Testable scenario for Relativity with minimum-length

Giovanni AMELINO-CAMELIA
Dipartimento di Fisica, Universitá “La Sapienza”, P.le Moro 2, I-00185 Roma, Italy

ABSTRACT

I propose a general class of space-times whose structure is governed by observer-independent scales of both velocity ($c$) and length (Planck length), and I observe that these space-times can naturally host a modification of FitzGerald-Lorentz contraction such that lengths which in their inertial rest frame are bigger than a “minimum length” are also bigger than the minimum length in all other inertial frames. With an analysis in leading order in the minimum length, I show that this is the case in a specific illustrative example of postulates for Relativity with velocity and length observer-independent scales.
The recent anniversary of Planck's introduction of his (reduced) constant \( \hbar \) \((\hbar \simeq 10^{-34}\text{Js})\) renders somewhat more disappointing the fact that we have not yet established which role (if any) should be played in the structure of space-time by one of the implications of the existence of \( \hbar \) which appeared to be most significant to Planck: the possibility to define the length scale now called Planck length \( L_p \) by combining \( \hbar \) with the gravitational constant \( G \) and the speed-of-light constant \( c \) \((L_p \equiv \sqrt{\hbar G/c^3} \sim 1.6 \times 10^{-35}\text{m})\). The fact that \( L_p \) is proportional to both \( \hbar \) and \( G \) appears to invite one to speculate that it might play a role in the microscopic (possibly quantum) structure of space-time, and in fact many “quantum-gravity” theories (theories attempting to unify general relativity and quantum mechanics), have either assumed or stumbled upon this possibility; however, a fundamental role for \( L_p \) in the structure of space-time appears to be conceptually troublesome for one of the cornerstones of Einstein's Special Relativity: FitzGerald-Lorentz length contraction. (It is noteworthy that length contraction had already been proposed by FitzGerald and Lorentz well before Planck’s introduction of his length scale.) The Relativity Principle demands that physical laws should be the same in all inertial frames, including the law that would attribute to the Planck length a fundamental role in the structure of space-time, whereas, according to FitzGerald-Lorentz length contraction, different inertial observers would attribute different values to the same physical length. The idea that the Planck length should play a truly fundamental role in the structure of space-time appears to be in conflict with the combined implications of the Relativity Principle and FitzGerald-Lorentz length contraction.

This conclusion does not depend on the specific role played by the Planck length in space-time structure. Let me clarify this by considering two popular ideas and showing that the issue emerges in both. A first popular example, which is encountered in many quantum-gravity approaches (including string theory), is

\footnote{While it is clear that a truly fundamental role for the Planck length in space-time structure is inconsistent with the combination of the Relativity Principle and ordinary FitzGerald-Lorentz length contraction, it is of course possible that the Planck length be associated with some sort of background in a way that is consistent with both the Relativity Principle and FitzGerald-Lorentz length contraction. This would be analogous to the well-known special-relativistic description of the motion of an electron in a background electromagnetic field. This physical context is described by different observers in a way that is consistent with the Relativity Principle, but only when these observers take into account the fact that the background electromagnetic field also takes different values in different inertial frames. The electric and the magnetic components of the background field are not observer-independent, but their combination affects the motion of the electron in a way that is of course consistent with the Relativity Principle. The Planck length could play a similar role in fundamental physics, \textit{i.e.} it could reflect the properties of a background, but then the presence of such a background would allow to single out a “preferred” class of inertial frames for the description of the short-distance structure of space-time. In the present study I show that in addition to this scenario, which introduces the Planck length together with a preferred class of inertial frames, it is also possible to follow another scenario for the introduction of the Planck length: this second option does not predict preferred inertial observers but does require a short-distance deformation of FitzGerald-Lorentz contraction.}

\footnote{It is of course not very significant for the point here being made that actually in string theory the minimum length might not be given exactly by the Planck length (it could be a few orders of magnitude bigger or smaller than \( L_p \).}
the one of the Planck length playing the role of “minimum length”, setting a limit on the localization of events. In this case the possibility to single out a preferred class of inertial frames for the description of space-time emerges as a result of the fact that an event localized with $L_p$ accuracy in one inertial frame, would be, according to FitzGerald-Lorentz contraction, localized with subPlanckian accuracy in some other inertial frames. A second example is provided by quantum-gravity approaches (see, e.g., Refs. [7, 8]) predicting new-physics effects that would be strong for particles of wavelength of the order of the Planck length but would be weak for particles of larger wavelengths, such as the ones associated with deformed dispersion relations of the type

$$E^2 = c^2p^2 \pm L_p^2 c^2 - n E_p^2.\] Assum ing the special-relativistic rules of transformation of energy and momentum, these dispersion relations would allow to select a preferred class of inertial frames.

In the present study I show that it is possible to formulate the Relativity postulates in a way that does not lead to inconsistencies in the case of space-times whose short-distance structure is governed by observer-independent scales of velocity and length, and that this new type of relativistic theories allows the introduction of a “minimum length” and/or a length-scale deformation of the dispersion relation, without giving rise to a preferred class of inertial frames for the description of space-time structure. The emerging picture provides a rather intuitive revision of FitzGerald-Lorentz contractions: boosts are essentially undeformed (Lorentz boosts) when acting on large lengths, but the contraction becomes “softer” when boosts act on short lengths. The scenario is also attractive in light of the fact that it makes predictions that are testable with planned experiments such as the GLAST gamma-ray space telescope [18], and it appears even plausible that certain outstanding experimental puzzles [19] in astrophysics, which have already been tentatively interpreted [20, 21, 22, 23, 24] as possibly representing a manifestation of a new fundamental length scale, could eventually be understood in terms of Relativity with two observer-independent scales.

The first step of my analysis is an acknowledgement of the central role that observer-independent scales (or absence thereof) already played in Galilean Relativity and Einstein’s Special Relativity. This will prove useful for my task of introducing an observer-independent length scale. The Relativity Principle demands that “the laws of physics are the same in all inertial frames” and clearly the implications of this principle for geometry and kinematics depend very strongly on whether the fundamental structure of space-time hosts fundamental scales of velocity and/or length. In fact, the introduction of a fundamental scale is itself a physical law, and therefore the Relativity Principle allows the introduction of such fundamental scales only if the rules that relate the observations performed by different inertial observers are structured in such a way that all inertial observers can agree on the value and physical interpretation of the fundamental scales. The Galileo/Newton rules of transformation between inertial observers can be easily obtained by combining the Relativity Principle with the assumption that there are no observer-independent scales for velocity or length. For example, without an observer-independent velocity scale, there is no plausible alternative to the simple Galilean law $v' = v_0 + v$ of composition of velocities.

Special Relativity describes the implications of the Relativity Principle for the case in which there is an observer-independent velocity scale. Einstein’s second postulate can be naturally divided in two parts: the introduction of an observer-independent velocity scale $c (c \simeq 3 \cdot 10^8 m/s)$ and the proposal of a physical interpretation of $c$ as

\[\text{Note that from this point onward I use conventions with } \hbar = 1.\]
the speed of light. This second postulate, when combined with the Relativity Principle (which is the first postulate of Special Relativity) and with the additional assumption that there is no observer-independent length scale leads straightforwardly to the now familiar Lorentz transformations, with their associated familiar formulation of FitzGerald-Lorentz contraction. The assumption that there is no observer-independent length scale plays a key role already in the way in which the second postulate was stated. Experimental data available when Special Relativity was formulated, such as the ones of the Michelson-Morley experiments, only concerned light of very long wavelengths (extremely long in comparison with the length scale $L_p$ introduced by Planck a few years earlier) and therefore the second postulate could have accordingly attributed to $c$ the physical role of speed of long-wavelength light (the infinite-wavelength limit of the speed of light); however, the implicit assumption of absence of an observer-independent length scale allowed to extrapolate from Michelson-Morley data a property for light of all wavelengths. In fact, it is not possible to assign a wavelength dependence to the speed of light without introducing a “preferred” class of inertial frames or an observer-independent length scale.

All the revolutionary elements of Special Relativity (in comparison with the Relativity of Galileo and Newton) are easily understood as direct consequences of the introduction of an observer-independent velocity scale This is particularly clear for the deformed law of composition of velocities, $v' = (v_0 + v)/(1 + v_0 v/c^2)$, and the demise of absolute time (which is untenable when an observer-independent velocity scale governs the exchange of information between clocks).

Within the perspective here being adopted it is clear that the Planck-length problem I am concerned with can be described as the task of showing that the Relativity Principle can coexist with the following postulate

- (L.1): The laws of physics involve a fundamental velocity scale $c$ and a fundamental length scale $L_p$.

The addition of an observer-independent length scale does not require major revisions of the physical interpretation of $c$, but, because of the mentioned connection between wavelength independence and absence of an observer-independent length scale, I shall not automatically assume that it is legitimate to extrapolate from our long-wavelength data:

- (L.1b): The value of the fundamental velocity scale $c$ can be measured by each inertial observer as the $\lambda/L_p \to \infty$ limit of the speed of light of wavelength $\lambda$.

While for $c$ we can at least rely on long-wavelength data, we basically have no experimental information on the role (if any) of $L_p$ in space-time structure. I can only use the intuition that is emerging from quantum-gravity approaches. I shall focus on the two mentioned popular ideas: $L_p$ could play the role of “minimum length” or the role of a reference scale for wavelengths, characterizing deformed dispersion relations. Among the results reported in the present study the one which appears to be most compelling to this author is the fact that the requirement of consistency with the Relativity Principle (and absence of a preferred frame for the description of the short-distance structure of space-time) can provide a connection between these otherwise unrelated intuitions: in some scenarios in which the Planck length is a reference scale of wavelengths characterizing deformed dispersion relations one can derive from consistency...
with the Relativity Principle that the Planck length is also the “minimum length”. Motivated by the objective of describing this connection and by the desire to provide an analysis that is relevant for planned experimental studies of the possibility of dispersion relations of type $E^2 = c^2p^2 + L_p cEp^2$, I consider here the following example of possible physical interpretation of the Planck length:

- (L.1c): Any inertial observer can establish the value of $L_p$ by determining the dispersion relation for photons, which takes the form $E^2 = c^2p^2 + f(E, p; L_p)$, where the function $f$ has leading $L_p$ dependence given by: $f(E, p; L_p) \simeq L_p cEp^2$.

The first task for establishing the logical consistency of this illustrative example of new Relativity postulates is the one of showing that there is a satisfactory deformation of Lorentz transformations such that the dispersion relation $E^2 \simeq c^2p^2 + L_p cEp^2$ holds in all inertial frames for fixed (observer-independent) value of $L_p$. It is actually quite easy (although it involves somewhat tedious mathematics) to construct these deformed transformations. I shall give a detailed technical description of the derivation and of the general properties of the deformed transformation rules elsewhere. For the analysis of the most significant physical implications of the new postulates it is sufficient to note here the transformation rules for boosts of photon momentum along the direction of motion. Specifically, let us consider a photon which, for a given inertial observer, is moving along the positive direction of the $z$ axis with momentum $p_0$ (and, of course, as imposed by the new dispersion relation, has energy $E_0 \simeq p_0 + L_p c p_0^2 / 2$). The new relativity postulates imply that for another inertial observer, which the first observer sees moving along the same $z$ axis, the photon has momentum $p$ related to $p_0$ by

$$p = p_0 e^{-\xi} + L_p p_0^2 e^{-\xi} - L_p p_0^2 e^{-2\xi} , \quad (1)$$

where $\xi$ is the familiar rapidity parameter of boosts. As manifest in (1), ordinary Lorentz boosts are of course obtained as the $L_p \to 0$ limit of the new boosts. The comparison between (1) and its $L_p \to 0$ limit also allows us to gain some insight on the type of deformation of FitzGerald-Lorentz contraction that characterizes the new postulates, and the associated emergence of a “minimum length”. As long as $p_0 < 1/L_p$ (wavelength $\lambda_0 > L_p$) and $e^{-\xi} \ll 1/(L_p p_0)$ the relation between $p$ and $p_0$ is well described by ordinary Lorentz transformations. Within the analysis in leading order in $L_p$ here reported it is not legitimate to consider the case $e^{-\xi} > 1/(L_p p_0)$ (which would require an exact all-order analysis of the implications of the function $f(E, p; L_p)$ introduced in the postulates), but we can look at the behaviour of the transformation rules when $e^{-\xi}$ is smaller but not much smaller than $1/(L_p p_0)$. While the transformation rules are basically unmodified when $e^{-\xi} \ll 1/(L_p p_0)$, as $e^{-\xi}$ approaches from below the value $1/(L_p p_0)$ the transformation rules are more and more severely modified: for large boosts, the ones that would lead to nearly Planckian wavelengths in the ordinary special-relativistic case, the magnitude of the wavelength contraction is sizably reduced. For example, for $e^{-\xi} \approx 1/(3L_p p_0)$ one would ordinarily predict $p \simeq 1/(3L_p)$ while the new transformation rules predict the softer momentum $p \simeq 2/(9L_p)$. This suggests that there should exist an exact all-order form of $f(E, p; L_p)$ (extending the present $f(E, p; L_p) \simeq L_p cEp^2$ leading-order analysis) such that when one inertial observer
assigns to the photon momentum smaller than $1/L_p$ (wavelength greater than $L_p$) all other inertial observers also find momentum smaller than $1/L_p$.

In order to gain some more direct intuition on the new type of length contraction which can emerge in theories with observer-independent scales of velocity and length, it is useful to analyse a gedanken length-measurement procedure. A key point for these analyses is the fact that the dispersion relation $E^2 \simeq c^2 p^2 + L_p c p^2$ corresponds to the deformed speed-of-light law

$$v_\gamma(p) = c \left(1 + L_p |p|/2\right). \quad (2)$$

The wavelength dependence of this speed-of-light law plays a key role in the emergence of a minimum length in measurement analysis. I show this in a simple context. Let us consider two observers each with its own (space-) ship moving in the same space direction, the $z$-axis, with different velocities (i.e. with some relative velocity), and let us mark “A” and “B” two $z$-axis points on one of the ships (the rest frame). The procedure of measurement of the distance $AB$ is structured as a time-of-flight measurement: an ideal mirror is placed at B and the distance is measured as the half of the time needed by a first photon wave packet, centered at momentum $p_0$, sent from A toward B to be back at A (after reflection by the mirror). Timing is provided by a digital light-clock: another mirror is placed in a point “C” of the rest frame/ship, with the same $z$-axis coordinate of A at some distance $AC$, and a second identical wave packet, again centered at $p_0$, is bounced back and forth between A and C. The rest-frame observer will therefore measure $AB$ as $AB' = v_\gamma(p_0)N\tau_0/2$, where $N$ is the number of ticks done by the digital light-clock during the A→B→A journey of the first wave packet and $\tau_0$ is the time interval corresponding to each tick of the light-clock ($\tau_0 = 2AC/v_\gamma(p_0)$). The observer on the second (space-) ship, moving with velocity $V$ with respect to the rest frame, will instead attribute to $AB$ the value

$$AB'' = \frac{v_\gamma(p)^2 - V^2}{v_\gamma(p)} N\tau_0/2, \quad (3)$$

$$AB'' = \frac{[v_\gamma(p)^2 - V^2]v_\gamma(p_0)}{v_\gamma(p)\sqrt{v_\gamma(p')^2 - V^2}} N\tau_0/2 = \frac{v_\gamma(p)^2 - V^2}{v_\gamma(p)\sqrt{v_\gamma(p')^2 - V^2}} AB', \quad (4)$$

where $p$ is related to $p_0$ through (1), while $p'$ is related to $p_0$ through the corresponding formula for boosts in a direction orthogonal to the one of motion of the photon. The derivation of (3)-(4) is completely analogous to the derivation of the familiar special-relativistic formulas in the analysis of the same measurement procedure assuming wavelength-independence of the speed of light, but the new ingredient of the wavelength dependence of the speed of light has important physics implications. In particular, (4) reflects time dilatation in the new relativistic theory ($\tau$ is the time interval which the second observer, moving with respect to the rest frame, attributes to each tick of the light-clock).  

The two wave packets are taken here to be identical only for simplicity. Nothing prevents one from considering a wave packet for the light-clock with (central) momentum $p_0^*$ ($p_0^* \neq p_0$). This more general case will be analysed elsewhere emphasizing the fact that the possibility $p_0^* \neq p_0$ plays a role in the difference between high precision measurements of large distances and low-precision measurements of short distances.
The implications of (4) for length contraction are in general quite complicated, but they are easily analysed in both the small-$V$ and the large-$V$ limits (examined here of course in leading order in $L_p$). For small $V$ and small momentum (large wavelength) of the probes Eq. (4) reproduces ordinary FitzGerald-Lorentz contraction. For large $V$ Eq. (4) predicts that $AB''$ receives two most important contributions: the familiar FitzGerald-Lorentz term $(AB' \sqrt{c^2-V^2})$ and a new term which is positive and of order $L_p |p| AB'/\sqrt{c^2-V^2}$. As $V$ increases the ordinary FitzGerald-Lorentz contribution to $AB''$ decreases as usual, but the new correction term increases. Imposing $|p| > |\delta p| > 1/AB''$ (the probe wavelength must of course be shorter than the distance being measured) one arrives at the result $AB'' > \sqrt{c^2-V^2}AB' + L_p AB'/(AB'' \sqrt{c^2-V^2})$, which clearly is such that $AB'' > L_p$ for all values of $V$. Again I must remind the reader that I am here reporting an analysis in leading order in $L_p$, and therefore the results cannot be trusted when $V$ is large enough that the correction term is actually bigger than the 0-th order contribution to $AB''$, but we can trust the indications of this analysis as long as the correction is smaller than the 0-th order term, and in that regime one finds that FitzGerald-Lorentz contraction is being significantly softened in the region corresponding to nearly Planckian contraction. This result clearly supports the hypothesis that there should exist a consistent all-order form of $f(E, p; L_p)$ such that when one inertial observer assigns to a length value greater than $L_p$ all other inertial observers also find that length to be greater than $L_p$. Such a form of $f(E, p; L_p)$ would provide a relativistic theory with observer-independent scales $c$ and $L_p$ in which $L_p$ has the intuitive role of “minimum length” described above.

For the future development of the general type of Relativity theories here proposed, it is important to understand whether this result is a general prediction of Relativity with observer-independent $c$ and $L_p$ or it requires the specific formulation of the postulates here explored. Even assuming that, as here proposed, space-time structure is characterized by observer-independent $c$ and $L_p$, and that it is consistent with the Relativity Principle (and that it does not involve some associated new background which singles out a preferred class of inertial observers for the description of space-time structure), one could contemplate a wide spectrum of possible roles for $L_p$, some involving different leading-order forms of the dispersion relation, some not even describable as deformations of the dispersion relation. Let me postpone to future studies this latter possibility, and consider here the former possibility: other scenarios for the leading-order form of the dispersion relation.

One first case to be considered is the one in which the leading-order form of the deformation is just the same as the one here examined but with opposite sign: $f(E, p; L_p) \sim -L_p cE p^2$. In that case all formulas here obtained would, of course, still be valid upon changing all the signs of the coefficients of $L_p$, and it is easy to see that a minimum length would not arise. (Changing the relevant signs one finds that the predicted contraction is even stronger than the FitzGerald-Lorentz one, and boosts reach subPlanckian lengths even more quickly than in Special Relativity.) So it appears that a minimum length requires that the wavelength dependence of the speed of light is such that bigger values of the wavelength lead to higher velocities. (It is perhaps worth noting that this author started, long ago, these studies with an unjustified but strongly felt intuition that the opposite situation should be favoured, an intuition which was changed by these results on minimum length.)

A second case which should be considered for illustrative purposes in the one in which $f(E, p; L_p)$ is such that the leading order is only quadratic in the Planck length,
e.g. \( f(E, p; L_p) \approx L_p^2 E^2 p^2 \) or \( f(E, p; L_p) \approx L_p^2 E^4 / c^2 \). In these cases (since the signs have been chosen positive) one does easily find the same qualitative results here obtained, with the only difference that the effects are weaker. In particular, these scenarios do lead to the emergence of a minimum length, but the region of good validity (validity to very good approximation) of ordinary FitzGerald-Lorentz contraction is somewhat extended with respect to the case on which I focused. The qualitative behaviour is however, exactly the same: the contraction is basically underformed for small velocities of the second observer with respect to the rest frame, but it gets softened for large velocities in a way that forbids access to subPlanckian lengths. It appears therefore that the overall sign of the leading-order deformation of the dispersion relation is fixed by the requirement of a minimum length, while the power of \( L_p \) appearing in the leading-order term cannot be fixed by such a requirement.

My closing remarks concern the important subject of the phenomenological implications of this proposal of new Relativity postulates. Since the Planck length is so tiny, and the effects here considered are inevitably suppressed by the ratio of the Planck length versus the wavelength of the photon or versus the length being measured, one might be tempted to assume that none of these effects could be tested in the near future. However, this is not true, at least not for equation (2). As emphasized in Refs. [7, 24, 26], experiments that will be done in a few years (e.g. by GLAST [18]) are expected to achieve sensitivity levels sufficient for tests of the possibility of \( L_p / \lambda \) wavelength dependence of the speed of light. The fact that linear effects can be studied in the near future is of course very significant for the Relativity proposal I am making, but a high-priority issue for the development of this research programme appears to be the one of finding experimental strategies for tests of quadratic effects (if the leading order of the deformation is quadratic in \( L_p \), the effect would be too small for presently-known strategies of experimental studies of the velocity law).

In this respect, while waiting for these tests of the \( L_p \)-linear scenario of velocity-law deformation, it appears most urgent to develop a general analysis of threshold energies in the new Relativity framework. As mentioned, certain outstanding experimental puzzles [19] in astrophysics appear [20, 21, 24, 23, 24] to invite one to introduce a deformation of special-relativistic threshold energies involving a new length scale. In fact, the relevant astrophysics data appear to be inconsistent with the special-relativistic evaluation of the (threshold) energies required by certain processes. Various authors [20, 21, 24, 23, 24] have observed that the paradox can be solved by introducing a new length scale in a way that would allow the identification of a preferred class of inertial frames. It would be exciting to discover that the general type of relativistic frameworks here proposed (in which a new length scale is introduced in a way that does not involve preferred inertial frames) also provides a solution of the threshold paradoxes. However, this requires a careful analysis of massive particles in the new framework, which is not easily done in full generality. For example, in the case on which I primarily focused here, where photons satisfy \( E^2 \simeq c^2 p^2 + L_p c E p^2 \), massive particles should consistently satisfy a dispersion relation of the type \( E^2 \simeq c^4 m^2 + c^2 p^2 + F(E, p; m, L_p) \), with \( F \) some function whose combined limit small-\( L_p, m = 0 \) is given by \( L_p c E p^2 \) (and, 5As mentioned, the scenario for wavelength dependence considered in Refs. [7, 25, 27] would not make room for \( L_p \) in the Relativity postulates, but the type of wavelength dependence is the one here considered (with the important physical/observable difference that there would be preferred inertial observers for the description of the dispersion relation).
of course, since $c^2m$ is the rest energy of the particle, $F(0, E; m; \tilde{L}_p) = 0$). Elsewhere \[9, 27\] I shall show that assuming that $F$ is $m$-independent one obtains a consistent relativistic theory, but I shall also show that within that assumption one does not obtain an explanation of the threshold paradoxes. I shall however observe that certain types of $m$ dependence of $F$ would provide a solution of the paradoxes, but these reacher structures of $F$ are not easily analyzed with respect to the consistency of the type of relativistic theory here proposed. Work on general criteria for the consistency of the function $F$ appear to be strongly motivated by the present analysis, since they might lead to the interpretation of the threshold paradoxes observed in astrophysics as the first manifestation of an observer-independent minimum length in Nature.

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