Over-modulation phenomena and its influence on the pulse width modulated single-phase inverter output voltage

This paper describes analysis of the pulse width modulated single-phase inverter output voltage. By using the over-modulation principle the low THD distortion of the output voltage appears but the first voltage harmonic is increased. The root mean square (RMS) component of the output voltage is also increased. Due to this phenomenon the dc converter input voltage can be decreased, which is always welcomed.

Key words: Over-modulation, single-phase voltage inverter, THD

1 INTRODUCTION

Single-phase inverters are used in battery powered systems (uninterruptible power system - UPS) and in active power factor correction single-phase AC rectifiers. In such circuits MOS-FETs and IGBTs have usually been used, depending on the presumed requirements.

The control and modulation of single-phase inverters have been studied in many references such as [1–5]. In [6] the author describes a survey of pulse width modulation techniques appropriate for single and three-phase systems. The well-known phenomena in inverter circuits due to dead-time introduction, in order to avoid short cuts according to input inverter terminals are described in [2]. Authors described an effective approach to compensate for this necessary but undesirable phenomenon. The appropriate PWM is always used in all these applications. The over-modulation phenomenon is widely used in three-phase systems in order to extend the output voltage range by introducing third voltage harmonics in the modulation procedure.

This paper deals with an analytical evaluation of the over-modulation phenomena in single-phase inverters circuits, where the main goal was to decrease the input DC voltage while keeping the RMS value and THD factor of output voltage at the same level and under prescribed limits. Full-bridge configuration [1] of a single-phase inverter is shown in Fig. 1. The input DC voltage \( U_{d} \) level is considered when switching elements (transistors) are selected in inverter design procedure. Consequently, the price of the inverter semiconductor switches is close connected with the \( U_{d} \) voltage level, therefore if voltage \( U_{d} \) is reduced, then the price of the whole inverter can be reduced and...
Over-modulation phenomena and its influence on the pulse width modulated single-phase inverter output voltage
D. Stojan, M. Milanović

at the same time the size and efficiency can be improved. Analytical evaluation of over-modulation phenomena was verified using simulation with MATLAB-Simulink software package, and experimentally.

2 MATHEMATICAL ANALYSIS OF ORDINARY AND OVER-MODULATION REGION

For each inverter leg PWM modulator is needed. In a PWM modulator a desired reference signal is compared with a triangular waveform as shown in Fig. 2. Outcome of the switching scheme shown on Fig. 2 is three-level voltage \( u_{AB} \) on the inverter output terminals.

2.1 Ordinary modulation region

In order to generate three-level output voltage \( u_{AB} \) the desired reference signal can be calculated by defining the converter output voltage \( u_{A0} \) and \( u_{B0} \) as a linear combination of the two events [3, 5]:

\[
u_{A0} = H_{11} \frac{U_d}{2} + H_{21} \left( -\frac{U_d}{2} \right) = (2H_{11} - 1) \frac{U_d}{2}, \tag{1}\]

where \( U_d \) represents the converter input voltage and the switching function \( H_{11} \) is defined as

\[
H_{11} = \begin{cases} 
1, & \text{when } S_{11} \text{ is ON, } S_{21} \text{ is OFF} \\
0, & \text{when } S_{11} \text{ is OFF, } S_{21} \text{ is ON}
\end{cases}
\]

Moreover, condition \( H_{11} + H_{21} = 1 \) must be fulfilled to avoid short-circuit in first converter leg. The same procedure follows for voltage \( u_{B0} \) evaluation:

\[
u_{B0} = H_{12} \frac{U_d}{2} + H_{22} \left( -\frac{U_d}{2} \right) = (2H_{12} - 1) \frac{U_d}{2}, \tag{2}\]

where the switching function \( H_{12} \) is defined as

\[
H_{12} = \begin{cases} 
1, & \text{when } S_{12} \text{ is ON, } S_{22} \text{ is OFF} \\
0, & \text{when } S_{12} \text{ is OFF, } S_{22} \text{ is ON}
\end{cases}
\]

and likewise condition \( H_{12} + H_{22} = 1 \) must be fulfilled in order to avoid short-circuit in second converter leg. Hence, voltage \( u_{AB} \) is evaluated as follows:

\[
u_{AB} = u_{A0} - u_{B0} = (H_{11} - H_{12}) U_d = (\Delta p_1 - \Delta p_2) U_d. \tag{3}\]

In (3) switching functions \( H_{12} \) and \( H_{22} \) are replaced by duty cycle functions \( \Delta p_1 \) and \( \Delta p_2 \) respectively (Fig. 3). The duty cycle functions can be computed from (1) and (2). In order to organize the three-level output voltage \( u_{AB} \) voltages \( u_{A0} \) and \( u_{B0} \) must be defined as:

\[
u_{A0} = \hat{U} \sin(\omega t), \quad u_{B0} = -\hat{U} \sin(\omega t), \tag{4}\]

such that from (1) and (2) the following is obtained:

\[
\Delta p_1(t) = \frac{U_{A0}}{U_d} = \frac{1}{2} + \frac{1}{T_s} \frac{\hat{U}}{U_d} \sin(\omega t), \quad \Delta p_2(t) = \frac{U_{B0}}{U_d} = \frac{1}{2} + \frac{1}{T_s} \frac{\hat{U}}{U_d} \sin(\omega t), \tag{5}\]

where \( \hat{m}_{iA}(t) \) and \( \hat{m}_{iB}(t) \) represent the reference signal or reference voltage which is sometimes also called the modulation function. Duty cycle functions in (5) also represent
the average switching function value during the interval $T_s$ as shown in Fig. 3. After using (4) in (5) reference voltages can be expressed as:

$$m_{IA} = \frac{U_d}{T_s} \cos(\omega t) = m_I \cos(\omega t),$$

$$m_{IB} = -\frac{U_d}{T_s} \cos(\omega t) = -m_I \cos(\omega t),$$

where $m_I = \hat{U} / U_d$ and represents the modulation index. By comparing the average duty cycle functions ($\Delta p_1$ and $\Delta p_2$, Fig. 2 (a)) with the triangular signal ($u_T$, Fig. 2 (a)) the PWM signal can be generated and applied to the switches (Fig. 3). The PWM signals are generated as follows:

$$\delta_{p1}(t) = \begin{cases} 1, & \text{when } H_{11} \geq u_T \text{ then } S_{11} \text{ is ON, } S_{21} \text{ is OFF}, \\ 0, & \text{when } H_{11} \leq u_T \text{ then } S_{11} \text{ is OFF, } S_{21} \text{ is ON}, \\ 1, & \text{when } H_{21} \geq u_T \text{ then } S_{21} \text{ is ON, } S_{22} \text{ is OFF}, \\ 0, & \text{when } H_{22} \leq u_T \text{ then } S_{21} \text{ is OFF, } S_{22} \text{ is ON}. \end{cases}$$

$$\delta_{p2}(t) = \begin{cases} 1, & \text{when } H_{11} \geq u_T \text{ then } S_{11} \text{ is ON, } S_{21} \text{ is OFF}, \\ 0, & \text{when } H_{11} \leq u_T \text{ then } S_{11} \text{ is OFF, } S_{21} \text{ is ON}, \\ 1, & \text{when } H_{21} \geq u_T \text{ then } S_{21} \text{ is ON, } S_{22} \text{ is OFF}, \\ 0, & \text{when } H_{22} \leq u_T \text{ then } S_{21} \text{ is OFF, } S_{22} \text{ is ON}. \end{cases}$$

(7)

When the instantaneous signals (7) of duty cycle functions $\delta_{p1}(t)$ and $\delta_{p2}(t)$ are applied at the switches (Fig. 4), the voltage $u_{AB}$ is generated at the converter output terminals. Voltage $u_{AB}$ is described as:

$$u_{AB} = (\delta_{p1}(t) - \delta_{p2}(t)) U_d$$

(8)

The PWM functions from (7) are depicted in Fig. 3. Both PWM functions $\delta_{p1}(t)$ and $\delta_{p2}(t)$ (7) can be approximated by using Fourier series [7]:

$$\delta_{p1}(t) = \Delta p_1(t) + \sum_{n=1}^{\infty} \frac{2}{\pi} \sin(n\pi \Delta p_1(t)) \cos(n\omega_T t),$$

$$\delta_{p2}(t) = \Delta p_2(t) + \sum_{n=1}^{\infty} \frac{2}{\pi} \sin(n\pi \Delta p_2(t)) \cos(n\omega_T t).$$

(9)

After some manipulation by (5), (6) and (9) and after the substitution of (9) into (8) the voltage can be described as follows:

$$u_{AB}(t) = U_d m_I \cos(\omega t) + \frac{U_d}{\pi},$$

$$\cdot \sum_{n=1}^{\infty} \frac{1}{n} \cos \left( \frac{n \pi}{2} \right) \sin \left( \frac{n \pi}{2} + \frac{n \pi}{2} m_I \cos(\omega t) \right) \cos(n\omega_T t).$$

(10)

Again, after some manipulation and by using the power of the Bessel series [4] (10) can be written as:

$$u_{AB} = U_d m_I \cos(\omega t) + \frac{U_d}{\pi},$$

$$\cdot \sum_{n=1}^{\infty} \frac{1}{n} \frac{\cos \left( \frac{n \pi}{2} \right) J_1 \left( \frac{n \pi m_I}{2} \right)}{J_0 \left( \frac{n \pi m_I}{2} \right)} \cos \left( n\omega_T + \omega \right) t +$$

$$\cdot + \cos \left( n\omega_T - \omega \right) t) -$$

$$\cdot \cos \left( \frac{n \pi}{2} \right) J_1 \left( \frac{n \pi m_I}{2} \right) \cos \left( n\omega_T + 3\omega \right) t) +$$

$$\cdot + \cos \left( n\omega_T - 3\omega \right) t) +$$

$$\cdot + \cos \left( \frac{n \pi}{2} \right) J_5 \left( \frac{n \pi m_I}{2} \right) \cos \left( n\omega_T + 5\omega \right) t).$$

(11)

Equation (11) was used in order to evaluate the voltage spectrum. The following modulation parameters have been used in order to calculate the voltage spectrum lines: DC voltage $U_d = 350$ V, frequency of synthesized signal $f = 50$ Hz ($\omega = 2\pi f$), $f_T = 2$ kHz ($\omega_T = 2\pi f_T$), and the modulation index $m_I = 1$. The dispositions of the spectral lines are evident from Tab. 1 and Tab. 2, and are depicted in Fig. 5. This shows that the frequency modulated signal $u_{AB}(t)$ has a fundamental component appearing at the frequency $\omega$ with a magnitude of $U_d m_I$ and, besides the carrier frequency $2nf_T$, also contains the frequencies $2nf_T \pm k\omega$, $n=1,2,3,...,\infty$, $k=\pm 1, \pm 2, \ldots, \infty$. It appears that in this case the fundamental component of the synthesized signal $\omega$ has a magnitude of $U_d m_I$.

|Table 1. Spectral lines around the second multiplier of carrier frequency ($2f_T$)|
|---|
|$a_{21}$ | $- \frac{U_d}{\pi} \cos \left( \frac{\pi}{2} \right) J_1 \left( \frac{2n \pi m_I}{2} \right)$ |
| | $-61.41$ |
|$a_{23}$ | $- \frac{U_d}{\pi} \cos \left( \frac{3\pi}{2} \right) J_1 \left( \frac{2n \pi m_I}{2} \right)$ |
| | $-74.3$ |
|$a_{25}$ | $- \frac{U_d}{\pi} \cos \left( \frac{5\pi}{2} \right) J_1 \left( \frac{2n \pi m_I}{2} \right)$ |
| | $-11.61$ |

|Table 2. Spectral lines around the fourth multiplier of carrier frequency ($4f_T$)|
|---|
|$a_{41}$ | $\frac{U_d}{\pi} \cos \left( \frac{\pi}{2} \right) J_1 \left( \frac{2n \pi m_I}{2} \right)$ |
| | $-23.66$ |
|$a_{43}$ | $\frac{U_d}{\pi} \cos \left( \frac{3\pi}{2} \right) J_1 \left( \frac{2n \pi m_I}{2} \right)$ |
| | $3.24$ |
|$a_{45}$ | $\frac{U_d}{\pi} \cos \left( \frac{5\pi}{2} \right) J_1 \left( \frac{2n \pi m_I}{2} \right)$ |
| | $-41.53$ |

Fig. 4. Triangle waveform ($u_{TA}$) and generation of the pulse train $\delta_x$

Fig. 5. Calculated spectrum of voltage $u_{AB}$
is also evident that the spectral line with multiplier of 3
\(k = 3, 6, 9, \ldots\), which belongs to the Bessel function
\(J_3(x)\) disappears from the voltage spectrum.

Such spectrum analysis can be done by using MATLAB (ft analysis) but has no benefit for a deeper understanding of the modulation phenomena. The magnitude of fundamental-frequency voltage varies linearly with \(m_I\) when \(m_I \leq 1\). When \(m_I\) is increased over 1.0 the amplitude of fundamental voltage component does not vary linearly with \(m_I\) any more. This phenomenon is called over-modulation.

2.2 Over-modulation region

“Single-phase” voltage \(u_{A0}(t)\) in the over-modulation region is shown Fig. 6. Both voltages \(u_{A0}(t)\) and \(u_{B0}(t)\) can be described by:

\[
\begin{align*}
  u_{A0}(t) &= \begin{cases} 
    U_d/2; & 0 \leq \omega t \leq \alpha, \\
    U_d \cos (\omega t); & \alpha \leq \omega t \leq \pi - \alpha, \\
    -U_d/2; & \pi - \alpha \leq \omega t \leq \pi + \alpha, \\
    U_d \sin (\omega t); & \pi + \alpha \leq \omega t \leq 2\pi - \alpha, \\
    -U_d/2; & 2\pi - \alpha \leq \omega t \leq 2\pi,
  \end{cases} \\
  u_{B0}(t) &= \begin{cases} 
    -U_d/2; & 0 \leq \omega t \leq \alpha, \\
    -U_d \cos (\omega t); & \alpha \leq \omega t \leq \pi - \alpha, \\
    U_d/2; & \pi - \alpha \leq \omega t \leq \pi + \alpha, \\
    -U_d \cos (\omega t); & \pi + \alpha \leq \omega t \leq 2\pi - \alpha, \\
    -U_d/2; & 2\pi - \alpha \leq \omega t \leq 2\pi,
  \end{cases}
\end{align*}
\]

(13)

where \(\alpha\) is indicated in Fig. 6 and can be calculated as follows:

\[
\alpha = \arccos \left( \frac{U_d}{2U} \right). 
\]

(14)

According to (5) the duty cycle function can now be represented slightly different:

\[
\Delta p_1(t) = \frac{1}{2} + \frac{U_d}{U} u_{A0},
\]

\[
\Delta p_2(t) = \frac{1}{2} - \frac{U_d}{U} u_{B0},
\]

(15)

where \(u_{A0}\) and \(u_{B0}\) must be considered as follows from (13). So the duty cycle functions are now described by expanding \(u_{A0}\) and \(u_{B0}\) using the Fourier series. It yields:

\[
\Delta p_1(t) = \frac{1}{2} + \frac{U_d}{U} \left[ b_1 \sin x + b_2 \sin 3x + b_3 \sin 5x + \cdots \right],
\]

\[
\Delta p_2(t) = \frac{1}{2} - \frac{U_d}{U} \left[ b_1 \sin x + b_3 \sin 3x + b_5 \sin 5x + \cdots \right],
\]

(16)

where coefficients \(b_1, b_3, b_5, \ldots\) are calculated by using formulas

\[
a_n = \frac{2U_d}{\pi} \int_0^{\pi/2} \left[ f(\omega t) + f(-\omega t) \right] \cos(n\omega t) dt,
\]

\[
b_n = \frac{2U_d}{\pi} \int_0^{\pi/2} \left[ f(\omega t) - f(-\omega t) \right] \sin(n\omega t) dt.
\]

After some mathematical operations, the general term for the arbitrary harmonic component is described by:

\[
b_n = \frac{2U_d}{\pi} \left[ \frac{1}{m_I} \cos((n - 1)\alpha) + \frac{1}{m_I + 1} \cos((n + 1)\alpha) \right].
\]

(17)

These particular coefficients can be calculated from (17) as follows:

\[
b_1 = \frac{2U_d}{\pi} \left[ m_I \cos\alpha + \frac{1}{m_I} \cos(\alpha) + \frac{1}{m_I + 1} \cos((n - 1)\alpha) \right],
\]

\[
b_3 = \frac{2U_d}{\pi} \left[ m_I \cos(3\alpha) + \frac{1}{m_I} \cos(3\alpha) + \frac{1}{m_I + 1} \cos((n - 1)\alpha) \right],
\]

\[
b_5 = \frac{2U_d}{\pi} \left[ m_I \cos(5\alpha) + \frac{1}{m_I} \cos(5\alpha) + \frac{1}{m_I + 1} \cos((n - 1)\alpha) \right],
\]

\[
\vdots
\]

(18)

Figure 7 shows the first voltage harmonic magnitude. It is evident that the first harmonic can be increased over the value \(U_d\). In order to evaluate the appropriate increase of the first harmonic in the over-modulation area, it is necessary to also calculate the total harmonic distortion factor (THD). According to the definition, THD can be evaluated as follows:

\[
THD = \sqrt{\sum_{n=2}^{\infty} \frac{U_n^2}{U_1^2}}.
\]

(19)
Thus by using the series as follows from (17) and (18), the particular harmonics and THD can be evaluated, as shown in Fig. 8. When a specific THD is chosen (usually this is load requirement) it is evident from the analysis and also from the THD calculation as to how the first voltage harmonic can be increased in order to keep the THD under the prescribed value. For example, when a THD of 10% has been chosen, the first harmonics can be increased app. for 16%. From this fact, it follows that the DC voltage can also be decreased. For example, in order to have 230 V RMS at the converter output in the case of ordinary modulation \((m_I = 1.0)\) \(U_d = 324\) V must be chosen, but by using the over-modulation region \((m_I = 1.4)\), the DC voltage can be decreased to \(U_d = 280\) V. The over-modulation phenomena is always welcome in cases where DC voltage \(U_d\) is produced from voltage sources as for solar panels or fuel cell systems, and need to be boosted by using the DC-DC boost converter.

3 SIMULATION

The harmonic analyzing method, analytical results and their consequences were verified by simulation. Simulations were done with input voltage \(U_d = 325\) V and ideal transistors were chosen. The dead-time effect was neglected. Considering (18) by choosing \(m_I = 1.133\) the first harmonic component can be increases by 8% and at \(m_I = 1.285\) by 13%.

Using expression \(U_d = 230 \cdot \sqrt{2}/1.08\) the voltage \(U_d\) can be reduced to 301 V at \(m_I = 1.133\) and to 290 V at \(m_I = 1.285\). According to the scheme in Fig. 9, the bandwidth of output filter was set at 10 kHz \((L = 250 \mu\text{H} \text{ and } C = 1 \mu\text{F})\), with \(R_L = 100\) Ω of load, the switching frequency was set to 50 kHz. Figure 10 and Fig. 11 show simulation results. It can be seen that input voltage can be decreased. Figure 11 shows filtered and non-filtered unipolar output voltage at modulation ratio \(m_I = 1.285\). The voltage \(u_{AB}\) was filtered so that over-modulation is visible.

4 EXPERIMENTAL RESULTS

To verify analytical and simulation results the preprogrammed PWM inverter was implemented with a dsPIC digital signal processor. The experiments were carried out.
Over-modulation phenomena and its influence on the pulse width modulated single-phase inverter output voltage

D. Stojan, M. Milanović

Fig. 11. Simulation results; filtered and non-filtered output voltage for \( m_I = 1.285 \), \( U_d = 290 \) V

within the following filter circuit parameters:

\[
L = 250 \, \mu\text{H},
\]

\[
C = 2.2 \, \mu\text{F},
\]

\[ P_{\text{load}} = 1 \, \text{kW}. \]

Fig. 12 shows measured results for the first experiment when a DC voltage of \( U_d = 330 \) V was applied. A modulation index of \( m_I = 0.98 \) was chosen in order to establish the 230 V RMS. Due to dead-time effect, which has been introduced into the modulation algorithm in order to avoid short-circuits in both of the converter’s legs, total harmonic distortion was 2.3%.

Figure 13 shows second experiment, when over-modulation algorithm was applied. The modulation index was 1.285 and \( THD \) was limited to 10%. In order to establish 230 V RMS on the output, the input DC voltage can be decreased to \( U_d = 295 \) V. These results were in accordance with the theoretical and simulation predicted results.

5 CONCLUSION

This paper describes over-modulation phenomena in a single-phase inverter. It has been shown that the mathematical and physical connections among modulation parameters (modulation index, desired THD) have an influence to the desired input and output voltages. It has also been shown that inverter input voltage can be reduced due to use of over-modulation, which is appropriate when low-voltage sources with a booster are applied during the power conversion process (solar panels, fuel cells...). The analytical approach to a modulation algorithm enables further investigation in the direction of harmonic minimization of components within the voltage spectra.

REFERENCES

[1] M. Milanović, Močnostna elektronika. FERI Maribor, 2007.

[2] N. Mohan, T. Undeland, and W. Robbins, Power Electronics Converters: Applications and Design. John Wiley & Sons, Inc., 1995.

[3] P. Krein, Elements of Power Electronics. Oxford: University Press, 1998.

[4] R. Ericson and D. Maksimovic, Fundamentals of Power electronics. Kluwer Academic Publisher, 2001.

[5] P. Wood, Switching Power Converters. New York: ??, 1981.

[6] H. Van der Broeck, “Analysis and realization of pulse width modulator based on voltage space vectors,” in Proceedings of the 1986 IEEE-IAS Annual Meeting, pp. 244–251, 1986.

[7] N. Bronstein, K. Semndljajev, G. Musiol, and H. Muhlig, Matematični priročnik. Tehniška založba Slovenije, 1997.
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