Vulnerability Assessment of Power Grids Based on Both Topological and Electrical Properties

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Abstract—In modern power grids, a local failure or attack can trigger catastrophic cascading failures, which make it challenging to assess the attack vulnerability of power grids. In this Brief, we define the \( k \)-link attack problem and study the attack vulnerability of power grids under cascading failures. Particularly, we propose a link centrality measure based on both topological and electrical properties of power grids. According to this centrality, we propose a greedy attack algorithm and an optimal attack algorithm. Simulation results on standard IEEE bus test data show that the optimal attack is better than the greedy attack and the traditional PSO-based attack in fracturing power grids. Moreover, the greedy attack has smaller computational complexity than the optimal attack and the PSO-based attack with an adequate attack efficiency. Our work helps to understand the vulnerability of power grids and provides some clues for securing power grids.

Index Terms—Vulnerability assessment, power grid, link centrality, cascading failure, network attacks.

I. INTRODUCTION

Nowadays, power grids in the real world face various kinds of risks such as natural disasters and attacks. Even worse, a local failure in power grids can result in large-scale blackouts [1], [2]. A recent example is the nationwide recurring electrical blackouts in Venezuela began in March 2019, which was supposed to be caused by a local vegetation fire and cyberattacks. The catastrophic cascades of failures pose a great threat to human life and national security. Thus, it is of great importance to understand and control the cascading failures of power grids.

In the past decade, the complex network theory has been widely applied to the study of cascading failures [3]. On the modelling side, power grids can be abstracted as interdependent networks, and then the percolation theory has been used to explore the dynamics of cascading failures [4]. Rich behaviors have been observed when taking the power grid as interdependent network. For instance, Buldyrev et al. [5] found a hybrid phase transition (HPT), where the order parameter has both a jump and a critical scaling.

On the controlling side, researchers proposed many optimization strategies against cascading failures in power grids. Tu et al. [6] used the simulated annealing method to optimize the network topology, and found that it is better to make the network sparsely connected, and place the generators as decentralized hubs. They further investigated the weak interdependency between networks of cyber-physical systems (CPS) and discussed how the failure propagation probabilities affect the robustness of CPS [7]. Chen et al. [8] performed the critical node analysis to identify the vital nodes in terms of network robustness. They found that assortative coupling of node destructiveness is more robust in densely coupled networks, whereas disassortative coupling of node robustness and node destructiveness is better in sparsely coupled networks. Zhong et al. [9] studied the repair process against cascading failures by considering the optimization of repair resource, timing and load tolerance, for different coupling strength and network topologies of interdependent networks. Zhu et al. [10] established two multiobjective optimization models that consider both the operational cost of links and the robustness of networks. Zhang et al. [11] employed the particle swarm optimization (PSO) algorithm to optimize the defense resource allocation to improve the network robustness.

To better understand and control cascading failures, we need to explore the role of individual nodes and links in power grids. When quantifying the importance of nodes or links, the complex network theory only focuses on network topology information [3], [12], [13]. However, the electrical features of nodes and links are profound [14]. Particularly, a link of small topological importance might have large current load. The broken of this kind of link has significant impact on the function of power grids. It is thus more reasonable to consider both topological and electrical features of power grids when characterizing the importance of nodes and links.

In this Brief, we study vulnerability assessment of power grids under cascading failures. We define the importance of links based on both topological and electrical features, and remove a few important links as the initial attack that triggers cascading failures. Our main contributions are as follows:

- We propose a link centrality measure, which combines link degree and link current. The weights of the two features are tunable and represented by two variables. This centrality is better than link degree and link current in quantifying the importance of links in power grids.
- According to the link centrality, we propose a greedy attack algorithm and an optimal attack algorithm. These attack algorithms are designed to cause large-scale cascading failures, based on which we can assess the vulnerability of power grids by simulation.

In the next section, we present the cascading failure model and related metrics. In section III, we introduce our link centrality measure and the parameter tuning method. In section IV, we define the link attack problem and provide our attack algorithms. In section V, we show the simulation results and...
related analysis. Section VI is our conclusion.

II. MODEL AND MEASUREMENT

In a power grid, power stations and transmission lines can be abstracted as nodes and links, respectively. Then, we obtain the network topology of power grid, denoted by \( G = (V, E) \), where \( V \) is the node set with \( N \) nodes, and \( E \) is the link set consisting of \( M \) links.

A. Current model

Usually, there are generator node, consumer node, distribution node, and transformer node in a power grid. Here, following Ref. [6] and for the purpose of simplification, we only consider two kinds of nodes: generator node \( i \) and consumer node \( j \). Then, the Kirchhoff’s current law equation for a power grid is written as

\[
Y \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_i \\ v_{j+1} \\ \vdots \\ v_N \end{bmatrix} = I_i \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},
\]

in which

\[
Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1,j-1} & y_{1,j+1} & \cdots & y_{1N} \\ y_{21} & y_{22} & \cdots & y_{2,j-1} & y_{2,j+1} & \cdots & y_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ y_{i1} & y_{i2} & \cdots & y_{i,i-1} & y_{i,i+1} & \cdots & y_{iN} \\ y_{j1} & y_{j2} & \cdots & -y_{j,j-1} & Y_{jj} & \cdots & y_{jN} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ y_{N1} & y_{N2} & \cdots & y_{N,j-1} & y_{N,j+1} & \cdots & y_{NN} \end{bmatrix},
\]

and \( v_i \) and \( v_j \) represent the voltages of generator node and consumer node, respectively. \( I_i \) is the current of consumer node \( j \). In matrix \( Y \), \( y_{ii} = 1 \). \( Y_{ij} \) is the admittance of link \((i, j)\), and \( Y_{ij} = 0 \) if nodes \( i \) and \( j \) are not connected. Also, we have \( Y_{jj} = -\sum_{\mu \neq j} Y_{j\mu} \). When the admittances of transmission lines, the voltages of generator nodes and the current consumptions of consumer nodes are given, the voltage of each node can be computed by Eq. (1). Then, the current flowing through link \((i, j)\) can be calculated as \( I_{ij} = (v_i - v_j) * Y_{ij} \).

B. Cascading failure model

Assume the load of node \( i \) is \( L(i) = u_i * I_{oi} \), where \( I_{oi} \) is the total current flowing out of node \( i \), and the load of link \((i, j)\) is the current flowing through it, i.e., \( I_{ij} \). The maximum load a node can bear is set to be \( 1 + \alpha \) times of its original load, and the maximum load of link \((i, j)\) is assumed to be \( 1 + \beta \) times of its original current, where parameters \( \alpha \) and \( \beta \) are the safety margins of nodes and links, respectively. Note that the original state of nodes or links corresponds to the case when the power grid operates normally, that is there is no attacks or failures. In the cascading failure model [5], there is usually an initial attack, e.g., randomly removing a node or link. This initial event will cause the load change of the other nodes and links, especially for those close to the area of initial attack. When the load of a node or link exceeds its maximum allowed value, it will break, which further causes the load change of the other nodes and links. The detailed steps in the cascading failure simulation process are as follows [6]:

i) Calculating the initial load and maximum load of each component (node and link) in the power grid.

ii) Randomly removing a component.

iii) The grid topology changes due to the removal, recalculating the load of each component. A component is set to be broken if its load exceeds its maximum.

iv) After removing the failed components, the network splits into several small subgraphs. If a subgraph does not contain a generator node, all nodes in this subgraph are set to be invalid nodes.

v) Repeating steps iii) -iv) until the load of all remaining components is no greater than the maximum.

C. Vulnerability measurement

In power systems, the outage scale is usually measured by the number of failed nodes. Following Ref. [6], the damage that is caused by component set \( i \) is quantified as

\[
\Phi(i) = \frac{\sum_{j=1}^{N} \delta(j)}{N},
\]

where \( \sum_{j=1}^{N} \delta(j) \) is the number of total failed nodes after the cascading failure caused by the initial removal of component set \( i \). Note that the failed nodes contain the overloaded ones and those in the subnetwork of no generator nodes. Obviously, the larger the damage, the more critical the component set is to the power grid.

III. LINK CENTRALITY MEASURE AND ITS SOLUTION

Nodes and links are the main components in power grids. Node centrality has been widely discussed in the literature. Here we study the link centrality which is relatively less discussed, but critical to the vulnerability assessment of power grids. Previously, many topology based link centralities were developed [6]. However, the joint effect of topological and electrical properties of links on the vulnerability of power grids is still not well understood.

A. Our link centrality measure

We quantify the centrality of links based on both topological and electrical features. Specifically, we consider the link degree and link current. The degree of a link is the number of links (except itself) incident to its two end nodes. The current of a link is the rate of flow of electric charge pasting it. Note that during the cascading failure, the degree and current of a link might change due to the broken of overloaded components. Since we focus on the initial attacks, we use the original state of power grid to quantify the centrality of links. Then, the centrality of link \((i, j)\) is defined as

\[
\Theta_{ij} = h_1 D_{ij} + h_2 I_{ij},
\]

where \( D_{ij} \) and \( I_{ij} \) are the initial degree and current of link \((i, j)\), respectively. For link current, we ignore its direction and use its absolute value. \( h_1 \) and \( h_2 \), ranging in \((-\infty, \infty)\), are the weights of link degree and link current. In real applications, we usually need to find the optimal values of \( h_1 \) and \( h_2 \), which is a NP-hard problem.

B. Parameter tuning based on PSO

We use the particle swarm optimization algorithm (PSO) [15], to search the optimal parameters of our link centrality measure. Compared with other heuristic algorithms, PSO has powerful global search ability and is easy to implement. In this
IV. APPLICATION OF LINK CENTRALITY IN VULNERABILITY ASSESSMENT

We apply our link centrality to the vulnerability assessment of power grids. In the assessment, we usually simulate network attacks and the consequent cascading failures, and then calculate the damage caused by the attacks. Different kinds of attacks result in different damage. Here, we consider the link attacks. The simplest link attack strategy is random attack, in which we randomly remove a certain fraction of links. More efficient link attack strategies are desired in the vulnerability assessment.

A. Problem Definition

Given an integer $K$, the problem is to find a set of $K$ links ($1 \leq K \leq M$), the removal of which will cause the maximum damage to the power grid (abbreviated as KLS problem). Note that the damage is measured as the percentage of failed nodes after the cascading failure (see Eq. (3)), which is triggered by the initial removal of the $K$-link set.

**Theorem 1.** The KLS problem is NP-Complete.

**Proof.** Given a set of $K$ links, we can calculate the percentage of failed nodes after the cascading failure triggered by the removal of the set of links in polynomial time, which means that the KLS problem is NP. Moreover, the KLS problem can be reduced to the 0/1 knapsack problem \[10\]. In this problem, given a set of items, each with a weight and a value, we determine the number of each item, 0 or 1, to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible. In the KLS problem, each link can cause some damage, and can be selected or not. The number of selected links is fixed to be $K$. The task is to determine the $K$-link set that achieves the largest damage. Since 0/1 knapsack problem is NP-Complete, so is the KLS problem.

B. Optimal attack based on PSO

Since the KLS problem can be reduced to the 0/1 knapsack problem and is NP-Complete, we employ the PSO to search the optimal $K$-link set. In the $m$ particles, particle $i$ is set to be $X_i = [x_{i1}, x_{i2}, \ldots, x_{im}]$, where $x_{ij}$ corresponds to the $j$th link of the power grid. If this link is selected to be removal, $x_{ij} = 1$; otherwise, $x_{ij} = 0$. The constraint is $\sum_{j=1}^{M} x_{ij} = K$. The particles update their velocity and position iteratively based on Eqs. (5)-(7) until finding the approximately optimal solution of the KLS problem. Since the duration of single cascade failure is unable to estimate, we only consider the number of cascading failures when discussing the time complexity. The PSO based optimal attack (PSO-OA) has the time complexity of $O(m \ast \text{iter}_\text{max})$. The pseudocode of this algorithm is in Algorithm 1.

**Algorithm 1 PSO based optimal attack (PSO-OA)**

**Require:** Adjacency matrix $G$, power generator node set $Q$, and integer $K$

**Ensure:** The total number of selected links is fixed to be $K$.

**Initialisation:** Randomly generate $m$ particles; each particle contains $M$ elements; randomly set $K$ elements to be 1 and the rest to 0.

**for** $i = 1, \text{iter}_\text{max}$ **do**

**for** $j = 1, m$ **do**

Calculate $\Phi(X_j)$ for each particle

end if

Update the optimal solution for each particle $p_{\text{best}}^i$, and the global optimal solution for the particle swarms $g_{\text{best}}$

end for

**for** $s = 1, m$ **do**

if $\Phi(X_s) > \Phi(p_{\text{best}}^s)$ then

$p_{\text{best}}^s \leftarrow \Phi(X_s), X_s$

end if

end if

end if

**end for**

**end for**

**return** the $K$ links of the largest damage

C. Greedy attack based on link centrality

In a simple greedy attack, we can calculate the damage of each link based on Eq. (3) and then select the $K$ links of the largest damage. However, it is time consuming to calculate the damage of all links. Therefore, we use our link centrality measurement to filter the links so that the relatively important links are left for consideration. Specifically, we rank all of the links based on their link centrality values. Then, we calculate the damage of the top $L$ (ML$\% \geq K$) percent of links in the ranking, and select the $K$ links of the largest damage.
Note that the parameters of our link centrality measure affect the ranking of links and therefore the selection of the $K$-link set. The better parameters correspond to a better ranking, and then $L$ can be much smaller. The time complexity of the link centrality based greedy attack (LC-GA) is $O(ML%)$ without considering the duration of single cascading failure. Note that we can also use PSO to search the optimal parameters in this algorithm, but the computational cost is very large. The pseudocode of LC-GA is given in Algorithm 2.

**Algorithm 2** Link centrality based greedy attack (LC-GA)

**Require:** Adjacency matrix $G$, power generator node set $Q$, and integer $K$

**Ensure:** The total number of selected links is fixed to be $K$.

**Initialisation:** Give a parameter set

Select the top $L\%$ links based on the ranking of link centrality values

for $j = 1, ML\%$ do

Calculate the damage $\Phi$ for each selected link

end for

return the $K$ links of the largest damage

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**D. Link centrality based optimal attack**

The PSO-OA algorithm does not use the topological and electrical properties of power grids. Thus, it is supposed to be not efficient. The LC-GA algorithm does use the topological and electrical properties, but it needs to calculate the link centrality and the damage of many links, which has large computational cost. Here, we employ PSO to search the optimal parameters so that removing the top $K$ links in the ranking of link centrality leads to the maximum damage. The set of the top $K$ links is thus an approximate solution of the $KLS$ problem. The time complexity of this link centrality based optimal attack (LC-OA) is $O(m*iter_{max})$ without considering the duration of single cascading failure. The pseudocode of this algorithm is provided in Algorithm 3.

**Algorithm 3** Link centrality based optimal attack (LC-OA)

**Require:** Adjacency matrix $G$, power generator node set $Q$, and integer $K$

**Ensure:** The total number of selected links is fixed to be $K$.

**Initialisation:** Randomly generate $m$ particles; each particle contains 2 elements which are randomly set to values in the range $[-1,1]$. For $i = 1, iter_{max}$ do

Calculate the centrality of each link based on Eq. (4)

Calculate the damage of removing the $K$ links of the largest centrality

Update $p_{best}$ and $g_{best}$ based on the PSO algorithm

end for

return the $K$ links of the largest centrality

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**V. SIMULATION RESULTS**

Based on MATLAB, we do simulation experiments to validate our link centrality measure and compare the performance of proposed attack algorithms. We use the standard IEEE bus test data [17] including IEEE 118 bus, 145 bus, and 162 bus. We randomly set 10 percent of nodes to be generator nodes. The related parameters of the experiments are given in Table 1.

| Parameter settings in the experiments |
|--------------------------------------|
| $m$ | $10$ |
| $c_2$ | $0.7$ |
| $iter_{max}$ | $30$ |
| $w_0$ | $0.96$ |
| $\beta$ | $0.2$ |
| $c_1$ | $0.7$ |
| $L$ | $50\%$ |

**A. Single link attack**

First, we study the single link attack, $K = 1$, which is removing the most critical link in terms of damage. The link degree, link currency, and our link centrality are used respectively to determine the critical link. For our link centrality measure, we use PSO to find the optimal solution (see LC-OA). The results are shown in Fig. 1, which are the average of 100 independent runs. We can see that for all the three IEEE bus test data, the random removal has the smallest damage, and the damage of our link centrality is larger than the link degree and link current. Moreover, link current is more efficient than link degree, which indicates that we should focus on electrical features more than topological features in the vulnerability assessment of power grids.

**B. Multiple link attack**

Furthermore, we study the multiple link attack, $k > 1$, to compare the performance of PSO-OA, LC-OA, and LC-GA. For LC-GA, parameters $h_1$ and $h_2$ are both set to 1. The results of damage are provided in Figs. 2 and 3. In Fig. 2, for all the IEEE bus data, the damage generally increases with the number of removed links for all the algorithms. This is because the more links removed at the beginning result in a wider range of cascading failure. An exception is that for LC-GA, the damage decreases when the number of removed links increases from 1 to 2. The reason is that LC-GA is essentially a greedy algorithm so that its solution of $K = 2$ is not the optimal one, while for $K = 1$, the solution could be the optimal one.
Moreover, we see that the damage of LC-OA is much larger than PSO-OA and LC-GA. In real situations, the numbers of particles and iterations are limited, as the parameter settings in our experiments. In this case, LC-OA can find a better solution than PSO-OA for the multiple link attack problem. The damage of LC-GA is smaller than LC-OA, since the former is a greedy algorithm, while the latter is based on global optimization essentially. Note that LC-GA is sometimes better than PSO-OA as shown in the results of K > 0 on IEEE 162 bus data. This further validates our link centrality measure. In Fig. 3, we see that for a given K, the damage of LC-OA increases and converges faster than PSO-OA with the growth of number of iterations. This further demonstrates that LC-OA is more efficient than PSO-GA. Moreover, we see that PSO-OA is prone to fall into the local optimum and needs many iterations to jump out of it.

VI. CONCLUSION

In summary, we study the vulnerability assessment of power grids under cascading failures. We define the K-link attack problem and prove that it is NP-complete. Particularly, we propose a link centrality measure by combining the link degree with link current. With this centrality, we develop two attack algorithms, which are the link centrality based greedy attack (LC-GA) and the link centrality based optimal attack (LC-OA). We evaluate our link centrality measure and its related attack algorithms on standard IEEE bus test data. Simulation results show that our link centrality measure performs better than the link degree and link current in identifying the optimal link in the single link attack scenario. Furthermore, in the multiple link attack problem, LC-OA is much more efficient than LC-GA and the traditional PSO based optimal attack (PSO-OA) algorithm. LC-GA has lower computational complexity than LC-OA and PSO-OA with a decent attack efficiency.

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