Aspherical Supernovae: Effects on Early Light Curves

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Received 2017 July 6; revised 2018 March 1; accepted 2018 March 1; published 2018 April 3

Abstract

Early light from core-collapse supernovae, now detectable in high-cadence surveys, holds clues to a star and its environment just before it explodes. However, effects that alter the early light have not been fully explored. We highlight the possibility of nonradial flows at the time of shock breakout. These develop in sufficiently nonspherical explosions if the progenitor is not too diffuse. When they do develop, nonradial flows limit ejecta speeds and cause ejecta–ejecta collisions. We explore these phenomena and their observational implications using global, axisymmetric, nonrelativistic FLASH simulations of simplified polytropic progenitors, which we scale to representative stars. We develop a method to track photon production within the ejecta, enabling us to estimate band-dependent light curves from adiabatic simulations. Immediate breakout emission becomes hidden as an oblique flow develops. Nonspherical effects lead the shock-heated ejecta to release a more constant luminosity at a higher, evolving color temperature at early times, effectively mixing breakout light with the early light curve. Collisions between nonradial ejecta thermalize a small fraction of the explosion energy; we will address emission from these collisions in a subsequent paper.

Key words: hydrodynamics – shock waves – supernovae: general

1. Introduction

The discovery volume for time-domain astronomy is expanding rapidly as new surveys come online. For core-collapse events, the early supernova (SN) light is a key target, as this carries information about the final moments of a star’s evolution—like its radius, wind state, and terminal velocity—that is difficult or impossible to glean from later epochs. Furthermore, the theory of early SN emission is fairly mature (Nakar & Sari 2010; Rabinak & Waxman 2011). Insofar as an explosion can be considered spherical, it evolves as follows. The shock front that defines the SN explosion speeds up as it crosses the thinning density zones of the star’s outer layers. Photons dominate the post-shock pressure; therefore, a shock of speed $v_s$ has a characteristic width, corresponding to the optical depth $\tau_s \approx c/v_s$, set by photon diffusion. At some point where $\tau_s$ falls below the optical depth to free space, photons leak away in a “breakout” flash (Klein & Chevalier 1978). While detailed predictions require simulations, the basic properties of this flash, as well as the ejecta’s density and temperature profiles in velocity, are all related to the original stellar structure and its explosion energy in a rather deterministic way (Matzner & McKee 1999; Tan et al. 2001). The light curve begins immediately after breakout; as soon as the ejecta have traveled a few times the original stellar radius, self-similar diffusion leads to a power-law decline of the total velocity (Chevalier 1992). Larger progenitors tend to make longer, redder, more energetic flashes and redder early light curves (with lower maximum speeds). But explosions within compact progenitors can become relativistic, leading in some cases to low-luminosity gamma-ray bursts (Tan et al. 2001; Nakar & Sari 2012). Behind fast shocks ($v_s/c \gtrsim 0.1$), emission and absorption do not have time to equilibrate (Katz et al. 2010, 2012; Sapir et al. 2011, 2013). Finally, massive stars produce winds, and the ejecta–wind collision leads to synchrotron and free–free emission (e.g., Fransson et al. 1996). However, an optically thick wind, like those surrounding Wolf–Rayet stars, will alter and enhance the breakout flash (Chevalier & Irwin 2011). Spherical theory has been applied widely, from the ionization of rings around SN 1987A to the radio shell around SN 1993J, the inference of relativistic ejecta from SN 1998bw, the X-ray flashes from SN 2008D and XRF 060218, the early light curve of SN 2011dh, and early SN observations from the CFHT-SNLS, GALEX, PTF, and Kepler surveys, to name a few.

However, spherical theory may be misleading if the underlying event is aspherical in an important way. This would not be surprising, considering that the central engine is thought to involve large-scale instability (Blondin et al. 2003) or magnetorotational energy extraction (Akiyama et al. 2003). Linear polarization of the line and continuum emission has long provided evidence of nonspherical ejecta, especially at low velocities (Leonard et al. 2001, 2006) and for stripped-envelope progenitors (Leonard & Filippenko 2005; Wang & Wheeler 2008). Likewise, young SN remnants like Cassiopeia A show a complicated distribution of ejecta (e.g., Lee et al. 2017). The stellar envelope may also be distorted by rapid rotation or tides from a companion. Moreover, there is a growing list of discrepancies between observations and spherical-theory expectations regarding early SN light (see Section 7), and it is important to consider the alternatives. This is especially clear when unexpected features of the early light curve are attributed to plumes of $^{56}$Ni, as any process that moves matter from the central engine to high ejecta velocities must be very strongly aspherical.

How might an aspherical explosion alter the early SN light? Previous studies offer somewhat disparate answers. Calzavara & Matzner (2004) considered only variations in the timing and intensity of shock breakout (SBO), effects calculated by Suzuki & Shigeyama (2010) within a simple model for the evolution of the breakout. Couch et al. (2009, 2011) performed adiabatic, axisymmetric simulations of jet-driven explosions and analyzed the results to infer the properties of the early light. Examining the breakout dynamics, Matzner et al. (2013;
hereafter M13) pointed out that aspherical explosions can undergo a transition to distinctly different behavior, predictions Salbi et al. (2014; hereafter S14) verified on the basis of simulations that address a small zone near the stellar surface. Most recently, Suzuki et al. (2016) simulated a mildly aspherical explosion using a radiation hydrodynamics calculation.

We present a new set of simulations and model light curves with two goals in mind. First, we wish to realize and test the predictions of M13 and S14 with global simulations. Arguing from a limiting case in which the stellar atmosphere is effectively planar, M13 and S14 made the following predictions.

(i) Outward acceleration of the explosion shock ceases at the depth \( \ell_c \), where the (radial) shock speed matches the (horizontal) pattern speed \( v_p \) of breakout across the stellar surface.

(ii) Matter from this region sprays out in a nonradial fashion (with a specific distribution in angle).

(iii) The maximum ejecta velocity is limited to \( 2v_p \) (when \( v_p \) is nonrelativistic).

(iv) Nonradial flows collide outside the star, providing a new energy source for early SN emission.

(v) “Oblique” breakout is hidden from the observer by an optically thick spray of ejecta, except, in some cases, from certain lines of sight.

(vi) Nonradial flows affect the early luminosity and polarization of the ejecta. (This prediction was also made by Couch et al. and Suzuki et al.)

(vii) None of these effects develop in regions so diffusive that SBO occurs below \( \ell_c \) or in adiabatic simulations that lack resolution of this scale. Therefore, nonradial effects should be much stronger for compact progenitors than for red supergiant explosions.

A second goal of this work is to make specific predictions about the early SN emission that reflect emission and absorption, as well as the scattering-dominated diffusion of photons. These effects are treated in the spherical case by Nakar & Sari (2010), and Couch et al. (2009, 2011) introduced an approximate treatment for their aspherical explosion simulations. We will introduce an improved approximation that applies to the expanding ejecta and take care to consider the ejecta collision zone separately. (We consider only the collisions’ dynamics and energetics here, saving the emission from this process for a future paper.)

We are mindful of two potential pitfalls. One concerns the many ways an explosion can be nonspherical: even considering only one model, like jet-driven explosions within spherical stars, there are many possible outcomes depending on parameters such as the jet’s structure and duration. Moreover, the physics of SBO imply that optical depth affects how the outermost ejecta respond to an aspherical explosion (see prediction (vii) above), in addition to the emission from these ejecta; this introduces still more parameters.

Another possible pitfall involves the complexity of the radiation processes at work. Robust predictions of SBO emission, for instance, require multigroup radiation hydrodynamics simulations (e.g., Tominaga et al. 2011), which are beyond our capability for aspherical flows. Likewise, the collision of nonradial ejecta may involve effects like cosmic-ray acceleration that are not easily accommodated within our numerical code.

For these reasons, we adopt an approach similar to that of Couch et al. but opt for simplicity in our initial model. Specifically, we perform axisymmetric, nonrelativistic, adiabatic explosions within the FLASH hydrodynamics code, ignoring details like self-gravity and the role of gas pressure in the equation of state. We adopt a simplified stellar structure (a spherical, \( n = 3 \) polytrope), and we make the explosion aspherical by fiat rather than driving it with a central jet.

Such simplifications have obvious drawbacks. Because our simulations do not include radiation transfer, they do not include the physics that limit ejecta speeds in spherical explosions or prevent the development of nonradial flows in aspherical explosions of red giants. Because we use a single polytrope, we cannot address shock ejection within realistic stellar atmospheres. Because we make no attempt to emulate a central engine, we cannot model its effect on the shape of the explosion.

But simplicity has advantages. A single simulation can be scaled to the radius, energy, and optical depth of a variety of SNe. Handling radiation transfer in post-processing allows us to specify exactly which aspects of a simulation (like its nonradial flow) are inconsistent with a given context, and in fact, our predicted light curves are not badly affected by this type of inconsistency. We are able to capture the overall differences between an aspherical event and its spherical counterpart that might otherwise be buried in the details.

This work is organized as follows. In Section 2, we describe the numerical approach and details of our axisymmetric model. In Section 3, we provide a general description of our simulation results, from the shock propagation to the circumstellar collisions. We also discuss the numerical effect of the resolution and shock speed derivation method. In Section 4, we comment on the conditions required for the formation of oblique SBO. Section 5 is dedicated to the observational implications of our work, including radiation diffusion, thermalization, and light-curve modeling. We briefly explore the potential effects of ejecta–ejecta collisions in Section 6. A general summary of our results is provided in Section 7. We conclude and set goals for future work in Section 8.

### 2. Physical Problem and Numerical Implementation

We construct a global numerical FLASH hydrodynamic (Fryxell 2000) simulation in a two-dimensional uniform grid in spherical polar coordinates. We ignore stellar gravity and hydrostatic pressure because the explosion energy is usually much higher than the binding energy of the stellar envelope. We also ignore the diffusion of radiation in the simulation; this leads to errors in the prediction of some of the fastest ejecta, which we account for in our post-processing. We comment on the radiation processes and diffusion in Section 4. Furthermore, we consider the case where the post-shock flow is radiation-dominated (i.e., adiabatic index \( \gamma = 4/3 \)) and the initial density profile is of an \( n = 3 \) polytrope, mainly because this reasonably represents the envelope structure of various progenitors and its density profile can be specified at an arbitrary high resolution. (While \( n = 3/2 \) polytropes are often used to represent the convective envelopes of red supergiants, we fix the structure to highlight the influence of radiation processes and aspherical geometry.) For simplicity, we assume purely nonrelativistic motion.
With all these simplifications, our simulations themselves are scale-free, apart from the numerical scales introduced by finite resolution and simulation volume. In physical terms, they are described by the arbitrary scales of energy \( E_\text{s} \), ejected mass \( M_\text{ej} \), and progenitor radius \( R_\text{p} \) and derived quantities like the scales of density, \( \rho_\text{s} = M_\text{ej}/R_\text{p}^3 \), velocity, \( v_\text{s} = \sqrt{E_\text{s}/M_\text{ej}} \); and time, \( t_\text{s} = R_\text{p}/v_\text{s} \). (Additional radiation parameters, such as the electron-scattering opacity \( \kappa \), will enter our post-processing step.) We list characteristic scales for red supergiants (RSGs), blue supergiants (BSGs), and compact Type Ic models in Table 1.

A difficulty that arises naturally while simulating asymmetric explosions is having many degrees of freedom in making axisymmetric explosions (e.g., finite-duration jet, off-axis explosion, directed momentum, prolate or oblate progenitor, etc.). The main focus of this work, however, is not exploring the parameter space of aspherical explosion but rather studying the dynamic of shock emergence and its observational impacts. We thus initiate the asymmetric simulations by adding axisymmetric momentum to the outcome of a purely spherical explosion.

We initialize the explosion in a two-step process. First, we model a point explosion within the progenitor, resolving its radius with 2000 zones. As shown in Figure 1, we modify the velocity profile at the moment this explosion reaches radius \( r_\text{v} \), the radius of velocity modification. Changing \( r_\text{v} \) allows us to vary the degree of asphericity (see Section 5.4); in our fiducial run, \( r_\text{v} = 0.12R_\text{p} \). Specifically, the velocity is modified by an angle-dependent scaling factor to induce nonspherical behavior,

\[
v(r, \theta) = \hat{v}(r) |\cos(\theta)|,
\]

where \( \hat{v}(r) \) is the spherical velocity profile. We used this specific formula to ensure that the velocity remains subsonic; i.e., \( v(r, \theta) \leq \hat{v}(r) \). The total energy of the modified explosion is then recorded for later scaling to the value of \( E_\text{s} \) desired.

For all of our simulations, the angular extent of our two-dimensional grid is \( \pi/2 \), and the outer grid boundary is set to allow outflow at \( 4R_\text{p} \). In this work, we do not attempt to simulate the effect of wind or mass-loss history. The ambient density is therefore set to a small value, about \( 10^{-14} \rho_\text{bo} \) or \( 10^{-5.6} \) times below the lowest stellar density, to prevent round-off numerical error. We run the code until most of the mass has left the grid and record 100 checkpoints at regularly spaced intervals along the way. The number of cells in both the radial and angular directions is equal. We refer to this number as \( \mathcal{R} \); keep in mind that there are \( \mathcal{R}/4 \) cells per stellar radius. We vary \( \mathcal{R}^2 \) from 128\(^2 \) to 2048\(^2 \) and designate the highest of these resolutions as our fiducial run.

For this work, FLASH was compiled with the Message Passing Interface and HDF5 parallel enabled on up to 256 cores of the GPC cluster at the SciNet facility located at the University of Toronto (Loken & Gruner 2010). We employed a directionally split piecewise parabolic method Riemann solver with a Courant number of 0.8.

### 3. Results

#### 3.1. Fiducial Run

Figure 2 shows the evolution of the density distribution of the simulation with \( \mathcal{R} = 2048 \) at four different snapshots, representing initial density distribution, SBO, transient evolution, and the end of shock progression inside the progenitor. In Figure 2(a), the initial density condition is depicted. The high-density shell inside the progenitor is the Sedov solution taken from the spherical explosion. As expected, the bipolar shock first progresses radially in the \( \theta = 0 \) direction. In Figure 2(b), the low-density regions in the post-shocked flow are formed by Kelvin–Helmholtz (KH) instabilities due to the velocity shear. In this figure, the SBO happens when the shock accelerates and reaches the progenitor pole, leading to sprays of ejecta that run into the ambient density. As shown in Figure 5, the shock pattern speed \( v_\text{s} \) decreases with distance from the pole, and when the condition \( v_\text{s} \sim v_\text{p} \) is met, the predictions for oblique flow (M13, S14) become relevant: the shock meets the stellar surface at a right angle, and the post-shock flow spreads out to fill the space above the stellar surface, as depicted in Figure 2(c). Finally, the deflected sprays of ejecta collide with each other and form an expanding wedge ofreshocked matter near the equatorial plane (Figure 2(d)).

#### 3.2. Resolution Study

We study finite resolution by changing the number of cells from \( \mathcal{R} = 128 \) to \( \mathcal{R} = 2048 \), with the results shown in Figure 3. Without unphysically large explicit viscosity, the outcome of the KH instability is unavoidably resolution-dependent, as demonstrated by Calder et al. (2002); this effect is clearly seen in our results.

More relevant to our study is that the speed and structure of the fastest ejecta also depend on resolution. We attribute this to...
several possible effects. First, there could be a difference in timing of SBO due to slightly different evolutions within the star; however, we see little evidence of this. Second, the highest possible shock speed (realized in a spherical model) depends on the shallowest resolvable depth, thanks to the scaling $v \propto (R_0 - r)^{-\beta}$, where $\beta \approx 0.2$ (Sakurai 1960).

Third, the formation of an oblique flow depends on resolving the scale $\rho_j$ at which the nonradial motions discussed in Section 4 become important. The consequence of this fact can be seen in the results of the $R = 256$ run in Figure 3, where the spray of ejecta is confined to a limited range of deflection angles, compared to higher-resolution runs.

In fact, this transition should be visible even within a well-resolved run, as $\rho_j$ is zero at the pole of a bipolar breakout and must therefore be of order the grid spacing at some angle. A radial feature is apparent in the high-velocity ejecta and appears at a smaller angle when $R = 2048$ than when $R = 1024$, so it is plausibly caused by this effect. (We note that S14 found subtle differences between resolutions even when $\rho_j$ is highly resolved).

4. Oblique Flow Formation

Our simulations neglect radiation diffusion, so they display nonradial flows where the resolution is sufficient to capture them. In real explosions, radiation diffusion limits the development of nonradial flows, just as radiation diffusion (or inadequate numerical resolution) limits the maximum ejecta velocity in a spherical explosion. This means that the fastest and most nonradial portions of a simulation may be unrealistic for a given progenitor model. However, with careful attention to the scales of oblique flow and the effects of radiation diffusion, these effects can be accounted for after the fact. For this purpose, it is important to compute the scales on which oblique or nonradial flows develop. These include a relevant length scale $\rho_j$, the density $\rho_j$ at depth $\rho_j$, and the pattern velocity $v_j$ at each location on the stellar surface.
4.1. Oblique Parameters

Oblique flow involves a rotation of the shock normal away from the radial direction, so the key to deriving these quantities is the shock velocity $v_s$. One can retrieve snapshots of $v_s$ from checkpoint data, but the limited number of such files demands a method with higher time resolution. For this reason, we record at each timestep the time of maximum compression rate, $t_s$, at each location. This is a viable proxy for the time at which the shock crosses each location. As shown in the left panel of Figure 4, $t_s$ captures the mushroom pattern of the shock progression very well and only fails in regions where the expanding ejecta along the stellar equator runs into the post-shocked lateral flow. The shock velocity vector is determined from $t_s$:

$$v_s = \frac{\nabla t_s}{||\nabla t_s||}.$$  (2)

The streamline plot of $v_s$ is depicted in the right panel of Figure 4. As expected, all streamlines are in the outward direction. The shock velocity is low inside the progenitor but considerably increases during breakout from the pole as the density steeply decreases. The shock streamline is then deflected toward the stellar surface and advances in the $\theta$ direction.
The shock pattern speed across the stellar surface, \( v_\varphi(\theta) \), can be derived using the same quantities by

\[
    v_\varphi(\theta) = \frac{1}{(\nabla t_\ell \cdot \hat{\theta})_{r=R_*}}.
\]

(The subscript \( \varphi \) refers to the lateral shock motion and should not be confused with the polar angle \( \theta \)). Figure 5 shows \( v_\varphi \) as a function of \( \theta \). The pattern speed diverges at the pole, because SBO is simultaneous there (as it is in a spherical explosion). However, as \( t_\ell \) and its gradient rise away from the pole, \( v_\varphi \) drops. The shock velocity at the stellar surface (\( \nu_s \) at \( r = R_* \)) is also plotted in Figure 5.

In a fully developed oblique breakout, the shock normal turns parallel to the stellar surface (M13, S14), implying \( \nu_r = v_\varphi \). Within the fiducial simulation, this occurs at \( \theta \approx 0.3 \) rad, so we interpret our results in terms of a resolved nonradial flow for \( \theta > 0.3 \) rad.

The oblique length scale \( \ell_\varphi \), defined in M13, is the depth where the outward shock velocity matches the pattern speed, so that a nonradial flow (oblique breakout) develops. Here it is measured as the depth at which contours of constant shock time \( t_\ell \) deflect 45° from the radial direction. This is plotted in Figure 6, along with a smoother version (a low-order polynomial fit) for the region \( 0.3 < \theta < 1.1 \) where \( \ell_\varphi \) is resolved. The increase of \( \ell_\varphi(\theta)/R_* \) toward the equator is expected in bipolar explosions, because \( v_\varphi(\theta) \) declines as the shock progresses away from the poles, and it thus matches the outward shock velocity at greater depths. Once \( \ell_\varphi \) is inferred from the \( t_\ell \) array, we retrieve the oblique density parameter \( \rho_\varphi \) as the progenitor density measured at the appropriate location, as shown in Figure 6.

We see in Figure 6 that the minimum resolvable value of \( \ell_\varphi \) is about \( 0.005R_* \), which corresponds to 2.5 zones at \( \beta = 2048 \). We infer that a few zones are required to resolve the oblique flow. Applying this rule to the lower-resolution runs, and supposing that changes in resolution do not appreciably change the timing of SBO, we expect oblique flow to develop at

\[
    \ell_\varphi > 0.005R_* = \beta \approx 2048,\quad \ell_\varphi \approx 2.5 \text{ zones}.
\]

\[\theta \approx (0.55, 0.35), \quad \text{where } \ell_\varphi = (0.02, 0.01)R_*, \text{ respectively, in the runs with } \beta = (512, 1024).\]

4.2. Obliquity and Radiation Diffusion

Sufficiently close to the stellar surface, oblique breakout has the property that each streamline (in a frame comoving with the breakout pattern) either traps photons, resulting in adiabatic near-surface flow, or releases them through diffusion (M13). The streamline in question can be parameterized by the comoving-frame deflection angle \( \alpha_f \), or, in the star’s frame, by the deflection angle \( \delta \theta_f = \theta_f - \theta \). The fluid’s initial and final directions of movement are \( (\theta, \theta_f, \delta \theta_f) \), where \( \delta \theta_f \) is the difference in azimuthal angle between the initial and final directions of the fluid. These angles are conveniently related: \( \alpha_f = 2 \delta \theta_f \).
Using high-resolution simulations of the adiabatic, planar, nonrelativistic limit of SBO, S14 measured the strength of diffusion on each streamline in terms of the local Péclet number,

\[
\mathcal{D} = \frac{3 \kappa \rho c P_\nu}{c},
\]

which is approximately constant on streamlines, as seen in the comoving frame (in the near-surface limit). Here the radiation pressure scale length is \( L_p = P_{\text{rad}}/|\nabla P_{\text{rad}}| \), \( D_1 \simeq 126 \), and \( \delta_\mathcal{D} \simeq 6 \). Diffusion is negligible when \( \mathcal{D} \gg 1 \) and strong for \( \mathcal{D} \ll 1 \). The dimensionless parameter \( \mathcal{D} \) scales with the progenitor parameters as

\[
\mathcal{D}(\alpha_f, \theta) \simeq \frac{D_1}{\alpha_f^5} \frac{\rho c \ell_\nu V_\nu}{\rho R_\nu V_\nu} = \frac{D_1}{\alpha_f^5} \frac{\kappa_0 E_{31} M_\odot}{R_\nu^2 c} f_\nu(\theta),
\]

where \( f_\nu(\theta) = (\rho c \ell_\nu V_\nu)/(\rho R_\nu V_\nu) \). In the second expression, the parameters in the parentheses are the scales of the explosion and the effective opacity \( \kappa_0 \) (usually dominated by electron scattering: \( 0.2(1 + X) \) cm\(^2\) g\(^{-1}\) for H mass fraction \( X \)), which are all progenitor-dependent, while the function \( f_\nu(\theta) \), shown in Figure 7, contains oblique parameters normalized to their characteristic values. These are directly related to the simulation output, as well as the degree of ellipticity,

\[
\varepsilon = 1 - \frac{\ell_\nu(\theta = 0, R_\nu)}{\ell_\nu(\theta = \pi/4, R_\nu)}.
\]

defined by M13. In the final line of Equation (5), progenitor parameters are scaled to their explosion values, where \( E_{31} = 10^{51} E_{31} \) erg and \( \kappa = 0.34 \) cm\(^2\) g\(^{-1}\). For our fiducial run with \( \nu_0 = 0.12 R_\nu \), we find \( \varepsilon = 0.26 \); but we also consider a spherical case (\( \varepsilon = 0 \)), as well as an intermediate case (\( \nu_0 = 0.06 R_\nu \) and \( \epsilon = 0.09 \)).

Diffusion begins to affect the most strongly nonradial streamlines when \( \mathcal{D} \leq 1 \), for \( \alpha_f / (\pi/2) \), and for strongly inhibited nonradial motion, one must have \( \mathcal{D} < 1 \) at \( \alpha_f = \pi/2 \). These criteria correspond to

\[
R_\nu \gtrsim (120, 940) \left( \frac{M_\odot}{10 M_\odot} \right)^{1/4} \left( \frac{E_{31} M_\odot}{10 M_\odot} \right)^{1/4} R_\odot,
\]

respectively. We evaluate \( \mathcal{D}(\theta) \) for the progenitor models of Table 1 in Figure 8. In the aspherical geometry considered here, early SN emission consists of three components: SBO radiation, glow from the expanding ejecta that have been heated by the SN shock, and cooling radiation from the zone of circumstellar ejecta collisions. Whereas a spherical SN produces the first two
in sequence and does not have colliding ejecta (in the absence of circumstellar matter), an aspherical SN can make all three simultaneously. (We assume there is no relativistic jet that pierces the surface or the jet cocoon and radioactive matter that would accompany one.)

5.1. SBO Emission

By “SBO emission,” we refer to light that is emitted as the shock arrives at the stellar surface and before the shock matter has been ejected any significant distance. In earlier analyses, Calzavara & Matzner (2004) and Suzuki & Shigeyama (2010) recognized that breakout emission would be extended by the time taken for the shock to cross the star’s visible surface and would change in intensity according to the local shock strength. However, as discussed in detail by M13 and S14, SBO emission becomes obscured by the ejecta spray as nonradial motions develop. As a crude approximation, we assume the SBO radiation is unaffected by these effects wherever Equation (5) predicts \( D(\alpha_f, \theta) = \pi/2 \) > 1.

Where this criterion is not satisfied, SBO emission is inhibited. But there still exists an outer layer of matter, with mass surface density \( \Sigma_{\text{diff}} \approx c/(3\kappa v_g) \), for which diffusion is important. In this layer, the diffusion speed, \( c/(3\kappa \Sigma_{\text{diff}}) \), equals the pattern speed, \( v_g \). The SBO energy per unit area is approximately the rate at which the kinetic energy of this layer (\( \Sigma_{\text{diff}} v_g^2/2 \) per unit area) is consumed in the frame of the advancing shock: \( L_{\text{SBO}} \approx \Sigma_{\text{diff}} v_g^2 \dot{A}/2 \), where \( \dot{A} \) is the rate at which the surface area is shocked. For a bipolar explosion, \( \dot{A} = 4\pi R^2 v_g \sin \theta \), so

\[
L_{\text{SBO}} \approx \frac{2\pi}{3} \frac{R^2 v_g^2 \dot{A}^2}{\kappa} \sin \theta \\
\approx 10^{40.33} \frac{E_{51}(R_*/50 R_{\odot})}{(M_*/15 M_{\odot})^{0.34}} \left( \frac{v_g}{v_\text{ap}} \right)^2 \sin \theta \text{ erg s}^{-1}. \quad (7)
\]

This radiation can escape through the outflow in directions for which \( D(\alpha_f) \lesssim 1 \) (if there are any).

Because \( D \) is conserved on each streamline, we note that much of this luminosity may diffuse out from streamlines with \( D \sim 1 \) at radii much larger than \( L_{\odot}^* \); for this reason, the SBO emission may blend, to some extent, into the early diffusion luminosity we consider below.

Note that \( L_{\text{SBO}} \) scales as \( R_{\text{ap}} E_{51}/\kappa M_{\odot} \) for explosions that share the same eccentricity, i.e., the same pattern of \( v_g/v_\text{ap} \).

5.2. Diffusion Front

Our simulations treat the explosion as adiabatic, i.e., as though the photons that generated the post-shock pressure are trapped in the flow and never get a chance to diffuse upstream. To make observational predictions and identify physically invalid features of the adiabatic approximation, we must identify where this condition breaks. For global simulations, we cannot rely on a constant flow speed (as S14 did), so we must be careful in parameterizing diffusion in Galilean-invariant way. For this purpose, we construct a new quantity \( D = t_{\text{diff}}/t_{\text{dyn}} \), where \( t_{\text{diff}} = 3\kappa R^2 v_g^2/c \) is the time for diffusion to act across the radiation pressure scale length \( L_p \), and \( t_{\text{dyn}} = |\nabla \cdot v|^1 \) is an estimate of the local dynamical time. With these definitions,

\[
D = 3\kappa R^2 |\nabla \cdot v|/c \quad (8)
\]

is a diffusion parameter (analogous to S14’s \( D \)) that we evaluate in our simulations. In terms of progenitor parameters,

\[
D = \frac{3\kappa(E_{51}M_*)^{-1/2}}{R_{\text{ap}}^2}, \quad (9)
\]

where \( g_\theta = L^2 p|\nabla \cdot v|/(R^2 R_{\text{ap}}^2 t_{\text{ap}}^{-1}) \) can be determined directly from the simulation outputs: \( g_\theta \) is a function of space and time (i.e., \( r/R_{\text{ap}}, \theta \), and \( t/t_{\text{ap}} \) that depends on the details of the simulation—most importantly, the ellipticity parameter \( \epsilon \). Normalized to characteristic values,

\[
D = 10^{5.0} \frac{R_{\text{ap}}^2}{R_\odot^2} \frac{E_{51}M_*/M_\odot}{R_\odot^2} g_\theta. \quad (10)
\]

The function \( g_\theta \) takes a wide range of values from \( 10^{-14} \) to \( 10^4 \) within our simulation volume (Figure 9). The condition \( D = 1 \) marks the diffusion surface or “luminosity shell” \( t_{\text{diff}}(\theta, t) \), outside of which photons stream through the ejecta to ultimately be released. In Figure 10, the diffusion front is shown for the progenitor models of Table 1. As expected from Equation (10), the diffusion front moves inward (relative to \( R_{\text{ap}} \)) as the progenitor becomes more extended.

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2 In an expanding flow with \( \rho(m, t) \sim (t - t_0)^q \), this definition corresponds to \( t - t_0 = qL_{\odot} \), so it would be reasonable to adopt \( q = 1, 2, \) or 3 for planar Hubble, wind or cylindrical Hubble, or spherical Hubble flow. We set \( q = 1 \) for simplicity.
5.3. Bolometric Light Curve

The radiation flux can be expressed from diffusion approximation (Chevalier 1992) as

\[
F_{\text{rad}}(t_{\text{diff}}, \theta, t) = -\frac{c}{\kappa \rho(t_{\text{diff}}, \theta, t)} \nabla P_{\text{rad}}(t_{\text{diff}}, \theta, t),
\]

which takes different values along the diffusion front for aspherical explosions. The total photon luminosity is the rate at which the photon energy is released from the diffusion front (Chevalier 1992; Nakar & Sari 2010) derived in Section 5.2.

The total luminosity \( L_{\text{tot}} \) at time \( t \) is given by

\[
L_{\text{tot}}(t) = \int_{S_{\text{diff}}} F_{\text{rad}}(t_{\text{diff}}, \theta, t) \cdot dA,
\]

where \( S_{\text{diff}} \) is the diffusion surface and \( dA = 2\pi \sin \theta d\theta / (\hat{r} \cdot \hat{n}) \). As shown in Figure 11, \( \hat{r} \) and \( \hat{n} \) are unit vectors along the radial and normal to the \( S_{\text{diff}} \) direction at location \( (t_{\text{diff}}, \theta) \), respectively. For spherical SNe, Equation (12) takes the familiar form \( L_{\text{tot}} = 4\pi r_{\text{diff}}^2 F_{\text{rad}} \).

The observer-dependent luminosity is generated by the geometry depicted in Figure 11. For a distant observer at \( D_{\text{obs}} \) whose angle from the symmetry axis is \( \Theta \), the observed flux is

\[
F_{\text{obs}} = \frac{1}{D_{\text{obs}}^2} \int_{S_{\text{diff}}} I dS,
\]

where \( I \) is the intensity, which we assume is isotropic outward at the source: \( I = F_{\text{rad}} / \pi \) for radiative flux \( F_{\text{rad}} \). The surface differential projected by the observer is

\[
dS = 2r_{\text{diff}}^2 \sin^2 \theta \phi_{\max} \cos \Theta \cos \theta \hat{n} + \sin \phi_{\max} \sin \Theta \sin \theta \hat{n} d\theta d\phi,
\]

where

\[
\phi_{\max} = \begin{cases} 
\pi & \text{if } \cos \theta_n > \sin \theta \& \theta_n > 0, \\
0 & \text{if } \cos \theta_n > \sin \Theta \& \theta_n < 0, \\
\arccos(-\cot \theta \cot \theta_n) & \text{if } \cos \theta_n < \sin \Theta,
\end{cases}
\]

and \( \theta_n \) is the angle from \( \hat{n} \) to the z-axis (i.e., \( \cos \theta_n = \hat{n} \cdot \hat{z} \)). According to Equation (14), the surface differential takes the simple form \( dS = 2r_{\text{diff}}^2 / (\hat{r} \cdot \hat{n}) \sin \theta \cos \theta_n d\theta d\phi \) for an observer along the symmetry axis (i.e., \( \Theta = 0 \)), while, for an equatorial observer (\( \Theta = \pi / 2 \)), \( dS = 2r_{\text{diff}}^2 / (\hat{r} \cdot \hat{n}) \sin \theta \sin \theta_n d\theta \).

The observed isotropic-equivalent luminosity at time \( t \) is calculated as

\[
L_{\text{obs}}(\Theta, t) = 4\pi D_{\text{obs}}^2 F_{\text{obs}}(\Theta, t) = 4\int_{S_{\text{diff}}} F_{\text{rad}}(t_{\text{diff}}, \theta, t) dS.
\]

In Figure 12, we compare the bolometric luminosity of a spherical explosion obtained from Equation (16) against the analytical light curves of Nakar & Sari (2010). We note that the peak bolometric luminosities are in good agreement for all three progenitors, but our light curves slowly diverge from the corresponding analytical result in the phase of homologous expansion.

Figure 13 depicts the bolometric light curves of Type Ic, BSG, and RSG models for a spherical explosion (thick black curve), a mildly aspherical explosion with a degree of ellipticity \( \epsilon = 0.09 \) according to Equation (6) (blue curves), and oblique SBO of \( \epsilon = 0.26 \) (red curves). In the aspherical cases, we consider several views: those of (a) an observer looking along the axis of symmetry (\( \Theta = 0 \)) and (b) an observer looking along the equator (\( \Theta = \pi / 2 \)), and these are compared with (c) the bolometric luminosity \( L_{\text{tot}} \) defined in Equation (12). In this section, we only discuss the fiducial case, leaving the mildly aspherical results for the next subsection. The time \( t = 0 \) is the beginning of the explosion. Note that the time axis is shifted for the light curves of the spherical and the mildly aspherical explosion for comparison purposes.

We first focus on the bolometric light curve, \( L_{\text{tot}} \). The peak luminosity occurs when the shock breaks from the poles. At this stage, the shock is normal to the surface and oblique breakout is not relevant, so the peak is due to radial diffusion, just as in the case of a spherical explosion. Only a small region of the progenitor is hit, however, so light travel time is relatively unimportant in the peak duration. Next, the luminosity rapidly drops as nonradial flows develop and the shock becomes oblique. As discussed in Section 5.1, when the shock is oblique, a fraction of photons with \( D(\alpha_l) > 1 \) cannot immediately escape, as they are engulfed in the spray of ejecta. They are released, along with the early diffusion luminosity, at much larger radii when \( D(\alpha_l) \approx 1 \). This can explain the slight increase in \( L_{\text{tot}} \) after the first peak.
Comparing the total aspherical luminosity with the spherical case shows that the oblique SBO luminosity is comparable to the cooling envelope emission of the expanding ejecta in the early light curve of the spherical SN. The light curve depends strongly on the direction to the observer. Except for the initial breakout peak, which is brightest when observed along the axis, the apparent luminosity is higher when viewed from the equator. This is because (a) the flux along the z-axis is smaller than that along the y-axis during the plateau phase of the light curve, as the regions close to the poles have already radiated much of their SBO luminosity, the emission from other regions being greatly suppressed due to viewing angle; and (b) the observer along the equator sees the breakout from both progenitor hemispheres, while the one along the z-axis can only observe the early light from one.

Finally, note that even though the RSG model does not develop strong oblique flow, we can still obtain its luminosity based on our adiabatic simulations, as the hydrodynamic solution must be valid within the diffusion front, where the luminosity is set.

5.4. Changing the Asphericity

As we discussed in the Introduction, nonradial effects should be weakest in RSGs, both because these appear to be the least aspherical (Leonard et al. 2001)—presumably due to the dampening effect of the hydrogen envelope on blastwave perturbations—and because more extended stars have stronger diffusion that limits the development of nonradial flows. While our analysis accounts for the effects of diffusion within a single simulation, changes in asphericity require a comparison between simulations. For this purpose, we consider an intermediate case in which the bipolar momentum is introduced at \( r = 0.06 \) (as opposed to \( r = 0.12 \) in our fiducial). In this case, the ellipticity parameter is reduced to \( \epsilon = 0.09 \). Our diffusion analysis shows that nonradial flows develop only for the compact Type Ic progenitor, while BSG and RSG models do not exhibit nonradial flows according to the criteria in Section 4. The absence of nonradial flows means that SBO is not hidden by an optically thick ejecta when the shock hits the equator, and—unlike previous sections—the blending of SBO into early cooling emission is not expected to happen. As shown in Figure 13, this change leads to a strong second peak in luminosity for BSG and RSG models as the shock breaks from the equator. Our results for the mildly aspherical explosion are similar to the light curves of aspherical but nonoblique simulations of Suzuki et al. (2016). The minimum \( \epsilon \) for producing oblique breakouts in certain progenitors is estimated in Table 1 of M13.

5.5. Color Temperature

The optical depth of the diffusion front is normally greater than unity; therefore, the energy of the released photons may change as they travel through the upper layers of ejecta. If there is time for absorption and emission to act, the population of photons will equilibrate with the local energy density; the color temperature will therefore reflect the blackbody relation where this has occurred. Otherwise, the photons from the diffusion front are out of thermalization, with energies reaching as high...
Figure 12. The bolometric light curves of our spherical simulation are plotted against the analytical light curve of Nakar & Sari (2010) for three progenitor models.

as hundreds of keV. Nakar & Sari (2010) defined the thermalization parameter

\[ \eta = \frac{n_{BB}}{n_{em}(T_{\text{fin}}) \min(t_{\text{diff}}, t_{\text{dyn}})}, \]  

(17)

where \( n_{BB} = aT_{BB}^3/(2.7k_B) \) is the photon number density at equilibrium temperature \( T_{BB} = (u_{rad}/a)^{1/4} \) and \( n_{em}(T) = C_{ff} \rho^2 T^{-1/2} \) is the effective free-photon production rate, where \( C_{ff} \approx 8 \times 10^{37} \times \{1, 0.5, 1.5, 2\} \) for \( \{\text{H, He, C, O}\} \) compositions is the appropriate coefficient. We refer to \( \eta \) as \( \eta_{\text{dyn}} \) when \( t_{\text{dyn}} < t_{\text{diff}} \) (which holds within the diffusion front) and as \( \eta_{\text{diff}} \) when \( t_{\text{dyn}} > t_{\text{diff}} \) (outside the diffusion front).

5.5.1. Case A: Weak Thermalization at \( t_{\text{diff}} \)

Radiation is rapidly brought into thermal equilibrium if \( \eta < 1 \), and thermal equilibrium is preserved by adiabatic expansion; this implies that \( T = T_{BB} \) at the diffusion front if matter has ever experienced \( \eta_{\text{dyn}} < 1 \), even if \( \eta(t_{\text{diff}}) \gtrsim 1 \) when photons are released.

However, if this matter has never been brought into thermal equilibrium, then \( T \approx \min(\eta_{\text{dyn}})^2 T_{BB} \), where the minimum refers to the peak time of photon production for the mass element in question. In fact, \( \eta_{\text{dyn}} \) is minimized at the transition to spherical flow (because for \( \rho(m, t) \propto t^{-d}, \eta_{\text{dyn}} \propto t^{2d/6} \); \( q \) is typically \( \approx 1 \) at early times, steepening to 3). This makes it sufficiently difficult to estimate the color temperature, especially in the nonspherical case, that we opt for a numerical evaluation.

In the Appendix, we show that the color temperature excess \( x = T/T_{BB} \) evolves in each mass element (while it is within the diffusion radius) according to

\[ \frac{d \ln x}{dt} = -\frac{n_{em}}{n_{ph}}(1 - x^{-4}). \]  

(18)

The time derivative is Lagrangian, i.e., evaluated along the fluid path. Using \( n_{ph} = n_{BB}/x \) and \( n_{em} = C_{ff} \rho^2 T^{-1/2} \), we find

\[ \int_{x_0}^{x} \frac{dx}{x^{3/2}(1 - x^{-4})} = \int_{t_e}^{t} \frac{C_{ff} \rho^2 T^{1/2}}{n_{BB}} dt, \]  

(19)

which we rewrite as

\[ G(x_0) - G(x) = \frac{C_{ff} \rho^2 T_{BB}^{-1/2}}{n_{*} T_{BB}} \int_{t_e}^{t} h(\tilde{\tau}) d\tilde{\tau} \equiv G_{\text{th}} \int dg, \]  

(20)

where \( G(x) \) is the integral on the left-hand side of Equation (19), \( \tilde{\tau} = t/t_e \) denotes normalized time, and \( h(\tilde{\tau}) = dg/d\tilde{\tau} \) contains normalized simulation parameters. Integration is along fluid trajectories. The final color temperature is determined by finding \( G(x) \) at \( t_{\text{diff}} \), then inverting to get \( x \). The function \( G(x) \) is analytical but cumbersome, so we do not write it out, but we plot it in Figure 14.

We integrate along fluid trajectories by making use of FLASH’s “mass scalar” capability, meant for passive advection of quantities with the fluid motion.\(^5\) We define \( g \) as a mass scalar and update it according to the differential equation \( dg = h d\tilde{\tau} \) between each hydro step. (We set \( dg = 0 \) for \( \tilde{\tau} < t_e \) to exclude the contribution to this integral before shock arrival, as this early contribution represents the initial thermal equilibrium of the star and is not involved in post-shock thermalization. Furthermore, Equation (20) assumes adiabatic flow, so it is invalid across the shock).

There are several points to note. First, \( G(x) \) has a logarithmic divergence at \( x = 1 \), and this feature effects thermalization \( T \rightarrow T_{BB} \) when \( G(x) \ll G(x_0) \). Second, we must estimate the initial value \( x_0 \) corresponding to the post-shock state, which we do in Section A.1 of the Appendix, although the end result is insensitive to this choice so long as \( x_0 \gg 1 \). Third, a single simulation suffices to determine the normalized thermalization field \( g \) in terms of other normalized variables; the color temperature at the diffusion front of a particular explosion can then be determined by setting appropriate values for \( G_{\text{th}} \) and \( x_0 \) and identifying \( t_{\text{diff}} \) as described in Section 5.2. However, this only represents the observed color temperature if thermalization is weak at \( t_{\text{diff}} \); we now consider the correction appropriate to thermalization outside the diffusion front.

5.5.2. Case B: Strong Thermalization at \( t_{\text{diff}} \)

In the alternate case, in which \( \eta < 1 \) at the diffusion front, photons continue to be produced in the zone of diffusing radiation, and the observed color temperature is set at the

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\(^5\) We thank Paul Ricker for suggesting this approach.
location where thermalization fails, i.e., where \( \eta_{\text{diff}} = 1 \). We treat this in an approximate fashion by examining the scaling of \( T \) with \( \eta_{\text{diff}} \) in a diffusive region. Between the diffusion radius and the photosphere, the diffusion equation \( dP_{\text{rad}} = -\phi \frac{d\tau}{d\tau} \) implies, for a relatively constant flux, \( P_{\text{rad}} \propto \tau \sim \kappa \rho L_\nu \), while \( L_\nu \sim \tau L_\rho / c \), where \( L_\rho \) is the local density scale length. Together with \( P_{\text{rad}} \propto T^4 \), this implies \( T \propto (\eta_{\text{diff}} / L_\rho)^{-2/17} \); we ignore the variation of \( L_\rho \). Since \( \eta_{\text{diff}} \) increases from its value at the diffusion front (which equals \( \eta_{\text{dyn}}(\tau_{\text{diff}}) \)) to unity, these approximations imply a color temperature \( T = \eta_{\text{diff}}(\tau_{\text{diff}})^{2/17} T_{\text{BB}}(\tau_{\text{diff}}) \). Note that, in this limit, \( T(\tau_{\text{diff}}) = T_{\text{BB}}(\tau_{\text{diff}}) \). This suggests that the single formula
\[
T_c \simeq \frac{T(\tau_{\text{diff}})}{\left[1 + 1/\eta_{\text{dyn}}(\tau_{\text{diff}})^2\right]^{1/17}}
\]  
(21)
captures the color temperature \( T_c \) in both regimes. Here \( T(\tau_{\text{diff}}) \) is meant to be evaluated according to the inversion of Equation (20), which includes thermalization, and any additional reduction relative to this value takes place only if thermalization is strong at \( \tau_{\text{diff}} \). We apply Equation (21) at every angle \( \theta \).

In our calculation, the early SN light is determined in its bolometric luminosity by radiation diffusion of shock-deposited heat, and the mean photon energy is determined by the color temperature that results from photon production in the ejecta (or in the diffusion front). Our post-processing of the adiabatic simulation is therefore similar in spirit to, but more accurate than, the analyses of Couch et al. (2009); blackbody emission from the photosphere) and Couch et al. (2011; blackbody emission from an estimated thermalization radius).

### 5.6. Average Color Temperature

With \( T_c \) calculated as above, the luminosity-weighted angle-averaged color temperature is
\[
T_{c,\text{tot}}(t) = \frac{1}{L_{\text{tot}}(t)} \sum_\theta \Delta L_{\theta,t} T_c(\theta, t),
\]  
(22)
where \( \Delta L_{\theta,t} \) is the total photon luminosity for a small patch along \( \theta \) at time \( t \). Here \( \Delta L_{\theta,t} = 2\pi \int_{\Delta \theta} \int_{d\hat{r} \cdot \hat{n}} |P_{\text{rad}}(\tau_{\text{diff}}, \theta, t)| \sin(\theta) d\theta \), as described in the integral of Equation (12).

In Figure 15, the total color temperature evolution of the oblique simulation is shown for the progenitors Ic, BSG, and RSG (solid color). Color temperature—like luminosity—depends on the position of the observer, which can be derived by substituting the luminosities in Equation (22) with the observed luminosities; the difference between observer-dependent color temperature and \( T_{c,\text{tot}} \), however, is found to be negligible. We also plot the color temperature predicted by Nakar & Sari (2010) for the spherical case (dash-dotted curve), along with the color temperature of our 1D simulation (dashed curve) for comparison; the methods agree well.

For \( T_{c,\text{tot}}(t) \), we first derive the location of \( \tau_{\text{diff}} \) and \( r_{\text{diff}} = 1 \) for the progenitors over time to check whether weak or strong thermalization regimes apply. For the compact Type Ic progenitor, we find \( \tau_{\text{diff}} > r_{\text{diff}} = 1 \) giving rise to a nonthermal spectrum, and thus the color temperature starts as high as 50 keV (hard X-rays) and rapidly cools down to 100 eV in a few seconds. We should, however, be cautious while comparing these results to observations; as pointed out by Couch et al. (2011), the thermalization front of compact progenitors—like Wolf–Rayet stars—is located in the region between the reverse
and the forward shock due to the thick winds that these progenitors normally launch. We do not address this possibility. Our analysis of the BSG model also shows that \( r_{\text{fin}} < r_{\text{eff}} \) during the evolution, as can be seen in Figure 16 for \( t = 6420 \) s. For this model, the color temperature peaks at 65 eV—slightly higher than its spherical counterpart—during the breakout from the poles and continues to closely follow the spherical limit as an oblique flow develops. Similarly, for the RSG model, the color temperature is initially as high as \( \sim 13 \) eV. The influence of asphericity on \( T_c \) is at most a factor of two. As \( T_c > 1 \) eV, hydrogen recombination will not significantly affect our light curves.

5.7. Multicolor Light Curves

Using color temperatures and bolometric luminosities, derived in Sections 5.5 and 5.3, respectively, we employ a blackbody photon distribution to find band-dependent light curves, ignoring complications such as Comptonization, finite photon chemical potential, and light travel time effects. In Figure 17, the multicolor light curves are shown for four different cases: (a) total luminosity \( L_{\text{tot}} \); (b) an observer along the axis of symmetry, \( \Theta = 0 \); (c) an observer along the equator, \( \Theta = \pi/2 \); and (d) the multicolor light curves of our 1D spherical explosion.

As shown in Figure 17, the early light curve of our Type Ic model is initially dimmer than the spherical case by several magnitudes in the optical (i.e., \( g^\prime \) filter) and far-ultraviolet (FUV) bands. There are two parameters determining the shape of these light curves: the bolometric light curve is less luminous for aspherical cases, and the color temperature is about 35 keV higher in aspherical cases (shown in Figure 15), shifting the optical and FUV bands more toward the Rayleigh–Jeans tail of the spectrum and hence reducing the flux in these bands. In the X-ray (0.2–20 keV), the evolution approximately follows the bolometric light curves of Figure 13(a), with minor differences. The earliest part of the light curve takes time to rise in \( g^\prime \) and FUV, so long that the observed band is in the Rayleigh–Jeans tail of the spectrum and the bolometric luminosity is decreasing. The rise stops when the band color matches the color temperature.

The early light curves of the aspherical BSG model have distinctive features in the optical and FUV, as shown in Figure 17. During the early nonoblique breakout phase, \( L_{\text{tot}} \) suddenly drops \( \geq 3 \) mag over \( \sim 100 \) s. As a result, the early aspherical light curves exhibit a peak, despite the fact that \( T_{\text{band}} \ll T_c \) and \( T_c \) is rapidly declining. The light curves then rise at a mostly decelerating rate. This evolution is somewhat different from that of the spherical case. First, the early part of the aspherical light curves (\( t < 10^{3.8} \) s) is \( \lesssim 4 \) mag dimmer than the spherical one. Second, the early peak in the aspherical cases is strong \( (\gtrsim 3 \) mag drop) compared to the spherical case \( (\lesssim 2 \) mag drop). The oblique phase \( (t > 10^{3.8} \) s) of the polar light curve is similar to but dimmer than the expanding ejecta phase of a spherical explosion. In the X-ray, the light curves closely follow their bolometric counterparts.

For the RSG model, the aspherical light curves are even more observer-dependent in the optical and FUV. For example, an observer looking along the axis of symmetry sees a shallower early peak followed by a slower rise. This is because the on-axis view of the isotropic-equivalent light curve,
obs \( Q = (\) \), declines more slowly than the equatorial view, \( L_{\text{obs}}(\Theta = \pi/2) \), and is monotonically decreasing. The early peak in the spherical model is associated with the SBO flash in the optical and FUV. The spherical model’s light curve then declines until \( t = 10^{4.7} \) and \( t = 10^{4.9} \text{s} \) in FUV and \( g' \), respectively, and rises afterward; this is when the planar phase ends and spherical evolution begins (Nakar & Sari 2010). The aspherical evolution does not exhibit these features, and the earliest light is much dimmer than in the spherical case. We must, however, be cautious when interpreting the band-dependent light curves of the RSG model. For this model, the simulation’s imposition of adiabatic flow is incorrect in the outermost zones. Because the diffusion front typically moves inward in mass coordinates, the adiabatic approximation tends to be valid up to the point that luminosity is generated; however, any imprint of the outermost matter must be taken with a grain of salt.

6. Circumstellar Collisions

Circumstellar collisions are a direct consequence of the formation of oblique flows. As shown in Figure 2(d), the collisions happen along the equator as oblique flows run into each other and form an expanding wedge of material with excess pressure and density. According to Figures 18(a) and (b), the wedge pressure \( P_c \) and wedge density \( \rho_c \) are both increasing within our limited simulation time. The horizontal in these plots represents \( \theta_c \equiv \sin^{-1}(x/R_*) \), which corresponds to the latitude of a point on the stellar surface tangent to a line crossing the equator at \( x \). This labels equatorial radii in a compact fashion. The tip of this axisymmetric wedge is roughly located at \( r/R_* = 1 \), equal to the original radius of the progenitor. The height of this wedge \( h_c \) increases with time, as shown in Figure 18(c), due to increasing internal energy in the region.

Figure 19(a) shows the energetics of the collision zone at different times and locations along the equator. In order to make this plot, the region where the fluid elements either enter the wedge or leave the simulation box between two consecutive temporal checkpoints is first identified. In this step, we assumed that the fluid elements move along straight lines at a constant speed. Next, the kinetic energy toward the equator (along \( \hat{z} \) for the upper/lower hemisphere) is computed for each cell in that region, and, subsequently, we obtained the location and time at which each fluid element hits the equator according to its velocity vector. This way, we ensure that the energy of a fluid element is not summed multiple times between temporal checkpoints and that all energy in the colliding region is taken into account, even the energy of material that enters the collisional wedge after the simulation end time. As shown in
is inversely related to the equatorial distance. In the middle panel, the normalized density in the collisional wedge is shown at different times. The right panel illustrates the height of the wedge at the border of the simulation box, i.e., $R/R_\text{e} = 4$, as a function of normalized time.

Figure 19(a), the input power peaks in the interval $\theta_c = (65^\circ, 90^\circ)$, i.e., $r < 1.1R_\text{e}$ over the period $0.4 < t/t_\text{s} < 1$. After this peak, the distance of the equatorial collision increases with collision time because the horizontal velocity is at most $2v_c$. Figure 19(b) depicts the radially integrated kinetic power into the equatorial wedge, as well as its cumulative energy. Its final value is $E_\text{c} = 10^{-3.17}E_\text{c} \approx 10^{47.8}$ erg, considering both hemispheres. Note that the characteristic energy for all three progenitors is $10^{51}$ erg, but the energy that ends up in nonradial flows depends on the progenitor model, as discussed in Section 4. The physical value of $E_\text{c}$ is thus mostly accurate for the Type Ic and BSG models. For RSG progenitors, $E_\text{c}$ is suppressed, as nonradial flows are inhibited by the early onset of radiation diffusion. (Furthermore, RSGs are likely to be less aspherical than our fiducial case; S14 found $E_\text{c} \propto \epsilon^{4.5}$ within adiabatic simulations.)

Deriving the emission from the collision zone is not trivial and is a subject for future work. Here we only describe possible solution regimes. As shown in Figure 20, the dividing lines for these regimes are set by the parameters $t_\text{diff}/t_\text{dyn}$ and $t_\text{cool}/t_\text{dyn}$, where $t_\text{cool}$ is the local cooling timescale. When $t_\text{diff} > t_\text{dyn}$, no emission is expected to be released, as the photons are trapped in the collision wedge and the adiabatic condition holds; this situation resembles an SN before SBO or ejecta before the arrival of the diffusion front. Adiabatic flow is also maintained when $t_\text{cool} > t_\text{dyn}$, so there is not enough time to cool. In the fast-cooling regime, i.e., $t_\text{cool} < t_\text{dyn}$ and $t_\text{diff} < t_\text{dyn}$, the luminosity is constrained by the rate at which the vertical kinetic energy $E_z$ enters the collision wedge. This rate is shown in Figure 19(b) in normalized units by a solid curve. For the other regimes, the kinetic luminosity is only an upper limit to the radiative luminosity. Scaling the normalized luminosity in Figure 19(b), we find that the peak of the collisional luminosity $L_{p,\text{col}} = \{10^{45.76}, 10^{43.14}\}$ erg $s^{-1}$ occurs at time $t_{p,\text{col}} = \{10^{4.45}, 10^{4.07}\}$ s for Type Ic and BSG progenitors, respectively. This potential luminosity source is significant relative to the diffusive light curve shown in Figure 13. However, we postpone any prediction of the actual emission to a future paper.

7. Summary and Discussion

We summarize this work and our major findings as follows.
—Goals and strategy. Our goals are to highlight the effects of aspherical geometry on the dynamics of SBO and SN early light, test theoretical predictions regarding oblique SBO in a global simulation, and advance the art of using adiabatic simulations to predict the outcome of radiation hydrodynamics and the band-dependent SN display. We do not attempt to study the mechanism that breaks spherical symmetry, nor do we survey realistic progenitors or include complicated physics (gravity, initial thermal energy, relativity, nuclear reactions, etc.) that are important for the central engine and remain important in certain classes of explosions. Instead, we consider strong, bipolar explosions within a single, simple, polytropic progenitor. We carefully study the effect of resolution on the explosion and breakout dynamics. We use the scale-free nature of adiabatic non-self-gravitating hydrodynamics to scale our results to several SN types. Then, we inspect our results to identify features that are invalid (for the chosen progenitor) due to the influence of photon diffusion; this is especially important in limiting the production of nonradial ejecta in diffuse RSG explosions. Next, we trace the progress of a photon diffusion front through the ejecta, acquiring the local energy flux and bolometric light curve. Finally, we track photon production within the ejecta in order to identify a color temperature for each patch of the diffusion front and hence derive band-dependent light curves. In each step, we rely on the fact that the rates of photon diffusion and production are power laws of the local fluid quantities and therefore can be scaled from the simulations through our variables $D$ (Pécellet number for nonradial flows; Equation (4)), $g_\text{e}$ (dimensionless, Galilean-invariant diffusion parameter; Equation (9)), and $G$ (photon thermalization parameter; Equation (20)).

—Dynamical evolution. We break spherical symmetry by adding a bipolar momentum to the hydrodynamic solution of an early spherical explosion. The shock first breaks out from the poles but then develops laterally, giving rise to a spray of ejecta in different directions. The asphericity in the simulation is sufficient to form highly nonradial flows during the SBO, limiting the ejection speed and strongly affecting the SBO emission and early light. It also engenders ejecta–ejecta
collisions in the equatorial disk. All of these features conform to the theoretical expectations of Matzner et al. (2013) and S14 and are consistent with the outcome of earlier numerical works, such as those by Couch et al. (2009). We note that reducing the asphericity can lead to mildly aspherical explosions, which are more realistic for larger progenitors such as RSGs. Nonradial motions are inhibited by radiation diffusion in such explosions.

—Resolution dependence. We investigate the effects of resolution on the results and find that the speed and structure of the fastest ejecta are relatively robust for sufficiently high-resolution runs, but KH instabilities affect the inner ejecta in an inevitably resolution-dependent fashion. We find that the creation of nonradial ejecta requires that the characteristic turning depth $\ell_c$ is resolved by about three zones. While this is far below the 683-zones-per-$\ell_c$ resolution of Salbi et al.’s fiducial run, it demands significant resolution of $R_\ast$ in a global simulation.

—Validity of adiabatic simulations. We use adiabatic simulations for a problem involving photon diffusion and radiative processes, so it is important to establish a regime of validity. This is especially important for the nonradial flow from oblique SBO; we find that diffusion significantly inhibits this flow ($D \ll 1$) for RSG progenitors (400 $R_\odot$), marginally affects BSG explosions (49 $R_\odot$), and is negligible for more compact progenitors. Again, this accords with the expectations set by M13 and S14.

—SBO emission. Whereas several prior works have assumed that SBO emission is extended but otherwise unaffected by the manner in which the shock reaches the stellar surface, M13 and S14 point out that strongly nonradial flow traps and hides this emission. We derive a simple estimate (Equation (7)) for the SBO luminosity in the case where photons are partially trapped. Those photons are trapped in optically thick ejecta until they reach radii at which diffusion is important; for this reason, the SBO emission blends into the early envelope-cooling luminosity.

—Bolometric light curve. We trace the progress of a photon diffusion front through the ejecta, acquiring the local energy flux at the diffusion radius and observer-dependent and spherically averaged bolometric light curves. We note that this method is robust against a breakdown of the adiabatic approximation, as the diffusion front moves inward relative to the matter—so the flow is essentially adiabatic until radiation is released. For our fiducial example (asphericity factor $\epsilon = 0.26$), we find that (1) the early peak in the luminosity is comparable to the spherical luminosity peak but briefer in time; (2) the peak is followed by a plateau phase for which the shock has become oblique, and this phase is comparable to the envelope-cooling phase of the spherical explosion but somewhat dimmer; and (3) geometrical effects make the light curves observer-dependent. In particular, the plateau is brighter for equatorial observers than polar observers.

—Thermalization, color temperature, and band-dependent light curves. Deviations from thermal equilibrium between
radiation and matter have a controlling influence on early SN brightness in observed bands. Photon production can either precede the arrival of the diffusion front or occur as radiation diffuses through the ejecta (Nakar & Sari 2010). We develop a technique to trace the thermalization of the photon population in each fluid element of the post-shock flow up to the arrival of the diffusion front and employ the results (along with an approximate treatment of the strongly thermalized case) to predict the color temperature of each sector of the ejecta. From this, we build band-dependent light curves. Our analysis indicates that the radiation field in Type Ic explosions is poorly thermalized, whereas thermalization is strong in RSG explosions. Again, BSGs represent an intermediate case. Because asphericity increases the color temperature, we see distinctive features in the optical and FUV. For example, the rise time is extended, and the first peak in magnitude is dimmer but deeper than that for the spherical case.

Our analysis is somewhat different from that of Wollaeger et al. (2017) and Barnes et al. (2017), who also conducted nonspherically symmetric hydrodynamical simulations and then predicted band-dependent light curves based on the result. Whereas these authors assumed a state of homologous free expansion and conducted Monte Carlo radiative transfer within the ejecta, accounting for radioactive heating, our approach addresses the emission of shock-deposited heat before homologous expansion is established. We anticipate that the two approaches will ultimately be combined.

**Circumstellar ejecta collisions.** A striking consequence of the nonradial nature of oblique breakout is the collision of ejecta outside the star. In the bipolar explosion considered here, ejecta–ejecta collisions occur in an expanding wedge around the equator of the explosion. We postpone a detailed analysis of radiative processes in this region to a subsequent paper, but we derive the energetics and constrain the luminosity in the fast-cooling regime to show the observational importance of these collisions. We find that a factor of \(10^{-3.17}\) of the total energy ends up in the collisions, and if this kinetic energy gets converted to radiation efficiently, the upper limit to the luminosity peak is \(L_{\text{p, col}} = \{10^{45.57}, 10^{43.14}\} \text{ erg s}^{-1}\) for Type Ic and BSG progenitors, respectively. We note that this estimate does not apply to the RSG model, as radiation diffusion breaks the adiabatic assumption made in the simulation.

**8. Conclusion**

Rich as it is, the parameter space of spherical models does not cover all of the dynamics and radiation processes that apply to SN explosions, and this holds for SN breakout emission, early light, and circumstellar interactions just as it does for the central engine. Nonspherical effects on the early light range from mild (extending the SBO emission but not altering it; Suzuki & Shigeyama 2010) to extreme (essential changes to the breakout emission and early diffusive light and a new source of dense circumstellar interactions), depending on the departure from spherical symmetry and the importance of radiative diffusion in the explosion. In general, one should suspect nonspherical effects, like those we consider here, if there is independent evidence of aspherical flow. Examples would include a declining early linear polarization (as seen in SN 2008ax by Chornock et al. 2011 and in SN 2011dh by Mauerhan et al. 2015) or high-velocity nickel/iron or other “inner” ejecta (as seen up to 3500 km s\(^{-1}\) in SN 1987A by Haas et al. 1990). In other cases, there have been discrepancies between modeling of the early light and constraints on the progenitor radius (SN 2011dh; Soderberg et al. 2012); extra \(u\)-band emission and transient narrow absorption features (SN 2013ge; Drout et al. 2016); and observed early light curves and what is expected from spherical theory (e.g., Taddia et al. 2015; Garnavich et al. 2016). While some of these features might be attributable to factors like extended stellar envelopes (Piro 2015) or intense pre-SN mass loss (e.g., Smith et al. 2011; Margutti et al. 2017), one cannot appeal to early radioactive heating from very high-velocity \(^{56}\)Ni (Taddia et al. 2015) without accounting for nonspherical effects as well.

Moreover, the central engine is itself likely to be nonspherical (Akiyama et al. 2003; Blondin et al. 2003), and binary interactions are likely to spin up or tidally distort a fraction of SN progenitors (Smith et al. 2011; Sana et al. 2012). Deviations from spherical symmetry will be weaker for more extended progenitors, both because their explosions are known to be more spherical and because rapid photon diffusion tends to prevent the development of nonradial motions. However, in the most compact progenitors, the early phase of emission (a few \(t_e\) after SBO) is optically dim and ends rapidly. We therefore posit that this particular signature of aspherical explosions will be most detectable in progenitors of intermediate radius, i.e., blue or yellow supergiant stars. Circumstellar collisions might then be the most notable hallmark of asphericity in compact Type Ibc events, but any firm conclusion awaits future study.

While this work demonstrates what types of changes one might attribute to an aspherical event, it is very limited in considering only a single structure for the progenitor and a restricted explosion geometry and in separating the dynamical problem from the radiation transfer problem (even when analyzing SBO, where the two are clearly linked). Radiation hydrodynamics codes, especially with multigroup radiation transfer, will offer far more sophisticated solutions. In this regard, we note that Suzuki et al. (2016) recently studied an aspherical BSG explosion within a radiation hydrodynamics code (M1 closure scheme) and did not see the development of strong nonradial flows. This is consistent with our theory, given the smaller departure from spherical symmetry in the Suzuki et al. model explosion.

We thank Paul Ricker for advice regarding the capabilities of the FLASH code and Stephen Ro, Yuri Levin, and Maria Drout for suggestions and comments. We also thank the anonymous referee for the helpful comments on the original draft of this work. This work was supported by an NSERC Discovery Grant (CDM) and a QEII-GSST Fellowship (NA). As the Appendix and parts of Section 5.3 were written at KITP Santa Barbara, we derive partial support from the US National Science Foundation under grant No. NSF PHY-1125915. Our simulations were carried out on Compute Canada resources. CDM thanks the Monash Centre for Astrophysics and the organizers of the KITP program The Mysteries and Inner Workings of Massive Stars for hospitality and support.

**Software:** FLASH4.2.
Appendix
Photor Production and Temperature Below the Diffusion Front

We find the color temperature evolution in a region of trapped matter and radiation by writing the conservation laws for photon number, internal energy, and mass in Lagrangian form,
\[
\frac{d n_{\text{ph}}}{dt} = -n_{\text{ph}} \nabla \cdot \mathbf{v} + \dot{n}_{\text{em}} - \dot{n}_{\text{abs}},
\]
(23)
\[
\frac{d u}{dt} = -(u + p) \nabla \cdot \mathbf{v} = -\gamma u \nabla \cdot \mathbf{v},
\]
(24)
\[
\frac{d \rho}{dt} = -\rho \nabla \cdot \mathbf{v},
\]
(25)
where \( n_{\text{ph}} \) is the photon number density; \( \dot{n}_{\text{em}} \) and \( \dot{n}_{\text{abs}} \) are the photon emission and absorption rates, respectively; \( u \) denotes the internal energy density; and \( p = (\gamma - 1)u \), according to the gamma-law equation of state. The operator \( d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla \) is the Lagrangian time derivative. For photon-dominated gas \( \gamma = 4/3 \) and \( u = 2.7n_{\text{ph}} kT \), we find
\[
\frac{d \ln T}{dt} = \frac{d \ln n_{\text{ph}}}{dt} + \gamma \frac{d \ln \rho}{dt} = (\gamma - 1) \frac{d \ln \rho}{dt} - \frac{\dot{n}_{\text{em}} - \dot{n}_{\text{abs}}}{n_{\text{ph}}},
\]
(26)
Equation (26) is obtained by plugging \( \nabla \cdot \mathbf{v} \) from Equation (25) into Equation (23). Taking \( T_{BB} \propto u^{1/4} \rho^{3/4} \) and normalizing the left-hand side of Equation (26) by \( T_{BB} \) gives
\[
\frac{d \ln (T/T_{BB})}{dt} = \left( \frac{3}{4} \gamma - 1 \right) \frac{d \ln \rho}{dt} - \frac{\dot{n}_{\text{em}} - \dot{n}_{\text{abs}}}{n_{\text{ph}}}.
\]
(27)
Using \( \dot{n}_{\text{em}} = \int 4\pi j_{\nu}/(h \nu) d\nu \) and \( j_{\nu} = \alpha_{\nu} B_{\nu}(T) \) from Kirchhoff’s law, we find \( \dot{n}_{\text{em}} = \alpha_{\text{em}} n_{\text{BB}}(T) \). Similarly, \( \dot{n}_{\text{abs}} = \int \alpha_{\nu} c_{\text{abs}}(\nu, T) d\nu = \alpha_{\text{abs}} n_{\text{BB}} \). Plugging \( \dot{n}_{\text{em}} \) and \( \dot{n}_{\text{abs}} \) into the right-hand side of Equation (27), we get
\[
\frac{\dot{n}_{\text{em}} - \dot{n}_{\text{abs}}}{n_{\text{ph}}} = \frac{\dot{n}_{\text{em}}}{n_{\text{ph}}} \left[ 1 - \frac{n_{\text{BB}}}{n_{\text{BB}}(T)} \right] - \frac{\dot{n}_{\text{abs}}}{n_{\text{ph}}} = \frac{\dot{n}_{\text{em}}}{n_{\text{ph}}} \left[ 1 - \left( \frac{T}{T_{BB}(u)} \right)^{-4} \right].
\]
(28)
where the last equation is found using \( u = 2.7n_{\text{ph}} kT = aT_{BB}^{2}(u) \) and \( n_{\text{BB}}(T) = aT^{3}/2.7k \). Combining Equations (27) and (28) and defining \( x = T/T_{BB}(u) \), we arrive at Equation (18) for the evolution of the temperature.

A.1. Initial Photon Starving Factor

Our analysis requires an initial value for the photon starving parameter \( x \). For this, we appeal to the theory of nonrelativistic, radiation-dominated shocks. For conditions relevant to core-collapse SNe, the shock jump is entirely controlled by radiation diffusion, with no hydrodynamic discontinuity (Tolstov et al. 2015). In a steady state, this diffusive shock has the form described by Weaver (1976), in which changes occur on a length scale \( c/(\eta_{\nu} \kappa_{\rho}) \) and timescale \( t_{\text{ph}} = c/(3\nu^{2} \kappa_{\rho}) \), where \( \rho_{0} \) is the upstream density and \( \nu_{\text{e}} \) is the shock speed.

Free–free photon production in such shocks has a diffusive shock transition in which the temperature reaches its maximum and a relaxation region in which diffusion is negligible. The relaxation zone is covered by Equation (18), so we use an estimate of the peak temperature to give \( x_{0} \). The time to build up a photon density \( n_{\text{ph}} \) is \( t_{\text{rel}} = n_{\text{ph}}/\dot{n}_{\text{em}} \). Therefore, a shock that builds up photons in time \( t_{\text{ph}} \), assuming that absorption and the initial photon population are both negligible, reaches a peak temperature \( T_{p} \) set by \( t_{\text{rel}} \sim t_{\text{ph}} \), given conditions at the shock: \( n_{\text{ph}} T_{p} \approx \rho_{0} v_{\text{ph}}^{2} \) and \( n_{\text{ph}} T_{p}^{1/2} / (C_{\text{ph}} \rho_{0}^{2}) \approx c/(\kappa_{\nu} \nu_{\text{e}}^{2}) \), implying \( kT_{p} \sim k\nu^{2} v_{\text{ph}}^{2}/(c^{2} C_{\text{ph}}^{2}) \), or using the prefactor estimated by Katz et al. (2010; which agrees with Weaver 1976),
\[
kT_{p} \approx 10 \text{ keV} \frac{A^{2}}{Z^{4}} \left( \frac{\nu_{\text{ph}}}{0.2 \, c} \right)^{8}.
\]
(29)
We have introduced the composition dependence, given by \( A^{2}/Z^{4} \approx \{1, 1, 1/9, 1/16\} \) for (H, He, C, O). To compute \( x_{0} \), we must compare \( T_{p} \) to the equilibrium temperature at the post-shock pressure:
\[
kT_{\text{eq}} = 15.4 \text{ keV} \left( \frac{E_{\text{em}}}{10^{51} \text{ erg}} \right)^{1/4} \left( \frac{R_{e}}{R_{s}} \right)^{3/4} \left( \frac{\rho_{0} v_{\text{ph}}^{2}}{\rho_{0} v_{\text{ph}}^{2}} \right)^{1/4}.
\]
(30)
In the case that \( T_{p} < T_{\text{eq}} \), thermal equilibrium is achieved within the shock itself, and so \( x_{0} = 1 \). Furthermore, an upper limit (\( x_{0} < x_{0,i} \)) is set by the pre-shock photon population. Therefore,
\[
x_{0} \approx \text{max} \left[ x_{0,i}, \frac{T_{p}}{T_{\text{eq}}} \right].
\]
(31)
As a practical matter, we do not track the initial density and shock velocity of the ejecta in the simulation; these must be reconstructed from conditions at the diffusion front to evaluate \( T_{p} \) and \( T_{\text{eq}} \). To estimate \( x_{i} \), we note that both planar self-similar breakouts and strongly oblique breakouts cast away matter at twice the shock velocity (2.03 and 2.13, respectively, to be precise; see Matzner & McKee 1999 and Matzner et al. 2013). So, we estimate \( v_{\text{ph}} = f_{v} \nu/v \) with \( f_{v} \approx 1 \) and adjust \( f_{v} \) to best fit the simulation results. Then, \( \rho_{0} \) can be obtained from entropy conservation: \( s = p/\rho v^{3/4} = (6/7^{3/4}) v^{2} k_{\nu}^{-1} \). Eliminating \( \rho_{0} \), we find
\[
\frac{T_{p}}{T_{\text{eq}}} = 0.25 \frac{A^{2}}{Z^{4}} \left( \frac{E_{\text{em}}}{10^{51} \text{ erg}} \right)^{15/4} \left( \frac{R_{e}}{R_{s}} \right)^{3/4}
\times \left( \frac{10 M_{\odot}}{M_{\text{ej}}^{2}} \right)^{4} \left( \frac{s}{s_{\text{e}}} \right)^{3/4} \left( \frac{f_{v} v}{10 v_{s}} \right)^{6}.
\]
(32)
We see that the deviation from thermal equilibrium is a strong function of the shock velocity. Therefore, \( x_{0} \) is quite uncertain. However, the band-dependent light curve is much less uncertain because of the insensitivity of the color temperature to \( x_{0} \).

For the upper limit, we assume that the pre-shock photons have time to be Compton scattered up to energy \( kT \) long before being released, even if this did not occur during the shock transition. (This is valid so long as \( \ln[n_{\text{ph}} c^{2}/(7.6kT)] < \int \kappa_{\nu} c \, dt \sim \kappa_{\rho} (5r)/c/v \), i.e., for layers not involved in a
breakout flash.) Then,

$$x_0 = \frac{(n/e)_{\text{eq,}2}}{(n/b)_{\text{eq,}2}} = \frac{(P_{\text{rad}}/P_{\text{gas}})_{\text{eq,}2}}{(P_{\text{rad}}/P_{\text{gas}})}. \quad (33)$$

Here the subscripts “i” and “eq, 2” mean the initial hydrostatic state and an ideal post-shock state of thermal equilibrium, respectively, and “r” and “b” mean photons and baryons. Equation (33) defines $x_0$ as the ratio between the equilibrium population of photons (expressed as photons per baryon) and the initial population; the latter expression involving pressures is valid if the mean molecular weight $\mu$ does not change across the shock front.

The denominator of this expression can be obtained by reference to the progenitor stellar model. It is particularly simple to evaluate within our $n=3$ polytrope progenitors, as $(P_{\text{rad}}/P_{\text{gas}})$, takes the uniform value $(1 - \beta_i)/\beta_i$, where $\beta_i$ is the solution to Eddington’s (1926) quartic equation,

$$\frac{(1 - \beta_i)^{1/4}}{\beta_i} = \frac{M}{0.618 (\chi/G)^{3/2}/\mu^2} = \frac{M}{50 (0.6 m_p/\mu^2 M_\odot)}^{1/2}. \quad (34)$$

With $\mu = (0.61, 0.61, 1.71) m_p$ for the (RSG, BSG, WR) progenitors, respectively, we calculate $(P_{\text{rad}}/P_{\text{gas}}) = (0.08, 0.09, 0.40)$.

In the numerator, the ratio $(P_{\text{rad}}/P_{\text{gas}})_{\text{eq,}2}$ can be easily inferred from the entropy profile within our simulations, as this ratio is conserved in adiabatic flow. Specifically, it is $(1 - \beta_i)/\beta_i$, where $\beta_i$ is the solution to

$$\frac{(1 - \beta_i)^{1/4}}{\beta_i} = \frac{\mu}{k_B} \left( \frac{a}{3} \right)^{1/4} \frac{p_n^{3/4}}{\rho_n} \left( \frac{s}{s_\odot} \right)^{3/4} = \frac{2.0}{0.6 m_p} \left( \frac{E_{in}}{10^{51} \text{ erg}} \right)^{1/4} \frac{R}{R_\odot} \left( \frac{s}{s_\odot} \right)^{3/4}. \quad (35)$$

In fact, as $\beta_i \ll 1$, the right-hand side of this expression is a good estimate for $(P_{\text{rad}}/P_{\text{gas}})_{\text{eq,}2}$.

All together, then, $x_0 \approx (10^{3.3}, 10^{2.4}, 8.4) \times (s/s_\odot)^{3/4}$ for the (RSG, BSG, Ic) progenitors, where $s$ must be sampled at the radiation diffusion front.

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