Detection of Apposite PSO Parameters using Taguchi Based Grey Relational Analysis: Optimization and Implementation Aspects on Manufacturing Related Problem

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Abstract

In this study Taguchi based grey relational analysis is used to find out optimum process parameters for Particle Swarm Optimization. PSO-type, population size and C values are selected as process parameters for this analysis. The parameters obtained by the analysis can generate better result within a preferable time. A supportive case study shows the effectiveness of the proposed approach.

1. Introduction

Particle swarm optimization (PSO) is a biological swarm intelligence based optimization. PSO algorithm was proposed by Kennedy and Eberhart in 1995. Particle Swarm Optimization is stochastic in nature and it search randomly to find optimum solution. It is an effective optimization technique to search for global optimization solution.
The idea of PSO originates from mimic bird flocking or fish schooling as described by Kennedy and Eberhart. "Particle swarm algorithm imitates human (or insects) social behavior. Individuals interact with one another while learning from their own experience and gradually the population members move into better regions of the problem space", [Kennedy and Eberhart (1995)]. The basic of PSO algorithm is to create a population of agents (called particles) uniformly distributes over x. Then evaluation of each particle’s position according to the objective functions. After that updating a particles current position if it is better than its previous best position and determine the best particle according to the particle’s best previous positions.

In modern manufacturing processes Taguchi method becomes very useful to optimize the productivity with increased product quality[Peace(1993)]. But Taguchi method basically optimizes single quality characteristics only and in case of multiple quality characteristics one improved characteristics can dampen other characteristics. So engineers have to compromise while taking decision thus increasing the uncertainty. To solve such problems Deng in 1982 proposed grey system theory [Deng (1982)]. This method normalizes multiple quality characteristics and converts them into single grey relational grade which make it easier to take proper decision.

In this paper using Taguchi based grey relational analysis, different process parameters of Particle Swarm Optimization such as PSO types, Population size, and ‘c’ values were evaluated to get better optimum parameters.

**Nomenclature**

| Abbreviation | Description |
|--------------|-------------|
| PSO          | particle swarm optimization |
| C value      | acceleration coefficient |
| μs           | micro second |

1. Types of PSO

Basically particle swarm optimization is three types

1.1. **PSO-O (PSO-original)**

PSO-O is the original PSO algorithm proposed by Kennedy and Eberhart in 1995 [Kennedy and Eberhart (1995)]. In this case the velocity and position of the particle changes according to the following equation

\[
v_{i}^{t+1} = v_{i}^{t} + c_{1}r_{1}^{t} \times (pb_{i}^{t} - x_{i}^{t}) + c_{2}r_{2}^{t} \times (gb_{i}^{t} - x_{i}^{t})
\]

(1)

\[
x_{i}^{t+1} = x_{i}^{t} + v_{i}^{t+1}
\]

(2)

Here i is the index of particles and \(v_{i}^{t}\) is the velocity and \(x_{i}^{t}\) is the position of particle i at time t. The parameters \(c_{1}\) and \(c_{2}\) are acceleration coefficient supplied by the user. The values \(r_{1}\) and \(r_{2}\) (0 ≤ \(r_{1}\) ≤ 1 and 0 ≤ \(r_{1}\) ≤ 1) are random values regenerated for each velocity update. The value pb_{i}^{t} is the individual best candidate solution for particle i at time t, and gb_{i}^{t} is the swarm’s global best candidate solution at time t. There are three terms of the velocity update equation and each of them have different roles in the PSO algorithm. The first term \(v_{i}^{t}\) is the inertia component, which is responsible for keeping the particle moving in the same direction it was originally heading. \(c_{1}r_{1}^{t}(pb_{i}^{t} - x_{i}^{t})\) is the second term called as cognitive component, acts as the particle’s memory, for which particles has the tendency to return to the region of search space where it has experienced high individual fitness. The cognitive coefficient \(c_{1}\) affects the step-size the particle takes toward its individual best candidate solution pb_{i}^{t}. And the third term \(c_{2}r_{2}^{t}\)
*(gbi\t-xi\t)* called the social component, causes the particle to move to the best region the swarm has found so far. The social coefficient \(c_2\) represents the step-size the particle takes toward the global best candidate solution \(gb_i\) the swarm has found up until that point. The random values \(r_1\) and \(r_2\) in the cognitive compound and social component respectively cause these components to have a stochastic influence on the velocity update. Due to this nature the particle moves in a semi-random manner heavily influenced in the directions of the individual best solution of the particle and global best solution of the swarm [Blondin (2009)].

1.2. PSO-IW (PSO-Inertia Weight)

PSO-IW is another kind of PSO algorithm proposed by and Shi and Eberhart [Shi and Eberhart (1998)].

\[
v_{i+1} = wv_i + c_1r_1 \times (pb_i + x_i) + c_2r_2 \times (gb_i + x_i)
\]  

Here \(w\) is the inertial coefficient which can either dampen the particle’s inertia or accelerate the particle in its original direction [Shi and Eberhart (1998)]. Generally, lower values of the inertial coefficient speed up the convergence of the swarm to optima, and higher values of the inertial coefficient encourage exploration of the entire search space [Blondin (2009)].

1.3. PSO-CF (PSO-Constriction Factor)

PSO-CF was proposed by Clerc and Kennedy [Clerc and Kennedy (2002)].

\[
v_{i+1} = \chi[v_i + c_1r_1 \times (pb_i - x_i) + c_2r_2 \times (gb_i - x_i)]
\]

\[
\chi = \frac{2k}{|2 - \phi - \sqrt{\phi^2 - 4\phi}|}
\]

Here the ‘constriction factor’ is denoted by ‘\(\chi\).

\(\phi = \phi_1 + \phi_2\)

\(\phi_1 = c_1r_1\) and \(\phi_2 = c_2r_2\), \(k=[0-1]\)

2. Grey Relational Analysis

Grey relational analysis widely used in the processes having multiple quality characteristics to find the optimum parameter [Deng. (1989)]. The fundamental idea of this method is to give single grey grade to every parameter by normalizing the different characteristics. By using Taguchi based grey relational analysis to find the optimum parameter for Particle Swarm Optimization need to follow some steps-

a) Identifying the process parameters and the quality characteristics which will be evaluated.

b) Levelling of parameters.

c) Depending upon the number of levels select the proper orthogonal array and assign the parameter to that.

d) With respect to the array conduct the analytical run.

e) Normalise the quality characteristics.

f) By grey relational analysis calculate the grey relational coefficient and grey relational grade.

2.1. Normalization

Normalisation of data can be done by three different way. When greater value of the characteristics improve the quality the “larger the better” method is used.
When lower value of the characteristics will ensure better quality then “smaller the better” method is used.

\[ x_i^*(k) = \frac{x_i^{(o)}(k) - \min x_i^{(o)}(k)}{\max x_i^{(o)}(k) - \min x_i^{(o)}(k)} \]  

(6)

Where \( x_i^*(k) \) is the grey relational generated value, \( \min x_i^{(o)}(k) \) and \( \max x_i^{(o)}(k) \) is the smallest and largest value of \( x_i^{(o)}(k) \) respectively for the \( k \)th response.

If targeted optimum value (OV) for quality characteristics is there then “nominal the better” method used.

\[ x_i^*(k) = 1 - \frac{|x_i^{(o)}(k) - OV|}{\max\{\max x_i^{(o)}(k) - OV, OV - \min x_i^{(o)}(k)\}} \]  

(7)

2.2. Grey Relational Coefficient

The grey relational coefficient \( \xi_i(k) \) can be calculated as:

\[ \xi_i(k) = \frac{\Delta_{\min} - \zeta \times \Delta_{\max}}{\Delta_{\min}(k) - \zeta \times \Delta_{\max}} \]  

(9)

Where \( \Delta_{\min}(k) = |x_i^*(k) - x_i^*(k)| \) is the deviation sequence from the reference sequence \( x_o^*(k) \) and comparability sequence \( x_i^*(k) \); \( \zeta \) is the distinguishing coefficient (0-1); \( \Delta_{\min} \) and \( \Delta_{\max} \) are the smallest and largest value of \( \Delta_{\min} \) respectively.

2.3. Grey Relational Grade

Grey relational grade can be obtain by averaging the Grey relational Coefficients.

\[ Y_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i(k) \]  

(10)

If the weight of factor is considered then

\[ Y_i = \sum_{k=1}^{n} \beta_k \xi_i(k), \sum_{k=1}^{n} \beta_k = 1 \]  

(11)

Where \( \beta_k \) is the given weight to the \( k \)th factor and \( n \) isthe number of characteristics.
3. Case Study

The following case study describes the optimum parameters for Particle Swarm Optimization. This study was done by using the fitness function of Tool Wear Ratio (TWR) of EDM process.

\[
    \text{TWR} = 0.059 - 0.0003X_1 + 0.0035X_3 + 0.0037X_4 + 0.0024X_1X_2 - 0.0034X_1X_3 - 0.0017X_4^2 + 0.0018X_2^2 + 0.0014X_3^2 + 0.0016X_1^2
\]  

(12)

This fitness function was proposed by El-Taweel in 2008 [El-Taweel (2009)]. In this fitness function $X_1$, $X_2$, $X_3$, and $X_4$ are the coded values of machining process parameters. Where

$X_1$ = Titanium Carbide percentage (wt.%) $[2 \leq X_1 \leq 18]$

$X_2$ = Peak current (A) $[2 \leq X_2 \leq 10]$

$X_3$ = Flushing pressure (MPa) $[0.4 \leq X_3 \leq 1.2]$

$X_4$ = Pulse on-time ($\mu$s) $[40 \leq X_4 \leq 200]$

4. Analytical Runs and Details

Analysis was carried out by changing acceleration coefficient values [Sun et al. (2010); Mahapatra et al. (2011)] PSO type and Population size. To start analysis first of all levels were given to every parameter shown in Table no 1.

| Symbols | Parameters  | Level 1 | Level 2 | Level 3 |
|---------|-------------|---------|---------|---------|
| A       | PSO-Type    | PSO-O   | PSO-IW  | PSO-CF  |
| B       | Pop size    | 50      | 100     | 150     |
| C       | C Values    | 1,1,1,1 | 2,2     | 2,8,1,3 |

Then to minimize the number of analytical run Taguchi’s $L_9$ orthogonal array was used where 9 sets of data were selected to find out the quality characteristics for every selected sets of data [Taguchi. (1990)]. In this study convergence time and average objective are considered as the quality characteristics of PSO. For every sets of data ten number of test runs were conducted and the average values of every quality characteristics were taken for optimization. The sets of data and the founded characteristics results are shown in Table 2.

| Run No | A | B | C | Convergence Time (sec) | Avg. Objective |
|--------|---|---|---|------------------------|---------------|
| 1      | 1 | 1 | 1 | 0.3104                 | 3.51E-02      |
| 2      | 1 | 2 | 2 | 0.3276                 | 6.68E-02      |
| 3      | 1 | 3 | 3 | 0.362                  | 1.06E-01      |
| 4      | 2 | 1 | 2 | 0.324                  | 2.51E-02      |
| 5      | 2 | 2 | 3 | 0.351                  | 4.95E-02      |
| 6      | 2 | 3 | 1 | 0.356                  | 7.38E-02      |
| 7      | 3 | 1 | 3 | 0.348                  | 2.48E-02      |
| 8      | 3 | 2 | 1 | 0.3492                 | 6.50E-02      |
| 9      | 3 | 3 | 2 | 0.359                  | 6.87E-02      |
The selected characteristics were ‘time of optimization’ and the ‘average objective’. The various parameters were different types of PSOs, Population size (pop size), acceleration coefficients (c values).

5. Result Analysis

The analytical computing results of two quality characteristics that are convergence time and average objective are shown in Table 2. As the lower value of time and average objective will give better result so for data pre-processing or normalization was done by “smaller-the-better” method. The calculation was done by the help of the equation (7). The normalized values are shown in Table 3.

| Run No | Normalised Values (“Smaller The Better”) |
|--------|----------------------------------------|
|        | Time | Avg. Objective |
| 1      | 1    | 0.873177307    |
| 2      | 0.666666667 | 0.484499449 |
| 3      | 0    | 0              |
| 4      | 0.736434109 | 0.996446514 |
| 5      | 0.213178295 | 0.696850876 |
| 6      | 0.11627907  | 0.399093248 |
| 7      | 0.271317829 | 1              |
| 8      | 0.248062016 | 0.506678103 |
| 9      | 0.058139535 | 0.461340522 |

The deviation sequences were measured by the equation \( \Delta_n(k) = |x_i^r(k) - x_i^s(k)| \). The deviation sequence is shown in Table 4.

| Comparability Sequence | Reference Sequence |
|------------------------|--------------------|
| Run NO                | \( \Delta t \) | \( \Delta A.O \) |
| 1                      | 1.000000000 | 1.000000000 |
| 2                      | 0.333333333 | 0.515500551 |
| 3                      | 1           | 1           |
| 4                      | 0.263565891 | 0.003553486 |
| 5                      | 0.786821705 | 0.303149124 |
| 6                      | 0.88372093  | 0.600906752 |
| 7                      | 0.72868217  | 0           |
| 8                      | 0.751937984 | 0.493321897 |
| 9                      | 0.941860465 | 0.538659478 |

distinguishing coefficient (\( \zeta \)) was taken as 0.5. The grey relational grades were calculated by equation (11). The weightages were given as 0.3 and 0.7 for grey relational coefficient of time and grey relational coefficient of average objective respectively. The values of grey relational coefficients and grey relational grades are shown in Table 5.
Table 5. Calculated Grey Relational Analysis and Grey Relational Grade

| Run No | Orthogonal Array L₉ (3³) | Grey relational coefficient | Grey relational Grade | Grey Order |
|--------|--------------------------|-----------------------------|-----------------------|------------|
|        | A B C ε₁(1) ε₂(2) Yᵢ O  |
| 1      | 1 1 1 1 0.797673 0.8583716 2 |
| 2      | 1 2 2 0.492368 0.5246576 5 |
| 3      | 2 3 3 0.333333 0.333333 9 |
| 4      | 3 1 2 0.654822 0.992943 27 |
| 5      | 1 1 1 0.492368 0.5246576 5 |
| 6      | 2 3 1 0.454171 0.463231 4 |
| 7      | 2 2 3 0.622549 0.5523508 41 |
| 8      | 2 2 3 0.622549 0.5523508 41 |
| 9      | 2 2 3 0.622549 0.5523508 41 |

The mean grey relational grades for each process parameters and levels are shown in Table 6.

Table 6. Mean Grey Grade for each Parameter

| Level | PSO-Type | Pop Size | C Values |
|-------|----------|----------|----------|
| 1     | 0.572121 | **0.583293** | 0.585621 |
| 2     | **0.623394** | 0.516392 | **0.619057** |
| 3     | 0.578418 | 0.400221 | 0.569255 |

Here delta is the difference between highest and lowest grey relational grade of each parameter as described in Taguchi method. The value of delta signifies the influence of the parameters on the process. Higher the delta value higher will be the influence [5].

According to the grade values shown in Table 6 following graphs are drawn for each factor (Fig.1.a,b,c).

![Graph a](image1.png)
![Graph b](image2.png)
![Graph c](image3.png)

Fig 1 (a) Grey relational grade graph for PSO type. (b) Grey relational grade graph for Population size (c) Grey relational grade graph for C values.

The graphs clearly indicate that the optimal parameter sets are A₂, B₁, and C₂, i.e PSO-IW, Population size 50 and C values 2,2. As the optimum parameters’ set is present in the selected sets of data, no confirmation test is done.
7. Conclusion

Particle Swarm Optimization is a very useful tool for optimization of various manufacturing processes. Its problem-solving ability, easy programmability and rapid result generation capability makes it very advantageous. In this study Taguchi based grey relational analysis is used to find out the comparatively more efficient and effective parameters for Particle Swarm Optimization. The results show the effectiveness of the proposed approach on selecting the optimum parameters for Particle swarm optimization by considering less number of data set. Moreover the delta values shown in the Table 6 implies that population size is the more influencing parameter than the other two.

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