Introduction.—Rotation of degenerate quantum gases has always been a subject of intense interest in cold atom physics [1, 2]. It plays an important role in the research of quantized vortices and superfluidity [3, 4], skyrmions and magnetism [5–8], as well as quantum Hall physics [9–12]. All of these aspects have significant overlap with the hot topics of superfluids [13], superconductors [14], magnetic materials [15, 16], and condensed matter physics [17, 18].

As a gauge system, the rotation properties of a SO-coupled Bose-Einstein condensate (BEC) can be rotated by an additional Ioffe-Pritchard (IP) magnetic field, whose coupling with spin equivalently imprints gauge angular momentum in the system. This demonstrates that a gauge system can be rotated by artificially imprinting gauge angular momentum instead of imprinting canonical angular momentum. It is also found that the gauge angular momentum is accompanied by spontaneous generation of equal and opposite canonical angular momentum in the system.

We consider two-dimensional (2D) Dresselhaus SO-coupled BECs in a IP magnetic field [41]. The Hamiltonian in the Gross-Pitaevskii mean-field approximation can be written as

\[ \mathcal{H} = \int dr \left( -\frac{\hbar^2 \nabla^2}{2M} + V + \mathcal{V}_{\text{so}} + \mu_B g_F \cdot \mathbf{B} \right) \Psi + \frac{1}{2} \int dr \sum_{i,j=\uparrow,\downarrow} g_{ij} \bar{\Psi}_{i}^{*}(r) \Psi_{j}^{*}(r) \Psi_{j}(r) \Psi_{i}(r), \]  

(1)

where \( \Psi = [\Psi_{\uparrow}(r), \Psi_{\downarrow}(r)]^{\top} \) with \( r = (x, y) \) denotes the spinor order parameter, and is normalized to satisfy \( \int dr \Psi^{\dagger} \Psi = N \). The harmonic potential for trapping the atoms is \( V = \frac{1}{2} M \omega_z^2 (x^2 + y^2) \), with \( M \) the atomic mass.
and $\omega_\perp$ the trapping frequency. The Dresselhaus SO coupling term is written as $V_{so} = -i\hbar\kappa(\sigma_x\partial_y + \sigma_y\partial_x)$ [28], where $\sigma_{x,y}$ are the components of the Pauli matrix vector $\sigma$ and $\kappa$ denotes the SO coupling strength. The IP magnetic field is often expressed as $B'$, $B'$ is the axial magnetic field in the gradient $\mathbf{B}$, and spin is related to the Bohr magneton $\mu_B$ and the Landé factor $g_F$. In real BEC experiments, the IP magnetic field has been successfully used in trapping atoms [42] and dynamically imprinting vortices [43] and spin textures [44, 45]. The Dresselhaus SO coupling may be experimentally created by Raman laser dressing [21, 40] or modulating gradient magnetic field [47, 49], and has been recently realized in $^{40}$K degenerate Fermi gases [23]. The 2D geometry can be realized by imposing a strong harmonic potential $V(z) = M\omega_\perp^2z^2/2$ along the axial direction with $\omega_\perp \gg \omega_\parallel$, in which case the effective contact-interaction strength is given by $g_{ij} = \sqrt{8\pi}(h^2/M)(a_{ij}/a_{hh})$ with $a_{ij}$ being the $s$-wave scattering length and $a_{hh} = \sqrt{\hbar/M\omega_\perp}$ the axial characteristic length [54].

Gauge rotation.—For atoms with a negative Landé $g$ factor, it is energy favored for the spin $\mathbf{S} = \Psi^\dagger\sigma\Psi$ of the condensate being parallel to the local magnetic field $\mathbf{B}$. At the same time, the gauge part of the particle current depends on the gauge potential $\mathbf{A}$ and is defined as $\mathbf{J}_g = -\frac{1}{\sqrt{\pi}}\Psi^\dagger\mathbf{A}\Psi$ [34, 51, 52]. For the Dresselhaus SO coupling, the gauge potential $\mathbf{A} = -\kappa M(\sigma_y, \sigma_z)$, and thus we have $\mathbf{J}_g = \kappa(S_y, S_z)$. This implies that the gauge particle current depends on the spin, which is dominated by the local magnetic field $\mathbf{B}$. In the case of IP magnetic field, a circulating gauge particle current will be induced as shown in Fig. 4.

According to the hydrodynamic theory, the contribution of the mechanical movement to the Hamiltonian can be written as $\mathcal{H}^{\text{mech}} = \int M(\mathbf{J}_c + \mathbf{J}_g)^2/2\rho d\mathbf{r}$ [51], where $\mathbf{J}_c = \rho\mathbf{v}$ represents the canonical particle current with $\mathbf{v} = (\rho_\uparrow\nabla\theta_\uparrow + \rho_\downarrow\nabla\theta_\downarrow)/\rho$ being the superflow velocity. Here, $\rho_\uparrow, \rho_\downarrow$ and $\theta_\uparrow, \theta_\downarrow$ denote the density and phase of each component, respectively, and $\rho$ denotes the total density. The energy minimization in the incompressible limit requires that the total particle current $\mathbf{J}^{\text{mech}} = \mathbf{J}_c + \mathbf{J}_g = 0$ [51]. This suggests that the emergence of circulating gauge particle current is accompanied by an equivalent canonical particle current in its opposite direction, as shown in Fig. 1. While the gauge particle current $\mathbf{J}_g$ depends on the spin, the canonical part $\mathbf{J}_c$ is related to the phase gradient, and thus may be sustained by the generation of quantized vortices, which will be discussed latter.

The mechanism of the gauge rotation can be understood by noting that the magnetic-field-spin-coupling term $\mathcal{H}^{\text{M-S}} = \mu_B \mathbf{g_F} \int d\mathbf{r} \Psi^\dagger(\mathbf{r} \times \mathbf{A})\Psi$ in the Hamiltonian equivalently imprints gauge angular momentum $L_z^e = -\int \Psi^\dagger(\mathbf{r} \times \mathbf{A})\Psi d\mathbf{r}$ in the system with Dresselhaus SO coupling. This causes gauge rotation of the condensates, and also induces equal and opposite canonical angular momentum in the incompressible limit. This is completely different from the traditional manner of rotation of a condensate by directly imprinting canonical angular momentum $L_z^e = \int \Psi^\dagger(\mathbf{r} \times \mathbf{p})\Psi d\mathbf{r}$ in the system [34, 41, 47].

Many-body ground states.—We next investigate the many-body ground states of SO-coupled BECs under gauge rotation, which can be calculated by numerically minimizing the Hamiltonian functional given by Eq. (1).
In the weakly interacting regime, it is found that the condensates are divided into several symmetrically placed domains, with radial vortex arrays playing the role of domain walls. The domain number increases with increasing the magnetic field gradient $B'$, as shown in Figs. 2(a)-2(d). The vortices in each component have winding number $n_w = 1$, except that in the center hole, which has winding number $n_w > 1$ and forms a multi-quantum vortex. This structure is similar to those obtained in systems under canonical rotation [37,38], but where the number of domains is dominated by the rotating angular frequency.

As the magnetic field gradient increases, all the vortices gather in the center hole, forming a giant vortex structure, as shown in Figs. 2(e)-2(f), which has been previously observed in SO systems with external rotation [37,38] or a toroidal trap [5,34]. Owing to the presence of SO coupling and IP magnetic field, the winding numbers of the giant vortices of the spin-up and -down components always differ by 1. This can be explained by representing the wave functions as density and phase $\Psi_j = \sqrt{\rho_j}e^{i\theta_j}$ in the polar coordinate $(r, \varphi)$ representation. The Hamiltonian related to the relative phase involves the SO coupling and IP magnetic field terms, and can be rewritten as

$$\mathcal{H}^{\text{IP}} = -2\kappa \int \sqrt{\rho \rho_a}{\partial \over \partial r} \left[\sqrt{\rho \rho_a} {\partial \theta \over \partial r} - \sqrt{\rho \rho_a} \frac{\partial \theta}{\partial \varphi} \right] \cos(\theta_\uparrow - \theta_\downarrow - \varphi) + 2B'\mu_B g_F \int r \sqrt{\rho \rho_a} r \cos(\theta_\uparrow - \theta_\downarrow - \varphi). \tag{2}$$

In order to satisfying energy minimization, it is required that

$$\theta_\uparrow - \theta_\downarrow - \varphi = 2\pi l, (l \in \mathbb{Z}) \tag{3}$$

with $\partial \theta_\uparrow / \partial \varphi < 0$. Thus, the giant vortex can be represented as $\Psi = [\sqrt{\rho}e^{-im\varphi}, \sqrt{\rho}e^{-i(m+1)\varphi}]^\top$ with $m \in \mathbb{Z}^+$. The IP magnetic field polarizes the spin in the $x$-$y$ plane with $S_z = \rho_\uparrow - \rho_\downarrow = 0$, so the circulation of the superfluid velocity along a closed path is $\oint \mathbf{v} \cdot d\mathbf{l} = -(m + \frac{1}{2})\mathbf{B}'$, implying that this state is essentially a half-integer giant vortex. According to the minimum of the real-space potential energy $E(r) = \frac{1}{2}M\omega_\perp^2 r^2 \pm \mu_B g_F B' r$ caused by the harmonic and IP magnetic traps, one can estimate the radius of the giant vortex as $r_0 = \frac{\mu_B g_F B'}{Ma^2 \omega_\perp}$. As a result, the corresponding gauge angular momentum can be expressed as $L^g_z = \kappa M \int rdr \approx \frac{N \mu_B g_F B'}{\omega_\perp^2}$, which induces equivalent canonical angular momentum $L^c_z = N(m + \frac{1}{2})\hbar$ with vortex winding number $m = \frac{\mu_B g_F B'}{Ma^2 \omega_\perp} - \frac{1}{2}$.

Numerical results of the gauge and canonical angular momenta as functions of the magnetic field gradient $B'$ are shown in Fig. 3. It is found that both the gauge and canonical momenta are asymptotically proportional to the SO coupling strength $\kappa$ and the magnetic field gradient $B'$, which is consistent with the above analytical analysis. In addition, we also observe slight jumps of the angular momentum (indicated by arrows in Fig. 3) where the domain number changes [35].

In the parameter region with relatively strong interatomic interactions, the ground-state vortex arrangement
FIG. 5: (Color online) Gauge angular momentum $\kappa$ and canonical angular momentum $L_z^c$ per particle as well as layer number as functions of the spin-orbit coupling strength $\kappa$. Green solid line plots the proportional function of $\kappa$ with slope $B' \mu_{\text{gr}} M_0^\text{h}/h^2$. Other parameters are fixed at $B' = 6 h^2/\mu_{\text{gr}} M_0^\text{h}$, $N_{\uparrow\uparrow} = N_{\downarrow\downarrow} = 5000 h^2/M$, and $N_{g\uparrow} = 4000 h^2/M$.

is very peculiar. It is found that the vortices prefer to arrange themselves as coaxially arranged annular arrays as shown in Fig. 4. The layer number of the annular vortex arrays increases with the SO coupling strength $\kappa$. This is not only different from the Abrikosov triangular lattice of superconductors $[39]$, but also from the square or hexagonal lattice discovered in traditional multi-component superfluids $[6, 57, 58]$. It should also be emphasized that in the coaxially annular arrays all the vortices take the same direction of circulation which different from those spontaneous vortex lattices in an irrotational SO-coupled system, where vortices and antivortices emerge in pairs $[59–64]$. Accompanying the vortex nucleation, equal and opposite canonical angular momenta are generated where vortices and antivortices emerge in pairs $[59–64]$. These, in fact, can also be explained from the point of gauge rotation by noting that the gradient magnetic field or spatially varying laser detuning $[33, 63, 66]$. Conclusions.—

In summary, we have developed a new method to rotate systems with spin-orbit coupling. It is suggested that a gauge system can be rotated by imprinting gauge angular momentum $L_z^g = -(\mathbf{r} \times \mathbf{A})_z$ instead of imprinting canonical angular momentum $L_z^c = \mathbf{r} \times \mathbf{p}_z$. In particular, we investigate the gauge rotation of Dresselhaus spin-orbit-coupled Bose-Einstein condensates, where the gauge angular momentum is imprinted by an additional Ioffe-Pritchard magnetic field. The many-body ground states are discussed, and it is found that equal and opposite canonical angular momentum is induced in the incompressible limit and accompanied by an exotic vortex arrangement that is different from the usual Abrikosov vortex lattice. The developed method circumvents the difficulties and challenges $[33, 34]$ faced by the traditional ways of rotation for a spin-orbit-coupled system, and brings new perspectives on the physics of ultra-cold atomic gases under gauge potentials.

Finally, the present investigation may be generalized to systems with general gauge potentials (including, but not limited to, spin-orbit coupling), where a possible scheme of gauge rotation may be realized by adding a term with $H^\text{add} = -\int \mathbf{Ψ}^\dagger (\mathbf{r} \times \mathbf{A}) \mathbf{Ψ} d\mathbf{r}$ in the Hamiltonian to imprinting non-zero gauge angular momentum. In addition, we note the vortex nucleation of condensates with other types of spin-orbit coupling induced by gradient magnetic field or spatially varying laser detuning $[33, 63, 66]$. This work was supported by the National Natural Science Foundation of China under Grants No. 11772177, No. 11704383, and No. 11547194; the West Light Foundation of the Chinese Academy of Sciences under Grant No. XAB2016B73; the Applied Basic Research Foundation of Shanxi Province under Grand No. 201701D121011.

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