Holographic phase transition in a non-critical holographic model

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Abstract

We consider a holographic model constructed from the intersecting brane configuration D4-\overline{D4}/D4 in noncritical string theory. We study the chiral phase diagram of this holographic QCD-like model with a finite baryon chemical potential through the supergravity dual approximation.
1 Introduction

The AdS/CFT correspondence [1]-[4] and [5] is one realization of the holographic principle [6]. It means that the IIB string theory on the $AdS_5 \times S^5$ background is equivalent to the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on the four-dimensional boundary. By adding some flavor branes into the D3 branes background, one can introduce the fundamental flavors into the low energy effective theory on the intersecting region of brane configuration [7]. Therefore, some more realistic effective theories can be constructed from string theory. The strong coupling physics in these effective boundary theories can be investigated by their classical supergravity duals. Recently, some holographic models and the corresponding holographic phase transitions are studied through the gauge/gravity correspondence in [8]-[22]. And one can see the reviews [23] for more related work.

Similar to the effective theories constructed from the brane configurations in critical string theory, one can also build some holographic models from the intersecting brane configurations in noncritical string theory. Recently, it is investigated in [24]-[30]. In [28], the authors consider a D4-$\overline{D4}$/D4 brane configuration. The low-energy effective theory on the intersecting region is a QCD-like theory. Through the dual supergravity investigation, the D4-$\overline{D4}$ pairs solution connected through a wormhole is preferred in the low temperature phase. It means the flavor branes induced chiral symmetry in gauge theory will be broken. After a confinement/deconfinement phase transition into the high temperature phase, there exists a first order chiral phase transition at a critical temperature in this high temperature phase. Below this temperature, the chiral symmetry is broken. However, this symmetry will be restored above it. All these results about the chiral symmetry breaking are similar to some holographic models being constructed in critical string theory [15]. In this brane configuration, the six-dimensional gravity background is the near horizon geometry of the color D4 brane. Compared with the D-brane gravity background in critical string theory, it has some good points since there is not compact sphere. But the fault is that such background is not very reliable in the gauge/gravity correspondence. The reason is now the ’t Hooft coupling constant and scalar curvature are almost of order one for these gravit backgrounds. So we can’t choose a large ’t Hooft limit in this holographic model.

In this paper, we extend to study the chiral phase structure of this holographic model with a chemical potential by using the methods in [17]-[20]. The chemical potential in the boundary gauge theory corresponds to turn on a zero-component of gauge field on the
flavor brane in the holographic brane configurations. We find, with a chemical potential, that a chiral symmetry breaking solution is preferred in the low temperature phase. This result is same as the case without a chemical potential. However, at high temperature, the chiral phase diagram is different with the no chemical potential case. And now the phase structure depends on some parameters, which are the chemical potential and the temperature of the black hole background.

The paper is organized as follows. In section two, we give a short introduction to this model. Then we will investigate the holographic phase structure with a chemical potential in sections three and four and the Appendix. In section five, we give our conclusion.

2 Brane configuration

We consider the holographic model constructed by the brane configuration D4-\overline{D4}/D4 in [28]. The coordinates of these branes are extended as follows

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
N_c & D4: & x & x & x & x \\
N_f & D4, \overline{D4}: & x & x & x & x
\end{array}
\tag{2.1}
\]

The number of the color and flavor branes satisfies the condition \( N_c \gg N_f \). In the quenching approximation, the backreaction of the flavor branes on the color branes can be ignored. Let the coordinate \( x_4 \) be compactified on a circle \( S^1 \), and then the adjoint fermions on the color D4 brane with anti-periodic boundary condition on this circle will be decoupled from the low energy effective theory. Similar to the Sakai-Sugimoto(SS) model [10], the low energy effective theory is QCD-like on the four dimensional intersecting region in this brane configuration. And this low energy theory has a global chiral symmetry \( U(N_f)_L \times U(N_f)_R \) induced by the flavor brane pairs D4-\overline{D4}.

The near horizon geometry of the color D4 branes in noncritical string theory is expressed as

\[
ds^2 = \left( \frac{U}{R} \right)^2 (-dt^2 + dx_i dx_i + f(U)dx_4^2) + \left( \frac{R}{U} \right)^2 \frac{1}{f(U)} dU^2,
\]

\[
F_5 = Q_c \left( \frac{U}{R} \right)^4 dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dU \wedge dx_4,
\]

\[
e^\phi = \frac{2\sqrt{2}}{\sqrt{3}Q_c}, \quad R^2 = 15/2, \quad f(U) = 1 - \left( \frac{U_{KK}}{U} \right)^5,
\]

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where the parameter $Q_c$ is proportional to the number of color brane $N_c$. From this background, the 't Hooft coupling constant and curvature scalar all are of order one. Because of the high order string corrections, this background is not very good at using the gauge/gravity correspondence. In order to avoid a singularity in this background, the coordinate $x_4$ will be periodic with a radius

$$x_4 \sim x_4 + \delta x_4 = x_4 + \frac{4\pi R^2}{5U_{KK}}.$$  \hfill (2.3)

And its corresponding Kluza-Klein (KK) mass scale of this compact dimension is

$$M_{KK} = \frac{2\pi}{\delta x_4} = \frac{5U_{KK}}{2R^2}. \hfill (2.4)$$

Passing into the finite temperature phase, there exist two supergravity backgrounds. One is that the geometry (2.2) does a Wick rotation $t_E = it$. So it is

$$ds^2 = \left( \frac{U}{R} \right)^2 \left( dt_E^2 + dx_i dx_i + f(U)dx_4^2 \right) + \left( \frac{R}{U} \right)^2 \frac{1}{f(U)}dU^2,$$

$$f(U) = 1 - \left( \frac{U_{KK}}{U} \right)^5,$$  \hfill (2.5)

where the Euclidean time satisfies the period $t_E \sim t_E + \beta$, and the coordinate $x_4$ still satisfies the periodic condition (2.3). Since the $\beta$ is arbitrary, the temperature $1/\beta$ of this background is undetermined.

Another finite temperature supergravity background is the black hole case

$$ds^2 = \left( \frac{U}{R} \right)^2 \left( \tilde{f}(U)dt_E^2 + dx_i dx_i + dx_4^2 \right) + \left( \frac{R}{U} \right)^2 \frac{1}{\tilde{f}(U)}dU^2,$$

$$\tilde{f}(U) = 1 - \left( \frac{U_T}{U} \right)^5,$$  \hfill (2.6)

where the Euclidean time satisfies

$$t_E \sim t_E + \delta t_E = t_E + \frac{4\pi R^2}{5U_T},$$  \hfill (2.7)

and the radius of the coordinate $x_4$ is arbitrary. After comparing the free energy between the background (2.5) and (2.6), one can find there exists a first order phase transition (corresponding to the confinement/deconfinement phase transition in the boundary theory) at a critical temperature $\beta = \delta x_4$. Below this temperature, the background (2.5) is dominated. Otherwise, the black hole background (2.6) will be dominated \cite{28}. These results are similar to the cases in the Sakai-Sugimoto model \cite{15}.  

3
In the following section, we consider a $U(1)$ baryon number symmetry. Similar to [17]–[22], in order to investigate this holographic model with a chemical potential, we only need open the zero component of the gauge field on the worldvolume of the flavor D4 branes. In the following, we assume that the zero component $A_0$ and the coordinate $x_4$ only depend on the coordinate $U$, and set $\alpha' \equiv 1$. And The Abelian effective action of the D4 branes is

$$S_{D4} = -T_4 \int d^5 \xi \ e^{-\phi} \sqrt{-\det( g_{MN} + 2\pi \alpha' F_{MN}) + \mu_5 \int C_5}. \quad (2.8)$$

By the arguments in [23], the Chern-Simons (CS) term $\int C_5$ need to be vanished in order to make the holographic duality to work. So we don’t consider the contributions of the CS term in the following.

3 Low temperature

In the low temperature phase, the induced metric on the flavor D4 branes is

$$ds^2 = \left(\frac{U}{R}\right)^2 (dt_E^2 + dx_i dx_i) + \left(\frac{U}{R}\right)^2 \left( f(U) \left( \frac{\partial x_4}{\partial U} \right)^2 + \left( \frac{R}{U} \right)^4 \frac{1}{f(U)} \right) dU^2. \quad (3.1)$$

Substituting this metric into the effective action (2.8), we get the D4 brane action as

$$S \sim \int dx_4 \ U^5 \sqrt{f(U) + \left( \frac{R}{U} \right)^4 (f(U)^{-1}U'^2 - (2\pi A_0')^2)}, \quad (3.2)$$

where $U' = dU/dx_4$ and $A_0'(U) = dA_0/dx_4$. Then the equation of motion of the coordinate $x_4(U)$ and the zero component $A_0(U)$ are derived as

$$\frac{d}{dx_4} \left[ \frac{U^5 f(U)}{\sqrt{f(U) + \left( \frac{R}{U} \right)^4 (f(U)^{-1}U'^2 - (2\pi A_0')^2)}} \right] = 0,$$

$$\frac{d}{dx_4} \left[ \frac{U^5 \left( \frac{R}{U} \right)^4 2\pi A_0'}{\sqrt{f(U) + \left( \frac{R}{U} \right)^4 (f(U)^{-1}U'^2 - (2\pi A_0')^2)}} \right] = 0. \quad (3.3)$$

After doing one integration, we get two equations

$$U'^2 = \frac{f(U)^2 \left[ f(U)(U_1^{10} + C_1^2 \left( \frac{U}{R} \right)^4) - f(U_0)(U_0^{10} + C_1^2 \left( \frac{U_0}{R} \right)^4) \right]}{(\frac{R}{U})^4 f(U_0) \left[ U_0^{10} + C_1^2 \left( \frac{U_0}{R} \right)^4 \right]}. \quad (3.4)$$
\[(2\pi A_0')^2 = \frac{U^8 f(U)^2}{R^8 f(U_0)(U_0^{10} + C_1^2 (\frac{U_0}{\pi})^4)} , \quad (3.4)\]

where the integrating constants $U_0$ and $C_1$ satisfy $U'_0|_{U=U_0} = 0$ and
\[
C_1 = \frac{U_0^5 \left( \frac{R}{U_0} \right)^4 2\pi A_0'(U_0)}{\sqrt{f(U_0) - \left( \frac{R}{U_0} \right)^4 (2\pi A_0'(U_0))^2}} . \quad (3.5)\]

Thus, we obtained a D4-D4 pairs solution connected through a wormhole $U = U_0$. And now the chiral symmetry $U(N_f)^L \times U(N_f)^R$ is broken to $U(N_f)^{\text{diag}}$. After inserting these solutions (3.4) into the action (3.2), we obtain the on-shell energy of this connected configuration
\[
S_1 \sim \int_1^\infty dy \frac{y^3}{\sqrt{f(y)(1 + ay^{-6}) - f(1)(1 + a)y^{-10}}} , \quad (3.6)\]

where $y = U/U_0$, $y_{KK} = U_{KK}/U_0$ and $a = C_1^2/R^4 U_0^6$. For the gravity background (2.5), the coordinate $x_4$ is periodic. The flavor branes don’t have any place to end in this background. So there doesn’t exist separated flavor D4 and $\overline{\text{D}4}$ solution. If the coordinate $x^4$ is not periodic, and there is not $f(u)$ factor in the metric (2.5), then the separated flavor solution will be existed [13]. Thus, the global chiral symmetry $U(N_f)^L \times U(N_f)^R$ is always broken to its diagonal part $U(N_f)^{\text{diag}}$ at low temperature.

From the equation of motion, the asymptotic distance along the direction $x^4$ between the D4 and $\overline{\text{D}4}$ pairs can be defined as
\[
\frac{L}{2} = \int_0^{L/2} dx_4 = \int_0^\infty dU \int_U^1 \frac{1}{U^t}
= \frac{R^2}{U_0} \int_1^\infty dy \int_0^{\infty} dy f(y) \int_U^1 \frac{1}{y^2 f(y)} \sqrt{f(y)(y^{10} + ay^4)} - 1
\]
\[
= \frac{R^2}{5U_0} \int_0^1 dz \frac{z^{1/5} \sqrt{(1 + a)(1 - y_{KK}^5)}}{(1 - y_{KK}^5 z) \sqrt{(1 - y_{KK}^5 z)(1 + az^{6/5})} - (1 - y_{KK}^5)(1 + a) z^2}
\]

It depends on the chemical potential and the parameter $y_{KK}$. If choosing $a = 0$, then we find it will be reduced to the no chemical potential case in [28].
4 High temperature

From the high temperature background (2.6), the induced metric on the worldvolume of the D4 flavor branes can be obtained as

\[ ds^2 = \left( \frac{U}{R} \right)^2 \left( \frac{\dot{f}(U)}{\dot{f}} \right) dt^2 + \left( \frac{U}{R} \right)^2 \left( \left( \frac{\partial x_4}{\partial U} \right)^2 + \left( \frac{R}{U} \right)^4 \frac{1}{f(U)} \right) dU^2. \] (4.1)

The same as in the low temperature phase, the D4 action in the background (2.6) is

\[ S \sim \int dx_4 U^5 \sqrt{f(U)} + \left( \frac{R}{U} \right)^4 \left( U'^2 - (2\pi A'_0)^2 \right). \] (4.2)

And the equations of motion is derived as

\[ \frac{d}{dx_4} \left[ \frac{U^5 \ddot{f}(U)}{\sqrt{\dot{f}(U)} + \left( \frac{R}{U} \right)^4 \left( U'^2 - (2\pi A'_0)^2 \right)} \right] = 0, \]

\[ \frac{d}{dx_4} \left[ \frac{U^5 (\frac{R}{U})^4 2\pi A'_0}{\sqrt{\dot{f}(U)} + \left( \frac{R}{U} \right)^4 \left( U'^2 - (2\pi A'_0)^2 \right)} \right] = 0. \] (4.3)

After integrating, we get the following two equations

\[ U'^2 = \frac{\ddot{f}(U) \left[ \ddot{f}(U)(U'^{10} + C_2^2 \left( \frac{U}{R} \right)^4) - \ddot{f}(U_0)(U'^{10}_0 + C_2^2 \left( \frac{U_0}{R} \right)^4) \right]}{(\frac{R}{U})^4 \ddot{f}(U_0) \left[ U'^{10}_0 + C_2^2 \left( \frac{U_0}{R} \right)^4 \right]}, \]

\[ (2\pi A'_0)^2 = \frac{U^8 \dddot{f}(U)^2}{R^8} \frac{C_2^2}{\dddot{f}(U_0)(U'^{10}_0 + C_2^2 \left( \frac{U_0}{R} \right)^4)}, \] (4.4)

where the integrating constants \( U_0 \) and \( C_2 \) satisfy \( U'|_{U=U_0} = 0 \) and

\[ C_2 = \frac{U_0^5 \left( \frac{R}{U_0} \right)^4 2\pi A'_0(U_0)}{\sqrt{\dddot{f}(U_0)} - \left( \frac{R}{U_0} \right)^4 (2\pi A'_0(U_0))^2}. \] (4.5)

The same as the cases in the low temperature phase, this solution denotes the connected configuration of the D4-D4 brane pairs.

Then, after substituting this solution into the action (4.2), we obtain the on-shell energy of this configuration

\[ S_3 \sim \int_{1}^{\infty} dy \frac{y^3 \sqrt{\dddot{f}(y)}}{\sqrt{\dddot{f}(y)(1 + by^{-6}) - \ddot{f}(1 + b)y^{-10}}}. \] (4.6)
where \( y = U/U_0 \), \( y_T = U_T/U_0 \) and \( b = C_2^2/R^4U_0^6 \).

And the separated solution of the D4-D\( \overline{4} \) brane pairs satisfies

\[
\frac{dx_4}{dU} = 0, \quad (2\pi)^2 \left( \frac{dA_0}{dU} \right)^2 = \frac{C_2^2}{C_2^2 + U^{10}(R/U)^4}. \tag{4.7}
\]

If \( U \to \infty \), then

\[
2\pi \left( \frac{dA_0}{dU} \right) \sim \frac{C_2}{R^2U^{-3}}, \tag{4.8}
\]

which is similar to the corresponding low temperature case. The on-shell action of the solution (4.7) is

\[
S_4 \sim \int_{y_T}^{\infty} dy \frac{y^3}{\sqrt{1 + by^{-6}}}. \tag{4.9}
\]

In order to determine which solution is more preferred, one needs to compare the on-energy of these two configurations. With the equations (4.6) and (4.9), the energy difference of these two configurations is

\[
\Delta S = S_3 - S_4 = \int_{1}^{\infty} dy \left( \frac{y^3 \sqrt{f(y)}}{\sqrt{f(y)(1 + by^{-6})} - f(1 + b)y^{-10}} - \frac{y^3}{\sqrt{1 + by^{-6}}} \right)
- \int_{y_T}^{1} dy \frac{y^3}{\sqrt{1 + by^{-6}}}
= \frac{1}{5} \int_{0}^{1} dz \frac{1}{z^{9/5}} \left( \frac{1 - y_T^5 z}{(1 - y_T^5)(1 + b z^{6/5}) - (1 - y_T^5)(1 + b) z^2} - \frac{1}{\sqrt{1 + b z^{6/5}}} \right)
- \frac{1}{5} \int_{y_T}^{5} dz \frac{1}{z^{9/5} \sqrt{1 + b z^{6/5}}}. \tag{4.10}
\]

After doing numerical calculations, we plotted the Fig. 1. From this figure, we find there exists a turning point at \( b = 0.16 \). If the parameter \( b \) is larger than it, then the separated configuration is dominated, and the chiral symmetry will be always unbroken. However, if smaller than this one, there exists two phases. One is the chiral symmetry breaking phase, the other is the chiral symmetry restoration phase. And in the \( b < 0.16 \) region, the chiral phase transition point \( y_T \) will become small with increasing the parameter \( b \). In all, the chiral symmetry will be broken at some smaller parameters \( b \) and \( y_T \) for this holographic model. These results about the chiral phase diagram here are similar to the cases for the Sakai-Sugimoto model with a chemical potential [17].

In the background (2.6), the asymptotic distance along the direction \( x^4 \) between the
Figure 1: The energy difference $\Delta S$ varies with the $y_T$ at various values $b = 0, 0.10, 0.16, 0.20$ and $0.30$ (from below to above). With increasing the parameter $b$, the energy difference $\Delta S$ is also increased for a fixed $y_T$. 

D4 and $\overline{D4}$ is

$$L = \frac{R^2}{U_0} \int_1^{\infty} \frac{dy}{y^2} \frac{1}{\sqrt{f(y)(y^{10} + b^4)}} - 1$$

$$= \frac{R^2}{5U_0} \int_0^1 dz \frac{z^{1/5}}{\sqrt{(1 - y_T^5 z)((1 - y_T^5 z)(1 + b z^{6/5}) - (1 - y_T^5)(1 + b)z^2)}}$$

$$\equiv \frac{R^2}{5U_0} F(y_T, b). \quad (4.11)$$

If one chooses $b = 0$, then it will be reduced to the case without a chemical potential in [28]. And using this distance $L$, we find the temperature of the black D4 background will be satisfied

$$T = \frac{5U_T}{4\pi R^2} = \frac{y_T}{2\pi L} F(y_T, b). \quad (4.12)$$

Since the zero component $A_0(y_T) = 0$ at the horizon, from the separated solution (4.7), the $A_0(U)$ satisfies

$$A_0(U) = \frac{U_0}{2\pi} \int_{y_T}^{y} dy \sqrt{\frac{b}{b + y^6}}. \quad (4.13)$$

By the gauge/gravity correspondence, the chemical potential $\mu$ corresponds to the zero
component \( A_0(U)|_{U \to \infty} \). Thus, we get

\[
\mu = A_0(\infty) = \frac{R^2}{5\pi L} F(y_T, b) \int_{y_T}^{\infty} dy \sqrt{\frac{b}{b + y^6}}.
\]  

(4.14)

From this expression, we can find the relation between the chemical potential \( \mu \) and the parameters \( y_T \) and \( b \). With the equation (4.12), the chemical potential can be expressed as

\[
\mu = T \frac{2R^2}{5y_T} \int_{y_T}^{\infty} dy \sqrt{\frac{b}{b + y^6}}.
\]  

(4.15)

It linearly depends on the temperature. And in the region of the smaller values of the temperature \( T \) and \( b \), the chiral symmetry is broken. However, the symmetry is restored at some larger values.

5 Conclusions

In this paper, we consider a holographic model constructed from the brane configuration D4-\( \overline{D4}/D4 \) in noncritical string theory \[28\]. We investigated the holographic chiral phase structure of this noncritical string model with a finite chemical potential \( \mu \). In the low temperature phase, just as the case without a chemical potential, the chiral symmetry breaking solution is always preferred.

In the high temperature phase, the results are different from the no chemical potential case. Now the chiral phase structure depends on some parameters, which are the chemical potential and temperature of the black hole background. From the Fig. 1, one can understand very clearly on the relations of the phase diagram with the parameters \( y_T \) and \( b \). Under the critical point \( b = 0.16 \), there exists two phases: the chiral symmetry breaking phase and the chiral symmetry restored phase. Which system the phase lies in depends on the value of the parameter \( y_T \). And above the value \( b = 0.16 \), the chiral symmetry will be restored. All results of this noncritical holographic model are similar to the corresponding cases of the SS model \[17\]. These results give some universal confirmations for some holographic models. In the appendix, we extend to give some simple calculations about the isospin chemical potential. One can generalize to study the chiral phase diagram of some other holographic models in noncritical string theory.
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Appendix

In this appendix, we generalize to consider the non-vanishing isospin chemical potential cases by following the method in [21] and [22]. It corresponds to a $\rho$ meson condensation. We list the results for the isospin chemical potential.

For the $N_f$ coincident flavor D4 branes, the global symmetry is $U(N_f)$. Its $U(1)$ part in the boundary corresponds to a baryon chemical potential. And the non-Abelian part $SU(N_f)$ produces a isospin chemical potential. For simplicity, we choose the flavor number $N_f = 2$. The baryon and isospin chemical potential are written as

$$
\mu_B = \frac{1}{2}(\mu_1 + \mu_2), \quad \mu_I = \frac{1}{2}(\mu_1 - \mu_2),
$$

where the chemical $\mu_1$ and $\mu_2$ are the boundary value of the zero component of Abelian gauge field $A_1$ and $A_2$ on two flavor D4-branes, respectively. If the chemical potentials satisfies $\mu_1 = \mu_2$, there only exists a baryon chemical potential, all chiral phases is discussed above. If $\mu_1 = -\mu_2$, then only the isospin chemical potential is non-vanished. For the other case, both these chemical potentials are existed. In the following, we mainly calculate the isospin chemical potential in the connected D4-$\overline{D4}$ brane phase with the condition $\partial_U A_{(1)0} = -\partial_U A_{(2),0}$.

In the low temperature phase, by using the equation (3.4), we get the isospin chemical potential

$$
\mu_I = \int_{U_0}^{\infty} dU \frac{U^2}{2\pi R^2 \int_{U_0}^{\infty} \sqrt{f(U)(U^{10} + C_1^2 U^4/R^4) - f(U_0)(U_0^{10} + C_1^2 U_0^4/R^4)}}.
$$

Defining $z = U_0^5/U_5^5$, $a = C_1^2/R^4 U_0^6$ and $y_{KK} = U_{KK}/U_0$, then the above equation becomes

$$
\mu_I = \frac{C_1}{10\pi R^2 U_0^2} \int_{0}^{1} \frac{dz}{z^{3/5} \sqrt{(1 - y_{KK}^2 z)(1 + a z^{6/5}) - (1 - y_{KK}^2)(1 + a) z^2}}.
$$

In the high temperature background, with the connected solution (4.4), the isospin chemical potential is derived as

$$
\mu_I = \int_{U_0}^{\infty} dU \frac{1}{\partial_U A_0}.
$$
\[
\begin{align*}
&= \frac{C_2}{2\pi R^2} \int_{U_0}^{\infty} \frac{U^2 \tilde{f}^{1/2}}{\sqrt{\tilde{f}(U)(U^{10} + C_2^2 U^4 / R^4) - \tilde{f}(U_0)(U_0^{10} + C_2^2 U_0^4 / R^4)}} \\
&= \frac{C_2}{10\pi R^2 U_0^2} \int_0^1 \frac{dz}{z^{3/5}} \sqrt{(1 - y_T^5 z)(1 + b z^{6/5}) - (1 - y_T^5)(1 + b) z^2},
\end{align*}
\]

where \( z = U_0^5 / U^5, y_T = U_T / U_0 \) and \( b = \frac{c_\mu^2}{R^4 U_0^6} \).

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