Three-Body Model Calculation of Spin Distribution in Two-Nucleon Transfer Reaction

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The differential cross sections of two-nucleon transfer reactions $^{238}\text{U}(^{18}\text{O},^{16}\text{O})^{240}\text{U}$ around 10 MeV per nucleon are calculated by one-step Born-approximation with a $^{16}\text{O}+^{2}n+^{238}\text{U}$ three-body model. The three-body wave function in the initial channel is obtained with the continuum-discretized coupled-channels method, and that in the final channel is evaluated with adiabatic approximation. The resulting cross sections have a peak around the grazing angle, and the spin distribution, i.e., the cross section at the peak as a function of the transferred spin, is investigated. The shape of the spin distribution is found not sensitive to the incident energies, optical potentials, and treatment of the breakup channels both in the initial and final states, while it depends on the excitation energy of the residual nucleus $^{240}\text{U}$. The peak of the spin distribution moves to the large-spin direction as the excitation energy increases. To fulfill the condition that the peak position should not exceeds 10ℏ, which is necessary for the surrogate ratio method to work, it is concluded that the excitation energy of $^{240}\text{U}$ must be less than 10 MeV.

Determination of neutron-induced reaction cross sections of unstable nuclei is one of the most important challenges for nuclear physics and its application. Systematic data of fission or neutron capture cross sections in the neutron-induced reaction with minor actinides (MAs) and long-lived fission products (LLFPs) are necessary for theoretical designs of the next generation nuclear plant. Such data also play a major role in discussing the nucleosynthesis of the s- and r-processes in nuclear astrophysics. It is, however, difficult to measure these reaction cross sections directly with currently available experimental techniques; both neutron and unstable nuclei cannot be used as a target because of their short lifetime.

The surrogate reaction method is an indirect technique to obtain the neutron cross section from an analysis of multi-nucleon transfer reactions or inelastic scatterings producing the same compound nucleus as that created by the desired (neutron-induced) reaction. This simplest approach, so-called the surrogate ratio method (SRM), is based on the Weisskopf-Ewing (WE) approximation, i.e., the decay branching ratio of the fission or capture process is assumed to be independent of the spin-parity $J^{π}$ of the compound nucleus populated. However, it is very difficult, or almost impossible, to find a surrogate reaction in which this assumption is satisfied. Recently, Chiba and Iwamoto proposed that only a weaker condition, which is called weak WE condition, was necessary and this idea made the SRM approach feasible. The weak WE condition is that a ratio of decay rate of a desired nucleus to that of another one must be constant with respect to $J^{π}$, which is fulfilled for $J \leq 10$ (in unit of ℏ) in the case of Uranium-isotopes. Therefore, a gross feature of

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the spin-parity distribution of the residual nucleus populated in the surrogate reaction is a key issue for the SRM approach; the spin-parity distribution should have a peak somewhat lower than $J = 10$. In previous studies\(^{10, 12}\), however, spin-parity distributions of compound nuclei are assumed rather arbitrarily.

In this Letter, we evaluate the differential cross section of the $^{238}\text{U}(^{18}\text{O},^{16}\text{O})^{240}\text{U}$ reaction at 180 MeV, with changing the spin transferred to $^{238}\text{U}$, and obtain the spin-parity distribution of $^{240}\text{U}$. We describe the transfer reaction with a $^{16}\text{O} + ^2\text{n} + ^{238}\text{U}$ three-body model. We neglect for simplicity the intrinsic spins of the three particles. The three-body wave function in the entrance channel is calculated by the Continuum-Discretized Coupled-Channels method (CDCC)\(^{13, 14}\), while that in the exit channel is described by adiabatic approximation\(^{18}\). Thus breakup effects of both $^{18}\text{O}$ and $^{240}\text{U}$ on the spin distribution are investigated. As for the transition matrix of the transfer process, we adopt one-step Born approximation (BA). This approach called CDCC-BA has been applied to many studies on transfer reactions; see, e.g., Refs.\(^{19, 20}\).

In the present study, two neutrons are treated as a bound particle (di-neutron) and the sequential two-nucleon transfer process is neglected. Nevertheless, this is the first calculation of the spin distribution of $^{240}\text{U}$ populated by a transfer process by means of a three-body reaction model and should be regarded as the starting point of the investigation on the spin distribution associated with the SRM approach.

In the calculation of the $^{16}\text{O}-^2\text{n}$ wave function in CDCC, we include the s-, p-, and d-waves calculated with a Woods-Saxon potential with the radial (diffuseness) parameter of $1.27 \times (16)^{1/3}$ fm (0.67 fm). We use the separation-energy method to determine the depth of the potential; we assume the ground state of the $^{16}\text{O}-^2\text{n}$ system is a s-wave state with the binding energy of 12.2 MeV. The maximum wave number of the $^{16}\text{O}-^2\text{n}$ continuum is taken to be 1.5 fm\(^{-1}\) and the width of the momentum-bin is 0.15 fm\(^{-1}\). As for the distorting potential of $^2\text{n}$ by $^{238}\text{U}$, we take the neutron global potential of Ref.\(^{21}\) with making the depth parameters of the real and imaginary parts twice. The distorting potential between $^{16}\text{O}$ and $^{238}\text{U}$ is evaluated by the double folding model with the Jeukenne-Lejeune-Mahaux (JLM) nucleon-nucleon interaction\(^{22}\) nuclear densities are obtained by Hartree-Fock method with finite-range Gogny D1S force\(^{23}\). The $^{18}\text{O},^{238}\text{U}$ distorted wave is evaluated up to 30 fm with an increment of 0.01 fm; we take the number of the partial waves to be 200.

The wave function of the exit channel is calculated with adiabatic approximation following Ref.\(^{18}\). As for the binding potential of $^2\text{n}$ by $^{238}\text{U}$, we adopt a Woods-Saxon form with the radial and diffuseness parameters of $1.27 \times (238)^{1/3}$ fm and 0.67 fm, respectively. The number of nodes of the $^2\text{n}-^{238}\text{U}$ wave function is set to be the same as that of the forbidden states. Note that the total spin $J$ (in unit of $\hbar$) of $^{240}\text{U}$ is equal to the orbital angular momentum $\ell$ between $^2\text{n}$ and $^{238}\text{U}$ in the present calculation.

The transfer process is described by one-step CDCC-BA as mentioned above. We made the zero-range approximation to the transition operator, i.e., the interaction between $^2\text{n}$ and $^{16}\text{O}$, in the calculation of the transition matrix. It is confirmed that inclusion of the finite-range correction\(^{24}\) never changes the conclusions below.
In Fig. 1, we show the differential cross section of $^{238}$U($^{18}$O,$^{16}$O)$^{240}$U at 180 MeV (10 MeV per nucleon) as a function of the outgoing angle $\theta$ of $^{16}$O in the center-of-mass (c.m.) frame. We put the binding energy $\epsilon_B$ between $^2n$ and $^{238}$U to be 10.74 MeV that corresponds to the ground state of $^{240}$U in the present three-body model. Cross sections for $0 \leq J \leq 10$ are plotted. They are localized well around $\theta \sim 35^\circ$. This “bell-shaped” angular distribution is a well-known feature of heavy-ion induced transfer reactions at energies that are above the Coulomb barrier but low so that the elastic scattering in each channel is still Fresnel-like. One sees that the peak of the cross section increases with $J$ for $J \leq 5$ and decreases afterwards. This is consistent with the assumption made in Ref. 9). In Fig. 2, we show the spin distribution; the solid line is the result of CDCC-BA and the dashed line is that obtained with neglecting breakup states of both $^{18}$O and $^{240}$U. One sees that breakup effects on the spin distribution are very small; the total breakup cross section

Fig. 1. (color online) Transfer cross sections of $^{238}$U($^{18}$O,$^{16}$O)$^{240}$U at 180 MeV for $J = 0$–10, as a function of the outgoing angle of $^{16}$O in the c.m. frame.

Fig. 2. (color online) Spin distribution corresponding to the result in Fig. 1. The solid (dashed) line shows the result with (without) breakup states of $^{18}$O and $^{240}$U.
Fig. 3. (color online) (a) Partial elastic cross sections for the entrance (dashed line) and exit (dotted and dashed lines) channels. The dotted and dashed lines correspond to $E_B = 10.74$ and 0.74 MeV, respectively. (b) Radial part of the $^2n-^{238}U$ wave function multiplied by $r$.

is very small ($\sim 2.8 \text{ mb}$). Although they cause change in the absolute values of the spin distribution by 20% at most, the gross features of present interest, i.e., the peak position and the shape, never change. The renormalization factors of the JLM interaction$^{22}$ are found to have the same effects on the spin distribution.

Because of the shortness of the wave number and strong absorption due to the target nucleus, the transfer process considered in this study can be interpreted with a simple picture. In the left panel of Fig. 3 we show the partial elastic cross sections (PEX) for the entrance (solid line) and exit (dotted line) channels. Each PEX shows a narrow peak at a grazing momentum $L_g$. The difference between the $L_g$ for the two channels, $\Delta L_g$, gives a constraint for $J$, i.e., a transfer process for $J \sim \Delta L_g$ is preferred. On the other hand, as shown in the right panel of Fig. 3 the bound state wave function between $^2n$ and $^{238}U$ at the grazing radius $r_g \sim 10 \text{ fm}$ decreases with $J$. Thus, these two features shown in Fig. 3 determine the spin distribution shown in Fig. 2.

Figures 1, 2, and 3 indicate some advantages to use $^{238}U(^{18}O,^{16}O)^{240}U$ at around 10 MeV per nucleon as a surrogate reaction. First, the bell-shaped angular distribution gives a clear criterion for the scattering angle to be measured. Second, a simple classical picture can be used to interpret the reaction process. Third, the spin distribution has a peak at a rather small value of $J$, i.e., $J = 5$. On the other hand, if one uses a ($^3\text{He},p$) reaction, the $J$ dependence of the cross section becomes complicated and the spin distribution is very sensitive to the detection angle. This can be seen in Fig. 4 in which the cross section of $^{238}U(^3\text{He},p)^{240}\text{Np}$ at 30 MeV (10 MeV per nucleon) calculated with the same framework is shown.

We show in Fig. 5 the energy dependence of the spin distribution. The solid, dashed, and dotted lines correspond to $E = 180$, 140, and 220 MeV, respectively;
breakup effects of $^{18}$O and $^{240}$U are not included. Although the peak is located at $J = 5$ for all these energies, the distribution seems to slightly shift to the smaller (larger) $J$ at lower (higher) energies. This suggests that $^{238}$U($^{18}$O,$^{16}$O)$^{240}$U at lower energies is more suitable for the SRM. It should be noted, however, that the incident energy should be larger than the Coulomb barrier height.

In the calculations shown above, we take $\epsilon_B = 10.74$ MeV and the $^2n$ transfer process to the ground state of $^{240}$U is investigated. In the surrogate reaction method, one aims to produce a compound nucleus $^{240}$U that has energy above the $n$-$^{239}$U threshold located at 5.93 MeV from the ground state of $^{240}$U. Thus, we take $\epsilon_B = 4.74$ and 0.74 MeV, which correspond to the excitation energy $E_{ex}$ of $^{240}$U of 6.0 and 10.0 MeV, respectively. Note that in the present three-body model calculation, the excited state of $^{240}$U is described by a bound state of $^2n$ by $^{238}$U. For $\epsilon_B = 0.74$ MeV, we set the maximum radius between $^{18}$O and $^{238}$U to be 45 fm in evaluation of the
transition matrix.

Figure 6 shows the spin distribution for $\epsilon_B = 4.74$ (dashed line) and 0.74 MeV (dotted line). Again, we neglect breakup effects of $^{18}\text{O}$ and $^{240}\text{U}$. Also shown by the solid line is the result for $\epsilon_B = 10.74$ MeV, i.e., the dashed line in Fig. 2. One sees that the spin distribution somewhat shifts to the high-$J$ direction as $E_{\text{ex}}$ increases. This is due to the increase in the reaction $Q$-value, hence $\Delta L_g$; see the dashed line in Fig. 3 that is the PEX in the exit channel corresponding to $\epsilon_B = 0.74$ MeV. It was conjectured in Ref. 9 that the SRM worked well unless a compound nucleus with $J > 10$ was populated by a surrogate reaction. Thus, we conclude from Fig. 6 that $E_{\text{ex}} = 10$ MeV is the upper limit of the SRM.

It should be noted that unlike schematic spin distributions of Escher in Ref. 12), the spin distribution of Fig. 6 is skewed so that low $J$-values are enhanced. Therefore, contributions for $J \geq 10$ are not the major part of the populated compound nuclei even in these heavy nuclear systems. Another remark is that we must multiply the above result by the level-density of the final state to evaluate the population cross section of different $J^\pi$ states. The $J$-dependence of the level-density calculated with the Fermi gas formula using the deformation parameter and mass of $^{240}\text{U}$ shown in Ref. 25 is very weak in the region of $J \leq 10$. This multiplication, therefore, gives no change in the conclusion above.

An important point of the SRM proposed in Ref. 9 is to find a pair of surrogate reactions, the spin distributions of which are equivalent (for $J \leq 10$). We show in Fig. 7 the comparison of the spin distributions for the $^{238}\text{U}(^{18}\text{O},^{16}\text{O})^{240}\text{U}$ (solid line) and $^{236}\text{U}(^{18}\text{O},^{16}\text{O})^{238}\text{U}$ (dashed line) reactions at 180 MeV; each of the residual nuclei is assumed to be a ground state. One clearly sees that the two results show good agreement. Therefore, the two reactions can be used as a pair of surrogate reactions required in the SRM in Ref. 9. It should be noted that we ignore the spectroscopic factors of the populated states in the present calculation. This can be justified by the fact that in the SRM two spectroscopic factors corresponding to two surrogate reactions are expected to cancel out by taking the ratio of the spin distributions; these two spectroscopic factors are considered to have similar $J^\pi$-dependence.
Fig. 7. (color online) Same as in Fig. 2 but for $^{236}\text{U}(^{18}\text{O},^{16}\text{O})^{238}\text{U}$ at 180 MeV (solid line). Result for $^{238}\text{U}(^{18}\text{O},^{16}\text{O})^{240}\text{U}$ at 180 MeV is also shown by the dashed line for comparison.

In summary, we present the differential cross sections of the two-nucleon transfer reaction $^{238}\text{U}(^{18}\text{O},^{16}\text{O})^{240}\text{U}$ and the spin distribution of the residue calculated with CDCC-BA based on a $^{16}\text{O}+2n+^{238}\text{U}$ three-body model. The angular distribution of the transfer reaction at 180 MeV with the binding energy $\epsilon_B = 10.74$ MeV, which corresponds to the ground state of $^{240}\text{U}$, is bell-shaped with the peak around the outgoing angle $\theta \sim 35^\circ$, while that of $^{238}\text{U}(^{3}\text{He},p)^{240}\text{Np}$ at 30 MeV is very complicated. The spin distribution of the $^{238}\text{U}(^{18}\text{O},^{16}\text{O})^{240}\text{U}$ cross section at the peak is explained by the difference between the grazing momenta in the initial and final channels, and has the peak at $J = 5$. The shape of the spin distribution hardly depends on the incident energy and the optical potential parameters. The breakup cross section of $^{18}\text{O}$ in the present system is very small and the breakup effect on the gross features of the spin distribution is negligible. With increasing the excitation energy of the $^{240}\text{U}$ produced, the peak of the spin distribution shifts to the larger $J$. Because the peak for $\epsilon_B = 0.74$ MeV corresponding to the neutron energy of 4.07 MeV is located at $J \sim 9$, this energy will be the upper limit that the SRM works well, according to the conclusion in Ref. [9].

For more detailed studies on the spin distribution of two-neutron transfer reaction, one should take into account the contribution of the sequential transfer of the two neutrons by a two-step process. This requires three-body description of $^{18}\text{O}$ and $^{240}\text{U}$, i.e., four-body CDCC. Effects of the deformation of $^{238}\text{U}$, which produces rotational states, will also be important. It is expected that the spin distribution spreads slightly because of the coupling with the rotational angular momentum, which is suggested to be around $2\hbar$ by the recent work with a dynamical model based on multi-dimensional Langevin equations in Ref. [26].

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