Reply to comment on “New apparatus design for high precision measurement of $G$ with atom interferometry”

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Received 28 June 2022 / Accepted 29 August 2022 / Published online 17 September 2022
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The scope of our work [1] is the determination of a source mass configuration suitable for a $G$ measurement in the 10 ppm range. Our Monte Carlo simulation calculates the gravitational field introduced by the source mass and uses it to evaluate the interferometer phase by integrating along the wave-packet trajectories. Therefore it fully includes the effects due to gravity gradient and higher order derivatives of the field both along the vertical axis and in the radial direction. We varied several cloud parameters with the purpose of identifying the most critical ones and evaluating the sensitivity of the $G$ measurement with respect to them. Simulation results clearly show that the radial extension of the atomic cloud can be a major issue, especially through the expansion dynamics caused by the radial temperature.

In the comment to our article [2], only the effect due to the initial spatial extension of the cloud ($\sigma_r$) has been taken into account and calculations have been performed considering the radial dependence of the gravity gradient at a given vertical position (the barycenter) of the source mass distribution, thus neglecting several other factors, i.e., the position of the apogees of both clouds, the vertical extension of the interferometers, the platform contribution to the gravitational field, etc. Moreover, the reported systematic error (RSE) of Eq. 9 in [2] is a shift over the single particle phase which is already included in our Monte Carlo simulation result. Instead, the $G$ measurement accuracy is determined by the errors on the knowledge of the systematic shift and not by the value of the shift itself. For $\sigma_r$ varied between 2.6 and 3.4 mm (see Tables 2, 5 and 6 in [1]) and for a compensation of the gravity gradient at the 1% level, we obtain a maximum variation of the relative phase of 18 ppm for configuration I and 3 ppm for configuration II. Please, notice that our values are fully compatible with the RSE intervals reported in (10) and (12) of [2].

Finally, we would like to point out that the suggested source mass configuration with a height of 0.701 m instead of 0.84 m is not suitable for the experiment since the independency of $\gamma$ from $r$ is only valid at the source mass barycenter. Therefore, a small offset is sufficient to turn on again the systematic shift (and error) caused by the finite radial size of the atomic cloud. In addition, a reduction of the source mass height degrades the gravity gradient homogeneity along the interferometer region, thus requiring a shorter gradiometer baseline and leading to a degraded instrument sensitivity.

In conclusion, the results presented in [2] are obtained from a simplified calculation that, although not in contradiction with our work, it is not sufficient for the optimization of the source mass geometry toward a $G$ measurement at the 10 ppm level.

**Corrigendum**

Through the review process, we have found a misprint in the values of $z_0$ reported in Table 1 of [1]. The correct ones are $−276.7$ mm and $−396.7$ mm.

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Acknowledgements We acknowledge financial support from INFN and the Italian Ministry of Education, University and Research (MIUR) under the aegis of Progetto Premiale “Interferometro Atomico” and PRIN 2015. GR acknowledges financial support from the European Research Council, Grant No. 804815 (MEGANTE).

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