Gauge Field Emergence from Kalb-Ramond Localization

G. Alencar, R. R. Landim, M. O. Tahim, and R. N. Costa Filho

Abstract: A new mechanism, valid for any smooth version of the Randall-Sundrum model, of getting localized massless vector field on the brane is described here. This is obtained by dimensional reduction of a five dimension massive two form, or Kalb-Ramond field, giving a Kalb-Ramond and an emergent vector field in four dimensions. In order to reach this we propose a Yukawa coupling with the Ricci scalar and fix the coupling constant such that we get the components of fields localized. This solution is obtained by decomposing the fields in transversal and longitudinal parts and showing that this provides decoupled equations of motion for the transverse vector and KR fields in four dimensions. To this end we prove some identities satisfied by the transverse components of the fields. This proof is valid for any smooth version of the RS model. With this we fix the coupling constant in a way that we obtain a localized zero mode for both components on the brane. Then we generalize all the above results to the massive $p$-form field. We show that in general we can not obtain effective $p$ and $(p-1)$-forms localized on the brane and we must choose one of them to localize. Therefore, for example, we can not have a vector and a scalar field localized by dimensional reduction of the five dimensional vector field. In fact we find the expression $p = (d-1)/2$ which determines, in function of $D$, what forms will give rise to both fields localized. For $D = 5$ as expected, this is valid only for the KR field.

Keywords: Brane-World, Localization, Resonances

This paper is dedicated to the memory of my wife Isabel Mara (R. R. Landim)
1 Introduction

In Kaluza-Klein models with extra dimensions (string theory and others) the most basic tool is the decomposition of fields depending on the dimensions they are embedded and its tensorial characteristics. For example, working in $D = 5$ and taking the important field as $g_{\mu\nu}$, the dimensional reduction to $D = 4$ will give us again a four dimensional gravitational field, a vector field and a scalar field (the dilaton) as dynamical actors. Enlarging the number of extra dimensions we can add Yang-Mills fields in the procedure of dimensional reduction to $D = 4$ [1]. The same can be made to $p$–form fields. For fermion fields there is the specific procedure to obtain in lower dimensions several kinds of fermionic fields (chiral or not, real or not). We present in this work a similar procedure that can be applied to localize $p$–form fields in the Randall-Sundrum scenario of extra dimensions [2, 3]. Interestingly, the results are similar to the fermion case and by dimensional reduction we generally have that some components of the lower dimensional fields are not localized. It is important to mention that this procedure actually provides a new mechanism to localize gauge vector fields: from a Kalb-Ramond field in $D = 5$ we can obtain the $4D$ Kalb-Ramond and an additional localized vector field. We can think the gauge field emerges in this mechanism.

The problem of gauge form field localization in several brane world scenarios has been studied along the last years. This is a necessary step to walk along since our four dimensional space-time presents us a propagating vector field, despite more possible signals which can be interpreted as coming from other tensor gauge fields. In this sense, it is already understood how to localize the zero modes of gravity and scalar fields [3, 4] in a positive tension brane. However, the conformal invariance of the basic vector model fall into serious problems for building a realistic model because the localization method gives no result. This problem has been approached in many ways. Some authors have introduced a dilaton coupling in order to solve it [5] and other propose that a strongly coupled gauge theory in five dimensions can generate a massless photon in the brane [6]. Modifications
of the model considering spherical branes, multiple branes or induced branes can be found in [7–16].

Beyond the gauge field (one form) other forms can be considered. In five dimensions we can have yet the two, three, four and five forms. In \( D \)-dimensions we can in fact think about the existence of any \( p \leq D \). However, as we will see, they can be considered in a unified way. The analysis of localizability of the form fields has been considered in [17] where it has been shown that in \( D \)-dimensions only the forms with \( p < (D - 3)/2 \) can be localized. However, it is well known that in the absence of a topological obstruction, the field strength of a \( p \)-form is dual to the the \( (D - p - 2) \)-form [18]. Using this the authors in [19] found that also for \( p > (D - 1)/2 \), the fields are localized. It is important to point that in the model proposed here the Hodge Duality is not valid since we consider mass terms in the action which, by the way, break the duality. Beyond the zero mode localization, recently the resonances of \( p \)-forms has also been studied [20–24].

By another hand, an interesting viewpoint is related to models where membranes are smoothed out by topological defects [25–33]. The advantage of these models is that the \( \delta \)-function singularities generated by the brane in the RS scenario are eliminated. This kind of generalization also provides methods for finding analytical solutions [34, 35]. This is a nice characteristic if we want to put forward the idea of considering a Yukawa coupling with the Ricci scalar. The Ricci scalar can inform about possible space-time singularities and, as we want avoid them, such a coupling is natural in this sense. We therefore consider this kind of coupling with the gauge field, the Kalb- Ramond field and \( p \)-form fields in models with smooth membranes. This kind of coupling has its origins in the DGP model and its consequences [36]. One of its consequences is a model of (quasi) localization of gauge fields [37] where the membrane is described by a delta function, i.e., a singular place that can be understood using the Ricci scalar: in fact we can get that function as coming from a smooth model. The Ricci scalar, when we make the limit to the RS model, give rise to a delta function and explain the geometrical coupling with the membrane.

Other studies using a topological mass term in the bulk were introduced, but without giving a massless photon in the brane [38]. Most of these models introduce other fields or nonlinearities to the gauge field [39]. As a way to circumvent this, the authors in [40] introduced a mass term in five dimensions and a coupling with the brane. This gives a localized massless photon in the brane with action given by

\[
S_A = \int d^5X \sqrt{-G} \left( -\frac{1}{4} G^{MN} g^{PQ} Y_{MP} Y_{NQ} - \frac{1}{2} M^2 G^{MN} X_M X_N \right) - c \int d^4x \frac{1}{2} \sqrt{-g} g^{MN} X_M X_N, \tag{1.1}
\]

where \( Y_{MN} = \partial_{[M} X_{N]} \). In this model the localization is obtained only for some values of the parameter \( c \) and for a range in \( M \). It is important to note that in this case the gauge symmetry is lost due the existence of a mass term but is recovered in the effective action of the zero mode. In this context, a model has been proposed in which the two couplings are replaced by a coupling with the Ricci scalar [41]. This is a very a natural way if we wants to consider smooth version of RS model. For obtaining their results the authors of
use the particular configuration of fields $\partial_\mu A^\mu = A_5 = 0$. This is the same gauge used in the massless case, however here we have a mass term and the gauge symmetry is lost. Therefore the result obtained by them is not generally valid. A solution to this problem was found by the present authors in [42]. We show there that the choice $\partial_\mu A^\mu = A_5 = 0$, yet being valid as a particular solution, is unnecessary. We show that upon dimensional reduction of the five dimensional vector field ($A_M$) we get decoupled equations for the scalar ($A_5$) and the transverse vector ($A_\mu$) fields in four dimensions. For this we prove some identities satisfied by the transverse component of the field $A_\mu$. Then we obtain that we just can localize the zero mode of the $A_\mu$ or of the scalar field.

In the present manuscript we consider the same procedure to the two form field, which by dimensional reduction give us a two and a one form fields in four dimensions. In this case we obtain that both fields are simultaneously localized on the four brane. Therefore, as commented before, we find that we can have to different situations: upon dimensional reduction some components of the lower dimensional fields are not localized. A special case happens for the KR field in $D = 5$. To have a better understanding of this we generalize our results to higher dimensions and consider $p$–forms filed on it. We find that for each space-time dimension $D$ we can have just one higher dimensional $p$–form which provides both components of lower dimensional form fields localized. In fact we find a relation, given by $p = (D - 1)/2$, in which this is valid.

The paper is organized as follows. In section two we review the results for the one form gauge field. In section three we study the generalization for the Kalb-Ramond, or two form field. After considering similar decomposition of the field we show that they are decoupled. By dimensional reduction it is also shown that we can localize both, the gauge and the Kalb-Ramond fields in four dimensions. In section four we generalize all the results to the $p$-form case.

2 The One Form Case

Here we must review the results found by the authors in a previous work [42]. First we consider the equations of motion

$$\partial_M (\sqrt{-g} g^{MO} g^{NP} Y_{OP}) = -\gamma_1 \sqrt{-g} R g^{NP} X_P; \quad (2.1)$$

$$\partial_N (\sqrt{-g} R g^{NO} X_O) = 0, \quad (2.2)$$

where Eq. (2.2) is obtained from the antisymmetry of Eq. Eq. (2.1). Then we split our field in two parts $X^\mu = X^\mu_L + X^\mu_T$, where $L$ stands for longitudinal and $T$ stands for transversal with

$$X^\mu_T = (\delta^\mu_\nu - \frac{\partial^\mu \partial_\nu}{\Box}) X^\nu; \quad X^\mu_L = \frac{\partial^\mu \partial_\nu}{\Box} X^\nu. \quad (2.3)$$

With this, Eq. (2.1) can be divided in two. We get , for $N = 5$

$$\partial_\mu Y^{5\mu} + \gamma_1 e^{2A} R \Phi = 0 \quad (2.4)$$

where $\Phi \equiv X_5$ and for $N = \nu$ we get

$$e^A \Box X^\nu_T + \partial (e^A \partial X^\nu_T + \gamma_1 e^{3A} RX^\nu_T + \partial (e^A Y^0_5) + \gamma_1 e^{3A} RX^\nu_L = 0. \quad (2.5)$$
Yet from Eq. (2.2) we get
\[ e^{3A} R \partial_{\mu} X^\mu = - \partial (e^{3A} R \Phi) \]  
(2.6)
and using (2.3) we can show the following identities
\[ \partial_{\mu} Y^{\mu \nu} = \Box X^\nu_T ; \quad Y^{5 \mu} = \partial X^\mu_T + \partial X^\mu_L - \partial^\mu \Phi \equiv \partial X^\mu_T + Y^5_{L \mu} ; \quad Y^5_{\mu 5} = \frac{\partial^\mu}{\Box} \partial_{\nu} Y^{\nu 5} . \]  
(2.7)

Using now (2.4), (2.6) and (2.7) we get
\[ \partial (e^{A} Y^\mu_L) = - \gamma_1 \frac{\partial}{\Box} \partial (e^{3A} R \Phi) = - \gamma_1 e^{3A} RX^\nu_L \]
and finally we obtain from Eq. (2.5) the equation for the transverse part of the gaugefield
\[ e^A \Box X^\nu_T + \partial (e^A \partial X^\nu_T) + \gamma_1 e^{3A} RX^\nu_L = 0 . \]

Finally, by using
\[ R = -4(2A'' + 3A'^2)e^{-2A} \]  
(2.8)
and performing the transformation \( \tilde{\psi} = e^{-A/2} \psi \) we get the desired Schrödinger equation with potential
\[ U = (\frac{1}{4} + 12\gamma_1) A'^2 + (\frac{1}{2} + 8\gamma_1) A'' \]  
(2.9)
which is localized for \( \gamma_1 = 1/16 \). For the scalar field we must be careful since we have
\[ \Box \Phi - \partial \partial_{\mu} A^\mu - \gamma_1 Re^{-2A} \Phi = 0 \]
and performing the separation of variables \( \Phi = \Psi(z) \phi(x) \) and the Eq. (2.6) we get the equation for the massive mode of the scalar field
\[ \partial (R^{-1} e^{-3A} \partial (Re^{3A} \Psi)) - \gamma_1 R e^{2A} g \Psi = m^2 \Psi . \]

Defining now \( g = e^{3A} R \) we get
\[ \partial (g^{-1} \partial (g \Psi)) - \gamma_1 e^{-A} g \Psi = m^2 \Psi . \]  
(2.10)

This kind of equation can be cast in a Schrödinger form by defining \( \Psi = g^{-1/2} \psi \) and we obtain the effective potential
\[ U = \frac{3}{4} \frac{g'^2}{g^2} - \frac{1}{2} \frac{g''}{g} + \gamma_1 e^{-A} g \]  
(2.11)
or
\[ U = \frac{1}{4} (3A' + (\ln R)')^2 - \frac{1}{2} (3A'' + (\ln R)'') + \gamma_1 R e^{2A} . \]

With this potential we see that the zero mode of the scalar field solution is localized for \( \gamma_1 = 9/16 \). This show us that we cannot have both fields localized.
3 The Kalb-Ramond Field Case

In this section we must work with the same approach used before in order to try to localize the zero mode of the Kalb-Ramond field. Upon dimensional reduction of the KR field we are left with two kinds of terms, namely a Kalb-Ramond in four dimensions $B_{\mu\nu}$ and a vector field $B_{\mu^5}$. We must remember that here we also do not have gauge symmetry and we cannot choose $B_{\mu^5} = 0$. However we can again show that the longitudinal and transversal parts of the field decouples and we get the desired results. The action in this case is given by

$$A = \int d^5x\sqrt{-g} \left[ -\frac{1}{24} Y_{M_1M_2M_3} Y^{M_1M_2M_3} - \frac{1}{4} \gamma_2 R X_{M_1M_2} X^{M_1M_2} \right]$$

and the equations of motion are given by

$$\frac{1}{2} \partial_{M_1} \left[ \sqrt{-g} g^{M_1M_2} g^{M_3M_4} Y_{M_2M_3M_4} \right] - \gamma_2 R \sqrt{-g} g^{M_1M_2} g^{M_3M_4} X_{M_1M_2M_3M_4} = 0. \quad (3.1)$$

and just like in the case for the one form field, the antisymmetry of the equation give us the identity

$$\partial_{M_1} \left[ R \sqrt{-g} g^{M_1M_2} g^{M_3M_4} X_{M_2M_3} \right] = 0. \quad (3.2)$$

Now we proceed to find the decoupled equations of motion. First of all the above equation must be expanded. For $M_2 = \mu_2$ and $M_3 = \mu_3$ we obtain

$$\frac{1}{2} e^{-A} \partial_{\mu_1} Y^{\mu_1\mu_2\mu_3} + \partial (e^{-A} Y^{5\mu_2\mu_3}) - \gamma_2 R e^{A} X^{\mu_2\mu_3} = 0; \quad (3.3)$$

and for $M_3 = 5$ we get

$$\frac{1}{2} \partial_{\mu_1} Y^{\mu_1\mu_25} - \gamma_2 R e^{A} X^{\mu_2} = 0. \quad (3.4)$$

The transverse equation (3.2), differently of the vector case, will give rise to two equations. For $M_4 = 5$ we get $\partial_\mu X^{\mu 5} \equiv \partial_\mu X^\mu = 0$, where we have used our previous definitions. Therefore we see that the transverse condition for our vector field is naturally obtained upon dimensional reduction. For $M_4 = \mu_4$ we get

$$\partial (Re^{A} X^{\mu_4}) + e^{A} R \partial_{\mu_1} X^{\mu_1\mu_4} = 0 \quad (3.5)$$

Just as in the case of the one form, here we have effective equations that couple the Kalb-Ramond and the Vector field. Before we proceed to solve the equations we can further simplify them if we take the longitudinal and transversal part of each field. As the vector field already satisfy the transverse condition we just need to perform this for the KR field by $X^{\mu_1\mu_2} = X_{L}^{\mu_1\mu_2} + X_{T}^{\mu_1\mu_2}$, defined as

$$X_{T}^{\mu_1\mu_2} = X^{\mu_1\mu_2} + \frac{1}{\Box} \partial^{[\mu_1} \partial_{\nu_1} X^{\mu_2]\nu_1}; \quad X_{L}^{\mu_1\mu_2} = -\frac{1}{\Box} \partial^{[\mu_1} \partial_{\nu_1} X^{\mu_2]\nu_1};$$

and observing that

$$\partial_{\mu_1} Y^{\mu_1\mu_2\mu_3} = 2\Box X_{T}^{\mu_2\mu_3}; \quad \partial_{\mu_1} Y^{\mu_1\mu_2} = 2\Box X_{T}^{\mu_2},$$
where $Y_{\mu\nu} = \partial_{[\mu} X_{\nu]}$. We see that the first term of eq. (3.3), just like in the case of the last section, is already decoupled from the longitudinal part. However the second term is not decoupled and we have

$$Y_{\mu\nu}^{5} = Y_{L}^{5\mu\nu} + 2\partial X^{\mu\nu}_{T},$$

then our equations become

$$e^{-A}\Box X^{\mu\nu}_{T} + \partial(e^{-A}\partial X^{\mu\nu}_{T}) - \gamma_{2}Re^{A}X^{\mu\nu}_{T} + \frac{1}{2}\partial(e^{-A}Y^{5\mu\nu}_{L}) - \gamma_{2}Re^{A}X^{\mu\nu}_{L} = 0 \quad (3.6)$$

and

$$\frac{1}{2}\partial_{\mu_{1}}Y^{\mu_{1}\mu_{2}}_{L} - \gamma_{2}Re^{2A}X^{\mu_{2}} = 0. \quad (3.7)$$

Therefore we see clearly from eq. (3.6) that we have a coupling between the transversal part of the field, the longitudinal part and the gauge field. From eq. (3.7) we see that the gauge field is coupled to the longitudinal part of the KR field. As in the case of the one form field we should expect that we have two uncoupled effective massive equations for the gauge fields $X^{\mu_{1}\mu_{2}}_{T}$ and $X^{\mu}_{\nu}$ since both satisfy the transverse condition in four dimensions. Let's prove this now. First of all note that using $\partial_{\mu}X^{\mu} = 0$ we can show that

$$Y^{\mu_{1}\mu_{2}5}_{L} = - \frac{1}{\Box} \partial_{[\mu_{1}} \partial_{\nu]Y^{\mu\nu}_{L}} = 2Re^{2A}\frac{\partial[\mu_{1}X^{\mu\nu}_{2}]}{\Box},$$

where in last equality we have used eq. (3.4). Now we can use eq. (3.5) to show that

$$\partial(e^{-A}Y^{\mu_{1}\mu_{2}5}_{L}) = 2Re^{A}\frac{\partial[e_{\mu_{1}}X^{\mu\nu}_{2}]}{\Box} = -2Re^{A}X^{\mu_{1}\mu_{2}}_{L}$$

and this term cancels exactly the longitudinal part of the mass term. Then we get the final form of the equation of motion

$$e^{-A}\Box X^{\mu_{1}\mu_{2}}_{T} + \partial(e^{-A}\partial X^{\mu_{1}\mu_{2}}_{T}) - \gamma_{2}Re^{A}X^{\mu_{1}\mu_{2}}_{T} = 0$$

Imposing the separation of variables in the form $X^{\mu_{1}\mu_{2}}_{T}(z, x) = f(z)\tilde{X}^{\mu_{1}\mu_{2}}_{T}(x)$ we obtain the following equations

$$\Box \tilde{X}^{\mu_{1}\mu_{2}}_{T} + m^{2}_{X}\tilde{X}^{\mu_{1}\mu_{2}}_{T} = 0, (e^{-A}f(z))' - \gamma_{2}Re^{A}f(z) = 2m^{2}_{X}e^{-A}f(z),$$

where primes means a derivative with respect to $z$. Performing them the transformation $f(z) = e^{A/2}\psi(z)$ we get the standard potential, plus the correction

$$U(z) = \left[\frac{A'^{2}}{4} - \frac{A''}{2} + Re^{2A}\right] = \left(\frac{1}{4} + 12\gamma\right)A'^{2} + (-\frac{1}{2} + 8\gamma)A''.$$

The zero mode solution is of the form $e^{bA}$ which if plugged in the above equation give us $\gamma = 5/16$ and we get the integrand $e^{5A/8}$ rendering a localized zero mode. Now we must analyze the localizability of the vector field. In order to decouple the vector field and the longitudinal part of KR field we can use eq.(3.5) in (3.7) we get

$$\Box X^{\mu\nu}_{T} + \partial[R^{-1}e^{-A}(\partial Re^{A}X^{\mu\nu})] - \gamma_{2}Re^{2A}X^{\mu\nu}_{T} = 0. \quad (3.8)$$
Performing now separation of variables $X^{\mu_1} = u(z) \tilde{X}^{\mu_1}(x)$ we get the set of equations
\begin{align}
\Box \tilde{X}^{\mu_2} + m_1^2 \tilde{X}^{\mu_2} &= 0, \\
(R^{-1} e^{-A(R^A u(z))})' - \gamma_2 R e^{2A} u(z) &= 2m_1^2 u(z). 
\end{align}

We can see that the form of the above equations is identical do Eq. (2.10) for $g = R^A$ and we get from Eq. (2.11) the solution
\[ U = \frac{1}{4} (A' + (\ln R)')^2 - \frac{1}{2} (A'' + (\ln R)'') + \gamma_2 R e^{2A}. \]

Therefore we see that for any smooth version of RS model the above potential is identical to that of the Kalb-Ramond case and we have a localized solution. In this sense, we can say that the vector field emerges en $D = 4$ from the localization of the Kalb-Ramond field. In the next section it will become clear why just for the KR field in five dimensions we can have both fields localized.

4 The $p$–form Field Case

In this section we further develop the previous methods in order to generalize our results to the $p$–form field case in a $(D - 1)$-brane. The action is given by
\[ A = \int d^D x \sqrt{-g} \left[ -\frac{1}{2p!(p+1)!} Y_{M_1...M_{p+1}} Y^{M_1...M_{p+1}} - \frac{1}{2p!} \gamma_p R X_{M_2...M_{p+1}} X^{M_2...M_{p+1}} \right], \]
\[ \text{(4.1)} \]
where $Y_{\mu_1...\mu_p} = \partial_{\mu_1} X_{\mu_2...\mu_p}$. The equations of motion are given by
\[ \frac{1}{p!} \partial_{M_1} \left[ \sqrt{-g} g^{M_1 N_1} ... g^{M_{p+1} N_{p+1}} Y_{N_1...N_{p+1}} \right] - \gamma_p R \sqrt{-g} g^{M_2 N_2} ... g^{M_{p+1} N_{p+1}} X_{N_2...N_{p+1}} = 0. \]
\[ \text{(4.2)} \]
Similarly to the one form case we get, from the above equation we get the identity
\[ \text{Re}(D-p)A \partial^{\mu_2} X_{\nu_2 N_3...N_{p+1}} + \partial \left[ \text{Re}(D-p)A X_{N_3...N_{p+1}} \right] = 0, \]
\[ \text{(4.3)} \]
where, like in previous sections, $\partial$ means a derivative with $z$ and, from now on, all $(D - 1)$-dimensional indices will be contracted with $\eta^{\mu\nu}$.

Now we can obtain the equations of motion by expanding eq. (4.2). We arrive at just two kinds of terms, where none of the indices is 5, giving
\[ \frac{1}{p!} e^{(D-2(p+1))A} \partial_{\mu_1} [Y^{\mu_1 \mu_2...\mu_{p+1}}] + \frac{1}{p!} \partial (e^{(D-2(p+1))A} Y^{5\mu_2...\mu_{p+1}}) - \gamma_p \text{Re}(D-2p)A X^{52...\mu_{p+1}} = 0, \]
\[ \text{(4.4)} \]
and when one of the indices is 5 we get
\[ \frac{1}{p!} \partial_{\mu_1} Y^{\mu_1 \mu_2...\mu_5} - \gamma_p \text{Re}^{2A} X^{\mu_2...\mu_5} = 0. \]
\[ \text{(4.5)} \]

Just like in the Kalb-Ramond case, the transverse equation (4.3) give rise to two equations. For the index with direction 5 we get $\partial_5 X^{\mu_1...\mu_{p-1}} = \partial_{\mu_1} X^{\mu_1...\mu_{p-1}} = 0$, where
we have used our previous definitions. Therefore we see that the transverse condition for our \((p - 1)\)-form field is naturally obtained upon dimensional reduction. For a index not equal to 5 we get

\[
\partial(\text{Re}(D-2p)A X^{\mu_1 \ldots \mu_{p-1}}) + \text{Re}(D-2p)A \partial_{\mu_p} X^{\mu_1 \ldots \mu_p} = 0. 
\]

(4.6)

First of all, we must split the field as done before in order to obtain

\[
X^{\mu_1 \ldots \mu_p}_T = X^{\mu_1 \ldots \mu_p} + \frac{(-1)^p}{\partial} \partial_{\mu_1} \partial_{\nu_1} X^{\mu_{2} \ldots \mu_{p} \nu_1 \ldots \nu_{l}}, \quad X^{\mu_1 \ldots \mu_p}_L = \frac{(-1)^{p-1}}{\partial} \partial_{\mu_1} \partial_{\nu_1} X^{\mu_{2} \ldots \mu_{p} \nu_1 \ldots \nu_{l}}, \quad s
\]

and observing that

\[
\partial_{\mu_1} Y^{\mu_1 \mu_2 \ldots \mu_p} = \Box X^{\mu_2 \ldots \mu_p}, \quad \partial_{\mu_1} Y^{\mu_1 \mu_2 \ldots \mu_p} = \Box X^{\mu_1 \mu_2 \ldots \mu_p-1},
\]

(4.7)

we see that the first term of eq. (4.4), just like in the last section, is already decoupled from the longitudinal part. However, the second term is not decoupled and if use the fact that

\[
Y^{5 \mu_1 \ldots \mu_p} = Y^{5 \mu_1 \ldots \mu_p} + p! \partial X^{\mu_1 \ldots \mu_p}
\]

(4.9)

we can write the equation (4.4) as

\[
e^{(D-2(p+1))A} \Box X^{\mu_1 \ldots \mu_p}_T + \partial(e^{(D-2(p+1))A} \partial X^{\mu_1 \ldots \mu_p}_T) - \text{Re}(D-2p)A X^{\mu_1 \ldots \mu_p}_T +
\]

\[+ \frac{1}{p!} \partial(e^{(D-2(p+1))A} Y^{5 \mu_1 \ldots \mu_p}_L) - \text{Re}(D-2p)A X^{\mu_1 \ldots \mu_p}_L = 0,
\]

(4.10)

and (4.5) as

\[
\frac{1}{p!} \partial_{\mu_1} Y^{\mu_1 \mu_2 \ldots \mu_p} - \text{Re}^{2A} X^{\mu_2 \ldots \mu_p} = 0.
\]

(4.11)

Therefore, we see clearly from eq. (4.10) that we have a coupling between the transversal part of the \(p\)-form field, the longitudinal part and the \((p - 1)\)-form field. From eq. (4.11) we see that the \((p - 1)\)-form is coupled to the longitudinal part of the \(p\)-form field. As in the case of the one form field, we should expect that we have to uncouple the effective massive equations for the gauge fields \(X^{\mu_1 \mu_2 \ldots \mu_p}_T\) and \(X^{\mu_2 \ldots \mu_p}_L\) since both satisfy the transverse condition in four dimensions. Lets walk along and prove this now. First of all note that using \(\partial_{\mu_2} X^{\mu_2 \ldots \mu_p} = 0\) we can show that

\[
Y^{\mu_1 \ldots \mu_p} = \frac{(-1)^{p-1}}{\partial} \partial_{\mu_1} \partial_{\nu_1} Y^{\mu_2 \ldots \mu_p | \nu}
\]

(4.12)

and we get an identity similar to that for the gauge field

\[
Y^{\mu_1 \ldots \mu_p}_L^5 = p!(-1)^{p-1} \partial X^{\mu_1 \ldots \mu_p}_L + p!(-1)^p Y^{\mu_1 \ldots \mu_p}_L = \frac{p!(-1)^p}{\partial} \left[ (-1)^{p-1} \partial \partial_{\mu_1} \partial_{\nu_1} X^{\mu_2 \ldots \mu_p | \nu} + \partial_{\mu_1} \partial_{\nu_1} Y^{\mu_2 \ldots \mu_p | \nu} \right]
\]

\[= \frac{1}{\partial} \partial_{\mu_1} \partial_{\nu_1} Y^{\mu_2 \ldots \mu_p | \nu} = \frac{p!}{\partial} \text{Re}^{2A} \partial_{\mu_1} X^{\mu_2 \ldots \mu_p},
\]

(4.13)

where in the last equation we have used equation (4.5). Using now the transverse equation (4.6) we obtain
\[ \partial \left( e^{(D-2(p+1))A}Y^\mu_1...\mu_pL^5 \right) = p! \partial^{[\mu_1}_L \partial \left( e^{(D-2p)A}RX^{\mu_2...\mu_p}\right) = -p! e^{(D-2p)A} R \partial^{[\mu_1}_L \partial_\mu X^{\mu_2...\mu_p]} \mu \]

And we get the equation of motion for the transversal part of p-form

\[ e^{(D-2(p+1))A} \square X^\mu_1...\mu_p + \partial(e^{(D-2(p+1))A} \partial X^\mu_1...\mu_p) - Re^{(D-2p)A} X^\mu_1...\mu_p = 0. \tag{4.15} \]

Imposing now the separation of variables in the form \( X^\mu_1...\mu_p(z, x) = f(z) \tilde{X}^\mu_1...\mu_p(x) \) we obtain

\[ \square \tilde{X}^\mu_1...\mu_p + m^2_X \tilde{X}^\mu_1...\mu_p = 0, \tag{4.16} \]

\[ (e^{(D-2(p+1))A} f'(z))' - Re^{(D-2p)A} f(z) = m^2_X p! e^{(D-2(p+1))A} f(z), \tag{4.17} \]

where the primes means derivative with respect to \( z \). Now, making \( f(z) = e^{-(D-2(p+1)/2}\psi, \) we can write the above equation in a Schrödinger form with potential given by

\[ U(z) = \left[ \frac{\alpha_p^2}{4} A'^2 + \frac{\alpha_p}{2} A'' + \gamma_p Re^{2A} \right] = \left( \frac{\alpha_p^2}{4} + 12 \gamma_p \right) A'^2 + \left( \frac{\alpha_p}{2} + 8 \gamma_p \right) A'' \tag{4.18} \]

with \( \alpha_p = D - 2(p + 1) \). The localized zero mode solution is given by \( e^{hA} \) with

\[ \gamma_p = \frac{3 - 2\alpha_p}{16} \tag{4.19} \]

For the \( (p - 1) \)-form we have, imposing the separation of variables \( X^{\mu_2...\mu_p}(z, x) = u(z) \tilde{X}^{\mu_2...\mu_p}(x) \) and from (4.5) and (4.6) the set of equations

\[ \square \tilde{X}^{\mu_2...\mu_p} + m^2_{p-1} \tilde{X}^{\mu_2...\mu_p} = 0, \tag{4.20} \]

\[ \left( Re^{-(D-2p)A}(Re^{(D-2p)A}u(z))' \right)' - \gamma_p Re^{2A}u(z) = m^2_{p-1} u(z). \tag{4.21} \]

Just as in the last two section we see that we just have to use \( g = Re^{(D-2p)A} \) in 2.11 to get the final potential

\[ U(z) = \frac{1}{4} \left[ (2\alpha_p + 1) A' + (\ln R)' \right]^2 - \frac{1}{2} \left( (2\alpha_p + 1) A'' + (\ln R)'' \right] + \gamma_p Re^{2A}. \tag{4.22} \]

From the above equation we see that we can recover all the previous cases. We also analyse the localizability of the field in a very simple way. For any metric which recovers the RS for large \( z \) we get the asymptotic potential

\[ U(z) = \frac{1}{4} \left( (2\alpha_p + 1) \right)^2 A'^2 - \frac{1}{2} \left( (2\alpha_p + 1) A'' \right) + \gamma_p Re^{2A}. \tag{4.23} \]

The solution to the above equation is found by fixing

\[ \gamma_p = \frac{7 + 2\alpha_p}{16} \tag{4.24} \]

Therefore we can see that the only case for localizing both fields happens for \( p = (D - 1)/2 \). Now it is clear why for \( D = 5 \) we have that KR field provides the localization of both fields. This is the result we want to stress here. This is possible due to the Yukawa geometrical coupling and the field splitting described.
5 Conclusions and Perspectives

In this paper we have developed the idea that a Yukawa coupling with the Ricci scalar can solve the problem of gauge field localization. We first shown that for any form field we can obtain decoupled equations of motion for the longitudinal and transverse components of the fields. We studied first the simplest cases, namely the Vector and Kalb-Ramond fields. From these we can understand how a generalization to $p-$forms can be obtained. Some points are worthwhile noting. First, we have found that for some specific value of coupling constant we can get the localization of any $p-$form. However, the $(p - 1)-$form obtained by dimensional reduction can not be simultaneously localized. Despite of this, something very interesting happens in the Kalb-Ramond case in $D = 5$. Here we get that through a dimensional reduction we naturally have the KR and the gauge field localized. This is a very important result since this gives a richer possibility of dynamics coming from a unique field in five dimensions. In fact, this can be seen as a new mechanism to localize the gauge vector field. It remains to analyze other characteristics like resonant modes in this situation. The question about fermions with similar couplings can be interesting to another study. We can ask here, because of the fact of non-localization at the same time of fields coming from the procedure explained, if there is some physical criteria to choose one field or another. These are good questions to think about and are left to future works.

Acknowledgment

We acknowledge the financial support provided by Fundação Cearense de Apoio ao Desenvolvimento Científico e Tecnológico (FUNCAP), the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and FUNCAP/CNPq/PRONEX.

References

[1] D. Bailin and A. Love, Rept. Prog. Phys. 50, 1087 (1987).
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [hep-ph/9905221].
[3] Lisa Randall and Raman Sundrum, Phys.Rev.Lett. 83 (1999) 4690–4693, arXiv:hep-th/9906064 [hep-th].
[4] B. Bajc and G. Gabadadze, Phys. Lett. B 474, 282 (2000) [hep-th/9912232].
[5] A. Kehagias and K. Tamvakis, Phys. Lett. B 504, 38 (2001) [hep-th/0010112].
[6] G. R. Dvali and M. A. Shifman, Phys. Lett. B 396, 64 (1997) [Erratum-ibid. B 407, 452 (1997)] [hep-th/9612128].
[7] M. Gogberashvili, Europhys. Lett. 49, 396 (2000) [hep-ph/9812365].
[8] I. C. Jardim, R. R. Landim, G. Alencar and R. N. Costa Filho, Phys. Rev. D 84, 064019 (2011) [arXiv:1105.4578 [gr-qc]].
[9] I. C. Jardim, R. R. Landim, G. Alencar and R. N. Costa Filho, Phys. Rev. D 88, no. 2, 024004 (2013) [arXiv:1301.2578 [gr-qc]].
[10] K. Akama, T. Hattori and H. Mukaida, arXiv:1109.0840 [gr-qc].
[11] N. Kaloper, Phys. Lett. B 474, 269 (2000) [hep-th/9912125].
[12] N. Kaloper, JHEP 0405, 061 (2004) [hep-th/0403208].
[13] K. Akama, Prog. Theor. Phys. 60, 1900 (1978).
[14] K. Akama and T. Hattori, arXiv:1403.5633 [gr-qc].
[15] K. Akama and T. Hattori, Class. Quant. Grav. 30, 205002 (2013) [arXiv:1309.3090 [gr-qc]].
[16] G. Alencar, R. R. Landim, M. O. Tahim and R. N. Costa Filho, Phys. Lett. B 726, 809 (2013) [arXiv:1301.2562 [hep-th]].
[17] N. Kaloper, E. Silverstein and L. Susskind, JHEP 0105, 031 (2001) [hep-th/0006192].
[18] M. J. Duff and P. van Nieuwenhuizen, Phys. Lett. B 94, 179 (1980).
[19] M. J. Duff and J. T. Liu, Phys. Lett. B 508, 381 (2001) [hep-th/0010171].
[20] G. Alencar, R. R. Landim, M. O. Tahim, K. C. Mendes, R. R. Landim, M. O. Tahim, R. N. C. Filho and K. C. Mendes, Europhys. Lett. 93, 10003 (2011) [arXiv:1009.1183 [hep-th]].
[21] G. Alencar, R. R. Landim, M. O. Tahim, C. R. Muniz and R. N. Costa Filho, Phys. Lett. B 693, 503 (2010) [arXiv:1008.0678 [hep-th]].
[22] R. R. Landim, G. Alencar, M. O. Tahim, M. A. M. Gomes and R. N. Costa Filho, Europhys. Lett. 97, 20003 (2012) [arXiv:1010.1548 [hep-th]].
[23] G. Alencar, M. O. Tahim, R. R. Landim, C. R. Muniz and R. N. Costa Filho, Phys. Rev. D 82, 104053 (2010) [arXiv:1005.1691 [hep-th]].
[24] C. E. Fu, Y. X. Liu, K. Yang and S. W. Wei, JHEP 1210, 060 (2012) [arXiv:1207.3152 [hep-th]].
[25] D. Bazeia and L. Losano, Phys. Rev. D73 (2006) 025016, arXiv:hep-th/0511193 [hep-th].
[26] Yu-Xiao Liu, Jie Yang, Zhen-Hua Zhao, Chun-E Fu, and Yi-Shi Duan, Phys. Rev. D80 (2009) 065019, arXiv:0904.1785 [hep-th].
[27] Zhen-Hua Zhao, Yu-Xiao Liu, and Hai-Tao Li, Class. Quant. Grav. 27 (2010) 185001, arXiv:0911.2572 [hep-th].
[28] Jun Liang and Yi-Shi Duan, Phys. Lett. B681 (2009) 172–178.
[29] Zhen-Hua Zhao, Yu-Xiao Liu, Hai-Tao Li, and Yong-Qiang Wang, Phys. Rev. D82 (2010) 084030, arXiv:1004.2181 [hep-th].
[30] Zhen-Hua Zhao, Yu-Xiao Liu, Yong-Qiang Wang, and Hai-Tao Li, JHEP 1106 (2011) 045, arXiv:1102.4894 [hep-th].
[31] R. R. Landim, G. Alencar, M. O. Tahim and R. N. Costa Filho, JHEP 1108, 071 (2011) [arXiv:1105.5573 [hep-th]].
[32] R. R. Landim, G. Alencar, M. O. Tahim and R. N. Costa Filho, JHEP 1202, 073 (2012) [arXiv:1110.5855 [hep-th]].
[33] G. Alencar, R. R. Landim, M. O. Tahim and R. N. C. Filho, JHEP 1301, 050 (2013) [arXiv:1207.3054 [hep-th]].
[34] Mirjam Cvetic and Marko Robnik, Phys. Rev. D77 (2008) 124003, arXiv:0801.0801 [hep-th].
[35] R. R. Landim, G. Alencar, M. O. Tahim and R. N. Costa Filho, Phys. Lett. B 731, 131 (2014) [arXiv:1310.2147 [hep-th]].

[36] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485, 208 (2000) [hep-th/0005016].

[37] G. R. Dvali, G. Gabadadze and M. A. Shifman, Phys. Lett. B 497, 271 (2001) [hep-th/0010071].

[38] I. Oda, hep-th/0103052.

[39] A. E. R. Chumbes, J. M. Hoff da Silva and M. B. Hott, Phys. Rev. D 85, 085003 (2012) [arXiv:1108.3821 [hep-th]].

[40] K. Ghoroku and A. Nakamura, Phys. Rev. D 65, 084017 (2002) [hep-th/0106145].

[41] Z. H. Zhao, Q. Y. Xie and Y. Zhong, arXiv:1406.3098 [hep-th].

[42] G. Alencar, R. R. Landim, M. O. Tahim and R. N. C. Filho, arXiv:1409.4396 [hep-th].