Event-Triggered State Estimation for Wireless Sensor Network Systems With Packet Losses and Correlated Noises

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ABSTRACT In this article, an event-triggered state estimation problem for wireless sensor network systems affected by random packet losses and correlated noises is considered. A set of independent Bernoulli variables are used to describe the random packet losses in the measurement transmission. An event-triggered transmission strategy is introduced to decrease limited network bandwidth consumption, and the measurement noise is correlated with the process noises of the same moment and the previous moment. Event-triggered estimator of process noises under the linear minimum variance criterion is derived. Then, an event-triggered state estimation algorithm related to the packet loss rate, noise correlation coefficient and triggering threshold is designed. Sufficient conditions are provided to guarantee convergence of the estimation error covariances of the proposed estimator. Finally, comparative simulation verifies the effectiveness of our algorithm.

INDEX TERMS Event-triggered mechanism, random packet losses, correlated noises, state estimation, estimator of process noise.

I. INTRODUCTION
In the past few years, due to its high accuracy, low cost, and flexible network settings, the state estimation of wireless sensor network systems (WSNs) has received extensive attention. With the high-speed development of network technology and sensor technology, the performance of sensors has been improved significantly, and the number and types of nodes in wireless sensor networks have shown explosive growth, which has brought great difficulties to network data transmission under limited bandwidth. Not to mention that the service life of the sensor is negatively related to its usage frequency. Therefore, how to design a reasonable transmission strategy under limited energy and bandwidth to send data to the estimator effectively is an urgent problem to be solved.

The event-triggered mechanism can determine whether to transmit measurements according to specific conditions. Compared with the traditional time-triggered mechanism, it reduces additional data transmission and energy consumption of sensor nodes, and extends the service life of the sensor effectively. After decades of development, a variety of event-triggered transmission mechanisms have emerged. The send-on-delta event-based mechanism is proposed by Miskowicz et al. for the first time [1]. Based on the available basic process model, Trimpe et al. propose the measurement-based triggering strategy by predicting the evolution of the measurement [2]. Few years later, based on the variance-based triggering law, they consider the problem of event-triggered state estimation [3]. Derived from variance-based triggering strategy, an event-triggered measurement data scheduler is designed by Wu et al. and the minimum squared error of the estimator is obtained in [4]. The three types of event-triggered mechanism mentioned above are contrasted and analyzed in [5]. Afterwards, stochastic event-triggered mechanisms are considered in [6]–[8]. By taking advantage of the “multiple point-and set-valued” filtering measurements, Shi et al.
solve event-triggered state estimation [9]. Li et al. conduct comparative experiments on the performance of the typical event-triggered state estimator mentioned above [10]. Xie et al. construct a joint switching mechanism to solve the state estimation problem of discrete-time fuzzy systems [11].

For WSNs, due to the inherent characteristics of the communication channel, random measurement losses are unavoidable [12], [13]. Based on a latest researched mechanism of packet losses compensation, the distributed and centralized fusion estimation algorithms for networked systems affected by random packet losses are proposed [14]. An event-triggered fusion estimation method is established by Li et al. for multi-sensor nonlinear system suffering from random transmission delays [15]. Considering network transmission delay, the event-triggered problem of finite time robust filtering is investigated by Sun et al. [16]. Considering the target tracking problem with packet loss, channel fading and switching topology, Qu et al. study the $H_{\infty}$ consensus filtering algorithm [17].

All the above researches are designed without considering the correlated noises. However, correlation of noises exists in many practical application systems, which may seriously affect the estimation property of sensor [18], [19]. Considering the influence of random noise and system communication capability, Wang et al. study the event-triggered tracking control problem for a class of nonlinear systems [20]. They also consider the exponential mean-square stability for a class of discrete-time nonlinear systems with multiplicative noise, and established a new control framework [21]. Considering that measurement noise and system noise are one-step cross-correlated, depending on triggering threshold and correlation parameter, an unscented Kalman filter-based filtering algorithm is proposed in [22]. In the same noise-correlated environment, Yan et al. design an event-triggered sequential fusion estimator [23]. Considering that measurement noise and process noise are correlated at the same moment, the event-triggered state estimator for the networked system subjected to packet losses and correlated noises is designed [24]. The event-triggered state estimator for networked systems with network-induced packet losses and bandwidth constraints is designed in [25]. However, if the discrete-time linear system is obtained based on the discretization of the continuous-time system, the measurement noise is not only related to the process noise of the previous moment, but also coupled with the process noise of the same moment [26]. While, the estimator of process noises is ignored in the above researches.

Motivated by the above analysis, the purpose of this article is to solve the problem of event-triggered state estimation for bandwidth-constrained wireless sensor network systems subject to random transmission packet losses and correlated noises. The major contributions of this article are summarized as follows:

(i) So as to avoid unnecessary data transmission, an event-triggered transmission mechanism is designed to determine whether measurements are transmitted to the estimator;

(ii) Based on the above strategy and iterative estimation of process noise estimator, an event-triggered state estimation algorithm associate to packet loss rate, triggering threshold and noise correlation coefficient is proposed;

(iii) Sufficient conditions associated to packet loss rate, event-triggered threshold and noise correlation coefficient are given to ensure the boundedness of the proposed algorithm.

This article is organized as follows. Section II explains the estimation problem influenced by packet losses and noise correlation, and the event-triggered scheduling mechanism is given. Section III presents an event-triggered state estimation algorithm considering packet losses and noise correlation. In Section IV, the boundedness of the state estimation error covariance is provided. In Section V, a simulation example is given to certificate the effectiveness of the designed algorithm. Section VI summarizes the full article.

II. PROBLEM FORMULATION

Consider the following linear discrete time-varying system

$$x_{k+1} = A_k x_k + B_k u_k, \quad k = 0, 1, \cdots$$

$$z_k = G_k x_k + v_k$$

where $z_k \in \mathbb{R}^m$ denotes the $k$-th measurement of sensor, $x_k \in \mathbb{R}^n$ is the system state vector, $A_k \in \mathbb{R}^{n \times n}$ is the system state matrix and $C_k \in \mathbb{R}^{n \times m}$ is the measurement matrix.

Assumption 1: The process noise $\omega_k$ and measurement noise $v_k$ are both assumed to be zero-mean Gaussian distribution, and

$$E\left(\begin{bmatrix} \omega_k \\ v_k \end{bmatrix} \right) = \begin{bmatrix} Q_k \\ S_k \end{bmatrix}, \quad E\left(\begin{bmatrix} \omega_k \mid v_k \end{bmatrix} \right) = \begin{bmatrix} Q_k \\ S_k \end{bmatrix} R_k \delta_{kl},$$

$E\{\omega_k \mid v_k \} = S_k^*$. Note that the process noises $\omega_k$ and $\omega_k$ are correlated with the measurement noises $v_k$ for $k = 1, 2, \cdots$, where $\delta_{kl}$ is the Kronecker delta function.

Assumption 2: The initial value of the system with mean $\hat{x}_0$ and covariance $P_0$ is assumed to be Gaussian distribution, and it is independent of $\omega_k$ and $v_k$.

Considering a scenario of remote estimation, sensor determines whether to transmit the measurement $z_k$ according to the event-triggered condition at each time $k$. On this basis, the unreliability of the network is considered. As the communication link is affected by some unstable factors, packet losses may occur during the transmission, which will affect the estimation accuracy of the sensor seriously.

A. EVENT-TRIGGERED MECHANISM

A communication strategy triggered by event similar to [23] is used to reduce the communication between the sensor and the estimator and prolong the lifetime of sensors. The innovation is

$$e_k = z_k - C_k \hat{x}_{k|k-1}$$

where $\hat{x}_{k|k-1}$ is the prediction of $x_k$. The covariance of $e_k$ is

$$P_{k} = E\{e_k e_k^T\}.$$
On account of $P_{e_k}$ is a positive semi-definite matrix, we can get a unitary matrix $U_k \in R^{m \times m}$ to obtain the following diagonalization matrix
\[
U_k^T P_{e_k} U_k = \Lambda_k
\]
where the matrix
\[
\Lambda_k = \text{diag}(\lambda_1^k, \cdots, \lambda_m^k) \in R^{m \times m}
\]
where $\lambda_1^k, \cdots, \lambda_m^k \in R$ are the eigenvalues of $P_{e_k}$. Define $H_k \in R^{m \times m}$ by
\[
H_k = U_k \Lambda_k^{-1/2}
\]
It can be seen that $H_k^T H_k = P_{e_k}^{-1}$. Then, $e_k$ is normalized as
\[
\tilde{e}_k = H_k^T e_k
\]
and all elements of the normalized innovation $\tilde{e}_k$ satisfy standard Gaussian distributed and are independent of each other. Hence, we define the event-triggering condition as [4].
\[
\gamma_k = \begin{cases} 
0, & \text{if } ||\tilde{e}_k||_\infty \leq \theta \\
1, & \text{otherwise}
\end{cases}
\]
(9)
where $||\cdot||_\infty$ represents the infinite-norm of the vector, $\theta \geq 0$ is a predetermined threshold that can determine the communication rate and will affect the estimation accuracy. $|\tilde{e}_k^1|, \cdots, |\tilde{e}_k^m|$ indicate the absolute values of the $1$-th $\sim m$-th dimensional elements of $\tilde{e}_k$, respectively. Therefore, it can be seen that when $\gamma_k = 1$, the accurate measurement $z_k$ will be transmitted to the remote estimator. Otherwise, when $\gamma_k = 0$, the state center cannot obtain the accurate measurement $z_k$, and we only know that $\max(|\tilde{e}_k^1|, \cdots, |\tilde{e}_k^m|) \leq \theta$.

B. PACKET LOSS

The measurements may be loss owing to sensor faults or communication problems. To simulate the packet losses phenomenon, a random variable is introduced [27].
\[
p(v_k | \lambda_k) = \begin{cases} 
N(0, R_k), & \lambda_k = 1 \\
N(0, \sigma^2 I), & \lambda_k = 0
\end{cases}
\]
(10)
where $N(0, \sigma^2)$ indicates zero-mean Gaussian distribution with variance $\sigma^2$, $\sigma \rightarrow \infty$. When $\lambda_k = 1$, it indicates that the measurement $z_k$ reaches normally, and if $\lambda_k = 0$ means $z_k$ is lost.

III. STATE ESTIMATION ALGORITHM WITH EVENT-TRIGGERED STRATEGY

In the sequel, we derive an event-triggered state estimator subjecting to packet losses and correlated noises.

After obtaining $z_k$, the event-triggered judgment condition $\gamma_k = 1$ or 0 is used to determine whether to send it to the remote estimator.

Define $I_k = \{0, \gamma_k, \gamma_k \circ \delta_0, \cdots, \gamma_k \circ \delta_0 \circ \delta_0, \cdots, \gamma_k \circ \lambda_k \circ \delta_0 \circ \delta_0 \}$ with $I_{-1} = \emptyset$, $I_k = I_{k-1} \cup \{\gamma_k = 0\}$. Define the set $\Omega \subset R^m$ as
\[
\Omega = \{\tilde{e}_k \in R^m : ||\tilde{e}_k||_\infty \leq \theta\}
\]
(11)
and $e_k$ satisfies a zero-mean Gaussian distribution with condition $I_{k-1}$. From
\[
e_k = C_k \tilde{x}_{k|k-1} + v_k,
\]
(12)
we obtain
\[
P_{e_k} = E \{e_k e_k^T | I_{k-1}\}
\]
\[
= E\{(C_k \tilde{x}_{k|k-1} + v_k)(C_k \tilde{x}_{k|k-1} + v_k)^T | I_{k-1}\}
\]
\[
= C_k P_{k|k-1} C_k^T + R_k + C_k E \{\tilde{x}_{k|k-1} v_k^T | I_{k-1}\}
\]
\[
+ E \{v_k \tilde{x}_{k|k-1}^T | I_{k-1}\} C_k^T
\]
\[
= C_k P_{k|k-1} C_k^T + R_k + C_k S_k^e + (S_k^e)^T C_k^T
\]
(13)
From (8) and (13), we obtain
\[
E \{\tilde{e}_k \tilde{e}_k^T | I_{k-1}\} = H_k^T P_{e_k} H_k = I_m
\]
(14)
where $\tilde{e}_k$ is a multivariate random variable with standard Gaussian distribution. $\tilde{e}_{k|k-1}^m$ represents the $m$-th element of $\tilde{e}_k$. If $m \neq m$, $\tilde{e}_{k|k-1}$ and $\tilde{e}_{k|k-1}$ are independent of each other. Notice that $\gamma_k = 0$ means the event $||\tilde{e}_k||_\infty \leq \theta$ happens.

Moreover, we can get the estimator gain as
\[
K_k = E \{\tilde{x}_{k|k-1} e_k^T | I_{k-1}\} P_{e_k}^{-1}
\]
\[
= (P_{k|k-1} C_k^T + S_k^e) [C_k P_{k|k-1} C_k^T + R_k + C_k S_k^e + (S_k^e)^T C_k^T]^{-1}
\]
(15)
Then, we draw the following conclusions.

Lemma 1 [4]: Let the variance of a zero-mean Gaussian random variable $x \in R$ be $E[x^2] = \sigma^2$ and $\Delta = \sigma \delta$, then
\[
E[x^2 | x] \leq \Delta = \sigma^2 (1 - \beta(\delta))
\]
where
\[
\beta(\delta) = \frac{2}{\sqrt{2\pi}} \delta e^{-\frac{\delta^2}{2}} [1 - 2Q(\delta)]^{-1}
\]
\[
Q(\delta) = \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx
\]
(16)
\[
\text{Lemma 2 [4]: Then, we have some preliminary results}
\]
\[
H_k^T E \{e_k e_k^T | \hat{I}_k\} H_k = E \{\tilde{e}_k^m \tilde{e}_k^m^T | \hat{I}_k\}
\]
\[
= [1 - \beta(\theta)] I_m
\]
(18)

Define the average sensor communication rate as
\[
\gamma = \lim \sup_{T \rightarrow \infty} \frac{1}{T+1} \sum_{k=0}^{T} E[\gamma_k]
\]
(20)
and the connection between the event-triggered threshold $\theta$ and the average communication rate $\gamma$ is obtained
\[
\gamma = 1 - [1 - 2Q(\theta)]^m
\]
(21)
A. EVENT-TRIGGERED KALMAN FILTER WITH RANDOM PACKET LOSSES AND CORRELATED NOISES

Theorem 1 (The Kalman filter (KFO)): The estimation of state $x_k$ for system (1) and (2) by the modified event-triggered Kalman filter is given as follows

$$
\begin{align*}
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + y_k \lambda_k K_k (z_k - C_k \hat{x}_{k|k-1}) \\
P_{k|k} &= P_{k|k-1} - [y_k \lambda_k + (1 - y_k)\beta(\theta)]K_k P_{k|k} K_k^T \\
\hat{x}_{k|k-1} &= A_k - 1 \hat{x}_{k-1|k-1} + \hat{\omega}_{k-1|k-1} \\
P_{k|k-1} &= A_k - 1 P_{k-1|k-1} A_k^T + A_k - 1 P_{k-1|k-1}^T + P_{k-1|k-1}^\omega \\
K_k &= (P_{k|k-1} C_k^T + S_k^2) C_k P_{k|k-1} C_k^T + \lambda_k R_k \\
&\quad+ (1 - \lambda_k)\sigma^2 I + C_k S_k^\gamma + (S_k^\gamma)^T C_k^\gamma [-1]
\end{align*}
$$

where, the calculation formulas of the process noise estimator are

$$
\begin{align*}
\hat{\omega}_{k|k} &= \gamma_k \lambda_k K_k^\omega (z_k - C_k \hat{x}_{k|k-1}) \\
K_k^\omega &= S_k [C_k P_{k|k-1} C_k^T + \lambda_k R_k + (1 - \lambda_k)\sigma^2 I \\
&\quad+ C_k S_k^\gamma + (S_k^\gamma)^T C_k^\gamma [-1] \\
P_{k|k}^\omega &= Q_k - \gamma_k \lambda_k K_k^\omega S_k^\gamma \\
P_{k|k}^\gamma &= -\gamma_k \lambda_k K_k S_k \gamma \\
P_{k|k}^\omega &= -\gamma_k \lambda_k S_k K_k \gamma
\end{align*}
$$

and

$$
\beta(\theta) = \frac{2}{\sqrt{2\pi}} e^{-\frac{\theta}{2}} \left\{ 1 - 2Q(\theta) \right\}^{-1}
$$

$$
Q(\theta) = \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx
$$

Proof: Projection theorem and induction are used to prove this theorem.

Step 1: First, the event-triggered state estimator under reliable measurement conditions is considered ($\lambda_k = 1$). So, the following derivation is divided into two cases for whether $z_k$ shall be sent or not.

1) $y_k = 1$: The measurement $z_k$ is received by the estimator. Under the circumstances, the event-triggered method degenerates into time-triggered, and we have

$$
\begin{align*}
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k (z_k - C_k \hat{x}_{k|k-1}) \\
P_{k|k} &= P_{k|k-1} - K_k P_{k|k} K_k^T \\
\hat{x}_{k|k-1} &= A_k - 1 \hat{x}_{k-1|k-1} + \hat{\omega}_{k-1|k-1} \\
P_{k|k-1} &= A_k - 1 P_{k-1|k-1} A_k^T + A_k - 1 P_{k-1|k-1}^T + P_{k-1|k-1}^\omega \\
K_k &= (P_{k|k-1} C_k^T + S_k^2) C_k P_{k|k-1} C_k^T + R_k \\
&\quad+ C_k S_k^\gamma + (S_k^\gamma)^T C_k^\gamma [-1]
\end{align*}
$$

where, the calculation formulas of the process noise estimator are

$$
\begin{align*}
\hat{\omega}_{k|k} &= K_k^\omega (z_k - C_k \hat{x}_{k|k-1}) \\
K_k^\omega &= S_k [C_k P_{k|k-1} C_k^T + R_k + C_k S_k^\gamma \\
&\quad+ (S_k^\gamma)^T C_k^\gamma [-1] \\
P_{k|k}^\omega &= Q_k - K_k^\omega S_k^\gamma \\
P_{k|k}^\gamma &= -K_k S_k^\gamma \\
P_{k|k}^\omega &= -S_k K_k^\gamma
\end{align*}
$$

2) $y_k = 0$: The measurement $z_k$ is unavailable for the estimator, yet the estimator knows that $||\hat{e}_k||_\infty \leq \theta$.

Given $I_{k-1}$, $e_k$ is a zero-mean standard Gaussian r.v., thus we define $p(\theta) = P(\{||\hat{e}_k||_\infty \leq \theta|I_{k-1}\})$. From the conditional probability density function

$$
f_{\hat{e}_k}(\hat{e}_k|I_{k-1}) = \begin{cases} 
\frac{f_{\hat{e}_k}(\hat{e}_k|I_{k-1})}{p(\theta)}, & \text{if } ||\hat{e}_k||_\infty \leq \theta \\
0, & \text{otherwise}
\end{cases}
$$

Based on conditional mathematical expectation

$$
E[X|Y \in D] = \frac{1}{P(Y \in D)} \int_{y \in D} E[X|Y = y] f_Y(y) dy
$$

we have

$$
\hat{x}_{k|k} = E[x_k|\hat{I}_k] = \frac{1}{p(\theta)} \int \Omega E[x_k|I_{k-1}, ||\hat{e}_k||_\infty \leq \theta] e_k d\hat{e}_k = \frac{1}{p(\theta)} \int \Omega \hat{x}_{k|k-1} + K_k (H_k^T)^{-1} \hat{e} \cdot f_{\hat{e}_k}(\hat{e}_k|I_{k-1}) d\hat{e}_k
$$

$$
\hat{x}_{k|k-1} = \hat{x}_{k|k-1} - K_k (H_k^T)^{-1} \hat{e} f_{\hat{e}_k}(\hat{e}_k|I_{k-1}) d\hat{e}_k = 0
$$

Denote

$$
\tilde{x}_{k|k-1} = x_k - \hat{x}_{k|k-1}
$$

then, according to Lemma 2, one can see that

$$
E[\tilde{x}_{k|k-1} e_k^T|\hat{I}_k] = \frac{1}{p(\theta)} \int \Omega E[\tilde{x}_{k|k-1}|I_{k-1}, ||\hat{e}_k||_\infty \leq \theta] e_k d\hat{e}_k
$$

$$
\hat{e}_k = (H_k^T)^{-1} \hat{e} = \hat{e}^T H_k^{-1} f_{\hat{e}_k}(\hat{e}_k|I_{k-1}) d\hat{e}_k = \frac{1}{p(\theta)} \int \Omega E[\tilde{x}_{k|k-1} e_k^T|I_{k-1}, ||\hat{e}_k||_\infty \leq \theta] e_k d\hat{e}_k
$$

$$
\hat{e}_k = (H_k^T)^{-1} \hat{e} - \tilde{x}_{k|k-1} = \tilde{x}_{k|k-1} e_k^T H_k^{-1} f_{\hat{e}_k}(\hat{e}_k|I_{k-1}) d\hat{e}_k
$$
then, from (33), we obtain

\[
K_k = (P_{k|k-1}C_k + S_k^a)(C_kP_{k|k-1}C_k^T + R_k + C_kS_k^a) + (S_k^a)^T C_k^T (34)
\]

then, from (33), we obtain

\[
E[\tilde{x}_{k|k-1} - K_k e_k]e_k^T | \tilde{I}_k = 0
\]

and

\[
E[\tilde{x}_{k|k-1} - K_k e_k](\tilde{x}_{k|k-1} - K_k e_k)^T | \tilde{I}_k
\]

where the estimator gain is

\[
K_k = (P_{k|k-1}C_k + S_k^a)\{C_kP_{k|k-1}C_k^T + R_k + C_kS_k^a + (S_k^a)^T C_k^T\}^{-1}
\]

For further, the estimator of process noise can be derived from the projection theory [28]

\[
\hat{o}_{k|k} = E[\omega_k | \hat{I}_k]
\]

the noise estimation error is obtained that

\[
P_{k|k} = E[\hat{o}_{k|k}^T \hat{o}_{k|k} | \hat{I}_k] = E[o_k^2]
\]

and the covariance matrix between the process noise and state can be derived as

\[
P_{\tilde{o}_{k|k}} = E[\tilde{o}_{k|k}^T | \tilde{I}_k] = E[\tilde{x}_{k|k-1}1_k | \tilde{I}_k]
\]

where, the noise estimator gain can be computed as

\[
K_k^o = S_k[ C_k P_{k|k-1}C_k^T + C_k S_k^a + (S_k^a)^T C_k^T + R_k]^{-1}
\]

the corresponding covariance matrix is calculated as:

\[
P_{\hat{o}_{k|k}} = E[\hat{o}_{k|k}^T | \hat{I}_k] = E[o_k^2]
\]

and the covariance matrix between the process noise and state under reliable measurement conditions

\[
\hat{x}_{k|k} = \tilde{x}_{k|k-1} + \gamma_k K_k(z_k - C_k \tilde{x}_{k|k-1})
\]

where, the value of \( \beta(\theta) \) is defined in Lemma 1, and the estimator of process noise can be derived as

\[
\hat{o}_{k|k} = -\gamma_k S_k^o k_{k|k-1}
\]

where, the value of \( \beta(\theta) \) is defined in Lemma 1, and the estimator of process noise can be derived as
Step 2: At last, we prove the event-triggered state estimation under unreliable measurement conditions. Note that unreliable measurements are available only during the measurement transmission \((\gamma_k = 1)\), then, motivated by [27], the event-triggered estimator for wireless sensor network systems subjected to packet losses and correlated noises can be derived as

\[
\begin{aligned}
\dot{x}_k|_{k-1} & = x_k|_{k-1} + y_k\lambda_k K_k (z_k - C_k \hat{x}_k|_{k-1}) \\
P_k|_{k-1} & = P_k|_{k-1} - [y_k\lambda_k + (1 - y_k)\beta(\theta)] K_k P_{ek} K^T_k \\
\dot{\hat{x}}_k|_{k-1} & = A_k \hat{x}_k|_{k-1} - \hat{\omega}_k|_{k-1} + \hat{\omega}_k|_{k-1} \\
P_k|_{k-1} & = A_k \dot{\hat{x}}_k|_{k-1} - A_k \hat{P}_{\hat{x}}|_{k-1} - A_k \hat{P}_{\hat{x}}|_{k-1} + \hat{\omega}_k|_{k-1} + \hat{\omega}_k|_{k-1} \\
K_k & = (P_k|_{k-1} - \hat{P}_{\hat{x}}|_{k-1} - \hat{\omega}_k|_{k-1} - A_k \hat{P}_{\hat{x}}|_{k-1} + \hat{\omega}_k|_{k-1}) \cdot (\lambda_k R_k + (1 - \lambda_k)\sigma^2 I + C_k S_k + (S_k^T)^T C_k)^{-1}
\end{aligned}
\]

where, the process noise estimator can be derived as

\[
\begin{aligned}
\dot{\hat{\omega}}_k|_{k-1} & = y_k\lambda_k K^o_\omega (z_k - C_k \hat{x}_k|_{k-1}) \\
K^o_\omega & = S_k [C_k P_{\hat{x}}|_{k-1} C_k^T + \lambda_k R_k + (1 - \lambda_k)\sigma^2 I \\
& + C_k S_k + (S_k^T)^T C_k]^{-1} \\
P^o_\omega & = Q_k - y_k\lambda_k K^o_\omega S_k \\
P^o_{\hat{\omega}} & = -y_k\lambda_k S_k K^T_k \\
P^o_{\hat{\omega}} & = -y_k\lambda_k S_k K^T_k
\end{aligned}
\]

and

\[
\begin{aligned}
\beta(\theta) & = \frac{2}{\sqrt{2\pi}} \theta e^{-\frac{\theta^2}{2}} [1 - 2Q(\theta)]^{-1} \\
Q(\theta) & = \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx
\end{aligned}
\]

This completes the proof. \(\square\)

**IV. BOUNDEDNESS OF EVENT-TRIGGERED STATE ESTIMATION ALGORITHM**

In this section, we analyze the boundedness of error covariance of the algorithm designed in this article.

**Lemma 4 [29]:** If there exists a symmetric positive definite matrix \(Z \in R^{m \times n}\), for two matrices \(N \in R^{n \times n}\) and \(M \in R^{m \times n}\), then

\[
-M^T Z^{-1} M \leq N^T Z N - M^T N - N^T M
\]

if and only if \(N = Z^{-1} M\), the equation holds.

**Lemma 5 [30]:** The following inequality exists for an arbitrary square matrix \(P\)

\[
\rho(P) \leq ||P||
\]

where, \(\rho(\cdot)\) denotes the spectral density of corresponding matrix.

**Theorem 2:** Suppose that the linearization of the discrete time-varying systems (1) and (2) meets the uniform observability condition, and \(C_k\) is nonsingular. There are real constants \(\bar{a}, \bar{c}, \bar{r}, \bar{\beta}, \bar{\gamma}, \bar{s}, \bar{s}\), namely

\[
||A_k|| \leq \bar{a}, \ ||C_k|| \leq \bar{c}, \ Q_k \leq \bar{q} I,
\]

\[
R_k \leq \bar{r} I, \ \theta \leq \bar{\theta}, \ S_k \leq \bar{s} I
\]

among them, \(|| \cdot ||\) represents the spectral norm of corresponding matrix or the Euclidean norm of the corresponding vector.

If

\[
\bar{\lambda} < 1 - \frac{\bar{a}^2}{2} - (1 - \bar{\gamma})\beta(\bar{\theta})[\bar{\gamma}(\bar{c})^2 \bar{c}^2 + \bar{a}^2 \bar{c} + (\bar{c})^2 \bar{c}]
\]

then

\[
E[tr(P_{k+1}|k+1)] \leq E[tr(P_{k+1}|k)] < \bar{p}
\]

where, \(tr(\cdot)\) represents the trace of a matrix

**Proof:** From the second equation of (22), it can be seen that

\[
E[\{P_{k+1}|k+1\}] \leq E[\{P_{k+1}|k\}]
\]

Therefore, it is only need to prove that \(P_{k+1}|k\) is bounded.

Based on the fourth equation of (22), we have

\[
P_{k+1}|k = A_k P_k|k A_k^T + A_k \hat{P}_{\hat{x}}|k + \hat{P}_{\hat{x}}|k A_k^T + P_{\hat{\omega}}|k
\]

\[
= A_k P_k|k - 1 A_k^T - [y_k\lambda_k + (1 - y_k)\beta(\theta)] \cdot A_k K_k P_{ek} A_k^T + A_k K_k S_k + (S_k^T)^T C_k^T
\]

\[
- y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + \gamma_k R_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T
\]

\[
- y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]

\[
\leq A_k P_k|k - 1 A_k^T - \gamma_k R_k + (1 - \gamma_k)\beta(\theta) A_k K_k - y_k\lambda_k S_k K^T_k + A_k K_k S_k^T + (S_k^T)^T C_k^T + \gamma_k R_k - y_k\lambda_k S_k K^T_k
\]
Calculating the Euclidean norm on both sides of the inequality, we have

$$\|P_{k+1|k}\| \\
< \|A_k P_{k+1|k-1} A_k^T\| + \|2[y_k \lambda_k + (1 - y_k) \beta(\theta)]\| \\
\cdot \{\|S_k^e\|^2 C_k P_{k+1|k-1} C_k^T S_k + (S_k^e)^T C_k^T S_k^e + (S_k^e)^T R_k S_k\| \\
+ |y_k \lambda_k + (1 - y_k) \beta(\theta)||A_k (P_{k+1|k-1} C_k^T + S_k) A_k^T| \\
+ |y_k \lambda_k + (1 - y_k) \beta(\theta)||A_k (P_{k+1|k-1} C_k^T + S_k)| \}
\cdot S_k^e + \| \bar{Q}_k \| + \| y_k \lambda_k \| |S_k S_k^e| + \| y_k \lambda_k \| (S_k^e)^T (S_k^e)^T |}
\| (60)$$

Note that $\gamma_k$ and $\lambda_k$ are independent of $P_{k|k-1}$, from Eq.(54), we obtain

$$E(\|P_{k+1|k}\|) \\
< \tilde{\gamma}^2 \tilde{\lambda}^2 (\tilde{\sigma}_k^2)^2 + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} \\
+ (\tilde{\sigma}_k^2)^2 \cdot E(\|P_{1|0}\|) + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 \\
+ (\tilde{\sigma}_k^2)^2 + 2(\tilde{\sigma}_k^2)^2 \cdot I_n + \tilde{\gamma} \tilde{\lambda} (2(\tilde{\sigma}_k^2)^2 + (\tilde{\sigma}_k^2)^2 I_n) \\
\cdot (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} + (\tilde{\sigma}_k^2)^2 \cdot I_n + \tilde{q}_n I_n \quad (61)$$

where $\tilde{\gamma} = E[\gamma_k]$, $\tilde{\lambda} = E[\lambda_k]$. Mathematical induction is employed to prove that $E(\|P_{k+1|k}\|)$ is bounded. Assume $E(\|P_{1|0}\|) > 0$, therefore, we obtain

$$E(\|P_{1|0}\|) < \tilde{\gamma}^2 \tilde{\lambda}^2 (\tilde{\sigma}_k^2)^2 + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} \\
+ (\tilde{\sigma}_k^2)^2 \cdot E(\|P_{1|0}\|) + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 \\
+ (\tilde{\sigma}_k^2)^2 + 2(\tilde{\sigma}_k^2)^2 \cdot I_n + \tilde{\gamma} \tilde{\lambda} (2(\tilde{\sigma}_k^2)^2 + (\tilde{\sigma}_k^2)^2 I_n) \\
\cdot (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} + (\tilde{\sigma}_k^2)^2 \cdot I_n + \tilde{q}_n I_n \quad (62)$$

where $\tilde{\beta} = \max\{E(\|P_{1|0}\|), \tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + (\tilde{\sigma}_k^2)^2 + 2(\tilde{\sigma}_k^2)^2 \cdot I_n + \tilde{\gamma} \tilde{\lambda} (2(\tilde{\sigma}_k^2)^2 + (\tilde{\sigma}_k^2)^2 I_n) + \tilde{q}_n I_n$. Next, assume that

$$E(\|P_{k+1|k-1}\|) < \tilde{\beta} \sum_{i=0}^{k-1} \{\tilde{\gamma}^2 \tilde{\lambda}^2 (\tilde{\sigma}_k^2)^2 + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} \\
+ (\tilde{\sigma}_k^2)^2 \cdot I_n \} I_n \quad (63)$$

Then, we obtain

$$E(\|P_{k+1|k}\|) \quad (60)$$

$$\quad < \{\tilde{\gamma}^2 \tilde{\lambda}^2 (\tilde{\sigma}_k^2)^2 + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} \\
+ (\tilde{\sigma}_k^2)^2 \} E(\|P_{k+1|k-1}\|) + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 \\
+ (\tilde{\sigma}_k^2)^2 + 2(\tilde{\sigma}_k^2)^2 \cdot I_n + \tilde{\gamma} \tilde{\lambda} (2(\tilde{\sigma}_k^2)^2 + (\tilde{\sigma}_k^2)^2 I_n) \\
\cdot (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} + (\tilde{\sigma}_k^2)^2 \cdot I_n + \tilde{q}_n I_n \} I_n \quad (64)$$

Setting $\tilde{\rho} \sum_{i=0}^{k} \{\tilde{\gamma}^2 \tilde{\lambda}^2 (\tilde{\sigma}_k^2)^2 + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} + (\tilde{\sigma}_k^2)^2 \} I_n = \tilde{p} I_n$. When

$$\tilde{\rho} \leq \frac{\tilde{\beta} \sum_{i=0}^{k} \{\tilde{\gamma}^2 \tilde{\lambda}^2 (\tilde{\sigma}_k^2)^2 + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} + (\tilde{\sigma}_k^2)^2 \} I_n}{\tilde{\gamma}^2 \tilde{\lambda}^2 (\tilde{\sigma}_k^2)^2 + \{\tilde{\gamma} \tilde{\lambda} + (1 - \tilde{\gamma}) \beta(\theta)\} (\tilde{\sigma}_k^2)^2 + \tilde{\alpha}^2 \tilde{c} + (\tilde{\sigma}_k^2)^2}$$

the designed filter converges is concluded. According to Lemma 5, we have

$$E(\|P_{k+1|k}\|) \leq n E(\rho(P_{k+1|k})) < n E(\|P_{k+1|k}\|) \quad (66)$$

where $n$ is state dimension. Then

$$E(\|P_{k+1|k+1}\|) = E(\|P_{k+1|k}\|) < n \tilde{p}$$

The proof is completed. □

V. SIMULATION

In this section, an example is introduced to illustrate the effectiveness of the proposed algorithm. Consider the following target tracking system

$$x_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + \Gamma_k \xi_k \quad (68)$$

$$z_k = C x_k + v_k, k = 1, 2, \cdots, L \quad (69)$$

$$v_k = \eta_k + \beta_1 \xi_k - 1 + \beta_2 \xi_k \quad (70)$$

The state vector $x_k = [s_k \; \xi_k]^T$, where $s_k$ is the position of the target and $\xi_k$ is the velocity of the target at time $kT$, where $T = 0.1$ is the sampling period. Suppose the covariance of zero-mean Gaussian white noise $\xi_k \in R$ is $\sigma^2_k$. $\Gamma_k = \{T\}$ is the noise transition matrix. $\xi_k$ is the observation of the sensor, where $C = [1 \; 0]$. $v_k$ is the observation noise, and is correlated with the process noises $\xi_{k-1}$ and $\xi_k$ at the previous moment and the current moment. $\beta_1$ and $\beta_2$ determine the correlation strength. Gaussian white noise $\eta_k$ of zero-mean and variances $\sigma^2_k$ is independent of $\xi_k$.

Based on (68), the covariance matrix of the process noise $\omega_k = \Gamma_k \xi_k$ is calculated as $Q_k = \Gamma_k T \sigma^2_k$. According to (69) and (70), we can obtain the measurement noise covariance matrix

$$R_k = \beta_1^2 + \beta_2^2 \sigma^2_k + \sigma^2_k \quad (71)$$

and, $S_k = \beta_1 \sigma^2_k \Gamma_k$ is the covariance matrix between $\omega_{k-1}$ and $v_k$, and $S_k = \beta_2 \sigma^2_k \Gamma_k$ is the covariance matrix between $\omega_k$ and $v_k$. $\sigma^2_k$ is the variance of $\xi_k$. $\Gamma_k$ is the noise transition matrix.
\( \omega_k \) and \( v_k \). Denote \( \sigma^2 = 0.3, \sigma_\eta^2 = 36, \beta_1 = 6, \beta_2 = 6, \bar{x}_0 = [1 \ 1]^T, \bar{P}_0 = I_2 \). The unreliable ratio of sensor is 0.1.

The event-triggered threshold of \( \theta \) is changing within \( \theta \in \{0, 0.5, 0.8, 1\} \) to explain its impact on the estimation performance, where, \( \theta = 0 \) means the filter can receive the observation at each moment, which is time triggered. \( L = 300 \) represents the length of samples, and 1000 Monte Carlo simulations are performed to observe the simulation results of the proposed algorithm which are shown in the Fig. 1 - Fig. 6 and Table 1.

From Fig. 1(a) and Fig. 1(b), the real position and real velocity signal (blue solid line) and the position and velocity estimates obtained by using the proposed modified Kalman filter algorithm (KFO, pink dashed line) are shown. One can see that the KFO algorithm derive good estimation.

The comparison curves of Root Mean Square Errors (RMSEs) of designed KFO algorithm under different triggered thresholds (\( \theta = \{1, 0.8, 0.5, 0\} \)) are shown in Fig. 2. From Fig. 2, it can be seen that the smaller the triggering threshold, the lower RMSE curves of the KFO algorithm, while the estimation performance is the best when \( \theta = 0 \). The average sensor communication rate of the sensor obtained through the 3 under different thresholds \( \theta \) is shown in Fig. 3. It can be seen that the larger the trigger threshold, the higher the requirement for event-triggered, and the lower the transmission rate \( \gamma \). The smaller the triggering threshold, the higher the data transmission rate, and therefore the better estimation performance. When the triggered-threshold is equal to \( \theta \), it means that all the measurements will be sent to the estimator, the system degenerates into the time-triggered. So, the data transmission rate is the highest and the estimation performance is the best. The time-averaged RMSEs of the KFO algorithm is shown in the first column of the table 1.

![FIGURE 1. The signal and the state estimates. (a) Position. (b) Velocity.](image)

We can draw the same conclusion as Fig. 2.

![FIGURE 2. RMSE curves of proposed KFO algorithm with different thresholds. (a) Position. (b) Velocity.](image)

![FIGURE 3. Scheduling parameter \( \theta \) versus sensor communication rate \( \gamma \).](image)

![FIGURE 4. RMSE curves of KFN and KFO with \( \theta = 0.5 \). (a) Position. (b) Velocity.](image)

![FIGURE 5. RMSE curves of KFD and KFO with \( \theta = 0.5 \). (a) Position. (b) Velocity.](image)

Considering the affect of data dropouts and noise correlation, the RMSEs (position and velocity) of different algorithms in this article are shown in Figs. 4–5, where blue dashed lines represent RMSEs obtained by KFO algorithm with consideration of noise correlation and unreliable measurements, which is proposed in Theorem 1, pink solid lines represent RMSEs obtained by KFN1 algorithm, green
dash-dotted lines represent RMSEs obtained by KFN2 algorithm, and red dash-dotted lines represent RMSEs obtained by KFD algorithm. The KFN1 algorithm means suboptimal KF algorithm that considers packet loss but does not consider the correlation between observation noises $v_k$ and the process noises of the previous moment and the same moment ($\omega_{k-1}$ and $\omega_k$). The KFN2 algorithm means suboptimal KF algorithm that considers packet loss and the correlation between observation noises $v_k$ and the process noises of the previous moment and the same moment ($\omega_{k-1}$ and $\omega_k$), but neglects data dropouts. These show that RMSEs of KFO algorithm are the smallest. It can be seen that state estimation algorithm in consideration of data dropouts and noise correlation is effective while neglecting data dropouts or noise correlation will reduce estimation precision. 

The upper bound of the estimation error covariances of the KFO algorithm with $\theta = 0.5$ is shown in Fig. 6, which verify the conclusion of Theorem 2.

VI. CONCLUSION

In this work, an event-triggered state estimator for wireless sensor network systems affected by random packet losses and correlated noises simultaneously has been proposed. By using the introduced event-triggered scheduling mechanism and process noise estimator, the designed state estimation algorithm can maintain satisfactory estimation performance even in the case of noise correlation interference, packet losses and less measurement transmission. Sufficient conditions related to data packet loss rate, noise correlation coefficient and triggering threshold have been given to ensure the boundedness of the designed state estimation algorithm. Finally, simulation result is illustrated to demonstrate the effectiveness of the proposed algorithms. An interesting future research direction consists of considering the stochastic trigger mechanism and extend the proposed algorithm to the non-linear system or multi-sensor systems.

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