Abstract. Cascade Joule-Thomson Microcoolers have been proposed in literature in which different compressors with low values of pressure ratio of order four using different working fluids are anticipated to drive the microcooler. A cascade of five stages is expected to provide cooling at a load temperature of 150 K. In this study a second-law analysis of such a microcooler is performed to quantify the effect of important design parameters representing the basic components and processes of the microcooler on its performance. The effects of several important design parameters including the effectiveness of all heat exchangers as well as the effect of possible pressure drop in the recuperative heat exchanger on cooling power and the exergetic efficiency of the microcooler are obtained. The inefficiency of the compressors is included using an exergetic efficiency parameter for the compressors. The heat transfer from each stage to other stages is modelled using an effectiveness parameter for the heat exchangers that can be varied to investigate their influence on cooling power and the efficiency of the microcooler.

1. Introduction
Micro Cryogenic Coolers (MCC) based on the Joule-Thomson method are of interest because of their high volumetric cooling capacity and consequently have been the subject of several investigations and development [1]. There are two general types of J-T cryogenic refrigerators. One type is based on an open cycle where the working fluid is stored in a high pressure storage that drives the refrigerator [2]. The second type is based on a closed cycle by introducing a compressor to provide the high pressure gas in a steady state operation of the cooler. The most important issue for successful operation of a closed cycle J-T microcooler is the design of a compressor to provide the required pressure ratio for the cycle operation. In order to reduce the required pressure ratio across the compressor for operation of microcoolers, working fluids made of a mixture of different components have been developed [3 and 4]. One disadvantage of using a mixture of refrigerants for J-T microcoolers has been the observed pulsation, temperature oscillation, and instability at the cold head of the J-T microcooler [5]. Another important consideration in designing a microcooler based on J-T refrigerators is the selection of refrigerants having high specific refrigeration power. Pure refrigerants operating in cascade can provide specific refrigeration power five to ten times the specific refrigeration power of the mixed refrigerants. However, design of cascade J-T microcoolers is more complicated than the J-T
refrigerator based on the mixed refrigerants. Recent advances in the development of microvacuum pumps using MEMs points to the possibility of the development of microcompressors for JT cascade microcoolers [6]. This opens the door for the development of cascade J-T microcoolers driven by reasonably low pressure ratios across each compressor using different refrigerants. In this study we develop a thermodynamic model including the effect of heat exchangers used in the microcooler to assess their impact on the performance of the cascade J-T microcooler. It should be pointed out that one of the goals of this study is to use second law analysis to assess the variation of design parameters on overall performance of the microcooler.

2. Thermal modeling of cascade J-T microcooler

2.1 Five-stage design of cascade J-T microcooler

Two layouts and geometry of cascade J-T microcoolers have been previously discussed in the literature and the advantages of the double layer arrangement over the basic layout have been noted [7 and 8]. The double-layer arrangement results in a more compact microcooler with a lower thermal resistance and is also the model used in this study. The most important components of the microcooler for thermal modelling are shown in figure 1 for two consecutive stages n and n+1. Important thermal resistances connecting the two stages are also shown in the figure. Each stage in general consists of four heat exchangers where two recuperators are used for heat recovery and to exchange heat between the high pressure and low pressure streams. The thermal interaction between the low and high pressure streams and the isothermal heat exchanger supported by the evaporator of the lower stage is also shown in the figure. The most important heat exchange process is the interaction of the high pressure stream of each stage with the isothermal heat exchanger supported by the evaporator of the lower stage. The system of the cascade J-T microcooler must be designed such that the input to the expansion valve is in saturated or compressed liquid state. The working fluids of a five-stage J-T microcooler requiring low pressure ratios of approximately four bars to one bar, previously reported in the literature, is also used in this study [7 and 8]. This cascade of five working fluids are isobutane, propane, R116, ethylene, and R14, requiring pressure ratios of approximately four bars to one bar except for R116 that requires at least a pressure ratio of five bars to one bar. It should be pointed out that the first stage using isobutane as the working fluid has only one recuperator and the last stage of the cascade is connected to the cold reservoir. The five stage cascade J-T microcooler used in this study is designed to provide 0.5 W of cooling power from a cold reservoir at a temperature of 150 K. In the thermal model of the five-stage cascade J-T microcooler a total of nine recuperators and eight heat exchange processes between the working fluids and the four isothermal heat exchangers should be considered.

2.2 Thermal Modeling of the heat exchangers

In general, each recuperator can be modeled using the temperature and pressure of the inlet of each stream. We assume these properties are either known or can be related to the components downstream and upstream of the recuperator. The recuperators are assumed to be adiabatic with no thermal interaction at their surface. For application to this problem, where the heat capacity of the high pressure fluid is larger than the low pressure fluid, we define the effectiveness of recuperators by the following equation.

$$\varepsilon = \frac{h(T_{le}, P_{le}) - h(T_{li}, P_{li})}{[h(T_{hi}, P_{le}) - h(T_{li}, P_{li})]}$$

(1)
Where $h$ is the enthalpy of working fluids and is as a function of temperature $T$ and pressure $P$ at the inlet and exit of recuperator. Subscripts H and L denote the high and low pressure streams, and sub enthalpy $h(T_{Hi},P_{Le})$ that is evaluated at scripts i and e denote the inlet and the exit streams, respectively. It should be pointed out that the enthalpy $h(T_{Hi},P_{Le})$ in Equation (1) is evaluated at the temperature $T_{Hi}$ and the pressure $P_{Le}$. Conservation of energy for the recuperators, assuming no heat loss or gain at the surfaces, can be written as

$$h(T_{Hi},P_{Hi}) - h(T_{Hi},P_{He}) = h(T_{Le},P_{Le}) - h(T_{Li},P_{Li})$$

(2)

For the isothermal heat exchangers it is assumed that the high pressure and low pressure streams at each stage interact with a constant temperature reservoir supported by the evaporator of the lower stage as given in figure 1. The exit temperature of the high pressure stream can be obtained from the following equation assuming infinite thermal capacity for the isothermal heat exchanger.

$$\varepsilon_{iso,H} = (T_{Hi} - T_{He})/(T_{Hi} - T_{iso})$$

(3)

Where $T_{iso}$ is the temperature of evaporator of the lower stage in the cascade. It should be pointed out that for the high pressure stream, high heat exchanger effectiveness close to one might be required for some working fluids so that the cascade J-T microcooler operates properly. The low pressure stream can be used to recover the enthalpy of the stream by passing it through the isothermal heat exchanger. The energy recovered is not expected to be significant. The exit temperature of the low pressure stream can be obtained from the following equation.

$$\varepsilon_{iso,L} = (T_{Le} - T_{Li})/(T_{iso} - T_{Li})$$

(4)

A value of $\varepsilon_{iso,L} = 0$ indicates that $T_{Le} = T_{Li}$ and no recovery of the enthalpy of low pressure fluid is implemented.
2.3 Second Law Analysis
When the inlet and exit temperature, pressure and mass flow rates of all streams for the cascade J-T microcooler are evaluated, the exergy flow rate for each stream can be calculated. The exergy balance for a control volume with one channel of heat transfer and several channels of mass transfer for steady state condition can be written as

\[ \dot{I} = \sum_i \dot{m}_i e_i - \sum_e \dot{m}_e e_e - W + (1 - \frac{T_0}{T}) \dot{Q} \]  

(5)

Where \( \dot{m} \) is the mass flow rate, \( e \) is the flow exergy per unit mass of the working fluid, \( T \) is appropriate temperature of the reservoir for the heat transfer interaction and the subscripts \( i \) and \( e \) indicate the inlet and exit, respectively. The flow exergy per unit mass is defined by

\[ e = (h - h_o) - T_o(s - s_o) \]  

(6)

Where \( h \) is the enthalpy, \( s \) is the entropy of the stream per unit mass and the subscript \( o \) denotes the environment. The enthalpy and entropy per unit mass are a function of temperature and pressure and must be calculated for each stream. The rate of irreversibility of each recuperator can be written as

\[ \dot{I}_{\text{rec}} = \dot{m}\left( [h(T_{Hi}, P_{Hi}) - h(T_{He}, P_{He})] + [h(T_{Li}, P_{Li}) - h(T_{Le}, P_{Le})] \right) - \dot{m}T_o \left( [s(T_{Hi}, P_{Hi}) - s(T_{He}, P_{He})] + [s(T_{Li}, P_{Li}) - s(T_{Le}, P_{Le})] \right) \]  

(7)

The thermodynamic quantities are calculated using EES® software [9]. Similar equations for the rate of irreversibility of the isothermal heat exchangers can be derived. For each isothermal heat exchanger there are four streams interacting with the heat exchanger as shown in FIGURE 1. The irreversibility of the expansion valve can be easily calculated using Equation (5). The quality at the exit of expansion valve is obtained using the isenthalpic process for the expansion valve. Since the process in the expansion valve is assumed to be isenthalpic, the rate of irreversibility for each expansion valve, can be written as

\[ \dot{I}_{\text{EV}} = \dot{m}T_o[s(T_{Hi}, P_{Hi}) - s(x_{Le}, T_{Le})] \]  

(8)

where \( x_{Le} \) is the quality at the exit of the expansion valve. One of the most important challenges in the design of cascade J-T microcooler for the operation with a low pressure ratio is to select the working fluids such that the exit of the valve at each stage is saturated liquid or slightly compressed liquid. This is challenging especially when pressure drop in heat exchangers is taken into account.

The most important source of irreversibility for each stage of a cascade J-T microcooler is expected to be the irreversibility of the compressor. In this study we define the efficiency of each compressor by its exergetic efficiency defined by

\[ \eta_{\text{comp}} = (e_e - e_i)/w_{\text{comp}} \]  

(9)

Where \( w_{\text{comp}} \) is the work input to the compressor per unit mass flowing through the compressor and \( e_e - e_i \) is the exergy change of the working fluid per unit mass across the compressor. It can be shown that the exergetic efficiency of the compressor defined by Equation (9) is close to the isothermal efficiency usually defined for the compressor. For the isothermal compressor operating at the temperature of the environment the exergy transfer to the environment is zero. The thermal exergy of the input stream to the compressor is small and it is lost in the compression process. It should be pointed out that the irreversibility for the heat exchanger at the cold end is taken into account by
assuming an appropriate thermal conductance between the refrigerator and the cold reservoir. It is important to recognize the irreversibility when an exergy balance for the second law analysis of the system is under taken.

3. Results and Discussion
TABLE 1 gives the summary of selected input parameters and the results of calculations for the important quantities of interest in the thermal design of a five stage cascade J-T microcooler. The cooling load is set at 0.5 watts and the cold reservoir temperature is set at a temperature of 150 K. TABLE 1 shows the results of calculations for five values of effectiveness of the recuperators ranging from 0.75 to 0.95 for all recuperators except the high temperature recuperator for the second stage (propane) where a fixed value of effectiveness equal to 0.8 is used in the calculations. This value of effectiveness for the high temperature recuperator of the second stage is necessary for proper operation of the second stage so that condensation of the working fluid occurs in the isothermal heat exchanger. The effectiveness of the isothermal heat exchangers at the high pressure side for all stages is set at 0.99. Such high values of the effectiveness at the high pressure side of the isothermal heat exchangers are necessary so that the working fluids at the inlet to expansion valves for all stages are either at a saturated liquid or compressed liquid state. It should be pointed out that the J-T refrigerators at each stage are designed to produce saturated liquid or compressed liquid states at the inlet of the expansion valve at each stage. The pressure of the working fluid at the exit of the compressor has to be increased if this condition is not satisfied. The inlet pressure of R116 refrigerant at stage 3 of the cascade is selected to be 5 bars [7]. In addition, the pressure at the exit of the compressor for R14 refrigerant is set to 4.8 bars while the pressure at the inlet of the compressor is taken to be 1.2 bars resulting in the pressure ratio of 4 across the compressor. These values are chosen so that the effect of pressure drop on the performance of stage five can be investigated and will be discussed later. The heat exchanger effectiveness of the isothermal heat exchangers at the low pressure side is set to a value of 0.9 for all heat exchangers. A high value of effectiveness at the low pressure side of isothermal heat exchanger is not required for proper operation of the refrigerator. No pressure drop in the heat exchangers is taken into account in the results shown in table 1. The thermodynamics software EES® is used to solve the equations generated for the five stage cascade J-T microcooler for quantities important in the design of the microcooler. These quantities include the cooling capacity for each stage in Watts (W),

| TABLE 1. Effect of the effectiveness of recuperators on five stage cascade J-T microcooler |
|---------------------------------------------|---------------------------------------------|
| Quantity (units)                           | Effectiveness of recuperators for each stage in the cascade of five stages |
|                                            | $\varepsilon=0.75$ | $\varepsilon=0.8$ | $\varepsilon=0.85$ | $\varepsilon=0.9$ | $\varepsilon=0.95$ |
| Cooling load; stage 1 (W)                  | 0.7004             | 0.6498             | 0.6030             | 0.5595             | 0.5192             |
| Cooling load; stage 2 (W)                  | 0.6920             | 0.6424             | 0.5965             | 0.5538             | 0.5142             |
| Cooling load; stage 3 (W)                  | 0.6239             | 0.5949             | 0.5670             | 0.5401             | 0.5143             |
| Cooling load; stage 4 (W)                  | 0.5806             | 0.5629             | 0.5454             | 0.5282             | 0.5113             |
| Cooling load; stage 5 (W)                  | 0.5000             | 0.5000             | 0.5000             | 0.5000             | 0.5000             |
| Mass flow rate; stage 1 (mg/s)             | 2.173              | 1.997              | 1.835              | 1.686              | 1.550              |
| Mass flow rate; stage 2 (mg/s)             | 1.778              | 1.641              | 1.515              | 1.399              | 1.291              |
| Mass flow rate; stage 3 (mg/s)             | 6.350              | 5.983              | 5.635              | 5.306              | 4.994              |
| Mass flow rate; stage 4 (mg/s)             | 1.307              | 1.262              | 1.218              | 1.175              | 1.133              |
| Mass flow rate; stage 5 (mg/s)             | 4.193              | 4.172              | 4.151              | 4.130              | 4.110              |
| Total input power; all stages with $\eta_{comp}=1$ (W) | 0.7360             | 0.711              | 0.6787             | 0.6484             | 0.6194             |
| Second law efficiency; all stages with $\eta_{comp}=1$ | 0.6793             | 0.7042             | 0.7367             | 0.7740             | 0.8073             |
the mass flow rate of the working fluid for each stage in milligrams per second (mg/s). Table 1 includes total input power of the compressors required for the operation of the cascade and its second law efficiency assuming no loss in the compressors.

3.1 Second Law Analysis of Stage Five for R-14 Working Fluid

Fluid flow and heat transfer processes in a heat exchanger influence the irreversibility of the heat exchanger. Usually a compromise exists between the two processes resulting in an optimum irreversibility for the heat exchanger. In this study we analyze the variation of effectiveness for the two recuperators and evaluate their influence on the performance of stage five of the cascade J-T microcooler. We include 1% pressure drop in high pressure stream and 2% pressure drop in the low pressure stream passing through the recuperators. The high pressure and low pressure streams are fixed at 4.8 bars and 1.2 bars at the exit and inlet of the compressor, respectively. This results in a pressure ratio of four across the compressor. The effectiveness of the isothermal heat exchangers at high and low pressure sides are fixed at 0.99 and 0.9 respectively. The working fluid for stage five is R14 and is cooled at the isothermal heat exchanger by the evaporator of stage 4 using ethylene as the working fluid at temperature of 169.1 K. Second law analysis of stage five quantifies the irreversibility distribution of components in the stage and the thermal exergy support that must be provided by stage four for its successful operation. It should be pointed out that each stage of the cascade is supported by its own compressor and the thermal exergy input from the preceding stage is as shown in figure 2.

![Figure 2](image)

**FIGURE 2** The effect of recuperator effectiveness on the irreversibility, exergy flow and the second law efficiency for the fifth state of the cascade J-T microcooler.

The results of the calculations for the variation of the effectiveness of recuperators in the range of 0.75 to 0.95 on the irreversibility of each components as well as the total irreversibility, are given on the left axis of figure 2. The results of calculations for the important exergy quantities, and the exergy efficiency values are given on the right axis of the figure. The total irreversibility of the components is reduced from approximately 0.12 watts to 0.06 watts when the effectiveness of recuperators is increased from 0.7 to 0.95. The most important component of irreversibility is due to the high
temperature recuperator followed by the isothermal heat exchanger. Irreversibilities of the expansion valve and the low temperature recuperator are not significant. The exergy input from stage four to stage five at the isothermal heat exchanger is significant and is reduced as effectiveness of the recuperators are increased. The second law efficiency of stage five is high and increases as the effectiveness of recuperators is increased. It is expected that the exergy efficiency of the compressors would be about ten percent reducing the efficiency of cascade J-T microcooler substantially.

4. Conclusions
Cascade J-T microcoolers have potential to provide specific cooling capacity an order of magnitude greater than conventional mechanical coolers. The effect of variation of the effectiveness of nine recuperators on the performance of a five stage cascade J-T microcooler was analyzed in this study. It was found that even though the recuperator losses are important, their effect on the efficiency of the microcooler is not very significant. The most important parameter influencing the second law efficiency of the microcooler is the exergetic efficiency of the compressor. More detailed second law analysis of the fifth stage of the microcooler using R-14 working fluid including the pressure drop in the recuperator was analyzed. It was shown that not only the recuperator losses are important for the performance of the stage but they also influence the exergy input required from the stages upstream necessary for the successful operation of the fifth stage. A figure of merit based on the second law was defined for the performance evaluation of each stage of cascade J-T microcoolers and the effect of recuperator losses on the figure of merit was evaluated. Assuming a second law efficiency of 0.1 for the compressors at each stage and including the internal irreversibility of the cold head, the second law efficiency of the cascade J-T microcooler is estimated to be 10 percent.

5. References
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