The wave-function is real but nonphysical: A view from counterfactual quantum cryptography

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Counterfactual quantum cryptography (CQC) is used here as a tool to assess the status of the quantum state: Is it real/ontic (an objective state of Nature) or epistemic (a state of the observer’s knowledge)? In contrast to recent approaches to wave function ontology, that are based on realist models of quantum theory, here we recast the question as a problem of communication between a sender (Bob), who uses interaction-free measurements, and a receiver (Alice), who observes an interference pattern in a Mach-Zehnder set-up. An advantage of our approach is that it allows us to define the concept of “physical”, apart from “real”. In instances of counterfactual quantum communication, reality is ascribed to the interaction-freely measured wave function ($\psi$) because Alice deterministically infers Bob’s measurement. On the other hand, $\psi$ does not correspond to the physical transmission of a particle because it produced no detection on Bob’s apparatus. We therefore conclude that the wave function in this case (and by extension, generally) is real, but not physical. Characteristically for classical phenomena, the reality and physicality of objects are equivalent, whereas for quantum phenomena, the former is strictly weaker. As a concrete application of this idea, the nonphysical reality of the wavefunction is shown to be the basic nonclassical phenomenon that underlies the security of CQC.

I. INTRODUCTION

Quantum superposition, as exemplified by the familiar double-slit experiment, lies at the heart of quantum weirdness. The experiment contains, according to Feynman, the “only mystery” in quantum mechanics [1]. The interference observed behind the double-slit indicates that the particle, when moving from the source, travels in some sense along both slits. Classical waves of water or sound show such interfering behavior, but they are collective phenomena, and can be attributed to distinct localized masses or molecules travelling through the two slits. In the quantum case, the interference can be observed even when the source is so attenuated that there is practically only one particle between the double-slit plane and the screen. Dirac summarized the situation by saying “Each photon then interferes only with itself” [2].

Here we revisit this phenomenon, prompted in part by recent interest in studying the nature of the quantum state. The basic, oldest and arguably most controversial question here is: Is the quantum state real, i.e., Does it objectively exist and represent a state of Nature (cf. eg., [3]); or, Is it epistemic, i.e., Does it represent the observer’s state of knowledge? In the framework of ontological models of quantum mechanics [4], which is based on the hidden variable theories of Bell [5] and Kochen-Specker [6], a model is $\psi$-ontic if every pair of pure quantum states corresponds to non-overlapping ontic supports, and is $\psi$-epistemic otherwise. Recently, a number of arguments have been put forth within this framework in favor of the ontic view [7–10] under certain assumptions [11–16].

Here we will assess the status of the quantum state from a very different perspective, one that differs from the above type of approach to wave function ontology in three basic ways: (1) our result is not based on the framework of realist or any other models of quantum theory, but instead one that is based on operational considerations about communication involving interaction-free measurements [17]; (2) The definition of “reality” is not explicitly tied to assumptions like independence, no-signaling, etc., but is direct and intuitive; (3) Our approach allows us to define not only a concept of “reality”, but also another, that of “physicality”. We think that this is crucial, because our work shows that the distinction between physicality and reality lies at the heart of what makes the quantum state nonclassical.

II. INTERACTION-FREE MEASUREMENTS AND COUNTERFACTUAL COMMUNICATION

Interaction-free measurements (IFMs) in quantum theory allow one to obtain knowledge of the presence or absence of an object without interrogating it directly. For example, an absorber placed in one of arms of a Mach-Zehnder interferometer is detected by its disturbance of the destructive interference that would have resulted in its absence [17]. IFM can be used as the basis for what are called counterfactual computation [18–19] and counterfactual cryptography [20–21], where the absorber is present or absent depending on the outcome of a computer or choice of a communicator. Here ‘counterfactual’ essentially means that Bob’s blockade in one of the arms

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of a Michelson interferometer causes a detection of the particle away from the blockade. Single-particle nonlocality lies at the heart of this phenomenon, since without this, the blockade would localize a particle near it.

Recent advances include counterfactual communication based on the quantum Zeno effect [22] (cf. also [23,24]) and the extension of the counterfactual bipartite protocols to the tripartite case [25,26]. The N09 protocol of [20] has been experimentally implemented [27], and its security proofs discussed by various authors [28–30].

For the purpose of this work, it suffices for any one outcome to be counterfactual, and we use the recently proposed semi-counterfactual quantum cryptographic scheme [21], where only one of the outcomes is counterfactual. This experimental set-up for counterfactual communication is based on a Michelson-type interferometer. One of the arms is taken to be an external one, connecting Alice and Bob, and the other arm is an internal arm within Alice’s station (Figure 1).

A single-photon from a source in Alice’s station hits her beamsplitter BS and is split along the two arms a and b. Alice and Bob each possesses a switch which is randomly in one of two modes: absorb (A) and reflect with a Faraday mirror (F). The state of the particle after the beamsplitter is:

$$|\Psi\rangle = \left(\frac{a^\dagger + ib^\dagger}{\sqrt{2}}\right)|0,0\rangle_{AB}, \quad (1)$$

where $a^\dagger$ ($b^\dagger$) is the creation operator for the light mode in the internal (external) arm, and $|0,0\rangle_{AB}$ is the vacuum state of those two modes. The operators for Alice’s detector modes, in terms of those for the arm modes, is given by:

$$d_j^\dagger = \frac{1}{\sqrt{2}} (a^\dagger + (-1)^j b^\dagger), \quad (j = 0, 1) \quad (2)$$

where $d_j^\dagger$ correspond to creation operators for modes corresponding to detector $D_j$ (Figure 1). The three possibilities of Alice’s and Bob’s choice are:

1. Alice and Bob both apply F: This results in an interference with a bright fringe at detector $D_1$ and a dark fringe at $D_0$:

$$P(D_1|A, B) = 1. \quad (3)$$

2. Exactly one of them applies A and the other F: This produces a click in detector $D_0$ or $D_1$ each equal probability:

$$P(D_0|A, B) = P(D_1|A, B) = \frac{1}{2}. \quad (4)$$

3. Both apply A: neither detector $D_0$ nor $D_1$ clicks:

$$P(D_0|A, B) = P(D_1|A, B) = 0. \quad (5)$$

Ideally, a bit is communicated when Alice observes $D_0$, for in this case she knows for certain that Bob applied an operation anti-correlated with hers. The efficiency of the protocol is $\frac{1}{8}$ [21].

The counterfactual situation arises in $D_0$ events when Bob applies $A_B$ and Alice applies $F_A$. In this case, if the detector $D_0$ clicks, then it carries 1-bit of information of Bob’s measurement choice (that it was $A_B$), even though the photon did not physically travel and interact with his detector, as evidenced by his lack of detection. It is in this sense that this 1-bit information is counterfactual, and the physical interpretation of this communication forms the objective of this work.

The other possibility for a $D_0$ event, that Bob applied $F_B$ and Alice $A_A$, is not counterfactual with respect to Bob (hence the characterization of the protocol of Ref. [21] as semi-counterfactual.)

### III. PHYSICAL INTERPRETATION

We distinguish two concepts to be used to describe the nature of the quantum state: the term “physical”, as used in counterfactual communication to qualify the travel of a particle, and the term “real”, as has been used recently in quantum foundations. We will define these two concepts based on simple, operational criteria inspired by the above experiment.
Consider the communication scenario described in Figure 2. This is an unwrapped version of the path taken by a particle from Alice’s source, via Bob, back to Alice in Figure 1. Alice knows that a certain entity $\psi$ is emitted from the source located at $a$ towards her, located at $a'$. Bob, located at the intermediate position $b$, may absorb the object (operation $A_B$) or forward it ($F_B$).

On the other hand, suppose $\psi$ is epistemic and not a real thing. Then it represents a probability distribution

$$P(\psi) \equiv (p_\psi, 1 - p_\psi),$$

as perceived by Alice, with $p_\psi$ being the probability that the particle travels from $a$ towards Alice through $b$, and $1 - p_\psi$, the probability that it does not travel from the source along this path towards Bob.

If Alice finds the particle at $a'$, then Bob clearly did not apply $A_B$. But if she does not recover the particle, it does not necessarily imply that Bob applied $A_B$, since there is a non-vanishing probability $\overline{p}_\psi$ that the particle did not travel from $a$ to $b$. In other words, an epistemic $\psi$ necessarily precludes Alice’s deterministic retrodiction of Bob’s blocking action.

If we denote by “no” Alice’s non-detection of the particle at $a'$, then $P(\text{no}|A_B) = 1$, and by Bayesian argument:

$$P(A_B|\text{no}) = \frac{P(A_B)}{\overline{p}_\psi + p_\psi P(A_B)},$$

where $P(A_B)$ is the probability that Bob applies $A_B$. Eq. (7) shows that $P(A_B|\text{no}) = 1$ only if $\overline{p}_\psi = 0$. Otherwise, $P(A_B|\text{no}) < 1$, implying that Alice cannot deterministically infer that Bob applied action $A_B$. More generally, “yes” indicates a specific type of outcome observed by Alice, conditioned on her any local operation. For example, if she turns on a knob, and a beep appears on a monitor, this could correspond to “yes”, while “no” would correspond to the non-appearance of such a beep.

This motivates the following definition of reality: We say that $\psi$ is real and not epistemic, if and only if there is some detection event $a'$ at $a'$ such that Alice can deterministically infer by local observation in her station that Bob applied the blocking action $A_B$ during the $\psi$’s transit:

$$\text{Real}(\psi) \equiv \exists_{a'}[P_{a'}(A_B|\text{no}) = 1],$$

where $\text{Real}(\psi)$ is the proposition that asserts the reality of $\psi$. Note that this condition for reality is quite operational: it is based on no interpretational framework for $\psi$ but is stated in terms of Bob’s actions and Alice’s observations in a laboratory.

We now apply the above definition to a classical object. If $\psi$ is (say) a physical ball transmitted along the indicated path in Figure 2 with $p_\psi = 1$ in Eq. (6). Then $\psi$ is real according to the above criterion, since every non-detection event at $a'$ allows Alice to infer Bob’s absorbing action. Further, if $\psi$ is epistemic (corresponding to $0 < p_\psi < 1$ in Eq. (6)), then it fails to be real according to our criterion (8).

Now consider the setup in Figure 1. To map the scheme of Figure 2 we let both coordinates $a$ and $a'$ to be with Alice, and $b$ with Bob. We denote as “yes” the $D_0$ events conditioned on Alice applying the operation $F$ locally. Thus the observation of $D_0$ conditioned on her applying $F$ corresponds to outcome “no” in Eq. (7). In such cases, Bob’s blocking action $A_B$ is deterministically inferred by Alice, since

$$P(A_B|D_0, F_A) = 1 - P(F_B|D_0, F_A)$$

$$= 1 - \frac{P(D_0|F_A, F_B)P(F_B|F_A)}{P(D_0|F_A)}$$

$$= 0,$$  

by Bayes’ theorem, and since $P(D_0|F_A, F_B) = 0$ by Eq. (3), while $P(D_0|F_A) > 0$. According to the above criterion, then the wave function $\psi$, that leaves the source and travels to Bob, must be real.

**B. Physicality**

It is in counterfactual quantum communication that we encounter the idea of “transfer of information without any physical travel of particles” or “non-physical transmission of information”. In the experiment of Figure 1 we saw that it arises naturally from the way communication occurs from Bob to Alice when he measures the photon via interaction-free measurement, and she observes $D_0$. 

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**FIG. 2:** Communication scenario based on a spread out version of the to-and-from particle travel along the external arm from Alice to Bob in Figure 1. Alice knows that an entity $\psi$ is transmitted from the source located at $a$ towards her, located at $a'$. Bob, located at the intermediate position $b$, may absorb the object (operation $A_B$) or forward it ($F_B$).
By contrast, in all foundational approaches to the status of the wavefunction, it is only the reality of the wavefunction, rather than its physicality, that is called into question. We want to stress that this is the significant contribution of quantum cryptography, in particular the counterfactual variety, to this discussion.

To be precise, the particle’s transmission is called nonphysical in counterfactual cryptography when Bob’s communication generates no detection on his apparatus. Generalizing this, the essence of physicality here, then, is that a physical object should be detected by an ideal detector.

This motivates us to identify physicality with physical detectability: We say that ψ is physical if and only if for every transmission event α, if Bob applies the absorption action A_B during ψ’s transit, then it necessarily leads to his detection (provided he has an ideal detector), else (i.e., he applies F_B), it leads to a detection by Alice. In other words, a physical ψ is always detected by an intercepting ideal detector:

\[ \text{Physical}(\psi) \equiv \forall_\alpha[P_\alpha(Y|A_B) = 1, \ P_\alpha(\text{yes}|F_B) = 1], \]

where “Y” indicates Bob’s detector registering a detection. Here Physical(ψ) is the proposition which asserts that ψ is physical. This condition, like Eq. (8) for reality, is operational: it is not based on any interpretational framework for ψ, but is stated in terms of Alice’s and Bob’s actions and observations.

Note that if ψ is physical, then from the second condition in (10), we find that ψ is also real. Thus physicality entails reality. On the other hand, the converse is not true. Real(ψ) only entails that there are some events such that if Bob applies F, Alice finds “yes”. If this does not hold true in all cases, then it fails to be physical, opening up the possibility of nonphysical but real ψ.

In Figure 2 consider ψ to be a classical real object. Then Ps ≡ (1, 0) and Bob’s action A_B always leads to a detection; while his action F_A leads to Alice’s observation of “yes”, thereby satisfying the physicality condition (10). A classical epistemic object fails (10) because in the probabilistic event when no particle travels down to Bob, his action A_B does not generate a detection for him. Thus, classical objects are either real and physical or epistemic and nonphysical. In other words, reality and physicality are equivalent in this formalism.

Now consider the set-up in Figure 1. In the D_0 events, Bob’s interaction-free action A_B does not generate any detection on his side, so that it fails the first condition in (10), making it nonphysical.

C. Nonphysical reality

The definitions Eq. (8) and (10) allow the existence of objects that are real-physical, real-nonphysical and epistemic-nonphysical in the present framework. As noted above, classical ψ conflates reality with physicality.

Thus the only classical possibilities are real-physical and epistemic-unphysical. Quantum mechanics allows the additional existence of real-nonphysical ψ.

Combining the results from the Subsections IIIA and IIIB, we conclude that the quantum wave function that propagates from Alice to Bob in the onward leg is real according to criterion (8) but nonphysical according to criterion (10). On the other hand, as noted above:

\[ \text{Physical}(\psi) \implies \text{Real}(\psi), \]

so that the class of real objects is strictly weaker than that of physical objects. Thus an epistemic (i.e., unreal) object is necessarily nonphysical, according to our criterion.

Since the equivalence of reality and physicality holds within the domain of classical objects, objects that are real but nonphysical are nonclassical. These ideas are depicted in Figure 3, where the set of real (physical) objects is called Real (Physical).

Note that classical waves (say water waves) show an interference behavior similar to the single-photon system. Yet water waves are both real and physical: ψ here can be taken to represent the relative mass of water moving down each arm, and “yes” can represent Alice observing a depletion of water mass at her end, while A_B represents Bob’s blocking action. It immediately follows that classical water waves are physical, and consequently real by criteria (8) and (10).

IV. SECURITY VIA NONPHYSICAL REALITY OF THE WAVE FUNCTION

We now claim that nonphysical reality underlies the security of the semi-counterfactual cryptography protocol of Section II in the precise sense that the security check...
for the protocol consists in verifying that the reality condition (8) holds, while the physicality (10) fails. Suppose an eavesdropper Eve attacks the exposed arm (b) in Figure 1 with the aim of testing whether the particle travelled along that path. To the extent she can detect the particle’s location, she commits the particle to one or the other arm, and thereby enforces classical behavior. The degree of failure of Alice to verify nonphysical-real behavior in the data, alerts Alice and Bob to Eve’s presence.

Let us consider this more quantitatively, a detailed study of which is presented in [21]. Eve attacks the particle in arm b at a point along the solid line in the onward leg, by first interacting it with a probe of hers prepared in the initial state |0⟩E using interaction V:

\[ V|\Psi⟩_{AB}|0⟩E = \frac{1}{\sqrt{2}}(a|0⟩_{AB}−|E⟩ + b|0⟩_{AB}+|⟩), \]

(12)

where |+⟩ and −⟩ represent non-orthogonal states, with |(−⟩ | +⟩⟩ = cos υ, where υ is a security parameter in the range [0, π/2]. Eve’s intention is to subsequently measure the probe, and use its outcome in conjunction with Alice’s announcement, to optimize her guess of the shared secret bit.

If Alice announces D0, then Eve measures her probe according to the following positive operator-valued measure (POVM) [31], which can be shown to be optimal:

\[ P_+ = \frac{1}{1 + \cos(υ)}(1 − |+⟩⟨+|), \]
\[ P_0 = 1 − P_+ − P_−, \]

(13)

where outcome P+ indicates a conclusive outcome, while outcome P0 is inconclusive. From (13), it follows that on an ensemble of |+⟩ and −⟩, the probability that Eve obtains a conclusive answer is 1 − (|+⟩⟨−| − |−⟩⟨+|) = 1 − cos(υ). Clearly, this is also Eve’s information IE on the (unprocessed) secret key.

Eve’s presence is therefore revealed through the departure from perfect interference, as observed by Alice, which from Eqs. (12) and (8) is

\[ P(D_0|F_A, F_B) = \frac{||⟨+⟩E−|−⟩E||^2}{\sqrt{2}} = \frac{1}{2}(1 − \cos(υ)), \]

(14)

implying a visibility of \( V = \cos(υ) \). If condition (8) is perfectly satisfied, then conditioned on Alice applying F, if Bob applies F, then a “yes” outcome, namely D1 detection, must arise. To the degree that \( P(D_0|F_A, F_B) > 0 \), the condition (8) fails, and the second condition in (10) also fails. To summarize, security arises because Eve’s attack physicalizes, and thus classicalizes the particle, which is detected by Alice and Bob who are monitoring nonphysical-real behavior.

To complete the discussion, the error \( \epsilon \) produced in the key is that Alice’s and Bob’s inputs are not anti-correlated when a \( D_0 \) click happens [21]:

\[ \epsilon = P(F_A, F_B|D_0) = \frac{P(D_0|F_A, F_B)P(F_A, F_B)}{P(D_0)} = \frac{1 − V}{2 − V}, \]

(15)

since \( P(D_0) = \sum yP(D_0|y)P(y) \) where \( y \in \{(A_A, A_B), (A_A, F_B), (F_A, A_B), (F_A, F_B)\} \), where \( P(D_0|A_A, A_B) = 0, P(D_0|A_A, F_B) = P(D_0|F_A, A_B) = \frac{1}{4} \) and \( P(A_A, A_B) = P(A_A, F_B) = P(F_A, A_B) = P(F_A, F_B) = \frac{1}{4} \). The mutual information between Alice and Bob is \( I_B = 1 − H(\epsilon) \), where \( H(\cdot) \) is the Shannon binary entropy. The condition for positive key rate is that \( I_B − I_E \geq 0 \) [32], or

\[ V \geq H \left( \frac{1 − V}{2 − V} \right), \]

(16)

which implies that the error rate \( \epsilon \) must be less than about 21% for security, and thence, for proof of nonphysicality of the wave function.

With more powerful models of Eve’s attack, this error threshold would be lowered. However, the basic idea—that Eve’s attack physicalizes, and thus classicalizes the particle—should intuitively be the same.

V. CONCLUSIONS AND DISCUSSIONS

We have used counterfactual quantum cryptography (CQC) as a tool to study the ontology of the quantum wave function. This stands in contrast to approaches based on ontological theories, where a concept of “reality” for Ψ is defined, and under certain arguable assumptions, arguments in favor of the quantum state being real are presented. Furthermore, they do not appear to throw sufficient light on what exactly is weird about quantum mechanics.

In the present approach, we gave operational definitions of “reality” and of “physicality” of Ψ. In particular, the reality of Ψ is inferred from the fact of genuine communication from Bob to Alice during counterfactual communication, while its nonphysicality is inferred from the fact that Bob makes no physical detection of the particle during that communication. Thereby we are able to draw conclusions that go beyond earlier approaches: (a) that the quantum state is real; (b) that it is not physical; (c) that this nonphysical reality is the mark of nonclassical phenomena, since reality and physicality coincide for classical phenomena.

Of course, the status of the wave function (as being real or epistemic) does not depend on Bob’s choice of \( A_B \) or \( F_B \). Nor does it depend on whether Bob is located at the end of arm a or b. What may conclude is that the effect of the superposed states in Eq. (1), \( Ψ_0 ≡ a|0⟩ + b|0⟩ \), is by itself real-nonphysical, and thus, so too the particle state state \( |Ψ⟩ = \frac{1}{√2}(ψ_a + ψ_b) \) in Eq. (1) is also real-nonphysical. We may therefore conclude that
the quantum state is quite generally real-nonphysical. In retrospect, we may reflect in this new light on the wisdom of Feynman’s observation with regard to the double-slit experiment, mentioned in the opening paragraph. Our approach suggests that in the production of fringes in the double-slit experiment, there is indeed some “real stuff” travelling down both slits, but it is not physical. This explication thus puts (or so we hope!) a name on the mystery alluded to by Feynman.

In counterfactual communication, Bob’s choice has a nonlocal influence on Alice’s local observation. This manifestation of single-particle nonlocality is consistent with relativity, since Alice must wait for the time required by Bob’s particle to return, before deciding whether he applied $A_B$ or $F_B$, and thus respects signal locality.

From the perspective presented here, the essential mark of non-classical behavior, and the ‘mystery’ of the quantum, is the non-vanishing gap between the real and the physical. An analogous situation has been reported also in the study of quantum correlations, where nonclassicality is identified with the gap between the communication cost $C$ of nonlocal correlations $P$ and the signal $S$ accessible within $P$ [33]. Now $C$ is just the signal required to simulate the correlations using resources from a deterministic hidden variable (DHV) theory. Thus the ‘mystery’ highlighted by this latter gap is that the signal that arises at the ontological level cannot be used for signaling at the physical level (cf. [34] [35]).

Our work showed that the non-physical reality of the wave function is not an abstruse philosophical notion, but has the concrete application of being responsible for security in CQC. Finally, we venture that it is the lack of distinction in the literature between the real and the physical aspect that is responsible for the historical difficulty in interpreting the physical significance of the quantum state. In the discussion pertaining to the double-slit experiment, at first one has the intuitive feeling that there is something real traveling down both slits. One then subconsciously maps this real thing to something physical. But clearly the possibility of the quantum wave as a physical entity is one that we would consciously reject. Thus, psychologically speaking, a person thinking about quantum foundations is caught in the perpetual dilemma of deciding whether or not the quantum state is real. It is our belief that our work resolves this dilemma.

Acknowledgments

RS acknowledges support from the DST project SR/S2/LOP-02/2012.

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