Fuzzy evaluation method of pipeline passing through dike based on Uncertain Analytic Hierarchy Process

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Abstract: In view of the characteristics of many factors influencing the safety risk of the pipeline through the embankment, which are hierarchical and uncertain, this paper combines the uncertain analytic hierarchy process (AHP) and fuzzy operation to establish a fuzzy comprehensive evaluation model. The uncertainty analytic hierarchy process (AHP) was used to determine the weight of the evaluation indicators, considering the uncertainty of the relative importance of the evaluation indicators at the same level, the credibility of experts and the degree of difference. According to the principle of maximum membership degree, the evaluation grade of the pipeline is determined. Finally, the feasibility of this method is verified by combining with specific engineering examples to reduce the potential loss and provide the safety management of the pipeline.

1. Introduction

The stress situation of the pipeline crossing project is complex, and it is greatly affected by uncertain risk factors. Once the pipeline system has an accident, it will cause serious harm. Therefore, based on the analysis of the characteristics of the dike crossing pipeline project and the research status at home and abroad[1-3], this paper established a complete safety risk assessment system for the dike crossing pipeline, taking full account of the systematic and complexity of the sluice project, using uncertain analytic hierarchy process (AHP) and fuzzy comprehensive assessment method[4] to evaluate the safety risk of the dike crossing pipeline, and obtains its safety level, this method was applied to an engineering example, and the effectiveness of the proposed method has been proved by an example.

2. Safety risk evaluation index system of pipeline through dike.

Based on the principles of comprehensiveness, scientificity, relative independence, hierarchy, and operability, and referring to relevant specifications and documents[5-8], a three-level safety risk evaluation index system for the pipeline through dike was established, which took the safety risk of the pipeline through dike as the overall goal, safety, applicability and durability as the sub goal, and the factors easy to describe and quantify as the basis for evaluation indexes. According to the subordinate relationship and hierarchical structure of each index, the safety risk assessment index system of the pipeline crossing the dike was established as shown in Figure 1:
3. **Specific steps of fuzzy comprehensive evaluation**

Fuzzy comprehensive evaluation mainly includes three aspects: the determination of weight vector, the establishment of fuzzy comprehensive evaluation matrix, and the selection of operator for fuzzy synthesis. The specific operation steps are as follows:

3.1. **Determine the main set of evaluation factors**

Firstly, the evaluated object was analyzed to find out the factors that affected the evaluated object, and then the evaluation factor set was established with corresponding measurement indexes \( D = \{d_1, d_2, \ldots, d_m\} \).

3.2. **Determine the weight vector of evaluation indicators**

According to the importance of each index factor, the uncertain analytic hierarchy process was used to realize the empowerment of the index. The weight was generally expressed as \( w_i (i = 1, 2, \ldots, m) \), where \( w_i (i = 1, 2, \ldots, m) \), \( c_i \) represents the importance degree of the index \( c_i \) \( (i = 1, 2, \ldots, m) \), meets

\[
\sum_{j=1}^{n} w_j = 1, 0 \leq w_j \leq 1
\]

In the traditional AHP method, the "1-9 scale scale" was used to determine the importance of the two evaluation indicators, and a certain value between 1/9 and 9 was used to represent the comparison results. The use of deterministic values to represent uncertain things was bound to be different from the actual situation, then unable to reflect this characteristic, and inconsistent with the actual situation. AHP used an interval number to replace the definite value in AHP to form an uncertain type judgment matrix,
which was consistent with the fuzziness and uncertainty of judgment and eliminated the defects of AHP.

3.2.1. Construct interval number judgment matrix
Similarly, the pairwise comparison method was adopted to create the interval judgment matrix, but the judgment result was no longer determined value but interval scale. Diagonal elements were specified as \([1,1]\), and the interval number judgment matrix was defined as:

\[
A = (A_{ij})_{n \times n}, \quad A_{ij} = [a_{ij}, b_{ij}],
\]

Then A is an interval number judgment matrix, and satisfies:

1) \[9 \geq b_{ij} \geq a_{ij} \geq \frac{1}{9};
\]
2) \[A_{ii} = [1,1], i = 1,2, \ldots, n ; \]
3) \[A_{ij} = \frac{1}{A_{ji}}, A_{ij} = [b_{ij}^{-1}, a_{ij}^{-1}].
\]

Generally, only the upper triangular elements of the judgment matrix need to be expressed. Due to the uncertainty and one-sidedness of the understanding of the objective things, a confused judgment matrix may be constructed which can not stand the scrutiny, resulting in the wrong judgment. Therefore, in order to ensure the reliability of the judgment results, it is necessary to test the consistency of the judgment matrix. Therefore, referring to [9], this paper uses an optimization algorithm to check the consistency of the judgment matrix.

3.2.2. Calculation of weight vector
From the interval judgment matrix, a deterministic judgment matrix satisfying the reciprocity can be calculated, as shown in formula (1).

\[
m_{ij} = \left( \prod_{k=1}^{n} \frac{a_{ik}b_{kj}}{a_{jk}b_{ik}} \right)^{1/n}, \quad i, j = 1,2, \ldots, n
\]

From this, the weight vector of the determinate judgment matrix can be obtained, as shown in formula (2).

\[
w_{ij} = \frac{\prod_{k=1}^{n} a_{ik}b_{kj}^{2}}{\sum_{i=1}^{n} \prod_{k=1}^{n} a_{ik}b_{kj}}, \quad i = 1,2, \ldots, n
\]

3.2.3. Weight correction value
There is an error between the weight vector \(w\) of M definite stereotyping judgment matrix and the weight of the interval number judgment matrix. The error transfer theory is used to calculate the modified weight value \(\overline{w}_{ij}\). The calculation formula is shown in formula (3). make

\[
\Delta_{1} m_{ij} = (m_{ij} - a_{ij}), \Delta_{2} m_{ij} = (b_{ij} - m_{ij})
\]

\[
\overline{w}_{ij} = (w_{ij} - \Delta_{1} w_{ij}, w_{ij} + \Delta_{2} w_{ij}) \quad j = 1,2, \ldots, n
\]

In formula, \((\Delta_{1} w_{ij})^2 = \frac{1}{(\sum m_{ij})^2} \sum_{i=1}^{n} \Delta_{1}^2 m_{ij}, (K = 1,2, \ldots; i = 1,2, \ldots, n)\)

3.2.4. Calculation of expert credibility [10-11]
(1) The judgment matrix constructed by experts can reflect the level of experts, so the credibility of experts can be expressed by the credibility of the judgment matrix. The closer the comprehensive result of an expert's judgment and group judgment is, the higher the credibility is, the greater the weight should be. The reliability of experts should be calculated from two aspects: the general similarity and the local difference.
(2) similarity. We used the angle between two vectors to indicate the similarity of expert evaluation. The smaller the angle between two vectors was, the larger the other chords were, the higher the similarity of two vectors was; on the contrary, the lower the similarity was. \( \eta_{ij} \) was the cosine value of the angle \( \theta \) between the vectors \( a_i \) and \( a_j \), which was called the geometric similarity coefficient:

\[
\eta_{ij} = \cos \theta = \frac{(a_i, a_j)}{\|a_i\|\|a_j\|}
\]

Make \( \eta^{(k)} = \sum_{j=1}^{m} \eta_{ij} - 1 \)

Formula (4) represents the sum of the similarity between vector \( a_k \) and other vectors. The larger the value of \( \eta^{(k)} \) is, the higher the similarity between \( A^{(K)} \) with other matrices is; otherwise, the lower the similarity is. For \( \eta^{(k)} \) normalization, there are

\[
u^{(k)} = \frac{\eta^{(k)}}{\sum_{i=1}^{m} \eta^{(i)}}
\]

That is to say \( e \) is the average value of each expert's evaluation value for the \( i \) evaluation index.

Make \( \sigma_{k} = \sum_{j=1}^{m} |a_{ij} - e_{i}| \) \((k = 1, 2, \ldots, m) \)

\( \sigma_{k} \) represents the sum of the difference between the evaluation value of each evaluation index by the \( K \) expert and the corresponding mean value.

Then the ratio \( \delta^{(k)} \) of the sum of the differences between the \( K \) expert and the sum of the differences judged by the \( m \) experts is defined as the degree of difference of the \( K \) expert, which is

\[
\delta^{(k)} = \frac{\sigma_{k}}{\sum_{k=1}^{m} \sigma_{k}} \quad (k = 1, 2, \ldots, m)
\]

(3) reliability calculation.

Taking the difference and similarity in group judgment as the variables of credibility, and determining the size of the two, the credibility of experts can be calculated according to the following formula,

\[
a_{k} = \begin{cases} 
\frac{\nu^{(k)}(1 - \delta^{(k)})}{1 - \sum_{i=1}^{m} \nu^{(i)} \delta^{(i)}} & \text{if} \ \sum_{i=1}^{m} \nu^{(i)} \delta^{(i)} \neq 1 \\
\nu^{(k)} & \text{if} \ \sum_{i=1}^{m} \nu^{(i)} \delta^{(i)} = 1 
\end{cases}
\]

\( a_{k} (k = 1, 2, \ldots, m) \) is the reliability of the judgment matrix given by the \( K \) expert.

3.2.5. Calculate the final weight

The final weight \( w_{i} \) can be obtained from the subjective and objective factors \( w_{i1} \) and \( w_{i2} \). The subjective weight \( w_{i1} \) represents the level of knowledge structure of experts, and the objective weight \( w_{i2} \) represents the degree of assurance and objectivity of all experts. The formula of subjective weight \( w_{i1} \) is shown in equation (7), and the formula of objective weight \( w_{i2} \) is shown in equation (9), the the formula of final weight \( w_{i2} \) is shown in equation (10),
\[ w_{ij} = G_i(U) = \frac{1}{2} \left( \frac{\sum_{k=1}^{m} a_k \left[ (u_{ik}^2) - (u_{kj}^2) \right]}{\sum_{k=1}^{m} a_k \left[ (u_{ik}^2) - (u_{kj}^2) \right]} \right) \]  \hfill (8)

\[ w_{i2} = \frac{b_{i2}}{\sum b_{i2}}, \quad i = 1, 2, \ldots, n \]

\[ b_{i2} = \frac{1}{1 + g}, \quad g = \frac{1}{3} \sum_{k=1}^{m} a_k \left[ \left( u_{i2}^2 - G_i(U) \right)^2 - \left( u_{i1}^2 - G_i(U) \right)^2 \right] \]

\[ w_j = \frac{w_{j1} w_{j2}}{\sum_{i=1}^{n} w_{j1} w_{j2}}, \quad i = 1, 2, \ldots, n \]  \hfill (9)

3.3. Establishing a Fuzzy Judgment Matrix

According to the established evaluation factor set at each level, each underlying factor is scored and dimensionless processed. Then, fuzzy processing is carried out through membership function to obtain the fuzzy matrix \( R = (r_{ij})_{m \times n} \) of the underlying evaluation, where \( r_{ij} \) represents the proportion of the \( i \) evaluation index corresponding to the grade \( j \) in the evaluation set.

3.4. Identifying Comment Sets

Determining the set of comments is the rating standard. According to the relevant regulations, comment set \( C = \{c_1, c_2, \ldots, c_n\} \) was established for the pipeline through the embankment.

3.5. Fuzzy synthesis operation

According to the determined weight vector and fuzzy matrix, the membership degree of the evaluated object to each evaluation level was obtained by using fuzzy synthesis operation. The result of fuzzy comprehensive evaluation was denoted as \( B = (b_1, b_2, \ldots, b_n) \), and the fuzzy operation formula was

\[ B = W \circ R = \left( w_1, w_2, \ldots, w_n \right) \circ (r_{ij})_{m \times n} \]

\[ = \left( w_1, w_2, \ldots, w_n \right) \circ \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & r_{m3} & r_{m4} \end{bmatrix} \]

\[ = (b_1, b_2, b_3, b_4) \]  \hfill (11)

Among them, “\( \circ \)” was the symbol of fuzzy synthesis operation, and weighted average operator was used in this paper. Finally, according to the principle of maximum membership degree, the corresponding safety risk level of the pipeline can be obtained.

4. Membership function of comprehensive evaluation

In combination with the safety risk assessment set of the pipeline across the embankment, the corresponding value range of each comment set for the safety risk assessment of the pipeline across the
embankment was given below.

| Grade          | security | Safer | Basic security | Unsafe |
|----------------|----------|-------|----------------|--------|
| Grading range  | 9~10     | 7~9   | 5~7            | 0~5    |

In this paper, the membership function as shown in equation (13) was adopted,

\[
g'_1(D) = \begin{cases} 
1 & \text{When } D \geq 10 \\
(D - 9) / (10 - 9) & \text{When } 9 \leq D \leq 10 \\
0 & \text{other} 
\end{cases} \\
g'_2(D) = \begin{cases} 
1 & \text{When } D \geq 10 \\
(10 - D) / (10 - 9) & \text{When } 9 \leq D \leq 10 \\
0 & \text{other} 
\end{cases} \\
g'_3(D) = \begin{cases} 
1 & \text{When } D \geq 10 \\
(D - 7) / (9 - 7) & \text{When } 7 \leq D \leq 9 \\
0 & \text{other} 
\end{cases} \\
g'_4(D) = \begin{cases} 
1 & \text{When } D \geq 10 \\
(9 - D) / (9 - 7) & \text{When } 7 \leq D \leq 9 \\
0 & \text{other} 
\end{cases} \\
\]

(12)

The index values of each evaluation index were substituted into the above equation to obtain the membership degrees of different grades, and the fuzzy comprehensive evaluation matrix was calculated at the same time.

In this paper, the evaluation indexes of each layer and the final safety risk evaluation indexes were divided into four grades: \( \omega = \{ \omega_1, \omega_2, \omega_3, \omega_4 \} = \{ \text{safety, relatively safety, basic safety, unsafe} \} \). In this paper, we used \([0,10]\) as the evaluation interval, and defined the corresponding score interval of four grades as better \([9,10]\), good \([7,9]\), general \([5,7]\), poor \([0,5]\). The higher the score is, the safety of the pipeline passing through the dike will be better.

5. Engineering example

An oil pipeline through dike was selected as a case for evaluation and calculation. The length of the pipe is 230km, which was built in 1972. The material is steel pipe, with a buried depth of 2.5m and a diameter of 450mm. The buried section of the pipeline is prosperous and has a large traffic flow.

According to the calculation principle of the uncertain analytic hierarchy process, the weight values of each index of the pipeline through the dike were calculated by MATLAB programming, as shown in Table 2. By referring to the field measured data and according to the given scoring criteria, the experts have given the index scores of the breakwater pipeline as shown in Table 2 below.
Then, the matrix of the underlying index was calculated by membership function, and the matrix was obtained by fuzzy synthesis with formula (12)

\[
B_1 = W_1 \circ R_1 = [0.393, 0.252, 0.193, 0.162] \odot \begin{bmatrix}
0 & 1 & 0 & 0 \\
0.8 & 0.2 & 0 & 0 \\
0.5 & 0.5 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix} = [0.4601, 0.5399, 0, 0]
\]

\[
B_2 = W_2 \circ R_2 = [0.687, 0.313] \odot \begin{bmatrix}
0 & 0.5 & 0.5 & 0 \\
0.85 & 0.15 & 0 & 0
\end{bmatrix} = [0.6095, 0.3905, 0]
\]

\[
B_3 = W_3 \circ R_3 = [0.382, 0.216, 0.157, 0.133, 0.112] \odot \begin{bmatrix}
0 & 0.35 & 0.65 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0.5 & 0.5 & 0 \\
0 & 0 & 0.86 & 0.14 \\
0 & 0.25 & 0.75 & 0
\end{bmatrix} = [0.2402, 0.7412, 0.0186]
\]

\[
B_4 = W_4 \circ R_4 = [0.596, 0.404] \odot \begin{bmatrix}
0 & 0.85 & 0.15 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} = [0.9106, 0.0894, 0]
\]

\[
B = W \circ R = [0.378, 0.261, 0.204, 0.157] \odot \begin{bmatrix}
0.4601 & 0.5399 & 0 & 0 \\
0 & 0.6095 & 0.3905 & 0 \\
0 & 0.2402 & 0.7412 & 0.0186 \\
0 & 0.9106 & 0.0894 & 0
\end{bmatrix} = [0.1739, 0.5551, 0.2672, 0.0038]
\]

According to the principle of maximum membership, it can be seen that the safety risk of the pipeline was relatively safe.

6. Conclusion
Considering the uncertainty and fuzziness of people's judgment, this paper introduces the uncertain type analytic hierarchy process (AHP) to calculate the weight of each index in the safety evaluation system. The comprehensive safety evaluation system of the pipeline passing through the dike was established, and the practicability of the method was verified by an example.

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