Modelling the effect of lockdown

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Abstract: We have modelled the effect of the lockdown during the first wave of COVID-19. We used SEIR type of model with a certain time lag between infection and becoming infectious. Firstly we confirmed the timing of the change of the coefficient of infection, growth rate of confirmed cases, corresponds to the change of mobility index, and secondly we assume the change of the coefficient of infection, activity index \( \beta \) (analogous to \( R_0 \)) and fit the parameter to reproduce the actual number of confirmed cases. Finally, we assume that the activity index \( \beta \) is roughly proportional to the square of the mobility and fit the parameters. The curves in various countries fits reasonably well in any cases, but estimating \( \beta \) from various parameters (including temperature) in the long term remains as an important task.

Keywords: COVID-19, Lockdown, SEIR model, Basic reproduction number

1. INTRODUCTION

Due to the worldwide pandemic of COVID-19, many countries are trying to slow down the spreading of the virus, and lockdown is a very powerful option for that. Therefore, it would be meaningful to estimate the effect of the lockdown on the dynamics of COVID-19 cases.

We use roughly two types of models for the number of patients, one is ordinary SEIR type of model as in [1], in which transfer from one state to another is proportional to the number of original state, and another is SEIR type of model which includes the effect of time delay[2]. In this model, once someone gets infected, they will become contagious and later quarantined after fixed period of time\(^1\). Using the actual number of patients[5] and the activity data from smartphones[6], we wish to know which type of model better describes the reality.

In the second section, we briefly describe the structure of two types of models, and in the third section we will see the overview of the model and fitting of the models with the real data. In the final section we will examine the result and possible future developments.

2. MODEL

Let me describe the model of contagion. The ordinary SEIR type of model consists of 5(or 4 for simplicity) states.

\[ S: \text{susceptible (not infected)}, \ E: \text{Exposed (infected but not yet contagious)} \]
\[ I: \text{Infected (infected and contagious)} \]
\[ Q: \text{Quarantined (infected but separated from others)} \]
\[ R: \text{Recovered (infected but recovered and no more contagious)} \]

\( \beta \) is the infection coefficient, and \( \beta N \) is the production number of infected cases. Therefore \( \beta \) can be estimated to \( \beta N \) which is the actual number of confirmed cases. Finally, we used SEIR type of model with a time lag between infection and becoming infectious.

\[ \frac{dS}{dt} = \beta SI - \beta N I \frac{N - I}{N} - \beta SI_{\text{delay}} \]
\[ \frac{dE}{dt} = \beta SI_{\text{delay}} - \beta E \frac{N - E}{N} \]
\[ \frac{dI}{dt} = \beta E - \beta I \frac{N - I}{N} - \beta I_{\text{delay}} \]
\[ \frac{dQ}{dt} = \beta I_{\text{delay}} - \beta Q \frac{N - Q}{N} \]
\[ \frac{dR}{dt} = \beta Q \frac{N - Q}{N} \]

Figure 1: Delayed SEIR model

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\[ \frac{dR}{dt} = \beta Q \frac{N - Q}{N} \]

1 In reality, we know that the time infected person becomes contagious is rather fixed[3], and also it will take a while since patients go to a doctor till they receive the result of PCR testing. Therefore we expect the model with time delay will describe the dynamics better.

2 The time from infection till one becomes symptomatic is around 5 days as in the study in Diamond Princess[3] and it is also known that one can be infectious 1 - 2 days before one becomes symptomatic[8]. Therefore time scale for this process is...
estimated to be around 1 week considering the situation in Japan[4], but expected to vary from country to country. The transfer parameter from state I to state R is expected to be around 1/14, because it is considered it takes about two weeks for COVID-19 to be cured (to become not infectious).

Another type of model is SEIR type of model with delay in time and the equations are given as below.

\[
\begin{align*}
\dot{S} &= -\beta(t)(S(t))I(t)/N(t) \\
\dot{E} &= \beta(t)(S(t))I(t)/N(t) - \beta(t - \tau)S(t-\tau)I(t-\tau)/N(t-\tau) \\
\dot{I} &= \beta(t - \tau)S(t-\tau)I(t-\tau)/N(t-\tau) - \alpha(t - \tau_1)S(t-\tau_1)I(t-\tau_1)/N(t-\tau_1) - \alpha(t - \tau_2)S(t-\tau_2)I(t-\tau_2)/N(t-\tau_2) \\
\dot{Q} &= \alpha(t - \tau_1)S(t-\tau_1)I(t-\tau_1)/N(t-\tau_1) - \alpha(t - \tau_2)S(t-\tau_2)I(t-\tau_2)/N(t-\tau_2) \\
\dot{R} &= (1 - \alpha)(t - \tau_1)S(t-\tau_1)I(t-\tau_1)/N(t-\tau_1) - \alpha(t - \tau_2)S(t-\tau_2)I(t-\tau_2)/N(t-\tau_2)
\end{align*}
\]

In reality, once you get infected with coronavirus, becoming symptomatic and being tested positive happens after certain amount of time rather than happening at certain probability. Thus, we have a heuristic reason to believe the latter model better describes the reality. In the later discussion in this note, we will focus on the delayed SEIR model. If transfer from S to E happens at time \( t \), transfer from state E to state I happens at time \( t + \tau \), where \( \tau \sim 4 \), and assuming that state I will proceed to state Q with probability \( \alpha \) and to state R with probability \( (1 - \alpha) \), we further assume that the transfer from state I to state Q will happen at time \( t - \tau_2 \) and I to R at time \( t - \tau_3 \).

3. DATA AND ANALYSIS

3.1 Data

We use the data for the confirmed cases from ourworlddata.org[5] and mobility data from apple mobility report[6]. The apple mobility report records how many times people searched for certain path using apple map, and the number of search is counted for car, walk and transit respectively.

3.2 First Analysis

In the first analysis, we wish to know the time lag from infection to quarantine (\( \tau_2 \)) heuristically. If the value of \( \beta \) and \( S \) is constant, the growth rate \( \dot{I} \) of \( I \) should be proportional to \( \beta \). Therefore, naively, we could expect that the growth rate of \( Q \) should be the same, so we might expect that the value of \( \beta \) and growth rate of \( \dot{Q} \) (\( = \) daily confirmed cases) is proportional. We have changed the value of \( \tau_2 \) and performed the linear regression of growth rate of \( \dot{Q}(t) \) with respect to \( \beta(t - \tau_2) \), and computed p-value for each regression. As an example of countries which had a successful lockdown, please take a look at figures for Austria as in Figure 2.

First figure shows the number of confirmed cases, and the second shows the growth rate of the number of confirmed cases and activity index taken by [6]. In the third figure, growth rate in confirmed cases and activity 12 days before was plotted. You can see that the growth rate changes in response to the change of activity index with around 10 days of delay. We have also performed linear regression on growth rate of confirmed cases and activity index, changing the time lag \( \tau \). As a result, we will have p-value for the regression and we have plotted the log(p-value) in the Figure 3. If we interpret the location of minimum p-value as the most probable model\(^6\), we can conclude \( \tau \sim 12 \).

3.2 Second Analysis

In the next step, let’s perform simulation with SEIR type model with delay. In this part, we simplify the model for ease of analysis. Here we ignore the state \( R \) because it

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\(^4\) We computed the slope of confirmed cases with respect to time as the growth rate

\(^5\) If possible we should have compared the model of different \( \tau \) by free energy or BIC.
is a bit hard to know the ratio of patients who will not quarantined but be cured without any treatment.

For the former model, we have variables the number of people for $S, E, I, Q$, and total number of people $N$, and transfer coefficients $\gamma, \delta$ are given by medical consideration. We will fit the model to the data by tuning $I_0$ (at certain time), and the value of $\beta$. We naively expect the value of $\beta$ to be proportional to activity index, but then it will not match the observation. We then assume that the time dependence of $\beta$ is similar to that of activity index. That is, as in Figure 4, we first approximate the dynamics of activity index as constant for $t < t_1$ and $t < t_2$, and linearly decrease for $t_1 < t \leq t_2$. We then assume $\beta$ is constant for $t < t_2$ and $t < t_2$, and linearly decrease for $t_1 < t \leq t_2$.

In the delayed SIR model, we instead have $N$ and $\tau$ from clinical consideration, because it is known that the time period from infection till one gets infectious is considered to be around 4 days. We then fit the model to daily confirmed cases and obtain $\tau_2, \beta_0, \beta_1, I_0$.

If we wish to see $\log(Q)$, $I_0$ corresponds to the intercept of the line, $\beta_0$ corresponds to the slope of the line, $\tau_2$ corresponds to the time of lockdown, and $\beta_1$ corresponds to the slope of the line after lockdown, so it is not so hard to estimate the value of the parameters. Here we adjust those parameters by hand and saw the value of $Q$ for actual and model. Here we show plot of those together with the plot of $\beta$.

For Australia, the parameter for the delayed model is, $\beta_0 = 1.0, \beta_1 = 0.04\beta_0, \tau = 4, \tau_2 = 12, I_0 = 0.01, t_0 = 2/2, t_1 = 3/12, t_2 = 3/23$. For Austria, the parameter for the delayed model is $\beta_0 = 0.7, \beta_1 = 0.07\beta_0, \tau = 4, \tau_2 = 14, I_0 = 6, t_0 = 3/1, t_1 = 3/7, t_2 = 3/15$. Here $t_0$ denotes the time when the number of confirmed cases became more than or equal to 10 for the first time (and we set the day to t=0 when we draw the graph for the number of patients). Considering the fact that the value of the activity became 20% of that due to lockdown in Australia, and 30% in Austria, the decline in $\beta$ were much more drastic compared to the decline in the value of the activity. From the fact that the ratio of $\beta_1$ to $\beta_0$ is close to the square of ratio of activity index, we can guess that the value of $\beta$ might just be proportional to square of activity index. Let’s try to fit the delayed model for the rest of the countries which experienced successful lockdown. With respect to each country, the estimated parameters would be as in table 1. The error between real value and theoretical value from delayed SEIR model is measured by $\sum_{days}(\log(Q_{real}) - \log(Q_{model}))^2$

| Table 1: parameters for fitting delayed SEIR model |
|---------------------------------|----------------|---------|---------|--------|--------|--------|--------|
|       | $B0$ | $\beta_1/B0$ | $\tau_2$ | $t_0$  | $t_0$  | $t_1$  | $t_2$  | Error |
| Japan  | 0.3  | 0.3        | 11      | 0.8    | 1/30   | 3/24   | 4/13   | 67    |
| Korea  | 1.0  | 0.2        | 7.2     | 1.5    | 2/1    | 2/19   | 2/26   | 35    |
| Czech  | 0.4  | 0.15       | 15      | 20     | 3/6    | 3/8    | 3/20   | 5     |
| Norway | 0.2  | 0.17       | 20      | 100    | 3/1    | 3/5    | 12/16  | 4     |
| Switzerland | 0.6  | 0.1        | 13      | 10     | 2/29   | 3/8    | 3/19   | 4     |
| Singapore | 0.29 | 0.17      | 16      | 0.04   | 1/20   | 3/19   | 4/10   | 649   |
| USA    | 1.0  | 0.1        | 13.7    | 0.004  | 2/3    | 3/8    | 3/22   | 334   |
| Luxembourg | 1.1  | 0.12       | 9       | 15     | 3/13   | 3/5    | 3/20   | 3     |
| Vietnam | 1.2  | 0.15       | 5.5     | 0.08   | 2/5    | 3/3    | 4/1    | 84    |

The ratios of activity index before and after the lockdown are respectively roughly 30% for Czech, 40 ~ 50% for Norway, and 40% for Switzerland, and square of those numbers roughly correspond to the ratio of $\beta$ before and after lockdown, again for those countries.

3.3 Third analysis

We will improve the analysis in the previous subsection in two ways; Firstly, we have seen that the contact coefficient $\beta$ is roughly proportional to (mobility)$^2$, and then we reverse the logic and assume this proportionality relationship to derive the value of $\beta$. We will again use [6], and use two possible mobility indices, one is average of mobility indices of three transportation (driving, transit, walking)$^8$, and the other is mobility index for transit. Secondly, in the previous subsection, we have assumed fixed period of time one requires from the time of infection till the period one gets infectious, $\tau$, and from the time of infection till the period one gets tested and quarantined, $\tau_2$. Here instead we assume Weibull accounts$^7$

$^7$ It is also claimed from purely analysing the data in Japan in some twitter

$^8$ Note there is no theoretical background why we should take the average of the three values, since it is NOT weighted average of those.
distribution for both time lag. From the analysis of [3], the distribution of the time period from the infection to starting to be symptomatic is, Weibull distribution with mean 6.4 and standard deviation 2.3. Considering the fact one becomes infectious 2–3 days before one becomes symptomatic[8], we could pretend that the distribution of the time period from the infection to starting to be infectious is Weibull distribution with mean 4 and standard deviation 2.3. This amounts to Weibull distribution with parameter \( k \sim 1.8 \) and \( \lambda \sim 4.5^9 \).

Furthermore, in case of Japan, the distribution from the time of becoming symptomatic and the time of getting tested is known [9], and adding typical time from infection till one gets tested, which is 6.4 days, to this distribution, the distribution of time from infection till getting tested can be approximated as another Weibull distribution, with \( k = 3.3 \) and \( \lambda = 13.3 \). We assume that for the corresponding distribution in other countries, the value of \( k \) is 3, and fit the \( \lambda \) parameter for this distribution (denote as \( \lambda_2 \)). As a result, fitting the curve by using \( \beta = (\text{mobility of public transportation})^2 \) seems to be the best fit. Moreover, we here assume \( a = 0 \), considering generalization to \( a = 0 \) will not change the dynamics qualitatively. The parameter for fitting is given as in table 2.

![Figure 5: Fitting delayed SEIR model with \( \beta \) given by transit mobility squared (Japan)](image)

| Country       | \( I_0 \) | \( \beta_0 \) | \( t_0 \) | \( \lambda_2 \) |
|---------------|---------|---------|-------|---------|
| Australia     | 0.008   | 0.7     | 2/2   | 11      |
| Japan         | 1.1     | 0.23    | 1/30  | 13.3    |
| Czech         | 80      | 0.4     | 3/6   | 24      |
| Norway        | 350     | 0.2     | 3/1   | 20      |
| Switzerland   | 100     | 0.6     | 2/29  | 15      |
| Singapore     | 4       | 0.35    | 1/30  | 30      |
| USA           | 0.04    | 0.4     | 2/3   | 24      |
| Luxembourg    | 60      | 1.3     | 3/13  | 12      |

### 3. CONCLUSION AND DISCUSSION

In the above discussion, as a crude data analysis, we have seen that growth rate of confirmed cases of COVID-19 is roughly linear in activity index, with time delay around 10 ± 5 days. In a more refined data analysis, we have tried to fit to the data with SEIR model with time delay and it can reproduce the data during lockdown. In that analysis, we saw that the ratio of the value of contact coefficient \( \beta \) before and after lockdown, \( \beta_1/\beta_0 \), is roughly equal to the ratio of square of mobility index before and after lockdown, so in the third analysis of this note, we have tried to fit the data with the assumption \( \beta = (\text{mobility})^2 \), using two different definition of mobility.

We have confirmed that the data roughly fits this assumption, although the true value of contact coefficient should be in between the result expected by using average mobility, and the one by transit mobility. We need further investigation to know more rigorous relationship between the mobility and the contact coefficient for modelling the number of confirmed cases of COVID-19. In particular, if we wish to fit the parameters for the number of confirmed cases, we might not be able to fit well with the data for the second wave, or third wave of covid-19, so it seems we need other parameters than mobility to explain the dynamics of confirmed cases. In Japanese case, temperature could be a promising candidate for the explanatory parameter, because in the mid-late summer we saw the decrease in the number of confirmed cases.

### REFERENCES

1. Hethcote, Herbert W. "The mathematics of infectious diseases." SIAM review 42, no. 4 (2000): 599-653.
2. COVID-19:Information sharing, Sato, Aki-Hiro, et al. "An epidemic simulation with a delayed stochastic SIR model based on international socioeconomic technological databases." 2015 IEEE International Conference on Big Data (Big Data). IEEE, 2015.
3. Mizumoto, Kenji, et al. "Estimating the asymptomatic proportion of coronavirus disease 2019 (COVID-19) cases on board the Diamond Princess cruise ship, Yokohama, Japan, 2020." Eurosurveillance 25.10 (2020).
4. "Suspected patients wait up to a week to get tested ", https://www3.nhk.or.jp/nhkworld/en/news/20200417_33/
5. [ourworldindata.org/coronavirus](https://ourworldindata.org/coronavirus)
6. [www.apple.com/covid19/mobility](https://www.apple.com/covid19/mobility)
7. [https://twitter.com/shunyielong](https://twitter.com/shunyielong), [https://twitter.com/ma_press](https://twitter.com/ma_press), [https://twitter.com/sken20k](https://twitter.com/sken20k)
8. Wei, Wycliffe E. "Presymptomatic Transmission of SARS-CoV-2—Singapore, January 23–March 16, 2020." MMWR. Morbidity and Mortality Weekly Report 69 (2020).
9. [https://datatstudio.google.com/u/0/reporting/c4d0f3e88-f72e-464e-a3b8-5e4e591c238d/page/ultJb?u=a3TzV-uQzaE](https://datatstudio.google.com/u/0/reporting/c4d0f3e88-f72e-464e-a3b8-5e4e591c238d/page/ultJb?u=a3TzV-uQzaE)