Reliable Detection for Spatial Modulation Systems

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Abstract—Spatial modulation (SM) is a promising multiple-input multiple-output system used to increase spectral efficiency. The maximum likelihood (ML) decoder jointly detects the transmitted SM symbol, which is of high complexity. In this paper, a novel reliable sphere decoder (RSD) algorithm based on tree-search is proposed for the SM system. The basic idea of the proposed RSD algorithm is to reduce the size of the tree-search, and then, a smart searching method inside the reduced tree-search is performed to find the solution. The proposed RSD algorithm provides a significant reduction in decoding complexity compared to the ML decoder and existent decoders as well. Moreover, the RSD algorithm provides a flexible trade-off between the BER performance and decoding complexity, so as to be reliable for a wide range of practical hardware implementations. The BER performance and decoding complexity analysis for the RSD algorithm are studied, and Monte Carlo simulations are then provided to demonstrate the findings.

Index Terms—Multiple-input multiple-output (MIMO), spatial modulation (SM), low complexity decoders, complexity analysis, error analysis.

I. INTRODUCTION

Spatial modulation (SM) is a promising technique that has been recently applied to many of the emerging technologies. It overcomes the inter-channel interference (ICI) problem that exists in multiple-input multiple-output (MIMO) systems. The SM system completely eliminates the ICI by delivering a phase-shift-keying (PSK) or quadrature amplitude modulation (QAM) symbol from only one transmit antenna at a time. A part of the input bit-stream determines an active antenna, while the rest determines the PSK/QAM symbol to be transmitted. The maximum likelihood (ML) decoder jointly detects the SM message. The accumulated distance metric vector of the i-th level, \( v(i) \), is given by

\[
v(i) = \left[ \sum_{n=1}^{i} |y_{n} - x_{n,1}|^2 \cdots \sum_{n=1}^{i} |y_{n} - x_{n,MN_{t}}|^2 \right].
\]

Typically, the last level of the tree-search is called the decision level. The ML decoder estimates \( \hat{x}_{\text{ML}} \) that corresponds to the minimum node in \( v(N_r) \). Note that the SM tree-search is quite different than the MIMO tree-search.

In this paper, the decoding complexity is defined as the total number of nodes that should be visited in the tree-search to estimate the transmitted SM message. The ML decoder
visits all nodes in the tree-search, its decoding complexity is $\Psi^{ML} = MN_1N_r$. The complexity of the ML decoder consequently becomes extensive, especially for higher SM-MIMO dimensions and/or QAM sizes. Several works in the literature have been proposed to reduce the ML complexity, which are based on tree-search and SD concepts. However, further complexity reduction can still be achieved, as well as progress towards its reliability to fit a wide range of hardware implementation.

**III. The Proposed RSD Algorithm**

The proposed RSD algorithm firstly reduces the size of the tree-search, and then performs a smart searching method to reach the solution. Let us define $\psi_{col}$ as the number of branches/SM message possibilities that most likely contains the optimum solution. The RSD algorithm performs its searching for the solution inside these $\psi_{col}$ branches and stops at the $\psi_{row}$-th level, where $1 \leq \psi_{row} \leq N_r$ is the maximum number of levels that can be visited by the RSD algorithm (i.e., the decision level at $\psi_{row}$). It is worth noting that the flexibility trade-off between the BER performance and complexity provided by the RSD algorithm comes from changing the value of $\psi_{row}$ within the range of 1 and $N_r$.

The steps of searching for the solution of the RSD algorithm inside the reduced tree-search are as follows:

**Step 1:** Expand all nodes of the first level, i.e., $v(1)$ in (3).

**Step 2:** Appropriately choose the smallest $\psi_{col}$ nodes that come from Step 1. It should be noted that the RSD algorithm searches for the solution inside the branches that correspond to the smallest $\psi_{col}$ nodes. Consequently, the RSD algorithm reduces the decoding complexity by at least $(MN_1 - \psi_{col})\psi_{row}$ nodes. The vector of distance metric nodes in (3) yields

$$v(i) = [v(i,1) \cdots v(i,j) \cdots v(i,\psi_{col})],$$

where $v(i,j)$ is the $j$-th node of level $i$, and given by

$$v(i,j) = \sum_{n=1}^{i} |y_n - x_{n,j}|^2.$$  

**Step 3:** Perform a single expansion to the minimum node in (4).

**Step 4:** Check if the expanded node from Step 3 still has a minimum value among the rest of $\psi_{col}$ nodes or not. If yes, perform another single expansion to that node. If no, find the new minimum node and expand it once.

**Algorithm 1** The proposed RSD algorithm pseudo-code.

- **Input $H$, $M$, and $N$;**
- **Compute** $v(1)$ in (3);
- **Choose** $\psi_{col}$ empirically, based on $M$ and $N_i$ to most likely include the optimum BER performance;
- **Store** the branches indices that corresponding to the smallest $\psi_{col}$ node of $v(1)$ into $\Xi_{\psi_{col}}$;
- **Choose** $\psi_{row}$ based on the system requirements from the BER and complexity points of views;
- **Define** $\text{Len}(j)$ as the length of the $j$-th branch and initiate it with one for $\forall j$;

1: While $n < \psi_{row}\psi_{col}$ do
2: Find $j_{\text{min}}$ that solves $\arg\min_{i_{\text{min}}} \{v(i_{\text{min}})\};$
3: Update $i_{\text{min}}$ as the level that corresponding to $j_{\text{min}}$;
4: if $\text{Len}(i_{\text{min}}) = \psi_{row}$
5: break and end the algorithm;
6: else
7: Expand $v(i_{\text{min}}, j_{\text{min}}) \leftarrow v(i_{\text{min}} + 1, j_{\text{min}})$;
8: Update $v(i_{\text{min}})$ based on $v(i_{\text{min}}, j_{\text{min}})$;
9: end if
10: Set $n \leftarrow n + 1;
11: end While

- **Output** $\hat{x}_{\text{RSD}} = \arg\min_{j \in \Xi_{\psi_{col}}} \{v(\psi_{row})\}.$

**Step 5:** Repeat Step 3 and Step 4 until the RSD algorithm obtains the minimum node at a branch with a length of $\psi_{row}$.

**Step 6:** Find the index corresponding to the node that comes from Step 5, and declare it as the solution of the RSD algorithm. The estimated SM message using the RSD algorithm, $\hat{x}_{\text{RSD}}$, can be given as

$$\hat{x}_{\text{RSD}} = \arg\min_{j \in \Xi_{\psi_{col}}} \sum_{i=1}^{\psi_{row}} |y_i - x_{i,j}|^2$$

where $\Xi_{\psi_{col}}$ denotes the set of branch indices that corresponds to the smallest $\psi_{col}$ metric node values of $v(1)$ (i.e., the first level at $i = 1$ in (3)). The RSD algorithm is summarized in Algorithm 1.

**IV. Theoretical Analysis**

The RSD algorithm provides the optimum BER performance with a significant reduction in the decoding complexity. In addition, by changing the value of $\psi_{row}$, a flexible trade-off between the BER performance and decoding complexity can be obtained to fit a wide range of hardware implementation. In this section, the BER performance and expected complexity are considered random variables, and their approximate expressions are derived using the probability theory.

**A. BER Upper Bound Analysis**

The general expression for the upper bound of the ML BER for SM is (6).
\[ \text{BER}_{\text{ML}} \leq \sum_{j=1}^{M_N} \sum_{j=1}^{M_N} \delta(x_j, \hat{x}_j) \mathbb{E} \left\{ \mathbb{P}_{\text{ML}}(x_j \to \hat{x}_j) \right\}, \]  

(7)

where \( \mathbb{P}_{\text{ML}}(x_j \to \hat{x}_j) \) is the pairwise error probability (PEP) of the ML algorithm, \( \mathbb{P}(\cdot) \) denotes the probability of an event, \( \mathbb{E} \{ \cdot \} \) represents the expectation operation, and \( \delta(x_j, \hat{x}_j) \) denotes the Hamming distance which measures the number of bits in error between \( x_j \) and \( \hat{x}_j \).

Since the RSD algorithm performs the search inside a portion of the tree-search with a size of \( \psi_{\text{row}} \times \psi_{\text{col}} \), the optimal solution may not be included in that portion of the tree-search. Thus, the PEP in (7) for the RSD algorithm can be written as

\[ \mathbb{P}_{\text{RSD}}(x_j \to \hat{x}_j) = \mathbb{P}(\hat{x}_{\text{opt}} \neq x_j | x_{\text{opt}} \in \Xi_{\text{col}}) + \mathbb{P}(\hat{x}_{\text{opt}} \notin \Xi_{\text{col}}), \]  

(8)

where \( \hat{x}_{\text{opt}} \) is the optimal solution. The conditional probability in (8) contains two independent events. The expected value of the QAM symbol of the \( \psi \) with \( \delta \) of an event, \( \rho \) is the optimal solution. The conditional probability \( \psi_j \) is the average signal to noise ratio (SNR), and \( s(j) \) represents the \( j \)-th element of the AWGN vector in (1). It is worth noting that this simplified assumption of \( \zeta \) is especially for high SNR. The distribution of \( \zeta \) is a central chi-square with \( 2\psi_{\text{row}} \) degrees of freedom and its probability density function, \( f_\zeta(\zeta) \), is (Ch. 2)

\[ f_\zeta(\zeta) = \frac{1}{\pi^{\frac{n}{2}} \sigma_{\text{row}}^{n+1}} \exp \left( -\frac{\zeta}{\sigma_{\text{row}}^2} \right) \frac{\zeta^{\frac{n}{2}-1}}{(\frac{\zeta}{\sigma_{\text{row}}^2})^{\frac{n}{2}}} F_{\frac{n}{2}, \frac{n}{2}} \left( 1, -\frac{n}{2}, \frac{\zeta}{\sigma_{\text{row}}^2} \right). \]

(15)

Hence, (14) yields

\[ \Pr \left( \psi_{\text{row}} \leq \zeta \left| x_i, H, \sigma_g^2, \zeta \right. \right) = 1 - \int_0^{\frac{\psi_{\text{row}}^2}{\sigma_g^2}} Q_i \left( \sqrt{\frac{\zeta}{\sigma_g^2}} \right) d\zeta. \]

(16)

The closed-form expression of (16) can be given as

\[ \Pr \left( \psi_{\text{row}} \leq \zeta \left| x_i, H, \sigma_g^2 \right. \right) = 2^{i} \exp \left( -\frac{\zeta}{\sigma_g^2} \right) \right) \]

\[ \times \sum_{n=0}^{\psi_{\text{row}} - 1} \left( \frac{i}{2^n n!} \right) F_1 \left( n + i; i; \frac{\zeta}{\sigma_g^2} \right), \]

(17)

where \( (i)_n \) represents the Pochhammer symbol and \( i F_1 \) is the Kummer hypergeometric function. Since the RSD algorithm searches for the solution inside a reduced tree-search with a size of \( \psi_{\text{row}} \times \psi_{\text{col}} \), the approximation of the expected complexity in (13) becomes

\[ \Psi_{\text{RSD}} \approx \psi_{\text{col}} + \sum_{i=1}^{\psi_{\text{row}}} 2^i \exp \left( -\frac{\zeta_{i,j}^2}{\sigma_g^2} \right) \]

\[ \times \sum_{n=0}^{\psi_{\text{row}} - 1} \left( \frac{i}{2^n n!} \right) F_1 \left( n + i; i; \frac{\zeta_{i,j}^2}{\sigma_g^2} \right), \]

(18)

where \( \Psi_{\text{RSD}} \) is the expected complexity of the RSD algorithm.

Alternatively, (16) can be numerically calculated using the Gauss–Laguerre quadrature [19]. Thus, (13) becomes
\[ \psi_{\text{RSD}} \approx \psi_{\text{col}} (\psi_{\text{row}} + 1) - \frac{1}{(\psi_{\text{row}} - 1)!} \times \sum_{j=1}^{\psi_{\text{col}}} \sum_{k=1}^{\beta} \sum_{i=1}^{\psi_{\text{row}}} w_k (z_k^{(i,j)}) (\psi_{\text{row}} - 1) Q_k \left( \frac{\sqrt{2^2 \gamma_i}}{\sigma_g}, \sqrt{2 z_k} \right), \quad (19) \]

where \( w_k \) and \( z_k \) are given values based on the order \( \beta \), which is given from (19) (Table 25.9). Note that (19) provides a close value to that in (18) with considerably lower execution time.

V. Simulation Results

In this section, the BER and ing complexity of the proposed RSD algorithm are assessed and compared with optimum algorithms in literature, such as [7], [8], and [10]. Two SM-MIMO systems are considered; 16-QAM for \( 8 \times 8 \) and \( 16 \times 16 \) SM-MIMO, respectively. As mentioned before, \( \psi_{\text{col}} \) is empirically chosen to provide the optimum BER performance (i.e., \( \Pr (\hat{x}_{\text{map}} \neq x) \approx 0 \) in (12) at \( \psi_{\text{row}} = N_r \), where \( \psi_{\text{col}} = 70 \) and 180 for the first and second SM-MIMO systems, respectively. The proposed RSD algorithm is denoted by RSD-(\( \psi_{\text{row}}, \psi_{\text{col}} \)) to show the values of \( \psi_{\text{row}} \) and \( \psi_{\text{col}} \). Monte Carlo simulations are used to obtain the results by running at least \( 10^6 \) Rayleigh flat fading channel realizations. The channel state information at the receiver is considered to be perfectly known.

A. Assessment of Expected Complexity for the RSD Algorithm

The expression in (19) is evaluated for the two considered SM-MIMO systems using \( \beta = 7 \). The expected complexity coming from (19) provides almost identical results to (18), however, with added speed. The corresponding \( w_k \) and \( z_k \) at \( \beta = 7 \) are given in (19) (Table 25.9).

Figures 2 and 3 depict the average number of visited nodes of the proposed RSD algorithm for 16-QAM with \( 8 \times 8 \) SM-MIMO and 16-QAM with \( 16 \times 16 \) SM-MIMO, respectively. By decreasing \( \psi_{\text{row}} \), the size of the tree-search decreases and the complexity decreases correspondingly, as shown in the figures. It is also notable that the RSD algorithm requires less complexity to find the solution as the SNR increases. As seen from these figures, the theoretical analysis in (19) (or in (18)) provides a tight expression for simulation results, for different values of \( \psi_{\text{row}} \). Note that (19) perfectly matches the simulation results in the higher SNR, which verifies the feasibility of the pruned radius simplification assumption mentioned in Section IV-B.

B. Comparisons with Literature Algorithms

In this subsection, the BER and complexity are compared with those of the literature algorithms (e.g., [7], [8], and [10]). The complexity comparison is assessed by calculating the complexity reduction ratio which is defined as

\[ \Psi_{\text{Reduction}}^\Omega = \frac{M_r N_r - \Psi^\Omega}{M_r N_r} = 1 - \frac{\Psi^\Omega}{M_r N_r}, \quad (20) \]

where \( \Psi_{\text{Reduction}}^\Omega \) is the complexity reduction ratio for the \( \Omega \in \{\text{RSD, SD-[7]}, \text{SD-[8]}, \text{SD-[10]}\} \) algorithm.

Figures 4 and 5 show the BER performance of the RSD algorithm compared to the optimum algorithms, for 16-QAM with \( 8 \times 8 \) SM-MIMO and 16-QAM with \( 16 \times 16 \) SM-MIMO, respectively. As shown from these figures, the RSD-(8,70) and RSD-(16,180) provide the same BER as the ML BER performance for 16-QAM with \( 8 \times 8 \) SM-MIMO and 16-QAM with \( 16 \times 16 \) SM-MIMO, respectively. It should be noted that the SD-[7] and SD-[8] algorithms provide the same BER performance as the ML and SD-[10] algorithms, and their results are omitted for the visibility of figures. Based on the reliable design of the RSD algorithm, sub-optimal BER performances can be obtained by varying the value of \( \psi_{\text{row}} \). The BER analysis in (12) is confirmed via simulation results.

Figures 6 and 7 depict the complexity reduction ratio of all algorithms for 16-QAM with \( 8 \times 8 \) SM-MIMO and 16-QAM with \( 16 \times 16 \) SM-MIMO, respectively. As seen from these figures, the RSD algorithm provides the best reduction in complexity compared to all existing algorithms. It also offers reliable decoding complexities that vary from 72\% to 92\% for 16-QAM with \( 8 \times 8 \) SM-MIMO and from 68\% to 95\% for 16-QAM with \( 16 \times 16 \) SM-MIMO. This reliability in the decoding can fit a wide range of practical application requirements.

VI. Conclusion

This paper proposes a novel reliable algorithm to decode SM transmitted messages. The BER performance and com-
BER comparison for the 16-QAM and 8 × 8 SM-MIMO.

BER comparison for the 16-QAM and 16 × 16 SM-MIMO.

Complexity of the proposed algorithm are theoretically derived. The proposed algorithm provides a significant reduction in the decoding complexity (e.g., up to 95%) compared to ML, without sacrificing the BER performance. A flexible trade-off between the BER performance and complexity is presented to demonstrate the reliability of the proposed algorithm.

Fig. 4. BER comparison for the 16-QAM and 8 × 8 SM-MIMO.

Fig. 5. BER comparison for the 16-QAM and 16 × 16 SM-MIMO.

Fig. 6. Complexity reduction comparison for the 16-QAM and 8 × 8 SM-MIMO system.

Fig. 7. Complexity reduction comparison for the 16-QAM and 16 × 16 SM-MIMO system.

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