The role of the ambipolar field in the formation of the EDF and the criteria of the local approximation

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Abstract. It is shown that in the situation when the ambipolar field exceeds the heating field the local approximation (LA) for determining the EDF is not applicable even if the energy relaxation length of the electron is relatively small. Therefore, accurate results can be obtained only solving the kinetic equation in energy and spatial variables.

1. Introduction
The electron distribution function is usually calculated using the local approximation. In this case the solution of the Boltzmann kinetic equation has a simplified form. The equation depends only on one variable (velocity \( v \) or kinetic energy \( w = \frac{mv^2}{2} \)) and can be written as

\[
- \frac{1}{\sqrt{w} \frac{\partial}{\partial w}} \left( \sqrt{w(eE)^2 D} \frac{\partial f_0}{\partial w} \right) = St(f_0) + St^*(f_0)
\]

In eq.(1) \( E \) – is the electric field at a given point in space, \( D = \frac{v \lambda_3}{3} \) - is the electron diffusion coefficient, \( St(f_0) \) and \( St^*(f_0) \) - the integrals of elastic and inelastic collisions.

Criteria of applicability of local approximation in the literature are usually produced in a standard way using a condition of smallness of spatial gradients in comparison with energy derivatives in the kinetic equation. The criteria is governed by the relation between the characteristic plasma dimension \( R \) and the energy relaxation length \( \lambda_e \) [1]

\[
\lambda_e = \sqrt{2D_r \tau_e}
\]

where \( D_r = \frac{v \lambda_3}{3} \) is the electron diffusion coefficient and the energy relaxation time has the following form

\[
\tau_e^{-1} = \delta \nu + \nu^*
\]

and determined by the energy losses due to elastic and inelastic collisions (the corresponding frequencies \( \nu \) and \( \nu^* \)), \( \delta << 1 \) is the energy transfer factor for elastic (and quasi-elastic) collisions. It is suggested that that if \( R > \lambda_e \), then we can use the local approximation (1) in the calculations. As shown by the results [2-4], the most disputable is the question of the electric field, which enters the
initial and the local kinetic equations. The electron reacts to the total field that exists at a given point in space, the kinetic equation includes the total electric field at a given location. As a rule, for numerical simulations by different computational codes [5, 6], the kinetic equation is substituted for the total field, which is found from the solution of the Poisson equation.

In contrast, in [7] it was noted the ambipolar field is also determining by spatial gradients. And when we use the local approximation in the local kinetic equation (1), the ambipolar electric field should be neglected. In other words, the ambipolar field should be removed (subtracted) from the total field when it is substituted into (1).

The question of which field should be substituted into the local kinetic equation (complete or minus ambipolar) is controversial, to which there is still no unequivocal opinion in the literature.

In this paper we present a continuation of the studies [2-4]. It is shown that at the periphery of the plasma, where the ambipolar field is dominant, the local approximation for determining the EDF and other characteristics of the electron gas is inapplicable. In condition when the $R > \lambda_e$ reliable results can be obtained only when solving the kinetic equation in both energy and spatial variables.

2. Results and discussion

The object of the research was a positive column (PC) of a DC discharge in argon in a tube of radius $R = 1$ cm in a wide range of pressures.

For carrying out the numerical experiments, a computational model was developed that includes a number of related modules in COMSOL Multiphysics [2-4]: a) the kinetic module for solving the 2D Boltzmann equation in coordinate variables and kinetic energy $(r, w)$; b) a diffusion-drift module that includes equations for finding the densities of all neutral and charged particles; c) Poisson equation for calculating the field and potential profiles. A set of plasmachemical reactions in argon includes direct ionization, excitation from the ground state, radiative excitation, stepwise ionization from the metastable state, and Penning ionization.

For example, fig. 1 shows normalized EDFs at different points of the radius for the average pressures of 12 Torr, when, by criterion (2.3), there must certainly be a local regime for the EDF ($\lambda_e > R$).

**Figure 1.** Comparison of EDF at different values of the radius for argon $p = 12$ Torr. $I = 3$ mA

**Figure 2.** Axial (heating) and transverse (ambipolar) fields in argon at different pressures
It can be seen that the EDF at different points of the radius do not coincide, i.e. depend not only on energy, but also on the spatial variable. It means that the EDF is nonlocal (meaning that when calculating the definition it is necessary to use the complete kinetic equation in the variables both the energies and the coordinates).

As preliminary studies have shown [2-4], the main reason for the appearance of the radial dependence of the EDF is the contribution of the ambipolar field, which, as is known [7], increases from the center to the periphery of the discharge.

Figure 2 shows the values of the ratio of ambipolar and heating fields at different pressures. It is seen that with increasing pressure (already from several Torr and higher), i.e. when the energy relaxation length of the electron (2) is small and \( R > \lambda_E \), the ambipolar field (sooner or later) begins to exceed the longitudinal field, at the periphery of the PC.

In order to determine which field should be substituted into the local kinetic equation, total or only heating field (as recommended in [1, 7]), were performed calculations of the local Boltzmann equation (1) with total and with only heating field (see Fig. 3.4).

![Figure 3 (a, b). Comparison of the EDF with the nonlocal and local approximation for argon at p = 12 Torr, I = 3 mA. a - on the tube axis \( r = 0 \). b - \( r = 0.8R \)](image)

It can be seen that on the axis (in the near-axis regions), where the ambipolar field \( E_a \) is small in comparison with the longitudinal \( E_z \) (see Fig. 3), the curve of the local EDF calculation are in good agreement with the complete solution of the kinetic equation. At the same time, local calculations with the total field \( E^2 = E_a^2 + E_z^2 \) practically coincide with the solution of the complete kinetic equation at the periphery of the discharge, where the ambipolar field dominates. In this case, large discrepancies occur when substituting in the local kinetic equation (1) only one longitudinal field \( E_z^2 \) (as recommended in [1, 7]).

For the argon, the elastic collision cross sections are growing with energy. In this case, as the energy increases, the friction ("resistance") of the electrons increases with their energy diffusion in the field. Therefore, it is of interest to consider the case in which the elastic scattering cross sections are incident. Figures 4 (a, b) show corresponding results, similar to fig. 3 but using the model cross section of elastic collisions of the type \( \sigma = 1/w \).
Figure 4 (a, b). A comparison of the EDF with a nonlocal and local approximation for a cross section of the type at 12 Torr (0r). The current is 3 mA. a) - on the tube axis r = 0, b) - r = 0.8R

From this comparison, it is seen that the opposite situation is observed in fig. 3: the calculations of local EDF in accordance with (1) and with the substitution of the longitudinal field (as recommended in [1, 7]) correspond to the results of modeling, while with a full field give large differences.

3. Conclusions

Consequently on the periphery of the plasma, where the ambipolar electric field exceeds the heating electric field, even if the condition $R > \lambda_e$ is satisfied, one can’t use the local approximation (1) to determine the EDF with the substitution of both the total field at a given point in space and only the heating field (minus the ambipolar field). Moreover, depending on the behavior of the cross section for the elastic scattering of electrons from energy (increasing or falling), errors occur both in the ambipolar field in the local kinetic equation (1) and in the use of the total field there. Therefore, reliable results can be obtained only when solving the kinetic equation in both energy and spatial variables.

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