Exact Solutions in SFT and Marginal Deformation in BCFT

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ABSTRACT: In this note we will study solution of open bosonic string field theory based on action of operators from chiral algebra of boundary conformal field theory on identity element of string field theory star algebra. We will demonstrate that the string field theory action for fluctuation fields around this classical solution can be mapped to the string field theory action defined through the new boundary conformal field theory that arises from the original one through the marginal deformation inserted on the world-sheet boundary.

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1. Introduction

Much work has focused on the search for classical solutions of cubic bosonic open string field theory (SFT) \[1\] (For review, see \[2, 3, 4, 5\]). Despite important technical progress in the understanding of the open string star product—notably the discovery of a new connections with non-commutative field theory \[6, 7, 8, 9, 10, 11, 12, 13, 14, 15\]—it is still very difficult to find analytic classical solutions. Exception is the special form of SFT, vacuum string field theory (VSFT), where exact results have been obtained \[16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\].

In our recent papers \[27, 28, 29, 30, 31\] we found some exact solutions of SFT equation of motion \[2\]. All these solutions have the property that when we expand the string field around this solution and insert it into the SFT action then the new BRST operator \(Q'\) suggests that SFT action for fluctuation modes is in some sense related to the SFT action defined around the new background boundary conformal field theory (\(BCFT''\)) that arises from the original \(BCFT\) by presence of marginal deformation on the world-sheet boundary. The possible relation between these two actions will be indication of the background independence of SFT \[51, 52, 53, 54, 55\] which unfortunately is not explicitly seen in its formulation. Let us say more about this issue.

This issue arises because SFT can be written after choosing what amounts to a classical solution, namely \(BCFT\) with central charge \(c = 0\). As a result the SFT actions \(S_1, S_2\) written using two different \(BCFT's\) \(BCFT_1\) and \(BCFT_2\) are not manifestly equivalent. However it was shown in \[51, 52, 53, 54, 55\] that for the case when \(BCFT_1\) and \(BCFT_2\) are nearby theories related by infinitesimal marginal

\[\footnote{For closely related papers, see \[34, 35, 36, 37\]. For other papers considering exact solutions in SFT, see \[18, 38, 39, 40\].} \]

\[\]
deformation, one can prove that $S_1$ and $S_2$ are the same action expanded around different solutions.

In this paper we will try to study this problem from slightly different point of view. In particular, we focus on the relation between solution of SFT equation of motion given in [27, 28, 29, 30, 31] and the marginal deformation in $BCFT$. Our goal is to show that when we expand string field around classical solution and insert it into the original SFT action $S$ which is defined on background $BCFT$, we obtain after suitable redefinition of the fluctuation modes the SFT action $S'$ defined on $BCFT''$ that is related to the original $BCFT$ by inserting marginal deformation on the boundary of the world-sheet (For more details about marginal deformations in $BCFT$, see [32]). To say differently, we will show that two SFT actions $S, S'$ written using two different $BCFT, BCFT''$ which are related by marginal deformation, are in fact two SFT actions expanded around different classical solutions.

In order to show this equivalence will consider operators, that determine exact solution of SFT, which belong to the chiral algebra $\mathcal{W}$ of $BCFT$ \(^3\). More precisely, we will construct the solution of SFT equation of motion that is based on an action of some operator $\mathcal{W}$ from $\mathcal{W}$ on the SFT star algebra identity element $I$. Then we will study the fluctuation modes around this solution. It turns out that after performing suitable redefinition of the fluctuation modes we will get correlation functions \(^4\) which have the same form as the correlation functions in the deformed $BCFT''$ that arises from the original one by insertion of marginal interaction $\mathcal{W}(x)$ on the world-sheet boundary. In order to be able to perform such a identification we took the operators $\mathcal{W}$ from $\mathcal{W}$ since only in that case we have well defined deformed correlation functions in $BCFT$ containing bulk and boundary operators as well as operator $\mathcal{W}(x)$ \([32]\). In other words we will show that the SFT action $S'$ for fluctuation modes is the same as the SFT action defined by the second $BCFT''$. This result can be considered as more precise extension of the analysis given in [27] in the sense that we explicitly show the equivalence of these two actions. We mean that this result could be considered as further indication of the background independence in SFT \([51, 52, 53, 54, 55]\).

The organization of this paper is as follows. In the next section (2) we review the basic facts about SFT, bulk an boundary CFT. Then we turn to the construction of the classical solution of SFT equation of motion following [27, 28, 29, 30, 31]. In section (3) we will study fluctuation modes above the classical solution and we will find the precise relation between the SFT $S'$ for fluctuation modes and SFT defined on $BCFT''$. In conclusion (4) we will outline our results.

2. General solutions

We begin this section with the review the basic facts about bosonic string field theory, \(^3\)For very nice review to the subject of $BCFT$ see for example [58, 59] and reference therein. \(^4\)These $BCFT$ correlation functions define SFT action $S$ in the CFT description [18, 19].
following mainly [4,3,1]. Gauge invariant string field theory is described with the full Hilbert space of the first quantized open string including b, c ghost fields subject to the condition that the states must carry ghost number one, where b has ghost number $-1$, c has ghost number 1 and $SL(2,C)$ invariant vacuum $|0\rangle$ carries ghost number 0. We denote $\mathcal{H}$ the subspace of the full Hilbert space carrying ghost number 1. Any state in $\mathcal{H}$ will be denoted as $|\Phi\rangle$ and corresponding vertex operator $\Phi$ is the vertex operator that creates state $|\Phi\rangle$ out of the vacuum state $|0\rangle$

$$|\Phi\rangle = \Phi |0\rangle . \quad (2.1)$$

Since we are dealing with open string theory, the vertex operators should be put on the boundary of the world-sheet $^5$. The string field theory action is given [1]

$$S = -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \langle I \circ \Phi(0) Q \Phi(0) \rangle + \frac{1}{3} \langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle \right) , \quad (2.2)$$

where $g_0$ is open string coupling constant, $Q$ is BRST operator and $<>$ denotes correlation function in the combined matter ghost conformal field theory in the upper half plane $\text{Im} z \geq 0$. $I, f_1, f_2, f_3$ are conformal mapping exact form of which is reviewed in [4] and $f_i \circ \Phi(0)$ denotes the conformal transformation of $\Phi(0)$ by $f_i$. For example, for $\Phi$ a primary field of dimension $h$, then $f_i \circ \Phi(0) = (f'(i)(0))^{h \Phi(f_i(0))}$. In our calculation we use convention from the very nice review [4]

$$T_m(z) = -\frac{1}{\alpha'} \partial_z X_L^\mu(z) \partial_z X_L^\nu \eta_{\mu\nu} ,$$

$$\bar{T}_m(\bar{z}) = -\frac{1}{\alpha'} \partial_{\bar{z}} X_R^\mu(\bar{z}) \partial_{\bar{z}} X_R^\nu(\bar{z}) \eta_{\mu\nu} ,$$

$$X_L^\mu(z) X_L^\nu(w) \sim -\frac{\alpha'}{2} \eta^{\mu\nu} \ln(z - w) ,$$

$$X_R^\mu(\bar{z}) X_R^\nu(\bar{w}) \sim -\frac{\alpha'}{2} \eta^{\mu\nu} \ln(\bar{z} - \bar{w}) ,$$

$$X_L^\mu(z) X_R^\nu(\bar{w}) \sim -\frac{\alpha'}{2} \eta^{\mu\nu} \ln(z - \bar{w})$$

$$\quad (2.3)$$

with the BRST operator

$$Q = \frac{1}{2\pi i} \int_C dz j_B(z) - \frac{1}{2\pi i} \int_C d\bar{z} \bar{j}_B(\bar{z}) ,$$

$$j_B(z) = c(z) \left[ T_m(z) + \frac{1}{2} T_{gh}(z) \right] ,$$

$$\bar{j}_B(\bar{z}) = \bar{c}(\bar{z}) \left[ \bar{T}_m(\bar{z}) + \frac{1}{2} \bar{T}_{gh}(\bar{z}) \right] . \quad (2.4)$$

$^5$Since these states describe open string they belong to the class so-named boundary operators in $BCFT$ that live on the real line $y = 0$ for $z = x + iy$ [4].
where \( j_B(z) \) is holomorphic and \( \overline{j_B}(\overline{z}) \) is anti-holomorphic current and where \( T_{\text{ghost}}, \overline{T}_{\text{ghost}} \) are holomorphic and anti-holomorphic stress energy tensor for the ghosts. In what follows we will not need to know the explicit form of the ghost contribution.

It turns out that the natural definition of the string field theory is in language of BCFT [18, 19]. BCFT are usually regarded as associated boundary theories to the bulk CFT theories. Recall that bulk CFT are defined on the whole complex plane and they appear in the world-sheet description of the closed strings. Their state spaces \( \mathcal{H}^P \) contain all the closed string modes and the coefficients \( C = C^P \) of their operator product expansions (OPE) encode closed string interactions. The space \( \mathcal{H}^P \) is equipped with the action of a Hamiltonian \( H^P \) and of field operators \( \phi(z, \overline{z}) \). According to state-field correspondence we have an identification

\[
\phi(z, \overline{z}) = \Phi^P (|\phi\rangle, z, \overline{z}) , \text{ for all } |\phi\rangle \in \mathcal{H}^P . \tag{2.5}
\]

Among the fields of a CFT one distinguishes so-called chiral fields which depend on only one of the coordinates \( z \) or \( \overline{z} \) so that they are either holomorphic, \( W = W(z) \), or anti-holomorphic, \( \overline{W} = \overline{W}(\overline{z}) \). The (anti)-holomorphic fields of given theory, or their Laurent modes \( W_n, \overline{W}_n \) defined through

\[
W(z) = \sum W_n z^{-n-h} , \overline{W}(\overline{z}) = \sum \overline{W}_n \overline{z}^{-n-\overline{h}} , \tag{2.6}
\]

generate two commuting chiral algebras \( W \) and \( \overline{W} \). The most important of these chiral fields, the Virasoro fields \( T(z), \overline{T}(\overline{z}) \) with modes \( L_n, \overline{L}_n \) are among the chiral fields and numbers \( h, \overline{h} \) are the (half-)integer conformal weights of \( W(z), \overline{W}(\overline{z}) \).

BCFT are conformal field theories on the upper half-plane \( \text{Im}z \geq 0 \) which in the interior \( \text{Im} > 0 \) are locally equivalent to the given bulk theory: The state space \( \mathcal{H}^H \) of the BCFT is equipped with the action of a Hamiltonian \( H^H \) and of bulk fields

\[
\phi(z, \overline{z}) = \Phi^H (|\phi\rangle, z, \overline{z}) , \tag{2.7}
\]

which are assigned to the same fields as were used to label filed in the bulk theory. However these fields \( \phi \) act on a different space of states \( \mathcal{H}^H \). We also demand that all the leading terms in the OPE’s of bulk fields coincide with the OPE’s in the bulk theory. Having the same singularities as in the bulk theory means that the boundary conditions do not affect the equation of motion in the bulk. We must also require the boundary theory to be conformal. This is guaranteed if the Virasoro field obeys the following gluing condition

\[
T(z) = \overline{T}(\overline{z}) , \quad z = \overline{z} . \tag{2.8}
\]

We also presume that all chiral fields \( W(z), \overline{W}(\overline{z}) \) can be extended analytically to the real line and that there exist a local automorphism \( \Omega \)-called the gluing map of the chiral algebra \( W \) such that

\[
W(z) = \Omega \left( \overline{W} \right)(\overline{z}) , \quad z = \overline{z} . \tag{2.9}
\]
Now the assumption of the existence of the gluing map $\Omega$ has powerful consequence that it gives rise to an action of one chiral algebra $W$ on the state space $H^H$ of the boundary theory. More precisely, we combine the chiral fields $W(z)$ and $\Omega W(\tau)$ into single object $W(z)$ defined on the whole complex plane such that

$$W(z) := W(z), \text{Im} z \geq 0, \quad W(z) := \Omega W(\tau), \text{Im} z < 0. \quad (2.10)$$

Thanks to the gluing condition along the boundary this field is analytic and we can expand it in a Laurent series

$$W(z) = \sum_n W_n z^{-n-h} \quad (2.11)$$

so that we introduce the modes $W_n$ that acts on the Hilbert space $H^H$. Then we can obtain the modes $W_n$ through the integration over the curve in the upper half complex plane

$$W_n = \frac{1}{2} \left( \frac{1}{2\pi i} \int_C dz z^{n+h-1} W(z) - \frac{1}{2\pi i} \int_C d\tau z^{n+h-1} \Omega(W)(\tau) \right), \quad (2.12)$$

where $C$ is curve in the upper half plane that is labeled as $z = -e^{-i\sigma + \tau}, \sigma \in (0, \pi)$. It is important to stress that there is just one such action of $W$ constructed out of the two chiral fields $W(z)$ and $\Omega W(\tau)$.

After this short review of $BCFT$ we return to the SFT and its equation of motion. Note that in the abstract language [1] the open string field theory action (2.2) is

$$S = -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \int \Phi \ast Q \Phi + \frac{1}{3} \int \Phi \ast Q \ast \Phi \right) \quad (2.13)$$

from which we immediately get an equation of motion

$$\frac{1}{\alpha'} Q \Phi_0 + \Phi_0 \ast Q \Phi_0 = 0 . \quad (2.14)$$

It is easy to see that the string field in the form

$$\Phi_0 = e^{-\lambda K_L(I)} \ast \frac{1}{\alpha'} Q(e^{\lambda K_L(I)}) , \lambda \in \mathbb{R} \quad (2.15)$$

is solution of (2.14) for any ghost number zero operator $K_L$ acting on the SFT star algebra $\ast$ identity element $I$ which is ghost number zero string field that obeys [33]

$$I \ast X = X \ast I = X, \quad (2.16)$$

for any string field $X$ 6. Let us consider operator $K$ in (2.13) from the chiral algebra $W$ of $BCFT$ in the form

$$K \equiv W = \frac{1}{2} \left( \frac{1}{2\pi i} \int_C dz W(z) - \frac{1}{2\pi i} \int_C d\tau \Omega(W)(\tau) \right) . \quad (2.17)$$

6For recent study of the identity element $I$, see [42, 43, 44, 45].
where \( W(z), \overline{W}(\overline{z}) \) are holomorphic, anti-holomorphic fields respectively of conformal weight \((1, 0), (0, 1)\) which transform under general conformal transformations \( z \to f(z) \) as

\[
U_f W(z) U_f^{-1} = \frac{df(z)}{dz} W(f(z)) \equiv f'(z) W(f(z)) ,
\]

\[
U_f \overline{W}(\overline{z}) U_f^{-1} = \frac{d\overline{f}(\overline{z})}{d\overline{z}} \overline{W}(\overline{f}(\overline{z})) \equiv \overline{f}'(\overline{z}) \overline{W}(\overline{f}(\overline{z})) .
\]

(2.18)

Then we immediately get that \( W \) is invariant under conformal transformations

\[
f \circ W \equiv U_f W U_f^{-1} = \frac{1}{2\pi i} \int_C dz f'(z) W(f(z)) - \frac{1}{2\pi i} \int_C d\overline{z} \overline{f}'(\overline{z}) \overline{W}(\overline{f}(\overline{z})) =
\]

\[
= \frac{1}{2\pi i} \int_{f(C)} dz f'(z) W(f(z)) - \frac{1}{2\pi i} \int_{f(C)} d\overline{z} \overline{f}'(\overline{z}) \overline{W}(\overline{f}(\overline{z})) = W
\]

(2.19)

using the fact that \( W \) does not depend on the integration contour \( C \) as a consequence of gluing conditions (2.9).

We can also define an action of \( W \) on identity field directly in CFT language, following [45, 46, 47]. Instead to taking \(|I⟩\) as identity element of star algebra we will define its through the relation

\[
⟨I| O⟩ = ⟨f_I \circ O⟩ \ , |O⟩ = O(0) |0⟩ \ ,
\]

(2.20)

where

\[
f_I = h^{-1}(h(z)^2) \ , h = \frac{1 + iz}{1 - iz} .
\]

(2.21)

In this approach the identity field is considered as state that belong to family of wedge states [48, 50, 21]. Wedge state \(|n⟩\) of an angle \(2\pi/n\) is defined

\[
⟨n| O⟩ = ⟨f_n \circ O⟩ \ , f_n = h^{-1}(h(z)^{2/n}) .
\]

(2.22)

It follows that \(|I⟩\) is the wedge state \(|n = 1⟩\) of an angle \(2\pi\). In this description we define action of the operator \( W \) on \( I \) as

\[
⟨I| W |φ⟩ = ⟨f_I \circ W f_I \circ φ(0)⟩ .
\]

(2.23)

Using invariance of \( W \) under conformal transformation we immediately get that \( W \) annihilates any wedge state since

\[
⟨n| W |φ⟩ = ⟨f_n \circ (W φ(0))⟩ = ⟨W f_n \circ φ(0)⟩ =
\]

\[
= \left\langle \frac{1}{2\pi i} \oint_C dz W(z) f_n \circ φ(0) \right\rangle = 0
\]

(2.24)
by deforming the contour $C$ until it shrinks to a point at infinity where there is no other operator. In the upper expression we used standard doubling trick to express $W$ through holomorphic current $W(z)$. Let us apply this result for identity field $I$ and write $W = W_L + W_R$, where subscripts $L, R$ denote the integrals of holomorphic and anti-holomorphic currents $W(z)$, $\overline{W(\tau)}$ over left and right side of the string respectively. Then we immediately get

$$W(I) = 0 \Rightarrow W_L(I) = -W_R(I). \quad (2.25)$$

For our next purposes it is important following “partial integration formula”

$$W_L(A) \star B + A \star W_R(B) = 0, \quad (2.26)$$

where $A, B$ are general string fields. Recent very nice discussion of the upper expression can be found in [45], where instead of operator $W$ the BRST operator $Q$ is considered. The proof given there can be easily applied for general operator that is invariant under conformal transformations so that (2.26) is valid for $W$ too.

Let us return to the solution of SFT (2.15). We observe that it has the form of pure gauge. This fact certainly deserves deeper explanation. As is well known the string field theory action (2.13) is invariant under small gauge transformations

$$\delta \Phi = Q\Lambda - \Lambda \star \Phi + \Phi \star \Lambda, \quad (2.27)$$

where $\Lambda$ is ghost number zero string field. On the other hand the action (2.13) is not generally invariant under the large gauge transformations

$$\Phi' = e^{-\Lambda} \star Q(e^\Lambda) + e^{-\Lambda} \star \Phi \star e^\Lambda. \quad (2.28)$$

As is well known there is a sharp distinction between the small gauge transformations and the large ones, for very nice discussion, see [56]. As was argued there, small gauge transformation describes redundancy in our description of the theory. On the other hand, large gauge transformations are true symmetries that relate different solutions in given gauge theory which in our case is the open bosonic string field theory. We will see that this interpretation of the large gauge transformation is the appropriate one in case of (2.15). To support this claim let us start to study fluctuation modes around $\Phi_0$. As usually we expand string field $\Phi$ as

$$\Phi = \Phi_0 + \Psi \quad (2.29)$$

and insert it in (2.13). Then we obtain an action for the fluctuation field $\Psi$ in the same form as the original one (2.13)

$$S' = -\frac{1}{g_0^2} \left( \frac{1}{2\alpha} \int \Psi \star Q' \Psi + \frac{1}{3} \int \Psi \star \Psi \star \Psi \right), \quad (2.30)$$
where the new BRST operator $Q'$ was introduced

$$Q'(X) = Q(X) + \alpha' \Phi_0 \ast X - \alpha'(-1)^{|X|} X \ast \Phi_0 .$$  \hspace{1cm} (2.31)

In order to obtain the new form of the BRST operator (2.31) we will follow the calculation given in [28]. We start with the function

$$F(t) = \frac{1}{\alpha'} e^{-t W_L(I)} \ast Q(e^{t W_L(I)}) , F(1) = \Phi_0 , F(0) = 0$$  \hspace{1cm} (2.32)

and perform Taylor expansion around the point $t = 1$

$$\Phi_0 = F(1) = F(0) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{d^n F}{d^n t}(0) ,$$  \hspace{1cm} (2.33)

where

$$\frac{dF}{dt} = \lambda e^{-t W_L(I)} \ast [Q, W]_L(I) \ast e^{t W_L(I)} ,$$

$$\frac{d^2 F}{d^2 t}(0) = \lambda \frac{d}{dt} [Q, W]_L(I) \equiv \lambda D_L(I) ,$$

$$\frac{d^2 F}{d^2 t}(0) = -\lambda W_L(I) \ast D_L(I) + D_L(I) \ast W_L(I) \ast e^{W_L(I)} ,$$

$$\frac{d^3 F}{d^3 t}(0) = \lambda^2 [W_L, [W_L, D_L]] , \ldots , \frac{d^n F}{d^n t}(0) = \lambda^n [W, [W, \ldots, [W, Q]]]_L(I)$$  \hspace{1cm} (2.34)

and consequently

$$\Phi_0 = \frac{1}{\alpha'} \sum_{n=1}^{\infty} \frac{\lambda^n}{n!} [W, [W, \ldots, [W, Q]]]_L(I) \equiv D_L(I) .$$  \hspace{1cm} (2.35)

It is important to stress that for validity of the calculation given above $W$ must obey the relation (2.29). We have also used $Q(I) = 0 \Rightarrow Q_R(I) = -Q_L(I) \Leftrightarrow [Q_R, W_L] = 0$. From (2.33) see that we can express $\Phi_0$ as a result of the action of the ghost number one operator $D_L$ acting on the identity field. Then we immediately obtain

$$Q'(X) = Q(X) + \alpha' D_L(I) \ast X - \alpha'(-1)^{|X|} X \ast D_L(I) =$$

$$= Q(X) - \alpha' I \ast D_R(X) - \alpha' D_L(X) \ast I = Q(X) - \alpha' D(X) =$$

$$= Q(X) + \sum_{n=1}^{\infty} \frac{\lambda^n}{n!} [W, [W, \ldots, [W, Q]]]_L(X) = e^{iW}(Q(e^{-iW}(X))) .$$  \hspace{1cm} (2.36)

This form of the shifted BRST operator $Q'$ is convenient for the analysis of fluctuation modes around solution (2.13) as we show in the next section.
3. Relation between SFT action $S'$ and the deformation in BCFT

In the previous section we have found an exact solution of the string field theory and also an action for fluctuation modes

$$S' = -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \int \Psi \star Q' \Psi + \frac{1}{3} \int \Psi \star \Psi \star \Psi \right) =$$

$$= -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \langle I \circ \Psi(0) Q' \Psi(0) \rangle + \frac{1}{3} \langle f_1 \circ \Psi(0) f_2 \circ \Psi(0) f_3 \circ \Psi(0) \rangle \right),$$

$$Q'(\Psi(0)) = e^\lambda W e^{-\lambda W}(\Psi(0)).$$

(3.1)

Upper expression implies that it is natural to consider following redefinition of fluctuation states

$$|\Psi\rangle = e^{\lambda W} |\Phi\rangle, \langle \Psi | = \Psi(x = 0) |0\rangle, \langle \Phi | = \Phi(x = 0) |0\rangle,$$

(3.2)

where $\Psi(x), \Phi(x)$ are boundary operators in BCFT which are localized at point $x$ on the real line. Using the fact that $W$ annihilates vacuum state $|0\rangle$ we can write

$$W |\Psi\rangle = W \Psi(0) |0\rangle = [W, \Psi(0)] |0\rangle$$

(3.3)

hence

$$|\Psi\rangle = e^{\lambda W} |\Phi\rangle = \sum_{N=0}^{\infty} \frac{\lambda^N}{N!} \underbrace{[W, [W, \ldots, [W, \Phi(0)]]]}_N |0\rangle, [W, \Phi(0)] = \frac{1}{2\pi i} \oint_C dz W(z) \Phi(0)$$

(3.4)

so that we can define the vertex operator for fluctuation field as

$$\Psi(x) = e^W (\Phi)(x) \equiv \sum_{N=1}^{\infty} \frac{\lambda^N}{2^N N!} \oint_{C_1} \ldots \oint_{C_N} \frac{dz_1}{2\pi i} \ldots \frac{dz_N}{2\pi i} W(z_1) \ldots W(z_N) \Phi(x),$$

(3.5)

where $C_i$ are small circles around the point $x$ and where their radii are given as $\epsilon_{i-1} > \epsilon_i$ and in the end of the calculation we take the limit $\epsilon_i \rightarrow 0$. In the previous expression we have slightly moved the insertion point $x$ above to real axis in order to perform contour integration. As a result the second term in $W$ has no singularity with $\Phi(x)$ and hence we can consider the holomorphic field $W(z)$ only.

When we insert (3.2) into (3.1) we get

$$S = -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \int e^{\lambda W}(\Phi) * e^{\lambda W} Q(e^{-\lambda W} e^{\lambda W}(\Phi)) + \frac{1}{3} \int e^{\lambda W}(\Phi) * e^{\lambda W}(\Phi) * e^{\lambda W}(\Phi) \right) =$$

$$= -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \langle I \circ (e^{\lambda W}(\Phi)(0)) e^{\lambda W}(Q\Phi(0)) \rangle + \frac{1}{3} \langle f_1 \circ (e^{\lambda W}(\Phi)(0)) f_2 \circ (e^{\lambda W}(\Phi)(0)) f_3 \circ (e^{\lambda W}(\Phi)(0)) \rangle \right).$$

(3.6)
Using (2.19) we obtain
\[ f_i \circ (e^{\lambda W}(\Phi)(0)) = U_f e^{\lambda W} U^{-1}_f \left( U_f \Phi(0) U^{-1}_f \right) = e^{\lambda W}(f_i \circ \Phi(0)) \] (3.7)
so that the SFT action \( S' \) for fluctuation modes can be written as
\[ S' = \frac{1}{g_0^2} \left( \frac{1}{2 \alpha'} \langle e^{\lambda W} (I \circ \Phi(0)) e^{\lambda W} (Q \Phi(0)) \rangle + \right.
\[ + \frac{1}{3} \langle e^{\lambda W}(f_1 \circ \Phi(0)) e^{\lambda W}(f_2 \circ \Phi(0)) e^{\lambda W}(f_3 \circ \Phi(0)) \rangle \right) = \]
\[ \frac{1}{g_0^2} \left( \frac{1}{2 \alpha'} \langle I \circ \Phi(0) Q \Phi(0) \rangle_{W,\lambda} + \frac{1}{3} \langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle_{W,\lambda} \right), \] (3.8)
where we have defined deformation of boundary correlators [32]
\[ \langle \Phi_1(x_1) \ldots \Phi_M(x_M) \rangle_{W,\lambda} = \langle e^{\lambda W}(\Phi_1)(x_1) \ldots e^{\lambda W}(\Phi_M)(x_M) \rangle . \] (3.9)
The form of the action (3.8) is the main result of our paper which says that when
we perform redefinition of fluctuation modes as in (3.5), then the SFT \( S' \) (3.1)
is the same as the SFT action defined on the background \( BCFT'' \) that arises from the
original one through marginal deformation inserted on the real line \( z = \overline{z} \). More
precisely, the general prescription of the boundary deformation in given \( BCFT \)
is as follows. We start with some \( BCFT \) with the state space \( \mathcal{H}_{\Omega,\alpha} \) where \( (\Omega, \alpha) \) denotes
the boundary condition along the real line. Boundary operators \( \psi(x) \in \Phi(\mathcal{H}) \)
may be used to define a new perturbed \( BCFT'' \) whose correlation functions are
constructed from the unperturbed ones by the formal expansion
\[ \langle \phi_1(z_1, \overline{z}_1) \ldots \phi_N(z_N, \overline{z}_N) \rangle_{\alpha,\lambda \psi} = Z^{-1} \langle I_{\lambda \psi} \phi_1(z_1, \overline{z}_1) \ldots \phi_N(z_N, \overline{z}_N) \rangle_{\alpha} = \]
\[ = Z^{-1} \sum_n \lambda^n \int \ldots \int_{x_1 < x_{n+1}} \frac{dx_1}{2\pi} \ldots \frac{dx_N}{2\pi} \langle \psi(x_1) \ldots \psi(x_N) \phi_1 \ldots \phi_N \rangle_{\alpha} , \] (3.10)
where \( \lambda \) is a real parameter. From the second line it is clear that the symbol \( I_{\lambda \psi} \)
in the first line should be understood as a path ordered exponential of the perturbing
operator
\[ I_{\lambda \psi} = P \exp (\lambda S_{\psi}) \equiv P \exp \left( \lambda \int_{-\infty}^{\infty} \frac{dx}{2\pi} \psi(x) \right) . \] (3.11)
We must mention that given expression is rather formal and suffers from UV diver-
gences and should be regularized. For more detailed discussion, see again [32]. And
finally (3.10) defines deformations of bulk correlators only. If there are extra bound-
dary fields present in the correlation function, these formulas have to be modified so
that these boundary fields are included in the path ordering. However for special
class the boundary deformations these formulas simplify considerably [32].
the boundary operator $\psi$ has conformal dimension $h$. For $h \neq 1$ the perturbation will automatically introduce length scale and we have to follow the renormalization group flow to come back $BCFT$. However as was stressed in [32] all these general perturbations have common feature that the new $BCFT$ is associated to the same bulk $CFT$. As a conclusion, the boundary perturbations can only induce changes of the boundary conditions. For marginal deformations with $h = 1$ this implies that the boundary deformation induces the change of the original $BCFT$ with the gluing condition $\Omega$ to the new $BCFT''$ with the new gluing condition $\Omega''$, where the precise form of the $\Omega''$ depends on the nature of $\psi$ [32].

Let us consider the deformation of correlators that contain boundary fields as well. It was shown in [32] that (3.10) admits for the obvious generalization

$$
\langle \psi_1(u_1) \ldots \psi_M(u_M) \phi_1(z_1, \overline{z}_1) \ldots \phi_N(z_N, \overline{z}_N) \rangle_{\alpha, \lambda \psi} =
$$

$$
Z^{-1} \sum_n \frac{\lambda^n}{n!} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{dx_1}{2\pi} \cdots \frac{dx_N}{2\pi} \langle \psi(x_1) \ldots \psi(x_N) \psi_1 \ldots \psi_M \phi_1 \ldots \phi_N \rangle_{\alpha} ,
$$

if and only if the boundary fields $\psi_1, \ldots, \psi_M$ are local with respect to the perturbing field $\psi$ [32], where two boundary fields $\psi_1(x_1), \psi_2(x_2)$ are said to be local if

$$
\psi_1(x_1) \psi_2(x_2) = \psi_2(x_2) \psi_1(x_1) , x_1 < x_2 .
$$

This equation is supposed to hold after insertion into arbitrary correlation functions and the right hand side to make sense it is required that there exists a unique analytic continuation from $x_1 < x_2$ to $x_1 > x_2$. We also say that a boundary field $\psi(x)$ is called self local or analytic if it is mutually local with respect to itself. For example, the OPE of a self-local boundary field $\psi$ with conformal dimension $h_\psi = 1$ is determined up to a constant to be

$$
\psi(x_1) \psi(x_2) = \frac{K}{(x_1 - x_2)^2} + \text{reg} , h_\psi = 1 .
$$

After appropriate renormalization [32] the correlation function (3.12) can be written as

$$
\langle \psi_1(u_1) \ldots \psi_M(u_M) \phi_1(z_1, \overline{z}_1) \ldots \phi_N(z_N, \overline{z}_N) \rangle_{\alpha, \lambda \psi} =
$$

$$
= \sum_n \frac{\lambda^n}{n!} \int_{\gamma_1} \cdots \int_{\gamma_n} \frac{dx_1}{2\pi} \cdots \frac{dx_N}{2\pi} \langle \psi(x_1) \ldots \psi(x_N) \tilde{\psi}_1 \ldots \tilde{\psi}_M \phi_1 \ldots \phi_N \rangle_{\alpha} ,
$$

where $\gamma_p$ is the straight line parallel to the real axis with $\text{Im} \gamma_p = i\epsilon/p$ and where the fields $\tilde{\phi}_i$ are given

$$
\tilde{\psi}_i(u_i) = \sum_{n=0}^{\infty} \frac{\lambda^n}{2^n n!} \oint_{C_i} \frac{dx_1}{2\pi} \cdots \oint_{C_n} \frac{dx_n}{2\pi} \psi_1(i) \psi(x_n) \ldots \psi(x_1) .
$$
Note that boundary fields $W$ from the chiral algebra $\mathcal{W}$ are local with respect to themselves to all other boundary and bulk fields in the theory so that (3.13) can be applied to correlators involving arbitrary bulk and boundary fields. Then we see that (3.9) is special case of (3.13) with no bulk operators inserted and that (3.5) has the same form as (3.16). Consequently we can claim that redefinition of fluctuation modes (3.5) in $S'$ (3.10) maps this action to the SFT action (3.8) that describes SFT action defined on background $BCFT''$.

4. Conclusion

In this note we have studied the solution of open bosonic SFT based on the existence of marginal operators $W$ from the chiral algebra $\mathcal{W}$ of $BCFT$, where $BCFT$ is the classical background on which given SFT is defined. We have mainly focused on the relation between the fluctuation field around the classical solution and the deformed $BCFT''$ that arises from the original $BCFT$ by inserting marginal interaction on the real line. We have seen that after an expansion of the string field around the classical solution and its insertion to the original action we obtain the SFT action $S'$ that after redefinition of fluctuation fields is written using $BCFT''$ correlators that are deformations of the correlators in $BCFT$ through introduction of perturbation $W(x)$ from the chiral algebra $\mathcal{W}$ on the real line. In other words, we have shown that two string field theory actions $S_1, S_2$ defined using two $BCFT$'s, $BCFT_1, BCFT_2$ where these two $BCFT$'s are related through marginal deformations from the chiral algebra (It is important that $\lambda$ is not infinitesimal), are in fact an expansion of SFT action around different classical solutions. We mean that this result could be considered as an additional evidence of the background independence of open bosonic string field theory [51, 52, 53, 54, 55] even in case of general deformation parameter $\lambda$. We also hope that this result could be helpful for recent application of SFT, for example for the study of the rolling tachyon solution. We hope to return to this problem in future.

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