Deviation from tri-bimaximal mixing and flavor symmetry breaking in a seesaw type $A_4$ model

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Abstract

We have studied the contribution of higher order corrections of the flavor symmetry breaking in the $A_4$ seesaw model with the supersymmetry. Taking account of possible higher dimensional mass operators, we predict the deviation from the tri-bimaximal lepton mixing for both normal hierarchy and inverted hierarchy of neutrino masses. We have found that the value of $\sin^2 \theta_{23}$ is larger than 0.96 and the upper bound of $\sin^2 \theta_{13}$ is 0.01. We have also examined the flavor changing neutral current of leptons from the soft SUSY breaking in slepton masses and $A$-terms within the framework of supergravity theory. Those magnitudes are enough suppressed to be consistent with experimental constraints.

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1 Introduction

Lepton flavor mixing provides us an important clues to understand the origin of the generation. Recent neutrino oscillation experimental data [1, 2] indicate the tri-bimaximal mixing for three lepton flavors [3]. Indeed, various types of models leading to the tri-bimaximal mixing have been proposed, e.g. by assuming several types of non-Abelian flavor symmetries. In particular, natural models realizing the tri-bimaximal mixing have been proposed based on the non-Abelian finite group $A_4$ [4]-[28]. Since neutrino experiments go into the new phase of precise determination of mixing angles and mass squared differences, it is important to study the $A_4$ flavor model in detail.

The $A_4$ flavor model considered by Alterelli et al [9, 10], which realizes the tri-bimaximal flavor mixing, can predict the deviation from the tri-bimaximal mixing. Actually, one of authors has investigated the deviation from the tri-bimaximal mixing including higher dimensional operators in the effective model without right-handed Majorana neutrinos [23]. In that paper, the effect on the alignment of vacuum from higher dimensional operators was taken account numerically.

In present paper, we discuss the $A_4$ flavor model with the supersymmetry including the right-handed neutrinos. We take into account higher dimensional operators of neutrino masses in the seesaw model, and then predict the deviation from the tri-bimaximal mixing. It is found that this deviation is dominated by the vacuum expectation value of $\phi_{T_1}$, which is the first component of an $A_4$ triplet scalar. Since the vacuum alignment is an important ingredient to reproduce the tri-bimaximal mixing of neutrinos, the effect of the shift of the vacuum alignment due to higher dimensional operators is also discussed. This effect is found to be negligibly small.

On the other hand, although squarks and sleptons have not been detected yet, their mass matrices are strongly constrained by experiments of flavor changing neutral current (FCNC) processes. Non-Abelian flavor symmetries and certain types of their breaking patterns are useful to suppress FCNCs. (See e.g. [29, 30, 31, 32].) In addition to flavor symmetries, their breaking patterns are important to derive lepton mass matrices and to predict slepton mass matrices. Therefore, we study which pattern of slepton mass matrices is predicted from the seesaw type $A_4$ flavor model including higher dimensional operators and to examine whether the predicted pattern of slepton mass matrices is consistent with the current FCNC experimental bounds [1].

In Section 2, we present the lepton superpotential including higher dimensional operators in the $A_4$ model [10]. We discuss the charged lepton mass matrix and the neutrino mass matrix in section 3. In section 4, the lepton mixing matrix is studied to find the deviation from the tri-bimaximal mixing matrix numerically. In Section 5, we discuss soft supersymmetry (SUSY) breaking terms of sleptons, i.e. soft scalar mass matrices and A-terms. Section 6 is devoted to the summary.

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\[1\] We have studied soft supersymmetry (SUSY) breaking terms of sleptons in the $A_4$ flavor model without right-handed Majorana neutrinos [32].
2 Lepton superpotential

We begin by discussing the supersymmetric seesaw type $A_4$ flavor model proposed by Alterelli et al [9 11]. In the non-Abelian finite group $A_4$, there are twelve group elements and four irreducible representations: 1, 1', 1'' and 3. The $A_4$ and $Z_3$ charge assignments of leptons and scalars are listed in Table 1. Under the $A_4$ symmetry, the chiral superfields for three families of the left-handed lepton doublet $l = (l_e, l_\mu, l_\tau)$ and right handed neutrino $\nu^c = (\nu^c_e, \nu^c_\mu, \nu^c_\tau)$ are assumed to transform as 3, while the right-handed ones of the charged lepton singlets $e^c, \mu^c$ and $\tau^c$ are assigned with 1, 1'', 1', respectively. The third row of Table 1 shows how each chiral multiplet transforms under $Z_3$, where $\omega = e^{2\pi i/3}$. The flavor symmetry is spontaneously broken by vacuum expectation values (VEV) of two 3’s, $\phi_T$, $\phi_S$, and by one singlet, $\xi$, which are $SU(2)_L \times U(1)_Y$ singlets. Their $Z_3$ charges are also shown in Table 1. Hereafter, we follow the convention that the chiral superfield and its lowest component are denoted by the same letter.

| $A_4$ | $(l_e, l_\mu, l_\tau)$ | $(\nu^c_e, \nu^c_\mu, \nu^c_\tau)$ | $e^c$ | $\mu^c$ | $\tau^c$ | $h_u$ | $h_d$ | $\xi$ | $\xi$ | $(\phi_{T_1, T_2, T_3})$ | $(\phi_{S_1, S_2, S_3})$ | $\Phi$ |
|-------|-----------------|-----------------|-------|-------|-------|-------|-------|-------|-------|-----------------|-----------------|-------|
| $A_4$ | 3               | 3               | 1     | 1''   | 1'    | 1     | 1     | 1     | 1     | 3               | 3               | 1     |
| $Z_3$ | $\omega$        | $\omega^2$     | $\omega^2$ | $\omega^2$ | $\omega^2$ | 1     | 1     | $\omega^2$ | 1     | $\omega^2$     | 1               |       |
| $U(1)_{FN}$ | 0 | 0 | 2 | $q$ | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ | | |

Table 1: $A_4$, $Z_3$ and $U(1)_{FN}$ charges

Allowed terms in the superpotential including charged leptons are written by

$$w_l = y_{0}^{e} e^{c} l^{c} \phi_{T} h_{d} \frac{\Phi \Phi}{\Lambda} + y_{0}^{\mu} \mu^{c} l^{c} \phi_{T} h_{d} \frac{\Phi \Phi}{\Lambda} + y_{0}^{\tau} \tau^{c} l^{c} \phi_{T} h_{d} \frac{\Phi \Phi}{\Lambda}$$

$$+y_{l}^{e} e^{c} l^{c} \phi_{T} h_{d} \frac{\Phi \Phi}{\Lambda} + y_{l}^{\mu} \mu^{c} l^{c} \phi_{T} h_{d} \frac{\Phi \Phi}{\Lambda}$$

$$+y_{l}^{\tau} \tau^{c} l^{c} \phi_{T} h_{d} \frac{\Phi \Phi}{\Lambda}$$

(1)

In our notation, all $y$ with some subscript denote Yukawa couplings of order 1 and $\Lambda$ denotes cut off scale of the $A_4$ symmetry. In order to obtain the natural hierarchy among lepton masses $m_e$, $m_\mu$ and $m_\tau$, the Froggatt-Nielsen mechanism [33] is introduced as an additional $U(1)_{FN}$ flavor symmetry under which only the right-handed lepton sector is charged. $\Lambda'$ is a cut off scale of the $U(1)_{FN}$ symmetry and $\Phi$ denotes the Froggatt-Nielsen flavon in Table 1. The $U(1)_{FN}$ charge values are taken as $2q, q$ and 0 for $e^c$, $\mu^c$ and $\tau^c$, respectively. By assuming that a flavon, carrying a negative unit charge of $U(1)_{FN}$, acquires a VEV $\langle \Phi \rangle / \Lambda' \equiv \lambda \ll 1$, the following mass ratio is realized through the Froggatt-Nielsen charges,

$$m_e : m_\mu : m_\tau = \lambda^{2q} : \lambda^{q} : 1.$$  

(2)

If we take $q = 2$, $\lambda \sim 0.2$ is required to be consistent with the observed charged lepton mass hierarchy. The $U(1)_{FN}$ charges are listed in the fourth row of Table 1.
The superpotential associated with the Dirac neutrino mass is given as
\[
 w_D = y_D^0 \nu^c l u + y_D^1 \nu^c l u \phi_T \frac{1}{\Lambda},
\]
and for the right-handed Majorana sector, the superpotential is given as
\[
 w_N = y_N^0 \nu^c \nu^c \phi_S + y_N^1 \nu^c \nu^c \xi + y_N^2 \nu^c \nu^c \phi_T \xi \frac{1}{\Lambda} + y_N^3 \nu^c \nu^c \phi_S \phi_T \frac{1}{\Lambda},
\]
where there appear $3 \times 3 \times 3$ and $3 \times 3 \times 3 \times 3$ products of $A_4$ triplets.

Vacuum alignments of $A_4$ triplet $\phi_T$ and $\phi_S$ are required to reproduce the tri-bimaximal mixing. These vacuum alignments are realized in the scalar potential of the leading order [10]. However, higher order operators shift these vacuum alignments, therefore we write vacuum expectation values (VEVs) as follows:
\[
\langle h_u \rangle = v_u, \quad \langle h_d \rangle = v_d, \quad \langle \xi \rangle = u, \\
\langle (\phi_{T1}, \phi_{T2}, \phi_{T3}) \rangle = v_T (1, \epsilon_1, \epsilon_2), \quad \langle (\phi_{S1}, \phi_{S2}, \phi_{S3}) \rangle = v_S (1, 1 + \delta_1, 1 + \delta_2),
\]
where $\delta_i \ll 1$ and $\epsilon_i \ll 1$. The parameters $\epsilon_i$ and $\delta_i$ are given in the model of [10] as
\[
\epsilon_1 = \epsilon_2 = C_0 \frac{u^3}{v_T^2} \frac{1}{\Lambda}, \quad \delta_1 = C_1 \frac{u^3}{v_T^2} \frac{1}{\Lambda}, \quad \delta_2 = C_2 \frac{u^3}{v_T^2} \frac{1}{\Lambda},
\]
where $C_i$s are coefficients of order one. We will estimate magnitudes of $\epsilon_i$ and $\delta_i$ in following numerical calculations.

3 Lepton mass matrices in $A_4$ flavor model

Inserting VEVs in the superpotential of the charged lepton sector in Eq.(1), we obtain the charged lepton mass matrix $M_E$ as
\[
 M_E = \alpha_T v_d \left( \begin{array}{ccc}
 y_0^e \lambda^q + \frac{2}{3} y_1^e \lambda^q \alpha_T & y_0^e \lambda^q \epsilon_1 & y_0^e \lambda^q \epsilon_2 \\
 y_0^\mu \lambda^q \epsilon_1 & y_0^\mu \lambda^q + \frac{2}{3} y_1^\mu \lambda^q \alpha_T & y_0^\mu \lambda^q \epsilon_2 \\
 y_0^\tau \lambda^q \epsilon_1 & y_0^\tau + \frac{2}{3} y_1^\tau \alpha_T & y_0^\tau \lambda^q \epsilon_2 \\
 \end{array} \right) + O(\alpha_T^2 \epsilon_i v_d),
\]
with
\[
\alpha_T = \frac{v_T}{\Lambda}.
\]
In this mass matrix, the off diagonal elements appear in order of $\epsilon_i$. Since we have
\[
 m^2_e = y_0^e \lambda^q \alpha_T^2 (1 - \epsilon_1^2 - \epsilon_1 \epsilon_2 - \epsilon_2^2)v_d^2, \\
 m^2_\mu = y_0^\mu \lambda^q \alpha_T^2 (1 - 2 \epsilon_1 \epsilon_2)v_d^2, \\
 m^2_\tau = y_0^\tau \alpha_T^2 (1 + \epsilon_1^2 + \epsilon_2^2)v_d^2,
\]

3
we can determine \( \alpha_T \) from the tau lepton mass by fixing \( y_0^\tau \):

\[
\alpha_T = \sqrt{\frac{m^2}{y_0^{\tau 2} v_d^2 (1 + \epsilon_1^2 + \epsilon_2^2)}}.
\]  

(10)

Since off diagonal elements of the charged lepton mass matrix are of order \( \epsilon_i \), the mixing is expected to be small. The mixing matrix is given as

\[
V_E = \begin{pmatrix}
1 & \theta_{12}^e & \epsilon_2 \\
-\theta_{12}^e & 1 & \epsilon_1 \\
-\epsilon_2 & -\epsilon_1 & 1
\end{pmatrix},
\]

(11)

the mixing angle \( \theta_{12}^e \) depends on the relative magnitude of \( \lambda^{2q} \) and \( \epsilon_i \) as

\[
\theta_{12}^e = \frac{y_0^{\mu 2} \lambda^{2q} + \frac{1}{3} y_0^{\tau 2} \epsilon_2}{y_0^{\mu 2} \lambda^{2q} + y_0^{\tau 2} (\epsilon_1^2 - \epsilon_2^2) - \frac{1}{3} y^2 \alpha_1^2} \epsilon_1.
\]

(12)

Now, we present the Dirac neutrino mass matrix as follows:

\[
M_D = v_u \begin{pmatrix}
y_0^D + \frac{2}{3} y_1^D \alpha_T & 0 & 0 \\
y_0^D - \frac{1}{3} y_1^D \alpha_T - \frac{1}{2} y_2^D \alpha_T & 0 & y_0^D - \frac{1}{3} y_1^D \alpha_T + \frac{1}{2} y_2^D \alpha_T
\end{pmatrix},
\]

(13)

where \( O(\alpha_T^2) \) terms are neglected. It is remarked that higher order terms come from \( \langle \phi \rangle \), which dominates leading terms of the charged lepton mass matrix in Eq. (11). In the same approximation, the right-handed Majorana mass matrix is

\[
M_N = 2 \Lambda \left( \begin{array}{ccccc}
\frac{2}{3} y_0^N \alpha_S + y_1^N \alpha_V & -\frac{1}{3} y_0^N \alpha_S (1 + \delta_1) & -\frac{1}{3} y_0^N \alpha_S (1 + \delta_2) \\
-\frac{1}{3} y_0^N \alpha_S (1 + \delta_1) & \frac{2}{3} y_0^N \alpha_S (1 + \delta_2) & -\frac{1}{3} y_0^N \alpha_S + y_1^N \alpha_V \\
-\frac{1}{3} y_0^N \alpha_S + y_1^N \alpha_V & -\frac{1}{3} y_0^N \alpha_S + y_1^N \alpha_V & -\frac{1}{3} y_0^N \alpha_S (1 + \delta_1)
\end{array} \right)
\]

\[
+2 \alpha_T \Lambda \left( \begin{array}{cccc}
y_3^N \alpha_S + \frac{4}{9} y_3^N \alpha_S + \frac{2}{9} y_2^N \alpha_V & y_3^N \alpha_S + \frac{4}{9} y_3^N \alpha_S - \frac{1}{9} y_2^N \alpha_S & y_3^N \alpha_S + \frac{4}{9} y_2^N \alpha_S - \frac{1}{9} y_3^N \alpha_S \\
y_3^N \alpha_S + \frac{4}{9} y_3^N \alpha_S - \frac{1}{9} y_2^N \alpha_S & y_3^N \alpha_S - \frac{2}{9} y_3^N \alpha_S + \frac{1}{9} y_2^N \alpha_S & y_3^N \alpha_S - \frac{2}{9} y_3^N \alpha_S + \frac{1}{9} y_2^N \alpha_S \\
y_3^N \alpha_S - \frac{2}{9} y_3^N \alpha_S - \frac{1}{9} y_2^N \alpha_S & y_3^N \alpha_S - \frac{2}{9} y_3^N \alpha_S + \frac{1}{9} y_2^N \alpha_S & y_3^N \alpha_S - \frac{2}{9} y_3^N \alpha_S + \frac{1}{9} y_2^N \alpha_S
\end{array} \right),
\]

(14)

where

\[
\alpha_S = \frac{v_S}{\Lambda}, \quad \alpha_V = \frac{u}{\Lambda}.
\]

(15)

By the seesaw mechanism \( M_D^T M_R^{-1} M_D \), we get the neutrino mass matrix \( M_\nu \), which is rather complicated. We only display leading matrix elements which correspond to the neutrino mass matrix in [10]:
which are suppressed in ~\(\Omega\) after rotating ~\(M_\nu\) as
\[
M_\nu = \frac{1}{3} \begin{pmatrix}
A + 2B & A - B \\
A - B & A + \frac{1}{2}B + \frac{3}{2}C & A + \frac{1}{2}B - \frac{3}{2}C \\
A - B & A + \frac{1}{2}B - \frac{3}{2}C & A + \frac{1}{2}B + \frac{3}{2}C
\end{pmatrix} + \ldots
\]
\[
= \frac{B + C}{2} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} + \frac{A - B}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix} + \frac{B - C}{2} \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix} + \ldots,
\]

where
\[
A = k_0(y_0^2 \alpha_S^2 - y_1^2 \alpha_V^2), \quad B = k_0(y_0^N y_1^N \alpha_S \alpha_V - y_1^{N^2} \alpha_V^2), \quad C = k_0(y_0^N y_1^N \alpha_S \alpha_V + y_1^{N^2} \alpha_V^2),
\]
\[
k_0 = \frac{y_0^D v_u^2}{(y_0^N y_1^N \alpha_S \alpha_V^2 - y_1^{N^2} \alpha_V^3)\Lambda}.
\]

At the leading order, neutrino masses are given as ~\(m_1 = B\), ~\(m_2 = A\), and ~\(m_3 = C\).

Our neutrino mass matrix is no more diagonalized by the tri-bimaximal mixing matrix ~\(U_{\text{tri}}\),
\[
U_{\text{tri}} = \begin{pmatrix}
2/\sqrt{6} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}.
\]

After rotating ~\(M_\nu\) as ~\(U_{\text{tri}}^T M_\nu U_{\text{tri}}\), diagonal components are
\[
(1,1) : \frac{y_0^D v_u^2}{2(y_0^N \alpha_S + y_1^N \alpha_V)\Lambda}(1 + \mathcal{O}(\alpha_S, \alpha_V, \epsilon_i, \delta_i)),
\]
\[
(2,2) : \frac{y_0^D v_u^2}{2y_1^N \alpha_V}(1 + \mathcal{O}(\alpha_S, \alpha_V, \epsilon_i, \delta_i))
\]
\[
(3,3) : \frac{y_0^D v_u^2}{2(y_0^N \alpha_S - y_1^N \alpha_V)\Lambda}(1 + \mathcal{O}(\alpha_S, \alpha_V, \epsilon_i, \delta_i)).
\]

Off diagonal elements are given as
\[
(1,2) : \frac{y_0^D (2\alpha_T \alpha_V (2y_1^D y_1^N - y_0^D y_2^N) + y_0^D y_0^N \alpha_S (\delta_1 + \delta_2) + (2y_1^D y_0^N - 2y_0^D y_3^N + y_0^D y_3^N)\alpha_S \alpha_T)}{6\sqrt{2}y_1^N \alpha_V (y_0^N \alpha_S + y_1^N \alpha_V)\Lambda} v_u^2,
\]
\[
(1,3) : \frac{\sqrt{3} y_0^D \alpha_S \alpha_T (-2y_0^D y_1^N - 3y_0^D y_3^N + 3y_0^D y_3^N)}{12(y_0^N \alpha_S^2 - y_1^N \alpha_V^2)\Lambda} v_u^2,
\]
\[
(2,3) : \frac{y_0^D y_0^N \alpha_S (y_0^D \delta_2 - y_0^D \delta_1 + y_1^D \alpha_T)}{2\sqrt{6}y_1^N \alpha_V (y_0^N \alpha_S - y_1^N \alpha_V)\Lambda} v_u^2,
\]

which are suppressed in ~\(\mathcal{O}(\alpha_T, \alpha_V, \delta_i)\) compared with diagonal elements. Therefore, mass eigenvalues are almost determined by Eq. (19). On the other hand, we can evaluate the deviation from
the tri-bimaximal mixing from the neutrino sector:

\[
\theta_{12}^\nu \approx \frac{2\alpha_T\alpha_V(2y_1^Dy_1^N - y_2^Dy_2^N) + y_0^Dy_0^N\alpha_S(\delta_1 + \delta_2) + (2y_1^Dy_1^N - 2y_0^Dy_3^N + y_0^Dy_3^N)\alpha_S\alpha_T}{3\sqrt{2y_0^Dy_0^N}\alpha_S},
\]

\[
\theta_{13}^\nu \approx \frac{-\alpha_S\alpha_T(2y_2^Dy_0^N + 3y_0^Dy_3^N - 3y_0^Dy_3^N)}{4\sqrt{3}y_1^N y_0^D\alpha_V},
\]

\[
\theta_{23}^\nu \approx \frac{y_0^N\alpha_S(y_0^D\delta_1 - y_0^D\delta_2 - y_0^D\alpha_T)}{\sqrt{6}y_0^D(y_0^N\alpha_S - 2y_1^N\alpha_V)}. \quad (21)
\]

Let us estimate magnitudes of \(\alpha_S\) and \(\alpha_V\). The squared mass differences are given by using Eq. (19) as,

\[
\Delta m^2_{\text{atm}} \approx \pm \frac{(y_0^Dv_u)^4}{\Lambda^2} \frac{y_0^N y_1^N \alpha_S\alpha_V}{[(y_0^N\alpha_S)^2 - (y_1^N\alpha_V)^2]^2}, \quad \Delta m^2_{\text{sol}} \approx \frac{(y_0^N v_u)^4}{4\Lambda^2} \frac{y_0^N\alpha_S(y_0^N\alpha_S + 2y_1^N\alpha_V)}{(y_1^N\alpha_V)^2(y_0^N\alpha_S + y_1^N\alpha_V)^2}, \quad (22)
\]

where the sign \(+(-)\) in \(\Delta m^2_{\text{atm}}\) corresponds to the normal (inverted) mass hierarchy. We can obtain \(\alpha_S\) and \(\alpha_V\) from these equations. In the case of the normal mass hierarchy, putting

\[
\alpha_S = k \alpha_V \quad (k > 0), \quad (23)
\]

we have

\[
\Delta m^2_{\text{atm}} \approx \frac{(y_0^Dv_u)^4}{\alpha^2\Lambda^2} \frac{y_0^N y_1^N k}{(y_0^N k + y_1^N)^2(y_0^N k - y_1^N)^2}, \quad \Delta m^2_{\text{sol}} \approx \frac{(y_0^Dv_u)^4}{4\alpha^2\Lambda^2} \frac{y_0^N k(y_0^N k + 2y_1^N)}{y_1^N(y_0^N k + y_1^N)^2}. \quad (24)
\]

The ratio of \(\Delta m^2_{\text{atm}}\) and \(\Delta m^2_{\text{sol}}\) is expressed in terms of \(k\) and Yukawa couplings as

\[
\frac{\Delta m^2_{\text{atm}}}{\Delta m^2_{\text{sol}}} \approx \frac{4(y_1^N)^3}{(y_0^N k + 2y_1^N)(y_0^N k - y_1^N)^2}. \quad (25)
\]

Yukawa couplings are expected to be order one since there is no symmetry to suppress them. Then, by using Eq. (25), we get

\[
k \approx 1 \pm \frac{2}{\sqrt{3}} \sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} \approx 1.2, \text{ or } 0.8. \quad (26)
\]

Thus, \(k\) is also expected to be order one, that is to say, \(\alpha_S \sim \alpha_V\), which indicates that symmetry breaking scales of \(\xi\) and \(\phi_S\) are same order in the neutrino sector. In the following numerical analyses, we take \(k = 1/3 \sim 3\).

We also obtain a typical value:

\[
\alpha_V \sim 5.8 \times 10^{-4}, \quad (27)
\]
where we put $\Lambda = 2.4 \times 10^{18}\text{GeV}$, $\Delta m^2_{\text{atm}} \sim 2.4 \times 10^{-3}\text{eV}^2$, $\Delta m^2_{\text{sol}} \sim 8.0 \times 10^{-5}\text{eV}^2$ and $\nu_u = 165\text{GeV}$. In following numerical calculations, we take magnitudes of Yukawa couplings to be $0.1 \sim 1$. It is found that $\alpha_V$ is lower than $10^{-3}$, which is much smaller than $\alpha_T \simeq 0.032$ in the charged lepton sector.

In the case of the inverted mass hierarchy, the situation is different from the case of the normal one. As seen in $\Delta m^2_{\text{atm}}$ of Eq. (22), the sign of $y_0^N$ is opposite against $y_1^N$. Therefore, $(y_0^N\alpha_S + 2y_1^N\alpha_V)$ should be suppressed compared with $(y_1^N\alpha_V)$ in order to be consistent with observed ratio $\Delta m^2_{\text{atm}}/\Delta m^2_{\text{sol}}$. In terms of the ratio $r$

$$r = \frac{y_1^N\alpha_V}{y_0^N\alpha_S + 2y_1^N\alpha_V},$$

we have

$$\frac{\Delta m^2_{\text{atm}}}{\Delta m^2_{\text{sol}}} = -r \frac{(y_1^N\alpha_V)^2}{(y_0^N\alpha_S - y_1^N\alpha_V)^2}.\tag{29}$$

Therefore, we expect $r \sim -100$ for $y_0^N\alpha_S \sim -2y_1^N\alpha_V$. Then, we obtain a typical value:

$$\alpha_V \sim 1.1 \times 10^{-4},\tag{30}$$

which is smaller than the one in the normal hierarchical case in Eq. (27). In the following numerical analyses, we take $r = -100 \sim -10$.

In both cases of normal and inverted mass hierarchies, $\alpha_V$ and $\alpha_S$ are much smaller than $\alpha_T$. Since $\epsilon_i \sim \delta_i \sim \mathcal{O}(\alpha_V^3/\alpha_T^2)$ in Eq. (6), magnitudes of $\epsilon_i$ and $\delta_i$ are expected to be $10^{-8}$. Therefore, $\epsilon_i$ and $\delta_i$ are negligibly small compared with $\alpha_T$, $\alpha_V$ and $\alpha_S$.

4 Deviation from the tri-bimaximal mixing

Let us discuss the deviation from the tri-bimaximal mixing. In terms of the charged lepton mixing matrix and the neutrino one, the MNS mixing matrix is written as

$$V_{\text{MNS}} = V_E^\dagger V_{\text{tri}} V_{\nu},\tag{31}$$

where we have estimated as

$$V_E = \begin{pmatrix} 1 & \theta_{12}^e & \epsilon_2 \\ -\theta_{12}^e & 1 & \epsilon_1 \\ -\epsilon_2 & -\epsilon_1 & 1 \end{pmatrix}, \quad V_{\text{tri}} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad V_{\nu} = \begin{pmatrix} 1 & \theta_{12}^\nu & \theta_{13}^\nu \\ -\theta_{12}^\nu & 1 & \theta_{23}^\nu \\ -\theta_{13}^\nu & -\theta_{23}^\nu & 1 \end{pmatrix}.\tag{32}$$

Then, the deviation from the tri-bimaximal mixing becomes

$$\delta V_{\text{MNS}} = \begin{pmatrix} \theta_{12}^e & \epsilon_2 & \theta_{23}^e - \epsilon_1 \\ \epsilon_2 & \epsilon_1 & -\theta_{12}^e - \theta_{13}^e \\ \theta_{12}^e - \epsilon_1 & -\epsilon_2 & \theta_{13}^e \end{pmatrix} \begin{pmatrix} \theta_{12}^\nu & \theta_{13}^\nu \\ -\theta_{12}^\nu & \theta_{23}^\nu \\ -\theta_{13}^\nu & -\theta_{23}^\nu \end{pmatrix} + \begin{pmatrix} \theta_{12}^e & \epsilon_2 & \theta_{23}^e - \epsilon_1 \\ \epsilon_2 & \epsilon_1 & -\theta_{12}^e - \theta_{13}^e \\ \theta_{12}^e - \epsilon_1 & -\epsilon_2 & \theta_{13}^e \end{pmatrix} \begin{pmatrix} \theta_{12}^\nu & \theta_{13}^\nu \\ -\theta_{12}^\nu & \theta_{23}^\nu \\ -\theta_{13}^\nu & -\theta_{23}^\nu \end{pmatrix}.$$
where $V_{\text{MNS}} = V_{\text{uri}} + \delta V_{\text{MNS}}$. Since magnitudes of $\epsilon_i$ is found to be $10^{-8}$, the charged lepton mass matrix is almost diagonal. Neglecting $\epsilon_i$ and $\delta_i$, and taking $\alpha_T \gg \alpha_V \sim \alpha_S$, neutrino mixing angles are simplified as

\[
\begin{align*}
\theta'_{12} &\approx \frac{4y_1^Py_1^N - 2y_0^Py_2^N + 2y_1^Py_0^N - 2y_0^Py_3^N + y_0^Py_3^N}{3\sqrt{2y_0^Ny_6^D}} \alpha_T, \\
\theta'_{13} &\approx \frac{-2y_2^Py_0^N + 3y_0^Py_3^N - 3y_0^Py_3^N}{4\sqrt{3y_0^Ny_1^N}} \alpha_T, \\
\theta'_{\nu 23} &\approx \frac{y_3^Py_0^N}{\sqrt{6y_0^N(2y_1^N - y_0^N)}} \alpha_T, \tag{34}
\end{align*}
\]

where these mixing angles are proportional to $\alpha_T$. Since $\delta_i$ and $\epsilon_i$ are $\mathcal{O}(10^{-8})$, the effect of the mixing from the charged lepton mass matrix is negligible. Therefore, the deviation from the tribimaximal mixing is of $\mathcal{O}(\alpha_T)$. Let us estimate typical mixing angles by taking Yukawa couplings to be 1. The typical values of $\alpha$'s are given as

\[
\alpha_T \sim \frac{m_\tau}{v_d} \simeq 3.2 \times 10^{-2}, \quad \alpha_V \sim \frac{\sqrt{3}v_u^2}{4\sqrt{\Delta m_{\text{sol}}^2} \Lambda} = 5.8 \times 10^{-4}, \quad \alpha_S \sim 7.0 \times 10^{-4}. \tag{35}
\]

Therefore, taking $\Lambda = 2.43 \times 10^{18}\text{GeV}$ and using experimental values of neutrino mass differences, we obtain

\[
\sin^2 \theta_{12} \sim 0.36, \quad \sin^2 \theta_{13} \sim 4.8 \times 10^{-6}, \quad \sin^2 \theta_{23} \sim 0.48, \tag{36}
\]

which is a typical prediction in our scheme.

We present the numerical results of neutrino mixing and $\alpha_V$ and $\alpha_S$ for both cases of normal and inverted hierarchies. Note that we neglect $\epsilon_1$, $\epsilon_2$, $\delta_1$, $\delta_2$, which are of order $10^{-8}$. The magnitude of $\alpha_T$ is given by the tau mass while $\alpha_S$ and $\alpha_V$ are related to neutrino mass squared differences. Yukawa couplings are randomly chosen from 0.1 to 1 with both plus and minus signs. Input data of masses and mixing angles are taken in the region of $3\sigma$ of the experimental data [1]:

\[
\Delta m_{\text{atm}}^2 = (2.07 \sim 2.75) \times 10^{-3}\text{eV}^2, \quad \Delta m_{\text{sol}}^2 = (7.05 \sim 8.34) \times 10^{-5}\text{eV}^2, \\
\sin^2 \theta_{\text{atm}} = 0.36 \sim 0.67, \quad \sin^2 \theta_{\text{sol}} = 0.25 \sim 0.37, \quad \sin^2 \theta_{\text{reactor}} \leq 0.056. \tag{37}
\]

In our numerical calculations, one million random parameter sets are produced and only the experimental consistent sets are plotted in our figures. Figure 1 shows our numerical results for the normal hierarchy of neutrino masses. In figures 1 (a) and (b), we plot the allowed region of mixing angles on planes of $\sin^2 \theta_{12} - \sin^2 2\theta_{23}$ and $\sin^2 \theta_{23} - \sin^2 \theta_{13}$, respectively, in the case of $\alpha_V = 10^{-4} \sim 10^{-3}$. The value of $\sin^2 2\theta_{23}$ is larger than 0.97. It is also found that the upper bound of $\sin^2 \theta_{13}$ is 0.01. In figure 1(c), we show the allowed region on the $\sin^2 \theta_{23} - \sin^2 \theta_{13}$ plane in the case of $\alpha_V = 10^{-3} \sim 5 \times 10^{-3}$. It is found that allowed points decrease much more in this region of $\alpha_V$. There are no allowed points in the region of $\alpha_V \geq 5 \times 10^{-3}$. Thus, $\alpha_V$ is expected
to be smaller than $\mathcal{O}(10^{-3})$. In figure 1(d), we plot the allowed region on the $\alpha_V-\alpha_S$ plane. It is found that $\alpha_V \simeq \alpha_S$ as expected in Eqs. (23) and (26).

Figure 2 shows our numerical results for the inverted hierarchy of neutrino masses. The value of $\sin^2 \theta_{23}$ is larger than 0.96 as seen in figure 2(a). It is also found that the upper bound of $\sin^2 \theta_{13}$ is 0.01 in figure 2(b). These are almost the same result as in the case of the normal hierarchy. In figure 2(c), we show the result on the $\sin^2 \theta_{23} \sin^2 \theta_{13}$ plane with $\alpha_V = 5 \times 10^{-4} \sim 10^{-3}$. Allowed points decrease considerably in this region of $\alpha_V$. There are no allowed points in the region of $\alpha_V \geq 10^{-3}$. Thus, $\alpha_V$ should be smaller than $\mathcal{O}(5 \times 10^{-4})$. As in the case of the normal hierarchy, $\alpha_V$ and $\alpha_S$ become the same magnitude. These values of $\alpha_V$ and $\alpha_S$ are important parameters to estimate the soft SUSY breaking in the next section.

5 Soft SUSY breaking terms

We discuss soft SUSY breaking terms, i.e. soft slepton masses and A-terms, which were discussed in detail the $A_4$ flavor model without three right-handed Majorana neutrinos [32]. We have
obtained the different result in our seesaw type model.

First let us study soft scalar masses. Within the framework of supergravity theory, the flavor symmetry $A_4 \times Z_3$ requires the following form of Kähler potential for left-handed and right-handed leptons

$$K^{(0)}_{\text{matter}} = a(Z, Z^\dagger)(L^\dagger_e L_e + L^\dagger_\mu L_\mu + L^\dagger_\tau L_\tau) + b_e(Z, Z^\dagger) R^\dagger_e R_e + b_\mu(Z, Z^\dagger) R^\dagger_\mu R_\mu + b_\tau(Z, Z^\dagger) R^\dagger_\tau R_\tau,$$

at the leading order, where $a(Z, Z^\dagger)$ and $b_I(Z, Z^\dagger)$ for $I = e, \mu, \tau$ are generic functions of moduli fields $Z$. However, the flavor symmetry $A_4 \times Z_3$ is broken to derive the realistic lepton mass matrices and such breaking introduces corrections in the Kähler potential and slepton masses. Because of $\langle \phi_{T_2} \rangle, \langle \phi_{T_3} \rangle \ll \langle \phi_{T_1} \rangle$, the most important correction terms would be linear terms of $\phi_{T_1}$. Precisely, the correction terms in the matter Kähler potential are obtained

$$\Delta K_{\text{matter}} = \frac{\phi_{T_1}}{\Lambda} \left[ a'_1(Z, Z^\dagger)(2L^\dagger_e L_e - L^\dagger_\mu L_\mu - L^\dagger_\tau L_\tau) + a'_2(Z, Z^\dagger)(L^\dagger_\mu L_\mu - L^\dagger_\tau L_\tau) \right] + h.c.,$$

Figure 2: Allowed regions on (a) $\sin^2 \theta_{12}-\sin^2 2 \theta_{23}$ and (b) $\sin^2 \theta_{23}-\sin^2 \theta_{13}$ planes for $10^{-4} < \alpha_V < 5 \times 10^{-4}$, (c) $\sin^2 \theta_{23}-\sin^2 \theta_{13}$ plane for $5 \times 10^{-4} < \alpha_V < 10^{-3}$, and (d) $\alpha_V - \alpha_S$ plane, where $-100 < r < 10$ is taken, in the case of the inverted hierarchy.
up to $\mathcal{O}(\tilde{a}^2)$, where $\tilde{a}$ is the linear combination of $\alpha_S$ and $\alpha_V$, and $a'_1(Z, Z^\dagger)$ and $a'_2(Z, Z^\dagger)$ are generic functions of moduli fields. All of off-diagonal Kähler metric entries for both left-handed and right-handed leptons appear at $\mathcal{O}(\tilde{a}^2)$.

Including these corrections, the slepton masses are written by

\begin{equation}
\begin{align*}
m_L^2 &= \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix} + m^2_{3/2} \begin{pmatrix} O(\alpha_T) & O(\tilde{a}^2) & O(\tilde{a}^2) \\ O(\tilde{a}^2) & O(\alpha_T) & O(\tilde{a}^2) \\ O(\tilde{a}^2) & O(\tilde{a}^2) & O(\alpha_T) \end{pmatrix}, \\
m_R^2 &= \begin{pmatrix} m_{R_1}^2 & 0 & 0 \\ 0 & m_{R_2}^2 & 0 \\ 0 & 0 & m_{R_3}^2 \end{pmatrix} + m^2_{3/2} \begin{pmatrix} O(\tilde{a}^2) & O(\lambda^q\tilde{a}^2) & O(\lambda^2\tilde{a}^2) \\ O(\lambda^q\tilde{a}^2) & O(\tilde{a}^2) & O(\lambda^2\tilde{a}^2) \\ O(\lambda^2\tilde{a}^2) & O(\lambda^2\tilde{a}^2) & O(\tilde{a}^2) \end{pmatrix},
\end{align*}
\end{equation}

where all of $m_L$ and $m_R_i$ for $i = 1, 2, 3$ would be of $\mathcal{O}(m_{3/2})$. Since the charged lepton mixing is of $\mathcal{O}(10^{-8})$, we can neglect its effect.

These forms would be obvious from the flavor symmetry $A_4$, that is, three families of left-handed leptons are the $A_4$ triplet, while right-handed leptons are $A_4$ singlets. At any rate, it is the prediction of the $A_4$ model that three families of left-handed slepton masses are almost degenerate.

We have a strong constraint on $(m_L^2)_{12}$ and $(m_R^2)_{12}$ from FCNC experiments \cite{35}. Since $\alpha_S$ and $\alpha_V$ are same order up to $10^{-3}$, we can estimate

\begin{equation}
\frac{(m_L^2)_{12}}{m^2_{\text{SUSY}}} \simeq \mathcal{O}(\tilde{a}^2) \leq \mathcal{O}(10^{-6}), \quad \frac{(m_R^2)_{12}}{m^2_{\text{SUSY}}} \simeq \mathcal{O}(\lambda^q\tilde{a}^2) \leq \mathcal{O}(10^{-7}),
\end{equation}

for $m_{\text{SUSY}} \sim 100$ GeV, where $m_{\text{SUSY}}$ denotes the average mass of slepton masses and it would be of $\mathcal{O}(m_{3/2})$. These predicted values are much smaller than the experimental bound $\mathcal{O}(10^{-3})$ \cite{35}.

Now, let us examine the mass matrix between left-handed and right-handed sleptons, which is generated by the so-called A-terms. The A-terms are trilinear couplings of two sleptons and one Higgs field \cite{32}, i.e.

\begin{equation}
\begin{align*}
h_{IJ} R_I L_J H_d &= h_{IJ}^{(Y)} R_I L_J H_d + h_{IJ}^{(K)} R_I L_J H_d, \quad (42)
\end{align*}
\end{equation}

The charged lepton mass matrix is diagonalized by $V_R^\dagger m_I V_L$, where

\begin{equation}
\begin{align*}
V_R &\sim \begin{pmatrix} 1 & \frac{m_\mu}{m_\mu} \epsilon_2 & \frac{m_\mu}{m_\tau} \epsilon_1 \\ \frac{m_\mu}{m_\mu} \epsilon_2 & 1 & \frac{m_\mu}{m_\tau} \epsilon_1 \\ \frac{m_\mu}{m_\tau} \epsilon_1 & \frac{m_\mu}{m_\tau} \epsilon_2 & 1 \end{pmatrix}, \quad V_L \sim \begin{pmatrix} 1 & \epsilon_1 & \epsilon_2 \\ -\epsilon_1 & 1 & \epsilon_1 \\ -\epsilon_2 & -\epsilon_1 & 1 \end{pmatrix}. \quad (43)
\end{align*}
\end{equation}

In the diagonal basis of the charged lepton mass matrix, we estimate the magnitude of $\tilde{m}_{RL}^2 \equiv V_R^\dagger m_{RL}^2 V_L$. By the parallel discussion in \cite{32}, the (2,1) entry of $\tilde{m}_{RL}^2$ from the second term $h_{IJ}^{(K)}$ in Eq. (42) is given as

\begin{equation}
(\tilde{m}_{RL}^2)_{21} = \mathcal{O}(m_\mu \epsilon_1 \alpha_T m_{3/2}),
\end{equation}

\begin{equation}
(\tilde{m}_{RL}^2)_{21} = \mathcal{O}(m_\mu \epsilon_1 \alpha_T m_{3/2}).
\end{equation}
which gives \((\tilde{m}_{RL}^2)_{21}/m_{\text{SUSY}}^2 = \mathcal{O}(10^{-12})\) for \(m_{\text{SUSY}} = 100\) GeV. On the other hand, the first term of (42) contributes to \((\tilde{m}_{RL}^2)_{21}\) as \[\[ (\tilde{m}_{RL}^2)_{21} = y_\mu v_4 \phi T_2 m_{3/2}/\Lambda \sim m_\mu \epsilon_1 m_{3/2}, \quad (45) \]
which gives \((\tilde{m}_{RL}^2)_{21}/m_{\text{SUSY}}^2 = \mathcal{O}(10^{-11})\) for \(m_{\text{SUSY}} = 100\) GeV. The predicted value is much smaller than the FCNC experimental upper bound \(\mathcal{O}(10^{-6})\).

6 Summary

We have studied the higher order corrections of the flavor symmetry breaking in the \(A_4\) seesaw model. We have discussed possible higher dimensional mass operators, which cause the deviation from the tri-bimaximal mixing. We have found the magnitude of deviation is dominated by the VEV of \(\phi_T_1\), which is determined by the tau lepton mass.

The model has 6 Yukawa couplings \((y_e^0, y_\mu^0, y_\tau^0, y_D^0, y_N^0, y_N^1)\) and 3 independent VEV’s devided by the scale factor \(\Lambda\), \((\alpha_T, \alpha_V, \alpha_S)\) at the leading order. In order to estimate the deviation from the tri-bimaximal mixing, we have discussed higher dimensional mass operators, in which additional 11 Yukawa couplings and 3 VEV parameters appear. Ratios of charged lepton masses are almost determined by the leading order Yukawa couplings as \(m_e/m_\tau \propto y_6^0/\tilde{y}_6^0, m_\mu/m_\tau \propto y_6^\mu/\tilde{y}_6^\mu\). Neutrino mass ratios are also determined by the leading order Yukawa couplings \(y_D^0, y_N^0, y_N^1\) and \(\alpha_S/\alpha_V\).

Since three shift parameters for alignment \((\epsilon_1 = \epsilon_2, \delta_1, \delta_2)\) are tiny, these effect is negligibly small both on mass eigenvalues and flavor mixing angles. On the other hand, the deviation from the tri-bimaximal mixing depends on additional 7 Yukawa couplings at the next leading order: \(y_1^D, y_2^D, y_2^N, y_3^2, y_3^3, y_4^N, y_5^N\). By varying these Yukawa couplings in the region \(|y_i^{D,N}| = 0.1 \sim 1\) at random, we can predict the deviation from the tri-bimaximal mixing.

We have obtained predictions of lepton mixing angles for both normal hierarchy and inverted hierarchy of neutrino masses. Since there is no symmetry to suppress the Yukawa couplings, we can expect them to be order one. After fixing them, mass matrices are determined so that neutrino masses and mixing angles can be calculated. As our result, the value of \(\sin^22\theta_{23}\) is larger than 0.96 and the upper bound of \(\sin^2\theta_{13}\) is 0.01. Therefore, we may expect the Double Chooz experiment observes the disappearance of \(\bar{\nu}_e \rightarrow \bar{\nu}_e\) process.

It is also found \(\alpha_V \sim \alpha_S \leq 10^{-3}\) while \(\alpha_T \simeq 0.03\). In terms of these values of \(\alpha_V\) and \(\alpha_S\), we have examined the soft SUSY breaking in slepton masses and A-terms within the framework of supergravity theory. Those magnitudes are enough suppressed to be consistent with experimental constraints from flavor changing neutral current processes. This suppression is stronger than that in the case of the effective neutrino mass matrix of \(A_4\) model, discussed in Ref. [32].

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