Numerical investigations on a framework for fracture prediction in metal forming with a material model based on stress-rate dependence and non-associated flow rule

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Abstract.
In this paper, the proposed framework for fracture prediction during sheet metal forming is explained and some numerical investigations are presented to demonstrate the effectiveness of this method. In particular, the role of parameters that affects the forming limit curves, which are represented by the initiation of local bifurcation in each stress rate, are examined; namely, anisotropic parameters and the $K_C$ value, which means a stress-rate dependence, and other material properties. The newly proposed concept of forming limit diagram is used to evaluate the formability of the concerned material and working process.

1. Introduction
To progress the sheet metal forming technology to the next stage, accuracy of forming simulation should be increased so that the prediction of the initiation of the fracture during the deformation is possible. This is difficult especially when difficult-to-form materials, such as high-strength steels and anisotropic metals, are used. To achieve this important but difficult task, the authors have engaged in developing a new framework for predicting sheet fracture in a forming process on the basis of bifurcation analysis.

The essential part of the proposed framework is a material model based on stress-rate dependency related with non-associate flow rule. This model is based on a non-associated flow rule with arbitrary higher order yield function and plastic potential function for any anisotropic materials [1][2][3]. And this formulation is combined with the stress-rate-depency plastic constitutive equation, which is known as the Ito-Goya plastic constitutive equation [4], to construct a generalized plastic constitutive model in which non-normality and non-associativity are reasonably incorporated. Then, by adopting the three-dimensional bifurcation theory [4], more accurate prediction of the initiation of shear band is realized, leading to general and reliable construction of forming limit diagram.

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2. Theory

In this section, a fracture prediction framework that has been developed by the authors is briefly introduced. The proposed fracture prediction method is based on the bifurcation analysis that is based on the Ito–Goya’s constitutive model, which is a plastic constitutive equation including stress-rate dependence, and a material model based on the non-associated flow rule. Details can be found in the previous papers.

2.1. Yield and plastic potential function

In the proposed model, we defined the yield function $f(\sigma)$ as being equal to the equivalent stress, namely,

$$f(\sigma) = \bar{\sigma} = 2^{m_y} \sqrt{\frac{3}{2(F + G + H)}} (s_{m_y} \cdot A \cdot s_{m_y}).$$

Here, the matrix $A$ has anisotropic parameters in its diagonal terms, and the pseudo vector $s_{m_y}$ is a set of deviatoric stress components raised to the power of $m_y$. This higher-order function preserves the form of Hill’s quadratic yield function, that is, it contains the same anisotropic parameters $F, G, H, L, M,$ and $N$. This feature is important because it is possible to construct a higher-order yield function by changing the power $m_y$ without increasing the number of undetermined variables.

In our non-associated flow-rule-based formulation, a function different from the yield function is adopted as the plastic potential function, which provides the direction of the plastic strain increment of the subsequent state of current stress. In this study, the previously introduced function $f(\sigma)$ is used as the yield function, and another function $g(\sigma)$, which takes the same form as $f(\sigma)$ but has different anisotropic parameters $F^*, G^*, H^*, L^*, M^*$, and $N^*$, is adopted as the plastic potential function. In this expression, asterisks are used to distinguish $f(\sigma)$ from $g(\sigma)$. For example, the anisotropy matrix $A$ is changed to $A^*$, in which the original parameters $F, G, H, L, M,$ and $N$ are also changed to $F^*, G^*, H^*, L^*, M^*$, and $N^*$, respectively. To express another order of the function, the power variable $m_p$ is used instead of $m_y$. Thus, the plastic potential function of the proposed model takes the form

$$g(\sigma) = \bar{\sigma}^* = 2^{m_p} \sqrt{\frac{3}{2(F^* + G^* + H^*)}} (s_{m_p} \cdot A^* \cdot s_{m_p}).$$

From the definition of plastic work, an explicit expression for the equivalent plastic strain increment is obtained as

$$d\varepsilon^p = \frac{m_p \bar{\sigma}^*}{\bar{\sigma}} \sqrt{\frac{2(F^* + G^* + H^*)}{3}} \left( D_{m_p}^{*\prime} \cdot d\varepsilon^p \right)^T \cdot \left\{ A^* \cdot \left( D_{m_p}^{*\prime} \cdot d\varepsilon^p \right) \right\}.$$

2.2. Ito–Goya plastic constitutive model

Local bifurcation abruptly changes the current strain rate direction. Since classical $J_2$ theory does not allow the rotation of the strain rate direction caused by the subsequent stress rate direction, it is not appropriate for use in bifurcation problems. Therefore, in this study, the
Ito–Goya plastic constitutive equation [4] is applied because it can take the dependence of the strain rate direction on the stress rate direction into account. The Ito–Goya plastic constitutive equation is expressed as

$$\dot{\varepsilon}^p = \Lambda (n_F : l_p) |\dot{\varepsilon}| [K_C l + (1 - K_C)n_N],$$

where $n_N$ is unit tensor called the natural direction. This tensor indicates the direction of the deviatoric stress rate, which is identical to that of the plastic strain rate. The unit tensor $n_F$ is the direction of the gradient of the yield function and $l$ is the direction of the current deviatoric stress. In Eq.(4), the parameter $K_C$, which takes a value between 0 and 1, indicates the dependence of the direction of the strain rate on the stress rate. When $K_C$ is equal to 1, the Ito–Goya constitutive equation becomes $J_2$ ow theory.

2.3. 3D local bifurcation theory

On the basis of Hill’s general bifurcation theory, bifurcation occurs when the following condition is satisfied:

$$I[\Delta v] = \int \Delta L : \Delta \dot{S}dV = 0,$$

where $\Delta v$ is the velocity field and $L$ and $\dot{S}$ are the velocity gradient tensor and the first Piola-Kirchhoff stress tensor rate, respectively. $\dot{S}$ can be represented using the Cauchy stress tensor as

$$\dot{S} = D : \dot{\varepsilon} + \omega \cdot \sigma - \sigma \cdot \omega - L \cdot \sigma = A : L,$$

where $\dot{\varepsilon}$, $\omega$, and $D$ are strain rate tensor, spin tensor, and tangent stiffness tensor, respectively. $A$ is a fourth-rank tensor that relates the nominal stress rate and the velocity gradient tensor $L$. To characterize the bifurcation mode, the velocity gradient tensor is allowed to be discontinuous when the velocity gradient tensor crosses the bifurcation border $\Gamma$. $L$ can be expressed in terms of the normal vector $n$ on the bifurcation border and the local deformation mode vector $m$ that is normal to $n$ as

$$L = m \otimes n.$$

The mode vector $m$ can be composed of two different vectors in the $\Gamma$ plane, namely, $m_{\text{SH}}$ and $m_{\text{SV}}$, which are vectors in the horizontal and vertical directions, respectively. These vectors are expressed using three angle parameters, $\phi$, $\psi$ and $\theta$. The expressions for these vectors are as follows:

$$n = (\sin\phi \cos\psi, \sin\phi \sin\psi, \cos\phi),$$

$$m_{\text{SH}} = (-\sin\phi, \cos\psi, 0),$$

$$m_{\text{SV}} = (\cos\phi \cos\psi, \cos\phi \sin\psi, -\sin\phi),$$

$$m = m_{\text{SH}} \cos\theta + m_{\text{SV}} \sin\theta.$$

Substituting Eqs.(6) and (7) into Eq.(5), we have the following bifurcation criterion:

$$I[m, n; \sigma] = hH[m, n; \alpha] - \sigma \Sigma[m, n; \alpha],$$

where the first and second terms of this functional are expressed as

$$H[m, n; \alpha] = m \cdot n \cdot \overline{D}(s) \cdot n \cdot m,$$

where $\overline{D}(s) \equiv D(s)/h,$

$$\Sigma[m, n; \alpha] = \frac{1}{2} [m \cdot \alpha \cdot m - n \cdot \alpha \cdot n].$$
In an elasto-plastic material subjected to large strain, ignoring elastic deformation, the tangent stiffness tensor can be assumed to be proportional to the hardening rate $h$.

Then, the current stress is expressed by

$$
\sigma = \sigma \alpha, \quad \sigma = \sqrt{\sigma \cdot \sigma}, \quad \alpha = \sigma / \sigma,
$$

where $\sigma$ and $\alpha$ are the norm of the current stress tensor and normalization tensor, which gives the stress ratio for each stress component, respectively.

On the basis of these relations, we have the following local bifurcation criterion.

$$
\left( \frac{\sigma}{h} \right)_{cr} = \min \left( \frac{H[m, n; \alpha]}{\sum|m, n; \alpha|} \right).
$$

The bifurcation criterion represented by Eq. (16) indicates that the local bifurcation, which is specified by the mode vectors $m$ and $n$ based on the current stress ratio tensor $\alpha$, should be identified by the ratio $\sigma/h$, which is the ratio of the stress level to the work-hardening. Mechanically, the stress $\sigma$ indicates the intensity of fracture initiation, and the hardening coefficient $h$ indicates the resistance of the material against fracture. Therefore, the formability represented in the $\sigma/h$ plane is free from the strain-path dependence that is usually observed in a typical FLD (forming limit diagram) represented in the strain space. Thus, because it is mechanically reasonable to exhibit forming limits in the $\sigma/h$ plane, this new expression is called the SHFLD ($\sigma/h$ FLD). Although the initiation of bifurcation is not completely identical to fracture, in sheet metal forming situation particularly, it is possible to interpret that they onset almost simultaneously. Because it is assumed that a bifurcation always initiates prior to the associated fracture, the fracture prediction in the SHFLD is in a safer region.

The fracture limits in the SHFLD can show 3D local bifurcation limits. The actual fracture is considered to lie between the lower bound represented by the S–R limit and the upper bound represented by the 3D local bifurcation limit.

3. Bifurcation analysis

In the previous studies[2][3], the authors have investigated the influence of the $K_C$ value, the hardening state, and the order of the yield function on the fracture limits that are predicted as the initiation of the three-dimensional local bifurcation. However, these analyses were under the isotropic situation; therefore, in this study, anisotropic situations were investigated.

In this analysis, assuming a plane stress condition, an isotropic material was considered at first with the following constants: $F = G = H = F^* = G^* = H^* = 1$, $L = M = N = L^* = M^* = N^* = 3$, Young's modulus $= 210$ GPa, Poisson’s ratio $\nu = 0.3$, $n = 0.2$, and $K = 5.0 \times 10^8$ in the $n$-power law for material hardening. To consider the anisotropic cases, the anisotropic constants $F$ and $G$ were changed under setting $H$ equals to 1. Namely, the following three cases were considered: (i) $F = G = 1$, (ii) $F = 0.5, G = 1$, and (iii) $F = 1, G = 0.5$. For the following analyses, the orders of the yield and potential functions were both set to 1.

The bifurcation analysis was conducted as follows. The minimum value of the functional on the right-hand side of Eq. (16) was searched for by changing the variables included in the fracture mode vectors $m$ and $n$. The simulated annealing algorithm was adopted in this optimization process. To calculate the yield function in the used equations, the stress ratio $\alpha$ was used to control the stress condition, for example, $\alpha = 0$ for uniaxial stress and $\alpha = 1$ for equi-biaxial stress. The obtained minimum values were used to show the initiation of bifurcation as a possible fracture onset in the fracture limit diagram in the $\sigma/h$ plane.

In practice, the $K_C$ value is determined by using a load and strain at where fracture occurred in an uniaxial tensile test. With a temporary $K_C$ value, bifurcation calculations in uniaxial direction are conducted. Then, the calculated fracture limit is assumed to be equivalent to
the experimental fracture point. Repeated evaluations with varied $K_C$ values are conducted until the error between the experiment and the calculation becomes sufficiently small. In the following numerical investigations, the $K_C$ values took common values that can be seen in ductile materials. Figure 1 shows the isotropic case, namely the case (i), for various $K_C$ values. As shown in this figure, the fracture limit lines descend with increasing $K_C$. This means that the bifurcation analysis carried out in this study was valid and shows physically reasonable behavior. In the next example, the effect of anisotropic parameters in the yield function on the limit curves was investigated. In other words, a case of stress anisotropy and deformation isotropy was tested. The $K_C$ values were 0.1 and 0.2, and the cases (ii) and (iii) were compared to the case (i). The results are shown in Figure 2, where no differences were observed for each case. The reason of this results can be related with the use of the non-associated flow rule. Owing to the separation of the potential function from the yield function, the influences of these functions can be examined. Based on the results shown in Figure 2, it is assumed that the shape of the yield function does not affect the bifurcation limits. Namely, when a material is deformation isotropy, the limit curves would be identical because strain ratio is unchanged. This interpretation seems physically reasonable; however, more investigations should be conducted.

![Figure 1. SHFLD for different $K_C$ values.](image1)

![Figure 2. SHFLD for anisotropic cases for $K_C = 0.1$ and $K_C = 0.2$ (in red curves). Curves were overlapped.](image2)

4. Conclusion
In this study, numerical investigations on a framework for fracture prediction in metal forming with a material model based on stress-rate dependence and non-associated flow rule have been conducted. Anisotropic cases were investigated, and the results represented reasonable effects of different anisotropic parameters on the limit curves.

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