Synchronized clusters in coupled map networks. II. Stability analysis

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We study self-organized and driven synchronization in some simple coupled map networks, namely globally coupled networks and complete bipartite networks, using both linear stability analysis and Lyapunov function approach and determine stability conditions for synchronization. The phase diagrams for the networks studied here have features very similar to the different kinds of structurally similar networks studied in Part I. Lyapunov function approach shows that when any two nodes are in driven synchronization, all the coupling terms in the difference between the variables of these two nodes cancel out, whereas when they are in self-organized synchronization, the direct coupling term between the two nodes adds an extra term while the other couplings cancel out. We also discuss the conditions for the occurrence of a floating node and suggest that the fluctuations of the conditional Lyapunov exponent about zero can be a criterion for its occurrence.

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I. INTRODUCTION

In Part I [1] of this paper we presented results of the numerical study of synchronization and cluster formation in coupled maps on different networks. Starting from random initial conditions the asymptotic behavior of these coupled map networks (CMNs) has revealed two interesting phenomena. First, there are two different mechanisms leading to two types of synchronized clusters. There are clusters with dominant intracluster couplings which are referred as self-organized clusters and there are clusters with dominant intercluster coupling which are referred as driven clusters [1,2]. The numerical studies reveal several clusters of both types as well as clusters of the mixed type where both mechanisms contribute. Second, there are floating nodes which show intermittent behavior between synchronous behavior with some cluster and an independent evolution. The residence times of a floating node in a synchronized cluster show an exponential distribution.

In the present paper we study the stability of synchronized dynamics of some simple coupled map networks (CMN) with a view to get a better understanding of the two mechanisms of cluster formation discussed above. For this study we use both linear stability analysis [3–9] and Lyapunov function approach [10,11]. As an example of networks showing self-organized synchronized clusters we take globally coupled maps [12] and for driven synchronized clusters we take complete bipartite coupled maps [13]. The Lyapunov function approach indicates that the self-organized behavior has its origin in the decay term arising due to intracluster couplings in the difference between the variables of any two synchronized nodes while the driven behavior has its origin in the cancellation of the intercluster couplings.

Synchronization of two dynamical variables is indicated by the appearance of some relation between the functionals of two variables [14,15,1,2]. In this paper we will mostly concentrate on exact synchronization, where the values of the dynamical variables associated with nodes are equal.

II. STABILITY ANALYSIS

We now present the stability analysis of synchronized states in globally coupled networks and complete bipartite networks. We also point out the similarity of these analytical results obtained for the simple networks with those observed numerically for complex networks in Part I and discuss possible reasons for this similarity. Section III discusses some aspects of the occurrence of the floating nodes. Section IV concludes the paper.

A. Globally coupled networks

Globally coupled networks have all pairs of nodes connected to each other, i.e., \( N_c = N(N-1)/2 \) where \( N \) is the total...