Comparison of wavelet analysis with velocity derivatives for detection of shear layer and vortices inside a turbulent boundary layer

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Abstract. Detection of highly energetic structure in the turbulent flow can be easily done by wavelet analysis, especially with Morlet function as a mother wavelet for detection of repetitious low-frequency structures and Mexican hat for detection of vortices. Since wavelet analysis can be simply applied on one-point measurement, the better understanding of its meaning is welcome. In this project, turbulent boundary layer was investigated with focus on intermittent dynamics. PIV snapshot were used for comparison with Wavelet analysis results, since PIV was performed with high sample frequency (500 Hz). Instantaneous snapshots of velocity and vorticity were studied also by POD and spectra of particular expansion coefficients were computed. Vorticity from planar velocity vectors was used as a suitable methods for recognition of strongly deformed streamlines. Nevertheless, to identify a trajectory of vortex and estimate the true advective velocity of vortex core in the flow, swirling strength or the second invariant could have been used as a proper tool.

1. Introduction

Ventilation and pedestrian wind comfort inside the street depends strongly on local flow conditions. Although the character of turbulent boundary layer is influenced by urban surface in integral way, a geometry of individual building essentially affects turbulent dynamics at given location.

Various dynamical structures are present in the flow. Some of them are formed as the flow separates on sharp edges of the buildings. The structures with high turbulent kinetic energy can be detect by spectral analysis. The Fourier transformation can reveal the characteristic frequency of the structure but with no temporal information. Wavelet analysis additionally provides the time of structure’s onset and cessation.

Wavelet analysis was introduced in turbulence thanks to Farge (1992). It begun to be extensively employed on velocity and scalar data. Gao (1989) and Watanabe (2004) investigated temperature and humidity signal in flow across the forest canopy. They detected ramp-like pattern within canopy which were apparently spatially correlated. Other studies were done on temperature and turbulent velocity signal (Gao, 1993; Feigenwinter, 2005; Xia, 2009; Watanabe,
2009; Huang, 2010). However, an explicit physical interpretation is often difficult to discover, since Wavelet analysis sometimes brings fuzzy and blurry results. In this paper we have focused on both testing Wavelet functions and afterwards, on decoding the physical meaning of local power spectrum in the flow.

Moreover, from 'frozen' snapshot of velocity vector field, the spatial derivatives can reveal position of shear layer and vortices. Comparison of these two methods is main goal of this contribution.

2. Experimental set-up

The boundary layer is generated in the wind-channel using a very rough terrain. The channel has the dimension of 0.25 m x 0.25 m in cross-section and 3 m in longitudinal direction. Reference wind speed in the axis of the channel is 5 m/s.

The roughness is built from long series of identical and parallel street canyons. Buildings of the canyons have triangle roofs (Figure 1). The model is manufactured in scale 1:400, so the 20 m high building in the street corresponds to the height of \( H = 50 \) mm in model. Such a big model provides a sufficient spatial resolution for PIV measurement. In the same time, this brings the inevitable and undesired blocking (higher than 20%) of the flow inside the channel. It has to be emphasize that due to technical limits it was the only way how to execute this PIV measurement. Reynolds building number based on velocity \( U_{2H} \) at the double level \( 2H \) of the building is \( Re_{2H} = 40 \, 000 \). The area of investigation lies as far as possible from the channel entrance (\( X = 2100 \) mm) in order to provide a sufficient fetch for development of well-balanced turbulent boundary layer (Cheng & Castro, 2002).

![Figure 1](image.png)

**Figure 1.** Side-view of the street canyon inside the wind-channel.

2.1. Hot-wire

At first, we measured velocity fluctuations with CTA hot-wire anemometry with high temporal resolution of 25 kHz (Dantec 55 P01). Acquisition time was 30 s. Wavelet analysis was then applied on these one-point velocity measurements.

For calculation, we adopted Matlab code developed by Torrence&Compo (1998), modified after recommendation of Ge (2007) and properly normalised for calculation. Since we work with velocity fluctuations, the square of modulus of wavelet coefficient represents a turbulent kinetic energy. In this study, the power, square of modulus divided by acquisition time, is carried out.

CTA hot-wire data helped to verify if measurement is chosen far enough from the channel beginning. The deviation among consecutive profiles appeared to be less then 1.5%. The wavelet analysis applied on the CTA data served for proper normalisation of total energy spectrum in a such way that Wavelet methods collapsed well with Fourier spectral analysis and theoretical curve from von Karman. High sample frequency of CTA allows us to estimate Kolmogorov scale of smallest eddies. The smallest vortices in this flow have 3-5 mm in diameter.
2.2. Particle Image Velocimetry

Secondly, Particle Image Velocimetry (PIV) with reasonably high repetition rate (500 Hz) is used to reveal 2-D instantaneous velocity values in vertical plane. Flow is filled by tracer particle of oily substance with mean radius of 1 \( \mu m \). Due to high intensity of turbulence, the oil droplets are evenly distributed in the chamber what produces image of good reliability. One run of PIV measurement consists of 1635 snapshots with spatial resolution of 1.2 mm x 1.2 mm. The acquisition time is reduced due to camera memory storage limit on 3.27 s. In table 1, the parameters of PIV set-up are published.

![Table 1.](image)

Diode pumped Nd:YLF laser

| Parameter                  | Value                                |
|----------------------------|--------------------------------------|
| Repetition rate            | 500 Hz                               |
| Resolution                 | 1280 x 1024 pks                      |
| Interrogation area          | 32 x 32 pks                          |
| Overlapping                | 50\% (80 x 64 vectors)               |
| Energy                     | 10 mJ                                |
| Area                       | 100 x 100 mm                         |
| Acquisition time           | 3.2 s                                |

Commercial software for PIV (DynamicStudio v3.00.) allows us to post-process data and reject possible spurious vectors. Afterwards, the time-series of wind speed in particular locations are extracted and wavelet analysis is applied on, too. Vector field serves also as a data source for calculation of vorticity. Moreover, to separate regions of shear from region of true vortical rotation, three methods which employ a velocity gradient tensor are implemented.

3. Results

3.1. Wavelet analysis

Wavelet analysis applied on time-series of signal reveals the inner frequency and time of its appearance in the signal. In this study we tested primarily the signal with velocity fluctuations.

Principle of wavelet transformation is to find the best convolution between signal and so-called mother wavelet. We tested two mother functions, Mexican hat and Morlet function with characteristic angular frequency \( \omega_0 = 6s^{-1} \). The description of these wavelets can be found in the guideline by Adisson (2002). Both are frequently employed in turbulent flow and have brought good results. Definition of functions (without normalisation) are in the table 2 below:

![Table 2.](image)

| Wavelet mother function | Definition                                      |
|-------------------------|-------------------------------------------------|
| Mexican hat             | \( \psi(t) = (1 - t^2)e^{-t^2/2} \)            |
| Morlet                  | \( \psi(t) = \pi^{-1/4}e^{i2\pi\omega_0 t}e^{-t^2/2} \) |

Mexican hat is popular for its good ability to find so-called ramp pattern in the flow. Since the passage of vortex over the measurement position imprints the pattern more or less similar to ramp-like event, it is useful tool for detailed detection of individual vortices.
Morlet function contains more waves and it is more sensitive in sinusoidal structure. Specially, the long-lasting waves or series of vortices are nicely recognised by Morlet function. It has also simpler image in the Fourier space what often results in more easily readable pictures.

We tested both functions and compared their local power spectra. In Figure 2 - upper can be seen the original signal, a simple sine with frequency of 20 Hz. Figure 2 - lower left depicts transformation into power spectra by Morlet function, whereas right picture displays wavelet transformation with Mexican hat. These representations are called scalogram. An abscissa indicates time in second, an ordinate axis displays frequency in Herz.

The Morlet energetic pattern is apparently smoother, better defined in frequency and more explicit then the one from the Mexican hat. On the other hand, the narrow gap between two Mexican hat images of simple sine helps to precisely localise the event in time. It should be noted that magnitudes of both powers reach the same value and furthermore, the total spectral characteristics collapse into each other (not shown).

Both mother wavelets are profitable for detection of a different flow event. To demonstrate this conclusion on the experimental data, the signal from PIV measurement is analysed by both mother functions. Signal is taken from middle of the canyon (X/H=0) slightly above the roof-top level (Z/H=1.2).

Firstly, the U-component is analysed by Morlet function (Figure 3 - upper) and Mexican hat (Figure 3 - lower). Morlet method brings relatively smooth patterns with energies located mainly in the lower frequencies, in the range between 4 and 16 Hz. Spots are rather horizontally

Figure 2. Left: Sinusoidal wave as a test function. Middle: Wavelet analysis with Morlet function. Right: Wavelet analysis with Mexican hat function.
elongated, so the demonstrative event lasts for longer time while keeping a constant frequency. Thanks to a detailed investigation, it was discovered that these patterns capture a wavy alternation of slow and fast wind speed. Variation of the speed is accompanied by variation in vertical direction in such a way that it forms together interchange of sweep and ejection events. That is why the negative momentum flux is dominant in turbulent boundary layer (speaking for the layer above the obstacles). In Kellnerova et al. (2010) is closely explained the flow dynamics of these structures and example of their wavy alternation.

Figure 3. Wavelet analysis of signal of U-component above the canyon (X/H=0, Z/H=1.2). Upper: Morlet function. Lower: Mexican hat function.

When one compares the Morlet and Mexican hat results, the latter one seems to be rather confusing. However, the real benefit from the Mexican hat function appears in analysis of vertical component. Figure 4 shows the both techniques applied on data of vertical velocity (W-component) in the same point as before, only for shorter time. Morlet analysis herein is much more complex. Unlike the U-component, this picture involves plenty figures stretched in vertical direction that exhibit a much higher frequency.

When we zoom into particular circular spot in Morlet analysis (Figure 5 - upper left) at time $\tau = 1.7s$, we can see that the compact event has frequency of 84 Hz. Since the Morlet function consists from several sine, it catches well the passage of several vortices in the line. In Figure 5 - upper right of wavelet analysis using Mexican hat, the individual vortices are visible as four patterns with three separation gaps.
Figure 4. Wavelet analysis of signal of W-component above the canyon (X/H=0, Z/H=1.2). Upper: Morlet function. Lower: Mexican hat function.

Mexican hat visualizes the only sine as two spots with narrow gap in between, so each half-wave corresponds to the one spot. The center of the gap is the place, where velocity changes the direction from positive to negative (or inversely). Every vortex consists form two half-waves, therefore from two spots in scalogram. Before-mentioned four spots supposedly represent two vortices. Deeper inspection revealed that vortices are actually three, just the very first and very last half-wave is to weak to result in visible blur. Only an extremely fine resolution of the levels uncover all six spots belonging to all three vortices (not shown).

Furthermore, to find out more about vortex, we can carry out a modulus as an output (Figure 5 - lower). Then we obtain an information about sign of half-wave of vertical velocity, if positive or negative. The positive illustrates a sweep event while the vortex is entering a measurement position (blue color). The negative describes the ejection, when the vortex has already passed the position (orange color). Typical temporal evolution is firstly the negative (sweep) event followed by the positive (ejection). This makes a clock-wise sense of rotation of each vortex, perfectly shown in $\tau = 1.7$ s. At time $\tau = 1.64$ s and frequency of 46 Hz we can see an example of inverse anti-clock wise rotation, when orange color comes first and is followed by blue color. The snapshot truly confirms the rare inverse sense of vortex.

Thus, inspection of the Mexican hat analysis informs us about every vortex that passes the measurement position. The only limitation is sample frequency of the signal and therefore the
The smallest vortices are not detectable. In this case, the time-life of the vortices is \( \tau = 0.010 \) s. Considering the advective velocity of the vortices this point, based on the Adrian (2000), the scale of the vortices is about 25 mm. This scale matches very well with the one visible in the snapshot (\( \approx 24 \) mm) of the flow.

![Wavelet analysis of W-component](image)

**Figure 5.** Detail of wavelet analysis of signal of W-component above the canyon (X/H=0, Z/H=1.2) in the time \( \tau = 1.7 \) s. Upper left: Circular shape on the Morlet analysis captures the passage of several vortices. Upper right: Mexican hat function decomposes circular pattern from Morlet analysis into individual vortices (four spots). Lower: Mexican hat function with modulus as an output. Sequence of three pairs with negative (blue) and positive (orange-violet) stripe in \( \tau = 1.7 \) s reveals all three vortices with clock-wise sense of rotation.

### 3.2. Fourier analysis

POD is linear method applied on complex data of turbulent flow. Description of this method can be found in Hertwig (2010). POD brings the most dominant modes of the flow dynamics. When working with velocity fluctuation, the measure of dominance is derived after TKE. The modes are function of space, not of time. Picture of the modes can be seen in Figure 6 and with detailed explanation in Kellnerova (2011). POD carries out also the temporal evolution of particular mode (so-called expansion coefficient). Fourier spectral analysis can be done on the coefficients. If do so, we can see what frequency is characteristic for each mode.

Generally, the spectrum of U-component velocity has characteristic frequency of 10 Hz. In Figure 7 is depicted smooth spectral density function for first four modes. Generally, the peaks belong into frequency interval between 10 and 30 Hz. The first mode is slower and lies near the lower edge of interval. This suggests that the first mode is robust and lasts for longer time. The
Figure 6. The first four POD modes ordered from the left to the right by the importance: Mode 1, Mode 2, Mode 3 and Mode 4.

second and third modes are shorter in duration and flip quickly from positive to negative values. Finally, the fourth mode can be the slowest one, with the energy in low frequencies starting at 3 Hz. This indicates that structure of fourth mode also lives for longer time and undergoes only slow changes with time. However, the structure exhibits energy in higher frequency as well. Thus the variety of life duration of this structure is wide.

Figure 7. Fourier spectral analysis of expansion coefficients for the first four modes.

3.3. Velocity derivatives
To find a physical interpretation of blurs in wavelet scalogram, each blur was tracked down on snapshots with velocity vectors. Thanks such a 2-D information about instantaneous velocity, individual vortices could have been viewed directly. From such a vector field we can calculate a vorticity as well. For computation the Stokes theorem was used. Many authors (Zhou et al., 1998; Adrian et al., 2000; Tomkins & Adrian, 2003; Coceal et al., 2007) use for detection of vortical motion some of the velocity derivatives. Namely vorticity does not distinguish the true vortex from the shear area. There are other three widely spread techniques for detection of vortex: lambda-2, swirling strength and second invariant Q. All are derived from a velocity gradient tensor. Two latter ones have brought very similar results in our study.
Since the Mexican hat can detect structure similar to vortex based on one-point velocity and whereas derivatives can reveal vortex based on gradients, it shall be desired to compare both methods.

The larger vortices, which bring large fluctuations, are perfectly recognised in the wavelet analysis. The same vortices however provides only a weak spatial derivative to be visible by vorticity or derivative techniques. Only the very small vortices with high frequency have both the temporal and spatial gradient sharp enough. However, this range of frequency is beyond the measuring capacity in our case.

From this point of view, the wavelet analysis of particular velocity component did not suit to much to the 2-D derivatives, since the harmonic event does not correspond to the shear layer, which is strong contributor to the vorticity. There is only a weak correlation between vorticity and vertical component reaching of -0.3.

4. Conclusion

Investigation of harmonic event based on wavelet analysis was done with experimental data. Two used mother wavelet revealed to be very helpful methods. However, each of them has to be applied for different flow event. Morlet function better recognises wavy alternation of acceleration and deceleration in wind speed, what is very often connected with sweep and ejection events. Mexican hat can ‘see’ even a minute vortex, so this function is proper for vortex detection an its of rotation.

Total energy spectrum was derived for POD expansion coefficient, since the particular modes represent some type of sweep/ejection or vortical event. Some of modes (1 and 4) last for longer time and have energetic peak in lower frequencies. The others are able to quickly flip their direction and keep fast alternation in sign and magnitude.

Comparison between Wavelet results and derivative revealed to be that both methods preferably detect the dissimilar kind of event. The temporal scale, where the both method could cross each other is unfortunately lower then the temporal resolution at disposal from experiment.

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