Model-Independent $Z'$ Constraints from Measurements at the $Z$ Peak

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Abstract

Model independent constraints on extra neutral gauge bosons are obtained from partial decay widths of the $Z_1$ and forward–backward and left–right asymmetries at the $Z_1$ peak. Constraints on the $ZZ'$ mixing angle in $E_6$ models are considered as special cases.

The measurements at the $Z_1$ resonance at LEP 1 are in very good agreement with the SM. This agreement can be interpreted as a constraint on new physics. In particular, the experiments at LEP 1 give the best present constraints on the $ZZ'$ mixing angle $\theta_M$ [1, 2]. Because these constraints depend on the couplings of the $Z'$ to SM fermions, they are usually given for selected $Z'$ models. Typical allowed regions for $\theta_M$ in $E_6$ models are $-0.005 < |\theta_M| < 0.003$ at the 95% CL, see [3]. In these model dependent analyses, all couplings of the $Z'$ to the SM fermions are linked by model assumptions.

Model dependent $Z'$ analyses have the advantage that many observables can be used as input for the fit. Furthermore, the resulting $Z'$ constraints can easily be compared with other experiments. They have the disadvantage that the output is a mixture of experimental data and theoretical assumptions. For every model, a separate fit has to be performed.

A model independent $Z'$ analysis does not rely on model assumptions. It can constrain only certain combinations of $Z'$ parameters. Model dependent $Z'$ constraints can be obtained from model independent $Z'$ constraints as special cases.

For a complete analysis, model dependent analyses should be complemented by a model independent analysis. Such a model independent $Z'$ analysis is done for off–resonance fermion pair production [4, 5] and $W$ pair production [6] at future $e^+e^-$ colliders. A model independent $Z'$ analysis for fermion pair production based on present LEP 2 data also exists [3]. It is not yet performed for LEP 1 data.

In this paper, we show how a model independent $Z'$ analysis based on LEP 1 data could be done using the partial decay widths of the $Z_1$ and forward–backward and left–right asymmetries at the $Z_1$ peak as input.

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To fix the notation, we repeat the neutral current interaction with SM fermions,

$$\mathcal{L} = e A_{\mu} J_{\gamma}^\mu + g Z_{\mu} J_{Z}^\mu + g' Z'_{\mu} J_{Z'}^\mu, \quad J_{\gamma}^\mu = \sum_i f_\gamma^a (v_f' + \gamma^a' a'_f) f. \quad (1)$$

Denoting the symmetry eigenstates by $Z$ and $Z'$, the mass eigenstates $Z_1$ and $Z_2$ are given by mixing,

$$Z_1 = Z \cos \theta_M + Z' \sin \theta_M, \quad Z_2 = -Z \sin \theta_M + Z' \cos \theta_M. \quad (2)$$

In the following, we are interested in the light mass eigenstate $Z_1$ precisely studied at LEP 1. Its axial vector and vector couplings to SM fermions depend on the $ZZ'$ mixing angle $\theta_M$,

$$a_f(1) = a_f \cos \theta_M + \frac{g'}{g} a'_f \sin \theta_M, \quad v_f(1) = v_f \cos \theta_M + \frac{g'}{g} v'_f \sin \theta_M. \quad (3)$$

The eigenvalue equation relates the mixing angle and the masses of the symmetry and mass eigenstates,

$$\frac{M_Z^2}{M_f^2} = \rho_{\text{mix}} = 1 + \sin^2 \theta_M \left( \frac{M_Z^2}{M_f^2} - 1 \right). \quad (4)$$

Consider the partial decay width $\Gamma_f$ of the $Z_1$ to $f \bar{f}$ and the forward–backward and left–right asymmetries at the peak in the limit of small $ZZ'$ mixing,

$$\Gamma_f = M_f \frac{g^2}{12\pi} \left[ v_f^2(1) + a_f^2(1) \right] \frac{N_f}{g} \approx \Gamma_f^0 \left\{ 1 + 2 \frac{v_f v'_f + a_f a'_f}{g(v_f^2 + a_f^2)} \right\}, \quad (5)$$

$$A_{FB}^f = 3 A_e A_f \approx A_{FB}^0 + \frac{3}{4} A_f^0 \Delta A_e + \frac{3}{4} A_e^0 \Delta A_f, \quad A_{LR}^f = A_f \approx A_f^0 + \Delta A_f$$

with $A_f \equiv \frac{2a_f(1)v_f(1)}{a_f(1)^2 + v_f(1)^2}$, and $\Delta A_f \approx 2 \frac{v_f a_f^M + a_f v_f^M}{g(v_f^2 + a_f^2)} - 4 \frac{v_f a_f v_f' + v_f' a_f v_f^M}{g(v_f^2 + a_f^2)^2}. \quad (6)$

The index zero denotes the observables without mixing. We see that measurements at the $Z_1$ peak constrain the combinations $a_f^M$ and $v_f^M$,

$$a_f^M \equiv \theta_M g a_f', \quad v_f^M \equiv \theta_M g v_f', \quad (7)$$

and not $a_f', v_f'$ and $\theta_M$ separately. For example, the observables with only leptons in the final state, $\Gamma_f^l$, $A_{FB}^l$ and $A_{LR}^l$, give model independent constraints on $a_f^M$ and $v_f^M$. Neglecting correlations, they are

$$|C_{v_f} v_f^M + C_{a_f} a_f^M| < \frac{\Delta \Gamma_f^l}{\Gamma_f^l}, \quad (8)$$

$$|D_{v_f} v_f^M + D_{a_f} a_f^M| < \Delta A_l, \quad |E_{v_f} v_f^M + E_{a_f} a_f^M| < \Delta A_{FB}^l.$$

The coefficients $C_{v,a}, D_{v,a}$ and $E_{v,a}$ depend on SM parameters only and can be easily deduced from equations (5). The right hand side of the inequalities (8) are experimental errors.

Model independent limits for $a_f^M, v_f^M, f = \bar{q}$ can be obtained in a similar way with the exception that $A_{FB}$ depends on the leptonic couplings too. We are interested in two–dimensional
constraints on \( a_f^M \) and \( v_f^M \). Therefore, we interpret the constraint from \( A_{FB}^f \) as a second measurement of \( A_f \),

\[
A_f = \frac{4 A_{FB}^f}{3 A_e}, \quad \Delta A_f = \frac{4}{3} \frac{\sqrt{(\Delta A_{FB}^f A_e^0)^2 + (A_{FB}^0 \Delta A_e)^2}}{(A_e)^2}.
\]

To get the connection to real data, radiative corrections have to be taken into account. They modify the relations (3) estimated at the Born level. Electroweak corrections can be included interpreting \( a_f \) and \( v_f \) in equation (3) contributing to \( a_f(1) \) and \( v_f(1) \) as effective couplings. Using the effective Weinberg angle \( \sin^2 \theta_W^{eff} = 0.2317 \), excellent agreement with the theoretical predictions in table 2 of [7] is obtained for \( \theta_M = 0 \):}

| Observable          | \( A_t \) | \( A_{FB}^t \) | \( A_c \) | \( A_{FB}^c \) | \( A_b \) | \( A_{FB}^b \) |
|---------------------|-----------|----------------|---------|---------------|---------|---------------|
| \( \sin^2 \theta_W^{eff} = 0.2317 \) | 0.1457    | 0.0159         | 0.667   | 0.0729        | 0.933   | 0.1020        |
| Experimental errors, [8] | 0.0067    | 0.0010         | 0.084   | 0.0048        | 0.049   | 0.0023        |

We included the electroweak [9] and QCD corrections for \( \Gamma_1^f \) following [10],

\[
\Gamma_1^f = \frac{N_f G_\mu \sqrt{2} M_1^3}{12 \pi} \rho_{mix} \rho_f^Z \mu R_{QED} R_{QCD}(M_F^1) \left\{ \left[ v_f(1)^2 + a_f(1)^2 \right] \left( 1 + 2 \frac{m_f^2}{M_1^2} \right) - 6a_f(1)^2 \frac{m_f^2}{M_1^2} \right\},
\]

where

\[
R_{QED} = 1 + \frac{3 \alpha}{4 \pi} Q_f^2, \quad \mu = \sqrt{1 - \frac{4 m_f^2}{M_1^2}},
\]

\[
R_{QCD} = \begin{cases} 1 + \frac{\alpha_s(M_1^2)}{\pi} & \text{for } f = l, \\ 1 + 1.405 \frac{\alpha_s^2(M_1^2)}{\pi^2} - 12.8 \frac{\alpha_s^3(M_1^2)}{\pi^3} - \frac{Q_f^2}{4} \frac{\alpha_s(M_1^2)}{\pi^2} & \text{for } f = q. \end{cases}
\]

In contrast to [11], we use the normalization \( a_l = -\frac{1}{2} \). For \( \Gamma_1^b \), additional contributions must be taken into account, see [4, 11] for original references.

Note that the coupling \( g^* \) in equation (5) has to be replaced by \( G_\mu \) to meet the experimental accuracy,

\[
g^* = \frac{4 \pi \alpha}{\sin^2 \theta_W \cos^2 \theta_W} = \frac{G_\mu \sqrt{2} M_Z^2}{1 - \Delta r} = \frac{G_\mu \sqrt{2} M_W^2}{(1 - \Delta r) \cos^2 \theta_W},
\]

where \( \Delta r \) is absorbed in \( \rho_f^Z \). The last of the above sequence of equations is valid only for restricted Higgs sectors. Through this replacement, a dependence on the Z mass, \( M_Z \), appears. Unfortunately, this induces, through equation (4), a dependence of \( \Gamma_1^f \) on \( M_2 \), which was absent at the Born level. For model independent limits on \( a_f^M \) and \( v_f^M \), this dependence can be eliminated by additional experimental input. The knowledge of \( M_W, G_\mu \) and \( \alpha(M_Z^2) \) together with equation (13) allows to calculate \( M_Z \) for models with restricted Higgs sectors. The main error of the calculated value of \( M_Z \) comes from the experimental error of the \( M_W \) measurement [8] and from the theoretical uncertainty of \( \Delta r \) due to the unknown Higgs and top mass [11]. We add these errors quadratically. As a result, we get \( \rho_{mix} = M_Z^2/M_1^2 = 1 \pm 0.003 \). We have \( \rho_{mix} > 1 \) in a theory with a \( Z' \). Note that the data used for the calculation of \( \rho_{mix} \) are independent of the measurements used for our further \( Z' \) analysis. We include the shift of

\[

\]
Fig. 1: Areas of \(a_1^M, v_1^M\), for which the extended gauge theory’s predictions are indistinguishable from the SM (95\% CL). Models between the long–dashed (short–dashed, dotted) lines cannot be detected with \(\Gamma^l_1, (A_{FB}^l, A_{LR}^l)\). The regions surrounded by the solid lines cannot be resolved by all three observables combined, see text. The numbers at the straight line indicate the value of \(\theta_M\) in units of \(10^{-3}\) for the \(\chi\) model. The rectangle is calculated from figure 1 of reference [6].

.. figure:: figure1.png
   :alt: Figure 1
   :caption: Figure 1: Areas of \(a_1^M, v_1^M\), for which the extended gauge theory’s predictions are indistinguishable from the SM (95\% CL). Models between the long–dashed (short–dashed, dotted) lines cannot be detected with \(\Gamma^l_1, (A_{FB}^l, A_{LR}^l)\). The regions surrounded by the solid lines cannot be resolved by all three observables combined, see text. The numbers at the straight line indicate the value of \(\theta_M\) in units of \(10^{-3}\) for the \(\chi\) model. The rectangle is calculated from figure 1 of reference [6].

\(\rho_{mix}\) from 1, which is within the experimental errors, in \(\rho^Z_f\). The error of \(\rho_{mix}\) is included in the error of \(\Gamma^f_1\).

Figure 1 illustrates the constraints on \(a_1^M\) and \(v_1^M\). The plotted regions correspond to \(\chi^2 = 5.99\). In our demonstration, the central value of the fit is assumed to be at the theoretical prediction but the experimental error quoted in (10) is used. We use \(\Gamma^l_1 = (83.91 \pm 0.11)\) MeV as input. Fixing \(\rho_{mix} = 1\), the region inside the ellipse cannot be excluded by the data. The constraint from every observable is shown separately for \(\rho_{mix} = 1\). The uncertainty of \(\rho_{mix}\) yields to a shift of the ellipse, which results to the larger solid region in figure 1. Future improved measurements of \(M_W\) and a determination of \(M_H\) would reduce the difference between the two regions.

The model independent constraints can be interpreted as constraints to the \(ZZ'\) mixing angle \(\theta_M\) for any fixed model. This is illustrated for the \(\chi\) model in figure 1. Graphically, one obtains \(|\theta_M| < 0.0035\).

Similarly, the constraints on \(\theta_M\) could be obtained for any other model. This is illustrated in figure 2, where \(a_1^M, v_1^M\) for \(E_6\) models [12, 13] and left–right models [13, 14] with \(\theta_M = 0.005\) are shown. A superposition with figure 1 allows to obtain the limits on \(\theta_M\) for any \(E_6\) and left–right model. The constraints for the \(\eta, \psi\) and LR model are \(|\theta_M| < 0.01, 0.0035\) and 0.0025, correspondingly.

Fig. 2: The vector and axial vector couplings \(a_1^M, v_1^M\) for \(\theta_M = 0.005\) in typical GUTs. For illustration, \(\theta_M\) for the \(\chi\) model is varied in units of \(10^{-3}\).

.. figure:: figure2.png
   :alt: Figure 2
   :caption: Figure 2: The vector and axial vector couplings \(a_1^M, v_1^M\) for \(\theta_M = 0.005\) in typical GUTs. For illustration, \(\theta_M\) for the \(\chi\) model is varied in units of \(10^{-3}\).

\[\rho_{mix}\] from 1, which is within the experimental errors, in \(\rho^Z_f\). The error of \(\rho_{mix}\) is included in the error of \(\Gamma^f_1\).
**Fig. 3:** Areas of \((a^M_q, v^M_q)\), for which the extended gauge theory’s predictions are indistinguishable from the SM (95% CL). Models between the dashed (dotted) lines cannot be detected with \(\Gamma_q^1 (A^q_{LR} + A^q_{FB})\). The ellipses are the combined regions, which cannot be resolved. Thick (thin) lines correspond to \(q = c(b)\).

Figure 3 shows our results similar to figure 1 but for \((a^M_e, v^M_e)\) and \((a^M_b, v^M_b)\). The weaker constraints reflect the larger systematic errors of the quark measurements. We used \(\Gamma_b^1 = R_b \Gamma_{had} = (379.7 \pm 1.9)\) MeV and \(\Gamma_c^1 = R_c \Gamma_{had} = (299.0 \pm 9.8)\) MeV as input [8]. In contrast to figure 1, the uncertainty of the exclusion region due to \(\rho_{mix}\) is negligible here. The missing constraint from \(A^b_{LR} + A^b_{FB}\) at one side is due to the theoretical prediction, which is too close to one to allow for a \(\chi^2 = 5.99\) even for \(A_b = 1\) with the given experimental error.

**Model dependent** fits constrain \(\theta_M\) using hadronic and leptonic observables simultaneously. Furthermore, there are less \(Z'\) parameters to be determined in such a fit. As a result, the model dependent \(Z'\) limits are better than those obtained for the same model from the model independent constraints as demonstrated above. Numerically, the difference can be estimated, for example, comparing the limit for the \(\chi\) model \(-0.006 < \theta_M < 0.008\) taken from figure 3 of the first reference in [2] for \(M_{Z'} = 700\) GeV with the range \(|\theta_M| < 0.04\), which would be obtained by our model independent analysis using \(\Gamma^l_1\) and \(A^l_{FB}\) from the same data set of the analysis [2].

The limits on \(a^M_e\) and \(v^M_e\) obtained from LEP 1 data can be compared with future constraints from \(e^+e^- \rightarrow W^+W^-\). The rectangle in figure 1 is calculated from figure 1 of reference [6] where the constraint on the anomalous couplings \(g_{WW\gamma}^* = 1 + \delta_\gamma\) and \(g_{WWZ}^* = \cot \theta_W + \delta_Z\) is plotted for \(\sqrt{s} = 500\) GeV and \(L_{int} = 50\) fb\(^{-1}\). For small \(ZZ'\) mixing and \(M_{Z'}^2 \gg s\), the values \(\delta_\gamma\) and \(\delta_Z\) are linearly related to \(a^M_e\) and \(v^M_e\),

\[
\delta_Z = \cot \theta_W \cdot \frac{a^M_e}{ag}, \quad \delta_\gamma = \cot v_e \theta_W \left( \frac{a^M_e}{ga} - \frac{v^M_e}{gv} \right) \chi, \quad \chi = \frac{s}{s - M_{Z'}^2},
\]

(14)

The constraint on \(a^M_q\) and \(v^M_q\) from the \(Z_1\) peak cannot be improved by \(W\) pair production.
The model independent limits in figure 1 can be interpreted as constraints on a weakly coupling $Z'$. Explore that figure 1 depends only on the combinations $\theta_M g' a'_l$ and $\theta_M g' v'_l$. Then, we obtain for sufficiently small $\theta_M$ the constraint on $\theta_M$ for a $Z'$ model with a “scaled” coupling strength $g' \rightarrow \lambda g'$ as $\theta_M \rightarrow \theta_M / \lambda$. In the simplest approximation where only the linear terms in $\theta_M$ are kept in equation (3), we obtain

$$|\theta_M| < \frac{\Delta c}{c} \frac{g c}{g' c'},$$

where $c = a_l, v_l$. \hspace{1cm} (15)

$\Delta c$ is the bound on $a'_l$ or $v'_l$ obtained from figure 1, i.e. $\Delta v_l = 0.0008$ and $\Delta a_l = +0.0003 - 0.0011$. The exact numerical result is shown in figure 4. The approximate bound (15) is recovered for large $g' c' / (g c)$. In contrast to (15), the exact calculation gives a bound on $\theta_M$ also for a $Z'$ with zero coupling, i.e. for $g' c' / (g c) = 0$. It is $|\theta_M| < 0.034$ for $c = a_l$. The existence of such a bound can easily be understood from equation (3), where the deviation of the coupling $a_f(1)$ or $v_f(1)$ from $a_f$ or $v_f$ with increasing $\theta_M$ eventually becomes larger than the experimental error, even in the case $g' = 0$. If one allows for a large $ZZ'$ mixing, there is one particular $Z'$ with all couplings proportional to those of the SM $Z$ boson and a proper overall coupling strength,

$$g' c'_f = g c_f \frac{1 - \cos \theta_M}{\sin \theta_M},$$

which would produce no deviation of $c_f(1)$ from $c_f$. The begin of this second region of insensitivity (besides the region $\theta_M \approx 0$) can be recognized for small $g' c' / (g c)$ in figure 4. Such a special model could be detected only by effects of the $Z_2$ propagator. If equation (16) is fulfilled for $a'_l$, but not for $v'_l$, the curve for $v'_l$ in figure 4 would constrain the $ZZ'$ mixing.

Considerations similar to figure 4 can be made for the couplings of the $Z'$ to quarks.

To summarize, we demonstrated in a simple analysis how model independent $Z'$ limits based on the data from the $Z_1$ peak could be obtained. The constraints on $a^M_c, v^M_c, a^M_b, v^M_b$ are unique, while the constraints on $a^M_e$ and $v^M_e$ could be improved by measurements of $e^+e^- \rightarrow W^+W^-$ at energies beyond LEP 2. The relation between model independent and model dependent $Z'$ constraints was discussed. It was pointed out that LEP data set bounds on the $ZZ'$ mixing angle even for a $Z'$ with arbitrary small couplings.

In our simple analysis, we neglected correlations between the different measurements and assumed that the SM parameters remain unchanged in a $Z'$ fit. In a more realistic analysis, the SM model parameters and the model independent $Z'$ parameters $a^M_f, v^M_f$ should be determined simultaneously and all correlations should be taken into account.

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