Impurities and Conductivity in a $D$-wave Superconductor

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Abstract

Impurity scattering in the unitary limit produces low energy quasiparticles with anisotropic spectrum in a two-dimensional $d$-wave superconductor. We describe a new quasi-one-dimensional limit of the quasiparticle scattering, which might occur in a superconductor with short coherence length and with finite impurity potential range. The dc conductivity in a $d$-wave superconductor is predicted to be proportional to the normal state scattering rate and is impurity-dependent. We show that quasi-one-dimensional regime might occur in high-$T_c$ superconductors with Zn impurities at low temperatures $T \lesssim 10$ K.

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In this short note I will address the role of a strongly scattering impurities with finite range on the dc conductivity in a short coherence length superconductor. This is report on the work, done in collaboration with A. Rosengren and B. Altshuler [1].

It is well known that scalar impurities are pair breakers in \(d\)-wave and any other nontrivial pairing state superconductor [2–4]. They produce a finite lifetime of the quasiparticles in the nodes of the gap, a finite density of states at low energy, and a finite low frequency conductivity at low temperatures, ignoring localization effects. For the special case of a 2D superconductor with a \(d\)-wave gap, a straightforward calculation yields the surprising result that \(dc\) conductivity \(\sigma(\omega \to 0)\) is a “universal” number [4], independent of the lifetime of quasiparticle (but dependent on the anisotropy ratio of the velocities of the quasiparticle in the node of the gap) [5]. However, recent experiments on microwave absorption in YBCO crystals with Zn impurities [6] show a linear temperature dependence of the conductivity for pure samples, evolving to the quadratic behavior for higher impurity concentration, and low-temperature conductivity, inversely proportional to the impurity concentration.

i) We find a new quasi-one-dimensional regime for \(dc\) conductivity in superconductors with a short coherence length \(\xi\), comparable to the range of impurity potential \(\lambda\). The quasiparticle contribution to \(dc\) conductivity is governed by self-energy \(\Sigma(\omega \to 0) = -i\gamma\) and by the phase space available for low-energy quasiparticles. The quasiparticle dispersion is strongly anisotropic in the vicinity of the nodes in a 2D \(d\)-wave superconductor: \(E_k = \sqrt{v_1^2 k_1^2 + v_F^2 k_3^2}\) and \(v_1/v_F \sim \Delta_0/\epsilon_F\). Here we linearized spectrum in the vicinity of the nodal point close to \((\frac{\pi}{2}, \frac{\pi}{2})\), so that \(k_1\) is the momentum along the Fermi surface and \(k_3\) is perpendicular. We find that the overall contribution to the conductivity depends on the ratio of the energy of the quasiparticle to the scattering rate \(v_1\lambda^{-1}/\gamma\), \(\lambda\) is the range of the impurity potential, and we get at \(T = 0\):

\[
\sigma(\omega \to 0) = \frac{e^2}{2\pi\hbar} \frac{2}{\pi^2} \frac{v_F}{v_1} \left(1 + \left(\frac{\gamma}{2v_1\lambda^{-1}}\right)^2\right)^{-1/2}.
\]

For \(v_1\lambda^{-1}/\gamma \ll 1\) quasiparticle dynamics is essentially quasi-one-dimensional and conductivity depends on the impurity concentration \(\sigma_{Q1D} \sim n_{imp}^{-1}\). Our model predicts that the dc
conductivity at low temperature should be proportional to the scattering rate in the normal state. This limit might occur in high-$T_c$ superconductors, for which we estimate $\lambda/a \sim 1 - 3$ and $\Delta_0/\epsilon_F \sim 10^{-1}$. In the limit $\lambda \to 0$ Eq. (1) gives the “universal” dc conductivity $\sigma_{2D} \sim v_F/v_1$, found in [4].

To explain this effective change of dimensionality we note that transverse momentum is limited by $k_1 < 2/\lambda$ and quasiparticle dispersion on such a small scale is irrelevant, compared to $\gamma$. The condition for this to occur is precisely $v_1\lambda^{-1}/\gamma \ll 1$. The transverse (along the Fermi surface) scattering does not contribute effectively to the conductivity; we call this case a quasi-one-dimensional limit. The existence of this limit is the result of the finite impurity range $\lambda$ [7]. In the opposite limit $v_1\lambda^{-1}/\gamma \gg 1$, which holds for “zero” impurity range, we recover standart unitary scattering results [8–10].

ii) Here we will explain the assumptions we made to calculate conductivity. We assume that impurities are strong scatterers with s-wave phase shift $\delta_0(q) \simeq \pi/2$, for $|q| < \lambda^{-1}$, where $q$ is wavevector, counted from the Fermi wavevector. This assumption is well supported by experiments on cuprates with Zn impurities. The origin of strong potential impurity scattering in high-$T_c$ superconductors is the highly correlated antiferromagnetic nature of the normal state. The second assumption is about the finite range $\lambda$ of the impurity potential, which plays role of the momentum cut off in the momentum dependence of phase shift $\delta_0(q)$. It is as well motivated by the fact that high-$T_c$ superconductors have a substantial antiferromagnetic coherence length $\xi_{AFM} \sim 3a$ at the transition temperature. A scalar impurity will produce distortions in magnetic correlations on the range of the $\xi_{AFM}$. On the other hand superconducting coherence length $\xi \sim 20 \, \text{Å}$ is comparable to this scale and thus, the range of the potential is finite on the scale relevant for superconductivity. This point should be contrasted to the case of heavy-fermion superconductors, where the coherence length is $\sim 10^3 \, \text{Å}$, and therefore, any potential impurity will have its range substantially shorter than the coherence length. We retain this cut off finite and on the order of few lattice constants ($\lambda \sim 2a$). This implies that impurities still are well screened and s-wave scattering is dominant.
iii) To calculate the quasiparticle conductivity we use lowest order bubble diagram with self-consistent Green functions with no vertex corrections, see for example [4]. For the dc conductivity we get [11]:

\[
\sigma(\omega \to 0) = \frac{e^2}{\hbar} \frac{4v_F^2}{\pi^2} \sum'_k \int d\epsilon (-\partial, n(\epsilon)) (|G''(k, \epsilon)|^2 + |F''(k, \epsilon)|^2),
\]

where, linearizing quasiparticle spectrum in the vicinity of nodes, 

\[G''(k, \omega = 0) = \frac{\gamma}{\gamma^2 + (v_1 k_1)^2 + (v_F k_3)^2}, \quad F''(k, \omega = 0) = 0.\]

The momentum integral in Eq. (2) is cut off at \(|k| \leq 2/\lambda\) and it yields the final formula Eq. (3) for \(T = 0\) with \(O(T^2)\) corrections.

For the particular case of strong disorder \(v_1 \lambda^{-1}/\gamma \ll 1\), considered in [1], relation between scattering rate in the superconducting state \(\gamma = i\Sigma(\omega \to 0)\) and scattering rate in the normal state \(\Gamma = n_{\text{imp}}/\pi N_0\) is \(\gamma = \pi/8\ p_F\lambda\ \Gamma\). Note that the scattering rate at low temperatures is \(linearly\) proportional to \(\Gamma\), as opposed to the \(\Gamma^{\frac{1}{2}}\) dependence in the standart unitary scattering case for \(v_1 \lambda^{-1}/\gamma \gg 1\). The assumption \(v_1 \lambda^{-1}/\gamma \ll 1\) is consistent at \(\lambda \sim 2a\) for \(\Gamma \geq 20K\). This estimate shows that the \(\text{quasi-one-dimensional}\) regime of quasiparticle scattering should occur in not too clean samples at \(T < \gamma\). In this limit scattering rate in the superconducting state is of the same order as the normal state scattering rate \(\gamma \sim 2\Gamma \sim 40\ K\) for \(p_F\lambda \sim 6\) and is similar to the scattering rate in the \(2D\) limit: \(\gamma/\tilde{\gamma} \sim \sqrt{\Gamma/\Delta_0} p_F\lambda \simeq 1\).

The finite density of states \(N(\omega \to 0)/N_0 = \Gamma/\Delta_0 \sim n_{\text{imp}}, \ linear\) in impurity concentration, is generated as well.

Using the above estimates we find the conductivity in \(\text{quasi-one-dimensional}\) regime

\[
\sigma(\omega \to 0) = \frac{e^2}{\pi \hbar} \frac{16}{\pi^3} \frac{\hbar}{m\lambda^2\Gamma}.
\]

It is smaller than the normal state conductivity \(\sigma(\omega \to 0)_{\text{normal}} = (e^2/\pi\hbar) (\epsilon_F/\Gamma)\) due to small factor \(\hbar/(m\lambda^2\epsilon_F) \lesssim 1\) with \(\lambda > a\). Conductivity is also impurity-\(dependent\), \(\sigma_{\text{Q1D}} \sim \Gamma^{-1} \sim n_{\text{imp}}^{-1}\). This model predicts that the dc \(\sigma\) at low temperatures should be \(\text{inversly proportional to the scattering rate in the normal state and to the impurity concentration}\).

We emphasize that both a higher value of the conductivity in the superconducting state as \(well as\) strong impurity dependence at low temperatures are observed experimentally in
microwave absorption of YBCO [6]. It should be pointed out that we are interested in only elastic scattering and strong inelastic contribution to the scattering rate above $T_c$ is not considered in this model.

iv) One can be almost certain that for dirty enough superconductors the *quasi-one-dimensional* regime will occur, since impurities will lead eventually to a very high scattering rate. The question remains about the competing phenomena, such as the localization of quasiparticles, which might occur earlier than *quasi-one-dimensional* regime. It is interesting to apply this model to the Zn impurities in the YBCO. The applicability of the results presented here to the different high-$T_c$ materials depends on the ratio of scattering rate to the relevant quasiparticle energy. For the same impurity concentration different cuprates may be in different regimes, depending on $\xi_{AFM}$, $\Delta_0/\epsilon_F$, and $\Gamma$.

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