DISTRIBUTED CONJUGATE GRADIENT STRATEGIES FOR PARAMETER ESTIMATION OVER SENSOR NETWORKS

Songcen Xu and Rodrigo c. de Lamare
Communications Research Group, Department of Electronics, University of York, U.K.
Email: sx520@ohm.york.ac.uk, rcdl500@ohm.york.ac.uk

ABSTRACT

This paper presents distributed adaptive algorithms based on the conjugate gradient (CG) method for distributed networks. Both incremental and diffusion adaptive solutions are all considered. The distributed conventional (CG) and modified CG (MCG) algorithms have an improved performance in terms of mean square error as compared with least-mean square (LMS)-based algorithms and a performance that is close to recursive least-squares (RLS) algorithms. The resulting algorithms are distributed, cooperative and able to respond in real time to changes in the environment.

Index Terms—Adaptive networks, distributed processing, incremental adaptive solution, diffusion adaptive solution.

1. INTRODUCTION

In recent years, distributed processing has become popular in wireless communication networks. This kind of processing can collect data at each node in a given area and convey the information to the whole network in a distributed way. For instance, each node can get the information from its neighbors, and then combine it with the use of distributed adaptive algorithms; each node has the ability to process the information. With distributed solutions, each node only requires the local information and its neighbors to process the information. This kind of communication needs the central processor to be powerful and reliable. With distributed solutions, each node only requires the local information and its neighbors to process the information. This kind of processing can significantly reduce the amount of processing and the communications bandwidth requirements.

There are three main cooperation modes: the incremental, the diffusion, and the probabilistic diffusion modes [1]. For the incremental mode, we can interpret it as a cycle in which the information goes through the nodes in one direction, which means each node passes the information to its adjacent node in a pre-determined direction. Because of its simple method, the need for communication and power is the least [1]. For the diffusion mode, each node transfers information to its whole neighbors. This kind of processing costs a huge amount of communication resources, but each node will get more information. To avoid the high communication cost, another kind of diffusion termed probabilistic diffusion is used. Instead of transferring information to all its neighbors, each node transfers data to a selected group of its neighbors, which can be chosen randomly.

Several algorithms have already been developed and reported in the literature for distributed networks. Steepest-descent, least mean square (LMS) [1] and affine projection (AP) [3] solutions have been considered with incremental adaptive strategies over distributed networks [1]. While the LMS and recursive least squares (RLS) algorithms have been reported using diffusion adaptive strategies [2, 4, 5]. Although the LMS-based algorithms have their own advantages, when compared with conjugate gradient (CG) algorithms, their shortages are obvious. First, for the LMS-based algorithms, the adaptation speed is often slow, especially for the conventional LMS algorithm. Second, when we are trying to increase the adaptation speed, the system stability may decrease significantly. Furthermore, the RLS-based algorithms usually have a high complexity. In order to develop a set of distributed solutions with a more attractive trade-off between performance and complexity, we focus on the CG algorithm. The CG algorithm has a faster convergence rate [7] than the LMS-type algorithms and a lower computational complexity than RLS-type algorithms. In this paper, the main contribution is to develop distributed CG algorithms for both incremental and diffusion adaptive strategies. In particular, we develop distributed versions of the conventional CG algorithm and of the modified CG algorithm for use in distributed estimation over sensor networks.

This paper is organized as follows. Section 2 describes the system model and states the problem. Section 3 presents the incremental distributed CG algorithms, whereas Section 4 considers the diffusion distributed CG algorithms. Section 5 presents and discusses the simulation results, whereas Section 6 gives the conclusions.

2. SYSTEM MODEL AND PROBLEM STATEMENT

In this part, we describe a system model of the distributed estimation scheme over sensor networks and introduce the problem statement.

2.1. System model

The basic idea of this system model is that for each node in a sensor network a designer deals with a system identification problem. Each node is equipped with an adaptive filter. We focus on the processing of an adaptive filter for adjusting the weight vector \( \omega \), with coefficients \( \omega_k (k = 1, \ldots , M) \), where \( M \) is the length of the filter. The desired signal of each node at time instant \( i \) is

\[
d^{(i)} = \omega^H (i) x^{(i)} + n^{(i)}, \quad i = 1, 2, \ldots , N,
\]

where \( d^{(i)} \) is the received signal sample, \( x^{(i)} \) is the \( M \times 1 \) input signal vector, \( \omega_0 \) is the \( M \times 1 \) system weight vector, \( n^{(i)} \) is the noise sample at each receiver, \(^H\) denotes Hermitian transpose and \( N \) is the number of time instants. At the same time, the output of the adaptive filter for each node is given by

\[
y^{(i)} = \omega^H (i) x^{(i)}, \quad i = 1, 2, \ldots , N,
\]

where \( \omega^{(i)} \) is the local estimator \( \omega \) for each node at time instant \( i \).

2.2. Problem statement

To get the optimum solution of the weight vector, we need to solve a problem expressed in the form of a minimization of a cost function. Consider a network which has \( N \) nodes, each node \( k \) has access to time realizations \( \{ d^{(i)}, u^{(i)} \} \) of zero-mean spatial data \( \{ d_k, u_k \}, \ k = 1, 2, \ldots , N \), where each \( d_k \) is a scalar measurement and each \( u_k \) is a row regression vector [1]. After that, two global matrices are built which are used to collect the measurement data and regression data that are expressed in the form of the matrices:

\[
X = [x_1, x_2, \ldots x_N], \quad (N \times M)
\]

\[
d = [d_1, d_2, \ldots d_N]^T, \quad (N \times 1)
\]

The data which these two equations collect cover all nodes. In order to design an algorithm to compute the optimum estimation value, we need to first define a cost function:

\[
J(\omega) = E[||d - X\omega||^2],
\]
where the $J(\omega)$ is used to calculate the MSE and our aim is to minimize the cost function. The optimal solution should satisfy [1]:

$$E[X^H(d - X\omega)] = 0.$$  

(6)

Meanwhile, the $\omega_0$ is also the solution to:

$$b = R\omega_0,$$  

(7)

where the $M \times M$ autocorrelation matrix is given by $R = E[X^H X]$ and $b = E[X^H d]$ is an $M \times 1$ cross-correlation matrix. In this work, we focus on incremental and diffusion CG-based algorithms to solve the equation and perform estimation in a distributed fashion.

## 3. PROPOSED INCREMENTAL DISTRIBUTED CG - BASED ALGORITHMS

For distributed estimation over sensor networks, we develop two CG-based algorithms which are the CCG and MCG with incremental distributed solution (IDCG). We derive the CG-based algorithms first, then we devise distributed versions of these algorithms for use in the network in an incremental and distributed way.

### 3.1. Derivation of the CG-based algorithms

When a CG algorithm is used in adaptive signal processing, it solves the following equation [8]:

$$R_k^{(j)} \omega_k^{(j)} (j) = b_k^{(j)}(j),$$  

(8)

where $R_k^{(j)}$ is the $M \times M$ correlation matrix for the input data vector, and $b_k^{(j)}(j)$ is the $M \times 1$ cross-correlation matrix between the input data vector and $d$ is the desired response. To solve this equation, we need to obtain:

$$\omega_k^{(j)} (j) = [R_k^{(j)}]^{-1}b_k^{(j)}(j).$$  

(9)

In the CG-based algorithm, the iteration procedure is introduced. For the $j$th iteration, we choose the negative direction as:

$$d_k^{(j)}(j) = b_k^{(j)}(j) - R_k^{(j)}(j)\omega_k^{(j)}.$$  

(10)

The CG-based weight vector $\omega_k^{(j)}(j)$ is defined as:

$$\omega_k^{(j)}(j) = \omega_k^{(j)}(j-1) + \alpha_k^{(j)}(j)p_k^{(j)}(j),$$  

(11)

where $p(j)$ is the direction vector with conjugacy and $\alpha(j)$ is calculated by replacing (11) in (9), then taking the gradient with respect to $\alpha(j)$ and using (11), we get:

$$\alpha_k^{(j)}(j) = \frac{\rho_k^{(j)}(j-1)}{p_k^{(j)}(j)^H c_k^{(j)}(j)},$$  

(12)

where

$$\rho_k^{(j)}(j) = g_k^{(j)}(j)^H g_k^{(j)}(j)$$  

(13)

and

$$c_k^{(j)}(j) = R_k^{(j)}(j)p_k^{(j)}(j).$$  

(14)

The direction vector $p_k^{(j)}(j)$ in (11) is defined as:

$$p_k^{(j)}(j + 1) = g_k^{(j)}(j) + \beta_k^{(j)}(j)p_k^{(j)}(j),$$  

(15)

where $\beta_k^{(j)}(j)$ is calculated by the Gram Schmidt orthogonalization procedure [8] for the conjugacy:

$$\beta_k^{(j)}(j) = \frac{\rho_k^{(j)}(j)}{\rho_k^{(j)}(j-1)}$$  

(16)

with

$$\rho_k^{(j)}(j) = g_k^{(j)}(j)^H g_k^{(j)}(j).$$  

(17)

Besides the basic CG algorithm, there are two ways to define the correlation and cross-correlation matrices which are 'finite sliding data window' and 'exponentially decaying data window' [9]. In this paper, we mainly focus on the 'exponentially decaying data window' because this approach employs the same correlation matrix as the RLS algorithm. The recursions are given by:

$$R_k^{(j)}(j) = \lambda_f R_{k-1}(j) + x_k^{(j)}(j)x_k^{(j)}(j)^H$$  

(18)

$$b_k^{(j)}(j) = \lambda_f b_{k-1}(j) + d_k^{(j)}(j)x_k^{(j)}(j)$$  

(19)

where $\lambda_f$ is the forgetting factor.

### 3.2. Incremental Distributed CG - Based Solutions

In the incremental distributed model of our algorithm, each node is only allowed to communicate with its direct neighbor at each time instant. To describe the whole process, we define a cycle, where each node in this network could only access its immediate neighbor in this cycle [11]. The quantity $\psi_k^{(j)}$ is defined as a local estimate of $\omega_k$ at time $i$. As a result, we assume that node $k$ has access to an estimate of $\omega_k$ at its immediate neighbor node $k - 1$ which is $\psi_{k-1}^{(j)}$ in the defined cycle. Fig 1 shows its processing.

![Fig 1. Incremental Distributed CG-Based Network Processing](image)

Based on the main steps of the CG algorithm, we propose two distributed adaptive filtering algorithms, namely, the CCG and the MCG for distributed estimating over sensor networks. The difference between these two strategies is that the CCG needs to run $k$ iterations while the MCG only needs one iteration. The implementation of Incremental Distributed CG Solution (IDCG) is showed in Table 1. Similarly to CCG algorithm lowercase, the Incremental Distributed MCG solution (IDMCN) only needs one iteration per time instant and the details are shown in Table 1. These two Incremental Distributed CG-Based Solutions can be summarised as:

1) assess local error
2) update its weight vector
3) pass the updated weight estimate $\psi_k^{(j)}$ to its neighbor node

The idea of the MCG algorithm is to redefine the gradient vector with a recursive expression [12]:

$$g_k^{(j)} = b_k^{(j)} - R_k^{(j)}\omega_k^{(j)} + \lambda_f g_{k-1}^{(j)} - \alpha_k^{(j)} R_k^{(j)} p_k^{(j)} + x_k^{(j)}[d_k^{(j)}(j) - x_k^{(j)} \omega_{k-1}].$$  

(20)

Premultiplying (17) by $p_k^{(j)^H}$ and considering $p_k^{(j)}$ uncorrelated with $x_k$, $d_k$, and $\omega_{k-1}$ and then taking the expectation, we get:

$$E[p_k^{(j)^H}g_k^{(j)}] = \lambda_f E[p_k^{(j)^H}g_{k-1}^{(j)}]$$  

(21)
Assuming the algorithm converges, then the last term of (18) could be neglected and we will get:

$$E[\alpha_k(i)] = \frac{E[p_{k}^{(i)} H g_{k-1}^{(i-1)}]}{E[p_{k}^{(i)} H R_{k}^{(i)} p_{k}^{(i)}]}$$

and

$$E[\alpha_k(i)] - \frac{E[p_{k}^{(i)} H g_{k-1}^{(i-1)}]}{E[p_{k}^{(i)} H R_{k}^{(i)} p_{k}^{(i)}]} \leq \frac{E[p_{k}^{(i)} H g_{k-1}^{(i-1)}]}{E[p_{k}^{(i)} H R_{k}^{(i)} p_{k}^{(i)}]} \leq E[\alpha_k(i)]$$

The inequalities in (19) are satisfied if we define:

$$\alpha_k(i) = \eta \frac{p_{k}^{(i)} H g_{k-1}^{(i-1)} p_{k}^{(i)} H R_{k}^{(i)} p_{k}^{(i)}}{p_{k}^{(i)} H g_{k-1}^{(i-1)} p_{k}^{(i)}},$$

where $(\lambda_f - 0.5) \leq \eta \leq \lambda_f$. The direction vector $p_k$ is defined by:

$$p_{k}^{(i+1)} = g_{k}^{(i)} + \beta_k(i) p_{k}^{(i)}$$

where $\beta_k$ is computed to avoid the residue produced by using the Polak-Ribiere approach which is given by:

$$\beta_k(i) = \frac{(g_{k}^{(i)} - g_{k-1}^{(i-1)}) H g_{k}^{(i)}}{g_{k-1}^{(i-1) H g_{k-1}^{(i-1)}}}.$$  

### 3.3. Computational Complexity

To analyse the proposed incremental distributed CG algorithms, we detail the computational complexity. Additions and multiplications are used to measure the complexity and listed in Table 2. It is obvious that the complexity of the incremental distributed CG algorithm depends on the iteration number $j$.

### Table 1. IDMCG Solutions

| Initialization: |
|----------------|
| $\omega_0 = 0, g_0 = b, p_1 = g_0$ |
| For each time instant $i = 1, 2, \ldots, n$ |
| For each node $k = 1, 2, \ldots, N$ |
| $\omega_0 = 0, g_0 = b, p_1 = g_0$ |
| For each time instant $i = 1, 2, \ldots, n$ |
| For each node $k = 1, 2, \ldots, N$ |

### Table 2. Computational Complexity of Algorithms

| Algorithm          | Additions                           | Multiplications          |
|--------------------|-------------------------------------|--------------------------|
| IDCCG              | $m^2 + 2m - 2 + J(2m^2 + 7m - 2)$ | $m^2 + 3m$               |
| IDMC$\tilde{G}$    | $3m^2 + 11m - 5$                    | $4m^2 + 11m - 2$         |
| IDLMS              | $4m - 1$                            | $3m + 1$                 |
| IDRLS              | $m^2 + 4m - 1$                      | $m^2 + 5m$               |

### 4. PROPOSED DIFFUSION DISTRIBUTED CG-BASED ALGORITHMS

#### 4.1. Network Structure

For the diffusion distributed CG-based strategy, we consider a network structure where each node from the same neighborhood could exchange information with each other at every iteration. For each node in the network, it can collect information from all its neighbors and itself, and then convey all the information to its local adaptive filter and update the estimation of the weight vector through our algorithms. Specifically, at any time instant $i - 1$, we define that node $k$ has access to a set of unbiased estimates $\{ \phi_k^{(i-1)} \}_{k \in N_k}$ from its neighborhood $N_k$ including itself. Then, these local estimates are combined at node $k$ as

$$\phi_k^{(i-1)} = \sum_{l \in N_{k, i-1}} c_{kl} \phi_l^{(i-1)}$$

where $c_{kl}$ should be satisfied

$$\sum_{l} c_{kl} = 1, l \in N_{k, i-1} \forall k$$

Among the strategies to choose the combiner C are the Metropolis, the Laplacian and the nearest neighbor rules[7]. For our proposed diffusion distributed CG-based algorithm, we choose the Metropolis
whose processing is shown in Fig. 2 and operates as follows:

\[
\begin{align*}
    c_{kl} &= \frac{1}{\text{max}(n_k, n_l)}, & \text{if } k \neq l & \text{are linked} \\
    c_{kl} &= 0, & \text{for } k \text{ and } l & \text{not linked} \\
    c_{kk} &= 1 - \sum_{l \in N_k} c_{kl}, & \text{for } k = l
\end{align*}
\]

(28)

4.2. Diffusion Distributed CG - Based Solutions

The CCG and MCG algorithms are also developed for the diffusion distributed CG-based solutions, the details for these two algorithms are shown in Table 3. To derive these two algorithms, we first use equation (23) to get the unbiased estimates \( \phi_k^{(i-1)} \) and substitute them into equation (11), which results in:

\[
\psi_k^{(i)}(j) = \phi_k^{(i-1)}(j) + \alpha_k^{(i)}(j) p_k^{(i)}(j)
\]

(29)

The rest of derivation is similar to the incremental CG-based solutions.

4.3. Computational Complexity

The computational complexity is used to analyse the proposed diffusion distributed CG-based algorithms where additions and multiplications are measured. The details are listed in Table 3. Similarly to incremental distributed CG-based algorithms, it is clear that the complexity of the incremental distributed CCG algorithm depends on the iteration number \( j \) and the number of linked nodes \( l \).

5. SIMULATION RESULTS

In this part, we test the proposed incremental and diffusion distributed CG-based algorithms in a sensor network and compare the results with LMS, RLS and AP algorithms based on the performance of excess MSE (EMSE). For each test, the number of repetitions is set to 1000, and we assume there are 20 nodes in the network. The number of taps of the adaptive filter is 10, the variance for the input signal and the noise are 1 and 0.001, respectively. Besides, the noise samples are modelled as complex Gaussian noise.

5.1. Performance of Proposed IDCG Algorithms

First, we give out the definitions of the parameters of our test for each algorithms and the network. After 1000 iterations, the performance of each algorithm has been showed in Fig. 3. We can see that, the performance of the IDMCG and IDCCG algorithm is better than IDLMS, while IDMCG is very close to the DRLS algorithm’s curve. The reason why the proposed IDMCG algorithm has a better performance is IDMCG defined the negative gradient vector \( g_k \) with a recursive expression and the \( \beta_k \) is computed to avoiding the residue produced by using the Polak-Ribiere approach. Comparing with the IDCCG algorithm, the IDMCG is a non-reset and low complexity algorithm with one iteration per time instant. Because of how often the algorithm is reset will influence the performance, the IDMCG introduce the non-reset method together with Polak-Ribiere method which is used for improved its performance.

5.2. Performance of Proposed DDCG Algorithms

For this group of Proposed DDCG Algorithms’ test, we use some similar definitions of parameters as in the last part. For the diffusion strategy, we build the link between each node randomly, and for the combiner C, we calculate it following the Metropolis rule. Fig. 4 shows the network structure. After 1000 iterations, the test result are showed in Fig. 5. We can see that, the proposed DDMCG and DDCCG still have a better performance than DDLMS algorithm and DDMCG is closer to the DDRLS’s curve. For the diffusion strategy, the network structure has a significant influence on the performance of our proposed DDCG Algorithms.

![Fig. 2. Diffusion Distributed CG-Based Network Processing](image)

![Fig. 3. Output EMSE against the number of iterations for Incremental Strategy with \( \alpha_{IDLMS}=0.005, \lambda=0.2, \lambda_{IDMCG} \neq \lambda_{IDCCG}=0.25, \eta_{IDCCG}=\eta_{IDMCG}=0.15, j=5, \alpha_{IDAP}=0.06, K=2 \)](image)

![Fig. 4. Network Structure](image)

![Fig. 5. Network Structure](image)
## Table 3. DDCG Solutions

| DDCCG Solution | DDMCG Solution |
|----------------|----------------|
| Initialization: | Initialization: |
| $\omega_0 = 0, g(0) = b, p(1) = g(0)$ | $\omega_0 = 0, g_0 = b, p_1 = g_0$ |
| For each time instant $i=1,2,\ldots,n$ | For each time instant $i=1,2,\ldots,n$ |
| For each node $k=1,2,\ldots,N$ | For each node $k=1,2,\ldots,N$ |
| $\phi^{(i-1)}(1) = 0$ | $\phi^{(i-1)}(1) = 0$ |
| For iterations $j=1,2,\ldots,J$ | | |
| $R_k^{(i)}(j) = \lambda_f R_k^{(i)}(j) + x_k^{(i)}(j)x_k^{(i)H}$ | $R_k^{(i)}(j) = \lambda_f R_k^{(i)}(j) + x_k^{(i)}(j)x_k^{(i)H}$ |
| $b_k^{(i)}(j) = \lambda_f b_k^{(i)}(j) + d_k^{(i)}(j)x_k^{(i)}(j)$ | $b_k^{(i)}(j) = \lambda_f b_k^{(i)}(j) + d_k^{(i)}(j)x_k^{(i)}(j)$ |
| $\alpha_k^{(i)}(j) = \eta [p_k^{(i)}(j) - \lambda_f R_k^{(i)}(j)p_k^{(i-1)}(j)]^{-1}[p_k^{(i)}(j) - \lambda_f R_k^{(i)}(j)p_k^{(i-1)}(j)Hg_k^{(i)}(j - 1)]$ | $\alpha_k^{(i)}(j) = \eta [p_k^{(i)}(j) - \lambda_f R_k^{(i)}(j)p_k^{(i-1)}(j)]^{-1}[p_k^{(i)}(j) - \lambda_f R_k^{(i)}(j)p_k^{(i-1)}(j)Hg_k^{(i)}(j - 1)]$ |
| where $\lambda_f = 0.25$, $\eta \leq \lambda_f$ | where $\lambda_f = 0.25$, $\eta \leq \lambda_f$ |
| $\phi^{(i-1)}(1) = \sum_{i(\in N_k, i-1)c_k}\psi^{(i-1)}(j)$ | $\phi^{(i-1)}(1) = \sum_{i(\in N_k, i-1)c_k}\psi^{(i-1)}(j)$ |
| $\psi^{(i)}(j) = \phi^{(i)}(1) + \alpha_k^{(i)}(j)p_k^{(i)}(j)$ | $\psi^{(i)}(j) = \phi^{(i)}(1) + \alpha_k^{(i)}(j)p_k^{(i)}(j)$ |
| $g_k^{(i)}(j) = g_k^{(i)}(j - 1) - \alpha_k^{(i)}(j)R_k^{(i)}(j)p_k^{(i)}(j)$ | $g_k^{(i)}(j) = g_k^{(i)}(j - 1) - \alpha_k^{(i)}(j)R_k^{(i)}(j)p_k^{(i)}(j)$ |
| $\beta_k^{(i)}(j) = [g_k^{(i)}(j - 1)Hg_k^{(i)}(j - 1)]^{-1}([g_k^{(i)}(j)Hg_k^{(i)}(j)] - g_k^{(i)}(j)Hg_k^{(i)}(j)]$ | $\beta_k^{(i)}(j) = [g_k^{(i)}(j - 1)Hg_k^{(i)}(j - 1)]^{-1}([g_k^{(i)}(j)Hg_k^{(i)}(j)] - g_k^{(i)}(j)Hg_k^{(i)}(j)]$ |
| $p_k^{(i)}(j + 1) = g_k^{(i)}(j) + \beta_k^{(i)}(j)p_k^{(i)}(j)$ | $p_k^{(i)}(j + 1) = g_k^{(i)}(j) + \beta_k^{(i)}(j)p_k^{(i)}(j)$ |
| $J = J + 1$ | |
| After J iterations | After N iterations |
| $K = K + 1$ | |
| After N iterations | |
| $\omega_i = \psi^{(i)}(1)$ | $\omega_i = \psi^{(i)}(1)$ |
| $i = i + 1$ | $i = i + 1$ |

### 6. CONCLUSIONS

We have developed distributed CG algorithms for incremental and diffusion type estimation over sensor networks. The CG-based strategies avoid the matrix inversion and numerical instability of RLS algorithms and have a faster convergence than LMS and AP algorithms. Simulation results have shown that the proposed IDMCG and DDMCG algorithms have a better performance than the LMS and AP algorithm, and a close performance to the RLS algorithm.

### 7. REFERENCES

[1] C. G. Lopes and A. H. Sayed, "Incremental adaptive strategies over distributed networks", *IEEE Trans. Sig. Proc.*, vol. 55, no. 8, pp. 4064-4077, August 2007.

[2] C. G. Lopes and A. H. Sayed, "Diffusion least-mean squares over adaptive networks: Formulation and performance analysis", *IEEE Trans. Sig. Proc.*, vol. 56, no. 7, pp. 3122-3136, July 2008.

[3] L. L. Li, J.A. Chambers, C.G. Lopes, and A.H. Sayed, "Distributed Estimation Over an Adaptive Incremental Network Based on the Affine Projection Algorithm", *IEEE Trans. Sig. Proc.*, vol. 58, issue. 1, pp. 151-164, Jan. 2010.

[4] F. Cattivelli, C. G. Lopes, and A. H. Sayed, "Diffusion recursive least-squares for distributed estimation over adaptive networks," *IEEE Trans. Sig. Proc.*, vol. 56, no. 5, pp. 1865-1877, May 2008.

[5] G. Mateos, I. D. Schizas, and G. B. Giannakis, "Distributed Recur- sive Least-Squares for Consensus-Based In-Network Adaptive Estimation," *IEEE Trans. Sig. Proc.*, vol. 57, no. 11, pp. 4583-4588, November 2009.

[6] P. Clarke and R. C. de Lamare, "Transmit Diversity and Relay Selection Algorithms for Multirelay Cooperative MIMO Systems" *IEEE Transactions on Vehicular Technology*, vol. 61, no. 3, pp. 1094-1098, October 2012.

[7] O. Axelsson, *Iterative Solution Methods*, New York: Cambridge Univ. Press, 1994.

[8] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 2nd Ed., Baltimore, MD: Johns Hopkins Univ. Press, 1989.

[9] P. S. Chang and A. N. Willson, Jr, "Analysis of Conjugate Gradient Algorithms for Adaptive Filtering", *IEEE Transactions on Signal Processing*, vol. 48, no. 2, pp. 409-418, February 2000.

[10] R. C. de Lamare and R. Sampaio-Neto, "Adaptive Reduced-Rank Processing Based on Joint and Iterative Interpolation, Decimation and Filtering", *IEEE Transactions on Signal Processing*, vol. 57, no. 7, July 2009, pp. 2503 - 2514.

[11] R. Fa, R. C. de Lamare and L. Wang, "Reduced-rank STAP schemes for airborne radar based on switched joint interpolation, decimation and filtering algorithm", *IEEE Trans. Sig. Proc.*, vol. 10, no. 8, pp.4182-4194.

[12] L. Wang, and R.C.de Lamare, "Constrained adaptive filtering algorithms based on conjugate gradient techniques for beamforming", *IET Signal Processing*, vol. 4, issue. 6, pp. 686-697, Feb. 2010.