The influence of spin-flip scattering on the preparation and detection of a single spin state in a quantum dot attached to a spin battery

Piotr Trocha\textsuperscript{1,}\textsuperscript{1}

\textsuperscript{1}Department of Physics, Adam Mickiewicz University, 61-614 Poznań, Poland

(Dated: January 20, 2013)

Recently, the possibility of an all electrical scheme of preparation and readout for a single spin state in a single quantum dot attached to spin biased leads has been shown [F. Chi \textit{et al} Phys. Rev. B 81, 075310 (2010)]. However, spin scattering mechanisms have been omitted. To remedy this lack we consider the influence of the spin-flip scattering process on the proposed preparation and readout scheme.

PACS numbers: 72.25.-b, 73.23.-b, 73.63.Kv, 85.35.Be

I. INTRODUCTION

Coherent manipulation of a single spin state in quantum dots (QDs) is crucial for spintronics and quantum information physics. A qubit, which is the basic requirement for quantum information processing, can be realized as an electron’s spin on a single-electron quantum dot. Quantum dots seem to be promising elements for spin-based quantum computer as their properties are rather easily and electrically controllable. Preparation and manipulation of a single spin state in a QD has been realized by various optical and magnetic means\textsuperscript{2–5} or spin to charge conversion\textsuperscript{6,7} techniques. However, to our knowledge, there is no experiment allowing all-electrical control of a single spin state in a QD (although such an attempt should reduce the complexity of the experiment).

Recently, Chi \textit{et al} have proposed a theoretical scheme for the preparation and readout of a single spin state in a QD utilizing spin battery realized in recent experiment\textsuperscript{8,9}. Manipulation and detection of a spin state have been also investigated in a double quantum dot system coupled to the spin\textsuperscript{10} or charge\textsuperscript{11,12} biased leads. However, spin scattering mechanisms in a single QD have not been taken into account. Intradot spin-flip processes can dramatically change a dot’s transport characteristics and lead to suppression of the tunnel magnetoresistance\textsuperscript{14,15}. At low temperature the two most important mechanisms in the QD responsible for spin-flip phenomena are spin-orbit interactions (SOI)\textsuperscript{25} and the hyperfine interaction (HI)\textsuperscript{12} which couples the electrons spin to an effective magnetic field induced by nuclear spins. However, for $B \gg B_{\text{nuclear}} \approx 3mT$ the HI mechanism is suppressed\textsuperscript{12} and the spin relaxation is then mainly due to spin-orbit interaction. Many low temperature experiments have been realized for measuring the spin relaxation time in semiconductor quantum dot\textsuperscript{16–18}. Although all the ingredients of the QD’s spin-based quantum computer are available, it is still a great challenge to build such a device, accessible for commercial applications.

The electron spin state can relax due to thermal decay, which prevents spin-based quantum computation at room temperature. Another dot’s spin-flip mechanisms can be induced by higher order tunneling events. Namely, when the dot is in the Coulomb blockade regime, cotunneling processes may flip the spin on a dot\textsuperscript{22}. However, incoherent processes, like inelastic transitions and co-tunneling, provide rather a minor contribution to the spin-flip mechanisms\textsuperscript{2}. It is also worth mentioning that in the case of strong coupling to the leads at low temperature significant impact on the spin scattering mechanism can originate from Kondo resonance.

In this paper we analyze the influence of the intradot spin-flip mechanism on the control of a single spin state in a quantum dot attached to a spin battery. A spin battery\textsuperscript{9,10,23} allows to inject pure spin current into the dot\textsuperscript{23}.

The paper is organized as follows. In section II we describe the model and theoretical formalism. Numerical results are presented and discussed in section III. A summary and final conclusions are gathered in section IV.

II. MODEL AND THEORETICAL FORMALISM

We consider a single-level quantum dot coupled to one spin battery and one normal lead. Then the Hamiltonian of the system is of the form:

\begin{equation}
\hat{H} = \sum_{\mathbf{k} \sigma} \varepsilon_{\mathbf{k} \sigma} \hat{c}^\dagger_{\mathbf{k} \sigma} \hat{c}_{\mathbf{k} \sigma} + \sum_{\sigma} \epsilon_{\sigma} \hat{q}^\dagger_{\sigma} \hat{q}_{\sigma} + U n_\sigma n_{\bar{\sigma}} + R (q^\dagger_\uparrow q^\dagger_\downarrow + q^\dagger_\downarrow q^\dagger_\uparrow) + \sum_{\mathbf{k} \sigma} (\hbar \nu_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k} \sigma} \hat{c}_{\mathbf{k} \sigma} + \text{h.c.}).
\end{equation}

The first term describes the left ($\alpha = L$) and right ($\alpha = R$) lead in the non-interacting quasi-particle approximation. Here, $\hat{c}^\dagger_{\mathbf{k} \sigma}$ ($c_{\mathbf{k} \sigma}$) is the creation (annihilation) operator of an electron with the wavevector $\mathbf{k}$ and spin $\sigma$ in the lead $\alpha$, whereas $\varepsilon_{\mathbf{k} \sigma}$ denotes the corresponding single-particle energy. The next three terms in the Hamiltonian \textsuperscript{1} describe the quantum dot. Here, $n_\sigma = \hat{q}^\dagger_\sigma \hat{q}_{\sigma}$ is the particle number operator ($\sigma = \uparrow, \downarrow$), $\epsilon_{\sigma}$ is the discrete energy level of the QD and $U$ is the intradot Coulomb integral. The parameter $R$ in the Hamiltonian \textsuperscript{1} describes the spin-flip transition amplitude. The
spin-flip term is assumed to be coherent, in the sense that the spin-flip strength \( R \) involves reversible transitions between up- and down-spin states on the dot. Such an effect may originate from a spin-orbit coupling in the Dirac distribution function in the leads and dot, where \( V^\alpha_\sigma \) are the relevant tunneling matrix elements. Coupling of the dots to external leads can be parametrized in terms of \( \Gamma^\alpha_\sigma(\epsilon) = 2\pi |V^\alpha_\sigma|^2 \rho_\alpha \), where \( \rho_\alpha \) is the density of states in the \( \alpha \) lead. In the wide band approximation \( \Gamma^\alpha_\sigma \) is constant within the electron band, \( \Gamma^\alpha_\sigma(\epsilon) = \Gamma^\alpha_\sigma = \text{const} \) for \( \epsilon \in (-W/2, W/2) \), and \( \Gamma^\alpha_\sigma(\epsilon) = 0 \) otherwise. Here, \( W \) denotes the electron bandwidth.

To find equations governing the dynamics of the system in the weak coupling regime we follow Ref.\(^27\) except as regards the derivation of the dot’s Green’s functions. In contrast to Ref.\(^27\) we do not restrict our model to weak spin-flip strengths by calculating the dot’s Green’s functions assuming vanishing \( R \). Here, the dot’s Green’s functions are obtained for finite \( R \). Details of the method are displayed in the Appendix. For the sake of simplicity we assume spin degenerate dot energy level (\( \epsilon_\uparrow = \epsilon_\downarrow \equiv \epsilon_0 \)). The rate equations for dot’s expectations values of the density matrix elements acquire the following form:

\[
\rho_{00} = \frac{1}{2} \sum_\sigma \{ [F^-_\sigma(\epsilon_0 + R) + F^-_\sigma(\epsilon_0 - R)]\rho_{\sigma\sigma} - [F^+_\sigma(\epsilon_0 + R) + F^+_\sigma(\epsilon_0 - R)]\rho_{\bar{\sigma}\bar{\sigma}} \}
\]

\[
\rho_{\sigma\sigma} = \frac{1}{2} \{ [F^+_\sigma(\epsilon_0 + R) + F^-_\sigma(\epsilon_0 - R)]\rho_{00} - [F^-_\sigma(\epsilon_0 + R) + F^+_\sigma(\epsilon_0 - R)]\rho_{\sigma\sigma} + [F^-_\sigma(\epsilon_0 + U + R) + F^-_\sigma(\epsilon_0 + U - R)]\rho_{22} + 2iR\rho_{\sigma\bar{\sigma}} \}
\]

\[
\rho_{\bar{\sigma}\bar{\sigma}} = \frac{1}{4} \{ [F^+_\bar{\sigma}(\epsilon_0 + R) - F^+_\bar{\sigma}(\epsilon_0 - R)]\rho_{00} - [F^-_\bar{\sigma}(\epsilon_0 + R) - F^-_\bar{\sigma}(\epsilon_0 - R) - 4iR\rho_{\bar{\sigma}\bar{\sigma}} \}
\]

\[
\rho_{22} = \frac{1}{2} \sum_\sigma \{ [F^+_\sigma(\epsilon_0 + U + R) + F^-_\sigma(\epsilon_0 + U - R)]\rho_{\bar{\sigma}\bar{\sigma}} - [F^+_\sigma(\epsilon_0 + U + R) - F^-_\sigma(\epsilon_0 + U - R)]\rho_{22} \}
\]

\[
\dot{J}_\alpha = -ie\langle \dot{N}_\alpha \rangle = -i\frac{e}{\hbar} \langle [H, N_\alpha] \rangle,
\]

where \( N_\alpha \) is a occupation number operator in \( \alpha \) lead. After performing onerous calculations the current formula, in the weak coupling approximation, acquires the following form:

\[
\dot{J}_\alpha = \frac{e}{2\hbar} \sum_\sigma \Re\{ [F^+_\alpha\sigma(\epsilon_0 + R) + F^+_\alpha\sigma(\epsilon_0 - R)]\rho_{00} - [F^-_\alpha\sigma(\epsilon_0 + R) + F^-_\alpha\sigma(\epsilon_0 - R)]\rho_{\sigma\sigma} + [F^+_\alpha\sigma(\epsilon_0 + U + R) + F^-_\alpha\sigma(\epsilon_0 + U - R)]\rho_{22} - [F^-_\alpha\sigma(\epsilon_0 + U + R) + F^-_\alpha\sigma(\epsilon_0 + U - R)]\rho_{\bar{\sigma}\bar{\sigma}} \},
\]
At the beginning we calculated of the Coulomb interaction parameter other parameters of the system are expressed in units occupation numbers of each spin component (values of the density matrix elements and dot occupation purity relation allow us to obtain relevant expectation steady state master equations together with the complementarity to the charge current.

III. RESULTS

In numerical calculations we take the parameters from Ref.8. As the charging energy is the largest energy in the QD we choose it as the energy unit, i.e., $U = 1$. Then, other parameters of the system are expressed in units of the Coulomb interaction parameter $U$. We set the dot-lead coupling to be $\Gamma_L = \Gamma_R = 0.02$. Moreover, we assume that only the left lead provides spin bias, whereas the right electrode is the normal lead. Specifically, we set $\mu_{L\uparrow} = eV_s$, $\mu_{L\downarrow} = -eV_s$ and $\mu_{R\uparrow} = \mu_{R\downarrow} = \mu_R = 0$, where $V_s$ is the applied spin bias. No (charge) bias voltage is applied.

Following Ref.8 we start from analyzing the preparation stage in the stationary limit. In this case we set the left hand sides of Eqs.2 equal to zero, i.e., $\dot{\rho}_{ij}(t) = 0$. Steady state master equations together with the completeness relation allow us to obtain relevant expectation values of the density matrix elements and dot occupation numbers of each spin component ($n_\sigma$ for $\sigma = \uparrow, \downarrow$). At the beginning we calculated $n_\sigma$ in the absence of spin-flip processes, which is equivalent to making the assumption that $R = 0$. Figure 1(a) shows dot’s occupation numbers as a function of the applied spin bias voltage calculated for the dot level situated symmetrically between $\mu_R$ and $\mu_{L\uparrow}$ ($e_0 = -eV_s/2$). In such a prepared system QD tends to be occupied by an electron with spin up when a sufficiently large spin bias is applied. With increasing temperature one needs to apply greater spin bias to obtain a QD in the state with a spin-up electron with probability close to unity. This indicates the possibility of preparation of a single spin state in a quantum dot by using a spin bias which has been shown in Ref.8. Moreover, a sufficiently high spin bias is now available in experiments. However, spin-flip processes may have a significant impact on the above picture. Thus, now we consider the influence of these processes on the preparation stage. In Fig.1(b) we plot dot occupation numbers for different values of the parameter $R$ (proportional to the strength of the spin-flip processes). Generally, when $R$ is nonzero the occupation number $n_\uparrow$ ($n_\downarrow$) is reduced (enhanced). However for small $R$ and sufficiently large spin bias ($eV_s \gtrsim 0.68$) this suppression of $n_\uparrow$ (enhancement of $n_\downarrow$) can be partially recovered. Thus, if the intradot spin relaxation is weak enough we are able to prepare dot in a given state. On the other hand, when the spin-flip processes are strong enough ($R > \Gamma$) the dot can be occupied with an electron with spin up or down with almost equal probability. Then, even increase in the spin bias voltage is not able to enforce the dot’s occupation by a spin-up electron. In contrast to the weak spin-flip case, now, further increase of the spin bias leads to increase of both $n_\uparrow$ and $n_\downarrow$. Moreover, additional features in the dot’s occupation number dependence can be noticed. These features are due to splitting of the dot’s energy level induced by a sufficiently strong spin-flip strength i.e., for $R \gg \Gamma$. After the initialization stage we lower the dot’s energy level in such way as to prohibit sequential tunneling events.
tem is then in the Coulomb blockade regime, because $\epsilon_0 < \mu_L \sigma, \mu_R$ and $\epsilon_0 + U > \mu_L \sigma, \mu_R$. In order to check how long a spin state can be maintained before a flip to another state we have derived numerically the rate equations \( \text{(2)} \) and calculated the time dependence of the occupation numbers \( n_\sigma \) for given initial conditions. In the Coulomb blockade the dot is occupied by one electron. Thus, \( \rho_{00}(0) = 0 = \rho_{22}(0) \), and we can write the initial dot’s state as \( |n_\uparrow(0), n_\downarrow(0)\rangle = \{|1,0\}, |0,1\rangle \) or as a superposition of spin-up and spin-down states. When the intradot spin-flip processes are neglected the state \( |1,0\rangle \) is very stable, whereas the spin-down state \( |0,1\rangle \) evolves into the state \( |0,1\rangle \) after some time. For a typical quantum dot at very low temperature this time can be long enough for performing many qubit operations. However, the spin-flip effect may greatly shorten this time scale.

In Fig. 2 we plot the time evolution of the dot’s occupation numbers for nonzero \( R \) and for the dot’s initial state \( |0,1\rangle \). One can notice that even weak spin-flip processes have a significant impact on these quantities and hence on the spin lifetime. We have also estimated the spin lifetime for different values of the parameter \( R \) assuming \( U = 10 \text{meV} \), \( k_B T = 0.01 U \) \( (T \sim 1K) \). The results obtained are shown in Tab. I. When the spin-flip strength is
TABLE I: Calculated spin lifetime corresponding to the given value of parameter $R$

| $R$[U] | 0 | 10$^{-6}$ | 10$^{-5}$ | 10$^{-4}$ | 10$^{-3}$ | 10$^{-2}$ | 10$^{-1}$ |
|--------|---|---------|---------|---------|---------|---------|---------|
| $\tau$[s] | 0.6 | 6 · 10$^{-9}$ | 6 · 10$^{-10}$ | 6 · 10$^{-12}$ | 6 · 10$^{-13}$ | 6 · 10$^{-14}$ |

larger than $R \gtrsim 0.01$ (0.1meV for the assumed parameters) the spin lifetime is too short for qubit manipulation even using (picosecond) optical methods. The time evolution of each component of the occupation numbers is a periodic function of the time. At the times $T = n\pi R^{-1}$ ($n = 1, 2, ...$) the initial dot’s state is restored. When the dot is initially occupied by a spin-up electron the time evolution is analogous to that when the initial state is $|0, 1\rangle$ after making the transition $n_\uparrow(0, 1) \to n_\downarrow(1, 0)$ and $n_\downarrow(0, 1) \to n_\uparrow(1, 0)$. With increasing $R$ the time evolution of the occupation numbers is qualitatively the same, however, the time scale is correspondingly changed (shorter).

Finally, we examine the influence of the spin-flip scattering on the read out process. After performing manipulations on a qubit state the dot’s energy level is further lowered until $\mu_{L\uparrow} > \epsilon_0 + U > \mu_R$. Now, spin-up electrons can take part in the transport. Regardless of the spin relaxation processes, when the dot’s level is initially occupied by a spin-down electron, an electron with spin up can tunnel through the dot (onto the dot from the left lead and further to the right lead) and a finite current arises. However, when a spin-up electron resides in the QD, the spin-down electron cannot tunnel onto the dot because $\epsilon_0 + U > \mu_{L\downarrow}$, whereas the spin-up electron cannot leave the dot because $\epsilon_0$ is beneath the transport window. As a result the current is suppressed. That is why measurements of the charge current should give us information about the dot’s state.

When $R = 0$ a finite current can be observed when dot was initially occupied by a spin-down electron, whereas the current vanishes for the state $|1, 0\rangle$. After a sufficiently long time the state $|0, 1\rangle$ finally evolves into the state $|1, 0\rangle$ and the charge current is suppressed. However the above scenario ceases to hold when spin-flip scattering is included. Due to the spin-flip process, now, the initial dot’s state $|1, 0\rangle$ ($|0, 1\rangle$) can be switched to the other spin state and the current can be partially enhanced (suppressed) as is shown in Fig. This switching mechanism leads to oscillations in occupation numbers (see Fig. as well as in the current and period inversely proportional to the strength of the spin-flip processes. These oscillations are damped due to the dot-lead coupling and the current saturates to a certain value.

IV. CONCLUSIONS

In conclusion, we have studied coherent transport through a single quantum dot subjected to spin bias in the presence of intradot spin-flip processes. We have found that sufficiently large spin-flip strength can prevent preparation and readout dot’s spin state by means of the present techniques. However, recent experiments have shown that the spin lifetime can reach millisecond or second time scales$^{6,17,18}$ which is enough for performing many spin operations. That is why, along with the authors of Ref.$^3$, we believe that a single spin state in the (all electrically controllable) quantum dot is a good candidate for being a qubit.

Acknowledgements

This work was supported partly by funds from the Polish Ministry of Science and Higher Education, as a research project N N202 169536 in the years 2009-2011.

Appendix: Rate Equations

To obtain the rate equations (2) we adopt the formalism presented in Ref.$^{28}$. Specifically, we express the dot’s operator in terms of Hubbard operators:

$$q_\sigma = |0\rangle\langle \sigma | + \sigma |\sigma \rangle.$$  

represented by four possible electron states in each dot which satisfy the following completeness relations:

$$|0\rangle \langle 0| + \sum_\sigma |\sigma \rangle \langle \sigma | + |2\rangle \langle 2| = 1. \quad (6)$$

Furthermore, the set of auxiliary operators is introduced and the dot operators are expressed by means of the slave-boson and pseudfermion operators$^{29}$. From the definitions of the Dirac brackets one is able to find the commutation (and anticommutation) rules for new operators$^{28}$. In the slave-boson representation the Hamiltonian of the system$^{[31]}$ acquires the form

$$\hat{H} = \sum_{k\alpha\sigma} \epsilon_\alpha \sigma c_{k\alpha\sigma}^\dagger c_{k\alpha\sigma} + \sum_\sigma \epsilon_\sigma (f_\uparrow^\dagger f_\uparrow + d^\dagger d)$$

$$+ R (f_\uparrow^\dagger f_\downarrow + H.c.) + U d^\dagger d$$

$$+ \sum_{k\alpha} \sum_\sigma [V_{\alpha\sigma} c_{k\alpha\sigma}^\dagger (e^\dagger f_\sigma + \sigma f_\sigma^\dagger d) + H.c.]. \quad (7)$$

Here, $d^\dagger$ is the slave-boson operator which creates an empty state in the dot, $f_\sigma^\dagger$ is a pseudofermion operator which creates a singly occupied state with an electron with spin $\sigma$, whereas $d^\dagger$ creates a doubly occupied state with an electron with spin $\sigma$ and another electron with spin $\bar{\sigma}$ in the dot. In the slave-particle representation the density matrix elements are written in the following way: $\rho_{00} = e^{\dagger} e_\sigma$, $\rho_{\sigma\bar{\sigma}} = f_\uparrow^\dagger f_\downarrow + \sigma f_\sigma^\dagger d$, $\rho_{22} = d^\dagger d$. Here, the statistical expectations of the density matrix elements $\langle \rho_{nn} \rangle$ with $n = 0, \sigma, 2$ give the occupation probabilities of the quantum dot being empty, singly occupied by an electron with spin $\sigma$, and doubly occupied, respectively.
To derive the rate equations we start from the von Neumann equation for the density matrix operator:

\[ \dot{\rho} = i[H, \rho], \]

(8)

where \( \dot{\rho} = (\dot{\rho}_{00}, \dot{\rho}_{\sigma \sigma}, \dot{\rho}_{\bar{\sigma} \bar{\sigma}}, \dot{\rho}_{22})^T \). Furthermore, the averaged equations for density matrix elements are expressed by means of dot-lead Green functions. and using the Langreth theorem\(^2\), the dot-lead Green functions can be set down by means of dot’s Green functions and free leads’ Green functions.\(^{27,30}\) The Green functions of the dot, in time space, are defined in the following way \( G_{\sigma \sigma}(t, t') = \langle \langle q_\sigma(t) q_\sigma^\dagger(t') \rangle \rangle = \langle \langle e^i t f_\sigma(t) f_\sigma^\dagger(t') e(t') \rangle \rangle + \sigma' \langle \langle f_\sigma^\dagger(t)d(t) d^\dagger(t') f_\sigma(t') \rangle \rangle = G_{\sigma \sigma'}(t, t') + G_{d \sigma}(t, t'). \)

Other parts of \( G_{\sigma \sigma'}(t, t') \) vanish for \( t' = t \), and thus are omitted as we are interested in the \( t' = t \) case. Now, we are able to express the rate equations by means of the dot’s Green’s functions and these can be written in the following form:

\[
\begin{align*}
\dot{\rho}_{00} &= -\frac{i}{2\pi} \int d\omega \sum_{\alpha} \left[ \Gamma^\alpha_\sigma f^{\alpha}(\omega) G^\omega_{\sigma \sigma}(\omega) + \Gamma^\alpha_\sigma (1 - f^{\alpha}(\omega)) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) \right] \\
\dot{\rho}_{\sigma \sigma} &= \frac{i}{2\pi} \int d\omega \sum_{\alpha} \left[ \Gamma^\alpha_\sigma f^{\alpha}(\omega) G^\omega_{\sigma \sigma}(\omega) + \Gamma^\alpha_\sigma (1 - f^{\alpha}(\omega)) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) - \Gamma^\alpha_\sigma f^{\alpha}(\omega) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) - \Gamma^\alpha_\sigma (1 - f^{\alpha}(\omega)) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) \right] \\
& \quad + iR(\rho_{\sigma \sigma} - \rho_{\bar{\sigma} \bar{\sigma}}) \\
\dot{\rho}_{\bar{\sigma} \bar{\sigma}} &= \frac{i}{4\pi} \int d\omega \sum_{\alpha} \left[ (\Gamma^\alpha_\sigma + \Gamma^\alpha_{\bar{\sigma}}) f^{\alpha}(\omega) G^\omega_{\sigma \sigma}(\omega) + (1 - f^{\alpha}(\omega)) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) - f^{\alpha}(\omega) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) - (1 - f^{\alpha}(\omega)) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) \right] \\
& \quad + iR(\rho_{\sigma \sigma} - \rho_{\bar{\sigma} \bar{\sigma}}) \\
\dot{\rho}_{22} &= \frac{i}{2\pi} \int d\omega \sum_{\alpha} \left[ \Gamma^\alpha_\sigma f^{\alpha}(\omega) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) + \Gamma^\alpha_\sigma (1 - f^{\alpha}(\omega)) G^\omega_{\bar{\sigma} \bar{\sigma}}(\omega) \right] \\
& \quad \text{from the corresponding equation of motion for the dot’s operators. Thus, we obtained}
\end{align*}
\]

(9)

Now, we only need to know the Green’s function of the dot. This is derived in the weak coupling approximation not limited to small values of the spin-flip strengths (\( R \ll \Gamma \)). Moreover, the form of these Green’s functions clearly shows that spin-flip processes lead to splitting of the dot’s energy level. Finally, connecting Eqs.\(^{11}\) with Eqs.\(^{3}\) we arrive at the coupled set of differential equations\(^2\).

---

\(^*\) Electronic address: pتروcha@amu.edu.pl

\(^1\) D. Loss, and D. P. DiVincenzo, Phys. Rev. A 57 (1998)
F. H. L. Koppens, C. Buizerk, K. J. Tielrooij, I. T. Vink, K. C. Nowack, T. Meunier, L. P. Kouwenhoven, and L. M. K. Vandersypen, Nature (London) **442** (2006) 766.

D. Press, T. D. Ladd, B. Y. Zhang, and Y. Yamamoto, Nature (London) **456** (2008) 218.

X. D. Xu, Y. W. Wu, B. Sun, Q. Huang, J. Chen, D. G. Steel, A. S. Bracker, D. Gammon, C. Emary, and L. J. Sham, Phys. Rev. Lett. **99** (2005) 196802.

K. C. Nowack, F. H. L. Koppens, Yu. V. Nazarov, and L. M. K. Vandersypen, Science **318** (2007) 1430.

J. M. Elzerman, R. Hanson, L. H. Willems van Beveren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouwenhoven, Nature (London) **430** (2004) 431.

R. Hanson, L. H. van Beveren Willems, I. T. Vink, J. M. Elzerman, W. J. M. Naber, F. H. L. Koppens, L. P. Kouwenhoven, and L. M. K. Vandersypen, Phys. Rev. Lett. **94** (2005) 196802.

F. Chi and Q.-F. Sun, Phys. Rev. B **81** (2010) 075310.

S. M. Frolov, A. Venkatesan, W. Yu, and J. A. Folk, and W. Wegscheider, Phys. Rev. Lett. **102** (2009) 11602.

H. Z. Lu and S.-Q. Shen, Phys. Rev. B **77** (2008) 235309.

T. Hayashi, T. Fujisawa, H. D. Cheong, Y. H. Jeong, and Y. Hirayama, Phys. Rev. Lett. **91** (2003) 226804.

J. M. Taylor, J. R. Petta, A. C. Johnson, A. Yacoby, C. M. Marcus, and M. D. Lukin, Phys. Rev. B **76** (2007) 035315.

W. Rudziński and J. Barnaś, Phys. Rev. B **64** (2001) 085318.

W. Rudziński, J. Phys.: Condens. Matter **21** (2009) 046005.

A. V. Khaetskii and Y. V. Nazarov, Phys. Rev. B **61** (2000) 12639.

A. C. Johnson, J. R. Petta, J. M. Taylor, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Nature **35** (2000) 925.

S. Amasha, K. MacLean, I. P. Radu, D. M. Zumbühl, M. A. Kastner, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **100** (2008) 046803.

R. Hanson, L. P. Kouwenhoven, J. R. Petta, S. Tarucha, and L. M. K. Vandersypen, Rev. Mod. Phys. **79** (2007) 1217.

T. Meunier, I. T. Vink, L. H. Willems van Beveren, K.-J. Tielrooij, R. Hanson R, F. H. L. Koppens, H. P. Tranitz, W. Wegscheider, L. P. Kouwenhoven, and L. M. K. Vandersypen, Phys. Rev. Lett. **98** (2007) 126601.

D. Heiss, S. Schaeck, H. Huebl, M. Bichler, G. Abstreiter, J. J. Finley, D. V. Bulaev, and D. Loss, Phys. Rev. B **76** (2007) 241306(R).

I. Weymann and J. Barnaś, Phys. Rev. B **73** (2006) 205309.

J.E. Hirsch, Phys. Rev. Lett. **83** (1999) 1834.

D.-K. Wang, Q.-F. Sun, and H. Guo, Phys. Rev. B **69** (2004) 205312.

A. V. Khaetskii and Y. V. Nazarov, Phys. Rev. B **61** (1999) 12639(R); C. F. Destefani, S. E. Ulloa, and G. E. Marques, Phys. Rev. B **69** (2004) 12502; E. Tsitsishvili, G. S. Lozano, A. O. Gogolin, Phys. Rev. B **70** (2004) 115316.

H.-A. Engel and D. Loss, Phys. Rev. Lett. **86** (2000) 4648.

B. Dong, H. L. Cui, and X. L. Lei, Phys. Rev. B **69** (2004) 035324.

J.C. Le Guillou, E. Ragoucy, Phys. Rev. B **52**, 2403 (1995).

H. Haug, A.-P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors*, Springer Berlin Heidelberg New York, Second Edition (2008).

P. Trocha, Phys. Rev. B **82** (2010) 115320.

L. I. Glazman and K. A. Matveev, Pisma Zh. Eksp. Teor. Fiz. **48** (1998) 403; JETP Lett. **48** (1998) 445.