Multi-Excitation Magnetoacoustic Tomography with Magnetic Induction (MAT-MI)

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Abstract. Magnetoacoustic tomography with magnetic induction (MAT-MI) is an approach proposed to do non-invasive electrical conductivity imaging of biological tissue using magnetic induction and ultrasound measurements. In the present study, based on the analysis of the relationship between the conductivity distribution and the generated MAT-MI acoustic source, we propose a new multi-excitation algorithm to achieve better reconstruction of the conductivity distribution. In this algorithm, multiple magnetic excitations using different coil configurations are employed and the ultrasound measurements corresponding to each excitation are collected to derive the conductivity distribution inside the sample. The corresponding modified algorithm is also proposed for MAT-MI imaging when only limited bandwidth acoustic measurements data are available. Computer simulations have been done to demonstrate the performance of the proposed algorithm. It is shown that using this algorithm and unlimited bandwidth data, we can accurately reconstruct the internal conductivity contrast of the object. With limited bandwidth data and using the modified algorithm we can also reconstruct the relative conductivity contrast of the object instead of only the boundaries at the conductivity heterogeneity. Benefits come with this new algorithm include better differentiation of different tissue types with conductivity contrast using MAT-MI approach, specifically for the potential breast cancer screening application in the future.

1. Introduction
Noninvasive electrical impedance imaging of biological tissues has drawn lots of research interest in recent years. Related imaging techniques include electrical impedance tomography (EIT) [1], magnetic induction tomography (MIT) [2] and magnetic resonance electrical impedance tomography (MREIT) [3]. An alternative approach is through the coupling between electromagnetic field and acoustic field as reported in magnetoacoustic tomography (MAT) [4], [5] and Hall Effect imaging (HEI) [6]. In order to avoid the “shielding effect” associated with surface electrode current injection, magnetoacoustic tomography with magnetic induction (MAT-MI) [7]-[10] was proposed. MAT-MI utilizes magnetic induction and the same Lorentz force coupling mechanism as in MAT/HEI.

2. Theory
We consider a sample with isotropic conductivity $\sigma(r)$ in a uniform static magnetic field $B_\phi = B_{0z}\hat{z}$. With $N$ different excitation coil setups, using the magnetic vector potential $A^j(r,t)$ and electrical scalar potential $\phi^j(r)$, magnetic induction in MAT-MI can be formulated as in equation (1) [8]
\[ \nabla \cdot (\sigma \left( \frac{\partial A^j}{\partial t} - \nabla \varphi^j \right)) = 0 \quad j = 1, \cdots, N \]  

(1)

With known \( \sigma \) and \( A^j \), we can solve for \( \varphi^j \) in the whole domain using finite element method [11].

In MAT-MI, the wave equation governing the generated pressure is given in equation (2) [5]

\[ \nabla^2 p_j - \frac{1}{c_s^2} \frac{\partial^2 p_j}{\partial t^2} = \nabla \cdot (J^j \times B_0) \quad j = 1, \cdots, N \]  

(2)

where \( p_j \) and \( J^j \) are the pressure and induced current density corresponding to the \( j \)th excitation and \( c_s \) is the acoustic speed in the media.

With the obtained acoustic measurements of \( p_j \), we can reconstruct the acoustic source map \( A S^j = \nabla \cdot (J^j \times B_0) \) using back projection method [7]. Expanding the acoustic source term, we have

\[ A S^j = (\sigma(-B_{iz}^j) + \left[ \frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right] \cdot (E_x^j, -E_y^j))B_0z \quad j = 1, \cdots, N \]  

(3)

where \( E_x^j \) and \( E_y^j \) are the x and y components of the induced electrical field vector \( E^j \), respectively. Using matrix form equation (3) becomes

\[ Ux = b \]  

(4)

where

\[ U = \begin{bmatrix} E_x^1 & -E_y^1 \\ \vdots & \vdots \\ E_x^N & -E_y^N \end{bmatrix} \quad x = \begin{bmatrix} \frac{\partial \sigma}{\partial x} \\ \frac{\partial \sigma}{\partial y} \end{bmatrix}^T \]  

and

\[ b = \begin{bmatrix} A S^1(r) \sigma B_{iz}^1 + \cdots A S^N(r) \sigma B_{iz}^N \end{bmatrix}^T \]

We can then solve \( x \) using the least square method. To computer \( \sigma \) from \( x \) in each imaging slice, an integration technique can be used [12]. The detailed algorithm description will be reported elsewhere.

One of the major technical limitations of the MAT-MI approach comes from the limited bandwidth acoustic measurements. Denoting the partly reconstructed acoustic source map as \( \tilde{A} S^j \), we have

\[ \tilde{A} S^j = (\left[ \frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right] \cdot (E_x^j, -E_y^j))B_0z \quad j = 1, \cdots, N \]  

(5)

A similar matrix equation can be obtained with a modified right hand side \( \tilde{b} \) and similar reconstruction procedure can be applied to estimate the conductivity distribution of the object.

**Figure 1.** (a) Target conductivity image. (b) Reconstructed conductivity image using unlimited bandwidth data and under noise free condition. (c) Conductivity profiles along \( y = 0.02 \) m.

3. Results
To validate the proposed multi-excitation MAT-MI method, we first did a computer simulation using unlimited bandwidth data and under noise free condition. Three groups of coils are used to sequentially send magnetic excitations, i.e. $N = 3$. The reconstruction result is shown in figure 1. The correlation coefficient (CC) between the target image and the reconstructed image is 99.1% and the relative error (RE) is 8.5%. This result shows that a much better performance can be obtained using the current algorithm as compared to other previous algorithms [8], [10].

![Simulated Impulse Response](image1)

![Reconstructed Conductivity (S/m)](image2)

Figure 2. (a) Simulated impulse response function. (b) and (c) are reconstructed conductivity images using limited bandwidth data with SNR to be 1000 and 100, respectively.

We have also conducted simulation study to test the performance of the modified reconstruction algorithm when only limited bandwidth measurement data are available. Figure 2 (a) shows the simulated impulse response function. We use it to simulate the bandwidth limited acoustic measurements. The reconstructed conductivity images with two different SNR levels of 1000 and 100 are shown in figure 2 (b) and (c), respectively. Note the different color scales used in these images as compared with figure 1 (a). Only the relative conductivity contrast was reconstructed in these images.

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