maintained during the radiotracer test, (2) a delta-Dirac tracer injection assuring that the tracer
concentration is function of time and axial position only, and (3) the coefficient of axial convective velocity and the axial dispersion are constant for stable operating conditions. For laboratory experimentation, these conditions are easily managed.

The concept of RTD was the first time proposed by Mac Mullin and Weber, but the extensive use of this concept was performed by Danckwerts to study flowing fluids in a non-ideal reactor such as tubes and mixers [33]. The RTD curve is obtained from injection chemical tracer or radioactive tracer into the system. The total amount of injected radiotracer can be calculated by normalizing the integration of the function of the fluid element from time equal zero to infinity [27]

\[
\int_0^\infty E(t)dt = 1
\]  

(8)

where \( E(t)dt \) is a fraction of fluid element at the outlet that has spent time between \( t \) and \( t + dt \). In many cases, the RTD data is expressed in dimensionless time, \( \theta = t/\bar{t} \), which has been defined in Eq. (6). The advantage of using dimensionless time is the MRT can be eliminated as a variable for different operating conditions. The relation of the RTD function has accordingly become [34]

\[
E(\theta) = \bar{t}E(t)
\]  

(9)

To generate the RTD model using the axial model is in dimensionless time was carried manually in excels spreadsheet of Microsoft Office 2010. The time interval of successive measurement was set for one second, same as its set up in data-logger instrumentation when the experiment is carried out. Whenever the RTD model is completely generated, it is then immediately fitted with the RTD experiment. The best-fitting is achieved when the Peclet number is equal to 13, as shown in Fig. 3. According to the definition of Peclet number, this number means that the flow due to bulk water flow is 13 times the flow due to diffusion in the axial direction. This number is also indicated that mixing to some extent has occurred in the water flow but the flow is still dominated by flow due to the convection process.

**Figure 3.** Fitting RTD-model to RTD-experimental in the dimensionless time domain. Best fitting is achieved when \( P_e = 13 \) with fitting error, RMS=0.003.

It has been known that simulation of water flow in pipe conduit using numerical axial dispersion model generated a fitting error which is termed as root mean square (RMS) and formulated as [10]

\[
RMS = \left\{ \frac{1}{N_r} \sum_{r=1}^{N_r} |E(\theta) - E_{model}(\theta, parameter)|^2 \right\}^{1/2} = \text{minimum}
\]  

(10)
where $E(\theta)$ is RTD function in dimensionless time, $E_{\text{model}}(\theta,\text{parameter})$ is RTD function in dimensionless time but for specific Peclet number and $N_T$ is the number of RTD data evaluated. The fitting error calculated by Eq. (6) is 0.003. This number is extremely small indicating that the proposed axial dispersion model was successfully validated and the model is appropriate to describe the flow dynamics of water in the pipe conduit.

5. Conclusion
A simple radiotracer experiment to simulate water flow in the PVC conduit has been carried out safely. The axial dispersion model was able to predict the dynamics of water flow in the conduit. The Numerical value of Peclet number, $P_e = 13$, calculated based on best fitting criteria indicated that mixing to some extent has occurred, but the water flow is still dominated by flow due to convection. Fitting error as low as 0.003 indicates that the proposed model is validated and the model is appropriate to describe water flow in a conduit.

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