A GLOBAL PHOTOMETRIC ANALYSIS OF 2MASS CALIBRATION DATA

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ABSTRACT

We present results from the application of a global photometric calibration (GPC) procedure to calibration data from the first 2 years of the Two Micron All Sky Survey (2MASS). The GPC algorithm uses photometry of both primary standards and moderately bright "tracer" stars in 35 2MASS calibration fields. During the first 2 years of the survey, each standard was observed on approximately 50 nights, with about 900 individual measurements. Based on the photometry of primary standard stars and secondary tracer stars and under the assumption that the nightly zero-point drift is linear, the GPC ties together all calibration fields and all survey nights simultaneously, producing a globally optimized solution. Calibration solutions for the Northern and Southern Hemisphere observatories are found separately and are tested for global consistency based on common fields near the celestial equator. Several results from the GPC are presented, including establishing candidate secondary standards, monitoring of near-infrared atmospheric extinction coefficients, and verification of the global validity of the standards.

The solutions give long-term averages of the atmospheric extinction coefficients (in magnitudes per air mass, \( A_J = 0.096, A_H = 0.026, A_K = 0.066 \) (north) and \( A_J = 0.092, A_H = 0.031, A_K = 0.065 \) (south), with formal error of 0.001. The residuals show small seasonal variations, most likely due to changing atmospheric content of water vapor. Extension of the GPC to ~100 field stars in each of the 35 calibration fields yields a catalog of more than 2000 photometric standards ranging from the 10th to 14th magnitude, with photometry that is globally consistent to ~1%.

Key words: methods: data analysis — standards — surveys

1. INTRODUCTION

The Two Micron All Sky Survey (2MASS) maps the entire sky in three near-infrared (NIR) bands, \( J (1.25 \mu m), H (1.65 \mu m) \), and \( K_s (2.16 \mu m) \). The survey operates in both hemispheres, with the northern facility at Mount Hopkins, Arizona, and the southern site at Cerro Tololo, Chile. Both sites acquire data with nearly identical 1.3 m Cassegrain equatorial telescopes optimized for efficient sky coverage. Each telescope is equipped with three NICMOS3 arrays capable of simultaneous observations in three NIR bands. The northern 2MASS facility began routine survey observations in 1997 June, while the southern facility started in 1998 March.

Nightly photometric calibration for 2MASS is derived using repeated observations of \( 1^\circ \times 8.5 \) calibration “fields” (see Table 1). Each hour, one of 40 calibration fields is scanned six times in the normal survey mode, providing six independent measurements of the standard stars in each field. The 2MASS calibration fields were selected initially to contain at least one “primary” calibration star drawn from the list of faint infrared standards published by Persson et al. (1998) and Casali & Hawarden (1992). One of the fields, 92409, does not contain an a priori standard star but was selected to fill a gap in the right ascension coverage of the fields. Photometry for the calibration stars in this field was developed using the techniques described in § 3.1 of this work. Table 1 contains a listing of the 2MASS calibration fields and the a priori catalog magnitudes for the primary calibration stars in each. The field centers listed are the coordinates of the primary calibration stars. The analysis described in this paper uses 2MASS pipeline source extractions and photometry from repeated observations of 35 of the calibration fields.

The paper has three major parts: § 2, which introduces the global photometric calibration method and compares it with nightly calibration; § 3, which presents the results of major applications of the procedure to 2MASS data; and § 4, which restates and summarizes the main results of the analysis. The major applications of the global photometric calibration (GPC) are (1) establishment of secondary standards (§ 3.1), (2) study of atmospheric extinction in the near-infrared (§ 3.2), and (3) analysis of global consistency (§ 3.3). Technical aspects of the method are given in Appendix A.

2. GLOBAL PHOTOMETRIC CALIBRATION

The purpose of nightly photometric calibration is to transfer instrumental magnitudes, \( m_{\text{inst}} \), onto a uniform photometric system \( m_{\text{cal}} \):

\[
m_{\text{cal}} = m_{\text{inst}} + C_1 - C_2 (X - 1) \tag{1}
\]

where \( C_1 \) is the photometric zero point, \( C_2 \) is the atmospheric extinction correction, and \( X \) is the air mass. Figure 1 plots a typical night of calibration data, showing both the data points (photometry of primary standards) and the nightly calibration solution \( C_1 \) (straight line). Each cluster of points in the nightly calibration plot is derived from one
set of calibration observations. Each cluster consists of six data points that plot the zero-point offset \([i.e., m_{cal} - m_{inst} + C_2(X - 1)]\) derived for each of the six apparitions of the primary calibration star. Because the survey’s 2" camera pixels are larger than the typical \(~1\"\) seeing disk, pixelization flux errors dominate the scatter of the bright primary standard observations and limit the rms uncertainty for multiple apparitions of bright stars in 2MASS observations to \(~0.02-0.03\) mag—underscoring the advantage of using several flux calibrators in each field to minimize the zero-point uncertainty of a single calibration observation.

Experience has shown that the nightly photometric zero point, \(C_1\), is usually either a constant or a linear function of time, \(C_1(t) = A + Bt\). Equation (1) is written for a set of primary calibrators with known \(m_{al}\) (Table 1), independently for each NIR band. Solving the calibration equation by a standard least-squares algorithm produces a nightly photometric calibration solution, i.e., \(C_1\) and \(C_2\).

The main difference between nightly and global calibration procedures is in the scope of the data upon which the solution is based: the former derives the calibration parameters \(C_1\) and \(C_2\) from a single night of data, while the latter uses all survey nights. The calibration equation is the same in both cases, but for GPC, it can be rewritten to underscore its global nature:

\[
m_{ijk}^{\text{inst}} = m_{ik}^{\text{al}} + a_n + b_n \Delta t_{jn} + A(X_{jn} - 1),
\]

where \(m_{ijk}^{\text{inst}}\) is the instrumental magnitude of the \(i\)th star in the \(j\)th calibration field, observed at time \(t_j\) during the \(n\)th survey night, \(m_{ik}^{\text{al}}\) is the true photometric magnitude of the
Fig. 1.—Nightly calibration solutions in three bands for a typical night. The x-axis units are hours. Data points represent calibrated photometry of primary standards, $m_{\text{inst}} - m_{\text{cal}} + C_j(X - 1)$; straight lines show the photometric zero point $C_j$ (see text). Calibration fields are indicated. Note the second-order effect in the $J$ band (nonlinear behavior of sensitivity), which is not addressed by the procedure but tends to be small.

star, $a_n$ and $b_n$ are nightly photometric constants (one pair for each night), and $\Delta t_{\text{on}}$ is the time offset between $t_j$ and midnight. The atmospheric extinction coefficient is now denoted by $A$. Equation (2) implicitly assumes that atmospheric extinction is constant. Seasonal variations in extinction coefficients (see § 3.2) are thereby smoothed and represented by their average values. The equation is applied to each photometric band separately and produces one solution for each band. Note that the global procedure does not rely on a priori knowledge of $m_{\text{cal}}$; the procedure can solve for these magnitudes as well. However, in that case the calibrated magnitudes will be determined to within an arbitrary constant offset.

The global calibration procedure (eq. [2]) provides a better photometric solution than nightly calibration, since it minimizes residuals globally. The benefit of global calibration is conveyed by Figure 2: each night's photometry is calibrated through nightly parameters $a$ and $b$, tying fields observed that night to each other, while different nights are tied together by global parameters $m_{\text{cal}}$ and $A$. For comparison, Figure 3 shows the actual survey sky coverage of calibration fields as a function of time.

In practice, the set of calibration equations is written for a group of primary standards and so-called tracer stars, selected among relatively bright field stars. The same number of tracers, $\mu$, is used in each calibration field. In principle, any number of tracers can be used to derive the global calibration solution. However, the large data sets associated with the problem prohibit using more than a few tracers per calibration field. The set of equations is solved by a standard linear least-squares algorithm (see Appendix A), producing a set of photometric nightly constants $(a_n, b_n)$ for all survey nights, values of atmospheric extinction $A$, and calibrated magnitudes $m_{\text{cal}}$ for primary standards and tracer stars. Once photometry is established for the primary and tracer stars in the field, it can be directly applied to any of the other stars in the field.

3. APPLICATIONS OF GLOBAL PHOTOMETRIC CALIBRATION

This section lists several applications of the global photometric calibration: establishment of secondary standards, study of atmospheric extinction, and demonstration of temporal and spatial consistency. The discussion in this section is based on global calibration solutions for northern data taken from 1997 May to 1999 May and southern data from 1998 March to 1999 February. Calibration solutions for north and south are calculated separately and compared.
through common calibration fields. The global solution was obtained in two steps. First, we obtained a preliminary calibration solution based on the entire set of nights. This preliminary solution was used to create nightly calibration plots similar to the one shown in Figure 1. The nightly plots were examined and the few with maximal deviations more than 0.05 mag from linear behavior were removed from the data set. We then reran the procedure on the new “clean” data to obtain the final calibration solution. Northern data consisted of a total of 14,833 calibration scans of 26 fields taken on 298 nights. Solving equation (2) by a linear least-squares algorithm amounts to inverting a sparse \(37,758 \times 675\) matrix. The dimensions of the matrix, \(N \times M\), are the total number of observations \(N\) and the number of free parameters \(M = k(\mu + 1) + 2n + 1\), where \(k\) is the number of calibration fields, \(\mu\) is the number of tracer stars in each field, and \(n\) is the number of survey nights. In the southern data, there are 227 observing nights for 26 fields (11,945 total calibration scans), and the solution is found by inverting a smaller, \(33,407 \times 533\) matrix. Both hemispheric solutions are obtained based on the photometry of fiducial tracers and two tracer stars in each calibration field (i.e., \(\mu = 2\)). In solving equation (2), we look for an unconstrained solution (i.e., calibrated magnitudes of primary standards are free parameters; see Appendix A), to ensure that no bias enters from the known magnitudes of primary standards. Because the solution is unconstrained, the GPC-
calibrated magnitude system will have an arbitrary constant offset in each band. The offset for each band is found by averaging the difference between GPC solution and a priori catalog magnitudes of the fiducial standards in Table 1. The GPC system magnitudes for the primary standards in each field are given in Table 3.

3.1. Secondary Standards

To establish candidate secondary standards in the calibration fields, we apply the GPC solution to additional field stars. To exclude variable stars, secondary standard candidates must have a low rms uncertainty,

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (m_i - \bar{m})^2}$$,

where $N$ is the number of individual observations (scans) of a source. To ensure the statistical significance, we keep only the candidates with $N > 100$. Figure 4 shows the pooled rms plots for all solutions. Each point in the plot represents a single star. Dashed lines, which indicate the adopted thresholds for rms error $\sigma$, were obtained by fitting a fourth-order polynomial $f(m)$ to the data and adjusting its intercept point so that the number of points below the curve was 66% (1 $\sigma$) of the total. Toward fainter magnitudes $f(m)$ rises sharply, so we modify our $\sigma$ threshold to pick sources with

$$\sigma < \min \{f(m), 0.05\}.$$

The above selection criteria produce a sample of low-variance stars, which are good candidates for secondary standards. The high-variance sources that fail our $\sigma$-criterion are often variable stars, which can be reliably identified by examining their light curves.

After applying the GPC solution to each hemisphere, we obtain two lists of candidate secondaries, one for each hemisphere. The magnitudes of stars in equatorial fields observed from both hemispheres are merged using a variance-weighted average:

$$m = \left( \frac{m_N + m_S}{\sigma_N^2 + \sigma_S^2} \right) \left( \frac{1}{\sigma_N^2} + \frac{1}{\sigma_S^2} \right),$$

where $m_N$ and $m_S$ are average magnitudes for the star in the northern and southern solutions, and $\sigma_N$ and $\sigma_S$ are the respective root variances. The list of 2189 candidate secondary standards for all calibration fields is given in Table 3, in Appendix B. This list is used by the 2MASS project to select a subset of unconfused point sources for actual calibration of 2MASS data.

3.2. Atmospheric Extinction Coefficients

The GPC solution provides an estimate of the atmospheric extinction in all three bands. The atmospheric extinction in the NIR is caused primarily by molecular bands of H$_2$O (Hall & Genet 1982; Mountain et al. 1985) and, to a lesser extent, by CO$_2$ (Manduca & Bell 1979). In equation (2), extinction enters through the coefficient $A$, which represents the average value of the atmospheric extinction over the period sampled (one number for each band). To decouple the atmospheric extinction parameters from photometric zero points (sensitivities), the observations during each night span a range of air masses ($X$). A more detailed model for atmospheric extinction, for example, involving a periodic function to mimic seasonal variations, could also be considered in equation (2). However, any extinction model can be devised and applied

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2 A third-order polynomial was used in the $K_s$ band.

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![Fig. 4](image_url) — Pooled rms error plots for both hemispheric solutions. Top row, northern solution; Bottom row, southern solution. Each dot represents one star. Dashed lines show the adopted thresholds (see text).
TABLE 2

ATMOSPHERIC EXTINCTION COEFFICIENTS FROM GPC

| Band | Hemisphere | 2MASS $A_j$ | Frogel $A_j$, CTIO |
|------|------------|-------------|------------------|
| $J$   | North      | 0.096 ± 0.001 | ...             |
|      | South      | 0.092 ± 0.001 | 0.100 ± 0.024   |
| $H$   | North      | 0.026 ± 0.001 | ...             |
|      | South      | 0.031 ± 0.001 | 0.055 ± 0.021   |
| $K_s$ | North      | 0.066 ± 0.001 | ...             |
|      | South      | 0.065 ± 0.001 | 0.085 ± 0.018   |

**NOTE.**—The extinction coefficients are in magnitudes per unit air mass. The CTIO values from Frogel 1998 are listed for comparison (note that CTIO value is given for $K$ band, not $K_s$).

a posteriori, once we compute the residuals between the regressed data and the observations, $\Delta m = m_{\text{cal}} - m_{\text{inst}}$. One such model is presented in § 3.3.

The atmospheric extinction coefficients for north and south produced by the GPC are given in Table 2, which also lists Cerro Tololo Inter-American Observatory (CTIO) values from Frogel (1998). The uncertainties in extinction coefficients are random errors, obtained by inversion of the Hessian matrix. The values listed in the table are consistent at the 2σ level. Figure 5 shows the residuals $m_{\text{inst}} - m_{\text{cal}} - a - b \Delta m$ as a function of air mass. The slopes of the linear regression lines are the mean extinction coefficients.

Given the time baseline of 2MASS and the homogeneity and accuracy of 2MASS photometric data, we have a great opportunity for studying atmospheric extinction in the NIR over the period of more than a year. Figure 6 shows the seasonal variations in atmospheric extinction. The monthly averages were derived by first calculating the residuals without the extinction term for each month separately and then by fitting a straight line to the distribution of residuals as a function of the air mass. While some scatter is present in Figure 6, the overall behavior of monthly extinction averages is in agreement with expectations. The water vapor content in the atmosphere peaks during northern spring and summer, which leads to higher atmospheric extinction in the north during that season. The amplitude of the seasonal changes is about 0.02 mag per air mass.

3.3. Global Consistency

The global consistency of the solution can be assessed from the analysis of residuals as a function of spatial coordinate (spatial uniformity) or time (temporal uniformity). In this section, we present the results of two such analyses. First, we examine the uniformity of the standards between the hemispheres, from looking at residuals $\Delta m = m_{\text{north}} - m_{\text{south}}$ for common calibration fields. Then, we test spatial and temporal uniformity by comparing the calibrated magnitudes of primary standards (derived from GPC) with literature photometry.

3.3.1. North versus South Uniformity

The linearity of the GPC solution is demonstrated in Figure 7, which shows the difference between catalog and GPC magnitudes for primary standards as a function of the catalog magnitude. The figure shows that the absolute value of the difference for most primaries is less than 0.02 mag, i.e., consistent with zero. The absence of any trends in these data demonstrates the linearity of the GPC solution. Note that the results of such a comparison are limited by the accuracy of the input catalog, since the errors in the fiducial magnitudes of primary standards are propagated unchanged. In fact, the good agreement demonstrates the high accuracy of the input catalog.

The comparison of the photometry between the hemispheres is carried out in equatorial fields (see Table 1). Data from each hemisphere were calibrated separately by unconstrained GPC. The solutions (nightly photometric constants $a$ and $b$ and extinction parameter $A$) were applied to the 100...
brightest stars in each calibration field. Figure 8 shows the difference between calibrated magnitudes of common stars as a function of the coordinate. Except for a few outliers, the largest excursions for individual secondary standards (crosses) are bounded by $\Delta m < 0.04$, while the largest excursions for the primary standards (squares) are less than 0.02 mag. The mean differences for the photometry of common standards (squares in Fig. 8) are $0.003 \pm 0.007$ (J), $-0.004 \pm 0.008$ (H), and $-0.004 \pm 0.008$ (Ks). The plot of north-south photometric offset versus declination displays a slight trend of unknown origin. The slopes of the corresponding regression lines in J, H, and Ks are $(-5.6 \pm 0.7) \times 10^{-4}$, $(-6.4 \pm 0.7) \times 10^{-4}$, and $(-3.5 \pm 0.5) \times 10^{-4}$ mag deg$^{-1}$, respectively. The same regression analysis using primary standards only gives slopes consistent with zero at the 2$\sigma$ level: the corresponding slopes are $(-4.9 \pm 2.8) \times 10^{-4}$ in J, $(-5.0 \pm 2.9) \times 10^{-4}$ in H, and $(-0.5 \pm 3.5) \times 10^{-4}$ in Ks. Figure 9 shows the same residuals as in Figure 8, except as a function of magnitude in the north. The slopes of the regression lines in all three bands are consistent with zero.

3.3.2. Temporal Stability

To analyze the temporal stability, we consider the difference between calibrated and fiducial magnitudes of primary standards as a function of time. Specifically, we calculate the difference “calibrated minus fiducial” for all primary standards observed on a given night and plot the average difference. Figure 10 shows the corresponding differences in all three bands for the Northern and Southern Hemispheres. The figure, which essentially indicates the long-term stability of the global calibration, shows a very small temporal variation in the photometry of the primary standards. The amplitude of the variations is less than 1%.

4. SUMMARY

We have presented a global photometric calibration procedure and applied it to more than a year of 2MASS calibration data. Exploiting the fact that the observed nightly zero-point drift is linear, one can simultaneously calibrate the photometry of fiducial standards and any number of tracer stars. The solution is found by using a standard least-squares algorithm and is globally optimized, in the sense that it is the best solution for all survey fields and nights. The solution does not presume a priori knowledge of the magnitudes for any of the stars but, instead, produces the optimal set of relative magnitudes. A constant offset is determined by averaging the differences between the calibrated and catalog magnitudes for the primary standards. The photometry of the primary and two tracer stars per field is transferred to the 100 brightest nonvariable stars in each field. With a $\sigma$ threshold, we select a low-dispersion sample of 2189 candidate secondary standards to accompany the set of primary 2MASS standards. The selected secondaries fall in the magnitude range $9.0 < J < 14.5$, $9.0 < H < 13.9$, $9.0 < K_s < 13.7$. 

Fig. 6.—Seasonal variations in the atmospheric extinction (in mag per air mass) in (a) the north and (b) the south. Monthly average coefficients in J, H, and Ks are plotted with triangles, squares, and circles, respectively. The errors are indicated. Dashed lines show global averages in three bands derived from the respective GPC solutions. No August data for the northern solution were available.

Fig. 7.—Differences between catalog and independently determined GPC magnitudes as a function of catalog magnitude for primary standards. Symbols represent J, H, and Ks magnitudes (triangle, squares, and circles respectively). Open symbols indicate Persson standards, filled symbols are for UKIRT standards. Most data points differ from zero by less than 0.02 mag. Note the absence of any trends over the entire magnitude range.
Using the global solution, we derive the atmospheric extinction. The global averages (in magnitudes per air mass) are $A_J/A_H/A_K = 0.096/0.026/0.066$ (north), and $A_J/A_H/A_K = 0.092/0.031/0.065$ (south). The averages are consistent with previous work at the 2σ level.

Seasonal variations in the atmospheric extinction are analyzed. The amplitude of the variations is approximately 0.02 mag per air mass and is consistent with the behavior expected from variations of the water vapor content in the atmosphere.

Fig. 8.—“North-south” difference between calibrated magnitudes of stars in common fields as a function of (a) right ascension and (b) declination. Crosses represent field stars, and squares represent primary standards. Clusters of points are calibration fields. Because of the elongated shape of the calibration fields in latitude, the clusters are more dispersed in the right panel. Dotted horizontal lines are drawn at $\Delta m = \pm 0.04$, dashed lines are at zero.

Fig. 9.—Difference (in magnitudes) between northern and southern solutions as a function of northern magnitude. The notations are the same as in Fig. 8.
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APPENDIX A

LEAST SQUARES

Equation (2) represents a linear least-squares (LLS) problem. The general LLS problem is well documented in the literature (Lawson & Hanson 1995; Greenbaum 1986) and has many professional codes available for its solution. Written in matrix form, the mathematical model for the GPC is

$$\Phi x = b,$$  

where $\Phi$ is the design matrix, $x$ is the parameter vector, and $b$ is the vector of observed (instrumental) magnitudes. The parameter vector, $x$, consists of the magnitudes of the fiducial standards in each field, followed by the magnitudes of the tracer stars for each field, followed by the photometric nightly constants $a$ and $b$ for each night and the atmospheric extinction coefficient $A$:

$$x^T = (m_1^{std}, \ldots, m_k^{std}, m_1^{0}, \ldots, m_k^{0}, m_1^{10}, \ldots, m_k^{10}, a_1, \ldots, a_n, b_1, \ldots, b_n, A),$$

where $k$ denotes the total number of calibration fields and $n$ denotes the total number of observation nights. The number of tracers in the field is $\mu$ (the same number of tracers in each field is used). The design matrix, $\Phi$, has the following structure:

$$\Phi = \begin{pmatrix} 
    k \text{ standards} & \mu \times k \text{ tracers} & n \text{ nights} & n \text{ nights} \\
    1, 0, \ldots, 0 & 0, 0, \ldots, 0 & 1, 0, \ldots, 0 & \Delta t_{jn}, 0, \ldots, 0 \\
    0, 1, \ldots, 0 & 0, 0, \ldots, 0 & 1, 0, \ldots, 0 & \Delta t_{jn}, 0, \ldots, 0 \\
    0, 1, \ldots, 0 & 0, 0, \ldots, 0 & 0, 1, \ldots, 0 & 0, \Delta t_{jn}, \ldots, 0 \\
    \vdots & \vdots & \vdots & \vdots \\
    0, 0, \ldots, 0 & 1, 0, \ldots, 0 & 1, 0, \ldots, 0 & \Delta t_{jn}, 0, \ldots, 0 \\
    \end{pmatrix} \quad X_{jn} - 1
$$

The tracer stars are grouped by fields; i.e., after the fiducial standards there are $n$ tracers from the first field, then there are $n$ tracers from the second field, and so on. There is a single unity in the first $k(\mu + 1)$ columns of each row of the design matrix,
representing the observation of either the fiducial standard or a tracer star. The subscript \( j \) denotes the time offset from midnight on night \( n \). The dimensions of the matrix are \( N \times M \), where \( N \) is the total number of observations of all fiducial standards and tracers and \( M = k(\mu + 1) + 2n + 1 \) is the number of free parameters.

### A1. Constrained Scheme

The least-squares solution of equation (2) is globally optimal (i.e., it minimizes \( \text{rms} \)) but, generally speaking, has an arbitrary offset from the true magnitude:

\[
(m_{ik}^{L,0})_{\text{true}} = (m_{ik}^{L,0})_{\text{sol}} + K_{ik}.
\]

To produce the optimal solution without the arbitrary additive constant \( K_{ik} \), one has to set the magnitudes of the fiducial standards as fixed points of the GPC. This results in a GPC with constraints, where the constraint equations are

\[
m_{ikn}^{\text{std}} = m_{ikn}^{\text{std,0}}
\]

(the superscript “std” denotes the fiducial standards in the fields). In matrix form, the calibration problem with constraints is written as

\[
\Phi x = b, \quad C x = d.
\]

For this particular problem, the constraint equations (eq. \([A4]\)) result in the identity constraint matrix \( C \), and the LLS problem with constraints is straightforward to solve using, e.g., an orthogonal basis for the null space of the constraint matrix (Lawson & Hanson 1995). The GPC with fixed points derives the globally optimal solution that is the closest to the listed magnitudes of the fiducial standards.

### Appendix B

#### Secondary Standards

Below we present a sample table of 2MASS secondary standards, which includes the primary and some of the secondaries for field 90021. The primary standard is the first star listed in each field in Table 3. Columns in the table are the J2000.0 equatorial coordinates of the stars, the globally calibrated magnitudes with their corresponding uncertainties, and the \( JHK_s \) number of observations (i.e., individual scans) in each band. The positions of the secondaries are accurate, with respect to the International Celestial Reference System, to better than \( 0.5 \) rms. Finding charts for the secondaries can be obtained by using the 2MASS visualizer tool. The full table of candidate secondary standards in all calibration fields is available from the FTP archives.

3 Finding charts are available only for calibration fields that are covered by the Second Incremental Data Release and are available at http://irsa.ipac.caltech.edu/applications/2MASS/ReleaseVis.

4 The FTP archives are available at ftp://nova.astro.umass.edu/pub/nikolaev/ and at ftp://anon-ftp.ipac.caltech.edu/pub/2mass/globalcal.

#### Table 3

| R.A. (s) | Decl. (deg) | J | \( \sigma_J \) | H | \( \sigma_H \) | K_s | \( \sigma_{K_s} \) | \( N_J \) | \( N_H \) | \( N_{K_s} \) |
|---------|-----------|---|-------------|---|-------------|-----|-------------|-----|-----|-------|
| Field 90021 |
| 6.1025  | -1.9728   | 11.860 | 0.016 | 11.079 | 0.016 | 10.559 | 0.018 | 793 | 823 | 817 |
| 6.0386  | -2.2663   | 13.858 | 0.029 | 13.253 | 0.032 | 13.146 | 0.044 | 450 | 464 | 463 |
| 6.0427  | -2.2817   | 11.918 | 0.026 | 11.552 | 0.025 | 11.483 | 0.026 | 740 | 767 | 762 |
| 6.0430  | -2.0241   | 11.891 | 0.019 | 11.240 | 0.019 | 11.088 | 0.019 | 748 | 775 | 770 |
| 6.0451  | -1.7189   | 11.365 | 0.019 | 10.878 | 0.019 | 10.797 | 0.019 | 768 | 798 | 792 |
| 6.0516  | -2.0484   | 13.700 | 0.022 | 13.209 | 0.023 | 13.134 | 0.030 | 781 | 811 | 805 |
| 6.0517  | -1.8626   | 13.550 | 0.022 | 13.238 | 0.024 | 13.188 | 0.033 | 781 | 811 | 805 |
| 6.0522  | -2.0179   | 13.259 | 0.019 | 12.843 | 0.021 | 12.765 | 0.025 | 781 | 811 | 805 |
| 6.0538  | -2.2769   | 13.051 | 0.025 | 12.580 | 0.029 | 12.496 | 0.032 | 785 | 815 | 809 |
| 6.0545  | -1.8331   | 13.206 | 0.021 | 12.991 | 0.024 | 12.821 | 0.028 | 787 | 817 | 811 |
| 6.0651  | -2.1682   | 13.883 | 0.025 | 13.402 | 0.027 | 13.326 | 0.033 | 793 | 823 | 817 |
| 6.0723  | -1.6285   | 13.779 | 0.022 | 13.135 | 0.025 | 12.939 | 0.027 | 793 | 823 | 817 |
| 6.0736  | -2.1658   | 13.475 | 0.023 | 12.861 | 0.022 | 12.628 | 0.025 | 793 | 823 | 817 |
| 6.0783  | -1.6903   | 14.194 | 0.026 | 13.495 | 0.029 | 13.327 | 0.032 | 793 | 823 | 817 |
| 6.0881  | -2.3185   | 14.015 | 0.031 | 13.361 | 0.035 | 13.177 | 0.041 | 793 | 823 | 817 |
| 6.0895  | -1.8796   | 13.327 | 0.020 | 12.748 | 0.021 | 12.509 | 0.024 | 793 | 823 | 817 |
| 6.0911  | -2.3433   | 12.411 | 0.023 | 11.865 | 0.025 | 11.770 | 0.026 | 793 | 823 | 817 |
| 6.0986  | -1.5541   | 12.304 | 0.018 | 11.969 | 0.022 | 11.914 | 0.022 | 793 | 823 | 817 |

**Note:** Columns list J2000.0 coordinates of the stars, \( JHK_s \) magnitudes with their rms errors, and the number of observations in each band.
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