Spin-orbit interaction and the 'metal-insulator' transition observed in two-dimensional hole systems

L.E. Golub and S. Pedersen

1. INTRODUCTION

Quantum phenomena such as weak localization and electron-electron interaction was for many years considered as a very well exploited field of condensed matter physics. Among other things it was firmly believed, that a so-called Metal-Insulator Transition (MIT) would not occur in a two dimensional system — for a review see Ref. 1. Hence it came quite unexpectedly when Kravchenko et al. presented experiments, suggesting the existence of such a MIT in dilute two-dimensional electron systems 2 in a series of recent works, similar anomalous temperature dependence of the conductance has been observed in a wide range of dilute electron and hole systems, for an extensive overview see Ref. 3. The results of these reports are all consistent in the sense, that they support the original observations of Kravchenko et al.

Several quantum phenomena are known to give rise to metallic behaviour, so-called weak anti-localization which occurs in the presence of magnetic impurities, spin-orbit scattering 4 or when bandstructure-induced spin relaxation is present 5. Very recently it has also been demonstrated theoretically for degenerate systems, that the electron-electron interaction to the conductivity can undergo a 'metal-insulator' transition, depending on the value of the Fermi-liquid constant $F_{\sigma}$ 6. Hence from a fundamental point of view a metallic phase is indeed possible in 2D. However in real 2D systems weak anti-localization is usually disregarded, because of its expected weakness in comparison to the conductivity corrections due to electron-electron interaction. In this paper we advocate the idea, that in the dilute $p$-type systems with a moderate mobility, localization due to hole-hole interaction is not significant. This is due to the fact, that the strength of hole-hole interaction induced conductivity correction is weakened when $k_{B}T$ becomes comparable with the Fermi energy $E_{F}$. Only when $k_{B}T/E_{F} \ll 1$, a logarithmic or linear hole-hole correction dominates over the weak localization correction. The systems in question are, due to their high effective mass, in a regime where $E_{F}/k_{B}T \approx 1$ at least for the highest range of experimentally accessible temperatures. This means that the corrections due to hole-hole interaction can be considered inferior in comparison to e.g. weak anti-localization in the diffusive regime.

We theoretically address a certain group of the dilute GaAs/AlGaAs $p$-type systems which have been experimentally studied in Refs. 13,14,15,16 and have in common that the samples have a moderate mobility (in contrast to the hole experiments presented by Mills et al. which were performed at very high mobilities). The results of these experiments can be summarized as follows: A transition between an isolating phase at low hole densities and a metallic phase at high hole densities is observed at a critically hole density $p_{c} \approx 0.3 \ldots 1.3 \times 10^{11} \text{cm}^{-2}$ — by isolating (metallic) phase we mean that the resistivity increases (decreases) with decreasing temperature.

The rest of this paper we divide in the following sections. In Section I we discuss the different types of quantum corrections to the conductivity in weakly disordered 2D systems. In Section II we derive Cooperon spin relaxation rates performing bandstructure calculations of the highly non-parabolic hole energy spectrum in $p$-type quantum wells. Finally we present in Section III the temperature dependence of the resistivity of a $p$-type quantum well at different hole densities and compare it with the performed experiments.
TABLE I: The resistance at which the transition occurs $\rho$, transport relaxation time $\tau_{tr}$, critical densities $p_c$, hole effective mass at the Fermi level $m^*$, transition temperature between ballistic and diffusive transport $T^*$ and Fermi temperature $T_F$ extracted from the studied dilute GaAs/AlGaAs $p$-type quantum wells.

| Author          | Resistance ($\rho$) (k$\Omega$) | Transport time ($\tau_{tr}$) (ps) | Critical density ($p_c$) ($10^{11}$ cm$^{-2}$) | Effective mass ($m^*/m_0$) | Transition temperature ($T^*$) (K) | Fermi temperature ($T_F$) (K) |
|-----------------|----------------------------------|----------------------------------|-----------------------------------------------|----------------------------|----------------------------------|-------------------------------|
| Hanein et al.   | 5                                | 8.3                              | 0.25                                          | 0.12                       | 2.3                              | 5.6                           |
| Simmons et al.  | 10                               | 2.0                              | 0.51                                          | 0.13                       | 8.7                              | 11.6                          |
| Yoon et al.     | 3                                | 27.2                             | 1.27                                          | 0.16                       | 0.5                              | 22                            |

II. CONDUCTIVITY CORRECTIONS IN 2D SYSTEMS

As mentioned in the Introduction, there exists several conductivity corrections in weakly disordered systems. These quantum corrections can be divided into two categories — interference effects and hole-hole interaction effects.

The quantum correction due to inter-particle interactions were recently revised in the paper by Zala et al.\textsuperscript{3} Their main result was that, when carefully taking into account both the exchange and Hartree interaction, the sign of the conductivity correction could change due to the competition between these two contributions. The sign of the conductivity correction depends on the value of the interaction constant $F_0^\phi$. As expected they also found that for diffusive systems ($k_B T \tau_{tr}/h < 1$) the temperature correction is logarithmic, whereas for ballistic systems ($k_B T \tau_{tr}/h > 1$) the correction is linear. However in case where the Fermi energy becomes comparable to the Fermi temperature $T_F$, the transition reduces to a temperature independent constant.

To estimate the value of the Fermi energy we have used $E_F = E(k_F)$, where $E(k)$ is the hole energy dispersion (its calculation is described in the next Section), and $k_F$ is the Fermi wavevector connected with the density at which the 'metal-insulator' transition occurs, $p_c$, via $k_F = \sqrt{2\pi p_c}$. In Table I we display the calculated values together with the characteristic temperature $T^* = h/k_B \tau_{tr}$ which determines the cross-over between ballistic and diffusive transport. The transport time $\tau_{tr}$ was estimated from the experimental data by using $\tau_{tr} = m^* \mu/e$, where $\mu$ is the Drude mobility, which should not to be confused with the somewhat misleading peak mobility $\mu_{\text{peak}}$ often given in the experimental reports. The effective hole mass is given by $m^* = h k_F/v_F$ where the Fermi velocity

$$v_F = \frac{1}{\hbar} \frac{dE}{dk} \bigg|_{k = k_F}.$$  

The interference effects known as weak localization arise from the constructive interference of pairs of time inversion symmetric scattering paths. In non-interacting two-dimensional systems their contribution to the conductivity can be expressed as\textsuperscript{11}

$$\Delta \sigma_{wl}(T) = \frac{e^2}{\pi \hbar} \ln (\omega_{\phi}^2).$$  

(2)

Here $\omega_{\phi}^2 = \tau_{tr}/\tau_{\phi}$ where $\tau_{\phi}$ is the phase coherence time given by\textsuperscript{11}

$$\frac{1}{\tau_{\phi}} = \frac{k_B T}{\hbar k_F l} \ln (k_F l),$$  

(3)

and $l$ is the mean free path. To summarize, we see that both interference and hole-hole interactions can give rise to isolating temperature dependences.

We now turn to the situation where the spin-orbit interaction is included in the conductivity corrections. In the case of $p$-type quantum wells, the conductivity correction has the form\textsuperscript{11}

$$\Delta \sigma_{wl}(T) = \frac{e^2}{2\pi \hbar} \left[ 2 \ln (\omega_{\phi}^2 + \omega_{\parallel}) + \ln (\omega_{\phi}^2 + \omega_{\perp}) - \ln (\omega_{\phi}^2) \right],$$  

(4)

where $\omega_{\parallel,\perp} = \tau_{tr}/\tau_{\parallel,\perp}$ are the relaxation rates of the Cooperon spin components parallel and perpendicular to the plane of the quantum well, respectively.

It is interesting to consider this expression in the limit of very weak spin-orbit interaction $\omega_{\perp,\parallel} \ll 1$ and in the regime where spin-orbit interaction is strong $\omega_{\perp,\parallel} \approx 1$. In the first case the weak localization correction reduces to Eq. (2), since we neglect spin-orbit interaction. In the case of strong spin-orbit interaction however we find

$$\Delta \sigma_{wl}(T) = -\frac{e^2}{2\pi \hbar} \ln (\omega_{\phi}^2).$$  

(5)

It is important to note the significant differences between the expressions (2) and (3). When introducing strong spin-orbit interaction, the sign of the conductivity correction changes, corresponding to so-called weak anti-localization. Hence if in the regime of strong spin-orbit interaction one only considers the interference correction, the temperature dependence of the conductivity indeed is metallic.

In experiments it is of cause an interplay of both the hole-hole correction $\Delta \sigma_{hh}(T)$ and the weak localization
correction $\Delta \sigma_{wl}(T)$ which is observed

$$\Delta \sigma(T) = \Delta \sigma_{wl}(T) + \Delta \sigma_{hh}(T),$$

(6)

hence the overall conductivity correction is dominated by the hole-hole interaction when $k_B T \ll E_F$. In situations where $k_B T \approx E_F$ the effect of hole-hole interaction however is small in comparison with weak localization or weak anti-localization. In this regime we find that a metallic temperature dependence of the conductivity indeed is possible when spin-orbit interactions are present.

III. SPIN AND PHASE RELAXATION IN p-TYPE QUANTUM WELLS

In symmetrical quantum wells, each hole level at a given wavevector is double-degenerate. The two states, $|1\rangle$ and $|2\rangle$, connected to each other by the time-inversion operation, have spin projections on the growth axis equal to $\pm 3/2$ at the bottom of the subband. At finite kinetic energy, mixing of the heavy- and light-hole states occurs due to strong spin-orbit interaction present in $p$-type structures. Hence even under spin-independent scattering, transitions $|1\rangle \rightarrow |2\rangle$ are allowed which leads to relaxation of hole spins. It was shown that the Cooperon spin relaxation in $p$-type quantum wells is anisotropic and the weak localization correction to conductivity is given by Eq. (4), with two independent parameters $\omega_{||}$ and $\omega_{\perp}$ determined by the matrix elements of hole scattering:

$$\omega_{||} = 1 - \frac{V_{11}^2}{|V_{11}|^2 + |V_{12}|^2},$$

(7)

$$\omega_{\perp} = \frac{2 |V_{12}|^2}{|V_{11}|^2 + |V_{12}|^2}.$$

(8)

The bar here means averaging over the scattering angle of holes at the Fermi surface. Eqs. (7), (8) show the rate $\omega_{\perp}$ is non-zero mainly due to spin-flip scattering and $\omega_{||}$ is finite because of the change of the spin-conserving scattering probability with increasing of the kinetic energy ($\text{Re}V_{11} \gg 3mV_{11,1}|V_{12}|$).

The value of $\omega_{\perp}$ is equal to $\tau_{tr}/\tau_s$, where $\tau_s$ is the hole pseudospin relaxation time, characterizing decay of the hole polarization. It could be obtained from both optical orientation and weak localization experiments. The value $\omega_{\perp}/2\tau_s = 1/2\tau_s$ has been calculated in Ref. [19] for different scattering potentials and quantum well widths.

The parameter $\omega_{||}$ is essentially different. It also describes decay of the zero harmonics of the density matrix, however the latter determines the Cooperon but not the pseudospin relaxation. Hence $\omega_{||}$ can not be obtained from optical experiments. It can however be extracted from magnetoresistance measurements as it has been done in Ref. [18].

As seen in Eq. (9) the dephasing rate $\omega_{\varphi}$ depends only logarithmically on the transport time. Therefore using the reasonable approximation $\ln(k_B l) \approx 2$, we get an expression for $\omega_{\varphi}$ which is independent of scattering parameters:

$$\omega_{\varphi} = \frac{2k_B T}{\hbar k_F v_F}.$$

(9)

Here it has been taken into account that the mean free path is connected to the hole velocity at the Fermi level via $l = v_F \tau_{tr}$.

FIG. 1: The Cooperon spin relaxation rates $\omega_{||,\perp}$ in a symmetrical $p$-type quantum well with a width $a = 200$ Å. Dashed lines in inset show the asymptotic dependences given by Eq. (11).

To obtain the spin relaxation and dephasing rates, we used the isotropic Luttinger Hamiltonian for calculation of hole states in a symmetrical quantum well. Assuming the barriers to be infinitely high, we get the hole energy dispersion in the first size-quantized subband, $E(k)$. The transcendental equation for finding $E(k)$ is given, e.g., in Ref. [20]. In this model, the dispersion is determined only by one parameter, namely by ratio of the bulk light- and heavy-hole effective masses, $m_l/m_h$. In calculations we use $m_l/m_h = 0.16$ corresponding to GaAs-based quantum wells. We take the four-component wavefunctions in the form proposed in Ref. [20] and choose the scattering potential as

$$V(r) = \sum_i V_0 \delta(r - R_i),$$

(10)

where $R_i$ are random positions of short-range scatterers (e.g., impurities). Averaging squares of matrix elements in Eqs. (7), (8) over $R_i$, we get $\omega_{||}$ and $\omega_{\perp}$ as functions of a dimensionless hole density, $p \times a^2$. Here $a$ is the width of the quantum well, and the density is connected to the Fermi wavevector as $p = k_F^2/2\pi$.

In Fig. 1 the Cooperon spin relaxation rates $\omega_{||,\perp}$ are presented by solid lines. It is important to note that for
short-range scattering $|t_0|$, $\omega_\parallel$ and $\omega_\perp$ are independent
of the strength of the potential, $V_0$. They are determined
only by the parameters $m_1/m_h$ and $p \times a^2$. In this sense
the dependences in Fig. 1 are universal.

In the lowest order in $p \times a^2 \ll 1$, the density
dependences of spin relaxation rates are as follows:

$$\omega_\parallel \sim (pa^2)^2, \quad \omega_\perp \sim (pa^2)^3. \quad (11)$$

These approximations are plotted by dashed lines in the
insets in Fig. 1. One can see that Eq. (11) is valid only
at extremely low density. The spin-orbit effects of higher
orders start to be important already at $p \geq 0.2/a^2$, and
the exact calculation of hole states has to be performed
for obtaining the spin relaxation rates.

The density dependence of the dephasing rate $\omega_\varphi$ is
presented in Fig. 2 for different quantum well widths
and $T = 1$ K. Because of strong non-parabolicity of
the hole energy dispersion, the Fermi velocity is a non-linear
function of $k_F$, and, hence, $\omega_\varphi$ changes with $p$ non-monotonically.

The inset in Fig. 2 shows the density dependence of
the parameter $k_Fl$ at $\tau_{tr} = 2$ ps. It is seen that $k_Fl$ also
reflects the complicated dependence of the density of hole
states at the Fermi level on concentration.

![Graph](image_url)

**FIG. 2:** The dephasing rate $\omega_\varphi$ at $T = 1$ K at different quantum well widths as a function of hole density. (1) $a = 100$ Å, (2) $a = 200$ Å, (3) $a = 300$ Å. The inset displays the density dependence of the parameter $k_Fl$ for $\tau_{tr} = 2$ ps.

**IV. THE METAL-INSULATOR TRANSITION IN $p$-TYPE SYSTEMS**

As seen in Table I the estimated Fermi temperatures
$T_F$ is comparable the highest temperatures obtained in

the experiments in question. This means that, when con-
considering the quantum correction to the conductivity, it is
possible to neglect the contribution from hole-hole inter-
actions. Hence the total resistivity in this temperature
regime is given by

$$\rho(T) = \frac{h}{e^2} \left[ \frac{1}{k_Fl} - \frac{1}{(k_Fl)^2} \frac{\Delta \sigma_{wl}(T)}{e^2/h} \right], \quad (12)$$

where the first term is the classical Drude expression and the second is the weak localization correction [see Eq. (11)].

In Fig. 3 we plot the calculated temperature dependence
of the resistivity of a 200 Å wide symmetrical quantum
well in units of $h/e^2 = 25.7$ kΩ for different densities $p$ in the range $0.8 \ldots 2 \times 10^{11}$ cm$^{-2}$. To obtain the pre-
sented curves we have used the values of $\omega_\varphi$, $\omega_\perp$ and $\omega_\parallel$
calculated in the previous section and a constant trans-
port time has been taken $\tau_{tr} = 2$ ps. Both the width of
the quantum well, $a$, and the transport time, $\tau_{tr}$, are
chosen to be close to the relevant values in the experiments
of Simmons et al.— see Table I.

From the curves it is seen that at relatively low hole
densities the resistivity increases with decreasing tempera-
tures (isolating behaviour). At higher hole densities the
opposite situation occurs (metallic behaviour). Hence
we attribute the metal/isolator transition with a weak
anti-localization/localization transition. This idea has
already been analysed in relation to MIT in electron sys-
tems. However the spin-orbit Hamiltonian in $p$-type
systems is significantly different from the linear in the
wavevector term in electron systems, therefore MIT in

hole systems cannot be described by the theory Ref. [2].
The theoretically observed transition occurs due to the density dependence of the spin scattering. The critical density where the cross-over between the two different temperature dependences occurs is approximately \( p_c \approx 1 \times 10^{11} \text{cm}^{-2} \), corresponding to \( p \times a^2 \approx 0.4 \). When comparing with the critical hole densities found in the experimental reports, we find good agreement between the observed and the predicted transition density — see Table I. Especially it is worthwhile to note the good agreement between theory and the experiments by Simmons et al. and Hanein et al. which were performed on a 200 Å wide symmetrical quantum well in accordance with our assumptions.

The calculated temperature dependences furthermore display a weak non-monotonic features close to the critical density — see the curves for \( p = 1.2 \times 10^{11} \text{cm}^{-2} \) and \( p = 2 \times 10^{11} \text{cm}^{-2} \). Hence close to the critical density the temperature dependence of the resistivity displays a local maxima. This behaviour is in agreement with experiments and was especially pronounced in the reports of E. Abrahams, S.V. Kravchenko and M.P. Sarachik, Rev. Mod. Phys. 73, 251 (2001) and Simmons et al. and Mills et al. This peculiarity of the interference correction is included into Eq. (2). Indeed, for very low temperatures the conductivity correction is metallic, but for high temperatures the system is insulating. For a given density this local maximum takes place when \( \partial \Delta \sigma_{wl} / \partial T = 0 \), i.e. at

\[
\omega_\rho = \sqrt{\left(\frac{\omega_\perp}{4}\right)^2 + \frac{\omega_\parallel \omega_\perp}{2} - \frac{\omega_\parallel}{4}}. \tag{13}
\]

This equation connects the density with temperature at the maximum. The corresponding dependence \( p_{\text{max}}(T) \) shown in Fig. 4 represents the ‘phase diagram’ of the diluted hole gas.

V. CONCLUSION

We have calculated the density dependence of the spin and phase relaxation rates in a p-type quantum well. By applying these results to the theory of weak-localization, we find that it is possible to explain the so-called ‘metal-insulator’ transition which is observed in a range of 2D hole systems at temperatures \( k_B T \approx E_F \) and at moderate mobilities. This transition is explained as a crossover between weak localization and weak anti-localization due to the strong density dependence of the spin and phase relaxation rates. We have made a direct comparison between our results and the experimental work by Simmons et al. and find a good agreement for the observed critical densities \( p_c \). For the sake of completeness we have in Table I listed the values found for several other relevant experiments.

We underline that our theory only is applicable in the regime \( k_B T \approx E_F \) and hence do not address situations where the system is degenerate, which normally is the case for electrons.

Finally we would like to encourage experimentalists to apply optical techniques and hence shine light on the connection between the metal-insulator transition and spin-relaxation. We also attract their attention to the non-monotonous temperature dependences of resistivity and the corresponding ‘phase diagram’ of the 2D hole system.

Acknowledgements

This work is financially supported by the RFBR, by the Programmes of Russian Ministry of Science and Presidium of RAS, and by INTAS.

---

1 P.A. Lee, T.V. Ramakrishnan, Rev. Mod. Phys. 57, 287 (1985).
2 S.V. Kravchenko, G.V. Kravchenko, J.E. Furneaux, V.M. Pudalov and M. D’Iorio, Phys. Rev. B. 50, 8039 (1994).
3 E. Abrahams, S.V. Kravchenko and M.P. Sarachik, Rev. Mod. Phys. 73, 251 (2001).
4 S. Hikami, A.I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. 63, 707 (1980).
5 W. Knap, C. Skierbiszewski, A. Zduniak, E. Litvin-Staszevska, D. Bertho, F. Kobbi, J.L. Robert, G.E. Pikus, F.G. Pikus, S.V. Iordanskii, V. Mosser, K. Zekentes, and Yu.B. Lyanda-Geller, Phys. Rev. B. 53, 3912 (1996).
6 N.S. Averkiev, L.E. Golub, and G.E. Pikus, JETP 86, 780 (1998) [Zh. Exp. Teor. Fiz. 113, 1429 (1998)].
7 N. S. Averkiev, L. E. Golub and G. E. Pikus, Solid State Commun. 107, 757 (1998).
8 N. S. Averkiev, L. E. Golub and G. E. Pikus, Semiconductors 32, 1087 (1998) [Fiz. Techn. Poluprov. 32, 1219 (1998)].
9 S. Pedersen, C.B. Sørensen, A. Kristensen, P.E. Lindelof, L.E. Golub, and N.S. Averkiev, Phys. Rev. B 60, 4880 (1999).
10 G. Zala, B.N. Narozhny, and I.L. Aleiner, Phys. Rev. B 64, 214204 (2001).
11 B.L. Altshuler, A.G. Aronov, D.E. Khmelnitskii and A.I. Larkin, in Quantum Theory of Solids, edited by I.M. Lifshits (MIR Publishers, Moscow, 1983)
12 B.L. Altshuler, D.L. Maslov, and V.M. Pudalov, Physica E 9, 209 (2000).
13 M.Y. Simmons, A.R. Hamilton, M. Pepper, E.H. Linfield, P.D. Rose, and D.A. Ritchie, Phys. Rev. Lett. 84, 2489 (2000).
14 M.Y. Simmons, A.R. Hamilton, M. Pepper, E.H. Linfield, P.D. Rose, D.A. Ritchie, A.K. Savchenko, and T.G. Griffiths, Phys. Rev. Lett. 80, 1292 (1998).
15 Y. Hanein, U. Meirav, D. Shahar, C.C. Li, D.C. Tsui, and H. Shtrikman, Phys. Rev. Lett. 80, 1288 (1998).
16 J. Yoon, C.C. Li, D. Shahar, D.C. Tsui, and M. Shayegan, Phys. Rev. Lett. 82, 1744 (1999).
17 A.P. Mills,Jr., A.P. Ramirez, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 83, 2805 (1999).
18 B.L. Altshuler, A.G. Aronov, and P.A. Lee, Phys. Rev. Lett. 44, 1288 (1980).
19 R. Ferreira and G. Bastard, Phys. Rev. B 43, 9687 (1991).
20 I.A. Merkulov, V.I. Perel, and M.E. Portnoi, Sov. Phys. JETP 72, 669 (1991) [Zh. Exp. Teor. Fiz. 99, 1202 (1991)].
21 Y. Lyanda-Geller, Phys. Rev. Lett. 80, 4273 (1998).
22 I.V. Gornyi, A.P. Dmitriev, and V.Yu. Kachorovskii, JETP Letters 68, 338 (1998) [Pis’ma Zh. Eksp. Teor. Fiz. 68, 314 (1998)].