Phenomenology Remarks in M-theory on $S^1/Z_2$

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Abstract

In the simplest compactification, we discuss the intermediate unification in M-theory on $S^1/Z_2$, and point out that we can push the eleven dimension Planck scale to the TeV range if the gauge coupling in the hidden sector is super weak, and the particles in the hidden sector might become candidates of dark matter. We also discuss the soft terms in non-standard embedding. To the next leading order, we compactify the perfect square and calculate the gravitino mass. Furthermore, we give the general Kähler potential, gauge kinetic function and superpotential if the next order correction is very large, i.e., $\frac{\alpha T}{\lambda}$ is close to 1.

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1 Introduction

M-theory on $S^1/Z_2$ suggested by Horava and Witten [1] is a 11-dimensional Supergravity theory with two boundaries where the two $E_8$ Yang-Mills fields live on respectively. Many studies in M-theory compactification and its phenomenology implications [2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36] suggest that this might be the good candidate for super unification. In addition, M-theory GUT Model was also built recently [36], so, it is possible to believe that we can make this theory more realistic in the future: constructing realistic GUT model in detail and comparing the low energy phenomenology with our future experiment at LHC and LEP.

Recently, more and more people discuss the low energy gravity by large extra dimension and TeV string scale [37, 38, 39], and their phenomenology in the current and future colliders. The intermediate unification has also been discussed. There are several ways to get intermediate unification: add more particles to the model which change the RGE running for the gauge couplings [35], or some models, for example: $SU(4) \times SU(2)_L \times SU(2)_R$ [39], have the unification scale at $10^{12}$ GeV order. We discuss the intermediate unification in M-theory on $S^1/Z_2$ [27], using the limit $(\pi\rho)^{-1}$ is larger than $10^{-3}$ eV, we obtain that the low bound on the eleven dimension Planck scale and GUT scale is about $10^7$ GeV if the correction $\alpha_T S$ is not very close to -1. If we define the GUT scale as the longest dimension in the Calabi-Yau manifold, we can push the realistic unification scale to $10^5$ GeV, but, the eleven dimension Planck scale is not changed. However, if $\alpha_T S$ is very close to -1 in nonstandard embedding, then, we can push the eleven dimension Planck scale and the GUT scale to the TeV range. The important structure for this scenario is that, in the hidden sector, the gauge coupling is super weak: from about $10^{-14}$ to $10^{-30}$, and the Calabi-Yau manifold is relatively large, therefore, the particles in the hidden sector might play the role of the dark matter. The major problem that might arise in this scenario are the gauge unification, SUSY breaking and proton decay.

We also discuss the soft terms in the non-standard embedding [26, 29], in other words, the gauge coupling in the hidden sector is weaker than that in the observable sector. We discuss the soft terms in two ways: fixing gauge couplings in the hidden sector and observable sector; combining the gaugino condensation with the F-term SUSY breaking. We find out that comparing to the gravitino mass, the magnitude of $M_1/2$, $A$ and $M_0$ in the non-standard embedding is larger than those in the standard embedding.

Moreover, in the eleven dimension metric, we compactify the perfect square to the 4-dimension, and also calculate the gravitino mass. The gaugino condensation scale is about the order of $10^{13}$ GeV. If we consider the gaugino condensation scale is just after the Calabi-Yau manifold's compactification, the gaugino condensation scale will be about $1.1 - 2.4 \times 10^{14}$ GeV, and the eleven dimension Planck scale and the physical scale of the Calabi-Yau manifold in the hidden sector will be about that scale, and the physical scale of the Calabi-Yau manifold in the observable sector will be about one half of that scale, and we can push the realistic unification scale to the
10^{12} \text{ GeV}. There is a realistic GUT model: $SU(4) \times SU(2)_L \times SU(2)_R$ \cite{39}, which satisfies this unification scale, and it has no problem on proton decay. We also point out that the goldstino might be the admixture of the super partner of $S$ and $T$.

Furthermore, we give the general kähler potential, gauge kinetic function and superpotential in the simplest compactification \cite{44}. Because in M-theory limit, $\alpha T/S$ is close to 1, we need to consider higher order terms which might be large and very important in phenomenology analysis because it will affect the soft terms, which are the boundaries in running RGE.

## 2 Intermediate Unification

Let us review the gauge couplings, gravitational coupling and the physical eleventh dimension radius in the M-theory \cite{15}. The relevant 11-dimensional Lagrangian is given by \cite{1}

$$
L_B = -\frac{1}{2\kappa^2} \int_{M^{11}} d^{11}x \sqrt{g} R - \sum_{i=1,2} \frac{1}{2\pi(4\pi\kappa^2)^{2/3}} \int_{M^{10}} d^{10}x \sqrt{g} \frac{1}{4} F^a_{AB} F^{aAB}. \quad (1)
$$

In the 11-dimensional metric \cite{1}, the gauge coupling and gravitational coupling in 4-dimension are \cite{2,11,15}:

$$
8\pi G_N^{(4)} = \frac{\kappa^2}{2\pi \rho_p V_p}, \quad (2)
$$

$$
\alpha_{\text{GUT}} = \frac{1}{2V_p(1+x)} (4\pi\kappa^2)^{2/3}, \quad (3)
$$

$$
[\alpha_H]_{W} = \frac{1}{2V_p(1-x)} (4\pi\kappa^2)^{2/3}, \quad (4)
$$

where $x$ is defined by:

$$
x = \pi^2 \rho_p V_p^{2/3} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_X \omega \wedge \frac{\text{tr} F \wedge F - \frac{1}{2} \text{tr} R \wedge R}{8\pi^2}, \quad (5)
$$

where $\rho_p, V_p$ are the physical eleventh dimension radius and Calabi-Yau manifold volume (which is defined by the middle point Calabi-Yau manifold volume between the observable sector and the hidden sector) respectively, and $V_p = V e^{3\varphi}$ where $V$ is the internal Calabi-Yau volume. From above formula, one obtains:

$$
x = \frac{\alpha_H \alpha_{\text{GUT}}^{-1} - 1}{\alpha_H \alpha_{\text{GUT}}^1 + 1}, \quad (6)
$$

\footnote{Because we think 11-dimension metric is more fundamental than string metric and Einstein frame, our discussion in this paper use 11-dimension metric.}

\footnote{In this paper, we do not consider the correction from Five-branes \cite{30}.}
The GUT scale $M_{\text{GUT}}$ and the hidden sector scale $M_H$ when the Calabi-Yau manifold is compactified are:

$$M_{\text{GUT}}^{-6} = V_p(1 + x) ,$$

(7)

$$M_H^{-6} = V_p(1 - x) ,$$

(8)
or we can express the $M_H$ as:

$$M_H = \left( \frac{\alpha_H}{\alpha_{\text{GUT}}} \right)^{1/6} M_{\text{GUT}} = \left( \frac{1 + x}{1 - x} \right)^{1/6} M_{\text{GUT}} .$$

(9)

Noticing that $M_{11} = \kappa^{-2/9}$, we have

$$M_{11} = \left[ 2(4\pi)^{-2/3} \alpha_{\text{GUT}} \right]^{-1/6} M_{\text{GUT}} .$$

(10)

And the physical scale of the eleventh dimension in the eleven-dimensional metric is:

$$[\pi p]\rho^{-1} = \frac{8\pi}{1 + x} (2\alpha_{\text{GUT}})^{-3/2} \frac{M_{\text{GUT}}^3}{M_{\text{Pl}}^2} ,$$

(11)

where $M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV. From the constraints that $M_{\text{GUT}}$ and $M_H$ is smaller than the scale of $M_{11}$, one obtain:

$$\alpha_{\text{GUT}} \leq \frac{(4\pi)^{2/3}}{2} ; \quad \alpha_H \leq \frac{(4\pi)^{2/3}}{2} ,$$

(12)
or

$$\alpha_{\text{GUT}} \leq 2.7 ; \quad \alpha_H \leq 2.7 ,$$

(13)

For the standard embedding, the upper bound on $x$ is 0.97 ($x < 0.97$), for $\alpha_{\text{GUT}} = \frac{1}{25}$. From the constraints that $[\pi p]\rho^{-1}$ is smaller than the scale of $M_{11}$, we obtain that:

$$M_{\text{GUT}}^{\alpha_{\text{GUT}}^{-2/3}} \leq \sqrt{1 + x} 2^{1/6} (4\pi)^{-4/9} M_{\text{Pl}} ,$$

(14)

which is obviously satisfied in the standard embedding. However, if we can consider non-standard embedding $x < 0$ [23, 24], i.e., the gauge coupling in the observable sector is larger than the gauge coupling in the hidden sector, the low bound on $x$ is:

$$x_{lb} \geq 2^{-1/3} (4\pi)^{8/9} (\alpha_{\text{GUT}})^{-4/3} \frac{M_{\text{GUT}}^2}{M_{\text{Pl}}^2} - 1 .$$

(15)

Now, let us discuss the intermediate unification [24]. We can write the eleven dimension Planck scale and the physical scale of the eleventh dimension in terms of the $\alpha_{\text{GUT}}$ and $M_{\text{GUT}}$:

$$M_{11} = 1.18 [\alpha_{\text{GUT}}]^{-1/6} M_{\text{GUT}} ,$$

(16)
\[
[\pi \rho_p]^{-1} = \frac{1.54}{1 + x} (\alpha_{\text{GUT}})^{-3/2} \left( \frac{M_{\text{GUT}}}{10^{16} \text{GeV}} \right)^3 \times 10^{12} \text{GeV} .
\] (17)

Because the gauge coupling \( \alpha_{\text{GUT}} \) in the observable sector can not be too large (at least from current model building as far as we know), and if the gauge coupling \( \alpha_H \) in the hidden sector was not too small, we might think that \( M_{\text{GUT}} \) is the important factor which determines the \( M_{11} \) and \([\pi \rho_p]^{-1}\).

First, we consider the standard embedding, i.e., \( x > 0 \). Assuming that \( \alpha_{\text{GUT}} = \frac{1}{25} \), we can write the eleven dimension Planck scale and the physical scale of the eleventh dimension:

\[
M_{11} = 2.02 M_{\text{GUT}},
\] (18)

\[
[\pi \rho_p]^{-1} = \frac{1.93}{1 + x} \left( \frac{M_{\text{GUT}}}{10^{16} \text{GeV}} \right)^3 \times 10^{14} \text{GeV} .
\] (19)

Noticing that in this case, \( x > 0 \), the key factor which will affect the eleven dimension Planck scale and the physical scale of the eleventh dimension is \( M_{\text{GUT}} \). And then we can obtain different unification scale. Using the limit that \([\pi \rho_p]^{-1}\) is larger than \(10^{-3} \) eV, we obtain that the low bound on \( M_{11} \) is about \(3.5 \times 10^7 \) GeV, and the low bound on \( M_{\text{GUT}} \) is about \(1.73 \times 10^7 \) GeV.

Second, we consider the non-standard embedding [20, 29], i.e., the gauge coupling in the observable sector is stronger than that in the hidden sector. Let us assume that \( \alpha_{\text{GUT}} = 0.15 \), we can write the eleven dimension Planck scale and the physical scale of the eleventh dimension:

\[
M_{11} = 1.62 M_{\text{GUT}},
\] (20)

\[
[\pi \rho_p]^{-1} = \frac{2.66}{1 + x} \left( \frac{M_{\text{GUT}}}{10^{16} \text{GeV}} \right)^3 \times 10^{13} \text{GeV} .
\] (21)

If we do not consider the factor \( \frac{1}{1+x} \), using the limit that \([\pi \rho_p]^{-1}\) is larger than \(10^{-3} \) eV, we obtain the low bound on \( M_{11} \) is \(5.4 \times 10^7 \) GeV, and the low bound on \( M_{\text{GUT}} \) is \(3.35 \times 10^7 \) GeV. However, we might get TeV unification scale or any scale unification (11-dimension Planck scale is at the same order) if theory is consistent, i.e., it can have that scale as the unification scale, and avoid the problem like proton decay, SUSY breaking etc., because we can set the \( x \) very close to -1, but \( x > x_{lb} \). For example, assuming that \([\pi \rho_p]^{-1}\) is \(10^{-3} \) eV, \( M_{11} = 5.4 \) TeV or \( M_{\text{GUT}} = 3.35 \) TeV, we obtain that \( M_H = 33.5 \) GeV and \( \alpha_H = 7.5 \times 10^{-14} \), therefore, the physical volume of Calabi-Yau manifold in the hidden sector is relative large and the gauge coupling in the hidden sector is super weak, but it is much stronger than gravity. And one might think that the particles in the hidden sector will play the role of dark matter. In addition, using \( \alpha_{\text{GUT}} = \frac{1}{25} \), the above argument will not change, assuming that \([\pi \rho_p]^{-1}\) is \(10^{-3} \) eV, \( M_{11} = 3.5 \) TeV or \( M_{\text{GUT}} = 1.73 \) TeV, we obtain that \( M_H = 17.3 \)
GeV and $\alpha_H = 2 \times 10^{-14}$. In addition, for $\alpha_{GUT} = 0.15$, $x = x_{lb} = 1.85 \times 10^{-28} - 1$, with $M_{11} = 5.4$ TeV or $M_{GUT} = 3.35$ TeV, we obtain that $M_H = 71$ MeV and $\alpha_H = 1.4 \times 10^{-29}$. For $\alpha_{GUT} = 0.04$, $x = x_{lb} = 2.86 \times 10^{-28} - 1$, with $M_{11} = 3.5$ TeV or $M_{GUT} = 1.73$ TeV, we obtain that $M_H = 40$ MeV and $\alpha_H = 5.7 \times 10^{-30}$. In short, $x_{lb} < x < 10^{-12} - 1$, and for $\alpha_{GUT} = 0.04$, and $M_{11} = 3.5$ TeV or $M_{GUT} = 1.73$ TeV, we have $33.5$ GeV $> M_H > 71$ MeV, $7.5 \times 10^{-14} > \alpha_H > 1.4 \times 10^{-29}$, for $\alpha_{GUT} = 0.15$, and $M_{11} = 5.4$ TeV or $M_{GUT} = 3.35$ TeV, we obtain $17.3$ GeV $> M_H > 40$ MeV, $2 \times 10^{-14} > \alpha_H > 5.7 \times 10^{-30}$. By the way, setting $x = x_{lb}$, we can obviously put the $M_{GUT}$ and $M_{11}$ ($\alpha_{11}^{-1} = M_{11}$) at any scale if the theory is consistent.

In the above discussion, one does not consider the detail of the length of each dimension in the Calabi-Yau manifold. If we just consider it as a 6-dimension manifold, we can define $M_{GUT} = l^{6-n} L^n$, i.e., we assume that $n$ dimension have relatively larger length and $6-n$ dimension have relatively smaller length. Assuming $l^{-1} = M_{11}$, and considering $L^{-1}$ as realistic GUT scale $M_{GUT}$, we obtain that:

$$M_{GUT}^r = \frac{\alpha_{GUT}^{1/n}}{1.18^{g/n}} M_{11}.$$  (22)

This is another way that we can push the realistic unification scale smaller (the eleven dimension scale and $M_{GUT}$ will not change), for $n=1$, $M_{GUT}^r = 0.015 M_{11}$ or $M_{GUT}^r = \frac{1}{33.5} M_{GUT}$ for $\alpha_{GUT} = 0.04$, $M_{GUT}^r = 0.056 M_{11}$ or $M_{GUT}^r = \frac{1}{11.13} M_{GUT}$ for $\alpha_{GUT} = 0.15$. And for $n=2$, $M_{GUT}^r = 0.12 M_{11}$ or $M_{GUT}^r = \frac{1}{4.1} M_{GUT}$ for $\alpha_{GUT} = 0.04$, $M_{GUT}^r = 0.24 M_{11}$ or $M_{GUT}^r = \frac{1}{2.6} M_{GUT}$ for $\alpha_{GUT} = 0.15$. However, using above low bound, we just put the low bound of $M_{GUT}^r$ to the order of $10^5$ GeV, which is relatively higher than the reach of LHC and LEP.

Even though we consider standard embedding where $0.97 > x > 0$, if we assume that we only observe the materials or physics at our boundary or the observable sector, and the fifth dimension is very large \[3\], then $M_{GUT}$ can be at any scale if the theory is consistent.

3 Soft Terms in Nonstandard Embedding

The kähler potential, gauge kinetic function and the superpotential in the simplest compactification of M-theory on $S^3/Z_2$ are \[16, 19\]:

$$K = \tilde{K} + \tilde{K}|C|^2,$$  (23)

$$\tilde{K} = - \ln [S + \bar{S}] - 3 \ln [T + \bar{T}],$$(24)

$$\tilde{K} = \left(\frac{3}{T + \bar{T}} + \frac{\alpha}{S + \bar{S}}\right)|C|^2.$$  (25)

\[3\] Of course, it is finite, considering $M_{GUT} = 1.73$ TeV and $\alpha_{GUT} = 0.04$ ( or $M_{GUT} = 3.35$ TeV and $\alpha_{GUT} = 0.15$), we obtain that $(\pi \rho)^{-1} = 10^{-24}$ GeV or $\pi \rho = 1.97 \times 10^8$ meter.
\[ Re f_{\alpha\beta}^O = Re(S + \alpha T) \delta_{\alpha\beta} , \]
\[ Re f_{\alpha\beta}^H = Re(S - \alpha T) \delta_{\alpha\beta} , \]
\[ W = d_{xyz} C^x C^y C^z , \]

where \( S, T \) and \( C \) are dilaton, moduli and matter fields respectively. \( \alpha \) is a next order correction constant which is related to the Calabi-Yau manifold. And the nonperturbative superpotential in the simplest model is \[ W_{np} = h \exp \left( - \frac{8\pi^2}{C_2(Q)} (S - \alpha T) \right) , \]

where the group in the hidden sector is \( Q \) which is a subgroup of \( E_8 \), and \( C_2(Q) \) is the quadratic Casimir of \( Q \). By the way, \( C_2(Q) = 30, 12 \) for \( Q \) is \( E_8 \) and \( E_6 \), respectively.

With those information, one can easily obtain the following soft terms \[ M_{1/2} = \frac{\sqrt{3} M_{3/2}}{1 + x} \left( \sin \theta + \frac{x}{\sqrt{3}} \cos \theta \right) , \]
\[ M_0^2 = M_{3/2}^2 - \frac{3 M_{3/2}^2}{(3 + x)^2} (x(6 + x) \sin^2 \theta + (3 + 2x) \cos^2 \theta - 2\sqrt{3}x \sin \theta \cos \theta) , \]
\[ A = -\frac{\sqrt{3} M_{3/2}}{(3 + x)} ((3 - 2x) \sin \theta + \sqrt{3} x \cos \theta) , \]

where \( M_{3/2} \) is the gravitino mass, the quantity \( x \) defined above is the same as that in the last section and can also be expressed as
\[ x = \frac{\alpha(T + \bar{T})}{S + \bar{S}} . \]

If one combined the gaugino condensation scenario with above soft terms, one obtain the angle \( \theta \):
\[ \tan \theta = \frac{1}{\sqrt{3}} \left( 1 + \frac{2\pi}{C_2(Q)} (-\alpha^{-1}_{\text{GUT}} + \alpha^{-1}_H) \right) . \]

so, the soft terms \( M_{1/2}, M_0, \) and \( A \) are the functions of the gravitino mass and the gauge coupling ( \( \alpha_H \) ) in the hidden sector when one consider the hidden sector gaugino condensation.

Now, we numerically evaluate those soft terms in nonstandard embedding \[ [23, 24]. \] We take the gauge coupling in the observable sector as: \( \alpha_{\text{GUT}} = 0.15 \). First, we just consider \( F \) term of \( S \) and \( T \) SUSY breaking but we do not consider the gaugino condensation, we draw the soft terms in the unit of gravitino mass versus \( \theta \) in fig. 1, 2, 3, 4 \[ 4 \] for \( \alpha_H = 0.025, 0.05, 0.075, 0.1 \) respectively, we can see that when \( \alpha_H \) is

\[ ^4 \text{Constraint that the scalar mass square } M_0^2 \text{ should be larger than zero is considered.} \]
small, the magnitude of $M_{1/2}$ and $A$ are larger than that of $M_0$, and comparing to the gravitino mass, the magnitude of $M_{1/2}$, $A$ and $M_0$ are relatively larger than those in the standard embedding. Combining the gaugino condensation and the F-term SUSY breaking, we draw the soft term versus $\alpha_H$ in fig. 4, 5 where the hidden sector group are $E_8$ and $E_6$ respectively, we obtain that, all the soft terms decrease when we increase the hidden sector gauge coupling, but the variation of $M_0$ is very small, and the variation of $M_{1/2}$ is very large at small $\alpha_H$. When $\alpha_H$ large enough, i.e., $\alpha_H > 0.1$, the variations of all the soft terms are very small. Noticing that in order to obtain the gravitino mass at the hundred GeV range, the hidden sector gaugino condensation scale can not be too small, i.e., we can not let $\alpha_H$ be too small. We pick two points as representative points: $\alpha_H = 0.05$ and 0.1 with $E_8$ as hidden sector gauge group. The soft terms for $\alpha_H = 0.05$ in the unit of gravitino mass are:

\[
M_0 = 1.23 ; \quad M_{1/2} = 2.64 ; \quad A = 2.20 ,
\]

the soft terms for $\alpha_H = 0.1$ in the units of the gravitino mass are:

\[
M_0 = 1.03 ; \quad M_{1/2} = 1.85 ; \quad A = 1.80 .
\]

4 Compactification of Perfect Square and Gravitino Mass

First, we would like to review Horava’s result [3]. The 10-dimensional and 11-dimensional conventions are as in [1]. The space time signature is $- + \ldots +$. Eleven dimensional vector indices are written as $I, J, K, L, \ldots$. The 11-dimensional $\Gamma$ matrices are $32 \times 32$ real matrices satisfying \( \{ \Gamma_I, \Gamma_J \} = 2g_{IJ} \), with $g_{IJ} = \eta_{mn} e^m_I e^n_J$, the eleven dimensional metric. Each boundary of 11-dimensional manifold surport one $E_8$ Yang-Mills supermultiplet. In the simplest compactification, one of the $E_8$’s will be broken by the spin connection embedding to a grand-unified $E_6$ group, while the other $E_8$ in the hidden sector will not be broken. The adjoint index of this hidden $E_8$ will be denoted by $a, b, \ldots$

On $R^4 \times S^1 / Z_2 \times X$, we will use four-dimensional vector indices $\mu, \nu, \ldots$ that parametrize the flat Minkowski space $R^4$, and vector indices $i, j, k, \ldots$ and their complex conjugates $\tilde{i}, \tilde{j}, \tilde{k}, \ldots$ that correspond to a complex coordinate system on the Calabi-Yau three-fold $X$. The ten-dimensional vector indices that parametrize $R^4 \times X$ will be written as $A, B, C, \ldots$. The other conventions on $X \times S^1 / Z_2$ are as in [1].

The effective Lagrangian for this theory was constructed in [1]. It contains the eleven-dimensional supergravity multiplet $e^m_I, \psi_I$ and $C_{IJK}$ in the bulk, coupled to each of two $E_8$ Yang-Mills supermultiplet $A^i_B, \chi^i$ at each of the two ten-dimensional boundaries.

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5 We use $E_8$ as an example, although it is not a proper subgroup of $E_8$. 
To order $\kappa^{2/3}$, the Lagrangian is given by

$$
\mathcal{L} = \frac{1}{\kappa^2} \int_{M_{11}} d^{11}x \sqrt{g} \left( -\frac{1}{2} R - \frac{1}{2} \bar{\psi} I \Gamma^{IJK} D_J \frac{\Omega + \bar{\Omega}}{2} \psi_K - \frac{1}{48} G_{IJKL} G^{IJKL} \right) - \frac{\sqrt{2}}{384} \left( \bar{\psi} I \Gamma^{IJKLMN} \psi_N + 12 \bar{\psi} J \Gamma^{KL} \psi^M \right) \left( G_{JKLM} + \hat{G}_{JKLM} \right) - \frac{\sqrt{2}}{3456} \epsilon^{I_1 I_2 \ldots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \ldots I_7} G_{I_8 \ldots I_{11}}
$$

$$
+ \frac{1}{2\pi (4\pi \kappa^2)^{2/3}} \int_{M_{10}} d^{10}x \sqrt{g} \left( -\frac{1}{4} F_{AB} F^{iAB} - \frac{1}{2} \bar{\chi} a \Gamma^{A} D_A (\hat{\Omega}) \chi^a - \frac{1}{8} \bar{\psi} A \Gamma^{BCD} \chi^a \right) \left( \Gamma^{BCD} \eta + \frac{\sqrt{2}}{48} \left( \bar{\chi} a \Gamma^{ABC} \chi^a \right) \hat{G}_{ABC} \right),
$$

where the definitions of $\hat{\Omega}$, $\hat{F}_{AB}^i$ and $\hat{G}_{IJKL}$ can be found in \cite{1} and $i$ in $F$, $\hat{F}$ and $M_{10}^{i}$ is boundary index, i. e., $i = 1, 2$.

The relevant supersymmetry transformations of the gravitino fields are:

$$
\delta \psi_A = D_A \eta + \frac{\sqrt{2}}{288} G_{IJKL} \left( \Gamma^{IJKL}_A - 8 \delta^I_A \Gamma^{JKL} \right) \eta
$$

$$
- \frac{1}{576\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{11}) \left( \bar{\chi} a \Gamma_{BCD} \chi^a \right) \left( \Gamma^{BCD}_A - 6 \delta^B_A \Gamma^{CD} \right) \eta + \ldots ,
$$

$$
\delta \psi_{11} = D_{11} \eta + \frac{\sqrt{2}}{288} G_{IJKL} \left( \Gamma^{IJKL}_{11} - 8 \delta^I_{11} \Gamma^{JKL} \right) \eta
$$

$$
+ \frac{1}{576\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{11}) \left( \bar{\chi} a \Gamma_{ABC} \chi^a \right) \Gamma^{ABC} \eta + \ldots ,
$$

where the ... denote terms of order $\kappa^{4/3}$, as well as known terms of order $\kappa^{2/3}$ bilinear in the gravitinos that we do not need to use it.

With another term at relative order of $\kappa^{4/3}$, Horava obtained the perfect square in the M-theory on $S^1/Z_2$ $\cite{3}$:

$$
- \frac{1}{12\kappa^2} \int_{M_{11}} d^{11}x \sqrt{g} \left( G_{ABC11} - \frac{\sqrt{2}}{16\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{11}) \bar{\chi} a \Gamma_{ABC} \chi^a \right)^2 ,
$$

this perfect square is similar to that in the weakly coupled heterotic string, in which one change $G_{ABC11}$ to $H_{ABC}$.

In the eleven dimension metric and simplest compactification, one can compactify this perfect square to 5-dimension, it is:

$$
L^{(5)} = - \frac{1}{12\kappa^2} \int_{M^{5}} d^{5}x V_p (1-x) \sqrt{g} \left( G_{ABC11} - \frac{\sqrt{2}}{16\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{5}) \bar{\chi} a \Gamma_{ABC} \chi^a \right)^2 ,
$$

(41)
and to the 4-dimension, it is:

$$L^{(4)} = -\frac{1}{12\kappa^2} \int_{M^4} d^4x 2\pi \rho_p V_p (1-x)(1-\frac{x}{3}) \sqrt{g} (G_{ABC11} - \frac{1}{1 - \frac{x}{3}} \frac{1}{2\pi \rho_p} \frac{2}{3} \prod_{C}^{2/3} \delta(x^5) \chi^a \Gamma_{ABC} \chi^a)^2 .$$ (42)

One can also write the transformation of the gravitino fields as:

$$\delta \psi_A = D_A \eta + \frac{\sqrt{2}}{288} G_{IJKL} \left( \Gamma_A^{IJKL} - 8\delta_A^I \Gamma^{JKL} \right) \eta - \frac{1}{1154\pi^2 \rho_p} \frac{1}{1 - \frac{x}{3}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left( \overline{\chi}^a \Gamma_{BCD} \chi^a \right) \left( \Gamma^B_{A} - 6\delta_B^A \Gamma^{CD} \right) \eta + \ldots ,$$ (43)

$$\delta \psi_{11} = D_{11} \eta + \frac{\sqrt{2}}{288} G_{IJKL} \left( \Gamma_{11}^{IJKL} - 8\delta_{11}^I \Gamma^{JKL} \right) \eta + \frac{1}{1154\pi^2 \rho_p} \frac{1}{1 - \frac{x}{3}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left( \overline{\chi}^a \Gamma_{ABC} \chi^a \right) \Gamma_{ABC} \eta + \ldots ,$$ (44)

define a modified field strength $\tilde{G}_{IJKL}$ by

$$\tilde{G}_{ABC11} = G_{ABC11} - \frac{\sqrt{2}}{32\pi^2 \rho_p} \frac{1}{1 - \frac{x}{3}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \chi^a \Gamma_{ABC} \chi^a ,$$ (45)

$$\tilde{G}_{ABCD} = G_{ABCD} ,$$ (46)

one can write the above supersymmetry transformation of the gravitino field in the hidden sector and in the 4-dimension as:

$$\delta \psi_A = D_A \eta + \frac{\sqrt{2}}{288} \tilde{G}_{IJKL} \left( \Gamma_A^{IJKL} - 8\delta_A^I \Gamma^{JKL} \right) \eta + \ldots ,$$ (47)

$$\delta \psi_{11} = D_{11} \eta + \frac{\sqrt{2}}{288} \tilde{G}_{IJKL} \left( \Gamma_{11}^{IJKL} - 8\delta_{11}^I \Gamma^{JKL} \right) \eta + \frac{1}{192\pi^2 \rho_p} \frac{1}{1 - \frac{x}{3}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left( \overline{\chi}^a \Gamma_{ABC} \chi^a \right) \Gamma_{ABC} \eta + \ldots .$$ (48)

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One should notice that after compactification, the remaining $\psi_A$ where $A$ is the Calabi-Yau manifold index will be the mixing of the $\psi_A$, which are singlets of SU(3) homonomy group. We are a little sloppy by just writing it as this way.
Therefore, as previously, the supersymmetry variations of $\psi_A$ do not include gaugino condensation term, but, the supersymmetry variations of $\psi_{11}$ do include gaugino condensation term. The $\psi_{11}$ will play the role of the goldstino in this scenario.

Now, let us calculate the gravitino mass by gaugino condensation [16]. The term which is relevant in the eleven dimension Lagrangian is:

$$- \frac{\sqrt{2}}{384 \kappa^2} \left( \bar{\psi}_I \Gamma^{JKLMN} \psi_N + 12 \bar{\psi}_J \Gamma^{KL} \psi^M \right) \left( G_{JKLM} + \hat{G}_{JKLM} \right),$$

(49)
after compactification and considering gaugino condensation, we obtain:

$$M_{3/2} = \frac{1}{64\pi \alpha_{GUT}} \frac{1 - x}{(1 + x)(1 - \frac{x}{3})} \Lambda^3 \frac{M_{pl}^2}{M_{3/2}},$$

(50)

where $\Lambda$ is defined by following equation:

$$< \bar{\chi}^a \Gamma_{ijk} \chi^a > = \Lambda^3 \epsilon_{ijk}.$$ 

(51)

It has been argued that $x$ can not be larger than 0.97 in order to let the hidden sector physical Calabi-Yau mass scale ($M_H$) less than the eleven dimension Planck scale $M_{11}$. In fact, if $M_H > M_{11}$, one need to worry about the anomaly at the scale between $M_H$ and $M_{11}$. Here, if we consider the gaugino condensation SUSY breaking, we can have another reason that we can not let $x = 1$, because we will have massless gravitino.

Now, let us discuss the gaugino condensation scale. First, taking $\alpha_{GUT} = 0.04$ and consider the standard embedding $x > 0$. For $M_{3/2} = 100$ GeV, we obtain that:

$$\Lambda = \left( \frac{(1 + x)(1 - \frac{x}{3})}{1 - x} \right)^{1/3} \times 1.67 \times 10^{13} \text{GeV},$$

(52)

for $M_{3/2} = 1$ TeV, we obtain:

$$\Lambda = \left( \frac{(1 + x)(1 - \frac{x}{3})}{1 - x} \right)^{1/3} \times 3.6 \times 10^{13} \text{GeV},$$

(53)

because $x$ is smaller than 0.97, the factor

$$\left( \frac{(1 + x)(1 - \frac{x}{3})}{1 - x} \right)^{1/3},$$

(54)

will be larger than 1 and smaller than 6.67. If we consider the case that the gaugino condensation occurs just at the physical Calabi-Yau compactification scale in the hidden sector, i. e., we consider intermediate unification with gaugino condensation SUSY breaking, we can obtain that in order to keep gravitino mass at hundreds GeV range, $\Lambda$ is about $1.1-2.4 \times 10^{14}$ GeV. $M_{11}$ and $M_H$ will be about the same as that
scale, and $M_{\text{GUT}}$ will be about one half of that scale. Of course, we can put the realistic GUT scale $M_{\text{GUT}}$ in the range $10^{12}$ GeV, which is the right unification scale for $SU(4) \times SU(2)_L \times SU(2)_R$ model \[39\], and this model has no problem on proton decay.

Second, taking $\alpha_{\text{GUT}} = 0.15$ and consider the non-standard embedding $x < 0$. For $M_{3/2} = 100$ GeV, we obtain that:

$$\Lambda = \left( \frac{(1 + x)(1 - \frac{x}{3})}{1 - x} \right)^{1/3} \times 1.62 \times 10^{13}\text{GeV} \, ,$$ \hspace{1cm} (55)

for $M_{3/2} = 1$ TeV, we obtain:

$$\Lambda = \left( \frac{(1 + x)(1 - \frac{x}{3})}{1 - x} \right)^{1/3} \times 3.5 \times 10^{13}\text{GeV} \, ,$$ \hspace{1cm} (56)

so, the variation is very small. Noticing that the $\Lambda$ is related to the non-perturbative superpotential as \[40\]:

$$\Lambda^3 \sim M_H^3 \exp \left( -\frac{8\pi^2}{C_2(Q)} S(1 - x) \right) \, ,$$ \hspace{1cm} (57)

we can not let $x$ close to -1. Therefore, we obtain in this case, the gaugino condensation scale will occur at $10^{13}$ GeV order. And if one consider the intermediate unification, $M_{\text{GUT}}$ will be in the range of $10^{11} - 10^{13}$ GeV.

We would like to comment on the superpartner of the goldstino \[16\]. Recalling the eleven dimension transformation of the supergravity multiplet \[1\]:

$$\delta e^m_I = \frac{1}{2} \tilde{\eta} \Gamma^m \psi_I \, ,$$ \hspace{1cm} (58)

$$\delta C_{IJK} = -\frac{\sqrt{2}}{8} \tilde{\eta} \Gamma_{[IJ}\psi_K] \, ,$$ \hspace{1cm} (59)

$$\delta \psi_I = D_I \eta + \frac{\sqrt{2}}{288} \left( \Gamma^L_{IJK} - 8 \delta^I_J \Gamma^{KLM} \right) \eta G_{JKLM} + \ldots \, ,$$ \hspace{1cm} (60)

and noticing that in the simplest compactification and non-deformed Calabi-Yau manifold, the eleven dimension metric can be written as \[11\]:

$$g^{11}_{\mu\nu} = e^{-\gamma} e^{-2\sigma} g^4_{\mu\nu} \, ,$$ \hspace{1cm} (61)

$$g^{11}_{ij} = e^\sigma g_{ij} \, ; \, g^{11}_{11,11} = e^{2\gamma} e^{-2\sigma} \, .$$ \hspace{1cm} (62)
And $S$ and $T$ are defined by:

$$S = e^{3\sigma} + i24\sqrt{2}D ,$$  \quad (63)

$$T = e^\gamma - i6\sqrt{2}C_5 + |C|^2 ,$$  \quad (64)

where $D$ and $C_5$ are defined by:

$$\frac{1}{4!}e^{\sigma}G_{11\mu\nu\rho} =  \epsilon_{\mu\nu\rho\delta}(\partial^\delta D) ,$$  \quad (65)

$$C_{5ij} = iC_5\delta_{ij} .$$  \quad (66)

Therefore, elfbein $e_{11,11}$ is a function of $S$ and $T$ and the $C_{IJ}^{11}$ are the imaginary parts of $S$ and $T$ in 4 dimension, we may conclude that the SUSY might be broken by F-term of $S$ and $T$. In addition, if one consider deformed Calabi-Yau manifold, $S$ and $T$ are mixed with each other in the two boundaries [16, 19], so, SUSY in this case will definitly be broken by the F-term of $S$ and $T$, although there exist the possibility that one of $F^S$ and $F^T$ might be zero.

## 5 Comments on General Kähler Function and Superpotential

When one mentions the difference between the M-theory on $S^1/Z_2$ and the weakly coupled heterotic string, one always points out the next order correction is large. But, in fact, if the next order correction is large, one need to consider the higher order terms (oder of $x^n$ for $n > 1$). Therefore, for the simplest compactification, we have the following general Kähler potentil, gauge kinetic function and non-perturbative superpotential as [14]:

$$K = \tilde{K} + \tilde{K}|C|^2 ,$$  \quad (67)

$$\tilde{K} = -\ln[S + \bar{S}] - 3\ln[T + \bar{T}] ,$$  \quad (68)

$$\tilde{K} = \left( 1 + \sum_{i=1}^{\infty} c_i \left( \frac{\alpha(T + T)}{S + \bar{S}} \right)^i \right) \left( \frac{3}{T + \bar{T}} \right)|C|^2 ,$$  \quad (69)

\textsuperscript{7}One of $F^S$ and $F^T$ might be zero.
\[ \text{Ref}^{\alpha\beta}_{\text{O}} = \text{Re}S \left( 1 + \sum_{i=1}^{\infty} d_i \left( \frac{s T}{S} \right)^i \right) \delta_{\alpha\beta}, \quad (70) \]

\[ \text{Ref}^{H}_{\alpha\beta} = \text{Re}S \left( 1 + \sum_{i=1}^{\infty} d_i \left( -\frac{s T}{S} \right)^i \right) \delta_{\alpha\beta}, \quad (71) \]

\[ W_{np} = h \exp(-\frac{8\pi^2}{C_2(Q)f^H}) . \quad (72) \]

So, if x is large, the higher order correction will also be large, which will change the soft terms, and then change the low energy phenomenology by RGE running.

Using standard method [41, 42], one can easily calculate the soft terms. And one need reconsider the 4-dimension Planck scale’s expressions, and also reconsider multiple moduli case, etc.. The detail of those discussions, will appear elsewhere [14].

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Figure 1: Soft terms versus angle $\theta$ with $\alpha_H=0.025$ in the unit of gravitino mass.
Figure 2: Soft terms versus angle $\theta$ with $\alpha_H=0.05$ in the unit of gravitino mass.
Figure 3: Soft terms versus angle $\theta$ with $\alpha_H=0.075$ in the unit of gravitino mass.
Figure 4: Soft terms versus angle $\theta$ with $\alpha_H=0.1$ in the unit of gravitino mass.
Figure 5: Soft terms versus angle $\alpha_H$ with $Q = E_8$ in the unit of gravitino mass.
Figure 6: Soft terms versus angle $\alpha_H$ with $Q = E_6$ in the unit of gravitino mass.