Eddington, Lemaître and the discovery of the expanding universe

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Abstract

One of the leading astronomers and theorists of his generation, Arthur Stanley Eddington was an important early proponent of the general theory of relativity in both theory and experiment. Yet when his former student Georges Lemaître suggested in 1927 that the well-known redshifts of the spiral nebulae could be explained in terms of a relativistic expansion of space, Eddington paid no attention for three years. In this paper, we consider the reasons Lemaître’s hypothesis attracted little attention when it was first articulated. We review several factors that have previously been discussed in the literature, from Lemaître’s status as an early-career researcher to his decision to publish in a lesser-known journal, from the language of the article to conceptual difficulties associated with time-varying cosmologies. We discuss two new factors that have not been previously been considered, namely the technical challenge presented by Lemaître’s analysis to contemporaneous readers and the preliminary nature of the observational data he used to support his model.
1. Introduction

Arthur Stanley Eddington was one of the most accomplished physicists, mathematicians and astronomers of his generation (Smart et al. 1945; McCrea 1982). An early and important proponent of the general theory of relativity, his 1915 ‘Report on the Relativity Theory of Gravitation’ for the Physical Society (Eddington 1915) provided an early authoritative exposition of the subject for English-speaking physicists (Vibert Douglas 1956 p42; Chandrasekhar 1983 p24). He played a leading role in the eclipse observations of 1919 that offered early astronomical evidence in support of the theory (Vibert Douglas 1956 pp 39-41; Chandrasekhar 1983 pp 24-29; Kennefick 2009) while his book ‘Space, Time and Gravitation’ (Eddington 1920) was one of the first popular treatises on general relativity for an English-speaking audience. In addition, Eddington’s textbook ‘The Mathematical Theory of Relativity’ (Eddington 1923) became a classic reference for English-speaking physicists with an interest in relativity (McCrea 1982; Chandrasekhar 1983 p32). Indeed, the book provided one of the first textbook accounts of relativistic models of the cosmos, complete with a discussion of possible links to one of the greatest astronomical puzzles of the age, the redshifts of the spiral nebulae (Eddington 1923 pp 155-170).¹

It is therefore quite surprising that, when Eddington’s former student Georges Lemaître suggested in a seminal article of 1927 (Lemaître 1927) that a universe of expanding radius could be derived from general relativity, and that the phenomenon could provide a natural explanation for the redshifts of the spiral nebulae, Eddington and others paid no attention. The oversight is particularly puzzling as it is almost certain that Eddington received a copy of the paper at the time of publication, as detailed below. Three years later, Eddington embraced Lemaître’s hypothesis with great enthusiasm, distributing it to colleagues and arranging for it to be republished in the *Monthly Notices of the Royal Astronomical Society*.

In this article, we first review some well-known explanations for the overlooking of Lemaître’s 1927 paper when it was first published. These include sociological factors such as Lemaître’s status as an early-career researcher and his decision to publish in a lesser-known journal in French (section 3). In section 4, we consider philosophical factors such as the conceptual difficulties associated with the transition from static to non-static cosmologies. In section 5, we discuss two aspects of Lemaître’s 1927 paper that have not previously been discussed in the literature; the technical complexity of the article and the preliminary nature of

¹ The For further information on Eddington’s contributions to relativity see (Smart e al. 1945; Vibert Douglas 1956 pp 42-53; McCrea 1982).
the astronomical data used to support his model. We provide a summary of our findings in section 6, but note that for the specific case of Eddington, there remains a possibility that the great astronomer simply did not read the contents of Lemaître’s paper until 1930.

2. Historical context: a delayed paradigm shift

On 10th January 1930, a landmark meeting took place at the Royal Astronomical Society in Burlington House in London (Kragh 1996 p 21, 31; Nussbaumer and Bieri 2009 p121). It was noted that the recent discovery of an approximately linear relation between the redshifts of the spiral nebulae and their radial distances\(^2\) could not be readily explained in the context of the standard mathematical models of the cosmos, i.e., in the context of the static cosmology of Albert Einstein (Einstein 1917) or the empty cosmology of Willem de Sitter (de Sitter 1917). During the course of the meeting, it was suggested that alternative cosmologies should be considered. This discussion was reported in the February issue of the Observatory (de Sitter 1930) and read by Georges Lemaître, who immediately wrote a letter to Eddington to remind him of his paper of 1927. In his letter, an excerpt of which is shown in figure 1(a), Lemaître wrote: “Dear Professor Eddington, I just read the February No of the Observatory and your suggestion of investigating non-statical intermediary solutions between those of Einstein and de Sitter. I made these investigations two years ago. I consider a universe of curvature constant in space but increasing in time. And I emphasize the existence of a solution in which the motion of the nebulae is always a receding one from time minus infinity to plus infinity” (Lemaître 1930). After giving some technical details of the model, Lemaître remarked: “I send you a few copies of the paper. Perhaps you find occasion to give it to de Sitter. I sent him also at the time but probably he didn’t read it” (figure 1(b)).

Eddington responded with enthusiasm, publicly acknowledging the importance of Lemaître’s 1927 paper in his next article: “Working in conjunction with Mr. G. C. McVittie, I began some months ago to examine whether Einstein’s spherical universe is stable. Before our investigation was complete we learnt of a paper by Abbé G. Lemaître which gives a remarkably complete solution of the various questions connected with the Einstein and de Sitter cosmogonies...I desire to review the situation from an astronomical standpoint, although my original hope of contributing some definitely new result has been forestalled by Lemaître’s brilliant solution” (Eddington 1930). Eddington brought Lemaître’s paper to the attention of his colleagues, including Willem de Sitter, who also publicly acknowledged the work: “It thus

\(^2\) Redshift/distance data from both Edwin Hubble and de Sitter were discussed (de Sitter 1930).
appears that a static solution of Einstein’s field equations cannot represent the observed facts. A non-static solution is contained in a paper by Dr. G. Lemaître, published in 1927, which had failed to attract my notice, but to which my attention was called by Professor Eddington only a few weeks ago” (de Sitter 1930b). Finally, Eddington arranged for Lemaître’s paper to be republished in English in the *Monthly Notices of the Royal Astronomical Society* (Kragh 1996 p32; Nussbaumer and Bieri 2009 pp 122-125), as will be discussed below.

Many years later, the British astronomer George McVittie, a former research student in Eddington’s group, recalled the day when Eddington received Lemaître’s 1930 letter: “I remember the day when Eddington, rather shamefacedly, showed me a letter from Lemaître which reminded Eddington of the solution to the problem which Lemaître had already given. Eddington confessed that although he had seen Lemaître’s paper in 1927 he had forgotten completely about it until that moment” (McVittie 1967). A second account was provided by McVittie in oral testimony to the historian David DeVorkin in 1978. He recalled Eddington saying: “I’m sure Lemaître must have sent me a reprint, he’s just sent me another, but I’d forgotten about it” (McVittie 1978). Unfortunately, both reported statements are slightly ambiguous; it is not clear whether Eddington is saying that he scanned the paper briefly and then forgot about the contents, or that he simply set the paper aside and forgot to read it. Considering Eddington’s great interest in general relativity, the highly specific title of Lemaître’s paper and the fact that the redshifts of the nebulae represented the outstanding astronomical puzzle of the age, we contend that either possibility requires some explanation.

3. **Standard explanations**

In this section, we reconsider some explanations for the delay in recognition of Lemaître’s 1927 paper that have previously been advanced in the literature.

(i) *Status of the researcher*

One possibility is that Lemaître’s status as a relatively junior researcher may have hindered the recognition of his seminal paper when it was first proposed. After all, busy professors do not always take the time to read every new paper by a former student or early-career researcher. More generally, a daring new scientific hypothesis tends to be more readily accepted if it is proposed by an eminent scientist rather than a comparative unknown (Merton 1973 p443; Zuckerman 1973 pp 96-144). However, we suggest a caveat to this observation, namely

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3 We first learnt of McVittie’s recollection from the work of Helge Kragh (Kragh 1996 p32).
Lemaître’s rapid rise as a young researcher. Having demonstrated a remarkable facility in relativity as a young student, Lemaître was awarded a prestigious government scholarship to study abroad for three years (Lambert 2015 p84; Mitton 2017). Thus he spent a year as a Research Associate at the University of Cambridge under the supervision of Eddington, followed by a sojourn as Research Associate at the Harvard College Observatory under Harlow Shapley; during this period he also enrolled in a PhD program at MIT, specializing in aspects of general relativity (Lambert 2015 pp 87-99; Mitton 2017; Kragh 2018). In 1925, Lemaître achieved his first major advance with a paper that demonstrated an inconsistency in de Sitter’s cosmology (Lemaître 1925). In this analysis, Lemaître showed that the de Sitter model was not homogeneous in its original formulation; when expressed in a co-ordinate frame in which it was strictly homogenous, the model was no longer static! While the non-static aspect of the de Sitter model had been suspected for some time, Lemaître’s analysis was an important contribution. Thus, while the young cleric was still an early-career researcher in 1927 in the literal sense, he had already established himself as a theoretician of note by that time (Mitton 2017; Kragh 2018).

(ii) Status of the journal
It has often been suggested that the journal in which Lemaître’s paper of 1927 was published, the *Annales de la Société Scientifique de Bruxelles*, was a rather obscure vehicle for the paper (Kragh 1996 p31; Nussbaumer and Bieri 2009 p99; Shaviv 2011). However, recent scholarship suggests that this point may have been somewhat exaggerated; in fact the *Annales* was a respected journal that could be found in the libraries of most European universities at the time. Articles in both French and English could be found in the journal and it was reasonably well-known throughout European academia (Lambert 2015 pp 132-133; Luminet 2013). We note that Lemaître may have felt a need to acknowledge his earlier governmental scholarship by publishing in a Belgian journal; the publication of his 1925 paper in the *Journal of Mathematics and Physics* was probably linked to the fact that the results were presented at the 1925 meeting of the American Physical Society (Mitton 2019).6

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4 During this period, Lemaître learnt of the recent astronomical discoveries of both Vesto Melvin Slipher and Edwin Hubble (Lambert 2015).
5 The model was a source of great confusion amongst some of the greatest mathematicians of the time, as detailed in section 4.
6 Many of the papers presented at that meeting were published in the *Journal of Mathematics and Physics* (Mitton 2019).
(iii) Language considerations

Many authors have noted that Lemaître’s decision to publish his landmark 1927 article in French, rather than English, may have impacted on the reception of the paper (Kragh 1996 p31; Shaviv 2011; Ostriker and Mitton 2013 p68).\textsuperscript{7} We note however that it was quite normal for English-speaking scientists to read scientific papers published in French at this time. Indeed, Eddington himself translated a number of articles and books on relativity from French to English (Smart et al. 1945).

We also note that the title of Lemaître’s paper is extremely specific. The first phrase, ‘Un Univers Homogène de Masse Constante et de Rayon Croissant’ makes clear the principal topic of the paper (A Homogeneous Universe of Constant Mass and Increasing Radius), even to a reader not fluent in French. Meanwhile the second phrase, ‘Rendant Compte de la Vitesses Radiale des Nébeleuses Extra-Galactiques’ highlights the link to astronomy (Accounting for the Radial Speeds of the Extra-Galactic Nebulae). Indeed, the entire contents of Lemaître’s paper are summarized in the title, at least to readers with a basic knowledge of standard terminology in astronomy and cosmology.

Notwithstanding the above, it is likely that Lemaître’s choice of journal and language had some impact on the reception of his paper when first published. After all, when Lemaître’s theory was accepted by Eddington three years later, the latter quickly arranged for the paper to be translated into English and republished in the Monthly Notices of the Royal Astronomical Society (Lemaître 1931a).

4. Philosophical considerations

It’s worth noting that conceptual and philosophical difficulties may also have affected the reception of Lemaître’s 1927 paper. As has been previously pointed out, the transition from static to dynamic cosmologies was no small, sudden transformation, but a paradigm shift of the type identified by Thomas Kuhn (North 1965 p111; Kragh and Smith 2003; Marx and Bornmann 2010).

We note first that, when discussing his 1917 model of the cosmos many years later, Einstein remarked that the hypothesis of cosmic stasis “appeared unavoidable to me at the time, since I thought that one would get into bottomless speculation without it” (Einstein 1946 p154). This attitude can also be seen in Einstein’s reaction to Alexander Friedman’s article on

\textsuperscript{7} This decision may also have been linked to governmental funding of Lemaître’s early research (Mitton 2019).
time-varying cosmology when it was published in the *Zeitschrift für Physik* (Friedman 1922). In a note to the same journal, Einstein declared “*the results obtained ... seem suspect to me*” and claimed that the solution was inconsistent with the field equations (Einstein 1922). While Einstein later retracted his technical correction (Einstein 1923a), an unpublished draft of Einstein’s retraction contains the revealing phrase “*to this solution a physical reality can hardly be ascribed*” (Einstein 1923b).\(^8\)

Historians have noted a similar reluctance to embrace time-varying cosmologies amongst many of Einstein’s contemporaries. For example, many of the early analyses of the de Sitter model by theorists such as Cornelius Lanczos (Lanczos 1922, 1923), Hermann Weyl (Weyl 1919, 1923) and Felix Klein (Klein 1924) were written within the paradigm of a static universe; yet the same papers can be read in retrospect as expanding cosmologies. Although the mathematical framework for non-static cosmologies was in place, these theorists did not make the conceptual transition to the expanding universe (North 1965 pp 110-113; North 1990; Ellis 1990; Kragh and Smith 2003; Nussbaumer and Bieri 2009 pp 77-85). It seems likely that this ‘conceptual factor’ also played a role in the reception of Lemaître’s paper of 1927.

Einstein’s reaction to Lemaître’s paper is possibly the clearest example of a reluctance to consider time-varying cosmologies. As is well-known, the young cleric travelled to the Solvay conference of 1927 in order to contrive a meeting with Einstein in person. The meeting took place in the Parc Léopold, close to the Institute in which the conference was taking place (Lambert 2015 pp 133-134). Many years later, Lemaître recalled that Einstein had already read his paper. However, after praising some technical aspects of the article, the great physicist dismissed expanding cosmologies as repugnant from the point of view of physics: “*Après quelques remarques techniques favorables, il conclut... du point de vue physique... cela lui paraissait tout à fait abominable*” (Lemaître 1958). This incident suggests once again that most physicists were not yet ready to embrace the reality of an expanding universe in 1927.\(^9\)

We note in passing that it was also on this occasion that Lemaître first learnt of Friedman’s work (Lemaître 1930, 1958). Thus the young cleric’s first meeting with Einstein must have been something of a setback; not only did the world’s most famous physicist consider Lemaître’s theory repugnant, a similar model had already been suggested a few years earlier! This setback may constitute one reason the Belgian cleric did not pursue any further work in cosmology until the 1930s.

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8 See (Nussbaumer and Bieri 2009 p92) for more on this revealing incident.

9 Lemaître gives a similar description of his meeting with Einstein in his 1930 letter to Eddington (Lemaître 1930).
5. New explanations

In this section, we consider two technical aspects of Lemaître’s 1927 paper that may have had an impact on the reception of the work. To our knowledge, neither of these ‘internal’ factors have been considered in the literature before now.¹⁰

(i) The issue of technical complexity
We note first that, when reading a classic scientific work, it is easy to underestimate the challenge presented to a contemporaneous reader. This occurs because the modern reader often possesses technical tools that were not available in former times (Jardine 2000; Loison 2016). In particular, modern physicists peruse classic papers in cosmology with the benefit of a mathematical framework that has been developed over many years.¹¹ No such framework was available to Lemaître’s contemporaries and one must attempt to read his paper through the eyes of a 1927 reader.

We recall first the technical difficulties faced by physicists when first confronted with the general theory of relativity; indeed, it took Einstein himself five long years to express the field equations in their final covariant form (Norton 1984; Janssen and Renn 2015). Similar difficulties were presented by relativistic cosmology; the complexities of the de Sitter model resulted in a rather confused debate between leading physicists such as Einstein, de Sitter, Hermann Weyl, Cornelius Lanczos and Felix Klein for many years,¹² while the instabilities associated with Einstein’s static solution of 1917 were not appreciated until 1930 (Eddington 1930; O’Raifeartaigh et al. 2017). We recall also that, when Einstein made a technical error in his criticism of Friedman’s paper of 1922, only Friedman himself noticed the error (Kragh 1996 p26; Belenkiy 2013). These observations suggest that relativistic cosmology presented a daunting challenge for the 1920s physicist, a state of affairs that is likely to have affected the reception of Lemaître’s time-varying cosmology of 1927.

Turning to the theoretical section of Lemaître’s 1927 paper, for reasons of clarity we make use of an English translation of the article recently published by Jean-Pierre Luminet.

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¹⁰ In historical studies, the term ‘internal history’ is sometimes used to denote discussions of the technical aspects of a scientific work, while the term ‘external history’ is used to denote the study of the sociological context of the work (Kuhn 1977 pp 105-126; Lakatos 1981).

¹¹ See (Kragh 1996 pp 36-38; Nussbaumer and Bieri 2009 pp 205-206) for a discussion of the Friedman-Lemaître-Roberston-Walker metric.

¹² See (Schulmann et al. 1998) for a review of the so-called Einstein-deSitter-Weyl-Klein debate on the meaning of the de Sitter model.
The first few paragraphs of Lemaître’s analysis are displayed in figure 2(a). Assuming the relativistic spacetime element

\[ ds^2 = -R^2 d\sigma^2 + dt^2 \]  \hspace{0.5cm} (1)

where \( R \) represents a cosmic radius that is variable in time, he proceeds directly to the differential equations

\[ \frac{3}{R^2} \frac{R'}{R^2} + \frac{3}{R^2} = \lambda + \kappa \rho \]  \hspace{0.5cm} (2)

\[ 2 \frac{R''}{R} + \frac{R''}{R^2} + \frac{1}{R^2} = \lambda - \kappa \rho \]  \hspace{0.5cm} (3)

Here \( R' \) and \( R'' \) denote the first and second derivatives of cosmic radius with respect to time, \( \kappa \) is the Einstein constant, \( \rho \) denotes the density of matter and \( \lambda \) is the cosmological constant.

In the next line, Lemaître asserts that four identities representing the conservation of momentum and energy reduce to the relation

\[ \frac{d\rho}{dt} + \frac{3R'}{R} (\rho + p) = 0 \]  \hspace{0.5cm} (4)

where \( p \) represents the pressure of radiation and, assuming a closed volume of space \( V \), proceeds to his classic conservation equation

\[ d(V\rho) + pdV = 0 \]  \hspace{0.5cm} (5)

Equations (2) to (5) give the key results of Lemaître’s cosmology, but the derivations are left entirely to the reader; we contend that this would have presented a formidable challenge for most physicists unfamiliar with time-varying cosmologies at the time.\(^{14}\)

A more prosaic challenge is seen on the next page of Lemaître’s analysis, displayed in figure 2(b). Given that \( R_0 \) denotes the initial value of cosmic radius and \( R_E \) denotes the radius of the static Einstein model, the equation

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\(^{13}\) This is an exact translation of Lemaître’s 1927 paper, published in General Relativity and Gravitation as one of their ‘Golden Oldie’ series of key papers in cosmology and relativity. It should not be confused with Lemaître’s abridged paper of 1931.

\(^{14}\) For comparison, Alexander Friedman (Friedman 1922) presents five pages of analysis in deriving similar equations to (2) and (3).
It seems puzzling on first perusal. It is only on repeated reading that it becomes clear that \( R \) denotes the \textit{present} radius of the universe in Lemaître’s equation (18), while it denotes a variable throughout the rest of the paper.\(^{15}\) One imagines this terminology may have further puzzled readers unfamiliar with time-varying cosmologies.

\( R^3 = R_0^2 R \)  \hspace{1cm} (18)

\((ii)\) \textit{The observational data of Lemaître’s paper}\n
There is little question that the kernel of Lemaître’s paper of 1927 was his brilliant exposition of a connection between an expansion of space derived from the field equations of relativity and the redshifts of the spiral nebulae. Indeed, it is generally accepted that it is this aspect of his paper that distinguishes his contribution from that of Friedman (Kragh and Smith 2003; Duerbeck and Seitter 2000; Nussbaumer and Bieri 2009 p113). However, while it is often assumed\(^{16}\) that the astronomical data used by Lemaître to support his theory were similar to those published by Hubble in 1929, this assumption is not quite correct.

The first part of the observational section of Lemaître’s paper is displayed in \textbf{figure 3(a)}. Having derived an approximate relation between the fractional expansion of the cosmic radius \( R'/R \) and the velocities \( v \) and distances \( r \) of the spiral nebulae according to

\[
\frac{R'}{R} = \frac{v}{cr}
\]  \hspace{1cm} (23)

Lemaître sets about obtaining an estimate for the rate of cosmic expansion using observational data for the redshifts and distances of the nebulae published by Gustav Strömberg and Edwin Hubble respectively (Strömberg 1925; Hubble 1926).\(^{17}\) He notes first that the radial distances of the nebulae have been established using the method of apparent magnitude: \textit{“The apparent magnitude \( m \) of these nebulae can be found in the work of Hubble. It is possible to deduce their distance from it, because Hubble has shown that extragalactic nebulae have approximately equal absolute magnitudes (magnitude = \(-15.2\) at \(10\) parsecs, with individual variations \(\pm 2\))”}.

\(^{15}\) In Lemaître’s model, the cosmic radius remains constant at an initial size \( R = R_0 \) for an indefinite length of time before increasing (Luminet 2013).

\(^{16}\) See for example (Bergh 2011a; Way and Nussbaumer 2011).

\(^{17}\) This is the first estimate of what is now known as the Hubble constant. Almost all of the redshift data in Strömberg’s paper was provided by Slipher.
the distance $r$ expressed in parsecs is then given by the formula $\log r = 0.2m + 4.04$”. Lemaître then calculates an approximate mean value for nebular distance, noting the error associated with the method: “One finds a mean distance of about $10^6$ parsecs, varying from a few tenths to 3.3 megaparsecs. The probable error resulting from the dispersion of absolute magnitudes is considerable. For a difference in absolute magnitude of $\pm 2$, the distance exceeds from 0.4 to 2.5 times the calculated distance. Moreover, the error is proportional to the distance”.

The crucial step can be seen in figure 3(b); taking redshifts and distances for 42 spiral nebulae from Strömberg and Hubble respectively, Lemaître extracts his estimate for cosmic expansion by dividing the mean velocity of the nebulae by the mean distance: “Using the 42 nebulae appearing in the lists of Hubble and Strömberg...and taking account of the proper velocity of the Sun...one finds a mean distance of 0.95 megaparsecs and a radial velocity of 600 Km/sec, i.e., 625 Km/sec at $10^6$ parsecs”. Inserting the last two figures into his equation (23) above he obtains

$$\frac{R'}{R} = \frac{v}{cr} = \frac{625 \times 10^5}{10^6 \times 3.08 \times 10^{18} \times 3 \times 10^{10}} = 0.68 \times 10^{-27} \text{cm}^{-1}$$ (24)

and proceeds to use his expansion parameter to calculate a value for $R_0$, the initial radius of his model.

Two aspects of this analysis are worth emphasizing. In the first instance, the nebular distances are taken from Hubble’s publication of 1926 and thus almost all of them were estimated using the method of apparent magnitude. In this paper, Hubble himself is mindful of the many uncertainties associated with the method. Indeed, he notes early in the work that “definite evidence as to distances and dimensions are restricted to six systems, including the Magellanic Clouds. The similar nature of the countless fainter nebulae has been inferred from the general principle of the uniformity of nature” (Hubble 1926). Throughout the paper, Hubble describes the method of measuring nebular distance by apparent magnitude as a ‘working hypothesis’ and stresses that “reliable values of distances, and hence of absolute magnitudes, are restricted to a very few of the brightest nebulae.....the number of known distance is too small to serve as a basis for estimates in the range in absolute magnitude among nebulae in general” (Hubble 1926).18 In the second instance, Lemaître is not offering an explanation for a linear relation between redshift and distance that is already established;

18 We note that most of the distances used in Hubble’s paper are taken from the catalogue of Holeschek-Hopmann (Hubble 1926; Hopmann 1921).
instead he predicts the existence of such a relation from theory.\textsuperscript{19} This is seen most clearly in an important footnote to the section, where Lemaître notes that recent attempts by Knut Lundmark and Gustave Strömberg to establish a relation between $v$ and $r$ indicate only a very weak correlation at best due to the uncertainties in nebular distances. He suggests that a systematic error may be avoided by considering the ratio $v/r$:

Some authors sought to highlight the relation between $v$ and $r$ and obtained only a very weak correlation between these two terms. The error in the determination of the individual distances is of the same order of magnitude as the interval covered by the observations and the proper velocity of nebulae (in any direction) is large (300 Km/sec according to Strömberg), it thus seems that these negative results are neither for nor against the relativistic interpretation of the Doppler effect. The inaccuracy of the observations makes only possible to assume $v$ proportional to $r$ and to try to avoid a systematic error in the determination of the ratio $v/r$. Cf. Lundmark… and Strömberg, l.c.

Unfortunately, this footnote was omitted from the 1931 translation of the paper, as discussed below.

In 1929, Hubble published the results of his own investigation of a redshift/distance relation for the nebulae, using the redshifts of Slipher and newly-derived estimates of nebular distances based on observations of individual stars within the nebulae (Hubble 1929). The results are reproduced in figure 4(a). As stated by Hubble: “the results establish an approximately linear relation between the velocities and distances among nebulae for which velocities have been previously published”.\textsuperscript{20} We note that the distances of the closest seven nebulae were estimated by observing Cepheid stars within the nebulae and employing Henrietta Leavitt’s period-luminosity relation to estimate their distance; the next thirteen distances were estimated by observing the most luminous stars in nebulae and assuming an upper limit of absolute magnitude $M = -6.3$; the remaining four objects had distances assigned on the basis of the mean luminosities of the nebulae in a cluster. Finally, the single cross represents a mean velocity/distance ratio for 22 nebulae whose distances could not be estimated individually (Hubble 1929). Even here, many commentators have noted that the quality and quantity of the data shown on Hubble’s graph only marginally support his conclusion of a linear relation between redshift and distance for the nebulae (Kragh 1996 p18; Longair 2006 p110; Nussbaumer and Bieri 2009 pp 115-116; Ostriker and Mitton 2013 p73; Peacock 2013). As pointed out by all these authors, it is likely that Hubble’s conclusion was influenced by an

\textsuperscript{19} See (Peebles 2015) for more on this point.

\textsuperscript{20} The filled circles and full line represent data corrected for solar motion using the nebulae individually; the open circles and dashed line are data corrected for solar motion by combining nebulae into groups.
important data point cited in the paper but not shown in the graph; namely Milton Humason’s measurement of an apparent velocity of 3779 km/s for a nebula at an estimated distance of 7.8 megaparsecs (Hubble 1929; Humason 1929). In any event, the graph marked an important turning point as it was soon accepted by many theorists as the first evidence of an expansion of space (Smith 1979; Nussbaumer and Bieri 2006 pp 118-119; Kragh and Smith 2003). This conclusion was strengthened with the publication of a paper that extended the results to much larger distances and redshifts (Hubble and Humason 1931) and the result later became known as Hubble’s law (Kragh and Smith 2003).

For comparison, a graph of the redshift/distance data used by Lemaître in 1927 is shown in figure 4(b); this graph was not shown in Lemaître’s paper but has been reconstructed by Hilmar Duerbeck and Waltraut Seitter from the observational data listed in the article (Duerbeck and Seitter 2000). As the authors point out, a roughly linear trend of redshift versus distance can also be discerned in this graph, although the scatter in the data is much larger. However, we find this comparison is a little misleading; a key difference between the two graphs is that most of the nebular distances of figure 4(b) were estimated using the method of apparent magnitude.

Indeed, it is interesting to compare the data of figure 4(b) with the graphs shown in figure 5(a) and (b). These graphs represent the earlier attempts at establishing a redshift/distance relation for the nebulae by Lundmark and Strömberg, using redshifts from Slipher and nebular distances obtained using the method of apparent magnitude (Lundmark 1924; Strömberg 1925). The graph of figure 4(b) can be viewed as a continuation of these investigations; indeed, we note that Duerbeck and Seitter calculate a correlation coefficient of 0.37, 0.23, 0.30 and 0.84 for the redshift/distance data of Lundmark, Strömberg, Lemaître and Hubble respectively (Duerbeck and Seitter 2000). Thus, we do not quite agree with statements such as “In 1929 Hubble repeats Lemaître’s work with essentially the same data and obtains similar results” (van der Bergh 2011a) or “Two years later Hubble found the same velocity-distance relationship on observational grounds from practically the same observations that Lemaître had used” (Way and Nussbaumer 2011). In our view, the observational data used by Lemaître in his paper of 1927 were of a preliminary nature because the nebular distances were established using a method that was prone to large errors; it is thus very possible that readers

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21 See also (Nussbaumer and Bieri 2009 p 108-109).
22 Note that Hubble’s graph is confined to more modest, if more accurate distances.
23 See (Smith 1979; Shaviv 2011) for a description of early attempts to establish a redshift/distance relation for the nebulae.
of the paper were not yet convinced of the reality of a linear redshift/distance relation for the nebulae.

Supporting evidence
Some supporting evidence for our thesis can be found by considering the republication of Lemaître’s 1927 paper in the *Monthly Notices of the Royal Astronomical Society* in 1931 (Lemaître 1931a). It has long been noted that the data section of Lemaître’s 1927 paper is missing in the 1931 translation (Peebles 1984, 1993 p82; Kragh 1996 p406; Nussbaumer and Bieri 2009 pp 126-127). Indeed, the two excerpts shown in figure 3 above, and the accompanying footnote, are replaced by a single sentence: “*From a discussion of available data, we adopt R'/R = 0.68x10^{-27} cm^{-1}*” (Lemaître 1931a). While this omission was until recently considered a great puzzle (Peebles 1993 p82; Nussbaumer and Bieri 2009 p126; Duerbeck and Seitter 2000; van de Bergh 2011b), it is now known that the translation and editing of the paper was carried out by Lemaître himself (Livio 2013). An excerpt from a letter from Lemaître (Lemaître 1931b) to William Marshall Smart, editor of the *Monthly Notices*, clarifies the reason for the edit:

> I highly appreciate the honour for me and for our society to have my 1927 paper reprinted by the Royal Astronomical Society. I send you a translation of the paper. I did not find advisable to reprint the provisional discussion of radial velocities which is clearly of no actual interest, and also the geometrical note, which could be replaced by a small bibliography of ancient and new papers on the subject.

We note that Lemaître almost certainly uses the word ‘actual’ in the sense of the French adjective ‘actuelle’ (contemporary), just as he uses the word ‘ancient’ in the next sentence for ‘old’. Thus, his letter supports our thesis that the astronomical data used in his paper of 1927 were of a preliminary nature. As more reliable estimates of the distances of the nebulae were available in the 1930s, why reproduce the preliminary data of his 1927 paper?

Many years later, Lemaître showed a similar attitude towards the data used in his 1927 paper. In a letter to *Les Annales d’Astrophysique* in 1950, he commented: “*Naturellement, avant la découverte et l’étude des amas de nébeleuses, il ne pouvait être question d’établir la loi de Hubble*” or “*Naturally, before the discovery and study of the clusters of nebulae, there was no question of establishing Hubble’s law*” (Lemaître 1950). Similarly, in a review article written in 1952, he wrote: “*Hubble and Humason established from observation the linear*
relation between velocity and distance which was expected for theoretical reasons and which is known as the Hubble velocity-distance relation” (Lemaître 1952).

5. Conclusions

In conclusion, we recall that Lemaître’s brilliant paper of 1927 was situated in the vanguard of both theory and observation. Yet it was ignored by the community for three long years. This oversight may have resulted from sociological factors such as Lemaître’s status as an early-career researcher, the journal he used and the language in which his paper was published. In addition, conceptual difficulties in making the transition from static to expanding cosmologies may have played a role. However, it is our view that two ‘internal’ aspects of Lemaître’s paper should not be overlooked; the technical challenge presented by his dense analysis to a contemporaneous reader and the preliminary nature of the observational data he used to support his model.

As regards the specific case of Eddington, one final factor presents itself. While it was noted above that Lemaître served as a Research Associate under Eddington in 1923, it is possible that this sojourn may have been a negative, rather than positive, factor. As pointed out by K.C. Wali in his biography of Subrahmanyan Chandrasekhar, Eddington did not always view his students and assistants as equals, or afford their theories the same regard as those of his colleagues (Wali 1991 pp 141-144). Indeed, the astronomical community in England was permeated at this time by a rather strict hierarchical system (Wali 1991 p115, 144). Thus, we must allow the possibility that Eddington did not glance at his former student’s paper when he first received it, despite the exciting title. We shall probably never know for sure; what is known is that by the time Eddington and his colleagues embraced Lemaître’s theory in 1930, the brilliant young cleric had already embarked on his second great advance, the hypothesis of a universe with a hot, dense origin (Lemaître 1931c; Lambert 2015 pp 145-154; Kragh 2018).

Coda

We note that the International Astronomical Union have recently voted to rename Hubble’s law as the Hubble-Lemaître law in order to recognise the seminal cosmological contributions of Lemaître.24 We find this change somewhat anachronistic as Hubble’s law was traditionally understood as an observationally-determined relation between the redshifts and distances of

24 See https://www.iau.org/news/pressreleases/detail/iau1812/
the spiral nebulae, a phenomenon that is quite distinct from Lemaître’s derivation of a general expansion of space from relativity. A separate article on this topic is currently underway.

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**Figure 1(a).** First page of a letter from Lemaître to Eddington, written in February 1930. © Archives Lemaître, Université Catholique de Louvain.

**Figure 1(b).** Second page of Lemaître’s 1930 letter to Eddington. © Archives Lemaître, Université Catholique de Louvain.
We identify $\rho$ and $-\rho$ with the components $T^4_4$ and $T^1_1 = T^2_2 = T^3_3$ of the material energy tensor, and $\delta$ with $T$. Working out the contracted Riemann tensor for a universe with a line-element given by
\[
d s^2 = -R^2d\sigma^2 + dt^2
\] (1)
where $d\sigma$ is the elementary distance in a space of radius unity, and the radius of space $R$ is a function of time, we find that the field equations can be written
\[
3\frac{\dot{R}^2}{R^2} + \frac{3}{R^2} = \lambda + \kappa \rho
\] (2)
and
\[
2\frac{R''}{R} + \frac{R'^2}{R^2} + \frac{1}{R^2} = \lambda - \kappa \rho
\] (3)

Accents denote derivatives with respect to $t$; $\lambda$ is the cosmological constant whose value is unknown, and $\kappa$ is the Einstein constant whose value is $1.87 \times 10^{-27}$ in C.G.S. units ($8\pi$ in natural units).

The four identities expressing the conservation of momentum and of energy reduce to
\[
\frac{d\rho}{dt} + \frac{3R'}{R}(\rho + p) = 0
\] (4)
which is the conservation of energy equation. This equation can replace (3). It is suitable for an interesting interpretation. Introducing the volume of space $V = \pi^2 R^3$, it can be written
\[
d(V\rho) + pdV = 0
\] (5)

**Figure 2(a).** Excerpt from the analysis section of Lemaître’s 1927 paper, translated by J.P. Luminet (Luminet 2013).
and put in (11) \( \beta = 0 \) and \( \alpha = 2R_0 \), it follows that

\[
t = R_0 \sqrt[3]{3} \int \frac{dR}{R - R_0} \sqrt{\frac{R}{R + 2R_0}}
\]

For this solution the two equations (13) are of course no longer valid. Writing

\[
r\delta = \frac{2}{R_E^2}
\]

we have from (14) and (15)

\[
R^3 = R_E^2 R_0
\]

The value of \( R_E \), the radius of the universe computed from the average density by Einstein’s equations (17), has been found by Hubble to be

\[
R_E = 8.5 \times 10^{28} \text{ cm.} = 2.7 \times 10^{19} \text{ parsecs}
\]

We shall see later that the value of \( R_0 \) can be computed from the radial velocities of the nebula; \( \dot{R} \) can then be found from (18). Finally, we shall show that a solution introducing a relation substantially different from (14) would lead to consequences not easily acceptable.

**Figure 2(b).** 2nd excerpt from the analysis section of Lemaître’s 1927 paper, translated by J.P. Luminet (Luminet 2013).
neighbourhood of the observer, because the period of the light emitted under the same physical conditions has the same value everywhere when reckoned in proper time. Therefore

\[ \frac{v}{c} = \frac{\delta t_2}{\delta t_1} - 1 = \frac{R_2}{R_1} - 1 \]  

(22)

measures the apparent Doppler effect due to the variation of the radius of the universe. It equals the ratio of the radii of the universe at the instants of observation and emission, diminished by unity. \( v \) is that velocity of the observer which would produce the same effect. When the source is near enough, we can write approximately

\[ \frac{v}{c} = \frac{R_2 - R_1}{R_1} = \frac{dR}{R} = \frac{R'}{R} \frac{dt}{R} = \frac{R'}{R} r \]

where \( r \) is the distance of the source. We have therefore

\[ \frac{R'}{R} = \frac{v}{cr} \]

(23)

Radial velocities of 43 extragalactic nebulae are given by Strömgren (6).

The apparent magnitude \( m \) of these nebulae can be found in the work of Hubble. It is possible to deduce their distance from it, because Hubble has shown that extragalactic nebulae have approximately equal absolute magnitudes (magnitude = \(-15.2\) at 10 parsecs, with individual variations \( \pm 2 \)), the distance \( r \) expressed in parsecs is then given by the formula \( \log r = 0,2m + 4,04 \).

One finds a distance of about \( 10^6 \) parsecs, varying from a few tenths to 3,3 megaparsecs. The probable error resulting from the dispersion of absolute magnitudes is considerable. For a difference in absolute magnitude of \( \pm 2 \), the distance exceeds from 0,4 to 2,5 times the calculated distance. Moreover, the error is proportional to the distance. One can admit that, for a distance of one megaparsec, the error resulting from the dispersion of magnitudes is of the same order as that resulting from the dispersion of velocities. Indeed, a difference of magnitude of value unity corresponds to a proper velocity of 300 Km/s, equal to the proper velocity of the sun compared to nebulae. One can hope to avoid a systematic error by giving to the observations a weight proportional to \( \frac{1}{\sqrt{1 + r^2}} \), where \( r \) is the distance in megaparsecs.

**Figure 3(a).** Excerpt from the observational section of Lemaître’s 1927 paper, translated by J.P. Luminet (Luminet 2013).
Using the 42 nebulae appearing in the lists of Hubble and Strömgberg (\(^7\)), and taking account of the proper velocity of the Sun (300 Km/s in the direction \(\alpha = 315^\circ, \delta = 62^\circ\)), one finds a mean distance of 0.95 megaparsecs and a radial velocity of 600 Km/sec, i.e. 625 Km/sec at \(10^6\) parsecs \(^8\).

We will thus adopt

\[
\frac{R'}{R} = \frac{v}{r c} = \frac{625 \times 10^5}{10^6 \times 3,08 \times 10^{18} \times 3 \times 10^{10}} = 0.68 \times 10^{-27} \text{cm}^{-1} \tag{24}
\]

This relation enables us to calculate \(R_0\). We have indeed by (16)

\[
\frac{R'}{R} = \frac{1}{R_0 \sqrt{3}} \sqrt{1 - 3y^2 + 2y^3} \tag{25}
\]

where we have set

\[
y = \frac{R_0}{R} \tag{26}
\]

On the other hand, from (18) and (26)

\[
R_0^2 = R_E^2 y^3 \tag{27}
\]

and therefore

\[
3 \left(\frac{R'}{R}\right)^2 R_E^2 = \frac{1 - 3y^2 + 2y^3}{y^3} \tag{28}
\]

With the adopted numerical data (24) for \(\frac{R'}{R}\) and (19) for \(R_E\), we have

\[
y = 0.0465.
\]

Figure 3(b). 2nd excerpt from the observational section of Lemaître’s 1927 paper, translated by J.P. Luminet (Luminet 2013).
Figure 4(a). Hubble’s graph of 1929, reproduced from (Hubble 1929).

Figure 4(b). Reconstruction of Lemaître’s data of 1927, reproduced from (Duerbeck and Seitter 2000).
Figure 5(a). Redshift vs distance graph for the nebulae, reproduced from (Lundmark 1924).

Figure 5(b). Redshift vs distance graph for the nebulae, reproduced from (Strömberg 1925).
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