Axionic Dirac seesaw and electroweak vacuum stability

J. T. Penedo\textsuperscript{a}\textsuperscript{1}, Yakefu Reyimuaji\textsuperscript{b}\textsuperscript{2}, Xinyi Zhang\textsuperscript{c}\textsuperscript{3}

\textsuperscript{a} Centro de Física Teórica de Partículas, CFTP, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais nr. 1, 1049-001 Lisboa, Portugal;

\textsuperscript{b} School of Physical Science and Technology, Xinjiang University, Urumqi, Xinjiang 830046, China

\textsuperscript{c} Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract

We explore the connection between tree-level Dirac neutrino masses and axion physics in a scenario where the PQ symmetry enforces lepton number conservation perturbatively. Requiring that the PQ scale $f_a$ is the only heavy scale to play a role in neutrino mass generation, we are led to the construction of a KSVZ-type model where Dirac neutrino masses are inversely proportional to $f_a$, provided a real scalar triplet (zero hypercharge) is added to the SM scalar sector. We analyse this extended scalar sector, focusing on the stabilisation of the electroweak vacuum. The contribution of the triplet VEV to the $W$ mass may also be responsible for the recent hint of beyond-the-SM physics by the CDF collaboration.

1 Introduction

Despite its many successes, the Standard Model (SM) cannot be a final description of Nature. It must be extended in order to clarify the origins of neutrino masses and dark matter. Moreover, the SM by itself does not offer an explanation to the non-observation of the neutron electric dipole moment. This so-called strong CP problem can be solved via the Peccei-Quinn (PQ) mechanism \cite{1,2,3}, whereby an axion is introduced in the theory (for a recent review see \cite{4}). From the point of view of a UV completion, this QCD axion arises as the pseudo-Nambu-Goldstone boson of a spontaneously broken, anomalous $U(1)_{\text{PQ}}$ symmetry.

Depending on its properties, the axion can provide the desired dark matter candidate. On the other hand, axion physics may directly connect to the generation of neutrino masses. In the case of Majorana neutrinos, the PQ scale $f_a$ is naturally identified with the type-I seesaw scale \cite{5,6,7,8,9,10}. The lepton-number-violating right-handed (RH) neutrino Majorana mass term is thus generated from a coupling of the type $\sigma v_R^c v_R^c$, where $\sigma$ is the PQ scalar field. In this case, light neutrino masses are suppressed by the PQ scale, $m_\nu \sim v^2/f_a$, with $v \approx 246$ GeV.

At present, the nature of neutrino masses is not known. Dirac neutrinos remain a viable and interesting possibility. In this context, however, the connection to axion physics is not so direct as in the Majorana case. This link has been explored in models where Dirac

\textsuperscript{1}Email: joao.t.n.penedo@tecnico.ulisboa.pt
\textsuperscript{2}Email: yreyi@hotmail.com
\textsuperscript{3}Email: zhangxinyi@ihep.ac.cn
neutrino masses are generated at the tree level \cite{11-16} and at the one-loop level \cite{14,17-19}.\footnote{The baryon asymmetry of the universe may be generated in such a setup by the neutrino genesis mechanism (aka Dirac leptogenesis) \cite{20,21}, see e.g. \cite{11}.} Focusing on the tree-level case, one typically finds $m_\nu \sim v f_a / \Lambda_{\text{UV}}$ \cite{13}, i.e. neutrino masses are proportional to the PQ scale and inversely proportional to an (in general) unrelated scale of new physics $\Lambda_{\text{UV}}$, e.g. the GUT scale. Here, the suppression of $m_\nu$ with respect to the electroweak scale arises from the smallness of Yukawa couplings and of the ratio $f_a / \Lambda_{\text{UV}}$. If an additional mass scale $\mu$ is present in the theory, one can instead obtain a relation of the type $m_\nu \sim \mu v f_a / \Lambda_{\text{UV}}^2$ \cite{11,12,14}, with the ratio $\mu / \Lambda_{\text{UV}}$ possibly providing a further source of suppression.

In this work, we look into the possibility of identifying the Dirac seesaw scale with the PQ scale $f_a$, so that the Dirac neutrino masses are suppressed by $f_a$ as in the Majorana case.\footnote{Recently Ref. \cite{16} appeared, where the relation $m_\nu \sim v^3 / f_a^2$ for Dirac neutrino masses is obtained.} To avoid introducing an independent heavy scale $\Lambda_{\text{UV}}$, we focus on the diagram of Figure 1 as the main contribution to neutrino masses, which effectively corresponds to a dimension-5 operator. In this case, one obtains a relation of the type $m_\nu \sim \mu v / f_a$, where $\mu$ corresponds to the vacuum expectation value (VEV) of a new neutral scalar. Such a relation was also found in the 3-3-1 setup of Ref. \cite{15}, with $\mu = 10^4$ GeV.

Moreover, we find that the PQ symmetry by itself is enough to explain the Dirac nature of neutrino masses in such a setup. Namely, one does not need to impose an additional lepton-number symmetry, since PQ charges forbid Majorana mass terms at all perturbative orders. Such an economical possibility was previously explored in Refs. \cite{13,14} for different classes of models. Unlike these models, which consider Dine-Fischler-Srednicki-Zhitnitsky (DSFZ)-type axions \cite{22,23}, we develop a scenario where the scalar fields (apart from $\sigma$) are not charged under $U(1)_{\text{PQ}}$. Thereby the SM Higgs doublet is not charged under this symmetry (neither are the SM quarks) and our axion is of the Kim–Shifman–Vainshtein–Zakharov (KSVZ) type \cite{24,25}. The considered extension of the scalar sector naturally warrants an analysis of the stability of the electroweak vacuum.

---

Figure 1: Tree-level Feynman diagram giving rise to PQ-suppressed Dirac neutrino masses. The seesaw partners $\Delta_{F_L}$ and $\Delta_{F_R}$ have masses proportional to $f_a$, enabling the identification of the (Dirac) seesaw and PQ scales.
In Section 2 we describe our framework, detailing the field content and neutrino masses. We further comment on the solution to the strong CP problem and on the possibility of explaining the recent CDF anomaly due to the contribution of the new scalar VEV to the W boson mass. In Section 3 we analyse the scalar sector of the theory. In particular, we look into the constraints imposed by vacuum stability on the discussed model. Finally, we present our conclusions in Section 4.

2 Framework

2.1 Axionic Dirac seesaw

2.1.1 Field content

We start by setting the field content. Aiming at identifying the seesaw scale with the PQ-breaking scale, i.e. \( \Lambda_{\text{seesaw}} \sim f_a \), first of all we introduce one complex singlet PQ field \( \sigma \).

Maintaining a minimal field content, we introduce 2 generations of RH neutrinos \( \nu_R \) and are led to the dimension-5 operator of the form \( \nu_R L H \chi / f_a \), where \( L \) and \( H \) are the SM lepton and Higgs doublets, respectively. Here, \( \chi \) can be either a singlet scalar or an \( SU(2)_L \) triplet, with zero hypercharge. In order to generate the Dirac neutrino masses at tree level, we open up the dimension-5 operator by introducing a vector-like fermion, \( \Delta_F = \Delta_F^R + \Delta_F^L \) which gains a mass after PQ-symmetry breaking, i.e. \( m_{\Delta_F} \sim f_a \). This corresponds to the seesaw diagram shown in Figure 1.

Going forward, we consider the case when \( \chi = \Delta_\chi \) is an \( SU(2)_L \) triplet, whereas the singlet possibility will be explored elsewhere \([26]\). The minimal choice then corresponds a real \( Y = 0 \) scalar triplet (see e.g. \([27–30]\)) instead of a complex one. It follows that the fermion fields \( \Delta_{FR} \) and \( \Delta_{FL} \) must also be triplets. We are thus dealing with a type-III Dirac seesaw, in the terminology of Refs. \([11,31]\). The triplets of the model are defined as

\[
\Delta_\chi \equiv \frac{1}{2} \left( \chi^0 \sqrt{2 \chi^+} - \chi^0 \right), \quad \Delta_{FR} \equiv \frac{1}{2} \left( F_R^0 \sqrt{2 F_R^+} - F_R^0 \right), \quad \Delta_{FL} \equiv \frac{1}{2} \left( F_L^0 \sqrt{2 F_L^+} - F_L^0 \right), \quad (1)
\]

where \( \chi^0 \) is real and \( (\chi^+)^* = \chi^- \). Notice that \( \Delta_\chi = \Delta_\chi^\dagger \). For \( \Delta_{FR} \) and \( \Delta_{FL} \), the component fields are all complex and \( (F_{RL,R,L}^+) \neq F_{RL,L}^- \).

The leading-order contribution to the light neutrino mass scale can be read from the considered diagram. One has

\[
m_\nu \sim \frac{v_\chi v}{v_\sigma}, \quad (2)
\]

where we assume that the neutral components of the scalars all acquire VEVs, i.e. \( \langle H^0 \rangle = v / \sqrt{2}, \langle \chi^0 \rangle = v_\chi, \) and \( \langle \sigma \rangle = v_\sigma / \sqrt{2} = \sqrt{2} N f_a \), with \( N \) being the QCD anomaly coefficient. Note that at least 2 copies of the vector-like fermions \( \Delta_F \) are required to generate both \( \Delta m^2_\odot \) and \( \Delta m^2_{\text{atm}} \) neutrino mass-squared differences. To obtain a sub-eV mass for the light

\[\text{Note that to preserve the structure of the diagram in Figure } 1 \text{ in the absence of extra symmetries, the singlet would have to carry a non-zero PQ charge and thus be complex, in order to forbid a direct Majorana-type } \overline{\nu}_R \Delta_{FR} \sigma \text{ coupling.} \]
neutrinos, we need $v_\chi/v_a \sim 10^{-12}$. It is curious that the experimental constraints on $v_\chi$, suggesting $v_\chi \sim \mathcal{O}(\text{GeV})$ at most (see Section 2.3), together with a typical scale of $f_a \sim 10^9-10^{12}$ GeV for axionic dark matter leads to viable neutrino mass scales for $\mathcal{O}(10^{-3}-1)$ Yukawa couplings.

Finally, to address the strong CP problem we introduce a vector-like quark $Q = Q_L + Q_R$, such that the SM quarks need not be charged under the PQ symmetry. Our model is therefore of the KSVZ-type (see also Section 2.2).

### 2.1.2 PQ as a lepton number symmetry

Having set the field content, we show that it is possible to impose no other symmetry aside from the PQ symmetry — especially no independent global lepton number symmetry — to guarantee Diracness. This requires that we charge the fields properly. To start, the PQ field is charged $\text{PQ}(\sigma) = 1$, while $\text{PQ}(Q_{L,R}) = \pm 1/2$ for the vector-like quark, as usual. Since we work in a KSVZ-type model, the SM Higgs is not charged under the PQ symmetry and, consequently, the SM lepton doublet needs to carry a charge $\text{PQ}(L) \equiv \alpha \neq 0$ to forbid a Weinberg-operator contribution to light neutrino masses. The charge assignments for all relevant fields are collected in Table 1. They follow from requiring that the interactions contained in the diagram of Figure 1 are allowed. One has $\text{PQ}(\Delta_\chi) = 0$, since it is a real scalar triplet.

Note that the direct Dirac coupling $\bar{L}H\nu_R$ is automatically forbidden. To ensure that light neutrino masses are generated by the described Dirac seesaw mechanism, one also needs to forbid other possible Majorana contributions. This puts some additional constraints on the PQ charge $\alpha$, namely

i) $\alpha \neq -1$ to avoid a direct RH neutrino Majorana mass term $\bar{\nu}_R^c \nu_R$, automatically forbidding higher-dimensional $\bar{\nu}_R^c \nu_R \Delta_\chi^n$ terms ($n$ even, to form $SU(2)_L$ singlets), and more generally

ii) $\alpha \neq k/2$ ($k \in \mathbb{Z}$) to forbid (possibly higher-dimensional) $\bar{\nu}_R^c \nu_R (\sigma^{(*)})^n$ Majorana terms and their variants with additional $H$ or $\Delta_\chi$ insertions.

This last requirement of $2\alpha \notin \mathbb{Z}$ contains the previous ones. It forbids Weinberg-operator contributions with any number of $\sigma^{(*)}$ insertions. Other possible Majorana-like contributions

| Field  | $L$ | $e_R$ | $\nu_R$ | $\Delta_{F_R}$ | $\Delta_{F_L}$ | $Q_R$ | $Q_L$ | $H$ | $\sigma$ | $\Delta_\chi$ |
|--------|-----|-------|---------|----------------|----------------|-------|-------|-----|----------|------------|
| $SU(3)_c$ | 1   | 1     | 1       | 1              | 1              | 3     | 3     | 1   | 1        | 1          |
| $SU(2)_L$ | 2   | 1     | 1       | 3              | 3              | 1     | 1     | 2   | 1        | 3          |
| $U(1)_Y$   | $-\frac{1}{2}$ | $-1$  | 0       | 0              | 0              | 0     | $\frac{1}{2}$ | 0   | 0        |
| $U(1)_{PQ}$ | $\alpha$ | $\alpha$ | $\alpha + 1$ | $\alpha$ | $\alpha + 1$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0   | 1        |

Table 1: Charge assignments of the considered Dirac seesaw model ($\alpha \neq 0$).
such as $\Delta^{-}_{F_{R}}\Delta_{F_{R}}, \Delta^{-}_{F_{L}}\Delta_{F_{L}},$ and $\Delta^{-}_{F_{R}}\nu_{R}\Delta^{n}_{\chi}$ ($n$ odd) are also not allowed, even with an arbitrary number of $\sigma^{(s)}$ insertions, since these carry integer PQ charge.

Making, for definiteness, the choice $\alpha = -1/3$ (in a parallel with SM quark e.m. charges), one finds that neutrinos are Dirac particles in this model. Lepton number conservation is hence enforced (perturbatively) by the PQ symmetry.

The relevant Lagrangian $\mathcal{L}$, extending the SM one, is

$$
\mathcal{L} = \mathcal{L}_{\text{kin}} - \mathcal{L}_{\text{Yuk}} - V(H, \Delta_{\chi}, \sigma), \quad (3a)
$$

$$
\mathcal{L}_{\text{kin}} = |\partial_{\mu}\sigma|^{2} + \text{Tr} |D_{\mu}\Delta_{\chi}|^{2} + \nu_{R}i\partial_{\mu}\nu_{R} + \bar{Q}i\partial_{\mu}Q + \bar{F}_{\mu}\partial_{\mu}F,
$$

$$
\mathcal{L}_{\text{Yuk}} = Y_{Q}\bar{Q}_{L}Q_{R}\sigma + \bar{L}\tilde{H}Y_{L}\Delta_{F_{R}} + \text{Tr} (\Delta_{F_{L}}Y_{F}^\dagger\Delta_{F_{R}})\sigma + \text{Tr} (\Delta^{2}_{F_{L}}\Delta_{\chi}^{n})R_{\nu_{R}} + \text{H.c.}, \quad (3c)
$$

where the covariant derivative for the zero-hypercharge triplets of Eq. (1) acts as $D_{\mu}\Delta = \partial_{\mu}\Delta + ig_{2}[W_{\mu}, \Delta]$, with $W_{\mu} = W^{a}_{\mu}T_{a}$ containing the $SU(2)_{L}$ gauge bosons $W^{a}_{\mu}$ and $g_{2}$ being the corresponding gauge coupling. Here, $Y_{Q}$ is a Yukawa coupling, while $Y_{L}, Y_{F},$ and $Y_{R}$ are Yukawa coupling matrices. In the minimal setup, $Y_{L}$ is a $3 \times 2$ matrix, while $Y_{F}$ and $Y_{R}$ are $2 \times 2$ matrices.

The scalar potential reads

$$
V(H, \sigma, \Delta_{\chi}) = -\mu_{H}^{2}H^{\dagger}H - \mu_{\chi}^{2}\text{Tr} \left(\Delta_{\chi}^{2}\right) - \mu_{\sigma}^{2}\sigma^{*}\sigma + \kappa H^{\dagger}\Delta_{\chi}H
$$

$$
+ \frac{\lambda}{2}(H^{\dagger}H)^{2} + \frac{\lambda_{H}}{2}\text{Tr} \left(\Delta_{\chi}^{4}\right) + \frac{\lambda_{\sigma}}{2}(\sigma^{*}\sigma)^{2}
$$

$$
+ \frac{\lambda_{H}}{2}(H^{\dagger}H)\text{Tr} \left(\Delta_{\chi}^{2}\right) + \frac{\lambda_{\sigma}}{2}(H^{\dagger}H)(\sigma^{*}\sigma) + \frac{\lambda_{\sigma}}{2}(\sigma^{*}\sigma)\text{Tr} \left(\Delta_{\chi}^{2}\right), \quad (4)
$$

where all the couplings are real. Note also that $\Delta_{\chi}^{2} = \left[\left(\chi^{0}/2 + \chi^{+}/2 - \chi^{-}/2\right)I\right], \text{directly implying that the terms} \left[\text{Tr} \left(\Delta_{\chi}^{6}\right)\right]^{2} = 2\text{Tr} \left(\Delta_{\chi}^{4}\right) \text{and} H^{\dagger}\Delta_{\chi}^{n}H = (H^{\dagger}H)\text{Tr} \left(\Delta_{\chi}^{2}\right)/2 \text{are not new. As is mentioned in Ref. [32], in the limit of vanishing} \kappa \text{the potential possesses a global symmetry} O(4)_{H} \times O(3)_{\Delta_{\chi}}; \text{and a discrete symmetry} \Delta_{\chi} \rightarrow -\Delta_{\chi},^{4} \text{Thus} \kappa \text{is protected by these symmetries and a small but non-vanishing} \kappa \text{corresponds to their soft breaking. One may fix the sign of} \kappa \text{via a sign flip of} \Delta_{\chi} \text{and} \nu_{R}. \text{We consider the convention} \kappa > 0 \text{in what follows.}

### 2.2 The axion and the solution to the strong CP problem

The solution to the strong CP problem in our model is identical to that of the KSVZ model $^{24}25$, which we briefly review here. Anticipating the spontaneous breakdown of the PQ symmetry, one can parameterise the PQ field as

$$
\sigma = \frac{1}{\sqrt{2}}(v_{\sigma} + \rho_{\sigma})e^{ia/v_{0}}, \quad (5)
$$

where $a$ is the Goldstone field, i.e. the axion, and $\rho_{\sigma}$ is the radial mode. The vector-like quark gets a mass from its interaction with the PQ field, $m_{Q} = Y_{Q}v_{\sigma}e^{ia/v_{0}}/\sqrt{2}$. By performing an axial transformation $Q \rightarrow e^{-i\gamma_{5}a/(2v_{0})}Q$, the field-dependent phase in $m_{Q}$ gets rotated away.
The $Q$ field is thus disentangled from the axion field and can be integrated out. The axial transformation is anomalous, leading to the $aG\tilde{G}/v_\sigma$ term, where $G$ is the $SU(3)_c$ field strength tensor. This term can be used to cancel the $\theta$ term, in a dynamical solution to the strong CP problem.

Comparing the generated $aG\tilde{G}$ term to the corresponding one in the axion effective Lagrangian, it follows that $v_\sigma = 2Nf_a$, as indicated in Section 2.1.1. Note that there are fields other than $\sigma$ that are charged under the PQ symmetry. The QCD anomaly coefficient is $N = 1/2$ in our model, and thus $v_\sigma = f_a$. It is more transparent to look back at Eq. (2) with $v_\sigma = f_a$. Since the domain wall number is $N_{DW} = 2N = 1$, this model is free from the cosmological domain wall problem.

The electromagnetic anomaly coefficient $E$ is also independent of the charge $\alpha$. We find it is given by $E = 2n_F$, where $n_F$ denotes the number of generations of vector-like seesaw partners $\Delta_F$. We consider the minimal case (recall Section 2.1.1), for which $n_F = 2$ and $E = 4$. This $E/N = 8$ value is safe from current axion experimental search bounds for...
large regions of the parameter space, as shown in Fig. 2. The allowed values of $f_a$ are constrained from below ($f_a \gtrsim 10^{8}$ GeV) due to the SN 1987A bound on the axion-nucleon couplings \cite{48}, which for our model implies the bound $g_{ap} \lesssim 3 \times 10^{-9}$ on the axion-proton coupling. On the other hand, one expects an upper bound on $f_a$ from the relation in Eq. (2). In particular, requiring perturbative Yukawa couplings of at most $\mathcal{O}(1)$ and taking a triplet VEV $v_\chi$ of at most $\mathcal{O}(5$ GeV), one sees that neutrino masses become suppressed beyond what is phenomenologically viable, i.e. $m_\nu < \sqrt{\Delta m^2_{\text{atm}}} \simeq 0.05$ eV, unless $f_a \lesssim 10^{13-14}$ GeV. Therefore, we take $f_a \in [10^8, 10^{13}]$ GeV as our viable range of interest. As shown in Fig. 2, large portions of the viable parameter space are expected to be probed by upcoming axion experiments.

2.3 A heavier W mass?

Due to the engagement of $\Delta_\chi$ in $SU(2)_L$ gauge interactions, the triplet extension of the Higgs sector can be constrained by electroweak precision measurements. In particular, a non-zero VEV $v_\chi$ in our model modifies the $\rho$ parameter, which at tree-level is calculated as

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 + \frac{v_\chi^2}{v^2},$$

given the tree-level expressions for the squared masses of weak gauge bosons,

$$M_W^2 = \frac{g_2^2}{4} \left( v^2 + 4v_\chi^2 \right), \quad M_Z^2 = \frac{g_2^2 v^2}{4 \cos^2 \theta_W}.$$  \hspace{1cm} (7)

Unlike in the SM case, custodial symmetry is not recovered in the limit $g' \rightarrow 0$. We keep our discussion at the tree level in order to arrive at a plausible and illustrative benchmark for $v_\chi$. At this level, the scalar triplet VEV does not affect $M_Z$.

Significant attention has recently been given to models with hyperchargeless triplet scalars in light of the new $W$ mass measurement given by the CDF collaboration \cite{49} (see e.g. \cite{50–64}). Taking the CDF II result as a hint for new physics, we re-express the $\rho$ parameter as

$$\rho = \left( \frac{M_W^{\text{CDF}}}{M_W^{\text{SM}}} \right)^2 \rho_{\text{SM}} \simeq \left( \frac{M_W^{\text{CDF}}}{M_W^{\text{SM}}} \right)^2 \rho_{\text{SM}},$$

where $M_W^{\text{CDF}} = 80433.5 \pm 9.4$ MeV is the CDF measurement, while the SM value is $M_W^{\text{SM}} = 80357 \pm 6$ MeV \cite{70}. Using central values, we obtain $\rho \simeq 1.0019$. Note that the change in $\rho$ has the correct positive sign in our model, at the tree level. Eq. (6) then gives $v_\chi \simeq 5.36$ GeV. In the next section, we analyse the scalar potential of the model, taking $v_\chi = 5.4$ GeV as a benchmark value.

3 Vacuum stability

The real and complex scalars introduced in the previous section will have non-trivial effects on the vacuum structure. In this section, we analyse the vacuum structure in detail, taking the potential in Eq. (4) as our starting point, with an emphasis on electroweak vacuum stability.
3.1 Mass spectrum

Assuming all the VEVs \((v, v_\chi, v_\sigma)\) are non-zero, we find the stationarity conditions as

\begin{equation}
\mu_H^2 = \frac{1}{4} \left( 2\lambda v^2 + \lambda_a v^2 + \lambda_b v^2 - 2\kappa v_\chi \right), \tag{9a}
\end{equation}

\begin{equation}
\mu_\chi^2 = \frac{1}{4v_\chi} \left( \lambda_a v^2 v_\chi + \lambda_c v^2 v_\chi + \lambda_\chi v_\chi^3 - \kappa v^2 \right), \tag{9b}
\end{equation}

\begin{equation}
\mu_\sigma^2 = \frac{1}{4} \left( \lambda_b v^2 + 2\lambda_\sigma v_\sigma^2 + \lambda_c v_\chi^2 \right), \tag{9c}
\end{equation}

under the parameterisations \(H = (\phi^+, (\phi^0 + v + iG_H)/\sqrt{2})^T\), \(\sigma = (\phi_\sigma + v_\sigma + iG_\sigma)/\sqrt{2}\). With these conditions, we find the mass matrix for the neutral scalars in the basis \((\phi^0, \chi^0, \phi_\sigma)\) to be

\[
M_{\text{CP-even}}^2 = \begin{pmatrix}
\lambda v^2 & \frac{1}{2}\lambda_a v v_\chi - \frac{1}{2}\kappa v & \frac{1}{2}\lambda_b v v_\sigma \\
\frac{1}{2}\lambda_a v v_\chi - \frac{1}{2}\kappa v & \frac{\kappa v^2}{4v_\chi} + \frac{\lambda_\chi v_\chi^2}{2} & \frac{1}{2}\lambda_c v_\sigma v_\chi \\
\frac{1}{2}\lambda_b v v_\sigma & \frac{1}{2}\lambda_c v_\sigma v_\chi & \lambda_\sigma v_\sigma^2
\end{pmatrix}, \tag{10}
\]

The VEVs \((v, v_\chi, v_\sigma)\) correspond to a minimum of the potential in Eq. (4) when this \(M_{\text{CP-even}}^2\) matrix is positive definite. The CP-odd mass matrix vanishes, corresponding to two Goldstone bosons. One of them becomes the longitudinal component of \(Z\) boson and the other one is the axion, which becomes massive when the chiral axion potential is considered, as in the KSVZ model.

The three CP-even mass eigenstates have masses \(m_{H_i} (i = 1, 2, 3)\). These obey \(m_{H_1} \sim m_{H_2} \ll m_{H_3}\), where the last one is much larger than the first two due to the large \(v_\sigma\) and the corresponding eigenstate effectively decouples. Indeed, from the considerations of the previous section, we have \(v_\chi/v_\sigma \lesssim 10^{-8}\), which indicates the hierarchy of the VEVs, \(v_\chi < v \ll v_\sigma\). As a result, we further decompose the \(3 \times 3\) mass matrix into four blocks

\[
M_{\text{CP-even}}^2 = \begin{pmatrix}
M_{\sigma}^2 & M_{h_\sigma}^2 \\
M_{h_\sigma}^2 & M_{h}^2
\end{pmatrix}, \tag{11}
\]

with

\[
M_h^2 \equiv \begin{pmatrix}
\lambda v^2 & \frac{1}{2}\lambda_a v v_\chi - \frac{1}{2}\kappa v \\
\frac{1}{2}\lambda_a v v_\chi - \frac{1}{2}\kappa v & \frac{\kappa v^2}{4v_\chi} + \frac{\lambda_\chi v_\chi^2}{2}
\end{pmatrix}, \quad M_{h_\sigma}^2 \equiv \begin{pmatrix}
\frac{1}{2}\lambda_b v v_\sigma \\
\frac{1}{2}\lambda_c v_\sigma v_\chi
\end{pmatrix}, \quad M_\sigma^2 \equiv \lambda_\sigma v_\sigma^2. \tag{12}
\]

The masses of the neutral scalars receive contributions from couplings to \(\sigma\). In the limit of vanishing \(M_h^2\), the couplings to \(\sigma\) contribute to the \(2 \times 2\) \(M_h^2\) block in the diagonalisation and can be calculated in a seesaw-like approximation as

\[
(M_h^2)^2 \equiv -\frac{1}{M_\sigma^2} M_{h_\sigma}(M_{h_\sigma})^T = -\frac{1}{4\lambda_\sigma} \begin{pmatrix}
\lambda_b v^2 & \lambda_b \lambda_\sigma v v_\chi \\
\lambda_b \lambda_\sigma v v_\chi & \lambda_\sigma v_\sigma^2
\end{pmatrix}. \tag{13}
\]
A rough estimate tells us that this contribution is of the same order as that of $M_2^2$. Consequently, the leading-order mass matrix for the two light scalars $(\phi^0, \chi^0)$ reads

$$M^2_{h\chi} \equiv M_h^2 + (M_h^\sigma)^2 = \begin{pmatrix}
\lambda v^2 - \frac{\lambda^2 v^2}{4\lambda_\sigma} & -\frac{\kappa v}{2} + \frac{\lambda_\sigma vv_\chi}{4\lambda_\sigma} - \frac{\lambda_b \lambda_c vv_\chi}{4\lambda_\sigma} \\
-\frac{\kappa v}{2} + \frac{\lambda_a vv_\chi}{4\lambda_\sigma} - \frac{\lambda_\sigma v_\chi^2}{4\lambda_\sigma} & \frac{\kappa v^2}{4v_\chi} - \frac{\lambda_c^2 v_\chi^2}{2} + \frac{\lambda_\chi v_\chi^2}{2}
\end{pmatrix},$$

which leads to an estimate for the neutral scalar mixing angle $\alpha$ of

$$\tan 2\alpha = \frac{2v vv_\chi}{\kappa v_\chi (\lambda_c^2 - 2\lambda_\sigma \lambda_\chi) - v^2 [\kappa \lambda_\sigma + v_\chi (\lambda_b^2 - 4\lambda_\lambda_\sigma)]},$$

such that $O^T M^2_{h\chi} O = \text{Diag}\{m_{h_1}^{\text{LO}}, (m_{h_2})^2\}$ with $O$ being a $2 \times 2$ rotation matrix parameterised by the angle $\alpha$. This approximation will be useful in understanding the parameter correlations discussed in Section 3.3. While one can solve for $m_{h_1}^{\text{LO}}$ and $m_{h_2}^{\text{LO}}$ starting from Eq. (14), the expressions are lengthy and we do not show them here. Although the block-diagonalisation procedure discussed so far is convenient to understand the leading-order contributions, in our numeric study we take into account the full $3 \times 3$ matrix as given in Eq. (10).

The mass matrix for the charged scalars in the basis $(\phi^\pm, \chi^\pm)$ is

$$M^2_{\text{charged}} = \begin{pmatrix}
\kappa vv_\chi & \frac{\kappa v}{2} \\
\frac{\kappa v}{2} & \frac{\kappa v^2}{4v_\chi}
\end{pmatrix}. $$

One of the two charged-scalar masses is zero, corresponding to the charged Goldstone boson that becomes the longitudinal component of $W^{\pm}$. The only non-zero squared mass is

$$m_{H^\pm}^2 = \frac{\kappa (v^2 + 4v_\chi^2)}{4v_\chi},$$

which grows with $\kappa$. The mixing is given by $\tan 2\beta = 4v vv_\chi/(v^2 - 4v_\chi^2)$. Inputting $v_\chi$ constrained by the CDF result, we find a value of $\beta \simeq 0.044$ for the mixing angle.

### 3.2 Constraints

The model parameter space is subject to many constraints. To start with, the potential should be bounded from below in any direction of large field values. This condition can be quantified by requiring the copositivity of the quartic coupling matrix [71]. In our model, the copositivity conditions read

$$\lambda \geq 0, \quad \lambda_\chi \geq 0, \quad \lambda_\sigma \geq 0, \quad \lambda_a \geq 0, \quad \lambda_b \geq 0, \quad \lambda_c \geq 0, \quad \lambda_\sigma \geq 0, \quad \lambda_\chi \geq 0, \quad \lambda_\lambda \geq 0, \quad \lambda_\sigma \geq 0, \quad \lambda_\chi \geq 0.$$
The perturbativity bound requires instead that all the quartic couplings remain perturbative at any scale, i.e. $|\lambda_i| < 4\pi$ (with $i$ being a pseudo-index running over all the quartic couplings). There are also constraints from requiring the unitarity of the $S$-matrix \[30,72\]:

$$|\lambda|, |\lambda_\sigma| < 8\pi, \quad |\lambda_a|, |\lambda_b|, |\lambda_c|, |\lambda_\chi| < 16\pi.$$ (19)

Unitarity gives three additional constraints, namely upper bounds on the quantities in Eq. (26) of Appendix A, where more details on the unitarity bounds can be found.

Additionally, we are interested in identifying the regions of parameter space where the desired vacuum configuration $(v, v_\chi, v_\sigma)$ with all VEVs non-zero is a global minimum. We therefore need to exclude deeper minima from alternative configurations $(v', v'_\chi, v'_\sigma)$ with one or more vanishing VEVs. Confining our attention to such charge-conserving VEVs, a direct minimum depth comparison results in a difference $\Delta V = V' - V$ with

$$\Delta V = \frac{1}{16} \left[ 2\lambda (v^4 - v'^4) + \lambda_\chi (v_\chi^4 - v'_\chi^4) + 2\lambda_\sigma (v_\sigma^4 - v'^4) + 2\lambda_c (v_\chi^2 v_\sigma^2 - v^2 v'^2) 
+ 2v^2 (\lambda_a v_\chi^2 + \lambda_b v_\sigma^2 - \kappa v_\chi) - 2v'^2 (\lambda_a v_\chi^2 + \lambda_b v_\sigma^2 - \kappa v'_\chi) \right],$$ (20)

which we require to be non-negative for all seven patterns $(\nu, \nu'_\chi, \nu'_\sigma) = (0, 0, 0), (\nu', 0, 0), (0, \nu'_\chi, 0), (\nu'_\chi, 0, 0), (0, 0, \nu'_\sigma)$, in case these lead to a positive definite mass-squared matrix. Note that the primed VEVs in this equation are constrained to satisfy stationarity conditions of their own, but with the same $\mu_H^2, \mu_\chi^2$ and $\mu_\sigma^2$ as given in Eq. (9) in terms of unprimed VEVs. We also check that the minima candidates are not locally destabilised by turning on charge-breaking VEVs (see [73] for an in-depth analysis in the $Y = 1$ triplet case).

Constraints from the experimental side arise mainly from two sources: electroweak precision measurements and collider experiments. For the former, we consider the constraint on the triplet VEV $v_\chi$ from the $\rho$ parameter, taking into account the latest measurement of the $W$ mass (see Section 2.3). The bounds on oblique parameters, i.e. on the Peskin-Takeuchi parameters $S, T$, and $U$ [74], also impose stringent constraints on models of new physics above the electroweak scale. The contributions beyond the tree level from the hyperchargeless triplet to a modified version of these parameters, adapted to this context, are \[28,29\].

$$S \simeq 0,$$ (21a)

$$T = \frac{1}{8\pi} \frac{1}{\sin^2 \theta_W \cos^2 \theta_W} \left[ \frac{m_{H_2}^2 + m_{H^\pm}^2}{m_Z^2} - \frac{2m_{H^\pm}^2 m_{H_2}^2}{m_Z^2 (m_{H_2}^2 - m_{H^\pm}^2)} \log \left( \frac{m_{H_2}^2}{m_{H^\pm}^2} \right) \right] \simeq \frac{1}{6\pi} \frac{1}{\sin^2 \theta_W \cos^2 \theta_W} \frac{(\Delta m)^2}{m_Z^2},$$ (21b)

$$U \simeq -\frac{1}{3\pi} \left[ m_{H_2}^4 \log \left( \frac{m_{H_2}^2}{m_{H^\pm}^2} \right) \frac{3m_{H^\pm}^2 - m_{H_2}^2}{(m_{H_2}^2 - m_{H^\pm}^2)^2} + \frac{5(m_{H_2}^4 + m_{H_2}^4) - 22m_{H_2}^2 m_{H^\pm}^2}{6(m_{H_2}^2 - m_{H^\pm}^2)^2} \right] \simeq \frac{\Delta m}{3\pi m_{H^\pm}},$$ (21c)

where $m_Z$ is the $Z$ boson mass, $\theta_W$ is the Weinberg angle, and $\Delta m \equiv m_{H_2} - m_{H^\pm}$. The last approximations hold for $|\Delta m| \ll m_{H^\pm}$. To be consistent with the updated fits of Refs. [50,75],
we require that $m_{H^\pm} \sim m_{H_2} \gg m_{H_1}$, where $m_{H_1}$ is assumed to be the SM Higgs. To be more specific, given that the benchmark $v_{\chi} = 5.4$ GeV already produces a large tree-level contribution $T_{\text{tree}} = \beta^2/\alpha_{\text{e.m.}} \simeq 0.25$, the fitted CDF value of $T = 0.27 \pm 0.06$ \cite{75} (under the assumption of negligible $U$) requires the additional loop-level contributions of Eq. (21) to be small, leading to the bound $|\Delta m| < 50$ GeV.

As for collider constraints, an important channel is that of Higgs decay into two photons, corresponding to a signal strength of $\mu_{\gamma\gamma} = 1.14^{+0.19}_{-0.18}$ \cite{76}. In our model, the novel contribution to $\mu_{\gamma\gamma}$ is dominated by $\lambda_a/m_{H^\pm}^2$ and can be made negligible if $m_{H^\pm} > 300$ GeV \cite{30}. Additionally, LEP provides the most stringent bound on the mass of a neutral scalar which is produced in association with the $Z$ boson, $m_h > 114$ GeV \cite{77,78}. However, it is possible to evade this bound in a hyperchargeless triplet model since the coupling of the new neutral scalar to the $Z$ may be suppressed \cite{72}. Therefore, in case there is a scalar lighter than the SM Higgs, we impose the constraint

$$|\sin \alpha| < 0.05,$$

which implies that $|\cos \alpha| \simeq 1$ and the LEP bound is not violated. If the lightest scalar has a mass below half the SM Higgs mass, it can contribute to the Higgs invisible decay rate and is subject to further constraints. Meanwhile, $\chi^0$ also couples to the $W$ boson and has the potential to be produced via such an interaction. A full analysis of the parameter space taking into consideration all Higgs search limits is beyond the scope of the current work. In a simplified analysis, we focus on Eq. (22) as the main constraint on a light scalar spectrum. Finally, the vector-like fermion triplets $\Delta_F$ acquire masses proportional to the PQ symmetry breaking scale and are thus safe from otherwise stringent low-energy limits (see e.g. \cite{79}).

### 3.3 Numerical results and discussion

Following the analysis of the mass spectrum and the above discussion on constraints, we are ready to search for the viable parameter space at the electroweak scale. We express the potential parameters $\mu^2_H$, $\mu^2_\chi$, $\mu^2_\sigma$ in terms of the non-zero VEVs ($v, v_\chi, v_\sigma$) and the quartic and trilinear couplings using the stationarity conditions in Eq. (9). The constraints to the quartic couplings can be directly applied. Meanwhile, there are constraints expressed in terms of the masses, which are also functions of the VEVs and the quartic and trilinear couplings, according to Eqs. (10) and (16). There are two possible mass spectra with some differences in constraints ($m_H$ is the SM Higgs mass):

- **“Heavy spectrum”**, with $m_{H_1} = m_H < m_{H_2}$, referring to the case where the new scalar has mass $m_{H_2}$ and is heavier than the SM Higgs. In this case, the oblique parameter constraints of Eq. (21) require $m_{H^\pm} \sim m_{H_2} \gg m_H$, as mentioned before.

- **“Light spectrum”**, with $m_{H_1} < m_{H_2} = m_H$, referring to the case where the new scalar has mass $m_{H_1}$ and is lighter than the SM Higgs. Oblique parameter constraints, which assume the scale of new physics to be large with respect to the electroweak scale, do not apply. Instead, we consider the bound of Eq. (22) to suppress the coupling to the $Z$ boson such that the LEP bound is not violated.
the vacuum of the type (0, 0) while for the light spectrum, it may only be a local one. In particular, for a light spectrum, we find that the desired vacuum can be the global one, the depth of the latter with that of the desired vacuum to guarantee they are not deeper, see Eq. (20). For the heavy spectrum, we find that the desired vacuum can be the global one. This is done by numerically checking whether the potential defined by the scanned parameters admits other types of vacua. If so, we compare the depth of the latter with that of the desired vacuum to guarantee they are not deeper, see Eq. (20). For the heavy spectrum, we find that the desired vacuum can be the global one, while for the light spectrum, it may only be a local one. In particular, for a light spectrum, the vacuum of the type (0, 0) is always deeper than the desired one. The difference ∆V is

$$V_{(0, v'_\chi, 0)} - V_{(v, v'_\chi, v_\sigma)} = \frac{1}{16} \left[ 2\lambda v^4 + 2v^2 (\lambda_\sigma v_\sigma^2 + \lambda_\alpha v_\alpha^2 - \kappa v_\chi) + 2\lambda_\sigma v'_\sigma^4 + 2\lambda_\chi v'_\chi v_\chi + \lambda_\chi (v'_\chi - v'_\chi) \right]$$

$$\approx \frac{1}{16} \left( 2\lambda_\sigma v'_\sigma^4 - \lambda_\chi v'_\chi \right). \quad (23)$$

Numerically, we find the points passing all the other constraints lead to $v'_\chi \sim O(10^{13})$ GeV, and thus to a negative value of the difference, given positive $\lambda_\sigma$ and $\lambda_\chi$. Although not being stable, it is still possible that the desired vacuum is meta-stable in the sense that the

|       | Light spectrum ($m_{H_2} = m_H$) | Heavy spectrum ($m_{H_1} = m_H$) |
|-------|----------------------------------|----------------------------------|
| $\lambda$ | [0.0011, 0.26]               | [0.26, 1.22]                   |
| $\lambda_\chi$ | $[1.01 \times 10^{-6}, 0.10]$      | $[1.01 \times 10^{-6}, 10.89]$              |
| $\lambda_\sigma$ | $[1.05 \times 10^{-6}, 8.99 \times 10^{-4}]$ | $[1.08 \times 10^{-6}, 11.53]$              |
| $\lambda_a$ | $[0.03, 3.15]$               | $[-1.27, 12.14]$               |
| $\lambda_b$ | $[-0.0044, -0.0001]$          | $[1.11 \times 10^{-6}, 5.40]$              |
| $\lambda_c$ | $[0.10, 3.68]$               | $[1.34 \times 10^{-6}, 12.17]$              |
| $\kappa$ [GeV] | $[32.06, 99.93]$           | $[32.07, 99.90]$               |
| $m_{H_1}(m_{H_2})$ [GeV] | $[0.25, 123.15]$           | $[258.34, 529.29]$          |
| $m_{H_2}$ [GeV] | $[1.00 \times 10^9, 3.00 \times 10^{10}]$ | $[1.04 \times 10^9, 3.40 \times 10^{12}]$          |
| $m_{H^\pm}$ [GeV] | $[300.01, 529.63]$           | $[300.04, 529.55]$          |

Table 2: The viable parameter space at the electroweak scale.

Numerically, we fix the VEVs to be

$$v = 246 \text{ GeV}, \quad v_\sigma = 10^{12} \text{ GeV}, \quad v_\chi = 5.4 \text{ GeV},$$

and randomly scan the trilinear and the quartic couplings in the ranges

$$\kappa \in [10, 100] \text{ GeV}, \quad \lambda, \lambda_\chi, \lambda_\sigma, |\lambda_a|, |\lambda_b|, |\lambda_c| \in [0, 4\pi].$$

In practice, for quartic couplings we take a flat logarithmic prior with a lower limit of $10^{-6}$. We also require that the mass of the SM Higgs-like scalar lies in the $3\sigma$ range of $125.25 \pm 0.51$ GeV [70], and scan for both possibilities, $m_{H_1} = m_H < m_{H_2}$ and $m_{H_1} < m_{H_2} = m_H$.

The parameter ranges for points satisfying both the theoretical and experimental constraints are shown in Table 2. For these points, we will further check if they allow the desired vacuum ($v, v_\chi, v_\sigma$) to be the global one. This is done by numerically checking whether the potential defined by the scanned parameters admits other types of vacua. If so, we compare the depth of the latter with that of the desired vacuum to guarantee they are not deeper, see Eq. (20). For the heavy spectrum, we find that the desired vacuum can be the global one, while for the light spectrum, it may only be a local one. In particular, for a light spectrum, the vacuum of the type (0, 0) is always deeper than the desired one. The difference ∆V is
Figure 3: Two-dimensional projections of the viable parameter space for the heavy spectrum, $m_{H_1} = m_H < m_{H_2}$, satisfying all the constraints while having the desired vacuum as a global minimum.

tunnelling time to other, deeper vacua is longer than the age of the Universe. We do not investigate this possibility here.

We require the couplings to remain perturbative and that the desired vacuum stays stable up to the PQ breaking scale, where other new physics is expected to come in. The evolution of the couplings is governed by the one-loop renormalisation group equations (RGEs), which are calculated using SARAH [80, 81], see Appendix B. As a first approximation, we analyse the RGE-improved tree-level potential (see also e.g. [82, 83]). The parameter space shown in Table 2 gets further constrained by perturbativity, copositivity and unitarity at the PQ scale. Roughly speaking, large values of the quartic couplings are ruled out.

The final viable parameter space of our model is presented in the form of two-parameter projections in Figures 3 and 4. For both spectra, we find regions of viable parameter space at the PQ scale, meaning that the desired vacuum can be stable at least up to this scale. For
the heavy spectrum (Fig. 3) several comments are in order:

- The top-left plot in Fig. 3 shows the mass spectrum with varying $\lambda_\sigma$. As one may expect from the discussion of Section 3.1, the approximate relation $m_{H_3}^2 \simeq \lambda_\sigma v^2_\sigma$ holds. Other scalar masses do not seem to be sensitive to $\lambda_\sigma$, even after taking into account all the discussed constraints, especially those on the mass spectrum (matching the SM Higgs mass and satisfying the upper limit on the charged-neutral mass splitting $\Delta m$). The upper bound of $m_{H_3}$ is set by the upper bound of $\lambda_\sigma$. There is no lower bound on $m_{H_3}$, which can be made smaller at the cost of tuning $\lambda_\sigma$ to very small values.

- The top-right plot shows the correlations between the Higgs quartic $\lambda$ and the $\sigma$-related quartic couplings $\lambda_b$ and $\lambda_c$. These are not affected by the requirement of having a global minimum. The bottom-left plot shows instead the dependence of the scalar mass $m_{H_2}$ on $\kappa$. Since the charged-scalar mass $m_{H^\pm}$ depends solely on $\kappa$ (see Eq. (17)) and $|\Delta m|$ is bounded, $m_{H_3}$ is expected to grow with $\sqrt{\kappa}$. We find this dependence becomes rather sharp, i.e. the numerically allowed values of mass splitting become quite small ($|\Delta m| \lesssim 0.1$ GeV), after excluding points not leading to a global minimum. The bottom-right plot shows the relation between $\lambda_\chi$ and $\lambda_c$. The $\lambda_\chi$-dependent upper bound on $\lambda_c$ arises only after applying the global minimum filter.

- Comparing the plotted parameter ranges with those in Table 2, we see that all the quartic coupling ranges shrink. Indeed, running up to the PQ scale and imposing the relevant constraints at that scale excludes the large-quartic portion of the parameter space, as previously mentioned. As we have seen in the previous point, asking for a global minimum also imposes non-trivial restrictions. This requirement further excludes the region $\lambda_\chi < 0.5$ for points with $\Delta m < 0$, and the upper bound on $\lambda_c$ becomes more stringent overall, going from $\lambda_c \lesssim 3$ to $\lambda_c \lesssim 0.8$.

For the light spectrum (Fig. 4), we also plot the final viable parameter space in terms of its projections in planes of two parameters, and several comments are in order:

- The top-left plot in Fig. 4 shows the mass spectrum with varying $\lambda$. Only $m_{H_1}$ grows with $\lambda$ while the other masses are mostly insensitive to it. In contrast to the case of the heavy spectrum, the heaviest neutral scalar $m_{H_3}$ now spans a much narrower range, roughly from $10^9$ GeV to $10^{10}$ GeV, which results from a much more stringent upper bound on $\lambda_\sigma$ (cf. Table 2 and the other subplots in the figure).

- The top- and bottom-right plots involve parameters directly related to $\sigma$ and can be read together to understand the upper limit on $m_{H_3}$. The mass $m_{H_3}$ grows with $\lambda_\sigma$ and thus with $|\lambda_b|$ and $\lambda_c$, due to the their correlations. However, $\lambda_c$ grows with $\lambda_\sigma$ rather fast, easily leading to exclusion when the RGE running is accounted for, bounding $\lambda_\sigma$ and consequently $m_{H_3}$ from above. The dependence of $m_{H_3}$ on $\lambda_\sigma$ is otherwise similar to that of the heavy mass spectrum and has no lower bound if we allow $\lambda_\sigma$, $|\lambda_b|$ and $\lambda_c$ to be vanishingly small.

- The bottom-left plot shows the relation between $\kappa$ and $\lambda_\sigma$. The lower bound on $\kappa$ is set by the lower bound on the charged Higgs mass. Besides the chosen cut at 100 GeV,
we see that there is an upper bound on $\kappa$, which becomes more stringent for larger $\lambda_\sigma$. This rough bound can be understood from the formula of Eq. (15) for the light scalar mixing and the requirement that said mixing is small, see Eq. (22). In particular, $\tan 2\alpha$ depends on $\kappa$ via the product $\kappa \lambda_\sigma$, which determines the observed exclusion.
4 Conclusions

In this work, we investigate the connection between tree-level Dirac neutrino masses and axion physics in a scenario where the PQ scale $f_a$ is the only heavy scale to play a role in neutrino mass generation. To realise such a connection, we focus on the diagram of Figure 1 as the main contribution to neutrino masses and build the model based on it. The minimal construction leads us to a KSVZ-type model, in which the SM scalar sector is extended by a real triplet $\Delta_\chi$ and by the PQ field $\sigma$. Scalars other than $\sigma$ are not charged under PQ. We find the PQ symmetry by itself is enough to explain the Dirac nature of neutrino masses in such a setup, i.e. the PQ symmetry enforces lepton number conservation perturbatively.

The scale $f_a$ suppresses Dirac neutrino masses and is consequently bounded from above, $f_a \lesssim 10^{13}$ GeV. The QCD axion in the model addresses the strong CP problem, while being a potential dark matter candidate. Future prospects for its detection have been discussed (see Fig. 2). In turn, the real scalar triplet contributes to the $W$ boson mass via its VEV and may be responsible for the recent hint of beyond-the-SM physics by the CDF collaboration. Finally, we look into the scalar sector of the model. We identify the regions in parameter space compatible with the desired VEV structure, taking into account electroweak precision constraints and the requirements of copositivity and perturbativity up to the PQ scale. Besides the SM-like Higgs, there is another light neutral scalar that can be either heavier or lighter than the former. The two possible spectra are dubbed “heavy” and “light”, respectively. We find that for the heavy spectrum the desired EW vacuum can be the global one up to the PQ scale, while it is only found to be a local one in the case of the light spectrum.

This work can be extended in many ways. On the one hand, a full survey of the light-spectrum parameter space, as well as the heavy-spectrum one, may lead to interesting collider phenomenology. On the other hand, there are rich Yukawa structures to be explored, which, working e.g. with flavour symmetry, have the potential to address the neutrino mixing pattern and enhance the predictive power. Last but not least, it would be interesting to examine whether neutrinogenesis is viable in this context.

Acknowledgements

X.Y.Z. would like to thank Prof. Shun Zhou for helpful discussions and comments. The work of J.T.P. was supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the projects PTDC/FIS-PAR/29436/2017, CERN/FIS-PAR/0004/2019, CERN/FIS-PAR/0008/2019, and CFTP-FCT Unit 777 (namely UIDB/00777/2020 and UIDP/00777/2020), which are partially funded through POCTI (FEDER), COMPETE, QREN and EU. The work of Y.R. was supported by the Doctoral Program of Tian Chi Foundation of Xinjiang Uyghur Autonomous Region of China under grant No. TCBS202128 and by the Natural Science Foundation of Xinjiang Uyghur Autonomous Region of China under grant No. 2022D01C52. The work of X.Y.Z. was supported in part by the National Natural Science Foundation of China under grant No. 11835013 and by the Key Research Program of the Chinese Academy of Sciences under grant No. XDPB15.
A Unitarity bounds on the quartic couplings

The unitarity of the scattering matrix for $2 \rightarrow 2$ process puts constraints on the model parameters. At high energies, according to the Goldstone boson equivalence theorem, scattering amplitudes of the longitudinal gauge boson can be well approximated by those of the corresponding Goldstone boson. Dominant contributions to the scattering amplitudes come from the quartic couplings of the scalars. In the following, we compute all possible $2 \rightarrow 2$ scattering matrices, classified by the total charges of the initial/final state particles.

Considering the total electric charge of the initial states is zero, the S-matrix can be written as a direct sum of the following two matrices:

$$\mathcal{M}^{(0)}_1 = \left( \begin{array}{cccccccccccc}
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_a}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_a}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\lambda_a}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\lambda_a}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_b}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_b}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_c}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_c}{2} & 0 & 0 \\
\end{array} \right), \quad (24)$$

for the initial state basis $(\phi^0 G_H, \phi_a G_\sigma, \chi^0 G_H, \phi^0 \chi^0, \phi^+ \chi^-, \chi^+ \phi^-, \phi^0 G_\sigma, \phi^0 \phi_\sigma, \phi_\sigma G_H, G_H G_\sigma, \chi^0 G_\sigma, \phi_\sigma \chi^0)$, and

$$\mathcal{M}^{(0)}_2 = \left( \begin{array}{cccccccccccc}
\frac{3\lambda}{2} & \lambda_b & \lambda & \lambda_b & \lambda & \lambda_a & \lambda_a & \lambda_a & \lambda_a & \lambda_a & \lambda_a & \lambda_a \\
\frac{\lambda_b}{3\lambda} & \frac{\lambda_b}{2} & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b \\
\frac{\lambda}{4} & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b \\
\frac{\lambda}{4} & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b \\
\frac{\lambda}{4} & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b \\
\frac{\lambda}{4} & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b \\
\frac{\lambda}{4} & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b & \lambda_b \\
\frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} & \frac{\lambda_b}{2\lambda_a} \\
\frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} \\
\frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} \\
\frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} & \frac{\lambda_b}{4\lambda_a} \\
\end{array} \right), \quad (25)$$
for the initial states \( \left( \phi^0 \phi^0 / \sqrt{2}, \phi \sigma \phi \sigma / \sqrt{2}, G_H G_H / \sqrt{2}, G_\sigma G_\sigma / \sqrt{2}, \phi^+ \phi^-, \chi^+ \chi^-, \chi^0 \chi^0 / \sqrt{2} \right) \). Here, the factor \( 1 / \sqrt{2} \) takes care of the statistics for identical particles. The eigenvalues of the matrix \( M_2(0) \) are \( \lambda \) (with multiplicity 2), \( \lambda_\sigma \), \( \lambda_\chi / 2 \), and

\[
\frac{1}{12} A + \frac{1}{2} \sqrt{ \frac{B}{3} } \cos \left[ \frac{1}{3} \cos^{-1} \left( \frac{3C}{2} \sqrt{ \frac{3}{B^3} } \right) + \frac{2k \pi}{3} \right], \quad \text{with } k = 0, 1, 2,
\]

where

\[
A = 12 \lambda + 8 \lambda_\sigma + 5 \lambda_\chi, \quad (27a) \\
B = \frac{A^2}{3} - 96 \lambda \lambda_\sigma - 60 \lambda \lambda_\chi + 12 \lambda_\sigma^2 + 8 \lambda_\chi^2 + 6 \lambda_\chi^2 - 40 \lambda \lambda_\chi, \quad (27b) \\
C = \frac{A}{3} \left( B - \frac{A^2}{9} \right). \quad (27c)
\]

For the charge +1 initial states \( \left( \phi^+ \phi^0, \phi^+ G_H, \chi^+ \chi^0, \phi^+ \phi^0, \chi^+ G_H, \phi^+ G_\sigma, \phi^+ \phi_\sigma, \chi^+ \phi_\sigma, \chi^+ G_\sigma \right) \), we obtain the following \( 10 \times 10 \) diagonal S-matrix

\[
\mathcal{M}^{(+1)} = \begin{pmatrix}
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_\chi / 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_\sigma / 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_\sigma / 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_\sigma / 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_\chi / 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_\sigma / 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_\chi / 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_\sigma / 2
\end{pmatrix}.
\]

Finally, the S-matrix of the charge +2 initial states \( \left( \phi^+ \phi^+ / \sqrt{2}, \chi^+ \chi^+ / \sqrt{2}, \phi^+ \chi^+ \right) \) is

\[
\mathcal{M}^{(+2)} = \begin{pmatrix}
\lambda & 0 & 0 \\
0 & \lambda_\chi / 2 & 0 \\
0 & 0 & \lambda_\sigma / 2
\end{pmatrix}.
\]

Unitarity constraints on the S-matrices force the absolute values of the eigenvalues of the matrices to be less than \( 8 \pi \). This implies the following upper bounds on the quartic couplings

\[
|\lambda|, \ |\lambda_\sigma| < 8 \pi, \quad |\lambda_\sigma|, \ |\lambda_\sigma|, \ |\lambda_\chi| < 16 \pi, \quad (30)
\]

and, on top of these, extra constraints are imposed by bounding the eigenvalues of the matrix \( \mathcal{M}_2^{(0)} \), given in Eq. (26).
B One-loop RGEs

In this work, we calculate the RGEs up to the one-loop level using the Mathematica package SARAH \cite{SARAH1,SARAH2}. The beta function of the coupling $X$ is defined as

$$\beta_X = \mu \frac{\partial X}{\partial \mu} = \frac{1}{16\pi^2} \beta^{(1)}_X. \quad (31)$$

The beta functions for the gauge couplings read

$$\beta^{(1)}_{g_1} = \frac{41}{10} g_1^3, \quad (32a)$$

$$\beta^{(1)}_{g_2} = \frac{5}{2} g_2^3, \quad (32b)$$

$$\beta^{(1)}_{g_3} = -\frac{19}{3} g_3^3. \quad (32c)$$

Here, $g_1 \equiv \sqrt{5/3} g'$. Note that the (high-energy) beta function for $g_2$ is modified not just by the new scalar triplet but also by the new triplet fermions $\Delta_F$. The beta function of $g_3$ is instead modified with respect to the SM one due to the presence of the vector-like quark $Q$. The beta functions for the Yukawa couplings are

$$\beta^{(1)}_{Y_F} = \frac{1}{8} \left(2Y_F Y_F^\dagger Y_F + 2Y_F Y_L^\dagger Y_L + 6Y_F \left(-16g_2^2 + 4|Y_Q|^2 + \text{Tr}(Y_F Y_F^\dagger)\right) + Y_R^T Y_R Y_F\right), \quad (33a)$$

$$\beta^{(1)}_{Y_L} = \frac{1}{8} \left(5 \left(4Y_e Y_e^\dagger Y_L + Y_L Y_L^\dagger Y_L\right) + Y_L Y_F^\dagger Y_F\right)$$

$$+ Y_L \left(3\text{Tr}(Y_d Y_d^\dagger) + 3\text{Tr}(Y_u Y_u^\dagger) - \frac{33}{4} g_2^2 + \frac{3}{4} \text{Tr}(Y_L Y_L^\dagger) - \frac{9}{20} g_1^2 + \text{Tr}(Y_e Y_e^\dagger)\right), \quad (33b)$$

$$\beta^{(1)}_{Y_u} = \frac{3}{2} \left(Y_u Y_u^\dagger Y_u - Y_d Y_d^\dagger Y_u\right) + Y_u \left(3\text{Tr}(Y_d Y_d^\dagger) + 3\text{Tr}(Y_u Y_u^\dagger)\right)$$

$$- 8g_3^2 - \frac{17}{20} g_1^2 + \frac{3}{4} \text{Tr}(Y_L Y_L^\dagger) - \frac{9}{4} g_2^2 + \text{Tr}(Y_e Y_e^\dagger)\right), \quad (33c)$$

$$\beta^{(1)}_{Y_Q} = 4Y_Q Y_Q^\dagger Y_Q^\dagger - 8g_3^2 Y_Q + \frac{3}{4} Y_Q \text{Tr}(Y_F Y_F^\dagger), \quad (33d)$$

$$\beta^{(1)}_{Y_R} = \frac{1}{8} \left(Y_R Y_R^* Y_R^T + Y_R \left(-6g_2^2 + \frac{1}{2} \text{Tr}(Y_R Y_R^\dagger)\right) + Y_R Y_R^\dagger Y_R\right), \quad (33e)$$

$$\beta^{(1)}_{Y_d} = \frac{1}{4} \left(6 \left(Y_d Y_d^\dagger Y_d - Y_u Y_u^\dagger Y_d\right) + Y_d \left(12\text{Tr}(Y_d Y_d^\dagger) + 12\text{Tr}(Y_u Y_u^\dagger) - 32g_3^2\right)$$

$$+ 3\text{Tr}(Y_L Y_L^\dagger) + 4\text{Tr}(Y_e Y_e^\dagger) - 9g_2^2 - g_1^2\right), \quad (33f)$$

$$\beta^{(1)}_{Y_e} = \frac{1}{8} \left(3 \left(4Y_e Y_e^\dagger Y_e + 5Y_L Y_L^\dagger Y_e\right) + Y_e \left(24\text{Tr}(Y_d Y_d^\dagger)\right.$$

$$\left. + 6 \left(-3(g_1^2 + g_2^2) + 4\text{Tr}(Y_u Y_u^\dagger) + \text{Tr}(Y_L Y_L^\dagger)\right) + 8\text{Tr}(Y_e Y_e^\dagger)\right)). \quad (33g)$$
The beta functions for the quartic scalar couplings are

\[ \beta_{\lambda}^{(1)} = \frac{27}{100} g_1^4 + \frac{9}{10} g_1^2 g_2 + \frac{9}{4} g_1^2 - \frac{9}{5} g_1^2 \lambda - 9 g_2^2 \lambda + 12 \lambda^2 + \frac{3}{4} \lambda_a^2 + \frac{1}{2} \lambda_b^2 + 12 \lambda \text{Tr}(Y_d Y_d^\dagger)
\]
\[ + 4 \lambda \text{Tr}(Y_e Y_e^\dagger) + 3 \lambda \text{Tr}(Y_e Y_L Y_L^\dagger) + 12 \lambda \text{Tr}(Y_u Y_u^\dagger) - 12 \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 4 \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger)
\]
\[ - 4 \text{Tr}(Y_e Y_L Y_L Y_e^\dagger) - \frac{5}{4} \text{Tr}(Y_L Y_L Y_L Y_L^\dagger) - 12 \text{Tr}(Y_a Y_a^\dagger Y_a Y_a^\dagger), \]
\[ (34a) \]

\[ \beta_{\lambda_a}^{(1)} = 10 \lambda_a^2 + 12 \lambda_a |Y_Q|^2 - 12 |Y_Q|^4 + 3 \lambda_a \text{Tr}(Y_F Y_F^\dagger) + \frac{3}{4} \lambda_e^2 - \frac{3}{4} \lambda_c^2 + \frac{19}{2} \lambda_a^2 + \lambda_c^2, \]
\[ (34b) \]

\[ \beta_{\lambda_b}^{(1)} = -24 g_2^2 \lambda_b + 2 \lambda_b^2 + 2 \lambda_b \text{Tr}(Y_R Y_R^\dagger) - 2 \text{Tr}(Y_R Y_R^\dagger Y_R Y_R^\dagger) + \frac{19}{2} \lambda_b^2 + \lambda_c^2, \]
\[ (34c) \]

\[ \beta_{\lambda_c}^{(1)} = -9 g_2^2 \lambda_c + 9 g_2^2 \lambda_b - 9 g_2^2 \lambda_b + 6 \lambda_b^2 + 2 \lambda_b^2 + \frac{3}{2} \lambda_a \lambda_c + 4 \lambda_b \lambda_a + 6 \lambda_b |Y_Q|^2 + 6 \lambda_b \text{Tr}(Y_d Y_d^\dagger)
\]
\[ + 2 \lambda_b \text{Tr}(Y_e Y_e^\dagger) + \frac{3}{2} \lambda_b \text{Tr}(Y_F Y_F^\dagger) + \frac{3}{2} \lambda_b \text{Tr}(Y_L Y_L^\dagger) + 6 \lambda_b \text{Tr}(Y_u Y_u^\dagger) - \frac{3}{2} \text{Tr}(Y_F Y_F Y_F^\dagger Y_L)
\]
\[ (34d) \]

\[ \beta_{\lambda_a}^{(1)} = -9 g_1^2 \lambda_a - \frac{33}{2} g_2^2 \lambda_a + 6 \lambda_a^2 + 2 \lambda_a^2 + \lambda_b \lambda_c + 6 \lambda_a \text{Tr}(Y_d Y_d^\dagger) + 2 \lambda_a \text{Tr}(Y_e Y_e^\dagger)
\]
\[ + \frac{3}{2} \lambda_a \text{Tr}(Y_L Y_L^\dagger) + \lambda_a \text{Tr}(Y_R Y_R^\dagger) + 6 \lambda_a \text{Tr}(Y_u Y_u^\dagger), \]
\[ (34e) \]

\[ \beta_{\lambda_c}^{(1)} = 2 \lambda_a \lambda_b - 12 g_2^2 \lambda_c + 2 \lambda_c^2 + 4 \lambda_c \lambda_a + \frac{5}{2} \lambda_c \lambda_x + 6 \lambda_c |Y_Q|^2 + \frac{3}{2} \lambda_c \text{Tr}(Y_F Y_F^\dagger) + \lambda_c \text{Tr}(Y_R Y_R^\dagger)
\]
\[ - \text{Tr}(Y_R Y_R Y_R^\dagger Y_F)
\]
\[ (34f) \]

Finally, the beta function for the trilinear scalar coupling is

\[ \beta_{\kappa}^{(1)} = \frac{1}{10} \kappa \left( -9 g_1^2 - 105 g_2^2 + 20 \lambda + 20 \lambda_a + 60 \text{Tr}(Y_d Y_d^\dagger) + 20 \text{Tr}(Y_e Y_e^\dagger) + 15 \text{Tr}(Y_L Y_L^\dagger)
\]
\[ + 5 \text{Tr}(Y_R Y_R^\dagger) + 60 \text{Tr}(Y_u Y_u^\dagger) \right). \]
\[ (35) \]

References

[1] R. D. Peccei and H. R. Quinn, *CP Conservation in the Presence of Instantons*, Phys. Rev. Lett. **38** (1977) 1440.

[2] R. D. Peccei and H. R. Quinn, *Constraints Imposed by CP Conservation in the Presence of Instantons*, Phys. Rev. D **16** (1977) 1791.

[3] F. Wilczek, *Problem of Strong P and T Invariance in the Presence of Instantons*, Phys. Rev. Lett. **40** (1978) 279.

[4] S. Weinberg, *A New Light Boson?*, Phys. Rev. Lett. **40** (1978) 223.

[5] L. Di Luzio, M. Giannotti, E. Nardi and L. Visinelli, *The landscape of QCD axion models*, Phys. Rept. **870** (2020) 1 [2003.01100].
[6] J. E. Kim, *Reason for SU(6) Grand Unification*, Phys. Lett. B **107** (1981) 69.

[7] P. Langacker, R. D. Peccei and T. Yanagida, *Invisible Axions and Light Neutrinos: Are They Connected?*, Mod. Phys. Lett. A **1** (1986) 541.

[8] M. Shin, *Light Neutrino Masses and Strong CP Problem*, Phys. Rev. Lett. **59** (1987) 2515 [Erratum: Phys.Rev.Lett. 60, 383 (1988)].

[9] A. G. Dias, A. C. B. Machado, C. C. Nishi, A. Ringwald and P. Vaudrevange, *The Quest for an Intermediate-Scale Accidental Axion and Further ALPs*, JHEP **06** (2014) 037 [1403.5760].

[10] G. Ballesteros, J. Redondo, A. Ringwald and C. Tamarit, *Unifying inflation with the axion, dark matter, baryogenesis and the seesaw mechanism*, Phys. Rev. Lett. **118** (2017) 071802 [1608.05414].

[11] P.-H. Gu, *Peccei-Quinn symmetry for Dirac seesaw and leptogenesis*, JCAP **07** (2016) 004 [1603.05070].

[12] S. Baek, *Dirac neutrino from the breaking of Peccei-Quinn symmetry*, Phys. Lett. B **805** (2020) 135415 [1911.04210].

[13] E. Peinado, M. Reig, R. Srivastava and J. W. F. Valle, *Dirac neutrinos from Peccei–Quinn symmetry: A fresh look at the axion*, Mod. Phys. Lett. A **35** (2020) 2050176 [1910.02961].

[14] L. M. G. de la Vega, N. Nath and E. Peinado, *Dirac neutrinos from Peccei-Quinn symmetry: two examples*, Nucl. Phys. B **957** (2020) 115099 [2001.01846].

[15] A. G. Dias, J. Leite, J. W. F. Valle and C. A. Vaquera-Araujo, *Reloading the axion in a 3-3-1 setup*, Phys. Lett. B **810** (2020) 135829 [2008.10650].

[16] M. Berbig, *S.M.A.S.H.E.D.: Standard Model Axion Seesaw Higgs Inflation Extended for Dirac Neutrinos*, 2207.08142.

[17] C.-S. Chen and L.-H. Tsai, *Peccei-Quinn symmetry as the origin of Dirac Neutrino Masses*, Phys. Rev. D **88** (2013) 055015 [1210.6264].

[18] C. D. R. Carvajal and O. Zapata, *One-loop Dirac neutrino mass and mixed axion-WIMP dark matter*, Phys. Rev. D **99** (2019) 075009 [1812.06364].

[19] C. D. R. Carvajal, R. Longas, O. Rodríguez and O. Zapata, *Singlet fermion dark matter and Dirac neutrinos from Peccei-Quinn symmetry*, Phys. Rev. D **105** (2022) 015003 [2110.15167].

[20] K. Dick, M. Lindner, M. Ratz and D. Wright, *Leptogenesis with Dirac neutrinos*, Phys. Rev. Lett. **84** (2000) 4039 [hep-ph/9907562].

[21] H. Murayama and A. Pierce, *Realistic Dirac leptogenesis*, Phys. Rev. Lett. **89** (2002) 271601 [hep-ph/0206177].
[22] M. Dine, W. Fischler and M. Srednicki, A Simple Solution to the Strong CP Problem with a Harmless Axion, *Phys. Lett. B* **104** (1981) 199.

[23] A. R. Zhitnitsky, On Possible Suppression of the Axion Hadron Interactions. (In Russian), *Sov. J. Nucl. Phys.* **31** (1980) 260.

[24] J. E. Kim, Weak Interaction Singlet and Strong CP Invariance, *Phys. Rev. Lett.* **43** (1979) 103.

[25] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Can Confinement Ensure Natural CP Invariance of Strong Interactions?, *Nucl. Phys. B* **166** (1980) 493.

[26] J. T. Penedo, Y. Reyimuaji and X. Zhang, in preparation.

[27] T. Blank and W. Hollik, Precision observables in $SU(2) \times U(1)$ models with an additional Higgs triplet, *Nucl. Phys. B* **514** (1998) 113 [hep-ph/9703392].

[28] J. R. Forshaw, D. A. Ross and B. E. White, Higgs mass bounds in a triplet model, *JHEP* **10** (2001) 007 [hep-ph/0107232].

[29] J. R. Forshaw, A. Sabio Vera and B. E. White, Mass bounds in a model with a triplet Higgs, *JHEP* **06** (2003) 059 [hep-ph/0302256].

[30] N. Khan, Exploring the hyperchargeless Higgs triplet model up to the Planck scale, *Eur. Phys. J. C* **78** (2018) 341 [1610.03178].

[31] S. Centelles Chuliá, R. Srivastava and J. W. F. Valle, Seesaw roadmap to neutrino mass and dark matter, *Phys. Lett. B* **781** (2018) 122 [1802.05722].

[32] P. Fileviez Perez, H. H. Patel, M. J. Ramsey-Musolf and K. Wang, Triplet Scalars and Dark Matter at the LHC, *Phys. Rev. D* **79** (2009) 055024 [0811.3957].

[33] CAST collaboration, V. Anastassopoulos et al., New CAST Limit on the Axion-Photon Interaction, *Nature Phys.* **13** (2017) 584 [1705.02290].

[34] ADMX collaboration, S. J. Asztalos et al., A SQUID-based microwave cavity search for dark-matter axions, *Phys. Rev. Lett.* **104** (2010) 041301 [0910.5914].

[35] ADMX collaboration, N. Du et al., A Search for Invisible Axion Dark Matter with the Axion Dark Matter Experiment, *Phys. Rev. Lett.* **120** (2018) 151301 [1804.05750].

[36] ADMX collaboration, T. Braine et al., Extended Search for the Invisible Axion with the Axion Dark Matter Experiment, *Phys. Rev. Lett.* **124** (2020) 101303 [1910.08638].

[37] ADMX collaboration, C. Bartram et al., Search for Invisible Axion Dark Matter in the 3.3–4.2 µeV Mass Range, *Phys. Rev. Lett.* **127** (2021) 261803 [2110.06096].

[38] S. Lee, S. Ahn, J. Choi, B. R. Ko and Y. K. Semertzidis, Axion Dark Matter Search around 6.7 µeV, *Phys. Rev. Lett.* **124** (2020) 101802 [2001.05102].
[39] J. Jeong, S. Youn, S. Bae, J. Kim, T. Seong, J. E. Kim et al., *Search for Invisible Axion Dark Matter with a Multiple-Cell Haloscope*, *Phys. Rev. Lett.* **125** (2020) 221302 [2008.10141].

[40] CAPP collaboration, O. Kwon et al., *First Results from an Axion Haloscope at CAPP around 10.7 µeV*, *Phys. Rev. Lett.* **126** (2021) 191802 [2012.10764].

[41] HAYSTAC collaboration, K. M. Backes et al., *A quantum-enhanced search for dark matter axions*, *Nature* **590** (2021) 238 [2008.01853].

[42] I. Shilon, A. Dudarev, H. Silva and H. H. J. ten Kate, *Conceptual Design of a New Large Superconducting Toroid for IAXO, the New International AXion Observatory*, *IEEE Trans. Appl. Supercond.* **23** (2013) 4500604 [1212.4633].

[43] ABRACADABRA collaboration, “A broadband/resonant approach to cosmic axion detection with an amplifying b-field ring apparatus.” [https://abracadabra.mit.edu](https://abracadabra.mit.edu).

[44] D. Alesini et al., *KLASH Conceptual Design Report*, 1911.02427.

[45] I. Stern, *ADMX Status*, *PoS* ICHEP2016 (2016) 198 [1612.08296].

[46] S. Beurthey et al., *MADMAX Status Report*, 2003.10894.

[47] C. O’Hare, “AxionLimits.” [https://cajohare.github.io/AxionLimits](https://cajohare.github.io/AxionLimits).

[48] P. Carenza, T. Fischer, M. Giannotti, G. Guo, G. Martínez-Pinedo and A. Mirizzi, *Improved axion emissivity from a supernova via nucleon-nucleon bremsstrahlung*, *JCAP* **10** (2019) 016 [1906.11844], [Erratum: JCAP 05, E01 (2020)].

[49] CDF collaboration, T. Aaltonen et al., *High-precision measurement of the W boson mass with the CDF II detector*, *Science* **376** (2022) 170.

[50] A. Strumia, *Interpreting electroweak precision data including the W-mass CDF anomaly*, 2204.04191.

[51] P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, *Oblique Lessons from the W Mass Measurement at CDF II*, 2204.05283.

[52] L. Di Luzio, R. Gröber and P. Paradisi, *Higgs physics confronts the MW anomaly*, *Phys. Lett. B* **832** (2022) 137250 [2204.05284].

[53] P. Perez Fileviez, H. H. Patel and A. D. Plascencia, *On the W-mass and New Higgs Bosons*, 2204.07144.

[54] D. Borah, S. Mahapatra, D. Nanda and N. Sahu, *Type II Dirac Seesaw with Observable ΔNeff in the light of W-mass Anomaly*, 2204.08266.

[55] O. Popov and R. Srivastava, *The Triplet Dirac Seesaw in the View of the Recent CDF-II W Mass Anomaly*, 2204.08568.
[56] A. Batra, S. K. A., S. Mandal and R. Srivastava, *W boson mass in Singlet-Triplet Scotogenic dark matter model*, 2204.09376.

[57] Y. Cheng, X.-G. He, F. Huang, J. Sun and Z.-P. Xing, *Dark photon kinetic mixing effects for CDF W mass excess*, 2204.10156.

[58] A. Addazi, A. Marciano, A. P. Morais, R. Pasechnik and H. Yang, *CDF II W-mass anomaly faces first-order electroweak phase transition*, 2204.10315.

[59] J.-W. Wang, X.-J. Bi, P.-F. Yin and Z.-H. Yu, *Electroweak dark matter model accounting for the CDF W-mass anomaly*, 2205.00783.

[60] J. L. Evans, T. T. Yanagida and N. Yokozaki, *W boson mass anomaly and grand unification*, 2205.03877.

[61] G. Lazarides, R. Maji, R. Roshan and Q. Shafi, *Heavier W-boson, dark matter and gravitational waves from strings in an SO(10) axion model*, 2205.04824.

[62] G. Senjanović and M. Zantedeschi, *SU(5) grand unification and W-boson mass*, 2205.05022.

[63] E. Ma, *Type III Neutrino Seesaw, Freeze-In Long-Lived Dark Matter, and the W Mass Shift*, 2205.09794.

[64] T. G. Rizzo, *Kinetic Mixing, Dark Higgs Triplets, M_W and All That*, 2206.09814.

[65] Y. Cheng, X.-G. He, Z.-L. Huang and M.-W. Li, *Type-II seesaw triplet scalar effects on neutrino trident scattering*, Phys. Lett. B 831 (2022) 137218 [2204.05031].

[66] S. Kanemura and K. Yagyu, *Implication of the W boson mass anomaly at CDF II in the Higgs triplet model with a mass difference*, Phys. Lett. B 831 (2022) 137217 [2204.07511].

[67] J. Heeck, *W-boson mass in the triplet seesaw model*, Phys. Rev. D 106 (2022) 015004 [2204.10270].

[68] N. Chakrabarty, *The muon g − 2 and W-mass anomalies explained and the electroweak vacuum stabilised by extending the minimal Type-II seesaw*, 2206.11771.

[69] H. Bahl, W. H. Chiu, C. Gao, L.-T. Wang and Y.-M. Zhong, *Tripling down on the W boson mass*, 2207.04205.

[70] PARTICLE DATA GROUP collaboration, P. Zyla et al., *Review of Particle Physics*, PTEP 2020 (2020) 083C01 and 2021 update.

[71] K. Kannike, *Vacuum Stability Conditions From Copositivity Criteria*, Eur. Phys. J. C 72 (2012) 2093 [1205.3781].

[72] M. Chabab, M. C. Peyranère and L. Rahili, *Probing the Higgs sector of Y = 0 Higgs Triplet Model at LHC*, Eur. Phys. J. C 78 (2018) 873 [1805.00286].
[73] P. M. Ferreira and B. L. Gonçalves, *Stability of neutral minima against charge breaking in the Higgs triplet model*, JHEP 02 (2020) 182 [1911.09746].

[74] M. E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, Phys. Rev. D 46 (1992) 381.

[75] C.-T. Lu, L. Wu, Y. Wu and B. Zhu, *Electroweak Precision Fit and New Physics in light of W Boson Mass*, 2204.03796.

[76] ATLAS, CMS collaboration, G. Aad et al., *Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at \( \sqrt{s} = 7 \) and 8 TeV*, JHEP 08 (2016) 045 [1606.02266].

[77] OPAL collaboration, G. Abbiendi et al., *Two Higgs doublet model and model independent interpretation of neutral Higgs boson searches*, Eur. Phys. J. C 18 (2001) 425 hep-ex/0007040.

[78] DELPHI collaboration, J. Abdallah et al., *Searches for neutral higgs bosons in extended models*, Eur. Phys. J. C 38 (2004) 1 hep-ex/0410017.

[79] CMS collaboration, S. Chatrchyan et al., *Searches for Long-Lived Charged Particles in pp Collisions at \( \sqrt{s} = 7 \) and 8 TeV*, JHEP 07 (2013) 122 [1305.0491].

[80] F. Staub, *SARAH 4 : A tool for (not only SUSY) model builders*, Comput. Phys. Commun. 185 (2014) 1773 [1309.7223].

[81] F. Staub, *Exploring new models in all detail with SARAH*, Adv. High Energy Phys. 2015 (2015) 840780 [1503.04200].

[82] J. Zhang and S. Zhou, *Electroweak Vacuum Stability and Diphoton Excess at 750 GeV*, Chin. Phys. C 40 (2016) 081001 [1512.07889].

[83] P. Ghorbani, *Vacuum stability vs. positivity in real singlet scalar extension of the standard model*, Nucl. Phys. B 971 (2021) 115533 [2104.09542].