Traditional Tracking with Kalman Filter on Parallel Architectures

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Abstract. Power density constraints are limiting the performance improvements of modern CPUs. To address this, we have seen the introduction of lower-power, multi-core processors, but the future will be even more exciting. In order to stay within the power density limits but still obtain Moore’s Law performance/price gains, it will be necessary to parallelize algorithms to exploit larger numbers of lightweight cores and specialized functions like large vector units. Example technologies today include Intel’s Xeon Phi and GPGPUs. Track finding and fitting is one of the most computationally challenging problems for event reconstruction in particle physics. At the High Luminosity LHC, for example, this will be by far the dominant problem. The most common track finding techniques in use today are however those based on the Kalman Filter. Significant experience has been accumulated with these techniques on real tracking detector systems, both in the trigger and offline. We report the results of our investigations into the potential and limitations of these algorithms on the new parallel hardware.

1. Introduction

Track finding and fitting is one of the most computationally challenging problems for event reconstruction in particle physics. At the High Luminosity LHC (HL-LHC), for example, this will be by far the dominant problem. The need for greater parallelism has driven investigations (e.g. [1, 2, 3]) of very different track finding techniques including Cellular Automata or returning to Hough Transform techniques originating in the days of bubble chambers. The most common track finding techniques in use today are based on the Kalman Filter [4]. Significant experience has been accumulated with these techniques on real tracking detector systems, both in the trigger and offline. They are known to provide high physics performance, are robust and are exactly those being used today for the design of the tracking system for HL-LHC. We report the results of our investigations into the potential and limitations of these algorithms on the new parallel hardware.

Kalman Filter tracking: We choose to attack charged particle tracking as it is one of the most CPU-time intensive tasks in the hadron collider event reconstruction chain. The CPU processing time is also acutely sensitive to increases in instantaneous luminosity and, therefore, pileup. Figure 1 shows the CPU time for a typical event reconstruction job. The growth term,
Figure 1. Timing vs instantaneous luminosity for $pp \rightarrow t\bar{t}$ simulated events, with overlaid pile-up events, with 25 ns bunch crossing. Samples with different average pile-up are used as reported on the plot. We show the timing of the full CMS reconstruction, and of the iterative tracking sequence by itself. As can be seen, the time required for tracking dominates the total time and increases sharply at higher values of the instantaneous luminosity.

i.e., the increase in CPU time as a function of instantaneous luminosity, is exponential and the dominant contribution clearly comes from track reconstruction [5].

We choose a Kalman Filter-based algorithm as this high-performance algorithm is the standard in modern HEP experiments. For instance, the CMS experiment uses a Kalman Filter in both its high-level trigger as well as its offline reconstruction application [6]. The algorithm has well-understood timing performance characteristics in serial computing environments. The physics performance is also well understood in terms of efficiencies and fake rates.

Additionally, the Kalman Filter algorithm is adaptable for the wide vector units of modern CPUs and Xeon Phi. The algorithm can be implemented via matrix manipulations, which are well-suited for these hardware platforms.

To be able to harness new computing platforms, event-level parallelization across cores is the key to gaining more speed. Due to limitations in memory and I/O bandwidth, it is insufficient to run the serial algorithm for many events at once in an ‘embarrassingly parallel’ approach.

Many-core: There has been a sea change in the process of computing in the past decade. Previously, one could expect that a given piece of code would run progressively faster with each generation of new processors with exponential gains consistent with Moore’s-law-like growth. These gains were largely achieved by increases in the processor clock frequency. However, these gains have stalled in the last decade as processors have started to hit scaling limits, as has been widely recognized [7, 8]. The limits are largely driven by constraints on power consumption.

Moore’s Law itself, which states that the number of transistors in an integrated circuit doubles approximately every two years, is still alive and kicking, but exists now in new forms. The first new form is the advent of so-called ‘multi-core’ processors, with more than one functional processor on a chip, however without increases in the main processor frequency. The next anticipated change is the advent of so-called ‘small core’ processors, such as ARM, GPUs, or MIC (Intel Xeon Phi). These small cores individually have less processing power than the big cores they replace, but in aggregate offer an immense amount of computing power. Sequential
applications, such as those favored by HEP experiments, do not run out of the box on these platforms, however, and must be adapted.

As an example of deploying a complex algorithm on a many-core system, we explore the implementation of the CMS Kalman Filter tracking code on an Intel Xeon Phi.

2. Challenges
New architectures are “easy” to deploy for simple problems and tailored algorithms. This is not the problem we are facing; we need to migrate tools from a running experiment such as CMS at the CERN LHC. The serial algorithms currently in use have been optimized by many person-years of effort and set a high bar for the physics performance that we must meet before being able to deploy a many-core system. Existing algorithms require new thinking to refactor and adapt them to parallel architectures. Additional constraints come from the heterogeneous environment that we expect to use. Software for modern HEP experiments must run on a diverse array of platforms (basically, whatever is available across countries and continents.) Our code must be usable on big cores, little cores, and GPUs; we cannot use algorithms that only work well on one architecture. We are exploring a family of algorithms consistent with these constraints. Our current attempt focuses on the Intel Xeon Phi.

To exploit the Intel Xeon Phi, we need to vectorize and multithread a Kalman Filter algorithm for track building and fitting. Some of the expected difficulties include the fact that the algorithm uses small matrices, and has multiple decision points along the building of each track. Details of cache occupancy, pipelining, and latencies must be understood to achieve maximum possible performance. An expected benefit is that vectorization and multithreading should improve the serial (big-core) performance as well, which would immediately be applicable to a running experiment without the need for new hardware purchases.

3. Detector, Geometry, Simulation
For this study, we used a simplified detector geometry consisting of equally spaced, fixed length concentric cylinders, with radii at fixed steps (ten layers, 2 m long, spaced every 4 cm in radius). This was implemented using USOLIDS, a light-weight geometry library with interfaces based on ROOT and GEANT4 libraries which was created as part of the AIDA project [9]. The primary advantage of using an existing geometry library is that more complex geometries (polygonal geometry, z-segmentation, etc.) will be straightforward to implement as algorithm development progresses. We are also investigating eventual use of VecGeom from GeantV for better vectorization when generating our simulated data sets. To provide input data for the Kalman Filter tests, a standalone simulation was written which uses this full geometry for track propagation and simulates the detector resolution via Gaussian smearing of the hit positions.

We generate sets of tracks with transverse momentum in the range \( 0.5 < p_T < 10.0 \) GeV. The tracks are limited to be within the central region of a barrel detector (\( |\eta| < 1.0 \)) and generated uniformly in \( \phi \). In this configuration, every track crosses every layer of our simplified detector. At the intersection of the generated tracks with the detector, we create a hit whose position is smeared in \( x - y \) by \( \sigma_{xy} = 0.01 \) cm and in \( z \) by \( \sigma_z = 0.1 \) cm.

4. Tracking
Track reconstruction can be divided into three steps: seeding, building, and fitting. While seeding is based on a different kind of algorithm, both track building and fitting are often based on a Kalman Filter. The Kalman Filter is an iterative procedure where a basic logic unit is applied repeatedly. The unit consists of the propagation of parameters and uncertainties (track state) from one layer to the next. At each layer, the track state is updated with the hit measurement information of the hits on that layer. (See Fig. 2 left.)
Figure 2. Left: Basic unit of the Kalman Filter algorithm. At each step, position information from hits is used to estimate the track parameters and their uncertainties. The red circle represents the measurement (a hit). The blue point on layer \( N \) represents the estimated state (position and direction) at layer \( N \) before taking into account information from hits on that layer. The yellow point is the updated state at layer \( N \), taking into account all hits from up to and including layer \( N \). Right: Cartoon representing the two stages of fitting: forward fit and backward smoothing. For this test we do not perform a smoothing step.

Figure 3. \( p_T \) and position pull plots for the fit. The performance of the fit is under control.

As can be seen in Fig. 2 (right), the track fit consists of the simple repetition of the basic logic unit for all the pre-determined track hits and therefore it is the easiest case to test. It is divided in two steps: a forward fit and a backward smoothing stage for optimal performance. For this test, only forward fit is run. The Kalman Filter requires initial estimation of track parameters to get started. In our test, the starting state is taken directly from simulation with 100% uncertainty. As a more realistic option, we also implemented a parabolic fit in the conformal space to define initial parameters. For the fitting test, the hits are attached to a track “by name” (i.e., no pattern recognition is performed) and a Kalman Filter fitting stage is performed. Figure 3 shows the \( p_T \) and position pulls for the resulting fit with Gaussian distributions consistent with
Figure 4. Memory layout for the new matrix library MATRIPLEX. The layout is optimized for our problem, which consists of matrix manipulations of low-dimensional matrices. The memory layout is matrix-major. In the Figure, the matrix dimension is $N \times N$ and the vector unit size is $n$.

unit width and demonstrates that the fit results are reasonable. The achieved $p_T$ resolution is roughly $\sigma_{p_T}/p_T = 0.005 \times p_T$.

5. Optimized Matrix Library MATRIPLEX

The computational problem of Kalman Filter-based tracking consists of a sequence of matrix operations on matrices of sizes from $N \times N = 3 \times 3$ up to $N \times N = 6 \times 6$. To allow maximum flexibility for exploring SIMD operations on small-dimensional matrices, and to decouple the specific computations from the high level algorithm, we have developed a new matrix library, MATRIPLEX. The MATRIPLEX memory layout is optimized for the loading of vector registers for SIMD operations on a set of matrices as shown in Fig. 4. MATRIPLEX includes a code generator for generation of optimized matrix operations supporting symmetric matrices and on-the-fly matrix transposition. Patterns of elements which are known by construction to be zero or one can be specified, and the resulting generated code will be optimized accordingly to reduce unnecessary register loads and arithmetic operations. The generated code can be either standard C++ or simple intrinsic macros that can be easily mapped to architecture-specific intrinsic functions.

6. Results

We present the results of this study in two stages: vectorization and parallelization. In the first step we restructure the code to allow use of the vector units in Xeon and Xeon Phi processors. In the second step we use OPENMP to parallelize the vectorized fitting procedure across the cores on the large-core and small-core devices.

Figure 5 shows the timing for fitting 1M tracks as a function of the vector size, using a single thread. Results are compared to scaling of serial processing time (“ideal scaling”), defined as the time with vector unit size=1 divided by the vector unit size. Both for Xeon and Xeon Phi, a significant vectorization speedup is achieved, with an effective utilization of the vector units of $\sim 50\%$.

Figure 6 shows the timing for fitting the same set of tracks as a function of the number of threads, in case all vector units are used. We test two approaches for distributing threads on the cores: filling every core with one thread or adding a second thread on the same core before moving to a different one. We compare to ideal parallelization performance (“ideal scaling”), assuming no hyperthreading (i.e. a maximum of 12 threads on Xeon, maximum 120 threads on Xeon Phi). Performance for one thread/core approach follows the ideal curve, with a small

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1 CentOS 6.5, 2 × 6 core Xeon E5-2620 @ 2GHz, 64 GB RAM, turbo off, hyperthreading enabled
2 Xeon Phi 7150, 16 GB RAM, 61 cores @ 1.24GHz
Figure 5. Timing results for vectorized code, as a function of the vector size, for host (left) and MIC (right). The results are compared to an ideal scaling described in the text. Significant speedups compared to serial code are observed.

Figure 6. Timing results for parallelization tests, as a function of the number of threads, for host (left) and MIC (right). Two different methods of distributing threads across cores is shown, and compared to an ideal scaling. We observe ideal scaling when distributing one thread/core.

overhead only when all cores are filled. The two threads per core approach shows deviation from ideal behavior immediately: on Xeon, this is due to the use of hyperthreading slots [10], while on Xeon Phi we surmise it is related to L1 cache contention issues.

7. Future Work: Track Building
Track building is by far the most time-consuming step in the whole event reconstruction and thus it is the most important next step of development. With respect to track fitting, track building adds significantly more complexity to the problem as hits are searched for within a compatibility window when moving to the next layer. The track candidate needs to branch in case of multiple matches, and the algorithm needs to be robust against missing (due to detector inefficiency) or outlier hits (Fig. 7). We implemented a preliminary serial version and tested it on events with 500 simulated tracks/event, finding a hit finding efficiency close to 100%. The inherent branching of the algorithm, along with the variable size of the combinatorial problem, make it difficult to vectorize and parallelize it efficiently; thus, specific design choices have to be made to boost its computing performance on the coprocessor.
Figure 7. Schematic representation of track building. Unlike in track fitting, the algorithm has many branch points, e.g. when missing hits on layers or when multiple hit candidates are encountered on a layer. This is among the challenges for achieving efficient vectorization of the track building code.

8. Progress and Plans
To summarize, we presented the start of a task of adapting a complex algorithm from running HEP experiments for use in new parallel hardware. We showed a preliminary implementation of Kalman Filter-based track fitting, vectorized and parallelized on Intel Xeon and Intel Xeon Phi. In both cases, we achieve \(\sim 50\%\) vectorization efficiency and \(\sim 100\%\) parallelization efficiency when running one thread per core.

We are currently working towards the implementation of more realistic geometries, particularly polygonal/planar geometries typical of pixel detectors, and of material effects, both in the simulation and reconstruction models, to complete the study of track fitting. The next major step is the implementation of a fully vectorized and parallelized track building algorithm. At the same time, we will continue work on optimizing fitting and matrix operations so that overheads are further reduced and computing efficiency is increased.

At this point the results shown encourage us to continue pursuing implementation of the Kalman Filter algorithm for charged particle tracking on parallel hardware, and demonstrate successful first steps towards our goal of exploiting parallel hardware for HEP experiments.

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