ETTORE MAJORANA: 
HIS WORK AND HIS LIFE

April 2014

Erasmo Recami
Facoltà di Ingegneria, Università statale di Bergamo, Dalmine (BG), Italy; INFN—Sezione di Milano, Milan, Italy; and DMO/FEEC, UNICAMP, Campinas, SP, Brazil

Abstract — In this paper we present a panoramic view of the main scientific articles published by Ettore Majorana, the brightest Italian theoretical physicist of the XX century (actually, Enrico Fermi regarded him as the brightest in the world of his time, and compared him to Galileo and Newton; even if to some people Majorana is often known mainly for his mysterious disappearance in 1938, when he was 31). Extensive information and comments are added with regard to the scientific manuscripts left unpublished by him. We also outline his life, the biographical data being based on letters, documents, testimonies discovered or collected by the author during more than four decades, and contained for instance in Recami’s 1987 book quoted in the text. Two pictures complete the paper.

1 Historical touch

Ettore Majorana’s fame[1] solidly rests on testimonies like the following one, by the mindful pen of Giuseppe Cocconi. At the request of Edoardo Amaldi[2], he wrote from CERN (July 18th, 1965):

“In January 1938, after having just graduated, I was offered, essentially by you, to come to the Institute of Physics at the University in Rome for six months as a teacher assistant, and once I was there I would have the good fortune of joining Fermi, Bernardini (who had been granted a chair at Camerino a few months earlier) and Ageno (he too, a neo-graduate), in the research of the products of disintegration of “mesons” $\mu$ (at that time called mesotrons or yuks) which are produced by cosmic rays. [...] It was actually while I was staying with Fermi in the little laboratory on the second floor, absorbed in our work, with Fermi working a piece of
Wilson’s chamber (which would help to reveal mesons at the end of their range) on a lathe and me constructing a jalopy for the illumination of the chamber, using the flash produced by the explosion of an aluminum ribbon shortcircuited on a battery, that Ettore Majorana came in search of Fermi. I was introduced to him and we exchanged few words. A dark face. And that was it. An easily forgettable experience if, after a few weeks while I was still with Fermi in that same workshop, news of Ettore Majorana’s disappearance in Naples had not arrived. I remember that Fermi busied himself with telephoning around until, after some days, he had the impression that Ettore would never be found.

It was then that Fermi, trying to make me understand the significance of this loss, expressed himself in quite a peculiar way: he, who was so objectively stern when judging people. And so at this point I would like to repeat his words, just as I can still hear them ringing in my memory: ‘Because, you see, in the world there are various categories of scientists; people of a secondary or tertiary standing, who do their best but do not go very far. There are also those of high standing, who come to discoveries of great importance, fundamental for the development of science’ (and here I had the impression that he had placed himself in that category). ‘But then there are geniuses like Galileo and Newton. Well, Ettore was one of them. Majorana had what no one else in the world had [...]’.

And with first-hand knowledge, Bruno Pontecorvo, added: “some time after his entry into Fermi’s group, Majorana already possessed such an erudition and had reached such a high level of comprehension of physics that he was able to speak on the same level with Fermi about scientific problems. Fermi himself held him to be the greatest theoretical physicist of our time. He often was astounded [...]. I remember exactly these words that Fermi spoke: ‘If a problem has already been proposed, no one in the world can resolve it better than Majorana’.” (see [3, 4, 5].)

Ettore Majorana disappeared rather mysteriously on March 26th, 1938, and nobody found him [1, 4, 5]. The myth of his “disappearance” has contributed to nothing more than the notoriety he was entitled to, for being a true genius and a genius before his time. All the serious documents known till now have been discovered or collected by us (starting with 1970), and published for the first time by us, for instance in our book [1].

In more recent times, after that his lecture notes had been published[6], we eventually published his notebooks Volumetti, namely, his study notes written in Rome between 1927, when he moved from his studies in engineering to take up physics, and 1931. Those manuscripts are a model not only of order, divided by argument and even supplied with an index, but also in conciseness, essentiality and originality. So much so that
the related notebooks can be regarded in a sense as a modern text of theoretical physics, even after ninety years, and a “mine” of new theoretical, physical, mathematical ideas and hints, quite stimulating and useful for present time research[7]. Even more interesting for present time physics are the pure research scientific manuscripts (like his Quaderni) left unpublished by Majorana: we made recently known part of them in a 500 pages Springer’s volume[8]. Incidentally, catalogues of the scientific manuscripts left by Majorana can be found in [9] [8].

Let us recall[1] that Majorana, after having switched to physics at the beginning of 1928, graduated with Fermi on July 6th, 1929, and went on to cooperate with the famous group created by Enrico Fermi and Franco Rasetti (at the beginning, with Orso M. Corbino’s important help); whose theory subgroup was formed mainly, in the order of their entrance into the Institute, by Ettore Majorana, Gian Carlo Wick, Giulio Racah, Giovanni Gentile Jr., Ugo Fano, Bruno Ferretti, and Piero Caldirola; while the members of the experimental subgroup were: Emilio Segré, Edoardo Amaldi, Bruno Pontecorvo, Eugenio Fubini, Mario Ageno, Giuseppe Cocconi, as well as the chemist Oscar D’Agostino. Afterwards, Majorana qualified for university teaching of theoretical physics (“Libera Docenza”) on November 12th, 1932; spent about six months in Leipzig with W. Heisenberg during 1933; and then for some unknown reasons stopped participating in the Fermi’s group activities and even in publishing his works. Except for his paper “Teoria simmetrica dell’elettrone e del positrone” that (even if ready since 1933) Majorana was happily convinced by his colleagues to take out from a drawer and publish at the threshold of the 1937 Italian national competition for three full-professorships: That is the paper dealing with the “Majorana’s neutrino”.

With respect to the last point, let us recall that in 1937 there were numerous Italian competitors and many of them were of exceptional valor; above all: Ettore Majorana, Giulio Racah, Gian Carlo Wick, and Giovanni Gentile Jr. (the son of the famous philosopher bearing the same name, and the inventor of “parastatistics” in quantum mechanics). The judging committee was presided by E. Fermi and had as members E. Persico, G. Polvani, A. Carrelli and O. Lazzarino. After a proposal by the judging committee, the Italian Minister of National Education nominated Majorana professor of theoretical physics at Naples University because of his “great and well-deserved fame”, independently of the competition itself (actually, “the Commission hesitated to apply the normal university competition procedures to him”). The attached report on the scientific activities of Ettore Majorana, sent by the committee, states[1]:

“Without listing his works, all of which are highly notable both for
their originality of methods utilized as well as for the importance of the achieved results, we limit ourselves to the following:

In modern nuclear theories the contribution made by this researcher with the introduction of the forces called “Forces of Majorana” is universally recognized as, among the most fundamental, the one that permits us to theoretically comprehend the reasons for nuclear stability. The work of Majorana today serves as a base for the most important research in the field.

In atomic physics, the merit of having resolved some of the most intricate questions on the structure of spectra through simple and elegant considerations of symmetry is due to Majorana.

Lastly, he devised a brilliant method that permits us to treat the positive and negative electron in a symmetrical way, finally eliminating the necessity of relying on the extremely artificial and unsatisfying hypothesis of an infinitely large electrical charge diffused in the space, a question that had been approached in vain by many other scholars.”

One of the most important works of Ettore Majorana, the one that introduces his “infinite components equation” was not mentioned: it had not been understood yet. It is interesting to note, however, that the right light was shed on his theory of electron and anti-electron symmetry (today at its height for its application to neutrinos and anti-neutrinos, as we mentioned); and in result of his capacity to eliminate the hypothesis called “of the Dirac sea; a hypothesis that was defined as “extremely artificial and unsatisfying”, regardless of the fact that in general it had been uncritically accepted.

The details of when Majorana and Fermi met the first time were narrated by E. Segré [10]: “The first important work written by Fermi in Rome [Su alcune proprietà statistiche dell’atomo (On certain statistical properties of the atom)] is today known as the Thomas-Fermi method... When Fermi found that he needed the solution to a non-linear differential equation characterized by unusual boundary conditions in order to proceed, in a week of assiduous work with his usual energy, he calculated the solution with a little hand calculator. Majorana, who had entered the Institute just a short time earlier and who was always very skeptical, decided that probably Fermi’s numeric solution was wrong and that it would have been better to verify it. He went home, transformed Fermi’s original equation into a Riccati equation, and resolved it without the aid of any calculator, utilizing his extraordinary aptitude for numeric calculation. When he returned to the Institute and skeptically compared the little piece of paper on which he had written his results to Fermi’s notebook, and found that the results coincided exactly, he could not hide his amazement.” We have taken the liberty of indulging in an anecdote since the pages in which Majorana solved that equation have been eventually found by us, and it has been
shown elsewhere that he did actually follow two independent (and quite original) mathematical methods to the same result: one of them leading to an Abel equation, rather than to a Riccati equation.

2 Ettore Majorana’s published papers

Majorana published few scientific articles: nine, actually, besides his semi-popular work entitled “Il valore delle leggi statistiche nella fisica e nelle scienze sociali” (The value of statistical laws in physics and social sciences), that was however published not by Majorana, but (posthumously) by G.Gentile Jr., in Scentia [36 (1942) 55-56]. We already know that Majorana changed over from engineering to physics in 1928 (the year in which he published his first article, written in collaboration with his friend G.Gentile jr.), and then went on publishing his works in theoretical physics only for very few years, practically until 1933 only. Nevertheless, even his published works are a goldmine of ideas and techniques of theoretical physics that still remains partially unexplored. Let us list his nine articles:

1) “Sullo sdoppiamento dei termini Roentgen ottici a causa dell’elettrone rotante e sulla intensità delle righe del Cesio”, in coll. with Giovanni Gentile, Jr.: *Rendiconti Accademia Lincei* 8 (1928) 229-233.

2) “Sulla formazione dello ione molecolare di He”: *Nuovo Cimento* 8 (1931) 22-28.

3) “I presunti termini anomali dell’Elio”: *Nuovo Cimento* 8 (1931) 78-83.

4) “Reazione pseudopolare fra atomi di Idrogeno”: *Rendiconti Accademia Lincei* 13 (1931) 58-61.

5) “Teoria dei tripletti $P$ incompleti”: *Nuovo Cimento* 8 (1931) 107-113.

6) “Atomi orientati in campo magnetico variabile”: *Nuovo Cimento* 9 (1932) 43-50.

7) “Teoria relativistica di particelle con momento intrinseco arbitrario”: *Nuovo Cimento* 9 (1932) 335-344.

8) “Über die Kerntheorie”: *Zeitschrift für Physik* 82 (1933) 137-145; and “Sulla teoria dei nuclei”: *La Ricerca Scientifica* 4(1) (1933) 559-565.

9) “Teoria simmetrica dell’elettrone e del positrone”: *Nuovo Cimento* 14 (1937) 171-184.
The first ones, written between 1928 and 1931, regard atomic and molecular physics: mainly questions of atomic spectroscopy or chemical bonds (within quantum mechanics, of course). As E. Amaldi has written [2], a profound examination of these works leaves one struck by their high class: they reveal both a deep knowledge of the experimental data even in the most minute details, and an uncommon ease, above all at that time, in the use of the symmetry properties of the quantum states in order to qualitatively simplify the problems and choose the most opportune way for quantitative resolution. As to these first articles, we confine ourselves to quote Arimondo et al.’s paper, which associated to them the birth, e.g., of autoionization.[11]

One cannot forget also the 6th one, “Atomi orientati in campo magnetico variabile” (Atoms oriented in a variable magnetic field). It is the article, famous amongst atomic physicists, in which the effect now known as the Majorana-Brossel Effect was introduced. In it, Majorana predicts and calculates the modification of the spectral line shape due to an oscillating magnetic field. This work has also remained a classic of the treatment of non-adiabatic spin-flip. His results—once generalized, as suggested by Majorana himself, by Rabi in 1937 and by Bloch and Rabi in 1945—established the theoretical base of the experimental method used to reverse the spin also of neutrons by a radiofrequency field: a method that is still used today, for example, in all polarized-neutron spectrometers. This article introduces moreover the so-called “Majorana Sphere” (to represent spinors by a set of points on the surface of a sphere), as it has been enthusiastically noticed, not long ago, by R. Penrose[12].

Majorana’s last three articles too are all of such importance that none of them can be set aside without comment[1].

The article “Teoria relativistica di particelle con momento intrinseco arbitrario” (Relativistic theory of particles with arbitrary spin), is a typical example of a work that is so far ahead of its time that it is understood and evaluated in depth only many years later. At that time it was common opinion that one could write relativistic quantum equations only in the case of particles with zero or half spin. Convinced of the contrary, Majorana—as we know from his manuscripts—began constructing suitable quantum-relativistic equations for the subsequent possible spin values (one, three-halves, etc.); and he even found out the method for writing down the equation for a generic spin-value. But he published nothing, until he discovered that a single equation could be written to represent an infinite series of cases, that is, an entire infinite family of particles of any spin (even if at that time the known particles could be counted on one hand). In order to carry his program out with these “infinite components”
equations, he invents a technique for the representation of a group several years before Eugene Wigner. And, what is more, Majorana gets the infinite dimensional unitary representations of the Lorentz Group that will be re-discovered by Wigner in his 1939 and 1948 works. The whole theory was re-invented by Soviet mathematicians (in particular Gelfand and collaborators) in a series of articles from 1948 to 1958, and finally applied by physicists in even later years. Majorana’s initial article, actually, remained in the shadows for a good 34 years until D.Fradkin[13] released, in 1966, what Majorana had accomplished so many years earlier.

As soon as the news of the Joliot-Curie experiments[14] reached Rome at the beginning of 1932, Majorana understood that they had discovered the “neutral proton” without having realized it. Thus, even before the official announcement of the discovery of the neutron, made just after by Chadwick[15], Majorana was able to explain the structure and the stability of atomic nuclei through protons and neutrons, preceding in this way also the pioneering work of D.Ivanenko[16]: as both Segré and Amaldi have recounted. His colleagues remember that even before Easter he had already concluded that protons and neutrons (indistinguishable with respect to the nuclear intercation) were bound by the “exchange forces” originating from the exchange of their spatial positions only (and not also of their spins, as Heisenberg will instead propose), so as to obtain the alpha particle (and not the deuteron) as saturated in respect of the binding energy.

Only after that Heisenberg had published his own article on the same argument, Fermi was able to persuade Majorana to meet the famous colleague in Leipzig; and finally Heisenberg is able to convince Majorana to publish his results in the paper “Über die Kerntheorie”. Majorana’s paper on the stability of nuclei was immediately recognized by the scientific community—a rare event, as we know, for his writings—thanks to that timely “propaganda” done by Heisenberg himself. Let us seize the present opportunity for quoting two brief passages from Majorana’s letters[1] from Leipzig. On February 14, 1933, he tells his mother: “The environment of the physics institute is very nice. I have good relations with Heisenberg, with Hund, and with everyone else. I am writing some articles in German. The first one is already ready...”. The work that is already ready is, naturally, the on nuclear forces that we are speaking about; which, however, remained the only one in German. Again: in the letter dated February 18th, he declares to his father[1]: “I will publish in German, after having extended it, also my latest article which appeared in Nuovo Cimento.” Actually, Majorana did not publish anything more, neither in Germany nor after his return to Italy, except for the article (in 1937) of which we are about to speak. It is therefore of notable importance to know that Ma-
jorana was writing other works: in particular, that he was expanding his article about the infinite components equations.

As we have said, from the manuscripts left it appears that Majo-
rana was also formulating the essential lines of his symmetrical theory of electrons and anti-electrons during those years (1932-1933). Even though Majorana published this theory years later, on the point of participating in the above mentioned competition for professorship: “Teoria simmet-
rica dell’eletrone e del positrone” (Symmetrical theory of the electron and positron); a publication that was initially noted almost exclusively for hav-
ing introduced the Majorana representation of the Dirac matrices in a real form. A consequence of this theory is that a neutral fermion can coin-
cide with its anti-particle: and Majorana suggests that neutrinos may be particles of this type. As with Majorana’s other writings, this article also started to have luck only decades later, beginning in 1957. Now expres-
sions like Majorana spinors, Majorana mass, and Majorana neutrinos are fashionable. As we already mentioned, Majorana’s publications (still little known, despite it all) can be regarded as a goldmine for physics. Recently, for example, it has been observed by C. Becchi how, in the first pages of this paper, a clear formulation of the quantum action principle appears: the same principle that in following years, for instance through Schwinger’s and Symanzik’s works, has brought about quite important developments in quantum field theory.

3 Ettore Majorana’s unpublished papers

Majorana also left us many unpublished scientific manuscripts (see, e.g., ref.[17, 18, 19]), also kept at Domus Galilaeana, which have been catalogued[9, 8]. The analysis of these manuscripts has allowed us to re-
veal that all the existing material seems to have been written by 1933 (even the rough copy of his last article, that Majorana will go on to publish in 1937 —as we already mentioned— seems to have been ready by 1933: the year in which he had the confirmation of the discovery of the positron). While nothing arrived to us of what he did in the following years from 1934 to 1938; except for a long series of 34 letters of response, written by Ma-
ajorana (between March 17th, 1931, and November 16th, 1937) to his uncle Quirino —a renowned experimental physicist, and a president of the Italian Physical Society— who had been pressing him for theoretical explanations of his own experiments. By contrast, his sister Maria recalled[1] that even in those years Majorana —who had reduced his visits to the Institute start-
ing from the beginning of 1933 (that is, from his return from Leipzig)—
continued to study and work at home many hours during the day and at night. Did he continue to dedicate himself to physics? From his letter to Quirino dated January 16, 1936, we find a first answer, because we come to learn that Majorana had been occupied “since some time, with quantum electrodynamics”; namely, knowing Majorana’s modesty and love for understatements, that in 1935 Majorana had profoundly devoted himself to original research in the field of quantum electrodynamics[1].

Do any other unpublished scientific manuscripts exist? The question raised by his letters from Leipzig to his family becomes of greater importance when one reads the letters sent to C.N.R. (the National Research Council of Italy) during that period. In the first one (dated January 21\textsuperscript{st}, 1933) Majorana specifies[1]: “At the moment, I am occupied with the elaboration of a theory for the description of arbitrary spin particles, that I began in Italy and of which I gave a summary notice in Nuovo Cimento...”. In the second one (dated March 3\textsuperscript{rd}, 1933) he even declares, referring to the same work: “I have sent an article on nuclear theory to Zeitschrift für Physik. I have the manuscript of a new theory on elementary particles ready and will send it to the same journal in a few days”. If we remember that the article considered here as a “summary notice” of a new theory was already of a very high level, one can understand how interesting it would be discovering a copy of the complete theory, which went unpublished. Perhaps, is it still in the Zeitschrift für Physik archives? And one must not forget, moreover, that the above-cited letter to Quirino Majorana, dated January 16, 1936, revealed to us that Majorana continued to work on theoretical physics even subsequently, occupying himself in depth—at least—with quantum electrodynamics.

Some other of Majorana’s ideas, when they did not remain in his mind, have left a trace in his colleagues’ memories. One of the testimonies we gathered is for instance of Gian Carlo Wick. Writing from Pisa on October 16, 1978, he says[1]: “...The scientific contact [between Ettore and me] mentioned by Segré happened in Rome on the occasion of the “A. Volta Congress” (a great deal before Majorana’s sojourn in Leipzig). The conversation took place in Heitler’s company at a restaurant, and therefore without a blackboard...; but even with the absence of details, what Majorana described in words was a ”relativistic theory of charged particles with zero spin based on the idea of field quantization’ (second quantization). When much later I saw Pauli and Weisskopf’s article [Helv. Phys. Acta 7 (1934) 709], I remained absolutely convinced that what Majorana had discussed was the same thing...”.

Other papers on Majorana may be found for instance in the volume
Majorana Legacy in Contemporary Physics[20]. Let us finally recall also the papers in ref.[21].

Figure 1: 38) Ettore Majorana (at the center) in Viareggio’s pinewood, Italy, August 1926, together with –from the left– his mother, his sisters Maria and Rosina, his friend and fellow student Gastone Piqué, and his maternal grandmother. Courtesy of F.Majorana, B.Russo, B.Piqué and the author. Reproduction forbidden.

4 Teaching theoretical physics

As we have seen, Majorana contributed significantly to some theoretical researches which were considered as the frontier topics in the 1930s. However, his own peculiar contribution ranged also on the basics and the applications of quantum mechanics, and Majorana’s lectures on theoretical physics[6] very effectively give evidence of this contribution.

As realized only recently, Majorana revealed a genuine interest in advanced physics teaching starting from 1933, just after that he obtained (at the end of 1932) the professorship degree of “libero docente” (analogous to the German Privatdozent). Due to this position, he proposed some academic courses at the University of Rome[22], as testified by the programs of three courses he would have given between 1933 and 1937. These documents are important, since they cover a period of time that has been referred to as Majorana’s gloomy years by the testimonies of that
epoch. Although Majorana never effectively delivered such three courses, probably due to the lacking of students, they are particularly interesting and informative due to a very careful choice of the topics he would have treated in his courses.

The first course (academic year 1933-34) proposed by Majorana was that of Mathematical Methods of Quantum Mechanics; the program for it contained the following topics\[22\]:

1) Unitary geometry. Linear transformations. Hermitian operators. Unitary transformations. Eigenvalues and eigenvectors.
2) Phase space and the quantum of action. Modifications to classical kinematics. General framework of quantum mechanics.
3) Hamiltonians which are invariant under a transformation group. Transformations as complex quantities. Non compatible systems. Representations of finite or continuous groups.
4) General elements on abstract groups. Representation theorems. The group of spatial rotations. Symmetric groups of permutations and other finite groups.
5) Properties of the systems endowed with spherical symmetry. Orbital and intrinsic momenta. Theory of the rigid rotator.
6) Systems with identical particles. Fermi and Bose-Einstein statistics. Symmetries of the eigenfunctions in the center-of-mass frames.
7) The Lorentz group and the spinor calculus. Applications to the relativistic theory of the elementary particles.

The second course (academic year 1935-36) was instead that of Mathematical Methods of Atomic Physics and the corresponding arguments are\[22\]:

Matrix calculus. Phase space and the correspondence principle. Minimal statistical sets or elementary cells. Elements of the quantum dynamics. Statistical theories. General definition of symmetry problems. Representations of groups. Complex atomic spectra. Kinematics of the rigid body. Diatomic and polyatomic molecules. Relativistic theory of the electron and the foundations of electrodynamics. Hyperfine structures and alternating bands. Elements of nuclear physics.

Finally, the third course (academic year 1936-37) was that of Quantum Electrodynamics, whose main topics were\[22\]:

Relativistic theory of the electron. Quantization procedures. Field quantities defined by commutability and anticommutability laws. Their kinematical equivalence with sets with an undetermined number of objects obeying the Bose-Einstein or Fermi statistics, respectively. Dynamical equivalence. Quantization of the Maxwell-Dirac equations. Study of the relativistic invariance. The positive electron and the symmetry of charges. Several applications of the theory. Radiation and scattering processes. Creation and annihilation of opposite charges. Collisions of fast electrons.

However, Majorana effectively lectured on theoretical physics only in 1938 when, as recalled above, he obtained a position as a full professor in Naples. He delivered his lectures starting on January 13 and ending with his disappearance (March 26), but his activity was intense, and his interest for teaching extremely high\[1\]. For the benefit of his students, and perhaps also for writing down a book, he prepared careful notes for his lectures. A recent analysis showed that Majorana’s 1938 course was very innovative for that time, and this has been confirmed by the retrieval (on September 2004) of a faithful transcription of the whole set of Majorana’s
lecture notes (the so-called “Moreno lecture notes”) comprising 6 lectures not included in the original collection[6].

The first part of his course on theoretical physics dealt with the phenomenology of the atomic physics and its interpretation in the framework of the old quantum theory of Bohr-Sommerfeld. This part presents strict analogy with the course given by Fermi in Rome (1927-28) followed by the student Majorana.

The second part starts, instead, with the classical radiation theory, reporting explicit solutions of the Maxwell equations, scattering of the solar light and some other applications. It then continues with the Theory of Special Relativity: after the presentation of the corresponding phenomenology, a complete discussion of the mathematical formalism required by the theory is given, ending with some applications as the relativistic dynamics of the electron. Then a discussion of important effects for the interpretation of quantum mechanics, such as the photoelectric effect, the Thomson scattering, the Compton effects and the Franck-Hertz experiment, follows[6].

The last part of the course, more mathematical in nature, treats explicitly on quantum mechanics, both in the Schrödinger and in the Heisenberg formulation. This part does not follow the Fermi approach, but rather refers to previous personal studies by Majorana, also following the original Weyl’s book on Group Theory and quantum mechanics.

5 The “Volumetti”

In 2003 we have reproduced and translated for the first time, in a Kluwer’s book[7], five orderly notebooks known, in Italian, as “Volumetti” (booklets). They were written in Rome by Ettore Majorana between 1927 and 1932. The original manuscript is kept at the Domus Galileana in Pisa. Each of them is composed of about 100–150 sequentially enumerated pages. A table of contents is given in the first page of each notebook. This was filled by the author when a particular section was completed. A date, written by the author at the initial blank page of each notebook, records when the notebook was finished. In fact, the last book, which is the smallest one, is probably unfinished and does not contain this information.

Each notebook was written during a period of time of about one year, starting from the years in which Ettore Majorana was completing his studies at the University of Rome. Thus the content of these notebooks goes from typical topics covered in academic courses to frontier research ones. Despite such a mixing between different arguments (which can be detected by looking at different notebooks as well as in a single notebook), the style with which a topic is treated is never obvious. As an example we refer here
to the study of the shift of the melting point of a given substance when it is placed in a region with a magnetic field or, more interestingly, that of heat propagation using the “cricket simile”. Also noticeable are the contemporary physics topics treated by Ettore Majorana in an original and very clear way, such as the Fermi explanation of the electromagnetic mass of the electron, the Dirac equation with its applications or the Lorentz group, revealing in some cases the preferred existing literature. As far as frontier research arguments are concerned, we here quote only two illuminating examples: the study of quasi-stationary states, anticipating the Fano theory\textsuperscript{[23]} of about 20 years (as initially pointed out to us by the late Prof.G.F.Bassani), and the Fermi theory of atoms, reporting analytic solutions of the Thomas-Fermi equation with appropriate boundary conditions in terms of simple quadratures which, to our best knowledge, were still lacking till our recent publication (see also refs.\textsuperscript{[24]}).

Figure 2: Ettore Majorana (the second from the right) together with his brother Salvatore, his mother (at the center), and his sisters Maria and Rosina. Karlsbad –at that time in Czechoslovakia–, autumn of 1931. Courtesy of F.Majorana, B.Russo, B.Piqué, and the author. Reproduction forbidden.

6 Majorana’s research notes: the “Quaderni”

The material presented in the Kluwer volume, as just mentioned, is a paragon of order, conciseness, essentiality and originality. So much so
that those notebooks can be partially regarded as an innovative text of theoretical physics, even after almost ninety years; besides they too being a gold-mine of seminal new theoretical, physical, and mathematical ideas and hints, stimulating and useful for modern research.

But Majorana’s most interesting scientific manuscripts —namely, his research notes— are represented by a host of loose papers and by the Quaderni, a selection of which, as announced in advance, formed the contents of a second book of ours, published by Springer in 2009. The topics treated in the Quaderni range from classical physics to quantum field theory, through the study of a number of applications in atomic, molecular and nuclear physics.

A particularly relevant attention is therein reserved to the Dirac theory and to quantum electrodynamics.

The Dirac equation describing spin-1/2 particles is usually considered in a lagrangian framework (in general, the canonical formalism is adopted), obtained from a least action principle. After an interesting preliminary study of the problem of the vibrating string, where Majorana obtains a (classical) Dirac-like equation for a two-component field, he then passes to consider a semiclassical relativistic theory for the electron, wherein the Klein-Gordon equation and the Dirac equation are deduced from a semiclassical Hamilton-Jacobi equation. Later on, the field equations and their properties (Lorentz invariance, issues related to the probabilistic interpretation, and so on) are considered in detail, and the quantization of the (free) Dirac field is discussed by means of the standard formalism with the use of annihilation and creation operators. Then, the electromagnetic interaction is introduced in the Dirac equation and the superposition of the Dirac and Maxwell fields is studied in a peculiar way, obtaining the expression for the quantized Hamiltonian of the interacting system after a normal mode decomposition.

Real (rather than complex) Dirac fields, introduced by Majorana in his famous paper on the symmetrical theory for electrons and positrons are considered in some points in the Quaderni, and by two slightly different formalisms (different decompositions of the field). The introduction of the electromagnetic interaction is performed in a rather interesting fashion, and an explicit expression for the total angular momentum carried by the real Dirac field is obtained starting from the Hamiltonian.

Some work behind Majorana’s important paper (7) can be found as well in the Quaderni. Here the author, starting from the usual Dirac equation for a 4-component spinor, obtains explicit expressions for the Dirac matrices in the cases of 6-component and 16-component spinors. Interesting enough, at the end of his discussion, Majorana also treats the case of spinors with an odd number of components, namely a 5-component field.
For what concerns quantum electrodynamics, again it is generally considered in a lagrangian and hamiltonian framework, with the use of a least action principle. As it is now usual, the electromagnetic field is decomposed in plane wave operators, and its properties are studied in a full Lorentz-invariant formalism by employing group-theoretical arguments. Explicit expressions for the quantized hamiltonian, creation and annihilation operators for the photons, as well as angular momentum operator, are deduced in several different basis, along with the appropriate commutation relations. The attitude of Majorana is near to that (more mathematical and more efficient) introduced by Heisenberg, Born, Jordan and Klein, rather than to the famous one by Fermi: once more anticipating the times.

In the Quaderni one also finds several studies, following an original idea by Oppenheimer, aimed at exploring the possibility of describing the electromagnetic field by means of the Dirac formalism. Some emphasis is given to consider the properties of the electromagnetic field as described by a real wavefunction for the photon [17, 18, 19], a study which is well beyond the contemporary works of other authors. Other interesting investigations concern the possibility to introduce an intrinsic time delay (as a universal constant) in the expressions for the electromagnetic retarded fields, and a study about the modification of the Maxwell equations in the presence of magnetic monopoles.

Besides the theoretical work about quantum electrodynamics, some applications are as well considered. This is the case of the free electron scattering, where Majorana gives an explicit expression for the transition probability, and the coherent scattering of bound electrons. Several other scattering processes are as well considered in the framework of perturbation theory, by means of the Dirac method or the Born method.

As remarked above, the most known (to the contemporary physicists community) contribution by Majorana in nuclear physics was his theory of nuclei made of protons and neutrons interacting through an exchange force of a particular kind (which corrected the Heisenberg model). In the research notes of the Quaderni several pages are also devoted to study possible forms of the nucleon potential inside a given nucleus, describing the interaction between neutrons and protons. Although generic nuclei are often considered in the discussion, some particular care is given by the author to light nuclei (deuteron, α-particles, etc.): the studies performed by Majorana on this subject were, at the same time, preliminary studies as well as and generalizations of what published by himself in his well-known paper (8), revealing a very rich, and peculiar, reasoning.

In addition to this, other nuclear physics topics were considered by our author (which appeared in the Volumetti too), and here we only mention the study of the problem of the energy loss of β particles in passing.
through a medium, where he deduced the Thomson formula by using classical arguments.

The largest number of pages in the Quaderni is, however, devoted to atomic physics, in agreement with the fact that such a topic was the main research argument faced by the Fermi group in Rome in the years 1928-1933. This is also testified by the papers published by Majorana in those years. Some echo of the work reported in those papers is present in the Quaderni, that point out that, especially in the case of the article (5) on the incomplete $P'$ triplets, some interesting material was not included by the author in the published work.

Several expressions for the wavefunctions and the different energy levels of two-electron atoms (and, in particular, of helium) are considered by Majorana, mainly in the framework of a variational method aimed at solving the related Schrödinger equation. Numerical values for the corresponding energy terms are usually reported by Majorana in large summary tables. Some approximate expressions are also obtained for three-electron atoms (and, in particular, for lithium) and for alkali, including the effect of polarization forces in hydrogen-like atoms. The problem of the hyperfine structure of the energy spectra of complex atoms is considered in some detail as well, revealing a careful attention of Majorana to the existing literature. A generalization of the Landé formula for the hyperfine splitting to non-Coulomb atomic field is given, together with a relativistic formula for the Rydberg corrections of the hyperfine structures. Such a detailed study by Majorana forms the basis of what discussed by Fermi and Segrè in a well-known paper of 1933 on this topic, as acknowledged by those authors themselves.

A smaller part of the Quaderni is devoted to several problems of molecular physics. Majorana studied in some detail, for example, the helium molecule and, then, considered the general theory of the vibrational modes in molecules, with particular reference to the $C_2H_2$ molecule of acetylene (which presents peculiar geometric properties).

Important are also some other pages, where the author considered the problem of ferromagnetism in the framework of the Heisenberg model with an exchange interaction. The approach used by Majorana in this study, however, is original, since it does follow neither the Heisenberg formulation, nor the subsequent van Vleck formulation in terms of the spin Hamiltonian. By using statistical arguments, Majorana calculated the magnetization (with respect to the saturation value) of the ferromagnetic system when an external magnetic field is applied on it, and the spontaneous magnetization. Several examples of ferromagnetic materials, with different geometries, were as well reported.

A number of other interesting topics, even dealing with what Majorana encountered in his academic studies at the University of Rome, may
be found as well in the Quaderni. This is the case, for example, of the problem of the electromagnetic and electrostatic mass of the electron (a problem considered by Fermi in one of his famous papers of 1924), or of his studies on tensor calculus, following Levi-Civita. We do not go further in our discussion, our aim being only that of drawing the attention of the reader on some specific points.

Let us add a comment, however. The method followed by Majorana for composing his notes was the following. When he reached a “semi-definitive” result in studying a given topic in his first-hand notes, he reported such a result into a Quaderno. Subsequently, after further research on the topic considered, if Majorana reached a “conclusive” result on this (according to his opinion), he reported this final result in a Volumetto. Clear expositions of particular topics can, then, be found only in the Volumetti.

The 18 Quaderni deposited at the Domus Galilaeana are booklets, composed of about 200 pages each, accounting for a total of about 2800 pages. Differently from what happens for the Volumetti, in the Quaderni no date is present, except for the Quaderni no.16 (“1929-30”), no.17 (“started on 20 June 1932”) and, probably, no.7 (“about year 1928”).

A Bibliography follows. Far from being exhaustive, it provides nevertheless some references about the topics touched upon in this paper.

7 Acknowledgments

This work has been partially supported by FAPESP, Brazil (under grant 2013/12025-8), and thanks are due to Michel Zamboni-Rached for his collaboration, and to Hugo Hernández-Figueroa for kind interest. We are moreover indebted to Dr. Carlo Segnini, the former curator of the Domus Galileana at Pisa (as well as to previous curators and directors), who facilitated our access to the original manuscripts, and to Salvatore Esposito for continuous collaboration. Last but not least, the author wishes to thank Dharam V. Ahluwalia, and Cheng-Yang Lee, for kind invitation to the “2nd International Workshop on Elko and Mass Dimension One Fermions” and subsequent quite useful editorial help.

Observation: Most of the material contained in book [1] is covered by copyright in favour of Maria Majorana, a late Ettore’s sister, and the present author. It includes letters and photos. It has been decided, however, that for scientific purposes the letter excerpts appearing above may be reproduced provided that also the original source (ref.[1]) is duly quoted. The photos
References

[1] E. Recami: *Il Caso Majorana: Epistolario, Documenti, Testimonianze* (Mondadori pub.; Milan, 1987 and 1991); subsequently published by Di Renzo pub., www.direnzo.it (Rome; 2000, 2002, 2008, 2011); the 6th enlarged revised edition (2011) consisting of 296 pages.

[2] E. Amaldi: *La Vita e l’Opera di E. Majorana* (Accademia dei Lincei; Rome, 1966); and “Ettore Majorana: Man and scientist”, in *Strong and Weak Interactions*, ed. by A. Zichichi (Academic Press; New York, 1966).

[3] B. Pontecorvo: *Fermi e la Fisica Moderna* (Editori Riuniti pub.; Rome, 1972); and Contribution to the “Proceedings of the International Conference on the History of Particle Physics, Paris, July 1982”, *Physique* 43 (1982).

[4] E. Recami: “Ricordo di E. Majorana a sessant’anni dalla sua scomparsa: L’opera edita ed inedita” (The published and unpublished work), *Quaderni di Storia della Fisica* (of the ‘Giornale di Fisica’) [Italian Phys. Soc., S.I.F.; Bologna], 5 (1999) 19-68.

[5] E. Recami: “New information about the physicist Ettore Majorana”, *Scientia* 110 (1975) 577-598.

[6] *Ettore Majorana – Lezioni all’Università di Napoli*, ed. by B. Preziosi (Bibliopolis pub.; Naples, 1987): book of 199 pages, containing the anastatic reproduction of the original (initial ten) notes for the lectures delivered by Majorana at the beginning of 1938 at the University of Naples. The complete set of the 16 lecture notes (including the Moreno Document) has been typewritten in *Ettore Majorana – Lezioni di Fisica Teorica*, edited by S. Esposito (Bibliopolis pub.; Naples, 2006).

[7] S. Esposito, E. Majorana Jr., A. van der Merwe and E. Recami: *Ettore Majorana - Notes on Theoretical Physics* (Kluwer Acad. Publs.; Dor-
drecht, Boston, London and New York, 2003); book of 512 pages. The original Italian version, instead, has been published in the book: S. Esposito and E. Recami: Ettore Majorana – Appunti inediti di Fisica teorica (Zanichelli pub.; Bologna, 2006).

[8] S. Esposito, E. Recami, A. van der Merwe and R. Battiston: E. Majorana – Unpublished Research Notes on Theoretical Physics (Springer; Berlin, 2009); book of 487 pages.

[9] M. Baldo, R. Mignani e E. Recami: “Catalogo dei manoscritti scientifici inediti di E. Majorana”, in E. Majorana – Lezioni all’Università di Napoli (Bibliopolis pub.; Napoli, 1987), p. 175. See also [4].

[10] E. Segré: Enrico Fermi, Fisico (Zanichelli Pub.; Bologna, 1971); and Autobiografia di un Fisico (Il Mulino; Roma, 1995). See also E. Segré: Enrico Fermi, physicist (University of Chicago Press; Chicago, 1970); and A mind always in motion (University of California Press; Berkeley, 1993).

[11] E. Arimondo, C. Clark and W. Martin: “Ettore Majorana and the birth of autoionization”, Rev. Mod. Phys. 82 (2010) 1947.

[12] R. Penrose: “Newton, quantum theory and reality”, in 300 Years of Gravity, ed. by S. W. Hawking & W. Israel (Cambridge Univ. Press; Cambridge, 1987); J. Zimba e R. Penrose: Stud. Hist. Phil. Sci. 24 (1993) 697; R. Penrose: Ombre della Mente (Shadows of the Mind) (Rizzoli; 1996), pp. 338-343 and 371-375. Cf. also ref. [23].

[13] D. Fradkin: “Comments on a paper by Majorana concerning elementary particles,” Am. J. Phys. 34 (1966) 314. Cf. also R. Casalbuoni: “Majorana and the infinite component wave equations”, arXiv: hep-th/0610252.

[14] I. Curie and F. Joliot: “The emissipon of high energy photons from hydrogenous substances irradiated with bery penetrating Alpha rays”, C. R. Acad. Sci. Paris 194 (1932) 273.

[15] J. Chadwick: “The existence of the neutron”, Proc. Roy. Soc. A136 (1932) 692.

[16] D. Iwanenko: “The neutron hypothesis”, Nature 129 (1932) 798.

[17] R. Mignani, M. Baldo and E. Recami: “About a Dirac–like equation for the photon, according to Ettore Majorana,” in Lett. Nuovo Cim. 11 (1974) 568.
[18] E. Giannetto: “Su alcuni manoscritti inediti di E. Majorana,” in Atti del IX Congresso Nazionale di Storia della Fisica, ed. by F. Bevilacqua (Milan, 1988) 173.

[19] S. Esposito: “Covariant Majorana formulation of Electrodynamics,” Found. Phys. 28 (1998) 231.

[20] Majorana Legacy in Contemporary Physics, ed. by I. Licata (Di Renzo pub.; Rome, 2006); this book appeared also in electronic form in Electron. J. Theor. Phys. 3 (2006), issue no.10.

[21] S. Esposito: “Hole theory and Quantum Electrodynamics in an unknown manuscript in French by Ettore Majorana”, Found. Phys., in press, [arXiv:physics/0609137]; “A peculiar lecture by Ettore Majorana”, Eur. J. Phys. 27 (2006) 1147; “Majorana and the path-integral approach to Quantum Mechanics”, Ann. Fond. Louis de Broglie, 31 (2006) 1; E. Di Grezia and S. Esposito: “Majorana and the quasi-stationary states in Nuclear Physics”, [arXiv:physics/0702179]; A. Drago and S. Esposito: “Following Weyl on Quantum Mechanics: the contribution of Ettore Majorana”, Found. Phys. 34 (2004) 871.

[22] A. De Gregorio and S. Esposito: “Teaching Theoretical Physics: the cases of Enrico Fermi and Ettore Majorana,” Am. J. Phys. 75 (2007) 781.

[23] C. Leonardi, F. Lillo, A. Vaglica and G. Vetri: “Majorana and Fano alternatives to the Hilbert space,” in Mysteries, Puzzles, and Paradoxes in Quantum Mechanics, ed. by R. Bonifacio (A.I.P.; Woodbury, N.Y., 1999), p.312.

[24] S. Esposito: “Majorana solution of the Thomas-Fermi equation”, Am. J. Phys. 70 (2002) 852; “Majorana transformation for differential equations,” in Int. J. Theor. Phys. 41 (2002) 2417; E. Di Grezia and S. Esposito: “Fermi, Majorana and the statistical model of atoms,” Found. Phys. 34 (2004) 1431.