Fuzzy Eqq Model with Shortages Using Kuhn-Tucker Conditions

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Abstract: The work involves purchase inventory model with shortages under fuzzy environment. An EOQ model is formulated in which the input parameters like order cost, demand rate, carrying cost and penalty cost and the decision variables like the maximum inventory level and the lot size are fuzzified using triangular fuzzy membership function. An optimum solution of the model is arrived by using Kuhn-Tucker conditions. The crisp values of the proposed model is obtained by defuzzifying the assumed model using Graded mean Integration (GMI) method. Finally the solutions are tabulated and an analysis of the crisp and fuzzy values of the total cost has been done in this paper.

Keywords: Defuzzification, Graded Mean Integration, Inventory model, Kuhn-Tucker conditions, Triangular fuzzy numbers.

I. INTRODUCTION

The basic question that arrives in an inventory management is that when and how much to order. Researches have been done for the past decades over inventory management describing numerous models with various conditions and assumptions. Harris developed various models based on economic order/production quantity [EOQ/EPQ] model.

An inventory model based on EOQ minimizes mainly the sum of the carrying cost and the set-up cost. The total inventory cost function are minimized by assuming that the input values and the decision variables are crisp in nature. Though the results of these models are obtained with various assumptions, they don’t represent real life situations and hence these models leads to erroneous decisions. In such optimization tasks, fuzzy set theory is more convenient. Hence fuzzy set theory is used in inventory model since uncertainty reflects reality.

Zadeh (2) first introduced the concept of fuzzy set theory in decision making. Following Zadeh, Zimmerman (3) introduced vast applications of fuzzy set theory. An inventory model with warehouse constraints has been solved using fuzzy dynamic programming approach by Sommer (4). An EOQ model with ordering cost and inventory holding cost as trapezoidal fuzzy numbers was proposed by Park (5). Roy and Maiti (6) fuzzified the annual total cost function and the available warehouse space after transforming an EOQ model into a nonlinear programming problem.

A multi item inventory model with uncertain warehouse space and number of orders with some fuzzy input parameters was developed by Mandal and Maiti(20). An EOQ model with total demand and unit carrying cost as triangular fuzzy numbers was proposed by Yao and Chiang (8) and defuzzified the parameters using signed distance and centroid method. Chang et al (9) proposed an inventory model with backorders as triangular fuzzy numbers. On the other hand, Bjork (13) investigated an EOQ model by fuzzifying the demand and lead time.

In this paper an EOQ model with input parameters like order cost, demand rate, holding cost and penalty cost and also the controlled variables like maximum inventory level and batch size are being considered as triangular fuzzy numbers. The proposed model is then defuzzified using Graded Mean Integration(GMI) method and optimal solution is obtained using extended Lagrangian method. Numerical solutions are obtained and a sensitivity analysis is done to compare the crisp and the fuzzy values.

II. FUZZY SETS AND FUZZY LOGIC

Fuzzy sets provide a mathematical way to represent impreciseness and fuzziness in real world situations. A fuzzy set is a set containing elements that have varying degrees of membership in the set.

A. Definition 1

A fuzzy set is any set that allows its members to have different degrees of membership, called membership function having interval (0,1).

B. Definition 2

Let X denotes the universe of discourse. Then the fuzzy subset of A of X is defined by the membership function $\mu_A(x) : X \rightarrow [0,1]$ which assigns a real number $\mu_A(x)$ in the interval [0,1], to each element $x \in X$, where the values of $\mu_A(x)$ of $x$ shows the grade of membership of $x \in X$.

C. Definition 3

The fuzzy number A is said to be triangular fuzzy number if it is fully determined by $(a_1, a_2, a_3)$ such that $a_1 < a_2 < a_3$ whose membership function, can be denoted by
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IV. NOTATIONS

Notations used in the proposed EOQ problem are as follows:

- Total Cost: \( TC \)
- Lot size or batch size: \( Q \)
- Maximum inventory level: \( I \)
- Order cost per cycle: \( S \)
- Demand rate per cycle: \( D \)
- Holding cost per unit per cycle: \( H \)
- Penalty cost per unit per cycle: \( m \)

Here, \( S, D, H, \) and \( m \) are input parameters and \( Q, I \) are decision variables.

Total Cost = Order cost + Holding cost + Penalty cost

\[
TC = \frac{SD}{Q} + \frac{I^2H}{2Q} + \frac{(Q-I)^2m}{2Q} + \frac{Qm}{2} - Im
\]

V. EOQ MODEL WITH BACKORDERS

The Economic order quantity (EOQ) is the number of units that a company should add to inventory with each order to minimize the total cost of inventory such as holding costs, order costs and penalty costs. This is a valuable tool that enable the entrepreneurs to decide the quantity of the lot size and to minimize the cost of uncontrolled parameters.

Total Cost = Order cost + Holding cost + Penalty cost

\[
TC = \frac{SD}{Q} + \frac{I^2(H+m)}{2Q} + \frac{Qm}{2} - Im
\]

VI. FUZZY EOQ MODEL WITH BACKORDERS

The proposed inventory model with backorders is completely fuzzified in this section using triangular fuzzy numbers.

E. Fuzzification of input parameters

Order cost: \( \hat{B} = (S - \varphi_i, S, S + \varphi_i), S > \varphi_i \)

Demand rate: \( \hat{B} = (D - \varphi_i, D, D + \varphi_i), D > \varphi_i \)
Holding cost: \( \ddot{H} = (H - \varphi_5, H + \varphi_5), H > \varphi_5 \)
Penalty cost: \( \ddot{m} = (m - \varphi_5, m + \varphi_5), m > \varphi_5 \)
where \( \varphi_5, i = 1, 2, 3, \ldots 8 \) is the amount by which a parameter arbitrarily deviates (above or below) from its base value, such that \( S > \varphi_1, D > \varphi_3, H > \varphi_3 \) and \( m > \varphi_7 \).

VII. FUZZIFICATION OF DECISION VARIABLES

Maximum Inventory level:
\( \ddot{F} = (I - \varphi_5, 1, I + \varphi_1), I > \varphi_9 \)
Lot size: \( \ddot{Q} = (Q - \varphi_1, Q, Q + \varphi_2), Q > \varphi_{11} \)
The values of \( \varphi_i, i = 1, 2, 3, \ldots 12 \) are determined by the decision makers. The fuzzy total annual cost function can be written as
\[
\ddot{C} = \frac{S \ddot{F} + \ddot{m} Q}{2 \ddot{Q}^2} \cdot \frac{\ddot{m} \ddot{F} + Q}{2}
\]
(5)
Where
\[
\frac{S \ddot{F} + \ddot{m} Q}{2 \ddot{Q}^2} = \frac{(S - \varphi_1)(D - \varphi_3)}{Q + \varphi_{12}}, SD
\]
(6)
\[
\frac{S \ddot{F} + \ddot{m} Q}{2 \ddot{Q}^2} = \frac{(S - \varphi_1)(I - \varphi_9)}{2Q + \varphi_{12}}
\]
(7)
\[
\ddot{m} \ddot{F} + Q = \frac{(H - \varphi_5, m - \varphi_7)}{2Q + \varphi_{12}}, \frac{Qm}{2}
\]
(8)
\[
\ddot{m} \ddot{F} + Q = \frac{-(I + \varphi_0)(m + \varphi_5), -(I - \varphi_0)(m - \varphi_7)}{2}
\]
(9)
Substitute equations (6) – (9) in equation (5),
\[
\ddot{C} = [C_1, C_2, C_3]
\]
where
\[
C_1 = \frac{(S - \varphi_1)(D - \varphi_3)}{Q + \varphi_{12}} + \frac{(H - \varphi_5 + m - \varphi_7)(I - \varphi_9)^2}{2(Q + \varphi_{12})}
\]
(10)
\[
C_2 = \frac{SD}{Q} + \frac{(H + m)^2}{2Q} + \frac{Qm}{2} - Im
\]
(11)
\[
C_3 = \frac{(S - \varphi_1)(D - \varphi_3)}{Q - \varphi_{11}} + \frac{(H + \varphi_5 + m + \varphi_7)(I + \varphi_{10})^2}{2(Q - \varphi_{11})}
\]
\[
+ \frac{(Q + \varphi_{12})(m + \varphi_7)}{2} - (I - \varphi_0)(m - \varphi_7)
\]
(12)
\[
\theta(\ddot{C}(\ddot{Q}, \ddot{F})) = \frac{1}{6} \left[ \frac{(S - \varphi_1)(D - \varphi_3)}{Q - \varphi_{11}} + \frac{(H - \varphi_5 + m - \varphi_7)(I - \varphi_9)^2}{2(Q - \varphi_{11})}
\]
\[
+ \frac{(Q + \varphi_{12})(m + \varphi_7)}{2} - (I - \varphi_0)(m - \varphi_7)
\]
\[
\right]
(13)
Let \( I = I_1, I = I_2, I + \varphi_{10} = I_3 \)
and \( Q - \varphi_1 = Q_1, Q = Q_2, Q + \varphi_{12} = Q_3 \), substitute this in equation (13) we get
\[
\theta(\ddot{C}(\ddot{Q}, \ddot{F})) = \frac{1}{6} \left[ \frac{(S - \varphi_1)(D - \varphi_3)}{Q_3} + \frac{(H - \varphi_5 + m - \varphi_7)I_1^2}{2Q_3}
\]
\[
+ \frac{(Q_3)(m - \varphi_7)}{2} - (I_3)(m + \varphi_5)
\]

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\[ \frac{2}{3}
\begin{bmatrix}
SD + (H + m)I_1^2
Q_2^2
+ \frac{Q_3}{2}m
- I_2m
\end{bmatrix}
\]

\[ \frac{1}{6}
\begin{bmatrix}
(S + \varphi_5)(D + \varphi_4)
Q_1
+ (H + \varphi_6 + m + \varphi_8)(I_3)^2
2(Q_1)
+ \frac{(Q_3)(m + \varphi_8)}{2}
- I_1(m - \varphi_7)
\end{bmatrix}
\]

where \( 0 \leq I_1 \leq I_2 \leq I_3 \) and \( 0 \leq Q_1 \leq Q_2 \leq Q_3 \)

The optimal solution of the total annual cost function given by the equation (14) subject to the inequality constraints \( I_1 - I_2 \leq 0 \), \( I_2 - I_3 \leq 0 \), \(-I_1 < 0\) and \( Q_1 - Q_2 \leq 0 \), \( Q_2 - Q_3 \leq 0 \), \(-Q_1 < 0\), is obtained using KKT conditions technique.

\[ \frac{1}{6}
\begin{bmatrix}
(H - \varphi_5 + m - \varphi_3)I_1
\end{bmatrix}
\]

\[-\lambda_1 + \lambda_3 \leq 0 \] (15.1)

\[ \frac{2}{3}
\begin{bmatrix}
(H + m)I_2
\end{bmatrix}
\]

\[-m
\]

\[+ \lambda_1 - \lambda_2 \leq 0 \] (15.2)

\[ \frac{1}{6}
\begin{bmatrix}
(m + \varphi_8) +

(H + \varphi_6 + m + \varphi_8)I_3
\end{bmatrix}
\]

\[Q_1
\]

\[+ \lambda_2 \leq 0 \] (15.3)

\[ \frac{1}{2}
\begin{bmatrix}
(H + \varphi_6 + m + \varphi_8)I_3^2
\end{bmatrix}
\]

\[2Q_1^2
\]

\[-(S + \varphi_5)(D + \varphi_4)
\]

\[Q_1
\]

\[-\lambda_4 + \lambda_6 \leq 0 \] (15.4)

\[ \frac{2}{3}
\begin{bmatrix}
SD
Q_2^2
- \frac{(H + m)I_1^2}{2}
+ \frac{m}{2}
\end{bmatrix}
\]

\[I_1 - I_2 \leq 0 \] (15.13)

\[I_2 - I_3 \leq 0 \] (15.14)

\[-I_1 < 0 \] (15.15)
After solving the equations (15.1) to (15.24), the solution is

\[
Q^* = \frac{1}{A} \left[ \frac{m - \varphi_4 + 4m + m + \varphi_5}{H - \varphi_1 + 4H + H + \varphi_0} \right]
\]

\[
I^* = \frac{1}{A} \left[ \frac{m - \varphi_4}{H - \varphi_1 + m - \varphi_4 + 4H + m + \varphi_5} \right]
\]

VIII. NUMERICAL EXAMPLES WITH OPTIMAL SOLUTION

Table I Triangular Fuzzy number for S, D, H and m.

| S    | D            |
|------|--------------|
| (15,150,165) | (6000,6000,78000) |
| (30,150,195)  | (12000,60000,84000) |
| (45,150,180)  | (30000,60000,72000) |
| (60,150,225)  | (36000,60000,90000) |
| (75,150,210)  | (42000,60000,93000) |
| H    | m            |
| (0.05,0.25,0.34) | (0.5,5,6.5) |
| (0.06,0.25,0.36) | (1.0,5,7.6) |
| (0.11,0.25,0.38) | (1.5,5,7.5) |
| (0.09,0.25,0.39) | (2.6,5,6.5) |
| (0.10,0.25,0.48) | (2.4,5,8.5) |

Table II Optimal policy for crisp and fuzzy values.

| Q*  | I*  | TC*(crisp) | TC*(Fuzzy) | Membership function Q* |
|-----|-----|------------|------------|------------------------|
| 8601.18 | 8185.17 | 1752.76 | 1896.22 | 0.5029 |
| 8830.05 | 8412.37 | 1875.88 | 1990.94 | 0.9248 |
| 8426.14 | 8014.37 | 1930.26 | 2790.28 | 0.0650 |
| 9107.50 | 8666.73 | 2068.38 | 2137.80 | 0.2312 |
| 8836.09 | 8406.26 | 2136.20 | 2213.60 | 0.9090 |

IX. CONCLUSION

An EOQ model with shortages is developed here under fuzzy environment by applying triangular fuzzy numbers.

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