Research Article

Cutting Force Modeling and Experimental Study for Ball-End Milling of Free-Form Surfaces

Zhaozhao Lei,1 Xiaojun Lin,1 Gang Wu,2 and Luzhou Sun1

1School of Mechanical Engineering, Northwestern Polytechnical University, Xi’an 710072, China
2Xi’an Zhongxin New Software Co., Xi’an 710072, China

Correspondence should be addressed to Zhaozhao Lei; 2561793018@qq.com

Received 26 July 2021; Revised 11 October 2021; Accepted 15 October 2021; Published 3 November 2021

1.Introduction

In the aerospace, automotive, marine, and other industrial fields, free-form surface parts occupy an important position [1]. Moreover, research on the cutting force of milling free-form surface parts has played an important role in improving machining accuracy and machining efficiency, improving machine tool utilization, and reducing energy loss, tool wear, and other fields [2]. Establishing the prediction model of cutting force during free-form surface machining and predicting the cutting force during machining can change the cutting parameters and modify the processing technology in time. This has an important guiding role in improving the above processing phenomenon. However, most of the researches focus on the cutting force in the process of non-ball-end milling, and the research on the cutting force of ball-end milling free-form surface is relatively less [3, 4].

In recent years, scholars at domestic and international organizations have carried out related research on the cutting force model for ball-end, mainly including the theoretical model and mechanical models. However, it mainly focuses on plane and regular surfaces, and there is little research on the cutting force model in milling free-form surfaces. The theoretical model is based on the friction angle theory and shear angle theory and using orthogonal cutting or oblique cutting analysis in basic theory to establish the milling force model [5]. Because the theoretical analytical modeling needs to know the exact geometric parameters of the tool and also has a close relationship with the properties of the tool and workpiece materials, there is a lot of simplification in the process of modeling, so the prediction accuracy of theoretical analytical modeling method is low [6].

Mechanical models are more extensive in practical applications. The mechanical model is a simplified model of the...
method based on the Z-map model, the cut-in and cut-out operation method based on the solid model, the discrete methods for calculating cutter engagement area: the Boolean calculate the cutting force for the ball-end. /__here are four formed chip thickness is solved by means of linear iteration. and the actual surface of machined workpiece, the unde-
section between the reference line of cutting microelement instantaneous undeformed chip thickness considering tool
milling. Zhang et al. [21] proposed an accurate model of face milling cutter, and milling cutter position angle as the
fluence of the angle between the workpiece and the tool on the cutting force during milling. Tsai and Liao [13] proposed the machining model under the condition of oblique feed. Kovacic et al. [14] established the milling force model using genetic algorithm. In order to establish the tool displacement model, Wojciechowski et al. [15] of Poznan University of Technology calculated the instantaneous orthogonal cutting force of face milling cutter during milling and selected the cutting length, contact area, effective number of cutting teeth of face milling cutter, and milling cutter position angle as the research object to study their influence on the instantaneous cutting force. Pimenov et al. [16, 17] of South Ural National University and others studied the coefficients in Guzeev and Pimenov’s cutting force model, obtained the value of \( k_1 k_2 k_3 \) through experiments, and completed the establishment of cutting force model.

The instantaneous undeformed chip thickness is an important parameter to calculate the cutting force. Huang et al. [18] proposed a method to calculate the chip thickness in the milling of five-axis ball head and studied the influence of the main and lateral deflection angles on the chip thickness in the milling of five-axis ball head. Tuysuz et al. [19] proposed a mechanical model to predict the cutting force in three directions during the milling process of ball-end cutters by simulating the chip thickness distribution, cutting, and indentation mechanics. Azeem et al. [20] proposed a method for characterizing the thickness of undeformed chips in three-dimensional tool motion and improved the cutting force model for multiaxis ball-end milling. Zhang et al. [21] proposed an accurate model of instantaneous undeformed chip thickness considering tool runout effect. By finding the approximate point of intersection between the reference line of cutting microelement and the actual surface of machined workpiece, the unde-
formed chip thickness is solved by means of linear iteration.

The exact solution of cutter engagement area is the key to calculate the cutting force for the ball-end. There are four methods for calculating cutter engagement area: the Boolean operation method based on the solid model, the discrete method based on the Z-map model, the cut-in and cut-out method, and the analytical geometry analysis method. The core idea of the Z-map method is discreteness. The Z-map method is robustness and efficiency so that it is widely used in CNC simulation. Lin et al. [22] analyzed the boundary curve of the contact area during plane cutting to obtain the contact area under the fixed tool posture and obtained the contact area under any tool posture through geometric transformation and then extended its analytical expression to a free-form surface.

Wang et al. [23] considered the influence of tool vi-
bration and runout on cutting force when predicting cutting force. Through experiments, the consistency of the results of the proposed predicted force and actual measured force was verified. In China, Ma and Lin [24] modeled the cutting force based on the idea of differential discrete and analyzed the eccentricity in the milling process. According to the milling characteristics of ball-end milling cutter, Shi et al. [25] established the milling force model of ball-end milling cutter. Tan [26] used the idea of axial microsegmentation to divide discrete cutting edges and used geometric analysis to judge the cutting in and cutting out conditions and estab-
lished a cutting microelement cutting force model. Wei et al. [27, 28] improved the cutting force model for the ball-end and the undeformed chip thickness model with the position angle of the cutting edge microelement as parameters, so that it can adapt to the changing cutting geometry condi-
tions in the surface machining, and established a cutting force prediction model of curved surface for ball-end. Guo et al. [29] proposed an analytical algorithm based on the space limited method for cutting edge contact interval for 5-axis wide row machining of free-form surface flat end milling cutter and established a milling force prediction model considering tool eccentricity. Wei et al. [27, 30] presented an integrated form error compensation approach for ball-end milling of sculptured surface with Z-level contouring tool path.

After analyzing and summarizing the literature at home and abroad, it is found that the research on the cutting force model of ball-end tool milling surface is less and incomplete. The existing models do not include the axial milling force component, so they are incomplete. At the same time, in the analysis of chip geometric parameters, the three-dimen-
Sional feed motion of the tool is simplified to a two-di-
Mensional case, which is a general case, which is difficult to be used for the continuous simulation of the three-di-
MenSional machining process of complex surfaces. In ad-
dition, in the determination of the important parameter of cutting edge segment involved in milling, most models adopt simple geometric analysis method, and the simulation accuracy is low. Therefore, it is necessary to further study the milling force of ball-end tool milling free-form surface.

Although there are many scholars studying ball-nose cutter cutting force modeling, most of them still stay in the plane field of simple rules in low-speed milling, and most of the model errors are large. There are relatively few studies on high-speed precision machining of surface, especially free surface, which has seriously restricted the actual needs. Therefore, it is necessary to further study the cutting force modeling of surface-type parts. Based on Lin et al.’s [22]
calculation of contact area and cutting force coefficient, this paper first calculates the instantaneous undeformed thickness in the cutting process and judges whether the microelement of cutting edge participates in cutting, then deduces the cutting force model of free-form surface of ball-nose cutter, and gives the cutting force prediction algorithm. Finally, the correctness of the model is verified by free-form surface and arc surface cutting experiments.

2. Materials and Methods

In this chapter, a cutting force model of free-form surface for ball-end is established. In the main work, relevant researches and calculations on instantaneous undeformed chip thickness are carried out, to judge whether the microelement cutting edge on the ball head cutter is involved in cutting, so that the cutting force model can be established more accurately. After that, the process and method of establishing the cutting force model are analyzed, and the modeling scheme is studied. Then the cutting force model of ball-end milling free-form surface is programmed; MATLAB is used to predict its value and change trend.

2.1. Instantaneous Undeformed Chip Thickness. The instantaneous undeformed chip thickness is defined as the distance between the cutting point on the current tool path and the machined workpiece surface along the microelement reference line of the cutting edge [29]. It is an important parameter for calculating the cutting force, which determines the size of the cutting load and is related to the current feed rate per tooth \( f \), the axial position angle \( \kappa \), and circumferential position angle \( \theta \) [30]. When studying the changing law of instantaneous undeformed cutting thickness, the main need is to consider the changing law in both horizontal and vertical directions, so the feed direction is equivalent to staying in the horizontal plane. Therefore, it is only necessary to analyze the horizontal change of the instantaneous undeformed chip thickness of the cutting microelement, as shown in Figure 1.

The thickness of the undeformed chips in the horizontal direction can be expressed by equation (1) [13].

\[
\begin{align*}
f (H) &= f_c \cdot \sin \theta \cdot \sin \kappa. \\
f_n &= f (H) = f_c \cdot \sin \theta \cdot \sin \kappa. \\
f (V) &= f_v \cdot \sin \phi \cdot \sin \theta \\
f_c &= K_a \cdot f_v.
\end{align*}
\]

2.2. Judgment of the Microelement Cutting Edge Participating in Cutting. If it is judged that a microelement edge of a ball-end cutter participates in cutting when modeling the cutting force, it will inevitably produce cutting force, which will affect the total cutting force. In actual machining process, if it is judged that it does not participate in cutting, the cutting force will be zero, and it will not need to be included in the total cutting force when modeling. Therefore, judging whether each microelement edge of the ball-end cutter participates (as shown in Figure 2) in cutting is an important issue in cutting force modeling.

Ding [2] calculated the cutter engagement area and calculated the limit values of the axial position angles \( \kappa_{\text{min}} \) and \( \kappa_{\text{max}} \) of the cutter engagement area and divided the contact area between \( \kappa_{\text{min}} \) and \( \kappa_{\text{max}} \) into m small microelement segments; the cut-in angle \( \theta_{\text{ci}} \) and cut-out angle \( \theta_{\text{co}} \) corresponding to each microelement segment are calculated. The cut-in angle and cut-out angle of the microelement limit the range of microelement involved in cutting. The geometric diagram of the microelement position angle and the circumferential microelement position angle is shown in Figure 2.

After judging whether each microelement on a single tooth participates in cutting separately, the cutting force of each tooth of the tool is calculated. Finally, each angle is calculated by this way so that we can obtain the results of microelement participation in cutting of the entire tool in one cycle. The calculation flow chart is shown in Figure 4.

\[ R \] represents the effective cutting radius.

After the judging whether the microedge is involved in cutting, the position angle \( \theta \) of each microedge of the tool needs to be calculated. If \( \theta_{\text{ci}} \leq \Psi \leq \theta_{\text{co}} \), the corresponding microedge of the cutting edge participates in cutting. Otherwise, the cutter edge microelement does not participate in cutting. At present, the ball-end cutter widely used in processing is the constant lead helix ball-end cutter, but its helix angle is not constant at the ball-end port. The spiral cutter edge of the head is shown in Figure 3.

In this diagram, \( \psi \) is position angle of cutting edge [rad]; \( \phi \) is the lag angle [rad]; and \( n \) is the spindle rotational speed [r/min].

After judging whether each microelement on a single tooth participates in cutting separately, the cutting force of each tooth of the tool is calculated. Finally, each angle is calculated by this way so that we can obtain the results of microelement participation in cutting of the entire tool in one cycle. The calculation flow chart is shown in Figure 4.

2.3. Cutting Force Modeling. To model cutting force prediction model of free-form surface for ball-end milling, first we determined the microelement cutting force of the free-form surface cutting edge and then integrated the microelement cutting force along the cutting edge of the ball-end cutter. By this way the cutting force on a cutting edge can be obtained. The cutter edge microelement cutting force prediction model of free-form surface is shown in

\[
\frac{dF_x}{dx} = R \cdot f_n \cdot R_t \left[ \begin{array}{c} K_{rs} \\ K_{rs} \\ K_{rs} \end{array} \right] + R \sqrt{\left( + \sin^4 \kappa \tan^3 \beta_0 \right)} \cdot R_t \left[ \begin{array}{c} K_{rc} \\ K_{rc} \end{array} \right] \cdot \frac{dF_y}{dx}.
\]
Substitute the undeformed chip thickness (equation (2)) into equation (3) to obtain

\[
\begin{bmatrix}
\frac{dF_x}{dF_y} \\
\frac{dF_y}{dF_z}
\end{bmatrix} = R \cdot f_c \cdot \sin \kappa \cdot \sin \theta \cdot R_t \begin{bmatrix} K_{ts} \\ K_{rs} \end{bmatrix} + R \sqrt{1 + \sin^4 \kappa \tan^2 \beta_0} \cdot \begin{bmatrix} K_{te} \\ K_{re} \end{bmatrix} \, d\kappa. \tag{4}
\]

In these equations, \(K_{ts}, K_{rs}, \) and \(K_{re}\) are the circumferential, radial, and axial shear force coefficients; \(K_{ts}, K_{re}, \) and \(K_{ae}\) are the circumferential, radial, and axial ploughing force coefficients; \(dF_x, dF_y, \) and \(dF_z\) are the circumferential, radial, and axial microcutting force. The feed per tooth is represented by \(f_c.\) \(\beta_0\) is the nominal helix angle. The integral can get the total cutting force of a certain tooth as shown in equation (5).

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = \int_{\theta_{min}}^{\theta_{max}} \begin{bmatrix} R \cdot f_c \cdot \sin \kappa \cdot \sin \theta \cdot R_t \begin{bmatrix} K_{ts} \\ K_{rs} \end{bmatrix} + R \sqrt{1 + \sin^4 \kappa \tan^2 \beta_0} \cdot \begin{bmatrix} K_{te} \\ K_{re} \end{bmatrix} \, d\kappa, \tag{5}
\end{bmatrix}
\]

The total cutting force of a cutter tooth during cutting is the total cutting force of the entire ball-end cutter, when precisely cutting a free-form surface. The total cutting force of a cutter tooth during cutting is calculated below; for an N-tooth cutter, the milling forces of all cutter teeth are added and summed to obtain the total milling force of the cutter. Its expression is

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = \sum_{j=1}^{N} \begin{bmatrix} F_{xj} \\ F_{yj} \\ F_{zj}
\end{bmatrix}. \tag{6}
\]

**2.4. Cutting Force Prediction Algorithm.** In order to represent the cutting force prediction algorithm, the pseudocode for the prediction of the cutting force prediction model of free-form surface for ball-end is given in Table 1. The user inputs cutting conditions, tool geometry parameters, tool posture, cutting force coefficients, axial position angle...
Figure 2: Diagram of geometric diagram of axial microelement position angle and circumferential microelement position angle.

Figure 3: Diagram of the ball cutting edge.
Input: Cut-in angle $\theta_{st}$ and cut-out angle $\theta_{ex}$, spiral lag angle of the selected cutting edge micro-element $\phi$, Number of micro-element per tooth $m$, $k=1$, $j=1$, $i=1$

$\theta_{st} \leq \psi_{ijk} \leq \theta_{ex}$

The micro-element is in cutting

The micro-element is not in cutting

$\begin{align*}
i &= i+1 \\
\text{if } i &\leq m \\
j &= j+1, i=1 \\
k &= k+1, j=1, i=1 \\
j &\leq 4 \\
k &\leq 360
\end{align*}$

Output result

The end

**Figure 4**: Flow chart of microelement cutting edge participation in cutting judgment.
Table 1: Pseudocode for milling force prediction algorithm.

| Input                                      | Cutting conditions  | Tool geometry |
|--------------------------------------------|---------------------|---------------|
| Cutting conditions                         | $ap$, $ae$, $n$, $fz$, $\theta_{as}$, $\theta_{ex}$ |
| Tool geometry                              | $R$, $N$, $\beta_0$ |
|                                            | $\alpha_z$, $\alpha_c$ |
|                                            | $K_{ts}$, $K_{rs}$, $K_{as}$, $K_{te}$, $K_{re}$, $K_{ae}$ |
|                                            | $\Delta\kappa$, $\Delta\lambda$ |
| Output                                     | Cutting force record |
|                                            | $FX$, $FY$, $FZ$ |

| Variable:                                  | $\phi_p = 2\pi/N$ |
|                                            | $m = |\kappa_{\text{max}} - \kappa_{\text{min}}|/\Delta\kappa$ |
|                                            | $n = 360/\Delta\lambda$ |
|                                            | $j = 1$ to $n$ |
|                                            | $Fx(j) = Fy(j) = Fz(j) = 0.0$ |
|                                            | $i = 1$ to $m$ |
|                                            | $\phi_i = (1 - \cos \kappa) \tan \beta_0$ |
|                                            | $\theta_i = \theta_{as} + \phi_i$ |
|                                            | $Fx(j) = Fy(j) = Fz(j) = 0.0$ |
|                                            | $k = 1$ to $N$ |
|                                            | $\theta_i = \theta_{as} + (k - 1) \phi_p$ |
|                                            | $\theta_i = \theta_{1}$ |
|                                            | $\text{if } \theta_{as} < \theta_i < \theta_{ex} \text{ then}$ |
|                                            | $f_n = f_c \cdot \sin \kappa \cdot \sin \theta_2$ |
|                                            | $dS = R \sqrt{(1 + \sin^4 \kappa \tan^2 \beta_0)} \cdot d\kappa$ |
|                                            | $db = R \cdot d\kappa$ |
|                                            | $\Delta F_t = K_{ts} f_n db + K_{as} dS$ |
|                                            | $\Delta F_r = K_{rs} f_n db + K_{re} dS$ |
|                                            | $\Delta F_a = K_{as} f_n db + K_{ae} dS$ |
|                                            | $\begin{bmatrix} \Delta F_t \\ \Delta F_r \\ \Delta F_a \end{bmatrix}$ |
|                                            | $\begin{bmatrix} Fx(j) \\ Fy(j) \\ Fz(j) \end{bmatrix}$ |
|                                            | $\begin{bmatrix} \Delta F_t \\ \Delta F_r \\ \Delta F_a \end{bmatrix}$ |
|                                            | $F(j) = \sqrt{Fx(j)^2 + Fy(j)^2 + Fz(j)^2}$ |
|                                            | $\text{Matrix transformation of tangential, radial, and axial microelement}$ |
|                                            | $\text{cutting forces is performed to obtain the cutting force in the tool}$ |
|                                            | $\text{coordinate system, and all microelement cutting forces are summed to}$ |
|                                            | $\text{obtain the total cutting force}$ |
|                                            | $\text{Cutting force}$ |
|                                            | $\text{Draw the cutting force in curve of one rotation of}$ |
|                                            | $FX$, $FY$, $FZ$, and $F$ |

Figure 5: Experimental data acquisition system.
intervals, and angle intervals within a cycle. After the program runs, the cutting force in a cycle of the entire cutter tooth is output.

3. Results and Discussion

3.1. Experimental Conditions and Equipment. In the process of verifying the accuracy of the free-form cutting force model of the ball cutter proposed in this paper, two sets of surface cutting experiments were carried out. Except that the shape of the curved surface to be cut is different in the experiment, the tools, specimen materials, and dynamometers used are the same. The equipment used in the whole experiment is as follows:

(1) Machine tool: Mikron hsm700 is used for the experiment
(2) Dynamometer: Kistler 9257b three-way dynamic piezoelectric dynamometer
(3) Data acquisition system: Kistler 5019b charge amplifier, dewesoft3010 data acquisition and processing system, Lenovo E430 notebook computer
(4) Sampling frequency: 20000 Hz

The schematic diagram of the experimental data acquisition system is shown in Figure 5, and the machine tool used in the experiment is shown in Figure 6.

The material used is TC4. Its chemical composition is shown in Table 2. In the comparative experiment, one set of cut surfaces is a free-form surface, and the other set is a cylindrical surface with a radius of 10 mm. The cutting parameters are shown in Table 3.

It can be seen from Table 4 that the variation range of the lead angle and the tilt angle between adjacent tool positions on the tool path is small, so the cutter engagement area is basically the same, and the actual measured cutting force is not much different. So we just select a point on each tool path, calculate the cutting force of its predicted force, and compare it with the actual cutting force to verify the correctness of the cutting force prediction model of free-form surface for ball-end proposed in this paper. Based on this, this paper selected the No. 2 tool position in Figure 7 to predict the cutting force and compared it with the actual cutting force, as shown in Figure 7. Figure 8(a) shows the change of the cutting force in the X direction during one revolution of the tool. Figure 8(b) shows the change of the cutting force in the Y direction during one revolution of the tool. Figure 8(c) indicates the change of the cutting force in the Z direction during one revolution of the tool. Figure 8(d) shows the change of the overall cutting force during one revolution of the tool.

It can be seen from Figure 8 that there is an error in the peak values of the predicted force curve and the actual cutting force curve in the X and Z directions. The measured cutting force is larger. However, the predicted cutting force and the predicted cutting force in the Y direction are realistic. In order to quantitatively analyze and predict the deviation of the cutting force, a calculation for the deviation is performed. The ratio of the difference between the peak predicted force and the measured peak force and the measured force is taken as the deviation. Through calculation, the average deviation in the X direction is 27.8%, the average deviation in the Y direction is 9.0%, the average deviation in the Z direction is 23.9%, and the deviation of the total force is 17.6%.

During the free-form cutting process, the change of the lead angle and the tilt angle of all the tool positions on the entire tool position path is small; it is not very accurate to show the rule that the cutting force changes when the cutter engagement area changes greatly. Therefore, this paper has carried out circular arc with a radius of 10 mm cutting experiment. Table 5 shows the lead angle and the tilt angle corresponding to each tool point of the circular arc cutting experiment. The change diagram of the cutter engagement area at each tool point is shown in Figure 9. The change rule of the maximum value and minimum value of the predicted cutting force and the actual measured cutting force during the entire cutting process is shown in Figure 10. Among them, Figures 10(a)–10(c) show the change of the predicted cutting force peak in the X, Y, and Z directions. Figures 10(d)–10(f) show the change of the measured cutting force peak in the X, Y, and Z directions, respectively.

As can be seen from Figure 10, the minimum value predicted by modeling is 0, that is, the cutting force when the cutting edge of the tool does not participate in cutting. The maximum value is the maximum value of the cutting force of the tool at each tool location. In order to clearly explain the change law of cutting force on the tool path, the curve is fitted and the maximum curve in Figure 11 is obtained. The predicted cutting force change trend is basically consistent with the measured cutting force change trend.

Although the experimental results of the above experiments show that the cutting force model established in this paper is basically consistent with the experimental results, it is not yet able to prove the advanced nature of the model established in this paper. Therefore, the comparative experiment between the traditional cutting force model for inclined plane and the cutting force model established in this paper is done. The inclined plane angle is 60°, as shown in Figure 11. The predicted force of the traditional predicted force model and the cutting force model in this paper are compared with the measured force, and Figure 12 can be obtained. Figures 12(a)–12(c) are the comparison diagrams with the speed of 5000 r/min, and Figures 12(d)–12(f) are the comparison diagrams with the speed of 8000 r/min.

It can be seen from Figure 12 that the cutting force predicted by the traditional cutting force model and the cutting force predicted by the cutting force model in this paper are in good agreement with the actually measured cutting force. Especially at the speed of 8000 r/min, the peak value of cutting force is obvious, and the measured cutting force is basically stable and small without cutter teeth.

In order to calculate the deviation between the predicted cutting force and the actual cutting force, the root mean square error is introduced to quantitatively analyze the consistency between the predicted cutting force and the measured cutting force with different cutting force coefficients. Through the measured cutting force data, the
maximum cutting force of each cutter tooth in the cutting process can be calculated, and the calculation formula is as follows:

\[
F_{x_{\text{max}}}=\sum_{j=1}^{j} F_{x_{i_{\text{max}}}}/j, \\
F_{y_{\text{min}}}=\sum_{j=1}^{j} F_{y_{i_{\text{min}}}}/j, \\
F_{z_{\text{max}}}=\sum_{j=1}^{j} F_{z_{i_{\text{max}}}}/j,
\]

where \(F_{x_{i_{\text{max}}}}, F_{z_{i_{\text{max}}}}\) are the measured maximum cutting force of the \(i\)th cutter tooth in \(X\) and \(Z\) directions, \(F_{y_{i_{\text{min}}}}\) is the measured minimum cutting force of the \(i\)th cutter tooth in the \(Y\) direction, and \(j\) is the number of teeth of the cutter.

The root mean square error is calculated as follows:

\[
\delta = \frac{\sqrt{\Delta F_{\text{sq}}}}{F_{x_{\text{max}}}^2 + F_{y_{\text{max}}}^2 + F_{z_{\text{max}}}^2},
\]

\[
\Delta F_{\text{sq}} = (F_{x_{\text{th}}}-F_{x_{\text{max}}})^2 + (F_{y_{\text{th}}}-F_{y_{\text{min}}})^2 + (F_{z_{\text{th}}}-F_{z_{\text{max}}})^2,
\]

where \(F_{x_{\text{th}}}, F_{z_{\text{th}}}, F_{y_{\text{th}}}\) are the predicted maximum cutting force in the \(X\) and \(Z\) directions and \(F_{y_{\text{th}}}\) is the predicted minimum cutting force in the \(Y\) direction.

According to the above calculation model, the root mean square error results can be obtained, as shown in Figure 11. Among them \(\delta_{\text{all}}\) is the root mean square error obtained by applying the traditional cutting force model, and \(\delta_{\text{sig}}\) is the root mean square error calculated by using the prediction results of the cutting force model proposed in this paper.

---

**Table 2: TC4 chemical composition.**

| Category         | Speed, \(n\) (r/min) | Cutting depth, \(ap\) (mm) | Spacing, \(ae\) (mm) | Feed rate, \(F\) (mm/min) | Tool radius, \(r\) (mm) |
|------------------|------------------------|----------------------------|----------------------|---------------------------|-------------------------|
| Free-form surface| 8000                   | 0.4                        | 0.4                  | 1500                      | 4                       |
| Cylindrical surface | 8000                 | 0.4                        | 0.4                  | 1500                      | 4                       |

**Table 3: Cutting experiment parameter table.**

| Element | Ti | Al   | V    | Fe   | Si   | C    | N    | H    | O    | Other elements |
|---------|----|------|------|------|------|------|------|------|------|----------------|
| Content/% | Allowance | 5.5–6.8 | 3.5–4.5 | ≤0.30 | ≤0.15 | ≤0.1 | ≤0.05 | ≤0.01 | ≤0.20 | 0.11            |

**Table 4: Lead angle and tilt angle corresponding to each tool point in free-form cutting experiment.**

| Tool point | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    |
|------------|------|------|------|------|------|------|------|------|
| Lead angle \(a_{z}\)/° | -0.32 | -0.31 | -0.29 | -0.27 | -0.27 | -0.27 | 0.25  | -0.23 | 0.22 |
| Tilt angle \(a_{c}\)/°  | -51.4 | -51.2 | -50.7 | -50.2 | -50.0 | -49.5 | -49.0 | -48.7 |      |

Figure 6: Machine tools and workpieces.
It can be seen from Figure 13 that when the speed is 5000 r/min, \( \delta_{\text{all}} \) is 13.4%, and \( \delta_{\text{sig}} \) was 11.9%. When the speed is 8000 r/min, \( \delta_{\text{all}} \) is 26.4%, and \( \delta_{\text{sig}} \) was 11.4%. At the same speed, \( \delta_{\text{sig}} \) is less than \( \delta_{\text{all}} \). In addition, as the speed increases, \( \delta_{\text{all}} \) increment is large, while the change of \( \delta_{\text{sig}} \) is very small.

Only one comparative experiment cannot verify the correctness of the calibration method of tooth cutting force coefficient. Therefore, two groups of verification experiments are carried out on the basis of this experiment. The cutting parameters of rotating speed \( n = 5000 \text{ r/min} \) are feed rate per tooth \( FC = 0.04 \text{ mm} \), cutting depth \( AP = 0.4 \text{ mm} \), and row width \( 0.8 \text{ mm} \). The cutting parameters of rotating speed \( n = 8000 \text{ r/min} \) are feed rate per tooth \( FC = 0.05 \text{ mm} \), cutting depth \( AP = 0.4 \text{ mm} \), and row width \( 0.8 \text{ mm} \). The experimental results are shown in Figure 14, and the root mean square error results are shown in Figure 15.

It can be seen from Figure 14 that the predicted cutting forces in the two groups of experiments verified are also in good agreement with the measured cutting forces. When the rotating speed changes from 5000 r/min to 8000 r/min, the oscillating cutting force in three directions of X, Y, and Z becomes larger when cutting without cutter teeth. It can be seen from Figure 15 that when the speed is 5000 r/min, \( \delta_{\text{all}} \) is 22.6%, and \( \delta_{\text{sig}} \) was 22.8%. When the speed is 8000 r/min, \( \delta_{\text{all}} \) is 32.4%, and \( \delta_{\text{sig}} \) was 22.7%. As the speed increases, \( \delta_{\text{all}} \) increment is large, while \( \delta_{\text{sig}} \) is basically unchanged.

3.2. Analysis and Discussion of Experimental Results. Observing Figures 7 and 9, it can be seen that when the ball-end cutter cuts free-form surface parts, the cutter engagement area is related with change of the shape of the free-form surface at the tool point. The smaller the absolute value of the lead angle and the tilt angle are, the closer the cutter engagement area is to the point of the tool tip, and, conversely, the farther the cutter engagement area is from the point of the knife point.

It can be seen from some of the graphs in Figures 8 and 10 that there are oscillation signals in the measured cutting force. It can be seen that the oscillation signal decreases with the decrease of the lead angle; the main reason is that the stiffness of the tool becomes better and the cutting force in the circumferential direction becomes smaller during the process of the lead angle from large to small, and the stability of the system becomes better. As for the situation where the cutting force is relatively large at the initial cutting and at the end of the cutting, it is because the cutting state at the beginning of the cutting and the final cutting is an unstable cutting state, which is not within the scope of this article.

This paper studies the cutting force under stable cutting state. In the process of just cutting in and finally cutting out, the cutting force is actually a sudden process, which is greater than the stable cutting force. The cutting forces calculated in this paper are all cutting forces under stable cutting state. In order to improve the unstable cutting...
condition, before the cutting experiment, firstly the tool is slotted along the normal direction of the workpiece in the tool cut-in area and tool cut-out area, which can greatly reduce the cutting in and cutting out process and prolong the time of stable cutting state. However, the whole process from the beginning of cutting to the completion of cutting is in a stable cutting state.

Through the analysis of the above chart, the following facts can be basically obtained. It can be seen from the measured cutting force curve that the peak value of the measured cutting force is obvious, the number of cutter teeth is clear, and the vibration value is relatively small. The measured data under the conditions of high speed, small cutting depth, and small total cutting force is enough to show that the measured cutting force is accurate. Secondly, the predicted cutting force is basically consistent with the measured cutting force. There is a small error in some parts, which is mainly due to the limitation of the defined cutting force coefficient. Using the cutting force coefficient calibration method cited in this paper, because the calculated cutting

| Tool point | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|------------|----|----|----|----|----|----|----|----|
| Lead angle $a_z$ (°) | 53.4 | 48.1 | 42.8 | 37.5 | 32.1 | 25.0 | 19.7 | 12.6 |
| Tilt angle $a_c$ (°) | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

Figure 8: Comparison of predicted cutting force and actual cutting force for free-form cutting. (a) The change of the cutting force in the $X$ direction. (b) The change of the cutting force in the $Y$ direction. (c) The change of the cutting force in the $Z$ direction. (d) The change of the overall cutting force.
contact area is small, the calculated cutting force coefficient curve is accurate near a fixed axial position angle, but the cutting force coefficient at other position angles is relatively inaccurate; for other position angles, the cutting force coefficient is relatively inaccurate. Then when the change range of contact area is large, there will be a small error between the prediction cutting force and the measure cutting force. After eliminating the above interference factors and other uncontrollable factors, the results of the predicted cutting force and the measured cutting force in this paper are in good agreement with small errors. This shows that the established model can more accurately predict the actual cutting force of free-form surface parts in the precision cutting process.

Under the condition of ball-end tool finish milling, compared with the traditional cutting force model, the prediction accuracy is high, and it is more in line with the actual cutting state.

Although the cutting force model established in this paper has high prediction accuracy in high-speed milling, it still has some limitations. And although the research on ball-nose cutter cutting force modeling has been mature, there is no commercialized cutting force prediction software. Because there is little research on unstable factors in cutting process, in most circumstances, the actual cutting state is not in the ideal cutting state. For example, first in this paper the cutting force coefficient calibrated is accurate at the axial position angle corresponding to the contact area because of the small cutting depth, while the relative accuracy in other areas is low. Secondly, the cutting force established in this paper is established without considering the stiffness of the tool and the cutting vibration, etc. However, in multi-axis machining, the stability and vibration amplitude of cutting system will change due to the change of tool axis and
stiffness of tool, which will directly or indirectly affect the cutting force. Finally, the cutting force under stable cutting state is studied in this paper. There is no research on cutting force under unstable cutting conditions such as first-cutter cutting, cut-in, and cut-out. In order to make the research on cutting force more complete, the cutting force under unstable cutting state mentioned above can be considered in further research.
Figure 11: Diagram of the experiment.

Figure 12: Continued.
Figure 12: Comparison of cutting force.

Figure 13: Root mean square error diagram.

Figure 14: Continued.
The predicted cutting force established in this paper

\( n = 5000 \text{r/min}; f_c = 0.04 \text{mm/tooth}; a_p = 0.4 \text{mm}; a_e = 0.8 \text{mm} \)

\( n = 8000 \text{r/min}; f_c = 0.05 \text{mm/tooth}; a_p = 0.4 \text{mm}; a_e = 0.8 \text{mm} \)

\( n = 8000 \text{r/min}; f_c = 0.41 \text{mm/tooth}; a_p = 0.4 \text{mm}; a_e = 0.6 \text{mm} \)

Figure 14: Cutting force comparison diagram.
4. Conclusion

In this paper, a calculation model of instantaneous undistorted cutting thickness under variable cutting conditions is derived, and the participation of cutting edge element in cutting at a certain cutting time is judged, and a free-form surface cutting force model of ball-nose cutter is obtained. Through free-form surface and arc cutting experiments, it is proved that the predicted cutting force and actual cutting force are in good agreement both at a cutter point and on the whole cutting path; it is verified that the established cutting force model is accurate. However, at the same time, the modeling in this paper also has some limitations, mainly reflected in the less research on unstable factors in cutting process, so the research on cutting force of ball-nose cutter milling free-form surface under unstable state should be carried out more deeply afterwards.

In order to prove the advancement of the model established in this paper, a series of inclined face comparison experiments were carried out. The experimental results show that the error of the cutting force prediction model established in this paper for ball-nose cutter milling free surface is reduced by 15% compared with the traditional cutting force model, and it is more suitable for high-speed and precise milling. It provides reliable and effective guidance for reducing errors and chatter in actual machining.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The research was supported by the National Science and Technology Major Projects of China (no. 2015ZX04001003) and the National Science and Technology Major Projects of China (2017-VII-0001-0095).

References

[1] Y. Zeng, S. H. Pei, and G. H. Yi, “The research status of surface quality of free-form surface based on NC Milling,” Internal Combustion Engine & Parts, no. 24, pp. 95-96, 2019.
[2] W. A. Kline and R. E. Devor, “The effect of runout on cutting geometry and forces in end milling,” International Journal of Machine Tool Design and Research, vol. 23, no. 2-3, pp. 123-140, 1983.
[3] H. Xiong and T. Limin, “Precise prediction of forces in milling circular corners,” International Journal of Machine Tools and Manufacture, vol. 44, no. 2-3, pp. 225-235, 2014.
[4] Z. C. Luo, W. X. Zhao, and J. Li, “Modeling and prediction of cutting forces in end milling of curved surfaces,” Acta Mechanica, vol. 36, no. 9, pp. 1727-1735, 2015.
[5] E. Budak, Y. Altintas, and E. J. A. Armarego, “Prediction of milling force coefficients from orthogonal cutting data,” Journal of Manufacturing Science and Engineering, vol. 118, no. 2, pp. 216-224, 1996.
[6] Y. P. Ding, Modeling and Simulation of Milling Force of Ball-End Mill in Multi-Axis Linkage NC Machining, Harbin University of Science and Technology, Harbin, China, 2013.
[7] I. Yellowley, “A note on the significance of the quasi-mean resultant force and the modelling of instantaneous torque and forces in peripheral milling operations,” Journal of Engineering for Industry, vol. 110, no. 3, pp. 300-303, 1988.
[8] Y. Altintas and P. Lee, “Mechanics and dynamics of ball end milling,” Journal of Manufacturing Science and Engineering, vol. 120, no. 4, pp. 684-692, 1998.
[9] M. Yang and H. Park, “The prediction of cutting force in ball-end milling,” International Journal of Machine Tools and Manufacture, vol. 31, no. 1, pp. 45-54, 1991.
[10] W. A. Kline, R. E. DeVor, and J. R. Lindberg, “The prediction of cutting forces in end milling with application to cornering cuts,” International Journal of Machine Tool Design and Research, vol. 22, no. 1, pp. 7–22, 1982.
[11] S. Engin and Y. Altintas, “Mechanics and dynamics of general milling cutters,” International Journal of Machine Tools and Manufacture, vol. 41, no. 15, pp. 2195-2212, 2001.
[12] M. Fontaine, A. Moufki, A. Devillez et al., “Modelling of cutting forces in ball-end milling with tool-surface inclination: part I: predictive force model and experimental validation,” Journal of Materials Processing Technology, vol. 189, no. 1–3, pp. 73–84, 2007.
[13] C. L. Tsai and Y. S. Liao, “Cutting force prediction in ball-end milling with inclined feed by means of geometrical analysis,” International Journal of Advanced Manufacturing Technology, vol. 46, no. 5–8, pp. 529–541, 2010.
[14] M. Kovacic, J. Balic, and M. Brezocnik, “Evolutionary approach for cutting forces prediction in milling,” Journal of Materials Processing Technology, vol. 155-156, pp. 1647–1652, 2004.
[15] S. Wojciechowski, T. Chwalczuk, fnm Twardowski, and G. M. Krolozyk, "Modeling of cutter displacements during ball end milling of inclined surfaces," Archives of Civil and Mechanical Engineering, vol. 15, no. 4, p. 798, 2015.

[16] D. Y. Pimenov, V. I. Guzeev, G. Krolozyk, M. Mia, and S. Wojciechowski, "Modeling flatness deviation in face milling considering angular movement of the machine tool system components and tool flank wear," Precision Engineering, vol. 54, pp. 327–337, 2018.

[17] D. Y. Pimenov, A. Hassui, S. Wojciechowski et al., "Effect of the relative position of the face milling tool towards the workpiece on machined surface roughness and milling dynamics," Applied Sciences-Basel, vol. 9, no. 5, p. 842, 2019.

[18] T. Huang, X. M. Zhang, and H. Ding, "Decoupled chip thickness calculation model for cutting force prediction in five-axis ball-end milling," International Journal of Advanced Manufacturing Technology, vol. 69, no. 5-8, pp. 1203–1217, 2013.

[19] O. Tuysuz, Y. Altintas, and H.-Y. Feng, "Prediction of cutting forces in three and five-axis ball-end milling with tool indentation effect," International Journal of Machine Tools and Manufacture, vol. 66, no. 2, pp. 66–81, 2013.

[20] A. Azeem and H. Y. Feng, "Cutting force prediction for ball-end mills with non-horizontal and; rotational cutting motions," International Journal of Advanced Manufacturing Technology, vol. 67, no. 5–8, pp. 1833–1845, 2013.

[21] X. Zhang, J. Zhang, B. Pang, and W. Zhao, "An accurate prediction method of cutting forces in 5-axis flank milling of sculptured surface," International Journal of Machine Tools and Manufacture, vol. 104, pp. 26–36, 2016.

[22] X. J. Lin and G. Wu, "The identification of the cutting force coefficients for ball-end in finish milling," International Journal of Advanced Manufacturing Technology, vol. 102, no. 9-12, pp. 4121–4135, 2019.

[23] S. B. Wang, L. Geng, Y. F. Zhang, K. Liu, and T. E. Ng, "Cutting force prediction for five-axis ball-end milling considering cutter vibrations and run-out," International Journal of Mechanical Sciences, vol. 96-97, pp. 206–215, 2015.

[24] W. T. Ma and Z. H. Lin, "Cutting force model of rigid ball end milling cutter," Mechanical Science and Technology, vol. 96-97, pp. 206–215, 2015.

[25] L. Shi, Y. J. Zhang, Z. B. Li et al., "Free -form surface machining with ball -end milling cutter under constraint of nearly constant cutting force," China Mechanical Engineering, vol. 20, no. 23, pp. 2773–2776, 2009.

[26] G. Tan, Study on the Cutting Force Modeling and Machined Surface Topography Simulation in Ball-end Milling Process, Northwestern Polytechnical University, Xi’an, China, 2007.

[27] Z. C. Wei, M. J. Wang, Y. J. Cai et al., "Prediction of cutting force in ball-end milling of sculptured surface; using improved Z-map," International Journal of Advanced Manufacturing Technology, vol. 68, no. 5–8, pp. 1167–1177, 2013.

[28] Z. C. Wei, M. L. Guo, M. J. Wang et al., “Force predictive model for five-axis ball end milling of sculptured surface,” International Journal of Advanced Manufacturing Technology, vol. 98, no. 1-2, 2018.

[29] Q. Guo, B. Zhao, Y. Jiang, and W. Zhao, "Cutting force modeling for non-uniform helix tools based on compensated chip thickness in five-axis flank milling process," Precision Engineering, vol. 51, pp. 659–681, 2018.

[30] Z. C. Wei, M. J. Wang, W. C. Tang et al., "Form error compensation in ball-end milling of sculptured surface with z-level contouring tool path," International Journal of Advanced Manufacturing Technology, vol. 67, no. 9-12, pp. 2853–2861, 2013.