Generation of longitudinal electric current by the transversal electromagnetic field in classical and quantum degenerate plasma

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Abstract

The analysis of nonlinear interaction of transversal electromagnetic field with degenerate collisionless classical and quantum plasmas is carried out. Formulas for calculation electric current in degenerate collisionless classical and quantum plasmas are deduced. It has appeared, that the nonlinearity account leads to occurrence of longitudinal electric current directed along a wave vector. This second current is orthogonal to the known transversal current, received at the classical linear analysis. Graphic comparison of density of electric current for classical degenerate Fermi plasmas and Fermi–Dirac plasmas (plasmas with any degree of degeneration of electronic gas) is carried out. Graphic comparison of density of electric current for classical and quantum degenerate plasmas is carried out. Also comparison of dependence of density of electric current of quantum degenerate plasmas from dimensionless wave number at various values of dimensionless frequency of oscillations of electromagnetic field is carried out.

Key words: collisionless plasmas, Vlasov equation, degenerate plasma, Wigner integral, quantum distribution function, longitudinal electrical current.

PACS numbers: 52.25.Dg Plasma kinetic equations, 52.25.-b Plasma properties, 05.30 Fk Fermion systems and electron gas

Introduction

In the present work formulas for calculation electric current in classical and quantum collisionless degenerate plasmas are deduced.

At the solution of the kinetic Vlasov equation describing behaviour of classical degenerate plasmas, we consider as in decomposition distribution functions, and in decomposition of quantity of the self-conjugate electromagnetic field the quantities proportional to square of intensity of an external electric field.

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At the solution of the kinetic equation with Wigner integral, describing behavior of quantum degenerate plasmas, we consider as in decomposition distribution functions, and in decomposition of Wigner integral the quantities proportional to square of potential of an external electromagnetic field.

At such nonlinear approximation it appears, that an electric current has two nonzero components. One component of electric current is directed along intensity of electric field (in classical plasma) or along potential of electromagnetic field (in quantum plasma). This component of electric field precisely the same, as well as in the linear analysis. It is "transversal" current. Thus, in linear approximation we receive the known expression of the transversal electric current.

The second nonzero of electric current component has the second order of infinitesimal concerning with quantity intensity of electric field (in case of classical plasma) or the second order of infinitesimal of quantity concerning with potential quantity of electromagnetic field (in case of quantum plasma). The second component of electric current is directed along the wave vector both in classical and in quantum plasma. This current is perpendicular to the first component. It is "longitudinal" current.

So, vector expression of an electric current consists from two composed, orthogonal each to other. The first composed, linear on intensity of electric field, is known classical expression of an electric current along the electric fields. The second composed represents an electric current, which is proportional to an intensity square of electric field. The second current is perpendicular to the first and it is directed along the wave vector. Occurrence of the second current comes to light the spent the nonlinear analysis of interaction of electromagnetic field with classical and quantum plasma.

Nonlinear effects in plasma are studied already long time [1] – [6].

In works [1] and [3] nonlinear effects are studied in plasma. In work [3] the nonlinear current was used, in particulars, in probability of questions of disintegration processes. We will note, that in work [2] it is underlined existence nonlinear current along a wave vector (see the formula (2.9) from [2]).

Quantum plasma was studied in works [8] – [16]. Collisional quantum plasma has started to be studied in work of Mermin [9]. Then quantum
collisional plasma it was studied in our works [10] – [14].

The collisional quantum plasma with variable frequency of collisions in our works [12] and [13] was studied. In works [15] and [16] was investigated generation of longitudinal current by the transversal electric field in classical and quantum Fermi–Dirac plasma [15] and in Maxwellian plasma [16].

1. The case of classical plasma

Let us show, that in case of the classical plasma described by the Vlasov equation, longitudinal current is generated and we will calculate its density. On existence of this current was specified more half a century ago [2]. We take Vlasov equation describing behaviour of collisionless plasmas

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + e \left( \mathbf{E} + \frac{1}{c} \mathbf{[v, H]} \right) \frac{\partial f}{\partial \mathbf{p}} = 0. \tag{1.1}$$

Electric and magnetic fields are connected with the vector potential by equalities

$$\mathbf{E} = -\frac{1}{c} \frac{\partial A}{\partial t}, \quad \mathbf{H} = \text{rot} A.$$

Let us consider, that the intensity of electric and magnetic fields vary harmoniously

$$\mathbf{E} = E_0 e^{i(kr - \omega t)}, \quad \mathbf{H} = H_0 e^{i(kr - \omega t)}.$$

The wave vector we will direct along an axis $x$: $\mathbf{k} = k(1, 0, 0)$, and intensity of electric field we will direct along an axis $y$: $\mathbf{E} = E_y(0, 1, 0)$.

Hence,

$$\mathbf{E} = -\frac{1}{c} \frac{\partial A_y}{\partial t} = \frac{i \omega}{c} A_y, \quad A_y = -\frac{ic}{\omega} E_y,$$

$$\mathbf{H} = \frac{ck}{\omega} E_y \cdot (0, 0, 1), \quad \mathbf{[v, H]} = \frac{ck}{\omega} E_y \cdot (v_y, -v_x, 0),$$

$$e \left( \mathbf{E} + \frac{1}{c} \mathbf{[v, H]} \right) \frac{\partial f}{\partial \mathbf{p}} = e \frac{1}{\omega} E_y \left[ k v_y \frac{\partial f}{\partial p_x} + (\omega - kv_x) \frac{\partial f}{\partial p_y} \right].$$

Let us operate with the method of consecutive approximations, considering as small parameter the quantity of intensity of electric field. Let us rewrite the equation (1.1) in the form

$$\frac{\partial f^{(k)}}{\partial t} + v_x \frac{\partial f^{(k)}}{\partial x} =$$
\[
= -\frac{eE_y}{\omega} \left[ kv_y \frac{\partial f^{(k-1)}}{\partial p_x} + (\omega - kv_x) \frac{\partial f^{(k-1)}}{\partial p_y} \right], \quad k = 1, 2. \tag{1.2}
\]

Here \( f^{(0)} \) is the absolute Fermi distribution,
\[
f^{(0)} = f_0 = \Theta(\mathcal{E}_0 - \mathcal{E}), \quad \mathcal{E} = \frac{mv^2}{2}, \quad \mathcal{E}_0 = \frac{mv_0^2}{2},
\]
\( \Theta(x) \) is the unit step of Heaviside, \( \mathcal{E} \) is the electron energy, \( \mathcal{E}_0 \) is the electron energy on Fermi surface, \( p_0 = mv_0 \) is the electron momentum on Fermi surface, \( v_0 \) is the electron velocity on Fermi surface.

We notice that
\[
[v, H] \frac{\partial f_0}{\partial p} = 0,
\]
because
\[
\frac{\partial f_0}{\partial p} \sim v.
\]

We search for the solution as first approximation in the form
\[
f^{(1)} = f_0 + f_1,
\]
where
\[
f_1 \sim E_y, \quad E_y \sim e^{i(kx - \omega t)}.
\]

In this approximation the equation (1.2) becomes simpler
\[
\frac{\partial f_1}{\partial t} + v_x \frac{\partial f_1}{\partial x} = -eE_y \frac{\partial f_0}{\partial p_y}.
\]

From here we receive
\[
-i(\omega - kv_x)f_1 = -\frac{eE_y}{m} \cdot \frac{\partial f_0}{\partial v_y}, \tag{1.3}
\]

From (1.3) we obtain
\[
f_1 = -\frac{eE_y}{m} \cdot \frac{\partial f_0}{\omega - kv_x}. \tag{1.4}
\]

Here
\[
\frac{\partial f_0}{\partial v_y} = -\delta(\mathcal{E}_0 - \mathcal{E})mv_y.
\]
In the second approximation for the solution of the equation (1.2) we search in the form
\[ f^{(2)} = f^{(1)} + f_2 = f_0 + f_1 + f_2, \]
where
\[ f_2 \sim E_y^2, \quad E_y^2 \sim e^{2i(kx-\omega t)}. \]
Let us substitute \( f^{(2)} \) in the equation (1.2). Considering the equation (1.3), we come to the equation
\[
\frac{\partial f_2}{\partial t} + v_x \frac{\partial f_2}{\partial x} = -\frac{eE_y}{\omega m} \left[ kv_y \frac{\partial f_1}{\partial v_x} + (\omega - kv_x) \frac{\partial f_1}{\partial v_y} \right].
\]
From this equation we obtain
\[
f_2 = -\frac{ieE_y}{2m\omega(\omega - kv_x)} \left[ kv_y \frac{\partial f_1}{\partial v_x} + (\omega - kv_x) \frac{\partial f_1}{\partial v_y} \right] =
-\frac{e^2E_y^2}{2m^2\omega(\omega - kv_x)} \left[ kv_y \frac{\partial}{\partial v_x} \left( \frac{\partial f_0/\partial v_y}{\omega - kv_x} \right) + \frac{\partial^2 f_0}{\partial v_y^2} \right]. \tag{1.5}
\]
The distribution function in second approximation is constructed
\[ f = f^{(2)} = f^{(0)} + f_1 + f_2, \tag{1.6} \]
where \( f_1, f_2 \) are given by equalities (1.4) and (1.5).

Let us find electric current density
\[
j = j_x \hat{i} + j_y \hat{j} + j_z \hat{k}
\]
\[
j = e \int v f \frac{2d^3p}{(2\pi\hbar)^3} = e \int v (f_1 + f_2) \frac{2d^3p}{(2\pi\hbar)^3}. \tag{1.7}
\]
From equalities (1.4) – (1.6) it is visible, that the vector of current density has two nonzero components
\[ j = (j_x, j_y, 0). \]
Here \( j_y \) is the density of transversal current,
\[
j_y = e \int v_y f \frac{2d^3p}{(2\pi\hbar)^3} = e \int v_y f_1 \frac{2d^3p}{(2\pi\hbar)^3}. \tag{1.8}
\]
This current is directed along an electric field, its density is defined only by the first approximation of distribution function. The second approximation of distribution function the contribution to current density does not bring.

The density of transversal current is defined by equality

$$j_y = \frac{2em^3}{(2\pi\hbar)^3} \int v_y f_1 d^3v.$$ 

For density of longitudinal current according to its definition it is had

$$j_x = e \int v_x f \frac{2d^3p}{(2\pi\hbar)^3} = e \int v_x f_2 \frac{2d^3p}{(2\pi\hbar)^3} = \frac{2em^3}{(2\pi\hbar)^3} \int v_x f_2 d^3v.$$ 

By means of (1.6) and (1.5) from here it is received, that

$$j_x = -\frac{e^2E_y^2 m}{(2\pi\hbar)^3 \omega} \int \left[ kv_y \frac{\partial}{\partial v_x} \left( \frac{\partial f_0}{\partial v_y} \omega - kv_x \right) + \frac{\partial^2 f_0}{\partial v_x^2} \right] \frac{v_x d^3v}{\omega - kv_x}. \quad (1.9)$$

In the second integral from (1.9) internal integral on $P_y$ is equal to zero

$$\int_{-\infty}^{\infty} \frac{\partial^2 f_0}{\partial v_y^2} dv_y = 0.$$ 

In the first integral from (1.9) internal integral on $P_x$ is calculated in parts

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial v_x} \left( \frac{\partial f_0}{\partial v_y} \omega - kv_x \right) \frac{v_x d^3v}{\omega - kv_x} = -\omega \int_{-\infty}^{\infty} \frac{(\partial f_0/\partial v_y) dv_x}{(\omega - kv_x)^3}.$$ 

Hence, equality (1.9) becomes simpler

$$j_x = \frac{e^3 E_y^2 mk}{(2\pi\hbar)^3} \int \frac{(\partial f_0/\partial v_y) v_y d^3v}{(\omega - kv_x)^3}. \quad (1.10)$$

We notice that

$$\int_{-\infty}^{\infty} v_y \frac{\partial f_0}{\partial v_y} dv_y = v_0 \Theta(v_0 - v) \bigg|_{v_y=+\infty}^{v_y=-\infty} - \int_{-\infty}^{\infty} \Theta(v_0 - v) dv_y.$$ 

Equality (1.10) is reduced now to integral

$$j_x = -\frac{e^3 E_y^2 mk}{(2\pi\hbar)^3} \int \frac{\Theta(v_0 - v) d^3v}{(\omega - kv_x)^3}. \quad (1.11)$$
The three-dimensional integral is equal

\[
\int \frac{\Theta(v_0 - v) d^3v}{(\omega - kv_x)^3} = \int \frac{d^3v}{(\omega - kv_x)^3} =
\]

\[
= \int_{-v_0}^{v_0} \frac{dv_x}{(\omega - kv_x)^3} \int \int d^2v_y = \pi \int_{-v_0}^{v_0} \frac{(v_0 - v_x^2) dv_x}{(\omega - kv_x)^3} =
\]

\[
= \frac{\pi}{k_0^3} \int_{-1}^{1} \frac{(1 - \tau^2) d\tau}{(\Omega - q\tau)^3}.
\]

Hence, the longitudinal current is equal

\[
j_x = -\frac{e^3 E_y^2 m k \pi}{(2\pi \hbar)^3 k_0^3} \int_{-1}^{1} \frac{(1 - \tau^2) d\tau}{(\Omega - q\tau)^3}. \tag{1.12}
\]

Here \( q \) is the dimensionless wave number, \( \Omega \) is the dimensionless frequency of oscillations of electromagnetic field,

\[
q = \frac{k}{k_0}, \quad \Omega = \frac{\omega}{k_0 v_0}.
\]

Let us find numerical density (concentration) of particles of the plasma, corresponding to Fermi—Dirac distribution

\[
N = \int \Theta(v_0 - v) \frac{2d^3p}{(2\pi \hbar)^3} = \frac{2m^3}{(2\pi \hbar)^3} \int \Theta(v_0 - v) d^3v = \frac{k_0^3}{3\pi^2},
\]

where \( k_0 \) is the Fermi wave number, \( k_0 = \frac{mv_0}{\hbar} \).

In expression before integral from (1.12) we will allocate the plasma (Langmuir) frequency

\[
\omega_p = \sqrt{\frac{4\pi e^2 N}{m}}
\]

and number density (concentration) \( N \), and last we will express through Fermi wave number. We will receive

\[
j_{x,\text{long}} = -E_y^2 \frac{3e\Omega_p^2}{k_0 p_0} \frac{k}{32\pi} \int_{-1}^{1} \frac{(1 - \tau^2) d\tau}{(\Omega - q\tau)^3},
\]
where \( \Omega_p = \frac{\omega_p}{k_0v_0} = \frac{\hbar \omega_p}{mv_0^2} \) is the dimensionless plasma frequency.

The previous equality we will copy in the form
\[
j_x^{\text{long}} = J_c(\Omega, q)\sigma_{1,\text{tr}} k E_y^2, \tag{1.13}
\]

where \( \sigma_{1,\text{tr}} \) is the longitudinal–transversal conductivity, \( J_c(\Omega, q) \) is the dimensionless part of current,
\[
\sigma_{1,\text{tr}} = \frac{e\hbar}{p_0^2} \left( \frac{\hbar \omega_p}{mv_0^2} \right)^2 = \frac{e}{p_0 k_0} \Omega_p^2,
\]
\[
J_c(\Omega, q) = -\frac{3}{32\pi} \int_{-1}^{1} \frac{(1 - \tau^2) d\tau}{(\Omega - q\tau)^3}.
\]

The integral from dimensionless part of current is calculated according to known Landau rule that is equivalent to equality
\[
J_c(\Omega, q) = -\frac{3}{32\pi} \lim_{\varepsilon \to +0} \int_{-1}^{1} \frac{(1 - \tau^2) d\tau}{(\Omega - i\varepsilon - q\tau)^3}. \tag{1.14}
\]

If we introduce the transversal field
\[
E_{\text{tr}} = E - \frac{k(Ek)}{k^2} = E - \frac{q(Eq)}{q^2},
\]
then equality (1.13) can be written down in invariant form
\[
j_{x}^{\text{long}} = J_c(\Omega, q)\sigma_{1,\text{tr}} k E_y^2 = J_c(\Omega, q)\sigma_{1,\text{tr}} \frac{\omega}{c} [E, H].
\]

Let us pass to consideration of the case of small values of wave number. From expression (1.12) at small values of wave number it is received
\[
j_{x}^{\text{classic}} = -\frac{4e^3}{24\pi^2\hbar^3 k_0^3 \Omega^5} \frac{E_y^2 m k}{E_y^2 k_0 p_0} = -\frac{e \omega_p^2 E_y^2 k}{8\pi(k_0 p_0)^2 k_0 p_0 \Omega^3} = -\frac{1}{8\pi \Omega^2} \cdot \sigma_{1,\text{tr}} k E_y^2. \tag{1.15}
\]

Let us notice, that in the case of degenerate plasmas and real and imaginary parts of longitudinal current beyond all bounds increase at \( q \to \Omega \). This singularity indicate the sharp vanishing of distribution function of degenerate plasmas at once behind Fermi’s surface.
At regimes of non-degenerate Fermi—Dirac plasmas, close to degenerate regime (the quantity $\alpha$ is large, but it is finite), i.e. at the temperatures close to zero, but not equal to zero, singularity in the point $q = \Omega$ is absent.

From Figs. 1 and 2 it is visible, that at increase of dimensionless chemical potential $\alpha$ velocity of growth of the real part of the longitudinal current increases also.

In the limit at $\alpha \to +\infty$ ($T \to +0$) the quantities $\text{Re} \ J_c(\Omega, q)$ and $\text{Re} \ J_c(\Omega, q)$ becomes explosive in the point $q = \Omega$. It also is the case of degenerate plasmas.

For graphics of the real and imaginary parts of the longitudinal current in Fermi—Dirac plasma on Figs. 1 – 4 the formula of longitudinal current from our work [15] is used. In this formula transition to dimensionless quantities by electron velocity $v_0$ on Fermi’s surface is spent. This formula looks like

$$j_{\text{quant}}^x = J_c(\Omega, q) \sigma_{1,\text{tr}} k E_y^2,$$

where

$$\sigma_{1,\text{tr}} = \frac{e \Omega_p^2}{p_0 v_0}, \quad \Omega_p = \frac{\omega_p}{k_0 v_0},$$

$$l_0(\alpha) = \int_0^\infty \ln(1 + e^{\alpha - \tau^2}) d\tau,$$

$$J_c(\Omega, q) = \frac{1}{16\pi l_0(\alpha)} \int_{-\infty}^{\infty} \ln(1 + e^{\alpha(1 - \tau^2)}) d\tau \frac{(q \tau - \Omega)^3}{(q \tau - \Omega)^3}.$$

Imaginary part of the dimensionless current in Fermi—Dirac plasma is calculated according to the previous equalities and Landau rules under the formula

$$\text{Im} \ J_c(\Omega, q) = -\frac{1}{32l_0(\alpha)q^3} [\ln(1 + e^{\alpha(1 - \tau^2)})]^{\tau = \Omega/q} =$$

$$= \frac{\alpha}{16l_0(\alpha)q^3} \cdot \frac{1 + (1 - 2\alpha\tau^2)e^{-\alpha(1 - \tau^2)}}{[1 + e^{-\alpha(1 - \tau^2)}]^2} \bigg|_{\tau = \Omega/q}.$$

With imaginary part of the dimensionless current in degenerate plasma things are more difficult. According to Lahdau rule imaginary part is calculated
under the formula

$$\text{Im } J_c(\Omega, q) = \frac{3}{32q^3} \begin{cases} 
0, & q < \Omega, \\
1, & q > \Omega 
\end{cases}. $$

On the other hand, if to integrate twice in parts integral from (1.14), we receive

$$\int_{-1}^{1} \frac{(1 - \tau^2)d\tau}{(q\tau - \Omega + i\varepsilon)^3} = \frac{2(\Omega - i\varepsilon)}{q^2[(q^2 - (\Omega - i\varepsilon)^2)] - \frac{1}{q^2} \left| \frac{q - \Omega}{q + \Omega} \right| - \frac{i}{q^3} \left[ \arctg \left( \frac{q - \Omega}{\varepsilon} \right) + \arctg \left( \frac{q + \Omega}{\varepsilon} \right) \right]. $$

Let us consider the quantity

$$A = \frac{2(\Omega - i\varepsilon)}{q^2[(q^2 - (\Omega - i\varepsilon)^2)] \sim \frac{2(\Omega - i\varepsilon)}{q^2[(q^2 - \Omega^2 + 2i\Omega\varepsilon]}. $$

If the parameter \( q \) runs to \( \Omega \), we receive

$$\text{Im } A = -\frac{1}{\Omega^2\varepsilon}. $$

If now the parameter \( \varepsilon \) runs to zero, we find, that

$$\text{Im } A \to \infty. $$

Let us consider that \( q \neq \Omega \), and then if the parameter \( \varepsilon \to 0 \) it is received, that

$$\text{Im } A \to 0. $$

It means, that limits in (1.14) on \( q \to 0 \) and \( \varepsilon \to 0 \) are non-commutativity.

As it was already specified, it mean that the sharp vanishing of distribution function of degenerate plasma at once behind Fermi’s surface. To eliminate this lack of the description of degenerate plasma is possible or "diffusion" Fermi–surface, or consideration such regimes, close to degenerate regime (at \( \alpha \to +\infty \)), or introduction in consideration of electron collisions and calculation so named "weakly collision" limit at \( q \to 0 \).

On Figs. 3 and 4 are presented graphics in which it is found out dependence of real (Fig. 3) and imaginary (Fig. 4) parts longitudinal current from quantity
of dimensionless wave number $q$ at various values of quantity of dimensionless frequency of oscillations of electromagnetic field $\Omega$.

It appears, that at increase $\Omega$ amplitude excursion of values of the real and imaginary parts of longitudinal current sharply decreases. A minimum of imaginary part $q_{min}$ is near to the point $q = \Omega$. With increase of quantity of chemical potential $\alpha$ this minimum moves in point $q = \Omega$: $q_{min} \to \Omega$.

**Fig. 1.** Real part of density of dimensionless current in classical Fermi–Dirac plasma (curves 1,2,3) and degenerate Fermi plasma (curve 4), $\Omega = 0.5$; curves 1, 2, 3 correspond to values of dimensionless chemical potential $\alpha = 1, 5, 9$. 
Fig. 2. Imaginary part of density of dimensionless current in classical Fermi–Dirac plasma (curves 1, 2, 3) and degenerate Fermi plasma (curve 4), $\Omega = 0.5$; curves 1, 2, 3 correspond to values of dimensionless chemical potential $\alpha = 1, 5, 9$. 
Fig. 3. Real part of density of dimensionless current in classical Fermi–Dirac plasma (curves 1,2,3) and degenerate Fermi plasma (curve 4) at $\Omega = 0.2$, $\alpha = 5$; curves 1, 2, 3 correspond to values of dimensionless oscillation frequency $\Omega = 0.2, 0.4, 0.6$. 
Fig. 4. Imaginary part of density of dimensionless current in classical Fermi–Dirac plasma (curves 1,2,3) and degenerate Fermi plasma (curve 4) at $\Omega = 0.2, \alpha = 5 \Omega = 0.5$; curves 1, 2, 3 correspond to values of dimensionless oscillation frequency $\Omega = 0.2, 0.4, 0.6$. 
2. Degenerate quantum plasma

In our previous work [15] the following expression for the longitudinal current has been received

\[
j_{\text{quadr}} = \frac{2e^3}{(2\pi\hbar)^3} \int \left[ \frac{e^2v_0^3\mathbf{P} (\mathbf{PA})^2}{2c^2\hbar^2(\omega - v_T\mathbf{k}\mathbf{P})} \left( f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P}) \right) + \right.
\]

\[
+ \frac{f_0(\mathbf{P} - \mathbf{q}) - f_0(\mathbf{P})}{\omega - v_T\mathbf{k}\mathbf{P}} - \frac{e^2v_0A\mathbf{P}^2}{4mc^2\hbar} \left( f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q}) \right) \left( \frac{\omega - v_0\mathbf{k}\mathbf{P}}{\mathbf{P} + \mathbf{q}/2} - \frac{\Omega}{q} \right) \right] d^3P. \tag{2.1}
\]

Let us consider the case of degenerate plasmas.

In the formula (2.1) we will carry out replacement of variables of integration

\[
\mathbf{P} \rightarrow \frac{v_0}{v_T} \mathbf{P},
\]

where \( v_0 \) is the electron velocity on Fermi–surface.

We receive the following expression for quantity of electric current in quantum plasma

\[
j_{\text{quadr}} = \frac{2e^3}{(2\pi\hbar)^3} \int \left[ \frac{e^2v_0^3\mathbf{P} (\mathbf{PA})^2}{2c^2\hbar^2(\omega - v_0\mathbf{k}\mathbf{P})} \left( f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P}) \right) - \right.
\]

\[
- \frac{f_0(\mathbf{P} - \mathbf{q}) - f_0(\mathbf{P})}{\omega - v_0\mathbf{k}\mathbf{P}} - \frac{e^2v_0A\mathbf{P}^2}{4mc^2\hbar} \left( f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q}) \right) \left( \frac{\omega - v_0\mathbf{k}\mathbf{P}}{\mathbf{P} + \mathbf{q}/2} - \frac{\Omega}{q} \right) \right] d^3P. \tag{2.2}
\]

As well as earlier, vector equality (2.2) contains one nonzero component

\[
j_x^{\text{quant}} = \frac{e^3}{(2\pi\hbar)^3c^2m^2v_0q^2} \int \left[ \left( f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P}) \right) - \right.
\]

\[
- \frac{f_0(\mathbf{P} - \mathbf{q}) - f_0(\mathbf{P})}{\mathbf{P} + \mathbf{q}/2 - \Omega/q} \right] \left( \frac{P_xP_y^2}{P_x - \Omega/q} \right)^2 P_x P_y \left( \frac{q}{2} f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q}) \right) \left( \frac{\omega - v_0\mathbf{k}\mathbf{P}}{\mathbf{P} + \mathbf{q}/2} - \frac{\Omega}{q} \right) \right] d^3P. \tag{2.3}
\]

Here

\[
f_0(\mathbf{P} \pm \mathbf{q}) = \left[ 1 + \exp \left( \left( P_x \pm q \right)^2 + P_y^2 + P_z^2 - \alpha \right) \right]^{-1}.
\]
In formula (2.2) \( p_0 = mv_0 \) is the electron momentum on Fermi–surface,

\[
f_0 = \frac{1}{1 + \exp \left( \frac{E_0 P^2 - \mu}{k_B T} \right)} = \frac{1}{1 + \exp \left( \frac{E_0 P^2 - \mu}{E_T} \right)},
\]

where \( E_0 \) is the electron energy on Fermi–surface, \( k_0 \) is the wave Fermi number, \( E_T = \frac{mv^2}{2} \),

\[
q = \frac{k}{k_0}, \quad \Omega = \frac{\omega}{k_0 v_0}, \quad k_0 = \frac{mv_0}{\hbar}.
\]

Let us notice, that in the limit of zero absolute temperature it is had

\[
\lim_{T \to 0} \mu = E_0, \quad E_0 = \frac{mv^2}{2},
\]

Hence, in the limit of zero temperature absolute Fermi–Dirac distribution function passes in the absolute Fermi’s distribution function for degenerate plasmas

\[
\lim_{T \to 0} f_0 = \lim_{T \to 0} \frac{1}{1 + \exp \left( \frac{E_0 P^2 - \mu}{E_T} \right)} = \Theta \left( \frac{E_0}{E_T} (1 - P^2) \right) = \Theta (1 - P^2).
\]

Thus, in the limit of zero temperature the formula (2.3) will be transformed to the form

\[
j_{x}^{\text{quant}} = \frac{e^3 p_0^3 A_y^2}{(2\pi\hbar)^3 c^2 m^2 v_0 q^2} \int \left[ \left( \frac{\Theta[1 - (P_x + q)^2 - P_y^2 - P_z^2]}{P_x + q/2 - \Omega/q} \right. \right.
\]

\[
- \left. \left. \frac{\Theta[1 - P^2]}{P_x - q/2 - \Omega/q} \right) \right] \frac{P_x P_y^2}{P_x - \Omega/q} 
\]

\[
+ \frac{q}{2} \frac{\Theta[1 - (P_x + q)^2 - P_y^2 - P_z^2]}{P_x - \Omega/q} \left. \right. \right] \frac{\Theta[1 - (P_x - q)^2 - P_y^2 - P_z^2]}{P_x - \Omega/q} \left. \right. \right] \frac{P_x P_y^2}{P_x - \Omega/q} \right] d^3P.
\]

or, having entered designations

\[
\Theta(P_x \pm q) = \Theta[1 - (P_x + q)^2 - P_y^2 - P_z^2], \quad \Theta(P) = \Theta (1 - P^2),
\]
we rewrite last equality in the form

\[ j_{x}^{\text{quant}} = \frac{e^{3}p_{0}^{3}A_{y}^{2}}{(2\pi\hbar)^{3}c^{2}m^{2}v_{0}} \int \left[ \frac{\Theta(P_{x} + q) - \Theta(P)}{qP_{x} + q^{2}/2 - \Omega} + \frac{\Theta(P_{x} - q) - \Theta(P)}{qP_{x} - q^{2}/2 - \Omega} \right] \frac{P_{x}P_{y}^{2}}{qP_{x} - \Omega} + \frac{\Theta(P_{x} + q) - \Theta(P_{x} - q)}{2(qP_{x} - \Omega)} P_{x} \] \, d^{3}P. \quad (2.4)

The expression facing in integral in (2.4), we will transform with the help of expression for numerical density and with use communications of potential and intensity of the electromagnetic field

\[ \frac{e^{3}p_{0}^{3}A_{y}^{2}}{(2\pi\hbar)^{3}c^{2}m^{2}v_{0}} = -\frac{3e\omega_{p}^{2}E_{y}^{2}}{32\pi^{2}p_{0}\omega^{2}} = -\frac{e\omega_{p}^{2}}{k_{0}p_{0}} \frac{3}{32\pi^{2}\Omega^{2}q} = -\frac{\sigma_{1,\text{tr}}kE_{y}^{2}}{32\pi^{2}\Omega^{2}q}. \]

Let us designate

\[ I = I(\Omega, q) = \int \left[ \frac{\Theta(P_{x} + q) - \Theta(P)}{qP_{x} + q^{2}/2 - \Omega} + \frac{\Theta(P_{x} - q) - \Theta(P)}{qP_{x} - q^{2}/2 - \Omega} \right] \frac{P_{x}P_{y}^{2}}{qP_{x} - \Omega} + \frac{\Theta(P_{x} + q) - \Theta(P_{x} - q)}{2(qP_{x} - \Omega)} P_{x} \, d^{3}P. \quad (2.5) \]

By means of a designation (2.5) we will copy equality (2.4) in the form

\[ j_{x}^{\text{quant}} = -\frac{3}{32\pi^{2}\Omega^{2}q} I(\Omega, q)\sigma_{1,\text{tr}}kE_{y}^{2}, \]

or

\[ j_{x}^{\text{quant}} = J_{q}(\Omega, q)\sigma_{1,\text{tr}}kE_{y}^{2}, \quad (2.4') \]

where

\[ J_{q}(\Omega, q) = -\frac{3}{32\pi^{2}\Omega^{2}q} I(\Omega, q), \]

and the quantity \( I(\Omega, q) \) is defined by equality (2.5).

Let us transform integrals from (2.5), using the obvious linear replacements of variables

\[ I_{1} = \int \frac{\Theta(P_{x} + q)P_{x}P_{y}^{2}d^{3}P}{(qP_{x} + q^{2}/2 - \Omega)(qP_{x} - \Omega)} = \int \frac{\Theta(P)(P_{x} - q)P_{y}^{2}d^{3}P}{(qP_{x} - q^{2}/2 - \Omega)(qP_{x} - q^{2} - \Omega)}, \]
\[ I_3 = \int \frac{\Theta(P_x - q)P_x P_y^2 d^3 P}{(qP_x - q^2/2 - \Omega)(qP_x - \Omega)} = \int \frac{\Theta(P)(P_x + q)P_y^2 d^3 P}{(qP_x + q^2/2 - \Omega)(qP_x + q^2 - \Omega)}. \]

Let us summarize the first and third integrals

\[ I_1 + I_3 = \int \frac{2P_x(qP_x - \Omega)^2 - 3q^3(qP_x - \Omega) + P_x q^4}{[(qP_x - \Omega)^2 - q^4/4][(qP_x - \Omega)^2 - q^4]} \Theta(P) P_y^3 d^3 P. \]

Let us summarize the second and fourth integrals

\[ I_2 + I_4 = -\int \left[ \frac{1}{qP_x - q^2/2 - \Omega} + \frac{1}{qP_x - q^2/2 - \Omega} \right] \Theta(P) P_x P_y^2 d^3 P \]
\[ = -\int \frac{2\Theta(P) P_x P_y^2 d^3 P}{[(qP_x - \Omega)^2 - q^4]}. \]

The sum of first four integrals is equal

\[ I_1 + I_2 + I_3 + I_4 = 3q^2 \Omega \int \frac{\Theta(P) P_y^3 d^3 P}{[(qP_x - \Omega)^2 - q^4/4][(qP_x - \Omega)^2 - q^4]}. \]

In the same way the fifth and sixth integrals give

\[ I_5 + I_6 = \int \frac{\Theta(P_x + q) - \Theta(P_x - q)}{2(qP_x - \Omega)} P_x d^3 P = q \Omega \int \frac{\Theta(P) d^3 P}{(qP_x - \Omega)^2 - q^4}. \]

Thus, the integral \( I \) is equal

\[ I = 3q^2 \Omega \int \frac{\Theta(P) P_y^3 d^3 P}{[(qP_x - \Omega)^2 - q^4/4][(qP_x - \Omega)^2 - q^4]} + q \Omega \int \frac{\Theta(P) d^3 P}{(qP_x - \Omega)^2 - q^4}. \]

Internal integrals in plane \((P_y, P_z)\) are equal

\[ \int \int_{P_y^2 + P_z^2 < 1 - P_x^2} P_y^2 dP_y dP_z = \frac{\pi}{4} (1 - P_x^2)^2, \]
\[ \int \int_{P_y^2 + P_z^2 < 1 - P_x^2} dP_y dP_z = \pi (1 - P_x^2). \]

Hence, the integral \( I \) is reduced to one-dimensional integral

\[ I = \frac{3\pi}{4} q^3 \Omega \int \frac{(1 - \tau^2)^2 d\tau}{((q\tau - \Omega)^2 - q^4/4)((q\tau - \Omega)^2 - q^4)} + \pi q \Omega \int \frac{(1 - \tau^2) d\tau}{(q\tau - \Omega)^2 - q^4} = \]
\[
= \frac{\pi q \Omega}{4} \int_{-1}^{1} \frac{(1 - \tau^2)\{q^2[3(1 - \tau^2) - q^2] + 4(q\tau - \Omega)^2\}}{[(q\tau - \Omega)^2 - q^4/4][(q\tau - \Omega)^2 - q^4]} d\tau.
\]

Thus, the longitudinal current in quantum plasma is equal

\[
j^\text{quant}_x = J_q(\Omega, q)\sigma_{1,\text{tr}}kE_y^2,
\]

where

\[
J_q(\Omega, q) = -\frac{3}{32\pi \Omega} \int_{-1}^{1} \frac{(1 - \tau^2)\{q^2[3(1 - \tau^2) - q^2] + 4(q\tau - \Omega)^2\}}{[(q\tau - \Omega)^2 - q^4/4][(q\tau - \Omega)^2 - q^4]} d\tau.
\]

Let us copy equality (2.6) in the invariant form

\[
j^\text{quant} = J_q(\Omega, q)\sigma_{1,\text{tr}}kE^2_{\text{tr}} = J_q(\Omega, q)\sigma_{1,\text{tr}}\frac{\omega}{c}[E, H],
\]

where \(E_{\text{tr}}\) is the transversal electrical field, entered above.

Let us consider the case of small values of the wave vector. We take equality (2.4'). We will notice, that at small values \(q\) the square bracket in (2.5) is equal to zero in linear approximation. Now we use the linear decomposition

\[
\Theta(P_x \pm q) = \Theta(1 - P^2) + \delta(1 - P^2)(\mp 2P_x q).
\]

Hence, in linear approximation we have

\[
\frac{\Theta(P_x + q) - \Theta(P_x - q)}{2qP_x - \Omega} = \frac{q}{\Omega}P_x\delta(P - 1).
\]

Now it agree (2.4') we receive

\[
j^\text{quant}_x = -\sigma_{1,\text{tr}}kE_y^2 \cdot \frac{1}{8\pi \Omega^3}.
\]

Let us notice, that expression (2.7) in accuracy coincides with expression of the longitudinal current (1.15) for classical degenerate plasma.

So, by us it is shown, that at small values of the wave vector longitudinal current in classical and quantum degenerate plasma is calculated under the same formula (1.14).

On Figs. 5 – 9 the behaviour of real and imaginary parts of degenerate classical and quantum plasma is graphically investigated. Let us notice, that the real part of a current in quantum plasma has at first minimum (at \(q < \Omega\),
and then has maximum (at \( q > \Omega \)). The imaginary part of the current in quantum plasma has the minimum at any values \( \Omega \).

On Figs. 5 and 6 we will result comparison accordingly of real (Fig. 5) and imaginary (Fig. 6) parts of the dimensionless part of density of the longitudinal current of classical and quantum plasma.

From Figs. 5 and 6 it is visible, that at small values dimensionless wave number \( q \) quantity of the real part of the longitudinal current of classical and quantum plasma the close friend to the friend and coincide in a limit at \( k \to 0 \). This fact is a consequence coincidence of equalities (1.14) and (2.7). At \( q \to \infty \) values and real, and imaginary parts of the longitudinal current in quantum and classical plasma approach.

On Figs. 7 and 8 we will represent behaviour of real (Fig. 7) and imaginary (Fig. 8) parts of the longitudinal current in quantum plasma in dependences on dimensionless wave number \( q \) at the various values of dimensionless frequency of an electromagnetic field \( \Omega \).

At decrease of quantity \( \Omega \) growth of values of quantity of real and imaginary parts of the longitudinal current is observed.

On Fig. 9 the imaginary part of density of the dimensionless current is represented in the case \( \Omega = 0.2 \); curves 1 and 2 answer accordingly to degenerate classical and quantum plasma.
Fig. 5. Real part of density of dimensionless current, $\Omega = 0.3$; curves 1 and 2 correspond to classical and quantum plasma.

Fig. 6. Imaginary part of density of dimensionless current, $\Omega = 0.5$; curves 1 and 2 correspond to classical and quantum plasma.
**Fig. 7.** Real part of density of dimensionless current of quantum plasma; curves 1, 2 and 3 correspond to values of dimensionless oscillation frequency of electromagnetic field $\Omega = 0.25, 0.3, 0.35$.

**Fig. 8.** Imaginary part of density of dimensionless current of quantum plasma; curves 1, 2 and 3 correspond to values of dimensionless oscillation frequency of electromagnetic field $\Omega = 0.25, 0.3, 0.35$. 
Fig. 9. Imaginary part of density of dimensionless current, \( \Omega = 0.2 \); curves 1 and 2 correspond to classical and quantum plasma.
3. Conclusion

In the present work the nonlinear analysis of interaction of electromagnetic wave with classical and quantum degenerate plasma is carried out. It is used square-law on intensity quantity of electric field the decomposition of distribution function and Vlasov equation in case of the classical degenerate plasmas. And also it is used square-law on vector potential quantity of electromagnetic field decomposition of quantum distribution function and Wigner equation in case of the quantum degenerate plasmas. It is shown, that besides the known transversal current both in classical and in quantum plasma it is generated the longitudinal current. Research real and imaginary parts of the longitudinal current is carried out.

In the following work we will consider generation of longitudinal current by transversal electromagnetic field in collisional classical plasma.

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