**Arbitrary polarization control by magnetic field variation**

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We propose a universal scheme for the construction of a device for arbitrary to arbitrary polarization transformation, which consists of two quarter-wave plates and two Faraday rotators. Using this device, one can continuously change the retardance and the rotation angle simply by changing the magnetic fields in each Faraday rotator.

**I. INTRODUCTION**

In many practical applications the ability to observe and control polarization is critical, as polarization is one of the basic characteristics of transverse light waves [1–5]. Two prominent optical devices for controlling light’s polarization state are optical polarization rotators and optical polarization retarders [1–5]. A polarization rotator rotates the plane of linear light polarization at a specified angle [6, 10], while a retarder (or a waveplate) introduces a phase difference between two orthogonal polarization components of a light wave [3, 7, 11–13].

Retarders are usually made from birefringent materials. Fresnel rhombs [14, 15] are also widely used and, based on total internal reflections, they achieve retardation at a broader range of wavelengths. Two common types of retarders are the half-wave plate and the quarter-wave plate. By introducing a phase shift of \( \pm \pi/2 \) between the two orthogonal polarization components for a particular wavelength, the half-wave plate effectively rotates the polarization vector to a predefined angle. The quarter-wave plate introduces a shift of \( \pm \pi/2 \), and thereby converts linearly polarized light into circularly polarized light and vice versa [1–5].

Although half-wave and quarter-wave plates clearly dominate in practice, retarders of any desired retardation can be designed and successfully applied. Tuning the retardance is a valuable feature because in some practical settings one may need a half-wave plate, while in others a quarter-wave plate is required. Tunable retardance can be achieved by liquid-crystals [16–18]. Alternatively, one can use the technique recently suggested by Messaadi et al. [19], which is based on two half-wave plates cascading between two quarter-wave plates. This basic optical system functions as an adjustable retarder that can be controlled by spinning one of the half-wave plates.

A polarization rotator may employ Faraday rotation or birefringence. The Faraday rotator consists of a magnetoactive material that is put inside a powerful magnet [1–5]. The magnetic field causes a circular anisotropy (Faraday effect), which makes left- and right-circular polarized waves “feel” different refraction indices. As a result, the linear polarization plane is rotated. A birefringent rotator can be achieved as a combination of two half-wave plates. Such a rotator would have an angle of rotation equal to twice the angle between the optical axes of the two half-wave plates [20].

Wave plates and rotators are basic building blocks for polarization manipulation. Indeed, every reversible polarization transformation, (a reversible change in the polarization vector from any initial state to any final state) can be achieved using a composition of a retarder and a rotator [21]. An arbitrary transformation requires one half-wave plate and two quarter-wave plates [22], or just two quarter-wave plates if the apparatus itself is rotated. Arbitrary polarization transformations, however, require one to perform rotations on individual plates which may turn out to be very impractical in particular applications where one needs to change the angles with certain frequency and speed.

In this paper we attempt to solve this problem by substituting mechanical rotations of the plates with variation of magnetic fields. The device we propose is a modified version of Simon-Mukunda’s controller and consists of two quarter-wave plates and two rotators. In this setting we can perform fast and continuous variation of the polarization vector simply by changing the magnetic fields in each Faraday rotator.

**II. PREFACES**

The Jones matrix that describes a rotator with rotation angle \( \theta \) is

\[
R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},
\]

while the Jones matrix that represents a retarder is

\[
J(\varphi) = \begin{bmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2} \end{bmatrix}.
\]

Here \( \varphi \) is the phase shift between the two orthogonal polarization components of the light wave. The most widely-used retarders are the half-wave plate \( (\varphi = \pi) \) and the quarter-wave plate \( (\varphi = \pi/2) \) [1–2].

Consider now a single polarizing birefringent plate of phase shift \( \varphi \), whose fast axis is rotated to an angle of \( \theta \) relative to the vertical axis (azimuth angle). In a 2-dimensional rectangular coordinate system whose axes are aligned with the horizontal and the vertical directions, the Jones matrix is given by

\[
J_\theta(\varphi) = R(-\theta)J(\varphi)R(\theta).
\]

If behind this plate one places another plate with azimuth angle \( \theta + \alpha/2 \), the resulting Jones matrix is given by the
It consists of one half-wave plate and two quarter-wave plates, which are oriented such that their fast optical axes are perpendicular to each other (cf. Eq. (11)).

Next we use that

$$J_{\theta_4}(\alpha) = J_{\theta_1}(\pi/2)J_{\theta_2}(\pi)J_{\theta_3}(\pi)J_{\theta_4}(\pi/2),$$

(7)

we obtain

$$J = J_{\theta_1}(\pi/2)J_{\theta_2}(\pi)J_{\theta_3}(\pi)J_{\theta_4}(\pi/2),$$

(8)

Next we use that

$$J_{\theta_4}(\alpha) = J_{\theta_1}(\pi/2)J_{\theta_2}(\pi)J_{\theta_3}(\pi)J_{\theta_4}(\pi/2),$$

(9)

Next we make use of Eq. (10) for the combination of two half-wave plates

$$R(2(\theta_1 - \theta_2)) = -J_{\theta_2}(\pi)J_{\theta_3}(\pi),$$

(10)

to get the final expression of the Jones matrix:

$$J = -J_{\theta_1}(\pi/2)R(\alpha)J_{\theta_4}(\pi/2),$$

(11)

where the rotator angle is $\alpha = 2(\theta_1 - \theta_2)$. Therefore the Simon–Mukunda polarization controller can be constructed as combination of two quarter-wave plates along with a rotator between them. This device would operate in a similar way as the one before: the first quarter-wave plate turns the input elliptical polarization into a linear polarization vector, which is then rotated by the rotator element, and is finally transformed into the required elliptical output polarization by the second quarter-wave plate.

IV. ARBITRARY RETARDER AS A SPECIAL CASE OF THE MODIFIED SIMON–MUKUNDA POLARIZATION CONTROLLER

In the special case when the two quarter-wave plates are oriented such that their fast optical axes are perpendicular to each other (cf. Eq. (11), $\theta_1 = \theta_2 = \pi/4$) we obtain a retarder with retardation $2\alpha$:

$$J_0(2\alpha) = J_{x/4}(\pi/2)R(\alpha)J_{x/4}(\pi/2) = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}. $$

(12)
If the two quarter-wave plates are achromatic (for example as in [11–13] or if Fresnel rhombs are used as quarter-wave plates) then one can achieve a wavelength tunable half-wave or quarter-wave plate that operates differently compared to previously suggested tunable wave plates [20, 27]. However, in contrast to previously used tunable wave plates, here we do not need to rotate the wave plates, but rather change the magnetic field in the Faraday rotator. Therefore, the suggested tunable retarder is not mechanical and can be used as a fast switcher, where the switching on/off time of the optical activity is in the order of microseconds with the state of the art approaching subnanosecond [28, 29].

V. ARBITRARY TO ARBITRARY POLARIZATION CONVERTER

Based on the fact that combining an arbitrary rotator with an arbitrary retarder allows to achieve any polarization transformation [21], we can combine the tunable retarder from Eq. (12) with an additional rotator to get an arbitrary-to-arbitrary polarization manipulation device. Its Jones matrix is

\[ \mathbf{J} = \mathbf{J}_0(2\alpha)\mathbf{R}(\beta), \quad (13) \]

or

\[ \mathbf{J} = \mathbf{J}_{\pi/4}(\pi/2)\mathbf{R}(\alpha)\mathbf{J}_{\pi/4}(-\pi/2)\mathbf{R}(\beta). \quad (14) \]

The proposed optical device, illustrated schematically in Fig. 1 b, has potential advantages over the Simon–Mukunda polarization controller [7, 22], where one has to adjust the spatial orientation of all three wave plates. The proposed device (cf. Eq. (14)) is more convenient to use in the sense that the rotator angle and the retardation are obtained by changing the magnetic field of the first and the second Faraday rotator, respectively. Furthermore, our scheme is fast switchable on and off, because in the absence of a magnetic field the polarization is not changed.

Finally, we investigate the possibility of achieving any pair of rotation angles \( \alpha \) and \( \beta \) (cf. (14)) for the proposed arbitrary-to-arbitrary polarization conversion device. The rotation angle for the Faraday rotator is given by

\[ \theta(\lambda) = V(\lambda)BL, \]

where \( B \) is the external magnetic field, \( L \) is the magneto-optical element length, and \( V(\lambda) \) is Verdet’s constant. We do the most common calculation involving the Terbium Gallium Garnet (TGG) crystal, as this crystal yields a high Verdet constant. So far, the dispersion of Verdet’s constant for the TGG crystal has been extensively studied [30–33] and the wavelength dependence has been shown to be described by the formula

\[ V(\lambda) = \frac{E}{\lambda_0^2 - \lambda^2}, \quad (15) \]

where \( E = 4.45 \cdot 10^7 \text{ rad m}^2 \text{mm}^{-2} \) and \( \lambda_0 = 257.5 \text{ nm} \) is the wavelength, often close to the Terbium ion’s 4f-5d transition wavelength. In the range of 400 — 1100 nm, excluding 470 — 500 nm (absorption window [33]), the TGG crystal has optimal material properties for a Faraday rotator. The Verdet constant decreases with increasing wavelength for most materials (in absolute value): for the TGG crystal it is equal to 475 rad m at 400 nm and 41 rad m at 1064 nm [33]. Our simulations were carried out for three different values for the magnetic field, \( B_1 = 0.5 \text{ T}, B_2 = 1 \text{ T} \) and \( B_3 = 2 \text{ T} \), at a fixed length \( L = 0.05 \text{ m} \) of the TGG crystal. As can be seen from Fig. 2 any pair of rotation angles \( \alpha \) and \( \beta \) in the interval \([0, 2\pi]\) can be achieved with magnetic field smaller than 1T for the visible spectrum. Therefore, with commercial Faraday rotators available on the market, the practical realization of the proposed polarization control device should be straightforward.

![FIG. 2. (Color online) The Faraday rotation angle \( \theta \) vs the light wavelength \( \lambda \) for three different magnetic fields \( B_1 = 0.5 \text{ T} \) (red dotted), \( B_2 = 1 \text{ T} \) (green solid) and \( B_3 = 2 \text{ T} \) (blue dashed).](image)

VI. CONCLUSION

In conclusion, we have suggested two useful polarization manipulation devices. The first device is the modified Simon–Mukunda polarization controller, which in contrast to the traditional controller is constructed as a combination of two quarter-wave plates and a Faraday rotator between them. Our second device for arbitrary to arbitrary polarization transformation is composed of two Faraday rotators and two quarter-wave plates, where the retardance and the rotation can be continuously modified merely by changing the magnetic fields of the two Faraday rotators. Because they use Faraday rotation, the suggested schemes are non-reciprocal. We hope the proposed methods for polarization control would be cost-
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