Non compound nucleus fission events and standard saddle-point statistical model

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Abstract

The large body of experimental data on the fission fragments anisotropies are analyzed in several heavy-ion induced fission reaction systems. The entrance channel mass asymmetry parameters of these systems put on the both sides of the Businaro-Gallone mass asymmetry parameters. The role of the mass numbers of the projectile and the target in the prediction of a normal or an anomalous behavior in angular anisotropy, as well as the validity of standard saddle-point statistical model are considered. The average contribution of non compound nucleus fission for the systems with an anomalous behavior in anisotropy are also determined.

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I. INTRODUCTION

During almost seven decades of researches, an immense body of experimental data on fission processes have been accumulated. In addition, tremendous effort has been invested on its theoretical understanding. Nevertheless, the full understanding of the fission process has still not been reached. The angular distribution of fission fragments in heavy-ion induced fission reaction is an effective probe to study the dynamics of fission reactions. Non compound nucleus (NCN) fission is an important area in the field of nuclear fission. In this process, target and projectile come in contact forming a composite system, in which the system reseparates before reaching to a compact compound nucleus (CN). Due to the presence of the NCN fission events, fission fragment anisotropies have been observed to be anomalous in comparison to the prediction of standard saddle-point statistical model (SSPSM), as well as the widths of the fission fragment mass distributions have been observed to be large in comparison to the compound nucleus fission events. In addition, the entrance channel properties, such as the mass asymmetry ($\alpha$) of the interacting systems with respect to the Businaro-Gallone mass asymmetry parameter ($\alpha_{BG}$), deformation of interacting nuclei, the bombarding energy relative to the fusion barrier, the nuclear orientation of the interacting nuclei such as the collision with the sides of the deformed target nucleus, and the product of $Z_P Z_T$ of the interacting systems (where, $Z_P$ and $Z_T$ are projectile and target atomic numbers, respectively) play an important role in the formation of CN. It is also reported that with deformed targets/projectiles shell effects play a major role in the survival probability of the CN [7, 54]. It is well known that the SSPSM as a standard theory of fission fragment angular distributions has been generally used to explain the observed anisotropy data and it is based on the assumption that the fission fragments are emitted along the symmetry axis of the fissioning nucleus and the K component of the total angular momentum I along the symmetry axis is conserved during the descent saddle to scission point [1]. Although, the SSPSM as the oldest model has had outstanding success for several induced fission reactions by lighter projectiles, but the angular anisotropies for several heavy-ion induced fusion-fission reactions are significantly higher than those expected from the SSPSM predictions. A majority of the existing models attribute the observation of anomalous behaviors in angular anisotropies of fission fragments to the presence of NCN fission (NCNF) mechanisms such as quasi fission (QF), fast fission (FF), and pre-equilibrium fission (PEF),
rather than to the breakdown of the SSPSM. It is reported that for the induced reactions by heavy projectiles \((A_P \geq 20)\), on various targets above the fusion barrier, the measured angular anisotropies were larger than the SSPSM predictions as well as, these anomalous behaviors in angular anisotropy were attributed to the contribution of NCNF (QF) events. Nevertheless, we observe normal behaviors in the measured anisotropies for many induced fission reactions by heavy projectiles \((A_P \geq 20)\) [4-6]. However, later experimental angular anisotropies were obtained for the reactions induced by light projectiles \((A_P \leq 20)\), on actinide targets in which the anisotropy could not be explained by SSPSM. For example, in the \(^{16}\text{O}+^{238}\text{U}\) reaction system, the contribution of NCNF was related to the deformed actinide target nucleus \([7,8]\). While, in a systematic study on the induced fission of \(^{238}\text{U}\) and \(^{232}\text{Th}\) targets by \(^{16}\text{O}\) and \(^{19}\text{F}\) projectiles, as well as for \(^{14}\text{N}+^{232}\text{Th}\) reaction system at energies near coulomb barrier, it is observed anomalous behaviors in the fission fragment anisotropies, it is found normal behaviors in measured anisotropies for several reactions induced by light projectiles \((A_P \leq 20)\) [4-6, 9-14]. In the literature, it is reported that for the systems with the entrance mass asymmetry \(\alpha = \frac{(A_T - A_p)}{(A_T + A_p)}\) greater than Businaro-Gallone critical mass asymmetry parameter \(\alpha_{BG}\) \(\alpha_{BG}\) is parameterized as \(\alpha_{BG} = 0\) for \(\chi < \chi_{BG}\); and \(\alpha_{BG} = 1.12 \sqrt{\frac{\chi - \chi_{BG}}{(\chi - \chi_{BG}) + 0.24}}\) for \(\chi > \chi_{BG}\), where \(\chi\) is fissility parameter, and \(\chi_{BG} = 0.396\) \([51]\) , the measured fragment anisotropies are in agreement with the SSPSM predictions, while in the case \(\alpha < \alpha_{BG}\), the experimental fragment anisotropies obviously deviate from the SSPSM calculations [12, 15]. However, for \(^{11}\text{B}+^{232}\text{Th}\) [9] reaction system having \(\alpha > \alpha_{BG}\), as well as for \(^{19}\text{F}+^{208}\text{Pb}\) [16], \(^{16}\text{O}+^{208}\text{Pb}\) [17], \(^{19}\text{F}+^{209}\text{Bi}\) [18] and \(^{16}\text{O}+^{209}\text{Bi}\) [19] reaction systems with \(\alpha < \alpha_{BG}\), the angular anisotropies show anomalous and normal behaviors, respectively. There are several heavy ion induced systems having \(\alpha > \alpha_{BG}\) with anomalous behaviors in angular anisotropies, as well as systems having \(\alpha < \alpha_{BG}\) with normal behaviors in angular anisotropies as indicated in Table I.

The model of Ramamurthy and Kapoor \([42]\) gives a quantitative estimate of the effect of NCNF on fission fragment angular distribution. According to this model, the probability of NCNF events \(P_{NCNF}\) is given by an approximate expression as follows

\[
P_{NCNF}(I) = \exp[-0.5B_f(I, K = 0)/T_{sad}],
\]

where, \(B_f\) and \(T_{sad}\) are the fission barrier height and the temperature at the saddle-point, respectively. Recently, the investigations of the fission fragment mass angle correlations and
TABLE I: Heavy ion induced fission systems with unexpected behaviors in angular anisotropies of fission fragments. These behaviors are not expected by the comparison between the entrance channel mass asymmetry (α) and the Businaro-Gallone critical mass asymmetry (α_{BG}).

Mass ratio distributions, as well as the analysis of the variance of the mass distributions as a function of temperature and angular momentum is used for the presence of QF in heavy ion induced fission reactions [46]. A sudden change in the standard deviation (a sudden increase in the standard deviation as energy decreases to below-barrier energies) of the fission fragments mass distribution as a function of $E_{c.m.}/V_b$ (where $E_{c.m.}$ and $V_b$ are the projectile energy in center of mass and the Coulomb barrier, respectively), the observation of an anomalous behavior in fission fragment anisotropies, the measurement of evaporation residue cross section and the measurement of prescission neutron multiplicity are known as the different probes for the presence of PEF and QF for several heavy ion induced fission systems [47]. It must be pointed that the product of $Z_PZ_T$ (where $Z_P$ and $Z_T$ are the

| Fission systems | Comparison between $\alpha$ and $\alpha_{BG}$ | NCNF contribution | References |
|-----------------|---------------------------------------------|-------------------|------------|
| $^{9}$Be + $^{232}$Th | $\alpha(=0.925)>\alpha_{BG}(=0.882)$ | Yes | [49] |
| $^{11}$B + $^{243}$Am | $\alpha(=0.913)>\alpha_{BG}(=0.903)$ | Yes | [23] |
| $^{12}$C + $^{232}$Th | $\alpha(=0.902)>\alpha_{BG}(=0.890)$ | Yes | [9] |
| $^{12}$C + $^{235}$U | $\alpha(=0.903)>\alpha_{BG}(=0.898)$ | Yes | [9, 27] |
| $^{12}$C + $^{236}$U | $\alpha(=0.903)>\alpha_{BG}(=0.897)$ | Yes | [9, 27] |
| $^{12}$C + $^{238}$U | $\alpha(=0.904)>\alpha_{BG}(=0.896)$ | Yes | [9, 27] |
| $^{16}$O + $^{182}$W | $\alpha(=0.838)<\alpha_{BG}(=0.840)$ | No | [25] |
| $^{16}$O + $^{186}$Os | $\alpha(=0.842)<\alpha_{BG}(=0.850)$ | No | [6] |
| $^{16}$O + $^{188}$Os | $\alpha(=0.843)<\alpha_{BG}(=0.849)$ | No | [6] |
| $^{16}$O + $^{194}$Pt | $\alpha(=0.848)<\alpha_{BG}(=0.863)$ | No | [22] |
| $^{16}$O + $^{197}$Au | $\alpha(=0.850)<\alpha_{BG}(=0.861)$ | No | [46] |
| $^{18}$O + $^{197}$Au | $\alpha(=0.833)<\alpha_{BG}(=0.860)$ | No | [20] |
| $^{19}$F + $^{184}$W | $\alpha(=0.813)<\alpha_{BG}(=0.843)$ | No | [50] |
| $^{19}$F + $^{188}$Os | $\alpha(=0.816)<\alpha_{BG}(=0.853)$ | No | [26] |
| $^{19}$F + $^{192}$Os | $\alpha(=0.820)<\alpha_{BG}(=0.849)$ | No | [26] |
| $^{19}$F + $^{194}$Pt | $\alpha(=0.822)<\alpha_{BG}(=0.861)$ | No | [24] |
| $^{19}$F + $^{197}$Au | $\alpha(=0.824)<\alpha_{BG}(=0.865)$ | No | [21] |
| $^{19}$F + $^{198}$Pt | $\alpha(=0.825)<\alpha_{BG}(=0.858)$ | No | [24] |
| $^{24}$Mg + $^{178}$Hf | $\alpha(=0.762)<\alpha_{BG}(=0.850)$ | No | [6] |
| $^{24}$Mg + $^{192}$Os | $\alpha(=0.778)<\alpha_{BG}(=0.865)$ | No | [28] |
| $^{24}$Mg + $^{197}$Au | $\alpha(=0.783)<\alpha_{BG}(=0.879)$ | No | [28] |
| $^{27}$Al + $^{186}$W | $\alpha(=0.764)<\alpha_{BG}(=0.861)$ | No | [46] |
| $^{28}$Si + $^{176}$Yb | $\alpha(=0.725)<\alpha_{BG}(=0.849)$ | No | [4] |
| $^{34}$S + $^{168}$Er | $\alpha(=0.663)<\alpha_{BG}(=0.850)$ | No | [5] |
projectile and target atomic numbers, respectively) of the interacting systems, play an important role in the formation of the CN. Although in the past, it is predicted that QF occurs when $Z_PZ_T \geq 1600$ but recent results show that the onset of QF starts at a $Z_PZ_T$ value equal to nearly 1000 and plays a dominant role at higher values of $Z_PZ_T$. With this motivation, the purpose of the present paper is to obtain a relation in terms of projectile and target mass numbers by analyzing the large body of experimental data on fission anisotropies for determination of the validity of SSPSM.
II. METHOD OF CALCULATIONS

A. Standard Saddle-Point Statistical Model, and the calculation of SSPSM predictions

According to statistical theory, fission fragments angular distribution \( W(\theta) \) for a spin zero projectile-target combination is given by the following expression \[29\]

\[
W(\theta) \propto \sum_{I=0}^{\infty} \frac{(2I+1)^2 T_I \exp[-(I + \frac{1}{2})^2 \sin^2 \theta/4K_0^2]J_0[i(I + \frac{1}{2})^2 \sin^2 \theta/4K_0^2]}{(2K_0^2)^{1/2}\text{erf}[(I + \frac{1}{2})/(2K_0^2)^{1/2}]}.
\] (2)

Where \( T_I, K_0^2, \) and \( J_0 \) are the transmission coefficient for fission, the variance of the \( K \) distribution ( \( K \) is the component of the angular momentum vector \( I \) on the symmetry axis of the fissioning nucleus ), and the zeroth order Bessel function, respectively. The variance of the \( K \) distribution is calculated by the following relation

\[
K_0^2 = \frac{3_{\text{eff}} T_{\text{sad}}}{a^2},
\] (3)

\( 3_{\text{eff}} \) and \( T_{\text{sad}} \) are the effective moment of inertia and the nuclear temperature of the compound nucleus at the saddle-point, respectively. The nuclear temperature of the compound nucleus at the saddle-point is given by

\[
T_{\text{sad}} = \sqrt{\frac{E_{\text{ex}}}{a}} = \sqrt{\frac{E_{\text{c.m.}} + Q - B_f - E_R - \nu E_n}{a}}.
\] (4)

In this equation, \( E_{\text{ex}} \) denotes the excitation energy of the compound nucleus at the saddle-point, while \( E_{\text{c.m.}}, Q, B_f, E_R, \nu, \) and \( E_n \) represent the center-of-mass energy of the projectile, the \( Q \) value, fission barrier height, rotational energy of the compound nucleus, the number of pre-fission neutrons, and the excitation energy lost due to evaporation of one neutron from the compound nucleus prior to the system reaching to the saddle-point. The quantity \( a \) stands for the level density parameter at the saddle-point. The fission fragment angular distributions are characterized by anisotropy \( (A) \), defined as the ratio of the yield at 180° or 0° to that at 90° \( (A = \frac{W(0^\circ \text{or } 180^\circ)}{W(90^\circ)}) \). The fission anisotropy in the SSPSM \( (A_{\text{SSPSM}}) \) is given by an approximate formula \[29\]

\[
A_{\text{SSPSM}} \approx 1 + \frac{<I^2>}{4K_0^2}.
\] (5)

In this work, \( a \) is taken \( \frac{A_{\text{CN}}}{8} \) (by accounting \( \frac{A_{\text{CN}}}{10} \) instead of \( \frac{A_{\text{CN}}}{8} \) in the calculations, the difference will be less than 10%), as well as \( 3_{\text{eff}}, B_f, \) and \( E_R \) are accounted by the
use of rotating finite range model (RFRM) \[30\], while \( < I^2 > \) quantities are taken from the literature \[2, 11, 31-34\]. We have used the values of the literature for \( \nu \) \[11, 34-38\] and \( E_n \) is taken 10 MeV \[18, 39\]. In the present work, the pre-fission neutrons are taken to be emitted before the saddle-point, since it is not straightforward to separate experimentally the contribution of neutrons emitted before the saddle-point and the ones emitted after the saddle-point but before the scission point.

**B. Calculation of the average contribution of NCNF anisotropies**

In recent years, heavy-ion induced fission fragments angular distribution measurements performed at below to above barrier energies have generated much interest due to the failure of the predictions of the SSPSM for heavy-ion induced fission of actinide targets \[12, 40, 41\]. The effects of entrance channel parameters such as mass asymmetry, target deformation, and target or projectile spins on fission fragment anisotropies have been identified in the past from systematic study of fission fragments angular distributions at energies around the Coulomb barrier energies in actinide targets \[12\]. Non-equilibrium fission (PEF, QF, and FF) was thought to be a probable cause of this anomaly. Almost 25 years ago, Ramamurthy and Kapoor \[42\] proposed the pre-equilibrium fission (PEF) model to explain the anomalous anisotropies in several heavy-ion induced fission reaction at above barrier energies. The main difference between CN fission and PEF is that in the latter case the K degree of freedom is not equilibrated but other degrees such as energy and mass-asymmetry are fully equilibrated. Therefore the assumption of symmetric mass division is justified in case of PEF. The K distributions of PEF will be the product of the entrance channel K distribution and the saddle point K distribution \[27, 43\], and the narrower of the above two K distributions governs the fragment anisotropy. This explains the observed larger anisotropies whenever the input K distribution is not fully equilibrated. According to this model, the final K distribution for fissioning nuclei is given by

\[
P_f(K) = P_{\text{initial}}(K)P_{\text{saddle}}(K),
\]

where \( P_{\text{initial}}(K) \) is the K distribution for the initial di-nuclear complex and \( P_{\text{saddle}}(K) \) is the Gaussian K distribution at the saddle-point. On the whole, the final K distribution is governed by the initial narrow K distribution populated in the formation phase. Following the work of \[27, 43\], the probability of a fissioning nucleus having the quantum numbers I and K, when populated from an entrance channel K-state distribution with peaks at \( \tilde{K} \) is
given by

\[ P(J, K, \tilde{K}) = \exp\left[ -\frac{(K - \tilde{K})^2}{2\sigma_K^2} \right] \times \exp\left[ -\frac{(K\hbar)^2}{2\Theta_{eff} T} \right] \]  \tag{6} \]

\( P(J, K, \tilde{K}) \) is obtained by taking the initial K-state distribution for each I value convoluted by a Gaussian with standard deviation \( \sigma_K \) and multiplied by the SSPSM K-state distribution at fission saddle-point.

It has been observed that at sub-barrier energies all the systems with actinide targets show anomalous anisotropies of varying extent with respect to the SSPSM. In order to explain such anomalous behaviors in angular anisotropies at sub-barrier energies, a few models such as dependent QF model, pre-equilibrium model, a model with considering the incorporation of target and projectile spins, and the entrance channel dependent K-state model (ECD-K) have been well recognized. It is obvious that, the prediction of SSPSM shows the anisotropy of compound nucleus fission (CNF) events, as well as the experimental values of anisotropies are due to CNF and NCNF events for the systems having anomalous behaviors in angular anisotropies. The average contribution of NCNF anisotropies over the energy range of projectile \( (A_{NCN}) \) for these systems is given by

\[ A_{NCN} = \frac{A_{exp} - A_{SSPSM}}{A_{exp}}. \]  \tag{7} \]

In this equation, \( A_{SSPSM} \) is the average contribution of SSPSM prediction over the energy range of the projectile, as well as \( A_{exp} \) stands the average experimental value of anisotropy over the same energy range.

III. RESULTS AND DISCUSSION

The experimental fission fragment angular anisotropies \( (A) \) along with the SSPSM predictions for the induced fission of \(^{208}\text{Pb}\) target by different projectiles \((^{24}\text{Mg}, \, ^{28}\text{Si}, \text{and} \, ^{32}\text{S})\) are shown in Fig. 1. Calculation of the average of NCNF contributions for these three systems over the \(1.05 \leq \frac{E_{cm}}{V_b} \leq 1.35\) energy range show that the NCNF contributions will increase as the mass number of the projectile increases. We also considered the induced fission of \(^{238}\text{U}\) target by \(^{16}\text{O}, \, ^{19}\text{F}, \, ^{27}\text{Al}\) projectiles, as well as the induced fission \(^{232}\text{Th}\) by \(^{16}\text{O}, \, ^{19}\text{F}, \text{and} \, ^{32}\text{S}\) projectiles. The average contributions of NCNF for the induced fission of \(^{208}\text{Pb}, \, ^{238}\text{U}, \text{and} \, ^{232}\text{Th}\) targets by different projectiles are shown in Fig. 2. These average contributions for the induced fission of \(^{238}\text{U}, \text{and} \, ^{232}\text{Th}\) targets by different projectiles are
FIG. 1: Experimental data of anisotropy (A) for the fission of $^{208}\text{Pb}$ target induced by several projectiles. (a) Solid and dashed curves are the SSPSM prediction and experimental value of anisotropy for the fission of $^{24}\text{Mg} + ^{208}\text{Pb}$ reaction system, (b) Solid and dashed curves are the SSPSM prediction and experimental value of anisotropy for the fission of $^{28}\text{Si} + ^{208}\text{Pb}$ reaction system, and (c) Solid and dashed curves are the SSPSM prediction and experimental value of anisotropy for the fission of $^{32}\text{S} + ^{208}\text{Pb}$ reaction system [2].

calculated over the $1.0 \leq \frac{E_{\text{c.m.}}}{V_b} \leq 1.2$ and $0.95 \leq \frac{E_{\text{c.m.}}}{V_b} \leq 1.15$ energy ranges, respectively. In this figure, the thick solid line shows the contributions of NCNF for induced fission of $^{208}\text{Pb}$ target by different projectiles. The points on this thick solid line are the average contributions of NCNF for the $^{24}\text{Mg}$, $^{28}\text{Si}$, and $^{32}\text{S} + ^{208}\text{Pb}$ reaction systems. It can be observed that in induced fission of $^{208}\text{Pb}$ nucleus, the contributions of NCNF for projectiles whose the mass number is 20 or less is zero, as well as there is the average contribution of NCNF for induced fission of this target by projectiles with the mass number more than 20.
This result are found to be in good agreement with experiments [2]. In this figure, the thin solid and dashed lines show the average contributions of NCNF for induced fission of $^{238}\text{U}$ target by different projectiles. The discrepancy on these two lines is because of the different values of $<I^2>$ for induced fission of $^{238}\text{U}$ by $^{16}\text{O}$ projectile that was taken from different references (Back et al., [2], and Nasirov et al., [34]).

![Diagram](image)

**FIG. 2:** The average contributions of NCNF ($A_{NCN}$) versus the mass number of projectile. Thick solid line for $^{208}\text{Pb}$, thin solid and dashed lines for $^{238}\text{U}$ nucleus by using Back data and by using Nasirov data, respectively, and dashed-dotted line for $^{232}\text{Th}$ target.

Dashed-dotted line in the Fig. 2 shows, the average contributions of NCNF for induced fission of $^{232}\text{Th}$ target by different projectiles over the $0.95 \leq \frac{E_{c.m.}}{V_b} \leq 1.15$ energy range. It is interesting to note that whatever the target is heavier, one can infer the onset of NCNF events occurs with lighter projectiles and vise versa. By considering several heavy-ion induced fission systems with anomalous behaviors in angular anisotropies, such as $^{24}\text{Mg}, ^{28}\text{Si}, \text{and } ^{32}\text{S} + ^{208}\text{Pb}$ in comparison with $^{16}\text{O}, ^{19}\text{F}, \text{and } ^{27}\text{Al} + ^{238}\text{U}$ systems, it can be observed that the contributions of NCNF in induced fission of heavier targets are more than the contributions of NCNF in induced fission of lighter target by the same projectile. While the calculated value of the average contributions of NCNF for the $^{32}\text{S} + ^{197}\text{Au}$ system over the $1.1 \leq \frac{E_{c.m.}}{V_b} \leq 1.2$ energy range is approximately 44%, this contribution for the $^{32}\text{S} + ^{208}\text{Pb}$ system over the same energy range is obtained 50%. The predicted value of average contributions of NCNF for the $^{40}\text{Ar} + ^{208}\text{Pb}$ system over the $1.05 \leq \frac{E_{c.m.}}{V_b} \leq 1.3$ energy range from Fig. 2 is approximately 87% where is a good agreement with the work of Keller *et al.* [52]. Itkis *et al.*, measured the mass and energy distributions of the $^{56}\text{Fe} + ^{208}\text{Pb}$ reaction...
system, as well as they reported that for this reaction, the QF process dominates at all measured energy \[53\]. This result is predicted very well from Fig. 2. In order to make a comparison between the average contributions of NCNF in induced fission of different targets by the same projectile, we have calculated these contributions for the induced fission of \(^{184}\)W, \(^{197}\)Au, and \(^{208}\)Pb targets by \(^{32}\)S projectile, as well as those of the induced fission of \(^{232}\)Th, \(^{238}\)U, and \(^{248}\)Cm targets by \(^{16}\)O projectile over the same energy range. These calculated contributions are shown in Fig. 3. It can be seen from this figure that the average contributions of NCNF for induced fission of different targets by the same projectile begin from a given target mass number, as well as this contributions also show a linear behavior in terms of the mass numbers of targets for a given projectile.

![Graph](image)

**FIG. 3**: The average contributions of NCNF (\(A_{NCN}\)) for induced fission of different targets by the same projectiles. Thick solid and thin solid lines shows \(A_{NCN}\) for induced fission of different targets by \(^{16}\)O and \(^{32}\)S projectiles, respectively.

Finally, as indicated in Fig. 4, for the heavy-ion induced fission reactions systems in which the mass numbers of target and projectile (\(A_T\), and \(A_P\), respectively) locate below the curve shown in this figure, fission fragment angular anisotropies exhibit normal behaviors, in addition, the predictions of SSPSM are in agreement with the experimental angular anisotropies data. However, for the reactions in which the \(A_T\) and \(A_P\) lying above this curve, there exist an admixture of CNF and NCNF events, so that the measured fission fragment anisotropies are anomalously large compared to the predictions based on the SSPSM, as well as this model is not valid.

It is interesting to note that Berriman et al., \[45\] reported a contribution of NCNF for
a very asymmetric reaction $^{19}$F + $^{197}$Au system by measuring of the width of the fission fragments mass distribution. However, a recent measurement of fission fragments angular distributions for the same reaction showed no evidence of NCNF [20, 21] as can be also seen from Fig. 4. In another work, evidence of NCNF has been found in the $^{34}$S + $^{168}$Er reaction system by considering of fission fragment mass distribution, but no evidence for NCNF was observed in the investigation of fission fragment angular distributions [5, 6]. Fig. 4 shows that the contribution of NCNF is not significant for induced fission of $^{168}$Er target by the projectiles with $A_P \leq 35$. Our results is in agreement with the experimental observations which have been obtained up to now [4-7, 9, 11, 16-21, 23, 27, 46, 47].

IV. SUMMARY AND CONCLUSIONS

The average contributions of NCNF have been calculated for several heavy-ion induced fission reaction systems with having an anomalous behaviors in fission fragments angular anisotropies by comparison between the experimental data of fission fragment angular distributions and the predictions of SSPSM. Although, it is reported that for the systems with $\alpha > \alpha_{BG}$, the measured fission fragments anisotropies are in general agreement with the expectation of the SSPSM, as well as for the systems with $\alpha < \alpha_{BG}$, the experimental fragment anisotropies are considerably greater than the predictions of the SSPSM at sub barrier and near barrier energies. However, there are the reaction systems on the different sides of Businaro-Gallone critical mass asymmetry with a unexpected behaviors in fission fragments
angular anisotropies as indicated in Table I. Our calculated NCNF contributions for the reaction systems with anomalous behaviors in angular anisotropies show that these contributions increase with increasing the mass number of projectiles for a given target, as well as this contribution exhibits a linear behavior as a function of the mass number of targets for a given projectile. Finally, the validity of SSPSM in the prediction of angular anisotropies for the reaction systems with normal behaviors in fission fragment angular anisotropies is also determined.

[1] C. Wagemans, ”The Nuclear Fission Process”, CRC Press, Boca Raton, Florida, USA, 1991.
[2] B. B. Back, R. R. Betts, J. E. Gindler, B. D. Wilkins, S. Saini, M. B. Tsang, C. K. Gelbke, W. G. Lynch, M. A. McMahan, and P. A. Baisden, Phys. Rev. C 32, 195 (1985).
[3] J. Toke, R. Bock, G.X. Dai, A. Gobbi, S. Gralla, K. D. Hildenbrand, J. Kuzminski, W. J. F. Muller, A. Olmi, and H. Stelzer, Nucl. Phys. A 440, 327 (1985).
[4] R. Tripathi, K. Sudarshan, S. K. Sharma, K. Ramachandran, A. V. R. Reddy, P. K. Pujari, and A. Goswami, Phys. Rev. C 79, 064607 (2009).
[5] C. R. Morton, D.J. Hinde, A. C. Berriman, R. D. Butt, M. Dasgupta, A. Godley, and J. O. Newton, Phys. Lett. B 481, 160 (2000).
[6] R. Rafiei, R. G. Thomas, D. J. Hinde, M. Dasgupta, C. R. Morton, L. R. Gasques, M. L. Brown, and M. D. Rodriguez, Phys. Rev. C 77, 024606 (2008).
[7] D. J. Hinde, M. Dasgupta, J. R. Leigh, J. P. Lestone, J. C. Mein, C. R. Morton, J. O. Newton, and H. Timmers, Phys. Rev. Lett. 74, 1295 (1995).
[8] D. J. Hinde, M. Dasgupta, J. R. Leigh, J. C. Mein, C. R. Morton, J. O. Newton, and H. Timmers, Phys. Rev. C 53, 1290 (1996).
[9] R. G. Thomas, R. K. Choudhury, A. K. Mohanty, A. Saxena, and S. S. Kapoor, Phys. Rev. C 67, 041601R (2003).
[10] S. Kailas, D. M. Nadkarni, A. Chatterjee, A. Saxena, S. S. Kapoor, R. Vandenbosch, J. P. Lestone, J. F. Liang, D. J. Prindle, A. A. Sonzogni, and J. D. Bierman, Phys. Rev. C 59, 2580 (1999).
[11] A. Karnik, S. Kailas, A. Chatterjee, A. Navin, A. Shrivastava, P. Singh, and M. S. Samant, Phys. Rev. C 52, 3189 (1995).
[12] S. Kailas, Phys. Rep. 284, 381 (1997).
[13] B. R. Behera, M. Satpathy, S. Jena, S. Kailas, R. G. Thomas, K. Mahata, A. Chatterjee, S. Roy, P. Basu, M. K. Sharan, and S. K. Datta, Phys. Rev. C 69, 064603 (2004).
[14] B. B. Back, P. B. Fernandez, B. G. Glagola, D. Henderson, S. Kaufman, J. G. Keller, S. J. Sanders, F. Videbaek, T. F. Wang, and B. D. Wilkins, Phys. Rev. C 53, 1734 (1996).
[15] B. P. Ajith Kumar, K. M. Varier, R. G. Thomas, K. Mahata, B. V. John, A. Saxena, H. G. Rajprakash, and S. Kailas, Phys. Rev. C 72, 067601 (2005).
[16] D. J. Hinde, A. C. Berriman, M. Dasgupta, J. R. Leigh, J. C. Mein, C. R. Morton, and J. O. Newton, Phys. Rev. C 60, 054602 (1999).
[17] H. Rossner, D. J. Hinde, J. R. Leigh, J. P. Lestone, J. O. Newton, J. X. Wei, and S. Elfstrom, Phys. Rev. C 45, 719 (1992).
[18] A. M. Samant, S. Kailas, A. Chatterjee, A. Shrivastava, A. Navin, and P. Singh, Eur. Phys. J. A 7, 59 (2000).
[19] E. Vulgaris, L. Grodzins, S. G. Steadman, and R. Ledoux, Phys. Rev. C 33, 2017 (1986).
[20] S. Appannababu, S. Mukherjee, N. L. Singh, P. K. Rath, G. K. Kumar, R. G. Thomas, S. Santra, B. K. Nayak, A. Saxena, R. K. Choudhury, K. S. Golda, A. Jhingan, R. Kumar, P. Sugathan and H. Singh, Phys. Rev. C 80, 024603 (2009).
[21] R. Tripathi, K. Sudarshan, S. Sodaye, A. V. R. Reddy, K. Mahata, and A. Goswami, Phys. Rev. C 71, 044616 (2005).
[22] E. Prasad, K. M. Varier, R. G. Thomas, P. Sugathan, A. Jhingan, N. Madhavan, B. R. S. Babu, R. Sandal, S. Kalkal, S. Appannababu, J. Gehlot, K. S. Golda, S. Nath, A. M. Vinodkumar, B. P. A. Kumar, B. V. John, G. Mohanto, M. M. Musthafa, R. Singh, A. K. Sinha, and S. Kailas, Phys. Rev. C 81, 054608 (2010).
[23] R. Tripathi, K. Sudarshan, S. Sodaye, S. K. Sharma, and A. V. R. Reddy, Phys. Rev. C 75, 024609 (2007).
[24] K. Mahata, S. Kailas, A. Shrivastava, A. Chatterjee, P. Singh, S. Santra, and B. S. Tomar, Phys. Rev. C 65, 034613 (2002). and S. K. Datta, Nucl. Phys. A 734, 249
[25] D. J. Hinde, W. Pan, A. C. Berriman, R. D. Butt, M. Dasgupta, C. R. Morton, and J. O. Newton, Phys. Rev. C 62, 024615 (2000).
[26] K. Mahata, S. Kailas, A. Shrivastava, A. Chatterjee, A. Navin, P. Singh, S. Santra, and B. S. Tomar, Nucl. Phys. A 720, 209 (2003).
[27] J. P. Lestone, A. A. Sonzogni, M. P. Kelly, and R. Vandenbosch, Phys. Rev. C 56, R2907 (1997).
[28] R. Tripathi, K. Sudarshan, S. Sodaye, A. Goswami, and A. V. R. Reddy, Int. J. Mod. Phys. E 17(2), 419 (2008).
[29] R. Vandenbosch and J. R. Huizenga, Nuclear Fission (Academic Press, New York, 1973).
[30] A. J. Sierk, Phys. Rev. C 33, 2039 (1986).
[31] A. K. Nasirov, A. I. Muminov, R. K. Utamuratov, G. Fazio, G. Giardina, F. Hanappe, G. Mandaglio, M. Manganaro, and W. Scheid, Eur. Phys. J. A 34, 325 (2007).
[32] W. Q. Shen, J. Albinski, A. Gobbi, S. Gralla, K. D. Hildenbrand, N. Herrmann, J. Kuzminski, W. J. Muller, H. Stelzer, J. Toke, B. B. Back, S. Bjornholm, and S. P. Sorensen, Phys. Rev. C 36, 115 (1987).
[33] N. Majumdar, P. Bhattacharya, D. C. Biswas, R. K. Choudhury, D. M. Nadkarni, and A. Saxena, Phys. Rev. C 51, 3109 (1995).
[34] H. Q. Zhang, C. L. Zhang, C. J. Lin, Z. H. Liu, F. Yang, A. K. Nasirov, G. Mandaglio, M. Manganaro, and G. Giardina, Phys. Rev. C 81, 034611 (2010).
[35] D. O. Eremenko, V. A. Drozdov, M. H. Eslamizadeh, O. V. Fotina, S. Yu. Platonov, and O. A. Yuminov, Physics of Atomic Nuclei 69, 1423 (2006).
[36] V. A. Drozdov, D. O. Eremenko, O. V. Fotina, S. Yu. Platonov, O. A. Yuminov, and G. Giardina, Physics of Atomic Nuclei 66, 1622 (2003).
[37] A. Saxena, A. Chatterjee, R. K. Choudhury, S. S. Kapoor, and D. M. Nadkarni, Phys. Rev. C 49, 932 (1994).
[38] D. J. Hinde, H. Ogata, M. Tanaka, T. Shimoda, N. Takahashi, A. Shinohara, S. Wakamatsu, K. Katori, and H. Okamura, Phys. Rev. c 39, 2268 (1989).
[39] R. Vandenbosch, J. D. Bierman, J. P. Lestone, J. F. Liang, D. J. Prindle, A. A. Sonzogni, S. Kailas, D. M. Nadkarni, and S. S. Kapoor, Phys. Rev. C 54, R977 (1996).
[40] R. K. Choudhury, Pramana, J. Phys. 57, 585 (2001).
[41] J. P. Lestone, A. A. Sonzogni, M. P. Kelly, and R. Vandenbosch, J. Phys. G 23, 1349 (1997).
[42] V. S. Ramamurthy and S. S. Kapoor, Phys. Rev. Lett. 54, 178 (1985).
[43] D. Vorkapic and B. Ivanisevic, Phys. Rev. C 52, 1980 (1995).
[44] B. K. Nayak, R. G. Thomas, R. K. Choudhury, A. Saxena, P. K. Sahu, S. S. Kapoor, R. Raghav Varma, and D. Umakanth, Phys. Rev. C 62, 031601R (2000).
[45] A. C. Berriman, D. J. Hinde, M. Dasgupta, C. R. Morton, R. D. Butt, and J. O. Newton, Nature (London) 413, 144 (2001).

[46] S. Appannababu, S. Mukherjee, B. K. Nayak, R. G. Thomas, P. Sugathan, A. Jhingan, E. Prasad, D. Negi, N. N. Deshmukh, P. K. Rath, N. L. Singh, and R. K. Choudhury, Phys. Rev. C 83, 034605 (2011).

[47] K. Banerjee, T. K. Ghosh, S. Bhattacharya, C. Bhattachary, S. Kundu, T. K. Rana, G. Mukherjee, J. K. Meena, J. Sadhukhan, S. Pal, P. Bhattachary, K. S. Golda, P. Sugathan, and R. P. Singh, Phys. Rev. C 83, 024605 (2011).

[48] W. J. Swiatecki, Phys. Scr. 24, 113 (1981).

[49] S. Appannababu, R. G. Thomas, L. S. Danu, P. K. Rath, Y. K. Gupta, B. V. Jhon, B. K. Nayak, D. C. Biswas, A. Saxena, S. Mukherjee, and R. K. Choudhury, Phys. Rev. C 83, 067601 (2011).

[50] S. Nath, K. S. Golda, A. Jhingan, J. Gehlot, E. Prasad, Sunil Kalkal, M. B. Naik, P. Sugathan, N. Madhavan, and P. V. Madhusudhana Rao2, EPJ Web of Conferences, 17, 16008 (2011).

[51] M. Abe, KEK preprint 86-26, KEK TH-28 (1986).

[52] H. Keller, K. Lützenkirchen, J. V. Kratz, G. Wirth, W. Brüchle, and K. Sümmerer, Z. Phys. A-Atomic Nuclei 326, 313 (1987).

[53] I. M. Itkis, E. M. Kozulin, M. G. Itkis, G. N. Knyazheva, A. A. Bogachev, E. V. Chernysheva, L. Krupa, Yu. Ts. Oganessian, and V. I. Zagrebaev, A. Ya. Rusanov, F. Goennenwein, O. Dorvaux, L. Stuttgé, F. Hanappe, E. Vardaci, and E. de Goês Brennand, Phys. Rev. C 83, 064613 (2011).

[54] A. Yu. Chiskov et al., Phys. Rev. C 67, 011603(R) (2003).