Hierarchical Mass Structure of Fermions in Warped Extra Dimension

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Abstract

The warped bulk standard model has been studied in the Randall-Sundrum background on $S^1/Z_2 \times Z_2'$ interval with the bulk gauge symmetry $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. With the assumption of no large cancellation between the fermion flavor mixing matrices, we present a simple analytic method to determine the bulk masses of standard model fermions in the almost universal bulk Yukawa coupling model. We also predict $U_{e3}$ element of MNS matrix to be near the experimental upper bound when the neutrino masses are of Dirac type.

PACS numbers: 11.10.Kk, 12.15Ff, 12.60.-i, 12.90.+b

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I. INTRODUCTION

It is a fascinating idea that some of the deep puzzles of particle physics may be attributed to the geometry of extra space dimensions. The most discussed one is the gauge hierarchy problem why the electroweak scale is much lower than the Planck scale. An attractive hypothesis to explain this hierarchy was proposed using large extra dimensions [1]. Soon later an alternative interesting idea was postulated by Randall and Sundrum [2]. In their first model (RS1), the compact extra dimension has a size not much larger than the Planck length, but with a warped metric. This warped extra dimension is also interesting in the context of AdS/CFT correspondence in string theories [3]. In fact, stringy realization of the warped extra dimension was considered in compactifications with non-vanishing fluxes of higher tensor fields. (See ref. [4] and references therein.)

Another puzzle which we would like to address here is the question of fermion masses and flavor mixing. An extra-dimensional explanation to this puzzle owes to the configuration of the wave functions of the quarks and leptons along the extra dimensions. In a field theory approach, the smallness of the Yukawa coupling and thus the fermion mass is due to the small overlap of the wave functions of the relevant fields in the extra dimensions. The idea was proposed in flat TeV$^{-1}$ size extra dimension [5], which was utilized to construct realistic models of the Yukawa sector [6, 7, 8, 9].

The geometrical approach to the Yukawa couplings can also be applied to the RS1 model. For this purpose, the standard model (SM) fields should reside in the warped bulk. Though this was recognized to be possible [10, 11], the electroweak (EW) precision test restricts the RS1 bulk SM strongly since the $t$ quark is much heavier than other quarks and can give a significant amount of shift to the weak gauge boson mass ratio $M_W/M_Z$ from the SM prediction due to $t$ quark and its Kaluza-Klein (KK) mode mixing [12]. Several attempts were made to resolve this problem [12, 13].

Recently, Agashe et al. [14] showed that the above problem of the too large Peskin-Takeuchi $T$ parameter is due to the absence of a custodial $SU(2)$ symmetry in the bulk, as is suggested by the AdS/CFT correspondence, and proposed a model which has the gauge symmetry $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. This model may be related with the warped Higgsless model which shows a possibility of EW symmetry breaking without a Higgs field in the RS1 SM [15, 16]. It must be stressed that the Higgs field in the model we consider must be confined on the brane in
order not to reincarnate the gauge hierarchy problem. Because of this peculiar property, the Higgs field acts as a boundary condition (BC) for the bulk field equations. If the Higgs couples to different bulk fermions with (more or less) universal strength at the boundary, the small masses and mixings of the SM fermions can be induced by the suppression of the zero-mode wave functions on the infra-red (IR) boundary.

In this paper, we shall consider the fermion mass structure of quarks and leptons (including neutrinos) in the framework of the warped bulk fermions. Under the situation that a fundamental principle to dictate the parameters in the 5D bulk theory is not known, it would be natural to take the hypothesis that the 5D Yukawa couplings do not have any particular textures. Thus we assume that the 5D Yukawa couplings are all around unity in magnitude. Under this “almost universal” hypothesis, the fermion mass structure is solely due to the configurations of the (zero-mode) wave functions of the bulk fermions. In the case at hand, they are controlled by bulk fermion masses. An attractive point of this approach is that the whole bulk structure may be revealed in future experiments to explore consequences of the Kaluza-Klein modes of the SM particles.

The purpose of the paper is two fold: First we present a simple analytic method which is useful to estimate the bulk fermion masses from known experimental data under the assumption of the almost universal 5D Yukawa couplings. Despite the fact that there are already many (numerical) analyzes on the fermion masses in the warped extra dimension in the literature, we believe that it is still worth presenting our analytic results because of simplicity and accessibility. The hierarchical structure in the quark mass matrices makes our analysis very robust. For the lepton sector, although it may suffer from some pollution of numerical coefficients because of its somewhat less hierarchical pattern of the masses and mixings, it is still possible to determine generic structure. The second purpose of the paper is to show that the $U_{e3}$ entry of the Maki-Nakagawa-Sakata (MNS) mixing matrix in the neutrino sector is typically close to the present experimental upper bound when the neutrino masses are of Dirac type.

II. THE STANDARD MODEL IN THE RS1 BULK

The basic framework of our study is a simple system where one flavor of fermion resides in the bulk of the RS1. We extend it to the three flavor system later in this paper. The RS1 metric is

\[
\sigma^2 = \frac{1}{2} \left( \frac{\sinh^2 \left( \frac{2\pi x}{L} \right)}{\sinh^2 \left( \frac{\pi y}{L} \right)} \right)
\]

where $L$ is the bulk scale length.
given by

\[ ds^2 = e^{-2\sigma(y)}(dt^2 - dx^2) - dy^2, \]

where \( y \) represents the warped coordinate for extra-dimension and \( \sigma(y) = k|y| \). The 5th dimension is bounded in the interval \((0, L)\). The gravity is confined at \( y = 0 \) boundary known as Planck (UV) brane, whereas our world is confined on the other end \((y = L)\), which is called the TeV (IR) brane. \( y \) coordinate can be converted to a conformally flat coordinate, \( z \equiv e^{\sigma/k} \), where the metric becomes

\[ ds^2 = \frac{1}{(kz)^2}(dt^2 - dx^2 - dz^2). \]

The interval becomes \( 1/k < z < 1/T \), where \( T = ke^{-kL} \sim O(1) \) TeV.

The 5D fermion action becomes

\[ S_{\text{fermion}} = \int d^4x \sum_n \left[ \bar{\Psi}_L^{(n)} i\gamma^\mu \partial_\mu \Psi_L^{(n)} + \bar{\Psi}_R^{(n)} i\gamma^\mu \partial_\mu \Psi_R^{(n)} - m^{(n)}(\bar{\Psi}_L^{(n)} \Psi_R^{(n)} + \bar{\Psi}_R^{(n)} \Psi_L^{(n)}) \right], \]

where \( \hat{\Psi} \equiv e^{-2\sigma} \Psi \). The bulk Majorana mass \( m_M \) is non-zero only if the fermion is neutral. This Lagrangian has the five-dimensional \( \mathbb{Z}_2 \times \mathbb{Z}_2' \) parity on each boundary (brane),

\[ \gamma_5 \Psi(x, -y) = \pm \Psi(x, y), \quad \gamma_5 \Psi(x, L - y) = \pm \Psi(x, L + y). \]

\( \mathbb{Z}_2 \) and \( \mathbb{Z}_2' \) represent UV and IR parity of bulk field and written in the form of (UV, IR). The bulk Dirac mass is defined by \( m_D = \sigma' = kc \text{ sign}(y) \). The bulk fermion can be divided into two chiral components, \( \Psi = \Psi_L + \Psi_R \), for \( \gamma_5 \Psi_L = -\Psi_L \), \( \gamma_5 \Psi_R = \Psi_R \). Each chiral field can be expanded to the KK modes

\[ \hat{\Psi}(x, y)_{L(R)} = \sqrt{k} \sum_n \psi^{(n)}_{L(R)}(x)f^{(n)}_{L(R)}(y). \]

After the mode expansion, the 4D effective action for KK modes becomes

\[ S_{\text{eff}} = \int d^4x \sum_n \left[ \bar{\psi}^{(n)}_{L} i\gamma^\mu \partial_\mu \psi^{(n)}_{L} + \bar{\psi}^{(n)}_{R} i\gamma^\mu \partial_\mu \psi^{(n)}_{R} - m^{(n)}(\bar{\psi}^{(n)}_{L} \psi^{(n)}_{R} + \bar{\psi}^{(n)}_{R} \psi^{(n)}_{L}) \right], \]

where \( m^{(n)} \) is a mass of nth KK excited mode. To generate the action (3), KK mode functions should satisfy the mode equations in \( z \) coordinate,

\[ \left( \partial_z \pm \frac{c}{z} \right) f^{(n)}_{L/R} = \mp m^{(n)} f^{(n)}_{R/L}. \]

A generic 5D bulk fermion can have four different forms according to the \( \mathbb{Z}_2 \times \mathbb{Z}_2' \) parity,

\[ \hat{\Psi}_i(x, y) = \sqrt{k} \sum_n [\psi^{(n)}_{iL}(x)f^{(n)}_{iL}(y) + \psi^{(n)}_{iR}(x)f^{(n)}_{iR}(y)]. \]
The indices \( i = 1, 2 \) represent the parallel conditions, where \( f_{iL} \) has \((\pm \mp)\) parity and \( f_{iR} \) has \((\mp \pm)\), and \( i = 3, 4 \) represent the crossed conditions, where \((\pm \mp)\) for \( f_{iL} \) and \((\mp \pm)\) for \( f_{iR} \), respectively. Each mode function except the zero modes can be written in the series of Bessel functions. For more details, see Ref. [19].

The Higgs field \( \phi(x) \) is confined on the IR boundary,

\[
S = - \int d^4xdy \frac{\lambda_5}{T} H(x) \left( \overline{\Psi}_1(x,y)\Psi_2(x,y) + \overline{\Psi}_2(x,y)\Psi_1(x,y) \right) \delta(y-L),
\]

where \( H = e^{-kL}\phi(x) \) is canonically normalized Higgs scalar and \( \lambda_5 \) is the Yukawa coupling. When the Higgs field get a vacuum expectation value \( \langle H \rangle = v_W \), the surviving zero modes give the SM fermion mass term,

\[
m_f = \frac{\lambda_5v_W}{T} \left. f_{1L}^{(0)} f_{2R}^{(0)} \overline{\psi}_{1L}^{(0)} \psi_{2R}^{(0)} \right|_{z=1/T},
\]

where the zero mode functions are

\[
f_{1L}^{(0)} = \frac{(kz)^{-c_1}}{N_1^{(0)}}, \quad f_{2R}^{(0)} = \frac{(kz)^{c_2}}{N_2^{(0)}},
\]

and the normalization becomes

\[
N_1^{(0)} = \sqrt{\frac{1 - \epsilon^{2c_1-1}}{2c_1-1}}, \quad N_2^{(0)} = \sqrt{\frac{\epsilon^{-2c_2-1} - 1}{2c_2 + 1}},
\]

with \( \epsilon = T/k = e^{-kL} \). We will drop the indices 1 and 2 from this point to avoid the confusion with family indices.

The SM requires that two \( SU(2)_L \) singlet right-handed fermions should exist for a corresponding left-handed doublet. To match the particle content, we set \((Q_i, U_i, D_i)\) and \((L_i, N_i, E_i)\) as bulk fields where \( i = 1, 2, 3 \) represent 3 generations. \( Q \) and \( L \) include \( SU(2)_L \) quark and lepton doublets. \( U, D, E, N \) include the SM fields \((u, d, e)_R\) and a right-handed neutrino \( N_R \), respectively.

If we expand the model to 3 generations, the mass term is written in \( 3 \times 3 \) matrix,

\[
M^f_{ij}/v_W = y^f_{ij} = \lambda^f_{5ij} F_L(c_i) \times F_R(c_j),
\]

where \( y^f_{ij} \) is a 4D effective Yukawa coupling of fermion \( f \) and \( \lambda^f_{5ij} \) is a 5D boundary Yukawa coupling, and

\[
F_L(c_i) = \epsilon^{c_i-1/2} \sqrt{\frac{2c_i - 1}{1 - \epsilon^{2c_i-1}}}, \quad F_R(c_i) = \epsilon^{-c_i-1/2} \sqrt{\frac{2c_i + 1}{\epsilon^{-2c_i-1} - 1}},
\]

\[5\]
where $c_i$ represents each mass of $(Q_i, U_i, D_i)$ and $(L_i, N_i, E_i)$. $F_{L(R)}(c_i) = 1$ when bulk fermion mass is zero $c_i = 0$. If we increase $(-)c_i$, $F_{L(R)}(c_i)$ decrease slowly until $(-)c_i = 1/2$. For $(-)c_i > 1/2$, it decrease fast in power of $\epsilon^{(-)c_i}$;

$$
F_{L(R)}(c_i) \approx e^{(-)c_i - 1/2} \sqrt{(-)2c_i - 1} \quad \text{for } (-)c_i - 1/2 \gg 1/kL
$$

$$
\approx (kL)^{-1/2} \quad \text{for } |(-)c_i - 1/2| \ll 1/kL
$$

$$
\approx \sqrt{1 - (-)2c_i} \quad \text{for } (-)c_i - 1/2 \ll -1/kL. \quad (15)
$$

The mass difference of bulk fermion gives the natural mass hierarchy between different SM fermions.

### III. FERMION MASSES AND MIXINGS

The bulk SM conflicts with the electro-weak precision test without some symmetry [12, 13]. The $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ bulk SM is a favorable candidate because its custodial isospin prevents the extra-contribution from the KK fermion modes to the gauge boson self-energy [14]. Also, this model draws interests due to the connection with the Higgsless model of electro-weak symmetry breaking [15, 16].

All SM fields except the Higgs field reside in the bulk [11]. There are some fields which have no SM counter part, e.g. $SU(2)_R$ charged gauge bosons. The “crossed” BC $(\pm \mp)$ assigned to these fields eliminates their zero modes, thus we will not see any light additional field. Among the bulk fermions we defined in previous section, $Q$ and $L$ fields are consisted with $(\pm \pm)$ fields only, while $SU(2)_R$ doublet contains one component with $(\pm \mp)$ parity, because their charged current should conserve $\mathbb{Z}_2 \times \mathbb{Z}_2'$ parity. Thus, for $U, N, D, E$ fields, only one component of the doublet can have a zero mode. If the SM is induced from this model, there should be at least one bulk $SU(2)_L$ doublet and two $SU(2)_R$ doublet fermions for each family.

To establish a simple but realistic model for 4D fermion masses, we choose the bulk mass matrices are real and diagonal. Also, for simplicity we ignore CP phase in the Yukawa couplings. Inclusion of the CP phase is straightforward. In this paper, we use a (almost) universal Yukawa coupling model that the Higgs scalar couples to all fermions with (almost) universal strength. In this model, the fermion mass hierarchy is generated only by the bulk fermion mass structures. For the case that the universality is exact, $3 \times 3$ matrix $M_{ij} = v_W F_L(c_{Qi}) F_R(c_{Aj})$ has only one
A. Quark Masses and Mixings

The bulk fields $Q_i, U_i$ and $D_i$ with bulk mass parameters $c_{Qi}, c_{Ui}, c_{Di}$, contain the zero modes which can be interpreted as the SM fermions. If we take all parameters to be real, the mass matrices can be diagonalized by bi-orthogonal transformation,

$$U_{qL}^T M_q U_{qR} = M_q^{\text{diag}} \quad \text{for } q = u, d. \quad (16)$$

The CKM matrix is defined as $K = U_{uL}^T U_{dL}$. With simplified Wolfenstein parametrization for $\lambda \simeq 0.22$, the CKM matrix $K$ can be written

$$K \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (17)$$

where the numerical coefficient of each entry is of order unity. A natural choice for $U_{uL}$ and $U_{dL}$ in this case is of the similar form as (17):

$$U_{uL} \simeq U_{dL} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \quad (18)$$

Here any number greater than $\lambda^{0.5}$ is replaced as unity. The above choice of mixing is reasonable since the $u_L$ and $d_L$ has the same bulk mass. The fermion masses can also be expressed in terms of $\lambda$,

$$M_u^{\text{diag}} = \text{diag}(m_u, m_c, m_t) \simeq v_W \text{ diag}(\lambda^8, \lambda^{3.5}, 1),$$
$$M_d^{\text{diag}} = \text{diag}(m_d, m_s, m_b) \simeq v_W \text{ diag}(\lambda^7, \lambda^5, \lambda^{2.5}). \quad (19)$$

If we consider the (almost) universal coupling, the quark mass matrices become

$$(M_a)_{ij} \simeq v_W F_L(c_{Qi}) F_R(c_{Aj}), \quad (20)$$
where \(a = u, d\) and \(A = U, D\). It follows from the above that

\[
(M_a M_a^T)_{ij} = (U_{aL}M_a^D)_{i2}U_{aL}^T F_{L}(c_{Qj})(\sum_k F_R(c_{Ak})^2).
\]

Let us now determine the mass parameters \(c\)'s. For \(u\) quark, we find

\[
M_u M_u^T \simeq (v_W C)^2 (F_L(c_{Q1})F_L(c_{Qj})) \simeq v_W^2 \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},
\]

where the last equality is obtained by substituting Eqs. (18) and (19) into (16). This leads

\[
F_L(c_{Q1}) \simeq C^{-1}\lambda^3, \quad F_L(c_{Q2}) \simeq C^{-1}\lambda^2, \quad F_L(c_{Q3}) \simeq C^{-1},
\]

where \(C \simeq F_R(c_{U3})\). Notice that this procedure works because of the hierarchical mass structure \(m_t \gg m_c, m_u\). This observation is crucial when discussing the neutrino masses. We will come back this point shortly.

If \(c_{U3}\) is too large, the mass of down sector quark from \(SU(2)_R\) doublet \(U_3\) becomes too small, giving too much contribution to Peskin-Takeuchi \(T\) parameter. It should be restricted, \(F_R(c_{U3}) \lesssim 1.2\). Also the constraint from \(Z \rightarrow b\bar{b}\) gives the allowed range \(F_L(c_{Q3}) \lesssim 0.7\). Since \(m_t/v_W \simeq F_L(c_{Q3})F_R(c_{U3}) \simeq 1\), for the range of our interest, \(2 \text{ TeV} < T < 8 \text{ TeV}\) and with the standard choice of curvature scale \(k \simeq M_{pl}\), the bulk top masses are almost fixed around the values \(c_{U3} \simeq 0.2\) and \(c_{Q3} \simeq 0.3\). Then from (14), we find

\[
c_{Q1} \simeq 0.61, \quad c_{Q2} \simeq 0.56, \quad c_{Q3} \simeq 0.3.
\]

If we assume that off-diagonal term in \(U_{qR}\) is small enough, with Eq. (16) and (19), the quark mass matrices can be written in the following form:

\[
M_u \simeq v_W \begin{pmatrix} \lambda^8 & \lambda^{4.5} & \lambda^3 \\ \lambda^9 & \lambda^{3.5} & \lambda^2 \\ \lambda^{11} & \lambda^{5.5} & 1 \end{pmatrix}, \quad U_{aR}^T \simeq v_W \begin{pmatrix} \lambda^8 & \lambda^{4.5} & \lambda^3 \\ \lambda^3.5 & \lambda^2 & \lambda^1 \end{pmatrix},
\]

for the \(U\) fields and,

\[
M_d \simeq v_W \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^{5.5} \\ \lambda^8 & \lambda^5 & \lambda^{4.5} \\ \lambda^{10} & \lambda^7 & \lambda^{2.5} \end{pmatrix}, \quad U_{dR}^T \simeq v_W \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^{5.5} \\ \lambda^5 & \lambda^{4.5} & \lambda^{2.5} \end{pmatrix},
\]
for the $D$ fields. The lower left components depend on the details of the mixing matrices and are redundant for the mass determination. Our hypothesis of the almost universal 5D Yukawa couplings implies that both of the mass matrices given above are expressed as Eq. (20), which can be achieved if one chooses

$$
\begin{align*}
    c_{U1} &\simeq -0.70, & c_{U2} &\simeq -0.52, & c_{U3} &\simeq 0.2, \\
    c_{D1} &\simeq -0.65, & c_{D2} &\simeq -0.60, & c_{D3} &\simeq -0.57.
\end{align*}
$$

(27)

(28)

There can be small modification for a different choice of initial parameter range. The bulk masses we obtained above are approximately in agreement with the previous calculations [24, 25].

B. Lepton Masses and Mixings

We now consider the mass matrices for charged leptons and neutrinos. It is required to use a more cautious analysis to the lepton sector, because the hierarchy between lepton masses and mixings is weaker than that of quarks.

An advantage of the extra-dimensional explanation for the fermion masses is that the small masses can easily be generated as a consequence of the separation of the wave functions. When the fermions are in the warped extra dimension, the zero-mode wave functions have exponential form so that this suppression mechanism is very effective. Thus the Dirac masses of the neutrinos can be very small, allowing us to discuss the case where the light neutrinos are Dirac ones.

Motivated by the aforementioned argument, let us first consider the Dirac neutrino case. We assume $SU(2)_L$ doublet bulk leptons $L_i$ with bulk mass $c_{Li}$ and $SU(2)_R$ doublets $E_i$ and $N_i$ with masses $c_{Ni}$ and $c_{Ei}$. Each of them contains the zero mode $l_{iL}$, $e_{iR}$ and $\nu_{iR}$, respectively. If we consider that the SM neutrinos are Dirac particles, the MNS mixing matrix for neutrino is equivalent to the CKM matrix, $U_{MNS} = U^\dagger_{\nu} U_{\nu}$, where

$$
M^\dagger_{\nu} M_{\nu} = U_{\nu} (M^\text{diag}_{\nu})^2 U^\dagger_{\nu}, \quad M^\dagger_{e} M_{e} = U_{e} (M^\text{diag}_{e})^2 U^\dagger_{e}.
$$

(29)

for Dirac neutrino mass. With the same approximation as the quark case, the MNS matrix can be approximated as

$$
|U_{MNS}| \sim \begin{pmatrix}
1 & 1 & \lambda^m \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix},
$$

(30)
where the experimental constraint on $U_{e3}$ gives $m > 1.3$.

Though the individual neutrino masses are not yet measured, the mass differences between them are determined by the neutrino oscillation data,

\[ \Delta m_{\text{sol}}^2 = m_2^2 - m_1^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{\text{atm}}^2 = |m_3^2 - m_2^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2. \]  

(31)

The WMAP result suggests that any of neutrino mass should be $m_i < 1.0 \text{ eV}$. With all known data, there exist three possible cases: (1) almost degenerate neutrinos, (2) the normal hierarchy (NH), (3) the inverse hierarchy (IH). If the neutrino masses are almost degenerate $m_i \lesssim 1 \text{ eV}$, then with the maximal mixing of the MNS matrix, we expect that all the left-handed mode functions have almost the same configurations. Also the right-handed neutrinos should have the same pattern, while the right-handed charged lepton should have the hierarchical form. This may be possible. However, the structure of the MNS matrix as well as the mass differences would be a consequence of some numerology. We will not discuss this case furthermore.

For the NH case, as $\nu_1$ is very light or even massless, the other neutrino masses are fixed as

\[ m_1 = 0, \quad m_2 = \sqrt{\Delta m_{\text{sol}}^2}, \quad m_3 = \sqrt{\Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2}. \]  

(32)

For the IH, $\nu_3$ is very light so that

\[ m_1 = \sqrt{\Delta m_{\text{atm}}^2 - \Delta m_{\text{sol}}^2}, \quad m_2 = \sqrt{\Delta m_{\text{atm}}^2}, \quad m_3 = 0. \]  

(33)

If we allow the random cancellation during the diagonalization of mass matrix, there can be too many possibilities. On the other hand, if we follow the first assumption of no-cancellation strictly, the mass matrix should have either of the following two forms

\[ M_{\nu}^T M_{\nu} \propto \begin{pmatrix} \lambda^2 & \lambda^n & \lambda^n \\ \lambda^n & 1 & 1 \\ \lambda^n & 1 & 1 \end{pmatrix} \quad \text{(NH)} \quad \text{or} \quad \begin{pmatrix} \lambda^2 k & 1 & 1 \\ 1 & \lambda^{2l} & \lambda^{2l} \\ 1 & \lambda^{2l} & \lambda^{2l} \end{pmatrix} \quad \text{(IH)}, \]  

(34)

where $k$, $l$ and $n$ are some positive numbers. The derivation of the above can be found in Refs. \[27, 28\]. In short, we utilize the fact that $U_{\text{MNS}}$ is almost tri/bi-maximal and the neutrino masses are close to $(0,0,1)$ for NH and $(1,-1,0)$ or $(1,1,0)$ for IH. We can derive \[34\] by adding a small perturbation to the solutions of the approximation. For the IH case, $k \sim l \gtrsim 1$ is favored to avoid too large $U_{e3}$. 

10
It is clear that the IH is not consistent with our almost universal Yukawa coupling approach, where the lepton mass matrices should be written as

\[(M_\nu)_{ij} \simeq v_W F_L(c_{Li}) F_R(c_{Nj}), \quad (M_e)_{ij} \simeq v_W F_L(c_{Li}) F_R(c_{Ej}).\] (35)

Therefore we consider only the NH case, where the lepton masses can be written as,

\[M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3) = v_W \text{diag}(\lambda^{20.5}, \lambda^{20.5}, \lambda^{19}),\]
\[M_e^{\text{diag}} = \text{diag}(m_e, m_\mu, m_\tau) = v_W \text{diag}(\lambda^{8.5}, \lambda^5, \lambda^3).\] (36)

Our hypothesis implies as in the quark sector

\[(M_a^T M_a)_{ij} \simeq (U_a (M_a^D)^2 U_a^T)_{ij} \simeq v_W^2 F_L(c_{Li}) F_L(c_{Lj}) \sum_k F_R(c_{Ak})^2\] (37)

with \(a = \{\nu, e\}\) and \(A = \{N, E\}\), which can accord with the NH neutrino masses. With Eq. (34), one finds

\[M_e^T M_e \simeq v_W^2 \lambda^6 \begin{pmatrix} \lambda^{2n} & \lambda^n & \lambda^n \\ \lambda^n & 1 & 1 \\ \lambda^n & 1 & 1 \end{pmatrix}, \quad M_\nu^T M_\nu \simeq \lambda^{32} M_e^T M_e.\] (38)

The bulk mass terms of \(SU(2)_L\) doublets are

\[F_L(c_{L1}) \simeq C_L^{-1} \lambda^{3+n}, \quad F_L(c_{L2}) \simeq C_L^{-1} \lambda^3, \quad F_L(c_{L3}) \simeq C_L^{-1} \lambda^3,\] (39)

where \(C_L \sim F_R(c_{E3}).\)

Unlike the quark case, we cannot simply set the mixing matrices \(U_e \simeq U_\nu\). The maximal mixing between 2 and 3 flavors together with (38) suggests the following left-handed mixing matrices

\[U_f \simeq \begin{pmatrix} 1 & \lambda^{a_f} & \lambda^n \\ \lambda^{b_f} & 1 & 1 \\ \lambda^{c_f} & 1 & 1 \end{pmatrix},\] (40)

with \(f = e, \nu\). Writing

\[(M_a^T M_a)_{ij} = m_{a3}^2 U_{a3i} U_{a3j} + m_{a2}^2 U_{a2i} U_{a2j} + m_{a1}^2 U_{a1i} U_{a1j},\] (41)

with \(m_{ai}\) being the \(i\)-th mass eigenvalue of species \(a\), one finds that the first term in the right-handed side should dominate over the rest to reproduce (38). This requires that the mass eigenvalues are more hierarchical than the mixings. In fact, one finds

\[1.5 + a_\nu \gtrsim n, 2 + a_e \gtrsim n.\] (42)
Next we consider the MNS matrix. The MNS matrix $U_{MNS} = U_e^T U_\nu$ can be evaluated by using (40). Then Eq. (30) implies

$$\lambda^a \simeq \lambda^b + \lambda^c \simeq 1, \quad \lambda^b + \lambda^c \lesssim \lambda^m, \quad \lambda^n \lesssim \lambda^m. \tag{43}$$

Eqs. (42) and (43) restrict the allowed values of $n$ and $m$ in a narrow range

$$1.3 \lesssim m \lesssim n \lesssim 1.5. \tag{44}$$

Thus, as a representative value, we expect

$$U_{e3} \simeq \lambda^m \simeq 0.10 - 0.14, \tag{45}$$

provided that the 5D Yukawa couplings are almost universal and no accidental cancellation takes place in the determination of the mass structure. This value is close to the present upper bound and should be explored by near future experiments. This observation may be one of the most important consequences of the present paper. For $m \simeq 1.5$, it is interesting to rewrite the above as follows:

$$U_{e3} \simeq \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}. \tag{46}$$

We should note that the numerical coefficient in front cannot be determined in our framework. The actual value would depend on the range of the 5D Yukawa couplings. We should also note that the interesting relation (46) may be polluted by possible cancellation among various contributions because of the less hierarchical structure of the neutrino masses and mixings.

The charged lepton bulk mass can be obtained with the similar method as the quark case. The relations,

$$F_L(c_{L1})F_R(c_{E1}) \simeq \lambda^{8.5}, \quad F_L(c_{L2})F_R(c_{E2}) \simeq \lambda^5, \quad F_L(c_{L3})F_R(c_{E3}) \simeq \lambda^3, \tag{47}$$

hold approximately if the right-handed lepton mixing is chosen to be small enough. However, there is a wider range of solution space in above equation than that of quarks. The lepton flavor violation limit from $\mu \to 3e$ and other experimental data restrict that $c_{L3}$ cannot be smaller than $0.5$ [25]. The lower bound on the bulk lepton masses are,

$$c_{L1} \simeq 0.59, \quad c_{L2} \simeq 0.5, \quad c_{L3} \simeq 0.5,$$

$$c_{E1} \simeq -0.74, \quad c_{E2} \simeq -0.65, \quad c_{E3} \simeq -0.55. \tag{48}$$
On the other hand, for $c_{E3} \approx 0$, one finds

$$c_{L1} \approx 0.68, \quad c_{L2} \approx 0.61, \quad c_{L3} \approx 0.61,$$

$$c_{E1} \approx -0.65, \quad c_{E2} \approx -0.55, \quad c_{E3} \approx 0.$$  \hspace{1cm} (49)

There is no strict upper bound on the bulk masses, but if $c_{E3}$ is much larger than 0.5, the first KK neutrino in $SU(2)_R$ doublet $E$, which has $(+\, -)$ BC, becomes too light. For instance, $c_{E3} \approx 0.7$ leads $m^{(1)}_N \sim 1$ GeV and if $c_{E3}$ approaches to the value 1, the mass become lower than MeV and can be considered as a sterile neutrino. This type of neutrino KK mode may conflict with experimental or cosmological data \[19, 30\].

Even though we fix $\nu_2$ and $\nu_3$ masses in the NH case, there is no data which determines the lightest neutrino mass. In other words, two right-handed neutrinos are just enough to explain all the existing experimental data. Using the similar method, we can determine the two bulk neutrino masses with the relations,

$$F_L(c_{L2})F_R(c_{N2}) \approx \lambda^{17.5}, \quad F_L(c_{L3})F_R(c_{N3}) \approx \lambda^{16}.$$  \hspace{1cm} (50)

In the valid range where $c_{L3} > 0.5$, the lower bound becomes,

$$c_{N2} \approx -1.2, \quad c_{N3} \approx -1.1.$$  \hspace{1cm} (51)

This value does not vary much in the range $0.5 < c_{L3} < 0.6$. Note that $c_{E3}$ is the most sensitive parameter and might be the easiest one to test at the near future high energy experiments.

Finally, we examine the case where the Majorana mass $m_M = \lambda_L \delta (z - 1/k)$ is present at the UV brane. It is known that the neutrinos can acquire a small Majorana mass via bulk seesaw mechanism even for a small $\lambda_L > 10^{-11}$ \[29\],

$$(M_\nu)_{ij} \approx v_W^2 \sum_{kl} h^\nu_{ik}h^\nu_{jl}F_L(c_{Li})F_R(c_{Nk})M^{-1}_{Rkl}F_L(c_{Lj})F_R(c_{Nl}).$$  \hspace{1cm} (52)

The Majorana mass matrix can be written as

$$M_{Rij} \approx \frac{\lambda_{Lij}}{2} k F_R(c_{Ni})F_R(c_{Nj}) \epsilon^{-c_{Ni}-c_{Nj}+1}.$$  \hspace{1cm} (53)

The assumption of the coupling universality for both boundaries, $\lambda_{Lij} \sim \lambda_L$ leads,

$$(M_\nu)_{ij} \approx \frac{F_L(c_{Li})F_L(c_{Lj})v_W^2}{\lambda_L T} \epsilon^{2c_{N1}},$$  \hspace{1cm} (54)
where $c_{N1} = \min \{ c_{N_i} \}$. The light neutrino mass is proportional to the charged lepton mass square,

$$M_\nu = \eta^2 \epsilon^{2c_{N1}} C_L^{-2} M_e^2,$$

where $C_L \simeq F_R(c_{E3})$ and $\eta^2 \equiv v_W / (\lambda_L T)$. The lightest bulk neutrino mass becomes

$$c_{N1} \simeq \frac{10 + \ln(\eta C_L^{-1})}{\ln(T/k)}.$$

If $\eta C_L^{-1} \sim 1$, the bulk neutrino mass is $c_{N1} \simeq -0.28$, which is quite different from the Dirac neutrino case. While the charged lepton bulk mass is the same, the Majorana neutrino case contains much larger bulk masses. However, to achieve the MNS matrix, $2n \lesssim 1.5$ is required in Majorana neutrino case even for the maximally hierarchical case ($m_1^\nu = 0$). The condition yields $U_{e3} \sim 0.3$ which is over the experimental bound 0.16. Even with the maximal ambiguity in the approximation, the value is marginally allowed. It is difficult to match the current experimental data with the Majorana neutrino in the almost universal Yukawa coupling model.

IV. CONCLUSIONS

In the warped bulk SM, the fermion mixing and mass hierarchy can be induced from the suppressed zero mode of KK field at the physical boundary. In the case where the Yukawa couplings are almost universal to all bulk fermions, we can determine the allowed regions of the bulk fermion masses through the data of mixings and masses of the SM particles with a simple analytic method.

If the Yukawa couplings of all SM fermions are universal and if there is no large cancellation in the multiplications between the different fermion mixing matrices, the current experimental data almost determine the bulk quark masses. For the bulk lepton masses, it yield a wide range of solutions. The existing data cannot narrow down the solution range much. Still, a few interesting predictions have been found in the lepton sector. One of them is that only the normal hierarchy is valid neutrino mass hierarchy in this model. Another is the fact that it is favorable to consider the light neutrinos are Dirac fermions. It is because that the seesaw mechanism in this model generates too large $U_{e3}$.

One of the most notable predictions is on the MNS matrix component $U_{e3}$ which is predicted to be $\sim 0.1$. This value is not so far from the current experimental upper bound and can be tested
by neutrino oscillation experiments in near future. The bulk quarks and lepton masses may be explored at future high energy colliders. The third generation of charged fermions has significantly different features from others and thus can be a probe for the bulk SM.

Acknowledgments

S.C. was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) No. KRF-2005-070-C00030. C.S.K. was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) No. R02-2003-000-10050-0. M.Y. was supported by the Scientific Grants from the Ministry of Education, Science, Sports, and Culture of Japan, No. 16081202 and 17340062.

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