Possible Ordered States
in the 2D Extended Hubbard Model

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Possible ordered states in the 2D extended Hubbard model with on-site ($U > 0$) and nearest-neighbor ($V$) interaction are examined near half filling, with emphasis on the effect of finite $V$. First, the phase diagram at absolute zero is determined in the mean field approximation. For $V < 0$, a state where $d_{x^2-y^2}$-wave superconductivity (dSC), commensurate spin-density-wave (SDW) and $\pi$-triplet pair coexist is seen to be stabilized. Here, the importance of $\pi$-triplet pair on the coexistence of dSC and SDW is indicated. This coexistent state is hampered by the phase separation (PS), which is generally expected to occur in the presence of finite-range attractive interaction, but survives. For $V > 0$, a state where commensurate charge-density-wave (CDW), SDW and ferromagnetism (FM) coexist is seen to be stabilized. Here, the importance of FM on the coexistence of CDW and SDW is indicated. Next, in order to examine the effects of fluctuation on each mean field ordered state, the renormalization group method for the special case that the Fermi level lies just on the saddle points, $(π,0)$ and $(0,π)$, is applied. The crucial difference from the mean field result is that superconductivity can arise even for $U > 0$ and $V ≥ 0$, where the superconducting gap symmetry is $d_{x^2-y^2}$-wave for $U > 4V$ and $s$-wave for $U < 4V$. Finally, the possibilities that the mean field coexistent states survive in the presence of fluctuation are discussed.

KEYWORDS: 2D extended Hubbard model, coexistent state, d-wave superconductivity, s-wave superconductivity, spin-density wave, charge-density wave, ferromagnetism, $\pi$-triplet pair, $\eta$-singlet pair, phase separation

§1. Introduction

In connection with the studies of the copper oxide high-$T_c$ superconductors with CuO$_2$ planes, the electronic states in two-dimensional systems has been intensively studied. Especially, the possibility of various ordered states has been discussed. One characteristic feature is the proximity of superconductivity and antiferromagnetism. In our previous work (hereafter referred to as I) we have shown in the mean field approximation that the coexistent state with d-wave superconductivity (dSC), commensurate spin-density-wave (SDW) and $\pi$-triplet pair can be stabilized near half filling by repulsive backward scattering ('Umklapp' and 'exchange') processes between electrons around the saddle points $(π,0)$ and $(0,π)$. As we shall show later, this model with such a particular type of interaction have similar features to those in a square lattice model with on-site repulsion $U > 0$ and nearest-neighbor attraction $V < 0$, i.e., an extended Hubbard model. Therefore, it is interesting to examine in more detail the possibility of the above coexistent state by use of this model. At the same time, the extended Hubbard model with both on-site and nearest-neighbor repulsion ($U > 0$ and $V > 0$), is also of physical interest. In the 2D extended Hubbard model for $U > 0$, it has been shown based on the mean field approximation that extended $s$-, $p$- and $d$-wave superconductivity can arise depending on the electron density $n$ for $V < 0$, and commensurate charge- and spin-density wave (CDW and SDW) can appear at half filling $n = 1$ for $V > 0$. However, the property of the ground state for finite carrier doping and the relationship among various order parameters, especially between dSC and SDW, have not been understood yet, even in the mean field approximation. From these points of view, we will study possible ordered states, especially possible coexistence of different orders, in the 2D extended Hubbard model on a square lattice near half filling for $U > 0$ and $V ≠ 0$, with emphasis on electrons around the saddle points $(π,0)$ and $(0,π)$.

In §2 the extended Hubbard model is introduced and its relationship to our previous model used in I is referred to. In §3 the phase diagram at absolute zero, $T = 0$, is determined in the mean field approximation. In §4 the effects of fluctuation on the mean field ordered states are examined based on the renormalization method applicable only for the special case that the saddle points $(π,0)$ and $(0,π)$ lie just on the Fermi surface.

§2. Extended Hubbard Hamiltonian

The extended Hubbard Hamiltonian, $H = H_0 + H_U + H_V$, is written as follows:

\begin{equation}
H_0 = \sum_{p \sigma} \xi_p c_{p \sigma}^\dagger c_{p \sigma},
\end{equation}

\begin{equation}
H_U = U \sum_i n_{i\uparrow} n_{i\downarrow} = U \sum_q n_{q\uparrow} n_{-q\downarrow},
\end{equation}

where $\xi_p$ is the dispersion relation, $c_{p \sigma}^\dagger$ and $c_{p \sigma}$ are creation and annihilation operators at site $p$ with spin $\sigma$, respectively. $U$ and $V$ are the on-site and nearest-neighbor repulsion, respectively.
\[ H_V = \frac{V}{2} \sum_{i\neq j} n_i n_{i\uparrow} - 1 \sum_{q} V_q n_q n_{-q}, \quad (2.1c) \]

where \( \sigma \) is the spin index taking a value of +1 (−1) for \( \uparrow (\downarrow) \) spin, and the opposite spin to \( \sigma \) is denoted by \( \sigma' \equiv -\sigma \). \( N \) is the total number of lattice sites, \( \xi_p = \epsilon_p - \mu \) is the one-particle energy dispersion relative to the chemical potential \( \mu \), including nearest-neighbor- (\( t \)) and next-nearest-neighbor- (\( t' \)) hopping integrals,

\[ \epsilon_q = -2t(\cos p_x + \cos p_y) - 4t' \cos p_x \cos p_y, \quad (2.1d) \]

\[ n_q = \sum_{\sigma} n_{q\sigma} = \sum_{q} c_{k\sigma}^\dagger c_{k+q\sigma}, \quad \hat{\rho} = \pm \hat{x}, \pm \hat{y} \]

is the unit lattice vector and

\[ V_q = V(\cos q_x + \cos q_y). \quad (2.1e) \]

The energy dispersion \( \epsilon_q \) has two independent saddle points, \((\pi, 0)\) and \((0, \pi)\). In this paper, we fix \( t'/t = -1/5 \) with \( t > 0 \), in which case the Fermi surface in the absence of interaction approaches \((\pi, 0)\) and \((0, \pi)\) as the hole doping rate, \( \delta \equiv 1 - n \), is increased from half filling, \( \delta = 0 \).

Here, we examine the relationship between the 'g-ology' model used in I and the present extended Hubbard model [11]. In I, we have treated the backward scattering with large momentum transfer between electrons around \((\pi, 0)\) and \((0, \pi)\), i.e., 'Unklapp' \((g_{1\perp})\) and 'exchange' \((g_{1\perp})\) processes, and considered three types of the scattering processes, i.e., (1) Cooper-pair, (2) density-wave and (3) \( \pi \)-pair channels. [12] (Here we denote the Hamiltonian for these channels as \( H_1, H_2 \) and \( H_3 \).) Especially for the repulsive case, \( g_{3\perp} > 0 \) and \( g_{1\perp} > 0 \), we have shown that the coexistent state with dSC, SDW and \( \pi \)-triplet pair can be stabilized near half filling at low temperature. However, the above effective interaction is too simplified in that (a) forward scattering processes are not included and (b) the \( k \)-dependence of interaction is ignored. Therefore, by transforming \( H_i \) \((i = 1, 2, 3)\), into real-space representation, we will obtain a well-defined model on a square lattice. If we keep on-site and nearest-neighbor density-density Coulomb interaction, we obtain

\[ H_1 = \frac{g_{3\perp}}{2N} \left\{ \sum_i n_{i\uparrow} n_{i\downarrow} - \alpha \sum_{<ij>\sigma} n_{i\sigma} n_{j\sigma'} \right\}, \quad (2.2a) \]

\[ H_2 = \frac{g_{1\perp}}{2N} \left\{ \sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{<ij>\sigma} n_{i\sigma} n_{j\sigma} \right\}, \quad (2.2b) \]

\[ H_3 = \frac{g_{1\perp}}{2N} \left\{ \sum_i n_{i\uparrow} n_{i\downarrow} - \alpha \sum_{<ij>\sigma} n_{i\sigma} n_{j\sigma} \right\}, \quad (2.2c) \]

where \( g_{\perp} = g_{3\perp} + g_{1\perp}, \quad <ij> > \) stands for a bond connecting site \( i \) and its nearest-neighbor site \( j \) and \( \alpha = 16/\pi^4 \sim 0.164 \). Each \( H_i \) is seen to describe on-site repulsion and nearest-neighbor attraction for \( g_{3\perp}, g_{1\perp} > 0 \). Therefore, also in the extended Hubbard model for \( U > 0 \) and \( V < 0 \), the coexistent state with dSC, SDW and \( \pi \)-triplet pair is expected to be stabilized.

### §3. Mean Field Analysis

First, we determine the phase diagram at absolute zero, \( T = 0 \), near half filling in the mean field approxima-

\[ \Delta_{dSC} = -2|V||s_0|, \quad \Delta_{\pi} = -2|V||q_0|, \quad \Delta_{SDW} = U m, \]

where \( \Delta_{dSC} \) and \( \Delta_{\pi} \) include only \( V \) because \( s_\beta \) and \( q_\beta \) are defined on a bond, and \( \Delta_{SDW} \) does not include \( V \) because \( <n_i>=\sum_{\sigma} n_{i\sigma}> \) is independent of \( m \).

The pure \( \pi \)-triplet pairing state with \( \Delta_{\pi} \neq 0 \) and \( \Delta_{dSC} = \Delta_{SDW} = 0 \) is always energetically unfavorable.

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**Fig. 1.** The Fermi surface in the absence of interaction for \( t'/t = -1/5 \) and \( n = 0.9 \).
compared with the pure dSC state with $\Delta_{dSC} \neq 0$ and $\Delta_{SDW} = \Delta_\pi = 0$. However, since the coexistence of dSC and SDW ($\Delta_{dSC} \neq 0$ and $\Delta_{SDW} \neq 0$) generally results in nonzero $t_0$ (and nonzero $\Delta_\pi$ here), the self-consistency of mean field calculation requires the consideration of $\pi$-triplet pair into account from the outset. The important fact that the coexistence of spin-singlet Cooper-pair and SDW always leads to nonzero spin-triplet pair amplitude with finite total momentum had been recognized by Psaltakis et al. in a slightly different context. The close relationship among the order parameters of dSC, SDW and $\pi$-triplet pair is discussed in Appendix A.

The mean field phase diagram in the plane of $U$ and $|V|$ is shown in Fig. 2. While the dSC state is stabilized for small $U/t$, the coexistent state with dSC, SDW and $\pi$-triplet pair is possible for large $U/t$, and the phase boundary between these two states is shown by solid line. Although the pure $\pi$-triplet pairing state cannot be stabilized, $\pi$-triplet pair can condensate as a result of the coexistence of dSC and SDW. Since there is attractive interaction for spin-triplet channel in the present model, the coexistent region of dSC and SDW is widened by the inclusion of $\pi$-triplet pair. We note that the Fermi surface remains in the SDW state near half filling. As we saw in I, when SDW appears first as the temperature is lowered, the coexistent state can be stabilized at lower temperature, especially at $T = 0$, near half filling.

Generally, in the presence of finite-range attractive density-density interaction, the system can be hampered by the phase separation (PS). In order to examine the PS transition, we calculate the charge compressibility, $\kappa$, given by static and uniform charge susceptibility, in the ground state, i.e., dSC or coexistent state. This phase boundary is determined from $\kappa^{-1} = 0$. Here we use the random phase approximation (RPA), and take the RPA diagram, shown in Fig. 3, into account. The explicit form of $\kappa$ in the RPA is shown in Appendix B. The PS transition line is shown in Fig. 2 by dotted line. It is seen that the coexistent state is severely suppressed by PS but survives. This PS is expected to be suppressed if we take the long-range Coulomb repulsion into account. Here, for simplicity, we consider the next-nearest-neighbor density-density repulsion $V' > 0$, $H_{V'} = \frac{V'}{2} \sum_{\langle i,j \rangle} n_i n_{i+j} = \frac{1}{N} \sum_q V'_q n_q n_{-q}$, (3.4a)

$$V'_q = 2V' \cos q_x \cos q_y, \quad (3.4b)$$

where $\hat{q} = \pm(\hat{x} + \hat{y}), \pm(\hat{x} - \hat{y})$, and incorporate $V'$ into the RPA calculation, i.e., we replace $V_q$ with $V_q + V'_q$. We note that this replacement, which does not alter the mean field equations, can bring about the nontrivial effect on $\kappa$ in the coexistent state, from eq. (B-10a). For $V' = |V|/2$, the PS transition line is shifted, as shown in Fig. 2, from dotted line to broken line. It is seen that the long-range Coulomb interaction does lead to the suppression of the PS, which is less prominent in the coexistent state than in the dSC state.

The calculation of $\kappa$ in the RPA had been carried over by Micnas et al. only in the normal state. Dagotto et al. have shown based on quantum Monte Carlo (QMC) simulation that (1) PS drastically reduces the size of the mean field dSC region and (2) the enhancement of $d_{x^2-y^2}$-pairing correlation itself is not found. This QMC result of (2) is different from the present mean field calculation. We note that this QMC calculation is limited to the case of half filling $n = 1$ and relatively high temperature $T = t/6$.

The coexistence of dSC and SDW near half filling has been found also in the $t-J$ model in the slave-boson mean field approximation and by use of variational Monte Carlo calculation (VMC) and in the repulsive Hubbard model ($V = 0$) by use of VMC. In these studies, however, $\pi$-triplet pair has not been taken into account. The effect of $\pi$-triplet pair on the coexistence of dSC and SDW has been recently examined by Arrachea et al. based on a generalized Hubbard model. In the repulsive Hubbard model, the nearest-neighbor hopping term is modified as the correlated one,

$$H_{ch} = - \sum_{<\langle i,j \rangle \sigma}> \left\{ c_{i\sigma}^+ c_{j\sigma} + c_{j\sigma}^+ c_{i\sigma} \right\} \times \left\{ t_{AA}(1 - n_{i\sigma})(1 - n_{j\bar{\sigma}}) + t_{BB} n_{i\sigma} n_{j\bar{\sigma}} \right. \right. + t_{AB} [n_{i\sigma}(1 - n_{j\bar{\sigma}}) + (1 - n_{i\sigma}) n_{j\bar{\sigma}}] \right\}, \quad (3.5)$$

where three hopping integrals, $t_{AA}$, $t_{BB}$ and $t_{AB}$, incorporate many-body effects into one-particle hopping processes phenomenologically. $t_{AA}$ and $t_{BB}$ do not change the number of doubly occupied sites, and $t_{AB}$ does, as
shown in Fig. 4. It is to be noted that \(H_{ch}\) can be rewritten as
\[
H_{ch} = \sum_{<ij>\sigma} \left\{ c_{i\sigma}^\dagger c_{j\sigma} + c_{i\sigma} c_{j\sigma} \right\} \\
\times \left\{ -t + t_2(n_{i\sigma} + n_{j\bar{\sigma}}) + t_3n_{i\sigma}n_{j\bar{\sigma}} \right\},
\]
where
\[
t \equiv t_{AA}, \ t_2 \equiv t_{AA} - t_{AB}, \ t_3 \equiv 2t_{AB} - t_{AA} - t_{BB}. \tag{3.6b}
\]
The \(t_2\) term can be also deduced from the bare Coulomb interaction or by including the effects of phonon in the antiadiabatic approximation \(M \to 0\) (where \(M\) is the phonon mass) and the \(t_3\) term describes the three-body interaction. Arrachea et al. have shown in the mean field approximation for \(t_{AB} > t_{AA} = t_{BB}\) (i.e., \(t_2 < 0\) and \(t_3 = -2t_2 > 0\)) that the coexistence of dSC and SDW is possible but prevented by \(\pi\)-triplet pair (and ruled out for large \(U\)), due to repulsive spin-triplet pairing interaction. Hence, the effect of \(\pi\)-triplet pair on the coexistence of dSC and SDW is different from that in the present extended Hubbard model.

\[
\begin{align*}
\Delta_{FM} &= U f, \tag{3.10a} \\
\Delta_{CDW} &= (8V - U)p, \quad \Delta_{SDW} = Um, \tag{3.10b} \\
\Delta_{OAF} &= 2Vg, \quad \Delta_{SN} = 2Vl, \tag{3.10c}
\end{align*}
\]
where \(\Delta_{OAF}\) and \(\Delta_{SN}\) include only \(V\) because \(g^\dagger\) and \(l^\dagger\) are defined on a bond, and \(\Delta_{FM}\) does not include \(V\) because \(\cos(QR_i)\) is independent of \(f\). The order parameters of CDW, SDW, OAF and SN are closely related to each other, which is shown in Appendix A.

The mean field phase diagram in the plane of \(U\) and \(V\) is shown in Fig. 6. We have found that a coexistent solution with nonzero \(\Delta_{CDW}\), \(\Delta_{SDW}\) and \(\Delta_{FM}\) can be stabilized for \(n \neq 1\). This state is ferrimagnetic, as shown in Fig. 7. We note that the coexistence of CDW and SDW (\(\Delta_{CDW} \neq 0\) and \(\Delta_{SDW} \neq 0\)) generally results in nonzero \(f\) (and nonzero \(\Delta_{FM}\) here), which had been indicated by Dzyaloshinskii based on a qualitative symmetry analysis. This is the reason why we take FM into account from the outset. With the present choice of parameters, pure FM state with only \(\Delta_{FM} \neq 0\) cannot be stabilized, but FM can arise as a result of the coexistence of CDW and SDW. In the present case, the coexistent region of CDW and SDW is widened by the inclusion of FM. We note that the Fermi surface remains in the CDW or SDW state near half filling.

It is to be noted that neither OAF nor SN can be stabilized solely in the mean field approximation, independent of \(t', U, V, T\) and \(n\). This conclusion is contrary to that of Chattopadhyay et al.\(^\dagger\) that pure OAF or SN state has lower ground-state energy than pure CDW or SDW state for the half-filled case by introducing finite
Moreover, a state where local-current (OAF or SN) and density-wave (CDW or SDW) coexist cannot be also stabilized.

For \( U/t = 4.0 \), the \( V \) dependences of \( \Delta_{CDW}, \Delta_{SDW}, \Delta_{FM} \) and the difference between the energy of the pure state (CDW for \( U < 4V \) or SDW for \( U > 4V \)), \( E_p \), and that of the coexistent state (CDW+SDW+FM), \( E_c \), are shown in Fig. 8. In the coexistent state with CDW, SDW and FM, \( |\Delta_{SDW}| \) is larger than \( |\Delta_{CDW}| \) for \( U > 4V \) and vice versa for \( U < 4V \), and the first-order phase transition occurs at \( U = 4V \). For fixed \( U, |\Delta_{FM}| \) rapidly saturates as a function of \( V \). It is seen that the energy gain in the coexistent state is very small.

The energy dispersion in the ferrimagnetic coexistent state is shown in Fig. 9. There are four energy bands, and the Fermi surface remains as in the CDW or SDW state. However, the lower band of electrons with majority spin (up spin for \( \Delta_{FM} > 0 \)) is fully occupied and the Fermi level crosses only the lower band of electrons with minority spin (down spin for \( \Delta_{FM} > 0 \)). Therefore, this coexistent state is half metallic.

The coexistence of CDW and SDW with same wave vectors has also been found in a 1D modified Hubbard model for a quarter-filled band in the mean field approximation. In this coexistent state, the wave vector of charge and spin density, \( q \), and the phase difference between CDW and SDW, \( \Delta \theta \), are equal to \( 2k_F = \pi/2 \) and \( \pi/2 \), respectively, and the magnitude of local spin moment is equal at each site, as shown in Fig. 10. On the other hand, in our coexistent state, \( q = Q \equiv (\pi, \pi) \) and \( \Delta \theta = 0 \), and the magnitude of local spin moment is different at each sublattice, as shown in Fig. 7. This ferrimagnetic coexistent state is the 2D version of that found in the 3D Hubbard model (\( V = 0 \)) which had been denoted as the special ferrimagnetic (S.F.) state. In the present 2D case, this coexistent state for \( V = 0 \) can be stabilized for \( 12 \leq U/t \leq 14 \) (not shown in Fig. 6).

\[ \frac{U}{t} = 4.0, \frac{V}{t} = 1.5, n = 0.9, t'/t = -1/5 \]

Fig. 9. The energy dispersion relative to the Fermi level in the coexistent state with CDW, SDW and FM for \( U/t = 4.0, V/t = 1.5, n = 0.9 \) and \( t'/t = -1/5 \). Full (dotted) lines stand for that of electrons with up (down) spin, respectively. The lower band of electrons with up spin is fully occupied.

Fig. 10. The coexistent state with \( 2k_F \) CDW and \( 2k_F \) SDW found in a quarter-filled 1D modified Hubbard model. Each lattice site is shaded according to electron density. The length of each arrow, proportional to the magnitude of local spin moment, is equal at each site.

### 4. Renormalization Group Analysis

In the last section, we examined possible ordered states for \( U > 0 \) and \( V \neq 0 \) in the mean field approximation. In this section, we examine the effects of fluctuation on these ordered states which are not taken into account in the mean field calculation. As a theoretical treatment beyond the mean field level, we adopt the renormalization group (RG) method for the saddle points which has been applied to the Hubbard model \((V = 0)\) and determine the most dominant correlation in the normal state. We also discuss the possibility of the coexistent states beyond the mean field approximation.

#### 4.1 Saddle Point Singularity

We consider the special case where the Fermi level in the absence of interaction lies just on the saddle points \( Q_A \equiv (\pi, 0) \) and \( Q_B \equiv (0, \pi) \), i.e., \( \mu = 4t' \) (\( n \sim 0.83 \)), and focus on electrons at these two saddle points on the Fermi surface, just as two Fermi points in 1D electron systems. The Fermi surface is shown in Fig. 11.

First, we examine the behavior of the following
for $\alpha, \alpha'$ functions, particle-particle ($K$) and particle-hole ($P$) correlation functions,

$$K_{\alpha\alpha'} = \lim_{q \to 0} \int_{k,e} G_\alpha(k,\epsilon)G_{\alpha'}(-k+q,-\epsilon), \quad (4.1a)$$

$$P_{\alpha\alpha'} = \lim_{q \to 0} \int_{k,e} G_\alpha(k,\epsilon)G_{\alpha'}(k+q,\epsilon), \quad (4.1b)$$

for $\alpha, \alpha' = A, B$, where

$$G_\alpha(k,\epsilon) \equiv \frac{1}{ie - \epsilon - Q_{\alpha} + k}, \quad (4.2)$$

is the one-particle Green function in the absence of interaction for electrons near the saddle point $Q_{\alpha}$ and

$$\int_{k,e} \equiv \int_{|k|<k_c} \frac{d^2k}{(2\pi)^2} \int \frac{d\epsilon}{2\pi}, \quad (4.3)$$

where the cutoff around the saddle points, $k_c$, is introduced. We note that

$$K_1 \equiv K_{AA} = K_{BB}, \quad K_2 \equiv K_{AB} = K_{BA}, \quad (4.4a)$$

stand for Cooper- and $\pi$-pair correlation, respectively, and

$$P_1 \equiv P_{AA} = P_{BB}, \quad P_2 \equiv P_{AB} = P_{BA}, \quad (4.4b)$$

for uniform and staggered density-density correlation, respectively. For $\mu = 4t'$, these correlation functions are logarithmically divergent,

$$K_1 \sim \frac{c}{8\pi^2 t} \log^2 \frac{E_c}{\omega}, \quad P_1 \sim \frac{c}{4\pi^2 t} \log \frac{E_c}{\omega}, \quad (4.5a)$$

$$K_2 \sim \left\{ \begin{array}{ll}
\frac{c'}{4\pi^2 t} \log \frac{E_c}{\omega} & \text{for } \omega \ll rE_c, \\
-K_1 & \text{for } \omega \gg rE_c,
\end{array} \right. \quad (4.5b)$$

$$P_2 \sim \left\{ \begin{array}{ll}
\frac{c'}{4\pi^2 t} \log \frac{E_c}{\omega} & \text{for } \omega \ll rE_c, \\
-K_2 & \text{for } \omega \gg rE_c,
\end{array} \right. \quad (4.5c)$$

where $E_c > 0$ and $\omega > 0$ ($\omega \ll E_c$) are the ultraviolet and infrared energy cutoff, respectively,

$$c \equiv \frac{1}{\sqrt{1 - 4r^2}}, \quad (4.6a)$$

$$c' \equiv \frac{\log 1 + \sqrt{1 - 4r^2}}{2r}, \quad (4.6b)$$

$$c'' \equiv \frac{1}{2r} \arctan \left( \frac{2r}{\sqrt{1 - 4r^2}} \right), \quad (4.6c)$$

and $r \equiv |t'|/t$. $c$, $c'$ and $c''$ as a function of $r$ are shown in Fig. 12. Especially for small $r$,

$$c, c'' \sim 1, \quad c' \sim -\log r. \quad (4.7)$$

For $r \gg r_c \sim 0.276$, $c > c'$ and $P_1$ is more divergent than $P_2$ for $\omega \to 0$. We note $c'' < \max\{c, c'\}$, i.e., $\pi$-pair susceptibility $K_2$ is always less divergent than particle-hole susceptibility. For $t'/t = -0.2$, $c'' < c < c'$ and they are comparable in magnitude.

### 4.2 Renormalization Group Method for the Saddle Points

In the last subsection, we saw that the saddle points on the Fermi surface lead to logarithmic divergence of particle-particle and particle-hole correlation functions. This implies that the fluctuation effect becomes strong. In the RG approach, we assume that the single renormalization group variable $x \equiv \log \frac{E_c}{t'}$ determine the behavior of the system. The increase of $x$ represents renormalization towards lower energy scale. For simplicity, we neglect (1) the deformation of the Fermi surface by in-
the interaction and (2) $k$-dependence of interaction for small $|k| < k_c,$ i.e., we consider only eight coupling constants, $g_{is}$ ($i = 1, 2, 3, 4$ and $s = <, >$). This interaction of the gology type is shown in Fig. 13. $g_1$ and $g_3$ ($g_2$ and $g_4$) stand for the backward (forward) scattering processes with large (small) momentum transfer, respectively. Especially, $g_1$ and $g_3$ describe 'exchange' and 'Umklapp' processes. In I, only $g_{1\perp}$ and $g_{3\perp}$ were treated and taken to be momentum-independent all over the magnetic Brillouin zone.

![Fig. 12. c', c' and c'' as a function of r.](image1)

![Fig. 13. The scattering processes. Solid and dashed lines stand for electrons near $Q_A = (\pi, 0)$ and $Q_B = (0, \pi)$, respectively.](image2)

We start with the renormalization of the couplings in the one-loop approximation. One-loop diagrams are shown in Fig. 14. The scaling equations are shown in Fig. 14. The scaling equations are

$$g_{1\perp} = -2g_{1\perp}K_2 - 2g_{1\perp}g_{4\perp}\hat{P}_1 + 2g_{1\perp}(g_{1\parallel} - g_{2\parallel})\hat{P}_2,$$

$$g_{1\parallel} = -2g_{1\parallel}K_2 - 2g_{1\parallel}g_{4\parallel}\hat{P}_1 + [2g_{1\parallel}(g_{1\parallel} - g_{2\parallel}) + (g_{1\perp}^2 - g_{1\parallel}^2)]\hat{P}_2 + (g_{1\perp}^2 - g_{3\perp}^2)\hat{P}_2,$$

$$g_{2\perp} = -(g_{2\perp}^2 + g_{3\perp}^2)K_2 - 2g_{1\perp}(g_{1\parallel} - g_{2\parallel})\hat{P}_1 - (g_{2\perp}^2 + g_{3\perp}^2)\hat{P}_2,$$

$$g_{2\parallel} = -(g_{2\parallel}^2 + g_{3\parallel}^2)K_2 - 2(g_{4\parallel}g_{1\parallel} - g_{4\perp}g_{2\perp})\hat{P}_1 - (g_{2\parallel}^2 + g_{3\parallel}^2)\hat{P}_2,$$

$$g_{3\parallel} = -2g_{3\parallel}g_{4\parallel}K_1 - 2g_{3\perp}(g_{2\parallel} + g_{2\perp} - g_{1\parallel})\hat{P}_2,$$

$$g_{3\perp} = -2g_{3\parallel}g_{4\perp}K_1 - 2g_{3\perp}(g_{2\parallel} + g_{2\perp} - g_{1\parallel})\hat{P}_2$$

which are to be solved with the initial conditions $g_{is}(x = x_i) = g_{is}^0$ ($\equiv d/dx$), where $g_{is}^0$ are the bare coupling constants. For example, $g_{1\perp}$ has the following form to one loop order,

$$g_{3\perp} = -2g_{3\perp}g_{4\perp}K_1 - 2g_{3\perp}g_{4\perp}(g_{2\parallel} + g_{2\perp} - g_{1\parallel})\hat{P}_2.$$  

By differentiating this equation by $x$ and replace $g_{is}^0$ by $g_{is}$, i.e., the bare coupling constants by the renormalized ones, we obtain the scaling equation eq. (4.8e).

If we take $g_{1\perp}^0 = g_{1\perp}^0$ as the initial conditions, the relation $g_{1\perp} = g_{1\parallel}$ holds all through the flow. Therefore, the above scaling equations are simplified as follows:

$$\dot{g}_1 = -2g_1g_2K_2 - 2g_1g_4\hat{P}_1 - 2g_1(g_2 - g_1)\hat{P}_2,$$

$$\dot{g}_2 = -(g_2^2 + g_3^2)K_2 + 2g_4(g_2 - g_1)\hat{P}_1 - (g_2^2 + g_3^2)\hat{P}_2,$$

$$\dot{g}_3 = -2g_3g_4K_1 - 2g_2(g_2 - g_1)\hat{P}_2,$$

$$\dot{g}_4 = -(g_3^2 + g_4^2)K_1 - g_2^2 + -2g_2(g_2 - g_1)\hat{P}_1.$$  

where $g_1 \equiv g_{1\perp} = g_{1\parallel}$. The divergence of $g_{is}(x)$ at a finite $x$ indicates the existence of the strong coupling fixed point, i.e., signals the development of an ordered state, at finite energy scale or finite temperature. (Strictly speaking, this finite onset temperature is an artifact of the present approximation in the 2D systems, and should be interpreted as a crossover temperature, or a critical temperature when finite three-dimensionality is assumed.) The properties of this strong coupling fixed point can be obtained qualitatively from various response functions. The response functions in the one-loop approximation are obtained from one-loop diagrams shown in Fig. 15. The response function,

$$R_\nu = \int_0^\beta d\tau e^{i\nu\tau} - \left\langle \frac{1}{N} < T_{\tau}\hat{O}_\nu(\tau)\hat{O}_\nu > \right\rangle,$$

where $\nu$ stands for the kind of correlation $\nu =$SC, SDW, · · ·), has the following form to one loop order,

$$R_\nu = R_\nu^0 + \frac{1}{4} g_\nu^0(R_\nu^0)^2,$$

where $g_\nu^0$ is the coupling constant (linear combination of $R_\nu^0$) and $R_\nu^0$ is the simple bubble. If we differentiate this equation by $x$ and replace $g_\nu^0$, $R_\nu^0$ by the renormalized ones, $g_\nu$ and $R_\nu$, we obtain

$$\tilde{R}_\nu = \tilde{R}_\nu^0 \left\{ 1 + \frac{1}{2} g_\nu R_\nu \right\}.$$  

This equation is to be solved with the initial condition that $R_\nu(x = x_i) \sim 0$. Since $\tilde{R}_\nu^0$ is positive, $R_\nu$ can
be divergent for $g_v > 0$ and are suppressed to zero for $g_v < 0$. In this paper, we consider the response functions shown in Fig 16, where $\hat{O}_{SC}$, $\hat{O}_\eta$ and $\hat{O}_{PS}$ stand for $s$-wave Cooper-pair, $\eta$-singlet pair with a total momentum $Q$ and total spin $S = 0$ and uniform charge density, respectively. The most divergent $R_{PS}$ is interpreted to describe the phase separation (PS). The relationship among these order parameters is discussed in Appendix A. We note that each correlation is treated independently in the above procedure. Therefore, we can determine the most dominant susceptibility in the normal state, and cannot assess the coexistence of different orders.

- **Fig. 14.** Diagrams contributing to the one-loop order correction to coupling constants.

  (a) \hspace{2cm} (b) \hspace{2cm} (c) \hspace{2cm} (d)

- **Fig. 15.** Diagrams contributing to the one-loop order correction to response functions.

  (a) \hspace{2cm} (b)

4.3 Phase Diagram

We solve the scaling equations, eq. (4.8) and (4.13), with the initial conditions,

- $g_{1\perp}^0 = g_{3\parallel}^0 = U + 2V_0 = U - 4V,
- g_{2\perp}^0 = g_{1\parallel}^0 = U + 2V_0 = U + 4V,
- g_{1\parallel}^0 = g_{3\perp}^0 = 2V_0 = -4V,
- g_{2\parallel}^0 = g_{4\perp}^0 = 2V_0 = 4V,

at $x = x_i$. Here, we take $x_i \equiv 0$ and $R_v(x_i) \equiv 0$ for simplicity, although the solution of the scaling equations depends on the value of $x_i$ and $R_v(x_i)$.

Before we show our results, we refer to previous results for $U > 0$ and $V = 0$ obtained by many authors. $H_U$

| $\nu$ | $R_v^0$ | $g_v^0$ |
|-------|---------|---------|
| dSC  | $\frac{1}{2} \sum_i \sigma_i c_i^\dagger c_i - 2c_i^\dagger c_i$ | $4K_1$ |
| sSC  | $\pi$ | $-g_{3\parallel} - g_{1\parallel}$ |
| $\eta$ | $\frac{1}{2} \sum_i \sigma_i c_i^\dagger c_i$ | $4K_2$ |
| CDW | $\sum_i \sigma_i c_i^\dagger c_i$ | $-4P_1$ |
| SDW | $\sum_i \sigma_i c_i^\dagger c_i$ | $4P_2$ |
| OAF | $\sum_i \sigma_i c_i^\dagger c_i$ | $4P_3$ |
| SN | $\sum_i \sigma_i c_i^\dagger c_i$ | $4P_4$ |
| FM | $\sum_i \sigma_i c_i^\dagger c_i$ | $4P_5$ |
| PS | $\sum_i \sigma_i c_i^\dagger c_i$ | $-4P_1$ |

$g_v^0$ for $U > 0$. In this paper, we consider the response functions $\eta_i$ obtained by many authors.

before we show our results, we refer to previous results for $U > 0$ and $V = 0$ obtained by many authors. $H_U$ can be rewritten as follows,

$$H_U = \frac{U}{2} \sum_i n_i n_i - \frac{U}{2} \sum_i n_i.$$ (4.15)

If we regard the second term in the r.h.s of eq. (4.13) as the chemical potential shift, we can take $g^0_{1\perp} = g^0_{3\parallel} = U$ as the initial conditions and therefore use eq. (4.10) as the scaling equations of the coupling constants. For the perfect nesting case $r \equiv |t'|/t = 0$, Schulz and Dzyaloshinski showed that SDW occurs, and pointed out that small deviations from half filling lead to dSC. Lederer et al. solved the flow equations with the initial condition $\sum_i \sigma_i c_i^\dagger c_i = 0$, the RG method for $U > 0$ and $V > 0$ favors small $V > 0$ as well as $V = 0$ near half filling is consistent with a recent calculation based on the fluctuation-exchange (FLEX) approximation. Except for superconductivity for $U > 0$ and $V > 0$, the RG phase diagram is qualitatively same as the mean field one when we do not take the coexistence of different orders into account, i.e., SDW and CDW appear for $U > 4V$ and $U < 4V$, respectively, and attractive $V < 0$ favors...
dSC for small $|V|$ and PS for large $|V|$, respectively. With regard to the correlation of $\pi$-triplet pair, our RG calculation has shown that it can be divergent for attractive $V < 0$ and large $|V|$ but is always subdominant. Similarly, FM cannot be the most dominant solely. These results are also consistent with our mean field ones.

Next, we discuss the possibility of the coexistence of different orders at low temperature, especially at $T = 0$. It is very important that our RG calculation shows the existence of a region where the onset temperature of SDW or CDW becomes highest, as shown in Fig. 17. Since our mean field calculation in (3.1) or (3.2) shows that the Fermi surface remains in the SDW and CDW states near half filling, we might expect to find a second phase transition at lower temperature in such SDW and CDW states. Therefore, at lower temperature in the SDW region in Fig. 17, (1) the coexistent state with dSC, SDW and $\pi$-triplet pair found for $V < 0$ in the mean field approximation might be expected to survive for not only $V < 0$ but also $V \geq 0$, and (2) the ferrimagnetic coexistent state with CDW, SDW and FM found for $U > 4V > 0$ in the mean field approximation might be expected to survive. In fact, as we have pointed out in (3.1), VMC calculations for $U > 0$ and $V = 0$ show the coexistence of dSC and SDW at low temperature, although $\pi$-triplet pair has been neglected. Similarly, at lower temperature in the CDW region in Fig. 17, (3) the ferrimagnetic coexistent state with CDW, SDW and FM found for $4V > U > 0$ in the mean field approximation might be expected to survive. Especially, $\pi$-triplet pair (FM), which cannot be stabilized solely in the parameter region considered in the present mean field and RG approximation, might be expected to survive as a result of the coexistence of dSC and SDW. In order to assess the effect of $\pi$-triplet pair (FM) on the coexistence of dSC and SDW (CDW and SDW), we need another theoretical treatment.

![Fig. 17. The RG phase diagram for $t'/t = -1/5$ and $\mu = 4t'$](image)

Here, we refer to the possibility of the coexistence of sSC and CDW. In the $g$-ology model in I, it can be easily shown that the coexistent state with sSC, CDW and $\eta$-singlet pair can be stabilized near half filling at low temperature for $g_{3\perp} < 0$ and $g_{1\perp} < 0$. As seen from eq. (2.2), this case corresponds to that of $U < 0$ and $V > 0$ in the present extended Hubbard model. Therefore, this coexistent state might be expected to be stabilized also in the extended Hubbard model for $U < 0$ and $V > 0$ in the mean field approximation. Moreover, based on the above discussion, it might be expected to survive in the presence of fluctuation for not only $U < 0$ but also $U \geq 0$, at lower temperature in the CDW region in Fig. 17. This will be reported elsewhere.

Finally, we refer to the ambiguities of the above RG method. Since there exist not only log- but also log$^2$-divergence in the particle-particle and particle-hole correlation functions, eq. (1.3), we cannot safely take the limit $\omega \to 0$ in the scaling equations of coupling constants and response functions, eqs. (1.8) and (1.13), i.e., it is not clear at all whether the above RG treatment is valid or not. In fact, the solution of eq. (1.8) and (1.13) depends on the initial value $x_i$. If we consider only the most singular log$^2$ term in $K_1$ (and $P_2$ for $r = 0$), and take $y \equiv x^2 = \log^2 \frac{\hbar}{\omega}$ as a new scaling variable, we can safely take $\omega \to 0$ limit in the scaling equations of coupling constants. In this case, the above RG method might correspond to a parquet summation of leading log$^2$ divergences, rather than renormalization procedure.

§5. Conclusion and Discussion

We have studied in detail possible ordered states, especially possible coexistence of different orders, near half filling in the 2D extended Hubbard model with on-site repulsion $U > 0$ and nearest-neighbor interaction $V$, with emphasis on electrons around the saddle points ($\pi, 0$) and $(0, \pi)$.

First, we have determined the phase diagram at $T = 0$ in the mean field approximation. For $V < 0$, we have shown that the coexistent state with dSC, SDW and $\pi$-triplet pair can be stabilized near half filling. Here, we have indicated the following important fact which has often been neglected in previous studies: when we discuss the coexistence of dSC and SDW, it is necessary to take $\pi$-triplet pair into account from the outset, because in general the coexistence of dSC and SDW results in $\pi$-triplet pair and is affected by $\pi$-triplet pair. Especially, $\pi$-triplet pair, which cannot condensate solely in the present model, can arise through the coexistence of dSC and SDW. Since the phase separation (PS) is generally expected to occur in the presence of finite-range attractive interaction such as $V < 0$, we have examined the effect of PS on the mean field ground state in the random phase approximation (RPA). The coexistent state with dSC, SDW and $\pi$-triplet pair is severely hampered by PS but survives, and that the long-range Coulomb repulsion such as next-nearest-neighbor density-density repulsion leads to the suppression of PS. On the other hand, for $V > 0$, we showed that a ferrimagnetic coexistent state with commensurate charge-density-wave (CDW), SDW and ferromagnetism (FM) can be stabilized near half filling. Here, we have indicated the following important fact: when we discuss the coexistence of CDW and SDW, it is necessary to take FM into account from the outset, because in general the coexistence of CDW and SDW results in FM and is affected by FM. Especially, FM, which cannot be stabilized solely with
the present choice of parameters, can arise through the coexistence of CDW and SDW. It is to be noted that the above mean field coexistent states near half filling can be stabilized at low temperature, especially at \( T = 0 \), when CDW or SDW, in which the Fermi surface remains near half filling, arises first at high temperature.

In order to examine the effects of fluctuation on the mean field ordered states, we have adopted the RG method for the special case that the Fermi level lies just on the saddle points. We have shown that the crucial difference from our mean field result is that superconductivity can arise even for \( U > 0 \) and \( V \geq 0 \); dSC and sSC for \( U > 4V \) and \( U < 4V \), respectively. Except for this difference, the RG phase diagram is qualitatively the same as the mean field one when we do not take the coexistence of different orders into account, e.g., SDW and CDW can arise for \( U > 4V \) and \( U < 4V \), respectively. Especially, the correlation of \( \pi \)-triplet pair or FM cannot be the most dominant solely. Here, it is very important that a region where the onset temperature of SDW or CDW becomes highest is found in the RG phase diagram. Since the Fermi surface remains near half filling in these SDW and CDW states, we might expect to find a second phase transition at lower temperature. In the RG method, however, we cannot assess such possibilities. On the other hand, the mean field approximation, which is often questionable for the 2D case, is of great advantage on the saddle points. We have shown that the crucial differences from our mean field result is that superconductivity can arise even for \( U > 0 \) and \( V \geq 0 \); dSC and sSC for \( U > 4V \) and \( U < 4V \), respectively. Except for the difference, the RG phase diagram is qualitatively the same as the mean field one when we do not take the coexistence of different orders into account, e.g., SDW and CDW can arise for \( U > 4V \) and \( U < 4V \), respectively. Especially, the correlation of \( \pi \)-triplet pair or FM cannot be the most dominant solely. Here, it is very important that a region where the onset temperature of SDW or CDW becomes highest is found in the RG phase diagram. Since the Fermi surface remains near half filling in these SDW and CDW states, we might expect to find a second phase transition at lower temperature. In the RG method, however, we cannot assess such possibilities. On the other hand, the mean field approximation, which is often questionable for the 2D case, is of great advantage.

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We note that similar relationships hold among dSC, CDW and \( \eta \)-singlet pair. We have shown that the crucial differences from our mean field result is that superconductivity can arise even for \( U > 0 \) and \( V \geq 0 \); dSC and sSC for \( U > 4V \) and \( U < 4V \), respectively. Except for this difference, the RG phase diagram is qualitatively the same as the mean field one when we do not take the coexistence of different orders into account, e.g., SDW and CDW can arise for \( U > 4V \) and \( U < 4V \), respectively. Especially, the correlation of \( \pi \)-triplet pair or FM cannot be the most dominant solely. Here, it is very important that a region where the onset temperature of SDW or CDW becomes highest is found in the RG phase diagram. Since the Fermi surface remains near half filling in these SDW and CDW states, we might expect to find a second phase transition at lower temperature. In the RG method, however, we cannot assess such possibilities. On the other hand, the mean field approximation, which is often questionable for the 2D case, is of great advantage.

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proximation that a state where dSC, OAF and \( \eta \)-singlet pair (sSC, SN and \( \pi \)-triplet pair) coexist can be stabilized in the g-ology model used in I for \( g_{\perp} > 0 \) and \( g_{\parallel} < 0 \) (\( g_{\perp} < 0 \) and \( g_{\parallel} > 0 \)).

Moreover, eq. (A.1)-(A.4) imply close relationships (1) between sSC and dSC, (2) between \( \eta \)-singlet pair and \( \pi \)-triplet pair, and (3) between density-wave (CDW, SDW) and local-current (OAF, SN). In fact, the hermite operators of uniform bond-charge and bond-spin (BC and BS) density with \( d_{x^2-y^2} \)-symmetry defined as

\[
\hat{O}_{BC} = \sum_{\sigma \sigma'} w_{p\sigma} c_{p\sigma}^\dagger c_{p\sigma'}, \quad \hat{O}_{BS} = \sum_{\sigma \sigma'} \sigma w_{p\sigma} c_{p\sigma}^\dagger c_{p\sigma'},
\]

satisfy similar commutation relations,

\[
[\hat{O}_{\alpha}^\dagger, \hat{O}_\beta] = 2i \hat{O}_\gamma, \quad [\hat{O}_\gamma, \hat{O}_\alpha^\dagger] = 2i \hat{O}_\beta^\dagger,
\]

(A.6a)

\[
[\hat{O}_\beta, \hat{O}_\gamma] = 2i \hat{O}_\alpha,
\]

(A.6b)

for \( (\alpha, \beta, \gamma) = (sSC, dSC, BC) \) and \( (\eta, \pi, BS) \), and

\[
[\hat{O}_\alpha^\dagger, \hat{O}_\beta] = 2i \hat{O}_\gamma, \quad [\hat{O}_\gamma, \hat{O}_\alpha^\dagger] = 2i \hat{O}_\beta^\dagger,
\]

(A.7a)

\[
[\hat{O}_\beta, \hat{O}_\gamma] = 2i \hat{O}_\alpha,
\]

(A.7b)

for \( (\alpha, \beta, \gamma) = (SDW, BC, SN) \), (CDW, BS, OAF), (SDW, BS, OAF) and (CDW, BS, SN). If \( w_p \propto \cos p_x - \cos p_y \), eq. (A.6b) and (A.7b) hold only approximately in each case.

The relationship among order parameters which are rotated into each other by \( \hat{O}_\eta \), \( \hat{O}_\sigma \), \( \hat{O}_{BC} \) and \( \hat{O}_{BS} \) is summarized as shown in Fig. 18, where the connection with \( \hat{O}_{dSC} \) and \( \hat{O}_{sSC} \) by \( \hat{O}_{BC} \) is not explicitly shown.

![Fig. 18. The relationship among various order parameters.](image)

Appendix B: Charge Susceptibility in the Random Phase Approximation

The charge compressibility \( \kappa \) is equal to static and uniform charge susceptibility \( \chi \),

\[
\kappa = \lim_{q \to 0} \chi^R(q, q, \omega = 0),
\]

(B-1)

where \( \chi^R(q, q', \omega) \) is the retarded density-density correlation function, which is obtained from the analytic continuation of \( \chi(q, q', \omega_l) \) through \( \omega_l \to \omega + i\delta \),

\[
\chi(q, q', \omega_l) = \int_0^\beta d\tau e^{i\omega_l\tau} \chi(q, q', \tau).
\]

(B-2a)

\( \chi(q, q', \tau) \) is the two-particle thermal Green function in imaginary time given by

\[
\chi(q, q', \tau) = \sum_{\sigma \sigma'} \chi_{\sigma \sigma'}(q, q', \tau),
\]

(B-3a)

\[
\chi_{\sigma \sigma'}(q, q', \tau) = \frac{1}{N} \langle T_{\tau} n_{\eta \sigma}(\tau) n_{-\eta \sigma} \rangle,
\]

(B-3b)

where \( n_{\eta \sigma} = \sum_k c_{\eta \sigma}^\dagger c_{\eta \sigma} \), \( n_{-\eta \sigma} = (n_{\eta \sigma})^\dagger \). We write \( \chi_{\sigma \sigma'} \) in the matrix form as follows,

\[
\hat{N} = \left( \begin{array}{cc} N_{1 \uparrow} & N_{1 \downarrow} \\ N_{\uparrow 1} & N_{\downarrow 1} \end{array} \right).
\]

(B-4)

For the mean field Hamiltonian with \( \Delta_{dSC}, \Delta_{SDW} \) and \( \Delta_{\pi} \), the one-particle thermal Green functions, defined by

\[
\chi_{\sigma \sigma'}(p, p', \tau) \equiv - < T_{\tau} c_{\sigma \parallel}(\tau)c_{\sigma' \parallel}^\dagger >,
\]

(B-5a)

\[
F_{\sigma \sigma'}^{(1)}(p, p', \tau) \equiv - < T_{\tau} c_{\sigma \parallel}(\tau)c_{\sigma' \parallel}^\dagger >,
\]

(B-5b)

\[
F_{\sigma \sigma'}^{(1)}(p, p', \tau) \equiv - < T_{\tau} c_{\sigma \parallel}(\tau)c_{\sigma' \parallel}^\dagger >,
\]

(B-5c)

have the following form,

\[
G_{\sigma \sigma'}(p, p', \tau) \equiv \delta_{p', p} G_{\sigma \parallel 1}(p, \tau) + \delta_{p', -p+Q} G_{\sigma \parallel 2}(p, \tau),
\]

(B-6a)

\[
F_{\sigma \sigma'}^{(1)}(p, p', \tau) \equiv \delta_{p', -p} F_{\sigma \parallel 1}^{(1)}(p, \tau) + \delta_{p', -p+Q} F_{\sigma \parallel 2}^{(1)}(p, \tau).
\]

(B-6b)

Therefore, it is seen that \( \chi \) has the following form,

\[
\hat{N}(q, q', \tau) = \delta_{q', q} \hat{N}_1(q, \tau) + \delta_{q', q+Q} \hat{N}_2(q, \tau),
\]

(B-7a)

\[
\hat{N}_1 = \left( \begin{array}{cc} N_{1 \uparrow} & N_{1 \downarrow} \\ N_{\uparrow 1} & N_{\downarrow 1} \end{array} \right),
\]

(B-7b)

\[
\hat{N}_2 = \left( \begin{array}{cc} N_{2 \uparrow} & N_{2 \downarrow} \\ -N_{\uparrow 2} & -N_{\downarrow 2} \end{array} \right),
\]

(B-7c)

The subscript 1 and 2 represent normal and Umklapp part, respectively. It is to be noted that there exists Umklapp part \( \hat{N}_2 \) for \( \Delta_{SDW} \neq 0 \) or \( \Delta_{\pi} \neq 0 \). With regard to \( \chi(q, q', \tau) \), it has only normal part,

\[
\chi(q, q', \tau) = \delta_{q', q} \cdot \chi_0(q, \tau),
\]

(B-8a)

\[
\chi_0(q, \tau) \equiv \frac{1}{N} \left[ N_{1 \parallel}(q, \tau) + N_{1 \perp}(q, \tau) \right].
\]

(B-8b)

Next, we treat the effect of interaction in the random phase approximation (RPA). The RPA equation is diagrammatically shown in Fig. 19. Here, we consider only (a) for interaction vertex in Fig. 19, which leads to the diagram shown in Fig. 3. In the frequency space, the RPA equation is written as follows \( (z \equiv \omega_l) \),

\[
\hat{N}_{RPA}(q, q', z) = \hat{N}(q, q', z) + \sum_{\eta_1} \hat{N}(q, q_1, z) \hat{g}(q_1) \hat{N}_{RPA}(q_1, q', z),
\]

(B-9a)

where

\[
\hat{g} \equiv \left( \begin{array}{cc} g_{\parallel} & g_{\perp} \\ g_{\perp} & g_{\parallel} \end{array} \right).
\]

(B-9b)
where $\chi_{RP A}$ is obtained as follows,

$$
\chi_{RP A}(q, q + Q) = 0, \quad (B.10b)
$$

and the variable $\kappa$ is not explicitly written. Therefore, we can obtain the charge compressibility $\kappa$ in the RPA, from eq. (B.1) and (B.10a). In the normal or dSC state, $\chi_{RP A}$ has a familiar form due to $u_+ \equiv 0$.

$$
\chi_{RP A}(q, q) = \frac{2n_+^2 X^+_{q+Q} + Y_q}{X^+_{q} X^+_{q+Q} - g_+ Y_q}, \quad (B.10a)
$$

where $n_+^\mp \equiv N_1\mp(q) \pm N_2(\pm q)$ and $n_+ \equiv N_2(q) \pm N_{2\perp}(q)$.

$$
g_+^\pm \equiv g_{\parallel}(q) \pm g_{\perp}(q), \quad (B.10d)
$$

and $X^+_{q} = 1 - g_+ n^+_q$, $Y_q = g_{+q} u_q u_{+q}$.

$$
x_{RP A} = 0 \quad \text{for } u_+ \neq 0 \text{ in the SDW or } \pi\text{-triplet pairing state, especially in the coexistent state with dSC, SDW and } \pi\text{-triplet pair, } \chi_{RP A} \text{ has a nontrivial form.}
$$

---

**Fig. 19.** The RPA equation for charge susceptibility. The square stands for interaction vertex. In our calculation, the diagram of (b) for interaction vertex is neglected. [3] E. Dagotto, J. Riera, Y. C. Chen, A. Moreo, A. Nazarenko, F. Alcaraz and F. Ortolani: Phys. Rev. B 49 (1994) 3548; A. Nazarenko, A. Moreo, E. Dagotto and J. Riera: Phys. Rev. B 54 (1996) R768.

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