Lower Limits on the Strengths of Gamma Ray Lines from WIMP Dark Matter Annihilation

Kevork N. Abazajian,1, 2 Prateek Agrawal,1 Zackaria Chacko,1 and Can Kilic3

1Maryland Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, MD 20742
2Department of Physics & Astronomy, University of California, Irvine, CA 92697
3Theory Group, Department of Physics and Texas Cosmology Center, The University of Texas at Austin, Austin, TX 78712

We study the spectra of gamma ray signals that arise from dark matter annihilation in the universe. We focus on the large class of theories where the photon spectrum includes both continuum spectrum of gamma rays that arise from annihilation into Standard Model states at tree level, as well as monochromatic gamma rays arising from annihilation directly into two photons at the one loop level. In this class of theories we obtain lower bounds on the ratio of the strength of the gamma ray line relative to the gamma ray continuum as a function of the dark matter mass and spin. These limits are obtained from the unitarity relation between the tree level amplitude of the primary annihilation channel and the imaginary part of the loop level amplitude for annihilation directly into photons, with the primary decay products running in the loop. These results are exact in the limit that dark matter annihilation is exclusively to a single Standard Model species, occurs through the lowest partial wave and respects CP. Away from this limit the bounds are approximate. Our conclusions agree with the known results in the literature in the case of the Minimal Supersymmetric Standard Model (MSSM). We use the Fermi-LAT observations to translate these limits into upper bounds on the dark matter annihilation cross section into any specific Standard Model state.

I. INTRODUCTION

Cosmological measurements have now established that about 80% of the matter in the universe is composed of non-luminous, non-baryonic dark matter [1]. However, the precise nature of the particles of which dark matter is composed remains a mystery. Weakly Interacting Massive Particles (WIMPs) - thermal relics with weak scale masses and weak scale cross sections with visible matter - constitute one well-motivated class of dark matter candidates. In these theories the relic abundance of the WIMP is set by its annihilation cross section into Standard Model (SM) particles, and turns out to be naturally of the right order to explain observations.

One approach to dark matter detection involves searching for the products of WIMP annihilation in the universe, such as photons. Dark matter is constrained to be neutral under electromagnetism, and therefore in renormalizable theories WIMPs cannot annihilate directly into photons at tree level. Nevertheless, a continuum spectrum of photons arises from decays of the primary annihilation products, and also from final state radiation off charged final states. The spectrum also includes monochromatic gamma rays that arise from annihilation directly into $\gamma\gamma$, $\gamma Z$ and $\gamma H$ final states, usually at loop level.

The continuum spectrum of photons arising from dark matter annihilation into any specific final state is to a large extent independent of the detailed form of the matrix element for the process, depending only on the SM quantum numbers of the particles in the final state and the dark matter mass. This allows limits on the observed diffuse gamma ray flux from regions of high dark matter density to be translated into robust bounds on the rate of dark matter annihilation into any such state.

Over the last few years the Large Area Telescope (LAT) aboard the Fermi Gamma-ray Space Telescope has been making precise observations of the gamma ray sky. Limits on the rate of dark matter annihilation into various final states have been placed by Fermi-LAT measurements of the gamma ray flux from nearby galaxy clusters [2], dwarf galaxies [3, 4], the Galactic center [5] and subhalos [6]. The diffuse gamma ray flux measurement from portions of the sky [7, 8] as well as the isotropic near full-sky [9, 10] have been used to set bounds on dark matter annihilation.

In typical models of WIMP dark matter, the relic abundance is set by tree level annihilation to final states consisting of two SM fermions or two weak gauge bosons. In this scenario monochromatic gamma rays corresponding to annihilation to the $\gamma\gamma$, $\gamma Z$ and $\gamma H$ final states are only generated at one loop. The strengths of the corresponding gamma ray lines are highly model dependent, being very sensitive to the detailed structure of the corresponding matrix element, which in turn depends on the details of the vertices in the theory. Furthermore, the amplitude will in general receive significant contributions from unknown non-SM states (associated with new physics) running in the loop. Therefore, the Fermi-LAT bounds on gamma ray lines [11] have primarily been used to probe specific well-motivated models. In particular, WIMP models of this type for which the one loop cross sections to the $\gamma\gamma$ and $\gamma Z$ final states have been computed include the Minimal Supersymmetric Standard Model (MSSM) [12–15], Universal Extra Dimensions (UED) [16, 17], the Littlest Higgs Model with T-Parity [18, 19] and the Inert Doublet Model [20]. In theories where dark matter is
a scalar or Majorana fermion, annihilation to the $\gamma H$ final state is suppressed in the non-relativistic limit. However, the cross section to this and to the $\gamma Z$ final state have been computed in a specific theory of Dirac dark matter [21] that arises in a class of Randall-Sundrum models. A model-independent approach based on effective operators that correlates the signal in direct detection experiments with the approximate line strength has also been developed [22], (see also [23]).

In this paper, we place lower bounds on the strengths of gamma ray lines from dark matter annihilation that are applicable to a large class of theories. We focus on models where the photon spectrum includes both continuum gamma rays that arise from annihilation into two body SM final states at tree level, as well as monochromatic gamma rays arising from annihilation directly into two photons at the one loop level. For this class of theories we obtain lower limits on the ratio of the strength of the gamma ray line relative to the gamma ray continuum as a function of the dark matter mass and spin. These limits are obtained from the unitarity relation between the tree level amplitude of the primary annihilation channel and the imaginary part of the loop level amplitude for annihilation directly into photons, with the primary decay products running in the loop. These bounds are exact in the limit that dark matter annihilates exclusively into a single SM species, that the annihilation is dominated by the $L = 0$ partial wave, so that the initial state $|i\rangle$ has $J = S$. While $J$ and $M$ are conserved in the annihilation process, this is not true of $L$ or $S$.

There is an alternative basis for labelling two particle states in the center of mass frame, the helicity basis, which will also prove useful. In this basis, in addition to the internal quantum numbers of the particles and their total energy, the states are labelled by the angular momentum quantum numbers $|J, M; \lambda_1, \lambda_2\rangle$, where $\lambda_1$ and $\lambda_2$ are the helicities of the two particles. Unlike $J$ and $M$, the helicities are not conserved in the annihilation process. The transformation that relates the $|J, M; \lambda_1, \lambda_2\rangle$ basis to the $|J, M; L, S\rangle$ basis may be found in the classic paper by Jacob and Wick [24].

Now, if the theory is invariant under time reversal (or equivalently $CP$), and the states $|i\rangle$ and $|f\rangle$ are eigenstates of angular momentum, labelled either by $|J, M; L, S\rangle$ or by $|J, M; \lambda_1, \lambda_2\rangle$, then, as shown in the appendix, $\langle f|T|i\rangle = \langle i|T|f\rangle$. Eq. (3) then simplifies to

$$2\text{Im}\langle f|T|i\rangle = \sum_X \langle f|T^\dagger|X\rangle \langle X|T|i\rangle.$$ (4)

At lowest order in perturbation theory, this equation relates the imaginary part of the loop amplitude for annihilation into the two photon final state to the tree amplitude for annihilation into two body SM states. Squaring, we get

$$4|\text{Im}\langle f|T|i\rangle|^2 = \sum_X |\langle f|T^\dagger|X\rangle \langle X|T|i\rangle|^2.$$ (5)

We now restrict ourselves to the case where dark matter annihilation at tree level is exclusively to a single SM species. If the SM state $|X\rangle$ is further characterized by fixed values of $L$ and $S$ (or fixed values of $\lambda_1$ and $\lambda_2$) for any given values $J$ and $M$ of the initial state $|i\rangle$, then

$$4|\text{Im}\langle f|T|i\rangle|^2 = |\langle f|T^\dagger|X\rangle \langle X|T|i\rangle|^2.$$ (6)

This equation implies that for all initial states $|i\rangle$ that annihilate at tree level exclusively to the SM state $|X\rangle$ characterized by fixed $L$ and $S$ (or fixed $\lambda_1$ and $\lambda_2$), the ratio

$$\frac{|\text{Im}\langle f|T|i\rangle|^2}{|\langle X|T|i\rangle|^2} = \frac{1}{4} |\langle f|T|X\rangle|^2.$$ (7)

II. LOWER LIMITS ON LINE STRENGTHS

In this section, we explain in detail how these limits are obtained. Unitarity implies that the $S$-matrix satisfies

$$S^\dagger S = 1.$$ (1)

Writing $S = 1 + iT$, where $T$ is the transition matrix, we obtain

$$-iT = T^\dagger T.$$ (2)

Consider the matrix element of this equation between the initial state $|i\rangle$ consisting of the two dark matter particles, and the final state $|f\rangle$ consisting of two photons,

$$-i\langle f|(T - T^\dagger)|i\rangle = \sum_X \langle f|T^\dagger|X\rangle \langle X|T|i\rangle,$$ (3)

where the sum over $|X\rangle$ runs over all final states into which the dark matter particle can annihilate.

In the center of mass frame, two particle states can be labelled by the internal quantum numbers of the particles, the total energy, and the angular momentum quantum numbers $|J, M; L, S\rangle$, where $J$ is the total angular momentum, $M$ is the component of total angular momentum along any fixed axis, $L$ is the orbital angular momentum and $S$ the total spin angular momentum. Dark matter annihilation in haloes occurs in the highly non-relativistic regime. We therefore focus on the case where annihilation occurs through the $L = 0$ partial wave, so that the initial state $|i\rangle$ has $J = S$. While $J$ and $M$ are conserved in the annihilation process, this is not true of $L$ or $S$.

There is an alternative basis for labelling two particle states in the center of mass frame, the helicity basis, which will also prove useful. In this basis, in addition to the internal quantum numbers of the particles and their total energy, the states are labelled by the angular momentum quantum numbers $|J, M; \lambda_1, \lambda_2\rangle$, where $\lambda_1$ and $\lambda_2$ are the helicities of the two particles. Unlike $J$ and $M$, the helicities are not conserved in the annihilation process. The transformation that relates the $|J, M; \lambda_1, \lambda_2\rangle$ basis to the $|J, M; L, S\rangle$ basis may be found in the classic paper by Jacob and Wick [24].

Now, if the theory is invariant under time reversal (or equivalently $CP$), and the states $|i\rangle$ and $|f\rangle$ are eigenstates of angular momentum, labelled either by $|J, M; L, S\rangle$ or by $|J, M; \lambda_1, \lambda_2\rangle$, then, as shown in the appendix, $\langle f|T|i\rangle = \langle i|T|f\rangle$. Eq. (3) then simplifies to

$$2\text{Im}\langle f|T|i\rangle = \sum_X \langle f|T^\dagger|X\rangle \langle X|T|i\rangle.$$ (4)

At lowest order in perturbation theory, this equation relates the imaginary part of the loop amplitude for annihilation into the two photon final state to the tree amplitude for annihilation into two body SM states. Squaring, we get

$$4|\text{Im}\langle f|T|i\rangle|^2 = \sum_X |\langle f|T^\dagger|X\rangle \langle X|T|i\rangle|^2.$$ (5)

We now restrict ourselves to the case where dark matter annihilation at tree level is exclusively to a single SM species. If the SM state $|X\rangle$ is further characterized by fixed values of $L$ and $S$ (or fixed values of $\lambda_1$ and $\lambda_2$) for any given values $J$ and $M$ of the initial state $|i\rangle$, then

$$4|\text{Im}\langle f|T|i\rangle|^2 = |\langle f|T^\dagger|X\rangle \langle X|T|i\rangle|^2.$$ (6)

This equation implies that for all initial states $|i\rangle$ that annihilate at tree level exclusively to the SM state $|X\rangle$ characterized by fixed $L$ and $S$ (or fixed $\lambda_1$ and $\lambda_2$), the ratio

$$\frac{|\text{Im}\langle f|T|i\rangle|^2}{|\langle X|T|i\rangle|^2} = \frac{1}{4} |\langle f|T|X\rangle|^2.$$ (7)
is a constant that depends only on SM parameters, and is otherwise independent of $|i\rangle$. Furthermore, under these conditions the quantity

$$\frac{|\langle f|T|i\rangle|^2}{|\langle X|T|i\rangle|^2} \geq \frac{1}{4} |\langle f|T|X\rangle|^2$$

is bounded from below. The numerator of the expression on the left hand side is proportional to the cross section for $|i\rangle \rightarrow |f\rangle$, while the denominator is proportional to the cross section for $|i\rangle \rightarrow |X\rangle$. The right hand side is proportional to the absolute value of the tree level matrix element for annihilation of the state $|X\rangle$, consisting of two SM particles, into two photons. This can be calculated within the SM. Therefore this expression can be translated into a lower bound on the strength of the gamma ray line relative to the continuum.

For the bound to apply, dark matter annihilation at tree level must occur exclusively into a single SM final state. Furthermore, this state must be characterized by definite values of $L$ and $S$ (or definite $\lambda_1$ and $\lambda_2$), for any given $J$ and $M$. Under what circumstances are these criteria satisfied? This turns out to depend on the dark matter spin, on whether the dark matter particle is its own anti-particle, and on the masses and spins of the SM particles in the state $|X\rangle$. We now consider the various possibilities in turn.

**Scalar dark matter**

Consider first the case where dark matter is a scalar, either real or complex. Then the initial state is $CP$ even, and has $J = 0$. Angular momentum conservation implies that annihilation into light fermions is helicity suppressed. Therefore we expect that annihilation will occur primarily to the heaviest SM fermion species that is kinematically accessible, or alternatively to $W^+W^-$ (excluding neutral states which obviously do not contribute to photon line signal at one loop). In the case of annihilation to fermions, angular momentum and $CP$ conservation imply that the final state must have $L = 1, S = 1$, while $L = 0, S = 0$ is forbidden. Therefore a bound can be obtained for this annihilation mode.

For annihilation to $W^+W^-$ on the other hand, the conservation laws allow both $L = 0, S = 0$ and $L = 2, S = 2$ final states, forbidding only $L = 1, S = 1$. Therefore our formalism is not directly applicable. Nevertheless, in the next section we shall see that in various kinematic limits a bound can indeed be obtained.

**Majorana fermion dark matter**

We move on to the case where dark matter is a Majorana fermion. Since Majorana fermions are identical particles, anti-symmetry of their wave function implies that if $L = 0, S$ is also then zero so that $J = 0$. However, unlike the case of scalar dark matter, the initial state is now $CP$ odd. As before annihilation to light fermions is disfavored by angular momentum
considerations, and so the heavy fermion and $W^+W^-$
final states are again expected to dominate. Consider
first annihilation to fermions. Angular momentum and
$CP$ conservation imply that the final state must have
$L = 1, S = 1$, while $L = 0, S = 0$ is forbidden. Therefore
our formalism applies to this annihilation channel. What
about the $W^+W^-$ channel? Now the conservation laws
allow only the $L = 1, S = 1$ final state, while forbidding
the $L = 0, S = 0$ and $L = 2, S = 2$ final states. Therefore
our formalism applies to this channel as well.

**Dirac fermion dark matter**

If dark matter is a Dirac fermion there are two
possibilities for the total angular momentum of the
initial state, $J = 0$ or $J = 1$. Annihilation will in general
proceed through both these channels, and since $J$ is a
conserved quantum number there is no interference. The
bound in each channel can be calculated independently.
The weaker of these two limits is then the true bound,
corresponding to the case where annihilation occurs
entirely through that channel. For $J = 0$, $CP$ is odd
and the analysis is identical to that of Majorana fermion
dark matter considered above. However, if $J = 1$,
annihilation to the $\gamma \gamma$ final state is forbidden by the
Landau-Yang theorem [25, 26]. Therefore dark matter
can annihilate to SM final states through this channel
without giving rise to a line signal from the two photon
final state. We conclude that in the case that dark
matter is a Dirac fermion, no general bound is possible.

**Vector boson dark matter**

Finally we consider the case where dark matter is a
real vector boson. Since the initial state is composed of
identical particles, the wave function must be symmetric,
allowing $J = 0$ or $J = 2$, but forbidding $J = 1$. Both
$J = 0$ and $J = 2$ are $CP$ even. For $J = 0$ the analysis
is identical to that of scalar dark matter considered
above. However, for $J = 2$ a separate analysis is needed.
Annihilation may occur to either light or heavy fermions,
or to $W^+W^-$. In particular, annihilation to light
fermions is no longer disfavored by angular momentum
conservation. Annihilation to fermions may occur either
through any of the $L = \{1, 2, 3\}, S = 1$ channels. Therefore,
even if annihilation occurs exclusively to a single fermion species, it is not possible to obtain a
general bound. However, it is possible to obtain a bound
in various kinematic limits.

Annihilation to $W^+W^-$ can also occur through mul-
tiple channels, including $L = 2, S = 0$, and $L =
\{0,1,2,3,4\}, S = 2$. Therefore, in general it is not
possible to constrain this channel either. However, we
shall see that it is possible to obtain a bound in the limit
that the dark matter mass is close to the $W$ mass, so that
the final state $W$ bosons are non-relativistic.

**III. COMPUTATION OF LIMITS**

In this section we place lower limits on line strengths
by explicitly calculating the ratio of the decay rates of
the boson $\Phi$:

$$\frac{\Gamma_{\text{Im}}(\Phi \rightarrow \gamma \gamma)}{\Gamma(\Phi \rightarrow X X)}.$$  \hfill (10)

As outlined earlier, different choices of the spin and $CP$
properties of the boson $\Phi$ map on to different dark matter
candidates. We choose the mass of $\Phi$ to be $2m_X$, to
ensure that the intermediate state particles have exactly
the same total energy as in dark matter annihilation.
This is required so that the quantum numbers of the
intermediate state can be exactly the same in the two
cases, which is required for the mapping to go through.
The final result will in general be seen to depend on the
velocity $\beta$ of the intermediate state particle $X$,

$$\beta = \sqrt{1 - \frac{m_X^2}{m_\chi^2}}.$$  \hfill (11)

**A. Scalar Dark Matter**

In the case where the dark matter particle is a scalar,
the incoming state is restricted to be in a $J = 0$ state
for non-relativistic annihilation. The initial state is then
a $CP$ even state with zero total angular momentum,
which is duplicated when the boson $\Phi$ is a scalar particle,
which we label by $\phi$.

**Scalar dark matter annihilation to fermions**

As explained in the introduction, symmetry consider-
ations require the fermions to be in the $L = 1, S = 1$
final state. We assume that dark matter annihilates
predominantly to a single species of SM fermions $f \bar{f}$.
In the absence of large new sources of chiral symmetry
breaking in the theory, this is a safe assumption, since the
matrix element for annihilation a given fermion species is
proportional to the fermion mass. Then annihilation to
heavier fermions such as tops and bottoms are preferred,
provided those channels are kinematically open.

We begin from the following interaction Lagrangian,
which is assumed to be valid below the scale of elec-
trowek symmetry breaking:

$$\mathcal{L}_{\text{int}} = \lambda \bar{f} f \phi.$$  \hfill (12)

From this Lagrangian it is straightforward to calculate the
ratio of the imaginary part of the decay to two photons to
the total decay rate to $f \bar{f}$. We obtain for the
bound

$$\frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma \gamma)}{\Gamma(\phi \rightarrow f \bar{f})} = \frac{N_c Q^4 e^4 m_\chi^2}{32\pi^2 m_X^2} \beta [\tanh^{-1} \beta]^2,$$  \hfill (13)

where $Q$ is the electric charge of the fermion (e.g. $Q = \frac{2}{3}$
for top quarks) and $N_c = 3$ is the color factor to be
included when the fermionic states are quarks.

**Scalar dark matter annihilation to W bosons**

As explained earlier, in this case annihilation can proceed through either the \( L = 0, S = 0 \) channel or through the \( L = 2, S = 2 \) channel. Therefore it is only possible to obtain model independent bounds in specific kinematic limits.

Consider first the limit that the dark matter mass is close to the \( W \) mass, \( m_\chi \sim m_\gamma \). Then the annihilation products will be non-relativistic and we expect that the \( L = 0, S = 0 \) final state will dominate. Therefore, in this limit a bound can be obtained. As before we model the annihilation by the decay of a scalar particle \( \phi \). We choose the coupling of \( \phi \) to \( W \) bosons to have the simple form

\[
\mathcal{L}_{\text{int}} = \frac{1}{\Lambda} \phi \, \text{Tr} \left[ F_{\mu\nu} F^{\mu\nu} \right],
\]

and calculate the decay rates to \( WW \) at tree level, and to \( \gamma \gamma \) at loop level. The required ratio of decay rates is given by

\[
\frac{\Gamma_{\text{im}}(\phi \rightarrow \gamma \gamma)}{\Gamma(\phi \rightarrow WW)} = \frac{3e^4}{64\pi^2\beta}
\]

(non-relativistic limit).

This is the bound in the non-relativistic limit.

In the opposite limit where the dark matter mass is much larger than the \( W \) mass, \( m_\chi \gg m_\gamma \), the \( W \) bosons are ultra-relativistic. Then the Goldstone boson equivalence theorem applies, and the longitudinal and transverse components of the \( W \) bosons correspond to distinct physical states. Annihilation can occur either to two longitudinal \( W \) bosons or to two transverse \( W \) bosons, and it is reasonable to assume that there is no large cancellation in the contributions to the \( \gamma \gamma \) amplitude from these different states. We therefore consider them separately. (Annihilation to one transverse and one longitudinal \( W \) is forbidden by angular momentum conservation, and so need not be considered.)

If annihilation is exclusively to longitudinal \( W \)'s, then there is a unique final state labelled by \( |0,0,0,0\rangle \) in the helicity basis \( |J,M,\lambda_1,\lambda_2\rangle \), and our formalism can immediately be applied. If annihilation is exclusively to transverse \( W \)'s, then angular momentum conservation allows both the \( |0,0,+,+\rangle \) and \( |0,0,-,-\rangle \) final states. However, only one linear combination of these states is \( CP \) even, while the other is \( CP \) odd. Since annihilation can only occur to the \( CP \) even state, once again our formalism is applicable.

We first consider annihilation to longitudinal \( W \) bosons. We couple the scalar \( \phi \) to the Higgs doublet, and consider decays into charged Higgses, and into two photons through a loop of charged Higgses. The Goldstone boson equivalence theorem guarantees that in the ultra-relativistic limit, the rates for these processes are identical to the rates for the corresponding processes involving \( W \) bosons. The interaction Lagrangian takes the form

\[
\mathcal{L}_{\text{int}} = \alpha \phi \, H^\dagger H.
\]

---

**Table I.** Summary of the bounds corresponding to the different dark matter candidates and annihilation channels. We have also indicated cases where a bound is only applicable when the tree-level annihilation products (on-shell intermediate states in the loop) are non-relativistic (NR) or ultra-relativistic (UR).

| Dark Matter      | Initial spin | Annihilation Channel | Mode     | Bound                          |
|------------------|--------------|----------------------|----------|-------------------------------|
| Scalar           | \( J = 0 \)  | \( WW \)             | \( L = 0, S = 0 \) | In NR / UR limits. |
|                  |              |                      | \( L = 2, S = 2 \) |                               |
|                  |              | \( f\bar{f} \)       | \( L = 1, S = 1 \) | ✓                             |
| Majorana Fermion | \( J = 0 \)  | \( WW \)             | \( L = 1, S = 1 \) | ✓                             |
|                  |              | \( f\bar{f} \)       | \( L = 0, S = 0 \) | ✓                             |
| Dirac Fermion    | \( J = 0 \)  | \( WW \)             | \( L = 1, S = 1 \) | ✓                             |
|                  |              | \( f\bar{f} \)       | \( L = 0, S = 0 \) | ✓                             |
|                  | \( J = 1 \)  |                      |           | Forbidden                      |
| Real Vector Boson| \( J = 0 \)  | \( WW \)             | \( L = 0, S = 0 \) | In NR / UR limits. |
|                  |              |                      | \( L = 2, S = 2 \) |                               |
|                  |              | \( f\bar{f} \)       | \( L = 0, S = 0 \) | ✓                             |
|                  | \( J = 2 \)  | \( WW \)             | \( L = \{0,1,2,3,4\}, S = 2 \) | In NR limit. |
|                  |              | \( f\bar{f} \)       | \( L = \{1,2,3\}, S = 1 \) | In NR / UR limits. |
The ratio of the decay rates in this case turns out to be suppressed. It scales roughly as the following,

$$\frac{\Gamma_{\text{Im}}(\phi \to \gamma\gamma)}{\Gamma(\phi \to WW)} \sim \frac{e^4}{16\pi^2} \frac{m_W^4}{m_chi^4} \left[ \log \left( \frac{4m_chi^2}{m_W^2} \right) \right]^2. \quad (17)$$

(ultra-relativistic limit).

Since this vanishes in the limit $m_W^2/m_chi^2 \to 0$, this shows that there is in fact no bound in the case of annihilation to longitudinal $W$ bosons in the ultra-relativistic limit.

We now consider annihilation to the transverse polarizations. We start from the interaction Lagrangian

$$L_{\text{int}} = \frac{1}{\Lambda} \phi \text{Tr} \left[ F_{\mu\nu} F^{\mu\nu} \right]. \quad (18)$$

In the ultra-relativistic limit, this interaction leads to decays of the scalar $\phi$ exclusively into transverse polarizations of the $W$ boson. The ratio of cross-sections is obtained as

$$\frac{\Gamma_{\text{Im}}(\phi \to \gamma\gamma)}{\Gamma(\phi \to WW)} = \frac{e^4}{32\pi^2} \left[ \log \left( \frac{4m_chi^2}{m_W^2} \right) \right]^2 \quad (19)$$

(ultra-relativistic limit).

This is then the lower bound in the case of annihilation exclusively into transverse polarizations of the $W$.

In general, annihilation will occur into both transverse and longitudinal polarizations. From the discussion above, it follows that if $F_T$ is the branching fraction into transverse polarizations of the $W$ in the ultra-relativistic limit, the bound is given by

$$\frac{\Gamma_{\text{Im}}(\phi \to \gamma\gamma)}{\Gamma(\phi \to WW)} = F_T \frac{e^4}{32\pi^2} \left[ \log \left( \frac{4m_chi^2}{m_W^2} \right) \right]^2 \quad (20)$$

(ultra-relativistic limit).

This formula is valid in the ultra-relativistic limit provided

$$F_T \gg \frac{m_W^2}{m_chi^2}. \quad (21)$$

If the branching fraction to transverse polarizations is smaller than this, then there is no bound in the limit $m_W^2/m_chi^2 \to 0$.

B. Majorana Fermion Dark Matter

In the case when the dark matter particles are Majorana fermions, the initial state again has total angular momentum $J = 0$. This is because in the non-relativistic limit $L = 0$ is picked out, and then $S = 0$ is required by the overall antisymmetry of the total wavefunction of the two fermion state. This configuration is $CP$ odd. The angular momentum and $CP$ quantum numbers of the initial state are exactly those of a pseudo-scalar particle. Therefore we can calculate the ratio by decaying a pseudo-scalar particle, which we denote by $\varphi$.

Majorana dark matter annihilation to fermions

We begin by considering annihilation to fermions. Once again we assume annihilation is exclusively to a single SM fermion species. As in the case of scalar dark matter this is a safe assumption, since annihilation to light fermions is chirality suppressed in the absence of large new sources of chiral symmetry breaking in the theory.

Angular momentum conservation and $CP$ symmetry require the final state fermions to be in the $L = 0, S = 0$ configuration rather than $L = 1, S = 1$. Our starting point is the following interaction Lagrangian

\begin{align*}
\end{align*}
which couples the pseudo-scalar $\varphi$ to the fermions $f\bar{f}$:
\[ \mathcal{L}_{\text{int}} = i\lambda f \gamma^5 f \varphi. \]  
(22)

From explicit calculation we obtain the bound in this case as
\[ \frac{\Gamma_{\text{im}}(\varphi \rightarrow \gamma\gamma)}{\Gamma(\varphi \rightarrow ff)} = \frac{N_c Q^2 e^4 m_f^2}{32\pi^2 m_\chi^2} \frac{1}{\beta} \left[\tanh^{-1} \beta\right]^2, \]  
(23)

where as before $Q$ denotes the electric charge, and $N_c$ denotes the color factor associated with the fermion. We have verified that this agrees with the result in the literature for the case of neutralino annihilation to fermions in the Minimal Supersymmetric Standard Model (MSSM).

Majorana fermion dark matter annihilation to $W$ bosons

We move on to considering annihilation to $W$ bosons. The only angular momentum quantum numbers for the $WW$ final state consistent with $J = 0$ and $CP$ conservation are $L = 1, S = 1$. As before, we model this annihilation by pseudo-scalar decay. The pseudo-scalar $\varphi$ is coupled to the SM in the following way:
\[ \mathcal{L}_{\text{int}} = \frac{1}{\Lambda} \varphi \text{Tr}(F_{\mu\nu} F^{\mu\nu}). \]  
(24)

This leads to the bound:
\[ \frac{\Gamma_{\text{im}}(\varphi \rightarrow \gamma\gamma)}{\Gamma(\varphi \rightarrow WW)} = \frac{e^4}{8\pi^2} \beta \left[\tanh^{-1} \beta\right]^2. \]  
(25)

This agrees with the result in the literature for neutralino annihilation to $W$ bosons in the MSSM.

Fig. 2 shows the lower limits on the strengths of the gamma ray line relative to the gamma ray continuum if Majorana fermion dark matter annihilates exclusively to the $b\bar{b}$ and $WW$ final states respectively. The widths of the emission lines are drawn so as to correspond with the resolution of the Fermi-LAT, at 11% of FWHM.

C. Real vector boson dark matter

Real vector boson dark matter annihilation can proceed through either the $J = 0$ channel or the $J = 2$ channel. The $J = 1$ channel is not allowed because the wavefunction is required to be symmetric under interchange of the two identical dark matter particles. Both the $J = 0$ and $J = 2$ initial states are $CP$ even.

If dark matter annihilation occurs primarily through the $J = 0$ channel, then the bounds from scalar dark matter annihilation to the corresponding SM states apply here as well. On the other hand, if the primary mode of annihilation is through the $J = 2$ channel, then obtaining the bound involves calculating the decays of a spin-2 $CP$ even particle. The model independent lower limit on the strength of the line corresponds to the weaker of the bounds obtained in the $J = 0$ and $J = 2$ cases.

We use the decays of a massive spin-2 graviton to model the $J = 2$ annihilation process. The massive graviton is taken to couple to the stress energy tensor of the matter fields, in analogy with the couplings of the Kaluza-Klein graviton in extra-dimensional theories [27, 28].

Vector dark matter annihilation to fermions

We begin by considering annihilation to fermions. The bounds we obtain assume annihilation exclusively to a single species of SM fermion. Away from this limit our bounds are approximate. The two fermion final states that can arise from annihilation of the $J = 2$ initial state, after taking into account angular momentum and $CP$ conservation, have angular momentum quantum numbers $L = 1, S = 1$, $L = 2, S = 1$ and $L = 3, S = 1$. This multiplicity of available states means that it is only possible to obtain a bound in specific kinematic limits.

We first consider the limit where the dark matter mass is close to the mass of the fermion species it annihilates into, so that the outgoing fermions are non-relativistic. Then the $L = 1, S = 1$ final state dominates, and our formalism is applicable.

Consider a massive Dirac fermion $f$ that couples to a massive graviton $h_{\mu\nu}$. The term in the Lagrangian which is relevant for the on-shell graviton decay is the following:
\[ \mathcal{L}_{\text{int}} = -\frac{\kappa}{2} h_{\mu\nu} \bar{f} i\gamma_\mu \partial_\nu f. \]  
(26)

The coupling constant $\kappa$ has dimensions of inverse mass. The mass of the fermion couples to the trace of the graviton $h^{\mu\nu}$, which vanishes for the on-shell decay. The limit in this case is obtained very weak because of the $p$-wave suppression of the amplitude:
\[ \left. \frac{\Gamma_{\text{im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow ff)} \right|_{J=2} = \frac{N_c Q^2 e^4 \beta^5}{120\pi^2}. \]  
(27)

This is significantly weaker than the corresponding bound on decays from the $J = 0$ state. We conclude that for real vector dark matter annihilation to heavy fermions in the non-relativistic limit, it is the bound from the $J = 2$ initial state that applies.

Next we consider the case of annihilation to very light SM fermions, such as electrons or muons. In particular, we work in the limit where the dark matter mass is much larger than the mass of the final state fermions. Then the final state fermions are ultra-relativistic, and can be treated as massless. In this limit, left- and right-handed fermions of the same flavor are effectively different SM species, and should be considered separately. We now show that in the ultra-relativistic limit, for annihilation exclusively to a single SM fermion species of definite chirality, it is possible to obtain a bound. To understand how this arises we work in the helicity basis, and consider annihilation exclusively to left-handed Weyl fermions (and right-handed anti-fermions). The unique
final state is then \( |J, M; \frac{1}{2}, -\frac{1}{2} \rangle \), and so our formalism can immediately be applied.

The couplings of a massless chiral Weyl fermion \( f \) to a massive graviton take the form:

\[
\mathcal{L}_{\text{int}} = -\frac{\kappa}{2} h^{\mu\nu} \bar{f} i \sigma_\mu \partial_\nu f.
\]  

(28)

We obtain the result

\[
\frac{\Gamma_{\text{im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow ff)} \bigg|_{J=2} = \frac{N_f Q^4 e^4}{144\pi^2} \quad \text{(ultra-relativistic limit)}.
\]  

(29)

Since annihilation to massless fermions from the initial state with \( J = 0 \) is forbidden, this is the applicable bound.

In this scenario, in contrast to the cases of scalar and Majorana fermion dark matter, the annihilation cross section to SM fermions is not chirality suppressed. It is then very natural for dark matter to couple identically to all three flavors of a SM (chiral) fermion species and annihilate with equal strength to all of them. In the absence of other annihilation modes, Eq. 29 gets modified to

\[
\frac{\Gamma_{\text{im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow ff)} \bigg|_{J=2} = \frac{N_f Q^4 e^4}{144\pi^2} \quad \text{(ultra-relativistic limit)}.
\]  

(30)

where \( N_f \) is the number of flavors.

### Vector dark matter annihilation to \( W \) bosons

For annihilation to \( W \) bosons from the \( J = 0 \) initial state, the earlier bounds from the case of scalar dark matter apply in the appropriate kinematic limits. In the case of the \( J = 2 \) initial state, even after imposing angular momentum and \( CP \) conservation, there are still several channels that can contribute. It is possible to obtain a bound in the limit that the dark matter mass is close to the \( W \) boson mass, \( m_\chi - m_W \ll m_W \), when annihilation to the \( L = 0, S = 2 \) final state dominates.

The relevant part of the Lagrangian, in unitary gauge, takes the form:

\[
\mathcal{L}_{\text{int}} = \frac{\kappa}{2} h^{\mu\nu} \left( [\partial_\rho W^+\rho - \partial^\rho W^+\rho] \partial_\nu W^- - \partial_\nu W^- \right) - m_W^2 W^+\rho W^- + \mu \leftrightarrow \nu
\]  

(31)

Explicit calculation leads to the bound:

\[
\frac{\Gamma_{\text{im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow WW)} \bigg|_{J=2} = \frac{e^4}{20\pi^2} \quad \text{(non-relativistic limit)}.
\]  

(32)

This bound is (slightly) stronger than that obtained for annihilation through the \( J = 0 \) channel (Eq. 15). We conclude that the \( J = 0 \) channel provides the appropriate bound for vector dark matter annihilation to \( WW \) in the non-relativistic limit.

## IV. CURRENT AND FUTURE EXPERIMENTAL LIMITS

In this section we combine these results with the current experimental constraints on gamma ray lines obtained by the Fermi-LAT collaboration [11] to place upper bounds on the rate of dark matter annihilation into various SM final states, and estimate potential future sensitivities.

We consider each of scalar, Majorana fermion and real vector boson dark matter in turn and consider annihilation into a few plausible SM final states. We have obtained a lower limit on the strength of the photon line given the annihilation cross section into each such final state. The null result in Fermi-LAT line search then puts an upper bound on this annihilation cross section.

Our results are then compared to the corresponding bounds on dark matter annihilation obtained from the Fermi-LAT constraints on continuum gamma rays. This allows us to compare the relative strength of the bounds from the line and the continuum photon spectra searches.

The flux in a line is given by

\[
\frac{d\Phi}{dE} = \frac{\langle \sigma_A \nu \rangle}{8\pi m_\chi^2} J_0 \frac{dN}{dE},
\]  

(33)

where

\[
\frac{dN}{dE} = 2\delta(E_\gamma - m_\chi)
\]  

(34)

for the given annihilation channel with annihilation cross section \( \langle \sigma_A \nu \rangle \), and \( J_0 = 1/[8.5 \text{ kpc}(0.3 \text{ GeV cm}^{-3})^2] \). The normalized integral of the mass density squared over the observational regions for the line search is

\[
\mathcal{J} = J_0 \int \rho^2 (r_{\text{gal}}(b, \ell, x)) \cos b \, dx \, db \, d\ell,
\]  

(35)

where \( r_{\text{gal}}(b, \ell, x) \) is the radial coordinate of the density distribution. The regions included in the line search are the Galactic caps at \( |b| > 10^\circ \) and the \( 20^\circ \times 20^\circ \) region in the Galactic center at \( |b| < 10^\circ \) and \( |\ell| < 10^\circ \). We adopt a minimal model of the Milky Way Galactic dark matter profile which minimizes the signal yet is consistent with constraints from Galactic dynamics and is consistent with profiles expected in cold dark matter halo formation. To do so, we take an Einasto profile for the dark matter density distribution with parameters that minimize \( \mathcal{J} \) and are consistent with the dynamical constraints in Ref. [29]. The Einasto profile is

\[
\rho_{\text{Einasto}}(r) = \rho_s \exp \left( -\frac{2}{\alpha} \left( \frac{r}{r_s} \right)^\alpha - 1 \right),
\]  

(36)

and the parameters from Ref. [29] within 68% confidence level (CL) that give a lower \( \mathcal{J} \) are \( \alpha = 0.22, r_s = 21 \text{ kpc}, \) with \( r_0 = 8.28 \text{ kpc} \) and \( r_S \) determined from the local solar dark matter density \( \rho_0 = 0.385 \text{ GeV cm}^{-3} \). The value of the normalized density squared toward the
Fig. 3. Upper, middle and lower rows of panels show current and future 95% CL constraints on dark matter annihilation channels in the scalar, Majorana, and vector dark matter cases respectively. The colored regions are from constraints from the isotropic DGRB on Galactic and extragalactic dark matter annihilation, as described in the text. The diagonally hatched regions are from current constraints from Fermi-LAT on the minimal line contribution in the respective channels, as discussed in the text. The vertically hatched regions are forecasts for the limits after 10 years of observation with Fermi-LAT, assuming Poisson statistics at the current energy resolution. In the case of scalar dark matter to $WW$ (upper right panel), the solid line indicates the true limit for ultra-relativistic $WW$ production, and the non-outlined hatched region is an interpolation of the limit to the non-relativistic regime at $m_\chi \sim 100$ GeV.

Galactic caps in this case is $J_{\text{cap}} = 28.6$ and toward the Galactic Center’s $20^\circ$ square is $J_{\text{GC}} = 37.0$. These limits depend on the adopted profile among dynamically consistent models at the level of $\sim 50\%$, as shown in Ref. [11]. Our adopted profile and parameters give a minimal $J$, and therefore provide a conservative limit, yet are consistent with cold dark matter halo structure formation.

In the case of Majorana fermion dark matter we consider annihilation to the $b\bar{b}$, $\tau^+\tau^-$ and $WW$ final states. For scalar dark matter we consider exactly the same final states, but with the added restriction in the WW case that the $W$-bosons are ultra-relativistic and transverse. In the case of real vector boson dark matter, we first consider the case where annihilation occurs equally to all three generations of ultra-relativistic right-handed down-type quarks $\bar{b}b$, $\bar{s}s$, and $\bar{d}d$, then the case where it occurs equally to all three generations of ultra-relativistic right-handed charged leptons $\tau\bar{\tau}$, $\mu\bar{\mu}$ and $e\bar{e}$ and finally the case where annihilation is to non-relativistic $W$-bosons. The 95% confidence level (CL) limits are shown in Fig. 3.

The continuum limits, as shown in Fig. 3, are derived as in Ref. [9] from Galactic and extragalactic contributions to the isotropic diffuse gamma ray background (DGRB) as measured by Fermi-LAT [30], with the exception that here we take the substructure boost to be a very conservative $B = 2.3$ within the Galactic halo component of the DGRB contribution. This boost value is found from the substructure enhancement as in Ref. [9], with minimal substructure parameters, and with a partial cancellation of the total luminosity boost.
contribution from our position within the Galactic halo.

The minimal line limits are in all cases weaker than continuum limits at this time from Fermi-LAT’s measurement of the isotropic DGRB. Note that limits from the stacking of dwarf galaxies are stronger than that from the DGRB by a factor of $\sim 10$ [4] and the isotropic DGRB sensitivity may be enhanced [31]. We compare to the isotropic DGRB constraints here because they are the most conservative among the annihilation constraints. In Fig. 3 we also show the expected future 10-year Fermi-LAT line limits assuming enhancement of the sensitivity with the exposure time $(t)$ of Fermi-LAT as $\sqrt{t}$. The line sensitivity for Fermi-LAT may improve substantially with systematic improvement at better than the Poisson count rate. For example, this may occur with enhanced energy resolution for lines over the lifetime of Fermi-LAT. Moreover, for certain cases of dark matter spins, any future gamma-ray experiment with even higher energy resolution (e.g. GAMMA-400 [32]) may prove to have line sensitivities or constraints on dark matter annihilation be more significant than that for the continuum.

V. CONCLUSIONS

In models where the dark matter particle has a primary annihilation channel to a unique final state in the SM, we have derived a robust lower limit on the cross section of a loop-induced annihilation mode to photons, for a variety of choices of spin of the dark matter particle as well as the SM states it annihilates into. This bound is based on unitarity considerations that relate the imaginary part of the loop induced amplitude for annihilation to photons to the cross section of the primary annihilation channel. Since the spectrum of continuum photons emitted through bremsstrahlung from the primary annihilation products can be reliably computed, this lower bound also relates the minimal strength of a gamma ray line from dark matter annihilation to the size of the continuum spectrum. While the very conservative bounds for a gamma ray line obtained in this way are less stringent than the bounds from the continuum photons, these results help identify combinations of initial and final state quantum numbers for which a model-dependent calculation of the full amplitude (rather than only its imaginary part) for annihilation into photons can easily give significantly stronger bounds than those derived from continuum emission.

ACKNOWLEDGMENTS

It is a pleasure to thank Kaustubh Agashe, Raman Sundrum and Neal Weiner for useful comments. PA and ZC are supported by the NSF under grant PHY-0801323 and PHY-0968854. KNA is supported by NSF Grant 07-57966 and NSF CAREER Award 09-55415. CK is supported by the NSF Grant Number PHY-0969020.

Appendix A: Time-Reversal Invariance and the $T$-Matrix

In this appendix we show that if the theory respects time-reversal invariance, then the $T$-matrix is symmetric in the angular momentum basis,

$$\langle f|T|i \rangle = \langle i|T|f \rangle , \tag{A1}$$

where $|i \rangle$ and $|f \rangle$ are angular momentum eigenstates.

The fact that the theory is time-reversal invariant implies that

$$\langle f|T|i \rangle = \langle \Theta i|T|\Theta f \rangle , \tag{A2}$$

where $\Theta$ is the time-reversal operator. The action of time-reversal on eigenstates of angular momentum is given by [24]

$$\Theta |J, M; L, S \rangle = (-1)^{J-M} |J, -M; L, S \rangle$$

$$\Theta |J, M; \lambda_1, \lambda_2 \rangle = (-1)^{J-M} |J, -M; \lambda_1, \lambda_2 \rangle . \tag{A3}$$

Now rotational invariance of the theory, in the form of the Wigner-Eckart theorem, implies that

$$\langle J', M'; L' S'|T|J, M; LS \rangle$$

$$= \delta_{J,J'} \delta_{M,M'} \langle L' S'|T^J|L S \rangle$$

$$= \langle J', -M'; L' S'|T|J, -M; L S \rangle . \tag{A4}$$

$$\langle J'M'; \lambda'_1, \lambda'_2|T|J, M; \lambda_1, \lambda_2 \rangle$$

$$= \delta_{J,J'} \delta_{M,M'} \langle \lambda'_1, \lambda'_2|T^J|\lambda_1, \lambda_2 \rangle$$

$$= \langle J', -M'; \lambda'_1, \lambda'_2|T|J, -M; \lambda_1, \lambda_2 \rangle . \tag{A5}$$

It follows from Eq. (A3), Eq. (A4) and Eq. (A5) that matrix elements between time-reversed states obey

$$\langle \Theta i|T|\Theta f \rangle = \langle i|T|f \rangle , \tag{A6}$$

where $|i \rangle$ and $|f \rangle$ are angular momentum eigenstates. It then immediately follows from Eq (A2) that the $T$-matrix is symmetric in this basis,

$$\langle f|T|i \rangle = \langle i|T|f \rangle . \tag{A7}$$

[1] E. Komatsu et al. (WMAP), Astrophys. J. Suppl. 192, 18 (2011), arXiv:1001.4538 [astro-ph.CO].

[2] M. Ackermann et al., JCAP 1005, 025 (2010),
