$W^+ - H^+$ Interference and Partial Width
Asymmetry in Top and Antitop Decays

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Abstract

We re-examine the question of a possible difference in the partial decay widths of $t$ and $\bar{t}$, induced by an intermediate scalar boson $H^+$ with $CP$-violating couplings. The interference of $W^+$ and $H^+$ exchanges is analysed by constructing the $2 \times 2$ propagator matrix of the $W^+ - H^+$ system, and determining its absorptive part in terms of fermion loops. Results are obtained for the partial rate difference in the channels $t \to bl^+\nu_l$ and $t \to bc\bar{s}$, which fulfil explicitly the constraints of $CPT$ invariance. These results are contrasted with those in previous work.

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1 Introduction

Recent literature [1, 2] has been witness to an interesting debate on the question of a possible $CP$-violating difference in the partial widths of $t$ and $\bar{t}$ decays, into conjugate channels such as $t \rightarrow b\tau^+\nu$ and $\bar{t} \rightarrow \bar{b}\tau^-\bar{\nu}$. This discussion has taken place in the context of a model in which the decays of the top quark are mediated, not only by $W^\pm$ bosons, but also by charged Higgs bosons $H^\pm$ with $CP$-violating couplings [3]. Two specific questions that have arisen in this regard are (i) the correct form of the propagator for an unstable $W$ boson [1, 2, 4, 5], and (ii) the implications of $CPT$ invariance and unitarity for partial rate asymmetries generated by absorptive parts of decay amplitudes [3].

In this paper, we present an analysis that, we believe, is more complete than that in Refs. [1, 2]. Central to our analysis is the derivation of the propagator matrix of the coupled $W^+ - H^+$ system, taking account of vacuum polarization effects induced by fermion loops. The propagator matrix includes off-diagonal transitions between $W^+$ and $H^+$, which turn out to be essential for obtaining a partial rate asymmetry that respects the constraints of $CPT$ invariance.

The model we use is defined by the Lagrangian [3]

\begin{align}
\mathcal{L} &= \mathcal{L}_W + \mathcal{L}_H, \\
\mathcal{L}_W &= -\frac{g}{\sqrt{2}} \left\{ \sum_{l=e,\mu,\tau} \left[ \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l W^+_{\mu} + \bar{\nu}_l \gamma^\mu (1 - \gamma_5) \nu_l W^-_{\mu} \right] \\
&\quad + \bar{\nu} \gamma^\mu (1 - \gamma_5) d W^+_{\mu} + \bar{d} \gamma^\mu (1 - \gamma_5) u W^-_{\mu} + (u, d) \rightarrow (c, s) + (u, d) \rightarrow (t, b) \right\}, \\
\mathcal{L}_H &= \frac{g}{\sqrt{2}} \left\{ - \sum_{l=e,\mu,\tau} \left[ H^+ Z \frac{m_l}{m_W} \bar{\nu}_l (1 + \gamma_5) l + H^- Z^* \frac{m_l}{m_W} \bar{\nu}_l (1 - \gamma_5) \nu_l \right] \\
&\quad + H^+ \bar{\nu} \left[ Y \frac{m_u}{m_W} (1 - \gamma_5) + X \frac{m_d}{m_W} (1 + \gamma_5) \right] d \right\},
\end{align}

(1)
\[ H - \mathcal{F}[X^* \frac{m_d}{m_W} (1 - \gamma_5) + Y^* \frac{m_u}{m_W} (1 + \gamma_5)] u \]
\[ + (u, d) \to (c, s) + (u, d) \to (t, b) \],

(3)

where we neglect quark-mixing. The parameters \( X, Y, Z \) appearing in \( \mathcal{L}_H \) are permitted to be complex relative to one another, so that this term is \( CP \)-violating. The interaction \( \mathcal{L}_H \) may be imagined to arise as a special case of the Weinberg model with three Higgs doublets \([7]\), in which the remaining charged scalars are sufficiently heavy to be disregarded.

2 The \( W^+ - H^+ \) Propagator

We are concerned with the propagator (in unitary gauge) of the coupled \( W^+ - H^+ \) system, which we describe by a \( 2 \times 2 \) matrix

\[ D = \begin{pmatrix} D^\mu_\nu & D^\mu_{W+H^+} \\ D^\nu_{W^+H^+} & D_H \end{pmatrix}. \]

(4)

The inverse of this matrix is defined by

\[ D^{-1} = \begin{pmatrix} g & 0 \\ 0 & 1 \end{pmatrix}, \]

(5)

where \( g \) is the metric tensor with elements \( g^\mu_\nu = diag(1, -1, -1, -1) \). The inverse matrix \( D^{-1} \) has the general form

\[ D^{-1} = -i \begin{pmatrix} (m^2_W - q^2 + F_1)g^\mu_\nu + q^\mu q^\nu(1 + F_2) & q^\mu F_3 \\ q^\nu F_4 & q^2 - m^2_H + F_5 \end{pmatrix}, \]

(6)

where the functions \( F_i(q^2), i = 1, \ldots, 5 \) are given by the one-particle-irreducible self-energies

\[ \Sigma^\mu_\nu(q^2) = i \left[ g^\mu_\nu F_1(q^2) + q^\mu q^\nu F_2(q^2) \right], \]
\[ \Sigma_{W^+H^+}(q^2) = iq^\mu F_3(q^2), \]
\[ \Sigma_{H^+W^+}(q^2) = iq^\mu F_4(q^2), \]
\[ \Sigma_H(q^2) = iF_5(q^2). \] (7)

Inversion of the matrix (6) yields the elements of the propagator matrix (4):

\[ D_{W^\mu W^\nu} = \frac{-g^{\mu\nu} + q^\mu q^\nu}{(m_W^2 + F_1 + q^2 F_2)(q^2 - m_H^2 + F_5)} \frac{(1 + F_2)(q^2 - m_H^2 + F_5) - F_3 F_4}{q^2 - m_W^2 - F_1}, \]
\[ D_{W^+H^+} = \frac{iq^\mu F_3}{q^2 F_3 F_4 - (q^2 - m_H^2 + F_5)(m_W^2 + F_1 + q^2 F_2)}, \]
\[ D_{H^+W^+} = \frac{iq^\mu F_4}{q^2 F_3 F_4 - (q^2 - m_H^2 + F_5)(m_W^2 + F_1 + q^2 F_2)}, \]
\[ D_H = \frac{i}{q^2 - m_H^2 + F_5 - \frac{q^2 F_3 F_4}{m_W^2 + F_1 + q^2 F_2}}. \] (8)

The corresponding propagator matrix for \( W^- - H^- \) is obtained by the replacement \( q^\mu \rightarrow -q^\mu, F_3(q^2) \leftrightarrow F_4(q^2) \). The above derivation is analogous to the description of the \( \gamma - Z \) system \[8\]. A graphical representation of Eqs. (7) and (8) is given in Figs. 1-3.

The function \( D_{W^\mu W^\nu} \), representing the \( WW \) element of the propagator matrix, can be decomposed into transverse and longitudinal pieces:

\[ D_{W^\mu W^\nu} = i(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2})G_T + \frac{q^\mu q^\nu}{q^2}G_L \] (9)

with

\[ G_T = \frac{1}{q^2 - m_W^2 - F_1}, \]
\[ G_L = \frac{1}{m_W^2 + F_1 + q^2 F_2 - \frac{q^2 F_3 F_4}{q^2 - m_H^2 + F_5}}. \] (10)

It will turn out that only the longitudinal part \( G_L \) contributes to the partial width asymmetry. If the term proportional to \( F_3 F_4 \) is dropped, the function \( G_L \) coincides
with that in Refs. [1, 2]. We work initially with the full expression in Eq. (8), in order to obtain results that are also valid for \( q^2 \simeq m^2_H \), a region that is physically accessible if \( m_H < m_t - m_b \).

3 Difference of Partial Widths

3.1 Asymmetry in Lepton Channels

The amplitude of the decay \( t \to b l^+ \nu_l \), including vacuum polarization effects in the \( W - H \) propagator, is given by the sum of the four diagrams shown in Fig. 4, and has the form

\[
M_l = \frac{ig^2}{8} \left\{ A_l \bar{u}_b \gamma^\mu (1 - \gamma_5) u_t \bar{u}_\nu \gamma_\mu (1 - \gamma_5) \nu_l + B_l \bar{u}_b (1 + \gamma_5) u_t \bar{u}_\nu (1 + \gamma_5) \nu_l + D_l \bar{u}_b (1 - \gamma_5) u_t \bar{u}_\nu (1 + \gamma_5) \nu_l \right\} \tag{11}
\]

with

\[
A_l = G_T, \\
B_l = \frac{m_t m_l}{m^2_W} \left\{ \frac{m^2_W}{q^2} (G_T + G_L) + Y^* ZG_5 + m_W N(Y^* F_4 + ZF_3) \right\}, \\
D_l = \frac{m_b m_l}{m^2_W} \left\{ -\frac{m^2_W}{q^2} (G_T + G_L) + X^* ZG_5 + m_W N(X^* F_4 - ZF_3) \right\}, \tag{12}
\]

where \( N \equiv [(m^2_W + F_1 + q^2 F_2)(q^2 - m^2_H + F_5) - q^2 F_3 F_4]^{-1} \) and \( G_5 \equiv N(m^2_W + F_1 + q^2 F_2) \). The corresponding decay amplitude for \( \bar{t} \to \bar{b} l^- \bar{\nu}_l \) is

\[
\overline{M}_l = \frac{ig^2}{8} \left\{ \overline{A}_l \bar{v}_b \gamma^\mu (1 - \gamma_5) v_l \bar{v}_\nu \gamma_\mu (1 - \gamma_5) \nu_l + \overline{B}_l \bar{v}_b (1 + \gamma_5) v_l \bar{v}_\nu (1 + \gamma_5) \nu_l + \overline{D}_l \bar{v}_b (1 - \gamma_5) v_l \bar{v}_\nu (1 + \gamma_5) \nu_l \right\} \tag{13}
\]

with

\[
\overline{A}_l = G_T, \\
\overline{B}_l = \frac{m_t m_l}{m^2_W} \left\{ \frac{m^2_W}{q^2} (G_T + G_L) + Y^* ZG_5 + m_W N(Y^* F_4 + ZF_3) \right\}, \\
\overline{D}_l = \frac{m_b m_l}{m^2_W} \left\{ -\frac{m^2_W}{q^2} (G_T + G_L) + X^* ZG_5 + m_W N(X^* F_4 - ZF_3) \right\}.
\]
\[ \bar{B}_l = \frac{m_t m_l}{m_W^2} \left\{ \frac{m_W^2}{q^2} (G_T + G_L) + Y Z^* G_5 + m_W N (Y F_3 + Z^* F_4) \right\}, \]

\[ \bar{D}_l = \frac{m_b m_l}{m_W^2} \left\{ \frac{m_W^2}{q^2} (G_T + G_L) + X Z^* G_5 + m_W N (X F_3 - Z^* F_4) \right\}; \quad (14) \]

The matrix elements \( M_l \) and \( \overline{M}_l \) yield the following asymmetry between the partial widths:

\[ \Delta_{\nu_l} \equiv \Gamma(\bar{t} \rightarrow b^{+}\nu_l) - \Gamma(t \rightarrow b^{+}\nu) \]

\[ = \frac{1}{2m_t} \int \frac{d^3p_b}{(2\pi)^3 2E_b} \frac{d^3p_t}{(2\pi)^3 2E_t} \frac{d^3p_\nu}{(2\pi)^3 2E_\nu} (2\pi)^4 \delta(4)(p_t - p_b - p_l - p_\nu) \]
\[ \left\{ |\overline{M}_l|^2 - |M_l|^2 \right\} \]
\[ = \frac{g^4}{211\pi^4 m_t^2} \int \frac{dq^2}{q^4} \lambda(q^2, m_t^2, m_b^2)(q^2 - m_t^2)^2 \left\{ (|\overline{B}_l|^2 - |B_l|^2 + |\overline{D}_l|^2 - |D_l|^2)q^2(m_t^2 + m_b^2 - q^2) \right. \]
\[ + 4Re(\overline{B}_l D_l - B_l^* D_l)m_t m_b q^2 \]
\[ - 2ReA^*(\overline{B}_l - B_l)m_t m_l(m_t^2 - m_b^2 - q^2) \]
\[ - 2ReA^*(\overline{D}_l - D_l)m_b m_l(m_t^2 - m_b^2 + q^2) \right\}, \quad (15) \]

where \( \lambda(a, b, c) = (a^2 + b^2 + c^2 - 2ab - 2ac - 2bc)^{1/2} \). Substituting the expressions for \( A_l, B_l, D_l, \overline{A}_l, \overline{B}_l, \overline{D}_l \) in the above integrand, we find (as anticipated) that terms proportional to the transverse propagator \( G_T \) cancel completely. Expressed in terms of the functions \( F_i(q^2) \), the asymmetry involves only the quantities \( \text{Im}(F_1 + q^2 F_2)(q^2) \), \( (F_3 - F_4^*)(q^2) \) and \( \text{Im}F_5(q^2) \). Representing the self-energies by fermion loops, these terms are

\[ \text{Im}(F_1 + q^2 F_2)(q^2) = \frac{g^2}{16\pi} \left\{ \frac{N_c \lambda(q^2, m_u^2, m_d^2)}{2q^4} \Theta[q^2 - (m_u + m_d)^2] \right. \]
\[ \cdot(-m_u^4 - m_u^4 + q^2 m_u^2 + q^2 m_d^2 + 2m_u^2 m_d^2) \]
\[ + (u, d) \rightarrow (c, s) + \sum_{l=e, \mu, \tau} \frac{(q^2 - m_l^2)^2}{2q^4} \Theta(q^2 - m_l^2) \right\}, \]
One finds that the contribution of the lepton loops (the pieces \(\sum_{l=e,\mu,\tau}^{\text{c, s}}\)) to the asymmetry vanishes identically, leaving as the final result

\[
(16) \quad \Delta_{l\nu} = \frac{g^6 N_c m_l^2}{214\pi^4 m_W^6 m_{\nu}^6} \int_{\max(m_{\nu}^2, m_{s}^2, m_{t}^2)}^{(m_{l} - m_{\nu})^2} dq^2 \frac{\lambda(q^2, m_{l}^2, m_{\nu}^2)\lambda(q^2, m_{l}^2, m_{\nu}^2)(q^2 - m_{l}^2)^2}{(q^2 - m_{\nu}^2)^2 + m_{H}^2 \Gamma_{H}^2} \bigg[ \left[ \begin{aligned} m_{l}^2 m_{s}^2 (m_{t}^2 &- m_{u}^2 - q^2)(q^2 + m_{c}^2 - m_{s}^2) \\ -m_{u}^2 m_{s}^2 (m_{t}^2 &- m_{u}^2 - q^2)(q^2 + m_{c}^2 - m_{s}^2) \end{aligned} \right] \\ \cdot \left[ \begin{aligned} (1 - \frac{m_{H}^2}{q^2}) Im(XY^* - XZ^* - YZ^*) \\ + |Y|^2 Im(XZ^*) - |Z|^2 Im(XY^*) + |X|^2 Im(YZ^*) \end{aligned} \right] \\ + 2ImYZ^* |X| + Y^2 m_{l}^2 m_{c}^2 \left[ \begin{aligned} q^2 (m_{s}^2 &- m_{u}^2) + m_{u}^2 m_{c}^2 - m_{t}^2 m_{s}^2 \\ + &m_{u}^2 m_{s}^2 \left[ q^2 (m_{l}^2 - m_{c}^2) + m_{l}^2 m_{c}^2 - m_{t}^2 m_{s}^2 \right] \end{aligned} \right] \bigg] \\ + (c, s) \rightarrow (u, d) \\ \equiv \Delta_{l\nu}(c, s) + \Delta_{l\nu}(u, d).
\]
in the form $ImF_5(q^2 = m_H^2) = m_H \Gamma_H$, and have neglected terms of relative order $q^2$.

### 3.2 Asymmetry in Quark Channels

In complete analogy to the lepton case, the matrix elements for the decays $t \to b c \bar{s}$, $\bar{t} \to \bar{b} c \bar{s}$ are

\[
M = \frac{ig^2}{8} \left\{ \begin{array}{l}
A \bar{u}_b \gamma^\mu (1 - \gamma_5) u_t \bar{u}_c \gamma_\mu (1 - \gamma_5) v_s \\
+ B \bar{u}_b (1 + \gamma_5) u_t \bar{u}_c (1 + \gamma_5) v_s + C \bar{u}_b (1 + \gamma_5) u_t \bar{u}_c (1 - \gamma_5) v_s \\
+ D \bar{u}_b (1 - \gamma_5) u_t \bar{u}_c (1 + \gamma_5) v_s + E \bar{u}_b (1 - \gamma_5) u_t \bar{u}_c (1 - \gamma_5) v_s
\end{array} \right\},
\]

\[
\overline{M} = \frac{ig^2}{8} \left\{ \begin{array}{l}
\bar{A} \bar{v}_t \gamma^\mu (1 - \gamma_5) v_b \bar{v}_s \gamma_\mu (1 - \gamma_5) v_c \\
+ \bar{B} \bar{v}_t (1 - \gamma_5) v_b \bar{v}_s (1 - \gamma_5) v_c + \bar{C} \bar{v}_t (1 - \gamma_5) v_b \bar{v}_s (1 + \gamma_5) v_c \\
+ \bar{D} \bar{v}_t (1 + \gamma_5) v_b \bar{v}_s (1 + \gamma_5) v_c + \bar{E} \bar{v}_t (1 + \gamma_5) v_b \bar{v}_s (1 + \gamma_5) v_c
\end{array} \right\},
\]

where

\[
A = G_T,
\]

\[
B = \frac{m_t m_s}{m_W^2} \left\{ -\frac{m_W^2}{q^2} (G_T + G_L) - X*YG_5 + m_W N(Y^*F_4 + XF_3) \right\},
\]

\[
C = \frac{m_t m_c}{m_W^2} \left\{ -\frac{m_W^2}{q^2} (G_T + G_L) - |Y|^2 G_5 - m_W N(Y^*F_4 + YF_3) \right\},
\]

\[
D = \frac{m_b m_s}{m_W^2} \left\{ -\frac{m_W^2}{q^2} (G_T + G_L) - |X|^2 G_5 + m_W N(X^*F_4 + XF_3) \right\},
\]

\[
E = \frac{m_b m_c}{m_W^2} \left\{ -\frac{m_W^2}{q^2} (G_T + G_L) - X*YG_5 + m_W N(YF_3 - X^*F_4) \right\}
\]

and

\[
\bar{A} = A, \quad \bar{C} = C, \quad \bar{D} = D, \quad \bar{B} = \frac{m_t m_s}{m_W^2} \left\{ -\frac{m_W^2}{q^2} (G_T + G_L) - X*YG_5 + m_W N(YF_3 - X^*F_4) \right\}.
\]

8
\[ E = \frac{m_b m_c}{m_W^2} \left\{ \frac{m_W^2}{q^2} (G_T + G_L) - X Y^* G_5 + m_W N(Y^* F_4 - X F_3) \right\}. \quad (20) \]

Once again, the transverse propagator term \( G_T \) makes no contribution to the asymmetry, which is given by

\[
\Delta_{cs} = \Gamma(t \rightarrow \bar{b}c) - \Gamma(t \rightarrow bc\pi) = \frac{N_c g^4}{2^{11} \pi^3 m_t^3} \int \frac{dq^2}{q^2} \lambda(q^2, m_t^2, m_b^2) \lambda(q^2, m_c^2, m_s^2) \left\{ (|\mathcal{B}_0|^2 - |B_0|^2 + |\mathcal{E}_0|^2)|E_0|^2)(m_t^2 + m_b^2 - q^2)(q^2 - m_c^2 - m_s^2) \right. \\
-4Re[C_0^* (B_0 - B_0) + D_0^* (E_0 - E_0)] m_c m_s (m_t^2 + m_b^2 - q^2) \\
+4Re[D_0^* (\bar{B}_0 - B_0) + C_0^* (E_0 - E_0)] m_t m_b (q^2 - m_c^2 - m_s^2), \quad (21) 
\]

where the subscript “0” means the expressions (19) and (20) without the terms proportional to \( G_T \). Expressed in terms of the functions \( F_i(q^2) \), the asymmetry involves only the combinations given in Eq. (16), yielding as the final result

\[
\Delta_{cs} = \frac{N_c g^6}{2^{13} \pi^4 m_t^3 m_b^6} \left\{ \frac{m_W^2}{q^2} X Y^* |X + Y|^2 \right\} \int \frac{dq^2}{q^2} \lambda(q^2, m_t^2, m_b^2) \lambda(q^2, m_c^2, m_s^2) \lambda(q^2, m_u^2, m_d^2) \\
\left( q^2 - m_H^2 \right) (q^2 - m_H^2) \right. \\
- \sum_{l=e,\mu,\tau} \Delta_{ln}(c, s), \quad (22) 
\]

where the last term follows from the relation \( \Delta_{cs}(l\nu) = -\Delta_{ln}(cs) \), which we have checked explicitly. The function \( f \) is defined by

\[
f(q^2, t, b, c, s, u, d) = q^4 (t b c d - t b s u - t c d s + t s u d + b c s u - b c u d) \\
+ q^2 (t^2 c s d - t^2 s u d - t b c^2 d - t b c d^2 + t b s^2 u + t b s u^2 \\
+ t c s d^2 - t s^2 u d - b^2 c s u + b^2 c u d + b c^2 u d - b c u^2) \\
+ (t - b c) (s u - c d) (t d - b u). \quad (23) 
\]
It has the remarkable property of being antisymmetric under any one of the following exchanges:

\[(u, d) \leftrightarrow (c, s) \quad ; \quad (u, d) \leftrightarrow (t, b) \quad ; \quad (c, s) \leftrightarrow (t, b).\] (24)

As a consequence of this asymmetry, we immediately see that (i) the \((c, s)\) loop does not contribute to \(\Delta_{cs}\), (ii) the analogous result for \(\Delta_{ud}\) is obtained by interchanging \((c, s)\) and \((u, d)\) in Eq. (22), and (iii) the asymmetries in the various channels satisfy the relation

\[\Delta_{cs} + \Delta_{ud} + \Delta_{\tau\nu_e} + \Delta_{\mu\nu_e} + \Delta_{\tau\nu_e} = 0,\] (25)

implying the equality of total width of \(t\) and \(\bar{t}\), mandated by \(CPT\) invariance.

4 Comments

(i) Our results fulfill all the general constraints on partial width asymmetries noted by Wolfenstein [6]. In particular, the asymmetry in a channel \(f\) associated with a loop \(n\) satisfies

\[\Delta_f(n) = -\Delta_n(f)\] (26)

and vanishes when \(n = f\).

(ii) A characteristic feature of \(W^+ - H^+\) interference is the result that the asymmetry \(\Delta_q(q')\) in the quark channel \(q\), arising from a quark loop \(q'\), is proportional to the function \(f(q^2, m_1^2, m_2^2, m_3^2, m_4^2)\) defined in Eq. (23), where \((m_1, m_2)\) and \((m_3, m_4)\) are the masses of the quark doublets contained in \(q\) and \(q'\). This implies that the specific asymmetry \(\Delta_q(q')\) vanishes when one of the masses \((m_1, m_2)\) and one of the masses \((m_3, m_4)\) is zero. For a similar reason, the asymmetry in a lepton
channel \( l \) due to a lepton loop \( l' \) vanishes, even for \( l \neq l' \), since the two doublets necessarily contain two massless neutrinos.

(iii) The fact that the asymmetries \( \Delta_{\mu\nu}, \Delta_{cs} \) and \( \Delta_{ud} \) given by Eqs. (17) and (22) satisfy the \( CPT \) condition (25) is a nontrivial test of the full \( W^+ - H^+ \) propagator constructed in Eq. (8). In particular, neglect of the off-diagonal terms \( F_3 \) and \( F_4 \) leads to conflict with \( CPT \) invariance. These terms have not been considered in previous work.

(iv) Our results for \( \Delta_{\tau\nu} \) and \( \Delta_{cs} \) do not coincide with those in Refs. [1, 2]. For instance, these earlier papers found an asymmetry \( \Delta_{\tau\nu} \) proportional to \( m_\tau^2 m_e^2 \). By contrast, the leading term of our result (Eq. (17)) is proportional to \( m_\tau^2 m_e^2 \). We have been able to trace the difference to the neglect of the off-diagonal part of the \( W^+ - H^+ \) propagator in Refs. [1, 2], which inevitably leads to a violation of the \( CPT \) condition (Eq. (25)).

(v) In the absence of any scalar interaction of the form \( L_H \), the transverse and longitudinal parts of the propagator \( D_W^{\mu\nu} \) obtained by us agree with those in Refs. [1, 2, 4].

(vi) Numerically, the partial width asymmetries resulting from \( W^+ - H^+ \) interference, in the models discussed here, are exceedingly small. As pointed out in Ref. [1], larger differences between \( t \to b\tau^+ \nu_\tau \) and \( \bar{t} \to \bar{b}\tau^- \bar{\nu}_\tau \) occur if one compares the spectra of these reactions, not only in the variable \( q^2 \) but also in the complementary Dalitz variable \( u = (p_\tau + p_b)^2 \) [8, 10]. Likewise, larger asymmetries are possible if one compares the \( \tau^+ \) and \( \tau^- \) polarization [11]. Whereas the partial width asymmetry discussed in this paper involves only the longitudinal part of the \( W \) propagator, these alternative effects involve the transverse part, and do not necessarily require absorptive phases associated with final state interactions.
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Figure Captions

Fig. 1. Diagonal and non-diagonal one-particle-irreducible self-energies of the $W - H$ system (Eq. (7)).

Fig. 2. Graphical representation of the “pure” $W$ and $H$ propagators, neglecting $W - H$ mixing.

Fig. 3. Graphical representation of the full $W - H$ propagator (Eq. (8)), in terms of the “pure” $W$ and $H$ propagators defined in Fig. 2.

Fig. 4. Feynman diagrams contributing to the reaction $t \rightarrow b \tau^+ \nu_{\tau}$. 
This figure "fig1-1.png" is available in "png" format from:

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Fig. 3.
Fig. 4.
\[
\Sigma_{W}^{\mu\nu}(q^2) = i[g^{\mu\nu}F_1(q^2) + q^\mu q^\nu F_2(q^2)]
\]

\[
\Sigma_{W+H}^{\mu}(q^2) = i q^\mu F_3(q^2)
\]

\[
\Sigma_{H+W}^{\mu}(q^2) = i q^\mu F_4(q^2)
\]

\[
\Sigma_{H}(q^2) = i F_5(q^2)
\]

Fig. 1.

\[
[D_W^{\mu\nu}]_0 = \cdots - + \cdots - + \cdots - + \cdots - + \cdots
\]

\[
[D_H]_0 = \cdots - + \cdots - + \cdots - + \cdots - + \cdots
\]

Fig. 2.