Acoustic phonon-limited resistivity of spin–orbit coupled two-dimensional electron gas: the deformation potential and piezoelectric scattering

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Abstract

We study the interaction between electron and acoustic phonons in a Rashba spin–orbit coupled two-dimensional electron gas using Boltzmann transport theory. Both the deformation potential and piezoelectric scattering mechanisms are considered in the Bloch–Grüneisen (BG) regime as well as in the equipartition (EP) regime. The effect of the Rashba spin–orbit interaction on the temperature dependence of the resistivity in the BG and EP regimes is discussed. We find that the effective exponent of the temperature dependence of the resistivity in the BG regime decreases due to spin–orbit coupling.

(Some figures may appear in colour only in the online journal)

1. Introduction

With the rise of the promising field of spintronics, two-dimensional electron gas (2DEG) with spin–orbit interaction (SOI) in semiconductor heterostructures has drawn much attention in recent years [1–3]. The importance of this field was first realized when Datta and Das gave a proposal of a spin field effect transistor [4]. Several studies on spintronics have been performed in recent years from both theoretical and experimental viewpoints. One major aim is to manipulate the spin degree of freedom of charge carriers in semiconductor nanostructures [5] so that spin-based device technology [6] and quantum information processing [7] can be developed in the near future. The SOI is an intrinsic phenomenon present in semiconductors. Mainly, there are two kinds of SOI present in the semiconductor heterostructures that we come across in the literature. One of them is the Rashba spin–orbit interaction (RSOI) [8] which originates from the inversion asymmetry of the confining potential in semiconductor heterostructures. The RSOI is proportional to the magnitude of the electric field generated internally due to the band bending or externally applied gate voltage. It can be tuned by applying a gate voltage [9, 10]. Another kind of SOI is the Dresselhaus spin–orbit interaction [11] which originates from the bulk inversion asymmetry of the host crystal. It entirely depends on crystal properties and it is not tunable.

Various electronic and transport properties of a 2DEG will be modified in the presence of the SOI. We mainly focus on the RSOI in 2DEG systems such as AlGaAs/GaAs heterostructures. The coupling between the electron and phonon becomes stronger when the SO coupling constant is large enough and due to this, the effective mass is also increased [12]. The critical temperature of superconductors can be controlled via the RSOI when the Fermi energy is small as compared to the characteristic energy scale of the RSOI [13]. The RSOI can produce an infinite number of bound states in a 2DEG with short-range impurity potentials [14]. At low frequency in the presence of the RSOI the universality of the spin Hall conductivity can be broken by taking into consideration the contribution of electron–phonon interaction to the spin-vertex correction [15]. The mobility, polaron mass correction and polaron binding energy can be changed significantly due to the presence of the RSOI [16, 17]. The relaxation time for various impurity potentials of low dimensional semiconductor structures with SOI has been studied [18].
The interaction between the electron and phonon plays a very crucial role in determining the transport properties of a 2DEG and it makes a finite contribution to the momentum relaxation time of the charge carriers. Other contributions [19–23] come from disorder, impurities, etc. There are three distinct temperature regimes: (a) the Bloch–Grüneisen one (BG), (b) equilibration (EP) and (c) inelastic phonon scattering. The BG temperature $T_{BG}$ can be defined [24] by the relation $k_B T_{BG} = 2 \hbar k_{FV}$, where $k_B$ is the Boltzmann constant, $v_F$ is the phonon velocity and $k_F$ is the Fermi wavevector. For typical electron density ($n_e \sim 10^{15}$ m$^{-2}$) in a 2DEG system, $T_{BG}$ is around 6.2 K. In the BG regime, a direct manifestation of the acoustic phonon-dominated transport property is the strong change in the temperature dependence of the resistivity. The existence of the BG regime has been confirmed for 2DEG experimentally [24]. The problem of electron–phonon interaction in a 2DEG confined in semiconductor heterostructures has been studied extensively [25–32]. Recently, the phonon-dominated transport properties in the BG regime in graphene have been studied both theoretically [33–35] and experimentally [36].

In the present work we would like to investigate the influence of the RSOI on the momentum relaxation time due to electron–phonon interaction and hence on the transport properties of 2DEG systems. We consider both the cases: the perfect 2DEG and the quasi-2DEG (usually found in semiconductor heterostructures). In the former case the 2D phonon wavevector $q$ couples with the 2D electron wavevector $k$ and in the latter case the coupling between the 3D bulk phonon wavevector $Q = (q, q_z)$ and the 2D electron wavevector $k$ is considered. We consider two mechanisms of electron–phonon interaction, namely the deformation potential (DP) and piezoelectric (PE) potential scattering, separately. In all the cases a linear temperature dependence of the inverse relaxation time (IRT) is found in the high temperature (EP) regime. But in the BG regime for the perfect 2DEG and quasi-2DEG, we find analytically that the resistivity is proportional to $T^4$ and $T^3$, respectively, in the case of the DP scattering mechanism. On the other hand, $\rho \sim T^3$ for the PE scattering mechanism in a quasi-2DEG. Our numerical calculations reveal that this exponent of $T$ strongly depends on the density and the SO coupling constant. In the SO coupling constant, the exponent of $T$ decreases due to the presence of spin–orbit coupling. We also discuss the resistivity as a function of the SO coupling constant in the BG and EP regimes.

This paper is organized as follows. In section 2 we derive all the theoretical results and discuss all the numerical analysis for the perfect 2DEG. In section 3 both DP and PE scattering mechanisms are taken into account for the quasi-2DEG and we discuss analytical and numerical results in detail. We summarize all the results in section 4.

2. Electron–phonon scattering in a perfect 2DEG

2.1. The theoretical model

We consider a 2DEG with the RSOI in the $xy$ plane in a semiconductor heterostructure. The single-particle Hamiltonian of this system is given by

$$ H = \frac{p_x^2}{2m^*} \sigma_0 + \frac{\hbar}{2} \left( \sigma_\mu p_\mu - \sigma_\lambda p_\lambda \right), $$

where $p$ is the two-dimensional momentum operator, $m^*$ is the effective mass of an electron, $\sigma_0$ is the unit 2 \times 2 matrix, $\sigma_{\mu} (\nu)$ is the Rashba spin–orbit coupling constant and $\sigma_{\mu (\nu)}$ are the Pauli matrices. The eigenenergies are given by

$$ \epsilon_s(k) = \frac{\hbar^2 k^2}{2m^*} + \lambda |\alpha| |k|, $$

with the corresponding normalized eigenspinors

$$ \psi_s(x, y) = \frac{1}{\sqrt{2A}} \left( \begin{array}{c} 1 \\ \lambda e^{-i\phi} \end{array} \right) e^{ikx}. $$

Here, $\lambda = \pm$ represents the upper and lower energy branches, $A$ is the area of the system and $\phi = \tan^{-1}(k_z/k_x)$. The densities of states for the two energy branches are given by [12]

$$ D_\pm(\epsilon) = \frac{D_0}{2} \left[ 1 \pm \sqrt{\epsilon - \epsilon_0 + \epsilon_\pm} \Theta(\epsilon) \right] + D_0 \sqrt{\epsilon - \epsilon_0 - \epsilon_\pm} \Theta(\epsilon - \epsilon_0). $$

Here, $D_0 = m^*/(\pi \hbar^2)$, $\Theta(x)$ is the unit step function and $\epsilon_\pm = m^* \alpha^2/(2\hbar^2)$ is the characteristic energy scale of the RSOI.

The Hamiltonian for electron–phonon interaction in the case of deformation potential coupling can be written as $H_{ep} = D \nabla \cdot \mathbf{u}(r)$, where $D$ is the deformation potential coupling constant and the lattice displacement vector $\mathbf{u}(r)$ is given by

$$ \mathbf{u}(r) = \sum_q \sqrt{\frac{\hbar}{2MNao_q}} [a_q e^{iq \cdot r} + a_q^\dagger e^{-iq \cdot r}]. $$

Here, $ao_q = v_s q$ is the phonon frequency with the wavevector $q$ and the sound velocity $v_s$, and $a_q$ and $a_q^\dagger$ are the phonon creation and annihilation operators, respectively. Also, $\mathbf{e}_q$ is a unit vector in the direction of the phonon polarization. The corresponding Hamiltonian can be written as

$$ H_{ep}(r) = \sum_q \left[ C_q a_q e^{iq \cdot r} + C_q^\dagger a_q^\dagger e^{-iq \cdot r} \right], $$

where $C_q = D \sqrt{\hbar/2MNao_q} (\mathbf{e}_q \cdot \mathbf{q})$.

The energy dependent relaxation time for electrons in a given energy branch $\lambda$ can be written as

$$ \frac{1}{\tau(\epsilon)} = \sum_{k,k'} (1 - \cos \theta_{kk'}) P_{kk'}^{\lambda \lambda'} \frac{1 - f(\epsilon_\lambda(k'))}{1 - f(\epsilon_\lambda(k))}, $$

where $\theta_{kk'}$ is the scattering angle between the two momentum vectors $k$ and $k'$, $\epsilon_\lambda$ is given by equation (2), $P_{kk'}^{\lambda \lambda'}$ is the transition rate for scattering of an electron from a state $| k, \lambda \rangle$ to $| k', \lambda' \rangle$ and $f(\epsilon) = [e^{\beta(\epsilon - \mu)} + 1]^{-1}$ is the Fermi–Dirac distribution function with $\beta = 1/(k_B T)$. The chemical potential $\mu$ at finite temperature $T$ can be
obtained self-consistently from the following normalization condition [37]:

\[ n_e = \int_{-\infty}^{\infty} \text{d} \epsilon \, D_0(\epsilon) + \int_{-\epsilon_a}^{0} \text{d} \epsilon \, D_0(\epsilon) \sqrt{\frac{\epsilon_a}{\epsilon + \epsilon_a}}, \]

where \( n_e \) is the electron density. At \( T = 0 \), the above equation reduces to \( \epsilon_F = \epsilon_a^0 = 2\epsilon_a \), where \( \epsilon_F \) and \( \epsilon_a^0 \) are the Fermi energies of a 2DEG in the presence and absence of the RSOI, respectively. Thus, the reduction in the Fermi energy due to the RSOI is \( 2\epsilon_a \).

We consider the interaction between the electron and acoustic phonon due to deformation potential coupling. The transition rate due to electron-phonon interaction can be written as

\[ P_{kk'}^{\lambda\lambda'} = \frac{2\pi}{\hbar} \rho \sum_q |C_{q}^{\lambda\lambda'}|^2 [N_q \delta(\epsilon'_{\lambda'} - \epsilon_{\lambda} + \hbar\omega_q) + (N_q + 1)\delta(\epsilon'_{\lambda'} - \epsilon_{\lambda} + \hbar\omega_q)], \]

where \( C_{q}^{\lambda\lambda'} \) is the matrix element for the acoustic phonon and is given by

\[ |C_{q}^{\lambda\lambda'}|^2 = \frac{D^2\hbar\omega_q}{2A\rho v_{\text{s}}} \frac{1 + \theta'_{\lambda\lambda'} \cos \theta}{2} \delta_{\lambda\lambda'}. \]

Here, \( \theta = \theta_{kk'} \), \( D \) is the deformation potential coupling constant and \( \rho = NM/A \) is the mass per unit area. The appearance of \( \delta_{\lambda\lambda'} \) is due to the fact that the electron-phonon interaction given by equation (6) is spin independent. Also, \( N_q = [\exp(\beta\hbar\omega_q) - 1]^{-1} \) is the phonon occupation number. The first and second terms on the right-hand side of equation (9) correspond to the absorption and emission of a phonon with energy \( \hbar\omega_q \), respectively. Within the small-angle scattering approximation \( (q = 2k\sin(\theta/2)) \), the matrix element for intra-cluster scattering \( (\lambda = \lambda') \) becomes

\[ |C_{q}^{\lambda\lambda'}|^2 = \frac{D^2\hbar\omega_q}{2A\rho v_{\text{s}}} (1 - \frac{\theta'_{\lambda\lambda'}}{4}). \]

A similar matrix element is obtained for single-layer graphene [33].

Before presenting the numerical results, we present how the IRT depends on \( T \) in the EP and BG regimes. At high temperature (the EP regime), the phonon energy is much smaller than the thermal energy, i.e. \( \hbar\omega_q \ll k_B T \). We neglect the \( \hbar\omega_q \) term in the delta functions, i.e. \( \delta(\epsilon' - \epsilon + \hbar\omega_q) \simeq \delta(\epsilon' - \epsilon) \). Again, in this temperature limit the Bose occupation factor \( N_q \) can be approximated as \( N_q \simeq N_q + 1 \simeq k_B T/\hbar\omega_q \). When \( \epsilon_F \geq \epsilon_a \), the total relaxation time at high temperature is

\[ \frac{1}{\tau(\epsilon)} \simeq \frac{m^* D^2}{2\rho_0 h^3 v_{\text{s}}^3} k_B T. \]

Therefore, in the EP regime, the IRT depends linearly on temperature.

Now we want to see how the resistivity depends on the temperature and \( \alpha \) at low temperature (the BG regime) where \( \hbar\omega_q \sim k_B T \). In the BG regime, the IRT strongly decreases because the phonon population decreases exponentially for phonon absorption and the sharp Fermi distribution prohibits phonon emission. To see the temperature dependence of the resistivity in the BG regime, it is convenient to calculate the IRT averaged over energy, as used for graphene [33], which is given by

\[ \frac{1}{\tau(\epsilon)} = \frac{\int \text{d} \epsilon' D_\lambda(\epsilon') \frac{\delta(\epsilon' - \epsilon) - \delta(\epsilon' + \epsilon)}{\int \text{d} \epsilon' D_\lambda(\epsilon')} \frac{\text{d} \epsilon'}{\delta(\epsilon')} \frac{\text{d} \epsilon}{\delta(\epsilon)} \delta(\epsilon' - \epsilon)}{\int \text{d} \epsilon' D_\lambda(\epsilon') \frac{\delta(\epsilon' - \epsilon) - \delta(\epsilon' + \epsilon)}{\int \text{d} \epsilon' D_\lambda(\epsilon')} \frac{\text{d} \epsilon'}{\delta(\epsilon')} \frac{\text{d} \epsilon}{\delta(\epsilon)}}. \]

Therefore, the resistivity of a 2DEG with the RSOI can be calculated from the following equation:

\[ \rho = \frac{m^*}{n_e \epsilon_a^2} \left( \frac{1}{\tau} \right). \]

When the temperature is very low we can make the following approximations: (i) the phonon energy is comparable with the thermal energy, i.e. \( k_B T \ll \hbar\omega_q \ll \epsilon_F \) and \( f(\epsilon)[1-f(\epsilon \pm \hbar\omega_q)] \simeq \hbar\omega_q(N_q + 1/2 \pm 1/2)\delta(\epsilon - \epsilon_F). \) After making these approximations, we obtain

\[ \frac{1}{\tau^{\pm}} \simeq \frac{2AD_0}{k_B T} \left( 1 \mp \frac{\epsilon_a}{\epsilon_F + \epsilon_a} \right) \times \int_0^{\pi} \text{d} \theta (1 - \cos \theta) |C_{q}^{\lambda\lambda'}\omega_q N_q(N_q + 1). \]

We can now convert the integration over \( \theta \) into an integration over \( q \) by using the relation (based on small-angle scattering) \( q = 2k_F \sin(\theta/2) \). Substituting this, after a straightforward calculation we finally obtain

\[ \frac{1}{\tau^{\pm}} \simeq \frac{D_0}{2} \left( 1 \mp \frac{\epsilon_a}{\epsilon_F + \epsilon_a} \right) D^2 4! \zeta(4) \left( k_F T \right)^4 \rho v_{\text{s}} (v_{\text{s}} c)^3 \left( k_F^2 \right)^3, \]

where \( k_F^2 = \sqrt{k_F^0^2 - k_\alpha^2} \gtrless k_\alpha, k_\alpha = m^* \alpha / \hbar^2 \) is the Rashba wavevector, \( k_F^0 = \sqrt{2\pi n_0} \) is the Fermi wavevector of a 2DEG without the RSOI and \( \zeta(4) = \pi^4 / (90) \). In this temperature regime the energy averaged IRT is proportional to \( T^4 \) and hence \( \rho \sim T^4 \). The \( T^4 \) scaling law is also found in other perfect 2D systems such as graphene [33]. As we will see in the numerical calculations, the exponent of the temperature dependence of the resistivity is reduced due to SOI coupling.

2.2. Numerical results

In this section we determine the IRT and the resistivity, and their dependences on energy, temperature, SOI coupling constant etc. We solve equation (7) numerically. For our numerical calculation, we use \( m^* = 0.067 m_0 \) with \( m_0 \) the free electron mass, \( \alpha = 0.9 \times 10^{-11} \text{eV m} \) and \( \epsilon_0 = 5 \times 10^3 \text{m}^{-1} \) and the electron density \( n_0 = 10^{15} \text{m}^{-2} \).

In figure 1 we have plotted IRT as a function of energy at a fixed temperature \( T = 1 \text{ K} \) for densities \( n_e = n_0 \) and \( n_e = 5n_0 \). It can be seen from figure 1 that there is a dip in the IRT. The dip occurs due to the sharpness of the Fermi distribution function at low enough temperature. It is clear from figure 1 that the dip occurs exactly at \( \epsilon = \epsilon_F^0 \) when \( \alpha = 0. As \alpha increases, the position of the dip starts to appear at energies lower than \( \epsilon_F^0 \). The dip occurs when \( \epsilon = \mu \), where \( \mu \) is the chemical potential which we evaluate numerically by solving equation (8). For the electron density \( n_0 \), the Fermi
energy without the RSOI is \( \epsilon_F^0 = 3.6 \text{ meV} \). When \( \alpha = 0 \), \( \mu = \epsilon_F^0 = 3.6 \text{ meV} \). When \( \alpha = 3\alpha_0 \), \( \epsilon_F = 2.8 \text{ meV} \) and \( \mu = 0.8\epsilon_F^0 \), so the dip occurs exactly at \( \epsilon = 0.8\epsilon_F^0 \). Similarly, for \( \alpha = 5\alpha_0 \), \( \epsilon_F = 1.4 \text{ meV} \) and \( \mu = 0.4\epsilon_F^0 \), and consequently the dip occurs exactly at \( \epsilon = 0.4\epsilon_F^0 \). The IRT of the 2DEG system with the RSOI is reduced compared to that for the case in the absence of SOI.

The effective exponent (\( \nu \)) of the temperature dependence of the resistivity strongly depends on the density and the RSOI coupling constant. We estimate the effective exponent from the log–log plot of the resistivity versus \( T \) in Figure 2 for \( \alpha = 0 \) and \( \alpha_0 \) with different densities. In the BG regime (\( T \sim 1–3 \text{ K} \)), we find \( \nu = 3.763, 4.111 \) and 4.235 for \( n_e = 3n_0, 5n_0 \) and 7\( n_0 \), respectively, when \( \alpha = 0 \). On the other hand, we find \( \nu = 2.829, 3.203 \) and 3.433 for \( n_e = 3n_0, 5n_0 \) and 7\( n_0 \), respectively, when \( \alpha = \alpha_0 \). The exponent \( \nu \) increases with \( \alpha \) in the high temperature limit.

In Figure 3, we plot \( \rho \) versus \( T \) at a fixed density for different values of \( \alpha \). In each panel of Figure 3, we consider the \( \alpha = 0 \) and \( \alpha_0 \) cases for densities \( n_e = 3n_0 \) and 5\( n_0 \). Figure 3 clearly shows that the slope of the curve for \( \alpha = 0 \) is greater than that for the \( \alpha = \alpha_0 \) case in the BG regime. When \( n_e = 3n_0 \), we estimate \( \nu = 3.763 \) and \( \nu = 2.829 \) for \( \alpha = 0 \) and \( \alpha = \alpha_0 \), respectively. Similarly, when \( n_e = 5n_0 \), we have \( \nu = 4.111 \) and \( \nu = 3.203 \) for \( \alpha = 0 \) and \( \alpha = \alpha_0 \), respectively, at very low temperature. The values of \( \nu \) differ significantly between \( \alpha = 0 \) and finite \( \alpha \) at very low temperature.

In Figure 4, we plot the resistivity of a perfect 2DEG versus \( \alpha \) for two different densities. In both the regimes, \( \rho \) increases with \( \alpha \). But the rate of increase of \( \rho \) is high for low electron density compared to that for high density. In the BG regime, the resistivity is saturated at high \( \alpha \).

3. Electron–phonon scattering in a quasi-2DEG

In this section we consider the three-dimensional bulk phonon with wavevector \( \mathbf{Q} = (q_x, q_z) \) interacting with the two-dimensional electron wavevector \( \mathbf{k} = (k_x, k_z) \). The component of \( \mathbf{Q} \) in the \( x-y \) plane obeys the conservation of momentum, i.e., \( \mathbf{k} = \mathbf{k}' = \mathbf{q} \) and the \( z \) component must be integrated out. In a semiconductor heterostructure the electrons move in the \( x-y \) plane in the presence of a triangular potential in the \( z \) direction. It is also assumed that the lowest energy level is occupied by the electrons. Generally the wavefunction can be written as \( \psi(\mathbf{r}) = \psi(x, y)z_0(z) \). Here the variational wavefunction \( z_0(z) \) is given by \( z_0(z) = \sqrt{b^3/2}e^{-bc/2} \). The variational parameter \( b \) is given by \( b = (48\pi m^*\epsilon^0/\epsilon_0\kappa_0\hbar^2)^{1/3}(n_c + 11n_0/32)^{1/3} \), where \( \kappa_0 = 12.9 \) is the static dielectric constant of GaAs, \( \epsilon_0 \) is the free space permittivity and \( n_c \) is the depletion charge density in the channel. In this section we will also discuss the piezoelectric scattering along with the deformation potential scattering. In this case the IRT can be written as

\[
\begin{align*}
\frac{1}{\tau^2(\epsilon)} &= \frac{1}{(2\pi)^3} \frac{2\pi}{\hbar} \sum_{k'} \int dk' \int d\theta (1 - \cos \theta) \\
&\times \int dq_z |l(q_z)|^2 \left[ C_{q_{0},q_z}^{\lambda,\lambda'} \right]^2 \left[ N_0 \delta(\epsilon_{k_z}' - \epsilon_{k_z} - \hbar\omega_Q) \\
&+ (N_0 + 1)\delta(\epsilon_{k_z}' - \epsilon_{k_z} + \hbar\omega_Q) \right] \frac{1 - f(\epsilon_{k_z}')} {1 - f(\epsilon_{k_z})},
\end{align*}
\]
where the term $|I(q_z)|^2$, called the form factor, is responsible for yielding the rate of transition from the three-dimensional bulk phonon state to two dimensions. The exact form of this term for a triangular potential is given by $|I(q_z)|^2 = (\frac{\int dz z_0^2 e^{i q_z z}}{|I(q_z)|^2}) = \frac{b_6}{(b_2 + q_z^2)^3}$.

### 3.1. Deformation potential scattering

The matrix element in this case is given by

$$|C_{q, q'}|_\lambda = \frac{D^2 hQ}{2 \rho_m v_s} \left( 1 + \lambda' \cos \theta \right),$$

where $\rho_m$ is the mass density. At high temperature, we have $N_Q \simeq N_0 + 1 \simeq k_BT/(\hbar v_s)$. Inserting this matrix element into equation (16), we get

$$\frac{1}{\tau(\epsilon)} \simeq \frac{2m^* D^2}{\pi^2 \hbar^4 \rho_m v_s^3} \left( 1 + \sqrt{\epsilon + \epsilon_{\alpha}} \right) k_BT \times \int_0^1 dx x^2 \sqrt{1 - x^2} \int_0^{\infty} dq_z |I(q_z)|^2.$$

with $x = q/2k$. Performing the integrations over $x$ and $q_z$, we finally obtain the total IRT in the EP regime for DP scattering as given by

$$\frac{1}{\tau(\epsilon)} = \frac{3}{32} \frac{m^* D^2}{\rho_m \hbar^3 v_s^3} k_BT.$$
On the other hand, in the BG regime, we obtain the following expression for the energy averaged IRT (see section A.1):

\[
\left( \frac{1}{\tau^+} \right)_{DP} \simeq \frac{D_{0}}{4} \left( 1 \mp \frac{\epsilon_{a}}{\epsilon_{F} - \epsilon_{a}} \right) \frac{D^2}{\rho_m v_s^2} \frac{5!}{5!} (k_B T)^5 \left( \frac{k_F^2}{\hbar} \right)^3.
\]

(20)

where \( \zeta(5) = 1.037 \).

### 3.2. Piezoelectric scattering

Piezoelectricity is nothing but generation of polarization due to the application of a strain to a crystal without inversion symmetry. Due to lattice vibration, a potential can be generated in such crystals and electrons are scattered by this kind of potential. To calculate the IRT due to the PE scattering, we can use equation (16). Following [25], the matrix elements of the Rashba system are obtained as

\[
|C_{q,l'}(\mathbf{q}, \mathbf{l})|^2 = \left( \frac{e h_{14}^q}{2 \rho_m v_{sl}(t)} \right) \frac{1 + \lambda \lambda' \cos \theta}{2 \sqrt{q^2 + q_z^2}} A_l(q, q_z).
\]

(21)

where \( A_l(q, q_z) = 9 q_c^2 q^4 / 2(q_z^2 + q^2)^3 \) and \( A_l(q, q_z) = (8 q_c^4 q^2 + q^6) / 4(q_z^2 + q^2)^3 \). In equation (21) the value of the PE tensor component \( h_{14} \) is 1.2 \times 10^9 \text{ V m}^{-1} \text{ and } v_{sl}(t) \) is the longitudinal (transverse) component of the sound velocity. By inserting this matrix element into equation (16) and doing some straightforward calculations in the high temperature regime we get

\[
\frac{1}{\tau^+_{T}} \simeq \frac{9}{8 \pi^2} \frac{m^* (e h_{14})^2}{\hbar^3} \left( 1 + \frac{\epsilon_{a}}{\epsilon + \epsilon_{a}} \right) k_B T \times \int_0^\pi \sin^2 \theta F_1(q) \sin \theta.
\]

(22)

and

\[
\frac{1}{\tau^+_{T}} \simeq \frac{3}{32} \frac{m^* (e h_{14})^2}{\hbar^3} \frac{1}{k^*} \left( 1 + \frac{\epsilon_{a}}{\epsilon + \epsilon_{a}} \right) k_B T.
\]

(26)

The following expression for the energy averaged IRT (see section A.1):

\[
\frac{1}{\tau^+_{T}} \simeq \frac{12}{192} \frac{m^* (e h_{14})^2}{\hbar^3} \frac{1}{k^*} \left( 1 + \frac{\epsilon_{a}}{\epsilon + \epsilon_{a}} \right) k_B T.
\]

(27)

Figure 4. Plots of \( \rho \) of a perfect 2DEG versus \( \alpha \) for two fixed densities. Here solid and dashed lines correspond to \( n_s = 3n_0 \) and \( n_s = 7n_0 \), respectively.

Here \( F_1(q) \) and \( F_1(q) \) are respectively given by

\[
F_1(q) = \int dq_z |I(q_z)|^2 \frac{q_z^2 q^4}{(q_z^2 + q^2)^2} = \frac{\pi}{16q} \frac{1 + 6\alpha + 2\alpha^3 + 2\alpha^3}{(1 + \alpha)^6}.
\]

(24)

and

\[
F_2(q) = \int dq_z |I(q_z)|^2 \frac{8q_z^4 q^4 + q^6}{(q_z^2 + q^2)^4} = \frac{\pi}{16q} \frac{1 + 72\alpha + 128\alpha^2 + 28\alpha^3 + 36\alpha^4 + 6\alpha^5}{(1 + \alpha)^6}.
\]

(25)

where \( \alpha = q/b \). Now, we are assuming that the quasi-2DEG is very thin, i.e. \( \alpha \ll 1 \). So \( F_1(q) \) and \( F_1(q) \) can be approximated as \( F_1(q) \approx \pi / 16q \) and \( F_1(q) \approx 3\pi / 16q \).

Substituting \( F_1(q) \), \( F_1(q) \) and \( q = 2k \sin(\theta/2) \) in equations (22) and (23) and integrating over \( \theta \), we obtain

\[
\frac{1}{\tau^+_{T}} \simeq \frac{3}{32} \frac{m^* (e h_{14})^2}{\hbar^3} \frac{1}{k^*} \left( 1 + \frac{\epsilon_{a}}{\epsilon + \epsilon_{a}} \right) k_B T \times \int_0^\pi \sin^2 \theta F_1(q) \sin \theta.
\]

(22)

and

\[
\frac{1}{\tau^+_{T}} \simeq \frac{12}{192} \frac{m^* (e h_{14})^2}{\hbar^3} \frac{1}{k^*} \left( 1 + \frac{\epsilon_{a}}{\epsilon + \epsilon_{a}} \right) k_B T.
\]

(27)
The total IRT for longitudinal and transverse cases are given as

\[
\frac{1}{\tau_l(\epsilon)} \simeq \frac{3}{16} \frac{m^*(e\hbar)^2}{\hbar^3} \frac{k_B T}{\rho_m v^2_{s\lambda}} \frac{\epsilon^0_F}{\epsilon + \epsilon_a} \tag{28}
\]

and

\[
\frac{1}{\tau_t(\epsilon)} \simeq \frac{13}{96} \frac{m^*(e\hbar)^2}{\hbar^3} \frac{k_B T}{\rho_m v^2_{s\lambda}} \frac{\epsilon^0_F}{\epsilon + \epsilon_a} \tag{29}
\]

The total IRT in the PE scattering case can be written as 
\[1/\tau_{PE} = 1/\tau_l + 1/\tau_t.\] In the high temperature regime, the IRT is proportional to \(k_B T\) and inversely proportional to \(\sqrt{\epsilon + \epsilon_a}\).

In the low temperature regime we calculate the IRT averaged over energy. The detailed calculations are given in section A.2. In this case, we have the following expressions:

\[
\left\langle \frac{1}{\tau_l(\epsilon)} \right\rangle \simeq \frac{45}{512} \frac{m^*(e\hbar)^2}{\hbar^3} \frac{3\zeta(3)}{\rho_m v^2_{s\lambda}} \left( \frac{k_B T}{\hbar^3} \right)^3 \left( \frac{\epsilon^0_F}{\epsilon + \epsilon_a} \right) \tag{30}
\]

and

\[
\left\langle \frac{1}{\tau_t(\epsilon)} \right\rangle \simeq \frac{59}{1024} \frac{m^*(e\hbar)^2}{\hbar^3} \frac{3\zeta(3)}{\rho_m v^2_{s\lambda}} \left( \frac{k_B T}{\hbar^3} \right)^3 \left( \frac{\epsilon^0_F}{\epsilon + \epsilon_a} \right). \tag{31}
\]

Here, \(\zeta(3) = 1.202\). Equations (30) and (31) show that the energy averaged IRT due to the PE scattering is proportional to \(T^3\) and hence the resistivity is also proportional to \(T^3\).

### 3.3. Numerical results

In this section we discuss the resistivity due to both DP and PE scattering mechanisms. We solve equation (16) numerically using the matrix elements for DP and PE scattering given in equations (17) and (21), respectively. For the numerical calculation we set \(v_{sl} = 5.31 \times 10^3\ \text{m/s}\), \(v_{sd} = 3.04 \times 10^3\ \text{m/s}\), \(n_e = 5 \times 10^{14}\ \text{m}^{-2}\), \(\rho_m = 5.12 \times 10^3 \text{kg/m}^3\) and \(D = 12\ \text{eV}\). The other parameters are the same as those given in section 2. The values of the numerical parameters considered in this section and also in section 2 are appropriate for GaAs/AlGaAs heterostructures. These material parameters are different for different kinds of heterostructures.

In figure 5, we show the temperature dependence of the resistivity of the quasi-2DEG due to the DP scattering for fixed values of \(\alpha = 0\) and \(\alpha_0\) with different densities. In the BG regime with \(\alpha = 0\), the effective exponents of \(\nu_{st}\) and \(\nu_{sl}\) are greater than the perfect 2DEG case discussed in section 3.2. This increase in \(\nu\) is due to the finite thickness of the quasi-2DEG in the \(z\) direction. In the EP regime, \(\nu\) becomes close to 1 as we increase \(n_e\).

In figure 6, we plot the resistivity of a quasi-2DEG due to DP scattering as a function of \(\alpha\) at fixed temperature and density. Figure 6 shows that \(\rho\) increases rapidly with \(\alpha\) in both the regimes but it is faster for low electron density and gets saturated after a certain value of \(\alpha\) in the BG regime.

In figure 7, we plot the temperature dependence of the resistivity of the quasi-2DEG due to the PE scattering for fixed values of \(\alpha = 0\) and \(\alpha_0\) with different densities. When \(\alpha = 0\), the exponents are \(\nu = 2.537, 2.830\) and 3.002 for densities \(n_e = 3n_0, 5n_0\) and \(7n_0\), respectively, in the BG regime. Similarly, the effective exponents are \(\alpha = 1.962, 2.208\) and
Figure 6. Plots of the resistivity of a quasi-2DEG due to DP scattering versus $\alpha$ for various densities. Here, solid and dashed lines correspond to $n_e = 3n_0$ and $n_e = 7n_0$, respectively.

Figure 7. Plots of the resistivity due to PE scattering versus temperature for different values of the density. Here, solid, dotted and dashed lines represent $n_e = 3n_0$, $n_e = 5n_0$ and $n_e = 7n_0$, respectively.

2.398 for densities $n_e = 3n_0$, $5n_0$ and $7n_0$, respectively, when $\alpha = \alpha_0$. This clearly shows that the exponent $\nu$ is decreased due to the presence of the RSOI.

For the PE scattering case we also calculate the resistivity of a quasi-2DEG as a function of $\alpha$ by solving equation (16) and we show the results in figure 8. Figure 8 shows that $\rho$ increases with $\alpha$ in both regimes. The other features are similar to the DP scattering mechanism.

Comparing figures 6 and 8, one can see that the resistivities due to DP and PE potentials are of the same order in the BG regime, but $\rho$ due to DP is dominant over PE in the EP regime.

4. Summary

In this work we have investigated the effect of the Rashba spin–orbit interaction on the momentum relaxation time due to the electron–phonon scattering in a 2DEG. We have considered both the perfect 2DEG and the quasi-2DEG. We have also considered both the deformation potential and piezoelectric scattering mechanisms responsible for the electron–phonon interaction separately. The temperature dependence of the resistivity has been calculated in both equipartition and Bloch–Grüneisen regimes. We have found through approximate calculations that the resistivity of a
perfect 2DEG is proportional to $T^4$ in the Bloch–Gruneisen regime for the deformation potential scattering. On the other hand, $\rho \sim T^3$ for deformation potential scattering and $\rho \sim T^5$ for piezoelectric scattering in a quasi-2DEG. We have also recovered the linear temperature dependence of the resistivity in the equipartition regime for all cases. Our numerical analysis showed that the effective exponent ($\nu$) of the temperature dependence of the resistivity strongly depends on the electron density $n_e$ and spin–orbit coupling constant $\alpha$. For the deformation potential and piezoelectric scattering, the values of the effective exponent of $T$ in the Bloch–Gruneisen regime for different values of $n_e$ and $\alpha$ are summarized in Table 1.

| Density ($n_e$) | $\alpha = 0$ | $\alpha = \alpha_0$ | $\alpha = 0$ | $\alpha = \alpha_0$ |
|----------------|-------------|-----------------|-------------|-----------------|
| $3n_0$         | 3.763       | 2.829           | 4.459       | 3.649           |
| $5n_0$         | 4.111       | 3.203           | 4.887       | 4.058           |
| $7n_0$         | 4.235       | 3.433           | 5.085       | 4.359           |

Table 1. The effective exponent of the temperature dependence of the resistivity in the Bloch–Gruneisen regime for various values of $n_e$ and $\alpha$.

There is a reduction in the exponent of the temperature dependence of the resistivity in the Bloch–Gruneisen regime due to the Rashba spin–orbit interaction at fixed electron density. We believe that the reduction in the exponent can be verified experimentally in near future.

The variation of $\rho$ with $\alpha$ has also been discussed for all cases. It is found that $\rho$ increases with $\alpha$ in both regimes. The rate of increase is faster in the low electron density case.

Appendix

In this appendix, we derive the energy averaged IRT due to DP and PE scattering in a quasi-2DEG in the BG regime. At very low temperature, following the approximations used in section 2.1 and using equation (16), we get

$$\left< \frac{1}{\tau^\pm} \right> \simeq \frac{m^* s}{2\pi^2 h^3} \left( 1 \mp \sqrt{\epsilon_\alpha \epsilon_\alpha^0} \right) \frac{1}{k_B T} \int_0^\pi d\theta \left( 1 - \cos \theta \right)$$

$$\times \int dq_q |I(q_q)|^2 |C_{q_q}\gamma|^2 \hbar \omega_0 N_Q(N_Q + 1). \quad (A.1)$$

When $T$ is very low, the phonon wavevector $q$ is very small compared to the Fermi wavevector, i.e. $q \ll 2k_F$. For a very thin quasi-2DEG, $|I(q_q)|^2$ can be approximated as $|I(q_q)|^2 \simeq 1$ since $q_q \ll b$. With all the approximations taken into account, equation (A.1) can be approximated further as

$$\left< \frac{1}{\tau^\pm} \right> \simeq \frac{m^* s}{2\pi^2 h^3} \left( 1 \mp \sqrt{\epsilon_\alpha \epsilon_\alpha^0} \right) \frac{1}{k_B T}$$

$$\times \int dq dq_q |C_{q_q}\gamma|^2 \hbar \omega_0 N_Q(N_Q + 1). \quad (A.2)$$

A.1. Deformation potential scattering

Inserting the matrix element given in equation (17) into equation (A.2), we get

$$\left< \frac{1}{\tau^\pm} \right>_{DP} \simeq \frac{m^* D^2}{4\pi^2 h^3 \rho_m v_F^2 k_F^3} \left( 1 \mp \sqrt{\epsilon_\alpha \epsilon_\alpha^0} \right) \frac{1}{k_B T}$$

$$\times \int dq dq_q q^2 (\hbar \omega_0)^2 N_Q(N_Q + 1). \quad (A.3)$$
Since the phonon dispersion relation is $\epsilon_p = \hbar v_s \sqrt{q^2 + q_z^2}$, we can carry out the following transformation: $dq dq_z \rightarrow \epsilon_p d\epsilon_p d\phi/(\hbar v_s)^2$ with $q = \epsilon_p \cos \phi/(\hbar v_s)$ and $q_z = \epsilon_p \sin \phi/(\hbar v_s)$. With these transformations, equation (A.2) reduces to
\[
\left\langle \mathbf{1}_{\pm \mathbf{1}} \right\rangle_{Down} \simeq \frac{m^* D^2}{4\pi^2 \hbar^3 \rho_m v_s^4 k_F^4} \left(1 \mp \frac{\epsilon_a}{\epsilon^0_F - \epsilon_a} \right) \frac{1}{(\hbar v_s)^4} \times \frac{1}{k_F} \int d\epsilon_p \epsilon_p^5 N_Q(N_Q + 1). \quad (A.4)
\]

Using the result $\int d\epsilon_p \epsilon_p^5 N_Q(N_Q + 1) = n!\xi(n)(k_F T)^{n+1}$ where $\xi(n)$ is the Riemann zeta function, we finally obtain
\[
\left\langle \mathbf{1}_{\pm \mathbf{1}} \right\rangle_{Down} \simeq \frac{D_0}{4} \left(1 \mp \frac{\epsilon_a}{\epsilon^0_F - \epsilon_a} \right) \frac{D^2}{\rho_m v_s^4 (v_s \hbar)^8} \frac{S(5) (k_F T)^5}{(k_F T)^3}. \quad (A.5)
\]

### A.2. Piezoelectric scattering

Using the matrix elements for PE scattering given in equations (21) and (A.2), we obtain the energy averaged IRT for longitudinal and transverse cases, respectively:
\[
\left\langle \mathbf{1}_{\pm \mathbf{1}} \right\rangle_{PE} \simeq \frac{9}{8\pi^2} \frac{m^* (e h_{14})^2}{\rho_m (v_s \hbar)^4 k_F^4} \left(1 \mp \frac{\epsilon_a}{\epsilon^0_F - \epsilon_a} \right) \frac{1}{k_F T} \times \int d\epsilon_p \epsilon_p^3 N_Q(N_Q + 1) \cos^6 \phi \sin^2 \phi d\phi
\]
and
\[
\left\langle \mathbf{1}_{\pm \mathbf{1}} \right\rangle_{PE} \simeq \frac{1}{8\pi^2} \frac{m^* (e h_{14})^2}{\rho_m (v_s \hbar)^4 k_F^4} \left(1 \mp \frac{\epsilon_a}{\epsilon^0_F - \epsilon_a} \right) \frac{1}{k_F T} \times \int (\cos^6 \phi + 8 \cos^4 \phi \sin^4 \phi) d\phi \times \int d\epsilon_p \epsilon_p^3 N_Q(N_Q + 1). \quad (A.6)
\]

After carrying out the integration over $\epsilon_p$ and $\phi$, we finally obtain the following expressions:
\[
\left\langle \mathbf{1}_{\pm \mathbf{1}} \right\rangle_{PE} \simeq \frac{45}{512} \frac{m^* (e h_{14})^2}{\pi \hbar} \frac{3\xi(3)}{\rho_m (v_s \hbar)^4} \frac{(k_F T)^3}{k_F^4} \times \left(1 \mp \frac{\epsilon_a}{\epsilon^0_F - \epsilon_a} \right) \quad (A.7)
\]
and
\[
\left\langle \mathbf{1}_{\pm \mathbf{1}} \right\rangle_{PE} \simeq \frac{59}{1024} \frac{m^* (e h_{14})^2}{\pi \hbar} \frac{3\xi(3)}{\rho_m (v_s \hbar)^4} \frac{(k_F T)^3}{k_F^4} \times \left(1 \mp \frac{\epsilon_a}{\epsilon^0_F - \epsilon_a} \right). \quad (A.8)
\]