Dispersive Approach to Abelian Axial Anomaly, Mixing of Pseudoscalar Mesons and Symmetries

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Abstract

We suggest a rigorous generalization of the pseudoscalar mesons mixing description in $SU(3)$ basis. It is shown that the appearance of extra massive state nicknamed in the paper as glueball is unavoidable in any scheme with more than one angle. In this framework we develop the dispersive approach to Abelian axial anomaly of isoscalar non-singlet current. Combining it with the analysis of experimental data of charmonium radiative decays ratio we get the number of quite strict constraints for mixing parameters. Our analysis favors the equal values of axial currents coupling constants which may be considered as a manifestation of $SU(3)$ symmetry and possible violation of chiral symmetry based predictions.

1 Introduction

The problem of pseudoscalar meson states mixing has been under intense theoretical and experimental study for many years. This topic attracts a lot of interest due to its close relation to such fundamental phenomena as quantum anomaly, chiral symmetry breaking and study of exotic states like glueball, which can elucidate the essential features of QCD.

During the last decade, a large amount of new experimental data on mesons has been collected. New data from the forthcoming experiments at COMPASS, BES-III, GlueX (JLAB) and at the upgraded facilities FAIR (GSI) and MAMI might allow a complete quantitative verification of the different mixing schemes and approaches.

A number of analyzes of mixing in $\eta - \eta'$ system based on different processes have been performed in the last decades and the mixing angles in the range $-10^\circ \div -24^\circ$ were obtained (see, for example [11 – 18]). The analysis of the axial anomaly generated decays $\eta(\eta') \to 2\gamma$ was also performed in [9] (in the framework of ChPT) and [10], and the estimation $\theta = -20^\circ \div 25^\circ$ was obtained.

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The approach with one mixing angle dominated for decades in the studies of the $\eta$-$\eta'$ system. However, in the recent years there was a rise of interest in the mixing schemes with two and three angles. The theoretical ground of this was based on the observation that taking into account the chiral anomaly through perturbative expansion in ChPT can lead to the introduction of two mixing angles in the description of the $\eta$-$\eta'$ system \cite{11, 12}. Some analyzes of various decay processes were also performed in this scheme \cite{13, 14, 15, 16, 17}.

The theoretical analysis of the mixing in the pseudoscalar sector base either on $SU(3)$ or quark basis. The last was introduced by T. Feldmann and P. Kroll \cite{13} and was widely used in the last years.

For us it happens to be more convenient to construct and use the rigorous generalization of $SU(3)$ basis similar to the mixing of massive neutrinos. This is because we use the dispersive approach to axial anomaly \cite{18} (see also \cite{19} for the review) to find some model-independent and precise restriction on the mixing parameters (angles and decay constants) in the different schemes (with one and more angles). The combination of our approach with certain processes leads to quite strict predictions. We consider this paper as a first step and application of our approach to the whole set of processes is to be done.

This paper is organized in the following way. In Section 2 we introduce the general mixing scheme in $SU(3)$ basis. More specifically, we derive the expression for (non-square) matrix of coupling constants generalizing the approach offered by B. Ioffe and M. Shifman \cite{20, 21}. In Section 3 we consider the dispersive approach to axial anomaly for the isoscalar non-singlet axial current $J^8_{\mu\delta}$ taking into account the possible contributions of higher mass states. In this way we derive our main equation while in Section 4 we supplement it by the analysis of the ratio of radiative decays of $J/\Psi$. These two constraints happen to be sufficient to provide strong bounds for mixing parameters. In Section 5 we consider the reduction of the general scheme with three angles to some currently popular particular (two-angle) schemes \cite{22} – \cite{28}. Finally, in Section 6 we summarize the results of numerical analysis and discuss the possible implications for $SU(3)$ and chiral symmetries.

## 2 Mixing

We start with a vector of physical pseudoscalar fields consisting of the fields of lightest pseudoscalar mesons and other fields:

$$
\tilde{\Phi} \equiv \begin{pmatrix}
\pi^0 \\
\eta \\
\eta' \\
G \\
\vdots
\end{pmatrix}
$$

We do not need to specify the physical nature of other components with higher masses, the lower of which $G$ is probably a glueball. Let us also introduce, following \cite{20, 21}, a set of $SU(3)$ fields
\[\varphi_k (k = 3, 8, 0) \text{ and other (singlet) fields } g_i: \]

\[
\Phi = \begin{pmatrix}
\varphi_3 \\
\varphi_8 \\
\varphi_0 \\
g \\
\vdots
\end{pmatrix}
\]  

(2)

and corresponding states \(|P_k\rangle\) and \(|P_{k'}\rangle\). The three upper fields \(\varphi_3, \varphi_8, \varphi_0\) diagonalize the matrix elements of axial currents \(J_{\mu 5}^I = \overline{q} \gamma_\mu \gamma_5 \frac{\lambda^I}{\sqrt{2}} q\):

\[
\langle 0\rangle|J_{\mu 5}^I|P_k\rangle = i \delta_{lk} f_k q_\mu, \ l, k = 3, 8, 0.
\]  

(3)

All other states are orthogonal to these currents:

\[
\langle 0\rangle|J_{\mu 5}^I|P_{k'}\rangle = 0.
\]  

(4)

At the same time all the corresponding fields enter the mass term in the effective Lagrangian with a generally speaking non-diagonal mass matrix \(M\):

\[
\Delta \mathcal{L} = \frac{1}{2} \Phi^T M \Phi
\]  

(5)

This formula immediately implies the generalized PCAC relation:

\[
\partial_\mu J_{\mu 5} = FM\Phi,
\]  

(6)

where

\[
\partial_\mu J_{\mu 5} \equiv \begin{pmatrix}
\partial_\mu J_{35}^3 \\
\partial_\mu J_{85}^8 \\
\partial_\mu J_{05}^0
\end{pmatrix}, \quad F \equiv \begin{pmatrix}
f_3 & 0 & 0 & 0 & \ldots & 0 \\
0 & f_8 & 0 & 0 & \ldots & 0 \\
0 & 0 & f_0 & 0 & \ldots & 0
\end{pmatrix},
\]  

(7)

\(F\) is a matrix of decay constants\(^1\) defined in (3).

In order to proceed from initial \(SU(3)\) fields \(\Phi\) to physical mass fields \(\widetilde{\Phi}\) the unitary matrix \(U\) is introduced

\[
\widetilde{\Phi} = U\Phi
\]  

(8)

which diagonalizes the mass matrix

\[
UMU^T = \widetilde{M} \equiv diag(m_{\pi^2}, m_{\eta^2}, m_{\eta'\eta}, m_G^2, \ldots),
\]  

(9)

where \(m_{\pi}, m_\eta, m_{\eta'}\) and \(m_G\) are the masses of the \(\pi, \eta, \eta'\) mesons and glueall state \(G\) respectively.

\(^1\)Note, that matrix of decay constants \(F\) is non-square expressing the fact that generally the number of \(SU(3)\) currents is less then the number of all possible states involved in mixing. The similar situation takes place (see e.g.\(^2\)) in one of the extensions of the Standard Model — neutrino mixing scenario involving sterile neutrinos.
Simple transformations of Eq.(6) reads:

\[ \partial_\mu J_{\mu 5} = F U^T \bar{\Phi} \]

This formula is close to those obtained in [20,21] (in the limit of small mixing). When the decay constants are equal this is reduced to formula (3.40) in [30].

Taking into account the well-known smallness of $\pi^0$ mixing with $\eta, \eta'$ sector [20,21,36] and neglecting all higher contributions we restrict our consideration to three physical states $\eta, \eta', G$ and two currents $J_{\mu 5}^8, J_{\mu 5}^9$. Then the divergencies of the axial currents (recall, that $G$ is a first mass state heavier than $\eta'$):

\[
\begin{pmatrix}
\partial_\mu J_{\mu 5}^8 \\
\partial_\mu J_{\mu 5}^9
\end{pmatrix} = 
\begin{pmatrix}
f_8 & 0 & 0 \\
f_0 & f_0 & 0
\end{pmatrix} U^T \begin{pmatrix}
m^2_\eta & 0 & 0 \\
0 & m^2_{\eta'} & 0 \\
0 & 0 & m^2_G
\end{pmatrix} \begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} \tag{11}
\]

Exploring the mentioned similarity of meson and lepton mixing, we use a well-known general Euler parametrization for the mixing matrix

\[
U = \begin{pmatrix}
c_3 & -s_3 & 0 \\
s_3 & c_3 & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & c_0 & -s_0 \\
0 & s_0 & c_0
\end{pmatrix} \begin{pmatrix}
c_8 & -s_8 & 0 \\
s_8 & c_8 & 0 \\
0 & 0 & 1
\end{pmatrix} = \begin{pmatrix}
c_8c_3 - c_0s_3s_8 - c_3s_8 - c_8c_0s_3 & s_3s_0 \\
c_3c_8 + c_0s_3s_8 - s_3s_8 + c_3c_8c_0 & -c_3s_0
\end{pmatrix} \tag{12}
\]

As soon as in the chiral limit $J_{\mu 5}^8$ should be conserved, Eq.(11) obviously implies, that the coefficients in front of the terms $m^2_\eta, m^2_{\eta'}, m^2_G$ should decrease at least as $(m_\eta/m_{\eta',G})^2$.

The generic matrix (12) can be reduced to different particular cases. The pure $\eta - \eta'$ mixing with no other admixtures corresponds to the case $\theta_0 = 0$ in Eq. (12). This is the so-called one-angle mixing scheme with the mixing angle $\theta = \theta_8 + \theta_3$. As it was mentioned above, that this scheme is not sufficient for description of the full set of experiments.

One can easily see from (11), (12) that the schemes with more than one mixing parameter unavoidably require introduction of new states for the mixing matrix to be unitary. Indeed, from these equations one can see that in general case we have 3 different angles and in some particular cases one can reduce the number of angles by one and to get the schemes with two different angles (put $\theta_3 = 0$ or $\theta_8 = 0$). Subsequently the schemes with 3 and 2 angles will be considered in details in sections 4 and 5.

In this work we sequentially use $SU(3)$ basis. For our purposes this basis is more preferable since in the next Section we will consider non-singlet isoscalar axial current $J_{\mu 5}^8$ which is free from non-Abelian anomaly. The transition to quark basis $\Phi_q = ((u\bar{u} + d\bar{d})/\sqrt{2}, s\bar{s}, g)$ which is also widely used in literature can be performed by multiplying by additional rotation matrix $V$:

\[
\Phi = V\Phi_q, \quad V = \begin{pmatrix}
\sqrt{1/3} & -\sqrt{2/3} & 0 \\
\sqrt{2/3} & \sqrt{1/3} & 0 \\
0 & 0 & 1
\end{pmatrix}. \tag{13}
\]

\[\text{We use notation } c_i \equiv \cos\theta_i, s_i \equiv \sin\theta_i.\]
3 Dispersive approach to axial anomaly

In our paper the dispersive form of the anomaly sum rule will be extensively used, so we remind briefly the main points of this approach (see e.g. review [19] for details).

Consider a matrix element of a transition of the axial current to two photons with momenta \(p\) and \(p'\)

\[
T_{\mu\alpha\beta}(p, p') = \langle p, p' | J_{\mu 5} | 0 \rangle .
\]  

(14)

The general form of \(T_{\mu\alpha\beta}\) for a case \(p^2 = p'^2\) can be represented in terms of structure functions (form factors):

\[
T_{\mu\alpha\beta}(p, p') = F_1(q^2) q_{\mu} \epsilon_{\alpha\beta\rho\sigma} p_{\rho} p'_{\sigma} + \frac{1}{2} F_2(q^2) \frac{p_{\alpha}}{p^2} \epsilon_{\mu\beta\rho\sigma} p_{\rho} p'_{\sigma} - \frac{p'_{\beta}}{p^2} \epsilon_{\mu\alpha\rho\sigma} p_{\rho} p'_{\sigma} - \epsilon_{\mu\alpha\beta \sigma} (p - p')_{\sigma} ,
\]  

(15)

where \(q = p + p'\). The functions \(F_1(q^2), F_2(q^2)\) can be described by dispersion relations with no subtractions and anomaly condition in QCD results in a sum rule:

\[
\int_0^\infty \text{Im} \ F_1(q^2) dq^2 = 2 \alpha N_c \sum e_q^2 ,
\]  

(16)

where \(e_q\) are quark electric charges and \(N_c\) is the number of colors. This sum rule was proved [31] for the general case \(p^2 \neq p'^2\) and earlier in [32] and [33] for the cases \(p^2 < 0, m = 0\) and \(p^2 = p'^2\) respectively. Notice that in QCD this equation does not have any perturbative corrections [34], and it is expected that it does not have any non-perturbative corrections also due to 't Hooft's consistency principle [35]. It will be important for us that at \(q^2 \to \infty\) the function \(\text{Im} F_1(q^2)\) decreases as \(1/q^4\). Note also that the relation (16) contains only mass-independent terms, which is especially important for the 8th component of the axial current \(J_{\mu 5}^{(8)}\) containing strange quarks:

\[
J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}} (\bar{u} \gamma_{\mu} \gamma_5 u + \bar{d} \gamma_{\mu} \gamma_5 d - 2 \bar{s} \gamma_{\mu} \gamma_5 s)
\]  

(17)

The general sum rule (16) takes the form:

\[
\int_0^\infty \text{Im} \ F_1(q^2) dq^2 = \frac{2}{\sqrt{6}} \alpha (e_u^2 + e_d^2 - 2 e_s^2) N_c = \sqrt{\frac{2}{3}} \alpha ,
\]  

(18)

where \(e_u = 2/3, \ e_d = e_s = -1/3, N_c = 3\).

Consider now a particular case of pure \(\eta - \eta'\) mixing, where no other mixing states are taken into account. Recall that this case has been studied for a long time, various approaches were considered and the numbers for the mixing angles in the range \((-10 \div 24)\) were obtained. The approach based on the dispersive representation of axial anomaly was introduced for \(\eta - \pi^0\) mixing in [36] and used in [37] where \(\eta - \eta'\) mixing was considered in assumption of small mixing angle).

In order to separate the form factor \(F_1(q^2)\), multiply \(T_{\mu\alpha\beta}(p, p')\) by \(q_{\mu}/q^2\). Then taking the imaginary part of \(F_1(q^2)\), using the expression for \(\partial_{\mu} J_{\mu 5}^{(8)}\) from Eq.(11) and saturating the matrix element with the \(\eta, \eta'\) states we get:
Figure 1: Mixing angle $\theta_1$ as a function of the decay constant $f_8$ in the one-angle mixing scheme. Dashed curves correspond to the uncertainties of the experimental data input. Horizontal dot-dashed line indicates the $f_8 = 1.28 f_\pi$ level.

\[ Im F_1(q^2) = Im q_{\mu} \frac{1}{q^2} (2\gamma | J^{(8)}_{\mu} | 0) = \]
\[ -\frac{f_8}{q^2} \langle 2\gamma | [m_\eta^2 \eta(c_1) + m_\eta' \eta'(s_1)] | 0 \rangle = \]
\[ \pi f_8 [A_\eta \delta(q^2 - m_\eta^2)(c_1) + A_\eta' \delta(q^2 - m_\eta')\rangle (s_1) \]  

(19)

Note, that if we have included higher resonances to this equation, they are expected to be suppressed as $1/m_{\text{res}}^4$ by virtue of the mentioned above asymptotical behavior of $F_1(q^2) \propto 1/q^4$. This (approximate) independence on higher resonances together with (exact) quark mass independence of the anomaly relation (16) may be an indication of some connection between these two effects.

If we employ the sum rule (18), we obtain a simple equation:

\[ c_1 + \beta s_1 = \xi, \]  

(20)

where

\[ \beta \equiv \frac{A_{\eta'}}{A_\eta} = \sqrt{\frac{\Gamma_{\eta'\to 2\gamma}}{\Gamma_{\eta\to 2\gamma}} \frac{m_\eta^3}{m_{\eta'}^3}}, \]  

(21)

\[ \xi \equiv \frac{\alpha^2 m_\eta^3}{96\pi^3 \Gamma_{\eta\to 2\gamma} f_8^2}, \quad \Gamma_{\eta\to 2\gamma} = \frac{m_\eta^3}{64\pi} A_\eta^2. \]  

(22)

For numerical evaluation of the $\eta - \eta'$ mixing angle in the latter case, put $f_8 = 1.28 f_\pi$, $f_\pi = 130.4$ MeV, $m_\eta = 547.85$ MeV, $m_{\eta'} = 957.78$ MeV, $\Gamma_{\eta\to 2\gamma} = 0.51$ keV, $\Gamma_{\eta'\to 2\gamma} = 4.30$ keV. The value for $f_8$ is taken from the chiral perturbation theory calculations, other numbers from PDG Review 2008 [38]. The mixing angle, responsible for the mixing of the $\eta - \eta'$ system appears to be $\theta_1 = -22.1^\circ \pm 1.5^\circ$. The dependence of the mixing angle on $f_8$ is shown on the Fig1. Remarkably, the anomaly sum rule fixes the mixing angle (provided we know $f_8$) in this case.

From our point of view one of the advantages of our approach is a high accuracy due to a high accuracy of the anomalous sum rule (18) for octet axial current. Let us stress that at this stage
we avoid consideration of the anomaly relation for the singlet axial current which contains the contributions of gluons, direct instantons and topological effects (see e.g. [39], [40]). As a result it appears to be unnecessary to include $f_0$ to our analysis.

The current result moderately agrees with our earlier analysis in the small mixing angle approximation [37]. It is not completely trivial that our result is also in a good agreement with previous analysis done in pioneering papers [2, 5]. This is because Eq.(20) happens to follow also from the (non-dispersive) anomaly equations used in [2, 5]. The similar result of dispersive and non-dispersive (local) approaches is in some sense natural taking into account that we omit the higher contributions with controlled accuracy $O(1/m^4)$. At the same time the mentioned approaches actually use the anomalous divergency for singlet current which we do not need.

Now let us consider a generic case involving glueball admixtures. Performing the same operations for the $\eta - \eta' - G$ system we get:

$$\text{Im} F_1(q^2) = \text{Im} q \frac{1}{q^2} \langle 2\gamma | J^{(8)}_{\mu 5} | 0 \rangle =$$

$$-\frac{f_8}{q^2} \langle 2\gamma | [m_\eta^2 \eta (c_8 c_3 - c_0 s_3 s_8) + m_{\eta'}^2 \eta' (s_3 c_8 + c_3 s_0 s_8) + m_G^2 G s_8 s_0] | 0 \rangle =$$

$$\pi f_8 [A_\eta \delta(q^2 - m_\eta^2)(c_8 c_3 - c_0 s_3 s_8) + A_{\eta'} \delta(q^2 - m_{\eta'}^2)(s_3 c_8 + c_3 s_0 s_8) + A_G \delta(q^2 - m_G^2)(s_8 s_0)] $$  (23)

The final equation following from the anomaly sum rule (18) for the $\eta - \eta' - G$ system is:

$$(c_8 c_3 - c_0 s_3 s_8) + \beta (s_3 c_8 + c_3 s_0 s_8) + \gamma (s_8 s_0) = \xi, $$  (24)

where

$$\beta \equiv \frac{A_{\eta'}}{A_\eta} = \sqrt{\frac{\Gamma_{\eta' \to 2\gamma}}{\Gamma_{\eta \to 2\gamma}} \frac{m_\eta^3}{m_{\eta'}^3}}, \quad \gamma \equiv \frac{A_G}{A_\eta} = \sqrt{\frac{\Gamma_G \to 2\gamma}{\Gamma_{\eta \to 2\gamma}} \frac{m_G^3}{m_\eta^3}}; $$  (25)

$$\xi \equiv \frac{\alpha^2 m_\eta^3}{96 \pi^3 \Gamma_{\eta \to 2\gamma}} \frac{1}{f_8^2}, \quad \Gamma_{\eta \to 2\gamma} = \frac{m_\eta^3}{64 \pi A_\eta^2}. $$  (26)

Let us summarize the situation with a theoretical accuracy of the Eq. (24). As we have pointed out, the anomaly sum rule (18) has no $\alpha_s$-corrections. The possible contributions from higher states may come as the additional terms in the l.h.s. of Eq. (24). As it was discussed before, the asymptotic behavior $F_1(q^2)$ is proportional to $1/q^4$ at large $q^2$ and a sort of quark-hadron duality implies that higher resonances should be suppressed as $(m_{\eta'}/m_{\text{res}})^4$.

### 4 $J/\Psi$ radiative decay ratio

Eq. (20) provide an exact constraint but contains too many free parameters. As an additional experimental constraint we, following [2], [5] use the data of decay ratio $R_{J/\Psi} = (\Gamma(J/\Psi) \to \eta'\gamma) / (\Gamma(J/\Psi) \to \eta\gamma)$.  

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3 However, this is not the case for the singlet axial current $J_{\mu 5}^0$. 
As it was pointed out in [41] the radiative decays $J/\Psi \to \eta(\eta')\gamma$ are dominated by non-perturbative gluonic matrix elements, and the ratio of the decay rates $R_{J/\Psi} = (\Gamma(J/\Psi) \to \eta')/\Gamma(J/\Psi \to \eta\gamma)$ can be expressed as follows:

$$R_{J/\Psi} = \left(\frac{\langle 0 \mid G\tilde{G} \mid \eta'\rangle}{\langle 0 \mid G\tilde{G} \mid \eta\rangle}\right)^2 \left(\frac{p_{\eta'}}{p_{\eta}}\right)^3,$$

where $p_{\eta(\eta')} = M_{J/\Psi}(1-m_{\eta(\eta')}^2/M_{J/\Psi}^2)/2$. The advantage of such a choice is an expected smallness of perturbative and non-perturbative corrections.

The divergencies of singlet and octet components of axial current in terms of quark fields can be written as:

$$\partial_\mu J_{\mu5}^3 = \frac{1}{\sqrt{6}}(m_u\bar{u}\gamma_\mu\gamma_5u + m_d\bar{d}\gamma_\mu\gamma_5d - 2m_s\bar{s}\gamma_\mu\gamma_5s),$$

$$\partial_\mu J_{\mu5}^0 = \frac{1}{\sqrt{3}}(m_u\bar{u}\gamma_\mu\gamma_5u + m_d\bar{d}\gamma_\mu\gamma_5d + m_s\bar{s}\gamma_\mu\gamma_5s) + \frac{1}{2\sqrt{3}}\frac{3\alpha_s}{4\pi}G\tilde{G}.$$  

Following [2], neglect the contribution of u- and d- quark masses, then the matrix elements of the anomaly term between the vacuum and $f$, $f'$ states are:

$$\frac{\sqrt{3}\alpha_s}{8\pi}\langle 0 \mid G\tilde{G} \mid \eta \rangle = \langle 0 \mid \partial_\mu J_{\mu5}^{(0)} \mid \eta \rangle + \frac{1}{\sqrt{2}}\langle 0 \mid \partial_\mu J_{\mu5}^{(8)} \mid \eta \rangle,$$

$$\frac{\sqrt{3}\alpha_s}{8\pi}\langle 0 \mid G\tilde{G} \mid \eta' \rangle = \langle 0 \mid \partial_\mu J_{\mu5}^{(0)} \mid \eta' \rangle + \frac{1}{\sqrt{2}}\langle 0 \mid \partial_\mu J_{\mu5}^{(8)} \mid \eta' \rangle.$$  

Using Eq. [11], (27), (30), (31) we deduce:

$$R_{J/\Psi} = \left[\frac{m_{\eta}^2 f_0(-s_3s_8 + c_3c_8 c_0) + \frac{1}{\sqrt{2}} f_8(s_3 c_8 + c_3 c_0 s_8)}{m_{\eta}^2 f_0(-c_3 s_8 - c_3 c_0 s_3) + \frac{1}{\sqrt{2}} f_8(c_3 s_8 - c_3 c_0 s_3)}\right]^2 \left(\frac{p_{\eta'}}{p_{\eta}}\right)^3.$$

Let us note that in obtaining this equation only operator relations for anomalies (28, 29) were used and one did not need to express the $\eta(\eta')$ mesons fields in the terms of divergencies of singlet (and octet) axial currents.

If we use Eq.(32) for the case of one-angle mixing scheme ($\theta_0 = 0$), for usual choice $f_8 = 1.28 f_\pi$, $f_0 = 1.1 f_\pi$ and corresponding angle value obtained from anomalous dispersive relation (20) ($\theta_1 \equiv \theta_3 + \theta_8 = -22.1^\circ$) we find $R_{J/\Psi} = 2.2$ which is in serious discrepancy with the experimental value $R_{J/\Psi} = 4.8 \pm 0.6$ [38]. Substituting the experimental value of $R$ into (32) and using anomaly equation (20) one can get the dependencies $f_8(\theta_1)$, $f_0(\theta_1)$ (Fig. 2).

As soon as we accept, that $f_8 \gtrsim f_0 \gtrsim f_\pi$ (for different kind of justification see e.g. [14], [11]), it follows from Fig. 2 that the only possibility is $f_8 \simeq f_0 \simeq 1$ ($\theta_1 \simeq -18^\circ$), which is quite far from the chiral perturbation theory expectations. Taking into account all experimental errors (the dominant contribution being provided by that of $R_{J/\Psi}$) one get Fig. 3 where the effects of these errors are indicated by shaded areas. From this figure it is clear that the maximal allowed value

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4These values appear to be preferable also in other mixing schemes as we will see later.
of $f_8$ is $f_8 = 1.2f_\pi (= f_0)$. Let us note that these values correspond to minimal allowed value of $R_{J/\Psi} = 4.2$. Here the importance of more accurate experimental value of $R_{J/\Psi}$ is already clear.

Let us pass to the more elaborated schemes with more than one mixing angle which were offered in \cite{11,13}. In the most general scheme with 3 angles introduced in Section 2 it is convenient to rewrite (32) in terms of $\theta_1 \equiv \theta_s + \theta_2$, $\theta_2 \equiv \theta_3 - \theta_3$:

\[
R_{J/\Psi} = \left[ \frac{m_\eta^2}{m_\eta^2} \frac{f_0(c_1 - c_2 + c_0(c_1 + c_2)) + \frac{1}{\sqrt{2}}f_8(s_1 - s_2 + c_0(s_1 + s_2))}{f_0(-s_1 - s_2 - c_0(s_1 - s_2)) + \frac{1}{\sqrt{2}}f_8(c_1 + c_2 - c_0(c_2 - c_1))} \right]^2 \left( \frac{p_{\eta'}}{p_\eta} \right)^3 \tag{33}
\]

The angle $\theta_1$ has an explicit physical sense. From the definition of the mixing matrix $U$ \cite{12} it is obvious that the angle $\theta_1$ describes the overlap in the $\eta - \eta'$ system with accuracy $\sim \theta_3^2/2$ and coincides with their mixing angle at $\theta_0 \to 0$. It is reasonable to suppose that the glueball contribution to the anomaly sum rule for non-singlet current \cite{24} is rather small. This doesn’t necessarily mean the extreme smallness of $\eta - G$ mixing angle itself but rather the cumulative effect of smallness of mixing angles and large mass $m_G$. That’s why we neglect the last term in the l.h.s. of (24) and rewrite it in terms of $\theta_1$, $\theta_2$:

\[
(c_1 + c_2 - c_0(c_2 - c_1)) + \beta(s_1 - s_2 + c_0(s_1 + s_2)) = 2\xi. \tag{34}
\]

The solutions of (33) (upper curves, red online), and (34) (lower curves, blue online) are shown on Fig. 4 for customary choice of decay constants $f_8 = 1.2f_\pi$, $f_0 = 1.1f_\pi$ and three different angles $20^\circ, 32^\circ, 40^\circ$. We see from these figures that at $\theta_0 = 20^\circ$ there is no intersection. The first common solution appears at larger angle $\theta_0 = 32^\circ$ when the curves touch each other. As $\theta_0$ grows, at $\theta_0 > 32^\circ$ two different solutions appear. While for these two solutions $\theta_2$ are significantly different, the solutions for $\theta_1$ are limited to relatively narrow region $\theta_1 \sim -13^\circ \div -18^\circ$. This
Table 1: Tree-angle mixing scheme. Minimal possible mixing angle $\theta_0$ (in degrees) for different values of decay constants $f_0$ and $f_8$.

| $f_0/f_\pi$ | $f_8/f_\pi$ | 1.28       | 1.1   | 1.0   |
|------------|-------------|------------|-------|-------|
| 1.28       | 24$^{+5}_{-6}$  |            |       |       |
| 1.1        | 32$^{+4.9}_{-4.5}$ | 17.5$^{+6}_{-12}$ |       |       |
| 1.0        | 35$^{+0.5}_{-5.5}$ | 23.5$^{+0.5}_{-6.5}$ | 10$^{+0}_{-10}$ |       |

is not surprising because of the physical sense of $\theta_1$ mentioned above. Taking into account the experimental uncertainties we see that the minimal allowed value of $\theta_0$ slightly decreases to $27^\circ$.

All this numerical analysis showed that for $f_8 = 1.28$, $f_0 = 1.1$ we need a substantial glueball admixture $\theta_0 > 27^\circ$. The minimal value of $\theta_0$ decreases only if $f_8$ decreases. The results of analysis for different $f_8$ and $f_0$ are presented in Table, from which one can make an important conclusion that relatively small glueball admixture even within experimental uncertainties is possible only for $f_8 = f_0$ (and most probably $f_8 = f_0 = f_\pi$).

Figure 4: General (tree-angle) mixing scheme. Solutions of $R_{J/\Psi}$ equation (33) (upper, red online curves) and anomaly condition (34) (lower, blue online curves) for $f_8 = 1.28 f_\pi$, $f_0 = 1.1 f_\pi$ and different choice of $\theta_0$. The dashed curves indicate experimental uncertainties.

5 Two-angle mixing schemes

Up to a moment our analysis was quite general. It is instructive to consider some particular cases which are currently discussed in literature. Starting from our general scheme, one can easily perform a reduction to partial mixing schemes with 2 angles. Here we have 2 distinct cases when

i) $\theta_3 = 0$ or

ii) $\theta_8 = 0$.

Other solutions not shown on the figures appear only at large $\theta_0 > 70^\circ$ which clearly has no physical sense.
1. The case $\theta_3 = 0$.

This case was introduced in [22], and was widely used by KLOE collaboration in a set of recent papers [23]-[25]. This choice clearly means that there is no mixing of $\eta$ with additional scalar state noted in (2) as $g$, i.e. $\eta$ is a mixture of $\phi_8$ and $\phi_0$ only. In this case our equations (24), (32) are simplified as follows:

$$c_8 + \beta c_0 s_8 + \gamma s_0 s_8 = \xi,$$

$$R_{J/\Psi} = \left[\frac{m_0}{m_0^2} f_0(c_8 c_0) + \frac{1}{\sqrt{2}} f_8(c_0 s_8)\right]^2 \left(\frac{p_{\eta'}}{p_\eta}\right)^3$$  \hspace{1cm} (36)

Note that $\theta_8$ in this scheme defines $\eta - \eta'$ mixing, as one can easily see from Eq. (12). In further analysis we suppose that $\gamma$ cannot exceed 2 for any reasonable values of $\Gamma_{G \to 2\gamma}$ (i.e. $\Gamma_{G \to 2\gamma}/m_G^3 \lesssim 4\Gamma_{\eta \to 2\gamma}/m_\eta^3$). This restriction corresponds to the assumption that 2-photon decay widths of pseudoscalar mesons grow like the third power of their masses, or in other words the glueball coupling to quarks to be of the same order as for the meson octet states.

The results of numerical analysis are shown on Fig. 5. These figures show the dependence of the glueball contribution $\gamma$ on the angle $\theta_8$ for different values of $f_8$ and $f_0$.

On Fig. 5(a) the dependence $\gamma(\theta_8)$ is shown for $f_8 = 1.28 f_\pi$ and $f_0 = (1.1, 1.28) f_\pi$. The dotted lines corresponds to experimental uncertainties, the uncertainty for R being dominant. From this figure one can see that for any $f_0 < f_8 = 1.28 f_\pi$ we get $\gamma > 2$. One can achieve $\gamma \simeq 2$ only for $f_0 \simeq 1.28 \simeq f_8$ (at the lower value of $R_{J/\Psi} = 4.2$ ). The corresponding values of mixing angles are $\theta_8 = -(14 \div 17)^\circ$, $\theta_0 = (12 \div 30)^\circ$.

On the Fig. 5(b) the case $f_8 = 1.1$ for two choices of $f_0 = (1.1, 1.0)$ is shown. Again, the reasonable values $\gamma \lesssim 2$ is achievable only for $f_0 \simeq 1.1 f_\pi \simeq f_8$. The corresponding values of mixing parameters are $\theta_8 = -(12 \div 18)^\circ$, $\theta_0 = (5 \div 35)^\circ$

And, at last, consider the choice $f_0 = f_8 = 1.0 f_\pi$ shown on Fig. 5(c). The corresponding values of mixing parameters are $\theta_8 = -(10 \div 18)^\circ$, $\theta_0 = (5 \div 37)^\circ$. Note that in this case the region of relatively small $\gamma \lesssim 1$ and $\theta_0 \sim 5$ are achievable.

Let us notice that for $f_8 > 1.28 f_\pi$ the minimal value of $\gamma$ is growing, e.g. for large $N_c$ value $f_8 = 1.34 f_\pi$ (and any $f_0$), $\gamma \gtrsim 2$ within experimental errors of $R_{J/\Psi}$.

We can conclude, that in this scheme the glueball admixture is bounded from below to minimal value $\theta_0 > 5^\circ$ for any $f_8 \gtrsim f_0 \gtrsim f_\pi$. The relatively small glueball admixture $\theta_0 \lesssim 10^\circ$ is possible only for $f_8 \simeq f_0 \simeq f_\pi$.

Let us stress, that here (like in general 3-angle mixing scheme) the values $f_8 \simeq f_0$ is much more preferable. All these results directly follow from the specific choice of mixing scheme, axial anomaly condition and $R_{J/\Psi}$.

It is instructive to compare our results with those obtained in [25], where the analysis of $BR(\phi \to \eta' \gamma)/BR(\phi \to \eta \gamma)$ together with several vector meson radiative decays to pseudoscalars

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6Note that in these papers authors use quark basis, see (13). The connection between notations of mixing angles in [25] and our notations: $\theta_0 = -\phi_G$, $\theta_3 = \phi_0 - \alpha$, where $\alpha = \arctan \sqrt{2/3} \approx 54.7^\circ$.

7If it grows with mass according to the naive dimensional arguments, the glueball width will be even smaller. We will discuss the relation between coupling constants and decay widths later.

8We limit ourselves to negative $\theta_8$ since this mixing angle in this scheme clearly have a sense of $\eta - \eta'$ mixing angle.
was performed in the same mixing scheme.

If we substitute their values of $\theta_8 = (-14.5 \pm 1), \theta_0 = 7.5^\circ$ to our dispersive relation and take into account possible sources of inaccuracy we can see that we directly get the dependence $\gamma(f_8)$ shown on Fig. 6. The large slope is due to small mixing angles $\theta_0$ and $\theta_8$.

![Graph of glueball contribution parameter $\gamma$ as a function of mixing angle $\theta_8$ for different decay constants](image)

(a) $f_8 = 1.28f_\pi$
(b) $f_8 = 1.1f_\pi$
(c) $f_8 = 1.0f_\pi$

Figure 5: Glueball contribution parameter $\gamma$ as a function of mixing angle $\theta_8$ for different decay constants. Dashed curves indicate errors arising from experimental data.

One can see that the reasonable value $\gamma < 2$ corresponds to $f_8 \sim 1((1.00 - 1.05)f_\pi)$. Thus anomaly constraint contradicts the preferred value $f_8 = 1.34f_\pi (f_8 = 1.28f_\pi)$ which is used in [25]. However, the constant $f_s$ enters only the expression for radiative decay width $\eta' \to 2\gamma$ (Eq.1.4 in [25]). The change of $f_s$ from $f_s = 1.34f_\pi(f_8 = 1.28f_\pi)$ to the value $f_s = f_\pi(f_8 = f_\pi)$ in this equation may be compensated by the change of $\Psi_G$ within the claimed accuracy. Therefore at the current level of accuracy the analysis of KLOE is compatible with ours. The combination of the KLOE results with both our constrains (i.e. anomaly and $R_{J/\Psi}$, [35, 36] leads to conclusion that $f_8 = f_0 = f_\pi$ which is in agreement with the analysis presented above.

2. The case $\theta_8 = 0$.

One can consider another particular case of general mixing supposing $\theta_8 = 0$ in [12]. This kind of a two-angle mixing scheme was used recently in [28]. This scheme implies no glueball coupling to $J_{\mu5}^s$, which means that the glueball contribution to anomaly relation is exactly zero. Therefore, the anomaly relation is exactly the same as for the one-angle mixing scheme so the dependence $f_8(\theta_3)$ is the same and shown on Fig. 1. Recall, that this relation is precise due to reasons, mentioned in Section 3.

The relation for $J/\Psi$ radiative decay ratio for this case can be directly obtained from [32] with $\theta_8 = 0$. This relation together with axial anomaly relation allow us to get the dependence of the ratio of decay constants $f_0/f_8$ as a function of $\theta_0$ (see Fig. 7).\footnote{As it was noted in Section 4 the ratio $f_0/f_8$ should not exceed 1.} Stripes bounded by dashed lines show the effects of experimental uncertainties. From Fig. 7 it is clear that the solution at $f_8 = 1.28f_\pi$ contradicts to the condition $f_0/f_8 < 1$ discussed above for any $\theta_0$. The only solution compatible with $f_0/f_8 \leq 1$ is the one at $f_8 = (1 - 1.05)f_\pi$ (the left stripe), the corresponding $\theta_0 = 0^\circ \div 14^\circ$, while the angle $\theta_3$ (having the sense of the $\eta - \eta'$ overlap) can vary in a very narrow region $\theta_3 = -17^\circ \div -18^\circ$.

So one can conclude that this scheme even within current experimental uncertainties demands $f_8 = f_0 = f_\pi$ and small glueball admixture $\theta_0 < 14^\circ$. 
Let us now examine the numerical results of [28]. Implementation of our procedure with their angle $\phi = 42.4^\circ$ ($\theta_3 = -12.3$) leads to $f_8 = 0.8$ and contradicts to $f_8/f_\pi > 1.0$ being the robust expectation from the ChPT.

As soon as the accuracy of the extracted [28] parameters is not available at the moment it is difficult to conclude whether this disagreement is statistically significant.

6 Summary

In this paper we presented the detailed analysis of the role of Abelian axial anomaly in the mixing of both light and heavy pseudoscalar states. We found that the anomaly imposes the severe constraints for the meson couplings and mixing angles. There is also a delicate interplay between pseudoscalar state, which we call glueball without specifying its nature, and light mesons.

We offered a (new, to our best knowledge) rigorous approach in $SU(3)$ basis to the consideration of mixing of pseudoscalar mesons. Our approach is quite similar to that for the mixing of massive neutrino, where the number of states with a definite mass may exceed the number of flavor states. In this sense the appearance of extra singlet mesons is similar to the role of sterile neutrinos.

We use the dispersive representation of axial anomaly for $J_{\mu5}^8$. The advantage of such representation is the high and controlled accuracy due to suppression of higher resonances and its independence on the quark masses. The use of $SU(3)$ basis allows us to limit ourself to more theoretically clear case of non-singlet current receiving contribution only from Abelian anomaly. Let us note, that our equation (20) may be formally derived in the other pioneering approaches using standard local anomaly relations (see e.g. [3] in the one-angle mixing scheme.

As a supplementary input we use experimental data for the ratio of radiative decays $R_{J/\Psi} \equiv (\Gamma(J/\Psi \to \eta'\gamma)/(\Gamma(J/\Psi \to \eta\gamma)$ which is theoretically safe and provides additional restriction.

We found that for the one-angle mixing scheme the only reasonable solution is $f_8 \simeq f_0 = f_\pi$, $\theta_1 = -18^\circ$. 

Figure 6: Two-angle mixing scheme, case $\theta_3 = 0$. Glueball contribution parameter $\gamma$ as a function of decay constant $f_8$ (in units of $f_\pi$) for $\theta_8 = (-14.5 \pm 1)^\circ$, $\theta_0 = 7.5^\circ$

Figure 7: Two-angle mixing scheme, case $\theta_8 = 0$. The dependence of the mixing angle $\theta_0$ on the ratio of decay constants $f_0/f_8$ together with experimental uncertainties.
We proved that any scheme with more than one angle unavoidably demands additional singlet admixture and considered the general scheme with 3 angles with one lowest additional singlet state, which we denoted as a glueball without any specification of its nature.

It was found, that the combination of our two inputs (anomaly relation and $R_{J/Ψ}$ data) leads to a rather strict constraints. The values $f_8 = 1.28 f_π$ inevitably leads to a large glueball admixture $θ_0 ≃ 30°$. At the same time, the reasonably small glueball admixture $θ_0 < 10°$ is possible only for $f_0 ≃ f_8 = 1.0 f_π$, which is far from expectations, based on chiral perturbation theory.

We checked this general observation by considering some particular cases, when the general mixing matrix (with 3 angles) is reduced to the $η - η' - G$ mixing scheme with two angles.

The first one is the mixing scheme with $θ_3 = 0$ which was investigated in a set of recent papers by KLOE collaboration. We concluded that application of our approach to this scheme unavoidably leads to $f_8/f_0 ≃ 1$ for any reasonable values of glueball two-photon decay width. Moreover, combining our constraints with the angles $θ_8 = 14.5°$, $θ_0 = 7.5°$ obtained in KLOE analysis based on the additional set of decays, we again found that $f_8 ≃ f_0 = f_π$. These values are still consistent with the results of their fits within their accuracy.

We also consider another two-angle mixing scheme where $θ_3 = 0$. In this case the only possible solution compatible with constraint $f_8 ≥ f_0$ is $f_8 = f_0 = f_π$ with high accuracy.

As a result, in all the considered schemes the relation $f_8 ≃ f_0(≃ f_π$ most likely) holds. This marks a sort of new manifestation of $SU(3)$ symmetry and at the same time the possible violation of chiral perturbation theory expectations. The possible origin of such a symmetry pattern may be the smallness of the strange quark mass (squared) with respect to nucleon one while it is still much larger than (genuine) higher twist parameters and may be treated sometimes as a heavy one [42].

The significant progress may be achieved by the more accurate determination of $R_{J/Ψ}$, in particular, at BES-III accelerator, complementing its vast program (see [43], Section 17.2.2). While our conclusion $f_8 ≃ f_0$ is more robust and is valid for all values of $R_{J/Ψ}$ within current experimental limits, the stronger result $f_8 = f_π$ may be questioned by the more accurate data. Therefore these measurements will provide a new test of chiral perturbation theory predictions.

Although our approach already provided the important new constraint for the analysis in pseudoscalar channel, the global fit exploiting the dispersive representation of axial anomaly remains to be done.

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