Abstract—We investigate the non-quasi-static (NQS) effects in graphene field-effect transistors (GFETs), which are relevant for the device operation at high frequencies as a result of significant carrier inertia. A small-signal NQS model is derived from the analytical solution of drift-diffusion equation coupled with the continuity equation, which can be expressed in terms of modified Bessel functions of the first kind. The NQS model can be conveniently simplified to provide an equivalent circuit of lumped elements ready to be used in standard circuit simulators. Taking into account only first-order NQS effects, accurate GFET-Equivalent circuit simulations up to several times the cutoff frequency \( f_T \) can be performed. Notably, it reduces to the quasi-static (QS) approach when the operation frequency is below \( f_T/4 \). To validate the NQS model, we have compared its outcome against simulations based on a multi-segment approach consisting of breaking down the channel length in \( N \) equal segments described by the QS model each one.

Index Terms—Field-effect transistor (FET), graphene, high frequency (HF), non-quasi-static (NQS), radio-frequency (RF) performance.

I. INTRODUCTION

GRAPHENE field-effect transistors (GFETs) have been demonstrated to operate within the millimeter-wave range showing intrinsic cutoff frequencies and maximum oscillation frequencies up to hundreds of gigahertz [1], [2]. The design of high-frequency (HF) circuits using these emerging devices requires an appropriate description of its behavior. Many GFET models have been proposed comprising large-signal and small-signal varieties [3]–[17], but all of them based on a quasi-static (QS) approximation, where the fluctuation of the varying terminal voltages is assumed to be slow, so the stored charge in the device could follow the voltage variations. Such an approximation is found to be valid when the transition time for the voltage to change is larger than the transit time of the carriers from source to drain. This approximation works well in many field-effect transistor (FET)-based circuits, although it could fail, especially with long-channel devices operating at HF or when the load capacitance is very small [18], [19]. This can present a serious problem in state-of-the-art GFET-based circuit designs, for example, in predicting phase margins and the stability of wideband amplifiers [20].

In this article, we present a small-signal non-QS (NQS) model for the intrinsic part of a four-terminal (4T) GFET. For such a purpose, we have used a dc-to-HF methodology [19], [20] to extract a general analytical solution of the drift-diffusion equation coupled with the continuity equation. This solution can be expressed in terms of the modified Bessel functions of the first kind forming the basis of the NQS model. Then, a first-order approximation of the analytical solution based on the intrinsic admittance parameters is provided, which can be transformed into an equivalent circuit with lumped elements, analogous to the one derived for conventional silicon CMOS technology. Such lumped elements can be conveniently described in terms of the graphene chemical potentials at the drain and source edges. Thus, the equivalent circuit can be straightforwardly embedded (for instance, using a description in Verilog-A) in standard circuit simulators, enabling the performance prediction of GFET-based circuits at HF. The assessment as well as the analysis of the region of validity of the complete charge-based QS, first-order NQS, and numerical NQS models are carried out for a prototype GFET.

II. MODEL FORMULATION

Based on the previous charge-based QS model for GFETs presented by Pasadas and Jiménez [7]–[9], the graphene transport charge per unit area under time-varying excitations can be expressed in a simple form in terms of the chemical potential, \( \nu_C(x, t) \) (the notation considered can be checked in Appendix A):

\[
q_T(x, t) = \frac{k}{2}(\nu_C^2(x, t) + \alpha)
\]

where \( q_T(x, t) \) is given as a function of position \( x \) along the channel and time \( t \) [positive \( x \)-direction is from source, \( x = 0 \),

...
\( \alpha \) is the reduced Planck’s constant; \( x \) is the channel length as shown in the device’s cross section depicted in Fig. 1(a).

In (1), \( k = 2q^2/(\hbar h^2v_F^2) \); \( q \) is the elementary charge; \( \hbar \) is the reduced Planck’s constant; \( v_F = 3a_{cc}g_0/2\hbar \) is the Fermi velocity; \( a_{cc} = 2.49 \) Å is the carbon–carbon distance of the honeycomb-like crystal lattice structure [21]; and \( g_0 = 3.16 \) eV is the interlayer coupling [22]. Moreover, \( \alpha = (\pi k_B T q)^{1/3} + 2\sigma_{pud}/k_B \) is the Boltzmann’s constant; \( T \) is the temperature; and \( \sigma_{pud} \) is the residual charge density due to electron-hole puddles [11].

Now, the current in the device is no longer considered the same everywhere along the channel because fast variations are allowed and, thus, should be taken into account. In this regard, we have considered the drift-diffusion mechanism to describe carrier transport coupled with the continuity equation which are given, respectively, by

\[
\begin{align*}
i_{DS}(x, t) &= \mu W q^2 T(x, t) \left( 1 + \frac{k|C(x, t)|}{C'} \right) \partial u C(x, t) \frac{\partial C(x, t)}{\partial x} = W \frac{\partial q^2 T(x, t)}{\partial t} = W k v C(x, t) \frac{\partial C(x, t)}{\partial t} \tag{2}
\end{align*}
\]

where \( i_{DS}(x, t) \) is the time-varying drain-to-source current; \( \mu \) is the effective carrier mobility for both electrons and holes (both assumed to be independent of the applied electric field, carrier density, or temperature); \( W \) is the channel width; and \( C' = C_1' + C_2' \), where \( C_1' \) (\( C_2' \)) is the geometrical capacitance per unit area of the top (back) gate oxide shown as TOX (BOX) in Fig. 1(a).

Considering that the time-varying excitations are small in amplitude, we can assume that only first-order terms in the ac components are retained. Moreover, if the excitations are sinusoidal, we can describe the ac terms with phasors and, therefore, rewrite (2) in the following form (a detailed explanation of the process followed to get (3) is given in Section A of [23]):

\[
\begin{align*}
i &= \mu W k \frac{2}{2} \left[ \frac{k^2}{\gamma} \left( \frac{|C'|}{C'} \right) + \left( 2 + 3b |C'| \right) \frac{\partial u}{\partial x} + (v^2 + \alpha) \left( 1 + \frac{k|C'|}{C'} \right) \frac{\partial u}{\partial x} \right] \\
\frac{\partial i}{\partial x} &= j \omega W k v u. \tag{3}
\end{align*}
\]

Applying some transformations to the differential equations describing the small-signal sinusoidal operation of a GFET in (3), the following second-order differential equation has been found to describe the NQS response of such a device:

\[
\frac{d^2 i}{dv^2} - \frac{1}{v} \frac{di}{dv} - j D v^3 = 0 \tag{4}
\]

where \( i_{DS} \) is the dc drain current. We have considered that \( v^2 \gg \alpha \), implying that the thermal charge density plus electron-hole puddle density is much lower than the charge density produced by the electrostatics (see (1)). That could be the case when the gate bias is far away from the Dirac voltage, which results in unipolar channels. This particular bias point is desirable when the GFET is used as a radio-frequency (RF) amplifier [24] and will be discussed in depth later.

The second-order differential equation expressed by (4) can be solved in two opposite limits:

**Case A:** \( k|v|/C' \ll 1 \).

In this case, (4) can be rewritten as follows:

\[
\frac{d^2 i}{dv^2} - \frac{1}{v} \frac{di}{dv} - j D v^3 = 0 \tag{5}
\]

where \( D = \omega \mu W k^2/(2I_{DS}^2) \). The general solution of (5) is given by

\[
\begin{align*}
i(v) &= \left[ n_1 v_{cs} (p_2 - p_3) n_2 + i_d p_3 (n_1 - n_3) p_1 \right] \\
&= \left[ \frac{n_1}{v_{cs}} \right] \left[ p_2 (n_1 - n_2) p_1 \right] + \frac{i_d}{v_{cd}} p_3 (n_1 - n_2) p_1 \tag{6}
\end{align*}
\]

with

\[
\begin{align*}
n_1 &= I_{-\frac{1}{2}} \left( \frac{2}{5} D_{1} v_{cs}^{5/2} \right) \quad p_1 = I_{-\frac{1}{2}} \left( \frac{2}{5} D_{1} v_{cs}^{5/2} \right) \\
n_2 &= I_{-\frac{1}{2}} \left( \frac{2}{5} D_{1} v_{cd}^{5/2} \right) \quad p_2 = I_{-\frac{1}{2}} \left( \frac{2}{5} D_{1} v_{cd}^{5/2} \right) \\
n_3 (v) &= I_{-\frac{1}{2}} \left( \frac{2}{5} D_{1} v_{cd}^{5/2} \right) \quad p_3 (v) = I_{-\frac{1}{2}} \left( \frac{2}{5} D_{1} v_{cd}^{5/2} \right) \tag{7}
\end{align*}
\]

where \( D_1 = (-1)^{1/4} D^{1/4} \) and \( I_{\lambda} (z) \) is the modified Bessel function of the first kind [25], where \( \lambda \) is the real order and \( z \) is the complex argument. \( I_{\lambda} (z) \) is a convergent series everywhere in the complex \( z \)-plane. The boundary conditions are given by ([23 Sec. B]) can be consulted to check the analytic computation of \( I_{\lambda} (z) \) and the procedure followed.
to determine the boundary conditions $i_s$, $i_d$, $v_{cs}$, $v_{cd}$, $u_{cs}$, and $u_{cd}$:

$$
\begin{align*}
    i_s &= v_{cs} D_2 u_{cd} \sqrt{v_{cd}} [n_1 n_{4s} - p_1 p_{4s}] + u_{cs} \sqrt{v_{cs}} [p_1 p_{4d} - n_1 n_{4d}] \\
    i_d &= v_{cd} D_2 u_{cd} \sqrt{v_{cd}} [n_2 n_{4s} - p_2 p_{4s}] + u_{cs} \sqrt{v_{cs}} [p_2 p_{4d} - n_2 n_{4d}] \\
    v_{cs} &= C' - \sqrt{C'^2 \pm 2k \left[ C'_0(V_G - V_G0 - V_S) + C'_b(V_B - V_B0 - V_S) \right]} \\
    v_{cd} &= C' - \sqrt{C'^2 \pm 2k \left[ C'_0(V_G - V_G0 - V_D) + C'_b(V_B - V_B0 - V_D) \right]} \\
    u_{cs} &= -\frac{C'_0(V_G - V_s) + C'_b(V_b - V_s)}{\sqrt{C'^2 \pm 2k \left[ C'_0(V_G - V_G0 - V_S) + C'_b(V_B - V_B0 - V_S) \right]}} \\
    u_{cd} &= -\frac{C'_0(V_G - V_d) + C'_b(V_b - V_d)}{\sqrt{C'^2 \pm 2k \left[ C'_0(V_G - V_G0 - V_D) + C'_b(V_B - V_B0 - V_D) \right]}}
\end{align*}
$$

with

$$
n_4(v) = I_{\frac{1}{3}} \left( \frac{2}{3} D_1 v^{5/2} \right) \quad p_4(v) = I_{\frac{1}{3}} \left( \frac{2}{3} D_1 v^{5/2} \right)
$$

where $D_2 = jD_1 D_S$; $n_{4s} = n_4(v_{cs})$; $n_{4d} = n_4(v_{cd})$; $p_{4s} = p_4(v_{cs})$; and $p_{4d} = p_4(v_{cd})$. The positive (negative) sign in (8) applies when $C'_0(V_G - V_G0 - V_S) + C'_b(V_B - V_B0 - V_S) < 0$ ($>0$), where the subscript $X$ stands for drain (D) and source (S).

**Case B:** $k|\nu|C' \ll 1$.

In this case, (4) can be rewritten as follows:

$$
d^2i + \frac{1}{v} \frac{di}{dv} - jD_1 v^4 i = 0
$$

where $D = \text{Sign}[\nu] \omega \mu W^2 k^3/(2C' I_{DS}^2)$. The general solution of (10) is also given by (6) but, in this case, we have to consider instead

$$
n_1 = I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right) \quad p_1 = I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right) \\
\begin{align*}
    n_2 &= I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right) \\
    p_2 &= I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right) \\
    n_3(v) &= I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right) \\
    p_3(v) &= I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right)
\end{align*}
$$

where $D_1 = (-1)\nu D^{1/2}$; $D_{1s} = D_1|\nu=v_{cs}$; $D_{1d} = D_1|\nu=v_{cd}$; and the boundary conditions, $i_s$ and $i_d$, as shown at the bottom of this page [see (12)], and

$$
n_4(v) = I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right) \quad p_4(v) = I_{\frac{1}{3}} \left( \frac{1}{3} D_1 v^{3/2} \right)
$$

where $D_2 = jD_1 D_S$; $n_{4s} = n_4(v_{cs})$; $n_{4d} = n_4(v_{cd})$; $p_{4s} = p_4(v_{cs})$; and $p_{4d} = p_4(v_{cd})$.

In Fig. 1(b), the schematic of the terminal currents of a 4T device is shown. In (8) and (12), expressions for computing the sinusoidal ac drain and source terminal currents are provided. To guarantee charge-conservation the sum of the ac terminal currents entering into a device must be zero, thus,

$$
i_s = v_{cs} D_1 D_2 u_{cd} u_{cd} [n_1 n_{4s} - p_1 p_{4s}] + D_1 D_2 u_{cd} u_{cs} v_{cs} [p_1 p_{4d} - n_1 n_{4d}] \\
i_d = v_{cd} D_1 D_2 u_{cd} u_{cd} [n_2 n_{4s} - p_2 p_{4s}] + D_1 D_2 u_{cd} u_{cs} v_{cs} [p_2 p_{4d} - n_2 n_{4d}]$

(12)
\[ i_g(t) + i_b(t) = -i_d(t) - i_s(t). \] As the top and back gate ac terminal currents are related to the top and back gate geometrical capacitances per unit area [9], the NQS ac terminal currents can be expressed as follows (see [23, Sec. B4] for further details):

\begin{align*}
I_s(\omega) & = -I_{ds}(0, \omega) = -i_s \\
I_d(\omega) & = I_{ds}(L, \omega) = i_d \\
i_g & = I_g(\omega) = -\frac{C'}{C}(i_d - i_s) \\
i_b & = I_b(\omega) = -\frac{C''}{C}(i_d - i_s). \quad (14)
\end{align*}

### A. NQS Short-Circuit Admittance Parameters of GFETs

An equivalent circuit based on the short-circuit admittance parameters can be provided by relating the terminal current phasors in (14) to the terminal voltage phasors. They can be easily calculated using the definition, taking the source as reference [20]:

\[ y_{jk} = \frac{i_j}{u_{ks}} \bigg|_{u_{ks} = 0, i \neq k} \quad (15) \]

where the subscripts \( j, k, \) and \( l \) stand for drain (d), top gate (g), and back gate (b). Only nine admittances out of 16 are independent, and hence sufficient to completely characterize the device [19]. The \( y \)-parameters are important since design of HF amplifiers is easily done in terms of these parameters. However, in such designs it is usual to consider that the back-gate terminal is also ac short-circuited to the source, thus, forming a two-port network in common-source configuration and resulting in the general form of the equivalent circuit depicted in Fig. 2(a), which is considered in the following derivation.

We have focused on the Case B. Reference [23, Sec. C] includes the full procedure followed to get the expressions of the admittance parameters for both cases A and B. The procedure begins by rewriting (6) as follows [26]:

\[ i(v) = k_1g_1(v) + k_2g_2(v) \quad (16) \]

where

\begin{align*}
& k_1 = \frac{i_d n_1 v_{cs} - i_s n_2 v_{cd}}{v_{cs} v_{cd}(p_{2n1} - n_{p1})} \left(\frac{D_1}{6}\right)^{1/3} \frac{1}{1/4(3)} \\
& k_2 = \frac{i_s p_{2o cd} - i_d p_{1o cs}}{v_{cs} v_{cd}(p_{2n1} - n_{p1})} \left(\frac{D_1}{6}\right)^{1/3} \frac{1}{1/2(3)}
\end{align*}

The positive (negative) sign in (21) applies when \( F_i(V_G - V_{G0} - V_X) + C_i'(V_B - V_{B0} - V_X) < 0 \) (>0), where the subscript \( X \) stands for drain (D) and source (S).

### B. First-Order NQS Equivalent Circuit of a GFET

If second- and higher-order terms in \( \omega \) are neglected when computing (20) to get the NQS \( y \)-parameters of a GFET, a first-order NQS model is obtained with the following admittance parameters:

\begin{align*}
y_{11} & = j \omega g_{m0} \frac{\tau_4}{1 + j \omega \tau_1} \\
y_{12} & = -j \omega g_{m0} \frac{\tau_2}{1 + j \omega \tau_1} \\
y_{21} & = g_{m0} \frac{1 - j \omega \tau_2}{1 + j \omega \tau_1} \\
y_{22} & = g_{d0} \frac{1 + j \omega \tau_3}{1 + j \omega \tau_1} \quad (22)
\end{align*}

\[
\begin{pmatrix}
i_1 \\
i_2
\end{pmatrix}
= \begin{pmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{pmatrix}
\begin{pmatrix}
v_1 \\
v_2
\end{pmatrix}
\]

\begin{align*}
y_{11} & = \frac{F_1 F_2 h_{GD} - F_2 h_{GS}(g_{1s} - g_{1d}) + (F_{id} h_{GS} - F_{1s} h_{GD}) (g_{2s} - g_{2d})}{B(F_{1s} F_{2d} - F_{1d} F_{2s})} \\
y_{12} & = \frac{h_{GD} F_2 (g_{1s} - g_{1d}) - F_{1s} (g_{2s} - g_{2d})}{B(F_{1s} F_{2d} - F_{1d} F_{2s})} \\
y_{21} & = \frac{(F_2 h_{GS} - F_2 h_{GD}) g_{1d} + (F_{1s} h_{GD} - F_{1d} h_{GS}) g_{2d}}{B(F_{1s} F_{2d} - F_{1d} F_{2s})} \\
y_{22} & = \frac{(h_{GD} + h_{BD}) F_{1s} g_{2d} - F_{2s} g_{1d}}{B(F_{1s} F_{2d} - F_{1d} F_{2s})}
\end{align*}

\[
g_1(v) = v^2 \left(1 + j \frac{D_0 b^6}{48} - j \frac{D_2 b^{12}}{8064} + \cdots \right)
\]

\[
g_2(v) = v^2 \left(1 + j \frac{D_0 b^6}{24} - j \frac{D_2 b^{12}}{1920} + \cdots \right) \quad (17)
\]

where \( \Gamma \) stands for the Gamma function [25]. If (16) is differentiated, the following equation is obtained:

\[ \frac{u}{B} = k_1 F_1(v) + k_2 F_2(v) \quad (18) \]

where \( B = 1/D_2 \) and

\begin{align*}
F_1(v) & = \frac{d g_1}{dv} / v^4 \\
F_2(v) & = \frac{d g_2}{dv} / v^4. \quad (19)
\end{align*}
to the one shown in Fig. 2(b) by adopting the following formulas:

\[ g_{m0} = -\mu \frac{W}{L} k^2 \left( h_{GD} |v_{cd}|^3 - h_{GS} |v_{cs}|^3 \right) \]

\[ g_{ds0} = \mu \frac{W}{L} \frac{k^2}{2} \left( h_{GD} + h_{BD} \right) |v_{cd}|^3 \]

\[ \tau_1 = -\frac{D'}{12 \left( v_{cs}^2 + v_{cd}^2 \right)} \]

\[ \tau_2 = \frac{D'}{24} \left( v_{cs}^6 - 3 v_{cs}^2 v_{cd}^4 + 2 v_{cs}^6 \right) h_{GD} v_{cd}^3 \]

\[ \tau_3 = \frac{D'}{24} \left( v_{cs}^6 - 3 v_{cs}^2 v_{cd}^4 + 2 v_{cs}^6 \right) h_{GS} v_{cd}^3 \]

\[ \tau_4 = \frac{D'}{24} \left( h_{GS} v_{cd}^3 (v_{cs}^2 + 2v_{cs}^2) + h_{GS} v_{cd}^3 (2v_{cs}^2 + 2v_{cd}^2) \right) \]

where \( D' = D/\omega \).

We can go from the equivalent circuit depicted in Fig. 2(a) to the one shown in Fig. 2(b) by adopting the following relationships:

\[ y_1 = y_{11} + y_{12} \]

\[ y_2 = -y_{12} \]

\[ y_m = y_{21} - y_{12} \]

\[ y_0 = y_{22} + y_{12}. \]

Therefore, the admittances in Fig. 2(b) can be expressed as

\[ y_1 = j \omega g_{m0} \left( \frac{\gamma \tau_4 - \tau_2}{1 + j \omega \tau_1} \right) \]

\[ y_2 = j \omega g_{m0} \frac{\tau_2}{1 + j \omega \tau_1} \]

\[ y_m = \frac{g_{ds0}}{1 + j \omega \tau_1} \]

\[ y_0 = g_{ds0} \frac{1 + \omega \tau_3 (1 - \gamma)}{1 + j \omega \tau_1}. \]

Equation (25) yields an equivalent circuit in which the gate–source and gate–drain admittances, \( y_1 \) and \( y_2 \), respectively, are simple \( RC \) networks, and the drain–source is formed by the parallel combination of a frequency-dependent current source \( y_m v_{GS} \) and the output admittance which is a simple lossy \( LC \) network. Fig. 2(c) shows the first-order NQS equivalent circuit of a GFET based on lumped elements, which are quantitatively described by the following formulas:

\[ C_1 = g_{m0} (\gamma \tau_4 - \tau_2) \]

\[ C_3 = g_{ds0} \tau_3 (1 - \gamma) \]

\[ r_1 = \frac{\tau_1}{C_1} \]

\[ r_3 = \frac{\tau_1}{C_3} \]

\[ C_2 = g_{m0} \tau_2 \]

\[ r_0 = \frac{1}{g_{ds0}} \]

\[ r_2 = \frac{\tau_1}{C_2} \]

\[ L_0 = \frac{\tau_1}{g_{ds0}}. \]

Note that an \( n \)-order NQS model of a GFET can be derived by taking higher order terms in \( \omega \) in both \( g_1(\omega) \) and \( g_2(\omega) \) in (17) (the detailed derivation of the zero- and first-order NQS models can be checked in [23, Sec. C1 and C2], respectively).

### III. Results and Discussion

As a convenient testbed to assess the NQS model, we have selected the prototype GFET described in Table I. The transistor is double-gated with 10-nm Al2O3 and 300-nm SiO2 dielectrics as top and back gates, respectively. First, Fig. 3(a) presents the dc transfer characteristics for \( V_D = 1 \text{ V} \), \( V_S = V_B = 0 \text{ V} \), and Fig. 3(b) shows the gate bias dependence of the chemical potentials \( v_{cs} \) and \( v_{cd} \) computed according to (8). The bias window in which the NQS model previously presented cannot be used is indicated by a shaded region, corresponding to biases where the graphene channel is bipolar and/or the consideration \( \nu^2 > \alpha = 1.2 \cdot 10^{-2} \text{ eV}^2 \), that is, \( |\nu| > 110 \text{ meV} \) is not satisfied. We considered that the device operates at room temperature (300 K) and the inhomogeneity of the electrostatic potential due to the presence of electron-hole puddles is 100 meV causing a residual density of \( \Delta \nu_{\text{paleq}} = 6.9 \cdot 10^{15} \text{ m}^{-2} \). The resulting gate bias region in which the NQS model cannot be applied is approximately from -0.3 to 1.3 V. Out of this bias window, case B applies for the specific device arrangement considered here.

Fig. 3(c) shows the chemical potential along the normalized channel length at \( V_G = 1.5 \text{ V} \). In doing so, assuming that the dc current \( I_{DS} \) is the same everywhere along the channel, the chemical potential along the channel can be described as...
Fig. 4. (a) Modulus of the ac current at the drain ($i_d$) and source ($i_s$) edges computed in an analytical (symbols) and numerical (dotted line) way. The former computed by truncating the modified Bessel function of the first kind to $n = 10$. For the numerical calculation, the function “besseli” of MATLAB software is used. (b) Modulus of the ac current along the normalized position in the channel for different frequencies. (c) Normalized magnitude and (d) phase of $y_m$ versus frequency at $V_G = 1.5$ V, $V_D = 1$ V, $V_B = V_S = 0$ V. Four kind of models are considered: zero-order model (dash-dotted lines); the complete QS model described by the parameters given in Table II (solid lines); the first-order NQS model described by the parameters given in Table III (dashed lines); and the numerical NQS model (dotted lines). (e) Small-signal current gain ($h_{21}$), unilateral power gain ($U$, Mason’s invariant [30]), and MSG/MAG versus frequency of the GFET under test predicted by the QS (solid lines), first-order NQS (dashed lines), and numerical NQS (dotted lines) models. The RF figures of merit $f_T$ and $f_{max}$ are gotten when such gains are reduced to unity (0 dB) (see Appendix B for more information). (f) Normalized magnitude of $y_m$ versus frequency under the operating bias point $V_G = 1.5$ V, $V_D = 1$ V, and $V_B = V_S = 0$ V for a single 1 µm-length GFET compared against a two-port configuration of a cascade of 20 GFETs, 50-nm-length each one, connected in series. (inset) Schematics of the multisegment approach applied to a GFET.

| TABLE II |
| --- |
| SMALL-SIGNAL ELEMENTS OF THE QS EQUIVALENT CIRCUIT [FIG. 2(D)] DESCRIBING THE GFET UNDER TEST |
| $g_m$ | 17.30 mS |
| $C_{gd}$ | 43.35 fF |
| $C_{gd}$ | 20.99 fF |
| $g_{ds}$ | 7.92 mS |
| $C_{ds}$ | 31.33 fF |
| $C_{cd}$ | -5.98 fF |

follows:

$$v = V_C(x) = \text{Sign}[v_{cd}] \left( n^4_{cd} - \frac{8C'I_D}{W\mu k^2}(L - x) \right).$$  \hspace{1cm} (27)

Both $v_{cs} = V_C(0)$ and $v_{cd} = V_C(L)$ have the same sign, meaning that the channel is occupied by the same kind of carrier, in this case, electrons.

Now, we superimpose a 4 mV sinusoidal time-varying signal at the drain terminal. Fig. 4(a) shows the absolute value of $i_s$ and $i_d$ for different frequencies, which have been computed according to (12) by truncating the modified Bessel function of the first kind to $n = 10$ [23, Sec. B3]. Table II describes the QS small-signal parameters describing the device at the considered bias point. For convenience, the equivalent circuit of the complete QS small-signal model of GFETs [27] has been depicted in Fig. 2(d). The parameters have been calculated according to the QS model presented by Pasadas and Jiménez in [7] and [8]. The cutoff frequency, $f_T = 38.8$ GHz, has also been marked in Fig. 4(a). It has been analytically calculated using the derivation carried out by Pasadas et al. in [27], with the small-signal parameters given in Table II and considering extrinsic resistances of $R_cW = R_dW = 100 \Omega \mu$m and $R_0 = 10 \Omega$. According to Fig. 4(a), the modulus of $i_s$ and $i_d$ is the same for frequencies lower than $\sim f_T/4$, which agrees with the QS assumption, similar to what happens in silicon-based FETs [19]. For frequencies higher than $f_T/4$, $|i_s|$ and $|i_d|$ depart from the QS value presenting different values, which indicate that the channel charge in the graphene layer cannot follow the voltage variations for such frequencies.

Fig. 4(b) shows the modulus of the ac current, $|i|$, along the normalized channel length for different frequencies. In doing so, (6) is computed by using the result of evaluating (27) [see Fig. 3(c)]. The solid orange line depicted in Fig. 4(b) corresponds to a frequency of $f_T/4$, which has been chosen to
TABLE III
SMALL-SIGNAL ELEMENTS OF THE FIRST-ORDER NQS EQUIVALENT CIRCUIT [FIG. 2(C)] DESCRIBING THE GFET UNDER TEST

| Parameter | Value 1 | Value 2 |
|-----------|---------|---------|
| $g_{m0}$  | 12.60 mS | 4.23 mS |
| $C_1$     | 44.18 fF | $L_0$   | 91.61 pH |
| $r_1$     | 8.78 Ω   | $r_9$   | 236.25 Ω |
| $C_2$     | 16.47 fF | $\tau_1$ | 387.76 fs |
| $r_2$     | 23.53 Ω | $\tau_2$ | 1.31 ps     |
| $C_3$     | 0.24 fF  | $\tau_3$ | 3.95 ps     |
| $r_3$     | 1.63 kΩ  | $\tau_4$ | 4.88 ps     |

Fig. 5. Gate bias dependence of the small-signal parameters of the first-order NQS model of GFET. The shaded region indicates the bias region where the NQS model developed in this article cannot be applied.

B. Comparison Among HF Models

A benchmarking of the NQS model against the QS model for the GFET under test is carried out. In doing so, Fig. 4(c) and (d) shows, respectively, the normalized magnitude and phase of the admittance $y_m$ given in (25). It is observed that going from the zero-order model to the first-order NQS model produces a drastic improvement in the region of validity. The region of validity for the complete QS model is limited by the fact that, at HFs, the error in the magnitude becomes severe. This is because $y_m$ contains a left-half-plane zero for this model [27] in contrast to the left-half-plane pole in $y_m$ for the first-order NQS model (see 25). The upward-going magnitude predicted by the QS model at HFs is clearly unrealistic, since it suggests an enhancement in the forward gate-to-drain action, contrary to the expectation that, at HFs, control of the gate on the drain current is gradually lost due to the carrier inertia in the graphene channel. However, the phase of $y_m$ is better predicted by the QS model than does the first-order NQS model, so a higher order correction would be needed if the phase of $y_m$ is crucial for the targeted range of frequency and intended application. This discussion as well as results shown in Fig. 4(c) and (d) are in agreement with the NQS studies carried out for conventional Si MOSFETs [19], [20].

Next, we have compared the frequency dependence of the current and power gains, namely $|h_{21}|$, $U$, and maximum stable gain/maximum available gain (MSG/MAG), as predicted by the different models. In Appendix B, we have given further details of RF performance calculation in FETs. The result is shown in Fig. 4(e). It can be seen from the plot that the differences between both QS and NQS model predictions are not significant below $f_T$ (38.8 GHz), but clearly the QS model overpredicts the gain in the frequency range 100 GHz–1 THz, highlighting the importance of including the NQS effects.

C. Multisegment Approach

One way to model a transistor at speeds where the QS model breaks down is to view it as consisting of several sections, each section being short enough to be modeled quasi-statically [19], [28], [29]. To test such an approach, we have run some simulations with the circuit simulator Advance Design Systems by using the intrinsic QS compact model developed by Pasadas and Jiménez [7]. Fig. 4(f) shows the normalized magnitude of $y_m$ for the intrinsic GFET described in Table I together with the result obtained by a cascade of 20 GFETs in series (connecting the drain of each GFET with the source of the next one and all the device gates are short-circuited), each having a length $L/20 = 50$ nm, and both examined cases at the same bias point previously considered ($V_G = 1.5$ V, $V_D = 1$ V, and $V_B = V_S = 0$ V). It is observed that the results agree with the ones shown in Fig. 4(c), which demonstrates the consistency of the NQS model presented here.

IV. Conclusion

An NQS model for the GFET has been proposed aiming to capture the delay between the charge change and voltage...
change when the device is operated at HFs. It has been derived following a dc-to-HF methodology that ends up in a general solution for the ac current, which can be numerically computed in terms of the modified Bessel function of the first kind and analytically by truncating the convergent series describing such a Bessel function. An analytical derivation of the first-order NQS model has been carried out to ultimately obtain an equivalent circuit based on lumped elements, which can be included in a circuit simulator to carry out simulations of arbitrary circuits based on GFETs operated at HFs.

A benchmarking of the first-order and numerical NQS models has been given against the QS model. The frequency region that the NQS model can manage has also been established which slightly depends on the bias point, the desired accuracy, and whether the magnitude or phase is of most interest. According to our simulations, the QS model works up to \( f_T/4 \), although a first-order NQS model does extend the range of considered frequencies up to \( f_T \) accepting an error in \(|y_{\text{m}1}/g_{\text{m}0}| < 1\% \) or even \( 2f_T \) accepting an error < 3.5\%. To further validate the NQS model, we have compared its outcome against simulations based on a multisegment approach where each segment is described by the QS model.

**APPENDIX A**

The following notation has been considered for the model formulation:

1. Charge per unit area: \( Q'(x) \).
2. Capacitance per unit area: \( C' \).
3. Total charge: \( Q = \int W \cdot Q'(x) \, dx \).
4. Total capacitance: \( C = WLC' \).
5. dc quantities (upper-case symbol with capital subscript): \( V_C(x) \).
6. Large-signal quantities (lower-case symbol with capital subscript): \( u_C(x, t) \).
7. Small-signal quantities (lower-case symbol with lower-case subscript): \( u_c(x, t) \).
8. Time-independent phasor quantities (upper-case symbol with lower-case subscript): \( V_c(x, \omega) \).

Besides, the following specific notation is also used when the time-varying excitation is considered small and sinusoidal:

1. dc channel current potentials:
   \[
   u = V_C(x) \quad u_{cs} = V_C(0) \quad u_{cd} = V_C(L).
   \]
2. Time-independent phasors of the chemical potential:
   \[
   u = V_c(x, \omega) \quad u_{cs} = V_c(0, \omega) \quad u_{cd} = V_c(L, \omega).
   \]
3. Time-independent phasors of the current:
   \[
   i = I_{ds}(x, \omega) \quad i_s = I_{ds}(0, \omega) \quad i_d = I_{ds}(L, \omega).
   \]
4. dc current:
   \[
   I_{DS} = I_{DS}(x).
   \]

**APPENDIX B**

To benchmark an RF technology, it is common to evaluate the cutoff frequency (\( f_T \)) and the maximum oscillation frequency (\( f_{\text{max}} \)). The computation of such figures of merit can be done from the \( y \)-parameter matrix. An extrinsic resistance matrix is added to account for the gate resistance (\( R_c \)) as well as the series combination of the contact and access resistances at the source (\( R_s \)) and drain (\( R_d \)) sides which are of utmost importance when dealing with low-dimensional FETs [27]. The resulting \( y \)-parameter matrix reads as follows:

\[
\begin{pmatrix}
y_{11,x} & y_{12,x} \\
y_{21,x} & y_{22,x}
\end{pmatrix} = \left( \begin{pmatrix} y_{11} & y_{12} \end{pmatrix}^{-1} + \begin{pmatrix} R_c + R_e & R_e \\
R_s & R_s + R_d \end{pmatrix} \right)^{-1}.
\]

(28)

The \( f_T \) is defined as the frequency at which the magnitude of the small-signal current gain (\( h_{21} \)) of the transistor is reduced to unity, while the \( f_{\text{max}} \) is defined as the frequency at which the magnitude of either the unilateral power gain [30] (\( U \) or Mason’s invariant) or the MSG/MAG of the transistor is reduced to unity:

\[
h_{21} = \frac{y_{21,x}}{y_{11,x}} \quad U = \frac{|y_{12,x} - y_{21,x}|^2}{4(\text{Re}[y_{11,x}] \text{Re}[y_{22,x}] - \text{Re}[y_{12,x}] \text{Re}[y_{21,x}])}
\]

\[
G_P^{\text{max}} = \begin{cases} 
\text{MSG} = \frac{y_{21,x}}{y_{11,x}}, & -1 < K < 1 \\
\text{MAG} = \frac{y_{21,x}}{y_{11,x}} (K - \sqrt{K^2 - 1}), & K \geq 1; \quad |\Delta| < 1
\end{cases}
\]

(29)

where \( K \) and \( \Delta \) are the factors used for the evaluation of the stability and are computed as follows:

\[
K = \frac{2\text{Re}[y_{11,x}] \text{Re}[y_{22,x}] - \text{Re}[y_{12,x}] y_{21,x}}{|y_{12,x} y_{21,x}|}
\]

\[
\Delta = \frac{(Y_0 - y_{11,x})(Y_0 - y_{22,x}) - y_{12,x} y_{21,x}}{(Y_0 + y_{11,x})(Y_0 + y_{22,x}) - y_{12,x} y_{21,x}}.
\]

(30)

where \( Y_0 = 20 \text{ mS} \) is the characteristic admittance.

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