The capacitance tomography image reconstruction algorithm based on sensitivity field sensitivity matrix

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Abstract. Successful applications of ECT technology depend on the speed and precision of the image reconstruction algorithms. The image reconstruction for electrical capacitance tomography (ECT) is an inherently nonlinear and ill-posed inverse problem. To solve the problems, in this paper a 12-electrode ECT system was set up to acquire real measurement capacitances under capacitance tomography that based on sensitivity field sensitivity matrix, which is T-LBP. In this paper four typical flow patterns are simulated, such as laminar flow, core flow, annular flow and bubbly flow, which in a circular section pipe. The simulation data indicate that the quality and the error, which was obtained in this paper, of the reconstructed images were better than the corresponding images obtained by the LBP and the Landweber algorithm. This new algorithm presents a feasible and effective way to research on image reconstruction algorithm for the industrial applications of Electrical Capacitance Tomography System.

1. Introduction

Electrical Capacitance Tomography (ECT) technology was developed in the 1980s. It performs online monitoring of multiphase fluids in industrial pipelines based on the capacitance sensitive mechanism. It is non-radiative, non-invasive, simple in structure, easy to install and carry, and cost effective. The advantages of low and fast response speed provide a new method for online measurement of multiphase fluids in modern industrial production such as petroleum, chemical, energy, and electric power [1-3].

Since the capacitance tomography inherent nonlinearity of capacitance measurements and independent (i.e., projection data) is very limited, far less than the number of pixels of the reconstructed image, the inverse problem is not present analytical solution [4-5]. At the same time, due to non-linear and soft field effects, the sensitivity of the sensitive field is not uniformly distributed, which makes the solution stability of the ECT system poor and has serious ill-conditions, so the image reconstruction is more difficult. Image reconstruction algorithm has always been the main difficulty in the practical application and further development of electrical capacitance tomography [6-7]. Exploring good image reconstruction algorithms is very important, so some ECT imaging algorithms have been proposed in recent years [8-10]. Among these algorithms, the linear back projection method (LBP) is characterized by simple algorithm, fast reconstruction speed but relatively poor imaging quality; Landweber algorithm, conjugate gradient algorithm, etc., have relatively high calculation accuracy, but the calculation process needs to be nonlinear Iterative, so it is not suitable for industrial online real-time imaging.
This paper is based on the 12-electrode electrical capacitance tomography system, focusing on the normalization method based on the sensitive field sensitivity matrix (T-LBP) electrical capacitance tomography technology method, and analyzes the four typical flow patterns (layered flow, core flow, annular flow, and bubbly flow) were analyzed and verified. Numerical calculation shows that the T-LBP electrical capacitance tomography algorithm proposed in this paper can significantly improve the stability and quality of image reconstruction. Experimental results show that this method has great advantages in terms of computational performance and calculation methods. At the same time, the image reconstruction quality obtained by this algorithm is better than that of LBP, Landweber and conjugate gradient algorithms, which provides a new way for ECT image reconstruction. Effective method. Due to the particularity of the ECT image reconstruction problem, it is of great significance to study more efficient and accurate reconstruction algorithms for complex dynamic reconstruction objects.

2. Mathematical model of electrical capacitance tomography

2.1. Principles of ECT imaging

The basic structure of the electrical capacitance tomography system is shown in Figure 1. The capacitance sensor is installed according to the pipeline to be measured. By measuring the capacitance value between the electrodes of different combinations of the capacitance sensor, the medium distribution inside the pipeline is preset according to the calculation needs. The structure of the electrical capacitance tomography system:

The ECT sensor model can be represented by a two-dimensional Laplace equation (assuming that there is no free charge inside), and the form of polar coordinates is:

\[ \nabla \cdot [\varepsilon(r, \theta) \nabla \varphi(r, \theta)] = 0 \]  

Among them, \( \varepsilon(r, \theta) \) is the dielectric constant of the measured sensitive area, and \( \varphi(r, \theta) \) is the potential distribution function.

![Fig.1 ECT imaging principle structure](image)

Applying Gauss's law, the mutual capacitance of each pair of excitation electrodes and measurement electrodes can be expressed as:

\[ C = \frac{Q}{U} = -\oint_{\Gamma} \varepsilon(r, \theta) \nabla \varphi(r, \theta) \cdot n \, dl \]  

In the formula, \( \Gamma \) is the curve surrounding the measuring electrode; \( n \) is the outer normal vector of \( \Gamma \); \( U \) is the potential difference between the electrodes; \( \nabla \varphi(r, \theta) \) is the potential gradient distribution.

The positive problem of ECT is that given the dielectric constant distribution \( \varepsilon(r, \theta) \) and boundary conditions, the capacitance of the electrode is determined by formulas (1) and (2); while the inverse problem is that the capacitance between the electrodes is known and the internal dielectric is inverted Constant distribution. Because \( \varepsilon(r, \theta) \) and \( \varphi(r, \theta) \) are non-linear functional relations, if the distribution
of dielectric constant is irregular, the analytical solution of the positive problem cannot be obtained. Therefore, numerical methods such as finite element method and finite difference are needed to solve the problem. Law and so on.

The commonly used approximation model to simplify the forward problem is the sensitivity model, which is a linear model based on the principle of superposition. For formula (2), if a small cell in the sensitive field changes from the background dielectric constant to a high dielectric constant, the change in electrode capacitance caused by the disturbance can be expressed as a first-order differential form (3):

$$\Delta C = \frac{dC}{d\varepsilon} \Delta \varepsilon + o[(\Delta \varepsilon)^2]$$  \hspace{1cm} (3)

2.2. Principle of imaging capacitance calculation

For an imaging system composed of N electrodes, the number of independently measured capacitance values is \(m=1/2(N(N-1))\). Since the capacitance value between the electrodes contains information related to the phase distribution in the pipeline, the measured capacitance value will be affected by the phase distribution. ECT uses a numerical calculation method to display the material distribution in the area according to the measured capacitance value, as shown in formula (4).

$$C_{i,j} = \int \int_D \varepsilon(x,y)S_{i,j}(x,y,\varepsilon(x,y))dxdy$$  \hspace{1cm} (4)

In the formula: \(\varepsilon(x, y)\) is the distribution of the dielectric constant in the pipeline, and \(C\) is the capacitance value between the electrodes. The most commonly used at present is to assume that there is a linear relationship between the material distribution and the capacitance, namely:

$$C_{i,j} = S_{i,j}(x,y) \cdot G(x,y)$$  \hspace{1cm} (5)

In the formula, \(S\) is the sensitive field, \(C\) is the capacitance between the electrodes, and \(G\) is the dielectric constant.

3. Image reconstruction algorithm

3.1. Principle of LBP algorithm

The simple linear back projection (LBP) imaging algorithm is the inverse process of formula (5), namely:

$$G = S^T \cdot C$$  \hspace{1cm} (6)

Among them, \(S\) is the normalized sensitive field matrix, and \(C\) is the normalized capacitance value.

This method is a fast imaging algorithm, the calculation speed of this method is faster, and the essence is to project all the capacitance values back to the pipe section after weighting, so the image construction quality is low.

3.2. The T-LBP algorithm

According to the finite element method, the capacitance change caused by the fluid medium change at each grid unit in the pipeline is calculated. The sensitive field is expressed in numerical form [11]:

$$S_{i,j}(x,y) = \int_{A(x,y)} \frac{E_i(x,y)}{V_i} \cdot \frac{E_j(x,y)}{V_j} dxdy$$  \hspace{1cm} (7)

Here \(E\) represents the field intensity distribution between the electrodes when the excitation point voltage is \(V\), and \(A\) represents the area of the pixel in \((x, y)\).

The T-LBP algorithm is based on the LBP algorithm. Before using the LBP algorithm to calculate the medium distribution in the electric field, the operator \(T\) is introduced to transform the sensitive field sensitivity matrix \(S\) to improve the image error caused by the uneven sensitivity matrix. The transformation process is as follows:

$$T(S) = A \cdot S = S^T$$  \hspace{1cm} (8)
Here, A represents the row normalization matrix of the sensitivity matrix, and $S^*$ represents the generated sensitivity matrix. The required sensitivity rectangle is as follows:

$$S'_i(x,y) = \frac{S(x,y)}{\sum_j S_{ij}(x,y)}$$, $j = 1, 2, \cdots N, i = 1, 2, \cdots M$  \hspace{1cm} (9)

The sensitive field sensitivity matrix obtained after the matrix transformation of equation (8) is put into equation (6) to obtain the formula for solving the dielectric constant:

$$G(x,y) = S'^* \cdot C$$  \hspace{1cm} (10)

4. Algorithm numerical verification
In order to verify the image reconstruction performance of the T-LBP algorithm, numerical experiments are used to evaluate, and the image reconstruction quality is compared with the LBP algorithm and the Landweber iterative algorithm. The numerical experiment is a 12-electrode sensor. The low-dielectric substance is air with $\varepsilon = 1$, and the high-dielectric substance is water with $\varepsilon = 3.5$. The other figures are the distribution of air and $\varepsilon = 2.5$ in the pipe. For ECT image reconstruction, the imaging area is divided into 4,215 pixels using a quadrilateral grid.

4.1. Image quality comparison
According to the capacitance value obtained, the image was reconstructed using the three algorithms described above. The results are compared for several typical flow patterns, as shown in Figure 2.

![Comparison of core flow calculation results](image-url)
It can be seen from Figures 2a and 2b that for the core flow and laminar flow, the T-LBP algorithm presented in this paper has no obvious difference in imaging compared with the LBP algorithm and the Landweber algorithm. For annular flow 2c and bubbly flow 2d, the image similarity of the T-LBP algorithm is significantly better than the latter two algorithms.
4.2. Image imaging error analysis
When comparing the image reconstruction quality, the spatial image error $E$ is selected as the image quality evaluation index [12], which is defined as follows:

$$E = \frac{\sum_{e=1}^{M} |G_t(e) - G_r(e)|}{\sum_{e=1}^{M} G_r(e)}$$  \hspace{1cm} (11)$$

Among them, $G_t(e)=1$ unit $e$ is located in the high dielectric constant area of the test mode; $G_r(e)=0$ unit $e$ is located in the low dielectric constant area of the test mode. $G_t(e)=1$, unit $e$ is located in the high dielectric constant area of the reconstructed image; $G_r(e)=0$, unit $e$ is located in the low dielectric constant area of the reconstructed image.

The calculation errors of the three algorithms in Table 1 show that the T-LBP algorithm has smaller calculation errors than the original graph, and shows greater calculation advantages as a whole, especially in the core flow and circulation.

5. Conclusion
This paper proposes an improved LBP electrical capacitance tomography algorithm (T-LBP). The algorithm introduces the operator $T$ to perform an equalization transformation on the sensitivity matrix of the sensitive field to improve the calculation of the image error caused by the uneven sensitivity matrix. The algorithm performs matrix model transformation based on the LBP algorithm, and the calculation and transformation process is simple and efficient. The calculation results show that the image reconstruction accuracy of this algorithm is higher than that of the LBP algorithm and the Landweber algorithm, and the reconstructed image is closer to the original flow pattern, which provides a new and effective method for the practical industrial application of ECT image reconstruction.

| Tab.1 Calculation error of each algorithm |
|------------------------------------------|
| Core flow E/% | Laminar flow E/% | Circulation flow E/% | Bubbly flow E/% |
|--------------|------------------|---------------------|-----------------|
| T-LBP        | 0.11891          | 0.81301             | 4.55844         |
| LBP          | 1.30913          | 0.82624             | 14.51948        |
| Landweber    | 1.18993          | 0.83632             | 14.48052        | 8.18414        |

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