Cavitating vortex characterization based on acoustic signal detection

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Abstract. In hydraulic turbines operating at part loads, a cavitating vortex structure appears at runner outlet. This helical vortex, called vortex rope, can be cavitating in its core if the local pressure is lower that the vaporization pressure. An actual concern is the detection of the cavitation apparition and the characterization of its level. This paper presents a potentially innovative method for the detection of the cavitating vortex presence based on acoustic methods. The method is tested on a reduced scale facility using two acoustic transceivers positioned in "V" configuration. The received signals were continuously recorded and their frequency content was chosen to fit the flow and the cavitating vortex. Experimental results showed that due to the increasing flow rate, the signal - vortex interaction is observed as modifications on the received signal's high order statistics and bandwidth. Also, the signal processing results were correlated with the data measured with a pressure sensor mounted in the cavitating vortex section. Finally it is shown that this non-intrusive acoustic approach can indicate the apparition, development and the damping of the cavitating vortex. For real scale facilities, applying this method is a work in progress.

1. Introduction

Due to the present conditions in the energy market, the hydraulic turbines are frequently required to operate far from the optimum operation regime. Usually, at partial flow rate operation, a helical vortex appears in the turbine draft tube which can be cavitating in its core. The effects are undesirable pressure fluctuations and vibrations which can affect the operation of the turbine and the entire hydraulic circuit. This is why is important to detect such phenomena before the effects produce irreversible damage to the machine or hydraulic circuit.

The present paper continues the research on the detection of the cavitating vortex based on non-intrusive active devises like the acoustic transducers. In previous works, the influence of the vortex rope presence over the acoustic signal bandwidth was analyzed \cite{1} and different acoustic
transducers configuration were used [2]. The results of both works mentioned above suggested that acoustic techniques could be robust and efficient methods for detection of cavitating vortex presence in flow through hydraulic machinery.

In the present work the objective is to emphasize the effect of the cavitating vortex upon the recorded signal in order to identify the different phases of evolution of the phenomenon using two pairs of acoustic transducers. Measurements were carried out on a laboratory facility which simulates the flow conditions downstream a hydraulic turbine. The Recurrence Plot Analysis (RPA) method was applied on the recorded signals in order to characterize the state of the flow in its evolution.

This paper is organized in five parts. After the introduction in section 1, section 2 presents the relation between the vortex flow and spectrum. Because the received Doppler spectrum is clearly related to the spatial velocity distribution of a Rankine vortex [3], the research will begin from this aspect. Further, section 3 describes the RPA method and highlights the quantification of the diagonal lines specific for the recurrence matrix. More, an interesting approach of the spectral analysis of the diagonal lines is presented. The experimental works and results are discussed in section 4, followed by the conclusions and perspectives in section 5.

2. The vortex flow and vortex spectrum
In literature the vortex vector is defined as \( \nabla \times u = 2\omega \), where \( \omega \) is the rotational speed of the fluid. The flow for an incompressible fluid of velocity \( u \) is described by the Navier-Stokes (1) and continuity equations (2) [3, 4]:

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho} \nabla p + v \nabla^2 u \tag{1}
\]

\[
\nabla \cdot u = 0 \tag{2}
\]

where \( p \) is the fluid pressure, \( \rho \) is the density of the fluid, \( u \) is the flow velocity vector, \( v \) is the flow viscosity and \( \nabla \) is del operator.

For vortex analysis, the Rankine model can be used. Analytically, the Rankine vortex is defined by the components from (3) [5]:

\[
u_r = 0; u_z = 0; u_\theta = \begin{cases} \frac{r}{r_{max}} v_{max}, r \leq r_{max} \\ \frac{r_{max}}{r} v_{max}, r > r_{max} \end{cases} \tag{3}
\]

where \( u_r \), \( u_z \) and \( u_\theta \) are the axial, radial and, respectively, tangential velocity components defined in a cylindrical coordinate system for the \( u \) flow velocity. It should be mentioned that, when \( r \leq r_{max} \), the tangential velocity \( u_\theta \) progresses in the forced vortex region, while, when \( r > r_{max} \), its progress happens in the free vortex region.

The frequency signature of the cavitating vortex is given by the solution (3) of equations (1) and (2) through the solution \( \omega \). Hereby, the relation between the vortex flow and its spectrum is:

\[
u_r = 0; u_z = 0; u_\theta = \begin{cases} \omega \cdot r, r \leq r_{max} \\ \frac{\omega}{r_{max}}, r > r_{max} \end{cases} \tag{4}
\]

In [5], there can be found more details about the spectral magnitude of Doppler velocity.
3. Recurrence Plot Analysis
The nonlinear time series analysis that characterizes the general dynamic system is used for the
signal processing of nonlinear systems. This method has its origins in the nonlinear time series
analysis which begins with a time series obtained from a measured set of data:
\[ x = \{x[1], x[2], \ldots, x[N]\} \quad (5) \]
which is represented in a \(m\)-dimensional space, named the phase space where \(m\) is the embedding
dimension. The following vectorial representation gives the trajectory represented in the phase
space:
\[ \vec{v}_i = \sum_{k=1}^{m} x[i + (k - 1)d] \cdot \vec{e}_k, i = 1, M \quad (6) \]
where \(x[i]\) is the sample from the time series \((5)\), \(\vec{e}_k\) are the unit vectors of the axis of the phase
space, \(M = N - (m - 1)d\) is the delay parameter. The values for parameters \(d\) and \(m\) are
important and chosen, in a manner which will be detailed.

Using the RPA method, the distance/recurrence matrix is determined after the trajectory is
obtained \[6\]. A certain type of distance between two points \(i\) and \(j\) on the trajectory is used
to obtain the matrix, each distance being compared with a threshold:
\[ D_{i,j} = \Delta(v_i, v_j) \quad (7) \]
\[ R_{i,j} = \Theta(\varepsilon - \Delta(v_i, v_j)) \quad (8) \]

where \(D(i, j)\) is the element of the distance matrix corresponding to the points \(i\) and \(j\), \(D(\cdot, \cdot)\) is the type of distance applied on the trajectory points (Euclidean distance, norm L1, norm
infinity, angular distance, etc.), \(R_{i,j}\) is the element of the recurrence matrix, \(\varepsilon\) is the threshold
which determines whether or not two points \(i\) and \(j\) are recurrent and \(\Theta\) is the Heaviside step
function. Generally, \(\varepsilon\) is constant.

The representation of the distance matrix is done by a continuous colored scale, while for the
representation of the recurrence matrix values 0 and 1 are used, colored in black and white \[7\].

The quality of the trajectory is directly influenced by the delay parameter, \(d\) \[8\]. A small
value of the delay will make the trajectory redundant (the trajectory is overly folded around
the first bisector of the phase space). A too large value of the delay, will make the trajectory
irrelevant (being too complicated).

The embedding dimension \(m\), must be considered in comparison to the real dimension of
the representation. If it is smaller, this trajectory would be a projection of the real trajectory
which would include points closer to each other than on the real trajectory. If the embedding
dimension is greater than the real one, then the computation resources and time would increase
significantly. So, in order to choose the optimal value, the False Nearest Neighbor (FNN)
method will be used \[6, 8, 9\], and this value will be determined by the first value of FNN equal
to zero.

It follows the quantification of the diagonal lines on the distance matrix, as presented by \[10\].
\[ sd[n] = \frac{1}{M - n} \sum_{i=1}^{M-n} D(i, j) \quad (9) \]
where \(M\) is presented below equation \(6\), \(n = 0, M - 1\) and \(D(v_i, v_{i+n})\) are the elements of the
\(n^{th}\) diagonal line of the recurrence matrix (the 0 diagonal corresponds to the main diagonal).

Furthermore, the Fourier transform is applied on the diagonal lines quantification:
\[ SD[k] = \sum_{n=0}^{M-1} sd[n] \cdot exp \left( -i \frac{2\pi kn}{M} \right) \quad (10) \]
This method has an important advantage, because it highlights several extra frequency components than the usual Fourier transform. In order to support this statement the mathematical operations will be presented in a general matter. Considering the following time series:

\[
x[n] = \cos(\Omega n) \cdot \cos(\omega n), \quad \Omega \gg \omega
\]  

(11)

the interaction between the acoustic wave characterized by the spectral parameter \( \Omega \) and a rotational vortex characterized by \( \omega \) is presented in a simple way.

The argumentation is done on a time series represented in two dimensions (so the embedding dimension, \( m \) is 2) and the evaluation is done with the distance matrix using the squared Euclidean distance:

\[
v_i^2 = x[i] \cdot \vec{e}_1 + x[i+d] \cdot \vec{e}_2
\]

(12)

\[
D(i, i+n) = (x[i] - x[i+n])^2 + (x[i+d] - x[i+d+n])^2 = E_i^2(n) + E_{i+d}(n)
\]

(13)

Equation (11) is introduced in equation (13) and it results:

\[
E_i(n) = 2\alpha \cdot \sin \frac{\Omega [2i + n]}{2} \cos \frac{\omega [2i + n]}{2}
\]

(14)

where \( \alpha = \frac{\sin(\Omega + \omega)}{2} \approx \frac{\sin(\Omega - \omega)}{2} \) considering that \( \Omega \gg \omega \).

The results from equation (14) are replaced in equation (13):

\[
sd[n] \propto D(i, i+n) \cdot \alpha^{-2} = 1 + \cos \left[2\omega \left( i + \frac{n}{2} \right) \right] - \cos \left[2\Omega \left( i + \frac{n}{2} \right) \right]
\]

(15)

\[
- \cos \left[\Omega (2i + n) \cdot \cos \left[\omega (2i + n) \right] + 1 + \cos \left[2\omega \left( i + d + \frac{n}{2} \right) \right] - \cos \left[2\Omega \left( i + d + \frac{n}{2} \right) \right]
\]

\[
- \cos \left[\Omega (2i + 2d + n) \cdot \cos \left[\omega (2i + 2d + n) \right] \right]
\]

Equation (15), shows that besides the classical components \( \Omega \pm \omega \), two extra components appear on the spectral representation of the diagonal lines quantification, one component at \( 2\Omega \) and the other at \( 2\omega \).

Further, the experimental part aims to emphasize the evolution of the cavitating vortex considering that the effect of the vortex can be attributed to the \( 2\omega \) spectral component.

4. Experimental Setup and Results

The measurements were done on the experimental set-up from [1, 2], but in a different configuration. A stator is pused to create a cavitating vortex rope, similar with vortex ropes often met in hydropower plant turbines, as presented in figures 1 and 2 [11]. The stator is placed 70 mm upstream the divergent section (previously, it was situated at 250 mm). For the acoustic analyse, two pairs of transceivers positioned in 'V' configuration were used (figures 1 and 2) instead of only one. Moreover, the transducers were placed further downstream from the cavitating vortex area, on the divergent section (figure 1), while in the previous studies they were placed on top of the cavitating vortex area. The acoustic transceivers have a wide band reception centered on 1MHz, but being able to receive signals in the range 800 – 1200kHz. The output signal varies from –20V to 20V . The transceivers were set in active configuration and the acquisition was made at a sampling frequency of 20MHz for bursts of 1ms with the repetition rate of 2.6ms. Also, besides the acoustic and discharge flow, the pressure was measured on the bottom side of the pipe, in the cavitating vortex area (right downstream the stator) using an absolute pressure transducer of 0...1 bar.

The experimental set-up is designed to create the conditions for a cavitating vortex to appear and develop while increasing the discharge to the maximum flow capacity of the set-up. The
The first set of data was obtained for the appearance and developing of the cavitating vortex during discharge increase. In order to cover the damping behavior of the cavitating vortex from a hydraulic turbine, the variation of the discharge was reversed in the set-up, so the effect of the cavitating vortex disappearance was created.

The acoustic transceivers operate around at $1 MHz$ and the emitted waveform is a wide band signal with the same central frequency. This signal, $u(\Omega)$ has a cubic characteristic law. As figure 2 shows, two of the transceivers were used as emitters and the other two as receivers. One emitted in the flow direction and is characterized by a cubic law, whereas the other emitted in the opposite direction and had an inverse cubic law.

The experiment was done while slowly varying the flow rate of the water. In other words, during the experiment the received signal is acquired and recorded in bursts of 2.6 ms simultaneously with the flow rate and the pressure of the water.

Considering that the cavitating vortex is characterized by an unknown law $v(\omega)$ and the wide band signals (that are continuously emitted) are characterized by the $u(\Omega)$, then, at the receiver, the signal carries the interaction between the emitted signal and the studied cavitating vortex in $w(\Omega, \omega)$. The purpose of the experiment is to highlight the signature of the cavitating vortex from the received signal and to compare the results with the ones obtained in different configuration.

The wide band acoustic signals transmitted during the experiment were chosen to characterize as good as possible the flow and the cavitating vortex in order to highlight the apparition, development and damping of the vortex. During the experiment, the water flow was varied from 6 to 19 l/s and backwards, in order to determine the critical flow rates for which the cavitating vortex appears/dampens.

The first set of data was obtained for water flow rate increase, when the apparition and developing of the cavitating vortex occurred. Based on the visual observation (figure 3), three stages of the vortex evolution were identified: $A_1$ region no vortex presence (up to 6l/s), $A_2$ region incipient vortex (from 6 to 7.2l/s) and $A_3$ region developed vortex (more than 7.2l/s).

Using the property of the RPA spectral analysis, the position of the $2\omega$ component from the
Figure 3: Cavitating vortex stages: $A_1$ no vortex, $A_2$ incipient vortex, $A_3$ developed vortex

The absolute value of $SD$ (the amplitude spectrum of $sd$ - eq. (9)) was determined for each burst of 2.6ms in each vortex region. Further, an example of $2\omega$ component identification is presented in figures 4 and 5. In figures 4 and 5 the main component at $1MHz$ corresponds to the central frequency of the emitted signal at the acoustic sensors. Other significant components can be found in the range of $0-100kHz$ (based on the demonstration from eq. (15)), and are attributed to $2\omega$ component, as presented in the zoomed details (figures 4b and 5b).

The frequencies corresponding to the $2\omega$ component (eg. in figures 4b and 5b for region $A_1$ at 20.14kHz, respectively for $A_3$ at 49.44kHz) are further presented as the time variation of the $2\omega$ spectral component in figures 6a and 6b. Also, the flow variation was represented simultaneous with time variation of $2\omega$ spectral component in order to identify the three cavitating vortex regions.

The normalization of the parameters presented in the following figures is done using the maximum value of each data vector.

(a) The amplitude spectrum of $A_1$ region  (b) Zoomed amplitude spectrum of $A_1$ region

Figure 4: $SD$ absolute value for an acquisition of 2.6ms in $A_1$ region

For the increasing flow rate configuration, it can be seen that, while there is no vortex ($A_1$ region), the $2\omega$ component has a reduced variation. When the vortex starts to appear in $A_2$ region, the deviation of the $2\omega$ is larger and is significantly increasing in $A_3$ region for the developed vortex. The standard deviations of the $2\omega$ component, $\sigma_{2\omega}$, for each region were computed, and cavitating vortex characterization criteria were defined according to them (table 1).

The validation of the criteria was firstly done using data recordings during decreasing water flow configuration. The $2\omega$ component was determined and its time variation was plotted
Figure 5: SD absolute value for an acquisition of 2.6ms in $A_3$ region

Figure 6: Time variation of $2\omega$ component (in blue) and the flow rate evolution (in red)

Table 1: Criteria limits for the cavitating vortex stages classification in increasing flow rate configuration

| Region | $\sigma_{2\omega}$[kHz] | Limits[kHz] | Vortex stage            |
|--------|--------------------------|-------------|-------------------------|
| $A_1$  | 0.76                     | $\sigma_{2\omega} < 1$ | No vortex               |
| $A_2$  | 2.26                     | $1 \leq \sigma_{2\omega} \leq 3$ | Incipient vortex        |
| $A_3$  | 9.71                     | $\sigma_{2\omega} > 3$  | Developed vortex        |

The cavitation vortex regions were identified by visual observations and fitted with the defined criteria (table 2). In this case, the incipient vortex stage ($A_2$ region) was not visible, which was also confirmed by the high value of standard deviation of the $2\omega$ component computed. This may be caused by the inertia of the fluid while decreasing the flow, which was done in a shorter period than in the increasing case (about 15s...
Table 2: Criteria validation for decreasing flow rate configuration

| Region | $\sigma_{2\omega}$ [kHz] | Vortex stage |
|--------|-------------------------|--------------|
| $A_1$  | 0.74                    | No vortex    |
| $A_2$  | –                       | –            |
| $A_3$  | 9.13                    | Developed vortex |

Table 3: Validation of vortex regions with experimental data obtained in different configuration [2]

| Region | $\sigma_{2\omega}$ increasing flow[kHz] | $\sigma_{2\omega}$ decreasing flow[kHz] | Vortex stage |
|--------|----------------------------------------|----------------------------------------|--------------|
| $A_1$  | 0.81                                   | 0.73                                   | No vortex    |
| $A_2$  | 2.15                                   | 2.29                                   | Incipient vortex |
| $A_3$  | 5.98                                   | 7.08                                   | Developed vortex |

Comparing to 30s).

Furthermore, experimental results obtained in our previous work for a different configuration of the cavitating vortex generator and acoustic sensors [2], were used for criteria validation. The vortex stages identified in [2] for increasing and decreasing flow variations fitted the limits defined in table 1, so confirming the criteria as presented in table 3.

Figure 7: The normalized amplitude spectrum in the increasing flow configuration

The spectral analysis of $2\omega$ and pressure variation showed the same spectral distribution of the components for the two main regions $A_1$ (figure 7) and $A_3$ (figure 8). Figures 7a and 8a clearly show that in the developed vortex regime the distribution of the spectral components is amplified for the low frequencies for both acoustic and pressure variations, which is characteristic for vortex flows. Therefore the new signal processing tool is able to characterize the vortex in its evolution.
5. Conclusions

This paper presents a new tool capable to characterize the presence of cavitating vortex flow (as in hydraulic turbines), using a non-invasive acoustic technique, easy to set up in experimental and industrial sites.

The experiment was done by varying the flow rate from 6.4 l/s, without cavitating vortex to a flow rate of 19.2 l/s, where the vortex is clear eye visible, as presented in figure 2.

This paper presents the RPA method, specific for chaotic systems, and, in order to detect the cavitating vortex signature, we have proposed the analysis of the spectral components of the diagonal lines quantification.

The diagonal lines reflect the fundamental frequency of a signal [6]. Moving forward, in this paper, it is described the spectral analysis of these diagonal lines quantification which brings into light the effect of an interaction between two different waves, in certain conditions.

The theory was confirmed by the experimental tests: the cavitating vortex signature is present in the spectral components of the diagonal lines quantification. This component describes the evolution of flow and of the cavitating vortex.

For the future research, a single pair of acoustic sensors will be used, as the use of two pairs induced interference in the received signals. Also, they will be set above the cavitating vortex area in order to try quantifying the vapor volume within the flow.

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