Mathematical Description of the Flow Dynamics of Sand, Crude Oil and Water in an Oil Well

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Abstract

The flow of particulates and fluids from a reservoir has been described using different models. However, obtaining one model that has the capacity to simulate all the phenomena existing in a horizontal oil well may be very difficult. The model described here is very flexible and has the capacity to integrate the essential parameters necessary to give a detailed description of the flow phenomenon. Furthermore, clear descriptions of the flow dynamics, threshold and critical velocity estimation of the components that make up the phases have been attempted in this paper. Within the model’s high level of accuracy, the unsafe limits for incipient sand deposition and bed load transport were estimated to be drag forces of $8.938 \times 10^{-8} - 10^{-10}$ kgm/s$^2$ as well as threshold and critical velocities of 0.0445 and 0.0134 m/s respectively for Reynolds number in the region of 300-1000. The findings of this study will aid upstream petroleum engineers in avoiding situations that will lead to critical/laminar flows which in turn cause sand bed formation or bed load transport which can in turn lead to pipeline blockage and loss of productivity.

Keywords: Concentration distribution; Flow dynamics; Threshold velocity; Critical velocity; Three-phase flow

1. Introduction

Pipelines conveying mixtures of crude oil, sand and associated water in oil wells have proven to be quite useful and promising till date. Often times two or more pipes joined together help to convey the reservoir fluid to the tubing head with the aid of pumps that supply the required energy to drive the fluid. In order that the mixture keeps flowing, the flow conditions must be such that the pressure drop is kept appreciably low so as to keep the mixture moving within the pipe [1-2]. An understanding of the principles of fluid mechanics and flow dynamics are essential in giving a clear description of the flow characteristics of the fluid’s rheology, concentrations, pressure, velocity and force distribution of the three components at different times and at the specified flow regimes. Evidence has shown that all three phases may behave as a continuum during turbulent flows, but will not move with the same velocity as the carrier fluid [3]. However, it is expected that the lightest of the three components of the mixture will flow with the highest velocity. Furthermore, based on their respective densities, the components will move with a fraction of the velocity of the carrier fluid. Capecelatro and Desjardins [4] carried out numerical evaluations of the complex dynamics of multiphase systems flowing through a slurry pipeline with intent on conditions above and below the critical deposit velocity.

Conditions leading to stratified and non-stratified flows for different flow regimes were investigated, and the model predictions were found to be in agreement with data gathered experimentally. They also asserted that for slurry flows below the critical particle settling regime/velocity, three layers exist namely, a bed-layer, a collisional shear flow layer as well as a freely-suspended particle layer. Research works have been carried out for fluid-particle transport where the solids are not sand. However, the findings have helped to gain deeper understanding on the subject. Previous attempts to model the laminar flow of sand-oil mixtures in horizontal oil wells were successfully achieved by Doan et al. [5-6]. Hahsemi et al. [7] determined particles velocities in highly concentrated slurry flows in a pipeline. Concentrations and velocities were measured for varying solid concentrations in water. From their results, they affirmed that solid concentrations fluctuations were high and localized near the pipe wall and tend to increase with increased slurry
velocity. They also added that the solids concentration fluctuations were largely contributed by the fluid-particle
turbulence rather than particle-particle transport.

Kai et al. [8] established a vibration technology detection of sand particles using time-frequency analysis and a
sand frequency digital band filter. Fluid-particle movements were tracked using the signals from a vibrator. The
sand particle size was about 325 mesh and the sand content was 0.3 % wt/wt of the mixture. From the findings,
the power spectrum amplitudes/peaks had a good correlation with sand-volume fractions in the multiphase
system. The work of Messa et al. [9] involves numerical estimations of the conditions that justify the safe
transportation of suspended particles in a slurry pipeline. Parameters of interest for the two-fluid model include
pressure gradient, solid-volume-fraction distribution and velocity. Despite the ability of the model to make
reasonable predictions, deviations were seen by comparing the model predictions with experimental data. The
model has the capacity to account for the mechanisms responsible for turbulent dispersion, interface and
mechanical friction. Particle sizes of 50 – 150 μm were considered here while the operational superficial
velocities were between 1 and 7 m/s with sand concentration maintained at 40%.

Narayan [10] discussed the simulation of particle and sand transport in pipelines. The work involved the use of
Transport phenomena Analysis Tool (TransAT) to predict particle flows in pipes alongside black powder
deposition in petroleum pipelines. The model combines the 3D transient mode of the Navier Stokes equations
with the Eulerian Lagrangian model. For the sand transport phase, the solution is based on the Eulerian approach
where the sand phase is characterized by a concentration field and sand settling velocity while taking into
cognizance the conditions that lead to sand re-suspension. Patankar et al. [11] carried out an investigation on the
lift-off of a particle in Newtonian and viscoelastic fluids. Circular particles were used where they added that a
particle denser than the fluid tends to be driven towards the bottom of a channel by poiseuille flow. The Sanni et
al. [12] model, is one that incorporates the effect of eddies in the flowing stream thus expanding the scope of its
application. Evaluations of the velocity of solid particles such as FeCO₃, FeO, FeS, Fe₃O₄, sand, weld spatter
and salt in horizontal pipes were carried out by Smart [13]. The findings showed that, for larger diameter pipes,
higher fluid velocities were required and the velocities of the iron-based compounds were very close due to their
slight difference in densities. Stevenson et al. [14] also investigated the movement of a small particle within the
viscous boundary layer region of a pipe wall.

After careful study, there seems to be substantial literature for particulate slurry flows but none clearly discusses
the flow dynamics of the three components in terms of threshold and critical velocity estimation for sand-
settling as well as the conditions leading to the increase in sand drag forces in relation to the formation of a sand
dune/fixed bed as well as those responsible for bed load transport, hence, this paper seeks to contribute on the
latter based on real-life data obtained from appropriate upstream flow facilities.

2. Data Generation

Data was generated using upstream flow facilities for an oil well in an oil field. It shows the different flow
parameters measured during the transport process. The flowing tubing pressure (FTP), choke size, barrels of oil
produced per day (BOPD), barrels of liquid per day (BLPD), gas flow rate, the amount of sand in parts per
thousand barrel (PTB) and base sediment and water (BSW) were all measured for the same adjustment of the
choke size. This was so as to see the possible variations that arise in the system when the same choke size is
maintained; see

Figure 1 for an illustration of the magnitude of the FTP, BOPD, BLPD and gas flow rate.

The model described in Sanni et al. [12] was modified for application considering the case of a three-phase
system of oil, water and sand flowing from a reservoir. The mass equations comprise mainly of volume fraction
time dependent compositions of the components alongside their velocities which were solved using finite
difference approach while the momentum balance for the system was used to estimate the pressure of the
components in the flowing stream.
2.1 Sand characteristics

According to International Standard Organization (ISO), fine sand is in the range of 0.063 – 0.2 mm, medium size sand ranges from 0.2 – 0.63 mm and coarse sand size lies between 0.63 – 2.0 mm while according to US standard (Krumbein scale), very fine sand sizes are from 0.0625 – 0.125 mm diameter, fine sand size lies within 0.125 – 0.25 mm, medium sand size is in the range of 0.25 – 0.5 mm, coarse sand size is from 0.5 – 1.0 mm while very coarse sand ranges in size from 1 – 2 mm. The US Krumbein scale was adopted for measuring the average sand size which was 0.375 mm with hardness of about 800 Vickers which was measured using a pycnometer. Sand density was measured as 1705.4 kg/m$^3$. Spherical medium sand particles of average size were considered for the simulation. The particle diameter is 0.325 mm (0.000375 m) and the sand particles were assumed to be of uniform size. This makes it easier to estimate the average / mean mix velocity at different points along the pipe.

$\frac{\partial}{\partial t} (\phi) + \frac{\partial}{\partial z} (\phi w_f) = 0$ \hspace{2cm} (1)

$\frac{\partial}{\partial t} (\sigma) + \frac{\partial}{\partial z} (\sigma w_f) = 0$ \hspace{2cm} (2)

$\frac{\partial}{\partial t} (\varepsilon) + \frac{\partial}{\partial z} (\varepsilon w_f) = 0$ \hspace{2cm} (3)

$\frac{\partial}{\partial t} (\theta) + \frac{\partial}{\partial z} (\theta w_o) = 0$ \hspace{2cm} (4)

$\frac{\partial}{\partial t} (\phi' w_f) + \frac{\partial}{\partial z} (\phi' w_{f'} w_{s'}) = -(\phi' g) - \frac{\phi'}{\rho_s} \frac{\partial P_s}{\partial z} + \frac{\beta}{\rho_s} (w_f - w_s) - \frac{P_s}{\rho_s} \frac{\partial \phi'}{\partial z}$ \hspace{2cm} (5)

$\frac{\partial}{\partial t} (\sigma w_f) + \frac{\partial}{\partial z} (\sigma w_{f'} w_{s'}) = -(\sigma g) - \frac{\sigma}{\rho_f} \frac{\partial P_f}{\partial z} + \frac{\beta}{\rho_f} (w_f - w_s)$ \hspace{2cm} (6)

$\frac{\partial}{\partial t} (\varepsilon w_f) + \frac{\partial}{\partial z} (\varepsilon w_{f'} w_{s'}) = -(\varepsilon g) - \frac{\varepsilon}{\rho_w} \frac{\partial P_w}{\partial z} + \frac{\beta}{\rho_w} (w_m - w_s)$ \hspace{2cm} (7)

Equations 1 & 2 are the solid phase mass conservation equations for suspension and deposition. Equations 3 & 4 are the mass conservation equations for the oil and water respectively while Equations 5, 6 and 7 are the momentum conservation equations for the three phases (i.e. sand, oil and water) respectively.
The parameter $\phi'$ is a total particle term i.e. $\phi' = \phi + \sigma$ (8)

Where $\phi$ and $\sigma$ are the suspended and deposited sand concentrations respectively.

Total fluid concentration $\xi = \theta + \epsilon$ (9)

According to Sanni et al. [15], equations (4) and (7) for the water phase are additional equations to 1, 2, 3, 5 and 6 constitute the Sanni et al. [12] model.

2.2 Pathways to model solution

Based on Sanni et al. [12], (equation 1) becomes:

$$\frac{\partial \phi_j}{\partial t} = - \frac{\partial}{\partial z} \left( \phi_j \frac{\partial w_j}{\partial z} \right)$$

Which gives:

$$\phi_{j+1} = \lambda (\phi_j - 2\phi_j + \phi_{j-1}) + \phi_j$$

Where:

$$\lambda = \frac{D_j \Delta t}{2 \Delta z^2}$$

Similarly, this gives:

$$\epsilon_{i+1} = \lambda (\epsilon_i - 2\epsilon_i + \epsilon_{i-1}) + \epsilon_i$$

$$\theta_{i+1} = \lambda (\theta_i - 2\theta_i + \theta_{i-1}) + \theta_i$$

For the oil and water phases respectively. Equation (10) is the finite difference algorithm for the combined equations (1) and (2) while (11) and (12) are the finite difference algorithms for (3) and (4) respectively.

According to Sanni et al. [12], the finite difference algorithms for (5), (6) and (7) are given as (13), (14) and (15):

$$k_{i+1} = \lambda (k_i - 2k_i + k_{i-1}) - \frac{1}{2} \{(\phi g) - \frac{\phi}{\rho_s} \frac{\partial P_f}{\partial z} - \frac{\beta}{\rho_s} (w_f - w_i) - \frac{P_f}{\rho_s} \frac{\partial \phi}{\partial z}\} + k_i$$

$$\gamma_{i+1} = \lambda (\gamma_i - 2\gamma_i + \gamma_{i-1}) - \frac{1}{2} \{(\epsilon g) - \frac{\epsilon}{\rho_f} \frac{\partial P_f}{\partial z} - \frac{\beta}{\rho_f} (w_f - w_i)\} + \gamma_i$$

$$\gamma_{i+1} = \lambda (\gamma_i - 2\gamma_i + \gamma_{i-1}) - \frac{1}{2} \{(\theta g) - \frac{\theta}{\rho_f} \frac{\partial P_f}{\partial z} - \frac{\beta}{\rho_f} (w_f - w_i)\} + \gamma_i$$

According to Sanni et al. [12], except that phase pressures were then determined here as the product of the volumetric concentrations and the flowing pressure of the mixture at different sections of the pipe (i.e. $x P_i$) where $x$ refers to the volume fraction of the component, $P$ is the pressure of the mixture and $i$ denotes to any of the three components (sand, oil and water). The essence was to limit errors and ensure high level of accuracy instead of adopting the pressure equation for estimating total and interfacial pressures [12,16], where the interfacial pressure approximates the sand phase pressure. Rather, the phase pressures of the components were direct functions of the components of the mixture at different points along the pipe axis. Since run 1 gives the least amount of sand produced and considered the safe injection pressure threshold, the simulations were based on data for that run i.e. run 1.

Measured fluid properties: oil density = 878 kg/m$^3$, mixture density = 762 kg/m$^3$, oil viscosity = 0.003441 kg/ms, water density = 1000 kg/m$^3$.

2.2.1 Boundaries for conservation equations

Based on the data generated by Sanni et al. [15], fluid concentration at the inlet or outlet implies: parameters measured at the inlet or outlet in m$^3$/day divided by the sum total of that parameter for each component at the pipe section.

$$\epsilon_{in} = \frac{volume of oil}{volume of oil + volume of sand + volume of water} = 0.528$$

$$\theta_{in} = \frac{volume of water}{volume of oil + volume of sand + volume of water} = \frac{177.376}{491.38} = 0.361$$
\[ \phi_{in} = 1 - (0.528 + 0.361) = 0.111 \text{ (sand concentration at pipe inlet)} = 0.1110 \]

Since \[ \theta + \varepsilon + \phi = 1 \]

At the outlet,
\[ \phi_{out} = \frac{\text{volume of sand}}{\text{volume of sand} + \text{volume of oil} + \text{volume of water}} \text{ at outlet} = \frac{43.97}{397.79} = 0.1105 \]

0.1105 is the sand concentration at the pipe outlet and the corresponding \( \varepsilon \) and \( \theta \) value (oil and water concentrations respectively) is 0.8805.

\[ \varepsilon_{out} = \frac{\text{volume of oil}}{\text{volume of sand} + \text{volume of oil} + \text{volume of water}} \text{ at outlet} = \frac{210.02}{397.79} = 0.5279670 \]

\[ \theta_{out} = \frac{\text{volume of water}}{\text{volume of sand} + \text{volume of oil} + \text{volume of water}} \text{ at outlet} = \frac{(186.96 - 43.97)}{397.79} = \frac{142.99}{397.79} = 0.3595 \]

### 2.2.2 Boundaries for the components’ momentum equations

Sand, oil and water-inlet velocities: 0.078 m/s, 0.371 m/s and 0.254 m/s respectively. These velocities were inlet boundary values for the momentum equations. The outlet boundary velocities are: \( 6.67 \times 10^{-5} \text{ m/s, 6.67} \times 10^{-5}, 3.24 \times 10^{-4} \text{ m/s and } 2.2 \times 10^{-4} \text{ m/s} \) [15].

### 2.2.3 Solutions to the conservation equations

Pipe length = 24 m, points marked out are 0, 6, 12, 18 and 24 m on the pipe as done by Sanni et al. [15]. The components’ concentrations varied slightly from the inlet through to the pipe’s exit. Adopted sand concentrations are: 0.111, 0.110875, 0.11075, 0.110625, 0.1105 from 0-24 m at 6m interval. Oil concentrations are: 0.52804347, 0.52802435, 0.52800523, 0.52798611, 0.5279670 while the volume fractions of water are: 0.361, 0.360625, 0.36025, 0.359875, 0.3595.

### 2.2.4 Estimation of drag forces, terminal and critical velocities

For particles subjected to a constant force e.g. say gravity, they experience a common acceleration and thereafter move with a uniform/terminal velocity within the flow conditions. This velocity depends on particle shape, size, density alongside fluid properties. During laminar flow, as particles move steadily through the mixture, two principal forces act on them namely, the external force which is responsible for the particles dynamics and the drag force arising from frictional/viscous forces of the fluid and in turn oppose motion.

The net force on the moving particle is given by equation 18.

\[ F_{net} = Vg(\rho_s - \rho_f) \]

Where: \( F_{net} = \) Net force, \( V = \) particle volume, \( g = \) gravitational acceleration, \( \rho_s = \) particle density, \( \rho_f = \) fluid density

The drag force \( (F_d) \) of the fluid on sand particles is the product of fluid pressure and the projected particle-area. i.e. for solids, pressure,

\[ P = \frac{F}{A} \]

Then,

\[ F_d = C_d \rho_f V^2 \frac{d^2}{2} \]

Where: \( F_d = \) the drag force, \( C_d = \) Drag coefficient, \( \rho_f = \) fluid density, \( V = \) particle volume, \( A = \) projected area of particle

Since the sand particles are considered spherical, the volume of a particle is given by

\[ \frac{1}{6} \pi d^3 \]

And \( A = \pi d^2 \)

Where: \( d = \) particle diameter

Now, equating the net force and the drag force gives an equation for estimating the terminal or critical velocity.

Note:
In the laminar region, \( C_d = 24/Re = 24 \mu/puvd \)
Re = Reynolds number of flow, \( u_t \) = critical/terminal velocity, \( \mu \) = fluid viscosity and \( d \) = particle diameter

Therefore, \( C_{d0} \frac{\nu^2}{2} = \frac{g\left(\rho_s - \rho_f\right)}{2} \)  

(24)

The above equation then becomes

\[ 24 - \frac{\nu^2}{2} = \frac{g\left(\rho_s - \rho_f\right)}{2} \]  

(25)

For \( Re = 1000 \) as discussed in Doan et al. [6] bed load transport of sand can take place hence, considering the Navier equation, the terminal/critical velocity is given as:

At critical conditions of flow i.e. 90 % drop in crude oil velocity, the threshold and critical deposition velocities were estimated using the Navier Stokes’ equation for laminar flow.

\[ u_t = \frac{1000 \times 0.003441}{761 \times 0.016} = 0.0445 \text{ m/s (threshold velocity)}; \text{ while the critical deposit velocity for the formation of a fixed bed is given by} \]

\[ u_c = \frac{300 \times 0.003441}{761 \times 0.016} = 0.0134 \text{ m/s} \]

Then the drag force for \( Re = 1000 \) implies:

\[ F_d = \frac{24 \times 761 \times 3.142 \times 3.142 \times 0.016 \times 3.142 \times 0.016}{1000 \times 2 \times 36} = 8.93 \times 10^{-6} \text{ (kgm/s}^2) \]

While for \( Re = 300 \)

\[ F_d = \frac{24 \times 761 \times 3.142 \times 3.142 \times 0.016 \times 3.142 \times 0.016}{300 \times 2 \times 36} = 8.93 \times 10^{-10} \text{ (kgm/s}^2) \]

3.0 Results and Discussions

(i) Operating Data

The flowing tubing pressure (Psia), the API gravity of the fluid, the gas produced, the base sediment and water (%), barrels of liquid per day (BOPD), barrels of oil produced per day (BOPD), gas-oil ratio, the gas-liquid rate, injection pressure (Psia) and the sand produced at the well head for the same choke size opening were measured and presented as shown in Figure 1. It is then obvious that maintaining the same valve opening while taking a set of data for the same time period would give changes in FTP, Gas recovery, and quantities of liquid (BLPD) and oil recovered per day (BOPD). Figure 1 gives an illustration of the variation of BOPD with BLPD for different experimental runs.

From the plot, it is clear that there exists a positive correlation between amount of gas produced and the gas oil ratio (GOR) while a positive trend is also observed for the gas-liquid flow rate (GL flow rate), the flowing tubing pressure (FTP) and the barrels of oil produced per day despite the slight variations in the GL rate. Also, the last run shows a drop in BOPD possibly because of the very high amount of gas recovered which could have resulted from flow stratification thus confining the gas to the upper and above the concentric section of the pipe which imparts significant momentum on the gas thus increasing its flow velocity and subsequently the gas flow rate. This is because, maintaining the FTP between 748 and 793 psia keeps sand production in the range of 1.22 – 1.33 PPTB. Below 748 and 550 Psia, and above 793 Psia, the sand production increases to 2.55 PPTB but efforts must be made to ensure that the sand phase velocity exceeds the threshold velocity as well as the critical velocity for sand deposition which helps to abate the possibility of having a stationary bed or bed load transport of sand.

(ii) Stream Velocities and Fluid-Particle Dynamics

Figure 2a gives an illustration of the velocity distribution of molecules and particles in the three phases. The calculated velocities are in the range of 0.078 m/s - 0.000067 m/s between the 0 - 24 m points respectively. The sand velocity difference was 24.9 % between the 0 and 6 m, it was 33.3% between 6 and 12 m points, 49.9% between the 12 and 18 m points while it was 99.6 % between the 18 m point and the exit. For oil, the velocities at the inlet and outlet are approximately 0.370 and 0.000032 m/s respectively giving a total change of 0.3697 m/s (about 99.9% change in oil velocity). Between the 0 and 18 m points, the percent-change in oil velocity from
pipe inlet to its exit ranges from 24.9% - 49.9%; the last two points (i.e. 18 and 24 m points), the estimated change in oil velocity was about 99.65%. The recorded velocity changes are due to the drop in pressure from the inlet through to the exit. For water, the velocity dropped from 0.253 m/s at the inlet to 0.00022 m/s at the exit. At the 6 m points, it changed by 24.9% from the previous value. At the 12 m point, it changed by 33.2% from the previous value. The velocity change is 49.9% between the 12 and 18 m points while the change in water velocity at the 18-24 points was 99.1% [15]. This happened due to the reduction in flow rate of crude oil which subsequently affected the water traversing through different points in the pipe.

The flow velocity for the mixed constituents in the pipe is shown in Figures 2b-i. At varying time which was simulated in days i.e. 0.1, 0.11, 0.12, 0.14, 0.16, 0.18, 0.20 and 0.22 with emphases on the collective effect of the fluid-particle interaction coefficient and the kinematic pressure. The ranges of the fluid-particle interaction coefficient and the kinematic pressure are given within the text. The following were the observations noted for the behaviour of the velocities of the mixed constituents (i.e. sand, oil and water). The longer the operation time under the given parameters (i.e. calculated within the text), the higher the flow rate (Figures 2b-i). The flow velocity is higher at the highest kinetic pressure and lowest fluid-particle interaction coefficient. This observation is very significant in pumping operations. However, it was deduced that the mixture-velocity may not be optimized beyond 0.22 m/s (i.e. 5 h 28 mins) of operation time, hence new parameters are required to optimize the surface pressure or FTP. One of the practical solutions could be the use of boosters or eRED valves if they can be integrated in the design of the lines in the wells because it improves on the flow velocity and ultimately saves time [17].

Recently, field-work has shown that the maximum hour attainable for flow between well heads and flow stations is 8 hours under certain conditions which are restricted at certain period of the year [18]; this is quite less for oil wells depending on the distance/depth covered. These observations are further proofs of the validity and capability of the three phase model discussed here owing to its ability to parameterize the flow rate transport situation in wells.

![Figure 2a: Sand, oil and water displacements per time along the pipe length](image-url)
Figures 2b-i: Influence of varying time performance on flow velocities of mixed constituents
Results were generated by comparing values for sand and crude oil throughput from the lines for 5 different runs. However, it was discovered that the Sanni et al. [12] and the new three phase model gave accurate predictions of the amount of crude oil recovered from the lines because the model has the advantage of treating the phases separately. In addition, the model cannot give accurate sand production data for high water cut reservoirs because, the water phase is not accounted for by the two phase model. The three phase model gave amount of predicted sand to 99% accuracy which proves the model to be reliable.

3.1 Critical and threshold velocities

The concentration distribution of sand dropped from the pipe inlet down to the exit at hourly production rate of the oil as shown in Figure 1; it varied between 0.528 and 0.527. In Figure 2, the hourly sand concentration distribution along the pipe axis was also seen to decrease from the pipe inlet to its exit. Similarly, as seen for the sand and oil phases, the volume fraction of water also decreased along the pipe for every one hour of operation; see Figure 3.

![Figure 3: Component Concentrations at Marked Points of the Pipe](Source: Adapted from Sanni et al. [15])

The conditions for reliable flow were altered by 90% drop in oil velocity (i.e. 960.24) in order to force the sand particles to withdraw from the suspension phase to the transition layer and laminar sub-layer [15]. At a Reynolds number of 1000, the particles were seen to form a mobile bed which corroborates the findings of Doan et al. [6] and Sanni et al. [12,19], however, the impact of the sand particles dragging along the pipe line was estimated in order to determine the magnitude of the impact of the force that can cause mechanical wear (fretting corrosion, stress corrosion cracking) since the sand particles were assumed to be crystalline in nature with very low solubility. At further drop in the flow velocity, the Reynolds number at which the fluid velocity had no impact on sand was found to be 300; at this condition, the sand particles agglomerated/clumped to form a stationary bed which is in turn surrounded by a film of water which is an electrolyte that stimulates sand dissolution thus electrochemical corrosion may set in. However, it is somewhat difficult to predict the point at which the electrochemical cell is set up but going by the mechanism of electrochemical corrosion, the water forms a silicic acid with the sand, the acid formed then serves as an electrolyte which stimulates some metallic/non-metallic interactions on the metal surface, the carbon steel alloy then becomes unstable and reacts with the acid to form pyroxene or a ferral silicate/an oxide which is the corrosion product and because the flow is laminar, the accumulated water and sand at a point on the pipe then begin to diffuse into the metal leaving traces of the silicate as marked out portions/scales on the pipe surface. Under this flow condition, the sand phase pressure forces and the viscous forces of the carrier oil dominate the inertia forces [15].

3.2 Implications of estimated velocities and drag forces for upstream petroleum engineers

Recently, reports from field studies show that the maximum hour attainable for flow between well heads and flow stations is 8 hours under certain conditions which are restricted at certain period of the year [18]; this is quite less for oil wells depending on the distance/depth covered. These observations are further proofs of the validity and capability of the three phase model discussed here owing to its ability to parameterize the flow velocities and drag forces in the wells under bed load condition. Therefore it is evident from calculations that the
estimated drag forces range from $8.938 \times 10^{-8}$ to $10^{-10}$ kgm/s$^2$ with threshold and critical velocities of 0.0445 and 0.0134 m/s respectively at Reynolds number in the region of 300-1000, which makes particles roll out of the suspension/carrier medium and drift towards the pipe wall. Thus, in order to control the flow to mimic flow in the transition or turbulent zone, the operating velocity must exceed 0.0445 m/s such that the Reynolds number is greater than 1000.

**Conclusion**

The three phase model discussed in this study is a very viable tool for estimating the drag forces, threshold and critical velocities responsible for the formation of a stationary bed or bed load transport. The dynamics of the components that make up the mix show that the velocities can be related to the interaction coefficients of the components as well as the kinematic pressures of the particles. The most critical condition to avoid, is the existence of the laminar sub-layer where the flow system can be characterized by a Reynolds number as low as 300.

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