Assessment of natural horizontal stresses and deformation characteristics of load-bearing elements in room-and-pillar mining by inverse problem solution using mine survey data

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Abstract. The authors discuss assessment of deformation characteristics of load-bearing elements in room-and-pillar mining system and natural horizontal stresses. This method is based on solution of mixed-type inverse problems using a two-dimensional elastic model. Numerical calculations for a typical mine geometry using synthetic input data (relative displacements of mine void boundary points, recorded using mine survey techniques) have found the required data amount sufficient to ensure unambiguous solvability of inverse problem.

1. Introduction
Evaluation of effective stresses in rock mass is required during mine planning and design, when optimizing flow charts of mineral access and in selecting mining technologies [1]. Vertical stress is governed by weight of overlying rocks, and horizontal stresses are connected to tectonics of the upper lithosphere [2]. The horizontal stresses of plates and micro plates of the lithosphere are found using indirect methods [3]. Within mine fields, the horizontal stresses are measured directly [4–6]. It is important to determine orientation of the maximal horizontal stress, for the main drifts to be stable for a long time are driven in parallel to this orientation [1]. The data on the effective stresses in the upper lithosphere are compared [7], and it is proposed to find orientation of the maximal horizontal stress from the analysis of seismotectonic strains in terms of the source mechanisms of earthquakes [8]. However, this procedure fails to determine specific values of stresses.

The in-situ stress estimation techniques consume much labor and money. In these methods, a stress field is subjected to perturbation, and the stresses of interest are estimated from the response of the medium. Amongst these methods, the hydraulic fracturing method of stress measurement [9, 10] is usually preferred as its results are independent of deformation characteristics of rocks.

The room-and-pillar method of mining of stratified deposits requires information on properties of pillars in order to predict their stability. Laboratory test data are distorted due to the natural or induced faulting of rock [11, 12]; therefore, the required parameters are to be found in mines [13]. Geometry of an underground mine space changes in the course of mining, and fields of strains and displacements alter as a result, both in adjacent rock mass and far beyond up to ground surface [14]. Nazarov et al [15] proposed to find natural stresses using the satellite geodesy data [16, 17] on strains and displacements of undermined ground surface.
This paper undertakes to evaluate deformation characteristics (Young’s modulus and Poisson’s ratio) of load-bearing elements of the room-and-pillar system and horizontal stresses by solving inverse problem on boundary displacements in mine roadways using a two-dimensional elastic model.

2. Direct problem

Figure 1 shows a typical geometry of the room-and-pillar system in a shallow horizontally stratified ore body with rib pillars. The length of the rooms along the strike is much more than their cross dimension; for this reason, it can be assumed in the first approximation that the computational domain experiences plane deformation state [4].

![Figure 1. Computational domain and boundary conditions.](image)

The geometrical parameters of the model are \( L_x = 400 \) m and \( L_z = 200 \) m along the coordinate axes, the occurrence depth of the stratum \( H = 100 \) m and its thickness is 10 m. Mine roadways are separated by safety pillars. The cross dimension of the rooms and pillars is 10×10 m. The computational domain experiences the action of vertical and horizontal compressive stresses. The vertical stress complies with the overlying rock weight, and the horizontal stress is characterized by the lateral earth pressure coefficient \( q \).

We describe deformation of rock mass using a system of linear elasticity equations, including equilibrium equation (1), Hooke’s law (2) and Cauchy’s relation for small strains (3):

\[
\sigma_{ij,j} + \rho g \delta_{ij} = 0, \\
\sigma_{ij} = \lambda \varepsilon_{ij} + 2\mu \varepsilon_{ij}, \\
\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i})
\]

where \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the components of the stress and strain tensors \((i, j = x, z)\); \( \varepsilon = \varepsilon_{xx} + \varepsilon_{zz} \) is the volumetric strain; \( u_{i} \) are the displacements; \( \lambda, \mu \) are the Lamé parameters; \( \delta_{ij} \) is the Kronecker delta.

The boundary conditions along the perimeter of the computation domain are given by:

\[
u_x(0,z) = 0, \quad \sigma_{xz}(0,z) = 0, \\
\sigma_{xx}(L_x,z) = q\sigma_{yy}(z), \quad \sigma_{xz}(L_x,z) = 0, \\
\sigma_{zz}(x,0) = 0, \quad \sigma_{xz}(x,0) = 0, \\
u_z(x,L_z) = 0, \quad \sigma_{xz}(x,L_z) = 0.
\]

The stratum and enclosing rocks interact via true cohesion. The boundaries of the rooms are stress-free.

System of equations (1)–(3) with boundary conditions (4)–(7) was solved by an original code [18] using the finite element method for structurally inhomogenous media with discontinuities. The rectangular finite element mesh 2 m in size contained 20301 nodes. The modeling assumed that enclosing rocks and the stratum represented an elastic medium with properties described in the table [19]. The values of \( \lambda \) and \( \mu \) were found in terms of Young’s modulus \( E \) and Poisson’s ratio \( \nu \).
Properties of model geomedium

| Geomedium     | $E$, GPa | $\nu$ | $\rho$, kg/m$^3$ |
|---------------|----------|-------|-----------------|
| Enclosing rocks | 2        | 0.3   | 2500            |
| Stratum       | 1.3      | 0.3   | 2400            |

Let us estimate horizontal displacements of walls in roadway 2 (Figure 1) induced by mining operations in new roadway 3. The wall displacements can be recorded using mine survey plug or high-precision surveyor’s levels [20], or borehole deformation gauges with resolution to 5 μm [21]. It is worth mentioning that the increase in the natural horizontal stresses results in the higher convergence of walls in the roadways.

Figure 2 illustrates convergence of sidewalls in roadway 2 ($x_1 = 190$ m, $x_2 = 200$ m) at various lateral earth pressure coefficients $q = 0.4–0.8$ (curves 1–5, respectively).

![Figure 2. Increment in horizontal displacement of sidewalls in roadway.](image)

3. Inverse problem

The load-bearing elements of the room-and-pillar mining systems are rib pillars. The deformation characteristics of the rib pillar may differ from the properties of enclosing rock mass because of the natural or induced faulting of strata. Stability assessment and life estimation of pillars need information on their geomechanical properties [22]. The model inverse problem is to determine the lateral earth pressure coefficient $q$, Young’s moduli $E_1$ and $E_2$, and Poisson’s ratios $\nu_1$ and $\nu_2$ of two neighbor pillars in terms of the horizontal $U_0(z_m)$ and vertical $V_0(x_n)$ displacement in the rooms ($x_n$, $z_m$—coordinates of recording gauges) after formation of a new roadway nearby (Figure 1).

We introduce an objective function such that its minimum yields the solution of the inverse problem:

$$
\Omega = \alpha \sum_m [U(q, E_1, \nu_1, E_2, \nu_2, z_m) - U_0(z_m)]^2 + (1 - \alpha) \sum_n [V(q, E_1, \nu_1, E_2, \nu_2, x_n) - V_0(x_n)]^2,
$$

where $\alpha$ is an empirical coordinate ($0 \leq \alpha \leq 1$); $U$ and $V$ are the relative horizontal and vertical displacements of the boundaries in the first and second roadways, calculated from (1)–(7); $U_0$ and $V_0$ are the in situ displacements measured in the mine.

The objective functions were calculated using synthetic input data: instead of the actual recorded displacements, we took the calculated displacements with superimposed random error:

$$
U_0(z_m) = (1 + \gamma)[u_{x1}(q^s, E_1^s, \nu_1^s, E_2^s, \nu_2^s, x_{11}, z_{m1}) - u_{x1}(q^s, E_1^s, \nu_1^s, E_2^s, \nu_2^s, x_{12}, z_{m2})] +
(1 + \gamma)[u_{x2}(q^s, E_1^s, \nu_1^s, E_2^s, \nu_2^s, x_{21}, z_{m2}) - u_{x2}(q^s, E_1^s, \nu_1^s, E_2^s, \nu_2^s, x_{22}, z_{m2})],
$$

where $q^s$, $E_1^s$, $\nu_1^s$, $E_2^s$, $\nu_2^s$ are the superimposed random errors.
\[ V_0(x_n) = (1 + \gamma)[u_{z1}(q^s, E_1^s, v_1^s, E_2^s, v_2^s, z_{11}, x_{11}) - u_{z1}(q^s, E_1^s, v_1^s, E_2^s, v_2^s, z_{12}, x_{12})] + \\
(1 + \gamma)[u_{z2}(q^s, E_1^s, v_1^s, E_2^s, v_2^s, z_{21}, x_{21}) - u_{z2}(q^s, E_1^s, v_1^s, E_2^s, v_2^s, z_{22}, x_{22})], \\
\]

(10)

where \( \gamma \) is an arbitrary value uniformly distributed over the interval \([-\eta, \eta]\); \( \eta \) is the amplitude of the error; \( u_{z1}, u_{z2} \) are the horizontal and vertical displacements of the roadway boundary; \( x_{i1}, x_{i2} \) and \( z_{j1}, z_{j2} \) are the coordinates of the measuring devices in the roadways ( \( i = 1, 2 \) — numbers of roadways).

In the formulated mixed-type inverse problem \([2]3\), the parameters \( q^s, E_1^s, v_1^s, E_2^s \) and \( v_2^s \) are found using the method of conjugate gradients \([15, 24]\) in its modification \([25]\). In this modification, partials derivatives are evaluated at each step of iteration with respect to each argument of the objective function.

Figure 3 shows contour line of the objective function \( \Omega \) in different cross sections, at the input data noise not higher that the limiting value of 20%. The squares mark the initial approximations of the unknown parameters; the dashed lines denote trajectories of iterations; the white circles are the exact solution.

![Figure 3. Contour lines of objective function in different cross sections: (a) \( v_1, v_2 \); (b) \( q, v_2 \); (c) \( E_1, E_2 \).](image)

Thus, the formulated inverse problem is solvable for the conditions set—the computational domain contains the single minimum of the objective function. The solution is greatly affected by the errors of the input data, and it is required to increase the number of measurement points in roadways in order to minimize this impact.

4. Conclusions

The method proposed for estimating horizontal natural stresses and elastic properties of load-bearing elements in the room-and-pillar system of mining is implementable using the mine survey data on relative displacements of open void boundaries in the course of mining. The numerical experiments show that at the input data noise of 20%, the lateral earth pressure coefficient and the elastic properties of pillars can be determined at permissible accuracy in case the increments in the boundary displacements are recorded not less than at 10 points in roadways.

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