S-wave bottom baryons

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Abstract

The masses of S-wave bottom baryons are calculated in the framework of coupled-channel formalism. The relativistic three-quark equations for the bottom baryons using the dispersion relation technique are found. The approximate solutions of these equations based on the extraction of leading singularities of the amplitude are obtained. The calculated mass values of S-wave bottom baryons are in good agreement with the experimental ones.

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I. Introduction.

Hadron spectroscopy has always played an important role in the revealing mechanisms underlying the dynamic of strong interactions.

The heavy hadron containing a single heavy quark is particularly interesting. The light degrees of freedom (quarks and gluons) circle around the nearby static heavy quark. Such a system behaves as the QCD analogue of familiar hydrogen bound by the electromagnetic interaction.

The heavy quark expansion provides a systematic tool for heavy hadrons. When the heavy quark mass $m_Q \rightarrow \infty$, the angular momentum of the light degree of freedom is a good quantum number. Therefore heavy hadrons form doublets. For example, $\Omega_b$ and $\Omega_b^*$ will be degenerate in the heavy quark limit. Their mass splitting is caused by the chromomagnetic interaction at the order $O(1/m_Q)$, which can be taken into account systematically in the framework of heavy quark effective field theory (HQET) [1 – 3].

Recently CDF Collaboration observed four bottom baryons $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ [4]. D0 and CDF have seen candidates for $\Xi_b^-$ [5, 6].

In the past two decades, various phenomenological models have been used to study heavy baryon masses [7 – 12]. Capstick and Isgur studied the heavy baryon system in a relativized quark potential model [7]. Roncaglia et al. predicted the masses of baryons containing one or two heavy quark using the Feynman-Hellman theorem and semiempirical mass formulas [8]. Jenkins studied heavy baryon masses using a combined expansion of $1/m_Q$ and $1/N_c$ [9]. Mathur et al. predicted the masses of charmed and bottom baryons from lattice QCD [10]. Ebert et al. calculated the masses of heavy baryons with the light-diquark approximation [11]. Stimulated by recent experimental progress, there have been several theoretical papers on the masses
of $\Sigma_b, \Sigma_b^*$ and $\Xi_b$ using the hyperfine interaction in the quark model [12 – 16]. Recently the strong decays of heavy baryons were investigated systematically using $^3P_0$ model in Ref. [17].

QCD sum rule has been applied to study heavy baryon masses [18 – 22].

In our papers [23 – 25] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting particles. The mass spectra of $S$-wave baryons including $u, d, s, c$ quarks were calculated by a method based on isolating of the leading singularities in the amplitude.

We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, the weaker ones being neglected. If we considered such an approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions at the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the present paper the relativistic three-particle amplitudes in the coupled-channels formalism are considered. We take into account the $u, d, s, c, b$ quarks and construct the flavor-spin functions for the 35 baryons with the spin-parity $J^p = \frac{1}{2}^+$ and $J^p = \frac{3}{2}^+$:

\[
\begin{align*}
J^p = \frac{1}{2}^+ & \quad J^p = \frac{3}{2}^+ \\
\Sigma_b & : uub, udb, ddb & \Sigma_b & : uub, udb, ddb \\
\Lambda_b & : udb & \Xi_{sb} & : usb, dsb \\
\Xi_{sb} & : usb, dsb & \Omega_{sbb} & : sshb \\
\Xi_{sb} & : usb, dsb & \Xi_{sb} & : usb, ddb \\
\Xi_{sb} & : usb, dsb & \Xi_{sb} & : usb, ddb \\
\Omega_{sbb} & : ssb & \Omega_{sbb} & : ssb \\
\Omega_{sbb} & : ccb & \Omega_{sbb} & : ccb \\
\Omega_{sbb} & : ccb & \Omega_{sbb} & : ccb \\
\Xi_{sb} & : ucb, dcb & \Xi_{sb} & : ucb, dcb \\
\Xi_{sb} & : ucb, dcb & \Xi_{sb} & : ucb, dcb \\
\Lambda_{sbb} & : scb & \Lambda_{sbb} & : scb \\
\Lambda_{sbb} & : scb & \Lambda_{sbb} & : scb \\
\Omega_{sbb} & : ccb & \Omega_{sbb} & : ccb \\
\Omega_{sbb} & : ccb & \Omega_{sbb} & : ccb \\
\Xi_{sbb} & : ubb, ddb & \Xi_{sbb} & : ubb, ddb \\
\Omega_{sbb} & : sbb & \Omega_{sbb} & : sbb \\
\Omega_{sbb} & : sbb & \Omega_{sbb} & : sbb \\
\Xi_{sbb} & : ubb, ddb & \Xi_{sbb} & : ubb, ddb \\
\Omega_{sbb} & : sbb & \Omega_{sbb} & : sbb \\
\Omega_{sbb} & : sbb & \Omega_{sbb} & : sbb
\end{align*}
\]

In the paper [25] the relativistic equations were obtained and the mass spectrum of $S$-wave charmed baryons was calculated.

In the present paper we will be able to use the similar method. In this case we consider 35 baryons with the spin-parity $J^p = \frac{1}{2}^+$ and $J^p = \frac{3}{2}^+$, which include one, two and three bottom quarks. We have considered the 23 baryons with different masses.

The paper is organized as follows. In Section II we obtain the relativistic three-particle equations which describe the interaction of the quarks in baryons. In Section III the coupled systems of equations for the reduced amplitudes are derived. Section IV is devoted to a discussion of the results for the mass spectrum of $S$-wave bottom baryons (Tables I, II). In the conclusion the status of the considered model is discussed.

In Appendix A we write down the three-particle integral equations for the $J^p = \frac{1}{2}^+$ and $J^p = \frac{3}{2}^+$ bottom baryon multiplets. In Appendix B the coupled systems of approximate equations for lowest bottom baryons are given.
II. The three-quark integral equations for the S-wave bottom baryons.

We calculate the masses of the bottom baryons in a relativistic approach using the dispersion relation technique. The relativistic three-quark integral equations are constructed in the form of the dispersion relations over the two-body subenergy.

We use the graphic equations for the functions $A_J(s, s_{ik})$ [23 – 25]. In order to represent the amplitude $A_J(s, s_{ik})$ in the form of dispersion relations, it is necessary to define the amplitudes of quark-quark interaction $a_J(s_{ik})$. The pair amplitudes $qq ightarrow qq$ are calculated in the framework of the dispersion $N/D$ method with the input four-fermion interaction with quantum numbers of the gluon [26]. We use results of our relativistic quark model [27] and write down the pair quark amplitudes in the following form

$$a_J(s_{ik}) = \frac{G_J^2(s_{ik})}{1 - B_J(s_{ik})}, \quad (2)$$

$$B_J(s_{ik}) = \frac{\Lambda_J(i,k)}{(m_i + m_k)^2} \frac{ds'_{ik} \rho_J(s'_{ik}) G_J^2(s'_{ik})}{\rho_J(s_{ik}) G_J^2(s_{ik})}, \quad (3)$$

$$\rho_J(s_{ik}) = \frac{(m_i + m_k)^2}{4\pi} \left( \alpha_J \frac{s_{ik}}{(m_i + m_k)^2} + \beta_J + \delta_J \right) \times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)}(s_{ik} - (m_i - m_k)^2)}{s_{ik}} \quad (4)$$

Here $G_J$ is the vertex function of a diquark, which can be expressed in terms of the $N$-function of the bootstrap $N/D$ method as $G_J = \sqrt{N_j}$; $B_J(s_{ik})$ is the Chew-Mandelstam function [28], and $\rho_J(s_{ik})$ is the phase space for a diquark. $s_{ik}$ is the pair energy squared of diquark, the index $J^p$ determines the spin-parity of diquark. The coefficients of Chew-Mandelstam function $\alpha_J, \beta_J$ and $\delta_J$ in Table III are given. $\Lambda_J(i,k)$ is the pair energy cutoff. In the case under discussion the interacting pairs of quarks do not form bound states. Therefore the integration in the dispersion integral (3) is carried out from $(m_i + m_k)^2$ to $\Lambda_J(i,k)$ (i,k=1,2,3). Including all possible rescatterings of each pair of quarks and grouping the terms according to the final states of the particles, we obtained the coupled systems of integral equations. For instance, for the $\Sigma^+_b$ with $J^p = \frac{1}{2}^+$ the wave function is $\varphi_{\Sigma^+_b} = \sqrt{\frac{2}{3}}\{u \uparrow d \uparrow b \downarrow\} - \sqrt{\frac{1}{6}}\{u \uparrow d \downarrow b \uparrow\} - \sqrt{\frac{1}{6}}\{u \downarrow d \uparrow b \uparrow\}$. Then the coupled system of equations has the following form:

$$
\begin{cases}
A_1(s, s_{12}) = \lambda b_1(s_{12}) L_1(s_{12}) + K_1(s_{12}) \left[ \frac{1}{4} A_{1\ell}(s, s_{13}) + \frac{3}{4} A_{0\ell}(s, s_{13}) \right] + \\
\quad \quad + \frac{1}{4} A_{1\ell}(s, s_{23}) + \frac{3}{4} A_{0\ell}(s, s_{23}) \\

A_{1\ell}(s, s_{13}) = \lambda b_{1\ell}(s_{13}) L_{1\ell}(s_{13}) + K_{1\ell}(s_{13}) \left[ \frac{1}{2} A_1(s, s_{12}) - \frac{1}{2} A_{1\ell}(s, s_{12}) \right] + \\
\quad \quad + \frac{3}{4} A_{0\ell}(s, s_{12}) + \frac{1}{2} A_1(s, s_{23}) - \frac{1}{4} A_{1\ell}(s, s_{23}) + \frac{3}{4} A_{0\ell}(s, s_{23}) \\

A_{0\ell}(s, s_{23}) = \lambda b_{0\ell}(s_{23}) L_{0\ell}(s_{23}) + K_{0\ell}(s_{23}) \left[ \frac{1}{2} A_1(s, s_{12}) + \frac{1}{2} A_{1\ell}(s, s_{12}) \right] + \\
\quad \quad + \frac{1}{4} A_{0\ell}(s, s_{12}) + \frac{1}{2} A_1(s, s_{13}) + \frac{1}{4} A_{1\ell}(s, s_{13}) + \frac{1}{4} A_{0\ell}(s, s_{13}) \right].
\end{cases}
$$
Here the function $L_j(s_{ik})$ has the form

$$L_j(s_{ik}) = \frac{G_j(s_{ik})}{1 - B_j(s_{ik})}.$$  
(6)

The integral operator $K_j(s_{ik})$ is

$$K_j(s_{ik}) = L_j(s_{ik}) \int_{(m_i + m_k)^2}^{\Lambda_j(s_{ik})} \frac{ds'_{ik} \rho_j(s'_{ik})G_j(s'_{ik})}{\pi s'_{ik} - s_{ik}} \int_{-1}^{1} \frac{dz}{2}.$$  
(7)

The function $b_j(s_{ik})$ is the truncated function of Chew-Mandelstam:

$$b_j(s_{ik}) = \int_{(m_i + m_k)^2}^{\infty} \frac{ds'_{ik} \rho_j(s'_{ik})G_j(s'_{ik})}{\pi s'_{ik} - s_{ik}},$$  
(8)

$z$ is the cosine of the angle between the relative momentum of particles $i$ and $k$ in the intermediate state and the momentum of particle $j$ in the final state, taken in the c.m. of the particles $i$ and $k$. Let some current produces three quarks with the vertex constant $\lambda$. This constant do not affect to the spectra mass of bottom baryons. By analogy with the $\Sigma_b^+$ state we obtain the rescattering amplitudes of the three various quarks for the all bottom states (Appendix A).

**III. Reduced equations for the S-wave bottom baryons.**

Let us extract two-particle singularities in $A_j(s, s_{ik})$:

$$A_j(s, s_{ik}) = \frac{\alpha_j(s, s_{ik})b_j(s_{ik})G_j(s_{ik})}{1 - B_j(s_{ik})},$$  
(9)

$\alpha_j(s, s_{ik})$ is the reduced amplitude. Accordingly to this, all integral equations can be rewritten using the reduced amplitudes. The function $\alpha_j(s, s_{ik})$ is the smooth function of $s_{ik}$ as compared with the singular part of the amplitude. We do not extract the three-body singularities, because they are weaker than the two-particle singularities. For instance, one considers the first equation of system for the $\Sigma_b^+$ with $J^p = \frac{1}{2}^+$:

$$\alpha_1(s, s_{12}) = \lambda + \frac{1}{b_1(s_{12})} \int_{(m_1 + m_2)^2}^{\Lambda_1(s_{12})} \frac{ds'_{12} \rho_1(s'_{12})G_1(s'_{12})}{\pi s'_{12} - s_{12}} \times$$

$$\times \int_{-1}^{1} \frac{dz}{2} \left( \frac{G_{1b}(s'_{13})b_{1b}(s'_{13})}{1 - B_{1b}(s'_{13})} \frac{1}{2} \alpha_{1b}(s, s'_{13}) + \frac{G_{\theta \phi}(s'_{13})b_{\theta \phi}(s'_{13})}{1 - B_{\theta \phi}(s'_{13})} \frac{3}{2} \alpha_{\theta \phi}(s, s'_{13}) \right).$$  
(10)

The connection between $s'_{12}$ and $s'_{13}$ is [29]:

$$s'_{13} = m_1^2 + m_2^2 - \frac{(s'_{12} + m_3^2 - s)(s'_{12} + m_1^2 - m_2^2)}{2s'_{12}} \pm$$

$$\pm \frac{z}{2s'_{12}} \times \sqrt{(s'_{12} - (m_1 + m_2)^2)(s'_{12} - (m_1 - m_2)^2)} \times \sqrt{(s'_{12} - (\sqrt{s} + m_3)^2)(s'_{12} - (\sqrt{s} - m_3)^2)}.$$  
(11)
The formula for $s'_{23}$ is similar to (11) with replaced by $z \to -z$. Thus $A_{1b}(s, s'_{13}) + A_{1b}(s, s'_{23})$ must be replaced by $2A_{1b}(s, s'_{13})$. $\Lambda_J(i, k)$ is the cutoff at the large value of $s_{ik}$, which determines the contribution from small distances.

The construction of the approximate solution of coupled system equations is based on the extraction of the leading singularities which are close to the region $s_{ik} = (m_i + m_k)^2$ [29].

We consider the approximation, which corresponds to the single interaction of the all three particles (two-particle and triangle singularities) and neglecting all the weaker ones.

The functions $\alpha_J(s, s_{ik})$ are the smooth functions of $s_{ik}$ as compared with the singular part of the amplitude, hence it can be expanded in a series at the singular point and only the first term of this series should be employed further. As $s_0$ it is convenient to take the middle point of physical region of the Dalitz plot in which $z = 0$. In this case we get from (11) $s_{ik} = s_0 = \frac{s + m_i^2 + m_k^2 + m_{2k}^2}{m_{i2} + m_{1k} + m_{2k}}$, where $m_{ik} = \frac{m_i + m_k}{2}$. We define functions $\alpha_J(s, s_{ik})$ and $b_J(s_{ik})$ at the point $s_0$. Such a choice of point $s_0$ allows us to replace integral equations (5) by the algebraic couple equations for the state $\Sigma_b^+$:

$$
\begin{align*}
\alpha_1(s, s_0) &= \lambda + \frac{1}{2} \alpha_{1b}(s, s_0) I_{11b}(s, s_0) \frac{b_{10}(s_0)}{b_{10}(s_0)} + \frac{3}{2} \alpha_{0b}(s, s_0) I_{10b}(s, s_0) \frac{b_{10}(s_0)}{b_{10}(s_0)} \\
\alpha_{1b}(s, s_0) &= \lambda + \alpha_1(s, s_0) I_{11b}(s, s_0) \frac{b_{10}(s_0)}{b_{10}(s_0)} \\
& \quad - \frac{1}{2} \alpha_{1b}(s, s_0) I_{11b}(s, s_0) \frac{b_{10}(s_0)}{b_{10}(s_0)} + \frac{3}{2} \alpha_{0b}(s, s_0) I_{10b}(s, s_0) \frac{b_{10}(s_0)}{b_{10}(s_0)} \\
\alpha_{0b}(s, s_0) &= \lambda + \alpha_1(s, s_0) I_{01b}(s, s_0) \frac{b_{10}(s_0)}{b_{10}(s_0)} \\
& \quad + \frac{1}{2} \alpha_{1b}(s, s_0) I_{01b}(s, s_0) \frac{b_{10}(s_0)}{b_{10}(s_0)} + \frac{1}{2} \alpha_{0b}(s, s_0) I_{01b}(s, s_0).
\end{align*}
$$

The function $I_{J_1J_2}(s, s_0)$ takes into account singularity which corresponds to the simultaneous vanishing of all propagators in the triangle diagram.

$$
I_{J_1J_2}(s, s_0) = \int \frac{dJ_J}{(m_i + m_k)^2} \frac{ds'_{ik} dJ_{J_1}(s'_{ik}) G_J^2(s'_{ik})}{\pi} \int \frac{dz}{2} \frac{1}{1 - B_{J_2}(s_{ik})}
$$

The $G_J(s_{ik})$ functions have the smooth dependence from energy $s_{ik}$ [27] therefore we suggest them as constants. The parameters of model: $g_J$ vertex constant and $\lambda_J$ cutoff parameter are chosen dimensionless.

$$
g_J = \frac{m_i + m_k}{2\pi} G_J, \quad \lambda_J = \frac{4\Lambda_J}{(m_i + m_k)^2}.
$$

Here $m_i$ and $m_k$ are quark masses in the intermediate state of the quark loop. We calculate the coupled system of equations and can determine the mass values of the $\Sigma_b^+$ state. We calculate a pole in $s$ which corresponds to the bound state of the three quarks.

By analogy with $\Sigma_b^+$-hyperon we obtain the systems of equations for the reduced amplitudes of all bottom baryons (Appendix B).

The solutions of the coupled system of equations are considered as:
\[ \alpha_J = \frac{F_J(s, \lambda_J)}{D(s)}, \]

where the zeros of the \( D(s) \) determinate the masses of bound states of baryons. \( F_J(s, \lambda_J) \) are the functions of \( s \) and \( \lambda_J \). The functions \( F_J(s, \lambda_J) \) determine the contributions of subamplitudes to the baryon amplitude.

**IV. Calculation results.**

The quark masses \( (m_u = m_d = m, m_s \text{ and } m_c) \) are given similar to the our paper ones [25]: \( m = 0.495 \text{ GeV}, m_s = 0.77 \text{ GeV} \) and \( m_c = 1.655 \text{ GeV} \). The strange quark mass is heavier than the strange quark mass in the some quark models [7 – 11, 23, 27]. This value of strange quark mass allows us to describe the spectroscopy of \( S \)-wave charmed baryons well [25]. We use only two new parameters as compared with the \( S \)-wave light baryons [23, 24]. The parameters of model are the cutoff energy parameters \( \lambda_q = 10.7, \lambda_c = 6.5 \) for the light \( u, d, s \) and charmed quarks, the vertex constants \( g_0 = 0.70, g_1 = 0.55 \), for the light diquarks with \( J^P = 0^+, 1^+ \) and \( g_c = 0.857 \) for the charmed diquarks. \( \lambda_q Q = \frac{1}{4} (\sqrt{\lambda_q} + \sqrt{\lambda_Q})^2 \) are chosen \( (q = u, d, s, Q = c) \).

In the present paper we have used two new parameters: the cutoff of the \( bb \) diquark \( \lambda_b = 5.4 \) and the coupling constant \( g_b = 1.03 \). These values have been determined by the \( b \)-baryon masses: \( M_{\Sigma_b^{q+}} = 5.808 \text{ GeV} \) and \( M_{\Sigma_b^{s+}} = 5.829 \text{ GeV} \). In order to fix \( m_b = 4.840 \text{ GeV} \) we use the \( b \)-baryon masses \( M_{\Sigma_b^{q+}} = 5.829 \text{ GeV} \). We represent the masses of all \( S \)-wave bottom baryons in the Tables I, II. The calculated mass value \( M_{\Lambda_b^{q+}} = 5.624 \text{ GeV} \) is equal to the experimental data [30], the mass value \( M_{\Xi_b^{q+}} = 5.761 \text{ GeV} \) is close to the experimental one [5]. But the more precise CDF mass (Table I) lies close to a prediction of Ref. [13]. We neglect with the mass distinction of \( u \) and \( d \) quarks. The estimation of the theoretical error on the bottom baryon masses is \( 2 - 5 \text{ MeV} \). This result was obtained by the choice of model parameters.

**V. Conclusion.**

In a strongly bound systems, which include the light quarks, where \( p/m \sim 1 \), the approximation by nonrelativistic kinematics and dynamics is not justified.

In our paper the relativistic description of three particles amplitudes of bottom baryons are considered. We take into account the \( u, d, s, c, b \) quarks. The mass spectrum of \( S \)-wave bottom baryons with one, two and three \( b \) quarks is considered. We use only two new parameters for the calculation of 23 baryon masses. The other model parameters in the our papers [23 – 25] are given. The charge-averaged hyperfine splitting between the \( J = \frac{1}{2} \) and \( J = \frac{3}{2} \) states predicted from that for charmed particles is similar to the Ref. [31].

In their paper the spin-averaged mass of the states \( \Xi_b \text{ and } \Xi_b^* \) is predicted to lie around \( 150 - 160 \text{ MeV} \) above \( M_{\Xi_b} \), while the hyperfine splitting between \( \Xi_b \text{ and } \Xi_b^* \) is predicted to lie in the rough range of 20 to 30 MeV.

We have predicted the masses of baryons containing \( b \) quarks using the coupled-channel formalism. We believe that the prediction for the \( S \)-wave bottom baryons based on the relativistic kinematics and dynamics allows as to take into account the relativistic corrections. In our consideration the bottom baryon masses are heavier than the masses in the other quark models [11, 31 – 34]. In our model the spin-averaged mass of the states \( \Xi_b \text{ and } \Xi_b^* \) is predicted to lie around to \( 250 \text{ MeV} \) above \( M_{\Xi_b} \). The
relativistic corrections are particularly important for the splitting between $\Omega_b^+$ and $\Omega_b$ baryons.

We will be able to calculate the $P$-wave bottom baryons in our approach [35, 36] using the new experimental data. The interesting opinions with the $S$-matrix singularities in Ref. [37] are given.

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Appendix A. Integral equations for three-particle amplitudes $A_{J^P}(s, s_{ik})$ of S-wave bottom baryons.

The $J^P = \frac{3}{2}^+$ multiplet.

1. Baryons $\Sigma_b, \Omega_{sb}, \Omega_{cbb}, \Xi_{bb}, \Omega_{sb}, \Omega_{cbb}$.
   The wave functions: $\varphi = \{x \uparrow x \uparrow y \uparrow \}$.
   \[
   \begin{align*}
   A_{1^+}(s, s_{12}) &= \lambda b_{1^+}(s_{12})L_{1^+}(s_{12}) + K_{1^+}(s_{12}) [A_{1^+}(s, s_{13}) + A_{1^+}(s, s_{23})] \\
   A_{1^+}(s, s_{13}) &= \lambda b_{1^+}(s_{13})L_{1^+}(s_{13}) + K_{1^+}(s_{13}) [A_{1^+}(s, s_{12}) + A_{1^+}(s, s_{23})].
   \end{align*}
   \]  
   (A1)

   Here: for the $\Sigma_{bb}$: $x = q$ ($q = u, d$), $y = b$; for the $\Omega_{sb}$: $x = s$, $y = b$; for the $\Omega_{cbb}$: $x = c$, $y = b$; for the $\Xi_{bb}$: $x = b$, $y = q$; for the $\Omega_{sb}$: $x = b$, $y = s$; for the $\Omega_{cbb}$: $x = b$, $y = c$.

2. Baryons $\Xi_{sb}, \Xi_{cb}, \Omega_{scb}$.
   The wave functions: $\varphi = \{x \uparrow y \uparrow z \uparrow \}$.
   \[
   \begin{align*}
   A_{1^+}(s, s_{12}) &= \lambda b_{1^+}(s_{12})L_{1^+}(s_{12}) + K_{1^+}(s_{12}) [A_{1^+}(s, s_{13}) + A_{1^+}(s, s_{23})] \\
   A_{1^+}(s, s_{13}) &= \lambda b_{1^+}(s_{13})L_{1^+}(s_{13}) + K_{1^+}(s_{13}) [A_{1^+}(s, s_{12}) + A_{1^+}(s, s_{23})] \\
   A_{1^+}(s, s_{23}) &= \lambda b_{1^+}(s_{23})L_{1^+}(s_{23}) + K_{1^+}(s_{23}) [A_{1^+}(s, s_{12}) + A_{1^+}(s, s_{13})].
   \end{align*}
   \]  
   (A2)

   Here: for the $\Xi_{sb}$: $x = q$, $y = s$, $z = b$; for the $\Xi_{cb}$: $x = q$, $y = c$, $z = b$; for the $\Omega_{scb}$: $x = s$, $y = c$, $z = b$.

3. Baryon $\Omega_{bbb}$.
   The wave functions: $\varphi = \{b \uparrow b \uparrow b \uparrow \}$.
   \[
   A_{1^+}(s, s_{12}) = \lambda b_{1^+}(s_{12})L_{1^+}(s_{12}) + K_{1^+}(s_{12}) [A_{1^+}(s, s_{13}) + A_{1^+}(s, s_{23})].
   \]  
   (A3)

The $J^P = \frac{1}{2}^+$ multiplet.

1. Baryons $\Sigma_b, \Lambda_b, \Omega_{sbb}, \Omega_{cbb}, \Xi_{bb}, \Omega_{sb}, \Omega_{cbb}$.
   The wave functions:
   for the $\Sigma_b$:
   $\varphi_{\Sigma^+_b} = \sqrt{\frac{1}{3}} \{u \uparrow d \uparrow b \downarrow\} - \sqrt{\frac{1}{6}} \{u \uparrow d \downarrow b \uparrow\} - \sqrt{\frac{1}{6}} \{u \downarrow d \uparrow b \uparrow\};$
   for the $\Lambda_b$:
   $\varphi_{\Lambda_b} = \sqrt{\frac{1}{2}} \{u \uparrow d \downarrow b \uparrow\} - \sqrt{\frac{1}{2}} \{u \downarrow d \uparrow b \uparrow\};$
   for the $\Omega_{sbb}, \Omega_{cbb}, \Xi_{bb}, \Omega_{sb}, \Omega_{cbb}$:
   $\varphi = \sqrt{\frac{1}{3}} \{x \uparrow x \uparrow y \downarrow\} - \sqrt{\frac{1}{3}} \{x \uparrow x \downarrow y \uparrow\};$
   here: for the $\Omega_{sbb}$: $x = s$, $y = b$; for the $\Omega_{cbb}$: $x = c$, $y = b$; for the $\Xi_{bb}$: $x = b$, $y = q$; for the $\Omega_{sb}$: $x = b$, $y = s$; for the $\Omega_{cbb}$: $x = b$, $y = c$.  


\[
\begin{aligned}
A_x(s, s_{12}) &= \lambda b_x(s_{12})L_x(s_{12}) + K_x(s_{12}) \left[ \frac{1}{4}A_y(s, s_{13}) + \frac{3}{4}A_z(s, s_{13}) + \frac{1}{4}A_y(s, s_{23}) + \frac{3}{4}A_z(s, s_{23}) \right], \\
A_y(s, s_{13}) &= \lambda b_y(s_{13})L_y(s_{13}) + K_y(s_{13}) \left[ \frac{1}{2}A_x(s, s_{12}) - \frac{1}{4}A_y(s, s_{12}) + \frac{3}{4}A_z(s, s_{12}) + \frac{1}{4}A_y(s, s_{23}) + \frac{3}{4}A_z(s, s_{23}) \right], \\
A_z(s, s_{23}) &= \lambda b_z(s_{23})L_z(s_{23}) + K_z(s_{23}) \left[ \frac{1}{2}A_x(s, s_{12}) + \frac{1}{4}A_y(s, s_{12}) + \frac{1}{4}A_z(s, s_{12}) + \frac{1}{4}A_y(s, s_{13}) + \frac{1}{4}A_z(s, s_{13}) \right].
\end{aligned}
\]  

(A4)

Here: for the $\Sigma_{bb}$: $x = 1^{qq}$, $y = 1^{qb}$, $z = 0^{qb}$; for the $\Lambda_{b}$: $x = 0^{qq}$, $y = 0^{qb}$, $z = 1^{qb}$; for the $\Omega_{ssb}$: $x = 1^{ss}$, $y = 1^{qb}$, $z = 0^{qb}$; for the $\Omega_{ccb}$: $x = 1^{cc}$, $y = 1^{cb}$, $z = 0^{cb}$; for the $\Xi_{bb}$: $x = 1^{bb}$, $y = 1^{qb}$, $z = 0^{qb}$; for the $\Omega_{sbb}$: $x = 1^{bb}$, $y = 1^{sb}$, $z = 0^{sb}$; for the $\Omega_{cbb}$: $x = 1^{bb}$, $y = 1^{cb}$, $z = 0^{cb}$.

2. Baryons $\Xi_{bb}^A$, $\Xi_{bb}^S$, $\Lambda_{sbb}^A$, $\Lambda_{sbb}^S$, $\Xi_{cb}^A$, $\Xi_{cb}^S$.

The wave functions:

for the $\Xi_{bb}^A$, $\Lambda_{sbb}^A$, $\Xi_{cb}^A$:

\[
\varphi = \sqrt{\frac{1}{2}} \{ x \uparrow y \uparrow z \downarrow \} - \sqrt{\frac{1}{2}} \{ x \uparrow y \downarrow z \uparrow \};
\]

here: for the $\Xi_{bb}^A$: $x = b$, $y = s$, $z = q$; for the $\Lambda_{sbb}^A$: $x = b$, $y = c$, $z = s$; for the $\Xi_{cb}^A$: $x = b$, $y = c$, $z = q$.

For the $\Xi_{bb}^S$, $\Lambda_{sbb}^S$, $\Xi_{cb}^S$:

\[
\varphi = \sqrt{\frac{1}{2}} \{ x \uparrow y \uparrow z \downarrow \} - \sqrt{\frac{1}{2}} \{ x \uparrow y \downarrow z \uparrow \} - \sqrt{\frac{1}{2}} \{ x \downarrow y \uparrow z \uparrow \};
\]

here: for the $\Xi_{bb}^S$: $x = q$, $y = s$, $z = b$; for the $\Lambda_{sbb}^S$: $x = s$, $y = c$, $z = b$; for the $\Xi_{cb}^S$: $x = q$, $y = c$, $z = b$. 
\[
\begin{aligned}
A_x(s, s_{12}) &= \lambda b_x(s_{12}) L_x(s_{12}) + K_x(s_{12}) \left[ \frac{1}{8} A_y(s, s_{13}) + \frac{1}{8} A_z(s, s_{13}) + \right.
\frac{3}{8} A_v(s, s_{13}) + \frac{3}{8} A_w(s, s_{13}) + \right.
\frac{3}{8} A_v(s, s_{23}) + \frac{3}{8} A_w(s, s_{23}) 
\end{aligned}
\]

\[
A_y(s, s_{13}) = \lambda b_y(s_{13}) L_y(s_{13}) + K_y(s_{13}) \left[ \frac{1}{2} A_x(s, s_{12}) - \frac{1}{4} A_y(s, s_{12}) + \right.
\frac{1}{2} A_v(s, s_{12}) + \frac{1}{2} A_x(s, s_{23}) - \frac{1}{4} A_y(s, s_{23}) + \frac{3}{4} A_v(s, s_{23}) \right]
\]

\[
A_z(s, s_{23}) = \lambda b_z(s_{23}) L_z(s_{23}) + K_z(s_{23}) \left[ \frac{1}{2} A_x(s, s_{12}) - \frac{1}{4} A_y(s, s_{12}) + \right.
\frac{1}{2} A_v(s, s_{12}) + \frac{1}{2} A_x(s, s_{23}) + \frac{1}{4} A_y(s, s_{23}) + \frac{3}{4} A_v(s, s_{23}) \right] \tag{A5}
\]

\[
A_v(s, s_{13}) = \lambda b_v(s_{13}) L_v(s_{13}) + K_v(s_{13}) \left[ \frac{1}{2} A_x(s, s_{12}) + \frac{1}{2} A_z(s, s_{12}) + \right.
\frac{1}{2} A_v(s, s_{12}) + \frac{1}{2} A_z(s, s_{23}) + \frac{1}{2} A_v(s, s_{23}) \right]
\]

\[
A_w(s, s_{23}) = \lambda b_w(s_{23}) L_w(s_{23}) + K_w(s_{23}) \left[ \frac{1}{2} A_x(s, s_{12}) + \frac{1}{4} A_y(s, s_{12}) + \right.
\frac{1}{2} A_v(s, s_{12}) + \frac{1}{2} A_x(s, s_{23}) + \frac{1}{4} A_y(s, s_{23}) + \frac{3}{4} A_v(s, s_{23}) \right]
\]

Here: for the Ξ^{A}_{bb}: x = 0^{qs}, y = 0^{qb}, z = 0^{sb}, v = 1^{qb}, w = 1^{sb}; for the Ξ^{A}_{cb}: x = 0^{qc}, y = 0^{qb}, z = 0^{sb}, v = 1^{qb}, w = 1^{cb}; for the Λ^{A}_{sbc}: x = 0^{qc}, y = 0^{sb}, z = 0^{sb}, v = 1^{qb}, w = 1^{sb}; for the Ξ^{S}_{sb}: x = 1^{qs}, y = 1^{qb}, z = 1^{sb}, v = 0^{qb}, w = 0^{sb}; for the Ξ^{S}_{cb}: x = 1^{qc}, y = 1^{qb}, z = 1^{cb}, v = 0^{qb}, w = 0^{cb}.

Appendix B. Couple systems of approximate equations for the S-wave bottom baryons.

The \(J^P = \frac{3^+}{2} \) multiplet.

1. Baryons \(\Sigma_b, \Omega_{sbb}, \Omega_{cbb}, \Xi_{bb}, \Omega_{sbb}, \Omega_{cbb} \).

\[
\begin{aligned}
\alpha_1^x(s, s_0) &= \lambda + 2 \alpha_1^v(s, s_0) I_{1^+1^+}(s, s_0) \frac{b_{1^+1^+}(s_0)}{b_{1^+1^+}(s_0)} \\
\alpha_1^v(s, s_0) &= \lambda + \alpha_1^x(s, s_0) I_{1^+1^+}(s, s_0) \frac{b_{1^+1^+}(s_0)}{b_{1^+1^+}(s_0)} + \alpha_1^v(s, s_0) I_{1^+1^+}(s, s_0) \tag{A6}
\end{aligned}
\]

Here: for the \(\Sigma_b\): \(x = q, y = b\); for the \(\Omega_{sbb}\): \(x = s, y = b\); for the \(\Omega_{cbb}\): \(x = c, y = b\); for the \(\Xi_{bb}\): \(x = b, y = q\); for the \(\Omega_{sbb}\): \(x = b, y = s\); for the \(\Omega_{cbb}\): \(x = b, y = c\).
2. Baryons \( \Xi_{sb}, \Xi_{cb}, \Omega_{sbb} \).

\[
\begin{align*}
\alpha_{1\pi}(s, s_0) &= \lambda + \alpha_{1\pi}(s, s_0) I_{1\pi1\pi}(s, s_0) \frac{b_{1\pi}(s_0)}{b_{1\pi}(s_0)} + \alpha_{1\pi}(s, s_0) I_{1\pi1\pi}(s, s_0) \frac{b_{1\pi}(s_0)}{b_{1\pi}(s_0)} \\
\alpha_{1\pi}(s, s_0) &= \lambda + \alpha_{1\pi}(s, s_0) I_{1\pi1\pi}(s, s_0) \frac{b_{1\pi}(s_0)}{b_{1\pi}(s_0)} + \alpha_{1\pi}(s, s_0) I_{1\pi1\pi}(s, s_0) \frac{b_{1\pi}(s_0)}{b_{1\pi}(s_0)} \\
\alpha_{1\pi}(s, s_0) &= \lambda + \alpha_{1\pi}(s, s_0) I_{1\pi1\pi}(s, s_0) \frac{b_{1\pi}(s_0)}{b_{1\pi}(s_0)} + \alpha_{1\pi}(s, s_0) I_{1\pi1\pi}(s, s_0) \frac{b_{1\pi}(s_0)}{b_{1\pi}(s_0)}.
\end{align*}
\] (A7)

Here: for the \( \Xi_{sb} \): \( x = q, y = s, z = b \); for the \( \Xi_{cb} \): \( x = q, y = c, z = b \); for the \( \Omega_{sbb} \): \( x = s, y = c, z = b \).

3. Baryon \( \Omega_{bbb} \).

\[
\alpha_{1\pi}(s, s_0) = \lambda + 2 \alpha_{1\pi}(s, s_0) I_{1\pi1\pi}(s, s_0).
\] (A8)

The \( J^P = \frac{1+}{2} \) multiplet.

1. Baryons \( \Sigma_b, \Lambda_b, \Omega_{sbb}, \Omega_{cbb}, \Xi_{bb}, \Omega_{sbb}, \Omega_{cbb} \).

\[
\begin{align*}
\alpha_x(s, s_0) &= \lambda + \frac{1}{2} \alpha_y(s, s_0) I_{xy}(s, s_0) \frac{b_x(s_0)}{b_y(s_0)} + \frac{3}{2} \alpha_z(s, s_0) I_{xz}(s, s_0) \frac{b_z(s_0)}{b_y(s_0)} \\
\alpha_y(s, s_0) &= \lambda + \alpha_x(s, s_0) I_{yx}(s, s_0) \frac{b_x(s_0)}{b_y(s_0)} - \frac{1}{2} \alpha_y(s, s_0) I_{yy}(s, s_0) + \frac{3}{2} \alpha_z(s, s_0) I_{yz}(s, s_0) \frac{b_z(s_0)}{b_y(s_0)} \\
\alpha_z(s, s_0) &= \lambda + \alpha_x(s, s_0) I_{xz}(s, s_0) \frac{b_x(s_0)}{b_z(s_0)} + \frac{1}{2} \alpha_y(s, s_0) I_{zy}(s, s_0) \frac{b_y(s_0)}{b_z(s_0)} + \frac{1}{2} \alpha_x(s, s_0) I_{zz}(s, s_0).
\end{align*}
\] (A9)

Here: for the \( \Sigma_b \): \( x = 1^q, y = 1^q, z = 0^b \); for the \( \Lambda_b \): \( x = 0^q, y = 0^q, z = 1^b \); for the \( \Omega_{sbb} \): \( x = 1^s, y = 1^b, z = 0^b \); for the \( \Omega_{cbb} \): \( x = 1^c, y = 1^b, z = 0^b \); for the \( \Xi_{bb} \): \( x = 1^b, y = 1^b, z = 0^b \); for the \( \Omega_{sbb} \): \( x = 1^b, y = 1^b, z = 0^b \); for the \( \Omega_{cbb} \): \( x = 1^b, y = 1^b, z = 0^b \).
2. Baryons $\Xi_{sb}^A$, $\Xi_{sb}^S$, $\Lambda_{scb}^A$, $\Lambda_{scb}^S$, $\Xi_{cb}^A$, $\Xi_{cb}^S$.

\[
\begin{align*}
\alpha_x(s, s_0) &= \lambda + \frac{1}{4} \alpha_y(s, s_0) I_{x\beta}(s, s_0) \frac{b_u(s_0)}{b_x(s_0)} + \frac{1}{4} \alpha_z(s, s_0) I_{x\beta}(s, s_0) \frac{b_u(s_0)}{b_x(s_0)} \\
&\quad + \frac{3}{4} \alpha_v(s, s_0) I_{x\nu}(s, s_0) \frac{b_u(s_0)}{b_x(s_0)} + \frac{3}{4} \alpha_w(s, s_0) I_{x\nu}(s, s_0) \frac{b_u(s_0)}{b_x(s_0)} \\
\alpha_y(s, s_0) &= \lambda + \alpha_x(s, s_0) I_{y\beta}(s, s_0) \frac{b_u(s_0)}{b_y(s_0)} - \frac{1}{2} \alpha_z(s, s_0) I_{y\beta}(s, s_0) \frac{b_u(s_0)}{b_y(s_0)} \\
&\quad + \frac{3}{2} \alpha_w(s, s_0) I_{y\nu}(s, s_0) \frac{b_u(s_0)}{b_y(s_0)} \\
\alpha_z(s, s_0) &= \lambda + \alpha_x(s, s_0) I_{z\beta}(s, s_0) \frac{b_u(s_0)}{b_z(s_0)} - \frac{1}{2} \alpha_y(s, s_0) I_{z\beta}(s, s_0) \frac{b_u(s_0)}{b_z(s_0)} \\
&\quad + \frac{3}{2} \alpha_v(s, s_0) I_{z\nu}(s, s_0) \frac{b_u(s_0)}{b_z(s_0)} \\
\alpha_v(s, s_0) &= \lambda + \alpha_x(s, s_0) I_{v\beta}(s, s_0) \frac{b_u(s_0)}{b_v(s_0)} + \frac{1}{2} \alpha_z(s, s_0) I_{v\beta}(s, s_0) \frac{b_u(s_0)}{b_v(s_0)} \\
&\quad + \frac{1}{2} \alpha_w(s, s_0) I_{v\nu}(s, s_0) \frac{b_u(s_0)}{b_v(s_0)} \\
\alpha_w(s, s_0) &= \lambda + \alpha_x(s, s_0) I_{w\beta}(s, s_0) \frac{b_u(s_0)}{b_w(s_0)} + \frac{1}{2} \alpha_y(s, s_0) I_{w\beta}(s, s_0) \frac{b_u(s_0)}{b_w(s_0)} \\
&\quad + \frac{1}{2} \alpha_v(s, s_0) I_{w\nu}(s, s_0) \frac{b_u(s_0)}{b_w(s_0)}.
\end{align*}
\]

(A10)

Here: for the $\Xi_{sb}^A$: $x = 0^{q_s}$, $y = 0^{q_b}$, $z = 0^{s_b}$, $v = 1^{q_b}$, $w = 1^{s_b}$; for the $\Xi_{cb}^A$: $x = 0^{q_c}$, $y = 0^{q_b}$, $z = 0^{s_b}$, $v = 1^{q_b}$, $w = 1^{s_b}$; for the $\Lambda_{scb}^A$: $x = 0^{s_c}$, $y = 0^{s_b}$, $z = 0^{c_b}$, $v = 1^{s_b}$, $w = 1^{c_b}$; for the $\Xi_{sb}^S$: $x = 1^{q_s}$, $y = 1^{q_b}$, $z = 1^{s_b}$, $v = 0^{q_b}$, $w = 0^{s_b}$; for the $\Xi_{cb}^S$: $x = 1^{q_c}$, $y = 1^{q_b}$, $z = 1^{s_b}$, $v = 0^{q_b}$, $w = 0^{s_b}$; for the $\Lambda_{scb}^S$: $x = 1^{s_c}$, $y = 1^{s_b}$, $z = 1^{c_b}$, $v = 0^{s_b}$, $w = 0^{c_b}$.
Table I. Bottom baryon masses of multiplet $\frac{1}{2}^+$.  
Parameters of model: quark masses $m_{u,d} = 495 \text{MeV}$, $m_s = 770 \text{MeV}$, $m_c = 1655 \text{MeV}$, $m_b = 4840 \text{MeV}$; cutoff parameters: $\lambda_q = 10.7 \ (q = u, d, s)$, $\lambda_c = 6.5$, $\lambda_b = 5.4$; gluon coupling constants: $g_0 = 0.70$, $g_1 = 0.55$ with $J^p = 0^+$ and $1^+$, $g_c = 0.857$, $g_b = 1.03$.

| Baryon | Mass (GeV) | Mass (GeV) (exp.) |
|--------|------------|-------------------|
| $\Sigma_b$ | 5.808 | 5.808 |
| $\Lambda_b$ | 5.624 | 5.624 |
| $\Xi_b$ | 5.761 | 5.793 |
| $\Xi_b^-$ | 6.007 | - |
| $\Omega_{s ub}$ | 6.120 | - |
| $\Omega_{c ub}$ | 6.789 | - |
| $\Xi_{cb}$ | 6.818 | - |
| $\Lambda_{s cb}$ | 6.798 | - |
| $\Lambda_{c sb}$ | 6.836 | - |
| $\Omega_{c cb}$ | 7.943 | - |
| $\Xi_{bb}$ | 10.045 | - |
| $\Omega_{s bb}$ | 9.999 | - |
| $\Omega_{c bb}$ | 11.089 | - |

Table II. Bottom baryon masses of multiplet $\frac{3}{2}^+$.  

| Baryon | Mass (GeV) | Mass (GeV) (exp.) |
|--------|------------|-------------------|
| $\Sigma_b$ | 5.829 | 5.829 |
| $\Xi_b$ | 6.066 | - |
| $\Omega_{s ub}$ | 6.220 | - |
| $\Xi_{cb}$ | 6.863 | - |
| $\Omega_{s cb}$ | 6.914 | - |
| $\Omega_{c cb}$ | 7.973 | - |
| $\Xi_{bb}$ | 10.104 | - |
| $\Omega_{s bb}$ | 10.126 | - |
| $\Omega_{c bb}$ | 11.123 | - |
| $\Omega_{c b}$ | 14.197 | - |

Table III. Coefficients of Gheu-Mandelstam functions.

| $J$ | $\alpha_J$ | $\beta_J$ | $\delta_J$ |
|-----|----------|----------|----------|
| $1^+$ | $\frac{1}{3}$ | $\frac{4 m_m m_k}{3 (m_i + m_k)^2} - \frac{1}{6}$ | $-\frac{1}{6} (m_i - m_k)^2$ |
| $0^+$ | $\frac{1}{2}$ | $-\frac{1}{2} (m_i - m_k)^2$ | $0$ |

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