Extending Binary Qualitative Direction Calculi with a Granular Distance Concept: Hidden Feature Attachment

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Abstract

In this paper we introduce a method for extending binary qualitative direction calculi with adjustable granularity like OPRAm or the star calculus with a granular distance concept. This method is similar to the concept of extending points with an internal reference direction to get oriented points which are the basic entities in the OPRAm calculus. Even if the spatial objects are from a geometrical point of view infinitesimal small points locally available reference measures are attached. In the case of OPRAm, a reference direction is attached. The same principle works also with local reference distances which are called elevations. The principle of attaching references features to a point is called hidden feature attachment.

1 Introduction

A qualitative representation provides mechanisms which characterize central essential properties of objects or configurations. A quantitative representation establishes a measure in relation to a unit of measurement which has to be generally available. Qualitative spatial calculi usually deal with elementary objects (e.g., positions, directions, regions) and qualitative relations between them (e.g., "adjacent", "on the left of", "included in").

The constant general availability of common measures is now self evident. However, one needs only remember the example of the history of technologies of measurement of length to see that the more local relative measures, which are qualitatively represented, (for example, "one piece of material is longer than another" versus "this thing is two meters long") can be managed by biological/epigenetic cognitive systems much more easily as absolute quantitative representations. Typically, in Qualitative Spatial Reasoning relatively coarse distinctions between configurations are made only. The two main trends in Qualitative Spatial Reasoning are topological reasoning about regions [11, 13, 2] and positional reasoning about point configurations [4, 6, 7, 12].
Applications exist in which finer qualitative acceptance areas are helpful. The possibility to use finer qualitative distinctions can be viewed as a stepwise transition to quantitative knowledge. The idea of using context dependant direction and distance intervals for the representation of spatial knowledge can be traced back to Clementini, di Felice, and Hernandez [1]. However, only special cases of reasoning were considered in their work. Here, we will propose calculi that make direct use of general purpose constraint propagation.

In this paper we introduce a method for extending binary qualitative direction calculi with adjustable granularity like $OPRA_m$ [7] or the star calculus [12] with a granular distance concept. This method is similar to the concept of extending points with an internal reference direction to get oriented points which are the basic entities in the $OPRA_m$ calculus [7]. Even if the spatial objects are from a geometrical point of view infinitesimal small points locally available reference measures are attached. In the case of $OPRA_m$, a reference direction is attached. The same principle works also with local reference distances which are called elevations. The principle of attaching references features to a point is called hidden feature attachment.

The newly proposed calculi are an extension of the $OPRA_m$ calculus called $EOPRA_m$ and an extension of the star calculus. The key principle to derive these new calculi is presented in the next section.

2 Hidden Feature Attachment with Point Calculi and Elevation of Points

There is a motivation about the use of oriented points in qualitative calculi. This motivation is necessary since qualitative calculi characterize central essential properties of objects or configurations (see section ??). A simple featureless point naturally would not have an internal reference direction. The conception which makes the internal reference direction plausible is a transition from a calculus which is based on straight line segments (dipoles) [9] with concrete length to line segments with infinitely small length. In this conceptualization the length of the objects no longer has any importance. Thus, only the direction of the objects is modeled. O-points, our term for oriented points, are specified as pair of a point and a direction on the 2D-plane [7].

This method attaches a feature which is used as a local reference to an object on the 2D-plane which is geometrically still a featureless point. We call this principle hidden feature attachment. This principle can be extended to other modalities than directions.

2.1 Qualitative o-point relations

In a coarse representation a single o-point induces the sectors depicted in Fig. ??.

In $OPRA_2$, for the general case where the two points have different positions, we use the following relation symbols (the abbreviations lf, lb, rb, rf stand for “left-front,” “left-back,” “right-back,” and “right-front,” respectively):

\[
\text{front lf, left lb, back rb, right rf, front lf, front lb, front rf, front lb, front rf, front lf, ... rf}
\]
Here, a qualitative spatial relative direction relation between two o-points is represented by two pieces of information:

- the sector (seen from the first o-point) in which the second o-point lies (this determines the lower part of the relation symbol), and
- the sector (seen from the second o-point) in which the first o-point lies (this determines the upper part of the relation symbol).

Altogether we obtain $8 \times 8$ base relations for the two points having different positions.

Then the configuration shown in Fig. 1 is expressed by the relation $A \, \text{if} \, B$. If both points share the same position, the lower relation symbol part is the word “same” and the upper part denotes the direction of the second o-point with respect to the first one, as shown in Fig. 2.

![Figure 1: Qualitative spatial relation between two oriented points at different positions. The qualitative spatial relation depicted here is $A \, \text{if} \, B$.](image1)

Altogether we obtain 72 different atomic relations (eight times eight general relations plus eight with the o-points at the same position). These relations are jointly exhaustive and pairwise disjoint (JEPD). The relation $\text{front}_\text{same}$ is the identity relation.

![Figure 2: Qualitative spatial relation between two oriented points located at the same position. The qualitative spatial relation depicted here is $A \, \text{vl}_{\text{same}} \, B$.](image2)

### 2.2 Elevated o-point relations

An extension of binary direction calculi with a local reference direction can also be motivated with the following conception. In this conception local basic perceptions of cognitive agent are the basis for the agent’s ability to distinguish relative distances. In this conception the point which represents the location is "elevated" above the 2D-plane (see Fig. 3). From this observer perspective Gibson’s insights about natural perspective [5] motivate the availability of depth clues which let the observer distinguish local distances based on it’s height as reference distance. In the case of the $\text{OPRA}_{\text{eo}}$ calculus in which the entities are oriented points called o-points, the elevated entities are called elevated o-points, or eo-points.
Figure 3: Qualitative spatial distance relation between two oriented points via elevation principle.

The granularity depicted in Fig. 3 corresponds to a granularity of $m = 2$ (for the concept of scalable granularity compare [10]). In this granularity we have the following binary distance relations for points which have different locations:

\[
\text{distant, equal, close } \quad \text{distant, equal, close } \quad \text{distant, equal, close } \quad \text{distant, equal, close }
\]

The distance relations can be combined with the binary relative direction relations of the OPRAM\(_m\) calculus. The relation depicted in Fig. 3 has then the following symbol:

\[A \text{ rf distant } B\]

To formally specify the eo-point relations we use two-dimensional continuous space, in particular $\mathbb{R}^2$. Every eo-point $S$ on the plane is an ordered triple of a point $p_S$ represented by its Cartesian coordinates $x$ and $y$, with $x, y \in \mathbb{R}$, and a direction $\phi_S$, and an internal reference distance $\delta_S$. The internal reference distance $\delta_S$ is the property which corresponds to the elevation height in the cognitive motivation.

\[S = (p_s, \phi_s, \delta_s), \quad p_s = ((p_s)_x, (p_s)_y)\]

The metrical distance between eo-points $A$ and $B$ is their euclidean distance:

\[|A - B| = \sqrt{((p_A)_x - (p_B)_x)^2 + ((p_A)_y - (p_B)_y)^2}\]

Then the nine base relations of the binary distance relation in configurations with $p_A \neq p_B$ are defined in the following way:

\[
\begin{align*}
\delta_A < |A - B| > \delta_B & \iff A \text{ distant } B \\
\delta_A < |A - B| = \delta_B & \iff A \text{ equal } B \\
\delta_A < |A - B| < \delta_B & \iff A \text{ close } B \\
\delta_A = |A - B| > \delta_B & \iff A \text{ distant } equal B \\
\delta_A = |A - B| = \delta_B & \iff A \text{ equal } equal B \\
\delta_A = |A - B| < \delta_B & \iff A \text{ close } equal B \\
\delta_A > |A - B| > \delta_B & \iff A \text{ distant } close B \\
\delta_A > |A - B| = \delta_B & \iff A \text{ equal } close B \\
\delta_A > |A - B| < \delta_B & \iff A \text{ close } close B
\end{align*}
\]
In OPRAm we have adjustable granularity with granularity parameter $m$ \cite{10}. Also the distance modality can have an adjustable granularity. The corresponding schema is demonstrated with granularity $m = 4$ on Fig. 4.

![Figure 4: Qualitative spatial distances with granularity $m = 4$ (e.g. observation angles in $22.5^\circ$ steps).](image)

We distinguish the relative distances and relative directions of the two eo-points $A$ and $B$ expressed by a calculus $EOPRA_m$ according to the following scheme. The distance relation symbols $k, l$ are attached to the corresponding direction relation symbols $i, j$:

$$A_m \angle_{ijl} B$$

There is also a schema for relative distances established by eo-point pairs. $A_B$ is an eo-point with $\delta_A = |A - B|$. The composition of relations follows the standard schema. As future work the $EOPRA_m$ composition is will be computed with condensed semantics, an enumeration of a small fragment of the domain (this is a broader concept than the one used in \cite{8}). In the case of $EOPRA_m$, a list of qualitative triangles is generated. This is a similar approach like the approach for $OPRA_m$ composition described in \cite{7}.

The star calculus can also be augmented with local distances. Then the same binary distance relations are combined with the absolute direction relations of the star calculus.

### 3 Conclusion

We presented a first draft of binary position calculi which combine direction and distance modalities. The method which we used is similar to the concept of extending points with an internal reference direction to get oriented points which are the basic entities in the $OPRA_m$ calculus. Local reference distances which are called elevations are attached to the basic entities of the calculi. The principle of attaching references features to a point is called hidden feature attachment.
References

[1] E. Clementini, P. Di Felice, and D. Hernandez. Qualitative Representation of Positional Information. *Artificial Intelligence*, 95:317–356, 1997.

[2] M. Egenhofer and R. Franzosa. Point-Set Topological Spatial Relations. *International Journal of Geographical Information Systems*, 5(2):161–174, 1991.

[3] A. Frank. Qualitative Spatial Reasoning with Cardinal Directions. In H. Kaindl, editor, *Proc. of 7th Österreichische Artificial-Intelligence-Tagung*, pages 157–167. Springer, 1991.

[4] C. Freksa. Using orientation information for qualitative spatial reasoning. In A. U. Frank, I. Campari, and U. Formentini, editors, *Theories and methods of spatio-temporal reasoning in geographic space*, volume 639 of *Lecture Notes in Comput. Sci.*, pages 162–178. Springer, 1992.

[5] J.J. Gibson. *The Ecological Approach to Visual Perception*. Boston: Houghton Mifflin, 1979.

[6] G. Ligozat. Reasoning about cardinal directions. *Journal of Visual Languages and Computing*, 9:23–44, 1998.

[7] Reinhard Moratz. Representing relative direction as a binary relation of oriented points. In Gerhard Brewka, Silvia Coradeschi, Anna Perini, and Paolo Traverso, editors, *ECAI*, volume 141 of *Frontiers in Artificial Intelligence and Applications*, pages 407–411. IOS Press, 2006.

[8] Reinhard Moratz, Dominik Lücke, and Till Mossakowski. Oriented straight line segment algebra: Qualitative spatial reasoning about oriented objects. *CoRR*, abs/0912.5533, 2009.

[9] Reinhard Moratz, Jochen Renz, and Diedrich Wolter. Qualitative spatial reasoning about line segments. In *Proceedings of ECAI 2000*, pages 234–238, 2000.

[10] Till Mossakowski and Reinhard Moratz. Qualitative reasoning about relative direction on adjustable levels of granularity. *CoRR*, abs/1011.0098, 2010.

[11] David A. Randell, Zhan Cui, and Anthony Cohn. A spatial logic based on regions and connection. In Bernhard Nebel, Charles Rich, and William Swartout, editors, *KR’92. Principles of Knowledge Representation and Reasoning: Proceedings of the Third International Conference*, pages 165–176. Morgan Kaufmann, San Mateo, California, 1992.

[12] J. Renz and D. Mitra. Qualitative direction calculi with arbitrary granularity. In *Proceedings PRICAI 2004 (LNAI 3157)*, September 2004.

[13] Jochen Renz and Bernhard Nebel. On the complexity of qualitative spatial reasoning: A maximal tractable fragment of the region connection calculus. *Artificial Intelligence*, 108(1-2):69–123, 1999.

[14] A. Scivos and B. Nebel. The finest of its class: The natural point-based ternary calculus for qualitative spatial reasoning. In C. Freksa, M. Knauff, B. Krieg Brückner, B. Nebel, and T.Barkowski, editors, *Spatial Cognition*, volume 3343 of *Lecture Notes in Comput. Sci.*, pages 283–303. Springer, 2004.