The Possibility of Checking the Equivalence Principle in a Null Gravitational Redshift by a Two-Resonator Laser System.

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A scheme of an optical detector is proposed for checking Einsteins equivalence principle (EEP) in a null gravitational redshift experiment and for testing methods for calculating the length of a resonator in a weak variable gravitational field by recording the variations of the difference frequency of resonators caused by lunisolar variations of the geopotential in a double or a two-resonator laser system.

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I. INTRODUCTION

Today, the general theory of relativity is a generally recognized metric theory of gravity and underlies our knowledge of the structure of spacetime. This theory is based on Einsteins equivalence principle (EEP). One of the consequences of this principle is the gravitational shift of spectral lines (the so-called gravitational redshift), which is equivalent to the dependence of the clock rate on a local gravitational potential $\varphi$. According to the local positional invariance (LPI) principle, which is a part of the EEP [1], the gravitational shift of the clock frequency is universal and must not depend on the type of clock used. On the other hand, attempts to develop a quantum theory of gravity sometimes suggest that the EEP should be violated at a certain level; thus, these attempts stimulate experimental verification of the EEP and, in particular, the LPI to an increasing degree of accuracy. The LPI principle can be tested by experiments on measuring the gravitational redshift. According to this effect, the dependence of the frequency $\nu$ of any clock on the potential $\varphi$ in weak gravitational fields $|\varphi/c^2| < 1/c$ is given by $\nu = \nu_0 (1 + \varphi/c^2)$, where $\nu_0$ is the frequency of a clock in the absence of the gravitational field (the proper frequency). If we assume that the LPI principle is violated, then this dependence can be represented as [1]

$$\nu_A = \nu_0 [1 + (1 + \beta_A) \varphi/c^2],$$

(1)

where the dimensionless quantity $\beta_A$ characterizes the deviation from the redshift law, which follows from the EEP, and the subscript $A$ indicates that the frequency $\nu_A$ may depend on the specific type of clock used.

There exist two methods for verifying the LPI principle. The first method (the classical experiment on the gravitational redshift) is based on the use of identical clocks situated at points with different values of the gravitational potential ($\varphi_1$ and $\varphi_2$) and on the measurement of the frequency difference

$$\nu_{A1} - \nu_{A2} = \nu_0 [1 + (1 + \beta_A) (\varphi_1 - \varphi_2) / c^2].$$

(2)

In [2], the authors carried out numerous experiments to determine possible values of $\beta_A$ with increasing accuracy. Comparing the oscillation frequencies of two separated hydrogen masers (H), they obtained the minimal upper limit $|\beta_H| < 7 \times 10^{-5}$. The second method for verifying the LPI principle (a null gravitational redshift experiment [1]) is based on the comparison of the rates of nonidentical clocks A and B situated at points with identical values of the gravitational potential under time variations of the potential $\varphi(t)$ itself. In this case, the violation of the LPI principle leads to a nonzero frequency difference

$$\nu_A - \nu_B = \nu_0 (\beta_A - \beta_B) \varphi(t) / c^2.$$  

(3)

A large number of experimental studies devoted to the verification of the LPI principle by formula (3) were based on the comparison of the frequencies of atomic transitions in different substances under the variation of the gravitational potential $\varphi(t)$ due to the orbital motion of the Earth around the Sun. The least upper bound for the possible values of $\beta_A - \beta_B$ was obtained by this method in [2] by comparing the frequency variations of a hydrogen maser and the cesium frequency standard (Cs): $|\beta_H - \beta_{Cs}| < 2.1 \times 10^{-5}$. In [4][5][6] a null gravitational redshift experiment was carried out by comparing the frequency of an atomic transition with the eigenfrequency of an electromagnetic resonator; i.e., the frequencies to be compared were of completely different physical natures. In [4], the measurement of the frequency variations of a hydrogen maser and of a superconducting electromagnetic resonator yielded an
upper bound of $|\beta_H - \beta_{res}| < 1.7 \times 10^{-2}$, whereas, in [3], the comparison of the variations of the electron transition frequency in iodine molecules ($I_2$) and the eigenfrequency of a cryogenic optical resonator yielded an upper bound of $|\beta_2 - \beta_{res}| < 4 \times 10^{-2}$.

Experiments with clocks of different physical natures yielded an upper bound that is three orders of magnitude greater than that obtained when verifying the LPI principle by comparing only atomic transition frequencies in different substances; this is associated with the large intrinsic noises of the experimental setups used in [4], [5]. However, it would be desirable to obtain a maximally accurate estimate for $\beta_A - \beta_B$ precisely in experiments with clocks of different physical natures. Below, we will show that this may provide an experimental verification of the relativistic generalization [6]-[7] of the classical theory of elasticity.

In the present paper, we consider the possibility of verifying the LPI principle by a stationary horizontal double laser system [9] one of resonators of which contains a cell with a nonlinearly absorbing gas.

II. OSCILLATION FREQUENCY OF A GAS LASER WITH A NONLINEARLY ABSORBING CELL IN A GRAVITATIONAL FIELD

In [9], the effect of the gravitational field of the Earth on the oscillation frequency of a gas laser was investigated on the basis of a covariant generalization of the Lamb theory. The investigation was carried out within a semiclassical approximation, when the electromagnetic and gravitational fields are described classically, while the active medium is described quantummechanically. The formalism of the Lamb theory [10] was applied to derive equations for a self-consistent field. As shown in [9], on the one hand, the formation of a macroscopic electromagnetic field in the active medium is determined by Maxwell’s equations, which are essentially relativistic and contain the gravitational field via metric coefficients and their derivatives; on the other hand, it is the mechanical subsystem, the resonator evolving in the gravitational field by different laws, that performs the selection of harmonics.

In [9], oscillation equations for a single-mode linear laser situated horizontally on the Earth’s surface were obtained with regard to Newton’s gravitational potential $\varphi$ at a given point:

$$E \left( \frac{d\Phi}{dt} + \omega - \Omega_m \right) = -2\pi\omega \left( 1 - \frac{2\varphi}{c^2} \right) C_m, \tag{4}$$

$$\frac{dE}{dt} + \frac{\Delta\Omega_R}{2} \left( 1 - \frac{2\varphi}{c^2} \right) E = -2\pi\omega \left( 1 - \frac{2\varphi}{c^2} \right) S_m. \tag{5}$$

Here, $E$ and $\Phi$ are the slowly varying amplitude and phase of the generated electromagnetic field; $c$ is the velocity of light; $\omega$ is the oscillation frequency; $\Omega_m$ is the natural longitudinal eigenfrequency of the $m$ the mode of the resonator in the gravitational field,

$$\Omega_m = \Omega_{m0} \left( 1 - \frac{2\varphi}{c^2} \right), \quad \Omega_{m0} = k_m c = \frac{\pi mc}{L}; \tag{6}$$

$\Delta\Omega_R$ is the linewidth of the empty resonator; and $L = L_0 \left( 1 + \xi \varphi / c^2 \right)$ is the length of the resonator, where the phenomenological parameter $\xi$ is determined by the solution of the elastodynamic problem of the evolution of the mechanical length of the resonator in the gravitational field and $L_0$ is the resonator length in the absence of the gravitational field. In the presence of a nonlinearly absorbing gas in the resonator, the polarization coefficients $C_m$ and $S_m$ can be represented as [10] $C_m = (\chi'_a + \chi'_b) E$ and $S_m = (\chi''_a + \chi''_b) E$, where the real ($\chi'_a$) and imaginary ($\chi''_a$) parts of the nonlinear susceptibility of the active medium, calculated by the method of [9], are given by the following expressions up to the second- order terms with respect to the field (under the assumption that the homogeneous amplification linewidth is small compared with the Doppler width and the detuning of the oscillation frequency from the center of the amplification line is small):

$$\chi'_a = d_a \frac{2(\omega - \omega_a)}{\alpha_a} \left( 1 + 2\xi \frac{\varphi}{c^2} \right) \times \left\{ 1 - \left[ 1 + (1 + \xi) \frac{\varphi}{c^2} \right] \frac{G}{2\alpha_a} (\Delta_a) \alpha_a E^2 \right\}, \tag{7}$$

$$\chi''_a = -d_a \left( 1 + \xi \frac{\varphi}{c^2} \right) \left( 1 - \left[ 1 + D (\Delta_a) \right] \alpha_a E^2 \right), \tag{8}$$
characterized by a narrow absorption linewidth
the nonlinear pulling of the oscillation frequency to the center of the absorption line (frequency self-stabilization) 

Indeed, setting $N$ frequency of the atomic transition $[7]$; $\gamma$ constants between operating levels 1 and 2, which are characterized (in the absence of the gravitational field) by the decay constants $\gamma_{1a}^0$ and $\gamma_{2a}^0$, respectively, and by the homogeneous linewidth $\Gamma_a^0, \omega_a^0$ is the proper (independent of gravitation) frequency of the atomic transition $[7]$; $N_a$ is the population inversion caused by pumping ($N_a > 0$); $k_B$ is the Boltzmann constant; and $T_a$ and $m_a$ are the temperature and the mass of the atoms of the active medium.

Expressions for the real ($\chi_a^0$) and imaginary ($\chi_a^\prime$) parts of the nonlinear susceptibility of the absorbing medium are obtained from formulas $[7]$ and $[8]$ by changing the subscript $a$ to $b$ (in this case, $N_b < 0$).

For a steady oscillation mode, we derive from $[5]$ an equation for the intensity of the electric field. Using this equation and introducing a dimensionless quantity $r = d_b / d_a > 0$, we obtain from $[11]$ the following equation for the oscillation frequency:

$$\begin{align*}
\omega - \Omega_m &= (\omega_a - \omega) \left\{ p_a \left[ 1 + (1 + \xi) \frac{\omega}{c^2} \right] - q_a D(\Delta_a) \right\} \\
&- (\omega_a - \omega) \left\{ p_b \left[ 1 + (1 + \xi) \frac{\omega}{c^2} \right] - q_b D(\Delta_b) \right\}, \\
p_a &= \frac{r \delta_a}{(1-r)(1-2aE^2)}, \\
q_a &= \frac{\delta_a \Omega_b a_e E^2}{2\pi a(1-r)(1-2aE^2)}, \\
p_b &= \frac{r \delta_b}{(1-r)(1-2bE^2)}, \\
q_b &= \frac{\delta_b \Omega_a b_e E^2}{2\pi b(1-r)(1-2bE^2)}, \\
\delta_{a,b} &= \frac{\delta_{a,b}}{\sqrt{a_b}}, \\
\alpha &= \frac{\chi_{a,b}^0}{2(1-r)}\left[ 1 + D(\Delta_a) - r \alpha_a [1 + D(\Delta_a)] \right].
\end{align*}$$

For a stable steady oscillation mode, it is required that $\alpha > 0$. Since the atoms of the absorbing cell are usually characterized by a narrow absorption linewidth ($\Gamma_b^0 \ll \Gamma_a^0$), we have $q_b \gg q_a$; i.e., the main nonlinear phenomenon is the nonlinear pulling of the oscillation frequency to the center of the absorption line (frequency self-stabilization) $[11]$. Indeed, setting $\Delta_{a,b} \ll 1$, we derive the following expression for the oscillation frequency from Eq. $[11]$: $\omega = \omega_b + \frac{\Omega_m \omega_b}{1+S} + \frac{\omega_b \omega_a}{1+S}$

$$\times \left\{ \left[ 1 + (1 + \xi) \frac{\omega}{c^2} \right] p_a - q_a \right\},$$

The quantity

$$S = \left[ 1 + (1 + \xi) \frac{\omega}{c^2} \right] (p_a - p_b) - (q_a - q_b)$$

$$d_a = \frac{D^2 \gamma_{1a}}{kk m_0 u_a}, \quad \alpha_a = \frac{D^2 \gamma_{1a}}{kk m_0 u_a},$$

$$D(\Delta_a) = \frac{1}{1+\Delta_a^2}, \quad \Delta_a = \frac{\omega_a - \omega}{\omega_a^2},$$

$$G_a = \sqrt{k m_0 u_a}, \quad u_a = \sqrt{2 k T_a m_a},$$

$$\Gamma_a = \Gamma_a^0 \left( 1 + \frac{\omega}{\omega_a^0} \right), \quad \omega_a = \omega_a^0 \left( 1 + \frac{\omega}{\omega_a^0} \right),$$

$$k_{m0} = \frac{2m}{a_a^0}.$$
is called a self-stabilization factor. For typical parameters of the absorbing cells \((q_b \gg p_{a,b})\), the inequality \(S \approx q_b\) holds; therefore, the oscillation frequency is determined by the absorption frequency \(\omega_b\) of the atoms of the cell in the gravitational field,

\[
\omega = \omega_b^0 \left(1 + \frac{\varphi}{c^2}\right).
\]

(15)

In the case of a laser without an absorbing cell in the resonator, the oscillation frequency is determined (see formula \((11)\) for \(r = 0\) and \(\delta_a \ll 1\)) \([9]\) by the eigenfrequency \(\Omega_{m0}\) of the resonator:

\[
\omega = \Omega_{m0} \left(1 + \frac{2\varphi}{c^2}\right).
\]

(16)

### III. COVARIANT PROPAGATION EQUATIONS FOR WAVES IN AN ELASTIC MEDIUM

The eigenfrequency

\[
\Omega_{m0} = \frac{\pi mc}{L} = \Omega_{m0}^0 \left(1 + \frac{\varphi}{c^2}\right).
\]

(17)

of a resonator depends on the geometrical dimension \(L(t)\) of the resonator; to determine this dimension in the gravitational field with potential \(\varphi(t)\), one should solve the elastodynamic problem, which predicts the value of the parameter \(\xi\).

At the classical level within Newton’s limit (i.e., in three-dimensional Euclidean space when the speed of light \(c \to \infty\)), the equation for the propagation of elastic waves is well known. Within this theory, the displacement \(U_i\) of an element of a body under a volume force \(F_i\) is described by the equation

\[
\ddot{U}_i + F_i = \frac{1}{\rho} \frac{\partial \sigma_{ik}}{\partial x^k}.
\]

(18)

Here and below, we assume that summation is performed over repeated indices; the Roman indices run over the values 1, 2, and 3, and the Greek indices run over 0, 1, 2, and 3; the dots denote differentiation with respect to time; \(\sigma_{ik}\) is the stress tensor; and \(\rho\) is the material density of the medium. In the case of a gravitational filed, the force \(\dot{F}_i\) can be represented as \(\dot{F}_i = \nabla_i \varphi\). Hooke’s law, which relates the stress tensor to the tensor of small deformations of the body \(\varepsilon_{mn}\), is expressed as \(\sigma_{ik} = C_{ikmn}\varepsilon_{mn}\) in the classical theory; here, \(C_{ikmn}\) are the elastic constants of the body and, due to Saint Venants condition for the consistency of deformations, the tensor of small deformations \(\varepsilon_{mn}\) is related to the vector of small displacements \(U_i\) as \(\varepsilon_{ij} = (U_{ij} + U_{ji})/2\). Henceforth, \(U_{i,j} = \partial U_i / \partial x^j\).

The classical theory of elasticity has a long history, whereas the development of the relativistic theory of elasticity began rather recently. The first attempt was made by Weber in the late 1950s \([12]\). The equations derived by him are used for describing the response of an elastic pig to a weak gravitational wave \([13],[14]\). In its modern form, the relativistic theory of elasticity was constructed only in 1972 by Carter and Quintana \([15]\). This theory is still being developed (see, for example, \([16],[17]\)). A significant contribution to the development of this field of science was made by Maugin. For instance, he proposed in \([18]\) a relativistic equation for the propagation of elastic waves.

To calculate the evolution of the resonator length \(L(t)\) in a variable gravitational field (see formula \((17)\)), we apply the propagation equation for elastic waves \([18]\). Within this approach, Hooke’s law and the equation of elastic vibrations in spacetime \(V_4\) are expressed as

\[
\sigma_{ik} = C_{ikmn}\varepsilon_{mn},
\]

(19)

\[
\ddot{\varepsilon}_{ik} = \frac{1}{\rho} \sigma_{(i[l]j[|k}\right) + c^2 R_{|0k0}, \ i,k = 1, 2, 3.
\]

(20)

The boundary conditions are written in the standard form, \(\langle \sigma_{ik} N^k \rangle |_{S} = F_i\), where \(F_i\) is an external surface force of nongravitational origin and \(N^k\) is a normal vector to the surface of the body \(S\). Everywhere below, we assume that forces of nongravitational origin do not act on the body, i.e., \(F_i = 0\).
For a homogeneous isotropic medium, the elasticity tensor is represented as

\[ C_{ikmn} = \lambda \delta_{ik} \delta_{mn} + \mu [\delta_{im} \delta_{kn} + \delta_{in} \delta_{km}], \tag{21} \]

where \( \lambda, \mu = \text{const} \) are the Lamé coefficients related to the longitudinal and transversal velocities of elastic waves in the medium by the formulas

\[ a_l = \sqrt{\frac{\lambda + \mu}{\rho}}, \quad a_t = \sqrt{\frac{\mu}{\rho}}. \tag{22} \]

Within this theory, in contrast to the classical one, the deformation of a medium consists of a background deformation \( h_{ik}/2 \) and a true deformation \( U_{ik}; i, k \) for which is obtained from \( \phi(24) \) by differentiation and symmetrization with respect to the spatial variables. For a tidal gravitational field at the observation point. Since the Earth moves along an elliptic orbit around the Sun, the gravitational field:

There exist a distinguished direction for the true deformations of the medium described by the tensor \( U_{ik} \) for the elastic medium under investigation, there cannot solve homogeneous elastodynamic equations \( \phi(25) \) with inhomogeneous boundary conditions \( \phi(26) \).

Consider the metric of a tidal action, i.e., the metric that describes a quasistatic uniform isotropic Newtonian gravitational field:

\[ dS^2 = \left( 1 + \frac{2\varphi}{c^2} \right) c^2 dt^2 - \left( 1 + \frac{2\varphi}{c^2} \right) \delta_{ij} dx^i dx^j. \tag{23} \]

The choice of the metric in this form has been motivated by the following facts. This metric has the form of the Schwarzschild metric expressed in isotropic coordinates up to terms on the order of \( M/r \), where \( M \) is the mass of the gravitating center and \( r \) is the distance to this center. In this case, the function \( \varphi \) has the sense of the potential of the gravitational field at the observation point. Since the Earth moves along an elliptic orbit around the Sun, \( r = r(t) \) for a terrestrial observer; therefore, \( \varphi = \varphi(t) \); however, at any moment in time \( t \), metric \( \phi(23) \) is the Schwarzschild metric expressed with the above-indicated accuracy. In addition, the variations of the gravitational potential at a given point of the Earth are also associated with the motions of Moon around the Earth. It is the varying gravitational potential at the observation point that we model by the function \( \varphi(t) \). The values of this function at a given point of space at any moment can be produced to a high degree of accuracy. Note that, compared with the rates of processes in a laser, the variation rate of the potential \( \varphi \) is extremely low. Therefore, for such processes, we can assume to a high degree of accuracy that the potential \( \varphi \) is constant. Further, we distinguish the constant \( \varphi_0 \) and variable \( \delta \varphi(t) \) parts of this function: \( \varphi(t) = \varphi_0 + \delta \varphi(t) \).

In this case, the elastodynamic equations \( \phi(20) \) can be rewritten as

\[ \ddot{U}_{ik} = \frac{1}{\rho} \left[ 3 \lambda \delta_{i[j]k} \Theta_{j} + 2 \mu U_{i[j],k} \right], \tag{24} \]

where \( \Theta \equiv U_{k,k} \). Equation \( \phi(21) \) is a differential consequence of another, simpler, equation

\[ \ddot{U}_{i} - \frac{3 \lambda}{\rho} \Theta_{,i} - \frac{\mu}{\rho} (U_{i,tt} + U_{i,tt}) = 0, \tag{25} \]

which is obtained from \( \phi(24) \) by differentiation and symmetrization with respect to the spatial variables. For a tidal action with metric \( \phi(23) \), the boundary conditions have the form

\[ 3 \lambda \Theta N^i + 2 \mu U_{ik} N_k = -\lambda \frac{3 \varphi(t)}{c^2} N^i - \mu \frac{2 \varphi(t)}{c^2} N^i. \tag{26} \]

Thus, in order to find the variations of the length of a rod due to variations of the gravitational potential, we should solve homogeneous elastodynamic equations \( \phi(24) \) with inhomogeneous boundary conditions \( \phi(25) \).

Under the above conditions for the gravitational field and for the elastic medium under investigation, there cannot exist a distinguished direction for the true deformations of the medium described by the tensor \( U_{ik} \). Therefore, we take the tensor of true deformations \( \dot{U}_{ik} \) in the form \( \dot{U}_{ik} = \delta_{ik} \Theta \).

For a one-dimensional rod of length \( L_0 \) situated along axis \( x \), the vibration equations and the boundary conditions are expressed as

\[ \ddot{U}_1 = \frac{3}{\rho} \lambda U_{1,xx} + \frac{2\mu}{\rho} U_{1,xx}; \tag{27} \]

\[ U_{1,x}(0,t) = U_{1,x}(L_0,t) = \frac{\varphi(t)}{c^2}. \tag{28} \]
In this case, the time variations in the variation of the rod length, \( \Delta L = U^1(L_0, t) - U^1(0, t) \), are described by

\[
\Delta L = \frac{\varphi L_0}{c^2} + \frac{8L_0}{\pi c^2} \sum \left\{ \frac{\varphi}{(2m-1)^2} + \frac{\pi \varphi}{2m+1} \right\} \\
\times \left\{ \sin \frac{\pi(2m-1)\omega t}{L_0} \int \varphi \cos \frac{\pi(2m-1)\omega t}{L_0} dt \right. \\
+ \cos \frac{\pi(2m-1)\omega t}{L_0} \int \varphi \sin \frac{\pi(2m-1)\omega t}{L_0} dt \right\} \\
\times \cos \frac{\pi(2m-1)x}{L_0}.
\]

When \( \delta \varphi(t) = A\cos(\Omega_g t) \), we have

\[
\Delta L = \frac{\varphi(t)}{c^2} L_0 + \frac{8L_0}{\pi c^2} \delta \varphi(t) \\
\times \sum \left\{ \Omega_{2m-1}^2 - \Omega_g^2 \right\} \\
\times \left\{ 1 + \frac{8}{\pi^2} \frac{\Omega_g^2}{\Omega_{2m-1}^2} \right\}.
\]

The length of the rod \( L(t) = L_0 + \Delta L(t) \) obtained under the above-mentioned assumptions can be calculated by the formula

\[
L(t) = L_0 \left[ 1 + \frac{\varphi(t)}{c^2} + \frac{8}{\pi^2} \frac{\varphi(t)}{c^2} \Omega_g^2 \frac{1}{\Omega_{2m-1}^2} \right].
\]

In experiments on the verification of the LPI principle, one deals with a nonresonance case \( (\Omega_1 \gg \Omega_g) \); hence, formula (32) contains a very small coefficient of order \( \Omega_g^2/\Omega_1^2 \) at a sufficiently small term \( \delta \varphi(t)/c^2 \). Therefore, in a practically important case for the experiment, we can neglect the extremely small terms and rewrite formula (32) as

\[
L(t) = L_0 \left[ 1 + \frac{\varphi(t)}{c^2} \right].
\]

Substituting these values into the formula \( \Omega_{m0} = \pi mc/L(t) \) and neglecting the terms quadratic in \( \varphi(t)/c^2 \), we obtain

\[
\Omega_{m0} = \Omega_0 \left[ 1 - \frac{\varphi(t)}{c^2} \right].
\]

Using formula (16), we obtain the following expression for the oscillation frequency of a laser without an absorbing cell in the resonator:

\[
\omega = \Omega_{m0} \left[ 1 + \frac{\varphi(t)}{c^2} \right];
\]

i.e., the phenomenological parameter \( \xi \) in formula (17) is equal to 1.

IV. A NULL GRAVITATIONAL REDSHIFT EXPERIMENT ON A TWO-RESONATOR LASER SYSTEM

To resolve the above-mentioned alternative experimentally, one can use a stationary double laser system, one of whose resonators contains a cell with a nonlinearly absorbing gas, which actually turns this resonator into a laser stabilized by a nonlinear absorption resonance. Figure 1 represents a possible optical scheme of the laser system (for simplicity, the electronic equipment necessary for tuning to the absorption peak of the cell is not shown in this figure). The basic optical elements of this scheme (the cavity end mirrors 1, the gas-discharge tubes 2, the absorbing cell 3, and
the partially transmitting mirrors 4) are rigidly fixed on the common base 5. The mirror 6 and the semitransparent dielectric plate 7 provide the mixing of light radiated from two resonators of the laser system, and the photodetector 8 serves for detecting the beat note. In a linear approximation in $\varphi/c^2$, one can obtain the following expression from (13) for the oscillation frequencies $\omega_{1,2}$ of the first and second resonators:

$$
\omega_{1,2} = \omega_0 \left[ 1 + \left( 1 - \frac{\varphi}{S_{1,2}} \right) \frac{c^2}{\varphi} \right].
$$

where $\omega_0 \approx \omega_0^0 \approx \Omega_0^0$ is the frequency of optical radiation, $S_1 = 1 + p_a - q_a \approx 1$ for the resonator without absorbing cell, and $S_2 = 1 + (p_a - p_b) - (q_a - q_b) \gg 1$ for the resonator with the absorbing cell.

A null gravitational redshift experiment by a double laser system consists in the following. Due to lunisolar tides, Newtons gravitational potential experiences a periodic time modulation, $\varphi(t) = \varphi_0 + \delta\varphi(t)$, where $\delta\varphi(t)$ has a frequency of $\Omega_g \sim 10^{-5}$ Hz and an amplitude of $\delta\varphi_0 \approx 2.88 \times 10^4 m^2/s^2$. Since the settling time of steady oscillations in the laser system is $\Delta t \ll 2\pi/\Omega_g$ ($\Delta t$ is on the order of $\Delta\Omega_R^{-1}$, where $\Delta\Omega_R \approx 10^6$-10$^7$rad/s), we can apply formula (36) to calculate the instantaneous oscillation frequency of the laser.

According to (36), the difference frequency of two resonators contains a periodically varying term

$$
\Delta\omega(t) = \omega_0 (1 - \xi) \left( \frac{S_2 - S_1}{S_2 S_1} \right) \frac{\delta\varphi(t)}{c^2} 
$$

$$
\approx \omega_0 (1 - \xi) \frac{\delta\varphi(t)}{c^2}.
$$

The value of this additional term essentially depends on the phenomenological parameter $\xi$. For example, when $\xi = 0$, the amplitude of the variations of the difference frequency $\Delta\omega(t)$ for the optical radiation ($\omega_0 \approx 10^{15}$ rad/s) is about 320 rad/s, which is much greater than the linewidth due to the spontaneous radiation of the atoms of the active medium in the case of gas lasers. Technical fluctuations of the difference frequency can be minimized by a single-block configuration of the double laser system and by placing it into a thermostabilized vacuum chamber shielded from external electric and magnetic fields. Moreover, since the detected signal is periodic, it can be accumulated during several months with the use of efficient methods for extracting a weak low-frequency signal from large noise [19]. The potential accuracy of determining $\Delta\beta = |\beta_A - \beta_B|$ (see formula (3) in the experiment is limited by natural fluctuations of the oscillation frequency of the laser that are associated with the spontaneous radiation of the atoms of the active medium and lead to a finite linewidth of the laser radiation. The latter parameter can be evaluated by the well-known Schawlow-Townes formula

$$
\Delta\nu_0 = \frac{4\hbar\nu_0}{P} (\Delta\Omega_R)^2,
$$

where $P$ is the power of laser radiation. For $P \approx 1$ mW and $\Delta\Omega_R \approx 10^6$ rad/s, we obtain the following estimate for the least possible value of $\Delta\beta$:

$$
\Delta\beta \geq \left( \frac{\delta\varphi_0}{c^2} \right)^{-1} \frac{4\hbar}{P} (\Delta\Omega_R)^2 \approx 10^{-6}
$$

Figure 1: Double laser system.
which is four orders of magnitude higher than the accuracy achieved in the experiments of [4], [5].

Figure 2 represents another possible scheme of the detector that is based on a two-resonator laser system in which a common active medium is used for generating oscillations in two spatially inequivalent resonators. The first (signal) resonator is enclosed between mirrors \( R \) and \( R_1 \), and the second (reference) resonator is placed between mirrors \( R \) and \( R_2 \). The following optical elements are common to both resonators: the active medium AM (a gas-discharge He-Ne tube without Brewster windows), quarter-wave plates \( L_1 \) and \( L_2 \) whose fast axes are mutually perpendicular and make an angle of 45° with the plane of the figure, and a Wollaston polarization prism \( P \). The Wollaston prism is oriented so that the extraordinary ray with TM polarization (the electric field vector lies in the plane of the figure) is directed to the autocollimation mirror \( R_1 \) and the ordinary ray with TE polarization (the electric field vector is perpendicular to the plane of the figure) is incident to the autocollimation mirror \( R_2 \).

Figure 2: Two-resonator laser system.

First, we consider the operation of the two-resonator laser system in the absence of quarter-wave plates \( L_1 \) and \( L_2 \). In this case, linearly polarized optical radiation with TE and TM polarizations is generated in the reference and signal resonators, respectively. The diaphragms \( D_1 \) and \( D_2 \) serve for separating the fundamental transverse TEM\(_{00}\) modes in each resonator. In addition, by displacing one of the diaphragms perpendicular to the initial position of the optical axis of the corresponding resonator, one can significantly change the spatial overlap of the generated modes with TE and TM polarizations in the active medium; this sharply reduces the competition and coupling between these modes [20]. Radiation from the reference resonator emitted through the partially transmitting output mirror \( R_2 \) is mixed, with the use of the beam splitter BS (see Fig. 2), with the radiation of the laser stabilized over the absorbing cell SL. The beat note detected by the photodetector PD\(_2\) is fed to the electronic unit of the automatic phase-locked frequency control device (PLFCD), which locks the oscillation frequency of the reference resonator to the oscillation frequency of the stabilized laser SL by means of a piezoelectric cell attached to the mirror \( R_2 \). Radiation from the signal and reference resonators transmitted through the common output mirror \( R \) passes through the linear polarizer \( P_0 \) whose transmission axis makes an angle of 45° with the plane of Fig. 2 and forms an interference pattern, which is detected by the photodetector PD\(_1\). The signal from the photodetector PD\(_1\) is fed to the signal-processing unit SP and to the electronic unit of the automatic frequency-control device (AFCD), which controls the operation of the piezoelectric cell attached to the mirror \( R_1 \). The AFCD enables one to fix the initial frequency difference between the signal and reference resonators or provides the operation within the frequency locking range when information on \( \Delta \omega(t) \) is transformed into the variation of the phase difference between the signal and reference resonators [20]. The AFCD can also be used for partially compensating the noise due to technical fluctuations of the difference frequency of two generated modes with TE and TM polarizations.

The minimal linewidth of the difference frequency of two laser modes is determined by the quantum noise level due to the spontaneous radiation of the atoms of the active medium. In [21], Scully theoretically predicted that the linewidth of the difference frequency of two modes with different upper excited atomic levels and a common lower level can be substantially narrowed down (virtually to zero) by producing an active (compulsory) correlation of populations of the upper levels. In [22], [23], this idea was realized experimentally and was used to suppress quantum phase noise at the difference frequency of the Zeeman \( \sigma^+ \) and \( \sigma^- \) components of optical radiation generated in a linear two-mirror single-mode He-Ne laser due to the splitting of the eigenmode of the resonator under a dc magnetic field \( B_0 \) applied.
along the active medium. The correlation of spontaneous radiation (the absence of the phase diffusion at the difference frequency) is achieved under the application of a transverse radio-frequency magnetic field $B_{rf}$ of a certain amplitude that couples the split upper laser levels due to a magnetic dipole transition between them. In the scheme shown in Fig. 2, such an effect can be realized by using quarter-wave plates $L_1$ and $L_2$ under the application of a longitudinal dc magnetic field $B_0$ and a transverse ac magnetic field $B_{rf}$ to the active medium AM. Applying the Jones matrix formalism to the investigation of the polarization state of eigenmodes in the two-resonator laser system, we can easily show that, as before, optical radiation with TE and TM polarizations is generated to the left of the plate $L_1$ and to the right of the plate $L_2$ in the reference and signal resonators, respectively, which correspond to clockwise and counterclockwise polarized waves in the space between the plates. In this operation mode of the detector, the time needed to extract a useful signal can be significantly reduced because of the absence of a phase drift at the difference frequency, which is associated with the spontaneous radiation of the atoms of the active medium.

V. CONCLUSIONS

Comparing formulas (1) and (11), we can easily see that the quantity $\Delta \beta = |\beta_A - \beta_B|$, which characterizes the deviation from the LPI principle in a null gravitational redshift experiment, restricts the possible values of the phenomenological parameter $\xi$ to $|1 - \xi| \leq \Delta \beta$. The experiments of [4], [5] show that $\Delta \beta$ is no greater than $10^{-2}$; therefore, the parameter $\xi$ must be close to unity. In the case of an experiment carried out according to the scheme proposed in this paper, one can obtain more accurate values of $\Delta \beta$ (to within $10^{-6}$ or less). This would allow one to verify the LPI principle for clocks of different physical natures more accurately than was done in [4], [5]. At the same time, one may experimentally verify the correctness of the relativistic generalization of the classical elasticity theory considered in this paper.

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