ABSTRACT. We propose two alternatives to Xu’s axiomatization of Chellas’s STIT. The first one simplifies its presentation, and also provides an alternative axiomatization of the deliberative STIT. The second one starts from the idea that the historic necessity operator can be defined as an abbreviation of operators of agency, and can thus be eliminated from the logic of Chellas’s STIT. The second axiomatization also allows us to establish that the problem of deciding the satisfiability of a STIT formula without temporal operators is NP-complete in the single-agent case, and is NEXPTIME-complete in the multiagent case, both for the deliberative and Chellas’s STIT.

KEY WORDS: axiomatization, complexity, logic of agency, STIT

1. INTRODUCTION

STIT theory is one of the most prominent accounts of agency in philosophy of action. It is the logic of constructions of the form ‘agent $i$ sees to it that $\varphi$ holds’. While STIT has played an important role in philosophical logic since the 80ies, its mathematical aspects have not been developed to the same extent. Most probably the reason is that STIT’s models of agency are much more complex than those existing for other modal concepts such as say necessity, belief, or knowledge: first, the ‘seeing-to-it-that’ modalities interact (or perhaps better: must be guaranteed not to interact) because the agents’ choices are supposed to be independent; second there is another kind of modality involved, viz. the ‘master modality’ of historic necessity. There are also temporal modalities, but just as most of the other proof-theoretic approaches to STIT, we do not investigate these here.

As a consequence, proof systems for STIT are rather complex, too. To our knowledge the following have been proposed in the literature.

- Xu provides Hilbert-style axiomatizations in terms of the historic necessity operator and Chellas’s STIT operator [4] or [16, Chap. 17], without considering temporal operators. As the deliberative STIT-operator can be expressed in terms of Chellas’ (together with the historic necessity operator), the axiomatization transfers to the deliberative STIT. Xu proves completeness by means of canonical models, and proves
decidability by means of filtration. Besides, Xu also gives a complete axiomatization of the one-agent achievement STIT [4 Chap. 16].

- Wölfl builds an axiomatics of STIT with instants in terms of historic necessity, Chellas’s STIT and tense operators [15]. However, completeness is obtained via the introduction of extra modal operators for ‘truth in all histories at the instant at hand’ and ‘truth in all histories (passing through the moment at hand) distinct from the history at hand’. Moreover, the more complex achievement STIT operator can be locally defined.

- Wansing provides a tableau proof system for the deliberative STIT [14]. The system is complete, but does not guarantee termination, and thus “is not tailored for defining tableau algorithms” [14].

- Dégremont gives a dialogical proof procedure for the deliberative STIT [7]. Again, the system is complete, but does not guarantee termination, and can therefore only be used to build proofs by hand.

In this note, we focus on the so-called Chellas’s STIT named after his proponent [5, 6]. The original operator defined by Chellas is nevertheless notably different since it does not come with the principle of independence of agents that plays a central role in STIT theory. Following its presentation in [9], we use the term CSTIT to refer to the logic of that modal operator. We show that Xu’s axiomatics of the logic of Chellas’s STIT can be greatly simplified.

The paper is organized as follows. After recalling Xu’s axiomatics (Section 2) we propose an alternative one and prove its completeness (Section 3). Based on the latter we show that in presence of at least two agents, the modal operator of historic necessity can be defined as an abbreviation (Section 4). This leads to a simplified semantics (Section 5), and to characterizations of the complexity of satisfiability (Section 6).

2. XU’S AXIOMS FOR THE CSTIT

In [4, Chap. 17], Ming Xu presents Ldm, an axiomatization for the deliberative STIT logic without temporal operators. As pointed out there, deliberative STIT logic and Chellas’s STIT logic are interdefinable and just differ in the choice of primitive operators. Following Xu we refer to these two logics as the deliberative STIT theories. We here mainly focus on Ldm with Chellas’s STIT operator as primitive.

2.1. LANGUAGE

The language of Chellas’s STIT logic is built from a countably infinite set of atomic propositions and a countable set of agents AGT. To simplify