High-resolution adaptive imaging of a single atom

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Optical imaging systems are used extensively in the life and physical sciences because of their ability to non-invasively capture details on the microscopic and nanoscopic scales. Such systems are often limited by source or detector noise, image distortions and human operator misjudgement. Here, we report a general, quantitative method to analyse and correct these errors. We use this method to identify and correct optical aberrations in an imaging system for single atoms and realize an atomic position sensitivity of $\sim 0.5$ nm Hz$^{-1/2}$ with a minimum uncertainty of 1.7 nm, allowing the direct imaging of atomic motion. This is the highest position sensitivity ever measured for an isolated atom and opens up the possibility of performing out-of-focus three-dimensional particle tracking, imaging of atoms in three-dimensional optical lattices or sensing forces at the yoctonewton ($10^{-24}$ N) scale.

The optical imaging of isolated emitters, such as individual molecules$^{1,2}$, optically active defects in solids$^3$, fluorescent dyes in a solution$^4$ or trapped atoms$^{5,6}$, relies on efficient light collection and excellent image quality$^7$. Such high-resolution imaging underlies many methods in quantum control and quantum information science$^6-8$, such as quantum networks$^8$, fundamental atom–light interactions$^9$ and the sensing of small-scale forces$^{10}$. Individual atoms in particular have been resolved and imaged for many such applications$^{11-18}$, with performance that depends critically on minimizing misalignments and optical aberrations from intervening optical surfaces such as a vacuum window.

In this Article we develop a general method for suppressing aberrations by characterizing and adapting the imaging system, and report the highest-performance optical imaging of an isolated atom to date. We image a single $^{174}$Yb$^+$ atomic ion with a position sensitivity of $\sim 0.5$ nm Hz$^{-1/2}$ for averaging times less than 0.1 s, observe a minimum uncertainty of $1.7(3)$ nm and obtain direct observations from intervening optical surfaces such as a vacuum window.

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The atomic imaging system is presented in Fig. 1 (see Supplementary Section I). A single $^{174}$Yb$^+$ ion is confined in vacuum using a linear Paul trap$^{6,19}$ with three-dimensional harmonic oscillation frequencies $(\omega_x, \omega_y, \omega_z)/2\pi = (1.1, 2, 0.8)$ MHz. Laser light at a wavelength of $\lambda = 369.5$ nm is incident on the ion and resonantly excites the $^2S_{1/2} \rightarrow ^2P_{1/2}$ cycling transition (radiative linewidth of $\gamma/2\pi = 20$ MHz) as shown in Fig. 1a. The ion is laser-cooled and localized in each of the three dimensions of position to $\Delta x = \sqrt{2}\hbar/(m\omega_x) \approx 5$ nm is the zero-point spread, $\hbar$ is the mean thermal vibrational occupation number along each of the dimensions of motion, and $m$ is the atomic mass$^{19}$. For Doppler laser cooling with the cooling laser at an oblique angle to all directions of motion, $\tilde{n} \approx y/2\omega_x \sim 10$, so $\Delta x \sim 20$ nm $\ll \lambda$, and the trapped ion acts as an excellent approximation to a point source.

The isotropic fluorescence from the atom at $\lambda = 369.5$ nm is transmitted through a vacuum viewport and collected by an objective lens of numerical aperture NA = 0.6 with 10x magnification (Fig. 1b). The intermediate image passes through a pinhole that spatially filters light from background sources. Additional magnification is provided by a second stage lens that forms an image at the face of an electron-multiplying charge-coupled-device (EMCCD) camera (Fig. 1c). The objective lens is mounted on a precision five-axis alignment stage to compensate for comatic aberrations, and cylindrical optics are inserted after the magnifier lens to compensate for astigmatic aberrations.

Aberration retrieval and suppression

The measured spatial distribution of the image is the point spread function (PSF)$^{20}$, which contains information about the ultimate resolution achievable in an imaging system and is the building block for more complex image formation through deconvolution techniques. The PSF can be decomposed into Zernike polynomials $Z_n^m(\rho, \theta)$ (see Methods) in space

$$
\text{PSF}(\rho, \theta) = \mathcal{F}\left\{ \exp\left(-ik\sum_{m,n} c_n^m Z_n^m(\rho, \theta) \right) \right\}^2
$$

where $\mathcal{F}\{(\cdot)\}$ is the Fourier transform operator, $k = 2\pi/\lambda$ is the wave-number and the $c_n^m$ coefficients are contributions of each Zernike component defined in the polar coordinates $\rho$ and $\theta$. The $c_n^m$ coefficients correspond to particular optical aberrations, so detailed characterization of the imaging system follows from the retrieval of the sign and magnitude of these coefficients.

Decomposing an image into Zernike polynomials relies on numerical algorithms$^{21,22}$ or semi-analytical calculations$^{23}$. Here, we obtain a full aberration characterization by using a least-squares fit to the measured data, using the $c_n^m$ coefficients and the exit pupil radius as fitting parameters. Although this method omits consideration of vector (polarization) effects, it remains a generally applicable technique because these effects can be neglected at numerical apertures above 0.6 NA (ref. 24).

Figure 2 shows six single-shot images of a single $^{174}$Yb$^+$ ion. The images in Fig. 2a–c were taken during alignment and in Fig. 2d–f were taken at different distances from the focal plane of the optimally aligned system. The images were integrated for $\sim 0.5$ s,
collecting \(\sim 7 \times 10^5\) photons and fitted according to equation (1) to a linear superposition of the first 12 Zernike polynomial basis functions. The overall fitting function was then smoothed by convolving with a Gaussian function that best fit the data and accounted for spatial drifts over long exposures. The Gaussian function parameters were added to the fitting algorithm and are only important for integration times longer than 0.2 s (see next section and Supplementary Section II). We found that the optimal image (Fig. 2f) has a characteristic radius of \(\rho_0 = 363(18)\) nm, consistent with the diffraction-limited Airy radius of \(\rho_0 = 0.61\lambda/\text{NA} = 375.1\) nm given the system numerical aperture.

Based on the one-to-one mapping of the Zernike polynomials to optical aberrations, we plotted an aberration budget, which shows the leading order aberration contributions to each of the images. For example, the contribution of the dominating negative (positive) defocus term of Fig. 2d (Fig. 2e) shows that we can map axial displacement on a transverse image distribution, with the position sensitivity in this experiment. The observed sensitivity of \(\sim 0.5\) nm Hz\(^{-1/2}\) at small integration times is somewhat higher than the expected level of shot noise (shown as the blue line in Fig. 3) and with simple mechanical improvements in the imaging set-up the resolution can probably be pushed well below 1 nm.

Sensing of RF-induced micromotion position

Confinement of atomic ions in a Paul trap is achieved by means of oscillating RF electric field gradients that create a harmonic ponderomotive potential. In the presence of a static uniform electric field \(E\), the ion acquires a ‘micromotion’ modulation in position \(x(t) = X_p\sin \Omega t\) to first order in the pseudopotential

\[
\delta_x = \sqrt{\frac{\tau_0}{2\rho^2} + \frac{b^2}{12 \tau_0^2} + \frac{16\rho^2 b}{R_0^3 \tau^2}}
\]

where \(b \approx 0.07\) is the mean background count rate per pixel, \(l_p \approx 33\) nm is the pixel size referred to the object (image pixel size divided by the magnification) and \(R_0 = \eta_b F/2\) is the maximum (saturated) measured fluorescence count rate from the atom, \(F \approx 10\)% is the solid angle fraction of fluorescence collected and \(\eta_b \approx 25\)% is the quantum efficiency of the camera. Finite pixel size and background counts have negligible impact on the measured position sensitivity in this experiment. The observed sensitivity of \(\sim 0.5\) nm Hz\(^{-1/2}\) at small integration times is somewhat higher than the expected level of shot noise (shown as the blue line in Fig. 3) and is consistent with observed super-Poissonian noise on the camera. We measured a minimum uncertainty of \(\delta_x \approx 1.7(3)\) nm at an integration time of \(\tau = 0.2\) s. For longer integration times, drifts in the relative position between the optical objective and the trapped ion degrade the position uncertainty as shown in Fig. 3 and with simple mechanical improvements in the imaging set-up the resolution can probably be pushed well below 1 nm.

Given this uncertainty in the position of the harmonically bound ion, the sensitivity to detecting external forces is \(\delta F = m\omega_x^2\delta_x\). For a single \(^{174}\text{Yb}^+\) ion with \(\omega_x/2\pi = 10\) kHz, this would correspond to a force sensitivity in the yoctonewton (10\(^{-24}\) N) scale, or an electric field at the \(\mu\text{V cm}^{-1}\) scale. Unlike earlier work\(^{10}\), this imaging force sensor applies to single ions and does not require the resolution of optical sidebands.
approximation\(^\text{19,31}\), where \(\Omega\) is the drive frequency of the RF trapping field (\(\sim 17\) MHz in this experiment) and \(X_\mu = \sqrt{2eE/(m\Omega\omega)}\) is the micromotion amplitude.

The conventional approach for sensing micromotion is based on the first-order Doppler modulation in the scattering of light from a laser beam of wavenumber \(k\) propagating along the micromotion velocity\(^\text{32}\) (Fig. 4a). The correlation between the photon arrival times (measured with a photomultiplier tube) and the micromotion velocity is obtained with a time-to-digital converter. With the excitation laser red-detuned from resonance of order \(\gamma\) and for small levels of micromotion \(kX_\mu \ll 1\), the measured fluorescence signal takes the form\(^\text{33}\)

\[
R(t) = \alpha R_0 + \beta R_0 \left( \frac{kX_\mu \Omega}{\gamma} \right) \cos \Omega t
\]

where \(a, b \leq 1\) are dimensionless constants that depend on the precise detuning and intensity of the excitation laser\(^\text{33}\).

Figure 2 | Aberration retrieval results. a–c, Single-shot images of the misaligned system. d–f, The optimally aligned system at various distances from the focal plane, with the best focus shown in f. In d,e, a high contribution from the defocus term is evident, with low contributions of astigmatism and coma (right). Large contributions of coma and astigmatism (a–c) are corrected with a five-axis stage and cylindrical lens (Supplementary Section I). The goodness of fit obtained for these examples approaches unity at coefficients of determination of 0.989, 0.965, 0.958, 0.957, 0.983 and 0.994 for images a–f, respectively. These images are integrated for \(\sim 0.5\) s. Further analysis of the coefficients of determination is provided in Supplementary Section II.
To also sense the direct position sensitivity to motion, we spatially mask the ion image with a sharp edge aperture, normal to the $x$ direction of motion. The mask position can be adjusted from effectively, $a = -\infty$ (completely exposed) to $a = +\infty$ (completely masked), with $a = 0$ covering exactly half of the image. The total fluorescence behind the mask is then the integrated fluorescence behind the exposed area:

$$R(a, t) = aF(a)R_0 + \beta F(a)R_0 \frac{kX_\mu \Omega}{\sqrt{2 \pi}} \cos \Omega t$$

$$+ a R_0 \frac{X_\mu}{\sigma \sqrt{\pi}} e^{-a^2 X_\mu^2} \sin \Omega t$$

(5)

where we assume a Gaussian image distribution in space with root-mean-square radius $\sigma = 0.36 \mu \lambda_0$ and the scale of the mask position $a$ is referred to the object. The cumulative distribution function is $F(x) = [1 - \text{erf}(x/\sigma \sqrt{2})]/2$.

We extract the two quadratures of the modulated fluorescence from equation (5) by performing sine and cosine transforms of the data. The phases of the modulated signal are calibrated by opening the aperture ($a = -\infty$) and taking the modulation as proportional to $\cos \Omega t$.

Figure 4b shows the position ($\sin \Omega t$) and velocity ($\cos \Omega t$) quadrature amplitudes (normalized to the amplitude at $a = -\infty$) as the mask position is scanned. Based on the observed velocity-induced modulation in the count rate with full exposure ($a = -\infty$), we infer a micromotion amplitude of $X_\mu \sim 20 \text{ nm}$. As the mask is scanned along $x$, a position-dependent modulation in the fluorescence rate arises, reaching a maximum level at $a = 0$. The absolute level of this position-dependent modulation is observed to be 15 times smaller than expected from equation (5). This may be due to slow drifts in the relative position of the ion with respect to the mask—a fluctuation of just 30 nm over the 300 s integration time required to obtain sufficient signal-to-noise ratio in the measurement would explain the observed reduction in the modulation.
Outlook

In the single-atom emitter presented here, ultimate position localization to the level of ångströms (10^{-10} m) should be possible for longer integration times, when drifts slower than 0.2 s are eliminated or actively corrected. Drifts may be reduced by further isolating the vacuum chamber from the environment and stabilizing the relative position between the sample and the objective. Because we obtain information of the centroid position, the zero-point ion motion does not affect these measurements.

More generally, the isolation and correction of wavefront distortions by lifting intensity images can be extended to the adaptive imaging of a variety of source objects. If the fluorescence from the emitter is incoherent, then imaging errors should not accumulate and images from multiple emitters localized in distinct regions of space could be corrected. This technique can also be used for the optimization of laser output cavity spatial modes. Most misalignments provide an unambiguous signature in the image decomposition, but for certain symmetric misalignments such as axial displacements about the focus for single atoms, introducing additional aberrations such as astigmatism or coma will yield a unique fit. Given sufficient emitter brightness, active shot-to-shot adjustments and feedback to appropriate optical elements should allow the continuous optimization of image quality. The bandwidth of this type of adaptive imaging would be limited by the speed of the correction elements and the computing time for numerically extracting the error signal (this is ~1 s in our experiment).

Methods

Methods and any associated references are available in the online version of the paper.

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Author contributions

All authors contributed to the design, construction and carrying out of the experiment, discussed the results and commented on the manuscript. J.D.W.-C. and K.G.J. analysed the data and performed the simulations. J.D.W.-C., K.G.J. and C.M. wrote the manuscript. B.N. and J.M. contributed equally to both the design and construction of the experiment.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to J.D.W.C.

Competing financial interests

The authors declare no competing financial interests.
Methods

Aberration characterization. Although optical aberrations can be described in terms of a Taylor expansion of the object height and pupil coordinates, Zernike polynomials $Z_m^n(\rho, \theta)$ are better suited for this because they form an orthogonal basis set of functions on a unit disk. Zernike polynomials are expressed in polar coordinates $\rho$ and $\theta$ as

$$Z_m^n(\rho, \theta) = \begin{cases} N_m^n R_m^n(\rho) \cos(m\theta) & \text{for } m \geq 0 \\ N_m^n R_m^n(\rho) \sin(m\theta) & \text{for } m < 0 \end{cases}$$

where $N_m^n$ is an integer number and $m$ can only take values $n, n-2, n-4, \ldots, -n$ for each $n$. The radial coordinate is scaled to the exit pupil radius $\rho_p$ (the radius of the image of the input aperture at the camera). Importantly, each term of this polynomial expansion has a one-to-one relation with a specific kind of aberration.

Given the Zernike expansion of a wavefront, we can calculate its deviation from a perfect wavefront using the $c_m^n$ coefficients of equation (1).

Dead time corrections. Dead times were corrected using the Allan B-functions (defined in Supplementary Section V):

$$\sigma^2(\tau) = \frac{\sigma^2(2, MT_0, M\tau_0)}{B_3(\mu)B_2(\mu)}$$

where $\mu$ is the noise model coefficient, in the range $-1 < \mu < 1$, $M$ is the binning parameter, $T_0$ is the time between data acquisitions, and $\tau_0$ is the sampling time. Dead times are then defined as $t_{\text{dead}} = T_0 - \tau_0$ for single acquisition times. The integration time for the Allan variance is $\tau = M \tau_0$. The noise model $\mu$ upon which the B-functions depend at each $\tau$ are found solving

$$\frac{B_1(\mu)}{B_3(\mu)} = \frac{\sigma^2(N, T, \tau)}{\sigma^2(2, MT_0, M\tau_0)}$$

for $\mu$, with $\sigma^2(N, T, \tau)$ defined as the standard variance.

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