Asynchronous resonance in the driven Ising model on adaptive networks

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Abstract. The stochastic resonance was investigated by the field-driven Ising model on adaptive networks. As the structure evolution probability $\eta$ increases, it is found that the resonance undergoes three states: normal resonance, transition state and abnormal resonance. This work reveals significant effects of the coevolution of the topology and the dynamical processes, which even leads to counterintuitive phenomenon. When the coevolution of the topological structure and states of the system is not strong (corresponding to small $\eta$) a weak external field will lead to a positive feedback. However, if the coevolution is strong enough (corresponding to large $\eta$) the external field will cause a negative feedback.

1. Introduction

The stochastic resonance (SR) as an important phenomenon was discovered for the first time by Benzi et al. [1]. After that SR has been established as an important mechanism throughout natural sciences [2-10]. In addition, as one of important but extremely simple model of statistical physics, recently, the research of Ising model on the network has become a hot topic [11,12]. In many real systems [13-16], the Ising model also has been used to show order-disorder transition. Brey and Prados have studied the stochastic resonance in a one-dimensional Ising model [17]. The field-driven Ising model has been used by Hong, Kim and Choi to investigate the stochastic resonance phenomena on small-world networks [5]. But instead of the small-scale networks without state evolution discussed in Ref. [5], we consider adaptive (coevolution) networks with both state and topological evolution, which is of high concern for many biological, social, and other applications.

Previous studies on SR in extended systems were based on static network structure. However, recent many studies have shown that real systems, such as computer networks, biochemical networks, neuronal networks, and social networks, possess quite complex structures. In these structures, network structures and dynamic processes can interact and progress together [18-24]. The coevolution of the topology and dynamical process can drive each other and produce complex feedback effects. Research on this coevolution is important, but still limited. In this paper, a field-driven Ising model based on adaptive network is proposed to study stochastic resonance. With this model we attempt to reveal how the coevolution of the topology and dynamical process influences the stochastic resonance.
2. Model and method

The field-driven Ising model on adaptive networks is used to describe the property of nodes and interaction among nodes; and the state and topological structure of the network coevolves under oscillating field and thermal agitation.

To construct this model the Small-World (SW) networks with the rewiring probability $\phi = 0.5$ and average degree $<k> = 10$ are taken as the initial networks. Certainly, scale-free networks and hierarchical networks et al. are also feasible. The nodes of the network are regarded as magnetic units of the Ising model. Indicating spin up and spin down, each node has two allowed states, expressed as +1 and -1. The states of each node are randomly selected from +1 and -1 in the initial network.

Then we make this network evolve with time. This evolution includes the state evolution and topology evolution.

We use the coupling energy $-J_0 \sigma_i \sigma_j$ to describe the coupling strength, where $\sigma_i = +1$ or -1. The coupling constant corresponding to the nearest nodes $J_{ij} = J$, otherwise $J_{ij} = 0$. Based on the Metropolis algorithm, Monte Carlo simulation method is used to evolve the state of nodes. The topological structure evolution stems from the selection-rewiring of links of networks.

The evolution process of state and topology is as follows. At a time the state and number of the nearest neighboring nodes are $\sigma_i$ and $k_i$, respectively, for the $i$-th node. We use the coupling energy to determine the state

$$\varepsilon_i = -J \sum_{j=1}^{k_i} \sigma_j$$

(1)

When $\varepsilon'_i > \varepsilon_i$, the node updates the state with a Boltzmann-type probability

$$\mu = \exp\left[-(\varepsilon'_i - \varepsilon_i)/k_B T\right]$$

(2)

\begin{enumerate}
  \item[(a)] $\varepsilon_i < \varepsilon'_i$
  \item[(b)] $\varepsilon_i > \varepsilon'_i$
\end{enumerate}

Figure 1. An adaptive network field-driven Ising model with coevolution of structure and state. (a) If $\varepsilon'_i > \varepsilon_i$, the probability of state updating is $\mu$, and the probability of state holding is $1 - \mu$. (b) If $\varepsilon'_i < \varepsilon_i$, the probability $1 - \eta$ is used to update the state, and the probability $\eta$ is used to reconnect to the node with opposite state at random.

and holds its own state with the probability $1 - \mu$, as shown in Figure 1(a). If $\varepsilon'_i < \varepsilon_i$, as shown in Fig. 1(b), the rewiring probability is $\eta$ and the updating probability is $1 - \eta$. Just the rewiring operation leads to the topological structure evolution. For this reason $\eta$ is named the structure evolution probability.

The system is placed in an oscillating magnetic field $h(t) = h_0 \cos(\omega t)$, where $h_0$ is the driving amplitude and $\omega$ is frequency. For the field-driven Ising model, the Hamiltonian has the form [4,5]
\[ H = -J \sum_{i<j} a_{ij}\sigma_i\sigma_j - h(t) \sum_{i} \sigma_i \]  

when nodes \(i\) and \(j\) are connected, \(a_{ij}=1\), otherwise \(a_{ij}=0\). The increment of the temperature is \(\Delta T = 0.02\) (in units of \(J/k_B\)). We set \(h_0 = 0.1\) and \(\omega = 0.01\) [5]. Measurements are made for \(10^4\) MC steps after the first \(4 \times 10^4\) MC steps. We set the network size \(N=6400\). The occupancy ratio \(R\) is defined to be the average fraction of the spins [3-5].

We use function \(m(t)\) to measure the magnetization

\[ m(t) = \frac{1}{N} \sum_{i=1}^{N} \sigma_i \]  

and take \(Q_n\) as the time average of \(m(t)\) [4,5]

\[ Q_n = \frac{\omega}{2\pi} \int_{t_n}^{t_{n+1}} m(t) dt \]  

with \(t_n = 2\pi n/\omega\). Then we can define the dynamic order parameter by

\[ Q = \lim_{n \to +\infty} Q_n \]  

When the time is long enough the resonance reaches stability and the dynamic order parameter consists of a time-independent part \(Q_0(\eta)\) and a periodic function \(Q_1(\eta, t)\) as

\[ Q(\eta, t) = Q_0(\eta) + Q_1(\eta, t) \]  

The period of \(Q_1(\eta, t)\) is the same as that of the oscillating field.

3. Results and discussions

Figure 2. (Color online) Influence of structure evolution upon the dynamic properties. (a) Change of \(Q\) with \(T\). (b) Occupancy ratio \(R\) vs \(T\). The structure evolution probability \(\eta = 0\) corresponds to the static networks Ising model, in which the structure evolution does not take place. The arrows indicate the transition temperature \(T_C\). The simulations are averaged over 100 realizations.

The influence of structural evolution on dynamic performance was studied. The results are shown in Figure 2. Figure 2(a) shows the relationship between the dynamic sequence parameter \(Q\) and the temperature \(T\). It can be seen that there exists a transition and the transition temperature \(T_C\) for the evolution network with the structure evolution probability \(\eta = 0.01\) is larger than that for the static network (corresponding to \(\eta = 0.01\)). i.e. a moderate amount of structure evolution promotes the magnetization. Figure 2(b) draws the occupancy ratio \(R\) vs \(T\) plot. It shows that there are two
resonance peaks. The resonance peaks shift to the right and their heights decrease when $\eta$ changes from 0 to 0.01.

Figure 3. (Color online) The influence of structure evolution upon the dynamic order parameter $Q$ and the occupancy ratio $R$ for three types of resonances: (a) the normal bimodal resonance (the structure evolution probability $\eta = 0.01$); (b) transition state ($\eta = 0.15$); (c) abnormal resonance ($\eta = 0.3$). The perpendicular line indicates the transition temperature $T_c$ which is determined by $Q$-$T$ relationship. (d) shows the plot of $R$ vs $T$ and $\eta$.

Further, the influence of the structure evolution upon the resonance behavior was investigated. The results are plotted in Figure 3. It can be seen from Fig. 3 that according to the structure evolution probability $\eta$ the resonance can be divided into three types: (a) the normal bimodal resonance ($\eta = 0.01$), (b) transition state ($\eta = 0.15$) and (c) abnormal resonance ($\eta = 0.3$). For both the normal bimodal and abnormal resonances there are two resonance peaks which lie in the two sides of the transition temperature $T_c$. In the normal bimodal resonance state the occupancy ratio $R$ takes maximum value and in the abnormal resonance state $R$ takes minimum. In the transition state there is no obvious resonance peak. To demonstrate how the resonance transits from the normal bimodal state to abnormal state a three-dimensional diagram of $R$ vs $T$ and $\eta$ is plotted in Fig. 3(d). In Figs. 3(a-c) the resonance temperatures are located in the vicinity of the transition temperature. It means that the origin of the resonance peaks comes from the time-scale matching condition and the external drive frequency matching the internal characteristic frequency [4].
Figure 4. (Color online) Change of the average path length $D$ with the probability $\eta$ at $T=3$. N, T and A correspond to three regions: the normal bimodal resonance, transition state and abnormal resonance, respectively.

In order to look into the cause of resonance temperature variation with $\eta$, we calculated the average path length $D$ (see Figure 4). As can be seen from Fig. 4, the average path length decreases with the increase of $\eta$ until $\eta = 0.03$. Small average path length is beneficial to system ordering, so that the increment of $\eta$ will lead to increase of the transition temperature $T_C$ before $\eta = 0.03$. Figure 4 also shows that $D$ continuously decreases as $\eta > 0.03$. However, the resonance temperatures and the transition temperature decrease in this region ($\eta > 0.03$). This fact shows that the average path length is not the dominant factor of the resonance temperature variation with $\eta$ after $\eta = 0.03$.

4. Conclusion

In this paper, the stochastic resonance was investigated by the field-driven Ising model on adaptive networks. Different from static networks, the stochastic resonance on adaptive networks undergoes three stages with increment of the structure evolution probability $\eta$. As $\eta < 0.03$ the synchronous resonance takes place and then a transition stage appears, at last ($\eta > 0.2$) the asynchronous resonance happens. This model shows that only if an external force is adaptable to the evolution of the system itself, one can attain expected objective. Otherwise, the external force can cause a negative feedback when the coevolution is strong enough (corresponding to larger $\eta$).

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