DOES DISK LOCKING SOLVE THE STELLAR ANGULAR MOMENTUM PROBLEM?

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ABSTRACT

We critically examine the theory of disk locking, which assumes that the angular momentum deposited on an accreting protostar is exactly removed by torques carried along magnetic field lines connecting the star to the disk. In this Letter, we consider that the differential rotation between the star and disk naturally leads to an opening (i.e., disconnecting) of the magnetic field between the two. We find that this significantly reduces the spin-down torque on the star by the disk. Thus, disk locking cannot account for the slow rotation ($\sim 10\%$ of breakup speed) observed in several systems and for which the model was originally developed.

Subject headings: accretion, accretion disks — MHD — stars: formation — stars: magnetic fields — stars: pre–main-sequence — stars: rotation

1. INTRODUCTION

We now know that most protostars undergo a phase in which they accrete mass from a disk and that a stellar magnetic field dominates this process near the star. However, the transport of angular momentum, while fairly well understood within the disk, remains a significant challenge for models of stellar spin (for a review, see Bodenheimer 1995). Classical T Tauri stars (CTTSSs) comprise a class of accreting protostars for which there is an abundance of observational data. For typical parameters in these systems, the angular momentum deposited by accretion can spin the star up to breakup speed in $\sim 10^5$ yr, assuming the star has not already formed at breakup speed. Since the accretion lifetime is often greater than $10^5$ yr, the stars must somehow rid themselves of this excess angular momentum. Furthermore, a significant number of CTTSSs (the so-called slow rotators) have spin rates as slow as $\sim 10\%$ of breakup speed (e.g., Bouvier et al. 1993), requiring significant torques to maintain.

Königl (1991, hereafter K91) proposed the currently accepted view that the slow rotators could be explained with a “disk-locking” (DL) mechanism (adapted from Ghosh & Lamb 1979), for which the star needs a kilogauss-strength dipole magnetic field. Indeed, CTTSSs are now known to possess kilogauss-strength fields (e.g., Johns-Krull, Valenti, & Koresko 1999b), and magnetic star-disk interaction models have also explained several observed spectral features of CTTSSs (e.g., K91). However, observations by Stassun et al. (1999) and Johns-Krull et al. (1999a) and theoretical considerations of Safier (1998) have called the standard DL scenario into question. According to DL theory, magnetic field lines causally connect the star to the disk (acting as “lever arms”) and carry torques that oppose and balance the angular momentum deposited by accretion. Uzdensky, Königl, & Litwin (2002, hereafter UKL) demonstrated that, although not usually considered in DL theory, differential rotation between the star and the disk leads to an opening of the field, drastically reducing the magnetic connection between the two. In this Letter, we consider the consequences of this reduced connection on DL. We find that the magnetic torque is significantly reduced and show that the DL model therefore cannot account for the angular momentum loss of the slow rotators.

2. TORQUES IN THE STAR-DISK INTERACTION

In order to discuss the disk-locked state, in which magnetic torques on the star exactly balance the angular momentum deposited by accreting material, we must first discuss the general case for an arbitrary state of the star-disk system but with the proper inclusion of field line opening. To this end, we mainly follow the formulation of Armitage & Clarke (1996, hereafter AC96), and a more detailed presentation of our model will be given in Matt & Pudritz (2004). According to the typical assumptions, the central star contains a rotation-axis–aligned dipolar magnetic field. This field is anchored in the stellar surface and also threads a thin, Keplerian accretion disk, in which the kinetic energy of orbiting gas is much greater than the magnetic field energy. The region above the star and disk contains low-density gas and is dominated by the magnetic field. The system is assumed to exist in a steady state, where the accretion rate $\dot{M}_a$ is constant in time and at all radii in the disk. Figure 1 illustrates the basic idea and identifies the location where the disk is truncated ($R_\star$) and the corotation radius ($R_{\text{co}}$), where the Keplerian angular rotation rate equals that of the star. The region in the disk threaded by closed stellar field lines is delimited by $R_{\text{out}}$ (the usual assumption is $R_{\text{out}} \gg R_{\text{co}}$).

The assumption of a steady state $M_\star$ requires that the net angular momentum carried away from each annulus of the disk (of width $\Delta r$ and vertically integrated) equals

$$\delta \tau_r = 0.5\dot{M}_a GM_\star r^{-1/2} \Delta r,$$  

where $M_\star$ is the stellar mass. If the torque anywhere differs from this, the disk will restructure itself on a dynamical timescale, reestablishing the steady state.

The star and disk rotate at different angular speeds, except at the singular location $R_{\text{co}}$. Thus, the magnetic field becomes twisted azimuthally by differential rotation, and magnetic forces act to restore the dipole configuration—conveying torques between the star and disk. The magnetic torque on the star exerted by a bundle of field lines threading an annulus in the disk can be written as (AC96)

$$\delta \tau_r = \gamma \mu^2 r^{-4} \Delta r, \quad \gamma \equiv B_\star B_\odot.$$  

This differential torque depends only on the strength of the dipole moment $\mu$ and on $\gamma$, which is the “twist” (or pitch angle) of the field. The sign of the torque, here and in all equations,
is relative to the star, so a positive torque spins the star up and thus spins the disk down.

The magnetic field cannot be perfectly frozen-in to the gas of the disk. Instead, it diffuses or reconnects through the disk azimuthally at some speed, \( \nu_d \approx \gamma \eta/h \) (UKL), where \( \eta \) is the magnetic diffusivity (we use the subscript ”r” to suggest a turbulent value) and \( h \) is the local, vertical scale height of the disk. For a standard \( \alpha \)-disk (Shakura & Sunyaev 1973), \( \nu_d = \beta v_{Kep} \), where \( v_{Kep} \) is the orbital speed, \( \beta = (\alpha/Pr)(h/r) \), and \( Pr \) is the turbulent Prandtl number (=turbulent viscosity divided by \( \eta \)). Since both \( h/r \) and \( \alpha/Pr \) are likely to have weak (though unknown) dependences on \( r \), we assume \( \beta \) is constant. The radial dependence of the magnetic torque is mainly dominated by the quick falloff of the dipole magnetic field (\( r^{-3} \)), and so a small radial dependence of \( \beta \) will not much affect our results (AC96).

In general, \( \beta \) is a simple scale factor that compares \( \nu_d \) to \( v_{Kep} \). It measures the coupling of the magnetic field to the gas. When \( \beta \ll 1 \) corresponds to strong coupling and \( \beta \gg 1 \) to weak. The value of \( \beta \) is unknown (AC96 used \( \beta = 1 \)). Typical \( \alpha \)-disk parameters give a limit of \( \beta \lesssim 1 \) and a likely value of a few orders of magnitude lower. For reasonable fiducial parameters, \( \beta = 10^{-3} \eta/10^{10} \text{ cm}^2 \text{ s}^{-1} (R_*/h)(100 \text{ km} \text{ s}^{-1} h_{Kep}) \). However, given the uncertainties, we keep \( \beta \) as a free parameter.

Where the field connects the star and disk, the magnetic twist (and so \( \nu_d \)) will increase, until the field can slip through the disk at a rate equal to the differential rotation rate. Thus, we expect a steady state in which

\[
\gamma = \beta^{-1}[(\nu_d/R_*)^{3/2} - 1].
\]

Note that \( \gamma(R_*) = 0 \), since the differential rotation is zero there. For \( r > R_\ast \), the twist increases to infinity.

Several recent theoretical and numerical studies of the star-disk interaction (see UKL and references therein; Lynden-Bell & Boily 1994; Agapitou & Papaloizou 2000) have shown with certainty that, as a dipole field is twisted this way, the magnetic torque reaches a maximum for finite \( \gamma \) but then reduces back to zero for larger values. This occurs because, when the magnetic field is twisted enough so that the magnetic pressure associated with \( B_0 \) overcomes the poloidal field lines, the latter will “inflate,” opening to infinity at midlatitudes. The star and disk then become causally disconnected because open field lines cannot convey torques between the two. The critical twist at which this happens is very nearly \( \gamma = \gamma_c \approx 1 \) (e.g., UKL).

The opening of field lines by this process can be added to the theory. For simplicity, we assume that the star is connected to the disk, as described above, at all locations where the twist is less than the critical value \( \gamma_c \). Where \( \gamma \geq \gamma_c \), the disk and star are disconnected, and so the differential magnetic torque there is zero. In other words, this process determines the outer radius of closed stellar field lines, \( R_{out} \). From equation (3), we find that 
\( R_{out} = (1 + \delta \gamma)^{-2} R_\ast \). AC96 assumed that the star is connected to the disk over a very large radial extent \( (R_{out} \to \infty) \), which is equivalent to the assumption that \( \gamma_c \to \infty \).

A combination of equations (2) and (3) reveals the radial dependence of the differential magnetic torque (\( \delta \tau_m \)). Figure 2 shows all differential torques (per \( R_\ast \)) in the disk, as a function of radius (normalized to \( R_\ast \)), assuming \( \beta = 1 \) and \( \gamma \to \infty \) (i.e., the AC96 solution). For the figure, we adopt representative system parameters from the well-studied CTTS BP Tau: \( M_\ast = 3 \times 10^{-8} M_\odot \text{ yr}^{-1} \), \( R_\ast = 2 R_\odot \), and \( M_\bullet = 0.5 M_\odot \) (Gullbring et al. 1998), and \( B_\ast = 2 \) kG (Johns-Krull et al. 1999). The stellar rotation period is 7.5 days (see § 3).

The lines in Figure 2 labeled “accretion” and “magnetic” represent the differential accretion and magnetic torques, \( \delta \tau_a \) and \( \delta \tau_m \), from equations (1) and (2), respectively. The steady state condition requires the disk to structure itself such that the net differential torque everywhere equals \( \delta \tau_a \). So when external magnetic torques \( \delta \tau_m \) act on the disk, there are torques internal to the disk (e.g., via the magnetorotational instability; Balbus & Hawley 1991) to counteract them and always provide a differential torque defined by \( \delta \tau = \delta \tau_a - \delta \tau_m \), represented by the dashed line in Figure 2. From the figure, it is evident that \( \delta \tau_a \) is strongest near the star, where the magnetic field is strong, and \( \delta \tau_m \) acts to spin up the star inside \( R_\ast \). At \( R_{out} \), \( \delta \tau_m \) goes to zero, since the field is not twisted there (eq. [3]). Beyond \( R_{out} \), the twist increases, and so \( \delta \tau_m \) becomes stronger, now acting to spin down the star. Since the dipole field strength decreases faster than the increase of \( \gamma \), \( \delta \tau_m \) reaches a maximum (in absolute value) and then approaches zero as \( r \to \infty \).

At the location where \( \delta \tau_m = \delta \tau_a \) (and \( \delta \tau = 0 \)), the torque from the stellar magnetic field is all that is necessary to provide \( M_\ast \). Thus, all of the specific angular momentum of the disk material at that location will end up on the star. This defines the radius \( R_\bullet \) (Fig. 2, dotted line), where the disk is truncated, and from where accretion will be magnetically channelled along field lines onto the star (K91; not shown in Fig. 1). So the net torque on the star from the accretion of disk material, \( \tau_a \), is obtained by integrating equation (1) from \( R_\bullet \) to the surface of the star. Similarly, the net magnetic torque, \( \tau_m \), is obtained by...
integrating equation (2) over the entire magnetically connected region of the disk, from $R_t$ to $R_{\text{out}}$, which gives

$$
\tau_m = \mu^2(3\beta)^{-1}R_{\text{out}}[2(1 + \beta \gamma_c)^{-1} - (1 + \beta \gamma_c)^{-2} - 2(R_{\text{out}}/R_c)^{1/2} + (R_{\text{out}}/R_c)^3].
$$

This is exactly the solution found by AC96 for the special case that $\beta = 1$ and $\gamma_c \to \infty$. Our formulation allows for an arbitrary value of the diffusion parameter $\beta$ and includes the effects of field line opening via twisting to a critical value of $\gamma_c$.

We determine the dependence of the spin-down torque on $\beta$ by setting $R_t = R_{\text{out}}$ in equation (4) and adopting $\gamma_c = 1$ (as justified by, e.g., UKL). We find that $\tau_c \propto \beta$ in the strong coupling limit ($\beta \ll 1$) has a maximal value for $\beta = 1$, and $\tau_c \propto \beta^{-1}$ in the case of weak coupling ($\beta \gg 1$). This behavior can be understood as a competition between two different effects: one is that, when $\beta$ is small, the size of the magnetically connected region in the disk ($R_{\text{out}}$) is small, reducing $\tau_c$; second, when $\beta$ is large, the field is less twisted (eq. [3]) at each radius, reducing $\delta\tau_c$. These two effects conspire to give a maximal value of $\tau_m$ for the critical value of $\beta = 1$, which thus represents the “best case” for DL theory. Even for this best case, the proper treatment of field line opening ($\gamma_c = 1$) gives a total spin-down torque that is a factor of 4 times less than if $\gamma_c \to \infty$.

To illustrate the effect of field line opening, Figure 3 shows the differential torques in the disk, for the same parameters as Figure 2, except that now $\gamma_c = 1$, and the spin period is 4.1 days (see § 3). The magnetic field is now open for $r \geq R_{\text{out}} \approx 1.6R_{\text{co}}$, and $\delta\tau_{\text{tor}}$ is zero there (by assumption). It is clear that the integrated torque $\tau_m$ will be less than if the magnetic field were everywhere closed (i.e., for $\gamma_c \to \infty$).

### 3. THE DISK-LOCKED STATE

For any given values of $M_*, R_*, B_0, \dot{M}_*$, and the stellar rotation period, the star-disk interaction theory presented in § 2 gives the net torque ($\tau_* + \tau_g$) on the star. The system is stable in that, for fast rotation, the net torque spins the star down, and for slow rotation, the star spins up. For typical CTTS parameters, the net torque will spin the star up or down in $\sim 10^5$ yr (e.g., AC96). So one expects that most systems older than this will exist in an equilibrium state, in which the net torque on the star is zero, and in which the system is “disk locked” with a period $T_{\text{eq}}$.

The equilibrium spin state is determined by combining the condition of net zero torque ($\tau_* + \tau_g = 0$) with the definition of $R_t$ (i.e., where $\delta\tau_g = \delta\tau_{\text{tor}}$). One finds that, in the DL state, there is a single predicted value of the truncation radius, $R_{\text{eq}}/R_{\text{co}}$ that depends on the quantity $\beta \gamma_c$ but not on any other system parameters. If $\beta \gamma_c$ approaches zero, $R_{\text{eq}}/R_{\text{co}}$ approaches $R_{\text{out}}$. The ratio $R_{\text{eq}}^m/R_{\text{co}}$ monotonically decreases for increasing $\beta \gamma_c$, but it never becomes smaller than 0.91, as $\beta \gamma_c \to \infty$.

We calculate that the equilibrium angular spin rate of the star is given by

$$
\Omega_{\text{eq}}^m = C(\beta, \gamma_c)M_*^{3/7}G^{5/7}R_*^{-6/7}R_{\text{co}}^{-18/7},
$$

where $C(\beta, \gamma_c)$ is a dimensionless function that depends on only $\beta$ and $\gamma_c$.

Figures 2 and 3 show the torques for stars that are in their equilibrium spin states, $T_{\text{eq}} = 7.5$ and 4.1 days, respectively. A comparison of these two figures reveals the effect of field line opening (see § 2). When this occurs, the net spin-down torque is decreased, so the star must spin faster for it to be in equilibrium. A faster spin reduces $R_{\text{co}}$, so the torques come from closer to the star, where the dipole field is stronger. Thus, a faster spin makes up for the decreased size of the magnetically connected region. Also, the truncation radius is nearer the rotation radius in Figure 3 than in Figure 2 ($R_{\text{eq}}^m \approx 0.97R_{\text{co}}$, compared to $\approx 0.91R_{\text{co}}$).

Equation (5) has the same dependences on system parameters as several DL models in the literature (e.g., K91 and Ostriker & Shu 1995), but the factor $C$ varies slightly from model to model. For example, both K91 and Ostriker & Shu (1995) use $C \approx 1.1$, while the AC96 value (i.e., for $\beta = 1$ and $\gamma_c \to \infty$) is $C \approx 1.6$. Our formulation of the problem includes the effects of field line opening and determines the dependence of the function $C$ on the diffusion parameter $\beta$. The solid line of Figure 4 reveals this dependence, for $\gamma_c = 1$. For comparison, if the field remains closed for arbitrary twist values ($\gamma_c \to \infty$), $C(\beta) \approx 1.6\beta^{3/7}$ (dotted line). The K91 value of $C$ is indicated by a dashed line. The effect of field line opening is significant. For the best case ($\beta = 1$), the predicted spin rate is 1.8 times faster than for $\gamma_c \to \infty$ and 2.6 times faster than predicted by K91. For a more reasonable value of $\beta = 10^{-2}$, the predicted spin is more than...
an order of magnitude faster than all other DL models, in which the field is typically assumed to remain closed while largely twisted.

To date, DL models have had some success in explaining the spin rates of slow rotators. For the example case of BP Tau (see § 2), when one assumes $\beta = 1$ and $\gamma_i \rightarrow \infty$, equation (5) predicts $T_{eq} = 7.5$ days, corresponding to $6\%$ of breakup speed and agreeing with the predictions of most models in the literature. This predicted period is remarkably similar to the observed value of 7.6 days (Vrba et al. 1986). However, when one properly includes the effect of field line opening (for $\gamma_i = 1$), the best case ($\beta = 1$) model predicts $T_{eq} = 4.1$ days, and the spin-up time is less than $10^5$ yr—significantly shorter than BP Tau’s age of $6 \times 10^7$ yr (Gullbring et al. 1998). Furthermore, for $\beta = 0.01$, $T_{eq} = 1.0$ days, with a spin-up time of $5 \times 10^5$ yr!

It is also important to note that we have thus far used parameters for BP Tau determined by Gullbring et al. (1998) and Johns-Krull et al. (1999b). If instead we use the higher $M_a$ derived by Hartigan, Edwards, & Ghandour (1995) of $1.6 \times 10^{-7} M_\odot$ yr$^{-1}$, even the “standard” ($\beta = 1$, $\gamma_i \rightarrow \infty$) model predicts $T_{eq} = 3.6$ days, with a spin-up time of $2 \times 10^4$ yr. Alternatively, if we use the upper limit on the dipole field strength of 200 G from Johns-Krull et al. (1999a), even the standard theory gives $T_{eq} = 1.0$ days.

We must conclude that either BP Tau and other slow rotators are rapidly spinning up or the DL picture, as an explanation of stellar spin, is incomplete. The former requires, as an example, a recent, dramatic increase in $M_a$—this seems unlikely, especially for BP Tau, for which the accretion disk is probably in the process of clearing out (Dutrey, Guilloteau, & Simon 2003). In order for BP Tau to currently be in an equilibrium spin state, there must be significant spin-down torques on the star other than those carried along field lines connecting the star to the disk.

4. SUMMARY OF PROBLEMS WITH DL

We have shown that a large portion of the magnetic field connecting the star to the disk will open up owing to the differential rotation between the two, resulting in a spin-down torque on the star that is much less than if the field is assumed to remain closed. The predicted disk-locked spin rate is therefore much faster, so that the DL scenario cannot explain the angular momentum loss of the slow rotators.

In addition, there are at least three, completely independent issues raised by other authors: (1) Stassun et al. (1999) found no correlation between accretion parameters and spin rates of TTSs in Orion. (2) CTTSs apparently do not have strong dipole fields (e.g., Safier 1998; Johns-Krull et al. 1999a), which are required for DL at slow spin rates. (3) Stellar winds are expected to open field lines that would otherwise connect to the disk (Safier 1998). A disk wind could have a similar effect.

We conclude that, in order for accreting protostars to spin as slowly as 10$\%$ of breakup speed, there must be spin-down torques acting on the star other than those carried by magnetic field lines connecting the star to the disk. The presence of open stellar field lines implies that excess angular momentum may be carried by a stellar wind (e.g., Tout & Pringle 1992), but this remains an open question. We are investigating the role of stellar winds, since then $T_{eq}$ will depend mostly on the stellar wind parameters, thereby explaining the lack of correlation between spin periods and accretion parameters.

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