We formulate EV charging as a feasibility problem that meets all EVs’ energy demands before departure under charging rate constraints and total power constraint. We propose an online algorithm, the smoothed least-laxity-first (sLLF) algorithm, that decides on the current charging rates based on only the information up to the current time. We characterize the performance of the sLLF algorithm analytically and numerically. Numerical experiments with real-world data show that it has significantly higher rate of generating feasible EV charging than several other common EV charging algorithms.

### Keywords
Online algorithm, online feasibility, resource augmentation, electric vehicle charging

### 1 Introduction
The electrification of transportation provides a great opportunity for energy efficiency and sustainability. There were over a million electric vehicles (EVs) worldwide as of 2015 [1], and accelerated EV proliferation is expected for many years to come. To charge a large number of EVs however presents a tremendous challenge, in terms of both its impact on power grid and management complexity. While the flexibility in charging time and rate can be exploited for proﬁt or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.

The existing EV charging algorithms can be categorized into either offline or online. The offline algorithms require complete information on all EVs to decide on the charging rates [4, 8, 16, 18, 23, 24]. However, information on future EV arrivals may not be available or very costly to obtain, which motivates online algorithms [2, 3, 5, 9, 13, 19, 22, 23, 26, 26]. The performance of the online algorithm is generally analyzed for the worst-case [5, 13] or average-case [9, 26]. Other desirable properties of charging algorithms are low complexity in computation and memory usage, which can be achieved by sorting or bisection based methods, such as earliest-deadline-first, least-laxity-first [21], Whittle’s index policy [25, 26], among others.

The multi-processor deadline scheduling problem [6, 7, 15] considers the scheduling of jobs on multiple processors. We can view the EV charging problem as a deadline scheduling problem by considering chargers as processors, and EVs with certain energy demand as jobs. Resource augmentation is a prominent analysis framework [10–12, 17] for analyzing the performance of online algorithms for multi-processor scheduling, we apply this framework to the EV charging problem. The main difference is that in our setting the power limit is time-varying, the maximum rates are heterogeneous, and the power limit may not necessarily be integer multiplication of the maximum rate.
2 MODELL AND ALGORITHM

2.1 System Model

Consider a system with one charging station that serves a set of EVs, indexed by \( i \in \mathcal{V} = \{1, 2, 3, \ldots \} \). We use a discrete-time model where time is divided into slots of equal sampling intervals, indexed by \( t \in \mathcal{T} = \{0, 1, 2, \ldots, T\} \). Each EV arrives at the charging station with an energy demand \( e_i \) at time \( a_i \), and departs from the station at time \( d_i \). During its stay at the station, the EV is charged at a rate (or power) of \( r_i(t) \geq 0 \), \( a_i \leq t < d_i \). For convenience, we extend this definition of \( r_i(t) \) to the entire temporal domain. The notations are summarized in Table 1.

To account for limitations in the charger or battery of an EV, each EV can only be charged up to a peak rate \( \bar{r}_i \), i.e.,

\[
\begin{align*}
\bar{r}_i(t) & \leq \bar{r}_i, & t \in [a_i, d_i], & i \in \mathcal{V} \\
\bar{r}_i(t) & = 0, & t \not\in [a_i, d_i], & i \in \mathcal{V}
\end{align*}
\]  

(1)

To account for limitations in the grid or power station, the charging station has a (possibly time-varying) power limit \( P(t) \) such that

\[
\sum_{i \in \mathcal{V}} r_i(t) \leq P(t), \ t \in \mathcal{T}.
\]  

(2)

Furthermore, the power limit and maximum charging rates fall within the following nominal ranges:

\[
P_{\text{min}} \leq P(t) \leq P_{\text{max}},
\]  

\[
\bar{r}_{\text{min}} \leq \bar{r}_i \leq \bar{r}_{\text{max}}, \ i \in \mathcal{V}.
\]

Finally, every EV's energy demands needs to be satisfied, i.e.,

\[
\sum_{i \in \mathcal{V}} r_i(t) = e_i, \ i \in \mathcal{V}.
\]  

(3)

Next, we define an EV charging problem instance as a quintuple \( \mathcal{I} = \{a_i, d_i, e_i, r_i; P(t)\}_{i \in \mathcal{V}, t \in \mathcal{T}} \). The primary goal of EV charging is to satisfy every EV's energy demands under the above power supply and peak rate constraints.

2.2 Online Scheduling

In practice, the energy demand and departure time of an EV are only informed after its arrival. Consequently, the charging station must use an online algorithm to determine an EV's current charging rate \( r_i(t) \) using only information up to the current time \( t \): \[
I_{\mathcal{T}} = \{a_i, d_i, e_i(t), \bar{r}_i; P(t)\}_{i \in \mathcal{V}, t \leq t},
\]  

(4)

where \( e_i(t) = e_i - \sum_{t=0}^{t-1} r_i(t) \) is the remaining energy demand of EV \( i \) at the beginning of time slot \( t \).

Definition 2.2 (Online algorithm). An online algorithm is a sequence of functions \( \mathcal{A} = \{\mathcal{A}_t\} \) where each function \( \mathcal{A}_t : I_{\mathcal{T}} \rightarrow r(t) \) maps the information up to the current time \( I_t \) to the current charging rates \( r(t) = \{r_i(t)\}_{i \in \mathcal{V}} \).

Definition 2.3 (Feasibility of the algorithm). An (online) algorithm \( \mathcal{A} \) is feasible (online feasible) on instance \( I \) if it gives charging rates that satisfy constraints (1)-(3).

2.3 The Smoothened Least-Laxity-First Algorithm

2.3.1 The Laxity. A measure for the flexibility (or urgency) in charging of an EV is its remaining time minus the minimum remaining time needed to fully charge it (time needed to fully charge it at the maximum rate). We refer to this measure as laxity.

Definition 2.4 (Laxity). The laxity of an EV \( i \in \mathcal{V} \) at time \( t \in \mathcal{T} \) is defined as

\[
l_i(t) = \begin{cases} 
[d_i - t]^+ - \frac{e_i(t)}{\bar{r}_i}, & t \geq a_i \\
+\infty, & t < a_i
\end{cases},
\]

where \( +^\ast \) denotes the projection onto the set \( \mathcal{R}_+ \) of non-negative real numbers.

Proposition 2.5 (Feasibility condition). The algorithm \( \mathcal{A} \) is feasible on an instance \( I \) if and only if \( \mathcal{A} \) gives charging rates that result in non-negative laxities for all EVs, i.e.,

\[
l_i(t) \geq 0, \ i \in \mathcal{V}, \ t \in \mathcal{T}.
\]  

(5)

Proposition 2.5 suggests that the minimum laxity among all EVs can serve as a measure of the distance from infeasibility. A naive approach—referred to as the least laxity first (LLF) algorithm—is to charge EVs starting from those with the least laxity to those with the most laxity. However, the LLF algorithm may compromise the feasibility of certain offline feasible instances (see Section 4).
and cause excessive preemptions and oscillations in the charging rate\(^7\), which may reduce the lifetime of certain batteries (e.g., Li-ion) \([13]\). Alternatively, we consider maximizing the minimum laxity among all EVs in order to maximize the feasibility margin, \(\max_r\min_{i\in\mathcal{V}}\ell_i(T)\). Although its solution may be non-unique, the following optimization problem produces an unique solution that is also a solution of \(\max_r\min_{i\in\mathcal{V}}\ell_i(T)\).\(^8\)

**Corollary 2.6 (Equivalent Problem).** Consider the optimization algorithm

\[
\max_r \sum_{i\in\mathcal{V}} \bar{r}_i f(\ell_i(T)) \quad \text{s.t.} \quad (1), (2), \sum_{i\in\mathcal{T}} r_i(t) \leq e_i, \ i \in \mathcal{V} \quad (6)
\]

where \(f\) is strictly increasing. Algorithm (6) is feasible for any offline feasible instance.

However, we cannot solve (6) because of the lack of future information of incoming EVs. Instead, we replace (6) with the following online algorithm: at each time \(t \in \mathcal{T}\), given \(\ell_i(t), i \in \mathcal{V}\), compute

\[
\max_{r(t)} \sum_{i\in\mathcal{V}} \bar{r}_i f(\ell_i(t + 1)) \quad \text{s.t.} \quad (1), (2), r_i(t) \leq e_i(t), \ i \in \mathcal{V}. \quad (7)
\]

The optimization problem (7) also maximizes the minimum laxity \(\min_{i\in\mathcal{V}}\ell_i(t + 1)\), and thus maximizes the feasibility margin at time \(t\).\(^9\) Next, we show the structure of the optimal solution, which will be used to construct a scalable algorithm.

**Proposition 2.7 (Valley-Filling Solution).** Assume that \(f\) is strictly concave, strictly increasing, and twice continuously differentiable. A solution to the optimization problem (7) is

\[
r_i^*(t) = [\bar{r}_i(L(t) - \ell_i(t) + 1)]_{\min(r_i,e_i)}(t), \quad i \in \mathcal{V} \quad (8)
\]

where \([x]_{a,b}\) denotes the projection of the scalar \(x\) on interval \([a, b]\), and the value \(L(t)\) satisfies

\[
\sum_{i\in\mathcal{V}} [\bar{r}_i(L(t) - \ell_i(t) + 1)]_{\min(r_i,e_i)}(t) = \sum_{i\in\mathcal{V}} \bar{r}_i(t) = \min_{P(t)} \left( P(t), \sum_{i\in\mathcal{V}} \min(r_i,e_i) \right) \quad (9)
\]

Observe that for \(EV \in \mathcal{V}\) with \(\bar{r}_i < \ell_i(t)\), the charging rates (8) result in \(\ell_i(t + 1) = [L(t)\ell_i(t)]_{\ell_i(t) - 1}\). Hence, \(L(t)\) can be considered as a threshold of \(\ell_i(t + 1)\), below which the energy is charged to \(EV\). Since \(r_i^*(t)\) in (8) is an increasing function of \(L(t)\), a binary search can be used to find the threshold \(L(t)\) in (9). Given \(L(t)\), the charging rates \(r_i^*(t)\) is then determined using (8). We formally state this procedure in Algorithm 1, and name it as the smoothed least-laxity-first (sLLF) algorithm.

The computational complexity of the sLLF algorithm is \(O(|\mathcal{V}| + \log(1/\delta))\), where \(\delta\) is the level of tolerable error. Lastly, we note that the sLLF algorithm has other useful properties such as Least-laxity-first property and fairness.

**Lemma 2.8 (Least-Laxity-First Property).** If there exist two EVs \(i, j \in \mathcal{V}\) under the sLLF algorithm such that

\[
\ell_i(t) \leq \ell_j(t), \quad (10)
\]

\[
\ell_i(t + 1) > \ell_j(t + 1), \quad (11)
\]

then either one of the following holds:

\[
t \geq d_i \text{ & } r_i(t) = 0, \quad (12)
\]

\[
t < d_i \text{ & } t < d_j \text{ & } \bar{r}_j(t + 1) = 0 \text{ & } r_i(t) \neq 0. \quad (13)
\]

Due to space constraints, the proofs are given in Appendix. The above properties will be useful in the analysis of feasibility conditions.

### 3 PERFORMANCE ANALYSIS

There are two extreme cases, \(\bar{r}_i \to \infty, i \in \mathcal{V}\) and \(P(t) \to \infty\), in which online algorithms can be feasible for any offline feasible instances. When \(\bar{r}_i \to \infty i \in \mathcal{V}\), or equivalently \(P(t) \leq \min_{i\in\mathcal{V}} \bar{r}_i\) for all \(t \in \mathcal{T}\), the charging problem is identical to the single processor preemptive scheduling problem where the processing capacity is time-variant. For this case, the earliest-deadline-first (EDF) algorithm is feasible for any offline feasible instances [20]. When \(P(t) \to \infty\), or equivalently \(P(t) \geq \sum_{i\in\mathcal{V}} \bar{r}_i(t)\) for all \(t \in \mathcal{T}\), the sLLF algorithm is feasible for any offline feasible instances. However, beyond the above two extreme cases, no online algorithm can be feasible on all offline feasible instances [23]. The hardness of finding feasible online algorithms motivates a quantitative measure to evaluate the likelihood of an algorithm being feasible. Observe that if more resources (e.g., \(P(t), \bar{r}_i\)) are allowed, an otherwise infeasible problem instance may become online feasible under the online algorithm. We use this (minimum) additional resource to analyze the performance of the sLLF algorithm, where either power \(P\) or both power \(P\) and peak rate \(\bar{r}_i\) are augmented. The former allows more EVs to be charged simultaneously, while the latter additionally allows EVs to be charged faster. As we will demonstrate, these two ways of resource augmentation are qualitatively different and provide complementary insights into the behavior of the sLLF algorithm.

#### 3.1 Power Augmentation

In the case of power augmentation, the online algorithm is allowed to use more power than the offline algorithm, i.e., \(P^{on}(t) = (1 + \epsilon)P(t), \bar{r}^{on}_i = \bar{r}_i\).

**Definition 3.1.** \([\epsilon\text{-power augmented instance}]\) Given an EV charging instance \(\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}}\), we define its \(\epsilon\)-power augmented instance as

\[
\{a_i, d_i, e_i, \bar{r}_i; (1 + \epsilon)P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}} \quad (14)
\]
Definition 3.2. [ε-power feasibility] An online algorithm 𝑈 is ε-power feasible if 𝑈 is feasible on the ε-power augmented instances 𝑃(𝑡) generated from any offline feasible instance 𝑃.11

Unfortunately, there is no ε-power feasible online algorithm for any finite 𝜀 > 0.11,15 However, under a mild assumption, ε-power feasibility condition can be obtained for a finite 𝜀. Assume that the energy demand of each EV is bounded by 𝑋 and the inter-arrival time between consecutive arrivals are greater than 𝑁, i.e.,

\[ e_1 \leq X, \quad i \in V, \]

\[ |a_i - a_j| > N, \quad i, j \in V. \]

We can characterize the relation between 𝑁 and the sufficient amount of resource augmentation 𝜀 as follows.

Theorem 3.3. Assume (15), (16). The sLLF algorithm is ε-power feasible with

\[ \varepsilon = \frac{P_{\max}}{P_{\text{min}}} \left\{ \log \left( \frac{\sqrt{5}X}{NP_{\max}} + \frac{1}{2} \right) + 2 \right\} - 1, \]

where \( \phi = 1.61803 \) is the golden ratio.

In particular, when \( N \geq X/P_{\max} \), we can further simplify the feasibility condition in Theorem 3.3.

Corollary 3.4. If \( N \geq X/P_{\max} \), then the sLLF algorithm is 3-power feasible.

3.2 Power and Rate Augmentation

In the case of power and maximum charging rate augmentation, the online algorithm is allowed to use more power and higher maximum rate than the offline algorithm: \( P_{\text{on}}(t) = (1 + \varepsilon)P(t) \), \( r^i_{\text{on}} = (1 + \varepsilon)r_i \).

Definition 3.5. [ε-augmented instance] Given an EV charging instance \( I\{a_i, d_i, e_i, r_i; P(t)\}_{i \in V, t \in T} \). We define its ε-augmented instance as

\[ \{a_i, d_i, e_i, (1 + \varepsilon)r_i; (1 + \varepsilon)P(t)\}_{i \in V, t \in T} \]

Definition 3.6. [ε-feasibility] An online algorithm 𝑈 is ε-feasible if 𝑈 is feasible on the ε-augmented instances \( P_{\text{on}}(t) \) generated from any offline feasible instance \( I \).

Contrary to the power augmentation, the sLLF algorithm is ε-feasible for a finite value of 𝜀 > 0 without any assumptions of the arrival patterns.

Theorem 3.7. The sLLF algorithm is ε-feasible with

\[ \varepsilon = \max_{i \in V} \left\{ \max_{t \in T} \frac{P(t_1)}{r_i} \right\} \]

As we demonstrate in the next section, the actual EV instance in ACN and others requires smaller amount of resource augmentation than the worse-case upper bound in practice.

4 SIMULATION

In this section, we show the performance of the sLLF algorithm using trace-base simulation on real EV datasets and compare it to that of several heuristic online EV charging algorithms.

4.1 Experimental Setup

Our simulations use datasets from the ACN deployment (CAGarage) and Google’s facilities in Mountain View (Google_mtv) and Sunnyvale (Google_svl). They include a total of 52,362 charging sessions over more than 4,000 charging days in 2016 at 104 locations. See Table 2 for a summary of the data. Each instance consists of one day of charging. We can see that there is a large degree of variation in the sojourn time and laxity of the vehicles in the instances.

For each instance, we compute the minimum power capacity in which the instance is feasible by using offline an LP, i.e., we minimize \( P = P(t) \), subject to (1)-(3). This corresponds to the minimum power supply in order for the instance to be offline feasible. We use this minimum power supply to generate an offline instance, and tested if the instance is feasible under online algorithms. Besides the sLLF algorithm, we also implemented some common (online) scheduling algorithms: earliest-deadline-first (EDF), least-laxity-first (LLF), equal share (ES), remaining energy proportional (REP) [20], and an online linear program (OLP) [3]. Due to space constraints, precise description of each algorithm is given in Appendix C.

Table 2: Statistics of the EV charging instances. Each entry is formatted as average (minimum, maximum), unit (m) denotes minutes.

|                | Instances | EV sojourn time (m) | Laxity (m) |
|----------------|-----------|---------------------|------------|
| CAGarage       | 92        | 321 (11, 720)       | 231 (0.1, 660) |
| Google_mtv     | 3793      | 149 (10, 720)       | 35 (0.001, 694) |
| Google_svl     | 246       | 152 (11, 720)       | 38 (0.02, 676) |

4.2 Results without Augmented Resources

We first evaluate the success rate of the online algorithms without resource augmentation. We define the success rate of an algorithm as the percentage of online feasible instances under the algorithm. The sLLF algorithm achieves uniformly high success rate for all datasets compared to other online algorithms considered. The EDF, ES, and REP algorithms perform much worse in terms of finding feasible schedules (Figure 1). This is not surprising as feasibility requires online algorithms to jointly consider deadline, maximum charging rate, and remaining energy of each EV. However, none of these (the EDF, ES and REP algorithms) consider all three factors simultaneously. The low success rate of the LLF algorithm, despite its similarity to the sLLF algorithm, suggests the importance of maximizing minimum laxity (see Section 2.3).

Next, we study what characteristics of the instances affect the success rate. We find that the minimum normalized laxity and the maximum ratio between EV sojourn times have high correlations with the success rate. The maximum ratio between EV sojourn times

|                | Instances | EV sojourn time (m) | Laxity (m) |
|----------------|-----------|---------------------|------------|
| CAGarage       | 92        | 321 (11, 720)       | 231 (0.1, 660) |
| Google_mtv     | 3793      | 149 (10, 720)       | 35 (0.001, 694) |
| Google_svl     | 246       | 152 (11, 720)       | 38 (0.02, 676) |
Smoothed Least-laxity-first Algorithm for EV Charging

**Figure 1:** Success rates of finding feasible online schedule without resource augmentation.

**Figure 2:** Success rate of finding feasible online schedule without resource augmentation.

![Success rate of finding feasible online schedule without resource augmentation.](image)

- (a) $\epsilon$-power augmentation
- (b) $\epsilon$-power and rate augmentation

**Figure 3:** Success rate of finding feasible online schedule under resource augmentation.

### 4.3 Results with Augmented Resources

While the sLLF algorithm has shown high success rate in finding feasible online EV charging schedules without resource augmentation, we further analyze the performance of online algorithms with resource augmentation in (a) power, and (b) both power and rate. Fig. 3 shows that the sLLF and OLP algorithm have the highest success rate of among other algorithms under various level of resource augmentation. We can see that to achieve 95% success rate for the sLLF algorithm, only 2% increase in resources is required. Table 3 shows that the minimum $\epsilon$ resource augmentation required for each algorithm to achieve 100% feasibility for all instances is smallest for the LLF and sLLF algorithms. Other algorithms (EDF, ES, REP and OLP) require significantly larger augmentation compared to the sLLF algorithm. While the OLP algorithm has high success rate without augmentation (Fig. 1), it requires much more resource augmentation to achieve 100% success rate (Table 3).

### 5 CONCLUSION

We have formulated EV charging as a feasibility problem that meets all EVs’ energy demands before departure under charging rate constraints and total power constraint, and proposed an online algorithm, the sLLF algorithm, that decides on the current charging rates based on only the information up to the current time. We characterize the performance of the sLLF algorithm analytically and numerically. Numerical experiments with real-world data show that it has significantly higher rate of generating feasible EV charging than several other common EV charging algorithms. By finding feasible EV charging schedules using only a small augmentation to the absolute minimum resource needed for offline feasibility, our proposed algorithm (sLLF) can significantly reduce infrastructural cost for EV charging facilities.

---

|            | REP | ES  | EDF | LLF | OLP | sLLF |
|------------|-----|-----|-----|-----|-----|------|
| power      | 4.61| 3.65| 1.39| 0.07| 0.28| 0.07 |
| power and rate | 4.61| 3.24| 0.54| 0.05| 0.28| 0.05 |

Table 3: Minimum $\epsilon$ resource augmentation for online feasibility for all instances. The LLF and sLLF algorithms have the smallest $\epsilon$ among algorithms considered.
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APPENDIX

A PROOFS FOR SECTION 2

Proof (Proposition 2.5). Observe that feasibility is equivalent with the condition
\[ e_i(d_i) = 0, \quad i \in V. \] (18)

Condition (5) implies that for any EV \( i \in V \), \( e_i(d_i) = -e_i(d_i)/\bar{f}_i \geq 0 \), which yields \( e_i(d_i) = 0 \). Next, notice that the laxity of EV \( i \) is monotonically decreasing at \( t < d_i \) and constant at \( t \geq d_i \), i.e.,
\[ \ell_i(t) = \ell_i(t+1) + 1 - r_i(t)/\bar{r}_i \geq \ell_i(t+1), \quad t < d_i \] (19)
\[ \ell_i(t) = \ell_i(t+1) \quad t \geq d_i \] (20)

Therefore, condition (21) implies that \( \ell_i(t) \geq 0 \) at any time \( t \in T \). □

Proof (Corollary 2.6). From constraint \( \sum_{i \in T} r_i(t) \leq e_i \) and \( f \) being strictly increasing, the objective function satisfies
\[ \sum_{i \in V} f(\ell_i(T)) \leq \sum_{i \in V} f(0). \]

Moreover, if an instance \( I \) is offline feasible, then there exists some charging rates that achieve \( \ell_i(T) = 0, \forall i \in V \). Since the laxity is monotonically decreasing at any \( t \in T \), such charging rates also satisfy \( \ell_i(t) \geq 0, i \in V, t \in T \). From Proposition 2.5, \( \ell_i(t) \geq 0, i \in V, t \in T \) implies that algorithm (6) is feasible on instance \( I \). Therefore, the cost \( \sum_{i \in V} f(\ell_i(T)) = \sum_{i \in V} f(0) \) is attainable. □

Proof (Proposition 2.5). Observe that feasibility is equivalent with the condition
\[ e_i(d_i) = 0, \quad i \in V. \] (21)

Condition (5) implies that for any EV \( i \in V \), \( e_i(d_i) = -e_i(d_i)/\bar{f}_i \geq 0 \), which yields \( e_i(d_i) = 0 \). Next, notice that the laxity of EV \( i \) is monotonically decreasing at \( t < d_i \) and constant at \( t \geq d_i \), i.e.,
\[ \ell_i(t) = \ell_i(t+1) + 1 - r_i(t)/\bar{r}_i \geq \ell_i(t+1), \quad t < d_i \] (22)
\[ \ell_i(t) = \ell_i(t+1) \quad t \geq d_i \] (23)

Therefore, condition (21) implies that \( \ell_i(t) \geq 0 \) at any time \( t \in T \). □

Proof (Corollary 2.6). From constraint \( \sum_{i \in T} r_i(t) \leq e_i \) and \( f \) strictly increasing, the objective function satisfies
\[ \sum_{i \in V} f(\ell_i(T)) \leq \sum_{i \in V} f(0). \]

If an instance \( I \) is offline feasible, then there exists certain charging rates that achieve \( \ell_i(T) = 0, \forall i \in V \), which yields \( \sum_{i \in V} f(\ell_i(T)) = \sum_{i \in V} f(0) \). Since the laxity is monotonically decreasing at any \( t \in T \), such charging rates also satisfy condition (5). From Proposition 2.5, condition (5) implies that algorithm (6) is feasible on instance \( I \). □
Proof (Proposition 2.7). From the Karush-Kuhn-Tucker (KKT) conditions for the optimization problem (7),

\[
\begin{align*}
  r_i(t) &\geq 0 & i &\in \mathcal{V}_t & (24) \\
  r_i(t) &\leq \min(e_i(t), r_i) & i &\in \mathcal{V}_t & (25) \\
  \sum_{i\in\mathcal{V}_t} r_i(t) &\leq P(t) & i &\in \mathcal{V}_t & (26) \\
  f'(l_i(t+1) + \hat{\lambda}_i - \hat{\lambda}_i + v) &\geq 0 & i &\in \mathcal{V}_t & (27) \\
  \hat{\lambda}_i &\geq 0 & & & (28) \\
  \hat{\lambda}_i r_i(t) &\geq 0 & i &\in \mathcal{V}_t & (29)
\end{align*}
\]

where \(\hat{\lambda}_i, \hat{\lambda}_i, v\) are the dual variables for constraints (24), (25), (26), respectively. We consider three mutually exclusive cases: \(r_i(t) = 0\), \(r_i(t) \in (0, \min(e_i(t), r_i))\), or \(r_i(t) = \min(e_i(t), r_i)\). When \(r_i(t) = 0\), \(\hat{\lambda}_i = 0\) and

\[
r_i(t)/r_i = f'^{-1}(-v) - l_i(t) + 1 - \hat{\lambda}_i \geq f'^{-1}(-v) - l_i(t) + 1,
\]

where the inverse of \(f'\) exists since \(f')\) is strictly concave, strictly increasing, and twice continuously differentiable. When \(r_i(t) \in (0, \min(e_i(t), r_i))\), then from (29) (complementary slackness), \(\hat{\lambda}_i = \hat{\lambda}_i = 0\). Substituting \(\hat{\lambda}_i = \hat{\lambda}_i = 0\) into (27), we obtain

\[
l_i(t) - 1 + r_i(t)/r_i = f'^{-1}(-v)
\]

When \(r_i(t) = \min(e_i(t), r_i)\), \(\hat{\lambda}_i = \hat{\lambda}_i = 0\) and

\[
r_i(t)/r_i = f'^{-1}(-v) - l_i(t) + 1 + \hat{\lambda}_i \geq f'^{-1}(-v) - l_i(t) + 1.
\]

Combining (30)-(32), we obtain

\[
r_i(t) = \frac{r_i(t)(f'^{-1}(-v) - l_i(t) + 1)_{0}^{\min(r_i, e_i(t))}}{(33)}
\]

Because the same value of \(f'^{-1}(-v)\) is shared for all EVs at the charging station, we can define an variable \(L(t) = f'^{-1}(-v)\). Since the optimal solution is attained at the boundary \(\sum_{i\in\mathcal{V}_t} r_i(t) = \min\left(P(t), \sum_{i\in\mathcal{V}_t} \min(r_i, e_i(t))\right)\), we obtain the optimal solution (8)-(9).

\[\Box\]

Proof (Lemma 2.8). First notice that, by Definition 2.4, it satisfies the following relation:

\[
\ell_i(t) - 1 \leq l_i(t+1) \leq \ell_i(t), \quad i \in \mathcal{V}.
\]

First, consider the case \(r_i(t) = 0\). The evolution of \(\ell_i\) satisfies

\[
\ell_i(t+1) = \begin{cases} 
\ell_i(t) - 1 & t < d_i \\
\ell_i(t) & t \geq d_i.
\end{cases}
\]

Suppose that \(t < d_i\), combining (10) and (34) gives

\[
\ell_i(t+1) \geq \ell_i(t) - 1 \geq \ell_i(t+1) = \ell_i(t+1),
\]

which contradicts (11). Therefore, \(t \geq d_i\), and (12) follows.

Next, consider the case \(r_i(t) \neq 0\). Non-zero \(r_i(t)\) implies \(t < d_i\). If \(t < d_i\), (10) and (11) jointly implies

\[
\frac{r_i(t)}{r_i(t)} < \frac{r_i(t)}{r_i(t)}.
\]

Under the sLLF algorithm, (36) happens only when \(e_i(t) = r_i(t)\), which leads to \(e_i(t+1) = 0\). If \(t \geq d_i\), then \(\ell_i(t+1) = \ell_i(t) \geq \ell_i(t) \geq \ell_i(t+1)\), which contradicts (11). Therefore, (13) follows.

\[\Box\]

B PROOFS FOR SECTION 3

Notations. Next we introduce some notation that will be used later. Denote by \(A_t = \{i \in \mathcal{V} : a_i \leq t\}\) the set of EVs that have arrived by time \(t\), \(D_t = \{i \in A_t : d_i \leq t\} \cup \{i \in A_t : e_i(t) = 0\}\) the set of EVs that have either departed or finished charging by time \(t\), \(V_t = \{i \in A_t : a_i \leq t < d_i\}\) the set of EVs remaining in the charging station at time \(t\), and \(U_t = \{i \in \mathcal{V} : e_i(t) > 0\}\) the set of EVs with unfulfilled energy demand at the beginning of time slot \(t\), where we reload the notation and use \(e_i(t)\) to denote the remaining energy demand of EV \(i\) at the beginning of time slot \(t\). In addition, denote by \(A_{[t_1,t_2]} = \{i \in \mathcal{V} : a_i \in [t_1, t_2]\}\) the set of EVs that arrive during time interval \([t_1, t_2]\), \(t_1, t_2 \in T\). See Table 4 for a summary of notation.

Denote the total energy supply to EVs in set \(S \subseteq \mathcal{V}\) during the interval \([t_1, t_2]\) under the (feasible) offline algorithm by

\[
\Psi^*_{[t_1,t_2]}(S; I) := \sum_{i \in S} r_i(t),
\]

and the total energy supply to EVs in set \(S \subseteq \mathcal{V}\) during the interval \([t_1, t_2]\) under the \(e\)-power augmentation (or \(e\)-augmentation) by

\[
\Psi^e_{[t_1,t_2]}(S; I) := \sum_{i \in S} r_i(t).
\]

We use superscript \('\) to indicate variables under an (feasible) offline algorithm with original power limit \(P(t)\) and maximum charging rates \(r_i\), and use superscript \('e\) to indicate variables under the augmented resources.

Table 4: Additional Notation

| Notation               | Description                              |
|-----------------------|------------------------------------------|
| \(A_t\)               | set of EVs arriving by time \(t\)        |
| \(A_{[t_1,t_2]}\)     | set of EVs arriving during interval \([t_1, t_2]\) |
| \(D_t\)               | set of EVs either departed or finished by time \(t\) |
| \(V_t\)               | set of EVs at the charging station at time \(t\) |
| \(U_t\)               | set of EVs unfulfilled charging at time \(t\) |
| \(\Psi_{[t_1,t_2]}(S; I)\) | total energy supplied to the set of EVs \(S\) during the interval \([t_1, t_2]\) under instance \(I \) |
| \(\Psi^e_{[t_1,t_2]}(S; I)\) | total energy supplied to \(S\) during \([t_1, t_2]\) under instance \(I \) with \(e\)-augmented resources |

B.1 Preliminaries

B.1.1 The infeasibility condition. For a charging instance \(I = (a_i, d_i, e_i, r_i; P(t))_{i\in\mathcal{V}, t\in T}\) that is not online feasible under the sLLF algorithm, there are times when some EV has negative laxity. Denote by \(t_\ast\) the earliest among such times. Let \(\mathcal{F} = \{i \in A_{t_\ast} : \ell_i(t_\ast) < 0\}\) denote the set of EVs arriving at the charging station by time \(t_\ast\) that have negative laxity, \(S_1 = \{i \in A_{t_\ast} : \ell_i(t_\ast) \geq 0 \text{ and } d_i \leq t_\ast\}\) the set of EVs with non-negative laxity that depart by time \(t_\ast\), and \(S_2 = \{i \in A_{t_\ast} : \ell_i(t_\ast) \geq 0 \text{ and } d_i > t_\ast\}\) the set of EVs with non-negative laxity that remain at the charging station at time \(t_\ast\). Sets \(\mathcal{F}, S_1\) and \(S_2\) are mutually exclusive, and \(A_{t_\ast} = \mathcal{F} \cup S_1 \cup S_2\).

Lemma B.1. When the sLLF algorithm is used on instance \(I\), for any \(i \in S_2\) and \(j \in \mathcal{F}\), the laxities satisfy

\[
\ell_i(t) > \ell_j(t), \quad t \in [\max(a_i, a_j), t_\ast].
\]
Proof (Lemma B.1). By the construction of $S_2$, relation (37) holds at $t = t_\ast$. By Lemma 2.8, a necessary condition for the inequality in (37) to flip at some time $t + 1 \leq t_\ast$ is (for $13$) to hold for EV $i$. We show below that this condition cannot hold for any EV in $\mathcal{F}$ or $S_1$. For EVs in $\mathcal{F}$, condition $e_i(t + 1) = 0$ in $\mathcal{F}$ cannot happen because negative laxity at some time implies the energy demand will not be fulfilled. For EVs in $S_1$, (37) holds only after $e_i(t + 1) = 0$ when they have energy demand fulfilled at time $t + 1$. Consequently, condition (37) holds for all $t \in [\max(a_i, a_j), t_\ast]$. Notice that the sLLF algorithm prioritizes EVs with smaller laxity so the presence of EVs with strictly greater laxity will not impact the charging of the EVs with smaller laxity. Let $\hat{V} = \mathcal{F} \cup S_1$, and use it to define another instance that does not contain the EVs in $S_2$: $\hat{I} = [a_i, d_i, e_i, r_i; P(t)]_{t \in \hat{V}, t \in \mathcal{T}}$. Following Corollary B.2 can be obtained as a consequence of Lemma B.1.

**Corollary B.2.** Regardless of the actual instance being $I$ or $\hat{I}$, the EVs in $\mathcal{F}$ are charged in exactly the same way under the sLLF algorithm by time $t_\ast$.

**B.1.2 The infeasibility condition of an augmented instance.** Let $I$ be an EV charging instance that are offline feasible. Consider using the sLLF algorithm with the $e$ augmented resources (power augmentation $p^\text{aug}(t) = eP(t)$, or power and rate augmentation $p^\text{aug}(t) = eP(t), r^\text{aug}(t) = r_i(t)$). Now, the above result from previous section, we derive a condition for the sLLF algorithm being infeasible on some online feasible instance, which holds for both power augmentation and power and rate augmentation.

Since the EVs in $S_1$ are fully charged by time $t_\ast$, under both the sLLF algorithm and the offline algorithm, we have

$$\Psi^e_{[0; t_\ast]}(S_1; \hat{I}) = \Psi^e_{[0; t_\ast]}(S_1; I),$$

where $S_1, \mathcal{F}$ are the sets defined above under the sLLF algorithm using augmented resources. Notice that $\ell_i(t) \geq 0, \forall t \in \mathcal{F}$ is a necessary condition for EV $i$ to be feasible. Thus, for $EV i \in \mathcal{F}$, the offline algorithm must maintain $\ell_i(t) \geq 0$. Given that laxity $\ell_i(t)$ is strictly decreasing in the remaining energy demand $e_i(t)$, the total energy fulfilled by $t_\ast$, under the offline algorithm must be strictly greater than that with the sLLF algorithm, i.e.,

$$\Psi^e_{[0; t_\ast]}((i); \hat{I}) < \Psi^e_{[0; t_\ast]}((i); I), \quad i \in \mathcal{F}$$

from which

$$\Psi^e_{[0; t_\ast]}(\hat{F}; \hat{I}) < \Psi^e_{[0; t_\ast]}(\mathcal{F}; I).$$

Recall that $\mathcal{F} \setminus \mathcal{F}_2$. Combining (38) and (40), we have

$$\Psi^e_{[0; t_\ast]}(\hat{V}; \hat{I}) < \Psi^e_{[0; t_\ast]}(\hat{V}; I).$$

**Corollary B.2 implies**

$$\Psi^e_{[0; t_\ast]}((i); \hat{I}) = \Psi^e_{[0; t_\ast]}((i); \hat{I}), \quad i \in \mathcal{F},$$

$$\Psi^e_{[0; t_\ast]}(\hat{V}; \hat{I}) = \Psi^e_{[0; t_\ast]}(\hat{V}; \hat{I}).$$

Further, since the charging instance $I$ is offline feasible, its sub-instance $\hat{I}$ is offline feasible too. Similar to equations (38)-(41), we can show that

$$\Psi^e_{[0; t_\ast]}(S_1; \hat{I}) = \Psi^e_{[0; t_\ast]}(S_1; I),$$

$$\Psi^e_{[0; t_\ast]}((i); \hat{I}) < \Psi^e_{[0; t_\ast]}((i); I), \quad i \in \mathcal{F},$$

$$\Psi^e_{[0; t_\ast]}(\hat{F}; \hat{I}) < \Psi^e_{[0; t_\ast]}(\mathcal{F}; I),$$

$$\Psi^e_{[0; t_\ast]}(\hat{V}; \hat{I}) < \Psi^e_{[0; t_\ast]}(\hat{V}; I).$$

**B.2 Proof of Theorem 3.3.** Consider the use of the sLLF algorithm on an offline feasible instance $I = [a_i, d_i, e_i, r_i; P(t)]_{t \in \mathcal{V}, t \in \mathcal{T}}$ under $e$-powered resources. Let

$$n = (1 + e) \frac{p^\text{min}}{P^\text{max}}$$

For $m \leq n$, we define the earliest time to charge at a power greater than $mp^\text{max}$ for the rest of the time until $t_\ast$ as

$$t_m = \min \left\{ t \in \mathcal{T} : \sum_{j \in \mathcal{V}_i} \min(r_j, e_j(t_\ast)) \geq mp^\text{max}, t \in [t_\ast, t_\ast] \right\}.$$  

Let $T_m = [t_m - 1, t_m]$ and $\hat{T}_m = [t_m, t_\ast]$ and denote their lengths by $|T_m|$ and $|\hat{T}_m|$. We first present a lemma that is used in the proof of Theorem 3.3.

**Lemma B.3.** For any integer $i \leq n - 1$, the following two relations hold:

$$\Psi^e_{[0; t_i]}(A_{T_i}; \hat{I}) - \Psi^e_{[0; t_i]}(A_{T_i}; I) > P^\text{max}|T_i|,$$

$$|T_i| > |\hat{T}_{i+1}|.$$  

**Proof (Lemma B.3).** On one hand, from definition (49),

$$\sum_{j \in \mathcal{V}_{(i+1)-1}} \min(r_j, e_j(t_\ast - 1)) < (i - 1)P^\text{max}.$$  

This implies that the EVs that have arrived before $t_{i-1}$ are charged at a total power of at most $(i - 1)P^\text{max}$ at $t_{i-1}$ and after. On the other hand, from definition (49), the total power supply is at least $iP^\text{max}$ during the interval $T_{i+1} = [t_i, t_{i+1}]$. Therefore, the total charging power to the EVs that arrive after $t_{i-1}$ is at least $P^\text{max}$ during $T_{i+1}$. Since the offline algorithm can only use a power of at most $P^\text{max}$, for the EVs that arrive after $t_{i-1}$ we obtain

$$\Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; \hat{I}) - \Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; I) < \Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; \hat{I}) - \Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; I).$$

The same argument can be applied to the interval $\hat{T}_{i+1} = [t_{i+1}, t_\ast]$. From definition (49), the total charging power is at least $(i + 1)P^\text{max}$ during $\hat{T}_{i+1}$. Therefore, during $\hat{T}_{i+1}$, the total charging power to the EVs that arrive after $t_{i-1}$ is at least $2P^\text{max}$. Since the offline algorithm can only use a power of at most $P^\text{max}$, the total energy supply to EVs in $\hat{T}_{i+1}$ under the augmented resources is greater than that without augmented resources, i.e.,

$$0 < \Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; \hat{I}) - \Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; I)$$

$$< \Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; \hat{I}) - \Psi^e_{[0; t_{i+1}]}(A_{T_{i+1}}; I) - P^\text{max}|T_{i+1}|.$$
Combining (52)-(53), we have
\[ \Psi^e_{[0:t]}(A_{T_i}; \tilde{I}) = \Psi^e_{[0:t]}(A_{T_i}; \tilde{I}) > P_{max}|\tilde{T}_i+1|, \] (54)
Since the set \( A_{T_i} \) is identical to the subset of \( A_{T_i} \) that contains only the EVs that have arrived by \( t_i \),
\[ \Psi^e_{[0:t]}(A_{T_i}; \tilde{I}) - \Psi^e_{[0:t]}(A_{T_i}; \tilde{I}) = \Psi^e_{[0:t]}(A_{T_i}; \tilde{I}) \] (55)
Combining (54) and (55) leads to relation (50).

Finally, as all EVs in \( A_{T_i} \) arrives after \( t_{i-1} \), during \( T_i \) the offline algorithm can charge a total energy of at most \( |T_i|P_{max} \), we obtain
\[ \Psi^e_{[0:t]}(A_{T_i}; \tilde{I}) - \Psi^e_{[0:t]}(A_{T_i}; \tilde{I}) \leq |T_i|P_{max}. \]
which together with (50) leads to (51). □

**Proof. (Theorem 3.3).** Suppose that there exists an offline feasible instance \( \tilde{I} = \{ a_i, d_i, e_i, \tilde{r}_i, P(t) \} \) such that the sLLF algorithm is not feasible with \( \epsilon \)-power augmented resources. Then, from Appendix B.1, there exists another offline feasible instance \( \tilde{I} = \{ a_i, d_i, e_i, \tilde{r}_i, P(t) \} \) such that
\[ \Psi^e_{[0:t]}(\tilde{I}; \tilde{I}) < \Psi^e_{[0:t]}(\tilde{I}; \tilde{I}), \ i \in \mathcal{V}. \] (56)
When \( m = 1 \), we obtain \( \sum_{j \in \mathcal{V}_{m-1}} \min(\tilde{r}_j, \tilde{r}_j(t_1-1)) < P_{max}. \) Let \( S = \{ i \in A_{T_1} : e_i(t_1) > 0 \} \) denote the set of EVs that arrive during \( T_1 \) and have not yet been fully charged by \( t_1 \). Because the number of EVs is upper bounded by \( P_{max}/r_{min} \) from (49), and the EVs in \( A_{T_1} \) are all fully charged,
\[ P_{max}(\tilde{T}_1) \leq \Psi^e_{[0:t]}(A_{T_1}; \tilde{I}) = \Psi^e_{[0:t]}(S; \tilde{I}) \leq X/r_{min}. \]
This leads to
\[ |\tilde{T}_1| < X/r_{min}. \] (57)
At time \( t < t_{m-1} \), we have
\[ \sum_{j \in \mathcal{V}_{m-1}} \min(\tilde{r}_j, \tilde{r}_j(t_{m-1} - 1)) < (m-1)P_{max}, \]
which implies that there are at most \((m-1)P_{max}/r_{min}\) EVs with unfulfilled energy demand by time \( t_f \). Meanwhile, at time \( t \geq t_m \), we have
\[ \sum_{j \in \mathcal{V}_{m-1}} \min(\tilde{r}_j, \tilde{r}_j(t_m)) \geq mP_{max}, \]
which implies that there are at least \( mP_{max}/r_{min} \) EVs with unfulfilled energy demand during \( T_{m-1} \). Therefore, the number of EVs that arrive during \( [t_{m-1}, t_m] \) is greater than the following:
\[ mP_{max}/r_{min} - (m-1)P_{max}/r_{min} \geq P_{max}/r_{min}. \] (58)
Since the inter-arrival periods of EVs are at least \( N \), the length of \( T_{m-1} \) satisfies
\[ |\tilde{T}_{m-1}| \geq P_{max}N/r_{min}. \] (59)
Now, consider the following recursion:
\[ |\tilde{T}_1| = |\tilde{T}_2| + |\tilde{T}_3| \]
\[ \geq |\tilde{T}_4| + |\tilde{T}_5| + 2|\tilde{T}_6| \]
\[ \geq 3|\tilde{T}_7| + 2|\tilde{T}_8| + 5|\tilde{T}_9| + 3|\tilde{T}_{10}| \]
\[ \geq \cdots \geq f_{k-2}|\tilde{T}_{m-1}| + f_{k-3}|\tilde{T}_{m}|. \]
where \( f_k \) is the Fibonacci sequence defined by \( f_1 = 1, f_2 = 1 \) and \( f_k = f_{k-1} + f_{k-2} \) for \( k \geq 3 \). From the above, we have
\[ |\tilde{T}_2| > f_{m-2}|\tilde{T}_{m-1}|. \]
Combining equations (57)-(59) gives
\[ X/r_{min} > |\tilde{T}_2| > f_{m-2}|\tilde{T}_{m-1}| > f_{m-2}P_{max}N/r_{min} \] (60)
From \( m \leq n \) for \( n \) defined in (48), we obtain
\[ \left( 1 + \epsilon \right) \frac{P_{min}}{P_{max}} - 2 = m - 2 \]
\[ \log \sqrt{2N}/N_{max} + 1/2 \]
which gives \( (1+\epsilon)P_{min}/P_{max} < \log \sqrt{2N}/N_{max} + 1/2 \). □

**Corollary 3.4.** Suppose that there exists an offline feasible instance \( \tilde{I} \) that is not feasible under the sLLF algorithm with \( \epsilon \)-power augmentation. Using the same argument of the proof for Theorem 3.3, we obtain inequality (60). However, from assumption
\[ f_1P_{max}N/r_{min} \leq X/r_{min}, \] which contradicts (60). □

**B.3 Proof of Theorem 3.7**

**Proof. (Theorem 3.3).** Suppose that there exists an instance \( \tilde{I} = \{ a_i, d_i, e_i, \tilde{r}_i, P(t) \} \) such that the sLLF algorithm is not feasible with \( \epsilon \)-augmented resources. We then have equation (47), repeated here for convenience:
\[ \Psi^e_{[0:t]}(V; \tilde{I}) < \Psi^e_{[0:t]}(V; \tilde{I}) \] for another instance \( \tilde{I} = \{ a_i, d_i, e_i, \tilde{r}_i, P(t) \} \).

Let \( S(V) \) be the set of EVs in the instance \( \tilde{I} \) that receive strictly less energy under the online algorithm than under the offline algorithm by some time \( t \) at which \( \Psi^e_{[0:t]}(V; \tilde{I}) < \Psi^e_{[0:t]}(V; \tilde{I}) \):
\[ S(V) = \{ i \in V : \exists t \in T \ s.t. \ \Psi^e_{[0:t]}(i; \tilde{I}) - \Psi^e_{[0:t]}(i; \tilde{I}) \} \] & \[ \Psi^e_{[0:t]}(V; \tilde{I}) < \Psi^e_{[0:t]}(V; \tilde{I}). \]
\[ \Psi^e_{[0:t]}(V; \tilde{I}) \leq \Psi^e_{[0:t]}(V; \tilde{I}). \]
In view of (47), \( S(V) \neq \emptyset \). Consider EV \( j = \arg \min_{i \in S(V)} a_i \) that arrives the earliest among those in \( S(V) \). There exists a time \( t \in [a_i, d_i] \) such that
\[ \Psi^e_{[0:t]}(i; \tilde{I}) < \Psi^e_{[0:t]}(i; \tilde{I}), \] (61)
\[ \Psi^e_{[0:t]}(V; \tilde{I}) < \Psi^e_{[0:t]}(V; \tilde{I}). \] (62)
Notice that \( \Psi^*_{[a_j, t]}(V; \tilde{I}) < \Psi^*_{[a_j, t]}(V; \tilde{I}) \) can only happen when there is another EV in \( S(V) \) that arrives before EV \( j \), which however contradicts the definitions of \( S(V) \) and \( j \). So,
\[
\Psi^*_{[a_j, t]}(V; \tilde{I}) \geq \Psi^*_{[a_j, t]}(V; \tilde{I}),
\]
which implies
\[
\Psi^*_{[a_j, t]}(\tilde{V}; \tilde{I}) < \Psi^*_{[a_j, t]}(\tilde{V}; \tilde{I}). \tag{63}
\]

Now, let us take a look at the energy demand fulfilled during the interval \([a_j, t]\) under the sLF algorithm with \( \epsilon \)-augmented resources. Define the overloaded times
\[
T_o = \left\{ t \in [a_j, t] : \sum_{i \in V} r_i(t) = (1 + \epsilon)P(t) \right\}
\]
and underloaded times
\[
T_u = \left\{ t \in [a_j, t] : \sum_{i \in V} r_i(t) < (1 + \epsilon)P(t) \right\},
\]
we have \(|T_o| + |T_u| = t + 1 - a_j\). The total energy demand fulfilled during the overloaded period is lower bounded by \(|T_o|/(1 + \epsilon) \) \( \min_{\tau \in [a_j, d_j]} P(\tau) \), while that during the underloaded period is at least \(|T_u|/(1 + \epsilon) \) \( \bar{\tau}_j \). Hence, the total and individual energy demands fulfilled during \([a_j, t]\) are lower bounded by
\[
(1 + \epsilon) \left( |T_o| \bar{\tau}_j + |T_u| \right) \min_{\tau \in [a_j, d_j]} P(\tau) \leq \Psi^*_{[a_j, t]}(\tilde{V}; \tilde{I}), \tag{64}
\]
\[
(1 + \epsilon)|T_u| \bar{\tau}_j \leq \Psi^*_{[a_j, t]}(j; \tilde{I}). \tag{65}
\]

Next, let us take a look at the energy demand fulfilled during the interval \([a_j, t + 1]\) by the offline algorithm without resource augmentation. The total energy fulfilled is upper bounded by
\[
\Psi^*_{[a_j, t]}(\tilde{V}; \tilde{I}) \leq (t + 1 - a_j) \max_{\tau \in [a_j, d_j]} P(\tau), \tag{66}
\]
and the energy fulfilled to EV \( j \) is upper bounded by
\[
\Psi^*_{a_j, j}(t) \leq (t + 1 - a_j) \bar{\tau}_j. \tag{67}
\]

By equations (61), (65) and (67), we have
\[
|T_u|(1 + \epsilon) < (t - a_j + 1). \tag{68}
\]

By equations (63) (64) and (66), we have
\[
(1 + \epsilon)(|T_u| \bar{\tau}_j + |T_o| \min_{\tau \in [a_j, d_j]} P(\tau))
< (t + 1 - a_j) \max_{\tau \in [a_j, d_j]} P(\tau).
\]

Combining equation (68) becomes
\[
(|T_o| + |T_u|)(1 + \epsilon) \min_{\tau \in [a_j, d_j]} P(\tau)
< (t - a_j + 1) \left( \max_{\tau \in [a_j, d_j]} P(\tau) + \min_{\tau \in [a_j, d_j]} P(\tau) - \bar{\tau}_j \right).
\]

Notice that \(|T_o| + |T_u| = t + 1 - a_j\), the above inequality leads to
\[
\epsilon < \max_{\tau, \tau_i \in [a_j, d_j]} \frac{P(\tau_i)}{P(\tau)} - \min_{\tau \in V, \tau_i \in [a_j, d_j]} \bar{\tau}_i.
\]