Exclusive photoproduction of $\phi$ meson in $\gamma p \to \phi p$ and $pp \to p\phi p$ reactions

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(Dated: April 2, 2010)

Abstract

The amplitude for $\gamma p \to \phi p$ is calculated in a pQCD $k_T$ - factorization approach. The total cross section for this process is compared with HERA data. Total cross section, as a function of photon-proton energy and photon virtuality, is calculated. We also discuss the ratio of $\sigma_L/\sigma_T$ and the dependence on the mass of the strange quark. The amplitude for $\gamma p \to \phi p$ is used to predict the cross section for exclusive photoproduction of $\phi$ meson in proton-proton collisions. Absorption effects are included. The results for RHIC, Tevatron and LHC energies are presented.

PACS numbers: 13.60.Le, 13.85.-t, 12.40.Nn, 14.40.Be
I. INTRODUCTION

It was shown recently how to calculate the cross section for exclusive production of heavy quarkonia in the $\gamma p \rightarrow V p$ [1, 2] as well as in $pp \rightarrow pV p$ [3, 4] collisions within the $k_{\perp}$-factorization approach. The same formalism was also used recently to calculate exclusive production of the $Z^0$ boson [5]. In the latter case the corresponding cross section is however very small and cannot be measured at present luminosities.

In order to describe the photoproduction process in a strict pQCD framework, at least a hard scale must be present, which could be for example either a quark mass for heavy quarkonia, or a large photon virtuality for deep inelastic diffractive processes. The $k_{\perp}$-factorization formalism described in [1, 2] allows, however, a smooth continuation from the hard to soft regime, and therefore can be also used to model the photoproduction of light vector mesons. This derives from the fact that the $k_{\perp}$-factorization is a momentum space version of the color–dipole approach, where the soft region is modelled by the behaviour of the dipole cross section at large dipole sizes [6].

The ZEUS collaboration at HERA has measured exclusive production of $\phi$ mesons in the $\gamma p \rightarrow \phi p$ reaction [7] and in $\gamma^* p \rightarrow \phi p$ reaction [8]. Here we wish to test the results of the approach against the HERA data as well as to make predictions for the $p\bar{p} \rightarrow p\phi \bar{p}$ reaction for the Tevatron and the $pp \rightarrow p\phi p$ reaction for RHIC and LHC.

II. PHOTOPRODUCTION PROCESS $\gamma p \rightarrow \phi p$

The amplitude for the reaction is shown schematically in Fig.1. This amplitude has been derived for the first time in Ref. [1, 2]. The imaginary part of the amplitude can be written as:

$$\Im m M = W^2 \frac{c_v}{4\pi^2} \alpha_{em} \int \frac{dk^2}{\kappa^4} \alpha_s(q^2) \mathcal{F}(x_1, x_2, k_1, k_2) \int \frac{dz d^2k}{z(1-z)} I(\lambda_V, \lambda_\gamma), \quad (2.1)$$
where $I(\lambda_V, \lambda_\gamma)$ have the form:

$$I(L, L) = 4QMz^2(1 - z)^2 \left[ 1 + \frac{(1 - 2z)^2}{4z(1 - z)} \frac{2m_q}{M + 2m_q} \right] \psi_v(z, k) \Phi_2, \quad (2.2)$$

$$I(T, T) = m_q^2 \psi_v(z, k) \Phi_2 + \left[ z^2 + (1 - z)^2 \right] (\psi_v(z, k) k \Phi_1) + \frac{m_q}{M + 2m_q} \left[ (k^2 \psi_v(z, k)) \Phi_2 - (2z - 1)^2 (k \Phi_1) \psi_v(z, k) \right]. \quad (2.3)$$

In the expressions above $\psi_v(z, k)$ is the meson light-cone wave function and $\Phi_1, \Phi_2$ are given by (ref. \cite{1}):

$$\Phi_2 = -\frac{1}{(r + \kappa)^2 + \epsilon^2} - \frac{1}{(r - \kappa)^2 + \epsilon^2} + \frac{1}{(r + \Delta/2)^2 + \epsilon^2} + \frac{1}{(r - \Delta/2)^2 + \epsilon^2}, \quad (2.4)$$

$$\Phi_1 = -\frac{r + \kappa}{(r + \kappa)^2 + \epsilon^2} - \frac{r - \kappa}{(r - \kappa)^2 + \epsilon^2} + \frac{r + \Delta/2}{(r + \Delta/2)^2 + \epsilon^2} + \frac{r - \Delta/2}{(r - \Delta/2)^2 + \epsilon^2}, \quad (2.5)$$

where: $r = k + (z - \frac{1}{2})\Delta$ and $\epsilon^2 = m_q^2 + z(1 - z)Q^2$.

The quantity $F(x_1, x_2, \kappa_1, \kappa_2)$ is the off-diagonal unintegrated gluon distribution, which for small $\Delta^2$, within the diffraction cone can be approximated as:

$$F(x_1, x_2, k_1, k_2) = F(x_{\text{eff}}, k^2) \exp \left( -\frac{B\Delta^2}{2} \right), \quad (2.6)$$

where $x_{\text{eff}} = c_s \left( \frac{M^2 + Q^2}{W^2} \right)$, $c_s = 0.41$ and the forward unintegrated gluon distribution is taken from Ref.\cite{9} where it was found in the analysis of the deep-inelastic scattering data.

In the forward scattering limit, i.e. for $\Delta = 0$ azimuthal integrations can be performed analytically (see \cite{4}), and one obtains the following representation for the imaginary part of the amplitude for the transverse photons:

$$\Im m \mathcal{M}_T(W, \Delta^2 = 0, Q^2) = W^2 \frac{c_w \sqrt{4\pi\alpha_{\text{em}}}}{4\pi^2} 2 \int_0^1 \frac{dz}{z(1 - z)} \int_0^\infty \pi dk^2 \psi_v(z, k^2) \quad (2.7)$$

$$\frac{\pi d\nu_k^2}{k^4} \alpha_s(q^2) F(x_{\text{eff}}, k^2) \left( A_0(z, k^2) W_0(k^2, \kappa^2) + A_1(z, k^2) W_1(k^2, \kappa^2) \right),$$

where

$$A_0(z, k^2) = m_q^2 + \frac{k^2 m_q}{M + 2m_q}, \quad (2.8)$$

$$A_1(z, k^2) = \left[ z^2 + (1 - z)^2 - (2z - 1)^2 \frac{m_q}{M + 2m_q} \right] \frac{k^2}{k^2 + \epsilon^2}, \quad (2.9)$$
\[ W_0(k^2, \kappa^2) = \frac{1}{k^2 + \epsilon^2} - \frac{1}{\sqrt{(k^2 - \epsilon^2 - \kappa^2)^2 + 4\epsilon^2k^2}} \]  

(2.10)

\[ W_1(k^2, \kappa^2) = 1 - \frac{k^2 + \epsilon^2}{2k^2} \left( 1 + \frac{k^2 - \epsilon^2 - \kappa^2}{\sqrt{(k^2 - \epsilon^2 - \kappa^2)^2 + 4\epsilon^2k^2}} \right) \].  

(2.11)

The imaginary part of the amplitude for longitudinal photons is given as:

\[ \Im M_L(W, \Delta^2 = 0, Q^2) = W^2 \frac{c_V \sqrt{4\pi \alpha_{em}}}{4\pi^2} 2 \int_0^1 \frac{dz}{z(1-z)} \int_0^\infty \pi dk^2 \psi_V(z, k^2) \int_0^\infty \frac{\pi dk^2}{k^4} \alpha_S(q^2) F(x_{eff}, k^2) \left( A_L(z, k^2) W_L(k^2, \kappa^2) \right), \]

(2.12)

where

\[ A_L(z, k^2) = 4Q^2 M z^2 (1 - z^2) \left( 1 + \frac{1 - 2z^2}{4z(1-z)} \frac{2m_q}{M + 2m_q} \right), \]

(2.13)

\[ W_L(k^2, \kappa^2) = \frac{1}{k^2 + \epsilon^2} - \frac{1}{\sqrt{(k^2 - \epsilon^2 - \kappa^2)^2 + 4\epsilon^2k^2}}. \]  

(2.14)

In our calculation we choose the scale of the QCD running coupling constant at:

\[ q^2 = \max\{\kappa^2, k^2 + m_q^2\}. \]

The full amplitude, at finite \( \Delta^2 \), for transverse and longitudinal polarization can be written as:

\[ M_T(W, \Delta^2 = 0, Q^2) = (i + \rho) \Im M_T(W, \Delta^2 = 0, Q^2) \exp \left( \frac{-B(W)\Delta^2}{2} \right), \]

(2.15)

\[ M_L(W, \Delta^2, Q^2) = (i + \rho) \Im M_L(W, \Delta^2 = 0, Q^2) \exp \left( \frac{-B(W)\Delta^2}{2} \right), \]

(2.16)

where \( \rho \) is a ratio of real to imaginary part of the amplitude

\[ \rho = \frac{\Re e M}{\Im m M} = \tan \left( \frac{\pi}{2} \frac{\partial \log \left( \Im m M/W^2 \right)}{\partial \log W^2} \right) = \tan \left( \frac{\pi}{2} \Delta \Psi \right). \]

(2.17)

Above \( B(W) \) is a slope parameter which in general depends on the photon-proton center-of-mass energy and is parametrized in the present analysis as:

\[ B(W) = B_0 + 2\alpha_{eff}' \log \left( \frac{W^2}{W_0^2} \right), \]

(2.18)

with: \( B_0 = 11 \text{ GeV}^{-2}, \alpha_{eff}' = 0.25 \text{ GeV}^{-2}, W_0 = 95 \text{ GeV} \). [10]

Assuming exponential dependence of the amplitude on \( \Delta^2 \) (see Eq. (2.16)) the total cross section for transverse and longitudinal polarization can be written as:

\[ \sigma_L(\gamma p \rightarrow \phi p) = \frac{1 + \rho^2}{16\pi B(W)} \left| \Im M_T(W, \Delta^2 = 0, Q^2) \right|^2, \]

(2.19)
\[ \sigma_T(\gamma p \rightarrow \phi p) = \frac{1 + \rho^2}{16\pi B(W)} \left| \mathcal{M}_L(W, \Delta^2 = 0, Q^2) \right|^2. \] (2.20)

The full cross section is a sum of these two components:

\[ \sigma_{\text{tot}}(\gamma p \rightarrow \phi p) = \sigma_T(\gamma p \rightarrow \phi p) + \epsilon \sigma_L(\gamma p \rightarrow \phi p). \] (2.21)

In HERA kinematics, \( \epsilon \approx 1 \).

Let us collect now the main formulas involving the \( q\bar{q} \) light–cone wave-function of the vector meson. The electronic decay width depends on the model of the \( \phi \) meson wave function as:

\[ \Gamma(V \rightarrow e^+e^-) = \frac{4\pi\alpha^2_{em}Ze}{3M_V} \cdot g_V^2. \] (2.22)

In our calculation we use leading-order approximation, i.e. we neglect a possible NLO \( K \)-factor. The parameter \( g_V \) can be expressed in terms of the \( \phi \) - meson wave function as [1, 2]

\[ g_V = \frac{8N_c}{3} \int \frac{d^3k}{(2\pi)^3} (M + m_q) \psi_V(k^2). \] (2.23)

Following [1, 2] we use two types of the wave function, the Gaussian one, representing a standard harmonic–oscillator type quark model:

\[ \psi_{1S}(k^2) = C_1 \exp \left( -\frac{k^2a_1^2}{2} \right), \] (2.24)

and a Coulomb wave function, representative of models in which the wave function has a long high–momentum tail:

\[ \psi_{1S}(k^2) = \frac{C_1}{\sqrt{M}} \frac{1}{(1 + a_2^2k^2)^2}. \] (2.25)

The parameters of the wave function are obtained by fitting to the decay width into \( e^+e^- \) and imposing the normalization condition:

\[ 1 = \frac{N_c4\pi}{(2\pi)^3} \int_0^\infty k^2dk 4M\psi_{1S}^2(k^2). \] (2.26)

III. EXCLUSIVE PHOTOPRODUCTION OF \( \phi \) IN \( pp \) AND \( p\bar{p} \) COLLISIONS

The bare amplitude (when absorption effects are ignored) can be written schematically [3] as:

\[ M^{(0)}(p_1, p_2) = e_1 \frac{2}{z_1 t_1} \mathcal{F}_{\lambda_1\lambda_1}(p_1, t_1) \mathcal{M}_{\gamma^*h_2 \rightarrow V h_2}(s_2, t_2, Q^2) \] (3.1)

\[ + e_2 \frac{2}{z_2 t_2} \mathcal{F}_{\lambda_2\lambda_2}(p_2, t_2) \mathcal{M}_{\gamma^*h_1 \rightarrow V h_1}(s_1, t_1, Q^2). \]

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Because of the presence of the proton from factors only small $Q_1^2$ and $Q_2^2$ enter the amplitude for the hadronic process. This means that in practise one can put $Q_1^2 = Q_2^2 = 0$.

The full amplitude for the $pp \rightarrow p\phi p$ reaction which includes elastic rescatterings reads:

$$M(p_1, p_2) = \int \frac{d^2k}{(2\pi)^2} S_{el}(k) M^{(0)}(p_1 - k, p_2 + k)$$

$$= M^{(0)}(p_1, p_2) - \delta M(p_1, p_2),$$

where

$$S_{el}(k) = (2\pi)^2 \delta^{(2)}(k) - \frac{1}{2} T(k), \quad T(k) = \sigma^{pp}_{tot}(s) \exp \left( - \frac{1}{2} B_{el} k^2 \right).$$

(3.3)

In practical evaluations we take $B_{el} = 14$ GeV$^{-2}$, $\sigma^{pp}_{tot} = 52$ mb for the RHIC energy $W = 200$ GeV, $B_{el} = 17$ GeV$^{-2}$, $\sigma^{pp}_{tot} = 76$ mb [11] for the Tevatron energy $W = 1.96$ TeV and $B_{el} = 21$ GeV$^{-2}$, $\sigma^{pp}_{tot} = 100$ mb for the LHC energy $W = 14$ TeV.

The absorptive correction to the amplitude can be written as the following convolution of the bare amplitude ($M^{(0)}$) and the amplitude for elastic $pp$ or $p\bar{p}$ scattering as:

$$\delta M(p_1, p_2) = \int \frac{d^2k}{2(2\pi)^2} T(k) M^{(0)}(p_1 - k, p_2 + k).$$

(3.4)

The differential cross section is expressed in terms of the amplitude $M$ as

$$d\sigma = \frac{1}{512\pi^4 s^2} |M|^2 dy dt_1 dt_2 d\varphi.$$ 

(3.5)

This formula is used below to calculate several differential distributions in $pp \rightarrow p\phi p$ and $p\bar{p} \rightarrow p\phi\bar{p}$ reactions.
IV. RESULTS

Let us start review of our results with the $\gamma p \rightarrow \phi p$ reaction. In Fig.4 we show the total cross section for $\gamma p \rightarrow \phi p$ as a function of photon-proton center-of-mass energy $W$ for $Q^2 = 0 \text{ GeV}^2$. Our results are compared with the corresponding HERA data [7]. In order to describe experimental data we treat the mass of the strange quark as a free parameter, although it was fixed at $m_S = 0.37 \text{ GeV}$ in Ref. [9]. We show results for three different values of the strange quark mass. The red solid (upper) line is for $m_S = 0.37 \text{ GeV}$, blue (lower) line for $m_S = 0.50 \text{ GeV}$ and the black line (which goes through the data points) for $m_S = 0.45 \text{ GeV}$. We can see that the result for $m_S = 0.45 \text{ GeV}$ gives the best description of the experimental data, so below we shall present results only for this value of the strange quark mass. Our results for $Q^2$-dependence look consistent with the ones previously obtained by Ivanov in his PhD thesis [2].

In Fig.5 we show the total cross section as a function of photon virtuality for photon-proton center-of-mass energy $W = 75 \text{ GeV}$. The data points are taken from Ref.[8]. The solid

FIG. 4: Total cross section for the photoproduction $\gamma p \rightarrow \phi p$ process as a function of the photon-proton center-of-mass energy. In this calculation the Gaussian wave function is used. The curves are described in the text.

FIG. 5: Total cross section as a function of photon virtuality for two different wave functions: Gauss (left) and Coulomb (right) panel respectively. Here $m_s = 0.45 \text{ GeV}$. The solid line is for transverse and longitudinal cross section and the dashed line is only for transverse cross section.
line is for the sum of longitudinal and transverse cross sections \((\sigma_L + \sigma_T)\) and the dashed line is for transverse cross section \((\sigma_T)\) alone. It is important that both these components are included in the calculation. In the left panel we show results for the Gaussian wave function and in the right panel for the Coulomb wave function. We can see that the Gaussian wave function much better describes the experimental data than the Coulomb wave function.

\[
\begin{array}{c}
\sigma_L/\sigma_T \text{ as a function of photon virtuality. The red solid line is for the Gaussian wave function and the black solid line is for the Coulomb wave function. Here } m_s = 0.45 \text{ GeV.}
\end{array}
\]

In Fig.6 we show the ratio of the longitudinal cross section \((\sigma_L)\) to the transverse cross section \((\sigma_T)\) as a function of photon virtuality, again for \(W = 75 \text{ GeV}\). The red solid line is for the Gaussian and the black line is for the Coulomb wave function. Our results are compared with the HERA data from Ref.[8].

\[
\begin{array}{c}
\text{FIG. 7: Rapidity spectrum of exclusive } \phi \text{ production at RHIC (left), Tevatron (middle) and LHC (right) energies. The thin black lines are without, the thick red lines with absorption included. In this calculation Gaussian wave function was used and } m_s = 0.45 \text{ GeV.}
\end{array}
\]

Let us come now to the presentation of our results for exclusive production of \(\phi\) meson in hadronic reactions. In Fig.7 we show distributions in rapidity for the \(p\bar{p} \to p\phi\bar{p}\) (Tevatron) and \(pp \to pp\phi\) (RHIC, LHC) reactions without (black thin solid) and with (grey thick solid) absorption effects. The absorption effects are rather small and will be not included in calculations of other observables.

In Fig.8 we show separate contributions of photon-pomeron and pomeron-photon fusion mechanisms in rapidity distribution of \(\phi\) in \(p\bar{p}\) (middle panel) and \(pp\) (left and right panels) collisions. Here we have shown only results without absorptive corrections. The green line is for photon-pomeron, the blue line is for pomeron-photon fusion mechanism and the red most
FIG. 8: The photon-pomeron and pomeron-photon contributions to the rapidity distribution for the RHIC (left), Tevatron (middle) and LHC (right) energies. Here Gaussian wave function was used, $m_s = 0.45$ GeV and absorptive corrections are not included.

_upper line is a sum of the two components. In rapidity distribution the two contributions add incoherently $^3$.

FIG. 9: The transverse momentum distribution for the RHIC (left), Tevatron (middle) and LHC (right) energies.

In Fig.9 we show distribution in transverse momentum of the exclusively produced meson $\phi$. In the left panel we present results for RHIC energy, in the middle panel for Tevatron energy and in the right panel for the LHC energy.

FIG. 10: Distribution in azimuthal angle for the RHIC (left), Tevatron (middle) and LHC (right) energies.

Finally in Fig.10 we show the distribution in relative azimuthal angle between outgoing protons. Quite different distributions are obtained for the RHIC, LHC ($pp$ collisions) and Tevatron ($\bar{p}p$ collisions). This effect is of the interference nature and was already discussed for the exclusive production of $J/\Psi$ $^3$. 

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V. CONCLUSIONS

In the present analysis we have performed the calculation of the cross section for the $\gamma p \rightarrow \phi p$ process as well as for exclusive production of $\phi$ meson in the $pp \rightarrow p\phi p$ reaction.

The cross section for the first process depends strongly on the model of the $\phi$ meson wave function as well as on the strange quark mass. Both Gaussian and Coulomb wave functions were used with parameters adjusted to reproduce the electronic decay width of $\phi$. The strange quark mass was adjusted to reproduce the $\gamma p \rightarrow \phi p$ cross section measured by the ZEUS collaboration at HERA in the Gaussian model of the wave function.

So-fixed parameters were used to make predictions for the $p\bar{p} \rightarrow p\bar{p}\phi$ and $pp \rightarrow p\phi p$ reactions. We have presented distributions in the $\phi$ meson rapidity, in the $\phi$ meson transverse momentum and in the relative azimuthal angle between outgoing protons. The azimuthal angle dependence found here is of purely interference nature. Quite different distributions have been predicted for the Tevatron and LHC due to the different sign of the interference term (different electric charge of proton and antiproton).

The cross sections found in the present analysis are fairly large. In practice one could measure the $\phi$ meson either in the lepton-antilepton or in kaon-antikaon decay channels. The later method could be used by the ALICE collaboration at the LHC where even small transverse momenta of charged kaons can be registered [12]. A feasibility of the measurements requires, however, detailed Monte Carlo studies.

Acknowledgments We are indebted to Aharon Levy for providing us with the ZEUS experimental data. This work was partially supported by the Polish grants of MNiSW No. N202 249235 and No. N202 191634.

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