One-loop Kähler potential in 5D gauged supergravity with generic prepotential

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Abstract

We calculate one-loop contributions to the Kähler potential in 4D effective theory of 5D gauged supergravity (SUGRA) on $S^1/Z_2$ with a generic form of the prepotential and arbitrary boundary terms. Our result is applicable to a wide class of 5D SUGRA models. The derivation is systematically performed by means of an $N = 1$ superfield formalism based on the superconformal formulation of 5D SUGRA. As an illustrative example, we provide an explicit expression of the Kähler potential in the case of 5D flat spacetime.

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1 Introduction

Higher-dimensional supergravities (SUGRA) have been attracted much attention and extensively studied in various aspects, such as the model building in the context of the brane-world scenario, effective theories of the superstring theory or M-theory, AdS/CFT correspondence, etc. Among them, five-dimensional (5D) SUGRA compactified on an orbifold $S^1/Z_2$ has been thoroughly investigated since it is the simplest setup for supersymmetric (SUSY) brane-world models, and it is shown to appear as an effective theory of the strongly coupled heterotic string theory \cite{1} compactified on a Calabi-Yau 3-fold \cite{2}. Besides, SUSY extensions of the Randall-Sundrum model \cite{3} are also constructed in 5D SUGRA on $S^1/Z_2$ \cite{4, 5, 6}.

Models with an extra dimension can easily realize the large hierarchy between the electroweak and the Planck scales or among the fermion masses in the standard model. The former is obtained by the warped geometry along the extra dimension \cite{3}, and the latter is by the wave function localization of matter fields in the extra dimension \cite{7, 8}. In both mechanisms, some mass scales have to be introduced in the 5D bulk. The warped geometry is induced by the 5D cosmological constant, and the wave function profiles are controlled by 5D masses of the matters. In SUGRA context, these mass scales are introduced by gauging some isometries with some 5D vector multiplets. Namely, we have to consider the gauged SUGRA. When the extra dimension is compactified on $S^1/Z_2$, the four-dimensional (4D) vector components in such vector multiplets must be $Z_2$-odd. Every 5D SUGRA model has this type of vector field, i.e., the graviphoton.\footnote{In this paper, the terminology “graviphoton” denotes a vector field in the gravitational multiplet of the on-shell formulation. It should be distinguished from the off-diagonal components of the 5D metric.} Therefore, most models based on 5D gauged SUGRA assume that the vector multiplet that gauges the isometries to induce the mass scales is the graviphoton multiplet. However this is not the only possibilities. There can be other vector multiplets whose 4D vector components are $Z_2$-odd. The 5D mass scales can also be obtained by gauging with these multiplets. Such 5D vector multiplets contain $Z_2$-even real scalar fields. These scalar fields have 4D zero-modes, and do not have any potential terms at least at tree level. Thus we refer to them as moduli in this paper.\footnote{These moduli are actually identified with the shape moduli of the compactified space for a 5D effective theory of the heterotic M-theory on the Calabi-Yau manifold \cite{2}, for example.} In fact, one linear combination of these moduli corresponds to the size modulus of the fifth dimension, i.e., the radion, which belongs to the same 5D supermultiplet as the graviphoton.
In the case that a model has more than one moduli, they generically mix with each other. Such mixing is characterized by a cubic polynomial, which is referred to as the norm function in this paper. This corresponds to the prepotential in 4D $N = 2$ SUSY gauge theories. As mentioned above, most models based on 5D SUGRA implicitly assumed a special form of the norm function such that the radion does not mix with the other moduli. In our previous works [9, 10], we derived 4D effective theory of 5D SUGRA with more than one moduli at tree level, and found that some terms appear in the Kähler potential, which do not exist in the single modulus case. We also showed those terms can significantly affect the flavor structure of the effective theory when the fermion mass hierarchy is realized by the wave function localization, and pointed out a possibility that the SUSY flavor problem is avoided. This indicates an importance of considering arbitrary form of the norm function with multi moduli when we construct a realistic model based on 5D SUGRA.

For a construction of realistic 5D SUGRA models, mediation of SUSY-breaking effects to our observable sector and stabilization of the radion to some finite value are indispensable issues. In some of the mechanisms for them, one-loop quantum corrections to the Kähler potential in 4D effective theory are relevant. For example, SUSY breaking at one of the boundaries of $S^1/Z_2$ can be transmitted to the other boundary where we live by the quantum loop effects of the bulk fields [11]-[14], and the radion can be stabilized by the vacuum energy through the Casimir effect [15]-[21]. The soft SUSY-breaking parameters and the radion mass are induced from the one-loop Kähler potential after taking into account the SUSY-breaking effects. These contributions are finite in spite of the non-renormalizability of 5D SUGRA. This is because each relevant loop diagram must touch both boundaries and cannot shrink to a point. Thus the inverse of the size of the extra dimension provides an effective cutoff in the momentum integral.

The one-loop corrections to the effective Kähler potential in the context of 5D SUGRA have already been discussed in Refs. [13, 14, 22, 23, 24]. However these works assume that the graviphoton multiplet (or the radion multiplet) is the only moduli multiplet which is relevant to the gauging of the isometries to induce the 5D mass scales. As mentioned above, this is only a special case among generic 5D SUGRA. Thus we extend the above works to more general class of theories in this paper. We calculate the one-loop Kähler potential for 5D SUGRA on $S^1/Z_2$ with an arbitrary form of the norm function. Our derivation is performed in an $N = 1$ superfield formalism based on the superconformal formulation of 5D SUGRA [25]-[28], which is developed in our previous works [29, 30]. This makes it
possible to deal with general 5D SUGRA in a systematic and transparent manner. Thus the result is applicable to a wide class of models based on 5D SUGRA.

The paper is organized as follows. In the next section, we briefly review our previous works, which provide an $N=1$ superfield description of 5D SUGRA on $S^1/Z_2$ with an arbitrary prepotential. In Sec. 3 we derive an expression of one-loop contributions to the 4D effective Kähler potential by means of the background field method and the superfield formalism. In Sec. 4, we apply the formula obtained in Sec. 3 in the case that 5D spacetime is flat as an illustrative example. Sec. 5 is devoted to the summary. In Appendix A we list the 5D superconformal transformation laws in terms of the $N=1$ superfields. In Appendix B we collect the definitions of useful projection operators in the $N=1$ superspace and their properties. In Appendix C we review the derivation of the effective Kähler potential at tree level. We show some detailed calculations to pick up quadratic terms for the bulk fluctuation superfields in Appendix D and to derive the boundary conditions for them in Appendix E. In Appendix F we provide an explicit expression of the one-loop Lagrangian in a simple case in terms of the bosonic components of the superfields.

## 2 Superfield description of 5D SUGRA

In this paper, we consider 5D SUGRA compactified on an orbifold $S^1/Z_2$. We take the fundamental region of $S^1/Z_2$ as $0 \leq y \leq L$, where $y$ is the coordinate of the extra dimension. The most general metric for the background spacetime that has the 4D Poincaré symmetry has a form of

$$ds^2 = e^{2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - \langle e^4_y \rangle^2 dy^2,$$

(2.1)

where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, $e^{\sigma(y)}$ is the warp factor, which is determined by solving 5D Einstein equation, and $\langle e^4_y \rangle$ is the background value of the component of the fünfbein $e^4_y$.

Notice that we can always absorb the warp factor in (2.1) by making use of the dilatation symmetry. In fact, the warp factor does not appear explicitly in our calculations since our formalism keeps the superconformal symmetries manifest. The information of the warped geometry is encoded in the gauging for the compensator hypermultiplets.

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3 We can always choose the coordinate $y$ so that $\langle e^4_y \rangle = 1$, but we leave it to be an arbitrary positive value in this paper.
In this section, we review our previous works [29, 30] that complete an $N = 1$ superfield description of 5D SUGRA on $S^1/Z_2$ (see also Refs. [32]-[35]). Our superfield description is based on the superconformal formulation developed in Refs. [25]-[28], and is considered as an extension of Ref. [36] to a generic system of vector multiplets and hypermultiplets.

2.1 Decomposition into $N = 1$ superfields

The 5D superconformal transformations are divided into two parts $\delta_{\text{sc}}^{(1)}$ and $\delta_{\text{sc}}^{(2)}$, where $\delta_{\text{sc}}^{(1)}$ forms an $N = 1$ subalgebra, and $\delta_{\text{sc}}^{(2)}$ is the rest part. As shown in Ref. [28], each 5D superconformal multiplet can be decomposed into $N = 1$ superconformal multiplets, which only respect $\delta_{\text{sc}}^{(1)}$ manifestly. We have explicitly shown in Ref. [37] how each $N = 1$ superconformal multiplet is expressed by an $N = 1$ superfield with the aid of the fields in the gravitational multiplet. We will consider the following three types of 5D superconformal multiplets in this paper.

Hypermultiplet

A hypermultiplet $H^a$ ($a = 1, 2, \cdots, n_C + n_H$) is decomposed into two chiral superfields $(\Phi^{2a-1}, \Phi^{2a})$, which have opposite $Z_2$-parities. We can always label the chiral superfields so that they have the $Z_2$-parities listed in Table I. The hypermultiplets are divided into two classes. One is the compensator multiplets $a = 1, 2, \cdots, n_C$ and the other is the physical matter multiplets $a = n_C + 1, \cdots, n_C + n_H$. The former is auxiliary degrees of freedom and eliminated by the superconformal gauge fixing. The Weyl and the chiral weights of the superfields are also listed in Table I.

Vector multiplet

A vector multiplet $V^I$ ($I = 1, 2, \cdots, n_V$) is decomposed into $N = 1$ vector and chiral superfields $(V^I, \Sigma^I)$, which have opposite $Z_2$-parities. The vector multiplets are also divided into two classes according to their $Z_2$-parities. One is a class of the gauge multiplets, which are denoted as $V^I_e$ ($I_e = 1, \cdots, n_V$). In this class, $V^I_e$ are $Z_2$-even and have zero-modes that are identified with the gauge superfields in 4D effective theory. The other is a class of the moduli multiplets, which are denoted as $V^I_o$.

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4 We do not consider the tensor multiplets, which are discussed in Ref. [38, 39], for simplicity.
5 The number of the compensator multiplets $n_C$ characterizes the hyperscalar manifold. For example, it is $USp(2, 2n_H)/USp(2) \times USp(2n_H)$ for $n_C = 1$, and $SU(2, n_H)/SU(2) \times SU(n_H)$ for $n_C = 2$.
6 The Weyl and the chiral weights are the charges of the dilatation and of $U(1)_A \subset SU(2)_U$, respectively. These weights of a superfield denote those of the lowest component in the superfield.
Table I: The decomposition of 5D superconformal multiplets into \( N = 1 \) superfields. The orbifold \( Z_2 \)-parities, the Weyl and the chiral weights of the \( N = 1 \) superfields are also shown.

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
5D multiplet & Hypermultiplet & Vector multiplet & Weyl multiplet \\
N = 1 superfield & \Phi^{2a-1} & \Phi^{2a} & V^I_o & \Sigma^I_o & V^I_e & \Sigma^I_e & U^\mu & U^y & V_E & \Psi^\alpha \\
Z_2-parity & - & + & - & + & + & - & + & - & + & - \\
Weyl weight & 3/2 & 3/2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
Chiral weight & 3/2 & 3/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\]

Weyl multiplet (Gravitational multiplet)

The 5D Weyl multiplet \( E_W \) is also decomposed into six real superfields \( U^\mu (\mu = 0, 1, 2, 3), U^y \) and \( V_E \) and a complex spinor superfield \( \Psi^\alpha \), which include components of the fünfbein, \( \tilde{e}_{\mu}^{\nu}, e_{\mu}^{4}, e_{y}^{4}, \) and \( e_{y}^{\nu} \), respectively. Here, \( \tilde{e}_{\mu}^{\nu} \equiv e_{\mu}^{\nu} - \delta_{\mu}^{\nu} \) is the fluctuation mode around the background \( \langle e_{\mu}^{\nu} \rangle = \delta_{\mu}^{\nu} \). Since the Weyl multiplet is the gauge multiplet for 5D superconformal symmetry, these superfields transform nonlinearly under \( \delta_{sc}^{(1)} \) and \( \delta_{sc}^{(2)} \) as shown in Appendix A. Hence we cannot assign the Weyl and the chiral weights for them, except for \( V_E \). In fact, \( V_E \) transforms under \( \delta_{sc}^{(1)} \) in a similar way to the vector superfields \( V^I \) because its components do not have 4D Lorentz indices.

2.2 5D SUGRA Lagrangian

5D SUGRA action is determined by 5D superconformal transformations \( \delta_{sc}^{(1)}, \delta_{sc}^{(2)} \) and the supergauge transformation \( \delta_{sg} \) [30]. In the following, we keep terms up to linear order in the

\footnote{The superfield \( U^y \) is related to \( U^4 \) in Ref. [30] by \( U^y = U^4/(V_E) \), where \( \langle V_E \rangle \) is the background value of \( V_E \) and was assumed to be 1 in Ref. [30].}
gravitational superfields for each interaction terms. Basically we use the two-component spinor notations of Ref. [40], except for the metric and the spinor derivatives. We take the convention of the 4D metric as \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) so as to match it to that of Ref. [41], and define the spinor derivatives \( D_\alpha \) and \( \bar{D}^{\dot{\alpha}} \) as

\[
D_\alpha \equiv \frac{\partial}{\partial \theta^\alpha} - i (\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \quad \bar{D}^{\dot{\alpha}} \equiv -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i (\theta \sigma^\mu)_{\dot{\alpha}} \partial_\mu, \tag{2.2}
\]

which satisfy \( \{D_\alpha, \bar{D}^{\dot{\alpha}}\} = 2i \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \). The spinor derivatives are understood as the left-derivatives. It is convenient to define the following differential operators.

\[
\hat{\partial}_y \equiv \partial_y - \left( \frac{1}{4} \bar{D}^2 \Psi D_\alpha + \frac{1}{2} \bar{D}^a \Psi \bar{D}_{\dot{\alpha}} D_\alpha + \frac{w + n}{24} \bar{D}^2 D^a \Psi + \text{h.c.} \right), \\
\Delta_\mu \equiv \frac{1}{4} \sigma^\alpha_{\mu\dot{\alpha}} \left( D_\alpha \bar{D}^{\dot{\alpha}} - \bar{D}^{\dot{\alpha}} D_\alpha \right), \tag{2.3}
\]

where \( w \) and \( n \) are the Weyl and the chiral weights of a superfield which \( \hat{\partial}_y \) acts on, and \( (w + n)^\dagger = w - n \). The spinor derivatives \( D_\alpha^R \) and \( \bar{D}^{\dot{\alpha}}_R \) are defined by the right-derivatives. Then \( \Delta_\mu \) satisfies the Leibniz rule on a product of bosonic superfields. On (anti-)chiral superfields, \( \Delta_\mu = -i \partial_\mu \) \((\Delta_\mu = i \partial_\mu)\). It should be noted that, for a chiral superfield \( \Phi \), \( \partial_y \Phi \) is not a chiral superfield in a superconformal sense because its \( \delta^{(1)}_{sc} \)-transformation law is no longer that of a chiral superfield [30]. Instead, \( \hat{\partial}_y \Phi \) transforms as a chiral superfield under \( \delta^{(1)}_{sc} \). Thus \( \hat{\partial}_y \) is understood as a covariant derivative for \( \delta^{(1)}_{sc} \). Similarly, \( D_\alpha \) and \( \bar{D}^{\dot{\alpha}} \) do not preserve the \( \delta^{(1)}_{sc} \)-transformation law of the \( N = 1 \) superfields, either. For them, however, there are no corresponding covariant derivatives for \( \delta^{(1)}_{sc} \).

In the \( d^4\theta \)-integral, which corresponds to the \( D \)-term formula in Ref. [41], a chiral superfield \( \Phi \) must appear through the combination of

\[
U(\Phi) \equiv (1 + i U^\mu \partial_\mu + i U^y \partial_y) \Phi. \tag{2.4}
\]

The first two terms correspond to an embedding of a chiral multiplet into a general multiplet in 4D superconformal formulation [41], and the third term is necessary for the \( \delta^{(2)}_{sc} \)-invariance of the action.

5D SUGRA is characterized by a cubic polynomial for the vector multiplets, which is referred to as the norm function in Refs. [25]-[28],

\[
\mathcal{N}(\Sigma) \equiv C_{IJK} \Sigma^I \Sigma^J \Sigma^K, \tag{2.5}
\]
where a real constant tensor \( C_{IJK} \) is completely symmetric for the indices. This corresponds to the prepotential of \( N = 2 \) SUSY gauge theories. For \( C_{IJK} \), there is a set of normalized anti-hermitian matrices \( \{ t_I \} \), which satisfies

\[
C_{IJK} = \frac{ic^3}{6} \text{tr} (t_I \{ t_J, t_K \}).
\]

(2.6)

where \( \text{tr} (t_I t_J) = -\frac{1}{2} \delta_{IJ} \), and a real constant \( c \) can take different values for each simple or Abelian group. Some of the gauge symmetries are broken by the orbifold projection, and \( t_{I_o} \) and \( t_{I_e} \) are the broken and the unbroken generators, respectively.

The supergauge transformation is expressed as

\[
e^V \rightarrow e^{\Lambda(\Lambda)} e^V e^{\Lambda(\Lambda)^\dagger}, \quad \Sigma \rightarrow e^\Lambda \left( \Sigma - \hat{\partial}_y \right) e^{-\Lambda};
\]

\[
\Phi_{\text{odd}} \rightarrow (e^{-\Lambda})^t \Phi_{\text{odd}}, \quad \Phi_{\text{even}} \rightarrow e^\Lambda \Phi_{\text{even}},
\]

(2.7)

where the transformation parameter \( \Lambda \) is a chiral superfield, and \( \Phi_{\text{odd}} \) and \( \Phi_{\text{even}} \) are \((n_C + n_H)\)-dimensional column vectors that consist of \( \Phi_{2a-1} \) and \( \Phi_{2a} \), respectively. We have used a matrix notation \((V, \Sigma) \equiv 2ig(V^I, \Sigma^I)t_I \). The gauge coupling \( g \) can take different values for each simple or Abelian factor of the gauge group. The gauge-invariant field strength superfields are defined as

\[
W_\alpha = \frac{1}{4} \bar{D}^2 \left\{ e^V D_\alpha e^{-V} - \frac{1}{2} \sigma_{\mu \beta} D_\alpha U^\mu \bar{D}_\beta \left( e^V D_\beta e^{-V} \right) \\
+ iD_\alpha U^\mu e^V \partial_\mu e^{-V} - iU^\mu \partial_\mu \left( e^V D_\alpha e^{-V} \right) \right\},
\]

\[
V = e^V \tilde{\partial}_y e^{-V} + U(\Sigma) + e^V U(\Sigma)^\dagger e^{-V} \\
+ i\partial_y U^y (\Sigma - e^V \Sigma^I e^{-V}) - \frac{i(V_E)^2}{2} (D^a U^y W_\alpha - \bar{D}_a U^y e^V (W^\dagger)_a e^{-V}),
\]

(2.8)

where

\[
\tilde{\partial}_y \equiv \partial_y - \frac{1}{4} \bar{D}^2 \Psi^\alpha D_\alpha - \frac{1}{4} D^2 \bar{D}_\bar{\alpha} \bar{D}^{\bar{\alpha}} \left( \bar{D}_\alpha \Psi^\alpha + D_\alpha \bar{\Psi}^{\bar{\alpha}} \right) \partial_\mu \\
+ \left\{ \partial_y U^\mu + \frac{1}{2} \sigma_{a \bar{a}}^\mu \left( \bar{D}_\alpha \Psi^\alpha - D_\alpha \bar{\Psi}^{\bar{\alpha}} \right) \right\} \Delta_\mu.
\]

(2.9)

They transform under (2.7) as

\[
W_\alpha \rightarrow e^\Lambda W_\alpha e^{-\Lambda}, \quad V \rightarrow e^{\Lambda(\Lambda)} V e^{-\Lambda(\Lambda)}.
\]

(2.10)

We can check that these field strength superfields follow the correct \( \delta_{sc}^{(1)} \)-transformation laws. The Weyl weights of \( W_\alpha \) and \( V \) are 3/2 and 0, respectively.

\footnote{Note that \( V \) is not hermitian, but \( e^{-\frac{i}{2}V} V e^{\frac{i}{2}V} \) is.}
Matter Lagrangian

The 5D SUGRA Lagrangian is expressed as
\[ L = L_{\text{kin}}^E - \int d^4 \theta \left( 1 + \frac{\Delta_\mu U^\mu}{3} \right) (2V_E \Omega_h + V^{-2}_E \Omega_v) \]
\[ + \left[ \int d^2 \theta \ (W_h + W_v) + \text{h.c.} \right] + 2 \sum_{y_*=0,L}^{L} \mathcal{L}_{\text{bd}}^{(y_*)} \delta(y - y_*), \] (2.11)

where \( L_{\text{kin}}^E \) denotes kinetic terms for the Weyl multiplet, \( \mathcal{L}_{\text{bd}}^{(y_*)} (y_*=0,L) \) are the boundary localized Lagrangians at \( y = y_* \), and

\[ \Omega_h \equiv \mathcal{U}(\Phi_{\text{odd}}) i \tilde{d}(c^V)^i \mathcal{U}(\Phi_{\text{odd}}) + \mathcal{U}(\Phi_{\text{even}}) i \tilde{d} e^{-V} \mathcal{U}(\Phi_{\text{even}}), \]
\[ \tilde{d} \equiv \text{diag}(1, -1), \]
\[ \Omega_v \equiv \mathcal{N}(\mathcal{V}) = -\frac{e^3}{24g^3} \text{tr} \left( \mathcal{V}^2 \right), \]
\[ W_h \equiv \Phi_{\text{odd}}^t \tilde{d} \left( \tilde{\partial}_y - \Sigma \right) \Phi_{\text{even}} - \Phi_{\text{even}}^t \tilde{d} \left( \tilde{\partial}_y + \Sigma^t \right) \Phi_{\text{odd}}, \]
\[ W_v \equiv \frac{e^3}{16g^3} \text{tr} \left[ \mathcal{W}^2 - \frac{1}{24} \tilde{D}^2 \left( \mathcal{Z}^\alpha \right) \left( \mathcal{W}_\alpha - \frac{1}{4} \mathcal{W}_\alpha^{(2)} \right) \right] + \cdots \] (2.12)

Here, \( W_v \) represents the supersymmetric Chern-Simons terms\(^9\) and a part of it provides the kinetic term for the vector superfield \( V \) after the superconformal gauge fixing. The ellipsis in \( W_v \) denotes terms that vanish in the Wess-Zumino gauge. \( \mathcal{W}_\alpha^{(2)} \) is a quadratic part of \( \mathcal{W}_\alpha \) in \( V \), and

\[ \mathcal{Z}_\alpha \equiv \{ X, \partial_y D_\alpha X \}_E - \{ \tilde{\partial}_y X, D_\alpha X \}_E, \] (2.13)

where \( X \equiv (1 + U^\mu \Delta_\mu) V - i U^\nu (\Sigma - e^V \Sigma^t e^{-V}) \), and

\[ \{ X, \mathcal{Y}_\alpha \}_E \equiv \{ X, [\mathcal{Y}_\alpha]_E \} - \frac{1}{2} \sigma^\beta_\mu \left( \{ D_\beta \mathcal{X}, \mathcal{Y}_\alpha \} + D_\alpha U^\mu \{ D_\beta \mathcal{X}, \mathcal{Y}_\beta \} \right), \]
\[ [\partial_y D_\alpha X]_E \equiv D_\alpha \tilde{\partial}_y X - \frac{1}{2} \sigma^\beta_\mu U^\mu D_\alpha D_\beta \tilde{\partial}_y X \\
+ \frac{1}{4} \left( \sigma^\mu_\alpha \partial_y U^\mu + \bar{D}_\beta \Psi_\alpha - \partial_y \bar{\Psi}_\beta \right) \tilde{D}^2 \tilde{D}_\beta X, \]
\[ [D_\alpha X]_E \equiv D_\alpha X - \frac{1}{2} \sigma^\beta_\mu U^\mu D_\alpha D_\beta \tilde{D}_\beta X. \] (2.14)

Kinetic terms for \( E_W \)

In contrast to the matter sector, \( L_{\text{kin}}^E \) is quadratic in the gravitational superfields.

\(^9\) The counterpart in the global 5D SUSY theory is shown in Refs. [42, 43].
It should be identified from the invariance of the action up to linear order in the gravitational superfields. This requires an extension of the 5D superconformal transformations (A.1) and (A.2) by including linear terms in the gravitational superfields. For the purpose of this paper, we only need terms in $L_{\text{kin}}$ that are independent of the quantum fluctuation of the matter superfields. Hence, we can treat the matter superfields in the corrections to (A.1) and (A.2) as the background values. The corrected transformations involving $U^\mu$ are listed in (A.5) in Appendix A. By requiring the invariance of the action under the corrected transformations, we find

$$L_{\text{kin}}^{E_W} = \int d^4 \theta \left\{ \left( \frac{2V_E \Omega_h + V_E^{-2} \Omega_v}{3} \right) E_2 + \left( \frac{V_E^{-1} \Omega_h - 4V_E^{-4} \Omega_v}{3} \right) C^\mu C_\mu \right\}, \quad (2.15)$$

where the symbol $\langle \cdots \rangle$ denotes the background value, and

$$E_2 \equiv - \frac{1}{8} U_\mu D^\alpha \bar{D}^2 D_\alpha U^\mu + \frac{1}{3} (\Delta_\mu U^\mu)^2 - (\partial_\mu U^\mu)^2,$$

$$C^\mu \equiv \partial_\mu U^\mu + \frac{1}{2} \sigma^\mu_{\alpha \dot{\alpha}} \left( \bar{D}^\dot{\alpha} \Psi^\alpha - D^\alpha \bar{\Psi}^\dot{\alpha} \right) + \langle V_E \rangle^2 \partial^\mu U^y. \quad (2.16)$$

In addition to the above terms, the following term is expected to appear in the 5D Lagrangian.

$$L_{\text{add}} = - \langle \Omega_v \rangle \frac{V_E}{4} \bar{D}^\alpha \Psi^\alpha D_\alpha \bar{\Psi}^\dot{\alpha}. \quad (2.17)$$

This term is necessary to obtain the correct kinetic terms for the vector superfields (3.15). In order to justify the existence of this term, we need to modify $\delta^{(1)}_{sc}$ and $\delta^{(2)}_{sc}$ further by including $\Psi_{\alpha}$-dependent terms in the right-hand sides of (A.1) and (A.2). Here we leave this task for future works, and just assume (2.17).

**Boundary localized terms**

We can introduce terms localized on the 4D boundaries of $S^1/Z_2$. The boundary actions are described by the action formulae of 4D superconformal formulation [41], and expressed in terms of the superfields as [37]

$$L_{\text{bd}}^{(y)} = \int d^4 \theta \left\{ - \frac{2}{3} \langle \Omega_{\text{bd}}^{(y)} \rangle E_2 + 2 \left( 1 + \frac{\Delta_\mu U^\mu}{3} \right) \Omega_{\text{bd}}^{(y)} \right\}$$

$$+ \left[ \int d^2 \theta \left\{ \phi^3 P^{(y)}(\chi) - \frac{1}{2} \text{tr} \left( f^{(y)}(\chi) W^{\alpha} W_\alpha \right) \right\} + \text{h.c.} \right], \quad (2.18)$$

where

$$\Omega_{\text{bd}}^{(y)} = - \frac{3}{2} |U(\phi_C)|^2 \exp \left\{ - \frac{K(y)}{3} \langle U(\chi), V_{4\text{d}} \rangle \right\}. \quad (2.19)$$
Chiral superfields $\phi_C$ and $\chi^a (a = 1, 2, \cdots)$ are the 4D compensator and the physical matter superfields, and $V_{4D}^I$ are 4D vector superfields. A real function $K^{(y_+)}$ is the Kähler potential, and holomorphic functions $P^{(y_+)}$ and $f^{(y_+)}$ are the superpotential and the gauge kinetic functions, respectively. Note that $U(\phi) = (1 + i U^a \partial_a) \phi$ in the above Lagrangian since $U^y$ is $Z_2$-odd and vanishes on the boundaries. In general, $\chi^a$ and $V_{4D}^I$ can be either boundary values of the $Z_2$-even bulk superfields or additional 4D superfields localized on the boundaries. In contrast to the 5D bulk action, we have only one compensator chiral multiplet. Thus, one combination of $Z_2$-even 5D compensators $\Phi^a_2 (a = 1, \cdots, n_C)$ plays its role.

In the case of $n_C = 1$, $\Phi^2$ is the only $Z_2$-even compensator superfield. Hence, the 4D chiral compensator superfield $\phi_C$ in $\mathcal{L}_{bd}^{(y_+)} (y_+ = 0, L)$ is identified as

$$\phi_C = \left( \Phi^2 \right)^{2/3} \bigg|_{y = y_*},$$

because $\phi_C$ must have $w = n = 1$. The bulk physical matter superfields can appear in $\mathcal{L}_{bd}^{(y_+)}$ in the forms of

$$\chi^a = \frac{\Phi^{2a+2}}{\Phi^2} \bigg|_{y = y_*}, \quad V_{4D} = V \big|_{y = y_*},$$

because the physical matter superfields must have zero Weyl (chiral) weight in the 4D superconformal formulation [41].

In the case of $n_C = 2$, there are two $Z_2$-even compensator superfields $\Phi^2$ and $\Phi^4$. In this case, we have to eliminate one combination of the 5D compensator multiplets. In Ref. [27], this is done by introducing a nondynamical (auxiliary) Abelian vector multiplet $\mathcal{V}_T = (V_T, \Sigma_T)$, and gauging a $U(1)$ subgroup of the isometries, which is referred to as $U(1)_T$, by it. The $U(1)_T$ charges $Q_T$ are chosen as $Q_T(\Phi^1) = Q_T(\Phi^4) = Q_T(\Phi^{2a+4}) = +1$ and $Q_T(\Phi^2) = Q_T(\Phi^3) = Q_T(\Phi^{2a+3}) = -1 \ (a \geq 1)$. Since the 4D superfields must be neutral for $U(1)_T$, they are identified as

$$\phi_C = \left( \Phi^2 \Phi^4 \right)^{1/3} \bigg|_{y = y_*}, \quad \chi^a = \frac{\Phi^{2a+4}}{\Phi^4} \bigg|_{y = y_*}.$$
As pointed out in Ref. [44], $V_E$ does not have a kinetic term and can be integrated out. From (2.11), $V_E$ is expressed as

$$V_E = \left( \frac{\Omega_v}{\Omega_h} \right)^{1/3}.$$  \hfill (2.23)

After integrating it out, the 5D Lagrangian becomes

$$\mathcal{L} = \int d^4 \theta \left\{ \left\langle \Omega_v^{1/3} \Omega_h^{2/3} \right\rangle E_2 - \left\langle \Omega_v^{-1/3} \Omega_h^{4/3} \right\rangle \left( C^\mu C_\mu + \bar{D}^\dot{\alpha} \Psi^\alpha D_\alpha \bar{\Psi}_{\dot{\alpha}} \right) \right. $$

$$\left. - 3 \left( 1 + \frac{\Delta \mu U_\mu}{3} \right) \Omega_v^{1/3} \Omega_h^{2/3} \right\}$$

$$+ \left[ \int d^2 \theta \left( W_h + W_v \right) + \text{h.c.} \right] + 2 \sum_{y_*=0,L} \mathcal{L}_{bd}^{(y_*)} \delta(y - y_*).$$  \hfill (2.24)

In our previous paper [30], we implicitly assumed that $\left\langle \Omega_v \right\rangle = \left\langle \Omega_h \right\rangle = 1$ (in the unit of the 5D Planck mass), but we need their explicit dependences on the background superfields of the matter for the derivation of the one-loop effective Kähler potential.

In order to obtain the Poincaré SUGRA, we have to impose the superconformal gauge-fixing conditions to eliminate the extra symmetries. For example, the dilatation symmetry will be fixed by the condition, $\Omega_v|_0 = \Omega_h|_0 = 1$ in the 5D Planck unit, where the symbol $|_0$ denotes the lowest component of the superfield. However, these gauge-fixing conditions are incompatible with the $N = 1$ off-shell structure. Thus we will add the gauge-fixing terms in the calculations in Sec. 3 instead of imposing such conditions.

## 3 One-loop effective Kähler potential

In our previous works [9, 10, 45], we derived the 4D effective action at tree level. We provide a brief review of the derivation in Appendix C. In this section, we calculate the one-loop contributions to the effective Kähler potential.

### 3.1 Background field method

We calculate the one-loop effective Kähler potential by using the background field method [46]. First we split each superfield into the background and the fluctuation parts.
Since we are interested in the effective theory for the zero-modes of the matter superfields, we only consider the background values of the $Z_2$-even matter superfields $\Phi_{\text{even}}, V^I_o,$ and $\Sigma^I_o$. We move to a gauge where $\langle \Sigma^I \rangle$ are zero by the supergauge transformation for the background superfields. This is accomplished by choosing the transformation parameter as (C.3) (or (D.7) in the Abelian case). Then $\langle V^I_o \rangle$ become discontinuous at $y = L$. In fact, in the case that the gauge group is Abelian, their boundary conditions are
\[
\begin{align*}
\lim_{y \to 0} \langle V^I_o \rangle &= 0, \\
\lim_{y \to L} \langle V^I_o \rangle &= -T^I_o - \bar{T}^I_o, \\
\langle V^I_o \rangle \big|_{y=0} &= \langle V^I_o \rangle \big|_{y=L} = 0,
\end{align*}
\]
where the limits are taken from the bulk region $0 < y < L$, and
\[
T^I_o \equiv \int_0^L dy \langle \Sigma^I_o \rangle.
\]
We refer to the chiral superfields $T^I_o$ as the moduli superfields in this paper. In order to take them into account, we also keep the background values of $V^I_o$ in addition to those of the $Z_2$-even matter superfields. Thus, each matter superfield is split as
\[
\Phi_{\text{odd}} = \tilde{\Phi}_{\text{odd}}, \quad \Phi_{\text{even}} = \Phi + \tilde{\Phi}_{\text{even}}, \\
V = V + \tilde{V}, \quad \Sigma = \tilde{\Sigma},
\]
where $\Phi$ and $V$ are the background values and the quantities with tilde denote the fluctuation parts. We neglect derivative terms in the effective Kähler potential, and thus we treat $\Phi$ and $V$ as functions of only $y$ in the following calculations. The gravitational superfields $U^\mu, U^y,$ and $\Psi_\alpha$ are considered as the fluctuation modes. ($V_E$ has already been integrated out.) As we have pointed out in Ref. [30], $U^y$ can be gauged away by $\delta^{(2)}_{sc}$ given in (A.2) in Appendix A. So we take the gauge where $U^y = 0$ in the following.

We expand the 5D Lagrangian (2.24) and pick up quadratic terms in the fluctuation superfields.
\[
L = \sum_F \int d^4 \theta \ F^\dagger \mathcal{O}_F F + \cdots,
\]
where $F$ runs over the fluctuation superfields, and $\mathcal{O}_F$ are differential operators that depend on $\Phi$ and $V$. Then the one-loop contribution to the effective action $\Delta_{\text{1loop}} S$ is calculated as
\[
\Delta_{\text{1loop}} S = \frac{i}{2(2\pi)^4} \sum_F \int d^4 p \text{ Tr} (\text{str} \ln \mathcal{O}_F),
\]
\[\text{13}\] In this gauge, we do not need to consider contributions from the ghost for $\delta^{(2)}_{sc}$ because it is decoupled from the background superfields $\Phi$ and $V$. 

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where $\text{str}$ is the supertrace over the functional space on the 16-dimensional graded vector space built from all combinations of $\theta$ and $\bar{\theta}$, and $\text{Tr}$ is the trace over the remaining space including the functional space of $y$. Here we denote an integrand of the $d^4\theta$-integral for $\text{str}$ as $\text{Istr}$ [48]. Namely, it follows that

$$\text{str \ ln} \mathcal{O}_F \equiv \int d^4\theta \ I\text{str} \ln \mathcal{O}_F.$$  \hfill (3.6)

Then the one-loop contribution to the Kähler potential $\Omega_{\text{eff}} = -3e^{-K_{\text{eff}}/3}$ is expressed as

$$\Omega_{\text{eff}}^{\text{loop}} = \frac{i}{2(2\pi)^4} \sum_F \int d^4p \ \text{Tr} \left( \text{Istr} \ln \mathcal{O}_F \right).$$  \hfill (3.7)

Since $\Omega_{\text{eff}}^{\text{loop}}$ is a function of the background superfields whose dependences on $x^\mu$ and $\theta \ (\bar{\theta})$ are now neglected, $\text{Istr}$ is calculated by

$$\text{Istr \ ln} \mathcal{O}_F = \left[ \frac{\ln \mathcal{O}_F}{\theta \bar{\theta}} \right]_{\theta=\bar{\theta}=0}.$$  \hfill (3.8)

Its values for various operators are collected in (B.10).

### 3.2 Quadratic terms for fluctuation modes

Here we pick up the quadratic terms in the fluctuation superfields, and find explicit forms of $\mathcal{O}_F$ in (3.4). Detailed calculations are shown in Appendix D.

#### 3.2.1 Bulk sector

Using the superspin projectors defined by (B.4) in Appendix B, $U^\mu$ is decomposed as [23, 49, 50]

$$U^\mu = \sum_s \Pi^\mu_{\nu s} U^\nu \equiv \sum_s U^\mu_s,$$  \hfill (3.9)

where $s = 0, 1/2, 1, 3/2$. We choose the gauge-fixing term for the superconformal symmetry $\delta^{(1)}_{\text{sc}}$ as

$$\mathcal{L}_{\text{gf}}^{\text{sc}} = \int d^4\theta \left( \frac{\Omega^{1/3}_v \Omega^{2/3}_h}{\xi_{\text{sc}}} \right) \hat{U}_\mu \Box_4 \Pi^\mu_{\nu s} (\xi_{\text{sc}}) \hat{U}_\nu,$$  \hfill (3.10)

where $\xi_{\text{sc}}$ is the gauge-fixing parameter, $\Box_4 \equiv \partial^\mu \partial_\mu$, and

$$\Pi^\mu_{\nu s} (\xi_{\text{sc}}) \equiv \eta^\mu_{\nu s} - \Pi^\mu_{\nu 3/2} - \frac{2\xi_{\text{sc}}}{3} \Pi^\mu_{0 s},$$

$$\hat{U}_\mu \equiv U_\mu + \frac{3i\xi_{\text{sc}}}{(3 - 2\xi_{\text{sc}}) \Box_4} \partial_\mu \left( \mathcal{T} + \bar{\Phi}_C - \mathcal{T} - \bar{\Phi}_C \right)$$

$$+ \frac{\xi_{\text{sc}}}{2 \Box_4} \left( \eta_{\mu \nu} + \frac{2\xi_{\text{sc}}}{3 - 2\xi_{\text{sc}}} \Pi_{0 \mu \nu} \right) \Delta^\nu \left( \bar{V}_v + \bar{V}_h \right).$$  \hfill (3.11)
Here, $T$, $\tilde{\Phi}_C$, $\tilde{V}_v$ and $\tilde{V}_h$ are defined as

$$T \equiv \frac{N_I}{3N} \langle \langle V \rangle \rangle \tilde{\Sigma}, \quad V_T \equiv \frac{N_I}{3N} \langle \langle V \rangle \rangle \tilde{\Sigma}_I, \quad \tilde{\Phi}_C \equiv \frac{2}{3} \tilde{\Phi}_{\text{even}}, \quad \Upsilon \equiv \frac{1}{\langle \Omega_h \rangle} \tilde{d} e^{-V} \Phi,$$

$$\tilde{V}_v \equiv -\frac{N_I}{3N} \langle \langle V \rangle \rangle \partial_y \tilde{V}_I, \quad \tilde{V}_h \equiv \frac{2}{3} \tilde{\Phi}^\dagger \Upsilon_I \tilde{V}_I, \quad \Upsilon_I \equiv \frac{1}{\langle \Omega_h \rangle} \tilde{\partial} \tilde{V}_I \tilde{d} e^{-V} \Phi,$$

where $N_I \equiv \partial \tilde{N} / \partial V^I$. Then the cross terms between $U_\mu$ and the other superfields are canceled, and we obtain

$$L + L_{\text{gf}}^{sc} = \int d^4 \theta \left\{ -U_\mu^{3/2} \mathcal{O}_{3/2} U_\mu + \bar{U}_\mu \mathcal{O}_U \bar{U}_\mu \right\} + \mathcal{O}(\xi_{\text{sc}})$$

$$+ \int d^4 \theta \frac{N_I N_J}{2N} \langle \langle V \rangle \rangle \tilde{V}_I \tilde{\square}_4 P_T \tilde{V}_J + \cdots, \quad \text{(3.13)}$$

where $P_T$ is a projection operator defined in (B.2), and

$$\bar{U}_\mu \equiv U_\mu - U_\mu^{3/2} = \left( \Pi_0^{\mu \nu} + \Pi_1^{\mu \nu} + \Pi_2^{\mu \nu} \right) U_\nu,$$

$$\mathcal{O}_{3/2} \equiv \left( \Omega_1^{1/3} \Omega_2^{2/3} \right) \left( \tilde{\square}_4 + \mathcal{D}_U \right), \quad \mathcal{O}_U \equiv \left( \Omega_1^{1/3} \Omega_2^{2/3} \right) \left( \frac{1}{\xi_{\text{sc}}} \tilde{\square}_4 + \mathcal{D}_U \right),$$

$$\mathcal{D}_U \equiv -\frac{\partial_y \left( \Omega_1^{1/3} \Omega_2^{2/3} \right) \partial_y}{\left( \Omega_1^{1/3} \Omega_2^{2/3} \right)} \quad \text{(3.14)}$$

The last term in (3.13) is combined with the quadratic terms in the vector sector shown in (D.1), and provides the kinetic terms for $\tilde{V}_I$,

$$L_{\text{kin}}^{sc} = \int d^4 \theta \left\{ \Omega_\nu \right\} a_{IJ} \tilde{V}_I \tilde{\square}_4 P_T \tilde{V}_J, \quad \text{(3.15)}$$

where

$$a_{IJ} \equiv -\frac{1}{2N} \left( N_{IJ} - \frac{N_I N_J}{N} \right), \quad N_{IJ} \equiv \frac{\partial^2 N}{\partial V^I \partial V^J}. \quad \text{(3.16)}$$

The arguments of the norm function and its derivatives are understood as $\langle V^I \rangle$ in this and the next subsections. These kinetic terms are consistent with those in Ref. \[25\].

In the following, we consider a case that the gauge group is Abelian for simplicity. We choose the gauge-fixing term for the supergauge symmetry $\delta_{\text{sg}}$ as

$$L_{\text{gf}}^{\text{sg}} = \int d^4 \theta \frac{\langle \Omega_\nu \rangle a_{IJ} \tilde{V}_I \tilde{\square}_4 P_C \tilde{V}_J,}{\xi_{\text{sg}}} \quad \text{(3.17)}$$

$^{14}$ $T$ and $V_T$ correspond to the 5D radion and the graviphoton superfields, respectively.
where \( \xi_{sg} \) is the gauge-fixing parameter, \( P_C \) is the chiral projection operator defined in (B.2), and

\[
\hat{V}^I \equiv \tilde{V}^I + \frac{\xi_{sg}}{\langle \Omega_v \rangle \Box_4} (\Xi_J + \Xi_J).
\]

(3.18)

The definition of \( \Xi_J \) is given in (D.11). Then the cross terms between \( \tilde{V} \) and the chiral superfields are canceled.

As a result, the quadratic terms for the fluctuation superfields in the 5D Lagrangian are summarized as

\[
\mathcal{L}_{\text{quad}} = \int d^4\theta U_{\mu} \left\{ -\mathcal{O}_{3/2} \Pi_{3/2}^\mu - \mathcal{O}_G \left( \eta^\mu - \Pi_{3/2}^\mu \right) \right\} U_{\nu} + \int d^4\theta \tilde{V}^I \left\{ (\mathcal{O}_T)_{IJ} P_T + (\mathcal{O}_C)_{IJ} P_C \right\} \tilde{V}^J
\]

\[
+ \int d^4\theta (\varphi^\dagger, \varphi) \left( \frac{K}{W} \frac{\Box_4^2}{\frac{3}{K}^2} \right) \left( \varphi \right),
\]

(3.19)

where \( \varphi \equiv (\tilde{\Sigma}^I, \tilde{\Phi}_{\text{even}}, \tilde{\Phi}_{\text{odd}})^t \), and

\[
(\mathcal{O}_T)_{IJ} \equiv \langle \Omega_v \rangle a_{IK} \left\{ \delta^K_J \Box_4 + (\mathcal{D}_V)^K_J \right\} + \mathcal{O}(\xi_{sg}),
\]

\[
(\mathcal{O}_C)_{IJ} \equiv \langle \Omega_v \rangle a_{IK} \left\{ \frac{\delta^K_J}{\xi_{sg}} \Box_4 + (\mathcal{D}_V)^K_J \right\} + \mathcal{O}(\xi_{sg}),
\]

\[
(\mathcal{D}_V)^I_J \equiv -\frac{a_{IK}}{\langle \Omega_v \rangle} \partial_y \left\{ \langle \Omega_v^{1/3} \Omega_h^{2/3} \rangle \left( (a \cdot \mathcal{P}_V)_{KJ} \partial_y + \frac{\mathcal{N}_K}{3N} \gamma^I \Phi \right) \right\}
\]

\[
+ \langle \Omega_h \rangle ^{2/3} a^{IK} \left( \frac{\mathcal{N}_J}{3N} \gamma^I \Phi \partial_y - \frac{\partial_I \partial_J \Omega_h}{\Omega_h} + \frac{\gamma^I \gamma^I}{3} + \frac{\gamma^I \gamma^I}{3} \right),
\]

\[
K \equiv \langle \Omega_v^{1/3} \Omega_h^{2/3} \rangle \left( \frac{(a \cdot \mathcal{P}_V)_{IJ}}{\langle \Omega_h \rangle} - \frac{\mathcal{N}_I}{3N} \gamma^I \Phi \partial_y - \frac{\partial_I \partial_J \Omega_h}{\Omega_h} + \frac{\gamma^I \gamma^I}{3} \right) + \mathcal{O}(\xi_{sg}),
\]

\[
W \equiv \begin{pmatrix}
0 & 0 & -\Phi^I d_I^J \\
0 & 0 & -\Phi^I d_I^J \\
-\Phi^I d_I^J & -\Phi^I d_I^J & 0
\end{pmatrix}.
\]

(3.20)

Here \( \hat{t}_I \equiv 2igt_I \) are hermitian generators, and

\[
(\mathcal{P}_V)^I_J \equiv \delta^I_J - \frac{\langle \mathcal{V}^I \rangle \mathcal{N}_J}{3N}.
\]

(3.21)

is a projection operator [25], which has a property,

\[
\mathcal{N}_I (\mathcal{P}_V)^I_J = (\mathcal{P}_V)^I_J (\mathcal{V}^J) = 0, \quad \mathcal{P}_V^2 = 1_{\mathcal{V}}.
\]

(3.22)

The definitions of \( \gamma \) and \( \gamma_I \) are given in (3.12). For the purpose of calculating the one-loop Kähler potential, it is convenient to choose the gauge-fixing parameters as \( \xi_{sc} = \xi_{sg} = 0 \).
3.2.2 Boundary sector

From (2.18), the quadratic terms for the fluctuation superfields in the boundary Lagrangians are found to be

$$\mathcal{L}_{\text{boundary}}^{(y_*)} = \mathcal{L}_{\text{bd}}^{(y_*)} + \mathcal{L}_{\text{gf}}^{(y_*)} + \mathcal{L}_{\text{gf}}^{(y_*)}$$

$$= \int d^4 \theta |\phi_C|^2 h^{(y_*)} \left\{ -U_\mu \square_4 \Pi_{\mu \nu}^{(y_*)} U_\nu + \frac{1}{\zeta_{\text{sc}}^{(y_*)}} U_\mu \square_4 \left( \eta_{\mu \nu} - \Pi_{\mu \nu}^{(y_*)} \right) U_\nu \right\}$$

$$+ \int d^4 \theta \left[ \text{Re} f_{I_e J_e}^{(y_*)} \left( \tilde{V}_e \square_4 \left( PT + \frac{1}{\epsilon_{\text{sg}}^{(y_*)}} P_C \right) \tilde{V}_e \right) - \frac{3}{2} |\phi_C|^2 h_{I_e J_e}^{(y_*)} \tilde{V}_e \tilde{V}_e \right]$$

$$+ \int d^4 \theta \left[ |\phi_C|^2 h_a^{(y_*)} \bar{\chi}^a \tilde{\chi}^b + \left( \tilde{\phi}_C h_a^{(y_*)} \bar{\phi}_C \tilde{\phi}^a + \text{h.c.} \right) + h^{(y_*)} \tilde{\phi}_C |^2 \right]$$

$$+ \left[ \int d^4 \theta \left( \frac{1}{2} \phi_C \bar{D}_{a b}^{(y_*)} \bar{\phi}^a \tilde{\phi}^b + 3 \phi_C \bar{D}_a \tilde{\phi}_C \bar{\phi}^a + 3 \phi_C \bar{D}^{(y_*)} \phi_C \phi_C \tilde{\phi}^a + \text{h.c.} \right) \right]$$

$$+ \mathcal{O} \left( \epsilon_{\text{sc}}^{(y_*)}, \epsilon_{\text{sg}}^{(y_*)} \right) + \cdots, \quad (3.23)$$

where $\phi_C$ and $\bar{\phi}_C$ are the background (fluctuation) parts of the compensator and the physical chiral superfields $\phi_C$ and $\bar{\chi}^a$, and $h^{(y_*)} \equiv -3 \exp \left(-K^{(y_*)}/3\right)$, $h_{I_e J_e}^{(y_*)} \equiv \partial h^{(y_*)}/\partial \chi^a$, $h_{I_e}^{(y_*)} \equiv \partial h^{(y_*)}/\partial \bar{V}_e$, $\cdots$, whose arguments are $(\chi, V)$. We have chosen the boundary gauge-fixing terms for the superconformal and the gauge symmetries as

$$\mathcal{L}_{\text{gf}}^{(y_*)} = - \int d^4 \theta \frac{|\phi_C|^2 h^{(y_*)}}{\zeta_{\text{sc}}^{(y_*)}} \check{U}_\mu \square_4 \Pi_{\mu \nu}^{(y_*)} (\zeta_{\text{sc}}^{(y_*)}) \check{U}_\nu,$$

$$\check{U}_\mu = U_\mu + \frac{3 \epsilon_{\text{sc}}^{(y_*)}}{(3 - 2 \epsilon_{\text{sc}}^{(y_*)}) \square_4} \partial_\mu \left( \frac{h^{(y_*)}}{h^{(y_*)}} \bar{\chi}^a + \frac{\tilde{\phi}_C}{\phi_C} - \text{h.c.} \right)$$

$$+ \frac{\zeta_{\text{sc}}^{(y_*)}}{2 \square_4} \left( \eta_{\mu \nu} + \frac{2 \epsilon_{\text{sc}}^{(y_*)}}{3 - 2 \epsilon_{\text{sc}}^{(y_*)}} \Pi_{\mu \nu} \right) \left( \frac{h_{I_e}^{(y_*)}}{h^{(y_*)}} \Delta^\nu \check{V}_e \right),$$

$$\mathcal{L}_{\text{gf}}^{(y_*)} = \int d^4 \theta \frac{\text{Re} f_{I_e J_e}^{(y_*)}(\chi)}{\zeta_{\text{sg}}^{(y_*)}} \check{V}_e \square_4 P_C \check{V}_e$$

$$\check{V}_e \equiv \check{V}_e + \frac{3 \epsilon_{\text{sg}}^{(y_*)}}{2 \square_4} \left\{ F_{I_e J_e} \left( \phi_C |^2 h_{I_e a}^{(y_*)} \bar{\chi}^a + \bar{\phi}_C h_{I_e}^{(y_*)} \phi_C + \text{h.c.} \right) \right\}, \quad (3.24)$$

where $F_{I_e J_e}$ is an inverse matrix of $\text{Re} f_{I_e J_e}^{(y_*)}(\chi)$. In the following, we will choose the gauge-fixing parameters as $\zeta_{\text{sc}}^{(y_*)} = \epsilon_{\text{sg}}^{(y_*)} = 0$.

In the case that $\phi_C$ and $\chi^a$ are the boundary values of the bulk superfields, the relations (2.20) and (2.21) (or (2.22)) in $\mathcal{L}_{\text{bd}}^{(L)}$ must be modified for the background superfields because we have performed the discontinuous gauge transformation at $y = L$. (See (C.16).)
In the case of \( n_C = 1 \), for example, the relations are modified as
\[
\phi_C = e^{-2k_{I_0} T^{I_0} (\Phi^1)^{2/3}} \bigg|_{y = L}, \quad \chi^a = \frac{\{ \exp (T^{I_0} \tilde{I}_{I_0}) \Phi \}^{a+1}}{\Phi^1} \bigg|_{y = L},
\]
where \( k_{I_0} \) and \( \tilde{I}_{I_0} \) are defined in (C.18).

The above boundary-localized terms affect the boundary conditions for the fluctuation modes of the bulk superfields, which are no longer determined only by the orbifold parities. We derive them in Appendix E.

### 3.3 Integration of fluctuation modes

In this subsection, we perform the integration of the fluctuation modes, and obtain formal expressions of the one-loop contributions to \( \Omega_{\text{eff}}^{1\text{loop}} \).

#### 3.3.1 Contribution from gravitational superfields

The contribution from the gravitational superfields is
\[
\Omega_U^U = \frac{i}{2(2\pi)^4} \int d^4p \, \text{Tr} \left\{ \text{Istr} \ln \mathcal{O}_U - \text{Istr} \ln \left( \frac{1}{\xi_{\text{sc}}} \left( \Omega_v^{1/3} \Omega_h^{2/3} \right) \Pi_{\text{gf}} \right) \right\},
\]
where
\[
\mathcal{O}_U^\mu^\nu \equiv -\mathcal{O}_{3/2} \Pi_{3/2}^\mu^\nu + \mathcal{O}_U \left( \eta^\mu^\nu - \Pi_{3/2}^\mu^\nu \right).
\]
The second term in (3.26) is a contribution from the ghost for \( \delta_{\text{sc}}^{(1)} \). (See the gauge-fixing term (3.10).) Since
\[
\ln \mathcal{O}_U = \Pi_{3/2}^\mu^\nu \ln (-\mathcal{O}_{3/2}) + \left( \eta^\mu^\nu - \Pi_{3/2}^\mu^\nu \right) \ln \mathcal{O}_U,
\]
it follows that
\[
\begin{align*}
\text{Tr} \left\{ \text{Istr} \ln \mathcal{O}_U - \text{Istr} \ln \left( \frac{1}{\xi_{\text{sc}}} \left( \Omega_v^{1/3} \Omega_h^{2/3} \right) \Pi_{\text{gf}} \right) \right\} \\
= \text{Istr} \Pi_{3/2} \text{Tr} \ln (-\mathcal{O}_{3/2}) + \text{Istr}(\eta - \Pi_{3/2}) \text{Tr} \ln \mathcal{O}_U - \text{Istr} \Pi_{3/2} \text{Tr} \ln \left( \left( \Omega_v^{1/3} \Omega_h^{2/3} \right) \right) \\
= \frac{4}{\square_4} \text{Tr} \ln \mathcal{O}_{3/2} - \frac{4\xi_{\text{sc}}}{3\square_4} \text{Tr} \ln \left( \left( \Omega_v^{1/3} \Omega_h^{2/3} \right) \right) + \cdots,
\end{align*}
\]
where the ellipsis denotes terms independent of the background superfields. Thus, when \( \xi_{\text{sc}} = 0 \), \( \Omega_U^U \) is calculated as
\[
\begin{align*}
\Omega_U^U &= \frac{i}{2(2\pi)^4} \int d^4p \, \frac{4}{-p^2} \text{Tr} \ln \mathcal{O}_{3/2}(p^2) + \cdots \\
&= -\int \frac{d^4p_E}{(2\pi)^4} \frac{2}{p_E^2} \text{ln Det}\mathcal{O}_{3/2}(-p_E^2) + \cdots,
\end{align*}
\]
where \( p'^{\mu} \equiv (p^1, p^2, p^3, -ip^0) \) is the Wick-rotated Euclidean momentum, and \( \text{Det} \) is the functional determinant, which is expressed as

\[
(\text{Det} \mathcal{O}_{3/2})^{-1/2} = \int \mathcal{D}F_U \exp \left\{ - \int_0^L dy \, F_U \mathcal{O}_{3/2} F_U \right\}. \tag{3.31}
\]

The integral variable \( F_U \) is a function of \( y \), and can be expanded as

\[
F_U(y) = \sum_k f_U(y; \mu_{(k)}^U) F^{(k)}_U,
\]

where \( f_U(y; \mu_U) \) is an eigenfunction of \( \mathcal{D}_U \) defined in (3.14) with an eigenvalue \( \mu_{(k)}^2 \), i.e.,

\[
\mathcal{D}_U f_U(y; \mu_U) = \mu_{(k)}^2 f_U(y; \mu_U). \tag{3.33}
\]

This has a form of the Sturm-Liouville equation. Thus the eigenfunctions satisfy the orthonormal condition,

\[
\int_0^L dy \left\langle \Omega^{1/3}_x \Omega^{2/3}_n \right| f_U(y; \mu_{(k)}^U) f_U(y; \mu_{(l)}^U) = \delta_{kl}. \tag{3.34}
\]

Then, (3.31) is rewritten as

\[
(\text{Det} \mathcal{O}_{3/2})^{-1/2} = \prod_k \mathcal{D}F^{(k)}_U \exp \left\{ - \sum_k F^{(k)}_U \left( p^2_E + \mu_{(k)}^2 \right) F^{(k)}_U \right\} = \prod_k \left( p^2_E + \mu_{(k)}^{(k)} \right)^{-1/2}, \tag{3.35}
\]

up to an irrelevant normalization constant. Therefore, (3.30) becomes

\[
\Omega_{\text{eff}}^U = \int \frac{d^D p_E}{(2\pi)^D} \frac{2}{p^2_E} \sum_k \ln \left( p^2_E + \mu_{(k)}^{(k)} \right) + \cdots = -\frac{2\Gamma(1 - \frac{D}{2})}{(4\pi)^\frac{D}{2} \left( \frac{D}{2} - 1 \right)} \sum_k \mu_{(k)}^{(k)D-2} + \cdots, \tag{3.36}
\]

where \( \Gamma(z) \) is the gamma function. We have used the dimensional reduction [51] to regularize the divergent momentum integral.\footnote{Since our formalism respects the superconformal symmetry, a momentum cutoff should not be introduced in contrast to Ref. [52].}
3.3.2 Contribution from vector superfields

The contribution from the vector superfields is

\[ \Omega_v^{\text{eff}} = \frac{i}{2(2\pi)^4} \int d^4p \, \text{Tr} \{ \text{Istr} \ln \mathcal{O}_V - \text{Istr} \ln (\langle \Omega_v \rangle a P_C) \}, \quad (3.37) \]

where \( a \) is the matrix defined in (3.16), and

\[ \mathcal{O}_V \equiv \mathcal{O}_T P_T + \mathcal{O}_C P_C. \quad (3.38) \]

The second term in (3.37) is a contribution from the ghost for \( \delta_{sg} \). (See the gauge-fixing term (3.17).) Since

\[ \ln \mathcal{O}_V = P_T \ln \mathcal{O}_T + P_C \ln \mathcal{O}_C, \quad (3.39) \]

it follows that

\[ \text{Tr} \{ \text{Istr} \ln \mathcal{O}_V - \text{Istr} \ln (\langle \Omega_v \rangle a P_C) \} \]
\[ = (\text{Istr} P_T) \text{Tr} \ln \mathcal{O}_T + (\text{Istr} P_C) \text{Tr} \ln \mathcal{O}_C - (\text{Istr} P_C) \text{Tr} (\langle \Omega_v \rangle a) \]
\[ = \frac{2}{\Box_4} \text{Tr} \{ \ln \mathcal{O}_T - \ln \mathcal{O}_C + \ln (\langle \Omega_v \rangle a) \}. \quad (3.40) \]

When \( \xi_{sg} \to 0 \),

\[ \ln \mathcal{O}_C \to \ln \left( \frac{\langle \Omega_v \rangle a \Box_4}{\xi_{sg}} \right) = \ln (\langle \Omega_v \rangle a) + \cdots, \quad (3.41) \]

where the ellipsis denotes terms independent of the background superfields. Therefore, \( \Omega_v^{\text{eff}} \) is calculated as

\[ \Omega_v^{\text{eff}} = \frac{i}{(2\pi)^4} \int d^4p \, \frac{1}{-p^2} \text{Tr} \ln \mathcal{O}_T(p^2) + \cdots \]
\[ = - \int \frac{d^4p_E}{(2\pi)^4} \frac{1}{p_E^2} \ln \text{Det} \mathcal{O}_T(-p_E^2) + \cdots. \quad (3.42) \]

Similarly to the derivation of (3.36), this can be rewritten as

\[ \Omega_v^{\text{eff}} = - \int \frac{d^Dp_E}{(2\pi)^D} \frac{1}{p_E^2} \sum_k \ln \left( p_E^2 + \mu_V^{(k)2} \right) + \cdots \]
\[ = \frac{\Gamma(1 - \frac{D}{2})}{(4\pi)^{\frac{D}{2}} (\frac{D}{2} - 1)} \sum_k \mu_V^{(k)D-2} + \cdots, \quad (3.43) \]

where \( \mu_V^{(k)2} \) are eigenvalues of \( D_V \) defined in (3.20), i.e.,

\[ (D_V)^I_{\,J} f_V^J(y; \mu_V) = \mu_V^2 f_V^I(y; \mu_V), \quad (3.44) \]

and the eigenfunctions satisfy the orthonormal condition,

\[ \int_0^L dy \, \langle \Omega_v \rangle a_{IJ} f_V^I(y; \mu_V^{(k)}) f_V^J(y; \mu_V^{(l)}) = \delta_{kl}. \quad (3.45) \]
3.3.3 Contribution from chiral superfields

The contribution from the chiral superfields is

\[ \Omega_{\text{ch}}^{\text{eff}} = \frac{i}{2(2\pi)^4} \int d^4p \, \text{Tr Istr} \left( \mathbb{P} \ln \mathcal{O}_{\text{ch}} \right), \]  

(3.46)

where

\[ \mathbb{P} \equiv \begin{pmatrix} P^+ \\ P^- \end{pmatrix}, \quad \mathcal{O}_{\text{ch}} \equiv \begin{pmatrix} \mathcal{K} & 0 \\ 0 & \mathcal{K}' \end{pmatrix}, \]  

(3.47)

The chiral projection operators \( P_{\pm} \) are defined in (B.1). In (3.46), \( \mathbb{P} \) is necessary because we have integrated the chiral fluctuation modes. Here, \( \mathcal{O}_{\text{ch}} \) is rewritten as

\[ \mathcal{O}_{\text{ch}} = \begin{pmatrix} \mathcal{K} & 0 \\ 0 & \mathcal{K}' \end{pmatrix} (1 + M_{\text{ch}}), \]  

(3.48)

where

\[ M_{\text{ch}} \equiv \begin{pmatrix} 0 & \mathcal{K}^{-1} \bar{W} \frac{\partial^2}{\partial^4} \\ (\mathcal{K}')^{-1} W \frac{\partial^2}{\partial^4} & 0 \end{pmatrix}. \]  

(3.49)

Notice that \( \mathcal{K} \) is a normal matrix when \( \xi_{\text{sg}} = 0 \) (see (3.20)), and

\[ \mathcal{K}^{-1} \equiv \left( \Omega^{-1/3} \Omega_{\text{ch}}^{-2/3} \right) \begin{pmatrix} a^{IJ} & -\langle \mathcal{V}^I \rangle \Phi^\dagger & 0 \\ -\Phi^\dagger \langle \mathcal{V}^J \rangle & -\langle \Omega_{\text{ch}} \rangle \bar{\Phi} \bar{d} + \Phi \Phi^\dagger & 0 \\ 0 & 0 & -\langle \Omega_{\text{ch}} \rangle (e^{-\mathcal{V}})^t \tilde{d} \end{pmatrix}, \]  

(3.50)

where \( \langle \mathcal{V}^I \rangle = -\partial_y \mathcal{V}^I \). When \( \xi_{\text{sg}} \neq 0 \), \( \mathcal{K} \) becomes a differential operator matrix and \( \mathcal{K}^{-1} \) must be understood as the Green’s function for it.

Since only even powers of \( M_{\text{ch}} \) contribute to the trace, it follows that

\[ \text{Tr Istr} \left( \mathbb{P} \ln \mathcal{O}_{\text{ch}} \right) = \text{Tr Istr} \left( \begin{pmatrix} P^+ \ln \mathcal{K} & 0 \\ 0 & P^- \ln \mathcal{K}' \end{pmatrix} \right) + \text{Tr Istr} \left\{ \mathbb{P} \ln \left( 1 + M_{\text{ch}} \right) \right\} \]

\[ = -\frac{2}{4} \text{Tr} \ln \mathcal{K} + \frac{1}{2} \text{Tr Istr} \left\{ \mathbb{P} \ln \left( 1 - M_{\text{ch}}^2 \right) \right\} \]

\[ = -\frac{1}{4} \text{Tr} \left\{ 2 \ln \det \mathcal{K} + \text{tr} \ln \left( 1 + \frac{\mathcal{K}^{-1} \bar{W} (\mathcal{K}')^{-1} W}{4} \right) \right\}, \]  

(3.51)

where \( \text{Tr} \) in the third line denotes the trace over only the functional space of \( y \). Therefore, (3.46) is calculated as

\[ \Omega_{\text{ch}}^{\text{eff}} = \frac{i}{2(2\pi)^4} \int d^4p \frac{1}{p^2} \text{Tr} \left\{ 2 \ln \det \mathcal{K} + \text{tr} \ln \left( -p^2 + \mathcal{D}_{\text{ch}} \right) \right\} + \cdots \]

\[ = \int \frac{d^4p_E}{2(2\pi)^4} \frac{1}{p^2_E} \text{Tr} \left\{ 2 \ln \det \mathcal{K} + \text{tr} \ln \left( p^2_E + \mathcal{D}_{\text{ch}} \right) \right\} + \cdots. \]  

(3.52)
where
\[ D_{\text{ch}} \equiv \mathcal{K}^{-1} \bar{W} (\mathcal{K}')^{-1} W. \]  
(3.53)

Similarly to the derivation of (3.36) or (3.43), this can be rewritten as
\[
\Omega_{\text{eff}}^\text{ch} = \frac{d^D p_E}{2(2\pi)^D} \frac{1}{p_E^2} \sum_k \ln \left( \frac{p_E^2 + \mu_{\text{ch}}^{(k)2}}{\mu_{\text{ch}}^{(k)2}} \right) + \cdots
\]
\[
= - \frac{\Gamma(1 - \frac{D}{2})}{2(4\pi)^{D/2} (\frac{D}{2} - 1)} \sum_k \mu_{\text{ch}}^{(k)D-2} + \cdots, \quad (3.54)
\]

where \( \mu_{\text{ch}}^{(k)2} \) are eigenvalues of \( D_{\text{ch}} \), i.e.,
\[
D_{\text{ch}} f_{\text{ch}}(y; \mu_{\text{ch}}) = \mu_{\text{ch}}^2 f_{\text{ch}}(y; \mu_{\text{ch}}).
\]  
(3.55)

### 3.3.4 Contribution from boundary actions

Here we calculate the contributions to \( \Omega_{\text{eff}}^{\text{1-loop}} \) from the boundary Lagrangians (3.23).

The contribution from the gravitational superfields is
\[
\Omega_{\text{eff}}^{(y_*)U} = \frac{i}{2(2\pi)^4} \int d^4 p \lim_{\zeta^{(y_*)} \to 0} \left\{ \text{Istr} \ln O_U^{\text{bd}} - \text{Istr} \ln \left( \frac{\phi_C}{\zeta^{(y_*)}} h^{(y_*)} \Pi_{\text{sc}}^{(y_*)} \right) \right\}
\]
\[
= - \int \frac{d^4 p_E}{8\pi^4} \frac{1}{p_E^3} \ln \left( |\phi_C|^2 h^{(y_*)} \right) + \cdots
\]
\[
= - \int \frac{d^4 p_E}{8\pi^2} \ln \left( |\phi_C|^2 h^{(y_*)} \right) + \cdots, \quad (3.56)
\]

where the ellipsis denotes terms independent of the background superfields, and
\[
O_U^{\text{bd}} = |\phi_C|^2 h^{(y_*)} \Box_4 \left\{ -\Pi^{\mu\nu}_{3/2} + \frac{1}{\zeta^{(y_*)}} \left( \eta^{\mu\nu} - \Pi^{\mu\nu}_{3/2} \right) \right\}.
\]  
(3.57)

Recall that our formalism respects the superconformal symmetry. Thus (3.56) is independent of the background superfields because their dependences can be absorbed by rescaling the momentum as \( p_E^2 \to p_E^2/\ln(|\phi_C|^2 h^{(y_*)}) \).

The contribution from the vector superfields is
\[
\Omega_{\text{eff}}^{(y_*)V} = \frac{i}{2(2\pi)^4} \int d^4 p \lim_{\zeta^{(y_*)} \to 0} \text{tr} \left\{ \text{Istr} \ln \left( O_T^{(y_*)} P_T + O_C^{(y_*)} P_C \right) - \text{Istr} \ln \left( \text{Re} f_{(y_*)} P_T \right) \right\}
\]
\[
= - \int \frac{d^4 p_E}{16\pi^4} \frac{1}{p_E^3} \text{tr} \left\{ \ln \text{Re} f^{(y_*)} + \ln \left( p_E^2 + M_{(y_*)}^2 \right) \right\} + \cdots, \quad (3.58)
\]

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where

\[(\mathcal{O}_T^{(y_\ast)})_{L_cJ_c} \equiv \text{Re} f_{L_cK_e}^{(y_\ast)} \left\{ \delta^{K_e}_{J_e} \square_4 P_T + \left( \mathcal{M}_V^{2(y_\ast)} \right)_{J_e} \right\},\]

\[(\mathcal{O}_C^{(y_\ast)})_{L_cJ_c} \equiv \text{Re} f_{L_cK_e}^{(y_\ast)} \left\{ \delta^{K_e}_{J_e} \square_4 P_T + \left( \mathcal{M}_V^{2(y_\ast)} \right)_{J_e} \right\},\]

\[\left( \mathcal{M}_V^{2(y_\ast)} \right)_{J_e} \equiv -\frac{3}{2} F^{(y_\ast)}_{L_cK_e} |\phi_C|^2 h_{K_e}^{(y_\ast)} , \quad (3.59)\]

and $F^{(y_\ast)}_{L_cK_e}$ is an inverse matrix of $\text{Re} f_{L_cK_e}^{(y_\ast)}$. The first term in the second line of $(3.58)$ is independent of the background superfields because they can be absorbed by the momentum rescaling. Thus, $(3.58)$ becomes

\[\Omega_{\text{eff}}^{(y_\ast)V} = -\int \frac{d^D p_E}{(2\pi)^D} \frac{1}{p_E^2} \text{tr} \ln \left( p_E^2 + \mathcal{M}_V^{2(y_\ast)} \right) + \cdots \]

\[= \frac{\Gamma(1 - \frac{D}{2})}{(4\pi)^{\frac{D}{2}} \left( \frac{D}{2} - 1 \right)} \text{tr} \left( \mathcal{K}_V^{2(y_\ast)} \right)^{\frac{D}{2} - 1} + \cdots . \quad (3.60)\]

Since the boundary Lagrangian in the chiral sector is written as

\[\mathcal{L}_{bd}^{(y_\ast)} = \int d^4 \theta \phi^\dagger \left( \mathcal{K}_{(y_\ast)} W_{(y_\ast)}^{D_{4\ast}} \mathcal{K}_{(y_\ast)}^{t} \right) \phi_{(y_\ast)} + \cdots, \quad (3.61)\]

where $\phi_{(y_\ast)} \equiv (\phi_C, \bar{\chi}^a)$, and

\[\mathcal{K}_{(y_\ast)} \equiv -3 \left( \begin{array}{cc} h_{(y_\ast)} & \phi_C h_{a}^{(y_\ast)} \\ \phi_C h_{b}^{(y_\ast)} & |\phi_C|^2 h_{ab}^{(y_\ast)} \end{array} \right),\]

\[W_{(y_\ast)} \equiv \left( \begin{array}{c} 3\phi_C P_{(y_\ast)} \frac{3}{2} \phi_C^3 P_{ab}^{(y_\ast)} \\
\frac{3}{2} \phi_C^3 P_{(y_\ast)} \phi_C \phi_C P_{ab}^{(y_\ast)} \end{array} \right), \quad (3.62)\]

the contribution from the chiral superfields is

\[\Omega_{\text{eff}}^{(y_\ast)\text{ch}} = \frac{i}{2(2\pi)^4} \int d^4 p \frac{2}{p^2} \text{tr} \left\{ \ln \mathcal{K}_{(y_\ast)} + \frac{1}{2} \ln \left( 1 - \frac{\mathcal{M}_V^{2(y_\ast)}}{p^2} \right) \right\} \]

\[= \int \frac{d^D p_E}{2(2\pi)^D} \frac{1}{p_E^2} \text{tr} \ln \left( p_E^2 + \mathcal{M}_V^{2(y_\ast)} \right) + \cdots \]

\[= \frac{\Gamma(1 - \frac{D}{2})}{2(4\pi)^{\frac{D}{2}} \left( \frac{D}{2} - 1 \right)} \text{tr} \left( \mathcal{M}_V^{2(y_\ast)} \right)^{\frac{D}{2} - 1} + \cdots , \quad (3.63)\]

where

\[\mathcal{M}_V^{2(y_\ast)} \equiv \mathcal{K}_{(y_\ast)}^{-1} W_{(y_\ast)} (\mathcal{K}_{(y_\ast)})^{-1} W_{(y_\ast)}. \quad (3.64)\]

We have dropped the first term in the first line of $(3.63)$ at the second equality because it can be absorbed by the momentum rescaling.
3.4 Eigenvalues of differential operators

Here we derive equations satisfied by the eigenvalues of $D_F$ ($F = U, V, \text{ch}$), which appear in (3.36), (3.43) and (3.54). Since we have already integrated out the fluctuation superfields, we rewrite the background superfields $\Phi$ and $V$ as $\Phi_{\text{even}}$ and $V$ in the following. From the procedure summarized in Appendix C, we see that $\Phi_{\text{even}}$ and $V_{I_{\text{e}}}$ are independent of $y$ while $V_{I_{\text{o}}}$ have nontrivial $y$-dependences. As explained in Appendix C.2, such $y$-dependences cannot be determined by the equations of motion [45]. Instead, their functional forms are determined when they are regarded as functions of $V_s$ defined by

$$V_s \equiv s_{I_{\text{o}}} V_{I_{\text{o}}}^s,$$

(3.65)

where $s_{I_{\text{o}}}$ are arbitrarily chosen constants [9, 10]. This has the following boundary conditions.

$$V_s |_{y=0} = 0, \quad \lim_{y \to L} V_s = \bar{V}_s \equiv -2s_{I_{\text{o}}} \text{Re} T_{I_{\text{o}}}.$$

(3.66)

As we will explicitly see in the next section, the $s_{I_{\text{o}}}$-dependences are canceled in the final result.

In order to rewrite the eigenvalue equations as differential equations for $V_s$, we rescale $\hat{\Sigma}^I_{I_{\text{o}}}$ as

$$\hat{\Sigma}^I_{I_{\text{o}}} \rightarrow \hat{\Sigma}^I_{I_{\text{o}}} \equiv \frac{\hat{\Sigma}^I_{I_{\text{o}}}}{s_{I_{\text{o}}} \langle V_{I_{\text{o}}}^s \rangle} = -\frac{\hat{\Sigma}^I_{I_{\text{o}}}}{\partial_y V_s}.$$

(3.67)

Then, (3.33), (3.44) and (3.55) are rewritten as

$$\tilde{D}_U \tilde{f}_U(V_s; \mu_U) = \mu_U^2 \tilde{f}_U(V_s; \mu_U),$$

$$\tilde{D}_V \tilde{f}_V(V_s; \mu_V) = \mu_V^2 \tilde{f}_V(V_s; \mu_V),$$

$$\tilde{D}_1 \tilde{D}_2 \tilde{f}_{12}(V_s; \mu_{\text{ch}}) = \mu_{\text{ch}}^2 \tilde{f}_{12}(V_s; \mu_{\text{ch}}),$$

$$\tilde{D}_2 \tilde{D}_1 \tilde{f}_{21}(V_s; \mu_{\text{ch}}) = \mu_{\text{ch}}^2 \tilde{f}_{21}(V_s; \mu_{\text{ch}}),$$

(3.68)

where

$$f_U(y; \mu_U) = \tilde{f}(V_s(y); \mu_U), \quad f_V(y; \mu_V) = \tilde{f}(V_s(y); \mu_V),$$

$$f_{\text{ch}}(y; \mu_{\text{ch}}) = \begin{pmatrix} \tilde{f}_{12}(V_s(y); \mu_{\text{ch}}) \\ \tilde{f}_{21}(V_s(y); \mu_{\text{ch}}) \end{pmatrix},$$

(3.69)
As explained in Appendix C.2, the equations of motion for the background superfields. Therefore, the arguments of the norm function and its derivatives are function \( \langle \Phi \rangle \) matrices, and the arguments of the norm function and its derivatives are functions of \( \chi^a \), which can be either the boundary values of the bulk superfields or 4D superfields localized on the boundaries.

and

\[
\tilde{\mathcal{D}}_V \equiv - \frac{1}{\langle \Omega_* \rangle} \partial_{V_*} \left( \frac{\langle \Omega_*^{4/3} \rangle}{\langle \Omega_*^{2/3} \rangle} \partial_{V_*} \right), \\
(\tilde{\mathcal{D}}_V)'_j \equiv - \frac{a_{IK}}{\langle \Omega_* \rangle} \partial_{V_*} \left\{ \langle \Omega_*^{1/3} \Omega_*^{2/3} \rangle \left( (a \cdot \mathcal{P}_V)_{KJ} \partial_{V_*} - \frac{N_K}{3N} \Phi_{even} \right) \right\} \\
- \frac{\langle \Omega_* \rangle}{\langle \Omega_* \rangle} 2^{3/2} a_{IK} \left( \frac{N_J}{3N} \Phi_{even} \partial_{V_*} + \left( \frac{\partial_K \partial_J \Omega_*}{\Omega_*} \right) - \frac{\Phi_{even} \Phi_{even} \partial_{V_*}}{3} \right), \\
\tilde{\mathcal{D}}_1 \equiv \frac{1}{\langle \Omega_*^{1/3} \Omega_*^{2/3} \rangle} \left( \Phi_{even} \Phi_{even} \partial_{V_*} - a_{IJ} \Phi_{even} \partial_{V_*} - v^t \Phi_{even} \partial_{V_*} \right), \\
\tilde{\mathcal{D}}_2 \equiv \frac{1}{\langle \Omega_* \rangle} (e^{-V})^t (\tilde{t}_I \Phi_{even}, \partial_{V_*}), \quad v^t \equiv \frac{\partial_{V_*} v^{I_*}}{\partial_{V_*} V_*}, \quad v \equiv v^{I_*} \Omega_{I_*}, \quad (3.70)
\]

Here, \( \tilde{\mathcal{D}}_1 \) and \( \tilde{\mathcal{D}}_2 \) are \((n_V + n_C + n_H) \times (n_C + n_H)\) and \((n_C + n_H) \times (n_V + n_C + n_H)\) matrices, respectively, and the arguments of the norm function and its derivatives are \((0_{n_{V_*}}, v^{I_*})\).

As explained in Appendix C.2, the \(V_*\)-dependencies of \(v^t\) and \(V^t\) are determined by the equations of motion for the background superfields. Therefore, the \(V_*\)-dependencies of \(\mathcal{D}_F\) \((F = U, V, 1, 2)\) are already known after deriving the tree-level Kähler potential.

The boundary conditions are obtained from \((E.2)\) and \((E.10)\) as

\[
\left\{ A_{(y_*)}^{(y_*)} \partial_{V_*} - B_{(y_*)}^{(y_*)} \right\} \tilde{f}_F \bigg|_{y=y_*} = 0, \quad (F = U, V, ch) \quad (3.71)
\]

where \(\tilde{f}_c \equiv (\tilde{f}_{12}, \tilde{f}_{21})^t\), \(A_{(y_*)}^{(y_*)}\) and \(B_{(y_*)}^{(y_*)}\) are defined in \((E.11)\), and

\[
A_{(y_*)}^{(y_*)} \equiv \langle \Omega_*^{-1/3} \Omega_*^{4/3} \rangle, \quad B_{(y_*)}^{(y_*)} \equiv \eta_{y_*} \left| \phi_C \right|^2 h_{(y_*)}^{(y_*)} \mu_{U_*}, \\
\left( A_{(y_*)}^{(y_*)} \right)_{I_c J_c} \equiv \langle \Omega_*^{1/3} \Omega_*^{2/3} \rangle a_{I_c J_c}, \quad \left( B_{(y_*)}^{(y_*)} \right)_{I_c J_c} \equiv \eta_{y_*} \left( \text{Re} \left( f_{I_c J_c}^{(y_*)} \right) \mu_{V_*}^2 + \frac{2}{3} \left| \phi_C \right|^2 h_{I_c J_c}^{(y_*)} \right), \\
\left( A_{(y_*)}^{(y_*)} \right)_{I_c J_o} \equiv \left( B_{(y_*)}^{(y_*)} \right)_{I_c J_o} = \left( A_{(y_*)}^{(y_*)} \right)_{I_o J_c} = 0, \quad \left( B_{(y_*)}^{(y_*)} \right)_{I_o J_c} \equiv \delta_{I_o J_c}, \quad (3.72)
\]

where \(\eta_0 = 1\) and \(\eta_L = -1\). We have used \(p^2 = \mu_{F}^2 \) \((F = U, V)\), which follows from the bulk equations of motion. The arguments of the norm function and its derivatives are understood as \((0_{n_{V_*}}, v^{I_*}|_{y=y_*})\), and \(h^{(y_*)}, P^{(y_*)}, f^{(y_*)}\) and their derivatives are functions of \(\chi^a\), which can be either the boundary values of the bulk superfields or 4D superfields localized on the boundaries.
Note that $\tilde{D}_V$, $\tilde{D}_1 \tilde{D}_2$ and $\tilde{D}_2 \tilde{D}_1$ can be expressed in the following forms.

\[
(\tilde{D}_V)^I_J = -\left\langle \frac{\Omega_h}{\Omega_v} \right\rangle^{2/3} \left\{ P_V \partial_{V_s}^2 + A_V \partial_{V_s} + B_V \right\}^I_J,
\]

\[
\tilde{D}_1 \tilde{D}_2 = -\left\langle \frac{\Omega_h}{\Omega_v} \right\rangle^{2/3} \left\{ \begin{pmatrix} 0 & \nu' \Upsilon^I \nline 0 & \mathcal{P}_ch \end{pmatrix} \partial_{V_s}^2 + A_{12} \partial_{V_s} + B_{12} \right\},
\]

\[
\tilde{D}_2 \tilde{D}_1 = -\left\langle \frac{\Omega_h}{\Omega_v} \right\rangle^{2/3} \left\{ (e^{-V})^I \mathcal{P}_ch (e^V)^J \partial_{V_s}^2 + A_{21} \partial_{V_s} + B_{21} \right\}^a_b,
\]

where matrices $A_F$ and $B_F$ ($F = V, 12, 21$) are functions of the background superfields, and

\[
\mathcal{P}_ch \equiv 1_{n_C+n_H} - \Phi_{\text{even}} \Upsilon^I
\]

is a projection operator that satisfies

\[
\mathcal{P}_ch \Phi_{\text{even}} = 0, \quad \Upsilon^I \mathcal{P}_ch = 0, \quad \mathcal{P}_ch^2 = \mathcal{P}_ch.
\]

Hence (3.68) is rewritten as

\[
\begin{align*}
\left\{ \partial_{V_s}^2 + \partial_{V_s} \ln \left\langle \frac{\Omega_v^{4/3}}{\Omega_v^{1/3}} \right\rangle \partial_{V_s} + \left\langle \frac{\Omega_v}{\Omega_h} \right\rangle^{2/3} \mu_U^2 \right\} \tilde{f}_U &= 0, \\
\left\{ P_V \partial_{V_s}^2 + A_V \partial_{V_s} + \tilde{B}_V \right\} \tilde{f}_V &= 0, \\
\left\{ P_{12} \partial_{V_s}^2 + \tilde{A}_{12} \partial_{V_s} + \tilde{B}_{12} \right\} \tilde{f}_{12} &= 0, \\
\left\{ (e^{-V})^I \mathcal{P}_ch (e^V)^J \partial_{V_s}^2 + A_{21} \partial_{V_s} + \tilde{B}_{21} \right\} \tilde{f}_{21} &= 0,
\end{align*}
\]

where

\[
\begin{align*}
\tilde{B}_V &\equiv B_V + \left\langle \frac{\Omega_v}{\Omega_h} \right\rangle^{2/3} \mu_V^2 \mathbf{1}, \quad \tilde{B}_{21} \equiv B_{21} + \left\langle \frac{\Omega_v}{\Omega_h} \right\rangle^{2/3} \mu_{ch}^2 \mathbf{1}, \\
\mathcal{P}_{12} &\equiv \begin{pmatrix} 0_{n_V} & 0 \\
0 & 1_{n_C+n_H} + \Phi_{\text{even}} \frac{N}{N_{\text{even}}} \end{pmatrix}, \quad \tilde{A}_{12} \equiv \begin{pmatrix} \mathcal{P}_V & 0 \\
\Phi_{\text{even}} \frac{N}{N_{\text{even}}} & 1_{n_C+n_H} \end{pmatrix} \mathcal{A}_{12}, \\
\tilde{B}_{12} &\equiv \begin{pmatrix} \mathcal{P}_V & 0 \\
\Phi_{\text{even}} \frac{N}{N_{\text{even}}} & 1_{n_C+n_H} \end{pmatrix} \left( B_{12} + \left\langle \frac{\Omega_v}{\Omega_h} \right\rangle^{2/3} \mu_{ch}^2 \mathbf{1} \right).
\end{align*}
\]

The appearance of the projection operators $\mathcal{P}_V$ and $\mathcal{P}_ch$ in (3.76) reflects the fact that the graviphoton and the compensator superfields are unphysical in the superconformal formulation, while that of $\mathcal{P}_{12}$ stems from the fact that $\tilde{\Sigma}^I$ do not propagate in the super-Landau gauge $\xi_{sg} = 0$. 

26
Solving (3.76) with the boundary conditions at \( y = 0 \) (i.e., \( V_s = 0 \)), we can express \( \tilde{f}_F(V_s) \) \((F = U, V, \text{ch})\) in the form of
\[
\tilde{f}_F(V_s) = C_F(V_s; \mu_F) \cdot N_F,
\] (3.78)
where \( C_F(V_s; \mu_F) \) are matrices that depend on the background superfields, and \( N_F \) is an integration constant vector. (See eq.(4.7) in the next section.) Then the boundary conditions at \( y = L \) (i.e., \( V_s = \bar{V}_s \)) are rewritten as
\[
Q_F(\mu_F) \cdot N_F = 0, \quad (F = U, V, \text{ch})
\] (3.79)
where \( Q_F(\mu_F) \equiv \left( B_F^{(L)} A_F^{(L)} \partial \bar{V}_s - 1 \right) C_F \bigg|_{V_s = \bar{V}_s} \). Due to the presence of the projection operators in (3.76), the constant vectors \( N_F \) \((F = V, \text{ch})\) belong to projected spaces \( \text{PS}_F \).

3.5 Expression of One-loop Kähler potential

Now we obtain the desired expression of the one-loop Kähler potential by summing up the contributions in Sec. 3.3.

\[
\Omega_{\text{eff}}^{1\text{loop}} = -\frac{\Gamma(1 - \frac{D}{2})}{(4\pi)^{\frac{D}{2}}} \sum_{y_* = 0, L} \sum_{F = V, \text{ch}} g_F \text{tr} \left( \mathcal{M}_F^{2(y_*)} \right)^{\frac{D}{2} - 1} - \frac{\Gamma(1 - \frac{D}{2})}{(4\pi)^{\frac{D}{2}}} \sum_{F = U, V, \text{ch}} g_F \sum_k \left( \mu_F^{(k)} \right)^{D - 2} + \cdots,
\] (3.81)
where \( g_U = -2, g_V = -1, g_{\text{ch}} = 1/2 \), and \( \mu_F^{(k)} \) \((F = U, V, \text{ch})\) are solutions of (3.80).

The first line of (3.81) is the contributions from the boundary actions, and is rewritten as
\[
\Omega_{\text{eff}}^{1\text{loop}} = \sum_{y_* = 0, L} \sum_{F = V, \text{ch}} g_F \frac{16\pi^2}{16\pi^2} \text{tr} \left[ \mathcal{M}_F^{2(y_*)} \left\{ \frac{2}{4 - D} - \gamma + \ln(4\pi) - \ln \mathcal{M}_F^{2(y_*)} + 2 \right\} \right] + \mathcal{O}((D - 4)^2) + \cdots,
\] (3.82)
where \( \gamma \) is the Euler’s constant, and the matrices \( \mathcal{M}_V^{2(y_*)} \) and \( \mathcal{M}_{\text{ch}}^{2(y_*)} \) are defined by (3.59) and (3.64), respectively. The divergence will be renormalized by local counterterms in the boundary Lagrangians \( \mathcal{L}_{\text{bd}}^{(y_*)} \) \((y_* = 0, L)\).
The summation of the eigenvalues in the second line of (3.81) can be performed by the technique of Refs. [16]-[21].

\[
\sum_k (\mu_F^{(k)})^{D-2} = \oint_{C_F} \frac{dz}{2\pi i} \frac{F'(z)}{F(z)} z^{D-2} = -\frac{D-2}{\pi} \sin \frac{\pi D}{2} \int_0^\infty d\lambda \lambda^{D-3} \ln \frac{F(i\lambda)}{F(asp)(i\lambda)} = -\frac{(D-2)}{\Gamma(1-\frac{D}{2})\Gamma(\frac{D}{2})} \int_0^\infty d\lambda \lambda^{D-3} \ln \frac{F(i\lambda)}{F(asp)(i\lambda)},
\]

(3.83)

where the functions \( F \) are defined by (3.80), \( C_F \) are contours that enclose the zeros of \( F(z) \), and \( F_{asp}(z) \) are some analytic functions that satisfy

\[
\frac{F(z)}{F_{asp}(z)} = 1 + O(z^{-1}),
\]

(3.84)

for \( \text{Im} \, z \gg 1 \). We can rescale \( z \) by a superfield-dependent factor \( C_{rs} \) so that \( \tilde{F}_{asp}(z) \equiv F_{asp}(C_{rs}z) \) become independent of the superfields. (See (4.14).)

Therefore, \( \Omega_{1\text{loop}}^{\text{eff}} \) is expressed as

\[
\Omega_{1\text{loop}}^{\text{eff}} = \Omega_0 + \Omega_L + \sum_{F=U,V,\text{ch}} \frac{g_F C_{rs}^2}{8\pi^2} \int_0^\infty d\lambda \lambda \ln \frac{\tilde{F}_F(i\lambda)}{\tilde{F}_{asp}(i\lambda)},
\]

(3.85)

where \( \tilde{F}_F(z) \equiv F_F(C_{rs}z) \), and \( \Omega_0 \) and \( \Omega_L \) are the contributions from the boundary actions at \( y = 0 \) and \( y = L \), which are expressed in (3.82). This is our main result. The explicit forms of \( \tilde{F}_F(z) \), \( \tilde{F}_{asp}(z) \), and \( C_{rs} \) are highly model-dependent, but we can easily find them once a model is specified. We will show their explicit forms in a specific case in the next section. The bulk contribution in (3.85) also contains divergent terms. Such terms originate from one-loop diagrams localized on the boundaries, and should be absorbed into \( \Omega_0 \) and \( \Omega_L \). (See the next section.)

### 4 Case of flat spacetime

In this section, we consider a case where the spacetime geometry is flat and \( n_C = 1 \) as an illustrative example. Namely, the compensator multiplet is neutral for the gauge symmetries, and the generators have the following form,

\[
\hat{t}_I = \begin{pmatrix} 0 \\ \hat{t}_I \end{pmatrix}.
\]

(4.1)
Notice that $\hat{t}_I$ contain the gauge coupling. In this case, from (C.29) and (C.26) in Appendix C we find

$$v^I_0(V_s) = \bar{v}^I_0 + \mathcal{O}(\chi^2), \quad V^I_0(V_s) = \bar{v}^I_0 V_s + \mathcal{O}(\chi^2),$$

(4.2)

where

$$\bar{v}^I_0 \equiv \frac{\text{Re} T^I_0}{s \cdot \text{Re} T} = \frac{2 \text{Re} T^I_0}{V_s}, \quad \chi^a \equiv \frac{\phi_{\text{even}}^{a+1}}{\Phi_{\text{even}}^1}.$$  

(4.3)

Therefore, we obtain

$$\langle \Omega_v \rangle = -\frac{N}{(2\text{Re} T)} \frac{1}{V_s^3} + \mathcal{O}(\chi^2), \quad \langle \Omega_h \rangle = |\phi_C|^3 + \mathcal{O}(\chi^2),$$

$$\hat{t}_I \Phi_{\text{even}} = \mathcal{O}(\chi^2), \quad \mathcal{P}_{\text{ch}} = \begin{pmatrix} 0 \\ 1_{n_H} \end{pmatrix} + \mathcal{O}(\chi^2),$$

(4.4)

where $\phi_C \equiv (\Phi_{\text{even}}^1)^{2/3}$.

In the following, we do not see the dependences on $\chi^a$ coming from the bulk hypermultiplets, for simplicity. Then (3.76) becomes simple.

$$\{ \partial^2_{V_s} + r_s^2 \mu^U \} \tilde{f}_U = 0,$$

$$\{ \mathcal{P}_V \partial^2_{V_s} + r_s^2 \mu^V \} \tilde{f}_V = 0,$$

$$\left( \begin{array}{cc} r_s^2 \mu^V \mathcal{P}_V \tilde{f}_U & 0 \\ r_s^2 \mu^V \Phi_{\text{even}}^{a+1} \frac{N}{2V_s} (\partial^2_{V_s} - \bar{v}^I_0 \partial_{V_s} + r_s^2 \mu^V \delta^a_b) \end{array} \right) \tilde{f}_{12} = 0,$$

$$\{ \mathcal{P}_{\text{ch}} \partial^2_{V_s} + \bar{v}^I_0 \partial_{V_s} + r_s^2 \mu^V \} \tilde{f}_{21} = 0,$$

(4.5)

where $\bar{v} \equiv \bar{v}^I_0 \hat{t}_I_0$ and

$$r_s \equiv \frac{\langle \Omega_v \rangle^{1/3}}{\langle \Omega_h \rangle} = -\frac{N^{1/3}(2\text{Re} T)}{|\phi_C| V_s}.$$  

(4.6)

Solutions of (4.5) that satisfy the boundary conditions at $y = 0$ are found to be

$$\tilde{f}_U(V_s; \mu_U) = \left\{ r_s \mu_U \cos (r_s \mu_U V_s) \mathcal{B}^{(0)-1}_U \mathcal{A}^{(0)}_U + \sin (r_s \mu_U V_s) \right\} N_U,$$

$$\tilde{f}_V(V_s; \mu_V) = \mathcal{P}_V \left\{ r_s \mu_V \cos (r_s \mu_V V_s) \mathcal{B}^{(0)-1}_V \mathcal{A}^{(0)}_V + \sin (r_s \mu_V V_s) \right\} N_V,$$

$$\tilde{f}_{\text{ch}}(V_s; \mu_{\text{ch}}) = \mathcal{P}_{\text{ch}} e^{U V_s} \left\{ \omega_{\text{ch}} \cos (\omega_{\text{ch}} V_s) \mathcal{B}^{(0)-1}_{\text{ch}} \mathcal{A}^{(0)}_{\text{ch}} + \sin (\omega_{\text{ch}} V_s) \left( 1 - U \mathcal{B}^{(0)-1}_{\text{ch}} \mathcal{A}^{(0)}_{\text{ch}} \right) \right\} N_{\text{ch}},$$

(4.7)
where \( N_F \) (\( F = U, V, \text{ch} \)) are constant vectors, \( \mathcal{A}_F^{(0)} \) and \( \mathcal{B}_F^{(0)} \) are defined in (3.72) and (E.11), and

\[
P_{\text{ch}} \equiv \begin{pmatrix} 0_{nV} & \mathcal{P}_{\text{ch}} \\ \mathcal{P}_{\text{ch}} & 0_{nV} \end{pmatrix}, \quad U \equiv \frac{1}{2} \begin{pmatrix} 0_{nV} \\ \bar{v} \end{pmatrix}, \quad \omega \equiv \{ r_s^2 \mu_{\text{ch}} P_{\text{ch}} - U^2 \}^{1/2}. \tag{4.8}
\]

Notice that \( \mathcal{P}_V \) and \( P_{\text{ch}} \omega \) commute with \( \mathcal{B}_V^{(0)-1} \mathcal{A}_V^{(0)} \), and \( \mathcal{B}_\text{ch}^{(0)-1} \mathcal{A}_\text{ch}^{(0)} \), respectively, and \( P_{\text{ch}} \omega U = U \omega \), in the present case. The explicit forms of \( \mathcal{B}_\text{ch}^{(0)-1} \mathcal{A}_\text{ch}^{(0)} \) and \( \mathcal{B}_\text{ch}^{(L)-1} \mathcal{A}_\text{ch}^{(L)} \) are calculated from (E.11) as

\[
\mathcal{B}_\text{ch}^{(0)-1} \mathcal{A}_\text{ch}^{(0)} = \frac{1}{r_s \mu_{\text{ch}}} \begin{pmatrix} 0 & 0 & -\frac{\bar{v}_d}{\phi_C} (1_{nH+1} - \mathcal{P}_{\text{ch}}) \\ 0 & -G^{(0)} \bar{d} & 0 \\ 0 & 1_{nH+1} & 0 \end{pmatrix},
\]

\[
\mathcal{B}_\text{ch}^{(L)-1} \mathcal{A}_\text{ch}^{(L)} = \frac{1}{r_s \mu_{\text{ch}}} \begin{pmatrix} 0 & 0 & -\frac{\bar{v}_d}{\phi_C} (1_{nH+1} - \mathcal{P}_{\text{ch}}) \\ 0 & G^{(L)} \bar{d} e^{-\bar{v}_s} & 0 \\ 0 & e^{-\bar{v}_s} & 0 \end{pmatrix}. \tag{4.9}
\]

where

\[
G^{(y_s)} \equiv \left( K_{\text{bd}}^{(y_s)} \mu_{\text{ch}} - W_{\text{bd}}^{(y_s)} \right)^{-1}. \tag{4.10}
\]

Using (4.7), we obtain the expressions of \( \mathcal{Q}_F(\mu_F) \) in (3.79) as

\[
\mathcal{Q}_U(\mu_U) = r_s \mu_U \left( \mathcal{B}_U^{(L)-1} \mathcal{A}_U^{(L)} - \mathcal{B}_U^{(0)-1} \mathcal{A}_U^{(0)} \right) \cos \left( r_s \mu_U \bar{V}_s \right),
\]

\[
-\left( r_s^2 \mu_U^2 \mathcal{B}_U^{(L)-1} \mathcal{A}_U^{(L)} \mathcal{B}_U^{(0)-1} \mathcal{A}_U^{(0)} \right) \sin \left( r_s \mu_U \bar{V}_s \right),
\]

\[
\mathcal{Q}_V(\mu_V) = \mathcal{P}_V \left\{ r_s \mu_V \left( \mathcal{B}_V^{(L)-1} \mathcal{A}_V^{(L)} - \mathcal{B}_V^{(0)-1} \mathcal{A}_V^{(0)} \right) \cos \left( r_s \mu_V \bar{V}_s \right),
\]

\[
-\left( r_s^2 \mu_V^2 \mathcal{B}_V^{(L)-1} \mathcal{A}_V^{(L)} \mathcal{B}_V^{(0)-1} \mathcal{A}_V^{(0)} \right) \sin \left( r_s \mu_V \bar{V}_s \right) \right\},
\]

\[
\mathcal{Q}_{\text{ch}}(\mu_{\text{ch}}) = \mathcal{P}_{\text{ch}} \left\{ \mathcal{B}_\text{ch}^{(L)-1} \mathcal{A}_\text{ch}^{(L)} e^{UV_s} \omega_{\text{ch}} \cos \left( \omega_{\text{ch}} \bar{V}_s \right) - e^{UV_s} \omega_{\text{ch}} \cos \left( \omega_{\text{ch}} \bar{V}_s \right),
\]

\[
-\left( r_s^2 \mu_{\text{ch}}^2 \mathcal{B}_\text{ch}^{(L)-1} \mathcal{A}_\text{ch}^{(L)} \mathcal{B}_\text{ch}^{(0)-1} \mathcal{A}_\text{ch}^{(0)} \right) \sin \left( \omega_{\text{ch}} \bar{V}_s \right),
\]

\[
+ \mathcal{B}_\text{ch}^{(L)-1} \mathcal{A}_\text{ch}^{(L)} e^{UV_s} U \sin \left( \omega_{\text{ch}} \bar{V}_s \right) + e^{UV_s} U \sin \left( \omega_{\text{ch}} \bar{V}_s \right) \mathcal{B}_\text{ch}^{(0)-1} \mathcal{A}_\text{ch}^{(0)} \right\}. \tag{4.11}
\]
Thus, $\mathcal{F}(\mu_F)$ in (3.80) are found to be

$$
\mathcal{F}_U(\mu_U) = \sin \left( \frac{N^{1/3} \mu_U}{|\phi_C|} \right) \left\{ 1 - \frac{|\phi_C|^2}{\mu_U^2 h^{(L)} h^{(0)}} - \frac{|\phi_C|}{\mu_U} \left( \frac{1}{h^{(L)}} + \frac{1}{h^{(0)}} \right) \cot \left( \frac{N^{1/3} \mu_U}{|\phi_C|} \right) \right\},
$$

$$
\mathcal{F}_V(\mu_V) = \sin^{n_V - 1} \left( \frac{N^{1/3} \mu_V}{|\phi_C|} \right) \det \left[ 1 - \frac{|\phi_C|^2}{\mu_V h^{(L)} - 1} - \frac{|\phi_C|}{\mu_V} (H_V^{(L)} - 1 + H_V^{(0)} - 1) \cot \left( \frac{N^{1/3} \mu_V}{|\phi_C|} \right) \right],
$$

$$
\mathcal{F}_{\text{ch}}(\mu_{\text{ch}}) = \det \left( e^{\frac{\pi}{4}} \sin \omega_T \right) \det \left[ 1 - \frac{|\phi_C|^2}{\mu_{\text{ch}} h^{(L)} - 1} - \frac{|\phi_C|^2}{\mu_{\text{ch}}} (H_{\text{ch}}^{(L)} - 1 + H_{\text{ch}}^{(0)} - 1) e^{\frac{\pi}{4}} \sin \omega_T (H_{\text{ch}}^{(0)} - 1) \right], \quad (4.12)
$$

where

$$
\left( H^{(y_s)}_V \right)_{J_e} \equiv \frac{a_{eJ_e}^{y_s}}{N^{2/3}} \left( \frac{\text{Re} f^{(y_s)}_{J_e}}{2 \mu^2_V} + \frac{2}{3} \left| \phi_C \right| h^{(y_s)}_{J_e} \right),
$$

$$
\left( H^{(y_s)}_{\text{ch}} \right)_{ab} \equiv \frac{1}{2} \left( \tilde{\Gamma}^{(y_s)}_{ab} - \frac{|\phi_C|}{\mu_{\text{ch}}} \Gamma^{(y_s)}_{ab} \right),
$$

$$
T_R \equiv \frac{\bar{V}^I l^I e \bar{Y}_e}{2 \text{Re} T^I l^I e},
$$

$$
\omega_T \equiv \left( \frac{N^{2/3} \mu^2_{\text{ch}}}{|\phi_C|^2} - \frac{T_R^2}{4} \right)^{1/2}. \quad (4.13)
$$

The arguments of $a_{eJ_e}$ and $N$ are $(0, 2\text{Re} T^I e)$. The determinants in the expressions of $\mathcal{F}_V(\mu_V)$ and $\mathcal{F}_{\text{ch}}(\mu_{\text{ch}})$ are taken over the $n_{V_s}$-dimensional space spanned by $\bar{V}^I e$ and the $n_H$-dimensional space projected by $\mathcal{P}_{\text{ch}}$, respectively. Notice that the $s_{I_e}$-dependences are completely canceled in (4.12) as mentioned in Sec. 3.4.

The contributions of the bulk superfields to $\Omega^{\text{loop}}_{\text{eff}}$ are calculated from the formula (3.85) with the functions in (4.12). Here we rescale the integral variable $\mu_F$ as $\mu_F \rightarrow C_{rs} \mu_F$, where $C_{rs} = |\phi_C| / N^{1/3}$. Then the analytic functions $\tilde{F}_U^{\text{asp}}$ in (3.85) can be chosen as

$$
\tilde{F}_U^{\text{asp}}(z) = \frac{i}{2} e^{-iz}, \quad \tilde{F}_V^{\text{asp}}(z) = \left( \frac{i}{2} e^{-iz} \right)^{n_V - 1}, \quad \tilde{F}_{\text{ch}}^{\text{asp}}(z) = \left( \frac{i}{2} e^{-iz} \right)^{2n_H}. \quad (4.14)
$$
and the bulk contribution in (3.85) is expressed as
\[
\Omega^{1\text{loop}} = \frac{|\phi_C|^2}{N^{2/3}} \left[ \frac{(n_V + 1)Q_1(0) - \text{tr} Q_1(T_R/2)}{8\pi^2} + Q_2 \text{tr} (T_R^3) \right.
\]
\[
+ \int_0^\infty \frac{d\lambda}{8\pi^2} \sum_{F=U,V,\text{ch}} g_F \lambda \ln g_F(\lambda) \bigg] + \cdots ,
\] (4.15)
where \( Q_1(x) \equiv -\int_{|x|}^\infty d\lambda \lambda \ln (2e^{-\lambda} \sinh \lambda) \), \( Q_2 \equiv \int_0^\infty \frac{d\lambda}{64\pi^2} \lambda^2 \left( \sqrt{1 + \lambda^2} - 1 \right) \), and
\[
G_U(\lambda) = 1 + \frac{N^{2/3}}{\lambda^2 h(L) h(0)} + \frac{N^{1/3}}{\lambda} \left( \frac{1}{h(L)} + \frac{1}{h(0)} \right) \coth \lambda,
\]
\[
G_V(\lambda) = \det \left\{ 1_{n_V} + \frac{N^{2/3}}{\lambda^2} \hat{H}_V^{(L)-1} \hat{H}_V^{(0)-1} + \frac{N^{1/3}}{\lambda} \left( \hat{H}_V^{(L)-1} + \hat{H}_V^{(0)-1} \right) \coth \lambda \right\},
\]
\[
G_{\text{ch}}(\lambda) = \det \left\{ 2e^{-\frac{T_R}{2}} e^{-\lambda} \sinh \hat{\omega}_T + \frac{2N^{2/3}}{\lambda^2} \hat{H}_{\text{ch}}^{(L)-1} \hat{H}_{\text{ch}}^{(0)-1} e^{-\frac{T_R}{2}} e^{-\lambda} \sinh \hat{\omega}_T \hat{H}_{\text{ch}}^{(0)-1}
\right.
\]
\[
+ \frac{2N^{1/3}}{\lambda^2} \hat{H}_{\text{ch}}^{(L)-1} e^{-\frac{T_R}{2}} e^{-\lambda} \left( \hat{\omega}_T \cosh \hat{\omega}_T - \frac{T_R}{2} \sinh \hat{\omega}_T \right)
\]
\[
+ 2e^{-\lambda} \left( \hat{\omega}_T \cosh \hat{\omega}_T + \frac{T_R}{2} \sinh \hat{\omega}_T \right) e^{-\frac{T_R}{2} N^{1/3}} \hat{H}_{\text{ch}}^{(0)-1} \right\}
\times \left\{ \det \left( 2e^{-\frac{T_R}{2}} e^{-\lambda} \sinh \hat{\omega}_T \right) \right\}^{-1}.
\] (4.16)
The argument of the norm function \( N \) is \((0_{n_V}, 2\text{Re} T^{I_e})\), and
\[
\left( \hat{H}_V^{(y_*)} \right)_{J_e}^{I_e} = a^{I_e}_{K_e} \left( \frac{\text{Re} f_{K_e J_e}^{(y_*)}}{N^{2/3}} - \frac{2}{3\lambda^2} \tilde{h}(y_*) \right),
\]
\[
\left( \hat{H}_{\text{ch}}^{(y_*)} \right)_{ab} = \frac{1}{2} \left( \tilde{h}(y_*) + i \frac{N^{1/3}}{\lambda} P_{ab}^{(y_*)} \right),
\]
\[
\hat{\omega}_T \equiv \left( \lambda^2 + \frac{T_R^2}{4} \right)^{1/2}.
\] (4.17)
In (4.15), we have used \( d\lambda \lambda = d\hat{\omega}_T \hat{\omega}_T \), and
\[
\int_0^\infty d\lambda \lambda \ln \det (e^{\hat{\omega}_T - \lambda}) = \int_0^\infty d\lambda \lambda \text{tr} (\hat{\omega}_T - \lambda) = Q_2 \text{tr} (T_R^3).
\] (4.18)
As mentioned in Sec. 3.5, Eq. (4.15) contain divergent terms. The constant \( Q_2 \) is divergent and will be renormalized by local counterterms. The last term in (4.15) also diverges in the presence of the boundary terms. In order to extract a finite part, we further rewrite it as
\[
\Omega^{1\text{loop}} = \frac{|\phi_C|^2}{N^{2/3}} \sum_{y_*=0, L} \int_0^\infty \frac{d\lambda}{8\pi^2} \sum_{F=U,V,\text{ch}} g_F \lambda \ln H_F^{(y_*)}(\lambda)
\]
\[
+ \frac{|\phi_C|^2}{N^{2/3}} \int_0^\infty \frac{d\lambda}{8\pi^2} \sum_{F=U,V,\text{ch}} g_F \lambda \ln \frac{G_F(\lambda)}{H_F^{(L)}(\lambda) H_F^{(0)}(\lambda)} + \cdots ,
\] (4.19)
where

\[
\mathcal{H}^{(y_\ast)}(\lambda) \equiv 1 + \mathcal{N}/\lambda^{1/3}, \quad \mathcal{H}_V^{(y_\ast)}(\lambda) \equiv \det \left( 1 + \mathcal{N}/\lambda H_V^{(y_\ast)-1} \right),
\]

\[
\mathcal{H}_{\text{ch}}^{(0)}(\lambda) \equiv \det \left( 1 + \mathcal{N}/\lambda^2 \left( \hat{\omega}_T + \frac{T_R}{2} \right) H_{\text{ch}}^{(0)-1} \right),
\]

\[
\mathcal{H}_{\text{ch}}^{(L)}(\lambda) \equiv \det \left( 1 + \mathcal{N}/\lambda^2 H_{\text{ch}}^{(L)-1} e^{T_R} \left( \hat{\omega}_T - \frac{T_R}{2} \right) \right).
\]

Now the second line of (4.19) is finite. Nonlocal effects such as the brane-to-brane mediation effects are contained in this part. We can also see that the divergent part, which is the first line of (4.19), does not depend on the parameters in \( \mathcal{L}_{\text{bd}}^{(0)} \) and those in \( \mathcal{L}_{\text{bd}}^{(L)} \) simultaneously. This indicates that the divergent terms originate from one-loop diagrams localized on the boundaries. Thus they should be combined with \( \Omega_{y_\ast} \) \((y_\ast = 0, L)\) as mentioned in Sec. 3.5.

As a result, the one-loop Kähler potential is expressed as

\[
\Omega^{\text{1loop}}_{\text{eff}}(\phi_C, \chi{a}, T^{I_o}, V^{I_o}) = \Omega_0 + \Omega_L + \frac{|\phi_C|^2}{\mathcal{N}^{2/3}} \left[ \frac{(n_V + 1)Q_1(0)}{8\pi^2} - \sum_{a=1}^{n_H} Q_1(c_{aI_o} \text{Re} T^{I_o}) \frac{8\pi^2}{\mathcal{N}} - Q_2 \sum_{a=1}^{n_H} (2c_{aI_o} \text{Re} T^{I_o})^3 \right.
\]

\[
\left. + \int_0^\infty d\lambda \frac{g_F^2}{8\pi^2} \sum_{F=U,V,\text{ch}} \text{Re} \ln \frac{G_F(\lambda)}{H_F^{(L)}(\lambda) H_F^{(0)}(\lambda)} \right] + O(\chi^2),
\]

where \( n_V - 1 \) and \( n_H \) are the numbers of the physical vector and hypermultiplets, respectively, \( \mathcal{N} = \mathcal{N}(0_{n_V}, 2\text{Re} T^{I_o}) \), \( Q_1 \approx 0.30 \), \((g_U, g_V, g_{\text{ch}}) = (-2, -1, \frac{1}{2})\), and \( \hat{\chi} \equiv \exp \left\{ \frac{1}{2} V^{I_o} \hat{t}_{I_o} \right\} \chi \). Here the generators \( \hat{t}_{I_o} \) are denoted as \( \hat{t}_{I_o} = -\text{diag}(c_{1I_o}, c_{2I_o}, \ldots, c_{n_H I_o}) \), where \( c_{aI_o} \) are \( \mathbb{Z}_2 \)-odd gauge couplings corresponding to the bulk masses for the hypermultiplets. The boundary contributions \( \Omega_{y_\ast} \) \((y_\ast = 0, L)\) are sum of (3.82) and the first line of (4.19), and are renormalized by local counterterms in the boundary Lagrangians \( \mathcal{L}_{\text{bd}}^{(y_\ast)} \).

The renormalized value of \( Q_2 \) cannot be predicted within the field theory. The last term in (4.21) involves the parameters both in \( \mathcal{L}_{\text{bd}}^{(0)} \) and \( \mathcal{L}_{\text{bd}}^{(L)} \), and becomes important when one of the boundary actions possesses some symmetries that are not held in the whole system. In such a case, terms prohibited by those symmetries are induced through loop diagrams involving the bulk superfields, and they are finite. The vector superfields \( V^{I_o} \) appear only through \( \hat{\chi} \), just like in the tree-level effective Lagrangian (C.17). The overall dependence on the moduli through \( \mathcal{N}^{-2/3} \) represents the volume suppression of the extra dimension, and the nontrivial dependence on them are induced through the gaugings accompanied by the hypermultiplet bulk masses \( c_{aI_o} \). The bosonic component expression of \( \int d^4\theta~\Omega^{\text{1loop}}_{\text{eff}} \) is shown in Appendix 5 in the absence of the boundary terms.
5 Summary

We derived one-loop contributions to the Kähler potential in 4D effective theory of 5D SUGRA on $S^1/Z_2$ with a generic form of the prepotential and arbitrary boundary-localized terms. Our work is regarded as an extension of Refs. [13, 14, 22, 23] to more general cases, and the result is applicable to a wide class of 5D SUGRA models, in which various isometries are gauged by arbitrary number of $Z_2$-odd vector multiplets (i.e., moduli multiplets). The calculations are performed by means of the $N = 1$ superfield formalism [10, 30], which is based on the superconformal formulation of 5D SUGRA [25]-[28]. Since the off-shell formulation of SUGRA contains unphysical modes, such as the compensator multiplet, some projection operators appear in the calculations. This makes the procedure somewhat complicated. Especially, due to the projection operator $P_V$, the ordinary Kaluza-Klein expansion of the vector superfields $V^I$ cannot be performed in a way that the $N = 1$ superfield structure is preserved [9, 45]. Instead, corresponding procedure becomes possible by changing the coordinate $y$ with $V_s$ defined in (3.65).

The one-loop effective Kähler potential $\Omega_{\text{loop}}^\text{eff}$ is relevant to the brane-to-brane communication of SUSY-breaking effects and the moduli stabilization by the Casimir effect. Our result makes it possible to discuss these issues in much wider class of 5D SUGRA models than ever. Although the explicit forms of $\tilde{F}_{\text{F}}(z), \tilde{F}_{\text{F}}^{\text{sup}}(z)$ ($F = U, V, \text{ch}$), and $C_{rs}$ in our formula (3.85) are highly model-dependent, we can easily find them once a model is specified. As an illustrative example, we provided an explicit expression of $\Omega_{\text{loop}}^\text{eff}$ in the case of 5D flat spacetime. In the case of a warped geometry, the expression becomes more complicated, and may not be expressed in an analytic form except in the Randall-Sundrum spacetime. Still, we expect that some properties can be extracted by means of a technique used in Ref. [53].

The one-loop Kähler potential is also relevant to gauge symmetry breaking by the Wilson line phase [54]. For example, we can discuss the gauge-Higgs unification scenario at the grand unification scale [55]-[58] in the context of 5D SUGRA after extending our result to non-Abelian gauge groups.

There are several ways to proceed. We plan to discuss the moduli stabilization and the SUSY-breaking mediation in 5D SUGRA models with a generic form of the prepotential.

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16 As we have pointed out in Ref. [10], we have to require a fine-tuning among the gauge couplings and the vacuum expectation values of the moduli in order to obtain the Randall-Sundrum spacetime when there are more than one moduli.
by making use of our result, and derive useful information for the phenomenological model-building. An extension of our result to higher-dimensional SUGRA is another direction for future works. Notice that an $N = 1$ superfield description of the action should be exist although such theories do not have a full off-shell formulation. Since our derivation in this paper is systematic, it can easily be extended to higher-dimensional SUGRA once we obtain the $N = 1$ superfield description.

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**A Superconformal transformations**

Here we list the 5D superconformal transformation laws expressed in terms of the $N = 1$ superfields. For the purpose of constructing the action up to linear in the gravitational superfields, it is enough to keep the transformations at the zeroth order in them.

The $N = 1$ part $\delta^{(1)}_{sc}$ is given by

\[
\begin{align*}
\delta^{(1)}_{sc} \Phi_{\text{odd}} &= \left( -\frac{1}{4} \bar{D}^2 L^\alpha D_{\alpha} - i \sigma^\mu_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu - \frac{1}{8} \bar{D}^2 D^\alpha L_\alpha \right) \Phi_{\text{odd}}, \\
\delta^{(1)}_{sc} \Phi_{\text{even}} &= \left( -\frac{1}{4} \bar{D}^2 L^\alpha D_{\alpha} - i \sigma^\mu_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu - \frac{1}{8} \bar{D}^2 D^\alpha L_\alpha \right) \Phi_{\text{even}}, \\
\delta^{(1)}_{sc} V &= \left( -\frac{1}{4} \bar{D}^2 L^\alpha D_{\alpha} - \frac{i}{2} \sigma^\mu_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu + \text{h.c.} \right) V, \\
\delta^{(1)}_{sc} \Sigma &= \left( -\frac{1}{4} \bar{D}^2 L^\alpha D_{\alpha} - i \sigma^\mu_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu \right) \Sigma, \\
\delta^{(1)}_{sc} U^\mu &= \frac{1}{2} \sigma^\mu_{\alpha\dot{\alpha}} \left( \bar{D}^{\dot{\alpha}} L^\alpha - D^\alpha \bar{L}^{\dot{\alpha}} \right), \\
\delta^{(1)}_{sc} \Psi^\alpha &= -\partial_y L^\alpha, \\
\delta^{(1)}_{sc} U^y &= 0,
\end{align*}
\]

where a complex spinor superfield $L^\alpha$ is the transformation parameter. The remaining
transformations $\delta^{(2)}_{sc}$ are given by

$$
\delta^{(2)}_{sc} \Phi_{\text{odd}} = \frac{Y}{\langle V_E \rangle} \partial_y \Phi_{\text{odd}} - \frac{i}{4} D^2 \left\{ \tilde{N}(e^{-V})' \Phi_{\text{even}} \right\},
$$

$$
\delta^{(2)}_{sc} \Phi_{\text{even}} = \frac{Y}{\langle V_E \rangle} \partial_y \Phi_{\text{even}} + \frac{i}{4} \tilde{D}^2 \left\{ \tilde{N} e^V \Phi_{\text{odd}} \right\},
$$

$$
\delta^{(2)}_{sc} e^V = \frac{Y + \bar{Y}}{2 \langle V_E \rangle} \partial_y e^V + \frac{i}{4} \tilde{D}^2 \left\{ \tilde{N} e^V \bar{D} \right\},
$$

$$
\delta^{(2)}_{sc} \Sigma = \partial_y \left( \frac{Y \Sigma}{\langle V_E \rangle} \right) - \frac{i \langle V_E \rangle}{8} \tilde{D}^2 \left( D^\alpha \tilde{N} \bar{D} e^V \bar{e}^{-V} \right),
$$

$$
\delta^{(2)}_{sc} U^\mu = 0, \quad \delta^{(2)}_{sc} V_E = \frac{1}{2} \partial_y (Y + \bar{Y}),
$$

$$
\delta^{(2)}_{sc} \Psi^\alpha = \frac{i \langle V_E \rangle}{2} D^\alpha \tilde{N}, \quad \delta^{(2)}_{sc} U^y = \frac{N}{\langle V_E \rangle},
$$

where a chiral and real superfields $Y$ and $N$ are the transformation parameters, and

$$
\tilde{N} \equiv N - \frac{i}{2} (Y - \bar{Y}).
$$

The components of $L_\alpha$,

$$
\xi^\mu \equiv - \text{Re} \left( i \sigma^\mu_{\alpha} \bar{D} L_\alpha \right) \big|_0, \quad \epsilon_\alpha \equiv - \frac{1}{4} \bar{D}^2 L_\alpha \big|_0,
$$

$$
\lambda_{\mu\nu} \equiv - \frac{1}{2} \text{Re} \left( \left( \sigma_{\mu\nu} \right)_{\beta} D^\alpha D^2 L_\beta \right) \big|_0, \quad \varphi_D \equiv \text{Re} \left( \frac{1}{4} D^\alpha D^2 L_\alpha \right) \big|_0,
$$

$$
\vartheta_A \equiv \text{Im} \left( - \frac{1}{6} D^\alpha \bar{D}^2 L_\alpha \right) \big|_0, \quad \eta_\alpha \equiv - \frac{1}{32} \bar{D}^2 \bar{D}^2 L_\alpha \big|_0,
$$

where the symbol $|_0$ denotes the lowest component of the superfield, are identified with the transformation parameters for the translation $P$, the supersymmetry $Q$, the Lorentz transformation $M$, the dilatation $D$, the R symmetry $U(1)_A$ and the conformal supersymmetry $S$, respectively. The components of $Y$ and $N$ are identified with the other transformation parameters that are $Z_2$-odd [30].

In order to determine the kinetic terms for the gravitational superfields $L^{E_W}_{\text{kin}}$, we need to extend the above transformations including linear order terms in the gravitational superfields. Since $L^{E_W}_{\text{kin}}$ is independent of the quantum fluctuation of the matter superfields, it is enough to focus on the background parts of the matter superfields in the extended parts of $\delta^{(1)}_{sc}$ and $\delta^{(2)}_{sc}$. We find the $U^\mu$-dependent part in the transformations as follows.
The \( \delta^{(1)}_{sc} \) does not receive any corrections at this order, but \( \delta^{(2)}_{sc} \) is modified as

\[
\delta^{(2)}_{sc}\Phi^{\text{odd}} = -\frac{i}{4} \bar{D}^2 \left\{ \left( \frac{\bar{N}}{3} \Delta U^\mu - Y \partial_\mu U^\mu \right) \left\langle (e^{-V})^\dagger \bar{\Phi}_{\text{even}} \right\rangle \right\} + \cdots ,
\]

\[
\delta^{(2)}_{sc}\Phi^{\text{even}} = \frac{i}{4} \bar{D}^2 \left\{ \left( \frac{\bar{N}}{3} \Delta U^\mu - Y \partial_\mu U^\mu \right) \left\langle e^V \bar{\Phi}^{\text{odd}} \right\rangle \right\} + \cdots ,
\]

\[
\delta^{(2)}_{sc} V^I = -i \partial_\mu U^\mu Y \left\langle \Sigma^I \right\rangle - \bar{Y} \left\langle \bar{\Sigma}^I \right\rangle \left\langle V^E \right\rangle + \cdots ,
\]

(A.5)

where the ellipses denote terms shown in (A.2). The other transformations are unchanged up to this order. Here we have considered in the Abelian case, for simplicity. Requiring the invariance of the action under this modified transformation, we can determine \( \mathcal{L}^{\text{kin}}_{\text{EW}} \) as (2.15).

B Projectors in superspace

The chiral and anti-chiral projection operators are defined as [40]

\[
P_+ \equiv -\frac{D^2 \bar{D}^2}{16 \Box}, \quad P_- \equiv -\frac{D^2 \bar{D}^2}{16 \Box}.
\]

We can divide a vector superfield \( V \) into a chiral and a transverse parts by the following projectors.

\[
P_C \equiv P_+ + P_-, \quad P_T \equiv \frac{D^\alpha \bar{D}^2 D_\alpha}{8 \Box}.
\]

(B.2)

These satisfy

\[
P_T + P_C = 1, \quad P_T^2 = P_T, \quad P_C^2 = P_C, \quad P_T P_C = P_C P_T = 0.
\]

(B.3)

Similarly, the gravitational superfield \( U^\mu \) can be divided by the following superspin projectors as [39] [23, 49, 50].

\[
\Pi^\mu_{0} \equiv \Pi^\mu_{L} P_C, \\
\Pi^\mu_{1/2} \equiv \frac{1}{3} Q^\mu + \Pi^\mu_{L} P_T + \frac{1}{3} \Pi^\mu_{L} P_C, \\
\Pi^\mu_{1} \equiv \Pi^\mu_{T} P_C, \\
\Pi^\mu_{3/2} \equiv -\frac{1}{3} Q^\mu + \eta^\mu P_T - \Pi^\mu_{L} + \frac{2}{3} \Pi^\mu_{L} P_C,
\]

(B.4)

where

\[
Q^\mu \equiv \frac{1}{16} \bar{\sigma}_\alpha \sigma^\nu \left[ D^\alpha, \bar{D}^\beta \right] \left[ D^\beta, \bar{D}^\delta \right],
\]

\[
\Pi^\mu_{T} \equiv \eta^\mu - \frac{\partial^\mu \partial^\nu}{\Box}, \quad \Pi^\mu_{L} \equiv \bar{\partial}^\mu \partial^\nu.
\]

(B.5)
These projectors satisfy
\[ \Pi^{\mu\nu}_0 + \Pi^{\mu\nu}_{1/2} + \Pi^{\mu\nu}_1 + \Pi^{\mu\nu}_{3/2} = \eta^{\mu\nu}, \]
\[ \Pi^{\mu\rho}_s \Pi^{\nu}_r = \delta_{rs} \Pi^{\mu\nu}_s, \] (B.6)
where \( r, s = 0, 1/2, 1, 3/2, \) and
\[ \partial_\mu \Pi^{\mu\nu}_0 = \partial_\nu P_C, \quad \partial_\mu \Pi^{\mu\nu}_{1/2} = \partial_\nu P_T, \quad \partial_\mu \Pi^{\mu\nu}_1 = \partial_\nu \Pi^{\mu\nu}_{3/2} = 0. \] (B.7)

Furthermore, \( Q^{\mu\nu} \) satisfies
\[ Q^{\mu\rho} Q_\rho^\nu = Q^{\mu\nu} (-4P_C + 3), \]
\[ Q^{\mu\nu} P_C = P_C Q^{\mu\nu} = -\Pi^{\mu\nu}_L P_C = -\Pi^{\mu\nu}_0, \]
\[ \partial_\mu Q^{\mu\nu} P_T = \partial_\mu P_T^T Q^{\mu\nu}. \] (B.8)

The supertrace integrand \( \text{Istr} \) in (3.8) satisfies the following relations.
\[ \text{Istr} 1 = 0, \quad \text{Istr} (\bar{D}^2 \bar{D}^2) = \text{Istr} (D^2 D^2) = \text{Istr} (\bar{D}^\alpha \bar{D}^2 D_\alpha) = 16, \] (B.9)
and thus,
\[ \text{Istr} P_\pm = -\frac{1}{\Box_4}, \quad \text{Istr} P_T = \frac{2}{\Box_4}, \quad \text{Istr} Q^{\mu\nu} = \text{tr} \left( \frac{2}{\Box_4} \eta^{\mu\nu} \right) = \frac{8}{\Box_4}, \]
\[ \text{Istr} \Pi^{\mu\nu}_0 = -\frac{2}{\Box_4}, \quad \text{Istr} \Pi^{\mu\nu}_{1/2} = \frac{4}{\Box_4}, \quad \text{Istr} \Pi^{\mu\nu}_1 = -\frac{6}{\Box_4}, \quad \text{Istr} \Pi^{\mu\nu}_{3/2} = \frac{4}{\Box_4}. \] (B.10)

## C Tree-level effective action

In this section, we briefly review the derivation of the 4D effective action at tree level. We have developed a systematic method to derive it in Ref. [45]. Explicit calculations in the flat and the warped spacetimes are performed in Refs. [9, 10].

The basic strategy is as follows. First, we drop the kinetic terms for \( Z_2 \)-odd superfields because they do not have zero-modes that are dynamical below the compactification scale. Then the \( Z_2 \)-odd superfields play a role of Lagrange multipliers, and their equations of motion extract zero-modes from the \( Z_2 \)-even superfields.

Since we are interested in the 4D effective action for the matter superfields, we neglect the gravitational superfields in this section. Namely, the 5D Lagrangian (2.24) reduces to
\[ \mathcal{L} = -\int d^4 \theta \, 3N^{1/3}(\mathcal{V}) \left\{ \Phi^\dagger_{\text{odd}} \tilde{d}(e^V)^t \Phi_{\text{odd}} + \Phi^\dagger_{\text{even}} \tilde{d} e^{-V} \Phi_{\text{even}} \right\}^{2/3} + \left[ \int d^2 \theta \left\{ 2 \Phi^\dagger_{\text{odd}} \tilde{d} (\partial_y - \Sigma) \Phi_{\text{even}} + W_v \right\} + \text{h.c.} \right] + 2 \sum_{y_*=0,L} \mathcal{L}_{\text{bd}}^{(y_*)} \delta(y - y_*), \] (C.1)
where we have performed the partial integral.
C.1 Gauge kinetic functions and superpotential

First, we divide $V$ into the $Z_2$-odd part $V_o$ and the $Z_2$-even part $V_e$ as

$$e^V = e^{V_o/2}e^{V_e/2}. \quad (C.2)$$

Before dropping the kinetic terms for the $Z_2$-odd superfields, we eliminate $\Sigma$ from the bulk action by means of the supergauge transformation (2.7) with the transformation parameter,

$$e^{-\Lambda(y)} = \exp \left\{ -\Lambda \Sigma(y) \right\} \equiv P \exp \left\{ \int_0^y dy' \Sigma(y') \right\}, \quad (C.3)$$

where $P$ denotes the path-ordering operator. Namely, this is a solution to $\partial_y e^{-\Lambda} = -\Sigma e^{-\Lambda}$.

Although the $Z_2$-odd superfields $\Sigma^I_e$ are completely gauged away, the zero-modes of the $Z_2$-even superfields $\Sigma^I_o$ remain in the theory as we will explain below. We define 4D superfields $T$ and $S$ as

$$e^S e^T \equiv \lim_{y \to L} \exp \left\{ -\Lambda \Sigma(y) \right\} = P \exp \left\{ \int_0^L dy \Sigma(y) \right\}, \quad (C.4)$$

where $\hat{t}_l \equiv 2igt_l$ are hermitian generators, and the limits are taken from the bulk region ($0 < y < L$). Then, the gauge-transformed vector superfields have the following boundary conditions.

$$\lim_{y \to 0} e^V = \left( e^{V'_e} \right)_{y=0}, \quad \lim_{y \to L} e^V = e^{-T}e^{-S} \left( e^{V'_o} \right)_{y=L} e^{-S^t}e^{-T^t}. \quad (C.5)$$

where $V'_e$ and $V'_o$ denote the vector superfields before the gauge transformation by (C.3).

Since $V_e$ corresponds to the gauge superfield for the 4D unbroken gauge group, it should vanish in $\mathcal{N}^{1/3}(\mathcal{V})$ in (C.1) because there is no corresponding term in 4D gauge theories. This implies that

$$\partial_y V_e = 0. \quad (C.6)$$

Then, $\mathcal{N}(\mathcal{V})$ reduces to

$$\mathcal{N}(\mathcal{V}) = \mathcal{N} \left( e^{V_o} \partial_y e^{-V_o} \right), \quad (C.7)$$

and the boundary conditions for $V_o$ in (C.5) becomes

$$V_o|_{y=0} = 0, \quad \lim_{y \to L} V_o = \tilde{V}_o \equiv -T - T^t + \frac{1}{2} \left[ V_e, T - T^t \right] + \cdots, \quad (C.8)$$

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where \( V_o \) is defined so that \( e^T e^V e^V e^T e^{T^\dagger} \) belongs to the unbroken gauge group. Notice that \( V_o \) is discontinuous at \( y = L \) since it is \( Z_2 \)-odd. This discontinuity stems from the discontinuous gauge transformation (C.3). (See (C.12).)

Now we impose constraints \( D_\alpha V^{I_o} = 0 \) to drop the kinetic terms for \( V^{I_o} \). To illustrate the procedure of deriving the gauge kinetic functions in (C.11), we consider a case that the gauge group is Abelian. Then, since \( \Sigma \) has been gauged away, \( W_v \) becomes

\[
W_v = \frac{c^3}{16g^3} \text{tr} \left\{ \frac{1}{12} \bar{D}^2 (\partial_y V_o D^\alpha V_e) W_{\alpha e} \right\} - \frac{c^3}{48g^3} \partial_y \text{tr} \{ \Lambda_\Sigma W^2_e \}
\]

\[
= \frac{c^3}{48g^3} \partial_y \text{tr} \left\{ \frac{1}{4} \bar{D}^2 (V_e D^\alpha V_e) W_{\alpha e} - \Lambda_\Sigma W^2_e \right\},
\]

where we have used (C.6) at the second equality, and

\[
\Lambda_\Sigma W^2_e = \frac{1}{4} \bar{D}^2 (e^V D^\alpha e^{-V_e}) = -\frac{1}{4} \bar{D}^2 D^\alpha V_e.
\]

We have also used that \( \text{tr} \{ \{ \hat{t}_{I_o}, \hat{t}_{J_e} \} \hat{t}_{K_e} \} = 0 \). (See the footnote [11]) The last term in the first line of (C.9) is induced by the supergauge transformation with \( \Lambda_\Sigma \). Thus,

\[
\int_0^L dy \left\{ \int d^2 \theta \ W_v + \text{h.c.} \right\} = \frac{c^3}{48g^3} \int d^4 \theta \left[ \text{tr} \left\{ \left( -2V_o - \Lambda_\Sigma - \Lambda^\dagger_\Sigma \right) D^\alpha V_e W_{\alpha e} \right\} \right]_0^L
\]

\[
= \frac{c^3}{16g^3} \int d^4 \theta \left\{ (T + T^\dagger) D^\alpha V_e W_{\alpha e} \right\}
\]

\[
= \frac{c^3}{16g^3} \int d^2 \theta \left( TW^2_e \right) + \text{h.c.}.
\]

We have performed the partial integrals, used the relations \( d^2 \bar{\theta} = -\frac{1}{4} \bar{D}^2, D^\alpha W_{\alpha e} = \tilde{D}_\alpha \tilde{W}_e^\alpha \), \( \text{tr} (\hat{t}_{I_o} \hat{t}_{J_e} \hat{t}_{K_e}) = 0 \), and

\[
\lim_{y \to 0} \Lambda_\Sigma = -S, \quad \lim_{y \to 0} V_o = 0,
\]

\[
\lim_{y \to L} \Lambda_\Sigma = -T - S, \quad \lim_{y \to L} V_o = -T - T^\dagger.
\]

The expression (C.11) is also valid in the non-Abelian case. In fact, it is invariant under the unbroken 4D gauge transformation

\[
T \to e^{\Lambda_0 T} e^{-\Lambda_0}, \quad e^{V_e} \to e^{\Lambda_0} e^{V_e} e^{\Lambda_0^\dagger},
\]

where \( \Lambda_0 = \sum_{I_o} \Lambda^I_o \hat{t}_{I_o} \) is y-independent.

\[\text{17 This gauge transformation preserves the gauge in which } \Sigma = 0.\]
Next we drop the kinetic terms for $\Phi_{\text{odd}}$ in the first line of (C.1). Then, from the equation of motion for $\Phi_{\text{odd}}$, we obtain
\[
\partial_y \Phi_{\text{even}} = 0, \tag{C.14}
\]
which means that $\Phi_{\text{even}}$ is $y$-independent for $0 \leq y < L$.

Recall that the gauge transformation parameter $e^{\Lambda \Sigma}$ is discontinuous at $y = L$,
\[
e^{\Lambda \Sigma} \bigg|_{y=L} = e^{-S}, \quad \lim_{y \to L} e^{\Lambda \Sigma} = e^{-T} e^{-S}, \tag{C.15}
\]
since the $\mathbb{Z}_2$-odd generators $\hat{t}_{I_o}$ vanish there. Hence, the boundary values of $\Phi_{\text{even}}$ and $e^{-V_e}$ that appear in $L_{\text{bd}}^{(L)}$ are related to their bulk values as
\[
\Phi_{\text{even}} \big|_{y=L} = e^T \Phi_{\text{even}},
V_e \big|_{y=L} = V_e^{(L)} \equiv \ln \left( e^{T \frac{V_e}{2}} e^{V_o} e^{\frac{T}{2} e}\right) = V_e - \frac{1}{2} [T, T^\dagger] + \cdots, \tag{C.16}
\]
where $\Phi_{\text{even}}$ and $V_e$ in the left-hand side denote the values in the bulk ($0 < y < L$).

Therefore, we obtain the expression of the 4D effective Lagrangian,
\[
L_{\text{eff}} = \int_0^L dy \, L = - \int d^4 \theta \int_0^L dy \, 3 \mathcal{N} \sqrt[3]{3} \left( e^{V_o} \partial_y e^{-V_o} \right) \left( \hat{\Phi}_{\text{even}}^\dagger e^{-V_o} \hat{\Phi}_{\text{even}} \right)^{2/3} + \left[ \int d^2 \theta \, \frac{e^3}{16 g^3} \text{tr} \left( T \mathcal{W}_e^2 \right) + \text{h.c.} \right] + L_{\text{bd}}^{(0)} \left( e^{-V_e}, \Phi_{\text{even}} \right) + L_{\text{bd}}^{(L)} \left( e^{-V_e^{(L)}}, e^T \Phi_{\text{even}} \right), \tag{C.17}
\]
where $\hat{\Phi}_{\text{even}} \equiv e^{V_e/2} \Phi_{\text{even}}$ is independent of $y$. From this expression, we can read off the gauge kinetic functions and the superpotential in the effective theory.

**C.2 Kähler potential**

In (C.17), the only $y$-dependent superfield is $V_o$. Since we have dropped its kinetic term, we can integrate it out by using its equation of motion.

In the following derivation, we focus on a subset of $\{ V_I o \hat{t}_{I_o} \}$, in which every generator commutes with each other. We also consider a single compensator case ($n_C = 1$), and the generators have the following form.
\[
\hat{t}_{I_o} = \begin{pmatrix}
-3k_{I_o} \\
-3k_{I_o} 1_{n_H} + \hat{t}_{I_o}
\end{pmatrix}, \tag{C.18}
\]

\[\text{Note that } \hat{t}_{I_o} \text{ include the } \mathbb{Z}_2 \text{-odd gauge couplings.}\]
Then (C.21) is rewritten as

$$\Omega_{\text{eff}}^\text{tree} = - \int_0^L dy \, (\phi_C)^2 \mathcal{N}^{1/3} (- \partial_y V_o) e^{2k \cdot V} \left( 1 - \chi^\dagger e^{-V_o} \chi \right)^{2/3},$$

(C.19)

where $k \cdot V \equiv \sum_{I_o} k_{I_o} V_{I_o}$, and

$$\phi_C \equiv \left( \hat{\Phi}_\text{even}^1 \right)^{2/3}, \quad \chi^a \equiv \hat{\Phi}_\text{even}^{a+1}, \quad \tilde{V}_o \equiv \sum_{I_o} V_{I_o} \dot{t}_{I_o}. \tag{C.20}$$

Then, from the equation of motion for $V_{I_o}$, we obtain

$$\left\{ \partial_y \left( \frac{N_{I_o}}{N^{2/3}} \right) + 6k_{I_o} N^{1/3} + \frac{2N^{1/3} \chi^\dagger e^{-V_o} \dot{t}_{I_o} \chi}{1 - \chi^\dagger e^{-V_o} \chi} \right\} (P_V)^{-1}_{I_o} = 0, \tag{C.21}$$

the arguments of the norm function and its derivative are $(0_{n_v} - \partial_y V_o)$, and the projection operator $(P_V)^{-1}_{I_o}$ is defined by \[\boxed{\text{(C.21)}}\]. The presence of $(P_V)^{-1}_{I_o}$ indicates that the number of independent equations is less than that of $V_{I_o}$. Thus we cannot solve $V_{I_o}$ as functions of $y$. Hence we need another method to integrate them out.

Let us define

$$V_s \equiv s_{I_o} V_{I_o}, \quad v_{I_o} \equiv \frac{\partial_y V_{I_o}}{\partial_y V_s}, \tag{C.22}$$

where $s_{I_o}$ are arbitrarily chosen constants, and $V_s$ satisfies the boundary conditions,

$$\lim_{y \to 0} V_s = 0, \quad \lim_{y \to L} V_s = \tilde{V}_s \equiv -2s_{I_o} \text{Re} T_{I_o}. \tag{C.23}$$

Then \[\boxed{\text{(C.21)}}\] is rewritten as

$$\left\{ \partial_y v_{I_o} a_{I_o, K_o}(v) + \left( 3k_{K_o} + \frac{\chi^\dagger e^{-V_o} t_{I_o} \chi}{1 - \chi^\dagger e^{-V_o} \chi} \right) \partial_y V_s \right\} (P_V)^{-1}_{I_o}(v) = 0. \tag{C.24}$$

From \boxed{\text{(C.22)}}$, $v_{I_o}$ satisfies $s_{I_o} v_{I_o} = 1$, and thus, $s_{I_o}(dv_{I_o}/dV_s) = 0$. Therefore, \boxed{\text{(C.24)}} is rewritten as

$$\frac{dv_{I_o}}{dV_s} = G^{I_o, I_o}(v) \left( 3k_{I_o} + \frac{\chi^\dagger e^{-V_o} t_{I_o} \chi}{1 - \chi^\dagger e^{-V_o} \chi} \right), \tag{C.25}$$

where $G^{I_o, I_o} = - (\partial_{K_o} \delta_{I_o} - v_{I_o} s_{K_o}) a_{K_o, I_o}$. Notice that these equations are solvable in contrast to \boxed{\text{(C.21)}}. Once $v_{I_o}(V_s)$ are obtained, $V_{I_o}$ are also expressed as functions of $V_s$ through

$$V_{I_o} = \int_0^y dy' \partial_y V_{I_o} = \int_0^y dy' v_{I_o} \partial_y V_s = \int_0^{V_s} dV' v_{I_o}(V'_s). \tag{C.26}$$
In the limit of \( y \to L \), this becomes

\[
-2\text{Re} T^{I_o} = \int_0^{V_s} dV_s \, v^{I_o}(V_s),
\]

which determines the integral constants for solutions of \( (C.25) \). Therefore, \( \Omega_{\text{eff}}^{\text{tree}} \) can be calculated as an integral for \( V_s \), instead of \( y \).

\[
\Omega_{\text{eff}}^{\text{tree}} = \int_0^{V_s} dV_s \, 3 |\phi_C|^2 \mathcal{N}^{1/3}(v(V_s)) e^{2k_V(V_s)} \left( 1 - \chi^\dagger e^{-V_o(V_s)} \chi \right)^{2/3}.
\]

We can solve \( (C.25) \) order by order in the matter chiral superfields \( \chi^a \). Here we consider a case of \( k_{I_o} = 0 \), which means that the background 5D spacetime is flat\(^{19} \). In this case, we find that

\[
v^{I_o}(V_a) = \bar{v}^{I_o} - \chi^\dagger G^{I_o}(\bar{v}) \tilde{v}^{-1} \left( e^{-\bar{v} V_s} - \frac{(\text{Re} \bar{T})^{-1}(e^{2\text{Re} T} - 1)}{2} \right) \tilde{t}_{I_o} \chi + \mathcal{O}(\chi^4),
\]

where

\[
\bar{v}^{I_o} \equiv \frac{\text{Re} T^{I_o}}{s \cdot \text{Re} T}, \quad \tilde{v} \equiv \sum_{I_o} \tilde{v}^{I_o} \tilde{t}_{I_o}, \quad \text{Re} \bar{T} \equiv \sum_{I_o} (\text{Re} T^{I_o}) \tilde{t}_{I_o}.
\]

Since \( \tilde{t}_{I_o} \) commute with each other, they can be diagonalized simultaneously.

\[
U \tilde{t}_{I_o} U^{-1} = -\text{diag}(c_{1I_o}, c_{2I_o}, \cdots, c_{n_{I_o}}), \quad \hat{\chi} \equiv U \chi.
\]

After some calculations, we obtain \([9, 10]\)

\[
\Omega_{\text{eff}}^{\text{tree}} = |\phi_C|^2 \mathcal{N}^{1/3} \left\{ -3 + \sum_a 2 Y(c_a \cdot \text{Re} T) |\hat{\chi}^a|^2 + \sum_{a,b} \Omega^{(4)}_{ab} |\hat{\chi}^a|^2 |\hat{\chi}^b|^2 + \mathcal{O}(|\chi|^6) \right\},
\]

where \( Y(x) \equiv \frac{1-e^{-2x}}{2x} \), and \(^{20} \)

\[
\Omega^{(4)}_{ab} \equiv -\frac{(c_a \cdot \mathcal{P}_V a^{-1} \cdot c_b)}{(c_a \cdot \text{Re} T)(c_b \cdot \text{Re} T)} \left\{ Y((c_a + c_b) \cdot \text{Re} T) - Y(c_a \cdot \text{Re} T) Y(c_b \cdot \text{Re} T) \right\}
\]

\[
+ \frac{Y((c_a + c_b) \cdot \text{Re} T)}{3}.
\]

The arguments of \( \mathcal{N} \) and \( \mathcal{P}_V \) are \((0_{n_V}, 2\text{Re} T^{I_o})\). Notice that the \( s_{I_o} \)-dependences are cancelled in the final result \( (C.32) \).

\(^{19} \) We calculated \( \Omega_{\text{eff}}^{\text{tree}} \) in the case of \( k_{I_o} \neq 0 \) in Ref. \([10]\).

\(^{20} \) The definitions of the moduli \( T^{I_o} \) and the gauge couplings \( c_a (a = 1, \cdots, n_H) \) are different from those of Ref. \([9, 10]\) by a factor 2.
D Quadratic terms for fluctuation superfields

Here we show the detailed derivation of the quadratic terms for the fluctuation superfields (3.19).

D.1 Gravitational sector

Notice that $\Psi_\alpha$ appears in the action only through $\bar{D}_\alpha \Psi_\alpha$ and its derivatives. Thus we define the following two real superfields,

$$V_+^\mu \equiv \frac{i}{2} \sigma_\alpha^{\mu \dot{\alpha}} (\bar{D}^{\dot{\alpha}} \Psi_\alpha + D^\alpha \bar{\Psi}_{\dot{\alpha}}), \quad V_-^\mu \equiv \frac{1}{2} \sigma^{\mu \dot{\alpha}}_\alpha (\bar{D}^{\dot{\alpha}} \Psi_\alpha - D^\alpha \bar{\Psi}_{\dot{\alpha}}) ,$$  \hspace{1cm} (D.1)

to describe the degree of freedom for $\Psi_\alpha$. Since

$$E_2 = -U_\mu \Box_4 \left( \Pi_{3/2}^{\mu \nu} - \frac{2}{3} \Pi_0^{\mu \nu} \right) U_\nu ,$$  \hspace{1cm} (D.2)

up to total derivatives, we can expand the integrand in (2.24) as

$$\left< \Omega_v^{1/3} \Omega^{2/3}_h \right> E_2 - \left< \Omega_v^{-1/3} \Omega_h^{4/3} \right> (C^\mu C_\mu + \bar{D}^{\dot{\alpha}} \Psi_\alpha D_\alpha \bar{\Psi}_{\dot{\alpha}}) - 3 \left( 1 + \frac{\Delta_\mu U_\mu}{3} \right) \Omega_v^{1/3} \Omega_h^{2/3}$$

$$= -\left< \Omega_v^{1/3} \Omega^{2/3}_h \right> \left\{ U_\mu^{\mu \nu} (\Box_4 + \mathcal{D}_U) U_\mu^{3/2} - \frac{2}{3} U_0^{\mu \nu} \Box_4 U_0^{\mu \nu} \right\}$$

$$- \left< \Omega_v^{-1/3} \Omega_h^{4/3} \right> \left( \partial_\mu \bar{U}_\mu \partial_\nu \bar{U}_\nu + 2 \partial_\mu \bar{U}_\mu V_{-\mu} + \frac{1}{2} V_{\mu}^\nu V_{-\mu} - \frac{1}{2} V_{+\mu}^\nu V_{+\mu} \right)$$

$$- \left< \Omega_v^{1/3} \Omega_h^{2/3} \right> \left\{ 2 i U_\mu^\mu \partial_\mu \left( T + \tilde{\Phi}_C - \bar{T} - \bar{\tilde{\Phi}}_C \right) + \frac{3}{2} (V_+^\mu \partial_\mu - V_-^\mu \Delta_\mu) V_T \right. \right.$$

$$\left. - 3 \partial_\mu U_\mu \Delta_\mu V_T + U_\mu \Delta_\mu \left( \bar{V}_0 + \bar{V}_h \right) + \cdots \right\} ,$$  \hspace{1cm} (D.3)

where we have performed the partial integrals, and $T$, $V_T$, $\bar{V}_0$, and $\bar{V}_h$ are defined in (3.12).

Since $V_\pm^\mu$ do not have kinetic terms, they are integrated out as

$$V_+^\mu = \left< \frac{3 \Omega_v^{2/3}}{2 \Omega_h^{2/3}} \right> \partial_\mu V_T , \quad V_-^\mu = -2 \partial_\mu \bar{U}_\mu + \left< \frac{3 \Omega_v^{2/3}}{2 \Omega_h^{2/3}} \right> \Delta_\mu V_T .$$  \hspace{1cm} (D.4)

After eliminating $V_\pm^\mu$, the 5D Lagrangian becomes

$$\mathcal{L} = \int d^4 \theta \left< \Omega_v^{1/3} \Omega_h^{2/3} \right> \left[ -U_\mu^{\mu \nu} (\Box_4 + \mathcal{D}_U) U_\mu^{3/2} + \frac{2}{3} U_0^{\mu \nu} \Box_4 U_0^{\mu \nu} + \left< \frac{\Omega_h^{4/3}}{\Omega_v^{1/3}} \right> \partial_\mu \bar{U}_\mu \partial_\nu \bar{U}_\nu \right. \right.$$

$$\left. - 2 i U_0^{\mu \nu} \partial_\mu \left( T + \tilde{\Phi}_C - \bar{T} - \bar{\tilde{\Phi}}_C \right) - U_\mu \Delta_\mu \left( \bar{V}_0 + \bar{V}_h \right) \right]$$

$$+ \int d^4 \theta \left< \frac{\Omega_v}{8} \right> V_T (\Delta_\mu \Delta_\mu + \Box_4) V_T + \cdots .$$  \hspace{1cm} (D.5)

Adding the gauge-fixing term (3.10), the cross terms between $U_\mu$ and the other superfields are canceled, and we obtain (3.13).
D.2 Matter sector

Since we have moved to the gauge where $\Sigma = 0$ by the supergauge transformation for the background superfields, $W_v$ in (2.12) is rewritten in terms of the gauge-transformed superfields as

$$W_v = \frac{c^3}{16g^3} \text{tr} \left\{ \tilde{\Sigma} \tilde{W}^2 - \frac{1}{12} \tilde{D}^2 \left( V \partial_y D^a \tilde{V} - \partial_y V D^a \tilde{V} \right) \tilde{W}_a \right\} - \frac{c^3}{48g^3} \partial_y \text{tr} \left( \Lambda_\Sigma \tilde{W}^2 \right),$$

where

$$\Lambda_\Sigma \equiv -\int_0^y dy' \Sigma(y').$$

Note that $V^I_0$ and $\Lambda_\Sigma^I$ have nontrivial boundary conditions at $y = L$ (see (C.12)). The quadratic terms for $\tilde{V}$ are read off as

$$\int d^2 \theta W_v + \text{h.c.}$$

$$= -\int d^4 \theta \frac{c^3}{16g^3} \text{tr} \left\{ \frac{1}{12} \left( V \partial_y D^a \tilde{V} - \partial_y V D^a \tilde{V} \right) \tilde{D}^2 D_\alpha \tilde{V} + \text{h.c.} \right\}$$

$$- \int d^4 \theta \frac{c^3}{192g^3} \partial_y \text{tr} \left( \Lambda_\Sigma D^a \tilde{V} \tilde{D}^2 D_\alpha \tilde{V} + \text{h.c.} \right) + \cdots$$

$$= -\int d^4 \theta \frac{c^3}{16g^3} \text{tr} \left\{ \frac{1}{4} \partial_y V \tilde{V} D^a \tilde{D}^2 D_\alpha \tilde{V} - \frac{1}{12} \partial_y \left\{ \left( V - \Lambda_\Sigma - \Lambda_\Sigma^I \right) \tilde{V} D^a \tilde{D}^2 D_\alpha \tilde{V} \right\} \right\}$$

$$+ \cdots$$

$$= -\int d^4 \theta \frac{c^3}{8g^3} \left[ \text{tr} \left( \partial_y V \tilde{V} \tilde{D}^4 P_T \tilde{V} \right) \right] + \cdots$$

$$= -\int d^4 \theta \frac{N_{IJ}(\langle \Omega \rangle)}{2} \tilde{V}^I \tilde{V}^J \tilde{D}^4 P_T \tilde{V}^J + \cdots,$$

where we have dropped total derivatives, and used (C.12). Combining this with the last term in (3.13), we find the kinetic terms for $\tilde{V}$ as (3.15).

Next we consider kinetic terms for the chiral superfields. We can expand $\Omega_v^{1/3} \Omega_h^{2/3}$ as

$$\Omega_v^{1/3} \Omega_h^{2/3}$$

$$= \langle \Omega_v^{1/3} \Omega_h^{2/3} \rangle \left[ \frac{N_{IJ}}{6N} \left\{ \partial_y \tilde{V}^I \partial_y \tilde{V}^J - 2 \left( \tilde{\Sigma} + \tilde{\Sigma}^I \right) \partial_y \tilde{V}^J + \left( \tilde{\Sigma} + \tilde{\Sigma}^I \right) \left( \tilde{\Sigma} + \tilde{\Sigma}^J \right) \right\} \right.$$

$$- \left. \left( \tilde{V}_v + \tilde{T} + \tilde{T}' \right)^2 + \left( \frac{\partial_I \partial_J \Omega_h}{3 \Omega_h} \right) \tilde{V}^I \tilde{V}^J + \frac{2}{3} \tilde{V}^I \left( \gamma^I \tilde{\Phi}_{\text{even}} + \text{h.c.} \right) \right.$$

$$+ \frac{2}{3} \langle \Omega_h \rangle \left( \tilde{\Phi}_{\text{odd}} \tilde{d} \left( e^V \right)^I \tilde{\Phi}_{\text{odd}} + \tilde{\Phi}_{\text{even}} \tilde{d} e^{-V} \tilde{\Phi}_{\text{even}} \right) - \frac{1}{4} \left( \tilde{V}_h + \tilde{\Phi}_C + \tilde{\Phi}_C \right)^2$$

$$+ \left( \tilde{V}_v + \tilde{T} + \tilde{T}' \right) \left( \tilde{V}_h + \tilde{\Phi}_C + \tilde{\Phi}_C \right) \right] + \cdots.$$ 

(D.9)
Thus the cross terms between $\tilde{V}$ and the chiral superfields are
\[
\mathcal{L}_{\text{cross}} = \int d^4 \theta \tilde{V}^I (\Xi_I + \tilde{\Xi}_I),
\]
where
\[
\Xi_I \equiv \partial_y \left\{ 2 \left( \frac{\Omega_v^{1/3} \Omega_h^{2/3}}{\Omega_v} \right) (a \cdot P_V)_{IJ} \tilde{\Sigma}^J \right\} - \frac{2}{3} \left( \frac{\Omega_h}{\Omega_v} \right)^{2/3} \Upsilon^I_N \gamma^I_{\tilde{\Sigma}}
\]
\[
- \partial_y \left( \frac{2}{3} \left( \frac{\Omega_h}{\Omega_v} \right)^{2/3} N_I \gamma^I_{\tilde{\Sigma}} \right) - 2 \left( \frac{\Omega_v^{1/3} \Omega_h^{2/3}}{\Omega_v} \right) T^I \left( 1 - \frac{\Phi \Upsilon^I}{3} \right) \tilde{\Phi}_{\text{even}}.
\]

Adding the gauge-fixing term (3.17), these cross terms are canceled, and we obtain
\[
\mathcal{L} + \mathcal{L}^{\text{gf}}_{\text{sg}} = \int d^4 \theta \tilde{V}^I \left[ \Omega_v \right]_{IJ} \square_4 \left[ \mathcal{P}_T + \frac{1}{\xi_{\text{sg}}} \right] \tilde{V}^J - \partial_y \left\{ \left( \frac{\Omega_v^{1/3} \Omega_h^{2/3}}{\Omega_v} \right) (a \cdot P_V)_{IJ} \partial_y \tilde{V}^J \right\}
\]
\[
- \partial_y \left( \frac{2}{3} \left( \frac{\Omega_h}{\Omega_v} \right)^{2/3} \right) \Upsilon^I_N \gamma^I_{\tilde{\Sigma}} + \left( \frac{\Omega_v^{1/3} \Omega_h^{2/3}}{\Omega_v} \right) \frac{N_I}{3N} \gamma^I_{\tilde{\Sigma}} \tilde{\Phi}_{\text{odd}}
\]
\[
- 3 \left( \frac{\Omega_v^{1/3} \Omega_h^{2/3}}{\Omega_v} \right) \left( \frac{\partial_I \partial_J \Omega_h}{3 \Omega_h} - \frac{\Upsilon^I_N \Omega_v^{1/3} \Omega_h^{2/3}}{9} \right) \tilde{V}^I
\]
\[
+ \int d^2 \theta \left( \frac{2}{\Omega_h} \right) \left[ (a \cdot P_V)_{IJ} \tilde{\Sigma}^J - \frac{2}{\Omega_h} \tilde{\Phi}_{\text{odd}} \tilde{d} (e^I) \tilde{\Phi}_{\text{odd}}
\]
\[
- \tilde{\Phi}_{\text{even}} \left( \frac{2 \tilde{d} e^I}{\Omega_h} - \frac{2}{3} \gamma^I \right) \tilde{\Phi}_{\text{even}} - \left( \frac{2 N_I}{3N} \gamma^I \tilde{\Phi}_{\text{even}} + \text{h.c.} \right) \right] + \mathcal{O}(\xi_{\text{sg}})
\]
\[
+ \left[ \int d^2 \theta \left( \tilde{\Phi}_{\text{odd}} \tilde{d} \partial_y \tilde{\Phi}_{\text{even}} - \tilde{\Phi}_{\text{even}} \tilde{d} \partial_y \tilde{\Phi}_{\text{odd}} - 2 \tilde{\Phi}_{\text{odd}} \tilde{d} \tilde{\Phi}_{\text{even}} \right) + \text{h.c.} \right] + \cdots.
\]

From (3.13), (D.8) and (D.12), the quadratic terms for the fluctuation superfields in the bulk Lagrangian are summarized as (3.19).

## E Boundary conditions for bulk fluctuation modes

Here we derive the boundary conditions for the fluctuation modes of the bulk superfields, which are determined by the orbifold parities and the boundary actions.

First, let us consider the boundary conditions of $U^\mu$ and $\tilde{V}^I$. Since we have chosen the gauge $\xi_{\text{sc}} = \xi_{\text{sg}} = \xi_{\text{sc}}^{(y^*)} = \xi_{\text{sg}}^{(y^*)} = 0$, only the transverse modes of $U^\mu$ and $\tilde{V}^I$ (i.e., $U^\mu_{3/2}$ and $\tilde{V}^I_T = P_T \tilde{V}^I$) can propagate. From (3.19) and (3.23), the equations of motion for them

21 In fact, $U^\mu_{3/2}$ and $\tilde{V}^I_T$ are gauge-invariant under $\delta_{\text{sc}}^{(1)}$ and $\delta_{\text{sg}}$, respectively. 

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are

\[
\left\{ \langle \Omega \rangle \Omega^{1/3} \Omega^{2/3}_h \right\} \left(-p^2 + D_U\right) - 2 \sum_{y_* = 0, L} \delta(y - y_*) \left| \phi_C \right|^2 h^{(y_*)} p^2 \right\} U^\mu_{3/2} = 0,
\]

\[
\langle \Omega \rangle a_{IK} \left\{ -\delta^K p^2 + (\mathcal{D}_V)_{J} \right\} \tilde{V}^j_T = 0,
\]

\[
-2 \sum_{y_* = 0, L} \delta(y - y_*) \left( \text{Re} f^{(y_*)}_{JcJd} p^2 + \frac{3}{2} \left| \phi_C \right|^2 h^{(y_*)}_{JcJd} \right) \tilde{V}^J_T = 0.
\]

(E.1)

By integrating these over infinitesimal intervals \([y_* - \epsilon, y_* + \epsilon]\) \((y_* = 0, L)\), we obtain

\[
\left\{ \langle \Omega \rangle \Omega^{1/3} \Omega^{2/3}_h \right\} \frac{\partial U^\mu_{3/2}}{y = y_* + \eta y_\epsilon} + \eta y_\epsilon \left| \phi_C \right|^2 h^{(y_*)} p^2 U^\mu_{3/2} \bigg|_{y = y_*} = 0,
\]

\[
\left\{ \langle \Omega \rangle \Omega^{1/3} \Omega^{2/3}_h \right\} a_{Lc} \frac{\partial \tilde{V}^J_T}{y = y_* + \eta y_\epsilon} + \eta y_\epsilon \left( \text{Re} f^{(y_*)}_{JcJd} p^2 + \frac{3}{2} \left| \phi_C \right|^2 h^{(y_*)}_{JcJd} \right) \tilde{V}^J_T \bigg|_{y = y_*} = 0,
\]

\[
\tilde{V}^J_T \bigg|_{y = y_* + \eta y_\epsilon} = 0,
\]

(E.2)

where \(y_* = 0, L\), \(\eta_0 = 1\) and \(\eta_L = -1\), and we have used that

\[
\mathcal{N}_{Ie} = 0, \quad (a \cdot \mathcal{P}_V)_{IeJ_e} = a_{IeJ_e}, \quad (a \cdot \mathcal{P}_V)_{IeJ_o} = 0,
\]

(E.3)

which follow from the fact that \(V^I_e\) are independent of \(y\). (See Appendix C)

Next we derive the boundary conditions for the chiral superfields. Since \(\tilde{\phi}_C\) and some of \(\tilde{x}^a\) are expressed in terms of \(\tilde{\Phi}_{\text{even}}\) as

\[
\begin{pmatrix}
\tilde{\phi}_C \\
\tilde{x}^a
\end{pmatrix} = \begin{pmatrix}
\frac{2}{3} \tilde{\phi}_C^{-1/2} \\
-\tilde{x}^a / \tilde{\phi}_C^{3/2} \quad 1 / \tilde{\phi}_C^{3/2}
\end{pmatrix}
\tilde{\Phi}_{\text{even}} + O(\tilde{\Phi}_{\text{even}}^2),
\]

(E.4)

the boundary Lagrangians \((5,23)\) are rewritten as

\[
L^{(y_*)}_{bd} = \int d^4 \theta \; 2\tilde{\Phi}_{\text{bd}}^{\dagger} \mathcal{K}^{(y_*)}_{\text{bd}} \tilde{\Phi}_{\text{bd}} + \left[ \int d^4 \theta \; \tilde{\Phi}_{\text{bd}}^{\dagger} W^{(y_*)}_{\text{bd}} \tilde{\Phi}_{\text{bd}} + \text{h.c.} \right] + \cdots,
\]

(E.5)

where

\[
\mathcal{K}^{(y_*)}_{\text{bd}} \equiv \frac{1}{2} \left| \phi_C \right|^2 \begin{pmatrix}
\left( h_{cd}^{(y_*)} \tilde{x}^c \tilde{x}^d - \frac{2}{3} \left( h_c^{(y_*)} \tilde{x}^c + h_{c}^{(y_*)} \tilde{x}^c \right) + \frac{4}{9} h^{(y_*)} - h_{eb}^{(y_*)} \tilde{x}^c - \frac{2}{3} h_{cb}^{(y_*)} \tilde{x}^c \\
- h_{ac}^{(y_*)} \tilde{x}^c - \frac{2}{3} h_{ab}^{(y_*)} \tilde{x}^c
\end{pmatrix},
\]

\[
W^{(y_*)}_{\text{bd}} \equiv \frac{1}{2} \begin{pmatrix}
P_{cd}^{(y_*)} \tilde{x}^c \tilde{x}^d - 4 P_{cd}^{(y_*)} \tilde{x}^c + \frac{8}{3} P^{(y_*)} - P_{eb}^{(y_*)} \tilde{x}^c + 2 P_{ab}^{(y_*)}
-P_{ac}^{(y_*)} \tilde{x}^c + 2 P_{ab}^{(y_*)}
\end{pmatrix}.
\]

(E.6)

Thus the equations of motion for the chiral superfields are read off from \((3,19)\) and \((E.5)\) as

\[
- \frac{1}{4} \mathcal{K} D^2 \varphi + \tilde{W} \varphi + \sum_{y_* = 0, L} \left\{ - \frac{1}{4} \mathcal{K}^{(y_*)}_{\text{bd}} D^2 \tilde{\Phi}_{\text{bd}} + \tilde{W}^{(y_*)}_{\text{bd}} \tilde{\Phi}_{\text{bd}} \right\} \cdot 2 \delta(y - y_*) = 0.
\]

(E.7)
By integrating this over \([y_\ast - \epsilon, y_\ast + \epsilon]\), we obtain
\[
- \eta y_\ast \tilde{\phi}_{\text{odd}} \bigg|_{y=y_\ast+\eta y_\ast \epsilon} + \left\{ -\frac{1}{4} \mathcal{K}^{(y_\ast)}_{\text{bd}} D^2 \Phi_{\text{even}} + \bar{W}^{(y_\ast)}_{\text{bd}} \tilde{\Phi}_{\text{even}} \right\} \bigg|_{y=y_\ast} = 0. \tag{E.8}
\]

Multiplying (E.7) by \(K^{-1}\) from the left and taking limits \(y \to y_\ast\) from the fundamental region \(0 < y < L\), we also obtain
\[
\left\{ -\frac{1}{4} D^2 \tilde{\Phi}_{\text{odd}} + \left( \frac{\Omega_h}{\Omega_v} \right)^{1/3} (e^{-V})^t \left( \tilde{\Phi} \bar{d} \Sigma - \partial y \tilde{\Phi}_{\text{even}} \right) \right\} \bigg|_{y=y_\ast+\eta y_\ast \epsilon} = 0. \tag{E.9}
\]

If we denote an eigenvalue of the differential operator \(K^{-1}W\) as \(\mu_{\text{ch}}\), the equation of motion in the bulk can be expressed as
\[
\frac{1}{4} D^2 \varphi = K^{-1} \bar{W} \varphi = \mu_{\text{ch}} \varphi. \tag{E.10}
\]

where
\[
A^{(y_\ast)}_{\text{ch}} \equiv \begin{pmatrix} 0 & 0 & - \frac{(V^t) \Phi^t \bar{d}}{\left( \Omega_v^{1/3} \Omega_h^{2/3} \right)} \\ 0 & 0 & 0 \\ \left( \frac{\Omega_h}{\Omega_v} \right)^{1/3} e^{-V} & 0 \end{pmatrix}, \quad y=y_\ast+\eta y_\ast \epsilon,
\]

\[
B^{(y_\ast)}_{\text{ch}} \equiv \begin{pmatrix} \mu_{\text{ch}} & 0 & s_j^t \Phi \\ 0 & \left( \mathcal{K}^{(y_\ast)}_{\text{bd}} \mu_{\text{ch}} - W^{(y_\ast)}_{\text{bd}} \right) 1_{n_C+n_H} & \eta y_\ast \bar{d} \\ -\left( \frac{\Omega_h}{\Omega_v} \right)^{1/3} e^{-V} t_j \Phi & 0 & \mu_{\text{ch}} 1_{n_C+n_H} \end{pmatrix} \bigg|_{y=y_\ast+\eta y_\ast \epsilon}. \tag{E.11}
\]

F Bosonic component expression of one-loop action

Here we provide an explicit expression of the one-loop Lagrangian in terms of the bosonic components in a simple case where 5D spacetime is flat and the boundary terms are absent. In this case, the one-loop Kähler potential (4.21) is reduced to
\[
\Omega_{\text{eff}}^{\text{loop}} = \frac{\left| \phi_C \right|^2}{N^{2/3}} \left\{ \bar{Q}_1 - Q_2 \sum_a (c_a \cdot \text{Re} T)^3 \right\} + \mathcal{O}(\chi^2), \tag{F.1}
\]
where $\tilde{Q}_1 \equiv (n_V - n_H + 1)Q_1/(8\pi^2)$, and $c_a \cdot \text{Re} T \equiv c_{aI_o} \text{Re} T^{I_o}$. Thus the one-loop Lagrangian is written as

$$\Delta^{\text{1-loop}} L = \int d^4 \theta \Omega^{\text{1-loop}}_{\text{eff}} + \cdots$$

$$= \frac{1}{N^2/3} \left\{ |F_{\phi C}|^2 L_{\tilde{\phi} \phi} + (F_{\phi C} F_{T^{I_o} \phi C} L_{\tilde{\phi} T^{I_o}} + \text{h.c.}) + F_{T^{I_o}} F_{T^{J_o}} |\phi C|^2 L_{T^{I_o} T^{J_o}} \right\}$$

$$+ O(F_x, D_{V^{I_o}}) + \cdots,$$

where $F_\varphi$ ($\varphi = \phi C, \chi, T^{I_o}$) and $D_{V^{I_o}}$ denote the $F$-component of a superfield $\varphi$ and the $D$-component of $V^{I_o}$ respectively, and $L_{\tilde{\phi} \phi} \equiv \tilde{Q}_1 - Q_2 \sum_a (c_a \cdot \text{Re} T)^3$,

$$L_{\tilde{\phi} T^{I_o}} \equiv -\frac{2N_{I_o}}{3N} \left\{ \tilde{Q}_1 - Q_2 \sum_a (c_a \cdot \text{Re} T)^3 \right\} - \frac{3Q_2}{2} \sum_a (c_a \cdot \text{Re} T)^2 c_{aI_o},$$

$$L_{T^{I_o} T^{J_o}} \equiv -\frac{3N N_{I_o J_o} - 5N_{I_o} N_{J_o}}{3N^2} \left\{ \tilde{Q}_1 - Q_2 \sum_a (c_a \cdot \text{Re} T)^3 \right\}$$

$$+ \frac{Q_2}{N} \sum_a (c_a \cdot \text{Re} T)^2 (N_{I_o} c_{aJ_o} + N_{J_o} c_{aI_o}) - Q_2 \sum_a (c_a \cdot \text{Re} T) c_{aI_o} c_{aJ_o}. \quad (F.3)$$

Here the arguments of $N$ and its derivatives are $(0_{n_{Ve}}, 2\text{Re} T^{I_o})$, and $\phi C$ and $T^{I_o}$ denote the lowest components of the corresponding superfields.

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