Lattice QCD Results on Strangeness and Quasi-Quarks in Heavy-Ion Collisions

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Abstract.
Fluctuations of conserved quantities in heavy-ion collisions have been argued to be diagnostic tools for the nature of the produced phase. These can be related to the predictions of quark number susceptibilities (QNS) from lattice QCD. Using the diagonal QNS, we extracted the Wróblewski parameter in a dynamical QCD computation. Our results on the cross correlations $\chi_{BQ}$, $\chi_{BY}$, $\chi_{BS}$ and $\chi_{QY}$ allow us to explore the charge and baryon number of objects that carry flavour. We present evidence that in the high temperature phase of QCD the different flavour quantum numbers are excited in linkages which are exactly the same as one expects from quarks.
1. Introduction

As exciting experimental results keep pouring in from the Relativistic Heavy Ion Collider (RHIC) in BNL, New York, USA, the task of detecting the quark-gluon plasma (QGP) and establishing its various properties becomes more and more pressing. One needs therefore to look at as many signatures in as different a variety of aspects as possible. This has, of course, been going on both theoretically and experimentally. Fluctuations in conserved quantities such as the baryon number $B$, electric charge $Q$ or strangeness $S$ have been proposed as promising signals of QGP. Similarly, since very early days of relativistic heavy ion collisions, enhancement of strangeness production has also been regarded as a useful indicator.

The original proposals were usually based on simple model considerations as was the case for many other signals. The key fact employed there was that the up/down and strange quark masses are smaller than the expected transition temperature whereas the masses of the low lying hadrons, except for pion, are significantly larger. While these arguments on the differences between the two phases are qualitatively appealing, one has to face quantitative questions of details for any meaningful comparison with the data. Since the temperature of the plasma produced in RHIC, or even LHC, may not be sufficiently high for simple ideal gas pictures to be applicable, numerical estimates or indeed even the degree of utility of the proposal, could be misleading. Information obtained directly from the underlying theory, quantum chromodynamics (QCD) appears desirable, and the lattice formulation of QCD can potentially deliver that.

A variety of aspects of the strangeness enhancement have been studied and many different variations have been proposed. One very useful way of looking for strangeness enhancement is the Wróblewski parameter. Defined as the ratio of newly created strange quarks to light quarks,

$$\lambda_s = \frac{2\langle s\bar{s} \rangle}{\langle u\bar{u} + d\bar{d} \rangle}$$

the Wróblewski parameter has been estimated for many processes using a hadron gas fireball model. An interesting finding from these analyses is that $\lambda_s$ is around 0.2 in most processes, including proton-proton scattering, but is about a factor of two higher in heavy ion collisions. An obvious question one can ask is whether this rise by a factor of two can be attributed to the strangeness enhancement due to quark gluon plasma and if yes, whether this can be quantitatively demonstrated. We show below how quark number susceptibilities, obtained from simulations of lattice QCD, may be useful in answering these questions.

An interesting property that the Wróblewski parameter possesses is that it is a robust quantity. Indeed, we have recently argued that the ratios of susceptibilities, $C_{K/L}$, defined by

$$C_{K/L} \equiv \frac{\chi_K}{\chi_L} = \frac{\sigma_K}{\sigma_L}$$

may be useful in answering these questions.
where $\chi_K$ and $\chi_L$ are quark number susceptibilities (QNS) for the conserved quantum numbers $K$ and $L$ and $\sigma_K$ and $\sigma_L$ are the corresponding experimentally measured variances, are robust variables in the high $T$ Phase. This is so both theoretically and experimentally, provided the two variances are obtained under identical experimental conditions and after removing counting (Poisson) fluctuations. We will show below our lattice QCD results in partial support of this claim.

We employed such robust variables to address the important outstanding question of the nature of plasma (QGP) excitations. From several lattice QCD investigations, it has been known for a long time that the straightforward perturbation theory fails to describe the lattice data for equation of state for $T \geq T_c$. Various resummation schemes, and phenomenological models have been tried to understand the nature of the QGP in this region. For $T/T_c \geq 3 - 5$, lattice results on entropy density seem to be in agreement with such modified weak coupling pictures. Quark number susceptibilities provide an independent check on them, and yielded a strong support for these ideas. Screening mass determinations also suggested an agreement with an ideal Fermi gas of quarks for $T \geq 2T_c$. Here we advocate a more direct approach. Creating an excitation of quantum number $K$, one can ask what else, e.g., like another quantum number $L$, does it carry. We address this by using the ratios of off-diagonal susceptibility $\chi_{KL}$ with $\chi_L$:

$$C_{(KL)/L} = \frac{\langle KL \rangle - \langle K \rangle \langle L \rangle}{\langle L^2 \rangle - \langle L \rangle^2}.$$  \hspace{1cm} (3)

### 2. Quark Number Susceptibilities

Let us first describe in this section the way one obtains various QNS from first principles using the lattice formulation of QCD. Assuming three flavours of quarks, and denoting by $\mu_f$ the corresponding chemical potentials, the QCD partition function is

$$Z = \int DU \exp(-S_G) \prod_{f=u,d,s} \text{Det} M(m_f, \mu_f).$$  \hspace{1cm} (4)

Note that the quark mass and the corresponding chemical potential enter only through the Dirac matrix $M$ for each flavour. We employ the standard Wilson form for the gluonic action $S_G$ and staggered fermions (with the usual square root method) to describe the quark matrix $M$. The formalism is, however, general and any change in either affects only the detailed expressions. The final physical results are expected to remain the same for sufficiently small lattice spacing $a$.

Defining $\mu_0 = \mu_u + \mu_d + \mu_s$ and $\mu_3 = \mu_u - \mu_d$, the baryon and isospin densities and the corresponding susceptibilities can be obtained as:

$$n_i = \frac{T}{V} \frac{\partial \ln Z}{\partial \mu_i}, \quad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_i \partial \mu_j}.$$  \hspace{1cm} (5)

Similarly, Charge ($Q$), Hypercharge ($Y$), Strangeness ($S$) susceptibilities can be defined. QNS in (5) are crucial for many quark-gluon plasma signatures which are based on fluctuations in globally conserved quantities such as baryon number or electric charge.
Theoretically, they serve as an important independent check on the methods and/or models which aim to explain the large deviations of the lattice results for pressure $P(\mu=0)$ from the corresponding perturbative expansion. Here we will be concerned with (i) extending our earlier quenched results on the Wróblewski parameter to full QCD with two dynamical quarks using our proposal \[6\] to estimate it from the QNS:

$$\lambda_s = \frac{2\chi_s}{\chi_u + \chi_d},$$  \hspace{1cm} (6)

and (ii) demonstrating the relevant degrees of freedom in QGP above $T_c$.

In order to use (6) to obtain an estimate for comparison with experiments or for (ii) above, one needs to compute the corresponding quark number susceptibilities on the lattice first and then take the continuum limit. All susceptibilities can be written as traces of products of $M^{-1}$ and various derivatives of $M$ with respect to $\mu$. With $m_u = m_d$, diagonal $\chi_{ii}$’s can be written as

$$\chi_{00} = \frac{T}{2V} [\langle O_2(m_u) + \frac{1}{2}O_{11}(m_u) \rangle]$$ \hspace{1cm} (7)

$$\chi_{33} = \frac{T}{2V} \langle O_2(m_u) \rangle$$ \hspace{1cm} (8)

$$\chi_{ss} = \frac{T}{4V} [\langle O_2(m_s) + \frac{1}{4}O_{11}(m_s) \rangle]$$ \hspace{1cm} (9)

Here $O_2 = Tr M_u^{-1}M_u' - Tr M_u^{-1}M_u'M_u'^{-1}M_u'$, and $O_{11}(m_u) = (Tr M_u^{-1}M_u')^2$, with the prime(s) denoting first(second) derivative of $M(\mu_i)$ with respect to $\mu_i$ at $\mu_i = 0$. The traces are estimated by a stochastic method: $\text{Tr} A = \frac{\sum_{i=1}^{N_v} R_i^\dagger A R_i}{2N_v}$, and $(\text{Tr} A)^2 = 2 \sum_{i>j=1}^{L} (\text{Tr} A)_i (\text{Tr} A)_{ij}/L(L-1)$, where $R_i$ is a complex vector from a set of $N_v$, subdivided further in $L$ independent sets.

Higher order susceptibilities are defined by

$$\chi_{fg\cdots} = \frac{T}{V} \frac{\partial^n \log Z}{\partial \mu_f \partial \mu_g \cdots} = \frac{\partial^n P}{\partial \mu_f \partial \mu_g \cdots}.$$ \hspace{1cm} (10)

These are Taylor coefficients of the pressure $P$ in its expansion in $\mu$. All of these can be written as traces of products of $M^{-1}$ and various higher derivatives of $M$. These too can be evaluated using the Gaussian noise technique described above, but with increasingly larger number of vectors for progressively higher orders. One can build the expansion order by order to obtain information on the critical point of QCD in the $T-\mu$ plane, as done, for example in \[7\].

Figure 1 displays our results \[5\] for the ratios $C_B/Q$ (left panel) and $C_{(QY)/Q}$ (right panel) as a function of (square of) the lattice spacing $a$ at $2T_c$, where $T_c$ is the transition temperature. The open symbols are for quenched QCD while the filled ones are for simulations with two flavours of light dynamical quarks. While the quenched results were obtained from a reanalysis of our earlier simulations \[6\], the data for the $N_f = 2$ full QCD, also denoted as partially quenched to distinguish from the real world case of two light and one heavy strange quark, were obtained \[7\] on $4 \times 16^3$ lattices. The configurations were generated with a bare sea quark mass $m = 0.1T_c$, which gives
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Figure 1. \( C_{B/Q} \) (left) and \( C_{(QY)/Q} \) (right) exhibited as a function of \( 1/N_t^{-2} \propto a^2 \) at \( T = 2T_c \), where \( a \) is the lattice spacing. Ratios of QNS are seen to be robust observables—being insensitive to both changes in lattice spacing \( a \) and the sea quark content of QCD.

\( m_\pi = 0.3m_\rho \). The setting of scale, the parameters employed and the statistics are detailed in Ref. [7]. To that set of data with \( T/T_c = 0.75 \pm 0.02, 0.8 \pm 0.02, 0.85 \pm 0.01, 0.9 \pm 0.01, 0.95 \pm 0.01, 1.00 \pm 0.01, 1.045 \pm 0.01, 1.25 \pm 0.02, 1.65 \pm 0.06 \) and \( 2.15 \pm 0.10 \), we added two more sets—55 configurations separated by more than two autocorrelation times at \( T/T_c = 0.975 \pm 0.010 \) (i.e., \( \beta = 5.2825 \)) and 86 configurations, similarly spaced, at \( T/T_c = 1.15 \pm 0.01 \) (i.e., \( \beta = 5.325 \)).

In both the quenched and the partially quenched case and for both figures, the light valence quark mass \( m_v \), appearing in (7)-(9), is \( 0.03T_c \) and the strange valence quark mass is \( T_c \). Both these ratios, and similar other ratios, display the property that \( r(a) = r + O(a^n) \) with \( n > 2 \), implying very good scaling behaviour. Furthermore, the results for the quenched and the partially quenched case on the \( N_t = 4 \) lattice differ by a few per cent only, although \( T_c \) in the two cases differ by a factor of 1.6. These two aspects of these ratios, very little dependence on the sea quark mass and the lattice spacing \( a \), prompted us to term them as robust observables. Such robust ratios can be reliably extracted on the smallest lattice already, which is very useful for the computationally expensive simulations with the dynamical quarks. One expects this robustness to persist at other temperatures above \( T_c \) as well. At lower temperatures, all susceptibilities, including the off-diagonal ones, are of similar order and such robustness may not persist. It would, nevertheless, be interesting to test it carefully.

3. Robust Predictions: Wróblewski Parameter

Figure 2 exhibits some robust predictions of fluctuation measures from QCD, obtained from our dynamical simulations with light \( u \) and \( d \) quarks. Shown are the ratios \( C_{X/Q} \) for the quantum number \( X \) indicated in the figure. The \( X = -S \) case is normalized by an extra factor of 2 to show it on the same plot. Note that the hierarchy seen in Figure 2 is itself a robust feature. Our results indicate that experimental studies of \( C_{S/Q} \) and \( C_{B/Q} \) are most promising in terms of distinguishing between the two phases of QCD,
because they exhibit the largest changes in going from the hadronic phase to QGP.

Recall that the strange quark susceptibility was obtained from the same simulations by simply choosing $m_v/T_c = 1$. Using (6), $\lambda_s(T)$ can then be easily obtained. Earlier we [6] had extrapolated $\lambda_s(T)$ in quenched QCD to $T_c$ by employing simple ansätze. This became necessary in order to avoid any effects of the order of the phase transition; quenched QCD has first order transition while the $N_f = 2$ has a lot smoother transition. While the resultant $\lambda_s(T_c)$ in quenched QCD displayed an impressive agreement [6] with the results obtained from the analysis of the RHIC and SPS data in the fireball model[4], clearly a cleaner determination with much better control of systematic errors needs its determination at $T_c$ in full QCD. Noting that $\lambda_s(T)$ is a robust observable as well, one can do so from our $N_t = 4$ simulations, at least for $T \geq T_c$.

Left panel of Figure 3 shows the Wróblewski parameter for full QCD as a function of $T/T_c$. Its value at $T_c$ is seen to be $\lambda_s \approx 0.4$, in agreement with the value of extracted from experiments, when the freeze-out temperature is close to $T_c$ [3]. It is also a pleasant fact that at lower temperatures the ratio keeps decreasing. It is rather sensitive to the quark masses at those temperatures. The data sets A, D and C in the right panel of Figure 3 display $\chi_{BY}/\Delta_{us}^2$, i.e, the susceptibility $\chi_{BY}$ divided by the square of the mass difference $\Delta_{us} = m_s - m_u$, for $m_s/T_c = 0.1, 0.75$ and 1 and $m_u/T_c = 0.03$. It is easy to show [5] that such cross susceptibilities are zero in the limit of equal quark masses, and are $\propto \Delta_{us}^2$. Strong kinematic effects are clearly visible near $T_c$. It remains to be seen whether a realistic set of quark of quark masses, corresponding to the physical pion and Kaon masses, brings $\lambda_s$ rapidly down by a factor of two or so below $T_c$.

The above mentioned nice agreement needs to be treated cautiously, however, in view of the various approximations made in equating the robust ratio determined on the lattice with the Wróblewski parameter extracted from the data. Let us list them in order of severity.
4. Flavour Carriers: Quasi-quarks?

Unlike the gluons, quarks carry flavour such as electric charge or strangeness. Flavour in quark sector can potentially assist in identification of relevant degrees of freedom in QGP just above $T_c$. One can look for correlations, by exciting one quantum number and looking for the presence of another. Choosing, baryon number and strangeness as these quantum numbers, Koch, Majumder and Randrup [9] introduced the variable

$$C_{BS} = -3C_{(BS)/S} = 1 + \frac{\chi_{us} + \chi_{ds}}{\chi_s},$$

in order to distinguish between bound state QCD -sQGP [10]– and the usual picture of the excitations in the plasma phase of QCD. This is expected to have a value of unity if strangeness is carried by (ideal) quarks, since $S = 1$ always comes linked with $B = -1/3$. In [10] it was shown that bound state QGP gives a value of $C_{BS} \approx 2/3$ (for $T > T_c$). Charge and strangeness correlation offers another similar possibility of being unity, if strangeness is carried by quarks:

$$C_{QS} = 3C_{(QS)/S} = 1 - \frac{2\chi_{us} - \chi_{ds}}{\chi_s}.$$

\begin{figure}
\centering
\includegraphics{figure3}
\caption{$\lambda_s$ as a function of temperature for QCD with two light dynamical quarks (Left panel) and various flavour symmetry breaking matrix elements (defined in the text) as a function of $T/T_c$ (Right panel).}
\end{figure}
In Figure 4, we show the first lattice QCD results [5] on $C_{BS}$ and $C_{QS}$ from our dynamical two light quark simulations. Note that while both are different from unity below $T_c$, they become close to unity immediately above $T_c$, strongly indicating that unit strangeness is carried in QGP by objects with baryon number $-1/3$ and charge $1/3$ very close to $T_c$ and beyond. The trends for $T < T_c$ as well as the order of magnitude estimated [9] in the hadron gas models are in agreement with our lattice results. We varied the strange quark mass, $m_s/T_c$, between 0.1 and 1.0 and found that for $T \geq T_c$ it does not alter the value, $\approx 1$, or the visible $T$-independence. A natural explanation of the $T$-behaviour is provided if strange excitations with nonzero baryon number become lighter at $T_c$. Furthermore, the $T$-independence suggests dominance of a single such excitation. Similar results were also obtained in [3] in the light quark sector, from e.g., $C_{(BU)/U}$ and $C_{(QU)/U}$, linking $u$-flavour carriers to $B = 1/3$ and $Q = 2/3$ objects.

A possible interpretation of the observed very small deviations from unity of $C_{BS}$ and $C_{QS}$ is as follows. Colour interactions merely dress up quarks. These effects can be computed in weak coupling limit, i.e., at high $T$, where the flavour linkages of quarks are natural. Close to $T_c$, the flavour linkages remarkably continue to persist as before, but they are no longer computable as the coupling is presumably not weak. This leads us to term them therefore as quasi-quarks.

5. Summary

Quark number susceptibilities which can be obtained from first principles using lattice QCD contribute substantially to the physics of RHIC signals. Since the ratios of quark number susceptibilities, $C_{A/B}$ were shown to depend rather weakly on the lattice spacing
and the sea quark content of QCD in the high temperature phase, they are theoretically robust variables. Consequently, one can compute them reliably relatively easily on small lattices, as we did. Being ratios, experimentally they are likely to be free of various systematic errors and thus again robust experimental observables. We presented first full (two light dynamical quarks) QCD results for the Wróblewski Parameter $\lambda_s(T)$. Near $T_c$, these are found to be in agreement with the RHIC and SPS results. Being a robust observable, small lattice cut-off effects expected in $\lambda_s$.

We demonstrated that the high temperature phase of QCD essentially consists of quasi-quarks by exploiting the flavour linkages of quarks. In particular, we showed that unit strangeness is carried by an object with baryon number $-1/3$ and charge $1/3$, as seen in Figure 4. Moreover, this correlation does not depend on the strange quark mass even when it is as large as $T_c$. Similarly, in the light quark sector one finds that $u$ and $d$ quantum numbers are not produced together, and that the $u$ flavour is carried by excitations with baryon number $+1/3$ and charge $+2/3$, whereas the $d$ flavour is carried by particles with baryon number $+1/3$ and charge $-1/3$.

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