Chiral spin symmetry and hot/dense QCD.

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Abstract

Above the chiral symmetry restoration crossover around $T_{ch} \sim 155$ MeV a new regime arises in QCD, a stringy fluid, which is characterized by an approximate chiral spin symmetry of the thermal partition function. This symmetry is not a symmetry of the Dirac Lagrangian and is a symmetry of the electric part of the QCD Lagrangian. In this regime the medium consists of the chirally symmetric and approximately chiral spin symmetric hadrons that are made of the chirally symmetric quarks connected into the color singlet compounds by a confining chromoelectric field. This regime is evidenced by the approximate chiral spin symmetry of the spatial and temporal correlators and by the breakdown of the thermal perturbation theory at the crossover between the partonic (the quark-gluon plasma) and the stringy fluid regimes at $\sim 3T_{ch}$. The chiral spin symmetry smoothly disappears above $\sim 3T_{ch}$ which means that the chromoelectric confining interaction gets screened. A direct evidence that the stringy fluid medium consists of densely packed hadrons is the pion spectral function that shows a distinct pion state and its first radial excitation above $T_{ch}$. Another direct evidence of the hadron degrees of freedom in the stringy fluid is the bottomonium spectrum with the $1S, 2S, 3S$ and $1P, 2P$ radial and orbital excitations that become broad with temperature. The hadrons between $T_{ch}$ and $\sim 3T_{ch}$ in the stringy fluid interact strongly which makes the stringy fluid more a liquid rather than a gas. We discuss how this chiral spin symmetric regime extends into the finite chemical potentials domain and present a qualitative sketch of the QCD phase diagram.

Keywords: chiral spin symmetry, hot and dense QCD, stringy fluid, QCD phase diagram
1. A little bit of history. Introduction.

Before the RHIC era there was a general belief that at some critical temperature $T_c$ there should happen a deconfinement phase transition from a hadron resonance gas (HRG) phase to a quark gluon plasma phase (QGP) [1, 2]. A crucial difference between two phases are degrees of freedom. While in the HRG phase the degrees of freedom are hadrons that practically do not interact, in the QGP phase these are partons - quarks and gluons. There is no spontaneous breaking of chiral symmetry in a system of free partons. Consequently it was expected that at the same temperature $T_c$ a chiral restoration happens, so the critical temperature $T_c$ should be a common temperature of both deconfinement and of chiral restoration phase transitions.

A lot of experimental efforts for the last 30 years were invested into a search of the quark-gluon plasma in heavy ion collisions at AGS (BNL), SPS (CERN), RHIC (BNL) and LHC (CERN). Experimental findings first at RHIC and then at LHC indicate that assuming local thermal equilibrium within the fireball the hot QCD matter is different from a dilute hadron resonance gas at low temperatures [3, 4, 5, 6, 7], indeed. The most prominent result is the observation of the elliptic flow. The fit of hadron spectra and elliptic flow at RHIC and LHC by means of viscous hydrodynamic, see [8, 9] and references therein, suggests rather small values of $\eta/s$ within the fireball at the RHIC and LHC temperatures, of the order 0.2, that is only slightly above the limiting value $1/4\pi$. This result tells that the system is strongly coupled with a small mean free path of the constituents, i.e., highly collective. It implies that the fireball cannot be a dilute gas of mesons like at low temperatures and zero net baryon density. Consequently, we can say that a new form of matter is seen experimentally. To answer the question about the origin and structure of this new form of matter one needs information about nature of the constituents. It was a priori assumed that these constituents should be strongly interacting quark and gluon quasiparticles as no other degrees of freedom would exist above the Hagedorn temperature. Another prominent indication of the strongly interacting matter within the fireball is a modification of a jet that propagates through the fireball. In a dilute meson gas one should not expect a significant modification of a jet as compared to the vacuum.

In parallel with the experimental activity the same field was developing on the lattice. On the lattice the only a priori ingredient is the QCD Lagrangian. In Euclidean space-time one can quantize the theory at vanishing baryon chemical potential in the equilibrium at a given temperature using existing Monte-Carlo algorithms and calculate observables. It was concluded that in reality there is no a phase transition and instead a smooth analytic crossover takes place [10]. The quark condensate decreases from its zero temperature value to practically zero at temperatures from 120 MeV to 180 MeV with a pseudocritical temperature of chiral symmetry restoration $T_{ch} \sim 155$ MeV [11] and approximately at the same temperature (or slightly above) the Polyakov loop, which is an order parameter for center symmetry ("deconfinement") in a pure glue theory, showed an inflection point. So the community took the point that in QCD there is a fast common deconfinement - chiral restoration crossover from hadron gas to QGP around the pseudocritical temperature $T_{pc} \sim 155$ MeV. This result was confirmed by a few lattice groups.

However, the inflection point of the non renormalized Polyakov loop was used to see a pseudocritical temperature for "deconfinement". Some time ago the evolution of the renormalized Polyakov was obtained, e.g., in Ref. [12], see Fig. 1.1. One clearly observes that there is no hint of a deconfinement crossover around $T_{ch} \sim 155$ MeV since the deconfinement transition should be accompanied by the Polyakov loop evolution from 0 to 1. The renormalized Polyakov loop evolves from 0 to 1 in a broad temperature interval up to temperatures of $T \sim 1$ GeV and its inflection point is around 300 MeV, as can be seen from Fig. 1.1. This suggests that above the chiral symmetry restoration crossover between 120 and 180 MeV QCD is still in the confining regime and there cannot be any (quasi)parton degrees freedom. Unfortunately this
circumstance was largely ignored by the authors and the community.

Another potential evidence for a deconfinement phase transition was suggested by Matsui and Satz in Ref. [13]. They argued that at a critical temperature the familiar linear + Coulomb confining potential between the static charges (infinitely heavy quarks) should be Debye screened and becomes weaker than a negative Coulomb potential. Such a potential, \( \sim -1/r \exp(-m_D r) \), does not support any bound state between heavy quarks and consequently signals a deconfining transition to QGP. A potential obtained from the correlators of the Polyakov loops at temperatures significantly above \( T_{ch} \) demonstrates a flattening of the linear part of the potential which is, however, not yet a Debye screening. In particular, no Debye screening is seen at the chiral restoration temperature, see Fig. 1.2, where such a potential is shown for \( N_f = 2 + 1 \) at physical quark masses [14]. The extracted potential with obvious flattening is exactly the same for temperatures below and above \( T_{ch} \). I.e., it cannot be related to "deconfinement" and simply signals that a process of production of two heavy-light mesons takes place. Actually the concept of an effective potential between static sources is a model dependent construction. If one assumes that an optical potential should take place instead of a pure real potential as in Fig. 1.2, then the real part of the potential turns out to be a linear confining potential up to large temperatures with rising with temperature imaginary part [15]. Recently it was concluded by the Bielefeld lattice group that no evidence of deconfinement from the Polyakov loop exists on the lattice in the vicinity of the chiral pseudocritical temperature [16].

There is no reliable and accepted definition and order parameter for deconfinement in QCD with light quarks. The only sensible question that should be answered is about degrees of freedom that drive the system. If it turns out that these degrees of freedom are (quasi)quarks and (quasi)gluons, then this would mean that it is a QGP. This situation is expected at a very large temperature where the asymptotic freedom forces the strong coupling constant to vanish [17]. However, there is no evidence either theoretical or experimental that above \( T_{ch} \sim 155 \) MeV in QCD the degrees of freedom are (quasi)quarks and (quasi)gluons. So it is a key problem to establish effective degrees of freedom above the chiral crossover.

Some time ago it was predicted that at finite temperatures above the chiral symmetry restoration crossover QCD should be still in the confining regime with hadron-like degrees of freedom [18]. Such a regime should be evidenced by a chiral spin symmetry [19, 20] of the QCD correlators above \( T_{ch} \). A year later first results on approximate chiral spin symmetry of spatial correlators above the chiral crossover were presented by a collaboration of theorists from Graz, Ljubljana and JLQCD [21]. Those results were limited by a temperature \( T \sim 400 \) MeV. In a subsequent study [22] the temperatures were extended up to \( T \sim 1 \) GeV and it was established that the approximate chiral spin symmetry smoothly disappears above \( T \sim 3T_{ch} \). Three regimes of QCD were identified with clearly distinguishable symmetries, with spontaneously broken chiral symmetry below \( T_{ch} \), with chiral symmetries and approximate chiral spin symmetry between \( T_{ch} \) and \( \sim 3T_{ch} \) and with chiral symmetry at higher temperatures, see Fig. 1.3. These regimes are different by symmetries and degrees of freedom. The stringy fluid regime is characterized by the approximate chiral spin symmetry of the thermal QCD partition function. The degrees of freedom are the color-singlet hadron-like states where chirally symmetric quarks are bound by the confining electric field. The chiral spin symmetry of the thermal partition function was verified in temporal correlators above \( T_{ch} \) [23]. These results have been summarized in a talk "Three regimes of QCD" [24].

Since then an important development in the field happened. Namely, an evidence for existence of such intermediate regime independent of symmetry arguments was obtained from published screening mass mass spectra which demonstrate the breakdown of partonic description of the system below \( 3T_{ch} \) [25]. Very recently another direct evidence for hadron-like degrees of freedom in the stringy fluid regime was presented [26]: The pseudoscalar spectral function extracted from the
spatial lattice correlators demonstrates a distinct pion state and its first radial excitation. Further evidence for hadron-like degrees of freedom above $T_{ch}$ was obtained for heavy quarks: The bottomonium spectral function above $T_{ch}$ is not flat and contains radial and orbital excitations $1S, 2S, 3S$ and $1P, 2P$ that become broader with temperature [27]. The stringy fluid medium consists of densely packed hadrons that interact strongly, in contrast to the dilute meson gas below $T_{ch}$ with a large mean free path of mesons. Hence the stringy fluid is more a liquid rather than a gas.

We also discuss a simple physical picture for chirally symmetric and approximately chiral spin symmetric mesons above $T_{ch}$. They can be presented as color-electric strings with massless chiral quarks at the ends. Such a view automatically explains the observed chiral spin symmetry.

Since the quark chemical potential in the QCD action is manifestly chiral spin symmetric [28] one should expect that the chiral spin symmetric regime extends at finite baryon density as a chiral spin symmetric band downwards across the QCD phase diagram [25]. In the cold and dense region the baryon parity doublet matter is proposed as a possible candidate for chiral spin symmetric matter.

Finally we discuss available experimental data on dileptons both at zero baryon density as well as at large baryon chemical potential and show that they are consistent with existence of the chiral spin symmetric band.

2. Chiral spin symmetry

The history of the chiral spin symmetry begins with the observation of an unexpected degeneracy of isovector $J = 1$ mesons seen on the lattice upon artificial truncation of the near-zero modes of the Dirac operator [29]. The quark condensate of the vacuum is connected with the density of the near-zero modes of the Euclidean Dirac operator via the Banks-Casher relation [30]

\[ \langle \bar{\psi} \psi \rangle = -\pi \rho(0), \]  
\[ \rho(0) = \lim_{m \to 0} \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{d\lambda \rho(\lambda)}{m - i\lambda}. \]

The hermitian Euclidean Dirac operator, $i\gamma_\mu D_\mu$, has in a finite volume $V$ a discrete spectrum with real eigenvalues $\lambda_n$:

\[ i\gamma_\mu D_\mu \psi_n(x) = \lambda_n \psi_n(x). \]  

Consequently, removing by hands $k$ lowest lying modes of the Dirac operator from the quark propagators,

\[ S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i| \langle \lambda_i |, \]

one a priori expects restoration of chiral $SU(2)_L \times SU(2)_R$ and possibly of $U(1)_A$ symmetries, if hadrons survive. This should be signalled by a degeneracy of hadrons connected by the $SU(2)_L \times SU(2)_R$ and $U(1)_A$ transformations. However, it turned out that all isovector $J = 1$ mesons get degenerate, not only those that are connected by the chiral transformations.
The symmetry groups that are responsible for this degeneracy, the transformation laws as well as their physical meaning were obtained in Refs. [19, 20].

In Ref. [19] the $SU(2)_{CS}$ chiral spin transformation was defined as a transformation that mixes the right- and left-handed Weyl quark spinors

$$\begin{pmatrix} R \\ L \end{pmatrix} \rightarrow \begin{pmatrix} R' \\ L' \end{pmatrix} = \exp \left( \frac{i \varepsilon n \sigma n}{2} \right) \begin{pmatrix} R \\ L \end{pmatrix} .$$

So the fundamental irreducible representation of $SU(2)_{CS}$ is two-dimensional. In terms of the Dirac spinors $\psi$ the same transformation can be written via four-dimensional $\gamma$-matrices [20]

$$\psi \rightarrow \psi' = \exp \left( \frac{i \Sigma^n}{2} \right) \psi,$$

where the generators $\Sigma^n$ of the four-dimensional reducible representation are

$$\Sigma^n = \{ \gamma_0, -i\gamma_5\gamma_0, \gamma_5 \}, \quad [\Sigma^n, \Sigma^p] = 2i\epsilon^{abc}\Sigma^c.$$

The $U(1)_A$ group is a subgroup of $SU(2)_{CS}$.

In Euclidean space with the $O(4)$ symmetry all four directions are equivalent and the $SU(2)_{CS}$ transformations can be generated by any Euclidean hermitian $\gamma$-matrix $\gamma_k$, $k = 1, 2, 3, 4$ instead of Minkowskian $\gamma_0$:

$$\Sigma^n = \{ \gamma_k, -i\gamma_5\gamma_k, \gamma_5 \},$$

$$\gamma_i\gamma_j + \gamma_j\gamma_i = 2\delta^{ij}; \quad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4.$$

The $su(2)$ algebra is satisfied with any $k = 1, 2, 3, 4$. A choice of $k$ is limited by the spatial $O(3)$ invariance: only those $k$ can be used that do not mix operators with different spatial $O(3)$ spins $J$.

Note that the chiral spin transformations mix quarks with different chiralities (i.e., they mix different irreducible representations of the Lorentz group) and consequently the $SU(2)_{CS}$ symmetry is not a symmetry of the Dirac Lagrangian.

The direct product of the $SU(2)_{CS}$ group with the flavor group $SU(N_F)$ can be embedded into a $SU(2N_F)$ group. This group contains the chiral symmetry $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ as a subgroup. The set of $(2N_F)^2 - 1$ generators of $SU(2N_F)$ is

$$\{ (\tau^a \otimes 1_D), (1_F \otimes \Sigma^n), (\tau^a \otimes \Sigma^n) \}$$

with $\tau$ being the flavor generators (with the flavor index $a$) and $n = 1, 2, 3$ is the $SU(2)_{CS}$ index. The fundamental vector of $SU(2N_F)$ at $N_F = 2$ is

$$\Psi = \begin{pmatrix} u_R \\ u_L \\ d_R \\ d_L \end{pmatrix} .$$

The chiral spin and $SU(2N_F)$ symmetries above should not be confused with the Pauli-Gürsey $SU(2N_F)$ symmetry [31, 32], which is a symmetry of the free Dirac Lagrangian and mixes the right quark with the left antiquark (and vice versa). It should also not be mixed up with the non-relativistic $SU(2N_F)$ symmetry with heavy quarks. The multiplets of the latter group contain states of only a given spatial parity.

While the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are not symmetries of the Dirac Lagrangian, they are symmetries of the Lorentz-invariant color charge

$$Q^a = \int d^3x \psi^\dagger(x) T^a \psi(x),$$

with $T^a$ the $SU(3)$ color generators. The color charge remains invariant under the unitary $SU(2)_{CS}$ and $SU(2N_F)$ transformations.

The latter important feature allows us to use the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries to distinguish the chromoelectric and chromomagnetic fields in a given reference frame because the chromoelectric field is defined through its interaction with the color charge while the chromomagnetic field is defined via its action on the spatial current. The latter current is not $SU(2)_{CS}$ and $SU(2N_F)$ symmetric. This can be made explicit as follows.
In Minkowski space in a given reference frame the electric and magnetic fields are different fields. Interaction of fermions with the gauge field in Minkowski space-time can be split in a given reference frame into temporal and spatial parts:

\[ \overline{\psi} \gamma^\mu D_\mu \psi = \overline{\psi} \gamma^0 D_0 \psi + \overline{\psi} \gamma^i D_i \psi, \]  

where the covariant derivative \( D_\mu \) includes interaction of the matter field \( \psi \) with the gauge field \( A_\mu \),

\[ D_\mu \psi = (\partial_\mu - i g T \cdot A_\mu) \psi. \]

The temporal term contains interaction of the color-octet charge density

\[ \tilde{\psi}(x) \gamma^0 T \psi(x) = \psi(x)^T T \psi(x) \]

with the electric part of the gluonic field. It is invariant under \( SU(2)_{CS} \) and \( SU(2N_F) \). Note that the \( SU(2)_{CS} \) transformations defined via the Euclidean Dirac matrices can be identically applied to Minkowski Dirac spinors without any modification of the generators. The spatial part contains the quark kinetic term and interaction with the chromomagnetic field. It breaks \( SU(2)_{CS} \) and \( SU(2N_F) \). We conclude that \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries are symmetries of the electric part of the QCD Lagrangian in a given reference frame and can be used to distinguish the electric and magnetic interactions: A symmetry of the electric part of the QCD Lagrangian is larger than the chiral symmetry of the QCD Lagrangian as a whole.\(^1\) Of course, in order to discuss the electric and magnetic components of the gauge field one needs to fix a reference frame. The invariant mass of the hadron is the rest frame energy. Consequently, to discuss physics of hadron mass it is natural to use the hadron rest frame. At high temperatures the Lorentz invariance is broken and the preferred frame is the medium rest frame.

This analysis suggests the necessary and sufficient conditions for emergence of approximate \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries: (i) both chiral symmetries must be at least approximately restored and (ii) the color-electric quark-gluon interaction must strongly dominate over the color-magnetic one and over kinetic terms. The latter condition implies that the color-electric field of an effective action must strongly dominate over the color-magnetic one. Within perturbative description this cannot happen since the symmetry of the perturbation theory is the symmetry of the Dirac Lagrangian, i.e. only chiral symmetry. In addition, the perturbative gluons, like photons, contain both electric and magnetic parts with equal magnitude.

3. Representations of chiral and chiral spin groups for mesons

Consider first spin \( J = 0 \) mesons within \( N_F = 2 \), which are the \( \pi(1,0^{-+}), f_0(0,0^{++}), a_0(1,0^{++}) \) and \( \eta(0,0^{-+}) \) mesons with the \( u, d \) quark content only. Their local interpolating fields are given as

\[ O_\pi(x) = \bar{q}(x) \tau \gamma_5 q(x), \]

\[ O_{f_0}(x) = \bar{q}(x) q(x), \]

\[ O_\eta(x) = \bar{q}(x) i \gamma_5 q(x), \]

\[ O_{a_0}(x) = \bar{q}(x) \tau q(x). \]

These four operators belong to an irreducible representation of the group \( U(2)_L \times U(2)_R \supset SU(2)_L \times SU(2)_R \times U(1)_A \). It is instructive to see how these interpolating fields transform under different subgroups of the group above.

The \( SU(2)_L \times SU(2)_R \) transformations consist of vectorial and axial transformations in the isospin space. The axial transformation

\[ q \rightarrow exp(i \gamma_5 \frac{\theta_A^a \tau^a}{2}) q \]

mixes fields of opposite parity. For instance,

\[ \bar{q}(x) q(x) \rightarrow \bar{q}(x) e^{i \gamma_5 \theta_A^a \tau^a} q(x) = \cos |\theta_A| \bar{q}(x) q(x) + \sin |\theta_A| \theta_A \cdot \bar{q}(x) \tau \gamma_5 q(x). \]  

\(^1\)Notice that it is a gauge-invariant statement since it is based on the gauge-invariant definition of the electric field, \( F = Q^a E^a \).
Hence, under the axial part of the $SU(2)_L \times SU(2)_R$ transformation the following fields get mixed

$$O_a(x) \leftrightarrow O_{fa}(x). \quad (3.6)$$

Similarly one obtains

$$O_{a0}(x) \leftrightarrow O_{\eta}(x). \quad (3.7)$$

The fields (3.6) form the basis functions of the $(1/2, 1/2)_a$ irreducible representation of the $SU(2)_L \times SU(2)_R$ group, while the fields (3.7) transform as $(1/2, 1/2)_b$. ²

The $U(1)_A$ transformation

$$q \rightarrow e^{\text{exp}(i\theta A\gamma_5)}q$$
mixes fields of the same isospin but opposite parity:

$$O_a(x) \leftrightarrow O_{a0}(x) \quad (3.8)$$

as well as

$$O_{fa}(x) \leftrightarrow O_{\eta}(x) \quad (3.9)$$

All four interpolators together belong to the representation $(1/2, 1/2)_a \oplus (1/2, 1/2)_b$ which is an irreducible representation of the groups $U(2)_L \times U(2)_R$ and $SU(2)_L \times SU(2)_R \times U(1)_A$.

With the spin $J = 0$ local fields it is impossible to construct irreducible representations of the chiral spin group (2.4). Indeed, applying the $SU(2)_{CS}$ transformations (2.5) to the fields (3.1),(3.2),(3.3) and (3.4) one obtains that these fields get mixed with $\bar{q}(x)\gamma_5 q(x)$ (and similar for the isovector operators), that represents the axial charge density. It does not create a physical state with $J = 0$ and consequently is not a proper $J = 0$ operator. This means that the $O(3)$ spatial invariance is not consistent with the $SU(2)_{CS}$ symmetry for $J = 0$ mesons (see also chapter 10 below). If the spatial rotational invariance is preserved, like it is in an isotropic medium, an approximate $SU(2)_{CS}$ symmetry of an effective action and of the thermal partition function cannot be observed with the $J = 0$ mesons. One needs the higher spin mesons to see this symmetry of an effective action.

The chiral as well as the chiral spin and $SU(4)$ multiplets for $J = 1$ are given in Fig. 3.1 [20]. The local fields presented in this figure are characterized by usual quantum numbers $I, J^{PC}$ and by a representation of the $SU(2)_L \times SU(2)_R$ group. All these fields are orthogonal since each of them has a unique set of quantum numbers. The $U(1)_A$ and $SU(2)_L \times SU(2)_R$ transformations of these fields obtained like for $J = 0$ fields are depicted in the upper part of the figure. The chiral spin transformations (2.5) connect operators for different $J = 1$ fields and the CS transformations are consistent with the $O(3)$ invariance. Note that these representations (2.5) are suited only for study of symmetries of the Hamiltonian, i.e. symmetries of the temporal Euclidean correlators. For the spatial correlators one needs to use multiplets discussed in detail in Ref. [22] and in Sec. 6.2 below.

A few important comments are in order. One observes from Fig. 3.1 that there are, e.g., two different $\rho$ operators. They both have the same spin, isospin as well as spatial and charge parities. They differ by the gamma-structure as well as by chiral representations. In vacuum with broken chiral symmetry both these operators create from the vacuum one and the same $\rho$-meson, though with different couplings. These couplings are determined by the chiral symmetry breaking in the physical $\rho$-meson wave function, i.e. by a mixture of $(1, 0) + (0, 1)$ and $(1/2, 1/2)_b$ components in the meson wave function. This issue is well understood on the lattice [35, 36]. However, in the chirally symmetric world above $T_c$ the index of the chiral representation becomes an exact and conserved quantum number of the physical state. This means that in the chirally symmetric world there are two different mesons with $(1, 1^{--})$ usual quantum numbers that differ by the chiral quantum number: one of them has the chiral quantum number $(1, 0) + (0, 1)$ while another one is described by $(1/2, 1/2)_b$. These are different orthogonal states. The $SU(2)_{CS}$ symmetry requires that the orthogonal states within an irreducible representation of $SU(2)_{CS}$ must be degenerate. Consequently a prediction of the chiral spin symmetry is existence of three degenerate states, two of them carry $(1, 1^{--})$ usual quantum numbers but differ by the chiral index and $b_1$ meson. The same situation takes place for the isoscalar mesons.

Similar transformation properties as well as representations of the chiral spin and $SU(4)$ groups can be obtained for $J = 2$ and higher spin meson operators. Note that the latter operators are necessarily nonlocal [33].

²The irreducible representations of the $SU(2)_L \times SU(2)_R$ group are described by the total isospins of the right and left-handed quarks, $(I_R, I_L)$. The total usual isospin of quarks can take values according to the standard angular momentum addition rules, $|I_R - I_L| \leq I \leq I_R + I_L$. The indices $a$ and $b$ distinguish two different representations $(1/2, 1/2)$.

³The Minkowskian $\gamma_0$ matrix coincides with the Euclidean $\gamma_4$.
Figure 3.1: Transformations between $J = 1$ operators, $i = 1, 2, 3$. The left columns indicate the $SU(2)_L \times SU(2)_R$ representation for every operator. Red and blue arrows connect operators which transform into each other under $SU(2)_L \times SU(2)_R$ and $U(1)_A$, respectively. Green arrows connect operators that belong to $SU(2)_CS$ irreducible triplets. Purple arrow shows the $SU(4)$ 15-plet. The $f_1$ operator is a singlet of $SU(4)$. From Ref. [20].
4. Observation of the chiral spin symmetry in truncation studies in mesons and its implications in vacuum and for hot QCD

The symmetry predictions from the $SU(2)_{CS}$ and $SU(2N_F)$ groups for $J=1,2$ mesons have been tested in $N_F = 2$ QCD in Refs. [33, 34], see as an example a degeneracy pattern of all $J = 1$ mesons [33] in Fig. 4.1. This large degeneracy, presumably only approximate, represents the $SU(2)_{CS}$ and the $SU(4)$ symmetries since it contains irreducible representations of both groups, see Fig. 3.1. These results imply, given the symmetry classification of the QCD Lagrangian, that while the confining chromoelectric interaction is distributed among all modes of the Dirac operator, the chromomagnetic interaction, which breaks both symmetries, is located at least predominantly in the near-zero modes. Consequently an artificial removal of the near-zero modes leads to the emergence of $SU(2)_{CS}$ and $SU(4)$ in hadron spectrum. Chiral symmetry breaking and confinement in QCD are not directly related phenomena. The highly degenerate level seen in Fig. 4.1 represents a $SU(4)$- symmetric level of the pure electric confining interaction. The hadron spectra could be viewed as a splitting of the level of the QCD string by means of dynamics contained in the near-zero modes of the Dirac operator, i.e., dynamics of $SU(2)_L \times SU(2)_R$ and $U(1)_A$ chiral symmetry breaking that also includes magnetic effects in QCD [19]. A possible candidate for latter dynamics could be local instanton or other topological fluctuations of the gluonic field [41, 42]. Note also that a confining interaction can also lead to the accumulation of the near-zero modes of the Dirac operator, i.e. to some contribution to the quark condensate. The present results seen in Fig. 4.1 imply, however, that chiral symmetry might be restored due to some specific reasons, e.g. in the QCD medium at some temperature or baryon density, but confinement would be still there.

Analytical studies [39, 40] conclude the following. Some specific gluonic dynamics leads to the accumulation of the near-zero modes of the Dirac operator and consequently to the breaking of $SU(2)_L \times SU(2)_R$ and $U(1)_A$ chiral symmetries. A gap in the Dirac spectrum provided by the artificial truncation of the near-zero modes in the Dirac operator necessarily implies restoration of both symmetries. The root of this statement is precisely the same as of the Banks-Casher relation. It is a general statement. We do not need to know which dynamics and why it leads to the accumulation of the near-zero modes. Emergence of larger approximate $SU(2)_{CS}$ and $SU(4)$ symmetries, seen in Fig. 4.1, requires that the electric confining interaction should be the most important for the higher-lying modes.

In reality the degeneracy of Fig. 4.1 represents a larger symmetry since both the 15-plet and the singlet of $SU(4)$ are also degenerate. What symmetry is it [20, 43]? The latter question was answered in Ref. [51]. It is a $SU(4) \times SU(4)$. Indeed the irreducible 16-plet of $SU(4) \times SU(4)$ is a direct sum of the 15-plet and of the singlet of $SU(4)$. A transparent physical reason for emergence of the larger $SU(4) \times SU(4)$ symmetry is a simple one. A confining electric flux tube binds
a quark and an antiquark and has two independent quark-gluon vertices. Each vertex has its own \( SU(4) \) symmetry.\(^4\)

The results of truncation studies, discussed above, have direct implications for QCD at temperatures above the pseudocritical temperature of chiral symmetry restoration around \( T_{ch} \approx 155 \text{ MeV} \). Here the quark condensate vanishes and consequently the near-zero modes of the Dirac operator are suppressed by temperature. There are strong indications from the lattice that the \( U(1)_A \) symmetry is also at least approximately effectively restored \([46, 47]\). Given these observations and given results on emerging symmetries obtained at \( T = 0 \) upon artificial truncation of the near-zero modes of the Dirac operator it was predicted that above the chiral restoration crossover the \( SU(2)_{CS} \) and \( SU(4) \) symmetries should naturally emerge, without any truncation, and QCD should still be in a confining mode with the hadron-like degrees of freedom \([18]\).

5. Representations of chiral spin group for nucleons

Emergence of the chiral spin and \( SU(4) \) symmetries in baryons upon truncation of the near zero modes was observed and studied in Ref. \([37]\), see degeneracy patterns of the correlators presented in this paper\(^5\). In the cited paper a complete classification of the chiral spin representations for nucleons was absent. Hence for future possible applications we present here such classification obtained in Ref. \([38]\).

Lorentz and Fierz-invariance of the local color-singlet three-quark operators restricts the number of such linear independent operators to be equal two \([45]\). However, the chiral spin symmetry is not a symmetry of the Dirac equation and the chiral spin transformations mix irreducible representations of the Lorentz group. Consequently if one discusses properties of operators under the chiral spin transformations we need a complete set of such operators with respect to \( SU(2)_{CS} \) and \( SU(4) \). A single-quark field transforms under a two-dimensional irreducible representation of the Lorentz group. Consequently if one discusses properties of operators under the chiral spin transformations we need a complete set of such operators with respect to \( SU(2)_{CS} \) and \( SU(4) \).

A complete set of local nucleon operators \((J = 1/2, I = 1/2)\) with positive and negative spatial parity with spin-zero and isospin-zero diquark has the following structure:

\[
N^{(i)}_{\pm} = \epsilon_{abc} \mathcal{P}_{\pm} \Gamma^{(i)}_1 u_a (u_b^T \Gamma^{(i)}_2 u_c - u_c^T \Gamma^{(i)}_2 u_b),
\]

with \( a, b, c \) being the color index and the parity projector \( \mathcal{P}_{\pm} = \frac{1}{2} (\mathbb{1} \pm \gamma_4) \). The matrices \( \Gamma^{(i)}_1 \) and \( \Gamma^{(i)}_2 \) \((i = 1, 2, 3, 4)\) for these four operators have the following explicit form: \( \mathbb{1} \) and \( C \gamma_{5\gamma_i} \) for \( i = 1 \); \( \gamma_5 \) and \( C \) for \( i = 2 \); \( i\mathbb{1} \) and \( C \gamma_5 \gamma_4 \) for \( i = 3 \) as well as \( \gamma_5 \) and \( C \gamma_4 \) for \( i = 4 \).

Notice that these three-quark fields are not orthogonal, in contrast to the \( J = 0, 1 \) meson interpolators, discussed earlier. This feature makes their classification with respect to \( U(1)_A, SU(2)_L \times SU(2)_R, SU(2)_{CS} \) and \( SU(4) \) more complicated. Applying the \( U(1)_A \) transformation on the given operator one obtains a linear combination of operators that are connected by blue arrows in Fig. 5.1. The irreducible representations of \( U(1)_A \) are one-dimensional, hence the operators that are connected by blue arrows form reducible representations of \( U(1)_A \). The irreducible representations can be obtained as linear combinations of operators linked by blue arrows.

The axial part of \( SU(2)_L \times SU(2)_R \) transforms the given operator into a superposition of operators connected by dashed red lines. This is true for both positive and negativeparity operators \( N^{(1)}_+ \) and \( N^{(2)}_+ \). For the operators \( N^{(3)}_+ \) and \( N^{(4)}_+ \) the situation is more complicated. In this case applying the axial part \( SU(2)_L \times SU(2)_R \) one obtains linear combinations of these operators and of \( I = 3/2 \) \( \Delta \)-operators with spin \( J = 1/2 \). This is because both nucleon and delta

---

\(^4\)Consider the Minkowski QCD Hamiltonian in Coulomb gauge in the hadron rest frame \([44]\):

\[
H_{QCD} = H_E + H_B + \int d^3x \bar{\Psi}(x) [-i \alpha \cdot \nabla] \Psi(x) + H_T + H_C,
\]

with the transverse and instantaneous "Coulombic" interactions to be:

\[
H_T = - g \int d^3x \bar{\Psi}(x) \alpha \cdot \tau^a A^a(x) \Psi(x),
\]

\[
H_C = \frac{g^2}{2} \int d^3x d^3y J^{-1} \rho^a(x) F^{ab}(x, y) J \rho^b(y).
\]

Here \( J \) is the Faddeev-Popov determinant, \( \rho^a(x) \) and \( \rho^a(y) \) are color-charge densities of quarks \((2.14)\) and gluons at the space points \( x \) and \( y \) and \( F^{ab}(x, y) \) is a "Coulombic" kernel.

The kinetic and transverse parts of the Hamiltonian are chirally symmetric. The confining "Coulombic" part \((4.3)\) carries the \( SU(2N_F) \) symmetry, because the quark color charge density operator is \( SU(2N_F) \) symmetric. The gluonic part of the color charge density is trivially \( SU(2N_F) \) invariant. However, both \( \rho^a(x) \) and \( \rho^a(y) \) are independently \( SU(2N_F) \) symmetric because the \( SU(2N_F) \) transformations at spatial points \( x \) and \( y \) can be completely independent, with different rotations angles. A contribution with \( x = y \) is absent because of Grassmannian nature of quarks. This means that the confining "Coulombic" interaction is actually \( SU(2N_F) \times SU(2N_F) \)-symmetric.

\(^5\)This Section is technically more involved and can be omitted at the first reading.
operators of the same spin form irreducible representations $(1, 1/2) + (1/2, 1)$ of the parity-chiral group. The latter $\Delta$-operators are not depicted in Fig. 5.1. The $SU(2)_{CS}$ transformations of the quark spinors (2.5) connect operators inside the green boxes. The $SU(4)$ transformations connect all eight operators of Fig. 5.1 along with the respective $\Delta$-partners.

A set of nucleon operators that transform under irreducible representations of $SU(2)_{CS}$ consists of linear combinations of nonorthogonal operators $N^{(i)}_{\pm}$ [38, 40]:

\[
\begin{align*}
B_{21}(-1/2) &= \frac{1}{4\sqrt{2}} \gamma_- \left[ -(N^{(1)}_+ - N^{(1)}_-) + (N^{(2)}_+ - N^{(2)}_-) - i(N^{(3)}_+ + N^{(3)}_-) + i(N^{(4)}_+ + N^{(4)}_-) \right], \\
B_{21}(1/2) &= \frac{1}{4\sqrt{2}} \gamma_- \left[ (N^{(1)}_+ + N^{(1)}_-) - (N^{(2)}_+ + N^{(2)}_-) + i(N^{(3)}_+ - N^{(3)}_-) - i(N^{(4)}_+ - N^{(4)}_-) \right], \\
B_{22}(-1/2) &= \frac{1}{8} \frac{\sqrt{2}}{3} \gamma_- \left[ -(N^{(1)}_+ - N^{(1)}_-) + (N^{(2)}_+ - N^{(2)}_-) - i(N^{(3)}_+ + N^{(3)}_-) + 3i(N^{(4)}_+ - N^{(4)}_-) \right], \\
B_{22}(1/2) &= \frac{1}{8} \frac{\sqrt{2}}{3} \gamma_- \left[ (N^{(1)}_+ + N^{(1)}_-) - (N^{(2)}_+ + N^{(2)}_-) + i(N^{(3)}_+ - N^{(3)}_-) + 3i(N^{(4)}_+ - N^{(4)}_-) \right], \\
B_{4}(-3/2) &= \frac{1}{4} \gamma_- \left[ (N^{(1)}_+ + N^{(1)}_-) + (N^{(2)}_+ + N^{(2)}_-) \right], \\
B_{4}(-1/2) &= \frac{1}{4} \gamma_- \left[ (N^{(1)}_+ - N^{(1)}_-) - (N^{(2)}_+ - N^{(2)}_-) - 2i(N^{(3)}_+ + N^{(3)}_-) \right], \\
B_{4}(1/2) &= \frac{1}{4} \gamma_- \left[ (N^{(1)}_+ + N^{(1)}_-) - (N^{(2)}_+ + N^{(2)}_-) - 2i(N^{(3)}_+ - N^{(3)}_-) \right], \\
B_{4}(3/2) &= \frac{1}{4} \gamma_- \left[ (N^{(1)}_+ - N^{(1)}_-) + (N^{(2)}_+ - N^{(2)}_-) \right].
\end{align*}
\]

Here $\gamma_\pm = \frac{1}{2}(1 \pm \gamma_5)$ and $B_r(\chi_z)$ is the nucleon interpolator in the irreducible representation of dimension $r = 2\chi + 1$ of $SU(2)_{CS}$ with $\chi_z$ being the $z$-projection of the chiral spin $\chi$. Upon the chiral spin transformation (2.4) only those nucleon operators $B$ are connected that belong to the same irreducible representation of $SU(2)_{CS}$.

The cross-correlation matrix is

\[
C(t)_{\chi_1,\chi_2,\chi_3} = \langle 0 | B_{r_1}(\chi_{z_1}; t) B_{r_2}(\chi_{z_2}; 0) | 0 \rangle.
\]
The $SU(2)_{CS}$ restoration requires the cross-correlators of operators from different representations of $SU(2)_{CS}$ to vanish. The cross-correlators of operators within a given representation of $SU(2)_{CS}$ that are diagonal in indices $\chi$ and $\chi'$ must coincide while the off-diagonal must vanish. In this case the diagonal correlators $C(t)_{N(\pm)}^{(i)}$ of nucleon interpolators $N^{(i)}_{\pm}$ are given as

$$C(t)_{N^{(\pm)}}^{(i)} = \frac{3}{4} C(t)_{3/2} + \frac{1}{12} C(t)_{1/2} + \frac{1}{4} C(t)_{1/2},$$

for $i = 1, 2, 3$.

Here $C(t)_{3/2}$ is a correlator $C(t)_{\chi_\pi \chi_\pi \chi_\pi \chi_\pi}$ with $\chi = \chi' = 3/2$ and with any $\chi_z = \chi_{t_z}$, and similar for $C(t)_{1/2}$ and $C(t)_{1/2}$. We conclude that in the $SU(2)_{CS}$-symmetric regime all correlators $C(t)_{N^{(i)}}$ with $i = 1, 2, 3$, inside the large green box in Fig. 5.1 should be degenerate. Such a degeneracy was indeed observed at zero temperature upon truncation of the near-zero modes of the Dirac operator in Ref. [37].

Note that the nucleon operators described in this section are appropriate only for study of temporal correlators.

6. Emergence of approximate chiral spin and $SU(4)$ symmetries above chiral restoration crossover

6.1. Correlators and spectral function

Symmetry properties of QCD can be studied via symmetries of correlators calculated at a given temperature. For meson operators $O_{\Gamma}(t, x, y, z) = \bar{\psi}(t, x, y, z) \Gamma \frac{\tau}{2} \psi(t, x, y, z)$ with $\Gamma \in \{1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu}, \gamma_5 \sigma_{\mu\nu}\}$, the Euclidean correlator functions,

$$C_{\Gamma}(t, x, y, z) = \langle O_{\Gamma}(t, x, y, z) O_{\Gamma}(0, 0) \rangle,$$

carry the full spectral information of all isovector excitations with $J = 0, 1$ in their associated spectral functions $\rho_{\Gamma}(\omega, p)$:

$$C_{\Gamma}(t, p) = \int_0^\infty \frac{d\omega}{2\pi} K(t, \omega) \rho_{\Gamma}(\omega, p),$$

$$K(t, \omega) = \frac{\cosh(\omega(t - 1/2T))}{\sinh(\omega/2T)}.$$

The spatial and temporal correlators are defined as

$$C_{\Gamma}^s(z) = \sum_{x, y, t} C_{\Gamma}(t, x, y, z),$$

$$C_{\Gamma}^t(t) = \sum_{x, y, z} C_{\Gamma}(t, x, y, z).$$

They collect the spectral information projected on the $(p_x = p_y = \omega = 0)$ and $(p_x = p_y = p_z = 0)$ axes, respectively. In thermal equilibrium the system is isotropic and momentum distributions are the same in all spatial directions. Consequently it is sufficient to study a propagation of the excitation only along one direction, e.g. $z$.

The temporal correlators reflect dynamics of the QCD Hamiltonian since $H$ translates states in Euclidean time

$$\langle \psi(t + 1; x, y, z) \rangle = \exp(-aH) \langle \psi(t; x, y, z) \rangle.$$

The spatial correlators are connected to the dynamics of the analogous operator $H_z$ translating states in $z$-direction

$$\langle \psi(t; x, y, z + 1) \rangle = \exp(-aH_z) \langle \psi(t; x, y, z) \rangle.$$

While the temporal correlator is completely determined by the spectral function of a hadron at rest $C_{\Gamma}^t(t) = C_{\Gamma}(t, p = 0)$, the spatial $z$-direction correlator requires integration of the spectral function over all possible spatial momenta

$$C_{\Gamma}^s(z) = \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} e^{ip_z z} \int_0^\infty \frac{d\omega}{\pi \omega} \rho_{\Gamma}(\omega, p_x = 0, p_y = 0, p_z = 0).$$

\textsuperscript{6}Note that at a finite temperature the correlation functions are automatically calculated in the medium rest frame which is the preferred reference frame.
that we can calculate on the lattice. Naively the spectral density of a hadron at rest and spatial Euclidean correlators. However, our goal is just opposite: to extract the spectral function from the correlators spectral functions. Given the continuous spectral function in Minkowski space one can directly calculate both temporal effective action, and hence of the thermal partition function of QCD.

Observing approximate chiral spin symmetry both in spatial and temporal correlators is sufficient to conclude that it is also a symmetry of the spectral function $\rho_T(\omega,p)$. Different quantum number channels at a given temperature are evaluated with the same effective action for QCD in the medium. Hence symmetries of the effective action that describes the medium in the rest frame at the temperature $T$ can be obtained from symmetries of the correlators. Observed degeneracy patterns reflect symmetries of the non-perturbative effective action, and hence of the thermal partition function of QCD.

Complete information about degrees of freedom in the thermal medium is contained in the experimentally measurable spectral functions. Given the continuous spectral function in Minkowski space one can directly calculate both temporal and spatial Euclidean correlators. However, our goal is just opposite: to extract the spectral function from the correlators that we can calculate on the lattice. Naively the spectral density of a hadron at rest $\rho_T(\omega,p = 0)$ could be obtained from the temporal correlators (6.5) via the inverse transform to the Eq. (6.2). However, on the lattice one calculates Euclidean correlators only on a finite number of discrete points. Then the inverse transform is ill-posed and extraction of spectral functions requires some additional assumptions, e.g. input from phenomenological modelling and the perturbation theory at large $\omega$ combined with statistical methods like the maximum entropy method, etc., for reviews see e.g. [48, 49, 50].

### 6.2. Spatial correlators and their symmetries

A complete set of all possible isovector local $J = 0,1$ operators relevant for spatial correlators is given in Table 6.1. This Table makes it also clear how these operators transform under $U(1)_A$ and axial part of $SU(2)_L \times SU(2)_R$. Restoration of these symmetries requires correlators of the corresponding operators to be degenerate. The $SU(2)_{CS}$ and $SU(4)$ transformation properties relevant to the spatial propagators were discussed Refs. [21, 22] and are shortly summarized below.

The chiral spin transformations (2.7) with $k = 1,2$ together with the $x \leftrightarrow y$ symmetry generate the following multiplets:

$$
(V_x, V_y): \ (A_x, A_y, T_t, X_t) , \ 
(V_t): \ (A_t, T_x, T_y, X_x, X_y) .
$$

(6.9) (6.10)

Considering $SU(4)$ one obtains larger multiplets of the isovector operators:

$$
(V_x, V_y, A_x, A_y, T_t, X_t) , \ 
(V_t, A_t, T_x, T_y, X_x, X_y) .
$$

(6.11) (6.12)

Complete $SU(4)$ multiplets contain also the isoscalar partners of the operators $A_x, A_y, T_t, X_t$ in Eq. (6.11) and isoscalar partners of the $A_t, T_x, T_y, X_x, X_y$ operators in Eq. (6.12).

In Fig. 6.1 we show spatial correlators (6.4) evaluated at different temperatures with chirally symmetric domain wall Dirac operator at physical quark masses using the $N_F = 2$ JLQCD ensembles [22]. Here a complete set of all possible isovector local $J = 0,1$ operators has been used. We see a distinct multiplet structure of the correlators. This multiplet structure reflects symmetry properties of the effective action at the given temperature.

The multiplet $E_1$ consists of isovector scalar (S) and pseudoscalar (PS) correlators. The degeneracy of S and PS correlators evidences restored $U(1)_A$ symmetry. If there is still a tiny breaking of $U(1)_A$ it should be too small to be seen in the present data.

The multiplet $E_2$ contains four approximately degenerate correlators obtained with $V_x, A_x, T_t, X_t$ $J = 1$ isovector operators. The $V_x$ and $A_x$ operators are connected by the axial part of the $SU(2)_L \times SU(2)_R$ transformation and their degeneracy is a signal of restored $SU(2)_L \times SU(2)_R$ symmetry. The $T_t$ and $X_t$ operators are connected by the $U(1)_A$ transformation and a degeneracy of the corresponding correlators is required by the restored $U(1)_A$ symmetry.

---

3This effective action is not known and should be eventually reconstructed; this is similar to classical electrodynamics, where the Maxwell equations in vacuum and in medium are different.
Figure 6.1: Spatial correlation functions of all possible isovector $J = 0, 1$ bilinears. For notations of the operators and their content see Table 6.1. From Ref. [22].
operators \((A_x, T_t, X_t)\) form a triplet of the \(SU(2)_{CS}\) group. An approximate degeneracy of the correlators indicates emerged approximate \(SU(2)_{CS}\) symmetry. All four operators \((V_x, A_x, T_t, X_t)\) are connected by the \(SU(4)\) transformation and a degeneracy of the corresponding correlators shows emergent approximate \(SU(4)\) symmetry.

The \(E_3\) multiplet consists of four approximately degenerate correlators obtained with \(V_t, A_t, T_x, X_x\) operators. Notice that the \(V_t, A_t\) operators represent the charge and axial charge densities, respectively. These operators do not create physical states. The current conservation connects the \(T_x, X_x\) to \(V_t, A_t\) so the former operators are not independent from the latter. If the \(V_t, A_t, T_x, X_x\) correlators are normalized, as it is in Fig. 6.1, then in the case of noninteracting quarks they must be identical [22]. Consequently a degeneracy of the normalized \(V_t, A_t, T_x, X_x\) correlators is consistent with both the \(SU(2)_L \times SU(2)_R \times U(1)_A\) symmetry alone and with the \(SU(4)\) symmetry. This is precisely the reason why the \(E_3\) multiplet persists at all temperatures. So it cannot be used as an indicator of emergent chiral spin symmetry and of its \(SU(4)\) extension.

We observe approximate emerged \(SU(2)_{CS}\) and \(SU(4)\) symmetry up to temperatures of about \(\sim 500\) MeV. At higher temperatures two different distinct multiplets \(E_1\) and \(E_2\) disappear. This happens because the full QCD correlators approach at high temperatures correlators of the free quark gas, as will become evident below.

In Fig. 6.2 we compare correlators from the \(E_1\) and \(E_2\) multiplets evaluated in full QCD with the corresponding correlators obtained with a free quark gas. The full QCD correlators are given by the solid lines while the correlators calculated with noninteracting quarks are described by the dashed curves. Note that the correlators calculated with noninteracting quarks reflect physics at a very high temperature and only chiral \(SU(2)_L \times SU(2)_R\) and \(U(1)_A\) symmetries are present in this case. No \(SU(2)_{CS}\) and \(SU(4)\) symmetries exist for free quarks.

The quark gluon plasma, which is a system of (quasi) free partons is characterized by chiral symmetries, i.e. symmetries of the Dirac equation. The presence of approximate \(SU(2)_{CS}\) and \(SU(4)\) symmetries below \(500\) MeV tells that the degrees of freedom should be the quark-antiquark systems with chirally symmetric quarks bound into color singlet objects by a confining electric field.

A dramatic difference between the \(S\) and \(PS\) correlators in full QCD and in free quark gas is obvious. This immediately tells us that there must be some color singlet resonances with pion and sigma quantum numbers below \(500\) MeV. This implies that the medium below \(500\) MeV is by far not a system are of quasi-free partons. This issue will be discussed below in chapter 8.

At the highest temperature of this study, \(T \sim 960\) MeV, the situation has changed significantly: All full QCD correlators are very close to the corresponding free correlators. Hence at \(T \sim 960\) MeV we have reached the region where only chiral \(U(1)_A\) and \(SU(2)_L \times SU(2)_R\) symmetries exist and the near coincidence with the free correlators suggests a gas of quasi-free quarks. Notice that this near coincidence is a consequence of the log scale used in Fig. 6.2. The QCD correlators are not identical to the free quark gas correlators, which is well seen in more detailed plots in Ref. [22].

### 6.3. Temporal correlators and their symmetries

On the right side of Fig. 6.3 we show temporal correlators (6.5) at \(T = 220\) MeV calculated with the domain wall Dirac operator at physical quark masses with \(N_F = 2\) JLQCD ensembles [23]. Transformation properties of the local \(J = 1\) quark-antiquark bilinears \(\mathcal{O}_F(x, y, z, t)\) with respect to \(U(1)_A, SU(2)_L \times SU(2)_R, SU(2)_{CS}\) and \(SU(4)\), relevant for temporal correlators, are given in Fig. 3.1. Emergence of the respective symmetries is signalled by degeneracy of the correlators (6.5) calculated with operators that are connected by the corresponding transformations.

On the l.h.s of Fig. 6.3 we demonstrate correlators calculated with noninteracting quarks on the same lattice. They represent a QGP at a very high temperature where due to asymptotic freedom the quark-gluon interaction can be neglected. Dynamics of free quarks are governed by the Dirac equation and only \(U(1)_A\) and \(SU(2)_L \times SU(2)_R\) chiral symmetries exist. A qualitative difference between the pattern on the l.h.s. and the pattern on the r.h.s of Fig. 6.3 is obvious. In the latter case we clearly see approximate \(SU(2)_{CS}\) and \(SU(4)\) symmetries. \(SU(2)_{CS}\) and \(SU(4)\) symmetries of the spatial and temporal correlators imply the same symmetries of spectral densities and of the thermal partition function.

### 6.4. Conclusions to symmetry studies

There are a few most important conclusions from the symmetry studies of the meson correlators.

The QCD effective action and thermal partition function above the chiral crossover have not only chiral symmetries but are approximately symmetric with respect to \(SU(2)_{CS}\) chiral spin group and its flavor extension \(SU(4)\) for \(N_F = 2\). This is true in the medium rest frame which is the preferred frame. (At a nonzero temperature there is no Lorentz invariance in the medium.) These groups are not symmetries of the Dirac Lagrangian. This implies that the medium is not a quark gluon plasma which is a system of weakly interacting partons and where only chiral symmetries exist.

The chiral spin group is a symmetry of the chromoelectric part of the QCD Lagrangian. The approximate chiral spin symmetry can emerge only when the quark-electric interaction strongly dominates over the quark-magnetic interaction and over the quark kinetic term. This symmetry is characteristic of quark-antiquark systems with chirally symmetric
Figure 6.2: Correlation functions of the bilinears $PS, S, V_x, A_x, T_t, X_t$. The solid curves represent full QCD calculation and the dashed lines are correlators calculated with free noninteracting quarks. From Ref. [22].

Figure 6.3: Temporal correlation functions for $12 \times 48^3$ lattices. The l.h.s. shows correlators calculated with free noninteracting quarks with manifest $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries. The r.h.s. presents full QCD results at a temperature 220 MeV, which shows multiplets of all $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(4)$ groups. From Ref. [23].
quarks bound by the chromoelectric field (presumably by a chromoelectric flux tube, that is why this regime was dubbed a stringy fluid). The emergent $SU(2)_{CS}$ and $SU(4)$ symmetries seen at $T_{ch} - 3T_{ch}$ suggest that the physical degrees of freedom at these temperatures are chirally symmetric quarks bound into color singlets by the chromoelectric field. The stringy fluid regime arises above $T_{ch}$ and extends to approximately $3T_{ch}$, as illustrated in Fig. 1.3. Above these temperatures the chiral spin symmetry smoothly disappears because the confining electric field gets screened and one observes a smooth transition to a quark gluon plasma.

6.5. Is $U(1)_{A}$ restored simultaneously with $SU(2)_{L} \times SU(2)_{R}$ in hot QCD?

The idea that above the chiral phase transition the $U(1)_{A}$ symmetry is still broken is an old one [52]. It was suggested that the instanton gas picture could be valid in the chirally restored and deconfined phase. It would still induce the $U(1)_{A}$ breaking. The quark condensate and the $U(1)_{A}$ breaking are differently sensitive to the near-zero modes of the Dirac operator:

$$\langle \bar{\psi} \psi \rangle = - \lim_{m \to 0} \int_{0}^{\infty} d\lambda \rho(\lambda, m) \frac{2m}{m^2 + \lambda^2} .$$

$$\int d^4x < O_\pi(x)O_\pi^\dagger(0) > = \int d^4x < O_{a0}(x)O_{a0}^\dagger(0) >= \lim_{m \to 0} \int_{0}^{\infty} d\lambda \rho(\lambda, m) \frac{4m^2}{(m^2 + \lambda^2)^2} .$$

Here $\rho(\lambda, m)$ is a density of modes of the Dirac operator with the quark mass $m$ (see (2.1)). It is possible to reconcile the vanishing quark condensate and a non vanishing difference of $\pi$ and $a_0$ correlators by assuming the non analytical form of the Dirac spectral density, $\rho(\lambda, m) \sim m^2\delta(\lambda)$. Here the $\delta(\lambda)$ reflects the near-zero modes arising from the well isolated instantons. Consequently, it is in principle possible that in the chirally symmetric regime with vanishing quark condensate the $U(1)_{A}$ susceptibility is not zero.

However, the existence of a confining electric field above $T_{ch}$, discussed in this review, rules out the instanton gas picture at temperatures below few times $T_{ch}$, because a dilute instanton gas cannot provide a confining electric field. Still, a possibility of a presence of rare topological fluctuations on top of a confining field is not excluded.

The modern topic and argument was initiated by Cohen [56] who insisted that in QCD in the chiral limit restoration of $SU(2)_{L} \times SU(2)_{R}$ at a critical temperature requires actually an effective restoration of $SU(2)_{L} \times SU(2)_{R}$, i.e., an effective restoration of anomalously broken $U(1)_{A}$ symmetry. This $U(1)_{A}$ effective restoration means that at least two-point correlation functions of operators connected by the $U(1)_{A}$ transformation must be identical. Cohen’s argument was challenged in Ref. [59]: Exact zero modes arising from the topological configurations with nonzero topological charge would violate the identity of the correlators connected by $U(1)_{A}$. However, it is known that in the thermodynamic limit $V \to \infty$ the contribution of the exact zero modes vanishes, see, e.g., Ref. [60]. Cohen further suggested that a finite gap in the Dirac spectrum might emerge above the chiral phase transition. Such a gap would automatically induce the effective restoration of $U(1)_{A}$ in meson and baryon two-point functions [39, 40].

A behavior of the near-zero modes and the question whether a finite gap in the Dirac spectrum arises or not is a delicate issue and can be answered only in nonperturbative lattice calculation. However, it is a rather complicated task that could not be completely accomplished so far, because it requires a Dirac operator with perfect chiral properties, a very large lattice volume, and a very small quark mass.

Existing lattice results related to this question could be grouped into three categories: (i) hybrid calculations that use staggered fermions for the vacuum configurations while the overlap Dirac operator for valence quarks [53, 54], (ii) the same chirally symmetric Dirac operator (either domain wall or overlap) is employed for both sea and valence quarks [61, 46, 47, 62, 63] and (iii) staggered sea and valence fermions [55]. One should also always keep in mind that the staggered fermions rely on the rooting procedure, which causes questions about its validity at small quark masses.

In works of categories (i) and (iii) a big peak near $\lambda = 0$ is seen in the Dirac eigenvalue spectrum at temperatures significantly above $T_{ch}$, which implies a serious violation of $U(1)_{A}$, such a peak is not observed in papers from the category (ii). A search of a possible gap in the Dirac spectrum above $T_{ch}$ was performed by the JLQCD collaboration in Refs. [46, 47, 62, 63]. In Ref. [46] the $N_{F} = 2$ QCD with the overlap Dirac operator in the trivial topological sector $Q = 0$ was studied. The Dirac spectrum in the quenched case (overlap valence quark Dirac operator on pure glue Q=0 vacuum configurations) showed a sharp peak at the smallest eigenvalues $\lambda$, in agreement with papers from the category (i). The full QCD

---

8The degree of $U(1)_{A}$ symmetry breaking at the chiral restoration point may have physical consequences. Analysis of Ref. [57] suggests that if this breaking is large in QCD with two massless flavors, then the transition might be second order. In the case of simultaneous restoration of both $SU(2)_{L} \times SU(2)_{R}$ and $U(1)_{A}$ it should be of first order. The same analysis tells, however, that the phase transition with $N_{F} = 3$ should be of first order, while recent lattice results with staggered fermions demonstrate second order phase transition at $N_{F} = 3, 4, 5, 6$ [58].

9Such a violation, if large, should be seen in spatial and temporal correlators. However, it is not observed, as discussed in the present chapter.
calculation demonstrates, however, absence of a peak and even a gap opens at $T > 200$ MeV. This suggests that the peak could be a quenching lattice artifact.

This issue was further investigated in Refs. [47, 62, 63] with $N_F = 2$. In Fig. 6.4 we show a typical result of these studies at $T = 203$ MeV with physical degenerate $u$ and $d$ quark masses. In the top panel a Dirac spectrum with the domain wall Dirac operator for both valence and sea quarks is shown. No peak at small values of $\lambda$ is visible. The domain wall operator still has small residual effects of violation of the Ginsparg-Wilson relation, i.e. of exact chiral symmetry. The authors suggest that a small non-vanishing density of Dirac eigenvalues near zero in the lowest bin could be connected with these small residual chiral symmetry breaking effects. To control the latter issue they reweight the domain wall eigenmodes with the overlap eigenmodes (bottom panel). Here the density of eigenmodes vanishes even with nonzero quark masses and a gap near zero is seen, as suggested in Ref. [60]. In the middle panel of Fig. 6.4 a partially quenched result is shown, where the valence overlap operator is combined with the domain wall sea quarks. A sharp peak is found in the lowest bin, in agreement with studies from the category (i). This result suggests that even a small partial quenching could induce spurious effects in the near zero Dirac modes.

While these results on restoration of $U(1)_A$ above the chiral crossover are interesting and convincing, they do not prove yet a simultaneous restoration of both $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries in the chiral limit. The precise temperature where $U(1)_A$ effectively restores, is not yet conclusively determined. We cannot exclude that it may be a bit larger than the $SU(2)_L \times SU(2)_R$ restoration temperature.

7. Screening masses and the equation of state

As we have discussed in Introduction, in QCD with light quarks there is no obvious definition of "deconfinement" and of the corresponding order parameter. The only sensible question is about effective degrees of freedom that drive the hot QCD matter at a given temperature. Emergence of approximate chiral spin symmetry rules out weakly interacting
(quasi)quarks and (quasi)gluons as the only symmetry of perturbation theory is chiral symmetry, that is the symmetry of the Dirac equation. The chiral spin symmetry points out a true nonperturbative regime where dynamics is dominated by the nonperturbative chromoelectric field. Since only the color-singlet states can survive the gauge averaging (and hence propagate) this means that these color-singlet states are chirally symmetric quark-antiquark systems bound by the chromoelectric field.

Still another observables that would be consistent with the above picture and that would discriminate degrees of freedom are highly welcome. Screening masses of spatial correlators are among such observables. Results of the present Section are based on Ref. [25]. The screening masses are defined as asymptotic exponential slope of spatial correlators (6.4) at $z \to \infty$ [64]:

$$C^z_T(z) \to \text{const} \cdot e^{-m_{scr}z}. \quad (7.1)$$

It is very well seen from Fig. (6.1) that indeed the spatial correlators are driven at large $z$ by the exponential asymptotic. This asymptotic determines the ground state of a "Hamiltonian" $H_z$ that acts on a Hilbert space defined over the $x,y,t$ Euclidean coordinates. $H_z$ generates translations in $z$-direction (6.7). If this asymptotic pure exponential, then this ground state corresponds to a bound state of $H_z$. A spectrum of $H_z$ is sensitive to the temperature as it is sensitive to the compactified Euclidean time direction $t$. The boundary conditions along the finite time direction, $T^{-1} = aN_t$, are fixed: periodic for gauge field and anti-periodic for fermionic fields. The thermodynamic limit is defined as $N_{x,y,z} \to \infty$ at the given temperature. Real lattice calculations are done on a finite lattice, hence either periodic or anti-periodic boundary condition should be imposed along the spatial axes $x,y,z$. In the thermodynamic limit results will not depend on a particular choice of spatial boundary conditions. In the limit $T = 0$ the spectrum of $H_z$ is identical to that of the Hamiltonian $H$ which provides translations in Euclidean time direction. In the opposite limit $T \to \infty$ a dimensional reduction to a 3d theory takes place and the spectrum of $H_z$ reduces to the spectrum of 3d QCD. For either unstable or multiparticle states the exponential in (7.1) gets multiplied by the inverse power law factors.

On a Euclidean lattice the thermal partition function can be represented in two equivalent ways, either via the spectrum of $H$, which is $T$-independent, or via the spectrum of $H_z$, which is explicitly $T$-dependent

$$e^{pV/T} = Z = \text{Tr}(e^{-aHN_t}) = \text{Tr}(e^{-aH_zN_z}) = \sum_i e^{-E^z_i aN_z}, \quad (7.2)$$

where $E^z_i$ is a spectrum of $H_z$. The full spectrum of $H_z$ defines the partition function and is directly related to the equation of state. Consequently the screening masses that represent the ground states of $H_z$ are also directly related to the equation of state.

Information about effective degrees of freedom at any temperature is encoded in the thermal partition function (7.2). If the thermal partition function and the equation of state are described by the parton dynamics one naturally speaks of the quark-gluon plasma. For a thermal equilibrium system, screening masses are accessible by perturbative and non-perturbative (lattice) calculations. If the non-perturbative lattice results for screening masses are well described with the perturbative parton language, one then concludes that effective degrees of freedom in the system are quarks and gluons.

Very recently the pseudo-scalar and vector screening masses have been calculated on the lattice at high temperatures $T = 1 - 160$ GeV which are shown in Fig. 7.1 [65]. Over two orders of magnitude in temperature the lattice data are well parameterized by

$$\frac{m_{PS}}{2\pi T} = 1 + p_2 \hat{g}^2(T) + p_3 \hat{g}^3(T) + p_4 \hat{g}^4(T),$$

$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4 \hat{g}^3(T), \quad (7.3)$$

where $\hat{g}^2(T)$ denotes the temperature-dependent running coupling constant renormalized in the $\overline{\text{MS}}$-scheme at $\mu = 2\pi T$. The value of $p_2$ is fixed by the EQCD calculation [66], while $p_3,p_4,s_4$ are not yet known analytically and consequently fitted to the lattice data. Note that $p_2,p_3,p_4,s_4$ are numbers and the temperature dependence of the screening masses resides in the coupling constant. The temperature dependence of the coupling constant is logarithmically slow which is the reason why the screening masses vary very little in the large temperature interval. A perturbative description of screening masses and of the equation of state suggests partonic degrees of freedom, which is a signal of the quark-gluon plasma.

Screening masses at lower temperatures above the chiral restoration crossover up to $T \sim 2.5$ GeV are shown in Fig. 7.2 [70]. One observes that above $T \sim 1$ GeV the screening masses in Figs. 7.1 and 7.2 match with each other and a temperature dependence of screening masses above $T \sim 1$ GeV is flat. What immediately attracts our attention is the rapid bending of curves within $T \sim 0.5 - 0.6$ GeV, from a steep increase with temperature to a flat behavior. Since the
Figure 7.1: Temperature dependence of the pseudoscalar and vector screening masses at large temperatures. The shadow bands represent a fit according to Eq. 7.3. From Ref. [65].

Figure 7.2: Screening masses of the lightest $\bar{u}d$ mesons. The solid line represents the screening masses at the order $\sim g^2$ obtained within EQCD [66]. From Ref.[70].
temperature dependence of the partonic description (7.3) is only in the coupling constant $\hat{g}^2(T)$, the partonic description cannot explain the nearly vertical parts of the plots. This feature is observed in all $J = 0, 1$ mesons with strangeness as well [70]. The screening masses in the $u, d, s$ sector are the dominant contributions to the partition function (7.2). We observe an apparent change of dynamics at $T \sim 0.5 - 0.6$ GeV from the parton dynamics at higher temperatures to another one at $T < 0.5$ GeV. The temperature at which we see a change of dynamics in the partition function coincides with the temperature where chiral spin symmetry disappears and the chromoelectric confining interaction gets screened. At $T \sim 0.5 - 0.6$ GeV and below the strong coupling constant is large, as can be concluded from Fig. 7.1, so it is not surprising that a non-perturbative confining dynamics is operative.

One should raise the question whether the description within the EQCD [67, 68, 69] is consistent or not with the chiral spin symmetric regime below $T \sim 500 - 600$ MeV. The EQCD is an effective bosonic description of QCD at high temperatures obtained upon perturbative dimensional reduction of the four dimensional QCD with both fermion and gluon degrees of freedom to effective bosonic degrees of freedom in a three dimensional space. At the asymptotically high temperatures it should be an accurate representation of QCD. The perturbative dimensional reduction at small coupling constants relies on the QCD Lagrangian where no approximate chiral spin symmetry can exist because within the perturbative description the quark kinetic term is of primary importance and which breaks the chiral spin symmetry. Consequently in a validity range of EQCD one would not expect an approximate chiral spin symmetry. Observation of the approximate chiral spin symmetry at $T < 500 - 600$ MeV restricts then application of EQCD to higher temperatures where the CS symmetry disappears. This simple consideration is consistent with results depicted in Fig. 7.1 with the $p_2$ term to be consistent with lattice results only at rather high temperatures, of the order of 1 GeV and larger.

We conclude that the behavior of meson screening masses from 12 different quantum number channels in $N_F = 2 + 1$ QCD provides an independent demonstration of the existence of the temperature window below 500 - 600 MeV in which chiral symmetry is restored but the dynamics is inconsistent with a partonic description.

The discussed behavior of screening masses below $T \sim 0.5 - 0.6$ GeV must also be reflected in the equation of state. Indeed, a very steep increase of $p/T^4$ with temperature in the same temperature interval is observed [71] which is shown in Fig. 7.3. Weakly interacting partons in the quark-gluon plasma require $p/T^4 \sim const$, which is detected at higher temperatures.

8. Pion states above the chiral crossover

A direct evidence for the hadron-like degrees of freedom in the stringy fluid should be observation of the corresponding states in spectral functions. A break-through in this direction was done in Ref. [26]. This section is devoted to the results obtained in this paper.

Typically attempts to reconstruct a spectral function at large temperatures relied on Euclidean temporal correlators that contain a small number of points. It is an ill-posed problem. It is not clear a-priori to which extent these reconstructions can be credible as there is no control of results. In Ref. [26] instead the pion spectral function was extracted from the spatial PS correlators depicted in Fig. 6.1. In this paper the approach was used which was developed in Refs. [72, 73, 74, 75, 76] and which is based on locality of QCD. Given the pion spectral function extracted from the spatial correlators, the temporal correlators can be directly predicted according to (6.2) and compared with the lattice results.

It is well known that locality (causality) of QFT at $T=0$ requires existence of the Källen-Lehmann spectral representation. In Ref. [72] this representation for scalar spectral density was generalized to arbitrary temperature and is

$$
\rho(\omega, p) = \int_0^\infty ds \int \frac{d^3 u}{(2\pi)^2} \delta(\omega) \delta(p_0^2 - (p - u)^2 - s) \bar{D}_\beta(u, s),
$$

(8.1)

with $\bar{D}_\beta(u, s)$ being the thermal spectral density which completely determines the properties of scalar particles in the medium. For stable particle that has in vacuum a discrete pole at $\sqrt{s} = m$ and which is well separated from the continuum contributions, the following Ansatz can be used for $\bar{D}_\beta(u, s)$ if one looks for this particle in the medium at a temperature $T$:

$$
\bar{D}_\beta(u, s) = \bar{D}_{m,\beta}(u, s) \delta(s - m^2) + \bar{D}_{c,\beta}(u, s),
$$

(8.2)

where $\bar{D}_{c,\beta}(u, s)$ is continuous in $s$. Refs. [72, 73, 74, 75, 76] discussed several reasons for why the discrete component in Eq. (8.2) (the first term) provides a natural description of a particle state in the medium. The damping factor $\bar{D}_{m,\beta}(u)$ causes $\rho(p_0, p)$ to have contributions outside of the mass shell $p^2 = m^2$, and hence the $T = 0$ peak of the spectral function gets broadened, which is a natural expectation. The precise nature of this broadening is controlled by the underlying interactions between the particle state and the constituents of the thermal medium [76]. The factorization of the $(u, T)$
Figure 7.3: The pressure calculated with HISQ action for $N_F = 2 + 1$ QCD. From Ref. [71].
Using these ideas Ref. [26] established a bridge between the spatial pion correlators (6.4) and the rest frame spectral density \( \rho(\omega, p = 0) \). From a two-exponent fit of the spatial correlator with a very good quality the pion spectral function was reconstructed at different temperatures. These two exponents are interpreted as contributions from two (quasi)discrete levels \( \pi, \pi' \) in the medium and the continuum part in Eq. (8.2) is neglected. The results are shown in Fig. 8.1. The spectral function demonstrates two distinct peaks that correspond to the pion and its first radial excitation in the medium. These peaks get broader with temperature and melt above \( T \sim 500 - 600 \) MeV out.

This spectral function extracted from the spatial correlators can be controlled since a temporal correlator can be calculated according to Eq. (6.2) and compared with the lattice results of Fig. 6.3. The output is shown in Fig. 8.2. It is a truly remarkable result. The large \( t \) part of the correlator is accurately reproduced. A deviation is seen only in the small \( t \) part that is sensitive to a contribution of the higher excited states, \( \pi'', \ldots \) and to contributions of neglected continuum. The latter contributions cannot be picked with the two exponential fit of the spatial correlators up. This test suggests that the low energy part of the spectral function with \( \pi, \pi' \) presented in Fig. 8.1 is close to reality, though the omitted contributions from \( \pi'', \ldots \) and continuum should influence a small \( t \) part of the correlator and the spectral function beginning from \( \omega \sim 1600 - 1700 \) MeV. Their effect should increase with temperature.

It is instructive to compare the spectral function of Fig. 8.1 with a typical result obtained earlier from the temporal correlators using the maximum entropy method with an additional constraint that at some critical temperature the spectral function is described by perturbative QCD [80]. While the latter spectral function also shows two distinct peaks at high temperatures, their position is proportional to the temperature and the width of these excitations remains constant with temperature, in contrast to the results presented in Fig. 8.1.

To summarize, the results of this section imply that degrees of freedom in the medium above the chiral crossover and below \( T \sim 500 - 600 \) MeV are hadrons. This is entirely consistent with the conclusions obtained in previous sections.
based on symmetries of correlators and on screening masses and the equation of state.

9. Bottomonium spectrum above $T_{ch}$

Another evidence that at temperatures significantly above $T_{ch}$ there is no "deconfinement", is the observation on the lattice of the 1S,2S,3S and 1P,2P radial and orbital excitations of bottomonium [27]. The states were obtained with the standard variational analysis of the corresponding correlators using non-relativistic QCD lattice framework. It is important to stress that no potential picture is assumed here, it is an output of QCD.

The results for the mass shifts of the 1S,2S,3S levels with respect to the vacuum masses of the corresponding states are shown in Fig. 9.1. We see that masses of the bottomonium states in the medium remain stable and agree with those in vacuum. The same feature was observed in previous Section for the pion spectral function. The widths of the corresponding states are shown in Fig. 9.2. The widths increase with temperature. Again the same feature is seen for pions.

The observation of the radial and orbital excitations in a heavy quark-antiquark system is an unambiguous evidence for confinement. According to the Matsui-Satz prediction [13], deconfinement at a critical temperature would mean that the confining Coulomb plus linear potential becomes Debye screened and gets weaker than the Coulomb potential,

$$\sim -\frac{1}{r}\exp(-m_D r).$$

(9.1)

Such potential does not support any bound state in a system of heavy quarks and would evidence a deconfinement. The Coulomb potential supports only the 1S state (positronium) and no P-levels. Above there would be a quark - antiquark continuum.

The survival of the radial and orbital excitations above the chiral restoration temperature points to a confining interaction at these temperatures. A phenomenological model picture consistent with these results is an optical potential that consists of a real linear confining potential, that is not modified with temperature, and an imaginary part, that increases with $T$ [15], see Fig. 9.3. The T-independent real part is responsible for the stability of the energy levels in the medium, and the rising with temperature imaginary part provides increasing widths.

As a conclusion, these results stress that degrees of freedom in the medium above $T_{ch}$ are color-singlet hadrons.
10. Why is the stringy fluid stringy?

It is not accidental that the name "stringy fluid" was given to a chirally symmetric and approximately chiral spin symmetric QCD matter above the chiral restoration crossover [28]. The reason is that a simple stringy picture of confined hadrons, provided that the chiral symmetry is restored [81], very naturally accommodates the chiral spin symmetry.

The celebrated approximately linear Regge trajectories

\[ M^2(n, L) = c_n n + c_L L + \text{corrections}, \]  

where \( n \) and \( L \) are the radial quantum number and angular momentum of the string, respectively, represent the most important achievement of the string description of hadrons. The slope of the angular trajectories, \( c_L \), is fixed by the string tension \( \sigma \) which is a fundamental parameter of the Nambu-Goto action.

What is missing in this description is a degeneracy of states with opposite parity, i.e., a presence of the chiral multiplets in the spectrum. This is because the spin degree of freedom of quarks at the ends of the string is missing in the standard open bosonic string description.

If chiral symmetry is restored, as it does above \( T_{ch} \), then one naturally views a string with massless quarks at the ends that have definite chiralities [81], see Fig. 10.1.

Explicitly all eigenvectors of chiral symmetry for mesons were constructed in Ref. [19]. The chirally symmetric \( \bar{q}q \) states are specified with: \( r; IJ^{PC} \), where \( r \) denotes a representation of the parity-chiral group and all other quantum numbers are isospin, spin, spatial and charge parities. The states fill out the following irreducible representations of the parity-chiral group \( SU(2)_L \times SU(2)_R \times C_i \), where \( C_i \) consists of the space inversion and identity. A product with the latter group is required to construct states of definite parity:

(i) \( (0,0) \):

\[ |(0,0); \pm J\rangle = \frac{1}{\sqrt{2}} |\bar{R}R \pm \bar{L}L\rangle_J. \]
Here $I = 0$, $R$ and $L$ denote the right-handed $SU(2)_R$ ($R^T = (u_R, d_R)$) and the left-handed $SU(2)_L$ ($L^T = (u_L, d_L)$) vectors. The subscript $J$ means that a definite spin and its projection ($J$ and $M$) are ascribed to the given quark-antiquark system according to the relativistic spherical helicity formalism [82]:

$$|\lambda_q \lambda\rangle_J = D^{(J)}_{\lambda_q \lambda\lambda_M}(n) \sqrt{\frac{2J + 1}{4\pi}} |\lambda_q\rangle - \lambda\rangle,$$

(10.3)

with $D^{(J)}_{\lambda_M \lambda_M}(n)$ being the Wigner $D$-function describing rotation from the quantization axis to the quark momentum direction $n = p/p$ and $\lambda_q (\lambda\rangle$ are the quark (antiquark) helicities. Note that the quark chirality and helicity coincide, while for the antiquark they are just opposite. The parity of the quark-antiquark state is then

$$\hat{P}|(0,0)\pm;J\rangle = \pm(-1)^J|(0,0)\pm;J\rangle.$$

(10.4)

(ii) $(1/2,1/2)_a$ and $(1/2,1/2)_b$:

$$|(1/2,1/2)_a; +; I = 0; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}L + \bar{L}R\rangle_J,$$

(10.5)

$$|(1/2,1/2)_a; -; I = 1; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}L - \bar{L}R\rangle_J,$$

(10.6)

and

$$|(1/2,1/2)_b; -; I = 0; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}L - \bar{L}R\rangle_J,$$

(10.7)

$$|(1/2,1/2)_b; +; I = 1; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}L + \bar{L}R\rangle_J.$$

(10.8)

Here $\tau$ are isospin Pauli matrices. The parity of all states in these representations is determined as

$$\hat{P}|(1/2,1/2); \pm; I; J\rangle = \pm(-1)^J|(1/2,1/2); \pm; I; J\rangle.$$

(10.9)

Note that a sum of the two independent $(1/2,1/2)_a$ and $(1/2,1/2)_b$ irreducible representations of $SU(2)_L \times SU(2)_R$ forms an irreducible representation of the $U(2)_L \times U(2)_R$ or $SU(2)_L \times SU(2)_R \times U(1)_A$ groups.

(iii) $(0,1)\oplus(1,0)$:

$$|(0,1) + (1,0); \pm; J\rangle = \frac{1}{\sqrt{2}} |\bar{R}R \pm \bar{L}L\rangle_J,$$

(10.10)

with $I = 1$ and parities

$$\hat{P}|(0,1) + (1,0); \pm; J\rangle = \pm(-1)^J|(0,1) + (1,0); \pm; J\rangle.$$

(10.11)

The $J = 0$ states are connected by the $SU(2)_L \times SU(2)_R$ and $U(1)_A$ transformations and cannot be constructed from the $(0,0)$ and $(0,1) + (1,0)$ representations, because the total spin projection onto the momentum direction of the quark is $\pm 1$ for the latter representations.

Now comes a key point. The states (10.2),(10.5),(10.6),(10.7),(10.8) and (10.10) with $J > 0$ transform into each other upon the $SU(2)_{CS}$ and $SU(4)$ transformations of the vector (2.10), for different irreducible representation of the latter groups see Fig. 3.1. We automatically incorporate the $SU(2)_L \times SU(2)_R \times U(1)_A$ chiral symmetry as well as the $SU(2)_{CS}$ and $SU(4)$ symmetries [19]. Namely, all hadrons with different chiral configurations of quarks at the ends of the string that belong to the same intrinsic quantum state of the string must be degenerate.

There are important implications. The spin-orbit interactions of quarks should vanish at the classical level. Indeed, if the quark has a definite chirality, then its spin is necessarily parallel (or anti-parallel) with its momentum. Hence the spin-orbit force, $\sim L \cdot S$, is necessarily zero. This is also true for the spin-orbit force due to the Thomas precession.

For a rotating $qq$ string the tensor force also vanishes. Indeed, the tensor force consists of the scalar products $S_i \cdot R_j$, where $R_j$ is the radius-vector of the given quark in the center-of-mass frame.

There is more than that. Both the spin-orbit and tensor forces are effects of the magnetic field. However, emergent chiral spin and $SU(4)$ symmetries indicate that the magnetic field in the medium is highly suppressed with respect to the confining electric field, as was discussed in chapter 2. Then the absence of the spin-orbit and tensor interactions between quarks is consistent with the latter emerged symmetries.

To summarize: Approximate chiral spin and $SU(4)$ symmetries seen above $T_{ch}$ are entirely consistent with a simple and intuitive picture that here hadrons are electric strings with chiral quarks at the ends.
11. Is the stringy fluid a gas or a liquid?

An ideal gas of quarks and gluons is characterized by the Stefan-Boltzmann behavior

\[ P \sim T^4. \]  

(11.1)

From Fig. 7.3 we can conclude that the system is close to this limit at temperatures above \( T \sim 1 \text{ GeV} \). However at lower temperatures, below 500-600 MeV (but above the chiral restoration temperature \( T_{ch} \)) the pressure rises with \( T \) much faster. This indicates that interaction between the constituents is very important. The same feature is also seen in Fig. 7.2. A flat temperature dependence of the screening masses is observed above \( T \sim 600 \text{ MeV} \). This is consistent with a perturbative HTL re-summation and is characteristic of a quark-gluon plasma. However, between \( T_{ch} \) and 500 - 600 MeV a steep increase is seen, that is inconsistent with the parton description.

From the chiral spin symmetry of an effective QCD action (and of the thermal QCD partition function) at \( T_{ch} - 3T_{ch} \) as well as from the pion and bottomonium spectral properties we have concluded that in this temperature window degrees of freedom in the medium should be chirally symmetric and approximately chiral spin symmetric hadron-like systems. Then the question arises, whether these hadrons interact strongly and what evidence exists for this? Here we will give a qualitative answer to this question and explain that the observed spectral properties of pions do imply a strong interaction between the hadrons in the medium. We stress that we do not attempt to construct an effective theory of pions in the stringy fluid. It is a task for future.

The key point is that the discrete pion level at \( \sqrt{s} = m \) in vacuum at zero temperature becomes a resonance with a finite width in the QCD medium at a temperature \( T > T_{ch} \), see Fig. 8.1. Let us simplify the system and assume that a hadron resonance gas at temperatures below \( T_{ch} \) consists only of noninteracting pions. Such a system is described by the Klein-Gordon Lagrangian:

\[ \mathcal{L} = \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi - \frac{1}{2} m^2 \phi^2. \]  

(11.2)

Naively the finite width of the pion seen in Fig. 8.1 can be connected to a decay of the pion into a quark-antiquark pair since the pion is the lightest hadron. If it were so this would point to absence of a confining interaction above \( T_{ch} \). However, it is not so and the presence of a finite width of the pion in Fig. 8.1 implies actually that above \( T_{ch} \) the medium cannot be a gas of noninteracting pions and instead the pions should strongly interact. The simplest possible interaction term in the effective Lagrangian above \( T_{ch} \) should be the \( \phi^4 \) term (or, some factors of \( \phi \) in the interaction term should be substituted by derivative of the pionic field according to the power counting). For example, the well known \( \phi^4 \) theory of interacting scalars is given by the following Lagrangian

\[ \mathcal{L} = \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4. \]  

(11.3)

The \( \phi^4 \) term in the Lagrangian describes a collision of four pions above \( T_{ch} \), respectively, and graphically corresponds to an interaction vertex with four pion legs. Such vertices imply that a strong decay of the pion into three pions is possible. Consequently this theory would require that the pion is not a stable particle and should be a resonance. We conclude that a finite decay width of the pion above \( T_{ch} \) points to a strong interaction of pions and to \( \phi^4 \)-like terms in an effective Lagrangian. We repeat, that it is not yet an attempt of an effective theory of interacting pions above chiral restoration temperature. Such a theory should rely on the power counting and be constrained by the emerged symmetry.

A presence of collisions of four particles in a system implies that the system is more a liquid rather than a gas, if the effective coupling constant \( \lambda \) is large enough\(^\text{10}\).

12. Baryonic parity doublets and chiral spin symmetry

Could a chiral spin symmetric regime exist in the baryon rich region at large chemical potentials and low temperatures? It turns out that the manifestly chirally symmetric free parity doublet Lagrangian \[83\] has precisely a \( SU(4) \) symmetry with a \( SU(2) \) subgroup that performs a rotation in the space of right-handed and left-handed fields \[86\].

Consider a Dirac Lagrangian for a massless fermion field

\[ \mathcal{L} = i \bar{\psi} \gamma_{\mu} \partial^{\mu} \psi = i \bar{\psi}_L \gamma_{\mu} \partial^{\mu} \psi_L + i \bar{\psi}_R \gamma_{\mu} \partial^{\mu} \psi_R, \]

(12.1)

where

\[ \psi_R = \frac{1}{2} (1 + \gamma_5) \psi, \quad \psi_L = \frac{1}{2} (1 - \gamma_5) \psi. \]

(12.2)

\(^{10}\)Recall that a condition for a condensation of vapor (gas) into water (liquid) at some temperature and pressure is a presence of collisions of at least three \( H_2O \) molecules. This can happen only if the system is rather dense and a typical distance between molecules is small.
This Lagrangian is obviously invariant upon independent $U(1)$ rotations of the $\psi_R$ and $\psi_L$ components, which represent the $U(1)_V \times U(1)_A$ symmetry.

If the fermion field $\psi$ is an isodoublet, this Lagrangian is also $SU(2)_L \times SU(2)_R$ invariant under two independent isospin rotations of the right-handed and left-handed components (12.2):

$$\psi_R \rightarrow \exp \left( i \frac{\theta^a_R \tau^a}{2} \right) \psi_R; \quad \psi_L \rightarrow \exp \left( i \frac{\theta^a_L \tau^a}{2} \right) \psi_L,$$

(12.3)

with $\tau^a$ being the isospin Pauli matrices and $\theta^a_R$ and $\theta^a_L$ parameterize rotations of the right- and left-handed components. The transformation (12.3) defines the $(0,1/2) \oplus (1/2,0)$ representation of the chiral group, where 0 and 1/2 represent isospins of the left- and right-handed components. A direct sum of two independent irreducible representations of the $SU(2)_L \times SU(2)_R$ group is required to get a field of a fixed spatial parity because under the spatial reflection one has $L \leftrightarrow R$.

It is known for a long time that it is possible to construct a chirally symmetric Lagrangian for a massive fermion field if there are two independent mass-degenerate fermions of opposite parity - parity doublets [83].

$$\theta^a \rightarrow \frac{\sigma^a}{2} \otimes \mathbb{1} \quad \sigma^a \rightarrow \frac{\sigma^a}{2} \otimes 1,$$

(12.4)

where independent Dirac bispinors $\Psi_+$ and $\Psi_-$ have positive and negative parity, respectively. The parity doublet is a spinor constructed from two independent Dirac bispinors and contains eight components. There is in addition an isospin index which is suppressed.

The right- and left-handed fields are directly connected with the opposite parity fields

$$\Psi_R = \frac{1}{\sqrt{2}} (\Psi_+ + \Psi_-); \quad \Psi_L = \frac{1}{\sqrt{2}} (\Psi_+ - \Psi_-).$$

(12.5)

Notice a difference with the definition of the right- and left-handed components (12.2) of a single massless Dirac field. The vectorial and axial parts of the $SU(2)_L \times SU(2)_R$ transformation under the $(0,1/2) \oplus (1/2,0)$ representation is

$$\Psi \rightarrow \exp \left( i \frac{\theta^a_R \tau^a}{2} \otimes \mathbb{1} \right) \Psi; \quad \Psi \rightarrow \exp \left( i \frac{\theta^a_L \tau^a}{2} \otimes \sigma_1 \right) \Psi,$$

(12.6)

where $\sigma_1$ is a Pauli matrix that acts in the space of the parity doublet. The axial part of the chiral transformation law (12.3) mixes the massless Dirac spinor $\psi$ with $\gamma_5 \psi$, while the chiral rotation of the parity doublet provides a mixing of two independent fields $\Psi_+$ and $\Psi_-$.

The chiral-invariant Lagrangian of the free parity doublet can be written in two equivalent forms as

$$\mathcal{L} = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m \bar{\Psi} \Psi$$

$$= i \bar{\Psi}_+ \gamma^\mu \partial_\mu \Psi_+ + i \bar{\Psi}_- \gamma^\mu \partial_\mu \Psi_- - m \bar{\Psi}_+ \Psi_+ - m \bar{\Psi}_- \Psi_-$$

(12.7)

or

$$\mathcal{L} = i \bar{\Psi}_L \gamma^\mu \partial_\mu \Psi_L + i \bar{\Psi}_R \gamma^\mu \partial_\mu \Psi_R - m \bar{\Psi}_L \Psi_L - m \bar{\Psi}_R \Psi_R.$$  

(12.8)

The latter form demonstrates that the right- and left-handed degrees of freedom are completely decoupled and the Lagrangian is manifestly chiral-invariant. It is also manifestly Lorentz-invariant.

This Lagrangian can also be written in another forms [84, 85]. Now we will demonstrate [86] its equivalence to the "mirror" assignment of Ref. [85]. The Lagrangian (2.36) of Ref. [85] with two Dirac fermions

$$\mathcal{L} = i \bar{\psi}_1 \gamma_\mu \partial^\mu \psi_1 + i \bar{\psi}_2 \gamma_\mu \partial^\mu \psi_2 - m (\bar{\psi}_1 \psi_2 + \bar{\psi}_2 \psi_1),$$

(12.9)

is invariant under chiral transformation with the "mirror assignment":

$$\psi_1 \rightarrow \exp \left( i \frac{\alpha^a \tau^a}{2} \gamma_5 \right) \psi_1, \quad \psi_2 \rightarrow \exp \left( -i \frac{\alpha^a \tau^a}{2} \gamma_5 \right) \psi_2.$$  

(12.10)

This is exactly equivalent to the chiral transformation law (12.6) of the doublet (12.4) where

$$\Psi_+ = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2),$$

$$\Psi_- = \frac{1}{\sqrt{2}} (\gamma_5 \psi_1 - \gamma_5 \psi_2).$$

(12.11)
Then upon the "mirror" transformation (12.10) the parity doublet transforms as:

\[
\begin{pmatrix}
\Psi_+
\Psi_-
\end{pmatrix}
\to
\exp\left(i\frac{\alpha^a\tau^a}{2}\otimes\sigma^1\right)
\begin{pmatrix}
\Psi_+
\Psi_-
\end{pmatrix}.
\]  
(12.12)

It turned out, however, that the free parity doublet Lagrangian (12.7-12.8) has a larger symmetry than the \(SU(2)_L \times SU(2)_R\) symmetry. It is manifestly \(SU(4)\) symmetric \cite{86}. Indeed, given Eq. (12.5), the parity doublet (12.4) can be unitarily transformed into a doublet

\[
\tilde{\Psi} = \begin{pmatrix}
\Psi_R
\Psi_L
\end{pmatrix}.
\]  
(12.13)

It is a two-component spinor composed of Dirac bispinors \(\Psi_R\) and \(\Psi_L\) (i.e., altogether there are eight components).

The Lagrangian (12.7-12.8) is obviously invariant under the \(SU(2)\) rotations that mix \(\Psi_R\) and \(\Psi_L\).

\[
\begin{pmatrix}
\Psi_R
\Psi_L
\end{pmatrix}
\to
\exp\left(i\frac{\varepsilon^a\sigma^a}{2}\right)
\begin{pmatrix}
\Psi_R
\Psi_L
\end{pmatrix}.
\]  
(12.14)

Then the parity doublet Lagrangian is not only chirally invariant under the transformation (12.6), but also \(SU(4)\)-invariant with the generators of \(SU(4)\) being

\[\{(\tau^a \otimes 1), (1 \otimes \sigma^n), (\tau^a \otimes \sigma^n)\} .\]
(12.15)

Since the rotation (2.4), that mixes the right- and left-handed Weyl spinors, defines the \(SU(2)\) rotations that mix \(\Psi_R\) and \(\Psi_L\). We conclude that a system of (quasi)free parity doublets, perhaps with a phenomenologically introduced short range repulsion (which would still preserve the \(SU(4)\) symmetry), is a good candidate for a chiral spin symmetric regime in a baryon rich medium at large chemical potentials and low temperatures.

This Lagrangian can be supplemented by the pion and sigma-fields of the linear sigma model \cite{84, 85}. A coupling of parity doublets to the \(\pi, \sigma\) field lifts the \(SU(4)\) symmetry and only chiral symmetry is left in the Lagrangian. This is because the \(\pi, \sigma\) Lagrangian is chirally invariant but not a \(SU(4)\)-singlet.

The chiral symmetry breaking order parameter, \(|0|\sigma|0\neq 0\), generates a mass splitting of the positive and negative parity baryons. I.e. the chiral symmetry of the Lagrangian (12.7-12.8) is lifted. This regime is reminiscent of nuclear matter, where physics at large distances is guided by a coupling of nucleons of positive parity with \(\pi, \sigma\) fields. However, a short range repulsion between nucleons is still missing in this model, which is important for properties of nuclear matter.

The parity doublets coupled to the \(\pi, \sigma\) fields have been used in baryon spectroscopy \cite{87, 88} and for study of chiral symmetry restoration at high temperature or density, where baryons with non-zero mass do not vanish upon a chiral restoration, see e.g. \cite{89, 90, 91, 92} and references therein. The chiral restoration transition can be either of first or second order \cite{89}. This Lagrangian is only chiral invariant since a coupling of the parity doublets to pion and sigma fields destroys the \(SU(4)\) symmetry. The latter symmetry would approximately persist only if the coupling to the \(\pi, \sigma\) fields were suppressed, i.e. there would be no baryon - baryon-hole excitations with pion quantum numbers. It is a very interesting question whether the approximately chiral spin symmetric matter at low temperatures and large baryon chemical potential realized in nature or not.

13. Chiral spin symmetric band of the QCD phase diagram

From the temperature dependence of the spatial correlators of Fig. 6.1, from the T-behavior of screening masses in Fig. 7.2 and pressure in Fig. 7.3, as well as from the pion spectral density in Fig. 8.1 it is naturally to assume that there is no critical line between the stringy fluid and QGP regimes and both regimes are connected by a smooth analytic crossover. This is precisely the reason why we call it regimes, but not phases. However, to rule out a non-analytic phase transition a finite size scaling study would be necessary to demonstrate that no discontinuity develops in the thermodynamic limit. At the moment our knowledge of the T-dependence of observables above and below \(3T_{ch}\) is not sufficiently detailed to claim a crossover or a phase transition. In the case of crossovers, there are no sharp phase boundaries and a position of the crossover line, that "separates" two regimes, necessary varies with its definition.

A possible definition could be a position of the bend of the vector screening masses at \(T_s \sim 500\) MeV seen in Fig. 7.2 \cite{25}. Then there are three different regimes in QCD at vanishing chemical potential, as illustrated in Fig. 1.3. At \(T < T_{ch}\) we have a hadron resonance gas with broken chiral symmetry. In the window \(T_{ch} < T < T_s\) the QCD medium is a stringy fluid with restored chiral symmetries and approximate chiral spin symmetry and still with hadron-like degrees of freedom. Above \(T_s\) the chiral spin symmetry disappears, the hadron degrees of freedom melt down and one can speak of a quark gluon plasma with parton degrees of freedom.
The next question is a fate of the chiral spin symmetric regime at non-vanishing baryon chemical potential. The quark chemical potential term in the quark-gluon part of the QCD action

$$S = \int_0^\beta d\tau \int d^3x \bar{\Psi} \left[ \gamma_\mu D_\mu + \mu \gamma_4 + m \right] \Psi,$$

(13.1)
is manifestly chiral spin and $SU(4)$ symmetric [28]. This suggests that these symmetries, observed at $\mu = 0$, should also persist at finite $\mu$.

We know from lattice simulations how the chiral crossover temperature, which constitutes a lower bound for the chiral spin symmetric regime, behaves for small $\mu_B < 3T$:

$$\frac{T_{ch}(\mu_B)}{T_{ch}(0)} = 1 - 0.016(5) \left( \frac{\mu_B}{T_{pc}(0)} \right)^2 + \ldots,$$

(13.2)

with the sub-leading term not yet statistically significant [93, 94, 95, 96]. The qualitative behavior of the upper boundary of the chiral spin symmetric band can be inferred from the value of a chosen vector meson screening mass at the temperature $T_s$,

$$\frac{m_V(T_s)}{T_s} = C_0.$$

(13.3)

Then, $CP$-symmetry requires that mesonic screening masses are even functions of $\mu_B/T$, and therefore

$$\frac{m_V(\mu_B)}{T} = C_0 + C_2 \left( \frac{\mu_B}{T} \right)^2 + \ldots.$$

(13.4)

Keeping this value constant as chemical potential is varied, $dm_V/d\mu_B = 0$, one finds

$$\frac{dT_s}{d\mu_B} = -\frac{2C_2 \mu_B}{C_0 T} - \frac{2C_2^2}{C_0} \left( \frac{\mu_B}{T} \right)^3 + \ldots.$$

(13.5)

We know from analytic calculations [97] as well as lattice simulations [98, 99] that $C_2 > 0$. Then the upper boundary of the chiral spin symmetric regime leaves the temperature axis with zero slope and negative curvature. This implies that a chiral spin symmetric band bends downwards with chemical potential, as sketched in Fig. 13.1 [25].

However, our expectations for the upper boundary of the CS symmetric band are based on sufficiently small $\mu_B/T$. Consequently we cannot exclude that at larger chemical potentials the upper and lower boundaries merge at some point. It could be expected, for example, at a possible critical end point of the first-order chiral phase transition at reasonably large chemical potential [100, 101], which is not yet excluded both by the lattice data and experiments. Then a CS symmetric band could be modified as sketched in Fig. 13.2 [25]. The trend for the upper boundary of the CS-symmetric band could be studied on the lattice similar to what was done for $T_{ch}(\mu)$ using the imaginary chemical potential, Taylor expansion, etc.

### 14. Dileptons and the chiral spin symmetric band

In vacuum the electron - positron annihilation into hadrons shows a powerful resonance peak from $\rho$- and $\omega$-mesons, then a sharp peak from $\phi$-meson. Above these peaks there are oscillations about perturbative $e^+ + e^- \to \bar{q} + q$ curve. These
oscillations arise from broad higher-lying resonances $\rho', \rho''$, .... The existence of the resonance peaks and oscillations around perturbative curve reflects the confining and chiral symmetry breaking properties of the QCD vacuum [102]. These properties should also persist in a dilute hadron resonance gas. The 30-years long experimental study in heavy ion collisions at different temperatures and chemical potentials employ the inverse process, with the final state being the electron-positron pair. This study intends to shed light on the question to what extent a hot or dense medium differs from the vacuum.

The dilepton production rate is determined by the spectral function of the electromagnetic current in the medium which is proportional to the imaginary part, $\text{Im}[\Pi_{\text{em}}(M, q; T, \mu_B)]$, of the two-point correlator of the electromagnetic current [103]:

$$
\frac{dN}{d^2q d^4x} = -\frac{\alpha^2}{\pi^3 M^2} f^B(q_0, T) \text{Im}[\Pi_{\text{em}}(M, q; T, \mu_B)].
$$

(14.1)

Here $M$ is the invariant mass of the $e^+e^-$ pair with the four-momentum $q = (q_0, \mathbf{q})$, $f^B(q_0, T)$ is the Bose-Einstein distribution in the thermalized medium and $\alpha$ the fine structure constant.

The absence of sharp resonance peaks within the fireball was usually taken as a signal of chiral symmetry restoration and deconfinement\(^{11}\). For correct interpretation some care is in order.

The finite temperature $\rho$- spectral function is encoded in the temporal $\rho_{(1,0)+(0,1)}$ correlators of Fig. 6.3 as well as in the spatial correlators $V_x$ of Fig. 6.2. If a Euclidean correlator evaluated in full QCD is essentially different from that calculated with non-interacting quarks, one can safely state that the spectral density will not be dual to a perturbative description, but should contain some remnant resonance structure. Comparing results for full QCD with those for free quark gas in Figs. 6.3 and 6.2 one notices such a difference, very clearly seen especially in spatial correlators, up to temperatures $T \sim 500$ MeV. One then expects some broad structure in the $\rho$- spectral function. Obviously, it should be essentially broader than in vacuum. This could be caused by a fast decay of the $J = 1$ excitation into $J = 0$ excitations, $\rho \to \pi + \pi$. This is consistent with the less pronounced $\rho$-peak in the spectral function representing the fireball above the chiral restoration temperature, as possibly observed at RHIC [104, 105], SPS [106, 107] and LHC [108]. We thus conclude that the absence of sharp $\rho$ and $\omega$ peaks in high temperature dilepton spectra coming from the fireball is entirely consistent with the hadronic description above the chiral restoration. It is an important task for future to establish quantitative experimental signatures for violation of the quark-hadron duality in dileptons, i.e. persistence of a broad $\rho$-like state above the chiral restoration crossover. This violation would imply a measurable difference between the perturbative $\bar{q}q + q \to e^+ + e^-$ contribution and total experimental result within the fireball.

Approximate $SU(4)$ symmetry requires the isoscalar $\omega_{(0,0)}$ correlator to be close to the isovector correlator $\rho_{(1,0)+(0,1)}$. Hence, what was said about the $\rho$ peak above chiral restoration line, should also be true with respect to the $\omega$ peak.

The dilepton production at essentially lower temperature $T \sim 72$ MeV and reasonably large baryon chemical potential $\mu_B \sim 900$ MeV has been studied by HADES collaboration in Au-Au collisions at $\sqrt{s_{NN}} = 2.42$ GeV [109]. The excess yield extracted by subtracting the $\eta, \omega$ contributions, which are produced beyond the fireball, is shown in Fig. 14.1. It exhibits a nearly exponential fall-off that can be well described by the black-body spectral distribution

$$
\frac{dN}{dM} \sim M^{3/2} e^{-M/T}.
$$

(14.2)

The latter fit allows HADES to extract the temperature $T \sim 72$ MeV.

\(^{11}\)Actually sharp $\rho, \omega$ peaks are seen in heavy ion collisions [104, 105, 108]. They are interpreted as arising from the $\rho, \omega$-decay into dileptons beyond the fireball, at the final stage of the heavy ion collision. The vacuum cross sections for dilepton production in different elementary reactions (the so-called cocktail) are subtracted from the full rate, and the sharp $\rho, \omega$ peaks become much smoother. It is not yet clear what is left in reality after such subtraction.
No pronounced $\rho$-structure is visible. The data are well described by the leading order $\bar{q} + q \rightarrow e^+ + e^-$ diagram (blue curve). A slight oscillation about the perturbative curve might also be visible in Fig. 14.1, that would hint at the quark-hadron duality violations. Notice that such violations can appear provided that there are still contributions from a very broad $\rho$-state. A broad $\rho$-state in the medium can exist in the chirally broken regime [110]. It can also exist in the chirally symmetric and chiral spin symmetric regime like at zero chemical potential.

The experimental results are also consistent with perturbative $\bar{q} + q \rightarrow e^+ + e^-$ curve without any oscillations. This might be explained by a perturbative quark matter. However, there is an alternative explanation. A possibility for a broad $\rho$-peak in a dense chirally symmetric baryonic medium or its absence is provided by the chiral spin and $SU(4)$ symmetric parity doublet matter. Within such matter the baryons of positive and negative parity to leading order decouple from pions and sigmas as discussed in Chapter 12. However, they can be coupled with the 15-plet of vector mesons from Fig. 3.1, what would keep the $SU(4)$ symmetry. Hence a baryonic medium becomes a Fermi gas, except, perhaps, possible corrections from a short range repulsion between baryons. Electromagnetic baryon - baryon hole excitations guarantee an equilibrium between the baryonic Fermi gas and the photonic Bose gas. This is consistent with the black-body radiation description of the excess shown in Fig. 14.1. The dilepton production of the chiral spin symmetric baryonic parity doublet matter is hence very similar to that of thermalized quark matter. This suggests that the HADES point at $T \sim 72$ MeV, $\mu_B \sim 900$ MeV might be just above the chiral restoration line, $T_{ch}(\mu_B)$, and could possibly be within the chiral spin and $SU(4)$ symmetric band.

15. Conclusions

In this review we have presented lattice evidences that above the chiral symmetry restoration crossover around $T_{ch} \sim 155$ MeV the QCD medium at vanishing baryon chemical potential is populated with the hadron-like (mostly mesons with $J = 0, 1$) degrees of freedom. This regime is dubbed a stringy fluid because these states are chirally symmetric quarks connected into color-singlet hadrons by a confining chromo-electric field. These hadrons are chirally symmetric and approximately chiral spin symmetric. The chiral spin and $SU(4)$ symmetries are symmetries of the electric part of the QCD Lagrangian which are larger than the chiral symmetries of the QCD Lagrangian as a whole. The chiral spin and $SU(4)$ symmetries can approximately emerge only if both $U(1)_A$ and $SU(2)_L \times SU(2)_R$ chiral symmetries are restored (at least approximately) and at the same time the chromo-electric contributions into energy strongly dominate over the chromomagnetic contributions and the quark kinetic terms. A direct evidence of approximate $SU(2)_{CS}$ and $SU(4)$ symmetries of the thermal QCD partition function and of an effective QCD action above $T_{ch} \sim 155$ MeV is a multiplet structure observed in spatial and temporal meson correlators calculated on the lattice with $N_F = 2$ QCD with a chirally symmetric Dirac operator at physical quark masses.

These $SU(2)_{CS}$ and $SU(4)$ symmetries smoothly disappear above $T \sim 500$ MeV and correlators of full QCD approach correlators calculated with a free quark gas. This can happen only if the contributions from the quark kinetic terms become dominant and the confining chromo-electric field gets screened. Consequently at zero baryon chemical potential we can distinguish three different regimes according to symmetries of the thermal partition function and degrees of freedom. Below $T_{ch} \sim 155$ MeV the QCD matter is a dilute meson gas with spontaneously broken chiral symmetry. Within the window $T_{ch} - 3T_{ch}$ the hot QCD is represented by the stringy fluid with restored chiral and approximate chiral spin symmetries. Above $\sim 3T_{ch}$ the chiral spin symmetry disappears and one observes a smooth transition to
partonic degrees of freedom, i.e. to a quark-gluon plasma.\textsuperscript{12} The symmetry arguments have been supported by the behavior of screening masses and of the equation of state. While the screening masses and the equation of state are compatible with the partonic description at temperatures of \(\sim 1\) GeV and above, at temperatures below \(\sim 500\) MeV they demonstrate a radically different behavior, not consistent with the perturbative description, indicating a truly non-perturbative regime.

A direct evidence of hadron degrees freedom in the stringy fluid is a pion spectral function extracted from the spatial lattice correlators using a generalized Källen-Lehmann representation. This spectral function demonstrates a distinct pion state and its first radial excitation at temperatures significantly above \(T_{ch}\). They become broader with temperature and melt above \(\sim 500\) MeV down. It is important that this spectral function allows one to predict temporal Euclidean correlators. The latter correlators can be compared with the lattice data and this comparison shows a satisfactory agreement. This test implies that the extracted spectral function is close to reality. Another direct evidence of hadron degrees of freedom in the stringy fluid is existence of 1S,2S,3S and 1P,2P radial and orbital bottomonium states seen on the lattice at temperatures above \(T_{ch}\), that become broader with temperature. This excitation spectrum is consistent with an optical potential that consists of its real part with a Coulomb plus linear confining potential, which is temperature independent, and an imaginary part that increases with temperature.

The very fact that the pion state, that is discrete in vacuum, becomes a broad state in medium above \(T_{ch}\), points to a strong interactions between hadrons in the stringy fluid. This strong interaction is induced by a small separation distance between the hadrons. The stringy fluid is a system of densely packed mesons mainly with \(J = 0,1\) and is closer to a liquid rather than to a gas.

The quark chemical potential in the QCD action is manifestly \(SU(2)_{CS}\) and \(SU(4)\) symmetric. This suggests that the chiral spin symmetric regime seen on the lattice at zero chemical potential between \(T_{ch}\) and \(3T_{ch}\) extends into the QCD phase diagram as a band that bends downwards with the chemical potential. In the cold and dense region a \(SU(4)\) symmetric parity doublet matter could be a good candidate for a chiral spin symmetric matter.

Finally we discussed available experimental data on dilepton production at temperatures above \(T_{ch}\). The lattice correlators hint at the existence of a very broad \(\rho\) state that fastly decays into two pions. We also discussed recent results of HADES at smaller temperature and reasonably large baryonic chemical potential.

\textsuperscript{12}After completion of the revised version of this review a new study of the chiral spin symmetry with domain wall fermions in 2+1+1 QCD has appeared with qualitatively similar results [111].
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