Unstable $g$-modes in proto-neutron stars

V Ferrari$^1$, L Gualtieri$^1$ and J A Pons$^2$

$^1$ Dipartimento di Fisica ‘G Marconi’, Sapienza Università di Roma and Sezione INFN ROMA1, piazzale Aldo Moro 2, I-00185 Roma, Italy
$^2$ Departament de Física Aplicada, Universitat d’Alacant, Apartat de correus 99, 03080 Alacant, Spain

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Abstract
In this paper we study the possibility that, due to nonlinear couplings, unstable $g$-modes associated with convective motions excite stable oscillating $g$-modes. This problem is of particular interest, since the gravitational waves emitted by a newly born proto-neutron star pulsating in its stable $g$-modes would be in the bandwidth of VIRGO and LIGO. Our results indicate that the nonlinear saturation of unstable modes occurs at relatively low amplitudes, and therefore, even if there exists a coupling between the stable and unstable modes, it does not seem to be sufficiently effective to explain, alone, the excitation of the oscillating $g$-modes found in hydrodynamical simulations.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

It is well known that proto-neutron stars (PNS) born in the aftermath of a gravitational core collapse are, during the first tens of seconds, convectively unstable [1]. In addition, dynamical simulations of core collapse have shown that a considerable fraction of the energy emitted in gravitational waves (GW) is actually due to this convective motion, rather than to the violent bounce [2, 3]. Recently, a new mechanism for the core collapse supernova explosion has been proposed, based on the excitation and sonic damping of core $g$-mode oscillations [4–6], and the same mechanism has also been pointed out to be relevant for GW emission [7]. The process begins as an advective-acoustic oscillation, and in [5] the authors suggest that the primary agent of the explosion is the acoustic power generated in the inner turbulent region; the accreting PNS is a self-excited oscillator, and the core oscillation acts as a transducer converting the accretion energy into the acoustic energy. Surprisingly, and despite the complexity of the nonlinear hydrodynamical effects involved in these simulations, the growth times, oscillation frequencies and temporal evolution of the modes are qualitatively and quantitatively similar to the predictions of a linear analysis done in [8]: there, the frequencies and damping times...
of \( g \)-modes were computed for an evolving PNS, modeled as sequences of quasi-stationary configurations.

These results put the \( g \)-modes in a somewhat different perspective with respect to the preceding literature on the subject; indeed, previous studies of \( g \)-mode oscillations in neutron stars had shown that their contribution to the gravitational wave signal was likely to be too weak to be detectable by current or near-future detectors [9–13].

Motivated by these considerations, in this paper we study the spectrum of stable and unstable \( g \)-modes of a newly born PNS, and we investigate how effective the coupling is, due to the nonlinearity of the equations of hydrodynamics, between unstable \( g \)-modes and oscillating \( g \)-modes. The aim is to understand whether this coupling induces \( g \)-mode oscillations which may significantly contribute to the gravitational wave emission during the early phases of a proto-neutron star’s life. For this purpose, we integrate the equations of relativistic stellar perturbations for the model of PNS considered in [8], which describes the quasi-stationary evolution of a PNS during the first minute after core collapse; we look for the eigenmodes the frequency of which, at some point of the evolution, becomes imaginary or acquires a negative imaginary part, a behavior which indicates a mode instability. We then discuss to what extent the unstable and the stable \( g \)-modes are coupled.

It should be mentioned that the existence of an unstable branch of \( g \)-modes is a natural consequence of the PNS thermal and chemical profiles, since some regions of the star are convectively unstable while other regions are not. Thus, even at the linear level one can see that both stable and unstable \( g \)-modes are present. The linear analysis allows us to determine which is the fastest growing mode and the frequencies of the oscillatory modes.

The PNS we consider, at a very early stage has a radius of about 100 km, and it gradually contracts in a few seconds. It has a fixed baryonic mass of \( 1.6M_\odot \), which corresponds to a final gravitational mass (after deleptonization and cooling) of \( 1.46M_\odot \). The radius of the PNS at \( t = 50 \) ms after bounce is 50 km but just 1 s later, due to neutrino losses from the envelope, the star contracts to a radius of less than 20 km. The final radius of the cold, catalyzed NS (after 30–40 s) is 12 km. In computing the \( g \)-mode eigenfrequencies at selected values of time, we have taken the corresponding radial profiles (mass, pressure, temperature, lepton content, etc) provided by the evolutionary models of [1], already used in [8].

Evaluating the effect of nonlinear couplings between different modes is a formidable task; however an estimate of the leading order contributions can be provided using a Newtonian approach based on the amplitude equation formalism developed in [14, 15].

This paper is organized as follows. In section 2 we describe the mathematical and numerical setup of our calculations; in section 3 we discuss the structure of the stable and the unstable \( g \)-modes in PNS; in section 4 we study the coupling between different modes in terms of appropriate integrals of the radial eigenfunctions; concluding remarks are given in section 5.

2. Equations and numerical setup

The equations governing the perturbations of spherical stars in general relativity have been derived within different approaches by many authors [16–22]. Here, we use the Lindblom–Splinter (LS) equations derived in [21], which describe dipolar \( (l = 1) \) perturbations, and the Lindblom–Detweiler (LD) equations derived in [20], for \( l \geq 2 \) perturbations. The metric and the fluid 4-velocity of the spherical background describing the unperturbed star are

\[
\begin{align*}
\text{d}s^2 &= -e^\nu \, \text{d}t^2 + e^\lambda \, \text{d}r^2 + r^2 (\text{d}\theta^2 + \sin^2 \theta \, \text{d}\phi^2) \\
u^\mu &= (e^{-\nu/2}, 0, 0, 0),
\end{align*}
\] (1)
The functions $v(r), \lambda(r)$ satisfy the TOV equations (see, for instance, [23]). We shall assume that matter in the star is a perfect fluid.

The metric perturbations and the Eulerian fluid perturbations are expanded in spherical harmonics $Y^{lm}(\theta, \phi)$, and Fourier expanded as described in [20, 21]. In particular, the radial component of the Lagrangian displacement is

$$\delta r = \delta r^{lm}(\omega, r) Y^{lm}(\theta, \phi) e^{i\omega t}. \quad (2)$$

The perturbed Einstein equations

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu}$$

provide a system of linear, ordinary differential equations (ODE) for the radial components of the harmonic expansion of the metric and fluid perturbations. Since the structure of the equations for $l = 1$ and $l > 1$ is quite different, we will treat the two cases separately.

For $l = 1$ perturbations are not associated with gravitational wave emission. Therefore, since we are neglecting further dissipative effects such as viscosity or heat transport, oscillation modes are undamped, normal modes and the corresponding eigenfrequencies, $\omega_k$, are real. Dipolar oscillations are described by the LS equations, which form a linear, third-order ODE system [21]. In order to find the mode eigenfrequencies, these equations have to be integrated from the center (where regularity conditions must be imposed) up to the stellar surface, for different values of $\omega$. The solutions satisfy the required boundary condition at the surface, i.e. the vanishing of the Lagrangian perturbation of the pressure, only for a discrete set of values of $\omega$: these are the normal mode eigenfrequencies. The equations depend on $\omega$ quadratically, and since all terms in these equations and all boundary conditions are real, there is no restriction in assuming that the perturbation functions and $\omega^2$ are real, i.e. in looking for normal modes. Unstable modes correspond to solutions belonging to $\omega^2 < 0$. They are non-oscillating, exponentially growing modes, with the growth time $\tau$ given by

$$\frac{1}{\tau} = \sqrt{-\omega^2}. \quad (3)$$

For $l \geq 2$, stellar perturbations are associated with gravitational wave emission; therefore the corresponding modes, said quasi-normal modes, are damped and the eigenfrequencies are complex

$$\omega = 2\pi \nu + \frac{i}{\tau}, \quad (4)$$

where $\nu$ is the mode frequency, and $\tau$ is its gravitational damping time. These perturbations are described by the LD equations, which form a linear, fourth-order ODE system [20]. These equations are integrated from the center, imposing regularity conditions, to the stellar surface, for different values of complex $\omega$. For each $\omega$ there exist two regular, independent solutions which have to be matched on the stellar surface to allow the vanishing of the Lagrangian perturbation of the pressure. This constraint reduces the number of independent solutions to one, modulo an overall scaling factor.

Outside the star, the variables associated with the fluid motion vanish and the equations reduce to a second-order ODE for the Zerilli function $Z^{lm}(\omega, r_*)$ [24]

$$\left(\frac{d^2}{dr_*^2} + \omega^2\right) Z^{lm}(\omega, r_*) = V^l(r) Z^{lm}(\omega, r_*), \quad (5)$$

where $V^l(r)$ is the Zerilli potential and $r_* = r + 2M \log \left(\frac{r}{r_0} - 1\right)$ is the tortoise coordinate. Far away from the star, the Zerilli function behaves as a superposition of ingoing and ingoing waves:

$$Z^{lm}(r) \simeq Z_{lm}^{in} e^{i\omega r} + Z_{lm}^{out} e^{-i\omega r}. \quad (6)$$
Quasi-normal modes are the solutions of the LD + Zerilli equations for which the ingoing component of the asymptotic solution (6), $Z_{lm}^{\infty}$, vanishes. The discrete set of frequencies corresponding to these solutions are the mode eigenfrequencies.

Like the LS equations, the LD equations have real coefficients; however the outgoing wave boundary condition $Z_{lm}^{\infty} = 0$ cannot be satisfied by a real $Z_{lm}^{\infty}$. Therefore, in order to represent a physical mode the harmonic components of the perturbations must belong to complex frequencies. For $l \geq 2$ unstable modes are the solution of the perturbed equations with $\Im(\omega) < 0$; an unstable mode corresponds to a perturbation which, while oscillating, is increasingly growing in amplitude; the oscillation frequency $\nu$ and the growth time $\tau(>0)$ of such an unstable mode are given by $\omega = 2\pi \nu - i/\tau$.

The relevant equations for $l \geq 1$ have been integrated using an adaptive stepsize fourth-order Runge–Kutta method. For $l \geq 2$ the solution has been continued outside the star by integrating Zerilli’s equation (5) using the continued fraction method [25]; the complex eigenfrequencies have been found as zeros of the function $Z_{lm}^{\infty}(\omega)$, using a Newton–Raphson method.

3. Normal and quasi-normal modes of proto-neutron stars

Before discussing our results, we shall briefly summarize what is known in the literature about the mode structure of non-rotating stars in Newtonian gravity. This will serve as a guideline to understanding our results for the relativistic PNS.

The normal mode spectrum of non-rotating stars has been studied in great detail in Newtonian gravity (see, for instance, [26, 27]). The main results can be summarized as follows.

- Since Newtonian stars do not emit gravitational waves, in the absence of viscous forces the square of the mode eigenfrequencies, $\nu_k^2$, is real for any value of $l$. The index $k$ denotes the order of the mode. If $\nu_k^2 > 0$, the mode corresponds to a stable oscillation, while if $\nu_k^2 < 0$ the mode is a pure exponential growth.
- Mode eigenfrequencies group into distinct classes, identified on the basis of the restoring force which dominates in bringing back to the equilibrium position the generic fluid element displaced by the perturbation; they are named $g_k$-modes if the restoring force is buoyancy, $p_k$-modes if it is due to pressure gradients. The $g$-mode eigenfrequencies are smaller than those of $p$-modes, and the two classes are separated by the single fundamental mode ($f$-mode) frequency.
- When $l = 1$, the $f$-mode frequency is identically zero: the $f$-mode does exist only for $l \geq 2$. If $l = 1$ $g$- and $p$-modes have an interesting characteristic: they correspond to a displacement of the geometrical center of the system, i.e. $\delta r(0) \neq 0$; as pointed out in [28], in the $l = 1$ case the geometrical center does not coincide with the center of mass of the star, which is not displaced.
- There can exist two classes of $g$-modes: the $g_k^{(+)}$ modes, which are stable, and the $g_k^{(-)}$, which are unstable. Whether both branches of modes do exist or not depends on the sign of the Schwarzschild discriminant

$$A = \frac{\rho'}{\rho} - \left( \frac{d\rho}{dp} \right)_s \frac{\rho''}{\rho}$$

- If $A = 0$ throughout the star (as it is for barotropic EOS), then there are no $g$-modes (or, more precisely, they are all degenerate to zero frequency).
- If $A < 0$ throughout the star (convective stability), there are only $g^{(+)}$-modes.
If $A > 0$ throughout the star (convective instability), there are only $g^{(-)}$-modes. If $A < 0$ in some region of the star, and $A > 0$ in some other region, then both $g^{(+)^{-}}$-modes and $g^{(-)}$-modes are present.

Thus, the sign of the Schwarzschild discriminant reveals the presence of convective instabilities, and of unstable $g$-modes. The $p$-modes and the $f$-mode are always stable.

• Typically, for $g_{\pm}^{(\pm)}$- and $p_{\pm}$-modes the number of nodes in the eigenfunction of $(\delta r)^{\pm}$ is equal to $k$ (not counting the node at $r = 0$ for $l \neq 1$). The $f$-mode eigenfunction has no nodes inside the star. However, such simple prescriptions may not hold in some cases [26].

Let us now consider our relativistic PNS. In order to find the frequencies of the quasi-normal modes, we have integrated the equations that describe the perturbations of a spherical star taking as a background the ‘quasi-stationary’ evolutionary models derived in [1]. ‘Quasi-stationary’ means that the stellar interior is described by a sequence of equilibrium configurations, which have been shown to adequately describe the evolution of a PNS for $t \gtrsim$ 0.1–0.2 s after bounce [1, 29]. These models have been used in [8] to compute how the quasi-normal mode frequencies of stable, quadrupole modes evolve in time. Here we consider the equation of state named GM3, with no quark matter in the core, and explore the time interval from $t = 0.2$ s up to $t = 25$ s after bounce. We find both stable and unstable $g$-modes. Unstable modes are present because the relativistic Schwarzschild discriminant [30]

$$S(r) = \frac{dp}{dr} - \left(\frac{dp}{d\epsilon}\right)_s \frac{d\epsilon}{dr}$$

(8)

is negative in some region inside the star, revealing the presence of convective instability3.

To compare our results with the literature on Newtonian stars (in particular, with figure 1 of [26]), we define a real, square frequency, $v^2$, as follows.

• For $l = 1$, $\omega^2_{k1}$ is always real; it is positive for stable modes, negative for unstable modes. Therefore we define $v^2_1 \equiv (\omega^2_{k1}/2\pi)^2$.

• For $l \gtrsim 2$, $\omega^2_k$ is complex. For stable fluid modes, in general $|\Re \omega_k| \gg |\Im \omega_k|$, whereas for unstable modes $|\Im \omega_k| \gg |\Re \omega_k|$. Therefore, we can define

$$(2\pi v^2_k)^2 \equiv |\Re \omega_k|^2 \simeq \omega^2_k, \quad \text{for stable modes}$$

$$(2\pi v^2_k)^2 \equiv -|\Im \omega_k|^2 \simeq \omega^2_k, \quad \text{for unstable modes.}$$

(9)

The mode structure of the considered stellar model is shown in figure 1, where $v^2_k$ is plotted for different values of the harmonic index $l$. $v^2_{k1}$ is plotted for the first two unstable $g$-modes, $g_1^{(-)}$ and $g_2^{(-)}$, respectively, for the first two stable $g$-modes, $g_1^{(+)^{-}}$ and $g_2^{(+)^{-}}$ and for the fundamental mode. The figure refers to $t = 2$ s after bounce.

We see that $v^2_k$ has a dependence on $l$ similar to that shown in the Newtonian case [26]. We recall that the main difference is that for $l > 1$ in the relativistic case $\omega^2_k$ is complex: the frequency of stable modes acquires a small imaginary part corresponding to the damping induced by the gravitational wave emission, while that of unstable modes acquires a small real part.

Let us now focus on the behavior of unstable $g$-modes. The lowest dipole mode, (i.e. $g_1^{(-)}$ for $l = 1$), at $t = 2$ s after bounce has a growth time $\tau_{g_1^{(-)}} = 0.51$ ms. This is very interesting because, since $\tau_{g_1^{(-)}}$ is extremely small, the amplitude of this mode may have enough time to grow significantly before being damped by other dissipative effects, or by nonlinear couplings.

3 Note that the sign convention for the relativistic discriminant (8) is the opposite to that assumed for the Newtonian discriminant (7).
For $l \geq 2$, the growth times of the lowest unstable $g$-modes are even smaller; for instance, for $l = 2$, $\tau_{g_{1,-}} = 0.3$ ms. In figure 2 we show how the growth time of the lowest unstable $g$-modes for $l = 1, 2$ changes during the first 25 s of the PNS life. We see that, after an initial decrease, all $\tau_{g_{i,-}}$s increase with time; however, they remain quite small, being their maximum values of the order of a few ms. After about 25 s, the star becomes convectively stable and unstable modes disappear.

For $l \geq 2$ unstable $g$-modes have a small oscillating part, which is of the order of $\nu \sim 10^{-8}$ Hz and can be neglected.

4. Mode couplings

The existence of rapidly growing, unstable $g$-modes indicates, at a linear level, the onset of an effective convective instability; this motivates our further investigation on mode coupling, to see whether unstable modes, coupling to stable ones, may trigger gravitational wave emission at frequencies which fall in the bandwidth of ground-based interferometers, as shown in [8].
A first thing one can do is to see whether the corresponding eigenfunctions overlap: if they do not, we would have a strong indication that the coupling is ineffective. In figure 3 we compare, as an example, the radial eigenfunctions of the lowest, stable and unstable $g$-modes belonging to $l = 1, 2$. We see that there is indeed an overlap\(^4\); however, this information is not sufficient to infer how effective the transfer of energy from the convective motion to the oscillatory modes can be.

In order to quantify the effectiveness of the couplings, we have used the amplitude equations formalism described in [14, 15], where couplings between different modes have been analyzed in the framework of Newtonian gravity. In this approach, the Lagrangian displacement $\vec{\delta}r(t, \vec{r})$ is expressed as a superposition of modes

$$\vec{\delta}r(t, \vec{r}) = \sum_i a^i(t) \vec{\xi}(i)(\vec{r}),$$  

(10)

where $\vec{\xi}(i)(\vec{r})$ are the mode eigenfunctions, normalized by imposing $\xi_r(i)(R_{\text{star}})/R_{\text{star}} = 1$, and $a^i(t)$ are the corresponding amplitudes. For non-resonant modes\(^5\), the nonlinear coupling between different modes is described by the equation

$$\dot{a}^i = i \omega_i a^i + Q_{ii}^i (a^i)^3 + Q_{ij}^i a^i (a^j)^2,$$  

(11)

where the frequency $\omega_i$ is in general complex, i.e. $\omega_i = 2\pi \nu_i \pm \frac{1}{\tau_i}$, if some dissipative process is in action. In our case, dissipation is due to gravitational waves and, as mentioned before, for stable modes we can assume $2\pi \nu \gg \frac{1}{\tau}$ while for unstable modes $2\pi \nu \ll \frac{1}{\tau}$.

The evaluation of the coupling coefficients $Q_{ij}^i$, the expressions of which are given in [14, 15], is a very hard task. Since we need an order of magnitude estimate, we shall compute only a leading term. In particular, we shall use the expressions of $Q_{ij}^i$ given in [15], but we

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\(^4\) Note that the eigenfunctions of the stable modes are defined mostly where $S(r) > 0$, while the eigenfunctions of the unstable modes are defined mostly where $S(r) < 0$.

\(^5\) Resonant modes are those for which the condition $\sum_j k_j \nu_j \geq 0$ is satisfied, with $k_j$ small integers [15].
have checked that, using those given in [14], the results are of the same order of magnitude. The leading term we consider is

\[ Q'_{ij} = iC'_{ij} \frac{3(2 - \delta_{ij})Z'_{ij}^3}{2\omega_i I'_i} + \cdots \]  

(12)

where

\[ Z'_{ij} = \int d\Omega Y^*_i Y_j, \]

\[ I'_i = \int dr \rho r^2 \left[ \frac{\left| \xi^i r \right|^2 + l(l+1)|\xi^i r|}{\rho} \right], \]

\[ C'_{ij} = \int dr \rho r^2 \frac{P'_{ij} r^3}{\rho} \left( \frac{\delta p_{ij}}{\rho} \right) \left( \frac{\delta p_{ij}}{\rho} \right)^2. \]  

(13)

\( Y_i \) is the spherical harmonic corresponding to the \( i \)th mode, and \( \rho, p, \delta \rho, \delta p \) are the density, the pressure, and their Lagrangian perturbations. If all quantities are expressed in geometrical units \((G = c = 1)\) as powers of \( \text{km} \), then \( Q'_{ij} \) has dimension \( \text{km}^{-1} \), which can be converted into \( s^{-1} \) multiplying by \( c \).

Let us first compute the coefficients \( Q'_{iii} \), i.e. the self-coupling coefficients (see equation (11)), for an unstable mode. Since in this case \( \omega_i \simeq -\frac{\tau_i}{2} \), equation (12) gives

\[ Q'_{iii} = -3 \tau_i^2 Z_{iii}^3 C_{iii} I'_i + \cdots, \]  

(14)

and, for the lowest unstable \( g \)-modes corresponding to \( l = 1, 2 \), we find

\[ |Q'_{l=1}| \sim 3.0 \times 10^{13} \text{ s}^{-1} \]

\[ |Q'_{l=2}| \sim 1.3 \times 10^{7} \text{ s}^{-1}. \]

Assuming that \( Q'_{iii} < 0 \), i.e. that the effect of self coupling is that of saturating the mode, using the values of \( |Q'_{l=1}| \) and \( |Q'_{l=2}| \) found above, we can estimate the saturation amplitude as the value of \( a' \) for which \( a' = 0 \), i.e.

\[ a_{i sat} \simeq \frac{1}{\sqrt{|Q'_{iii}| \tau_i}}. \]  

(15)

We find

for \( l = 1 \) \( \delta r(R_{\text{star}}) \sim 8.2 \text{ cm} \)

for \( l = 2 \) \( \delta r(R_{\text{star}}) \sim 150 \text{ m}. \)

Next we want to check whether unstable modes can trigger stable modes, eventually making them unstable. Therefore, we need to evaluate the coefficients which describe the coupling between the \( i \)th stable mode and the \( j \)th unstable mode. If the \( i \)th mode is stable, equation (11) becomes

\[ \dot{a'} \simeq i2\pi v_i a' - \frac{1}{\tau_i} a' + Q'_{ij}(a')^3 + Q'_{ij}(a')^2. \]  

(16)

The purely imaginary terms on the right-hand side of equation (16) determine a shift of the frequency \( v_i \) and are not relevant for the mode stability; therefore, we are interested only in the real part of the right-hand side of equation (16). If we define

\[ \tilde{Q}'_{ij} = C'_{ij} \frac{3Z'_{ij}^3}{(2\pi v_i)^2 I'_i}, \]  

(17)
then equation (11) can be written as
\[ \dot{a}_i \simeq i(\ldots) - \frac{1}{\tau_i} a^j \left[ 1 + \tilde{Q}_{ijj}(a^j)^2 \right]. \] (18)

We must then evaluate the dimensionless quantity \( \tilde{Q}_{ijj}(a^j)^2 \), where \( a^j_{\text{sat}} \) is the saturation amplitude of the unstable mode \( j \), previously computed. In particular, we want to check whether the sign of the term in the square brackets in equation (18) can be reversed.

It should be noted that the stable \( i \)th mode must belong to \( l \geq 2 \), because modes belonging to \( l = 1 \) are not associated with gravitational wave emission, and for them \( 1/\tau_i = 0 \). This means that the mechanism of mode enhancing we are studying is triggered by gravitational wave emission, as in the case of the well-known CFS instability [31].

In Table 1 we show the values of \( \left| \tilde{Q}_{ijj}(a^j)^2 \right| \); the index \( j \) refers to unstable modes \( g^{(\pm)}_nl \), where \( n = 1, 2 \) is the order of the mode, and \( l = 1, 2 \); the index \( i \) refers to stable modes \( g^{(\pm)}_nl \) and \( f_l \) (fundamental mode), with \( l = 2 \). In all cases we find
\[ \left| \tilde{Q}_{ijj}(a^j)^2 \right| \ll 1; \] (19)
this means that the term in the square brackets cannot change sign and that the effect of unstable modes on stable modes is too weak to induce a significant growth of their oscillation amplitudes.

5. Concluding remarks

We have determined the frequencies of stable and unstable modes of a newly born proto-neutron star during the early phases of its life. Our goal was to understand whether the coupling between stable and unstable modes could provide a mechanism to convert the energy of convective motion into g-mode oscillations, enhancing gravitational wave emission.

Our results indicate that the existing coupling does not seem large enough to explain, alone, the excitation of the g-mode oscillations found in hydrodynamical simulations. Thus, other processes should be considered as driving the excitation of these modes, as, for instance, accretion from the mantle [4, 32].

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