NUCLEOSYNTHESIS CONSTRAINTS ON DEFECT-MEDIATED ELECTROWEAK BARYOGENESIS

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Abstract

In the defect-mediated electroweak baryogenesis scenario, baryons are produced in well separated regions of space. It is shown that between the electroweak phase transition at a temperature of $T \sim 100\text{GeV}$ and the end of nucleosynthesis at $T \sim 1\text{KeV}$ the baryon inhomogeneities dissipate, and that no constraints on defect-mediated electroweak baryogenesis can be derived from considerations of inhomogeneous nucleosynthesis.
1. Introduction

Recently there has been a lot of interest in the possibility that the observed baryon to entropy ratio was generated at the electroweak scale [1-3] (for recent reviews see e.g. Refs. 4 and 5). Most electroweak baryogenesis models assume that the electroweak phase transition was first order and proceeded via the nucleation, expansion and subsequent percolation of critical bubbles. In this case, the baryon distribution after the bubble percolation is essentially homogeneous since the regions of net baryon production, the bubble walls, sweep out all of space.

The order of the electroweak phase transition, however, is not known. Even if the transition is first order, its dynamics may be driven by thermal fluctuations. Recent simulations in scalar field theories [6] and evidence from condensed matter systems (see e.g. Ref. 7 and references therein) argue against nucleation of critical bubbles as the mechanism driving the phase transition.

In Refs. 8 and 9, a new mechanism of baryogenesis was proposed which is independent of the order and detailed dynamics of the electroweak phase transition. In this theory, baryon production is mediated by topological defects produced at an energy scale $\eta$ equal or higher than the scale $\eta_{EW}$ of electroweak symmetry breaking. Provided that the electroweak symmetry is restored in the core of the defects, baryons are produced there during the out-of-equilibrium motion of the defects. In this scenario, baryons are produced inhomogeneously.

There are severe constraints on inhomogeneous nucleosynthesis. The success of homogeneous big bang nucleosynthesis in explaining the observed abundances of light elements[10] (in particular $^4$He, $^3H$e and $^7$Li) makes it hard to allow for any inhomogeneities in the baryon distribution at the time when nucleosynthesis begins. This leads to possible constraints on scenarios with inhomogeneous baryogenesis [11,12].

Consider this issue in more detail. The predictions of homogeneous big bang nucleosynthesis for the light elements $^4$He, $^3$H$e$ and $^7$Li are compatible with the observed abundances only in a narrow interval of the baryon to entropy ratio $\hat{\eta} = n_b/n_\gamma$ ($n_b$ and $n_\gamma$
are the baryon and photon number densities, respectively):

\[ 3 \times 10^{-10} < \hat{\eta} < 10^{-9} \quad . \tag{1} \]

Any significantly higher or lower values of \( \hat{\eta} \) lead to an overproduction of \(^7\text{Li}\). A lower value of \( \hat{\eta} \) leads to an overproduction of \( d + ^3\text{He} \) but to a deficit of \(^4\text{He} \), and a higher value of \( \hat{\eta} \) induces overproduction of \(^4\text{He} \) but a deficit of \( d + ^3\text{He} \). Hence, if at the onset of nucleosynthesis the baryons are inhomogeneously distributed, with regions of \( 0 < \hat{\eta} < 3 \times 10^{-10} \) surrounding regions with \( \hat{\eta} < 10^{-9} \) but with the same space-averaged value of \( \hat{\eta} \), then an overproduction of \(^7\text{Li} \) will result. This issue has recently been analysed in detail in Ref. 13 (see Ref. 14 for a selection of early references). Note that it is possible to construct special inhomogeneous nucleosynthesis scenarios. For example,[15], if there are no baryons at all in the “low baryon density” regions, then agreement with observations can be obtained by lowering the overall value of \( \hat{\eta} \) such that \( \hat{\eta} \) lies in the range (1) in the high baryon density regions, thus leading to a low \( \Omega_B \) Universe.

In this letter we investigate whether the defect-mediated electroweak baryogenesis scenario is constrained by nucleosynthesis considerations. In order to answer this question one must study the dissipation of baryon inhomogeneities between \( T \sim 100\text{GeV} = T_{EW} \) (the electroweak phase transition scale) and \( T \sim 1\text{KeV} \) (the end of nucleosynthesis). Our conclusion is that for most parameter values of the defect-mediated baryogenesis scenarios studied, the combined effects of neutrino inflation and baryon diffusion are sufficiently strong to homogenize the baryon distribution by the temperature \( T \sim 100 \text{ KeV} \). Hence, no constraints can be derived.

2. Dissipation of Baryon Inhomogeneities

The dissipation of baryon inhomogeneities at temperatures between 100 GeV and 1 KeV has recently been studied in great detail in Refs. 16 & 17 (see also Ref. 12). The most important processes are baryon diffusion and neutrino diffusion.

At temperatures above about 1 MeV, neutrinos are in thermal equilibrium with the plasma. Since they are the particles with the longest mean free path, they are the most efficient ones at transporting energy.
In an inhomogeneous electroweak baryogenesis model, entropy perturbations are produced during the electroweak phase transitions. By the equation of pressure equilibrium, regions with an overdensity of baryons must have a lower than average temperature. Neutrino heating of such a cold baryon-rich region will cause it to expand in order to maintain pressure equilibrium [16,17]. This effect is called “neutrino inflation”.

Neutrino inflation has the effect of lowering the amplitude of the inhomogeneities, but not washing them out entirely. The key length scale in electroweak baryogenesis is the Hubble radius at \( T_{EW} \) which is

\[
t_{EW} \simeq \frac{1}{(10g^*)^{1/2} m_{pl} \eta_{EW}^{-2}} \simeq 0.3 \text{cm}.
\] (2)

Here, \( g^* \) is the number of spin degrees of freedom of the thermal bath, and \( m_{pl} \) is the Planck mass. If \( \lambda \) is the diameter of a region with baryon excess, then neutrino inflation will between 100 GeV and 1 MeV reduce the amplitude of such a baryon perturbation to a value \( A \) which depends on \( \lambda \)[16, 17]

\[
A \simeq 10^4 \text{ for } \lambda \in [10^{-7}, 10^{-1}] \text{cm}
\]

\[
A \simeq \left( \frac{\lambda}{10^{-15}} \right)^{1/2} \text{ for } \lambda \in [10^{-15}, 10^{-7}] \text{cm}
\] (3)

\[
A = 1 \text{ for } \lambda < 10^{-15} \text{cm}
\]

In the above, \( \lambda \) is the physical size at \( T_{EW} \). For defect-mediated baryogenesis, only values of \( \lambda \) smaller than \( t_{EW} \) are of interest. We have implicitly assumed that the initial value of \( A \) exceeds the value given in (3).

After the neutrinos fall out of thermal equilibrium, neutrino inflation ceases to be an effective energy transport mechanism. It has been shown [17] that for \( T < 1 \text{MeV} \), baryon diffusion is the dominant way of dissipating entropy fluctuations. The baryon diffusion length \( l_{diff} \) depends on the initial value of \( A \) (at 1 MeV). An approximate expression for \( l_{diff} \) is [17]

\[
l_{diff} \simeq 0.1 \left( \frac{A}{A_0} \right)^{1/2} \text{ cm}, \quad A \geq A_0,
\] (4)
with $A_0 = 10^2$ (note that $l_{	ext{diff}}$ is expressed in terms of physical size at 100 GeV of a given comoving scale). Baryon inhomogeneities on scales smaller than $l_{	ext{diff}}$ get erased by diffusion. As is evident from Figs. 14 - 16 of Ref. 17, the distribution of baryons has become essentially homogeneous already at $T = 0.1$ MeV, the onset of nucleosynthesis.

To summarize, the evolution of baryon inhomogeneities produced during the electroweak phase transition proceeds in two stages. Between 100 GeV and 1 MeV, the amplitude of the perturbations decreases by neutrino diffusion. Below 1 MeV, baryon diffusion becomes dominant and spreads out the baryons.

In order to study possible constraints on defect-mediated baryogenesis from nucleosynthesis considerations, we must know the initial amplitude $A$ of the entropy perturbations, and the mean separation $d$ of the defects inducing baryogenesis. If

$$d < l_{	ext{diff}}(A) \quad ,$$

then the baryons have homogenized by the time of the onset of nucleosynthesis, and the scenario is safe.

In this paper we will assume that independent of the scale $\eta$ at which the defects are produced and independent of the type of defects considered, the mean baryon to entropy ratio $\hat{\eta}$ lies in the interval (1).

3. Constraints on Defect-Mediated Electroweak Baryogenesis

In the defect-mediated electroweak baryogenesis scenario of Refs. 8 and 9, baryons are produced inside of moving topological defects in which the electroweak symmetry is restored. Baryon number violating electroweak sphaleron processes are unsuppressed in the defect cores, CP violation is enhanced in the defect walls (in models with extra CP violation in the Higgs sector such as the two doublet model, a theory commonly used to study electroweak baryogenesis), and the defect dynamics is out of equilibrium. Thus, all of Sakharov’s criteria [18] for baryogenesis are satisfied.

In the baryogenesis scenario of Refs. 8 and 9, baryons are thus only produced in regions swept out by the topological defects following the electroweak phase transition. The size of the baryon-rich regions depends on the spatial extent of the particular type
of defect which is catalyzing baryogenesis, and the mean separation of these regions is
determined by the mean separation of the defects. Thus, we have a realization of the
situation studied in Refs. 16 and 17 in which baryons are produced inhomogeneously.

If $V_{BG}$ is the volume participating in electroweak baryogenesis, and $V$ is the total
volume, then the amplitude of the baryon perturbation is

$$A = \frac{V}{V_{BG}}.$$  \hspace{1cm} (6)

Knowing this amplitude and the mean separation of the regions where baryons are pro-
duced, it is possible to verify whether the condition (5) is satisfied, i.e. whether the baryon
inhomogeneities can dissipate sufficiently or not to effect nucleosynthesis.

In the following we shall investigate domain wall and cosmic string mediated baryo-
genesis. Domain walls moving with velocity $v_D$ in their normal direction will sweep out a
fraction of the order $v_D$ of space. Hence

$$A \sim v_D^{-1}. \hspace{1cm} (7)$$

For domain walls forming at a scale $\eta$ close to $\eta_{EW}$, the mean separation $d$ will be micro-
scopic

$$d(t_{EW}) \sim \eta^{-1} \left( \frac{\eta}{\eta_{EW}} \right)^p,$$  \hspace{1cm} (8)

where $p$ is some power determined by the evolution of the domain wall network between
$T = \eta$ and $T = \eta_{EW}$. For domain walls, $\eta$ must be close to $\eta_{EW}$, otherwise the Universe
would be domain wall dominated at $\eta_{EW}$. Hence, generically

$$d(t_{EW}) \ll l_{diff}(A), \hspace{1cm} (9)$$

and no additional constraints on the model result from the considerations of this work.
Note that even if domain walls are produced at $\eta$ close to $\eta_{EW}$, they must disappear by
the present time in order to ensure that the Universe is not dominated by walls today.

For model building, a theory in which cosmic strings mediate electroweak baryogenesis
is less constrained. The evolution of a network of cosmic strings has been studied in great
detail. For
\begin{equation}
\eta < (m_{pl}\eta_{EW})^{1/2} \simeq 3 \times 10^{10} \text{ GeV} \tag{10}
\end{equation}

the evolution of the string network is still friction-dominated, for larger values of $\eta$ the strings are in their scaling regime.

For friction-dominated strings, the mean separation is [19]

\begin{equation}
d = \xi(t_{EW}) \sim (G\mu)^{1/2} m_{pl}^{1/4} t_{EW}^{5/4} \tag{11}
\end{equation}

where $\mu \simeq \eta^2$ is the mass per unit length of the string. The length $\xi(t_{EW})$ also determines the volume in which baryogenesis takes place. Thus the initial amplitude $A_{in}$ of the baryon perturbation is

\begin{equation}
A_{in} \sim \left( \frac{t_{EW}}{\xi(t_{EW})} \right)^2 t_{EW} \frac{t_{EW}}{R_s} \gg 10^4, \tag{12}
\end{equation}

where $R_s$ is the radius about the string to which the electroweak symmetry is restored. Except for values of $\eta$ very close to $\eta_{EW}$

\begin{equation}
\frac{d}{t_{EW}} > 10^{-7}. \tag{13}
\end{equation}

Hence, from (3) it follows that after neutrino diffusion, the amplitude of the baryon perturbations will be

\begin{equation}
A \simeq 10^4, \tag{14}
\end{equation}

the value we will use for all strings in the friction era. With the above value for $A$, one can easily evaluate the criterion (5). We find that for

\begin{equation}
\eta < 10^9, \tag{15}
\end{equation}

equation (5) is satisfied.

We thus conclude that for $\eta < 10^9$, the baryon distribution at the onset of nucleosynthesis in cosmic string-mediated electroweak baryogenesis is homogeneous.
For $\eta > 3 \times 10^{10}$, strings are in their scaling regime. In the scaling regime, the string network consists of long strings with curvature radius and mean separation proportional to the Hubble radius, and of a distribution of loops with number density [20]

$$n(R, t) = \begin{cases} 
\nu R^{-2} t^{-2} & \gamma G\mu t < R < \alpha t \\
\nu(\gamma G\mu)^{-2} t^{-4} & R < \gamma G\mu t = R_c 
\end{cases}$$  \hspace{1cm} (16)

where $\alpha$, $\gamma$ and $\nu$ are constants which must be determined in numerical simulations. The constant $\alpha$ is a measure of the scale $\alpha t$ at which loops are produced at time $t$. Present simulations[21] indicate $\alpha < 10^{-2}$. The value of $\nu$ is at present also poorly determined, but is [21] of the order 1, and $\gamma$ measures the strength of gravitational radiation ($\gamma \sim 50$ according to Ref. 22).

The separation $d$ between baryogenesis volumes is given by the separation of small loops which is

$$d \simeq (R_c \nu(\gamma G\mu)^{-2} t^{-4})^{-1/3} = \nu^{-1/3}(\gamma G\mu)^{1/3} t_{EW}.$$  \hspace{1cm} (17)

From (3) it follows that

$$A \simeq 10^4$$

(the initial amplitude $A_{in}$ for scaling strings is much larger than $A$ because the strings occupy only a small volume of space). We can now investigate under which conditions the criterion (5) is satisfied. The result is

$$G\mu < \nu \gamma^{-1}$$  \hspace{1cm} (19)

or

$$\frac{\eta}{m_{pl}} < (\nu \gamma^{-1})^{1/2}$$  \hspace{1cm} (20)

This implies that for all particle physics motivated cosmic strings in the scaling regime, baryons have homogenized by the onset of nucleosynthesis.

At first sight it seems that for a narrow range of values of $\eta$

$$10^9 \text{GeV} < \eta < 3 \times 10^{10} \text{ GeV},$$  \hspace{1cm} (21)
defect-mediated electroweak baryogenesis is constrained by nucleosynthesis. However, for
this range of values of $\eta$ there will already be many loops as part of the string distribution.
Hence, the mean separation of baryogenesis sites will be smaller than that given in Eq.
(11), and the criterion (5) will in fact be satisfied.

4. Conclusions

We have shown that the combined effects of neutrino inflation and baryon diffusion are
efficient enough to homogenize the baryon distribution in the defect-mediated electroweak
baryogenesis scenarios we have considered, in spite of the initial large amplitude of the
baryon perturbations.

Neutrino inflation is crucial in order to obtain this result, since it leads to a decrease of
the amplitude of entropy inhomogeneities to a value $A \simeq 10^4$ between 100 GeV and 1 MeV.
With this value of $A$ the comoving baryon diffusion length is comparable to the Hubble
radius at $t_{EW}$ (when translated to physical length at $t_{EW}$). In cosmic string-mediated
models of electroweak baryogenesis the mean separation of the defects turns out to be much
smaller than the baryon diffusion length, unless $\eta$ is very close to the Planck scale. Note
that without taking neutrino diffusion into account, the effective baryon dissipation length
would have been so small as to render most of the models in conflict with nucleosynthesis.

We have shown that the only defect-mediated models of electroweak baryogenesis
which are endangered by the inhomogeneous nucleosynthesis constraints are theories in
which the mean separation of the defects at $t_{EW}$ exceeds about 3% of the Hubble radius
at $t_{EW}$ (see Eqs. (4) & (5)).

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