Two-Stage Method Applied for Approximate Calculations
of Selected Types of Statically Indeterminate Trusses

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Received 25 August 2018
Revised 19 November 2019
Accepted 10 December 2019
Published 6 February 2020

The paper presents examples of approximate calculations of force values in members of selected types of trusses, which are at the same time an internal and external statically indeterminate systems. The two-stage method makes possible the approximate calculation of such trusses by help of, for example, the Cremona’s method. In each stage, a statically determinate truss is considered, pattern of which is defined by removing from the basic truss a suitable number of members. There are also presented results of calculations of the same trusses done by means of suitable computer software together with analyses and comparison of outcomes.

Keywords: Statically indeterminate truss; calculus of vectors; principle of superposition.

1. Introduction

The values of forces in members of the statically indeterminate trusses have to be determined by a suitable computational method, which among others takes into consideration the stiffness differences between members connected to the truss nodes. For this purposes are applied such methods like for instance, the force method, the displacement method, the iteration methods like the method of successive approximations, and the finite elements method, etc. [Timoshenko (1966); Allen et al. (2010); Makowski (1981); Kolendowicz (1993); Hibbeler (1995); Niezgodziński and Niezgodzińska (1979); Przewłocki and Górski (2006); Dylag et al. (1989); Cywiński (1976)]. Mathematic concepts of these methods are adapted in modern types of various computer software [Makowski (1981); Zienkiewicz and Taylor (2000)]. The two-stage method has been invented during initial static analysis of a certain type
of a tension-strut structure [Rebielak (2014)]. If the basic structure is overloaded then a certain number of members are excluded from process of the force transmission. Number of these members is equal to the statically indeterminacy of the basic truss system. The point of the two-stage method is to carry out static calculations in two independent stages for statically determinate trusses, shapes of which are determined by removing from area of the basic truss the number of members equal to its statically indeterminacy [Rebielak (2018a)]. In each stage it is considered an appropriate statically determinate truss, therefore values of forces in its members can be calculated by means of, for example, the Cremona’s method. In both stages, suitable trusses having the same clear span and construction depth like the basic truss are calculated, but they are appropriately loaded by forces having half values in comparison to values of forces applied to the basic truss. Final values of forces acting in all members of the basic truss are resultants of the force values determined in each stage for the suitable member. The calculation procedure of the two-stage method is justified by rules of calculus of vectors and principle of superposition. It is one of the approximate methods of calculation of the statically indeterminate systems [Tan et al. (2018)].

2. Definition of Research Problem

The correctness of the basic theoretic assumption of the two-stage method has been verified by calculating the internal statically indeterminate trusses supported on two supports, while the one is the pivot bearing and the second one is the pivot sliding bearing. It implies that there are only three unknown bearing reactions. Two forms of trusses were computed in order to prove feasibility study that the two-stage method can be useful for calculations of the internal statically indeterminate trusses being at the same time the external statically indeterminate trusses. The first one is the vertically positioned truss loaded by the means of horizontally applied forces and supported by two pivot bearings. The second one is the horizontally located truss loaded by the means of vertically and optionally applied forces, and which is supported by two pivot bearings. All the calculations are made for the appropriate geometric, structural and load conditions. Results obtained in this way are compared with outcomes gained by the application of suitable computer software for calculation of the force values in a truss having the same geometric, structural and load conditions.

3. First Subject of Static Calculation and Analysis

The assumed form of the basic truss is built by the usage of three square strut modules located vertically on each other, see Fig. 1. The truss is supported by two pivot bearings A and B and it is loaded by three concentrated forces \( F \) horizontally applied to nodes of its left vertical chord. All calculations have been made for the single load forces \( F \), each of value equal to 1 kN, applied to nodes of a basic truss. It was assumed that the construction depth of the truss tower “H” equals 1.00 meter,
while its height “$L$” is equal to 3 m. The number of nodes was assumed to denote by symbol “$w$”, while symbol “$p$” to define number of the members. The condition for the internal statically determinacy of the plane truss is determined by the following equation:

$$p = 2 \cdot w - 3.$$  \hspace{1cm} (1)

The considered basic truss system is built by number of nodes $w = 11$, what implies that the statically determinate truss created by the means of this number of nodes has to be constructed by the following number of members:

$$19 = 2 \cdot 11 - 3.$$  \hspace{1cm} (2)

The basic truss system is built by the number of members $p = 22$, what implies that the calculated structure is the threefold statically indeterminate system. It implies further that in order to create an appropriate statically determined system it is necessary to exclude three members from area of the basic truss.

According to the rules of the two-stage method in its first stage one should remove three members, for instance from the left vertical chord of the basic truss, and then to apply load forces of half values ($F/2$) to suitable nodes of this chord. In the second stage it is necessary to remove three members from the right vertical cord and, like previously, to apply load forces of half values ($F/2$) having the same senses, like in the basic truss, to corresponding nodes of the left vertical chord. Because the basic truss is supported by two pivot bearings A and B, therefore it is also the external statically indeterminate system. That is why similar operations have to be undertaken regarding the changes of statuses of the supports. In the first stage it is proposed to keep support A as the pivot bearing and to consider support B as the pivot sliding bearing, see Fig. 2. In the second stage support A is considered as the pivot sliding bearing, while support B remains the pivot bearing, see Fig. 3.
Fig. 2. The values of the forces determined in the first stage of calculations for the truss of a tower configuration with suitable Cremona’s polygon of forces.

Fig. 3. The values of the forces determined in the second stage of calculations for the truss of a tower configuration with suitable Cremona’s polygon of forces.
of forces acting in members of the basic truss are resultants of forces calculated in corresponding members at each stage. The concept of the two-stage method is compatible with the rules of calculus of vectors, principle of superposition and respects the three fundamental conditions of equilibrium presented below:

\[ \sum_{i=1}^{n} F_{ix} = 0, \quad (3) \]
\[ \sum_{i=1}^{n} F_{iy} = 0, \quad (4) \]
\[ \sum_{i=1}^{n} M_i = 0. \quad (5) \]

The results of the calculations of the basic truss, having a tower configuration, see Fig. 1, obtained in each stage of the two-stage method by application of the Cremona’s method, are presented respectively in Figs. 2 and 3. Final values of the forces, defined in this method, in all members of the basic truss are shown in Fig. 4(a).

The same vertical, tower configuration of the basic truss has been subjected to the static calculation carried out by the application of the Autodesk Robot Structural Analysis Professional 2019. The computer software is considered to be the very precise tool for calculation of the force values acting in members of the statically indeterminate systems. Static calculations were made by the assumption that the truss consists of the steel tubular members having diameter of 30 mm,

![Diagram](https://example.com/diagram.png)

Fig. 4. Comparison of the values of forces in members for the tower truss configuration calculated: (a) in the two-stage method and (b) by the means of computer software.
the thickness of the section equals to 4 mm and the steel material has the Young’s modulus equal to 210 GPa. The material and cross-sectional parameters are identical to those considered in papers [Rebielak (2018a,b)]. In these calculations, as well as previously, only the values of the load forces are taken into account, while the weight of the nodes and members are omitted. The results of the computer calculations of the same truss system are presented in Fig. 4(b).

In the two-stage method the final values of forces acting in particular members are calculated according to the rules of the calculus of forces and to principle of superposition. For instance the final value of a compressive force in the cross brace located between node 3 and node 6 is equal to $-1.41 \text{kN}$, see Fig. 4(a). It is a resultant of the tensile force $+0.707 \text{kN}$ determined in the first stage in corresponding member located between nodes of the same numbers, see Fig. 2 and the compressive force $-2.121 \text{kN}$ determined in the second stage, see Fig. 3. In similar way the final force is defined in vertical member placed between node 3 and node 2. In the first stage of calculation this member has been rejected from the basic truss, see Fig. 2, therefore it is assumed, that in this case the appropriate force value equals 0.000 kN.

In the second stage the value of tensile force defined in corresponding member is equal to $+2.000 \text{kN}$, see Fig. 3. Therefore the final force value in member located between nodes 2 and 3 equals $+2 \text{kN}$, see Fig. 4(a).

From the comparison of the force values gained in both methods for the same truss members follows that in general the results are congruent to each other. For instance value of a compressive force determined in the member placed between node 3 and node 9 by application of the two-stage method is equal to $-0.50 \text{kN}$. The force value defined in the same member by application of the computer software mentioned above equals $-0.23 \text{kN}$. The relative difference equals up to 54% of the bigger value. One can notice a smaller differentiation between values of forces calculated in two different methods for the same member e.g. in vertical member placed between node 2 and node 1. The value of the tensile force calculated for this member in the two-stage method equals $+4.50 \text{kN}$, while by applying of the computer software it is equal to $+4.55 \text{kN}$. In this case, the relative difference is equal to only ca. 1% of the bigger value. It is worth noting that that values of all types of the suitable bearing reactions determined in both compared methods are the same or they are of very approximated values.

4. Second Subject of Static Calculation and Analysis

The scheme of the basic truss presented in Fig. 5 has been selected as an object of further comparative investigation in order to estimate the usefulness of the two-stage method for calculation of all types of the planar internal and external statically indeterminate systems. The basic truss is of similar structure like the previous one but it is located horizontally and supported by two supports, both being the pivot bearings. That is why four unknown bearing reactions have to be considered in these supports. It implies that the basic truss, like the previous one, is the threefold
Fig. 5. The initial concept of the two-stage method applied for the calculation of horizontally positioned statically indeterminate truss supported in two pivot bearings.

 internal indeterminate system and once-fold external indeterminate system. Moreover, one should be aware that the horizontal components of the bearing reactions for the basic truss will have mostly the vector senses shown in Fig. 5, this follows from the principles of equilibrium of the outer statically indeterminate systems.

The basic truss is loaded by the concentrated forces, which may be applied at any directions, what can be represented by the direction of force $F_1$. This force is inclined at an angle of $45^\circ$ towards the horizontal line. According to the rules of the two-stage method presented above, in its first stage it is calculated truss, which form is obtained by removing three members of e.g. the bottom chord from the pattern of the basic truss. The shape of truss considered in the second stage is a result of the cancelation of three members of the top chord from the basic truss. In both stages the calculated truss is loaded by forces of the half values applied in appropriate way to the suitable truss system. The results of calculations obtained in the first stage are shown in Fig. 6. The values of the forces defined in the second stage of calculations are presented in Fig. 7. The final values of forces calculated in the two-stage method are presented in Fig. 8(a). The results obtained by the application of suitable computer software are shown in Fig. 8(b).

One can notice significant differences of the values and even of senses of the force determined in the same members in two compared methods of calculations. First, the values and senses of the horizontal components of the bearing reactions

Fig. 6. The values of the forces defined in the first stage of the calculations for horizontally positioned truss with suitable Cremona’s polygon of forces.
Fig. 7. The values of the forces defined in the second stage of the calculations for horizontally positioned truss with suitable Cremona’s polygon of forces.

determined in the two-stage method, see Fig. 8(a), are completely different than calculated in the exact method, see Fig. 8(b). Moreover by application of the two-stage method value of tensile force in a member located between nodes 3 and 6 equals +0.10 kN, while by applying of the computer software it was defined as a compressive force of value equal to −0.15 kN. Much smaller difference is noticed between compressive force value defined in a member placed between nodes 3 and 2 by the help of the two-stage method, which equals −1.18 kN, while the outcome of computer software defines it as a compressive force having value of −1.03 kN. One can observe the substantial differentiations in values of forces calculated by the application of compared methods in numerous members of the truss. For example, in the cross brace placed between nodes 2 and 5 in the two-stage method it is calculated as a tensile force of value equal +0.30 kN, while in the same member a force calculated by means of computer software is defined as the compression force of the value equal to −0.54 kN. The values and senses of the vertical components of the bearing reactions calculated in both methods are equal. To the most significant differences one has to count the differentiation of values and senses of the horizontal components of the bearing reactions. These reactions, defined in the
two-stage method, are of equal values and both have same senses, see Fig. 8(a), what is directly determined by the basic principles of this method. Because the investigated truss is also the once-fold external indeterminate system, therefore the real horizontal components of the bearing reactions are of the values and senses presented in Fig. 8(b). It implies that the horizontal components of these reactions have to be calculated in another way by application of the two-stage method.

5. Proposal of Approximate Calculation of the Horizontal Components of Bearing Reactions

In order to recognize more precisely the features of the two-stage method it is necessary to carry out a static calculation of the horizontally positioned truss built by the means of a larger number of members and loaded by the forces applied at various directions. The scheme of such a truss is shown in Fig. 9(a) which implies, that it is a once-fold external statically indeterminate system. It is built from a number of members \( p = 36 \) and number of nodes \( w = 17 \). The internal statically determined truss containing 17 nodes has to be built by the means of 31 members which defines Eq. (1). Therefore the basic truss is the fivefold internal statically indeterminate system. It consists of five horizontally positioned square modules located inside a rectangular field bordered by the corner nodes A–D, its clear span equals 5 m, its construction depth like previously is equal to 1 m. The truss is supported by the pivot bearings A and B, while it is loaded by forces applied to the top chord. The two concentrated forces are inclined at an angle of 45° towards the horizontal line and they are separately applied to the second and the third node of the top chord, counting from left-hand side towards the node D. Two other load forces are directed vertically, respectively to the forth and the fifth node of this chord. All the load forces have the same absolute value equal to 1 kN. The values of the vertical components of bearing reactions, \( V_a \) and \( V_b \) can be easily determined by the application of the three fundamental conditions of equilibrium (3)–(5), which have to be obeyed in the two-stage method.

\[
\sum M_a = 0, \quad (6)
\]

\[
F_{1x} \cdot 1.0m + F_{1y} \cdot 1.0m + F_{2x} \cdot 1.0m + F_{2y} \cdot 2.0m + F_3 \cdot 3.0m + F_4 \cdot 4.0m - 5V_b = 0, \quad (7)
\]

\[
V_b \approx 2.11 \text{kN}. \quad (8)
\]

If the same condition of equilibrium is applied towards the support B, then one can calculate, in an equally simple and easy way, the vertical component of the bearing reaction acting in pivot A:

\[
\sum M_b = 0. \quad (9)
\]

\[
F_{1x} \cdot 1.0m - F_{1y} \cdot 4.0m + F_{2x} \cdot 1.0m - F_{2y} \cdot 3.0m - F_3 \cdot 2.0m + F_4 \cdot 1.0m + 5V_a = 0, \quad (10)
\]

\[
V_a \approx 1.31 \text{kN}. \quad (11)
\]
Fig. 9. (a) Structural configuration of the calculated truss, (b) schemes of the computation procedure of the calculation of the horizontal components of bearing reactions, (c) system of horizontal components of forces real applied to the basic truss, and (d) analysis of equilibrium of horizontal components of the forces.

From analysis of all vertical components of forces applied to the investigated truss follows, that the second condition of equilibrium (4) is fulfilled.

As it is initially pointed out in the two-stage method it is also obligatory to keep the three fundamental conditions of the equilibrium in calculations of horizontal components of the bearing reactions. Two different ways of their calculation were considered, in both of them the horizontally positioned truss supported in two extreme nodes, like it is shown in Figs. 8 and 9, has been pondered as appropriately supported and loaded cantilever truss. In the first one there is considered a
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balance of only horizontal components of all the load forces and the bearing reactions. The second one is more general because it gives more exact results and it can be applied for all types of application of the load forces. Below is presented a proposal of the second way of calculation of the horizontal components of the bearing reactions. The investigated structural configuration of the appropriate truss system, see Fig. 9(b), resembles to some degree the truss schemes shown in Fig. 1. Taking similar assumptions following from the principle of superposition it is considered that the basic truss is at the same time supported in all the corner nodes A, B, C and D. In the basic truss the corner nodes A and B are the pivot bearing supports, see Fig. 9(a), therefore in the investigated structural configuration they remain the pivot supports. In the basic truss the corner nodes C and D are not the support nodes, see Fig. 9(a), that is why in the investigated structural configuration they are considered as the pivot sliding bearings, where are applied only horizontal components of reactions $H_c$ and $H_d$, see Fig. 9(b). Because at the corner nodes C and D of the basic truss, being nodes of the top chord, are not applied any horizontal forces therefore for a selected point placed on line of the top chord there will be defined the suitable equation of equilibrium (5). For instance, the point G situated in the middle of the top chord, see Fig. 9(b), can be an appropriate point. The equilibrium equation for the right side of point G is as follows:

$$\sum M_{Gr} = 0,$$

$$F_3 \cdot 0.5m + F_4 \cdot 1.5m - V_b \cdot 2.5m + H_b \cdot 1.0m = 0,$$

$$H_{br} = 3.275 \text{kN}.$$  

(12)  
(13)  
(14)

Similar equilibrium equation, defined now for the left-hand side of point G, is as follows:

$$\sum M_{Gl} = 0,$$

$$-H_a \cdot 1.0m + V_a \cdot 2.5m - F_{1y} \cdot 1.5m - F_{2y} \cdot 0.5m = 0,$$

$$H_{ar} = 1.275 \text{kN}.$$  

(15)  
(16)  
(17)

From the analysis of the values of all the horizontal components of forces, see Fig. 9(c), applied to the nodes of the basic truss follows, that the first condition of equilibrium (3) is not fulfilled. In order to keep this condition an additional horizontal force $F_s$, see Fig. 9(d), has to be taken into consideration:

$$H_{ar} + F_{1x} + F_{2x} + F_s - H_{br} = 0.$$  

(18)

The value of the unbalanced horizontal force $F_s$ is determined by solving of the below equation:

$$F_s = H_{br} - H_{ar} - F_{1x} - F_{2x},$$  

$$F_s = 3.275 \text{kN} - 1.275 \text{kN} - 0.707 \text{kN} - 0.707 \text{kN},$$  

$$F_s = 0.586 \text{kN}.$$  

(19)  
(20)  
(21)
Taking into assumption that the unbalanced force $F_s$ is in an equally way distributed to supports A and B, then its half value ($F_s/2$) which in this particular case equals 0.293 kN, will be suitably added to values of $H_{ar}$ and $H_{br}$. Keeping the rules of calculus of vectors the final values of horizontal components of the bearing reactions will be determined in the following way: One has to point out again that the values of horizontal components of the bearing reactions calculated in this way are the approximate values.

$$H_a = H_{ar} + F_s = 1.275 \text{kN} + 0.293 \text{kN} = 1.568 \text{kN}, \quad (22)$$

$$H_b = H_{br} - F_s = 3.275 \text{kN} - 0.293 \text{kN} = 2.982 \text{kN}. \quad (23)$$

One has to point out again that the values of horizontal components of the bearing reactions calculated in this way are the approximate values.

6. Static Calculations and Comparative Analysis of Received Results

The values of horizontal and vertical components of the bearing reactions computed in this way are applied in the two-stage method in a procedure of calculation of

Fig. 10. Force values computed in the first stage of calculations for suitably defined values of bearing reactions of the basic truss together with Cremona’s polygon of forces.
Fig. 11. Force values computed in the second stage of calculations for suitably defined values of bearing reactions of the basic truss together with Cremona’s polygon of forces.

the basic truss of the static scheme presented in Fig. 9(a). The values of forces defined in particular members are compared with values of forces computed for the same truss by the means of a suitable computer software. As previously it was assumed to use the Cremona’s polygon of forces in the two-stage method. Intermediate results gained in this method are presented in Figs. 10 and 11. Final values of forces obtained in this approximate method of calculation are presented in Fig. 12(a). Results received in an exact method by application of software Autodesk Robot Structural Analysis Professional 2019 are presented in Fig. 12(b).

Values of forces calculated in members of the investigated basic truss are defined for the same material parameters like for the truss presented in Fig. 1 by application the computer software Autodesk Robot Structural Analysis Professional 2019. Values of vertical components of the bearing reactions in the both compared methods are the same, \( V_a = 1.31 \) kN, \( V_b = 2.11 \) kN, and they are the exact values of such forces. Horizontal components of the bearing reactions \( H_a = 1.30 \) kN and \( H_b = 2.72 \) kN defined by means of computer software, see Fig. 12(b), have
to be considered also as the exact values. In the two-stage method there are applied approximate values of the horizontal components of the bearing reactions $H_a = 1.568\,\text{kN}$ and $H_b = 2.982\,\text{kN}$. In case of $H_a$ the differentiation of the force values is up to ca. 17% towards the bigger value. From analysis of all the force values determined in the both compared methods for the same members one can notice significant differences.

The degree of differentiation is even bigger than in the results of calculations carried out by the direct application of the simple rules of the two-stage method. For instance the compressive force value computed by means of the computer software in the member located between nodes 3 and 4 equals $-2.96\,\text{kN}$, see Fig. 12(b). In the same member due to the application of the two-stage method, value of the compressive force is defined as equal to $-1.80\,\text{kN}$, see Fig. 12(a), what implies that the second value is much smaller than the first one. The member is subjected to act of the biggest compressive force in the whole truss. The relative difference of the calculated force values is equal to ca. 39% towards the bigger value and this difference can be considered as significant and moreover as too big. Similar remarks refer also regarding values and various senses of the vector forces calculated in both compared methods for the same members. For instance by application of the two-stage method in member located between nodes 5 and 10 there is defined the
compressive force equals $-1.30\, \text{kN}$, see Fig. 12(a), while in the same member by usage of a computer software there is determined the tensile force of the value equal to $+0.35\, \text{kN}$. From comparative analysis of the presented results calculated for the horizontal positioned trusses loaded by forces applied at optional directions to the truss nodes follows, that somewhat better approximate force values to the exact values of forces acting in members of the truss, one can obtain by application of the simple kind of the two-stage method.

7. Conclusion

The two-stage method is an approximate method of calculation of the statically indeterminate trusses because in both stages it applies the rules, which are appropriate for the calculation procedures of the statically determinate trusses. The degree of approximation of the obtained force values to the values of forces defined by means of the exact methods in general is good enough when the two-stage method is applied for calculation of the inner indeterminate trusses.

One can notice significantly differences between the exact and appropriate force values calculated for members, where are acting the smallest forces, especially having absolute values close to zero. However one should be aware that members subjected to the act of such forces are designed mostly according to instructions of the building codes. In these cases the cross-sectional areas of such members are much larger than they are determined directly on basis of the results of static calculations. The accuracy of the force values calculated by application of the two-stage method can be significantly improved by taking into account the differences between the stiffness of members connected in the truss nodes. It can be made by defining a set of appropriate coefficients defining differences of members connected to each particular node. When the two-stage method is applied for calculation of the external and internal statically indeterminate truss and if directions of the applied load forces are parallel to the line determined by positions of two pivot bearings, then the approximate force values are almost in exact accordance with outcomes gained by usage of suitable computer software. If this method is used in the computation processes of such trusses loaded by forces applied at optional directions then one can notice quite big differences between the force values defined in this way and the values of forces calculated by application of an exact computer method. Various possible applications of the two-stage method for calculations of different types of statically indeterminate trusses are planned to be subjects of further research in order to estimate more closely the features and the practical usefulness of this method for structural analysis.

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