Compression Buckling Analysis of Flat Panels with Z-section Stiffeners

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NACA conducted extensive tests of z-stiffened panels under compressive load in the 1940s and 1950s in order to create direct-reading design charts. Because of the asymmetry of z-stiffened panels, the buckling behavior is complex and buckling analysis of the NACA data has not been conducted. In order to analyze the buckling of z-stiffened panels, a new tool for the buckling analysis of thin-walled structures is developed. This paper presents the results of the buckling analysis of the NACA data and investigates the buckling behavior of z-stiffened panels. It is found that the lateral buckling of the stiffener is critical in the short column region, while the torsional buckling of the stiffener is not critical.

Key Words: Buckling Analysis of Z-stiffened Panel, Local Buckling, Stiffener Lateral Buckling, Column Buckling

Nomenclature

- \( a \): support span of stiffened panel
- \( b \): width of strip element
- \( b_{M} \): width of attached flange
- \( b_{F} \) or \( d \): width of free flange
- \( b_{S} \): stiffener pitch
- \( b_{W} \): width of stiffener web
- \( C \): end-fixity of column
- \( [d] \), \( [d_{m}] \), \( [d_{s}] \): grid point displacements
- \( [D_{M}] \), \( [D_{B}] \): membrane stiffness and bending stiffness matrices
- \( E, E_{s}, E_{t} \): Young’s modulus
- \( E_{s} \): secant modulus
- \( E_{t} \): tangent modulus
- \( F \): Farrar’s structural efficiency
- \( F_{c} \): buckling stress
- \( F_{y} \): compression yield stress
- \( F_{f} \): failure stress
- \( G, G_{s} \): shear modulus
- \( h \): height of stiffener free flange
- \( k \): Rayleigh quotient
- \( [K], [K_{M}], [K_{B}], \cdots \): stiffness matrices of strip element
- \( L' \): length of test panel
- \( L \): effective column length
- \( n \): shape parameter of Ramberg-Osgood
- \( N \): compression load per width
- \( p \): number of half wave
- \( P \): compressive load per inch of panel width
- \( t_{s} \): skin thickness
- \( t_{w} \): stiffener thickness
- \( T \): compression force per width
- \( u, v, w \): displacements
- \( U \): strain energy
- \( V_{P} \): work done by external load
- \( [\varepsilon], [\varepsilon_{s}, \varepsilon_{t}, \gamma_{xy}], \cdots \): strain
- \( \eta \): plasticity correction factor
- \( [\kappa], [\kappa_{s}, \kappa_{t}, \cdots] \): curvature
- \( \nu_{c}, \nu \): elastic Poisson’s ratio, Poisson’s ratio
- \( \theta \): rotation
- \( \rho \): radius of gyration of area

Subscripts

- \( B \): bending
- \( M \): membrane

1. Introduction

Z-stiffened panels are used in major parts of aircraft structures, such as wing skins and fuselage skins. The advantages of z-stiffened panels are high structural efficiency for compressive load, and easy assembly and inspection. A cross-section of a flat z-stiffened panel is shown in Fig. 1.

Buckling and failure modes of z-stiffened panels under compressive load are complex because of the geometrical asymmetry of the panel. There are many dimensional parameters to design z-stiffened panels: skin thickness, stiffener thickness, stiffener pitch, etc. The buckling modes and the buckling loads depend on the dimensional parameters. NACA conducted extensive compressive tests of flat z-stiffened panels made of aluminum alloys in the 1940s and 1950s. The test results were published in NACA TNs.1–3 NACA also published design charts of z-stiffened panels which were derived from the test data.4,5

Cross-sectional deformation should be considered in the analysis of the local buckling of z-stiffened panels. A chart of the local buckling stress for the skin and the stiffener of z-stiffened panels was produced by Gallaher and Boughan.6 ESDU also published a chart of the local buckling and twist buckling stress,7 but the accuracy is not as good as Gallaher and Boughan.

The buckling analysis of z-stiffened panels for skin local buckling, stiffener torsion buckling, and column buckling was conducted by Argyris.8 He assumed that the cross-sec-
tional shape of the z-stiffeners did not deform, and considered the flexure-torsion deformation of the stiffeners. Yusuff derived equations of buckling loads of stiffened panels which include the torsional and flexural rigidity of the stiffeners. He also neglected the cross-sectional deformation of the stiffeners.

Farrar investigated the optimum design of z-stiffened panels under compression using the ESDU chart. He created a contour map of the dimensions of the optimized panels, and the map is still used in the textbook of aircraft structures written by Niu.

Van der Neut pointed out the importance of cross-sectional deformation on the buckling of z-stiffened panels. He derived equations for “overall buckling,” which is the lateral buckling of the stiffener (see Fig. 2 for the deformation).

Finite element analysis or finite strip analysis are necessary to analyze the buckling of the z-stiffened panels considering cross-sectional deformation. An example of the analyses is a paper by Bushnell. However, the effect of z-stiffened panel dimensions on the buckling load has not been investigated. Even though there is a wide variety of test data by NACA, no detailed buckling analyses have been made for the test data. The purpose of this paper is to investigate the effect of the panel dimensions on the buckling behavior of z-stiffened panels. Detailed buckling analysis of the NACA test specimens is conducted.

2. Tool for Buckling Analysis

Finite element analysis is not suitable to analyze the NACA test panels because the dimensions of the NACA test panels vary specimen by specimen. A new tool to analyze the buckling of thin-walled structures was developed. The tool is based on the energy principle and finite strip modeling. “Solver” in MS-Excel is used to conduct the calculation.

2.1. Finite strip formulation

One unit of a simply supported z-stiffened panel with length of $a$ is divided to strips as shown in Fig. 3. The finite strip formulation and the derived stiffness matrices are based on Li.

2.1.1. Displacement of strip element

The displacement of a strip is expressed by the grid point displacement as follows:

\[
\begin{align*}
    u &= \left(1 - \frac{x}{b} \right) \left[ \begin{array}{c}
    u_i \\
    u_{i+1}
    \end{array} \right] \frac{\sin \frac{p \pi y}{t}}{a} \\
    v &= \left(1 - \frac{x}{b} \right) \left[ \begin{array}{c}
    v_i \\
    v_{i+1}
    \end{array} \right] \frac{\cos \frac{p \pi y}{t}}{a} \\
    w &= \left[1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3} \right] x \left(1 - \frac{2x}{b} + \frac{x^2}{b^2} \right) \frac{3x^2}{b^2} - \frac{2x^3}{b^3} - x \frac{x^2}{b^2} \frac{x}{b} \left[ \begin{array}{c}
    w_i \\
    \theta_i \\
    w_{i+1} \\
    \theta_{i+1}
    \end{array} \right] \frac{\sin \frac{p \pi y}{t}}{a}
\end{align*}
\]
where \( u, v, w \): displacements in an element
\( u_i, u_{i+1}, v_i, v_{i+1}, w_i, w_{i+1} \): grid point displacements
\( \theta_i, \theta_{i+1} \): rotation of a grid point

Equation (1) is expressed in a simpler form as follows

\[
\begin{bmatrix}
\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}
\end{bmatrix}_M = \begin{bmatrix}
\frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\end{bmatrix}_M = [B_M][d_{uv}]
\]

(2)

where \([N_{uv}]\) and \([N_u]\) are shape functions.

### 2.1.3. Strain energy

The strain energy of the strip element is

\[
U = \frac{1}{2} \int_{\text{element}} [\varepsilon_M]^T [\sigma_M] dV + \frac{1}{2} \int_{\text{element}} [\varepsilon_b]^T [\sigma_b] dV
\]

(4)

The constitutive equation of an orthotropic plate is expressed as follows.
where

\[
I_1 = \frac{a}{2}, \quad I_2 = I_3 = -\frac{\pi^2 p^2}{2a}, \quad I_4 = \frac{\pi^3 p^4}{2a^3}
\]

\[
I_5 = \frac{\pi^2 p^2}{2a}, \quad c_1 = c_2 = \frac{p\pi}{a}
\]

\[
E_1 = \frac{E_x}{1 - v_x v_y}, \quad E_2 = \frac{E_y}{1 - v_x v_y}
\]

\[
K_{B,11} = \frac{1}{420b^3} \left( 5040 D_x I_1 - 504b^2 D_1 I_2 - 504b^2 D_1 I_3 \right)
\]

\[
K_{B,12} = \frac{1}{420b^3} \left( 2520b D_x I_1 - 462b^3 D_1 I_2 - 42b^3 D_1 I_3 \right)
\]

\[
K_{B,13} = \frac{1}{420b^3} \left( -5040 D_x I_1 + 504b^2 D_1 I_2 + 504b^2 D_1 I_3 \right)
\]

\[
K_{B,14} = \frac{1}{420b^3} \left( 2520b D_x I_1 - 42b^3 D_1 I_2 - 42b^3 D_1 I_3 \right)
\]

\[
K_{B,21} = \frac{1}{420b^3} \left( 1680b^2 D_1 I_1 - 56b^4 D_1 I_2 - 56b^4 D_1 I_3 \right)
\]

\[
K_{B,22} = \frac{1}{420b^3} \left( 1680b^2 D_1 I_1 - 56b^4 D_1 I_2 - 56b^4 D_1 I_3 \right)
\]

\[
K_{B,23} = \frac{1}{420b^3} \left( -2520b D_x I_1 + 42b^3 D_1 I_2 + 42b^3 D_1 I_3 \right)
\]

\[
K_{B,24} = \frac{1}{420b^3} \left( 840b^2 D_1 I_1 - 14b^4 D_1 I_2 - 14b^4 D_1 I_3 \right)
\]

\[
K_{B,31} = \frac{1}{420b^3} \left( 5040 D_x I_1 - 504b^2 D_1 I_2 - 504b^2 D_1 I_3 \right)
\]

\[
K_{B,32} = \frac{1}{420b^3} \left( -2520b D_x I_1 + 42b^3 D_1 I_2 + 42b^3 D_1 I_3 \right)
\]

\[
K_{B,33} = \frac{1}{420b^3} \left( 5040 D_x I_1 - 504b^2 D_1 I_2 - 504b^2 D_1 I_3 \right)
\]

\[
K_{B,34} = \frac{1}{420b^3} \left( -2520b D_x I_1 - 42b^3 D_1 I_2 - 42b^3 D_1 I_3 \right)
\]

\[
K_{B,41} = \frac{1}{420b^3} \left( 1680b^2 D_1 I_1 - 56b^4 D_1 I_2 - 56b^4 D_1 I_3 \right)
\]

\[
K_{B,42} = \frac{1}{420b^3} \left( 1680b^2 D_1 I_1 - 56b^4 D_1 I_2 - 56b^4 D_1 I_3 \right)
\]

\[
K_{B,43} = \frac{1}{420b^3} \left( +4b^6 D_1 I_1 + 224b^4 D_1 I_3 \right)
\]

\[
K_{B,44} = \frac{1}{420b^3} \left( +4b^6 D_1 I_1 + 224b^4 D_1 I_3 \right)
\]

The partial derivatives in Eq. (9) are expressed using the grid

\[
\left( \frac{\partial u}{\partial y} \right) + \left( \frac{\partial v}{\partial y} \right) + \left( \frac{\partial w}{\partial y} \right)
\]

The point displacements as follows

\[
\left( \begin{array}{c}
\theta_i \\
\theta_i+1 \\
\theta_i \\
\theta_i+1
\end{array} \right) = \left( \begin{array}{c}
w_i \\
w_i+1 \\
v_i \\
v_i+1
\end{array} \right) \left[ G_B^T [G_B] \right] \left( \begin{array}{c}
w_i \\
w_i+1 \\
v_i \\
v_i+1
\end{array} \right)
\]

\[
\sum_{i=1}^{n} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \text{dxdy}
\]

Equation (10) is substituted in Eq. (9), and we obtain

\[
V_P = \frac{1}{2} \int_{T_i}^{T_i+1} \int_{T_j}^{T_j+1} \left[ T_i - (T_i - T_{i+1}) \right] \left[ G_B^T [G_B] \right] \text{dxdy}
\]

2.1.5. Geometric stiffness matrices

The compressive load per width T on the loaded edge of a strip element varies linearly as shown in Fig. 3. The work done by this external load is expressed as follows\(^{(5)}\)

\[
K_{B,M,11} = \frac{(3T_i + T_{i+1})bI_5}{12}, \quad K_{B,M,12} = 0
\]

\[
K_{B,M,22} = \frac{(3T_i + T_{i+1})b^2 I_4}{12\pi^2 p^2}, \quad K_{B,M,23} = 0
\]

\[
K_{B,M,24} = \frac{(T_i + T_{i+1})b^3 I_4}{12\pi^2 p^2}, \quad K_{B,M,33} = \frac{(T_i + 3T_{i+1})b I_5}{12}
\]

\[
K_{B,M,34} = 0, \quad K_{B,M,44} = \frac{(T_i + 3T_{i+1})b^2 I_4}{12\pi^2 p^2}
\]
2.1.6 Rayleigh quotient and buckling load

The Rayleigh quotient is defined as the total strain energy of all strip elements divided by the work done by the external load:

\[ k = \frac{U}{V_p} = \frac{\sum_{\text{all elements}} \frac{1}{2} (d^T [K] d)}{\sum_{\text{all elements}} \frac{1}{2} (d^T [K] d)} \]  

The set of grid point displacements which minimizes the Rayleigh quotient is the buckling mode, and the minimum Rayleigh quotient is the buckling load factor. The buckling load is obtained by multiplying the Rayleigh quotient to the applied external load, \( T_i \).

2.2 Minimization of Rayleigh quotient

“Solver” in MS-Excel is used to minimize the Rayleigh quotient. The merit of MS-Excel for the buckling analysis is as follows:

- Programming is not necessary.
- Matrix manipulation is not necessary to obtain eigenvalues.
- Post-processing of results, such as drawing graphs and producing tables, is easy in MS-Excel.
- MS-Excel is suitable for parametric study because the change in dimensions of stiffened panels is easy.

3. Buckling Analysis of NACA Test Data

Buckling analysis of the test data of NACA TN-1829, TN-1978, and TN-3431 is conducted. The representative results of the analysis are presented in this paper.

3.1 NACA test data

The typical cross-section of the NACA test specimens is shown in Fig. 4.

3.1.1 Material

The material used for the test specimens is 7075-T6 aluminum alloy. The material properties of the test specimens are as follows:

- Young’s modulus, \( E_c = 10,500 \text{ ksi} \)
- Poisson’s ratio, \( \nu = 0.33 \)
- Compression yield stress, \( F_{cy} = 74.6 \text{ ksi} \) (average value of skin)
- Shape parameter of Ramberg-Osgood: \( n = 25 \)

3.1.2 Panel dimensions and structural efficiency

The \( z \)-stiffened panels of the NACA tests cover a wide range of dimensions as shown in Fig. 5. Farrar derived the structural efficiency equation (Eq. (15)) for \( z \)-stiffened panels. Using the square root of the structural index in Eq. (15) as an abscissa, the NACA test data are plotted in Fig. 6. The solid lines in Fig. 6 show Eq. (15) with \( E_t \) of the Ramberg-Osgood shape parameter, \( n = 25 \).

\[ F_f = F \sqrt{\frac{E_t N}{L}} \]  

where \( F_f \): failure stress
\( N \): load per width
\( E_t \): tangent modulus
\( L \): support span

Fig. 4. Cross-section of NACA specimen.  
Fig. 5. Dimensions of NACA test specimens.  
Fig. 6. Structural index and strength of NACA test data.
### Table 1. NACA test data of representative panels.

| ID          | $t_w$ | $t_w/t_S$ | $b_S/t_S$ | $b_W/t_S$ | $b_F/t_S$ | $b_A/t_S$ | $h/b_S$ | $b_F/h$ | $L/b_W$ | $F_{cr}$ | $F_f$ | $N/L = F_p/(L'/\sqrt{C})$ | $h$ | $L'$ | $\sqrt{N/L}$ |
|------------|-------|-----------|-----------|-----------|-----------|-----------|---------|---------|---------|---------|------|--------------------------|-----|------|---------------|
| TN-1829-3-4 | 0.0974 | 0.942 | 20.1 | 12.6 | 4.88 | 6.73 | 0.64 | 0.36 | 8.0 | 75.2 | 3.32 | 1.328 | 9.82 | 5.07 | 57.6 |
| TN-1829-3-13| 0.1024 | 0.998 | 29.6 | 20.0 | 7.87 | 6.78 | 0.71 | 0.37 | 8.1 | 54.6 | 60.0 | 1.56 | 2.151 | 16.59 | 8.57 | 39.5 |
| Average    | 0.1003 | 0.973 | 29.4 | 20.4 | 7.96 | 6.64 | 0.71 | 0.37 | 14.1 | 56.4 | 58.0 | 0.86 | 2.148 | 28.85 | 14.90 | 29.3 |
| TN-1978-4-3 | 0.0667 | 1.043 | 50.0 | 27.7 | 11.56 | 5.64 | 0.60 | 0.40 | 7.9 | 18.8 | 40.1 | 0.66 | 1.913 | 14.60 | 7.54 | 25.7 |
| Average    | 0.1010 | 0.981 | 39.3 | 30.3 | 12.00 | 6.71 | 0.78 | 0.38 | 8.2 | 32.4 | 42.3 | 0.76 | 3.153 | 25.02 | 12.92 | 27.6 |
| TN-1829-3-18 | 0.1016 | 0.970 | 29.3 | 20.3 | 7.83 | 6.63 | 0.71 | 0.37 | 22.1 | 55.8 | 57.7 | 0.55 | 2.166 | 45.58 | 23.54 | 23.5 |
| Average    | 0.1001 | 0.942 | 29.6 | 20.5 | 7.90 | 7.61 | 0.70 | 0.37 | 34.4 | 42.0 | 0.26 | 2.137 | 70.03 | 36.16 | 16.1 |
| TN-1978-4-8 | 0.1007 | 0.969 | 39.2 | 30.3 | 12.17 | 6.84 | 0.77 | 0.39 | 34.7 | 50.7 | 32.7 | 0.14 | 3.154 | 105.88 | 54.67 | 11.8 |
| Average    | 0.1014 | 0.992 | 39.6 | 30.1 | 11.82 | 6.75 | 0.70 | 0.37 | 51.2 | 20.9 | 0.06 | 3.154 | 156.27 | 80.70 | 7.7 |
| TN-1978-4-4 | 0.1010 | 0.981 | 39.3 | 30.3 | 11.95 | 6.74 | 0.78 | 0.38 | —   | —   | —   | —   | 3.161 | —   | —   | —   |
| Average    | 0.1059 | 0.939 | 30.6 | 28.8 | 11.66 | 6.51 | 0.61 | 0.39 | —   | —   | —   | —   | 1.964 | —   | —   | —   |
| TN-1829-4-3 | 0.0646 | 0.980 | 58.4 | 39.6 | 15.76 | 6.63 | 0.68 | 0.39 | 8.0 | 14.3 | 32.3 | 0.41 | 2.623 | 20.47 | 10.57 | 20.2 |
| Average    | 0.0656 | 0.928 | 58.2 | 39.2 | 15.71 | 6.60 | 0.61 | 0.39 | 13.8 | 21.3 | 36.9 | 0.35 | 1.980 | 26.43 | 13.65 | 18.7 |
| TN-1829-4-8 | 0.0653 | 1.027 | 60.7 | 39.5 | 15.56 | 6.55 | 0.69 | 0.39 | 21.8 | 14.6 | 26.8 | 0.12 | 2.644 | 56.23 | 29.04 | 11.0 |
| Average    | 0.0631 | 0.918 | 56.0 | 40.5 | 16.13 | 6.78 | 0.68 | 0.39 | 34.0 | 12.6 | 24.2 | 0.07 | 2.621 | 86.89 | 44.87 | 8.4 |
| TN-1978-4-4 | 0.0648 | 0.939 | 56.6 | 39.6 | 15.70 | 6.60 | 0.67 | 0.39 | 49.9 | 15.9 | 19.4 | 0.04 | 2.166 | 45.58 | 23.54 | 16.1 |
| Average    | 0.0648 | 0.972 | 58.0 | 39.6 | 15.74 | 6.59 | 0.68 | 0.39 | —   | —   | —   | —   | 2.632 | —   | —   | —   |

ID “NACA TN-xxxx-z” means, xxxx: NACA report No., z: Table No., z-th cross-section in table y.

**Fig. 7. Structural index and strength of representative panels.**

$F$: Farrar’s structural efficiency factor

$N/L$: structural index

According to Farrar, the minimum weight design for a given loading condition is achieved when local buckling and column buckling coincide. While the theoretical value of maximum efficiency is 0.95, the maximum efficiency by the NACA tests is:

- About 0.80 for $\sqrt{N/L} \geq 15 \text{ lb/in.}$: skin not buckled
- About 0.90 for $\sqrt{N/L} \leq 15 \text{ lb/in.}$: buckled skin

#### 3.1.3. Representative panels and their test data

Buckling analysis was conducted for all test panels of NACA TN-1829 and some of the test panels of NACA TN-1978. The results of the buckling analysis of some representative panels are presented in this paper. Structurally efficient panels are selected as the representative panels as shown in Fig. 7 and Table 1. The dimensions of the representative panels are indicated in Fig. 5.

#### 3.2. Buckling analysis model

The buckling analysis model is shown in Fig. 8. The rivet joint between skin and stiffener is modeled by a strip element with equivalent shear stiffness, and the rotations of the grid points of the skin and the stiffener on the joint are equated.

#### 3.3. Results of buckling analysis and correlation with test data

The correction of plasticity for buckling stress is applied after the buckling analysis. The following plasticity correction factor for the plate is used:\(^{17,18}\):

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where $\eta$: plasticity correction factor

$E_i$: secant modulus

$E_t$: tangent modulus

$\nu$: elastic Poisson’s ratio

$\bar{\nu}$: plastic Poisson’s ratio

$F_{cr}$: buckling stress

Tangent modulus theory is applied for Euler buckling stress.

### 3.3.2. Buckling modes

Many buckling modes appear in the buckling of the z-stiffened panels because of the asymmetry of the cross-sectional shape (Fig. 9). Couplings of two buckling modes also appear and the couplings cause a reduction of the buckling stress.

- Column buckling is a symmetric mode.
- Stiffener lateral buckling mode appears as both symmetric and anti-symmetric modes. The free flanges of the stiffeners move sideways. (Nomenclature “stiffener lateral buckling” is used in this paper instead of “over-all buckling” used by van der Neut.)
- Stiffener torsion buckling is an anti-symmetric mode. The cross-sectional shape of the stiffener does not deform in this mode.
- Skin local buckling appears as both symmetric and anti-symmetric modes.
- Stiffener local buckling is a symmetric mode.
- Wrinkling buckling is a symmetric mode. It resembles stiffener local buckling and skin symmetric local buckling, but it accompanies large deformations of the attached flange.

### 3.3.3. Representative buckling curves

Figures 10 to 14 show the buckling curves of the representative panels. The abscissa of the figures is the slenderness ratio $L^*/\rho \sqrt{C}$, where $\rho$ is the radius of gyration of area and $C$ is end-fixity. Euler buckling stresses are plotted in the figures for reference. Correction of plasticity is applied. Skin symmetric local buckling or stiffener lateral buckling is critical in the short column region. Column buckling is critical in the long column region. Stiffener torsion buckling is not critical in the entire region for the representative panels.

As the thicknesses of the stiffener and the skin increase, the skin local buckling stress increases. Then the stiffener lateral buckling becomes critical. The skin local buckling stress and the stiffener lateral buckling stress are nearly equal in the NACA TN-1829-3-13 panel, and this panel is the most efficient as seen in Fig. 7.

Note that the column buckling stress is always lower than the Euler buckling stress. The reasons are:

- The coupling of the column buckling and the stiffener lat-
eral buckling

- The effect of the shear deformation of the stiffener web (not in the scope of this paper)

Both the stiffener lateral buckling and the local buckling exist in the short column region for all the representative panels. The stiffener lateral buckling appears in the panels of NACA TN-1829 with $h/b_S > 0.4$, as shown in Fig. 15.

If the stiffener lateral buckling stress is smaller than the local buckling stress, the failure occurs at the stiffener lateral buckling stress. Strength is determined by the minimum (cut-off) value of the stiffener lateral buckling stress and the column buckling as shown in Fig. 11. A region of slen-

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Table 2. NACA TN-3431 test data.

| ID          | Stiffener type | $t_w$ | $t_w/t_s$ | $b_s/t_s$ | $b_w/t_w$ | $b_r/b_w$ | $b_r/t_w$ | $b_A/t_w$ | $r_A/t_w$ | Buckling stress | Failure stress | $F_{cr}$ (ksi) | $F_{cr}/\eta$ | $F_{cr}$ (ksi) |
|-------------|----------------|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------------|----------------|----------------|----------------|----------------|
| TN-3431-1-1 | Sheet          | 0.0660| 0.640     | 24.6      | 18.7      | 0.4       | 7.48      | 5.20      | 12.50     | 3.0       | 45.2           | 45.2           | 0.0625         | 14             | 0.490          |
| TN-3431-1-2 | Sheet          | 0.0663| 0.636     | 24.5      | 18.5      | 0.4       | 7.40      | 5.00      | 12.50     | 3.0       | 49.5           | 52.0           | 0.0938         | 9.3             | 0.444          |
| TN-3431-1-3 | Sheet          | 0.0680| 0.650     | 24.8      | 19.2      | 0.4       | 7.68      | 6.20      | 12.50     | 3.0       | 48.0           | 48.5           | 0.0938         | 9.3             | 0.484          |
| TN-3431-1-4 | Sheet          | 0.0663| 0.630     | 24.3      | 19.4      | 0.4       | 7.76      | 7.20      | 12.50     | 3.0       | 41.0           | 44.3           | 0.0938         | 9.3             | 0.501          |
| TN-3431-1-5 | Sheet          | 0.0658| 0.640     | 24.8      | 19.6      | 0.4       | 7.84      | 8.60      | 12.50     | 3.0       | 38.8           | 41.2           | 0.0938         | 9.3             | 0.546          |
| TN-3431-1-6 | Sheet          | 0.0666| 0.645     | 24.7      | 18.2      | 0.4       | 7.28      | 5.00      | 12.50     | 3.0       | 54.0           | 55.0           | 0.1250         | 7              | 0.406          |
| TN-3431-1-7 | Sheet          | 0.0660| 0.628     | 24.2      | 19.2      | 0.4       | 7.68      | 6.10      | 12.50     | 3.0       | 51.3           | 53.0           | 0.1250         | 7              | 0.432          |
| TN-3431-1-8 | Sheet          | 0.0666| 0.635     | 24.2      | 19.2      | 0.4       | 7.68      | 7.10      | 12.50     | 3.0       | 45.4           | 48.1           | 0.1250         | 7              | 0.466          |
| TN-3431-1-9 | Sheet          | 0.0662| 0.635     | 24.4      | 19.4      | 0.4       | 7.76      | 8.30      | 12.50     | 3.0       | 42.1           | 45.3           | 0.1250         | 7              | 0.510          |
| TN-3431-1-10| Sheet          | 0.0664| 0.641     | 24.6      | 18.5      | 0.4       | 7.32      | 5.10      | 12.50     | 3.0       | 55.4           | 56.2           | 0.1563         | 5.6             | 0.369          |
| TN-3431-1-11| Sheet          | 0.0664| 0.630     | 24.2      | 19.4      | 0.4       | 7.76      | 6.20      | 12.50     | 3.0       | 53.2           | 54.6           | 0.1563         | 5.6             | 0.405          |
| TN-3431-1-12| Sheet          | 0.0660| 0.638     | 24.6      | 19.5      | 0.4       | 7.80      | 7.40      | 12.50     | 3.0       | 49.2           | 51.0           | 0.1563         | 5.6             | 0.446          |
| TN-3431-1-13| Sheet          | 0.0664| 0.633     | 24.3      | 19.2      | 0.4       | 7.68      | 8.20      | 12.50     | 3.0       | 45.8           | 47.6           | 0.1563         | 5.6             | 0.485          |
| TN-3431-1-14| Sheet          | 0.0663| 0.634     | 24.4      | 19.0      | 0.4       | 7.60      | 5.40      | 12.50     | 3.0       | 58.7           | 59.2           | 0.1875         | 4.7             | 0.351          |
| TN-3431-1-15| Sheet          | 0.0666| 0.636     | 24.4      | 19.2      | 0.4       | 7.68      | 6.10      | 12.50     | 3.0       | 55.2           | 57.0           | 0.1875         | 4.7             | 0.383          |
| TN-3431-1-16| Sheet          | 0.0666| 0.636     | 24.4      | 19.3      | 0.4       | 7.72      | 7.20      | 12.50     | 3.0       | 50.3           | 52.8           | 0.1875         | 4.7             | 0.420          |
| TN-3431-1-17| Sheet          | 0.0662| 0.631     | 24.4      | 19.4      | 0.4       | 7.76      | 8.20      | 12.50     | 3.0       | 46.8           | 50.2           | 0.1875         | 4.7             | 0.463          |
| TN-3431-1-18| Sheet          | 0.0647| 0.620     | 24.5      | 18.2      | 0.4       | 7.28      | 3.80      | 12.50     | 1.0       | 60.7           | 62.8           | 0.1563         | 5.6             | 0.324          |
| TN-3431-1-19| Sheet          | 0.0663| 0.640     | 24.5      | 19.3      | 0.4       | 7.72      | 6.60      | 12.50     | 4.0       | 53.1           | 54.6           | 0.1563         | 5.6             | 0.418          |
| TN-3431-1-20| Sheet          | 0.0665| 0.639     | 24.5      | 19.3      | 0.4       | 7.72      | 7.20      | 12.50     | 5.0       | 49.6           | 52.0           | 0.1563         | 5.6             | 0.442          |

- Fig. 15. Stiffener lateral buckling—NACA TN-1829 specimens.
- Fig. 16. Buckling curve and test data—NACA TN-3431-1-24.

Slenderness ratio of 40 to 50 is considered as the transition region and the actual strength is much smaller than in the analysis. If the local buckling stress is smaller than the stiffener lateral buckling stress, the panel resists higher stress than the local buckling stress. The local buckling stress is accurately predicted by the buckling analysis, but the failure stress is not predicted by the buckling analysis.
3.3.4. Wrinkling

Wrinkling test data of z-stiffened panels are published in NACA TN-3431 (see Table 2). The buckling analysis of this paper can be applied to the wrinkling. In the analysis, the effective rivet offset shown in Fig. 8 is used in place of the actual rivet offset dimension. Rivet offset is determined by Fig. 8 in NACA TN-3431. An example of the buckling curves of the wrinkling analysis is shown in Fig. 16. Plasticity correction is not applied to the figure. The comparison between the wrinkling stresses predicted by the analysis and the test data is shown in Fig. 17. The analysis is accurate and conservative.

4. Conclusions

(1) A new buckling analysis tool on MS-Excel for the thin-walled structures was developed. The tool is based on the energy principle.

(2) Buckling analysis of all test panels in NACA TN-1829 and some of the test panels in NACA TN-1978 was conducted. It is found that the stiffener lateral buckling is important in the short column region.

(3) Wrinkling buckling is accurately predicted by the buckling analysis using the effective rivet offset in NACA TN-3431.

References

1) Hickman, W. A. and Dow, N. F.: Data on the Compressive Strength of 75S-T6 Aluminum-Alloy Flat Panels with Longitudinal Extruded Z-Section Stiffeners, NACA TN-1829, 1949.
2) Hickman, W. A. and Dow, N. F.: Data on the Compressive Strength of 75S-T6 Aluminum-Alloy Flat Panels Having Small, Thin, Widely Spaced, Longitudinal Extruded Z-Section Stiffeners, NACA TN-1978, 1949.
3) Semonian, J. W. and Peterson, J. P.: An Analysis of the Stability and Ultimate Compressive Strength of Short Sheet-Stringer Panels with Special Reference to the Influence of Riveted Connection between Sheet and Stringer, NACA TN-3431, 1955.
4) Schuette, E. H.: Charts for the Minimum-Weight Design of 24S-T Aluminum-Alloy Flat Compression Panels with Longitudinal Z-section Stiffeners, NACA ARR No. L5F15, 1945.
5) Hickman, W. A. and Dow, N.: Direct-Reading Design Charts for 75S-T6 Aluminum-Alloy Flat Compression Panels Having Longitudinal Extruded Z-section Stiffeners, NACA TN-2435, 1952.
6) Gallaher, G. L. and Boughan, R. B.: A Method of Calculating the Compressive Strength of Z-Stiffened Panels That Develop Local Instability, NACA TN-1482, 1947.
7) Initial Buckling Stress of Flat Panels with Z-section Stringers under Compression, ESDU, 02.01.25, 1947.
8) Argyris, J. H.: Flexure-Torsion Failure of Panels—A Study of Instability and Failure of Stiffened Panels under Compression When Buckling in Long Wavelength, Aircraft Engineering, 26 (1954), pp. 174–184, 213–219.
9) Yusuff, S.: Buckling Phenomena of Stiffened Panels, J. Aerospace Sciences, 25, 8 (1958), pp. 507–514.
10) Farrar, D. J.: The Design of Compression Structures for Minimum Weight, J. Royal Aeronautical Soc., 53 (1949), pp. 1041–1052.
11) Niu, M. C.: Aircraft Structural Design, Comnilt Press, Hong Kong, 1989.
12) van der Neut, A.: Overall Buckling of Z-stiffened Panels in Compression, LR-303, Delft University of Technology, 1980.
13) Bushnell, D.: Optimization of Panels with Riveted Z-shaped Stiffeners via PANDA2, AIAA Paper 98-1990, 1998.
14) Taki’s Home Page, www.geocities.jp/toshiimi.taki/structure/buckling/ buckling.htm (accessed 2017/4/12)
15) Li, Z.: Buckling Analysis of the Finite Strip Method and Theoretical Extension of the Constrained Finite Strip Method for General Boundary Conditions, Research Report, Johns Hopkins University, 2009.
16) Washizu, K.: Variational Methods in Elasticity and Plasticity, Pergamon Press, 1968.
17) Gerard, G. and Becker, H.: Handbook of Structural Stability, Part I—Buckling of Flat Plates, NACA-TN-3781, 1957.
18) Bruhn, E. F.: Analysis and Design of Flight Vehicle Structures, S. R. Jacobs & Associates, 1973.

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