Squeezing in the interaction of radiation with two-level atoms

Abir Bandyopadhyay* and Jagdish Rai†

Department of Physics, Indian Institute of Technology,
Kanpur - 208 016, INDIA.
(November 3, 2018)

Abstract

We propose a simple experimental procedure to produce squeezing and other non-classical properties like photon antibunching of radiation, and amplification without population inversion. The method also decreases the uncertainties of the angular-momentum quadratures representing the two-level atomic system in the interaction of the two-level atoms with quantized radiation.

PACS no. 42.50.Dv, 32.80.-t

*Electronic address : abir@iitk.ernet.in
†Electronic address : jrai@iitk.ernet.in
Squeezed states of quantum systems have been an active area of interest for more than a decade [1]. Nonclassical states of radiation fields showing squeezing properties have been experimentally produced in four wave mixing procedures or by passing a coherent beam through optically nonlinear medium [2]. It is now well established that these states have many potential applications in increasing the sensitivity of interferometers [3] and in noise free transmission of information [4]. In case of radiation, squeezing redistributes the equal quantum noises of the quadratures present in the minimum uncertainty or coherent states (lasers) unequally to retain the minimum uncertainty nature. The attempt to generate squeezed radiation is still on as a technological problem [5]. In this letter we propose a new procedure to generate squeezing in the interaction of a coherent radiation field with a coherent atomic beam. Our method does not need any non-linear medium for this purpose and thus has a much simpler theory as well as experimental requirement. The procedure also enables to produce amplification in the radiation without population inversion. We also show that the uncertainties of the Bloch vector (representing atomic dipole moment) can also be reduced through the same interaction.

Schwinger developed an abstract SU(2) representation for angular momentum systems in a Hilbert space [6]. Atkins and Dobson constructed angular momentum coherent states using Schwinger representation containing both integer and half-integer angular momentum states in the SO(3) space [7]. We call these states as Schwinger angular-momentum coherent (SAMC) states. These states were successfully applied by Fonda et.al to nuclear and molecular coherent rotational states [8] mentioning the problem with the system of real spin-\(\frac{1}{2}\) particles (fermions). However, this problem does not arise in the case of any two-level quantum system consisted of both bosonic and fermionic harmonic oscillator. The examples of such systems are : an admixture of bosonic and fermionic ultracold ensemble of two-level atoms [9] and, two-mode radiation system [10]. We use the first example for the present purpose.
In figure 1 we sketch the schematic experimental setup for the generation of radiation squeezing. A hole in the mirror, deflecting the coherent radiation (laser), injects an ultracold beam consisted of both bosonic and fermionic atoms described by the SAMC states. The atomic beam interacts with the radiation on the way. The Hamiltonian of the combined atom field system in Jaynes-Cummings model \[1\] as (in units of $\hbar$)

$$H = H_A + H_F + V_I$$

where $H_A = \omega_0 J_z$ is the Hamiltonian for the free two-level atom with energy difference of $\omega_0$ between the levels, $H_F = \omega a a^\dagger$ is the Hamiltonian of the free field with frequency $\omega$. $V_I = g(a J_+ + a^\dagger J_-)$ is the interaction term under rotating wave approximation (RWA) with interaction strength of $g = \sqrt{\frac{\hbar^3 c A}{\omega^2}} \[2\]$. $A$ is the $A$-coefficient of the two-level atoms in the mode volume $V$. We also take $\omega = \omega_0$ for simplicity in the calculation.

Due to the simplicity and resemblance with the classical perception we work in the Heisenberg picture, where any operator $\hat{O}(0)$ is transformed to $\hat{O}(t) = \exp(i H t) \hat{O}(0) \exp(-i H t)$ as time evolves. Using Baker-Campbell-Housdroff relation for operators $e^{\xi A} \hat{O} e^{-\xi A} = \hat{O} + \xi [A, \hat{O}] + \frac{\xi^2}{2!} [A, [A, \hat{O}]] + \cdots$ and retaining the terms up to the order of $g^2$ for field operators and $g$ for angular momentum projection operator, we were able to close the infinite series occurring in the transformed operators. The transformed linear operators of the field up to the order of $g^2$ are

$$a(t) = e^{-i\omega t} \left[a(0) - i gt J_-(0) - g^2 t^2 a(0) J_z(0)\right]$$

$$a^\dagger(t) = e^{i\omega t} \left[a^\dagger(0) + i gt J_+(0) - g^2 t^2 a^\dagger(0) J_z(0)\right]$$

and of the angular-momentum representing the atomic system up to the order of $g$ are

$$J_+(t) = e^{i\omega t} \left[J_+(0) - 2igt a^\dagger(0) J_z(0)\right]$$

$$J_-(t) = e^{-i\omega t} \left[J_-(0) + 2igt a(0) J_z(0)\right]$$

$$J_z(t) = J_z(0) - ig t a J_z(0)$$

\[2\]
Instead of calculating the transformation of the quadratic operators we can introduce the unity operator \( \exp(-iHt) \exp(iHt) \) between the linear operators. Thus simply multiplying the transformed linear operators one can find the product operators up to the order of the term in the transformed linear operators. These transformed product operators are used to calculate the different variances discussed below.

Now, we define the initial state of the whole system in the Schrödinger picture, i.e. the stationary state in the Heisenberg picture, as a product of the free field and the free atomic states \( |\Psi\rangle = |\psi_F\rangle \otimes |\phi_A\rangle \). Choosing both the field and the atomic states to be coherent states as required by the experimental proposal we can write

\[
|\Psi\rangle = |\alpha\rangle \otimes |\beta_+, \beta_-\rangle
\]

where \( \alpha \) is the complex parameter of the coherent radiation field and \( \beta_\pm \) are the complex parameters for the angular-momentum coherent state. Calculating the matrix elements of the time evolved operators for the above mentioned coherent-coherent states of the field and the atomic system we found the variances of the quadratures of the field variables up to the order of \( g^2 \) as

\[
\Delta X_1^2(t) = \frac{1}{2} - 2gt|\alpha||\beta_+||\beta_-|[\sin(\theta_\alpha - \theta_{\beta_+} + \theta_{\beta_-}) + \sin(2\omega t - \theta_{\beta_+} + \theta_{\beta_-})] - 2\cos(\omega t - \theta_\alpha)\cos(\omega t - \theta_{\beta_+} + \theta_{\beta_-})] \\
+ g^2t^2[|\beta_+|^2 + (|\beta_+|^2 - |\beta_-|^2)[|\alpha|^2\cos 2(\omega t - \theta_\alpha) - \frac{\alpha}{2}\cos(2\omega t - \theta_\alpha)] \]
\]

\[
\Delta X_2^2(t) = \frac{1}{2} - 2gt|\alpha||\beta_+||\beta_-|[\sin(\theta_\alpha - \theta_{\beta_+} + \theta_{\beta_-}) - \sin(2\omega t - \theta_{\beta_+} + \theta_{\beta_-})] + 2\sin(\omega t - \theta_\alpha)\cos(\omega t - \theta_{\beta_+} + \theta_{\beta_-})] \\
+ g^2t^2[|\beta_+|^2 - (|\beta_+|^2 - |\beta_-|^2)[|\alpha|^2\cos 2(\omega t - \theta_\alpha) - \frac{\alpha}{2}\cos(2\omega t - \theta_\alpha)] \]
\]

where \( \theta_\alpha \)s are the phases or arguments of the complex parameters \( \alpha \) and \( \beta_\pm \). For simplicity and a better understanding of the results we set all the phases (\( \theta_\alpha \)s) to be zero, which does not reduce the importance of our results except that the linear dependencies on \( gt \) drops
out. This dependency will be reported in a detailed parametric study with all the phases. As we are interested in the generation of non-classical properties, the second order term fulfills the present purpose. Under this simplification the uncertainties reduce to

\[
\Delta X_1^2(t) = \frac{1}{2} + g^2 t^2 (|\beta_+|^2 + (|\beta_+|^2 - |\beta_-|^2)) (|\alpha|^2 - \frac{|\alpha|}{2}) \cos 2\omega t \]

(5a)

\[
\Delta X_2^2(t) = \frac{1}{2} + g^2 t^2 (|\beta_+|^2 - (|\beta_+|^2 - |\beta_-|^2)) (|\alpha|^2 - \frac{|\alpha|}{2}) \cos 2\omega t \]

(5b)

We have plotted the time development of the uncertainties in Fig. 2 for \( \nu = \frac{\omega}{2\pi} = 6 \times 10^{14} \text{Hz} \), \( g = \omega \times 10^{-5} \), \( |\alpha| = 5.0 \), \( |\beta_+| = 1.0 \), \( |\beta_-| = 10.0 \). The uncertainties in the quadratures show oscillations due to the sinusoidal term after a certain time and start to show squeezing properties. The amount of squeezing become more and more over time due to the \((gt)^2\) dependency. However, the squeezing property also oscillates between the quadratures and at certain time intervals they return back to the coherent state.

We have also calculated the mean of the number of photons and the bunching parameter \( \langle B \rangle = \langle a^\dagger a^2 \rangle - \langle a^\dagger a \rangle^2 \) for the above simplified choice of phases to check the amplification and non-classical behavior of the statistics of the photon number distribution. The mean number of the photons and the bunching parameter up to the order of \( g^2 \) were calculated to be

\[
\langle n(t) \rangle = |\alpha|^2 + g^2 t^2 (|\beta_+|^2 (1 + |\beta_-|^2) - |\alpha|^2 (|\beta_+|^2 - |\beta_-|^2))
\]

(6)

\[
\langle B \rangle = g^2 t^2 (|\beta_+|^2 (3 + |\beta_-|^2) - (|\alpha|^2 - |\alpha|(|\beta_+|^2 - |\beta_-|^2)))
\]

(7)

The expression of the mean number of photons has an extra term of the order of \((gt)^2\) and thus if the quantity in the square bracket is chosen to be positive, amplification of the optical signal is possible through the interaction. The bunching parameter depends only on \((gt)^2\) with a constant factor. If this constant factor, dependent only on the mean number of initial photons and the mean numbers of atoms in the two states, is chosen positive or negative, then it is possible to have non-Poissonian (bunched for positive and antibunched
for negative) statistics of the radiation field.

The angular momentum quadratures are actually the measure of the Bloch vector or the dipole moment of the atoms. We are interested in the position uncertainty of the Bloch vector and the population inversion in the atomic system. So we have similarly calculated the matrix elements for the atomic variables up to the order of $g$ but the results came out to be very much complicated in phase dependency and can not be understood directly. For this reason we present the results of the angular-momentum matrix elements of present interest for the same choice of phases as in the last section.

\[
\Delta J_x^2 = \frac{1}{4}(|\beta_+|^2 + |\beta_-|^2) + 2gt|\alpha||\beta_+||\beta_-||(|\beta_+|^2 - |\beta_-|^2) \sin 2\omega t \tag{8a}
\]

\[
\Delta J_y^2 = \frac{1}{4}(|\beta_+|^2 + |\beta_-|^2) - 2gt|\alpha||\beta_+||\beta_-||(|\beta_+|^2 - |\beta_-|^2) \sin 2\omega t \tag{8b}
\]

\[
\langle J_z \rangle = \frac{1}{2}(|\beta_+|^2 - |\beta_-|^2) \tag{8c}
\]

Unlike the case of the radiation, the linear dependencies of the quadratures of the atomic system on $gt$ does not drop out for the simplified choice. However, the angular momentum projection or the population difference has linear dependency on $gt$ which drops out for the choice of phases to be zero. The correction term present there for any phase is $+2gt|\alpha||\beta_+||\beta_-|\sin(\theta_\alpha - \theta_{\beta_+} + \theta_{\beta_-})$. If the phases are chosen to make this term non-zero, then population difference can also be controlled through the interaction. This means that the population inversion can also be controlled by the choice of the phases. However, as we are interested in optical amplification, it is shown in the previous subsection that the simplified choice of the phases can amplify the optical signal even without any population inversion.

We have plotted the normalized quadratures $A_+ = \frac{\Delta J_x^2}{|\langle J_z \rangle|}$ and $A_- = \frac{\Delta J_y^2}{|\langle J_z \rangle|}$ in the angular momentum uncertainty relation $\frac{\Delta J_x^2}{|\langle J_z \rangle|} \frac{\Delta J_y^2}{|\langle J_z \rangle|} \geq \frac{1}{4}$ in fig 3 for the same choice of parameters as in
the case of field. It is noticed that the uncertainties oscillate in time with opposite phase and come back to the initial value with a periodicity like the field case. One more difference in the plots is that the field starts showing the interaction effect only after some time whereas the atoms are affected as soon the interaction starts. Also notice that we have not started from the minimum uncertainty (though coherent by definition) angular-momentum states where all the atoms are coherently either in ground ($\beta_\parallel =0$) or in excited state ($\beta_\perp =0$) as this will drop all effects on the quadratures of the atomic system and the it will remain at minimum uncertainty state throughout.

In connection with the experiment proposed in figure 1, the atomic beam coming from the hole in the mirror copropagate with the coherent radiation. The parameter $gt$ is related to the length of interaction. The time of interaction is controlled by deflecting the atomic beam using an atomic beam-splitter [13] after a desired interval of time effectively defined by the length of interaction. The atomic beam-splitter deflects the coherent upper state atoms and the coherent lower state atoms in two directions while the radiation passes through without deflection. The properties of the radiation after interaction can now be verified. Two atomic beams can be recombined at some other place to study the properties of the atomic system.

In conclusion, we have designed a simple experimental setup and calculated the change in the field and atomic variables using the rotating wave approximation (RWA) in Jaynes-Cummings Model for the Hamiltonian retaining the terms of the order of square of the interaction strength for the field variables and the terms of the order of the interaction strength for the atomic variables. We show that under these approximations the interaction produces squeezing in the initially coherent radiation field. It is shown that the statistics of the photon number of the radiation field can be made non-Poissonian through the interaction. We have also shown amplification in the radiation field without any population inversion in the atomic system. It is observed that the uncertainties of the atomic system
show similar oscillatory dependence over time. We have calculated the effect of the interaction on the atomic system in coherent angular-momentum state and prescribed a method to reduce the uncertainties in the atomic quadrature in expense of the other. However, it is not possible to generate squeezed atomic systems by only increasing the time as large time of interaction will increase the product $gt$ where the perturbation theory breaks down.
REFERENCES

[1] D. F. Walls, Nature (London), 306, 141(1983).

[2] D. F. Walls and G. J. Milburn, in Quantum Optics (Springer-Verlag, 1994); M. C. Teich and B. A. E. Saleh, Quantum Optics, 1, 151(1989).

[3] M. Hillery and L. Mlodinow, Phys. Rev. A 48, 1548(1993); Abir Bandyopadhyay and Jagdish Rai, Phys. Rev. A 51, 1597(1995).

[4] B. L. Schumaker, Opt. Lett. 9, 189(1984); B. A. E. Saleh and M. C. Teich, Phys. Rev. Lett. 58, 2656(1987).

[5] A. M. Fox, J. J. Baumberg, M Dabbico, B. Huttner and J. F. Ryan, Phys. Rev. Lett. 74, 1728(1995); J. Kitching and Y. Yariv, Phys. Rev. Lett. 74, 3372(1995).

[6] J. Schwinger, in Quantum Theory of Angular Momentum, edited by L. Beidenharn and H. van Dam (Academic Press, New York, 1965), p. 229.

[7] P. W. Atkins and J. C. Dobson, Proc. R. Soc. London Ser. A 321, 321(1971).

[8] L. Fonda, N. Mankoc-Borstnik and M. Rosina, Phys. Rep. 158, 160(1988).

[9] W. Zhang and D. F. Walls, Phys. Rev. A49, 3799(1994).

[10] B. Yurke, S. L. McCall, J. R. Klauder, Phys. Rev. A33, 4033(1986).

[11] E. T. Jayens and F. W. Cummings, Proc. IEEE 51, 89(1963).

[12] M. Butler and P. D. Drummond, Optica Acta, 33, 1(1986).

[13] R. Grimm, J. Söding and Yu. B. Ovchinnikov, Opt. Lett. 19, 658(1994).