Reversing type II migration: resonance trapping of a lighter giant protoplanet

F. Masset and M. Snellgrove

Astronomy Unit, School of Mathematical Sciences, Queen Mary & Westfield College, Mile End Road, London E1 4NS, England

Received *****; in original form ******

ABSTRACT

We present a mechanism related to the migration of giant protoplanets embedded in a protoplanetary disc whereby a giant protoplanet is caught up, before having migrated all the way to the central star, by a lighter outer giant protoplanet. This outer protoplanet may get captured into the 2:3 resonance with the more massive one, in which case the gaps that the two planets open in the disc overlap. Two effects arise, namely a squared mass weighted torque imbalance and an increased mass flow through the overlapping gaps from the outer disc to the inner disc, which both play in favour of an outwards migration. Indeed under the conditions presented here, which describe the evolution of a pair of protoplanets respectively Jupiter and Saturn sized, the migration is reversed, while the planets semi-major axis ratio is constant and the eccentricities are confined to small values by the disc material. The long-term behaviour of the system is briefly discussed, and could account for the high eccentricities observed for the extrasolar planets with semi-major axis \( a > 0.2 \) AU.

Key words: Accretion, accretion discs – Hydrodynamics – Solar system: formation – Planetary systems

1 INTRODUCTION

In the past few years a number of extrasolar giant planets have been discovered around nearby solar-type stars. These objects masses range from 0.17 \( M_J \) to 11 \( M_J \) (where \( M_J \) is Jupiter’s mass) and their orbital semi-major axis range from 0.038 AU to 3.3 AU (Marcy, Cochran & Mayor, 1999). Although many uncertainties remain about planet formation, it is now commonly accepted that planets have formed in and from protoplanetary discs. Necessarily, there must be some time interval over which a giant planet and the surrounding gaseous disc material coexist. The planet and the disc exchange angular momentum through tidal interactions which generally make the planet lose angular momentum. This mechanism is known as migration. It can roughly be divided in two regimes:

- If the planet mass is small enough, the disc response is linear. The migration rate, in that regime, is proportional to the planet and disc masses, is independent of the viscosity and weakly dependent of the disc surface density and temperature profiles. This is the so-called type I migration (Ward, 1997).
- When the protoplanet mass is above a certain threshold, the torques acting locally on the surrounding disc material open a gap (Papaloizou & Lin, 1984), whose width and depth are controlled by the balance between the tidal torques, which tend to open the gap, and the viscous torques which tend to close it. The disc response is significantly non-linear, and most of the protoplanet Lindblad resonances fall in the gap and therefore cannot contribute to the planet-disc angular momentum exchange. The migration rate slows down dramatically compared to type I migration. Furthermore, the tidal truncation splits the disc into two parts and the planet is locked to the disc viscous evolution (Nelson et al. 2000). This is the type II migration, which describes the orbital evolution of giant protoplanets.

In this letter we consider the coupled evolution of a system of giant protoplanets consisting of two non-accreting cores with masses 1 \( M_J \) and 0.29 \( M_J \), which are going to call from now on respectively “Jupiter” and “Saturn”. Attempts have already been made to describe the behaviour of a system of planets embedded in a disc. Melita & Woolfson (1996) and Haghighipour (1999) considered an embedded Jupiter and Saturn system orbiting a solar mass star, and showed how resonance trapping would affect their evolution. However the dissipative force in these works was due to the dynamical friction with a uniform density interplanetary medium, hence type II migration effects were not taken into account.
into account. Resonance trapping of planetesimals by a fixed orbit Jupiter sized protoplanet has also been investigated by Beaugé et al. (1994), and shown to be able to build up a single planetary core with orbital characteristics close to Saturn’s ones. Kley (2000) studied the orbital evolution of two maximally accreting giant cores embedded in a minimal mass protosolar disc, and showed how the migration of the inner core could be halted by the presence of the outer one, and how the eccentricity of the inner core is pumped up by the outer one.

2 RESULTS

2.1 Numerical codes description

In order to investigate the long-term behaviour of the embedded Jupiter and Saturn system, we have used two independent hydrocodes, which have been described elsewhere in full detail (Nelson et al. 2000). These two codes are fixed Eulerian grid based codes, one of them is NIRVANA (Ziegler & Yorke, 1997) and the other one has been written by one of us (FM). Both have been endowed with the fast advection FARGO algorithm (Masset, 2000), and can run either with this algorithm or with a standard advection algorithm. They gave very similar results. They consist of a pure N-body kernel based on either a fourth (NIRVANA) or fifth order adaptive timestep Runge-Kutta solver (sufficient for the short time-scales involved in this dissipative problem) embedded in a hydrocode which provides a tidal interaction with a 2D non self-gravitating gaseous disc. The simulations are performed in the non-inertial non-rotating frame centered on the primary. The grid outer boundary does not allow inflow nor outflow and is chosen sufficiently far from the planets in order for the spiral density waves that they launch to be damped before they reach it, while the grid inner boundary only allows outflow (inwards), so that the disc material can be accreted on to the primary. Failing to do so may lead to overestimate the inner disc density and artificially favours an outwards migration. In the following our length unit is 5.2 AU, the mass unit is one solar mass, and the time unit is the initial orbital period of Jupiter (the actual period may vary as Jupiter migrates).

We present in fig. 1 the central star–planet distance curves as a function of time. The outer dashed curve represents the nominal position of the 1:2 resonance with Jupiter, while the inner dashed curve is the nominal position of the 2:3 resonance. The zoomed plot enables one to closely compare Jupiter’s orbital evolution against a test run without Saturn.

Figure 1. Primary–planet distances as a function of time. The outer dashed curve represents the nominal position of the 1:2 resonance with Jupiter, while the inner dashed curve is the nominal position of the 2:3 resonance. The zoomed plot enables one to closely compare Jupiter’s orbital evolution against a test run without Saturn.

The mass of Jupiter is sufficient to open a deep gap and hence it settles in a type II migration (Nelson et al. 2000), whereas Saturn is unable to fully empty its coorbital region because: (i) its mass is smaller; (ii) The planet is in a regime known as the inertial limit (Ward & Hourigan, 1989) where the inwards migration speed is so high that it makes the planet pass through what would be the gap inner edge before it had time to actually open it.

Therefore Saturn does not clear a deep gap initially, and its migration rate is typical of type I migration, since all its Lindblad resonances can still contribute to the angular momentum exchange with the disc.

2.3 Run results

We present in fig. 1 the central star–planet distance curves as a function of time. We see how initially Jupiter migrates as if it was the only planet in the disc (see test run). In the meantime, Saturn starts a much faster migration (the obvious initial acceleration of its migration will be discussed elsewhere), and reaches the 1:2 resonance with Jupiter at time \( t \approx 110 \). The eccentricities at that time are small (see fig. 1), and in particular Saturn’s eccentricity is much smaller than the eccentricity threshold below which the capture into resonance is certain if the “adiabatic” condition on the migration rate is satisfied (Malhotra 1993): \( |\dot{a_j}|/(a_j\Omega \mu) \ll 0.5j(j + 1)\mu_e e_s \) for the \( j:j+1 \) resonance, where \( \mu_e \) is the mass ratio of Jupiter to the central object, and where \( e_s \) is Saturn’s eccentricity. This condition is not satisfied when Saturn reaches the 1:2 resonance, and it passes through.

The planets then obtain higher eccentricities, and Sat-
In this figure we see the planets eccentricities as a function of time. They simultaneously increase as Saturn passes through the 1:2 and 3:5 resonances with Jupiter. Once Saturn is trapped into the 2:3 resonance with Jupiter, both eccentricities settle at a roughly constant level, which results of a balance between the migration rate which pumps them up and the eccentricity damping by the disc coorbital material.

Saturn’s migration rate is reduced. Saturn’s eccentricity increases again rapidly as it passes through the 3:5 resonance with Jupiter at $t \approx 220$. Eventually the adiabatic condition on the migration rate is satisfied for the 2:3 resonance and Saturn’s eccentricity is still below the corresponding critical threshold, so it gets trapped into the 2:3 resonance with Jupiter (both $e$ and $e'$ resonances, since the two critical angles $\phi = 3\lambda_s - 2\lambda_J - \dot{\omega}_s$ and $\phi' = 3\lambda_s - 2\lambda_J - \dot{\omega}_J$ librate, where $\lambda$ is the mean longitude and $\dot{\omega}$ the longitude of perihelion). At that time both planets steadily migrate outwards.

### 2.4 Interpretation

We define the system of interest as the system composed of the two planets. This resonance locked system interacts with the inner disc through torques proportional to $M_J^2$, at Jupiter’s inner Lindblad resonances (ILR), whereas it interacts with the outer disc through torques proportional to $M_S^2$ at Saturn’s outer Lindblad resonances (OLR), as indicated on fig. 3. It can be seen that Saturn’s ILR fall in Jupiter’s gap and Jupiter’s OLR fall in Saturn’s gap so their effect is weakened compared to the situation where Jupiter is alone. As $M_J^2/M_S^2 \sim 10$, the torque imbalance does not favour an inwards migration as strongly as in a one planet case, and may even lead to a positive differential Lindblad torque on the two planet system. Actually one can estimate what the maximum mass ratio of the outer planet to the inner one should be to get a migration reversal, if one neglects the Inner Lindblad torque on the outer planet and the Outer Lindblad torque on the inner planet. The Inner Lindblad torque on the inner planet reads as:

$$T_{ILR} = C_{ILR} \Sigma_0 a_J^3 (a_J \Omega_J)^2 h^{-3}$$  \hspace{1cm} (1)

where $C_{ILR}$ is a dimensionless coefficient which is a sizable fraction of unity (Ward, 1997), and where $h^c$ is the disc aspect ratio. There is a similar formula for the Outer Lindblad torque on the outer planet (obtained by substituting the ILR and $J$ indices in Eq. (1) respectively with OLR and $S$). The resulting torque imbalance will be positive if:

$$\frac{\mu_S}{\mu_J} < \left( \frac{C_{ILR}}{C_{OLR}} \right) \left( \frac{2}{3} \right)^{1/3}$$  \hspace{1cm} (2)

If we assume that $C_{ILR} = C_{OLR}$ then we get: $\mu_S/\mu_J < 0.87$, whereas if we make the conservative assumption that $C_{ILR} = \frac{1}{3} C_{OLR}$, we have: $\mu_S/\mu_J < 0.62$. This threshold is much bigger than the actual ratio, therefore if the common gap is deep enough to shut off Jupiter’s OLR torques (and Saturn’s ILR torques) then the net Lindblad torque on the two planet system is positive. As the two planet system proceeds outwards in the disc, it does not act on the gas as a snow-plough, but rather it allows the material from the outer disc to travel across the common gap and eventually feed the inner disc. We can find the gap “permeability” condition by requiring that the rate of angular momentum change of the ring of material lying immediately outside Saturn’s gap that is required to expand accordingly to Saturn’s orbit (snow-plough effect) is greater than the torque available from Saturn (at most the sum of its outer Lindblad torques, in which case we need to assume that the waves excited at their OLR are damped locally). In our case, this turns out not to be the case and most of the outer disc material flows through the common gap to the inner disc. We find that in all our runs it is possible to check that the rate of mass flow through the common gap (see fig. 3) can be expressed as:

$$M \approx 3\pi \Sigma_0 \mp 2\pi r_s \Sigma_0$$  \hspace{1cm} (3)

with a reasonable precision (10–20 %). Furthermore we have performed many “restart runs” which consist in restarting a
torque is the so-called coorbital corotation torque (Goldreich & Tremaine 1979, Ward 1991 and 1992). To the best of our knowledge, an analytical evaluation of the corotation torque in this two planet problem is far beyond the scope of this paper. We will just comment that the corotation torque in our case might not be negligible compared to the differential Lindblad torque at some stage.

3 DISCUSSIONS

We have performed a series of restart runs (see section 2.4) in order to check for a variety of behaviours.

3.1 Differential Lindblad torque sign

The one sided Lindblad torque has been shown to be proportional to $h^{-3}$ (Ward 1997). We have performed two restart runs ($h' = 0.04 \rightarrow 0.03$ and $h' = 0.04 \rightarrow 0.05$) in order to check that the migration rate variation is consistent with this dependence. This is indeed the case. We note in passing that the migration rate varies as $h'^{-3}$, and not as $h'^{-2}$ as it would be the case in a one planet problem, since the Outer/Inner Lindblad torque asymmetry does not vanish as the disk thickness tends to zero (the OLRS would pile-up at Saturn’s orbit, whereas the ILRS would pile-up at Jupiter’s orbit). These results confirm that the behaviour we observe occurs mainly due to the differential Lindblad torque and shows as well that this latter quantity is positive, as expected from Eq. (3).

3.2 $\alpha$-viscosity vs. uniform viscosity

So far we have only considered a uniform viscosity. Switching to a uniform-$\alpha$ viscosity of the form $\nu = \alpha \nu_c H$ makes $\nu$ scale here as $r^{1/2}$, so the viscosity at the outer edge of the common gap is higher, whereas it is smaller in the inner disc. This has the following effect, which plays in favour of enhancing the migration reversal mechanism: the viscous time-scale of the inner disc is higher and therefore its surface density increases accordingly, since the material brought through the gap piles-up in the inner disc for a longer time before being accreted on the primary. This has been checked with a restart run.

3.3 Accretion on to the planets

The cores considered above do not accrete gas from the disc. One can wonder what would be the effects of accretion. We have performed a number of restart runs in order to investigate the effect of accretion on the mechanism presented here. We have only considered accretion on to Jupiter, as it is likely that the accretion rate on Saturn can be regarded as being negligible (i.e. its mass doubling time is much longer than the timescale of the outwards migration, see e.g. Pollack et al. 1996). The prescription we used to model accretion on to Jupiter consists in removing a proportion of the material which lies in the inner Roche lobe (i.e. a sphere with a radius of half the Hill radius). The amount which is removed in one timestep is calculated from the half emptying time of the inner Roche lobe $t_{1/2}$. We have performed four different restart runs, corresponding to the following values...
of \( \tau_{1/2}: \tau_{1/2} = T_0 \) (maximally accreting core, see Kley 1999), \( \tau_{1/2} = 3T_0 \), \( \tau_{1/2} = 107T_0 \) and \( \tau_{1/2} = 30T_0 \), where \( T_0 = 2\pi/\Omega_J \) is Jupiter’s orbital time. In each of these cases, turning on accretion had no impact on the system migration rate, at least in the early stages: in the first case, the mass doubling time for Jupiter is relatively short, and when Jupiter’s mass is significantly larger than its initial mass some additional effects, which will be presented in much greater detail elsewhere, affect the migration rate which then differ from the non-accreting case.

3.4 Smoothing

The smoothing parameter of the potential can have a dramatic impact on Saturn’s initial migration rate. This rate is controlled by a subtle balance between outer disc and inner disc torques. In the case of Saturn, all the Lindblad resonances play a role, since there is no gap. Many prescriptions for the smoothing are unable to give trustworthy results for the balance between the outer and inner torques since, depending on the prescription, these two quantities are affected in a different way. On the other hand Jupiter’s migration rate is much more robust, since the presence of the gap prevents high-\( m \) Lindblad resonances playing a role in the migration, which is therefore controlled only by remote, low \( m \) resonances and thus almost insensitive to the smoothing parameter. For this reason we have adopted an approach which involves choosing a smoothing prescription which endows Saturn with a migration velocity of the order of magnitude of the linear analytical predictions (type I migration), which is needed to give correct results for the capture into resonance. Once Saturn is trapped into resonance with Jupiter, it is dynamically slaved by the latter and the system evolution is only very weakly affected by the exact value of the outer disc torque exerted on Saturn. We have found that using either of the two prescriptions below satisfactorily preserves the analytical torque imbalance on Saturn and therefore gives it a type I migration rate:

- The potential of a planet acting on the disc is smoothed over the length \( \varepsilon = 0.4R_H \) where \( R_H \) is the Hill radius of the planet under consideration, whereas the potential of the disc acting back on the planet is smoothed over \( \varepsilon' = \sqrt{H^2 + d^2} \) where \( H \) and \( d \) are respectively the local disc thickness and zone diagonal. Since \( \varepsilon' \neq \varepsilon \) the action-reaction law is not fulfilled and the numerical biases which arise favour an inwards migration, as can be easily checked.

- The potential of a planet acting on the disc and the potential of the disc acting on the same planet are smoothed over \( \varepsilon = 0.4R_H \). This prescription does fulfill the action-reaction law.

In both these two cases, as in any other which gives Saturn a type I migration rate, including runs performed with a uniform radial spacing, the migration gets reversed. The run presented here corresponds to the first prescription.

3.5 Impact of mass ratio and Long-term behaviour

One can wonder about the size of the interval of “Saturn”’s mass which causes the migration to be significantly slowed down or reversed. If “Saturn” is not massive enough it will not significantly affect Jupiter’s evolution (the common “gap” will be too full on Saturn’s side, and therefore Jupiter’s OLR torques will not be shut off), whereas if it is too massive, the torque imbalance will be negative again. Work is in progress to accurately determine which range of parameters leads to a migration reversal. It should be noted that the results presented here depend on the artificial initial conditions. We have performed other runs in which Saturn is initially very close either to the 1 : 2 or 3 : 5 resonance, and it turns out that neither of these resonances is able to struggle against the strong Lindblad torques on Saturn: no resonance angle can be found which provides a resonant torque on Saturn which counteracts the tide. Therefore a trapping into the 2 : 3 resonance is the most likely outcome when the system is still embedded in a massive disc, whatever the initial conditions; catching-up of “Saturn” or in-situ assembling from smaller, type I migrating bodies.

The long-term behaviour of the system is twofold:

- The system is locked into resonance as long as :
  - The two-planet system can adjust its resonance angle in order to prevent the planets being “pushed” towards each other by the Lindblad torques exerted by the disk on each of them. In all our runs we have never observed this behaviour. Now, given the small eccentricities involved here, and given the fact that the adiabatic criterion threshold increases as \( j(j + 1) \), the most probable outcome is that Saturn would then be captured in the next order resonance, that is to say 3 : 4, and all the physics exposed in this paper would still be valid (presence of a common gap, sharing of the coorbital material by the two planets, mass-weighted torque imbalance, etc.)
  - The planets are not pulled apart by any other torques. Now we have mentioned the possibly important role of the coorbital corotation torque in this problem, which may be sufficient to move the planets apart at some stage, in which case we may ultimately get a low eccentricity double giant planet system when the disc disappears. This will be presented in greater detail elsewhere.

- If the planets happen to be locked into resonance at the time that the gas effects become negligible, then the system is likely to be unstable (we mentioned already that at least two angles librate simultaneously, which strongly suggests a possible chaotic behaviour; see also Kley 2000), and the most likely outcome is that one planet will be ejected whereas the other planet will end up on an eccentric orbit. This could account for the observed eccentricities of the extrasolar planets which are not orbiting close to their host star, i.e. which have not migrated all the way to the star.

4 ACKNOWLEDGEMENTS

We wish to thank J.C.B. Papaloizou, R.P. Nelson, C. Terquem, J.D. Larwood, A.A. Christou and an anonymous referee for useful comments and criticism. This work was partially supported (for F.M.) by the research network “Accretion onto black holes, compact stars and protostars”
funded by the European Commission under contract number ERBFMRX-CT98-0195, and additionally supported (for M.S.) by funding from a PPARC research studentship. Computational resources of the Grand HPC consortium were available and are gratefully acknowledged. We thank Udo Ziegler for making a FORTRAN version of his code NIRVANA publicly available.

REFERENCES

Balbus S.A., Hawley J.F., 1991, ApJ, 376, 214
Beaugé C., Arsesth S.J., Ferraz-Mello S., 1994, MNRAS, 270, 21
Goldreich, P., Tremaine, S., 1979, ApJ, 233, 857
Haghighipour N., 1999, MNRAS, 304, 185
Kley W., 1999, MNRAS, 303, 696
Kley W., 2000, MNRAS, 313, 47
Malhotra, R., 1993, Icarus, 106, 264
Marcy G.W., Cochran W.D., Mayor M., 1999, Protostars and Planets IV, Tucson: University of Arizona Press; eds Mannings, V., Boss, A.P., Russell, S., p. 1285.
Masset F., 2000, A&AS, 141, 165
Melita M.D., Woolfson M.M., 1996, MNRAS, 280, 854
Nelson R.P., Papaloizou J.C.B., Masset F., Kley W., 2000, MNRAS, 318, 18
Papaloizou J.C.B., Lin D.N.C., 1984, ApJ, 285, 818
Pollack, J.B., Hubickyj, O., Bodenheimer, P., Lissauer, J.J., Podolak, M., Greenzweig, Y., 1996, Icarus, 124, 62
Ward W.R, Hourigan K. 1989, ApJ, 347, 490
Ward, W.R., 1991, Abstracts of the Lunar and Planetary Science Conference, 22, 1463
Ward, W.R., 1992, Abstracts of the Lunar and Planetary Science Conference, 23, 1491
Ward W.R., 1997, Icarus, 126, 261
Ziegler U., Yorke H.W., 1997, Comp. Phys. Comp., 101, 54