DISTRIBUTION OF ACCRETING GAS AND ANGULAR MOMENTUM ONTO CIRCUMPLANETARY DISKS

Takayuki Tanigawa1,2, Keiji Ohtsuki1,3,4, and Masahiro N. Machida5
1 Center for Planetary Science, Kobe University, Kobe, Japan; tanigawa@cps-jp.org
2 Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan
3 Department of Earth and Planetary Sciences, Kobe University, Kobe, Japan
4 Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, CO, USA
5 Department of Earth and Planetary Sciences, Graduate School of Sciences, Kyushu University, Fukuoka, Japan

Received 2011 July 21; accepted 2011 December 15; published 2012 February 13

ABSTRACT

We investigate gas accretion flow onto a circumplanetary disk from a protoplanetary disk in detail by using high-resolution three-dimensional nested-grid hydrodynamic simulations, in order to provide a basis for a discussion of satellite formation. Processes of satellites around giant planets have been seen. They are thought to have formed in circumplanetary disks, which are believed to have existed around giant planets during their gas capturing growing stage. In earlier works, the formation process of satellite systems has been considered based on a minimum mass subnebula (MMSN) model, in which satellites form from a disk that contains sufficient solid mass with solar composition for reproducing the current satellite systems. As an analog of the minimum mass solar nebula model (Hayashi 1981), the MMSN model had one order of magnitude lower gas surface density than the original solar nebula model (Lunine & Stevenson 1982), as an analog of the minimum mass solar nebula model (Hayashi 1981). However, it was suggested that the MMSN model has difficulty in reproducing current satellite systems around Jupiter and Saturn (Canup & Ward 2002). One of the severe problems is that the model leads to much higher temperature than that of H2O ice sublimation at the current regular satellite region, which means that ice, which is the main component of the satellites, cannot be used as building material of the satellites.

In order to overcome the difficulties of the MMSN-type models which assume a closed and static disk, alternative models have been developed. Canup & Ward (2002) proposed a model in which gas accretion disk with a continuous supply of gas and solid is considered as a proto-satellite disk. This model is based on results of hydrodynamic simulations of the gas capturing process of giant planets (e.g., Lubow et al. 1999; D’Angelo et al. 2002), which demonstrated that gas accretion from the protoplanetary disk toward the parent planets occurs through a circumplanetary disk. In this model, the surface density and temperature of the circumplanetary disk are kept lower than assumed in the MMSN model, and thus H2O ice can be used as a solid building material of satellites. Based on a comparison among timescales of different processes, they concluded that formation of the Galilean satellites can be best explained if the circumplanetary disk had one order of magnitude lower gas surface density than the MMSN disk, with slow gas accretion rate corresponding to Jupiter’s growth timescale longer than a few ×106 yr (see also Sasaki et al. 2010). On the other hand, Mosqueira & Estrada (2003) proposed another disk model which consists of two components, i.e., inner MMSN-type massive disk and outer low-density extended disk. The model reproduced, for example, three inner Galilean satellites as well as the only partially differentiated Callisto. Although these models seem to reproduce the current satellite systems of the giant planets in our solar system, they needed to assume parameters for the disk structure such as surface density profile as a basis of physical processes of satellite formation.

The structure of a circumplanetary disk is closely related to the gas accretion process of giant planets. Gas flow around protogiant planets in protoplanetary disks has been studied using hydrodynamic simulations. Earlier two-dimensional simulations showed that the gas in the Hill sphere rotates in the prograde direction (Miki 1982; Sekiya et al. 1987; Korycansky & Papaloizou 1996) and a pair of spiral shocks stands in the circumplanetary disks (e.g., Kley 1999; Lubow et al. 1999; Tanigawa & Watanabe 2002; D’Angelo et al. 2002). However, the scale height of the protoplanetary disk for Jupiter-sized planets is comparable to the Hill radius; thus simulations with two-dimensional approximation cannot capture the feature of the accretion flow. Recent three-dimensional hydrodynamic
simulations revealed that the two-dimensional picture for circumplanetary disks is not appropriate for the flow in the Hill sphere (e.g., D’Angelo et al. 2003; Bate et al. 2003; Machida et al. 2008; Paardekooper & Mellema 2008; Ayliffe & Bate 2009; Coradini et al. 2010). One of the most important features newly found in three-dimensional calculations is that, in the region of circumplanetary disks, the gas accretes nearly vertically downward toward the midplane from high altitude.

Although some studies using hydrodynamic simulations were able to obtain the structures of circumplanetary disks, the direct use of results of hydrodynamic simulations for the structure, such as surface density, is problematic. One of the reasons is that disk-like shear-dominant flow is susceptible to numerical viscosity, which is an intrinsic nature of numerical hydrodynamic calculation, no matter whether the smoothed particle hydrodynamics (SPH) method or the mesh-based method is used, and thus it is difficult to obtain reliable structure of the disk directly from such simulations. In particular, the orbital radius of the rotating gas in disks basically changes only slightly with time; thus numerical error tends to accumulate easily in the simulation of long-term evolution. Also the timescale required for low viscous disks such as circumplanetary disks to reach a steady state is much longer than the typical dynamical time of the fluid. While high-resolution hydrodynamic simulations are needed to resolve the structure of circumplanetary disks, their long-term evolution is difficult to follow with such time-consuming simulation. In addition, physical (non-numerical) viscosity in the disk is not well understood, and we need to assume specific viscosity models that are hard to justify.

As an alternative approach, in the present work, we examine gas accretion flow onto circumplanetary disks from protoplanetary disks, in order to determine gas accretion rate as a function of distance from parent planets. Unlike the structure in a rotation disk, the accretion flow onto circumplanetary disks is not susceptible to numerical viscosity. Also, because the circumplanetary disks are located at the downstream of supersonic accretion flow, the accretion flow itself is hardly affected by the circumplanetary disk structure, which depends on poorly known effective viscosity. In Section 2, we describe our settings of hydrodynamic simulations. In Section 3, results and analyses of the simulations are shown. We discuss the implication of our analyses of the accretion flow in Section 4. We summarize our results in Section 5.

2. METHODS

2.1. Settings and Basic Equations

We consider a situation in which a growing giant planet embedded in a protoplanetary disk has induced the nucleated instability and the gas of the disk accretes dynamically onto the planet. We take local Cartesian coordinates rotating with the planet, x-axis corresponding to the radial direction, y-axis to the revolving direction of the planet, and z-axis normal to the disk midplane. The planet is located at the origin.

We adopt the local approximation, in which tidal potential is linearized and curvature of the protoplanetary disk is neglected. This approximation is valid as long as the Hill radius is much smaller than the orbital radius of the planet. The orbit of the planet is assumed to be fixed circular and coplanar with the disk midplane. The gas is assumed to be inviscid and isothermal. The magnetic field and self-gravity of the gas are neglected.

We employ the equation of continuity, the equation of motion for compressive inviscid gas, and the equation of state for isothermal gas to simulate the gas motion. In order to understand the physics clearly, we use these equations in a non-dimensional form. We normalize time by the inverse of the Keplerian angular velocity $\Omega^{-1} \equiv (GM_\ast/a^3)^{-1/2}$, length by scale height $h \equiv c\Omega^{-1}$, and mass by unperturbed surface density of the protoplanetary disk gas $\Sigma_0$ times $h^2$, where $G$ is the gravitational constant, $M_\ast$ is the solar mass, $a$ is the semimajor axis of the planet, and $c$ is the sound speed of the disk gas on the planet orbit. The sound speed $c$ is unity in our normalization. Normalized quantities are denoted with tilde on the variables in this paper.

The normalized equations can be written as

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\tilde{\rho} \tilde{\mathbf{v}}) = 0, \quad (1)$$

$$\frac{\partial \tilde{\mathbf{v}}}{\partial t} + (\tilde{\mathbf{v}} \cdot \nabla)\tilde{\rho} = -\frac{1}{\tilde{\rho}} \tilde{\nabla} \tilde{\Phi} - 2\epsilon_c \times \tilde{\mathbf{v}}, \quad (2)$$

$$\tilde{\rho} \tilde{\mathbf{v}} = \tilde{\rho}, \quad (3)$$

$$\tilde{\Phi} = \tilde{\Phi}_{\text{tidal}} + \tilde{\Phi}_p + \frac{9}{2} \frac{r^2}{H}, \quad (4)$$

where

$$\tilde{\Phi}_{\text{tidal}} = -3 \frac{3}{2} x^2 + \frac{1}{2} z^2, \quad (5)$$

$$\tilde{\Phi}_p = -\frac{3\bar{\rho}^3}{\bar{r}}. \quad (6)$$

In the above $\bar{r} = |\tilde{r}|$ is distance from the origin, $\tilde{r}_H$ is the normalized Hill radius, $\tilde{r}_{\text{tidal}} \equiv \tilde{r}_{\text{tidal}}/h = (M_p/3M_\ast)^{1/3}(a/h)$, $M_p$ is the planet mass, and $\epsilon_c \equiv (0, 0, 1)$ is a unit vector in the $z$-direction. The third term of the right-hand side in Equation (4) is added so that $\tilde{\Phi} = 0$ at the two Lagrangian points $L_1$ and $L_2$ (i.e., $(\tilde{x}, \tilde{y}, \tilde{z}) = (\pm \tilde{r}_H, 0, 0)$). The normalized quantities can be written, for example, as $\tilde{\mathbf{v}} = \mathbf{v}/(h\Omega^{-1}) = \mathbf{v}/c$, $\tilde{\rho} = \rho/(\Sigma_0/(\sqrt{2\pi h}))$, $\tilde{P} = P/(c^2(\Sigma_0/(\sqrt{2\pi h})))$, and $\tilde{\Phi} = \Phi/c^2$ for velocity, density, pressure, and potential, respectively. The full equation set before the normalization can be found in Machida et al. (2008). In the normalized system, $\tilde{r}_{\text{tidal}}$ is the only characteristic physical parameter of the system, if we do not consider physical radius of the planet. In this paper, we focus on the case with $\tilde{r}_H = 1$, which roughly corresponds to Jupiter mass at 5 AU.

2.2. Numerical Modeling

We employ a three-dimensional nested-grid hydrodynamic simulation code (e.g., Machida et al. 2005, 2006), which was originally developed to explore the star formation process by a collapse of the molecular cloud core (Matsumoto & Hanawa 2003). The size of the whole computational domain $(L_x, L_y, L_z)$ is $(24, 24, 6)$. The domain has a symmetry of about $\tilde{z} = 0$ plane, and the covered region in simulation is $\tilde{x} = [-12, 12]$, $\tilde{y} = [-12, 12]$, and $\tilde{z} = [0, 6]$. We set 11 levels for the nested-grid system, and the numbers of grids in each level are $(n_x, n_y, n_z) = (64, 64, 16)$. The level of the nested grid is denoted by $l$ and $l = 1$ is the largest grid level. An increment of $l$ by 1 reduces the size of the computational domain to half in all directions, keeping its center at the planet. The finest grid size is thus $L_x/64/2^{11-l} = 0.000366$, which corresponds to about one-fourth of the present Jovian radius at 5.2 AU.

We adopt two types of artificial gravitational weakening, which allows us to avoid a non-physical crash of the computation without essentially changing results. One is the widely used
weakening for the region adjacent to the planet to avoid singularity of the planet gravity. The modified potential is given by

$$\Phi_p = -\frac{3\tilde{r}_{sm}^3}{(\tilde{r}^2 + \tilde{r}_{sm}^2)^{3/2}},$$

(7)

where $\tilde{r}_{sm}$ is the so-called smoothing length and we set $\tilde{r}_{sm} = 0.00073$, which corresponds to a two grid size of the finest grid level ($l = 11$). The other gravitational weakening is also applied for the $\tilde{r}$-component of the tidal force in the high-$\tilde{r}$ region. In this region, the tidal force in the $\tilde{z}$-direction (toward the midplane) is very strong and causes too large of a density gradient to be described by the grid size we set in the code. Thus we connect two parabolic curves smoothly at $\tilde{z} = \tilde{L}_z^*$ as

$$\Phi_{\text{tidal},z} = \begin{cases} \frac{1}{2} \frac{\tilde{L}_z^2}{\tilde{L}_z - \tilde{L}_z^*} (\tilde{L}_z - \tilde{L}_z^*)^2 + \tilde{L}_z \tilde{L}_z^* & \text{if } \tilde{L}_z^* \leq \tilde{z} \\ \frac{1}{2} \frac{\tilde{L}_z^2}{\tilde{L}_z^*} (\tilde{L}_z^* - \tilde{L}_z^*)^2 + \tilde{L}_z \tilde{L}_z^* & \text{if } \tilde{L}_z^* \leq \tilde{z} \leq \tilde{L}_z, \end{cases}$$

(8)

where we set $\tilde{L}_z^* = 4$. The initial density profile in the $\tilde{z}$-direction is modified accordingly as

$$\tilde{\rho}(\tilde{z}) = \begin{cases} \frac{1}{\sqrt{2\pi}} \exp\left[-\tilde{z}^2/2\right] & \text{if } \tilde{z} \leq \tilde{L}_z^* \\ \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} \frac{\tilde{L}_z^2}{\tilde{L}_z^* - \tilde{L}_z^*} (\tilde{L}_z^* - \tilde{L}_z^*)^2 + \tilde{L}_z \tilde{L}_z^* \right] & \text{if } \tilde{L}_z^* \leq \tilde{z} \leq \tilde{L}_z, \end{cases}$$

(9)

for hydrostatic equilibrium under the potential given by Equation (8), and $\partial \tilde{\rho}/\partial \tilde{z} = \partial \tilde{\rho}/\partial \tilde{y} = 0$. This weakening does not cause any significant effect for the region where we need to observe because the mass above $\tilde{L}_z^*$ is negligible (0.0063% of the total mass for the hydrostatic structure).

As boundary conditions, we set the flow at $\tilde{x} = \pm \tilde{L}_x/2$ to be the unperturbed one, and the mirror condition is applied at $\tilde{z} = 0$ and $\tilde{L}_z$. As for the boundaries at $\tilde{y} = \pm \tilde{L}_y/2$ and $\tilde{x} = 0$ (inflow region), we use mixed boundary conditions as follows. When $\tilde{t} = 0$, we set unperturbed condition, which is the same as the initial condition, and change it gradually to the periodic boundary condition until $\tilde{t} = 100$ based on linear interpolation with respect to time. When $\tilde{t} \geq 100$, the periodic boundary condition is applied to all regions at $\tilde{y} = \pm \tilde{L}_y/2$. For the initial condition, we adopt unperturbed velocity: $\tilde{v}_x = \tilde{v}_z = 0$, $\tilde{v}_y = -(3/2)\tilde{x}$.

In order to obtain high-resolution results efficiently, calculations are started with only the largest grid level and the number of levels is increased with time. The second largest level ($l = 2$) starts at $\tilde{t} = 20$, $l = 3$ at $\tilde{t} = 50$, and the higher levels ($l \geq 4$) are started in order from $\tilde{t} = 150$ by considering the relaxation degree in each level. In this paper, we will show a snapshot of the flow at $\tilde{t} = 160.7$, when the accretion flow is nearly in the steady state.

We set sink cells around the origin in order to see the pure effect of gas accretion flow without the effect of the planet body as well as to mimic the gas accretion phase onto the planet that follows the nucleated instability (e.g., Mizuno 1980; Bodenheimer & Pollack 1986; Ikoma et al. 2000). In the sink cells, gas is removed at a rate that corresponds to $\tilde{\rho}/\tilde{\rho} = 10^{-4}$. We set $\tilde{r}_{\text{sink}} = \tilde{r}_{sm}$.

3. RESULTS

3.1. Path to Circumplanetary Disks

First, we examine the overall structure of accreting gas flow onto a circumplanetary disk. Figure 1 shows streamlines from outside of the Hill sphere (i.e., from a protoplanetary disk) toward the planet. The starting point of a streamline is defined by $(\tilde{x}_0, \tilde{y}_0, \tilde{z}_0)$. Four panels show streamlines starting from four different heights: $\tilde{z}_0 = 0.0, 0.5, 1.0, \text{ and } 1.5$. Based on the destination of streamlines, we divide the flow into three regions: the passing region where gas approaches the planet and passes by it without making a U-turn, the horseshoe region where gas crosses the planet orbit with a U-turn during the encounter, and the accreting region where gas is accreted onto the circumplanetary disk. We find that there are no accreting streamlines on the midplane ($\tilde{z} = 0$), while some streamlines starting from the off-midplane region accrete to the circumplanetary disk. This implies that the gas near the midplane in protoplanetary disks is harder to accrete to circumplanetary disks and planets. This is confirmed by Figure 2, which shows streamlines starting from three different radii $\tilde{R} \equiv \sqrt{\tilde{x}^2 + \tilde{y}^2} = 0.05, 0.1, 0.2$, that is, $(\tilde{x}_0, \tilde{y}_0, \tilde{z}_0) = (0.05, 0, 0), (0.1, 0, 0), (0.2, 0, 0)$. We can see that gas on the midplane in $0.2 \lesssim \tilde{R} \lesssim 1$ spirals outward and escapes from the Hill sphere within a short timescale through one of the two Lagrangian points ($L_1$ or $L_2$), and that the outward radial velocity decreases with decreasing distance from the planet (see also Figure 7).

The vertical structure of the flow is more clearly demonstrated in Figure 3. In this figure, initial positions of streamlines on the $\tilde{x}-\tilde{z}$ plane at $\tilde{y} = \tilde{L}_y/2$ are classified into three regions by their destinations. We define the passing region where streamlines reach the boundary at $\tilde{y} = -\tilde{L}_y/2$ with $\tilde{x} > 0$, the horseshoe region where streamlines reach the boundary at $\tilde{y} = \tilde{L}_y/2$ with $\tilde{x} < 0$, and the accreting region where streamlines end up within a sphere with radius $\tilde{r}_b$: we set $\tilde{r}_b = 0.2$. We can clearly see that there is no accretion band in the midplane, while the off-midplane site has an accretion band with a significant width. Note that the classification of the three regions does not depend on $\tilde{r}_b$ as long as $0.1 \lesssim \tilde{r}_b \lesssim \tilde{L}_y/2$. This implies that, even in the Hill sphere, the gas in the region $\tilde{r} \gtrsim 0.1$ stays there temporarily and is not necessarily captured by the planet. We will discuss the flow in this region in Section 4.1.

3.2. Disk Structure and Gas Motion

Figure 4 shows the azimuthally averaged density, velocity, and specific angular momentum of the gas with three different scales. We first recognize that the gas clearly forms a disk-like structure; the density distribution is concentrated near the midplane, almost Keplerian rotation is realized in the high-density region (judging from the contour lines of specific angular momentum), and the flow velocity in the $\tilde{R}-\tilde{z}$ direction of the region is very low. The density profile in the $\tilde{z}$-direction can be roughly described by hydrostatic equilibrium in the region where $\tilde{z} \lesssim \tilde{h}_p$ and $\tilde{R} \lesssim 0.2$, where $\tilde{h}_p$ is the scale height of the circumplanetary disk defined by

$$\tilde{h}_p \equiv \sqrt{\tilde{R}^3/(3\tilde{r}_p^3)} \propto \tilde{R}^{3/2}. \quad (10)$$

From this dependence a flare-up disk is expected, which is seen indeed in Figure 4. Above the disk surface, large downward velocity, which is almost free-fall velocity, is observed; this shows that gas is indeed accreted directly onto the disk surface. The accreting gas forms a shock surface, which stands at $\tilde{z} \sim 5\tilde{h}_p$. By analyzing streamlines, we confirmed that these gas elements are actually originated from off-midplane gas (mostly
Figure 1. Streamlines starting from four different heights ($\tilde{z}_0 = 0.0, 0.5, 1.0, 1.5$), with $\tilde{x}_0 = [\pm 2, \pm 3]$ and $\tilde{y}_0 = \pm \tilde{L}_y/2$. The interval of the starting points is 0.05 in the $x$-direction. Green, blue, and red curves show streamlines in the horseshoe, accretion, and passing regions, respectively.

(A color version of this figure is available in the online journal.)

Figure 2. Streamlines in the midplane of a circumplanetary disk. The left panel shows a streamline starting from $\tilde{R} = 0.2$ in the midplane ($\tilde{x}_0, \tilde{y}_0, \tilde{z}_0) = (0.2, 0, 0)$.

The dashed line shows the contour line of $\tilde{\Phi} = 0$, which passes the two Lagrangian points $L_1 (-1, 0, 0)$ and $L_2 (1, 0, 0)$. Colors show potential $\tilde{\Phi}$, red regions are $\tilde{\Phi} > 0$, and blue regions are $\tilde{\Phi} < 0$. The right panel shows a closer view of three streamlines starting from $\tilde{R} = 0.2$ (same as the left panel), 0.1, and 0.05, respectively.

(A color version of this figure is available in the online journal.)
Figure 3. Classification of starting points of streamlines on the $x$–$z$ plane at $\tilde{y} = \tilde{L}_y/2$. Streamlines starting from the red region reach the boundary $\tilde{y} = -\tilde{L}_y/2$ with $\tilde{x} > 0$ (passing region), those in the green region make a U-turn and reach the boundary $\tilde{y} = \tilde{L}_y/2$ with $\tilde{x} < 0$ (horseshoe region), and those in the blue region become trapped in $\tilde{r} < 0.2$ (accretion region).

(A color version of this figure is available in the online journal.)

Figure 5 shows radial mass flux $\tilde{\rho}\tilde{v}_r$ (where $\tilde{v}_r \equiv \tilde{v} \cdot \tilde{r}/\tilde{r}$), as a function of the azimuth and elevation angles at spheres with four different radii $\tilde{r} = 1.0, 0.3, 0.1, 0.03$. This figure clearly shows that the accretion manner strongly depends on both of these angles and is not spherically symmetric at all. For example, the flux at the $\tilde{r} = 1.0$ sphere shows that the mass flux can be both inward and outward near the midplane (where $|\theta| \lesssim 40^\circ$, where $\theta$ is elevation angle) depending on the azimuth angle $\phi$, while it is always inward (i.e., negative flux) at high $|\theta|$. The two maxima near $\phi = 0$ and 180 on the midplane ($\theta = 0$) in the case of $\tilde{r} = 1.0$ correspond to the outflow from the Hill sphere through the two Lagrangian points $L_1$ and $L_2$ shown in Figure 2.

The flux on the other three spheres also shows that there are two positive maxima and negative minima near the midplane, which implies the formation of a two-arm spiral structure in the circumplanetary disk. The range of the elevation angle $\theta$ that from $\tilde{z} > 0.5$) in the protoplanetary disk. Also, the contour lines above the shock surface are aligned with the velocity vectors, which means that specific angular momentum does not change very much before the gas elements hit the disk surface in this inner region, where the three-body effect is weak enough to be neglected.

Figure 4. Circumplanetary disk structure in the $R$–$z$ plane with three different spatial scales. Log density is shown with colors, specific angular momentum is shown with contour lines, and gas velocity is expressed with arrows. All quantities are averaged in the azimuthal direction.

(A color version of this figure is available in the online journal.)
corresponds to outward flux shrinks with decreasing $\tilde{r}$, which indicates the change of the disk aspect ratio with $\tilde{r}$.

Although Figure 5 shows that the gas at a high elevation angle is always falling radially inward, it is difficult to judge the direction of net mass flux in the disk (corresponding to low $|\theta|$) from these plots. In order to examine the net mass flux, next we calculate azimuthally integrated mass flux at spheres with several radii as a function of $\theta$ (Figure 6(a)):

$$\tilde{F}_{\tilde{r}}(\tilde{r}, \theta) = \int_0^{2\pi} (\tilde{\rho} \tilde{v}_r \cos \theta) d\phi.$$  \hspace{1cm} (11)

We find that the direction of the net radial flux near the midplane ($|\theta| \lesssim 20^\circ$ in the case of $\tilde{r} = 1.0$), where the gas moves both inward and outward through the sphere in the region depending on the longitude, is actually outward, while it moves inward for higher elevation angle ($|\theta| \gtrsim 20^\circ$ in the case of $\tilde{r} = 1.0$), which is obvious in Figure 5. This basic profile does not change with $\tilde{r}$, but where $\tilde{r} \lesssim 0.1$, the profile becomes sharper, that is, the region where gas moves outward is limited in the narrower elevation angle, which corresponds to the dependence of the disk aspect ratio on the radius. This clear outward motion around the midplane for a wide range of $\tilde{r}$ shows more evidence of the unbound state of the gas where $\tilde{r} \gtrsim 0.1$, which is already shown above (Figure 2). In the cases with $\tilde{r} = 0.1$ and 0.03, there is a narrow band where the mass flux is inward (negative) with a peak at $\theta \sim \pm 25^\circ$ and $\pm 15^\circ$, respectively. Figure 6(b) shows azimuthally averaged radial velocity:

$$\bar{\tilde{v}}_r(\tilde{r}, \theta) = \frac{\int_0^{2\pi} \tilde{\rho}(\tilde{r}, \theta, \phi) \tilde{v}_r(\tilde{r}, \theta, \phi) d\phi}{\int_0^{2\pi} \tilde{\rho}(\tilde{r}, \theta, \phi) d\phi}. \hspace{1cm} (12)$$

This quantitatively shows that the velocity of the gas elements accreting toward the disk nearly vertically at a high elevation angle ($|\theta| \gtrsim 40^\circ$ in the case with $\tilde{r} = 0.1$) is nearly free-fall velocity ($\sim \sqrt{\frac{G M}{r}}$), which is faster than the sound speed (=1). Near the midplane, the radial velocity is very small, which corresponds to the hydrostatic disk region described above (Figure 4). Between the two regions, there is a transition region, which is under the shock standing at the disk surface, and the gas in the region is not in a dynamically equilibrium state as a part of the disk. The range of $\theta$ of the transition region corresponds to the narrow band with inward mass flux described above. Thus, this is a kind of layered accretion and its mechanism will be discussed in Section 3.3.
Figure 6. Left panel shows azimuthally integrated mass flux at the radii \( \tilde{r} = 1.0 \) (solid red line), 0.3 (dashed green line), 0.1 (dot-dashed blue line), and 0.03 (dotted purple line) as a function of elevation angle \( \theta \). The right panel shows the azimuthally averaged radial velocity at the same radii. (A color version of this figure is available in the online journal.)

Figure 7. Azimuthally averaged radial velocity in the midplane normalized by local Keplerian velocity \( V_R \), which corresponds to the angle between the velocity vector of the flow and the circular orbit at the point.

We find that radial velocity is very small but takes on positive values for a wide range of radii \( 0.005 \lesssim \tilde{R} \lesssim 0.2 \). Significant positive values at \( \tilde{R} \gtrsim 0.2 \) correspond to the outward motion which is already shown in Figure 2. Non-negligible positive values of \( V_R \) at \( \tilde{R} \lesssim 0.005 \) may reflect non-steady-state small-scale structure in the region or an artifact of an averaging operation over discrete grids whose size may not be sufficiently small compared to the distance from the origin. The formation of the circumplanetary disk with nearly Keplerian motion is also demonstrated in Figure 8, which shows azimuthally averaged specific angular momentum at the midplane. We clearly see that the rotation velocity is nearly Keplerian for a wide range of radii \( \tilde{R} \lesssim 0.1 \). Keplerian rotation is realized when the ratio of the thermal energy of the gas to the potential energy of the planet is much smaller than unity (or the pressure force is much weaker than the gravity force). Equation (10) indicates that \( h_0/\tilde{R} \), which is the square root of the ratio, becomes smaller with decreasing \( \tilde{R} \), which is consistent with our result.

Figure 9 shows the plots of azimuthally averaged density at the midplane and surface density. We can see that the profiles at \( \tilde{R} \gtrsim 0.02 \) can be fitted roughly by power-law functions as \( \tilde{\rho} \propto \tilde{R}^{-3} \) and \( \tilde{\Sigma} \propto \tilde{R}^{-3/2} \), respectively. In an equilibrium state, we have \( \tilde{\rho} = \Sigma/(\sqrt{2\pi} h_0) \). The above two functions imply that this relationship is approximately satisfied and the hydrostatic

\[
V_R(\tilde{R}) = \frac{\tilde{v}_r(\tilde{r} = \tilde{R}, \theta = 0)}{\sqrt{3\tilde{R}^3/\tilde{R}}}. \tag{13}
\]
The time required to reach a steady state can be estimated as compared to the time required to reach a steady state or due to the effect of sink cells around the center. Here we emphasize again that the distributions of mass and angular momentum of accreting gas that we present in Section 3.3 are more important and useful quantities less affected by numerical procedures, as we mentioned in Section 1. Note that Machida (2009) introduced the idea of a centrifugal barrier to explain the peak of surface density. However, we do not find any physical reason for the existence of such a barrier because, as we will show in Section 3.3, distributions of accreting mass and angular momentum onto the circumplanetary disk are well described by power-law functions, which do not have typical lengths, such as centrifugal radius.

Figures 10 and 11 show variation of physical quantities along two streamlines starting from different heights that correspond to the passing and accreting regions. Overall variation is shown in Figure 10, while a close-up view near the shock front is shown in Figure 11. The key quantity in this figure is the Bernoulli integral $\tilde{B}$ given by

$$\tilde{B} = \frac{1}{2} |\tilde{\psi}|^2 + \log \tilde{\rho} + \Phi,$$

where we included potential energy. This quantity is conserved along each streamline except shock surfaces.

First we examine a streamline in the midplane plane (Figures 10(a) and 11(a)). The starting point of the streamline is $(x_0, y_0, z_0) = (2.37, \tilde{L}_r/2, 0)$, which is in the passing region (red region in Figures 1 and 3); $x$ keeps positive values not far from 2, and $y$ monotonically decreases with increasing $\tilde{d}$, where $\tilde{d}$ is the distance from the starting points of the streamlines. While approaching to the planet (corresponding to $\tilde{d} \lesssim 11.6$), the gas element climbs the tidal potential slope, decreasing its kinetic energy, and density slightly decreases. Although these quantities change with $\tilde{d}$, Bernoulli integral $\tilde{B}$, which is the sum of the three quantities, is constant until $\tilde{d} \sim 11.6$, which shows that there are no shocks along the path until this point. However, there is a shock surface at $\tilde{d} \sim 11.6$ where kinetic energy decreases and density increases discontinuously, and $\tilde{B}$ decreases as a result of shock dissipation. This shock surface stands from the Hill sphere toward both inside and outside of the planet orbit, forming spiral density waves around the central star. After passage of the shock, $\tilde{B}$ remains nearly constant, although other quantities change with increasing $\tilde{d}$. This means that the gas element on the streamline underwent only one shock with modest strength.

Next we examine an off-midplane streamline (Figures 10(b) and 11(b)). The starting point is $(\tilde{x}_0, \tilde{y}_0, \tilde{z}_0) = (2.37, \tilde{L}_r/2, 1)$, which is just above the previous starting point with the same horizontal position, and this corresponds to the accreting region (blue region in Figures 1 and 3). The quantities change in a similar way to the case in the midplane before $\tilde{d} \sim 12.0$, where the gas element hits the shock surface forming around the Hill sphere. We have non-zero $\tilde{z}$ values in this case but it is kept almost constant until it encounters the shock, which shows that the flow before the shock is laminar. When the gas element passes through the shock surface at $\tilde{d} \sim 12.0$, the basic behavior is the same as the $\tilde{z}_0 = 0$ case, but the gas element changes direction slightly upward (positive $\tilde{z}$-direction) at the shock (see the line showing the change in $\tilde{z}$ in Figure 11(b)). This is because the shock surface forms a bow shock and is curved upward, and thus the gas element of the off-midplane, which passed an oblique shock, gains upward momentum. After that, the gas element falls steeply toward the midplane and hits the surface of the circumplanetary disk, forming another shock at $\tilde{d} \sim 13.9$. The point of fall is at $\tilde{r} \sim 0.1$, which is sufficiently close to the planet to make use of its gravity for acceleration. Thus the Mach number of the gas when it hits the disk surface is much larger than unity, and a strong shock dissipation is caused. Owing to this strong dissipation, the gas becomes captured within the Hill sphere and merged as a part of disk. Note that the gradual decrease of $\tilde{B}$ before $\tilde{d} \sim 13.9$ arises presumably from collisions between falling gas elements on the way to the surface, especially between two gas elements originated from $\tilde{x} > 0$ and $\tilde{x} < 0$ in the protoplanetary disk (see also Figure 12 and the corresponding description in

![Figure 9](image_url). Azimuthally averaged density at the midplane (left). Azimuthally averaged surface density (right), both as a function of the radial distance in the midplane.

---

6 The time required to reach steady state can be estimated as $(\pi R^2 \Sigma/\dot{M})_{t=0.02} \sim 30$, which is longer than the duration with full level calculation. Here we used $\dot{M} = \dot{\Sigma}(\tilde{R})$ (see Figure 14).
Figure 10. Quantities along two streamlines starting from two different heights. The horizontal axis $\tilde{d}$ is the distance along each streamline from the starting point. Starting points for the upper and lower panels are $(2.37, \tilde{L}_y/2, 0.0)$ and $(2.37, \tilde{L}_y/2, 1.0)$, respectively. The thick solid (red) line shows the Bernoulli integral, the thick dashed (green) line shows the kinetic energy, the thick dotted (blue) line shows the potential energy, and the thick dot-dashed line (purple) shows the logarithm of density. The thin dotted, dashed, and dot-dashed lines show $\tilde{x}$, $\tilde{y}$, $\tilde{z}$, respectively. The streamline shown in the lower panel is accreted onto the circumplanetary disk, while that in the upper panel is not. Note that the streamline in the case of $\tilde{z}_0 = 1.0$ is truncated at a point where the gas element starts rotating around the planet in the circumplanetary disk.

(A color version of this figure is available in the online journal.)

Figure 11. Close-up view of Figure 10. In addition to the quantities shown in Figure 10, the azimuth angle $\phi$ divided by $\pi$ (thin dashed blue), specific angular momentum (thin dot-dot-dashed green), and logarithm of $r$ (thin solid red) are also shown.

(A color version of this figure is available in the online journal.)
the text). After the passage of the shock forming above the circumplanetary disk ($d \gtrsim 14$), $\tilde{r}$ becomes almost constant and the azimuth angle changes with a constant rate, which means that the gas is now rotating around the planet, i.e., the gas element has become a part of the circumplanetary disk. Note that specific angular momentum is not constant along the streamlines at all, although it is sometimes assumed to be a conserved quantity.

Now we consider why the gas in the midplane cannot accrete to the circumplanetary disk. As described above, effective shock dissipation is required for gas elements to become accreted onto the circumplanetary disk, and the planet gravity is essential for the effective acceleration. When the gas element consumes the potential energy without a significant increase of gas density, the kinetic energy of the gas increases effectively (see Equation (14)) and the gas element can have a high Mach number, which leads to strong shock dissipation. However, in order for the gas in the midplane to consume the planet gravitational energy, the gas element has to pass through the high-density region of the circumplanetary disk. The increase of density absorbs potential energy and prevents gas from effective acceleration. The gas near the midplane thus has difficulty forming strong shocks and dissipating energy effectively, which prevents it from being captured by the planet gravity. Therefore, the gas in the midplane has difficulty being accreted onto the circumplanetary disk.

One may think that passing many weak shocks during rotating around the planet in the circumplanetary disk would contribute to the inward migration of the rotating gas. However, the decrease in $\tilde{B}$ is a third-order small quantity with respect to $(M - 1)$ (where $M$ is Mach number), so such weak shocks are not effective for energy dissipation and inward migration. Note also that acceleration by the tidal force in the $z$-direction alone is not sufficiently strong to form strong shocks. This is because the gas is in hydrostatic equilibrium supported by thermal pressure, so that the resultant accelerated velocity should not be much larger than the sound speed, being able to produce only weak shocks. We therefore conclude that the strong shock produced by direct infall onto the surface of the inner part of the circumplanetary disk and associated energy dissipation is essential for the accretion of the gas from the protoplanetary disk onto the circumplanetary disk.

### 3.3. Distribution of Accreting Mass and Angular Momentum onto the Disk

Next, we examine the accretion rate of mass and angular momentum as a function of distance from the planet. First we

---

**Figure 12.** Distribution of the quantities associated with the accretion flow at the surface defined by $\tilde{z} = \tilde{z}_s(R)$. The density $\tilde{\rho}_s$, mass flux through the disk surface $\tilde{f}_s$, specific angular momentum $\tilde{j}_s$, and angular momentum flux through the surface $\tilde{f}_{s}\tilde{j}_s$. (A color version of this figure is available in the online journal.)
define the surface at which physical quantities immediately prior to the accretion onto the surface of a circumplanetary disk are measured. The surface is axisymmetric about the z-axis and the height of the surface from the midplane is denoted by \( \tilde{z}_s(\tilde{R}) \). We define the height of the surface by a linear function of the local scale height of the circumplanetary disk as

\[
\tilde{z}_s(\tilde{R}) = f_h \tilde{h}_p + \tilde{z}_{s0},
\]

where \( f_h \) and \( \tilde{z}_{s0} \) are constants. We set \( f_h = 6 \) and \( \tilde{z}_{s0} = 0.004 \) so that the surface \( \tilde{z}_s \) is above but not far from the shock surface of the circumplanetary disk in the region with \( \tilde{R} < 0.2 \).

Next we define mass flux onto the circumplanetary disk by

\[
\tilde{f}_s(\tilde{R}, \phi) \equiv -\tilde{\rho}_s \tilde{\nu}_s \cdot \tilde{n}_s \cos \theta_s,
\]

where \( \tilde{\rho}_s \) and \( \tilde{\nu}_s \) are the density and velocity vector at the surface defined by Equation (15), \( \tilde{n}_s \) is an upward unit vector normal to the surface, \( \theta_s = \tilde{d} \tilde{z}_s / \tilde{d} \tilde{R} \), and the negative sign is added so that \( \tilde{f}_s \) becomes positive when the gas passes the surface downward. Note that \( \cos \theta_s \) appears in the denominator in Equation (16) because \( \tilde{f}_s \) is defined as a flux at the surface per unit area of the \( x-\tilde{y} \) plane, not a flux per unit area of the surface.

Figure 12 shows several quantities on the surface \( \tilde{z} = \tilde{z}_s \) projected on the \( \tilde{x}-\tilde{y} \) plane. We find that the density of the accreting gas at the surface \( \tilde{\rho}_s \) has peaks at \( (\tilde{x}, \tilde{y}) \simeq (\pm 0.1, \pm 0.08) \), while the mass flux \( \tilde{f}_s \) has peaks at \( (\tilde{x}, \tilde{y}) \simeq (\pm 0.04, \pm 0.04) \), which are located closer to the planet than the density maxima. This is because accreting velocity, which is almost free-fall velocity, is faster at the inner region. These two peaks of the two-arm structure correspond to the places where gas elements coming from interior \( (\tilde{x} < 0) \) and exterior \( (\tilde{x} > 0) \) to the planet orbit collide with each other. As for the \( z \)-component of specific angular momentum around the planet measured in the inertial frame \( \tilde{\nu}_s \equiv (\tilde{\nu} \times \tilde{e}_z)_{\tilde{x} = \tilde{z}_s} \), \( \tilde{e}_z + \tilde{R} \tilde{\nu} \) (where the \( \tilde{R}^2 \) term arises from the rotation of the coordinate system), its value is positive (i.e., prograde) in the whole region. The distribution is roughly axisymmetric with some elongation in the x-direction within \( \tilde{R} \lesssim 0.1 \), and the value increases with increasing radius. As for the angular momentum flux, the above axisymmetric property breaks and has peaks at \( (\tilde{x}, \tilde{y}) \simeq (\pm 0.12, \pm 0.08) \), which simply corresponds to the two-arm structure observed in the density or mass flux. Note that the negative values observed in \( \tilde{f}_s \) in the outer region \( (\tilde{R} \gtrsim 0.1) \) are due to the fact that the gas passes through the \( \tilde{z} = \tilde{z}_s \) surface from beneath. This is because, in this region \( (\tilde{R} \gtrsim 0.1) \), the scale height to the radial distance of the circumplanetary disk is not much smaller than unity anymore and \( \tilde{z}_s \) is comparable to \( \tilde{R} \) accordingly. This leads to negative \( \tilde{f}_s \) for a gas element whose horizontal velocity is comparable to or larger than the vertical velocity even when the vertical velocity is negative (downward). The surface of the disk in this region is hard to define; thus the way to measure the mass flux described above may not be appropriate in this region.

For a better understanding of the distribution of accreting gas, we next show azimuthally averaged quantities at the surface defined by Equation (15). First we examine mass accretion rate. Figure 13 shows the plots of azimuthally averaged mass flux onto the disk, defined by

\[
\bar{\tilde{f}}_s(\tilde{R}) \equiv \frac{1}{2\pi} \int_0^{2\pi} \tilde{f}_s(\tilde{R}, \phi) d\phi.
\]

We can see from Figure 13 that \( \bar{\tilde{f}}_s(\tilde{R}) \) is almost constant from \( \tilde{R} \sim 0.1 \) all the way to the very center \( (\tilde{R} \sim 0.001) \). Cumulative mass accretion rate \( \bar{M}_s \) is thus roughly proportional to \( \tilde{R}^2 \), which is a power-law function and thus does not have any typical length except for the outer end of the downward accretion on the disk surface \( (\tilde{R} \sim 0.1) \) in this case. This distribution shows that the main contribution of mass accretion comes from the outer region \( (\tilde{R} \sim 0.1) \) and there is no specific radius where gas accretion is concentrated somewhere in between. Note that dotted lines in Figures 13 and 14 show quantities calculated by Equations (17) and (18), respectively, integration being performed over only regions with \( \tilde{f}_s > 0 \). The deviation from each solid line can only be seen at \( \tilde{R} \gtrsim 0.15 \), where gas can pass through the \( \tilde{z} = \tilde{z}_s \) surface even upward depending on the azimuth angle (see Figure 12). This is the reason why \( \bar{M}_s \) (solid line) decreases with increasing radius at \( \tilde{R} \gtrsim 0.2 \) although it is a cumulative quantity. The treatment of our analysis in this region is a difficult problem and will be discussed later (Section 4.1).

The distribution of \( \tilde{f}_s \) (or \( \bar{M}_s \)) provides us with useful information but it is not sufficient for an understanding of the mass distribution in a circumpolar disk. In general, the radial distance from the planet of the location where gas is first accreted on the disk is different from its final orbit after it settles in as a part of the Keplerian rotating disk. What connects the two is the angular momentum of the gas. Figure 15 shows the azimuthally
averaged specific angular momentum \( \tilde{j}_{z,s} \) at the \( \tilde{z} = z_s \) surface given by

\[
\tilde{j}_{z,s}(\tilde{R}) = \frac{\int_0^{2\pi} \tilde{j}_{z,s} \tilde{\rho}_i d\phi}{\int_0^{2\pi} \tilde{\rho}_i d\phi}.
\]

We can see that \( \tilde{j}_{z,s} \) is always smaller than the specific angular momentum for Keplerian rotation \( \tilde{j}_{\text{Kep}} = \sqrt{3\tilde{M}_s / \tilde{R}} \). This means that the gas does not have enough angular momentum for Keplerian rotation at the radius, which would lead to inward migration of the gas after the fall on the disk surface, while the gas near the midplane moves outward, as mentioned above (Figure 7). This explains the inward stream of the layer just under the shock of the disk surface (Figure 6). In addition, \( \tilde{j}_{z,s} \) is nearly proportional to \( \tilde{R} \), which is steeper than \( \tilde{j}_{\text{Kep}} \propto \tilde{R}^{1/2} \), and thus the ratio of \( \tilde{j}_{z,s} \) to \( \tilde{j}_{\text{Kep}} \) decreases with decreasing \( \tilde{R} \). This suggests that the gas accreted in the outer region of the disk \( (\tilde{R} \sim 0.1) \) tends to keep the position when it rotates as a part of the circumplanetary disk, whereas the gas accreted in the inner region tends to move further inward in order to achieve balance between centrifugal and gravitational forces.

We now have the mass flux and angular momentum in the accretion flow, which allows us to estimate the effective distribution of accreting gas elements, assuming their redistribution to radial distances where their specific angular momentum matches that of the local Keplerian rotation. Let \( \tilde{R}_{\text{Kep}} \) denote the radius where the gas with specific angular momentum \( \tilde{j}_{z,s} \) is rotating with the Keplerian velocity, i.e.,

\[
\tilde{R}_{\text{Kep}}(\tilde{R}, \phi) = \frac{\tilde{j}_{z,s}(\tilde{R}, \phi)}{3\tilde{M}_s / \tilde{R}}.
\]

Then the effective distribution of the azimuthally averaged mass flux after redistribution of the gas with the angular momentum conservation can be written as

\[
\tilde{j}_{\text{Kep}}(\tilde{R}) d\tilde{R} = \frac{1}{2\pi \tilde{R}} \int_0^{2\pi} \tilde{\rho}_i(\tilde{R}', \phi) d\phi 
\]

\[
\int_0^{\tilde{R}'} d\tilde{R}' \tilde{R}' \tilde{j}_{\text{Kep}}(\tilde{R}', \phi) \delta(\tilde{R}', \phi),
\]

where

\[
\delta(\tilde{R}', \phi) = \begin{cases} 1 & \text{if } \tilde{R}' - d\tilde{R}'/2 < \tilde{R}_{\text{Kep}}(\tilde{R}', \phi) < \tilde{R}' + d\tilde{R}'/2 \text{ and } \tilde{s}_s > 0, \\ 0 & \text{otherwise}. \end{cases}
\]

The corresponding cumulative mass accretion rate is

\[
\tilde{\dot{M}}_{\text{Kep}}(\tilde{R}) = \int_0^{\tilde{R}} 2\pi \tilde{R}' \tilde{j}_{\text{Kep}}(\tilde{R}') d\tilde{R}'.
\]

The plots of \( \tilde{j}_{\text{Kep}} \) and \( \tilde{\dot{M}}_{\text{Kep}} \) are shown in Figures 13 and 14. We can see that \( \tilde{j}_{\text{Kep}} \) is roughly proportional to \( \tilde{R}^{-1} \) and \( \tilde{\dot{M}}_{\text{Kep}} \) is to \( \tilde{R} \). By comparison with \( \tilde{s}_s \), we find that \( \tilde{j}_{\text{Kep}} \) is more center-concentrated distribution, but still the outer region is a dominant source of gas accretion in the sense that \( \partial \ln \tilde{j}_{\text{Kep}} / \partial \ln \tilde{R} > -2 \). Since angular momentum of a gas element after the shock of the disk surface is not necessarily conserved until the gas reaches the radius where it rotates in the Keplerian velocity, the distributions of \( \tilde{j}_{\text{Kep}} \) and \( \tilde{\dot{M}}_{\text{Kep}} \) should be regarded as the case of the most center-concentrated limit.

4. DISCUSSION

4.1. Size of Circumplanetary Disks

The size of a circumplanetary disk is closely related to the location of satellite formation. However, it is not easy to define the outer edge of the disk, and there have been several attempts. One natural way would be to define the disk edge based on its density distribution. But the density is monotonically

![Figure 14](image1.png)

**Figure 14.** Mass accretion rate onto the disk surface within a circle of radius. The solid line shows \( \dot{M}_s \) (Equation (18)) and the dashed line shows \( \dot{M}_{\text{Kep}} \) (Equation (23)). The dotted line shows \( \dot{M}_s \) but excluding the region where \( \tilde{s}_s < 0 \) from the integration of Equation (18).

![Figure 15](image2.png)

**Figure 15.** Azimuthally averaged specific angular momentum of the accretion flow onto circumplanetary disks denoted by \( \tilde{j}_{z,s} \) in the text (solid line). The dashed line is \( \tilde{j}_{\text{Kep}} = \sqrt{3\tilde{M}_s / \tilde{R}} \), which is the specific angular momentum for Keplerian rotation around the planet.
and smoothly decreasing with increasing radius toward the Hill sphere, so it is difficult to define the edge from density distribution. Ayliffe & Bate (2009), who performed three-dimensional hydrodynamic simulations, suggested a criterion for the disk edge based on the angular momentum distribution. Their simulations show that the specific angular momentum of the disk gas has a peak at a certain radial location, whereas the specific angular momentum of a Keplerian disk increases monotonically. Their Figure 2 suggests that the turnover point is about one-third of the Hill radius, and they defined the disk edge by the radial location of this peak. Martin & Lubow (2011) examined periodic orbits of a particle around a planet under the influence of the gravitational force from a central star and the planet, and found that, as the size of the orbit is increased, the orbits start crossing with each other at \( r \sim 0.4 R_{\text{H}} \). They inferred that this corresponds to the location of the disk’s outer edge where the tidal torque of the central star’s gravity becomes strong and called it the tidal truncation radius \( (r_{\text{trunc}}) \). The above value of \( r_{\text{trunc}} \) is in agreement with the point of turnover of the specific angular momentum found in their two-dimensional SPH simulation as well as in the result of Ayliffe & Bate (2009).

Our Figure 8 shows that the turnover point is \( \tilde{R} \sim 0.3 \tilde{r}_{\text{trunc}} \), which is also roughly in agreement with \( r_{\text{trunc}} \).

We here consider an alternative criterion by examining \( V_R \) defined by Equation (13) to define the position of the outer edge. We observed that azimuthally averaged radial velocity in the midplane is positive (outward) in almost all of the regions, although the value is very small for \( \tilde{R} \lesssim 0.2 \) (Figure 7), and the outward velocity significantly increases at \( \tilde{R} \gtrsim 0.2 \) (see also Figure 2). Since gas at \( \tilde{R} \gtrsim 0.2 \) moves outward and escapes from the Hill sphere quickly, the region \( \tilde{R} \gtrsim 0.2 \) can be regarded as outside of the circumplanetary disk. We thus define the disk edge as the radial location where \( V_R \) starts increasing significantly and has a non-negligible positive value; \( \tilde{R} \sim 0.2 \) in our case as shown in Figure 7. Note that a two-dimensional SPH simulation shows that negative torque is exerted on the outer disk (Martin & Lubow 2011), which gives rise to inward flow and is contrary to our results. This might be due to the fact that two-dimensional simulations, which create clearer spiral structure than three-dimensional ones, tend to enhance torque density. In particular, the disk thickness in the outer region \( (\tilde{R} \gtrsim 0.2) \), at which the outflow is observed, is very thin (thickness is comparable to radius). Thus, it would be unlikely that our three-dimensional calculation is significantly affected by the negative torque suggested by Martin & Lubow (2011). The distribution of the accreting angular momentum also seems to indicate a similar radius for the edge. As we see in Figure 15, \( \tilde{j}_k \) and \( \tilde{j}_{\text{c, c}} \) have different dependence on \( \tilde{R} \). If we fit each profile as a single power-law function, they cross each other at \( \tilde{R} \sim 0.3 \) in the present case. This roughly agrees with the location of the disk edge defined above based on the significant increase of outward velocity. The gas exterior to this radius has too large of an angular momentum to achieve Keplerian rotation, and thus moves radially outward.

Once the angular momentum of accreting gas is obtained, one can calculate the mean specific angular momentum of the accretion flow, which is sometimes used in calculating the so-called centrifugal radius \( \tilde{r}_c \equiv \tilde{L}^2/(3\tilde{M}_d \tilde{R}) \) to infer the disk size (e.g., Mosqueira & Estrada 2003; Ward & Canup 2010). The mean specific angular momentum of the accretion flow within a radius \( \tilde{R} \) is given by

\[
\tilde{\ell}(\tilde{R}) = \frac{\tilde{j}_s(\tilde{R})}{\tilde{M}_d(\tilde{R})},
\]

where

\[
\tilde{j}_s(\tilde{R}) = \int_0^\tilde{R} d\tilde{R}' \int_0^{2\pi} \tilde{R}' d\phi \tilde{j}_s(\tilde{R}', \phi) \tilde{j}_s(\tilde{R}', \phi).
\]

We can see from Figure 16 that \( \tilde{\ell}(\tilde{R}) \) increases nearly linearly with radius and levels around \( \tilde{R} \sim 0.2 \), where \( \tilde{\ell} \sim 0.2 \). This corresponds to \( \tilde{r}_c \sim 0.013 \), which might give us a rough estimation of the disk size. However, it is practically difficult to determine the upper bound of the integral range with respect to \( \tilde{R} \), which affects the value of \( \tilde{\ell} \), because the circumplanetary disks are smoothly connected to the protoplanetary disks. Also, unlike the case of a particle, the angular momentum of a gas element is not a conserved quantity along a flow in principle, because the gas is the continuum medium which can transfer the angular momentum through waves. This also makes it difficult to determine \( \tilde{\ell} \) precisely, and thus \( \tilde{r}_c \) as well.

### 4.2. Picture of Gas Accretion Flow onto Circumplanetary Disks

Figure 17 shows a schematic picture of gas accretion flow onto a circumplanetary disk based on the results obtained in the present work. Gas accretion occurs mostly downward from high altitude with a high incident angle (see Figure 4). The accreting gas is accelerated by the planet gravity to have almost free-fall velocity of the planet (Figure 6(b)). The value of the angular momentum of the accreting gas normalized by the local Keplerian angular momentum is lower in the inner region of the disk and higher in the outer region (Figure 15). The falling gas reaches the shock surface formed on the top of the circumplanetary disk.

Gas near the midplane (especially where \( \tilde{R} \lesssim 0.1 \)) is almost in Keplerian rotation and hydrostatic equilibrium in the...
Figure 17. Schematic picture of the flow structure of circumplanetary disks. (A color version of this figure is available in the online journal.)

In the $z$-direction. In this region, radial velocity is very small and it is difficult for the gas to accrete inward through the disk midplane, as mentioned above. On the other hand, the gas at $\tilde{R} \gtrsim 0.2$ in the midplane shows significant outflow, which eventually escapes from the Hill sphere (see Figure 2). Thus the inflow from high altitude and the outflow near the midplane coexist, and they do not interfere with each other. This means that there is a circulation across the Hill sphere and fresh (protoplanetary) gas is always supplied in the $\tilde{R} \gtrsim 0.2$ region. In addition, the two Lagrangian points $L_1$ and $L_2$, which are often thought to be the most likely points of inflow, are actually the points of outflow, even in the gas accretion stage (Figure 5). This seems consistent with Klahr & Kley (2006), who also performed a three-dimensional hydrodynamic simulation with no explicit physical viscosity, but with lower resolution and with radiation. They showed that accretion mainly occurs via the poles of the planet and no inflow along the equatorial plane, which is quite similar to ours. But they explained that the circulation in their simulation is driven by accretion heating, which we do not consider. Thus, it is not clear if the driving mechanisms of the circulation are the same. Note that Ayliffe & Bate (2009) show that the vast majority of the mass flows into the Hill sphere near the equator, which seems to be inconsistent with our result. There are several differences in setting between their analysis and ours, so it is not easy to judge which factor causes the difference. Disk thickness would be one of the reasons for that, even though our disk thickness is not very different from theirs. Another possibility is gap formation. When a deep gap is formed, the contribution of the vertically accreting gas would become less significant, which reduces the difference. However, here we think that viscosity is the main reason for the difference. When there is explicit viscosity, the circumplanetary disk gas should transfer its angular momentum outward and most of the gas would move inward accordingly, but this is not necessarily true in the inviscid limit. Actually, inviscid simulations by Klahr & Kley (2006) showed similar results to ours. Also, the inviscid limit might not be bad because circumplanetary disks are likely to be MRI inactive in most cases (Fujii et al. 2011).

Since dust particles tend to settle down toward the midplane, gas accretion flow from high altitude is likely dust-poor gas, which diminishes the dust-to-gas ratio in the circumplanetary disk. On the other hand, since gas in the outflow region ($\tilde{R} \gtrsim 0.2$) rotates significantly slower than the Keplerian velocity around the planet, dust particles would migrate inward quickly. Thus the outflow in the midplane is also likely dust-poor gas, which would be the source of solid material in the circumplanetary disk and would enhance the dust-to-gas ratio in the disk. The dust-to-gas ratio in the circumplanetary disk is one of the most important factors for satellite formation processes...
As we see in Figure 6, there is a layer just under the shock at the disk surface where gas moves inward. This *layered accretion* is explained by the fact that the angular momentum of the accreting gas onto the disk surface is lower than that of the Keplerian rotation at the radius (Figure 15). The inward stream in the layer might play an important role for the net mass accretion toward the planet in circumplanetary disks. This accretion picture should be explored more quantitatively by further high-resolution simulations, using a code with lower artificial viscosity such as the one with polar coordinates for numerical grids.

4.3. Effects of Gap Formation

One important issue to be addressed is the effect of the gap, which is a lower density annulus region near the planet orbit in the protoplanetary disk. We observed the gap as a slightly lower density band formed around \( \tilde{\chi} = 0 \) in our simulations. However, in the last stage of giant planet formation, a giant planet would become massive enough to create a deep gap, which would be able to truncate its growth. Since the deep gap is associated with a steep density gradient at the edge, gap formation may affect accretion flow and circumplanetary disk formation. However, at the edge of such a deep gap, the gas density changes over a radial distance comparable to the disk scale height, while the width of the accretion band onto the circumplanetary disk is much narrower than the scale height (Figure 3). Therefore, we think that the gap formation would not affect the qualitative feature of the accretion flow. Another possible effect of gap formation is the disk size. As described in Section 4.1, several mechanisms are proposed to explain the disk size. If the disk size is determined by the tidal effect as suggested by Martin & Lubow (2011), it should not be affected by gap formation, except when the Bondi radius is smaller than the Hill radius. On the other hand, if the disk size is determined by the radially outward velocity of the gas as described in Section 4.1, lower density around the Hill sphere and thus a stronger radial pressure gradient near the disk edge may enhance the outflow from the circumplanetary disk, and the disk size may become smaller. In any case, effects of gap formation should be examined in future works to check the validity of the results shown in this paper.

5. SUMMARY

In order to understand the structure of circumplanetary disks, we performed a high-resolution hydrodynamic simulation and analyzed gas accretion flow in detail. We confirmed that gas accretion onto circumplanetary disks occurs in a manner that the gas is accreted from high altitude toward the disk surface downward with a large incident angle, which was suggested by previous studies (e.g., D’Angelo et al. 2003; Bate et al. 2003), whereas Ayliife & Bate (2009) showed that, in terms of mass flux, the downward accretion flow is not significant, which is inconsistent with our results. We found that the gas that has passed through the shock surface moves inward because its specific angular momentum is smaller than that of Keplerian rotation, whereas gas accretion through the midplane does not occur. While the net gas accretion across the Hill sphere is inward, outflow through near the two Lagrangian points is observed, although the regions around the two Lagrangian points have been, from the point of view of the potential energy, thought to be a main accretion channel from protoplanetary disks. This outflow was not observed in previous hydrodynamic simulations for Jupiter-sized planets (e.g., Bate et al. 2003; Ayliife & Bate 2009). The outward radial velocity of the gas near the midplane significantly increases at a radial distance of about 0.2 times the Hill radius from the planet, and the gas can escape from the Hill sphere within a short period of time.

We also obtained the distribution of mass and angular momentum of accreting gas onto the surface of circumplanetary disks. We found that the accretion rates of mass and angular momentum can be well described by power-law functions. This distribution would be useful in the study of satellite formation, for example, a radially one-dimensional viscous-evolution model for circumplanetary disks, such as Ward & Canup (2010) and Martin & Lubow (2011). Recent development of viscous modeling for MRI turbulence in protoplanetary and circumplanetary disks (Okuzumi & Hirose 2011; Fujii et al. 2011) would also contribute to construct more realistic models for the circumplanetary disks. However, in order to understand satellite formation processes, long-term evolution of circumplanetary disks, which is determined by the evolution of the accretion rate from protoplanetary disks to circumplanetary disks, is necessary and thus the global evolution of protoplanetary disks with embedded giant planets until the complete dissipation of protoplanetary disks needs to be examined. Together with such global models (e.g., Tanigawa & Ikoma 2007; Ward & Canup 2010), we will be able to obtain the long-term evolution of circumplanetary disks, which would provide a better understanding of satellite formation processes.

Our results demonstrate that gas accretion toward and within circumplanetary disks has a complicated vertical structure. The width of the accretion band toward a circumplanetary disk depends on the initial height of gas elements (Figure 3). Also, after accretion onto the disk, the gas just below the shock surface migrates inward, while the gas near the midplane moves radially outward (Figure 17). Such vertical heterogeneity of the flow may have significant influence on the dynamical evolution of solid bodies in the circumplanetary disk, which we will examine in our future work. We also need to address the effect of viscosity and local calculation, which would affect the flow quantitatively.

We are grateful to Hidekazu Tanaka, Shu-ichiro Inutsuka, Hiroshi Kobayashi, and Satoshi Okuzumi for valuable comments. T.T. also thanks Sei-ichiro Watanabe and Ryuuji Morishima for continuous encouragement to advance this research. This work was supported by Center for Planetary Science running under the auspices of the MEXT Global COE Program entitled “Foundation of International Center for Planetary Science.” We are also grateful for the support by JSPS and NASA’s Origins of Solar Systems Program. Numerical calculations were carried out on NEC SX-9 at the Center for Computational Astrophysics, CfCA, of the National Astronomical Observatory of Japan. A part of the figures were produced by the GFD-DENNOU Library.

REFERENCES

Ayliife, B. A., & Bate, M. R. 2009, MNRAS, 397, 657
Bate, M. R., Lubow, S. H., Ogilvie, G. I., & Miller, K. A. 2003, MNRAS, 341, 215
Bodenheimer, P., & Pollack, J. B. 1986, Icarus, 67, 391
Canup, R. M., & Ward, W. R. 2002, AJ, 124, 3404
Canup, R. M., & Ward, W. R. 2006, Nature, 441, 834
Coradini, A., Magni, G., & Turrini, D. 2010, Space Sci. Rev., 153, 411
D’Angelo, G., Henning, T., & Kley, W. 2002, A&A, 385, 647
D’Angelo, G., Kley, W., & Henning, T. 2003, ApJ, 586, 540
Fujii, Y. I., Okuzumi, S., & Inutsuka, S. 2011, ApJ, 743, 53
Hayashi, C. 1981, Prog. Theor. Phys. Suppl., 70, 35
Ikoma, M., Nakazawa, K., & Emori, H. 2000, ApJ, 537, 1013
Klahr, H. H., & Kley, W. 2006, A&A, 445, 747
Kley, W. 1999, MNRAS, 303, 696
Kobayashi, H., & Tanaka, H. 2010, Icarus, 206, 735
Koryczansky, D. G., & Papaloizou, J. C. B. 1996, ApJS, 105, 181
Lubow, S. H., Seibert, M., & Artymowicz, P. 1999, ApJ, 526, 1001
Lunine, J. I., & Stevenson, D. J. 1982, Icarus, 67, 1053
Machida, M. N. 2009, MNRAS, 392, 514
Machida, M. N., Kokubo, E., Inutsuka, S., & Matsumoto, T. 2008, ApJ, 685, 1220
Machida, M. N., Matsumoto, T., Hanawa, T., & Tomisaka, K. 2006, ApJ, 645, 1227
Machida, M. N., Matsumoto, T., Tomisaka, K., & Hanawa, T. 2005, MNRAS, 362, 369
Martin, R. G., & Lubow, S. H. 2011, MNRAS, 413, 1447
Matsumoto, T., & Hanawa, T. 2003, ApJ, 595, 913
Miki, S. 1982, Prog. Theor. Phys., 67, 1053
Mizuno, H. 1980, Prog. Theor. Phys., 64, 544
Mosqueira, I., & Estrada, P. R. 2003, Icarus, 163, 198
Okuzumi, S., & Hirose, S. 2011, ApJ, 742, 65
Paardekooper, S.-J., & Mellema, G. 2008, A&A, 478, 245
Sasaki, T., Steward, G. R., & Ida, S. 2010, ApJ, 714, 1052
Sekiya, M., Miyama, S., & Hayashi, C. 1987, Earth Moon Planets, 39, 1
Tanigawa, T., & Ikoma, M. 2007, ApJ, 667, 557
Tanigawa, T., & Watanabe, S. 2002, ApJ, 580, 506
Ward, W. R., & Canup, R. M. 2010, AJ, 140, 1168