Visco-elastic coupling between a linear two-dof system and a rocking rigid block to improve the dynamic response

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Abstract. A two-degree of freedom linear system is used as a model for a multi-degree of freedom frame structure. Such a structure is coupled with a rocking rigid block to improve its dynamic response. The lowest part of the structure is connected to the block through a linear visco-elastic device. The nonlinear equations of motion of the couple system frame-structure-block are obtained by a Lagrangian approach. Such equations are numerically integrated to analyze the behaviour of the coupled system. The coupling is considered effective if it reduces the displacements of the structure. Simulations are performed using a harmonic excitation as forcing term. An extensive parametric analysis is performed, and the results are summarized through behaviour maps. Such maps provide the ratio between the maximum displacements of the structure with and without the coupling with the rigid block for several combinations of system’s parameters. When the ratio of the displacements is less than unity, the coupling is effective. Results show that the presence of the rocking rigid block improves the dynamics of the structure in large ranges of the parameters that characterize the coupled system.

1. Introduction

After the pioneering work of Housner [1], several papers analysed the dynamics of rigid blocks. A general formulation for the rocking and slide-rocking motions of freestanding symmetric rigid blocks was proposed in [2] and [3]. Some formulations removed the simplifying assumption of symmetric bi-dimensional blocks considering either non-symmetric rigid blocks [4], or three-dimensional blocks [5-7]. In the last two decades, passive protection techniques were examined by different authors. Specifically, the effectiveness of base anchorages in preventing the overturning was studied in [8] and [9], while in [10-12] researchers analysed the possibility of using base isolation for the same purpose. In particular, in [12] the authors investigated the effects of base isolation on the dynamic response of a rigid block placed on a multi-story frame. In [13, 14] the authors demonstrated the effectiveness of a mass dynamic absorber in the shape of a pendulum in improving the dynamics of rigid blocks. Differently, in [15-17] a mass-damper modelled as a single degree of freedom and running on the top of the block was considered as a safety device. While traditionally, in the literature, the block is coupled with other devices to protect it from the overturning, in recent papers, a rocking block is used as a secondary structure to improve the behaviour of frames. In [18, 19] a block is rigidly connected to a frame structure to improve the response to seismic excitation.
This paper analyses the linear visco-elastic coupling of a frame structure and a rigid block aimed at improving the dynamic behavior of the frame. A two-degree of freedom linear system is used as model for a multi-story frame structure. A first visco-elastic device connects the top of the block to the lowest part of the frame structure. A second visco-elastic device connects the block directly to the ground. Such devices are respectively called coupling and external device in the following sections. The nonlinear equations of motion of the coupled-system are obtained by a Lagrangian approach and successively numerically integrated to analyze the behaviour of the coupled system. Simulations are performed using a harmonic excitation as forcing term. An extensive parametric analysis is performed, and the results are summarized in behaviour maps. The maps show the ratio between the maximum displacements or the drifts of the coupled and uncoupled systems in different planes of the system’s parameters.

2. Motivation
Frame structures can be coupled with other mechanical systems (i.e., mechanical devices or other structures) to improve their behaviour under external loads. Some examples of such mechanical systems are oscillating masses working as tuned mass dampers, dynamic mass absorbers, and elasto-plastic dampers. Among others possibilities, a rocking rigid block can be used to improve the dynamic and seismic behaviour of a frame structure. For example, the authors in [18, 19] consider a rigid coupling between a frame and a rocking wall that has the same height as the frame.

This paper considers a non-rigid connection between the frame and the block. Figure 1 shows the scheme of the coupled mechanical system. The block can be shorter than the frame and can be placed partially below the foundation level of the structure to minimize the interference with the frame. Contrarily to [18], where there is a connection at each story of the frame, in the presented model the connection between the block and the frame is at a single level, the lowest one.

The paper aims to investigate the possibility to improve the dynamics of the part of the structure above the connection point by using the external rocking block.

![Figure 1. Coupling between the frame structure and the rocking wall (CD: Connecting Device).](image)

3. Mechanical model
A two degree of freedom model is considered representative of a frame structure. The frame structure is coupled with a rocking rigid block through a visco-elastic device. Such a device connects the lower
part of the structure and a point on the vertical side of the block. A second visco-elastic device connects the block to the ground (Fig.1). The block has a mass \( M = \rho \times 2b \times 2h_b \times s \), where \( \rho = 25 \text{kN} / \text{m}^3 \) and \( s \) is the dimension orthogonal to the plane of the figure. Figure 1 shows the geometrical configuration and characteristics of the coupled mechanical system.

The frame structure is modelled as a two degree of freedom system. The distinction between the two parts of the frame structure obtained using two degrees of freedom is needed because the study aims to mainly improve the dynamics of the upper part of the structure that is not connected to the rigid block.

![Diagram of mechanical system](image)

**Figure 2.** Mechanical system: (a) Geometrical characterization of the system; (b) Lagrangian parameters.

### 3.1. Equations of motion

It is assumed that the block cannot slide, hence only rocking motions can occur. As a consequence, three Lagrangian parameters fully describe the motion. Such parameters are the displacements of the two d.o.f. system \( u_1 \) and \( u_2 \), and the rotation of the block \( \theta \) (Fig.2b). Two sets of three equations of motion, which describe the motion of the system when the block rocks around either the left corner \( A \) or the right corner \( B \), are derived. Figure 2b shows the positive directions of the Lagrangian parameters \( u_1 \), \( u_2 \), and
The positions of the mass centers of the bodies are evaluated to write the equations of motion of the mechanical system via a Lagrangian approach. For this purpose, an inertial reference frame with origin in \( O \), initially coincident with the left base corner \( A \) of the block, is considered (Fig.2a). For the sake of brevity, in this section, only the relationships needed to describe the motion of the system when the block is rocking around the left corner \( A \) are reported. The positions of the mass centers \( G_1 \) and \( G_2 \) of the two d.o.f. structure are:

\[
x_{G1}(t) = \begin{pmatrix} x_g(t) - d - d_{h_1} + u_1(t) \\ d_{h1} + h_1 \\ 0 \end{pmatrix}; \quad x_{G2}(t) = \begin{pmatrix} x_g(t) - d - d_{h2} + u_2(t) \\ d_{h2} + h_2 \\ 0 \end{pmatrix}
\]

The position of the mass center \( C \) of the block, during a rocking around the left corner \( A \), reads:

\[
x_C(t) = \begin{pmatrix} x_g(t) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \cos \theta(t) & -\sin \theta(t) & 0 \\ \sin \theta(t) & \cos \theta(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b \\ h_1 \\ 0 \end{pmatrix}
\]

where the matrix in Eq.(2) is the rotation tensor of the block, \( R_b \). The kinetic energy of the mechanical system during a rocking motion of the block around the left corner \( A \) reads:

\[
T = \frac{1}{2} \sum_{i=1}^{2} m_i (\dot{x}_{Gi}(t) \cdot \dot{x}_{Gi}(t)) + J_C (\dot{\theta}(t) \cdot \dot{\theta}(t)) + M (\dot{x}_C(t) \cdot \dot{x}_C(t))
\]

where \( \dot{\theta}(t) = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}(t) \end{pmatrix} \) and \( J_C \) is the polar inertia of the block with respect to its mass center. To evaluate the potential energy for a rocking motion around the corner \( A \), the distance vectors between the couple of points \( W, K \) and \( Z, Y \) have to be evaluated. They are required to compute the potential energy associate to the elastic devices that have stiffness \( k_c \) and \( k_e \). The distance vectors between the previous couples of points, read:

\[
x_{WK}(t) = x_k(t) - x_w(t) = \begin{pmatrix} d - \sin \theta(t)(d_{h1} + h_1) \\ -d_{h1} - h_1 + \cos \theta(t)(d_{h1} + h_1) \\ 0 \end{pmatrix}
\]

\[
x_{ZY}(t) = x_y(t) - x_z(t) = \begin{pmatrix} a + 2b - 2b \cos \theta(t) + d_{h1} \sin \theta(t) \\ d_{h1} - d_{h2} \cos \theta(t) - 2b \sin \theta(t) \\ 0 \end{pmatrix}
\]

where

\[
x_w(t) = \begin{pmatrix} -d + x_g(t) + u_1(t) \\ d_{h1} + h_1 \\ 0 \end{pmatrix}; \quad x_k(t) = \begin{pmatrix} x_g(t) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \cos \theta(t) & -\sin \theta(t) & 0 \\ \sin \theta(t) & \cos \theta(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ d_{h1} + h_1 \\ 0 \end{pmatrix};
\]

\[
x_y(t) = \begin{pmatrix} x_g(t) + 2b + a \\ d_{h1} \\ 0 \end{pmatrix}; \quad x_z(t) = \begin{pmatrix} x_g(t) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \cos \theta(t) & -\sin \theta(t) & 0 \\ \sin \theta(t) & \cos \theta(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2b \\ d_{h1} \\ 0 \end{pmatrix}
\]

The potential energy of the system is given by Eq. 6 as

\[
U = \text{...}
\]
where $g$ is the gravity acceleration, $j = \{0, 1, 0\}^T$ is the unity vector of the $y$-axis and $\bar{x}_c = \{b, h_b, 0\}^T$ is the initial position of the mass center of the block. Due to the presence of dashpots with damping coefficients $c_1, c_2, c_3$ and $c_e$ (Fig. 2a), the virtual work of the non-conservative viscous forces has to be considered to obtain the Lagrangian equations of motion; it reads

\[
\partial W_A = -\left[ c_1 \dot{u}_1(t) \dot{u}_1(t) + c_2 (\dot{u}_2(t) - \dot{u}_1(t)) (\delta u_2(t) - \dot{u}_1(t)) \right] - \left[ c_e (\bar{x}_{wk}(t) \cdot \delta \bar{x}_{wk}(t)) + c_e (\bar{x}_{zy}(t) \cdot \delta \bar{x}_{zy}(t)) \right]
\]

Finally, the equation of motion can be obtained by

\[
\left[ \frac{\partial}{\partial \theta} \left( \frac{\partial L_A}{\partial \dot{\theta}} \right) - \frac{\partial L_A}{\partial \theta} \right] \dot{\theta} = \partial W_A(\theta, \dot{\theta}), \quad \forall \dot{\theta} \neq 0; \quad (i = 1, 2, 3)
\]

where $(q_1, q_2, q_3) = (u_1, u_2, \theta)$ and $(\delta q_1, \delta q_2, \delta q_3) = (\delta u_1, \delta u_2, \delta \theta)$. They read

\[
k_c \left( d - (d_1 + h) \sin \theta - u_1 \right) \sqrt{\left( -d + (d_1 + h) \sin \theta + u_1 \right)^2 + (d_1 + h)^2 \cos \theta - 1} - d
\]

\[
k_c \left( (2b(a + 2b) + d_1^2) \sin \theta + a d_1 \cos \theta \right) \sqrt{\left( a - 2b \cos \theta + 2b + d_1 \sin \theta \right)^2 + (2b \sin \theta + d_1 \cos \theta - 1)^2 - a}
\]

\[
g M \cos \theta - h_b \sin \theta \right) + \frac{\partial \dot{\theta}}{\partial \dot{\theta}} + \frac{\partial \ddot{\theta}}{\partial \dot{\theta}} + \frac{\partial \dddot{\theta}}{\partial \dot{\theta}} + \frac{\partial \dddot{\theta}}{\partial \theta}
\]

\[
k_c \left( d_1 \right) \left( u_1 - d \right) \cos \theta + \left( d_1 + h \right) \sin \theta \right) \sqrt{\left( -d + (d_1 + h) \sin \theta + u_1 \right)^2 + (d_1 + h)^2 \cos \theta - 1} - d
\]

\[
k_c \left( d_1 \right) \left( u_1 - d \right) \cos \theta + \left( d_1 + h \right) \sin \theta \right) \sqrt{\left( -d + (d_1 + h) \sin \theta + u_1 \right)^2 + (d_1 + h)^2 \cos \theta - 1} - d
\]

where the dependence on time, $t$, is removed to make the equation more readable. The equations of motion referring to a block that rocks around the right corner $B$ can be obtained similarly and are not reported here for the sake of brevity.

3.2. Uplift and impact conditions of the block

The uplift of the block around point $A$ takes place when the resisting moment $M_A = Mg b$ due to the weight of the block is smaller than the overturning moment $M_O = -M \ddot{x}_g(t) h_b$.
\[ k_c u_i(t) + c_v \ddot{u}_i(t) \] due to the inertial force and the visco-elastic forces of the internal coupling device. All these forces are evaluated with respect to the base point \( A \) (Fig. 1). By vanishing the sum of the two previous moments, it is possible to obtain the external acceleration \( \ddot{x}_g = a_{up} \) able to uplift the block. Such an acceleration reads

\[ a_{up} = \frac{g}{\lambda} + \frac{[k_c u_i(t) + c_v \ddot{u}_i(t)](d_s + h_i)}{M h_b} \]  

(10)

where \( \lambda = h_b / b \) is the slenderness of the block. In the absence of the coupling with the device, the uplift condition is the same as the one of a stand-alone block.

During the rocking motion, when the rotation \( \dot{\theta}(t) \) approaches zero, an impact between the block and the support occurs. Post-impact conditions of the rocking motion can be found assuming that the impact happens instantly, the body position remains unchanged, and the angular momentum is maintained. This condition can be expressed as \( (J_o - BS_i)\dot{\theta}^+ = J_o \dot{\theta}^- \), where \( J_o \) is the polar inertia of the block with respect to one of the two base corners (\( A \) or \( B \)); \( S_i = M b \) is the static moment of the block with respect to a vertical axis passing through one of the two base corners. Superscript (\( - \)) and superscript (\( + \)) denote pre- and post-impact quantities, respectively. From the conservation of the angular momentum, the maximum value of the post-impact angular velocity \( \dot{\theta}^+ \) can be obtained as a function of the pre-impact angular velocity \( \dot{\theta}^- \). The post-impact angular velocity that is considered in the numerical simulations is equal to \( \dot{\theta}^+ = \eta (J_o - BS_i / J_o) \dot{\theta}^- \), where \( \eta \) is a coefficient lesser than unity, introduced to include a further loss of mechanical energy. In the analyses, the value of \( \eta \) is fixed (\( \eta = 0.9 \)).

4. Parametric analysis

An extensive parametric analysis is performed to investigate the behaviour of the coupled system by numerically integrating the equations of motion. Special care is devoted to the detection of the impacts of the block. In the analyses performed, the block never exceeds the critical angle \( \arctan(b/h) \).

In the parametric analyses, the two d.o.f. model refers to a small building of three similar stories. Each of the stories has a surface of 100,000m\(^2\), and the inter-story height is 3.0m. The displacement \( u_1 \) refers to the first story of the frame structure (sub-structure), where there is the connection with the block. The displacement \( u_2 \) represents the displacement of the upper stories of the structure (super-structure). The values of the stiffness, \( k_1 \) and \( k_2 \), are evaluated by using the procedure in [20-22]. In particular, the values of mass and stiffness used in the analyses are, \( m_1 = 120.6 \times 10^5 \text{kg} \), \( m_2 = 2m_1 = 241.2 \times 10^5 \text{kg} \), \( k_1 = 2.194415 \times 10^6 \text{kN/m} \), and \( k_2 = 9.404636 \times 10^6 \text{kN/m} \). The damping ratios of the structure are assumed constant and equal to \( \xi_1 = \xi_2 = 0.05 \). The following parametrization is used to improve the understanding of the results

\[ k_c = \beta k_1, k_e = \gamma k_1 \]  

(11)

The displacement \( u_i \) and the drift \( u_2 - u_1 \) are used as indicators to evaluate the dynamic performance of the system. The smaller \( u_i \) and \( u_2 - u_1 \) are, the greater is the effectiveness of the coupling with the block. As done in [20], two gain parameters are then introduced,

\[ \alpha_1 = \frac{\max |u_i(t)|}{\max |\dot{u}_i(t)|}, \quad \alpha_2 = \frac{\max |u_2(t) - u_1(t)|}{\max |\dot{u}_2(t) - \dot{u}_1(t)|} \]  

(12)

where the displacements (\( \bullet \)) refer to the uncoupled frame structure (i.e., not coupled with the block). If the parameters of Eq. (12) are lesser than unity, the coupling between the frame structure and the rocking block is beneficial for the frame structure. An extensive parametric analysis is performed with the aim to build gain maps that represent the values of \( \alpha_1 \) and \( \alpha_2 \) in specific parameters planes. The following
values of geometrical and mechanical parameters are assumed: \( h_t = 3.0m, d_h = 5.0m, s = 5.0m, h_{over} = 0, \) and \( c_k = 0 \) (Fig.2).

4.1. Base excitation

The mechanical system is excited by a harmonic input. Specifically, the excitation used in the analyses is \( x_b(t) = A_s \sin(\omega t), \) \( 0 \leq t \leq t_{max} \), where \( \omega = 2\pi / T_s \) is the circular frequency of the excitation, \( T_s \) is the period of the harmonic cycle, \( A_s \) is its amplitude and \( t_{max} \) is the maximum time used in the numerical integrations (\( t_{max} = 10T_s \)). Since the mechanical system is nonlinear, its behavior depends on the amplitude \( A_s \) of the excitation. However, some preliminary tests showed that, for values of the block rotation sufficiently smaller than the critical angle \( \alpha_c \), the dependence of the motion on the amplitude itself is minimal. Hence, in the numerical simulations, the amplitude is fixed at \( A_s = 25.0 \) m/s.

4.2. The role of the coupling visco-elastic device

The first parametric analysis investigates the influence of the coupling visco-elastic device (i.e., the internal device) on the motion of the structure. For this analysis, the external device connecting the block with the ground is neglected \( (k_g = c_k = 0, \) see Fig.2). The parameters that are varied in the analysis are the base of the block, \( 2b \), and the stiffness ratio \( \beta \) of the coupling device (Eq.(11)). The viscous damping is initially neglected \( (\zeta = 0) \).

In Fig.3, the gain maps of the parameters \( \alpha_1 \) and \( \alpha_2 \) are organized in matrix form. Each row refers to a different circular frequency of the harmonic excitation. Inside the light grey regions, the gain parameters are less than unity. Hence, these regions, which are named gain regions, represent combinations of the parameters for which the coupling with the rigid block is beneficial for the structure. The extent of such regions depends on the frequency of the excitation. Inside this regions, the efficiency of the coupling with the block, measured by the gain parameters, strongly depends on the frequency. Lower values of the gain parameters are obtained for high circular frequencies. In particular, the lowest values (i.e., the best performance of the protection device) occurs for \( \Omega = 15 \) rad/s.

Figure 4 shows the time-histories of \( u_1, u_2 - u_1, \) and \( \theta, \) referring to point \( A, \) appearing in the in Fig.3. Specifically, Fig. 4a shows the comparison between the displacement \( u_1 \) and drift \( u_2 - u_1 \) of the coupled (thick line) and uncoupled (thin line) systems. The coupled system largely outperform the uncoupled one (i.e., it has smaller displacements). Instead, Fig.4b shows the displacement \( u_1 \) (thin line) and the rotation \( \theta \) (thick line). After the first oscillations, the motions of the mass \( m_1 \) and the block synchronize. Given the positive directions of the displacement \( u_1 \) and rotation, \( \theta, \) which are shown in Fig.2b, the mass \( m_1 \) and the block synchronize in counter phase. In this case, the block works as a tuned mass damper for the frame structure.

Finally, the sensitivity of the motion to the value of the viscous damping of the coupling device is analysed. Figure 5a shows the gain maps of \( \alpha_1 \) and \( \alpha_2 \) for a circular frequency \( \Omega = 15 \) rad/s. They can be compared with the maps shown in the third row of Fig.3 that are obtained neglecting the viscous damping. The main difference is that the damping regularizes both the gain regions and the gain surfaces. Figure 5b and 5c show the gains surfaces of the parameter \( \alpha_2, \) whose projections are the corresponding gain regions for the undamped and the damped case, respectively.

4.3. The role of the external visco-elastic device

The following section investigates the effects of the external device on the motion of the frame structure. In the analyses related to this section, the viscous damping of such device is neglected \( (c_k = 0). \) The first parametric analysis is performed in the range of small values of the stiffness of both the devices. For this analysis, the length of the block’s base is the same that characterizes point \( A \) in Fig.3. The analysis is performed by varying both the stiffness ratios \( \beta \) and \( \gamma \) (Eq.(11)). In Fig.6, the maps of \( \alpha_1 \) and \( \alpha_2 \) show that a variation in the stiffness of the external device does not lead to any improvement in the behaviour of the system (i.e., it does not sensibly reduces the displacements of the frame structure).
In fact, by moving along the vertical dashed line passing through the referring point \( A \) (i.e., increasing the stiffness \( k_x \)), the values of both the gain parameters increase above the unity.

Figure 7 shows gain maps related to \( \alpha_2 \) that are similar to the map presented in Fig. 6. In Fig. 7 the maps are obtained for different circular frequencies of the excitation and larger ranges of the stiffness values (i.e., both the ranges of \( \beta \) and \( \gamma \) are increased with respect to the maps in Fig. 6). For low frequencies of the excitation, the gain region (light grey region) covers almost all the parameter’s plane, but it shrinks when \( \Omega \) increases. The gain region of Fig. 6b is too small to be seen in Fig. 7 since it is confined in a very small area around the origins of the maps. The small gain region of Fig. 6b and the large gain regions of Fig. 7 do not connect for any value of \( \Omega \). In the corresponding \( \alpha_1 \) maps, not shown in the paper, the gain regions also cover almost all the parameter plane for all of the considered frequencies. As the frequency of the excitation increases, this analysis shows a behaviour that is opposite to the one presented in Fig. 3. If only the first internal device is modeled, and low values of its stiffness are considered, the area of the gain regions increases (i.e., in the results shown in Fig. 3). When both the internal and external devices are modeled, and high values of their stiffness are considered, the area of the gain regions tends to decreases when the circular frequency \( \Omega \) increases.

Figure 8 shows the time-histories of the drift \( u_2 - u_1 \) and rotation \( \theta_1 \), referring to the point labelled as \( B \) in the gain maps of Fig. 7. Specifically, Fig. 8a shows the comparison between the drift \( u_2 - u_1 \) of the coupled (thick line) and the uncoupled (thin line) systems. Also in this case, the presence of the block improves the dynamics of the frame structure. Figure 8b shows the corresponding displacement \( u_1 \) (thin line) and rotation \( \theta_1 \) (thick line). In this case, for the given positive directions of the displacements, the mass \( m_1 \) and the block oscillate in phase. Hence, the main contribution to the improvement of the dynamics of the coupled system is provided by the external device. The dynamics of the coupled system is similar to the dynamics of a two d.o.f. system representing the frame structure where the lower degree of freedom \( u_1 \) has a higher mass and stiffness than the lowest part of the frame structure not coupled with the block. Such kind of results is already observed in [20].

5. Conclusions

In the paper, a frame structure is coupled with a rocking rigid block to improve the dynamics of the frame. The multi-story frame is modeled as a two degree of freedom system that is connected to the block through a linear visco-elastic device. Such a device connects the block to the lowest part of the structure. The model also includes a second visco-elastic device that connects the block to the ground. The nonlinear equations of motion are obtained by a Lagrangian approach and successively numerically integrated to analyze the behavior of the coupled system. The coupling with the block is considered beneficial for the frame structure when there is a reduction in the displacements of the structure. Simulations are performed considering a harmonic excitation. The results of an extensive parametric analysis are summarized in behaviour maps plotted in different planes of the system’s parameters. The maps provide the ratio between the maximum displacement (or the drift) of the coupled system and the maximum displacement (or the drift) of the frame structure not coupled with the block. A ratio less than unity highlights the effectiveness of the block in improving the dynamics of the frame structure. Results show that the presence of the rocking rigid block improves the dynamics of the structure in two cases. In the first case, when only the internal coupling device is present, the block works as a tuned mass damper for the structure. This happens for very low values of the stiffness of the internal coupling device. In the second case, when both the devices are present, the block oscillates in phase with the bottom part of the structure. The resulting dynamics is similar to the one that would be obtained increasing the mass and stiffness of the degree of freedom related to the bottom part of the frame structure. When high values of stiffness are considered for both the devices, it is possible to largely improve the behaviour of the frame structure in wide ranges of the parameters’ values.
Figure 3. Gain maps ($k_E = c_c = c_E = 0$).
Figure 4. Time-histories: (a) absolute displacement $u_i$ and drift $u_2 - u_1$ (thin line: no-coupled structure, thick line: coupled structure); (b) displacement $u_i$ (thin line) and rotation $\theta$ (thick line); $(b = 0.75 m, k_c = 0.04 k_b, k_E = c_c = c_E = 0, \Omega = 15 \text{ rad/s})$.

Figure 5. Effects of the viscous damping: (a) $\alpha_1$ and $\alpha_2$ gain maps ($c_c = 0.35 c_b$); (b) $\alpha_2$ gain surface ($c_c = 0$); (b) $\alpha_2$ gain surface ($c_c = 0.35 c_b$); $(b = 0.75 m, k_E = c_E = 0, \Omega = 15 \text{ rad/s})$. 
Figure 6. Effects of the external device for small values of its stiffness: $\alpha_1$ and $\alpha_2$ gain maps; $(b = 0.75m, c_c = c_e = 0, \Omega = 15 rad / s)$.

Figure 7. Effects of the devices for large values of their stiffness; $\alpha_2$ gain maps; $(b = 0.75m, c_c = c_e = 0, \Omega = 15 rad / s)$.
Figure 8. Time-histories: (a) drift $u_2 - u_1$ (thin line: no-coupled structure, thick line: coupled structure); (b) displacement $u_1$ (thin line) and rotation $\theta$ (thick line); $(b = 0.75m, k_c = 3k_1, k_E = 3k_1, c_c = c_E = 0, \Omega = 15\text{rad/s})$.

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