Resonance Conversion as a Catalyser of Nuclear Reactions

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\textbf{Abstract}

It is shown that resonance internal conversion offers a feasible tool for mastering nuclear processes with laser or synchrotron radiation. Physics of the process is discussed in detail in historical aspect. Possible way of experimental application is shown in the case of the $M$ \textsuperscript{170.6-keV transition in nuclei of 169\textsuperscript{Yb}. Nuclear transition rate in hydrogenlike ions of this nuclide can be enhanced by up to four orders of magnitude.

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1 Introduction

In 1939, Bohr and Wheeler proposed the version of nuclear theory, which since that time is applied for description of this wonderful process \cite{1}. Ten years later, Wheeler told out an idea that fission of $^{238}\text{U}$ nucleus in the muonic atom can be induced by a radiationless muon transition $2s \rightarrow 1s$\cite{2}. Later investigation showed that the population of the $2s$ level is less probable, of a few percent of that for the $2p$ state, and additionally, the radiative $2s \rightarrow 2p$ transition makes a strong competition. However, it was shown that the radiationless transition probability for higher multipoles, $E1$ for the $2p,3p \rightarrow 1s$ transition\cite{3}, $E2$ for the $3d \rightarrow 1s$\cite{4,5}, and even $E3$ for the $3d \rightarrow 2p$ transition\cite{6,7} are all of approximately the same probability. Experiments fully confirmed the Wheeler’s conjecture and the further predictions\cite{8}.

In ref.\cite{6,7} special attention was brought to the fact that the radiationless transition is a reverse conversion process. It was considered in terms of the radiative nuclear width and internal conversion coefficients(ICC). That explained the reason why higher multipole transitions turn out to be of about the same probability. In 1958, Morita proposed a similar process of Nuclear Excitation in Electron Transition, which is since known as NEET\cite{9}. Calculations\cite{6,7} revealed the strong resonance coupling arising in atoms between the nucleus and the shell, or the muon in the case of muonic atom. This coupling offers a real tool of mastering nuclear electromagnetic transitions through resonance with the electron shell, using laser or synchrotron radiation.

The present paper is built as follows. (i)We remind in main features general internal conversion(IC) theory. (ii)We present the theory of the resonance internal conversion and outline experiments where it was discovered. (iii)We propose a way of mastering the rate of the resonance conversion by applying external laser or synchrotron radiation, and consider concrete examples.

2 Internal conversion

As a result of prompt fission, the muon is entrained on one of the fragments, mostly on the heavy one. The fragment is excited, and emits $\gamma$’s, neutrons, protons, alphas. Emitted $\gamma$’s can be absorbed by the muon, which leaves the atom (Fig.1). Such a process is well known in electronic atoms. It is called internal conversion (IC). The necessary condition is

$$\omega_n > I \quad (1)$$

where $\omega$ is the nuclear transition energy and $I$ is the ionization potential.

A very useful value is ICC, which is defined as the ratio of the conversion and radiative transition probabilities,

$$\alpha(\tau L) = \frac{\Gamma _\gamma}{\Gamma ^ {\{\alpha\}}}, \quad (2)$$
\[ \tau, L \text{ being the type and multipole order of the transition. } \alpha \text{ is nearly independent of the nuclear model. This allows one to reverse eq.}(2) \text{ and put down} \]
\[ \Gamma_c = \alpha \Gamma^{(n)} \gamma. \]  
(3)

Values of \( \alpha \) are tabulated. Eq. (3) hence allows one to estimate the conversion transition probability, as soon as the radiative nuclear width is known. IC is one of the principal tools for nuclear spectroscopy.

### 3 Resonance or Bound IC (BIC)

If condition (2) is broken, i.e. \( \omega_n < I \), the conversion electron cannot leave the atom. It occupies an excited electron level, forming an unstable intermediate state (Fig.2). This state then undergoes decay. This is mainly performed by the radiative electron transition, filling the hole formed in the place of the conversion electron. We still can formally calculate the \( \alpha \) value by means of the formula, used for the traditional ICC calculations, inserting the wave function of the related discrete atomic state as the conversion electron function:

\[ \alpha_d(\tau L) = \sum \kappa |M^{(\tau L)}_\kappa|^2, M^{(\tau L)}_\kappa = Q^{(L)}_\kappa R^{(\tau L)}_\kappa \]  
(4)

\[ Q^{(L)}_\kappa = -\sqrt{\frac{\alpha \pi \omega}{L(L+1)}} C_{\alpha j}^{\frac{1}{2}l, \frac{1}{2}L, 0}; \]

\[ R^{(ML)}_\kappa = (\kappa_1 + \kappa_2)(R_1 + R_2) \]

\[ R^{(EL)}_\kappa = L(R_3 + R_4) + (\kappa_1 - \kappa_2 - L)R_5 + \]

\[ + (\kappa_1 - \kappa_2 + L)R_6 \]

with \( R_i \) — the radial integrals

\[ R_1 = \int_0^\infty G_i F_j X_{Ld} dr \quad R_2 = \int_0^\infty F_i G_j X_{Ld} dr \]

\[ R_3 = \int_0^\infty F_i F_j X_{Ld} dr \quad R_4 = \int_0^\infty G_i G_j X_{Ld} dr \]

\[ R_5 = \int_0^\infty G_i F_j X_{L-1d} dr \quad R_6 = \int_0^\infty F_i G_j X_{L-1d} dr. \]

Here \( \alpha_d(\tau L) \) is the discrete ICC, \( \tau L \) is the type and multipole order of the transition, \( EL \) and \( ML \) stand for the electric and for the magnetic types, respectively. Subscripts \( i \) and \( f \) denote the initial and final states, respectively; \( \kappa = (l - j)(2j + 1) \) is the relativistic quantum number, with \( l, j \) for the orbital and total angular moments; \( G \) and \( F \) are the big and small components of the radial wave function, normalized at

\[ \int_0^\infty [G^2 + F^2(r)] dr = 1 \]  
(5)

Furthermore, \( \alpha \approx 1/137 \) is the fine structure constant, \( M^{(\tau L)}_\kappa \) are the conversion matrix elements, \( \omega \) is the nuclear transition energy. \( R_1 \ldots R_6 \) are the radial integrals, with \( X \nu \) the interaction potential of the nuclear and electron transition current. With account of the finite nuclear size, the latter becomes

\[ X_\nu = h_\nu(\omega r) \]  
(6)

for \( r \geq R_0 \), with \( R_0 \) — the nuclear radius. For \( r < R_0 \), nuclear model of the surface transition current provides an adequate description:\n
\[ X_\nu = \frac{h_\nu(\omega R_0)}{f_\nu(\omega R_0)} j_\nu(\omega r). \]  
(7)

In the case of discrete conversion, however, the \( \alpha_d \) value acquires dimension of energy, due to another normalization of the wavefunction. Therefore, it cannot serve as ICC anymore. There is an evident way, to form a dimensionless factor \( R \) by multiplying \( \alpha_d \) by the resonance Breit-Wigner factor. We add a subscript \( d \) to the sign of \( \alpha_d \), to distinguish it from a traditional ICC. Then the expression for \( R \) becomes as follows:

\[ R = \frac{\alpha_d \Gamma/2\pi}{\Delta^2 + (\Gamma/2)^2} \]  
(8)

where \( \Gamma \) is the full width of the intermediate atomic state, and \( \Delta \) is the defect of the resonance of the nuclear and electron transitions. With the account of BIC, resulting lifetime of the nuclear level will be

\[ \lambda = \frac{\lambda_\gamma}{1 + \alpha_{tot} + R}, \]

where \( \lambda_\gamma \) is the radiative lifetime, and \( \alpha_{tot} \) is the total ICC.

### 4 Tuning BIC

It follows from eq.(4) that the BIC probability can be enhanced in the case of resonance by the value of

\[ \frac{R_{res}}{R} \simeq \left( \frac{\Delta}{\Gamma} \right)^2. \]  
(9)
For nuclei, typical values of $\Delta \simeq 1\text{keV}$. Typical value of $\Gamma$ is $\sim 20$ eV (which is a typical $K$-hole width), or $\sim 10^{-6} - 10^{-5}$ eV in the case of BIC in the outer electron shells. Therefore, expected effect can be around ten orders of magnitude and more [12, 13].

The idea is to arrange a two-photon resonance. Consider atom in an external field of a plane electromagnetic wave. Some atomic electron can go to an excited state by absorption of one or several photons from the field. The probability of multiphoton absorption increases drastically if the total energy of the absorbed photons approaches the difference of the electron levels. This effect is used by RIS — resonance ionization spectroscopy. Let us consider the two-photon resonance, and replace one photon of the field by the nuclear photon [15]. This is laser assisted nuclear BIC transition, as the electron makes a two-photon transition to an excited state, one photon being from the nucleus, and the other from the field. Necessary condition is then

$$\omega_n \pm \omega_l = \omega_a.$$  \hspace{1cm} (10)

The two signs in (6) correspond to either an absorption, or induced emission of a photon of the field. The both probabilities are of the same value.

5 Nuclei in electromagnetic field

Typical nuclear sizes are of the order of $R \simeq 10$ Fm. Typical scale for their transition energies is of $\sim 1$ MeV. Therefore, meaningful effect of the electromagnetic field on nuclei can be only expected for fantastic electrical strength of the field $\varepsilon \simeq 10^{18}$ V/cm. At such strength, spontaneous generation of $e^+e^-$ pairs already takes place (Klein’s paradox[10]). Such a simple estimate helps to realize that all the photo-nuclear reactions are due to resonance effect. Only resonance quanta can be absorbed by the nuclei, with the energy which exactly equals the nuclear level separation energy.

Another lesson is why the only isomer was probably triggered up to now, that of $^{180\text{m}2}$Hf, in spite of tremendous experimental efforts applied in this field[13].

Finally, atomic size is by four orders of magnitude larger than the nuclear one. This means that it is much easier to affect the nucleus by electromagnetic field through mediation of the electron shell, which plays a role of resonator[15]. In view of the above-mentioned difficulties related with triggering the isomers, one must not afford a neglecting of several orders of magnitude gain which can be benefited from making use of the resonance properties of BIC. Further experimental study in the field must be directed in this way. We show in a separate paper a concrete example of this idea as applied to the $^{178\text{m}2}$Hf isomer.

6 Resonance conversion in experiments

A bright example of resonance effect of BIC is provided by characteristic muonic X-rays from heavy prompt fission fragments. This radiation arises due to resonance excitation of the bound muon to the $2p$ state, with reemission in the succeeding back muon transition $2p \to 1s$. This resonance effect predicted in ref.[10] is shown in Fig.3. The effect was experimentally studied

![Fig.3. Spectrum of $\gamma$ rays from prompt fission fragments, calculated with the account of the $\mu$ in the orbit.](image)
it is expected that the nuclear lifetime shall be considerably shortened in the field of electromagnetic wave of such frequency\cite{18}. This effect is demonstrated in Fig.5. As one can see, the effect can achieve up to four orders of magnitude.

A very promising field of application of the resonance BIC is the laser produced plasma. Then NEET, which is a reverse BIC process, is one of the possible mechanisms which can lead to formation of nuclear isomers in a heat bath\cite{19, 20}. This mechanism is under experimental investigation at the time being, specifically, with respect to the 76-eV $^{235m}$U isomer\cite{21}, 1.5-keV $^{201}$Hg isomer\cite{22}, 6-keV $^{178}$Ta isomer\cite{23}, and others. Further peculiarities of NEET arising in plasma are considered in paper\cite{24}.

Some of the results presented above were delivered on the conference\cite{25} held under honorary patronage of J. Wheeler. He was satisfied to know about development of his idea.

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**Fig. 4.** Scheme illustrating how traditional internal conversion transforms into the resonance one in ionized atoms of $^{125}$Te.

**Fig. 5.** Values of the resonance enhancement factor $R$ calculated for the hydrogenlike ions of $^{169}$Yb versus intensity of the external field.

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**References**

[1] Bohr N and Wheeler J A 1939 Phys Rev 56 426