Dynamical Theory of Disoriented Chiral Condensates at QCD Phase Transition

Sang Pyo Kim†

Department of Physics, Kunsan National University, Kunsan 573-701, Korea

(Dated: June 23, 2018)

Abstract

We apply the canonical quantum field theory based on the Liouville-von Neumann equation to the nonequilibrium linear sigma model. Particular emphasis is put on the mechanism for domain growth of disoriented chiral condensates due to long wavelength modes and its scaling behavior. Scattering effects, decoherence and emergence of order parameter are also discussed beyond the Hartree approximation.

† Electronic address: sangkim@kunsan.ac.kr
I. INTRODUCTION

The high density and temperature state of hadronic matter consists of quark-gluon plasma and would have occurred in the early universe or may be realizable in heavy ion collision experiments. In QCD with two massless quarks, the chiral symmetry $SU(2)_L \times SU(2)_R$ at high temperatures spontaneously breaks down to $SU(2)_{L+R}$ at lower temperatures by the quark-antiquark condensate $\langle \bar{q}_L^i q_R^j \rangle = \sigma \delta_{ij}^{\tau} \cdot \tau^i_j$, an order parameter. The field $\phi_a = (\sigma, \vec{\pi})$ respects $O(4)$ rotations and thus belongs to the universality class of a four component isotropic Heisenberg antiferromagnet \[1\]. The effective theory of the QCD phase transition is described by the linear sigma model [2] or equally by a mean-field theory of Polyakov loops and/or by the glueball fields for the hadronic states of QCD [3].

In QCD the $SU(2)$ phase transition would probably be a second order whereas the $SU(3)$ phase transition would be a weakly first order. In relativistic heavy ion collisions, hot and high dense regions are made, where the chiral symmetry would be restored, and as the regions cool, the quark-gluon plasma would undergo the second order phase transition. In second order phase transitions, as temperature approaches a critical temperature, the correlation length cannot grow indefinitely and must be frozen due to causality [4]. However, the cooling process may be fast enough for the kinematic time scale of quench to be smaller than thermal relaxation time scale. The rapid expansion of the quark-gluon plasma enforces a rapid quench and results in a phase transition far from equilibrium (nonequilibrium). In such a nonequilibrium phase transition domains (regions of misaligned vacuum) develop due to the instability of long wavelength modes.

Another possible candidate for the QCD phase transition is Centauri events in comic rays [5]. Anomally large event-by-event fluctuations have been observed in the ratio of charged to neutral pions and thus require a new theory or interpretation. The disoriented chiral condensate (DCC) of classical pion fields was introduced as one of the proposed mechanisms for coherent emission of pions from a large domain [6]. If the QCD phase transition proceeds in equilibrium, all directions of $\vec{\pi}$ are equally probable and domains of size $1/T_c$ cannot explain the anomalous production of pions of a certain kind. Rajagopal and Wilczek advocated the nonequilibrium QCD phase transition through a quench for DCC domain growth, where unstable long wavelength modes of pions are exponentially amplified [2].
In this talk, we adopt a recently introduced canonical field method to elaborate the nonequilibrium QCD phase transition based on the linear sigma model. The quench process is imitated by a mass squared that changes signs during a finite quench time \([7, 8, 9]\). We particularly focus on the dynamical process of domain formation from long wavelength modes of pions growing exponentially during an instability period and on the scaling behavior of domains size. The scaling behavior of domains has been found through simulations in condensed matter systems and cosmology. We further discuss the effects of direct scatterings among each pion field modes on domains size and on decohering long wavelength modes and emergence of an order parameter.

II. NONEQUILIBRIUM LINEAR SIGMA MODEL

The effective theory of QCD phase transition with two massless quarks is described by the quark-antiquark condensate \(\langle \bar{q}q \rangle\), which, in turn, defines \(\phi_a = (\sigma, \pi)\), \(\pi\) being the pion field. The quark mass \(m_q\) provides a symmetry breaking external field to an, otherwise, \(O(4)\) symmetric field theory at high temperatures. To include a quench, we may write the linear sigma model \([10]\) in the form

\[
L = \int d^3x \left[ \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{\lambda}{4} (\phi^a \phi_a)^2 - \frac{1}{2} m^2(t) \phi^a \phi_a + H\sigma \right].
\]  

(1)

The time-dependent quench process has been imitated by the symmetry breaking mass squared \(m^2(t)\), which changes signs during the phase transition. In this sense Eq. (1) is the nonequilibrium linear sigma model. As \(H\) does not affect much the instability of long wavelength modes during the phase transition, we assume \(H = 0\) for simplicity reason.

To illustrate how nonequilibrium phase transitions in general affect formation of domains, we consider a simple field model with a quench time scale \([7]\). The simple model motivated by the nonequilibrium linear sigma model is given by the potential

\[
V(\phi_a) = \frac{m^2(t)}{2} \phi^a \phi_a + \frac{\lambda}{4} (\phi^a \phi_a)^2,
\]  

(2)

with the mass squared

\[
m^2(t) = m^2_1 - m^2_0 \tanh \left( \frac{t}{\tau} \right), \quad (m^2_0 > m^2_1).
\]  

(3)

In the limit of zero quench time \((\tau = 0)\), we obtain the instantaneous (sudden) quench. Now finite temperature field theory \([11]\) cannot be applied to this model when the quench
time scale $\tau$ is smaller than relaxation time scale. Further, in the second order phase transitions, long wavelength modes grow exponentially during rolling over the barrier. Finite temperature field theory does not properly take into account dynamical processes of phase transitions.

One therefore needs some nonequilibrium quantum field theory when the kinematic time scale is smaller than thermal relaxation time scale so that finite temperature field theory cannot be applied. There are several methods for nonequilibrium quantum fields such as the closed-time path integral [12] and the functional Schrödinger-picture [13]. Recently there has been developed a canonical method based on the quantum Liouville-von Neumann equation, which provides all time-dependent Fock states at the leading order [7, 8]. The new canonical method is equivalent, at leading order, to the time-dependent Hartree approximation, but it can go beyond the Hartree approximation since any perturbation method can be readily applied to these Fock states.

As a nonlinear theory, the linear sigma model has defied yet any nonperturbative solution in a closed form. We can, at best, rely on perturbation methods. The Hartree approximation, though being a perturbation scheme, includes some part of nonperturbative effects at the lowest order [14, 15]. In the Hartree approximation, dividing the $\sigma$ field into a background field $\phi(t)$ and its quantum fluctuation $\chi(x, t)$, and using the Hartree factorization [14], we obtain the truncated Hamiltonian for the linear sigma model

$$H_0 = \int d^3x \left[ \frac{\pi^2}{2} + \frac{(\nabla\phi)^2}{2} + \frac{(\nabla\pi)^2}{2} + h_\phi(t)\chi + \frac{m_\chi(t)}{2} \chi^2 + \frac{m_\pi(t)}{2} \pi^2 \right],$$

where the effective couplings are

$$h_\phi(t) = \phi(t)[m^2(t) + 4\lambda\phi^2(t) + 4\lambda\langle\pi^2\rangle],$$

$$m_\chi(t) = m^2(t) + 4\lambda\phi^2(t) + 4\lambda\langle\pi^2\rangle,$$

$$m_\pi(t) = m^2(t) + 12\lambda\phi^2(t) + 4\lambda\langle\pi^2\rangle.$$  

III. DCC DOMAIN GROWTH

Quantum dynamics of DCC can be further approximated by an exactly solvable model now motivated by the linear sigma model (1) or the Hartree approximation (4). The model Hamiltonian for the pion field $\phi_a$ is [16]

$$H_0(t) = \int d^3x \left[ \frac{1}{2} \pi^a \pi_a + \frac{1}{2} (\nabla\phi^a \cdot \nabla\phi_a)^2 + \frac{1}{2} m^2(t) \phi^a \phi_a \right].$$
Note that all pion fields are decoupled from each other since the nonlinear term is neglected at this moment. In terms of the Fourier cosine and sine modes
\[
\phi_k^{(+)}(t) = \frac{1}{2}[\phi_k(t) + \phi_{-k}(t)], \quad \phi_k^{(-)}(t) = \frac{i}{2}[\phi_k(t) - \phi_{-k}(t)],
\]
and with a compact notations \(\alpha = \{(\pm), k\}\), the Hamiltonian becomes a sum of decoupled time-dependent oscillators
\[
H_0(t) = \sum_{\alpha a} \frac{1}{2} \pi_{aa}^2 + \frac{1}{2} \omega_\alpha^2(t) \phi_{aa}^2, \quad (\omega_\alpha^2(t) = k^2 + m^2(t)).
\]
Then quantum states of the pion field are found by the time-dependent creation and annihilation operators [7, 8]
\[
\hat{a}_{aa}^\dagger(t) = -i[\varphi_{aa}(t)\hat{\pi}_{aa} - \dot{\varphi}_{aa}(t)\hat{\phi}_{aa}],
\hat{a}_{aa}(t) = i[\varphi^*_{aa}(t)\hat{\pi}_{aa} - \dot{\varphi}^*_{aa}(t)\hat{\phi}_{aa}],
\]
where \(\hat{\pi}_{aa}\) and \(\hat{\phi}_{aa}\) are Schrödinger operators. Note that these operators do not diagonalize the Hamiltonian (8), but satisfy the Liouville-von Neumann equations
\[
i \frac{\partial}{\partial t} \hat{a}_{aa}^\dagger(t) + [\hat{a}_{aa}^\dagger(t), \hat{H}_{aa}(t)] = 0, \quad i \frac{\partial}{\partial t} \hat{a}_{aa}(t) + [\hat{a}_{aa}(t), \hat{H}_{aa}(t)] = 0,
\]
which lead to mean-field equations for the auxiliary field variables
\[
\ddot{\varphi}_{aa}(t) + \omega_\alpha^2(t)\varphi_{aa}(t) = 0.
\]
In fact, these operators satisfy the equal time commutation relations
\[
[\hat{a}_{aa}(t), \hat{a}_{b\beta}^\dagger(t)] = \delta_{ab}\delta_{\alpha\beta},
\]
with the aid of the Wronskian condition
\[
\varphi^*_{aa}\varphi_{aa} - \varphi^*_{aa}\varphi_{aa} = i.
\]
The Fock space for each pion field mode consists of number states defined as
\[
\hat{N}_{aa}(t)|n_{aa}, t\rangle_0 \equiv \hat{a}_{aa}^\dagger(t)\hat{a}_{aa}(t)|n_{aa}, t\rangle_0 = n_{aa}|n_{aa}, t\rangle_0.
\]
We should note that these are exact quantum states of the time-dependent Schrödinger equation for the Hamiltonian (6) or (8). The quantum state of the pion field itself is then
a product of each mode state for each pion field. Of a particular interest is the Gaussian vacuum state of the pion field

$$|0, t\rangle_0 = \prod_{aa} |0_{aa}, t\rangle_a.$$  \hspace{1cm} (15)

Now the Green function for the pion field is simply given by

$$G_0(x, t; x', t') = \prod_{aa} G_{0aa}(\phi_{aa}, t; \phi'_{aa}, t'),$$  \hspace{1cm} (16)

where the \((aa)\)-mode Green function takes the form

$$G_{0aa}(\phi_{aa}, t; \phi'_{aa}, t') = \sum_{n_{aa}} \langle \phi_{aa} | n_{aa}, t\rangle_0 \langle n_{aa}, t' | \phi'_{aa} \rangle.$$  \hspace{1cm} (17)

To study formation of domains during a nonequilibrium quench process, we consider the smooth finite quench (3). The free field theory (6) is then exactly solvable [7], and is a good approximation for the linear sigma model as long as \(|m^2(t)|\) is larger than \(|\phi(t)|\) and \(\langle \vec{\pi} \rangle^2 = \sum_{b\beta} \varphi_b^\dagger \varphi_{b\beta} \). Far before the phase transition, each mode is stable and oscillates around the true vacuum with the solution

$$\varphi_{aa\alpha}(t) = \frac{1}{\sqrt{2\omega_{aa\alpha}}} e^{-i\omega_{aa\alpha} t}, \quad \omega_{aa\alpha} = \sqrt{k^2 + m^2_0}.$$  \hspace{1cm} (18)

The two-point correlation function for each component of pion field is the Green function at equal times

$$G_{0a}(x, x', t) = \langle \hat{\phi}_a(x, t) \hat{\phi}_a(x', t) \rangle_0 = G_{0a}(x, t; x', t)$$  \hspace{1cm} (19)

with respect to the Gaussian vacuum or thermal equilibrium.

On the other hand, after the phase transition \((m^2 = -m^2_f = -(m^2_0 - m^2_1))\), the long wavelength modes with \(k < m_f\) become unstable and exponentially grow, whereas short wavelength modes with \(k > m_f\) are still stable and oscillate around the false vacuum. Far after the phase transition, the unstable long wavelength modes have the asymptotic solutions

$$\varphi_{aa\alpha}(t) = \frac{\mu_k}{\sqrt{2(m^2_f - k^2)^{1/2}}} e^{(m^2_f - k^2)^{1/2} t} + \frac{\nu_k}{\sqrt{2(m^2_f - k^2)^{1/2}}} e^{-(m^2_f - k^2)^{1/2} t}.$$  \hspace{1cm} (20)

Here \(\mu_k\) and \(\nu_k\) depend on the quench process and satisfy the relation \(|\mu_k|^2 - |\nu_k|^2 = 1\). The correlation function is then dominated by the exponentially growing part

$$G_{0af}(x, t; x', t) \simeq \int_0^{m_f} \frac{d^3 k}{(2\pi)^3} e^{i k \cdot (x - x')} |\mu_k|^2 e^{2 \sqrt{m^2_f - k^2} t} \frac{\mu_k^2}{2(k^2 - m^2_f)}.$$  \hspace{1cm} (21)
Using the exact solutions [7], we obtain the two-point thermal correlation function for each pion field at the intermediate stage of the quench \((-\tau < t < \tau)\)

\[ G_{0aT}(r, t) \simeq G_{0aT}(0, t) \sin\left(\frac{\sqrt{\tau}r}{m_0} \right) \exp\left(-\frac{r^2}{8\sqrt{\tau}m_0} \right). \] (22)

It follows that domains obey a scaling relation for the correlation length

\[ \xi_D(t) = 2\left(\frac{2\tau t}{m_0^2}\right)^{1/4}. \] (23)

The power 1/4 has been found in numerical simulations [17]. At the later stage far after the quench \((t \gg \tau)\), domains still show the Cahn-Allen scaling behavior but with a different power

\[ \xi_D(t) = 2\left(\frac{2\tilde{t}}{m_f^2}\right)^{1/2}, \quad \tilde{t} = t - \frac{\tau^3}{8}[\zeta(3) - 1](m_i^2 + m_f^2). \] (24)

The scaling relation for the instantaneous quench is obtained by letting \(\tau = 0\) in Eq. (24). A kind of resonance has also been observed in the correlation function with simple poles at

\[ \tau = \frac{n}{(m_f^2 - k^2)^{1/2}}, \quad (n = 1, 2, 3, \ldots). \] (25)

This structure implies certain adjusted quench rates \(\tau\) may lead to sufficiently large domains [7].

**IV. SCATTERING, DECOHERENCE AND ORDER PARAMETER**

The tree level approximation in Sec. 3 does take into account neither any interaction among different modes of each pion field nor the interaction between pions. Similarly the Hartree approximation includes only mean-field effects among modes and pion fields and thus neglects any direct scatterings among modes. To go beyond the Hartree approximation we may use the formalism in Ref. 8. The wave functional for the Schrödinger equation can be expressed in terms of the Green function (kernel or propagator) as

\[ \Psi(x, t) = \int G(x, t; x_0, t_0)\Psi(x_0, t_0)d\mathbf{x}_0dt_0. \] (26)

As the linear sigma model is nonlinear, we use a perturbation method. We divide the Hamiltonian

\[ H(t) = H_0(t) + \lambda H_P(t), \] (27)
into an exactly solvable (quadratic) Gaussian and a perturbation part

$$H_0 = \frac{1}{2}\pi_\phi^a\pi_\phi^a + \frac{1}{2}(\nabla\phi^a \cdot \nabla\phi^a)^2 + \frac{1}{2}(m^2 + 9\lambda\langle\phi^b\phi^b\rangle)\phi^a\phi^a,$$

$$H_P = \frac{1}{4}(\phi^a\phi^a)^2 - \frac{9}{2}(\phi^b\phi^b)\phi^a\phi^a. \tag{28}$$

Then in terms of the Green function for $\hat{H}_0$ in Sec. 3,

$$\left(i\frac{\partial}{\partial t} - \hat{H}_0(x, t)\right)G_0(x, t; x', t') = \delta(x - x')\delta(t - t'), \tag{29}$$

we write the wave functional as

$$\Psi(x, t) = \Psi_0(x, t) + \lambda \int G_0(x, t; x', t')\hat{H}_P(x', t')\Psi(x', t')dx'dt', \tag{30}$$

and finally obtain the wave functional of the form

$$\Psi(1) = \Psi_0(1) + \lambda \int G_0(1, 2)\hat{H}_P(2)\Psi_0(2)$$

$$+ \lambda^2 \int \int G_0(1, 2)\hat{H}_P(2)G_0(2, 3)\hat{H}_P(3)\Psi_0(3) + \cdots, \tag{31}$$

where $(i)$ denotes $(x_i, t_i)$ and $\Psi_0$ is a wave functional for $\hat{H}_0$. This method goes beyond the Hartree approximation and gives us the wave functional in a series of $\lambda$

$$\Psi(x, t) = \Psi_0(x, t) + \sum_{n=1}\lambda^n\Psi_0^{(n)}. \tag{32}$$

The term $\Psi_0^{(n)}$ comes from $n$th order correction of $H_P$.

We now give a few remarks on the effects on DCC formation of the non-Gaussian (beyond the Hartree) approximation. First, the nonlinear correction due to $\Psi_0^{(n)}$ enhances the correlation length for domains by a factor \cite{9}

$$\frac{\xi_D}{\xi_D} = (2n + 1)^{1/2}. \tag{33}$$

Higher order correction terms begin to grow provided that the duration of instability is long enough before crossing the inflection point. This condition may be provided by the quench time scale that is comparatively large but still smaller than relaxation time scale. Under this condition, significantly large DCC domains may lead to observable effects in heavy ion collisions or high energy cosmic rays. Second, the higher order correction terms in Eq. (32) include direct scattering effects. The direct scattering can be shown obviously in Eq. (31), where the Green function is simply given by

$$G_0(x, t; x', t') = \sum_{q=0}^\infty \Psi_q(x, t)\Psi_q^*(x', t') \tag{34}$$

8
with $\Psi_q$ being the Fock states of $H_0$. Thus the nonlinear perturbation $H_P$ scatters $\Psi_{\alpha\beta\gamma}$ into $\Psi_{\delta\epsilon\zeta}$, and vice versa. In particular, direct scatterings with short wavelength modes (environment or noise) would lead to decoherence of long wavelength modes. Therefore, long wavelength modes achieve not only classical correlation but also decoherence [18] and long wavelength modes emerge as a classical order parameter.

V. CONCLUSION

The QCD with two massless quarks would undergo a second order phase transition. The effective theory for the QCD phase transition is the linear sigma model for the quark-antiquark condensate, that is, the sigma and pion fields. Under a rapid cooling process, the QCD phase transition proceeds far from equilibrium (nonequilibrium). In this talk we focused on the dynamical process of the nonequilibrium phase transition and its implications on DCC domain growth.

The most prominent feature of nonequilibrium second order phase transitions is the instability of long wavelength modes. These modes begin to grow exponentially at the onset of phase transition while rolling over the barrier from the false vacuum to the true vacuum. Therefore, this nonequilibrium dynamical process leads to large domains of disoriented chiral condensate, regions of misaligned vacuum. On the contrary, domains formed from thermal equilibrium have small sizes determined by thermal energy and are randomly oriented in isospin space.

Using the nonequilibrium linear sigma model with a smooth finite, we observed that DCC domains grow according to some power-law scaling relations. We found the power $t^{1/4}$ for the scaling behavior in the intermediate stage and the Cahn-Allen scaling power $t^{1/2}$ at the later stage of phase transition. However, in the Hartree approximation, exponentially growing long wavelength modes contribute $\lambda\langle\vec{\pi}^2\rangle$, which in turn attenuates the instability. The instability completely stops when $\lambda\langle\bar{\pi}^2\rangle$ dominates over $m_f^2$. Therefore, domains grow for a limited time and reach a typical size of $1 \sim 2$ fm in the Hartree approximation [14]. This size of DCC domain may not be large enough to lead to any significant observation.

It was shown that higher order quantum corrections increase the domains size by additional factors $(2n+1)^{1/2}$ if the duration of instability is long enough to make higher order terms grow. The longer is the quench time, the larger domains are. However, relaxation
time scale gives a limitation on domain growth through instability, because thermal equili-
bra tion competes with instability when the quench time is comparable to relaxation time.
This non-Gaussian effects on DCC domains may lead to observations in heavy ion collisions
and high energy cosmic rays. This mechanism should be distinguished from the anomaly
enhanced domain formation [19].

Acknowledgments

The author would like to thank K. Rajagopal for useful discussions. This work is sup-
ported by Korea Research Foundation under grant No. KRF-2003-041-C20053.

[1] R. Pisarski and F. Wilczek, Phys. Rev. D29, 338 (1984); F. Wilczek, Int. J. Mod. Phys. A7,
3911 (1992).
[2] K. Rajagopal and F. Wilczek, Nucl. Phys. B399, 395 (1993); B404, 577 (1993).
[3] R. D. Pisarski, Nucl. Phys. A702, 151 (2002); Phys. Rev. D62, 111501(R) (2000); F. Sannino,
Phys. Rev. D66, 034013 (2002).
[4] T. W. B. Kibble, J. Phys. A9, 1387 (1976); W. H. Zurek, Nature 317, 505 (1985).
[5] C. M. Lattes, Y. Fujimoto, and S. Hasegawa, Phys. Rept. 65, 151 (1980).
[6] A. Anselm, Phys. Lett. B217, 169 (1988); A. Anselm and M. G. Ryskin, Phys. Lett. B266,
482 (1991).
[7] S. P. Kim and C. H. Lee, Phys. Rev. D62, 125020 (2000); S. P. Kim, S. Sengupta, and F. C.
Khanna, Phys. Rev. D64, 105026 (2001).
[8] S. Sengupta, F. C. Khanna, and S. P. Kim, Phys. Rev. D68, 105014 (2003); S. P. Kim, Dyn-
amical Theory of Phase Transitions and Topological Defect Formation in the Early Universe,
hep-ph/0401095.
[9] S. P. Kim and F. C. Khanna, Non-Gaussian Effects on Domain Growth, hep-ph/0011115
(unpublished).
[10] M. Gell-Mann and M. Levy, Nuovo Cimento 16, (705) (1960).
[11] L. Dolan and R. Jackiw, Phys. Rev. D9, 3320 (1974).
[12] J. Schwinger, J. Math. Phys. 2, 407 (1961); L. V. Keldysh, Sov. Phys. JETP 20, 1018 (1965).
[13] K. Freese, C. T. Hill, and M. Mueller, *Nucl Phys.* B 255, 693 (1985); A. Ringwald, Phys. Rev. D 36, 2598 (1987); O. Éboli, R. Jackiw, and S.-Y. Pi, *Phys. Rev.* D37, 3557 (1988).

[14] D. Boyanovsky, D.-S. Lee, and A. Singh, *Phys. Rev.* D48, 800 (1993); D. Boyanovsky, H. J. de Vega, and R. Holman, *Phys. Rev.* D51, 734 (1995).

[15] F. Cooper, Y. Kluger, E. Mottola, and J. P. Paz, *Phys. Rev.* D51, 2377 (1995).

[16] R. D. Amado and I. I. Kogan, *Phys. Rev.* D51, 190 (1995); R. Randrup, *Phys. Rev. Lett.* 77, 1226 (1996); *Phys. Rev.* D55, 1188 (1997); *Heavy Ion Phys.* 9, 289 (1999); *Phys. Rev.* C62, 064905 (2000).

[17] G. Holzwarth and J. Klomfass, *Phys. Rev.* D66, 045032 (2002).

[18] F. C. Lombardo, F. D. Mazzitelli, and R. J. Rivers, *Phys. Lett.* B523, 317 (2001); S. P. Kim and C. H. Lee, *Phys. Rev.* D65, 045013 (2002). F. C. Lombardo, F. D. Mazzitelli, and R. J. Rivers, *Nucl. Phys.* B672, 462 (2003).

[19] M. Asakawa, H. Minakata, and B. Müller, *Phys. Rev.* D58, 094011 (1998); *Phys. Rev.* C65, 057901 (2002).