Saint-Venant principle for kinematic boundary conditions

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Abstract. A variant of the extended formulation of the Saint-Venant principle is formulated, including its generalization to kinematic parameters. Using the LIRA CAD software package, a numerical experiment is carried out, during which the zones of tangential stress disturbances in the support zone of a rigidly clamped I-beam under various kinematic boundary conditions are determined.

1. Introduction
It is known that in the resistance of materials the principle of Saint-Venant is formulated only in relation to loads. It is believed that in sections sufficiently remote from the places of application of the load, deformation and stress, it does not depend on the specific type of load and is determined only by its static equivalent. Thus, this principle allows one to replace some boundary conditions (acting forces) with others (convenient for static calculation), provided that the resultant and main point of the new given system of forces does not change. The attenuation of the disturbance of the stress-strain state in the calculation of the rods is usually taken to be one transverse dimension. The validity of the principle of Saint-Venant has no theoretical proof, but it is confirmed by numerous experimental data and the results of numerical calculations.

In previously published works [1, 2], the authors investigated the applicability of the Saint-Venant principle for rods of rectangular cross section. The authors also performed a numerical calculation of an I-beam, in which the behavior of normal and shear stresses near the places of load application was studied [3]. It was found that, in fact, the attenuation of stress disturbances, both normal and tangential, occurs approximately at a distance of one transverse dimension from the places of application of the loads. Outside of this local zone of perturbation, the validity of the application of the formulas of the resistance of materials is established. However, the question of the distribution of normal and shear stresses near kinematic inhomogeneities (boundary conditions) remains open both in the linear and plastic stages of the construction. In the present work, the task was to study the behavior of tangential stresses in the supporting zones of an I-beam in order to verify the assumption of a fast decay of their disturbance. The research tool was the LIRA-SAPR software package. The convergence criterion was the coincidence of the numerical solution with the results of calculations according strength of materials formulas up to 5% for shear stresses $\tau_{xy}$ and $\tau_{xz}$. 
2. Research

2.1. Computational model

In all calculations, welded I-beam supported in various ways was considered, 4.8 m long, the cross section of which is shown in figure 1. A finite element mesh of 5x5 cm was used. Since all the elements were modeled by the centers of gravity of the cross section, the LIRA CAD software package used rigid inserts in the calculations to bring the plate elements to the middle surface of the beam elements.

In order to simplify the design calculation scheme, a welded beam was considered, in which the web and flanges consisted of rectangles, and it was assumed that there were no residual stresses when welding the wall with the shelves.

In the calculations, a load is accepted that does not exceed the limit of proportionality (the linear Hooke law is valid). This is due to the fact that with nonlinear behavior of the material, the formation of plastic zones is possible, which blur the picture of the stress distribution.

![Figure 1. Cross-section of the beam](image)

2.2. Calculation No. 1

As kinematic boundary conditions, a rigid support of the left edge of the beam was adopted, implemented through the prohibition of all linear and angular movements (UX = UY = UZ = ROTX = ROTY = ROTZ = 0).

![Figure 2. Shear stress field (calculation No. 1)](image)
A study was also made of the influence of the method of applying the load on the type of perturbation of shear stresses. It was found that due to the removal of the zone of kinematic boundary conditions from static ($L > 2h$), the load application methods did not change the shape of the shear stress diagrams at the other end of the rod. This is due to the fact that the perturbation of stresses from static boundary conditions has time to decay at a distance of approximately one transverse dimension of the rod, corresponding to the height of the wall, and already in zones closer to the left edge of the rod they again take the form of disturbances caused already by the influence of kinematic boundary conditions.

This can be clearly seen on stress isofield highlighted in blue. Figure 3 shows the distribution of shear stresses along the section height (without linear approximation by elements) for calculation No. 1.

![Figure 3. Shear stress distribution diagrams in sections 1 - 4 (calculation No. 1).](image)

2.3. Calculation No. 2
In calculation No. 2, as kinematic boundary conditions, it was accepted rigid support of the left edge of the beam, implemented through the prohibition of 3 nodes from linear and angular movements ($UX = UY = UZ = ROTX = ROTY = ROTZ = 0$) of nodes at the upper and lower edges, as well as at the level of the neutral axis.

The shear stress field is shown in figure 4. As can be seen, the pattern of the distribution of tangential stresses near the kinematic boundary conditions has changed dramatically compared to 2 both in shape and in the value of shear stresses (the difference is about 5.5 times).

And in the case of fastening at individual points of the flanges to the web in the local zone near the kinematic boundary conditions, the tangential $\tau_{xz}$ are greater than $\tau_{xy}$, which must be taken into account in structural engineering.
Figure 4. Shear stress field (calculation No. 2)

Figure 5. Shear stress distribution diagrams in sections 1 - 4 (calculation No. 2).

Figure 5 shows the distribution of shear stresses along the section height (without linear approximation by elements) for calculation No. 2.

We remind you that only elastic work of the structure is accepted in the calculation, which means that stress values can take on an arbitrarily large value without redistributing stresses over the cross section (lack of plastic work). Since the task was to consider the distribution of shear stresses over the cross section during elastic work, the load bearing conditions were not taken into account during the
study, although the fact of disturbance of the stress-strain state should be taken into account in design calculations, since this circumstance can strongly affect the bearing capacity real designs.

2.4. Calculation No. 3

In calculation No. 3, the hinged support was adopted as kinematic boundary conditions, implemented by prohibiting the lower nodes of the left and right edges of the rod from linear displacements (UX = UY = UZ = 0), thereby simulating a real bearing.

![Figure 6. Shear stress field (calculation No. 3)](image1)

![Figure 7. Shear stress distribution diagrams in sections 1 - 4 (calculation No. 3).](image2)
The shear stress field is shown in figure 6. As can be seen, the distribution pattern of the perturbation of the tangential stresses near the kinematic boundary conditions has shifted towards the supporting part.

The perturbation of the stress-strain state is shifted to the fastening region along the I-beam web. This allows us to conclude that not only the shape of the perturbation, but also the displacement of the peak value of the tangential stresses depends on the type of fastening. Figure 7 shows the distribution of shear stresses along the section height (without linear approximation by elements) for calculation No. 3.

2.5. Calculation No. 4
In calculation No. 4, as a kinematic boundary condition, a hinge support was adopted, implemented by prohibiting all nodes of the left and right edges of the rod from linear vertical movements (UY = 0).

The shear stress field is shown in figure 8. As can be seen, the pattern of the distribution of tangential stresses near the kinematic boundary conditions practically does not have much perturbation, which is due to the fact that the tangential stresses are evenly distributed over the support nodes of the beam and do not cause perturbations to shift in one direction or another, while remaining distributed over the entire reference section.

Figure 9 shows the distribution of shear stresses over the section height (without linear approximation by elements) for calculation No. 3.
3. Conclusion

The studies confirmed the hypothesis of a fast decay of the disturbance of the stress state caused by various inhomogeneities of not only static (associated with the loads), but also kinematic parameters, which led to the possibility of an extended interpretation of the Saint-Venant principle, according to which, in places quite remote from static and the kinematic inhomogeneities of the stress-strain disturbances decay quickly enough, which makes it possible to use the well-known strength of materials formulas for them.

References

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