Turbulent transport of finite sized material particles

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Abstract. Turbulent transport of material inclusions plays an important role in many natural and industrial situations. In the present study, we report an exhaustive experimental investigation of Lagrangian dynamics of material particles in a turbulent air flow, over a wide range of sizes and densities. For fixed carrier flow conditions, we find that (i) velocity statistics are not affected by particles inertia; (ii) acceleration statistics have a very robust signature, where only acceleration variance is affected by inertia; (iii) inertial particles always have an intermittent dynamics; (iv) intermittency signature depends on particles inertia; (v) particles actual response time to turbulent forcing remains essentially of the order of the carrier flow dissipation time rather than any particles dependent time (as the Stokes time for instance). These observations are in contrast with usual predictions from Stokesian models for point particles.

1. Introduction

Predicting the dynamics of material particles dispersed and transported in a turbulent flow remains a challenge with important applications in industrial and natural systems: dust and pollutants dispersion, industrial mixers, sediments in rivers, dispersion of gametes of marine animals, water droplets in clouds, atmospheric balloons, etc. One of the main difficulties lies in the intrinsic multi-scale nature of turbulence: depending on their size and density, particles will interact with structures of the carrier flow at different time and spatial scales. For instance neutrally buoyant particles much smaller than the dissipation scale \( \eta \) of the turbulent field are expected to behave as tracers, with the same dynamics as fluid particles. On the contrary particles with mismatch density and/or with size comparable to turbulent eddies, will not follow the flow exactly. We refer to these particles as inertial. How the coupling between particle dynamics and fluid dynamics is influenced by size and density effects remains an open question and a very active field of research. Several inertial effects have been known for a long time and can be interpreted qualitatively in terms of particle interaction with turbulent eddies. For instance the well known preferential concentration effect leading to the formation of clusters of inertial particles separated by depleted regions can be understood as the centrifugal expulsion of denser particles from the turbulent eddies. However an accurate quantitative description of such effects is still lacking. One of the reasons is our inability to write a proper equation of motion for inertial particles in a turbulent environment. A relatively rigorous analytical approach only exists for the limit case of point particles, for which the BBOT (Basset-Boussinesq-Oseen-Tchen) equation - revisited in 1983 by Maxey & Riley [14] and Gatignol [8] - offers a suitable model. In a more general case (finite size particles with arbitrary density), apart from some first order corrections to the BBOT equation (as Faxén corrections for instance [6]), a valid equation of motion remains to be found. In the limit of very small particles, the equation of motion for the advected particles can be simplified as:

\[
\frac{d\vec{v}}{dt} = \frac{1}{\tau_p}(\vec{u} - \vec{v}),
\]

with \( \vec{v} \) the particle velocity, \( \vec{u} \) the carrier flow velocity at the position of the particle and \( \tau_p \) the viscous response time (Stokes time) of the particle. This model is particularly attractive because of its simplicity and has been extensively investigated in the past decades. In such model,
Figure 1. Particle size and density parameter space explored in previous investigations of acceleration statistics. Present studies (blue circles) concern particles which are both large and dense. Background contours indicate corresponding particle Stokes number, estimated following Xu and collaborators [27] as $St = \tau_p / \tau_D$, with $\tau_D$ the eddy turnover time at the scale of particle size, and $\tau_p$ the particle viscous response time, including finite particulate Reynolds number corrections [4].

the Stokesian coupling between the fluid and the particle, mainly acts as a first order low-pass filter, where particles turbulent energy decreases with increasing response time (hence increasing inertia)[21]. Recent simulations have also shown that in such Stokesian model, the same filtering mechanisms results in a continuous transition of particles acceleration statistics from highly non-gaussian for tracers to gaussian for highly inertial particles [3].

The goal of the present work is to investigate experimentally how size and density affect the turbulent dynamics of particles. Comparison with predictions from the point particle limit is particularly enlightening in order to catch the range of validity of this limit case, which is by far the most documented from theoretical studies and numerical simulations.

In the first section we briefly summarize main previous results concerning experimental investigation of inertial particles acceleration, which is particularly enlightening as it directly reflects the turbulent exerted by the flow. In the second section we describe our experimental setup (an acoustical Lagrangian tracking technique for particles transported in a grid generated wind tunnel turbulent flow). In the third section we present the results on particle dynamics. To finish, a brief discussion and comparison of experimental results with existing models is proposed.

2. Acceleration of inertial particles : a brief review

Particles acceleration is a direct image of the turbulent forcing experienced by the particles, what is a key ingredient in any dynamical model for particles motion. For these reason recent experimental and numerical investigations have shown much interest in studying this quantity, whose measurement have always been a challenge.

Particles are characterized by their diameter $D$ and density $\rho_p$. In the following we will consider the dimensionless size and density parameters: $\Phi = D / \eta$ and $\Gamma = \rho_p / \rho_f$, where $\eta$ is the carrier flow dissipation scale and $\rho_f$ is the carrier fluid density. In the point particle limit, particle inertia is generally parametrized by the so-called particle response time $\tau_p$, which for a real particle in a smooth flow is given by the Stokes time $\tau_p = D^2(\rho_f + 2\rho_p)/(36\nu\rho_f)$ – the dimensionless Stokes number $St = \tau_p / \tau_D$ is commonly used. In this limit, particle response time therefore combines in a single parameter the role of size and density. During the past
Figure 2. Sketch of our acoustical Lagrangian tracking facility in grid generated turbulence.

decade, high resolution Lagrangian particle tracking techniques have emerged [23, 26, 17] which have allowed a detailed characterization of particle acceleration statistics. Figure 1 presents particle classes, in the $(\Phi, \Gamma)$ parameter space, whose acceleration statistics in turbulent conditions have been recently investigated experimentally. It can be noted that before the present work, most existing studies have considered either tracer particles ($\Phi \ll 1, \Gamma = 1$) or particles much denser than the carrier fluid ($\Gamma >> 1$) but small ($\Phi << 1$), or particles over a wider range of size (including $\Phi > 1$) but weakly inertial ($\Phi > 1, \Gamma < 4$).

In the case of tracers, highly intermittent Lagrangian dynamics and highly non-Gaussian acceleration statistics (with high acceleration events occurring with a probability orders of magnitude higher than a Gaussian distribution with the same variance) have been reported [11, 16]. For inertial particles, numerical simulations in the point particle limit predict a reduction of these non-Gaussian statistical tails and a trend of acceleration probability density function (PDF) to a Gaussian shape as particles Stokes number is increased [3, 25], consistent with a filtering mechanism due to particles response time. This trend seems to be confirmed by recent experiments [1, 2] for very small inertial particles ($\Phi << 1$ and $\Gamma >> 1$, red circles in figure 1). Beyond the point particle limit, experiments had mainly considered only the case of weakly inertial particles ($\Phi > 1, \Gamma < 4$). In this situation, no clear evidence of gaussianization of acceleration PDF with increasing Stokes number has been observed. In all cases however, a trend of acceleration variance to decrease with increasing Stokes number has been reported.

In this work we present the first measurements of particles which are both much larger than the dissipation scale of the carrier flow and significantly denser than the carrier flow (we cover the range $10 < \Phi < 30$ and $1 < \Gamma < 70$, corresponding to the largest Stokes numbers reported).

3. Experimental setup

Our experiment runs in a large wind tunnel with a measurement section of 75x75 cm downstream of a grid (with a 7.5 cm mesh size) which reproduces almost ideal isotropic turbulence (figure 2b). The mean velocity of the fluid is $U = 15$ ms$^{-1}$ and the turbulence level is $u_{rms}/U \sim 3\%$. The corresponding Reynolds number, based on Taylor microscale, is of the order of $R_\lambda = 175$. The Kolmogorov dissipation scale is $\eta = (\nu^3/\epsilon)^{1/4} \sim 240 \mu$m (where $\nu = 1.5 \cdot 10^{-5}$ m$^2$s$^{-1}$ is the air viscosity at working temperature and $\epsilon \sim 1.0$ m$^2$s$^{-3}$ is the turbulent energy dissipation rate per unit mass) and the energy injection scale is $L \sim 6$ cm. Particles are individually tracked by one component Lagrangian acoustic Doppler velocimetry [17, 20, 19]. We measure the streamwise velocity component $v_z$ of the particles as they are tracked along their trajectory. Acceleration component $a_z$ is obtained by differentiation of the velocity by convolution with a gaussian kernel [15]. As particles we use soap bubbles which we can inflate with different gases (we use helium to produce neutrally buoyant bubbles and
4. Particle dynamics

In this section we present results on the Lagrangian dynamics of particles. The first subsection deals with single time statistics of velocity and acceleration while the second one presents some results on two time statistics, with a main focus on Lagrangian intermittency and acceleration Lagrangian correlation.

4.1. Single time statistics

For all particle classes investigated, velocity statistics are found to have Gaussian fluctuations (figure 3a shows a typical probability density function (PDF) of particles Lagrangian velocity).
We define the particle fluctuation rate as the ratio of the root mean square velocity \( \sigma_{rms} \) and the average velocity \( < V > \) (which is identical to the mean streamwise velocity \( U_l \) of the carrier flow). Interestingly, this fluctuation level is found to be independent of particle properties and does not present any trend either with particle size or with particle density (figure 3b). Moreover the corresponding fluctuation level, of the order of 3\%, is identical to the turbulence level of the carrier flow itself measured from classical hot-wire Eulerian anemometry. While Eulerian and Lagrangian velocity fluctuations are indeed expected to coincide for tracer particles, this result is in contrast with predictions from Stokesian models for inertial point particle, where a monotonic decrease of velocity fluctuations is predicted as particle inertia increases (as the result of a low-pass filtering effect from the particle response time scale as mentioned in the introduction). Such a filtering effect on the global energy of the particles is clearly not observed experimentally for finite size particles.

This suggest that if there is any significant change in particles dynamics, it should better be looked for in the small scales than in the large scales. A typical small scale quantity is particle acceleration. Figure 4 shows acceleration PDFs for all the particle classes we have investigated. Note that in this plot PDFs have been normalized to unity variance. As already noted in [20, 19], it is striking to observe that all such normalized PDFs collapse onto almost a single curve, indicating a very robust statistical signature of acceleration fluctuations that is independent of particle size and density. This observation is again in contrast with predictions based on point particle models where the filtering Stokes number effect has been shown to induce a Gaussianization of acceleration PDF with increasing particle inertia. In recent experiments, Volk and collaborators [24] have shown that a change in acceleration PDFs might though appear for much smaller particles than the sizes investigated here. In our measurements, a clear influence of particle size and density can be observed when PDFs are not normalized to unity variance (not shown here). PDFs tend then to narrow and to peak as particle density increases. As a consequence, it appears that for finite size particles, the evolution of acceleration PDFs is entirely coded by acceleration variance only, while the global PDF shape remains essentially unchanged when normalized to unity variance.

Size and density effects on particles acceleration statistics can therefore be entirely characterized by the single investigation of acceleration variance \( < a^2 > \). Figure 4b represents the evolution of \( < a^2 > \) as a function of particle size and density. Several important features need to be stressed. For neutrally buoyant particles, a monotonic decrease of \( < a^2 > \sim \Phi^{-2/3} \) is observed for particles larger than \( \Phi \sim 10 \) (acceleration variance for smaller particles saturates at value for fluid tracer). Such a \( \Phi^{-2/3} \) trend has already been reported by Voth et al. [26] and confirmed more recently by other experiments [5, 24]. We have shown that this trend can be interpreted as the leading role of pressure in the fluid acting at the scale of particle size [20] and that it can be directly related to classical K41 scaling for pressure increments at the scale of particle size. This interpretation seems to be confirmed by recent numerical simulations with fully resolved finite size particles (though at lower Reynolds number) by Homann and Bec [10], what confirms that finite size effects of neutrally buoyant particles are indeed not inertial mechanisms associated to particle’s response time, but the result of integrating carrier fluid’s pressure at the particle scale.

If we consider now the evolution for a fixed particle size \( \Phi, < a^2 > \) is found to decrease monotonically when particle density is increased and to reach a finite limit \( a^2_0 \) for large density values. The physical origin of this saturation remains unclear. A possibility could be a balance between inertial effects which tend to decrease particle acceleration variance and the enhancement of settling as particles density increases which would contribute to increase the fluctuation level due to crossing trajectories effect. Another interesting finding is the non trivial size dependence of \( a^2_0 \) which is found to increase abruptly for increasing particle size around \( \Phi \sim 18 \). This counter-intuitive observation (it shows an increase of fluctuations with increasing particle size and therefore with increasing particle Stokes number) needs to be understood and will be discussed later.

4.2. Two time statistics

Previous single time statistics analysis has shown that large scales (as represented by velocity fluctuations) and small scales (as represented by acceleration fluctuations) are affected differently by particle inertia. A classical way to investigate scale by scale turbulent dynamics is to consider velocity increments. Figure 5a presents PDFs of Lagrangian velocity increments...
Figure 4.  (a) Particle acceleration probability density function.  (b) Acceleration variance as a function of particle size and density ($<a^2>$ is presented in the dimensionless form $A_0 = <a^2> \epsilon^{-3/2} \nu^{1/2}$ known as the Heisenberg-Yaglom normalization).

$(\delta v(t) = v(t + \tau) - v(t))$ for one class of particle ($\Phi = 16.6, \Gamma = 1$). We observe a continuous deformation from highly non-Gaussian fluctuations at small scale increments (which reflect acceleration) to Gaussian fluctuations at large scale increments (which reflect velocity itself). This evolution of increments statistics with the inertial scales is representative of the intermittent nature of particle Lagrangian dynamics. An interesting finding of the present work is that intermittency is observed for all the particles which have been investigated. However, the fine signature of this intermittent dynamics is found to depend on particle inertia. To illustrate this point, figure 5b presents the evolution of increment flatness $F$ (which measures the extent of PDF tails) as a function of time lag $\tau$ for two different classes of particles with fixed size but different density. Within error bars, flatness is comparable for both at small sub-Kolmogorov scales (this only reflects the robustness of acceleration PDF shape, already discussed above) and at large scales (flatness tend to a value of 3, which corresponds to large scale Gaussian fluctuations, as already reported for velocity itself). However the evolution between small and large scales is strongly particle class dependent, in particular for timescales
near the dissipation time scale $\tau_\eta$ of the carrier flow. At such scales, a clear drop $\Delta \mathcal{F}$ of the flatness (corresponding to a sudden reduction of increments PDF tails) can be observed, with an increasing amplitude as particle density is increased. Subsequently to this drop flatness smoothly tends to its large scale Gaussian limit as larger time scales are considered. This suggests that the important effects associated to particle inertia occur for time scales of the order of $\tau_\eta$.

Such effects are usually expected to be related to the particle response time $\tau_p$. Therefore, a natural question at this point is: what is the actual response time of the particle? As already mentioned, usual estimations of $\tau_p$ are based on particle Stokes time. In the limit of vanishing particles Reynolds number it is given by $\tau_p = D^2(\rho_f + 2\rho_p)/(36\nu \rho_f)$. For finite particulate Reynolds number, empirical corrections to this relation can be found in the literature \[4\] (these corrections have been used here to estimate the Stokes time used to calculate Stokes numbers shown in figure 1 for instance). In addition to this a priori estimation of $\tau_p$, an experimental measurement of actual particle response time to the turbulent forcing $\tau_p^{exp}$ can be obtained from the analysis of the particle acceleration Lagrangian correlation function $R_a$. Such a typical correlation function is shown in figure 6a. We define the experimental particle response time as $\tau_p^{exp} = \int_{t_0}^{N} R_a(t) \, dt$, where $t_0$ is the first zero crossing of $R_a$, as suggested by Calzavarini and

\[\begin{align*}
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Figure 6. (a) Typical acceleration correlation function $R_a$ and definition of the experimental particle response time $\tau_{\text{exp}}$. (b) A priori estimated Stokes time $\tau_p$ (stars) and experimentally measured particle time $\tau_{\text{exp}}$ (circles) as a function of particle size (color codes particles density).

collaborators [6]. Figure 6b shows the evolution of a priori estimated Stokes time $\tau_p$ (stars) and experimentally measured response time $\tau_{\text{exp}}$ (circles) as a function of particle size and density. It is striking to observe that, while the estimated $\tau_p$ varied over more than one order of magnitude among the different particles we have investigated, the actual measured response time does not exhibit any significant change with particle size and density and remains of the order of $\tau_\eta$. Not only is the actual response time different to the estimated one, but also it appears to be determined mainly by the carrier flow itself and not to change significantly with particle properties. The fact that the typical correlation time of acceleration does not significantly change, shows that the decrease of acceleration variance previously reported for instance when increasing particle density ratio, does result from the filtering of the acceleration spectrum of the particle but from the global decrease of the amplitude of this spectrum.
5. Discussion and Conclusions

Our results for the investigation of finite sized inertial particles indicate that: (i) velocity statistics are not affected by particle inertia; (ii) acceleration statistics have a very robust signature, where only acceleration variance is affected by inertia; (iii) inertial particles always have intermittent dynamics; (iv) intermittency signature depends on particle inertia; (v) particle actual response time to turbulent forcing remains of the order of the carrier flow dissipation time rather than any particle dependent time (the Stokes time for instance). Most of these observations cannot be interpreted by a simple filtering mechanism as suggested by existing Stokesian models in the limit of point particles. A first correction to this limit can be made by including so called Faxén corrections in the point particle equation of motion, to account for local inhomogeneity of flow velocity field around a finite size particle. A recent numerical investigation including these corrections [6] has shown to improve qualitative trends of the model when compared to experiments, in particular regarding the shape of acceleration statistics, though quantitative discrepancies remain as particles become larger than a few dissipation scale.

The usual filtering mechanism related to particles response time seems therefore not relevant to describe the dynamics of finite size particles. We believe that our observations can be better understood in terms of a sampling mechanism (rather than a filtering effect), related to the preferential sampling by particles of certain turbulent structures in the carrier flow. Such sampling is expected due to the preferential concentration phenomenon of inertial particles. Two main possible scenarios are usually considered where heavy particles preferentially sample either low-vorticity/high-strain regions [13], or low-acceleration regions [9] of the carrier flow. The latter scenario, known as sweep-stick mechanism, exhibits several features shared with our observations [7]: (i) numerical simulations of stick-sweep mechanisms show that inertial particles tend to cluster near low acceleration points of the carrier flow and that velocity statistics of such points are identical to overall velocity statistics of the carrier flow as observed in our experiments; (ii) as a result, inertial particles stick preferentially to low acceleration regions, level of acceleration fluctuations is also reduced, as observed experimentally for heavy particles; (iii) when the size of such heavy particles increases and become larger than the typical size of these quiet regions, an increase of fluctuations can be expected as particles experience again the influence of more active surrounding structures, which could explain for instance the increase of $<a_\infty>$ we have observed for $\Phi \sim 18$.

Several points still need to be investigated further and important questions remain. The overall relevance of this sampling scenario could be further probed by a systematic investigation (both experimental and numerical) of acceleration statistics of the carrier flow restricted to such low-acceleration regions. Can acceleration statistics of the inertial particles be simply related to that of the carrier flow so conditioned? Then, how can all these information help to build an accurate and simple model for particles turbulent dynamics? Future experiments investigating simultaneously the dispersed particles and the surrounding carrier flow, hence exploring more precisely the fluid-particle coupling, will be helpful in addressing these questions. From the numerical point of view, simulations with fully resolved particles are in a promising and encouraging take off stage [10, 18, 22, 12] and should be extremely enlightening. All these experimental and numerical advances are indeed very promising.

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References

[1] S. Ayyalasomayajula, A. Gylfason, L. Collins, E. Bodenschatz, and Z. Warhaft. Lagrangian measurements of inertial particle accelerations in grid generated wind tunnel turbulence. Physical Review Letters, 97:144507, 2006.

[2] S. Ayyalasomayajula, A. Gylfason, and Z. Warhaft. Lagrangian measurements of fluid and inertial particles in decaying grid generated turbulence. In Y. Kaneda, editor, RITAM Symposium on Computational Physics
and New Perspectives in Turbulence}, volume 4, pages 171–175, Nagoya, Japan, September, 11-14 2006. Nagoya University, Springer.

[3] J. Bec, L. Biferale, G. Boffetta, A. Celani, M. Cencini, A. Lanotte, S. Musacchio, and F. Toschi. Acceleration statistics of heavy particles in turbulence. Journal of Fluid Mechanics, 550:349–358, 2006.

[4] P. Brown and D. Lawler. Sphere drag and settling velocity revisited. Journal of Environmental Engineering-Asce, 129(3):222–231, Mar. 2003.

[5] R. D. Brown, Z. Warhaft, and G. A. Voth. Acceleration Statistics of Neutrally Buoyant Spherical Particles in Intense Turbulence. PHYSICAL REVIEW LETTERS, 103(19), NOV 6 2009.

[6] E. Calzavarini, R. Volk, M. Bourgoin, E. Lévêque, J.-F. Pinton, and F. Toschi. Acceleration statistics of finite-sized particles in turbulent flow: the role of faxen forces. Journal of Fluid Mechanics, in press, 2009.

[7] S. W. Coleman and J. C. Vassilicos. A unified sweep-stick mechanism to explain particle clustering in two- and three-dimensional homogeneous, isotropic turbulence. Physics of Fluids, 21(11):113301, Nov. 2009.

[8] R. Gatignol. The faxen formulas for a rigid particle in an unsteady non-uniform stoke flow. Journal de Mécanique théorique et appliquée, 2(2):143–160, 1983.

[9] S. Goto and J. C. Vassilicos. Sweep-stick mechanism of heavy particle clustering in fluid turbulence. Physical Review Letters, 100(5):054503, 2008.

[10] H. Homann and J. Bec. Finite-size effects in the dynamics of neutrally buoyant particles in turbulent flow. JOURNAL OF FLUID MECHANICS, 651:81–91, MAY 25 2010.

[11] A. LaPorta, G. A. Voth, A. M. Crawford, J. Alexander, and E. Bodenschatz. Fluid particle accelerations in fully developed turbulence. Nature, 409:1017, February 2001.

[12] F. Lucci, A. Ferrante, and S. Elghobashi. Is stokes number an appropriate indicator for turbulence modulation by particles of taylor-length-scale size? Physics of Fluids, 23(2):025101, Feb. 2011.

[13] M. Maxey. The gravitational settling of aerosol-particles in homogeneous turbulence and random flow-fields. Journal of Fluid Mechanics, 174:441–465, Jan. 1987.

[14] R. Maxey and J. J. Riley. Equation of motion for a small rigid sphere in a nonuniform flow. Physics of Fluids, 26(4):883–889, 1983.

[15] N. Mordant, A. M. Crawford, and E. Bodenschatz. Experimental lagrangian acceleration probability density function measurement. Physica D, 193:245–251, 2004.

[16] N. Mordant, J. Delour, F. Lévêque, O. Michel, A. Arnéodo, and J.-F. Pinton. Lagrangian velocity fluctuations in fully developed turbulence: scaling, intermittency and dynamics. Journal of Statistical Physics, 113:701–717, 2003.

[17] N. Mordant, P. Metz, O. Michel, and J.-F. Pinton. Measurement of lagrangian velocity in fully developed turbulence. Physical Review Letters, 87(21):214501, 2001.

[18] A. Naso and A. Prosperetti. The interaction between a solid particle and a turbulent flow. New Journal of Physics, 12:033040, Mar. 2010.

[19] N. M. Qureshi, U. Arrieta, C. Baudet, Y. Gagne, and M. Bourgoin. Acceleration statistics of inertial particles in turbulent flow. European Physical Journal B, 66:531–536, 2008.

[20] N. M. Qureshi, M. Bourgoin, C. Baudet, A. Cartellier, and Y. Gagne. Turbulent transport of material particles: an experimental study of finite size effects. Physical Review Letters, 99:184502, 2007.

[21] C.-M. Tchen. Mean value and correlation problems connected with the motion of small particles suspended in a turbulent fluid. PhD thesis, Delft University of Technology, 1947.

[22] M. Uhlmann. An immersed boundary method with direct forcing for the simulation of particulate flows. Journal of Computational Physics, 209(2):448–476, Nov. 2005.

[23] M. Virant and T. Dracos. 3D PTV and its application on lagrangian motion. Measurement science and technology, 8:1539–1552, 1997.

[24] R. Volk, E. Calzavarini, E. Lévêque, and J.-F. Pinton. Dynamics of inertial particles in a turbulent von karman flow. Journal of Fluid Mechanics, 668:223–235, Feb. 2011.

[25] R. Volk, E. Calzavarini, G. Verhille, D. Lobse, N. Mordant, J. F. Pinton, and F. Toschi. Acceleration of heavy and light particles in turbulence: Comparison between experiments and direct numerical simulations. Physica D, 237(14-17):2084–2089, 2008.

[26] G. A. Voth, A. LaPorta, A. M. Crawford, J. Alexander, and E. Bodenschatz. Measurement of particle accelerations in fully developed turbulence. Journal of Fluid Mechanics, 469:121–160, 2002.

[27] H. Xu and E. Bodenschatz. Motion of inertial particles with size larger than Kolmogorov scale in turbulent flows. Physica D, 227:2095–2100, 2008.