Throughput Optimal and Fast Near-Optimal Scheduling with Heterogeneously Delayed Network-State Information
(Extended Version)

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Abstract—We consider the problem of distributed scheduling in wireless networks where heterogeneously delayed information about queue lengths and channel states of all links are available at all the transmitters. In an earlier work (by Reddy et al. in Queueing Systems, 2012), a throughput optimal scheduling policy (which we refer to henceforth as the $R$ policy) for this setting was proposed. We study the $R$ policy, and examine its two drawbacks—(i) its huge computational complexity, and (ii) its non-optimal average per-packet queueing delay. We show that the $R$ policy unnecessarily constrains itself to work with information that is more delayed than that afforded by the system. We propose a new policy that fully exploits the commonly available information, thereby greatly improving upon the computational complexity and the delay performance of the $R$ policy. We show that our policy is throughput optimal. Our main contribution in this work is the design of two fast and near-throughput-optimal policies for this setting, whose explicit throughput and runtime performances we characterize analytically. While the $R$ policy takes a few milliseconds to several tens of seconds to compute the schedule once (for varying number of links in the network), the running times of the proposed near-throughput-optimal algorithms range from a few microseconds to only a few hundred microseconds, and are thus suitable for practical implementation in networks with heterogeneously delayed information.

Index Terms—Distributed Scheduling, Heterogeneous Delays, Network State Information, Throughput Optimality

I. INTRODUCTION

A central problem in any wireless network is one of scheduling the different users in the network with the objective of optimizing some desired metric—for example, maximizing the system throughput—in the presence of challenges that are unique to the wireless medium—namely, channel fading and interference due to transmissions from other users in the network. This problem has been studied extensively in the literature. A highly influential and often cited work in this area is the work by Tassiulas and Ephremides [2], who proposed the Back-Pressure scheduling algorithm (a version of the Max-Weight algorithm [3, 4]), which is a centralized algorithm that schedules the links in the network based on global knowledge of the instantaneous queue lengths at all the links. Even though this algorithm is provably throughput optimal (see Sec. II-E), it is a centralized algorithm that requires solving a global optimization problem in each time slot, and it also requires knowledge of instantaneous queue lengths at all links in the network to determine the schedule [1, 2].

The Max-Weight algorithm, being a centralized policy, involves the computationally costly task of finding a maximal independent (i.e., non-interfering) set of links that can be activated simultaneously and whose summation of link-weights is maximum [7]. To circumvent this limitation, researchers have considered two broad approaches [7]—namely, design of random-access algorithms in which access probabilities are dependent on queue sizes [7] or on arrival rates [8]–[13], and design of distributed implementations of the Max-Weight algorithm [14], [15]. Some of these approaches require knowledge of instantaneous queue lengths and/or instantaneous channel states to attain their objective. Even though it may be reasonable to assume that each node has knowledge of instantaneous queue lengths and channel states (at all times) for its own links (links emanating from itself), it is less pragmatic to assume that any node possesses instantaneous information about any link in the network other than its own links (at any time instant). This could be because, for example, these quantities vary quickly with time (e.g., fast fading), or because the propagation delay of the feedback channel is large [16]. In [17], the authors consider networks where each node possesses knowledge of instantaneous queue lengths and channel states for its own links, but only has this information from other links in the network with some globally fixed delay (commonly referred to as homogeneous delay). The assumption of homogeneous delays, however, is not satisfied in many networks where there is often a mismatch in the delays with which each node can acquire information about queue lengths and channel states of other links in the network [1]. An example of such a case is a network where the “quality” of information that a node possesses, of queue lengths and channel states of links other than its own direct links, is a decreasing function of their “distances” from this node [1]. Non-homogeneous delays are commonly referred to as heterogeneous delays.

One serious issue with heterogeneous delays is that the nodes could potentially have different views of the state of the network [1]. In [1], the authors consider distributed scheduling in a wireless network where information about the queue lengths and channel states of links in the network, available at the nodes, are heterogeneously delayed. They characterize the
system throughput region for this setting, propose a scheduling policy, and show that their policy is throughput optimal. We study the limitations of the policy proposed in [1].

As in [1], we consider the problem of distributed scheduling in wireless networks where delayed queue length and delayed channel state information of each wireless link in the network, are available at all the transmitters in the network, but with possibly different delays. We refer to these as heterogeneously delayed queue state information (QSI) and heterogeneously delayed channel state information (CSI) respectively, and collectively refer to them as heterogeneously delayed network state information (NSI). We refer the reader to Sec. [D] for detailed information about the structure of heterogeneously delayed NSI that we have considered in this work. Our contributions are summarized below.

A. Our Contributions

Our primary contribution in this work is the design, analysis and performance evaluation of two fast and near-throughput-optimal scheduling policies for the case of networks with heterogeneously delayed NSI. Starting with the policy proposed in [1] (which we refer to henceforth as the R policy), we arrive at our fast and near-throughput-optimal policies through the following steps of identifying issues with the R policy and making progressive refinements to address these issues:

1) First, we study the limitations of the R policy. The R policy is formulated as a functional optimization problem that searches for optimal threshold functions (see Sec. [III]) that maximize the queue-length weighted aggregate expected throughput of the system. We study the computational complexity of the R policy and note that the R policy is computationaly very costly – (i) it needs to evaluate a number of functions in its optimization domain that grows as a double exponential in the number of links in the network, and grows as an exponential in the number of levels into which the channel states on the wireless links are quantized, and (ii) it needs to consider, in computing the expected rate, a number of sample paths that is exponential in the number of links in the network and in the maximum of the heterogeneous delay values.

2) We show that the delay performance of the R policy is non-optimal, and that it can be improved upon significantly.

3) Next, we show that the structure of heterogeneously delayed NSI as defined in the system model in [1] affords each node access to NSI that is less delayed than that used in [1], and yet commonly available at all nodes in the network. We propose two modifications to the R policy to make use of this less delayed NSI (we call the resulting policy the H policy). We show that the H policy hugely improves upon the computational complexity of the R policy, and numerically show that the H policy also improves upon the delay performance of the R policy substantially. We establish that, like the R policy, the H policy too is throughput optimal.

4) Finally, we show that despite the huge leap that the H policy takes in reducing the computational complexity in comparison to that of the R policy, the computational complexity of the H policy still remains impractically large, making the case for low-complexity scheduling policies. Taking design cues from the H policy, we propose two low-complexity near-throughput-optimal algorithms that are several orders of magnitude faster than the R and H policies. We obtain explicit analytical expressions for the expected saturated system throughputs of these policies, evaluate their performance and show that these policies closely approximate the optimal throughputs of the R and H policies.

II. SYSTEM MODEL

Our system model is precisely the same as that used in [1]. We use a slotted-time system and restrict our attention to single-hop transmissions. We borrow heavily in definitions, notations and nomenclature from [1] to keep our presentation easily cross-referenced and compared with [1].

A. Network Model

Our model of the wireless network has L transmitter-receiver pairs (or links); the set of links is denoted \( \mathcal{L} \). We abstract the channel condition on each of the wireless links by the link’s capacity. We model the time-varying capacity of each link \( l \) as a separate discrete-time Markov chain (DTMC) denoted \( \{C_l[t]\} \) on the same state space \( C = \{c_1, c_2, \ldots, c_M\} \), where \( c_1 < c_2 < \ldots < c_M \) are non-negative integers, and with the same transition probabilities. We assume that the channel conditions on the wireless links are all independent of each other, but identically distributed, with transition probabilities \( p_{ij} := \text{Pr}[C_l[t+1] = c_j \mid C_l[t] = c_i] \). We assume that these DTMCs are irreducible and aperiodic, and therefore have a stationary distribution, with the stationary probability of being in state \( c_j, j \in [1, 2, \ldots, M] \) denoted \( \pi(c_j) \).

B. Interference Model

For each link \( l \), we let \( I_l \) denote the set of links in the network that interfere with transmissions on link \( l \); thus, \( I_l \cup \{l\} \) is an interference set. We say that a collision occurs with a transmission scheduled on link \( l \) if, in the same time slot, a transmission is scheduled on at least one link \( l' \in I_l \). When a link \( l \) successfully transmits (i.e. when the packets transmitted on link \( l \) do not encounter a collision), a number of packets equal to \( C_l[t] \) in the case of saturated queues, and equal to \( \min(C_l[t], Q[t]) \) in the case of non-saturated queues, are successfully received by the receiver on the other side of the link. However, in the case of a collision, \( \gamma_l C_l[t] \) packets in the case of saturated queues, and \( \min(\gamma_l C_l[t], Q[t]) \) in the case of non-saturated queues, where \( \gamma_l \in [0, 1] \), are received at the intended receivers.

1 We use \( \mathcal{L} \) to denote both the set and its cardinality.
2 We remark that the above channel model is assumed for making notations simpler and to enhance clarity. Our results hold even for the case of networks where each link is modeled as a separate DTMC (with different state spaces and/or different transition probabilities).
3 \( I_l \) can capture arbitrary interference constraints.
4 An interference set is a set of wireless links such that if there is transmission on more than one link in the set in the same time slot, then these transmissions interfere with one another, possibly resulting in only a part of the transmission being received successfully at (any of) the intended receiver(s).
5 We assume that \( \{\gamma_l\}_{l \in \mathcal{L}} \) are such that \( \{\gamma_l c_1, \ldots, \gamma_l c_M\}_{l \in \mathcal{L}} \) for \( \gamma_l \in [0, 1] \) are all integers.
TABLE I: An instance of (small) delay values for a wireless network with three links

| Delay in obtaining NSI of link $l_1$ | Delay in obtaining NSI of link $l_2$ | Delay in obtaining NSI of link $l_3$ |
|--------------------------------------|--------------------------------------|--------------------------------------|
| At TX $A$ | 0 | 1 | 3 |
| At TX $B$ | 2 | 0 | 4 |
| At TX $C$ | 1 | 2 | 0 |

$^1$A, B, C are transmitter nodes of links $l_1$, $l_2$, and $l_3$ respectively.

C. Traffic Model and Queue Dynamics

As noted earlier, we use a slotted-time model. Packets arriving at a transmitter, depending on their intended destination, are assigned a wireless link. Prior to transmission on this link, these packets are buffered in the queue associated with this link. We denote by $Q_l(t)$ the length of the queue associated with link $l$ at time $t$. We model the packet arrivals into the queue associated with link $l$, by an arrival process denoted $A_l(t)$, that describes the number of packets that arrive into the queue, at time $t$. For every link $l$, we assume that $A_l(t)$ is an integer-valued process, independent across time slots $t$, with $0 \leq A_l(t) \leq A_{\text{max}} < \infty$ almost surely, with $\lambda_l := E[A_l(t)] < \infty$. The queue lengths are governed by the update equation:

$$Q_l[t + 1] = (Q_l[t] + A_l[t] - S_l[t])^+, \quad (1)$$

where $S_l[t] := C_l[t]$ is the maximum possible service rate of link $l$ at time $t$, and $x^+ := \max(0, x)$.

D. Information Model and Structure of Heterogeneously Delayed NSI

We use precisely the same structure for heterogeneously delayed NSI that each node has access to, as that used in [1]. In our model, at time $t$, the transmitter node of each link $l$ has information of the current queue length and the current channel state of link $l$, but only has delayed queue length and delayed channel state information of other links in the network, where these delays are possibly heterogeneous but time-invariant. As an example, consider the delay values in Table I for a wireless network with three links $l_1$, $l_2$, $l_3$ with $A$, $B$, $C$ as the transmitter nodes of these links respectively, such that they form a network with complete interference.

From the first column of this table, transmitter $A$ has NSI of link $l_1$ (a link emanating from transmitter $A$) with a delay of 0 time slots, NSI of links $l_2$ and $l_3$ with delays of 2 and 1 time slots respectively. Similarly, transmitter $B$ has NSI of links $l_1$, $l_2$ (a link emanating from transmitter $B$), and $l_3$ with delays of 1, 0, and 2 time slots respectively, and transmitter $C$ has NSI of links $l_1$, $l_2$, and $l_3$ (a link emanating from transmitter $C$) with delays of 3, 4, and 0 time slots respectively. Analogously, transmitters $A$, $B$ and $C$ possess the NSI of link $l_1$ with delays of 0, 1, and 3 time slots respectively, of link $l_2$ with delays of 2, 0, and 4 time slots respectively, and of link $l_3$ with delays of 1, 2, and 0 time slots respectively. Additionally, the entire table of time-invariant delay values is assumed to be available at all transmitters in the network.

We will need some notations. Let $\tau_l(h)$ denote the time-invariant delay with which the NSI of link $h$ is available at the transmitter node of link $l$. For example, referring to the (2, 3) entry of Table I, $\tau_{l_2}(l_2)$ is 4. Let $\tau_{\text{max}}$ denote the maximum of the delay values across the whole network, i.e., $\tau_{\text{max}} := \max_{l \in L, h \in L} \tau_l(h)$. Also, let $\tau_l$ denote the maximum of the delays with which the NSI of link $l$ is available at the transmitter node of any link in the network, i.e., $\tau_{l,\text{max}} := \max_{h \in L, \tau_l(h) \neq \tau_l} \tau_l(h)$. In other words, referring to the table of delay values, $\tau_{\text{max}}$ refers to the maximum of all the entries in the table ($\tau_{\text{max}} = 4$ in Table I), and $\tau_{l,\text{max}} \forall l \in L$ refers to the maximum of the entries in row $l$ ($\tau_{l,\text{max}} = 3$, $\tau_{l_2,\text{max}} = 4$, and $\tau_{l_3,\text{max}} = 2$ in Table I). We note that $\tau_l(l) = 0 \forall l \in L$ in our model (the diagonal entries in Table I).

Let $C_l[t]((0 : \tau_l) := \{C_l[t - \tau], C_l[t - \tau + 1], \ldots, C_l[t]\}$, $C_l[t]((0 : \tau_{\text{max}}) := \{C_l[t]((0 : \tau_{\text{max}})) \in L\}$ and $C_l[t]((0 : \tau_{\text{max}}) := \{C_l[t]((0 : \tau_{\text{max}})) \in L\}$. Let $Q_l[(0 : \tau_{\text{max}})$ and $Q_l[(0 : \tau_{\text{max}})$ be similarly defined. We denote the NSI available at the transmitter node of link $l$ by $\{P_l(Q_l((0 : \tau_{\text{max}}), P_l(C_l((0 : \tau_{\text{max}}))\}$, where

$$P_l(Q_l((0 : \tau_{\text{max}}) := \{P_l(Q_l((0 : \tau_{\text{max}})) \in L\}, P_l(C_l((0 : \tau_{\text{max}}) := \{P_l(C_l((0 : \tau_{\text{max}})) \in L\},$$

$$P_l(C_l((0 : \tau_{\text{max}}) := \{P_l(C_l((0 : \tau_{\text{max}})) \in L\}$$

As an example, with reference to the delay values in Table I, the NSI available at transmitter $A$ (the transmitter node of link $l_1$), $\{P_l(Q_l((0 : \tau_{\text{max}}), P_l(C_l((0 : \tau_{\text{max}}))\}$ is the set: $\{(Q_{l_1}[t - 3], Q_{l_1}[t - 4], Q_{l_1}[t - 2], Q_{l_2}[t - 1]), Q_{l_2}[t - 3], Q_{l_2}[t - 4], Q_{l_2}[t - 2], Q_{l_3}[t - 3], Q_{l_3}[t - 4], Q_{l_3}[t - 2] \}$, and is the set: $\{(Q_{l_2}[t - 3], Q_{l_2}[t - 4], Q_{l_2}[t - 2], Q_{l_3}[t - 1]), (Q_{l_3}[t - 3], Q_{l_3}[t - 4], Q_{l_3}[t - 2], Q_{l_3}[t - 1]) \}$. We note that, in Table I, the NSI available at link $l$ is defined as $\{P_l(Q_l((0 : \tau_{\text{max}}), P_l(C_l((0 : \tau_{\text{max}}))\}$, and as a consequence, even though the structure of NSI that we use in our work is the same as that in Table I, the NSI at link $l$ in our model is a subset of the corresponding NSI used in Table I since for each $l$, $\tau_{l,\text{max}} \leq \tau_{l,\text{max}}$. It is crucial to note that $\{Q_l((0 : \tau), C_l((0 : \tau))\}$ are all common information available at the transmitter node of each link in the network, as are $\{Q_l((0 : \tau), C_l((0 : \tau))\} \in L$. It is intuitively appealing that $\{Q_l((0 : \tau), C_l((0 : \tau))\}$, being less delayed (i.e., more recent) compared to $\{Q_l((0 : \tau), C_l((0 : \tau))\}$, and also being commonly available at all transmitters in the network, it is preferable that each transmitter in the network base its transmit/no-transmit decision on this information rather than on $\{Q_l((0 : \tau), C_l((0 : \tau))\} \in L$. In Sec. [V] we show that this intuition is indeed correct.

E. Performance Metric

The metric of interest to us is the throughput region of the saturated system (except in Sec. [V]), where we are interested in the saturated system throughput). For this, we define the state of the network at time $t$ as the process $Y[t] := \{Q_l[t]((0 : \tau_{\text{max}}), C_l[t]((0 : \tau_{\text{max}}))$.
\( \tau_{l, \text{max}} \}) \in \mathcal{L} \). We denote \( \mathbf{Y}[\tau] \) under the scheduling policy \( \mathbf{f}^* \) by \( \mathbf{Y}^{f*}[\tau] \). It is easily seen that this process is a DTMC.

Given an arrival rate vector \( \{ \lambda_l \}_{l \in \mathcal{L}} \), we say that the network is stochastically stable under the scheduling policy \( \mathbf{f}^* \) if the DTMC \( \mathbf{Y}^{f*}[\tau] \) is positive recurrent. We say that an arrival rate vector \( \{ \lambda_l \}_{l \in \mathcal{L}} \) is supportable if some scheduling policy makes the network stochastically stable for this arrival rate vector. We say that a scheduling policy is throughput optimal if it stabilizes the network for any arrival rate vector that is supportable.

### F. Characterization of System Throughput Region

Consider the collection of functions \( \{ f_l \}_{l \in \mathcal{L}} \), where \( f_l = \mathcal{P}_l(\mathbf{C}[\tau]([0 : \tau_{l, \text{max}}])) \) has the following semantics – in each time slot \( t \), the transmitter associated with each link \( l \) computes the binary value \( f_l(\mathcal{P}_l(\mathbf{C}[\tau]([0 : \tau_{l, \text{max}}]))) \) and attempts to transmit on link \( l \) if and only if the outcome is 1. Note that the outcome of \( f_l \) is independent of all queue-length information.

Let \( S_l(c, f) \) be the expected data transmission rate (i.e. the expected number of bits or packets transmitted per time-slot) at time \( t \) that the transmitter associated with link \( l \) would receive if it applies the scheduling policy \( f_l \) (where \( f := \{ f_l \}_{l \in \mathcal{L}} \) when the delayed CSI at time \( t \) is \( \mathbf{C}[\tau_{l, \text{max}}] = c \). That is,

\[
S_l(c, f) = \mathbb{E}
\left[ C_l[t] f_l(P_l(\gamma \tau \text{ arg max}_{l', l} \text{C}_l[t] \text{max})) \right | \mathbf{C}[\tau_{l, \text{max}}] = c \bigg],
\]

where \( \text{C}_l[t] \) denotes the channel state of link \( l \) at time \( t \), \( \mathbb{E} \) denotes the expected value, and \( \text{arg max}_{l', l} \text{C}_l[t] \text{max} \) represents the channel state of the link that maximizes the throughput.

\[
\text{Step 1:} \quad \text{all the transmitters compute threshold functions based on common CSI available at all transmitters. These threshold functions, one for each transmitter, map the respective transmitter’s critical NSI to a corresponding threshold value, and are computed by solving the following optimization problem:}
\]

\[
\arg \max_{l, \tau} \sum_{l \in \mathcal{L}} Q_l[t, \tau] \times R_{l, \tau}(T),
\]

where

\[
R_{l, \tau}(T) := \mathbb{E}
\left[ C_l[t] \times 1_{\{C_l[t] \geq T_{l}(\cdot)\}} \right | \mathbf{C}[\tau_{l, \text{max}}] = c \bigg],
\]

\[
\text{Step 2:} \quad \text{each transmitter observes its current critical NSI, evaluates its threshold function (found in Step 1) at this critical NSI, and attempts to transmit if and only if its current channel rate exceeds the threshold value, i.e., when \( C_l[t] \geq T_l(C_l[t]) \)).
\]

To briefly illustrate the key idea behind the \( R \) policy and the difficulty that the problem of scheduling with heterogeneously delayed CSI entails, consider a network with three links – \( l_1, l_2 \) and \( l_3 \) with perfect collision and complete interference (illustration is easier for this setting). First, let us consider the case where the transmitter node of each link has the \( \tau \)-unit delayed CSI of each link (the case of homogeneous delays), and no link possesses the NSI of any link (including that of itself) with a delay lesser than \( \tau \) units (this of course does not conform to the heterogeneously delayed CSI setting). In this case, the \( R \) policy can be thought of as optimally partitioning the space of all sample paths (so as to maximize the achieved system throughput) into \( L \) partitions (where some partitions

\[\text{1)}\text{We remark that all the quantities that pertain to the } R \text{ policy that we show with a superscript } R \text{ (e.g., } CS^{R}(\cdot), \mathcal{P}^{R}(\cdot) \text{) appear without superscripts in } 1.\]

}\[\text{1)}\text{We remark that all the quantities that pertain to the } R \text{ policy that we show with a superscript } R \text{ (e.g., } CS^{R}(\cdot), \mathcal{P}^{R}(\cdot) \text{) appear without superscripts in } 1.\]
are possibly empty) with the connotation that the ith partition contains sample paths for which link $l_i$ will carry transmission when any of these sample paths is realized. To see this, consider the 3-tuple $(C_{l_1}[t−τ], C_{l_2}[t−τ], C_{l_3}[t−τ])$ which give rise to eight possibilities when $C = \{1, 2\}$. We can think of all the sample paths as grouped into eight classes corresponding to these eight possibilities of $(C_{l_1}[t−τ], C_{l_2}[t−τ], C_{l_3}[t−τ])$. These eight classes are then partitioned optimally into three partitions such that when any sample path in the first partition is realized, link $l_1$ will carry transmission (and links $l_2$ and $l_3$ won’t), and so on. When a particular sample path is realized, the transmitter nodes of links $l_1$, $l_2$ and $l_3$ look at the tuple $(C_{l_1}[t−τ], C_{l_2}[t−τ], C_{l_3}[t−τ])$ and determine the partition to which it belongs and thereby decide which of the three links will carry transmission. Now, this is true when the delays are homogeneous. In the case of heterogeneous delays, the problem of partitioning the sample paths is slightly more complicated by the fact that the different transmitters in the network have disparate views of the state of the network. We continue this illustration for the heterogeneously delayed NSI setting after introducing the example in Sec. III-A.

A. Computational Complexity

From Expr. 5, we see that the domain of optimization is the set of threshold function vectors $T$, where $T = \{T_l(\cdot)\}_{l \in L}$. From Expr. 6 we see that, while computing the conditional expectation, given a threshold function vector $T$, each sample path in the evaluation of the conditional expectation is subjected to evaluation by the threshold functions $\{T_l(\cdot)\}_{l \in L}$. As a consequence, there are two aspects to the computational complexity of the $R$ policy – namely, (i) functional evaluation complexity – the number of threshold function vectors $T$ over which the optimization is to be done, and (ii) sample-path complexity – the number of sample paths that are to be considered in evaluating the conditional expectation. We first characterize the structure on the delay values that produces the worst-case functional evaluation complexity in the $R$ policy. We then consider the complexity of functional evaluation in this worst-case setting, and obtain an expression for the number of threshold function vectors required in the domain of optimization in the worst case.

**Lemma 1.** The worst-case complexity of functional evaluation in the $R$ policy is realized when the delays in the table of delay values are all distinct, and the delay values at positions $(i, j)$ in row $i$ of the table of delay values, for $j = 1, 2, \ldots, i−1, i+1, \ldots, L$ appear in descending order, for all $i$.

We relegate the proof of this lemma to Appendix A. We now characterize the functional evaluation complexity of the $R$ policy for the worst-case setting noted in Lemma 1.

**Lemma 2.** For the $R$ policy, the total number of threshold functions that are needed to be considered in the domain of optimization in Expr. 5, for the worst-case scenario noted in Lemma 1, is $2^{\sum_{l \in L} (l−2)+l−1}$, where $C$ is the number of channel states and $L$ is the number of links in the network.

From this lemma, the number of threshold functions required in the domain of optimization in Expr. 5 is doubly exponential in the number of links in the network, and exponential in the number of levels into which the channel states on the wireless links are quantized. We present a proof of this lemma in Appendix B. Next, we characterize the sample-path complexity of the $R$ policy.

**Lemma 3.** For the $R$ policy, the number of sample paths that are needed to be considered in the evaluation of the conditional expectation in Expr. 6 is given by $C^{L_{\max}}$, where $C$ is the number of channel states and $L$ is the number of links in the network.

From this lemma, the number of sample paths involved in the evaluation of the conditional expectation in Expr. 6 is exponential in the number of links in the network and in the maximum of the heterogeneous delay values. We present a proof of this lemma in Appendix C. As noted earlier, given a threshold function vector $T = \{T_l(\cdot)\}_{l \in L}$, each of these $C^{L_{\max}}$ sample paths is subjected to evaluation by the $L$ threshold functions in $T$. We now illustrate the functional evaluation and sample-path complexities with an example.

**Example 1** Consider a three node network with three links – $l_1$, $l_2$, and $l_3$ with heterogeneous delays $\tau_2(l_1) = 7, \tau_3(l_1) = 11, \tau_1(l_2) = 8, \tau_3(l_2) = 9, \tau_1(l_3) = 12, \tau_2(l_3) = 6$. Rearranging these delays as noted in Lemma 1 we have $\tau_1(l_1) = 11, \tau_1(l_2) = 7, \tau_1(l_3) = 9, \tau_1(l_2) = 8, \tau_1(l_1) = 12, \tau_1(l_3) = 6$. From these delays, we get $CS^{R}_{l_1}(\cdot) = \{C_{l_1}[t−11], C_{l_1}[t−7], C_{l_1}[t−9], C_{l_1}[t−12], C_{l_3}[t−8], C_{l_3}[t−6]\}$, and $CS^{R}_{l_3}(\cdot) = \{C_{l_3}[t−11], C_{l_3}[t−7], C_{l_3}[t−9], C_{l_3}[t−12], C_{l_3}[t−6]\}$. Therefore, given $\{C[t−\tau_{\max}]\}_{l \in L}$, $T_{l_1}(\cdot), T_{l_2}(\cdot), T_{l_3}(\cdot)$ are functions of 3,4 and 5 variables respectively. Taking $C = \{1, 2\}$, $T_{l_1}(\cdot)$ maps the $2^3$ values in its domain to two real numbers, say 0.5 and 2.5, independently. Thus, the number of choices of threshold functions for $T_{l_1}(\cdot)$ is $2^3$. Similarly, the number of choices for $T_{l_2}(\cdot)$ and $T_{l_3}(\cdot)$ are $2^{16}$ and $2^{32}$ respectively. Therefore, the total number of threshold function vectors $T$ in the domain of optimization is $2^{56}$. The number of sample paths to be considered in evaluating the conditional expectation is $2^{36}$ since $\tau_{\max} = 12$ and $L = 3$.

Continuing the discussion on the key idea behind the $R$ policy and the difficulty that the problem of scheduling with heterogeneously delayed NSI entails from the last paragraph in Sec. III for the case of heterogeneous delays in the above example, when a particular sample path is realized, the transmitter node of link $l_1$ looks at $T(CS^{R}_{l_1}(\cdot))$ (i.e., at $T(C_{l_1}[t−11], C_{l_1}[t−7], C_{l_1}[t−9], C_{l_1}[t−12])$) and decides whether link $l_1$ will carry transmission depending on whether $C_{l_1}[t] \geq T(CS^{R}_{l_1}(\cdot))$, the transmitter node of link $l_2$ looks at $T(CS^{R}_{l_2}(\cdot))$ (i.e., at $T(C_{l_2}[t−11], C_{l_2}[t−9], C_{l_2}[t−8], C_{l_2}[t−12], C_{l_3}[t−6])$) to decide, and so on. Even though $CS^{R}_{l_1}(\cdot)$ and $CS^{R}_{l_2}(\cdot)$ do not coincide, it is easy to convince...
oneself that the sample paths will be partitioned into three partitions as before, since we are dealing with perfect collision interference (i.e., $\gamma_l = 0$, $\forall l$).

**B. Delay Performance**

The $R$ policy, in Expr. (9), uses $\tau_{\text{max}}$-delayed queue-length information to stabilize the queues. As noted in Sec. II-D, $\{Q_l[t-\tau_{\text{max}}], C_l[t-\tau_{\text{max}}]\} \in L$ are all common information available at all the transmitters. The values $\{Q_l[t-\tau_{\text{max}}]\} \in L$ being less delayed than $\{Q_l[t-\tau_{\text{max}}]\} \in L$, it is intuitively appealing to use the lesser-delayed queue lengths $\{Q_l[t-\tau_{l},\text{max}]\} \in L$ in place of $\{Q_l[t-\tau_{\text{max}}]\} \in L$ to stabilize the queues. In the following, we show through numerical simulation that using the lesser-delayed queue lengths $\{Q_l[t-\tau_{l},\text{max}]\} \in L$ in place of $\{Q_l[t-\tau_{\text{max}}]\} \in L$ to stabilize the queues significantly reduces the average per-packet queueing delay, and hence that the $R$ policy is not delay-optimal.

We consider a network with two links $l_1$ and $l_2$ that carry transmission from their respective transmitter nodes $A$ and $B$ to their respective destinations (with $I_{l_1} = \{l_2\}$ and $I_{l_2} = \{l_1\}$), with the channels on these links modeled as independent DTMCs on the state space $C = \{1, 2\}$ with crossover probability 0.1. We consider the heterogeneous delays $\tau_{l_1}(l_1) = 1, \tau_{l_1}(l_2) = x$, where $x \geq 2$ is a parameter we vary. For this heterogeneous delay setting, we see that $\tau_{\text{max}} = x, \tau_{l_1,\text{max}} = 1, \tau_{l_2,\text{max}} = x$.

With the objective of examining the effect of using $\tau_{\text{max}}$-delayed queue lengths in place of $\tau_{\text{max}}$-delayed queue lengths on the delay performance of the system, we consider two scheduling policies both of which have access to instantaneous CSI but differ (only) in the delays that they use to access the queue lengths. The first policy, DQIC1 (for Delayed Queue lengths and Instantaneous Channel states), uses $\tau_{\text{max}}$-delayed queue lengths to stabilize the queues, whereas the second policy, DQIC2, uses $\tau_{l_1,\text{max}}$-delayed queue lengths. Specifically, the two policies are:

**DQIC1:**

$$\text{arg max} \{Q_{l_1}[t-\tau_{\text{max}}] \times C_{l_1}[t], Q_{l_2}[t-\tau_{\text{max}}] \times C_{l_2}[t]\}$$

**DQIC2:**

$$\text{arg max} \{Q_{l_1}[t-\tau_{l_1,\text{max}}] \times C_{l_1}[t], Q_{l_2}[t-\tau_{l_2,\text{max}}] \times C_{l_2}[t]\}$$

That is, DQIC1 schedules link $l_1$ if $Q_{l_1}[t-x] \times C_{l_1}[t] \geq Q_{l_1}[t-x] \times C_{l_2}[t]$ and link $l_2$ otherwise, whereas DQIC2 schedules link $l_1$ if $Q_{l_1}[t-1] \times C_{l_1}[t] \geq Q_{l_2}[t-x] \times C_{l_2}[t]$, and link $l_2$ otherwise.

Packets arrive into the two queues at the two transmitters as Poisson processes with rates $\lambda_1 = \lambda_2 = 0.5$ (for this setting, it is easily seen that sum rates (i.e., $\lambda_1 + \lambda_2$) of up to 1.75 are supportable). Packets are time-stamped on arrival and on exit, and on servicing link $l$ successfully, a number of packets equal to $\min(C_l[t], Q_l[t])$ are removed from the queue of link $l$.

**Step 1:** using the common NSI available at all transmitters, each transmitter determines the optimal threshold function vector, by solving the following optimization problem:

$$\text{arg max}_T \sum_{l \in L} Q_l[t-\tau_{l,\text{max}}] R_{l,\tau_{\text{max}}}(T), \quad (10)$$

where

$$R_{l,\tau}(T) := \mathbb{E} \left[ C_l[t] \cdot \mathbf{1}_{\{C_l[t] \geq T_l(t)\}} \left( \gamma_l + (1 - \gamma_l) \prod_{m \in I_l} \mathbf{1}_{\{C_m[t] < T_m(t)\}} \right) \right]. \quad (11)$$

Analytical characterization of the delay performances of these policies appears hard.

IV. The $H$ Policy

In Sec. II-D we conjectured that $\{C_l[t-\tau_{l,\text{max}}]\} \in L$, being less delayed compared to $\{C_l[t-\tau_{\text{max}}]\} \in L$, and being commonly available at all transmitters in the network, it may be preferable that each transmitter base its transmit/no-transmit decision on this information rather than on $\{C_l[t-\tau_{\text{max}}]\} \in L$.

In addition to testing this hypothesis, we wish to reduce the computational complexity and the average per-packet queueing delay in comparison with that of the $R$ policy. Motivated by these, we define the $H$ policy to be the following scheduling policy. In each time slot,

- **Step 1:** using the common NSI available at all transmitters, each transmitter determines the optimal threshold function vector, by solving the following optimization problem:

  $$\text{arg max}_T \sum_{l \in L} Q_l[t-\tau_{l,\text{max}}] R_{l,\tau_{\text{max}}}(T), \quad (10)$$

  where

  $$R_{l,\tau}(T) := \mathbb{E} \left[ C_l[t] \cdot \mathbf{1}_{\{C_l[t] \geq T_l(t)\}} \left( \gamma_l + (1 - \gamma_l) \prod_{m \in I_l} \mathbf{1}_{\{C_m[t] < T_m(t)\}} \right) \right]. \quad (11)$$

15Considering the four possibilities of data rates for $C_l[t]$ and $C_{l_2}[t]$, a data rate of 1 unit is supportable when $C_1[t] = C_{l_2}[t] = 1$ which happens with probability 0.25, and a data rate of 2 units are supportable in all the other three cases since when the queues are saturated, both the DQIC1 and DQIC2 policies would choose the link with the largest data rate. These three cases happen with probability 0.25 each.
\[ C[t - \tau_{\text{max}}] := \{C_l[t - \tau_{l, \text{max}}] \}_{l \in L}, \]
\[ T_l : CS_l(C[t](0 : \tau_{\text{max}})) \to \mathbb{R}, \]
\[ CS_l(C[t](0 : \tau_{\text{max}})) := CS(C[t](0 : \tau_{\text{max}})) \cap P_l(C[t](0 : \tau_{\text{max}})), \quad (12) \]
\[ \text{and } CS(C[t](0 : \tau_{\text{max}})) := \{\{C_l[t - \tau_{k}(l)]\}_{k \in L, k \neq 1}\}_{l \in L}. \quad (13) \]

- **Step 2:** Each transmitter observes its current critical NSI, evaluates its threshold function (found in Step 1) at this critical NSI, and attempts to transmit if and only if its current channel rate exceeds the threshold value, i.e., when \( C_l[t] \geq T_l(CS_l(C[t](0 : \tau_{\text{max}}))). \)

**Remark:** It is to be noted that the \( R \) and \( H \) policies differ only in the delay values that they use to access queue lengths and channel state information, with the \( R \) policy using the delay value \( \tau_{\text{max}} \) and the \( H \) policy using the delay value \( \tau_{l, \text{max}} \) to access the NSI of link \( l \).

### A. Computational Complexity

In this section, similar to the way that we dealt with the \( R \) policy in Sec. III-A, we first characterize the structure on the delay values (in the table of delay values) that produce the worst-case functional evaluation complexity in the \( H \) policy. We then consider the complexity of functional evaluation in this worst-case setting, and obtain an expression for the number of threshold function vectors required in the domain of optimization in the worst case in the \( H \) policy. First, we will need the following result.

**Lemma 4.** \( CS_{\text{H}}(\{C[t](0 : \tau_{\text{max}})\}) = CS_l(\{C[t](0 : \tau_{\text{max}})\}) \).

We provide a proof of this lemma in Appendix D. We next characterize the structure on the delay values that brings out the worst-case functional evaluation complexity in the \( H \) policy.

**Lemma 5.** The worst-case complexity of functional evaluation in the \( H \) policy is realized when the delays in the table of delay values are all distinct, and the delay values at positions \( (i, j) \) in row \( i \) of the table of delay values, for \( j = 1, 2, \ldots, i - 1, i + 1, \ldots, L \), appear in descending order, for all \( i \).

We highlight a minor difference in the proof of this lemma compared to that of Lemma 1 in Appendix E. We now characterize the functional evaluation complexity of the \( H \) policy for the worst-case setting noted in Lemma 5.

**Lemma 6.** For the \( H \) policy, the total number of threshold functions that are needed to be considered in the domain of optimization in Expr. (7), for the worst-case scenario noted in Lemma 3 is \( 2^{\sum_{l \in L} \tau_{l, \text{max}}} \), where \( C \) is the number of channel states and \( L \) is the number of links in the network.

We present a proof of this lemma in Appendix F. Next, we characterize the sample-path complexity of the \( H \) policy.

**Lemma 7.** For the \( H \) policy, the number of sample paths that are needed to be considered in the evaluation of the conditional expectation in Expr. (7) is given by \( C^{\sum_{l \in L} \tau_{l, \text{max}}} \), where \( C \) is the number of channel states.

The proof of this lemma is similar to the proof of Lemma 3 and is therefore omitted. We now illustrate with an example the reduction in computational complexity that the \( H \) policy achieves over that of the \( R \) policy.

**Example:** Consider the same example as in Sec. III-A. After computing \( CS_{\text{H}}(\cdot) \) using Expr. (12), we see that given \( \{C_l[t - \tau_{l, \text{max}}]\}_{l \in L} \), \( T_{l_1}(\cdot), T_{l_2}(\cdot), T_{l_3}(\cdot) \) are functions of 1, 2, and 3 variables respectively. Using the same arguments as in the proofs of Lemmas 2 and 6 in Appendices B and F, we find that the number of choices of threshold functions for \( T_{l_1}(\cdot), T_{l_2}(\cdot) \) and \( T_{l_3}(\cdot) \) are \( 2^2, 2^4 \) and \( 2^8 \) respectively. Therefore, the total number of threshold function vectors \( T \) in the domain of optimization is \( 2^{14} \) (the \( R \) policy required \( 2^{36} \) threshold functions) and the number of sample paths required to be considered in evaluating the conditional expectation is \( 2^{32} \) (the \( R \) policy required considering \( 2^{36} \) sample paths), yielding an enormous reduction in computational complexity over that of the \( R \) policy. We note that this vast reduction is mainly due to the fact that for the \( H \) policy, the exponent in the double exponential in the expression in Lemma 6 is smaller than that of the \( R \) policy.

### B. Delay Performance

Fig. 4 shows the average per-packet queueing delay (in units of time-slots) in the \( H \) policy for the setting of the example in Sec. III-B. The average per-packet queueing delay of the \( R \) policy grows linearly with \( \tau_{\text{max}} \) whereas that of the \( H \) policy tends to flatten out. Comparing the \( R, H, DQIC1 \) and \( DQIC2 \) policies and their delay performances, it is clearly evident that the use of \( \tau_{\text{max}} \)-delayed queue lengths in the \( H \) and \( DQIC2 \) policies is what gives these policies their better delay performances in comparison to those of the \( R \) and \( DQIC1 \) policies which use \( \tau_{\text{max}} \)-delayed queue lengths.

### C. Throughput Optimality

In this section we show that, like the \( R \) policy, the \( H \) policy too is throughput optimal. First, we prove that if an arrival process is supportable, then the expected arrival rate of this process should lie within the system throughput region \( \Lambda \) defined in Sec. II-D. This would then mean that the system throughput region \( \Lambda \) is the region that encompasses all supportable arrival rates given the NSI structure in Sec. II-D. Next, we show that the \( H \) policy stabilizes all arrival rate vectors in the system throughput region \( \Lambda \). These two together would then imply that the \( H \) policy is throughput optimal.

**Lemma 8.** Under the NSI structure noted in Sec. II-D if the traffic arrival process \( \{A[t]\}_t \) is supportable, then \( E[A[t]] \in \Lambda \).

The proof of this lemma, which is similar to the proof of Lemma 4.1 in [1], is available in Appendix C.

**Corollary 1.** The system throughput regions defined in Eqn. (5), and in Sec. 4.1 in [1], are identical.

This is an immediate consequence of Lemma 4.1 in [1] and Lemma 8 stated above. See Sec. 4.1 in [1] for the definition of the system throughput region considered in [1].
Theorem 1. The $H$ policy is throughput optimal.

The proof of this theorem, which is similar to the proof of the corresponding theorem for the $R$ policy in [1] (but with significantly more details) is available in Appendix [2]. An important implication of this theorem is that using $\tau_{l,max}$-delayed NSI instead of $\tau_{max}$-delayed NSI (for each link $l$) does not harm the achievable system throughput, a fact that we exploit crucially in designing our computationally efficient near-throughput-optimal policies in Sec. 6.

V. LOW COMPLEXITY SCHEDULING POLICIES

The $H$ policy, despite the immense reduction in computational complexity that it achieves in comparison to the $R$ policy, is still computationally complex, and therefore impractical. In this section, we propose and evaluate two fast and near-throughput-optimal scheduling policies – namely, LC-ELDR (for Eliminate link with Least Data Rate), and LC-ERD,%MC (for Eliminate link that Reduces Delays for Maximum number of Channels) – for the heterogeneously delayed NSI setting, confining our attention to the more pragmatic case of perfect collision interference (i.e., $\gamma_i = 0, \forall l \in L$). Initially, we consider the case of complete interference (i.e., $I_l = L \setminus \{l\} \forall l \in L$), and consider extension to the case of multiple interference sets subsequently. These low-complexity scheduling policies derive their main idea from the $H$ policy; the idea being that the common delay value that all the contending transmitters (i.e., transmitters corresponding to contending links) can use to access the NSI of some link $l$ (of at least one link), could be reduced (and hence more recent [i.e., more reliable] delayed link-statistics could be made use of in computing the schedule) by carefully choosing to eliminate one link from among the contending links. These policies are iterative (we call each iteration, a "round") and they operate by choosing to eliminate one link in each round; the two policies differ only in the criterion they use for selecting the link that they eliminate in each round. Further, eliminating one link in each round accords these policies polynomial running times as we show in Sec. V-C. We list the LC-ELDR policy in Algorithm 1 and demonstrate its working in detail in Sec. V-A. The LC-ERD,%MC policy is identical to the LC-ELDR policy listed in Algorithm 1 except for step 16 which is listed separately in Algorithm 2. We derive analytical expressions for the exact expected saturated system throughputs of the LC-ELDR and LC-ERD,%MC policies in Sec. V-B and show these expressions in Sec. VI and show that these policies are near-optimal.

A. Dynamics of the LC-ELDR Policy

We now demonstrate the working of the LC-ELDR policy for a network with four wireless links – $l_1$, $l_2$, $l_3$, $l_4$ (with $A$, $B$, $C$, $D$ as the transmitter nodes on these links, respectively), all contending for transmission in the current time slot. We will assume that the queues at the transmitter node of these links are saturated, and that the heterogeneous delays are as in Table I. Let each of the wireless links be modeled as a

Algorithm 1

1: procedure LC-ELDR(ActiveSet)
2: $I \leftarrow$ all links in the network for which this node is the transmitter
3: while $|ActiveSet| > 2$ do
4: Compute $\tau_{l,max} \forall l \in ActiveSet$ from the delay table after suppressing the rows and columns corresponding to links not in ActiveSet
5: Compute the queue-length weighted expected data rate that will be realized if link $l$ is allowed to carry transmission given the $\tau_{l,max}$-delayed channel state of link $l$, for all links $l$ in ActiveSet, as follows: $Q_l[t – \tau_{l,max}] \times \sum_{i \in \text{H}} C_i[t] | C_i[t – \tau_{l,max}] = c_i, \tau_{l,max}$, where $c_i, \tau_{l,max}$ is the channel-state realization of link $l$ at time $t – \tau_{l,max}$
6: Let $H$ be the link with the largest queue-length weighted expected data rate computed in Step 5 (arbitrarily chosen if more than one satisfy this criterion)
7: Let $EC$ (for Elimination Candidates) be the subset of $ActiveSet \setminus \{H\}$ such that if link $K \in ActiveSet \setminus \{H\}$ then link $K \in EC$ if, on recomputing the delays $\tau_{l,max} \forall l \in ActiveSet$ after masking the rows and columns corresponding to link $K$ and the links not in ActiveSet from the table of delay values, there is a reduction in the $\tau_{l,max}$ value of at least one link $l \in ActiveSet \setminus \{K\}$
8: if $EC = \phi$ (the empty set) then
9: if $H \in I$ then
10: Set transmit decision of $H = \text{TRANSMIT}$; For all $l \in I \setminus \{H\}$, set transmit decision of $l = \text{NOTRANSMIT}$
11: else
12: For all $l \in I$, set transmit decision of $l = \text{NOTRANSMIT}$
13: end if
14: Exit.
15:
else
16: Let $S$ be the link in $EC$ with the lowest queue-length weighted expected data rate computed in Step 5 (arbitrarily chosen if more than one satisfy this criterion)
17: if $S \in I$ then
18: Set transmit decision of $S = \text{NOTRANSMIT}$. $I \leftarrow I \setminus \{S\}$
19: if $I = \phi$ then Exit.
20: end if
21: $ActiveSet \leftarrow ActiveSet \setminus \{S\}$
22: end if
23: end while
24: Recompute the delays $\tau_{l,max}$ as in Step 2. Recompute the queue-length weighted expected data rates as in Step 5. Let $T \in ActiveSet$ be the link with the largest expected data rate (arbitrarily chosen if more than one satisfy this criterion)
25: if $T \in I$ then
26: Set transmit decision of $T = \text{TRANSMIT}$. For all $l \in I \setminus \{T\}$, set transmit decision of $l = \text{NOTRANSMIT}$
27: else
28: For all $l \in I$, set transmit decision of $l = \text{NOTRANSMIT}$
29: end if
30: end procedure

18 the node where this algorithm is being run
Markov chain on the state space $\mathcal{C} = \{1, 2\}$ with crossover probability 0.1 (the case of very slow varying channel). The $n$-step transition probability matrices for this Markov chain are as below (shown for only those delays [n-values] that are required in our computation):

$$P(1) = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}, \quad P(2) = \begin{bmatrix} 0.82 & 0.18 \\ 0.18 & 0.82 \end{bmatrix},$$

$$P(3) = \begin{bmatrix} 0.756 & 0.244 \\ 0.244 & 0.756 \end{bmatrix}, \quad P(4) = \begin{bmatrix} 0.7048 & 0.2952 \\ 0.2952 & 0.7048 \end{bmatrix},$$

$$P(5) = \begin{bmatrix} 0.6638 & 0.3362 \\ 0.3362 & 0.6638 \end{bmatrix}$$

We assume the following delayed channel-realizations for our illustration: $C_{l_1}[t-4] = 2, C_{l_1}[t-1] = 1, C_{l_1}[t-2] = 2, C_{l_2}[t-1] = 1, C_{l_2}[t-5] = 2, C_{l_3}[t-1] = 2, C_{l_3}[t-3] = 2, C_{l_3}[t-1] = 1$. We first illustrate the working of the LC-ELDR policy when it is executed at transmitter $C$, and towards the end we mention the minor differences in the dynamics when it is executed at the other transmitters. Since we are initially concerned with the case of complete interference, we set $ActiveSet = \{l_1, l_2, l_3, l_4\}$ before invoking Algorithm 1, where $ActiveSet$ is the set of links that are (still) contending for transmission in the current time slot.

**ROUND 1**

(i) first, transmitter $C$ temporarily masks the row and column pertaining to link $l_1$ (shown by hatching the corresponding row and column, in Fig. 2(b)), computes the $\tau_{i,\text{max}}$ values for links $l_1, i = 2, 3, 4$ (i.e., for all links other than link $l_1$), compares them to the previously computed $\tau_{i,\text{max}}$ values for these links, finds that upon masking the row and column pertaining to link $l_1$, the $\tau_{i,\text{max}}$ value for at least one link (namely, that of link $l_4$) reduces (from 3 to 1; see Fig. 2(b)), and hence decides that link $l_1$ is a candidate for elimination, and adds it to the set of elimination candidates, $EC$. What this means is that if transmitter $C$ does choose to eliminate link $l_1$, each of the transmitters corresponding to the remaining links would then be able to access the delayed channel state of link $l_4$ at a reduced common delay value of 1 unit (instead of 3 units, as was required previously) since the other transmitters assuredly possess the delayed channel-state of link $l_4$ at this new reduced delay of 1 unit

(ii) transmitter $C$ skips this procedure for link $l_2$ since $l_2$, being the link with the largest expected data rate, was set aside in the previous step

(iii) next, transmitter $C$ temporarily masks the row and column pertaining to link $l_3$ (i.e., its own link; shown again by hatching the row and column of link $l_3$, in Fig. 2(c)), computes the $\tau_{i,\text{max}}$ values for links $l_3$ for $i = 1, 2, 4$ (i.e., for all links other than link $l_3$), compares them to the previously computed $\tau_{i,\text{max}}$ values for these links, finds that upon masking the row and column pertaining to link $l_3$, the $\tau_{i,\text{max}}$ value for none of the links $l_i$ for $i = 1, 2, 4$ reduce, and hence decides that link $l_3$ (i.e., its own link) is not a candidate for elimination

(iv) lastly, transmitter $C$ temporarily masks the row and column pertaining to link $l_4$ (shown by hatching the row and column of link $l_4$, in Fig. 2(d)), computes the $\tau_{i,\text{max}}$ values for links $l_i$ for $i = 1, 2, 3$ (i.e., for all links other than link $l_4$), compares them to the previously computed $\tau_{i,\text{max}}$ values for these links, finds that upon masking the row and column pertaining to link $l_4$, the $\tau_{i,\text{max}}$ value for at least one link (namely, that of links $l_2$ and $l_3$) reduces (from 2 to 1 for link $l_2$, and from 5 to 1 for link $l_3$; see Fig. 2(d)), and hence decides that link $l_4$ is a candidate for elimination, and adds it to the set of elimination candidates, $EC$. This means that if transmitter $C$ does choose to eliminate link $l_4$, it is assured that each of the links remaining in $ActiveSet$ would then be able to access the channel state of links $l_2$ and $l_3$ at reduced delays of 1 unit and 1 unit respectively (instead of 2 and 5 units respectively, as was required previously)

Thus, transmitter $C$ has computed the set of elimination candidates to be $EC = \{l_1, l_4\}$

**ROUND 2**

(i) transmitter $C$ sets $I = \{l_3\}$

(ii) transmitter $C$ computes $\tau_{i,\text{max}}$, for $i = 1, 2, 3, 4$, which results in $\tau_{i,\text{max}} = 4$, $\tau_{2,\text{max}} = 2$, $\tau_{3,\text{max}} = 5$, $\tau_{4,\text{max}} = 3$. This is illustrated in the last column of the table in Fig. 2(a)

(iii) transmitter $C$ computes the expected data rate on each link $l_i$ in $ActiveSet$ given the $\tau_{i,\text{max}}$ delayed channel-state realization of that link, as follows 99 $E[|C_i[t]| \mid \tau_{i,\text{max}}] = 1.7048, E[|C_3[t]| \mid \tau_{3,\text{max}}] = 1.82, E[|C_4[t]| \mid \tau_{4,\text{max}}] = 2] = 1.7566, E[|C_1[t]| \mid \tau_{1,\text{max}}] = 1.6638, E[|C_2[t]| \mid \tau_{2,\text{max}}] = 2]\] = 1.7566.

(iv) transmitter $C$ sets aside link $l_2$ by setting $H = l_2$, since link $l_2$ has the largest expected data rate in this round, as computed in the previous step (this is done so that the link with the largest expected data rate in this round is not eliminated). This is shown by highlighting transmitter $C$’s (B) is the transmitter node on link $l_2$) column in green in Fig. 2(b)

(v) transmitter $C$, to decide on the link it wants to eliminate in this round, computes $EC$ as follows:

We will call the “while” loop body from line 3 to line 23, and also the computation in lines 24 – 29 in Algorithm 1, a “round”.

20 We ignore the multiplication of the delayed queue-lengths with the expected data rates since we are considering saturated queues for this illustration.

21 Transmitter $C$ removes link $l_1$ from $ActiveSet$
Fig. 2: Illustration of the dynamics of the LC-ELDR policy. In the beginning, ActiveSet = \{l_1, l_2, l_3, l_4\}. Subfigure (a) shows the original table of delay values and \(\tau_{i,\text{max}}\) values computed in step 4 of Algorithm 1 in round 1. Subfigures (b), (c), (d) show the computation of the set EC in round 1 (step 7). In subfigure (b), after setting aside the link with the largest expected data rate (link \(l_2\), shown by highlighting in green the column of transmitter B [the transmitter node of link \(l_2\)], we see whether eliminating link \(l_1\) reduces the \(\tau_{i,\text{max}}\) delays for any other link in ActiveSet. This is done by temporarily masking the row and column pertaining to link \(l_1\) (shown hatched), recomputing the \(\tau_{i,\text{max}}\) delays and comparing with previously computed values. The value of \(\tau_{i,\text{max}}\) reduces (from 3 to 1), and hence link \(l_1\) is a candidate for elimination, and therefore belongs to EC. Similarly, in subfigure (c), we determine that link \(l_3\) does not belong to EC, and in subfigure (d), we determine that link \(l_4\) belongs to EC. In subfigure (e), we eliminate link \(l_1\) (shown by striking-through in red the row and column pertaining to link \(l_1\)) since only \(l_1\) and \(l_4\) belong to EC and \(l_1\) has smaller expected data rate than \(l_4\) (steps 16, 21). This reduces ActiveSet to \{\(l_2, l_3, l_4\)\}. Subfigures (f), (g), (h) show similar calculations for round 2, at the end of which link \(l_3\) is eliminated, leaving behind only \(l_2\) and \(l_3\) in ActiveSet.

(round 2)

Note: For all purposes, the table of delay values for this round is the original table minus the row and column corresponding to link \(l_1\) since it was eliminated in the previous round.

(step 4) transmitter \(C\) sets \(\tau_{1,\text{max}} = X\) (don’t care) since link \(l_1\) does not belong to ActiveSet anymore. Transmitter \(C\) then calculates the new values of \(\tau_{i,\text{max}}\), for \(i = 2, 3, 4\). All these values could potentially be smaller than the corresponding values in round 1, since when calculating these values in round 2, transmitter \(C\) ignores the delay values in the row and column pertaining to link \(l_1\) which was eliminated in the previous round. The new values are: \(\tau_{2,\text{max}} = 2, \tau_{3,\text{max}} = 5, \tau_{4,\text{max}} = 1\). Note that \(\tau_{4,\text{max}}\) has reduced from 3 (in the previous round) to 1 (see Fig. 2(c)).

(step 5) as in round 1, transmitter \(C\) computes the expected data rate on each link \(l_i\) in ActiveSet given the \(\tau_{i,\text{max}}\)-delayed channel-state realization of that link (as computed in step 4 above), as follows: \(E[C_{l_2}[t] \mid C_{l_2}[t - 2] = 2] = 1.82\), \(E[C_{l_3}[t] \mid C_{l_3}[t - 5] = 2] = 1.6638\), \(E[C_{l_4}[t] \mid C_{l_4}[t - 1] = 1] = 1.1\).

(step 6) as in round 1, transmitter \(C\) sets aside the link that has the largest expected data rate in this round, as computed in the previous step. Coincidentally, link \(l_3\) turns out to be that link in this round also. Therefore, transmitter \(C\) sets \(H = l_2\). This is shown by highlighting transmitter \(B\)’s column in green in Fig. 2(f).

(step 7) once again, transmitter \(C\) goes about computing \(EC\) (to decide on the link it wants to eliminate in this round) as follows:

(i) first, transmitter \(C\) decides to skip this procedure for link \(l_1\) (since it is not in ActiveSet anymore), and for link \(l_2\) (since \(l_2\), being the link with the largest expected data rate, was set aside in step 6 above)

(ii) next, following similar arguments as in round 1, transmitter \(C\) decides that link \(l_3\) (i.e., its own link) is not a candidate for elimination (see Fig. 2(f))

(iii) lastly, again following similar arguments as in round 1, transmitter \(C\) decides that link \(l_4\) is a candidate for elimination, and adds it to the set of elimination candidates, EC (see Fig. 2(g)).

Thus, transmitter \(C\) has computed the set of elimination candidates to be \(EC = \{l_4\}\).

(step 16) since there is only one elimination candidate (namely, link \(l_4\)) in set \(EC\), transmitter \(C\) chooses to eliminate link \(l_4\) by setting \(S = l_4\). This is shown by striking out the row and column pertaining to link \(l_4\) in Fig. 2(h).

(step 21) Transmitter \(C\) removes link \(l_4\) from ActiveSet (since link \(l_4\), having been eliminated in this round, is no longer in contention with the other links in ActiveSet for carrying transmission in the current time slot). Thus, ActiveSet = \{\(l_2, l_3\)\} is the revised set of links still contending.

(round 3)

(step 24) transmitter \(C\) sets \(\tau_{1,\text{max}}\) and \(\tau_{4,\text{max}}\) to \(X\) (don’t care) since links \(l_1\) and \(l_4\) do not belong to ActiveSet anymore. Transmitter \(C\) then calculates the new values of \(\tau_{i,\text{max}}\), for \(i = 2, 3\). To be \(\tau_{2,\text{max}} = 1, \tau_{3,\text{max}} = 1\). Transmitter \(C\) computes the expected data rate on each link \(l_i\) in ActiveSet given the \(\tau_{i,\text{max}}\)-delayed channel-state realization of that link (where the new \(\tau_{i,\text{max}}\) value is as computed only just), as follows: \(E[C_{l_2}[t] \mid C_{l_2}[t - 1] = 1] = 1.1\), \(E[C_{l_3}[t] \mid C_{l_3}[t - 1] = 2] = 1.9\). Note that \(\tau_{2,\text{max}}\) and \(\tau_{3,\text{max}}\) have changed from 2 and 5 respectively (in the previous round) to 1 each (see Fig. 2(h)). Transmitter \(C\) sets \(T = l_3\) since link \(l_3\) (i.e., its own link) has the largest expected data rate as computed above.

(step 26) Transmitter \(C\) sets the transmit decision for link \(l_3\) (i.e., for its own link) to TRANSMIT

(step 30) Transmitter \(C\) terminates this run of the LC-ELDR policy.
Algorithm 2

procedure LC-ERDMC(ActiveSet)
16: Let $S$ be the link in $EC$ with the lowest expected data rate among those links that, upon their elimination, reduce the maximum of the delay values in a row, for the largest number of rows (i.e., channels) [corresponding to links in ActiveSet] in the table of delay values [after suppressing the rows and columns corresponding to links that are not in ActiveSet]
end procedure

policy.

It is easy to convince oneself that all the transmitters running the LC-ELDR policy in the current time slot would arrive at the same decision – links other than $l_3$ would set their transmit decision to NOTTRANSMIT while link $l_3$ sets its transmit decision to TRANSMIT. To illustrate this in short, transmitter $D$ for example, when running the LC-ELDR policy in the current time slot, would make the same calculations as shown above till step 16 in round 2, whereupon transmitter $D$ would execute step 18 (note that $I = \{l_4\}$ in this case) and thereby set the transmit decision for link $l_4$ (i.e., its own link) to NOTTRANSMIT, and terminate this run of the algorithm at transmitter $D$ at step 19.

So far, we have considered the case of networks with complete interference. Extension to the case of networks with multiple interference sets is as follows: (i) Let $ActiveSet$ contain all the links in the network. Set $M \leftarrow \phi$. Set $\delta_{ij} \leftarrow 0 \forall i \notin I_j \forall i, j$ where $\delta_{ij}$ is the delay value in row $i$ and column $j$ of the delay table (ii) Run the LC-ELDR policy. Let $l$ be the link chosen for transmission by the LC-ELDR policy. Set $M \leftarrow M \cup \{l\}$. Set $ActiveSet \leftarrow ActiveSet \setminus \{l\}$.
(iii) Repeat step (ii) while $ActiveSet$ is non-empty. When $ActiveSet$ becomes empty, the links in $M$ are the set of links that will be allowed to carry transmission in the current time slot.

The policy LC-ERDMC is a minor variation of LC-ELDR, where only step 16 of LC-ELDR is modified as noted in Algorithm 2. In the example above, in step 16 of round 1, the LC-ELDR policy chose to eliminate link $l_1$ since it had a lower expected data rate compared to that of link $l_3$, whereas the LC-ERDMC policy chooses to eliminate link $l_4$ since eliminating link $l_4$ reduces the delays with which the channel state of two channels (namely, $l_2$ and $l_3$) can be accessed, whereas eliminating link $l_1$ would reduce the delay of only one channel (namely, $l_4$).

B. Throughput Near-Optimality

In this section, we derive analytical expressions for the expected saturated system throughputs of the LC-ELDR and LC-ERDMC policies. We evaluate these expressions in Sec. VI for various network settings, and demonstrate that these expressions approximate the optimal throughput values very closely. We need a few definitions, which we introduce here rather informally, and make these definitions precise mathematically later in Appendices J and K. Let $N$ be the number of links and $T := \{1, 2, \ldots, N\}$ be the set of links in $ActiveSet$ before calling the LC-ELDR policy. Further, let

$$1\{a\}_{i \in I} := \begin{cases} 1 & \text{if } a \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

We will call the “while” loop body from line 3 to line 23, and also the computation in lines 24–29 in the algorithm listing of the LC-ELDR policy in Sec. [I-D] a “round”. Thus, if the LC-ELDR policy terminates at line 26 or at line 28, then it would have executed $N - 1$ rounds (specifically, $N - 2$ rounds in the body of the “while” loop, and the last round [round $N - 1$] in lines 24–29), and it would have executed $r \ (r < N - 1)$ rounds if it terminates at line 14 or 19. Thus, the LC-ELDR policy can terminate after $r$ rounds, $1 \leq r \leq N - 1$.

Let $\tau^{(r)}_i$ (not to be confused with $\tau_i(h)$ defined in Sec. [I-D]) be equal to $\tau_i, \max$ (see Sec. [I-D]) at the beginning of round $r$, where $\tau_i, \max$ is calculated after masking the rows and columns pertaining to the links that have been eliminated in rounds 1 to $r - 1$ (as illustrated in Figs. 2(a), 2(e), and 2(h)). Let $c_{k, r}$ be the realization of channel-state on link $l_k$ at time $t - \tau$.

$e^{(r)}$: Let $e^{(r)}$ be the link that was (or that will be) eliminated (by the LC-ELDR policy) in round $r$ (thus, “$e^{(r)} = k$” [for $k \in T$] implies that link $k$ was (or will be) eliminated in round $r$, and “$e^{(r)} = 0$” implies that there is no candidate link to eliminate, and hence that the LC-ELDR policy terminates). Let $i \in T$ be the link chosen by the LC-ELDR policy in a particular time slot, say $t$, when the LC-ELDR policy terminates after executing $r$ rounds, $1 \leq r \leq N - 1$. We now consider the working of the LC-ELDR policy when it is executed at the transmitter node of link $i$ in time slot $t$. Consider a particular round $\bar{r} \ (1 \leq \bar{r} \leq r)$ of the LC-ELDR policy. If link $i$ has survived (i.e., was not eliminated) in the first $\bar{r} - 1$ rounds, then it will survive round $\bar{r}$ if one of the following happens:

(i) $s(i, \bar{r}, 1)$: link $i$ has the largest expected data rate in round $\bar{r}$ (hence $i = H$ and therefore $i \notin EC$; see steps 6 and 7 of Algorithm 1). We denote this condition as $s(i, \bar{r}, 1)$.

(ii) $s(i, \bar{r}, 2)$: link $i$ does not have the largest expected data rate in round $\bar{r}$ (hence $i \neq H$), and link $i$ does not reduce the maximum of the delay values in a row, for any row (corresponding to a link in $ActiveSet$) in the table of delay values (after suppressing the rows and columns corresponding to links that have been eliminated up to round $\bar{r}$) if it is eliminated in round $\bar{r}$ (hence $i \notin EC$; see step 7 of Algorithm 1), and some link (other than the one with the largest expected data rate in round $\bar{r}$) reduces the maximum of the delay values in a row, for at least one row if it is eliminated in round $\bar{r}$ (i.e., $EC \neq \phi$). We denote this condition as $s(i, \bar{r}, 2)$.

(iii) $s(i, \bar{r}, 3)$: link $i$ does not have the largest expected data rate in round $\bar{r}$ (hence $i \neq H$), and link $i$ reduces the maximum of the delay values in a row, for at least one row (corresponding to a link in $ActiveSet$) in the table of delay values (after suppressing the rows and columns corresponding to links that have been eliminated up to
round \( \tilde{r} \) if it is eliminated in round \( \tilde{r} \) (hence \( i \in EC \)), and
link \( i \) does not have the smallest expected data rate among
the links that reduce the maximum of the delay values in
a row, for at least one row if that link is eliminated in
round \( \tilde{r} \) (hence \( i \neq S \); see step 16 of Algorithm \[1\]). We
denote this condition as \( s(i, \tilde{r}, 3) \).

With the required definitions in place, we are now ready to
demonstrate that these expressions approximate the optimal
throughput very closely and hence that the expressions in Lemmas 9 and 10 numerically
evaluate the expressions in Lemma 9, except that the expression for
the transition probability of reaching state \( \pi \)
in the channel-state Markov chain of link \( c \).

**Lemma 9.** The expected saturated system throughput of the
LC-ELDR policy is exactly

\[
\sum_{i=1}^{N} \sum_{r=1}^{N-1} \sum_{q \in \{1,2,\ldots,r\}} \sum_{j,\tau(q)} \mathbb{E} \left[ C_l[t] \prod_{r=1}^{r-1} \left(1 + \{s(i, \tilde{r}, n)\}_{n \in \{1,2,3\}} \right) \right] \\
\times 1\{e^{(r)} \neq 0\} \left\{ \left( s(i, r, 1) \right) \left( e^{(r)} = 0 \right) \right\} \\
\left\{ C_t[t - \tau_i^{(r)}] = c_i, \tau_i^{(r)}, \right\} \\
\left\{ C_t[t - \tau_i^{(q)}] = c_i, \tau_i^{(q)}, \right\} \\
\left\{ C_t[p[t - \tau_p^{(q)}] = c_p, \tau_p^{(q)}, \right\} \\
\left\{ C_t[p[t - \tau_p^{(q)}] = c_p, \tau_p^{(q)}, \right\} \\
\times \prod_{n=1}^{N} \left( \pi_m \left( c_{m,r(1)} \right) \prod_{n=1}^{r-1} \left( n \pi_m \left( c_{m,r(n)} \right) \right) \right) \\
\times \prod_{r=1}^{r-1} \left( n \pi_m \left( c_{m,r(n)} \right) \right)
\]

where \( \pi_m(c) \) is the steady-state probability of being in state \( c \) in the channel-state Markov chain of link \( m \), and \( p_{m,i,j}^{(k)} \) is
the transition probability of reaching state \( j \) from state \( i \) in
\( k \) steps in the channel-state Markov chain of link \( m \), with
\( p_{m,i,j}^{(k)} = 1 \forall i, j, m \).

We present mathematically precise definitions of the symbols that were defined informally in the paragraphs preceding
Lemma 9 and the proof itself in Appendix K.

**Lemma 10.** The expression for the expected saturated system throughput of the LC-ERDMC policy is the same as the expression in Lemma 9 except that the expression for \( e^{(r)} \) is
redefined to be as noted in Appendix K.

The proof of this lemma is essentially the same as that for
Lemma 9; the minor departures are noted in Appendix K.

We evaluate the expressions in Lemmas 9 and 10 numerically
and plot them for various network settings in Sec. VI and
demonstrate that these expressions approximate the optimal
throughput values very closely and hence that the LC-ELDR
and LC-ERDMC policies are near-throughput-optimal.

C. Computational Complexity

We have the following result:

**Lemma 11.** The running times of both of the LC-ELDR and
LC-ERDMC policies are \( O(CL^2 + L^4) \).

We provide a proof of this lemma in Appendix \[1\] Thus from
this lemma, for the case of a network with complete interference,
the computational complexities of the LC-ELDR and
LC-ERDMC policies are polynomial in the number links in
the network and linear in the number of channel states.

We emphasize that the policies in a custom C++ simulator. All the numbers from
simulation that we quote in this section were averaged over
10^7 trials.

VI. NUMERICAL RESULTS AND DISCUSSION

In this section, we present numerical results comparing
the saturated system throughputs of the \( R, H, \) LC-ELDR
and LC-ERDMC policies. In addition, for purposes of comparison, we define two new policies that we call \( O \) and \( IC \):

Policy \( O \): \( \max \{Q_l[t - \tau_{l,max}] \times E[C_l[t] | C_l[t - \tau_{l,max}] = C_l, \tau_{l,max}\} \} t \in \mathcal{L} \).

Policy \( IC \): \( \max \{Q_l[t] \times C_l[t]\} t \in \mathcal{L} \).

The \( O \) policy chooses the link \( l \) that has the largest value of queue-length weighted expected data rate on the
link, computed using the \( \tau_{l,max} \)-delayed NSI of that link.

Note that the \( O \) policy corresponds to stopping after round 1
in the LC-ELDR and LC-ERDMC policies and declaring
link \( H \) the “winner” (see step 6 of Algorithm \[1\]). The
\( IC \) policy chooses the link \( l \) that has the largest value of
queue-length weighted data rate on the link, computed using
instantaneous NSI of that link, with the assumption that each
link has access to the instantaneous NSI for all channels.

We emphasize that the \( IC \) policy does not conform to the
structure of delayed NSI noted in Sec. II-D but it serves
as an upper bound on the system throughput that can be
achieved in the delayed NSI regime.

A. Methodology

As in \[1\], we consider networks with complete interference
(i.e., \( I_l = \mathcal{L} \setminus \{l\} \), \( \forall l \in \mathcal{L} \) and perfect collision (i.e., \( \gamma_l = 0, \forall l \in \mathcal{L} \)). The channel state on each link is modeled as
a DTMC on the state space \( \mathcal{C} = \{1, 2\} \). We only consider
single-hop transmissions in the network. We implemented all
the policies in a custom C++ simulator. All the numbers from
simulation that we quote in this section were averaged over
10^7 trials.

B. Results and Discussion

Fig. \[3\] shows the expected saturated system throughputs of
the various policies for a network with three links, for the very
small delay (VSD) delay profile (see Table \[3\]), for different
channel profiles. For this purpose, we define five channel
TABLE III: The Very Small Delays (VSD) delay profile for a wireless network with three links

| Link | TX A | TX B | TX C |
|------|------|------|------|
| l₁  | 0    | 1    | 1    |
| l₂  | 1    | 0    | 1    |
| l₃  | 1    | 2    | 0    |

Fig. 3: Expected saturated system throughputs (in data units transmitted) per time slot of the various policies for a network with three links, VSD delay profile and different channel profiles. The curves of all but the IC policy overlap.

profiles – namely, very slow varying channel (V SVC), slow varying channel (SVC), medium varying channel (MVC), fast varying channel (FVC) and very fast varying channel (VFVC), with channel-state crossover probabilities of 0.1, 0.3, 0.5, 0.7 and 0.9 respectively. We observe that all the policies perform equally well when the delay values are very small (the plots for all but the IC policy overlap in Fig. 3).

Fig. 4 shows the expected saturated system throughputs of the LC-ELDR, LC-ERDMC, O and IC policies for a network with three links and VSVC channel profile, for different delay profiles. For this purpose, in addition to the VSD delay profile noted in Table III, we define four other delay profiles – namely, small delays (SD), medium delays (MD), large delays (LD), and very large delays (VLD) as shown below:

\[
\begin{align*}
SD : & \begin{bmatrix}
0 & 1 & 3 \\
2 & 0 & 4 \\
1 & 2 & 0
\end{bmatrix}, \\
MD : & \begin{bmatrix}
0 & 7 & 11 \\
8 & 0 & 9 \\
12 & 6 & 0
\end{bmatrix}, \\
LD : & \begin{bmatrix}
0 & 20 & 15 \\
24 & 0 & 17 \\
12 & 28 & 0
\end{bmatrix}, \\
VLD : & \begin{bmatrix}
0 & 78 & 36 \\
59 & 0 & 88 \\
45 & 92 & 0
\end{bmatrix},
\end{align*}
\]

Similar to the entries in Table III the entry at position \(ij\) in each of the delay profiles is to be interpreted as the least delay with which the NSI of link \(l_i\) is available at transmitter \(j\) (i.e., at the transmitter node of link \(l_j\)). We do not show the system throughputs of the \(R\) and \(H\) policies in Fig. 4 since the delay values for all but the VSD profile are such that it is computationally hard to evaluate the saturated system throughput with increasing delay values. We note that the LC-ELDR and LC-ERDMC policies perform almost equally well, and both of them outperform the \(O\) policy. Fig. 4 also depicts the degradation of system throughput with increasing delay values.

Fig. 5 shows the expected saturated system throughputs of the \(R\), \(H\), LC-ELDR, LC-ERDMC, O and IC policies for a network with VSVC channel profile and VSD delay profile, for different number of links in the network (analysis curves of LC-ELDR and LC-ERDMC are shown only up to \#links = 5). The analysis and simulation curves of each policy overlap.

TABLE IV: Table of heterogeneous delay values for the Very Small Delays (VSD) delay profile for a network with ten links

| Link  | T₁ | T₂ | T₃ | T₄ | T₅ | T₆ | T₇ | T₈ | T₉ | T₁₀ |
|-------|----|----|----|----|----|----|----|----|----|-----|
| l₁    | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1   |
| l₂    | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1   |
| l₃    | 1  | 2  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1   |
| l₄    | 2  | 1  | 1  | 0  | 1  | 1  | 2  | 1  | 1  | 2   |
| l₅    | 1  | 1  | 1  | 1  | 0  | 1  | 2  | 1  | 1  | 2   |
| l₆    | 1  | 2  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 1   |
| l₇    | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 2  | 1  | 1   |
| l₈    | 1  | 2  | 1  | 1  | 1  | 2  | 1  | 0  | 2  | 1   |
| l₉    | 2  | 1  | 1  | 1  | 1  | 1  | 1  | 2  | 0  | 1   |
| l₁₀   | 1  | 1  | 2  | 1  | 1  | 2  | 1  | 1  | 1  | 0   |
TABLE V: Table of heterogeneous delay values for the Medium Delays (MD) delay profile for a network with ten links

| Link | 0 | 7 | 11 | 6 | 5 | 3 | 9 | 7 | 11 | 4 |
|------|---|---|----|---|---|---|---|---|----|---|
| 1    |   |   |    |   |   |   |   |   |    |   |
| 2    |   |   |    |   |   |   |   |   |    |   |
| 3    |   |   |    |   |   |   |   |   |    |   |
| 4    |   |   |    |   |   |   |   |   |    |   |
| 5    |   |   |    |   |   |   |   |   |    |   |
| 6    |   |   |    |   |   |   |   |   |    |   |
| 7    |   |   |    |   |   |   |   |   |    |   |
| 8    |   |   |    |   |   |   |   |   |    |   |
| 9    |   |   |    |   |   |   |   |   |    |   |
| 10   |   |   |    |   |   |   |   |   |    |   |

VSD delay profile for a network with \( n \) links, \( 2 \leq n \leq 10 \), is derived from this table by taking the \( n \times n \) sub-matrix hinged at the top-left corner of this table. We show analysis plots of the \( LC-ELDR \) and \( LC-ERDMC \) policies only up to \#links = 5 since computing these values for networks with larger number of links is computationally costly. We note that the throughputs of both the \( LC-ELDR \) and \( LC-ERDMC \) policies overlap with the optimal throughput values of the \( R \) and \( H \) policies.

Fig. 6 shows the expected saturated system throughputs of the \( LC-ELDR \), \( LC-ERDMC \), \( O \) and \( IC \) policies for a network with VSVC channel profile and MD delay profile, for different number of links in the network. The MD delay profile for a network with ten links is shown in Table V. As before, the MD delay profile for a network with \( n \) links, \( 2 \leq n \leq 10 \), is derived from this table by taking the \( n \times n \) sub-matrix hinged at the top-left corner of this table. We do not show the system throughputs of the \( R \) and \( H \) policies in Fig. 6 since the MD delay profile is such that it is computationally hard to evaluate the saturated system throughputs of these policies. Again, as before, we show analysis plots of the \( LC-ELDR \) and \( LC-ERDMC \) policies only up to \#links = 5 since computing these values for networks with more links is computationally costly. We note that both of the \( LC-ELDR \) and \( LC-ERDMC \) policies outperform the \( O \) policy. We also note, more importantly, that there are situations (#links = 3, 4) where \( LC-ELDR \) outperforms \( LC-ERDMC \) and other situations (#links = 6 to 10) where \( LC-ERDMC \) outperforms \( LC-ELDR \), implying that none of these policies could be throughput optimal.  

Fig. 7 shows the expected saturated system throughputs of the \( R \), \( H \), \( LC-ELDR \) and \( LC-ERDMC \) polices for networks with four links, VSVC channel profile and different delay profiles (DP for short; see below). We first comment that the high computational cost of the \( R \) and \( H \) policies hinder us from measuring their performances for very many delay profiles, and for all the delay profiles that we could measure, the expected saturated system throughputs of the \( LC-ELDR \) and \( LC-ERDMC \) policies were less than 1.1% (and in many cases less than 0.5%) away from the optimal throughput values of the \( R \) and \( H \) policies, thus demonstrating the near-optimality of throughputs of the \( LC-ELDR \) and \( LC-ERDMC \) policies. The following are the definitions of the six delay profiles referred to in Fig. 7 (the entry at position \( i,j \) has the same interpretation as before):  

\[
\begin{align*}
DP_1 & : [0\ 1\ 1\ 1], \quad DP_2 : [0\ 1\ 1\ 2], \\
DP_3 & : [0\ 1\ 1\ 2], \quad DP_4 : [0\ 1\ 2\ 2] \\
DP_5 & : [0\ 2\ 2\ 2], \quad DP_6 : [0\ 4\ 4\ 4]
\end{align*}
\]

22With reference to the low-complexity scheduling policies in Sec. V in the process of obtaining a link that would maximize the expected system throughput, in each iteration, the \( LC-ELDR \) policy discards the link with the lowest expected data rate, and the \( LC-ERDMC \) policy discards the link that, upon its elimination, reduces the delay values for the largest number of channels (see step 10 of the algorithms). Even though taking recourse to eliminating one link in each iteration gives the \( LC-ELDR \) and \( LC-ERDMC \) policies their short running times, the approaches that these policies take to achieve this, as alluded to in Fig. 4, are not optimal. It remains unresolved as to whether there is an optimal strategy to isolate a link that would be eliminated in each iteration, and if one exists, what its structure should be. This needs further exploration.
TABLE VI: Comparison of the running times of the $R$, $LC$-ELDR and $LC$-ERDMC policies. All times are in microseconds.

| # Links | $R$ Policy $^\dagger$ | $LC$-ELDR Policy $^\ddagger$ | $LC$-ERDMC Policy $^\ddagger$ |
|---------|------------------|-----------------------------|-----------------------------|
| 5       | $4 \times 10^3$  | 11                          | 10                          |
| 10      | $1.1 \times 10^5$| 102                         | 96                          |
| 15      | $3.9 \times 10^6$| 165                         | 151                         |
| 20      | $8.14 \times 10^7$| 247                         | 237                         |

$^\dagger$ As reported in [1].

$^\ddagger$ Information about the delay profile used in [1] is not available. We are reasonably sure that the delay profile used in [1] uses smaller delay values than those in the MD delay profile that we have used here for the $LC$-ELDR and $LC$-ERDMC policies, since evaluating the $R$ policy for the MD delay profile is prohibitively time consuming. Thus the running time values that we mention here for the $LC$-ELDR and $LC$-ERDMC policies are relatively pessimistic, and provide a more-than-fair comparison.

$^\ddagger$ As measured on a computer with Intel Core 2 Duo processor clocked at 2 GHz, with 2 GB RAM.

$^\ddagger$ For the MD delay profile whose delay values for #links = 5, 10 are shown in Table VII, the delay values in the MD delay profile for #links = 15, 20 are similar to those shown in Table VII.

In Table VII, we compare the running times of the $R$, $LC$-ELDR and $LC$-ERDMC policies for a network with varying number of links, V SVC channel profile, and MD delay profile. Evidently, in stark contrast to the $R$ policy, the $LC$-ELDR and $LC$-ERDMC policies are computationally very efficient, validating the computational complexity analyses in Lemmas 2, 3 and 7.

VII. CONCLUSION

In this work, addressing the problem of distributed scheduling in wireless networks with heterogeneously delayed NSI, we proposed, analyzed and evaluated the performances of two fast and near-throughput-optimal scheduling policies. En route, we identified and dealt with two deficiencies (namely, non-optimal delay performance and high computational complexity) in an earlier work in [1]. We showed that the information afforded by the system model could be exploited more aggressively, and that by doing so, immense reduction in computational complexity and substantial reduction in the expected per-packet queueing delay could be obtained. We proposed a provably throughput-optimal scheduling policy that embodies these ideas. The proposed fast and near-throughput-optimal scheduling policies champion these ideas further, possess desirable queueing delay characteristics, and have running times that are of the order of microseconds, sufficiently small for use in practical deployments.

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Appendices

APPENDIX A

PROOF OF LEMMA 1

\( T_i(.) \) is a function of \( CS^R(.) \), and hence the number of choices of threshold functions for \( T_i(.) \) is a function of the number of random variables in \( CS^R(.) \), which in turn is dependent on \( CS^R(.) \) through Expr. (7). Evidently, the number of random variables in \( CS^R(.) \) is maximized when the delays in the table of delay values are all distinct.

The number of random variables in the set \( CS^R(.) \) pertaining to each link \( l_k \) (i.e., entries of the form \( C_{l_k}[t-\tau] \) in the set \( CS^R(.) \)) is maximized when \( \tau_i(l_k) \) is the smallest of all the delay values in row \( k \). This is because, the transmitter node of link \( l_1 \), having the CSI of link \( l_k \) with the delay \( \tau_i(l_k) \) (which is the smallest delay in row \( k \)), also has the CSI of link \( l_k \) with delays larger than \( \tau_i(l_k) \), and hence the random variables \( \{C_{l_k}[t-\tau] \}_{\tau=\tau_i(l_k),j=1,...,L,j\neq k} \) are all present in the set \( CS^R(.) \). Similarly, after fixing the delay value at position \((k,1)\) to be the smallest value in row \( k \) as noted above, the number of random variables in the set \( CS^R(.) \) pertaining to link \( l_k \) (i.e., entries of the form \( C_{l_k}[t-\tau] \) in the set \( CS^R(.) \)) is maximized when \( \tau_i(l_k) \) is the second-smallest of all the delay values in row \( k \) (second-smallest since, \( \tau_i(l_k) \) being the smallest in row \( k \), it can’t be that \( \tau_i(l_k) < \tau_i(l_k) \)).

Following the structure noted above, we rearrange all the delay values in each row \( i \) in ascending order (except the delay value of 0 at position \((i,i)\) which is left as is). We note that this structure on the delays in the table of delay values characterizes a worst-case scenario for the \( R \) policy. Finally, the structure on the table of delay values as needed in the statement of the lemma is obtained by swapping columns \( j \) and \( L-j+1 \) for \( j=1,...,\lfloor L/2 \rfloor \), which is legitimate since the transmitters are symmetric for purposes of calculating functional evaluation complexity.

\( \square \)

APPENDIX B

PROOF OF LEMMA 2

\( T_i(.) \) is a function of \( CS^R(.) \). Given the structure of worst-case delay values noted in Lemma 1, we see that for transmitter 1, the set \( CS^R(.) \) has a number of random variables that is the sum of the values in the \( L \)-tuple \((L-1, 1, \ldots, 1)\), since transmitter 1 has NSI of link \( l_1 \) with zero delay and hence possesses NSI of link \( l_1 \) with delays equal to all other \( L-1 \) values in row 1 of the delay table, and since transmitter 1 has NSI of link \( l_i \) \((2 \leq i \leq L)\) with a delay that is the maximum for link \( l_1 \) across all transmitters. Similarly, for transmitter 2, the set \( CS^R(.) \) has a number of random variables that is the sum of the values in the \( L \)-tuple \((1, L-1, 2, 2, \ldots, 2)\), since transmitter 2 has NSI of link \( l_1 \) with a delay that is the maximum for link \( l_1 \) across all transmitters, and since transmitter 2 has NSI of link \( l_2 \) with zero delay and hence possesses NSI of link \( l_2 \) with delays equal to all other \( L-1 \) values in row 2 of the delay table, and since transmitter 2 has NSI of link \( l_i \) \((3 \leq i \leq L)\) with a delay that is second-maximum for link \( l_1 \) across all transmitters. Generalizing, we see that for transmitter \( i \), the set \( CS^R(.) \) has a number of random variables that is the sum of the values in the \( L \)-tuple \((i-1, i-1, \ldots, i-1, L-1, i, i, \ldots, i)\) (where the term \( L-1 \) is at position \( i \) in the \( L \)-tuple). Also, given \( \{C_{l}[t-\tau]\}_{l \in \mathcal{L}} \), the number of random variables in each of the sets \( CS^R(.) \), \( 1 \leq i \leq L \), reduce by 1. Therefore, for each of the sets \( CS^R(.) \), \( 1 \leq i \leq L \), \( \tau_i(.) \) are respectively \((\lfloor L-1 \rfloor+1)-1\), \((1+ \lfloor L-1 \rfloor-2)\), and \((i-1)^2 + \lfloor L-1 \rfloor + i(L-1)\) which simplifies to \((i-1)^2 + (L-1)\).

Each of the random variables in \( CS^R(.) \) can take any of the \( C \) values where \( C \) is the number of channel states. Thus, for \( T_i(.) \), there are \( C^{(L-2)+\lfloor L-1 \rfloor} \) different values in the domain of \( T_i(.) \). Each of these \( C^{(L-2)+\lfloor L-1 \rfloor} \) in the domain of \( T_i(.) \) can be independently mapped to a real number. We see in Step 1 of the \( R \) policy (Expr. (9)), that the result of \( T_i(.) \) is compared with \( C_i[t] \) (and that of \( T_m(.) \) with \( C_m[t] \)). For any value of \( C_i[t] \), the comparison operation \( C_i[t] \geq T_i(.) \) can be made to evaluate either to false or to true by mapping each of the \( C^{(L-2)+\lfloor L-1 \rfloor} \) different values in the domain of \( T_i(.) \) independently to only two values in the range of \( T_i(.) \) – one value less than \( c_1 \) (say \( c_1 - 0.5 \)) and another greater than \( c_M \) (say \( c_M + 0.5 \)), where \( c_1 \) and \( c_M \) are such that \( C = \{c_1, c_2, \ldots, c_M\} \), \( c_1 < c_2 < \cdots < c_M \) is the state space of the channel DTMCS. Thus, to allow for all combinations of possibilities, we need only two values in the range of \( T_i(.) \). Therefore, there are \( 2^{C^{(L-2)+\lfloor L-1 \rfloor}} \) possible choices of threshold functions for \( T_i(.) \). Hence, the total number of threshold functions that are needed to be considered in the domain of optimization in Expr. (5) is...
The proof of this lemma is similar to the proof of Lemma 4.1 in [1]. We are given that {A[t]} is supportable. This implies that there exists a policy \( \mathcal{F} \) that has the following properties: (i) \( \mathcal{F} \) uses \( k_i \) time units delayed NSI of link \( l_i \) in making its scheduling decisions at time \( t \) (where \( \tau_{\max} \leq k_i \leq t \)), (ii) \( \mathcal{F} \) stabilizes the Markov chain \( Z^\mathcal{F}[t] = \{Q[t](0 : \tau_{k,m}), C[t](0 : \tau_{k,m})\} \), where \( Q = \{Q[t](0 : \tau_{k,m}), C[t](0 : \tau_{k,m})\}_{i \in \mathcal{L}}. \) We note that \( \mathcal{F} \) could possibly use queue-state information in making its scheduling decisions.

First, from \( \mathcal{F} \), we construct a new policy \( \mathcal{F}' \), on the system state Markov chain \( Y[t] = \{Q[t](0 : \tau_{k,m}), C[t](0 : \tau_{k,m})\}_{i \in \mathcal{L}}. \) We construct next \( \mathcal{F} \) as noted next. Consider \( z \in Z^\mathcal{F}[t] \) where \( z = \{q(\tau_{\max}+1 : k)q(0 : \tau_{\max}), c(\tau_{\max}+1 : k)c(0 : \tau_{\max})\} \), where \( q(\tau_{\max}+1 : k) := \{q_i(\tau_{\max}+1 : k_i)\}_{i \in \mathcal{L}}, \) \( q(0 : \tau_{\max}) := \{q_i(0 : \tau_{\max})\}_{i \in \mathcal{L}}, \) \( c(\tau_{\max}+1 : k) \) and \( c(0 : \tau_{\max}) \) are similarly defined. We “collapse” \( Z^\mathcal{F}[t] \) onto \( Y^\mathcal{F}[t] \) as follows. Let \( l \) be the time that \( \mathcal{F} \) schedules in state \( z \). Then \( z \) is collapsed onto state \( y \in Y^\mathcal{F}[t] \) where \( y = \{q(0 : \tau_{\max}), c(0 : \tau_{\max})\} \), with \( \mathcal{F}' \) scheduling link \( l \) in state \( y \) with probability \( \sum_z \hat{\pi}(z') \) where \( \hat{\pi}(z') \) is the stationary probability of being in state \( z' \) under the policy \( \mathcal{F} \). Let \( \hat{\pi}(y) = \sum_l \hat{\pi}(y,l) \). In Lemma 12 we show that the probability with which link \( l \) is scheduled in the policies \( \mathcal{F} \) and \( \mathcal{F}' \) is the same.

For \( y = (q,c) \), we let \( r(y) = \Pr(q,c) \) where \( \Pr(q,c) = \hat{\pi}(y) \). From \( \mathcal{F}' \), we construct a new policy \( \mathcal{F}_s \), whose scheduling decisions are independent of all queue-state information, on the system-state Markov chain \( Y[t] = \{Q[t](0 : \tau_{k,m}), C[t](0 : \tau_{k,m})\} \) at each time \( t \), when the delayed CSI \( C[t](0 : \tau_{\max}) = c \), the policy \( \mathcal{F}_s \) probabilistically schedules link \( l \) with probability \( \sum_y \hat{\pi}(y,l)/\hat{\pi}(y) \times r(y) \). In Lemma 13 we show that the probability with which link \( l \) is scheduled in the policies \( \mathcal{F}' \) and \( \mathcal{F}_s \) is the same. This would then imply that \( \mathcal{F}_s \) supports \( A[t] \). Since \( \mathcal{F}_s \) makes its scheduling decisions based on the delayed CSI \( C[t](0 : \tau_{\max}) = c \) (oblivious of queue state information), and since the service rates of all policies using the delayed CSI \( C[t](0 : \tau_{\max}) = c \) are considered in defining \( \Lambda \), we have \( E[A[t]] \in \Lambda \).

**Lemma 12.** The policies \( \mathcal{F} \) and \( \mathcal{F}' \) schedule a link \( l \) with the same probability.

Proof: Policy \( \mathcal{F} \) schedules link \( l \) with probability \( \sum_z \hat{\pi}(z) \), where \( z \) is all the states in \( Z^\mathcal{F}[t] \) such that \( \mathcal{F} \) schedules link \( l \) in state \( z \). Policy \( \mathcal{F}' \) schedules link \( l \) with probability \( \sum_y \sum_z \hat{\pi}(z') \), where \( z' \) is all the states in \( Z^\mathcal{F}[t] \) that “collapse” into state \( y \in Y^\mathcal{F}[t] \) such that \( \mathcal{F} \) schedules link \( l \) in 24we write \( Z^\mathcal{F}[t] \) as \( Z^\mathcal{F}[t] \) to improve legibility.
state $z'$. This quantity is equal to $\sum_{y \in Y} \pi(y,l)$ (alternatively, in state $y \in Y[t]$, the policy $\mathcal{F}'$ schedules link $l$ with probability $\hat{\pi}(y,l)/\pi(y)$, and hence, overall, policy $\mathcal{F}'$ schedules link $l$ with probability $\sum_{y} \hat{\pi}(y,l)/\pi(y) \times \pi(y) = \sum_{y} \hat{\pi}(y,l)$. The proof is complete by noting that $\sum_{z} \hat{\pi}(z) = \sum_{y} \hat{\pi}(y,l)$ since states $z$ include all states in $Z^{\mathcal{F}}[t]$ such that $\mathcal{F}$ schedules link $l$ in state $z$, not just those that “collapse” into a particular state $y \in Y^{\mathcal{F}}[t]$.

Lemma 13. The policies $\mathcal{F}'$ and $\mathcal{F}_s$ schedule a link $l$ with the same probability.

Proof: From the proof of Lemma 12 we see that the policy $\mathcal{F}'$ schedules link $l$ with probability $\sum_{y \in (q,c)} \hat{\pi}(y,l) \times r(y)$. Given that the delayed CSI $C[t](0 : \tau_{,\text{max}}) = c$, the policy $\mathcal{F}_s$ schedules link $l$ with probability:

$$\sum_{y \in (q,c)} \hat{\pi}(y,l) \times \pi(y) = \sum_{q} \hat{\pi}(y,l) \times \pi(y), \text{ where } y = (q,c).$$

$$= \sum_{q} \hat{\pi}(y,l) \times \pi(y) \times \text{Pr}(c) = \sum_{q} \hat{\pi}(y,l) \times \sum_{q} \pi(y) \times \text{Pr}(c) = \sum_{q} \hat{\pi}(y,l) \times \frac{\pi(y)}{\text{Pr}(c)} \times \text{Pr}(c) \times \sum_{q} \pi(y) \times \text{Pr}(c)$$

Now, averaging over all $c$, we see that the policy $\mathcal{F}_s$ schedules link $l$ with probability

$$\sum_{c} \sum_{q} \hat{\pi}(y,l) \times \text{Pr}(c) = \sum_{(q,c)} \hat{\pi}(y,l) = \sum_{y} \hat{\pi}(y,l).$$

APPENDIX H
PROOF OF THEOREM 1

Consider the optimization formulation

$$\arg \max_{\mathbf{F}_t} \sum_{l \in \mathcal{L}} Q_l[t - \tau_{,\text{max}}] R_{l,\tau_{,\text{max}}} (\mathbf{F}_t),$$

where

$$R_{l,\tau} (\mathbf{F}_t) := \mathbb{E}[C_l[t] F_l(.) (\gamma_l + (1 - \gamma_l) \prod_{m \in l} (1 - F_m(.) \mid C[t - \tau])],$$

$$C[t - \tau_{,\text{max}}] := \{ C_l[t - \tau_{,\text{max}}] \}_{l \in \mathcal{L}},$$

and

$$F_l : \mathcal{P}_l(C[t](0 : \tau_{,\text{max}})) \rightarrow \{0, 1\} \forall l \in \mathcal{L},$$

with the semantics that, at time $t$, the transmitter node of each link $l$ is allowed to transmit if and only if $F^*_l(P_l(C[t](0 : \tau_{,\text{max}}))) = 1$. From Lemma 14 the optimizing solution (namely, $\mathbf{F}^*_t(.) \in \mathcal{F}^*$) satisfies the threshold property $F^*_l(P_l(C[t](0 : \tau_{,\text{max}}))) = 1 \mathbb{1}_{[C_l[t] \geq \tau^*_l(P_l(C[t](0 : \tau_{,\text{max}})))]} \text{ from Lemma 15, the optimizing solution depends only on the critical set NST for each link } l, \text{ i.e. } T^*_l(P_l(C[t](0 : \tau_{,\text{max}}))) = T^*_l(CS_l(C[t](0 : \tau_{,\text{max}})))$, and from Lemma 16 the $H$ policy supports any mean arrival rate vector $\lambda$ that satisfies $(1 + \epsilon)\lambda \in \Lambda$ for any $\epsilon > 0$, but from Lemma 16 $\lambda$ is the region that encompasses all supportable arrival rates given the NSI structure in Sec. 11.

Lemma 14. $F^*_l(P_l(C[t](0 : \tau_{,\text{max}}))) = 1 \mathbb{1}_{[C_l[t] \geq \tau^*_l(P_l(C[t](0 : \tau_{,\text{max}})))]}$

Proof: The proof of this lemma is similar to the proof of Lemma 4.2 (part one) in [1], and is therefore omitted.

Lemma 15. $T^*_l(P_l(C[t](0 : \tau_{,\text{max}}))) = T^*_l(CS_l(C[t](0 : \tau_{,\text{max}})))$

Proof: Even though a proof similar to the proof of Lemma 4.2 (part two) in [1] applies, we give an alternate more intuitive proof here. First, consider the NSI of link $l_1$ at time $t - \tau$, where $\tau < r_{l_1,\text{min}}$, $\tau_{l_1,\text{min}} = \min_{l \in \mathcal{L}} r_{l_1}$. Since no transmitter has NSI of link $l_1$ at time $t - \tau$, none of the transmitters can make their thresholding decision based on $C_{l_1}[t - \tau]$, and therefore, $C_{l_1}[t - \tau]$ is absent from $CS_{l_1}(C[t](0 : \tau_{,\text{max}}))$ for all $l_1 \in \mathcal{L}$. Next, consider two links $l_i, l_j, i,j \neq 1$ such that $\tau_{l_i}(l_j) > \tau_{l_j}(l_i) + 1$ and $\tau_{l_i}(l_j)$ is the next larger delay value after $\tau_{l_i}(l_j)$ in the row corresponding to link $l_1$ in the table of delay values. Consider a delay value $\tau$ such that $\tau_{l_i}(l_j) < \tau < \tau_{l_j}(l_i)$, i.e. $\tau$ does not appear in the row corresponding to link $l_1$ in the table of delay values, or in other words, no transmitter has the NSI of link $l_1$ with delay $\tau$. Now, transmitter node of link $l_i$ does not need to make its thresholding decision based on $\tau$ since it has NSI of link $l_1$ with a delay smaller than $\tau$ (since $\tau_{l_i}(l_j) < \tau$) and the channel is Markovian. Therefore, $CS_{l_i}(C[t](0 : \tau_{,\text{max}}))$ need not contain $C_{l_i}[t - \tau]$. Lastly, consider two links $l_i, l_j, i,j \neq 1$ as above, and a delay value $\tau$ such that $\tau_{l_i}(l_j) < \tau = \tau_{l_j}(l_i)$. While it may appear that $CS_{l_i}(\cdot)$ need not contain $C_{l_i}[t - \tau]$ since $\tau_{l_i}(l_j) < \tau$ and the channel is Markovian, $C_{l_i}[t - \tau]$ is nonetheless required to be in $CS_{l_i}(\cdot)$ since transmitter node of link $l_i$ makes its thresholding decision based on $\tau$, and $C_{l_i}[t - \tau]$ helps in arbitrating when comparing the thresholding functions $T_{l_i}(\cdot)$ and $T_{l_j}(\cdot)$. From the above arguments it follows that the optimal solution does not depend on any channel state information that is not critical NSI.

Lemma 16. The $H$ policy supports any mean arrival rate vector $\lambda$ that satisfies $(1 + \epsilon)\lambda \in \Lambda$ for any $\epsilon > 0$.

Proof: We first define a Lyapunov function of the queue lengths available in the system state $Y^{\mathcal{F}}[t]$ as follows:

$$V[t] := V[Y^{\mathcal{F}}[t]] := \sum_{l \in \mathcal{L}} Q^*_l[t]$$

For a given arrival rate vector $\lambda$ that satisfies $(1 + \epsilon)\lambda \in \Lambda$ such that $\epsilon > 0$, we show that the expected change, from one time slot to the next, in the sum of the squares of the queue lengths at all the links in the network, is negative for all but a finite number of states. This implies, from Foster’s theorem (see [18]), that the system state Markov chain $Y^{\mathcal{F}}[t]$
is positive recurrent, establishing that the network remains stochastically stable for this arrival rate vector.

From Equation (15), we have
\[ E[ V[t + 1] - V[t] | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] = E[ \sum_{l \in L} (Q_l[t + 1] - Q_l[t])Q_l[t + 1] + Q_l[t]) | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

where \( Q[t - \tau_{\text{max}}] = \{ Q_l[t - \tau_{\text{max}}] | l \in L \} \) and \( C[t - \tau_{\text{max}}] = \{ C_l[t - \tau_{\text{max}}] | l \in L \} \)

Since in each time slot, the arrivals into a queue and departures out of this queue are both bounded, we have
\[ E[ \sum_{l \in L} (Q_l[t + 1] - Q_l[t])Q_l[t + 1] + Q_l[t]) | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

Next, using the update equation we show that
\[ E[ \sum_{l \in L} (Q_l[t + 1] - Q_l[t])Q_l[t + 1] + Q_l[t]) | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

where \( T^* \) is the optimal \( T \) resulting from the optimization formulation in the scheduling policy.

Using the update equation (Eq. (1)), we have
\[ Q_l[t + 1] - Q_l[t] \leq A_l[t] + S_l[t]. \]

Multiplying both sides of this inequality by \( 2Q_l[t - \tau_{\text{max}}] \) and taking conditional expectation with respect to \( Q[t - \tau_{\text{max}}] \) and \( C[t - \tau_{\text{max}}] \), we have
\[ E[ (Q_l[t + 1] - Q_l[t])2Q_l[t - \tau_{\text{max}}] | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

We note that the first term on the RHS is a constant, since the value of the term \( Q_l[t - \tau_{\text{max}}] \) is given in \( Q[t - \tau_{\text{max}}] \). We let \( k_l \) represent this constant, and also note that for the \( H \) policy, the factor \( S_l[t] \) in the second term is the same as \( R_l[t, \tau_{\text{max}}] (T^*) \). Therefore, we have
\[ E[ (Q_l[t + 1] - Q_l[t])2Q_l[t - \tau_{\text{max}}] | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

noting that expectation is a linear operator, we have
\[ E[ \sum_{l \in L} (Q_l[t + 1] - Q_l[t])2Q_l[t - \tau_{\text{max}}] | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

From Inequalities (16) and (19), we have
\[ E[ \sum_{l \in L} (Q_l[t + 1] - Q_l[t])Q_l[t + 1] + Q_l[t]) | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

That is,
\[ E[ V[t + 1] - V[t] | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] ] \]

From the optimization formulation of the \( H \) policy, we also have
\[ \sum_{l \in L} (E[ R_l[t, \tau_{\text{max}}] (T^*) | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] = c ] ) \]

This is because, \( T^* \) is an optimal threshold, and for a given \( c \), the term \( \bar{\eta}(c) \) in the RHS is the \( \ell \)th component of the point \( \bar{\eta}(c) \in \mathbb{R}^L \), and this point \( \bar{\eta}(c) \) arises from a specific weighted combination of \( T \)s (the weights being the fraction of times these \( T \)s are chosen), where some of these \( T \)s may not be optimal. The expectation term in the LHS results from giving a weight of 1 to \( T^* \). Also, given \( T^* \), the term \( Q[t - \tau_{\text{max}}] \) in the conditional expectation on the LHS is redundant.

Let \( K \) be an upper bound on the LHS of Inequality (24) such that this upper bound is at least twice as large as the value of the expression on the LHS. Then, the quantity \( K \) minus the value of the expression on the RHS serves as an upper bound on the value of the expression on the LHS. That is,
\[ \sum_{l \in L} (E[ R_l[t, \tau_{\text{max}}] (T^*) | Q[t - \tau_{\text{max}}], C[t - \tau_{\text{max}}] = c ] ) \]

(25)
From Inequality (23), we have
\[ \sum_{e \in C^L} \pi(e) (-\bar{\eta}(e)) \leq -(1 + \epsilon)\lambda_t \]
(26)

Taking expectation on both sides of Inequality (25) over \( C[t - \tau_{\text{max}}] \), and noting that expectation is a linear operator, we get
\[
\sum_{e \in C^L} \{ (E) \sum_{l \in I} R_{l, \tau_{\text{max}}} (T^*) \mid Q[t - \tau_{\text{max}}], \]
\[ C[t - \tau_{\text{max}}] = e \} \times (2Q[t - \tau_{\text{max}}]) \} \pi(e) \]
\[ \leq \bar{K} + \sum_{l \in I} \{ (2Q[t - \tau_{\text{max}}]) \} \sum_{e \in C^L} \pi(e) (-\bar{\eta}(e)) \]
\[ \leq \bar{K} + \sum_{l \in I} \{ (2Q[t - \tau_{\text{max}}]) \} (- (1 + \epsilon)\lambda_t) \text{ (using (26))} \]
\[ \leq \bar{K} - 2\bar{\epsilon} \sum_{l \in I} \{ Q[t - \tau_{\text{max}}] \} \lambda_t \]
(27)

That is,
\[ \sum_{\in C^L} \{ (E) \sum_{l \in I} R_{l, \tau_{\text{max}}} (T^*) \mid Q[t - \tau_{\text{max}}], \]
\[ C[t - \tau_{\text{max}}] = e \} \times (2Q[t - \tau_{\text{max}}]) \} \pi(e) \]
\[ \leq \bar{K} - 2\bar{\epsilon} \sum_{l \in I} \{ Q[t - \tau_{\text{max}}] \} \lambda_t \]
(28)

Since \( K \) is a constant, the RHS in the above inequality is greater or equal to zero only for a finite number of states in the system state Markov chain \( Y^F[t] \), and lesser or equal to zero for the rest of the states in \( Y^F[t] \). It now follows from Foster’s theorem (see [18]), that the system state Markov chain \( Y^F[t] \) is positive recurrent. Hence the \( H \) policy preserves the stochastic stability of the network for the given arrival rate vector \( \lambda \). Since \( \Lambda \) is any arbitrary vector in \( \Lambda \) such that \( (1 + \epsilon)\lambda \in \Lambda \), it follows that the \( H \) policy preserves the stochastic stability of the network for all arrival rate vectors in the interior of \( \Lambda \).

Appendix I

Proof of Lemma 11

We first count the number of comparison operations in steps 4, 6, 7, 16, and 24 when the number of links in \( \text{ActiveSet} \) is \( n \). We note that \( \text{ActiveSet} \) has \( L \) links in the first round, and that its size reduces by 1 in each subsequent round. In step 4 there are \( n - 1 \) comparisons for each of the \( n \) links in \( \text{ActiveSet} \). Therefore, the total number of comparisons in step 4 (for all rounds together) is \( L(L - 1) + (L - 1)(L - 2) + \ldots + 2^2 = O(L^3) \). Step 6 can be accomplished using \( n - 1 \) comparisons yielding a total of \( (L - 1) + (L - 2) + \ldots + 2 = O(L^2) \) comparisons. In step 7 after suppressing the row and column of any one of the \( n - 1 \) links in \( \text{ActiveSet} \setminus \{ H \} \), we need \( n - 2 \) comparisons for each of the remaining \( n - 1 \) links in \( \text{ActiveSet} \setminus \{ K \} \) to compute the values \( \{ \tau_{\text{max}} \} \text{ActiveSet} \setminus \{ K \} \) where \( K \) is the link whose row and column we suppress. We further need another \( n - 1 \) comparisons in the worst case to find whether eliminating this link reduces the delay for at least one channel, by comparing the \( n - 1 \) newly computed delays with those computed earlier in step 4. This gives us \((L - 1)((L - 1)(L - 2) + (L - 1)) + (L - 2)((L - 2)(L - 3) + (L - 2)) + \ldots + 3(3.2 + 3) + 2(2.1 + 2) = O(L^4) \) comparisons.

In the \( LC-ELDR \) policy, the set \( EC \) has \( n - 1 \) elements in the worst case, and hence step 16 can be accomplished with \( n - 2 \) comparisons for a total of \( (L - 2) + (L - 3) + \ldots + 1 = O(L^2) \) comparisons. In step 16 of the \( LC-ERDMC \) policy, the set \( EC \) has \( n - 1 \) elements in the worst case, and hence, as in step 7 after suppressing the row and column for each link in \( EC \), we need \( n - 2 \) comparisons for each of the remaining \( n - 1 \) links in \( EC \) to compute the values \( \{ \tau_{\text{max}} \} \text{ActiveSet} \setminus \{ K \} \). We further need another \( n - 1 \) comparisons to find the number of links for which the newly computed delays are lesser than the corresponding delays computed earlier in step 4. This gives us \((L - 1)((L - 1)(L - 2) + (L - 1)) + (L - 2)((L - 2)(L - 3) + (L - 2)) + \ldots + 3(3.2 + 3) + 2(2.1 + 2) = O(L^4) \) comparisons. Step 24 contributes a constant to the runtime since there would only be two links left in \( \text{ActiveSet} \).

Next, we count the number of multiplications and additions required in steps 5 and 24 in computing the queue-length weighted conditional expected values. Evaluation of the conditional expectation for each of the \( n \) links in \( \text{ActiveSet} \) requires \( C \) multiplications and \( C \) additions, and one additional multiplication is required for multiplying with queue-length. Hence, in all, \( L(2C^2 + 1) + (L - 1)(2C^2 + 1) + \ldots + 2(C^2 + 1) = O(CL^2) \) multiplications and additions are required. Therefore, the cost of comparisons, multiplications and additions together is \( O(CL^2 + L^4) \).

Appendix J

Proof of Lemma 9

First, we need to set up some notations. Let \( N \) be the number of links in \( \text{ActiveSet} \) and \( T := \{1, 2, \ldots, N\} \) be the set of links in \( \text{ActiveSet} \) before the call to Algorithm 1 (recall that in the case of a network with complete interference, and for the first call to Algorithm 1 in the case of a network with multiple interference sets, \( \text{ActiveSet} \) will have all the links in the network). Let \( \delta_{ij} \) for \( 1 \leq i, j \leq N \) be the delay value in row \( i \) and column \( j \) of the delay table (note that \( \delta_{ii} = 0 \) \( \forall i \) in our model). Let \( c_{k,T} \) be the realization of channel-state on link \( k \) at time \( t - \tau \). Further, let
\[ 1\{a\} := \begin{cases} 1 & \text{if } a \text{ is true} \\ 0 & \text{otherwise} \end{cases} \]
\[ 1\{a_i\}_{i \in I} := \prod_{i \in I} 1\{a_i\} \]
\[ 1 + 1\{a_i\}_{i \in I} := \max_{i \in I} \{1\{a_i\}\}_{i \in I} \]
\[ \mathbb{E}(k, \tau_k) := \mathbb{E}\left[C_{l,k} | C_{l,k} - t - \tau_k = c_{k,T}\right] \]
We will call the “while” loop body from line 3 to line 23, and also the computation in lines 24–29 in Algorithm 7 in Sec. V a “round”. Thus, if the \( LC-ELDR \) policy terminates
the set of contending links, an
loop, and the last round [round 1
Note that
r
and columns pertaining to the links that have been eliminated
the required analytical expression for the expected saturated
system throughput of the LC-ELDR policy. Let \( i \in T \) be
the link chosen by the LC-ELDR policy in a particular time
slot, say \( t \). We now consider the working of the LC-ELDR
policy when it is executed at the transmitter node of link \( i 
in time slot \( t \). From the listing of the LC-ELDR policy in
Sec. \( \text{V} \) we see that link \( i \) can emerge as the “winner” (i.e.,
as the link chosen by the LC-ELDR policy for carrying
transmission) in time slot \( t \) in two ways – (i) from lines
25–26 (i.e., round \( N-1 \) is the last round), or (ii) from lines
8–10, 14 (i.e., an intermediate round \( r < N-1 \) is the last
round). Thus, link \( i \) can emerge as the “winner” when the
last round is any of 1 to \( N-1 \). Now, link \( i \) will emerge as
the “winner” when the last round is \( r, \ 1 \leq r \leq N-1 \), if
it “survives” (i.e., if it not eliminated in) round 1, survives
round 2, ..., survives round \( r \).

Let us first fix a particular last round \( r \), \( 1 \leq r \leq N-1 \)
(i.e, the LC-ELDR policy runs from round 1 to round \( r 
and terminates immediately after round \( r \)). When the last
round is \( r \), if link \( i \) emerges as the “winner”, its contribution to
the total expected saturated system throughput is given by

\[
\sum_{j \in T} \sum_{q \in \{1,2,\ldots,r\}} \mathbb{E} \left[ C_{i,t} \right] \times \prod_{r=1}^{r-1} \left( 1 \{ \text{link } i \text{ is not eliminated in round } \tilde{r} \} \right) \times 1 \{ \text{round } \tilde{r} \text{ is not the last round} \} \times 1 \{ \text{link } i \text{ is not eliminated in round } r \} \times 1 \{ \text{round } r \text{ is the last round} \} \times \Pr \left\{ C_{i,t} \left[ t - \tau_i^{(1)} \right] = c_{i,\tau_i^{(1)}} \right\} \times \Pr \left\{ C_{i,t} \left[ t - \tau_i^{(2)} \right] = c_{i,\tau_i^{(2)}} \right\} \times \ldots \times \Pr \left\{ C_{i,t} \left[ t - \tau_i^{(r)} \right] = c_{i,\tau_i^{(r)}} \right\}.
\]

Now, consider a particular round \( \tilde{r} \) of the LC-ELDR policy.

If link \( i \) has survived (i.e., has not been eliminated in) the
first \( \tilde{r} \)-1 rounds, than it will survive round \( \tilde{r} \) if one of the

at line 26 or at line 28, then it would have executed \( N-1 \)
rounds (specifically, \( N-2 \) rounds in the body of the “while”
loop, and the last round [round \( N-1 \)] in lines 24–29).

Let us call the link that has not yet been eliminated from
the set of contending links, an active link. Let \( M(r) \) be the
active link with the largest expected data rate in round \( r \). That
is,

\[
M(r) := \arg \max_{k \in T \setminus \left\{ \bigcup_{1 \leq n \leq r-1} e(n) \right\}} \left\{ \mathbb{E}(k, t_k(r)) \right\},
\]

where \( e(r) \) and \( t_k(r) \) are as defined below.

Let \( e(r) \) be the link other than \( M(r) \) that has the smallest
expected data rate among all links that reduce the common
delay value with which the channel state of at least one link can
be accessed by the transmitter nodes of all the contending
links, if this link is eliminated in round \( r \) (i.e., among all
links that reduce the maximum of the delay values in a row
for at least one row [corresponding to an active link] in the
delay values [after suppressing the rows and columns
corresponding to links that have been eliminated up to round
\( r \), if this link is eliminated in round \( r \)]. Equivalently, \( e(r) \)
is the link that was eliminated in round \( r \) (if \( r \) is less than
the current round number), or the link that will be eliminated
in round \( r \) (if \( r \) is equal to the current round number). We
can write \( e(r) \) as in Expr. (29). The computation of \( e(r) \) is
illustrated in Figs. 2(b) – (e) for \( r = 1 \), and in Figs. 2(f) – (h)
for \( r = 2 \), for the setting in Sec. \( \text{V-A} \). Note that the condition
“\( e(r) \neq 0 \)” indicates that some link (other than the one with
the largest expected data rate in round \( r \) is eliminated in round
\( r \), and hence that the LC-ELDR policy can proceed to round
\( r+1 \) (i.e., round \( r \) is not the last round).

We let \( \tau_j^{(r)} \) (not to be confused with \( \tau_j(h) \) defined in Sec. \( \text{II-D} \)) be equal to \( \tau_{j,\max} \) (see Sec. \( \text{II-D} \)) at the beginning of
round \( r \), where \( \tau_{j,\max} \) is calculated after masking the rows and columns pertaining to the links that have been eliminated in rounds \( 1 \) to \( r-1 \) (as illustrated in Figs. 2(a), 2(e), and
2(h)). That is,

\[
\tau_j^{(r)} := \max_{k \in T \setminus \{ j \} \cup \left\{ \bigcup_{1 \leq n \leq r-1} e(n) \right\}} \left\{ \delta_{jk} \right\}
\]

Note that \( \tau_j^{(1)} \geq \tau_j^{(2)} \geq \cdots \geq \tau_j^{(N-1)} \).

With these definitions in place, we are now ready to derive
the required analytical expression for the expected saturated
following happens:

1) link \( i \) has the largest expected data rate in round \( \tilde{r} \) (hence \( i = H \) and therefore \( i \notin EC \); recall that we set aside the link with the largest expected data rate in each round so that it will not be eliminated in that round [see steps 6 and 7 of Algorithm \([I]\)]. We denote this condition as \( s(i, \tilde{r}, 1) \).

2) link \( i \) does not have the largest expected data rate in round \( \tilde{r} \) (hence \( i \neq H \)), and link \( i \) does not reduce the maximum of the delay values in a row, for any row (corresponding to a link in ActiveSet) in the table of delay values (after suppressing the rows and columns corresponding to links that have been eliminated up to round \( \tilde{r} \)) if it is eliminated in round \( \tilde{r} \) (hence \( i \notin EC \); see step 7 of Algorithm \([I]\)), and some link (other than the one with the largest expected data rate in round \( \tilde{r} \)) reduces the maximum of the delay values in a row, for at least one row (i.e. link) if it is eliminated in round \( \tilde{r} \) (i.e., \( EC \neq \phi \)). We denote this condition as \( s(i, \tilde{r}, 2) \).

3) link \( i \) does not have the largest expected data rate in round \( \tilde{r} \) (hence \( i \neq H \)), and link \( i \) reduces the maximum of the delay values in a row, for at least one row (corresponding to a link in ActiveSet) in the table of delay values (after suppressing the rows and columns corresponding to links that have been eliminated up to round \( \tilde{r} \)) if it is eliminated in round \( \tilde{r} \) (hence \( i \in EC \), and link \( i \) does not have the smallest expected data rate among the links that reduce the maximum of the delay values in a row, for at least one row (i.e. link) if that link is eliminated in round \( \tilde{r} \) (hence \( i \neq S \); see step 16 of Algorithm \([I]\)). We denote this condition as \( s(i, \tilde{r}, 3) \).

Now, \( s(i, r, 1) \) can be written mathematically as:

\[
1\{ s(i, r, 1) \} = \begin{cases} 1 & \text{if } s(i, r, 1) \text{ holds} \\ 0 & \text{otherwise} \end{cases}
\]

\[
1\left\{ \mathbb{E}(i, \tau^r_i) > \max_{k \in T \backslash \bigcup_{1 \leq n \leq r-1} e(n)} \{ \mathbb{E}(k, \tau^r_k) \} \right\} \times 1\left\{ \delta_{ji} \leq \max_{m \in T \backslash \{i,j\}} \{ e(m) \} \right\} \times 1\left\{ e(r) \neq 0 \right\}
\]  \hspace{1cm} (31)

In the expression for \( s(i, r, 1) \) above, we note that splitting the comparison of the expected data rates into two – namely, (i) comparison with the (maximum of the expected data rates of) transmitters that are less than \( i \), and (ii) comparison with transmitters that are greater than \( i \), creates a lexicographic ordering among the transmitters. This is required to consistently resolve the “winner” in case there is a tie in the expected data rates of multiple transmitters – we always resolve in favor of the smallest numbered transmitter (as a convention) in case of a tie.

Next, \( s(i, r, 2) \) can be written mathematically as:

\[
1\left\{ \mathbb{E}(i, \tau^r_i) > \max_{k \in T \backslash \bigcup_{1 \leq n \leq r-1} e(n)} \{ \mathbb{E}(k, \tau^r_k) \} \right\} \times 1\left\{ \delta_{ji} > \max_{m \in T \backslash \{i,j\}} \{ \delta_{jm} \} \right\} \times 1\left\{ e(r) \neq 0 \right\}
\]  \hspace{1cm} (32)

Finally, \( s(i, r, 3) \) can be written mathematically as:

\[
1\left\{ \mathbb{E}(i, \tau^r_i) \leq \max_{k \in T \backslash \bigcup_{1 \leq n \leq r-1} e(n)} \{ \mathbb{E}(k, \tau^r_k) \} \right\} \times 1\left\{ \delta_{ji} \leq \max_{m \in T \backslash \{i,j\}} \{ e(m) \} \right\} \times 1\left\{ e(r) \neq 0 \right\}
\]  \hspace{1cm} (33)

From \([I], (31), (32) \) and \((33)\), the expression for the contribution of link \( i \) to the total expected saturated system throughput when link \( i \) emerges as the “winner” when round \( r, 1 \leq r \leq N - 1 \), is the last round becomes

\[
\sum_{q \in \{1,2,...,r\}} \sum_{j=r_{q-1}}^{r_q} \sum_{i=1}^{N} \mathbb{E}(i, \tau^r_i) \left[ \mathbb{E}(i, \tau^r_i) \right]
\]

where \( \pi_m(c) \) is the steady-state probability of being in state \( c \) in the channel-state Markov chain of link \( m \), and \( \tau^r_{m,ij} \) is the probability of reaching state \( j \) from state \( i \) in \( k \) steps in the channel-state Markov chain of link \( m \).

Now, summing over all possible last rounds \( r = 1 \) to \( N - 1 \), and over all links \( i = 1 \) to \( N \), we get the expression for the expected saturated system throughput of the \( LC-ELDR \)

\[25 \pi_m(c) = \pi(c) \text{ and } p_m^{(k)} \]
Appendix J, we need the following notations: addition to carrying over all the notations we established in Lemma 9, except that we need to redefine $LC$ of the $M$ other than $E$.

For the $LC$ channel state of a link can be accessed by the transmitters all links that reduce the common delay value with which the $LC$ expression for the expected saturated system throughput

$\sum_{i=1}^{N} \sum_{r=1}^{N-1} C_i [t] \prod_{\tau=1}^{r-1} \{ 1 \{ s(i, r, n) \} \}_{n \in \{1,2,3\}}$

$\times 1 \{ e^{(r)} \neq 0 \} 1 \{ s(i, r, 1) \} 1 \{ e^{(r)} = 0 \}$

$\left\{ C_i \left[ t - \tau_i^{(r)} \right] = c_{i,\tau_i^{(r)}} \right\}_{q \in \{1,2,\ldots, r-1\}}$

$\left\{ C_i \left[ t - \tau_i^{(q)} \right] = c_{i,\tau_i^{(q)}} \right\}_{p \in T \setminus \{ i \}}$

$\times \prod_{m=1}^{N} \left( p_{m,\tau_m^{(n)}} \prod_{\tau=1}^{r-1} \{ p_{m,\tau_m^{(n)}} - p_{m,\tau_m^{(n+1)}} \} \right)$

as required.

APPENDIX K

PROOF OF LEMMA 10

The expression for the expected saturated system throughput of the $LC$ policy is the same as the expression in Lemma 9 except that we need to redefine $e^{(r)}$. For that, in addition to carrying over all the notations we established in Appendix 9 we need the following notations:

$1_{\infty} \{ a \} := \begin{cases} 1 & \text{if } a \text{ is true} \\ \infty & \text{otherwise} \end{cases}$

$s_+ \{ a_i \} \in I := \sum_{i \in I} 1 \{ a_i \}$

$s_+ \{ k, r \} := s_+ \{ \delta_{jk} > m \} \max_{m \in T \setminus \{ j, k, 1 \leq n \leq r-1 \}} \{ e^{(n)} \}_{m \in T \setminus \{ j, 1 \leq n \leq r-1 \}}$

For the $LC$ policy, we redefine $e^{(r)}$ to be the link other than $M^{(r)}$ that has the smallest expected data rate among all links that reduce the common delay value with which the channel state of a link can be accessed by the transmitters.

\[ e^{(r)} := \begin{cases} 1 \{ e^{(r)} \neq 0 \} 1 \{ s(n, r, 1) \} 1 \{ e^{(r)} = 0 \} \\ \prod_{m=1}^{N} \left( p_{m,\tau_m^{(n)}} \prod_{\tau=1}^{r-1} \{ p_{m,\tau_m^{(n)}} - p_{m,\tau_m^{(n+1)}} \} \right) \end{cases} \]