On the Penetrating Nature of Electromagnetic Probes

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Abstract

We establish the extent to which photons and dileptons are penetrating probes of hot hadronic matter by computing mean free paths. Absorption is indeed negligible for real photons as mean free paths are roughly tenths of Ångströms. However, vector dominance effects, which provide rich structure in virtual photon mass spectra, introduce nonnegligible reabsorption and rehadronization resulting in mean free paths of the order tens of femtometers. Consequences are illustrated by considering the net dilepton yield near the rho peak via thermal ππ annihilation from a short-lived hadronic fireball. Results suggest that disappearance of rho in heavy-ion collisions can be due in part to reabsorption/rehadronization rather than solely due to collective effects.
Ever since the suggestion more than twenty years ago\cite{1, 2, 3} that photons and dileptons would make excellent probes of hadronic matter owing to their expected long mean free paths, there has grown a wide body of literature both on the theoretical side\cite{4} assessing various features of production rates and model calculations for yields, and on the experimental side documenting a variety of results in heavy-ion collisions spanning a wide range of energies and systems\cite{5}. Electromagnetic signals have small cross sections of the order $\alpha$ for real photons ($\alpha^2$ for dileptons) times the square of some characteristic hadron size, i.e. 1 fm$^2$, and are therefore rarely produced as compared to hadronic signals with cross sections of order 1–10 times the same characteristic 1 fm$^2$. Once produced, they reinteract with the same small probabilities and consequently travel through and indeed escape the nuclear matter carrying with them valuable kinematical fingerprints of the interior of the fireball. Electromagnetic probes are today considered one of the premier assessment tools for studying possible phase crossover from hadronic matter to prehadronic matter, the so-called quark-gluon plasma. Eventual success of this endeavor is contingent in part on the purity of electromagnetic probes. Up to now, only back-of-the-envelope estimates have been made, so we undertake a more quantitative study with particular emphasis on virtual photons with masses near vector mesons.

We begin by discussing the general features of thermal photon production mechanisms in hadronic matter and later consider dileptons. Real photons in the energy range 500 MeV to a few GeV are produced in thermalized matter via hadronic reactions, most notably, $\pi + \rho \rightarrow \pi + \gamma$ supporting either pion exchange or a resonant $a_1$ contribution\cite{6, 7}. Bremsstrahlung mechanisms dominate the energy region below a few hundred MeV. Given that the photons are on shell, they will neither hadronize nor decay electromagnetically and we need only focus on absorption. Their mean free path, assuming dominance from the reaction $\gamma + \pi \rightarrow a_1 \rightarrow \rho + \pi$, is

$$\langle \lambda \rangle = \frac{1}{\Gamma},$$

(1)

where

$$\Gamma = \frac{1}{8\pi^2 T} \int_{z_0}^{\infty} dz \lambda(s, 0, m_{\pi}^2) K_1(z) \sigma(s)$$

(2)

and

$$\sigma(s) = \frac{3\pi}{k^2} \frac{m_{a_1}^2 \Gamma_{a_1}^2}{(s - m_{a_1}^2)^2 + (m_{a_1} \Gamma_{a_1})^2} \left( \frac{4\pi\alpha}{f_{\rho}^2} \right),$$

(3)
\(k\) is the \(\pi\rho\) center-of-mass momentum, \(z = \sqrt{S/T}\), \(z_0 = (m_\pi + m_\rho)/T\), \(\lambda(x, y, z) = x^2 - 2x(y + z) + (y - z)^2\) — a function which accounts for relative velocity and other kinematical effects, \(f_\rho\) is the “universal” vector-dominance coupling constant for rho \(\simeq 5.1\), and \(\mathcal{K}_1\) is a modified Bessel function. Branching ratios and spin degeneracies have been included to give the particular prefactor appearing in Eq. (3). An energy-dependent \(a_1\) width is used having 400 MeV at the centroid, which gives the following results for the mean free paths: 1.80, 0.08, and 0.02 Å at 100, 150 and 200 MeV temperature respectively. Other contributions can be introduced; for example, \(t\)-channel reactions involving \(a_1\), \(\pi\)-exchange and \(\omega\)-exchange graphs are next-leading candidates, but the inevitable result would be a smaller \(\langle \lambda \rangle\) by at most a factor of 2–3 since the cross sections for these additional processes are somewhat smaller than the one considered here. With mean free paths several orders of magnitude larger than any nuclear system, we safely conclude that real photons penetrate completely the entire spacetime extent of the nuclear reaction.

Photons off their mass shell (virtual photons) are thermally produced most abundantly by \(\pi\pi\) annihilation for masses a few hundred MeV to about 1 GeV and via bremsstrahlung \((\pi\pi \rightarrow \pi\pi\gamma^*)\) as well as other hadronic reactions \((e.g. \pi\rho \rightarrow \pi\gamma^*)\) for invariant mass below about 300 MeV. Virtual photons eventually decay into lepton pairs or possibly back into hadrons. We consider a virtual photon of mass \(M\) produced in the interior of the fireball. Where does it decay? The answer clearly depends on its own lifetime against electromagnetic and/or hadronic decays. The rate for decay comes from two pieces: a purely electromagnetic rate plus a hadronic decay (reverse vector dominance) as follows

\[
\Gamma_{\gamma^*} = \Theta(M - 2m_\ell) \frac{\alpha M}{3} \sqrt{1 - \frac{4m_\ell^2}{M^2}} \left(1 + \frac{2m_\ell^2}{M^2}\right) + \Theta(M - 2m_\pi) \left(\frac{4\pi\alpha}{f_\rho^2}\right) \frac{m_\rho^4}{(M^2 - m_\rho^2)^2 + (m_\rho \Gamma_\rho)^2} \Gamma_\rho, \tag{4}
\]

and we use an energy-dependent \(\rho \rightarrow \pi\pi\) width

\[
\Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{4\pi} \frac{p^3}{3m_\rho^4}, \tag{5}
\]

with \(g_{\rho\pi\pi} = 6.0\) and \(p\) being the center-of-mass momentum of the decay products. Other resonances can also be included in Eq. (4), \(e.g. \omega\) and \(\phi\) open up at the three-pion threshold, \(\rho'\) at the four-pion threshold, and so on. In Fig. [4], we show the inverse width (lifetime) for
FIG. 1: Virtual photon lifetime as a function of invariant mass. Dashed curve includes electromagnetic decays only, while solid curve includes in addition, contributions from $\rho$, $\omega$, $\phi$, and one of the $\rho'$ states.
the virtual photon. We conclude that once produced, the virtual photon carries kinematical information tens of femtometers at least before decaying. It therefore decays, on average, outside the fireball! This does not mean that in-medium hadron properties are lost in favor of vacuum properties (unless, of course, it re-hadronizes outside), it merely sets the scale for decay.

Since virtual photon propagation persists during the entire fireball evolution and beyond, we must ask about the chances that along the way it gets reabsorbed or possibly rehadronizes. The free path as a function of temperature, invariant mass and momentum is the following

\[ \lambda = \frac{v}{\Gamma}, \]  

where \( v = p/E \), and the differential absorption rate \( \Gamma \) is

\[ d\Gamma = g_\pi \frac{d^3p_\pi}{(2\pi)^3} f(E_\pi) \sigma(s) v_{rel} \delta \left( (p + p_\pi)^2 - s \right) ds, \]  

where

\[ v_{rel} = \sqrt{\left( p \cdot p_\pi \right)^2 - M^2} \frac{m_\rho}{E_\pi}, \]

\( g_\pi = 2 \) reflecting the two possible charged states for the entrance-channel pseudoscalar in \( \gamma^*\pi \rightarrow \rho\pi \), \( f \) is taken to be a Boltzmann and, where again, dominance of the \( s \)-channel \( a_1 \)-exchange process is assumed. The cross section is therefore simply obtained by multiplying Eq. (3) by a vector dominance model (VDM) form factor

\[ |F_\rho(M)|^2 = \frac{m_\rho^4}{\left( M^2 - m_\rho^2 \right)^2 + (m_\rho \Gamma_\rho)^2}, \]

and by 2/3 to reflect the fact that a virtual photon has three polarization states as compared with two for an on-shell photon. From Fig. [1] it is clear that maximal effect occurs near the vector poles. So, in Fig [2] we carry out the necessary integration in Eq. (7) and plot the free path for a virtual photon at the rho mass at fixed temperature \( T = 170 \text{ MeV} \) as a function of its momentum. Low momentum \( \gamma^* \)'s have reduced survival probabilities due to the moderate mean free paths ranging from 1 fm to a few tens of femtometers. In experiment, where cuts on dilepton three-momenta are made and bins with a few hundred MeV momentum are analyzed, such reabsorption could play an important role.

Next we average over virtual photon momentum (assumed to be distributed according to a Boltzmann) and arrive at a mean free path at fixed temperature as a function of mass.
FIG. 2: Free path for $\gamma^*$ having mass $m_\rho$ at fixed temperature $T = 170$ MeV.

The expression can be written as

$$\langle \lambda \rangle = \frac{2 e^{-M/T} T (M + T)}{M^2 K_2(M/T)} \frac{1}{T^{\text{abs}}},$$

(10)
where $K_2$ is again a modified Bessel function and the absorption rate is
\[
\Gamma_{\text{abs}} = \frac{g_\gamma T}{8\pi^2 M^2 K_2(M/T)} \int_{z_0}^\infty dz K_1(z) \lambda\left(s, M^2, m_\pi^2\right) \sigma(s). \tag{11}
\]
The lower limit on the integral is $z_0 = \text{MAX}\left[\left(\frac{m_\pi + m_\rho}{T}, \frac{m_\pi + M}{T}\right\right]$ and the cross section is the same as that which was used in Eq. (7). Numerical results are displayed in Fig. 3 as the dashed curve. The limit $M \to 0$ (and $m_\ell \to 0$) provides a consistency check for the results. Since the virtual photon’s spin-averaged cross section is smaller by one-third due to polarization-state differences, the mean free path is greater by the same factor as compared with real photon results. Effects from the $\rho$ have been emphasized, but a straightforward generalization would lead to structure for $\omega$ and $\phi$.

Fig. 3 also includes the mean free path which results from adding to the absorption rate another term which is due to thermal reconversion to hadrons and subsequent hadronic decay. The thermal decay rate is
\[
\Gamma(\gamma^* \to \pi^+\pi^-) = \left(\frac{4\pi\alpha}{f_\rho^2}\right) \frac{m_\rho^4}{(M^2 - m_\rho^2)^2 + (m_\rho\Gamma_\rho)^2} \Gamma_\rho K_2(M/T). \tag{12}
\]
The net mean free path is reported as the solid curve which, at the rho peak, is about 10 fm. This is small enough to have important consequences for dilepton production.

We next illustrate the consequences of reabsorption and rehadronization of virtual photons by considering the net yield from thermal $\pi^+\pi^- \to e^+e^-$ production near the rho peak. Since the sought-after $\gamma^*$ propagates for 10 fm/c or longer, partly within the fireball and partly in free space, its survival probability could be diminished. The net (thermal) yield from a fireball can be approximated as the “bare” production rate at average temperature $\langle T \rangle = 170$ MeV, times an overall four-volume $V\tau = 3 \times 10^4$ fm$^4$, times a survival probability $N$,
\[
\left(\frac{dN}{dM}\right)_{\text{net}} = \left(\frac{dR}{dM^2}\right)_{\text{bare}} \left[2MV\tau\right] N. \tag{13}
\]
This approximation is only intended to benchmark the size of the effects and not for direct comparison to heavy-ion data. A dynamical model would be useful.

The bare rate describes dilepton production via $\pi^+\pi^- \to \rho^0 \to \gamma^* \to e^+e^-$.
\[
\left(\frac{dR}{dM^2}\right)_{\text{bare}} = \frac{\sigma(M)}{2(2\pi)^4} \sqrt{1 - \frac{4m_\pi^2}{M^2}} T M K_1(M/T), \tag{14}
\]
FIG. 3: Mean free path for $\gamma^*$ as a function of mass at fixed temperature $T = 170$ MeV.

The survival probability is approximated (using a fixed temperature for simplicity) as

$$\sigma(M) = \frac{4\pi}{3} \frac{\alpha^2}{M^2} \sqrt{1 - \frac{4m^2}{M^2}} \sqrt{1 - \frac{4m^2}{m_f^2}} \left(1 + \frac{2m^2}{M^2}\right) |F_\rho(M)|^2. \quad (15)$$

The survival probability is approximated (using a fixed temperature for simplicity) as
\[ N = \begin{cases} \left( \frac{1}{t_h} \int_0^{t_h} dt e^{-\Gamma_{\text{tot}} t} \right) \left[ e^{\frac{r_{\text{d}} t_d}{(t_d - t_h)}} \int_{t_h}^{t_d} dt e^{-\Gamma_{\text{d}} t} \right] & t_h < t_d \\ \left( \frac{1}{t_d} \int_0^{t_d} dt e^{-\Gamma_{\text{tot}} t} \right) \left[ e^{\frac{r_{\text{d}} t_h}{(t_d - t_h)}} \int_{t_h}^{t_d} dt e^{-\Gamma_{\text{d}} t} \right] & t_d < t_h, \end{cases} \]  

(16)

where \( \Gamma_{\text{tot}} \) is the sum of the thermal absorption and thermal decay rates from Eqs. (11) and (12), \( t_h \) is the time spent by the virtual photon within the hot hadronic matter, \( t_d \) is the lifetime (in the frame of the fireball) of the virtual photon \( [t_d = \gamma \tau_d] \), where \( \tau_d \) is the lifetime (plotted in Fig. 1) and the mean Lorentz \( \gamma \) factor can be written as

\[ \gamma = \frac{3T}{M} + \frac{K_1(M/T)}{K_2(M/T)}. \]  

(17)

Finally, \( \Gamma_{\text{d}} \) is the free-space decay rate into \( \pi^+\pi^- \)—the primary reason for loss.

We plot in Fig. 4 the yield of thermal lepton pairs emphasizing the rho region. The bare-yield curve results from setting the survival probability to unity while the net yield includes loss effects and uses \( t_h = 10 \text{ fm}/c \) in the calculation of the survival probability. Owing to the relative smallness of the absorption rate as compared with the hadronization rate, the result is not sensitive to the value of \( t_h \). One can see that loss effects account for a 40% suppression at the rho peak.

Typical model calculations up to now have used “bare” production alone, assuming that losses of any kind were completely negligible. As models evolve and become more sophisticated, tens of percent effects must be included. The message here is that vector dominance helps as it provides rich structure in the invariant mass spectra and provides opportunity to do spectroscopy looking for collective effects, but at the same time it hurts in the sense that part of the rho peak is removed. Drawing conclusions from heavy-ion dilepton production data regarding the extent to which the rho is melted or disappeared due to collision broadening[15, 16, 17] or other collective nuclear effects is unfortunately much more difficult in light of these results.

We remark that this sort of suppression would not be seen in muon pair spectra near the \( J/\psi \) mass region since the \( D\bar{D} \) threshold is too high to allow rehadronization.

Electromagnetic probes are exceptional in terms of their ability to penetrate and probe hot hadronic matter. We have argued that real photon signals suffer practically no disturbance after being produced. Dilepton signals, which originate from intermediate virtual (off-shell) photons, exhibit mean free paths of the order ten fm at the rho mass, and therefore can
FIG. 4: Dilepton yield from $\pi\pi \rightarrow e^+e^-$ assuming an average temperature $\langle T \rangle = 170$ MeV and a four-volume $3 \times 10^4$ fm$^4$. The suppression observed in the net yield is discussed in detail in the text—it is due to reabsorption and rehadronization.
suffer reabsorption and rehadronization effects. The net dilepton yield from thermal $\pi\pi$ annihilation in a fireball has been shown to be reduced at the rho peak with its apparent structure modified. Future models and model calculations for dilepton production near vector meson mass regions would be incomplete without accounting for possible loss. This work provides motivation to study the problem in greater detail, going beyond Boltzmann approximations for particle occupation, including Bose-enhancement effects, and with many hadronic processes included within, say, an effective chiral Lagrangian description for the hadronic matter.

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