Comment on “On physical interpretation of the Poynting-Robertson effect”

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Abstract

In response to the comments of astro-ph/0006426 on Srikanth (Icarus 1999), we wish to clarify, and thereby justify, the definition of Poynting-Robertson drag adopted in the latter. We confirm that dust absorption is a necessary condition while re-emission is neither necessary nor sufficient for the inspiralling of a rest-isotropically emitting dust.

1 Introduction

Poynting-Robertson (P-R) effect causes small bodies in circum-solar orbit, such as dust, assumed to totally absorb intercepted radiation and re-emit isotropically in the bodies’ rest-frame (“rest-isotropically”), to inspiral with orbits of decreasing eccentricity (Harwit 1985; Burns 1979). The paradoxical, and hence interesting, element of the effect is that outward-directed radiation from a primary (the Sun or a star) can cause an otherwise stably orbiting dust to infall. Although the effect is familiar enough through treatment in standard textbooks, a physical understanding of its origin has been unclear (for a historical introduction, see Robertson 1937; Klačka 1993; Srikanth 1999).

More often than not the drag has been attributed to the front-back asymmetry of the dust re-emission as seen in the heliocentric frame. The valid objection to this is that rest-symmetric emission should not alter the inertial state of the center-of-mass, as is evident in the instantaneous rest-frame of the dust. In response to this, the aberration of sunlight is invoked to explain the drag in the dust frame. But to this we raise the objection that it is inconsistent to invoke two different causes (dust-emission asymmetry / sunlight aberration) in the two different frames (heliocentric frame / instantaneous dust rest-frame).

The fact of a given cause engendering an observed effect is absolute, and not relative to the choice of reference frame. The covariance of the force equations automatically enforces this: the status of a given tensor as vanishing or non-vanishing is itself absolute, even though its components transform covariantly. The belief that the cause of the P-R effect is also frame-dependent is a popular fundamental misconception surrounding the effect. A subtlety to be aware of is frame-dependent manifestations of a single cause (e.g., the magnetic force between two current carrying wires transforms

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into an electric in the rest frame of the valence electrons; this happens basically because these two fields are part of a single Maxwell tensor field). However, in the P-R effect, absorption and re-emission are in principle distinct processes and transform independently of each other. Strangely, this curious state of impasse doesn’t seem to have attracted sufficient mainstream attention.

In Srikanth (Icarus 1999), the P-R “paradox” was resolved by carefully distinguishing the contributions to the dust dynamics from the absorption and re-emission. We showed that solar radiation absorption plays an explicit role in the drag (i.e., azimuthal slow-down) of the dust, while re-emission, assumed rest-isotropic, does not. The slow-down may be visualized as a consequence of dust absorption and angular momentum conservation.

The present paper is intended to further clarify this result, and in specific to respond to comments made in Klačka (astro-ph/0006426; hereafter Klačka). This is worth the effort, since history affirms that the paradoxical nature of this effect can cause confusion.

2 The equations of motion

It is sufficient for our purpose to consider a simple model of a spherical, completely absorbing dust, with the added generalization that the absorption and re-emission parameters of the dust are mutually distinct. The generally covariant equations governing the motion of the dust can be written as-

\[
\frac{Dp}{D\tau} = \frac{d}{d\tau}u^\mu + m \frac{Du}{D\tau} = f_{\text{ext}}^\mu,
\]

where \( p^\mu = mu^\mu \) is four-momentum of the dust, \( \tau \) proper time, \( f_{\text{ext}}^\mu \) is the external four-force, and the operator \( D/D\tau \) is the “total” covariant derivative in the General Relativistic sense and includes gravitational effects. Thus gravitation is not part of \( f_{\text{ext}}^\mu \). The mass change term accounts for possible change in the dust’s internal energy due to heating or cooling (Srikanth 1999).

The primary radiation is considered as a plane-parallel beam flowing radially outward, represented by the force \( f_{\text{rad}}^\mu = \epsilon l^\mu \), where \( l^\mu \) is a dimensionless null-vector, with its spatial part being purely radial (Robertson 1937). The scalar \( \epsilon (>0) \) is the rest-rate of absorption of solar radiation by the dust, with dimension momentum over time. We assume that the dust absorbs all the incident radiation and re-emits rest-isotropically. The relativistic four-force associated with a rest-isotropic emission is the time-like four-vector \( f_{\text{emit}}^\mu = -(\xi/c^2)u^\mu \), where \( u^\mu \) is four-velocity, and \( \xi (>0) \) is the rest-frame energy emission rate. Accordingly, Eq. (1) becomes:-

\[
\frac{dm}{d\tau}u^\mu + m \frac{Du}{D\tau} = f_{\text{rad}}^\mu + f_{\text{emit}}^\mu,
\]

\[
= \epsilon l^\mu - (\xi/c^2)u^\mu.
\]

This can be split into scalar and spacelike components as follows.

Contracting Eq. (2) by \( u_\mu \), we get the scalar equation:

\[
\epsilon^2 \frac{dm}{d\tau} = \epsilon u_\mu u_\mu - \xi.
\]

Substituting this back into Eq. (2), we get the true equations of motion:-

\[
m(\tau) \frac{Du}{D\tau} = \epsilon \left( u^\mu - \frac{l^\mu u_\mu}{c^2} u^\mu \right),
\]

2
where the bracketed term is just the part of radiation orthogonal to $u^\alpha$. The significance of this split is that Eq. (3) describes the internal energy change of the dust but not its motion, while Eq. (4) describes its motion. Both equations are in general coupled because of the time-dependence of mass. Implementing the metric for a weak static gravitational field, and letting $v \ll c$, Eq. (4) can be shown to lead to the usual non-relativistic equations of motion for the P-R effect (Srikanth 1999).

3 Identifying the P-R drag

The first term on the right-hand side of Eq. (4) is the radiation pressure term. The second term, which is formally the relativistic generalization of a friction force, is responsible for the azimuthal deceleration of the dust. Hence it is called the drag term. Switching it off, as it were, leads to a stably orbiting dust with the gravitational field modified by radiation pressure. To check that this is the case, in Eq. (2), we put $\epsilon = 0$, which leads to:

$$c^2 \frac{dm}{d\tau} = -\xi.$$  \hspace{1cm} (5)

$$m(\tau) \frac{D u^\mu}{D\tau} = 0.$$ \hspace{1cm} (6)

Eq. (5) shows that in the absence of absorption, the dust moves along a geodesic. The mass of the dust is time-dependent, but does not affect its motion. Hence absorption is a necessary condition for dust slow-down, while re-emission is not a sufficient condition for slow-down.

On the other hand, associating the four-momentum carried away from the dust, which is the second term in the r.h.s of Eq. (2), with the drag, as Kláčka recommends, is misleading. In Eq. (6), putting $\xi = 0$ leads to:

$$c^2 \frac{dm}{d\tau} = \epsilon l^\alpha u_\alpha.$$ \hspace{1cm} (7)

while Eq. (6) remains unchanged. Thus, we still have the slow-down of the dust in the absence of re-emission, though the trajectory is modified by the mass change. This shows that re-emission is not a necessary condition for slow-down, while absorption is a sufficient condition (assuming all the usual non-P-R aspects of problem as given). If $dm/d\tau \neq 0$ then re-emission contributes implicitly via the time-dependent mass. However, this is not a friction or drag-like contribution. Depending on whether the dust heats up or cools down, this process renders the dust less or more susceptible to drag.

In summary, re-emission is neither a necessary nor sufficient condition for the drag, while absorption is necessary. The association of re-emission with the drag is erroneous except in the case of an isothermal ($dm/d\tau = 0$) dust, as seen via Eq. (3). Because historically the P-R effect had been studied with an implicit assumption of isothermality, this equality led to a general confusion of the relative roles played by these two processes in the drag, as discussed in the Introduction. Indeed, the two-parameter model of the P-R effect that we have adopted was motivated by this observation.
4 Isothermality

As seen from its rest frame, an isothermal dust emits as much as it absorbs (Srikanth 1999). However, in any other frame, as seen from Eq. (2), the excess of absorbed energy over re-emission is balanced by kinetic energy. Kláčka argues that this distinction between the reference frames is obvious, and no more necessary to make explicit than to state that the laws of reflection must be referred to the rest frame of the reflecting surface.

However, in fact, this analogy is not apt. Here, the rest-frame of the reflector is a natural choice for reference frame. The simplest statement of the law of reflection also refers to this frame. But, in the P-R effect, while the convenient choice of frame is the heliocentric frame, the simplest possible satisfaction of isothermality (by the equality of emitted to absorbed radiation) holds good in the dust rest-frame. This has lead in some extant literature to the erroneous association of the simplest version of isothermality with the heliocentric reference frame. The purpose of mentioning this point in Srikanth (1999) was of pedagogical and historical interest.

5 Conclusions

Some clarifications on an earlier result reached by us in Srikanth (1999) are given. We find that in the P-R effect dust absorption is a necessary condition for drag, whereas re-emission, assumed to be rest-isotropic, is neither necessary nor sufficient. This result is independent of the reference frame used for the description of the effect.

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