Estimating of higher order velocity moments and their derivatives in boundary layer by Smoke Image Velocimetry

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Abstract. The results of an experimental evaluation of the third-order moments profiles of velocity fluctuations and their partial derivatives in a zero pressure-gradient turbulent boundary layer are presented. Profiles of characteristics are estimated on the basis of the dynamics of two-component instantaneous velocity vector fields measured by the optical method Smoke Image Velocimetry (SIV). Comparison SIV-measurements with the results of measurements by a thermoanemometer and DNS data with similar Re\(_0\) and Re\(_\theta\) showed good agreement between the profiles of \(\langle u'^2 v' \rangle^*, \langle u' v'^2 \rangle^*, \partial <u'^2 v'> */\partial y^*\) and \(\partial <u' v'^2>/\partial y^*\) obtained by SIV and DNS.

1. Introduction

The physical interpretation of third-order velocity moments are the convective transport of second-order moments by the turbulent velocity fluctuation [1, 2]. Triple correlations play a defining role in the diffusion term of the turbulent energy balance equation which is the basis for analytical and numerical methods of investigating the boundary layer. The importance of estimating the triple correlations \(\langle u'^3 \rangle, \langle u'^2 v' \rangle, \langle u' v'^2 \rangle, \langle v'^3 \rangle\) was noted in the work of Townsend [3] long enough. In describing of complex unsteady flows by the Reynolds-averaged Navier-Stokes equations, the order of the moments in the closing equations has a significant effect on the uncertainty of the solution. As an example would be flows with dominant convective forces [4]. Nevertheless, the inclusion of higher order moments in the empirical turbulence models is hampered by the insufficient volume of the accumulated experimental database. The results of estimating of triple correlations in zero pressure-gradient boundary layer obtained from the results of measurements by a thermoanemometer are presented in [5-13]. The point method of laser Doppler anemometry for this purpose was used much less often [14-15]. In the case of estimating higher-order moments from the results of measurement with a thermoanemometer the uncertainty of the estimate depends essentially on the scale of the measurement, which is determined by the distance between the wires and their length [16]. Obviously, one of the possible solutions to this problem is the use of thermoanemometric probes with reduced wire sizes, in particular, for the estimation of third-order moments in work [13] a thermoanemometer with platinum wire of 60 \(\mu\)m in length and 2 \(\mu\)m in diameter was used. The second approach to reducing the uncertainty of estimates of small-scale characteristics is the use of empirical-based algorithms for filtering the signal from...
measurement errors [16-21]. Nevertheless, the question of the uncertainty of estimating higher-order moments from the results of measurement with a thermoanemometer is still open [22].

However, if we return to the main reason for the relevance of the experimental estimate of triple correlations, namely, to the rate of advection of turbulent energy by turbulent motion

$$- \frac{\partial}{\partial x_j} \left( \frac{1}{2} u_i u_j \right),$$

(1)

there are new obvious problems and uncertainties associated with the determination of spatial gradients from the measurement results by point methods, so this estimate is potentially more correct to be performed on the basis of the measurement results by field methods.

The profiles of triple correlations estimated from measurements of the field optical method Particle Image Velocimetry (PIV) were performed in [23]. Unfortunately, according to the authors of this paper the profiles of spatial gradients of the third-order moments were not reliable and were not performed. This can be explained by the limitation of the PIV method on the balance between spatial resolution and uncertainty in the measurement of small-scale turbulence structures [24-36]. In part, this problem was solved in two new spatial measurement techniques "Shake-The-Box" (STB) [37] and VIC+ [38]. However, these two approaches have not yet been tested to estimate higher-order moments of the velocity fluctuations.

The use of the high-speed optical method for measuring the instantaneous velocity fields Smoke Image Velocimetry (SIV) [39, 40] due to the multiply higher tracer concentration compared to PIV and PTV (which helps reduce measurement noise with increasing spatial resolution) allows estimating the fields of small-scale characteristics with a sufficient degree of accuracy, for example, [40] presents an estimate of the dissipation term for the conservation equation of turbulent energy with a spatial resolution to 1.6-2.0 of the Kolmogorov scale and temporal resolution of 7 kHz. The results of the evaluation of the second and third orders moments and their derivatives in a turbulent boundary layer obtained on the basis of SIV-measurements are presented in this paper.

2. Experimental setup

The structure of air flow in a smooth plane channel has been studied using the optical method of Smoke Image Velocimetry (SIV). Experimental setup is shown in figure 1. The setup arrangement and flow parameters were very close to the ones in the experiments by Willert [41], in which turbulence characteristics in the developed turbulent boundary layer were studied using PIV technique. The test section 1 was a rectangular 75×150 mm² channel with the length of 1 m and a smooth inlet 10 with 6:1 contraction. A turbulence generating grid 9 with 5 mm cell size, 1.2 mm steel wire diameter and 36% solidity was mounted downstream of the smooth inlet 1. A 50-mm long strip of abrasive P24 (ISO 6344-2) 12 was glued onto the channel perimeter. This provided fully developed turbulent boundary layer in the measurement area during the experiments. Channel walls were made of transparent materials (glass and polycarbonate). Stable air flow rate downstream of the test section was provided by a regulating gate 11 and a 1.3 m³ receiver tank 2 mounted upstream of the latter. Flow rate was measured by an ultrasonic flowmeter 3 IRVIS RS4-Ultra mounted downstream of the receiver tank. The relative error in flow rate did not exceed 1%.

To visualize the flow pattern, the air-aerosol mixture (MT-Gravity fluid with medium fog density and average particle size of 0.1…5 μm; Safex aerosol generator 5) was supplied from the preparation chamber 4 to the channel inlet. The measurement area 6 was illuminated by a continuous diode-pumped solid-state laser KLM-532/5000-h 7. The flow pattern in the channel symmetry plane at the distance of $L = 0.7$ m from the turbulence generating grid 9 (figure 1) was recorded by a monochrome high-speed camera Fastec HiSpec 8 with the frame resolution of 665×110 pixel (scaling factor of 0.0625 mm/pixel), frame rate $f = 7083$ 1/s, and recording time of 3.5 s. The camera was equipped with a Navitar 1″F/0.95 lens (focal length 25 mm, manual focus).

Flow velocity fields were measured by optical SIV technique based on digital processing of flow pattern video recordings. Here, velocity vector fields were estimated by the analysis of turbulent structure displacements visualized by smoke. Profiles of velocity and turbulent fluctuations were
estimated from 16×16 pixel windows as well. Maximum displacement of turbulent structures between two consecutive frames was 10 pixels (at the boundary layer edge). Image resolution in y⁺ coordinates was 1 pixel = 0.8 y⁺.

![Figure 1. Experimental setup](image)

3. Results and discussion

Table 1 presents the main characteristics of the investigated turbulent boundary layer. Profiles of turbulent fluctuation intensity and triple correlation were written in wall coordinates:

\[
y⁺ = \frac{yu_τ}{v}, \quad \frac{u_μu_μ}{u_τ} = \frac{u_μu_μ}{u_τ},
\]

where dynamic velocity, uₜ, was estimated by using the results of velocity measurement at a point within the viscous sublayer.

Figure 2 shows the profiles of triple correlations <u²v²> and <u'v'²> at close values of Reₐ and Reₜ estimated by the results of SIV measurements, by the thermoanemometer [7] and obtained by the results of DNS [42]. The profile <u²v²> estimated from the results of SIV-measurements qualitatively and quantitatively agrees well with the results of DNS. In the viscous sublayer negative values of <u²v²> monotonically decreasing at a distance from the wall are observed. In part of transition the viscous sublayer to the buffer region near the coordinate y⁺≈8, the local extremum is clearly observed on the <u²v²> profiles. In the buffer layer, the profile <u²v²> monotonically increases to the coordinate y⁺≈30 where the second local extremum is observed. Further from this coordinate to the edge of boundary layer, <u²v²> decreases monotonically to an asymptotic zero value. A similar behavior is observed for the profile <u'v'²> but both the SIV and DNS results indicate a shift of the second extremum into the depth of the logarithmic sublayer. The described features of the evolution of the profiles along the thickness of boundary layer are not found on the profiles estimated from the results of measurements by the thermoanemometer [7], however, triple correlations have a similar order. The graphs also clearly demonstrate the advantage of the field optical method over the point method with respect to the spatial frequency of measurements.

The profiles of the triple correlation derivatives <u²v²>' and <u'v'²>' (figure 3) taken along the normal to the wall describe the transfer of turbulent energy by turbulent velocity fluctuations. The quantities \(\partial\frac{<u²v²>}{\partial y⁺}\) and \(\partial\frac{<u'v'²>}{\partial y⁺}\) describe respectively the transport (diffusion) of the double correlations u² and u'v' over the thickness of boundary layer [23]. The profiles of the partial differentials <u²v²>' and <u'v'²>' in figure 3 are obtained by approximating the DNS, SIV, and thermoanemometric measurements using a three-point central difference scheme. The trends of profiles \(\partial\frac{<u²v²>}{\partial y⁺}\) and \(\partial\frac{<u'v'²>}{\partial y⁺}\) estimated by the results of SIV-measurements, in spite of the relatively large spread of discrete values, are generally agree satisfactorily with DNS results. The application of the SIV-measurement technique made it possible to estimate the location of the extreme point and the sign of \(\partial\frac{<u²v²>}{\partial y⁺}\) and \(\partial\frac{<u'v'²>}{\partial y⁺}\) with good agreement with the DNS results. At the same time, the profiles \(\partial\frac{<u²v²>}{\partial y⁺}\) and \(\partial\frac{<u'v'²>}{\partial y⁺}\) estimated from the results of measurements by the thermoanemometer [7] showed a satisfactory agreement with DNS results only in order of magnitude.

4. Conclusions

For the considered turbulent boundary layer, the optical method of SIV made it possible to estimate the profiles of <u²v²>' and <u'v'²>' with a reasonable degree of accuracy in comparison with the results of DNS [42] with similar Reₜ. Profiles \(\partial\frac{<u²v²>}{\partial y⁺}\) and \(\partial\frac{<u'v'²>}{\partial y⁺}\) estimated by the results of SIV-measurements, in spite of the available scatter of values, on the whole describe DNS results well enough.
Table 1. Characteristics of turbulent boundary layer measured

| Characteristic                                      | Value | Unit |
|----------------------------------------------------|-------|------|
| Shear velocity at the wall (s⁻¹)                   | 2767  |      |
| Dynamic velocity, \( u_c \) (m/s)                  | 0.204 |      |
| Velocity at the channel axis, \( U_\infty \) (m/s) | 4.06  |      |
| Turbulent boundary layer thickness, \( \delta_{99\%} \) (mm) | 15.88 |      |
| Displacement thickness, \( \delta^* \) (mm)       | 2.44  |      |

Figure 2. Profiles of triple correlations; - DNS [42] (Re\( \tau \)=180); - hot wire [7] (Re\( \theta \)=706); - SIV (Re\( \tau \)=214, Re\( \theta \)=425)

Figure 3. Profiles of triple correlation derivatives; - approximation of DNS results [42] (Re\( \tau \)=180); - approximation of hot wire results [7] (Re\( \theta \)=706); - SIV (Re\( \tau \)=214, Re\( \theta \)=425)

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