Enabling direct kinetic simulation of dense plasma plume expansion for laser ablation plasma thrusters

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Abstract

Laser ablation plasma thrusters are an emerging space propulsion concept that provides promise for lightweight payload delivery. Predicting the lifetime and performance of these thrusters hinges on a comprehensive characterization of the expansion dynamics of the ablated plasma plume. While state-of-the-art techniques for simulating plasmas are often particle-based, a grid-based direct kinetic solver confers advantages in such a transient and inhomogeneous problem by eliminating statistical noise. A direct kinetic solver including interparticle collisions is employed on a plume expansion model problem spanning one dimension each in configuration and velocity space. The high degree of thermodynamic nonequilibrium inherent in plume expansion is characterized, justifying the need for a kinetic rather than a hybrid or fluid solver. Thruster-relevant metrics such as the momentum flux are also computed. The plume dynamics are observed to be highly inhomogeneous in space with insufficient time for thermalization in the region preceding the expansion front, and the theoretical possibility of reducing the local grid resolution by up to two orders of magnitude at the far end of the domain is established. These grid-point requirements are verified via the employment of nonuniform grids of various expansion ratios, several of which also employ coarsening in velocity space. Longer domain lengths are explored to characterize thruster-scale phenomena and larger ambient pressures are simulated as a testbed to probe facility effects due to collisions with background particles.

Keywords: Laser ablation propulsion, Direct kinetic solver, Vlasov solver, Nonuniform grid, Facility effects

Introduction

The laser ablation plasma thruster is a novel advanced space propulsion concept that has emerged as a promising candidate at, but not limited to, microthrust levels [1–6]. These thrusters can be associated with high specific impulses and have the potential to be accelerated by laser sources external to the spacecraft, thus decreasing payload sizes and masses. A schematic of a laser ablation plasma thruster is provided in Fig. 1. The accelerating laser impinges on a propellant block behind the spacecraft, ablating the propellant and generating a plume normal to the propellant surface. Thruster design requires a detailed understanding of the physics of ablation and the ensuing
plasma plume expansion. In particular, the dynamics of plume expansion directly influence thruster efficiency and lifetime. The plume expansion process is sensitive to ambient conditions and inhomogeneities in the ablation process, both at the laser beam and propellant substrate levels [3, 7–9]. A robust computational tool that predicts thruster performance in a variety of plume expansion scenarios is key to the development and characterization of reliable and durable thrusters. We lay the foundations to this end by developing a direct kinetic (DK) solver that computes plume expansion in an accurate manner with increased efficiency and the ability to prototype a variety of component models for the underlying physics, such as interparticle collisions and ablated particle distributions, in a broad range of ambient operating conditions.

The setup of plume expansion is deceptively simple, for the challenge of simulating ablation plasma plume expansion lies in the broad range of physical regimes that it spans. Near the substrate, the plume is dense, often approaching atmospheric number densities at typical laser powers, and the corresponding plasma plume is strongly coupled. The plasma parameter is large and collisions dominate long-range interactions. At the expansion front, the plume expands into a sparsely populated or evacuated ambient, and the corresponding plasma plume is weakly coupled. The plasma parameter is small and electromagnetic fields become important for particle dynamics. Any instabilities that emerge in the domain can propagate quickly between the two regions at the electron thermal speed, far exceeding the expansion front speed that characterizes the baseline plume dynamics [10].

The state-of-the-art technique for simulating plasmas is the particle-in-cell method, where ions and/or electrons are represented by superparticles interacting in configuration space. Particle methods are susceptible to statistical noise since a sufficiently large number of superparticles is required in each computational cell to adequately sample the particle distribution function in the configuration and velocity spaces [11–13]. This is exacerbated in plume expansion problems that span a broad range of baseline number densities due to the spatial inhomogeneity of the setup, as well as mean axial speeds due to ambipolar acceleration by fast electrons. Such statistical noise is eliminated in DK methods, where the distribution function is directly simulated in these spaces without discretization into superparticles. This work focuses on justification and setup of efficient application of the DK method to a simplified plume expansion problem.
Grid-based kinetic solvers, also known as DK, continuum kinetic, discrete velocity, and Vlasov solvers, are an increasingly relevant method of high-fidelity plasma simulation. Since they eliminate the statistical noise associated with particle-in-cell methods, they are inherently more suitable for transient, multiscale, and spatially inhomogeneous problems with low-noise-floor requirements, which constitute many complex natural systems and modern engineering challenges. These solvers involve the direct Eulerian simulation of the particle distribution function according to the Boltzmann or Vlasov equations (depending on whether interparticle collisions and the self-consistent electric field are included), which describe the kinetic transport of this probability density function in the configuration (physical) and velocity spaces, as reviewed in detail by Refs. [14, 15]. In particular, the DK method differs from typical Vlasov solvers in that interparticle collisions are included. To date, DK solvers have been implemented both in a hybrid sense with fluid schemes [16], as well as in a fully kinetic setting [17], with applications to nuclear fusion [18], space physics and astrophysics [19], and space propulsion and materials processing [15]. The DK method is suitable for unsteady problems where time averaging cannot be used, as well as quasisteady problems with nascent instabilities where the amount of time averaging required to resolve the instabilities can become prohibitive. This grid-based method is also superior to particle methods in regions with lower particle densities, such as the expansion front in ablation plumes. The tradeoff is the higher dimensionality associated with direct solution of the Vlasov and Boltzmann equations. This work highlights the nonequilibrium nature of plume expansion in order to justify the use of a kinetic solver, and explores avenues to reduce the computational cost of the DK method through the employment of a suitable nonuniform grid, building on the grid-point requirements identified in our earlier work [20]. A nonuniform grid enables the employment of a DK solver to thruster-scale domains. In addition, thruster-relevant investigations such as the computation of momentum flux and the nascent characterization of facility effects are carried out.

With these objectives in mind, the paper is structured as follows. In the “Methodology” section, the methodology adopted in this work is introduced, including details of the employed solver, numerical setup of the expanding plume, and a brief description of the key plume dynamics. Results are discussed in the “Results & Discussion” section, including characterization of thermodynamic nonequilibrium and spatial inhomogeneity, computation of the momentum flux, and discussion of nonuniform grid design accounting for previously identified grid-point requirements, which are used as reference points in a comparison of uniform and nonuniform grid results. Preliminary investigations into thruster-scale domain lengths and the characterization of facility effects through variations in the ambient pressure are also discussed! Conclusions are provided in the “Conclusions” section.

**Methodology**

**Solver description: Vlasov-Poisson-BGK equations**

The grid-based DK solver employed here was originally developed at the University of Michigan with extensive verification and validation against canonical and complex plasma problems [15, 21–26]. Under the electrostatic approximation, the solver
computes the time evolution of the probability density function, \( f_\ast \), for some particle type \( \ast \) according to the following transport equation

\[
\frac{\partial f_\ast(x, v; t)}{\partial t} + v \cdot \nabla_x f_\ast(x, v; t) + \frac{q_\ast E}{m_\ast} \cdot \nabla_v f_\ast(x, v; t) = S(f_\ast),
\]

(1)

where \( x, v, \) and \( E \) respectively denote the spatial displacement, velocity, and electric field vectors, \( m_\ast \) and \( q_\ast \) respectively denote the mass and charge of the simulated particle type, \( t \) denotes the time coordinate, and \( S \) represents a collision term that is set to zero in a collisionless simulation, yielding the Vlasov equation, and is otherwise modeled. Multiple particle types and species may be simultaneously simulated with appropriate cross-collision terms. The computational domain is discretized in \( x-v \) space with a finite-volume method, and advection is performed to second-order accuracy using the monotonic upstream-centered scheme for conservation laws (MUSCL) with Strang splitting and a modified Arora-Roe limiter to preserve positivity. Gauss’s law is expressed using \( E = -\nabla_x \phi \) as a Poisson equation for the electric potential \( \phi \)

\[
\nabla_x^2 \phi = -\frac{e(n_i - n_e)}{\varepsilon_0},
\]

(2)

where \( \varepsilon_0 \) and \( e \) respectively denote the vacuum permittivity and electric charge, and \( n_i \) and \( n_e \) respectively denote the ion and electron number densities

\[
n_i(x; t) = \int_v f_i(x, v'; t) \, dv',
\]

(3)

\[
n_e(x; t) = \int_v f_e(x, v'; t) \, dv'.
\]

(4)

All collisions in this work are modeled using the Bhatnagar-Gross-Krook (BGK) operator

\[
S(f_\ast) = \frac{f^\ast_{eq} - f_\ast}{\tau},
\]

(5)

where \( f^\ast_{eq} \) is the equilibrium (Maxwellian) distribution for the simulated particle type and \( \tau \) is a characteristic collision time. The simulations are parallelized using the Message Passing Interface, with domain decomposition performed solely in configuration space to avoid communication during the computation of Eqs. (3) and (4), and the SuperLU_DIST library implemented with the help of the PETSc toolkit for direct solution of Eq. (2). Electron advection and collisions are substepped to speed up the simulation runtime. The current version of the solver has been tested with the physical plume setup used in this work on up to 432 computational cores with at least 80% weak scaling efficiency over an order-of-magnitude variation in core count. For these weak scaling tests, the spatial-cell-per-core ratio and the number of velocity cells per spatial location both exceed 100. The maximum baseline simulation size tested is about 40 million cells per particle type with a wall time of about 16 hours on 288 Intel Skylake cores [20], and the chief scaling bottleneck is the parallel Poisson solver. While the longer-domain simulations in the “Extended domain length with spatial coarsening” section have fewer cells, their wall time can approach 2 weeks on up to 432 Skylake cores due to correspondingly
longer simulation durations $t_{\text{sim}}$. Our DK solver has the capability to handle a nonuniform grid in all dimensions to alleviate costs as discussed in the “Comparison of uniform and nonuniform grid simulations” section.

**Plume setup**

This work primarily involves an unsteady, spatially nonperiodic, and weakly collisional plume expansion model problem involving one dimension each in space and velocity (1D1V) for ease of numerical prototyping. A schematic of the physical and computational setups corresponding to the model problem is depicted in Fig. 2. In a physical setting, an impinging laser ablates material from a target surface, forming a warm dense plume that is often almost as dense as the standard atmosphere, which then expands away from the surface. The plume expansion often occurs in ambient surroundings with low particle density, such as rarefied or evacuated outer space. The left boundary of the computational domain is set in the middle of this plume near the target surface, such that the left end of the domain contains a high-density partially ionized quasineutral gas at local thermodynamic equilibrium, and inlet boundary conditions are imposed on the left with gas of the same composition entering the domain. The computational boundary is offset some distance from the physical wall to obviate the need to resolve the wall sheath. The right boundary of the computational domain is set sufficiently far away from this plume, such that the right end of the domain contains a low-density gas of a cooler temperature, and outlet boundary conditions are imposed on the right with no backflow permitted. These boundaries are modeled by homogeneous Neumann conditions at both domain boundaries for the electric potential with a fixed value on the left boundary. While the expanding and ambient gases are typically of different species in a physical setting, they are modeled by the same gas species (aluminum) in this work for simplicity.

The quantitative initial and boundary conditions of the simulation are as follows. The domain is initialized with a partially ionized aluminum gas that has a higher particle density on the left near the inlet. The initial ($t = 0$) spatial variation of the particle distribution function for each of the considered particle types (neutral, ion, electron) is given by

![Fig. 2](image_url) Schematic of the physical and computational setups for the plume expansion problem in configuration (physical, $x$) space
where \( x \) and \( v \) respectively denote the axial and velocity coordinates, the 1 and 2 subscripts respectively denote the reference distributions at the left and right ends of the domain, \( L \) denotes the domain length, and \( x_c = 0.2L \) denotes the location where the transition between these two reference distributions occurs. A hyperbolic tangent smoothing profile is introduced to allow the underlying grid to resolve the transitions in particle density and temperature, and prevent the generation of numerical instabilities at the transition location. All reference particle distributions are Maxwellian, i.e.,

\[
f_{s,1}(v) = n_{s,1} \frac{n_{s,1}}{\sqrt{2\pi \nu_{th,s,1}} \exp \left( -\frac{v^2}{2 \nu_{th,s,1}^2} \right)},
\]

\[
f_{s,2}(v) = n_{s,2} \frac{n_{s,2}}{\sqrt{2\pi \nu_{th,s,2}} \exp \left( -\frac{v^2}{2 \nu_{th,s,2}^2} \right)},
\]

where \( \nu_{th,s,j} = \sqrt{k_B T_j/m_s} \) denotes the reference thermal velocity of each particle type, \( k_B \) is the Boltzmann constant, and \( n_{s,j} \) and \( T_j \) respectively denote the reference number density and temperature at both ends of the domain. All considered cases are initialized with the following baseline reference number density and temperature ratios

\[
\frac{n_{i,1}}{n_{i,2}} \sim \frac{n_{e,1}}{n_{e,2}} \sim \frac{n_{n,1}}{n_{n,2}} = 10^{10},
\]

\[
\frac{T_1}{T_2} = 20,
\]

\[
\frac{n_{i,1}}{n_{n,1}} = \frac{n_{e,1}}{n_{n,1}} = 3.24437 \times 10^{-2},
\]

where the left reference temperature is \( T_1 = 6,000 \text{ K} \), which is commonly associated with nanosecond laser pulses traditionally used in ablation experiments. The ratio (11) is chosen to match the ionization state for a gas with neutral number density \( n_{n,1} = 10^{25} \text{ m}^{-3} \) at temperature \( T_1 \), and the neutral–neutral ratio in Eq. (9) corresponds in turn to a characteristic ambient density \( n_{n,2} \) for low Earth orbit. For the densities, baseline domain length, and baseline simulation duration considered in this work, \( \lambda_{mfp}/L \sim t_{mfp}/t_{sim} \sim 1 \), where \( \lambda_{mfp} \sim (n_{n,1} d_{\text{atom}}^2)^{-1} \) and \( t_{mfp} \sim \lambda_{mfp}/\nu_{th,n,1} \) respectively denote the neutral–neutral collisional mean free path and time, and \( d_{\text{atom}} \sim 10^{-10} \text{ m} \) is an estimate for the atomic diameter.

The initial number density profiles of the various particle types, normalized by the left reference neutral number density \( n_{n,1} \), are plotted in Fig. 3. Number densities and lengths are hereinafter nondimensionalized by their left reference value \( n_{s,1} \) and the Debye length \( \lambda_{D,1} = \sqrt{\varepsilon_0 k_B T_1/(2n_{i,1} e^2)} \) associated with the relevant reference values, respectively. A list of these and other reference quantities used for nondimensionalization is provided in
Table 1. All variables and plot axes are nondimensional from this point on unless otherwise stated, but reference dimensional values are provided in the corresponding figure captions for ease of interpretation.

For the baseline simulations of this work, \( L = 4,000 \) and thus 99\% of the dip in the hyperbolic tangent function occurs across about \( O(100) \) Debye lengths. We discuss the motivation behind this initial condition in more detail in Ref. [20]. The baseline total simulation duration is \( t_{\text{sim}} = 288 \). At the inlet, Maxwellian particle distributions are injected at \( T_1 \) matching the interior temperature. However, the following inlet particle number densities \( n_{i,0} \) are used instead of \( n_{i,1} \) to maintain quasineutrality near the inlet

\[
n_{n,0} = n_{n,1}, \quad \tag{12}
\]

\[
n_{i,0} = n_{i,1}, \quad \tag{13}
\]

Table 1. Summary of reference quantities used for nondimensionalization. \( \phi_{\text{st},1} \) denotes the left reference thermal potential

| Quantity             | Symbol | Characteristic scale |
|----------------------|--------|----------------------|
| Number density       | \( n \) | \( n_{n,1} \) |
| Velocity             | \( \nu \) | \( \nu_{\text{st},1} = \sqrt{k_B T_1/m_n} \) |
| Position, length     | \( x, L \) | \( \lambda_{\text{D},1} = \sqrt{\varepsilon_0 k_B T_1/(2n_i e^2)} \) |
| Time                 | \( t \) | \( \lambda_{\text{D},1}/\nu_{\text{st},1} = \sqrt{m_i\varepsilon_0/(2n_i e^2)} \) |
| Electric potential   | \( \phi \) | \( \phi_{\text{st},1} = k_B T_1/e \) |
| Electric field       | \( E \) | \( \phi_{\text{st},1}/\lambda_{\text{D},1} \) |
where $\Delta t_{\text{sub}}$ is the simulation time step used for electron substepping. The expression (14) mimics the action of a proportional-integral controller with integral gain $K_i = 0.1$ and is further discussed in Ref. [20]. The extent of the velocity domain is $[-6, 20]$ for neutrals and ions, and $[-6, 6]$ for electrons. The extended positive range for ions is to account for ion acceleration. Otherwise, six thermal velocities in each direction can capture the bulk of the distribution of a particle population with zero mean velocity. Neutral–neutral, ion–neutral, and electron–neutral momentum-exchange collisions are included using a BGK collision model. Charge-exchange collisions are not considered here since the model problem is meant to mimic a physical setup where the ablated and background gases are different. The initial spatial Courant number based on the maximum resolved ion speed is set at 0.9. The Courant number is further adjusted on the fly through appropriate substepping to account for large instantaneous electric fields, which restrict the numerical stability condition for explicit advection along the velocity dimension. All mass ratios employed in this work take their actual physical values. Simulation parameters are summarized in Table 2.

Plume dynamics

Figure 4 plots the number density and electric field profiles at two time instances ($t = 72$ and 288). In the simulations corresponding to these plots, the nondimensional grid size is $\Delta x = 1/2$ in configuration space and $\Delta v = 1/72$ in velocity space for each particle type. At this grid resolution, both low-order macroscopic quantities and particle distribution functions are converged to $O(0.1\%)$ [20]. Right after initialization, electrons expand more quickly due to their higher thermal velocity and accelerate ions forward as a result. A quasineutral region of ions and electrons forms behind the expansion front. After some time has elapsed, a double layer forms at the expansion front as evidenced in Fig. 4(b). The secondary peak in the electric field profile to the left of the primary peak at late times is a direct consequence of interparticle collisions and is not present when collisions are reduced (not shown here) [20]. The dip in the density profiles at the right edge of the domain is due to the lack of backflow and does not significantly impact the results in the interior of the domain at the presented times.

Table 2  Summary of simulation parameters

| Quantity                              | Value        |
|---------------------------------------|--------------|
| Baseline simulation length $L$        | 4000         |
| Baseline simulation duration $t_{\text{sim}}$ | 288         |
| Velocity domain for ions and neutrals | $[-6, 20]$  |
| Velocity domain for electrons         | $[-6, 6]$   |
| Initial spatial Courant number        | 0.9          |
| Left reference temperature $T_1$, K   | 6,000        |
| Ion mass, amu                         | 26.98        |
| Electron mass, amu                    | $5.486 \times 10^{-4}$ |
Figures 5 and 6 plot the ion and electron velocity distribution functions (VDFs) at the same two time instances. Hereinafter, all particle distributions are divided by the magnitude of their left reference charged-particle number density, \(n_{i,1}\) = \(n_{e,1}\). The ion distributions are highly non-Maxwellian, particularly close to the expansion front,
whereas the electron distributions seem to largely retain their Maxwellian behavior throughout the simulation upon a first inspection. A more detailed analysis of the distributions will be carried out in the “Thermodynamic nonequilibrium” section.

Results & Discussion

Thermodynamic nonequilibrium

The determination of thermodynamic nonequilibrium begins with an assessment of the deviation of the particle distribution function from a Maxwellian. Figures 7 and 8 plot $x$-slices of the ion and electron distribution functions at an upstream axial location, near the double-layer location, and slightly downstream of the double layer at the same two time instances considered in the “Plume dynamics” section. On these linear–logarithmic axes, a Maxwellian distribution presents as a parabola. Panels (a) and (b) of both figures indicate that the distribution remains largely Maxwellian in the plume interior as expected. Panels (c) and (e) of both figures confirm that the downstream ion distribution is highly non-Maxwellian, with the left and right peaks arising from the background and fast ion populations, respectively, alongside the presence of broadened tails due to interparticle collisions. Panels (d) and (f) of both figures indicate that while the
electron distribution remains close to a Maxwellian as suggested by Figs. 5 and 6, one may observe non-Maxwellian features arising from the preference for forward-moving electrons at the expansion front, as well as the retardation of fast electrons by the fast ions close behind. The presence of these nonequilibrium ion and electron distribution features suggests that a simulation approach that is not completely kinetic in nature may not fully predict such nuances in the ion and electron dynamics, as well as their downstream physical implications.

To gain more insight into the nonequilibrium nature of plume expansion, we examine the higher-order moments of the ion distribution function. Figure 9 plots the ion mean axial velocity and temperature profiles at the same two time instances. Bearing in
mind the non-Maxwellian nature of the ion distribution beyond the plume interior, the “temperature” is more rigorously interpreted as a quantity proportional to the difference between the total and bulk (coherent) kinetic energies of the ions and thus represents their random (thermal) kinetic energy. Their mean axial velocity increases downstream and reaches a value of $O(10)$ thermal velocities near the expansion front. Conversely, their random kinetic energy decreases downstream by one to three orders of magnitude, although the ion “temperature” then reaches a peak of $O(10)$ times the left reference temperature beyond the expansion front. These observations suggest that preceding the expansion front, the increased coherent kinetic energy due to ambipolar acceleration has not had sufficient time to be thermalized into random kinetic energy through
interparticle collisions, bearing in mind that $\dot{\lambda}_{\text{mfp}} \sim L$ and $t_{\text{mfp}} \sim t_{\text{sim}}$, and a fluid approximation assuming local thermodynamic equilibrium is evidently inappropriate in this region. Ahead of the expansion front, ambipolar acceleration is no longer sufficient to accelerate a significant number of ions to a speed of the magnitude of several thermal
velocities, while the remaining accelerated ions still constitute a nonnegligible amount of incoherent kinetic energy that is considerable relative to their coherent kinetic energy. Note that panels (c) and (d) in Figs. 7 and 8 are close to the location of the velocity peak and temperature trough, while panels (e) and (f) in the same figures are beyond the velocity peak and close to the temperature peak. Panel (e), in particular, suggests that the large ratio of incoherent to coherent kinetic energy may be a consequence of the locally bi-Maxwellian nature of the ion velocity distribution function. Once again, the artifacts at the right domain edge are due to the lack of backflow and do not significantly impact the results in the domain interior at the presented times.

We conclude this discussion of thermodynamic nonequilibrium with an analysis of the complete energetics of the system. The ion and electron bulk and random kinetic energy profiles, as well as the electrostatic potential energy profile, are plotted in Fig. 10. Three observations may be made. First, the bulk kinetic energy of the ions is much larger than their random kinetic energy in a region between the inlet and expansion front, corresponding approximately to the region of thermal nonequilibrium identified earlier. Second, the bulk kinetic energy of the electrons never overwhelms their random kinetic energy unlike in the case of ions, which corroborates the greater tendency of electrons to thermodynamic equilibrium throughout the domain represented by their near-Maxwellian distribution. Third, the electrostatic potential energy is relatively insignificant prior to the expansion front but is a dominant contributor after the front, suggesting that electrostatic forces are a primary driver of the energetics as one would expect. This energy is matched by the electron random kinetic energy immediately ahead of the front as these forces are themselves a direct consequence of the faster thermal velocity of electrons relative to ions. This congruence diverges downstream as one departs further from the location of maximum charge separation and the most dominant electrostatic forces become more long-range in nature. The rich interplay of different energies at various plume locations lends itself naturally to a discussion of the spatial inhomogeneity of the system, which is also reflected in the momentum flux.

**Momentum flux and spatial inhomogeneity**

In addition to the particle distribution function, mean axial velocity, and various energies of the system discussed above, the momentum flux also exhibits significant inhomogeneity. This flux is of especial interest since it is directly related to the produced thrust and thus thruster performance. The ion momentum flux, \( n_i v_i^2 \), is plotted in Fig. 11 at the same two time instances considered earlier, dropping the ion mass in the flux expression since all ions are of the same identity. It reaches a peak magnitude of about \( 10^{-2} \) in the region of thermodynamic nonequilibrium but otherwise spans several orders of magnitude in spatial variation. This suggests that thruster performance is intimately related to the characteristic time taken for bulk kinetic energy, which provides the basis for thruster acceleration, to relax to random kinetic energy and thus increase the internal energy of the expelled ions. The computed momentum flux provides an upper bound to the achievable momentum flux in actual thrusters since plume divergence necessitates some of this flux to occur radially in a physical multidimensional plume.

It is instructive to compare the simulated momentum flux with comparable experimental measurements made with laser ablation thruster prototypes. By multiplying
the observed nondimensional momentum flux of \( O(10^{-2}) \) by the dimensional reference value of \( 1.8 \times 10^{31} \text{ m}^{-1} \), the mass of an aluminum atom \( (4.5 \times 10^{-26} \text{ kg}) \), and the area covered by a laser spot of radius 25 \( \mu \text{m} \), we obtain a dimensional thrust of \( O(10 \mu \text{N}) \). This may be compared with the microlaser-ablation plasma thruster of
Ref. [7], where thrusts of $O(10$ to $100\, \mu N)$ were realized, as well as the laser ablation plasma thruster of Ref. [27], which exhibits a comparable specific impulse and impulse bit to the thruster of Ref. [7] in its pure laser propulsion mode.

The spatial inhomogeneity identified in the “Thermodynamic nonequilibrium” and “Momentum flux and spatial inhomogeneity” sections suggests that the grid resolution...
may be relaxed downstream in locations with lower number density, although the relaxation cannot be performed indiscriminately since the major contributions to thrust occur throughout the region of thermodynamic nonequilibrium. This leads to corresponding considerations in the design of a nonuniform spatial grid to enable thruster-scale simulations.

**Nonuniform grid design based on grid-point requirements**

In Ref. [20], we observed that the grid-point requirements of a uniform-grid DK simulation scale with the thermal velocity associated with the coldest characteristic temperature, as well as the most pertinent Debye length. The former is dictated in this case by Eq. (10), in particular by the background temperature of 300 K, which is a constant of the system. Hence, in our considerations for nonuniform grid design, we focus on the requirements due to spatial variations in the latter, which is itself a function of the number density profiles outputted by the solver. Noting from the previous section that the bulk ion kinetic energy derived from electrostatic origins has mostly not had the time to be thermalized into random kinetic energy, we assume that the characteristic temperature governing the ion dynamics is effectively $T_1$ preceding the expansion front and $T_2$ ahead of it. A gross estimate of the spatial variation of the local Debye length then chiefly depends on the corresponding variation of $\sqrt{n_i}$, which is plotted in Fig. 12. The relative charge density, $(n_i - n_e)/n_i$, is also plotted in Fig. 12 for reference, since it dictates the axial location where consideration of the local Debye length is most pertinent. Figure 12(a) suggests that the local Debye length increases by about an order of magnitude relative to the left reference value at the location of peak charge density, with Fig. 12(b) indicating a slight increase to around two orders of magnitude as one proceeds downstream. Hence, in this work, we target a nonuniform grid whose spatial grid size increases by one to two orders of magnitude from the left end of the computational domain to the right for the baseline domain length.

**Comparison of uniform and nonuniform grid simulations**

While the solver grid could be made nonuniform along both the spatial and velocity coordinates, we focus on nonuniformity in space in this work for simplicity, with a brief discussion of nonuniformity in velocity in the “Baseline domain length with spatial and velocity coarsening” section.

**Baseline domain length with spatial coarsening**

A nonuniform grid is constructed by increasing the grid size beyond the halfway point of the computational domain. The grid size successively increases by a fixed compounded percentage downstream beyond this point. Two expansion ratios, defined as $(\Delta x_{i+1} - \Delta x_i)/(\Delta x_i)$, are considered: 0.1% and 0.5%. The number of spatial grid points for each expansion ratio is 5,534 and 4,614, respectively, compared to 8,000 for the corresponding uniform grid. The ratio of the maximum to the minimum grid size for each expansion ratio is 5.4 and 20.6, respectively. These are of the same orders of magnitude as the corresponding Debye-length ratio discussed in the “Nonuniform grid design based on grid-point requirements” section. Visual comparisons
of key macroscopic quantities are provided for these two ratios in Figs. 13 and 14, respectively. Visual convergence of all macroscopic quantities is satisfactory at this resolution, apart from the ion temperature in the vicinity of the double layer for the 0.5% grid, which fundamentally arises from resolution of the ion distribution peaks.
Fig. 13 Comparison of ion number density (a, normalized by $n_{i1} = 3.2 \times 10^{23} \text{ m}^{-3}$), electric field (b, normalized by $5.5 \times 10^7 \text{ V/m}$), ion mean axial velocity (c, normalized by $1.4 \times 10^3 \text{ m/s}$), ion temperature (d, normalized by 6,000 K), relative charge density (e), and ion momentum flux (f, normalized by $1.8 \times 10^{31} \text{ m}^{-1}/\text{s}^2$) for the baseline domain length ($L = 4,000 \lambda_{D,1} = 27 \mu\text{m}$) and a grid expansion ratio of $0.1\%$, all at $t = 288\lambda_{D,1}/v_{th,i,1} = 1.4 \text{ ns}$

Fig. 14 Comparison of electric field (a, normalized by $5.5 \times 10^7 \text{ V/m}$) and ion temperature (b, normalized by 6,000 K) for the baseline domain length ($L = 4,000 \lambda_{D,1} = 27 \mu\text{m}$) and a grid expansion ratio of $0.5\%$, all at $t = 288\lambda_{D,1}/v_{th,i,1} = 1.4 \text{ ns}$
**Extended domain length with spatial coarsening**

The dimensional domain length of the baseline simulations is 27 µm, which is somewhat removed from actual device dimensions. The nonuniform grid capability discussed above enables domain lengths that approach practical thruster dimensions more closely. Here, the domain length is extended to \( L = 4 \times 10^4 \), \( L = 4 \times 10^5 \), and \( L = 4 \times 10^6 \), keeping the transition location at \( x_c = 800 \) for the first two cases and \( x_c = 8,000 \) for the third case, and continuing to increase the grid size from \( x = 2,000 \) onwards. These correspond to dimensional lengths of 0.27 mm, 2.7 mm, and 2.7 cm, and bring the domain length up to \( O(10^1) \), \( O(10^2) \), and \( O(10^3) \) mean free paths, respectively.

To ensure that the corresponding uniform-grid simulation is tractable, a comparison between uniform and nonuniform grid results is performed only for the domain length of \( L = 4 \times 10^5 \). In addition, the uniform spatial grid resolution is relaxed to \( \Delta x = 4 \), at which low-order macroscopic quantities remain converged to \( O(0.1\%) \) but not the particle distribution function itself. A visual comparison of various macroscopic quantities is provided in Fig. 15 only for the expansion ratio of 0.1%, as the simulation corresponding to a 0.5% ratio did not converge. The number of spatial grid points is 2,854 in this case, compared with 10,000 for the uniform grid, and the ratio of the maximum to the minimum grid size is 10.5. The corresponding Debye-length ratio, discussed in the “Nonuniform grid design based on grid-point requirements” section for the baseline domain length, takes a value of \( O(10^2) \) at early times (not shown here) and increases to \( O(10^3) \) as evidenced in Figs. 15(a) and (e). Visual convergence of most of the quantities remains satisfactory, apart from visible oscillations in the electric field, as well as deviations in the temperature within the region of thermodynamic nonequilibrium, which are expected due to the grid-point requirements necessary for convergence of these higher-order quantities [20].

With this satisfactory convergence in hand for an extended domain, we proceed to extend the domain further to \( L = 4 \times 10^6 \), also with an expansion ratio of 0.1%. The number of spatial grid points is 5,109 in this case, compared with what would have been 100,000 for the corresponding uniform grid, and the ratio of the maximum to the minimum grid size is 100.4. The corresponding Debye-length ratio takes a value between \( O(10^2) \) and \( O(10^3) \) at early times and up to \( O(10^4) \) at late times (not shown here). Key macroscopic quantities are plotted in Fig. 16. Several distinctive features
are present relative to the results from the baseline domain length simulations. While panel (a) indicates bunching of all particles in the left one-tenth of the domain where collisionality is expected to be most pronounced, panel (b) reflects the sustained depletion of ion random kinetic energy throughout a significant portion of the domain up to the inlet, indicating that incomplete thermalization is occurring even tens to hundreds of mean free paths behind the expansion front. The inlet mean axial velocity is nonzero as evidenced in panel (c), and the corresponding ion momentum flux also peaks near the inlet in panel (d), rather than near the expansion front in the baseline simulations. The flux still peaks at a magnitude of about $10^{-2}$, but it almost monotonically decays downstream by several orders of magnitude, since the growth of the mean axial velocity is linear while the decay of the number density is near-exponential. Although the broad strokes of the profiles in Figs. 15 and 16 are similar, the varying importance of collisionality across the domain precludes complete self-similarity of the process.

Finally, we proceed to extend the domain further to $L = 4 \times 10^6$, this time with an expansion ratio of 0.02% for the first half of the domain and then 0.01% for the next half. The number of spatial grid points is 27,597 in this case, compared with what would have been $10^6$ for the corresponding uniform grid, and the ratio of the maximum to the minimum grid size is 127.8. The corresponding Debye-length ratio takes a value between $O(10^3)$ and $O(10^4)$ at early times, and between $O(10^4)$ and $O(10^5)$ at late times (not shown here). Key macroscopic quantities are plotted in Fig. 17.
trends in Figs. 16 and 17 are qualitatively similar, but subtle quantitative differences exist, again precluding complete self-similarity of the process even in the far field of the thruster.

**Baseline domain length with spatial and velocity coarsening**

In addition to spatial coarsening, velocity coarsening may also be used to reduce the number of computational cells. While the characteristic velocity scale to be resolved is predetermined by the prescribed species temperatures, the velocity grid spacing can be coarsened at the domain edges since the distribution typically falls off quickly in magnitude at these extremes and comparable resolution often becomes less crucial. Here, the electron grid size is increased in both directions beyond the zero-velocity centerline, while the ion grid size is increased for all negative velocities, as well as positive velocities above six thermal velocities. An expansion ratio, defined as $(\Delta v_{i+1} - \Delta v_i) / (\Delta v_i)$, of 5% is considered, reducing the number of electron grid points from 432 to 102 and the number of neutral and ion grid points from 936 to 403. The ratios of the maximum to the minimum grid size for electrons and neutrals/ions are 10.7 and 20.1, respectively. Visual comparisons of key macroscopic quantities are provided in Fig. 18. Visual convergence of all macroscopic quantities is mostly satisfactory at this resolution with observable deviations in the momentum flux near the inlet, as well as the electric field and particularly the ion temperature near the expansion front.
Characterization of facility effects

An important step in reconciling field and laboratory observations is determining the influence of facility effects, in particular the role of background pressure in thruster performance. Computational solvers can bridge the two through the development of suitable numerical experiments. Here, we consider the direct modification of the ambient pressure in the baseline simulations using

$$\frac{n_{e,1}}{n_{i,1}} \sim \frac{n_{e,2}}{n_{i,2}} \sim \frac{n_{n,1}}{n_{n,2}} = 10^4. \quad (15)$$

Due to the higher ambient pressure, the modified simulations enforce (12)–(14) at the right boundary as well, replacing \(n_{e,1}\) by \(n_{e,2}\) in order to prevent the development of an
outlet sheath. Figure 19 plots a comparison of several macroscopic quantities for the two considered sets of density ratios. As one would expect, more frequent collisions with the background gas retard plume expansion and suppress charge separation, as seen in panels (a) and (c), respectively. In addition, increased collisionality increases the tendency to thermodynamic equilibrium, and the ion “temperature” does not dip as severely. The momentum flux near the expansion front is also modified with potential impact on the resulting thrust measurements.

While largely beyond the scope of this work, it is instructive to surmise the physical implications of the suppression of plume expansion at longer time scales. A clue to these implications is provided by Fig. 20, which illustrates the change in the electron mean axial velocity profile with increased collisionality. In the presence of facility effects, the mean electron speed is sustained at $O(1)$ over a larger portion of the domain. Since the mean ion speed never exceeds 5% of the electron thermal velocity as evidenced in Fig. 19, the relative electron drift speed is significant and can be responsible for the localized onset of current-driven instabilities. (Recalling that the temperature loses its significance in regions where the particle distribution function is non-Maxwellian, it is not straightforward to predict the onset of these instabilities with linear theory in this setting.) While preliminary simulations of longer domains with facility effects have suggested the presence of such instabilities (not shown here), they will need to be confirmed with a grid refinement study and are deferred to future work.

![Fig. 19](image.png)

**Fig. 19** Comparison of ion mean axial velocity (a, normalized by $1.4 \times 10^3$ m/s), ion temperature (b, normalized by 6,000 K), electric field (c, normalized by $5.5 \times 10^7$ V/m), and ion momentum flux (d, normalized by $1.8 \times 10^{11}$ m$^{-1}$/s$^2$) for the baseline simulations ($\zeta = 4,000\lambda_{D,1} = 27$ µm) with a variation in the ambient pressure, all at $t = 286\lambda_{D,1}/v_{th,1} = 1.4$ ns
Conclusions

Laser ablation plasma thrusters are a promising space propulsion concept for lightweight payload delivery. Determining their lifetime and performance requires a detailed understanding of the dynamics of expansion of the ablated plasma plume. Grid-based DK solvers are an increasingly relevant method to characterize such transient, inhomogeneous, and multiscale flows due to their elimination of statistical noise. This work justifies the employment of kinetic solvers for plume expansion and explores avenues to reduce the cost of DK solvers by drawing on grid-point requirements derived in previous work [20]. Thruster-relevant metrics are discussed for a range of domain lengths.

Employment of a DK solver to a 1D1V plume expansion model problem reveals that the underlying particle distribution functions are in a highly nonequilibrium state. Contributing processes include ion ambipolar acceleration and electron retardation, as well as the preference for forward-moving particles at the expansion front. A fully kinetic code is necessary to capture the resulting non-Maxwellian features and their downstream physical implications. The ion distribution function moments indicate relatively slow thermalization. The baseline simulation plots may thus be revealing of the internal structure of the expansion front of a quasineutral plasma plume or ball. The system energetics highlight an intricate interplay between the bulk and random kinetic energies of the ions and electrons, as well as the electrostatic potential energy of the corresponding electric field, all of which vary significantly with downstream distance. The ion bulk kinetic energy is representative of the ion momentum flux, which is crucial to predicting thrust. Thruster performance can thus be sensitive to the degree of thermalization at various plume locations.

The highly inhomogeneous plume leads one to question if the DK simulation cost may be reduced by a nonuniform grid. Analysis of the local Debye length and plume charge density suggests that the grid size may be relaxed by up to two orders of magnitude for the baseline simulation length. Nonuniform grid simulations relaxing the grid size by an order of magnitude achieve satisfactory convergence. The viability of a nonuniform grid enables the simulation of longer domain lengths approaching typical thruster dimensions, and simulations of up to 1,000 times the baseline domain length and simulation duration were performed with cost savings of up to 40 times. Velocity coarsening can add further savings of 2 to 4 times. While rapid thermalization is absent in a wide

![Fig. 20](image-url) Comparison of electron mean axial velocity, normalized by $3.0 \times 10^5$ m/s, for the baseline simulations ($L = 4,000 \lambda_{D,1} = 27$ µm) with a variation in the ambient pressure at $t = 72 \lambda_{D,1} / \nu_{th,i,1} = 0.35$ ns (a) and $t = 216 \lambda_{D,1} / \nu_{th,i,1} = 1.1$ ns (b)
region behind the expansion front in these extended simulations, the varying degree of collisionality across the domain can still impede a fully self-similar description of the complete solution, especially in the highly collisional upstream region where coherent particle advancement may be enhanced.

Besides the momentum flux, a DK solver can also be valuable in determining the influence of facility effects. An increased ambient pressure decreases the peak electric field and ion mean axial velocity. The ion momentum flux near the expansion front is also modified by increased collisions with the background gas. Increased collisionality creates a sustained region with significant electron drift that may excite observable instabilities at longer time scales.

The simulations of this work suggest that thruster-scale DK simulations can be viable with the deployment of a nonuniform grid, and extensions to more realistic configurations of higher dimensionalities (e.g., 2D2V and 3D3V) are in order. The general feasibility of a nonuniform DK solver increases its viability of application to a broad range of plasma problems of interest of varying dimensionalities, with the potential to yield crucial physical insights into nonequilibrium processes and their associated energetics that elude traditional analysis methods and numerical solvers with significant statistical noise.

Acknowledgements
The authors would like to acknowledge M. Petrusky, C. Cui, A. Vazsonyi, A. Raisanen, R. Chaudhry, and P. Moin for discussions helpful to this work. This work utilized the Blanca condo computing resource of the University of Colorado, as well as the Summit supercomputer, which is supported by the National Science Foundation (awards ACI-1532235 and ACI-1532236), the University of Colorado, and Colorado State University.

Authors’ contributions
All authors contributed to the study conception and design. Code preparation, data collection, and analysis were performed by W.H.R. Chan. The first draft of the manuscript was written by W.H.R. Chan and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Funding
Not applicable.

Availability of data and materials
Available upon request.

Code availability
In-house code (see “Solver description: Vlasov-Poisson-BGK equations” section)

Declarations

Competing interests
The authors declare no competing interests.

Received: 26 September 2022   Accepted: 10 November 2022

Published online: 21 November 2022

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