How do Indonesian elementary school mathematics textbooks introduce fractions?

T Rahmawati¹, S R Pangesti¹, I Nuriadin¹, M D Kurniasih¹ and Y W Purnomo²

¹ Universitas Muhammadiyah Prof. Dr. HAMKA
² Universitas Negeri Yogyakarta

E-mail: yoppy.wahyu@uny.ac.id

Abstract. This research aimed to analyze and describe the approach that was used to introduce the concept of fractions in elementary school mathematics textbooks. There were six 3rd grade of elementary school mathematics textbooks that represented two curriculums which were the 2006 curriculum and the 2013 curriculum that were included in the analysis. Vertical analysis of textbook research was conducted to focus on how the concept of fractions was introduced in the textbooks for each curriculum. The results of the research indicated that all the books included in the analysis only used one construct of fractions, which was as part-whole, to introduce fractions. This could cause the students to be narrow-minded about fractions and its implementations for further materials and in everyday life. Further discussion about the results can be seen in this article.

1. Introduction

Curriculum implementation needs an instrument that can bridge the implementation and the intended curriculum. Textbooks are one of the bridging instruments which include certain learning materials and are arranged according to the goals of learning and refer to the currently implemented curriculum [1], [2]. Furthermore, textbooks have an important role in the teaching and learning process and would mainly determine what would be taught and what the students would learn [3]–[5]. However, the importance of the textbooks needs to be supported by conducting research on how the presentation of content and features are related to the desired demand. Research is helpful in improving textbooks in the future and in turn would give a chance for students to have a meaningful learning.

According to research conducted by Chang and Silalahi [6] that analyzed 44 papers related to mathematics textbook research stated that fractions is a topic that is popular among textbook researchers along with numbers and their operations. As an example, Alajmi [5] who focused on physical characteristics, lessons structure, and nature of the problems on the topic of fractions in three different countries; Charalambous and colleagues [7] who compared the additions and subtractions of fractions between textbooks in three different countries; and similar research that focused on the division of fractions by Li and colleagues [8]. However, in some of the research above and what can be found in literature, there is only a few textbook research that focused on comparing textbooks between their own curriculums that changed over time. This is important as an effort of identifying what has been improved from the previous curriculum and what was good in the previous curriculum that went
unnoticed by the newest curriculum. In addition, there is still a lack of research that focuses on the approach chosen to introduce the concept of fractions in textbooks. Introducing the fraction meaning is important in building a strong foundation for further learning of fractions. Wijaya [9] had initiated this before, but only focused on textbooks from one curriculum.

Fractions became one of the challenging and difficult topics in elementary mathematics [10]–[12] and many students have a limited perspective about fractions and that has become one of the causes of their difficulties [13]. Limited perspective in this context is knowing basic concepts of fractions only as part-whole.

In Tsai and Li [13], Kieren classifies fractions into five constructs which are part-whole, measure, quotient, operator, and ratio (see also in [14]–[16]). Fraction as part-whole refers to the division of quantity into equal-sized parts. Fraction as measure shows that, in the example of \( \frac{3}{4} \), the whole is partitioned into 4 parts, in which \( \frac{3}{4} \) is used to measure 3 units of the measurement of \( \frac{1}{4} \). Fraction as quotient refers to the result of partitioning from division operations. Fraction as operator refers to fraction as the multiplier of a quantity, for example \( \frac{1}{2} \) of 10, in which \( \frac{1}{2} \) is the operator; and fraction as the ratio expresses the multiplicative relationship between two quantities. Many literature have showed that these five constructions play an important role in the student's understanding of fractions and the development of their knowledge that is connected and representative [13]. Because of this, the five constructs have an effect on success and have an important role in achieving desired goals of learning.

This research focused on analyzing Indonesian textbooks regarding the approach used to introduce fractions. Various approaches for introducing the concept of fractions are preferred to facilitate the student's understanding for further materials about fractions. Because of this, to achieve that aim, this research was guided by using the following questions.

1. How is the concept of fractions introduced and developed in elementary school mathematics textbooks?
2. What are the similarities and differences of the concept of fractions that were introduced and developed between the last two curriculums?

2. Methods

This research used textbook research to achieve the research’s aim. There are various types of textbook research [17]–[19]. In this research we used the comparison of textbooks, which is comparing the similarities and differences between one or more mathematics textbooks.

2.1. Materials

There are six elementary school mathematics textbooks with two different curriculums that were included in the analysis. The details of the books and the curriculums can be seen in Table 1. We decided that there should at least be 3rd grade textbooks published by the government and private publishers.

The researchers chose the textbooks from these publishers because of many reasons, (1) the electronic school books published by the Center of Books or Pusat Perbukuan of the Department of National Education are textbooks provided by the government as learning resources in schools and are compulsory for each school to have; and (2) the books published by Erlangga and Yudhistira are high quality books with excellent review management and editing. They are also the biggest and most trusted school textbook publishers as a resource of learning in school.

| Curriculum | Textbook Code | Title | Publisher | Year |
|------------|---------------|-------|-----------|------|
|            | KE2006        | Cerdas Berhitung Matematika untuk SD/MI Kelas 3 | Kemendikbud | 2008 |
|            |               | [Smart in Mathematics for 3rd Grade of Elementary School] | | |
|            | ER2006        | Pandai Berhitung Matematika untuk Sekolah Dasar | Erlangga | 2008 |

Table 1. Identity of each analyzed textbook
2.2. Data Analysis
Charalambous and colleagues divide the analysis into two dimensions which are the Horizontal and Vertical Analysis. Horizontal analysis is used to give a big picture about textbooks with a few lessons in certain topics and in which they are placed according to the grade level and vertical analysis offers an in-depth understanding of mathematics content [7]. In this paper, we only reported the vertical analysis results. The vertical analysis focused on the approach of introducing the concept of fractions in regards to the material, examples, and practice problem sections. We classified each concept approach into 5 constructs proposed by Kieren [16], [20], which were part-whole, measure, ratio, quotient, and operator. Analysis and coding were conducted by the second author, then to guarantee reliability, the two other authors reviewed the coding. The results of the review were then discussed between the researchers and revision was carried out.

3. Results

3.1. Approach to the Concept of Fractions in 2006 Curriculum Textbooks

3.1.1. Introduction to material section. In the introduction to material section, the three textbooks analyzed in the 2006 curriculum, all used the part-whole approach to introduce fractions. The KE2006 book gave explanations by using illustrations of circles, which was comparing the shaded area and the whole area. This textbook introduced fractions by directly using examples of problems. ER2006 also presented similar illustrations, however, rectangles were used as the pictorial model. Different from both of these textbooks, YU2006 explained the concept of fractions by using context, in which pizza was divided equally into two parts. The YU2006 book also presented more context compared to the last two textbooks. The following is a sample of the presentation from the YU2006 textbook.

![Figure 1. Sample lesson on teaching fractions as Part-whole in YU2006](image)

*Translate:*  
Rendi Bought Pizza.  
Rendi’s pizza was cut into two equal-sized pieces.  
The pizza will be given to his two younger siblings,  
Rina and Rani.  
Rina and Rani will both receive half of the pizza.*
3.1.2. Example of Problems Section. In this section, the three analyzed 2006 curriculum textbooks were dominated by non-word problems and all of them (100%) used the part-whole concept approach. The KE2006 book used an illustration of a square being divided into three equal-sized parts with one of them being shaded to indicate \( \frac{1}{3} \). This example also guided readers to read the fractions and showed the placement of the numerator and denominator. The presentation of similar examples was also given in the ER2006 and YU2006 books, which used two-dimensional shapes as an example of fractions as part-whole.

Figure 2. Example of a Problem Sample from the KE2006 Textbook

Translate:
The colored area is 1 part out of 3. Because of this, that area shows a fraction of 1/3. The fraction of 1/3 is read as one third or a third. 1 is the numerator and 3 is referred to as the denominator.

3.1.3. Practice problems section. Based on the results of the practice problems of the 2006 curriculum textbooks, the most practice problems are in the KE2006 textbook which has 11 practice problems, along with ER2006 which has 8 practice problems and YU2006 which has 7 practice problems. The practice problems in these three books have a similarity which is that they all used illustrations of shapes. The KE2006 and ER2006 books asked students to state the fractions of the shaded area out of the whole shape’s known area. The practice problems in both books included non-word problems and not problem-solving types, because it did not demand the student to think analytically in solving problems. The YU2006 textbook also did not have the word problems and problem-solving types. The practice problems from the YU2006 book asked the students to choose which two-dimensional shape had the same size and which ones did not. Therefore, all of the practice problems from the 2006 curriculum textbooks used the part-whole construct to demand the understanding of the concept of fractions. The following is a practice problem from one of the 2006 curriculum books.

Figure 3. Practice problem in the KE2006 textbook
Translate:
State the colored area according to the fraction (solve it in your workbook!)

3.2. Approach to the Concept of Fractions in 2013 Curriculum textbooks

3.2.1. Introduction to material section. The part-whole concept approach was also a main choice that was presented in the three analyzed 2013 curriculum textbooks. During the introduction to the concept of fractions, the three textbooks used pictorial and context representation. KE2013 used an illustration of two donuts that were cut to show an example and not an example of a half. The first donut illustrated two equal-sized pieces, meanwhile the second donut was cut unequally. Similarly, the ER2013 used an illustration of an apple cut into two equal-sized pieces and showed that the apple pieces were \( \frac{1}{2} \) with 1 as the numerator and 2 as the denominator. Lastly, the YU2013 textbook used the context of bread that was modified into story form. The narration was similar with the ER2013 textbook.

3.2.2. Example of Problem. According to the three analyzed textbooks, there was one textbook that did not have example of problems which was the YU2013 textbook. KE2013 and ER2013 used the part-whole construct and was the pictorial representation type. KE2013 used the same type of problem examples as the ones in the introduction to material section, meanwhile ER2013 provided an opportunity for the students to shade the half of the two shapes.

3.2.3. Practice Problems. Practice problems in the KE2013 textbook were divided into two which were the discussion problems and the general practice problems. For the discussion problems, the students were asked to discuss to find the reason why each donut piece was referred to as half and not
half. This type of problem demanded problem-solving skills because the students were urged to think critically by analyzing the problem. Other types of practice problems in this book were more or less similar to the ER2013 and YU2013 textbooks, which were completing the correct fractions in the given context. The practice problems in ER2013 asked the students to shade the shape's area that represented \( \frac{1}{2} \), meanwhile YU2013 asked the students to find which was half and which was not. Even with the representation and different context, the three 2013 textbooks only used one construct to introduce fractions which was as part-whole. The following is a sample of a practice problem from the KE2013 textbook.

![Figure 5. A practice problem in the KE2013 textbook](image)

Translate:

**Part 1: Let's discuss!**

1. Why are each of the two donut pieces on the side referred to as a half?
2. Why are each of the two donut pieces on the side not referred to as a half?

**Write down the results of your discussion!**

**Part 2: Let's practice!**

Complete the following statements correctly!

1. A donut is split into two equal-sized pieces. Each piece is referred to as....
2. Siti has a paper in the shape of a rectangle. The paper is then cut into two equal-sized pieces. Each piece of paper is....
3. Siti has a donut. Siti wants half of the donut. Siti has to cut the donut into... equal-sized pieces.
4. To have \( \frac{1}{2} \) of the piece of cake, we have to split it into.... sizes

**4. Discussion**

The results of this research indicated that there is no significant difference between the 2006 curriculum textbooks published both by the government and private publishers in regards to the introduction to material, example of problems, and practice problems. All of them used fractions as part-whole approach. We also found this result in the 2013 curriculum textbooks, in which all textbooks used the approach of fractions as part-whole concept. This result is also in accordance to the results by Wijaya [9] who analyzed two 2013 curriculum mathematics textbooks published by the government that the Indonesian mathematics textbooks that were used as the sample only provided one construct to introduce the concept of fractions, which was the concept of fractions as part-whole. The concept of part-whole is the most simple and commonly used concept to use when introducing fractions for the first time [9], [16]. However, students will have a deeper understanding of the fraction meanings by emphasizing on other fraction meanings [21], [22]. Moreover, the researchers also think
that introducing fractions that only uses one construct will cause difficulties in the future learning of fractions [9].

It is important for the future research, authors of textbooks, and/or developers of elementary school mathematics curriculums to develop a concept of fractions using other various constructs. An example of this is as applied in the mathematics curriculum in Japan, in which the concept of fractions are introduced for the first time using the ratio and measure construct in 4th grade, meanwhile fractions as part-whole is introduced in 5th grade [23]. There are many urgencies to provide various concepts for introducing fractions, for example, (1) fractions as part-whole is useful for comparing and ordering fractions; (2) fractions as measure plays a role in student's understanding of the addition of fractions and the density property of fractions; (3) fractions as quotient plays a role in students understanding the concepts of proportional division, the division of fractions, and understanding algebraic fractions; (4) fraction as ratio helps students in understanding the fraction’s meaning by its quantity; and (5) fraction as density property is the fundamental base for developing proportional reasoning [13], [14].

In addition to the results above, the present research found that 2006 curriculum textbooks had more pictorial representations in the form of two-dimensional shapes as objects to introduce the part-whole concept, meanwhile 2013 curriculum textbooks used a combination of context and pictorial representation, especially in the section of practice problems. Because of this, 2013 curriculum textbooks provided more chances to introduce fractions relating to student's everyday experiences. A significant difference could also be found on context-based tasks that could be found more in 2013 curriculum textbooks but could not be found in 2006 curriculum textbooks. The 2006 curriculum textbooks emphasized more on practice related more on different representation from the concept of fractions as part-whole. The number of practice problems varied for each textbook. The 2006 curriculum textbooks had more practice problems compared to the 2013 curriculum textbooks. However, the large amount only provided one demand in the form of a variation of pictorial representation from the part-whole concept. On the other hand, this research also identified that in both the 2006 and 2013 curriculum, the textbooks published by the government gave more chances of working on context-based practice problems for the introduction to fractions material compared to textbooks published by private publishers. The importance of context embedded in mathematical learning is one of the important means of the goal of mathematics education [24], [25]. Because essentially, the goal of mathematics education is to bring students to be able to apply mathematics in the context of problems in everyday life. Because of this, it is important that the development of mathematics textbooks in the future provides context-based tasks.

5. Conclusion
This research analyzed Indonesian mathematics textbooks which included the 2006 and 2013 curriculum. The analysis focused on the approach of the construct used to introduce the concept of fractions in the introduction to materials, example of problems, and practice problems sections. The results of this research indicated that the elementary school mathematics textbooks that were the samples for this research only used one construct which was the part-whole construct to introduce fractions. We also concluded that the 2013 curriculum textbooks had more variety of representations that introduced the concept of fractions compared to the 2006 curriculum textbooks. In addition, the mathematics textbooks published by the government were richer in context-based tasks and were HOTs-orientated compared to the textbooks published by private publishers. Because of this, it is important for researchers, authors, and developers of curriculums to emphasize various constructs to introduce the concept of fractions and context-based tasks to improve the awareness and knowledge of students about how to apply fractions in everyday life.

Acknowledgments
We would like to thank the Directorate of Research and Community Service, the Directorate General of Strengthening for Research and Development, Ministry of Research, Technology, and Higher
Education Republic of Indonesia (Direktorat Riset dan Pengabdian Masyarakat, Direktorat Jenderal Penguatan Riset dan Pengembangan, Kementerian Riset, Teknologi dan Pendidikan Tinggi) for the supporting and facilitation of this research. We also thank Thahira Hanum Sekarmewangi for improving the draft of this article.

References
[1] Usiskin Z 2013 Studying textbooks in an information age-a United States perspective ZDM - Int J Math Educ vol 45 no 5 p 713–723
[2] Gracin D G 2018 Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples Int J Math Educ Sci Technol p 1–22
[3] Hong D S and Choi K M 2014 A comparison of Korean and American secondary school textbooks: The case of quadratic equations Educ Stud Math vol 85 no 2 p 241–263
[4] Purnomo Y W, Mastura F S and Perbowo K S 2019 Contextual Features of Geometrical Problems in Indonesian Mathematics Textbooks, in Journal of Physics: Conference Series vol 1315 p 012048.
[5] Alajmi A H 2012 How do elementary textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait Educ Stud Math vol 79 no 2 p 239–261
[6] Chang C C and Silalahi S M 2017 A Review and Content Analysis of Mathematics Textbooks in Educational Research Probl Educ 21st Century vol 75 no 3 p 235–251
[7] Charalambous C Y, Delaney S, Hsu H Y and Mesa V 2010 A comparative analysis of the addition and subtraction of fractions in textbooks from three Countries Math Think Learn vol 12 no 2 p 117–151
[8] Li Y, Chen X and An S 2009 Conceptualizing and organizing content for teaching and learning in selected Chinese, Japanese and US mathematics textbooks: The case of fraction division ZDM - Int J Math Educ vol 41 no 6 p 809–826
[9] Wijaya A 2017 The Relationships between Indonesian Fourth Graders’ Difficulties in Fractions and the Opportunity to Learn Fractions: A Snapshot of TIMSS Result Int J Instr vol 10 no 4 p 221–236
[10] Purnomo Y W, Widowati C and Ulfah S 2019 Incomprehension of the Indonesian Elementary School Students on Fraction Division Problem Infinity J vol 8 no 1 p 57
[11] Purnomo Y W, Widowati C, Aziz T A and Pramudiani P 2017 Fractions division knowledge of elementary school student: The case of Lala, in AIP Conference Proceedings vol 1868
[12] National Council of Teachers of Mathematics 2000 Principles and Standards for School Mathematics (Reston, VA: National Council of Teachers of Mathematics)
[13] Tsai T L and Li H C 2017 Towards a framework for developing students’ fraction proficiency Int J Math Educ Sci Technol vol 48 no 2 p 244–255
[14] Lamon S J 2012 Teaching Fractions and Ratios for Understanding (Routledge)
[15] J. A. Van de Walle, K. S. Karp, and J. M. Bay-William, Elementary and Middle School Mathematics: Teaching Developmentally (Ninth Edition). 2015.
[16] Purnomo Y W 2015 Pembelajaran Matematika untuk PGSD: Bagaimana Guru Mengembangkan Penalaran Proporsional Siswa (Jakarta: Erlangga)
[17] Fan L, Zhu Y and Miao Z 2013 Textbook research in mathematics education: development status and directions ZDM vol 45 no 5 p 633–646
[18] Fan L 2018 Research on Mathematics Textbooks and Teachers’ Resources.
[19] Fan L 2013 Textbook research as scientific research: towards a common ground on issues and methods of research on mathematics textbooks ZDM vol 45 no 5 p 765–777
[20] Charalambous C Y and Pitta-Pantazi D 2007 Drawing on a Theoretical Model to Study Students’ Understandings of Fractions Educ Stud Math vol 64 no 3 p 293
[21] Clarke D M, Roche A and Mitchell A 2008 Ten Practical Tips for Making Fractions Come Alive and Make Sense Math Teach Middle School vol 13 no 7 p 372–380
[22] Siebert D and Gaskin N 2006 Creating, Naming, and Justifying Fractions *Teach Child Math* vol 12 no 8 p 394–400

[23] Watanabe T 2006 The teaching and learning of fractions A Japanese perspective *Teach. Child. Math* vol 12 no 7 p 368–374

[24] Wijaya A, van den Heuvel-Panhuizen M, Doorman M and Robitzsch A 2014 Difficulties in solving context-based PISA mathematics tasks: An analysis of students’ errors *Math Enthus* vol 11 no 3 p 555–584

[25] Wijaya A, , van den Heuvel-Panhuizen M and Doorman M 2015 Teachers’ teaching practices and beliefs regarding context-based tasks and their relation with students’ difficulties in solving these tasks *Math Educ Res J* vol 27 no 4 p 637–662