Mathematical Model of Solid Municipal Waste Management

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Abstract. The research subject is a mathematical model of municipal solid waste management (hereinafter – MMMSWM). Relevance: the growth of the world's population and its consumption (primarily, in the Golden Billion countries) leads to a corresponding increase in the amount of solid municipal waste (hereinafter – SMW). This increase is an additional global warming factor. Also, growing SMW requires additional land areas. Since the time of Galileo, science has been the application of mathematics to study any object. In the case of SMW, building and solving a mathematical model of SMW management will reduce the anthropogenic load on Earth. The research objective is to build a mathematical model of municipal solid waste management and propose ways to solve it. The research goal is to perform a literature review of existing mathematical models of municipal solid waste management and suggest a new MMMSWM. The research techniques include retrospective analysis, synthesis, comparison, and methods of the theory of differential equations. The research result is building a new mathematical model of solid municipal waste management, which ensures an acceptable control over the filtrate concentration.

1. Introduction
The research relevance. In Russia, 40-50 million tons of municipal solid waste (hereinafter – SMW) are annually generated, and less than 2 % of them are recycled. Accordingly, every year, 500 thousand hectares of land (a square with an area of 70x70 km) are allocated for SMW landfills [1]. Herein, SMW means human waste generated in households (solid household waste – SHW) and at service enterprises (solid service industry waste – SSIW).

\[
\text{SMW} = \text{SHW} + \text{SSIW} \quad (1)
\]

Accordingly, waste of industry will be a solid industrial waste (SIW). The SMW landfills are growing 3-4 times faster than the population [2]. Moreover, not 100 % of SMW is removed to controlled landfills. The consuming society leaves behind more and more waste. The Earth's ecosystem cannot cope with the anthropogenic load. This is manifested in climate change, the emergence of new types of pathogens, etc. the issue of reducing the anthropogenic load to values sparing the Earth is worth thinking about.

The research objective is a solid municipal waste.
The research subject is a mathematical model of SMW management (hereinafter - MMMSWM).
The research goal is to analyze the existing mathematical models of SMW management, highlight the main ways of building mathematical models of SMW management, and propose a new mathematical model of SMW management.

2. Literature review

Theoretical and methodical aspects of municipal solid waste management are studied in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

In [11], Voronin P.M. proposes to start the SMW management with forecasting future SMW volumes. Voronin forecasting models are based on regression models. According to the authors, the regression model requires additional research for the presence (absence) of cointegration. [12]

Before further considering mathematical models of municipal waste management, let us highlight the resources that may be extracted from SMW (Table I)

| Item No. | Resource | Share, d* | Application |
|----------|----------|-----------|-------------|
| 1        | Wastepaper, textiles | 33 | Production of cardboard, roofing materials (e.g., onduline), thermal insulation materials. |
| 2        | Plastics PE, PP, PET, PVC, PS (approximately 8% of all SMW) | 4 | Recycling of plastics. E.g., PET recycling. Obtaining composite polymer materials. |
| 3        | Wood waste | 2 | Manufacturing industrial chips |
| 4        | Construction waste | 2 | Road construction |
| 5        | Glass cullet | 7 | Road construction, manufacturing glass ceramic tiles |
| 6        | Tires, rubber | 3 | Road construction, manufacturing waterproofing materials |
| 7        | Metals | 3 | Re-melting |
| 8        | Food waste | 46 | |

* - according to [13]

In [14] Kornilov A.M. and Pazyuk K.T. have built a mathematical model for the processing of tires based on a linear optimization problem. The objective function (profit) is maximized; there are three groups of limitations: by suppliers, processors, and consumers. According to the authors, the request to maximize profits may lead to a solution supposing an increase in tire production, and as a consequence, an increase in SMW (in terms of tires).

In [15], Podchashinskiy Y. et al. propose to estimate the environmental load from SMW landfills using polynomial equations (3, 4 degrees) reflecting the time dependence of the concentration of harmful substances (chlorides, sulfates, phosphates, nitrates, nitrites, alkalis, Fe, Cu, Co, Sn, Zn). E.g., for chlorides, the dependence has the form (1)
According to the authors, the econometric model can give a simplified picture of the phenomenon and not consider significant factors. When studying SMW in [13], Volynkina E.P. and Domnin K.I. analyze the formation of landfill gas ((40-60) % CH4 + (30-45) % CO2) since it is one of the warming factors. Researchers analyze exponential relationship (2)

\[ C = C_0 e^{kt} \]

where \( C \) is the reactant concentration, \( k \) is the reaction coefficient.

Probably, in this model, it is assumed to control landfill gas generation through \( k \). According to the authors, not one but the interaction of several agents should be considered.

In [16], Ismagilova G.V. et al. propose to apply a variant of the Lotka-Volterra Model (3)

\[
\begin{align*}
\frac{dx}{dt} &= -ax + bxy \\
\frac{dy}{dt} &= cy - dxy
\end{align*}
\]

where \( x \) is the annual mass of SMW, 
\( y \) is the free area dedicated for SMW,
\( a \) is the relative decrease in SMW due to lack of areas,
\( b \) is the increase in the storage area per 1 ton of SMW,
\( c \) is the increase in areas in the absence of waste,
\( d \) is the annual increase in the area under waste.

According to the authors, in (3), the main lever to control SMW is the landfill area. This does not reflect the real situation since the SMW volume primarily depends on the consumption of households.

In [17], Igoshev A.K. et al. propose a mathematical model of SMW generation as a regression model with four predictors: population, accumulation rate, SMW density, and SMW generation mass. Accordingly, SMW can be controlled through the accumulation rate. According to the authors, this is a bad practice supposing a continuous growth of the accumulation rate.

3. Techniques
Basic research techniques include retrospective MMMSWM analysis, synthesis, comparison, and methods of the theory of differential equations.

4. Results
The main negative SMW landfill result is filtrate, which is a highly toxic liquid penetrating the soil and polluting groundwater (salinity from 11 to 17 g/l) [18]. Let us build a mathematical model of the filtrate generation. MM is based on water balance. The model assumptions are:

1) \( H_1 = hxS \) is the atmospheric water
   \( h \) is the annual rate of precipitation in a given area, mm/m²,
   \( S \) is the landfill area.
2) \( H_2 = wxdM \) is the water from waste
   \( w \) is the food waste moisture content (assuming the bound water in the rest of the waste),
   \( d \) is the share of food waste in SMW,
   \( M(t) \) is the SMW mass.
3) \( H_3 = pS \) is the moisture evaporation

\[ C_{Cl} = 1.74x^4 - 30.37x^3 + 168.06x^2 - 38.61x + 8250.80 \]
p is the annual moisture evaporation coefficient, mm/m²
4) \(H_4 = m(t)(1-C)/C\) is the water content in the filtrate
m(t) is the mass of the substance dissolved in the filtrate,
C is the concentration (mass fraction of a substance)
The balance equation has the form
\[
H_1 + H_2 = H_3 + H_4
\]
\[
h x S + w d M(t) = p S + m(t)(1-C)/C
\]
Accordingly, the mass of the substance passed into the filtrate is calculated by the formula
\[
m(t) = \frac{(h S + w d M(t) - p S) C}{1-C}
\]
and the mass concentration by the formula
\[
C = \frac{m(t)}{h S + w d M(t) - p S + m(t)}
\]
If we tie M(t) to the population (according to the Malthus model, e.g.), then the formula for M(t)
\[
M(t) = q N_0 e^{rt}
\]
where q is the annual SMW rate per person, kg/person,
N₀ is the population at the time instant \(t=0\),
r is the annual population growth rate.
Now, let us estimate the amount of substance m passing into the filtrate. This amount depends on the temperature inside the landfill since the reaction rate depends on T (regardless of the reaction order) and the concentrations of the reactants
\[
m(t) = A e^{-\frac{E_a}{R T}} [B][C]
\]
where \(E_a\) is the activation energy,
A is the frequency factor,
R is the universal gas constant,
[B] is the concentration of the first reagent,
[C] is the concentration of the second reagent.
In (8), a bimolecular reaction is considered; this is a certain simplification since groups of aerobic and anaerobic reactions sequentially run at the SMW landfill. [19] But if a simplified mathematical model of SMW management is formalized, then its complication will not be difficult.
To calculate the SMW landfill temperature, we use the heat transfer equation [20]
\[
\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}
\]
where k is the thermal conductivity coefficient of the landfill content,
x is the landfill depth from its surface.
If SMW management means limitation of the filtrate concentration
\[
C < L
\]
where L is the maximum permissible concentration (MPC) of the filtrate, then MMMSWM has the form
\[ C = \frac{m(t)}{hS + w dqNe^{rT} - pS + m(t)} < L \]

\[ m(t) = Ae^{-\frac{z}{vC}}[B][C] \]

\[ \frac{\partial T}{\partial t} = k \frac{\partial T}{\partial x} \]

5. Conclusions

1) A retrospective analysis of mathematical models of SMW management has been performed.

2) A mathematical model of SMW (in particular, filtrate concentration) management has been built.

The further research area is solving the resulting model. The authors consider it possible to apply the variable separation method and search for T in the form of a trigonometric function.

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