Resonance Contributions to $\eta$ Photoproduction on the Nucleon in the Isobaric Model

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(Received 11 July 2017; published 29 November 2017)

Abstract
The contributions of the resonances $S_{11}(1535)$, $S_{11}(1650)$, and $P_{11}(1710)$ to gamma nucleon $\rightarrow$ eta nucleon are found. In order to estimate model dependence of the obtained resonance characteristics, an isobaric models used for analysis. For the resonances considered, our analysis yields mass values compatible with those advocated by the Particle Data Group. We emphasise, however, that cross-section data alone are unable to pin down the resonance parameters and it is shown that the beam and or target asymmetries impose more stringent constraints on these parameter values. The amplitude transitions of Feynman diagrams at the center of mass frame used to find amplitude squares involving the s-channel, u-channel, and t-channel on Born term and resonance. The value of the differential cross section on the energy system used by 1.685 MeV up to 2.795 MeV.

Keywords: photoproduction, resonances, amplitude transitions

DOI: 10.31758/OmegaJPhysPhysEduc.v3i2.40

Introduction
Understanding the structure of the proton and its excited states is one of the key questions in hadronic physics. Known as the missing-baryon problem, quark models based on three constituent quark degrees of freedom predict many more states than have been observed experimentally. Baryon resonances are broad and widely overlap, especially at higher energies, imposing challenges on the interpretation of experimental data in terms of resonance contributions. Without precise data from many decay channels, it will be difficult or even impossible to accurately determine the properties of well-established resonances, or to confirm or rule out the existence of weakly established resonances or new, so far unobserved states [1].

Although theoretical research in the field of particle physics has reached the standard model, the current model is believed to be a theory that explains the interaction between the constituent particles of matter, quarks and leptons, with the intermediate particles of the intermediate bosons (gauge boson), but there are still many phenomena in the field of particle physics that can not be fully explained.

One of the areas of research which is the link between nuclear physics and particle physics is the intermediate energy physics (intermediate energy physics). This field is related to the explanation of the phenomenon of nuclear physics and hadrons approach to particle physics, which is not just looking at the core and hadrons as a collection of nucleons (which is a composite particle), but furthermore looked nucleus and hadrons as a collection of elementary particles, namely quarks, that interact...
with the gluon intermediary as the physicist understood the theory of quantum chromodynamics [2].

The meson particles $\eta$ were discovered in 1961 by Pevsner [3]. The meson particles $\eta$ represent isospin with condition 0, so only resonance with isospin $\frac{1}{2}^+ \text{ contributes to the s-channel and u.}$ Since the $\eta$ particle has a neutral charge, it causes the seagull term that has an important role in meson production does not contribute anything. Thus, there is an interesting interest in the meson phenotype $\eta$, both theoretically and experimentally.

In general, Mainz group data provides information about the more systematic nature of the production of $\eta$ at the threshold area, which has better angular resolution allowing us to study the more $S_{11}(1535)$ resonance structure good. Thus, the $\eta$ particle is interesting to note that the amplitude $A_{\frac{1}{2}}^P$ extracted from Mainz data [4] gives closer results to the quark model [5, 6]. From theoretical side, the theoretical study on the $\eta$ photoproduction almost corresponds to the Breit-Wigner parameterisation framework [7] or isobar model with the coupled channel [8].

### Methods

Eta belongs to the meson group because it is a combination of a quark and antiquark. Where, eta is a composite of one quark strange and one antiquark up or down, or one antiquark strange and one quark up or down. Eta has a mass of 1070 times the mass of an electron, with a silent mass $547.853.862 \pm 0.018 \text{ MeV/c}^2$, and average life time $(5.02 \pm 0.19) \times 10^{-19} \text{ s}$, and is a zero-loaded elementary particle type. The quantities required in the calculations are shown in Table 1.

| Particle | $Q$ | $K$ | $S$ | $J^P$ | $I$ | Mass (MeV)        |
|----------|-----|-----|-----|-------|----|------------------|
| $\eta$   | 0   | -1.91 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 939.565379       |
| $n$      | 0   | -1.91 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 939.565379       |

Where $Q$, $K$, $S$, $J$, $P$, and $I$ are the number of charge, magnetic moment, spin, total spin, parity, and isospin, respectively.

The isobar model is often used to analyse the process of eta photoproduction produced from neutrons. The reason, this model is easier to use for the production of nuclei. This method is based on a Feynman Diagram appropriate for the s-, u-, t-, and resonances channels with an unknown parameter to match the model generated by the experimental data. In this case, the calculation process in the $\eta$ meson photoproduction reaction uses relativistic kinematics. The general reaction equation corresponding to this study is

$$\gamma(k) + N(p_1) \rightarrow \eta(q) + N(p_2) \quad (1)$$

for next of the above equation we will consistently use $k_1$, $k_2$, $p_1$, and $p_2$ respectively as the four momentum of the incoming real photons, $\eta$ generated, the initial state nucleons and nucleons in the final state.

In explaining the mechanisms for the particle interaction process we use two terms of reference. The first frame of reference is the central mass frame in which both particles are moving at the same speed but in opposite directions so that the total momentum in this reference frame is equal to zero (see Figure 1).

![Figure 1: Scattering of two particles in the framework of the center of mass where the direction of the vector is opposite.](image1)

The second frame of reference is the laboratory frame in which the particles are subjected to collisions or are in a state of silence (see Figure 2). In order to facilitate the calculation of all quantities referring to the laboratory frame it will be marked tilde (for example, the four photon momentum in the laboratory frame and the mass center is denoted by $\tilde{k}_1$ dan $\tilde{k}_1$, respectively).

![Figure 2: Scattering of two particles in the laboratory frame where one of them becomes the target.](image2)

Calculations of kinematics that include energy, momentum, angle can be written in some relativistic invariant form (Lorentz-invariant), which does not depend on the system reference frame defined by [9].
\[ s = (p_A + p_B)^2 = (p_C + p_D)^2 \]
\[ t = (p_C - p_A)^2 = (p_D - p_B)^2 \]
\[ u = (p_C - p_B)^2 = (p_D - p_A)^2 \]

where \( s, t, \) and \( u \) also known as Mandelstam variables, which also satisfies the identity

\[ s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2. \] (3)

In the photoproduction reaction \( \eta \), if we use the calculation in terms of the center of mass, Figure 1, then the four momentum of each particle:

\[ p_{\gamma} = (E_{\gamma}, p_{\gamma}) \]
\[ p_N = (E_N, p_N) \]
\[ p_\eta = (E_\eta, p_\eta) \]
\[ p_{N'} = (E_{N'}, p_{N'}) \]

where \( p_{\gamma} = -p_N = \frac{k}{2} \) and \( p_\eta = -p_{N'} = q \). Then Equation (2) becomes

\[ s = (p_{\gamma} + p_N)^2 = (E_{\gamma} + E_N)^2 = W^2 \]
\[ t = (p_\eta - p_N)^2 \]
\[ = m_\eta^2 + m_N^2 - 2E_\eta E_N + 2|p_\eta| |p_\gamma| \cos(\eta, \gamma) \]
\[ u = (p_\eta - p_{N'})^2 \]
\[ = m_\eta^2 + m_{N'}^2 - 2E_\eta E_{N'} + 2|p_N| |p_\eta| \cos(N, \eta). \] (5)

Since the transition amplitudes will be calculated in the framework of C.M., we need to find the values of photon and meson-\( \eta \) momentum in the framework. Photon momentum in C.M. is obtained from the conservation law of momentum four where the total invariant mass of the system is as follows

\[ W^2 = (p_{\gamma} + p_N)^2 = (p_{\gamma} + p_1)^2 = (p_\eta + p_{N'})^2 = (p_\eta + p_2). \] (6)

This quantity is the invariance of the Lorentz transform, then we can recalculate it within the lab. Where on the nucleos lab framework is targeted or in silence, \( p_{N'} = p_1 = (m_N, 0) \), and photons as projectiles, \( p_{\gamma} = \vec{E}_{\gamma}, \vec{p}_{\gamma} \). So that the invariant mass becomes

\[ W = \sqrt{(E_{\gamma} + m_N)^2 - \vec{p}_{\gamma}^2} \] (7)

as a function of particle momentum at the initial state, that is a photon. After we recalculate the first line of Equation (6) the C.M. frame, it will generate

\[ |p_\gamma| = \sqrt{\frac{[W^2 - (m_\gamma + m_N)^2][W^2 - (m_\gamma - m_N)^2]}{4W^2}} \] (8)

for the momentum of photon particles in the framework of C.M. The same treatment for the second row in Equation (6) so as to generate the particle momentum \( \eta \) in the framework of C.M.

\[ |p_\eta| = \sqrt{\frac{[W^2 - (m_\eta + m_N)^2][W^2 - (m_\eta - m_N)^2]}{4W^2}}. \] (9)

The energy of the photon threshold in the laboratory frame is

\[ E_{\gamma}^{th} = \frac{(m_\gamma + m_N)^2 - m_\eta^2}{2m_N}. \] (10)

While the photon threshold energy in the C.M. framework is

\[ E_\gamma = \frac{m_\eta^2 + 2m_\eta m_N}{2(m_\eta + m_N)}. \] (11)

By substituting \( m_\eta = 547.853 \text{ MeV} \) and \( m_N = 939.565 \text{ MeV} \) into Equation (10), then for this photoproduction reaction we use threshold energy the photon is \( E_{\gamma}^{th} = 707.577 \text{ MeV} \).

To calculate the cross section according to this study we used the equation according to [8] can be written as follows

Cross section = \( \frac{W_{fi}}{\text{flux first}} \) (number in the last condition)

with the transition rate \( W_{fi} \),

\[ W_{fi} = \frac{|T_{fi}|^2}{\tau} \] (13)

where \( \tau \) is an interval of time of the interaction, while the amplitude of the transition \( T_{fi} \), i.e.

\[ T_{fi} = -iN_A N_B N_C N_D (2\pi)^4 \delta^4 \langle \bar{P} C + P_D - P_A - P_B \rangle M \] (14)

and then Equation (13) becomes

\[ W_{fi} = (2\pi)^4 \delta^4 \langle \bar{P} C + P_D - P_A - P_B \rangle |M|^2 \] (15)

with

\[ N = \frac{1}{\sqrt{V}}. \] (16)

Once calculated then we will get the results for cross-section differentials in the center of mass framework i.e.

\[ \frac{d\sigma}{d\Omega_C} = \frac{|M|^2}{64\pi^3 s} |q| \] (17)
with the value \( S = (E_A + E_B)^2 = W^2 \) and \(|M|^2\) is the transition amplitudes total of all channel.

The amplitude of the transition is one of the important variables in the calculation of the cross section. Calculation of the amplitude of the transition is usually calculated based on Feynman diagrams. Some interactions in the Feynman diagram are depicted in s-channel, t-channel, u-channel, and resonance. The contribution of these channels and resonance is used to match the model to the experimental data.

**Results and Discussion**

For the first step, we use a single resonance in doing fitting experimental data. Some resonance we investigate are \( S_{11}(1535) \), \( S_{11}(1650) \), and \( P_{11}(1710) \). The resulting graph fitting for resonance \( S_{11}(1535) \) can be seen in Figure 3. The results show that for large angles (backward angles) between graphs and experiment data coincide well, it is just a little different for small angles (forward angles) that gives quite a wide difference, but overall graph of the results fitting is consistent with experimental data. This is understandable because of the small \( \chi^2 \) value, which is about 17.36320, it can be concluded that the \( S_{11}(1535) \) resonance has an important contribution to the photoproduction reaction process \( \eta \).

### Table 2: Parameters fitting \( S_{11}(1535) \).

| Parameters | Value       |
|------------|-------------|
| \( \Lambda_N \) (MeV) | 1466.4402 |
| \( g_{\eta NN} \) | -0.0567    |
| \( \Lambda_{N^*} \) (MeV) | 1814.5824 |
| \( g_{\eta NN^*} \) | 1.8523     |
| \( \Lambda_{\rho} \) (MeV) | 1140.2047 |
| \( g_{\rho \eta} \) | 0.0623     |

While for the parameter values fitting obtained can be seen in Table 2. From Table 2 it is known that the value of the coupling constant \( g_{\eta NN^*} \) and mass cut off dominant resonance nucleons.

![Figure 3: The resulting graph fitting differential cross section with resonance \( S_{11}(1535) \) for different variations of the scattering angle.](image-url)
For contributions of resonant $S_{11}(1650)$ can be seen in Figure 4. For the large angle, visible experimental data can be well explained starting from the value of $\cos \theta = -0.35$, less then the resultant graph fitting is still widening from the experimental data. But the overall results of the fitting in accordance with experimental data. From the graph in Figure 4 it generally gives the same result as the $S_{11}(1535)$ resonance. As for the parameter values fitting obtained can be seen in Table 3. From the table it is known that the value of the coupling constants $g_{\eta NN}$ and mass cut off dominant resonance nucleons. This is because the value of the cross section of the resonant $S_{11}(1650)$ has a value which is almost equal to $S_{11}(1535)$ for all values of the scattering angle variation. Thus the results of fitting using resonant $S_{11}(1650)$ will give a value of $\chi^2$ sizable unlike the resonance $S_{11}(1535)$, which is about 29.25026.

The parameter value fitting we obtain in this case can be seen in Table 3, and note that the value of the coupling constant $g_{\eta}$ and mass $NN$ cut off the more dominant resonance nucleons. Based on the graph of the result fitting by using a $S_{11}(1650)$ resonant, this indicates that the $S_{11}(1650)$ resonance also has an important contribution in the $\eta$ photoproduction reaction process.

Table 3: Parameters fitting $S_{11}(1650)$.

| Parameters     | Value        |
|----------------|--------------|
| $\Lambda_N$ (MeV) | 1345.7216 |
| $g_{\eta NN}$               | -0.1023     |
| $\Lambda_N^*$ (MeV)         | 1860.5997   |
| $g_{\eta NN^*}$             | 0.8607      |
| $\Lambda_\rho$ (MeV)        | 1149.1165   |
| $g_{\rho\eta}$              | 0.0775      |

The resulting graph fitting for resonant $P_{11}(1710)$ can be seen in Figure 5. These results show that between graphs and experimental data do not occur crushes. This is understandable because of the large $\chi^2$ value, which is about 68.86677 whereas for the parameter values fitting obtained
can be seen in Table 4. From the table it is known that the coupling constant value $g_{\eta NN}$ and mass cut off dominant resonance nucleons.

![Table 4: Parameters fitting $P_{11}(1710)$.

| Parameters | Value |
|------------|-------|
| $\Lambda_N$ (MeV) | 225.7498 |
| $g_{\eta' NN}$ | -20.0000 |
| $\Lambda_{N^*}$ (MeV) | 1372.3486 |
| $g_{\eta NN^*}$ | 4.3235 |
| $\Lambda_p$ (MeV) | 741.6699 |
| $g_{\rho \eta}$ | -0.2906 |

**Conclusion**

From the results obtained in the study, on the graph for low energy then the amount of $\eta$ scattered on the proton is always more, is inversely proportional to the higher energy where the number $\eta$ scattered on the protons is always less. With a large contribution of $u$-channel then $\eta$ is more scattered when in the $u$-channel.

The graph also shows that $\eta$ scattered on theoretical results is also slightly higher than the $\eta$ sums scattered on the protons in the experiment. The introduction of the hadronic form factor into the reaction calculation $\gamma p \rightarrow \eta p$ also gives results that are more in line with the experimental data than the result without using the hadronic form factor.

The use of resonant $S_{11}(1,535)$ and $S_{11}(1650)$ generate considerable contribution both to generate numerical data trends for the fitting process in order to compare it with the experimental results. The more resonance used then the better the model we have. The cut off mass of the nucleon and resonant nucleon also affects the results obtained.

In this research is not too much difference theoretical study between photoproduction meson $\eta$ on the neutron with model isobar and photoproduction meson $\eta$ on a proton with model isobar that distinguishes the difference in charge and the mass
of protons and neutrons.

References

[1] V. Crede et al. (CBELSA/TAPS Collaboration), Photoproduction of \( \eta \) and \( \eta' \) mesons off protons, Phys. Rev. C 80, 055202 (2009).

[2] V. Credé et al. (CB-ELSA Collaboration), Photoproduction of \( \eta \) mesons off protons for 0.75 GeV < \( E_\gamma \) < 3 GeV, Phys. Rev. Lett. 94, 012004 (2005).

[3] A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein, R. Strand, T. Toohig, M. Block, A. Engler, R. Gessaroli, and C. Meltzer, Evidence for a three-pion resonance near 550 MeV, Phys. Rev. Lett. 7, 421 (1961).

[4] B. Krusche, J. Ahrens, G. Anton, R. Beck, M. Fuchs, A. R. Gabler, F. Härter, S. Hall, P. Harty, S. Hlavac, D. MacGregor, C. McGeorge, V. Metag, R. Owens, J. Peise, M. Röbig-Landau, A. Schubert, R. S. Simon, H. Ströher, and V. Tries, Near threshold photoproduction of \( \eta \) mesons off the proton, Phys. Rev. Lett. 74, 3736 (1995).

[5] F. E. Close and Z. Li, Photo- and electroproduction of \( N^* \) in a quark model, Phys. Rev. D 42, 2194 (1990).

[6] R. Koniuk and N. Isgur, Baryon decays in a quark model with chromodynamics, Phys. Rev. D 21, 1868 (1980).

[7] H. R. Hicks, S. R. Deans, D. T. Jacobs, P. W. Lyons, and D. L. Montgomery, Isobar analysis of \( \gamma p \rightarrow \eta p \), Phys. Rev. D 7, 2614 (1973).

[8] C. Bennhold and H. Tanabe, Low-energy \( \eta \) photoproduction from nucleons and nuclei, Phys. Lett. B 243 (1-2), 13-17 (1990).

[9] W. Greiner and J. Reinhardt, Quantum Electrodynamics, (Springer-Verlag GmbH, Heidelberg, 2009).