Magnetic Moments of the Octet Baryons in the Colour-Dielectric Model

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Abstract

Baryon magnetic moments are calculated in the colour-dielectric model with pion and kaon loops. The only free parameter of the model is determined from the nucleon isoscalar radius, and all SU(3) symmetry breaking, including that in the quark sector, is determined by mesonic masses and decay constants. Good agreement with experiment is obtained for the ratios of the magnetic moments, but the inclusion of kaons does not improve the results. The results obtained in this approach are significantly better than any that have been obtained in hedgehog-based models.
1 Introduction

A satisfactory description of the magnetic moments of baryons within the framework of a dynamical model has proved harder to obtain than might have been expected. Although a naive additive quark model with different strange and non-strange magnetic moments reproduces the overall pattern reasonably well, there are some notable problems, such as fitting $\mu_p - \mu_{\Sigma^+}$ at the same time as $\mu_\Lambda$ and $\mu_\Omega$, or obtaining the correct sign of $\mu_{\Xi^-} - \mu_\Lambda$. However very few dynamical models come close to doing even as well. The problem of the magnetic moments is interesting precisely because it appears to be a sensitive test of the inclusion of SU(3)-breaking effects beyond first order. Numerous attempts in Skyrme and other hedgehog-based models have all failed to improve on the additive quark model. [1, 2, 3, 4, 5]

One of the best results within the framework of a relativistic quark model was obtained twelve years ago by Théberge and Thomas [6] in the cloudy bag model with pion loops included perturbatively. A more recent model which, like the cloudy bag model, has a relatively weak quark-pion coupling, but which unlike it is fully dynamical, is the colour-dielectric model [7]. In this model the role of the static bag is taken by a scalar, chiral singlet field which confines the quarks. Chiral invariance is restored by implementing the coupling through the $\sigma$ field of the linear $\sigma$ model, with a matching coupling to pions. Though hedgehog solutions can be found, the $\sigma$ and pion fields remain near their vacuum values [8]. Thus a cloudy bag approach to the inclusion of mesons is more natural [9]. This approach, including pion and kaon fields, has been successfully used to calculate the octet and decuplet mass spectra, in an approach in which the strength of the SU(3) symmetry breaking was governed entirely by the physical pion and kaon masses and coupling constants [10]. The current paper applies the same method to the calculation of the magnetic moments.

The paper is organised as follows. In section 2 the colour-dielectric model is introduced and the perturbative inclusion of mesons explained. In section 3 the various contributions to the magnetic moments are derived. In these two sections excessive overlap with refs. [3, 10] has been avoided where possible. In section 4 the results are presented and a comparison with other models and with the results of chiral perturbation theory is given.

2 The colour-dielectric model

The version of the colour-dielectric model used in this paper is the three-flavour one developed by McGovern [10]. The original model involves quarks and a $\chi$ field representing a glueball or glueball-meson hybrid, with an unusual interaction term:

$$\mathcal{L} = \bar{\psi} \left[ i\gamma^\mu D_\mu + \frac{g}{\chi} \right] \psi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi) + \kappa(\chi) F^\mu\nu F_{\mu\nu}. \quad (1)$$

The potential $U(\chi)$ has a global minimum at $\chi = 0$, so $\chi$ vanishes in the vacuum. The covariant derivative and the field tensor $F^\mu\nu$ involve “coarse-grained” gluons, which are to be treated as Abelian and only included to lowest order, since non-perturbative effects are included through the $\chi$ field. The colour dielectric, $\kappa$, is proportional to $\chi^4$ and thus vanishes in the vacuum, ensuring that colour is confined. The inverse quark-$\chi$
coupling provides confinement even for colourless quarks, as their effective mass in the vacuum would be infinite.

Many groups have now obtained soliton solutions in this model, either ignoring the gluons or including one-gluon exchange self-consistently or perturbatively. It has been shown [10] that, at least in the case of a simple quadratic $\chi$ potential, first-order perturbation theory works well.

This model is of course not chirally symmetric. It can be made so by allowing the quarks to couple to the inverse $\chi$ field only through an appropriate combination of meson fields—in the SU(2) case, $g/\chi \rightarrow g(\sigma + i\gamma_5\tau \cdot \phi)/(f_\pi\chi)$ where $\phi$ is the pion isotriplet and $\sigma$ its scalar, isoscalar chiral partner; in this linear sigma model the hidden nature of chiral symmetry is reflected in a vacuum expectation value of $f_\pi$ for the $\sigma$ field. When these mesons are included in a hedgehog ansatz the deviation from their vacuum expectation values within the soliton is small; it is therefore possible to ignore them in the construction of the soliton and include pion contributions to observables perturbatively, in the manner of the cloudy bag model.

Three flavours and broken SU(3) symmetry can also be incorporated in the model. (For details, see ref. [10].) Formally this requires two nonets, of scalars and pseudoscalars; the meson potential is a generalised mexican hat with explicit symmetry-breaking terms which generate separate pion and kaon masses, and two of the scalar fields, the singlet and the eighth member of the octet, develop vacuum expectation values which are determined by the pion and kaon decay constants. If the same strategy of ignoring deviations of the meson fields from their vacuum expectation values is adopted, one is left with a very simple model in which strange and non-strange quarks couple to the inverse $\chi$ field with different coupling constants, the ratio of which is fixed:

$$c_g \equiv \frac{g_s}{g_n} = \frac{2f_K}{f_\pi} - 1.$$  \hspace{1cm} (2)

Light mesons, that is pions, kaons and the $\eta$, can be included perturbatively. The interest of this model is that SU(3) symmetry breaking effects in the baryon sector are not dependent on some arbitrary strange-quark mass, but on well determined experimental quantities from the meson sector, $f_K/f_\pi$ and $m_K$.

The basic quark-$\chi$ Lagrangian is as follows:

$$\mathcal{L} = \bar{\psi} \left( i\gamma^\mu \partial_\mu + \frac{G}{\chi^p} \right) \psi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} M_\chi^2 \chi^2,$$  \hspace{1cm} (3)

where $G$ is a matrix in flavour space, $G = \text{diag}(g_n, g_n, g_s)$, and the possibility of varying the power $p$ of the inverse $\chi$-field which couples to the quarks has been included. (Theoretical arguments can be advanced for several values of the power $p$, but we will only consider $p = 1$ or 2.) This Lagrangian has two free parameters, $M_\chi$ and the dimensionless parameter $\beta^2 = g_n M_\chi^{-(p+1)}$. The standard procedure is to find the solution for three non-strange quarks in the same s-wave orbital, and to fit $M_\chi$ by requiring that the r.m.s quark radius equal the nucleon isoscalar radius, since that will be least affected by mesonic corrections. It is then found that the energy and other properties are remarkably insensitive to the value of $\beta$, and that $M_\chi$ scales as $\beta^2/2^{p+1}$ at least for $M_\chi$ greater than around 1 GeV. This scaling has been shown to be exact if the $\chi$ kinetic energy can be ignored [11].
The average of the nucleon and $\Delta$ energy, as given by the flavour symmetric calculation, is extremely high: $E = 1906$ MeV for $p = 1$ and $E = 1773$ MeV for $p = 2$. However estimates of the mesonic, gluonic and centre of mass corrections bring these energies down considerably.

Baryons with non-zero strangeness are based on bare solitons with one or more of the three quarks in an s-wave strange quark orbital. The full octet and decuplet mass spectrum has been calculated in ref. [10], and good agreement with experiment obtained. This paper explores whether the same formalism can reproduce the magnetic moments.

2.1 Hamiltonian and vertex functions

Although as a first approximation baryons can be treated as containing only three quarks in the appropriate SU(6) wavefunction, the coupling to mesons in the full Lagrangian means that there is a certain probability that, say, a $\Delta$ will be made up of a bare nucleon plus a pion. Since the coupling is weak, it should be a good approximation to ignore contributions with two or more mesons, and also to ignore the effect of the meson fields on the soliton quark and $\chi$ fields. The pion contributions have traditionally been included in the cloudy bag model. Kaon contributions however are also not negligible, and they also contribute to mass splittings within the octet and decuplet, as was shown in ref. [10]. On the other hand the heavy mesons ($\eta'$ and the scalar) should have little effect, and they will be ignored. The interaction Lagrangian is then equivalent to one with a non-linear realisation of the light mesons, and the formalism of the cloudy bag can be adopted intact. The reader is referred to ref. [6, 12] for details; the final baryon-meson interaction Hamiltonian is

$$H_I = \sum_{a=1}^{8} \int \frac{d^3 k}{(2\pi)^3 2 \omega_a(k)} e^{ik \cdot r} \langle A_0 | : \bar{q} \gamma_5 q : | B_0 \rangle$$

where $c_a^\dagger(k)$ and $c_a(k)$ are pseudoscalar octet meson creation and annihilation operators and $A_0^\dagger$ creates a bare (three-quark) baryon $A$. The vertex function $v^{AB}_a(k)$ is given by

$$v^{AB}_a(k) = \frac{i g_\pi}{f_\pi} \int \frac{d^3 r}{[(2\pi)^3 2 \omega_a(k)]^{\frac{3}{2}}} e^{ik \cdot r} \langle A_0 | : \bar{q} \lambda_a \gamma_5 q : | B_0 \rangle$$

where $|A_0\rangle$ is the wavefunction of bare baryon $A$, $\omega_a(k) = \sqrt{m_a^2 + k^2}$ and $m_a = m_\pi$, $a = 1 - 3$, $m_K$, $a = 4 - 7$ or $m_\eta$, $a = 8$. In order to calculate the matrix element in Eq. 4 correctly one would need a full quantum wavefunction for the quarks and $\chi$ field in baryons $A$ and $B$ (a coherent state, for instance). Such sophistication would be wasted on a perturbative calculation such as this. There is in any case no problem for the dominant, pion contributions, where $A$ and $B$ have the same hypercharge; there the basic quark-$\chi$ structure of each is the same and the matrix element can be calculated in the mean-field approximation. The problem arises with the kaons. These are however already less important than the pions, and it is unlikely to cause major errors if we take the overlap of the $\chi$ part of the wavefunctions to be unity here also.

With the quark spinor for the $i$th quark given by

$$q_i(r) = \left( \begin{array}{c} G_i(r) \\ i\sigma \cdot r F_i(r) \end{array} \right)$$

4
Eq. 5 becomes
\[
v^A_{aB}(k) = -\frac{i}{[(2\pi)^32\omega_a(k)]^{1/2}} u_a(k)\zeta_a \langle A_{sf} \rangle \sum_{i=1}^3 \lambda_a^{(i)} \sigma^{(i)} \cdot k | B_{sf}\rangle. \tag{7}
\]

In the above $| A_{sf}\rangle$ is the SU(6) spin-flavour wavefunction and $u_a$ is a normalised form-factor
\[
u_a(k) = \frac{3 \int dr r^3 (G_i F_i + G_j F_j)/\chi^p \ j_1(kr)/kr}{\int dr r^3 (G_i F_i + G_j F_j)/\chi^p}.
\tag{8}
\]
i and j are both non-strange for pions ($a = 1 - 3$), one strange and one non-strange for kaons ($a = 4 - 7$), and the appropriate combination of both strange and both non-strange for $\eta$ ($a = 8$). In addition, the factor $\zeta_a$ is given by
\[
\zeta_a = \frac{4\pi g_n}{3f_\pi} \int dr r^3 (G_i F_j + G_j F_i)/\chi^p.
\tag{9}
\]
Using the Wigner-Eckart theorem the vertex function can be written
\[
v^A_{aB}(k) = -\frac{i}{[(2\pi)^32\omega_k]^{1/2}} \sum_\nu \zeta_\nu u_\nu(k) f^A_\nu \sum_{q,i} \langle S_B S_B T_{3B} T_{3}\nu | S_A S_A T_{3A}\rangle e^{*}_{q_i} e_{q_i}
\times \langle T_B T_{3B} T_{3}\nu | T_A T_{3A}\rangle c^{*}_{\nu a}.
\tag{10}
\]
where
\[
f^A_\nu = \sum_\gamma \left( \frac{d_B}{T_B Y_B} \frac{8}{T_\nu Y_\nu} \right) (d_A, d_B, d_B, \gamma).
\tag{11}
\]
and the reduced matrix elements are calculated to be [11]
\[
R(8,8)_1 = 2\sqrt{5} \ , \quad R(8,8)_2 = 4 \ , \quad R(8,10) = -2\sqrt{10} \ , \quad R(10,8) = -4 \ , \quad R(10,10) = 2\sqrt{10}.
\tag{12}
\]
The matrices $[e]_{iq}$ and $[c]_{a\nu}$ are, respectively, the transformation matrix between the cartesian and spherical bases, and that between the basis in the SU(3) regular representation and the basis in the isospin classification of SU(3) multiplets. They satisfy the orthogonality relations
\[
\sum_i e_{qi} e^{*}_{q'i} = \delta_{qq'}, \quad \sum_a c_{va} c^{*}_{v'a} = \delta_{\nu\nu'}.
\tag{13}
\]
As an example of a meson contribution to an observable, the mesonic self-energy of baryon $A$ is then given by
\[
\Sigma_A = \sum_B \sum_a \int d^3k \frac{v^A_{aB}(k) v^{BA}_{a}(k)}{\omega_a(k)(M_A - M_B - \omega_a(k))}.
\tag{14}
\]
In cloudy bag calculations [3, 12] it is customary to take $M_A$ and $M_B$ as the physical baryon masses. This is equivalent to summing some self-energy diagrams to all orders, but without even including the first order change in the quark wavefunctions due to the presence of the mesons. It is more consistent in a first-order perturbative calculation to use the bare (or soliton) masses.
3. Perturbative Contributions to the Magnetic Moment

Théberge and Thomas have calculated the magnetic moments of the octet baryons in the cloudy bag model \[6\]. Our calculation mirrors theirs, with two exceptions. One is that we include kaon and $\eta$ loops. The other is that we calculated according to strict second-order perturbation theory, so that our vertex functions are “unrenormalised” and the terms $\omega_{AB} = M_A - M_B$ which occur in denominators are taken as bare (soliton), rather than physical, mass differences. Henceforth whenever reference is made to equations in their paper, this difference should be borne in mind.

In terms of the electromagnetic current operator $\hat{j}^\mu(x)$, the magnetic moment of baryon $A$ defined as

$$\mu_A \equiv \langle A, S_A = S_A | \int d^3r \; r \times \hat{j}(r) | A, S_A = S_A \rangle,$$

(15)

that is the magnetic moment is defined in terms of the spin-stretched state. It is implied that the center-of-mass system of baryon $A$ is taken. Since it is impossible to define a momentum eigenstate of a spatially localized soliton, there arises a need to remove spurious center-of-mass motion. However, center-of-mass corrections are not taken into account in the present work.

The electromagnetic current may be split into quark and meson terms. Accordingly, the magnetic moment has three separate contributions, as shown diagrammatically in Fig. 1, corresponding to the coupling of a photon to a bare baryon ($\mu^A_0$), to a charged meson “in flight” about a baryon ($\mu^A_\phi$), and to a baryon with a meson in flight ($\mu^A_Q$), and the total is given by $\mu_A = Z_A (\mu^A_0 + \mu^A_\phi + \mu^A_Q)$. $Z_A$ is the wavefunction normalisation factor, given to the same order in perturbation theory by

$$Z_A^{-1} = 1 + \frac{1}{12 \pi^2} \sum_{B, \nu} (\zeta_\nu f^{AB}_\nu)^2 \int_0^{\infty} \frac{dk \; k^4 u^2_\nu(k)}{\omega_\nu(k) (\omega_B + \omega_\nu(k))^2}.$$

(16)

Of course, within the framework of second order in perturbation theory there is no need to multiply the second order contributions by a normalisation factor different from one, but we chose to do so so that the coupling of a time-like photon calculated in the same approximation does indeed give a proton electric charge of $+e$. Convergence of the perturbation expansion is also improved.

We will deal with the diagrams in which the photon couples to a meson, and those in which it couples to a quark (with or without a meson in flight) separately in the next two sections.

3.1 Meson-photon coupling

We start with examining the effect of a virtual pion coupling to a photon corresponding to Fig. 1c. This mirrors exactly the treatment in section 4.5 of ref. \[6\], with
the V-spin labelled charged kaon vertex functions: take on a form identical to Eqns. (4.45) and (4.46) of [6], written however in terms of annihilation operators $a^\dagger$. The spin and isospin Clebsch-Gordan coefficients are summarized in $s_A(B)$ which is given by

$$s_A(B) = \begin{cases} 1 & \text{if } S_B = \frac{1}{2} \\ \frac{1}{2} & \text{if } S_B = \frac{3}{2} \end{cases} \quad \text{and } s_B = \frac{1}{2},$$

and $t_A(B)$:

$$t_A(B) = \langle T_B T_B 1 1 | T_A T_A 3 \rangle^2 - \langle T_B T_B 1 - 1 | T_A T_A 3 \rangle^2$$

$$= \begin{cases} T_A 3 / T_A & \text{if } T_B = T_A - 1 \\ T_A 3 / [T_A (T_A + 1)] & \text{if } T_B = T_A \\ - T_A 3 / (T_A + 1) & \text{if } T_B = T_A + 1 \end{cases}$$

The coupling to kaons can be evaluated in a similar way. Since the charged kaons and a combination of the neutral pion and $\eta$ form a V-spin triplet, an expression for the kaon current very similar to Eq. (4.32) of ref. [6] is obtained:

$$j^\mu_k (r) = - \frac{i e}{2} \int \frac{d^3k d^3k'}{(2\pi)^3 \sqrt{\omega_k \omega_{k'}}} e^{-i(k' - k) \cdot r} k^\mu \sum_{j'j=1}^2 \epsilon_{j'j3} \hat{S}_k (k', j, \mu),$$

where $\omega_k \equiv \omega_k (k)$ and

$$\hat{S}_k (k', j, \mu) = (a_{j'}(-k') + a_{j'}^\dagger(k'))(a_j(k) - g_{\mu\nu} a_{j'}^\dagger(-k)).$$

but where the labels on $a_1$ and $a_2$ correspond to V-spin, and in terms of an octet of annihilation operators $c_a$, we have $a_1 \equiv c_4$ and $a_2 \equiv c_5$. The expectation values $S^A_K$ take on a form identical to Eqns. (4.45) and (4.46) of [6], written however in terms of the V-spin labelled charged kaon vertex functions:

$$v_{K_j}^{AB}(k) = - \frac{i}{[(2\pi)^3 2 \omega_k]^{1/2}} f_K^{AB} \sum_{q, i, r, =\pm \frac{1}{2}} \zeta_r u_r(k) \langle S_B S_B 1 q | S_A S_A e \rangle e_{q_i} k_i$$

$$\times \langle T_B T_B 1 r | T_A T_A 3 \rangle e_{2r, j},$$

Hence the magnetic moment due to the kaon-photon coupling acquires a form similar to Eq. [17]:

$$\mu^A_\pi = \frac{e}{18\pi^2} \sum_{B, r=\pm \frac{1}{2}} (\zeta_r f^{AB})^2 s_A(B) t_A^e(B) \int_0^\infty \frac{k^4 u^2_B(k)(\omega_{BA} + 2\omega_k)}{2\omega_k^3(\omega_{BA} + \omega_k)^2} dk.$$
The form is more complicated than the pion case because the form factors for positive and negatively charged kaons are not quite identical, and also because the mass differences between solitons of different strangeness do not vanish. If such differences were ignored the sum over $r$ could be done analytically, and the results expressed in terms of “V-scalar factors” rather than the isoscalar factors $f^{AB}$ and “$v_A(B)$” which depends on the V-spin of A and B exactly as the pionic $t_A(B)$ does on the isospin.

3.2 Quark-photon coupling

As the final perturbative contribution, we discuss the quark-photon coupling, Figs. 1a and 1b, starting from the definition of the magnetic moment, Eq. 15. The difference from the pion-only case is that whereas in that case, for instance, the nucleon magnetic moments depended only on the bare nucleon, delta and nucleon-delta transition magnetic moments, hyperon magnetic moments and transition magnetic moments now enter. Those bare moments are listed in Table 2 of [3] (see later for comments on errors and sign conventions).

The contribution to the magnetic moment from photon-quark coupling is

$$
\mu_Q^A = \mu^{(0)}(A) + \mu_{QM}^A,
$$

(25)

where $\mu^{(0)}(A)$ is the bare magnetic moment, corresponding to Fig. 1a, and $\mu_{QM}^A$ is the magnetic moment corresponding to Fig. 1b in which a photon couples to the quarks with a virtual meson in flight.

Both parts of $\mu_Q^Q$ involve the quantity

$$
\mu_Q(a, \frac{1}{2} B) = \langle A, S_{A z} = \frac{1}{2} | \hat{\mu}_{Q z} | B, S_{B z} = \frac{1}{2} \rangle.
$$

(26)

The bare magnetic moment $\mu^{(0)}(A)$ in Eq. 25 is related to $\mu_Q(a)$ as follows. For the octet members

$$
\mu^{(0)}(A) = \mu_Q(a, A),
$$

(27)

while

$$
\mu^{(0)}(\Omega) = 3 \mu_Q(\Omega, \Omega),
$$

(28)

as a consequence of the definition of $\mu_Q$ in terms of the $S_z = \frac{1}{2}$ state rather than the spin-stretched state.

Using the form of the quark spinors in Eq. 3 we obtain

$$
\mu_Q(a, B) = \langle A_{s f}, S_{A z} = \frac{1}{2} | \sum_{i=1}^{3} Q_i \sigma_i^{(i)} \frac{8\pi}{3} \int_0^\infty dr r^3 G_i F_i | B_{s f}, S_{B z} = \frac{1}{2} \rangle;
$$

(29)

there are no ambiguities in the choice of wavefunction since the magnetic moment operator does not change flavour. Note that

$$
\mu_Q(a, B) = \mu_Q(B, A).
$$

(30)
The \( \mu_{Q0}(A, B) \), are expressed in terms of

\[
\mu_f \equiv \frac{8\pi e}{3} \int_0^\infty dr \ r^2 G_f F_f, \tag{31}
\]

where the quark flavour \( f \) is either \( n \), non-strange, or \( s \), strange. They are given in Table 2 of ref. [8]. However it should be noted that according to the sign convention used there, the signs of \( \mu_{Q0}(\Sigma^-, \Sigma^0) \) and \( \mu_{Q0}(\Xi^-, \Xi^0) \) are given wrongly. Furthermore, to obtain agreement with the the sign conventions of most tables of isoscalar factors [13], all the \( \mu_{Q0}(\Sigma, \Sigma^0) \), \( \mu_{Q0}(\Sigma, \Lambda) \) and \( \mu_{Q0}(\Xi, \Xi^0) \) should be multiplied by \(-1\). The set is completed by \( \mu_{Q0}(\Omega, \Omega) = -\frac{1}{3} \mu_s \).

The second order piece \( \mu_{QM}^A \) is given by

\[
\mu_{QM}^A = \sum_{a, B, C} \int d^3k \frac{v_{a}^{AB}(k)}{(\omega_{BA} + \omega_a(k))} (B_0 | \mu_{Qz} | C_0) \frac{v_{a}^{AC}(k)}{(\omega_{CA} + \omega_a(k))} \tag{32}
\]

(equation 4.72 of [8], but allowing kaon and \( \eta \) loops). Substituting the vertex functions, Eq. [11], and manipulating the Clebsch-Gordan coefficients yields

\[
\mu_{QM}^A = \frac{e}{36\pi^2} \sum_{\nu, B, C} \zeta^2 f^{AB}_\nu f^{AC}_\nu S_{BC} \int_0^\infty \frac{dk k^4 u_{\nu}^2(k)}{\omega_{\nu}(k)(\omega_{BA} + \omega_{\nu}(k))^2} \times \\
\sum_{T_{BA}} (T_{BA} T_{\nu} | T_A T_{A3} ) (T_{CA} T_{\nu} | T_A T_{A3} ) \mu_{Q0}(B, C), \tag{33}
\]

where the relation \( \omega_{BA} = \omega_{CA} \) has been used, and \( S_{BC} \) is

| \( s_A \) | \( s_B \) | \( s_C \frac{= S_C = \frac{1}{2}}{} \) |
|------------------|------------------|------------------|
| \( \frac{1}{2} \) | \( -1 \) | \( -2 \) |
| \( \frac{3}{2} \) | \( 3 \) | \( -\frac{3}{2}\sqrt{10} \) |

As was mentioned in sect. (2.1), appropriate combinations of non-strange and strange form factors, \( u_n(k) \) and \( u_s(k) \), must be taken for the \( \eta \)-meson, since it couples to both types of quarks:

\[
\zeta_\eta u_\eta(k) = C^{AB}_n \zeta_n u_n(k) + C^{AB}_s \zeta_s u_s(k), \tag{34}
\]

where the coefficients \( C^{AB}_n \) and \( C^{AB}_s \) depend on the transition being considered. They are given in Table 1 of ref. [8].

### 3.3 Summary of the formalism

To summarize, we write down a formula for the perturbative corrections to the magnetic moment of baryon \( A \)

\[
\mu^{(1)}(A) = Z_A [\mu^A_\pi + \mu^A_\kappa + \mu^A_{Q\pi} + \mu^A_{Q\kappa} + \mu^A_{Qn}]
\]

\[
= \frac{Z_A e}{36\pi^2} \left[ D^A_\pi I^A_\pi + \sum_B D^A_\kappa I^A_\kappa + (D^A_\pi (\pi n) \mu_n + D^A_\pi (\pi s) \mu_s) I^A_{\pi n} + \right.
\]

\[
\left. \sum_B [(D^A_\pi (Kn) \mu_n + D^A_\pi (Ks) \mu_s) I^A_{Kn} + (D^A_\pi (\eta n) \mu_n + D^A_\pi (\eta s) \mu_s) I^A_{\eta n}] \right] ,
\]
where taken out and
\[ I_A^\pi = \zeta_n^2 \int_0^\infty dk \frac{2k^4u_n^2(k)}{\omega_n^2(k)}, \quad I_{A\pi}^k = \zeta_n^2 \int_0^\infty dk \frac{k^4u_{ns}^2(k)(\omega_{BA} + 2\omega_K(k))}{\omega_k^2(k)(\omega_{BA} + \omega_K(k))^2}, \]
\[ I_{Q\pi}^A = \zeta_n^2 \int_0^\infty dk \frac{k^4u_n^2(k)}{\omega_n^2(k)}, \quad I_{Q\pi}^{AB} = \zeta_n^2 \int_0^\infty \frac{dk k^4u_{ns}^2(k)}{\omega_K(k)(\omega_{BA} + \omega_K(k))^2}, \]
\[ I_{Q\pi}^{AB} = \int_0^\infty dk \frac{k^4}{\omega_k^2(k)}(D_{n\pi}^{AB}\zeta_nu_n(k) + D_{s\pi}^{AB}\zeta_su_s(k))^2. \quad (36) \]

For ease of notation, the dependences of the meson masses on charge and of the form factors on hypercharge are not explicitly shown, though taken into consideration in actual calculations. In particular, transitions induced by kaons involve intermediate states \( B \) which have different values of hypercharge, so that \( \omega_{BA} \) can vary over a range of a few hundred MeV. For a check of numerical calculation it is useful to examine the coefficients
\[ D_k^A \equiv \sum_B D_k^{AB}, \quad D_Q^K(Kf) \equiv \sum_B D_Q^{AB}(Kf), \quad D_Q^K(\eta f) \equiv \sum_B D_Q^{AB}(\eta f) \quad (37) \]
and \( D_\pi^A, D_Q^K(\pi f) \), which we tabulate in Table 1.

Some relationships between \( D_\pi \) and \( D_k \) are worth pointing out. As was noted in sect. 3.1 there is a close analogy between isospin in the pion-photon coupling and \( V \)-spin in the kaon-photon coupling. This leads to the equality
\[ (D_{\pi}^p, D_{\pi}^n) = (D_{k}^{\Sigma^+}, D_{k}^{\Xi^0}), \quad (38) \]
which may be viewed as the relation between the isospin-\( \frac{1}{2} \) pair \( (p, n) \) and the \( V \)-spin-\( \frac{1}{2} \) pair \( (\Sigma^+, \Xi^0) \). Similarly the following relations are observed:
\[ (D_{\pi}^{\Sigma^+}, D_{\pi}^{\Sigma^-}) = (D_{k}^p, D_{k}^n), \quad (D_{\pi}^{\Xi^0}, D_{\pi}^{\Xi^-}) = (D_{k}^n, D_{k}^p). \quad (39) \]

Note, however, that the \( \Lambda \) and \( \Sigma^0 \) are not eigenstates of \( V \)-spin but the appropriate combinations of \( V = 0 \) and \( V = 1 \) states, and therefore no straightforward relations between \( D_\pi \) and \( D_k \) exist in these cases.

The final result for the magnetic moment is
\[ \mu(A) = Z_A \mu^{(0)}(A) + \mu^{(1)}(A). \quad (40) \]

One of the goals of the present work is to reveal the way in which the predictions of the model are influenced by the effects of kaons and eta. In fact, it has been shown \[ [10] \] that the inclusion of those effects are crucial for reproducing the observed mass spectrum of the octet and decuplet baryons. It is therefore useful to study the two classes of magnetic moment:
\[ \mu_{\pi \text{on}}(A) = Z_{\pi \text{on}}^A(\mu^{(0)}(A) + \mu_{\pi}^A + \mu_{Q\pi}^A) \quad (41) \]
\[ \mu_{\text{all}}(A) = Z_{\text{all}}^A(\mu^{(0)}(A) + \mu_{\pi}^A + \mu_{k}^A + \mu_{Q\pi}^A + \mu_{Qk}^A + \mu_{Q\eta}^A), \quad (42) \]

where \( Z_{\pi \text{on}}^A \) and \( Z_{\text{all}}^A \) correspond to the cases in which intermediate states in Eq. \[ [13] \] include pions alone and all types of mesons, respectively.
4 Results and Discussion

4.1 CDM results and input parameters

Armed with the formalism developed in the preceding two chapters, we now investigate numerically how the magnetic moments of the octet baryons and \( \Omega \) can be described in our theoretical framework. The present model is successful in reproducing the mass spectrum of low-lying baryons [10], and we wish to calculate the magnetic moments with the same parameters. It is also of interest to determine the sensitivity to the model parameters, which are as follows. The parameter \( p \) is the power of the \( \chi \) field in Eq. 3, which is taken to be one or two. \( \beta \) is the dimensionless coupling constant of a non-strange quark to the \( \chi \) field. \( c_g \) of Eq. 2 is the ratio of the non-strange to strange coupling strengths. The experimental values of \( f_\pi \) and \( f_K \) give \( c_g = 1 \). It has been suggested [10] that, if the approximation of holding the scalar, isoscalar at their vacuum value is lifted and they are allowed to interact with the quark and \( \chi \) fields, the effective coupling ratio inside the soliton is enhanced to \( c_g = 1.58 \). We therefore study this ratio too.

As was described in sect. 2, the \( \chi \)-field mass \( M_\chi \) is determined by fitting the r.m.s. quark radius of the nucleon to its isoscalar charge radius \( r_I \). Once determined, \( M_\chi \) fixes the overall energy scale of the theory prescribed by the solitonic Lagrangian, Eq. 3. The literature shows a considerable uncertainty in the experimentally extracted value of the charge radius of the proton [14] as well as that of the neutron [15], and thus \( r_I \) ranges over \( 0.74 < r_I < 0.85 \text{ fm} \). Furthermore it has been shown [10] that the correction of spurious center-of-mass motion associated with a localized soliton reduces the isoscalar radius \( r_I \) by a factor of some 10%. In view of these uncertainties in \( r_I \), we have considered a range of values from 0.72 (used in ref. [10]) to 1 fm. It turns out the ratios of magnetic moments are very insensitive to this parameter. It should be stressed that \( M_\chi \) is determined once \( p, \beta \) and \( r_I \) are specified and is therefore not an additional input parameter.

There is one source of ambiguity still left, namely the scale of the magnetic moment. The non-relativistic reduction gives results expressed in terms of the nuclear magneton \( \mu_N = e/2m_N \), but it is not obvious what value of the nucleon mass \( m_N \) to take. (This problem is common to many models [17]) Ideally, \( m_N \) ought to be calculated within the framework of our model. However, former work on the same model has shown [10] that the best-fit input parameters, set (II) in Table 5 with \( r_I = 0.72 \text{ fm} \), yield a nucleon mass of 1227 MeV, about 30% above the experimental value. Although the level scheme of mass of the octet and decuplet baryons is reproduced reasonably well, it should be kept in mind that there is an uncertainty of up to 30% in the absolute energy scale. We use the experimental value of \( \mu_N \) to calculate what we term the ‘absolute magnetic moment’, which is subject to uncertainty originating from the ambiguity in scale. The absolute value of the proton magnetic moment varies from 1.96 at \( r_I = 0.72 \text{ fm} \) to 2.61 at \( r_I = 1.0 \text{ fm} \). This uncertainty may be removed by setting the absolute magnetic moment of one specific baryon, for example the proton, to the corresponding experimental value and by scaling the absolute magnetic moments of other baryons accordingly. The results thus obtained are called ‘scaled magnetic moments’. In what follows we will discuss the implications of the present theory based primarily on the scaled magnetic moments.
As a representative case of the $p = 1$ model, we select $r_I = 0.90$ fm, $c_g = 1.44$ and $\beta = 0.028$. In Table 2 we show the various components of magnetic moment defined in Eqs. 27 and 35, the comparison between the effects of pions alone and those of all mesons, i.e. $\mu_{\text{pion}}$, $\mu_{\text{all}}$, $Z_{\text{pion}}$, $Z_{\text{all}}$ in Eqs. 41 and 42, and the experimental moments $\mu_{\text{exp}}$ with their errors. All magnetic moments are given in terms of absolute values, except scaled magnetic moments $\tilde{\mu}_{\text{pion}}$ and $\tilde{\mu}_{\text{all}}$.

Fig 2 shows the scaled magnetic moments for these parameters (ii), and then with $r_I = 0.7$ fm and other parameters as before (iii), $\beta = 0.1$ (iv) and $c_g = 1.58$ (v). Finally, the results for the $p = 2$ model, with $r_I = 0.90$ fm, $c_g = 1.44$ and $\beta = 0.00245$ (vi) are shown.

The most striking feature of the results is that the colour-dielectric model does a remarkably good job of describing the magnetic moments, with a maximum deviation of 0.27 and an average deviation of only 0.11 nuclear magnetons. It can also be seen that changing $r_I$ and $\beta$ and $c_g$ has essentially no effect on the scaled magnetic moments. As was discussed in sect. 2, the bare level of our theory does not depend on the parameter $\beta$ once the overall energy scale is fixed. It is evident that the meson masses hardly spoil the scaling property: the pion masses are small on the typical energy scale of low-energy phenomena. Though the kaon masses are much larger, the values of the kaon contributions are too small to spoil the scaling.

However fit for the $p = 2$ case is distinctly worse. In particular there is inadequate SU(3) breaking, as can be seen by the reduced $\mu_p - \mu_{\Sigma^+} - |\mu_\Omega|$, $\mu_n - \mu_{\Sigma^0}$ and $\mu_{\Sigma^-} - \mu_{\Xi^-}$ splittings. (All are zero in an SU(3) symmetric model.) This agrees with the results of ref. [10], where the octet and decuplet mass splittings were found to be unsatisfactorily small in the $p = 2$ model.

A notable feature of the results in Table 6 is the small influence on the magnetic moments of the effects of kaon and $\eta$ mesons. Columns (ii) to (iv) of Fig. 3, show the scaled magnetic moments without mesons, with pions, and with kaons and the $\eta$ in addition. With pions alone the SU(3) splittings referred to above are almost perfect; including kaons decreases them slightly. The contributions of all the diagrams involving kaons are small, and they are almost offset by the increased value of the inverse normalisation constant $Z^{-1}$. Similar results have been found in the CBM [18]. The origin of these small contributions can be traced back to small values of the integrals in Eq. 24. A typical denominator of the integrand contains at least the third power of the meson mass, which, together with the form factor, greatly suppresses the integral value. In contrast, the self-energy contribution of mesons involve the second-power of the meson mass, so that the integral is not expected to be reduced so much. In fact, the inclusion of kaons and $\eta$ is essential for describing the mass spectrum of low-lying baryons [10].

The $\Omega$ contains three strange and no non-strange valence quarks, so that $\mu_\pi = 0$ and $Z_{\text{pion}} = 1$. The successful reproduction, as can be seen in Fig. 3, of the $\Omega$ magnetic moment at the pion level without the inclusion of kaon and $\eta$ effects can thus be regarded as evidence the bare solitons are a good starting point for a description of baryons. We should particularly recall that our model provides a natural mechanism for determining the ratio between the non-strange and strange coupling strengths. It is an experimentally fixed parameter in the hidden-symmetry realization of an approximate $SU(3)_R \times SU(3)_L$ symmetry of our model Lagrangian.
4.2 Comparison with other Models

Direct comparison with the results of ref. [6] for the cloudy bag model is complicated by the use there of the physical masses, renormalised coupling constants and the exclusion of kaons, as well as the inclusion of estimates of the centre-of-mass corrections. Thus we have repeated our calculation to produce results for the CBM treated in the same manner as the CDM. These are shown in the last two columns of Fig. 3. (The bare values are indistinguishable, so that column (ii) could be either model). Pions only are included in (v), while kaons and the $\eta$ are added in (vi). The strange quark mass has been taken as 144 MeV and the bag radius as 1 fm, the preferred parameters of Théberge and Thomas [6]. There is a strong resemblance between the two models, as might be expected from the common methods used. The fit in the CBM is poorer, but would probably be improved by a larger strange quark mass. There too, kaons play very little role.

It is worth noting that in the cloudy bag model, the inclusion of one-gluon exchange improves overall agreement and is vital for recovering the correct order of $\mu(\Lambda)$ and $\mu(\Xi^-)$ [19]. Furthermore, center-of-mass corrections can in principle be carried out unambiguously by projecting a soliton on to an eigenstate of momentum. By eliminating spurious center-of-mass motion, a significant improvement of nucleon properties has been achieved in two-flavor versions of the color-dielectric model [16, 20]. This is an advantage of a model which consists entirely of dynamical fields. Although the same technique has been applied to the MIT bag model [21], ambiguities associated with the static bag are unavoidable.

The magnetic moments of baryons have now been calculated in many models, most of which claim to give a reasonable fit. To the extent that most (but not all) correctly reproduce the order of the moments (except $\mu(\Lambda)$ and $\mu(\Xi^-)$) this may be true. However all models are certainly not equal in the quality of fit, as can be seen from Fig. 4, where published results from a number of models are shown (all scaled to give the experimental proton moment). As a rough guide, $\chi^2$ values for all the fits have been calculated, excluding the $\Omega$ and in each case allowing the overall scale to vary to minimise $\chi^2$. The theoretical values have been assigned an arbitrary error of 0.11 so that the $\chi^2$ per degree of freedom is about 1 (1.1) for the CDM.

First, it can be seen from column (iii) that lattice calculations (Leinweber [22]) now rival most dynamical models. With $\chi^2 = 1.5$ the results are only slightly worse that the CDM. A non-relativistic model with pion-exchange currents (Wagner et al [23]) does not give sufficient SU(3) breaking. It is shown in column (iv), and with $\chi^2 = 2.2$ is not as good as a simple additive quark model with $\mu_s = 0.6\mu_n$ ($\chi^2 = 1.6$, not shown). These models however all do very much better that the remaining class of models, including the Skyrme model, the NJL model and the chiral bag model, all of which are based on the hedgehog. Column (viii) is the SU(3) Skyrme model calculation of Park, Schechter and Weigel [5], ($\chi^2 = 8.3$) which clearly has inadequate SU(3) breaking. Park and Weigel [4] also included vector mesons which actually reverses the order of two of the SU(3) splittings ($\chi^2 = 9.9$, not shown). The bound state treatment of the Skyrme model, (vii) (Oh, Min and Rho [3]; $\chi^2 = 6.8$) has the same failing. At zero bag radius the chiral bag model (Hong and Brown [24]; $\chi^2 = 11.2$) resembles the Skyrme model; at $r_{bag} = 1$ fm the fit is a little better ($\chi^2 = 10.1$; column (vi)) but the $\Sigma^-$ is particularly bad, falling below the $\Lambda$. The NJL model (Kim et al [25]; $\chi^2 = 6.0$, not shown) is the
best of the projected hedgehogs. It is clear that all these models have problems in the treatment of kaon fields. In the SU(3) Skyrme model, for instance, kaons appear when the embedded SU(2) pionic hedgehog is rotated in flavour space. As a result, the kaons have a profile governed by the pion mass. Schwesinger and Weigel [17] have performed calculations in the Skyrme model in which they have allowed the kaon profile to vary (the “slow rotor approach”), with much improved results. (Column (v) shows their results for the usual Skyrme term; $\chi^2 = 2.5$ dominated by the poor neutron moment).

Finally, it is worth comparing our results with those of Jenkins et al in chiral perturbation theory [26]. Expanding meson loop diagrams in powers of the quark masses, they obtain predictions for the SU(3) splittings which are all too large by factors of 2.5 to 4.8 if kaons are included, and poor results still without kaons. In contrast, the worst in the CDM with kaons is the $\mu_{\Sigma^-} - \mu_{\Xi^-}$ splitting, at 60% of its experimental value, while with pions alone this rises to 80%, with the other two near perfect. On the other hand the relations which hold to $O(m_q^{1/2})$ are also extremely well satisfied in the CDM:

$$\mu_{\Sigma^+} = -2\mu_\Lambda - \mu_{\Sigma^-},$$
$$\mu_{\Xi^0} + \mu_{\Xi^-} + \mu_n = 2\mu_\Lambda - \mu_p;$$

the first is satisfied to within 2% and the second to within 0.3% with or without kaons. Thus it is clear that the fact that the experimental moments approximately satisfy these relations is not related to the convergence or otherwise of the perturbation expansion, and that loop diagrams treated to all orders agree much better with experiment that the first term in their chiral expansion. Similar behaviour has also been noted in the context of baryon self energies. [27]

5 Conclusion

We have calculated the octet and $\Omega$ magnetic moments in the colour-dielectric model, in which pion and kaon fields have been included perturbatively. Good agreement with experimental values are obtained for the scaled magnetic moments, in an approach in which $\mu_p$ is effectively the only free parameter. In particular the strength of SU(3) breaking is governed entirely by the pion and kaon masses and decay constants. However the absolute values are subject to some uncertainty. This work confirms that models based on the perturbative inclusion of mesons give very much better results for the magnetic moments than those based on hedgehogs, and while the colour-dielectric model and the cloudy bag model give very similar results, the former has the advantage of being fully dynamical. Finally the comparison with chiral perturbation theory suggests that in the work which has been done so far, the expansion in powers of the strange quark mass has not converged.
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Table 1: $D^A$

| $A$ | $D_\pi$ | $D_K$ | $D_{Q(\pi n)}$ | $D_{Q(\pi s)}$ | $D_{Q(Kn)}$ | $D_{Q(Ks)}$ | $D_{Q(\eta n)}$ | $D_{Q(\eta s)}$ |
|-----|---------|-------|-----------------|-----------------|-------------|-------------|----------------|----------------|
| $p$  | 22      | 8     | 145             | 0               | 36          | 2           | -1            | 0              |
| $n$  | -22     | -2    | -100            | 0               | -24         | 2           | $\frac{2}{3}$ | 0              |
| $\Sigma^+$ | 8 | 22 | $\frac{88}{3}$ | $\frac{20}{3}$ | 102 | $\frac{40}{3}$ | $\frac{256}{9}$ | $\frac{20}{9}$ |
| $\Sigma^-$ | -8 | 2 | $-\frac{56}{3}$ | $\frac{20}{3}$ | -50 | $\frac{40}{3}$ | $-\frac{128}{9}$ | $\frac{20}{9}$ |
| $\Lambda$ | 0 | -12 | 0 | -36 | -2 | -24 | 0 | $\frac{4}{3}$ |
| $\Xi^0$ | -2 | -22 | 0 | -12 | -32 | $-\frac{154}{3}$ | $-\frac{94}{9}$ | $-\frac{140}{9}$ |
| $\Xi^-$ | 2 | -8 | 1 | -12 | 12 | $-\frac{154}{3}$ | $\frac{47}{9}$ | $-\frac{140}{9}$ |
| $\Omega$ | 0 | -18 | 0 | 0 | -6 | -72 | 0 | -44 |

The coefficients $D^A$ defined in Eq. 35. Note the relations between $D_\pi$ and $D_K$, Eqns. 38 and 39.
Table 2: Various components of the magnetic moment

|                | $p$   | $n$   | $\Sigma^+$ | $\Sigma^-$ | $\Lambda$ | $\Xi^0$ | $\Xi^-$ | $\Omega$ |
|----------------|-------|-------|------------|------------|-----------|---------|---------|---------|
| $\mu^{(0)}$    | 1.99  | −1.33 | 1.96       | −0.69      | −0.58     | −1.22   | −0.56   | −1.76   |
| $\mu_\pi$      | 0.79  | −0.79 | 0.29       | −0.29      | 0.00      | −0.07   | 0.07    | 0.00    |
| $\mu_K$        | 0.04  | −0.01 | 0.09       | 0.01       | −0.05     | −0.09   | −0.04   | −0.08   |
| $\mu_{Q\pi}$   | 0.96  | −0.66 | 0.24       | −0.09      | −0.21     | −0.07   | −0.06   | 0.00    |
| $\mu_{QK}$     | 0.05  | −0.03 | 0.17       | −0.06      | −0.03     | −0.12   | −0.04   | −0.12   |
| $\mu_{Qq}$     | 0.00  | 0.00  | 0.04       | −0.01      | 0.00      | −0.03   | −0.01   | −0.04   |
| $Z_{\pi\pi}^{-1}$ | 1.57  | 1.57  | 1.20       | 1.20       | 1.36      | 1.09    | 1.09    | 1.00    |
| $Z_{\text{all}}^{-1}$ | 1.61  | 1.61  | 1.32       | 1.32       | 1.45      | 1.24    | 1.24    | 1.13    |
| $\tilde{\mu}_{\pi\pi}$ | 2.38  | −1.78 | 2.07       | −0.89      | −0.58     | −1.25   | −0.50   | −1.76   |
| $\tilde{\mu}_{\text{all}}$ | 2.38  | −1.76 | 2.11       | −0.86      | −0.60     | −1.29   | −0.52   | −1.78   |
| $\tilde{\tilde{\mu}}_{\pi\pi}$ | 2.79  | −2.08 | 2.42       | −1.04      | −0.68     | −1.46   | −0.59   | −2.06   |
| $\tilde{\tilde{\mu}}_{\text{all}}$ | 2.79  | −2.07 | 2.47       | −1.01      | −0.71     | −1.52   | −0.61   | −2.09   |
| $\tilde{\mu}_{\exp}$ | 2.79  | −1.91 | 2.42       | −1.16      | −0.61     | −1.25   | −0.65   | −1.94   |

The breakdown of the magnetic moments of the octet baryons and $\Omega$ which are calculated for the parameter set $r_I = 0.90$ fm, $\beta = 0.028$, $c_g = 1.44$, $p = 1$. See Eqns. [17, 24, 27, 33] and [34] for the definitions of the various components of magnetic moment. $\tilde{\mu}_{\pi\pi}$ and $\tilde{\mu}_{\text{all}}$ are scaled magnetic moments while the rest are absolute magnetic moments.
Figure captions

Fig. 1  Four types of contributions to the magnetic moment: (a) bare, (b) quark-photon coupling, (c) charged-pion-photon coupling, (d) charged-kaon-photon coupling. The solid, dashed and wavy lines represent a bare baryon, a virtual meson dressing it and a probing photon, respectively.

Fig. 2  The scaled magnetic moments calculated for various input parameters.
(i) Experimental magnetic moments;
(ii) $p = 1$, $c_g = 1.44$, $r_I = 0.9$ fm and $\beta = 0.028$ (hence $M_\chi = 1883$ MeV);
(iii) as (ii) but $r_I = 0.72$ fm ($M_\chi = 2354$ MeV);
(iv) as (ii) but $\beta = 0.1$ ($M_\chi = 805$ MeV);
(v) as (ii) but $c_g = 1.58$;
(vi) $p = 2$, $c_g = 1.44$, $r_I = 0.9$ fm and $\beta = 0.00254$ ($M_\chi = 1676$ MeV);

Fig. 3  The effects of mesons and comparison with the cloudy bag.
(i) Experimental magnetic moments;
(ii) no mesons (CBM or CDM);
(iii) CDM, parameter set as Fig. 2(iii), pions only;
(iv) CDM with all mesons;
(v) CBM, $r_{bag} = 1.0$ fm, $m_s = 144$, pions only;
(vi) CBM, all mesons.

Fig. 4  Comparison with other models. For more details, see text.
(i) Experimental magnetic moments;
(ii) CDM, as Fig. 3(iii);
(iii) Lattice QCD [22];
(iv) Non-relativistic quark model with pion exchange currents [23];
(v) Skyrme model (4th order) in the slow rotor approximation [17];
(vi) Chiral bag model, $r_{bag} = 1.0$ fm [24];
(vii) Skyrme model in the bound state approximation [3];
(viii) Skyrme model (Yabu-Ando diagonalisation) [4].
Fig. 1
Fig. 2

(i) (ii) (iii) (iv) (v) (vi)
Fig. 3
Fig. 4