SM Extension With Gauged Flavor $U(1)_F$

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Abstract

Extension of the Standard Model with anomaly free $U(1)_F$ flavor symmetry is studded. With this extension and addition of the right handed neutrino states the solution of anomaly free charge assignment is found, which gives appealing texture zero and hierarchical Yukawa matrices. This gives natural understanding of hierarchies between charged fermion masses and CKM matrix elements. Neutrino Dirac and Majorana coupling matrices also have desirable structures leading to successful neutrino oscillations with inverted neutrino mass ordering. Other interesting implications of the presented scenario are also discussed.

1 Introduction

Although being very successful, the Standard Model (SM) is unable to resolve some puzzles. Among them is a problem of fermion flavor. The origin of hierarchies between charged fermion masses and CKM mixing angles is unexplained. Moreover, the SM is unable to accommodate the neutrino data [1]. In this work we consider extension which give simultaneous resolution of these problems. The extension, we consider is the flavor $U(1)_F$ symmetry, which will be gauged. Besides this, we augment the fermion sector with right handed neutrinos (RHN) which will be responsible for generation of light neutrino masses and mixings.

While the Abelian flavor $U(1)_F$ is a simplest candidate for the flavor symmetry [2], it’s gauging is a challenging task, because the anomaly cancelation conditions give severe constraints for realistic model building. Below we present our finding of such $U(1)_F$ charge assignment.

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2 Anomaly Free Flavor $U(1)_F$

Earlier attempts to find anomaly free setup, with $U(1)_F$ symmetry, exist in a literature [3–5]. These have been either within minimal supersymmetric (SUSY) extension of the SM [3] or within SUSY Grand Unified Theories (GUT) [4, 5]). In [5] for finding of the anomaly free have been either within minimal supersymmetric (SUSY) extension of the SM [3] or within SUSY extended GUT symmetry groups [unifying $SU(5)$ GUT and $U(1)_F$ (or latter’s some part)] has been used. Although approach is very attractive, unification putting additional constraints, disallow to have much texture zeros and predictions. Besides these, GUTs usually suffer other problems which are not directly related to the flavor symmetry. Since we feel that finding of anomaly free constructions are too far from being fully explored, our study here will be SM extension with gauged $U(1)_F$ symmetry and RHN states. 

The non-trivial states under the SM gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, we introduce, will be just those of SM. These are the Higgs doublet $\varphi$ and three families of matter $\{q, u^c, d^c, l, e^c\}_{i=1,2,3}$ where $i = 1, 2, 3$ is the family index.2 As for the extension is concerned, fermionic sector will be augmented with RHNs $N_{i=1,2,3}$. As already emphasized, the extra gauge symmetry $U(1)_F$ is considered, with the scalar field $X$ - the ‘flavon’ - needed for the $U(1)_F$ breaking.

For finding anomaly free $U(1)_F$ charges we will use several simple observations. First of all, recall that simplest anomaly free $U(1)$ symmetry is the hypercharge symmetry $U(1)_Y$ - the part of the SM gauge sector. So, in principle for $U(1)_F$ the family dependent (hyper-) charges can be used. Furthermore, introducing the right handed neutrinos one can also build the gauged $(B-L)$ symmetry, which is also anomaly free. Obviously, with family dependent $(B-L)$ charges, anomalies will still vanish. So, one option is to have $U(1)_F$’s charges $Q_i(f)$ as the following superposition $\bar{a}_i Y(f) + \tilde{b}_i Q_{B-L}(f)$, where $\bar{a}_i, \tilde{b}_i$ are some constants. With this superposition, all anomalies of the $G_{SM}$ remain intact and also additional and mixed anomalies

\[(U(1)_F)^3: \quad A_{111} = \sum_i^3 Q_i^3, \quad (1a)\]
\[U(1)_Y \times (U(1)_F)^2: \quad A_{1Y1} = \sum_i Y_i Q_i^2, \quad (1b)\]
\[(U(1)_F)^2 \times U(1)_F: \quad A_{YY1} = \sum_i Y_i^2 Q_i, \quad (1c)\]
\[(SU(2)_L)^2 \times U(1)_F: \quad A_{221} = \sum_i [Q_i(l_i) + 3Q_i(q_i)], \quad (1d)\]
\[(SU(3)_c)^2 \times U(1)_F: \quad A_{331} = \sum_i [2Q_i(g_i) + Q_i(u^c_i) + Q_i(d^c_i)], \quad (1e)\]
\[(\text{Gravity})^2 \times U(1)_F: \quad A_{GG1} = \sum_i Q_i, \quad (1f)\]

automatically vanish. Although for the $Q_i(f)$’s assignments the superposition $\bar{a}_i Y(f) + \tilde{b}_i Q_{B-L}(f)$ can be considered, one immediate outcome is that by requiring that the top quark has the renormalizable Yukawa coupling $(\lambda_t \sim 1)$ with the Higgs doublet $\varphi$, also bottom quark and the tau lepton Yukawas will be allowed at renormalizable level - with the expectancy $\lambda_b, \lambda_\tau \sim 1$. Besides

\[2\]Here and below for fermionic $f$ states we use a two-component Weyl spinor in $(\frac{1}{2}, 0)$ representation of the Lorentz group.
this unpleasant fact, only with \(\tilde{a}_i, \tilde{b}_i\) we can not get Yukawa coupling matrices with much texture zeros. Thus, for the \(U(1)_F\)’s charges we will consider the modified superposition

\[
Q_i(f) = \tilde{a}_i Y(f) + \tilde{b}_i Q_{B-L}(f) + \Delta Q_i(f),
\]

(2)

were the additions \(\Delta Q_i(f)\) will be selected in such a way that the anomalies \(A_{YY1}, A_{221}, A_{331}, A_{GG1}\) stay intact. However, for vanishing of the anomalies \(A_{111}\) and \(A_{Y1}\) additional constraints on the charge prescriptions need to be imposed. It turns out that for this goal, instead of tree RHNs, we will need four of them - \(N_{1,2,3,4}\). The additions satisfying these and give desirable fermion pattern are:

\[
\Delta Q_i(q) = \tilde{\delta}_3 \{0, 1, -1\} + \tilde{\delta}_8 \{1, 1, -2\},
\]

(3a)

\[
\Delta Q_i(u^c) = \tilde{u}_3 \{0, 1, -1\} + \tilde{u}_8 \{1, 1, -2\},
\]

(3b)

\[
\Delta Q_i(d^c) = \tilde{d}_3 \{1, -1, 0\} + \tilde{d}_8 \{1, 1, -2\},
\]

(3c)

\[
\Delta Q_i(l) = \tilde{l}_3 \{1, -1, 0\} + \tilde{l}_8 \{1, 1, -2\},
\]

(3d)

\[
\Delta Q_i(e^c) = 0,
\]

(3e)

\[
\Delta Q_i(N) = \tilde{n} \{1, 1, 1, -3\}.
\]

(3f)

Note that, being traceless, these additions coincide with diagonal (Cartan) generators of \(SU(3)\) [in Eqs. (3a)-(3d)] and \(SU(4)\) unitary groups [in Eq. (3f)]. Thus the notations for the constants \((\tilde{q}_{3,8}, \cdots, \tilde{l}_{3,8})\) become obvious. These constants, together with \(\tilde{a}_i, \tilde{b}_i\) will be enough for our purposes. Upon selecting these constants we will bare in mind some requirements which need to be satisfied in order to obtain desirable and phenomenologically viable model. These requirements are:

(i) In order to have top quark Yukawa coupling \(\lambda_t \sim 1\), the \(U(1)_F\) symmetry should allow coupling \(q_3 u^c_3 \varphi\) at renormalizable level. At the same time, all other Yukawa terms (responsible for charged fermion masses) should emerge by spontaneous breaking of the \(U(1)_F\). So, the adequate mass hierarchies and CKM mixings will be expressed by powers of \(\langle X \rangle / M_{Pl}\).

(ii) Dirac and Majorana-type couplings involving RHN \(N\)-states should be such that naturally generate light neutrino masses and mixings in order to accommodate recent neutrino data [1].

(iii) While the \(U(1)_F\) charge assignment ansatz of Eqs. (2), (3) automatically insure zero anomalies of (1c)-(1f), an additional constraints need to be imposed for canceling anomalies of (1a) and (1b).

(iv) Finally, ratios of the states’ charges should be rational, in order to allow (phenomenologically required) couplings between them.

Guided by these, in (2) we use normalization such that \(Y(l) = 1\) and \(Q_{B-L}(q) = 1/3 = -3Q_{B-L}(l)\). Also, without loss of any generality, for the scalar \(X\), we will select \(Q_X = 1\). With these and requirements listed above, the best selection which we find is the following:

\[
\tilde{a}_i = \frac{1}{3} \{46, 43, 10\}, \quad \tilde{b}_i = \frac{1}{3} \{-91, 35, 38\},
\]

\[
\{\tilde{q}_3, \tilde{u}_3, \tilde{d}_3, \tilde{l}_3\} = \frac{1}{3} \{-16, 7, -67/2, -3/2\},
\]

\[
\{\tilde{q}_8, \tilde{u}_8, \tilde{d}_8, \tilde{l}_8\} = \frac{1}{9} \{38, -41, 23/2, 51/2\}, \quad \tilde{n} = -\frac{5}{3}.
\]

(4)
Table 1: $U(1)_F$ charge ($Q$) assignment for the states. $Q_X = 1$, $Q_\varphi = -7$.

| $q_1, q_2, q_3$ | $u_1^c, u_2^c, u_3^c$ | $d_1^c, d_2^c, d_3^c$ | $l_1, l_2, l_3$ | $e_1^c, e_2^c, e_3^c$ | $N_1, N_2, N_3, N_4$ |
|------------------|------------------------|------------------------|-----------------|------------------------|------------------------|
| $\{-11, -2, 0\}$| $\{26, 13, 7\}$       | $\{-10, -1, -9\}$     | $\{48, 6, -15\}$| $\{-61, -17, 6\}$     | $\{-32, 10, 11, 5\}$  |

With these, by using (2) and (3a)-(3f) we obtained the charges given in Table 1. One can readily check out that all anomalies given in (1a)-(1f) vanish. Note that after all charges are fixed, since whole Lagrangian respect $U(1)_Y$ symmetry, by making family universal charge shift for the states $Q \rightarrow Q + \alpha Y$, all couplings and anomalies will remain intact. The constant $\alpha$ can be selected to have convenient form of the charges. We have already exploited this by setting $Q(q_3) = 0$ (see Tab. 1). Presented charge assignment give interesting textures for charged fermion mass matrices and neutrinos as well. These we discuss in the following sections.

3 Quark and Charged Lepton Yukawa Textures

As mentioned, for the $U(1)_F$ gauge symmetry braking, the SM singlet scalar $X$ - the flavon field - is introduced and it’s $U(1)_F$ charge is taken to be $Q_X = 1$. The VEV $\langle X \rangle$ brakes the $U(1)_F$ and also forms fermion mass matrices. Since in the Yukawa couplings the appropriate powers of $\frac{X}{M_{Pl}}$ and $\frac{\overline{X}}{M_{Pl}}$ will appear, it is convenient to introduce notations

$$
\frac{X}{M_{Pl}} \equiv \varepsilon , \quad \frac{\overline{X}}{M_{Pl}} \equiv \overline{\varepsilon} .
$$

Note that, $M_{Pl} \simeq 2.4 \cdot 10^{18}$ GeV is reduced Planck scale, which will be treated as natural cut off for all higher dimensional non-renormalizable operators.

With the $U(1)_F$ charges of the Higgs doublet $\varphi$ of $Q_\varphi = -7$, and of the fermion states given in Table 1, the $qu^c\varphi, qd^c\overline{\varphi}$ and $le^c\overline{\varphi}$ type couplings, involving different powers of $\varepsilon$ and $\overline{\varepsilon}$, will be:

$$
\left( q_1, q_2, q_3 \right) \left( \begin{array}{ccc}
\varepsilon^6 & \varepsilon^5 & \varepsilon^{11} \\
\varepsilon^{17} & \varepsilon^4 & \varepsilon^2 \\
\varepsilon^{19} & \varepsilon^6 & 1
\end{array} \right) \left( \begin{array}{c}
q_1^c \\
q_2^c \\
q_3^c
\end{array} \right) \varphi + \left( q_1, q_2, q_3 \right) \left( \begin{array}{ccc}
\varepsilon^{14} & \varepsilon^5 & \varepsilon^{13} \\
\varepsilon^5 & \varepsilon^4 & \varepsilon^4 \\
\varepsilon^3 & \varepsilon^6 & \varepsilon^2
\end{array} \right) \left( \begin{array}{c}
d_1^c \\
d_2^c \\
d_3^c
\end{array} \right) \overline{\varphi} + \left( l_1, l_2, l_3 \right) \left( \begin{array}{ccc}
\varepsilon^6 & \varepsilon^{38} & \varepsilon^{61} \\
\varepsilon^{48} & \varepsilon^4 & \varepsilon^{19} \\
\varepsilon^{69} & \varepsilon^{25} & \varepsilon^2
\end{array} \right) \left( \begin{array}{c}
e_1^c \\
e_2^c \\
e_3^c
\end{array} \right) \overline{\varphi} + \text{h.c.}
$$

(6)

In front of each operator of (6) the dimensionless coupling (omitted here) should stand. Substituting the VEVs $\langle \varepsilon \rangle = \langle \overline{\varepsilon} \rangle \equiv \varepsilon$, and omitting those terms, with high powers of $\varepsilon$, which are irrelevant in practice, the Yukawa matrices $Y_U, Y_D, Y_E$ corresponding to up, down quarks and charged leptons respectively, are (in a basis we comment below):

$$
Y_U \simeq \begin{pmatrix}
a_1^e \varepsilon^8 & a_2 \varepsilon^5 & 0 \\
0 & a_2 \varepsilon^4 & \varepsilon^2 \\
0 & 0 & 1
\end{pmatrix} \lambda^0_t ,
$$

(7)
$$Y_D \simeq \begin{pmatrix} e^{-i\eta_1} & 0 & 0 \\ 0 & e^{-i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & b_1\epsilon^3 & 0 \\ b_1^*\epsilon^3 & b_2\epsilon^2 & b_2^*\epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \kappa_\ell \epsilon^2. \quad (8)$$

$$Y_E \simeq \begin{pmatrix} c_1\epsilon^4 & 0 & 0 \\ 0 & c_2\epsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \kappa_\tau \epsilon^2. \quad (9)$$

We have made field phase redefinitions in such a way that, in this basis, CKM matrix remains unity. Also, we have performed $1-3$ and $2-3$ rotation of $d^c$ states in such a way that $3-1$ and $3-2$ entries of $Y_D$ vanishes (this transformation of the $d_{1,2,3}^c$ states is unobservable in the SM). Moreover, $Y_U$ is real, two phases $\eta_{1,2}$ appear in $Y_D$, while $Y_E$ is real. The phases $\eta_{1,2}$ will not contribute to the quark masses, but will be important for the CKM matrix elements.

Starting with the quark sector, with proper (and fully natural) selection of input parameters we can get desirable values for fermion masses and CKM mixing angles. Since the Yukawa matrices are hierarchical, in a pretty good approximation we can derive the following analytic expressions:

$$\lambda_t = \lambda_t^0[1 + \mathcal{O}(\epsilon^4)], \quad \frac{\lambda_u}{\lambda_t} \simeq \frac{a_1^0\epsilon^8}{\sqrt{1 + (a_1\epsilon/a_2)^2}}, \quad \frac{\lambda_c}{\lambda_t} \simeq a_2\epsilon^4 \sqrt{1 + (a_1\epsilon/a_2)^2}, \quad (10)$$

$$\lambda_b = \kappa_b\epsilon^2[1 + \mathcal{O}(\epsilon^4)], \quad \frac{\lambda_d}{\lambda_b} \simeq \frac{b_1b_2^*\epsilon^4}{\sqrt{b_2^2 + (b_1^2 + b_2^2)\epsilon^2}}, \quad \frac{\lambda_s}{\lambda_b} \simeq \epsilon^2 \sqrt{b_2^2 + (b_1^2 + b_2^2)\epsilon^2}. \quad (11)$$

For writing down expression of the CKM matrix elements, it is useful to introduce two angles

$$\tan \theta_u = \frac{a_1}{a_2} \epsilon \sqrt{1 + \epsilon^4}, \quad \tan 2\theta_d = \frac{2b_1b_2\epsilon \sqrt{1 + b_2^2\epsilon^4}}{b_2^2 - (b_1^2 + b_2^2)\epsilon^2},$$

and notations $\sin \theta_{u,d} \equiv s_{u,d}$ and $\cos \theta_{u,d} \equiv c_{u,d}$. With these we have

$$|V_{us}| = |c_u s_d e^{i\eta_1} - s_u c_d| \frac{(e^{i\eta_2} + b_2^*\epsilon^4)}{\sqrt{1 + \epsilon^4} \sqrt{1 + b_2^2 \epsilon^4}} + \mathcal{O}(\epsilon^7),$$

$$|V_{cb}| = c_u \epsilon^2 \frac{|1 - e^{i\eta_2} b_2(1 + b_2^2 \epsilon^4)|}{\sqrt{1 + \epsilon^4} \sqrt{1 + b_2^2 \epsilon^4}} + \mathcal{O}(\epsilon^8), \quad \frac{|V_{ab}|}{|V_{cb}|} = \tan \theta_u. \quad (13)$$

For the parameter

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}, \quad (14)$$

related to the CP violation and defined in a phase convention independent way [10], we obtain

$$\overline{\rho} + i\overline{\eta} \simeq \frac{c_u c_d e^{i\eta_1} + s_u s_d e^{i\eta_2}}{c_d s_u e^{i\eta_1} - c_u s_d e^{i\eta_2}} \tan \theta_u. \quad (15)$$

Upon parametrization of the Yukawa matrices we have taken away the factors $\lambda^0_t$ and $\kappa_b$. These will be selected in such a way as to get observed values of masses $M_t$ and $m_b$. Remaining parameters (i.e. $\epsilon, a_{1,2}, a_{1}', b_{1,2}, b_{1,2}', \eta_{1,2}$) will determine light quark masses and CKM matrix elements. Relations (10)-(13) and (15) will help to find parameters giving desirable fit. Before going to that, let us
These, by performing renormalization [using (16)-(19) and input 
\[ M_4 \Lambda = 2 \]
Good fit is obtained for the following values of input parameters (values are given at high scale \( \Lambda \) which we take close to the GUT scale - few\( \times 10^{16} \) GeV), need to be renormalized at low energies. For this we perform the renormalization and calculate these quantities at low scales. We have

\[
\frac{\lambda_{u,c}}{\lambda_t} \bigg|_{M_t} = \eta_{u,c} \frac{\lambda_{u,c}}{\lambda_t} \bigg|_{\Lambda}, \quad \frac{\lambda_{d,s}}{\lambda_b} \bigg|_{M_t} = \eta_{d,s} \frac{\lambda_{d,s}}{\lambda_b} \bigg|_{\Lambda},
\]

\[
V_{\alpha\beta} \big|_{M_Z} = \eta_{mix} V_{\alpha\beta} \big|_{\Lambda}, \quad \text{if} \quad (\alpha, \beta) = (ub, cb, td, ts),
\]

\[
V_{\alpha\beta} \big|_{M_Z} = \eta_{mix} V_{\alpha\beta} \big|_{\Lambda}, \quad \text{if} \quad (\alpha, \beta) = (ud, us, cd, cs, tb).
\]

In 1-loop approximation we have \( \eta_{u,c} \simeq 1/\eta_{d,s} \simeq 1/\eta_{mix} \simeq \exp \left( \frac{3}{32 \pi^2} \lambda_0^2 d \ln \mu \right) \). However, we will perform more accurate calculations. For the renormalization of the light family Yukawa couplings and \( \lambda_{b, \tau} \) we use 2-loop RG equations, while the runnings of \( \lambda_t \) and \( g_3 \) are performed through 3-loop RGs. For the running of the CKM matrix elements the 2-loop RG’s [6] will be used. Upon the running between \( M_t \) (the pole mass of the top quark) and the scale \( \Lambda \), for boundary values of the couplings at \( \mu = M_t \) we use values given in [7].

Doing so, for \( M_t = 172.5 \) GeV and \( \alpha_3(M_Z) = 0.1179 \) (the values we use throughout of this work) we get

\[
\eta_{u,c} \simeq 1.1262 + 0.00187 \cdot \ln \left( \frac{\Lambda}{2 \cdot 10^{16} \text{GeV}} \right),
\]

\[
\eta_{d,s} \simeq 0.8916 - 0.00143 \cdot \ln \left( \frac{\Lambda}{2 \cdot 10^{16} \text{GeV}} \right),
\]

\[
\eta_{mix} \simeq 0.89157 - 0.001433 \cdot \ln \left( \frac{\Lambda}{2 \cdot 10^{16} \text{GeV}} \right),
\]

- the interpolated expressions which work pretty well for \( 10^{15} \) GeV < \( \Lambda < M_{Pl} \).

Also, for light quark masses, the running from \( M_t \) down to low scales need to be performed by the standard technics [7-9].

**Fit for Charged Fermion Masses and CKM Elements**

Good fit is obtained for the following values of input parameters (values are given at high scale \( \Lambda = 2 \cdot 10^{16} \) GeV):

\[
\epsilon = 0.21, \quad \{a_1, a'_1, a_2\} = \{0.6974, 1.7065, 1.6606\}, \quad \{\eta_1, \eta_2\} = \{3.01985, -1.3954\},
\]

\[
\{b_1, b'_1, b_2, b'_2\} = \{0.47834, 0.54541, 0.45448, 0.59088\}.
\]

These, by performing renormalization [using (16)-(19) and input \( M_t = 172.5 \) GeV, \( m_b(m_b) = 4.18 \) GeV], at low scales give:

\[
(m_u, m_d, m_s) (2 \text{ GeV}) = (2.16, 4.67, 93) \text{ MeV}, \quad m_c(m_c) = 1.27 \text{ GeV},
\]

at \( \mu = M_Z \) : \( |V_{us}| = 0.225, \quad |V_{cb}| = 0.04182, \quad |V_{ub}| = 0.00369, \quad \overline{\rho} = 0.159, \quad \overline{\eta} = 0.3477 \)

where definitions for \( \overline{\rho}, \overline{\eta} \) are given in Eq. (14). All results given above are in perfect agreement with experiments [10].
As far the charged lepton masses are concerned, from (9) with the input $M_\tau = 1.777$ GeV and at $\mu = \Lambda$, \{c_1, c_2\} $\simeq \{0.1437, 1.335\}$, (22) and taking into account that $\frac{\lambda_{\tau \mu}}{M_\tau} \simeq \frac{\lambda_{\tau \mu}}{\Lambda}$, we obtain:

$$M_e = 0.511 \text{ MeV}, \quad M_\mu = 105.66 \text{ MeV},$$

(23) - also in agreement with experiments.

### 4 Neutrino Sector

For building the realistic neutrino sector, the singlet states $N_{1,2,3}$ will be used as right handed neutrinos. Since the $N_4$ is not really needed for these purposes, its couplings to the leptons and also to $N_{1,2,3}$ can be easily avoided by imposing the reflection symmetry $N_4 \rightarrow -N_4$ (about this symmetry and its possible implication will be commented below). This will make analysis simpler. Thus, with $U(1)_F$ charges given in Table 1 the $lN\varphi$ and $N_iN_j$ type couplings ($i, j = 1, 2, 3$) will be:

$$m_D \simeq \begin{pmatrix} A\epsilon^9 & 0 & 0 \\ 0 & B_1\epsilon^9 & C_1\epsilon^{10} \\ 0 & B_2\epsilon^{12} & C_2\epsilon^{11} \end{pmatrix}v, \quad M_R \simeq \begin{pmatrix} 0 & ae^2 & de \\ ae^2 & b & ce \\ de & ce & \epsilon^2 \end{pmatrix} \bar{c}M_{Pl}\epsilon^{20}.\quad (25)$$

In these operators the dimensionless couplings are still omitted. Substituting the VEVs $\langle \varphi \rangle = \langle \bar{\varphi} \rangle = \epsilon, \langle \varphi \rangle = v$ and omitting irrelevant small entries, for neutrino Dirac and Majorana matrices we get:

$$M_\nu \simeq -m_DM_R^{-1}m_D^T \simeq \begin{pmatrix} \beta & \gamma & \gamma' \\ \gamma & \alpha^2 & \alpha \\ \gamma' & \alpha & 1 \end{pmatrix} \tilde{m},$$

(26) with $\tilde{m}$ and the dimensionless couplings $\alpha, \beta, \gamma, \gamma'$ expressed by the scales and couplings appearing in Eq. (25). Note that, $M_\nu$’s 2 – 3 block’s determinant is zero:

$$M_\nu^{(2,2)}M_\nu^{(3,3)} - (M_\nu^{(2,3)})^2 = 0.\quad (27)$$

The origin of this relation can be understood as follows. Because of $M_R^{(1,1)} = 0$, the determinant of the lower $2 \times 2$ block of $M_R^{-1}$ is zero. Moreover, since lower $2 \times 2$ block of $m_D$ decouples (i.e. (1, 2) and (1, 3) entries in $m_D$ are zero), the see-saw formula $M_\nu \simeq -m_DM_R^{-1}m_D^T$ gives the relation of Eq. (27). The latter give specific predictions, on which we will focus now.
Since the charged lepton mass matrix $Y_E$ is essentially diagonal, the whole lepton mixing matrix $U$ comes from the neutrino sector. Therefore, we have

$$M_{\nu} = P U^* P' M^\text{diag}_{\nu} U^\dagger P,$$

(28)

where in a standard parametrization, $U$ has the form:

$$U = \begin{pmatrix}
    c_{13} s_{12} & c_{13} s_{12} & s_{13} e^{-i\delta} \\
    -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\
    s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13}
\end{pmatrix}$$

(29)

with $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The phase matrices $P, P'$ are given by:

$$P = \text{Diag} \left( e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3} \right), \quad P' = \text{Diag} \left( 1, e^{i\rho_1}, e^{i\rho_2} \right),$$

(30)

where $\omega_{1,2,3}, \rho_{1,2}$ are some phases.

As was investigated in details (see second Ref. in [5]), the relation (27) excludes the possibility of the normal ordering of the neutrino masses. Using in (27) the (28)-(30) we obtain

$$e^{i\rho_1 m_1 m_2 s_{13}^2} + e^{i(\rho_2 + 2\delta)} (m_{1 s_{12}^2} + e^{i\rho_1} m_2 c_{12}^2) m_3 c_{13}^2 = 0,$$

(31)

which in turn gives:

$$\cos \rho_1 = \frac{m_1^2 m_2^2 \tan^4 \theta_{13} - m_3^2 (m_{1 s_{12}^4} + m_2^2 c_{12}^4)}{2 m_1 m_2 m_3^2 s_{12}^2 c_{12}^2},$$

(32)

$$2\delta = \pm \pi - \rho_2 + \text{Arg} \left( s_{12}^2 e^{i\rho_1} + \frac{m_2}{m_1} c_{12}^2 \right).$$

(33)

Using recent results from the neutrino experiments [1], we can easily verify that the relation of Eq. (32) is incompatible with normal ordering of neutrino masses. On the other hand, inverted ordering (IO) of neutrino masses is possible. Using the best fit values (bfv) of $\theta_{ij}, \Delta m^2_\text{sol} = m_2^2 - m_1^2$, $\Delta m^2_\text{atm} = m_2^2 - m_3^2$, expressing $m_{1,2}$ by $m_3$ as $m_1 = \sqrt{\Delta m^2_\text{atm} - \Delta m^2_\text{sol} + m_3^2}$, $m_2 = \sqrt{\Delta m^2_\text{atm} + m_3^2}$, from Eq. (32) we get allowed region for $m_3$:

$$0.001129 \text{ eV} \lesssim m_3 \lesssim 0.002833 \text{ eV}.$$

(34)

This imply $0.1002 \text{ eV} \lesssim \sum m_i \lesssim 0.1021 \text{ eV}$, satisfying the current upper bound $\sum m_i < 0.12 \text{ eV}$ [11], obtained from cosmology.

Moreover, for neutrino less double $\beta$-decay ($0\nu\beta\beta$) parameter $m_{\beta\beta} = \left| \sum U_{ai}^2 m_i P_i^a \right|$ we obtain:

$$m_{\beta\beta} = \left| c_{12}^2 s_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{i\rho_1} + s_{13}^2 m_3 e^{i(2\delta + \rho_2)} \right|,$$

(35)

which taking into account Eqs. (32), (33) and bsv’s of the oscillation parameters leads to:

$$0.01864 \text{ eV} \lesssim m_{\beta\beta} \lesssim 0.0483 \text{ eV}.$$

(36)

This range is also compatible with limits provided by $0\nu\beta\beta$ experiments [12]. In fact, due to predictive relations in Eqs. (32), (33) both parameters $\sum m_i$ and $m_{\beta\beta}$ are unequivocally determined
Figure 1: Correlation between $\sum m_i$ and $m_\beta\beta$. Solid (middle) blue line corresponds to the bfv's of the oscillation parameters [1]. Green (wider) area corresponds to the cases with oscillation parameters within the 1$\sigma$ deviations.

by the $m_3$'s values. Thus, there is correlation between $\sum m_i$ and $m_\beta\beta$, which is given in Fig. 1. Hopefully, future experiments will be able to test viability of this scenario [13].

Now we give one selection of the parameters, appearing in (25), which blends well with this neutrino scenario and then discuss some implications and outcomes. With the choice

$$\{a, b, c, d, \bar{c}\} \simeq \{3.2672e^{1.5473}, 0.79405e^{0.003733}, 0.89097e^{-0.0028735}, 0.15853e^{-1.5586}, 0.56333e^{2.9194}\}$$

$$\{A, B_1, B_2, C_1, C_2\} \simeq \{2.0236, 2.0236, 1.6189, 2.4283, -0.8094\}, \quad (37)$$

for the light neutrino masses and mixing angles we obtain:

$$\{m_1, m_2, m_3\} = \{0.049197, 0.049942, 0.0015\}eV, \quad (38)$$

$$\{\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13}\} = \{0.3035, 0.57, 0.02235\}. \quad (39)$$

From (38) we get

$$\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2 = 7.39 \cdot 10^{-5}eV^2, \quad \Delta m_{\text{atm}}^2 = m_2^2 - m_3^2 = 2.492 \cdot 10^{-3}eV^2. \quad (40)$$

Results of (39) and (40) correspond to the bfv's of the IO neutrino scenario [1]. Moreover, for the passes we get

$$\{\delta, \rho_1, \rho_2\} = \{276^\circ, 91.69^\circ, 11.49^\circ\}, \quad \omega_{1,2,3} = 0. \quad (41)$$
For this case we have $m_{\beta\beta} \approx 0.0362$ eV and $\sum m_i \simeq 0.101$ eV. These, certainly blend with the discussed predictions [of Eqs. (32), (33) and Fig. 1].

From the input (37) for the heavy right handed neutrino (RHN) masses we get:

$$\{M_{N_1}, M_{N_2}, M_{N_3}\} \simeq \{1.6, 953.5, 32480\}\text{GeV}. \quad (42)$$

Few remarks about the heavy RHN sector are in order.

The state $N_1$ (with $M_{N_1} \simeq 1.6$ GeV) can be produced in decays of heavy mesons, however the corresponding mixing $|U_{eN_1}|^2 \simeq 2.76 \cdot 10^{-11}$ is bit below (by factor $\approx 3$) of sensitivity of SHiP experiment [14]. As a separate study, would be interesting to do more detailed investigation/exploration of the model’s parameters from the perspective of this experiment.

Since the lightest RHN’s mass is $M_{N_1} \simeq 1.6$ GeV, and it mixes with $\nu_e$, there will be additional contribution to the $02\beta\beta$ parameter, which is given by [15]:

$$\left| \sum_{i=1}^{3} U_{ei}^2 m_{i} P_{i}^k + \frac{M_{N_i}}{1 + M_{N_i}^2/(p^2)} U_{eN_1}^2 \right| = e^{-0.42i}0.0362 \text{eV} + \frac{e^{-0.15i}2.76 \cdot 10^{-11} M_{N_i}}{1 + M_{N_i}^2/(p^2)} = 0.0368 \text{eV}, \quad \text{(for } \langle p^2 \rangle = (200 \text{MeV})^2\text{)}.$$  

The second term in the absolute values of Eq. (43) is the contribution from the $N_1$. The $\langle p^2 \rangle$ is averaged momentum squared corresponding to this process. As can be seen, for $\langle p^2 \rangle = (100–200\text{MeV})^2$ [15,16] the correction from the $N_1$ state is within $(0.5–1.8)\%$, i.e. negligible. Therefore, the predictions, made from the light neutrino sector (and correlation of Fig. 1) are robust.

With $N_1$’s mass within the GeV scale, we need to insure it’s sufficiently fast decay (within $\lesssim 0.3$ sec.) in order to not affect the standard Big Bang nucleosynthesis (BBN). Dominant decays of $N_1$ are three body decays via neutral and charged currents (i.e. via $Z^*$ and $W^*(\pm)$ exchange). These are leptonic $N_1 \rightarrow \nu_i \nu_j \bar{\nu}_j$, $\nu_i e_j^+ e_j^-$, $e_i^- e_j^+ \nu_j$ and semi-leptonic $N_1 \rightarrow \nu_i \bar{q}_j q_j$, $e_i^- u_j \bar{d}_k$ decays. For the leptonic decay widths we use expressions given in Ref. [17]. For the semi-leptonic decays, taking into account all inclusive decays into the quarks, by proper use of the matching RG factor [18] one can get quite reasonable estimate. Summing by all kinematically allowed channels of $N_1$’s decays and using proper expressions [17,18], for the total width (i.e. for inverse lifetime) we obtain:

$$\Gamma(N_1) = \frac{1}{\tau_{N_1}} \simeq \frac{G_F^2 M_{N_1}^5}{16 \pi^3} (1.37|U_{1N_1}|^2 + 1.35|U_{2N_1}|^2 + 0.487|U_{3N_1}|^2) \simeq \frac{1}{0.0038 \text{s}}, \quad (44)$$

which is compatible with BBN. In Eq. (44), for the squared mixing matrix elements we have used values obtained within our model:

$$|U_{iN_1}|^2 \simeq \{2.76, 1.29, 1.09\} \cdot 10^{-11}, \quad (45)$$

obtained from the inputs of (37). The states $N_{2,3}$ will decay much rapidly via the $N_{2,3} \rightarrow \varphi \ell$ channel (with lifetimes $\approx 7 \cdot 10^{-3}$ ps and $2 \cdot 10^{-4}$ ps respectively). As far as the state $N_4$ (which presence is important for anomaly cancelation) is concerned, because of the reflection symmetry $N_4 \rightarrow -N_4$ (we have introduced), it’s mixing with $N_{1,2,3}$ and couplings to the SM leptons are forbidden. However,
it will gain the mass via the $\frac{1}{2} M_{Pl} \bar{\varepsilon} N_4 N_4$ operator: $M_{N_4} \sim M_{Pl} \varepsilon^{10} \approx 4 \cdot 10^{11}$ GeV. For its decay would be responsible the operators

$$\frac{\lambda_1 \varepsilon}{M_{Pl}} (N_4 u_3^c) (d_1^c d_2^c) + \frac{\lambda_2 \varepsilon}{M_{Pl}} (N_4 u_2^c) (d_1^c d_2^c) + \text{h.c.}$$

(46)

allowed if all quarks also change sign [i.e. $(q, u^c, d^c) \rightarrow -(q, u^c, d^c)$ under reflection symmetry. This do not affect the charged fermion and neutrino sectors]. These operators will give decays $N_4 \rightarrow u_3^c d_1^c d_2^c$, $u_2^c d_2^c d_3^c$. Since $N_4$ is a Majorana state, also $N_4 \rightarrow \bar{u}_3^c d_1^c d_2^c$, $\bar{u}_2^c d_2^c d_3^c$ decays will proceed. All these give

$$\Gamma(N_4) = \frac{(|\lambda_1|^2 + |\lambda_2|^2)M_{N_4}^5 \varepsilon^2}{128 \pi^3 M_{Pl}^4} = \frac{1}{10^{-5}\text{sec.}} \left(\frac{M_{N_4}}{4 \cdot 10^{11}\text{GeV}}\right)^5 \text{ (with } \lambda_{1,2} = 1), \text{ and therefore making } N_4 \text{ harmless for the BBN. It would have been interesting to have scenario with } N_4 \text{ having proper value of mass and needed couplings for serving as a dark matter candidate. This turned out impossible with presented } U(1)_F \text{ charge assignment. Perhaps separate study is worth to be focused also on this issue.}

In summary, exploring possibility of anomaly free gauged $U(1)_F$ flavor symmetry offered attractive pattern for the charged fermion masses, neutrino oscillations and also interesting phenomenological implications. These motivate to think more and try to find other possibilities within the framework discussed in this work.

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