COOPER-MESONS IN THE COLOR-FLAVOR-LOCKED
SUPERCONDUCTING PHASE OF DENSE QCD

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QCD superconductors in the color-flavor-locked (CFL) phase sustain excitations
(“Cooper” mesons) that can be described as pairs of particles or holes around a
gapped Fermi surface. In weak coupling and to leading logarithm accuracy the
masses, decay constants and form factors of the scalar, pseudoscalar, vector and
axial-vector excitations, which explicitly are of finite size, can be calculated ex-
actly. Furthermore, the constraints of this microscopic calculation on the effective-
lagrangian description and the computation of the generalized triangle anomaly
are discussed.

1 Introduction

Quantum chromodynamics (QCD) at high density, relevant to the physics
of the early universe, compact stars and relativistic heavy ion collisions, is
presently attracting a renewed attention from both nuclear and particle physi-
cists. For large chemical potential, \( \mu \gg \Lambda_{QCD} \), quarks at the edge of the
Fermi surface interact weakly, although the high degeneracy of the Fermi sur-
face causes perturbation theory to fail. Thus particles can pair as diquarks
and condense at the boundary of the Fermi surface leading to energy gaps and
therefore to a superconducting ground state

\[ \text{1}, \text{2}, \text{3}, \text{4}, \text{5}, \text{6}, \text{7}\]. The resulting QCD su-
perconductor breaks color and flavor symmetry spontaneously. Therefore, the
ground state exhibits Goldstone modes that are either particle-hole excitations
(ordinary pions) or particle-particle and hole-hole excitations (BCS pions \( \tilde{\pi} \))
with a mass that vanishes in the chiral limit.

Effective-lagrangian approaches to QCD in the color-flavor-locked (CFL)
superconducting phase\[s\] provide a convenient description of the long-wavelength
physics structured by global flavor-color symmetries (incl. anomalies)\[a\].

\[ \mathcal{L} = \frac{F_T^2}{4} \text{Tr} \left( \partial_0 \tilde{U} \partial_0 \tilde{U}^\dagger \right) - \frac{F_T^2}{4} \text{Tr} \left( \partial_i \tilde{U} \partial_i \tilde{U}^\dagger \right) + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{WZW}} + \mathcal{L}_{\text{h.o.}}, \quad (1) \]

where \( \tilde{U} = \exp(i \sigma^a \tilde{\pi}^a / F_T) \) is the pertinent unitary chiral matrix of the pseudo-
scalar Goldstone modes \( \tilde{\pi}^a \) which are of diquark nature here. The nonlinear
sigma-model part of this effective lagrangian is of the nonrelativistic antiferromagnetic type introduced by Leutwyler. Note that the space-like pion decay constant \( F_S \) is smaller than the time-like one \( F_T \). This is a common property for the propagation of Goldstone modes in a matter background. \( L_{\text{mass}} \) is the explicitly symmetry-breaking part, \( L_{\text{WZW}} \) is the (generalized) Wess-Zumino-Witten term, and \( L_{h.o.} \) stands for all higher order derivative or mass contributions.

However, such an effective-lagrangian approach is intrinsically based on a point-like description and does not allow a direct calculation of the underlying parameters that are important for a quantitative description of the bulk (thermodynamic and transport) properties of the QCD superconductor. This requires a more microscopic description. In this talk, I report about such a work where the excitations of QCD superconductors in the CFL phase are described microscopically as composite (finite size) pairs of quasi-particles or quasi-holes around the gaped Fermi surface. The results of microscopic calculations in the leading logarithm approximation of the masses and decay constants of these generalized (alias “Cooper”) mesons are reviewed.

2 QCD Superconductor in the CFL Phase

In the CFL phase of the QCD superconductor, the quarks are gapped. Their propagation in the chiral limit is conveniently described in the Nambu-Gorkov formalism where the fermi field \( \psi \) is doubled to \( (\psi, \psi^C) \) with \( \psi^C \equiv \bar{C} \gamma^5 \psi \). For large chemical potential \( \mu \), the antiparticles decouple: the particle/hole energies are \( \epsilon_q \approx \mp (q_0^2 + G(q)\Lambda^\perp)^{1/2} \), whereas the antiparticle/antiholes ones are \( \bar{\epsilon}_q \approx \mp 2\mu \), with \( q_0 = |q| - \mu \) the momentum measured parallel to the Fermi momentum. Thus the quark-propagator in the Nambu-Gorkov space reads:

\[
S \approx \begin{pmatrix} \gamma^0 (q_0 + q_1) \Lambda^-(q) & -M^\dagger G^*(q) \Lambda^+(q) \\ M G(q) \Lambda^-(q) & \gamma^0 (q_0 - q_1) \Lambda^+(q) \end{pmatrix} \frac{1}{q_0^2 - \epsilon_q^2} \]  

(2)

in terms of the projectors \( \Lambda^\pm(q) = \frac{1}{2}(1 \pm \alpha \cdot \hat{q}) \) onto positive and negative energies and the color-flavor-locking matrix \( M_{f,f'} = \epsilon_{f_f'} \epsilon_{c_c} \gamma_5 \) of the CFL phase. \( G(q) \) is the gap function (not a constant) that satisfies the gap equation of Fig.:

\[
G(p_||) \approx \frac{\hbar^2}{6} \int_0^\infty dq_|| \frac{G(q_||)}{\sqrt{q_||^2 + |G(q_||)|^2}} \ln \left\{ \left( 1 + \frac{\Lambda^2}{(p_|| - q_||)^2 + m_E^2} \right)^3 \right. \\
\left. \times \left( 1 + \frac{\Lambda^2}{|p_|| - q_||^3 + \frac{\pi}{4} m_E^2 |p_|| - q_||^2} \right)^2 \right\}, \]  

(3)
Figure 1: BCS gap equation. The thin and thick lines are the free and dressed quark propagator, respectively, whereas the wiggly line is the gluon propagator. The indices refer to the off-diagonal components in the Nambu-Gorkov space

where $h^2 = \frac{4g^2}{\sqrt{3}}$ in terms of the strong coupling constant $g$. The gap equation is not of a conventional BCS-type because of the long-range structure of the gluon propagator, i.e.

$$D(q) = \frac{1}{2} \frac{1}{q^2 + m_E^2} + \frac{1}{2} \frac{1}{q^2 + m_M^2}$$

in Euclidean space. The gluon propagator is Debye screened ($m_E^2 = m_D^2 \approx N_f (g\mu)^2 / 2\pi^2$) and Landau damped ($m_M^2 \approx \pi m_E^2 |q_4| / |4q|$), where $m_D$ is the Debye mass, $m_M$ is the magnetic screening generated by Landau damping and $N_f$ the number of flavors. In weak coupling, the upper limit for the transverse momenta is $\Lambda_\perp = 2\mu > m_E, m_M$ and the logarithms in Eq. (3) cannot be expanded. Instead logarithmic scales $x = \ln(\Lambda_\perp / q_{||})$ and $x_0 = \ln(\Lambda_\perp / G_0)$ (with $\Lambda_\perp = (4\Lambda_0^6 / \pi m_E^5)$) have to be introduced. To leading logarithm accuracy, the gap function $G(q)$ is then solely a real-valued function of $q_{||}$ given by

$$G(x) = G_0 \sin \left( \frac{\pi x}{2x_0} \right) = G_0 \sin \left( h_* x / \sqrt{3} \right) \quad \text{with} \quad G_0 \approx \frac{4\Lambda_0^6}{\pi m_E^5} e^{-\sqrt{3}h_*/2}. \quad (5)$$

3 Generalized Scalar and Pseudoscalar Mesons

The generalized mesons of the CFL phase are excitations in $qq$, whereas the conventional mesons are excitations in $\bar{q}q$. Their wave-functions follow form the Bethe-Salpeter equation pictorially described in Fig. 2. In particular, in leading logarithm approximation, the scalar vertex operator of ‘isospin’ $A$ for a pair of particles or holes with momenta $P/2 \pm p$ is given in the Nambu-Gorkov space by

$$\Gamma_A^S(p, P) = \frac{1}{F_T} \left( \begin{array}{cc} 0 & i \Gamma^*(p, P) M A \\
0 & 0 \end{array} \right) \left( M A \right)^1$$

with $M A = M^{i\alpha} (\tau^A)^{i\alpha}$ and $M^{i\alpha} = \epsilon_f \epsilon_c \gamma_5$, where $(\epsilon^a)^{bc} = \epsilon^a_{bc}$ is the totally antisymmetric tensor in flavor $(f)$ and color $(c)$ space, respectively. The
corresponding composite pseudoscalar vertex reads

\[ \Gamma_{\pi}^A(p, P) = \frac{1}{F_T} \left( \begin{array}{ccc} 0 & -i\gamma_5 \Gamma_{PS}(p, P) M^A \end{array} \right) . \]  

In the chiral limit, the scalar and pseudoscalar vertex reduces to the gap, i.e. \( \Gamma_{(P)S}(p, 0) = G(p) \). Therefore the scalar and pseudoscalar modes are massless Goldstone modes. In Ref. 16 the generalized PCAC relation

\[ \langle BCS|\mathbf{A}_\mu^a(0)|\tilde{\pi}_B^\beta(P)\rangle \equiv iF \mu \delta^{a\beta} \]  

was studied. It was found there (see also Ref. 10) that the temporal decay constant of the generalized pion is given by \( F_T = \mu/\pi \) whereas the spatial one, \( F_S \), is smaller by a factor \( 1/\sqrt{3} \). Thus the velocity of the Goldstone modes, \( v = F_S/F_T = 1/\sqrt{3} \), is less than the speed of light \( (c \equiv 1) \). The latter property mirrors the propagation of ordinary pions in nuclear matter. The decay constants of the generalized pions are very large compared with the gap function which is bounded by \( G_0 \). Thus the Cooper-pions have a very small (spatial) size (of the order of the inverse momentum exchanged between quark pairs at the Fermi surface), irrespectively of any screening.

The dependence of the mass of the generalized pion on the current quark mass can be estimated with similar methods and was studied in Refs. 10, 16.

4 Higgs Mechanism

The generalized scalar mesons mix with the longitudinal gluons in the CFL phase. In fact, all the 8 generalized scalars are eaten up by the longitudinal gluons, such that the 8 originally massless gluons (with two transverse polarizations) acquire masses (Meissner effect). This Meissner mass refers to the inverse penetration length of static colored magnetic fields in the QCD superconductor which is unexpectedly small, i.e. \( 1/g\mu \). In weak coupling, the Meissner mass is of the order of the electric screening mass \( m_E \approx g\mu \). It
is not of the order of $gG_0$ as in a conventional superconductor with a constant (energy independent) gap. Note, however, that the nonstatic gluonic modes with energy $Q_0 > G_0$ sense “free quarks” for which there is electric screening, but no magnetic screening.

5 Generalized Vector and Axial-Vector Mesons

The pairs of (quasi-)quarks or (quasi-)holes at the Fermi surface can also form generalized vector and axial-vector meson-states in the CFL phase. These composites have a similar form factor as the Cooper-pions and therefore finite size. Because Lorentz invariance is absent, there are electric and magnetic vector-composites. The difference to the (pseudo-)scalar case is the reduced strength of the coupling in the Bethe-Salpeter (BS) equation. Thus the (pseudo-)scalar BS equation admits massless modes, while the (axial-)vector one does not. The mass of the composite vector excitations is close to (but smaller than) twice the gap in weak coupling, but goes asymptotically to zero with increasing coupling thereby realizing Georgi’s vector limit in cold and dense matter. Both the vector and axial-vector Cooper-octets are degenerate in leading-log approximation, in spite of the chiral symmetry breaking of the CFL phase. Furthermore, it is shown in Ref. that the composite vector mesons decouple from the Noether currents and that they do not decay to pions in leading logarithm accuracy, contrary to their analogues in the QCD vacuum. Moreover, they decouple from the gluons and scalars in the CFL phase as well.

6 Finite Size and Hidden Gauge

The quark pairs of the mesons in the QCD superconductor have finite size. Though the space like separation is of the order $1/\mu$ and therefore small, the time-like separation is governed by the scale of the “magnetic mass”

$$1/m_M \approx 1/(m^2_{E}G_0)^{1/3} \gg 1/\mu.$$  \hspace{1cm} (9)

Hidden gauge symmetry and vector meson dominance (VMD) can therefore be only approximate concepts and do not hold in weak coupling and to leading-log approximation.

7 Modified Triangle Anomaly

In the CFL phase the ordinary photon is screened, and the gluons are either screened or higgsed. However, it was pointed out in Ref. that the CFL phase
is transparent to a modified or tilde photon,

\[ \tilde{A}_\mu = A_\mu \cos \theta + H_\mu \sin \theta, \]  

(10)

where \( A_\mu \) is the ordinary photon field, which couples to the charge matrix \( e Q_{em} = e \text{diag}(2/3, -1/3, -1/3) \) of the quarks, and \( H_\mu \) is the gluon field for \( U(1)_Y \) where \( g Y = g \text{diag}(-2/3, 1/3, 1/3) \) is the color-hypercharge matrix. Furthermore, \( \cos \theta = g/\sqrt{e^2 + g^2} \) and \( \sin \theta = e/\sqrt{e^2 + g^2} \). \( \tilde{A}_\mu \) carries color-flavor and tags to the charges of the Goldstone modes. The quark coupling to the tilde photon is in units of \( \tilde{e} = e \cos \theta \). As a result, the CFL phase is characterized by generalized flavor-color anomalies. In Ref. it was shown how the triangle anomaly emerges from a direct calculation of the decay of the generalized pion into two generalized photons, \( \tilde{\pi}^0 \to \tilde{\gamma} \tilde{\gamma} \), in the leading logarithm approximation, i.e.

\[ T^{\tilde{A}}_{\mu
u}(p_1, p_2) = i \frac{\tilde{e}^2}{F_T} \frac{1}{4\pi^2} \text{Tr} \left( \tau^A \tilde{Q}^2 \right) \epsilon_{\mu \nu \alpha \beta} p_1^\alpha p_2^\beta \]  

(11)

(with \( \tilde{Q} = Q_{em} \)). This result is consistent with the modified triangle anomaly

\[ \partial^\mu A_\mu = \frac{\tilde{e}^2}{96\pi^2} \epsilon_{\mu \nu \rho \sigma} \tilde{F}^{\mu \nu} \tilde{F}^{\rho \sigma} \]  

(12)

suggested by the generalized WZW term. Here \( \tilde{F}^{\mu \nu} \) is the field strength associated to Eq. (10). Note that the conventional triangle anomaly is larger by a factor \( N_c \). This factor is absent here, because of the color-flavor locking of the quarks. Furthermore, the internal line between the two generalized-photon vertices in Fig. 3 has to correspond to an antiparticle. Therefore, the amplitude (11) does not have an explicit \( \mu^2 \) dependence which naively would be expected from the momentum integration around the Fermi surface. In fact, because of the dependence on \( F_T = \mu/\pi \), the radiative decay of the “Cooper” pion (11) vanishes as \( 1/\mu \) in dense matter.
8 Conclusion

In this talk, I have reported about the so-called Cooper-mesons: generalized scalar, pseudoscalar, vector and axial-vector particle-particle or hole-hole excitations in the CFL superconductor of QCD in the weak coupling limit. The octet scalar and pseudoscalar excitations are both massless, but only the pseudoscalars survive as Goldstone modes, while the scalars ones are higgsed by the gluons leading to the Meissner effect. The vectors and axial vectors are bound and degenerate irrespectively of their polarization. Their mass is less than twice the gap. Chiral symmetry is explicitly realized in the vector spectrum in the CFL phase in leading logarithm approximation, in spite of its breaking in general. In the CFL superconductor the vector mesons are characterized by form factors that are similar but not identical to those of the generalized pions. These self-generated form factors provide a natural cutoff to regulate the effective calculations at the Fermi surface.

The composite vector mesons can only be viewed as hidden-gauge excitations if their size is ignored (their form factor set to one). Only in this approximate limit the effective lagrangian description, vector dominance and universality are valid. The zero-size limit is not compatible with the weak-coupling limit, because of the long-range pairing mechanism at work at large quark chemical potential. It is an open question whether going beyond the weak-coupling and leading-log approximations would render the hidden-gauge or vector-meson-dominance concepts of effective field theories more appropriate. The finite-size of the generalized pion, however, does not upset the geometrical normalization of the triangle anomaly in the generalized WZW form of Ref.8. Much like in the vacuum, the radiative decay of the generalized pions is dictated by geometry in leading order. Furthermore, it vanishes at asymptotic densities.

The existence of bound light vector and axial-vector mesons in QCD at high density may have interesting consequences on dilepton and neutrino emissivities in dense environments: e.g., in young and hot neutron stars neutrino production via quarks in the superconducting phase can be substantially modified if the vector excitations are deeply bound with a non-vanishing coupling. These excitations may be directly seen by scattering electrons off compressed nuclear matter (with densities that allow for a superconducting phase to form) and may cause substantial soft dilepton emission in the same energy range in “cold” heavy-ion collisions.
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