Radiative Effects on a Free Convective MHD Flow past a Vertically Inclined Plate with with Heat Source and Sink

P Sambath, BapujiPullepu* and R M Kannan
Department of Mathematics, S R M Institute of Science and Technology, Kattankulathur, Tamil Nadu, India- 603 203
sambathsrnm@yahoo.co.in; kannanrm19@yahoo.in; bapujip@yahoo.com
*Corresponding author: bapujip@yahoo.com

Abstract. The impact of thermal radiation on unsteady laminar free convective MHD flow of incompressible viscous fluid passes through a vertically inclined plate under the persuade of heat source and sink is presented here. Plate surface is considered to have variable wall temperature. The fluid regarded as gray absorbing / emitting, but non dispersing medium. The periphery layer dimensionless equations that administer the flow are evaluated by a finite difference implicit method called Crank Nicolson method. Numerical solutions are carried out for velocity, temperature, local shear stress, heat transfer rate for various values of the parameters (Pr, λ, ∆M, Rγ) are presented.

1. Introduction

Numerical investigations of convection stimulated by the effect of buoyancy forces ensuing from thermal diffusion have always been significant attention to many researchers. A laminar free convection flow in the presence of a magnetic field of an electrically conducting fluid was investigated by many researchers because of its industrial and technological relevancelike plastics extrusion in the production of Rayon and Nylon, crude oil distillation, magnetic materials processing, control processes in glass manufacturing.

Hamadet al [1] described laminar two dimensional magneto hydrodynamic incompressible viscous flow over a porous flat plate in steady state. Palani and Kim [2] examined the attributes of natural convection heat transfer past a vertical cone with the impacts of radiation and magnetic field. Ahmed [3] discussed the persuade of chemical reaction and thermal radiation on the steady magneto hydrodynamic mass transfer mixed convective flow of an incompressible, sticky fluid from a vertical porous plate. The effects of radiation on a transient magneto hydrodynamic natural convection flow past a semi infinite vertical plate in the existence of a transverse magnetic field were studied by Abd El-Naby et al. [10]. Bhattacharyya and Layek [6] discussed the problem with Similarity solution of magneto hydrodynamic periphery layer flow over a porous flat plate with chemical reaction absorption and blustering. Chamkhaet al [5] studied transient magneto hydrodynamic free convection from a heated permeable plate. Mohamed et al [7] discussed transient magneto hydrodynamic doublediffusive periphery layer flow of a hot vertical surface which is radiating in porous media in the subsistence of heat sink. Makindeet al [8, 9] studied in transient state the consequences of viscosity which is varying on thermal periphery layer over a permeable plate and porous plate with chemical reaction through a binary mixture. Muthucumaraswamy and Ganesan discussed the impacts of radiation with variable temperature on spontaneously started infinite vertical plate. BapujiPullepu and
Sambath [12, 13] obtained numerical solutions for free convection stream of a sticky dissipative fluid with uneven surface temperature and surface heat flux through a non-isothermal vertical cone.

2. Mathematical Modeling

An axi-symmetric transient free convective MHD flow of a viscous incompressible electrically conducting fluid past a vertically inclined plate has been considered here. Let $\phi$ be the angle between the plate surfaces inclined with the horizontal axis. Rosseland approximation is applied in the energy equation for the radiative heat flux term. The bounding fluid at rest and the surface of the plate are at the equal temperature $T_\infty'$. Then at time $t' > 0$, the plate surface temperature is suddenly raised to $T_w'(x) = T'_\infty + ax^p$ and preserved at this same level. The coordinate system is taken in such a way that $x$ quantifies the distance alongside the exterior of the plate and $y$ axis taken perpendicular to it.

The fluid characteristics are considered to be stable with the exception of density differences. The basic equations governing the flow such as ‘Continuity, Momentum and Energy’ under Boussinesq approximation are given below.

**Basic Equations**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \beta (T' - T'_\infty) \cos \phi + \nu \frac{\partial^2 u}{\partial y^2} + g \beta_c (C' - C'_\infty) \cos \phi - \frac{\sigma B^2 u}{\rho} \tag{2}
\]

\[
\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} + \frac{Q_0}{\rho c_p} (T' - T'_\infty) - \frac{1}{\rho c_p} \frac{\partial q_y}{\partial y} \tag{3}
\]

\[
\frac{\partial C'}{\partial t'} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} - k_c (C' - C'_\infty) \tag{4}
\]

The initial and boundary conditions are as follows:

- $t' \leq 0 : u = 0, v = 0, T' = T'_\infty, C' = C'_\infty$ for all $x$ and $y$,
- $t' > 0 : u = 0, v = 0, T'(x) = T'_\infty + ax^p, C'(x) = C'_\infty + bx^p$ at $y = 0$,
- $u = 0, T' = T'_\infty, C' = C'_\infty$ at $x = 0$,
- $u \to 0, T' \to T'_\infty, C' \to C'_\infty$ as $y \to \infty \tag{5}$
The expression in the energy equation \( \frac{\partial q_r}{\partial y} \) is made simpler by employing the Rosseland approximation as explained by Brewster [14, 15] is

\[
q_r = \frac{-4\sigma * \partial T'^4}{3k * \partial y}.
\]  

(6)

It should be noted that we bound our investigation to optically thick fluids by using the Rosseland approximation. If the term \( T' - T'_w \) inside the flow is adequately small, then the linear form of equation (6) can be obtained by escalating \( T'^4 \) by using the Taylor series expansion about \( T'_\infty \) by omitting higher-order terms.

We obtain \( T'^4 \approx T'^4_{\infty} T' - 3 T'^4_{\infty} \).

(7)

Using the following dimensionless quantities:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}(Gr_L)^{\frac{1}{4}}, \quad R = \frac{r}{L},
\]

\[
V = \frac{VL}{\nu}(Gr_L)^{-\frac{1}{4}}, \quad U = \frac{UL}{\nu}(Gr_L)^{-\frac{1}{2}}, \quad t = \frac{UL}{\nu}(Gr_L)^{\frac{1}{2}},
\]

\[
T = \frac{(T' - T'_w)}{(T'_w - T'_\infty)}, \quad Gr_L = \frac{g\beta(T'_w - T'_\infty)L^2 \cos \phi}{\nu^2}, \quad Pr = \frac{\nu}{\alpha},
\]

\[
C = \frac{(C'_w - C'_\infty)}{(C'_w - C'_\infty)}, \quad GrC = \frac{g\beta(C'_w - C'_\infty)L^2 \cos \phi}{\nu^2}, \quad Sc = \frac{\nu}{D},
\]

\[
\mathcal{N} = \frac{GrC}{Gr_L}, \quad \Delta = \frac{Q}{CP\mu}(Gr_L)^{-\frac{1}{2}}, \quad \lambda = \frac{k_1 L^2}{\nu}(Gr_L)^{-\frac{1}{2}}, \quad M = \frac{\sigma B_c^2 L^2}{\mu}Gr_L^{-\frac{1}{2}}, \quad R_j = \frac{K*K}{4\sigma * T'^4_{\infty}}.
\]  

(8)

The non-dimensional form of the governing equations are given below

\[
\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} = 0
\]

(9)

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + (T + NC) - MU
\]

(10)

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \left( 1 + \frac{4}{3R_j} \right) \frac{\partial^2 T}{\partial Y^2} + \Delta T
\]

(11)

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - \lambda C
\]

(12)

The initial and boundary conditions in dimensionless form

\[ t \leq 0 : U = 0, V = 0, T = 0, C = 0 \text{ for all } X \text{ and } Y, \]

\[ t > 0 : U = 0, V = 0, T = X^n \text{ and } C = X^m \text{ at } Y = 0, \]

\[ U = 0, T = 0, C = 0 \text{ at } X = 0, \]

\[ U \to 0, T \to 0, C \to 0 \text{ as } Y \to \infty. \]  

(13)


3. Method of Solution

The transient, nonlinear coupled pde’s (9)-(12) along with (13) are evaluated by Crank Nicolson method. This method is unconditionally stable and rapidly convergent as discussed by Soundalgekar and Ganesan [4]. After applying the method, the dimensionless equations converted to the system of tri-diagonal equations. We solve them by using well known Thomas algorithm by which we attain the desired solution.

The integral area is treated as a square or with $X_{\text{max}} (=1)$ and $Y_{\text{max}} (=20)$ where $Y_{\text{max}}$ corresponds to $Y=\infty$ which is located away from the momentum and thermal periphery layers. The value for $Y$ is taken to be 20 by analyzing in detail and considered in order to satisfy the ultimate and penultimate conditions of (13) and we observed that it is fulfilled with accuracy up to the tolerance limit of $10^{-5}$.

4. Discussion of Results

Velocity, temperature portfolios for various numerical quantities of Prandtl number $Pr$ are shown through Figure 2. It is found that the momentum boundary layer thickness is more with $Pr = 0.71$ and lesser for $Pr = 6.7$. Figure 3 displays the impact of $\lambda$, the chemical reaction parameter, on the momentum profiles. As $\lambda$ increases, there is a significant reduction in the momentum is seen, the existence of $\Delta$ increases the momentum profiles.
Figure 4 show that the enhanced radiation and magnetic field generates overturn movement to the flow and which is known as “Lorentz force”. In Figure 5 we perceive that there is a reduction in temperature where as the numerical values of Pr increases. Higher values of Pr reduces the thickness of the thermal periphery layer and thermal conductivity.

Figure 6 indicate that the temperature is more for larger quantities of $\lambda$ and lesser quantities of $\Delta$ and also the thermal periphery layer thickness rises. Figure 7 demonstrates that the higher quantities of Rd makes the fluid thicker and reduces the temperature and the thickness of the thermal periphery layer. Hence we notice that the reduction in the temperature for the presence of $M$ and $Rd$. 
Figure 8 illustrates the effects of $Pr$ on the local skin friction. The shear stress decreases for the increase in $Pr$. Figure 9 shows the consequences of $\lambda$, $\Delta$, $M$ and $Rd$ on the shear stress. Stronger thermal radiation accelerates the flow but reduces heat transfer rate hence the local skin friction got decreased due to the presence of $M$ and $Rd$ where as it amplifies for the higher values of $\Delta$ and $\lambda$.

Figure 10 shows the effects of $Pr$ on heat transfer rate. As $Pr$ raises the rate of heat transfer decreases. Figure 11 indicates the effects of $\lambda$, $\Delta$, $M$ and $Rd$ on the heat transfer rate. The local Nusselt number decreases for higher values $\Delta$, $\lambda$, $M$ and $Rd$. 
5. Conclusions

1. The momentum and temperature increases for lesser quantities of $Pr$, $M$, $\lambda$ and $Rd$. but larger quantities of $\Delta$.
2. The shear stress reduces with the rising quantities of $Pr$, $M$, $\lambda$ and $Rd$ but rises for $\Delta$.
3. The heat transfer rate decreases for the rising quantities of $Pr$, $\Delta$, $M$, $\lambda$, and $Rd$.

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