Melting temperature of heavy quarkonium with a holographic potential up to sub-leading order

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Abstract: A calculation of the melting temperatures of heavy quarkonium states with the holographic potential was introduced in the work of\cite{1}. In this paper, we consider the holographic potential including its sub-leading order, we find this correction lowers the dissociation temperatures of heavy quarkonium.

Key words: Melting temperature, Heavy quarkonium, Holographic potential, Correction

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1 Introduction

Heavy quarkonium dissociation is an important signal of the formation of Quark-Gluon Plasma (QGP) in heavy ion collisions at RHIC and LHC. However, much experiment data indicates that the QGP is strongly coupled. Thus, the study of heavy quarkonium and its dissociation requires non-perturbative techniques, such as Lattice QCD and potential models\cite{2}. The lattice simulation of the quark-antiquark potential and the spectral density of hadronic correlators yield consistent picture of quarkonium dissociation as well as the numerical value $T_d$. On the other hand, the heavy quarkonium dissociation can be studied within the potential models, for instance the energy levels and the dissociation temperature can be carried out with the aid of a non-relativistic Schrodinger equation with a temperature dependent effective potential when we neglect the velocity ($v \ll 1$) of the constituent quarks\cite{3–5}, and if we consider the relativistical correction, the two-body Dirac equation can be employed\cite{6}. The holographic potential addressed in this paper is one example of potential models.

Holographic potential at finite temperature at strong coupling relies on the AdS/CFT duality which can explore the strongly coupled $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) plasma through the correspondence between the type IIB superstring theory formulated on $\text{AdS}_5 \times S^5$ and $\mathcal{N}=4$ SYM in four dimensions\cite{7–11}.

In a previous work\cite{1}, the melting temperatures of heavy quarkonium states is studied with the holographic potential. In this paper we consider the holographic potential including its sub-leading order and obtain the correction to the melting temperatures.

The paper is organized as follows. In the next section, the work in Refs\cite{1} will be reviewed and the melting temperatures under the holographic potential is presented. We will add the sub-leading order to the holographic potential and calculate the corrections to the dissociation temperatures in section 3. The section 4 concludes the paper.

2 The holographic potential model

The heavy quarkonium, $J/\psi$ or $\Upsilon$ can be modelled as a non-relativistic bound state of a heavy quark and its antiparticle, the wave function of their relative motion satisfies the Schrodinger equation

\begin{equation}
\left[-\frac{1}{2\mu} \nabla^2 + U(r,T)\right] \psi = -E(T) \psi \tag{1}
\end{equation}

where $E(T)$ is the binding energy of the bound state and $U(r,T)$ is related to the free energy $F(r,T)$ by

\begin{equation}
U(r,T) = -T^2 \left[\frac{\partial}{\partial T} \left(\frac{F(r,T)}{T}\right)\right]_r \tag{2}
\end{equation}

and $F(r,T)$ can be extracted from the Wilson loop operator between a static pair of $q\bar{q}$

\begin{equation}
e^{-\frac{1}{4} F(r,T)} = \frac{\text{tr} < W^\dagger(L_\beta) W(L_\beta) >}{\text{tr} < W^\dagger(L_\beta) > < W(L_\beta) >} \tag{3}
\end{equation}

where $L_\beta$ denotes the Wilson line running in the Euclidian time direction at spatial coordinates $(0,0,\pm \frac{1}{2} r)$ and is closed by the periodicity $\beta = \frac{4\pi}{r}$ and

\begin{equation}
W(L_\pm) = Pe^{-i f_{L_\pm} dx^\mu A_\mu(x)} \tag{4}
\end{equation}
with $A_\mu$ the gauge potential and the symbol $P$ enforcing the path ordering along the loop $C$. The thermal expectation value $\langle W(C) \rangle$ can be measured for QCD on a lattice and the heavy quark potential is defined with F-ansatz or U-ansatz.

According to the holographic principle which places the $L_\pm$ on the boundary ($z \to \infty$) of the Schwarzschild-AdS$_5 \times S^5$ whose metric can be written as
\[
d s^2 = \pi^2 T^2 z^2 (f dt^2 + d\vec{x}^2) + \frac{1}{\pi^2 T^2 z^2} dz^2
\]
where $f = 1 - \frac{1}{x_1}$, $d\vec{x}^2 = dx_1^2 + dx_2^2 + dx_3^2$ with $x_1 = x_2 = 0$ and $x_3$ a function of $z$.

In the case of $\mathcal{N} = 4$ SYM, the AdS/CFT duality relates the Wilson loop expectation value to the path integral of the string-sigma action developed in [12] of the worldsheet in the AdS$_5 \times S^5$ bulk. To the leading order of strong coupling, the path integral is given by its classical limit, which is the minimum area of the world sheet
\[
F(r, T) = -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)} \frac{\sqrt{\lambda}}{r} \min\{g_0(rT), 0\},
\]
with
\[
-\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)} \frac{\sqrt{\lambda}}{r} g_0(rT) = \frac{1}{\pi\alpha'} \left[ \int_0^{\rho_0} dz \frac{\sqrt{J_z} z^2}{z^2 \sqrt{z_0^4 - z^4}} - 1 \right] - \int_{\rho_0}^{\rho_{c0}} dz \frac{\sqrt{J_z} z^2}{z^2}
\]
(7)

where $g_0(rT)$ is a monotonically decreasing function with $g_0(0) = 1$, $g_0(r_0 T) = 0$ and $r_0$ is the screening length.

As a matter of convenience, we introduce a dimensionless radial coordinate $\rho = \pi Tr$, and we find
\[
F(r, T) = -\frac{\alpha}{r} \phi(\rho) \theta(\rho_0 - \rho),
\]
with $\alpha = \frac{\pi^2}{\Gamma^4\left(\frac{1}{4}\right)} \sqrt{\lambda} \approx 0.2285 \sqrt{\lambda}$.

The analytical small $\rho$ expansion of $\phi(\rho)$ and numerical results both suggest that $\phi(\rho)$ can be well approximated by a linear function
\[
\phi(\rho) \approx 1 - \frac{\rho}{\rho_0},
\]
with $\rho_0 = 0.7359$.

We define the dissociation temperature $T_d$ as the temperature when the bound energy $E(T_d)$ becomes zero, and the corresponding Schrödinger equation reduces to
\[
\frac{d^2 R}{d\rho^2} + 2 \frac{dR}{d\rho} - \left[ \frac{l(l+1)}{\rho^2} + U \right] R = 0
\]
with $U = \frac{m V_{eff}}{\pi^2 T^2}$.

Here we consider the U-ansatz, we have
\[
U = -\frac{\eta^2}{\rho_0 \rho} [\phi(\rho) - \rho \frac{d\phi}{d\rho}] \theta(\rho_0 - \rho)
\]
with
\[
\eta = \sqrt{\frac{3 \rho_0 m}{\pi T}}.
\]

It follows from (9) and (11) that
\[
U = -\frac{\eta^2}{\rho_0 \rho} \theta(\rho_0 - \rho).
\]

Actually, one should consider the exact holographic potential. But it has been found in [6] that the comparison with the dissociation temperature obtained from (13) is very close to that one from the exact holographic potential. So we stay with the truncated Coulomb potential for the rest of the paper.

To proceed, substituting (13) into (10) one gets
\[
R(r) = \frac{1}{\sqrt{\rho}} J_{2l+1}(2\eta \sqrt{\rho/\rho_0}), \quad \rho \leq \rho_0,
\]
\[
R(r) = \text{const.} \rho^{-l-1}, \quad \rho > \rho_0
\]
with $J_n(x)$ the Bessel function. Then the threshold $\eta_d$ can be related to the matching condition at $\rho = \rho_0$
\[
d \frac{d}{d\rho} (\rho^{l+1} R(r)) \big|_{\rho = \rho_0} = 0,
\]
this yields the correspond secular equation for $\eta_d$
\[
2l + 1 - \eta_d = 0.
\]

Knowing the values of $l$, $\eta_d$ can be carried out from (16).

Thus, Eq. (12) implies
\[
T_d = \frac{\alpha \rho_0 m}{\pi \eta_d^2}.
\]

The numerical results for $T_d$ of $J/\Psi$ and $\Upsilon$ are presented in Table 1, where we have chosen $m = 1.65 GeV, 4.85 GeV$ for c and b quarks.

Table 1. $T_d$ in MeV’s for $J/\Psi$ and $\Upsilon$ under the holographic potential.

|          | $T_d(\lambda = 5.5)$ | $T_d(\lambda = 6\pi)$ |
|----------|----------------------|-----------------------|
| $J/\Psi(1s)$ | 143                  | 265                   |
| $J/\Psi(2s)$ | 27                   | 50                    |
| $J/\Psi(1p)$ | 31                   | 58                    |
| $\Upsilon(1s)$ | 421                  | 780                   |
| $\Upsilon(2s)$ | 80                   | 148                   |
| $\Upsilon(1p)$ | 92                   | 171                   |
3 Melting temperature with sub-leading order potential

Now we add the sub-leading order term to the holographic potential and explore its contribution.

As was shown in Ref.[13] that the strong coupling expansion of $F(r,T)$ can be written as

$$F(r,T) = \frac{4\pi^2}{\Gamma^3(\frac{1}{4})} \min \left[ g_0(rT) - \frac{1.3346g_1(rT)}{\sqrt{\lambda}} + O(\frac{1}{\lambda}), 0 \right]$$

(18)

where $g_0(rT)$ has been discussed in the previous section and $g_1(rT)$ is a monotonically decreasing function, which reaches 0.92 at $r_0$ where $g_0(r_0T) = 0$.

Likewise, we write

$$F(r,T) = -\frac{\alpha}{\rho} \phi_1(\rho) \theta(\rho_1 - \rho),$$

(19)

where

$$\phi_1(\rho) = g_0(\rho) - \frac{1.3346g_1(\rho)}{\sqrt{\lambda}} \approx 1 - \frac{\rho}{\rho_0} - \frac{1.3346g_1(\rho)}{\sqrt{\lambda}},$$

(20)

where $\rho_0 = 0.7359$ and $\rho_1$ is determined by $\phi_1(\rho_1) = 0$.

As the temperature correction to the sub-leading term of the heavy quark potential is numerically small, in another word $g_1(\rho)$ decreases monotonically from 1 to 0.92 as $\rho \in (0, \rho_0)$, we can fit $g_1(\rho) = 1 - 0.11\rho$.

Then we have

$$U_1 = -\eta^2 \rho_1(1 - \frac{1.3346}{\sqrt{\lambda}})\theta(\rho_1 - \rho),$$

(21)

with

$$\eta_1 = \sqrt{\frac{\alpha \rho_1 m}{\pi T_1}}.$$  

(22)

On writing

$$\eta_x = \eta_1 \sqrt{1 - \frac{1.3346}{\sqrt{\lambda}}},$$

(23)

We have

$$U_1 = -\frac{\eta^2}{\rho_1} \theta(\rho_1 - \rho).$$

(24)

Parallel to (18) $\sim$ (19), one can readily verify that $\eta_x = \eta_1$, and this yields

$$\eta_d = \eta_{d1} \sqrt{1 - \frac{1.3346}{\sqrt{\lambda}}},$$

(25)

where $\eta_{d1}$ refers to the threshold value of $\eta_1$ and the counterpart $\eta_d$ has been discussed in Eq. (19). Thus, the rectified melting temperature $T'_d$ can be carried according to

$$T'_d = \frac{\alpha \rho_1 m}{\pi \eta^2_{d1}}$$

(26)

Here we show the curve about $T'_d/T_d$ vs $\lambda$ in Fig. 1. Note that owing to the $\lambda$ correction to the holographic potential, $T'_d$ is smaller than $T_d$. We find this correction gives rise to 47% reduction of $T_d$ when $\lambda = 6\pi$, but it vanishes for $\lambda \to \infty$ as it must, since the sub-leading order to the potential vanishes in that limit. The physical meaning will be discussed in the next section.

4 Conclusion

In this paper, we have reviewed the melting temperatures studied with the truncated holographic potential and obtained its correction from the contribution of the sub-leading term of such potential. The dissociation temperature is lowered, leaving the corrected values further below the lattice result. This disagreement can be attributed to some reasons: Firstly, the short screening length $r_a \simeq 0.25 fm$ at $T = 200 MeV$ of the AdS/CFT potential and sharp cutoff nature of the screening. Secondly, one should take into account the different number of degrees of freedom in $N_c = 3$ SYM and 3 flavor QCD, the similar problem has been figured out in the calculation of jet quenching parameter beyond AdS/CFT correspondence in [14], where they have matched the corresponding entropy density to obtain $T^3 \simeq 3T_{YM}^3$. Moreover, one should also bear in mind that the particle of $N' = 4$ SYM is quite different to that of QCD. In particular it does not include particles in the fundamental representation, but only in the adjoint representation.

To summarize, the melting temperature studied in this work relies on the holographic quark potential. How-
ever, in gauge-gravity duality, heavy quark potential at finite temperature is usually calculated with the pure AdS background, and the potential also does not contain any confining term in the deconfined phase. This led some authors to consider the potential closer to QCD, for instance heavy quark potential in strongly-coupled $\mathcal{N} = 4$ SYM in a magnetic field[15] and with some deformed AdS$_5$ model[16]. Applying these rectified potential, one could obtain the melting temperature as well. We hope to report our progress in this regard.

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