Small-$x$ Resummations for the Structure Functions $F_{2p}^p$, $F_{pL}^p$ and $F_{2\gamma}^\gamma$

J. Blümlein* and A. Vogt†

*DESY–Zeuthen, Platanenallee 6, D–15735 Zeuthen, Germany
† Institut für Theoretische Physik, Universität Würzburg, Am Hubland, D–97074 Würzburg, Germany

Abstract. The numerical effects of the known all-order leading and next-to-leading logarithmic small-$x$ contributions to the anomalous dimensions and coefficient functions of the unpolarized singlet evolution are discussed for the structure functions $F_{2p}^p(x, Q^2)$, $F_{pL}^p(x, Q^2)$, and $F_{2\gamma}^\gamma(x, Q^2)$.

Introduction

The evolution kernels of the deep-inelastic scattering (DIS) structure functions contain large logarithmic contributions for small Bjorken-$x$. The effect of resumming these terms to all orders in $\alpha_s$ can be consistently studied in a framework based on the renormalization group (RG) equations, which describes the mass factorization. In this framework, the evolution equations of fixed-order perturbative QCD are generalized by including the resummed small-$x$ contributions to the respective anomalous dimensions and Wilson coefficients [1–4] beyond next-to-leading order in $\alpha_s$ (NLO). The numerical impact of these higher-order contributions has been investigated for the non-singlet nucleon structure functions $F_{2p}^{p-n}$ and $F_{3\nu}^{N}$ [5], $g_{1\nu}^{p-n}$ [5,6] and $g_{5}^{\gamma Z}$ [7]; for the polarized singlet quantity $g_{1S}^{p}$ [8], and for the unpolarized singlet structure functions $F_{2S}$ [9–11] and $F_{pLS}$ [10]. $F_{2S}$ and $F_{pLS}$ have been studied using different RG-based approaches as well [12].

In the present note we extend a previous account [7] by considering, besides the resummed next-to-leading logarithmic small-$x$ (NL$x$) quark terms of ref. [2], also the recently derived NL$x$ contributions $\propto N_f$ to the anomalous

1) Talk presented by J. Blümlein. To appear in: Proceedings of the 5th International Workshop on Deep Inelastic Scattering and QCD (DIS97), Chicago, April 1997

© 1997 American Institute of Physics
dimension $\gamma_{gg}$ [3] and their impact on $F_2^p$. Furthermore, we briefly discuss the numerical resummation effects on the evolution of $F_L^p$ and the photon structure function $F_2^\gamma$. Details of the calculations may be found in ref. [10].

The NL$x$ Contributions $\propto N_f$ to $\gamma_{gg}$

These terms were calculated in ref. [3]. In the $\overline{\text{MS}}$–DIS scheme they read [10]

$$
\gamma_{gg,NL}^q = \gamma_{gg,NL}^{Q_0} + \frac{\beta_0}{4\pi} \alpha_s d \ln R(\alpha_s) + \frac{C_F}{C_A} [1 - R(\alpha_s)] \gamma_{gg,NL}^{Q_0} \\
\equiv \alpha_s \sum_{k=1}^{\infty} \left[ \frac{N_f}{6\pi} \left( d_{gg,k}^{q(1)} + \frac{C_F}{C_A} d_{gg,k}^{q(2)} \right) + \frac{\beta_0}{4\pi} \hat{r}_k \right] \left( \frac{\alpha_s}{N - 1} \right)^{k-1},
$$

with $\gamma_{gg,NL}^{Q_0}$ being the $N_f$ contribution in the $Q_0$ scheme [13]. $N$ denotes the usual Mellin variable, $\overline{\alpha_s} \equiv C_A \alpha_s / \pi$, and $R(\alpha_s)$ is defined in ref. [2]. $\gamma_{gg,NL}^q$ contains terms $\propto C_F/C_A$ in both schemes, whereas the $\beta_0$-contribution originates in transformation from the $Q_0$ scheme to the $\overline{\text{MS}}$–DIS scheme. Numerical values for the coefficients $d_{gg,k}^{q(1,2)}$ and $\hat{r}_k$ are given in Table 1.

| $k$ | $d_{gg,k}^{q(1)}$ | $d_{gg,k}^{q(2)}$ | $\hat{r}_k$ |
|-----|------------------|------------------|--------------|
| 1   | -1.000000000 E+0 | 0.000000000 E+0 | 0.000000000 E+0 |
| 2   | -3.833333333 E+0 | 0.000000000 E+0 | 0.000000000 E+0 |
| 3   | -2.299510376 E+0 | 0.000000000 E+0 | 0.000000000 E+0 |
| 4   | -5.065605818 E+0 | 3.205485075 E+0 | 9.61645224 E+0 |
| 5   | -3.523670351 E+1 | 8.56870514 E-0  | -3.246969702 E+0 |
| 6   | -3.218245315 E+1 | 1.83544765 E+1  | 2.281241061 E+1 |
| 7   | -1.060268680 E+2 | 8.63283009 E+1  | 1.654162989 E+2 |
| 8   | -4.835159484 E+2 | 1.92408636 E+2  | -2.469139930 E+0 |
| 9   | -5.806186371 E+2 | 4.96234497 E+2  | 7.458249428 E+2 |
| 10  | -2.176371931 E+3 | 1.79474281 E+3  | 2.784859262 E+3 |
| 11  | -7.553679737 E+3 | 4.02332019 E+3  | 1.50501272 E+3 |
| 12  | -1.158215080 E+4 | 1.13655938 E+4  | 1.81320928 E+4 |
| 13  | -4.328579102 E+4 | 3.58963882 E+4  | 4.899274185 E+5 |
| 14  | -1.269309428 E+5 | 8.41252988 E+4  | 6.109247725 E+5 |
| 15  | -2.392549581 E+5 | 2.45609713 E+5  | 3.984470167 E+5 |
| 16  | -8.49557573 E+5  | 7.16857201 E+6  | 9.205515787 E+5 |
| 17  | -2.262541206 E+6 | 1.76458723 E+6  | 1.783326920 E+6 |
| 18  | -4.974873276 E+6 | 5.16784417 E+6  | 8.347774614 E+6 |
| 19  | -1.648990863 E+7 | 1.44300988 E+7  | 1.842662795 E+7 |
| 20  | -4.222994214 E+7 | 3.70226358 E+7  | 4.535538189 E+7 |

Table 1: Numerical values of the expansion coefficients for $\gamma_{gg,NL}^{DIS}$ in eq. (1).
Less Singular Small-$x$ Contributions to $\gamma$

The small-$x$ resummed anomalous dimension matrix $\hat{\gamma}^{\text{res}}$ does not comply with the energy-momentum sum rule for the parton densities. Several prescriptions have been imposed for restoring this sum rule beyond NLO [9–11], e.g.,

\begin{align}
A : \hat{\gamma}^{\text{res}}(n, \alpha_s) & \rightarrow \hat{\gamma}^{\text{res}}(n, \alpha_s) - \hat{\gamma}^{\text{res}}(0, \alpha_s) \\
B : \hat{\gamma}^{\text{res}}(n, \alpha_s) & \rightarrow \hat{\gamma}^{\text{res}}(n, \alpha_s) (1 - n) \\
D : \hat{\gamma}^{\text{res}}(n, \alpha_s) & \rightarrow \hat{\gamma}^{\text{res}}(n, \alpha_s) (1 - 2n + n^3).
\end{align}

The difference between the results obtained with these prescriptions allows for a rough estimate of the possible effect of the presently unknown higher-order terms less singular at small-$x$ ($n \equiv N-1 \rightarrow 0$).

The Resummed Evolution of $F_2^{ep}$ and $F_L^{ep}$

The numerical effect of the known small-$x$ resummations on the behavior of the proton structure functions $F_2$ and $F_L$ is illustrated in Fig. 1. For both the NLO and the resummed calculations, the MRS(A') DIS-scheme parton densities have been employed as initial distributions at $Q_0^2 = 4$ GeV$^2$, together with $\Lambda_{\overline{MS}}^{(4)} = 231$ MeV [14]. They behave like $xg, xq \sim x^{-0.17}$ at small $x$, with the quark part rather directly constrained by present HERA $F_2$ data.

Figure 1: The resummed small-$x$ evolutions of the proton structure functions $F_2$ and $F_L$ compared to the NLO results. The dotted curve in the $F_2$ part represents the contribution of $\gamma_{gg,\text{DIS}}^{\text{res}}$ only. The possible impact of (presently unknown) less singular higher-order terms is indicated, cf. eq. (2) and the discussion in the text.
The resummation effects on $F_2(x, Q^2)$ at small $x$ are displayed in Fig. 1 (a). Note the huge effect arising from the NL$x$ quark anomalous dimensions [2] and its large uncertainty due to unknown less singular terms. The impact of $\gamma_{gq, NL}^\gamma$ [3] is displayed separately. It amounts to less than 3% over the full $x$-range shown. It will be interesting to see to which extent the forthcoming complete NL$x$ anomalous dimensions [15] will modify these results.

The longitudinal structure function $F_L(x, Q^2)$ is considered in Fig. 1 (b). Obviously substantial contributions can also be expected from subleading small-$x$ terms in the coefficient functions $C_L$. In fact, these uncertainties are large. Thus both for the small-$x$ resummed contributions to anomalous dimensions and coefficient functions further subleading terms need to be calculated. Further insight into the interplay of leading and less singular terms in $N$ may also be gained from the structure of the fixed-order anomalous dimensions and coefficient functions. Besides the known NLO result, particularly the yet unknown 3-loop anomalous dimensions are of interest here.

**The Resummation of the Small-$x$ Contributions to $F_2^\gamma$**

The evolution of the photon structure functions is, at the lowest order in $\alpha_{em}$ considered here, governed by an inhomogeneous generalization of the hadronic evolution equations. At the present resummation accuracy [1,2] the additional anomalous dimensions $\gamma_{q\gamma}$ and $\gamma_{g\gamma}$ do not receive any non-vanishing higher-order small-$x$ contributions [10]. Hence the resummation effect on the photon-specific inhomogeneous solution originates solely from the resummed homogeneous evolution operator.

![Figure 2: The small-$x$ evolution of the photon structure function $F_2^\gamma$ in NLO and using the NL$x$ resummed anomalous dimensions.](image)
The resummed evolution of the structure function $F_2^\gamma$ is compared to the NLO results in Fig. 2. The NLO GRV parametrization has been used for the initial distributions at $Q_0^2 = 4 \text{ GeV}^2$, together with $\Lambda_{\overline{\text{MS}}}^{(4)} = 200 \text{ MeV}$ [16]. The overall small-$x$ behavior, presented in Fig. 2 (a), is rather similar to the hadronic case, due to the dominance of the homogeneous solution. Note, however, the significantly enhanced resummation effect in the inhomogeneous solution separately shown in Fig. 2 (b). This behavior is dominated by the convolution of the resummed hadronic evolution operator with the leading-order photon-quark anomalous dimension, which, unlike the hadronic initial distributions, is large for $x \to 1$.

**Acknowledgement:** This work was supported in part by the German Federal Ministry for Research and Technology (BMBF) under contract No. 05 7WZ91P (0).

**REFERENCES**

1. Y. Balitsky and L. Lipatov, *Sov. J. Nucl. Phys.* **28** 822 (1978).
2. S. Catani and F. Hautmann, *Nucl. Phys.* **B427** 475 (1994).
3. G. Camici and M. Ciafaloni, *Phys. Lett.* **B386** 341 (1996); *hep-ph/9701303*.
4. R. Kirschner and L. Lipatov, *Nucl. Phys.* **B213** 122 (1983);
   J. Bartels, B. Ermolaev, and M. Ryskin, *Z. Phys.* **C72** 627 (1997).
5. J. Blümlein and A. Vogt, *Phys. Lett.* **B370** 149 (1996);
   *Acta Phys. Polonica* **B27** 1309 (1996).
6. J. Kiyo, J. Kodaira, and H. Tochimura, *hep-ph/9701365*.
7. J. Blümlein, S. Riemersma, and A. Vogt, *Nucl. Phys.* **B** (Proc. Suppl.) **51C** 30 (1996).
8. J. Blümlein and A. Vogt, *Phys. Lett.* **B386** 350 (1996).
9. R.K. Ellis, F. Hautmann, and B. Webber, *Phys. Lett.* **B348** 582 (1995);
   R. Ball and S. Forte, *Phys. Lett.* **B351** 313 (1995), **B358** 365 (1995).
10. J. Blümlein and A. Vogt, DESY 96–096.
11. I. Bojak and M. Ernst, *Phys. Lett.* **B397** 296 (1997); *hep-ph/9702282*.
12. J. Forshaw, R. Roberts, and R. Thorne, *Phys. Lett.* **B356** 79 (1995);
   R. Thorne, *Phys. Lett.* **B392** 463 (1997); *hep-ph/9701241*.
13. M. Ciafaloni, *Phys. Lett.* **B356** 74 (1995).
14. A.D. Martin, R.G. Roberts, and W.J. Stirling, *Phys. Lett.* **B354** 155 (1995)
15. M. Ciafaloni et al., in preparation.
16. M. Glück, E. Reya, and A. Vogt, *Phys. Rev.* **D46** 1973 (1992).