Direct evidence for two-fluid effects in molecular clouds

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ABSTRACT

We present a combination of theoretical and simulation-based examinations of the role of two-fluid ambipolar drift on molecular linewidths. The dissipation provided by ion–neutral interactions can produce a significant difference between the widths of neutral molecules and the widths of ionic species, comparable to the sound speed. We demonstrate that Alfvén waves and certain families of magneto-sonic waves become strongly damped on scales comparable to the ambipolar diffusion scale. Using the RIEMANN code, we simulate two-fluid turbulence with ionization fractions ranging from $10^{-2}$ to $10^{-6}$. We show that the wave damping causes the power spectrum of the ion velocity to drop below that of the neutral velocity when measured on a relative basis. Following a set of motivational observations by Li & Houde, we produce synthetic linewidth–size relations that show a difference between the ion and neutral linewidths, illustrating that two-fluid effects can have an observationally detectable role in modifying the MHD turbulence in the clouds.

Key words: MHD – turbulence – methods: numerical – stars: formation – ISM: clouds.

1 INTRODUCTION

Most of the star formation in our Galaxy is thought to take place in molecular clouds, and a majority of it take place in giant molecular clouds (GMCs). While GMCs are observed to be turbulent on the largest scales of parsecs, the theoretical models for how stars might be formed are still under debate since observations have not reached the point where they can help fully distinguish between competing models. One school of thought (see the reviews of Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2007, and the references therein) suggests that the turbulence can stir the GMC, preventing global collapse, while converging shocks induced in the turbulence can provide sites for star formation. An alternative line of thought (Fiedler & Mouschovias 1993) asserts that cores collapse to form stars quasi-statically with the physics of the collapse being dominated by microphysical processes associated with a two-fluid plasma (Tassis & Mouschovias 2005). An intermediate viewpoint consists of acknowledging the importance of turbulence while recognizing the dissipative effects that occur in a partially ionized plasma (Zweibel & Josafatsson 1983; Klessen, Heitsch & Mac Low 2000; Mac Low & Klessen 2004; Basu et al. 2009). Li & Houde (2008, hereafter LH) presented some very interesting line profile data that suggest that the important role of ambipolar diffusion in modifying the turbulence can indeed be distinguished observationally. The interpretation provided by LH was qualitative. If the results of LH could be supported by detailed dynamical calculations, it would indicate that two-fluid effects indeed play a very important role in modifying the turbulence on the smaller (inner) scales of the turbulence, thus setting the stage for core collapse on yet smaller scales. The goal of the present paper is to provide dynamical calculations that confirm LH’s interpretation of their data.

Molecular clouds are primarily composed of neutral gas, but a small fraction of the material can remain ionized. This ionized component can couple to the magnetic field through the Lorentz force, and to the neutral cloud material via a frictional force. The intermediary role of the ions serves as a source of dissipation for turbulence, and can be characterized by an ambipolar dissipation scale $L_{\text{AD}} = v_A/y\rho_i$ (where $v_A$ is the Alfvén speed in the coupled fluid, $y$ is a frictional coupling coefficient between the neutral and ionized fluids and $\rho_i$ is the density of the ions). This in turn allows one to define an ambipolar Reynolds number on the scale $L_{\text{AD}}/\sqrt{v_m L/v_A L_{\text{AD}}}$ (introduced by Balsara 1996 and used in Zweibel & Brandenburg 1997). Near the ambipolar dissipation scale, some MHD wave families may become damped or modified by the frictional coupling. There have been several attempts to analytically study the propagation of these waves in two-fluid magnetized plasmas that occur in molecular clouds (Langer 1978; Balsara 1996). In particular, the Alfvén waves are always strongly damped on scales smaller than the ambipolar dissipation scale, while either the fast magneto-sonic or the slow magneto-sonic waves will be damped below the dissipation scale, depending on whether the fluid is pressure or magnetically dominated. Measurements of the strengths of magnetic fields via the Zeeman effect or the Chandrasekhar–Fermi method suggest that the magnetic pressure is typically comparable to the total gas pressure (Crutcher 1999, 2004; Jijina, Myers & Adams 1999; Crutcher, Hakobian & Troland 2004). As a result, in a highly turbulent cloud pressure and magnetic field fluctuations will

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lead to some regions that are pressure dominated and others that are magnetically dominated.

LH examined the possibility that the existence of the ambipolar dissipation scale might be inferred from the intercomparison of optically thin neutral and ion line profiles. They measured the linewidths of HCO\(^+\), a tracer of the ionized component, and HCN, a tracer of the neutral component, at different spatial resolutions in the massive star-forming region M17. These two species have similar molecular weights and similar critical densities for emission, simplifying their comparison. LH reported that the minimum linewidths of HCO\(^+\) observed at any scale were systematically smaller than the linewidths of HCN by approximately 0.5 km s\(^{-1}\). They further conjectured that the minimum linewidths on any given scale is related to a power law that connects the turbulent linewidths to the ambipolar diffusion Reynolds number.

On the largest scales in any turbulent plasma, one should expect turbulent driving to produce equipartition between the ionized and neutral components. Consequently, line profiles of ions and neutrals measured on these scales should have the same velocity broadening. On the smaller scales, where ambipolar diffusion becomes significant, the dispersion analysis for wave propagation (Balsara 1996) says that certain families of waves in the ionized fluid should be damped. As a result, the rms velocity in the neutral fluid should be larger than the rms velocity in the ionized fluid on smaller length scales. LH relied on this fact in order to understand their data within the context of a turbulence model. To understand these observations, one needs to carry out a driven two-fluid turbulence simulation, and examine the rms velocities in the ions and neutrals on a range of length scales. Ideally, one would like to have a large separation of scales between the turbulent driving and the dissipation. It is not currently feasible to carry out such computations with a large dynamic range and a realistic ionization fraction, however, two-fluid calculations with some dynamic range and an ability to fully capture the dissipation scales are now possible. We present such calculations and verify that the trends noticed by LH are indeed seen in the simulations. A more detailed examination of the linewidths on a range of scales is, therefore, carried out showing that the systematics observed by LH are in fact verified. As a result, we have a direct probe of the ambipolar dissipation scales which would otherwise be unobservable.

Numerical methods that can correctly capture the dissipation are essential in order to track all of the different wave families on scales larger and smaller than the dissipation scale. The numerical investigation of partially ionized magnetized fluids has proceeded primarily on two algorithmic paths.

(1) A two-fluid treatment, which evolves the ionized and neutral components separately and utilizes a friction term to couple them. The large Alfvén velocities are usually compensated for by invoking a ‘heavy ion approximation’ (HIA; Oishi & Mac Low 2006; Li et al. 2008a), which inflates the ion masses in order to reduce the Alfvén velocity while simultaneously reducing the coupling coefficient to keep the friction term unchanged. Tilley & Balsara (2008) have calibrated the two-fluid treatment when the HIA is dispensed with and have shown that it is possible to saliently simulate plasmas with ionization fractions of \(10^{-6}\) using their riemann code.

(2) A single-fluid treatment, in which the advection terms are removed from the ion momentum equation in order to express the ion–neutral friction force in terms of the magnetic stresses. The resulting equations closely resemble the ideal MHD equations, but with an additional diffusive term in the induction equation; see O’Sullivan & Downes (2006, 2007) for numerical work along this direction. In this approach one cannot extract density fluctuations for the ions. Furthermore, the plasma effects below the ambipolar diffusion scale are obliterated by the numerical approximation (Balsara 1996). As a result, this latter approach is not suited for extracting line profiles in the ions.

Both the single-fluid approach and the HIA for the two-fluid method either strongly modify or ignore the ion momentum equation, preventing either of these strategies from producing separate diagnostics of the ions and neutrals. The two-fluid method without the HIA can produce this information if one is willing to absorb the cost of a restrictive time-step that results from the Alfvén and magnetosonic waves. Furthermore, the HIA and the single-fluid treatment will both modify the propagation and dissipation characteristics of the MHD waves (Balsara 1996).

The theoretical background used in the analysis of LH relied on the well-ionized, single-fluid calculations of Ostriker, Stone & Gammie (2001). A similar set of simulations incorporating two-fluid turbulence was performed by Oishi & Mac Low (2006), using a relatively large ionization fraction of 0.1. Oishi & Mac Low (2006) and Li et al. (2008a) discovered that turbulent structures could still be maintained at scales smaller than the ambipolar diffusion scale via the propagation of slow magnetosonic waves. This would appear to undermine the proposed explanation of LH. However, the dispersion analysis of Balsara (1996) and the results from Tilley & Balsara (2008) show that as the ionization fraction becomes much smaller, there is a large range of wavenumbers where the only propagating waves are sound waves in the neutral fluid. One might expect that if two-fluid turbulence calculation were performed at ionization fractions of \(10^{-4}\) or less, a more pronounced effect due to ambipolar diffusion might be observed.

In this paper, we present a series of isothermal two-fluid MHD simulations with turbulent forcing, performed at ionization fractions ranging from \(10^{-2}\) to \(10^{-6}\). We demonstrate that there is a pronounced difference in the linewidths of the ionized and neutral fluids that is enhanced at smaller ionization fractions. Section 2 presents our methods, Section 3 presents the results and Section 4 presents our conclusions.

2 METHODS

We use the riemann code (Balsara 1998a,b, 2004; Balsara & Spicer 1999a,b) to update the hydrodynamic and MHD fluid variables. We incorporate ion–neutral friction via an operator-split method (Tilley & Balsara 2008).

Our initial velocity distribution is generated in Fourier space, with an initial spectrum of \(\exp(-k^2)k^{-5/3}\) and complex amplitudes drawn from a Gaussian random distribution. We chose our initial conditions such that the dissipation scale is found within the computational domain for ionization fraction of \(10^{-4}\), as the computational expense increases dramatically at lower ionization fractions. In the context of dense molecular cloud cores, the physical scale of the ambipolar dissipation scale is extremely small at this ionization fraction, on the order of \(\mu\)pc. This is much smaller than the scales observed by LH, who assumed a much smaller ionization fraction of \(\sim 10^{-7}\) in their estimations of the dissipation scale. Those densities are computationally unfeasible at present. We are free, however, to rescale our simulation to mean total densities of \(10^3\), with a corresponding increasing of the ambipolar dissipation scale to \(\mu\)pc ranges. We describe our initial neutral densities, molecular weights, sound speeds and ambipolar diffusion drag coefficients for both density scalings in Table 1. The initial ionization fractions are in
Table 1. Initial conditions for the simulations.

| Neutral density (cm\(^{-3}\)) | Sound speed (km s\(^{-1}\)) | Drag coefficient (cm\(^3\) g\(^{-1}\) s\(^{-1}\)) | Ion molecular weight | Neutral molecular weight | \(v_{rms}\) | \(L_{box}\) |
|-------------------------------|-----------------------------|-----------------------------|---------------------|------------------------|------------|------------|
| \(10^6\)                     | 0.424                       | \(3.5 \times 10^{13}\)      | 29.0                | 2.3                    | \(3c_s\)  | 0.31 mpc   |
| \(10^7\)                     | 0.424                       | \(3.5 \times 10^{13}\)      | 29.0                | 2.3                    | \(3c_s\)  | 0.31 mpc   |

Table 2. List of simulations.

| Run | R2       | R3       | R4       | R5       | R6       |
|-----|----------|----------|----------|----------|----------|
| \(\xi\) | \(10^{-2}\) | \(10^{-3}\) | \(10^{-4}\) | \(10^{-5}\) | \(10^{-6}\) |
| \(L_{AD}/L_{box}\) | 2.64 \(\times 10^{-3}\) | 2.64 \(\times 10^{-2}\) | 0.264 | 2.64 | 26.4 |
| \(R_{AD}/L_{box}\) | 1.13 \(\times 10^{3}\) | 1.13 \(\times 10^{2}\) | 11.3 | 1.13 | 0.113 |

Table 2, along with the ambipolar dissipation scale for each simulation. The initial magnetic field is set so that the Alfvén velocity of the neutral+ionized fluid is \(v_A = (1 + \rho_i/\rho_n)^{1/2} c_s\), and is uniform in the x-direction.

We provide forcing with a solenoidal vector field to the kinetic energy of the neutral fluid with a tapered spectrum centred on \(kL/2\pi = 4\). For the simulations at ionization fractions of \(10^{-4}\) and larger, the forcing scale is larger than the dissipation scale and we have a clear separation between the forcing and dissipation scales.

For ionization fractions of \(10^{-5}\) and \(10^{-6}\), it is not possible to have this requisite scale separation. These simulations were performed on meshes with 192\(^3\) zones. Because the problem demands a very large dynamic range, we do not make hard claims about spectra, but only extract the information that robustly permeates all of our simulations.

We will show that this is sufficient to reproduce the results of LH on scales near the dissipation scale. The results presented here are taken after the turbulence has reached steady state at one turnover time.

3 RESULTS

This section is divided into three parts. We first discuss the dispersion relation for a two-fluid plasma and show its consequences for the turbulent spectra. Next we show simulated line profiles for the ions and neutrals. Finally, we show that our simulations reproduce the results of LH.

3.1 Dispersion analysis and spectra

Balsara (1996) and Tilley & Balsara (2008) studied the dispersion analysis for a two-fluid, self-gravitating, isothermal system. They demonstrated that below the ambipolar dissipation scale, only one family of magnetosonic waves remains undamped (the fast magnetosonic waves persist if the Alfvén speed is less than the sound speed of the fluid; the slow magnetosonic waves persist if the Alfvén speed is greater than the sound speed). The other magnetosonic waves persist if the Alfvén and magnetosonic waves are strongly damped.

Balsara (1996) also demonstrated that there is a smaller scale on which the neutral and ionized fluids are sufficiently decoupled that the Alfvén and magnetosonic waves can re-emerge in the ionized fluid only. In this weakly coupled regime, the Alfvén and magnetosonic velocities depend only on the ion density; this is in contrast to the strongly coupled regime on scales larger than the dissipation scale, where the relevant velocities depend on the combined density of the two fluids. The effects of the HIA on the propagation of MHD two-fluid waves have not been studied, so we briefly present that here.

We draw on the analysis of Balsara (1996), which presents the eigenvalue equation for the two-fluid isothermal system, to compare the dispersion relation for a gas with an ionization fraction of \(10^{-6}\) with and without the HIA. We increase the molecular weight of the ions by factor of \(10^4\), corresponding to the HIA. The HIA correspondingly decreases the coupling coefficient by the same factor of \(10^4\). Since the ambipolar dissipation scales varies with the product of the ion density and the coupling coefficient, it remains unchanged under the HIA. However, the ion momentum equation is modified due to the increased ion inertia, and this modification leads to a change in the propagation of the MHD waves. In Fig. 1(a), we show the phase velocity of MHD waves on scales above and below the dissipation scale for an ionization fraction of \(10^{-6}\) without the HIA, for a wave travelling at an angle of \(31^\circ\) with respect to the mean magnetic field which has a strength set by \(B = 1.5 c_s\). We plot the slow magnetosonic waves that turn into sound waves with solid lines, the slow magnetosonic waves that appear in the ions only with dotted lines, the Alfvén waves with dashed lines and the fast waves with dot–dashed lines. Because of the large range wave speeds present, we split the plot into three sections that show the three key ranges in speed: the range \([-2c_s, 2c_s]\) in the centre, \([300c_s, 450c_s]\) above and \([-450c_s, -300c_s]\) on the bottom. We clearly see that the fast and Alfvén waves disappear on the dissipation scale \(kL_{AD} \sim 1\). An examination of the damping rate (Balsara 1996) shows that these waves propagate with strong damping on length scales that are almost an order of magnitude larger than \(L_{AD}\). As a result, it is not surprising that LH detect a velocity difference between the ions and neutrals on scales that are an order of magnitude larger than their conjectured dissipation scales. At \(kL_{AD} \sim 100\), we see the re-emergence of these two waves, now unhindered by the neutral inertia. In contrast, Fig. 1(b) shows the results for the HIA. Here, there is no dissipation range to be found in between where the MHD waves die off and where they re-appear in the weakly coupled regime. The dissipation scale in Fig. 1(b) is the correct value for an ionization fraction of \(10^{-6}\), but the pattern of the dispersion relation in fact looks very much like the dispersion relation for a fluid at an ionization fraction of \(10^{-2}\). The HIA in this case will not correctly capture the MHD waves within the dissipation range.

Fig. 2 shows the turbulent spectrum for the specific kinetic energy \(v^2\) of the neutrals (dotted line), specific kinetic energy of the ions (solid line) and magnetic energy (dot–dashed line). Fig. 2(a) shows this data for our simulation with an ionization fraction of \(10^{-3}\). Fig. 2(b) shows the same for an ionization fraction of \(10^{-4}\) and Fig. 2(c) shows the same for an ionization fraction of \(10^{-5}\). The spectra have been normalized so that they have the same value at the largest scale in the simulation. We see from Fig. 2(b) that on scales comparable to the dissipation scale (i.e. on length scales that have \(R_{AD} \sim 1\)), the kinetic energy in the ions is smaller than the kinetic energy in the neutrals on a relative basis. On the smaller scales...
of any high-Mach-number MHD calculation, much of the energy is expected to be in the form of waves, not shocks. Since our dispersion analysis shows that the Alfvén waves as well as one family of magnetosonic waves is strongly damped, the magnetic energy also shows a rapid decrement below the dissipation scale. Conversely, the neutral kinetic energy is not strongly damped because the neutral sonic waves continue to propagate on scales below the dissipation scale. Fig. 2(c), which pertains to an ionization fraction of $10^{-5}$, but is similar to Fig. 2(b) in all other respects, shows a larger range of scales where the kinetic energy of the ions as well as the magnetic energy is damped relative to the kinetic energy of the neutrals.

Figure 1. (a) Real part of the dispersion relation for an ionization fraction of $10^{-6}$. In the weakly coupled regime ($kL_{AD} > 100$), the Alfvén speed depends solely on the ion density, and hence is significantly larger than the Alfvén speed in the strongly coupled regime. As a result, we show these in the upper and lower panels. (b) Real part of the dispersion relation for the heavy-ion approximation. Because of resolution limitations, we had to force this simulation on scales that are smaller than the dissipation scale. Had it been forced on larger scales, the spread between the ion and neutral kinetic energies would have been even more pronounced. The simulation in Fig. 2(a) with an ionization fraction of $10^{-3}$, by contrast, has a very small $L_{AD}$. Consequently, the velocity spectra for ions and neutrals as well as that of the magnetic field closely track each other. This is consistent our wave propagation model from Fig. 1(a) and our prior demonstration that the dissipation scale has a larger value at smaller ionization fractions. On going to even smaller ionization fractions, this trend becomes even more pronounced.
Figure 2. (a) Spectrum of the simulation with an ionization fraction of $10^{-3}$. (b) Spectrum of the simulation with an ionization fraction of $10^{-4}$. (c) Spectrum of the simulation with an ionization fraction of $10^{-5}$. 
3.2 Simulated line profiles

The shapes of spectral line profiles provide one of the few methods to probe the kinematics within a cloud because they directly track the velocities in the turbulent fluid. In Fig. 3 we plot simulated line profiles for our runs, integrated over the entire domain. As we do not include self-gravity in these calculations, the maximum density that is achieved arises only through the density jump caused by shocks, a factor of \( \sim 30 \) in the neutral fluid and \( \sim 7 \) in the ions. As a result, we do not expect that effects due to optical thickness or freeze-out of our molecular tracers onto dust grains to have a significant role in modifying the line profiles, and we are thus justified in using a very simple model for line emission. We calculate the profiles in Fig. 3 by summing individual lines from every cell with an emissivity proportional to the density, a linewidth equal to the isothermal sound speed, and centred on the mean velocity parallel to the line-of-sight within that zone. Figs 3(a)–(c) show the line profiles for our simulations with ionization fractions of \( 10^{-3} \), \( 10^{-4} \) and \( 10^{-5} \). We see that the equivalent width of the neutral profile (marked by a dashed line) is wider than the equivalent width of the ionized profile (solid line), by 0.33\( c_s \) for Fig. 3(a), 1.2\( c_s \) for Fig. 3(b) and 1.9\( c_s \) for Fig. 3(c). For ionization fractions of \( 10^{-2} \) and \( 10^{-6} \) we find that the equivalent widths of the neutral and ionized line profiles are larger than that of the ions by 0.016\( c_s \) and 2.1\( c_s \), respectively. The claim in LH is that they are looking at the ambipolar diffusion scales for two-fluid turbulence, and our simulations focus on the dissipation scales. Our spectra have shown that on those scales we should expect a smaller rms velocity in the ions and this is supported by the dynamics of wave propagation. The simulated line profiles that we get are also consistent with that expectation. We thus find that the results reported by LH of a systematic shift between the widths of ionized and neutral lines could be justified by our simulations.

3.3 Reproducing the results of LH via simulations

In order to test the hypothesis of LH, we calculate individual line profiles along each column of zones in our simulations using the same prescription as we used in Section 3.2. We then combined the projected line profiles by binning adjacent profiles together, in order to get a distribution of line profiles on different scales. In analogy to LH, we then plot the square of the linewidths as a function of scale for ionization fractions of \( 10^{-4} \) and \( 10^{-5} \) in Fig. 4. Figs 4(a) and (c) plot all of the linewidths we extracted at each scale at ionization fractions of \( 10^{-4} \) and \( 10^{-5} \), respectively. Figs 4(b) and (d) plot the mean value of the square of the linewidth, as well as the dispersion above and below the mean. The smaller plotting symbols mark the minimum linewidth at each resolution. The ionized component linewidths are marked by squares, and the neutral component linewidths are marked by diamonds. As the ion and neutral line profiles are measured on exactly the same scales as each other, we introduce a slight offset to the neutral linewidths in the x-axis in order to the differences in the distributions of the two sets of data to become apparent. We have a larger range of scales to work with as well, so the x-axis is logarithmic in Fig. 4.

LH claim that the lower envelope of linewidths for the neutral fluid is systematically \( \sim 0.5 \) km/s \( \times \) larger than the lower envelope of linewidths for the ionized fluid. We see that the lower envelopes differ typically by \( \sim 1-2c_s^2 \) for ionization fractions of \( 10^{-4} \), and up to \( 3c_s^2 \) on the largest scales at ionization fractions of \( 10^{-5} \). We see a similar trend in the mean values at each length scale. The sound speed of the gas at a temperature of 50 K is 0.42 km/s; the differences in the lower envelope of the linewidths thus works out to about 0.54 km/s. The difference in the linewidths is more pronounced at lower ionization fractions. Our simulations produce a result very similar to LH, thus confirming that two-fluid turbulence can explain the observational results.

4 CONCLUSIONS

The large observed linewidths in the ions and neutrals are most conveniently attributed to MHD turbulence in molecular clouds. Since the plasma in a molecular cloud is partially ionized, it will display two-fluid effects. The low levels of ionization in the plasma ensure that the ionized and neutral fluids become partially uncoupled on subparsec scales. This process causes certain families of magnetoacoustic waves and all families of Alfvén waves to be strongly damped on scales comparable to the ambipolar diffusion scale, \( L_{AD} \). As a result, the power spectrum in the ions dips below that of the neutrals when measured on a relative basis. Consequently, we show that line profiles for optically thin neutral lines are wider than the line profiles for optically thin ion lines. The above statement holds true as long as the ion and neutral species have comparable molecular weights. We use this to produce synthetic linewidth–size relations that reproduce the trends in LH’s observed data. The observations of LH, taken along with our simulations, therefore provide direct evidence that two-fluid effects play an important role in modifying the MHD turbulence in molecular clouds. This modification is scientifically significant because it takes place on the same scale that cores are formed. As a result, models for star formation should include two-fluid effects without resorting to computationally expedient simplifications.

The Atacama Large Millimeter Array (ALMA) instrument will be an even more powerful and precise tool for measuring linewidths as well as magnetic fields when compared to the capabilities of current observatories. As a result, future observational studies should be able to demonstrate the importance of two-fluid effects in several
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systems using the methods pioneered by LH and computationally supported by this paper. Future observational studies should be able to study ion and neutral lines at a range of densities and temperatures (Bergin & Tafalla 2007). This makes a very good case for including chemical networks in computational models of molecular cloud turbulence.

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