On self-synchronization of inertial vibration exciters in a chain-type oscillatory system

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Abstract. A model of a vibratory jaw crusher in the form of four-mass chain-type oscillating system, which oscillations are excited by self-synchronizing inertial vibration exciters installed on the working bodies is considered. Modes of the crusher motion without separating cheeks from processed medium are analyzed. Possible types of synchronization of the vibration exciters rotation and corresponding modes of the system’s oscillations depending on the inertia and elasticity characteristics of the processed medium are identified.

1. Introduction
For operation of vibratory jaw crushers with two movable cheeks, which oscillations are excited by self-synchronizing inertial exciters, it is necessary to ensure their synchronous oscillations in opposite direction. Type of synchronization of cheeks oscillations and of vibration exciters rotation in such a system is ensured by the effect of self-synchronization, and is determined by the ratio between mechanical parameters of the system, characteristics of the processed medium, parameters of electric drive of the vibration exciters, and frequency of their rotation [1, 2]. It is especially difficult to account for the interaction of the cheeks with the material being processed when studying the dynamics of such crushers. When developing mathematical models, force interaction is usually taken into account in the form of equivalent viscous friction forces [2, 3]. It usually does not take into account the mass of the processed medium, its elastic characteristics, as well as the possible asymmetry of the forces of interaction of the cheeks with the processed medium, due to the heterogeneity of its properties distribution in the occupied volume.

In the work in order to identify possible influence of the processed medium properties on the synchronization of the debalances rotation and oscillations of the crusher’s cheeks, the dynamics of a jaw crusher with two translationally movable cheeks excited by unbalance exciters installed on the cheeks are analyzed based on the model of a four-mass chain type oscillating system. It is assumed that the crusher operates in the non-impact mode, i.e. without breaking contact between the cheeks and processed medium. Such modes are characteristic of a number of technological processes which requires creation of a shock-free high-frequency force effect on the material being processed.

2. Mathematical model and simulation methods
The design scheme of the crusher model with translational oscillations of the cheeks is shown in figure 1. The model consists of a rigid frame modeled by a rigid body of mass $m_1$, and two movable cheeks modeled by rigid bodies of mass $m_2$ and $m_4$, respectively. The frame is attached to the fixed base by a
linear spring with stiffness $c_{01}$. The left-hand-side and right-hand-side cheeks are attached to the frame by means of linear springs with stiffness $c_{12}$ and $c_{14}$, respectively. The processed medium is modeled by a rigid body of mass $m_3$, which interacts with the cheeks by means of linear viscoelastic elements with stiffness and viscosity coefficients, respectively, $c_{23}$ and $b_{23}$ for the left-hand-side element, and $c_{34}$ and $b_{34}$ for the right-hand-side element. Each of these solids has one degree of freedom corresponding to the translational motion along the horizontal axis $Ox$.

Oscillation of the mechanical system are excited by two unbalance exciters with imbalanced mass $m_{ej}$ and eccentricity $r_j$ ($j = 1, 2$ – the exciter number), rigidly fixed on the cheeks. Rotation of each of the debalances is provided by an asynchronous electric motor, with the moment of inertia $J_j$ reduced to the debalance shaft. The torque $L_j$ of each motor is described by its static characteristic [4]. Friction in the supports of the debalance shafts is taken into account in the form of the moments $R_j$ of the dry frictional forces (not shown in figure 1).

Displacements of the bodies $m_i$ ($i = 1,2,.4$) are described by coordinates $x_i$ of their mass centers, respectively, measured from their equilibrium position. Angular positions of the imbalances are described by the angles of rotation $\varphi_j$ ($j = 1,2$), measured from the negative direction of the axis $Ox$.

![Figure 1. Design scheme.](image)

The equations of system’s motion in dimensionless form are:

\[
\begin{align*}
\mu_5 \ddot{y}_1 + (1 + \zeta_{14} + \zeta_{01})y_1 - y_2 - \zeta_{14}y_4 &= 0 \\
\mu_3 \ddot{y}_2 - 2\eta_3 y_2 - y_1 + (1 + \zeta_{23})y_2 - \zeta_{23}y_3 &= \bar{\varphi}_1 \sin \varphi_1 + \bar{\varphi}_2 \cos \varphi_1 \\
\mu_3 \ddot{y}_3 - 2\eta_3 y_2 + 2\eta_3 y_3 - y_1 + (1 + \zeta_{23})y_2 + (1 + \zeta_{34})y_3 - \zeta_{34}y_4 &= 0 \\
p_4 \ddot{y}_4 - 2\eta_3 y_2 y_3 + 2\eta_3 y_4 - \zeta_{14} y_1 - \zeta_{23} y_3 + (\zeta_{14} + \zeta_{16})y_4 = \bar{\varphi}_3 \sin \varphi_2 + \bar{\varphi}_4 \cos \varphi_2 \\
\bar{\varphi}_1 &= d_1 ((\bar{L}_1 - \bar{R}_1) L_1 + \dot{y}_2 \sin \varphi_1) \\
\bar{\varphi}_2 &= d_2 ((\bar{L}_2 - \bar{R}_2) L_2 + \dot{y}_4 \sin \varphi_2)
\end{align*}
\]

where $\mu_s = m_s / (m_2 + m_{e1})$ ($s = 1,3$), $\mu_4 = (m_4 + m_{e2}) / (m_2 + m_{e1})$, $\zeta_{01} = c_{01} / c_{12}$, $\zeta_{23} = c_{23} / c_{12}$, $\zeta_{34} = c_{34} / c_{12}$, $\zeta_{14} = c_{14} / c_{12}$, $2\eta = b_{23} m_2 + m_{e1} T_*$, $\beta_{34} = b_{34} / b_{23}$, $y_i = x_i / X_* -$ dimensionless coordinates, $X_* = \frac{m_{e1} T_1}{c_{12} T_2}$, $T_* = \sqrt{\frac{m_2 + m_{e1}}{c_{12}}}$ - time scale factor, $d_j = \frac{m_{ej} X_*}{(J_j + m_{ej} r_j^2)}$, $L_* = \frac{\tau_*^2}{m_{ej} r_j X_*}$ - denoted differentiation by dimensionless time $\tau = t / T_*$. The system of equations allows to analyze the modes of the crusher motion taking into account possible asymmetry of the forces of interaction of the processed medium with its cheeks.

Consider the problem of simple self-synchronization of the exciters in case of non-resonant oscillations of the crusher. The problem solution will be obtained using the method of separation of motions, which is described in detail in [5]. Following this method, we will look for solutions corresponding to almost uniform rotation of the exciter’s debalances with the same angular velocity $\omega_j = \omega_e = \omega$ in the form $\varphi_j = \sigma_j (\sigma_j + \alpha_j + \psi_j (t, \omega t))$ and $x_i = x_i (t, \omega t)$ ($k = 1,2..4$), where $\alpha_j$ are slow, and $\psi_j$ and
are fast $2\pi$ period components by $\omega \tau$ periodic, the average of which is zero over a period, $\sigma = 1$ when the debalance rotates in the positive direction of the angle $\varphi$ measurement and $\sigma = -1$ when rotating in the opposite direction. In this case, the stationary values of the slow variables $\alpha_j$ correspond to the synchronous motions. The rotation frequency and the phase shift $\alpha = \alpha_1 - \alpha_2$ are determined from the system of transcendental equations:

\begin{align*}
    P_1 &= d_1((L_1(\omega) - R_1(\omega))L_1' - W_1) = 0 \\
    P_2 &= d_2((L_2(\omega) - R_2(\omega))L_2' - W_2) = 0
\end{align*}

(2)

where $W_1$ and $W_2$ – vibrational moments, determined as follows:

\begin{align*}
    W_1 &= -\frac{1}{2} \omega^2 (A_{22} \sin \alpha + B_{22} \cos \alpha + B_{21}) \\
    W_2 &= -\frac{1}{2} \omega^2 (-A_{41} \sin \alpha + B_{41} \cos \alpha + B_{42})
\end{align*}

which coefficients $A_{ij}$ and $B_{ij}$ are determined from the solution of the problem of forced oscillations of system (1) under conditions of the vibration exciters uniform rotation with constant angular velocity $\omega$, which is sought in the form $
    x_s = \sum_{j=1}^4 (A_{sj} \cos(\omega \tau + \alpha_j) + B_{sj} \sin(\omega \tau + \alpha_j)), \quad (l = 1.2; \quad j = 1,2), \quad \text{where index } j \text{ corresponds to the number of the vibration exciter.}
$

System (2) allows the solution $\alpha = \pi/2$, at which the debalances rotate in opposite directions synchronously in antiphase, thus exciting synchronous antiphase oscillations of the cheeks, that corresponds to the normal operation of the crusher. The stability of the solutions found is determined from the analysis of the roots of the equation:

\begin{equation}
    \left| \frac{d(P_1 - P_2)}{d\alpha} \right|_{\alpha = \pi/2} = 0,
\end{equation}

wherein the stable solutions correspond to negative values of $\chi$.

Consider the effect of the processed medium parameters of elasticity $\zeta_{23}$ and $\zeta_{34}$ as well as damping $\beta_{34}$ on the stability of the solution $\alpha = \pi / 2$. In this case, we will assume that the parameters of the motors, the debalances, inertia of the cheeks and elasticity of their suspension are identical, i.e. $(L_1 - R_1)L_1' = (L_2 - R_2)L_2', J_1 = J_2, m_{e1} = m_{e2} = m_e, r_1 = r_2 = r, m_2 = m_4, \quad \zeta_{14} = 1$. Then:

\begin{equation}
    \chi = \frac{x_s m e r \omega^2}{2} (A_{22} + A_{41}),
\end{equation}

and the stability of the solution will be determined by the sign of the expression in parentheses $\chi_s = A_{22} + A_{41}$. The difference value of $\chi_s$ from zero characterizes the magnitude of the stability margin of the synchronous antiphase mode of the debalances rotation.

3. Simulation results
Fig. 2 shows the change of $\chi_s$ depending on the debalances rotational speed for different ratios of the processed medium stiffness $\lambda = \zeta_{34}/\zeta_{23}$. The following values of the system parameters were taken for the calculations: $\mu_1 = 5, \mu_3 = 0.1, \mu_4 = 1, \eta = 0.05, \zeta_{01} = 0.1, \zeta_{23} = 10, \beta_{34} = 1$. One can see that the stability condition of synchronous antiphase rotation is satisfied between the first and second eigenfrequencies of the system in the frequency range $(p_1, p_*)$, as well as in the frequency range bounded by the second and third eigenfrequencies of the system. Note that the stability condition is also satisfied in the frequency range above the fourth eigenfrequency, which, however, is not usually used due to low energy efficiency, and therefore is not shown in the figure. Decrease in $\lambda$ leads to a decrease in the third eigenfrequency and, as a consequence, to a narrowing the frequency range of stability of the debalances synchronous antiphase rotation.
Figure 2. Change of $\chi_*$ depending on the frequency at different ratios of the processed medium stiffness.

Figure 3. Change of $\chi_*$ depending on the frequency at different dissipation of the processed medium.

Figure 3 shows the change of $\chi_*$ depending on the debalances rotational speed for different values of the parameter $\beta_{34}$. The calculations were carried out at $\lambda_1 = 1$ and the same values of the other parameters as indicated above. One can see that a change in the dissipative properties of the processed medium leads to a change in the stability margin near the third natural frequency of the oscillations of the system.

Analysis of the effect of the processed medium mass on the stability regions of synchronous antiphase rotation of the debalances showed that an increase in mass by 10 times leads to a slight expansion of the stability region between the second and third natural frequencies due to a decrease in the second natural frequency.
4. Conclusion
The proposed model of a jaw crusher with unilateral movements of the cheeks allows to take into account possible asymmetry in the properties of the forces of interaction between the processed medium and the cheeks at impactless operational mode. It is found that a decrease in the contact stiffness of the treated medium with one of the crusher's cheeks leads to a narrowing the frequency range of the stability of the debalances synchronous antiphase rotations. Herewith a change in dissipative characteristics of the material leads to a change only in the stability margin near the third natural frequency of the system.

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