The effect of social inequality on the growth of COVID-19 death case

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Abstract. World Health Organization (WHO) declared COVID-19 has become a pandemic on March 11 2020, and counted as a dangerous disease including in Indonesia since it is causing immune drop after an infection that could emerge other diseases. Therefore, many people, both poor or wealthy, are worried. We formulate a modified Susceptible Infected Recovered compartmental model (SIR), where the Infected compartment could be dead because of the disease. Assuming that the poor have less access to excellent health facilities, this population is more likely to have more diseases that can recur after being infected with COVID-19. From pandemic history, life expectancy and death rates are disproportionate between the wealthiest and most deprived populations. It is based on previous pandemic cases that the life expectancy and death rates between the wealthy and poor communities are excessive. We show the effect of social inequality on the growth of the COVID-19 death case using the SIR model for the COVID-19 outbreak considering that the reinfection of COVID-19 could happen in some cases knowing that immune could be waning between people that got recovered from the virus. By numerical calculations and illustrating it in the graph, the results show that cases with high social inequality tend to have higher death rates and cases with low social inequality levels tend to have lower death rates. Thus, social inequality could affect the death rate caused by COVID-19 cases.

1. Introduction
At the end of 2019, it was reported that there was a disease spreading rapidly in Wuhan, China. It is suspected that the disease originated from an animal market which is the characteristic of Wuhan. After being analyzed, the virus was a novel coronavirus (2019-nCoV). It was then officially named Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) or also known as COVID-19 by the International Committee on Taxonomy of Viruses. On March 11, 2020, World Health Organization officially declared that COVID-19 was a pandemic and is then counted as a dangerous disease worldwide, including in Indonesia since it is causing immune drop after an infection that could emerge other diseases [1].

Most if not all previous coronaviruses that also infect humans seem to originate from bat CoV that transmitted to humans directly or indirectly through an intermediate host. In the past 15 years, we’ve seen several zoonotic events such as SARS (severe acute respiratory syndrome)-CoV which originated from civets cats, MERS (Middle East respiratory syndrome)-CoV which is presumed to be from dromedary camels, and many others [2]. Recently at the end of 2019, Wuhan experienced an outbreak of a novel coronavirus, more than eighteen hundred fell victim and died to its infection while seventy thousand other individuals got infected within the first fifty days of the epidemic. The International
Committee on Taxonomy of Viruses (ICTV) named the virus as SARS-CoV-2 and the disease as COVID-19 [3].

After the announcement of COVID-19 as a pandemic by the WHO, we can’t deny the gravity of its impact on our society. Although not every people are affected in the same way. The social inequality in a certain nation will determine the effect seen on its inhabitant on different level of social status [4]. This is an important phenomenon to be discussed because of the different approaches we should take on different social status.

COVID-19 isn’t the first pandemic human being has faced, around the 8th century, exactly from 1347 until 1351, Black Death took over 40 to 50 percent of the whole European inhabitant only within four years. This pandemic can also be our prime example since there exists a lot of writings that thoroughly describes the luxurious environment the “riches” enjoyed during the chaos that befalls the “poor” who ended up being the ones that filled the pandemic’s list of victims [5]. One of the researches that examines the effect of social inequality on the death caused by COVID-19 is the research done by Vida Abedi, et al. with the title Racial, Economic, and Health Inequality and COVID-19 Infection in the United States. Abedi concluded that nations with the highest poverty rate have the highest death rate. Abedi also explained that nations with the most diversified income rate have a higher risk of getting infected by COVID-19 [6].

Other than that, a research done by Calderón et al. with the title COVID-19: Risk Accumulation Among Biologically and Socially Vulnerable Populations also quoted that the risk of getting infected is higher in the elderly with low income [7].

Goutte et al. in their research titled The Role of Economic Structural Factors in Determining Pandemic Mortality Rates: Evidence from the COVID-19 outbreak in France concluded that area with diversity based on a lot of economical factor (income, jobless subsidy, poverty rate, minimum wage and productive age’s education) have bigger risk on getting infected by COVID-19 [8].

Aldila by his research about the impact of the media campaign and rapid testing for COVID-19 as an optimal control problem in East Java, Indonesia also stated that for now there is no vaccine or medicine to cure COVID-19. Policymakers are forced to use an intervention that could be used such as physical distancing, facemask required in all activity, self-quarantined, periodic rapid testing, hospitalizing etc [9]. Because of that too people who are infected required a lot of money to get a high-quality treatment or intervention. All of these early findings led us to research the effect of social inequality on the death rate caused by COVID-19. We formulated a Modified SIR model (where the infected compartment can become a deceased because of the infection) which will then be simulated through computation.

2. Method

Assumption and Dynamic Model on the effect of social disparity on the growth of total death caused by COVID-19.

The dynamic model on the effect of social disparity on the growth of total death caused by COVID-19 will have several assumptions, including:

1. Susceptible opportunities exist in newborn;
2. Natural death exists in uninfected individual;
3. When a person got infected by COVID-19, they can be either cured or died due to the disease;
4. Natural death exists in the cured individual;
5. An area impacted by COVID-19 with low social disparity will easily recover from the situation and have a low number of death cases;
6. An area impacted by COVID-19 with high social disparity will be having difficulties to recover from the situation and have a high number of death cases;
7. The time duration is within one month;
8. Gini ratio represents the social disparity and becomes the parameter on how many cured patient cases timed by the cases of death patient, within this assumption the higher the Gini Ratio means the higher the social disparity in the area is and also the higher total death cases.
As stated before, we’ll formulate a modified SIR model into SEIRF model consisting of five subs population which are \(S(t), E(t), I(t), R(t), F(t)\) to describe the changes in the population within time \(t\) and we’ll assume that social inequality gives an effect on the growth of COVID-19 death case, which is the bigger the social inequality happens in a country, the bigger the growth of COVID-19 death case. Each of which can be defined as \(S(t)\) represents Susceptible Cases, \(E(t)\) represents Exposed Cases, \(I(t)\) represents Infectious Cases, \(R(t)\) represents Recovered Cases, and \(F(t)\) represents Dead Cases.

This model is also made of 6 parameters that are as defined in Table 1.

| Parameter | Description |
|-----------|-------------|
| \(\beta\) | Transmission rate |
| \(\alpha\) | Social inequality rate (based on Gini ratio) |
| \(\mu_1\) | Removed rate |
| \(\sigma\) | Incubation rate |
| \(p\) | Death probability |
| \(1 − p\) | Cured probability |
| \(L\) | Natural Birth Rate |
| \(\mu\) | Natural Death Rate |

The infected population can either be recovered or dead with a certain probability [10], thus we multiply \((1 − p)\), which is the cured probability, with the Infected compartment that is going to the Recovered compartment. Otherwise, we multiply \(p\), which is the death probability, with the Infected compartment that is going to the Death compartment caused by COVID-19.

To show the effect of social inequality on the death case caused by COVID-19, we use the Gini ratio in the system. We assume that the higher the social inequality happens in a nation is, the higher the death case would be, thus we multiply \(\alpha\), which is the social inequality rate, with \(\mu_1\) for the Infected compartment that are going to the death compartment caused by COVID-19. On the other hand, we also assume that the higher the social inequality happens in a nation is, the lower the recovered case would be, thus we multiply \(\alpha^{-1}\), which is the inverse of social inequality rate, with \(\mu_1\) for the Infected compartment that are going to the recovered compartment. The completed system can be written in a differential equation and illustrated within a flowchart as we can see in Figure 1.

![Figure 1. Transmission diagram of COVID-19 spreads with the effect of social inequality rate and concludes the Death Cases as a new compartment.](image)

Based on the transmission diagram above, we could define the model as follow:
\[
\begin{align*}
\frac{dS(t)}{dt} &= L - \beta S(t)I(t) - \mu S(t) \\
\frac{dE(t)}{dt} &= \beta S(t)I(t) - \sigma E(t) - \mu E(t) \\
\frac{dI(t)}{dt} &= \sigma E(t) - \frac{\mu_1}{\alpha}(1-p)I(t) - \mu \alpha p I(t) - \mu I(t) \\
\frac{dR(t)}{dt} &= \frac{\mu_1}{\alpha}(1-p)I(t) - \mu R(t) \\
\frac{dF(t)}{dt} &= \mu_1 \alpha p I(t)
\end{align*}
\]

3. Result and Discussion

3.1. Non-endemic equilibrium point

To make the equation simple, then variable \(F\) doesn’t need to be included in the equations (1) to (4) this system can be done by setting \(\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0\), thus, we obtain the non-endemic equilibrium point of the model as follow:

\[
E_0 = (S^*, E^*, I^*, R^*) = \left( \frac{L}{\mu}, 0, 0, 0 \right).
\]

3.2. Basic Reproduction Number \((R_0)\)

Refer to [11], we find the eigenvalue and then pick the highest eigen \((\mu + \sigma)(\mu_1 \alpha^2 p + \alpha \mu - \mu_1 p + \mu_1)\) which is mean we obtain the non-endemic equilibrium point.

After obtaining it, we get the basic reproduction number by using the Next Generation Matrix approaches, as follow:

\[
R_0 = \frac{\alpha \sigma L \beta}{(\alpha \mu + \mu_1 (\alpha^2 p - p + 1))(\mu + \sigma)\mu}
\]

3.3. Endemic Equilibrium Existence

The endemic equilibrium exists if \(R_0 > 1\). With the help of Maple 18 application, we can simply obtain:

\[
\begin{align*}
S^* &= \frac{(\alpha \mu + \mu_1 (\alpha^2 p - p + 1))(\mu + \sigma)}{\alpha \beta \sigma}, \\
E^* &= \frac{\mu (\alpha \mu + \mu_1 (\alpha^2 p - p + 1))(R_0 - 1)}{\sigma \alpha \beta}, \\
I^* &= \frac{\mu (R_0 - 1)}{\beta}, \\
R^* &= -\frac{(\mu_1 \alpha p) \mu_1 (R_0 - 1)}{\alpha \beta}.
\end{align*}
\]

As previously explained, we have simplified the model by not including the F compartment in the equation as we obtain the equilibrium point. That is why, we also didn’t obtain the endemic equilibrium point for \(F\). But, on the other hand, because \(I^*\) is a unique positive solution of the equation, we could obtain the endemic equilibrium point for the \(F\) compartment \((F^*)\) by substituting the \(I^*\) value to equation (5) and we obtain:

\[
F^* = \frac{\mu_1 \alpha p \mu (R_0 - 1)}{\beta}.
\]

3.4. Non-endemic stability

First, we form the Jacobian matrix within a non-endemic point \((E_0)\) as follow
Then, based on the matrix above, the characteristic equation (with the other eigenvalues squared) is:

$$-\frac{1}{a\mu} (\lambda + \mu)^2 \lambda P_1(\lambda) = 0,$$

where:

$$a = -\alpha \mu,$$
$$b = -\mu_1 p(a^2 - 1) - 2\alpha \mu^2 - \alpha \mu \sigma - \mu_1 \mu,$$
$$c = (R_0 - 1)\mu(\mu + \sigma)((p(a^2 - 1) + 1)\mu_1 + \alpha \mu).$$

Since the $a$ and $b$ coefficients above are negative, the system will be stable if the $c$ coefficient is also negative as follow:

$$c = (R_0 - 1)\mu(\mu + \sigma)((p(a^2 - 1) + 1)\mu_1 + \alpha \mu) < 0.$$

Thus, we obtain that $R_0 < 1$. It shows that the non-endemic equilibrium of the system is locally asymptotically stable.

### 3.5 Endemic Stability

By using the same method that we use to analyze the non-endemic equilibrium point, which is the method to determine local stability by finding the determinant of the Jacobian matrix, we obtained the characteristic equation

$$-\frac{1}{a} (\lambda + \mu)^2 P_2(\lambda) = 0,$$

where $P_2(\lambda) = k_{11} \lambda^3 + k_{22} \lambda^2 + k_{33} \lambda + k_{44}$, with coefficients as follow:

$$k_{11} = (a\mu + \mu_1 (\alpha^2 p - p + 1)) \alpha (\mu + \sigma),$$
$$k_{22} = (a\mu + \mu_1 (\alpha^2 p - p + 1))(\mu_1 (\alpha^2 p - p + 1) + ((R_0 + 2)\mu + \sigma)\alpha)(\mu + \sigma),$$
$$k_{33} = R_0 \mu(a\mu + \mu_1 (\alpha^2 p - p + 1))(\mu + \sigma)(\mu_1 (\alpha^2 p - p + 1) + \alpha(\sigma + 2\mu),$$
$$k_{44} = \mu(a\mu + \mu_1 (\alpha^2 p - p + 1))^2(\mu + \sigma)^2(R_0 - 1).$$

Using the Routh Hurwitz stability criteria that say to determine the system is stable or not, all roots must contain a negative real part which means all coefficient must be positive [12]. Since $k_{11}, k_{22},$ and $k_{33}$ are positive, the system will be stable if $k_{44}$ is positive. So that the $k_{44}$ is positive, $R_0 - 1$ must be above 0 and we can obtain that $R_0 > 1$. It shows that the endemic equilibrium of the system is locally asymptotically stable.

### 3.6 Numerical Simulation

For the numerical calculation, we will need data from actual cases, thus, the cases in Indonesia as written in Table 2 shall be our example, here are the data up until 6th September 2020 from the Indonesian government that we use as the initial values in the numerical calculation and to determine the probability parameters.
Table 2. Total positive cases, total recovered cases, total death cases and total population in Indonesia

| Country | Positive Case | Recovered Case | Death Case | Total Population |
|---------|---------------|----------------|------------|------------------|
| Indonesia | 190,665       | 136,401        | 7,940      | 274,045,297      |

Source: https://www.worldometers.info/coronavirus

After that, we are going to need the estimated number from each parameter on the model just as written in Table 3. As we stated before, we use some values from Table 2, the total death case value, and the total population value, to get the death probability caused by COVID-19 or the parameter \( p \) as written in Table 3.

Table 3. The parameter values

| Parameter | Value | Source |
|-----------|-------|--------|
| \( \beta \) | \( 1.1 \cdot \frac{1}{7} \approx 0.16 \) | [13] |
| \( \alpha \) | 0.381 | https://www.bps.go.id/website/materi_ind/materiBrslnd-20200715120937.pdf |
| \( \mu_1 \) | \( \frac{1}{7} \) | [13] |
| \( \sigma \) | \( \frac{1}{5} \) | [13] |
| \( p \) | \( \frac{7,940}{274,045,297} \approx \frac{1}{34514} \) | https://www.worldometers.info/coronavirus/ |
| \( L \) | \( \frac{1}{3} \) | Assumed |
| \( \mu \) | \( \frac{1}{6} \) | Assumed |

Then, using the parameter values above, a numerical simulation is done with the graphical results as we can see in Figure 2 and Figure 3.

Figure 2. The dynamic of the case.

Figure 3. Comparison of the three cases (red curve = case with \( \alpha = 0.381 \), blue curve = case with \( \alpha = 0.24 \), green curve = case with \( \alpha = 0.62 \))

Based on Figure 2, we could assume that the model we have in this paper can represent the dynamic of COVID-19 in Indonesia. Thus, we can use this model to determine whether the Gini ratio, which represents the social inequality value could affect the death number caused by COVID-19 or not. As for Figure 3, the values listed are in per million, and referring to the blue curve in Figure 3, we could see that the case with the lowest Gini ratio (\( \alpha = 0.24 \)) has the lowest total death caused by COVID-19. On the other side of the spectrum, referring to the red curve in Figure 3, we could see that the case with the
highest Gini ratio ($\alpha = 0.62$) has the highest total death caused by COVID-19. Thus, we could conclude that the lower the social inequality on a nation, the lower the total death number by COVID-19 on the nation is, vice versa.

4. Conclusion
From the data on the graphic and the table, we can see that social inequality has a huge effect on the total death rate caused by COVID-19. There is a difference in the effect of the pandemic on inhabitants with different social status.

Based on the graphic that was calculated by the Maple, we can see that the red Gini ratio increase the largest within times, meanwhile the green and blue increase similarly with green having a slightly larger increase within the same period. From this graphic, we can conclude that the higher the social inequality on a certain nation is, the higher the total of death case caused by COVID-19, therefore we hope that the social inequality on the nation can be decreased because it is proven that nation with lower social inequality have lower total of death case caused by COVID-19.

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