Study on Macro-fiber Composite Coupled-Plate Structures

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Abstract. In this paper, the dynamic and vibration control effect of macro-fiber composite (MFC) were studied on the plate and shell structure, proposing a new calculation method of MFC coupled-plate structure actuating equation. Based on the first kind of piezoelectric equation, this paper deduces the equation of P1-type MFC composite plate structure, considering the influence of the Poisson's ratio of the controlled structure and the effect of the MFC plane strain on MFC's actuating force and actuating moment. In this paper, the vibration control experiment of the MFC coupled-plate structure is completed. The deviation between the experimental results and the simulation results is less than 7.5%, indicating that the P1-type MFC actuating equation is correct and can be used in the simulation calculation of vibration control for the MFC coupled-plate structure.

1. Introduction

As the most common piezoelectric materials so far, piezoelectric ceramics are characterized by high sensitivity and temperature stability. Some deficiencies, such as poor flexibility and piezoelectric effect, restrict its application into practical projects, though. Macro Fiber Composite (MFC), a new type of piezoelectric composite, is compounded of piezoelectric ceramic fibers, polymer matrix and interdigitated electrodes. Compared with traditional piezoelectric ceramics, polymer matrix in MFC enhances the material flexibility [1]. By making piezoelectric ceramics fibrous phase and coordinating them with finger electrodes, it tremendously improves piezoelectric effect of piezoelectric materials. Therefore, MFC is of the best overall performance at present.

With its “inverse piezoelectric effect”, a main application of MFC is to change its actuating force through adjusting input voltage, so as to implement active control over the structural vibration. Therein, a crucial step is establishing an accurate MFC actuating equation which many scholars have studied in recent years. Considering the influence of polymer matrix on the composite, S. Sreenivasa Prasath et al. have made further researches on the influence of the volume fraction represented by piezoelectric ceramic fibers on MFC elasticity modulus, piezoelectric coefficients, stress strain, etc. by establishing the MFC mechanical model with equivalent stratification [2]. Kateřina Steiger et al. have established the MFC finite element model based on the hypothesis of approximating MFC into clintheriform well-distributed piezoelectric material [3]. Only with the mechanical analysis of MFC material, those researches have not obtained the relationship between the actuating force and input voltage, remaining a certain gap to MFC implementing the active control. Guo Yi et al. have simplified the constitutive equation of MFC composite to be two-dimensional, ignored the transformation of through-thickness direction, and deduced the one-dimensional stress and electricity domino effect formula of MFC [4]. It is considered that the thickness direction is the polarization direction of the piezoelectric fiber, not the longitudinal direction. And he has not considered the Poisson’s ratio influence of the controlled structure when calculating the activation force and activation bending moment, which produces a great error in the result.
Hereby, this paper not only considers the influence of Poisson’s ratio of the controlled structure on MFC activation force and activation bending moment, but the place strain produced by MFC actuation, deduces the MFC two-dimensional actuating equation for plate structures, and obtains the accurate MFC strength and electricity relationship, on the basis of which the active control simulation is performed on the vibration of a plate structure and the active control experiment is conducted.

2. The actuating equation of MFC coupled-plate structures

The actuating equation of MFC coupled-plate structures is deduced on the basis of the first kind of piezoelectric basic equations. The stress in MFC is obtained by transformation of the first kind of piezoelectric basic equation; the actuating equation of MFC coupled-plate structures is deduced by the stress equation; and then the MFC activation force and activation bending moment are obtained for plate structures.

2.1. MFC piezoelectric equation

The active layer of MFC is treated as homogeneous piezoelectric fibers, as is shown in figure 1. The constitutive equation of MFC linear-elastic piezoelectric material with orientation polarization on x is:

$$
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yx} \\
\gamma_{xz} \\
\gamma_{zy}
\end{bmatrix} =
\begin{bmatrix}
s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\
s_{21}^E & s_{22}^E & s_{23}^E & 0 & 0 & 0 \\
s_{31}^E & s_{32}^E & s_{33}^E & 0 & 0 & 0 \\
s_{44}^E & 0 & 0 & 0 & 0 & 0 \\
s_{55}^E & 0 & 0 & 0 & 0 & 0 \\
s_{66}^E & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yx} \\
\tau_{xz} \\
\tau_{zy}
\end{bmatrix} +
\begin{bmatrix}
d_{31} \\
d_{32} \\
d_{33}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
$$

In the equation, $s_{ij}^E$ is the flexibility coefficient, in which $i$ represents the MFC strain direction, $j$ the MFC stress direction; $d_{ii}$ is the piezoelectric strain coefficient, in which $i$ represents the electric field action direction, $j$ the strain direction of piezoelectric material under electric field action, the numbers 1, 2, 3 respectively correspond to $z$, $y$, $x$ directions in Cartesian coordinate system, and 4, 5, 6 the shear deformation directions $yx$, $xz$, $zy$; $E_x$ is the electric field intensity in $x$ direction.

MFC is lamelliferous with the thickness of only 0.3mm. According to the vibration theory of thin shells, stress and strain in the through-thickness direction can be ignored. Since MFC shear strain $\gamma_{xz}$ is irrelevant to the electric field intensity $E_x$ and its own shear strain $\tau_{xz}$ is rather small, the influence of shear strain can be ignored. Therefore, equation (1) can be simplified as:

$$
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_z
\end{bmatrix} =
\begin{bmatrix}
s_{11}^E & s_{13}^E \\
s_{31}^E & s_{33}^E
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_z
\end{bmatrix} +
\begin{bmatrix}
d_{31} \\
d_{33}
\end{bmatrix}
E_x
$$

The electric field intensity $E_x$ in MFC is:

$$
E_x = \frac{U}{t}
$$

in which $U$ is the applied voltage amplitude, and $t$ is the spacing of interdigital electrodes.

2.2. The actuating equation of MFC coupled-plate structures

MFC coupled-plate structure is shown in figure 2, in which $M_x$ is the MFC equivalent moment for the controlled structure; $F_x$ is the MFC equivalent acting force for the controlled structure; $H$ is the plate thickness; $a$ is the effective length of MFC, $b$ is the width, and $h$ is the thickness; $L$ is the position of neutral axis. Because its thickness is extremely small, $L=0.5H$. 

2
Figure 1. The construction of MFC.

Figure 2. The construction of plate equipped with MFC.

The two-dimensional MFC stress is obtained through formula (2):

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z
\end{bmatrix} =
\begin{bmatrix}
\frac{E_{33} - E_{13}}{E_{11}} & \frac{E_{13}}{E_{11}} & \frac{E_{13}}{E_{11}} \\
\frac{E_{13}}{E_{11}} & \frac{E_{33} - E_{13}}{E_{33}} & \frac{E_{13}}{E_{33}} \\
\frac{E_{13}}{E_{33}} & \frac{E_{13}}{E_{33}} & \frac{E_{33} - E_{13}}{E_{11}}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x - d_{31}E_b \\
\varepsilon_y - d_{31}E_b \\
\varepsilon_z - d_{33}E_b
\end{bmatrix}
\] (4)

The two-dimensional flexibility matrix can be represented as:

\[
\begin{bmatrix}
E_{11} & E_{13} \\
E_{13} & E_{33}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & -v_f/E_{33} \\
0 & 1 & -v_f/E_{11}
\end{bmatrix}
\] (5)

in which \( E_{11}, E_{33} \) are elasticity modulus of MFC in x direction and in y direction respectively; \( v_f \) the Poisson’s ratio of MFC.

\[
\varepsilon_{11} = \frac{d_{31}U}{t}, \varepsilon_{33} = \frac{d_{33}U}{t}
\] (6)

in which \( \varepsilon_{11}, \varepsilon_{33} \) are the strain of MFC created by voltage in z direction and in x direction respectively.

The actuating force and actuating moment of MFC for the plate structure are:

\[
\begin{align*}
F_z &= -a \int_{L}^{L+h} \sigma_{z} \, dy = a \int_{L}^{L+h} \left[ Q_{31}^{E}(\varepsilon_{11} - \varepsilon_{z}) + Q_{33}^{E}(\varepsilon_{33} - \varepsilon_{z}) \right] \, dy \\
F_x &= -b \int_{L}^{L+h} \sigma_{x} \, dy = b \int_{L}^{L+h} \left[ Q_{11}^{E}(\varepsilon_{11} - \varepsilon_{x}) + Q_{13}^{E}(\varepsilon_{33} - \varepsilon_{x}) \right] \, dy \\
M_z &= a \int_{L}^{L+h} (-\sigma_{z}) \, dy = a \int_{L}^{L+h} \left[ Q_{31}^{E}(\varepsilon_{11} - \varepsilon_{z}) + Q_{33}^{E}(\varepsilon_{33} - \varepsilon_{z}) \right] \, dy \\
M_x &= b \int_{L}^{L+h} (-\sigma_{x}) \, dy = b \int_{L}^{L+h} \left[ Q_{11}^{E}(\varepsilon_{11} - \varepsilon_{x}) + Q_{13}^{E}(\varepsilon_{33} - \varepsilon_{x}) \right] \, dy
\end{align*}
\] (7)

Since \( \varepsilon_{11}, \varepsilon_{33} \) in those equations are unknown, \( \varepsilon_{11}, \varepsilon_{33} \) are assumed to be in linear distribution along the through-thickness direction of the structure and the constitutive relation is obtained between the plate and MFC, as is shown below [5].

\[
\varepsilon_y = \alpha_x y + \varepsilon_{y0}, \varepsilon_z = \alpha_z y + \varepsilon_{z0}
\] (9)
\[\int_{L}^{L+h} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x - v_f \varepsilon_y \right) dy + \int_{L}^{L+(H-L)} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x \right) dy = 0\]

\[\int_{L}^{L+h} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x - v_f \varepsilon_y \right) dy + \int_{L}^{L+(H-L)} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x \right) dy = 0\]

\[\int_{L}^{L+h} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x - v_f \varepsilon_y \right) dy + \int_{L}^{L+(H-L)} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x \right) dy = 0\]

\[\int_{L}^{L+h} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x - v_f \varepsilon_y \right) dy + \int_{L}^{L+(H-L)} \frac{E_p}{1-v_f^2} \left( \varepsilon_z + v_f \varepsilon_x \right) dy = 0\]

\[\alpha_z, \alpha_x\] are slopes of strain distribution; \(\varepsilon_{x0}, \varepsilon_{z0}\) are the membrane strain in \(x\) direction and in \(z\) direction respectively; \(\sigma_{fx}, \sigma_{fz}\) are the stress in MFC, and \(E_p\) is the elasticity modulus of the plate; \(\varepsilon_{px}, \varepsilon_{pz}\) are the strain of the plate; \(\nu_p\) is the Poisson’s ratio of MFC. Then the equilibrium equation is written to solve \(\alpha_x, \alpha_z, \varepsilon_{x0}, \varepsilon_{z0}\).

To sum up

\[\begin{align*}
F_x &= \frac{bE_pH\left[2CT-2DS+(H-2L)(CU-DT)\right]}{2(T^2-SU)(1-v^2)} \left(\varepsilon_{x0}+v_{x0}\varepsilon_{x0}\right) \\
F_z &= \frac{aE_pH\left[2QA-2BP+(H-2L)(AR-BQ)\right]}{2(Q^2-PR)(1-v^2)} \left(\varepsilon_{z0}+v_{z0}\varepsilon_{z0}\right)
\end{align*}\]

\[\begin{align*}
M_x &= \frac{bE_pH}{6(T^2-SU)(1-v^2)} \left[2(H^2-3HL+3L^2)(DT-CU)+3(H-2L)(DS-CT)\right] \\
M_z &= \frac{aE_pH}{6(Q^2-PR)(1-v^2)} \left[2(H^2-3HL+3L^2)(QB-RA)+3(H-2L)(BP-AQ)\right]
\end{align*}\]

in which

\[\begin{align*}
A &= \frac{E_p}{2(1-v^2)} \left[\frac{(L+h)^2-L^2}{1-v^2}\right], \\
B &= \frac{E_p}{1-v^2}, \\
C &= \frac{AE_p}{E_p}, \\
D &= \frac{BE_p}{E_p}, \\
P &= \frac{E_p}{3(1-v^2)} \left[\frac{(L+h)^3-L^3}{1-v^2}\right] + \frac{E_p}{3(1-v^2)} \left[\frac{(H-L)^3+L^3}{1-v^2}\right]
\end{align*}\]

Because the Poisson’s ratios hardly differ among regular materials, it is assumed that the Poisson’s ratios of the plate and MFC are equal and the minimum value is taken to calculate the action of MFC over the controlled structure and solve the simultaneous the mechanical equilibrium equation.

3. The actuating equation of MFC coupled-plate structures
To verify the correctness of the equation, the model is verified through vibration control experiment and analogue simulation.

In the analogue simulation, modeling is conducted through ANSYS, and the modeling parameters of the plate model are illustrated in table 1. Then the exciting force and the actuating force of MFC are
defined, with exciting position of the exciting force in the center of the plate and the paste position of MFC shown by the red lines in figure 3. The actuating force and actuating bending moment provided by MFC and the plate structure are obtained by the actuating equation deduced in the first section of the paper, which are exerted into the node located at the red lines in figure 3. In simulation, the exciting force and the MFC working voltage are both in simple harmonic form, and the exciting force are made opposite to the displacement phase created by MFC actuating force and actuating bending moment, so that the MFC actuating force and actuating bending moment can decrease vibration. The exciting forces in simulation analysis are in two working conditions, 20Hz, 10N and 50Hz, 10N respectively, and MFC working voltage is 300V.

### Table 1. Material Properties.

| Material          | Elasticity Modulus (GPa) | Density (kg m\(^{-3}\)) | Poisson’s Ratio | Length (mm) | Width (mm) | Thickness (mm) |
|-------------------|--------------------------|--------------------------|-----------------|-------------|------------|----------------|
| Plate (aluminum)  | 70.5                     | 2700                     | 0.31            | 400         | 400        | 2              |

**Figure 3.** The ANSYS model of plate.  
**Figure 4.** The model of plate experiment.

In the vibration control experiment of MFC coupled-plate, the plate parameters are the same as in table 1. The experimental system is composed of sensing, controlling, and exciting, in which the acceleration sensor is adopted as the sensor part, MFC the control part, and the exciter the exciter part, shown in figure 4. In the experiment, two sinusoidal signals, 20Hz and 50Hz, output by dSPACE provide exciting signals for the exciter and MFC. The figure 5 shows the acceleration comparison of simulation and experiment before and after the control with 20Hz and 50Hz excitation frequency and 300V working voltage. It is evident that the deviation of simulation and experiment under two working conditions is less than 7.5%, proving that the actuating equation deduced in this paper is accurate and applicative. Furthermore, MFC can decrease vibration of the plate structure effectively. When the exciting force is 20Hz, 10N, and the working voltage is 300V, MFC in the experiment can decrease the acceleration amplitude by 7%; when the exciting force is 50Hz, 10N, and the working voltage is 300V, MFC in the experiment can decrease the acceleration amplitude by 13%.
4. Conclusion
This paper studies plate structures, deduces a new actuating force and actuating bending moment equation of MFC coupled-plate structures, and conducts the comparison validation about simulation and experiment, reaching conclusions as follows:

The deduced actuating equation of two-dimensional MFC coupled-plate structures considers the influence of membrane strain produced by actuation between plate-structural Poisson's ratio and MFC on MFC actuating force. This paper obtain MFC's actuating force and actuating moment on the plate structure.

The plate vibration control simulation and experiment is accomplished. Through analogue simulation and experimental verification, the comparison results are obtained before and after the control under 20Hz, 10N and 50Hz, 10N excitation. Under the working conditions of 20Hz, 10N and 50Hz, 10N, the deviation of simulation and experiment is less than 7.5%, which proves that the MFC actuating equation in this paper is correct and applicative to the plate structure, and that MFC produces good effect on vibration control of plate structures.

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