Simulations of Galactic polarized synchrotron emission for Epoch of Reionization observations

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ABSTRACT

The detection of the redshifted cosmological 21 cm line signal requires the removal of the Galactic and extragalactic foreground emission, which is orders of magnitude brighter anywhere in the sky. Foreground cleaning methods currently used are efficient in removing spectrally smooth components. However, they struggle in the presence of not spectrally smooth contamination that is, therefore, potentially the most dangerous one. An example of this is the polarized synchrotron emission, which is Faraday rotated by the interstellar medium and leaks into total intensity due to instrumental imperfections. In this work we present new full-sky simulations of this polarized synchrotron emission in the 50−200 MHz range, obtained from the observed properties of diffuse polarized emission at low frequencies. The simulated polarized maps are made publicly available, aiming to provide more realistic templates to simulate the effect of instrumental leakage and the effectiveness of foreground separation techniques.

Key words: polarization – cosmology: observations – dark ages, reionization, first starts

1 INTRODUCTION

The study of the first luminous sources and the consequent epoch of reionization (EoR) occupies a central place in modern cosmology. Amongst the various probes of this phase of the Universe, the redshifted 21-cm line is expected to be the most promising one, potentially allowing us to observe even before the first stars started to shine (see Furlanetto 2016; McQuinn 2016, for recent reviews).

Measurements of the redshifted 21-cm line are plagued by foregrounds that are a few orders of magnitude brighter than the 21-cm signal anywhere in the sky (e.g., Bernardi et al. 2009; Parsons et al. 2014) and can be only separated by leveraging upon their different spectral coherence (e.g., Santos et al. 2005; Dillon et al. 2014; Ali et al. 2015; Chapman et al. 2016; Patil et al. 2017; Liu et al. 2014a,b; Wang et al. 2013). Over the last decade, 21-cm upper limits have steadily improved both from sky-averaged (Bernardi et al. 2016; Singh et al. 2017; Monsalve et al. 2017) and power spectrum (Dillon et al. 2015; Jacobs et al. 2015; Ali et al. 2015; Ewall-Wice et al. 2016; Beardsley et al. 2016; Patil et al. 2017) observations.

As upper limits become more and more stringent, systematic effects need to be known and modelled more precisely. One of these effects has been recognized since early on to be the leakage from polarized foregrounds into the total intensity where the cosmological signal is measured (Bernardi et al. 2010; Jelieć et al. 2010). Although usually small (a few percent or less) this leakage can be quite important as 1) the sky polarization can be much larger than the cosmological signal and 2) the polarized signal can have a not smooth frequency dependence which makes this leakage much harder to clean. This frequency dependence is due to Faraday rotation, a rotation of the polarization angle in linearly polarized radiation as it traverses the Galaxy due to its interaction with the Galactic magnetic field (see Rybicki & Lightman 1986 for further details).

The most prominent mechanism that leads to polarization leakage into the EoR signal is likely due to the intrinsic polarization response of low frequency receptors (“polarized beams”). Asad et al. (2015, 2016) studied the case of EoR observations with the LOFAR telescope that has a relatively narrow field of view (∼6° at 150 MHz) and can be pointed to sky regions with relatively faint Galactic polarized emission (Bernardi et al. 2010). Under those specific conditions, they found that the polarization leakage may be kept below the expected 21-cm signal. Moore et al. (2013), conversely, simulated EoR observations with very wide field of view in-
struments and found that polarization leakage may be well above the EoR signal, in particular due to the population of polarized extragalactic sources. Current observations have only placed upper limits on the level of all-sky polarized foreground emission (Kohn et al. 2016; Moore et al. 2017).

Recently, Nunhokee et al. (2017) modelled the contamination due to polarization leakage from wide-field polarized beams and found that it is likely to be non-negligible, although its exact magnitude strongly depends upon the properties of all-sky polarized foregrounds. An accurate understanding and modelling of polarized foregrounds is therefore crucial to quantify both the amount of leakage to the 21-cm power spectrum and the effectiveness of foreground separation techniques in the presence of such a leakage.

In this paper we present new all-sky simulations of Galactic polarized emission at frequencies below 200 MHz, aimed at improving the accuracy of polarization leakage simulations. Unlike previous efforts, our simulation do not rely on intensity data but are built from the statistics of observed polarized foregrounds at low frequencies. Moreover, they can be further improved with the inclusion of upcoming observations.

The paper is organized as follows: we give a brief summary of the polarization framework in section 2, we describe our simulation method in section 3; the simulation results are presented in section 4 and we conclude in section 5.

2 THEORETICAL BACKGROUND

The intensity of the linearly polarized synchrotron emission can be written in a complex form as:

\[ P = Q + iU = I_P e^{2i\chi}, \]

where \( I_P = \sqrt{Q^2 + U^2} \) is the polarization intensity (\( Q \) and \( U \) are the standard Stokes parameters) and the polarization angle is:

\[ \chi = \frac{1}{2} \arctan(U/Q). \]

As polarized synchrotron emission travels through the interstellar medium (ISM), its polarization angle \( \chi \) rotates as a function of the square of the wavelength \( \lambda \):

\[ \chi(\lambda^2) = \chi_0 + \psi \lambda^2 \]

where \( \chi_0 \) is the intrinsic polarization angle at the source and \( \psi \) is the Faraday depth along the line of sight towards the source (i.e. Burn 1966):

\[ \psi \propto \int_0^\infty \frac{1}{n_e} B_\parallel \, d\lambda, \]

where \( n_e \) is the thermal electron density and \( B_\parallel \) is the magnetic field component along the line of sight. The integral is carried out between the observer’s location and the source distance. Faraday rotation therefore imprints a specific frequency \( \psi \)-dependent coherence on the Stokes \( Q \) and \( U \) parameters for any given line of sight \( n \):

\[ Q(n, \nu) = Q_0(n) \cos(2 \psi \lambda^2) + U_0(n) \sin(2 \psi \lambda^2), \]

\[ U(n, \nu) = -U_0(n) \sin(2 \psi \lambda^2) + Q_0(n) \cos(2 \psi \lambda^2), \]

where it is implicit that the frequency \( \nu = c/\lambda \) and \( Q_0 \) and \( U_0 \) are measured at a given reference frequency, \( \nu_0 \). Note that the expressions above are only valid for the emission from one single source. The observed synchrotron radiation will be an integral over many emission sources along each line of sight.

The diffuse polarized emission is often analysed using the rotation measure (RM) synthesis technique, presented in Burn (1966) and extended in Brentjens & de Bruyn (2005). This technique is useful in cases where there is a superposition of emitting regions along the line of sight, with different values of Faraday depth or of faint highly rotating emission. In this section, we briefly outline this technique since this will be helpful to describe our simulation strategy in section 3.2. For a more detailed review we refer to the original papers or Heald (2009). The RM synthesis takes advantage of the formal Fourier relation between the polarized emission and the intrinsic polarized flux as a function of Faraday depth. Indeed, the complex polarized intensity \( P \) as a function of wavelength \( \lambda^2 \) and its Faraday dispersion \( \hat{P}(\psi) \) form a Fourier pair:

\[ P(\lambda^2) = \int_{-\infty}^{+\infty} \hat{P}(\psi) e^{2i\psi \lambda^2} d\psi \]

\[ \hat{P}(\psi) = \int_{-\infty}^{+\infty} P(\lambda^2) e^{-2i\psi \lambda^2} d\lambda^2, \]

where \( \hat{P}(\psi) \) can be formally re-written as:

\[ \hat{P}(\psi) = \hat{Q}(\psi) + i\hat{U}(\psi), \]

with

\[ \hat{I}_P(\psi) = \sqrt{\hat{Q}(\psi)^2 + \hat{U}(\psi)^2}. \]

Since \( \lambda^2 \) is positive by definition and, in practice, the sampling in \( \lambda^2 \) space is always incomplete, this formula has been corrected in Brentjens & de Bruyn (2005) and expressed as:

\[ P_{\text{obs}}(\psi) = K \int_{-\infty}^{+\infty} P_{\text{obs}}(\lambda^2) e^{-2i\psi (\lambda^2 - \lambda_0^2)} d\lambda^2 = \hat{P}(\psi) \ast R(\psi) \]

where the sampling (or window) function, \( W(\lambda^2) \), which sets the frequency range, is included in \( P_{\text{obs}} \) and \( K \) is the inverse of the integral over this sampling function. The RM transfer function (RMTF):

\[ R(\psi) = K \int_{-\infty}^{+\infty} W(\lambda^2) e^{-2i\psi (\lambda^2 - \lambda_0^2)} d\lambda^2 \]

determines the resolution in Faraday depth. Note the factor \( \lambda_0^2 \) (the weighted average of the observed \( \lambda^2 \), in equation 7 and 8, that has been introduced to improve the behaviour of the RMTF.

As with standard Fourier transforms, the FWHM \( \delta \psi \) of the main peak of the RMTF is inversely proportional to the full width of the \( \lambda^2 \) space covered by observations. The largest scale in \( \psi \) space to which one is sensitive is inversely proportional to the shortest wavelength square \( \lambda_{\text{min}}^2 \), while the maximum observable Faraday depth \( \psi_{\text{max}} \) depends on the channel width.

The output of the RM synthesis is a cube of polarized maps, \( \hat{P}(\psi) \) at selected values of Faraday depth \( \psi \). In the next sections we will model it in order to produce simulated Stokes \( Q \) and \( U \) maps at the frequencies of interest through equation 6.
3 SIMULATIONS

In this section we describe in detail our simulation method for low frequency polarised synchrotron emission. We start by reviewing some of the approaches that can be found in literature in section 3.1, which mostly rely on intensity data at higher frequencies. We then discuss the limitation of these methods and present our simulation recipe based on polarized data in section 3.2. Section 3.3 is devoted to the statistical analysis of the available observations whose properties will be extended to full sky maps in section 3.4.

3.1 Differences with previous literature

Before entering in the details of the description of our method, we briefly describe some of the other techniques to model polarized synchrotron emission at low frequencies that have been carried out in the literature and their limitations. Geil et al. (2011) and Jelić et al. (2008) both use the total intensity synchrotron emission $I$ as a template for polarized emission:

\[ Q(\hat{n}, \nu_0) = pI(\hat{n}, \nu_0) \cos(2\chi(\hat{n})) \]
\[ U(\hat{n}, \nu_0) = pI(\hat{n}, \nu_0) \sin(2\chi(\hat{n})) \]

where $p$ is the polarization fraction, function of the total intensity spectral index $\alpha_{\text{syn}}$ (Le Roux 1961; Cortiglioni & Spoelstra 1995):

\[ p = \frac{3\alpha_{\text{syn}} - 3}{3\alpha_{\text{syn}} - 1}. \]  

In Jelić et al. (2008), $\alpha_{\text{syn}}$ is drawn from a 4D (3 spatial plus 1 frequency) Gaussian distribution while Geil et al. (2011) uses a running spectral index drawn from a Gaussian distribution. In Geil et al. (2011) the propagation through the ISM that leads to the Faraday modulation of the Stokes $Q$ and $U$ parameters is modelled using a single Faraday screen with $1 \lesssim \psi \lesssim 5$ rad m$^{-2}$ and an angular distribution with a small gradient over the sky-plane $\nabla \psi(x, y) \propto \hat{x} + \hat{y}$. Jelić et al. (2008) use two Faraday screens with zero mean, standard deviation 0.3 and a power law power spectrum with power law index arbitrarily set to $-2$.

Alonso et al. (2014) and Shaw et al. (2015) use a different approach to the problem and simulate polarized emission directly in Faraday space, assuming its angular correlation function to be a power law with a correlation length $\xi_\psi$. The Faraday depth $\psi$ is chosen to be a normally distributed variable around zero with variance $\sigma_\psi(\hat{n})$ determined from the full-sky Faraday rotation map of Oppermann et al. (2012). Simulated maps are normalised to have a 20 – 30% average polarization fraction at high Galactic latitudes, according to the 23 GHz WMAP results (Kogut et al. 2007).

The aforementioned approaches have limitations in reproducing the characteristics of Galactic polarized emission below 200 MHz. Low frequency observations show that Galactic polarized emission is essentially ubiquitous at the Kelvin rms level both in selected, small sky patches (e.g. Bernardi et al. 2009, 2010; Iacobelli et al. 2013; Jelić et al. 2014) as well as in large scale surveys (e.g. Bernardi et al. 2013; Lenc et al. 2016). Its spatial structure is often patchy and extends from arcmin to a few degree scales, although filamentary structures in the form of narrow, elongated “canals” have been occasionally observed (Jelić et al. 2015). Its spatial structure is found to be statistically well described by a power spectrum in the usual multipole $\ell$ space (e.g. Bernardi et al. 2009; Jelić et al. 2014; Iacobelli et al. 2013), i.e. $C_\ell \propto \ell^\beta$ with $\beta \sim -1.5$, flatter compared to measurements at cm-wavelengths (e.g. Carretti et al. 2005; La Porta et al. 2008).

One of the key features of the observed polarized emission at low frequencies that is not captured in the current simulation approaches is the almost complete lack of spatial correlation between total intensity and polarized emission due to a combination of observational effects: interferometric observations intrinsically filter out the large scale emission and are more sensitive to small scale thermal structure in the ISM (e.g. Wieringa et al. 1993; Gaensler et al. 2001; Bernardi et al. 2003). Low frequency polarized emission is also structured along virtually any line of sight, showing emission peaks at different $\psi$ values (e.g. Schnitzeler et al. 2009; Bernardi et al. 2013; Lenc et al. 2016) - unlike the cm-wavelength regime. Such structures can be represented using the complex polarization $P$ as a function of Faraday depth $\psi$ described in section 2.

The spatial co-location of synchrotron emitting and Faraday rotating plasma - a common situation in the ISM - may originate multiple peaks in the Faraday spectrum $\tilde{P}(\psi)$ that can be separated at low frequencies due to the high $\psi$ resolution. Most of the Faraday peaks are observed at small $\psi$ values, consistent with a local (within a few hundreds pc) origin of the polarized emission (Haverkorn et al. 2004; Bernardi et al. 2013; Lenc et al. 2016). Current simulation methods fall short, in general, to account for such Faraday structures. The use of the all-sky rotation measure (RM) map derived from extragalactic radio sources Alonso et al. (2014) is likely correct at GHz-frequencies but substantially overestimates the distribution of Faraday depths at low frequencies as it is integrated over the whole Galactic halo. Jelić et al. (2010) take into account various structures in Faraday depths, although they limit themselves to a few empirical models.

For our simulations, we will relax the assumption that total and polarized emission are spatially correlated and we use a realistic statistical representation of the angular and Faraday properties of the polarized emission driven by the available data.

3.2 Simulation recipe

The goal of our simulations is to generate Stokes $Q$ and $U$ parameters at any sky direction $\hat{n}$ and frequency $\nu$.

In the previous section we discussed how modelling $P(\hat{n}, \lambda^2)$ is difficult because of our limited knowledge of the detailed processes that occur in the ISM at low frequencies. We therefore decide to take advantage of the Fourier relationship that exists between $P(\lambda^2)$ and $\tilde{P}(\psi)$ (equation 6) and generate $\tilde{P}(\hat{n}, \psi)$ maps whose statistics is constrained

\[ 1 \text{ In this section we will interchangeably use } \nu \text{ and } \lambda^2 \text{ to indicate the frequency dependence of the Stokes parameters as } \nu \text{ is the observer's variable and } \lambda^2 \text{ is the proper variable to describe the properties in Faraday space.} \]
mysql error
Figure 2. Top: average polarized emission (solid line) and its 1σ standard deviation (shaded region) as a function of ψ for the B13 data. The average peaks at ≈ 7 K around ψ ∼ 0 rad m⁻² and flattens out at |ψ| > 40 rad m⁻². Middle: histogram of |I_P(ψ)| for some selected ψ values. Bottom: signal-to-noise ratio of the best fit Rayleigh parameter as a function of ψ. The grey region indicates the 2σ limit below which the polarized intensity slices are consistent with noise-like emission.

\[ a_{\ell m} = \frac{4\pi}{N_{\text{pix}}} \sum_{p=0}^{N_{\text{pix}}-1} Y^*_{\ell m}(\hat{n}_p) f(\hat{n}_p). \]  

(17)

We correct for the effect of incomplete sky coverage using the MASTER algorithm (Hivon et al. 2002). Figure 3 shows an example of power spectra computed from the data. It fairly follows a power-law behaviour in ℓ at any observed ψ value. Given the limited angular scales sample by the data, the power spectrum slope fits were limited to the 50 < ℓ < 700 range. In the fit, we associated to the raw points calculated by MASTER an error at large scales due to cosmic variance and at small scales due to the thermal noise (Tegmark 1997). We use for the r.m.s noise the estimated value \( \bar{\sigma}_N \) from equation 15. We find slopes in the \(-1.5 < \beta < -1.1\) range (figure 4), consistent with what was observed in much smaller sky patches (e.g., Bernardi et al. 2009; Jelić et al. 2014).

We note that the power spectrum slope tends to steepen at high |ψ| values, although, given the error bars, it is consistent with an average value of \( \beta = -1.3 \) across the observed range. In the next section we describe how we construct full-sky maps that preserve the observed statistics.

3.4 Full-sky extrapolation

We now want to construct full-sky Stokes Q and U simulated maps whose statistics is consistent with what we derived.
from the polarized intensity maps in the previous section. In particular, we need to generate Stokes $\hat{Q}$ and $\hat{U}$ maps according to two independent Gaussian distributions (assumed to have the same mean and variance) that lead to a polarized intensity map with a spatial correlation that follows $C_{\ell}^{IJ}(\beta, \psi_i) \propto \delta(\psi - \psi_i)$. This is not a trivial problem so we address it through a Monte Carlo approach, assuming also $\hat{Q}$ and $\hat{U}$ have a power law power spectrum. This allows us to determine the best value of $\alpha(\psi)$ (equation 11) that leads to the observed power spectrum slope $\beta(\psi)$. Each Monte Carlo realization includes the following steps:

(i) for each $\beta_i$ value, corresponding to the $i$-th $\beta$ in figure 4, we scan a set of values $\{\alpha_{ij}\}$ carefully chosen around $\beta_i$ and generate, for every $\alpha_{ij}$ a $\hat{Q}_{ij}$ and a $\hat{U}_{ij}$ map with $C_{ij}^\alpha = E^{\alpha_{ij}}$;

(ii) we calculate the polarized intensity maps $I_{F,ij} = \sqrt{\hat{Q}_{ij}^2 + \hat{U}_{ij}^2}$;

(iii) we fit a power law to the polarized intensity power spectrum $C_{\ell}^{IP}(\alpha_{ij}) \propto \ell^{\beta(\alpha_{ij})}$ to obtain the best fit slope $\beta(\alpha_{ij})$ of the realization.

The above procedure is repeated for $N = 10$ realizations for every value of $\alpha_{ij}$. For each given $\beta_i$, we construct the estimator:

$$\chi^2_{\ell}(\alpha_{ij}|\beta_i) = \sum_\ell \frac{(C_{\ell}^{IP}(\alpha_{ij}) - C_{\ell}^{IP}(\beta_i))^2}{\sigma_{C_{\ell}^{IP}(\beta_i)}}$$

as a function of $\alpha_{ij}$ and for the $N$ different realizations. As an estimation of $\sigma_{C_{\ell}(\beta)}$ we consider only the cosmic variance contribution, i.e. $\sigma_{C_{\ell}^{IP}(\beta_i)} = \sqrt{\frac{2}{2\ell+1}C_{\ell}^{IP}(\beta_i)}$. We then select, for every $\beta_i$, the value of $\alpha_{ij}$ that minimizes the mean over the $N$ realizations of the value of $\chi^2_{\ell}(\alpha_{ij}|\beta_i)$. This allows us to map the values of $\beta_i$, i.e. the spectral index of the $I_F$, into values of $\alpha$, i.e. the corresponding spectral index for the $\hat{Q}$ and $\hat{U}$ maps, as shown in figure 5.

At this point, the desired maps can be obtained from the coefficients of the spherical harmonic expansion:

$$a_{\ell,m}^\hat{Q},\hat{U} = \sqrt{\frac{C_{\ell}}{2}} N(0,1) + i \sqrt{\frac{C_{\ell}}{2}} N(0,1).$$

As a function of $\psi$, the maps simulated in this way have the desired angular power spectrum and the pixel values follow, by construction, a Gaussian distribution with zero mean and variance $4\pi \sum_{\ell} C_{\ell}$. We renormalised them to the variance $\sigma_\psi$ in order to produce the values observed in the B13 data (section 3.3).

We can calculate the polarisation angle $\chi$ using equation 2, pixel by pixel. By construction, with generic simulated Stokes $\hat{Q}$ and $\hat{U}$ maps that follow a Gaussian distribution, one obtains a uniform distribution in the polarization angle. However, when one imposes a spatial structure with a steep power law to $\hat{Q}$ and $\hat{U}$, the spatial behaviour of the phase deviates from just the flat power spectrum. In figure 6 we show that, for the power law values considered in this analysis, the polarization angle spatial structure is still reasonably flat and thus uninformative. Note that the knowledge of the absolute polarization angle is not necessary for the purpose of our simulations, but only if we were interested to simulate the orientation of the magnetic field.

Since we extract constraints from polarized intensity without any information on the phase, we can always argue that this information can be added from external data. We can easily include a phase map $\chi$ constructing a new $P = I_F e^{2i\chi} = Q' + i U'$. The resulting $Q'$ and $U'$ maps will not be the original one of the procedure, but the total polarized intensity structure will still hold and will show the desired power spectrum.

4 RESULTS AND DISCUSSION

In section 3 we described the details of our simulations to the production of full-sky $\hat{Q}$ and $\hat{U}$ cubes that cover the $-18 < \psi < 23$ range in steps of $\delta \psi \simeq 3$ rad $m^{-2}$. In this
where $\delta \psi$ is the range in $\psi$, $\langle \cdot \rangle$ is the average over the different lines of sight, and $\Delta \psi$ is the range in $\psi$ covered by the simulations. Both tests confirm that our simulated maps follow the data statistical properties.

The simulated cube is converted to frequency space through equation 13. The simulated maps at 160 MHz are shown in figure 9 as an example. We blanked out the regions of brightest emission in the 23 GHz WMAP polarized intensity map where real observations may show a significant deviation from the statistics used in our simulations. However, as discussed in section 1, the low global level of polarization measured by MWA in comparison with the expectation obtained from a standard power-law extrapolation from higher frequencies, reveals that the emission has a local origin. Indeed, the polarization horizon (Landecker et al. 2001), i.e. the maximal distance from the observer beyond which the emission is depolarized, seems to diminish consistently going to the low frequencies of interest (e.g. Bernardi et al. (2003); Brouw & Spoelstra (1976)). Both Bernardi et al. (2013) and Lenc et al. (2016), from their measured values of $\psi$ and using equation 4 with simplistic assumptions on the thermal electron density and the magnetic field strength, found that the polarization horizon is not farther than $\sim$ 120 pc. Lenc et al. (2016) refines this estimation using $\psi$ measures from pulsar, bringing the polarization horizon down to $\sim$ 50 pc. These values implies that the structure of the Galactic plane\(^3\) in polarization, should be very difficult to see at low frequencies.

Our simulations allow us to quantify the expected frequency coherence of the Galactic polarized foreground: the Stokes $Q$ parameter for two arbitrary lines of sight is displayed in figure 10, where we can see that its sign changes on scales of a few MHz, not much different than the scale of coherence of the 21 cm signal. Figure 11 offers a view of the coherence scale directly in $k_\perp$ space. We use simulations of a 10 MHz bandwidth (where no significant cosmological evolution is expected to happen) centred around 160 and 180 MHz refining the frequency resolution to 0.1 MHz. We calculated the power spectrum along the line of sight as:

$$
\langle \hat{Q}(k_\parallel)\hat{Q}^*(k_\parallel) \rangle = P(k_\parallel)\Delta r/(\delta \nu)^2
$$

where $\hat{Q}(k_\parallel)$ is the FFT of $Q(\nu)$ re-sampled to be equispaced in the conoving distance $r(z)$. $\langle \cdot \rangle$ is again the average over all the lines of sight. $\Delta r$ is the interval in Mpc corresponding to the frequency range and $\delta \nu$ is converted to cosmological distance $\delta r$ using:

$$
\delta r = cH^{-1}(z)(1+z)^2\nu_{21}^{-2}\delta \nu,
$$

where $r(z)$ and $H(z)$ are obtained assuming Planck 2015 cosmology (Planck Collaboration XIII 2016) and $\nu_{21} = 1420$ MHz is the rest frequency of the 21 cm line. Note that since we are using a Healpix pixelization with $N_{\text{side}} = 512$, the power spectrum of figure 11 corresponds to a smoothing scale in the perpendicular direction of about $k_\perp = 0.055$ Mpc$^{-1}$, where $k_\perp = 1/(r(z)\Delta \theta)$ with $\Delta \theta$ the pixel size.

Power spectra at both frequencies are fairly similar, as expected, and both show a fast decline beyond $k_\parallel \sim 0.05$ Mpc$^{-1}$. As there is a linear relationship between the Faraday depth $\psi$ and $k_\parallel$ (Pen et al. 2009; Moore et al. 2017; Numhokee et al. 2017), the decrease in power at high $k_\parallel$ is consistent with the fact that the simulated maps have most of the power at small $\psi$.

\(^3\) We remind that the Galactic center is $\sim$ 8 Kpc distant from us.

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**Figure 7.** Mean (blue solid line) and 1σ fluctuations (blue shaded region) as a function of $\psi$ for the full-sky simulated maps of polarized intensity. The mean agrees with the average computed from the B13 data (red line, same as figure 2). We recall we only use and then simulate the interval $-18 < \psi < 23$ rad m$^{-2}$.

**Figure 8.** Comparison between the correlation in the data and those computed as:

$$(\hat{I}_P(u)\hat{I}_P(u)) = \hat{D}(u)\Delta \psi/(\delta \psi)^2$$

where $\hat{I}_P(u)$ is the FFT of $I_P(\psi)$, $\langle \cdot \rangle$ indicates the average over the different lines of sight, $\delta \psi$ is the bin width in $\psi$ and $\Delta \psi$ is the range in $\psi$ covered by the simulations. Both tests confirm that our simulated maps follow the data statistical properties.
5 CONCLUSIONS

In this paper we presented full-sky simulations of the Galactic polarized synchrotron emission in the 50 – 200 MHz frequency range, relevant for 21 cm observations from the EoR. Unlike previous simulation methods, we did not use total intensity data as a proxy since there is a lack of correlation between total and polarized diffuse emission at low frequencies. We derived, instead, the statistical properties of large-scale observations of diffuse polarized emission at low frequencies (Bernardi et al. 2013) and used them to simulate polarization maps directly in Faraday space that are then Fourier transformed to frequency space (Burn 1966; Brentjens & de Bruyn 2005). The simulated Stokes $Q$ and $U$ maps spanning the 50 – 200 MHz interval with 1 MHz resolution, are publicly available at UWC - Center for Radio Cosmology Google Drive Directory.

The polarized power spectra derived from simulations shows a steep declining power at $k_P > 0.05$ Mpc$^{-1}$, consistent with the fact that polarized emission is mostly located at small Faraday depth values. At a qualitative level, this result is consistent with the simulations in Nunhokee et al. (2017) and, therefore, leave open the possibility that there is a power spectrum region where the polarization leakage contamination may just be below the 21 cm signal, mitigating the requirements for its modelling and subtraction. In our case, however, the simulations predict the power spectrum amplitude and the shape more realistically than in previous literature and represent, therefore, better inputs for instrumental simulations.

It is worth underlying that our simulation extrapolation to 50 MHz has been based on the statistics from the 189 MHz
data and, therefore, did not account for possible depolarization effects occurring at lower frequencies or for the spectral dependence of the polarized emission. As pointed out earlier, however, both these effects are largely unknown and our choice of extrapolating the properties of the 189 MHz Faraday space to lower frequencies is, therefore, somewhat conservative. As data at lower frequencies become available, they can be directly included in our simulation framework.

Future work will focus on using our results in realistic simulation pipelines of experiments such as the Hydrogen Epoch of Reionization Array (DeBoer et al. 2017) or the upcoming Square Kilometre Array (Koopmans et al. 2015). This will allow to predict the actual contamination level for different leakage terms and test foreground separation methods.

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