Matter-Antimatter Asymmetry in the Universe and an Arrow for Time

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0.1 Introduction

In the Big Bang model, the Universe starts out as very hot with all particle species in equilibrium. In these circumstances, creation and annihilation of matter and antimatter is equally probable. As the Universe expands it cools down and when the temperature of the Universe goes below a temperature of \( O(T \sim 1\text{GeV}) \) the inverse annihilation process \( 2\gamma \rightarrow p + \bar{p} \) is energetically blocked. Hence, around this temperature, the direct process \( p + \bar{p} \rightarrow 2\gamma \) begins to turn all protons and antiprotons into photons. This process is very efficient and, if one starts in a matter - antimatter symmetric Universe, one expects the Universe now to have very little matter (or antimatter) left. In detail, one predicts a ratio of the baryon to photon densities now of order

\[
\eta_{\text{Matter}} = \frac{n_B}{n_\gamma} \equiv \eta_{\text{Antimatter}} \simeq 10^{-18}.
\] (1)

In fact, the Universe appears to be matter dominated, with little or no antimatter: \( \eta_{\text{Matter}} \gg \eta_{\text{Antimatter}} \). Furthermore, one finds that the Universe now has much more matter than expected. The ratio \( \eta \) of baryons to photons is very well determined by data from the cosmic background anisotropy measured by WMAP and by Big Bang Nucleosynthesis (BBN):

\[
\eta = (6.097 \pm 0.206) \times 10^{-10}.
\] (2)

This result for \( \eta \) makes sense only if this parameter is not a measure of \( n_B/n_\gamma \), but rather is a measure of some primordial asymmetry:

\[
\eta \equiv \frac{n_{\text{Matter}} - n_{\text{Antimatter}}}{n_\gamma} \bigg|_{\text{Primordial}}.
\] (3)

Then, the question becomes one of whether one can explain the origin of this primordial matter-antimatter asymmetry from the laws of physics?

0.2 The Sakharov Conditions

Remarkably, in 1967 Andrei Sakharov identified the physical conditions needed for a matter-antimatter asymmetry to be established in the Universe. These three conditions are:

i) The theory must violate fermion number. To establish a matter-antimatter asymmetry there must exist processes where matter can disappear, otherwise \( n_{\text{Matter}} - n_{\text{Antimatter}} \) is a constant and \( \eta \) is set by the initial condition assumed for the Universe, and thus is not determined by physical processes.
0.3. FERMION NUMBER VIOLATION

ii) The laws of nature must violate C and CP. If either C or CP is a good symmetry, the rates of decay of matter and antimatter are the same \( \Gamma_{\text{Matter}} = \Gamma_{\text{Antimatter}} \). As a result no asymmetry can result.

iii) The fermion number violating processes must be out of equilibrium in the Universe. In equilibrium processes that create baryon number [e.g., \( e^+ + \pi^0 \rightarrow p \)] or destroy baryon number [e.g., \( p \rightarrow e^+ + \pi^0 \)] are equally effective, so no asymmetry can be established.

I would like to discuss in some detail the first two requirements above, as there are some subtleties involved.

0.3 Fermion Number Violation

In the Standard Model (SM), classically, both Baryon number (B) and Lepton number (L) are conserved. However, because the fermions are in chiral asymmetric representations of \( SU(2) \times U(1) \), both the B- and L-symmetry currents have a chiral anomaly. These anomalies are the same for both currents, so that the (B-L) combination survives as an exact symmetry at the quantum level, while for the (B+L) current one finds

\[
\partial_\mu J_{B+L}^\mu = \frac{N_g}{16\pi^2} \left[ g_2^2 W_{a\mu\nu} W_a^{\mu\nu} + g_1^2 Y_{\mu\nu} Y^{\mu\nu} \right],
\]

(4)

where \( N_g \) is the number of generations. Even though the (B+L)-current is not conserved, 't Hooft showed that the actual rate for (B+L)-violating processes at \( T=0 \) is tiny. However, sphaleron processes in the thermal bath provided by an expanding Universe can substantially alter this rate, and at temperatures near where the electroweak phase transition takes place (B+L)-violating processes can occur at a significant rate. This observation by Kuzmim, Rubakov and Shaposhnikov has significant implications for the matter-antimatter asymmetry in the Universe.

Before considering these implications in detail, let me comment briefly on the physics behind this result. The processes at \( T=0 \) which contribute to (B+L)-violation involve gauge field configurations which are topologically distinct from each other at \( t = -\infty \) and \( t = +\infty \). Effectively, these processes require quantum tunneling between the EW vacuum states and, because of this, the rate for (B+L)-violating processes is suppressed exponentially. In the thermal bath of an expanding Universe, on the other hand, the transitions between EW vacuum states can occur through thermal fluctuations. Furthermore the energy barrier between the vacuum states, which is given by the Boltzmann factor \( \exp[E_{\text{sph}}(T)/T] \) decreases with
T, since the sphaleron energy $E_{\text{sph}}(T)$ vanishes at the electroweak phase transition, leading to ubiquitous (B+L)-violation in the early Universe.

One can show that, at temperatures above the electroweak phase transition, the rate for (B+L)-violating processes remains significant. \[10, 11\] Indeed, this rate scales as

$$\Gamma_{(B+L)\text{violation}} \sim \alpha^5 \ln \alpha \cdot T,$$

and is faster than the Universe’s expansion rate $H \sim T^2/M_P$ for

$$T_{\text{EW}} \sim 10^2 \text{GeV} \leq T \leq T_{\text{max}} \sim 10^{12} \text{GeV}.$$  

This has 2 consequences:

i) Any primordial (B+L) asymmetry generated before $T_{\text{max}}$ gets erased, as (B+L)-violating process go into equilibrium.

ii) As a result of sphaleron re-processing, a primordial (B - L) asymmetry generates both a B and an L asymmetry. Thus $\eta$ is naturally associated with a primordial (B - L) asymmetry.

### 0.4 An Arrow for Time

Because of the CPT Theorem, \[12\] CP-violation is equivalent to a violation of Time reversal (T). Because CP-violation is a necessary ingredient for generating a primordial matter-antimatter asymmetry, our own existence is predicated on having an arrow for time. Typically, under CP, operators are replaced by their Hermitian adjoints. \[13\] Thus, for example, under CP a $W^+$ field goes into:

$$W^{+\mu}(\vec{x}, t) \rightarrow \eta(\mu)W^{-\mu}(-\vec{x}, t),$$

where $\eta(0) = -1; \eta(i) = 1$. Thus, schematically, under CP:

$$O(\vec{x}, t) \rightarrow O^\dagger(-\vec{x}, t)$$

However, because Lagrangians are Hermitian structures, if a Lagrangian contains an operator O it also contains $O^\dagger$ and one has the structure:

$$L = aO + a^*O^\dagger,$$

where $a$ is a c-number. Hence, one sees that, under CP,

$$L \rightarrow L \text{ only if } a = a^*.$$
Thus CP-violation and T-violation require having complex structures in the theory.

The origin of this complexity is not understood, but an intriguing idea is that perhaps it might be the result of compactification of a higher-dimensional theory to 4-dimensional space-time. Indeed, it was noted long ago by Dine, Leigh and MacIntire [14] and by Choi, Kaplan and Nelson [15] that in 10-dimensional heterotic string theory, ordinary 4-dimensional CP is an exact symmetry, being the product of a gauge transformation and a 10-dimensional Lorentz transformation. This is easily understood by noting that in 3-space dimensions Parity can be viewed as a reflection in one dimension and a rotation by \( \pi \) in the plane orthogonal to the axis that was reflected. That is,

\[
\{ \vec{x} \rightarrow -\vec{x} \} \equiv \{ x_1 \rightarrow -x_1 \text{ and } R_{x_2x_3}[\pi] \}.
\]  

(11)

In 4-space dimensions, however, one can invert all the coordinates simply by performing two rotations:

\[
\{ \vec{x} \rightarrow -\vec{x} \} \equiv \{ R_{x_1x_2}[\pi]R_{x_3x_4}[\pi] \}.
\]  

(12)

So it is clear that Parity in 4-dimensional space time can be part of a Lorentz transformation in ten dimensions.

Charge conjugation interchanges fermions with anti-fermions. Because in the heterotic string theory the fermions and anti-fermions transform both as members of the adjoint representation of the local \( E_8 \) symmetry of the theory, charge conjugation is equivalent to a gauge rotation. Hence, in these theories, 4-dimensional CP is an exact symmetry, being a product of a Lorentz rotation and a gauge transformation. So, if our 4-dimensional world originates from theories of this sort, where 4-dimensional CP is a good symmetry in the higher dimensional theory, the complexity which gives rise to CP-violation arises as a result of the compactification of the extra dimensions and may well be characterized by a simple geometrical phase.

Regardless of whether this speculation has merit or not, in 4-dimensions CP violation is naturally associated with the presence of scalar fields. This is because a theory involving only fermions and gauge fields is CP-conserving, up to \( \theta \)-terms, since the gauge couplings are real and the gauge fields sit in (real) adjoint representations. Furthermore, at least in the Standard Model (SM), the \( \theta \)-terms do not appear to be effective sources for CP violation. The weak \( \theta \)-angle can be rotated to zero because \( SU(2) \) is a chiral theory and the strong \( \theta \)-angle must be very small \( [\theta_s \leq 10^{-10}] \) to satisfy the stringent bound that exists on the electric dipole moment of the neutron.
If there are no elementary scalars, one could imagine that CP is broken spontaneously by the dynamical formation of some complex fermion condensates: $<\bar{f}f>\sim e^{i\delta}$. However, it is difficult to reconcile spontaneous CP-breaking with cosmology. As Zeldovich, Kobzarev and Okun [16] pointed out, if CP is broken spontaneously, domains of different CP form in the Universe, separated by walls which dissipate very slowly as the Universe cools as it expands. Typically, the energy density in the domain walls only decreases linearly with temperature $\rho \sim \sigma T$, where $\sigma$ is the surface energy density and is directly related to the scale of the CP-violating condensates. If this scale, say, is of order the Fermi scale $v_F \simeq 250$ GeV so that $\sigma \sim v_F^3$, the energy density in the domain walls now would vastly exceed the closure density for the Universe.

Because of these considerations, it is natural to assume that the CP-violating phenomena observed experimentally are due to the presence of a scalar sector. Indeed, all data in K and B decays are perfectly consistent [17] with CP violation being due to the CKM paradigm, [18] where CP violation originates from complex Yukawa couplings. However, it is unlikely that the CKM phase $\gamma$ is directly related to the CP phase associated with the generation of the observed matter-antimatter asymmetry in the Universe. Let me turn to a more detailed discussion of this matter next.

0.5 Scenarios for Generating $\eta$

In the years since Sakharov’s seminal paper many scenarios have been proposed for generating the matter-antimatter asymmetry in the Universe. Here I will describe briefly the elements of four of these scenarios, focusing on their viability and their connection to some other physics.

0.5.1 GUT-scale Baryogenesis

Grand Unified Theories (GUTS) [19] provided the first realistic models for baryogenesis. [20] In GUTs the SM forces are unified into a larger group $G$ which contains the $SU(3) \times SU(2) \times U(1)$ symmetry group of the SM. As a result, quarks and leptons are in same representations, leading naturally to nucleon instability. The proton lifetime, however, is very long because proton decay is mediated by the very heavy force carriers $M_X \sim 10^{16}$GeV which lead to the unification of forces.

In GUTs all 3 Sakharov’s conditions are satisfied. First of all, as remarked already above, the heavy GUT states couple to channels with different baryon number. For instance, the charge 4/3 gauge boson $X$ of $SU(5)$
couples to both two u-quarks and to an anti d-quark and a positron. Hence X-decays violate baryon number:

\[ X \rightarrow uu \quad (B = 2/3) \quad ; \quad X \rightarrow e^+ \bar{d} \quad (B = -1/3). \]  

(13)

Second, in GUTs the rates for these decays for particles and anti-particles are not the same, since there are CP-violating phases in these theories. Hence, for example:

\[ \Gamma_X(X \rightarrow uu) \neq \Gamma_{\bar{X}}(\bar{X} \rightarrow \bar{u} \bar{u}). \]  

(14)

Third, in the early Universe, at temperatures of order the mass of the heavy GUT states \( T \sim M_X \), the baryon number violating decays are out of equilibrium since their decay rates are slow compared to the Universe’s expansion rate.

Although it is possible in GUTs to obtain \( \eta \simeq 6 \times 10^{-10} \), \[21\] this is difficult to do since, in general, the rate difference \( \Gamma_X(X \rightarrow uu) - \Gamma_{\bar{X}}(\bar{X} \rightarrow \bar{u} \bar{u}) \) which drives \( \eta \) involves highly suppressed CP-violating processes. However, the main difficulty of GUT Baryogenesis is that it creates a (B+L)-asymmetry. \[^1\] As pointed out by Weinberg \[22\] and by Wilczek and Zee \[23\], the dominant dimension six SM operators that violate B, actually conserve (B-L). For example, the operator

\[ O_1 = \epsilon_{\alpha \beta \gamma} \epsilon_{ij} (d_{\alpha R}^T C u_{\beta R})(Q_{\gamma i L}^T C L_{j L}) \]  

(15)

has B-L=0. Thus GUT baryogenesis really produces a (B+L)-asymmetry. However, as we discussed earlier, sphaleron processes will erase any primordial (B+L)-asymmetry as the Universe cools below \( T \sim 10^{12} \) GeV. So, if GUT baryogenesis only produces a (B+L)-asymmetry, this cannot be the origin of the observed \( \eta \).

### 0.5.2 Electroweak Baryogenesis

A very interesting alternative to GUT baryogenesis is to imagine that baryogenesis occurs at the EW phase transition. \[24\] Unfortunately, this scenario also has significant difficulties. The advantages of electroweak baryogenesis, if it were possible, is that if the electroweak phase transition is a first order transition, then fermion number violating processes would naturally be out of equilibrium. Furthermore, the resulting asymmetry (which is really a (B+L)-asymmetry) would be driven by CP-violating processes at the

\[^1\]There are further troubles for GUT baryogenesis due to the fact that the matter-antimatter asymmetry is created at a very high temperature. Such temperatures are difficult to achieve in the re-heating that occurs after inflation.
weak scale and so the asymmetry would be proportional to the CKM phase: 
\[ \eta \sim \gamma. \]

Unfortunately, for the SM the dynamical conditions for electroweak baryogenesis do not work out in detail. First of all, the CP-violating processes which contribute to \( \eta \) are very suppressed by GIM factors and the smallness of the Jarlskog determinant \[25\] which result in a calculated value for \( \eta < 10^{-10} \). Second, because the mass of the Higgs boson in the SM is known experimentally to be rather heavy (\( M_H > 114 \text{ GeV} \) \[26\]), it follows that the electroweak phase transition is not a very strong first order transition. This also has dire consequences, since in this case it turns out that (B+L)-violating processes are still fast enough to be in equilibrium after the phase transition and serve to erase the produced asymmetry. In detail, for the asymmetry created at the electroweak phase transition not to be erased after the transition, one needs

\[ \Gamma_{(\text{B+L})\text{violation}}(< \Phi^* >) < H, \]  

where \(< \Phi^* >\) is the Higgs VEV after the transition. This inequality holds if \(< \Phi^* > / T^* \geq 1. \] \[27\] In more physical terms, the condition on the jump of the Higgs vacuum expectation value can be translated into an upper bound on the Higgs mass \( M_H < 45 \text{ GeV} \) \[28\] which, unfortunately, is in contradiction with experiment.

It is possible to ameliorate both these troubles in supersymmetric theories, \[29\] but the resulting scenarios for the supersymmetric matter are quite constrained and \( \eta \) is no longer directly related to the CKM phase \( \gamma \). In fact, realistic supersymmetric models of electroweak baryogenesis in general need to introduce additional singlet fields to produce the asymmetry. \[30\] Other extensions of the SM also can produce the desired asymmetry, but they all need to introduce additional unwarranted assumptions. For instance, a recent paper by Fromme, Huber and Seniuch, \[31\] in a 2 Higgs model, finds a strong first order transition for heavy extra Higgs states [\( m_H > 300 \text{ GeV} \)]. In this case, the CP phase that drives \( \eta \) is that of the dimension two mass term which connects the two Higgs fields

\[ V_{\text{break}} = \mu_{12} e^{i\delta} (\Phi_1^T C \Phi_2) + \mu_{12} e^{-i\delta} (\Phi_1^T C \Phi_2)^\dagger \]  

This term breaks CP and a discrete symmetry which protects this theory from flavor changing neutral currents (FCNC). However, there appears to be no motivation for introducing this term, except for helping generate a large enough \( \eta \). Furthermore, to generate a large enough asymmetry one needs parameters which produce an electric dipole moment for the neutron
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and the electron very close to the present bounds, and the strong first order phase transition occurs near the perturbative breakdown of the model.

0.5.3 Baryogenesis through Leptogenesis

A much more promising scenario for baryogenesis is the leptogenesis scenario suggested by Fukugita and Yanagida [32] twenty years ago. [33] In this scenario, a primordial lepton-antilepton asymmetry is generated from the out of equilibrium decays of heavy Majorana neutrinos into light neutrinos and Higgs bosons [$N \rightarrow \ell\Phi; N \rightarrow \ell\Phi^\dagger$]. Because the (B+L)- current is anomalous, this lepton number asymmetry, is transmuted through sphaleron processes [9] into a baryon number asymmetry. After a simple calculation, [34] one finds in the SM:

$$\eta = -\frac{[8N_g + 4]}{22N_g + 13}\eta_L \simeq -0.35\eta_L,$$

(18)

where in the above $\eta_L$ is the leptonic asymmetry produced in the heavy neutrino decays.

The nicest feature of this scenario is that baryogenesis is related to another physical phenomena, since the heavy neutrino states $N$ with masses $M_N >> v_F$ whose decays produce the asymmetry explain also, via the seesaw mechanism, [35] why neutrino masses are tiny:

$$m_\nu \sim \frac{m^2}{M_N},$$

(19)

Thus, $\eta_L$ depends on properties of the light neutrino spectrum and so does the matter-antimatter asymmetry $\eta$.

To produce the desired baryon asymmetry, one needs a lepton number asymmetry of order $\eta_L \simeq 1.7 \times 10^{-9}$. Now,

$$\eta_L \simeq \frac{7\epsilon\kappa}{g^*},$$

(20)

where $\epsilon$ is a measure of the CP-asymmetry in the decay of the heavy neutrino $N$, $\kappa$ takes into account of a possible washout of the asymmetry after it is established, and $g^* \sim 100$ is the number of effective degrees of freedom at the temperature where the asymmetry is produced: $T \sim M_N$.

The parameter $\epsilon$ vanishes if there is no CP violation in $N$ decays and is related to the neutrino spectrum:

$$\epsilon = -\frac{3M_1}{16\pi^2v_F^2}\frac{\text{Im}(\Gamma^*M_\nu\Gamma^\dagger)}{(\Gamma\Gamma^\dagger)_{11}}.$$  

(21)
Here $M_\nu$ is the light neutrino mass matrix and $\Gamma$ is the Yukawa coupling matrix of the heavy neutrinos to the light neutrinos and the Higgs boson. Here we have assumed the case of hierarchical heavy neutrinos, where the asymmetry is generated by the decay of the lightest such neutrino (whose mass we have denoted by $M_1$). For $T \sim M_1 \sim 10^{10}$ GeV, one needs very light neutrino masses ($m_\nu < \text{eV}$) to obtain the typical parameters needed for the CP asymmetry [$\epsilon \sim 10^{-6}$] and the washout [$\kappa \sim 10^{-2}$] to get $\eta_L \sim 2 \times 10^{-9}$.

One of the nice results that emerges is that, if the light neutrino masses lie in the range $10^{-3} \text{eV} \leq m_\nu \leq \text{eV}$, then the washout factor $\kappa$ is independent of the initial abundance of the heavy neutrinos and/or any pre-existing lepton asymmetry. \cite{36}

Because the CP-asymmetry parameter $\epsilon$ cannot be too small for leptogenesis to work, this provides a lower bound on $M_1$. \cite{37} The analysis of Buchm"uller, Di Bari and Pl"umacher \cite{36} yields the bound $M_1 > 2 \times 10^9$ GeV. \cite{36} It turns out that leptogenesis also provides an upper bound on the light neutrinos masses, since the washout rate increases proportionally to the sum of the light neutrino masses squared, $W \sim \sum_i m_i^2$. Buchm"uller et al \cite{38} find the bound:

$$m_i < 0.1 \text{ eV}.$$ \hspace{1cm} (22)

Obviously, the realization that leptogenesis occurs as the result of the out-of-equilibrium decays of heavy neutrinos with masses in the $10^{10}$ GeV range, and that these heavy neutrinos are associated with a sub eV light neutrino spectrum is very encouraging!

Leptogenesis is a triumph for neutrino physics. Indeed, if $\eta$ is generated this way, we owe our own existence to the CP phases in the neutrino sector! However, the fact that leptogenesis occurred at temperatures $T$ of order $T \sim M_1 > 2 \times 10^9$ GeV has significant import for supersymmetric theories. In particular, in supergravity theories if the reheating temperature after inflation $T_R$ is too high, one overproduces gravitinos. This has catastrophic consequences for the evolution of the Universe, since the decay products of gravitinos end up by destroying the light elements produced in Big Bang Nucleosynthesis. To avoid these troubles, one requires that the reheating temperature be bounded by $T_R < 10^7$ GeV. \cite{39} However, leptogenesis argues that $T_R > M_1 > 2 \times 10^9$ GeV!

There are solutions to this, so-called, gravitino problem, but these in general alter the "normal" SUSY expectations coming from supergravity. For instance, if $m_{3/2} > 100$ TeV, \cite{40} then the gravitinos decay before BBN ameliorating the problem. Or perhaps the gravitinos are the LSP, but then one must insure that the NLSP does not give rise to the same troubles. \cite{41}
Lepton flavor violation (LFV) provides another example of tension between SUSY and leptogenesis. For instance, the predictions for $\mu \to e\gamma$, although model dependent, are sensitive to the mass of the heaviest neutrinos $[BR \sim (M_3 \ln M_X/M_3)^2]$ and, in general, to satisfy the present bounds on $\mu \to e\gamma$ one needs $M_3 < 10^{13}$ GeV. Thus LFV provides constrains from above on the spectrum of heavy neutrinos, while leptogenesis provides constraints from below. These examples suggest that seeking compatibility between SUSY and leptogenesis provides interesting testable experimental predictions and insights into neutrino physics.

0.5.4 Affleck-Dine Baryogenesis

There is another interesting mechanism in the early Universe for baryogenesis, which was first discussed by Affleck and Dine. The Affleck-Dine scenario most naturally ensues in SUSY theories where:

i. Some scalar fields carry B or L (e.g. squarks or sleptons).

ii. These scalar fields lie along flat directions of the potential, which are eventually lifted when one includes higher dimensional operators in the theory.

In these circumstances, it is possible for the scalar fields lying in the flat direction to acquire large initial values, $\phi = \phi_0$. Eventually, when the expansion rate of the Universe $H$ gets to be of $O(m_\phi)$, the field $\phi$ begins to oscillate about the minimum of the potential. These coherent oscillations of $\phi$ generate $\eta$.

It is useful to illustrate how the Affleck-Dine mechanism works by means of a simple toy example. Consider a Lagrangian involving a complex scalar field

$$L_0 = -\partial_\mu \phi \partial^\mu \phi^\dagger - m^2 \phi \phi^\dagger.$$  \hfill (23)

Clearly $L_0$ is invariant under the phase transformation $\phi \to e^{i\alpha} \phi$, which plays the role of B in the model. It is also invariant under the discrete symmetry $\phi \to \phi^\dagger$, which plays the role of CP in the model. If one adds to this Lagrangian the perturbation

$$L_1 = \epsilon \phi^3 \phi^\dagger + \epsilon^* \phi^3 \phi^\dagger$$  \hfill (24)

both B and CP are broken.

The evolution of $\phi(t)$ in the Universe is determined by

$$\ddot{\phi} + 3H \dot{\phi} + m^2 \phi = \epsilon \phi^3 + 3\epsilon^* \phi^3 \phi.$$

(25)
If one neglects the perturbation on the RHS, and considers for definitiveness the Hubble parameter appropriate for a matter dominated Universe, $H = 2/3t$, the result is a damped oscillation for $\phi$ for $H << m$:

$$\phi = \phi_0 \frac{\sin mt}{mt}. \quad (26)$$

When one includes the effect of the small perturbation $L_1$, besides $\text{Re}\phi \equiv \phi_t = \phi_0 \sin mt/mt$, one develops also a small $\text{Im}\phi \equiv \phi_i$, which obeys the equation:

$$\ddot{\phi}_i + \frac{2\dot{\phi}_i}{t} + m^2 \phi_i \simeq 4\text{Im} \epsilon \phi^3_t. \quad (27)$$

The RHS of the above equation is negligible for large $t$, but sets the size of $\phi_i$. One finds

$$\phi_i \simeq 4A \text{Im} \epsilon \phi^3_0 \frac{\sin(mt + \delta)}{m^3 t}, \quad (28)$$

where $A$ and $\delta$ are parameters of $O(1)$. The presence of $\phi_i$ allows a baryon number to develop

$$n_B = i\left[ \phi \frac{\partial \phi^\dagger}{\partial t} - \phi^\dagger \frac{\partial \phi}{\partial t} \right] = \phi_t \dot{\phi}_i - \phi_i \dot{\phi}_t \quad (29)$$

and $n_B \simeq \text{Im} \epsilon \phi^4_0/m$ can be large even if $\text{Im} \epsilon$ is small, provided $\phi_0$ is large.

After this toy model discussion, I want to illustrate how the Affleck-Dine mechanism for baryogenesis works in a more realistic scenario. The scenario involves an interesting flat direction which arises in the minimal supersymmetric extension of the SM, the MSSM. [45] The scalar field in question is the product of the scalar fields associated with the Higgs and lepton doublets: $\Phi^2 \equiv (\phi_u L)$.

For this field the seesaw mechanism induces a quartic term in the superpotential

$$W = \left[ \frac{1}{\Lambda} \right] (\phi_u L)^2 \quad (30)$$

which, in turn, gives rise to a potential

$$V = \left[ \frac{1}{\Lambda} \right]^2 |\Phi|^6. \quad (31)$$

In the above, $\Lambda$ is a heavy scale. In addition to this term in $V$, one expects SUSY breaking contribution to the scalar potential involving $\Phi$ with the generic form:

$$\delta V = m^2 |\Phi|^2 + \left[ \frac{m_{\text{SUSY}}}{\Lambda} \right] (a \Phi^4 + a^* \Phi^4) \quad (32)$$
where the mass $m \sim m_{SUSY} \sim \text{TeV}$.

The Affleck-Dine scenario ensues by assuming that during inflation the field $\Phi$ acquires a negative mass-squared contribution $-H^2 \Phi^2$ which is balanced by $V$. Thus, the field $\Phi$ settles initially at $\Phi_0 \sim [H \Lambda]^{1/2}$. After inflation $\Phi_0$ keeps decreasing and begins to oscillate when the Hubble parameter is of order $H \sim m$, at which point $\Phi_0 \simeq [m \Lambda]^{1/2}$. At this time lepton-number is also generated, because $\Phi$ acquires an imaginary component as a result of the quartic perturbation in $\delta V$. One has:

$$\frac{\partial n_L}{\partial t} + 3H n_L = 4\left[\frac{m_{SUSY}}{\Lambda}\right] \text{Im}(a \Phi_0^4).$$

(33)

The generated lepton number is of the order of

$$n_L \simeq \left[\frac{m_{SUSY}}{\Lambda}\right] \Phi_0^4 \delta_{\text{osc}} \simeq m^2 \Lambda \delta_{\text{osc}}.$$ 

(34)

In the above the effective CP-violating phase $\delta_{\text{osc}} = \sin(\arg a + 4\arg \Phi_0)$ and we have used that $t_{\text{osc}} \sim 1/m$.

Taking into account the entropy that is produced during inflaton decay

$$s \sim \frac{m^2 M_G^2}{T_R},$$

(35)

where $M_G$ is the reduced Planck mass $M_G = M_P/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV, gives

$$\eta_L = \frac{\tau n_L}{s} \simeq 7 \frac{\Lambda T_R}{M_G^2} \delta_{\text{osc}},$$

(36)

which for $T_R \sim 10^7$ GeV , after sphaleron re-processing, needs $\Lambda \sim M_G$ to get the desired $\eta$.

### 0.6 Concluding Remarks

The present situation regarding the matter-antimatter asymmetry in the Universe is somewhat unsettled. Because $\eta$ is just one number, it is difficult to prove any one scenario correct. From my own perspective, perhaps the most promising scenario is leptogenesis, which connects the magnitude of $\eta \sim 10^{-9}$ with the existence of sub-eV neutrinos. However, the conflict of leptogenesis with "standard" SUSY should not be ignored and, if supersymmetry is found, will have to be resolved.

From our discussion, it seems pretty clear that there is little hope to connect $\eta$ to the CP violating CKM phase $\gamma$. For instance, in Affleck-Dine
baryogenesis the CP phase is connected with the SUSY breaking terms in the potential, while in leptogenesis the CP phase is connected to the heavy neutrino Yukawa couplings. These "high energy" CP-violating phases are generally different than those measured at low energy.

For example, in the case of leptogenesis, one can easily enumerate the 21 parameters of the lepton sector. Of these, 12 are relevant at low energy: 3 light neutrino and 3 charged lepton masses; 3 mixing angles; and 3 CP violating phases $\alpha_1, \alpha_2$ and $\delta$, where the last phase is the analogue of the CKM phase $\gamma$ and the other 2 phases arise because neutrinos are Majorana particles. The other 9 parameters in the lepton sector are related to physics at high scales. They involve the 3 heavy neutrino masses; 3 additional mixing angles; and 3 "high scale" CP-violating phases $\phi_i$.

In a basis where the heavy neutrino mass matrix $M$ and the charged lepton mass matrix $m_\ell$ are diagonal, all 6 phases appear in the Dirac mass matrix $m_D$ which connects the left-handed neutrino $SU(2)$ doublets with the right-handed neutrino singlets. The $3 \times 3$ matrix $m_D$ can always be written as a product of a unitary matrix $U$ times a triangular matrix $Y$, $m_D = U Y$, where

$$Y = \begin{bmatrix} y_{11} & 0 & 0 \\ y_{21} e^{i \phi_{21}} & y_{22} & 0 \\ y_{31} e^{i \phi_{31}} & y_{32} e^{i \phi_{23}} & y_{33} \end{bmatrix}. \quad (37)$$

The low energy CP violating phases $\alpha_1, \alpha_2, \delta$ in this parametrization depend on all 6 of these phases (3 in the unitary matrix $U$ and 3 in $Y$). On the other hand, the CP-violating parameter $\epsilon$ which enters in leptogenesis, because it depends on $m_D^\dagger m_D$, involves only the 3 phases in $Y$.

Thus, without further assumptions, also in leptogenesis there is no direct link between the matter-antimatter asymmetry and potentially measurable CP-violating phenomena in the neutrino sector. This is a more general problem. Without an understanding of what is the correct underlying theory, there is no hope of identifying the phase(s) which are responsible for $\eta$ and hence our existence, much less of relating these phases to the ur-phase arising from the compactification from $d=10$ to $d=4$ dimensions, presumably responsible for the arrow of time.

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Bibliography

[1] See, for example, M. S. Turner, in Intersection between Particle Physics and Cosmology, Jerusalem Winter School for Theoretical Physics, Vol. 1, p. 99, eds. T. Piran and S. Weinberg (World Scientific, Singapore, 1986).

[2] A. Dolgov and J. Silk, Phys. Rev. D47, 4244 (1993); M. Y. Khlopov, M. G. Rubin, and A. S. Sakharov, Phys. Rev. D62, 083505 (2000); A. Cohen, A. De Rujula and S. Glashow, Astrophys. J. 495, 539 (1998).

[3] WMAP Collaboration, C. L. Bennett, et al, Astrophys. J. Suppl. 148, 1 (2003); D. N. Spergel, et al, Astrophys. J. Suppl. 148, 175 (2003).

[4] See, for example, G. Steigman, Int. J. Mod. Phys. E15, 1 (2006).

[5] A. D. Sakharov, JETP Lett. 5, 24 (1967).

[6] S. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60, 47 (1969).

[7] G. t’Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D14, 4332 (1976).

[8] F. R. Klinkhamer and N. S. Manton, Phys. Rev. D30, 2212 (1984).

[9] V. A. Kuzmin, V. A. Rubakov and M. A. Shaposhnikov, Phys. Lett. B155, 36 (1985).

[10] P. Arnold and L. Yaffe, Phys. Rev. D62, 125014 (2000).

[11] D. Bödeker, Phys. Lett. B426, 351 (1998); Nucl. Phys. B559, 502 (1999).

[12] W. Pauli, in Niels Bohr and the Development of Physics, ed. W. Pauli (Pergamon Press, New York, 1955); J. Schwinger, Phys. Rev. 82, 914 (1951); G. Lüders, Dansk. Mat. Fys. Medd. 28, 5 (1954); G. Lüders and B. Zumino, Phys. Rev. 110, 1450 (1958).
[13] See, for example, R. D. Peccei, in *Broken Symmetries*, eds. L. Mathelitsch and W. Plessas, Springer Lecture Notes in Physics 521 (Springer Verlag, Berlin, 1999).

[14] M. Dine, R. G. Leigh and D. A. MacIntire, Phys. Rev. Lett. **69**, 2030 (1992).

[15] K. Choi, D. B. Kaplan and A. E. Nelson, Nucl. Phys. **B391**, 515 (1993).

[16] Y. B. Zeldovich, I. B. Kobzarev and L. Okun, Phys. Lett. **B50**, 340 (1974).

[17] P. Burchat, these Proceedings.

[18] N. Cabibbo, Phys. Rev. Lett. **12**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[19] For a review see, Grand Unified Theories by S. Raby in the Particle Data Group Report, W. -M. Yao, *et al*, J. Phys. G. **33**, 1 (2006).

[20] M. Yoshimura, Phy. Rev. Lett. **41**, 281 (1978); *ibid.* **42**, 746(E) (1979);
D. Toussaint, S. B. Treiman, F. Wilczek and A. Zee, Phys. Rev. **D19**, 1036 (1979); S. Weinberg, Phys. Rev Lett. **42**, 850 (1979); S. Dimopoulos and L. Susskind, Phys. Rev. **D18**, 4500 (1978).

[21] See, for example, P. Langacker, Phys. Rept **72C**, 185 (1981).

[22] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[23] F. Wilczek and A. Zee, Phys. Rev. Lett. **43**, 1571 (1979).

[24] V. A. Rubakov and M. S. Shaposhnikov, Phys. Usp. **39**, 461 (1996).

[25] M. E. Shaposhnikov, Nucl. Phys. **B287**, 757 (1987); *ibid.*, **B299**, 797 (1988).

[26] Particle Data Group, W. -M. Yao, *et al*, J. Phys. G. **33**, 1 (2006).

[27] M. E. Shaposhnikov, JETP Lett. **44**, 465 (1986).

[28] K. Jansen, Nucl. Phys. (Proc. Supp.) **B47**, 196 (1996).

[29] B. de Carlos and J. R. Espinosa, Nucl. Phys. **B503**, 24 (1997); M. Carena, M. Quiros and C. E. M. Wagner, Phys. Lett. **B380**, 81 (1996); M. Carena, M. Quiros, M. Seco and C. E. M. Wagner, Nucl. Phys. **B650**, 24 (2003); K. Kainulainen, T. Prokopec, M. G. Schmidt and S. Weinstock, JHEP **0106**, 031 (2001).
[30] J. Kang, P. Langacker, T. Li and T. Liu, Phys. Rev. Lett. 94, 061801 (2005); S. J. Huber, T. Konstandin, T. Prokopec and M. G. Schmidt, hep-ph/0606298.

[31] L. Fromme, S. J. Huber and M. Seniuch, hep-ph/0605242.

[32] M. Fukugita and T. Yanagida, Phys. Lett. B174, 45 (1986).

[33] For an extensive review of this scenario, see for example, W. Buchmüller, R. D. Peccei and T. Yanagida, Ann. Rev. Nucl. Part. Sci. 55, 311 (2005).

[34] J. A. Harvey and M. S. Turner, Phys. Rev. D42, 3344 (1990); R. N. Mohapatra and X. Zhang, Phys. Rev. D45, 2699 (1992); S. Yu. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B308, 885 (1988).

[35] T. Yanagida, in Proc. of the Workshop on ”The Unified Theory and the Baryon Number in the Universe”, Tsukuba, Japan, Feb. 13-14, 1979, p. 95, eds. O. Sawada and S. Sugamoto, (KEK Report KEK-79-18, 1979, Tsukuba); Progr. Theor. Phys. 64, 1103 (1980); P. Ramond, in Talk given at the Sanibel Symposium, Palm Coast, Fla., Feb. 25-Mar. 2, 1979, preprint CALT-68-709. See also S. Glashow, in Proc. of the Cargese Summer Institute on ”Quarks and Leptons”, Cargese, July 9-29, 1979, p. 707 eds. M. Levy, et al (Plenum, New York, 1980).

[36] W. Buchmüller, P. Di Bari and M. Plümacher, Ann. Phys. 315, 303 (2005).

[37] S. Davidson and A. Ibarra, Phys. Lett. B535, 25 (2002).

[38] W. Buchmüller, P. Di Bari and M. Plümacher, Nucl. Phys. B643, 367 (2002); Phys. Lett. B547, 128 (2002).

[39] M. Kawasaki, K. Kohri and T. Moroi, Phys. Rev. D71, 083502 (2005).

[40] M. Ibe, R. Kitano, H. Murayama and T. Yanagida, Phys. Rev. D70, 075012 (2004).

[41] M. Bolz, W. Buchmüller and M. Plümacher, Phys. Lett. B443, 209 (1998); M. Fujii, M. Ibe and T. Yanagida, Phys. Lett. B579, 6 (2004); J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B588 7 (2004); J. L. Feng, S. Su and F. Takayama, Phys. Rev. D70, 075019 (2004); L. Roszkowski, R. Ruiz de Austri and K.-Y. Choi, JHEP 0508
080 (2005); J. L. Feng, A. Rajaraman and F. Takayama, Phys. Rev. Lett. 91, 011302-1 (2003).

[42] S. T. Petcov and T. Shindou, [hep-ph/0605151] and [hep-ph/0605204].

[43] I. Affleck and M. Dine, Nucl. Phys. B249, 361 (1985).

[44] M. Dine and A. Kusenko, Rev. Mod. Phys. 76, 1 (2004).

[45] H. Murayama and T. Yanagida, Phys. Lett. B322, 349 (1994).

[46] M. Dine, L. Randall and S. Thomas, Nucl. Phys. B458, 291 (1996).

[47] G. C. Branco, L. Lavoura and M. N. Rebelo, Phys. Lett. B180, 264 (1986).

[48] J. Hashida, T. Morozumi and A. Purwanto, Prog. Theor. Phys. 101, 379 (2000).