The hybrid meson: new results from the updated $m_g$ and $\alpha_s$ parameters.

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Abstract
We present new results concerning the masses and the decay widths of the most interesting hybrid meson states taking as inputs the gluon mass $m_g$ and the non-perturbative QCD running coupling constant $\alpha_s(0)$ coming from both LQCD and SDE recent estimations.

1 Introduction
Hybrid meson is one of the most promising new species of hadrons allowed by QCD and subject of lot of works both in the theoretical and experimental levels.

The hybrid mesons are studied from different models: lattice QCD\cite{1,2}, flux tube model\cite{3-8}, bag model\cite{9-15}, QCD sum rules\cite{16-28}, constituent gluon models\cite{29-32} and from Effective Hamiltonian model\cite{34-36}. Some of them can perform both estimations of mass and decay widths.

The nature of gluonic field inside hybrid is not yet be clear because the gluon plays a double role: it propagates the interaction between color sources and being itself colored it undergoes the interaction. Whereas, LQCD and Sum rules QCD make no assumptions about it, two important hypothesis can be retained from literature. The first one consider gluonic degrees of freedom as "excitations" of the "flux tube" between quark and antiquark, which leads to the linear potential, that is familiar from quark model (flux-tube model).

The second issue, which are supported by the present work, assumes that hybrid is a bound state of quark-antiquark and a constituent glue which interact through an adequate phenomenological potential. We can adapt our interaction scheme with the idea of confined and confining gluons\cite{37} (In the Landau and Coulomb gauges and in interpolating gauges between them).

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A confining gluons establish an area law behavior of the Wilson loop and the linearly rising interquark confinement. The confined gluons do not propagate over long distances. We can accommodate confined (massive, constituent) gluon in coexistence with an effective quark interaction which is confining[39].

The quark model is still necessary for describing most of the available spectroscopic data and their free parameters are fitted to the experimental values. Further to these parameters (quark masses, potential), our generalized Quark Model with Constituent Glue[30] uses two free parameters: the constituent gluon mass $m_g$ and the decay parameter $\alpha_s$. They must be set otherwise, since no (confirmed) spectroscopic data are available.

Several studies[38], [39]-[41] support the hypothesis that the gluon may develop a dynamical mass which is intrinsically related to an infrared finite gluon propagator[42]. This is consistent with QCD lattice simulations[43]-[50]. From the theoretical point of view, a non-vanishing gluon mass is welcome to regularize infrared divergences and solve some problems related with unitarity. From the phenomenological point of view, a non-vanishing gluon mass is welcome by diffractive phenomena[51] and inclusive radiative decays of $J/\Psi$ and $\gamma$. For the glueball states, color-singlet bound states of gluons, are considered to be fairly massive e.g., about 1.5 GeV for the lowest $0^{++}$ and about 2 GeV for the lowest $2^{++}$, as indicated in lattice QCD calculations[53],[54], a simple constituent gluon picture may be approximately obtained as $M_{GB} \simeq 2m_g$ for the glueball mass $M_{GB}$.

There are many theoretical evidences that the QCD effective charge freezes at small momenta[39],[40], [55]-[63]. The infrared finiteness of the effective charge can be considered as one of the manifestation of the phenomenon of dynamical gluon mass generation[39],[40],[42].

Phenomenology sensitive to infrared properties of QCD gives[64]-[66] $\alpha_s(0) \simeq 0.7 \pm 0.3$. The phenomenological evidences for the strong coupling constant freezing in the infrared (IR) are much more numerous. Models where a static potential is used to compute the hadronic spectra make use of a frozen coupling constant at long distances[67]-[70].

2 Update the parameters of the model

The gluon mass

In earlier estimations[30] we have used $m_g \sim 800$ MeV to the best fit of our results to the ones obtained by lattice calculations of $1^{-+}c\bar{c}$ and $b\bar{b}$ masses. This value is also compatible with those obtained from different works (see table 1).

Recent lattice data[69],[70] estimate $m_g$ around 600 MeV. This value has been already obtained many years ago in the context of Schwinger-Dyson equations (SDEs)[59], this is also compatible with the effective QCD Coulomb gauge Hamiltonian approach[52],[56]. We choose this (process independent) value as an input for our model.
The decay width parameters

The decay of an hybrid state $A$ into two ordinary mesons $B$ and $C$ is represented by the matrix element of the Hamiltonian annihilating a gluon and creating a quark pair (QPC model)\[29\]:

$$\langle BC | H | A \rangle = g f(A, B, C) (2\pi)^3 \delta^3(p_A - p_B - p_C);$$ (1)

where $f(A, B, C)$ is the decay amplitude involving the flavor, the color, the spatial and the (non-relativitic) spin overlaps.

The partial width is given by:

$$\Gamma (A \to BC) = 4\alpha_s |f(A, B, C)|^2 \frac{P_B E_B E_C}{M_A};$$ (2)

where $\alpha_s$ represents the only free parameter in this decay model. We have always chosen $\alpha_s \sim 1$ in our previous works, but now we can use a more convincing values available from recent Schwinger-Dyson equations (SDEs) and Lattice QCD data.

There are two characteristic definitions of the effective charge, frequently employed in the literature. The first definition is obtained within the pinch technique (PT) framework\[39\]-\[40\] (which can be appropriately extended to the Taylor ghost-gluon coupling\[71\],\[72\]). This effective charge, to be denoted by $\alpha_{PT}(q^2)$, constitutes the most direct non-abelian generalization of the familiar concept of the QED effective charge. The second definition of the QCD effective charge, to be denoted by $\alpha_{gh}(q^2)$, involves the ghost and gluon self-energies, in the Landau gauge, and in the kinematic configuration where the well-known Taylor non-renormalization theorem becomes applicable. $\alpha_{gh}(q^2)$ has been employed extensively in lattice studies (see for instance\[73\]-\[76\] and references therein), where the Landau gauge is the standard choice for the simulation of the gluon and ghost propagators, as well as in various investigations based on Schwinger-Dyson equations (SDEs)\[77\]-\[79\]. The two charges are identical not only in the deep UV, where asymptotic freedom manifests itself, but also in the deep IR, where they “freeze” at the same non-vanishing value\[71\].

Using recent (quenched) lattice data on the gluon and ghost propagators, as well as the Kugo-Ojima function, authors of reference\[80\] extract the non-perturbative behavior of QCD effective charges.

They have offered a plausible explanation for the observed discrepancy in the freezing values of the effective charges obtained from the lattice ($\alpha_s(0) \sim 2 - 2.5$\[81\]\[82\]) and those derived from the fitting of various QCD processes, sensitive to non-perturbative physics ($\sim 0.7 \pm 0.3$). They claim that the underlying reason for the discrepancy is the difference in the gauges (Landau vs Feynman) used in the two approaches.

Our decay model is obtained in the Feynman gauge\[29\], so it’s natural to choose $\alpha_s \sim \alpha_{PT}(0) \simeq 0.85$ for $m_g = 600$ MeV\[83\].
3 Results and discussion

We present in table 2, the updated hybrid masse estimates using these recent LQCD constituent gluon masses. The gluon energy $\omega$ appearing in the decay’s formulas, can be set to $M_g \simeq 1 \text{ GeV}$, the energy of the confined gluon. So the old decay widths must be multiplied by the factor:

$$\frac{(\alpha_s)_{\text{New}}}{(\alpha_s)_{\text{Old}}} \cdot \frac{(\omega)_{\text{Old}}}{(\omega)_{\text{New}}} = 0.85 \cdot 0.8 = 0.68$$

The results are summarised in tables 3 à 8.

Note that LQCD\cite{83} gives the 2.2(2) GeV hybrid partial decay widths: $\Gamma_{b_1\pi} = 400 \pm 120 \text{ MeV} > \Gamma_{f_1\pi} = 90 \pm 60 \text{ MeV}$ which are in agreement with our results. The experimental results for $\pi_1(2000)$ are: $\Gamma_{b_1\pi} = 230 \pm 32 \pm 73 \text{ MeV} \lesssim \Gamma_{f_1\pi} = 333 \pm 52 \pm 49 \text{ MeV}$\cite{85}. It’s difficult to reconcile our partial decay width $\Gamma_{1^-+n_{\pi}(2000)-\rightarrow \rho}\pi$ with the experimental one.

The same remark holds for $\Gamma_{1^-+n_{\pi}(1600)-\rightarrow \rho}\pi$ which disagrees with the experimental data $240 \pm 60 \text{ MeV}$\cite{86} and for $\Gamma_{1^-+n_{\pi}(1600)-\rightarrow \rho}\pi$ which is very far from the experimental values $269 \pm 21 \text{ MeV}$\cite{87} and $168 \pm 20 \text{ MeV}$\cite{88}.

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| Authors                  | Method                      | $m_g$     |
|-------------------------|-----------------------------|-----------|
| Parisi, Petronzio       | $J/\psi \to \gamma \chi$   | 800 MeV   |
| Donnachie, Landshoff    | Pomeron parameters          | 687-985 MeV |
| Hancock, Ross           | Pomeron slope               | 800 MeV   |
| Nikolaev et al.         | Pomeron parameters          | 750 MeV   |
| Spiridonov, Chetyrkin   | $\Pi_{\mu\nu}^{m} \cdot (TrG_{\mu\nu})^2$ | 750 MeV |
| Field                   | $J/\psi \to \gamma \chi$   | $0.72 \pm 0.016 \text{GeV}$ |

Table 1. Some estimations of the $m_g$.

| Masse                     | $M_q$ | $M_g \simeq \omega$ |
|---------------------------|-------|---------------------|
| $1^{-+} n\bar{n}g$        | 1.70  | 0.87                |
| $1^{-} c\bar{c}g$         | 4.10  | 1.95                |

Table 2. $1^{-+}$ light and $1^{-}$ charmed hybrid mesons masses.

|                                  | $\Gamma_{\rho\pi}$ | $\Gamma_{K^*K}$ | $\Gamma_{\rho\omega}$ |
|----------------------------------|---------------------|------------------|-----------------------|
| $\rho\pi$                        | $\sim 36 \text{ MeV}$ | $\sim 10 \text{ MeV}$ | $\sim 1 \text{ MeV}$ |
|                                  | $\rho\omega$        | $\rho(1450)$     | $K^*(1410)$           |
|                                  | $\sim 12 \text{ MeV}$ | $\sim 3 \text{ MeV}$ |

Table 3. Partial decay widths of the ($M=1.6$) hybrid in (S+S)-standard mesons.

| $L$ | $\Gamma_{B_0^0}^{-\pi^-}$ | $\Gamma_{B_{-1}^+}^{-\pi^0}$ | $\Gamma_{D^0}^{-\pi^-}$ | $\Gamma_{D^*}^{-\pi^-}$ |
|-----|---------------------------|-------------------------------|--------------------------|--------------------------|
| 0   | 98                        | 294                           | 491                      |                          |
| 1   | 79                        | 60                            | 100                      |                          |
| 2   | 26                        | 20                            | 33                       |                          |

Table 4. Partial decay widths of the ($M=2.0$) hybrid in (S+S)-standard mesons.

| $L$ | $\Gamma_{B_0^0}^{-\pi^-}$ | $\Gamma_{B_{-1}^+}^{-\pi^0}$ | $\Gamma_{D^0}^{-\pi^-}$ | $\Gamma_{D^*}^{-\pi^-}$ |
|-----|---------------------------|-------------------------------|--------------------------|--------------------------|
| 0   | 106                       | 317                           | 528                      |                          |
| 1   | 96                        | 73                            | 122                      |                          |
| 2   | 67                        | 51                            | 85                       |                          |

Table 5. Partial decay widths of the ($M=1.6$) hybrid in (L+S)-standard mesons (in MeV).

| $L$ | $\Gamma_{D^0}^{+D^0}$ | $\Gamma_{D^+}^{+D^-}$ | $\Gamma_{D^0}^{+D^-}$ |
|-----|-----------------------|-----------------------|-----------------------|
| 0   | 88                    | 264                   | 440                   |
| 1   | 92                    | 276                   | 460                   |
| 2   | 97                    | 291                   | 486                   |

Table 6. Partial decay widths of the ($M=2.0$) hybrid in (P+S)-standard mesons (in MeV).

| $L$ | $\Gamma_{D^0}^{+D^0}$ | $\Gamma_{D^+}^{+D^-}$ | $\Gamma_{D^0}^{+D^-}$ |
|-----|-----------------------|-----------------------|-----------------------|
| 0   | 21                    | 63                    | 1                     |
| 1   | 0                     | 0                     | 0                     |
| 2   | 34                    | 251                   | 17                    |

Table 7. Partial decay widths of the ($M=4.26$) hybrid in (S+S)-standard mesons (in MeV).
\[ \Gamma_{D_1(2420)D^\pm} = \Gamma_{D_1(2420)D^0} \simeq \Gamma_{D_1^+(2420)D^\mp} = \Gamma_{D_1^+(2420)D^\pm} \sim 78^{+312} \]

Table 8. Partial decay widths of the \((M=\pm 4.3)\) hybrid in \((P+S)\)-standard mesons (in \(MeV\)).
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1 Introduction
Hybrid meson is one of the most promising new species of hadrons allowed by QCD and subject of lot of works both in the theoretical and experimental levels.

The hybrid mesons are studied from different models: lattice QCD[1,2], flux tube model[3-8], bag model[9-15], QCD sum rules[16-28], constituent gluon models[29-32] and from Effective Hamiltonian model[34-36]. Some of them can perform both estimations of masses and decay widths.

The nature of gluonic field inside hybrid is not yet be clear because the gluon plays a double role: it propagates the interaction between color sources and being itself colored it undergoes the interaction. Whereas, LQCD and QCD Sum rules make no assumptions about it, two important hypothesis can be retained from literature. The first one consider gluonic degrees of freedom as ”excitations” of the ”flux tube” between quark and antiquark, which leads to the linear potential, that is familiar from quark model (flux-tube model).

The second issue, which are supported by the present work, assumes that hybrid is a bound state of quark-antiquark and a constituent glue which interact through an adequate phenomenological potential. We can adapt our interaction scheme with the idea of confined and confining gluons[37] (In the Landau and Coulomb gauges and in interpolating gauges between them).

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A confining gluons establish an area law behavior of the Wilson loop and the linearly rising interquark confinement. The confined gluons do not propagate over long distances. We can accommodate confined (massive, constituent) gluon in coexistence with an effective quark interaction which is confining.

The quark model is still necessary for describing most of the available spectroscopic data and their free parameters are fitted to the experiment. In addition, our generalized Quark Model with Constituent Glue uses two free parameters: the constituent gluon mass $m_g$ and the decay parameter $\alpha_s$ (the effective quark-gluon vertex coupling). They must be set otherwise, since no (confirmed) hybrid mesons data are available.

Several studies support the hypothesis that the gluon may develop a dynamical mass which is intrinsically related to an infrared finite gluon propagator. This is consistent with QCD lattice simulations. From the theoretical point of view, a non-vanishing gluon mass is welcome to regularize infrared divergences and solve some problems related with unitarity. From the phenomenological point of view, a non-vanishing gluon mass is welcome by diffractive phenomena and inclusive radiative decays of $J/\Psi$ and $\gamma$. For, the glueball states, color singlet bound states of gluons, are considered to be fairly massive e.g., about 1.5 GeV for the lowest $0^{++}$ and about 2 GeV for the lowest $2^{++}$, as indicated in lattice QCD calculations. A simple constituent gluon picture may be approximately obtained as $M_{GB} \simeq 2m_g$ for the glueball mass $M_{GB}$.

There are many theoretical evidences that the QCD effective charge $\alpha_s$ freezes at small momenta. The infrared finiteness of the effective charge can be considered as one of the manifestation of the phenomenon of dynamical gluon mass generation.

Phenomenology sensitive to infrared properties of QCD gives $\alpha_s(0) \simeq 0.7 \pm 0.3$. The phenomenological evidences for the strong coupling constant freezing in the infrared (IR) are much more numerous. Models where a static potential is used to compute the hadronic spectra make use of a frozen coupling constant at long distances.

## 2 Update the gluon mass

In earlier estimations, we have used $m_g \sim 800$ MeV to the best fit of our results to the ones obtained by lattice calculations of $1^{-+}c\bar{c}$ and $b\bar{b}$ masses. This is also compatible with other works (table 1).

| Authors           | Method          | $m_g$     |
|-------------------|-----------------|-----------|
| Parisi, Petronzio | $J/\psi \rightarrow \gamma X$ | 800 MeV   |
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| Spiridonov, Chetyrkin | $\Pi_{\mu\nu}^{em} \langle \text{Tr} G_{\mu\nu}^{em} \rangle$ | 750 MeV   |
| Field             | $J/\psi \rightarrow \gamma X$ | $0.721_{-0.068}^{+0.016}$ GeV |

Table 1. Some estimations of the $m_g$ around 800 MeV
Recent lattice data\(^{49,50}\) estimate \(m_g\) around 600 MeV. This value has been already obtained many years ago in the context of Schwinger-Dyson equations (SDEs)\(^{39}\), this is also compatible with the effective QCD Coulomb gauge Hamiltonian approach\(^{35,36}\). We choose this value as an input for our model, described by the binding Hamiltonian

\[
H = \sum_{i=q,\bar{q},g} \left( \frac{\tilde{p}_i^2}{2M_i} + \frac{M_i}{2} + \frac{m_i^2}{2M_i} \right) + V_{eff}
\]

\(V_{eff}\) is the average over the color space of chromo-spatial potential:

\[
V_{eff} = \left\langle V \right\rangle_{color} = \sum_{i<j=1}^{N} \alpha_{ij} v(r_{ij}) \tag{2}
\]

where \(v(r_{ij})\) is the phenomenological potential term.

We take a potential which has the form:

\[
v(r_{ij}) = v_{CL}(r_{ij}) + v_{SD}(r_{ij}) \tag{3}\]

where the QCD-motivated "Coulomb.+Linear" term reads:

\[
v_{CL}(r_{ij}) = -\frac{\alpha_s}{r_{ij}} + \sigma r_{ij} + c \tag{4}\]

The spin-dependent term can split into Spin-Spin, Spin-Orbit and Tensor terms:

\[
v_{SD} = v_{SS} + v_{SO} + v_{T} \tag{5}\]

\[
(v_{SS})_{ij} = \frac{8\pi\alpha_s}{3M_iM_j \sqrt{2}} \sigma_i^2 \exp(-\sigma_i^2 r_{ij}^2) \frac{s_i \cdot s_j}{\sqrt{2}} \ (\text{light sector}) \tag{6}\]

\[
(v_{SO})_{ij} = \frac{\alpha_s}{2r_{ij}} \left( \frac{s_i \cdot r_{ij} \times p_i}{M_i^2} - \frac{s_j \cdot r_{ij} \times p_j}{M_j^2} \right) - \frac{2s_i \cdot r_{ij} \times p_j - 2s_j \cdot r_{ij} \times p_i}{M_iM_j} \tag{7}\]

\[
(v_{T})_{ij} = \frac{\alpha_s}{M_iM_j r_{ij}^3} (3 s_i \cdot \tilde{r}_{ij} s_j \cdot \tilde{r}_{ij} - s_i \cdot s_j) \tag{8}\]

Using the variational method, one can find the mass and the wavefunction of any \(J^{PC}\) hybrid state\(^{39}\).

The energy of the constituent gluon inside the hybrid \(\omega \simeq M_g\) can be evaluated using the condition

\[
\frac{\partial E}{\partial M_i} = 0 \tag{9}\]

The value of \(\omega\) and the hybrid wavefunctions are used to evaluate the spatial overlaps in the decay width calculations.
3 Update the decay width parameters

The decay of an hybrid state $A$ into two ordinary mesons $B$ and $C$ is represented by the matrix element of the Hamiltonian annihilating a gluon and creating a quark pair (QPC model)\cite{29}:

$$\langle BC | H | A \rangle = g f(A, B, C) (2\pi)^3 \delta_3 (p_A - p_B - p_C); \quad (10)$$

where $f(A, B, C)$ is the decay amplitude involving the flavor, the color, the spatial and the (non-relativistic) spin overlaps. The spatial overlap is given by:

$$I = \int \int \frac{d\vec{p} \, d\vec{k}}{(2\pi)^6 \sqrt{2\omega}} \Psi_{q_1 q_2 m}^{l m_{q_1} l_{q_2} m_{q_2}} (\vec{P}_B - \vec{p}, \vec{k}) \times \Psi_{\bar{q}_1 \bar{q}_2 m_B}^{l m_B} \Psi_{q_3 q_4 m_C}^{l m_{q_3} l_{q_4} m_{q_4}} (\vec{P}_2 - \vec{p}_1) Y_{l}^{m} \ast (\Omega_B) d\Omega_B, \quad (11)$$

and the partial width by:

$$\Gamma (A \rightarrow BC) = 4\alpha_s |f (A, B, C)|^2 \frac{P_B E_B E_C}{M_A}; \quad (12)$$

where $\alpha_s$ represents the infrared quark-gluon vertex coupling. We have always chosen $\alpha_s \approx 1$ (and $\omega \approx 0.8 \text{ GeV}$) in our previous works, but now we can use a more convincing values available from recent Schwinger-Dyson equations (SDEs) and Lattice QCD data.

There are two characteristic definitions of the effective charge, frequently employed in the literature. The first definition is obtained within the pinch technique (PT) framework\cite{39,40} (which can be appropriately extended to the Taylor ghost-gluon coupling\cite{71,72}). This effective charge, to be denoted by $\alpha_{PT}$, constitutes the most direct non-abelian generalization of the familiar concept of the QED effective charge. The second definition of the QCD effective charge, to be denoted by $\alpha_{gh}$, involves the ghost and gluon self-energies, in the Landau gauge, and in the kinematic configuration where the well-known Taylor non-renormalization theorem becomes applicable. $\alpha_{gh}$ has been employed extensively in lattice studies (see for instance \cite{73,74,75} and references therein), where the Landau gauge is the standard choice for the simulation of the gluon and ghost propagators, as well as in various investigations based on Schwinger-Dyson equations (SDEs)\cite{77,78}. The two charges are identical not only in the deep UV, where asymptotic freedom manifests itself, but also in the deep IR, where they “freeze” at the same non-vanishing value\cite{71}.

Using recent (quenched) lattice data on the gluon and ghost propagators, as well as the Kugo-Ojima function, authors of reference \cite{80} extract the non-perturbative behavior of QCD effective charges.

They have offered a plausible explanation for the observed discrepancy in the freezing values of the effective charges obtained from the lattice ($\alpha_s(0) \sim 2 - 2.5\pm0.5$\cite{81,82}) and those derived from the fitting of various QCD processes, sensitive to non-perturbative physics ($\sim 0.7\pm0.3$). They claim that the underlying reason
for the discrepancy is the difference in the gauges (Landau vs Feynman) used in the two approaches.

Since our decay model is obtained in the Feynman gauge \[29\], it’s natural to choose \(\alpha_s \approx \alpha_{\text{PT}}(0)\) corresponding to the pinch technique gluon propagator, i.e. the background field propagator calculated in the Feynman gauge. For \(m_g = 600 \text{ MeV}\) we have \(\alpha_s \approx \alpha_{\text{PT}}(0) \approx 0.85\) \[23\].

We present in table 2, the updated hybrid masses using the recent LQCD constituent gluon mass \(m_g = 600 \text{ MeV}\). The gluon energy \(\omega\) appearing in the decay’s formulas, can be set to \(M_g \approx 1 \text{ GeV}\), the energy of the confined gluon.

| \(1^{-+} m\bar{q}g\) | \(M_g\) | \(M_g \approx \omega\) |
|-----------------|------|------|
| 1.70            | 0.87 | 1.08 |
| 4.10            | 1.95 | 1.00 |

Table 2. \(1^{-+}\) light and \(1^{--}\) charmed hybrid mesons masses.

Since \(\Gamma(A \rightarrow BC) \sim \frac{\alpha_s}{\omega}\), the old decay widths must be multiplied by the factor:

\[
\frac{(\alpha_s)_{\text{New}}}{(\alpha_s)_{\text{Old}}} \frac{(\omega)_{\text{Old}}}{(\omega)_{\text{New}}} = 0.85 \cdot 0.8 = 0.68
\]

The results are summarized in tables 3-8.

| \(\Gamma_{\rho\pi}\) | 36 \(\text{MeV}\) |
| \(\Gamma_{K^*K}\)  | 10 \(\text{MeV}\) |
| \(\Gamma_{\rho\omega}\) | 1 \(\text{MeV}\) |

Table 3. Partial decay widths of the \((M=1.6)\) hybrid in \((S+S)\)-standard mesons.

| \(\Gamma_{\rho\pi}\) | 47 \(\text{MeV}\) |
| \(\Gamma_{K^*K}\)  | 36 \(\text{MeV}\) |
| \(\Gamma_{\rho\omega}\) | 21 \(\text{MeV}\) |
| \(\Gamma_{\rho(1450)\pi}\) | 12 \(\text{MeV}\) |
| \(\Gamma_{K^*(1410)K}\) | 3 \(\text{MeV}\) |

Table 4. Partial decay widths of the \((M=2.0)\) hybrid in \((S+S)\)-standard mesons.

| \(L\) | 0 | 1 | 2 |
|------|---|---|---|
| \(\Gamma_{b^+\pi^-} \approx \Gamma_{b^-\pi^0}\) | 98 | 294 | 491 |
| \(\Gamma_{f^0(1285)\pi^-}\) | 79 | 60 | 100 |
| \(\Gamma_{f^0(1420)\pi^-}\) | 26 | 20 | 33 |

Table 5. Partial decay widths of the \((M=1.6)\) hybrid in \((L+S)\)-standard mesons \((\text{in MeV})\).

| \(L\) | 0 | 1 | 2 |
|------|---|---|---|
| \(\Gamma_{b^+\pi^-} \approx \Gamma_{b^-\pi^0}\) | 106 | 317 | 528 |
| \(\Gamma_{f^0(1285)\pi^-}\) | 96 | 73 | 122 |
| \(\Gamma_{f^0(1420)\pi^-}\) | 67 | 51 | 85 |

Table 6. Partial decay widths of the \((M=2.0)\) hybrid in \((P+S)\)-standard mesons \((\text{in MeV})\).
### Table 7: Partial decay widths of the \((M=4.26)\) hybrid in \((S+S)\)-standard mesons (in MeV).

| \(L\)   | 0      | 1      | 2      |
|---------|--------|--------|--------|
| \(\Gamma_{D^0 D^0}\) | 88     | 264    | 440    |
| \(\Gamma_{D^+ D^-}\)   | 92     | 276    | 460    |
| \(\Gamma_{D^+ D^-}\)   | 97     | 291    | 486    |

Note that \(LQCD^{[83]}\) gives the \(2(2)\) GeV hybrid partial decay widths: \(\Gamma_{b_1 \pi} = 400 \pm 120 \text{ MeV} > \Gamma_{f_1 \pi} = 90 \pm 60 \text{ MeV}\) which are in agreement with our results. The experimental results for \(\pi_1(2000)\) are: \(\Gamma_{b_1 \pi} = 333 \pm 52 \pm 49 \text{ MeV}^{[85]}\). It’s difficult to reconcile our partial decay width \(\Gamma_{1^-+ n\pi(2000)\to f_1 \pi}\) with the experimental one.

The same remark holds for \(\Gamma_{1^-+ n\pi(1600)\to f_1 \pi}\) which disagrees with the experimental data \(240 \pm 60 \text{ MeV}^{[85]}\) and for \(\Gamma_{1^-+ n\pi(1600)\to \rho \pi}\) which is very far from the experimental values \(269 \pm 21 \text{ MeV}^{[87]}\) and \(168 \pm 20 \text{ MeV}^{[88]}\).

### Table 8. Partial decay widths of the \((M=4.3)\) hybrid in \((P+S)\)-standard mesons (in MeV).

| \(S\)   | 0      | 1      | 2      |
|---------|--------|--------|--------|
| \(\Gamma_{D^0(2420)D^0} = \Gamma_{D^0(2420)D^0} = \Gamma_{D^0(2420)D^0} = \Gamma_{D^0(2420)D^0}\) | \(78 = 312.4\) |

4 Conclusion

In the framework of the quark model with constituent glue, and taking into account the new values of \(m_g\) and \(\alpha_s(0)\) parameters, available from recent LQCD and DSE calculations, we have updated our previous estimations\[^{[30]}\] of masses and decay widths of the more interesting hybrid meson states.

We found that \(M_{1^-+ n\pi g} \sim 1.7 \text{ GeV}\) and the decay widths \(\Gamma_{1^-+ n\pi g(1600)\to \rho \pi}\) and \(\Gamma_{1^-+ n\pi g(1600)\to f_1 \pi}\) are in disagreement with the experimental ones. So we disregard the hypothesis of the hybrid meson structure of the candidate \(\pi_1(1600)\).

The estimated mass of the \(1^-+ n\pi g\) GE-hybrid is around 1.9 GeV, but we can’t interpret this state as the candidate \(\pi_1(2000)\), because it’s difficult to reconcile our partial decay widths with the experimental results.

In the charm sector, the \(1^-+ c\bar{c}g\) is estimated to have a mass around 4.1 GeV which is consistent with the candidate \(Y(4260)\), and the partial decay widths show that the \(1^-+ c\bar{c}g\) can generate observable resonances in both channels ”S+S” and ”S+P”. Note that the mixing of \(1^-+ c\bar{c}g\) and the corresponding \(1^-+ c\bar{c}\) is excluded\[^{[97]}\].

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