Dual Anti-de Sitter Superalgebras from Partial Supersymmetry Breaking

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Abstract

The partial breaking of supersymmetry in anti-de Sitter (AdS) space can be accomplished using two of four dual representations for the massive $OSp(1,4)$ spin-3/2 multiplet. The representations can be “unHiggsed,” which gives rise to a set of dual $N = 2$ supergravities and supersymmetry algebras.
1 Introduction

Most supersymmetry phenomenology is based on the minimal supersymmetric standard model, which is assumed to be the low energy limit of a more fundamental theory in which an \( N = 8 \) supersymmetry is spontaneously broken to \( N = 1 \). The Goldstone fermions from the seven broken supersymmetries are eliminated by the superHiggs effect: they become the longitudinal components of seven massive gravitinos.

The physics that underlies this superHiggs effect is most easily illustrated by the simpler case of \( N = 2 \) supersymmetry, spontaneously broken to \( N = 1 \). Indeed, in Ref. [1] a set of theories were constructed that realized this partial supersymmetry breaking in flat Minkowski space. These theories circumvent a naive argument that forbids partial breaking by abandoning one of its assumptions, namely that of a positive definite Hilbert space. In covariantly-quantized supergravity theories, the gravitino \( \psi_{\alpha a} \) is a gauge field with negative-norm components, so the Hilbert space is not positive definite.

The Minkowski-space theories were based on \( N = 2 \) super-Poincaré algebras with certain central extensions. In anti-de Sitter (AdS) space, however, the \( N = 2 \) supersymmetry algebra is different. The algebra is known as \( OSP(2,4) \); the relevant parts are (see appendix A),

\[
\begin{align*}
\{ Q_i^\alpha, \bar{Q}_j^\dot{\beta} \} &= 2 \sigma^\alpha_\beta R_a \delta^i_j \\
\{ Q_i^\alpha, Q^{\beta j} \} &= 2i \Lambda \sigma^{ab}_\alpha \beta M_{ab} \delta^{ij} + 2i \delta^{\beta}_\alpha T^{ij} \\
[T^{ij}, Q^k] &= i \Lambda (\delta^{jk} Q^i - \delta^{ik} Q^j).
\end{align*}
\]

In this expression, the \( Q_i^\alpha (i \in \{1,2\}) \) denote the two supercharges, while \( M_{ab} \) and \( R_a \) are the generators of \( SO(3,2) \). The antisymmetric matrix \( T^{ij} \) is the single hermitian generator of an additional \( SO(2) \). As the cosmological constant \( \Lambda \to 0 \), the algebra contracts to the usual \( N = 2 \) Poincaré supersymmetry algebra with at most one real central charge. (The generator \( R_a \) contracts to the momentum generator \( P_a \), while \( T^{ij} \) contracts to zero or to a single real central charge, depending on the rescaling of the operators.)

In Minkowski space, partial supersymmetry breaking was found to require super-Poincaré algebras with two central extensions [1]. The \( OSP(2,4) \) algebra contracts to a super-Poincaré algebra with at most one central charge. This suggests that if partial breaking is to occur, the AdS algebra must be modified.

In this paper we will study this question using the same approach as in [1]. We will see that partial breaking in AdS space occurs for two of four dual representations of the \( OSP(1,4) \) massive spin-3/2 multiplet. We will find that the two dual representations give rise to new AdS supergravities with appropriately modified \( OSP(2,4) \) supersymmetry algebras\[1\]. As the cosmological constant \( \Lambda \to 0 \), the new algebras contract to the \( N = 2 \) Poincaré algebras with the required set of central extensions.

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\[1\] A theory exhibiting partial supersymmetry breaking in AdS space was derived in [2]. However, this construction contains more fields because it has complete \( N = 2 \) multiplets.
2 The SuperHiggs Effect in Partially Broken AdS Supersymmetry

2.1 Dual Versions of Massive $OSp(1, 4)$ Spin-3/2 Multiplets

The starting point for our investigation is the massive $OSp(1, 4)$ spin-3/2 multiplet. This multiplet contains six bosonic and six fermionic degrees of freedom, arranged in states of the following spins,

$$\begin{pmatrix}
\frac{3}{2} \\
1 \\
1 \\
\frac{1}{2}
\end{pmatrix}.$$

(2)

It contains the following AdS representations (see e.g. [3] and references therein),

$$D(E + \frac{1}{2}, \frac{3}{2}) \oplus D(E, 1) \oplus D(E + 1, 1) \oplus D(E + \frac{1}{2}, \frac{1}{2})$$

(3)

where $D(E, s)$ is labeled by the eigenvalues of the diagonal operators of the maximal compact subgroup $SO(2) \times SU(2) \subset SO(3, 2)$ and unitarity requires $E \geq 2$. (The eigenvalue $E$ is the AdS generalization of a representation’s rest-frame energy. As $E \to 2$, the first two representations in (3) become “massless,” with eigenvalues $(s + 1, s)$. The massless representations are short representations of $OSp(1, 4)$.)

As in Minkowski space, a massive spin-1 field can be represented by a vector or by an antisymmetric tensor. For the case at hand, there are four possibilities. The Lagrangian with two vectors is given by

$$e^{-1} \mathcal{L} = e^{-\epsilon \sigma_n \partial_n \psi^m - i \sigma^m \nabla_m \psi - \frac{1}{4} A_{mn} A^{mn} - \frac{1}{4} B_{mn} B^{mn}$$

$$- \frac{1}{2} (m^2 - m \Lambda) A_m A^m - \frac{1}{2} (m^2 + m \Lambda) B_m B^m$$

$$+ \frac{1}{2} m \zeta \zeta + \frac{1}{2} m \bar{\zeta} \bar{\zeta} - m \psi_m \sigma^{mn} \psi_n - m \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n$$

(4)

where $\Lambda \geq 0$ and $\nabla_m$ is the AdS covariant derivative. Here $\psi_m$ is a spin-3/2 Rarita-Schwinger field, $\zeta$ a spin-1/2 fermion, and $A_{mn}$ and $B_{mn}$ are the field strengths of the real vectors $A_m$ and $B_m$. This Lagrangian is invariant under the following supersymmetry transformations,\(^2\)

$$\delta_\eta A_m = \sqrt{1 + \epsilon} \left( \psi_m \eta + \bar{\psi}_m \bar{\eta} \right)$$

$$+ \frac{1}{\sqrt{1 - \epsilon}} \left( i \frac{1}{\sqrt{3}} (1 - \epsilon) (\bar{\eta} \sigma_m \zeta - \bar{\zeta} \sigma_m \eta) - \frac{1}{\sqrt{3}} \partial_m (\zeta \eta + \bar{\zeta} \bar{\eta}) \right)$$

$$\delta_\eta B_m = \sqrt{1 - \epsilon} \left[ (-i \psi_m \eta + i \bar{\psi}_m \bar{\eta}) + \frac{1}{\sqrt{1 + \epsilon}} \right]$$

$$+ \frac{1}{\sqrt{1 + \epsilon}} \left( i \frac{1}{\sqrt{3}} (1 + \epsilon) (\bar{\eta} \sigma_m \zeta + \bar{\zeta} \sigma_m \eta) + \frac{1}{\sqrt{3}} \partial_m (\zeta \eta - \bar{\zeta} \bar{\eta}) \right).$$

\(^2\)Here, and in all subsequent rigid supersymmetry transformations, the parameter $\eta$ is covariantly constant but $x$-dependent (see (28) in appendix A).
\[ \delta_\eta \zeta = \sqrt{1 - \epsilon} \left( \frac{1}{\sqrt{3}} A_{mn} \sigma^{mn} \eta - \frac{i}{\sqrt{3}} \sigma^m \bar{\eta} A_m \right) \]

\[ + \sqrt{1 + \epsilon} \left( - \frac{i}{\sqrt{3}} B_{mn} \sigma^{mn} \eta + \frac{m}{\sqrt{3}} \sigma^m \bar{\eta} B_m \right) \]

\[ \delta_\eta \psi_m = \frac{1}{\sqrt{1 + \epsilon}} \left( \frac{1}{3m} \nabla_m (A_{rs} \sigma^{rs} \eta + 2im \sigma^n \bar{\eta} A_n) - \frac{i}{2} (H^A_{+mn} \sigma^n + \frac{1}{3} H^A_{-mn} \sigma^n) \bar{\eta} \right. \]

\[ - \frac{2}{3} m (\sigma^m \eta + A_m \eta) - \frac{i}{2} \epsilon H^A_{+mn} \sigma^n \bar{\eta} - \epsilon m A_m \eta \]

\[ + \frac{1}{\sqrt{1 - \epsilon}} \left( - \frac{i}{3m} \nabla_m (B_{rs} \sigma^{rs} \eta - 2im \sigma^n \bar{\eta} B_n) + \frac{1}{2} (H^B_{+mn} \sigma^n \eta + \frac{2}{3} im (\sigma^m \eta + B_m \eta) - \frac{1}{2} \epsilon H^B_{+mn} \sigma^n \bar{\eta} - i \epsilon m B_m \eta) \right). \]

(5)

where \( H^A_{\pm mn} = A_{mn} \pm \frac{i}{2} \epsilon_{mnrs} A^r s \) and \( \epsilon = \Lambda / m \). Note that the “mass” \( m \) is defined to be \( m = (E - 1) \Lambda \). This definition is consistent with the AdS representations in (3). The fact that \( E \geq 2 \) implies that \( 0 \leq \epsilon \leq 1 \).

In Minkowki space, other field representations of the massive spin-3/2 multiplet can be derived \([1]\) using a Poincaré duality which relates massive vector fields to massive anti-symmetric tensor fields of rank two. The same duality also holds in AdS space where, for example, the vector \( B_m \) can be replaced by an antisymmetric tensor \( B_{mn} \). The Lagrangian for the dual theory is then:

\[ e^{-1} \mathcal{L} = e^{-1} \epsilon^{mnrs} \bar{\psi}_m \sigma_n \nabla_r \psi_s - i \bar{\zeta} \sigma^m \nabla_m \zeta - \frac{1}{4} A_{mn} A^{mn} + \frac{1}{2} v^B_m v^B_m \]

\[ - \frac{1}{2} (m^2 - m \Lambda) A_m A^m - \frac{1}{4} (m^2 + m \Lambda) B_{mn} B^{mn} \]

\[ + \frac{1}{2} m \zeta \bar{\zeta} + \frac{1}{2} m \bar{\zeta} \zeta - m \psi_m \sigma^{mn} \psi_n - m \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n \]

(6)

where \( A_{mn} \) is the field strength associated with the real vector field \( A_m \) and \( v_m \) is the field strength for the antisymmetric tensor \( B_{mn} \). This Lagrangian is invariant under the following supersymmetry transformations:\(^3\)

\[ \delta_\eta A_m = \sqrt{1 + \epsilon} (\psi_m \eta + \bar{\psi}_m \bar{\eta}) \]

\[ + \frac{1}{\sqrt{1 - \epsilon}} \left( i \frac{1}{\sqrt{3}} (1 - \epsilon) (\bar{\eta} \sigma_m \zeta - \bar{\zeta} \sigma_m \eta) - \frac{1}{\sqrt{3m}} \nabla_m (\zeta \eta + \bar{\zeta} \bar{\eta}) \right) \]

\[ \delta_\eta B_{mn} = \sqrt{1 + \epsilon} \left( - \frac{1}{m} \nabla_m (\eta \psi_n) - i m \sigma_m \bar{\psi}_n) - \frac{2}{\sqrt{3}} \left( \bar{\eta} \sigma_m \zeta + \frac{1}{2m} \nabla_m (\bar{\zeta} \sigma_n \eta) \right) \right. \]

\[ + h.c. \]

\[ \delta_\eta \zeta = \sqrt{1 + \epsilon} (\frac{1}{\sqrt{3}} A_{mn} \sigma^{mn} \eta - \frac{i}{\sqrt{3}} \sigma^m \bar{\eta} A_m) \]

\[ + \frac{m}{\sqrt{3}} (1 + \epsilon) B_{mn} \sigma^{mn} \eta + \frac{1}{\sqrt{3}} \sigma^m \bar{\eta} v^B_m \]

\(^3\)Here, the square brackets denote antisymmetrization, without a factor of 1/2.
\[ \delta_\eta \psi_m = \frac{1}{\sqrt{1 + \epsilon}} \left( \frac{1}{3m} \nabla_m (A_{rs} \sigma^{rs} \eta + 2 m \sigma^n \eta A_n) - \frac{i}{2} (H_{+mn}^A \sigma^n + \frac{1}{3} H_{-mn}^A \sigma^n) \eta \right) \\
- \frac{2}{3} m (\sigma_m^n A_n \eta + A_m \eta) - \frac{i}{2} H_{+mn}^A \sigma^n \eta - \epsilon m A_m \eta \right) \\
+ \frac{1}{\sqrt{1 - \epsilon}} \left( \frac{1}{3m} \nabla_m (m \sqrt{1 + \epsilon} B_{rs} \sigma^{rs} \eta - 2 - \frac{1}{\sqrt{1 + \epsilon}} \sigma^n \eta v_n^B) \right) \\
+ \sqrt{1} + \epsilon ((\frac{1}{3} - \frac{\epsilon}{2}) B_{mn} \sigma^n \eta + i (\frac{1}{3} - \frac{\epsilon}{4}) \epsilon B_{mnrs} \sigma^n \eta \\n+ \frac{2}{3} \sqrt{1 + \epsilon} (\sigma_m^B \eta + v_n^B \eta - i \frac{\epsilon}{\sqrt{1 + \epsilon} v_m^B \eta}) \right) .
\]

Two more representations can be found by dualizing the vector \( A_m \). The derivations are straightforward, so we will not write the Lagrangians and transformations here. Each of the four dual Lagrangians describe the dynamics of free massive spin-3/2 and 1/2 fermions, together with their supersymmetric partners, massive spin-one vector and tensor fields.

In what follows we shall see that the first two representations are special because they can be regarded as “unitary gauge” descriptions of theories with a set of additional symmetries: a fermionic gauge symmetry for the massive spin-3/2 fermion, as well as additional gauge symmetries associated with the massive gauge fields.

### 2.2 UnHiggsing Massive \( OSp(1, 4) \) Spin-3/2 Multiplets

To exhibit the superHiggs effect, we will unHiggs these Lagrangians by first coupling them to \( N = 1 \) supergravity and including a Goldstone fermion and its superpartners, and then gauging the full \( N = 2 \) supersymmetry. In this way we will construct theories with a local \( N = 2 \) supersymmetry nonlinearly realized, but with \( N = 1 \) represented linearly on the fields. The resulting Lagrangians describe the physics of partial supersymmetry breaking well below the scale \( v \) where the second supersymmetry is broken.

In flat space, a massive representation can be unHiggsed by passing to its massless limit, where the Goldstone fields become physical degrees of freedom. In AdS space, the “massless” limit corresponds to \( E \to 2 \). In this limit the massive spin-3/2 multiplet splits into a massless spin-3/2 multiplet, plus a massive \( \text{vector/tensor multiplet of spin one} \) (see also appendix B):

\[
\begin{align*}
\text{massive spin-3/2 multiplet} & \\
D(E, 1) & \oplus D(E + \frac{1}{2}, \frac{3}{2}) & \oplus D(E + \frac{1}{2}, \frac{1}{2}) & \oplus D(E + 1, 1) \\
E \to 2 & \\
\text{massless spin-3/2 multiplet} & \\
D(2, 1) & \oplus D(\frac{5}{2}, \frac{3}{2}) & \text{and} & \quad D(\frac{5}{2}, \frac{1}{2}) & \oplus D(3, 1) & \oplus D(3, 0) & \oplus D(\frac{7}{2}, \frac{1}{2})
\end{align*}
\]
Figure 1: The degrees of freedom of the unHiggsed $OSp(1, 4)$ massive spin-3/2 multiplet coupled to gravity. The massive spin-1 field can be represented by either a vector or an antisymmetric tensor.

The spin-one multiplet with $\hat{E} = 5/2$ (The normalization of $E$ differs for different multiplets; see appendix B and Ref. [3].) cannot itself be unHiggsed because that would require $E \to 1$. For the case at hand, this would spoil the unitarity of the spin-3/2 field.

In Figure 1 the physical fields of the massive spin-3/2 multiplet coupled to gravity are arranged in terms of $N = 1$ multiplets. The fields of lowest spin form a massive $N = 1$ vector/tensor multiplet. They may be thought of as $N = 1$ “matter.” The remaining fields are the gauge fields of $N = 2$ supergravity. In unitary gauge, the massless vector eats the scalar, while the Rarita-Schwinger field eats one linear combination of the spin-1/2 fermions. This leaves the massive $N = 1$ spin-3/2 multiplet coupled to $N = 1$ supergravity. In contrast to the superHiggs effect in a flat background, where an $N = 2$ multiplet emerges in the massless limit of the unHiggsed theory [4], the $E \to 2$ limit in AdS space gives rise to $N = 1$ multiplets only.

To find the Lagrangian, let us introduce a set of Goldstone fields by the following Stückelberg redefinitions. For the case with two vectors, we include Goldstone fields by replacing

$$
A_m \rightarrow A_m - \frac{1}{\sqrt{1 - \epsilon m}} \partial_m \phi_A
$$

$$
B_m \rightarrow B_m - \frac{1}{\sqrt{1 + \epsilon m}} \partial_m \phi_B .
$$

(7)

For the dual representation, we take

$$
A_m \rightarrow A_m - \frac{1}{\sqrt{1 - \epsilon m}} \partial_m \phi
$$

$$
B_{mn} \rightarrow B_{mn} - \frac{1}{\sqrt{1 + \epsilon m}} \partial_{[m} B_{n]} .
$$

(8)

In each case, the introduction of the Goldstino $\nu$ requires an additional shift

$$
\psi_m \rightarrow \psi_m - \frac{1}{\sqrt{6} \sqrt{1 - \epsilon m}} (2 \nabla_m \nu + im \sigma_m \bar{\nu})
$$

(9)
to obtain a proper kinetic term for $\nu$.

For the case with two vectors, the Lagrangian is as follows,

$$ e^{-1} \mathcal{L} = $$

$$ - \frac{1}{2\kappa^2} \mathcal{R} + \epsilon^{mnrs} \vec{\psi}_m \sigma_n D_r \psi^s - i\lambda \sigma^m D_m \lambda - i\lambda \sigma^m D_m \chi $$

$$ - \frac{1}{4} A_{mn} A^{mn} - \frac{1}{4} B_{mn} B^{mn} - \frac{1}{2} D_m \phi_A D^m \phi_A - \frac{1}{2} D_m \phi_B D^m \phi_B $$

$$ - \left( \frac{1}{\sqrt{2}} m \sqrt{1 - \epsilon^2 \psi^2_m \sigma^m \chi} + m \sqrt{1 - \epsilon^2 \psi^2_n \sigma^m \chi} \right) $$

$$ + \sqrt{2} m i \lambda \chi + \frac{1}{2} m \lambda \chi + m \psi^2_m \sigma^m \psi^2_n - \epsilon m \psi^1_m \sigma^m \psi^1_n $$

$$ + \frac{\kappa}{4} \epsilon_{ij} \psi^i_j \psi^j_\phi (\sqrt{1 + \epsilon H_{A-}^{mn} - i\sqrt{1 - \epsilon H_{B-}^{mn}}}) $$

$$ + \frac{\kappa}{2} \lambda \sigma^m \psi^1_m (D_n \phi_A - iD_n \phi_B) $$

$$ + \frac{\kappa}{2\sqrt{2}} \bar{\lambda} \Psi_m \psi^1_n (\sqrt{1 - \epsilon^2 H_{A+}^{mn} - i\sqrt{1 - \epsilon H_{B+}^{mn}}}) $$

$$ + \frac{\kappa}{2} \epsilon^{mnrs} \sqrt{1 - \epsilon^2 \psi^2_m \sigma^r \psi^1_r} (\partial_s \phi_A - i\partial_s \phi_B) $$

$$ - \frac{\kappa}{2} m \epsilon^{mnrs} \bar{\psi}_m \sigma^r \psi^1_r (\sqrt{1 + \epsilon A_s - i\sqrt{1 - \epsilon B_s}}) $$

$$ - 2\kappa m \epsilon \sqrt{1 - \epsilon^2 \psi^2_m \sigma^m \psi^1_n \phi_A + \frac{\kappa m \epsilon}{\sqrt{2}} \bar{\lambda} \sigma^m \psi^1_m \phi_A} $$

$$ + i\kappa m \bar{\chi} \sigma^m \psi^1_m \phi_A + h.c.) + 3 \frac{\epsilon^2 m^2}{\kappa^2}. \quad (10) $$

In this expression, $\kappa$ denotes Newton’s constant, $m = \sqrt{\Lambda^2 + \kappa^2 \nu^4}$ and $D_m$ is the full covariant derivative. The scalar-field gauge-invariant derivatives are as follows,

$$ D_m \phi_A = \partial_m \phi_A - m \sqrt{1 - \epsilon \nu^2_m} $$

$$ D_m \phi_B = \partial_m \phi_B - m \sqrt{1 + \epsilon \nu^2_m}, \quad (11) $$

while the supercovariant derivatives take the form

$$ \hat{D}_m \phi_A = \partial_m \phi_A - m \sqrt{1 - \epsilon \nu^2_m} - \frac{\kappa}{2} (\bar{\psi}_m \chi + \psi^1_m \bar{\chi}) $$

$$ \hat{D}_m \phi_B = \partial_m \phi_B - m \sqrt{1 + \epsilon \nu^2_m} i + \frac{\kappa}{2} (\bar{\psi}_m \chi - \psi^1_m \bar{\chi}) $$

$$ \hat{A}_{mn} = A_{mn} + \frac{\kappa}{2} \sqrt{1 + \epsilon (\psi^2_m \psi^1_n + \bar{\psi}^2_m \bar{\psi}^1_n)} $$

$$ - \frac{\kappa}{2 \sqrt{2}} (\bar{\lambda} \sigma^m \psi^1_m + \psi^1_m \sigma^m \lambda) $$

$$ \hat{B}_{mn} = B_{mn} - \frac{\kappa}{2} \sqrt{1 - \epsilon (\psi^2_m \psi^1_n - \bar{\psi}^2_m \bar{\psi}^1_n)} $$

$$ + \frac{\kappa}{2 \sqrt{2}} (\bar{\lambda} \sigma^m \psi^1_m - \psi^1_m \sigma^m \lambda) \cdot \quad (12) $$
This Lagrangian is invariant (to lowest order in the fields) under the following supersymmetry transformations,

\[
\begin{align*}
\delta \psi_m^a &= i \kappa \eta^i \bar{\sigma}^m \psi_m^i + i \kappa \bar{\eta}^i \sigma^m \psi_m^i \\
\delta \eta^1 &= \frac{2}{\kappa} D_m \eta^1 + i \frac{e m}{\kappa} \sigma_m \eta^1 \\
\delta \eta A_m &= \sqrt{1 + \epsilon \epsilon_{ij} (\psi_{m}^i \eta^j + \psi_{m}^j \eta^i)} + \sqrt{1 - \frac{1}{\sqrt{2}} (\bar{\eta}^1 \bar{\sigma}_m \lambda + \bar{\lambda} \bar{\sigma}_m \eta^1)} \\
\delta \eta B_m &= \sqrt{1 - \epsilon \epsilon_{ij} (-i \psi_{m}^i \eta^j + i \psi_{m}^j \eta^i)} + \sqrt{1 + \epsilon \frac{1}{\sqrt{2}} (\bar{\eta}^1 \bar{\sigma}_m \lambda - \bar{\lambda} \bar{\sigma}_m \eta^1)} \\
\delta \eta \lambda &= i \sqrt{1 - \frac{1}{\sqrt{2}} \hat{A}_{mn} \sigma^{mn} \eta^1} + \sqrt{1 + \epsilon} \frac{1}{\sqrt{2}} \hat{B}_{mn} \sigma^{mn} \eta^1 \\
&+ \sqrt{2} \frac{i e m}{\kappa} \phi_A \eta^1 - i \sqrt{2} \frac{m}{\kappa} \sqrt{1 - \epsilon^2} \eta^1 \\
\delta \eta \chi &= i \sigma^m \eta^1 \hat{D}_m \phi_A - \sigma^m \eta^1 \hat{D}_m \phi_B - 2 \epsilon \frac{e m}{\kappa} \phi_A \eta^1 + 2 \frac{m}{\kappa} \sqrt{1 - \epsilon^2} \eta^1 \\
\delta m^2 &= \frac{2}{\kappa} D_m \eta^2 + i \frac{m}{\kappa} \sigma_m \eta^2 - \frac{i}{2} \sqrt{1 + \epsilon} \hat{A}_{mn} \sigma^{mn} \eta^1 - m \sqrt{1 + \epsilon} A_m \eta^1 \\
&+ \frac{1}{2} \sqrt{1 - \epsilon} \hat{B}_{mn} \sigma^{mn} \eta^1 + \sqrt{1 + \epsilon} \frac{1}{2} \delta (\partial_m \phi_A - i D_m \phi_B) \eta^1 \\
&- \frac{\kappa}{2} \sqrt{1 - \epsilon} \bar{\psi}_m^1 (\delta \eta \phi_A - i \delta \eta \phi_B) - i \frac{e m}{\kappa} \sqrt{1 + \epsilon} \phi_A \sigma m \eta^1 \\
\delta \eta \phi_A &= \chi \eta^1 + \bar{\chi} \eta^1 \\
\delta \eta \phi_B &= -i \chi \eta^1 + i \bar{\chi} \eta^1
\end{align*}
\]

(13)

This result holds to leading order, that is, up to and including terms in the transformations that are linear in the fields. Note that this representation is irreducible in the sense that there are no subsets of fields that transform only into themselves under the supersymmetry transformations. The Lagrangian \([10]\) describes the spontaneous breaking of \(N = 2\) supersymmetry in AdS space. It has \(N = 2\) supersymmetry and a local \(U(1)\) gauge symmetry. In unitary gauge, it reduces to the massive \(N = 1\) Lagrangian of eq. \([9]\).

Let us now consider the dual case with one massive tensor. The degree of freedom counting is as in Figure 1. Note that the massive \(N = 1\) "vector" multiplet now contains a massive antisymmetric tensor.

The Lagrangian and supersymmetry transformations for this system can be worked out following the procedures described above. They can also be derived by dualizing first the scalar \(\phi_B\) and then the vector \(B_m\) using the method described in \([10]\). The Lagrangian is given by

\[
e^{-1} \mathcal{L} =
- \frac{1}{2 \kappa^2} \mathcal{R} + \epsilon_{mnrs} \psi_m^i \sigma_n D_r \psi_s^j - i \bar{\chi} \sigma^m D_m \lambda - i \bar{\sigma}^m D_m \chi
- \frac{1}{4} A_{mn} A^{mn} - \frac{1}{4} F_{mn} F^{mn} - \frac{1}{2} D_m \phi_A D_m \phi_A + \frac{1}{2} u^B_{mn} u^B_m
- \frac{1}{\sqrt{2}} m \sqrt{1 - \epsilon^2} \psi_m^2 \sigma^m \bar{\lambda} + m \sqrt{1 - \epsilon^2} \psi_m^2 \sigma^m \bar{\chi}
\]
The supersymmetry transformations are as follows:

\[
\delta \bar{\eta}_m = \frac{i}{\kappa} \epsilon_{ij} \psi^i_m \bar{\psi}^j_n + \frac{i}{\kappa} \epsilon_{ij} \sigma^a_{mn} \psi^j_n - \epsilon m \psi^1_m \sigma^{mn} \psi^1_n + \frac{\sqrt{2} m \lambda \chi + \frac{1}{2} m \chi \chi + m \psi^2_m \sigma^{mn} \psi^2_n - \epsilon m \psi^1_m \sigma^{mn} \psi^1_n}{2} + \frac{\kappa}{4} \epsilon_{ij} \psi^i_m \bar{\psi}^j_n (\sqrt{1 + \epsilon H^{mn}_{A-} + \sqrt{1 - \epsilon F^{Bmn}_-}) + \frac{\kappa}{2} \chi \sigma^m \bar{\sigma}^m \psi^1_n (D_m \phi_A + iv^B_n) + \frac{\kappa}{2 \sqrt{2}} \chi \sigma^m \bar{\sigma}^m \psi^1_n (\sqrt{1 - \epsilon H^{mn}_{A+} - \sqrt{1 + \epsilon F^{Bmn}_+})\]
\[
+ \frac{\kappa}{2} \epsilon^{mnr} \sqrt{\frac{1 - \epsilon}{1 + \epsilon}} \psi^1_m \bar{\sigma}^m \bar{\psi}^1_n (\partial_s \phi_A + iv^B_s) - \frac{\kappa}{2} m \epsilon^{mnr} \sqrt{\frac{1 + \epsilon}{1 - \epsilon}} \psi^1_m \bar{\sigma}^m \bar{\psi}^1_n \sqrt{1 + \epsilon A_s}
\]
\[
- 2 \kappa \epsilon m \epsilon_{mnr} \sqrt{\frac{1 - \epsilon}{1 + \epsilon}} \psi^1_m \bar{\sigma}^m \bar{\psi}^1_n \sqrt{1 + \epsilon} \bar{\phi}_A + \frac{\kappa m \epsilon \bar{\lambda} \sigma^m \psi^1_m \bar{\phi}_A}{\sqrt{2}}
\]
\[
+ i \kappa m \bar{\chi} \sigma^m \psi^1_m \bar{\phi}_A + h.c.) + 3 \frac{\epsilon^2 m^2}{\kappa^2} . \quad (14)
\]

where

\[
D_m \phi_A = \partial_m \phi_A - m \sqrt{1 - \epsilon} A_m
\]
\[
F^{Bmn} = \partial_{[m} B_{n]} - m \sqrt{1 + \epsilon} B_{mn}
\]

and

\[
\hat{D}_m \phi_A = \partial_m \phi_A - m \sqrt{1 - \epsilon} A_m - \frac{\kappa}{2} (\psi^1_m \chi + \bar{\psi}^1_m \bar{\chi})
\]
\[
\hat{v}_m = \psi_m - \left( i \kappa \psi^1_n \sigma^m \chi - \frac{i \kappa}{2} \sqrt{\frac{1 - \epsilon}{1 + \epsilon}} m \epsilon^{mnr} \psi^1_n \sigma^r \bar{\psi}^2_s + h.c. \right)
\]
\[
\hat{A}_{mn} = A_{mn} + \frac{\kappa}{2} \sqrt{1 + \epsilon} (\psi^2_m \psi^1_n + \bar{\psi}^2_m \bar{\psi}^1_n)
\]
\[
- \sqrt{1 - \epsilon} \frac{\kappa}{2 \sqrt{2}} (\bar{\lambda} \sigma^m \psi^1_m + \bar{\psi}^1_m \sigma^m \lambda)
\]
\[
\hat{F}^{B}_{mn} = F^{B}_{mn} + \frac{\kappa}{2} \sqrt{1 - \epsilon} (\psi^2_m \psi^1_n + \bar{\psi}^2_m \bar{\psi}^1_n)
\]
\[
+ \sqrt{1 + \epsilon} \frac{\kappa}{2 \sqrt{2}} (\bar{\lambda} \sigma^m \psi^1_m + \bar{\psi}^1_m \sigma^m \lambda) . \quad (16)
\]

The supersymmetry transformations are as follows:

\[
\delta e^a_m = i \kappa \eta^1 \sigma^a \bar{\psi}^i_m + i \kappa \bar{\eta} \sigma^a \psi^i_m
\]
\[
\delta \eta^1_m = \frac{2}{\kappa} D_m \eta^1 + i \frac{\epsilon m}{\kappa} \sigma_m \eta^1
\]
\[
\delta \eta A_m = \sqrt{1 + \epsilon} \epsilon_{ij} (\psi^i_m \eta^j + \bar{\psi}^i_m \bar{\eta}^j) + \sqrt{1 - \epsilon} \frac{1}{\sqrt{2}} (\bar{\eta}^1 \sigma_m \lambda + \bar{\lambda} \sigma_m \eta^1)
\]
\[
\delta \eta B_m = \sqrt{1 - \epsilon} \epsilon_{ij} (\psi^i_m \eta^j + \bar{\psi}^i_m \bar{\eta}^j) - \sqrt{1 + \epsilon} \frac{1}{\sqrt{2}} (\bar{\eta}^1 \sigma_m \lambda + \bar{\lambda} \sigma_m \eta^1)
\]
\[
\delta \eta B_{mn} = -2 \eta^1 \sigma_{mn} \chi - \sqrt{1 - \epsilon} \left( \eta^1 \sigma_{[m} \bar{\psi}^2_{n]} + i \eta^2 \sigma_{[m} \bar{\psi}^1_{n]} \right) + h.c.
\]
\[
\delta_\eta A = i \sqrt{1 - \epsilon} \frac{1}{\sqrt{2}} A_m \eta^1 - \sqrt{1 + \epsilon} \frac{i}{\sqrt{2}} \hat{F}^B_{mn} \sigma^{mn} \eta^1 \\
+ \sqrt{2} \epsilon m \phi_A \eta^1 - i \sqrt{2} \frac{m}{\kappa} \sqrt{1 - \epsilon^2} \eta^2
\]
\[
\delta_\eta \chi = i \sigma^m \bar{\eta} D_m \phi_A + \bar{v}_m \sigma^m \eta^1 - 2 \epsilon m \phi_A \eta^1 + 2 \frac{m}{\kappa} \sqrt{1 - \epsilon^2} \eta^2
\]
\[
\delta_\eta \psi_m^2 = \frac{2}{\kappa} D_m \eta^2 + i \frac{m}{\kappa} \sigma_m \bar{\eta} - \frac{i}{2} \sqrt{1 + \epsilon} \hat{H}^A_{+mn} \sigma^m \eta^1 - m \sqrt{1 + \epsilon} A_m \eta^1 \\
+ \sqrt{\frac{1 - \epsilon}{1 + \epsilon}} \phi_A \eta^1
\]
\[
\delta_\eta \phi_A = \chi \eta^1 + \bar{\chi} \bar{\eta}^1
\]  

(17)

These fields form an irreducible representation of the \( N = 2 \) algebra.

Each of the two Lagrangians has a full \( N = 2 \) supersymmetry (up to the appropriate order). The first supersymmetry is realized linearly\(^4\). The second is realized nonlinearly: it is spontaneously broken. In each case, the transformations imply that

\[
\zeta = \frac{1}{\sqrt{3}} (\chi - i \sqrt{2} \lambda)
\]  

(18)

does not shift, while

\[
\nu = \frac{1}{\sqrt{3}} (\sqrt{2} \chi + i \lambda)
\]  

(19)

does. Therefore \( \nu \) is the Goldstone fermion for \( N = 2 \) supersymmetry, spontaneously broken to \( N = 1 \).

We do not know how to unHiggs the other two representations of the massive spin-3/2 multiplet. If we use Stückelberg redefinitions as in (8, 9), the supersymmetry transformations are singular as \( \epsilon \to 1 \). If we try to dualize the above representations, the procedure is thwarted by the bare \( \phi_A \) fields in the Lagrangians and transformation laws.

### 3 The Supersymmetry Algebras

To find the supersymmetry algebras, let us compute the closure of the first and second supersymmetry transformations to zeroth order in the fields. This will allow us to identify the Goldstone fields associated with any spontaneously broken bosonic symmetries.

In the case with two scalars \((13)\), the algebra is as follows,

\[
[\delta_\eta^2, \delta_\eta] \phi_A = 2 \frac{m}{\kappa} \sqrt{1 - \epsilon^2} (\eta^1 \eta^2 + \bar{\eta}^1 \bar{\eta}^2)
\]

\(^4\)In AdS supergravity, the gravitinos undergo a shift even for linearly realized supersymmetry \((8)\); see \((13,17)\) and \((23)\) in appendix A.
\[ [\delta_{\eta^2}, \delta_{\eta^1}] A_m = \sqrt{1 + \epsilon} \frac{2}{\kappa} \partial_m (\eta^1 \eta^2 + \bar{\eta}^1 \bar{\eta}^2) \]
\[ [\delta_{\eta^2}, \delta_{\eta^1}] \phi_B = -2i \frac{m}{\kappa} \sqrt{1 - \epsilon^2} (\eta^1 \eta^2 - \bar{\eta}^1 \bar{\eta}^2) \]
\[ [\delta_{\eta^2}, \delta_{\eta^1}] B_m = -i \sqrt{1 - \epsilon} \frac{2}{\kappa} \partial_m (\eta^1 \eta^2 - \bar{\eta}^1 \bar{\eta}^2) \] (20)

From these expressions we see that \( \phi_A \) and \( \phi_B \) are Goldstone bosons associated with nonlinearly realized \( U(1) \) symmetries that are gauged by the vectors \( A_m \) and \( B_m \).

In the case with one scalar and one antisymmetric tensor, (17), the last two lines in (20) are replaced by
\[ [\delta_{\eta^2}, \delta_{\eta^1}] B_m = \frac{2}{\kappa} \sqrt{1 - \epsilon} \frac{m}{\kappa} \partial_m (\eta^1 \eta^2 + \bar{\eta}^1 \bar{\eta}^2) + 2i \frac{m}{\kappa} \sqrt{1 - \epsilon} (\eta^1 \sigma_m \eta^2 - \eta^2 \sigma_m \bar{\eta}^1) \]
\[ [\delta_{\eta^2}, \delta_{\eta^1}] B_{mn} = -2i \frac{1}{\kappa} \sqrt{1 - \epsilon} \frac{m}{1 + \epsilon} \partial_m (\eta^2 \sigma_n \bar{\eta}^1 - \eta^1 \sigma_n \bar{\eta}^2) \] (21)

In this case \( \phi_A \) and \( B_m \) are the Goldstone bosons of nonlinearly realized \( U(1) \)'s gauged by \( A_m \) and \( B_{mn} \).

To find the symmetry algebra, let us consider these algebras in the limit \( \kappa \to 0 \), with fixed \( v^2 \neq 0, \Lambda \neq 0 \). This limit corresponds to a fixed AdS background, in which central charges can be identified. For the case with two scalars, we find
\[ [\delta_{\eta^2}, \delta_{\eta^1}] \phi_A = 2v^2 (\eta^1 \eta^2 + \bar{\eta}^1 \bar{\eta}^2) \]
\[ [\delta_{\eta^2}, \delta_{\eta^1}] A_m = 0 \] (22)
\[ [\delta_{\eta^2}, \delta_{\eta^1}] \phi_B = -2iv^2 (\eta^1 \eta^2 - \bar{\eta}^1 \bar{\eta}^2) \]
\[ [\delta_{\eta^2}, \delta_{\eta^1}] B_m = -\sqrt{2}iv^2 \frac{\partial_m}{\Lambda} (\eta^1 \eta^2 - \bar{\eta}^1 \bar{\eta}^2) \] (23)

For the case with one scalar and one antisymmetric tensor, the last two lines are replaced by
\[ [\delta_{\eta^2}, \delta_{\eta^1}] B_m = 2iv^2 (\eta^1 \sigma_m \bar{\eta}^1 - \eta^2 \sigma_m \bar{\eta}^2) \]
\[ [\delta_{\eta^2}, \delta_{\eta^1}] B_{mn} = -\sqrt{2}i \frac{v^2}{\Lambda} \partial_m (\eta^2 \sigma_n \bar{\eta}^1 - \eta^1 \sigma_n \bar{\eta}^2) \] (24)

Equation (22) implies that the real scalar \( \phi_A \) is the Goldstone boson associated with the \( U(1) \) generator of the AdS algebra. (It is this generator which contracts to a real central charge in flat space.) Equation (23) (24) indicates that the scalar \( \phi_B \) (vector \( B_m \)) is the Goldstone boson associated with a spontaneously-broken \( U(1) \) symmetry, one which is gauged by the vector field \( B_m \) (tensor field \( B_{mn} \)).

These results imply that when \( v \neq 0 \) and \( \Lambda \neq 0 \), the full current algebra is actually \( OSp(2,4) \times_s U(1) \), nonlinearly realized. The symbol \( \times_s \) is a semi-direct product; it is appropriate because the supersymmetry generators close into the local \( U(1) \) symmetry. This construction evades the AdS generalization of the Coleman-Mandula/Haag-Lopuszański-Sohnius theorem because the broken supercharges do not exist. The \( OSp(2,4) \times_s U(1) \) symmetry only exists at the level of the current algebra; the \( U(1) \) symmetry is always spontaneously broken.
The supergravity theories that we have found depend on three dimensionful parameters: $\kappa$, $\Lambda$, and $v^2$. Since we are interested in partial supersymmetry breaking, we shall keep $v^2 \neq 0$. We then consider the Lagrangians (10,14) as a function of $\kappa$ and $\Lambda$ only. The dimensionless variable $\epsilon = \Lambda/\sqrt{\Lambda^2 + \kappa^2 v^4}$ is a particularly useful parameter, because the limit $\epsilon \to 0$ corresponds to the case of partially broken $N = 2$ supergravity in Minkowski space, while $\epsilon \to 1$ approaches the “massless” limit of partially broken supersymmetry in a fixed AdS background. The full manifold of $N = 2$ supergravities, described by the parameter $\epsilon$, is plotted in Figure 2. The center region corresponds to the new AdS supergravities described above.

A prominent feature in Figure 2 is the vertical line at ($\kappa = 0$, $\Lambda = 0$). This line connects theories in a Minkowski background ($\epsilon = 0$, $\Lambda = 0$) with the “massless” limit of theories in a fixed AdS background ($\kappa = 0$, $\epsilon = 1$). The line suggests that there should be a family of globally supersymmetric theories in Minkowski space, only one representative of which ($\epsilon = 0$) can be deformed to a partially broken supergravity theory in a Minkowski background. In contrast, a continuum of theories ($0 < \epsilon < 1$) can be deformed to partially broken supergravity theories in an AdS background.

Indeed, let us consider the limit $\kappa \to 0$, $\Lambda \to 0$ such that $\epsilon$ remains finite. If we write $\epsilon = \sin(2\theta)$, we find the following $N = 1$ transformations for the case with two scalars (13)

$$
\begin{align}
\delta \psi_m^2 &= - \frac{i}{2} \cos \theta H_{-mn} \sigma^n \eta^1 - \frac{i}{2} \sin \theta H_{+mn} \sigma^n \bar{\eta}^1 + \sqrt{2} \partial_m \tilde{\phi} \eta^1 \\
\delta A_m &= 2 \cos \theta \overline{\psi}_m \eta^1 + 2 \sin \theta \psi_m \bar{\eta}^1 \\
&\quad + \sqrt{2} \sin \theta \chi \sigma_m \eta^1 + \sqrt{2} \cos \theta \lambda \sigma_m \bar{\eta}^1 \\
\delta \chi &= i \sqrt{2} \sigma^m \partial_m \tilde{\phi} \eta^1 \\
\delta \lambda &= \frac{i}{2\sqrt{2}} \sin \theta \overline{(H_{-mn} \sigma^{mn})} \eta^1 - \frac{i}{2\sqrt{2}} \cos \theta (H_{+mn} \sigma^{mn}) \eta^1 \\
\delta \phi &= \sqrt{2} \chi \eta^1.
\end{align}
$$

Here, $\phi = (\phi_A + i \phi_B)/\sqrt{2}$ and $A_m = A_m + i B_m$; $A_{mn}$ is its corresponding field strength.
(The case with one scalar and one antisymmetric tensor can be obtained by dualization of $\phi_B$ and $B_m$; it is not presented here.)

The angle $\theta$ can be interpreted in terms of models with a full $N = 2$ multiplet structure\[6, 7\]. In these models, a necessary ingredient for partial supersymmetry seems to be presence of at least one vector- and one hyper-multiplet, as well as the non-existence of a prepotential for the special Kähler manifold\[7\] parametrized by the complex scalars $z_i$ of the $i$ vector multiplets. It was shown in\[8\] that such models can always be obtained by a symplectic transformation from a model with a prepotential.

In\[7\] the symplectic vector $\Omega$ for the special Kähler manifold $SU(1,1)/U(1)$ with one complex scalar $z_1 = z$ takes the form

$$\Omega = \begin{pmatrix} 0 & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ iz & \frac{1}{2} & 0 \end{pmatrix}. \tag{26}$$

If we assume that the scalar $z$ acquires a vacuum expectation value $\langle z \rangle \sim \kappa^{-1}$, and we expand the supersymmetry transformations\[9\] around this vacuum expectation value, we find that the angle $\theta$ parametrizes the symplectic transformation that maps this model with no prepotential continuously to the case of the so-called “minimal coupling models”\[10\]:

$$\Omega \rightarrow \begin{pmatrix} 0 & -\frac{1}{2} \sin \theta & 0 & -\frac{1}{2} \sin \theta \\ -\frac{1}{2} \cos \theta & 0 & -\frac{1}{2} \sin \theta & 0 \\ -\frac{1}{2} \sin \theta & -\frac{1}{2} \cos \theta & 0 & -\frac{1}{2} \\ iz & 2 \sin \theta & 2 \sin \theta & 0 \cos \theta & 0 \end{pmatrix} \begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \\ iz & 0 \cos \theta \\ \frac{1}{2} & 2 \sin \theta \cos \theta \end{pmatrix}. \tag{27}$$

Of course, this identification of the angle $\theta$ only holds to linear order in the fields; at higher order, the model in\[7\] cannot be consistently truncated to our field content.

4 Conclusion

In this paper we have examined the partial breaking of supersymmetry in anti-de Sitter space. We have seen that partial breaking in AdS space can be accomplished using two of four dual representations of the massive $N = 1$ spin-3/2 multiplet. During the course of this work, we found new $N = 2$ supergravities and new $N = 2$ supersymmetry algebras based on the semi-direct product $OSp(2,4) \times_s U(1)$, where the $U(1)$ is always nonlinearly realized for finite $\Lambda$.

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5 Appendix

A Geometry of AdS space

Anti-de Sitter space is the space of constant curvature $R < 0$. It has the topology $S^1 \times R^3$ and can be represented as the hyperboloid

$$x^A x^B \eta_{AB} = -\frac{1}{\Lambda^2}$$

with $\eta_{AB} = \text{diag}(-1 1 1 1 -1)$ in flat five-dimensional space. It contains closed time-like curves; therefore its universal covering space is taken as the physical space.

In the spirit of nonlinear realizations, the coset spaces of anti-de Sitter space and $OSp(1,4)$ are parametrized by the coset elements \[11\]

$$g(z) = O(3,2)/O(3,1) = e^{-iz^m R_m}$$

$$G(z, \theta, \bar{\theta}) = O(3,2)/O(3,1) \cdot OSp(1,4)/O(3,2) = g(z)e^{i(1-\frac{1}{4}\Lambda(\theta\theta + \bar{\theta}\bar{\theta}))(\theta Q^{O(3,2)} + \bar{\theta} Q^{O(3,2)})}$$

with $m \in \{0, ..., 3\}$. Subsequent expressions are facilitated by a transformation to new bosonic co-ordinates $x^m$

$$x^m = z^m \tanh(\frac{1}{2}\Lambda z)$$

In these co-ordinates, the vierbein takes the form

$$e_m^a = a(x) \delta_m^a \text{ with } a(x) = \frac{1}{1 - \frac{\Lambda^2}{x^2}}$$

With the above choice of the fermionic co-ordinate system, the co-ordinates $\theta$ transform like a Lorentz-spinor and not like an $O(3,2)$ spinor. Therefore, the components of $OSp(1,4)$ superfields do not transform like $O(3,2)$ fields.

The generators $Q^{O(3,2)}$, however, are $O(3,2)$ spinors. They can be transformed into $O(3,1)$ spinors $Q$ by shifting the factor \[11\] $\Lambda(x)$ to the transformation parameter $\epsilon$:

$$e^{i\epsilon Q^{O(3,2)}} = e^{i\eta Q}$$

Hence the supersymmetry transformation parameter $\eta$ becomes $x$-dependent \[11, 12\]:

$$\nabla_m \eta(x) = -i\frac{\Lambda}{2} \sigma_m \bar{\eta}(x) \quad (28)$$

The supersymmetry transformations of the gravitinos (see eqs. \[13, 17\]) is also modified by the transition from $O(3,2)$ to $O(3,1)$ spinors:

$$\delta \psi^{O(3,2)}_m = \partial_m \epsilon^s(x)$$

$$\delta(\Lambda(x) \psi_m) = \partial_m (\Lambda(x) \eta(x))$$

$$\delta \psi_m = \nabla_m \eta(x) + \Lambda^{-1}(x)(\nabla_m \Lambda(x))\eta(x)$$

$$= \nabla_m \eta(x) + i\frac{\Lambda}{2} \sigma_m \bar{\eta}(x) \quad (29)$$

\[5\]Here, $\Lambda(x)$ is the group-theoretical factor that maps $O(3,1)$-spinors to $O(3,2)$-spinors \[11\]; it is not the cosmological constant.
The algebra of $OSp(2, 4)$ \((i, j \in \{1, 2\})\) reads:

\[
\begin{align*}
[M_{ab}, M_{bc}] &= -i\eta_{bb}M_{ac} \\
[M_{ab}, R_c] &= -i(\eta_{bc}R_a - \eta_{ac}R_b) \\
[R_a, R_b] &= -i\Lambda^2 M_{ab} \\
[T^{ij}, Q^k] &= i\Lambda(\delta^{ik}Q^j - \delta^{kj}Q^i) \\
\{Q^i_\alpha, Q^j_{\bar{\beta}}\} &= 2\sigma^{\alpha\beta}R_\delta \delta^i_j \\
\{Q^i_\alpha, Q^j_\beta\} &= 2i\Lambda\sigma^{ab\alpha\beta}M_{ab}\delta^{ij} + 2i\delta^{\alpha\beta}T^{ij} \\
[R_a, Q^i] &= \frac{1}{2}\Lambda\sigma_a \bar{Q}_i \\
[M_{ab}, Q^i] &= -i\sigma_{ab}Q^i \\
\end{align*}
\]

Here $T^{ij}$ is the hermitian generator of $SO(2)$. The $N = 2$ Minkowski-algebra is recovered in the limit $\Lambda \to 0$. Note that the $N = 2$ Poincaré algebra with one central charge is recovered in the limit $\Lambda T^{ij} = X^{ij}$, with $\Lambda \to 0$.

**B The massive $OSp(1, 4)$ spin-1 multiplet**

In section 2.2, the unHiggsing of the massive spin-$\frac{3}{2}$ multiplet led to the appearance of a massive spin-1 multiplet with $\hat{E} = 5/2$. Here, the Lagrangian and transformations for general $\hat{E}$ will be presented.

The massive spin-1 multiplet contains the following AdS representations (see [3]):

\[
D(\hat{E}, \frac{1}{2}) \oplus D(\hat{E} + \frac{1}{2}, 1) \oplus D(\hat{E} + \frac{1}{2}, 0) \oplus D(\hat{E} + 1, \frac{1}{2}) \quad \text{with } \hat{E} \geq \frac{3}{2}
\]

The corresponding Lagrangian is:

\[
\mathcal{L} = -\frac{1}{4}v_{mn}v^{mn} - \frac{1}{2}\partial^mC\partial_mC \\
-\frac{1}{2}\bar{\sigma}^m\nabla_m\lambda - \frac{1}{2}\bar{\chi}\bar{\sigma}^m\nabla_m\chi \\
-\frac{1}{2}\mathcal{D}_m\phi\mathcal{D}^m\phi - \frac{1}{2}m^2(1 - \epsilon)(1 + 2\epsilon)\chi^2 \\
-\frac{1}{2}m\lambda\lambda + \frac{1}{2}m(1 + \epsilon)\chi\chi + \text{h.c.}
\]

where $m = (\hat{E} - \frac{3}{2})\Lambda \geq 0$ and $\epsilon = \Lambda/m$. The Stückelberg redefinition $\mathcal{D}_m\phi = \partial_m\phi - m\sqrt{1 + \epsilon}v_m$ has already been performed.

This Lagrangian is invariant under the supersymmetry transformations:

\[
\delta_\eta v_m = \frac{1}{\sqrt{1 + \frac{3}{2}\sqrt{2}}}\left(\bar{\eta}\bar{\sigma}_m\chi + \bar{\chi}\bar{\sigma}_m\eta\right) \\
+ \frac{1 + \epsilon}{\sqrt{1 + \frac{3}{2}\sqrt{2}}}\left(\bar{\eta}\bar{\sigma}_m\lambda + \bar{\lambda}\bar{\sigma}_m\eta\right)
\]
In the limit \( \hat{E} \to \frac{3}{2} \) \((m \to 0)\) this Lagrangian reduces to that of a massless spin-1 multiplet and a chiral multiplet \([11]\):

It is only this chiral multiplet with \( \hat{E} = 2 \) that can be dualized to a linear multiplet in AdS; other chiral multiplets with \( \hat{E} \neq 2 \) have bare \( \phi \)-terms (without derivatives) in its transformations and cannot be dualized.

The Lagrangian of the dual linear multiplet is

\[
\mathcal{L} = -\frac{1}{2} \partial_m C \partial^m C + \frac{1}{2} v_m v^m - i \bar{\chi} \sigma^m \nabla_m \chi \\
- \frac{1}{2} \Lambda \chi \chi - \frac{1}{2} \Lambda \bar{\chi} \bar{\chi} + \Lambda^2 C^2
\]

and its transformations are given by

\[
\begin{align*}
\delta_\eta B_{mn} &= -2 \eta \sigma_{mn} \chi - 2 \bar{\eta} \sigma_{mn} \bar{\chi} \\
\delta_\eta C &= -\chi \eta - \bar{\chi} \bar{\eta} \\
\delta_\eta \psi &= -i \sigma^m \bar{\eta} \partial_m C + 2 \Lambda C \eta + \sigma^m \bar{\eta} v_m
\end{align*}
\]

where \( v_m = \frac{1}{2} \epsilon_{mnr} \partial^n B^{rs} \).
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