High-energy reactions with microscopic wave functions

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A new method is proposed to investigate high-energy reactions on heavy targets. It combines the eikonal model with a microscopic cluster description of the projectile. This approach is based on a nucleon–target interaction, and does not require projectile–target optical potentials, which are in general poorly known. We discuss the general formalism, and apply it to α scattering on $^{58}$Ni and $^{208}$Pb at $E_{\text{lab}} = 288$ MeV. The cross sections are shown to be very sensitive to the radius of the α particle. This method opens new perspectives in the description of high-energy reactions involving halo nuclei, where the long-range part of the projectile wave function is a fundamental issue.

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1. Introduction

High-energy reactions are widely used to investigate the structure of exotic nuclei [1]. In particular, halo nuclei are unusual objects [2] since they present a large spatial extension, and can be regarded as a core surrounded by one or two distant nucleons [3]. Typical examples are $^6$He or $^{11}$Li which are considered as a core surrounded by one or two distant nucleons [3].

In recent years, much effort has been done to develop microscopic [4] or non-microscopic [5] three-body models. The former are based on a nucleon–nucleon interaction and take an exact account of the Pauli principle, and therefore have a fairly strong predictive power. In non-microscopic models, antisymmetrization effects are simulated by an appropriate choice of the nucleus–nucleus interactions [6].

The theoretical treatment of reaction cross sections involving exotic nuclei is rather complicated, since the collision mechanism should also include their specific structure. Several models have been developed to describe elastic-scattering and breakup reactions: in particular, the eikonal model [7–9] and its dynamical extension [10], continuum-discretized coupled channel (CDCC) approximations [11], or numerical solutions of the time-dependent Schrödinger equation [12]. All these methods are based on a two or three-body description of the projectile. Interactions between the target and the constituents of the projectile must be known at the corresponding energy. If nucleon–nucleus interactions are available for many nuclei, and in a wide energy range (see e.g. Ref. [13]), the availability of nucleus–nucleus optical potentials may be a difficult issue, in particular when they involve a radioactive nucleus such as $^{8}$Li. This problem can be addressed by using a microscopic description of the projectile. In that case, only nucleon–target interactions are necessary.

2. Microscopic theory of elastic scattering

Merging high-precision microscopic wave functions with a description of the reaction dynamics is a challenge for nuclear theory in the next years. In a microscopic model, the Hamiltonian of the projectile with a nucleon number $A_p$ is defined as

$$H_0 = \sum_{i=1}^{A_p} T_i + \sum_{i>j=1}^{A_p} V_{ij},$$

(1)

where $T_i$ is the kinetic energy of nucleon $i$, and $V_{ij}$ a nucleon–nucleon interaction. The goal of the present work is to use the wave function of the projectile $\Psi_0$, solution of the Schrödinger equation associated with Eq. (1), in a reaction theory. Various methods are used in the literature to solve this Schrödinger equation. For small nucleon numbers, exact methods become available [14]. However, these methods rely on very long computer times and, in general, do not provide scattering states, required to deal with breakup cross sections.

We use here the cluster approximation [15], where the wave function $\Psi_0$ is written in terms of $N$ internal antisymmetric wave functions $\phi_i$ as

$$\Psi_0 = A \phi_1 \cdots \phi_N g,$$

(2)

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where $A$ is the $A$-nucleon antisymmetrizer, and $g$ a relative function, depending on the $N - 1$ relative coordinates between the clusters. The antisymmetrization operator ensures the Pauli principle to be exactly taken into account, not only inside the clusters, but also between the different clusters. Functions $\phi_i$ are in general Slater determinants based on shell model orbitals [15]. Essentially two-cluster and three-cluster models have been developed. In particular, a microscopic description of the $^6$He has been proposed in Ref. [4] based on the hyperspherical formalism [16]. The use of a cluster description allows an accurate treatment of the long-range part of the projectile wave function. This is crucial for reactions involving halo nuclei where the asymptotics of the wave function are known to play an important role.

The aim of this Letter is to introduce a microscopic cluster description of the projectile in the eikonal formalism of the reaction. The Hamiltonian associated with the projectile–target system reads

$$H = T_R + H_0(r_i) + \sum_i V_{TI}(r_i - R),$$

(3)

where $r_i$ are the nucleon coordinates inside the projectile, and $R = (b, Z)$ the relative distance between the target and the projectile (see Fig. 1); $b$ is the impact parameter, defined in the plane perpendicular to the projectile direction. As usual, the structure of the target is neglected. In (3), $T_R$ is the kinetic energy term, and $V_{TI}(s)$ a target–nucleon interaction. In contrast with standard eikonal calculations [7], the model is based on a nucleon–target optical potentials. These potentials are known for many nuclei in a wide energy range.

In the eikonal theory, the total wave function $\psi$ associated with Hamiltonian (3) is factorized as

$$\psi = \exp(ikZ)\hat{\psi},$$

(4)

where $k$ is the wave number, assumed to be along the $Z$ axis. The eikonal function $\hat{\psi}$ is obtained [7,8] from

$$\hat{\psi}(R, r_i) \approx \psi_0(r_i) \exp \left[ -\frac{i}{\hbar v} \sum_i \int_{-\infty}^{\infty} V_{TI}(b, Z', r_i) dZ' \right].$$

(5)

In the eikonal approximation, the relative velocity $v = \hbar k/\mu$ is assumed to be large enough to neglect $[T_R \hat{\psi}]$, which provides the wave function (5). From Eq. (5), one derives the scattering amplitude

$$f(\Omega) = \frac{i k}{2\pi} \int d\mathbf{b} \exp(-i q \cdot b) \left[ 1 - S(b) \right],$$

(6)

where $q = k' - k$ (vector $k'$ is obtained by rotating $k$ by an angle $\theta$). The eikonal scattering amplitude reads

$$S(b) = \langle \psi_0 | \exp[i \chi(b)] | \psi_0 \rangle,$$

(7)

where the Dirac notation stands for integrals over the nucleon coordinates $r_i$ inside the projectile. In this definition, $\chi(b)$ is the eikonal phase shift, depending on the internal coordinates $r_i$.

$$\chi(b) = \sum_i \chi_i(b, r_i),$$

(8)

The eikonal phase shift can be therefore written as a sum over the individual nucleon–target phase shifts. The Coulomb convergence problem is addressed as explained in [17,18], and does not need the introduction of any cutoff on the impact parameter. From the eikonal phase shift (8), and the scattering amplitude (6), the elastic cross section is directly derived after integration over the impact parameter $b$ [8].

Until here, the presentation is general, and can be, in principle, applied to a projectile wave function with any cluster number (2). The usual method (see for example Ref. [19]) is to expand the eikonal phase shift (8) in multipoles, which allows to evaluate the scattering amplitude (7). In the cluster model, the eikonal phase shift is a sum of one-body operators. Matrix elements of these operators between microscopic cluster wave functions can be determined in a systematic way [20].

The application of the cluster model is rather heavy since the wave function (2) must be projected on angular momentum [15]. This represents, for the eikonal phase shift (8), the evaluation of many-dimensional integrals. In this exploratory work, we start with a simple cluster, the $\alpha$ particle. In such a case, the projectile wave function (2) is a Slater determinant with four 0 $\uparrow$ and 0 $\downarrow$ nucleon–target phase shifts. The Coulomb convergence problem is addressed as explained in [17,18], and does not need the introduction of any cutoff on the impact parameter.

$$\chi_0 = \chi_\uparrow \chi_\downarrow,$$

(9)

where notation $(n, p)$ refers to the isospin and $(\downarrow, \uparrow)$ to the spin projection. The use of harmonic-oscillator functions for the individual orbitals in (9) provides an exact factorization of the center of mass (c.m.) motion [15]. The calculation of matrix elements (7) can therefore be done by removing spurious c.m. effects. Simple and exact treatment of the c.m. motion stems from the harmonic-oscillator orbitals, and remains valid for multi-cluster wave functions if the oscillator parameters of the clusters are identical. The one-cluster wave function (9) has a spin zero, and further angular-momentum projection is not required, as in multi-cluster variants. This provides, for the eikonal scattering amplitude,

$$S_j(b) = \prod_{j=1}^{4} S_j(b),$$

(10)

where index $j$ corresponds to the four different spin/isospin combinations. The factorization (10) stems from the fact that all orbitals in (9) are orthogonal to each other. In Eq. (10), each individual contribution is given by

$$S_j(b) = \langle \psi(r) | \exp \left( -\frac{i}{\hbar v} \int_{-\infty}^{\infty} V_{TI}(b, Z', r) dZ' \right) | \psi(r) \rangle,$$

(11)

and therefore represents a four-dimensional integral. In practice, the three integrals associated with $r$ are determined by Gauss–Hermite quadratures, while the integral over $Z'$ is obtained from a Simpson approximation. In general the nucleon–target potential does not depend on the nucleon spin. In that case, the factorization (10) only involves two different terms, related to the neutron–target and proton–target interactions.
3. Application to \( \alpha \)-nucleus scattering

The formalism is applied to the \( \alpha + ^{58}\text{Ni} \) and \( \alpha + ^{208}\text{Pb} \) collisions at \( E_\alpha = 288 \text{ MeV} \). The microscopic eikonal cross sections are presented in Fig. 2 and 3, respectively, with the experimental data of Ref. [21]. The neutron–target and proton–target optical potentials are taken from [13] at \( E_{\text{lab}} = 72 \text{ MeV} \). Fig. 2 and 3 show the cross section for various choices of the \( \alpha \) oscillator parameter \( B \). We start with \( B = 0.1 \text{ fm} \), which nearly corresponds to a structureless \( \alpha \) particle (the \( 0s \) orbitals are equivalent to delta functions). In that case the experimental cross section is strongly overestimated at \( \theta \gtrsim 15^\circ \). The sensitivity with \( B \) is rather weak at forward angles \( (\theta < 10^\circ) \), but the \( \alpha \)-particle structure clearly plays a role at larger angles.

It is interesting to notice that, for both systems, the optimal oscillator parameter is near \( B \approx 1 \text{ fm} \), which is smaller than the value deduced from the \( \alpha \)-particle rms radius \( (B \approx 1.4 \text{ fm}) \). The theoretical cross sections with \( B = 1.05 \text{ fm} \) \( (\alpha + ^{58}\text{Ni}) \) and \( B = 0.95 \text{ fm} \) \( (\alpha + ^{208}\text{Pb}) \) are in good agreement with the data. The minima in the cross sections are consistent with experiment. However, the relatively small \( B \) value suggests that dynamical effects are important, and that the \( \alpha \) particle is distorted during the collision. The radius deduced from high-energy collisions would then be smaller than the actual radius of the \( \alpha \) particle.

4. Conclusion and outlook

In summary, the present work opens new perspectives in high-energy reactions. We have extended the eikonal theory to a microscopic description of the projectile. The present approach is based on nucleon–target interactions, and does not need any optical potential between the target and projectile. We started with a simple example, the \( \alpha \) particle defined by four \( 0s \) orbitals.

The application to \( \alpha + ^{58}\text{Ni} \) and \( \alpha + ^{208}\text{Pb} \) elastic cross sections shows a clear evidence for structure effects. Neglecting the structure of the \( \alpha \) particle provides a strong overestimation of the data at forward angles. Optimizing the \( \alpha \) oscillator parameter gives a fairly good agreement with experiment. The optimal value \( B \approx 1 \text{ fm} \) is smaller that the oscillator parameter deduced from the radius \( (B \approx 1.4 \text{ fm}) \). This suggests distortion effects in high-energy reactions.

Future works should go beyond the \( \alpha \) particle, and use two- or three-cluster microscopic wave functions for the projectile. Typical examples are \( ^3\text{Li} (= \alpha + \text{t}) \) or \( ^6\text{Li} (= \alpha + d) \) for a two-cluster model, and \( ^4\text{He} (= \alpha + n + n) \) for a three-cluster description. This would allow to extend the present calculation to breakup cross sections. These calculations are highly time-consuming but should be feasible in the future. They represent a challenge for theoretical models of high-energy reactions.

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References

[1] P.G. Hansen, A.S. Jensen, B. Jonson, Annu. Rev. Nucl. Sci. 45 (1995) 591.
[2] B. Jonson, Phys. Rep. 389 (2004) 1.
[3] I. Tanigata, J. Phys. G 22 (1996) 157.
[4] S. Korennyov, P. Descouvemont, Nucl. Phys. A 740 (2004) 249.
[5] M.V. Zhukov, B.V. Danilin, D.V. Fedorov, J.M. Bang, I.J. Thompson, J.S. Vaagen, Phys. Rep. 231 (1993) 151.
[6] B. Buck, C.B. Dover, J.P. Vary, Phys. Rev. C 11 (1975) 1803.
[7] R.J. Glauber, High energy collision theory, in: Lectures in Theoretical Physics, vol. 1, 1959.
[8] Y. Suzuki, R.G. Lovas, K. Yahama, K. Varga, Structure and Reactions of Light Exotic Nuclei, in: Taylor & Francis, London, 2003.
[9] J.S. Al-Khalili, R. Crespo, R.C. Johnson, A.M. Moro, I.J. Thompson, Phys. Rev. C 75 (2) (2007) 024608.
[10] G. Goldstein, D. Baye, P. Capel, Phys. Rev. C 73 (2006) 024602.
[11] M. Yahiro, Y. Iseri, H. Kaneyama, M. Kaminura, M. Kawai, Prog. Theor. Phys. Suppl. 89 (1986) 32.
[12] V.S. Melezhik, D. Baye, Phys. Rev. C 59 (1999) 2322.
[13] A.J. Koning, J.P. Delaroche, Nucl. Phys. A 713 (2003) 231.
[14] P. Navrálík, S. Quaglioni, I. Stetcu, B.R. Barrett, J. Phys. G 36 (2009) 083101.
[15] K. Wildermuth, Y.C. Tang, A Unified Theory of the Nucleus, Vieweg, Braunschweig, 1977.
[16] C.D. Lin, Phys. Rep. 257 (1995) 1.
[17] J. Margueron, A. Bonaccorso, D.M. Brink, Nucl. Phys. A 703 (2002) 105.
[18] P. Capel, D. Baye, Y. Suzuki, Phys. Rev. C 78 (2008) 054602.
[19] D. Baye, P. Capel, P. Descouvemont, Y. Suzuki, Phys. Rev. C 79 (2009) 024607.
[20] D. Brink, in: Proc. Int. School “Enrico Fermi” 36, Varenna 1965, Academic Press, New York, 1966, p. 247.
[21] B. Bonin, N. Alamanos, B. Berthier, G. Bruge, H. Faraggi, J.C. Lugol, W. Mittig, L. Papineau, A.I. Yavin, J. Arvieux, L. Farvacque, M. Buenerd, W. Bauhoff, Nucl. Phys. A 445 (1985) 381.