Spin light of neutrino in neutron star matter

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Abstract. In this paper we discuss the possibility of SL\nu radiation by a neutrino moving in dense matter of a neutron star. We determine a set of conditions inside neutron stars for which the radiation is possible and might have a considerable efficiency.

1. Introduction

The proof of existing of nonzero neutrino mass, that has been recently awarded by the Nobel Prize [1, 2], supports the confidence that neutrino must possess non-trivial electromagnetic properties [3]. The existence of neutrino diagonal magnetic moment makes it possible the process

\[ \nu \rightarrow \nu + \gamma \]  

with direct neutrino coupling to photons and without change of neutrino type. Being forbidden in the vacuum it can proceed under external conditions which can be represented by electromagnetic fields or background matter. The latter case was considered for the first time in [4] on the quasiclassical grounds and the corresponding radiation was termed there as “the spin light of neutrino” (SL\nu) (see also [5, 6]). The quantum theory of the SL\nu in the limit of vacuum photon dispersion was developed in [7, 8] and in [9].

From the general grounds, the phenomenon is of interest due to its direct connection to the neutrino magnetic moment. In above cited papers it was shown that the radiation possesses some peculiar features that makes it also possibly interesting for astrophysical applications where the high density of matter is met. In particular, at high neutrino energies and matter densities the SL\nu rate and the corresponding radiation intensity are strongly dependent on the matter density (to the second power). Moreover, SL\nu photons on average carry away the major part of the initial neutrino energy and acquire almost total circular polarization. In this light it seems important to investigate the phenomenon of SL\nu under conditions more appropriate to the real.

The necessity of taking into account the influence of plasma on the SL\nu was first indicated in [8]. The SL\nu mechanism and similar processes with a special focus on the medium influence on the emitted SL\nu photon were discussed in [10, 11] (see also references therein). It was
shown that due to the nontrivial dispersion of the photon (which should be termed as massive plasmon in this case) the process acquires a threshold, and conditions under which the process is kinematically open were discussed. The recent studies of these effects can be also found in [12] where the process was considered at different degrees of closeness to the threshold. However, to assess the real possibility for the $SL\nu$ to occur a more thorough analysis is needed that would consider other competing neutrino processes.

As it was shown in [7] the neutron star (NS) matter is relevant for $SL\nu$ production by antineutrinos, so that below we discuss the propagation of an antineutrino in such a kind of matter though still refer to it as neutrino if the misunderstanding does not arise.

2. Plasma effect on $SL\nu$

On the quantum level, the $SL\nu$ is described by the standard amplitude of the dipole electromagnetic interaction where the initial and final neutrino states are helicity-dependent in matter and have different helicities. The corresponding wave functions are the solutions of the modified Dirac equation in which the matter influence on neutrino is accounted for via the additional potential [7]. This equation gives also the dispersion relation for antineutrinos

$$E_\nu = \sqrt{(p + s\tilde{n})^2 + m^2_\nu + \tilde{n}},$$

in which the dependence on the helicity $s = \pm 1$ is explicitly present. The matter density parameter $\tilde{n}$ characterizes the strength of the interaction with matter and for the case of electron antineutrino moving in uniform matter composed of neutrons is given by

$$\tilde{n} = \frac{1}{2\sqrt{2}} G_F n_n \simeq 3.2 \times \left( \frac{n_n}{10^{38} \text{ cm}^{-3}} \right) \text{ eV},$$

where the last expression is brought to the density values peculiar for the neutron star interior.

The kinematics for the process (1) accounting for neutrino dispersion (2) and under assumption of the vacuum photon dispersion is easily resolved with respect to photon momentum $k$ and leads to unique solution for which neutrino helicity transits as $s = 1 \rightarrow s' = -1$. The correspondent angular dependence of $k$ is a single-valued function, defined for every angle. The solution exists for any value of the initial neutrino momentum, so that no threshold arise for the process in this case.

In the real conditions of the neutron star matter a considerable fraction of electrons is always present in the form of ideal degenerate Fermi gas [13]. The density of this gas is nevertheless considerably lower than the neutron density and according to various estimates may be of the order $n_e \simeq 0.05 - 0.1 n_n$. The gas is ultra-relativistic because its chemical potential is

$$\mu_e \simeq \left( \frac{3\pi^2 n_e}{2} \right)^{1/3} \simeq 130 \times \left( \frac{n_e}{10^{37} \text{ cm}^{-3}} \right)^{1/3} \text{ MeV} \gg m_e \simeq 0.51 \text{ MeV}. \quad (4)$$

Since electrons much less abundant we will ignore their influence on the neutrino motion. On the contrary, they are the key factor affecting the photon propagation in matter that result in appearance of the plasmon mass. Under the discussed above conditions it is calculated as [14]

$$m_\gamma = \left( \frac{2\alpha}{\pi} \right)^{1/2} \mu_e \simeq 8.87 \times \left( \frac{n_e}{10^{37} \text{ cm}^{-3}} \right)^{1/3} \text{ MeV}. \quad (5)$$

The energy-momentum conservation equations with the photon dispersion $\omega = \sqrt{k^2 + m_\gamma^2}$ taken into account can be reduced to quadratic equation with respect to the final neutrino momentum $p'$. The solution to the equation exist [12] when the initial neutrino momentum

$$p > \frac{(m_\gamma^2 + 2 m_\gamma m_\nu)/4\tilde{n}}{m_\nu + m_\gamma}.$$ 

(6)
Using the relevant numbers and introducing the threshold neutrino energy $E_{th}$ we have

$$p > p_{th} \simeq E_{th} \simeq 28.5 \times \frac{Y_e^{2/3}}{1 - Y_e} \left(\frac{10^{38} \text{cm}^{-3}}{n_n}\right)^{1/3} \text{TeV},$$

where $Y_e = n_e / n_n \simeq n_e / n_b$ is the number of electrons per baryon.

As it follows from Eqs. (3), (5) and (7), the neutrino mass is smaller than any of the parameters in the problem (the current upper limit for neutrino mass is $m_\nu \sim 1 \text{ eV}$) and thus can be omitted in the further analysis. In this case the photon momentum is found as

$$k_{1,2} = \frac{\left[4 (p + \bar{n}) \bar{n} + m_\gamma^2\right] p \cos \theta \pm (p + 2 \bar{n}) \sqrt{[4 (p + \bar{n}) \bar{n} - m_\gamma^2]^2 - 4 p^2 m_\gamma^2 \sin^2 \theta}}{2 [(p + 2 \bar{n})^2 - p^2 \cos^2 \theta]},$$

where $\theta$ is the angle between vectors $\mathbf{k}$ and $\mathbf{p}$. It has two branches defined on the same range of angles and thus the angular dependence of the photon momentum is two-valued. This feature of the $SL\nu$ is analogous to on-flight decays of single particles with generation of massive particles. The radiation is confined within the boundary angle $\theta_0$ which is determined by the radicand in Eq. (8). The angular dependence of the photon energy $\omega_{1,2} = \sqrt{k_{1,2}^2 + m_\gamma^2}$ calculated using (8) is presented in Fig. 1. As it is displayed, the range of the photon energy can span several orders of magnitude. The boundary angle is very small so that the radiation is strongly collimated.

![Figure 1](image)

**Figure 1.** The angular dependence of $SL\nu$ photon energy. The solid and dashed lines correspond to branches $\omega_1$ and $\omega_2$, respectively. The dash-dotted line correspond to the boundary angle $\theta_0$. A set of parameters used: $p = 3 \times 10^2 \text{ TeV}$, $m_\gamma = 8.87 \text{ MeV}$ and $\bar{n} = 3.2 \text{ eV}$.

Before turning our consideration to radiation characteristics we introduce the parameter

$$a = \frac{m_\gamma^2}{4 \bar{n} p},$$

with which the threshold condition reads simply as $a < 1$ (one should keep in mind also that $a > 0$). By this means, the parameter $a$ describes the closeness to the process threshold and thus expressions for $SL\nu$ dynamical parameters can be classified according to its value.

For relativistic neutrinos ($m_\nu / p \ll 1$) and $a$ not approaching 1 (the process is not close to the threshold) the $SL\nu$ rate and power are given by [12, 10]

$$\Gamma = 4 \mu^2 \bar{n}^2 p \left[(1 - a)(1 + 7a) + 4a(1 + a) \ln a\right],$$

$$I = \frac{4}{3} \mu^2 \bar{n}^2 p^2 \left[(1 - a)(1 - 5a - 8a^2) - 12a^2 \ln a\right].$$

We note that these formulae covers the most wide range of the parameter $a$ variation.
For $a \ll 1$ or $a \to 0$ (the “far above-threshold” regime) these expressions transform into the ones obtained for relativistic neutrinos and with no account for the plasmon mass [8]

$$\Gamma = 4\mu^2 n^2 \rho, \quad I = \frac{4}{3} \mu^2 n^2 \rho^2.$$  \hspace{1cm} (12)

We therefore conclude that in the case of ultrarelativistic neutrino energies the effects of the nontrivial photon dispersion in plasma on the $SL\nu$ become insignificant.

For $1 - a \ll 1$ or $a \to 1$ (the “near-threshold” regime) we have [12]

$$\Gamma = 4\mu^2 n^2 (1 - a) [(1 - a) \rho + 2\tilde{n}], \quad I = 4\mu^2 n^2 \rho (1 - a) [(1 - a) \rho + 2\tilde{n}].$$  \hspace{1cm} (13)

These quantities tend to zero as $a \to 1$ (when approaching the threshold).

3. $SL\nu$ radiation in neutron star matter

Substituting $n_n \simeq 10^{38} \text{ cm}^{-3}$ and $Y_e = 0.1$ into Eq. (7) we obtain $E_{th} \simeq 6.82 \text{ TeV}$. At such high energies the (anti)neutrino can also scatter on the electrons of matter with creation of $W$ boson, $\bar{\nu}_e + e^- \to W^-$. The threshold energy $\varepsilon_W$ for this process is given by

$$\varepsilon_W = \frac{m_W^2}{4\mu_e} \zeta 5.77 \times \left( \frac{10^{38} \text{ cm}^{-3}}{Y_e n_n} \right)^{1/3} \text{ TeV}.$$  \hspace{1cm} (14)

For the neutrino energies greater than this value the probability of $SL\nu$ is considerably lower than that of the $W$ boson production. Moreover, for neutrino energies around $\varepsilon_W$ the propagator effect in the neutrino equation of motion should be accounted for due to the Glashow resonance in the $W$ boson propagator [11, 16]. With the known procedure of handling the resonance amplitude behavior [15] it is possible to show that for $Y_e = 0.1$ the $SL\nu$ process is closed, $\varepsilon_W < E_{th}$. But already for $Y_e = 0.09$ it becomes kinematically open and a “window” of allowed energies expands rather fast with the decrease of the electron fraction $Y_e$. For $Y_e = 0.05$ it extends from $E_{th} \simeq 4.6$ TeV to $\varepsilon_W \simeq 15.8$ TeV, see Figure 2 [16].

![Figure 2](image_url)

**Figure 2.** The allowed range (green area) of electron antineutrino energies for the $SL\nu$ in the NS matter depending on the neutron density. Solid and dash-dotted lines: $E_{th}$ without and with the propagator effect, respectively; dashed line: $\varepsilon_W$ (see the text). (a) $Y_e = 0.09$; (b) $Y_e = 0.05$.

Concerning other antineutrino species, since they scatter on electrons only via neutral currents in the t-channel the competing boson production and Glashow resonance do not occur, and
accordingly, no shift in $SL\nu$ threshold is expected [16]. Thus the $SL\nu$ is possible for muon and tau antineutrinos with energies above the threshold given by Eq. (6) (in which the proper expression for $\tilde{n}_n$ must be inserted). In numerical evaluation it is about $2 - 7$ $TeV$ depending on the neutron density and electron fraction.

In order to assess the process efficiency, we calculate the neutrino lifetime in NS matter with respect to $SL\nu$ photon radiation for muon and tau-antineutrinos far from the threshold:

$$\tau_{SL\nu} \simeq 2.17 \times 10^6 \left( \frac{10^{-11} \mu_B}{\mu} \right)^2 \left( \frac{10^{38} \text{ cm}^{-3}}{n_n} \right)^2 \left( \frac{10 \text{ TeV}}{E_\nu} \right) \text{ s.} \quad (15)$$

Considering the optimistic case when $\mu \simeq 2.9 \times 10^{-11} \mu_B$, $n_n = 10^{38} \text{ cm}^{-3}$ and $E_\nu \simeq 10 \text{ PeV}$ we obtain $\tau_{SL\nu} \simeq 320 \text{ s}$. The corresponding radiation length $\ell = c\tau_{SL\nu} \simeq 9.6 \times 10^{12} \text{ cm}$ is much larger than the typical NS radius $R \sim 10^6 \text{ cm}$. The smaller values for the radiation length are accessible within hypothetical models of the so-called third family stars (quark and hybrid stars) where the baryon density can possibly reach values much greater than that of the neutron stars. Then assuming $n_b \sim 10^{41} \text{ cm}^{-3}$ [13] we have $\tau_{SL\nu} \simeq 2.6 \times 10^{-4} \text{ s}$ and $\ell \simeq R$.

Thus, we show that under certain conditions for the neutron star matter the $SL\nu$ radiation is allowed. For ultra-relativistic neutrinos the radiation can be quite efficient, however to suit the real conditions either extra-dense background matter or extra-high neutrino energy is needed. As the most promising relevant environment we can point at galaxy clusters [17] where energetic charged particles can be accelerated due to various mechanisms and bound for very long periods of time at the same time producing high-energy neutrinos (see [16] for detailed discussion on this topic).

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