Calculation of the axially symmetric eigenfunctions of the finite propagation operator in the near-field diffraction

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Abstract. The propagation of axially symmetric laser beams in the near diffraction (at a distance in the order of the wavelength) can be described by means of an expansion in plane waves, which after considering axial symmetry reduces to an axisymmetric propagation operator involving Fourier-Hankel transforms. The eigenfunctions of the operator, when eigenvalues are close to one, determine the characteristics of the signals (information) transmitted lossless (without distortion). The beam propagation distance and the region of spatial frequency limitation are parameters of the operator and essentially change the set of eigenvalues and functions. We calculate the axisymmetric eigenfunctions of the finite propagation operator in the near diffraction zone and investigate their qualitative and quantitative characteristics depending on the propagation distance and the constraints imposed in the object and spectral domains.

1. Introduction

The basic concept of near field optics is evanescent electromagnetic waves, whose contribution becomes significant when the size of an object or a distance equals to the wavelength or smaller. The value of evanescent waves was ignored in optics until the appearance of near-field microscopes [1-4]. However, near-field optics is not limited to the near-field microscopy. Various theoretical approaches [5-12] and calculation algorithms [13-17] have been developed taken into account evanescent waves. The main idea of the near field optics is to increase the interval of spatial frequencies that ensures the conservation of evanescent components of the source field and thus overcomes the diffraction limit [18-26].

It should be noted that overcoming the diffraction limit can be achieved outside the near-field diffraction zone [27]. In particular, compact localization of laser radiation is achieved by sharp focusing. However, in this case, some additional means can be required: the amplitude or phase apodization of the pupil of the focusing system [28-33], the use of special types of polarization [34-37]...
or the introduction of a phase singularity into the beam [38, 39], and the combination of all these factors [40-43] with the purpose of optimizing the formed field [44-47]. However, a decreased size of the light spot outside the near-field zone is usually accompanied by a significant increase in the side lobes [27, 47-49], while there is no restriction on the size of the light spot at a distance less than the wavelength. The localization of laser radiation can be arbitrarily small, although it depends essentially on the size of the details of the focusing element [50-52] or of the acting beams [53-55].

In this paper we consider the propagation of axially symmetric laser beams in the near diffraction zone (at a distance in the order of the wavelength) by means of a plane wave decomposition involving Fourier-Hankel transforms. The eigenfunctions of such an operator with eigenvalues are close to one, determining the characteristics of signals (data) transmitted lossless (without distortion). The boundedness of the propagation operator in both the spatial and spectral regions leads to numerical eigenfunctions calculation [56-61] being required. We calculated the axisymmetric eigenfunctions of the finite propagation operator in the near diffraction zone and investigated their qualitative and quantitative characteristics depending on the propagation distance and the constraints imposed in the object and spectral domains.

2. Theoretical background

A scalar propagation nonparaxial operator using the plane wave decomposition is as given below [16, 17]:

\[
E(u,v,z) = \iint F(\xi,\eta) \exp \left( ikz \sqrt{1 - \xi^2 - \eta^2} \right) \exp \left[ i k (\xi u + \eta v) \right] d\xi d\eta,
\]

\[
F(\xi,\eta) = \frac{1}{\lambda^2} \iint E_0(x,y) \exp \left[ -ik(x\eta + y\xi) \right] dx dy,
\]

where \( F(\xi,\eta) \) is the spectrum of the input field expansion by plane waves, \( \Sigma_x : \sigma_x \leq \sqrt{\xi^2 + \eta^2} \leq \sigma_z \) is area of spatial frequencies taken into account. With \( \sigma_x = 0, \sigma_z = 1 \) only propagating waves are considered, and with \( \sigma_x = 1, \sigma_z > 1 \) only evanescent waves are considered.

In the case where the input field is axisymmetric:

\[
E_0(x,y) = E_0(r),
\]

the expression (1) can be simplified:

\[
E(\rho,z) = -ik^2 \int_0^{\sigma_0} \int_0^\rho E_0(r) J_0(k\sigma r) r dr \exp \left( ikz \sqrt{1 - \sigma^2} \right) J_0(k\sigma \rho) \sigma d\sigma,
\]

where \( \rho \) is radial coordinate in output plane, \( \sigma \) is radial coordinate in spectrum plane, \( \sigma_0 \) is radius of spatial frequencies taking into account.

Based on the Nyquist theorem, for numerical realization \( \sigma_0 \) is determined by input field \( \Delta r \) sampling:

\[
\sigma_0 \leq \frac{\lambda}{2\Delta r}.
\]

Propagating waves correspond to the spatial frequencies located in a radius of a circle \( \sigma_0 \leq 1 \). In order to take into account evanescent waves, which contribute at distances less than the wavelength, it is necessary to increase the radius of the considered spatial frequencies to a certain value \( \sigma_z > 1 \), which depends on the distance \( z \) from the aperture. Let us estimate the value.

The integral from equation (1) is considered in polar coordinates only in the region of evanescent waves:
\[ E(\rho, \theta, z) = \int_1^\infty \exp \left( -kz\sqrt{\sigma^2 - 1} \right) \left\{ \int_0^{2\pi} F(\sigma, \phi) \exp \left[ ik\rho \sigma \cos(0 - \phi) \right] \, d\phi \right\} \sigma \, d\sigma. \] (5)

Let us analyze the part of equation (5), which depends on the polar angle. The exponential factor is equal to unity in absolute value. Spectrum function \( F(\sigma, \phi) \) (for affixed \( \phi \)) decreases not slower than \( 1/\sigma \), otherwise Parseval equality is violated. Thus, integration over the angle gives a function that does not increase with increasing \( \sigma \), and for further analysis it can be replaced by a constant:

\[ I = \int_1^\infty \exp \left( -kz\sqrt{\sigma^2 - 1} \right) \sigma \, d\sigma = \frac{1}{(kz)^2}. \] (6)

Absolute error after replacing the upper limit by the final value \( \sigma_z \) is:

\[ \Delta = \sqrt{\frac{\sigma_z^2 - 1}{kz}} + \frac{1}{(kz)^2} \exp \left( -kz\sqrt{\sigma_z^2 - 1} \right), \] (7)
relative error is:

\[ \varepsilon = \frac{\Delta}{I} = \left( kz\sqrt{\sigma_z^2 - 1} + 1 \right) \exp \left( -kz\sqrt{\sigma_z^2 - 1} \right), \] (8)
The value \( \varepsilon \) monotonically decreases with increasing \( \sigma_z \), what can be can be proved by taking the derivative.

With replacement \( t = kz\sqrt{\sigma_z^2 - 1} \), we obtain a function that does not depend on definite values \( \lambda \) and \( z \). To find the admissible cut-off boundary, it is necessary to specify a certain error \( \varepsilon \) and solve the equation (8).

In particular, for \( \varepsilon = 0.04 \) we obtain \( t = 5 \), which is the choice as the upper frequency limit:

\[ \sigma_z = \sqrt{\frac{5}{kz}} + 1, \] (9)
provides error of equation (5) calculation not more than 5%.

We rewrite the operator (3) to the form of:

\[ E(\rho, z) = \int_0^\infty E_0(r) K(r, \rho, z) r \, dr, \] (10)
where

\[ K(r, \rho, z) = -ik^2 \int_0^1 \exp \left( ikz\sqrt{1 - \sigma^2} \right) J_0 \left( \frac{2\pi}{\lambda} \sigma \rho \right) J_0 \left( \frac{2\pi}{\lambda} \sigma r \right) \sigma \, d\sigma. \] (11)

Then the problem of calculating axisymmetric eigenfunctions in the near diffraction zone reduces to the search for the eigenfunctions of the following finite operator:

\[ b_n(z) \psi_n(\rho, z) = \int_0^\infty \psi_n(r) K(r, \rho, z) r \, dr, \] (12)
where \( z \) is distance, \( b_n(z) \) are eigenvalues, \( \psi_n(\rho, z) \) are eigenfunctions.

It is clear, the characteristics of the eigenfunctions will depend not only on the propagation distance \( z \), but also on the constraints imposed on the field in the object and spectral domains.

3. Calculation of the axisymmetric eigenfunctions of the finite propagation operator in the near diffraction zone

The calculation of the eigenvalues and eigenfunctions was performed for various values of the parameters at the test wavelength of the laser radiation \( \lambda = 1 \mu m \).
The Figure 1 shows the form of the matrices (11), which are the core of the transformation (12), and the Figure 2 shows the view of the calculated matrices of ordered eigenvectors for various parameters.

![Figure 1](image1.png)  \( \text{(a)} \) \( \text{(b)} \)

**Figure 1.** The amplitude of matrices (11), which are the core of the transformation (12) with (a) \( r_0 = 10 \lambda, \quad z = 0.5 \lambda, \quad \sigma_0 = 10 \), (b) \( r_0 = 10 \lambda, \quad z = 20 \lambda, \quad \sigma_0 = 1 \).

![Figure 2](image2.png)  \( \text{(a)} \) \( \text{(b)} \)

**Figure 2.** The view (amplitude, negative) of the calculated matrices of ordered eigenvectors for various parameters (a) \( r_0 = 10 \lambda, \quad z = 0.5 \lambda, \quad \sigma_0 = 10 \), (b) \( r_0 = 10 \lambda, \quad z = 20 \lambda, \quad \sigma_0 = 1 \).

As can be seen in Figures 1 and 2, constriction of the region of spatial frequencies (decreasing the value of \( \sigma_0 \)) leads to the transformation kernel matrix being filled with nonzero values. In this case, the calculation of eigenvectors becomes more complicated (the Figure 2b).

The Figure 3 shows the graphs of the calculated eigenvalues. It can be seen that the eigenvalue graph has a classical form close to the step function at distances shorter than the wavelength for evanescent waves are taken into account \( |\alpha| > 1 \) (the Figure 3a). If the distance is significantly increased when only propagating waves taken into account \(|\alpha| < 1 \) (the Figure 3b), then a number of eigenvalues absolute values close to unity becomes much smaller.

![Figure 3](image3.png)  \( \text{(a)} \) \( \text{(b)} \)

**Figure 3.** The graphs of eigenvalues absolute values \( b_n(z) \) with (a) \( r_0 = 10 \lambda, \quad z = 0.5 \lambda, \quad \sigma_0 = 10 \), (b) \( r_0 = 10 \lambda, \quad z = 20 \lambda, \quad \sigma_0 = 1 \).

The Figure 4 shows the normalized graphs of the obtained eigenfunctions. Since the functions are generally complex, it shows only the real part of the functions. It can be seen in Fig. 4 that the eigenfunctions in the first case have a classical form, and in the second case they have the "degenerate" one. To improve the situation, it is necessary to increase the size of the input field.
Figure 4. The graphs of the normalized eigenfunction real parts of $\psi_n(r, z)$ (for $n=1$ black thick line, for $n=2$ grey thick line, and for $n=15$ black thin line) with (a) $r_0 = 10\lambda$, $z = 0.5\lambda$, $\sigma_0 = 10$, (b) $r_0 = 10\lambda$, $z = 20\lambda$, $\sigma_0 = 1$.

4. Propagation modeling for fields matched with the calculated eigenfunctions

For modeling propagation of fields in free space we used expression (3). The simulation results for fields matched with the calculated eigenfunctions are shown in Tables 1 and 2.

Table 1. The simulation results for fields matched with the eigenfunctions calculated for parameters $r_0 = 10\lambda$, $z = 0.5\lambda$, $\sigma_0 = 10$.

| $n$ | Input amplitude | Spatial spectrum, $\sigma < 1$, $y \in [-10\lambda, 10\lambda]$, $z \in [0.1\lambda, 5\lambda]$ | Longitudinal distribution at distance $5\lambda$ | Transverse distribution at distance $5\lambda$ |
|-----|-----------------|------------------------------------------|------------------------------------------|------------------------------------------|
| 1   |                 |                                          |                                          |                                          |
| 2   |                 |                                          |                                          |                                          |
| 15  |                 |                                          |                                          |                                          |

As can be seen from the results given in Table 1, the fields matched with the eigenfunctions propagate in free space with preservation of its structure. For low-order eigenfunctions, a longer conservation distance is characteristic than for higher-order functions. In particular, for $n = 1, 2$, the complete conservation of the structure at a distance of $5\lambda$ is seen, while for $n = 15$ the peripheral part...
of the field is lost at the same distance. We note that the generated fields and their spatial spectra are similar to the Bessel modes [62].

Table 2. The simulation results for fields matched with the eigenfunctions calculated for parameters $r_0 = 10\lambda$, $\zeta = 20\lambda$, $\sigma_0 = 1$.

| $n$ | Input amplitude | Spatial spectrum, $\sigma < 0.5$ | Longitudinal distribution, $y \in [-10\lambda, 10\lambda]$, $z \in [5\lambda, 50\lambda]$ | Transverse distribution at distance $20\lambda$ |
|-----|-----------------|----------------------------------|---------------------------------------------|-----------------------------------------------|
| 1   | ![Input amplitude](image1) | ![Spatial spectrum](image2) | ![Longitudinal distribution](image3) | ![Transverse distribution](image4) |
| 2   | ![Input amplitude](image1) | ![Spatial spectrum](image2) | ![Longitudinal distribution](image3) | ![Transverse distribution](image4) |
| 5   | ![Input amplitude](image1) | ![Spatial spectrum](image2) | ![Longitudinal distribution](image3) | ![Transverse distribution](image4) |

As can be seen from the results given in Table 2, the fields matched with the “degenerate” eigenfunctions propagate in free space with preservation of its structure only in the central part.

5. Conclusion

In this paper, the calculation of axially symmetric eigenfunctions of finite propagation operator in the near-field diffraction was performed. It is proved that the qualitative and quantitative characteristics of eigenfunctions depend on the propagation distance and the constraints imposed in the object and spectral domains.

The simulation results show that fields matched with the calculated eigenfunctions propagate in free space with preservation of its structure. For low-order eigenfunctions, a longer conservation distance is characteristic than for higher-order functions.

6. References

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