Automated calculation of matrix elements and physics motivated observables

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Abstract. The central aspect of my personal scientific activity, has focused on calculations useful for interpretation of High Energy accelerator experimental results, especially in a domain of precision tests of the Standard Model. My activities started in early 80’s, when computer support for algebraic manipulations was in its infancy. But already then it was important for my work. It brought a multitude of benefits, but at the price of some inconvenience for physics intuition. Calculations became more complex, work had to be distributed over teams of researchers and due to automatization, some aspects of the intermediate results became more difficult to identify.

In my talk I will not be very exhaustive, I will present examples from my personal research only: (i) calculations of spin effects for the process $e^+e^-\rightarrow \tau^+\tau^-\gamma$ at Petra/PEP energies, calculations (with the help of the Grace system of Minami-tateya group) and phenomenology of spin amplitudes for (ii) $e^+e^-\rightarrow 4f$ and for (iii) $e^+e^-\rightarrow \nu_e\bar{\nu}_e\gamma\gamma$ processes, (iv) phenomenology of CP-sensitive observables for Higgs boson parity in $H\rightarrow \tau^+\tau^-$, $\tau^\pm \rightarrow \nu 2(3)\pi$ cascade decays.

1. Introduction

Once computers equipped with algebraic languages became available, approaches to phenomenology of High Energy accelerator experiments changed substantially. For me personally, it all started in 1980. It was clear that numerous benefits appeared. Control of large expression became easier, in many cases it simply became possible for the first time. Nearly immediately some drawbacks appeared as well. For example, some seemingly obsolete expertise, like methods for special function expansions, started to disappear. At least for some years and for some communities.

This was all part of a complex and generally very fruitful development. In this presentation I will concentrate on my personal experience. I do not have any intentions to be systematic or balanced. A more balanced picture will hopefully appear together with other talks collected in the proceedings. That is also why, I think, I do not need to focus on successes of the field. These are well known. I will rather review difficulties or traps I have encountered myself. In fact traps, once resolved, turned out to be rewarding, often in an unexpected way.

The presentation is organized as follows. Section 2 will discuss computer algebra techniques which were applied in our work for Monte Carlo program KORALB [1] for the $e^+e^-\rightarrow \tau^+\tau^-\gamma$ process at Petra/PEP energies. The following Section 3, is devoted to work on spin amplitudes for KKMC Monte Carlo [2]. In particular to studies of spin amplitude sub-structure for the $e^+e^-\rightarrow \nu_e\bar{\nu}_e\gamma\gamma$ process, necessary to accommodate resulting expressions to Yennie Frautschi Suura exponentiation [3]; variant of exclusive exponentiation of initial and final state radiation.
The t-channel (even though non-singular) $W$ exchange required careful attention. The following Section 4 is devoted to phenomenology of four-fermion state production at high energy and the study of unexpected formation of peaks due to conspiracy between spin and selection cut effects [5]. Finally, in Section 5, we turn our attention to methods of Machine Learning in applications for evaluation of (massively multi-dimensional) observables for Higgs parity [6]. Section 6 closes the paper with a Summary.

2. Year 1981: KORALB Monte Carlo for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ at Petra/PEP energies

At the time, Poland seemed to be an isolated place, but with enormous in-flow of young talents to research. In reality a lot of contacts existed, but it was not to be seen by me. Limited, and in fact quite awkward, access to computing existed. It looked like a hopeless loss of time, but a lot of bright minds were attracted to the computing center of Jagellonian University. I had access to algebraic manipulation language: Shoonschip [7] too.

One of my first projects was to evaluate the spin density matrix for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ at Petra/PEP energies [1, 8] This work was performed under the guidance of Prof. S. Jadach. This was quite an experience in looking at spin amplitudes as (reducible) representations of (Lorentz×gauge) groups. It was a great opportunity, programs were queuing for execution time. We could then concentrate and understand the details of what we were actually doing. Later, in all my work, it came as an enormous benefit: how to represent complicated formulas (moderately complicated for today standards) of spin amplitudes in a compact form. We have used a tree of reference frames to obtain the goal, see Fig. 1. Instead of spin projections of individual fermions, we used differences (or sums) of such projections for incoming and/or outgoing leptons. In this way we could visualize spin properties of intermediate (formally virtual) photons. Simplifications of lowest order amplitudes, remained for bremsstrahlung as well. Nearly all calculations were done by hand, but every step was cross checked with the help of algebraic calculation with Shoonschip. This was an enormous help. We could also observe how some features of amplitudes could be identified if calculations were done by hand, but how easily these features were overlooked if we relied too much on automatization. Only if we knew what we were looking for, could we confirm the patterns with the algebraic tool. It is important to keep this in mind. Similar problems are encountered in modern times with applications of Machine Learning as well. I will return to this point in Section 5.

3. The t-channel contribution to spin amplitudes of s-channel Exclusive Exponentiation and double gluon emission in QCD.

There is a multitude of factorization schemes available for Field Theory calculations. Over years, I was checking, on particular examples, if hints for that could be identified already at the spin amplitude level. The general principle of the searches was rather simple. One tries to identify in amplitudes, gauge invariant sub-structures (parts) responsible for Matrix Element enhancements: in some regions of the phase-space (collinear-soft etc.). This is fundamental, especially from the point of view of Monte Carlo algorithm construction. Discussions with Shimizu-sensei were important for some stages of that work. The principle was to start from the complete expression, coded in a numerical program, to identify the most singular term and then group some other terms necessary to obtain gauge invariant part of spin amplitude. Then, for the remaining part, the search was repeated, until all interesting parts were identified. Starting point for that work had to be amplitudes guaranteed to be correct. In the two cases $e^+e^- \rightarrow \nu_\ell \bar{\nu}_\ell \gamma\gamma$ [9] and $q\bar{q} \rightarrow l^+l^-gg$ [10] expressions obtained with the help of algebraic programs were used for that purpose.

Even though in principle there were no general rules to follow, separation of amplitudes into gauge invariant parts was a straightforward and seemingly unique procedure. However I was not able to perform the task automatically. In fact, only some of the patterns appeared
Figure 1. Tree of reference frames used to define amplitudes and Monte Carlo algorithm for $e^+e^- \rightarrow \tau^+\tau^-$ (panel a) and $e^+e^- \rightarrow \tau^+\tau^-\gamma$ (panel b) processes used and defined with all necessary details in refs. [1, 8]. Let us recall that CMS mean laboratory frame, QMS rest frame of outgoing lepton pair, RS rest frame of the indicated lepton, $B_3$ boost along third axis with parameter $\eta$ and $R_i$ rotation around $i$-axis by the given angle.
as a consequence of ordering singular terms. Feynman diagrams 1 and 2 of Fig. 2, combined, complete the amplitude for $e^+e^- \rightarrow \nu_e \bar{\nu}_e \gamma$ production, that is why they form gauge invariant part of amplitude for $e^+e^- \rightarrow \nu_e \bar{\nu}_e \gamma$ consisting of all diagrams of Fig. 2.

![Feynman diagrams](image)

**Figure 2.** The Feynman diagrams for $e^+e^- \rightarrow \nu_e \bar{\nu}_e \gamma$.

Further, because these two diagrams represent initial state QED bremsstrahlung amplitude, they must divide into parts, corresponding to $\beta_0, \beta_1$ of Yennie-Frautshi-Suura exponentiation [3]. This has long been known, but the question was, whether it can be expanded to other cases, to higher orders or to terms of different singularities/enhancements than in case of QED photon emissions. Unexpectedly, from my experience the answer seemed to be always “yes”. I could observe it not only in QED and QCD cases but also for example, in scalar QED [11]. I was encouraged by Shimizu-sensei to follow this path, also in case of complete electroweak effects, and for loop contributions, but so far I have not found the solution.

Instead, let me present first, single photon emission amplitude for $e^+e^- \rightarrow \nu_e \bar{\nu}_e \gamma$ (formula 1) and later for double gluon emission in $q\bar{q} \rightarrow l^+l^-$ (formula 2). For notation conventions references [9, 10] are used.

\[
\mathcal{M}_{1}^{(I)} \left( p_{k_1}^{l}, \sigma_1 \right) = \mathcal{M}^0 + \mathcal{M}^1 + \mathcal{M}^2 + \mathcal{M}^3,
\]

\[
\mathcal{M}^0 = eQ_e \bar{v}(p_b, \lambda_b) M_{bd}^{\text{nd}}(I) \left( \frac{\not{p}_{a} + m - \not{k}_1}{-2k_1 p_a} \right) e^*_{\sigma_1}(k_1) \cdot u(p_a, \lambda_a)
\]

\[
+ eQ_e \bar{v}(p_b, \lambda_b) \frac{-\not{p}_{b} + m + \not{k}_1}{-2k_1 p_b} M^{ac}_{(I)} \cdot u(p_a, \lambda_a),
\]

\[
\mathcal{M}^1 = \mathcal{M}^1 + \mathcal{M}^{1''},
\]

\[
\mathcal{M}^{1'} = e \bar{v}(p_b, \lambda_b) M^{bd,ac}_{(I)} u(p_a, \lambda_a) e^*_{\sigma_1}(k_1) \cdot (p_c - p_a) \frac{1}{t_a - M_W^2} \frac{1}{t_b - M_W^2},
\]

\[
\mathcal{M}^{1''} = e \bar{v}(p_b, \lambda_b) M^{bd,ac}_{(I)} u(p_a, \lambda_a) e^*_{\sigma_1}(k_1) \cdot (p_c - p_d) \frac{1}{t_a - M_W^2} \frac{1}{t_b - M_W^2}.
\]
\( \mathcal{M}^2 = +e \bar{v}(p_b, \lambda_b)g_{k_3 \lambda_3}W_{e^+}^{\nu}(k_1) \, v(p_d, \lambda_d)\bar{u}(p_c, \lambda_c)g_{k_4 \lambda_4}W_{e^+}^{\nu} \, f_{\sigma_1}(k_1) \, u(p_a, \lambda_a) \frac{1}{t_a - M_W^2} \frac{1}{t_b - M_W^2}, \)

\( \mathcal{M}^3 = -e \bar{v}(p_b, \lambda_b)g_{k_3 \lambda_3}W_{e^+}^{\nu} \, f_{\sigma_1}(k_1) \, v(p_d, \lambda_d)\bar{u}(p_c, \lambda_c)g_{k_4 \lambda_4}W_{e^+}^{\nu} \, u(p_a, \lambda_a) \frac{1}{t_a - M_W^2} \frac{1}{t_b - M_W^2}. \)

Once manipulations are completed, we separate the spin amplitude into six individually QED gauge invariant parts (\( \mathcal{M}^0 \) can be separated into 2 or 3 sub-parts). This is rather easy to check, replacing photon polarization vector with its four-momentum. Each of the obtained parts has a well defined physical interpretation. It is also possible to verify that the gauge invariance of each part can be preserved in the case of the extrapolation, necessary in case of QED exclusive exponentiation (see Refs. [2, 4]). Then, because of additional photons, the condition \( p_a + p_b = p_c + p_d + k_1 \) is not valid.

Let us now turn to double gluon emission amplitude, that is for the process \( q\bar{q} \to l^+l^- gg \):

\[
\mathcal{M}^{a,b} = \frac{1}{2} \bar{v}(p) \left( T^a T^b I^{(1,2)} + T^b T^a I^{(2,1)} \right) u(q). \tag{2}
\]

For the \( T^a T^b \)-part, we find

\[
I^{(1,2)} = \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{k_1 \cdot k_2}{2p \cdot k_1} \right) J \left( \frac{k_2 \cdot k_2}{2q \cdot k_2} + \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{q \cdot e_2}{q \cdot k_2} \right) + \left( \frac{p \cdot k_2}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} \right) \left( \frac{p \cdot e_1}{p \cdot k_1} - \frac{k_2 \cdot e_1}{k_2 \cdot k_1} - \frac{k_1 \cdot k_2}{2p \cdot k_1} \right) \left( \frac{p \cdot e_2}{p \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{k_2 \cdot e_2}{k_2 \cdot k_1} \right) J \left( \frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} - \frac{k_2 \cdot e_2}{k_2 \cdot k_1} \right) J + \left( \frac{1}{p \cdot k_1 + p \cdot k_2 - k_1 \cdot k_2} \right) \left( \frac{k_1 \cdot k_2}{q \cdot k_1} - \frac{k_1 \cdot k_2}{q \cdot k_2} - \frac{k_1 \cdot k_2}{q \cdot k_2} \right) \left( \frac{k_1 \cdot k_2}{k_1 \cdot k_2} - \frac{k_1 \cdot k_2}{k_1 \cdot k_2} \right) + \left( \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \right) \left( \frac{k_1 \cdot k_2}{k_1 \cdot k_2} - \frac{k_1 \cdot k_2}{k_1 \cdot k_2} \right) \left( \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \right) J + \left( \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \right) \left( \frac{1}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \right) J \left( \frac{k_1 \cdot k_2}{k_1 \cdot k_2} - \frac{k_1 \cdot k_2}{k_1 \cdot k_2} \right) J. \tag{3}
\]

The part proportional to other order of SU(3) group generators \( T^b T^a \), is obtained by a permutation of the momenta and polarization vectors of the gluons. Each line of the above expression is individually gauge invariant. Also, this expression for double gluon emission is rather compact too.

4. The \( e^+e^- \to 4f \) process

The main purpose of my visit to KEK MinamiTateya group in 1996, was to work on Grace spin amplitudes [5] and to prepare them for use in our KORALW Monte Carlo [12] for the \( e^+e^- \to 4f \) fermion processes at LEP II energies. In this work, because of Monte Carlo integration, phase space regions of collinear configurations, resulted in numerical difficulties. This required careful and painful work to avoid ‘trivial’ mistakes due to rounding errors damaging gauge cancellations. On the margins of this work, kinds of fake ‘New Physics’ phenomena appeared. Let me show, for myself at first rather unexpected example, see Ref. [13]. It illustrates that the interplay of theoretical effects and selection cuts can be confusing. In \( e^+e^- \to WW \to q\bar{q}qq \) two jets were requested to be lost in the beam pipe the invariant mass of the other two was monitored. If all diagrams of the lowest order Standard Model amplitudes were taken into account, a clear Z peak is present (Fig. 3 left side). But there is another peak at a much higher mass, which do not disappear, even if only \( W \)-pair production and decay amplitudes are taken
The differential distribution of the "visible" $s\bar{s}$ jets where $c\bar{c}$ jets escape detection. The centre-of-mass energy is 195 GeV. Input parameters of type 2: CC-03 (thick line); and type 4: CC-43 (thin line).

The differential distribution of the "visible" $s\bar{s}$ jets where $c\bar{c}$ jets escape detection. The centre-of-mass energy is 195 GeV. Input parameters of type 2: CC-03 no spin correlation (thin line); and type 2: CC-43 spin correlations switched on (thick line).

Figure 3. Figure taken from Ref. [13]

into account (see Fig. 3 right side). It is a consequence of the veto cut, $W^+$ and $W^-$ peaks, and spin correlations as explained in Ref. [13].

Such phenomena need to be included in generators used for simulation of backgrounds. Otherwise intuition may fail to recognize importance for background estimation effects such as generally small spin correlations. Cross-check with Grace spin amplitudes was helpful to confirm the origin of the phenomenon.

5. Machine Learning for Higgs parity measurement in $H \rightarrow \tau\tau$ decay

Let us now turn to another example, where complex observables need to be defined and computer techniques of the Neural Network or Machine Learning (ML) type [14, 15, 16, 17, 18] are useful or even irreplaceable. Fifteen years ago, in Ref. [19], we have proposed to measure Higgs boson parity in its $H \rightarrow \tau\tau$ decay with the help of acoplanarity angle for planes built on visible decay products for consecutive $\tau^\pm \rightarrow \pi^\pm \pi^0 \nu_\tau$ decays. A variant of the observable where impact parameter was used, is presented in [20]. In attempt to extend the method to $\tau \rightarrow 3\pi \nu$ decays, one can construct 4 or 16 such angles for each event. Each of these distributions provide some CP sensitivity, but distributions are correlated and possible backgrounds will complicate future measurements even further. Such observables may be controlled today with ML techniques. All correlations between angles and $y_i$ variables (defined later) can be automatically identified and explored in an optimal way. The task nearly impossible to perform by hand. An attempt in this direction was presented in Ref. [6]. In the following, let us recall some details of this work.

The $H$ or $A$ (scalar or pseudo-scalar), thanks to the sign difference in the formula below, can be extracted from the correlations between $\tau^+$ and $\tau^-$ transverse spin components. The decay probability

$$\Gamma(H/A \rightarrow \tau^+ \tau^-) \sim 1 - s^+ s^- + s^+ s^-$$

is sensitive to the $\tau^\pm$ polarization vectors $s^\pm$ and $s^\mp$ (defined in their respective rest frames). The symbols $\parallel, \perp$ denote components parallel/transverse to the Higgs boson momentum as seen from the respective $\tau^\pm$ rest frames.
Because of the narrow $\tau$ width, cross-section for the process $f \bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to X^-\nu$ reads:

$$d\sigma = \sum_{\text{spin}} |M|^2 d\Omega = \sum_{\text{spin}} |M|^2 d\Omega_{\text{prod}} d\Omega_{\tau^+} d\Omega_{\tau^-}. $$

With only $\tau$ spin indices explicit $M$ reads:

$$M = \sum_{\lambda_1\lambda_2=1}^2 M^\text{prod}_{\lambda_1\lambda_2} \cdot M_{\lambda_1}^{\tau^+} M_{\lambda_2}^{\tau^-}. $$

The $d\sigma$ can be re-written into core formula of spin algorithms of $\tau$-pair production and decay:

$$d\sigma = \left( \sum_{\text{spin}} |M^\text{prod}|^2 \right) \left( \sum_{\text{spin}} |M^{\tau^+}|^2 \right) \left( \sum_{\text{spin}} |M^{\tau^-}|^2 \right) wt d\Omega_{\text{prod}} d\Omega_{\tau^+} d\Omega_{\tau^-} $$

To complete explanations, we need to first recall, details of general formalism of semileptonic $\tau$-lepton decays. The Matrix Element used in TAUSTLA Monte Carlo [21] for semileptonic decay $\tau(P, s) \to \nu_{\tau}(N)X$ can be written as follows: $M = \frac{G_F}{\sqrt{2}} \bar{u}(N)\gamma^\mu(v + a\gamma_5)u(P)J_\mu$, the current $J_\mu$ depends on the momenta of all hadrons. Then

$$|M|^2 = G_F^2 \frac{v^2 + a^2}{2} (\omega + H_\mu s^\mu), $$

$$\omega = P^\mu (\Pi_\mu - \gamma_{\nu a} \Pi_\nu), $$

$$H_\mu = \frac{1}{M} (M^2 \delta^\mu_\nu - P^\mu P^\nu)(\Pi_\nu - \gamma_{\nu a} \Pi_\nu), $$

$$\Pi_\mu = 2[(J^* \cdot N)J_\mu + (J \cdot N)J^*_\mu - (J^* \cdot J)N_\mu], $$

$$\Pi^\nu_\mu = 2 \Im \epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho N_\sigma, $$

$$\gamma_{\nu a} = -\frac{2}{v^2 + a^2}, $$

$$\hat{\omega} = \frac{2v^2 - a^2}{v^2 + a^2} m_\nu M(J^* \cdot J), $$

$$\hat{H}_\mu = \frac{2v^2 - a^2}{v^2 + a^2} m_\nu \Im \epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho P_\sigma. $$

In the following we will use $h_i = H^i/H_0$.

The Higgs decay probability in the formalism of Refs. [22, 23] in case when both scalar and pseudo-scalar $H \tau\tau$ coupling are allowed $\bar{\tau}N(\cos \phi^{CP} + i \sin \phi^{CP}\gamma_5)\tau$, reads

$$\Gamma(h_{\text{mix}} \to \tau^+\tau^-) \sim 1 - s_{||}^{h_{||}} s_{||}^{h_{||}} + s_{\perp}^{h_{\perp}} R(2\phi^{CP}) s_{\perp}^{h_{\perp}}. $$

The $R(2\phi^{CP})$ — denotes the operator for the rotation by angle $2\phi^{CP}$ around the $||$ direction.

As a consequence, spin weight $wt$ for the combined production and decay of tau pair reads as:

$$wt = 1 - h_{||}^{h_{||}} h_{||}^{h_{||}} + h_{\perp}^{h_{\perp}} R(2\phi^{CP}) h_{\perp}^{h_{\perp}}. $$

Naturally, the Higgs parity should reflects itself in some kind of correlations between the $\tau^+$ and $\tau^-$ decay products in the directions transverse to the $\tau^+\tau^-$ axes.

Let us recall first the case when both $\tau$ leptons decay to $\pi^\pm\pi^0\nu_\tau$. Then $h^i = N\left[2(q \cdot N)q^i - q^2 N^i\right]$, where $q \cdot N = (E_{\pi^+} - E_{\pi^0})m_\tau$ and four momentum $q = p_{\pi^+} - p_{\pi^0}$ is build from
four momenta of $\pi$'s. The $N$ denote four momentum of neutrino again in the $\tau$ lepton rest-frame. Because $q \cdot N$ may be either positive or negative, corresponding regions of phase-space would contribute canceling out parity effects. To separate we may use $y_1$, $y_2$ variables. The $y_1 = \frac{E_{\pi^+} - E_{\nu}}{E_{\pi^+} + E_{\nu}}$; $y_2 = \frac{E_{\pi^-} - E_{\nu}}{E_{\pi^-} + E_{\nu}}$ can be calculated directly from $\pi$'s energies as measured in the laboratory frame. It is enough to take the sign of the $y_1y_2$ only. The sensitivity to parity can be seen in Fig. 4. Acoplanarity angle (see Fig. 5 for definition) was used. Already in this case, the observable is in principle of 3-dimensional nature.

![Figure 4](image1)

**Figure 4.** The $\rho^+\rho^-$ decay products' acoplanarity distribution in $\rho^+\rho^-$ pair rest-frame. Thick line denote the case of the scalar Higgs and thin line the pseudo-scalar.

![Figure 5](image2)

**Figure 5.** Acoplanarity angle $0 < \varphi^* < 2\pi$ between oriented half-planes (or $0 < \varphi^* < \pi$ between oriented planes) spanned respectively on $\rho^\pm - \pi^\pm$ decay products and in the rest frame of $\rho^+\rho^-$ pair.

In case of $\tau \to 3\pi\nu$ decay products, four distinct planes can be spanned on its visible decay products, thus if both $\tau^+$ and $\tau^-$ decay in this Chanel, 16 acoplanarity angles can be defined, see Fig. 6, and also 8 variables similar to $y_{1,2}$. The observable is thus built on 24 dimensional space. Fortunately, with the modern techniques, such as used in Ref. [6] an overall sensitivity can be evaluated. On the other hand, as it may be difficult to develop intuition, the risk of misinterpretation can not be ignored. Difficulties of pattern recognition projects are known, see Fig. 7. These challenges, were explored by Giuseppe Arcimboldo (1572 - 1593) in his paintings.
Figure 6. Acoplanarity angles of oriented half decay planes: $\varphi^*_{\rho\rho}$ (left), $\varphi^*_{a_1\rho}$ (middle) and $\varphi^*_{a_1a_1}$ (right), for events grouped by the sign of $y^{+}_{\rho\rho}y^{-}_{\rho\rho}$, $y^{+}_{a_1\rho}y^{-}_{a_1\rho}$ and $y^{+}_{a_1}y^{-}_{a_1}$ respectively. Three CP mixing angles $\phi^{CP}$ = 0.0 (scalar), 0.2 and 0.4. Note that physics model depends on 1 parameter only and effect of $\phi^{CP}$, the Higgs mixing scalar pseudo-scalar angle, is a linear shift.

Figure 7. Artificial Neural Networks have spurred remarkable recent progress in image classification and speech recognition. But even though these are very useful tools based on well-known mathematical methods, we actually understand surprisingly little of why certain models work and others don’t.

6. Summary
Inspired by Shimizu-sensei conference, I have focused on an aspect of work for Monte Carlo generators and on phenomenology of High Energy Physics experiments related to the use of
algebraic manipulation programs. My aim was to show several simple examples of challenges, resulting from complexity: how automated calculations were of help, but also sources of difficulties. Some of the important examples originate from my work in Minami Tateya group I was visiting over the last 25 years. Each example deserve substantial introduction. This was impossible for a short talk. Whenever possible, I delegate the reader to references. Inescapably, I have presented only scattered projects, where use of computer algebraic methods or pattern recognition techniques (Machine Learning) were necessary. Collecting material for my talk was inspiring and educative for myself as I could look back at particular context of my scientific activities over all these years.

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