Research on Ultrasound Image of Interventional Catheterization Processing Method Based on Wavelet Transform and Fuzzy Theory

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Abstract. Interventional catheterization can help patients to accurately assess the condition, early diagnosis and intervention. Confirming the location of catheter by ultrasound has the advantages of real-time imaging, non-invasive, radiative, fast and convenient. Due to speckle noise and similar acoustic impedance, ultrasound images are not clear. In this paper an ultrasonic image processing algorithm based on wavelet transform and fuzzy theory is proposed. First, logarithmic transformation of ultrasound images is used to convert multiplicative noise into additive noise. Then the wavelet coefficients of the image are obtained by multiscale wavelet transform. The high frequency wavelet coefficients of the image are denoised by thresholding, and the low-frequency wavelet coefficients of the image are processed by fuzzy enhancement. Finally, the processed image is obtained through wavelet reconstruction and exponential transformation. Experiments show that this proposed method can effectively improve the visual effect of images.

1. Introduction

Ultrasonic image denoising has always been a hot research topic in ultrasound image. The method of ultrasonic image denoising is mainly divided into two categories: One is the spatial domain method, mainly mean filtering, Wiener filtering, median filtering, Lee filtering and anisotropic diffusion algorithm. Another is frequency domain method based on Fourier transform and wavelet transform. The main difficulty of the spatial domain filtering method is that how to achieve denoising while protecting the edge details of the image. In recent years, some scholars, such as Loupas, proposed the adaptive spatial domain filtering method\cite{1}. But it is still difficult to protect the edge details of the image when the noise is removed\cite{2}. In the frequency domain denoising method, wavelet denoising method has developed rapidly. In 1995, the wavelet threshold denoising method proposed by Donoho was widely used in image processing\cite{3}. With the unique time-frequency analysis characteristics, it has well solved the problem of protecting the edge of the image at the same time, and has been praised by scholars as “mathematical microscope”. On the basis of anisotropic diffusion equation and median filtering, recently, some scholars have designed a medical ultrasonic image filtering method based on edge preserving, also achieved good results\cite{4}.
Image enhancement means purposeful enhancement or suppression of some image information so as to enhance the readability of the image. At present, the main image enhancement methods include histogram modification, high pass filtering, filtering algorithm based on mathematical morphology, enhancement algorithm based on wavelet transform and enhancement algorithm based on fuzzy mathematics. Fundamentally speaking, image enhancement does not have a universal judgement standard. Observers are the most powerful judges of image enhancement. In recent years, fuzzy enhancement method has introduced fuzzy mathematical theory into the image enhancement method. It has achieved good results in dealing with the fuzziness of the image enhancement evaluation standard from the mathematical point of view. The enhancement algorithm based on wavelet transform can suppress noise while enhancing the image, the enhancement efficiency is greatly improved.

Based on the discussion as above, in the first section of this paper, we propose an algorithm of ultrasonic image processing based on wavelet transform and fuzzy theory. In the second section, we introduce the noise model of the image and transform the multiplicative noise into additive noise. In the third section, the multi-scale wavelet transform is applied to the image and threshold denoising performed on the high frequency part. The fourth section is fuzzy enhancement of the low frequency part of the image. In the fifth section, the specific algorithm flow is given. Finally, in the sixth section the experiment results prove that the proposed algorithm can not only effectively remove the noise, but also protect and enhance the image edge details, thus greatly improving the “readability” of the image.

2. Noise Model of Ultrasonic Image

Because of the imaging principle, medical ultrasound images will inevitably generate some noise during the imaging process, mainly speckle noise. Karamra M and other studies show that the common speckle noise obeys Rayleigh distribution, and its mean is proportional to the standard deviation, which indicates that the speckle noise is multiplicative[5]. Therefore, in 1989, Jain proposed an ultrasonic image noise model that combines multiplicative noise and additive noise,Formula (1) can be rewritten as:

\[ F(x, y) = h(x, y) \ast m(x, y) + a(x, y) \]  \hspace{1cm} (1)

Formula (1) can be rewritten as:In the formula, \( F \) is pixel gray value, \( h \) is a noise free image that needs to be restored. \( m \) is multiplicative noise, and \( a \) is additive noise, \( x \) and \( y \) is pixel coordinates. In medical ultrasonic images, the effect of additive noise is rather small in multiplicative noise, so the effect of additive noise can often be ignored in practical applications.

\[ F(x, y) = h(x, y) \ast m(x, y) \]  \hspace{1cm} (2)

Logarithm for both sides of formula (2) can transform the multiplicative noise model into additive noise model, such as formula (3):

\[ F(x, y) = h(x, y) + m(x, y) \]  \hspace{1cm} (3)

3. Wavelet Threshold De-noising

According to the Mallat pyramid decomposition algorithm, the fast wavelet decomposition of two-dimensional images can be realized, and its decomposition structure is shown in Figure 1.

In Figure 1, \( A_i \) is the approximate component of each decomposition level, \( H_i \) is the horizontal directional detail component, \( V_i \) is the vertical detail component, and \( D_i \) is the diagonal direction detail component. The low frequency coefficient \( A_i \) is enhanced and the high frequency coefficients \( H_i, V_i \) and \( D_i \) in all directions are de-noised, and the enhanced and de-noised images can be obtained through the reconstruction of the wavelet inverse transform.

The algorithm idea of wavelet threshold de-noising is that the image and noise have different statistical characteristics after the wavelet transform, and the energy of the image itself corresponds to the large
amplitude of the wavelet coefficient, which is mainly concentrated in the low frequency section. The noise energy corresponds to the wavelet coefficients with smaller amplitudes, mainly concentrated in the high frequency band. According to this feature, a threshold is set up. It is considered that the main component of the wavelet coefficients larger than that of the threshold is a useful signal, which is reserved. The wavelet coefficients less than the threshold value, the main component of which is noise, is filtered, thus the denoising purpose can be achieved.

There are two kinds of threshold function: hard threshold function and soft threshold function.

The expression of the hard threshold function is as formula (4), and as shown in the figure 2:

\[
U = \begin{cases} 
W, |W| \geq \lambda \\
0, |W| < \lambda
\end{cases}
\]  

(4)

In the formula, \( \lambda \) is the threshold, \( W \) is the value of the wavelet coefficient, and \( U \) is the size of the wavelet coefficient after the threshold function is applied. When the absolute value of the wavelet coefficient is greater than the threshold value, it remains unchanged; when the absolute value of the wavelet coefficient is less than the threshold value, set it as 0.
The expression of the soft threshold function is as formula (5), and as shown in the figure 3:

\[ U = \begin{cases} 
   \text{Si}_1(n(W)(|W| - \lambda)), & |W| \geq \lambda \\
   0, & |W| < \lambda 
\end{cases} \quad (5) \]

\( \text{Si}_1(n) \) represents a symbolic function. When the absolute value of the wavelet coefficient is greater than the threshold value, it subtracts the threshold value; when the absolute value of the wavelet coefficient is less than the threshold value, set it as 0.

The hard threshold method can preserve the edge and other local features of the image better, but the estimated wavelet coefficients are poor in continuity, which may cause the shock of the reconstructed signal and make the image appear ringing, pseudo Gibbs effect and other visual distortion[6]. Soft thresholding usually makes the denoised signal smooth, but it also loses some features, making the edge fuzzy. In view of the characteristics of the soft and hard thresholding, a compromise method is selected in this paper. Compared with the soft and hard threshold method, the compromise threshold can achieve an ideal effect. In this method, we need to set two thresholds, and the high threshold \( \lambda_2 \) is two times of the low threshold value, that is, \( \lambda_2 = 2\lambda_1 \), and its specific threshold function is as formula (6), and as shown in the figure 4:

\[ U = \begin{cases} 
   W, & |W| \geq \lambda_2 \\
   \text{sign}(W) \frac{\lambda_2(|W| - \lambda_1)}{\lambda_2 - \lambda_1}, & \lambda_2 < |W| < \lambda_1 \\
   0, & |W| < \lambda_1 
\end{cases} \quad (6) \]
In the Figure, $W$ is wavelet coefficients, $U$ is its amplitude.

The key of wavelet threshold denoising is threshold selection. If the threshold selection is too small, it will not achieve the effect of denouncing. If the threshold selection is too large, it will remove useful ingredients, cause distortion. Because the unified threshold value $E = \sigma$ is too large for $N$, it tends to stifle wavelet coefficients, which will lead to relatively large reconstruction errors[7]. This algorithm uses the Bayes Shrink threshold:

$$\lambda = \frac{\sigma^2}{\sigma_x}$$  \hspace{1cm} (7)

$\sigma$ is the standard deviation of noise, and $\sigma_x$ is the standard variance of the generalized Gauss distribution.

$$\sigma_x = \sqrt{\frac{1}{M \times N} + \sum_{i,j} Y_{i,j}^2}$$  \hspace{1cm} (8)

Among them, $M \times N$ is the number of sub-band wavelet coefficients, $Y_{i,j}$ are the wavelet coefficients of the image at a certain scale. The threshold of BayesShrink is different due to the different subscales of different subscales and directions. The corresponding threshold is also different, that is, the threshold is adjusted adaptively with $\sigma_x$. It can not only achieve good denoising effect, but also need not calculate too much.

### 4. Fuzzy Enhancement Algorithm

The fuzzy set theory was first proposed by L.A. zadeh of California University in the United States in 1965. Pal and King first introduced the fuzzy theory into the image enhancement processing research in 1981[9]. They transform the image from the gray level of the space domain to the feature plane of the fuzzy domain (the plane of membership degree), and then make fuzzy enhancement of the transformed image, and then get the enhanced image by inverse transform of the fuzzy domain. In the algorithm of Pal and King[10], an image $X$ with a size of $M \times N$ and a gray level $L$ can be represented as a fuzzy matrix of $M \times N$.

According to the fuzzy theory, it can be expressed as:

$$X = U^M_{i=1} U^N_{j=1} u_{ij} x_{ij}$$  \hspace{1cm} (9)

$u_{ij}$ is the membership grade of the gray $x_{ij}$ of the pixels $(i, j)$ of the image relative to a particular gray level, which is a fuzzy distribution. In the algorithm of Pal and King, the membership function is defined as:

$$u_{ij} = T(x_{ij}) = \left[1 + \frac{(L - 1) - x_{ij}}{F_d}\right]^{-F_e}$$  \hspace{1cm} (10)

$F_d$ and $F_e$ are reciprocal fuzzy factor and exponential fuzzy factor respectively. Usually $F_e=2$. At the same time, the definition is: when the special gray level of $u_{ij} = T(x_{ij}) = 0.5$ is called the transition point, $F_d$ is determined by the transition point. After obtaining the $u_{ij}$, the transformation function $G_r(\cdot)$ is used to enhance the image in the fuzzy domain ($r$ is the number of iterations). The fuzzy enhancement operator is defined as:

$$u_{ij} = G_r(u_{ij}), r = 1, 2, 3, \ldots$$  \hspace{1cm} (11)

$$G_r(u_{ij}) = \begin{cases} 
2(u_{ij})^2, & 0 \leq u_{ij} < 0.5 \\
1 - 2(1 - u_{ij})^2, & 0.5 \leq u_{ij} < 1
\end{cases}$$  \hspace{1cm} (12)

Finally, the inverse transformation of the enhanced $u_{ij}$ is performed, and get the image $Y$ after the fuzzy enhancement. The gray value of the pixels $(i, j)$ in the $Y$ is:

$$x_{ij} = T_{u_{ij}}^{-1}$$  \hspace{1cm} (13)
However, in the classical Pal algorithm, the selection of transition points is obtained only by experience or multiple attempts is random. Choosing different thresholds has great influence on image enhancement effect. After the image fuzzy membership transformation, the negative value obtained from the inverse transformation formula of membership degree. In the Pal algorithm, the gray value of the negative value is set to 0, which causes some information loss and distortion of the image. The algorithm uses power exponent function as its fuzzy membership function, and its computation is large and time-consuming when it transforms the image from gray space to fuzzy space and its inverse transformation.

Based on the algorithm of Pal and King, a sine membership function is selected to transform the image into the fuzzy domain. The function is defined as follows:

$$u_{ij} = [\sin(\frac{\pi}{2} \times \frac{x_{ij} - x_{\min}}{x_{max} - x_{\min}})]^k$$

(14)

$u_{ij}$ represents the membership of the gray $x_{ij}$ of pixels $(i, j)$ relative to the maximum gray level $x_{max}$, $x_{\min}$ is the minimum gray value of the image, and $k$ is the adjustment parameter. Compared with the power exponent function in the Pal algorithm, the membership function improves the speed of operation and does not appear to be hard to cut off.

The fuzzy enhancement operator (12) is used to enhance the image quality and transform the data from the fuzzy domain to the spatial domain of the image. The inverse transformation formula is as follows[9]:

$$x_{ij} = x_{\min} + (x_{max} - x_{\min}) \cdot \arcsin(u_{ij}^{1/k}) \cdot \frac{2}{\pi}$$

(15)

By adjusting the parameters $k$, the image can be processed by fuzzy enhancement.

5. Algorithm Flow

A. The algorithm flowchart is shown as shown in the diagram

![Flow Chart](image)

Figure 5: flow chart

B. The algorithm flow is as follows:

- 1. Taking logarithmic transformation of the original image to convert the multiplicative noise in the image to additive noise.

- 2. Transforming the image to the wavelet transform and select the bior3.3 wavelet function basis which has biorthogonality to decompose the image into three layers and get the corresponding wavelet coefficients.
3. Fuzzy domain enhancement for low frequency coefficients after wavelet decomposition, the sinusoidal membership function is used to transform between the spatial domain and the fuzzy domain, which effectively avoids the gray value being cut off in the classical Pal algorithm.

4. Wavelet threshold de-noising for high frequency coefficients after wavelet decomposition, and Bayes Shrink threshold is used to de-noise wavelet coefficients by semi soft threshold.

6. Experimental Results and Analysis

In figure 6a is an international catheter ultrasound image, size 251 × 494, gray level of 0–255. There are a lot of speckle noise in the original image, and the impedance is similar to that of the contrast of the image is poor. First, the traditional de-noising and enhancement methods are applied to image processing to verify the effectiveness of the algorithm proposed in this paper. Figure 6b is obtained by wiener filter, the image can be seen in the noise is eliminated to a certain extent, but the contrast is not enhanced, and some image details is destroyed. Figure 6c is histogram equalization image, which is a traditional method for enhancing contrast, we can see the image contrast was increased significantly, but in the enhancement of the image also enhance the noise, and it affects the visual effect of the image. Figure 6d is only processed by fuzzy enhancement, because this algorithm directly enhances the image, and most of the gray values of pixels are similar, it can not achieve a good enhancement effect, in addition, the noise of the image is also enhanced. Figure 6e is only processed by wavelet de-noising, the image is not enhanced. Figure 6f is the image after this algorithm is processed, it can be seen in the image contrast enhancement and noise suppression, making the image details clear, strong sense of hierarchy, making the image obtain better visual effect.

![Figure 6: Experimental image](image_url)

From the analysis as above, we can draw the following conclusions: In the ultrasound image of interventional catheterization, not only the contrast is low, but also has a lot of speckle noise. Therefore, we not
only need for image noise reduction processing, but also to enhance the image, improve the image contrast. The key in the image processing is that can effectively remove the noise while enhancing the image, and can protect the edge details of the image while removing noise. The algorithm used in this paper combines wavelet de-noising and fuzzy enhancement, can take the advantages of both, when the image is enhanced, the noise can be effectively removed and the edge details of the image can be protected. Experiments show that the algorithm can effectively improve the visual effect of interventional catheter ultrasound image and lay the foundation for future processing.

References

[1] Loupas T, Mcdicken, Allan P L. An adaptive weighted median filter for speckle suppression in medical ultrasonic images[J]. IEEE Transactions on Circuits and Systems, 1989, 36(1): 129-135.

[2] HAN Han. Research on ultrasonic image filtering and enhancement algorithm[D]. Harbin: Harbin Institute of Technology, 2010.

[3] Donoho DL. De-noising by soft-thresholding[J]. IEEE Trans Inform Theory, 1995, 41(3): 613-627.

[4] CHEN Wenshan. Filtering method for medical ultrasonic images based on edge preserving[J]. Manufacturing automation, 2018, 40(06): 154-156.

[5] Karamra M., Kutay MA, Bszdagi G. An adaptive speckle suppression filter for medical ultrasound imaging[J]. IEEE Transactions on Medical Imaging, 1995: 14(2): 283-292.

[6] GUO Min, MA Yuan-liang, ZHU Ting. A method of medical ultrasonic image denoising and enhancement based on wavelet transform[J]. Chin J Med Imaging Technol, 2006, 22(9): 1435-1437.

[7] Donoho D L, Johnstone I M, Kerkyacharian G, et al. Wavelet shrinkage: Asymptopia[J]. Journal of the Royal Statistical Society, 1995, 57(2): 301-369.

[8] Chang S G, Yu B, Vetterli M. Adaptive wavelet thresholding for image denoising and compression[J]. IEEE Transactions on Image Processing, 2002, 9(9): 1532-1546.

[9] Pal S K, King R A. Image enhancement using smoothing with fuzzy sets [J]. IEEE Trans, Syst, Man, Syst, Man, Cybern, 1981, SMC-11: 494-501.

[10] CAI Yan-yan, XU Li-ping, GUO Chang-ying, et al. Application of fuzzy logic to the enhancement of pneumoconiosis X-ray films[J]. Microcomputer Information, 2008, 24(4-3): 290-292.

[11] LI Ping, Research on medical image enhancement algorithm based on Fuzzy Theory[D]. North University of China, 2017.