de Sitter space from M-theory?

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Abstract

In this note we study a massive IIA supergravity theory obtained in hep-th/9707139 by compactification of M-theory. We point out that de Sitter space in arbitrary dimensions arises naturally as the vacuum of this theory. This explicitly shows how de Sitter space can be embedded into eleven-dimensional supergravity. In addition we discuss the novel way in which this theory avoids various ‘no-go theorems’ which assert that de Sitter space is not a consistent vacua of eleven-dimensional supergravity theory. We also point out that the eight-branes of this theory, which couple electrically to the ten-form, will typically sweep out de Sitter world-volumes.

1 Introduction

de Sitter space has recently become the focus of much attention (see, for example, [1, 2, 3, 4, 5]). From the observational point of view there is some evidence that there is a non-vanishing cosmological constant. In addition any inflationary scenario requires de Sitter space or some close cousin. From the theoretical point of view de Sitter space has become somewhat of an anomaly because, of all the things you can get out of string theory, de Sitter space doesn’t seem to be one of them.

Although little is known about M-theory it has provided us with a remarkable understanding of many phenomenon in string theory. In particular M-theory allows us to explore regions beyond the reach of string theory. In this paper we wish to discuss a slight extension of eleven-dimensional supergravity (that still preserves eleven-dimensional supersymmetry). Furthermore it is natural to assume that this extension reflects an underlying, modified (or massive) M-theory which we call MM-theory. The main focus here will be to show that de Sitter space is the natural ground state of

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MM-theory, contrary to the case in standard M-theory and string theory. We also discuss how various no-go theorems are avoided.

de Sitter space has been obtained from supergravity by other methods [6, 7], which involve reducing on noncompact internal spaces or supergravities with negative norm fields. Here we wish to discuss another mechanism that is intriguingly related to standard M-theory. Indeed it suggests that the standard M-theory moduli space should be extended. However, even if the discussion presented here turns out to have no relation to M-theory or string theory, it nevertheless provides a natural embedding of de Sitter space into eleven-dimensional supergravity.

2 MM-theory

In [9] it was observed that the equations of motion of eleven-dimensional supergravity [8] admit a slight modification. Rather than using the standard spin connection $D$ it is possible to include a conformal spin connection $\hat{D} \sim D + 2k$, provided that the conformal part of the curvature vanishes, i.e. $dk = 0$. In simply connected spacetimes this implies that $k$ is exact and the modification is simply a field redefinition. In particular, if $k = d\theta$ then the redefinition that takes the equations of motion defined with the connection $\hat{D}$ back to the usual ones is

$$
e^M_N \rightarrow e^{-2\theta} e^M_N, \quad \psi_M \rightarrow e^{-2\theta} \psi_M,$$

(2.1)

where $M, N = 0, ..., 10$. However if the spacetime is non-simply connected then this modification is non-trivial [10].

At present the underlying principles behind M-theory are unclear. Most of what we understand is based on the principle that M-theory has eleven-dimensional supersymmetry and by compactification on a circle it can be related to type IIA string theory. From this point of view there is no reason not to consider the most general set of equations with this property. To lowest order in a derivative expansion these equation are just those of [9]. Thus we introduce the notion of modified (or massive) M-theory or MM-theory, which includes the conformal spin connection.

The simplest example of a non-simply connected manifold, on which we may compactify MM-theory, is $M_9 \times S^1$. One can then choose $k = mdy$, where $dy$ is the tangent vector to the circle. The resulting ten-dimensional supergravity was constructed in [10] by the usual ansatz that none of the fields depend on the coordinate $y$. If we turn off the four-form field strength and Fermions then the equations of motion of the compactified theory are, in ten dimensions,

$$R_{ab} - \frac{1}{2} g_{ab} R = -2(D_a D_b \phi - g_{ab} D^2 \phi + g_{ab} (D\phi)^2)$$

$$+ \frac{1}{2} (F^{ac} F^b_c - \frac{1}{4} g_{ab} F^2) e^{2\phi} - 18m (D (A_b A_c) - g_{ab} D^2 A_c)$$

$$- 36m^2 (A_a A_b + 4 g_{ab} A^2) - 12mA (a \partial_b \phi)$$

$$- 30mg_{ab} A^c \partial_c \phi - 144m^2 g_{ab} e^{-2\phi}$$

(2.2)

$$D^b F_{ab} = 18mA_b F_a^b + 72m^2 e^{-2\phi} A_a - 24me^{-2\phi} \partial_a \phi$$

(2.3)

$$6D^2 \phi - 8(D\phi)^2 = -R + \frac{3}{4} e^{2\phi} F^2 + 360m^2 e^{-2\phi} + 288m^2 A^2 + 96mA^b \partial_b \phi - 36mD^b A_b$$

(2.4)
where $F_{ab}$ is the R-R sector vector and $F_{ab} = \partial_a A_b - \partial_b A_a$ as usual. Note that we have also changed the convention for the Ricci curvature from that used in [10] to agree with the standard literature and have corrected some miss-prints.

The equations of motion 2.2, 2.3 and 2.4 are certainly rather odd and cannot be obtained from an action [10]. We also note here that these equations of motion are just the eleven-dimensional equations but written in a manner that only ten-dimensional Poincare symmetry is manifest (and with no dependence on $y$), i.e. any solution of these equations is a solution of the MM-theory equations of motion. Only in the case $m = 0$ does one recover the standard massless type IIA supergravity and the relation of M-theory to perturbative string theory. Finally we note naively the vector field $A_a$ has a tachyonic mass. However the complicated form of the equations, and the lack of an action formulation (and probably any suitable notion of energy) suggests that this is not as problematic as it may at first seem. This is further supported by the fact locally these equations of motion are the same as those of ordinary M-theory. Nevertheless it is compelling to assume that MM-theory (i.e. $m \neq 0$) is a sector of M-theory that should be seriously considered.

The same equations of motion can also be obtained through a sort of ‘Scherk-Schwarz’ dimensional reduction of eleven dimensional supergravity over a noncompact dimension [11]. These authors also include the four-form and hence give the complete equations of motion. One can see that the vector $A_a$ has become massive by eating the scalar $\phi$. In addition the four-form becomes massive by eating the three-form. While the construction presented in [11] is certainly very interesting, we would like to emphasize that it is distinct from the derivation of this massive IIA supergravity presented in [10]. In particular, in [11] there is no Weyl connection, and the theory is obtained by reducing ordinary M-theory on a noncompact direction. More precisely, one may foliate eleven-dimensional Minkowski space with ten-dimensional de Sitter hyperboloids, and in the ‘generalized’ dimensional reduction of [11] one reduces along the direction orthogonal to the hyperboloids. Presumably, any attempt to compactify the direction transverse to the de Sitter hyperboloids will lead to some metric discontinuity. On the other hand, in [10] the theory is obtained by first introducing a non-trivial Weyl connection, as described above, and reducing MM-theory along a smooth circle. It would be interesting to better understand how these two constructions are related.

3 de Sitter space

If we turn off all the gauge potentials, it is straightforward to show that the only remaining equation is the Einstein equation:

$$R_{ab} = 36m^2 e^{-2\phi} g_{ab}$$

(3.1)

together with the “Maxwell” and “scalar” equations 2.3, 2.4, which simply imply that the dilaton $\phi$ is a constant. Thus, if we turn off all of the fields in this theory except gravity, we recover ten-dimensional de Sitter space. The effective cosmological constant is then given explicitly in terms of the mass and scalar vev as

$$\Lambda = 576m^2 e^{-2\phi}$$

(3.2)

It is of interest to further compactify this theory to four-dimensions. A first attempt might be to employ a sort of Freund-Rubin compactification that is familiar in the AdS cases. However an examination of the equations of motion found in [11] soon shows that there are no four-dimensional compactifications that are Poincare invariant unless the four-form and three-form vanish. This is effectively because the four-form has eaten the three-form to become massive. Therefore it is no longer possible to set the three form to zero without also setting the four-form to zero.

On the other hand, since we already have a positive cosmological constant in ten dimensions, it is not necessary to include additional fields to induce one. Indeed, if we consider the simplest vacuum
ten-dimensional equation 3.1, corresponding to a constant $\phi$ with all other fields vanishing, then we may solve it by compactifying on any $(10 - D)$-dimensional manifold $\mathcal{M}$ with the ansatz

$$ g_{ab} = \begin{pmatrix} g_{\mu \nu} & 0 \\ 0 & g_{ij} \end{pmatrix}, $$

where $\mu, \nu = 0, ..., D - 1$ and $i, j = 1, ..., 10 - D$. The equations of motion now split into two independent conditions: $R^{(D)}_{ij} = 36m^2 e^{-2\phi} g_{ij}$ and $R^{(10-D)}_{\mu \nu} = 36m^2 e^{-2\phi} g_{\mu \nu}$, i.e. the internal space has constant scalar curvature $R^{(10-D)}_{ij} = 36(10 - D)m^2 e^{-2\phi}$ and the spacetime has constant curvature $R^{(D)} = 36Dm^2 e^{-2\phi}$. Thus in particular the direct product of $D$-dimensional de Sitter space with a $(10 - D)$-dimensional sphere is a solution to the equations of motion.

We may lift these solutions to eleven dimensions following the compactification ansatz used in [4]. In this case the four-form vanishes and the metric is

$$ ds^2_{11} = e^{-2\phi/3} ds^2_{dS} + e^{-2\phi/3} ds^2_{(10-D)} + e^{4\phi/3} dy^2, $$

where $\phi$ is constant, $ds^2_{dS}$ is the $D$-dimensional de Sitter metric, $ds^2_{(10-D)}$ the $(10 - D)$-dimensional sphere metric and $y$ is a compact coordinate around $S^1$. In other words $dS^D \times S^{(10-D)} \times S^1$ is a solution to $\mathcal{M}$-theory for any $D$. In the limit that $m \to 0$ we simply recover flat eleven-dimensional Minkowski space with a single compact direction. On the other hand we could keep $m \neq 0$ and decompactify the $S^1$. This does not lead to a solution of ordinary eleven-dimensional supergravity. However by performing the field redefinition 2.1 (i.e. rescaling the metric by $e^{-4m y}$) we obtain flat eleven-dimensional Minkowski space constructed as a foliation by ten-dimensional de Sitter spaces [4], which certainly is a solution of $\mathcal{M}$-theory. Note that now we may no longer compactify the $y$ dimension without introducing singularities. Thus $\mathcal{M}$-theory and $\mathcal{M}$-theory are physically distinct.

4 No no-go theorems

It has been strongly argued that de Sitter space cannot be embedded into eleven-dimensional supergravity [1, 2, 3, 4]. In the case presented here the no-go theorem of [3] does not apply because it assumes an action formulation in uncompactified eleven dimensions. The theorem of [3] also does not apply since it assumes that the starting point is the standard ($k = 0$) eleven-dimensional supergravity [8]. There have also been doubts raised as to whether or not de Sitter space could ever arise from string theory or M-theory [3]. It has long been known that there is no de Sitter superalgebra (at least not in [8]). There have also been doubts raised as to whether or not de Sitter space could ever arise from string theory or M-theory [5]. It has long been known that there is no de Sitter superalgebra (at least not in [8]).

Even though the equations of motion 2.2, 2.3 and 2.4 are, by construction, supersymmetric, the fact that there is no action means that Noether’s theorem can not be directly applied. Thus it is perfectly consistent that there is no conserved supercharge, even though the equations of motion are invariant under a continuous symmetry group.

This kind of obstruction is not uncommon. For example something rather similar occurs in $N = 2$ gauge theories, such as the Seiberg-Witten effective action. In this case the equations of motion are invariant under $SL(2, \mathbb{R})$ modular transformations that rotate $a$ and $a_D$, $F_{\mu \nu}$ and $\star F_{\mu \nu}$, i.e S-duality. However one does not expect that there is a “modular” charge because the action (which exists in this case) is not invariant under the modular group.

We expect something similar to occur here. Indeed in the construction of the theory the covariance of the eleven-dimensional equations of motion under Weyl transformations was used. However the
eleven-dimensional action is certainly not Weyl invariant and there are no “Weyl” charges. For $m \neq 0$ the supersymmetries become mixed with the Weyl transformations. Therefore one does not expect conserved supercharges to arise since there is no conserved “Weyl” charge.

To see this in more detail we consider the usual supergravity supercharge obtained as an integral over a spacelike hypersurface of the supercurrent (for a recent discussion see [13])

$$\bar{\epsilon}Q = \int d\Sigma M S^M = -i \int d\Sigma M \epsilon \Gamma^{MNP} \partial_N \psi_P \ldots , \quad (4.1)$$

where $M, N = 0, \ldots , 10$ and the ellipsis denotes less important terms. For our purposes it is sufficient to consider the linearised theory where $\epsilon$ and $\Gamma^M$ are constant and hence $\partial_M S^M = 0$ is trivially satisfied.

Locally the equations of motion of the massive theory of section two differ only by the field redefinition 2.1 from those of M-theory. In this way we find that the linearised supercurrent in M-theory is

$$\hat{S}_M^\epsilon = e^{\alpha y} \left( i \epsilon \Gamma^{MNP} \partial_N \psi_P + 2m \epsilon \Gamma^M y^P \psi_P \right) \quad (4.2)$$

and it is easy to see that conservation implies that $\alpha = 2$. However, even though $\partial_M \hat{S}_M^\epsilon = 0$, the supercharge $\bar{\epsilon}Q$ is not conserved since

$$\int \partial_y \hat{S}_M^\epsilon dy \neq 0 , \quad (4.3)$$

due to the fact that if $y$ is compact, $\hat{S}_M^\epsilon$ is multivalued. Thus M-theory compactified on $S^1$ with a topologically non-trivial conformal connection has no globally conserved supercharge.

For example one can look for Killing spinors in the $dS_{10} \times S^1$ solution of section three, i.e. spinors satisfying $\hat{\epsilon}D_M \psi = 0$. Indeed locally such spinors exist and, in a coordinate system where the metric is $ds_{11}^2 = -dt^2 + e^{4mt}(dx_1^2 + \ldots + dx_9^2) + dy^2$, satisfy $\partial_y \epsilon = -2me$ and $\Gamma^y_0 \epsilon = -\epsilon$. However there are no Killing spinors that are globally defined around the $S^1$ and hence no supercharge.

5 de Sitter space on the world-volume of eight-branes

A D8-brane in IIA string theory couples to the ten-form field strength $F_{10}$ of the R-R sector. As noted by Polchinski [14], this ten-form is not dynamical - it is just a constant field which generates a uniform energy density, or ‘cosmological constant’. This cosmological term is proportional to the square of the mass term of the massive IIA supergravity theory derived by Romans [15]. In the Romans theory, the mass term arises from a Higg’s mechanism in which the two-form “eats” the vector. In the massive theory of [10] the two-form is eaten by the the three-form and the scalar is eaten by the vector.

As described in [17], we may dualise the conformal connection $k$ to a ten-form $F_{10}$ using the eleven-dimensional Hodge star operator. This is analogous to the ten-form formulation of Romans theory in [16]. When we do this we obtain a ten-form $F_{10}$ which is covariantly constant:

$$\star_{11} F_{10} = k . \quad (5.1)$$

We can then use this ten-form to write down a (truncated) action for the theory, in the sector where the dilaton is constant:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( e^{-2\phi} R - \frac{1}{2} e^{-4\phi} F_{10}^2 \right) . \quad (5.2)$$

Note first of all the factor of $e^{-4\phi}$ which appears in front of the 10-form. Thus the ten-form of this theory is not a R-R sector field and consequently the eight-branes are not D-branes. Furthermore, if
we match dimensions in (1.6) we find that the tension \(T\) of a brane which couples electrically to \(F_{10}\) must scale as
\[
T \sim e^\phi
\] (5.3)
Thus if we assume that \(\phi\) is very large, i.e the M-theory limit of type IIA string theory, then it makes sense to think of an eight-brane as a ‘domain wall’, separating two phases (i.e., separating two bubbles of ten-dimensional Schwarzschild-de Sitter space). In [17], the trajectories of these eight-branes were worked out; furthermore, it is straightforward to see that the metric induced on the world-volumes of these eight-branes can typically be de Sitter (or de Sitter with a radiation term). Thus, we see that it is possible to get de Sitter both ‘on the eight-brane’ and ‘in the bulk’ of the theory. Presumably these results also apply to compactifications to lower dimensions.

6 Conclusion: Is inflation ‘natural’ in M-theory?

We have shown that de Sitter space can be obtained in a straightforward fashion by compactifying eleven-dimensional supergravity, which is the low-energy limit of ‘MM-theory’. As a cosmological theory the supergravity discussed here has other interesting features. For example it is possible to find a spacetime that admits half of the thirty-two supersymmetries and describes an inflating universe [10]. Furthermore this theory leads to the novel suggestion that the universe is the solution to a supersymmetric system, but the lack of the supercharges has prevented us from seeing the supersymmetry, i.e., without supercharges it is not clear that there needs to be a bose-fermi degeneracy or any Goldstone fermions of broken supersymmetries. However, does this really mean that de Sitter space is a natural vacua of string theory?

In order to answer this, first note that we have focussed on a particular IIA supergravity theory, where the three-form has eaten the two-form. Hence there is no field to which F-strings can couple to electrically. In this sense, there are no ‘strings’ in this theory! Thus, it would seem that this theory has avoided any potential conflict between strings and de Sitter vacua simply because it is not a theory of strings. On the other hand, to the best of our knowledge, this theory is a perfectly respectable corner of the M-theory moduli space. We should always remember that p-brane democracy teaches us to respect all of the diverse and varied degrees of freedom which we may encounter in M-theory, no matter how bizarre these may seem at first glance.

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