Radiation damage thin coating of silicon carbide

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Abstract. This paper presents computer experiments studies interaction of radiation (ions Xe++ gas with keV energy) with crystal lattice of “SiC/metal” substrate. Properties of substrate are altered due to porosity and stresses formation into thin layers covering silicon carbide as a result of inert gas flux penetration. First, the stochastic model of non-equilibrium phase transitions (or nucleation of defects) in nanomaterials is presented. Behavior of gaseous bubbles in thin layers are tested and corresponding details computer modeling are discussed. Secondly, the analytical models of Gibbs free energy implanted microdefects as well as their Brownian motion in long-range potentials of indirect interaction between gaseous bubbles into lattice through acoustic phonons are concretized. These models predict stress from gas penetration as a result of damaging in form of gaseous bubbles (named here “blisters”). New emphasis is on numerical modeling includes stochastic model clustering of defects as the diffusion of Markov processes. Finally, the mechanism of porosity formation in materials such as silicon carbide can be accounted as the ways to improve solidity and fracture toughness and hardening of materials.

1. Introduction
Transformations in solids at ion irradiation are accompanied strongly nonequilibrium processes. Large amplitude of atoms oscillations, acoustic and nonlinear waves appear due to radiation defects formation. It significantly limits the possibilities to apply the classical theories of phase transitions. Known to help in understanding the nature of those or other transformations, as well as in their forecasting can give a solution of the kinetic equations in the partial derivatives describing phase transitions in solids. Study of radiation stimulated damaging can be carried out in computer simulations of models of bilayers, for example, Si/Mo which for metal mirrors of future X-Ray lithography [1-3]. Damaging of silicon carbide to action of radiation fluxes can be considered for prediction of materials properties for plasma engine and the thermonuclear reactor divertor as well as in case of interaction radiation fluxes with new kind semiconductors in the junction with metal nanowire (such as SiC/Mo). Modeling blistering (stretched gaseous implanted structures arising in solids) was carried out by stochastic kinetic code [1-8]. Peculiarities clustering non-point wise defects are follows: alteration of micro structure of porosity into thin layers of devices under irradiation as well as formation tension within anti-diffusion “barrier” layer into multilayer structure of mirrors. Instability phase transition in “open physical systems” (under constant flux of gas) during impulse time into bilayer SiC/Mo is discussed. New kind of self-organization defects in bilayer appears from indirect elastic interaction between defects through perturbations acoustic phonons which can be complemented by the accounting of interaction of defects with the internal interfaces. We are able to put attention on the kinetic description of defects formation (voids or blisters) structures under
condition of many interfaces between metal layers and dielectric layers but here we study only bilayer. The processes of blistering are characterized by the kinetic distribution function /DF/ of defects which depends on Cartesian coordinates and velocities as well defects sizes. The blistering in substrate SiC/Mo is considered under Xe$^{++}$ fluxes similar to parameters of plasma engine. We put main attention on the kinetic description of defects formation (voids or blisters) structures as the heterogeneous first-order phase transition at non-equilibrium stage. It can be represented by model of vacancy-gaseous clustering. Many problems of probability nature described by statistical mechanics, kinetic theory and stochastic dynamics are also in good agreement with models governed by Fokker-Planck-Kolmogorov and Smolukhivskyi-Einstein equations. Non-steady state stochastic model of gas nucleation in lattice bilayer takes into consideration influence on growing (or degradation) sizes of defects the elastic forces in model of Gibbs free energy for cluster formation. This process of phase transition can be complemented by their Brownian motion into lattice. The interaction between the surface of bilayer and flow Xe$^{++}$ can be described by the equations mathematical physics [1-8], and by their stochastic analogues equations Ito-Stratonovich [9] in the presenting numerical models. We show the relation of the both problems in one-, two- and three-dimensional cases the inference of coefficients stochastic differential equations /SDEs/. The solution of SDEs gives evaluation of mathematical expectation of average values of cluster's sizes and defects coordinates as well their dispersions. We additionally are able calculate density of transient probabilities of stochastic processes /DTPSP/ as well find macroscopic characteristics of damaging SiC in Xe$^{++}$ fluxes, such as porosity SiC and stress of their lattice. DTPSP reflect (in form of DF) the evolution of clustering and Brownian motion. Modification of the surface layers properties is happening at the atomic level as a result of formation of spatial structures, for example, in the material thin film coating. Change of stress in thin surface layers in result of the development of porosity under ion implantation can be calculated. The nonlinearity of DF of blisters from their sizes is presented in this paper. This phenomenon consists in appearance distribution defects on sizes which is different from lognormal during initial non-equilibrium stage of phase transition (during several microseconds).

2. Kinetic and stochastic equations
General physical and physic-chemical problems of ion implantation and radiation physics of solid body related with physical problems of inert gases ions implantation in crystal lattice are considered in computer experiments. This models are focused on research of radiation-thermal modification of properties of materials. Kinetic equations of Kolmogorov–Feller and Einstein–Smoluchowskii had been formulated in general. The radiation damaging are able to be described in terms of probability density of distribution defects into lattice and this approach is applied to description of initial stage of first-order phase transition. The equations in partial derivative are solved using Stochastic Simulation Method /SSM/ which is similar to molecular dynamics but different from the approach of imitation real particles of lattice and Newtonian dynamics of motion. We have used set of stochastic differential equations Ito-Stratonovich which equivalent to kinetic equations in partial derivatives. Gaseous defects (pores) appear as a result of penetration into lattice of inert gas ions of high energy (about several keV). This model is used for prediction of radiation damage (in case of cover silicon carbide). Also the same model is considered for creation of new materials using radiation stimulated structures of porosity [3]. Method of stochastic analogue [9] is based on theory according to which the kinetic equations of parabolic type are uniquely linked with stochastic differential equations /SDE/ Ito and with the density of transition probabilities of Markov random process, the solution of SDE's Ito can be interpreted as the distribution function /DF/ of the corresponding kinetic equation [1-9]. Coefficients of the kinetic equations depend on probability density defects distribution namely the thermodynamic potential of nucleation (or Gibbs free energy) and long-range potentials of indirect elastic interaction of lattice defects each with other (occurs through the perturbation of acoustic phonons lattice defects and Friedel oscillations of electrons density). Diffusion in the phase space of defects sizes \{G\} and diffusion in phase space \{R\} of crystalline lattices are accounted also as coefficients depended on DF. So, Brownian motion of radiation defects in thin layers of silicon carbide occurs under the influence
of long-range forces. Stable solution of linear SDE's Ito-Stratonovich [10] is modified on a case quasilinear equations of model as stable numerical method the second order of accuracy with infinite area of sustainability (according to the new definitions and theorems of [10]) on a regular grid of time (without the limitations on the time step).

The scheme of numerical experiment is presented on fig.1.

![Fig.1 The scheme of numerical experiment is presented.](image)

The kinetic Fokker-Plank-Kolmogorov equation is

\[
\frac{\partial f(X,t)}{\partial t} = -\sum_{i=1}^{N} \frac{\partial}{\partial X_i} a_i(X,t)f(X,t) + \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{\partial^2}{\partial X_j \partial X_k} b_{jk}(X,t)f(X,t) \\
f(X,t)\big|_{t=0} = f_0(X) \\
\int \left[ a_i(X,t)f(X,t) + \frac{1}{2} \sum_{j=1}^{N} \frac{\partial}{\partial X_j} b_{ij}(X,t) \right] d\Gamma = 0, \quad i = 1, \ldots, m
\]

Where \( f(X,t) \) is probability density of a Markovian diffusion process \( X(t) \). The coefficients \( a(X,t) \) and \( b(X,t) \) are drift and diffusion coefficients or local characteristics of the process.

The trajectories of \( n \)-dimensional diffusion process with the one-dimensional probability density satisfying the Fokker-Planck-Kolmogorov equation, can be found as solution of the initial value problem for SDEs in the Ito sense

\[
X(t) = X(t_0) + \int_{t_0}^{t} \underbrace{H(\tau,X(\tau))}_{\text{drift}} d\tau + \int_{t_0}^{t} \underbrace{\sigma(\tau,X(\tau))}_{\text{stochastic}} dW(\tau),
\]

\[
X\big|_{t=0} = X_0 \quad t \in [t_0,T_{\text{end}}]
\]

\( dW \) is increment of stochastic Wiener process.

The association of kinetic and stochastic equation is following for one-dimensional case:
\[ a_i = l_i m_r \frac{1}{\Delta t} \mathbb{E}[X(t+\Delta t) - X(t) | X(t) = x] \]
\[ b_{ij} = l_i m_r \frac{1}{\Delta t} \mathbb{E}[X(t+\Delta t) - X(t) | X(t) = x] \]
and
\[ H(X,t) = a(X,t) - \frac{1}{2} \frac{\partial \sigma(X,t)}{\partial X} \sigma(X,t), \]
\[ \left( \frac{\partial \sigma(X,t)}{\partial X} \sigma(X,t) \right)_i = \sum_{k=1}^{n} \sum_{j=1}^{n} \frac{\partial \sigma_y(X,t)}{\partial y_k} \sigma_y(X,t) \]
\[ \sigma(E(X),t) = \sqrt{b_{11}} \]
\[ \sigma \text{ in two-dimensional case is } \sigma = \begin{pmatrix} b_{11} & 0 \\ \sqrt{b_{21}} & 0 \\ \sqrt{b_{22}} - \frac{b_{21}^2}{b_{11}} \end{pmatrix} \]
in three-dimensional case
\[
\sigma = \begin{pmatrix}
\sqrt{b_{11}} & 0 & 0 \\
\frac{b_{21}}{\sqrt{b_{11}}} & \sqrt{b_{22} - \frac{b_{21}^2}{b_{11}}} & 0 \\
\frac{b_{31}}{\sqrt{b_{11}}} & \sqrt{b_{22} - \frac{b_{21}^2}{b_{11}}} & \sqrt{b_{33} - \frac{b_{31}^2}{b_{11}} - \frac{b_{21}^2}{b_{11}}} - \frac{(b_{32} - b_{31}b_{21})^2}{b_{11}} \\
\end{pmatrix}
\]
So kinetic equations for examined problem are presented below.

The Kolmogorov–Feller equation for blister size evolution at the point with the coordinate \( r \) is as follows:
\[
\frac{\partial f_r(g,t)}{\partial t} = \frac{\partial}{\partial g} D_g(g,t) \frac{\partial f_r(g,t)}{\partial g} + \frac{1}{kT} \frac{\partial}{\partial g} D_g(g,t) f_r(g,t) \frac{\partial [\Phi(g,\tilde{r},t)]}{\partial g} + S_{\alpha},
\]
\[ f_r(g,0) = f_{0g}, \quad \left. \frac{df_r(g,t)}{dg} \right|_{g=2} = 0, \quad f_r(g,t) \big|_{g<2} = 0. \]
Here \( S_{\alpha} \) is the source of particles forming a nucleus; \( S_{\alpha} \) is the source of “monomers” (vacancy or gas particles) with \( f_{\alpha} \). Maxwellian DF, \( f_{\alpha}(g, t) \) is the size of nuclei’s DF at the point \( r \) of the lattice volume; \( T \) is the sample’s temperature; \( D_g \) is the diffusion coefficient in the phase space defined.
according to nucleus sizes; $\Delta \Phi (g, r, t)$ is the thermodynamic potential of the nucleus formation, $\Delta \Phi$ is measured in $kT$, $k$ is Boltzmann constant. Total potential energy of cluster formation $\Delta \Phi$ may consist of a number of additive terms:

$$
\Delta \Phi = \Delta \Phi_\mu + \Delta \Phi_\sigma + \Delta \Phi_{el} + \Delta \Phi_b + \Delta \Phi_t \tag{2}
$$

$\Delta \Phi_\mu$ corresponding to difference of chemical potentials of the phases (monomers of vacancy/gas in lattice and blisters); $\Delta \Phi_\sigma$ taking into account the surface tension at the bubble–lattice interface and spherical form of blister; $\Delta \Phi_{el}$ which takes into consideration elastic site bonds in the lattice; the rupture of relations of the atoms in the lattice around a blister is accounted by $\Delta \Phi_b$, $\Delta \Phi_t$ is a part taking into account the difference between the nodes and internodes in the array; $\Delta \Phi_p$ which takes into consideration place of blister in tension fields occurring because of disparity of parameters of lattices Mo and SiC. $D_g$ is functional-coefficient of diffusion in space of cluster sizes, which depends from the frequency of oscillations of the lattice atom, lattice parameter of material, radius of injected gas atom, modulus of rigidity of lattice material and expectation of blister size in lattice point $\{x,y,z\}$.

The common form of Gibbs energy (kJoul/mole) of blister located in SiC on distance from SiC/Mo surface 2.5 lattice parameters is presented in Fig. 2.

**Fig. 2** The dependence of Gibbs energy (kJoul/mole) of cluster of nuclei from blister size is shown. Blister size is measured in unit volums ($Xe$ atom volum). Blisters locate in 3C-SiC. The distance from SiC/Mo surface equals 2.5 lattice parameters of SiC.

The Einstein–Smoluchowski equation for BPs with the mass $M_g$ obtained from Eq. (1)

$$
\frac{\partial f_g(\vec{r},t)}{\partial t} = \frac{\partial}{\partial \vec{r}} D_r(\vec{r},t) \frac{\partial f_g(\vec{r},t)}{\partial \vec{r}} - \frac{\partial}{\partial \vec{r}} \left[ \frac{\vec{F}(\vec{r},t)}{M_g \gamma} f_g(\vec{r},t) \right] - Q, \tag{3}
$$

$$
f_g(\vec{r},t)|_{t=0} = f_{0_g},
$$

$$
f_g(\vec{r},t)|_{x=x_{left}} = f_g(\vec{r},t)|_{x=x_{right}}, \quad f_g(\vec{r},t)|_{y=y_{left}} = f_g(\vec{r},t)|_{y=y_{right}},
$$

here $f_g(r, t)$ is the distribution function of a BP with mass $M_g$ over the substrate lattice coordinates. Blister mass $M_g$ is measured in injected atom mass. Drains of the monomers and blisters are designated in (3) as $Q$; their location depends on model, here it had been chosen boundary between layers and boundaries of bilayer; $\gamma$ is coefficient of friction. Projection on the z-axis of the total force acting on BP from other particles and boundaries is located in the right part of the equation (3) and
calculated as \( F_z = -\frac{\partial U(x, y, z)}{\partial z} \), where \( U(x, y, z) \) is the superposition of long-range and sign-variable potentials of the indirect elastic interaction of BPs (through the perturbation of the lattice acoustic phonons and Friedel oscillations of the lattice electron density) among themselves, with layer boundaries, and so on. The superposition of potentials of the indirect elastic interaction of BPs only through the perturbation of the lattice acoustic phonons is presented on fig. 3.

\[
\left( \frac{N_e V_F (2k_F)}{2\pi \varepsilon^2} + W_{\text{elastic}} \right) \frac{\Omega}{kT}
\]

\( \Omega \) - volume of elementary cell, \( N_e \) - electron density, \( \varepsilon \) - permittivity, \( W_{\text{elastic}} \) - elastic moduli of the material, \( k_F \) is Fermi momentum of electron, \( V_F(2k_F) \) is the Fourier component of the potential interaction of the electron with a point defect, \( k \) is Boltzman constant, \( T \) is temperature.

\( D_r \) is functional-coefficient of diffusion in space of crystal lattice which depending from physical parameters of problem and potential energy of cluster formation \( \Delta \Phi(g, \vec{r}, t) \), cluster mass and expectation of blister size in lattice point \( \vec{r} \). Numerical method of non-linear SDE solution is based on Taylor series expansion of exact solution of Ito SDE [11-13]. The generalized type Rozenbrok methods are used for the numerical solution of the SDEs in the Stratonovich form. In this work SDEs in Ito-Stratonovich form is solved using a modified method having a second-order (or higher) accuracy [10]. Every step of time functional-coefficients (diffusion in space of blister size, diffusion in lattices space, Gibbs free energy, interaction potential) of problem are iterated taking account data about DF on previous step of time.

Example of numerical scheme for calculation of blister size and its depth for \( i \)-trajectory of Markovian random process is following:
\[
\begin{align*}
(g)_n^i &= \left(g_{n+9}ight)_n^i + \left[\frac{h_g}{2} \frac{\partial H_{n+9}^i}{\partial g} + \frac{h_z}{2} \frac{\partial H_{zn}^i}{\partial z_i}\right]^{-1} \left(K_{(n+9)g}^i \right) \\
K_{(n+9)g}^i &= h_g H_{(n+9)g}^i + \sqrt{h\sigma_{(n+9)g}^i} \xi_{(n+9)g}^i + \frac{h_g}{2} \frac{\partial \sigma_{(n+9)g}^i}{\partial g} \xi_{(n+9)g}^2 \\
K_{zn}^i &= h H_{zn}^i + \frac{h_z}{2} \frac{\partial \sigma_{zn}^i}{\partial z} \xi_{zn}^2
\end{align*}
\]

The time step for bubble size calculation is \( h_g = 0.1 h_z \), here \( h_z = h_x = h_y \) is time step for calculation position in bilayer, \( I \) – the identity matrix.

\[
H_{(n+9)g}^i = -\frac{1}{kT} D_g^i \left(g_{n+9}, t_n\right) \frac{\partial \Delta \Phi^i \left(g_{n+9}, x_n^i, y_n^i, z_n^i, t_n\right)}{\partial g_{n+9}} - \frac{1}{2} \frac{\partial D_{n+9}^i}{\partial g_{n+9}}
\]

\[
\sigma_{(n+9)g}^i = \frac{1}{q} \sqrt{2D_{n+9}^i \left(g_{n+9}, t_n\right)}
\]

\[
H_{zn}^i = \frac{1}{M \left(g_{n+1}^i\right)} \frac{\partial U^i \left(x_n^i, y_n^i, z_n^i\right)}{\partial z} - \frac{1}{2} \frac{\partial D_{zn}^i}{\partial z} \xi_{zn}^2, \quad \sigma_{zn}^i = \frac{1}{q} \sqrt{2D_{zn}^i}
\]

\( q \)- noise intensity, \( \alpha_1 \) and \( \alpha_1 \) are random numbers uniformly distributed in the interval \( (0; 1] \).

### 3. Results of numerical simulation

The color visualization of distribution function of blisters from its radiuses and time are presented on fig 4. The initial bubble radius in silicon carbide is 10Å approximately, temperature is 1600 K, irradiation dose is \( 10^{15} \) cm\(^{-2}\), Xe\(^{++}\) energy is 5 keV. As can see from this picture, DF is not lognormal distribution during fluctuation stage of first order phase transition. The blisters with radiuses from 3 Å to 28 Å develop. The expectation of blister size in lattice point is \( \langle g \rangle = \int g f(g, \vec{r}, t) \, dg \). The thermodynamic critical blister size \( \left(g_{cr}\right) \) is calculated from \( \frac{\partial \Delta \Phi}{\partial g} \big|_{g=g_{cr}} = 0 \). Despite the fact that initial blister size is bigger that critical blister size, two branch of DF are develop (see fig. 4). The initial bubbles radiuses locate in the region of instability in sizes of first order phase transition. The domain of instability in sizes is determined by the equation \( \Delta \Phi^i \left(g_{min}^i\right) = \Delta \Phi^i \left(g_{max}^i\right) = \Delta \Phi^i \left(g_{cr}^i\right) - kT \). Blisters with radius from 3 Å to 6 Å form one branch of DF. Big blisters form second branch of DF. Number of trajectories is \( 10^6 \). The time step for calculation is \( 5 \times 10^{-7} \) c, number of time step is \( 10^4 \), finish time is 0.5 mc. In future, so different typical sizes of nucleus will lead to the formation of large and small blisters which are observed in microscope.
The important characteristic is the stress resulting from the ions implantation of poorly soluble gases into bilayer. The stress resulting from blistering can influence on efficiency of devices under plasma irradiation (for example, surfaces of the plasma engines and nano-optical devices). Stress corresponding vacancy-gas pores formation and mismatch lattices layers is presented on fig. 5. Temperature is 1530 K, dose is $10^{15}$ cm$^{-2}$.

Porous silicon carbide is recognized promising material for nanooptics now. Therefore, the formation and development of porosity in a layer of silicon carbide are examined in this article. The porosity of bilayer changes during fluctuation stage. First, single blisters and vacancies form and develop/degrade. Next, blisters and vacancies clusters form long-sized structures of defects (nuclei cracks),
after cracks develop. These cracks can have output on the irradiated surface and can be filled in other nano-materials for obtaining the required optical properties. The development of long-sized structures of defects for several time moments is presented on fig. 6.

![Image of crack development](image)

**Fig.6** The development of long-sized structures of defects is presented. Red line on left callout corresponds possible directions of development of long-sized structure. The realization of this script of crack formation is presented on right callout.

The correspondence between porosity due to the development of vacancy-gas defects and stresses in the sample is shown from the comparison of fig. 5 and 6. Stress corresponding blisters formation is comparable to stress corresponding mismatch lattices layers during fluctuation stage of blistering. So, this stress must be taken into account for creation of disordered porous medium as a basis for the one-dimensional photonic crystal [14]. It should be noted that porosity of layer of silicon carbide increases at 19 times during fluctuation stage, while the average stress in this layer changes only 4 times.
4. Conclusions
Calculations have shown, that the porosity formation in a layer of silicon carbide depends on its thickness, doses, temperature and degree of discrepancy of parameters lattices layer of the coating and the substrate. Elastic stress from defects in the layers of mkm- thickness can reach value of stress corresponding discrepancy between lattice parameters of layers «coating-substrate» during of initial stage of nucleation (ms). Study of the mechanisms of phase transition which are non-equilibrium at short time (about 1 mc) carry out by SSM. Calculations are important in fusion reactor materials science, electrical propulsion engines of the spacecraft, and also in the creation of porous semiconducting and dielectric materials [14].
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5. References
[1] Bondareva A.L., Zmievskaia G.I. 2010 Computer simulation of blistering in multylayer mirrors for EUV lithography Journal of Surface Investigation. X-ray, Synchrotron and Neutron Techniques vol. 4 No 3 pp 480-487
[2] Bondareva A.L., Levchenko T.V., Zmievskaia G.I. 2010 Computer simulation model for first-order phase transition fluctuation stage Defect and Diffusion Forum vol. 297–301: Diffusion in Solids and Liquids V (Switzerland: Trans Tech Publications) pp 502-507.
[3] Zmievskaia G.I., Bondareva A.L 2012 Numerical simulation of porosity development kinetics into multy-layer media Selected scientific works of the M.V. Keldysh Institute of applied mathematics of Russian Academy of Sciences “Physical and mathematical models of plasma and plasma-like media” Edited by Giuseppe Maino and Galina I. Zmievskaia pp 16-30 (rus) (ISBN 978-5-98354-009-5 )
[4] Zmievskaia G.I., Bondareva A.L., Levchenko V.D., Levchenko T.V. 2007 A kinetic stochastic model of blistering and nanofilm islands deposition: self organization problem Journ. of Phys. D: Appl.Phys. vol. 40 pp 4842–4849.
[5] Bondareva A.L., Zmievskaia G.I., Levchenko V.D. 2006 Stochastic model applied to plasma-surface interaction's simulation Eur.Phys.J.D vol. 38, pp 143-149
[6] Bondareva A.L., Zmievskaia G.I. 2005 Stochastic simulation of fluctuation stage of thin films formation Doklady Akademii Nauk. vol. 401. № 4. (Moscow: “Nauka”) p. 471-475
[7] Zmievskaia G.I., Bondareva A.L. 2011 Crystalline islands of semiconductors films Plasma Physics Reports vol. 37, No. 1, pp. 87–95.
[8] Zmievskaia G.I., Bondareva A.L. 2010 Thin film semiconductors islands and numerical experiment Journal of Surface Investigation. X-ray, Synchrotron and Neutron Techniques vol.4 No 5 pp 836-844
[9] Zmievskaia G.I. 1997 Plasma Physics Reports vol. 23 No. 4 p 45.
[10] Artem'ev S.S., Averina T.A. 1997 Numerical analysis of systems of ordinary and stochastic differential equations (The Netherlands: Utrecht) pp. 176.
[11] Kloeden P.E., Platen E. 1992 Numerical Solution of Stochastic Differential Equations (New York: Springer) p. 36.
[12] Kloeden P.E., Platen E., Schurz, H. 1994 Numerical Solution of SDE Through Computer Experiments (Berlin: Springer) p. 292.
[13] Pugachev V., Sinitsin I. 1987 Stochastic Differential Systems: Analysis and Filtering (New York: Wiley)
[14] Diederik S. Wiersma 2013 Disordered photonics Nature Photonics vol. 7. pp 188–196