Friction torque analysis of high-speed spindle bearing based on the quasi-static improved model

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Abstract. In order to solve the problem that the quasi-static model cannot calculate the friction torque of the high-speed spindle bearing, a new improved quasi-statics analysis model which can be used to calculate the friction torque of high-speed bearing was established, in which the force balance condition of ball bearing in rotary plane was added by comprehensively considering the role of the cage and the oil films. And its correctness was verified by both theory and experiment. Based on the improved quasi-static model, the friction torque which affects the internal characteristic and service performance of high-speed spindle bearing is analyzed systematically. Results indicate that the rotation speed and axial pre-load and installation are the key factors influencing the spindle bearing internal friction torque and the friction torque inside the bearing can be effectively reduced by selecting ceramic balls and more small balls structure as well as optimizing the curvature parameters of the raceway of the bearing rings, etc. In addition, the improved quasi-static analysis method provides a new and effective analysis method for the performance analysis, structural optimization design and practical application of high-speed spindle bearing.

1. Introduction

Angular contact ball bearing (also known as spindle bearing) is used as the support of the rotor system in high-speed rotating machinery such as machine tool spindle. The supporting performance of the spindle bearing, especially the dynamic performance parameters such as friction torque and heat in the bearing during high-speed operation, directly affects the ultimate speed and service life of the bearing, and affects the overall performance of the rotating machinery[1]. Therefore the performance analysis method of high-speed spindle bearing has attracted the attention of relevant scholars and engineers.

Many scholars have studied problems regarding the analysis method of rolling bearing and friction torque of high-speed spindle bearing. A regression formula of bearing friction torque was obtained by palmgren[2] through radial and axial force analysis of ball bearing. Based on Hertz theory, the static analysis model of the ball bearing was established by Stribeck. Considering the centrifugal force and gyroscopic torque of the ball as the external force in the balance The centrifugal force and gyroscopic torque of the ball were introduced into the equilibrium equation of the ball rolling body as external forces, and the quasi-static model was established by Jones[3] by using the raceway control theory of the ring. Harris[4] considered the elastohydrodynamic effect of lubricants and the effect of the cage,
and the quasi-dynamic model was built. And then the quasi-dynamic analysis model was improved by Poplawski and Rumbarger[5-6]. Gupta[7] put forward the dynamic analysis model. Houpert[8] considered the cage as a plane model for stress analysis. After that, the influencing factors of friction torque in low speed and high speed running of the bearing were emphatically analyzed[9]. A fitting formula with higher accuracy than Palmgren’s was proposed by Tao Run et al.[10]. Lu Chunyu and Gao Feng et al.[11-12] respectively established a quasi-static model of angular contact ball bearing for solving the problems such as the difficulty in determining the initial value, non-convergence of iteration process and slow convergence of traditional algorithm in numerical solution, different Newton-Raphson improved algorithm were proposed. The variation law of the friction torque was explored for the friction torque of low-speed and light-load aerospace bearing[13]. Above all, the basic theory and method system of bearing force analysis and dynamic stiffness calculation of rolling axle are built.

In the existing theoretical analysis model, the static model is too simple to suit for high-speed spindle bearing. Many factors of quasi-dynamics and dynamic models cannot be accurately simulated, so the calculation results often have some errors or even do not converge, and the computational workload is large, which are mostly used for academic research. However, the traditional quasi-static model has high precision, and the model is relatively simple and easy to calculate, so it has been widely used in engineering practice. However, it ignores the role of the retainer and the influence of the elastohydrodynamic film between the ball and the ring. There are still problems in the analysis of the friction torque and other important performance parameters in the bearing during high-speed operation.

Therefore, based on the classical theory of ring control, on the basis of the traditional quasi-statics model, using some viewpoints of quasi-dynamics, the effect of cage and the influence of lubricating oil film are taken into account, and the force balance conditions of rolling body, cage and ring in the rotation plane of bearing are improved in this paper. Then the improved quasi statics model is established, which is more in line with the engineering practice and suitable for high speed spindle bearing. It provides a practical and effective method for the performance analysis of high-speed spindle bearing, especially for the analysis of friction torque.

Therefore, in this paper, based on the traditional quasi-statics model, using some viewpoints of quasi-dynamics, the force balance condition of the bearing in rotary plane was perfected by taking the role of internal bearing cage and the influence of oil film into account. And the new improved quasi-statics analysis model, which is more in line with the engineering practice and suitable for high speed spindle bearing was established. It provides a practical and effective method for the performance analysis of high-speed spindle bearing, especially for the analysis of friction torque.

2. Improvement of quasi-statics model

2.1. The geometric relationship of bearing internal deformation considering lubricating oil film

The existence of lubricating oil film between the ball and the inner and outer race and its influence on the force balance of the rolling body were ignored in the traditional quasi-static model[9]. But in practical engineering, especially in high-speed operation, there is an elastic hydrodynamic lubrication film between the raceway and the rolling body. Elastic lag and hydrodynamic pressure are two important factors that affect the friction torque, and affect the normal contact elastic deformation between the ball rolling body and the inner and outer raceways. According to the oil film thickness formula proposed, the equation of the dimensionless central oil film thickness between the inner ball rolling body and the inner and outer raceways of the main shaft bearing is created, as follows:

$$H_0 = 2.69U^{0.67}G^{0.53}W^{-0.067}(1 - 0.61e^{-0.73L})$$

Where, dimensionless parameter $H_0 = h_0/R_0$, $h_0$ is the centre oil film thickness. $U$ is a dimensionless speed parameter, $U = \frac{\eta_0 U}{2E_0R_0}$. $W$ is a dimensionless load parameter, $W = \frac{Q}{E_0R_0}$. $k_c$ is the ellipticity of the contact surface. $G = \frac{1}{\lambda E_0}$ is a dimensionless material parameter, where $\lambda$ is the Viscosity pressure.
coefficient (Pa⁻¹). \( \eta_0 \) is the dynamic viscosity (Pa·s) of the lubricating oil under normal pressure. \( R_\xi \) is the equivalent radius of curvature along the moving direction of the rolling body. \( Q \) is the contact load. \( E_0 \) is the equivalent modulus of elasticity. \( u \) is the average speed between the rolling body and the ferrule (that is, the fluid speed between the raceway and the ball in the contact area).

According to the theory of ring control, for the high-speed spindle bearing, due to the obvious centrifugal force, the ball rolling will tend to press to the outer raceway and break away from the inner raceway, so it belongs to the outer raceway control. After adding the lubricating oil film between the ball in the bearing and the inner and outer rings, the geometric deformation diagram of the interaction between the ball and the inner and outer rings at the angular position \( \Psi_j \) is shown in Figure 1.

According to the positional relationship about deformation and the geometry in figure 1, the following equation can be obtained:\(^{[14]}\)

\[
A_j = BD \sin \alpha_o + \delta_x + R_i \left( \theta_j \sin \Psi_j + \theta_z \cos \Psi_j \right) \tag{2}
\]

\[
A_{2j} = BD \cos \alpha_o + \delta_x \cos \Psi_j + \delta_z \sin \Psi_j \tag{3}
\]

\[
X_{ij}^2 + X_{j2}^2 = \left( (f_i - 0.5) D_u + \delta_x + h_y \right)^2 \tag{4}
\]

\[
(A_j - X_{ij})^2 + (A_{2i} - X_{j2})^2 = \left( (f_i - 0.5) D_u + \delta_x + h_y \right)^2 \tag{5}
\]

\[
\delta_x = \sqrt{X_{ij}^2 + X_{j2}^2} - (f_i - 0.5) D_u - h_y \tag{6}
\]

\[
\delta_y = \sqrt{(A_j - X_{ij})^2 + (A_{2i} - X_{j2})^2} - (f_i - 0.5) D_u - h_y \tag{7}
\]

Where, BD is the distance between the curvature centre points of the inner and outer raceways of the bearing under no load. \( \alpha_o \) is the initial contact angle of the angular contact ball bearing, rad. \( R_i \) is the circumference radius of the curvature centre of the inner raceway, mm. \( \Psi_j \) is the ball azimuth, rad.

\[
R_i = \frac{d_u}{2} (f_i - 0.5) D_u \cos \alpha_o \tag{8}
\]

2.2. The interaction between ball and cage
During the actual operation of the main shaft bearing, the ball centre \( O_b \) does not coincide with the cage pocket centre \( O_p \), as shown in figure 2. According to the viewpoint of quasi dynamics, there are two kinds of relative movements between all balls in the bearing and the cage pocket. One is that the cage pocket centre \( O_p \) is ahead of the ball centre \( O_b \) (for example, \( O_{p1} \) is ahead of \( O_{b1} \)), that is, the cage speed is greater than the ball speed, and \( Z_c \) is positive. The other is that the centre speed of the cage is less than that of the ball.
The forces exerted by the ball on the cage are mainly the normal force $F_{bcj}$ and tangential friction $f_{bcj}$ of the ball on the cage pocket, while the axial friction $f_{tbcj}$ is ignored. (Assuming the cage drives the ball "+".) The following equations can be obtained.

$$Z_{cj} = Z_{c(j-1)} + \pi D_{i} \frac{1}{Z} \left(1 - \frac{\omega_{mj(j-1)} + \omega_{mj}}{2\omega_{j}}\right), \quad F_{sc} = K_{c} \cdot Z_{c}$$

(9)

Where, $\omega_{mj}$ is the rotation speed of the $j$th ball around the $x$-axis of the fixed coordinate system. $Z$ is the total number of balls. $\omega_{c}$ is the rotation speed of the cage around the $x$-axis of the inertial coordinate system. $K_{c} = 11/C_{p}$ is the linear approximation constant determined by the test, N/mm. $C_{p}$ is the radius pocket clearance, mm.

2.3. Force analysis of ball with cage

In the traditional quasi-static model, the effect of cage was not considered, and the force and torque on the ball are balanced, as shown in figure 3(a). After considering the effect of cage, the force of the improved model ball in $y_{bo}z_{bo}$ plane and $x_{bo}z_{bo}$ plane is shown in figure 3(b) and 3(c).

Where, $Q_{ij}$ is the normal contact load between the steel ball and the raceway of the inner and outer rings; $F_{ij}$ are the components of the friction force on the long and short axis of the contact ellipse on the rolling body and the inner and outer rings respectively; $M_{zgj}$, $F_{c}$ are the inertia effects of the ball such as gyroscopic torque and centrifugal force, and $g$ is the mass of the ball.

The following equations can be obtained from figure 3:

$$Q_{ij} \sin \alpha_{ij} - Q_{ij} \sin \alpha_{ij} F_{ij} \cos \alpha_{ij} + F_{ij} \cos \alpha_{ij} - f_{tij} = 0$$

(10)

$$Q_{ij} \cos \alpha_{ij} - Q_{ij} \cos \alpha_{ij} F_{ij} \sin \alpha_{ij} + F_{ij} \sin \alpha_{ij} + F_{ij} + g \cos \psi_{j} = 0$$

(11)

$$F_{ij} F_{ij} - F_{ij} F_{ij} = 0$$

(12)

$$F_{ij} D_{ij} = 0$$

(13)

$$F_{ij} D_{ij} = 0$$

(14)

$$D_{ij} F_{ij} - M_{ij} = 0$$

(15)

2.4. Force analysis of improved model cage.

If the contact point between the cage and the guide surface of the outer ring is at point $a$, and the contact angle between the cage and the outer ring is $\phi$, then the normal force $F_{ctl}$ is outward and inward along the radial direction of the cage, and the friction $f_{ctl}$ is perpendicular to the normal force direction, as shown in figure 4[2][15].

The formula of friction force between guide ring and cage is as follows:
\[ f_{cl} = -\frac{\eta \pi \cdot D_c \cdot B_{cl}}{1 - d_1/d_2} (\omega_{cl} - \omega) \]  

Where, \( \eta \) is the dynamic viscosity of lubricating oil, \( D_c \) is the diameter of cage center surface, \( B_{cl} \) is the total width of cage guiding part, \( d_1 \) is the smaller diameter of guiding surface and cage center surface, \( d_2 \) is the larger diameter of guiding surface and cage center surface.

It can be obtained from figure 5 that the force balance equations in Y-axis and Z-axis direction and torque balance equation of the cage are as follows.

\[ f_{cl} \cdot \sin \phi - \sum_{j=1}^{z} (F_{yj} \sin \psi_j + F_{zj} \cos \psi_j) = 0 \]  

(17)

\[ f_{cl} \cdot \cos \phi - \sum_{j=1}^{z} (F_{yj} \cos \psi_j - F_{zj} \sin \psi_j) = 0 \]  

(18)

\[ f_{cl} \cdot D_c = D_c \cdot \sum_{j=1}^{z} F_{zj} \]  

(19)

Figure 5. Force diagram of cage. Figure 6. Force diagram of ball on inner ring.

2.5. Stress analysis of inner ring of improved model

When the high-speed spindle bearing works, the outer ring is fixed and the inner ring rotates. The force of the ball on the inner ring is shown in figure 6.

Considering the effect of all the balls of the bearing on the inner ring, the radial load \( F_r \), \( F_z \), axial preload \( F_x \), torque \( M_x \), \( M_y \) and driving torque \( M_x \) in the rotation plane of the bearing inner ring on the rotor are balanced with them, the force balance equations of the inner ring are as follows.

\[
\begin{align*}
F_x - \sum_{j=1}^{z} \left[ Q_y \sin \alpha_j + F_{wqj} \cos \alpha_j \right] &= 0 \\
F_y - \sum_{j=1}^{z} \left[ Q_y \cos \alpha_j + F_{wqj} \sin \alpha_j \sin \psi_j + F_{wqj} \cos \psi_j \right] &= 0 \\
F_z - \sum_{j=1}^{z} \left[ Q_y \cos \alpha_j - F_{wqj} \sin \alpha_j \cos \psi_j \right] &= 0 \\
M_x - \sum_{j=1}^{z} \left[ R_y \left( Q_y \sin \alpha_j + \cos \alpha_j \right) \cos \psi_j \right] &= 0 \\
M_y - \sum_{j=1}^{z} \left[ R_y \left( Q_y \sin \alpha_j + \cos \alpha_j \right) \sin \psi_j \right] &= 0 \\
M_z - \sum_{j=1}^{z} \left[ R_y \left( Q_y \sin \alpha_j + \cos \alpha_j \right) \right] &= 0 \\
\end{align*}
\]  

(20)

(21)

(22)
Where, \( A = \frac{1}{2} D_a - \frac{1}{2} D_a \cos \alpha \) is the position of the ball to cone friction to the bearing centreline, mm.

Here, the driving torque \( M_x \) in the rotating plane will be balanced with the friction torque inside the bearing. A new improved quasi-static model can be obtained by combining all the above equations which consider the effect of cage and the effect of elastohydrodynamic lubrication film between ball and raceway.

3. Theoretical verification of the improved model

3.1. Verification of force analysis

In order to verify the correctness of the improved model established in this paper, the typical examples of Jones quasi-statics model and reference [15] are used to verify. The basic parameters of the example bearing are: Ball Number: 19, ball diameter: 15.081 mm, pitch diameter: 105 mm, contact angle: 25°, groove curvature coefficient: 0.52, contact angle: 25°, groove curvature coefficient: 0.52, elastic modulus: \( 2.06 \times 10^{11} \) Pa, Poisson's ratio: 0.3. The initial axial preload is 889.84 N and the working speed is 15000 r/min. In the aspect of bearing capacity, the comparison between the improved model and the simulation results of classical examples is shown in table 1. It can be seen that the calculation results of the main parameters of the improved quasi-statics model established in this paper are consistent with the analysis results of Jones quasi-statics model and the experimental results in reference [15]. The results show that the correctness of force analysis results of the improved quasi-static model which considered the effect of the internal cage of the bearing and the influence of the EHL film is intuitively clear.

| Dynamic characteristic parameters | \( \omega_m \) (r/min) | \( \omega_s \) (r/min) | \( \omega_r \) (r/min) | \( \alpha \) (°) | \( \alpha_e \) (°) | \( Q_i \) (N) | \( Q_e \) (N) |
|-----------------------------------|----------------|----------------|----------------|---------|---------|---------|---------|
| Jones quasi-statics model results \[3\] | 7255 | 0.689 | 0 | 37.70 | 4.074 | 76.49 | 488.44 |
| Experimental results of reference \[15\] | 7252 | 0.688 | 0 | 37.61 | 4.031 | 76.75 | 487.02 |
| Results of Gupta kinetic model\[7\] | 7200 | 0.67 | 0 | 37.6 | 4.01 | 76.5 | 477.00 |
| The results of improved model | 7249 | 0.623 | 0 | 37.59 | 4.086 | 76.62 | 483.84 |

When 889.84N axial preload is applied to the example bearing, the improved model and the traditional Jones quasi-statics model are simulated to analyze the influence of rotational speed on the contact angle and contact stress on the inner and outer race raceway of the bearing. The results are compared as shown in figure 7 and 8. It can be seen that the trend of the curve is basically consistent, which further verifies the correctness of the improved quasi-statics model in this paper. Moreover, according to Tab. 1, the calculation results of various dynamic characteristic parameters of the improved quasi-statics model which considered the influence of lubricating oil film and cage are basically the same as those of the mature Jones quasi-statics model, Gupta dynamic model and the experimental results in reference [15], and the calculation accuracy is higher. The comparison results show that the improved quasi-statics model in this paper is theoretically correct and reliable.
3.2. Test verification of friction torque

The friction torque is one of the most important characteristic parameters of the main shaft bearing in high-speed operation, and it is the key influencing factor of the internal heating, temperature rise and friction and wear of the bearing. Take B7005C bearing as an example, its basic parameters are shown in Table 2, and the friction torque of high-speed spindle bearing is tested by series tests using a special test platform[9]. GCr15 material is used for the bearing. 4109 aviation lubricating oil of which dynamic viscosity is 0.033 Pa·s and a viscosity pressure coefficient is $1.28 \times 10^{-8}$ Pa$^{-1}$ is selected. The bearing cage is guided by the outer ring. The inner ring rotates and the outer ring is stationary under operation condition. The preloading mode is constant pressure preloading.

Table 2. Parameters of B7005C bearing structural.

| Parameters                        | Values  |
|-----------------------------------|---------|
| outer diameter $D_o$/mm           | 47      |
| inner diameter $D_i$/mm            | 25      |
| Ball diameter $D_w$/mm             | 5.5     |
| Number of balls $Z$                | 15      |
| Initial design contact angle $\alpha_o/\degree$ | 15 |
| Curvature radius coefficient of outer channel | 0.55 |
| Bearing width $B$/mm               | 12      |
| $C$/mm                            | 35.1    |
| Outside diameter of cage $C_o$/mm  | 39.06   |
| Cage center surface width $B_c$/mm | 11.4    |
| Cage guide clearance $C_1$/mm      | 0.12    |
| Cage pocket diameter $D_c$/mm      | 5.77    |
| Curvature radius coefficient of inner channel | 0.53 |

Figure 7. Simulated results comparison of the relationship between rotational speed and contact angle in different models.

Figure 8. Simulated results comparison of the relationship between rotational speed and contact stress in different models.

Figure 9. Results Comparison of simulated and experimental when axial preload 51 N.

Figure 10. Results Comparison of simulated and experimental when axial preload 121 N.
Apply 51N and 121N axial preloads to the bearing respectively, and compare the results with the calculation results of the improved model in this paper, as shown in figure 9 and figure 10. It can be seen that with the increase of running speed and axial preload, the friction torque inside the bearing increases correspondingly, and the calculation value and test of the improved model in this paper The change trend of the measured values is consistent, and the values are basically consistent, which verifies the correctness of the improved model in the analysis of bearing friction torque.

4. Friction torque analysis of high-speed spindle bearing

The factors that affect the friction torque of high-speed spindle bearing can be divided into external factors and internal factors. The external factors are the external working conditions of the bearing during operation, such as speed, axial preload and radial load. The internal factors are related to the design and manufacturing parameters of the bearing, such as the curvature radius coefficient of the inner and outer raceways, the ball material, the ball diameter and the number of balls, etc. Take the above design parameters of B7005C main shaft bearing as an example, apply the axial preload of 100N and the radial outward load of 10N, and use the quasi-statics model in this paper to analyze the influence of different external working conditions and internal factors of the bearing on the friction torque of the high-speed main shaft bearing. The simulation results are shown in figure 11-16.

It can be seen from figure 11 and 12 that the friction torque increases significantly with the increase of rotating speed and axial preload, while the radial preload has little influence on the friction torque. In figure 13 and 14, the smaller the curvature radius coefficient of the inner channel is, the larger the friction torque value is, while the larger the curvature radius coefficient of the outer channel is, the larger the friction torque value is, and the greater the influence of the curvature radius coefficient of the outer channel is. In figure 15, the larger the number of balls and the smaller the diameter, the smaller the friction torque value. In figure 16, Si3N4 ceramic is used instead of conventional GCr15 bearing steel as the ball material, which can greatly reduce the friction torque of the bearing.

![Figure 11. Simulated results of friction torque with different axial preloads.](image1)

![Figure 12. Simulated results of friction torque with different radial preloads.](image2)
5. Conclusion

(1) In this paper, based on the classical theory of ring control and the viewpoint of pseudo dynamics, the effect of cage and lubricating oil film is taken into account in the traditional quasi-statics model, the force balance condition of bearing is improved, and a new quasi-statics improved model is established. And it can also research the friction torque which affects the high-speed performance of the bearing, and expand the function and application scope of the traditional quasi-statics model.

(2) The results of the improved model are compared with the classical quasi-static model. In addition, the calculation results of the friction torque of the bearing are compared with the test results by using the improved model, which verifies the correctness of the improved model.

(3) Using the improved quasi-statics model established in this paper, the influencing factors and changing rules of internal friction torque of high-speed spindle bearing are analyzed systematically. The results show that the rotation speed, axial preload and installation mode are the key factors affecting the internal friction torque of the main shaft bearing. The friction torque inside the bearing can be effectively reduced by selecting ceramic balls and more small balls structure, also optimizing the curvature parameters of the raceway of the bearing ring. The improved quasi static analysis method provides a new and effective analysis method for the performance analysis, structural optimization design and practical application of high-speed spindle bearing.
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