Contribution of sea quarks to the electromagnetic decay of decuplet baryons

Gustavo Guerrero-Navarro and Roelof Bijker
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Ciudad de México 04510, México
E-mail: gustavo.guerrero@correo.nucleares.unam.mx

Abstract. In this work, we study briefly the electromagnetic dipole transitions from decuplet to octet S-wave baryons, where the current experimental data for the radiative decay width shows that these values are approximately two times larger than the predictions of the constituent quark model. The main aim is to solve this problem with the approach of effective degrees of freedom for quarks in the quark model, considering also the sea quarks, obtaining the valence and the sea quarks contributions. In order to do this, an unquenched quark model has been considered where the effects of the quark-antiquark pairs are incorporated as a perturbation with the quantum numbers of vacuum in the wave function for baryons as baryon-meson intermediate coupled states with the quantum numbers of the initial baryon through the $^3P_0$ model. Also, an extended expression has been obtained to the M1 electromagnetic transition that gives a closer value to the actual experimental data due to the sea quark core contribution.

1. Introduction
In nuclear and particle physics the electromagnetic transitions are a fundamental process for the understanding of the baryon structure. The constituent quark model (CQM) has had a very good achievement by describing the radiative transitions and a good agreement with the spatially excited resonances, the magnetic moments of baryons and charmonium in its non-relativistic approximation [1]-[10], even in the prediction of the decuplet to octet baryon transitions [11]. There have been many attempts in the frame of QCD like effective models of hadrons to predict the experimental data, but there is still a lack of models that can explain the differences between them. The main approach of effective models is taking account degrees of freedom QCD allowed by breaking symmetries. Those can be taken away considering intermediate states of sea quarks that can contribute to the electromagnetic amplitude transitions [12]-[21]. In this context, we consider the S-wave magnetic dipole (M1) transition between decuplet and octet baryons and we investigate the contributions of the sea core to this term.

In the case of the electromagnetic transitions from decuplet to octet, the transition amplitude used, even for the photoproduction or the decay transitions, takes into account the models that describe the wave function of baryons but it can’t reproduce the available experimental data. The current values of the electromagnetic decay widths, $\Gamma_{A\rightarrow A'\gamma}$, are $703 \pm 61$ keV, $251 \pm 61$ keV and $449 \pm 78$ keV for $\Gamma_{\Delta\rightarrow N\gamma}$, $\Gamma_{\Sigma^+\rightarrow \Sigma^*\gamma}$ and $\Gamma_{\Sigma^0\rightarrow \Lambda\gamma}$ respectively which corresponds to the $(0.60 \pm 0.05)\%$, $(0.70 \pm 0.17)\%$ and $(1.25 \pm 0.13)\%$ of the total Breit-Wigner width [22]. Meanwhile, the result of the CQM is almost one half of the cited experimental data (tables 1
On the other hand, we have extensions of the CQM (χQM, MCM, UQM) where the effects of sea quarks are considered. Those are motivated by the Gottfried sum rule violation measurements for the proton that probes the flavor asymmetry on it (a violation of the basic assumption of the CQM). Using the parton distribution functions (PDFs) of quarks and antiquarks, the Gottfried sum rule is given by

\[ S_G = \frac{1}{3} + \frac{2}{3} \Delta P \]  

where \( \Delta P = \int_0^1 dx (\bar{u}_p(x) - \bar{d}_p(x)) \) and \( x \) the Bjorken variable. This means that if \( \bar{d} \) and \( \bar{u} \) in the proton are symmetric, the Gottfried sum rule would give \( 1/3 \) but the measurements of the NMC experiment at CERN measured this by deep inelastic muon scattering \cite{23} finding a smaller value than \( 1/3 \), \( S_G = 0.2281 \pm 0.0065 \), i.e. \( \bar{u} < \bar{d} \) gives the flavor asymmetry. It has also measured (experiment NA51) the \( x \) dependence of the radio \( (\bar{d}(x)/\bar{u}(x)) \) at \( 0.004 < x < 0.8 \) with the Drell-Yan process \cite{24}. Another experiment (E866) at Fermilab measured it at regions of between \( 0.015 < x < 0.35 \) with a final value of \( S_G = 0.255 \pm 0.008 \) \cite{25} and the Hermes collaboration \cite{26}, \( S_G = 0.23 \pm 0.02 \) at values of \( 0.020 < x < 0.30 \).

By this the understanding of the flavor asymmetry of the antiquarks in the proton can be tested with the EM transitions of baryons by using QCD effective and hadron models such as meson cloud models that contains virtual meson cloud states like \( |p⟩ = α|p_0⟩ + β|nπ^+⟩ + γ|Δ^+π^-⟩ + \ldots \). Those states are described in the Unquenched Quark Model (UQM) \cite{27} through the \( ^3P_0 \) pair quark creation model.

2. The Unquenched Quark Model

The UQM is an extension to the CQM inspired by the \( ^3P_0 \) model originally used to describe the meson strong decays \cite{28} including the degrees of freedom allowed by the QCD. On this formalism, and taking only the effective degrees of freedom from the constituent quarks, the hadron states expands in the different order Fock states as follows

\[ |qqq⟩ \sim |qqq⟩ + |qqq q\bar{q}⟩ + |qqq q\bar{q} q\bar{q}⟩ + \cdots \]  

In the UQM the wave states for baryons are obtained from the quark level introducing the \( T^\dagger(3P_0) \) operator \cite{27, 29} to generate the sea pair quarks and coupling this to the zeroth order three quark state to generate the first order Fock states, \( |qqq q\bar{q}⟩ \). These states are generated as a sum of the quantum number of the possible higher Fock states in the form of the baryon-meson first order state, \( |BC⟩ \), coupled with the vacuum quantum numbers from the sea pair and the zeroth order state \( |A⟩ \). The UQM state for a baryons reads

\[ |Ψ_A⟩ = \mathcal{N}_A \left[ |A⟩ + \sum_{BCIJ} \int dk d\vec{k} k^2 |BCIJ; \vec{K}⟩ ⟨BCIJ; \vec{K} k ^T | A⟩ \right] \]  

where the \( \mathcal{N}_A \) is the normalization factor, \( |A⟩ \) is the valence baryon state which corresponds to the CQM state, \( \vec{I} \) is the orbital relative angular momentum of the baryon-meson state, \( |BC⟩ \), and \( \vec{J} = \vec{J}_B + \vec{J}_C + \vec{I} \) their total angular momentum, and in the same way \( \vec{k} \) and \( \vec{K} \) are the relative and total momentum of the baryon-meson state, \( m_A, E_B \) and \( E_C \) are the corresponding energies (\( A \) baryon is in the rest frame). The strength factor for the sea quark pair creation within \( T^\dagger \) actually can be fitted in the UQM through the flavor content to the proton asymmetry flavor, \( \Delta P \) (1).
3. Electromagnetic transitions

The decuplet to octet electromagnetic transitions have been widely studied for a variety of quark models [12]-[21]. In the UQM frame to determine the electromagnetic decay width for the \( A \rightarrow A' \gamma \) process in the form of the matrix element we use the Fermi Golden rule
\[
\Gamma_{A \rightarrow A' \gamma} = \frac{\alpha E_A p_\gamma^2}{2m_A \mu_N^2} |\langle \Psi_A'| \hat{\mu} | \Psi_A \rangle|^2
\]
\[(4)\]
in unit of the nuclear magneton \( \mu_N \), where \( \alpha = 1/137 \) is the electromagnetic coupling constant, \( \vec{p}_\gamma \) is the photon momentum, \( m_N \) is the nucleon mass and
\[
\hat{\mu} = \hat{\mu}_S + \hat{\mu}_l = \sum_i \mu_i \left( 2\hat{s}_i + \hat{l}_i \right),
\]
\[(5)\]
where \( \mu_i, \hat{s}_i \) and \( \hat{l}_i \) are the magnetic moments, the spin operator and the orbital operator of the \( i \)th quark respectively, \( \hat{\mu}_S \) is the spin part magnetic moment operator and \( \hat{\mu}_l \) is the orbital magnetic moment operator where, in the ground state, only contributes the relative orbital momentum between the baryon, \( B \), and the meson, \( C \), taking the expectation value as the matrix element of
\[
\hat{\mu}_l = \sum_i \mu_i \hat{l}_i \rightarrow \left( \alpha_{BC} \frac{\mu_1 + \mu_2 + \mu_3}{3} + \left( 1 - \alpha_{BC} \right) \frac{\mu_4 + \mu_5}{2} \right) \hat{l},
\]
\[(6)\]
with \( \alpha_{BC} = m_C/(m_B + m_C) \) a function of the baryon and meson masses, \( m_B \) and \( m_C \), \( \mu_1, \mu_2 \) and \( \mu_3 \) the magnetic moments of the constituent quarks in the \( B \) baryon; in the same way \( \mu_4 \) and \( \mu_5 \) corresponds to the quarks in the \( C \) meson.

In the UQM the matrix element of the total magnetic moment operator in (4) results as a function of the widths, \( \alpha_A, \alpha_B, \) of the baryon and meson state functions as well of the quark-antiquark pair width, \( \alpha_D \), fixed approximately to 0.35 fm. Considering the isospin symmetry for the case of the \( \Delta \rightarrow N \gamma \) we are able to compute the expectation value for the spin part transition magnetic moment operator, \( \hat{\mu}_S \) (5), obtaining an expression in the form
\[
\langle \Psi_N | \hat{\mu}_S | \Psi_\Delta \rangle = \mu_{\Delta N}^{val} + \mu_{\Delta N}^{sea} = \mu_{\Delta N}^{N \text{spin}},
\]
\[(7)\]
where \( \mu_{\Delta N}^{val} \) indicates the valence spin contribution that corresponds to the CQM expression including the normalization factor given in (3) and \( \mu_{\Delta N}^{sea} \) is the baryon-meson coupled state, \( |BC \rangle \), contribution. As mentioned above, another contribution to the transition magnetic moment in the UQM for the ground states of baryons is the orbital relative magnetic momentum from the intermediate states \( |BC \rangle \). As always, considering the Jacobi coordinates for the constituent quarks coordinates and the operator \( \hat{l} \), the matrix element of (6) results
\[
\langle \Psi_N | \hat{\mu}_l | \Psi_\Delta \rangle = \mu_{\Delta N}^{l \text{orb}},
\]
\[(8)\]
Table 1. Meson cloud contributions for the decuplet to octet transition magnetic moments (in unit of $\mu_N$). In each group, the first line indicates the valence quark contribution, $\mu_{BB'}^{val}$, and the sum of all meson cloud contributions (boldface), the second line the pion cloud contributions, the third line the kaon cloud, and the fourth the $\eta\eta'$ mixed cloud. The column $\mu_{BB'}^{spin}$ presents the sum of the valence quark contribution, $\mu_{BB'}^{val}$, and the total spin sea quark core, $\mu_{BB'}^{sea\ spin}$. The column $\mu_{BB'}^{sea\ orb}$ shows the relative orbital contributions. The column $\mu_{BB'}^{total}$ shows respectively the total meson cloud contributions and the final results (boldface).

| Process | cloud | $\mu_{BB'}^{val}$ | $\mu_{BB'}^{sea\ spin}$ | $\mu_{BB'}^{spin}$ | $\mu_{BB'}^{sea\ orb}$ | $\mu_{BB'}^{total}$ |
|---------|-------|-------------------|--------------------------|-------------------|-----------------------|-------------------|
| $\Delta \rightarrow N\gamma$ | 1.02±0.10 | 2.26±0.14 | 3.28±0.04 | 0.33±0.02 | 3.62±0.06 |
| $\pi$ | 2.11±0.13 | 0.33±0.02 | 2.43±0.15 |
| $K$ | 0.08±0.01 | 0.01±0.00 | 0.09±0.01 |
| $\eta\eta'$ | 0.08±0.01 | | 0.08±0.01 |

that contains only the sea contribution given the zero orbital contribution of the ground state for the valence part.

4. Fitting Procedure
In order to calculate the dipole transition magnetic moment and the EM decay width taking into account the experimental data we need to determine the $^3P_0$ sea amplitude coefficients in equation (3) which depends on the experimental masses, the charge radii of proton and pion [22] that depends (through the oscillator harmonic model) of the baryon, meson and sea quark pair widths, $\alpha_B, \alpha_C, \alpha_D$, respectively (into the expression for $\langle BC|\tilde{K}k|T\dagger|A\rangle$). The experimental flavor asymmetry in the proton, $\Delta P$ [23, 25], fits the strength coupling factor for the operator $T\dagger$, as well the Color-Spin-Flavor coefficients [27] employing numerical methods. Once the sea coefficients have been fitted to the data, we follow by fitting the magnetic moments of the constituent quarks using the magnetic moments for $p$, $n$ and $\Lambda^0$ octet baryons [22] needed in the equations (7) and (8).

In order to calculate the error propagation from the experimental data to all the parameters into the UQM expressions until the final results, we use a Monte-Carlo simulation [30] using random gaussian distributed values for each experimental data to ensure the correct correlation among parameters into the simulation. The fit was made by computing 10000 random points having enough statistical information finding that, in this particular case, the transition magnetic moment (and the EM decay widths) has also a gaussian frequency distribution. For that reason, we are able to calculate in good approximation the error bars values at the $1\sigma$ for this proper gaussian function. In this way, we have been considered all the correlations between the parameters in the UQM.

5. Results
The final fit in the UQM to the data give us the valence and the sea contribution to the transition magnetic moment of the $\Delta \rightarrow N\gamma$ process. On the sea quark core contributions, we have the spin and the relative orbital baryon-meson contributions, while the valence contribution is only spin dependent due to the treatment of the ground state of the baryons. In the table we show the numerical values of this giving the sea cloud contribution of each meson, $C$, corresponding to the intermediate $\langle BC \rangle$ channel states as showed in (3).
Table 2. Electromagnetic transition of the Δ baryon in the UQM. Error bars are reported at the $1\sigma$ and take into account all the correlations among parameters.

| $\Delta \rightarrow N\gamma$ | CQM | UQM spin | UQM total | Exp [22] |
|-----------------------------|-----|----------|-----------|----------|
| $\mu_{BB'}$ ($\mu_N$)      | 2.66±0.00  | 3.28±0.04 | 3.62±0.06 | 3.53±0.16 |
| $\Gamma_{B \rightarrow B'\gamma}$ (keV) | 399±7 | 608±20 | 738±29 | 703±61 |

In the table 2 we give the corresponding electromagnetic decay width for the transition magnetic moment for the CQM case and the UQM spin contribution following by the spin plus orbital (total) contribution and the experimental comparison. Figures 1 and 2 show the plots where the sea contributions in the UQM increase the values in comparison with CQM, due to the valence contribution is the only one contribution for it.

6. Discussion and conclusions

Our results show a valence and sea core explicit contribution to the transition magnetic moment for $\Delta/N$, as well to the electromagnetic decay width. We can see directly in the figure 3, corresponding to the values in the table 1, that the UQM spin results are closer to the experiment result than the CQM. At the same time in the UQM, the sum of the spin sea cloud contributions become largest than the valence (spin) quark core while the orbital sea contribution is the smaller one. It seems more important the pion cloud contribution to the electromagnetic transition because of the pion smaller mass as we expected. Similarly, the lower sea contribution is given by the Kaon and $\eta\eta'$ cloud with larger mass for these mesons. The orbital sea contribution it’s not so important at least it is added to the total spin contribution in the sense that the total, spin plus orbital, contribution reach the experimental data results, as a consequence that the
total spin contribution results very close to the experimental value.
It’s worth to mention that the valence contribution in the UQM and the CQM value only differs on the normalization factor, although the expressions are in that sense the same. By this, the normalization factor is close to 1/2.

About the total spin contribution, we found that the valence and the sea part are correlated through the $^3P_0$ strength factor giving us a lower error bar for this fit because of the experimental error bar of the proton flavor asymmetry.

Finally, the UQM results given in the table 2 shows the effective contribution of the meson sea cloud by a unquenching in the CQM. Those results are in agreement with the experimental value for the $\Delta \rightarrow N\gamma$ and we found that the sea part has an important role as the valence core do it.

Acknowledgments
This work is supported by SNI-CONACYT, Mexico (Exp. Ayte. 13928).

References
[1] Eichten E, Gottfried K, Kinoshita T, B Kogut J, Lane D K and M Yan T 1975 Phys. Rev. Lett. 34 369; Eichten E, Gottfried K, Kinoshita T, D Lane K and M Yan T 1978 Phys. Rev. D 17 3090; 1980 Phys. Rev. D 21 203
[2] Isgur N and Karl G 1978 Phys. Rev. D 18 4187; 1979 Phys. Rev. D 19, 2653; 1979 Phys. Rev. D 20 1191
[3] Godfrey S and Isgur N 1985 Phys. Rev. D 32 189
[4] Capstick S and Isgur N 1986 Phys. Rev. D 34 2809
[5] Ferraris M, Giannini M M, Pizzo M, Santopinto E and Tiator L 1995 Phys. Lett. B 364 231; Santopinto E, Iachello F and Giannini M M 1998 Eur. Phys. J. A 1 307; Santopinto E and Giannini M M 2012 Phys. Rev. C 86 065202; Giannini M M, Santopinto E 2015 Chin. J. Phys. 53 020301; Aiello M et al 1996 Phys.Lett. B 387, 215; Aiello et al 1988 J. of Phys. G 24 753; Bijker R, Iachello F, Santopinto E 1998 J. of Phys. A 31 9041; M De Sanctis et al 2007 Phys. Rev. C 6 062201
[6] Glozman L Y, Riska D O 1996 Phys. Rept. 268 263; 1998 Glozman L Y, Plessas W, Varga K, Wagenbrunn R F, Phys. Rev. D 58 094030
[7] Loring U, Metsch B C and Petry H R 2001 Eur. Phys. J. A 10 395
[8] Santopinto E 2005 Phys. Rev. C 72 022201; Ferretti J, Vassallo A and Santopinto E 2011 Phys. Rev. C 83, 065204; De Sanctis M, Ferretti J, Santopinto E and Vassallo A (2014); De Sanctis M et al 2011 Phys. Rev. C 84 055201
[9] Galata G and Santopinto E 2012 Phys. Rev. C 86 045202
[10] Bijker R, Iachello F and Leviatan A 1994 Ann. Phys. (N.Y.) 236 69; 1996 Phys. Rev. C 54 1935; 1997 Phys. Rev. D 55 2862; 2000 Ann. Phys. (N.Y.) 284 89
[11] Beg M A B, Lee B W and Pais A 1964 Phys. Rev. Lett. 13 514
[12] Leinweber D B, Draper T and Woloshyn R M 1993 *Phys. Rev.* D 48 2230
[13] Kim H C, Polyakov M, Przaszalowicz M, Yang G S and Goeke K 2005 *Phys. Rev.* D 71 094023
[14] Ramalho G and Tsushima K 2013 *Phys. Rev.* D 87 9-093011
[15] Dhir R and Verma R C 2009 *Eur. Phys. J.* A 42 243
[16] Ramalho G and Tsushima K 2013 *Phys. Rev.* D 88 053002
[17] Keller D and Hicks K 2013 *Eur. Phys. J.* A 49 53
[18] Wang L and Lee F X 2009 *AIP: Conf. Proc.* 1182 532
[19] Hong S T 2007 *Phys. Rev.* D 76 094029
[20] Jenkins E E 2012 *Phys. Rev.* D 85 065007
[21] Lebed R F and Martin D R 2004 *Phys. Rev.* D 70 016008
[22] Patrignani C *et al* 2016 (Particle Data Group) *Chin. Phys.* C 40 100001
[23] Amaudruz P *et al* 1991 *Phys. Rev. Lett.* 66 2712; Arneodo M *et al* 1997 *Nucl. Phys.* B 487 3
[24] Baldit A *et al* 1994 *Phys. Lett.* B 332 244; Towel R S *et al* 2001 *Phys. Rev.* D 64 052002
[25] Hawker E A *et al* 1998 *Phys Rev. Lett.* 80 3715
[26] Ackerstaff K *et al* 1998 *Phys. Rev. Lett.* 81 5519
[27] Bijker R, Ferretti J, Galata G, Garcia-Tecocoatzi H, Santopinto E 1996 *Phys. Rev.* D 94 074040
[28] Micu L 1969 *Nucl. Phys.* B 10 521
[29] Roberts W and Silvestre-Brac B 1992 *Few-Body Syst.* 11, 171
[30] Fernández-Ramírez C, Danilkin I V, Mathieu V, and Szczepaniak A P (JPAC Collaboration) 2016 *Phys. Rev.* D 93 074015