I. INTRODUCTION

While the Higgs discovery [1, 2] at the Large Hadron Collider (LHC) finds the last missing piece in the Standard Model (SM), the dynamics governing particle interactions is still not fully understood. It has been shown that though the SM Higgs can generate a first order electroweak phase transition (EWPhT), it is not strong [3–5], as it must be in order to successfully account for the observed imbalance between baryons and anti-baryons in the universe [6–8], i.e., observed dark matter relic abundance [14]. While an additional scalar multiplet over and above the Standard Model can lead to a strong electroweak phase transition, depending on its quantum numbers, it also potentially confronts crucial constraints from theory and experiments. Should there exist more than one copy of such a multiplet, it is possible to predict that number from the requirements of a strong electroweak phase transition and agreement with the latest constraints. We aim to probe this specific issue in this study in the context of $N$ degenerate scalar triplets governed by a global $O(N)$ symmetry. We fold in important constraints from $h \to \gamma \gamma$ signal strength, dark matter direct detection and Landau pole behaviour. A combined analysis reveals $N \gtrsim 70$ for a strong phase transition and consistency with the constraints. We also look into possible gravitational wave signals in the parameter regions of interest.

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We now try to zero in on the choice of quantum numbers for the scalar multiplet. The most minimal one confronting $h \to \gamma \gamma$ is a radiatively corrected $hWW$ or $hZZ$ vertex with the latter’s strength directly connected to Higgs production at the proposed International Linear Collider [29–33]. Since $h \to \gamma \gamma$ is a $\gamma\gamma$ signal strength, dark matter direct detection and Landau pole behaviour. A combined analysis reveals $N \gtrsim 70$ for a strong phase transition and consistency with the constraints. We also look into possible gravitational wave signals in the parameter regions of interest.

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The lagrangian consistent with the gauge and radiative effects to measurable amplitudes having gauge bosons in the final states. A foreexample is a radiatively corrected $hWW$ or $hZZ$ vertex with the latter’s strength directly connected to Higgs production at the proposed International Linear Collider [29–33]. In fact, there could be more copies of such a scalar multiplet. While a strong first order phase transition would favour a larger number of multiplets, other constraints such as the absence of a Landau pole below a certain cutoff scale, the $h \to \gamma \gamma$ constraint and the dark matter direct detection bound would tend to limit that number. And this opens the possibility of an interesting interplay. In this study, we aim to determine the minimum number of a scalar multiplet that catalyses strong EWPhT and is also simultaneously consistent with miscellaneous restrictions from both theory and experiments. We also explore if the number for the multiplet so obtained can lead to observable GW signals at the proposed detectors.

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The common mass for \( m \) from
\[ m_{T^0}^2 = \mu_\Delta^2 + \frac{\lambda_H \Delta}{2} v^2 \quad \text{for} \ i = 1, \ldots, N; \quad (3a) \]
\[ m_{T^+}^2 = \mu_\Delta^2 + \lambda_H \Delta \frac{v^2}{2} \quad \text{for} \ i = 1, \ldots, N; \quad (3b) \]
The common mass for \( m_{T^0} \) on account of radiative effects \cite{39, 40}. That is
\[ m_{T^+} - m_{T^0} = \frac{\alpha m_{T^0}}{4\pi} \left[ f\left( \frac{M_W}{m_{T^0}} \right) - c_0^2 f\left( \frac{M_Z}{m_{T^0}} \right) \right]. \quad (4) \]

Here, \( \alpha \) is the fine-structure constant and \( f(x) = -\frac{1}{4} \left[ 2x^3 \ln(x) + (x^2 - 4)^{3/2} \ln\left( \sqrt{x^2 - 4} + 2 \right) \right] \). One derives this splitting to be \( \approx 166 \text{ MeV} \) for 100 GeV < \( m_{T^0} < 2 \text{ TeV} \). Eliminating \( \mu_\Delta \) in favour of \( m_{T^0} \), we use \( \{ m_{T^0}, \lambda_H \Delta, \lambda_\Delta, N \} \) to describe the triplet sector. That the number of Lagrangian parameters for \( N \) triplets remains the same as the \( N = 1 \) case is an artefact of the \( O(N) \) invariance.

This study is organised as follows. We discuss the theoretical constraints on this setup in section II. Discussions on oblique parameters and the diphoton signal strength are to be found in sections III and IV. We elaborate the dark matter constraint, especially direct detection in section V. The dynamics of a SFOEWPhT and the generation of GW in context of the present framework is described in section VI. The same section also contains an analysis combining the SFOEWPhT and the aforesaid constraints. We finally conclude in section VII.

II. THEORY CONSTRAINTS

A. Perturbativity

We first discuss possible bounds on the framework from perturbativity, albeit in a somewhat heuristic fashion. We do a naive power counting analysis of \( \Delta_i \Delta_j \rightarrow \Delta_i \Delta_i \) using the first few terms of the perturbation series. It shows that the successive terms get smaller in magnitude for \( \frac{\mathcal{N} \lambda_\Delta}{16\pi^2} \lesssim 1 \). In fact, a similar exercise in case of \( \Delta_i \Delta_j \rightarrow \phi \phi \) also leads to the same condition. Thus, the bound on \( \lambda_\Delta \) is
\[ \lambda_\Delta \lesssim \frac{16 \pi^2}{N}. \quad (5) \]

However, when it comes to \( \phi \phi \rightarrow \phi \phi \), we prima facie end up with two conditions, i.e., \( \frac{\mathcal{N} \lambda_\Delta}{16\pi^2} < \lambda_H \) and \( \frac{\mathcal{N} \lambda_\Delta}{16\pi^2} < 1 \).

That is,
\[ \lambda_H \Delta \lesssim 4\pi \sqrt{\frac{\Lambda_H}{N}} \quad \text{and} \quad \lambda_\Delta \lesssim \frac{16 \pi^2}{N}. \quad (6) \]

We obey the more stringent of the two bounds in our analysis. One finds that \( \lambda_H \Delta \lesssim 4\pi \sqrt{\frac{\Lambda_H}{N}} \) is more constraining for \( N \lesssim 1200 \).

B. Location of Landau poles

An \( O(N) \)-multiplet can lead to interesting predictions on the location of Landau poles of the theory. Since the present scenario involves only two additional scalar quartic couplings \( \lambda_H \) and \( \lambda_T \) in addition to the SM-like Higgs self coupling \( \lambda_H \), it is straightforward to understand such dynamics in terms of the one-loop beta functions. We quote below the one-loop beta functions for this framework that are dependent on \( N \).

\[ 16\pi^2 \beta_{\lambda_H} = 24 \lambda_H^2 + \frac{3}{2} N \lambda_H^2, \quad (7a) \]
\[ 16\pi^2 \beta_{\lambda_H} = 12 \lambda_H^2 + 4 \lambda_H^2 + \left( \frac{2 + 3N}{3} \right) \lambda_T \lambda_H, \quad (7b) \]
\[ 16\pi^2 \beta_{\lambda_T} = 12 \lambda_H^2 + \left( \frac{8 + 3N}{3} \right) \lambda_T^2, \quad (7c) \]
\[ 16\pi^2 \beta_{g_2} = \left( \frac{19}{6} + \frac{N}{3} \right) g_2^3. \quad (7d) \]

The beta functions for the rest of the parameters coincide with the corresponding SM expressions and hence are not given here for brevity. One reckons that the Landau pole shifts to lower energy scales with increasing \( N \) for fixed values of the couplings at the input scale. For illustration, the Landau pole for \( g_2 \) at one-loop can be analytically determined to be \( \Lambda' = M_t \exp \left[ -\frac{8 \pi^2}{3} \left( \frac{g_2(M_t)}{M_t} \right)^2 \right] \).

We then find that \( \Lambda' \gtrsim 1 \text{ TeV} \) (10 TeV) requires \( N \lesssim 330 \) (148). The choice of \( \Lambda \) is driven by the expectation that some hitherto unknown additional dynamics takes over beyond this scale. In this work, we ensure the absence of Landau poles below \( \Lambda \) by demanding the more stringent condition of perturbativity, i.e., the magnitudes of the quartic (gauge) couplings remain smaller than \( 4\pi \) \( (\sqrt{4\pi}) \) at all intermediate scales up to \( \Lambda \) under renormalisation group evolution. This is a more stringent restriction. For example, demanding a perturbative \( g_2 \) up to 1 TeV (10 TeV) necessitates \( N \lesssim 320 \) (144).
III. OBLIQUE PARAMETERS

A function that appears while calculating the contribution of a BSM scalar sector to the oblique parameter $T$ is $F(x,y) = \frac{x^{2}}{2} - \frac{y}{2y} \ln\left(\frac{y}{x}\right)$. We then quote below the contribution of the $O(N)$-symmetric $Y = 0$ triplet sector to $T$.

$$\Delta T = \frac{N}{16\pi s_{W}^{2} M_{W}^{2}} F(m_{T^{+}}^{2}, m_{T_{0}}^{2}),$$  

$$\simeq \frac{N(m_{T^{+}} - m_{T_{0}})^{2}}{24\pi s_{W}^{2} M_{W}^{2}} \simeq 2.458 \times 10^{-7} N.$$  

It is immediately seen that an $N$ consistent with the perturbativity constraint, for $g_{2}$ for instance, $\Delta T$ is well within the latest PDG bound [38]. There thus is no constraint from the $T$-parameter in this framework.

IV. $h \rightarrow \gamma \gamma$ SIGNAL STRENGTH

The $O(N)$-symmetric scalar triplet sector does not modify the tree level couplings of $h$ with other SM particles on account of zero mixing between $\phi$ and $\Delta_{i}$. However, it does modify the loop-induced $h \rightarrow \gamma \gamma$ amplitude through additional loops involving the charged scalars $T_{i}^{\pm}$. The $O(N)$ symmetry ensures that the $T_{i}^{+} - T_{i}^{-} - h$ coupling is $-\lambda_{i} H\Delta v$ for all $i = 1, ..., N$. The $h \rightarrow \gamma \gamma$ amplitude induced by the $O(N)$-symmetric sector therefore reads

$$M_{h \rightarrow \gamma \gamma}^{O(N)} = N \frac{\lambda_{H} \Delta v^{2}}{2m_{T^{+}}^{2}} A_{0}\left(\frac{m_{h}^{2}}{4m_{T^{+}}^{2}}\right).$$  

Here $A_{0}(x)$ is the amplitude for the spin-0 particles in the loop [41, 42] and is expressed as

$$A_{0}(x) = -\frac{1}{x^{2}} (x - f(x)),$$  

with

$$f(x) = \arcsin^{2}(\sqrt{x}) ; x \leq 1$$

$$= -\frac{1}{4} \left[ \log \frac{1 + \sqrt{1 - x^{-1}}}{1 - \sqrt{1 - x^{-1}}} - i \pi \right]^{2} ; x > 1.$$  

The $O(N)$ symmetry expectedly makes the contribution of the triplets to scale with $N$. The amplitude and the decay width for this model is thus given by

$$M_{h \rightarrow \gamma \gamma} = M_{h \rightarrow \gamma \gamma}^{SM} + M_{h \rightarrow \gamma \gamma}^{O(N)}$$

$$\Gamma_{h \rightarrow \gamma \gamma} = \frac{G_{F}^{2} s_{W}^{2} m_{h}^{3}}{128\pi^{2}} |M_{h \rightarrow \gamma \gamma}|^{2},$$  

where $G_{F}$ and $\alpha_{em}$ denote respectively the Fermi constant and the QED fine-structure constant.

Since the $pp \rightarrow h$ production rate remains unchanged in this model and the modification to the total width is negligible, the signal strength becomes

$$\mu_{\gamma \gamma} \simeq \frac{\Gamma_{h \rightarrow \gamma \gamma}^{SM}}{\Gamma_{h \rightarrow \gamma \gamma}^{T_{0}}}. \quad (14)$$

The latest 13 TeV results on the diphoton signal strength from the LHC read $\mu_{\gamma \gamma} = 0.99^{+0.14}_{-0.14}$ (ATLAS [43]) and $\mu_{\gamma \gamma} = 1.18^{+0.17}_{-0.14}$ (CMS [44]). We obtain $\mu_{\gamma \gamma} = 0.99 \pm 0.14$ upon combining the individual central values and uncertainties and impose this constraint at 2$\sigma$.

V. DARK MATTER

The degenerate neutral scalars $T_{bi}$ are rendered cosmologically stable by virtue of the $O(N)$ symmetry and hence they become potential DM candidates. In fact, the $N = 1$ (a single $Y = 0$ triplet) case is widely studied where a $Z_{2}$-like discrete symmetry is required to stabilise the neutral scalar. The tiny difference between the masses of $T^{+}$ and $T_{0}$ compulsorily triggers coannihilation processes of the type $T^{+}T_{0} \rightarrow W^{+}Y, W^{+}Z, W^{+}h$. Over-all, the single triplet case is seen to yield the observed relic density, i.e., $\Omega_{\text{DM}}^{obs}h^{2} = 0.1200 \pm 0.001 [14]$, only for $m_{T_{0}} \lesssim 1.8$ TeV.

In the case of multiple DM candidates, the total relic density is always a sum of the individual relic densities. Given the individual $T_{bi}$ are mass-degenerate in this setup having identical interaction strengths, the total relic density simply becomes

$$\Omega_{\text{total}}h^{2} = N \Omega_{T_{0}}h^{2}, \quad (15)$$

where $\Omega_{T_{0}}h^{2}$ is the value in case of $N = 1$. In this study, we do not impose the prediction of the observed relic as a binding requirement and impose rather the relaxed condition $\Omega_{\text{total}}h^{2} \leq 0.12$. That is, we keep alive the possibility that there are other DM sources in the universe that account for the rest of the requisite relic.

We discuss next our treatment of the direct detection bounds in this analysis. Let us assume DM entity, say $X$; predicts a relic density $\Omega_{X}h^{2} \leq \Omega_{\text{DM}}^{obs}h^{2}$ and a direct detection cross section $\sigma_{X}^{DD}$. Then, the effective direct detection cross section is computed as

$$\sigma_{X}^{DD, \text{eff}} = \left( \frac{\Omega_{X}h^{2}}{\Omega_{\text{DM}}^{obs}h^{2}} \right) \sigma_{X}^{DD}. \quad (16)$$

The direct detection constraints are obeyed by stipulating that $\sigma_{X}^{DD, \text{eff}}$ stays within the bound set by the experiments. The fraction in the prefactor accordingly scales down the theoretical cross section in case of underabundance. Applying this to the $O(N)$ scalars, one writes the effective direct detection cross section for a given $T_{bi}$ as

$$\sigma_{i}^{DD, \text{eff}} = \left( \frac{\Omega_{T_{0}}h^{2}}{\Omega_{\text{DM}}^{obs}h^{2}} \right) \sigma_{i}^{DD} \simeq \left( \frac{\Omega_{T_{0}}h^{2}}{0.12} \right) \sigma_{T_{0}}^{DD}. \quad (17)$$

We use $\sigma_{i}^{DD} = \sigma_{T_{0}}^{DD}$ in the last step in Eq.(17). We compute $\Omega_{T_{0}}h^{2}$ and $\sigma_{T_{0}}^{DD}$ using the publicly available tool.
micrOMEGAs [45] and demand that $\left(\frac{\Omega_{\nu} h^2}{0.12}\right)_{\sigma_{ID}}$ stays below the bound from the XENON-1T experiment [46].

On the other hand, indirect detection experiments have put upper bounds on DM annihilation cross section to SM states such as $b\bar{b}$ and $\gamma\gamma$. Since we allow for abundance, the quantity of interest is an effective indirect detection cross section defined as

$$\sigma_{ID}^{\text{eff}} = \left(\frac{\Omega_{\nu} h^2}{0.12}\right)^2 \sigma_{ID} \simeq \left(\frac{\Omega_{\nu} h^2}{0.12}\right)^2 \sigma_{ID}.$$  \hspace{1cm} (17)

One notes that the ratio $\left(\frac{\Omega_{\nu} h^2}{0.12}\right)$ gets squared for indirect detection since there now are two DM particles in the initial state.

VI. FINITE TEMPERATURE SCALAR POTENTIAL AND FIRST ORDER PHASE TRANSITIONS

We discuss in detail the $T \neq 0$ dynamics leading up to a first order phase transition for this setup. Investigations of phase transition in scenarios involving scalar triplets can be found in [47–49]. The background field $\phi$ in terms of which the scalar potential is expressed is chosen to be along the Higgs direction, i.e., one substitutes $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}$. Note that $\phi$ is distinct from the physical Higgs boson $h$. The tree level scalar potential as a function of the background field is given by

$$V_0(\phi) = -\frac{\mu_H^2}{2}\phi^2 + \frac{\lambda_H}{4}\phi^4. \hspace{1cm} (18)$$

Next, we quote the one-loop Coleman-Weinberg correction to the scalar potential following [Quiros].

$$\Delta V_{\text{CW}}(\phi) = \frac{1}{64\pi^2} \sum_a n_a \left[ m_a^2(\phi) \left( \log \frac{m_a^2(v)}{m_a^2(\phi)} - \frac{3}{2} \right) + 2m_a^2(v)m_a^2(\phi) \right], \hspace{1cm} (19)$$

where $n_a$ denotes the degrees of freedom of the $a$th particle. This particular form for $V_{\text{CW}}(\phi)$ ensures that the electroweak vacuum $v$ and the Higgs mass $m_h$ do not change upon the inclusion of one-loop corrections. The field-dependent squared masses $m_a^2(\phi)$ for this framework are given below.

$$m_W^2(\phi) = \frac{1}{4}g_1^2\phi^2, \hspace{0.5cm} m_Z^2(\phi) = \frac{1}{4}(g_1^2 + g_2^2)\phi^2, \hspace{1cm} (20a)$$

$$m_t^2(\phi) = \frac{1}{2}g_1^2\phi^2, \hspace{0.5cm} m_h^2(\phi) = 3\lambda_H\phi^2 - \mu_H^2, \hspace{1cm} (20b)$$

$$m_{\nu_i}^2(v) = m_{\nu_i}^2 + \frac{1}{2}\lambda_H\phi^2. \hspace{1cm} (20c)$$

The one-loop correction to the scalar potential induced in presence of $T \neq 0$ reads

$$\Delta V_1(\phi, T) = \frac{T^4}{2\pi^2} \sum_{a=bosons} n_a J_B[m_a^2(\phi)/T^2]$$

$$+ \sum_{a=fermions} n_a J_F[m_a^2(\phi)/T^2]. \hspace{1cm} (21)$$

Here, $J_B/F(y) = \int_0^\infty dx x^2 \log [1 \mp e^{-\sqrt{x^2 + y}]}$. Further, the infrared effects are included by the daisy renormalization technique [50–58]. We adopt the Arnold-Espinosa prescription [57] for the daisy term. That is,

$$\Delta V_{\text{ring}}(\phi, T) = -\frac{T}{12\pi} \sum_{a=bosons} \left[ m_a^2(\phi, T) - m_a^2(\phi) \right]. \hspace{1cm} (22)$$

The total potential is a sum of the individual contributions,

$$V(\phi, T) = V_0(\phi) + \Delta V_{\text{CW}}(\phi) + \Delta V_1(\phi, T) + \Delta V_{\text{ring}}(\phi, T). \hspace{1cm} (23)$$

We develop a semi-analytic treatment of $V_{\text{total}}(\phi, T)$ in order to gain insight on the $T \neq 0$ dynamics. Neglecting the contributions coming from the SM bosons, one finds

$$V(\phi, T) \simeq A(T)\phi^2 + C(T)(\phi^2 + K(T))^{\frac{3}{2}} + B(T)\phi^4. \hspace{1cm} (24)$$

A similar approach is followed in [59, 60]. The various coefficients are computed to be

$$A(T) = -\frac{1}{2} \mu_H^2 + \frac{1}{32\pi^2} \left[ -6g_2^2 M_W^2(v) + \frac{3N\lambda_H^2}{2} \left( m_{\nu_i}^2 + \mu_H^2 \log \frac{T^2}{m_{\nu_i}^2(v)} \right) \right]$$

$$+ \frac{1}{16} (2g_l^2 + N\lambda_H) T^2, \hspace{1cm} (25a)$$

$$B(T) = \frac{1}{4} \left[ \lambda_H + \frac{1}{16\pi^2} \left( -6g_2^2 \log \frac{T^2}{m_{\nu_i}^2(v)} \right) + N \frac{3\lambda_H^2}{4} \log \frac{T^2}{m_{\nu_i}^2(v)} \right], \hspace{1cm} (25b)$$

$$C(T) = -\frac{N}{4\pi} \left( \lambda_H \Delta \right)^{\frac{3}{2}} T, \hspace{1cm} (25c)$$

$$K(T) = \frac{2\mu_\Delta^2}{\lambda_H^2} T$$

$$+ \left[ 2\lambda_H^2 + (3N + 2)\lambda_H + 3g_2^2 \right] \frac{T^2}{6\lambda_H^2}. \hspace{1cm} (25d)$$

Eq.(24) throws light on how the shape of the potential changes with $T$. An extremum at $\phi = 0$ is readily identified and it is a minima (maxima) for $A(T) > 0$ ($A(T) < 0$). One thus identifies a threshold temperature $T_0$ determined through $A(T_0) = 0$ such that $\phi = 0$ is a minima only when $T > T_0$. Another temperature threshold above $T = T_0$ is $T = T_{\text{inf}}$ in
which case \( V_{\text{total}}(\phi, T) \) develops an inflection point. One solves for \( T = T_{\text{inf}} \) through \( I^2(T_{\text{inf}}) \equiv 9C^2(T_{\text{inf}}) - 32A(T_{\text{inf}})B(T_{\text{inf}}) + 64B^2(T_{\text{inf}})K(T_{\text{inf}}) = 0 \). For a temperature range \( T_0 < T < T_{\text{inf}} \), the inflection point disappears and a maxima and a minima appear at

\[
\phi_{\text{max,min}}(T) = \left[ \frac{1}{32}(B(T))^2 [9C^2(T) - 16A(T)B(T)] \pm 3(C(T)I(T)) \right]^{1/2}
\]

The temperature range \( T_0 < T < T_{\text{inf}} \) is the most interesting from the perspective of first order phase transitions on account of coexisting vacua at \( \phi = 0, \phi_{\text{min}}(T) \). One finds a critical temperature \( T_c \) in the aforesaid temperature range for which the two vacua are degenerate, i.e., \( V(0, T_c) = V(\phi_{\text{min}}(T_c), T_c) \). A strong first order phase transition is identified by \( \phi_{\text{min}}(T_c) > 1 \).

Next, we demonstrate the interplay of the various constraints and SFOEWPhT. The publicly available tool PhaseTracer [61] is used for this part. Fig. 1 shows regions in the \( m_{\tau_1} - \lambda_{H_\Delta} \) plane that are consistent with \( \phi_{\text{min}}(T_c) > 1 \) and the \( h \to \gamma\gamma \) and dark matter constraints. The multiplicity \( N \) and the parameter \( \lambda_\Delta \) are held fixed. The semi-analytic treatment shows that the lower is \( \lambda_\Delta \), the stronger is the phase transition. We thus choose \( \lambda_\Delta = 10^{-4} \) in the subsequent analysis. In addition, the framework is perturbative up to a given scale in the region below the corresponding horizontal line.

Fig. 1 shows the results for \( N = 50, 60, 70, 80, 90 \) and 100. The diphoton constraint leads to an allowed band in the \( m_{\tau_1} - \lambda_{H_\Delta} \) plane for the \( \lambda_{H_\Delta} \gtrsim 0.1 \) region. As \( N \) increases, the band moves downward in the parameter plane. This trend can be understood from the expression for the \( h \to \gamma\gamma \) amplitude in Eq. (9). The maximum scale for perturbativity, i.e., \( \Lambda_{\text{pert}} \), is decided by the choice of \( N \) as well as the initial values of \( \lambda_{H_\Delta}, \lambda_\Delta \). It is independent of the triplet mass and this explains the horizontal line. Therefore, the line corresponding to a given \( \Lambda_{\text{pert}} \) expectedly moves downward in the parameter plane as \( N \) increases. On another part, a SFOEWPhT in a typical Higgs portal setup is known to demand an \( O(1) \) magnitude for the portal coupling. However, this value decreases upon increasing the multiplicity \( N \). This is also corroborated by Fig. 1. In all, it is seen that there is always an overlap between the parameter spaces compatible with SFOEWPhT and the diphoton constraint. And these overlapping regions are perturbative till scales between 1 TeV and 100 TeV.

However, the direct detection constraint hugely restricts the possibilities. And this is where the present study differs from [34, 60]. In fact, in intervals of 10 for \( N \), the lowest value for the multiplicity for which a SFOEWPhT becomes compatible with \( h \to \gamma\gamma \) as well as direct detection is \( N = 70 \). The common parameter patch remains perturbative up to a scale between 10-100 TeV. As \( N \) increases, the overlapping patch grows and the maximum scale of perturbativity accordingly gets lowered. In all, that we can approximately identify \( N \approx 70 \) as the minimum number of hyperchargeless scalar triplets is the major upshot of this analysis.

Gravitational waves (GW) are inevitably associated with a strong first order phase transition [15–18]. The three sources that contribute to such a spectrum are, (a) bubble collision [62–67], (b) sound waves [68–72], and, (c) magnetohydrodynamic turbulence [73–77]. The net GW amplitude thus reads,

\[
\Omega_{\text{GW}} h^2 = \Omega_{\text{coll}} h^2 + \Omega_{\text{sw}} h^2 + \Omega_{\text{tur}} h^2.
\]

We now list some important parameters that appear in the individual amplitudes. First, the nucleation temperature \( T_n \) is determined through \( \frac{\Delta V(T_n)}{T_n^4} = 140 \). Here, \( S_3 \) refers to the Euclidean action in \( d = 3 \) or simply “bounce”. The parameter \( \beta \) is defined as

\[
\beta = \frac{\beta}{H} = \left[ T \frac{d}{dT} \left( \frac{S_3}{T} \right) \right]_{T_n}.
\]

One must note that value of the Hubble parameter \( H \) in Eq. (28) must be computed at \( T = T_n \). Next, the energy budget of the phase transition during the bubble nucleation is given by

\[
\epsilon = \Delta V_{\text{tot}}(T_n) - \left[ T \frac{d}{dT} \Delta V_{\text{tot}}(T) \right]_{T_n}.
\]

In the above, \( \Delta V_{\text{tot}}(T) = V_{\text{tot}}(0, T) - V_{\text{tot}}(\phi_{\text{min}}(T), T) \) measures the difference in the depths of the potential at the two vacua at a temperature \( T \). In addition energy density during nucleation expressed as \( \rho_n = \frac{g_s \pi^2 T_n^4}{90} \), where \( g_s \) denotes the number of degrees of freedom. One subsequently defines

\[
\alpha = \frac{\epsilon}{\rho_n}.
\]

The quantities \( \alpha \) and \( \beta \) are of paramount importance in determining the strength of the GW amplitude. The bubble wall velocity \( v_b \) used in the above equation is estimated as [78–80]

\[
v_b = 1/\sqrt{3} + \sqrt{\alpha^2 + 2/3\alpha}.
\]

The GW amplitude of a given source peaks at a given frequency. Such frequencies for the contributions coming from bubble collisions, sound waves and turbulence are expressed as

\[
\begin{align*}
\omega_{\text{coll}} &= 1.65 \times 10^{-5} \left( \frac{0.62}{1.8 - 0.1 v_b + v_b^2} \right) \\
\omega_{\text{sw}} &= 1.9 \times 10^{-5} \left( \frac{1}{v_b} \right) \left( \frac{\beta}{H} \right) \left( \frac{T_n}{100} \right) \left( \frac{g_s}{100} \right) \frac{1}{\beta} \\
\omega_{\text{tur}} &= 2.7 \times 10^{-5} \left( \frac{1}{v_b} \right) \left( \frac{\beta}{H} \right) \left( \frac{T_n}{100} \right) \left( \frac{g_s}{100} \right) \frac{1}{\beta}.
\end{align*}
\]
Finally, we express the GW amplitudes from the three sources as a function of frequency $f$ below.

\[
\Omega_{\text{coll}}(f) = 1.67 \times 10^{-5} \left( \frac{\beta}{H} \right)^{-2} \left( \frac{0.11v_b^3}{0.42 + v_b^2} \right) \left( \frac{\kappa \alpha}{1 + \alpha} \right)^2 \\
\left( \frac{g_*}{100} \right)^{-\frac{1}{4}} \left( \frac{f}{f_{\text{SW}}} \right)^3 \left( \frac{7}{4 + 3(f/f_{\text{SW}})^2} \right)^2
\]

(33)

\[
\Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left( \frac{\beta}{H} \right)^{-2} v_b \left( \frac{\kappa \alpha}{1 + \alpha} \right)^2 \\
\left( \frac{g_*}{100} \right)^{-\frac{1}{4}} \left( \frac{f}{f_{\text{sw}}} \right)^3 \left( \frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^2
\]

(34)
VII. CONCLUSIONS AND FUTURE OUTLOOK

The $Y = 0$ triplet is one of the most minimal extensions of the SM furnishing a scalar DM candidate. In this study, we have generalised such a framework by postulating $N$ such triplets governed by a global $O(N)$ symmetry. This global invariance ensures that the framework is describable in terms of the same Lagrangian parameters as the $N = 1$ case, that is, the triplet mass $m_{T_0}$, the Higgs-triplet portal coupling $\lambda_{H\Delta}$, and the triplet self coupling $\lambda_\Delta$. The multiplicity $N$ is an a priori unknown integer of this framework. It follows that several quantities of interest are sensitive to the value chosen for $N$ in such a scenario.

In this study, we probe SFOEWPht in the $O(N)$-symmetric scalar triplet scenario. While multi-step phase transitions are permissible in principle, we look for a single-step one for minimality in the direction of the SM Higgs doublet. In the process, we also discuss how this scenario confronts the crucial constraints coming from $h \to \gamma\gamma$ signal strength, DM direct detection and an absence of Landau poles up to a given cutoff. For a given $N$, we identify the area in the $m_{T_0}$ – $\lambda_{H\Delta}$ plane compatible with the single step SFOEWPht and the aforementioned constraints. This analysis reveals $N \gtrsim 70$ for the SFOEWPht to be compatible with the constraints. The direct detection constraints turn out to be hugely restricting in this scenario given the economy of parameters in this setup. We also estimate the strength of the GW signals associated with the SFOEWPht parameter regions. We find that for GW signals to be observable in the proposed space-based detectors ultimate-DECIGO and BBO, one must have $N \gtrsim 90$. Therefore, the present analysis also zeroes-in on the minimum multiplicity of the triplets required to produce observable GW signatures.

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TABLE I: Benchmark points chosen to study the GW spectrum.

| $N$ | $m_{T_0}$ (GeV) | $\lambda_H\Delta$ | $\lambda_\Delta$ | $T_L$ | $\frac{\rho_\Lambda}{\rho}$ | $T_0$ | $\alpha$ | $\frac{\rho_\Lambda}{\rho}$ |
|-----|-----------------|--------------------|-------------------|------|---------------------|------|---------|---------------------|
| BP1 | 70 127.533 GeV  | 0.264752           | $10^{-4}$         | 106.698 GeV | 1.69719             | 106.423 GeV | 0.0379803 | 112744               |
| BP2 | 80 134.717 GeV  | 0.257330           | $10^{-4}$         | 113.801 GeV | 2.21102             | 113.108 GeV | 0.0600576 | 50244.3              |
| BP3 | 90 141.867 GeV  | 0.265049           | $10^{-4}$         | 123.315 GeV | 3.57403             | 119.376 GeV | 0.140045   | 10481.3              |
| BP4 | 100 150.448 GeV | 0.271032           | $10^{-4}$         | 140.876 GeV | 4.73334             | 129.447 GeV | 0.219615   | 4562.59              |

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