A Quantum Electromagnetic Theory of the Pions, 
Muons and Their Emitting Particles (I)

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Abstract. In direct accordance to the overall relevant experimental demonstrations, we represent the charged pion \( \pi^- \) as a heavy electron \( h^- \) in precessional-orbital (P-O) motion at essentially the light speed \( c \) about \( \bar{\nu}_c \)-orbit of a normal at quantised angle \( \pi - \theta_{1/2} = -\arccos(1/\sqrt{3}) \) to the z axis. \( h^- \) is the level \( N = 1 \) oscillation of charge \(-e\) and its electromagnetic radiation originally generated in the weak potential field of another particle. The P-O kinetic energy current and two additional opposite ones created upon \( \pi^- \) decay represent confined neutrinos \( \bar{\nu}_c, \bar{\nu}_\mu, \bar{\nu}_\mu \). The muon \( \mu^- \) is a \( xy \)-projected \( h^- \) in two superposing P-O motions along \( \bar{\nu}_c, \bar{\nu}_\mu \)-orbits of normals at angles \( \pi - \theta_{1/2}, \theta_{1/2} \) to \( z \). The \( \mu^- \) (rest) mass is a geometric projection of the reduced \( \pi^- \) mass, \( M_\mu = (M_\pi - M_\mu)/\cos \theta_{1/2} = 105.86\) MeV. The \( \mu^- \) mass is fundamentally predetermined by the mixed two states \( m_2 = -1, +1 \) of level \( n = 2 \) of a double heavy positronium in the CM frame produced in a relativistic \( e^-e^+ \) collision, and is \textit{ab initio} predicted to be \( M_\mu = (\frac{3}{4} + 1)M_\pi = 105.549 \) MeV, where \( \alpha = e^2/4\pi\epsilon_0hc \). The un-projected \( n = 2 \) level gives the bound \( \pi^- \) mass containing a friction term \( \Omega_\mu, M_\pi + \Omega_\mu = (\frac{3}{4} + 1)M_\pi = 140.525 \) MeV. Their antiparticles \( \pi^+, \mu^+ \) and the tauons \( \tau^- \) can be similarly represented. The remaining unstable elementary particles can be constructed as composites of two or more single charged ones in certain spatial quantised P-O motions.

1. Introduction
In the Standard Model (SM) description[1, 2], elementary particles are divided into \( 2 \times 6 \) leptons, baryons / mesons made of \( 2 \times 6 \) quarks, and five or so force mediators. Regarding their weak decays, such as the neutron \( \beta \) decay \( n \rightarrow e^-\bar{\nu}_e\), the SM quark model assumes that the decay product particles \( e^-, p, \bar{\nu}_e \) do not exist until \( n \) decays, and that they are instantaneously produced and emitted upon the \( n \) decay. The SM has been successful in reproducing the charges, spins, C, P, T symmetries of the hundreds of observational elementary particles, and predicting their decay rates - absolute or comparative. But the SM has unnatural aspects, concerning the quarks and weak decays in particular. The baryons and mesons are made of quarks rather than the particles they decay into, which is an abrupt departure from the atomic and nuclear descriptions. Free quarks are never observed in experiment. The SM weak interaction Hamiltonian \( H = G/(4\pi)^2 \) is a phenomenological construction. The weak force is not predicted. The weak, strong, gravitational and electromagnetic forces are not unified. The basic questions common to the regular particles \( e^-, p \) (being the two only permanent particles - why?) remain outstanding, including the origin of mass, the nature of matter waves, and the cause of gravity.

The Internally Electrodynamic (IED) particle model (see a review and the original references given in [3]) is a complementary approach to the SM and is beyond the SM. The IED model was developed using overall observational properties of particles as input information, and
aimed from the beginning to achieve a unification of the three basic mechanics and of the four fundamental forces. In terms of this model, the electron $e^-$ and the proton $p$ are composed of oscillatory charges $-e$ and $+e$ and their electromagnetic radiation (EMR) fields produced in a polarisable dielectric vacuum filled of vacuumons[4, 5]. Their distinct masses are uniquely determined [6] by the quantum oscillation levels of $-e, +e$ in their vacuum potentials that are asymmetric to $-e, +e$. The masses of their antiparticles $e^+, \bar{p}$ are determined accordingly by pair productions. The remaining single charged particles, including the six manifestly free forms $\pi^+, \mu^+, \tau^+$ and ones in (confined) higher excited states such as comprising $\rho(770)$, are the main subject of investigation of this paper. We shall show that, in accordance to the overall experimental demonstrations, these can be generally described each as a charge $-e$ or $+e$ oscillation and the resulting EMR originally generated in the weak potential field of another particle, that are as a whole in certain precessional orbital (P-O) motions. The remaining unstable elementary matter particles such as $n, \Lambda, \pi^0, \rho(770)$ are composites of two or more single charged ones. A formal $e^-, p$ model of the neutron $n$ and first principles solutions for spin, magnetic-hence weak-interaction force, coupling constant, and (unpublished) mixing angle have been achieved in [7]. The $\Lambda$ hyperon is the simplest baryon emitting a pion $\pi^-$, but otherwise a direct analogue of $n$, and will serve a prototype system for the model constructions in this paper.

The formal representation divides in three sections. In Sec 2, using the relevant observations as input information we propose the existence of mass states "heavy electrons (or positrons)", originally generated by charges $-e$’s (or $+e$’s) in the weak potential fields of other particles. In terms of this we propose the structures of the pions $\pi^\pm$ and muon $\mu^\pm$. In Sec 3, we present a generalised first principles description of the masses of the single charged particles such as $\pi^\pm, \mu^\pm, e^-, p$, and the composite particles such as $n, \Lambda$. In Sec 4, we predict the existence of a double heavy positronium preceding the rudimentary productions of $\mu^\pm, \pi^\pm$, etc. and, based on its eigen level $n = 2$ solutions, we ab initio predict the masses of $\mu^\pm$, $\pi^\pm$. Representation of the remaining unstable composite elementary particles will be described in a separate paper.

2. Structures of the charged pions and the muons
2.1 The charged pions $\pi^-, \pi^+$

The simplest composite baryon emitting a charged pion $\pi^-$, and secondarily a muon $\mu^-$, is the $\Lambda$ hyperon. Observationally, $\Lambda$ has an identical charge (0), spin $\frac{1}{2}$, and analogous decay reaction ($\Lambda \rightarrow \pi^- p \rightarrow \mu^- \bar{\nu}_\mu p$) to those of the neutron $n$ ($0, \frac{1}{2}$, $n \rightarrow e^- \bar{\nu}_e p$), but $\Lambda$ has a heavier mass 1115.6 MeV. These suggest an analogous structure of $\Lambda$ to $n$ [7] (Fig 1a, Inset), except in place of $e^-$ of $n$, $\Lambda$ apparently has a "heavy electron" ($h^-$) in spatial quantised P-O motion relative to $p$ (Fig 1a). In accordance to the implication of $\Lambda$, the observational $\pi^-$ charge $-e$, spin $s_\pi = 0$, mass $M_\pi = 139.569$ MeV, decay reaction $\pi^- \rightarrow \mu^- \bar{\nu}_\mu \rightarrow (e^- \bar{\nu}_e \nu_\mu) \bar{\nu}_\mu$ and rudimentary direct production reaction $e^- e^+ \rightarrow \rho(770) \rightarrow \pi^- \pi^+$, we propose a two-step description of the $\pi^-$ structure: (1) There presents a (confined) particle state called heavy electron, $h^-$, that has a charge $-e$, spin $\frac{1}{2}$ as the electron $e^-$, and that is yet a heavier mass state, the charge $-e$ oscillation and its resultant EMR of a mass $M_{h^-}$ (originally) generated in the weak potential field $V \frac{1}{2}$ of another particle. (2) The pion $\pi^-$ is a heavy electron $h^-$, of spin $S_{h^\pm} = \frac{1}{2} h$ in $z$ direction, in P-O (precessional-orbital) motion at essentially the light speed, $v_\perp = c$, along $\bar{\nu}_e$-orbit of a normal $(-z')$ at quantised angle $\pi - \theta_{\perp}$ to the $z$ axis (Fig 1b). The P-O kinetic energy current along $\bar{\nu}_e$-orbit resembles a confined antineutrino $\bar{\nu}_e$, of an apparent rest mass $M_{\bar{\nu}_e}$. Two equal but opposite P-O momentum currents along $\nu_\mu$, $\nu_\mu$-orbits (of confined neutrinos $\nu_\mu, \nu_\mu$) are generated momentarily upon $\pi^-$ decay in an explosive collision (Fig 1b, Inset), these do not contribute to the $\pi^-$ dynamical variables. By virtual of its antisymmetric properties, $\pi^+$ is a heavy positron $h^+$ in P-O motion along orbit $\nu_\mu$. The rest mass of $\pi^-$ (or $\pi^+$) is formally $M_\pi = M_{h^-} + M_{\bar{\nu}_e}$.

The total and the $z$ component angular momenta of the P-O motion of $h^-$ in the lab frame
are given by the eigen solutions of bound state $n = 2, \, l = 1, \, j = l - i'^{\pi} = \frac{1}{2}$ in the original $V_{n}^{*}$ field (e.g. of $p$ as in A) similarly as for $n(7)$. $J_{2} = |r_{2}^{A}|(m_{e}u_{2}^{A}) = \sqrt{j(j+1)}h = \sqrt{\frac{3}{2}}h, \, J_{2}z = J_{2}^{z} = J_{2}^{z}(\cos(\pi - \theta_{2}^{z}) = -j\hbar = -\frac{1}{2}\hbar), \, \text{where} \cos(\pi - \theta_{2}^{z}) = -1/\sqrt{3}, \, m_{e} = \gamma_{2}M_{\gamma}, \, \gamma_{2} = (1 - (u)^{2}/c^{2})^{-1/2} (\mathcal{M}_{A}, \text{is the reduced mass and } r_{1A}, \text{the orbit radius of } \pi, \, p \text{ comprising } A). \, \text{Those of the total system } \pi^{*}(c_{2}^{2}[u_{2}]) \text{ are } S_{2z} = J_{2z} + S_{h_{z}} = \left(-\frac{1}{2} + \frac{1}{2}\right)\hbar = s_{2}\hbar = 0, \, s_{2} = 0, \, \text{and } S_{z} = \sqrt{s_{n}(s_{n} + 1)\hbar} = 0. \, \text{The kinetic energy is } T_{\pi} = \frac{\gamma}{\gamma^{2}}m_{\pi}v_{\pi}^{2} = \frac{\gamma}{\gamma^{2}}J_{2}/r_{1}.

2.2 The muons $\mu^{+}$ - $\mu^{-}$ In direct accordance to the observational charge $-e$, spin $s_{\mu} = \frac{1}{2}$, mass $M_{\mu} = 105.658 \, \text{MeV}$ and decay reaction $\mu^{+} \rightarrow e^{+}\nu_{e}\mu_{\mu}$ of $\mu^{+}$, to its specific production from $\pi^{-} \rightarrow \mu^{-}\bar{\nu}_{\mu}$ and the $\pi^{-}$ structure of Sec 2.1, we propose: By an apparent loss of mass energy projected in the $y'z$ plane(s), $h^{*}$ transforms to a $xy$-projected mass state $h_{x'y}$ of mass $M_{h_{xy}}$, and of an unchanged charge $-e$ and spin $s_{h_{xy}} = \frac{1}{2}$ = $s_{h_{xy}}/h$ in $+z$ direction here. The muon $\mu^{*}$ is a $h_{x'y}$ in rotational motion along two coinciding elliptics $\nu_{y'z}, \nu_{y''z}$ projected from the $\nu_{e}, \nu_{\mu}$-orbits in the $y'z$ (or $y''z$) plane. The $\nu_{y'z}$- (or $\nu_{y''z}$-) motion is the supposition of two P-O motions at speed $v_{z} = c$ each along $\nu_{e}, \nu_{\mu}$-orbits of equal radius $r_{1\mu}$, and of normals $z', z''$ at quantised angles $\pi - \theta_{1}^{z}$ and $\pi - \theta_{2}^{z}$ to the $z$ axis (Fig 2a).

The projections of the $\nu_{e}, \nu_{\mu}$- or $-e$ orbits in the $y'z$ plane, $\nu_{y'z}, \nu_{y''z}$, or in the $xy$ plane, $\nu_{xy}$'s, necessarily coincide. The component kinetic motions of $-e$ along $\nu_{xy}$'s are thus equal and opposite and cancel out, leaving $h_{x'y}$ at rest on $\nu_{xy}$ at a random angle $\varphi$ to the $x$ axis; and the motions along $\nu_{y'z}, \nu_{y''z}$ add up. Similarly as for $n_{1}[\pi]$ and $\pi^{-}$ (Sec 2.1), the angular momenta of the P-O motions along $\nu_{e}, \nu_{\mu}$ in the lab frame are each given by the eigen solutions in the states $n = 2, \, l = 1, \, j = l - i'^{\pi} = \frac{1}{2}$ in the original $V_{2}^{*}$ field: $J_{2}^{z} = r_{1}\gamma_{2}M_{\gamma}c = \sqrt{j(j+1)}\hbar = \frac{3}{2}\hbar$. The projections along $z$ (with $m_{j}^{1}, m_{j}^{2} = -\frac{1}{2}, \frac{1}{2}$) are $J_{2}^{z} = J_{2}^{z} = J_{2}^{z}(\cos(\pi - \theta_{2}^{z}) = -\frac{1}{2}\hbar), \, \text{and in the} \, \text{xy plane are } J_{2}^{xy} = J_{2}^{y} = J_{2}^{x} = J_{2}^{x} = J_{2}^{x} = J_{2}^{z} = J_{2}^{z} = J_{2}^{z} = \frac{1}{2}\hbar = 0 \text{ along } z, \, \text{and } J_{2}^{xy,\mu} = J_{2}^{xy} + J_{2}^{z} = \sqrt{2}\hbar \text{ at angle } \varphi \text{ to } x \text{ in the } xy \text{ plane, with a time average } \langle J_{2}^{xy,\mu} \rangle = 0. \, \text{The spin (angular momentum) of the total system } \mu^{*}(h_{x'y}[\nu_{e}, \nu_{\mu}]) \text{ is } S_{\mu z} = S_{h_{xy},z} + J_{2}^{z,\mu} = \frac{1}{2}\hbar \text{ in } +z \text{ direction.}

**Figure 1.** Schematic structures of (a) $A$ in analogy to $n$ (Inset.a), except in place of $e^{-}$, $A$ is a heavy electron $h^{-}$ in P-O motion shown in the $p$ rest frame, and (b) stationary $\pi^{-}$: (Inset.b) shows $\pi^{-}$ just before decay.

**Figure 2.** (a) Schematic structure of $\mu^{+}$, which is a $h_{x'y}$ of charge $-e$, spin $\frac{1}{2}$, and in two superposing P-O motions along orbits $\nu_{\mu}, \nu_{e}$ of normals $z', z''$ at angles $\pi - \theta_{1/2}, \theta_{1/2}$ to the $z$ axis. (b) $e^{-}$, $e^{+}$ near head-on collision at separation $r_{2,1}^{*}$ to form a double heavy positronium, preceding to $\mu^{+}\mu^{+}$ production.
3. A generalised IED model extending to weak potential field

In the IED picture, the mass of a single charged permanent particle $\alpha$ (e.g. $e^-$) is generated by the oscillation of its charge $q (= -e)$, of a displacement $u_q$ (at the CM of the minute but extensive $q$), in the vacuum potential field $V_v$ (mainly the electrostatic field produced by the surrounding vacuum-dipoles) [3, 6]. In absence of other particles, $V_v = \tilde{V}_v + V_{\partial\xi} = \frac{1}{2}\beta_v u_q^2 + V_{\partial\xi}$, where $\beta_v = 2\Omega_v^2$ is the force constant, $\partial\xi$ is the (sub-vacuum) mass of the charge $q$, and $\Omega_v$ is the angular frequency; $\partial\xi$ represents a friction against the $q$ motion ordinarily in its own $V_v$ field. Assume $q$ oscillates about a fixed position and hence $\alpha$ is at rest here. If another particle $\alpha'$ of charge $q'$ presents in the vicinity and exerting on the vacuum a potential $\tilde{V}_v$, then $\partial\xi, (\alpha$ friction), hence $\beta_v, \tilde{V}_v$ of $q$ of $\alpha$ would in general be modified, by a factor $\eta_v$ to $\partial\xi, \beta_v$, and

$$
\tilde{V}_v'(u_q) = \frac{1}{2} \beta_v' u_q^2 + \frac{1}{2} \beta_v u_q^2 + \frac{1}{2} \partial\xi_q e^2, \quad \beta_v' = \beta_v (1 + \frac{\partial\xi q}{2\langle V_v \rangle}), \quad \partial\xi_q = \frac{\eta_v D_v}{D_q' c^2},
$$

where $D, D'$ are the dimensions of the $q, q'$ oscillations; $\langle \cdot \rangle$ indicates the time average.

Consider a particle $\alpha$ of charge $q$ is in P-O motion along orbit $\tilde{\nu}$ (of a normal $z'$ relative to $\alpha'$ in the central magnetic or weak potential field of $\alpha'$, $V'_\phi (r_1) = -G/((4\pi r_1^4))$, at an equilibrium separation $r_1 (\sim 10^{-18} \text{ m})$, $G$ being the coupling constant. Attach the local co-ordinates $\xi, \zeta$ to the moving $\alpha$, which origin is fixed at $r_1$ and $\xi, \zeta$ axes are parallel to $r, z'$ (the radius and normal of orbit $\tilde{\nu}$). The oscillation of the (CM of) $q$, of displacement $u_q = r - r_1 = \xi_q + \zeta_q$ in $\tilde{V}_v$ is now in the (instantaneous) $\xi, \zeta$ plane of dimension $D = 2$, at a random angle $\vartheta$ to $\zeta$, and of a rest amplitude $A_{\vartheta, \phi} = A_{\vartheta} | \sin \vartheta \phi + \cos \vartheta \phi | = | A_{\vartheta} \xi_q + A_{\vartheta} \zeta_q |$; $\vartheta$ varies randomly in $2\pi$ over time. Consider further $q$ is at initial time driven by an external force ($\mathbf{F}_{\text{ext}}$) into displacement $\xi_q$ away from $r_1$ along radial $r$- or $\zeta$- direction. After a relaxation time, $q$ maintains an oscillation of displacement $\xi_q(t) = r - r_1 = A_{\vartheta, \phi} \psi_q(t)$, expressed for a free $\alpha$ at rest here, under the restoring force $\mathbf{F}_h = -\frac{\partial\xi_q}{\partial t} \dot{\psi}_q$ produced by one half of the difference weak potential,

$$
V_h(\xi_q) = \frac{1}{2} \left[ V'_\phi (r_1 + \xi_q) - V'_\phi (r_1) \right] = \frac{9G \xi_q^2}{8\pi r_1^4 A_h} \approx \frac{9G \xi_q^2}{8\pi r_1^4 A_h} = \frac{1}{2} \beta_h \xi_q^2, \quad \beta_h = \frac{9G}{4\pi r_1^4 A_h} = \frac{2K_{\mathbb{h}} c^2}{A_h},
$$

where the second of Eqs (2a) is given for $\xi/ r_1 \ll 1$; the third is a quadratic approximation but gives the exact energy level at $\xi_q = A_h$. The other half $V_h(\xi_q') = V_h(\xi_q)$ is shared by $\alpha'$. $\alpha'$ similarly presents to $V_h$ a friction term $\partial\xi_q = \eta_q V_h = A_h \partial\xi_q$.

Setting the friction effect aside, the total restoring forces on $q$ in $\xi, \zeta$ directions are then $F(\xi_q) = F_v(\xi_q) + F_h(\xi_q) = -(\beta_v + \beta_h) \xi_q, F(\zeta_q) = F_v (\zeta_q) = -\beta_v \zeta_q$. The Newtonian equations of motion (com's) of the CM of $q$ along $\xi, \zeta$ are thus $-\left( \beta_v + \beta_h \right) \xi_q = (\partial\xi_v + \partial\xi_h) \frac{\partial^2 \xi_q}{\partial t^2}$, $-\beta_v \zeta_q = (\partial\xi_v + \partial\xi_h) \frac{\partial^2 \zeta_q}{\partial t^2}$. The general solutions are $\xi_q = \alpha e^{i\Omega_v t} + A \alpha e^{i\Omega_h t} | \epsilon_0 - t |$, $\zeta_q = \alpha e^{i\Omega_v t}$, where $\Omega_v = \beta_v / \partial\xi_v, \Omega_h = \beta_h / \partial\xi_h, \epsilon_0 - t = 0$ unit vectors for the regular and excited $h$ mass states and assumed orthogonal. $A_v$ is the time average of $A_{\vartheta, \phi}, A_v = \frac{4\pi}{2\pi} = \frac{1}{2} A_{\vartheta, \phi}$. The total mechanical energies of the $q$ oscillations in $\tilde{V}_v$, $V_h$ including friction are $E_\xi = E_v + \partial\xi_q e^2, E_v = \frac{1}{2} \partial\xi_v \dot{u}_q^2 + \tilde{V}_v = \frac{1}{2} \beta_v | u_q - c |^2 = \frac{1}{2} \beta_v A_q^2$, $E_\xi = E_h + \partial\xi_q e^2, E_h = \frac{1}{2} \partial\xi_h \dot{u}_q^2 + V_h = \frac{1}{2} \beta_h | u_q - c |^2 = \frac{1}{2} \beta_h A_q^2$. For simplicity of discussion we consider the extreme case that no (internal) radiation is being generated by the charge oscillation: $E_\xi, \xi_q$ thus equal the originally imparted energies.

The charge $q$ is point like to its ERM field and yet is extensive at the scale $10^{-18} \text{ m}$ and hence in $V'_\phi (r_6)$ and $V_v(7)$. Denote by $\rho_c = | \psi_c(u, t) |^2$ the dimensionless charge density; $\psi_c$ is a complex function, and $\mathbf{u} = r - r_1 = \xi + \zeta$ is the displacement a volume element on $q$ makes at time $t$. The $\rho_c$ current is thus $j_c = \rho_c \frac{\partial u}{\partial t} = -D_q | \psi_c^* (\nabla \psi_c) - (\nabla \psi_c^*) \psi_c |; D = \frac{4\pi}{2\pi} A_q^2$ is an imaginary diffusion constant. The stationary state is described by the continuity equation.
\[ \partial_t \rho_c + \nabla_j \rho_c + \Omega \rho_c = 0, \] 

where \( \Omega = \frac{V(u) - V(0)}{4\pi} \equiv 0. \) This decomposes to two equations of a Schrödinger form, given for \( \psi \) as \( \text{i} \hbar \partial_t \psi_c = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(u) \right) \psi_c. \) Using in it (1a) for

\[ V(\zeta) = \frac{1}{2} \beta^2 \zeta^2 \] 

and (2a) for \( V(\xi) = \frac{1}{2} \beta^2 \xi^2 \) for the two pure orthogonal oscillation states along \( \zeta, \xi, \) one obtains the standard solutions as for a usual harmonic oscillator: The eigen functions \( \psi \) are hermitian polynomials. The eigen energies are quantised,

\[ \varepsilon^i_{c1} = \hbar \Omega^i_{c1} = \frac{1}{2} \beta_c A^2_{c1} + \Omega_c c^2 = M^2_{c1} - \frac{1}{4} \left( \frac{V(h)}{c^2} \right), \] 

\[ \varepsilon^i_{hN} = N \varepsilon_h + \Omega_h c^2, \] 

\[ \varepsilon_h = \hbar \Omega_h = \frac{1}{2} \beta_h A^2_{h1} = M^2_h, \] 

where based on comparison with experiment [6] only the \( n_c \) is 1th excited state is permissible in \( V_c, \) and \( N = 0, 1, 2, \ldots \) in \( V_2; \) the terms \( \frac{1}{2} \hbar \Omega_2, \frac{1}{2} \hbar \Omega_4 \) are judged unphysical and dropped. The second of Eqs (3a), (c) identify with the classical energies, except that \( A_{c1} = h A_{h1} \) are here quantised. The last equation \( \varepsilon^i_{c1}, \varepsilon_{hN} \) with the rest-mass energies, and \( M^2_{c1}, M^2_h \) are the rest masses of the particles \( \alpha, \) as a generalised basic assumption of the IED model.

Assuming \( q' \) is not in excited mass state in \( V_2, \) and for the averaged \( A_c \) used, \( (3a) \) gives also the actual total hamiltonian of \( q \) oscillation in \( V_c \) (the vacuum ground mass state), and (3b) that of the \( q \) oscillation - the excited mass state - in \( V_2. \) The respective particle rest masses are

\[ M^2_{c1} = M_c + \Omega_c, \quad M^2_{hN} = N M_h + \Omega_h, \quad M_h = M_p - \Omega_h = \frac{\varepsilon_{h1}}{c^2} = \frac{2 \langle V_h \rangle}{c^2} = \frac{9 G A_{h1}}{8 \pi \eta_c c^2} = K_h A_{h1} \] 

The second of Eqs (4c) is given by the definition for \( \pi^- \) rest mass (Sec 2) for \( q = - \) (cf Sec 4).

Indicated by experiment [1] and in theory (Sec 4), the charge \( q \) generates a particle \( \alpha \) either in a pure \( n_c = 1 \) state in \( V_c \) (i.e. \( \alpha' = \) or a \( N \geq 1 \) excited state in \( V_{hN} \) (e.g. \( h^- \), giving \( \pi^- \)), and not a mixed state of both. Suppose that \( q \) is in \( N \geq 1 \) excited mass state, and \( q' \) is in \( N' \) state.

So \( q' \) is not in the \( N \geq 1 \) kinetic oscillation but shares the other half of the potential energy of (2), of a projected average \( \frac{D}{D'} \langle V_{hN} \rangle, \) \( \langle V_{hN} \rangle = \langle V_{hN} \rangle = \frac{1}{2} \beta_h (\xi_h(t)^2)_{N} = \frac{1}{2} \Omega_{hN}, \) \( D_h/D'_1 = \frac{1}{2}, \) reflects the fractional time that \( q' \), oscillating regularly in a \( D' = 2 \) plane, spends along the \( D_h = 1 \) -axis. Including the shared potential in (3a) applied to \( q' \), the (rest) mass of \( \alpha' \) is

\[ M^2_{\alpha'} = M_{\alpha'} + \frac{D_h V_{hN}}{D'_1 c^2} + \Omega'_{\alpha'} = M_{\alpha'} + \frac{1}{4} N M_h + \Omega'_{\alpha'}, \] 

\[ \Omega'_{\alpha'} = \frac{\eta \langle D'_1 V_{hN} \rangle}{D'_1 c^2 h}. \] 

Eqs (4), (5) above, and (6) later, describe a common rest mass formation scheme due to quantised oscillation in \( V_c, V_h \) or \( V_{hxy}, \) as contrasted to relativistic mass due to translation. Until Sec 4, we can predict the mass value, for in (4) \( A_{h1} \) is not known, also \( G, r_1 \) are not universal.

As we illustrate we apply \( (4a, b, c), (5) \) to two composite particles for evaluating the \( \eta_p \) or mass. First, the neutron \( n(\bar{\nu}_e, p)[\bar{T}] \) composed of \( \alpha, \alpha' = \epsilon^e, p \) both in the \( n_c, n' = 1, 1 \) states; \( D, D' = 2 \), in their mutual presence the \( \epsilon^e \), \( p \) masses are each given by (4a), \( M^2_{\alpha} = M_c + \frac{\eta \langle D'M_h \rangle}{2 M_p}, \) \( M^2_{\alpha'} = M_p + \frac{\eta \langle D'M_{\bar{\alpha}} \rangle}{2 M_p}, \) where \( M_c, M_p \) are the free \( \epsilon^e, p \) rest masses; their P-O current has an apparent rest mass \( M_{\bar{\nu}_e}(\equiv 0). \) The \( n \) mass is the sum \( M_n = M_{\alpha'} + M_{p} + M_{\bar{\nu}_e} \equiv M_c + M_p + \Omega_n \), \( \Omega_n = \frac{\eta \langle D'M_{\bar{\nu}_e} \rangle}{2 M_p} = 1.67 \times 10^{-3}. \) Second, the \( \Lambda(h^- [\bar{\nu}_e, p] \) hyperon (Sec 2) composed of \( \alpha = h^- \) in the \( N = 1 \) state in \( V_h \), and \( \alpha' = p \) in \( n' = 1 \) state in \( V_{h'}; \) \( D_h = 1, D'_1 = 2. \) To the \( h' \), \( p \) masses (4b-c), (5) apply: \( M^2_h = M_p + \Omega_n, \) \( M^2_p = \frac{\eta \langle D'M_{\alpha'} \rangle}{2 M_p} = M_p + \Omega'_{\alpha'}, \) \( M_{\bar{\nu}_e}(\equiv 0). \) The \( \Lambda \) mass is thus \( M_{\Lambda} = M_h + M_p + \Omega_n = \frac{1}{2} M_p + \Omega_n = \frac{1}{2} M_p + \Omega_n = \frac{1}{2} M_p + \Omega_n \), \( \Omega_n = \eta \left( \frac{\eta \langle D'M_{\alpha'} \rangle}{2 M_p} + \frac{\eta \langle D'M_{\alpha} \rangle}{2 M_p} \right) = \eta \left( \frac{1}{2} M_p + M_{\alpha'} \right) \equiv 0.623 \) MeV estimated using the \( \eta \) value of \( n; \) the experimental values for \( M_{\pi}, M_p, M_{\bar{\nu}_e}(\equiv 0) \) are used.

Finally we derive the \( \mu^- \) mass from the projection of (4c). Consider a \( \mu^- \) produced from \( \pi^- \rightarrow \mu^- \bar{\nu}_\mu \), of the original mass \( M^2_{\mu} = M_{\pi} - M_{\bar{\nu}_\mu} = M_h + M_{\nu_\mu} + M_{\bar{\nu}_\mu}, \) generated by \(-e \) along
\(\nu'_{\gamma z}\) (Sec 2.2). Its total EMR field has one component travelling on the continually reorienting \(\nu'_{\gamma z}\), which will not be readily re-absorbed by \(e^{-}\) and hence lost to the kinetic energies of \(\nu'_{\gamma z}\), \(\nu_{e}\), \(\nu_{\mu}\), and one (as two opposite but non-cancelling standing waves) on \(\nu'_{\gamma z}\), which can be more readily re-absorbed by \(e^{-}\). So the \(xy\)-projection of \(M_{\nu}'\) gives the \(\mu^{-}\) mass, \(M_{\mu}(\rho) = (M_{\nu}'_{\rho})_{xy} = [M_{h}(1 + \rho)]_{xy}\), where \(\rho = r_{xy}\) is the radius of the elliptic \(\nu'_{\gamma z}\), \(M_{h} = K_{h}A_{h1}\) [Eq (4e)], and \(K_{h}\) is a constant. Provided taking \(\Theta\) as a constant of \(\rho\), the \(xy\)-projection of the mass \(M_{\nu}'\) reduces to a pure geometric \(xy\)-projection of \(A_{h1}\), \(A_{h1\gamma z}(\rho)\), which is dependent on \(\rho\). At the semi-major and semi-minor axes \(\rho = a, b\), \(A_{h1\gamma z}(a) = A_{h1}\) and \(A_{h1\gamma z}(b) = A_{h1}\cos\theta_{z}\). Using their mean \(\bar{A}_{h1\gamma z} = (A_{h1\gamma z}(a)A_{h1\gamma z}(b))^{1/2}/A_{h1} = (M_{\pi} - M_{\mu})\sqrt{\cos\theta_{z}}\) from earlier, gives

\[
M_{\mu} = M_{\nu}'_{\rho}(A_{h1\gamma z}(a)A_{h1\gamma z}(b))^{1/2}/A_{h1} = (M_{\pi} - M_{\mu})\sqrt{\cos\theta_{z}} = 105.860\text{ MeV.} \tag{6}
\]

4. *Ab initio* predictions of the muon and charged pion masses

Consider first two particles \(a, b\) of charges \(-e, +e\) and masses \(m_{a} = \gamma_{a}M_{a}, m_{b} = \gamma_{b}M_{b}\), moving at relative speed \(v\) under their Coulomb force \(F_{c} = -\nabla V_{c} = -\hbar c/\alpha r^{2}\) at a separation \(r = r^{0}/\gamma\), where \(V_{c} = -\hbar c/\alpha r^{2}\), \(\alpha = \frac{e^{2}}{4\pi\epsilon_{0}c}\) = 1/137.036. In the CM frame, the relativistic com

\[
\mathcal{M} \mathcal{E}^{2} = F_{c} \vec{r}_{c}, \text{ where } \mathcal{M} = \gamma, \mathcal{E} = 1/\sqrt{1 - v^{2}/c^{2}}, \mathcal{M}^{0} = M_{a}M_{b}/M_{a} + M_{b}; \quad r_{c} = \gamma_{a} - \gamma_{b}\text{; and }
\]

\[
r_{a} = \frac{m_{a}r}{m_{a} + m_{b}}, \quad r_{b} = -\frac{m_{b}r}{m_{a} + m_{b}}.
\]

The wavelength is \(\lambda = \frac{\hbar}{\gamma_{a}\gamma_{b}}\). Its total EMF field has one component travelling on the continually reorienting \(\nu'_{\gamma z}\). Its total EMF field has one component travelling on the continually reorienting \(\nu'_{\gamma z}\), which will not be readily re-absorbed by \(e^{-}\) and hence lost to the kinetic energies of \(\nu'_{\gamma z}\), \(\nu_{e}\), \(\nu_{\mu}\), and one (as two opposite but non-cancelling standing waves) on \(\nu'_{\gamma z}\), which can be more readily re-absorbed by \(e^{-}\). So the \(xy\)-projection of \(M_{\nu}'\) gives the \(\mu^{-}\) mass, \(M_{\mu}(\rho) = (M_{\nu}'_{\rho})_{xy} = [M_{h}(1 + \rho)]_{xy}\), where \(\rho = r_{xy}\) is the radius of the elliptic \(\nu'_{\gamma z}\), \(M_{h} = K_{h}A_{h1}\) [Eq (4e)], and \(K_{h}\) is a constant. Provided taking \(\Theta\) as a constant of \(\rho\), the \(xy\)-projection of the mass \(M_{\nu}'\) reduces to a pure geometric \(xy\)-projection of \(A_{h1}\), \(A_{h1\gamma z}(\rho)\), which is dependent on \(\rho\). At the semi-major and semi-minor axes \(\rho = a, b\), \(A_{h1\gamma z}(a) = A_{h1}\) and \(A_{h1\gamma z}(b) = A_{h1}\cos\theta_{z}\). Using their mean \(\bar{A}_{h1\gamma z} = (A_{h1\gamma z}(a)A_{h1\gamma z}(b))^{1/2}/A_{h1} = (M_{\pi} - M_{\mu})\sqrt{\cos\theta_{z}}\) from earlier, gives

\[
M_{\mu} = M_{\nu}'_{\rho}(A_{h1\gamma z}(a)A_{h1\gamma z}(b))^{1/2}/A_{h1} = (M_{\pi} - M_{\mu})\sqrt{\cos\theta_{z}} = 105.860\text{ MeV.} \tag{6}
\]
\[ \mathcal{M}_{su}^* = \mathcal{M}_{sn}^\prime \frac{2l + 1}{n^2} = 2 \gamma_n^* \mathcal{M}_0^* = 2 \mathcal{M}_{nl}^*; \quad \gamma_n^* = \frac{\gamma_n(2l + 1)}{n^2} = \frac{(2l + 1)}{n^2} \alpha^2, \]  

(10)
given using (7); \( g_n = \sqrt{\frac{\gamma_n}{2\gamma}}} \). To achieve such dynamics for both the charges \(-e, +e\) of \( e^+, e^{-}\), there requires in the CM frame two fictitious particles, designating by \( \gamma = \mu, \bar{\mu}, \) each having a mixed total wave \( \psi_n = \psi_\mu \psi_\bar{\mu} e^{iJ\bar{\mu}} \), with \( \psi_\mu, \psi_\bar{\mu} \) satisfying the SQR-KGE. Accordingly \( \mathcal{M}_{su}^* \) is double the \( \mathcal{M}_{su}^* \) of a single relativistic positronium. The \( \mathcal{M}_{su}^* \)'s are actually each moving at the speed \( v_n^* \approx \bar{v}_n^* \approx c \); they have therefore each a relativistic wavelength

\[ \lambda_n^* = \frac{\hbar}{\mathcal{M}_{su}^*c} = \frac{\lambda_n}{2}, \quad \lambda_n^* = \frac{\hbar}{\mathcal{M}_{sn}^*c} = \frac{\lambda_n}{\mathcal{M}_{nl}^*c} = \frac{(\alpha \sqrt{2l + 1}) \cdot h \cdot n^2}{(\alpha \sqrt{2l + 1}) \cdot (\alpha \sqrt{2l + 1})} = \frac{\alpha^2}{n^2 \gamma_n^*}. \]  

(11)
The above system can be obtained by accelerating \( e^-, e^+ \) (from rest) to \( e^-, e^+ \) of the CM frame linear momenta \( p_n = -M_{su}^* v_n^* \bar{z}, p_b = -p_a \) in \(-z, \bar{z}\) directions at time \( t = 0 \), and then subjecting them to a near head-on “quantum collision” at \( r_a, r_b \), \( -\bar{r}_n^*, -\bar{r}_n \), at a separation

\[ r_n^* = \frac{1}{2} r_n^* = r_a = r_b, \quad r_n^* = |r_a - r_b| = \frac{n^2 \lambda_n}{\sqrt{2l + 1} \pi} \]  

(12)

for \( v_n^* \approx c \); \( \sin \theta_1 = 1/\sqrt{(l + 1)} \), \( \cos \theta_1 = 1/\sqrt{(l + 1)} ; \) \( \theta_n^* \) is equal to \( \theta_{su}^* \) of the usual positronium. The orbits \( \bar{\nu}, \nu \) undergo Thomas precession (TP) each about \( z \) in the lab frame (for a usual system the TP effect gives a small energy correction through \( g \) factor(8)); their normals \( \bar{z}', \bar{z} '' \) are thus turned to angles \( \pi - \theta_j, \theta_j \) to the \( z \) axis (cf Fig 2a), so that \( J^\alpha_{\mu\sigma \nu} = |J^\alpha_{\mu\sigma \nu} + J^\alpha_{\mu\sigma \nu} | = \sqrt{J^2 + \pi} \) \( J^\alpha_{\mu\sigma \nu} \)

for \( \eta = \bar{\nu}, \nu \) and \( j = 1 \). For the superposed total motion of \( \mu, J_{\mu\sigma\nu} = J_{\mu\sigma\nu}^\alpha + J_{\mu\sigma\nu}^\beta = 2 \sqrt{J^2 + \pi} \sin \theta_{\nu} \hat{r}_{xy}, J_{\mu\nu\lambda} = J_{\mu\nu\lambda}^\alpha = 0. \) The total motion of \( \mu \) is along the elliptic \( \nu_{\nu z} \) in a \( y'z \) plane at a random angle \( \mu \) to the \( x \) axis, and is at rest on the elliptic \( \nu_{xy} \) in the \( xy \) plane. The \( xy \)-projected \( \nu, \bar{\nu} \) motions cancel out; these also do not present for the incident \( e^+ \) (and \( e^- \)) but (when actually decomposed) can be produced in an explosive collision. For \( (n = 2) \) \( t = 1 \), the angular motion of \( \mu \) resembles directly that of the muon \( \mu^* \) (\( \mu^* \)) in Sec 2.2.

\[ e^+, e^{-}, \] or \( \bar{\mu}, \bar{\mu} \) in the CM frame, are energetically unstable, since at \( t > 0 \) their separation \( (r_n^* \rightarrow) \) along \( y'z \) is variant. Under action \( V^\prime(<0) \), \( e^+, e^- \) will continue to move closer, switching to magnetic or weak interaction \( V^\prime \) dominant at \( r' \) comparable to the charge size \( a \) \((\approx 10^{-18} \sim 10^{-17} \text{ m}) \). Here the two quanta of the total apparent rest mass energy \( M_{su}^* c^2 \) each are able to in situ \(- \) at the same positions and velocities \(- e, +e \) convert to the more stable quantum oscillation energies of the charges \(- e, +e \) in their \( V^\prime \) field. Relative to the CM and \( V^\prime \), the total mass energy and (instantaneous) four momentum (for the rotation) of \( s = \mu \) (or \( \bar{\mu} \)) before the conversion are \( E^\prime = T^\prime + M_{su}^* c^2 = \gamma_{n} M_{su}^* c^2 = \gamma_{n} M_{su}^* c^2, \) \( p_{n}^\prime = (\gamma_{n} M_{su}^* v_{n}) \), where \( \gamma_{n} = 1/\sqrt{1 - (v_{n}/c)^2} \) \( v_{n} \) is the apparent velocity of \( \mathcal{M}_{su}^* \).
after the conversion are $E^\prime = \gamma^\prime M^\prime v^2$, $p^\prime = (\gamma^\prime)^{-1/2}p^\prime_0$, where $M^\prime = M_{N,\gamma^\prime} + \sum M_i$, $\gamma^\prime = 1/\sqrt{1 - (v^\prime/c)^2}$, and $v^\prime$ is the velocity of $M^\prime$. For the in situ conversion, $v_* = v^\prime$, $\gamma_* = \gamma^\prime$. Four momentum invariance $p_i^\prime p_i^\prime = p_i^0 p_i^0$ gives $M^\prime_\gamma c^2 \gamma^\prime t^2 (1 - (v^\prime/c)^2) = M_{\gamma\gamma} c^2 \gamma_* t^2 (1 - (v_*/c)^2)$, or $M^\prime = M_{\gamma\gamma} = \mu_{\gamma\gamma} + M_e$.

In sum, $\mu, \bar{\mu}$ can be assigned with charges $-e, +e$, (the lab-frame) spins $S_{\mu z} = J_{\mu z} + S_{\mu e} = \frac{1}{2} \hbar$ for $l = 1, j = \frac{1}{2}$, and have for $n = 2$ the masses $M^\prime_{2,1}$ (at rest in the $xy$ plane) given by (14), which are overall identifiable to those of the muons $\mu^+, \mu^-$. Thus when in situ converted to the $N$ = 1th quantum oscillations of the charges $-e, +e$ in $V_2^\prime$, the converted resemble in all respects the muons $\mu^+, \mu^-$ (Sec 2.2). The un-projected $N = 1$ oscillation states of $-e, +e$ converted from the un-projected $n = 2$ states (of mass $M^\prime_{sn}$ each) resemble then the pions $\pi^+, \pi^+$ (Sec 2.1) in theory. Using (10) for $M^\prime_{sn}$, $n = 2, l = 1$, gives in the CM frame, and in the lab frame too where the $\nu, \bar{\nu}$ precessions cancel out, the rest mass of the muon $\mu^-$ (or $\mu^+$), $M^\prime = \mu_{\mu\mu}$.

$$M_{\mu^\prime} = M^\prime_{2,1} = \mathcal{M}^\prime_{2,1} + M_e = \frac{3}{4} \mathcal{M}^\prime_{2} + M_e = \frac{3}{4} \left( \frac{2Me}{\alpha} \right) + M_e = 105.549 \text{ MeV.}$$ (14)

$M_e$ = 0.511 MeV, $\alpha = 1/137.036$ are used. Here $M^\mu_{\mu^\prime} = M_{\mu^\prime} - \hbar_\eta = M_{\mu^\prime}$, apparently $\hbar_\eta = 0$, since the $M^\mu_{\mu^\prime}, M^\mu_{\mu^+}$ are defined in the $xy$ plane and they have no relative motion in this plane. For the pion $\pi^\prime$ ($M^\prime \equiv M^\prime_{\pi^\prime} = \mathcal{M}^\prime_{\pi^\prime} + M_e = \frac{2Me}{\alpha} + M_e = 140.525 \text{ MeV}$ in theory); $\hbar_\eta$ is a friction term from an actual $\pi^+\pi^-$ production, not the $n = 2$ states here. In experiment, an $e^+, e^+$ collision can directly produce $\mu^-, \mu^+$, apparently owing to the symmetric partial $\bar{\nu}, \nu$ orbits for each charge, whilst for producing $\pi^+, \pi^-$, only one $\bar{\nu}$ or $\nu$ can be attributed to each charge. It instead takes an intermediate bound state ($e^+e^- \rightarrow \rho^0(770)$ to produce a $\pi^+\pi^+$ pair. $\rho$ can be represented as consisting primarily of a $h_{\gamma}^- h_{\gamma}^+$ pair generated by $-e, +e$ in $N = 2$ oscillation states in $V_2^\prime$, converted from $n = 4, l = 1$ states of the DHP. Indicated by its still larger experimental mass, $\rho$ also contains two $e^+, e^+$ pairs, which are present apparently to provide the $\bar{\nu}, \nu$ form of symmetry to each charge. Equations (9)-(13) are general. The $n > 2$ levels can be expected to give rise to all the higher masses of unstable leptons ($\tau^\pm$ are the only observed ones) and composite mesons, typically stabilised in presence of secondary $e^+, e^+$ pair(s).

The author expresses thanks to Chairman Professor C Burdik and the Organising Committee for the opportunity of presenting this work at the ISQS26, Tech Univ, Prague, 2019, to emeritus scientist P-I Johansson for continued moral support and private financial support to the author’s unification research, to Professors B Johansson and I Lindgren for giving moral support to the author’s unification research, and to Dr R Dahm and Professor S Catto for useful discussions.

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