Gluonic Pole Matrix Elements in Spectator Models

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We investigate the gluonic pole matrix element contributing to the first $p_T$ moment of the distribution and fragmentation functions in a spectator model. By performing a spectral analysis, we find that for a large class of spectator models, the contribution of gluonic pole matrix elements is non-zero for the distribution correlators, whereas in fragmentation correlators they vanish. This outcome is important in the study of universality for fragmentation functions.

Keywords: Distribution function, fragmentation function, gluonic pole

1. Introduction

In order to access intrinsic transverse momenta, one needs to do a careful study of the azimuthal dependencies in high energy processes. Azimuthal imbalance can generate single spin asymmetries. These effects are not suppressed by powers of hard scale in comparison with the leading order terms. But it requires measuring hadronic scale quantities (transverse momenta) in a high momentum environment. Various symmetries, in particular, time reversal invariance play a key role here.

The proper gauge invariant definitions of transverse momentum dependent correlators involves path ordered exponentials (gauge links).\textsuperscript{1} Recently, it has been found that the part of the path due to the transverse gluon field at lightcone $\pm \infty$ are not necessarily suppressed in light-cone gauge.\textsuperscript{2,3} It is to be noted that the gauge invariance of collinear correlators also require a gauge link, however, in this case the bilocality in the operator is only in the lightcone direction and the gauge link is universal, independent of the process. Moreover, the link can be set to unity by choosing an axial gauge. This is not the case for in the transverse momentum dependent (TMD) correlators, as the bilocality is not restricted to the lightcone direction but exists
also in the transverse direction. The resulting full color gauge invariant matrix element becomes process dependent due to the process dependent gauge link.

As QCD is time reversal invariant, it is possible to distinguish quantities and observables according to their T-behavior. For distribution correlators \( \Phi(x) \), involving plane wave hadronic states in their definition, T-reversal and Hermiticity implies that the collinear correlators are T-even. However the TMD distribution correlators \( \Phi(x, p_T) \) are process dependent due to the gauge link as stated above and time reversal relates \( \Phi^{+}(x, p_T) \) and \( \Phi^{-}(x, p_T) \), where the link is in two opposite light cone direction. Thus one can construct T-even and T-odd combinations and in general, TMD correlators can be parametrized in terms of both T-even and T-odd functions. The T-odd distribution functions, such as Sivers function, are non-zero due to the presence of the gauge link in the correlator.

The situation regarding T-invariance is different for the fragmentation TMD correlator. These involve hadronic out state in the definition and thus one cannot use the T-invariance as a constraint while parametrizing them. One always has to allow T-even and T-odd parts of the correlator. For spin 0 and spin 1/2 hadrons, in the collinear case, no T-odd functions appear at leading twist. Including TMD, for fragmentation functions there are now two mechanisms for T-odd terms:

- That due to the operator including the gauge link;
- That due to the final state.

A nice feature, however, is that the two mechanisms leading to T-odd functions can be distinguished. The T-odd operator structure can be traced back to the color gauge link that necessarily appears in correlators to render them color gauge-invariant and it is process dependent. That due to the final state is independent of the process.

Relating T-odd effects in different processes, requires azimuthal weighting, which projects out the transverse momentum weighted parts of the correlators \( \Phi \) and \( \Delta \), referred to as (first) transverse moments \( \Phi_\theta \) and \( \Delta_\theta \), respectively. After azimuthal weighting of the cross sections, one simply finds that the T-odd features originating from the gauge link lead to specific factors with which the T-odd functions appear in observables. Comparing T-odd effects in distribution functions in semi-inclusive DIS (SIDIS) and Drell-Yan (DY) processes, one finds a relative minus sign, which means that the Sivers function in these two processes differ by a sign. The T-odd operator parts of \( \Phi_\theta \) and \( \Delta_\theta \) are precisely the soft limits \( (k_1 \to 0) \) of the
gluonic pole matrix elements, denoted by $\phi_G(k_1 = 0)$ and $\Delta G(k_1 = 0)$. In this work, we investigate the spectral properties of these matrix elements, which play an important role in the universality issue of the TMD distribution and fragmentation functions.

![Graphical representation of the correlator in the case of distributions of partons with momentum $k$ in a hadron with momentum $P$ and the spectator model description.](image1)

![Graphical representation of the correlator in the case of fragmentation of partons with momentum $k$ into a hadron with momentum $P$ (a) and the spectator model description (b).](image2)

For the correlators, depicted in Figs 1 and 2, the expressions in terms of bilocal matrix elements are frequently used as a starting point in modeling distribution and fragmentation functions. In particular the spectator model has become fairly popular, because it is easy, flexible and intuitively attractive. On the other hand, one should be very careful, because the predictive power depends on limiting oneself in the choice of spectator (e.g. a diquark with fixed mass for the nucleon) and using simple vertices. In fact making a spectral analysis of the spectator and allowing for the most general vertices one would lose all predictive power. Here we will investigate differences between distribution and fragmentation functions using a spectral analysis and using physical intuition in restricting the momentum dependence and asymptotic behavior of the vertices.
Rather than looking at the full process in the model and carefully studying the cuts,\textsuperscript{4,5} we look at the soft part only,\textsuperscript{6} but do this for the quark-quark-gluon correlators $\Phi_G$ and $\Delta_G$, as shown in Figs 3 and 4, respectively. Our approach starting directly with the multi-parton correlator has the advantage that we can work at tree-level and just do a spectral analysis, unlike a one loop calculation.\textsuperscript{7}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The graphical representation of the quark-quark-gluon correlator $\Phi_G$ in the case of distributions including besides the quark a gluon with momentum $k_1$ and the possible intermediate states (a) and (b) in a spectator model description.}
\end{figure}

2. Gluonic pole matrix elements

We make a Sudakov decomposition of the momenta of active partons, $k = xP + \sigma n + k_T$. The Sudakov vector $n$ is an arbitrary light-like four-vector $n^2 = 0$ that has non-zero overlap $P \cdot n$ with the hadron’s momentum $P$. We will simply choose $P \cdot n = 1$.

The TMD distribution correlators are given by :

$$\Phi^{ij}_{\alpha}(x,k_T) = \int \frac{d(\xi \cdot P) d^2\xi_T}{(2\pi)^3} e^{ik\cdot \xi} \langle \psi_j(0) U_{\{0;\xi\}} \psi_i(\xi) \mid P \rangle_{\text{LF}}. \quad (1)$$

The Wilson line or gauge link $U_{\{\eta;\xi\}} = \text{P} \exp[-ig\int_{C} ds \cdot A^a(s) t^a]$ is a path-ordered exponential along the integration path $C$ with endpoints at $\eta$ and $\xi$.

In azimuthal asymmetries one needs the transverse moments contained in the correlator

$$\Phi^{[TL]}_\alpha(x) = \int d^2 k_T \ k_T^\alpha \Phi^{TL}(x,k_T). \quad (2)$$
This can be written as

\[
\Phi^\alpha_{[\ell]}(x) = \tilde{\Phi}^\alpha_{[\ell]}(x) + C^\alpha_{[\ell]} \pi \Phi_G^\alpha(x, x),
\]

with calculable process-dependent gluonic pole factors \(C^\alpha_{[\ell]}\) and process (link) independent correlators \(\tilde{\Phi} \) (T even) and \(\Phi_G\) (T-odd). The latter is precisely the soft limit \(x_1 \to 0\) of a quark-gluon correlator \(\Phi_G^\alpha(x, x_1)\) of the type

\[
\Phi_G^\alpha(x, x-x_1) = n_\mu \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{ix_1(\eta \cdot P)} e^{i(x-x_1)(\xi \cdot P)} e^{i \int \phi(0) U_{\eta}^n g^{\mu\alpha}(\eta) U_{\xi}^n \psi(\xi) |P|} \langle P \psi(0) |U_{[\eta]} |\xi, \xi|\psi, \psi(P) \rangle |P| \Bigg|_{\text{LC}}.
\]

The TMD fragmentation correlator is given by\(^2\)

\[
\Delta_{ij}^{\ell}(z, k_T) = \sum X \int d\xi \frac{d^2 \xi}{(2\pi)^3} e^{ik\xi} \langle 0 |U_{[\xi]} \psi_i(\xi) |P, X \rangle \langle P, X |\tilde{U}_j(0) |0 \rangle |_{\text{LF}}.
\]

In the transverse moments obtained after \(k_T\)-weighting,

\[
\Delta_{ij}^{[\ell]}(z) = \int d^2 k_T k_T^\alpha \Delta_{ij}^{[\ell]}(z, k_T) = \tilde{\Delta}_{ij}^{[\ell]} \left( \frac{1}{2} \right) + C^\alpha_{[\ell]} \pi \Delta_G^{\alpha} \left( \frac{1}{2}; \frac{1}{2} \right);
\]

the two link independent correlators \(\tilde{\Delta}_{ij}\) and \(\Delta_G\) contain again a T-even and T-odd operator combination, respectively, however, parametrizations
of both of them contain T-odd functions as the final state is involved in both. The gluonic pole correlator is the soft limit, $z^{-1} = x_1 \to 0$, of the quark-gluon correlator

$$\Delta_{G,ij}^\alpha(x, x-x_1) = \sum_X \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{i x_1 (\eta \cdot P)} e^{i (x-x_1) (\xi \cdot P)}$$

$$\times \langle 0 | U_n^{\alpha \alpha} (\eta) U_{ij}^\rho(\xi) | P, X \rangle \langle P, X | \bar{\psi}_j(0) | 0 \rangle \bigg|_{LC} \right)$$

(7)

With $\Delta_{G}(x, x)$ being zero, T-odd FFs in $\Delta_{G}$ only come from the T-even operator combination in $\tilde{\Delta}_{G}$, which is due to the hadron-jet state and they are process independent, for instance the T-odd Collins function. In contrast T-odd DFs in $\Phi_{G}$ only can come from $\Phi_{G}(x, x)$. They can still be universal but acquire process dependent gluonic pole factors.\(^8,9\) We calculate $\Phi_{G}$ and $\Delta_{G}$ in a spectator model approach.

3. The spectator model approach

In a typical spectator model approach to distribution or fragmentation correlators one considers a spectator with mass $M_s$. The result for the cut, but untruncated, diagrams, such as in Figs. 1 and 2 are of the form

$$\Phi(x,k_T) \sim \int d(k \cdot P) \frac{F(k^2,k \cdot P)}{(k^2 - m^2 + i\epsilon)^2} \frac{\delta ((k - P)^2 - M_s^2)}{((k - P)^2 - M_s^2)}.$$  \hspace{1cm} (8)

The details of the numerator function depend on the details of the model, including the vertices, polarization sums, etc. These must be chosen in such a way as to not produce unphysical effects, such as a decaying proton if $M \geq m + M_s$. Thus $m$ in Eq. 8 must represent some constituent mass in the quark propagator, rather than the bare mass. The useful feature of the model is its ability to produce reasonable valence and even sea quark distributions using the freedom in the model connected to an intuitive picture. The results for the fragmentation function in the spectator model is identical upon the substitution of $x = 1/z$.

The quark-gluon correlators as shown in Figs. 3 and 4 can be calculated in the spectator model and the gluonic pole matrix elements can be extracted by taking the limit $x_1 \to 0$. Assuming that the numerator does not grow with $k_1^{-}$ one can easily perform the $k_1^{-}$ integrations.\(^6\) We obtain (for $x \geq 0$)

$$\Phi_{G}(x, x) = - \int d^2 k_T d^2 k_{1T} \frac{(1-x) F_1(x,0,k_T,k_{1T}) \theta(1-x)}{(\mu^2 - k_T^2)(x B_1 + (1-x) A_2) A_1}.$$ \hspace{1cm} (9)
and for fragmentation functions \((x = 1/z \geq 1)\)

\[ \Delta_G(x, x) = 0. \]  

(10)

4. Conclusion

We have investigated the gluonic pole contributions to the distribution and fragmentation functions. Instead of doing a quantitative analysis involving details of a phenomenological model, we limit ourselves to a spectral analysis within the spectator framework, in order to understand the basic features of these quantities. We simply assumed that masses and vertices do not spoil our analysis. We find that under realistic assumptions, the gluonic pole contributions for fragmentation correlators vanish whereas these contributions do not vanish for distribution correlators. The result for fragmentation correlators at nonzero gluon momentum is nonzero. A full proof that the gluonic pole contributions to the fragmentation correlators vanish is important as it eliminates a whole class of matrix elements parameterized in terms of T-odd fragmentation functions besides the T-odd fragmentation functions in the parameterization of the two-parton correlators.

5. Acknowledgments

AM would like to thank the organizers of Transversity 2008 for the kind invitation and support.

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