Parsec-scale Constraints on the Ionized Interstellar Medium with the Terzan 5 Pulsars

Scott M. Ransom
National Radio Astronomy Observatory, 520 Edgemont Rd., Charlottesville, VA 22901

Abstract. Over the past two years, we used a series of GBT observations to uncover at least 33 millisecond pulsars in the globular cluster Terzan 5 located in the Galactic bulge. We now have 32 timing solutions for the pulsars which give us precise positions and dispersion measures (DMs) and indicate that the DMs are dominated by variations in the integrated electron density along the slightly different sight lines towards the pulsars. At a distance of $\sim 8.7$ kpc, angular separations between the pulsars range from $0.4'' - 100''$ and correspond to projected physical separations of $0.01 - 4$ pc, giving us a unique probe into the ionized ISM properties on these scales. Our measurement of the DM structure function toward Terzan 5 is not inconsistent with Kolmogorov-like electron density fluctuations in the ISM on scales ranging from at least $0.2 - 2$ pc.

1. Introduction

Terzan 5 is a rich globular cluster (GC) located in the Galactic bulge ($l=3^\circ.8$, $b=1^\circ.7$) at a distance $d=8.7\pm2$ kpc (Cohn et al. 2002). Deep and repeated observations of Terzan 5 using the Green Bank Telescope (GBT) at 2 GHz since mid-2004 have uncovered 30 new millisecond pulsars (MSPs), bringing the total known in the cluster to 33, by far the most of any GC (Ransom et al. 2005; Hessels et al. 2006). Many individual GC MSPs are in interesting exotic binary systems, yet ensembles of MSPs in the same cluster can provide unique science as well. Since the pulsars are at identical (within $\sim 0.1\%$) and known distances, and are separated by small angles on the sky ($\lesssim 1'' - 2''$), they are unique probes of pulsar luminosities and masses (e.g. Anderson 1992), cluster dynamics (e.g. Phinney 1992), and ionized gas — both within the cluster (Freire et al. 2001) and in the intervening interstellar medium (ISM; Anderson 1992).

We have recently determined timing solutions for 32 of the 33 pulsars in Terzan 5. These solutions provide (among other things), positions on the sky (errors are typically $\sim 0.01''$ in right ascension and $\sim 0.1'' - 0.4''$ in declination; see Figure 1), dispersion measures (DMs; the integrated number of electrons along the line-of-sight to the pulsar) with relative errors $\lesssim 0.01$ pc cm$^{-3}$ and absolute errors $\lesssim 0.1$ pc cm$^{-3}$, and the apparent spin period derivative, $\dot{P}_{\text{obs}}$. The latter values are usually dominated by the gravitational acceleration of the cluster itself, such that values of $\dot{P}_{\text{obs}}$ less than or greater than zero indicate positions behind or in front of the cluster respectively (Phinney 1992). In this work, we use the accurate positions and DMs to compute the DM structure function, $D_{\text{DM}}(\theta)$, over a range of angles $1'' \lesssim \theta \lesssim 100''$, and to constrain the intervening ionized ISM on linear scales from $10^{17} \lesssim l \lesssim 10^{19}$ cm.
Figure 1. Positions and Dispersion Measures (DMs) of the 32 Terzan 5 pulsars with timing solutions. Positional errors are typically \( \sim 0.01'' \) in right ascension, and \( \sim 0.1 - 0.4'' \) in declination. Relative errors on the DMs are \( \lesssim 0.01 \) pc cm\(^{-3} \), while the absolute DMs are known to \( \lesssim 0.1 \) pc cm\(^{-3} \). The grey circle represents the cluster core radius \( r_c = 7.9'' \). The two pulsars Terzan 5 O and Terzan 5 R are separated by only 1.2'' on the sky, yet their DMs are different by 1.1 pc cm\(^{-3} \).

2. The DM Structure Function

Variations in ISM electron density, \( \delta n_e \), have been shown to be consistent with a Kolmogorov-like wavenumber spectrum of the form \( P_{\delta n_e}(q) = C_2^2 q^{-\beta} \), with \( \beta \sim 11/3 \) over a very wide range of wavenumber \( q = 2\pi/l \) (corresponding to scales \( l \) from \( 10^8 - 10^{15} \) cm; e.g. Armstrong, Rickett, & Spangler 1995). Measurements of pulsar scintillation, scattering, and DM time variations have been important throughout this range in \( l \).

For simultaneous observations of multiple pulsars separated by small angles \( \delta \theta \ll 1 \) rad (i.e. GC MSPs), and assuming isotropic electron variations, the electromagnetic phase structure function (e.g. Cordes & Rickett 1998) depends
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only on the separation of the sources in the plane of the sky, \( \delta r = d \delta \theta \), and can be written

\[
D_{\phi}(\delta r) = (r_e \lambda)^2 f_\alpha (d \delta \theta)^\alpha \int_0^d \left( \frac{s}{d} \right)^\alpha C_n^2(s) \, ds,
\]

where \( \lambda \) is the radio frequency, \( r_e \) is the classical electron radius, \( \alpha = \beta - 2 \) and \( f_\alpha \) is a constant (when \( \alpha = 5/3 \), \( f_\alpha = 88.3 \)). Following Cordes (private comm), if we assume that the \( C_n^2 \) are constant out to distance \( d_{\text{eff}} \), and recognize \( C_n^2 d_{\text{eff}} \) as the scattering measure \( \text{SM}(d_{\text{eff}}) \),

\[
D_{\phi}(\delta r) = (r_e \lambda)^2 \frac{f_\alpha}{\alpha + 1} \left( \frac{d}{d_{\text{eff}}} \right)^{\alpha} \text{SM}(d_{\text{eff}})^\alpha.
\]

For GC MSPs, we observe DM variations as a function of \( \delta \theta \) rather than phase variations. But we can relate the DM variations to phase variations using \( \delta \text{DM} = \delta \phi / \lambda r_e \). Therefore, we effectively measure the phase structure function by determining the squared DM differences as a function of angular separation \( \delta \theta \), which is the DM structure function:

\[
D_{\text{DM}}(\delta \theta) = \frac{f_\alpha}{\alpha + 1} \left( \frac{d}{d_{\text{eff}}} \right)^{\alpha} \text{SM}(d_{\text{eff}})^\alpha.
\]

For Terzan 5 (or other sources at large distances in the Galactic plane within the \( \sim 1 \) kpc scale height of \( n_e \)), \( d \sim d_{\text{eff}} \). This means that a good measurement of \( D_{\text{DM}}(\delta \theta) \) over a range of \( \delta \theta \) will determine the power-law index of the electron density variations (from its slope \( \alpha = \beta - 2 \) on a log-log plot) and an estimate of the scattering measure and \( C_n^2 \) (from the \( y \)-intercept). To be useful for ISM constraints, though, measured DM variations must be dominated by the intervening ISM rather than an ionized intracluster medium as has been detected in 47 Tucanae (Freire et al. 2001).

As noted recently by Freire et al. (2005), the total DM spread, \( \Delta \text{DM} \), of the pulsars in GCs with large average DMs (\( \gtrsim 50 \) pc cm\(^{-3} \)) scales roughly linearly with DM. If DM variations were dominated by intracluster gas, \( \Delta \text{DM} \) would be roughly independent of distance and the average DM. However, Terzan 5 has one of the largest average DMs (\( \sim 239 \) pc cm\(^{-3} \)) and the largest \( \Delta \text{DM} \) (almost 10 pc cm\(^{-3} \)) of any known cluster. If intracluster gas was the cause, its density would be a factor 10–20 greater than that found in 47 Tucanae (Freire et al. 2005). Another striking indication of the dominance of the ISM contribution to \( \Delta \text{DM} \) over an intracluster gas contribution comes from the pulsars Terzan 5 O and Terzan 5 R (see Figure 1). These pulsars are separated by only 1.2" on the sky, yet their DMs are different by 1.1 pc cm\(^{-3} \) in the “wrong” way to be explained by gas within the cluster. Terzan 5 O, which has the smaller DM, is located behind the cluster (as determined by its negative \( P_{\text{obs}} \)), while Terzan 5 R has the larger DM and is likely in front of the cluster. Such a situation can only occur if the DM variations are dominated by an intervening ISM that is highly structured on linear scales between 0.01–1 pc.

3. Data Analysis

With \( n = 32 \) independent timing solutions for the MSPs in Terzan 5, we made \( n(n - 1)/2 = 496 \) measurements of \( (\delta \text{DM})^2 \), one for each pair of MSPs (see
Figure 2. The DM structure function, $D_{\text{DM}}(\delta \theta)$, for the 32 Terzan 5 MSPs with timing solutions. The 496 measurements of $(\delta \text{DM})^2$ and their errors are the thin crosses. The grey histogram show the number of measurements in 40 evenly spaced bins. For bins with $\geq 6$ measurements, the $\langle (\delta \text{DM})^2 \rangle$ are shown as black boxes with error bars. The thick black line shows the linear fit to the bins with $\geq 20$ measurements, which determined the power-law index $\alpha = 1.38 \pm 0.26$. The grey dotted line shows the best fit model assuming a Kolmogorov spectrum of electron density variations with $\alpha = 5/3$.

Figure 2). To determine $D_{\text{DM}}(\delta \theta)$, we binned the full range of separations, projected to the assumed $d=8.7 \text{ kpc}$ distance to Terzan 5 using $d \delta \theta$, into 40 logarithmically spaced bins. If there were at least 6 measurements of $(\delta \text{DM})^2$ within a bin, we computed the mean-squared DM difference $\langle (\delta \text{DM})^2 \rangle$. Errors on $\langle (\delta \text{DM})^2 \rangle$ were estimated using the measured DM distribution for Terzan 5 and the number of measurements per bin, according to normal sampling theory.

For bins with $\geq 20$ measurements of $(\delta \text{DM})^2$, we fit a simple linear (in log-log space) model to the data: $\log_{10} \langle (\delta \text{DM})^2 \rangle = \alpha \log_{10} (d \delta \theta) + b$. For the first model, we allowed both $\alpha$ and $b$ to vary and measured $\alpha=1.38\pm0.26$. Since the measured power-law index of the turbulence is only $\sim 1-\sigma$ away from the value for Kolmogorov fluctuations, we determined $b$ by assuming $\alpha=5/3$, and re-fitting to get $b=-29.893\pm0.030$. Applying appropriate unit conversions,

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1Since we are probing the intervening ISM, an argument can be made to use a fraction of the distance to Terzan 5 rather than the total distance, $d$. 
SM = 10^{(b+29.166)} = 0.188 \pm 0.013 \text{ kpc m}^{-20/3}, \text{ and therefore } C_n^2 = \frac{\text{SM}}{d_{\text{eff}}} \sim 0.022 \text{ m}^{-20/3}.

4. Discussion and Conclusions

It is interesting to compare our estimate of the SM towards Terzan 5, $0.188 \pm 0.013 \text{ kpc m}^{-20/3}$, to values determined in other ways. \cite{Nice_Thorsett_1992} measured a pulse broadening timescale due to scattering, $\tau_p$, for pulsar Terzan 5 A at 685 MHz of 0.70 ms. This can be converted to a pulse broadening based SM estimate, $\text{SM}_\tau$, using $\tau_p = 0.90 \text{ ms} \left( \text{SM}_\tau \right)^{5/2} \nu^{-22/5} d$, with $\nu$ in GHz and $d$ in kpc \cite{Cordes_2002}, which gives $\text{SM}_\tau = 0.033 \text{ kpc m}^{-20/3}$. This value is smaller by a factor of $\sim 6$ than our measurement, although we should expect some systematic variation due to the different weighting factors used by the methods over what is almost certainly an inhomogeneous electron distribution along the line-of-sight \cite{Cordes_2002}. It could also be that there is an upturn in the power spectrum between the small scales probed by $\tau_p$ and the much longer scales relevant to the DM structure function.

Similarly, we can use either the known $\langle \text{DM} \rangle \sim 239 \text{ pc cm}^{-3}$ or distance to Terzan 5, $d=8.7 \text{ kpc}$, as inputs to the NE2001 Galactic electron density model, which can estimate SM \cite{Cordes_Lazio_2002}. If we specify $d$, NE2001 predicts $\text{SM}_d=0.79 \text{ kpc m}^{-20/3}$, while if we specify DM, $\text{SM}_{\text{DM}}=0.29 \text{ kpc m}^{-20/3}$. The latter value is quite close to our measurement, and significantly closer than the estimate based on the distance to the cluster.\footnote{If the distance to Terzan 5 is really 8.7 kpc, NE2001 greatly overestimates the integrated electron density towards the cluster. For that distance, NE2001 predicts $\text{DM} \sim 530 \text{ pc cm}^{-3}$.}

It is quite interesting that the measured power-law index of the DM structure function for linear scales ranging from $\sim 0.2-2 \text{ pc}$ (or wavenumbers, $10^{-16} < q < 10^{-15} \text{ m}^{-1}$) is not inconsistent with Kolmogorov fluctuations of the ISM. These scales are $10^2-10^3$ times longer than those probed by most other pulsar techniques. When we use our estimates of $C_n^2$ and assume $\beta = 11/3$ to determine $P_{\text{rms}}(q)$, the resulting values are amazingly close to extrapolations based on scintillation and other measurements at wavenumbers between $10^{-13} < q < 10^{-4} \text{ m}^{-1}$ \cite{Armstrong_1995}, implying that Kolmogorov-like density fluctuations are present in the ISM over size scales covering almost ten decades. It is intriguing to note the possible downturn of $D_{\text{DM}}(\delta \theta)$ at linear scales of 2–3 pc. While this is likely due to the poor statistics at the large-angle end of the structure function, it could also indicate saturation of the structure function upon reaching the outer scale (i.e. the largest size scale where power is input into the turbulent process governing the ISM).

Given the large number of pairwise measurements provided by the Terzan 5 MSPs, one of our next tasks will be to check for evidence of anisotropic electron density fluctuations over the same size scales. In addition, ongoing searches of GCs using the GBT and other telescopes have resulted in several other bulge GCs with substantial numbers of pulsars where these techniques might be useful in the future (for instance, M28 currently has 11 known pulsars).
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References

Anderson, S. B. 1992, PhD thesis, California Institute of Technology
Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Cohn, H. N., Lugger, P. M., Grindlay, J. E., & Edmonds, P. D. 2002, ApJ, 571, 818
Cordes, J. M. 2002, in ASP Conf. Ser. 278: Single-Dish Radio Astronomy: Techniques and Applications, ed. S. Stanimirovic, D. Altschuler, P. Goldsmith, & C. Salter, 227–250
Cordes, J. M. & Lazio, T. J. W. 2002, ArXiv Astrophysics e-prints [astro-ph/0207156]
Freire, P. C., Kramer, M., Lyne, A. G., Camilo, F., Manchester, R. N., & D’Amico, N. 2001, ApJ, 557, L105
Freire, P. C. C., Hessels, J. W. T., Nice, D. J., Ransom, S. M., Lorimer, D. R., & Stairs, I. H. 2005, ApJ, 621, 959
Hessels, J. W. T., Ransom, S. M., Stairs, I. H., Freire, P. C. C., Kaspi, V. M., & Camilo, F. 2006, Science, 311, 1901
Nice, D. J. & Thorsett, S. E. 1992, ApJ, 397, 249
Phinney, E. S. 1992, Phil. Trans. Roy. Soc. A, 341, 39
Ransom, S. M., Hessels, J. W. T., Stairs, I. H., Freire, P. C. C., Camilo, F., Kaspi, V. M., & Kaplan, D. L. 2005, Science, 307, 892