Effect of Electron-electron Interaction on Surface Transport in Three-Dimensional Topological Insulators

H. K. Pal, V. I. Yudson, and D. L. Maslov
1 Department of Physics, University of Florida, Gainesville, FL 32611-8440, USA
2 Institute for Spectroscopy, Russian Academy of Sciences, Troitsk, Moscow Region, 142190, Russia
(Dated: April 28, 2013)

We study the effect of electron-electron interaction on the surface resistivity of three-dimensional (3D) topological insulators. In the absence of umklapp scattering, the existence of the Fermi-liquid ($T^2$) term in resistivity of a two-dimensional (2D) metal depends on the Fermi surface geometry, in particular, on whether it is convex or concave. On doping, the Fermi surface of 2D metallic surface states in 3D topological insulators of the Bi$_2$Te$_3$ family changes its shape from convex to concave due to hexagonal warping, while still being too small to allow for umklapp scattering. We show that the $T^2$ term in the resistivity is present only in the concave regime and demonstrate that the resistivity obeys a universal scaling form valid for an arbitrary 2D Fermi surface near a convex/concave transition.

PACS numbers: 72.10.-d, 73.20.-r

Topological insulators (TI) are characterized by a gapped bulk spectrum with conducting surface states extending across the entire gap. The surface states contain an odd number of Dirac cones and are protected against any perturbation that preserves time-reversal symmetry $[1]$. A wide variety of interesting physics resulting from these surface states is expected to be observed ranging from Majorana fermions $[2]$ to magnetic monopoles $[3]$. Although photoemission and tunneling microscopy $[4]$ have convincingly established the presence of such surface states in these materials, signatures of these states in transport measurements are more difficult to observe, mainly because of strong conduction in the bulk $[5]$. With recent experimental progress, however, in the ability to tune the number of surface charge carriers $[6]$, it is now possible to see more clearly evidence of surface transport.

In light of this progress, it is timely to ask what is the effect of the electron-electron ($e-e$) interaction on surface transport. Indeed, including $e-e$ interaction is crucial for explaining the observed field and temperature dependences in quantum magnetotransport $[7]$. In this Letter, we address the manifestation of the $e-e$ interaction in semiclassical transport within a model of a two-dimensional Fermi liquid relevant for surface states doped away from the Dirac point.

An archetypal signature of the Fermi-liquid behavior in metals is the $T^2$ dependence of the resistivity ($\rho$). With an exception of compensated semi-metals $[8]$, this dependence in clean conductors arises due to a special type of scattering processes—“umklapps” $[9][10]$, in which the total momentum of an electron pair is changed by an integer multiple of the reciprocal lattice vector. Umklapps are possible if certain conditions are met, namely, if the Fermi surface (FS) is large enough (the band is more than quarter full) and if the matrix element of the interaction has sufficient weight at large momentum transfers. Otherwise, the umklapp contribution to the resistivity is suppressed. In this case, the $T^2$ contribution to $\rho$ may still occur due to the combined effect of the momentum-conserving (“normal”) interaction among electrons on a lattice and electron-impurity ($e-i$) scattering. Whether this really happens, turns out to depend crucially on the dimensionality. While the $T^2$ term is allowed for an anisotropic FS with a non-parabolic spectrum in three dimensions (3D), the conditions in two dimensions (2D) are much more stringent $[11]$. In particular, a $T^2$ term occurs in 2D only if the FS is either concave or multiply-connected; otherwise, the leading $e-e$ contribution scales as $T^4$ $[12]$. Likewise, the frequency dependence of the $ac$ resistivity scales as $\Omega^4$ instead of $\Omega^2$ $[13]$. The reason for such a behavior is that the $T^2$ ($\Omega^2$) term arises from electrons confined to the FS contour. For a convex and singly-connected contour, the momentum and energy conservations are similar to the 1D case, where no relaxation is possible.

We propose the surface state of a 3D TI as a testing ground for the theoretical results outlined above. Photoemission shows that the surface states of the Se, Te based compounds (Bi$_2$Te$_3$, Bi$_2$Se$_3$, and Sb$_2$Te$_3$) have a small, singly-connected FS at the center of the Brillouin zone (BZ). Following Fu $[14]$, the electronic dispersion in these systems can be described by

$$\epsilon_k^\pm = \pm \sqrt{v^2 k^2 + \lambda^2 k_x \cos^2(3\theta)}, \quad (1)$$

where $\theta$ is the azimuthal angle, $v$ is the Dirac velocity, and $\lambda$ is a constant. Corresponding isoeenergetic contours are presented in Fig. 1. As the Fermi energy increases, the FS changes rapidly from a circle to a hexagon and then to a hexagram. At some critical value of the Fermi energy $\epsilon_F = \epsilon_c = 0.16$ eV for Bi$_2$Te$_3$, for example $[14]$, the shape changes from convex to concave. Theory $[12][13]$ predicts, therefore, that the $e-e$ contribution to the resistivity scales as max{$T^4, \Omega^4$} on the convex side and as max{$T^2, \Omega^2$} on the concave side. The main result of
this Letter is that, near the convex/concave transition, the resistivity obeys a universal scaling form
\[ \rho = \rho_0 + A \left( \frac{\Delta}{\epsilon_F} \right)^{9/2} \Theta(\Delta)T^2 + BT^4, \]
where \( \rho_0 \) is the residual resistivity, \( \Delta = \epsilon_F - \epsilon_c, \theta(x) \) is the step function, and \( A, B \) are material-dependent parameters.

The exponents of 2, 4, and 9/2 in Eq. (2) are universal, i.e., they are the same for an arbitrary 2D Fermi surface with a non-quadratic energy spectrum near a convex/concave transition. We emphasize, however, that the surface states of 3D TIs present a unique case of a small yet strongly warped 2D FS, where the predicted effects can be seen best. The drawback of 3D TIs is that they have a large background dielectric constant (\( \epsilon \sim 29 - 85 \)), and hence, the electron-phonon (e-ph) interaction is expected to dominate the T-dependence of the resistivity down to very low \( T [10] \). This drawback can be circumvented by measuring the frequency dependence of the optical conductivity at frequencies above the Bloch-Gruneisen frequency, where the e-e contribution dominates over the e-ph one [17].

As in Ref. [12], we adopt an approach based on the semiclassical Boltzmann equation (BE) [18] and neglect quantum corrections to the conductivity. For simplicity, the e-i interaction is accounted for within the 1\( \tau \) approximation. First, we consider the dc case (\( \Omega = 0 \)). For low enough \( T \), when \( \tau_{ee} \gg \tau_{ei} \), we solve the BE to leading order in the e-e interaction and obtain the correction to the residual conductivity as
\[ \delta \sigma_{jj} = -\frac{e^2 \tau_i^2}{2 T} \int \frac{d^2q}{(2\pi)^2} \int \int d\epsilon_k d\epsilon_p \int \int \frac{d\epsilon_k}{\tau_k} \frac{d\epsilon_p}{\tau_p} \times |M_{k,p}(q)|^2 (\Delta v_j)^2 n(\epsilon_k)n(\epsilon_p)[1 - n(\epsilon_k - \omega)] \times [1 - n(\epsilon_p + \omega)] \delta(\epsilon_k - \epsilon_q - \omega) \delta(\epsilon_p - \epsilon_q + \omega). \]

Here, \( \tau_i \) is the mean free time due to impurity scattering, \( \delta \sigma_{jj} \) is the momentum and energy transfers, \( \Delta v = v_k + v_p - v_{k-q} - v_{p+q} \), \( d\epsilon_k \) is the FS element, \( n(\epsilon) = (\exp(\epsilon/T) + 1)^{-1} \), and \( M_{k,p}(q) \) is the matrix element of the e-e interaction. To obtain the lowest in \( T \) term in \( \delta \sigma_{jj} \), we project electrons onto the FS, which amounts to neglecting \( \omega \) in the arguments of the \( \delta \) functions. Since the typical values of \( \epsilon_k \) and \( \omega \) are of order \( T \), while the typical values of other variables are \( T \) independent, \( \delta \sigma_{jj} \) scales as \( T^2 \), which is the expected Fermi-liquid behavior. However, whether the prefactor of the \( T^2 \) term is non-zero depends on whether \( \Delta \) is non-zero for all \( k, p \), and \( q \) satisfying energy conservation. On relabeling \( p \) to \( -p \) and invoking time-reversal symmetry, the arguments of both \( \delta \) functions become the same. The problem of finding the allowed initial states \( k \) and \( p \) at fixed \( q \) now reduces to finding the solutions of the equation \( \epsilon_k = \epsilon_{k-p} \) (and the same for \( k \to p \)). Geometrically, this is equivalent to shifting the FS by a vector \( q \) and finding \( k \) and \( p \) as the points where the original and shifted FSs intersect.

Consider first the case \( \epsilon_F < \epsilon_c \), when the FS is convex. As shown in Fig. 2(a), there are only two points of intersection. If \( k \) is one of these intersection points then, by symmetry, the other point is \(-k + q\). Since solutions for \( p \) must belong to the same set \( \{k, -k + q\} \), the scattering process either occurs in the Cooper channel (\( \{k, -k \} \to \{k - q, -k + q\} \)) or corresponds to swapping of initial momenta (\( \{k, k - q \} \to \{k - q, k\} \)). In both cases, \( \Delta v = 0 \), and thus the \( T^2 \) correction to the conductivity vanishes. The first nonvanishing term in this case is \( T^4 \), which can be obtained from Eq. (3) by expanding the product of the \( \delta \) functions to second order in \( \omega \). On the other hand, if the FS is concave \( \epsilon_F > \epsilon_c \), there are six possible points of intersection yielding six solutions for each \( k \) and \( p \) [cf. Fig. 2(b)]. The total set of thirty six pairs for \( \{k, p\} \) contains processes other than Cooper channel and swapping, and \( \Delta v \) is non-zero for these processes.

The analysis presented above is valid either well below or well above the convex/concave transition, i.e., when \( |\Delta| \sim \epsilon_F \). We now turn to the vicinity of the transition, where \( |\Delta| \ll \epsilon_F \). First, we focus on the most interesting case of \( \Delta \gg T \), when the isoenergetic contours near the Fermi energy are concave, and then discuss the case of \( |\Delta| \ll T \), when both convex and concave contours near the FS are thermally populated. Near the transition, several quantities in Eq. (3) exhibit a critical dependence on \( \Delta \). First, it is \( \Delta v \) which is zero on the convex side and non-zero on the concave side. Additionally, there are two other quantities which also show a critical behavior. As Figs. 2(c) and 2(d) illustrate, even if the FS is concave, it has more than two self-intersection points only if it is shifted along one of the special directions and the magnitude of the shift is sufficiently small. [These special directions are high-symmetry axes that intersect the FS at points with positive curvature, as in Fig. 2(b).]

Therefore, the width of the angular interval near a special direction (\( \Delta \theta_q \)) and the maximum value of \( q \) (\( q_{\text{max}} \)) also depend on \( \Delta \) in a critical manner. Approximating \( \int d^2q \) by \( \Delta \theta_q q_{\text{max}}^2 \), resolving the \( \delta \) functions, and integrating

\[ \text{FIG. 1. (color online). Isoenergetic contours for the spectrum in Eq. (1). The dashed line corresponds to the critical energy for the convex/concave transition.} \]
so is a set of invariant points, i.e., common points for all contours. Even if self-intersections exist, the non-monotonic part is centered around certain points - the second term in Eq. (2) accounts for this contribution. A crossover between the $T^4$ and $T^2$ behaviors occurs at $T \sim \epsilon_F (\Delta / \epsilon_F)^{3/4} \ll \epsilon_F$. A large value of the exponent $(9/2)$ indicates that one needs to go sufficiently high above the convex/concave transition in order to see the $T^4$ term.

Now we return to the range of energies $\Delta \lesssim T$, when both convex and concave contours are populated. Instead of a $\Delta^{3/2}$ factor, we get a factor of $T^{3/2}$, leading to a
part continues to grow either as $\Omega^2$ or $\Omega^4$, depending on the sign of $\Delta$. This advantage was used in the past to detect the $e$-$e$ contribution to $\Gamma(\Omega)$ in noble metals [17], and we propose to apply the same technique to TIs.

This work was supported by NSF-DMR-0908029 (H.K.P. and D.L.M.) and RFBR-09-02-1235 (V.Y.). D.L.M. acknowledges the hospitality of MPI-PKS (Dresden), where a part of this work was done.

[1] O. A. Pankratov, S. V. Pakhomov, and B. A. Volkov, Solid State Commun. 61, 93 (1987); C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005); L. Fu and C. L. Kane, Phys. Rev. B 76, 045302 (2007); König, et al., Science 318, 766 (2007); D. Hsieh, et al., Nature (London) 452, 970 (2008); Y. Xia, et al., Nat. Phys. 5, 398 (2009); H. Zhang, et al., Nat. Phys. 5, 438 (2009).

[2] L. Fu and C. L. Kane, Phys. Rev. Lett. 100, 096407 (2008).

[3] X. Qi, et al., Science 323, 1184 (2009).

[4] D. Hsieh, et al., Science 323, 919 (2009); Y. L. Chen, et al., Science 325, 178 (2009); Z. Alpichshev, et al., Phys. Rev. Lett. 104, 016401 (2010); P. Cheng, et al., Phys. Rev. Lett. 105, 076801 (2010); T. Hanaguri, et al., Phys. Rev. B 82, 081305 (2010).

[5] J. G. Checkelsky, et al., Phys. Rev. Lett. 103, 246601 (2009); D. X. Qu, et al., Science 329, 821 (2010); N. P. Butch, et al., Phys. Rev. B 81, 241301 (2010).

[6] J. G. Analytis, et al., Nat. Phys. 6, 960 (2010); Y. S. Hor, et al., arXiv:1006.0317 (2010); H. Steinberg, et al., Nano Lett. 10, 5032 (2010).

[7] J. Wang, et al., arXiv:1012.0274v2 (2011).

[8] W. G. Baber, Proc. R. Soc. London Ser. A156, 383 (1937).

[9] R. Peierls, Ann. Phys. 3, 1055 (1929).

[10] L. D. Landau and I. J. Pomeranchuk, Phys. Z. Sowjetunion 10, 649 (1936); Zh. Eksp. Teor. Fiz. 7, 370 (1937).

[11] R. N. Gurzhii, A. N. Kalinenko, and A. I. Kopeliovich, Phys. Rev. B 52, 4744 (1995).

[12] D. L. Maslov, V. I. Yukson, and A. V. Chubukov, Phys. Rev. Lett. 106, 106403 (2011).

[13] A. Rosch and P. C. Howell, Phys. Rev. B72, 104510 (2005); A. Rosch, Ann. Phys. 15, 526 (2006).

[14] L. Fu, Phys. Rev. Lett. 103, 266801 (2009).

[15] W. Richter, H. Köhler, and C. R. Becker, Phys. Stat. Sol. (b) 84, 619 (1977).

[16] S. Giraud and R. Egger, arXiv:1103.0178v1 (2011).

[17] See V. F. Gantmakher and Y. B. Levinson, Scattering in metals and semiconductors (North-Holland, 1987) and references therein.

[18] Although we use a scalar BE instead of a spinor one suitable for helical states, it does not affect any of our conclusions. Away from the Dirac point, the effect of the phase factors in the spinor wavefunctions is only to suppress backscattering probability of all processes considered.

[19] In light of this, it is now clear why a convex contour does not allow for more than two solutions and why a concave contour allows for more than two solutions only for special directions of $\mathbf{q}$.

[20] E. G. Mishchenko, M. Y. Reizer, and L. I. Glazman, Phys. Rev. B 69, 195302 (2004).