Supersolid of indirect excitons in electron-hole quantum Hall systems

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We investigate the ground state of a balanced electron-hole system in the quantum Hall regime using mean-field theory and obtain a rich phase diagram as a function of interlayer distance $d$ and the filling factor within a layer. We identify an excitonic condensate phase, an excitonic supersolid phase, as well as uncorrelated Wigner crystal states. We find that balanced electron-hole system exhibits a supersolid phase a wide range of filling factors, with different crystal structure ground states. We obtain the ground state stiffness in the excitonic phases and show that the phase transitions from a uniform condensate to a supersolid is accompanied by a marked change in the stiffness. Our results provide the first semi-quantitative determination excitonic supersolid phase diagram and properties.

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Introduction: Over the past decade, Bose-Einstein condensation of indirect excitons \cite{1} in optically pumped and doped electron-hole double quantum wells has been extensively explored \cite{2, 3, 4, 5, 6}. The concurrent theoretical analysis has largely focused on either a uniform condensate, where a macroscopic number of excitons occupy the zero-momentum state, or the condensation in a condensate, where a macroscopic number of excitons occupy the zero-momentum state, or the condensation in a trap where the translational symmetry is broken explicitly \cite{7, 8, 9, 10, 11}. On the other hand, recent observations of the supersolid phase in $^4$He \cite{12, 13} have revived the interest in and questions about excitonic Bose-Einstein condensates with spontaneously broken symmetries and their properties \cite{14, 15}. (“Supersolid” phase of cold atoms in optical lattices has been extensively discussed, although, in that case, the translational symmetry is explicitly broken by the optical lattice.)

Lozovik and Berman \cite{16} first discussed the coherent charge-density-wave (CCDW) ground state of indirect excitons in electron-hole double quantum wells. Recently, based on general principles, it was shown that electron-hole systems must support a supersolid phase of excitons due to their dipolar repulsion \cite{17}. Although a simple qualitative analysis implies the existence of a supersolid phase in electron-hole bilayer systems, the quantitative determination of the phase-boundary and supersolid properties is made difficult by the dispersion of electron (hole) bands.

In this paper, we study an electron-hole system in a strong magnetic field, where the kinetic energy of carriers is quenched \cite{18}. We consider a system with equal electron and hole filling factors $\nu_e = \nu_h = \nu_T/2 \leq 1/2$. CCDW ground states of such a system have been investigated \cite{19, 20}; however, such states are destabilized by fluctuations \cite{21, 22}. We obtain the ground state phase diagram in the $(d, \nu_T)$ plane (Fig. 1). Particle-hole symmetry in the lowest Landau level maps the electron-hole system at $\nu_T$ onto an electron-hole system at $2 - \nu_T$ and implies that the phase diagram is symmetric around $\nu_T = 1$. We verify that this exact relation is satisfied by our mean-field results. The phase diagram of a closely related system - bilayer quantum Hall system where carriers in both layers have the same polarity, near total filling factor $\nu_T = 1$ - has been extensively explored \cite{18}. These systems have provided signatures of excitonic condensation in interlayer tunneling and counterflow experiments at small $d$ \cite{18} and in the presence of a bias voltage \cite{23}.

For an electron-hole quantum Hall system, we find that (Fig. 1): i) for small $d \leq d_{c_1}$ (region I), the ground state is a uniform excitonic condensate; at large $d > d_{c_2}$ (region IV), the ground state is uncorrelated triangular Wigner crystals. For $d_{c_1} \leq d \leq d_{c_2}$ the ground state is a supersolid of excitons, with triangular (region II) and anisotropic (region III) lattice structures respectively. This state has spontaneous interlayer phase coherence as well as spontaneously broken translational symmetry. The inset shows that ground state lattice anisotropy $\gamma(\nu_T)$ changes discontinuously at the boundary between regions II and III.

![Phase Diagram](chart.png)
To study the robustness of phase coherence, we calculate the ground state stiffness \( \rho_s(d, v_T) \) by considering the mean-field energy dependence on an in-plane magnetic field. We find that the phase transitions with increasing \( d \) - from a uniform excitonic condensate to a supersolid to Wigner crystals - are accompanied by marked changes in the stiffness.

In the following section, we briefly sketch the details of mean-field calculations. Then we discuss the phase diagram (Fig. 1) focusing on the excitonic suprasolid phases, and the dependence of stiffness \( \rho_s(d) \) on the interlayer distance. We conclude the section with a comment on electron-hole quantum Hall systems at total filling factors \( v_T \) and \( 2 - v_T \). In the last section, we summarize our results.

**Microscopic Model:** Let us consider a bilayer system with electrons as carriers in the top layer and holes in the bottom layer, in a uniform magnetic field \( \mathbf{B} = B_z \hat{z} + B_{\perp} \hat{\mathbf{p}} \). The magnetic field normal to the layers \( B_{\perp} \) quenches the kinetic energy of the carriers, whereas the in-plane magnetic field \( B_z \) allows us to study mean-field states with a vanishing interlayer phase. We assume a vanishing interlayer tunneling amplitude, reminding us that the in-plane magnetic field could not be gauged away for any nonzero interlayer tunneling. The Hamiltonian for the system in the lowest Landau level approximation is given by

\[
\hat{H} = \frac{1}{2A} \sum_{\sigma_1\sigma_2,q} V_{\sigma_1\sigma_2}(q) \hat{\rho}_{\sigma_1\sigma_2}(q) \hat{\rho}_{\sigma_1\sigma_2}(-q),
\]

where \( \sigma = (e(h)) \) denotes electron (hole), \( A \) is the area of the sample, \( V_{ee}(q) = V_{hh}(q) = 2\pi e^2/(cq) \) is the repulsive intralayer Coulomb interaction \( (e \sim 10 \) is the dielectric constant of the semiconductor) and \( V_{eh}(q) = -V_{ee}(q) \exp(-qd) \) is the interlayer attractive Coulomb interaction. The momentum-space density operator in second quantization is

\[
\hat{\rho}_{\sigma}(q) = \frac{1}{N_\sigma} \sum_{k_1k_2} \langle k_1|e^{-iq\cdot r}|k_2\rangle c_{\sigma,k_1}^\dagger c_{\sigma,k_2}
\]

where \( c_{\sigma,k} \) \( (c_{\sigma,k}^\dagger) \) is the annihilation (creation) operator for a particle in the lowest Landau level state \( |n\rangle = |n, 0, k\rangle \). \( N_\sigma = A/(2\pi l_B^2) \) is the degeneracy of a single Landau level, and \( l_B = \sqrt{\hbar c/\epsilon B} \) is the magnetic length. We define the excitonic condensate operator as

\[
\hat{\rho}_{eh}(q) = \frac{1}{N_\sigma} \sum_{k_1k_2} \langle k_1|e^{-iq\cdot r}|k_2\rangle c_{h,k_1}^\dagger c_{e,k_2}
\]

and recall that single-particle wave-functions for holes are obtained by complex-conjugation from the single-particle wavefunctions for electrons, \( \langle \tau|n,k\rangle_h = \langle \bar{\tau}|n,k\rangle_e^\ast \). Following standard procedure, we obtain the Hartree-Fock Hamiltonian

\[
\hat{H}_{HF} = \frac{N_\sigma}{e\epsilon B} \sum_{\sigma q} [U_{\sigma\sigma}(q) \hat{\rho}_{\sigma\sigma}(q) + U_{\sigma\bar{\sigma}}(q) \hat{\rho}_{\sigma\bar{\sigma}}(q)].
\]

The first term, \( U_{\sigma\sigma}(q) = [V_\sigma(q) - V_h(q)] \rho_{\sigma\sigma}(-q) - V_e(q) \rho_{\sigma\sigma}(-q) \) contains the intralayer Hartree and exchange, and interlayer Hartree contributions respectively \( (\epsilon = h) \), and second term \( U_{\sigma\bar{\sigma}}(q) = -V_\sigma(q) \rho_{\sigma\bar{\sigma}}(q) \) denotes the excitonic condensate contribution. These dimensionless contributions are given by \( V_{\sigma}(q) = 1/4l_B \), \( V_h(q) = \sqrt{i\pi/2l_0(q^2l_B^2/4)} \), \( V_e(q) = -e^{-qd}V_\sigma(q) \), and

\[
V_\sigma(q) = \int \frac{d^2p}{(2\pi)^2} V_\sigma(p)e^{-qd}\hat{\rho}_{eh}(p)\hat{\rho}_{eh}(-p)\hat{\rho}_{eh}(p)\hat{\rho}_{eh}(-p).
\]

To obtain the self-consistent density matrices, we introduce a two-component operator \( a_{k}^\dagger = [c_{\uparrow,k}^\dagger, c_{\downarrow,-k}] \) and define the \( 2 \times 2 \) matrix Green’s function \( [24] \)

\[
G(Q; \tau) = -\frac{1}{N_\sigma} \sum_{k_1k_2} \langle k_1|e^{-iQ\cdot r}|k_2\rangle \langle \bar{\tau}a_{k_1}(\tau)a_{k_2}^\dagger(0)\rangle.
\]

The electron and hole density matrices \( \langle \hat{\rho}_{\sigma\sigma}(Q) \rangle \) as well as the (complex) excitonic order parameter \( \langle \hat{\rho}_{eh}(Q) \rangle \) are then determined from the equal-time limit \( (\tau \rightarrow 0^-) \) of this Green’s function matrix. In the Hartree-Fock approximation, the equation of motion for \( G \) matrix in the frequency space is given by \( [19, 25, 26] \)

\[
\delta_{Q,0} I = \begin{bmatrix}
i\omega_n + \mu & 0 \\
0 & i\omega_n - \mu
\end{bmatrix} G(Q; i\omega_n)
- \sum_j \left[ \frac{\Sigma_{ee}(Q, j)}{\Sigma_{he}(Q, j)} \Sigma_{ee}(Q, j) - \Sigma_{he}(Q, j) \Sigma_{hh}(Q, j) \right] G(Q; i\omega_n),
\]

where \( I \) is the identity matrix, \( Q^\pm = Q \pm k_B \), \( k_B l_B^2 = \tilde{g} d B/|B| \) represents the relative displacement of single-particle wavefunctions in the two layers due to the Lorentz drift caused by the in-plane field \( [26] \), and we have defined a modified Green’s function

\[
G(Q; \omega_n) = \begin{bmatrix}
G_{ee}(Q) & G_{eh}(Q) e^{ik_B\cdot Q}\times\hat{z}/2 \\
G_{he}(Q) e^{ik_B\cdot Q}\times\hat{z}/2 & G_{hh}(Q)
\end{bmatrix}.
\]

The self-energies in Eq. (7) are given by \( \Sigma_{\sigma_1\sigma_2}(Q, j) = U_{\sigma_1\sigma_2}(Q, j) \exp(iQ\times\hat{z}/2) \) with \( Q_{j} = Q - Q_j \). The modified Green’s function can be expressed as

\[
G(Q; i\omega_n) = \sum_k \lambda_k(Q) \chi_k^\dagger(0)\chi_k, \quad \lambda_k(Q) = \omega_k - \sum_j \left[ \frac{\Sigma_{ee}(Q, j) - \mu \delta_{ij}}{\Sigma_{he}(Q, j)} \Sigma_{eh}(Q, j) - \delta_{ij} \Sigma_{hh}(Q, j) \right] \lambda_k(Q_j)\chi_k^\dagger(0)\chi_k.
\]

where \( \lambda_k(Q) = [V_{k}^*(Q), U_{k}^*(Q)] \) are the eigenvectors of the self-energy matrix with eigenvalue \( \omega_k \).

\[
\sum_j \left[ \frac{\Sigma_{ee}(Q, j) - \mu \delta_{ij}}{\Sigma_{he}(Q, j)} \Sigma_{eh}(Q, j) - \delta_{ij} \Sigma_{hh}(Q, j) \right] \lambda_k(Q_j)\chi_k^\dagger(0)\chi_k = \omega_k \lambda_k(Q),
\]

(10)
We notice that, in contrast to a similar analysis for bilayer quantum Hall systems, - the chemical potential appears with opposite signs in the electron and hole self-energies. The Hartree-Fock ground state energy \( E_{HF}(k_B) \) is calculated using the self-consistent density matrix using Eq. (11).

Equations (30)-(39) provide the set of equations that are iteratively solved. We check that the resulting density matrix satisfies the sum-rule (27)

\[
\sum_Q \left[ |\rho_{ee}(Q)|^2 + |\rho_{eh}(Q)|^2 \right] = \rho_{ee}(0) = \nu_T/2.
\]  

Results: In order to determine the phase diagram in the \( d - \nu_T \) plane, we compare the energies of uniform excitonic condensate (\( \rho_{eh} \neq 0, \rho \propto \delta_{Q,0} \)), Wigner crystal in each layer (\( \rho_{eh} = 0, \rho_{ee} = \rho_{hh} \propto \delta_{Q,0} \)), and supersolid (\( \rho_{eh} \neq 0, \rho_{ee} = \rho_{hh} \propto \delta_{Q,0} \)) in the absence of an in-plane magnetic field \( B_1 = 0 \). We use simplified anisotropic lattice with two primitive lattice vectors cibrey2000 \( a_1 = (a, b/2) \) and \( a_2 = (0, b) \), and define the lattice anisotropy \( \gamma = b/a \). The lattice constant \( a \) is determined by the constraint that the unit cell contains one electron-hole pair, and the optimal value of \( \gamma \) is obtained by minimizing the mean-field energy. Note that the triangular lattice (\( \gamma = 2/\sqrt{3} \approx 1.155 \)) and a stripe lattice (\( \gamma \to 0 \)) are its special cases. (We have found that square lattices have higher energy in the supersolid and uncorrelated Wigner crystal phases.)

Figure 1 shows the ground state phase diagram. At small values of \( d \leq d_c_1 \), we find a uniform excitonic condensate as the ground state over the entire range of total filling factor (region I). For such a state, the mean-field equations can be analytically solved and we obtain the layer densities \( \rho_{ee}(0) = \rho_{hh}(0) = \nu_T/2 \) and the excitonic condensate order parameter \( \rho_{eh}(0) = \sqrt{\nu_T(2 - \nu_T)/2} \). These analytical results satisfy Eq. (11) and imply that the uniform phase coherence order parameter is the same for systems with filling factors \( \nu_T \) and \( 2 - \nu_T \). The monotonc increase in \( d \) with decreasing \( \nu_T \) is consistent with observed strengthening of excitonic condensate phase in bilayer quantum Hall systems with layer imbalance (22). At large values of \( d \geq d_c_2 \) (which is the same as \( d_c_1 \) for \( \nu_T \leq 1/3 \)), the ground state is uncorrelated triangular Wigner crystals in the electron layer and the hole layer (region IV).

For \( \nu_T \geq 1/3 \), we find that the ground state is a supersolid at intermediate values of distance \( d_c_1 \leq d \leq d_c_2 \). In region II, the supersolid has a triangular lattice structure (\( \gamma = 2/\sqrt{3} \)) whereas in region III, the lattice is anisotropic. (The anisotropic supersolid with small \( \gamma \) is similar to the CCDW solutions discussed in Ref. [19].) The boundary between two regions corresponds to a line of first-order phase transitions from a triangular lattice to an anisotropic lattice. The inset in Fig. 1 shows the ground state lattice anisotropy \( \gamma(\nu_T) \) along the line \( d_c_1 \). For \( \nu_T \leq 0.6 \), \( \gamma = 2/\sqrt{3} \) is constant, as expected for a triangular lattice; near \( \nu_T \approx 0.6 \), \( \gamma \) drops discontinuously and then reduces monotonically with \( \nu_T \). This discontinuity in \( \gamma \) indicates a first order transition from a triangular to an anisotropic supersolid. The reverse transition, from region III to region II, can be induced by increasing the interlayer distance \( d \) at a given value of \( \nu_T \). Figure 2 shows the mean-field energy as function of lattice anisotropy \( \gamma \) for different values of \( d/l_B \) at total filling factor \( \nu_T = 0.62 \). For \( d/l_B = 1.26 \), the optimal value of \( \gamma \approx 0.89 \) corresponds to an anisotropic supersolid, whereas increasing \( d/l_B \) shifts the optimal value to \( \gamma = 2/\sqrt{3} = 1.155 \) corresponding to a triangular supersolid.

An important property of a Bose-Einstein condensate, uniform or otherwise, is the energy cost associated with spatial variations of its phase. A uniform in-plane magnetic field induces such a variation in a uniform excitonic condensate, whereas in region IV, the lattice structure in regions II and IV suggests that the phase transition between them is a continuous phase transition. On the other hand, different lattice structures in region III and IV imply that the transition between them will be a first order transition.

The stiffness, defined as the variation of ground state energy with an in-plane magnetic field (28)

\[
\rho_s = \frac{1}{A} \left. \frac{\partial^2 E_{HF}(k_B)}{\partial k_B^2} \right|_{k_B=0}
\]

from the ground state energy. Figure 3 shows dependence of the stiffness on interlayer distance \( \rho_s(d) \) at \( \nu_T = 0.4 \). We see a change in the stiffness slope at \( d_c_1 \), where the system changes from a uniform excitonic condensate to a triangular supersolid and \( d_c_2 \), where it changes from the supersolid to uncorrelated Wigner crystals, along with the expected discontinuous decrease with \( d \). Since the transport properties (in superfluid or supersolid phase) depend on the stiffness, generically, we expect that the phase transitions will be visible in transport measurements.

An electron-hole system at \( \nu_T \) is mapped on to a system at \( 2 - \nu_T \) after a particle-hole transformation on both layers. Figure 4 shows that the mean-field density matrices, obtained separately for \( \nu_T = 0.5 \) and \( \nu_T = 1.5 \),
existence of the supersolid phase in the intermediate distance regime and the accompanying change in the phase stiffness - are generically valid.

Our results present an example of a supersolid whose properties can be investigated starting from a microscopic Hamiltonian: in particular, the study of low-energy excitations and transport properties in the supersolid phase is possible. We point out that this excitonic system, although analytically tractable, is significantly different from the zero-field system in which, at low exciton densities, the excitons behave as non-interacting bosons. Nonetheless, an experimental verification (or falsification) of our predictions will provide a better understanding of the supersolid phase and the properties of excitons in the quantum Hall regime.

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**Discussion:** The supersolid phase, with two spontaneously broken continuous symmetries, has been a source of extensive investigations in $^4$He,[12][13][14], but has not been experimentally realized in any other system. Our results predict that electron-hole quantum Hall systems exhibit a rich phase diagram including a supersolid phase robust over filling factors $\nu_T \gtrsim 1/3$. We show that the supersolid phase exhibits a nonzero stiffness and obtain the dependence of stiffness $\rho_s(d)$ on interlayer distance. It is well known that mean-field approach overestimates the stability of one type of order (excitonic condensation) over another (broken translational symmetry); therefore, we believe that our conclusions -

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[28] This stiffness, for a spatially homogeneous system, corresponds to the cost of phase fluctuations. Our calculations, with a supersolid ground state, include contributions from the phase variations and relative density displacements.