Passivity-based filtering for networked semi-Markov robotic manipulators with mode-dependent quantization and event-triggered communication

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Abstract
In this article, the networked filtering problem for a class of robotic manipulators with semi-Markov type parameters is investigated under the passivity framework. In particular, the mode-dependent quantization and event-triggered communication scheme are both proposed for increasing the network transmission efficiency. Sufficient stability conditions are first derived by choosing mode-dependent Lyapunov–Krasovskii functionals. Then, the mode-dependent filter gains and the event-triggering parameters are further designed with the help of matrix convex optimization. In the end, a simulation example is provided such that the effectiveness of the proposed filtering method can be well demonstrated.

Keywords
Semi-Markov manipulator, networked filtering, mode-dependent quantization, mode-dependent event-triggered communication

Date received: 22 October 2020; accepted: 21 December 2020

Introduction
Robotic manipulators have been widely utilized in various applications during the past years.¹,² In particular, with the rapid development of network technology, novel networked control schemes have been developed for the stability and state estimation problems of manipulators. It is worth mentioning that there still exist certain constraints brought by the communication network, such as limited bandwidth, transmission delay, data packet dropout, and so on.³–⁶ These constraints would lead to instability of control systems or system control performance degradation. To deal with these issues, many effective networked analysis and synthesis strategies have been reported. Recently, the so-called event-triggered communication methods have been extensively studied.⁷–¹² To name a few, a novel event-trigger-based adaptive control approach is proposed for nonlinear systems with switching threshold strategy in the literature.¹¹ Moreover, by applying the improved event-triggered adaptive backstepping control method, the stability of uncertain networked control system has been successfully solved with desired results in the literature.¹² In contrast to traditional time-triggered mechanisms, the information transmission architecture of event-triggered schemes is waken-up by certain event

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rather than time sequences, which can further decrease network load and increase information exchange effectiveness.\textsuperscript{13} Since robotic manipulators are always with networked communication environment, it is necessary to apply the event-triggered schemes to improve the communication efficiency. Meanwhile, it is noted that many digital communication networks have bandwidth limitations. As a result, another useful method that can reduce the communication burden is the quantization strategy. By reducing the data packet size to be transmitted, the network bandwidth utilization efficiency can be correspondingly improved.\textsuperscript{8,14–16}

On another active research area, much effort has been paid to Markovian jump systems (MJSs), for the reason that MJSs have a powerful ability to model control systems with jumping parameters. Model examples of MJS in practical applications can be found with power systems, neural systems, robotic systems, and other actual examples that have taken advantage of MJS.\textsuperscript{17–20} Under this context, the transition probability of MJS plays a key role in conducting the system dynamics. As a result, there are many research merits of MJS with fixed or partially unknown transition probability cases.\textsuperscript{21,22} However, it should be pointed out that sometimes the transition probability may be time varying, which gives rise to a growing research interest in the semi-Markov jump systems (SMJSs) owing to the superiority of SMJS.\textsuperscript{23–25} With this regard, the payloads of manipulators could be varying in an unstructured and complicated environment, which has a significant impact on stable con-

The rest of the article is arranged as follows: In the second section, the networked filtering problem for single-link semi-Markov manipulator is formulated and the novel mode-dependent quantizer with event-triggered mechanism is introduced. In the third section, the main theoretical results are presented with proven details. In the fourth section, the usefulness of our proposed method is demonstrated by a simulation example. In the final section, we summarize the article and consider further study.

Notations: $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space. $A - B > 0$ implies that $A - B$ is positive definite. $\text{diag}\{\cdots\}$ denotes the block-diagonal matrix. $\mathbb{E}(\cdot)$ represents the expectation operator.

**Preliminaries and problem formulation**

Firstly, fix a probability space $(\mathcal{Q}, \mathcal{F}, \mathcal{P})$ and let $\{\sigma(t), t \geq 0\}$ denote a continuous-time discrete-state semi-Markov process on $(\mathcal{Q}, \mathcal{F}, \mathcal{P})$ taking values in a finite set $\mathcal{I} = \{1, \ldots, N\}$. The transition probability matrix $\Theta := (\pi_{ij}(h)), h > 0, \forall i, j \in \mathcal{I}$ is defined by

$$
\Pr(\sigma(t + h) = j|\sigma(t) = i) = \begin{cases} 
\pi_{ij}(h) + o(h), & i \neq j \\
1 + \pi_{ii}(h) + o(h), & i = j
\end{cases}
$$

with $\lim(o(h)/h) = 0$, $\pi_{ij}(h) \geq 0$, $i \neq j$ is the transition rate from mode $i$ at time $t$ to mode $j$ at time $t + h$, which satisfies $\pi_{ii}(h) = - \sum_{j \neq i}^N \pi_{ij}(h), \forall i \in \mathcal{I}$.

For simplicity, the following single-link manipulator model depicted in Figure 1 is considered.
\[
\dot{\theta}(t) = \psi(t)
\]
\[
J(\sigma(t)) \dot{\psi}(t) = -D(\sigma(t)) \psi(t) - M(\sigma(t)) g L \sin(\theta(t)) + u(t) + E_w(\sigma(t)) w(t)
\]
(2)

where \(\theta(t)\) denotes the angular position, \(\psi(t)\) denotes the angular velocity, \(J(\sigma(t))\) denotes the total moment of inertia, \(D(\sigma(t))\) denotes the coefficient of viscous friction, \(M(\sigma(t))\) denotes the mass of the payload, \(g\) denotes the acceleration of gravity, \(L\) denotes the length of the manipulator, \(u(t)\) denotes the feedback control input, \(E_w(\sigma(t))\) is the disturbance gain, and \(w(t)\) denotes the external disturbances.

Let \(x(t) = [x_1(t), x_2(t)]^T\), where \(x_1(t) = \theta(t)\) and \(x_2(t) = \dot{\psi}(t)\). Then, it can be obtained that
\[
\begin{align*}
\dot{x}(t) &= A(\sigma(t)) x(t) + f(\sigma(t), x(t)) + B(\sigma(t)) u(t) + A_w(\sigma(t)) w(t) \\
y(t) &= C(\sigma(t)) x(t) \\
z(t) &= L(\sigma(t)) x(t) \\
x(0) &= x_0
\end{align*}
\]
(3)

where \(y(t)\) denotes the measured output, \(z(t)\) denotes the objective signal for estimation, and
\[
A(\sigma(t)) = \begin{bmatrix} 0 & 1 \\ 0 & -D(\sigma(t))/J(\sigma(t)) \end{bmatrix}
\]
(4)
\[
f(\sigma(t), x(t)) = \begin{bmatrix} 0 \\ -M(\sigma(t)) g L \sin(\theta(t))/J(\sigma(t)) \end{bmatrix}
\]
(5)
\[
B(\sigma(t)) = \begin{bmatrix} 0 \\ 1/J(\sigma(t)) \end{bmatrix}
\]
(6)
\[
A_w(\sigma(t)) = \begin{bmatrix} 0 \\ E_w(\sigma(t)) \end{bmatrix}
\]
(7)

By denoting \(\sigma(t)\) as \(i\) index, system (3) can be rewritten as follows
\[
\begin{align*}
\dot{x}(t) &= A_i x(t) + f_i(x(t)) + B_i u(t) + A_{wi} w(t) \\
x(0) &= x_0
\end{align*}
\]
(8)

Under the networked environment, it is assumed that the sensor is time driven with sampling period \(h_i\) according to mode \(i\) and there is no Zeno behavior. To estimate the system state, the following mode-dependent observer is designed
\[
\begin{align*}
\dot{\hat{x}}(t) &= A_i \hat{x}(t) + f_i(\hat{x}(t)) + B_i u(t) + K_i (y(t) - q_i(y(t \beta h_i))) \\
\dot{\hat{y}}(t) &= C_i \hat{x}(t) \\
\dot{\hat{z}}(t) &= L_i \hat{x}(t)
\end{align*}
\]
(9)

where \(\hat{x}(t)\) denotes the estimation of \(x(t)\), \(K_i\) denotes the mode-dependent filter gains, and \(q_i(y(t \beta h_i))\) denotes the quantized output by the communication network. The event generator updates the released signals with \(t_k h_i\), \(k = 0, 1, 2, \ldots\). In addition, the event-triggering function is designed by
\[
e_k^T(t) W_1 e_k(t) \geq \varepsilon \dot{y}^T(t \beta h_i + j h_i) W_2 \dot{y}(t \beta h_i + j h_i)
\]
(10)

where
\[
e_k(t) = y(t \beta h_i) - y(t \beta h_i + j h_i),
\]
(11)

where \(0 \leq \varepsilon, \beta < 1\) and \(W_{1i} > 0, W_{2i} > 0\) is a mode-dependent scale matrix.

**Remark 1.** It is worth mentioning that the above-formulated system model can also be applied to common nonlinear SMJSs with Lipschitz nonlinear characteristics, such that our developed results have broad applicability for SMJSs.

**Remark 2.** In comparison with mode-independent event-triggered strategy, the mode-dependent strategy can lead to less conservatism and is more applicable for the semi-Markov systems, since the precise mode information can be effectively used by mode-dependent strategy.

Recently, some novel hybrid-triggered schemes have been investigated with satisfying results, which can further combine the advantages of event-triggered and time-triggered mechanisms.

Once the event-triggering function is satisfied, the latest data will be transmitted to the corresponding mode-dependent quantizer
\[
\Gamma_i = \{w_i = \mu_i w_0, k = 0, 1, 2, \ldots \} \cup \{0\}, w_0 > 0
\]
(12)

where \(\mu_i \in (0, 1]\) and \(q_i(\cdot) : \mathbb{R} \rightarrow \Gamma\) is defined as follows
\[
q_i(y(t \beta h_i)) = \begin{cases}
\frac{1}{1 + \kappa_i} w_i, & \text{if } \frac{1}{1 + \kappa_i} w_i < \frac{1}{1 - \kappa_i} w_i \\
0, & \text{if } \frac{1}{1 - \kappa_i} w_i \geq 0, y(t \beta h_i) < 0
\end{cases}
\]
(13)

where \(\delta_i = \frac{1 - \mu_i}{\mu_i (1 + \kappa_i)}\) denotes sector bound. The quantization density for quantizer (13) is defined as \(\frac{\rho}{\delta_i}\). Then, it follows that
\[
q_i(y(t \beta h_i)) = (I + \Delta_i) y(t \beta h_i), \quad \Delta_i \in [-\delta_i, \delta_i]
\]
(14)

**Remark 3.** The logarithmic quantizer strategy is adopted with mode-dependent features in this article, such that each corresponding mode-dependent quantizer can effectively deal with the system jumping behaviors accordingly.

In addition, the mode-dependent network-induced transmission delay is assumed to be bounded by \(\bar{\tau}_i\). Then, by letting
\[ \tau_i(t) = t - t_k h_i - j h_i \]  
\[ e_k^T(t) W_1 e_k(t) \geq \varepsilon_1 \gamma(t - \tau_i(t)) W_2 e_k^T(t - \tau_i(t)) \]  
where \( 0 \leq \tau_i(t) \leq \bar{\tau} \).

As a result, by letting filtering error be \( \tilde{x}(t) = x(t) - \bar{x}(t) \), \( \bar{z}(t) = z(t) - \bar{z}(t) \), and \( f_i(\tilde{x}(t)) = f_i(x(t)) \), it follows that

\[
\begin{cases}
\tilde{x}(t) = A_i \tilde{x}(t) + f_i(\tilde{x}(t)) + A_{wi} w(t) \\
-K_i C_i x(t) + K_i C_i \tilde{x}(t) \\
+K_i(I + \Delta_i) e_k(t) + C_i x(t - \tau_i(t)) \\
\bar{z}(t) = L_i \tilde{x}(t)
\end{cases}
\]

Then, one can obtain that

\[
\begin{cases}
\dot{\zeta}(t) = A_i \zeta(t) + A_{ei} e_k(t) + A_{di}(t - \tau_i(t)) \\
+ F_i(\zeta(t)) + A_{wi} w(t) \\
\bar{z}(t) = L_i \zeta(t)
\end{cases}
\]

where \( \zeta(t) = [x^T(t), \tilde{x}^T(t)]^T \) and

\[
A_i = \begin{bmatrix}
A_i & 0 \\
-K_i C_i & A_i + K_i C_i
\end{bmatrix}
\]

\[
A_{ei} = \begin{bmatrix}
0 \\
K_i(I + \Delta_i)
\end{bmatrix}
\]

\[
A_{di} = \begin{bmatrix}
K_i(0 & 0 & 0 \\
K_i(I + \Delta_i) C_i & 0
\end{bmatrix}
\]

\[
F_i(\zeta(t)) = \begin{bmatrix}
f_i(\tilde{x}(t)) \\
f_i(x(t)) - f_i(\tilde{x}(t))
\end{bmatrix}
\]

\[
L_i = \begin{bmatrix}
0 & L_i
\end{bmatrix}
\]

The structure of mode-dependent filter is shown in Figure 2. For the filtering problem, the passivity performance is adopted with the following definition.

**Definition 1.**\(^{32,33}\) If there exists a positive constant \( \gamma \), such that

\[
2 \mathbb{E} \left\{ \int_0^T \tilde{z}^T(t) w(t) \, dt \right\} \geq -\gamma \int_0^T w^T(t) w(t) \, dt
\]

then the system is said to satisfy the passivity performance.

**Remark 4.** Different from the common \( H_\infty \) performance for disturbance attenuation, the passivity performance is concerned with the system input and output for complex systems from an energy perspective. Passivity performance implies that the energy increment must be less than that supplied and thus ensures the stability at the same time.\(^{32,34,35}\)

To this end, the following lemma is provided for deriving the main results.

**Lemma 1.**\(^{36}\) Let \( \mathcal{L}^T = \mathcal{L} \), \( \mathcal{H} \) and \( \mathcal{E} \) be real matrices of appropriate dimensions with \( \mathcal{F}(t) \) satisfying \( \mathcal{F}^T(t) \mathcal{F}(t) \leq I \). Then, \( \mathcal{L} + \mathcal{H} \mathcal{E} + \mathcal{E}^T \mathcal{H} \mathcal{E}^T T < 0 \), if and only if there exists a scalar \( \varepsilon > 0 \) such that \( \mathcal{L} + \varepsilon^{-1} \mathcal{H} \mathcal{E} + \varepsilon \mathcal{E}^T \mathcal{H} \mathcal{E} < 0 \), or equivalently

\[
\begin{bmatrix}
\mathcal{L} & \mathcal{H} & \varepsilon \mathcal{E}^T \\
* & -\varepsilon I & 0 \\
* & * & -\varepsilon I
\end{bmatrix} < 0
\]

**Main results**

In this section, the mode-dependent filter design procedure will be given in detail.

**Theorem 1.** Based on the event-triggered function and designed mode-dependent filter gains \( \bar{K}_i \), the passivity performance of augmented system (18) can be satisfied according to Definition 1 such that the filtering problem of semi-Markov robotic manipulator can be solved, if there exist mode-dependent matrices \( \mathcal{P}_i > 0, \mathcal{W}_1 > 0, \mathcal{W}_2 > 0 \) and matrices \( \mathcal{Q} > 0, \mathcal{R} > 0 \), and it holds that \( \Pi_{i,k} < 0 \), where \( i \in \mathcal{N} \) and \( k = 1, 2, \ldots, \mathcal{K} \), and

\[
\Pi_{i,k} = \begin{bmatrix}
\Pi_{i1,k} & \Pi_{i2,k} \\
* & \Pi_{i3,k}
\end{bmatrix}
\]

\[
\Pi_{i1,k} = \begin{bmatrix}
\Pi_{i11,k} & \mathcal{P}_i \mathcal{A}_{di} + \mathcal{R} \\
* & \Pi_{i12,k}
\end{bmatrix}
\]

\[
\Pi_{i11,k} = \mathcal{P}_i \mathcal{A}_i + \mathcal{Q} - \mathcal{R} + i^2 + \sum_{j=1}^\mathcal{K} \pi_{ij,k} \mathcal{P}_j
\]

\[
\Pi_{i12,k} = -2\mathcal{R} + \varepsilon_i \mathcal{C}_i^T \begin{bmatrix}
I & 0 \\
0 & \mathcal{W}_2 \end{bmatrix} \mathcal{C}_i
\]

\[
\Pi_{i2,k} = \begin{bmatrix}
0 & \mathcal{P}_i \mathcal{A}_{ei} & \mathcal{P}_i \mathcal{A}_{wi} - \mathcal{L}_i^T \mathcal{H} \tau \mathcal{A}_{ei}^T \mathcal{R} \\
\mathcal{R} & 0 & 0 \\
-\mathcal{R} \tau \mathcal{A}_{ei}^T \mathcal{R}
\end{bmatrix}
\]

\[
\Pi_{i3,k} = \begin{bmatrix}
-\mathcal{Q} - \mathcal{R} & 0 & 0 & 0 \\
0 & \mathcal{R} & 0 & \bar{\tau} \mathcal{A}_{ei}^T \mathcal{R} \\
-\mathcal{Q} - \mathcal{R} & 0 & \bar{\tau} \mathcal{A}_{ei}^T \mathcal{R} \\
0 & \mathcal{R} & 0 & \bar{\tau} \mathcal{A}_{ei}^T \mathcal{R}
\end{bmatrix}
\]
It can be verified that
\[
\tau^2 \zeta^T(t) R \zeta(t) = \begin{bmatrix}
\tau A^T_{di} \\
\tau A^T_{di} \\
\tau A^T_{di} \\
\tau A^T_{di}
\end{bmatrix}
\begin{bmatrix}
\tau A^T_{di} \\
\tau A^T_{di} \\
\tau A^T_{di} \\
\tau A^T_{di}
\end{bmatrix}^T
\begin{bmatrix}
0 \\
0 \\
\tau I \\
\tau I
\end{bmatrix} \delta(t)
\]
where \( \delta(t) = [\zeta^T(t), \zeta^T(t - \tau_i(t)), \zeta^T(t - \bar{\tau}), F_i^T(\zeta(t)), e_i^T(t), w^T(t)]^T \).

From the event-triggering function, one has
\[
-2 \zeta^T(t) L_i^T w(t) - \gamma w^T(t) w(t) + \varepsilon_i \zeta^T(t - \tau_i(t)) C_i^T[I \ 0]^T W_{2i} + [I \ 0] C_i \zeta(t - \tau_i(t)) - e_i^T(t) W_{1i} e_i(t) \geq 0
\]
Furthermore, it holds that
\[
-f^T(i, x(t)) f(i, x(t)) + \zeta^T(t) x(t) \geq 0
\]
where \( i \equiv M_g L / D_i J_i \).

Consequently, if it holds that \( \Pi_i < 0 \), where
\[
\Pi_i = \begin{bmatrix}
\Pi_{i1} & \Pi_{i2} \\
* & \Pi_{i3}
\end{bmatrix}
\]

\[
\Pi_{i1} = \begin{bmatrix}
\Pi_{i11} & \mathcal{P}_i A_{di} + \mathcal{R}
\end{bmatrix}
\]

\[
P_{i11} = 2 \mathcal{P}_i A_i + Q - R + i^2 + \sum_{j=1}^{N_i} \pi_i(j) \mathcal{P}_j
\]

\[
P_{i12} = -2R + \varepsilon_i C_i^T[I \ 0]^T W_{2i} + [I \ 0] C_i
\]

\[
P_{i2} = \begin{bmatrix}
0 & \mathcal{P}_i & \mathcal{P}_i \mathcal{A}_{ei} \mathcal{P}_i \mathcal{A}_{wi} - L_i^T \tau A_{di}^T \mathcal{R} \\
R & 0 & 0 \\
* & -I & 0 \\
* & * & -\tau R
\end{bmatrix}
\]

\[
P_{i3} = \begin{bmatrix}
* & -W_{1i} & 0 & \tau A_{wi}^T \mathcal{R} \\
* & * & 0 & -R
\end{bmatrix}
\]
then, one has \( \mathcal{L} V(i, t) < 0 \).

In addition, by taking into account the time-varying dwell time \( h(t) \), one has \( \tau_i(h) = \sum_{k=1}^{K} \tau_k \pi_k(h) \), \( \sum_{k=1}^{K} \zeta_k = 1 \), and \( \zeta_k \geq 0 \). This implies that if \( \Pi_{i3} < 0 \) holds, the filtering error of networked semi-Markov robotic manipulator can achieve the passivity performance in the mean-square sense. This completes the proof.
Based on the derived conditions in Theorem 1, the following theorem can be given for calculating the desired filter gains.

**Theorem 2.** Based on the event-triggered function, the passivity performance of augmented system (18) can be satisfied according to Definition 1 such that the filtering passivity performance of augmented system (18) can be calculated.

The following theorem can be given for calculating the desired filter gains.

\[
\Xi_{i,\kappa} := \begin{bmatrix} \Xi_{i,\kappa} & \Xi_{i,\kappa} \\ \ast & \Xi_{i,\kappa} \end{bmatrix}
\]

(52)

\[
\Xi_{i1,\kappa} := \begin{bmatrix} \Xi_{i11,\kappa} & -\tilde{K}_{i},C_{i} & R_{1} & 0 \\ * & \Xi_{i12,\kappa} & \tilde{K}_{i},C_{i} & R_{2} \\ * & * & \Xi_{i13,\kappa} & 0 \\ * & * & * & -2R_{2} \end{bmatrix}
\]

(53)

\[
\Xi_{i11,\kappa} := 2P_{1i}A_{i} + Q_{i} - R_{1} + \sum_{j=1}^{N} \pi_{j}P_{1j} + \varepsilon^{2}
\]

(54)

\[
\Xi_{i12,\kappa} := 2P_{2i}A_{i} + \tilde{K}_{i},C_{i} + Q_{i} - R_{2} + \sum_{j=1}^{N} \pi_{j}P_{2j} + \varepsilon^{2}
\]

(55)

\[
\Xi_{i13,\kappa} := -2R_{1} + \varepsilon_{i}C_{i}^{T}W_{2i}C_{i} - \Xi_{i21,\kappa}
\]

(56)

\[
\Xi_{i21,\kappa} := \begin{bmatrix} 0 & 0 & P_{1i} & 0 & 0 & P_{1i}A_{wi} \\ 0 & 0 & 0 & P_{2i} & \tilde{K}_{i} & P_{2i}A_{wi} - L_{i}^{T} \\ R_{1} & 0 & 0 & 0 & 0 & 0 \\ R_{2} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

(57)

\[
\Xi_{i22,\kappa} := \begin{bmatrix} \tau A_{i}^{T}P_{1i} & -\tilde{K}_{i}^{T} & 0 & 0 \\ 0 & \tau A_{i}^{T}P_{2i} + C_{i}^{T}\tilde{K}_{i} & 0 & 0 \\ 0 & \tilde{K}_{i}^{T} & \tau C_{i} & \varepsilon C_{i} \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

(58)

\[
\Xi_{i3,\kappa} := \begin{bmatrix} \Xi_{i31,\kappa} & \Xi_{i32,\kappa} \\ \ast & \Xi_{i33,\kappa} \end{bmatrix}
\]

(60)

\[
\Xi_{i31,\kappa} := \begin{bmatrix} -Q_{1} - R_{1} & 0 & 0 & 0 & 0 \\ * & -Q_{2} - R_{2} & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -W_{1i} \end{bmatrix}
\]

(61)

When the above conditions are satisfied, the mode-dependent filtering gains can be calculated by

\[
K_{i} = P_{2i}^{-1}\tilde{K}_{i}
\]

(64)

**Proof.** Based on Lemma 1 and matrix transformation, the proof follows directly from Theorem 1 by letting \(\tilde{K}_{i} = P_{2i}K_{i}\).

**Remark 5.** The established convex optimization conditions are with strict LMIs, which can be easily solved by MATLAB LMI toolbox or YALMIP with feasible solutions. It can be seen that the computation complexity is mainly related with the number of system modes \(N\) and the convex combination form of time-varying transition probability matrix with \(\mathcal{K}\), such that the number of LMIs is \(N \times \mathcal{K}\).

**Illustrative example**

In this section, a numerical simulation is given to validate our proposed filter design.

Consider the formulated semi-Markov robotic manipulator model (2) with two modes, where \(M_{1} = 2\) kg, \(M_{2} = 2.5\) kg, \(J_{1} = 5\) kg m\(^{2}\), \(J_{2} = 6.25\) kg m\(^{2}\), \(D_{1} = 2\) Nm rad/s, \(D_{2} = 2\) Nm rad/s, \(L = 0.16\) m, \(E_{1} = 0.5\), \(E_{2} = 0.8\), \(C_{1} = \text{diag} \{2, 2\}\), and \(C_{2} = \text{diag} \{1.8, 2.1\}\).

Furthermore, the transition rates are supposed to be \(\pi_{11}(h) \in (-1.6, -1.4)\) and \(\pi_{22}(h) \in (-1.7, -1.3)\), which implies that \(\pi_{11,1} = -1.4\), \(\pi_{11,2} = -1.6\), \(\pi_{22,1} = -1.3\), and \(\pi_{22,2} = -1.7\) with \(\mathcal{K} = 2\).

In the simulation, it is assumed that \(h_{1} = 0.1\) s and \(h_{2} = 0.2\) s. The passivity performance index is set by \(\gamma = 1\) and the disturbance is supposed to be \(w(t) = 0.1\sin(t)\). With these parameters, the event-triggered scalar matrices and the desired mode-dependent filter gains are obtained as follows

\[
W_{11} = \begin{bmatrix} 1.6505 & -0.0335 \\ -0.0335 & 1.0426 \end{bmatrix}
\]

\[
W_{12} = \begin{bmatrix} 0.9867 & -0.1470 \\ 0.9867 & -0.1470 \end{bmatrix}
\]
From Figures 3 and 4, it can be seen that our designed mode-dependent filters can well estimate the interested states with disturbances under the passivity framework. More precisely, Figure 3 shows that the mode-dependent filter can well obtain the true state of manipulator, such that the filtering errors can converge to zero. Figure 4 depicts that the objective signal $z(t)$ can be correspondingly estimated with desired filtering performance since the error signal $\tilde{z}(t)$ can converge to zero with disturbances. Moreover, Figures 5 to 7 show that the developed event-triggered approach can effectively decrease the numbers of communications compared with tradition time-triggered schemes. One can found that the mode-dependent event-triggered communication can also deal with bandwidth limitations, where the network burden can be reduced considerably. Thus, the numerical simulation can support our theoretical design and demonstrate the effectiveness. In addition, the relation between passivity performance index $\gamma$ and different quantization density with $\mu_i$ is given in Table 1. It can be seen that the larger $\mu_i$ can lead to larger minimum passivity performance.

**Conclusion and discussions**

This article is concerned with the filtering problems of networked semi-Markov robotic manipulators with...
quantization and event-triggered communication. Furthermore, a new design strategy with mode-dependent characteristics is firstly developed for this problem. Then, the passivity performance is adopted to deal with the external disturbance. Sufficient filtering criterion is established for ensuring the prescribed performance of the augmented filtering system, based on which the desired mode-dependent filtering gains are designed in correspondence with these derived conditions via matrix transformation. Finally, the simulation results are presented to illustrate the effectiveness and availability of our design scheme. It can be found that our developed filtering method can well estimate the semi-Markov manipulator with disturbances. Meanwhile, it should be pointed out that the mode information is needed as a prior knowledge, which has certain conservatism in practical applications. In our future research, we will focus on extending our current theoretical results to the case with asynchronous semi-Markov processes and relevant experiments, which means that the modes of the observer could be asynchronous to the modes of the manipulator and is more practical for real-world applications.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

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