ABOUT LIMIT MASSES OF ELEMENTARY PARTICLES

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Abstract
The simple examples of spontaneous breaking of various symmetries for the scalar theory with fundamental mass have been considered\textsuperscript{1}. Higgs’ generalizations on ”fundamental mass” that was introduced into the theory on a basis of the five-dimensional de Sitter space. The connection among ”fundamental mass”, ”Planck’s mass” and ”maximons” has been found. Consequently, the relationship among G- gravitational constant and other universal parameters can be established.

Key words: Quantum field theory, fundamental mass, spontaneous breaking, ultra-high energies.

1. Introduction

The concept of mass having its root from deep antiquity (including Galileo’s Pisans experiment, theoretical research of the connection of a mass with the Einstein’s energy etc.) still remains fundamental. Every theoretical and experimental research in classical physics and quantum physics associated with mass is a step to the discernment of Nature. Besides mass, the other fundamental constants such as Plank’s constant and the speed of light also play the most important role in the modern theories. The first one is related to quantum mechanics and the second one is related to the theory of relativity. Nowadays the properties and interactions of elementary particles can be described more or less adequately in terms of local fields that are affiliated with the lowest representations of corresponding compact groups of symmetry. It is known that mass of any body is composed of masses of its comprising elementary particles. The mass of elementary particles is the Kasimir’s operator of the noncompact Paunkare group, and those representations of the given group, that are being used in Quantum Field Theory (QFT), and it can take

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any values in the interval of . Two particles, today referred to as elementary particles, can have masses distinct one from another by many orders. For example, the vectorial bosons with the mass of $\sim 10^{15}$ GeV take place in GUT modules, whereas the mass of an electron is only $\sim 10^{-4}$ GeV. Formally, the standard QFT remains logical in a case when the mass of particles can be compared with the mass of automobiles. The modern QFT does not forbid such a physically meaningless extrapolation. Perhaps, it is a principal defect of the theory? In 1965 Markov put forward a hypothesis [1], according to which the spectrum of masses of elementary particles must jump into discontinuity on the ”Plank’s mass”, well-known universal constants, and G-gravitational constant take place in this expression. Markov named particles of a limiting mass as ”maximums”. In works V.G.Kadyshevsky and his students[2, 3] in the quantum theory of a field on a strict mathematical basis the parameter ”fundamental mass” which alongside with parameters of the standard quantum theory to play an essential role in physics at high energy is entered. In the given work, we have strived for the connection of the universal constants ($\hbar, c, G$), ”Plank’s mass”, ”maximon” and ”fundamental mass” on a basis of the spontaneous breaking of symmetry.

2. On Quantum Field Theory with a new parameter fundamental mass

Here we will cite briefly the major ideas of quantum field theory (QFT), in which the four-dimensional momentum space is a constant curvature space with a large radius $M$.

The construction of the sequential Quantum Field Theory (QFT) with a new, universal scale in the superhigh energy region- Fundamental Mass $M$ emanates to V.G. Kadyshevsky’s work [2] and his students [3]. The parameter $M$ is called fundamental mass and its inverse fundamental length. The fundamental mass fixes a new universal scale of theory in the high-energy region. The standard QFT is recovered in the ”flat limit” . Recall that fundamental mass $M$ is a new hypothetical parameter of mass dimension, which should be as universal as or Newtonian gravitational constant $G$ and serve as characteristic scale in the region of high energies. A key role in the approach developed belongs to the five-dimensional configuration representation. Being four-dimensional in its essence the theory admits a specific local Lagrangian formulation in which the dependence of fields on auxiliary fifth coordinate is local too. Internal symmetries in this formalism generate gauge transformations localized in the same five-dimensional configuration space. The de Sitter space has a constant curvature. Depending on its sign there are two possibilities The de Sitter space has a constant curvature. Depending on its sign there are two possibilities
\[ p_0^2 - p_1^2 - p_2^2 - p_3^2 + p_5^2 \equiv g^{KL}P_KP_L = M^2; \]
\[ K, L = 0, 1, 2, 3, 5 \]
(positive curvature : \( g^{00} = -g^{11} = -g^{22} = -g^{33} = g^{55} = 1 \))
\[ p_0^2 - p_1^2 - p_2^2 - p_3^2 - p_5^2 \equiv g^{KL}P_KP_L = -M^2; \]
\[ K, L = 0, 1, 2, 3, 5 \]
(negative curvature : \( g^{00} = -g^{11} = -g^{22} = -g^{33} = -g^{55} = 1 \)).

It is natural that QFT based on momentum representation of the form (1)-(2) must predict new physical phenomena at energies. In principle, the parameter M may turn out to be close the Planck mass GeV. Then, the new scheme should include quantum gravity. The standard QFT corresponds to the "small" 4-momentum approximation and which formally can be performed by letting ("flat limit"). The formulation of QFT with fundamental mass discussed in this paper is based on the quantum version of the de Sitter equation (2), i.e. on the five-dimensional field equation

\[ \left[ \frac{\partial^2}{\partial x^\mu \partial x_\mu} - \frac{\partial^2}{\partial x_5^2} - \frac{M^2c^2}{\hbar^2} \right] \Phi(x, x^5) = 0 \]
\[ \mu = 0, 1, 2, 3. \]  
(3)

All the fields independently of their tensor dimension must obey (3) since similar universality was inherent in the "classical" prototype of (3)- de Sitter p-space (2). As applied to scalar, spinor, vector and other fields we shall write down the five-dimensional wave function \( \Phi(x, x^5) \) in the form \( \varphi(x, x^5), \psi_\alpha(x, x^5) \) and \( A_\mu = (x, x^5) \) and the decision (3) in view of the relevant Cauchy problem, which is correctness on \( x^5 \), looks like:

\[ \Phi(x, x^5) \leftrightarrow \left( \begin{array}{c} \Phi(x, 0) \\ \frac{\partial \Phi(x, 0)}{\partial x^5} \end{array} \right) \equiv \left( \begin{array}{c} \Phi(x) \\ \chi(x) \end{array} \right). \]  
(4)

In other words, the statement that to each field in the 5-space there corresponds its wave function \( \Phi(x, x^5) \) obeying (3), implies that each of these fields in the usual space-time is described by the wave function with a doubled number of components. Then, it is natural to assume that the initial date (4) obey the Lagrangian equations of motion following from the action principle

\[ S = \int d^4xL \left[ \Phi(x, 0), \frac{\partial \Phi(x, 0)}{\partial x^5} \right]. \]  
(5)

A key role in the approach developed belongs to the five-dimensional configuration representation. Being four-dimensional in its essence the theory admits a specific local lagrangian formulation in which the dependence of fields on an auxiliary fifth coordinate is...
local too. Internal symmetry’s in this formalism generate gauge transformations localized in the same five-dimensional configuration space.

3. Calculations

Let’s consider simple examples of spontaneous breaking of various symmetries for the scalar theory with a fundamental mass. The real scalar field, whose Lagrangian can be obtained from the beneath expression:

\[ L(x, M) = \frac{1}{2} \left[ \left( \frac{\partial \Phi(x)}{\partial x_n} \right)^2 + m^2 \Phi^2(x) + M^2 \left[ \chi(x) - \cos \mu \Phi(x) \right]^2 \right] \]  

Using (4) and it can be expressed as follows:

\[ L(x, M) = \frac{1}{2} \left[ \frac{\partial \varphi(x)}{\partial x_\mu} \right]^2 - \frac{1}{2} m^2 \varphi(x)^2 - \frac{1}{2} M^2 (\chi(x) - \cos \mu \varphi(x))^2 - \chi(x) U(\varphi(x)) \]  

Here \( \cos \mu = \sqrt{1 - \frac{m^2}{M^2}} \) where \( m \) - mass of particles, described by fields \( \varphi, U(\varphi) \) - an unknown function characterizing interactions among these particles. Is it possible to select the interaction \( L_{int} \) between the fields \( \varphi \) and \( \chi \) in order that under the exclusion of the field \( \chi \), the Higgs potential is appropriate to the field \( \varphi \)?

The free Lagrangian is invariant relatively the transformations \( \varphi \rightarrow -\varphi \) and \( \chi \rightarrow -\chi \). It is necessary to require that \( U(\phi) \rightarrow U(-\phi) \), i.e. \( U(\phi) \)-an odd function of \( \varphi \), an action for (7) will be written as:

\[ S(x, M) = \int \left\{ \frac{1}{2} \left[ \frac{\partial \varphi(x)}{\partial x_\mu} \right]^2 - \frac{1}{2} m^2 \varphi(x)^2 - \frac{1}{2} M^2 (\chi(x) - \cos \mu \varphi(x))^2 + \chi(x) U(\varphi(x)) \right\} d\varphi \]

Taking a derivative of (8) on \( \chi \) we obtain:

\[ \chi = \cos \mu \varphi + \frac{U(\varphi)}{M^2} \]

Substituting (9) to (7), we have:

\[ L_{tot} = \frac{1}{2} \left[ \left( \frac{\partial \varphi(x)}{\partial x_\mu} \right)^2 - m^2 \varphi(x)^2 + \frac{U^2(\varphi)}{M^2} + 2U(\varphi(x)) \cos \mu \varphi(x) \right] \]

that is invariant relatively \( \varphi \rightarrow -\varphi \). We know from spontaneous breaking of the discrete symmetry for the usual scalar field, that the Higgs potential will have a view:

\[ V(\varphi) = -\frac{1}{2} m^2 \varphi(x)^2 + \frac{1}{4} \lambda^2 \varphi(x)^4 \]
where $\lambda$ - a mass less constant, characterizing interactions among particles.

Let’s find a form of the function $U(\varphi)$, in order for the Higgs potential to figure in (10). Let’s consider the Lagrangian (10) under $m \to -m$, then

$$\cos \mu = \sqrt{1 - \frac{m^2}{M^2}} \to \sqrt{1 + \frac{m^2}{M^2}}$$

The potential energy (11) will have a view:

$$V(\varphi) = -\frac{1}{2} m^2 \varphi(x)^2 - \frac{U^2}{2M} - U(\varphi) \cos \mu \varphi$$

Comparing (12) and (11) for $U(\varphi)$ we have two different roots (real and imaginary) under

$$\varphi^2 < \frac{2 M^2 \cosh^2 \mu}{\lambda^2}$$

and

$$U = -M^2 \cosh^2 \mu \varphi$$

under

$$\varphi^2 = \frac{2 M^2 \cosh^2 \mu}{\lambda^2}$$

This results in this that $L_{\text{tot}}(\varphi)_{\text{Higgs}} = L_{\text{maximon}}^0(\varphi)$ i.e.

$$L_{\text{maximon}}^0(\varphi) = \frac{1}{2} (\frac{\partial \varphi}{\partial x_\mu})^2 - \frac{1}{2} M^2 \varphi^2$$

Now let’s consider a case when

$$L_{\text{tot}}(\varphi) = -\frac{\lambda^2}{4} \varphi^2 \chi^2$$

Under $m \to -m$, the Lagrangian (7) will have a view:

$$L_{\text{tot}}(\varphi) = \frac{1}{2} (\frac{\partial \varphi}{\partial x_\mu})^2 - M^2 \sinh^2 \mu \varphi^2 - M^2 (\chi - \cosh \mu \varphi)^2 - \lambda^2 \varphi^2 \chi^2$$

(14)

Taking a derivative of (14) $\chi$ on

$$\chi = \frac{M^2 \cosh \mu \varphi}{M^2 + \frac{\lambda^2}{2} \varphi^2 \chi^2}$$

we obtain:

$$L_{\text{tot}}(\varphi) = \frac{1}{2} (\frac{\partial \varphi}{\partial x_\mu})^2 + M^2 \sinh^2 \mu \varphi^2 - \frac{\lambda^2}{2} \varphi^2 (\frac{\lambda^2 \varphi^2}{2M^2} + 1) \frac{M^4 \cosh^2 \mu \varphi^2}{(M^2 + \frac{\lambda^2}{2} \varphi^2 \chi^2)^2}$$

(15)
This is one of the Higgs’ generalizations on a fundamental mass. We will obtain the usual Higgs’ Lagrangian from (15) under \( M \to \infty \):

\[
\lim_{M \to \infty} L_{\text{tot}}(\phi) = \frac{1}{2}[(\partial \phi / \partial x_\mu)^2 + m^2 \phi^2 - \frac{\lambda^2 \phi^4}{2}]
\] (16)

If taking into account in (11) the strong connection \( \lambda \to \infty \) then we come to a Lagrangian:

\[
L_{\text{tot}}(\varphi) = \frac{1}{2}(\partial \varphi / \partial x_\mu)^2 - M^2 \varphi^2,
\]

i.e.

\[
L_{\text{tot}}(\varphi)_{\text{Higgs}} = L^0_{\text{maximon}}(\varphi)
\]

Spontaneous breaking of the global \( U(1) \)-symmetry results in:

\[
L_{\text{tot}} = |\partial \varphi / \partial x|^2 - M^2 |\varphi|^2 - M^2 |\chi - \cos \mu \varphi|^2 - \frac{\lambda^2}{2} |\chi|^2 |\varphi|^2
\]

under \( m \to im \), then

\[
L_{\text{tot}} = |\partial \varphi / \partial x|^2 - M^2 |\varphi|^2 - M^2 |\chi|^2 + M^2 \cos \mu |\chi \overline{\varphi} + \varphi \overline{\chi}| - \frac{\lambda^2}{2} |\chi|^2 |\varphi|^2
\] (17)

This Lagrangian differs from (17) by a sign before \( m^2 \), but it is still invariant relatively the group of global transformations:

\[
\begin{align*}
\varphi(x) &\to \varphi(x) = \exp(ig\gamma)\varphi(x), & \varphi(x)^* &\to \varphi(x)^* = \exp(-ig\gamma)\varphi(x)^* \\
\chi(x) &\to \chi(x) = \exp(ig\gamma)\chi(x), & \chi(x)^* &\to \chi(x)^* = \exp(-ig\gamma)\chi(x)^*
\end{align*}
\] (19)

Taking the derivative of \( L_{\text{tot}}(x) \) on \( \chi \) and \( \overline{\chi} \), we will find the equation of motion for \( \overline{\chi} \) and \( L_{\text{tot}}(x) \) respectively

\[
-M^2 \overline{\chi} + M^2 \chi \mu \varphi - \frac{\lambda^2}{2} |\varphi|^2 \overline{\chi} = 0
\]

and

\[
-M^2 \overline{\chi} + M^2 \chi \mu \varphi - \frac{\lambda^2}{2} |\varphi|^2 \chi = 0
\]

From here we get:

\[
\overline{\chi} = \frac{\chi \mu \varphi}{1 + \frac{\lambda^2}{2M^2 |\varphi|^2}}
\]
and
\[ \chi = \frac{ch\mu \varphi}{1 + \frac{\lambda^2}{2M^2} |\varphi|^2} \]
\[ L_{tot} = \left| \frac{\partial \varphi}{\partial x} \right|^2 - V(\varphi) \]
(20)
where \( V(\varphi) \)-Higgs potential
\[ V(\varphi) = M^2 |\varphi|^2 - \frac{M^2 ch^2 \mu'}{1 + \frac{\lambda^2}{2M^2} |\varphi|^2} \]
under \( |\varphi|^2 = \frac{2M^2}{\chi^2}(ch\mu' - 1) \) has its \( V_{min}(\varphi) = -\frac{2M^4}{\chi^4}(ch\mu' - 1)^2 \). In the flat limit \( M \to \infty \), (20) will have a usual view. If we write \( V(\varphi) = V_{min}(\varphi) \), then
\[ V_{New}(|\varphi|) = \frac{\lambda^2}{2} \left[ |\varphi|^2 - \frac{h^2}{2} \right] \]
(21)
where \( \frac{h^2}{2} = \frac{2M^2}{\chi^2}(ch\mu' - 1) \) under \( M \to \infty \) equals \( \frac{m^2}{\chi^2} \).

Finally, we obtain
\[ L_{tot}(\varphi) = \left| \frac{\partial \varphi}{\partial x} \right|^2 - \frac{\lambda^2}{2} \left[ |\varphi|^2 - \frac{h^2}{2} \right]^2 \]
(22)
\[ L_{tot}(\varphi, \varphi_1, \varphi_2) = \]
(24)

The system, described by the Lagrangian (18), has the spontaneous broken \( U(1) \)-symmetry. Now a point is not appropriate to the minimum of energy \( \varphi(x) = \overline{\varphi}(x) \), but any point on a circle of the radius
\[ R = \sqrt{2} \frac{M^2}{\chi} \sqrt{ch\mu' - 1} \]

Let’s write \( \varphi(x) \) in a view of the real and imaginary parts. As a stable vacuum, we can choose any state, lying on a circle of the radius of \( R \), i.e. all the states are equivalent due to invariation relatively transformations (19).

Let’s pick out the magnitude of a gauge phase to be \( \beta = 0 \), identical for the whole world, and let’s write \( \varphi \) in a view
\[ \varphi(x) = \frac{1}{\sqrt{2}} (h + \varphi_1(x) + i\varphi_2(x)) \]
(23)
Substituting (23) into (22), we obtain
\[ L_{tot}(\varphi) \Rightarrow L_{tot}(\varphi_1, \varphi_2) = \]
(24)
The Goldstone scalar mass less particle $\varphi_2$ has appeared as a result of the spontaneous breaking of the symmetry, and a real scalar particle with a mass

$$m_1 = \frac{\lambda h}{\left[1 + \frac{\lambda^2 h^2}{16 M^2}\right]^{1/2}}$$  \hspace{1cm} (25)$$

Under $M \to \infty$ from (25) we have $m_1 = \sqrt{2} m$. If we consider the real scalar particle as a maximon, then from (25) we obtain:

$$m_{1\text{maximon}} = \frac{2 M}{[\sqrt{2} + 125]^{1/2}}$$

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