Spatial coherence singularities and incoherent vortex solitons

Kristian Motzek
Institute of Applied Physics, Darmstadt University of Technology, D-64289 Darmstadt, Germany

Yuri S. Kivshar
Nonlinear Physics Center, Research School of Physical Sciences and Engineering, Australia National University, Canberra ACT 0200, Australia

Ming-Feng Shih
Physics Department, National Taiwan University, Taipei, 106, Taiwan

Grover A. Swartzlander, Jr.
Optical Sciences Center, University of Arizona, Tucson, Arizona 85721, USA

We study spatially localized optical vortices created by self-trapping of partially incoherent light with a phase dislocation in a biased photorefractive crystal. In contrast to the decay of coherent self-trapped vortex beams due to the azimuthal instability, the incoherent vortices are stabilized when the spatial incoherence of light exceeds a certain threshold. We analyze the spatial coherence properties of the incoherent optical vortices and reveal the existence of ring-like singularities in the spatial coherence function of a vortex field that can characterize the stable propagation of vortices through nonlinear media.

I. INTRODUCTION

Vortices are the fundamental objects in physics, and they can be found in different types of linear and nonlinear coherent systems. A typical scalar vortex has the amplitude vanishing at its center and a well-defined phase associated with the circulation of momentum around the helix axis [1]. In optics, vortices are associated with phase dislocations (or phase singularities) carried by optical beams [2]. The last decade has seen a resurgence of interest in the study of optical vortices [3], owing in part to readily available computer-generated holographic techniques for creating phase singularities in laser beams.

In a self-focusing nonlinear medium, the singular optical beam undergoes self-focusing, and it becomes self-trapped creating a stationary ring-like structure with zero intensity at the center and a phase singularity [4]. However, such an optical vortex soliton is known to be highly unstable in self-focusing nonlinear media [5], it decays by splitting into several fundamental (no nodes) solitons flying off the soliton ring [6]. This effect has been observed experimentally in different nonlinear systems, including the saturable Kerr-like nonlinear media [7], biased photorefractive crystals [8], and quadratic nonlinear media [9] operating in the self-focusing regime. This effect is also expected to occur in other physical systems including the attractive Bose-Einstein condensates [10].

A number of recent theoretical studies [11, 12, 13, 14, 15, 16], including the rigorous analysis of linear stability of a self-trapped vortex beam [17, 18], suggest that the stable propagation of spatial and spatiotemporal vortex-like stationary structures may become possible in the models with competing nonlinearities in the presence of large higher-order defocusing nonlinearity; however such materials are not yet known and no stable coherent vortex solitons have been observed in experiment so far.

Recently, stable propagation of spatially localized optical vortices in a self-focusing biased nonlinear photorefractive crystal has been observed experimentally in the case when the vortices are created by partially incoherent light carrying a phase dislocation [19]. In particular, it was shown, both experimentally and theoretically, that single- and double-charge optical vortices can be stabilized in self-focusing nonlinear media when the spatial incoherence of light exceeds a certain threshold.

The successful experimental observation of stable self-trapped vortex beams created by partially incoherent light call for additional studies of the specific properties of partially coherent light carrying phase singularities and propagating in a nonlinear medium. Indeed, if a vortex-carrying beam is partially incoherent, the phase front topology is not well defined, and statistics are required to quantify the vortex phase. In the incoherent limit neither the helical phase nor the characteristic zero intensity at the vortex center can be observed.

However, several recent studies have shed light on the question how phase singularities can be unveiled in incoherent light fields propagating in linear media [20, 21]. In particular, Palacios et al. [21] used both experimental and numerical techniques to explore how a beam transmitted through a vortex phase mask changes as the transverse coherence length at the input of the mask varies. Assuming a quasi-monochromatic, statistically stationary light source and ignoring temporal coherence effects, they demonstrated that robust attributes of the vortex remain in the beam, most
prominently in the form of a ring dislocation in the cross-correlation function.

The purpose of this paper is twofold. First, we study numerically the effect of vortex stabilization through the analysis of the spatial coherence function of a vortex beam propagating in a self-focusing nonlinear medium. We reveal the specific features of the coherent function and demonstrate its importance for the study of singular beams in nonlinear media. Second, by applying the modal theory approach, we provide a deeper physical insight into the effect of the vortex stabilization by partially coherent light observed in experiment.

The paper is organized as follows. In Sec. 2 we introduce our numerical model that is based on the coherent density approach and describes the propagation of partially incoherent light in a slow-response nonlinear medium such as a biased photorefractive crystal. Section 3 demonstrates some examples of the stable partially incoherent vortex solitons, including the experimental results. In Sec. 4 we introduce the spatial coherence function and analyze its properties, whereas Sec. 5 is devoted to a simplified approach based on the truncated modal expansion. Finally, Sec. 6 concludes the paper.

II. MODEL AND NUMERICAL APPROACH

In order to study numerically the propagation of partially incoherent optical vortices in a biased photorefractive nonlinear medium, we employ the coherent density approach [22]. The coherent density approach is based on the fact that an incoherent light source can be thought of as a superposition of (infinitely) many coherent components \( E_j \) that are mutually incoherent, having slightly different propagation directions:

\[
E(r, t) = \sum_j E_j(r) e^{i k_{\perp j} \cdot r} e^{i \gamma_j(t)},
\]

where \( k_{\perp j} = k (\alpha_j e_x + \beta_j e_y) \) is the transverse wave vector of the \( j \)-th component, having direction cosines \( \alpha_j \) and \( \beta_j \), \( r = x e_x + y e_y \), \( \gamma_j(t) \) is a random variable that changes on the time scale of the coherence time of the light source, and \( k = 2\pi/\lambda \) is the wavenumber. The vortex is introduced via a phase mask at the input face \((z = 0)\) of the medium.

To avoid complexities that may arise from incoherent light sources having abrupt boundaries, we assume the source has a Gaussian profile

\[
E_j(r) = \left( \frac{1}{\sqrt{\pi} \theta_0} e^{- (\alpha_j^2 + \beta_j^2)/\theta_0^2} \right)^{1/2} A(r),
\]

where

\[
A(r) = (r/w_0)^2 e^{i m \varphi} e^{-r^2/\sigma^2}
\]

is the complex vortex profile, \( \varphi \) is the angular variable, and \( \theta_0 \) is a parameter that controls the beam’s coherence, i.e. less coherence means larger value of \( \theta_0 \).

Scaling the lengths in the transverse directions to \( x_0 = 1 \mu m \) and the length in propagation direction to \( z_0 = 2k x_0^2 \), where we chose \( k = 2\pi/(230 \text{ nm}) \), the propagating field \( E_j(r, z) \) can be described by the nonlinear Schrödinger equation:

\[
i \frac{\partial E_j(r, z)}{\partial z} + \nabla_\perp^2 E_j(r, z) + \eta(r, z) E_j(r, z) = 0,
\]

where \( \eta(r, z) \) accounts for the nonlinear refractive index change in the material. We assume a photorefractive medium with a saturable nonlinearity having a response time much larger than the coherence time of the light source. In this case \( \eta \) depends on the time-integrated intensity, \( I = \sum_j |E_j|^2 \), and it can be written as

\[
\eta(r, z) = \frac{I(r, z)}{1 + s I(r, z)},
\]

where \( s \) is the saturation parameter. Whereas numerical solutions of Eq. (4) may be readily computed using the coherence density approach, later we adopt also the equivalent multi-mode theory [23] to provide a physical insight for our findings.
III. PARTIALLY INCOHERENT VORTEX SOLITONS

The experimental results, first reported in Ref. [19], were obtained for a vortex beam generated in a self-focusing biased photorefractive SBN crystal. The rotating diffuser was used to introduce random-varying phase and amplitude of the input light beam on the time scales much shorter than the response time of the crystal. By adjusting the position of the diffuser to near (away from) the focal point of the lens in front the diffuser, the degree of the light coherence was increased (decreased). The light after the rotating diffuser was sent through a computer-generated hologram to imprint a vortex phase on the light beam. Such a partially coherent vortex beam was sent into the photorefractive crystal.

The experimental results are summarized in Fig. 1(lower row), and they are compared with the corresponding numerical results [see Fig. 1(upper row)] obtained in the framework of the theoretical model introduced in Sec. 2 above. First, both numeric and experiment reproduce the well known result that the coherent single-charge \((m = 1)\) vortex beam cannot propagate stably in a self-focusing nonlinear medium (left plots). Indeed, when the diffuser is removed from the experimental setup and a 2.5 kV biasing voltage is applied on the photorefractive crystal creating a Kerr-type self-focusing nonlinear medium, the vortex beam breaks up into two pieces. This vortex break-up observed in a self-focusing medium is due to the azimuthal instability, and it has been observed previously.

When the rotating diffuser is used, the degree of coherence of the vortex beam varies, and we observe clearly that the vortex beam can be stabilized by the reduction of the degree of coherence, as is summarized in Fig. 1. Above a certain value of the coherence parameter \(\theta_0\), the generated stable partially incoherent vortex soliton is observed at the output face of the crystal.

IV. SPATIAL COHERENCE FUNCTION

In order to quantify the second-order coherence properties of the singular beam propagating in a nonlinear medium, we calculate the mutual coherence function

\[
\Gamma(r_1, r_2; z) = \langle E^*(r_2, z, t)E(r_1, z, t) \rangle,
\]

where the brackets stand for averaging over the net field \(E(r, z, t) = \sum_{j=1}^{N} E_j(r, z) \exp(i\gamma_j(t))\). Again, we assume that for the photorefractive nonlinearities the random phase factors \(\gamma_j(t)\) vary on a timescale much faster than the response time of the medium. For the linear propagation, Palacios et al. demonstrated that the phase singularities occur in the cross-correlation \(\Gamma(-r, r)\) of an incoherent vortex beam, where the origin of the coordinate system is chosen to coincide with the vortex center.

In Figs. 2, 3, we show the numerical results for the stable and unstable nonlinear evolution of an incoherent vortex and the corresponding evolution of the vortex cross-correlation function. In these examples, we simulate the model with \(N = 1681\) components, with the parameters \(w_0 = 1.8, \sigma = 1.5\) and \(s = 0.5\). The size of the numerical simulation domain corresponds to the domain \(35 \times 35\mu m\).

First, we notice that in the nonlinear case the beam intensity has a local minimum in the center of the vortex, even after propagating many diffraction lengths. This is contrary to the case of the linear propagation where a beam with the same degree of coherence \(\theta_0\) has maximum intensity in the center of the vortex after only a few diffraction lengths. Also, if we had chosen to propagate an incoherent ring of light without topological charge instead of an incoherent vortex, we would also observe a maximum in the beam’s center. Thus we can state that the coherence function of the vortex manifests itself in the intensity distribution of the light beam after propagating through a nonlinear medium.

In fact, the intensity profile remains reminiscent of a vortex, even if the intensity does not quite drop to zero in the center of the beam.

Analyzing the structure of the beam cross-correlation function, we clearly observe, similar to the case of the linear propagation, a ring of phase singularities in the cross-correlation function \(\Gamma(-r, r)\) that is preserved when the vortex is stabilized (see Fig. 2) or disintegrates and decays when the vortex breaks up (see Fig. 3).

Thus, as the first result of our numerical studies we state that the phase singularities in cross-correlation predicted for the incoherent vortices propagating in linear media also survive the propagation through a nonlinear medium. This is not self-evident, considering that in the nonlinear case the separate components that form an incoherent light beam do interact, contrary to the linear case. A physically intuitive explanation how this ring of phase singularities develops under linear propagation is given in Ref. [21]. However, this issue becomes more complicated for the propagation in a nonlinear medium.

In addition, in Fig. 4 we show the situation in the far field. All parameters are identical to those used in Fig. 2. In the far field we observe as well a ring-like structure of the cross-correlation function \(\Gamma(-f, f)\), where \(f\) stands now for the spatial coordinates in the far field. The intensity distribution in the far field can also show a local minimum in
the center of the beam, contrary to what one would obtain if the vortex is propagating through a linear medium [21], and also in contrast to the result we would obtain if we were propagating a light beam without topological charge. This emphasizes the importance of the interaction between the beam coherence function and the nonlinearity.

V. MODAL THEORY APPROACH

Although the coherence density approach can be used to simulate the propagation of partially incoherent light with an arbitrary accuracy, it is of a little use when it comes to finding an explanation for the results obtained from the numerical simulations such as those presented above. A deeper physical insight can be obtained by using the modal theory of incoherent solitons [24]. According to the modal theory, the incoherent solitons can be regarded as an incoherent superposition of guided modes of the waveguide induced by the total light intensity. Since the incoherent vortices that we are dealing with induce circularly symmetric waveguides, the guided modes we have to consider are also circularly symmetric. To explain our numerical findings, we construct numerically, using a standard relaxation technique, a partially incoherent vortex soliton that consists of the circularly symmetric modes with the topological charges \( m = 0, 1 \) and \( 2 \): 

\[
E(r) = \sum_{m=0}^{2} E_m(r) \exp(i m \varphi) \exp(i \gamma_m(t)).
\]

A more precise modelling of incoherent vortices would require more modes. Here, we restrict ourselves to three modes only, assuming that for a partially incoherent vortex the \( m = 1 \) component should be dominant and that the next strongest components should be those with topological charge \( m' = m \pm 1 \), i.e. \( m' = 0, 2 \). Indeed, we find that the main features of incoherent vortex solitons can be explained qualitatively using only these three modes.

For this three-mode composite vortex soliton, the relative intensity of the \( m = 0 \) and \( m = 2 \) modes, controls the overall beam coherence, as compared to the \( m = 1 \) main vortex mode. However, in order to assure that the total topological charge of the beam

\[
m_{\text{tot}} = \text{Im}\{ \left( \int E^*(r \times \nabla E) \, dr \right) e_z / \int I \, dr \},
\]

is equal to one, we have to chose the \( m = 0 \) and \( m = 2 \) components of equal intensity. In order to check whether this simple approach yields the results that agree at least qualitatively with the full numerical model of an incoherent vortex soliton, we calculate the resulting shape of the vortex components, the total intensity, and cross-correlation \( \Gamma(-r, r) \) shown in Fig. 3. Comparing Fig. 2 and Fig. 3, we notice the presence of two similar features: (i) the local minimum of the intensity in the center of the beam, and (ii) the ring-like structure of the cross-correlation. Hence, these two phenomena can be explained by considering a simple modal representation of the incoherent vortex consisting of only three modes with the topological charges \( m = 0, m = 1, \) and \( m = 2 \).

First, the local minimum in the center of the beam can be explained by the fact that the waveguide induced by the \( m = 1 \) and \( m = 2 \) components affects the \( m = 0 \) mode in such a way, that it also develops a local intensity minimum in its center, a fact well known from the vortex-mode vector solitons [26]. Second, the ring-like structure of the cross-correlation comes from the different radial extent of the single components. As is known from the physics of vortex-mode vector solitons [5], the \( m = 0 \) component has the smallest radial extent, whereas the \( m = 1 \) and \( m = 2 \) components have larger radii. Hence the cross-correlation given by

\[
\Gamma(-r, r) = \sum_{m,m'=0}^{2} \langle E_m^*(-r) E_m(r) \rangle = \sum_{m=0}^{2} E_m^*(-r) E_m(r),
\]

is dominated for small \( r \) by the auto-correlated \( m = 0 \) component, whereas the \( m = 1 \) component dominates for larger \( r \). For even larger \( r \), the \( m = 2 \) component can also come into play which can eventually result in a second ring of auto-correlation.

VI. CONCLUSIONS

We have demonstrated stable propagation of optical vortices in a self-focusing nonlinear medium when the vortices are created by self-trapped partially incoherent light with a phase singularity propagating in a slow-response nonlinear medium such as a photorefractive crystal. The vortex azimuthal instability is found to be suppressed for the light incoherence above a critical value. In order to get a deeper physical insight into the effect observed in both numerics and experiment, we have studied the phase singularities in the spatial coherence function employed earlier in the linear optics and demonstrated that they survive the propagation through nonlinear media when the singular beam creates an incoherent vortex soliton. Our results emphasize the importance of the spatial coherence function in the studies of the propagation of incoherent singular beams. Not only the phase structure, but also the intensity distribution strongly depends on the initial form of the coherence function of the light beam as it enters a nonlinear medium.
Acknowledgements

This work was supported by the Australian Research Council and the Alexander von Humboldt Foundation. Kristian Motzek thanks Nonlinear Physics Center of the Australian National University for a warm hospitality.
[1] J. F. Nye and M.V. Berry, Proc. R. Soc. London A 336, 165 (1974).
[2] See, e.g., a comprehensive review paper, M.S. Soskin and M.V. Vasnetsov, in Progress in Optics, Vol. 42, Ed. E. Wolf (Elsevier, Amsterdam, 2001).
[3] See an extensive list of references on optical vortices in G. A. Swartzlander, Jr., Singular Optics / Optical Vortex References, http://www.u.arizona.edu/~grovers/SO/so.html.
[4] Such self-trapped singular beams were first suggested in V.I. Kruglov and R.A. Vlasov, Phys. Lett. A 111, 401 (1985).
[5] See, e.g., Yu. S. Kivshar and G. P. Agrawal, Optical Solitons: From Fibers to Photonic Crystals (Academic, San Diego, 2003), Chap. 8.
[6] W.J. Firth and D.V. Skryabin, Phys. Rev. Lett. 79, 2450 (1997); D.V. Skryabin and W. Firth, Phys. Rev. E 58, 3916 (1998).
[7] V. Tikhonenko, J. Christou, and B. Luther-Davies, J. Opt. Soc. Am. B 12, 2046 (1995); Phys. Rev. Lett. 76, 2698 (1996).
[8] Z. Chen, M. Shih, M. Segev, D.W. Wilson, R.E. Muller, and P.D. Maker, Opt. Lett. 22, 1751 (1997).
[9] D.V. Petrov, L. Torner, J. Nartorell, R. Vilaseca, J.P. Torres, and C. Cojocaru, Opt. Lett. 23, 1444 (1998).
[10] H. Saito and M. Ueda, Phys. Rev. Lett. 89, 190402 (2002).
[11] M. Quiroga-Teixeiro and H. Michinel, J. Opt. Soc. Am. B 14, 2004 (1997).
[12] A. Desyatnikov, A. Maimistov, and B.A. Malomed, Phys. Rev. E 61, 3107 (2000).
[13] H. Michinel, J. Campo-Táboas, M.L. Quiroga-Teixeiro, J.R. Salqueiro, and R. Gracía-Fernández, J. Opt. B: Quantum Semiclass. 3, 314 (2001).
[14] V. Skarka, N.B. Aleksić, V.I. Berezhiani, Phys. Lett. A 291, 124 (2001).
[15] B.A. Malomed, L.-C. Crasovan, and D. Mihalache, Physica D 161, 187 (2002).
[16] T.A. Davydova, A.I. Yakimenko, and Yu.A. Zaliznyak, Phys. Rev. E 67, 026402 (2003).
[17] I. Towers, A.V. Buryak, R.A. Sammut, B.A. Malomed, L.-C. Crasovan, and D. Mihalache, Phys. Lett. A 288, 292 (2001).
[18] D. Mihalache, D. Mazilu, L.-C. Crasovan, I. Towers, A.V. Buryak, B.A. Malomed, L. Torner, J.P. Torres, and F. Lederer, Phys. Rev. Lett. 88, 073902 (2002).
[19] C.-C. Jeng, M.-F. Shih, K. Motzek, and Yu.S. Kivshar, Phys. Rev. Lett. 92, 043904 (2004).
[20] H.F. Schouten, G. Gbur, T.D. Visser, and E. Wolf, Opt. Lett. 28, 968 (2003).
[21] D.M. Palacios, I.D. Maleev, A.S. Marathay, and G.A. Swartzlander, Jr., Phys. Rev. Lett. 92, 143905 (2004).
[22] D.N. Christodoulides, T.H. Coskun, M. Mitchell, and M. Segev, Phys. Rev. Lett. 78, 646 (1997).
[23] D.N. Christodoulides, E.D. Eugenieva, T.H. Coskun, M. Segev, and M. Mitchell, Phys. Rev. E 63, 035601 (2001).
[24] M. Mitchell, M. Segev, T. Coskun, and D.N. Christodoulides, Phys. Rev. Lett. 79, 4990 (1997).
[25] K. Motzek, A. Stepken, F. Kaiser, M.R. Belić, M. Ahles, C. Weilnau, and C. Denz, Opt. Comm. 197 161 (2001).
FIG. 1: Comparison between numerical (upper row) and experimental (lower row) results for the vortex stabilization effect. Numerical results are shown for the vortex after 9mm of propagation for (from left to right): the coherent case and for the partially incoherent cases at $\theta_0 = 0.14$, $\theta_0 = 0.29$, $\theta_0 = 0.38$, respectively.
FIG. 2: Contour plots of the intensity (left column) and the modulus of the cross-correlation (right column) of an incoherent vortex with $\theta_0 = 0.64^\circ$ (strong incoherence). Contrary to the case of the linear propagation, there is a local intensity minimum in the beam’s center. The cross-correlation, however, shows the same ring of phase singularities as predicted in the linear theory.
FIG. 3: Contour plots of the intensity (upper row) and the modulus of the cross-correlation (lower row) for the breakup of an incoherent vortex at $\theta_0 = 0.37$ (weak incoherence), when the ring is not preserved.
FIG. 4: The intensity (left column) and the cross-correlation (right column) of the far field. The effects of the nonlinearity on the intensity distribution can be clearly seen, whereas the cross-correlation maintains more or less the structure one would expect in the case of linear propagation.
FIG. 5: A composite soliton calculated by using the three modes with the topological charges $m = 0, 1$ and 3: (a) profiles of the three components, (b) total intensity of the vortex soliton, and (c) vortex cross-correlation $\Gamma(-r, r)$. 