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Mode-filtering technique based on all-fiber-based external cavity for fiber-based optical frequency comb

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Abstract: We developed a mode-filtering technique based on the all-fiber-based external cavity for a fiber-based optical frequency comb for high repetition rate \( f_{\text{rep}} \) frequency comb, and the carrier envelope offset frequency \( f_{\text{ceo}} \) can be detected and stabilized and is robust to environmental fluctuations. To achieve multiplication of the \( f_{\text{rep}} \) with a high multiplication factor using the fiber ring cavity, a long fiber was developed to mitigate the physical limitation inhibiting the shortening of the cavity length. In this study, the length of the fiber cavity was set to 6.7 m (free spectral range = 44.7 MHz) as the fiber-based comb length was 6.1 m. We were able to demonstrate a multiplication factor of 11, i.e., \( f_{\text{rep}} \) increased from 48.7 MHz to 536.0 MHz with a side mode suppression ratio of about 25 dB using the double-pass configuration.

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References and links

1. R. Holzwarth, T. Udem, T. W. Hänsch, J. C. Knight, W. J. Wadsworth, and P. St. J. Russell, “Optical frequency synthesizer for precision spectroscopy,” Phys. Rev. Lett. 85(11), 2264–2267 (2000).
2. D. J. Jones, S. A. Diddams, J. K. Ranka, A. Stentz, R. S. Windeler, J. L. Hall, and S. T. Cundiff, “Carrier-envelope phase control of femtosecond mode-locked lasers and direct optical frequency synthesis,” Science 286(5466), 635–640 (2000).
3. S. A. Diddams, L. Hollberg, and V. Mbele, “Molecular fingerprinting with the resolved modes of a femtosecond laser frequency comb,” Nature 445(7128), 627–630 (2007).
4. M. C. Stowe, M. J. Thorpe, A. Pe’er, J. Ye, J. E. Stalnaker, V. Gerginov, and S. A. Diddams, “Direct frequency comb spectroscopy,” Adv. At. Mol. Opt. Phys. 55, 1–60 (2008).
5. D. Hillerkuss, R. Schmogrow, T. Schellinger, M. Jordan, M. Winter, G. Huber, T. Vallaitis, R. Bonk, P. Kleinow, F. Frey, M. Roeger, S. Koenig, A. Ludwig, A. Marcelesca, J. Li, M. Hoh, M. Dreschmann, J. Meyer, S. Ben Ezra, N. Narkiss, B. Nebendahl, F. Parmigiani, P. Petropoulos, B. Resan, A. Oehler, K. Weingarten, T. Ellermeyer, J. Lutz, M. Moeller, M. Hübner, J. Becker, C. Koos, W. Freude, and J. Leuthold, “26 Tbit s⁻¹ line-rate super-channel transmission utilizing all-optical fast Fourier transform processing,” Nat. Photonics 5(6), 364–371 (2011).
6. S. T. Cundiff and A. T. Weiner, “Optical arbitrary waveform generation,” Nat. Photonics 4(11), 760–766 (2010).
7. S. A. Diddams, M. Kirchner, T. Fortier, D. Braje, A. M. Weiner, and L. Hollberg, “Improved signal-to-noise ratio of 10 GHz microwave signals generated with a mode-filtered femtosecond laser frequency comb,” Opt. Express 17(5), 3331–3340 (2009).
8. X. Xie, R. Bouchand, D. Nicodemi, M. Giunta, W. Hänsel, M. Lezias, A. Joshi, S. Datta, C. Alexandre, M. Lours, P.-A. Tremblin, G. Santarelli, R. Holzwarth, and Y. L. Coq, “Photonic microwave signals with zeptosecond-level absolute timing noise,” Nat. Photonics 1(1), 44–47 (2017).
9. T. Wilken, G. L. Curto, R. A. Probst, T. Steinmetz, A. Manescau, L. Pasquini, J. I. González Hernández, R. Rebolo, T. W. Hänsch, T. Udem, and R. Holzwarth, “A spectrograph for exoplanet observations calibrated at the centimetre-per-second level,” Nature 485(7400), 611–614 (2012).
10. A. G. Glenday, C.-H. Li, N. Langellier, G. Chang, L.-J. Chen, G. Furesz, A. A. Zibrov, F. Kärtner, D. F. Phillips, D. Sasselov, A. Szentgyorgyi, and R. L. Holzwarth, “Operation of a broadband visible-wavelength astro-comb with a high-resolution astrophysical spectrograph,” Optica 2(3), 250–254 (2015).
11. K. Minoshima and H. Matsumoto, “High-accuracy measurement of 240-m distance in an optical tunnel by use of a compact femtosecond laser,” Appl. Opt. 39(30), 5512–5517 (2000).
12. M. Endo, I. Ito, and Y. Kobayashi, “Direct 15-GHz mode-spacing optical frequency comb with a Kerr-lens mode-locked Yb:Y2O3 ceramic laser,” Opt. Express 23(2), 1276–1282 (2015).
13. I. Hartl, R. Ruehl, and M. Ferrmann, “Yb Fiber-Based Frequency Combs,” in Optical Fiber Communication Conference OSA Technical Digest (CD) (Optical Society of America, 2010), paper OTuJ5.
14. C. Li, Y. Ma, X. Gao, F. Niu, T. Jiang, A. Wang, and Z. Zhang, “1 GHz repetition rate femtosecond Yb:fiber laser for direct generation of carrier-envelope offset frequency,” Appl. Opt. 54(28), 8350–8353 (2015).
15. B. Xu, H. Yasui, Y. Nakajima, Y. Ma, Z. Zhang, and K. Minoshima, “Fully stabilized 750-MHz Yb: fiber frequency comb,” Opt. Express 25(12), 13332 (2017).
16. H.-W. Chen, G. Chang, S. Xu, Z. Yang, and F. X. Kärnér, “3 GHz, fundamentally mode-locked, femtosecond Yb-fiber laser,” Opt. Lett. 37(17), 3522–3524 (2012).
17. H. Inaba, Y. Daimon, F.-L. Hong, A. Onae, K. Minoshima, T. R. Schibli, H. Matsumoto, M. Hirano, T. Okuno, M. Onishi, and M. Nakazawa, “Long-term measurement of optical frequencies using a simple, robust and low-noise fiber based frequency comb,” Opt. Express 14(12), 5223–5231 (2006).
18. Y. Nakajima, H. Inaba, K. Hosaka, K. Minoshima, A. Onae, M. Yasada, T. Kohno, S. Kawato, T. Kobayashi, T. Katsuyama, and F.-L. Hong, “A multi-branch, fiber-based frequency comb with millihertz-level relative linewidths using an intra-cavity electro-optic modulator,” Opt. Express 18(2), 1667–1676 (2010).
19. J. Chen, J. W. Sickler, P. K. Pavlov, E. P. Ippen, F. X. Kärnér, T. Wilken, R. Holzwarth, and T. W. Hänsch, “Generation of low-timing-jitter femtosecond pulse trains with 2 GHz repetition rate via external repetition rate multiplication,” Opt. Lett. 33(9), 959–961 (2008).
20. C. A. Diddams, M. Kirchner, T. Fortier, D. Braje, A. M. Weiner, and L. Hollberg, “Improved signal-to-noise ratio of 10 GHz microwave signals generated with a mode-filtered femtosecond laser frequency comb,” Opt. Express 17(5), 3331–3340 (2009).
21. A. Nishiyama, S. Yoshida, T. Hariki, Y. Nakajima, and K. Minoshima, “Sensitivity improvement of dual-comb spectroscopy using mode-filtering technique,” Opt. Express 25(25), 31730–31738 (2017).
22. J. Lee, S.-W. Kim, and Y.-J. Kim, “Repetition rate multiplication of femtosecond light pulses using a phase-locked all-pass fiber resonator,” Opt. Express 23(8), 10117–10125 (2015).
23. L. Yang, H. Yang, H. Zhang, H. Wei, and Y. Li, “Repetition rate multiplication of frequency comb using all-pass fiber resonator,” Rev. Sci. Instrum. 87(9), 093101 (2016).
24. M. S. Kirchner, D. A. Braje, T. M. Fortier, A. M. Weiner, L. Hollberg, and S. A. Diddams, “Generation of 20 GHz, 1.96 nm via repetition-rate multiplication,” Opt. Lett. 34(7), 872–874 (2009).
25. B. Edvold and L. Grüner-Nielsen, in Proceedings of 22nd European Conference on Optical Communication, (IEEE, 1996), pp. 245–248.
26. P. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, “Laser phase and frequency stabilization using an optical resonator,” Appl. Phys. B 31(2), 97–105 (1983).
27. T. W. Hänsch and B. Couillaud, “Laser frequency stabilization by polarization spectroscopy of a reflecting reference cavity,” Opt. Commun. 35(3), 441–444 (1980).

1. Introduction

Optical frequency combs have become indispensable tools in various science and technology fields [1,2]. High-repetition-rate ($f_{rep}$) optical frequency combs possess the higher available power per mode compared to lower $f_{rep}$ comb with $f_{rep}$ of around 100 MHz, and each comb tooth can be resolved by a wavemeter or spectrometer. Therefore, they are required in various applications such as high-precision spectroscopy [3,4], optical communications [5], arbitrary waveform generation [6], high-precision microwave generation [7,8], astronomical spectrograph calibration [9,10], and absolute distance measurements [11]. Many of these applications require phase-stabilized frequency combs, that is, the detection and stabilization of $f_{rep}$ and carrier envelope offset frequency ($f_{ceo}$). A Kerr-lens mode-locked Yb:Y2O3 ceramic laser has been reported [12], and the direct generation of a $f_{rep} = 17$ GHz frequency comb has been reported. However, for mode spacings above a few gigahertz, it is difficult to achieve used self-referencing scheme because of the low pulse energy. In contrast, the fiber-based frequency comb has been widely used in various applications involving phase-stabilized frequency comb generation as $f_{ceo}$ can be directly detected, provides systems with long-term stability, and is compact and easy to operate at various wavelengths. In the case of a mode-locked Yb:fiber laser, the generation of a frequency comb with $f_{rep}$ near 1 GHz and the direct detection [13,14] and stabilization of $f_{ceo}$ using the self-referencing scheme have been reported [15]. However, for higher fundamental $f_{rep}$, it is difficult to shorten the cavity length of the laser owing to the physical limitation as it reduces the pulse peak power, which should be maintained above a certain threshold to maintain the nonlinearity required for mode...
locking. The generation of $f_{\text{rep}} = 3$-GHz frequency combs has been reported with a Yb:fiber laser; however, the direct detection of $f_{\text{ceo}}$ has not yet been demonstrated [16]. Er:fiber-based frequency combs are most commonly used for phase stabilized frequency comb generation with an all-fiber setup [17,18]. However, their $f_{\text{rep}}$ is typically around 100 MHz and it is difficult to shorten the cavity length of the laser because of the physical limitation.

In order to overcome these limitations, the mode-filtering of low-$f_{\text{rep}}$ frequency combs has been widely used. In the conventional method, the multiplication of low-$f_{\text{rep}}$ frequency combs, whose $f_{\text{rep}}$ and $f_{\text{ceo}}$ are stabilized, has been widely studied using Fabry–Perot cavities (FPCs) [19–21]. This technique is capable of spacing modes up to several tens of gigahertz, and improving in the signal detection sensitivity in microwave generation [20] and dual-comb spectroscopy [21]. However, an FPC consists of two mirrors and free space, so it requires critical beam alignment and suffers from environmental fluctuations. Therefore, it can be detrimental to the robustness of the Er:fiber-based comb during environmental fluctuations. In contrast, fiber-based ring cavities consisting of single fiber couplers (FCs) have recently begun to attract attention as a robust and compact system for $f_{\text{rep}}$ multiplication [22,23]. Unfortunately, these cavities were only able to demonstrate a multiplication factor of 2 because of the physical limitation inhibiting the shortening of the cavity length, which is required to expand the spacing of the filtered comb modes. In addition, the finesse of the fiber cavity was very low ($\sim 6$) because of the large coupling loss ($\sim 74\%$) of the free-space optical delay line inside the cavity and the large splice loss between the dispersion compensating fiber (DCF) and single-mode fiber (SMF).

In this study, we propose a mode-filtering technique based on an all-fiber-based long ring cavity whose length is longer than the laser cavity length, i.e., the pulse separation. Because the physical limitation regarding the cavity length is greatly mitigated, it can be utilized for the high multiplication of $f_{\text{rep}}$. The length of the fiber cavity was set to 6.7 m (free spectral range (FSR) = 44.7 MHz) as the pulse separation was 6.1 m, and we were able to demonstrate a multiplication factor of 11, i.e., $f_{\text{rep}}$ increased from 48.7 to 536.0 MHz with a side mode suppression ratio (SMSR) of about 25 dB owing to the all-fiber-based double-pass configuration. The developed technique can be applied to high-$f_{\text{rep}}$, all-fiber-based frequency comb generation that is able to detect $f_{\text{ceo}}$ directly, so is strongly desired in various applications.

2. Experimental setup for mode-filtering technique based on an all-fiber-based long ring cavity

The optical length of the external fiber-ring cavity $nL$ ($n$: refractive index of fiber, $L$: cavity length) was set to $(M \pm m)/M$ times the pulse separation $c/f_{\text{rep}}$, where $c$ is the speed of light in a vacuum, $M$ is the multiplication factor, and $m$ is an integer, i.e., $M \times f_{\text{rep}} = (M \pm m) \times \text{FSR}$ of the fiber cavity. By applying two FCs to the fiber cavity, the transmission spectrum exhibits sharp maxima at the resonant frequencies, therefore, comb modes with a separation of $M \times f_{\text{rep}}$ can pass into the cavity. Moreover, in the case of $(M + m)$, the fiber cavity length is longer than the laser cavity length, therefore, the physical limitation inhibiting the shortening of the length of the fiber ring cavity is greatly mitigated.

Figure 1 illustrates the experimental setup of the all-fiber-based mode filter for $f_{\text{rep}}$ multiplication. A mode-locked Er:fiber laser is used as a light source. The $f_{\text{rep}}$ is tunable from 48.55 to 48.95 MHz using a fiber delay line in the laser cavity, with a central wavelength of 1560 nm. The output is passed into the fiber ring cavity via a circulator, an Er:fiber amplifier (EDFA1), and a polarization controller (PC1). The fiber cavity consists of an SMF, DCF, and two FCs with a 99/1 split ratio. The input and output of the fiber cavity are connected to 1% ports, and the inside of the cavity is connected to the 99% ports of the FCs. By applying two FCs to the fiber cavity, the transmission spectrum exhibits sharp maxima at the resonant frequencies, similar to a free-space FPC; therefore, comb modes with a separation of $M \times f_{\text{rep}}$ can pass into the cavity. After passing through the cavity, the output of the cavity is sent into
the fiber loop mirror via PC2, EDFA2, and the coupler for the single-pass cavity output. Then, the output is reflected from the fiber loop mirror and sent into the same cavity via the same fiber path. After passing through the cavity again, the double-pass cavity output is emitted from the circulator. From this, the all-fiber-based double-pass configuration [24] is realized.

The DCF is included inside the cavity to compensate for a net dispersion in the cavity of zero around the wavelength of 1550 nm ($\lambda_0$). Owing to the large difference between the mode field diameters (MFDs) of the SMF (SMF28e +, Corning, MFD = 10.4 μm at $\lambda_0$) and DCF (DCF38, Thorlabs, MFD = 6.0 μm at $\lambda_0$), the splice loss is large (our optimized value: 0.8 dB). To reduce the loss by using an intermediate fiber (IMF) between the SMF and DCF [25], we used a 1-μm SMF (980HP, Nufern, MFD = 6.8 μm at $\lambda_0$) and improved the splice loss by about 0.3 dB.

![Experimental setup](image)

Fig. 1. Experimental setup. LD: Laser diode, WDM coupler: Wavelength division multiplexing coupler, ISO: Isolator, EDF: Erbium-doped fiber, PZT: Piezoelectric transducer, PC: Polarization controller, ODL: Optical delay liner, EDFA: Erbium-doped fiber amplifier, DCF: Dispersion compensating fiber, IMF: Intermediate fiber. The black line represents the single-mode fiber (SMF).

In this experiment, we demonstrated mode filtering with $M = 11$, thus, the optical length of the fiber cavity was set to 6.7 m with $m = 1$ ($nL = (M + 1)/M \times c/f_{rep}$) for $f_{rep} = 48.7$ MHz and, therefore, the FSR of the fiber cavity was 44.7 MHz ($(11 + 1) \times FSR = 11 \times f_{rep} = 536.0$ MHz). As depicted in Fig. 2, the frequencies of the comb modes with a separation of $11 \times f_{rep}$ match the transmission peaks of the fiber cavity with a separation of $12 \times$ FSR in this case. The physical lengths of the SMF, IMF, and DCF were 3.0, 0.2, and 1.4 m, respectively. For the adjustment of the transmission peaks and comb modes, we tuned the $f_{rep}$ of the fiber-based frequency comb.

![Mode-filtering effect](image)

Fig. 2. Depiction of mode-filtering effect with a long-fiber-based ring cavity. In this case, the multiplication factor $M = 11$ and the integer $m = 1$, i.e., $11 \times f_{rep} = 12 \times$ FSR.
3. Experimental results

3.1 Transmission peaks of the long-fiber-based ring cavity

First, the basic performance of the developed fiber ring cavity was evaluated with a single-frequency CW laser as a light source. Figure 3 shows the transmission peaks of the cavity output when the laser frequency is swept. As shown in Fig. 3 (a), the FSR and the full-width at half-maximum (FWHM) linewidth of the transmission peak (Δν) are measured to be about 44.7 and 2.4 MHz, respectively, corresponding to a cavity finesse value of about 18.9 (FSR/Δν). The finesse of the fiber cavity can be expressed as π√K/(1 – K), thus, the coupling ratio K of the cavity is approximately 84%. This value corresponds to the estimated internal loss of the cavity (~81%) owing to the splice loss between the DCF and SMF. The splice loss can be improved by parameter optimization. Figure 3 (b) shows the transmission peaks of the double-pass cavity output, and the FSR is the same as the single-pass case, however, Δν decreases because of the double-pass. From this result, an improvement in the SMSR of the unwanted comb modes is expected.

![Fig. 3. Transmission peaks of the fiber cavity output when a single-frequency CW laser is used as a light source. (a) Single-pass cavity output and (b) double-pass cavity output.](image)

3.2 Optical and RF spectra of single and double-pass cavity outputs

Figure 4 presents the optical and RF spectra of the input (Figs. 4 (a) and (b)), single-pass cavity output (Figs. 4 (c) and (d)), and double-pass cavity output (Figs. 4 (e) and (f)). In this evaluation, we did not stabilize the cavity length and environmental temperature around the cavity, however, it was not a major issue in the range of the measurement results described in this paper to demonstrate our proposed mode-filtering technique. Further cavity stabilization can be implemented to improve performances. The FWHM of the optical spectra of the single and double-pass cavity outputs are 18.3 and 16.0 nm, respectively. Because of the dispersion compensation by using the DCF, almost no spectral narrowing is observed. In Fig. 4 (c) and (e), the center wavelength of the optical spectra shifts to shorter wavelengths. The dispersion compensated bandwidth is 1530-1560 nm. In contrast, the center wavelength of the input is about 1560 nm and is at the edge of the compensated bandwidth; therefore, it shifted. The RF
spectra shown in Figs. 4 (b), (d) and (f) were measured using a fast photodiode (F7096FC, Hamamatsu), and the incident powers were 0.6 mW. In Fig. 4 (d), strong peaks at 536.0 MHz and its harmonics are observed with an SMSR of ~9 dB in the single-pass case. On the contrary, in Fig. 4 (f), the SMSR improves to ~25 dB because of the double-pass cavity output. The SMSR of ~25 dB is already sufficient for various applications in frequency domain, e.g., sensitivity improvement of dual-comb spectroscopy [21]. By decreasing the splicing loss between DCF and SMF, we expect to improve the SMSR. A frequency of 536.0 MHz is obtained for the set value of $M = 11$ and $m = 1$ for $f_{\text{rep}} = 48.7$ MHz, i.e., $11 \times f_{\text{rep}} = 536.0$ MHz.

We also evaluated time-domain characteristics. The output of the cavity was detected using a 1 GHz photodiode (1611FC-AC, Newfocus) and displayed on a fast oscilloscope. Figure 5 presents the time domain signals of the input (Fig. 5 (a)), single-pass cavity output (Fig. 5 (b)), and double-pass cavity output (Fig. 5 (c)). As shown in Figs. 5 (b) and (c), the single-pass cavity output shows more than 95% amplitude modulation at the original 48.7 MHz repetition rate, while the double-pass cavity output shows less than 35% amplitude modulation. These results demonstrate that the all-fiber-based double-pass configuration is a powerful technique for the mode-filtering of the fiber-based frequency comb also in time-domain.
3.3 Evaluation in optical frequency domain

In addition to RF domain evaluation, we measured the heterodyne beat signal between the filtered comb mode and single-frequency CW laser (ORION, RIO) at a wavelength of 1560 nm (Fig. 6 (a)). As shown in Figs. 6 (b) and (c), the single-pass cavity exhibits an SMSR of approximately 9 dB, while the double-pass cavity provides an SMSR of about 25 dB. Theoretically, the SMSRs should be 6 dB larger than the RF domain evaluations, however, here we obtain similar values because the cavity length was not stabilized. We expect higher SMSRs in optical frequency domain when the cavity is stabilized. In this study, our goal is to demonstrate the mode-filtering technique with a long-fiber-based ring cavity. Further study will include the cavity stabilization by using the Pound-Drever-Hall (PDH) technique [26] or the Hänsch-Couillaud method [27]. From these results, mode-filtered comb modes and an improvement in the SMSR in the optical frequency domain are confirmed. To the best of our knowledge, this is the highest multiplication factor ever achieved for $M = 11$ and $f_{\text{rep}}$ using a fiber-based ring cavity.
4. Conclusions

In this paper, we demonstrated an all-fiber-based mode-filtering technique for the $f_{\text{rep}}$ multiplication of a low-$f_{\text{rep}}$ fiber-based frequency comb. The length of the external fiber ring cavity was longer than that of the fiber laser cavity. Using such a long fiber cavity scheme greatly mitigates the physical limitation to achieve high $f_{\text{rep}}$ multiplication. The cavity length was set to 6.7 m (FSR = 44.7 MHz) as the pulse separation was 6.1 m ($f_{\text{rep}} = 48.7$ MHz), and $f_{\text{rep}}$ multiplication with $M = 11$ was demonstrated. In addition, the all-fiber-based double-pass configuration generated comb modes with $11 \times f_{\text{rep}} = 536.0$ MHz and an SMSR of 25 dB. The SMSR is already sufficient for various applications in frequency domain, e.g., sensitivity improvement of dual-comb spectroscopy [21]. On the other hand, in time domain, the residual interleaved pulse train causes amplitude modulation. Therefore, higher SMSR is demanded for applications especially in time domain. In the range of the current configuration, further multiplication factor will cause degradation of SMSR. However, by decreasing the splice loss between DCF and SMF, we expect to improve the SMSR, and we obtained high SMSR of >40 dB in preliminary experiment. Moreover, already with the demonstrated multiplication factor of 11, we can generate mode-filtered comb modes with 2.75 GHz $(= 11 \times 250$ MHz) with our proposed technique by using a commercially available fiber-based frequency comb with $f_{\text{rep}}$ of 250 MHz. Therefore, even higher repetition rate should be available with our proposed technique. This technique does not impair the features of the original fiber-based comb, and $f_{\text{CEO}}$ can be detected and stabilized and is robust to environmental fluctuations.
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