Quantum Quench for inhomogeneous states in the non-local Luttinger model

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In the Luttinger model with non-local interaction we investigate, by exact analytical methods, the time evolution of an inhomogeneous state with a localized fermion added to the non interacting ground state. In absence of interaction the averaged density has two peaks moving in opposite directions with constant velocities. If the state is evolved with the interacting Hamiltonian two main effects appear. The first is that the peaks have velocities which are not constant but vary between a minimal and maximal value. The second is that a dynamical ‘Landau quasi-particle weight’ appears in the oscillating part of the averaged density, asymptotically vanishing with time, as consequence of the fact that fermions are not excitations of the interacting Hamiltonian.

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Introduction

Recent experiments on cold atoms \cite{1} have motivated an increasing interest in the dynamical properties of many body quantum systems which are \textit{closed} and isolated from any reservoir or environment \cite{2}. Non equilibrium properties can be investigated by quantum quenches, in which the system is prepared in a state and its subsequent time evolution driven by a many body Hamiltonian is observed. As the resulting dynamical behaviour is the cumulative effect of the interactions between an infinite or very large number of particles, the computation of local observables averaged over time-evolved states poses typically great analytical difficulties; the problem is then mainly studied in one dimension, see for instance \cite{3,4,5,6,7,8,9,10,11,12,13,14}.

A major difference with respect to the equilibrium case relies on the fact that in such a case a form of universality holds, ensuring that a number of properties are essentially insensitive to the model details; for instance a large class of one dimensional system, named \textit{Luttinger liquids} \cite{17}, have similar equilibrium properties irrespectively from the exact form of the Hamiltonian, and this fact can be even proven rigorously under certain hypothesis using constructive RG methods \cite{18}. Universality and independence from the details explain also why even crude approximations are able to capture the essential physics of such systems. At non-equilibrium the behavior depends instead on model details; for instance integrability in spin chains dramatically affect the non equilibrium behavior \cite{19} while it does not alter the $T = 0$ equilibrium properties \cite{20}. This extreme sensitivity to details or approximations asks for a certain number of analytical exact results at non-equilibrium, to provide a benchmark for experiments or approximate computations.

One of the interacting fermionic system where non-equilibrium properties can be investigated is the Luttinger model, which provides a great number of information in the equilibrium case. In this model the quadratic dispersion relation of non relativistic fermions is replaced with a linear dispersion relation, with the idea that the properties are mainly determined by the states close to the Fermi points, where the energy is essentially linear; a Dirac sea is introduced, filling all the states with negative energy. It is important to stress that there exist two versions of this model, the local Luttinger model (LLM) and the non local Luttinger model (NLLM); in the former a local delta-like interaction is present while in the latter the interaction is short ranged but non local. At equilibrium such two models are often confused as they have similar behavior, due to the above mentioned insensitivity to model details; there is however no reason to expect that this is true also at non equilibrium. It should be also stressed that the LLM is plagued by ultraviolet divergences typical of a QFT and an ad-hoc regularization is necessary to get physical predictions; the short time or distance behavior depends on the chosen regularization.

The quantum quench of homogeneous states in the LLM was derived in \cite{4,5} and in the NLLM in \cite{11}; the predictions agree for long times but are rather different for short times. Regarding the quantum quench of inhomogeneous states, in \cite{9} the dynamical evolution in the LLM of a domain wall state was considered, as an approximate description for the analogous problem in the spin XXZ spin chain. It was found in \cite{9} that the evolution in the free or interacting case is the same up to a finite renormalization of the parameters; in particular the front evolves with a constant velocity. Non constant velocities appear, from numerical simulations, in more realistic models like the XXZ chain \cite{13}.

In this paper we consider the evolution of inhomogeneous states in the NLLM, using exact analytical methods in the infinite volume limit. In particular we will consider the state obtained adding a particle to non in-
Interacting ground state or the vacuum. In the absence of the interaction the particle moves with a ballistic motion with a constant velocity, showing a typical “light cone” dynamics. In presence of interaction, the dynamics is still ballistic (in agreement with the fact that the conductivity computed via Kubo formula is diverging), but the evolution is not simply the free one with a renormalized velocity; on the contrary, the evolution is driven by velocities which are non constant and energy dependent. Moreover the interaction produces a dynamical ‘Landau quasi-particle weight’ in the oscillating part, asymptotically vanishing with time; no vanishing weight is instead present in the non oscillating part. Note also that the expressions we get do not require any ultraviolet regularization, and correctly captures also the short time dynamics.

The plan of the paper is the following. We introduce the NLLM in §II and in §III we derive by this method the ground state 2-point function and the average over an homogeneous quenched state. §IV contains our main result, namely the time evolution of an inhomogeneous state. In the Appendices the analytical derivation of our results is exposed.

### The non-local Luttinger model

The non-local Luttinger model (NLLM) Hamiltonian is

\[
H = H_0 + V = \sum_{k>0} k[(a^+_{k,1}a^-_{-k,1} + a^-_{-k,1}a^+_{k,1}) + (a^+_{k,2}a^-_{-k,2} + a^-_{-k,2}a^+_{k,2})] + \frac{2\lambda}{L} \sum_{p>0} [\rho_1(p)\rho_2(-p) + \rho_1(-p)\rho_2(p)] + \frac{\lambda}{L} \bar{\nu}(0) N_1 N_2
\]

where

\[
\rho_{\omega}(p) = \sum_k a^+_{k,p,\omega}a^-_{k,\omega}
\]

\[
N_\omega = \sum_{k>0} (a^+_{k,\omega}a^-_{k,\omega} - a^-_{-k,\omega}a^+_{k,\omega})
\]

The regularization implicit in the above expressions is that \(\rho_{\omega}(p)\) must be thought as \(\lim_{\Lambda \to \infty} \sum_k \chi_k(k)\chi_\Lambda(k + p)a^+_{k,p,\omega}a^-_{k,\omega}\) where \(\chi_k(k)\) is 1 for \(|k| \leq \Lambda\) and 0 otherwise. The Hamiltonian \(H_\Lambda\) as well as the \(\rho_{\omega}(p)\) can be regarded as operators acting on the Hilbert space \(\mathcal{H}\) constructed applying finitely many times creation or annihilation operators on

\[
|0> = \prod_{k<0} a^+_{k,1}a^-_{-k,2}|vac>
\]

We define

\[
\hat{\psi}^\pm_x = e^{iH_0t}\psi^\pm_{\omega,x} e^{-iH_0t} = \frac{1}{\sqrt{L}} \sum_k \hat{a}^\pm_{k,\omega} e^{i(kx - \omega kt - 0^+|k|)} ,
\]

where \(\varepsilon_1 = +, \varepsilon_2 = -\) so that

\[
<0|\psi^\mp_{\omega,x}\psi^\pm_{\omega,y}|0> = \frac{(2\pi)^{-1}}{i\varepsilon_\omega(x-y) - i(t-s) + 0^+} .
\]

The basic property of the Luttinger model is the validity of the following anomalous commutation relations, first proved in [15]

\[
[\rho_1(-p), \rho_1(p')] = [\rho_2(p), \rho_2(-p')] = \frac{pL}{2\pi} \delta_{p,p'}
\]

Moreover one can verify that

\[
\rho_2(p)|0> = 0 \quad \rho_1(-p)|0> = 0.
\]

Other important commutation relations are the following

\[
[H_0, \rho_1(\pm p)] = \pm \rho_1(\pm p) \quad [H_0, \rho_2(\pm p)] = \mp \rho_2(\pm p)
\]

\[
[\rho_{\omega,\varepsilon}, \hat{\psi}^\pm_{\omega,x}] = e^{ipx}\hat{\psi}^\varepsilon_{\omega,x}
\]

It is convenient, see [15], to introduce an operator \(T = \frac{2\pi}{L} \sum_{p>0} \rho_1(p)\rho_1(-p) + \rho_2(p)\rho_2(-p)\) and write \(H_\Lambda = (H_0 - T) + (V + T) = H_1 + H_2\). Note that \(H_1\) commutes with \(\rho_{\omega}\) and \(H_2\) can be written in diagonal form by the following transformation

\[
e^{iS}H_2e^{-iS} = \tilde{H}_2
\]

\[
= \frac{2\pi}{L} \sum_p \text{sech}2\phi_p[\rho_1(p)\rho_1(-p) + \rho_2(-p)\rho_2(p)] + E_0
\]
so that
\[ e^{iS} e^{iH_xt} e^{-iS} = e^{i(H_0 + D)t} \] (12)
where
\[ S = \frac{2\pi}{L} \sum_{p \neq 0} \phi_p p^{-1} \rho_1(p) \rho_2(-p), \quad \tanh(\phi(p)) = -\frac{\lambda \nu(p)}{2\pi} \]

Defining \( D = T + \tilde{H}_2 \), we can write:
\[ D = \frac{2\pi}{L} \sum_{p} \sigma_p [\rho_1(p) \rho_1(-p) + \rho_2(-p) \rho_2(p)] + E_0 \] (13)
where \( \sigma_p = \text{sech}2\phi(p) - 1 \) and and \([H_0, D] = 0\).

The ground state of \( H \) is
\[ |\text{GS} > = e^{iS} |0 > \] (14)
while \( |0 > \) is the ground state of \( H_0 \).

The relation between the creation or annihilation fermionic operators and the quasi-particle operators can be defined as
\[ \psi_x = e^{ip_{F}x} \psi_{x,1} + e^{-ip_{F}x} \psi_{x,2} \] (15)
and we call \( e^{ip_{F}x} \psi_{x,1} = \tilde{\psi}_{x,1} \) and \( e^{-ip_{F}x} \psi_{x,-1} = \tilde{\psi}_{x,2} \), where \( p_{F} \) is the Fermi momentum. In momentum space this simply means that the momentum \( k \) is measured from the Fermi points, that is \( c_k,\omega = c_{k+c_{F},p_{F},\omega}, \omega = \pm \). Finally we recall that the XXZ spin chain model can be mapped in an interacting fermionic system; when the interaction in the third direction of the spin is missing (XX chain) the mapping is over a non interacting fermionic system with Fermi momentum \( \cos p_{F} = h \). Therefore \( |0 > \) corresponds to the ground state of the XX chain with magnetization \( m \) such that \( p_{F} = \pi(\frac{1}{2} - m) \).

In the NLLM the average of the 2-point function over the ground state is [15], in the \( L \rightarrow \infty \) limit, see App. C
\[ < \text{GS}|\psi_{\omega,x}^{+}\psi_{\omega,y}^{-}|\text{GS} > = \frac{1}{2\pi} \frac{1}{\epsilon_{\omega,x} + \frac{1}{v^2}} \]
\[ \exp \int_{0}^{\infty} dp \frac{1}{p} \{ 2 \sinh 2\phi_p (\cos px - 1) \} \]
Asymptotically for large distances
\[ < \text{GS}|\psi_{\omega,x}^{+}\psi_{\omega,y}^{-}|\text{GS} > \sim O(|x|^{-1-\eta}), \eta = \sinh^2 \phi_0 \] (17)

implying that the average of the occupation number over the interacting ground state is \( n_{k'+c_{F},p_{F}} \sim a + O(k'^{\eta}) \).

We now consider a quantum quench in which the interaction is switched on at \( t = 0 \). An interesting quantity is the non interacting ground state evolved the the interacting Hamiltonian [4]
\[ < O_t |\psi_{\omega,x}^{+}\psi_{\omega,y}^{-}|O_t > = |0 > e^{-iH_t} |\psi_{\omega,x}^{+}\psi_{\omega,y}^{-}| e^{iH_t} |0 > \] (18)

One finds, see App. D, in the limit \( L \rightarrow \infty \)
\[ < O_t |\psi_{\omega,x}^{+}\psi_{\omega,y}^{-}|O_t > = \frac{1}{2\pi} \frac{1}{\epsilon_{\omega,x} + 1} \]
\[ \exp \int_{0}^{\infty} dp \frac{\gamma(p)}{p} \{ (\cos px - 1) - \cos 2p(\sigma_p + 1)t \} \]
where \( \gamma(p) = 4 \sinh^2 \phi_p \cos^2 \varphi_p \). Keeping \( x \) fixed, see App.
\[ \lim_{t \rightarrow \infty} < O_t |\psi_{\omega,x}^{+}\psi_{\omega,y}^{-}|O_t > = \frac{1}{2\pi} \frac{1}{\epsilon_{\omega,x} + 1} \]
\[ \exp \int_{0}^{\infty} dp \frac{1}{p} \{ \gamma(p) (\cos px - 1) \} \]

The 2-point function over \( |0 > \) reaches for \( t \rightarrow \infty \) a limit, similar but different with respect to the average over the ground state (16); thermalization does not occur and memory of the initial state persists. The difference between the limit of the quench and the ground state average is that the prefactor in the integrand (related to the critical exponent) is in one case \( \gamma(p) = 4 \sinh^2 \phi_p \cos^2 \varphi_p \) and in the other \( 2 \sinh^2 \phi_p \).

The value of \( < O_t |\psi_{\omega,x}^{+}\psi_{\omega,y}^{-}|O_t > \) in the LLM can be obtained from (20) replacing \( \gamma(p), \sigma_p \) with \( \gamma(0), \sigma_0 \). Doing that the integral in the exponent of (16) becomes ultraviolet divergent and it requires a regularization; it is found, see [4]
\[ \frac{1}{2\pi} \frac{1}{\epsilon_{\omega,x} + 1} \frac{1}{(x(\gamma(0) + 1 - \frac{v^2t^2}{v^2t^2 - \frac{1}{\gamma(0)}}) \frac{1}{\gamma(0)}} \]
(21)
where \( v = 1 + \sigma_0 \). Comparing (19) with (21) we see that the expressions in the LLM and the NLLM are rather different at short times; in the LLM there is a divergence at \( t = 0 \) due to the ad hoc regularization which is of course absent in the NLLM. The expressions qualitatively agree if the limit \( t \rightarrow \infty \) is performed first but only if we consider the large distance behavior; on the contrary for small distances the behavior is radically different. In the NLLM one sees that the interaction has no effect at small distances (the integral in (20) is \( 1 = x = 0 \)); this is what one expects in a solid state model, as there are no high energy processes altering the short distances (or high momentum) behavior. On the contrary, from (21) we see that the interaction has a strong effect even for small \( x \), as a singularity \( O(x^{-1-\gamma}) \) is present, which is a consequence of the absence of an intrinsic cut-off in such a model.

**Quantum quench for the single particle state**

Let us consider now an inhomogeneous state obtained adding a particle to the non interacting ground state with Fermi momentum \( p_{F} \); the case in which the particle is added to the vacuum is obtained setting \( p_{F} = 0 \).
state is the evolution of $\psi^+_\lambda|0>$ which by (15) can be written as

$$\langle I_{\lambda,t}\rangle = e^{iHt}(e^{ipF\psi^+_\lambda} + e^{-ipF\psi^+_\lambda}|0>$$ \hspace{1cm} (22)

and we consider the average of the number operator $n(z)$

$$\langle I_{\lambda,t}|n(z)|I_{\lambda,t}\rangle$$ \hspace{1cm} (23)

where $n(z)$ is the regularized version of the particle number $\psi^+_\lambda\psi^-_\lambda$, namely $n(z) =$

$$\frac{1}{2} \sum_{\rho=\pm} (\psi^+_{1,z+\rho}\psi^-_{1,z+\rho} + \psi^+_{2,z+\rho}\psi^-_{2,z+\rho} + \psi^+_{1,z+\rho}\psi^-_{2,z+\rho} + \psi^+_{2,z+\rho}\psi^-_{1,z}).$$ \hspace{1cm} (24)

One needs to introducing a point splitting (the sum over $\rho = \pm$) playing the same role as Wick ordering, and as the end the limit $\varepsilon \to 0$ is taken. Note that using the correspondence with the XXZ spin modes, the state $|I_t>$ corresponds to adding an excitation to the ground state of the XX chain with total magnetization $m = 1/2 - pF/\pi$. It turns out that $\langle I_t|n(z)|I_t\rangle$ is sum of several terms

$$\langle 0|\psi^-_{1,z}e^{iHt}\psi^+_{1,z+\rho}\psi^-_{1,z+\rho}e^{iHt}\psi^+_{1,z}|0> + \langle 0|\psi^-_{2,z}e^{iHt}\psi^+_{2,z+\rho}\psi^-_{2,z+\rho}e^{iHt}\psi^+_{2,z}|0> + \langle 0|\psi^-_{1,z}e^{iHt}\psi^+_{1,z+\rho}\psi^-_{2,z+\rho}e^{iHt}\psi^+_{2,z}|0> + \langle 0|\psi^-_{2,z}e^{iHt}\psi^+_{2,z+\rho}\psi^-_{1,z+\rho}e^{iHt}\psi^+_{1,z}|0>$$

In the non-interacting case $\lambda = 0$ the first term can be written as

$$\langle 0|\psi^-_{1,z}\psi^+_{1,z+\rho}|0>\langle 0|\psi^-_{2,z}\psi^+_{2,z}|0>$$ \hspace{1cm} (25)

so that in the limit $\varepsilon \to 0$ this term is equal to $e^{2ipF(x-y)}(4\pi^2)^{-1}(2\pi)^{-1}[(x-z)^2-t^2]^{-1}$; a similar result is found for the second term. The third and fourth terms are vanishing as $\sum_{\rho=\pm} = 0$; similarly the last two term give $(4\pi^2)^{-1}(2\pi)^{-1}[(x-z)\pm t]^{-2}$. Therefore in the absence of interaction one gets

$$\lim_{L\to\infty} \langle I_{0,t}|n(z)|I_{0,t}\rangle = \frac{1}{\pi^2} \frac{\cos 2pF(x-y)}{(x-z)^2-t^2} + \frac{1}{4\pi^2} \frac{1}{((x-z) - t)^2 + ((x-z) + t)^2}$$ \hspace{1cm} (26)

The average of the density is sum of two terms, an oscillating and a non oscillating part (when the particle is added to the vacuum there are no oscillations $pF = 0$). At $t = 0$ the density is peaked at $z = x$, where the average is singular. With the time increasing the particle peaks move in the left and right directions with constant velocity $v_F = 1$ (ballistic motion); that is, the average of the density is singular at $z = x \pm t$ and a "light cone dynamics" is found.

The interaction addresses in a quite non trivial way on the above dynamics. We get in the $L \to \infty$ limit, see App. C

$$\lim_{L\to\infty} \langle I_{\lambda,t}|n(z)|I_{\lambda,t}\rangle = \frac{1}{4\pi^2} \frac{1}{((x-z) - t)^2 + ((x-z) + t)^2} + \frac{1}{4\pi^2} \frac{1}{(x-z)^2 - t^2}$$

$$e^{Z(t)} \left[ e^{2ipF(x-z)}e^{Q_\lambda(x,z)} + e^{-2ipF(x-z)}e^{Q_\lambda(x,z)} \right],$$ \hspace{1cm} (27)

where

$$Z(t) = \int_0^\infty \frac{dp}{p^2}\gamma(p)(\cos 2p(\sigma + 1)t - 1)$$ \hspace{1cm} (28)

and $\gamma(p) = \frac{e^{\sigma(p)-1}}{2}$; moreover $Q_\lambda =$

$$\int_0^\infty \frac{dp}{p^2} \left[ e^{-p\sigma}\left( e^{ip(x-z)+ip(\sigma+1)t} - e^{ip(x-z)+ipt} \right) + \left( e^{-ip(x-z)+ip(\sigma+1)t} - e^{-ip(x-z)+ipt} \right) \right]$$

and $Q_b =$

$$\int_0^\infty \frac{dp}{p^2} \left[ e^{-p\sigma}\left( e^{-ip(x-z)+ip(\sigma+1)t} - e^{-ip(x-z)+ipt} \right) + \left( e^{-ip(x-z)-ip(\sigma+1)t} - e^{-ip(x-z)-ipt} \right) \right].$$

By looking at (27) we see first that the interaction does not modify the non oscillating part. Regarding the oscillating part, it produces two main effects. First of all the velocity of the peaks is not anymore constant but varies between a maximal and minimal value. This is an effect which is absent in the LLM; indeed if we replace $\sigma_F$ with $\sigma_0$ we have

$$\frac{1}{(x-z)^2 - t^2} e^{Q_\lambda} = \frac{1}{(x-z)^2 - (1 + \sigma_0)^2t^2},$$ \hspace{1cm} (29)

so that is one gets the same expression as in the free case with a renormalized velocity (a similar expression is valid for $Q_b$). The presence of non constant velocity is in agreement with the result of numerical simulations in the XXZ chain [13].

The interaction has also another non trivial effect: it introduces a dynamical "Landau quasi-particle weight" in the oscillating part, asymptotically vanishing with time. Indeed for large $t$

$$\exp Z(t) = O(t^{-\gamma(0)})$$ \hspace{1cm} (30)

while $Z(0) = 1$. This vanishing weight can be physically interpreted as a consequence of the fact that fermions are not excitations of the interacting Hamiltonian. Finally note that the quasi-particle weight is $= 1$ at $t = 0$ and decreases at large $t$. 
Conclusions

We have computed by exact analytical methods the time evolution of an inhomogeneous state with a localized fermion added to the non interacting ground state in the non local Luttinger model. The interaction does not produce a simple renormalization of the parameters of the non interacting evolution; on the contrary it generates non constant velocities and a dynamical ‘Landau quasi-particle weight’ appears in the oscillating part of the averaged density, asymptotically vanishing with time. We believe that similar phenomena would be present also in the evolution of more complex initial states like a domain wall profile, and we plan to extend our methods to such a case.

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Appendix A

In order to prove (20) we set \( z = p(1 + \sigma) \) and we note that \( \partial_p z(p) = H_p \) is bounded and different from zero: moreover \( z \) is an increasing function of \( p \) such that \( p/z \) tends to a constant for \( p \to 0 \) and \( p \to \infty \). Integrating by parts and using \( \frac{\sin px}{p} \sim x^2 \), hence (20) follows.

In order to evaluate the large distance behavior of \( Z(t) \) we use that \( \gamma(p) = \frac{v(p)}{2} \) and we write \( \int_0^{\infty} \frac{dp}{p} \gamma(p) \cos 2\omega(p)pt - 1 \) as \( \int_1^{\infty} \frac{1}{p} \) where the second integral is bounded by a constant; in the first term we can write \( \gamma(p) = \gamma(0)e^{-xp} + r(p) \) with \( r(p) = O(p) \), and the integral containing \( r(p) \) is again bounded by a constant. Note that

\[
\gamma(0) \int_0^1 \frac{e^{-xp}dp}{p} \cos 2\omega(0)pt - 1 = \frac{\gamma(0)}{2} \log \frac{1}{\kappa^2 + 4\omega(0)^2p^2} \tag{31}
\]

Moreover we can write \( \omega(p) = 1 + \sigma_p = \omega(0) + f(p) \) with \( f(p) = O(p) \) and

\[
\int_0^1 \frac{e^{-xp}dp}{p} \left[ \cos(2\omega(0)pt) - \cos(2\omega(0)p)t \right] = \int_0^1 \frac{e^{-xp}dp}{p} \left[ \cos(2\omega(0)p)(f(p)pt - 1) + \sin(2\omega(0)p)t \sin(f(p)pt) \right] \tag{32}
\]

Integrating by parts and dividing the integrals from 0 to \( t^{-1} \) ad from \( t^{-1} \) to 1 we get that both integrals are bounded by a constant.

Appendix B

In order to derive (16) we write \( \langle GS | \psi_{1,x}^+ \psi_{1,y}^- | GS \rangle \) as

\[
< 0 | e^{iS} \psi_{1,x}^+ e^{-iS} \psi_{1,y}^- | 0 > \tag{33}
\]

and

\[
e^{iS} \psi_{1,x}^+ e^{-iS} = W_{1,x} R_{1,x} \psi_{1,x} \tag{34}
\]

with \( c(\phi) = \cosh \varepsilon \phi - 1, \ s(\phi) = \sin \varepsilon \phi \)

\[
W_{1,x} = \exp \left\{ -\frac{2\pi}{L} \sum_{p > 0} \frac{e^{-0^+p}}{p} |p_1(-p)e^{ipx} - p_1(p)e^{-ipx}| c(\phi) \right\}
\]

\[
R_{1,x} = \exp \left\{ -\frac{2\pi}{L} \sum_{p > 0} \frac{e^{-0^+p}}{p} |p_2(-p)e^{ipx} - p_2(p)e^{-ipx}| s(\phi) \right\}
\]

so that (33)

\[
< 0 | e^{iS} \psi_{1,x}^+ W_{1,x}^{-1} R_{1,x}^{-1} R_{1,y} W_{1,y} \psi_{1,y}^- | 0 > \tag{35}
\]

By using the commutation relations (8) and \( e^{Ae^B} = e^B e^A e^{[A,B]} \) to carry \( p_1(p) \) \( p_2(p) \) to the left (right) and \( p_1(-p) \) \( p_2(-p) \) to the right (left) and using (9), we get (16).

Appendix C

Let us consider now the interacting case starting from

\[
\langle 0 | \psi_{1,x}^- e^{iHt} \psi_{1,x}^+ \psi_{2,x}^- e^{-iHt} \psi_{2,x}^+ | 0 \rangle \tag{36}
\]

which can be rewritten as

\[
\langle 0 | \psi_{1,x}^- e^{-iS} \{ e^{iS} e^{iHt} e^{-iS} e^{iS} e^{iHt} e^{-iS} \} \psi_{2,x}^- e^{-iS} e^{iHt} e^{-iS} e^{iHt} e^{-iS} | 0 \rangle
\]

We use the relation

\[
e^{i(H_0 + D)t} e^{iS} \psi_{1,x}^+ e^{-i(H_0 + D)t} = \psi_{1,x}^+ W_{1,x,t}^{-1} R_{1,x,t}^{-1}
\]

where \( e^{i(H_0 + D)t} \psi_{1,x}^+ e^{-i(H_0 + D)t} = \psi_{1,x}^+ \) and, calling \( c(\phi) = \cosh \phi - 1, \ s(\phi) = \sinh \phi \)

\[
W_{1,x,t} = \exp \left\{ -\frac{2\pi}{L} \sum_{p > 0} \frac{1}{|p_1(-p,t)\epsilon_p|} \right\}
\]

\[
R_{1,x,t} = \exp \left\{ -\frac{2\pi}{L} \sum_{p > 0} \frac{1}{|p_2(-p,t)\epsilon_p|} \right\}
\]
where $\rho_1(\pm p,t) = e^{\pm i(\sigma_\pi t+1)}\rho_1(\pm p)$, $\rho_2(\pm p,t) = e^{\pm i(\sigma_\pi t)}\rho_2(\pm p)$; moreover

$$\tilde{\psi}_{1,x} = z_0 \psi_{1,x} B_{1,+,x} B_{1,+,x} = z_0 B_{1,+,x} B_{1,-,x} \psi_{1,x}$$  \hspace{1cm} (37)$$

where $B_{1,+,x} =$

$$\exp \frac{2\pi}{L} \left \{ \begin{array}{l}
\int_0^\infty \int_0^\infty \rho_1(\pm p) \left ( e^{-ipx + ip(\sigma_\pi t) + \epsilon} - e^{-ipx + ip(\epsilon-\epsilon)} \right ) \\
\int_0^\infty \int_0^\infty \rho_2(\pm p) \left ( e^{-ipx + ip(\epsilon-\epsilon)} \right )
\end{array} \right \}$$

and $z_0 = \exp \frac{2\pi}{L} \sum_{\rho \neq 0} \frac{1}{\rho} \left ( e^{ip\sigma_\pi t - 1} \right )$.

We write

$$e^{-iS} \tilde{\psi}_{1,x} W_{1,x,t}^{-1} R_{1,x,t}^{-1} e^{IS} = e^{-iS} \psi_{1,x} W_{1,y,t}^{-1} R_{1,y,t}^{-1}$$  \hspace{1cm} (38)$$

where $W_{y,t}, R_{y,t}$ are equal to $W_{y,t}, R_{y,t}$ in (37) with $\rho(p)$ replaced by $e^{-iS} \rho_1(\pm p)e^{iS} = \rho_1(\pm p) \cos \phi(\pm p) - \rho_2(\pm p) \sin \phi(\pm p)$.

Note that $W_{y,t} R_{y,t} = W_{1,y,t} R_{1,y,t}$ so that $W_{1,y,t} R_{1,y,t} = \psi_{1,x}^{\pm}$. It remains to evaluate $e^{-iS} \psi_{1,x} e^{iS}$; we use (37) so that it can be written as

$$z_0 B_{1,+,x,t} B_{1,-,x,t} e^{-iS} \psi_{1,x} e^{iS}$$  \hspace{1cm} (40)$$

where $B_{1,+,x,t}, B_{1,-,x,t}$ are equal to $B_{1,+,x,t}, B_{1,-,x,t}$ with $\rho(p)$ replaced by (39); moreover

$$e^{-iS} \psi_{1,x} e^{iS} = \psi_{1,x} W_{1,0,0,t} R_{1,0,0,t}^{-1}$$  \hspace{1cm} (41)$$

and $W_{1,0,0,t}, R_{1,0,0,t}$ are equal to $W_{1,x,t}, R_{1,x,t}$ with $\sigma_\pi = 0$. In conclusion (18) is given by which can be rewritten as

$$\langle 0 | e^{-iS} \psi_{1,x} e^{iS} \{ e^{iS} e^{iH} e^{-iS} \psi_{1,x} e^{iS} e^{iH} e^{-iS} \} \} e^{-iS} \psi_{2,x} e^{iS}$$

and proceeding as above

$$\langle 0 | \psi_{1,x} | B_{1,+,x,t} B_{1,+,x,t} \psi_{1,x} W_{1,0,0,t} R_{1,0,0,t}^{-1}$$

and $W_{1,x,t} R_{1,0,0,t}^{-1} \psi_{1,x} e^{iS} \psi_{2,x} e^{-iS}$ from which we finally obtain

$$\langle 0 | \{ e^{-iS} \psi_{1,x} e^{iS} W_{1,1,0,t} R_{1,1,0,t}^{-1} \} \times \{ e^{-iS} \psi_{1,x} e^{iS} \psi_{2,x} e^{-iS} \}$$

As in the previous case, we now use the commutation relations (8) and the relation $e^{iA} B e^{iA} = B e^{i[A,B]}$ to carry $\rho_1(\pm p) \rho_2(\pm p)$ to the left (right) and $\rho_1(\pm p) \rho_2(\pm p)$ to the right (left) and using (9) and we get (16). The final expression coincides with the one found in [11] by a different method, namely using a bosonization identity expressing the fermionic field in terms of bosons and Majorana operators.

**Appendix D**

We can write (18) as

$$\langle 0 | e^{-iS} \psi_{1,x} e^{iS} e^{iS} e^{iH} e^{-iS} \psi_{1,x} e^{iS} e^{iH} e^{-iS} \} e^{-iS} \psi_{2,x} e^{iS}$$

which is equal to

$$\langle 0 | e^{-iS} e^{iH} e^{-iS} \psi_{1,x} e^{iS} e^{iH} e^{-iS} \} e^{-iS} \psi_{2,x} e^{iS}$$

and by (37)

$$\langle 0 | e^{-iS} \psi_{1,x} e^{iS} W_{1,1,0,t} R_{1,1,0,t}^{-1} \times \{ e^{-iS} \psi_{1,x} e^{iS} \psi_{2,x} e^{-iS} \}$$

References:

[1] I. Bloch, J. Dalibard and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008).
[2] Polkovnikov A., Sengupta, K., Silva A and Vengalattore M Rev. Mod. Phys. 83 863 (2011)
[3] T. Antal, Z. Raczk, G.M. Schutz, Phys. Rev. E 59, 5 4912 (1999)
[4] M. Cazalilla Phys. Rev. Lett. 97 150403 2006
[5] Iucci A and Cazalilla MA 2009 Phys. Rev. A 80 063619
[6] S.R. Manmana, S. Wessel, R.M. Noack, A. Muramatsu Phys. Rev. Lett.98.210405 (2006)
[7] P. Calabrese, J. Cardy Phys.Rev.Lett. 96, 136801 (2006)
[8] D.Fiorotto, M. Mussardo New J.Phys.12:055015,2010
[9] J. Lancaster A. Mitra Phys. Rev. E 81, 061134 (2010)
[10] A. Mitra and T. Giamarchi, Phys. Rev. Lett. 107, 150602 (2011).
[11] C. Karrasch, J. Rentrop, D. Schuricht, V. Meden Phys. Rev. Lett. 109, 126406 (2012)
[12] W. Liu, N. Andrei Phys. Rev. Lett 2014
[13] T. Sabetta, G. Misguich Phys. Rev. B 88, 245114 (2013)
[14] L. Bonnes, H. L. Eisler, A. M. Luchli Phys. Rev. Lett. 113, 187203 (2014)
[15] D. C. Mattis and E. H. Lieb J. Math. Phys. 6, 2304 (1965)
[16] C. N. Yang and C. P. Yang, Phys. Rev. 150, 321 (1966)
[17] D. Haldane. Phys. Rev. Lett. 45, 1358 (1980)
[18] G.Benfatto, P. Falco V. Mastropietro. Phys. Rev. Lett. 104 075701 (2010).
[19] J. Sørbye, R.G. Pereira, I. Affleck Phys. Rev. B 83, 035115 (2011)
[20] V. Mastropietro. Phys. Rev. E 87, 042121 (2013)