SCHEMES OF NEUTRINO MIXING FROM THE RESULTS OF NEUTRINO OSCILLATION EXPERIMENTS

S.M. Bilenky(a), C. Giunti(b) and W. Grimus(c)

(a) Joint Institute for Nuclear Research, Dubna, Russia

(b) INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino, Via P. Giuria 1, I–10125 Torino, Italy

(c) Institute for Theoretical Physics, University of Vienna, Boltzmannasse 5, A–1090 Vienna, Austria

Abstract

The mixing of three and four massive neutrinos is considered. It is shown that the neutrino oscillation data are not compatible with a hierarchy of couplings in the three-neutrino case. In the case of four neutrinos, a hierarchy of masses is not favored by the data. Only two schemes with two pairs of close masses separated by a gap of the order of 1 eV can accommodate the results of all experiments. If the existing indications in favor of neutrino oscillations will be confirmed, it will mean that the general features of neutrino mixing are quite different from those of quark mixing.

Forty years after its proposal by B. Pontecorvo [1], neutrino oscillations (see Ref.[2]) are considered today as one of the most interesting phenomena in high energy physics and one of the most promising methods for the search of new physics beyond the Standard Model (see Ref.[3]).

In this report we discuss which information on the neutrino mass spectrum and mixing parameters can be obtained from the results of neutrino oscillation experiments. No evidence of neutrino oscillations was found in many experiments and their results are useful in order to constrain the allowed values of the neutrino masses and mixing parameters. These are all short-baseline (SBL) neutrino oscillation experiments with reactor and accelerator neutrinos. In particular, we use the exclusion plots obtained from the data of the Bugey [4] $\bar{\nu}_e$ disappearance experiment, of the CDHS and CCFR $\bar{\nu}_\mu$ disappearance experiments and of the BNL E734, BNL E776 and CCFR $\nu_\mu \rightarrow \nu_e$ appearance experiments.

1 Talk presented by C. Giunti at the 9th International School Particles and Cosmology, Kabardino Balkaria, Baksan Valley, Russia, April 15–22, 1997.
There are three experimental indications in favor of neutrino oscillations. They come from the existence of the solar neutrino problem, the atmospheric neutrino anomaly and from the results of the LSND experiment. The solar neutrino problem is the oldest and most widely believed indication in favor of neutrino oscillations: the event rates measured by all solar neutrino experiments (Homestake, Kamiokande, GALLEX, SAGE and Super-Kamiokande) are significantly smaller than those predicted by the Standard Solar Model. This deficit can be explained with oscillations of solar $\nu_e$'s into other states and indicates a mass-squared difference $\delta m^2 \approx 10^{-5} \text{eV}^2$ in the case of resonant MSW transitions or $\delta m^2 \approx 10^{-10} \text{eV}^2$ in the case of vacuum oscillations.

The atmospheric neutrino anomaly has been found in the Kamiokande, IMB, and Soudan experiments. It can be explained by $(\nu_\mu \rightarrow \nu_x)$ oscillations ($x \neq \mu$) with a mass-squared difference $\delta m^2 \approx 10^{-2} \text{eV}^2$. However, the existence of an atmospheric neutrino anomaly is a controversial issue, because no anomaly was observed in the ratio of contained muon-like to electron-like events measured in the Fréjus and NUSEX experiments and in the flux of upward-going muons measured in the Kamiokande, IMB, Baksan and MACRO experiments. The existence of an atmospheric neutrino anomaly will be checked in the near future by the Super-Kamiokande experiment. Moreover, several long-baseline (LBL) experiments will search for neutrino oscillations due to $\delta m^2 \approx 10^{-2} \text{eV}^2$: the CHOOZ and Palo Verde LBL $\bar{\nu}_e$ disappearance experiments with reactor anti-neutrinos and the accelerator KEK–Super-Kamiokande (K2K), Fermilab–Soudan (MINOS) and CERN–Gran Sasso (ICARUS) LBL $\nu_\mu$ disappearance and $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\tau$ appearance experiments.

Finally, indications in favor of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations have been found in the LSND experiment. This is the only SBL experiment which presently claims an evidence in favor of neutrino oscillations. The analysis of the data of this experiment, taking into account the negative results of other SBL experiments (in particular, the Bugey and BNL E776 experiments), indicate a value of $\delta m^2$ in the range $0.3 \lesssim \delta m^2 \lesssim 2.2 \text{eV}^2$.

The three indications in favor of neutrino oscillations need three different scales of $\delta m^2$, which can be obtained with at least four massive neutrinos. From the LEP measurements of the invisible width of the $Z$-boson (see Ref.) we know that there are three light flavor neutrinos: $\nu_e$, $\nu_\mu$ and $\nu_\tau$. These are called flavor neutrinos because they take part in weak interactions and each of them couples to the corresponding charged lepton through the charged-current weak interaction. However, if the neutrino Lagrangian has a mass term, in general the left-handed flavor neutrino fields $\nu_{\alpha L}$ are superpositions of the left-handed components $\nu_{kL}$ of the fields of neutrinos.

\[ \delta m^2 \] Here the symbol $\delta m^2$ indicates a generic difference between the squares of two neutrino masses.
with definite mass \((k = 1, 2, 3, \ldots, n)\):

\[
\nu_{\alpha L} = \sum_{k=1}^{n} U_{\alpha k} \nu_{kL}.
\]  

(1)

Here \(U\) is a unitary mixing matrix. The number \(n\) of massive neutrinos can be three or more, without any experimental upper limit. If \(n > 3\) there are \(n - 3\) sterile neutrino fields, i.e., fields of neutrinos which do not take part in weak interactions. Hence, in Eq. (1) we have \(\alpha = e, \mu, \tau, s_1, s_2, \ldots, s_{n-3}\), with \(n - 3\) sterile neutrino fields \(\nu_{s_1}, \nu_{s_2}, \ldots, \nu_{s_{n-3}}\).

In the following we will consider mixing schemes in which only the largest mass square difference \(\Delta m^2_{n1} \equiv m_{s_1}^2 - m_{s_2}^2\) is relevant for SBL oscillations [18, 20, 23]. These schemes are based on mass spectra with two groups of massive neutrinos with close masses, \(\nu_1, \ldots, \nu_{r-1}\) and \(\nu_r, \ldots, \nu_n\), separated by a mass difference in the eV range: \(m_1 < \ldots < m_{r-1} < m_r < \ldots < m_n\). The general expression for the transition probability of relativistic neutrinos (see Refs. [2, 3])

\[
P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_{k=1}^{n} U_{\beta k} U_{\alpha k}^* e^{-i \frac{\Delta m^2_{\alpha k} L}{2E}} \right|^2
\]

(2)

can be written as

\[
P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_{k=1}^{r-1} U_{\beta k} U_{\alpha k}^* e^{-i \frac{\Delta m^2_{\alpha k} L}{2E}} + e^{-i \frac{\Delta m^2_{s_1} L}{2E}} \sum_{k=r}^{n} U_{\beta k} U_{\alpha k}^* e^{i \frac{\Delta m^2_{s_1} L}{2E}} \right|^2, \quad (3)
\]

where \(L\) is the distance between the neutrino source and detector, \(E\) is the neutrino energy and \(\Delta m^2_{\alpha k} \equiv m_{\alpha k}^2 - m_{\alpha 1}^2\). In SBL experiments we have

\[
\frac{\Delta m^2_{\alpha 1} L}{2E} \gtrsim 1, \quad \frac{\Delta m^2_{\alpha k} L}{2E} \ll 1 \quad \text{for} \quad k < r \quad \text{and} \quad \frac{\Delta m^2_{\alpha k} L}{2E} \ll 1 \quad \text{for} \quad k \geq r, \quad \text{if} \quad \Delta m^2 \equiv \Delta m^2_{\alpha 1} \quad \text{and} \quad \text{the oscillation amplitudes}
\]

\[
P_{\nu_{\alpha} \rightarrow \nu_{\beta}}^{(\text{SBL})} = \frac{1}{2} A_{\alpha;\beta} \left(1 - \cos \frac{\Delta m^2 L}{2E}\right), \quad P_{\nu_{\alpha} \rightarrow \nu_{\alpha}}^{(\text{SBL})} = 1 - \frac{1}{2} B_{\alpha;\alpha} \left(1 - \cos \frac{\Delta m^2 L}{2E}\right), \quad (5)
\]

with \(\Delta m^2 \equiv \Delta m^2_{\alpha 1}\) and the oscillation amplitudes

\[
A_{\alpha;\beta} = 4 \left| \sum_{k=r}^{n} U_{\beta k} U_{\alpha k}^* \right|^2 = 4 \left| \sum_{k=1}^{r-1} U_{\beta k} U_{\alpha k}^* \right|^2, \quad (6)
\]

\[
B_{\alpha;\alpha} = 4 \left( \sum_{k=r}^{n} |U_{\alpha k}|^2 \right) \left(1 - \sum_{k=r}^{n} |U_{\alpha k}|^2 \right) = 4 \left( \sum_{k=1}^{r-1} |U_{\alpha k}|^2 \right) \left(1 - \sum_{k=1}^{r-1} |U_{\alpha k}|^2 \right). \quad (7)
\]
The equalities of the two expressions in Eq. (6) and in Eq. (7) are due to the unitarity of the mixing matrix. The formulas (5) have the same form of the standard expressions for the oscillation probabilities in the case of two neutrinos (see Refs. [2, 3]). This fact is very important, because the data of all the SBL experiments have been analyzed by the experimental groups under the assumption of two-generation mixing, obtaining constraints on the possible values of the mixing parameters $\Delta m^2$ and $\sin^2 2\theta$ ($\theta$ is the mixing angle). Hence, we can use the results of the analyses of the neutrino oscillation data made by the experimental groups in order to constraint the possible values of the oscillation amplitudes $A_{\alpha;\beta}$ and $B_{\alpha;\alpha}$.

First, we consider the scheme with three neutrinos and the mass hierarchy (see also Ref. [21])

\[
\begin{array}{c}
\text{solar} \\
\begin{array}{c}
m_1 \ll m_2 \ll m_3 \\
\text{LSND}
\end{array}
\end{array}
\]

This scheme (as all the schemes with three neutrinos) provides only two independent mass-squared differences, $\Delta m^2_{21}$ and $\Delta m^2_{31}$, which we choose to be relevant for the solution of the solar neutrino problem and for neutrino oscillations in the LSND experiment.

Let us emphasize that the mass spectrum (8) with three neutrinos and a mass hierarchy is the simplest and most natural one, being analogous to the mass spectra of charged leptons, up and down quarks. Moreover, a scheme with three neutrinos and a mass hierarchy is predicted by the see-saw mechanism for the generation of neutrino masses [22], which can explain the smallness of the neutrino masses with respect to the masses of the corresponding charged leptons.

In the case of scheme (8) we have $n = r = 3$ and from Eqs. (6) and (7) it follows that

\[
A_{\alpha;\beta} = 4 |U_{\alpha 3}|^2 |U_{\beta 3}|^2, \quad B_{\alpha;\alpha} = 4 |U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right).
\]

Hence, neutrino oscillations in SBL experiments depend on three parameters: $\Delta m^2 \equiv \Delta m^2_{31}$, $|U_{e 3}|^2$ and $|U_{\mu 3}|^2$ (the unitarity of $U$ implies that $|U_{\tau 3}|^2 = 1 - |U_{e 3}|^2 - |U_{\mu 3}|^2$).

From the exclusion plots obtained in reactor $\bar{\nu}_e$ and accelerator $\bar{\nu}_\mu$ disappearance experiments it follows that, at any fixed value of $\Delta m^2$, the oscillation amplitudes $B_{e;e}$ and $B_{\mu;\mu}$ are bounded by the upper values $B^0_{e;e}$ and $B^0_{\mu;\mu}$, respectively. The values of $B^0_{e;e}$ and $B^0_{\mu;\mu}$ given by the exclusion plots obtained in the Bugey [4] $\bar{\nu}_e$ disappearance experiment and in the CDHS and CCFR [3] $\bar{\nu}_\mu$ disappearance experiments are small for any value of $\Delta m^2$ in the wide interval $0.3 \lesssim \Delta m^2 \lesssim 10^3 \text{eV}^2$. From Eq. (9) one can see that small upper bounds for $B_{e;e}$ and $B_{\mu;\mu}$ imply that the parameters $|U_{e 3}|^2$ and $|U_{\mu 3}|^2$ can be small or large (i.e., close to one):

\[
|U_{\alpha 3}|^2 \leq a^0_\alpha \quad \text{or} \quad |U_{\alpha 3}|^2 \geq 1 - a^0_\alpha \quad (\alpha = e, \mu),
\]

with

\[
a^0_\alpha = \frac{1}{2} \left(1 - \sqrt{1 - B^0_{\alpha;\alpha}}\right).
\]

\[4\]
Both $a_0^e$ and $a_0^\mu$ are small ($a_0^e \lesssim 4 \times 10^{-2}$ and $a_0^\mu \lesssim 2 \times 10^{-1}$) for any value of $\Delta m^2$ in the range $0.3 \lesssim \Delta m^2 \lesssim 10^3 \text{eV}^2$ (see Fig. 1).

Since large values of both $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ are excluded by the unitarity of the mixing matrix ($|U_{e3}|^2 + |U_{\mu 3}|^2 = 1$), at any fixed value of $\Delta m^2$ there are three regions in the $|U_{e3}|^2 - |U_{\mu 3}|^2$ plane which are allowed by the exclusion plots of SBL disappearance experiments: Region I, with $|U_{e3}|^2 \leq a_0^e$ and $|U_{\mu 3}|^2 \leq a_0^\mu$; Region II, with $|U_{e3}|^2 \leq a_0^e$ and $|U_{\mu 3}|^2 \geq 1 - a_0^\mu$; Region III, with $|U_{e3}|^2 \geq 1 - a_0^e$ and $|U_{\mu 3}|^2 \leq a_0^\mu$.

In Region III $|U_{e3}|^2$ is large and $\nu_e$ has a large mixing with $\nu_3$ and a small mixing with $\nu_1$ and $\nu_2$. Since the squared-mass difference $\Delta m^2_{21}$ is assumed to be responsible for the oscillations of solar neutrinos, a small mixing of the electron neutrino with $\nu_1$ and $\nu_2$ implies that the oscillations of solar $\nu_e$’s are suppressed and the solar neutrino problem cannot be solved by neutrino oscillations. Indeed, the survival probability of solar $\nu_e$’s, $P_{\nu_e \rightarrow \nu_e}$, is bounded by $P_{\nu_e \rightarrow \nu_e}^\text{sun} \geq |U_{e3}|^2$ (see Ref. [18]). If $|U_{e3}|^2 \geq 1 - a_0^e$, we have $P_{\nu_e \rightarrow \nu_e}^\text{sun} \geq 0.92$ at all neutrino energies, which is a bound that is not compatible with the solar neutrino data. Hence, Region III is excluded by solar neutrinos.

The Region I is disfavored by the results of the LSND experiment. Indeed, in Region I we have

$$A_{\mu; e} \leq 4 a_0^e a_0^\mu. \quad (12)$$

This inequality implies that $\nu_\mu \leftrightarrow \nu_e$ transitions in SBL experiments are strongly suppressed. The upper bound obtained with the inequality (12) from the 90% CL exclusion plots of the Bugey [4] $\bar{\nu}_e$ disappearance experiment and of the CDHS and CCFR [7] $\nu_\mu$ disappearance experiments is represented in Fig. 2 by the curve passing trough the circles. The shadowed regions in Fig. 2 are allowed at 90% CL by the results of the LSND experiment. Also shown are the 90% CL exclusion curves found in the BNL E734, BNL E776 and CCFR [5] $\nu_\mu \rightarrow \nu_e$ appearance experiments and in the Bugey experiment. One can see from Fig. 2 that the bounds obtained from the direct experiments on the search for $\nu_\mu \rightarrow \nu_e$ oscillations and the bound (12) obtained in Region I are not compatible with the allowed regions of the LSND experiment [13]. Therefore, we come to the conclusion that the Region I is not favored by the existing experimental data. This is an important result, because the Region I is the only one in which it is possible to have a hierarchy of the elements of the neutrino mixing matrix analogous to the one of the quark mixing matrix.

Having excluded the Regions I and III of the scheme [8], we are left only with the Region II, where $\nu_\mu$ has a large mixing with $\nu_3$, i.e., $\nu_\mu$ (not $\nu_\tau$) is the “heaviest” neutrino.

The scheme [8] does not allow to explain the atmospheric neutrino anomaly with neutrino oscillations. However, it must be noticed that only the fit of the Kamiokande multi-GeV data sample requires a definite value of $\delta m^2$ of the order of $10^{-2} \text{eV}^2$, whereas the Kamiokande sub-GeV data and the IMB and Soudan data can be fitted also with higher values of $\delta m^2$. Hence, in the three neutrino scheme [8] a $\Delta m^2_{31} \simeq 0.3 \text{eV}^2$, which is the lowest value allowed by the results of the
Bugey and LSND experiments, could be responsible also for the oscillations of atmospheric neutrinos [19]. Indeed, the CDHS exclusion curve implies that $B_{\mu,\mu}^0 \simeq 0.7$ at $\Delta m_{31}^2 \simeq 0.3 \text{eV}^2$. For the sub-GeV events we have $P_{\nu_{\mu} \rightarrow \nu_{\mu}}^{\text{atm}} = 1 - \frac{1}{2} B_{\mu,\mu}$. Therefore, we get the lower bound $P_{\nu_{\mu} \rightarrow \nu_{\mu}}^{\text{atm}} \gtrsim 0.65$. One can show [23] that the double-ratio $R = (\mu/e)_{\text{data}}/(\mu/e)_{\text{MC}}$ of sub-GeV muon and electron events in the Kamiokande detector ($(\mu/e)_{\text{MC}}$ is the Monte-Carlo calculated ratio of muon and electron events without neutrino oscillations) satisfies the inequality $R \geq P_{\nu_{\mu} \rightarrow \nu_{\mu}}^{\text{atm}}$. Therefore, for $\Delta m_{31}^2 \simeq 0.3 \text{eV}^2$ we have $R \gtrsim 0.65$, which is compatible with the experimental value $R^{\text{exp}} = 0.60^{+0.06}_{-0.05} \pm 0.05$. At higher values of $\Delta m_{31}^2$ the CDHS exclusion curve gives lower values for $B_{\mu,\mu}^0$, leading to an incompatibility with the Kamiokande sub-GeV data. For example, at $\Delta m_{31}^2 \simeq 0.4 \text{eV}^2$ from the CDHS exclusion curve we have $B_{\mu,\mu}^0 \simeq 0.4$, which yields the lower bound $R \gtrsim 0.80$ that is incompatible with the experimental value.

Another scheme with three neutrinos and a mass hierarchy in which $\Delta m_{21}^2 \simeq 10^{-2} \text{eV}^2$ is responsible for the oscillations of solar and atmospheric neutrinos has been proposed in Ref. [24]. This scheme corresponds to Region I, which is marginally allowed for $\Delta m_{31}^2 \simeq 1.7 \text{eV}^2$, as can be seen from Fig 2. However, the fact that in this scheme the survival probability of solar $\nu_e$’s does not depend on the neutrino energy is at odds with the present solar neutrino data [25].

Let us now consider the schemes with four neutrinos, which provide three independent mass-squared differences and allow to accommodate in a natural way all the
three experimental indications in favor of neutrino oscillations (see also Ref. [26]). We consider first the scheme with four neutrinos and the mass hierarchy

\[
\frac{\mathrm{solar}}{\mathrm{atm}}
\begin{align*}
\hspace{1cm}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
m_1 \ll m_2 \ll m_3 \ll m_4
\end{array}
\end{array}
\end{array}
\end{align*}
\] (13)

The three independent mass-squared differences, \(\Delta m_{21}^2\), \(\Delta m_{31}^2\) and \(\Delta m_{41}^2\), are taken to be relevant for the oscillations of solar, atmospheric and LSND neutrinos, respectively. In the case of scheme (13) we have \(n = r = 4\) and Eqs. (6) and (7) imply that the oscillation amplitudes are given by

\[
A_{\alpha;\beta} = 4 |U_{\alpha 4}|^2 |U_{\beta 4}|^2, \quad B_{\alpha;\alpha} = 4 |U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2) .
\] (14)

In this case, neutrino oscillations in SBL experiments depend on four parameters: \(\Delta m^2 \equiv \Delta m_{31}^2\), \(|U_{e3}|^2\), \(|U_{\mu 3}|^2\) and \(|U_{\tau 3}|^2\). From the similarity of the amplitudes (14) with the corresponding ones given in Eq. (9), it is clear that, replacing \(|U_{e3}|^2\) with \(|U_{\alpha 3}|^2\), we can apply to the scheme (13) the same analysis presented for the scheme (8) with three neutrinos and a mass hierarchy. At any fixed value of \(\Delta m^2\) we have three regions in the \(|U_{e4}|^2 - |U_{\mu 4}|^2\) plane which are allowed by the exclusion plots of SBL disappearance experiments:

- Region I, with \(|U_{e4}|^2 \leq a_e^0\) and \(|U_{\mu 4}|^2 \leq a_{\mu}^0\);
- Region II, with \(|U_{e4}|^2 \leq a_{e}^0\) and \(|U_{\mu 4}|^2 \geq 1 - a_{\mu}^0\);
- Region III, with \(|U_{e4}|^2 \geq 1 - a_e^0\) and \(|U_{\mu 4}|^2 \leq a_{\mu}^0\). The Regions III and I are excluded, respectively, by the solar neutrino problem and by the results of the LSND experiment, for the same reasons discussed in the case of the scheme (8). Furthermore, the purpose of considering the scheme (13) is to have the possibility to explain the atmospheric neutrino anomaly, but this is not possible if the neutrino mixing parameters lie in Region II. Indeed, in Region II \(|U_{\mu 4}|^2\) is large and the muon neutrino has a large mixing with the heaviest massive neutrino, \(\nu_4\), and a small mixing with the light neutrinos \(\nu_1\), \(\nu_2\) and \(\nu_3\). Since the atmospheric neutrino oscillations are assumed to be due to the phase generated by \(\Delta m_{31}^2\), a relatively large mixing of \(\nu_\mu\) with the three light neutrinos is necessary in order to explain the observed deficit of atmospheric muon neutrinos. In Ref. [23] it has been shown quantitatively that the small mixing of \(\nu_\mu\) with \(\nu_1\), \(\nu_2\) and \(\nu_3\) in Region II is incompatible with the atmospheric neutrino data.

Hence, in the framework of scheme (13) all the regions of the mixing parameters are incompatible with the results of neutrino oscillation experiments and we conclude that this scheme is not favored by the experimental data. It is possible to show that, for the same reasons, all possible schemes with four neutrinos and a mass spectrum in which three masses are clustered and one mass is separated from the others by a gap of about 1 eV (needed for the explanation of the LSND data) are not compatible with the results of all neutrino oscillation experiments. Therefore, there are only two possible schemes with four neutrinos which are compatible with the results of all the
neutrino oscillation experiments:

\[(A) \quad \begin{array}{lll}  \text{atm} & \quad \begin{array}{ll} m_1 < m_2 & \ll m_3 < m_4 \end{array} \end{array} \quad \text{and} \quad \begin{array}{ll} \text{solar} & \quad \begin{array}{ll} m_1 < m_2 & \ll m_3 < m_4 \end{array} \end{array} \] (15)

In these two schemes the four neutrino masses are divided in two pairs of close masses separated by a gap of about 1 eV. In scheme A, $\Delta m^2_{21}$ is relevant for the explanation of the atmospheric neutrino anomaly and $\Delta m^2_{43}$ is relevant for the suppression of solar $\nu_e$’s. In scheme B, the roles of $\Delta m^2_{21}$ and $\Delta m^2_{43}$ are reversed.

From Eq. (15), the oscillation amplitudes $B_{\alpha \alpha}$ in the schemes (15), with $n = 4$ and $r = 3$, are given by

$$B_{\alpha \alpha} = 4 c_\alpha (1 - c_\alpha),$$ (16)

with the following definitions of the parameters $c_\alpha$ in the two schemes A and B:

\[(A) \quad c_\alpha \equiv \sum_{k=1,2} |U_{\alpha k}|^2 \quad \text{and} \quad (B) \quad c_\alpha \equiv \sum_{k=3,4} |U_{\alpha k}|^2.\] (17)

The expression (16) for $B_{\alpha \alpha}$ has the same form as the one in Eq. (14), with $|U_{\alpha 4}|^2$ replaced by $c_\alpha$. Therefore, we can apply the same analysis to the results of SBL disappearance experiments as that presented for the case of scheme (13) and we obtain four allowed regions in the $c_e-c_\mu$ plane (now the region with large $c_e$ and $c_\mu$ is not excluded by the unitarity of the mixing matrix, which gives the constraint $c_e + c_\mu \leq 2$): Region I, with $c_e \leq a^0_e$ and $c_\mu \leq a^0_\mu$; Region II, with $c_e \leq a^0_e$ and $c_\mu \geq 1-a^0_\mu$; Region III, with $c_e \geq 1-a^0_e$ and $c_\mu \leq a^0_\mu$; Region IV, with $c_e \geq 1-a^0_e$ and $c_\mu \geq 1-a^0_\mu$. However, following the same reasoning as in the case of scheme (13), one can see that the Regions III and IV are excluded by the solar neutrino data and the Regions I and III are excluded by the results of the atmospheric neutrino experiments [23]. Hence, only the Region II is allowed by the results of all experiments.

If the neutrino mixing parameters lie in Region II, in the scheme A (B) the electron (muon) neutrino is “heavy”, because it has a large mixing with $\nu_3$ and $\nu_4$, and the muon (electron) neutrino is light. Thus, the schemes A and B give different predictions for the effective Majorana mass $\langle m \rangle = \sum_k U^2_{\alpha k} m_k$ in neutrinoless double-beta decay experiments: since $m_3 \simeq m_4 \gg m_1 \simeq m_2$, we have

\[(A) \quad |\langle m \rangle| \leq (1-c_e)m_4 \simeq m_4, \quad (B) \quad |\langle m \rangle| \leq c_e m_4 \leq a^0_e m_4 \ll m_4.\] (18)

Thus, if scheme A is realized in nature the experiments on the search for neutrinoless double-beta decay can reveal the effects of the heavy neutrino masses $m_3 \simeq m_4$. Furthermore, the smallness of $c_e$ in both schemes A and B implies that the electron neutrino has a small mixing with the neutrinos whose mass-squared difference is responsible for the oscillations of atmospheric and LBL neutrinos (i.e., $\nu_1$, $\nu_2$ in scheme A and $\nu_3$, $\nu_4$ in scheme B). Hence, the probability of transitions of atmospheric and LBL electron neutrinos into other states is suppressed [27].
In conclusion, the analysis presented here shows that, if the experimental indications in favor of neutrino oscillations are confirmed, the mixing of leptons is very different from the mixing of quarks.

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