REVIEW

Advances in atomic physics
Four decades of contribution of the Cairo University – Atomic Physics Group

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GRAPHICAL ABSTRACT

In this review article, important developments in the field of atomic physics are highlighted and linked to research works the author was involved in himself as a leader of the Cairo University – Atomic Physics Group. Starting from the late 1960s – when the author first engaged in research - an overview is provided of the milestones in the fascinating landscape of atomic physics.

ARTICLE INFO

Article history:
Received 26 May 2013
Received in revised form 19 August 2013
Accepted 19 August 2013
Available online 26 August 2013

Keywords:
Atomic physics
Laser physics
Plasma physics
Astrophysics

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http://dx.doi.org/10.1016/j.jare.2013.08.004

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**Tharwat El-Sherbini** is the leader of the atomic physics group at Cairo University. He acquired a Ph. D. in atomic and molecular physics from Leiden University (The Netherlands) in 1972 and a D. Sc. degree in atomic and laser physics in 1984. He has published more than 160 publications in the field of atomic, molecular and laser physics and has established the research Laboratory of Lasers and New Materials (LLNM) at the physics department of Cairo University.

Professor El-Sherbini has received several awards and honors, among which: the “State Award for Scientific Appreciation”, the “NILE Award” and the “State Decoration Of The First Order For Sciences and Arts”.

**Introduction**

During the last decades, we witnessed a continuous development in the field of atomic physics that had direct impact on other fields of research such as astrophysics, plasma physics, controlled thermonuclear fusion, laser physics, and condensed matter physics.

The landscape is vast and cannot possibly be covered in one review article, but it would require a complete book. Therefore, I will confine myself to the research works I was involved in and those that have direct connections with the work I have done.

The review is structured around five main topics:

- Electron–atom collisions.
- Ion–atom collisions.
- Atomic structure calculations and X-ray lasers.
- Laser-induced breakdown spectroscopy (LIBS).
- Laser cooling and Bose–Einstein condensation.

**Electron–atom collisions**

The physics of electron–atom collisions originated in 1930 by the work of Ramsauer and Kollath [1,2] on the total scattering cross-section of low energy electrons against noble gases, which contributed so much to the development of quantum theory. This work was followed by Tate and Smith [3] on inelastic total cross-sections for excitation of noble gases. Several well known physicists, e.g., Bleakney and Smith [4], Hughes and Rojansky [5], and Massey and Smith [6], at this period gave important contributions in the field of electron collision physics. The theory was developed by Stueckelberg [7], Landau [8], and Zener [9]. In 1952, Massey and Burhop’s book [10] appeared on “Electronic and Ionic Impact phenomena,” which provided the basis for any scientist who wants to start the work on the subject.

Multiple ionization of noble gases by low energy electrons (below 600 eV) has been studied extensively in mass spectrometers [3,11,12]. However, total electron impact cross-sections were determined by Van der Wiel et al. [13] and El-Sherbini et al. [14] for the formation of singly and multiply charged ions of He, Ne, Ar, Kr, and Xe by fast electrons (2–16 keV). The ion selection was performed in a charge analyzer with 100% transmission, and consequently, it was possible to avoid the discrimination effects in the measurement of the relative abundances of the multiply charged ions. Therefore, the data were more reliable than those obtain in low transmission mass spectrometers. The ionization cross-section of large electron impact energies is given by

$$\sigma_{ni} = \frac{4\pi a_0^2}{R} E_{el} \ln \frac{E_{el}}{R} + C_{ni}$$

where $\sigma_{ni}$ is the cross-section for formation of $n^+$ ions, $E_{el}$ is the electron energy corrected for relativistic effects, $a_0$ is the first Bohr radius, $R$ is the Rydberg energy, $M_{ni}^2$, and $C_{ni}$ are constants.

The constant $M_{ni}^2$ is given by

$$M_{ni}^2 = \int \frac{d\sigma^{n^+}}{dE} \frac{R}{E} dE$$

where $d\sigma^{n^+}/dE$ is the differential dipole oscillator strength for an ionization to $n^+$ continuum at excitation energy $E$.

In 1970, an experiment was developed by van der Wiel [15], in which fast electrons (10 keV), scattered by He, Ne, and Ar are detected in coincidence with the ions formed (Fig. 1). It was possible from the measurements of the scattering intensity at small angles to calculate optical oscillator strengths. The differential scattering of fast electrons is given by Bethe et al. [16] (in au):

![Fig. 1 Schematic view of the scattered electron–ion coincidence apparatus.](image)
\[
\sigma(\theta, E) = \frac{2}{E} \frac{k_0}{E_0} \frac{1}{K^2} \frac{d \langle K \rangle}{dE}
\]

where \( \theta \) is the scattering angle, \( E \) the energy loss, \( k_0 \) and \( k_n \) are the magnitudes of the momenta of the primary electron before and after collision, \( K \) is the magnitude of the momentum transfer \( (K = k_0 - k_n) \), and \( (d \langle K \rangle/dE) \) is the generalized oscillator strength. This last quantity may be expanded in terms of \( K^2 \):

\[
\frac{d \langle K \rangle}{dE} = \frac{df}{dE} + aK^2 + bK^4 + ...
\]

where \( \frac{df}{dE} \) is the optical oscillator strength, as defined in the dipole approximation.

The work was closely connected to that where ion charge distribution is measured after irradiation of atoms with photons at a number of selected wavelengths [17,18]. However, the use of photon source is simulated by measuring the small-angle, inelastic scattering of 10 keV electrons in coincidence with the ions formed. The simulation is based on the fact that measured energy lost by the scattered electron in the coincident experiment corresponds to the photon energy absorbed in the photon experiments for the same process. Moreover, the incident electron energy of 10 keV is large compared to the energy losses studied \( \leq 400 \) eV, and also, the incident momentum (370 au) is much larger than the momentum transfer \( \leq 0.5 \) au. Under these conditions, the first Born approximation holds. By making use of the first Born approximation for inelastic electron scattering at small momentum transfer, the measured intensities of scattering were converted into optical oscillator strengths.

**Fig. 2** Block diagram of the coincidence circuit. Signal from the ion detector (channel 1). Signal from the electron detector (channel 2).
by electron impact [23]. The study of the ionization of N₂ and CO by 10 keV electrons as a function of the energy loss was done by El-Sherbini and van der Wiel for the valence electrons [24] as well as for inner-shell electrons [25].

Our results on electron–atom ionization were the first of its type and corresponded well with those of photo-ionization by real and big synchrotron devices, but our apparatus was much faster and easier to operate. Our device was a sort of model synchrotron and in fact was considered to be the first tabletop synchrotron.

**Ion–atom collisions**

Collision processes between fast heavy atoms and ions can be simply described by the interactions between relatively fast protons and alpha particles with neutral atoms. Besides the normal excitations and ionizations which are analogous to what happens in electron–atom collisions, an extra phenomenon occurs, named charge exchange. The best way to describe both types of phenomena is in treating the three particles involved, viz the point charge projectile, the target atom, and the electron with one Hamiltonian. It is one closed system in which kinetic energy of the projectile is transferred into electronic excitation energy. The impact parameter treatment has proven very useful, see Bates [26]. It gave a semiclassical description of the collision process, with the external motions classically and the internal motions quantum mechanically. Due to the heavy mass of the proton or alpha particle, the kinetic energy of the projectile is much bigger than the electronic excitations concerned. Therefore, the trajectory of the projectile is considered rectilinear during the whole collision event. The projectile keeps constant velocity, approximately. The impact parameter \( p \) is defined as the distance between the trajectory and the target nucleus. The cross-section \( \sigma \) for transition of the electronic system from state \( i \) to state \( f \) is given by

\[
\sigma(\varepsilon) = 2\pi \int_0^\infty \rho P(\rho) d\rho
\]

where \( \varepsilon \) is the kinetic energy of the projectile in the center of mass system, and

\[
P(\rho) = |a_f(\rho, t = \infty)|^2
\]

with

\[
i\frac{d}{dt}a_f(\vec{R}, t) = \sum_k a_{ik}(\vec{R}, t) V_{jk}(\vec{R}) \exp(-i\Delta E_{jk}t)
\]

Fig. 3 Oscillator-strength spectra of Kr²⁺ and Kr³⁺. The inset of the upper figure shows the direct ejection of two N electrons below the 3d⁹ threshold in the Kr²⁺ spectrum. The inset of the lower figure shows the threshold for discrete triple ionization in the Kr³⁺ spectrum.
ΔE_{if} = E_k - E_f \tag{8} \label{eq:1}

\widetilde{R} is the distance between both nuclei; \( V_{jk}(\widetilde{R}) \) is the matrix element of the potential field of target particle scaled by \( \frac{2\pi}{\hbar^2} \) between the target eigen states \( f \) and \( k \); \( E_k \) and \( E_f \) are eigen energies of target particle; and \( a_{jk} \) is the amplitude of the target eigen functions. For kinetic energies \( E \) far above the threshold, we can apply the Dirac condition, which assumes that the most dominant transition is from the initial to the final state i.e.

\[ a_{ik} = \delta_{ij} a_{ik} \tag{9} \label{eq:2} \]

This leads to the integral equation

\[ ia_{if}(\widetilde{R}, t) = \int_{-\infty}^{+\infty} a_{if}(\widetilde{R}, \tau) V_{if}(\widetilde{R}) \exp(-i\Delta E_{if}\tau) d\tau \tag{10} \label{eq:3} \]

In the first order Born approximation, we obtain

\[ ia_{if}(\rho, t = \infty) = \int_{-\infty}^{+\infty} V_{if}(\widetilde{R}) \exp(-i\Delta E_{if}) d\rho \tag{11} \label{eq:4} \]

see Merzbacher [27]. Replacing \( t \) by \( \frac{z}{u} \) where \( u \) is the velocity, one gets

\[ ia_{if}(\rho, t = \infty) = \frac{1}{iu} \int_{-\infty}^{+\infty} V_{if}(\widetilde{R}) \exp\left(-i\Delta E_{if}\frac{z}{u}\right) dz \tag{12} \label{eq:5} \]

From this relation, the dependence of \( P(\rho) \) on \( u \) can be deduced. Therefore, it will depend on

\[ \frac{a\Delta E_{if}}{hu} \text{ (in atomic units)} \tag{13} \tag{13} \]

One measures the effective interaction length “\( a \)” along the trajectory \( z \), if the projectile passes by the target particle. This is the Massey Criterion. For large values of \( u \), we see

\[ \frac{a\Delta E_{if}}{hu} < 2\pi \tag{14} \tag{14} \]

which means that

\[ P(\rho) \sim |a_\rho|^2 \sim \frac{1}{u^2} \sim \frac{1}{E} \tag{15} \tag{15} \]

Decreasing speed coming from large values of \( u \), one expects a maximum in \( P(\rho) \) if \( (a\Delta E_{if}/u) \approx 2\pi \), following the oscillatory behavior of \( \exp(-i\Delta E_{if}/u) \) as a function of \( u \). This type of behavior has been studied by Hasted [28,29] who measured total cross-sections for exchange between various kinds of ions and neutral targets. Differential cross-sections, not only velocity dependent but also as a function of the scattering angle, have been measured by Morgan and Everhart [30] and by Kessel and Everhart [31].
Advances in this field were made by measuring electron capture by multiply charged ions. It attracted attention of many physicists in various fields of physics such as astrophysics, plasma physics, controlled thermonuclear fusion research, and X-ray laser production. When multiply charged ions collide with neutral particles (at low to intermediate impact velocities $u \leq 1$ au), capture reactions populating excited states in the projectile are very probable, see, for instance, Niehaus and Ruf[32] and Winter et al.[33]. For singly excited states in the projectile are very probable, see, for instance, Niehaus and Ruf[32] and Winter et al.[33]. For single-electron capture, these reactions may lead to population inversion and are of importance in several schemes for the production of XUV and soft X-ray lasers. However, in these collisions, non-radiative (i.e. auto-ionizing) processes can be important, and competition with radiative processes occurs. Measurements of these non-radiative processes by Winter et al.[34] showed that the corresponding total cross-sections for the production of slow electrons were large and strongly charge state dependent. These results were interpreted by them to be the result of capture ionization, i.e., an Auger ionization in the short-lived quasi-molecule.

Let $X^{z+}$ be the multiply charged ion and $Y$ is the target atom, then the reactions can be followed by radiative emission

$$X^{z+} + Y \rightarrow X^{(z-1)+} + Y^+ \rightarrow X^{(z-1)+} + Y^+ + h\nu$$

or by electron emission through one of the following channels

$$X^{z+} + Y \rightarrow X^{(z-1)+} + Y^{2+} + e$$

Auger ionization of the quasi-molecule formed during collision,

$$X^{z+} + Y \rightarrow X^{(z-1)+} + Y^+ \rightarrow X^{(z-1)+} + Y^+ + h\nu$$

Penning ionization after single electron capture,

$$X^{z+} + Y \rightarrow X^{(z-1)+} + Y^{2+} \rightarrow X^{(z-1)+} + Y^{2+} + e$$

double electron capture into autoionizing states of the projectile,

$$X^{z+} + Y \rightarrow X^{(z-1)+} + Y^+ \rightarrow X^{(z-2)+} + Y^{2+}$$

$$\rightarrow X^{(z-1)+} + Y^{2+} + e$$

The measurements of Winter et al. [34] yielded only total cross-sections for $Ne^{z+}$ ($z = 1–4$) and $Ar^{z+}$ ($z = 1–8$) colliding at energies 100 keV and 200 keV, respectively, with noble gas atoms. However, data on the energy spectrum of the electrons are still needed to investigate these phenomena in more detail. Woerlee et al.[36] have extended the work by measuring energy spectra of electrons produced in collisions of multiply charged neon ions with noble gas atoms. Fig. 5 shows the experimental results for 100 keV $Ne^{1+}$ on Ar. The spectrum consists of a continuous background on which peaks are superimposed. The spectra for $Ne^{1+}$ and $Ne^{2+}$ are almost identical, but large changes are seen when the projectile charge state is increased from 2+ to 3+ and 3+ to 4+. The largest changes are an increase in the continuum below ±20 eV, and an increasing number of peaks superimposed on the continua. The increase in the continuum below 20 eV is the result of capture ionization in the short-lived quasi-molecule [37]. The bars in Fig. 5 indicate the positions of calculated transition energies corrected for a Doppler shift of $-2.7$ eV. The peaks observed in 100 keV $Ne^{3+}$ on Ar shift to lower energies when the projectile energy is increased. This shift is equal to the kinematical shift, which would be expected, when the corresponding electrons are emitted by the projectile. Therefore, we concluded that the peaks originate from auto-ionizing states in the projectile, which decay after the collision has taken place. Since no photoabsorption data exist on the auto-ionizing states of multiply charged neon ions, we tried to calculate energy levels of doubly excited neon ions with a single configuration HF method. In order to determine the energies of the various levels, we included the electrostatic energy splitting due to the core electrons, see El-Sherbini and Farrag [38]. The energy splitting caused by the excited electrons is small and was not taken into account. We found that for $Ne^{2+}$–Ar, the peaks occur in the region for the calculated peak energies of $Ne^{1+++}$, $Ne^{2++}$, and $Ne^{1+++}$, but $Ne^{2++}$ seems to cover most of the data. For $Ne^{3+}$–Ar, calculated energies of $Ne^{1+++}$ and $Ne^{2+++}$ appear in the region of the observed peaks.

![Fig. 5](image_url)

Fig. 5 Electron spectra for 100 keV $Ne^{n+}$ on Ar ($\theta = 90^\circ$), $\cdots \cdots \cdots \cdots \cdots n = 1$; $\cdots \cdots \cdots \cdots \cdots n = 2$; $\cdots \cdots \cdots \cdots \cdots n = 3$; $\cdots \cdots \cdots \cdots \cdots n = 4$. The bars in the figure indicate the positions of calculated transition energies corrected for a Doppler shift of $-2.7$ eV.
Further developments in this field were done by El-Sherbini et al. [39], where they measured target dependence of excitation resulting from electron capture in collisions of 200 keV Ar\textsuperscript{6+} ions with noble gases. The study shows strongly rising total capture excitation cross-sections and shifts in the post-collision projectile excited-state distributions to higher n levels with the increase in the target atomic number. Energy dependence of excitation and ionization resulting from electron capture in Ar\textsuperscript{b+}–H\textsubscript{2} collision in the range of ion projectile energies 200–1200 keV was measured by El-Sherbini et al. [40]. These studies indicate that single electron charge transfer into excited states of the product ion is the most important inelastic process. Photon emission between 20 and 250 nm and slow electron and ion production cross-sections have been measured. The capture occurred mainly into n = 4 levels with the excitation of the higher angular momentum states dominating over most of the projectile energy range. The capture ionization cross-section is appreciable, amounting to 30–40% of the total excitation cross-section. These results are extremely valuable for the developments of controlled thermonuclear fusion reactors (see El-Sherbini [41]).

These studies indicate that single electron charge transfer into excited states of the product ion is the most important inelastic process. Photon emission between 20 and 250 nm and slow electron and ion production cross-sections have been measured. The capture occurred mainly into n = 4 levels with the excitation of the higher angular momentum states dominating over most of the projectile energy range. The capture ionization cross-section is appreciable, amounting to 30–40% of the total excitation cross-section. These results are extremely valuable for the developments of controlled thermonuclear fusion reactors (see El-Sherbini [41]). To obtain more information about the coupling mechanisms, which gives rise to capture into excited states in ion–atom collisions at intermediate energies (u \sim 0.5 au), El-Sherbini and de Heer [42] measured photon emission in the spectral region between 60 and 100 nm in the collision of Ar\textsuperscript{q+} (q = 1, 2, and 3) with He and Ne at impact energies between 15 and 400 keV. The experimental results were explained qualitatively by considering the MO correlation diagram (Fig. 6). The emission cross-section for the collision of Ar\textsuperscript{q+} with He is shown in Fig. 7. It was often found that the cross-section for excitation decreases with the increase in the number of intermediate transitions required in order to reach the excited state. When there is a mechanism involving radial coupling leading from initial to final states, then it was found that the measured emission cross-section decreases with energy, whereas, as mechanisms involving rotational coupling lead to cross-sections that increase with increasing energy up to 200 keV or more. The results have been of particular importance in evaluating theoretical models and have provided a valuable check of the range of validity of existing theories.

**Atomic structure calculations and X-ray lasers**

In the field of atomic collisions, as we noticed in the previous sections, much attention was paid to the excitation of noble gas atoms. A systematic study of the excitation process...
requires the knowledge of accurate dipole transition probabilities for spontaneous emission between the various configurations of the ions. Laser physics and astrophysics are other branches, which have stimulated more accurate atomic line strengths and transition probabilities calculations. Garstang [43,44] performed the first intermediate coupling calculations for Ne II. On this basis, Wiese et al. [45] composed their data compilations. However, the previously tabulated line strengths were in need of revision. In his work, Luyken [46,47] performed new calculations of line strengths and transition probabilities for Ne II and Ar II where specific configuration interactions were investigated and some effective operators were included. The results showed that the agreement with the experimental data was improved as compared with the earlier calculations. El-Sherbini [48–50] has extended the work of Luyken to the calculation of transition probabilities and radiative lifetimes for Kr II and Xe II. He used “exact” intermediate coupling wave functions to describe the various states [48] :

$$\Psi(J, M) = \sum \alpha_i |p^J L^j S^j_i > ,$$

where \( \alpha_i \) is the expansion coefficient, \( J \) is the total angular momentum, \( M \) is the magnetic quantum number, \( L^j \) and \( S^j \) are the orbital and spin angular momentum of the core electrons, \( l^j \) is the orbital angular momentum of the running electron, and \( L^l \) and \( S^l \) are the orbital and spin angular momentum of the core–L–S bases states on which the “exact” \( \Psi(J, M) \) is expanded. The transition probability between two states with summation indices \( i \) and \( j \) refer to the upper and lower level, respectively, is given by

$$A(J_u, J_l) = \frac{64\pi^2}{3\hbar^2(2J_u + 1)} S(J_u, J_l)$$

where \( S(J_u, J_l) \) is the line strength and \( J_u, J_l \) are the total angular momentum of the upper and lower states, respectively. The line strength is given by El-Sherbini [48]

$$S(J_u, J_l) = e^2 \left\{ \sum \eta \left[ \begin{array}{c} \delta(S_u, S_l) \delta(L_u, L_l) \delta(L_{u\ell}, L_{l\ell}) \delta(J_u, J_l) \delta(J_{u\ell}, J_{l\ell}) \end{array} \right] \right\}$$

$$\left\{ \begin{array}{c} L_u \ 1 \ L_{u\ell} \\ 1 \ L_l \end{array} \right\} \left\{ \begin{array}{c} J_u \ 1 \ \ L_{u\ell} \ \ 1 \ J_{u\ell} \\ 1 \ J_{u\ell} \end{array} \right\} |(2J_u + 1)(2J_l + 1)2J_{u\ell} + 1|^2 \int_0^\infty R_{u\ell}(r)|R_{l\ell}(r)|dr$$

where \( J_u, J_l \) and \( l_{u\ell}, l_{l\ell} \) are, respectively, the total angular momentum of the states and the orbital angular momentum of the running electron in the upper and lower states. \( R_{u\ell}(r) \) and \( R_{l\ell}(r) \) are the one electron radial wavefunctions in the two different states.

The lifetime \( \tau_u \) of the upper state is given by El-Sherbini [49]

$$\tau_u = \frac{1}{\sum \frac{1}{A(J_u, J_l)}}$$

The parametric potential method was used to calculate the radial part of the wave function [51], while the method of least squares fit of energy levels [52] was applied in obtaining the angular part of the wave function. The results obtained in intermediate coupling showed a much better agreement with the experimental data than those using pure LS-coupling wave functions. Further improvements in the atomic structure calculations of Kr II were obtained by El-Sherbini and Farrag [38] when including configuration interaction effects. The results showed that the \( 4s^24p^4(D)4d\) level is strongly perturbed through interaction with the \( 45d^5(S)1^2\) level, in agreement with the earlier predictions from the Kr II analysis. Theoretical investigations of the \( 5s^25p^55d + 5s5p5^65s + 5s5p5^6 \) level structure in Xe II were performed by El-Sherbini and Zaki [53]. Taking into account, configuration-interaction effects in the calculations showed that some observed energy levels of the \( 5p^55d \) configuration were not correctly designated. A strong interaction between the \( 5p^55d \) and \( 5s5p^6 \) configurations was also reported. Moreover, the calculated energies of the \( 6s \) and \( 5d \) levels were improved considerably by introducing configuration interactions into the calculations. The presence of strong configuration interaction between the \( 45p^6 \), \( 4p^54d \), and \( 4p^55s \) configurations in singly ionized krypton [38] makes it difficult to perform accurate calculations for the energies, pumping rates, and lifetimes of levels in these configurations. Therefore, it was important to improve upon the previous calculations, see El-Sherbini [54,55], on the low lying \( 4p^54d \) and \( 4p^55s \) laser levels in this ion. Therefore, multi-configuration Hartree–Fock (MCHF) calculation in order to determine the lifetimes of these laser levels was done by El-Sherbini [56]. The results showed that some of these levels are metastable. They also suggest a two-step excitation from the ground state of the ion to the \( 4p^55p \) level involving some intermediate metastable states as a possible laser excitation mechanism.

Further developments in the field of atomic structure calculations were done by the studies of excitation of electrons in atomic isoelectronic sequences [57–59]. These studies are essential not only for better understanding of atomic structure and ionizing phenomena, but also they provide new laser lines which could be extended into the X-ray spectral region [60,61]. This in turn will help in the development of X-ray laser devices. Once X-ray lasers become reliable, efficient, and economical, they will have several important applications. First and foremost, their short wave lengths, coherence, and extreme brightness should allow the exploration of living structures much smaller than one can see with optical methods. They will also have important applications in high resolution atomic spectroscopy, diagnostics of high density plasmas, radiation chemistry, photolithography, metallurgy, crystallography, medical radiology, and holographic imaging. Shortly after the demonstration of the first soft X-ray amplification in neon-isoelectronic selenium by Mathews et al. [62], extensive work was done both theoretically and experimentally on other systems [63,64]. Progress toward the development of soft X-ray lasers with several plasma-ion media of different isoelectronic sequences was achieved at many laboratories [65,66]. A soft X-ray laser transitions in the Be-isoelectronic sequence were proposed by Krishnan and Trebes [67]. They suggested that intense line radiation from plasmas of Mn VI, P IV, Al V, Al V III, Al IX, and Al XI may be used to selectively pump gain at a number of different temperatures and electron densities for the \( 3p \rightarrow 3s \) laser transition in the highly charged ions of Be-sequence. Al-Rabban [69] has extended both the work of Krishnan and Trebes [67] and Feldman et al. [68], to the higher members of the Be-isoelectronic sequence and to more transition states (which are promising for X-ray laser emission). She
carried out an ab initio multi-configuration Hartree–Fock calculations of energy levels, atomic oscillator strengths, and radiative lifetimes for singly and doubly excited states in Be I and Be-like ions. Configuration interaction effects between the various configurations were included using the computer program code CIV3 described by Hibbert [70]. In this code, the N-electron energy and eigenfunctions are obtained by diagonalizing the Hamiltonian matrix, which may have quite large dimensions. The choice for the spatial (radial) part of the single particle wave functions is based on expansions in Slater-type orbitals [71]:

\[
P_{nl}(r) = \sum_{j=1}^{k} C_{jnl} r^{j-1} \exp(-\xi_{jnl}r)
\]

The coefficients in the expansion \(C_{jnl}\) as well as \(\xi_{jnl}\) in the exponents are treated as variational parameters.

Investigations of the possibilities of obtaining population inversion and laser emission could be achieved by calculating the level population of the excited states. These calculations were done by the group of atomic physics at the Physics Department of the Faculty of Science – Cairo University, solving the coupled rate equations [72]

\[
N_j = N_e \left( \sum_{i<j} A_{ij} + N_e \left( \sum_{i} C_{ij}^e + \sum_{j} C_{ij}^d \right) \right)
= N_e \left( \sum_{i<j} N_j C_{ij}^e + \sum_{i} N_i C_{ij}^d \right) + \sum_{i<j} N_i A_{ij}
\]

where \(N_j\) is the population density of level \(j\), \(A_{ij}\) is the spontaneous decay rate from level \(j\) to level \(i\), \(C_{ij}^e\) is the electron collisional excitation rate coefficient, \(C_{ij}^d\) is the electron collisional de-excitation rate coefficient, and \(N_e\) is the plasma electron density. The gain coefficient \(\alpha\) for Doppler broadening of the various transitions is given by Elton [73]:

\[
\alpha = \frac{j^2}{8\pi} \left( \frac{M}{2nkT_i} \right)^{1/2} A_{ud} N_e F
\]

where \(M\) is the ion mass, \(j\) is the transition wave length in cm, \(T_i\) is the ion temperature in K, \(u\) and \(l\) represent the upper and lower transition levels, respectively, \(N_j\) is the population of the upper level, and \(F\) is the gain factor.

Vriens and Smeets [74] gave empirical formulas for the calculation of rate coefficients in hydrogen atom. Their work was extended by Allam [75] to be valid for atoms with one electron outside a closed shell and also for two-electron atoms (ions). Allam [75] adopted the method of Palumb and Elton [76] for modeling plasmas of helium-like and carbon-like ions, and he has developed a computer program (CRMOC) in order to calculate excitation and de-excitation rate coefficients for two-electron system. In his program which was developed for collisional radiative model calculations, the principal quantum numbers of the excited states were replaced by effective quantum numbers. Using the above theoretical schemes, the atomic physics group was able to extensively investigate the possibility of X-ray laser emission in several isoelectronic systems, see for example Figs. 8 and 9. The studies include helium isoelectronic sequence [77], beryllium isoelectronic sequence [69,78], boron isoelectronic sequence [79–81], magnesium isoelectronic sequence [82], sodium isoelectronic sequence [83–85], silicon isoelectronic sequence [86–88], aluminum isoelectronic sequence [89], silicon isoelectronic sequence [90–92], sulfur isoelectronic sequence [93], potassium isoelectronic sequence [94], scandium isoelectronic sequence [95], and nickel isoelectronic sequence [96]. Most of the heavy members of the isoelectronic sequences studied radiate in the XUV and Soft X-Ray spectral regions (\(\lambda\) between 50 and 1000 Å). The reported stimulated emission transitions in these ions indicate that some of the transitions are promising and could lead to progress toward the development of XUV and Soft X-Ray lasers.

**Laser-induced breakdown spectroscopy (LIBS)**

Laser-induced breakdown spectroscopy is a form of optical (atomic) emission spectroscopy [97]. It is a technique based on utilizing light emitted from plasma generated via interaction of a high power lasers with matter (solids, liquids or gases). Assuming that light emitted is sufficiently influenced by the characteristic parameters of the plasma, the atomic spectroscopic analysis of this light shows considerable information about the elemental structure and the basic physical processes in plasmas. There is a growing interest in LIBS, particularly in the last 20 years because of its applications in the laboratory and in industry, art, environment, medicine, and forensic sciences [98–100]. Most commonly, LIBS has been applied to sensitive elemental analysis of solids, conductors

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**Fig. 8** Reduced fractional population for selected levels of Ni\(^{14+}\) ions at electron temperature 3/4 the ionization potential.

**Fig. 9** Gain coefficient of laser transitions against electron density at temperature 2 keV in E\(^{35+}\) ions.
and non-conductors, as well as liquid and gaseous samples [101]. It has many practical advantages over more conventional elemental analysis techniques. LIBS has been utilized to analyze thin metal films [102], and it has found more and more applications in monitoring of industrial processes [98], characterization of jewellery products [103], soil studies [104], pulsed laser thin film deposition [105], quality control of pharmaceutical products [100], cleaning [106], and in situ planetary exploration [107].

An enhancement of the LIBS sensitivity was achieved by introducing the double pulse technique [108]. The double pulse (DP)-LIBS configuration, which makes use of two laser pulses separated by a suitable temporal delay instead of a single pulse for inducing the plasma, was reported to give a substantial enhancement of the signal to noise ratio with respect to single pulse (SP)-LIBS configuration with a corresponding improvement of the limits of detections [109]. The double pulse laser ablation (DPLA) approach in relation to the spectral analysis was first reported by Piepmeyer and Malmöstad [110]. However, the first systematic investigation of (DPLA) was reported by Sattmann et al. [111]. They performed a quantitative microchemical analysis of low-alloy steels with single and double laser pulses, where they found that the analytical performance was considerably improved by the double pulse technique. The great contribution to the development of (DPLA) for practical analysis was made by Petukh et al. [112]. They compared radiation of plasma flares produced on exposure of metals to laser radiation in a monopulse generation mode in the case of single and double pulses with change in air pressure. They observed in the case of double pulses increases in the duration and the intensity of the radiation of the spectral lines. For elucidation of the double pulse laser ablation (DPLA) mechanisms, see, for instance, St-Onge et al. [113] and Noll [114]. DP-LIBS technique was also applied for the fabrication of nanosize particles, Tarasenko et al. [115] studied and analyzed the capabilities of laser ablation in liquids for fabrication metallic and composite nanoparticles. The technique offers the better controle over the particle formation process. They found that the mean size of the nanoparticles and their stability could be controlled by proper selection of the parameters of laser ablation such as temporal delays between pulses, laser fluence, and the sort of liquid used. Therefore, the optimal conditions favoring the formation of nanoparticles with a desired structure could be reached.

Parallel to the work on atomic structure calculations by our atomic physics group at the physics department, the group was also involved in the study of the physical parameters of plasmas generated by high power laser irradiation of solid targets (plasma diagnostics), applying the (LIBS) technique. The spectroscopic plasma diagnostics which is essentially based on the measurements of the optical radiation emitted from the plasma enables the group to obtain simultaneously a large amount of information about the plasma without disturbing it. Spectral fingerprints of optical plasma emission provide information about the physical and chemical processes that occur in the plasma. The spectra can contain individual spectral lines, band, or continuum radiation. Plasma emits line radiations resulting from bound–bound electronic transitions and continuum radiations resulting from free-bound and free–free electronic transitions. However, utility of spectroscopic diagnostics depends upon the knowledge about radiative behavior of atomic and molecular species and type of equilibrium attained in the plasma. It is assumed that the plasma in our laboratory (laboratory of lasers and new materials at the physics department) is in local thermodynamic equilibrium (LTE). In local thermodynamic equilibrium, all the species in the plasma, i.e., electrons, ions, and neutrals are in thermodynamic equilibrium except the radiation. This condition generally is observed to be valid in a collision dominated plasma such as high-pressure arc plasma produced in plasma torches. Small size of such plasmas allows radiation to escape to the surroundings. In (LTE) plasmas, the number of electronic transitions due to collisions between the first excited states and the fundamental level is 10 times larger than the number of transitions due to spontaneous emission. Collisions are mainly responsible for excitation and deexcitation, ionization, and recombination. The two main parameters that characterized the state of the plasma are namely the plasma temperature and the electron density. Knowledge of the temperature leads to understand the plasma processes occurring such as vaporization, dissociation, ionization, and excitation. The optical emission spectroscopic (OES) method for the determination of the plasma temperature is based on the measurements of the intensity of the spectral lines. In optically thin plasma, the integrated intensity of an atomic emission line is related to excitation energy, population density of upper state and transition probability as given by

\[ I_u = \frac{1}{4\pi} A_u n_u h \nu_{ul} L \text{ (W/m}^2\text{-ster)} \]  

(23)

where \( I_u \) is the line intensity of transition from upper level \( u \) to lower level \( l \) integrated over the plasma length \( L \), \( A_u \) is the spontaneous transition probability, \( n_u \) is the density of atom excited in the upper energy level \( u \), and \( h \nu_{ul} \) is the energy of each emitted quantum. The measurement of \( I_u \) gives only the population of upper level \( u \). When the thermal plasma is in (LTE), the density of atoms excited to the upper level is given by the Boltzmann distribution function:

\[ n_u = \left( \frac{n_0}{Z_0} \right) g_u \exp \left( -\frac{E_u}{kT} \right) \]  

(24)

where \( n_0 \) is the total density of atoms, \( g_u \) is the statistical weight of the upper state, \( E_u \) is the energy of upper state, \( k \) is Boltzmann constant, and \( Z_0 \) is the partition function defined by

\[ Z_0 = \sum_u g_u \exp \left( -\frac{E_u}{kT} \right) \]  

(25)

Substituting the value of \( n_u \) into Eq. (23), we get

\[ I_u = \frac{1}{4\pi} \frac{h c A_u}{\lambda_{ul}} \frac{n_0 L}{Z_0} g_u \exp \left( -\frac{E_u}{kT} \right) \]  

(26)

In case of the evaluation of absolute line intensity, one should know the initial composition, pressure and wave length of the emission line. The values of \( A_u, \lambda_{ul}, \text{ and } E_u \) can be obtained from spectroscopic tables. However, one must also know the plasma length, and an absolute spectral radiance calibration must be performed using a standard source. For relative line intensities measurement of the same species and stage of ionization, one needs not to know the values of partition function, \( n_0 \) and plasma emitting length. The ratio of two emission lines \( I_1 \) and \( I_2 \) is given by

\[ \frac{I_1}{I_2} = \frac{g_1 A_1 \lambda_2}{g_2 A_2 \lambda_1} \exp \left( \frac{E_2 - E_1}{kT} \right) \]  

(27)
The terms in Eq. (26) can be arranged as

\[
\frac{I_{\text{ul}}}{g_s A_s} = \frac{hcn_0 L}{4\pi Z_0} \exp \left( -\frac{E_n}{kT} \right)
\]

and therefore, we can write

\[
\ln \left( \frac{I_{\text{ul}}}{g_s A_s} \right) = B - \frac{E_n}{kT}
\]

This is an equation of straight line where \( B = \ln \left( \frac{hcn_0 L}{4\pi Z_0} \right) \) is a constant. If \( \ln \left( \frac{I_{\text{ul}}}{g_s A_s} \right) \) values are plotted against \( E_n \), the temperature is given by the reciprocal of the slope of the straight line. This is called the atomic Boltzmann plot method. The other key parameter in the diagnostics of plasma is the electron density. Determination of the electron density \( n_e \) is based on the broadening of emission lines from the plasma. It is assumed that the Stark effect is the dominant broadening mechanism, in comparison with Doppler broadening and the other pressure broadening mechanisms, due to collisions with neutral atoms. The validity of this assumption was generally admitted in works on (LIBS) and is justified in various studies [116,117]. For the linear Stark effect (hydrogen and hydrogenic ions), the following equation is valid [116]

\[
n_e = C(T, n_e) \Delta \lambda_s
\]

where \( \Delta \lambda_s \) is the full width at half maximum (FWHM) of the spectral line and \( C(T, n_e) \) is a coefficient that is only a weak function of electron density and temperature. For elements other than hydrogen and hydrogen-like ions, the quadratic Stark effect acts on the total half width at half maximum (HWHM), due to collisions with electrons and ions, \( \omega_{\text{total}} \) is approximated given by Griem [117]

\[
\omega_{\text{total}} = \left[ 1 + 1.75A(1 - 0.75R) \right] \omega, \frac{n_e}{n_e^{ref}}
\]

where \( \omega_{\text{total}} \) is the electron impact (half) width, \( A \) is the ion broadening parameter, \( \omega \) is the Stark width, \( n_e^{ref} \) is a reference electron density, usually of the order of \( 10^{16} \) or \( 10^{17} \) cm\(^{-3} \), and \( R \) is the ratio of the mean distance between ion and Debye radius. Eq. (31) is used in LIBS to determine the electron density from experimentally measured line widths of selected lines. For accurate measurements of the electron density, spectral lines as isolated as possible and emitted in optically thin conditions have to be selected. Stark broadening of isolated spectral lines of non-hydrogenic neutral atoms and ions is due mainly to electrons. As a consequence, the contribution of quasi-static ions was generally neglected, and hence, the Lorentzian FWHM can be approximated by

\[
\omega_{\text{total}} = \omega, \frac{n_e}{n_e^{ref}}
\]

Our group at the laboratory of lasers and new materials at the physics department has studied emission spectra from laser-induced titanium plasma [118] and measured population density and temperature of argon metastable \( ^1S_\text{I} \) state using tunable diode laser-absorption diagnostic technique [119]. An elemental analysis of some minerals using laser-induced breakdown spectroscopy (LIBS) was performed at the laboratory by El-Sergany et al. [120].

Self-absorption effect can distort the spectral line shape and therefore produces apparently an increase in the line width and a decrease in the line intensity, Griem [116]. It is originated mostly from cooler boundary layer of the plasma which contains much lower population density [117]. This effect will mislead investigators and can give in accurate results for the plasma parameters. Therefore, for the work on LIBS, one has to check the presence of self-absorption. Our group has evaluated self-absorption coefficient of Aluminum emission lines in laser-induced breakdown spectroscopy (LIBS) measurements, see El-Sherbini et al. [121]. Hydrogen lines, especially those of the Balmer series exhibit linear Stark effect, are the most strongly broadened lines, and they are easy to measure. Therefore, they are oftenly used to determine the electron density. Our group, El-Sherbini et al. [122], has measured the electron density in a laser produced plasma experiment using the Stark broadening of \( \text{H}_\alpha \)-line at 656.27 nm. This line is produced from the interaction of a Q-switched Nd-Yag laser beam at the fundamental wave length of 1.06\( \mu \)m with a plane solid aluminum target in a humid air Fig. 10. In this experiment, light emitted from the plasma plume is collected by a lens and optical fiber arrangement using an imaging spectograph with ICCD camera, see Fig. 10. The wave length scale was calibrated with a low pressure Hg-lamp. The emitted light was collected in the wave length region from 200 to 1000 nm. The gain of the camera was kept fixed at a maximum level of 250. The measurement was confirmed by observing the spectra emitted at wave length regions from plasma at delay times from 30 to 50\( \mu \)s which gave the same band width. Identification of the different spectral lines was carried out using a software spectrum analyzer (version 1.6). The agreement between the measured electron density from both the \( \text{H}_\alpha \)-line and the Al II-line at 281.62 nm confirms the reliability of utilizing the \( \text{H}_\alpha \)-line as an electron density standard reference line in LIBS experiments.

Our group has also measured the Stark broadening of atomic emission lines in non-optically thin plasmas by of laser-induced breakdown spectroscopy, El-Sherbini et al. [123]. An assessment of LIBS diagnostics of plasma using the hydrogen \( \text{H}_\alpha \)-line at different laser energies in air was performed by the group, El-Sherbini et al. [124]. Moreover, the atomic physics group has applied the diode laser atomic absorption spectroscopy (DLAAS) technique to assess the degree of optical opacity of plasma at the wave length of \( \text{H}_\alpha \)-line, El-Sherbini et al. [125]. They found that the plasma associated with metallic targets is almost optically thin at the \( \text{H}_\alpha \)-line over all fluencies and at delay times \( \approx 1 \)\( \mu \)s, but rather thick for hydrogen-rich targets (plastic and wood) over all delay times and fluencies.

Recent measurements of plasma electron temperature utilizing magnesium lines appeared in laser produced aluminum plasmas were done by the group, El-Sherbini et al. [126]. This work shows that the Mg I and Mg II lines appeared at the short
wave length region of the LIBS spectrum are good candidates for measuring the temperature of the plasma in LIBS experiments, but after correction against self-absorption. The spectrometric measurements of plasma parameters utilizing the target ambient gas O I and N I atomic lines show the reliability in the values of the electron density and the temperature of the plasma generated by the interaction of laser beam with solid targets, El-Sherbini et al. [127].

Advances in the LIBS measurements at our laboratory of lasers and new materials in the physics department at the faculty of science – Cairo University were achieved by studying the X-ray/particle emission from plasmas produced by laser irradiating nano-structured targets, Hegazy et al. [128]. In this experiment, nano-copper structures evaporated onto copper bulk disks and nano-gold structures evaporated onto gold ones were used. An ion collector and X-ray semiconductor diode were used to study the ion and X-ray emission, respectively. A comparison of both studies in the case of nano-structured targets and bulk targets was performed at different laser fluencies ($1 \times 10^9 - 1 \times 10^{15}$ W/cm$^2$) on the target. A 20% increase in the X-ray emission for nano-gold with respect to the bulk gold was observed; however, the X-ray emission in the nano-copper and copper was the same. At high laser intensities, the presence of non-linear processes in the preformed plasma may significantly increase the temperature of the fast electrons, and therefore mainly, the hard components of X-ray radiation at the fast ion emission are produced that does not happen in low energy nano-second experiments. Another progress was also recently achieved at our laboratory, by observing enhancements in LIBS signals from nano vs. bulk ZnO targets and nano-based targets, El-Sherbini et al. [129]. The study revealed that the signal enhancement cannot be attributed to the plasma temperature difference or the difference in the electron density. The signal enhancement depends only on the relative atomic concentration in the plasma created from the nano-based material with respect to the bulk-based plasma. This can be qualitatively explained in terms of the collisional radiative modeling; for an electron density in the order of $10^{17}$ cm$^{-3}$, the collision processes are the dominant ones. Therefore, the enhanced emission of Zn I-lines from the nano-based target Fig. 11 could be attributed to the higher concentration of neutral atoms in the nano-based material plasma with respect to the corresponding bulk-based ZnO material.

The evolution of Al plasma generated by Nd-YAG laser radiation at the fundamental wave length was studied at our laboratory by Hegazy et al. [130]. The results showed that the plasma temperature and the electron density are strongly dependent on the time and on the laser energy. More recently, another study was done at our laboratory by the same authors on titanium targets [131]. They investigated the spectral-evolution of nano-second laser interaction with Ti target in air. In this study, time resolved optical emission spectroscopy (OES) has been successfully employed to investigate the evolution of plasma produced by IR- and visible-pulsed laser beams irradiating a titanium target in ambient air at atmospheric pressure. The characterization of the plasma-assisted pulsed laser ablation of the titanium target was discussed. The obtained temperature was in a good agreement with the one obtained from Ti II spectral lines previously suggested by Herman et al. [132]. Moreover, the Stark broadening method has been employed in the experiment for electron density measurements.

In conclusion, plasma diagnostic technique with LIBS is essential for accurate determination of plasma parameters such as temperature and electron density. It can give more

![Fig. 11](image)

(a) experimental setup; (b) TEM image of the ZnO-20 nm size; (c) enhanced emission of the Zn I – line from the nano-based target (red color) in comparison to that from the bulk-based material (black color).
information about the atomic processes and the dynamic nature of the plasma. This information plays a major role in astrophysics and in controlled thermonuclear fusion research.

**Laser cooling and Bose–Einstein condensation**

One of the research highlights in atomic physics in recent years is the study of laser cooling and Bose–Einstein condensation. As early as 1917, Einstein [133] had predicted that momentum is transferred in the absorption and emission of light, but it was not until the mid eighties that such optical momentum transfer was used to cool and trap neutral atoms [134,135]. By properly tuning laser light close to atomic transitions, atomic samples can be cooled to extremely low temperatures [136]. In 1985, National Bureau of standards group [137] cooled atoms in a thermal atomic beam in the range of 50–100 m K by irradiating them with a beam of counter-propagating against their motion. If the laser is tuned to the low frequency side of an atomic resonance, an atom moving against the direction of a laser beam will see the beam Doppler-shifted into resonance, while the beam co-propagating with the atom will be Doppler-shifted out of resonance. Thus, the atom will preferentially scatter photons from the beam opposing the direction of motion. A step further was cooling in three dimensions, and this was accomplished by creating three sets of counter-propagating beams along the x, y, and z axes. The idea was suggested by Hansch and Schawlow [138] and was demonstrated by Chu et al. [139]. Because the cooling force is viscous (linearly proportional to the velocity of atoms for low velocities), the laser beams that generate the drag force was named “optical molasses.”

A substantial improvement in laser cooling was achieved with optical molasses (OM), where three intersecting orthogonal pairs of oppositely directed laser beams are used to severely restrict the movement of atoms in the intersection region. With this technique, atoms could be cooled to temperatures between 1 and 0.1 μK. The first cooling and trapping of neutral atoms (sodium atoms) was accomplished with optical molasses in combination with magneto-static fields Fig. 12, and the technique was named magneto-optical trapping MOT [140].

When atoms in a gas are cooled to extremely low temperatures, they will (under the appropriate condition) condensate into a single quantum mechanical state known as a Bose–Einstein condensate (BEC). This phenomenon was predicted by Nath Bose and Albert Einstein in 1925 when they pointed out that at low temperatures particles in a gas could all reside in the same quantum state. The Bose–Einstein condensation exhibits a new state of matter which occurs at extremely low temperatures when the de Broglie wave length of atoms becomes comparable to the average distance between them Fig. 13. The temperatures reached by laser cooling are impressively low, but they are not low enough to produce BEC in gases at the densities that are realizable experimentally. In the experiments performed to obtain BEC of alkali gases, evaporative cooling which was first suggested by Hess [141] was used after laser cooling. With this additional cooling, the temperature reached about 2 nK, which was sufficiently low to form BEC. The basic physical effect in evaporative cooling is that if particles escaping from a system have energy higher than the average energy of particles in the system, the remaining particles are cooled. If one makes a hole in the magneto-optical trap (MOT), only atoms with an energy at least equal to the energy of the trap at the hole will be able to escape. In practice, one can make such a hole by applying radio-frequency radiation that flips the spin state of an atom from a low-field seeking one to a high field seeking one, thereby expelling the atom from the trap [142]. In order to diagnose the dense and cold samples of trapped atoms, the time of flight (TOF) method is used [143]. The (TOF) method was capable of mapping out the velocity distribution for both hyperfine and ground states of dilute gases of alkali atoms along their path through the trap, see Fig. 14.

Fig. 12 The technique of magneto-optical trapping MOT, Scientific American (January 1998).

by Nath Bose and Albert Einstein in 1925 when they pointed out that at low temperatures particles in a gas could all reside in the same quantum state. The Bose–Einstein condensation exhibits a new state of matter which occurs at extremely low temperatures when the de Broglie wave length of atoms becomes comparable to the average distance between them Fig. 13. The temperatures reached by laser cooling are impressively low, but they are not low enough to produce BEC in gases at the densities that are realizable experimentally. In the experiments performed to obtain BEC of alkali gases, evaporative cooling which was first suggested by Hess [141] was used after laser cooling. With this additional cooling, the temperature reached about 2 nK, which was sufficiently low to form BEC. The basic physical effect in evaporative cooling is that if particles escaping from a system have energy higher than the average energy of particles in the system, the remaining particles are cooled. If one makes a hole in the magneto-optical trap (MOT), only atoms with an energy at least equal to the energy of the trap at the hole will be able to escape. In practice, one can make such a hole by applying radio-frequency radiation that flips the spin state of an atom from a low-field seeking one to a high field seeking one, thereby expelling the atom from the trap [142]. In order to diagnose the dense and cold samples of trapped atoms, the time of flight (TOF) method is used [143]. The (TOF) method was capable of mapping out the velocity distribution for both hyperfine and ground states of dilute gases of alkali atoms along their path through the trap, see Fig. 14. The peculiar BEC gaseous state was created and diagnosed in the laboratory for the first time in 1995, by Carl Wieman, Cornell, and co-workers [144] at the JILA laboratory in Boulder and by Ketterle and co-workers [145] at the MIT, using the powerful laser cooling method together with evaporative cooling method. Nowadays, the phenomenon of BEC has become an increasingly active area of research both experimentally and theoretically. Research work on this form of matter is relevant to many different areas of physics – from atomic clocks and
quantum computing to superfluidity, superconductivity, atom lasers, and quantum phase transition.

The pioneering work on BEC by Wieman, Cornell and Ketterle and others [144–146] was centered on atoms that were bosons, particles with integer spins. But atomic physicists have since extended their work to create Fermi gases from atoms that are fermions, particles with half odd integer spins. The advances came in 2003 by Deborah Jin and co-workers at the university of Colorado [147], who created a BEC of weakly bound molecules from ultra-cold Fermi gas of potassium-40 atoms. This new form of matter may help in a better understanding of superconductivity and could pave the way toward a superconductor that works at room temperature. Recently, research efforts in the field of BEC were directed toward storing ultra-cold bosonic and fermionic quantum gases in artificial periodic potentials of light, i.e., in optical lattices [148]. This has opened innovative manipulation and control possibilities, in many cases creating structures far beyond those currently achievable in typical condensed matter physics systems. Trapping ultra-cold quantum gases in optical lattices will open the door to a wide interdisciplinary field of physics ranging from non-linear dynamics to strongly correlated quantum phases and quantum information processing.

Around 2004, the group of atomic physics at the Physics Department of the Faculty of Science – Cairo University started theoretical work on BEC, and a brief review on the subject was given by El-Sherbini [149]. The group adopted in their studies the semiclassical approximation, i.e., the density of state (DOS) approach [150]. In this approach, the sum over the discrete spectrum for the thermodynamic quantities of the Bose–Einstein condensate was replaced by an integral weighted of a piecewise DOS. The latter was calculated via the technique of the high temperature expansion for the partition function [151]. This approximation has been widely used in variety of problems in statistical physics [152]. These studies showed that the resulting thermodynamic parameters depend crucially on the choice and construction of the DOS [153].

A generalization of the semiclassical approximation was suggested by Hassan and El-Badry [154], allowing for an essential extension of its region of applicability. The parameterized DOS has considered the effects of finite size, anisotropic of the harmonic potential, and the positive chemical potential; all of them simultaneously. The latter effect is similar to the effect of repulsive interaction provided by the mean field theory approach [155]. The outcome results provide a solid theoretical formulation for the existing experiments [155]. The generalized semiclassical approximation was also applied by Hassan and El-Badry [156] in the study of thermodynamic properties of quasi-equilibrium magnons in crystalline bulk materials and thin films and by Hassan [157] in the calculations of effective area and expansion energy of trapped Bose gas in a combined magnetic-optical potential. Hassan et al. [158] were able to calculate the critical temperature of a Bose–Einstein condensate in a 3D non-cubic optical lattice. Moreover, Hassan et al. [159] have studied recently the thermodynamic properties of condensed 39 K Bose gas in a harmonic trap. In particular the critical atoms number and its corresponding temperature are predicted via the graphical representation [160]. A step further in the advancement of research on BEC was carried out by the group, where they determined the thermodynamic properties of rotating Bose gas in a harmonic trap [161].

Nowadays, the work on fast rotating gases in Bose Einstein condensates which was initiated by the group of Dalibard at the Ecole Normale Superieure (ENS) laboratory in Paris [162] was recently under investigation by our group. Usually, the effective trapping potential \( V_{\text{eff}} \) for fast rotation regime is approximated by harmonic plus weak quartic potential [163]:

\[
V_{\text{eff}} = \frac{m}{2} \left[ \omega_x^2 x^2 + \omega_y^2 y^2 + \frac{1}{4} \Omega^2 r^2 - \frac{m}{2} \Omega^2 r^2 \right] \tag{33}
\]

where \( m \) is the mass of the atom and \( r^2 = x^2 + y^2 \) is the perpendicular radius to the rotation axis \( z \). The parameter \( x \) classifies the strength of the quartic term and, moreover, plays an important role in exploring the fast rotation regime. The parameter \( x \) must be greater than zero; otherwise, the effective potential felt by the atom \( V_{\text{eff}} \) would tend to \(-\infty \) for \( r \rightarrow \pm \infty \).

In a recent paper by the group, El-Sherbini et al. [164], modified semiclassical approximation is provided to study the effects of finite size, the positive chemical potential, and anisotropy of the trap, on the thermodynamical properties of a rotating gas in a harmonic plus quartic trap. Fig. 15 presents the characteristic dependence of the condensate fraction \( N_0/N \) on the reduced temperature \( T/T_0 \) and the rotation rate \( z \). It shows the monotonically decreasing nature of the condensate fraction due to the increase in the reduced temperature everywhere. This decrease is minor in the slow rotation range and rapid in the fast rotation range monotonically in agreement with the experimental observations and the numerical calculation [165]. Our results also provide a correction due to the finite size and positive chemical potential effects (interaction effect) for the results of Kling and Pelster [165]. Both of them show a significant quenching of the condensate fraction and a shift of the critical temperature toward the lower values.

Fig. 14 The density profile of the condensate after time of flight expansion, giving emphasis to the velocity distribution, Ref. [144].

Fig. 15 The condensate fraction \( N_0/N \) as a function of the reduced temperature \( T/T_0 \) and the rotation ratio \( z \).
The study of BEC in a rotating optical lattice was done in the atomic physics group by Abdel-Gany et al. [166]. The effective trapping potential $V_{\text{eff}}$ for rotating optical lattice trap is considered to be,

$$V_{\text{eff}} = \frac{m}{2} \left[ \omega_x^2 x^2 + \omega_z^2 z^2 \right] + V_0 \left[ \sin^2 \left( \frac{\pi x}{d} \right) + \sin^2 \left( \frac{\pi z}{d} \right) \right] - \frac{m}{2} \Omega r_z^2,$$

(34)

where $d$ is the lattice spacing. The dependence of the condensate fraction, critical temperature, and the heat capacity on the recoil frequency, optical potential depth, and rotating frequency is investigated. The results show that the normalized recoil frequency can control the characteristic shape of the condensate fraction. Fig. 16. Furthermore, our results show that the rotating BEC in optical lattice is accompanied by a peak in the critical temperature $T_c$ at defined value for the normalized recoil frequency $S_z$, Fig. 17. This remarkable behavior was observed experimentally by Burger et al. [167] and numerically detected for non-rotating boson gas in optical lattice by Blakie et al. [168].

Bose–Einstein condensates in optical lattices represent model systems for solid state physics with unprecedented level of control. They can be used for exploring a wide range of fundamental problems in condensed matter physics such as Mott (metal-insulator) transitions, Anderson localization, type II superconductivity, and quantum Hall effect.

Exactly solvable models are very important in physics since they enable physicists to estimate the accuracy of the different approximate methods. In condensed atomic systems, the major problem involves solving many body interacting systems. The group of atomic physics at the Physics Department of the Faculty of Science – Cairo University has started to work on an exactly solvable model for a system composed of two species of identical Bosons in three-dimensional space interacting via harmonic potential. Aboul-Seoud et al. [169] have studied a system of two N-particles of identical Bosons with equal numbers, assuming that particles belonging to the same species repel each other and particles belong to different species attract each other. It was realized that the system is condensed in one channel when the coupling strengths are identical and in two channels when the coupling strengths are different. This study will enable us to understand the behavior of the miscibility of atomic species as a function of the variations in the coupling strengths between the Bose species.

Further research activities in the field of BEC are being pursued by members of our atomic physics group. The research focuses on the study of the thermodynamics of Bose systems at finite temperature and inter-particle interactions. A system of equations describes trapped and condensed Bose systems at finite temperature by being investigated. The description of weakly interacting Bosons at zero temperature saw a breakthrough in 1947 with the seminal work of Bogoliubov [170], who derived the mean field equations that govern the statics, as well as the dynamics, of the condensed state. Bogoliubov’s work correctly describes the microscopic low energy spectrum of the condensed state of a uniform condensate. Further progress was achieved through the work of Gross and Pitaevskii [171,172], who derived the zero temperature equations of a non-uniform condensate. Their approach has become the basis for much of the theoretical work on degenerate Bose gases. This work saw a surge of activity after the experimental realization of Bose–Einstein Condensation in electromagnetic-optical traps containing metastable vapors of alkali atoms [144,145].

More work is needed, however, to address both dynamical and thermodynamic aspects of the condensed phase at finite temperatures. At finite temperature, the system contains bosonic quasi-particle excitations. As the temperature is raised, the density of such excitations increases and their interactions with the condensate particles, as well as among themselves, may have a pronounced effect on the physics of the trapped system. Another aspect comes about because of the experimental possibility to fine-tune the inter-particle interactions using Feshbach resonances. This means that the interactions between the atoms in the condensate are not weak anymore. The validity of many of the approaches that have been developed with both finite temperature and strong inter-particle interactions has been questioned by Yukalov [173]. In order to treat these effects consistently, he derived a system of coupled equations for the main quantities that govern the macroscopic properties of degenerate trapped gases. These quantities include the condensate density, the density of non-condensate particles, the local speed of sound, and the anomalous density of the trapped system. We are comparing the solutions we are obtaining with known experimental results [174]. This approach is promising since it appears that it can be verified experimentally.

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**Fig. 16** The condensate fraction $N_0/N$ as a function of the normalized recoil frequency $S$ and the reduced temperature $T/T_0$.

**Fig. 17** The normalized recoil frequency $S$ as function of the critical temperature $T_c$ and the optical depth $V_0$. 
easily generalized to include non-uniformities due to vortices or other special excitations, as well as dynamic phenomena at finite temperature. The results of our work are currently being prepared for submission for publication.

**Concluding,** I would like once more to stress that this review article was by no means an attempt to cover the whole of developments of atomic physics of the last half century. Rather, it was aimed at outlining the contributions of the atomic physics group of Cairo University that I had the honor to lead for the past four decades, as well as the research work I conducted during my scientific visits to the FOM-Institute for Atomic and Molecular Physics in Amsterdam.

With future publications, our group hopes to continue participating in the worldwide edifice of atomic physics.

**Conflict of interest**

The author has declared no conflict of interest.

**Compliance with Ethics Requirements**

This article does not contain any studies with human or animal subjects.

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